Adversarial Turing Patterns from Cellular Automata

Nurislam Tursynbek, Ilya Vilkoviskiy, Maria Sindeeva, Ivan Oseledets

1 Skolkovo Institute of Science and Technology
nurislam.tursynbek@gmail.com, reminguk@gmail.com, maria.sindeeva@skoltech.ru, i.oseledets@skoltech.ru

Abstract
State-of-the-art deep classifiers are intriguingly vulnerable to universal adversarial perturbations: single disturbances of small magnitude that lead to misclassification of most inputs. This phenomenon may potentially result in a serious security problem. Despite the extensive research in this area, there is a lack of theoretical understanding of the structure of these perturbations. In image domain, there is a certain visual similarity between patterns, that represent these perturbations, and classical Turing patterns, which appear as a solution of non-linear partial differential equations and are underlying concept of many processes in nature. In this paper, we provide a theoretical bridge between these two different theories, by mapping a simplified algorithm for crafting universal perturbations to (inhomogeneous) cellular automata, the latter is known to generate Turing patterns. Furthermore, we propose to use Turing patterns, generated by cellular automata, as universal perturbations, and experimentally show that they significantly degrade the performance of deep learning models. We found this method to be a fast and efficient way to create a data-agnostic quasi-imperceptible perturbation in the black-box scenario.

Introduction
Deep neural networks have shown success in solving complex problems for different applications ranging from medical diagnoses to self-driving cars, but recent findings surprisingly show they are not safe and vulnerable to well-designed negligibly perturbed inputs (Szegedy et al. 2013; Goodfellow, Shlens, and Szegedy 2015), called adversarial examples, compromising people’s confidence in them. Moreover, most of modern defenses to adversarial examples are found to be easily circumvented (Athalye, Carlini, and Wagner 2018). One reason why adversarial examples are hard to defend against is the difficulty of constructing a theory of the crafting process of them.

Intriguingly, adversarial perturbations can be transferable across inputs. Universal Adversarial Perturbations (UAPs), single disturbances of small magnitude that lead to misclassification of most inputs, were presented in image domain by Moosavi-Dezfooli et. al (Moosavi-Dezfooli et al. 2017), where authors proposed iterative strategy of gradually pushing a data point to the decision boundary. However, to construct a successful perturbation thousands of images were needed, whereas Krulkov et. al (Krulkov and Oseledets 2018) proposed an efficient algorithm of constructing UAPs with a very small number of samples. The proposed universal perturbations construct complex and interesting unusual patterns. Studying how these patterns emerge will allow better understanding the nature of adversarial examples.

We start from an interesting observation that patterns generated in (Krulkov and Oseledets 2018) visually look very similarly to the so-called Turing patterns (Figure 1) which were introduced by Alan Turing in the seminal paper “The Chemical Basis of Morphogenesis” (Turing 1952). It describes the way in which patterns in nature such as stripes and spots can arise naturally out of a homogeneous uniform state. The original theory of Turing patterns, a two-component reaction-diffusion system, is an important model in mathematical biology and chemistry. Turing found that a stable state in the system with local interactions can become unstable in the presence of diffusion. Reaction–diffusion systems have gained significant attention and was used as a prototype model for pattern formation.

In this paper, we provide an explanation why UAPs from (Krulkov and Oseledets 2018) bear similarity to the Turing patterns using the formalism of cellular automata (CA): the iterative process for generating UAPs can be approximated by such process, and Turing patterns can be easily generated by cellular automata (Young 1984). Besides, this gives a very simple way to generate new examples by learning the parameters of such automata by black-box optimization. We also experimentally show this formalism can produce examples very close to so-called single Fourier attacks by studying Fourier harmonics of the obtained examples.

Main contributions of the paper are following:

• We show that the iterative process to generate Universal Adversarial Perturbations from (Krulkov and Oseledets 2018) can be reformulated as a cellular automata that generates Turing patterns.

• We experimentally show Turing patterns can be used to generate UAPs in a black-box scenario with high fooling rates for different networks.
Background

Universal Adversarial Perturbations

Adversarial perturbations are small disturbances added to the inputs that cause machine learning models to make a mistake. In (Szegedy et al. 2013) authors discovered these noises by solving the optimization problem:

\[
\min_{\varepsilon} \|\varepsilon\|_2 \quad \text{s.t.} \quad \mathcal{C}(x + \varepsilon) \neq \mathcal{C}(x),
\]

where \(x\) is an input object and \(\mathcal{C}(\cdot)\) is a neural network classifier. It was shown that the solution to the minimization model (1) leads to perturbations imperceptible to human eye.

Universal adversarial perturbation (Moosavi-Dezfooli et al. 2017) is a small (\(\|\varepsilon\|_p \leq L\)) noise that makes classifier to misclassify the fraction \((1 - \delta)\) of inputs from the given dataset \(\mu\). The goal is to make \(\delta\) as small as possible and find a perturbation \(\varepsilon\) such that:

\[
P_{x \sim \mu} |\mathcal{C}(x + \varepsilon) \neq \mathcal{C}(x)| \geq 1 - \delta \quad \text{s.t.} \quad \|\varepsilon\|_p \leq L,
\]

In (Khrulkov and Oseledets 2018) an efficient way of computing such UAPs was proposed, achieving relatively high fooling rates using only small number of inputs. Consider an input \(x \in \mathbb{R}^d\), its \(i\)-th layer output \(f_i(x) \in \mathbb{R}^d\), and Jacobian matrix \(J_i(x) = \frac{\partial f_i(x)}{\partial x}|_{x} \in \mathbb{R}^{d_i \times d}\). For a small input perturbation \(\varepsilon \in \mathbb{R}^d\), using first-order Taylor expansion \(f_i(x + \varepsilon) \approx f_i(x) + J_i(x)\varepsilon\), authors find that to construct a UAP, it is sufficient to maximize the sum of norms of Jacobian matrix product with perturbation for a small batch of inputs \(X_b\), constrained with perturbation norm \(\|\varepsilon\|_p = L\) is obtained by multiplying the solution by \(L\):

\[
\sum_{x_j \in X_b} \|J_i(x_j)\varepsilon\|_q^p \rightarrow \text{max}, \quad \text{s.t.} \quad \|\varepsilon\|_p = 1.
\]

To solve the optimization problem (3) the Boyd iteration (Boyd 1974) (generalization of the power method to the problem of computing generalized singular vectors) is found to provide fast convergence:

\[
\varepsilon_{t+1} = \frac{\psi_p(J^T f_i(X_b)\psi_q(J_i(X_b)\varepsilon_t))}{\|\psi_p(J^T f_i(X_b)\psi_q(J_i(X_b)\varepsilon_t))\|_p},
\]

where \(\frac{1}{p} + \frac{1}{q} = 1\) and \(\psi_r(z) = \text{sign}(z)|z|^{r-1}\), and \(J_i(X_b) \in \mathbb{R}^{bd_i \times d}\) for a batch \(X_b\) with batch size \(b\) is given as a block matrix:

\[
J_i(X_b) = \begin{bmatrix}
J_i(x_1) \\
\vdots \\
J_i(x_b)
\end{bmatrix}.
\]

For the case of \(p = \infty\) (4) takes the form:

\[
\varepsilon_{t+1} = \text{sign}(J^T f_i(X_b)\psi_q(J_i(X_b)\varepsilon_t)).
\]

Equation (6) is the first crucial point in our study, and we will show, how they connect to Turing patterns. We now proceed with describing the background behind these patterns and mathematical correspondence between Equation (6) and emergence of Turing patterns as cellular automata is described in next Section.

Turing Patterns as Cellular Automata

In his seminal work (Turing 1952) Alan Turing studied the emergence theory of patterns in nature such as stripes and spots that can arise out of a homogeneous uniform state. The proposed model was based on the solution of reaction-diffusion equations of two chemical morphogens (reagents):

\[
\begin{align*}
\frac{\partial n_1(i,j)}{\partial t} & = -\mu_1 \nabla^2 n_1(i,j) + a(n_1(i,j), n_2(i,j)), \\
\frac{\partial n_2(i,j)}{\partial t} & = -\mu_2 \nabla^2 n_2(i,j) + b(n_1(i,j), n_2(i,j)).
\end{align*}
\]

Here, \(n_1(i,j)\) and \(n_2(i,j)\) are concentrations of two morphogens in the point with coordinates \((i,j)\). \(\mu_1\) and \(\mu_2\) are scalar coefficients. \(a\) and \(b\) are nonlinear functions, with at least two points \((i,j)\), satisfying \((i,j) : \{a(n_1(i,j), n_2(i,j)) = 0 \}

\[
\begin{align*}
b(n_1(i,j), n_2(i,j)) & = 0.
\end{align*}
\]
Turing noted the solution presents alternating patterns with specific size that does not depend on the coordinate and describes the stationary solution, which interpolates between zeros of \( a \) and \( b \).

Young et. al. [Young 1984] proposed to generate Turing patterns by a discrete cellular automata as following. Let us consider a 2D grid of cells. Each cell \((i, j)\) is equipped with a number \( n(i, j) \in \{0, 1\} \). The sum of values of cells, neighbouring with the current cell \((i, j)\) within the radius \( r_1 \) is multiplied by \( w \), while the sum of values of cells, neighbouring with the current cell \((i, j)\) between radii \( r_{in} \) and \( r_{out} \), is multiplied by \(-1\). If the total sum of these two terms is positive, the new value of the cell is 1, otherwise 0. This process can be written by introducing the convolutional kernel \( Y(m, l) \):

\[
Y(m, l) = \begin{cases} 
   w & \text{if } |m|^2 + |l|^2 < r_{in}^2, \\
   -1 & \text{if } r_{out}^2 > |m|^2 + |l|^2 > r_{in}^2. 
\end{cases} 
\]  
(8)

The coefficient \( w \) is found from the condition that the sum of elements of kernel \( Y(m, l) \) is set to be 0:

\[
\sum_m \sum_l Y(m, l) = 0. 
\]  
(9)

The update rule is written as:

\[
n_{k+1}(i, j) = \frac{1}{2} \left[ \text{sign}(Y \ast n_k(i, j)) + 1 \right], 
\]  
(10)

where \( \ast \) denotes a usual 2D convolution operator:

\[
Y \ast n(i, j) = \sum_m \sum_l Y(m, l) n(i + m, j + l). 
\]  
(11)

For convenience, instead of \( \{0, 1\} \), we can use \( \pm 1 \) and rewrite (10) (considering condition (9)) as:

\[
n_{k+1}(i, j) = \text{sign}(Y \ast n_k(i, j)). 
\]  
(12)

Note that (12) is similar to (6), and lets investigate this connection in more details.

**Connection between Boyd iteration and cellular automata**

We restrict our study to the case \( q = 2 \). Then \( \psi_2(z) = z \) and Equation (6) can be rewritten as:

\[
e_{l+1} = \text{sign}(J^T_i(X_b)J_i(X_b)e_i). 
\]  
(13)

In Khrulkov and Oseledets [2018] it was shown that perturbations of the first layers give the most significant results. Initial layers of most of the state-of-the-art image classification neural networks are convolutional followed by nonlinear activation functions \( \sigma \) (e.g. \( \text{ReLU}(z) = \max(0, z) \)). If the network does not include other operations (such as pooling or skip connections) the output of the \( i \)-th layer can be written as:

\[
f_i(x) = \sigma(M_i f_{i-1}(x)), 
\]  
(14)

where \( M_i \in \mathbb{R}^{d_i \times d_{i-1}} \) is a matrix of the 2D convolution operator of the \( i \)-th layer, corresponding to the convolutional kernel \( K_i \). The Jacobian in (6) can be then written using the chain rule as:

\[
J_i(x) = \frac{\partial f_i(x)}{\partial x} = \frac{\partial f_i(x)}{\partial f_{i-1}(x)} \cdots \frac{\partial f_1(x)}{\partial x} = D_i(x)M_i \cdots D_1(x)M_1, 
\]  
(15)

where \( D_j(x) = \text{diag}(\theta(M_j f_{j-1}(x))) \in \mathbb{R}^{d_j \times d_j} \) and \( \theta(z) = \frac{\partial \text{ReLU}(z)}{\partial z} = \begin{cases} 1, & \text{if } z > 0 \\ 0, & \text{if } z < 0 \end{cases} \).

The update matrix from Equation (13) is then:

\[
J_i^T(X_b)J_i(X_b) = \begin{bmatrix} J_i^T(x_1) & \cdots & J_i^T(x_b) \\ \vdots & \ddots & \vdots \\ J_i^T(x_b) & \cdots & J_i^T(x_1) \end{bmatrix} = \sum_{x \in X_b} J_i^T(x)J_i(x). 
\]  
(16)

Performance of the UAPs from Khrulkov and Oseledets [2018] increases with the increase of batch size. Then considering the limit case of sufficiently large batch size \( b \) we can approximate the averaging in (16) by the expected value:

\[
\sum_{x \in X_b} J_i^T(x)J_i(x) \approx \mathbb{E}_x [J_i^T(x)J_i(x)] = \mathbb{E}_x [M_i^T D_i^T(x) \cdots M_1^T D_1^T(x)D_1(x)M_1 \cdots D_1(x)M_1]. 
\]  
(17)

It should be noted that \( D_i^T(x)D_i(x) = D_i^T(x) = D_i(x) \). To simplify Equation (17), we make additional assumption that the elements of the diagonal matrices have the same mean value:

\[
\mathbb{E}_x [D_i(x)] \approx c_j I, 
\]  
(18)

where \( c_j \) is scalar. This is based on the fact that we consider universal attack, which is input-independent, we can assume \( x \) as random variable and \( D(x) \) as random matrix, which is diagonal and have elements 0 or 1. Then our assumption is not unrealistic, as we assume that all diagonal elements of random diagonal matrix over expectation converge to the same number \( c \) between 0 and 1. Then, taking into consideration (16), (17), (18) for the first layer:

\[
J_i^T(X_b)J_i(X_b) \approx b \mathbb{E}_x [M_i^T D_i^T(x)D_1(x)M_1] = b M_i^T \mathbb{E}_x [D_1(x)]M_1 = b c_1 M_i^T M_1, 
\]  
(19)

i.e. taking the expected value has removed the term corresponding to the non-linearity!

Two subsequent convolutions does not result into one convolution, however the only source of error is boundary effects. Since the size of convolutional kernels \( d < 10 \) is much smaller than the dimension of image \( N = 224 \) (for Imagenet), the error is small and is up to \( d/N \approx 0.5 \times 10^{-2} \). It definitely could be neglected for our purposes. Moreover, as shown in Miyato et al. [2018], this is common in convolutional neural networks for images. One can advocate this by the asymptotic theory of Toeplitz matrices [Böttcher and Grudsky 2000]. Thus, we have shown that \( J_i^T(X_b)J_i(X_b) \) has an approximate convolutional structure. To show that \( J_i^T(X_b)J_i(X_b) \) is also convolutional, we introduce additional assumptions:
Figure 2: UAPs for different layers of VGG-19 classifier. According to the theory, the specific feature size of the patterns increases with the depth of the attacked layer. The numbering of images is row-wise. First row corresponds to Layers 1-5, second row - Layers 6-10, third row - Layers 11-15.

Assumption 1. Matrices $D_j(x)$ and $D_k(x)$ are uncorrelated for all pairs of $j$ and $k$.

Assumption 2. Diagonal elements $d_{ij} = \theta(M_j f_{i-1}(x))$ of matrix $D_j(x)$ have covariance matrix of convolutional structure $C_j$ as linear operators of typical convolutional layers:

$$\text{Cov}(d_{ij}, d_{il}) = C_{jl}.$$  \hspace{1cm} (20)

These two assumptions might seem to be strong, so here we discuss how realistic and legit they are.

Regarding assumption 1, we should point that matrix $D$ is not the outputs of ReLU activation function, but is indicator ($0$ and $1$) whether the feature is positive or negative, and this is reasonable to assume the independence between elements $D_i$ and $D_j$. Further, let us consider covariance of matrix elements of diagonal matrices $\text{Cov}(D_{i,j}, D_{k,l})$.

First of all, for a large enough image batch it is natural to assume that this covariance if translationally invariant, i.e. there is no selected points. This means $\text{Cov}(D_{i,j}, D_{k,l}) = C(i - k, j - l)$, further, because of finite receptive field of convolutional network - far elements $D_{i,j}, D_{k,l}$ are uncorrelated for $|i - k|^2 + |j - l|^2 \gg 1$. Combining these two statements together we constructed assumption 2.

These assumptions need additional conditions for satisfactory boundary effects, however we find them natural, and realistic to hold in the main part of features.

**Theorem 0.1** Given $x$ is a random variable, $A = A(x)$ is a square matrix, such that $\mathbb{E}_x[A] = B$ is convolutional, and $D = D(x)$ is a diagonal matrix, such that $\mathbb{E}_x[D_{il} D_{mm}] = C_{lm}$. Then, $\mathbb{E}_x[DAD]$ is a convolutional matrix.

**Proof:**

$$\mathbb{E}_x[DAD(x)]_{lm} = \mathbb{E}_x[A_{lm} D_{il} D_{mm}] = \mathbb{E}_x[A_{lm}] \mathbb{E}_x[D_{il} D_{mm}] = B_{lm} C_{lm} = (B \circ C)_{lm},$$

where $\circ$ denotes element-wise product.

Applying Theorem 0.1 for each of the layers, and using $D_j = D_j(x)$ for all $j$ in (16) and (17):

$$J_i^T(X_b) J_i(X_b) = b \mathbb{E}_x \left[ D_i^T D_i M_1 M_2 \cdots M_i \cdots M_i^T D_i M_1 M_2 \cdots M_i^T D_i M_1 \right] _{lm}$$

$$= b \mathbb{E}_x \left[ (\cdots M_i^T ((M_i^T C_i M_1) \circ C_2) M_2 \cdots ) \circ C_i) M_i \right] _{lm}$$

which concludes that matrix $J_i^T(X_b) J_i(X_b)$ is convolutional matrix and can be written to cellular automata [12].

The structure of patterns by [Khrulkov and Oseledets 2018] contains some interesting elements, which has specific size. We empirically find that changing the train batch
Turing Patterns as Black-box Universal Adversarial Perturbations

Rigorous connection in previous section suggests that we can use Turing patterns to craft UAPs in a black-box setting, i.e. we can learn the parameters of CA by maximizing the fooling rate directly.

For a classifier $f$ and $N$ images, the fooling rate of Turing pattern $\varepsilon(\theta)$ with parameters $\theta$ is:

$$\text{FR} = \frac{1}{N} \sum_{i=1}^{N} [\arg \max f(x_i) \neq \arg \max f(x_i + \varepsilon(\theta))] . \quad (22)$$

There are several options how to parametrize CA, and we discuss them one-by-one. For all approaches the target function is optimized on 512 random images from ImageNet (Russakovsky et al. 2015) train dataset, and perturbation is constrained as $\|\varepsilon\|_\infty \leq 10$. Following Meunier, Atif, and Teytaud (2019), as a black-box optimization algorithm we use standard evolutionary algorithm CMA-ES (Hansen, Müller, and Koumoutsakos 2003) by the Nevergrad gradient-free optimization package (Rapin and Teytaud 2018). Fooling rates are calculated for 10000 images from ImageNet validation dataset. Tested classifiers: VGG-19 (Simonyan and Zisserman 2014), InceptionV3 (Szegedy et al. 2016), MobilenetV2 (Sandler et al. 2018).

**Simple CA.** In this scenario, we select the kernel $Y$ in Equation (12) to be $L \times L$ (we find that $L = 13$ produces best results), with elements filled by $-1$, except for the inner central rectangle of size $l_1 \times l_2$ whose elements are equal to constant such that the sum of elements of $Y$ is 0. Besides $l_1$ and $l_2$, initial maps of $n(i, j)$ from Equation (12) are added as parameters. To reduce black-box queries, initial maps are chosen to be $7 \times 7$ square tiles (size $32 \times 32$) (Meunier, Atif, and Teytaud 2019) for each of the 3 maps representing each image channel. $\theta = (l_1, l_2, \text{init maps})$. Resulting UAPs are shown in Fig. 3.

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Table 1: Fooling rates of a random Turing pattern vs. the one optimized in $l_1, l_2$ and initialization

| Target model       | Random | Optimized |
|--------------------|--------|-----------|
| MobileNetV2        | 52.79  | 56.15     |
| VGG-19             | 52.56  | 55.4      |
| InceptionV3        | 45.32  | 47.18     |

Table 2: Fooling rates of patterns with and without $\text{init maps}$ optimization (250 and 500 queries)

| Target model | With $250 \ q$ | Without $250 \ q$ | With $500 \ q$ | Without $500 \ q$ |
|--------------|----------------|-------------------|----------------|-------------------|
| MobileNetV2  | 85.62          | 90.91             | 91.67          | 94.47             |
| VGG-19       | 79.73          | 75.9              | 76.94          | 77.61             |
| InceptionV3  | 52.9           | 54.2              | 53.71          | 53.23             |

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Figure 3: Turing patterns generated using optimized parameters, and initial maps.
Turing pattern (Turing pattern with random not optimized parameters) is shown in Table 1 and full results are in Table 5. Here, we should mention our method is black-box, i.e. without access to the architecture and weights, and we do not compare it to (Khrulkov and Oseledets 2018) as authors consider white-box scenario with full access to the network. Moreover, even if we consider black-box scenario, our results using more advanced optimization significantly better in terms of fooling rate (see Tables 6-7) even comparing to the white-box (Khrulkov and Oseledets 2018).

**CA with kernel optimization.** In this scenario, all elements of the kernel $Y$ are considered as unknown parameters. Optimizing both over $Y$ and the initialization maps we got a substantial increase in fooling rates (see Fig. 4 for perturbations and Table 6 for fooling rate results). Results for different kernel sizes are shown in Fig. 5a.

**Optimization with random initialization map.** Next is the sensitivity of the patterns to the initialization. We remove the initialization map from the optimized parameters select it randomly. As experiments show (Table 2), performance of patterns generated without optimized initialization maps on average does not differ significantly from the pattern with optimized initialization maps. Thus, the initialization maps for pattern generation can be randomly initialized, and less queries can be made without significant loss in fooling rates. Results for different kernel sizes are shown in Fig. 5b.

Another resource for optimization are different strategies for dealing with the channels of the image (This might produce color patches). The detailed experiments are given in the Appendix, the results for the best case are in Table 7.

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**Table 3:** Experiments with Fourier frequencies of DFT for VGG-19

| Threshold Mixing | max  | max - 1 | 0.9 * max |
|------------------|------|---------|-----------|
| Summation        | 77.5 | 78.0    | 77.9      |
| Pointwise        | 78.0 | 77.7    | 78.0      |

**Table 4:** Experiments with Fourier frequencies of DFT for InceptionV3

| Threshold Mixing | max  | max - 1 | 0.9 * max |
|------------------|------|---------|-----------|
| Summation        | 53.1 | 50.6    | 51.0      |
| Pointwise        | 53.3 | 53.6    | 53.5      |

---

**Similarity to Fourier attacks**

We noticed that some of the resulting patterns look similar to the ones generated as single Fourier harmonics, as in (Tsuzuku and Sato 2019). To check how they are related, the following is done: We took the discrete Fourier transform (DFT) of the pattern and cut all the frequencies with the amplitude below some threshold (we used 2 cases of thresholds: $\text{max} - 1$ and $0.9 \times \text{max}$) and then reversed the DFT for all three pattern channels. We applied this transformation to the patterns acquired using different channel mixing approaches (2 (summation) and 4 (pointwise) described above). As a result, in most cases neither the fooling rates (see Table 3-4) nor the visual representation of the pat-
Table 5: Turing patterns optimized in $l_1$, $l_2$, and initialization maps

| Target Trained on | MobileNetV2 | VGG-19 | InceptionV3 |
|-------------------|-------------|--------|-------------|
| MobileNetV2       | 60.15       | 56.96  | 45.16       |
| VGG-19            | 50.49       | 55.4   | 49.98       |
| InceptionV3       | 49.55       | 48.76  | 47.18       |

Table 6: Patterns optimized over kernel $Y$ and initialization maps

| Target Trained on | MobileNetV2 | VGG-19 | InceptionV3 |
|-------------------|-------------|--------|-------------|
| MobileNetV2       | 90.59       | 61.2   | 35.15       |
| VGG-19            | 62.15       | 76.64  | 31.91       |
| InceptionV3       | 65.69       | 76.40  | 53.93       |

Table 7: Summation channel mixing

| Target Trained on | MobileNetV2 | VGG-19 | InceptionV3 |
|-------------------|-------------|--------|-------------|
| MobileNetV2       | 94.78       | 57.90  | 35.04       |
| VGG-19            | 73.15       | 77.50  | 43.06       |
| InceptionV3       | 63.90       | 64.46  | 53.10       |

Table 8: Performance of Turing Patterns (2 parameters) and Single Fourier Attack \((\text{Tsuzuku and Sato} 2019)\) (224*224*3 parameters) in black-box scenario

| Model                  | ResNet | GoogleNet | VGG-16 |
|------------------------|--------|-----------|--------|
| Our attack             | 61.4   | 58.2      | 81.2   |

Related Work

Adversarial Perturbations. Intriguing vulnerability of neural networks in (Szegedy et al. 2013; Biggio et al. 2013) proposed numerous techniques to construct white-box (Goodfellow, Shlens, and Szegedy 2015; Carlini and Wagner 2017; Moosavi-Dezfooli, Fawzi, and Frossard 2016; Madry et al. 2017) and black-box (Papernot et al. 2017; Brendel, Rauber, and Bethge 2017; Chen and Pock 2017; Ilyas et al. 2018) adversarial examples. Modern countermeasures against adversarial examples have been shown to be brittle (Athalye, Carlini, and Wagner 2018). Interestingly, input-agnostic Universal Adversarial Perturbations were discovered recently (Moosavi-Dezfooli et al. 2017). Several methods were proposed to craft UAPs (Khrulkov and Osledets 2018; Mopuri, Garg, and Babu 2017; Tsuzuku and Sato 2019; Mopuri et al. 2017) used activation-maximization approach. Khrulkov et al. (Khrulkov and Osledets 2018) proposed efficient use of \((p, q)\)—singular vectors to perturb hidden layers. Tsuzuku et al. (Tsuzuku and Sato 2019) used Fourier harmonics of convolutional layers without any activation function. Though UAPs were first found in image classification, they exist in other areas as well, such as semantic segmentation (Metzen et al. 2017; Tsuzuku and Sato 2019). Mopuri et al. (Mopuri et al. 2019) showed that training cellular automata using neural networks can organize cells into shapes of complex structure.

Conclusion

The key technical point of our work is that a complicated Jacobian in the Boyd iteration can be replaced without sacrificing the fooling rate by a single convolution, leading to a remarkably simple low-parametric cellular automata that generates universal adversarial perturbations. It also explains the similarity between UAPs and Turing patterns. It means that the non-linearities are effectively averaged out from the process. This idea could be potentially useful for other problems involving the Jacobians of the loss functions for complicated neural networks, e.g. spectral normalization.

This work studies theory of interesting phenomena of Universal Adversarial Perturbations. Fundamental susceptibility of deep CNNs to small alternating patterns, that are easily generated, added to images was shown in this work and their connection to Turing patterns, a rigorous theory in mathematical biology that describes many natural patterns. This might help to understand the theory of adversarial perturbations, and in the future to robustly defend from them.
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Appendix

Effect of different filter sizes. Another experiment is the dependence of the fooling rate on the filter size. Here we show results for both patterns with and without specifically optimized initialization maps (attacking MobileNetV2). In both cases small filter size does not provide good results, however surprisingly there is a non-monotonic dependence on the filter sizes once they are larger than 7.

Effect of channel mixing. Different channel mixing strategies within the pattern generation algorithm are used. We compared the following approaches to mixing channel maps during pattern generation:

- One 2D-filter for all 3 independent image channels, no channel mixing
- One 2D-filter for all image channels, after convolution the sum of each two channels becomes the remaining channel (i.e. channel3 = channel1 + channel2)
- One 3D-filter for all 3 image channels
- 3 separate 2D-filters (one for each channel), and a $3 \times 3$ matrix for channel mixing, like pointwise convolution

One can expect for 3D-filter to be a generalization of all of the other cases, thus it would be the most optimal to optimize over. However, as can be seen in Table 9, the results suggest that it is not definitively better than the other approaches. It can be seen summation, 3D and pointwise approaches give similar results, with pointwise performing slightly better on average over the other models.

Transferability

We tested several patterns discussed earlier for their transferability among the target networks. The results can be seen in Table 10. It seems that the attacks generally transfer rather well to MobileNetV2 and VGG-19, while InceptionV3 seems harder to transfer this types of attacks to. The results show that while optimization over cellular automata parameters seems to significantly boost fooling rates for target networks, some transferability loss can be expected. Note that for MobileNetV2 we get almost 95% fooling rate.

Algorithm 1: $(\mu, \lambda)$-CMA-ES algorithm

```
initialize $<x>$
$C \leftarrow I$
repeat
    $B, D \leftarrow \text{eigendecomposition}(C)$
    for $i = 1 \leftarrow \lambda$ do
        $y_i \leftarrow B \cdot D \cdot N(0, I)$
        $x_i \leftarrow <x> + \sigma y_i$
        $f_i \leftarrow f(x_i)$
    end
    $<y> \leftarrow \sum_{i=1}^{\mu} w_i y_{i:\lambda}$
    $<x> \leftarrow <x> + \sigma <y> = \sum_{i=1}^{\mu} w_i x_{i:\lambda}$
    $\sigma \leftarrow \text{update}(\sigma, C, y)$
    $C \leftarrow \text{update}(C, w, y)$
until Termination criterion fulfilled;
```
| Target model | Channel mixing | Independent | Summation | 3D filter | Pointwise |
|--------------|----------------|-------------|-----------|-----------|-----------|
| MobileNetV2  |                | 60.87       | **94.78** | 81.97     | 94.26     |
| VGG-19       |                | 60.74       | 77.50     | **78.87** | 78.00     |
| InceptionV3  |                | 47.45       | 53.10     | 51.31     | **53.33** |

Table 9: Fooling rates for different channel mixing approaches

Figure 6: Pointwise channel mixing patterns attacking InceptionV3 after Fourier modification

Figure 7: Summation channel mixing patterns attacking InceptionV3 after Fourier modification

Figure 8: Examples of images misclassified after the perturbation with UAPs and Turing patterns.
Table 10: Transferability of different types of attacks between the target models

| Trained on    | MobileNetV2 | VGG-19 | InceptionV3 |
|---------------|-------------|--------|-------------|
| MobileNetV2   | 56.15       | 56.96  | 45.16       |
| VGG-19        | 50.49       | 50.06  | 49.98       |
| InceptionV3   | 49.55       | 48.76  | 47.18       |

Table 11: Turing patterns optimized in $l_1, l_2$

| Trained on    | MobileNetV2 | VGG-19 | InceptionV3 |
|---------------|-------------|--------|-------------|
| MobileNetV2   | 90.59       | 61.27  | 35.15       |
| VGG-19        | 62.15       | 75.64  | 31.91       |
| InceptionV3   | 65.69       | 76.40  | 53.93       |

Table 12: Patterns optimized over filter $Y$ and initialization maps

| Trained on    | MobileNetV2 | VGG-19 | InceptionV3 |
|---------------|-------------|--------|-------------|
| MobileNetV2   | 60.87       | 45.00  | 37.49       |
| VGG-19        | 61.99       | 60.74  | 47.76       |
| InceptionV3   | 53.22       | 54.54  | 47.45       |

Table 13: Independent channels

| Trained on    | MobileNetV2 | VGG-19 | InceptionV3 |
|---------------|-------------|--------|-------------|
| MobileNetV2   | 94.78       | 57.90  | 35.04       |
| VGG-19        | 73.15       | 77.50  | 43.06       |
| InceptionV3   | 63.90       | 64.46  | 53.10       |

Table 14: Summation channel mixing

| Trained on    | MobileNetV2 | VGG-19 | InceptionV3 |
|---------------|-------------|--------|-------------|
| MobileNetV2   | 81.97       | 68.30  | 48.24       |
| VGG-19        | 76.15       | 78.87  | 43.83       |
| InceptionV3   | 61.36       | 61.62  | 51.31       |

Table 15: 3D filter

| Trained on    | MobileNetV2 | VGG-19 | InceptionV3 |
|---------------|-------------|--------|-------------|
| MobileNetV2   | 94.26       | 57.80  | 35.05       |
| VGG-19        | 74.79       | 78.00  | 43.38       |
| InceptionV3   | 59.29       | 67.70  | 53.33       |

Table 16: Pointwise channel mixing