Interface Stress Element Method, Rigid Body Spring Model and Applied Element Method

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Abstract. Interface Stress Element Method (ISEM), Rigid Body Spring Model (RBSM) and Applied Element Method (AEM) are typical connected discrete element methods derived from the same origin. Although there are many differences in their model compositions, expression forms, development paths and application situations, the three methods are closely related. In this paper, the commonalities and differences among ISEM, RBSM and AEM were expounded from five aspects: model composition, stiffness matrix for interface, formula for inter-element stress, theoretical system and analysis procedure. They discretize the domain with rigid blocks, but connect them using interface layers, distributed spring pairs and multiple spring pairs respectively. For homogeneous isotropic materials, the stiffness matrixes for RBSM are equal to that for ISEM, and there are only four items different in the stiffness matrix for AEM. The formula for inter-element stress of RBSM and AEM can be expressed by that for ISEM. The relationships among ISEM, RBSM and AEM were further clarified. RBSM and AEM are both special cases of ISEM. They can be incorporated into the theoretical system of Interface Stress Element Method. The three constitute a unified method family.

Keywords. Interface stress element method, rigid body spring model, applied element method, ISEM, RBSM, AEM.

1. Introduction
Interface Stress Element Method (ISEM, also known as Interface Element Method or Rigid Body-Interface Element Method) is a numerical method developed on the base of Rigid Body Spring Model (RBSM). This method expanded and improved the main idea of RBSM, then formed a rather mature theoretical system gradually [1].

Kawai first proposed RBSM in 1976 [2]. When he studied the bending of beams and plates under ultimate load, he came up with the idea of using the combination of rigid bar (rigid plate) and torsion spring to characterize the beam (plate) element, based on the observation of plastic hinge phenomenon. By calculating the strain energy of torsion spring, the formula for element stiffness matrix can be obtained using the Castigliano theorem, and the stiffness coefficient of torsion spring can be determined from classical beam (plate) bending theory. With the element stiffness matrix, the remaining work is to integrate the global stiffness matrix, deal with the boundary conditions and solve the equations. The original RBSM was proposed to solve some specific problems. It was an engineering model based on experiences.

In 1977, based on the slip line theory in plastic mechanics, Kawai established a rigid body spring model for plane stress problems and plane strain problems [3]. In this model, the centroid of the element is the primary node, and the rigid displacement expression of any point on the element is
given based on the assumption of small angular displacement. Several spring pairs, including normal spring, tangential spring and torsion spring, are set at the midpoint of the interface between adjacent elements. The determination of spring stiffness based on the constant strain assumption of elements. The derivation method of stiffness matrix for plane element is same as beam (plate) element. Then, Kawai and his collaborators studied several numerical examples [4-7] to verify the accuracy and applicability of RBSM. The results show that the bending model (beam or plate element) is always convergent and accurate. However, the shape of solution domain, boundary conditions, mesh scheme and other factors influence convergence and accuracy of the plane model. In reference [3], Kawai had noticed that the stiffness matrix for plane model is imprecise without finding out the reason.

From 1978 to 1979, Kawai and his collaborators began to modify and improve the plane model while they were expanding the theory and application of the bending model [8-10]. In 1980, Kawai discussed the theory and application of RBSM comprehensively [11]. He gave a three-dimensional RBSM, and used an integral scheme to calculate spring strain energy for the first time. This means that distributed springs replaced the concentrated springs in original RBSM, and an accurate stiffness matrix can be obtained. This is the most important modification to RBSM. At the same time, Kawai proved that RBSM is a simplified form of hybrid displacement model through the generalized variational principle. Since the basic frame of RBSM had formed, Kawai and his collaborators began to focus on its applications in engineering [12].

However, there were some problems unsolved in the theory system of RBSM. In 1991, Qian and Zhang gave the mathematical expression and finite element formulation of the plane model for RBSM [13]. The authors proved the existence and uniqueness of the solution. They also found that the stress accuracy is usually better than the displacement accuracy in RBSM. In this paper, they pointed out that RBSM is in essence a special finite element, and called it as Rigid Body Finite Element Method. In 1993, Zhuo and Zhao comprehensively discussed the mechanical and mathematical theory of RBSM [14]. Based on the continuous condition of interface stress, they gave a general expression of interface stress for the first time, which is applicable to the interface with different materials. Based on the principle of virtual work, the weighted residual method and the Hamiltonian principle, they established a relatively intact theory system, and extended its application to field problems [26-27], engineering stability problems [28-29] and material fracture problems [30-31]. The main contributions of team Zhuo include: 1) They expanded and improved RBSM with laying a solid theoretical foundation for it; 2) They expanded the application scope of this method as an effective tool for discontinuous problems.

In 1994, Zhao and Zhuo first proposed Interface Stress Element Method [15]. In 1995, Zhao and Zhuo gave the interface stress formula for transversely isotropic materials and established a three-dimensional nonlinear ISEM analysis model for rock mass structures [16]. Since then, the team headed by Zhuo carried out a series of research on ISEM [17-25]. Gradually, they established a relatively intact theory system, and extended its application to field problems [26-27], engineering stability problems [28-29] and material fracture problems [30-31]. The main contributions of team Zhuo include: 1) They expanded and improved RBSM with laying a solid theoretical foundation for it; 2) They expanded the application scope of this method as an effective tool for discontinuous problems.

In the development of ISEM, other scholars made their own contributions. In 1996, Gao pointed out that the physical equation of ISEM is based on either the assumption of zero tangential normal strain or the assumption of zero tangential normal stress [32]. In 1997, Zhang comprehensively discussed the basic theory of rigid body spring element [33]. In 2004, Chen and Zhou introduced a crack and damage gradient model into ISEM to analyse the stability and cracking process of arch dam structures [34]. In 2004, Qi discussed the programming implementation of interface stress element [35]. In 2006, Yin and others studied soil-structure interaction using a mixed finite element-interface element model [36].

With the rapid development of ISEM, the research team led by Meguro also proposed Applied Element Method (AEM) [37], which shows great power in the analysis of cracking and collapse of concrete structures [38-40]. The rigid body and spring system is also applied to discretize the domain, but many pairs of springs are used to describe the stress and deformation characteristics at the
interface of elements. The applied element method is very similar to the rigid body spring model. In fact, it is a variant of RBSM.

In brief, ISEM, RBSM and AEM are closely related. The commonalities and differences among them is a problem worthy of attention. It is of great theoretical significance to answer this question. If we can prove that there is inclusion relationship among the three methods, then the three methods can be integrated into the same theoretical framework to form a unified method family.

In this paper, the author compared and analysed the similarities and differences of the interface stress element method, rigid body spring model and applied element method from five aspects: model composition, stiffness matrix for interface, formula for inter-element stress, theoretical system and analysis procedure. He clarified the relationships among them subsequently.

2. Relationships between Interface Stress Element Method and Rigid Body Spring Model

2.1. Model Composition

Both RBSM and ISEM use rigid blocks to discretize the domain. The centroid of each element is a primary node, to which rigid displacement mode is applied. The difference between them is that RBSM uses a distributed spring system to describe the deformation and connection characteristics of an element (figure 1), while ISEM uses an interface layer. As shown in figure 2, a rigid sub-domain and the deformable interface layer surrounding it constitute the alternative model of a deformable element.

![Figure 1. Model composition of RBSM.](image1)

![Figure 2. Rigid body-interface element in ISEM.](image2)

The interface layer is a more clear conception in mechanics than spring system, which can describe the behaviors of a sub-domain more distinctly. 1) The interface layer represents the accumulated deformation inside the element. The relative displacement of any point on the interface layer can be considered the deformation of the differential strip (column) from the point to the centroid of the element. 2) The interface between adjacent elements can be regarded as the part of interface layers cemented together, each of which belongs to its own element. It is easy to deal with the solution domain composed of various materials. 3) The interface layer is composed of numerous points, which has obvious distribution characteristics. To calculate the stiffness matrix for interface layer, an integral scheme is naturally needed. 4) Since the deformation accumulates in the interface layer, the strain energy of the element should also accumulates here. The assumption of strain energy accumulation ensures the energy balance in a sub-domain.

Compared with the spring system, the main advantage of interface layer is that the material properties of a solution domain need not to be assumed in advance. Therefore, it can be elastic element, plastic element, viscous element, crack element, contact element, etc. ISEM can model a variety of problems, with a wider range of applications.

2.2. Stiffness Matrix for Interface

The formula for element stiffness matrix of RBSM is [12]:

\[
[k] = \int [B]^T [D] [B] \, dS
\]  

where, $k_d$ and $k_s$ are the stiffness of normal spring and tangential spring respectively. $l$, $m$ and $n$ are cosines of the normal vector.

For ISEM, the formula for interface stiffness matrix is (in global coordinate system):

$$ [D] = \begin{bmatrix}
-k_d l^2 + k_s (m^2 + n^2) & (k_d - k_s) l m & (k_d - k_s) n l \\
(k_d - k_s) l m & k_d m^2 + k_s (l^2 + n^2) & (k_d - k_s) m n \\
(k_d - k_s) n l & (k_d - k_s) m n & k_d n^2 + k_s (m^2 + l^2)
\end{bmatrix} $$

(3)

This shows that equation (1) and equation (4) is equal to each other. They calculate element stiffness in the same way.

2.3. Formula for Inter-Element Stress

In RBSM, the expression of inter-element stress is [12]:

$$ \tau_s = k_s \delta_s $$

(8)

Where, $k_d = E'/h, k_s = G/h, h = h_l + h_m, E' = (1 - \mu)E / (1 + \mu)(1 - 2\mu), G = E / 2(1 + \mu)$. In ISEM, the expression of interface stress is [1]:

$$ T_l = D^{im} \delta_l $$

(9)

This shows that equation (1) and equation (4) is equal to each other. They calculate element stiffness in the same way. The two methods have the same mechanical nature.

2.4. Theoretical System

The theoretical derivation of RBSM is quite simple. Formula for the relative displacement between two elements was obtained firstly, and strain energy on the interface were calculated sequentially. Then the interface stiffness equations and the formula for the interface stiffness matrix were derived based on Castiglione's theorem [12]. As for the geometric equation, constitutive equation and the interface stress mode, characteristics of springs were utilized simply without systematic and rigorous deduction.
In IESM, the establishment of the element displacement mode and interface stress model is based on several basic assumptions (rigid block assumption, basic assumptions for mechanical properties of interface layers, constant strain assumption, zero tangent strain assumption). Then the governing equation is derived based on the principle of virtual work or the dynamic universal equation, followed by the stiffness matrix, load matrix, mass matrix and damping matrix [1]. Obviously, theoretical derivation in ISEM is more rigorous than RBSM, and the theoretical system formed is more mature and universal.

2.5. Analysis Procedure
The governing equation and calculation formulas of ISEM are similar to those of finite element method. Its analysis procedure is also nearly the same as that of FEM. In both methods, the element analysis is carried out first, and the primary unknowns are obtained by solving the global equations. Then all the displacement (deformation) and stress solutions are calculated out finally. ISEM can be undertook in several standard steps. Therefore, not only the general ISEM program codes can be composed easily, but also the coupling analysis with FEM can be utilized smoothly. Although the analysis process of RBSM is similar to that of ISEM, it is difficult to develop general programs without the support of general calculation formulas.

According to the analysis above, we drew the following conclusions: 1) Rigid Body Spring Model and Interface Stress Element Method have the same mechanical nature; 2) The former is an engineering model, oriented to application; the latter is a general model, with more rigorous theory. In other words, ISEM is a superset of RBSM, and RBSM is a special case of ISEM. The relationship between them is just like that between matrix displacement method and finite element method.

3. Relationships between Interface Stress Element Method and Applied Element Method

3.1. Model Composition
AEM also models the structure as an assembly of rigid elements connected by springs. This is same to RBSM. The main difference between RBSM and AEM is that the former uses distributed spring system, while the latter replaces it with multiple spring pairs (as shown in figure 3).

![Figure 3. Modelling of structure to AEM (Meguro & Hatem, 1999).](image)

AEM is mainly employed to analyse the cracking and collapse of structures. In the process of solving, it is necessary to judge whether the cracking occurs according to the interface stress state. However, the interface is a region composed of numerous points, so it is impossible to calculate the stress of each point in practical analysis. Therefore, a representative point is usually selected to investigate whether its stress state meets the cracking criterion. This one point criterion is utilized in RBSM. Once an interface cracks, it is considered failed, and the corresponding part should be deleted from the global stiffness matrix. However, the interface cracking is a gradual process. The ability of an interface to resist deformation also loses gradually. In order to describe the process of interface cracking more accurately, AEM introduced a multi-point criterion: the interface is divided into several...
regions, and the one point criterion is applied in each region. In order to achieve this, we need to give the representative point position of each region, and calculate the stiffness matrix of each region separately. A simple way is to arrange representative points in the centroid of each area, and the stiffness of each area is represented by a set of concentrated springs. Therefore, the rigid body spring model is transformed into the model of AEM. It is why AEM is more applicable than RBSM in analysis of structural failure behavior.

As shown in figure 4, each interface in ISEM model can be divided into many sub-interfaces, on each of which there is a stress point. According to the status of stresses and deformations on stress points, we can also follow the progressive failure behavior of material gradually. In fact, the stiffness matrix corresponding to each region on the interface can be obtained directly by integrating according to formula (1) or (4). The multi-point criterion can be implemented in ISEM more naturally.

![Figure 4. Model compositions of ISEM.](image)

### 3.2. Stiffness Matrix for Interface

In AEM, the stiffness matrix (in local coordinate system) corresponding to each pair of springs is [41]:

\[
K_w^* = \begin{bmatrix}
K_n & 0 & -K_n A_y & -K_n & 0 & K_n B_y \\
0 & K_s & K_s A_x & 0 & -K_s & -K_s B_x \\
-K_n A_y & K_s A_x & K_n A_y + K_s A_x^2 & K_n A_y & -K_s A_x & -K_n A_y B_y - K_s A_x B_x \\
-K_n & 0 & K_n A_y & K_n & 0 & -K_n B_y \\
0 & -K_s & K_s A_x & 0 & K_s & -K_s B_x \\
-K_n B_y & -K_s B_x & -K_n A_y B_y - K_s A_x B_x & -K_n B_y & K_s B_x & K_n B_y + K_s B_x^2
\end{bmatrix}
\]

(10)

where, \( A_x = x'_c - x'_g \), \( A_y = y'_c - y'_g \), \( B_x = x'_c - x'_gm \), \( B_y = y'_c - y'_gm \). \((x'_c, y'_c)\) is the coordinate of the position of the pair of springs \( w \) in the local coordinate system; \((x'_g, y'_g)\), \((x'_gm, y'_gm)\) are the centroid coordinates of block elements on both sides of the interface in local coordinate system respectively. \( K_n, K_s \) are the stiffness coefficients of the normal spring and the tangential spring in the spring pair \( w \); \( K_n = E \cdot d \cdot t / a \), \( K_s = G \cdot d \cdot t / a \). Where \( t \) is the thickness of the element, \( D \) is the representative width of the spring, and \( a \) is the distance between the centroids of two adjacent elements (as shown in figure 3).

In ISEM, the stiffness matrix (in local coordinate system) of the region represented by the pair of springs can be calculated as follows:

\[
[k'_w] = \iint_{S'_w} C_e^T N^* N^* D^{im} N^* C_e \, dS
\]

(11)

Corresponding to the interface division scheme of AEM,
\[
K' \mathbf{w} = \begin{bmatrix}
    d_n & 0 & -d_n A_y & -d_n & 0 & d_n B_y \\
    0 & d_s & d_s A_x & -d_s & 0 & d_s B_x \\
    -d_n A_y & d_s A_x & d_n A_y^2 + d_s A_x^2 + \frac{d_n d_s^2}{12} & d_n A_y & -d_n A_y B_y - d_s A_x B_x - \frac{d_n d_s^2}{12} \\
    -d_n & 0 & d_n A_y & d_n & 0 & -d_n B_y \\
    0 & -d_s & -d_s A_x & 0 & d_s & d_s B_x \\
    d_n B_y & -d_s B_x & -d_n A_y B_y - d_s A_x B_x - \frac{d_n d_s^2}{12} & -d_n B_y & d_s B_x & d_n B_y^2 + d_s B_x^2 + \frac{d_n d_s^2}{12}
\end{bmatrix}
\]  

\[S = d \cdot t\] 

where, \(S = d \cdot t\). Other parameters are same as those in equation (10).

Obviously, if \(d_n = E/\alpha, d_s = G/\alpha\), then \(K'_w\) and \(K''_w\) have only four elements inconsistent, and the difference is all \(d_n d_s^2 S/12\). If the value of \(d\) is small enough (enough pairs of springs), the error caused by this term can be ignored. This proved that the formula for stiffness matrix in AEM is a simplified form of that in ISEM.

### 3.3. Formula for Inter-Element Stress

AEM uses the formula for inter-element stress of RBSM, and zero tangent normal stress assumption is adopted. In ISEM, the assumption that the tangential normal strain of differential strip (column) is zero can also be used. The inhomogeneous and anisotropic elastic matrix is also applicable to equation (9).

### 3.4. Theoretical System

Like RBSM, the theory of AEM is relatively immature, especially in the mathematical and mechanical foundation. Due to the lack of rigorous theoretical support, the selection of some parameters and calculation formulas depends on practical experience. From this point of view, AEM is also an engineering model.

### 3.5. Analysis Procedure

The analysis flow of AEM is similar to that of ISEM. Because the interface is segmented, the calculation method of interface stiffness and the integration method of global stiffness matrix are slightly different from those of conventional ISEM. In fact, ISEM can be treated in the same way, and the calculation results are more accurate and applicable. It is worth noting that this treatment is only meaningful for cracking and collapse analysis. For the analysis of deformation stage, this method is not only unnecessary, but also inefficient.

In a word, the applied element method is also a special case of the interface stress element method. It is a simplified form of the interface stress element method, which slices the interface into pieces.

### 4. Conclusion

As typical connected discrete element methods, the fundamental ideas of RBSM, AEM and ISEM derived from the same origin. Although there are some differences in the model composition, expression form, development path and application situation, the mechanical nature described by them is essentially identical.

From the point of view of connotation, RBSM is a special case of ISEM applied to homogeneous and isotropic materials, and AEM is a simplified form of ISEM with numerous sub-interfaces. Both AEM and RBSM can be incorporated into the theoretical system of Interface Stress Element Method. The three methods constitute a unified method family.

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References

[1] Zhuo J S and Zhang Q 2000 Interface Stress Element Method for Discontinuous Medium Problems (Beijing: Science Press, China).
[2] Kawai T and Kondou K 1976 New beam and plate elements in finite element analysis Seisan Kenkyu 28 409-412.
[3] Kawai T 1977 New element models in discrete structural analysis Journal of the Society of Naval Architects of Japan 141 174-180.
[4] Kawai T and Toi Y 1977 A discrete analysis on dynamic collapse of a beam under impulsive transverse load Seisan Kenkyu 29(5) 288-290.
[5] Toi Y and Kawai T 1977 A new discrete model for analysis of visco-elastic problems Seisan Kenkyu 29(8) 430-434.
[6] Toi Y and Kawai T 1977 Transient response analysis of an elasto-visco plastic beam subjected to transverse impact Seisan Kenkyu 29(12) 662-665.
[7] Kawai T 1978 New discrete models and their application to seismic response analysis of structures Nuclear Engineering and Design 48(1) 207-229.
[8] Kawai T and Chen C N 1978 A discrete element analysis of beam bending problems including the effects of shear deformation Seisan Kenkyu 30(5) 165-168.
[9] Toi Y and Kawai T 1978 A new discrete analysis on dynamic collapse of structures Journal of the Society of Naval Architech of Japan (143) 257-263.
[10] Toi Y and Kawai T 1979 A new discrete analysis on dynamic collapse of structures (Further report) Journal of the Society of Naval Architects of Japan (145) 112-119.
[11] Kawai T 1980 Some consideration on the finite element method International Journal for Numerical Methods in Engineering 16(1) 81-120.
[12] Kikuchi A, Kawai T and Suzuki N 1992 The rigid bodies-spring models and their applications to three dimensional crack problems Computers & Structures 44(1-2) 469-480.
[13] Qian X X and Zhang X 1991 Rigid body finite element method in structural analysis Computational Structural Mechanics and Applications 8(1) pp 1-14.
[14] Zhao J S and Zhao N 1993 Piecewise rigid body-interface spring method for problems of discontinuous medium Journal of Hohai University 21 34-43.
[15] Zhao N and Zhuo J S 1994 Coupling method of rigid-body interface stress element with finite element in dynamic analysis Journal of Hohai University 22(6) 8-15.
[16] Zhao N and Zhuo J S 1995 3-D nonlinear interface element analysis of structures Journal of Hohai University 23(2) 23-31.
[17] Zhuo J S, Zhang Q and Zhao N 1995 Interface stress element method for deformable body with discontinuous medium such as rock mass Proceedings of 8th ISRM (Tokyo, 25-29 Sept., 1995) p 939-941.
[18] Fang Y L, Zhuo J S and Zhang Q 1998 Interface stress element method for discrete model with elements of arbitrary shape Engineering Mechanics 15(2) 27-37.
[19] Fang Y L and Zhuo J S 1998 Rigid-body surface element method for plate bending and vibration problems Journal of Hohai University 26(2) 70-74.
[20] Zhang Q and Zhuo J S 1998 The interface stress element model for anchored rock mass Chinese Journal of Geotechnical Engineering 20(5) 50-53.
[21] Wang W B, Wu Q X and Zhuo J S 1999 Stochastic interface-stress element method for 2-D problems Journal of Hohai University 27(4) 111-115.
[22] Xu Z 1999 Interface-Stress Element Method for Structural Analysis and Applications (Nanjing: Hohai University).
[23] Zhang Q 2000 Theoretical Model, Analysis Method and Application of Discontinuous Medium Mechanics in Engineering Structure (Nanjing: Hohai University).
[24] Wang W B, Wu Q X, Jin W L and Zhou J S 2003 Stochastic interface-stress element method for 3-D problems Acta Mechanica Sinica 35(1) 105-109.
[25] Zhang Q, Zhou Z B and Zhuo J S 2005 Mixed model for partitioned interface stress element method-finite element method-infinite element method Chinese Journal of Computational Mechanics 22(1) 8-12.
[26] Fang Y L and Zhuo J S 1998 Rigid-body surface element method in analysis of thermal field problems Journal of Hohai University 26(4) 87-91.
[27] Fang Y L and Zhuo J S 1998 Rigid-body interface element method (RIEM) in analysis of thermal field thermal stress and creeping stress Journal of Hydraulic Engineering 29(7) 50-54.
[28] Zhang Q and Zhuo J S 2000 Interface element method and criterion for stability analysis of high slopes in three gorges shiplock Chinese Journal of Rock Mechanics and Engineering 19(3) 285-288.
[29] Wang Y F and Zhang Q 2009 Analysis of anti-sliding stability in deep foundation of Xiangjiaba gravity dam based on interface element method Rock and Soil Mechanics 30(9) 2691-2696.
[30] Wu F 2004 Meso Numerical Simulation of Failure Process of Concrete Materials (Nanjing: Hohai University).
[31] Du P R, Zhuo J S, and Meng W Y 2007 Analysis model for reinforced concrete interface problem Journal of Hohai University 35(3) 312-314.
[32] Gao P Z 1996 Rigid body-interface element method in dynamic plasticity Engineering Mechanics 13(4) 135-144.
[33] Zhang J H, Fan J W and Hu D 1997 Theory and Application of Rigid Body-Spring Element (Chendu: Chengdu University of science and Technology Press).
[34] Chen X 2004 Fracture and Gradient Enhanced Damage Model in Interface Stress Element Method and Its Application in Engineering (Beijing: Tsinghua University).
[35] Qi L L, Zhao H T and Liang R Y 2004 Numerical method of the interface stress element method Journal of Xi’an University of Architecture and Technology 36(2) 152-155.
[36] Yin H W, Yi W J and Liu Y 2006 Mixed interface stress element-finite element model with its application Journal of Hunan University of Science & Technology (Natural Science Edition), 21(4) 41-46.
[37] Meguro K and Tagel-Din H 1997 A new efficient technique for fracture analysis of structures Bulletin of Earthquake Resistant Structure 30 103-116.
[38] Tagel-Din H 1998 A New Efficient Method for Nonlinear, Large Deformation and Collapse Analysis of Structures (Tokyo: The University of Tokyo).
[39] Meguro K and Tagel-Din H 1999 Simulation of buckling and post-buckling behavior of structures using applied element method Bulletin of Earthquake Resistant Structure 32 125-135.
[40] Tagel-Din H and Meguro K 1999 Simulation of collapse process of a scaled RC building subjected to base excitation Proceedings of the 25th Japan Society of Civil Engineers Earthquake Engineering Symposium (Tokyo: July 1999) p 949-952.
[41] Mayorca P 2003 Strengthening of Unreinforced Masonry Structures in Earthquake Prone Regions (Tokyo: The University of Tokyo)