FOUR PARTICLES AND EIGHT PARTICLES IN a SINGLE j- SHELL-THE BACKBONE of the SHELL MODEL

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Abstract

Previous results of single -j shell calculations for ⁴⁴Ti and ⁴⁸Cr are compared to experiment. In the single j-shell approximation for these N=Z nuclei we have respectively four particles in the f⁷/₂ shell. Agreement is not perfect but is sufficiently good to lend support to the validity of the shell model. We compare B(E2)'s in the Shell, Rotational and Vibrational Models for ⁴⁴Ti and ⁴⁸Cr. We discuss the I=0 ground state wave functions of even-even Ti isotopes using isospin considerations.

1 Single -j energy Levels

We consider the spectra of ⁴⁴Ti and ⁴⁸Cr in a single j-shell calculation with the interaction taken from ref [1]. This is based on earlier works [2,3,4] but the T=0 matrix elements have been updated. The interaction matrix elements are taken from the spectrum of ⁴²Sc. The values of the matrix elements <(jj)J | V | (jj)> (j=f⁷/₂) from J=0⁺ to J=7⁺ are respectively 0.0000, 0.6111, 1.5863, 1.4904, 2.8153, 1.5101, 3.240 and 0.6163 MeV. The resulting spectra for ⁴⁴Ti and ⁴⁸Cr are shown in Table 1 and comparison with experiment is also shown.
Table 1. Spectra of $^{44}$Ti (four particles) and $^{48}$Cr (eight particles).

| $J$ | $^{44}$Ti (theo) | $^{44}$Ti (exp) | $^{48}$Cr (theo) | $^{48}$Cr (exp) |
|-----|-----------------|----------------|-----------------|----------------|
| 0   | 0.000           | 0.000          | 0.000           | 0.000          |
| 2   | 1.163           | 1.083          | 1.206           | 0.752          |
| 4   | 2.790           | 2.454          | 2.233           | 1.858          |
| 6   | 4.062           | 4.015          | 3.489           | 3.445          |
| 8   | 6.084           | 6.508          | 5.010           | 5.188          |
| 10  | 7.384           | 7.671          | 6.453           | 7.063          |
| 12  | 7.702           | 8.040          | 7.888           | 8.410          |
| 14  |                 |                | 10.255          | 10.280         |
| 16  |                 |                | 13.573          | 13.308         |

We see that although the agreement is not perfect is is reasonably good and shows that the shell model has validity in this region. Of course large-space shell model calculations should also be performed not only to improve the yrast spectrum but also to obtain additional energy levels. But it is good to remember that the singly j shell calculations have some validity.

2 Comparison of shell model B(E2)'s with those of rotational and vibrational models.

Large space shell model calculations of energy levels and B(E2)'s in the f$_{7/2}$ were performed in the past by Robinson, Escuderos and Zamick [5]. The FPD6 interaction was used. Figures of energy levels are contained in that paper and will not be repeated here. We will however make a comparison of the B(E2)'s calculated in the shell model with those of the rotational and vibrational models. In that reference the B(E2)'s were calculated from J to J+ (going up). Here we will present the reverse—from J+2 to J (going down). Comparison with the vibrational model is easier that way. The relation between down and up is:

$$B(E2,J+2–J) = B(E2,J–J+2)* (2J+1)/(2J+5)$$

In the vibrational model the B(E2) for the yrast sequence J=0,2,4,6,8, e.t.c. is given by

$$B(E2,J+2–J) = (J+2) B(E2,2–0)$$

i.e. the B(E2) is proportional to the number of quanta.

In the rotational model B(E2, J+2–J) is proportional to $(J+2) 20KJK^2$ i.e. to the square of a Clebsch-Gordan coefficient. We assume a K=0 band. We normalize the collective values so that they give the same results for the J=2 to J=0 transition.

The results for B(E2) are given in Table 2.

We note that the shell model B(E2)'s start to grow with increasing J from J=2 to J=4 in agreement with the rotational model but then they get smaller while the rotational values continue to increase, although at a very slow rate. The
vibrational values increase multiplicity and get much larger than not only the shell model values but also the rotational values. It is very doubtful that such behaviour corresponds to reality.

It has also been pointed out by Cederwall et al. [6] that in the $g_{9/2}$ shell near $N=50, Z=50$, the shell model and rotational model are qualitatively similar up to $J=6^+$ as far as $B(E2)$’s are concerned (but not energy levels).

Note that in $^{48}$Cr there is no dramatic drop in $B(E2)$ $J=10^+$ to $J=8^+$ that was commented in the context of $^{50}$Cr by Zamick, Fayache and Zheng[7]. They asserted that in the rotational model the $J=10^+$ state of $^{50}$Cr does not belong to the K=0 ground state band. Indeed it could belong to a K=10$^+$ band. They used static quadrupole calculations to support their claim. This was also discussed by a dominantly experimental group, Brandolini et al. [9].

3 Ground state wave functions of even-even Ti isotopes in the single j shell-Isospin considerations.

We use as a basis states $[J_p, J_n]^{I}$. Hence the wave functions are of the form $\Psi^{I,\alpha} = \sum D(J_p, J_n)^{I,\alpha} [J_p, J_n]^{I}$. Here $D$ is the probability amplitude that in the state $I, \alpha$ the protons couple to $J_p$ and the neutrons to $J_n$. We here address the issue of how $D(0,0)$ varies as we go through the even-even Ti isotopes. For $^{42}$Ti and $^{50}$Ti the wave functions are of course unique in the single j ($f_{7/2}$) model. For each of $A=44, 46$ and 48 there are 3 states with isospin $T=(N-Z)/2$ and one with $T=(N-Z)/2+1$. Let us focus on the higher isospin states. By virtue of the fact that they are double analogs of unique states in Calcium we see that the wave function components are 2 particle coefficients of fractional parentage. That is $D(J_p, J_n) = |(j^2(J_p) j^n(J_n)) | j^{(n+2)} I |$. A recursion relation for these is given in De-Shalit and Talmi [10] (page 529) which for $J_p=0$ and $J_n=0 I=0$ is given by

$$
|j^{(n+2)}(0) | j^{(n+2)}(0) | = |j^{(n-2)}(0) j^2(0) | |j^n(0) | * \sqrt{(n+1)(2j+1-n)} / \sqrt{(n+1)(2j+3-n)}.
$$

The values shown in ref [1] for $D(0,0)$ for higher isospin states of $^{44}$Ti, $^{46}$Ti and $^{48}$Ti, $T=2, 3, 4$ respectively are -0.50000, 0.31623 and 0.18898. This is in agreement with the above formula. The fact that some of the $J_p=0, J_n=0$ strength is eaten up by the higher isospin states means that on cannot have a lower isospin state with 100%, even with a J=0 pairing interaction. $J_p=0J_n=0$ strength. For $^{44}$Ti (T=0) one can only have 75% of this strength; more for $^{46}$Ti (T=1, 90%) and even more for $^{48}$Ti (T=2, 96.4%). If one uses the pairing interaction the entire $T$($lower$) strength resides in one state e.g. in $^{44}$Ti the lowest $T=0$ state gets 75% of the overall strength whilst the unique $T=2$ state gets the other 25%.

There have been many discussions of how $T=0$ and $T=1$ pairings affect ground state wave functions. It is good to remember that simple isospin con-
siderations also play a role.

Table 2. B(E2)'s (Q) in the shell model, rotational model and vibrational model for $^{44}$Ti and $^{48}$Cr (e$^2$fm$^4$)

| $^{44}$Ti  | Shell | Rot | Vib |
|-----------|-------|-----|-----|
| J         | B(E2) | Q   | B(E2) | B(E2) |
| 2         | 140   | −21.6 | 121  | 121  |
| 4         | 190   | −29.0 | 173  | 242  |
| 6         | 161   | −33.4 | 191  | 364  |
| 8         | 112   | −27.1 | 200  | 486  |
| 10        | 109   | −25.6 | 205  | 607  |
| 12        | 63    | −28.5 | 209  | 729  |

| $^{48}$Cr | J       |     |     |     |
|-----------|---------|-----|-----|-----|
| 2         | 312     | −35.4 | 312 | 312 |
| 4         | 436     | −45.5 | 445 | 624 |
| 6         | 452     | −47.9 | 490 | 936 |
| 8         | 426     | −48.9 | 514 | 1248|
| 10        | 341     | −41.5 | 527 | 1562|
| 12        | 152     | −8.0  | 537 | 1874|
| 14        | 138     | −9.4  | 543 | 2186|
| 16        | 68      | −8.7  | 549 | 2498|
| 18        | 2       | −34.0 | 552 | 2810|
| 20        | 8       | −46.7 | 556 | 3122|

Table 3. B(E2)'s (Q) in the shell model, rotational model and vibrational model for $^{96}$Cd and $^{92}$Pd(e$^2$fm$^4$)

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