Taking into account constraints on magnitude and first derivative of the Global Navigation Satellite Systems differential corrections during their forecasting

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Abstract. Algorithms for predicting differential corrections to GNSS measurements have been developed, the low labor intensity of which allows them to be applied in real time. The processing of corrections before the break is performed using the Kalman filter in reverse time. The interval of processed corrections gradually expands as the forecast duration increases. Based on the results of experiments with field GNSS data, a comparison of the algorithms efficiency was made.

1. Introduction

High accuracy of Global Navigation Satellite Systems (GNSS) measurements processing in the differential mode is in demand in many areas, including: positioning car transport, mapping, geodesy, compensating for gravimeter vertical accelerations, etc. In these and some other tasks, extended gaps and a rare occurrence of GNSS data in conditions of limited satellites visibility and reflected signals reception are a fairly common phenomenon. So, for example, in real time, one has to face gaps in the receipt of differential corrections (DC) for GNSS code measurements due to failures at the base station and in the radio communication channel [1,2]. Thus there is a need for the DC forecast, the accuracy of that substantially depends on the adequacy of the model describing the predicted process. It is worth mentioning that the GNSS errors and DC description using the stochastic model is problematic due to the unstable state of the ionosphere and troposphere. In addition, the standard method for DC prediction with the coefficients of their linear trend, provided in the RTCM format, is inefficient due to the low accuracy of these coefficients, that are determined only by the latest values of the DC [3].

The aim of this work is to obtain effective solutions for satellite data forecasting in the absence of the stochastic models.

2. Problem statement

For discrete moments of time \( t_k = t_0 + k\Delta t \), \( k = -m, 0 \) with discrete interval \( \Delta t \) there are DC values for the pseudorange of the same satellite. DC have the form

\[
d_k = g(t_k) + \delta_k,
\]

(1)

where \( g(t_k) \) is a slowly changing component of the DC, \( \delta_k \) is a discrete white noise with a Gaussian probability distribution and variance \( \sigma^2 \), \( g(t_k) \) is a low-frequency component of DC.

Component \( g(t_k) \) at time-interval \([t_m; t_n]\) is presented in the form

\[
g(t) = c\rho(t),
\]

(2)
where
\[ \rho(t) = b_0 + b_1(t - t_n) + \frac{b_2}{T} + \frac{(b_1 - b_0 - b_2)(t - t_n)^2}{T^2}, \]

(3)

For \( \rho(t) \) the following inequalities are true
\[ |\rho(t)| \leq 1, \quad |\dot{\rho}(t)| \leq \ddot{\rho}, \]

(4)

Here \( b_0, b_1, b_2 \) are the constant coefficients, \( T = (m + n)\Delta t \), \( b = (b_0, b_1, b_2)^T \), \( B(t) \) is a row-matrix, \( c \) is and \( \ddot{\rho} \) are given parameters. In these conditions, it is required to predict \( g(t) \). There are three main approaches to solving: deterministic [4], stochastic [5,6], quasideterministic [7]. The novelty of the work consist in the forecasting DC based on a quasi-deterministic approach. The results of the study are described in more detail in [8,9].

3. Forecast algorithms
The paper considers 4 algorithms for predicting DC with varying degree of originality:

1) The quadratic prediction algorithm with constraints (CQP), which involves estimating \( g(t) \) by measurements (1) taking into account expressions (2), (3) and inequalities (4) that limit the quantity and its derivative.

2) The quadratic prediction (QP) algorithm, which implies a parabolic approximation \( g(t) \) in the interval \([t_m; t_n]\) without taking into account restrictions (4).

3) The linear prediction algorithm with constraints (CLP) performs a forecast \( g(t) \) at the moment \( t_n \) according to the formula
\[ g(t_n) = g(t_0) + \dot{g}(t_0)(t_n - t_0), \quad n=1,2,\ldots \]

(5)

assuming \( g(t) \) that for on the interval \([t_m; t_n]\) (2), (3), (4) are valid.

4) The LP algorithm operates according to the same formula (5), where \( g(t) \) in the interval \([t_m; t_n]\) is represented as an unlimited parabola.

In addition, as the simplest forecast option, the fixation of the last DC is considered (FP), i.e.
\[ g(t_n) = d_0, \quad n=1,2,\ldots \]

Consider the features proposed by the author of the CQP and CLP algorithms. Firstly, the slowly changing component of the corrections is presented in the form:
\[ g(t) = g(t_0 - \tau) = a_0 + a_1\tau + a_2\tau^2 = A(\tau)a, \]

(6)

where \( \tau = t_n - t \) is a reverse time, \( A(\tau) \) is a row-matrix.

In this case, the DC are processed in the reverse order \(- d_n, d_{n-1}, \ldots \) etc. They are processed recursively using a Kalman filter that evaluates the vector \( a = (a_0, a_1, a_2)^T \) of polynomial coefficients (6). It is assumed that on a short forecast interval using the second-order polynomial it is possible to depict local features of the DC behavior, while using the longer accumulated background of the DC helps to extrapolate corrections over a long interval. It is assumed here that for the forecast \( g(t) \) on \( n \) steps is used \([m] \) DC, i.e. \( m + 1 = [m] \), where \( r > 0 \) is a given parameter, \([\cdot] \) is the operation of rounding to the nearest integer up.

At each step, according to the known relations of the Kalman filter, an estimate \( \hat{a} \) and a covariance matrix \( \hat{P}_a \) of the estimation error are formed. Further, \( \hat{a} \) and \( \hat{P}_a \) are recalculated in \( \hat{b} \) and \( \hat{P}_b \) for representation (3), and for each step vector \( b = (b_0, b_1, b_2)^T \) is formed. At the final stage, it is required to clarify the estimate of the vector \( b \) due to the information on the observance of inequalities (4) for the function \( \rho(t) \) and its derivative and the relation of this function with \( g(t) \) provided in (2). Clarification is reduced to the problem of conditional minimization
\[ \hat{b} = \arg \min_{\hat{b} \in \hat{a}} (b - \hat{b})^T \hat{P}_b^{-1}(b - \hat{b}), \]

(7)
where $\mathcal{B}$ is a region including those and only those values for $b$ which constraints (4) are satisfied.

It was also shown in [7] that problem (7) is equivalent to maximizing the likelihood function $f(d_0,...,d_m | b)$ over the same domain $\mathcal{B}$. Obviously, the use of (7) is preferable, since direct maximization $f(d_0,...,d_m | b)$ with a large number of DC, it turns out to be excessively timeconsuming, but in real time it may turn out to be completely unrealizable. As a result, forecast estimates using CQP and CLP are formed according to the updated estimate in accordance with expressions (2), (3), and in the case of CLP also taking into account (5).

4. Results of processing experimental data

By processing the DC to code measurements in the RTCM format - RTCM21 messages - in the camera mode, the algorithms of QP, LP, CQP, CLP, and also FP were compared. Measurement were received from NovAtel DL-V3 [12] and processed in Matlab. We used two samples of DC implementations, obtained at daily intervals at different times of the year, which excluded the same state of the ionosphere and reduced the similarity of the behavior of DC in two samples. Each sample includes 55 continuous DC implementations. The first sample was used to determine the best parameters c and used in the CQP and CLP algorithms from the point of view of RMS, and the second one was used to test algorithms with these parameters. The DC forecast RMS graphs for both samples are presented in Figure 1.

It follows from the Figure 1 that for each algorithm, the results obtained from the training and test samples do not have fundamental differences. The most accurate are CQP and CLP, as well as the simplest method FP. With a forecast of more than 700 s, the advantage of CLP becomes more noticeable, reaching 10-15% per 1000 s of the forecast in relation to the RMS of CQP and FP.

![Figure 1](image-url)

**Figure 1.** RMS QP (blue color), RMS LP (purple color), RMS FP(red color), RMS CQP (green color) and RMS CLP(orange color).

| Forecast algorithm | $r=1$ | $r=2$ | $r=3$ |
|-------------------|-------|-------|-------|
|                   | 500   | 1000  | 500   | 1000  | 500   | 1000  |
| QP                | 2.75  | 2.9   | 1.48  | 2.15  | 0.9   | 2.3   |
| LP                | 1.48  | 1.8   | 1.1   | 1.9   | 0.8   | 1.95  |
| FP                | 0.95  | 2     | 0.95  | 2.14  | 0.95  | 2.14  |
| CQP               | 0.93  | 2.2   | 0.96  | 2.14  | 0.97  | 2.15  |
| LQP               | 0.88  | 1.7   | 0.87  | 1.71  | 0.88  | 1.7   |

Further, the forecast algorithms were tested with an increased measurement interval duration relative to the forecast time. Here, options with $r=2$ and $r=3$ were considered and, again, for a
forecast of up to 1000 s. The results are given in Table 1 only for test samples of DC with constraints determined by the training sample for the extended interval \( T \) relative to those obtained with \( r = 1 \).

From the Table 1 it follows that only a three time increase in the measurement duration allows the QP algorithm to approach the results of CQP and CLP by 500 s from the forecast, which they demonstrated at \( r = 1 \). For the LP algorithm, a two time increase in the measurement duration is sufficient to compete with CLP and CQP for 500 s of the forecast. Speaking about the forecast for 1000 s, it is worth noting that for \( r = 2 \) and \( r = 3 \), the QP and LP algorithms show results close to the results of LPO and KPO at \( r = 1 \).

![Figure 2](image_url)

**Figure 2.** The difference between RMS FP and RMS CQP (left) and between RMS FP and RMS CLP (right) when \( m+1=n \) (\( r=1 \)) for \( n=m_{\text{max}}+1 \), and \( m=m_{\text{max}} \) for \( n>m_{\text{max}}+1 \).

At the same time, the algorithms were tested under conditions when \( m \) can only increase to a certain \( m_{\text{max}} \), i.e. to the left of the start of the prediction \( t_0 \), only a limited number of DC is available. In this case, the QP and LP algorithms turned out to be ineffective, since already starting \( m_{\text{max}} = 500 \) their RMS exceeded 10 m. Figure 2 shows the results of CQP and CLP algorithms, from which it follows that these algorithms are resistant to measurement reduction.

**5. Conclusion**

Algorithms have been developed for predicting differential corrections to GNSS measurements, the low complexity of which allows them to be applied in real time. Algorithms estimate the slowly changing component the correction by its values accumulated before the break. Corrections before the break are processed using the Kalman filter in the inverse time. The interval of processed corrections gradually expands as the duration of the forecast increases. Based on the results of experiments with GNSS full-scale data, it was found that the best from the point of view of the standard error of the forecast is the algorithm in which the quadratic representation of the component \( g(t) \) applied before the break, taking into account the restrictions on its value and derivative, and in the forecast section, the linear representation \( g(t) \) CLP turns out to be 10-15% more accurate than FP, 2-3 times better than LP and QP and a limited number of received DC before the break does not significantly impair its accuracy.

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