Full counting statistics of Kondo-type tunneling in a quantum dot: the fluctuation effect of Slave-Boson field

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We study the full counting statistics (FCS) of electron tunneling through a multi-terminal quantum dot in the Kondo regime within the slave-boson mean field theory. By employing the A.O. Gogolin and A. Komnik’s method of calculating the FCS generating function based on the nonequilibrium Green’s function [Phys. Rev. B 73, 195301 (2006)], we obtain the counting field $\lambda$-dependent self-consistent equations for the mean values of the slave-boson fields and the explicit expression for the derivative of the adiabatic potential of the system with respect to the counting fields. Performing perturbative expansion to the first order of $\lambda$, we find an extra contribution to the shot noise due to the bias-induced Bose field fluctuation, and then confirm that the nonequilibrium particle number fluctuation plays an important role in the current noise of the Kondo dot: enhancement of the current auto-correlation and a positive current cross-correlation.

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I. INTRODUCTION

Recently, the full counting statistics (FCS)\textsuperscript{3–8} of charge transport in mesoscopic system has become an active topic of experimental\textsuperscript{3–8} and theoretical\textsuperscript{12–24} investigation. This concept was first proposed by Levitov and Lesovik\textsuperscript{25} to describe the whole probability distribution of transmitted charge during a fixed time interval in a non-interacting mesoscopic conductor. It is therefore believed that the FCS contain full information about electron correlations that can not be obtained via measuring the average current alone.\textsuperscript{26} Since then FCS has been studied using the scattering matrix theory in a variety of systems, for example, normal and superconductor hybrid tunneling junctions\textsuperscript{16,17} chaotic quantum dots (QDs)\textsuperscript{18} entangled electrons\textsuperscript{19} and spin current\textsuperscript{20}. Moreover, for taking account of the effect of electron-electron interaction, a general scheme has been developed to evaluate the FCS of the Coulomb-blockade QD in the framework of a quantum master equation approach.\textsuperscript{21} This scheme has been then applied to analyze the FCS in a multilevel QD\textsuperscript{22} a coupled QDs\textsuperscript{18,19} and a single-molecule magnet.\textsuperscript{20} In combination with standard Rayleigh-Schrödinger perturbation theory, this approach has been further improved for investigation of the FCS in nano-electro-mechanical systems\textsuperscript{23} and even for consideration of non-Markovian dynamics.\textsuperscript{24} Very recently, finite-frequency FCS has been explored in an interacting QD.\textsuperscript{25} Besides, one of the authors has extended the famous MacDonald’s formula to calculate the FCS for the nonequilibrated-vibration-assisted tunneling in a molecular QD.\textsuperscript{26}

Over last decade, since its experimental discovery in nanoscale devices the Kondo effect and related physics have attracted enduring attention.\textsuperscript{27} This is owing to the impressive advantage of an artificial atom, easy tunability of system parameters in a considerably wide range and full controllability of external circumstances, which facilitate a new realm of research, the nonequilibrium Kondo physics in transport measurements.\textsuperscript{28,29} For instance, a bias-voltage-induced splitting of the Kondo peak in the local density of states of the QD has been experimentally observed in a three-terminal transport setup.\textsuperscript{30} It is therefore desirable to explore the FCS of electron tunneling passing through a QD connected to multi-terminal in the Kondo regime. However, those approaches mentioned above are not applicable for this purpose.

Actually, many attempts have been made in the literature to study the zero-frequency shot noise, i.e. the second current cumulant of the FCS, in a Kondo-QD\textsuperscript{31} Hershfield\textsuperscript{32} calculated the zero-frequency current noise using perturbation theory in the Hartree approximation based on the Green function (GF) approach. He found the interaction can either enhance or reduce the shot noise. Yamaguchi and Kawamura\textsuperscript{33} performed a complementary analysis by choosing the tunneling term of the Hamiltonian as the perturbation parameter and revealed a large suppression as comparison with the Poisson value. Ding and Ng\textsuperscript{34} calculated the frequency-dependent shot noise by employing the equation-of-motion method and Ng’s ansatz for the correlation GF. Their results also demonstrated suppression of the shot noise below the noninteracting value. However these results are all only valid at high temperature, $T > T_K$ ($T_K$ is the Kondo temperature), and weak Kondo correlations. For analyzing the shot noise in the case of strong Kondo correlation, Dong and Lei\textsuperscript{35} employed the finite Coulomb interaction $U$ slave-boson mean-field theory (SBMFT) to calculate the current-current correlation function at zero temperature based on the nonequilibrium GF (NGF) technique and Wick’s theorem. Later, López and his coworkers utilized the same theoretical framework with the infinite-$U$ version of SBMFT to study the shot noise in a single QD with ferromagnetic leads\textsuperscript{36} and multi-terminals\textsuperscript{37} and a coupled QD system.\textsuperscript{35} Very recently, this method has been used to evaluate the shot noise in the parallel coupled QDs to distinguish between the spin and orbital Kondo effect.\textsuperscript{38} It should be noted that the fluctuations of slave Bose fields are all neglected in the calculation.
of current correlation function. Meir and Golub performed a exhaustive calculation of the shot noise based on the noncrossing approximation (NCA). However, they just substituted the resulted NCA propagators into the current correlation function derived from the noninteracting electron presumption.

For applying the SBMFT to explore the FCS, one must overcome a technical difficulty: how to evaluate all higher-order terms of current correlation functions one-by-one according to Wick’s theorem. Even though this is not impossible from theoretical point of view (for example, the third cumulant has been derived using Wick’s theorem), it is actually not practically executable. Fortunately, a successful solution to this problem has been given in Ref. done by Gogolin and Konnik. In their seminal work, they introduced a fictitious measuring field \( \lambda \) in the tunneling Hamiltonian to count an electron when it goes through the system under studied in the direction of current and assumed it as a slowly-varying quantity in time (this is valid since it is set to be equal to zero in the final concrete calculations). This presumption facilitates an adiabatic expansion of the generating function of the cumulants of charge current distribution to the first order. By doing so, the calculation of generating function is then transferred to the calculation of a so-called adiabatic potential, which can be completely determined by the well defined single-particle Keldysh GFs. At the end, they obtained a generic formula for the generating function of the cumulants expressed only in terms of the local Keldysh GFs of the central region (and the tunnel-coupling and Fermi functions of the leads), as done in the Meir-Wingreen current formula, except for the presence of the counting field. The advantage of this Hamiltonian approach is that (1) it provides a systematic and easy method to evaluate the cumulants of FCS by avoiding the tedious application of Wick’s theorem; (2) more importantly, the derivation makes no assumptions about interaction between electrons inside the central region; and (3) we can directly use the whole power of the Feynman diagram technique and connect to many known results of the NGF. Of course, a proper knowledge of the self energy is still indispensable for interacting systems. With respect to this consideration, Gogolin and his coworkers first applied their approach to investigate the FCS for a Kondo-QD in the multiterminal regime at the Toulouse limit, where the single-impurity Anderson Hamiltonian can be mapped to a quadratic form by performing a canonical transformation. Moreover, they analyzed the FCS of charge and spin currents at the strong-coupling fixed point within the framework of the Nozières-Fermi-liquid theory. Besides, they also studied the current cross-correlation (CC) correlations of a multiterminal Kondo-QD in the strong-coupling limit. Another application of this approach is to explore the FCS of a molecular QD with strong electron-phonon interaction.

However, it is known that the theories of the Toulouse limit and the strong-coupling limit are both valid only at the deep Kondo region, where charge fluctuation is totally quenched and the dynamic properties of the system are completely determined by spin fluctuation. So in the present paper we will perform a complementary investigation of the FCS for a Kondo-QD in the multi-probe case at the deep and intermediate Kondo regions by means of the Coleman’s infinite-\( U \) SBMFT which is believed to provide a proper description of the Kondo correlation at these regions even under nonequilibrium situation at zero temperature. In combination of the SBMFT with the Gogolin and Konnik’s approach, we find that the self-consistent equations, which determine the expectation values of the slave-boson fields, becomes counting field \( \lambda \)-dependent. It is argued that these \( \lambda \)-dependent terms of the mean values describe the Bose field fluctuation (it is equivalent to the charge fluctuation due to the completeness relation between fermions and bosons) induced by transport measurement. More interestingly, we find that these \( \lambda \)-related parts of slave-boson fields generate an additional contribution in the zero-frequency shot noise formula, in contrast to our previous result without consideration of the Bose field fluctuations. Numerical calculations show that these additional term results in obvious enhancement of the current auto-correlation in the two-terminal setup and even a sign change of the current cross-correlation in the three-terminal setup of the Kondo dot in the intermediate Kondo regions.

The outline of the paper is as follows. In Sec. II, we give the model Hamiltonian describing electron tunneling through an interacting QD attached to three leads in the presence of the three respective counting fields and explain the theoretical formulation of the SBFMT, the counting-field-dependent self-consistent equations. In Sec. III, we provide the explicit expressions of the first-order differential equation of the adiabatic potential with respect to the counting fields, and the zero-frequency current auto- and cross-correlations at zero temperature. In particular, we discuss the additional terms due to fluctuation of slave-boson field. In Sec. IV, we perform concrete numerical calculations and discussions for the current auto- and cross-correlations based on the formulae given in Sec. III. Finally, our conclusions are given in section V.

II. MODEL HAMILTONIAN AND THEORETICAL FORMULATION

A. Model

We model the electronic transport through a single-level QD coupled to multi-terminal using the infinite-\( U \) Anderson Hamiltonian as:

\[
H = \sum_{\eta k \sigma} \epsilon_{\eta k} c_{\eta k \sigma}^\dagger c_{\eta k \sigma} + \sum_{\sigma} \epsilon_d d_{\sigma}^\dagger d_{\sigma} + Un_d n_d^\dagger
\]
where $c_{\eta\kappa\sigma}^\dagger$ ($c_{\eta\kappa\sigma}$) creates (destroys) an electron with momentum $k$, spin $\sigma = \{\uparrow, \downarrow\}$, and energy dispersion $\epsilon_{\eta\kappa\sigma}$ in the lead $\eta$, with $\eta = \{1,2,3\}$ in the three-terminal case: $d_{\eta}^\dagger$ ($d_{\eta}$) creates (destroys) a spin-$\sigma$ electron on the QD with energy level $\epsilon_d$; $\gamma_{\eta}$ is the coupling matrix element between the dot and lead $\eta$; and $\lambda_{\eta}(t)$ is the artificially introduced measuring field with respect to the lead $\eta$ on the Keldysh contour: $\lambda_{\eta}(t) = \lambda_{\eta-}\theta(t)\theta(T-t)$ on the forward path and $\lambda_{\eta}(t) = \lambda_{\eta+}\theta(t)\theta(T-t)$ on the backward path ($T$ is the measuring time during which the counting fields are non-zero and $\lambda_{\eta-} = -\lambda_{\eta+} = \lambda$)\textsuperscript{11,20,42}. Under the framework of the infinite-$U$ slave-boson approach, the ordinary electron operators on the QD are decomposed into a boson operator $\hat{b}$ (describing the empty state on the QD) and a fermion operator $f_{\sigma}$ (denoting the singly occupied state with electron spin-$\sigma$), $d_{\sigma} = f_{\sigma}\hat{b}$ and $d_{\sigma}^\dagger = f_{\sigma}^\dagger\hat{b}$\textsuperscript{24}. In addition, a constraint, $\hat{b}^\dagger\hat{b} + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} = 1$, must be imposed on these auxiliary operators as requirement of no double occupancy in the $U \to \infty$ limit. Then, the effective Hamiltonian becomes

$$H_{\text{eff}} = \sum_{\eta\kappa\sigma} \epsilon_{\eta\kappa\sigma} c_{\eta\kappa\sigma}^\dagger c_{\eta\kappa\sigma} + \sum_{\sigma} \epsilon_{d}\ f_{\sigma}^\dagger f_{\sigma} + \sum_{\eta\kappa\sigma} \left[\gamma_{\eta} e^{i\lambda_{\eta}(t)/4} f_{\sigma}^\dagger b c_{\eta\kappa\sigma} + \text{H.c.}\right] + \xi \left(\hat{b}^\dagger \hat{b} + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} - 1\right),$$

(2)

with a Lagrange multiplier $\xi$ to guarantee satisfaction of the constraint.

### B. Self-consistent Equations of Slave-Boson Mean Field Theory

In the mean field approximation, the slave Bose operators, $\hat{b}^\dagger$ and $\hat{b}$, can be assumed as $c$-number and replaced by their corresponding expectation values, $b^\dagger$ and $b$, in the Hamiltonian Eq. (2). To determine the unknown parameters, $b$ and $\xi$, we start from the constraint, and the equation of motion of the slave-boson operator using the Keldysh technique for systems out of equilibrium:

$$\sum_{\sigma} \gamma_{\eta} G_{\sigma,\sigma,\eta}(t, t) = 0,$$

(4)

where $n_b = \langle \hat{b}^\dagger \hat{b} \rangle$ is the unoccupied-state number, $G_{\sigma,\sigma,\eta}^+(t', t) = i(f_{\sigma}^\dagger(t') f_{\sigma}(t))$ is the dot non-equilibrium correlation GF, and $G_{\sigma,\eta\kappa\eta}(t, t') = i(c_{\eta\kappa\sigma}(t') f_{\sigma}(t))$ denotes the dot-lead correlation GF. With the Hamiltonian Eq. (2), the mixed correlation GF can be readily cast in terms of $G_{\sigma,\sigma,\eta}^+(t, t')$ with the help of the equation of motion of the operators and then applying the Langreth analytical continuation rules in a complex time contour\textsuperscript{45}. In this end, we obtain a self-consistent set of equations in terms of the QD’s correlation GF in Fourier space:

$$\frac{1}{2\pi i} \sum_{\sigma} \int d\omega G_{\sigma,\sigma}^+(\omega)(\omega - \epsilon_{\sigma} - \xi) = 0,$$

(6)

Therefore, our next step is to calculate the dot correlation GF, $G_{\sigma,\sigma}^+(\omega)$. It is evident that the mean-field Hamiltonian Eq. (2) is effectively a noninteracting resonant level model. By taking the counting field $\lambda_{\eta}(t)$ to be opposite constant on the forward and backward Keldysh branches as $\lambda_{\eta-} = -\lambda_{\eta+} = \lambda_{\eta}$\textsuperscript{11} one can therefore readily evaluate the NGF of pseudo-fermion operator $f_{\sigma}$ in terms of the original notation of Keldysh for GFs (the time-ordered GFs):

$$G_{\sigma}(\omega) = \frac{1}{D_0(\omega)} \left( \omega - \epsilon_{\sigma} - \xi + i n_b \sum_{\eta} \Gamma_{\eta}(2 f_{\eta} - 1) - 2 i n_b \sum_{\eta} e^{-i\lambda_{\eta}/2} \Gamma_{\eta}(1 - f_{\eta}) - (\omega - \epsilon_{\sigma} - \xi) + i n_b \sum_{\eta} \Gamma_{\eta}(2 f_{\eta} - 1) \right),$$

(7)

with $(\Gamma = \sum_{\eta} \Gamma_{\eta})$

$$D_0(\omega) = (\omega - \epsilon_{\sigma} - \xi)^2 + (n_b \Gamma)^2 + 4 n_b^2 \sum_{\eta,\eta'} \Gamma_{\eta} \Gamma_{\eta'} f_{\eta}(1 - f_{\eta'}) \left(e^{i(\lambda_{\eta} - \lambda_{\eta'})/2} - 1\right),$$

(8)

where $f_{\eta} = [1 + \exp(\omega - \mu_{\eta})/k_BT]^{-1}$ is the Fermi distribution function at temperature $T$ and chemical potential $\mu_{\eta} = E_F + eV_{\eta}$ of lead $\eta$ ($E_F$ is the Fermi energy and $V_{\eta}$ is the bias-voltage applied to lead $\eta$), and
\[ \Gamma_n = \pi \sum_k |V_{nk}|^2 \delta(\omega - \epsilon_{nk}) \] is the coupling strength between the QD and lead \( \eta \). In the wide band limit, we neglect the energy dependence of \( \Gamma_n \) and take it as a constant. Note that the GF formulæ is similar to that of a noninteracting system, except with the effective energy level \( \epsilon_d + \xi \) and the effective tunnel-coupling constant \( n_b \Gamma_n \) instead, which renormalize the GF of the QD due to Kondo correlation under the approximation employed here.

Substituting the resulting GF Eq. (7) into the self-consistent equations (4) and (5), we can obtain the two parameters, \( n_b \) and \( \xi \), under a finite bias voltage. Obviously, they are both functions of the counting fields \( \lambda_n \). For the sake of analysis and calculation of the shot noise, we can expand \( n_b \) and \( \xi \) to the first order of \( \lambda_n \) as:

\[
\begin{align*}
n_b &= n_b^{(0)} + i \sum_n \frac{1}{n} \lambda_n n_b^{(1)} + O(\lambda_n^2), \\
\xi &= \xi^{(0)} + i \sum_n \frac{1}{n} \lambda_n \xi^{(1)} + O(\lambda_n^2),
\end{align*}
\]

where the superscripts \((0)\) and \((1)\) denote the zeroth and first order terms of the expansion coefficients, respectively. The zeroth order terms, \( n_b^{(0)} \) and \( \xi^{(0)} \), are the original variational parameters of the SBMFT and irrespective of the counting fields; while the first order terms, \( n_b^{(1)} \) and \( \xi^{(1)} \), are new here depicting fluctuations of the two parameters around their respective expectation values due to measurement.

The zeroth order self-consistent equations become

\[
\begin{align*}
n_b^{(0)} &= \int \frac{d\omega}{2\pi} \frac{2 \sum_n \Gamma_n f_n}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2} + 1 = 1, \\
\xi^{(0)} &= \int \frac{d\omega}{\pi} \frac{2(\omega - \tilde{\epsilon}_d)(\sum_n \Gamma_n f_n)}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2} = 0,
\end{align*}
\]

with \( \tilde{\epsilon}_d = \epsilon_d + \xi^{(0)} \) and \( \tilde{\Gamma} = n_b^{(0)} \Gamma \). It is noted that these two nonlinear equations are exactly the same as the previous results without the presence of the counting fields \( \lambda_n \) and they constitute a closed set of equations to completely determine the unknown parameters, \( n_b^{(0)} \) and \( \xi^{(0)} \), for a particular QD system, \( \epsilon_d \), at a given bias voltage. The parameter \( \xi^{(0)} \) characterizes the location of the Kondo peak in the quasiparticle density of states, and the parameter \( n_b^{(0)} \) mimics the width of the Kondo peak (i.e. the Kondo temperature \( T_K \)) and renormalizes the tunnel-coupling of the QD to the external leads by Kondo correlation. It is known that the two parameters give the correct qualitative behavior of Kondo physics at zero temperature and low bias voltages \( (eV \sim T_K) \), and thus define the current-bias voltage characteristics and the differential conductance of a Kondo-QD.\[42,43,45,47\]

Furthermore, we can obtain the first order self-consistent equations as

\[
(I^{(0)} - I^{(1)}_{a}) \frac{n_b^{(0)}}{n_b^{(1)}} + I^{(1)}_{b} \xi^{(1)} + I^{(1)}_{c} = 0, \quad (13)
\]

\[
K^{(1)}_{a} \frac{n_b^{(1)}}{n_b^{(0)}} - K^{(1)}_{b} \xi^{(1)} + K^{(1)}_{c} = 0, \quad (14)
\]

where the integrals are as follows:

\[
\begin{align*}
I^{(0)} &= \int \frac{d\omega}{2\pi} \frac{2 \sum_n \Gamma_n f_n}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2}, \\
I^{(1)}_{a} &= \int \frac{d\omega}{2\pi} \frac{4 \tilde{\Gamma}^2 \sum_n \Gamma_n f_n}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2}, \\
I^{(1)}_{b} &= \int \frac{d\omega}{2\pi} \frac{4(\omega - \tilde{\epsilon}_d)(\sum_n \Gamma_n f_n)}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2}, \\
I^{(1)}_{c} &= \int \frac{d\omega}{2\pi} \frac{4 \sum_n \Gamma_n f_n}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2}, \\
K^{(1)}_{a} &= 2 \tilde{\Gamma}^2 I^{(1)}_{b}, \\
K^{(1)}_{b} &= 1 + \int \frac{d\omega}{2\pi} \frac{4[(\omega - \tilde{\epsilon}_d)^2 - \tilde{\Gamma}^2](\sum_n \Gamma_n f_n)}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2}, \\
K^{(1)}_{c} &= \int \frac{d\omega}{2\pi} \frac{16(\sum_n \Gamma_n f_n)^2(\omega - \tilde{\epsilon}_d)}{[(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2]^2}. \quad (15)
\end{align*}
\]

These equations are one of the central results of this paper. Once \( n_b^{(0)} \) and \( \xi^{(0)} \) are known, \( n_b^{(1)} \) and \( \xi^{(1)} \) can be evaluated by solving the set of linear equations, Eqs. (13) and (14). The two first order terms have no influence on the current but do affect the shot noise and of course the higher cumulants (see below). One can argue that the two terms depict the fluctuations of both the boson field \( b \) and the renormalization of the resonant peak \( \xi \), which can be ascribed to the charge fluctuation of the QD due to its attachment to external electrodes and application of bias voltage. Consequently, we will find in the following calculation that these terms have a nonzero contribution to the shot noise of the QD at a finite bias voltage. Moreover, they play a more important role at the intermediate Kondo regions, where the charge fluctuation is more profound, than at the deep Kondo regions.
III. FULL COUNTING STATISTICS BY SLAVE-BOSON MEAN FIELD THEORY

In this section, we will first study the cumulant generating function (CGF) \( \chi(\lambda) \equiv \chi(\lambda_1, \lambda_2, \lambda_3) \) for the FCS of a Kondo-QD with three terminals within SBMFT. Then we will derive the formula for the current auto-correlation and cross-correlation taking into account the charge fluctuation effect in the two- and three-terminal configurations, respectively.

The CGF can be calculated as a Keldysh partition function:

\[
\chi(\lambda) = \langle T_C e^{-i \int_C T_\lambda(t) dt} \rangle ,
\]

where \( T_C \) is the Keldysh contour ordering operator and

\[
T_\lambda(t) = \sum_{\eta \kappa \sigma} [\gamma_{\eta} e^{i \lambda_n(t)/4} f_{\sigma}^\dagger \hat{b}_{\eta \kappa} + \text{H.c.}] \]

is the electron tunneling operator in the Hamiltonian Eq. (2). According to Refs. [40-42], to calculate the CGF \( \chi(\lambda) \) it is technically more convenient employing the adiabatic potential method: \( \ln \chi(\lambda) = -i \mathcal{U}(\lambda_-, \lambda_+) = -i \mathcal{U}(\lambda, -\lambda) \), where the adiabatic potential \( \mathcal{U}(\lambda, -\lambda) \) is defined due to the nonequilibrium Feynman-Hellmann theorem as

\[
\frac{\partial \mathcal{U}(\lambda_-, \lambda_+)}{\partial \lambda_{\eta^-}} = \langle \frac{\partial T_\lambda(t)}{\partial \lambda_{\eta^-}} \rangle_{\lambda} = \frac{i}{4} \sum_{\kappa \sigma} \langle \gamma_{\eta} e^{i \lambda_{\eta^-}/4} f_{\sigma}^\dagger \hat{b}_{\eta \kappa} - \text{H.c.} \rangle_{\lambda} .
\]

Therefore, the evaluation of the adiabatic potential amounts to a calculation of the mixed GFs, which can be cast into a combination of the dot GF \( G_{f\sigma}(\omega) \) and bare lead GFs. By using the NGF obtained in last section and taking the counting field \( \lambda_{\eta^-} = -\lambda_{\eta^+} = \lambda_\eta \), the derivative of the adiabatic potential is given by

\[
\frac{\partial \mathcal{U}(\lambda, -\lambda)}{\partial \lambda_{\eta}} = -2 \int \frac{d\omega}{2\pi} \frac{\sum_{\eta'} T_{\eta\eta'}(\omega) \left[ f_{\eta}(1 - f_{\eta'}) e^{i(\lambda_n - \lambda_{\eta'})/2} - f_{\eta'}(1 - f_{\eta}) e^{-i(\lambda_n - \lambda_{\eta'})/2} \right]}{1 + \sum_{\eta, \eta'} T_{\eta\eta'}(\omega) \left[ f_{\eta'}(1 - f_{\eta}) e^{i(\lambda_n - \lambda_{\eta'})/2} - 1 \right]} ,
\]

where \( T_{\eta\eta'}(\omega) \) is the transmission coefficient of electron between leads \( \eta \) and \( \eta' \):

\[
T_{\eta\eta'}(\omega) = \frac{4 n_k^2 \Gamma_{\eta} \Gamma_{\eta'}}{(\omega - \epsilon_d - \xi)^2 + (n_k \Gamma)^2} .
\]

It is clear to see that the transmission coefficient (1) has a simple Lorentzian line shape centered around the Fermi level (the Abrikosov-Suhl resonance) with a renormalized width due to Kondo correlation, and (2) is symmetric for exchanging \( \eta \) and \( \eta' \): \( T_{\eta\eta'}(\omega) = T_{\eta'\eta}(\omega) \). It should be noted that due to the counting field dependence of the parameters \( n_k \) and \( \xi \) involved in the transmission coefficient \( T_{\eta\eta'}(\omega) \), an explicit analytic expression for the CGF cannot be recovered here from Eq. (18). However, one can still obtain the \( m \)-th cumulant of the charge distribution involving lead \( \eta \) by executing the \( m \)-th derivative of the CGF, with respect to the corresponding counting field \( \lambda_\eta \) at \( \lambda_1 = \lambda_2 = \lambda_3 = 0 \) (we denote this condition as \( \lambda = 0 \) for shorthand in the following). In particular, the first two cumulants give the average current and the zero-frequency shot noise, respectively, which constitute our main objects in the present paper. For example, the current \( I_\eta \) from lead \( \eta \) to the QD is evaluated as follows:

\[
I_\eta = \sum_{\sigma} I_{\eta\sigma} = \frac{2e}{h} \int \frac{d\omega}{2\pi} \frac{\sum_{\eta} T_{\eta\eta'}(\omega) \left[ f_{\eta}(1 - f_{\eta'}) e^{i(\lambda_n - \lambda_{\eta'})/2} - f_{\eta'}(1 - f_{\eta}) e^{-i(\lambda_n - \lambda_{\eta'})/2} \right]}{1 + \sum_{\eta, \eta'} T_{\eta\eta'}(\omega) \left[ f_{\eta'}(1 - f_{\eta}) e^{i(\lambda_n - \lambda_{\eta'})/2} - 1 \right]}_{\lambda=0} = \frac{2e}{h} \int \frac{d\omega}{2\pi} \frac{\sum_{\eta} T_{\eta\eta'}(\omega) \left( f_{\eta} - f_{\eta'} \right)_{\lambda=0}} .
\]

It is clear that the three-terminal current formula Eq. (20) is in perfect agreement with the previous result [32,34] and reduces to the transparent form of the famous Meir-Wingreen current formula in the two-terminal case (for instance, by setting \( \Gamma_3 = 0 \)) [42].
We calculate now the zero-frequency current fluctuations of lead $\eta$:

$$S_{\eta\eta}(0) = \frac{4e^2}{h} \frac{1}{T} \frac{\partial^2 \ln \chi(\lambda)}{\partial (i\lambda \eta/2)^2} \bigg|_{\lambda=0} = S_{\eta\eta}^{(0)}(0) + S_{\eta\eta}^{(c)}(0),$$

(21)

where $S_{\eta\eta}^{(0)}(0)$ is the mean-field result of the current correlation

$$S_{\eta\eta}^{(0)}(0) = \frac{4e^2}{h} \int d\omega \left\{ \sum_{\eta'} \left[ \sum_{\eta'' \neq \eta} T_{\eta\eta'}(\omega)[f_\eta(1-f_{\eta''}) + f_{\eta''}(1-f_\eta)] - \left[ \sum_{\eta'' \neq \eta} \sum_{\eta'''} \right] T_{\eta\eta'}(\omega)(f_\eta - f_{\eta''}) \right\} \bigg|_{\lambda=0},$$

(22)

and $S_{\eta\eta}^{(c)}(0)$ is the correction to the mean-field result of the shot noise

$$S_{\eta\eta}^{(c)}(0) = \frac{4e^2}{h} \int d\omega \sum_{\eta'} \left[ \frac{\partial T_{\eta\eta'}(\omega)}{\partial (i\lambda \eta/2)} \bigg|_{\lambda=0} (f_\eta - f_{\eta'}) \right].$$

(23)

It is clear that $S_{\eta\eta}^{(0)}(0)$ depends on the zeroth order terms of the two parameters, $n^{(0)}$ and $\xi^{(0)}$ only, while $S_{\eta\eta}^{(c)}(0)$ is relevant to the first order terms, $n^{(1)}_{\eta\eta}$ and $\xi^{(1)}_{\eta\eta}$, and can be explicitly expressed as

$$S_{\eta\eta}^{(c)}(0) = \frac{4e^2}{h} \int d\omega \sum_{\eta'} \left\{ \frac{8n^{(1)}_{\eta\eta} (\omega - \tilde{\epsilon}_d)^2 \tilde{T}_{\eta\eta}^d \tilde{T}_{\eta\eta}'^d}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2} + 8\xi^{(1)}_{\eta\eta} (\omega - \tilde{\epsilon}_d) \tilde{T}_{\eta\eta}^d \tilde{T}_{\eta\eta}'^d \right\} (f_\eta - f_{\eta'}).$$

(24)

The zero-frequency CC of currents through the different leads $\eta \neq \eta'$ can be calculated as

$$S_{\eta\eta'}(0) = \frac{4e^2}{h} \frac{1}{T} \frac{\partial^2 \ln \chi(\lambda)}{\partial (i\lambda \eta/2) \partial (i\lambda \eta'/2)} \bigg|_{\lambda=0} = S_{\eta\eta'}^{(0)}(0) + S_{\eta\eta'}^{(c)}(0).$$

(25)

Likewise, the CC $S_{\eta\eta'}(0)$ can be separated into two parts, the mean field result $S_{\eta\eta'}^{(0)}(0)$ and its correction $S_{\eta\eta'}^{(c)}(0)$:

$$S_{\eta\eta'}^{(0)}(0) = -\frac{4e^2}{h} \int d\omega \left\{ T_{\eta\eta'}(\omega)[f_\eta(1-f_{\eta'}) + f_{\eta'}(1-f_\eta)] + \left[ \sum_{\eta_1} T_{\eta\eta_1}(\omega)(f_\eta - f_{\eta_1}) \right] \left[ \sum_{\eta_2} T_{\eta_2\eta'}(\omega)(f_{\eta'} - f_{\eta_2}) \right] \right\} \bigg|_{\lambda=0},$$

(26)

$$S_{\eta\eta'}^{(c)}(0) = \frac{4e^2}{h} \int d\omega \sum_{\eta_1} \frac{\partial T_{\eta\eta_1}(\omega)}{\partial (i\lambda \eta/2)} \bigg|_{\lambda=0} (f_\eta - f_{\eta_1})$$

$$= \frac{4e^2}{h} \int d\omega \int \sum_{\eta_1} \left\{ \frac{8n^{(1)}_{\eta\eta} (\omega - \tilde{\epsilon}_d)^2 \tilde{T}_{\eta\eta}^d \tilde{T}_{\eta\eta_1}^d}{(\omega - \tilde{\epsilon}_d)^2 + \Gamma^2} + 8\xi^{(1)}_{\eta\eta} (\omega - \tilde{\epsilon}_d) \tilde{T}_{\eta\eta}^d \tilde{T}_{\eta\eta_1}^d \right\} (f_\eta - f_{\eta_1}).$$

(27)

For illustration, we give the explicit expression for the mean-field shot noise, i.e., the current auto-correlation measured at the same lead (e.g., lead 1):

$$S_{11}^{(0)}(0) = \frac{4e^2}{h} \int d\omega \left\{ T_{12}[f_1(1-f_1) + f_2(1-f_2)] + T_{13}[f_1(1-f_1) + f_3(1-f_3)] + T_{12}(1 - T_{12})(f_1 - f_2)^2 + T_{13}(1 - T_{13})(f_1 - f_3)^2 - 2T_{12}T_{13}(f_1 - f_2)(f_1 - f_3) \right\} \bigg|_{\lambda=0},$$

(28)

and the explicit expression for the mean-field current CC between leads 2 and 3:

$$S_{23}^{(0)}(0) = -\frac{4e^2}{h} \int d\omega \left\{ T_{23}[f_2(1-f_2) + f_3(1-f_3)] + T_{23}(1 - T_{23})(f_2 - f_3)^2 + T_{23}S_{23}(f_2 - f_3)(f_2 - f_3) + T_{23}T_{23}(f_3 - f_1)(f_2 - f_3) \right\} \bigg|_{\lambda=0}. $$

(29)

Furthermore, in order to compare the present results with the previous formulae, we discuss the symmetrized
shot noise in the case of a QD connected to two reservoirs. By setting \( \Gamma_2 = 0 \), the model employed above becomes a two-terminal system and the symmetrized shot noise is then defined as \( S(0) = (S_{11}(0) + S_{22}(0) - 2S_{12}(0))/4 = S^{(0)}(0) + S^{(c)}(0) \), whose first part can be calculated via Eqs. (22) and (25) as

\[
S^{(0)}(0) = \frac{4e^2}{h} \int d\omega \{ T_{12}\{ f_1(1 - f_2) + f_2(1 - f_1) \}
\]
\[
- T_{12}^2(f_1 - f_2)^2 \} |_{\lambda=0} . \quad (30)
\]

Obviously, this formula is exactly the same as our previous derivation applying Wick theorem within the framework of the SBFMT. This is the reason that we call it and Eqs. (22) and (25) as mean-field terms of the current correlations. For reference, we briefly recall our previous derivation. The current operator \( \hat{I}_\eta \) flowing from the lead \( \eta \) to the QD is defined as

\[
\hat{I}_\eta(t) = -\frac{i e}{\hbar} [H_{\text{eff}}, \sum_{\eta\sigma} c_{\eta\sigma}^\dagger c_{\eta\sigma}]
\]
\[
= \frac{i e}{\hbar} \sum_{\eta\sigma} (V_{\eta} b_{\eta\sigma}^c f_\sigma - V_{\eta}^* f_\sigma^c b_{\eta\sigma}), \quad (31)
\]

and the power spectrum of the CC correlator reads

\[
S_{\eta\eta'}(\omega) = 2 \int dr e^{i\omega r} \langle \{ \{ \hat{I}_\eta(t), \hat{I}_{\eta'}(t') \} \} - \langle \hat{I}_\eta \rangle \langle \hat{I}_{\eta'} \rangle \rangle, \quad (32)
\]

with \( \tau = t - t' \). Calculations of the correlator involves a pair of creation and destruction operators for each of the current operators at the two time variables. According to the conventional diagrammatic expansion technique of GF, the statistical expectation of the product of any two operators \( \hat{A} \) and \( \hat{B} \) can be divided into two parts

\[
\langle \hat{A}\hat{B} \rangle = \langle \hat{A} \rangle \langle \hat{B} \rangle + \langle \hat{A}\hat{B} \rangle_c,
\]

where \( \langle \cdots \rangle_c \) means the connected part of the statistical average, which is vanishing for noninteracting systems. As a result, under the framework of SBFMFT and neglecting the fluctuations of both \( n_b \) and \( \xi \) (i.e. only considering the leading terms, \( n_b^{(0)} \) and \( \xi^{(0)} \)), we can eliminate the connected part of the above statistical expectation, and entirely contract the ensemble averaging with a product of four operators in \( \langle \{ \hat{I}_\eta(t), \hat{I}_{\eta'}(t') \} \rangle \) as a product of ensemble averagings of two pairs of operators, such as

\[
\langle c_{\eta\sigma}^\dagger c_{\eta'\sigma}(t) f_\sigma(t') c_{\eta'\sigma}^\dagger c_{\eta\sigma}(t) \rangle
\]
\[
= \langle c_{\eta\sigma}^\dagger c_{\eta'\sigma}(t) f_\sigma(t') \rangle \langle c_{\eta'\sigma}(t) c_{\eta'\sigma}^\dagger(t') \rangle
\]
\[
+ \langle c_{\eta\sigma}^\dagger c_{\eta'\sigma}(t) c_{\eta'\sigma}^\dagger c_{\eta\sigma}(t') \rangle \langle f_\sigma(t) c_{\eta'\sigma}(t) \rangle, \quad (34)
\]

and

\[
\langle c_{\eta\sigma}^\dagger c_{\eta'\sigma}(t) f_\sigma(t') f_\sigma(t') c_{\eta'\sigma}^\dagger c_{\eta\sigma}(t') \rangle
\]
\[
= \langle c_{\eta\sigma}^\dagger c_{\eta'\sigma}(t) f_\sigma(t') \rangle \langle f_\sigma(t') c_{\eta'\sigma}^\dagger c_{\eta\sigma}^\dagger(t') \rangle
\]
\[
+ \langle c_{\eta\sigma}^\dagger c_{\eta'\sigma}(t) c_{\eta'\sigma}^\dagger c_{\eta\sigma}(t') \rangle \langle f_\sigma(t) f_\sigma(t') \rangle. \quad (35)
\]

All first terms in the left-hand side cancel out the term \( \langle I_\eta \rangle \langle I_{\eta'} \rangle \) of Eq. (22), whereas the second terms result in two lesser GFs running antiparallel in time (one starts at \( t \) going to \( t' \), the other goes from \( t' \) to \( t \)), according to the Langreth theorem: the object with two antiparallel GFs \( C(t, t') = A(t', t)B(t, t') \) has the lesser component of \( C^<(t, t') = A^<(t', t)B^<(t, t') \). In succession, the hybrid lesser GFs can further simplified as a proper combination of the local Keldysh GFs of the QD with the tunnel-coupling and Fermi functions of the leads, which leads in the end to the mean-field formula of the shot noise, Eq. (30).

Now we concentrate on the second part of the symmetrized shot noise, \( S^{(c)}(0) \). Without loss of generality, we can assume \( \lambda_1 = -\lambda_2 \) (as a matter of fact, in the two-terminal case, although two counting fields \( \lambda_1 \) and \( \lambda_2 \) are introduced, the final result depends only on the difference \( \lambda = \lambda_1 - \lambda_2 \) due to the charge conservation), and define \( n_b^{(1)} - n_b^{(2)} = n_b^{(1)} \) and \( \xi^{(1)} - \xi^{(2)} = \xi^{(1)} \). Then from Eqs. (24) and (27), we yield

\[
S^{(c)}(0) = \frac{4e^2}{h} \int d\omega \left\{ \begin{array}{l}
8n_b^{(1)}(0) \frac{(\omega - \tilde{\epsilon}_d)^2 \tilde{\Gamma}_1 \tilde{\Gamma}_2}{(\omega - \tilde{\epsilon}_d)^2 + (\tilde{\Gamma})^2} \\
+ 8\xi^{(1)}(\omega - \tilde{\epsilon}_d) \tilde{\Gamma}_1 \tilde{\Gamma}_2 \end{array} \right\} (f_1 - f_2). \quad (36)
\]

From technical point of view, this term is a direct result of the connected part in Eq. (33), and consequently constitutes a correction to the mean-field result, Eq. (30). Meanwhile, bearing in mind that in above mean-field derivation the fluctuations of the boson field and the renormalization of the resonant level are both neglected, one can immediately conclude that this term \( S^{(c)}(0) \) is physically an account of the bias-voltage-induced fluctuation effect of the boson field. It is clear to see that \( S^{(c)}(0) \) is proportional to both \( n_b^{(1)} \) and \( \xi^{(1)} \), and is vanishing under equilibrium situation.

IV. RESULTS AND DISCUSSION

In this section, we present the results of numerical calculations of the zero-frequency auto- and cross-current noises for the QD in the two- and three-terminal cases in the Kondo regime at zero temperature, paying special attention to the effect of boson field fluctuation on the current correlations, i.e., the correction terms of the shot noise formulae. The self-consistent parameters, \( n_b^{(0)} \) and \( \xi^{(0)} \), \( n_b^{(1)} \) and \( \xi^{(1)} \), can be obtained by solving Eqs. (11) and (12), (13) and (14), respectively, for each bias voltage and a given system \( \epsilon_d \). Using these calculated parameters, we can then compute the mean-field results of shot noises and the total shot noises as functions of bias voltage according to Eqs. (20), (27), (33), and (35), respectively. In particular, we choose \( \Gamma_1 = \Gamma_2 = \Gamma/2 \) as a
We depict in Fig. 1 the calculated self-consistent parameters, $n_b^{(0)}$ and $n_b^{(1)}$, $\xi^{(0)}$ and $\xi^{(1)}$ as functions of bias voltage $V$ for the two-terminal QD with the energy level $\epsilon = -4.0$.

Typical example and set $\Gamma$ as the energy unit. The reference energy will be always set at $E_F = 0$. For the multiterminal Kondo-QD, the width of the Kondo peak at the Fermi energy is characterized by the Kondo temperature, $k_BT_K = D\exp(-\pi|\epsilon|/2\Gamma)$, at equilibrium ($D$ is the energy cutoff and is set to $D = 100\Gamma$ here). In the following calculations, we confine the bias voltage from 0 to $3k_BT_K/e$ ($eV \leq 3k_BT_K$) to ensure that the SBFMT is applicable for the Kondo correlation at nonequilibrium.

First, we discuss the auto-current correlation for the two-terminal case, in which the external bias voltage is applied symmetrically, $V_1 = -V_2 = V/2$. For illustration, we depict in Fig. 1 the calculated self-consistent parameters, the zeroth order terms $n_b^{(0)}$, $\xi^{(0)}$, and the first order terms $n_b^{(1)}$, $\xi^{(1)}$ versus the bias voltage $V$ for a QD with the discrete energy level $\epsilon = -4.0$. It is easy to see that (1) these first order terms are nearly two orders of magnitude smaller than their corresponding zeroth order terms at the whole region of the bias voltages; (2) they are increasing with increase of applied bias voltage; (3) they are vanishing at equilibrium, $V = 0$. These features are in good agreement with the consideration that the first order terms describe the fluctuations of the corresponding parameters stemming from the bias-voltage-induced charge fluctuations even in the Kondo regime. However, these small fluctuations do cause an additional positive contribution to the mean-field shot noise as shown in Fig. 2(a,b), where the mean-field results $S(0)$ and the total results $S(0)$ of the symmetrized shot noise are both plotted as functions of bias voltage for two QDs with $\epsilon = -5.0, -4.0$. Meanwhile, it is clear that the enhancement is more obvious in the system with $\epsilon = -4.0$ than that of the system with $\epsilon = -5.0$. This observation is consistent with the fact that the charge fluctuation effect is entirely quenched in the deep Kondo regime but begins to play a unambiguous role in determining dynamic properties of the Kondo system when the system departs away from the deep Kondo regime. In Fig. 2(c,d), we plot the Fano factor $\gamma = S(0)/2eI$ [$I$ is the current calculated according to Eq. (20)] as well.

We now turn to the current CC in the case of three-terminal QD for $V_1 = -V_2 = -V_3 = V/2$ and equal tunnel-couplings $\Gamma_3 = \Gamma/2$. In Fig. 3, we show the zero-frequency CCs with and without the contribution of the Bose field fluctuation, $S_{23}(0)$ and $S_{23}^{(0)}(0)$, and the corresponding Fano factors defined as $\gamma_{23} = S_{23}(0)/2e\sqrt{T_2T_3}$. It is well-known that in a noninteracting system, the sign of the current CC between different normal-metallic leads is always negative (antibunching) due to the Pauli exclusion principle of Fermionic statistics of electrons which has been confirmed experimentally in a Hanbury-Brown-Twiss setup (HBT). Nevertheless, a positive current CC (bunching) has been predicted in certain situations: a hybrid superconductor-normal system spin-dependent sequential tunneling through a QD with three ferromagnetic leads, a Coulomb interaction coupled system. Very recently, a sign change of the current CC due to dynamical channel blockade has been experimentally observed in a subtle measurement of the CC between two output terminals in sequential tunneling through two capacitively coupled QDs connected
to four independent electrodes. What is interesting in the present calculations is that we indeed find a positive current CC for the QD with $\epsilon_d = -4.0$ under higher bias voltages $V > 2.0T_K$, as shown in Fig. 3(c,f) by the dash-blue line, if the charge fluctuation is taken into account, even though the system under studied involves no ferromagnetic electrode. But, the current CC remains negative for the systems with $\epsilon_d = -6.0$ and $-5.0$, since the charge fluctuation is weaker in the two systems than that in the system with $\epsilon_d = -4.0$. Generally, the overall role of charge fluctuation is to reduce the fermionic HBT type correlation at higher bias voltage region, which is consistent with the analytical conclusion in Ref. 43.

V. CONCLUSIONS

In conclusion, we have analyzed the effects of charge fluctuation on the current statistics in a Kondo QD connected to multi-leads at zero temperature on the basis of the infinite-$U$ SBMFT to demonstrate the Kondo correlation. By introducing counting fields with respect to each electrodes and employing the NGF technique, we have derived an explicit analytical expression for the first-order differential equation of the adiabatic potential with respect to the counting fields. Meanwhile, we have found that the SBMFT self-consistent equations become dependent on the counting fields, which provides a powerful instrument for taking account of the effect of the Bose field fluctuation, i.e. the charge fluctuation effect, at nonequilibrium on the current correlation. In particular, we have performed derivations for the concrete analytical expressions for the symmetrized shot noise in the case of two-terminal QD and the current CC in the case of three-terminal QD. It has been found that the consideration of the effect of charge fluctuation generates an additional contribution to the mean-field result, which leads to enhancement of the symmetrized shot noise in comparison to the mean-field result and a positive current CC for certain system parameters.

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