Evading the astrophysical limits on light pseudoscalars

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Abstract: We study the possibility of evading astrophysical bounds on light pseudoscalars. We argue that the solar bounds can be evaded if we have a sufficiently strong self coupling of the pseudoscalars. The required couplings do not conflict with any known experimental bounds. We show that it is possible to find a coupling range such that the results of the recent PVLAS experiment are not in conflict with any astrophysical bounds.

1 Introduction

In a recent paper, Zavattini et al. [1] have reported a rotation of polarization of light in vacuum in the presence of a transverse magnetic field. If we interpret this rotation in terms of the coupling of a light pseudoscalar particle to photons,

$$\mathcal{L}_{\phi\gamma\gamma} = \frac{1}{4M_\phi} \phi F_{\mu\nu} \tilde{F}^{\mu\nu},$$

we find that the allowed range of parameters is $1 \times 10^5$ GeV $\leq M_\phi \leq 6 \times 10^5$ GeV and $0.7$ meV $\leq m_\phi \leq 2$ meV [1], where $m_\phi$ is the mass of the pseudoscalar. This range of parameters does not conflict with any laboratory bounds [2]. However these values are ruled out by astrophysical considerations [3] if we assume that the pseudoscalar is an axion. For a review of axion physics see, for example, Ref. [4]. The most stringent astrophysical limit comes from SN1987A which suggests that the mass of the axion cannot be greater than 0.01 eV [5]. Assuming that this particle is an invisible axion, this in turn implies an upper bound on the coupling, $g_{\phi\gamma\gamma} < 2 \times 10^{-12}$ GeV$^{-1}$, upto a model dependent factor of order unity. The limit on the axion mass arises by demanding that axion emission should not lead to too much energy loss from the core. Similar but less stringent limits can be obtained by considering energy loss from the core of the sun. The coupling, Eq. [1] also leads to...
several interesting astrophysical polarization effects \cite{6}. For standard axion, its mass and coupling to photons are both related to the Peccei-Quinn scale \cite{7}. It is clearly of interest to see if the astrophysical bounds can somehow be evaded \cite{8}. All the astrophysical bounds assume that the pseudoscalar couplings are so small that once produced it will freely escape from the source, which may be the sun or a red giant or a supernova. In the present paper we examine whether the pseudoscalar particles might be trapped inside the sun.

2 Trapping Pseudoscalars

We assume the following interaction lagrangian of pseudoscalars,

\[ \mathcal{L}_I = \frac{1}{4M_\phi} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\lambda}{4!} \phi^4 \]  

(2)

Here we have also included the self coupling of the pseudoscalars. The value of the pseudoscalar coupling to photons will be taken from data \cite{1}. We do not assume \( \phi \) to be an axion and use only experimental data to determine its couplings. We point out that the self couplings are completely unconstrained experimentally.

We propose the following mechanism for trapping pseudoscalars. We assume that the self coupling \( \lambda \) is of order unity. The photons in the sun’s core convert into pseudoscalars due to the Primakoff process shown in fig. \cite{1}. The temperatures are such that almost the entire flux of photons emerging from the core is converted into pseudoscalars. The pseudoscalars begin accumulating inside the core due to scattering on other pseudoscalars through the process \( \phi \phi \rightarrow \phi \phi \). This is possible if the self coupling of pseudoscalars \( \lambda \) is sufficiently strong. We can also have a significant cross section for the process \( \phi \phi \rightarrow \phi \phi \phi \phi \) through the loop diagram, fig. \cite{2} if the coupling \( \lambda \) is strong. This process leads to a degradation of the energy per particle for pseudoscalars. Hence as the pseudoscalars accumulate they start producing pseudoscalars of lower energy through this process and their density increases rapidly. This process stops when the energy per particle approaches the mass of the pseudoscalars. As we show the final density of pseudoscalars is sufficient to trap pseudoscalars inside the star. We also find that the mean free path of pseudoscalars becomes so small that radiative transport \cite{9} occurs primarily through photons.
Figure 1: Feynman diagram for the Primakoff process for the conversion of photons $\gamma(k)$ into pseudoscalars $\phi(k')$.

Figure 2: The process $\phi\phi \rightarrow \phi\phi\phi\phi$ through a loop diagram.

Figure 3: Feynman diagram for the inverse Primakoff process for the conversion of pseudoscalars $\phi(k)$ to photons $\gamma(k')$. 
We first compute the pseudoscalar density in the core of the sun assuming the current equilibrium conditions. We assume the core radius to be one-fifth of the solar radius $R_\odot$. The mean density in this region is approximately $60$ gm/cm$^3$. The average temperature in the sun’s core is approximately $1.7 \times 10^7$ K. The pseudoscalars are produced by the Primakoff like process shown in fig. 1 and have mean energy roughly equal to $800$ eV. We assume that the production rate of photons inside the core is equal to their rate of conversion into pseudoscalars. This is a reasonable assumption since the energy loss due to pseudoscalar production by this process is found to be larger than luminosity of sun. The solar luminosity is $3.9 \times 10^{33}$ erg/s. Hence the mean photon energy production rate inside the sun is roughly $2$ erg/(gm sec). It is clear that the production rate will be larger inside the core due to its higher temperature. For a conservative estimate we take the photon energy production rate inside the core to be $1$ erg/(gm sec). The mean thermal energy per photon inside the core is roughly $800$ eV ($\approx 1.3 \times 10^{-9}$ ergs). Hence the number of photons produced per second is approximately $5.21 \times 10^{41}$. Since we have assumed that all the photons produced convert into pseudoscalars we find that this is also the number of pseudoscalars produced per second. Assuming that these particles escape freely from the sun we find that the flux of pseudoscalars from the sun’s core is $2.14 \times 10^{20}$ cm$^{-2}$s$^{-1}$. If the pseudoscalars are in a steady state the number density of pseudoscalars inside the core is $7.1 \times 10^9$ cm$^{-3}$.

We next determine the mean free path of a pseudoscalar propagating through this medium. The cross section of the process $\phi(p_1)\phi(p_2) \to \phi(p_3)\phi(p_4)$ due to the $\phi^4$ coupling at leading order in perturbation theory is given by

$$
\sigma_{\phi\phi} = \frac{\lambda^2}{32\pi E_{cm}^2} = 9.94 \times 10^9 \lambda^2 \left[ \frac{10^{-6} \text{ GeV}}{E_{cm}} \right]^2 \text{ GeV}^{-2}
$$

With $\lambda = 1$ and $E_{cm} = 1$ KeV, the mean free path $l_{\phi\phi} = 1/(n_\phi\sigma_{\phi\phi}) = 3.6 \times 10^7$ cm. This is almost three orders of magnitude smaller than the core radius $0.2R_\odot = 1.4 \times 10^{10}$ cm. This implies that due to self interaction pseudoscalars will not be able to escape freely from the sun but will instead start accumulating inside the core. Hence our initial assumption that the pseudoscalars escape freely from the sun is not valid.

We next consider the contribution due to higher order processes. We focus on the process $\phi\phi \to \phi\phi\phi\phi$ for which the leading order diagram is shown in fig. 2. This process is suppressed compared to the leading order
process by two powers of \( \alpha \lambda = \lambda^2/4\pi \). With \( \lambda = 1 \), this process is down only by a factor of 100 and can contribute significantly. This process leads to fragmentation of \( \phi \) particles and hence will degrade their energy per particle. The fragmentation stops when the energy per particle approaches their mass, which is of order 1 meV. Since the initial energy is 1 KeV, this will lead to an increase in the number of particles by a factor of \( 10^6 \) and hence will result in a considerable enhancement of the number density of \( \phi \) particles. The center of mass energy is now equal to \( \sqrt{2m_\phi E_\phi} \). This further enhances the cross section for the process \( \phi \phi \rightarrow \phi \phi \) which becomes \( 2.5 \times 10^{15} \text{ GeV}^{-2} \).

The above analysis shows that, during the main sequence phase, the pseudoscalars will accumulate inside the sun due to self interaction. As they accumulate, processes which deplete pseudoscalars also start contributing significantly. The dominant process contributing to depletion is the inverse Primakoff process shown in fig. 3. The mean energy of the pseudoscalars is much smaller than that of photons. Hence the photon produced by the inverse Primakoff process will have much lower energy, of the order of meV. The photons produced will be quickly absorbed by electrons by the inverse Bremsstrahlung process and thermalize in the medium. Eventually the energy deposited into pseudoscalars will equal the energy released by pseudoscalars due to their conversion back into photons. We next determine the density profile of the pseudoscalars in this equilibrium situation.

The cross section for the inverse process (fig. 3) is approximately \( 10^{-12} \text{ GeV}^{-1} \) for \( E_\phi << m_e \), assuming the target particles to be electrons. Their number density will reach steady state only when the rate at which energy is gained by the pseudoscalars becomes equal to the rate at which they loose energy. The mean energy of photons is about \( 10^6 \) times larger than that of pseudoscalars. Hence a pseudoscalar produced by the primakoff like process (fig. 1) will have roughly \( 10^6 \) times the energy of the photon produced by the inverse process (fig. 3). The production rate of pseudoscalars per unit volume is \( \sigma_{\gamma X \rightarrow \phi X} n_\gamma n_X v \) where \( v = c \) is the relative velocity and \( n_\gamma \) is the photon density of the medium. We set this equal to \( 10^{-6} \) times the conversion rate of pseudoscalars into photons \( \sigma_{\phi X \rightarrow \gamma X} n_X n_\phi v \). This gives us the final number density of pseudoscalars when the system reaches steady state \( n_\phi = 10^6 n_\gamma \sigma_{\gamma X \rightarrow \phi X} / \sigma_{\phi X \rightarrow \gamma X} \). The number density of photons in the core is roughly \( 10^{23} \) per cm\(^3\). Since the two cross sections are approximately equal, we find that the pseudoscalar number density required to achieve steady state is roughly \( 10^{29} \) per cm\(^3\). This leads to a mean free path of pseudoscalars of order \( 10^{-17} \) cm inside the core of the sun. This is much smaller than the
mean free path of photons inside the core. Hence the contribution of \( \phi \) particles to radiative transport inside sun would be negligible compared to the contribution of photons, once the sun becomes a main sequence star.

So far we have only considered pseudoscalars inside the sun’s core assuming steady state conditions. We expect the pseudoscalar density to be significant outside the core, extending much beyond the solar radius. This pseudoscalar halo will be gravitationally bound to the sun. We assume it to be spherically symmetric since we expect its rotational speed to be small. Due to the fragmentation process the pseudoscalar kinetic energies after emerging from the core can be at most as large as their mass. Hence their velocity may still be comparable to the velocity of light, which is much larger in comparison to the escape velocity from the surface of the sun, \( v_\odot \approx 6 \times 10^5 \text{ m/s} \). We assume that the pseudoscalars loose energy as they propagate inside the sun such that their velocities become nonrelativistic. There are many processes by which they can loose energy. For this we assume additional terms in the interaction lagrangian

\[
\Delta L_I = g_A \phi \bar{\psi} \gamma_5 \psi + \lambda_1 \phi^3 .
\]  

where the term proportional to \( \lambda_1 \) is parity violating. We are allowed to add such terms since parity is not a symmetry of nature. The coupling \( g_A \) is also constrained by astrophysical processes. However, due to pseudoscalar self coupling and accumulation inside the sun the limits on \( g_A \) are also much less stringent in comparison to the limit, \( g_A < 0.80 \times 10^{-10} \) \cite{10}, imposed by assuming that the pseudoscalar is an invisible axion. Laboratory bounds on this coupling also exist if we assume a very small mass of the pseudoscalar, i.e. if \( m_\phi \leq 10^{-6} \text{ eV} \) \cite{11}. The coupling \( \lambda_1 \) is unconstrained. Some of the processes which can contribute to energy loss are shown in fig. 4. The diagram, fig. 4a, by itself can contribute significantly to energy loss if the coupling \( g_A \sim 10^{-7} \), which is not ruled out by any laboratory experiments. Furthermore the diagram, fig. 4b, is only first order in the coupling \( g_A \) and may lead to sizeable energy loss due to scattering on non-relativistic electrons.

We may estimate the density profile of the pseudoscalar halo for \( r > R_\odot \) by imposing steady state conditions. The pressure of pseudoscalars at distance \( r \) from center of sun is \( P = \rho_\phi < v^2 > /3 \). The mean velocity at distance \( r \) is given by

\[
<v^2> = c + \frac{2G(M_\odot + M_r)}{r}
\]  


Figure 4: Some of the diagrams which contribute to energy loss of pseudoscalars as they propagate through sun. Here the pseudoscalar, electron and photon are represented by dashed, solid and wavy lines respectively.
where $c = < v^2 > -2G(M_{\odot} + M_{R_\odot})/R_\odot$ is a constant, $M_{\odot}$ is the mass of the sun and $M_r$ is the total mass of pseudoscalar particles within a sphere of radius $r$ centered at the center of sun. We also have

$$\frac{dM_r}{dr} = 4\pi r^2 \rho_\phi(r) ,$$  
(6)

and

$$\frac{dP}{dr} = -\frac{G(M_{\odot} + M_r)\rho}{r^2} .$$  
(7)

Using equations 5, 6 and 7 and the equation $P = \rho_\phi < v^2 > /3$, we find,

$$\frac{d\rho_\phi}{dr} = -\frac{1}{< v^2 >} \left[ \frac{G(M_{\odot} + M_r)\rho_\phi}{r^2} + 8\pi G\rho_\phi^2 r \right] .$$  
(8)

Since the right hand side is negative for all $r$, we find that $\rho_\phi$ decreases with $r$. In the small $r$ regime where $M_r << M_{\odot}$ and $8\pi \rho_\phi r^3 << M_{\odot}$ we find that $\rho_\phi(r)/\rho_\phi(r_0) \approx \sqrt{v^2(r)/v^2(r_0)}$. In the large $r$ regime, $M_r >> M_{\odot}$ we find that $\rho_\phi \propto 1/r^2$ and hence $M_r \propto r$.

In fig. 5 we show results of the numerical integration of coupled equations 5 and 8. All masses are expressed in units of solar mass and distances in units of astronomical unit (AU). We have set the mass of the pseudoscalar equal to 1 meV. The results are shown for different values of the parameter $c/G$. We integrate the equations starting from the core of the sun. The equations given above can be easily generalized for $R_\odot < r < R_{\text{core}}$. For $c/G \geq 0$ the pseudoscalars have enough energy to escape the gravitational attraction of the sun and hence the halo extends to infinite distance. For values of $c/G \lesssim -1$ we find that the radius of the pseudoscalar halo is quite small and contributes negligibly to the mass of the solar system. Hence we find that for a wide range of parameters we can evade all the astrophysical bounds on light pseudoscalars.

For $c/G \geq 0$, we find that $M_r >> M_{\odot}$ for $r >> 1$ AU. Very large values of $M_r$ are clearly unphysical and suggest that in this case the pseudoscalars never reach equilibrium. In such cases pseudoscalars may rapidly cool the star which may never reach the main sequence. We also find that the large $r$ behavior, $M_r >> M_{\odot}$, of the density of pseudoscalars is precisely the same as that required for the galactic dark matter. This raises the possibility that the galactic dark matter might be identified with these light pseudoscalars. If these pseudoscalars populate the galaxy then they will also exert pressure.
Figure 5: The mass contribution of pseudoscalars, $M_r$, as a function of the distance from the center of the sun.
on the pseudoscalars emitted by the sun. This may also have considerable influence on the pseudoscalar density profile and will be investigated in a future publication.

So far we have shown that, assuming the current equilibrium conditions inside sun’s core, the pseudoscalars contribute negligibly to radiative transport. Since all the bounds imposed in the literature consider perturbations around the equilibrium conditions, our analysis show that such bounds are not valid in the presence of self couplings of pseudoscalars.

We next consider the approach of the star to the main sequence phase. Since we are considering a relatively large pseudoscalar photon coupling the pseudoscalar flux can be significant even when the star has not reached the main sequence. When the temperature is lower than the current temperature the pseudoscalar flux and hence their density inside the core will be smaller. Hence for a certain temperature range the pseudoscalars will escape freely. We consider the phase when the star undergoes quasi-static collapse due to its gravitational attraction. By virial theorem the total energy of a star in equilibrium is equal to half its potential energy. Hence its energy loss is limited by virial theorem and pseudoscalar production cannot change its evolution drastically. We point out that the star will evolve quasi-statically even in the presence of pseudoscalars since the processes leading to equilibrium proceed at a much faster rate in comparison to the production rate of pseudoscalars. As the core temperature increases, the rate of photon and hence pseudoscalar production also increases. Once the photon production rate reaches values close to what is observed today, pseudoscalars will start accumulating and their contribution to radiative transport will rapidly decrease.

3 Conclusions

In conclusion we find that it is possible to evade all astrophysical limits on pseudoscalar photon coupling if the self coupling of pseudoscalars is sufficiently strong. The pseudoscalars start accumulating inside the sun due to scattering with other pseudoscalars. The energy per pseudoscalar degrades due to higher order processes as well as by energy loss due to scattering on electrons, nucleons and pseudoscalars. The pseudoscalar number density eventually reaches steady state due to conversion of pseudoscalars back into photons. We find that the mean free path of these particles inside the core is much smaller in comparison to the mean free path of photons. Hence
they contribute almost negligibly to radiative transport. In our analysis we have used the pseudoscalar coupling and mass extracted by the PVLAS experiment. Our results show that the results of the PVLAS experiment are consistent with astrophysical limits if we allow strong self couplings.

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