GENERALIZED ACTION PRINCIPLE and SUPERFIELD EQUATIONS OF MOTION for D=10 D–p–BRANES

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ABSTRACT

The action for the $D = 10$ type II Dirichlet super–p–branes, which has been obtained recently, is reconstructed in a more geometrical form involving Lorentz harmonic variables. This new (Lorentz harmonic) formulation possesses $\kappa$–symmetry in an irreducible form and is used as a basis for applying a generalized action principle that provides the superfield equations of motion and clarifies the geometrical nature of the $\kappa$–symmetry of these models. The case of a Dirichlet super–3–brane is considered in detail.
1 Introduction

Recently $\kappa$–invariant actions for the $D = 10$ type IIB Dirichlet super–3–brane [1] and then for all $D = 10$ type II Dirichlet super–p–branes (D–p–branes) [2]–[4] were obtained. These actions consist of the sum of a Dirac–Born–Infeld (DBI) action and a Wess–Zumino (WZ) term. The role of fermionic $\kappa$–symmetry in these models is to reduce half the number of components of the target space spinors. Therefore the $\kappa$–symmetry variation of the spinors involves a projector given in terms of a traceless matrix $\bar{\Gamma}$ that squares to unity. A remarkable property of the models [1]–[4] is that the $\kappa$–symmetry variation of the DBI action can be written as the integral of a $(p + 1)$–form and hence can compensate the variation of the WZ term. This property looks miraculous from the point of view of the papers [1]–[4].

In the present paper, by use of Lorentz harmonics [6]–[11] as auxiliary variables, we rewrite the full action of these models as the integral of a Lagrangian $(p + 1)$–form over a $d = p + 1$ dimensional worldvolume (see [10]–[11] for superstrings and type I super–p–branes) and verify its $\kappa$–invariance. In this way the remarkable property mentioned above appears quite naturally and its geometrical nature is clarified.

The formulation we propose is a generalization to the case of super–D–p–branes of a geometrical twistor–like approach to describing supersymmetric extended objects developed in [10]–[13].

Since our action is written in terms of differential forms without any use of Hodge operation it extends in a straightforward way to a group manifold (or generalized) action [14]–[11] that describes the embedding of the brane superworldvolume in the target superspace [10]–[13].

This is done simply by replacing the purely bosonic worldvolume with an arbitrary $(p+1)$–dimensional surface in the whole superworldvolume and regarding the coordinate functions and supervielbein components as worldvolume superfields restricted to this arbitrary surface. Then a generalized action principle produces the superspace field equations typical to the twistor–like formulations and, as in the case of superparticles, superstrings and type I super–p–branes, gives the geometrical meaning of the $\kappa$–symmetry in these models as a manifestation of worldvolume superdiffeomorphisms [17]–[31]. Thus our approach provides a bridge between the formulation of refs. [1]–[4] and the superspace approach of ref. [12]–[13].

For simplicity the details will be worked out only for a super–D–3–brane but the generalization to other super-D-branes is straightforward.

The paper is organized as follows. In Section 2 we fix our notation and introduce Lorentz harmonics [3]–[11]. In Section 3 we describe the DBI action for super–D–p–branes [1]–[4]. In Section 4 we propose the new formulation for Dirichlet super–p–branes which involves Lorentz–harmonic variables. Section 5 is devoted to the proof of the $\kappa$–
invariance of the Lorentz–harmonic action. In Section 6 we demonstrate that by use of the Lorentz harmonics $\kappa$–symmetry can be rewritten in an irreducible form and present the set of 16 covariant parameters of irreducible $\kappa$–symmetry for the type II$B$ super–3–brane.

In Section 7 we use the Lorentz–harmonic formulation for the construction of the generalized action for super–D-p–branes in $D = 10$ type $II$ supergravity background and obtain superfield equations of motion for these objects. Equations of motion for a super–3–brane in flat $D = 10$ type II$B$ superspace are analyzed in more detail in Section 8.

2 Notation and conventions

First of all let us describe our notation. We shall use underlined indices for target (super)space and not underlined ones for world (super)surface. Latin and Greek letters denote vector and spinor indices respectively and letters from the beginning or the middle of the alphabet refer respectively to tangent space or curved spaces. The hat and/or tilde over the index denote its reducible structure with respect to the Lorentz group, namely, a 32–dimensional type $II$ spinor index \cite{1, 2} and a composite spinor index of $SO(1, p) \times SO(9 − p)$ respectively (see below).

The supervielbeins of $D = 10$ $N = 2$ target superspace are denoted as

$$ E_{\hat{\alpha}} = dZ^\mathcal{M} E^\mathcal{M}_{\hat{\alpha}}(Z) \equiv (E^a, \hat{E}_{\hat{\alpha}}), $$

where $Z^\mathcal{M} \equiv (x^\mathcal{M}, \theta^\mathcal{\tilde{m}})$ ($\mathcal{\tilde{m}} = 0, ..., 9$, $\mathcal{\tilde{\mu}} = 1, ..., 32$) are the local coordinates of the type $II$ superbrane, $a = 0, 1, ..., 9$ is a $D = 10$ vector index, and $\hat{\alpha}$ is a 32–valued Majorana index of $SO(1, 9)$ for II A models or a composite spinor index of $SO(1, 9) \times SO(2)$ for II$B$ models.

The spinor index is reducible with respect to the $SO(1, 9)$ Lorentz group and can be decomposed into two 16–valued indices. In the type $IIA$ case this decomposition corresponds to the splitting of a $D = 10$ Majorana spinor into two Majorana–Weyl spinors of opposite chiralities

$$ \hat{E}_{\hat{\alpha}} = (E^{a1}, E^{a2}), \quad \alpha = 1, ..., 16 \quad (1) $$

For type $IIB$ case it is convenient to use splitting

$$ \hat{E}_{\hat{\alpha}} = (E^a, \tilde{E}^\alpha) \equiv (E^{a1} + iE^{a2}, E^{a1} − iE^{a2}), \quad \alpha = 1, ..., 16 \quad (2) $$

which possesses a complex structure inherent to II$B$ superspace. (Note that in refs. \cite{1, 2} real splitting of the spinor components

$$ \hat{E}_{\hat{\alpha}} = (E^{a1}, E^{a2}), \quad \alpha = 1, ..., 16 \quad (3) $$
was implied).

The representation of gamma matrices corresponding to the decomposition (2) is

\[(\Gamma^\alpha_{\hat{\alpha}\hat{\beta}}) = \sigma^a_{\alpha\beta} \otimes K \]  

(4)

where the matrix \( K \) belongs to a set \((K, J, I)\) introduced in \([1, 2]\). These matrices have the form

\[K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad J = i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad I = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\]

(5)

in the representation (3), which corresponds to

\[K = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\]

(6)

in the real representation (3). In (4) \( \sigma^a_{\alpha\beta} \) are 16 × 16 Majorana–Weyl γ–matrices whose \( SO(1, p) \otimes SO(9 - p) \) invariant representation can be chosen in a form which reflects a complex structure inherent in the worldvolume superspace of the \( D-p \)-brane. For instance, for \( p = 3 \) it is convenient to choose the following representation

\[\sigma^a_{\alpha\beta} = \begin{pmatrix} \sigma^a_{\alpha\beta} & \sigma^i_{\alpha\beta} \\ \sigma^i_{\alpha\beta} & \sigma^a_{\alpha\beta} \end{pmatrix}, \quad a = 0, \ldots, 3 \quad i = 1, \ldots, 6 \quad a = 0, \ldots, 9\]

(7)

\[\sigma^a_{\alpha\beta} = \begin{pmatrix} 0 & \sigma^a_{\alpha\beta} \delta^q_p \\ \sigma^a_{\alpha\beta} \delta^q_p & 0 \end{pmatrix}, \quad (\tilde{\sigma}^a)_{\alpha\beta} = \begin{pmatrix} 0 & (\tilde{\sigma}^a)_{\beta\alpha} \delta^p_q \\ (\tilde{\sigma}^a)_{\beta\alpha} \delta^p_q & 0 \end{pmatrix},\]

\[\alpha, \beta = 1, 2 \quad \hat{\alpha}, \hat{\beta} = 1, 2 \quad q, p = 1, \ldots, 4\]

where

\[\sigma^a_{\alpha\beta} = \epsilon_{\alpha\beta} \epsilon_{\beta\alpha} \gamma^q_p \quad (\tilde{\sigma}^a)_{\beta\alpha} = -\epsilon_{\alpha\beta} \epsilon_{\beta\alpha} \gamma^r_s \]

are relativistic Pauli matrices, \( \epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}, \quad \epsilon_{12} = -1 = -\epsilon_{12} \), and

\[\gamma^i_{pq} = -\gamma^i_{qp} = -((\tilde{\gamma}^i)^{qp})^* = \frac{1}{2} \epsilon_{qprs} (\tilde{\gamma}^i)^r_s\]

are Klebsh–Gordan coefficients for the group \( SU(4) = SO(6)\) [14].

The worldvolume supervielbeins are

\[e^A = (e^a, e^{\hat{a}}) = dz^M e^A_M(z) = d\xi^m e^A_m + d\eta^{\hat{m}} e^A_{\hat{m}}\]

(8)

where \( z^M = (\xi^m, \eta^{\hat{m}}) \) \( (m = 0, \ldots, p; \quad \hat{m} = 1, \ldots, 16) \) are local coordinates of the worldvolume superspace of a super–\( D-p \)-brane, \( a \) is a \( d = p + 1 \) tangent space vector index and \( \alpha \) is a composite 16–valued spinor index of \( SO(1, p) \times SO(9 - p)\).

Again in the \( IIB \) case the representation with complex structure

\[e^{\hat{a}} = (e^{\hat{a}}, e^{\hat{\alpha}, q})\]

(9)
is convenient, where the indices $\alpha, \dot{\alpha}$ stand for spinor representations of $SO(1, p)$ and $q$ is a spinor index of $SO(9 - p)$. For the Dirichlet 3–brane

$$p = 3, \quad \alpha = 1, 2, \quad \dot{\alpha} = 1, 2, \quad q = 1, ..., 4.$$ 

To construct a super–$D$–brane action we introduce vector and spinor Lorentz harmonics $[6]$–$[10]$ given by a $10 \times 10$ matrix $u_{\dot{\alpha}}^\alpha$ and a $16 \times 16$ matrix $v_{\alpha}^{\dot{\alpha}}$ such that

$$||u_{\dot{\alpha}}^\alpha|| \in SO(1, 9) \Rightarrow u_{\dot{\alpha}}^\alpha \eta_{\alpha\beta} u_{\beta}^\gamma = \eta_{\dot{\alpha}\dot{\beta}} \quad \text{and} \quad ||v_{\alpha}^{\dot{\alpha}}|| \in \text{Spin}(1, 9) \quad (10)$$

$u$ and $v$ matrices are related to each other by the following condition of the invariance of the $\gamma$–matrices under the Lorentz rotations

$$u_{\dot{\alpha}}^\alpha \sigma_{\dot{\alpha}\dot{\beta}} = v_{\alpha}^{\dot{\alpha}} \sigma_{\alpha\beta} v_{\beta}^\dot{\alpha}, \quad u_{\dot{\alpha}}^\alpha \sigma_{\dot{\alpha}\dot{\beta}} = v_{\alpha}^{\dot{\alpha}} \sigma_{\alpha\beta} v_{\beta}^\alpha. \quad (11)$$

When adapted to the superbrane worldvolume, $u_{\dot{\alpha}}^\alpha$ splits covariantly into

$$u_{\dot{\alpha}}^\alpha = (u_a^\alpha, u_i^\alpha), \quad (12)$$

where $u^a$ and $u^i$ are respectively tangent and orthogonal vectors to the worldvolume [1].

In a similar way $v_{\alpha}^{\dot{\alpha}}$ splits into

$$v_{\alpha}^{\dot{\alpha}} = (v_{aq}^\alpha, \bar{v}_{\alpha}^q) \quad (13)$$

In the $p = 3$ IIB case $\alpha = 1, 2, \dot{\alpha} = 1, 2, q = 1, ..., 4$ and bar denotes complex conjugation. The representation $[10]$ can be used to specify (11) as follows

$$u_{\dot{\alpha}}^\alpha \sigma_{\dot{\alpha}\dot{\beta}} = v_{\alpha}^{\dot{\alpha}} \sigma_{\alpha\beta} v_{\beta}^\alpha,$$

$$u_{\alpha} a \sigma_{\alpha\beta} = v_{aq}^\alpha \sigma_{\alpha\beta} v_{bp}^\alpha - \bar{v}_{q}^{\alpha \dot{\alpha}} \sigma_{\alpha\beta} \bar{v}_{\beta}^{\dot{\alpha}}. \quad (14)$$

The role of the Lorentz harmonics is to adapt the supervielbeins $E^A$ to the super–$p$–brane worldvolume $[10]$ as follows

$$E^A \rightarrow \tilde{E}^\dot{\alpha} = (\tilde{E}_{\dot{\alpha}}, \tilde{E}_{\dot{\alpha}}).$$

\[1\] The decomposition (12) and (13) are invariant under local $SO(1, p) \times SO(9 - p)$ transformations, which form a natural gauge symmetry of the $p$–brane embedded into $D = 10$ space–time. This provides the possibility of treating Lorentz harmonics (12) and (13) as coordinates of a coset space $SO(1, 9)/SO(1, p) \times SO(9 - p)$. $\text{[10]}$ $\text{[1]}$ $\text{[10]}$. 


\[ E^\alpha \equiv E^\alpha_u a^\alpha = (E^\alpha, E^i) \] (15)

\[ \dot{E}^\alpha = \left\{ \begin{array}{l}
(E^{\alpha 1}, E^{\alpha 2}) = (E^{\alpha 1}, E^{\alpha q 1}; E^{\alpha q 2}, E^{\alpha q}) \quad \text{IIA} \\
(E^\alpha, \dot{E}^\alpha) = (E^\alpha, E^{\alpha q}; E^{\alpha q}, \dot{E}^{\alpha q}) \quad \text{IIB}
\end{array} \right. \] (16)

where

\[ E^\alpha = E^\alpha_u a^\alpha, \quad E^i = E^i u^i, \] (17)

\[ \begin{array}{l}
\text{IIA}: \quad E^{\alpha 1} = E^{\alpha 1} v^{\alpha q}, \quad E^{\alpha q 1} = E^{\alpha q 1} v^{\alpha q}, \quad E^{\alpha q 2} = E^{\alpha q 2} v^{\alpha q}, \quad E^{\alpha q} = E^{\alpha q} v^{\alpha q}, \\
\text{IIB}: \quad E^\alpha = E^\alpha v^{\alpha q}, \quad \dot{E}^\alpha = \dot{E}^\alpha v^{\alpha q}, \quad \dot{E}^{\alpha q} = \dot{E}^{\alpha q} v^{\alpha q}.
\end{array} \] (18)

In addition to the superspace coordinates \( Z^M \), super \( D \text{-}p \text{-} \text{branes} \) have a worldvolume (super) one-form

\[ A = d z^M A_M(z^M). \]

In what follows the worldvolume supervielbeins (8) and (9) will be regarded as ones induced by embedding and hence related to pullbacks into the superworldvolume of (17) and (18) (see below).

### 3 Original Super \( D \text{-}p \text{-} \text{brane actions} \)

In the \( \kappa \)-invariant formulation of refs. [1]–[4] the worldvolume \( M_0 \) is purely bosonic (not supersymmetric). Therefore \( \eta^a \) and the spinor vielbeins \( e^\alpha \) are absent and

\[ A = d \xi^m A_m(\xi) \]

is a one–form and \( Z^M(\xi) \) are functions on \( M_0 \).

The action functional obtained in [1]–[4] for super \( D \text{-}p \text{-} \text{branes} \) propagating in a background of \( D = 10 \) type \( \text{II} \) supergravity has the form

\[ S^{[1]} = I_{DBI} + I_{WZ} \] (19)

where \( I_{DBI} \) is the Dirac–Born–Infeld action

\[ I_{DBI} = - \int_{M_0} d^{p+1} \xi \sqrt{-\det(g_{mn} + e^{-\frac{1}{2} \phi} F_{mn})}, \] (20)

\( \phi = \phi(Z^M(\xi)) \) is the dilaton superfield, \( g_{mn} = E^a_m \eta_{ab} E^b_n \) is the induced metric and the field \( F_{mn} \) are the components of the 2–form

\[ F = da - B_{(2)}. \]

In (5) \( B_{(2)} \) is the NS–NS (background) super 2–form (see [3], [4], [2] and refs. therein) with the field strength

\[ H_{(3)} = dB_{(2)} \] (21)
The Wess–Zumino term $I_{WZ}$

$$I_{WZ} = - \int_{M_0} L_{p+1}^{WZ} = \int_{M_0} e^F \wedge C \quad (22)$$

is the integral over the worldvolume $M_0$ of the Wess–Zumino form $L_{p+1}^{WZ}$ expressed in terms of the form $F$ and the formal sum of the RR super–n–forms $C$ ($n$ are even for the $IIB$ and odd for the $IIA$ case)

$$C = \oplus_{n=0}^9 C_{(n)} \quad (23)$$

The field strengths of $C_{(n)}$ are

$$R = e^{B(2)} \wedge d(e^{-B(2)} \wedge C) = \oplus_{n=1}^{10} R_{(n)} \quad (24)$$

The invariance of the action (39) under fermionic $\kappa$–transformations is stipulated by the existence of a $32 \times 32$ traceless matrix $\overline{\Gamma}$ acting on the tangent spinors and satisfying the condition $\overline{\Gamma}^2 = 1$. In refs. [1, 2, 4] it was proved that this matrix exists for any D–p–brane and is given by the formal sum

$$d\xi^{p+1} \overline{\Gamma} = - \frac{e^{\frac{1}{2} (p-3) \phi}}{L_{DBI}} \exp(e^{-\frac{1}{2} \phi} F) \gamma |_{M_0} \quad (25)$$

where

$$\left\{ \gamma = \oplus_n \gamma^{(2n)}(K)^n I \quad \text{in the IIB case} \right. \left. \gamma = \oplus_n \gamma^{(2n+1)} \gamma^{11} \quad \text{in the IIA case} \right\} \quad \gamma^{(n)} \equiv \frac{1}{n!} E_{2a} \cdots E_{2^{2n}} \Gamma_{\underline{a}_1 \cdots \underline{a}_n}$$

and $K$ and $I$ are the $2 \times 2$ matrices given by Eq. (3) (or (4) in the real representation) and $L_{DBI} \equiv \sqrt{-\det(g + e^{-\frac{1}{2} \phi} F)}$.

As usual $\Gamma_{\underline{a}}$ are the Dirac matrices in $D = 10$ times a charge conjugation matrix (see (2) for the $IIB$ case) and $\Gamma_{\underline{a}_1 \cdots \underline{a}_n}$ is the antisymmetrized product of $\Gamma_{\underline{a}}$ with unit weight.

The infinite reducible $\kappa$–symmetry transformations of $Z^M$ and $A(z)$ which leave the action (19) invariant are

$$\delta_{\kappa} Z^M = i_{\kappa} E^M = 0, \quad \delta_{\kappa} Z^M \hat{E}_M = i_{\kappa} \hat{E}_M = \kappa \hat{\underline{a}} \quad (26)$$

$$\delta A = i_{\kappa} B_{(2)} \quad \leftrightarrow \quad \delta F = i_{\kappa} H_{(3)} \quad (27)$$

where

$$\kappa \hat{\underline{a}} = \kappa \hat{\underline{a}}(\overline{\Gamma})^2 \hat{\underline{a}} \quad (28)$$

For instance, for the 3–brane $L_4^{WZ}$ and $\overline{\Gamma}$ are given by

$$L_4^{WZ} = (C(4) + F \wedge C(2) + \frac{1}{2} F \wedge F C(0)) |_{M_0}, \quad (29)$$

$$dL_4^{WZ} = (R(5) + F \wedge R(3) + \frac{1}{2} F \wedge F \wedge R(1)), \quad (30)$$
\[ d^4 \xi \Gamma = - \frac{1}{L_{DBI}} (\gamma^{(4)} + e^{-\phi} \mathcal{F} \wedge \gamma^{(2)} K + \frac{1}{2} e^{-\phi} \mathcal{F} \wedge \mathcal{F}) I|_{\mathcal{M}_0}. \] (31)

The superspace constraints for \( H_3 \), \( R_5 \), \( R_3 \) and \( R_1 \) are

\[ H_{(3)} = e^{\phi} \left[ \frac{i}{2} E^a \wedge E^b \wedge E^c (\Gamma_{abc}) K_{\alpha\beta} \right] + \frac{1}{2} E^a \wedge E^b \wedge E^c (\Gamma_{abc} K) \Lambda^{\alpha\beta}_3 \right] + \frac{1}{3!} E^a \wedge E^b \wedge E^c H_{\alpha\beta\gamma} \] (32)

\[ R_{(5)} = \frac{i}{2} E^a \wedge E^b \wedge E^c \wedge E^d \wedge E^e (\Gamma_{abcd} I) \Lambda^{\alpha\beta\gamma}_5 \] + \frac{1}{5!} E^a \wedge E^b \wedge E^c \wedge E^d \wedge E^e R_{\alpha\beta\gamma\delta} \ (33)

\[ R_{(3)} = e^{-\phi} \left[ \frac{i}{2} E^a \wedge E^b \wedge E^c (\Gamma_{abc}) K I \Lambda^{\alpha\beta}_3 \right] + \frac{1}{3!} E^a \wedge E^b \wedge E^c R_{\alpha\beta\gamma} \] (34)

\[ R_{(1)} = 2 e^{-\phi} \left[ E^a (I \Lambda_1) \right] \Lambda_2 + E^b R_b \] (35)

where

\[ \Lambda_3 = \frac{1}{2} \nabla_\alpha \phi (Z) \] (36)

The \( D = 10 \) type \( II \) supergravity torsion constraints are

\[ T^a = dE^a = - \frac{i}{2} \hat{E}^a \wedge \hat{E}^b \wedge \hat{E}^c \Gamma_{abc}^{\alpha\beta}_3 = - \frac{i}{2} (\hat{E} \Gamma^a \hat{E}) \] (37)

### 4 Super–\( D–p \)–brane action functional in terms of differential forms

Instead of (19) we propose the following action functional

\[ S = I_0 + I_{WZ} = \int_{\mathcal{M}_0} \left( \mathcal{L}_{p+1}^0 + \mathcal{L}_{p+1}^{WZ} \right) \] (38)

where the Wess–Zumino term \( I_{WZ} \) is the same as before (Eq.(23)) and

\[ I_0 = \int_{\mathcal{M}_0} \mathcal{L}_{p+1}^0 \equiv \int_{\mathcal{M}_0} \left( \frac{1}{(p+1)!} E^{a_0} \wedge E^{a_1} \wedge ... \wedge E^{a_p} \epsilon_{a_0a_1...a_p} e^{-\frac{1}{2} \phi} \sqrt{-\det(\eta_{ab} + F_{ab})} \right) \] (39)

\[ + Q_{p-1} \wedge \left( e^{-\frac{1}{2} \phi} (dA - B_{(2)}) - \frac{1}{2} E^b \wedge E^a F_{ab} \right) \]

Here \( E^a \) are defined in (13)–(18), \( F_{ab} \) is an auxiliary antisymmetric tensor field with tangent space (Lorentz group) indices and \( Q_{p-1} \) is a Lagrange multiplier which produces the algebraic equation

\[ F_2 = 1/2 E^{b} \wedge E^{a} F_{ab} = e^{-\phi/over2} (dA - B_{(2)}) \equiv e^{-\phi/over2} \mathcal{F} \] (40)

and identifies the auxiliary field \( F_{ab} \) with the components of the form \( \mathcal{F} \) of the original action (20).

The introduction into (38) of the term with \( Q_{p-1} \) and the use of \( E^a \) (17) enabled us to rewrite the DBI action (20) as the integral of a differential \( (p + 1) \)–form over \( \mathcal{M}_0 \) and,
therefore, to consider it on an equal footing with the WZ term. This explains why the \( \kappa \)-variation of the DBI functional (20) is an integral of a \((p + 1)\)-form [1, 2].

The Lagrange multiplier form \( Q_{(p+1)} \) does not contain propagating degrees of freedom because the variation of (39) with respect to the auxiliary field \( F_{ab} \) yields the equation

\[
Q_{p-1} \wedge E^b \wedge E^a = \frac{1}{(p+1)!} E^{a_0} \wedge E^{a_1} \wedge \ldots \wedge E^{a_p} \epsilon_{a_0a_1\ldots a_p} e^{-\frac{e}{2}\phi} \frac{\partial}{\partial F_{ab}} (\sqrt{-det(\eta + F)}) \quad (41)
\]

which is algebraic and can be easily solved

\[
Q_{p-1} = \frac{1}{4} \sqrt{-det(\eta + F)} e^{-\frac{e}{2}\phi} E^{a_1} \wedge \ldots \wedge E^{a_{p-1}} \epsilon_{a_0a_1\ldots a_{p-1}ab} (\eta + F)^{-1}.
\]

The variation of the Lorentz harmonics \( u_{\hat{a}} \) (contained in \( E^a \)) for getting field equations requires some comments. Since \( u_{\hat{a}} \) must satisfy the constraints [11] one should add to the action (38) the term

\[
I_c = \int L_{\hat{a}ab}(u_{\hat{a}} \eta_{ab} u_{\hat{b}} - \eta_{\hat{a} \hat{b}})
\]

where \( L_{\hat{a}ab} \) are Lagrange multiplier \((p + 1)\)-forms. Then (38) would extend to \( S' = S + I_c \). The field equations

\[
\frac{\delta S'}{\delta u_{\hat{a}}} = 0
\]

lead to

\[
L^{ij} = 0 = L^{ai},
\]

while the field equations

\[
u_{\hat{a}}^a \frac{\delta S'}{\delta u_{\hat{a}}} = 0
\]

specify \( L_{ab} \) and

\[
u_{\hat{a}}^i \frac{\delta S'}{\delta u_{\hat{a}}^i} = 0
\]

imply a so-called rheotropic condition [11]

\[
E^i \equiv dZ^M E^i_M = 0 \quad (43)
\]

which reads that the pullback of \( E^i \) into the worldvolume is zero.

Alternatively one can avoid adding the term \( I_c \) but perform the variation with respect to \( u_{\hat{a}} \) according to the rule

\[
\delta u_{\hat{a}} = u_{\hat{b}} \Omega_{\hat{a}\hat{b}}(\delta) \equiv u_{\hat{b}} i_\delta \Omega_{\hat{a}\hat{b}}
\]

where \( \Omega_{\hat{a}\hat{b}} \) is the \( SO(1,9) \)-valued Cartan 1–form

\[
\Omega_{\hat{a}\hat{b}} = -\Omega_{\hat{b}\hat{a}} = \begin{pmatrix} \Omega^{ab} & \Omega^{ai} \\ -\Omega^{bi} & \Omega^{ij} \end{pmatrix}
\]

(45)
and $\Omega(\delta) = i_\delta \Omega$ (For the details see, for instance, [9, 10]). Then the variation of $S$ with respect to $u^a_\underline{a}$ gives again (43). Taking into account Eq. (43) we get

$$g_{mn} \equiv E^a_m \eta_{ab} E^b_n = E^a_m \eta_{ab} E^b_n$$

(46)

so that $E^a_m$ can be regarded as induced worldvolume vielbeins.

Using the algebraic equation (40), (42), (43) and (46) we can reduce the functional $I_0$ (39) to $I_{DBI}$ (20). This proves that at the classical level the formulation under consideration is equivalent to that of refs. [1–4].

5 \hspace{1cm} \kappa–Invariance

Since the action (38) is equivalent to (19) its invariance under $\kappa$–symmetry is guaranteed. However it is instructive, in view of the consideration in the next section, to verify it explicitly.

Our action functional is the integral of a $(p+1)$–form $\mathcal{L}_{p+1}$ over the world volume. If this form was the pullback of a target space form its variation could be obtained from the Lie derivative of the Lagrangian density $\mathcal{L}_{p+1}$

$$\delta \mathcal{L}_{p+1} = i_\kappa d \mathcal{L}_{p+1} - d(i_\kappa \mathcal{L}_{p+1})$$

so that, neglecting boundary terms, we would have

$$\delta S = \int i_\kappa d \mathcal{L}_{p+1}. \tag{47}$$

Note that Eq. (43) allows to identify the worldvolume vielbeins $e^a$ with a linear combination of the pullbacks of $E^a$ tangent to the worldvolume. The basic field variations defined by contraction of the forms $E^a$ with the $\kappa$ parameter $E^a(\delta_\kappa) \equiv i_\kappa E^a$ vanishes due to the definition of the $\kappa$–symmetry (26). Hence the contraction of $Q$ vanishes as well (see (42)).

But since $\mathcal{L}_{p+1}$ also contains genuine worldvolume fields which are not the pullbacks of target space objects (such as Lorentz harmonics), we must add to (47) the $\kappa$ variations of these fields. However these variations (which are still undefined) are multiplied by the algebraic field equations (10), (11) and (13) and, therefore, they can be appropriately chosen to compensate possible terms proportional to the algebraic equations that arise from the variation of other terms. It means, in particular, that when computing $\delta S$ we can freely use these algebraic equations and, at the same time, drop the $\kappa$ variations of these genuine worldvolume quantities if we are not interested in their specific form. Also notice that from (27) the $\kappa$–variation of $\mathcal{F}$ is

$$\delta \mathcal{F} = i_\kappa d \mathcal{F} = i_\kappa H$$
which can be also viewed as the Lie derivative of $\mathcal{F}$ provided we formally assume $i_\kappa \mathcal{F} = 0$.

Thus in order to check the $\kappa$–invariance of $S$ one has to compute the differential of $\mathcal{L}_{p+1}^{WZ}$ and $\mathcal{L}_p^0$ (modulo the algebraic equations (10), (11) and (13)).

We shall explicitly compute the differential of $\mathcal{L}_{(p+1)}$ only for the 3–brane. The other cases can be treated in the same way. From (22) and the definition of the curvatures one has

$$d\mathcal{L}_4^{WZ} = R_5 + \mathcal{F} \wedge R_3 + \frac{1}{2} \mathcal{F} \wedge \mathcal{F} \wedge R_1 = \frac{i}{2}(\hat{E} \gamma(3) \hat{E}) + (\hat{E} \hat{\gamma}(4) \hat{\Lambda}),$$

where

$$\gamma(3) = \left[ \frac{1}{3!} E^a \wedge E^b \wedge E^c \right] \Gamma_{cba},\quad \hat{\gamma}(4) = \left[ F_2 \wedge E^a \wedge E^b \Gamma_{ba} K - F_2 \wedge F_2 \right] I$$

and

$$\Gamma^a = \Gamma^a \gamma^a_{\Lambda}.$$

On the other hand the differential of $\mathcal{L}_0$ is

$$d\mathcal{L}_4^0 = \sqrt{-\det(\eta + F)} \left\{ \frac{1}{3!} \epsilon_{a_1 a_2 a_3} E^{a_1} \wedge E^{a_2} \wedge E^{a_3} \wedge T^a - \frac{1}{4} E^{a_1} \wedge E^{a_2} \epsilon_{a_1 a_2 a_3 a_4} (\eta + F)^{-1} a^{a_4} \wedge \left[ e^{-\frac{a}{4}H_3} + \frac{1}{2} F_2 \wedge d\phi + T^a \wedge E^b F_{ba} \right] \right\},$$

where $T^a = T^a \gamma^a_{\Lambda}$. Using the constraints (32) and (37) and making some algebraic manipulations we can rewrite $d\mathcal{L}_4^0$ as

$$d\mathcal{L}_4^0 = \sqrt{-\det(\eta + F)} \left\{ -\frac{i}{2} \epsilon_{a_1 a_2 a_3 b} E^{a_1} \wedge E^{a_2} \wedge E^{a_3} \wedge \left( \hat{E} \Gamma_a (\eta + K F)^{-1} a^{ab} \hat{E} \right) + \epsilon_{a_1 a_2 a_3 a_4} E^{a_1} \wedge \ldots \wedge E^{a_4} \wedge \left\{ \frac{1}{16} \left( \hat{E} \Gamma_b (\eta + K F)^{-1} a^{ba} \hat{E} \right) - \frac{1}{4} (\hat{E} \hat{\Lambda}) \right\} \right\},$$

where

$$(\eta + K F)^{-1} a^{ba} \equiv (\eta + F)^{-1} a^{ba} = (\eta + F)^{-1} a^{ba} K = (\eta^{ba} - F^{c a} F_{cd} + \ldots) 1 + (F^{ba} - F^{bc} F_{cd} F^{da} + \ldots) K$$

Inserting the unit matrix $\hat{\Gamma}^2$ and using the remarkable identity

$$\hat{\Gamma}_a = \frac{1}{\sqrt{-\det(\eta + F)}} \left[ \frac{1}{3!} \Gamma_{a_1 a_2 a_3} + \frac{1}{2} F_{a_1 a_2} \Gamma_{a_3} K \right] e^{a_1 a_2 a_3 b} (\eta + F K)_{ba}$$

one gets

$$d\mathcal{L}_4^0 = -\frac{i}{2}(\hat{E} \hat{\Gamma} \gamma(3) \hat{E}) - (\hat{E} \hat{\Gamma} \hat{\gamma}(4) \hat{\Lambda})$$

$\gamma(3)$ and $\hat{\gamma}(4)$ are given in (49) and (50).

In conclusion

$$d\mathcal{L}_4 = d\mathcal{L}_4^0 + d\mathcal{L}_4^{WZ} = i(\hat{E}(-) \gamma(3) \hat{E}(-)) - 2(\hat{E}(-) \hat{\gamma}(4) \hat{\Lambda}),$$

(52)
where

\[ \hat{E}^{(-)\hat{\alpha}} = \frac{1}{2}(\hat{E}(1 - \Gamma))\hat{\alpha}. \] (53)

(Going from (48) and (50) to (52) we used the properties \( \bar{\Gamma}^T = -\bar{\Gamma} \) and \( \bar{\Gamma}\gamma^{(3)} = -\gamma^{(3)}\bar{\Gamma}^T \).)

At this point the \( \kappa \)–invariance of \( S \) becomes obvious (see (26), (28) and (53)) since

\[ i_\kappa \hat{E}^{(-)\hat{\alpha}} = 0 = i_k E^{\hat{\alpha}}, \] (54)

and hence

\[ i_k d\mathcal{L} = 0. \] (55)

6 Irreducibility of \( \kappa \) symmetry in the Lorentz harmonic formulation

It should be stressed that the use of the Lorentz harmonics provides us with the possibility of extracting the covariant set of 16 independent parameters of the \( \kappa \)–symmetry. This means that passing from the original functional (19), (20), (22) to the classically equivalent action (38), (39), (22) we achieve an irreducible description of the \( \kappa \)–symmetry (see [9, 17]–[32] for superparticles, superstrings and type I super–p–branes).

As an example let us consider the 3–brane case. In 32–component spinor notations used in the previous Section the \( \text{SO}(1,9)/\text{SO}(1,3) \times \text{SO}(6) \) Lorentz harmonics (11) are represented by the reducible matrix

\[ v^{\hat{\alpha}}_{\hat{\beta}} = \begin{pmatrix} v_{\hat{\alpha}} & 0 \\ 0 & v_{\hat{\alpha}} \end{pmatrix}. \] (56)

To extract the irreducible part from the \( \kappa \)–symmetry parameter (28)

\[ \kappa^{\hat{\alpha}} = i_\delta \hat{E}^{(+)}\hat{\alpha} = i_\delta \hat{E}^{\hat{\alpha}}(\frac{1 + \bar{\Gamma}}{2})\hat{\alpha}, \] (57)

it is necessary first of all to contract \( \kappa^{\hat{\alpha}} \) with the \( 32 \times 32 \) Lorentz–harmonic matrix (76). As a result we get the parameter

\[ \kappa^{\hat{\alpha}} = \kappa^{\hat{\alpha}} v^{\hat{\alpha}}_{\hat{\beta}} = (\kappa^{\hat{\alpha}}; \kappa^{\hat{\beta}}) = (\kappa^{\alpha}; \kappa^{\hat{\alpha}}; \kappa^{\hat{\alpha}}; \kappa^{\hat{\alpha}}) \] (58)

covariantly splitted into four pieces.

The set of the parameters \( \kappa^{\hat{\alpha}} \) (58) satisfies the condition

\[ \kappa^{\hat{\alpha}} = \kappa^{\hat{\alpha}}(\bar{\Gamma}')\hat{\alpha} \] (59)

where (in the complex representation (2))

\[ (\bar{\Gamma}')\hat{\alpha} \equiv v_{\hat{\alpha}}\gamma^{(3)} v_{\hat{\alpha}} = \]
\[
\frac{1}{\sqrt{-\det(\eta + F)}} \left( (\sigma(4))^{\tilde{\alpha}}_{\tilde{\beta}} + \frac{i}{8} \epsilon^{abcd} F_{ab} F_{cd} \delta^{\tilde{\alpha}}_{\tilde{\beta}} - 2i (\sigma(2))^{\tilde{\alpha}}_{\tilde{\beta}} - (\sigma(4))^{\tilde{\alpha}}_{\tilde{\beta}} - \frac{1}{8} \epsilon^{abcd} F_{ab} F_{cd} \delta^{\tilde{\alpha}}_{\tilde{\beta}} \right)
\]

\[
(\sigma(4))^{\tilde{\beta}}_{\tilde{\alpha}} = \frac{i}{4} \epsilon^{abcd} (\sigma_{abcd})^{\tilde{\alpha}}_{\tilde{\beta}} = \left( \begin{array}{cc}
\delta_{\tilde{\alpha}}^{\tilde{\beta}} & 0 \\
0 & -\delta_{\tilde{\alpha}}^{\tilde{\beta}}
\end{array} \right)
\]

\[
(\sigma(2))^{\tilde{\beta}}_{\tilde{\alpha}} = \frac{1}{8} \epsilon^{abcd} F_{ab} (\sigma_{cd})^{\tilde{\alpha}}_{\tilde{\beta}} = \left( \begin{array}{cc}
\frac{i}{4} F^{ab} (\sigma_a \overline{\sigma}_b)_{\alpha}^{\beta} & 0 \\
0 & -\frac{i}{4} F^{ab} (\overline{\sigma}_a \sigma_b)_{\alpha}^{\beta}
\end{array} \right).
\]

The solution of Eq. (59) has the form

\[
\kappa_{\tilde{\alpha}} = i \delta E^{(+)} \overline{\tilde{\alpha}} = \left( \begin{array}{c}
\kappa_{\tilde{\alpha}} \\
2b_- \overline{\kappa}_{\tilde{\beta}} f_{\tilde{\beta}}^{\tilde{\alpha}} \\
2b_+ \kappa_{\tilde{\alpha}} f_{\tilde{\beta}}^{\tilde{\alpha}} \\
\overline{\kappa}_{\tilde{\alpha}}
\end{array} \right)^T
\]

where

\[
f_{\alpha \beta} = f_{\beta \alpha} = \frac{1}{4} F^{ab} (\sigma_a \overline{\sigma}_b)_{\alpha \beta}, \quad \overline{f}_{\alpha \hat{\beta}} = \overline{f}_{\hat{\beta} \alpha} = \frac{1}{4} F^{ab} (\overline{\sigma}_a \sigma_b)_{\hat{\beta} \alpha},
\]

\[
b_{\pm} = \frac{1}{1 \pm \frac{i}{8} \epsilon^{abcd} F_{ab} \sigma_{cd} + \sqrt{-\det(\eta + F)}}.
\]

The solution (61) singles out 16 independent covariant parameters \(\kappa_{\alpha}^{\alpha}, \overline{\kappa}_{\alpha}^{\alpha}\) of the irreducible \(\kappa\)-symmetry of the super–D–3–brane.

Remember that the infinite reducibility of the \(\kappa\)-symmetry in the Green–Schwarz formulation of superstring theory \[15\] is a main problem which hampers the covariant quantization. The same problems appear in the DBI like formulation of the super–D–p–branes \[1\]–\[3\], which have \(\kappa\)-symmetry realized in the infinitely reducible form.

In this respect it is remarkable that, as has been proved in this Section, the \(\kappa\)-symmetry of the Lorentz–harmonic formulation of the super–D–p–branes is realized in irreducible form.

### 7 Generalized action functional and superfield equations of motion

The action (38), (39) and (22) is written in terms of differential forms without any use of Hodge operation \(*\) and, hence, can be used for the construction of the generalized action \[11\] (see \[14\] for supergravity) for Dirichlet super–p–branes in a \(D = 10\) type \(II\) supergravity background.
This generalized action can be regarded as a dynamical basis for deriving superfield
equations of motion of the super–$D$–$p$–branes as geometrical conditions of embedding
their superworldvolumes into a target superspace \[10, 11, 12, 13\].

Suppose the integration surface in the functional (38) to be an arbitrary surface
\[\mathcal{M}^{p+1} = \{(\xi^m, \eta^\mu(\xi))\}\] (63)
in a worldvolume superspace
\[\Sigma^{(p+1)8+8} = \{(\xi^m, \eta^\mu)\}\] (64)
of the type II super–$p$–brane specified by 16 Grassmann functions (Goldstone fermions \[33\])
\[\eta^\mu = \eta^\mu(\xi^m).\]

Henceforth suppose all the coordinates of the target superspace and Lorentz harmonics
involved into (39) and (22) to be superfields on \[\Sigma^{(p+1)8+8}\]
\[Z^M = Z^M(\xi, \eta), \quad u^a_m = u^a_m(\xi, \eta), \quad ...\] (65)
but restricted to an arbitrary surface \[\mathcal{M}^{(p+1)} : \eta = \eta(\xi)\]
\[Z^M = Z^M(\xi, \eta(\xi)), \quad u^a_m = u^a_m(\xi, \eta(\xi)), \quad ...\] (66)

In this way we get a generalized action for super-D-p-branes (see \[11\] for superstrings and
type I super–$p$–branes) in \[D = 10\] type II supergravity background
\[S = \int_{\mathcal{M}^{p+1} = \{(\xi, \eta = \eta(\xi))\}} (\mathcal{L}_0^{p+1} + \mathcal{L}^{WZ}_{p+1}),\]
\[\mathcal{L}_0^{p+1} \equiv (\frac{1}{(p+1)!} E^{a_0} \wedge E^{a_1} \wedge \ldots \wedge E^{a_p} \epsilon_{a_0 a_1 \ldots a_p} e^{-\frac{p-3}{2} \phi} \sqrt{-\det(\eta_{ab} + F_{ab})} + Q_{p-1} \wedge [e^{-\frac{1}{2} \phi}(dA - B_{(2)}) - \frac{1}{2} E^b \wedge E^a F_{ab}])|_{\mathcal{M}^{p+1}},\]
\[\mathcal{L}^{WZ}_{p+1} = e^F \wedge C|_{\mathcal{M}^{p+1}}.\]

In (67) the formal sum of the forms \(C\) is defined by (23) and all the variables should be
regarded as superfields (65) restricted to the bosonic surface \[\mathcal{M}^{p+1}\] (66). All the forms
are defined on the whole worldvolume superspace (64) and pulled back into \[\mathcal{M}^{p+1}\] (this
is denoted by \(|\mathcal{M}^{p+1}\)). For example, the external differential is
\[d = d\xi^m \partial_m + d\eta^\mu \partial_\mu = e^A \nabla_A = e^a \nabla_a + e^\alpha \nabla_\alpha,\]
and its pullback is
\[d = d\xi^m (\partial_m + \partial_m \eta^\mu(\xi) \partial_\mu) = d\xi^m (e^A_m + \partial_m \eta^\mu(\xi) e^A_\mu) \nabla_A.\]
Note that the difference in construction of the generalized action (67) from the generalized action for ordinary type I super–p–branes [11] is that (67) does not contain intrinsic worldvolume supervielbeins (8) and (3) as independent auxiliary fields 2. As we shall see below and in the next Section worldvolume supergeometry is induced and completely specified by conditions of embedding into target superspace.

A reason why one finds more convenient to construct the action without intrinsic worldvolume supervielbeins is the presence of the worldvolume 1–form gauge (super)field and the nonlinear nature of the $D$–brane theories reflected in the form of the DBI functional in the original formulation [1-5].

The generalized action principle [14, 11] is based on the requirement that the equations of motion originate from the vanishing of the variation of the functional (67) with respect to the variation of the superfields involved as well as under arbitrary variations of the surface $\mathcal{M}^{(p+1)}$ itself which can be regarded as a variation with respect to the Goldstone fermion field $\eta^\mu(\xi)$.

For the Lagrangian form under consideration it can be proved (see Refs. [11] ) that the variation with respect to the surface $\mathcal{M}^{(p+1)}$, i.e.

$$\frac{\delta S}{\delta \eta(\xi)} = 0,$$

does not lead to new equations of motion. (The letter are consequences of the equations of motion for other fields). This implies the superdiffeomorphism invariance of the generalized action [14, 11].

In the same way as it was done in refs. [11] for superstrings and type I super–p–branes one can show that this superdiffeomorphism invariance is related to the irreducible $\kappa$–symmetry of the Lorentz–harmonic formulation (38) as well as to the infinitely reducible $\kappa$–symmetry (26) – (28) of the original DBI–like formulation. Thus, as in the case of ordinary super–p–branes (see [11]–[31]), the generalized action clarifies the origin of the $\kappa$–symmetry of the super–$D$–p–brane theories as a manifestation of the local worldvolume supersymmetry.

The fact that the surface $\mathcal{M}^{(p+1)}$ (33) is arbitrary and that the whole set of such surfaces spans the whole worldvolume superspace (54) ensures the possibility of considering equations of motion ($\delta S/\delta Z^\alpha = 0$, etc. ) as superfield ones, i.e. as equations for the superforms and superfields defined in the whole worldvolume superspace $\Sigma^{(p+1)8+8}$. These equations are formally the same as ones obtained from the action (38) (see, for instance, (43), (41) and (41)):

$$E^i \equiv dZ^\alpha E^i_M = 0,$$

$$i\hat{\gamma}^{(p)}(\hat{\alpha})\hat{E}^{(-)\hat{\beta}} - \left(\frac{1}{2}(1 - \hat{\Gamma})\hat{\gamma}^{(p+1)}\hat{\alpha}\hat{\beta}\hat{\Lambda}_{\hat{\beta}}\right) = 0,$$

2In this sense the action (67) is closer to a formulation of bosonic strings with auxiliary vector fields proposed in [34] than to the Lorentz–harmonic formulation of refs. [10, 11].
but now these are the equations for superforms and hence they should be expanded in
the whole basis (8) of the supervielbeins

\[ e^A = (e^a, e^{\dot{\alpha}}) = (e^a, e^{\alpha}^q, e^{\dot{\alpha}q}) \]  \hspace{1cm} (72)

of the worldvolume superspace (64), the external differential being determined in (68).

As a result (70) contains now a spinor component

\[ E^i \equiv dZ^M E^i_M = e^a E^i_a + e^{\dot{\alpha}} E^i_{\dot{\alpha}} = 0 \implies E^i_a = 0, \quad E^i_{\dot{\alpha}} = 0. \]  \hspace{1cm} (73)

The vanishing of the vector component of (70) \((E^i_a = 0)\) implies that the worldvolume
bosonic vielbein superform can be identified with the induced vielbein \(E^a\) up to a non-
singular matrix \(m^a_b \equiv E^a_b\) \(\hspace{1cm} (74)\)

(which is a conventional rheotropic condition \(\hspace{1cm} (74)\)). Eqs. (73) and (74) result in the
geometrodynamic condition

\[ E^a_{\dot{\alpha}} = 0 \]  \hspace{1cm} (75)

being a basic point of the superfield (twistor–like) description of superstrings and the
\textit{type I} superbranes \[17 \text{ - } 31\], \(10\) as well as of the \(D = 11\) super–5–brane \(13\) and
Dirichlet–p–branes in the linearized approximation \(12\).

Decomposing Eq. (71) in the basic worldvolume \((p + 1)–\)forms we find that the second
term contains the input proportional to the form \(e^{a_1} \wedge ... \wedge e^{a_{p+1}} \epsilon_{a_1...a_{p+1}}\) only, while the
input proportional to the basic form \(e^{a_1} \wedge ... \wedge e^{a_p} \wedge e^{\dot{\alpha}}\) comes from the first term which
\[ \text{gives rise to an independent geometrical equation for the components of the Grassmann} \]
\[ \text{supervielbein } E^{\dot{\alpha}} \text{ (a fermionic rheotropic condition \(\hspace{1cm} (74)\)).} \]

Upon omitting a nonsingular matrix multiplier one reduces this equation to

\[ \hat{E}^{(-)\dot{\alpha}}_{\dot{\alpha}} = 0, \]  \hspace{1cm} (76)

or, in terms of differential forms, to

\[ \hat{E}^{(-)\dot{\alpha}} = e^a \hat{\psi}^{(-)\dot{\alpha}}_a. \]  \hspace{1cm} (77)

\[ \text{Eq. (76) (or (77)) together with (70) and (74) form the complete set of} \]
\[ \text{superfield equations for the super–D–p–branes in } D = 10 \text{ type II supergravity} \]
\[ \text{background.} \]

The superfield equation being the coefficient of the basic \((p + 1)–\)form \(e^{a_1} \wedge ... \wedge e^{a_{p+1}} \epsilon_{a_1...a_{p+1}}\) in (71) expresses the gamma trace of the superfield \(\hat{\psi}^{(-)\dot{\alpha}}_a \) (77) through the

\[\text{3The choice of the matrix } m^a_b \text{ is a matter of convenience and can be used to get the main spinor–spinor} \]
\[\text{component of the worldvolume torsion in the standard form (see below).}\]
derivatives of the dilaton superfield $\Lambda_\mathbf{A} = 1/2 D_\mathbf{A} \phi$. For example, for the case of 3–brane we get

$$
\sigma^a_{\alpha \dot{\alpha}} \psi^{(-) \dot{a} q} = 4i(\sqrt{-\det(\eta + F)} - 1) \Lambda^\alpha_q + 8i b_+ f^\beta \Lambda^q_{\beta},
$$

$$
\psi^{(-) \alpha}_{a \ q} \sigma^a_{\alpha \dot{\alpha}} = -4i(\sqrt{-\det(\eta + F)} - 1) \Lambda_{\dot{\alpha} q} - 8i b_+ f^\beta \Lambda^q_{\beta}.
$$

This equation is the same as the equation of motion of $\Theta \hat{\mathbf{A}}$ derived from the “component” action (78), but with worldvolume superfields instead of fields. We should stress that these fermionic equations can be obtained from the selfconsistency conditions for Eqs. (74), (74) and (74).

8 Superfield equations for type IIB super–D–3–brane

As an instructive example let us consider the superfield equations for a super–D–3–brane in flat $D = 10 N = IIB$ superspace.

As it was done for the parameter of the $\kappa$–symmetry in the section 4, using the explicit form of the projector $\Gamma \ (61)$ in the complex representation (2) we can express two independent 8–component forms $E^{(-) \dot{a} q}$ and $E^{(-) q}_{\dot{a}}$ of the 32–component form

$$
\hat{E}^{(-) \hat{\mathbf{A}}} = \hat{E}^{(-) \hat{\beta} q} \hat{\mathbf{A}}^\beta = \begin{pmatrix} 2b_+ \hat{E}^{(-) q}_{\dot{a} \beta} f^{\dot{a} \beta} \\
\hat{E}^{(-) \dot{a} q} \\
2b_\beta (-\hat{E}^{(-) \beta \dot{a}} \hat{f}^{\beta \dot{a}}) \end{pmatrix} T
$$

(78)

(for the definition of $f$, $\bar{f}$ and $b_\pm$ see (62)) in terms of the covariant components $E^a_q$, $E^{\dot{a} q}$, $E^\alpha_q$, and $\bar{E}^{\dot{a} q}$ of the complete pullback into the worldvolume superspace of the Grassmann vielbein 1–form

$$
\hat{E}^{\hat{\mathbf{A}}} = \hat{E}^{\hat{\beta} q} \hat{\mathbf{A}}^\beta = \begin{pmatrix} E^a_q \\
E^{\dot{a} q} \\
E^\alpha_q \\
\bar{E}^{\dot{a} q} \end{pmatrix} T.
$$

In this way we get

$$
E^{(-) \dot{a} q} = \frac{1}{2b_- \sqrt{-\det(\eta + F)}} (\hat{E}^{\dot{a} q} - 2b_- \bar{E}^{\beta \dot{a}} \hat{f}^{\beta}),
$$

$$
E^{(-) q}_{\dot{a}} = \frac{1}{2b_+ \sqrt{-\det(\eta + F)}} (\hat{E}^\alpha_q - 2b_+ E^{\beta \dot{a}} f_{\beta}^\alpha)
$$

In flat $D = 10$ IIB superspace (where, in particular, $\Lambda_\mathbf{A} = 0$) after some algebra Eq. (71) for the super–D–3–brane takes the form

$$
\hat{E}^{(-)} \wedge \gamma^{(3)} = 0 \quad \Leftrightarrow \quad \left\{ \begin{array}{l}
\frac{1}{3!} E^{a_1} \wedge E^{a_2} \wedge E^{a_3} \wedge \epsilon_{a_1 a_2 a_3 a_4} m^a_{a_4} \wedge \bar{E}^{(-) q}_{\dot{a}} \sigma^a_{\alpha \dot{\alpha}} = 0,
\frac{1}{3!} E^{a_1} \wedge E^{a_2} \wedge E^{a_3} \wedge \epsilon_{a_1 a_2 a_3 a_4} m^a_{a_4} \wedge \sigma^a_{\alpha \dot{\alpha}} E^{(-) \dot{a} q} = 0
\end{array} \right. (80)
$$
In (80)
\[ m^b_a = \delta^b_a + b_+ b_- \text{Sp}(\bar{\sigma}_a f \sigma^b \bar{f}) \] (81)
where \( f, \bar{f} \) are spinor representations for the self–dual and the anti–self dual part of the tensor \( F_{ab} \) defined in Eq.(62) together with \( b_\pm \). Choosing the worldvolume vielbein as in (74) with the matrix \( m \) given by (81) we get from (80)
\[
\begin{cases}
\frac{1}{3!} e^{a_1} \wedge e^{a_2} \wedge e^{a_3} \wedge \epsilon_{a_1 a_2 a_3 a} \wedge \bar{E}^{(-)}_q \sigma^a_{\alpha \dot{\alpha}} = 0, \\
\frac{1}{3!} e^{a_1} \wedge e^{a_2} \wedge e^{a_3} \wedge \epsilon_{a_1 a_2 a_3 a_4} \wedge \sigma^a_{\alpha \dot{\alpha}} E^{(-)}_{\dot{\alpha}q} = 0
\end{cases}
\] (82)

Using the expressions (79) we can represent the geometrical Grassmann equations (rheotropic conditions) (77) in the form
\[
\bar{E}^q = 2b_+ E^q \beta_\alpha + e^a \psi^a_q, \\
E^{\dot{\alpha}q} = 2b_- \bar{E}^{\dot{\alpha}q} \bar{\beta}_\dot{\alpha} + e^a \bar{\psi}^{\dot{\alpha}a}_q,
\] (83)
where (see (77))
\[
\psi^{a}_q = 2b_- \sqrt{\text{det}(\eta + F)} \bar{\psi}^{(-)\alpha}_q, \\
\bar{\psi}^{\dot{\alpha}q} = 2b_+ \sqrt{\text{det}(\eta + F)} \psi^{(-)\dot{\alpha}}_q.
\]
Equations (83) together with (73) and (74)
\[
E^i \equiv dZ^i M_\alpha E^a \hat{u}_{\frac{a}{2}} = 0,
\] (84)
\[
E^a \equiv dZ^i M_\alpha E^a \hat{u}_{\frac{a}{2}} = e^b m^a_b
\] (85)
form the complete set of the superfield equations (rheotropic conditions [11]) for the type IIB super–3–brane in flat \( D = 10, N = 2B \) superspace.

To completely specify the worldvolume superspace geometry we should add to the above equations conventional rheotropic conditions determining the Grassmann worldvolume supervielbeins \( e^{\dot{\alpha}} = (\dot{e}^q_a, \bar{\psi}^{\dot{\alpha}q}) \) which are not present in the generalized action. It can be proved (in a way similar to one described in the refs. [11] for superstrings and ordinary super–p–branes) that \( e^{\dot{\alpha}} \) can be identified with a linear combination of \( E^q_a, E^{\dot{\alpha}q} \) and \( E^a \) pulled back into superworldvolume
\[
E^a_q - E^a \chi^a_{aq} = \dot{e}^q_a, \\
\bar{E}^{\dot{\alpha}q} - E^{\dot{\alpha}q} \chi^{\dot{\alpha}q} = \bar{\psi}^{\dot{\alpha}q},
\] (86)

The dynamical equations of motion of \( \Theta^L \) contained in (82)
\[
\sigma^a_{\alpha \dot{\alpha}} \bar{\psi}^{\dot{\alpha}q} = 0, \\
\psi^a_{aq} \sigma^a_{\alpha \dot{\alpha}} = 0,
\] (87)
can be obtained from selfconsistency conditions of the equations (83), (84) and (85) as described in [10, 11] for type I super–p–branes and superstrings.
The selfconsistency conditions also leads to worldvolume supergravity torsion constraints \cite{11,13}. In this respect it should be stressed that choosing in \cite{85} the $m$ matrix in the form \cite{81} we get the main torsion constraint in the standard form

$$ T^{q \ c}_{\alpha \ \beta p} = -i\delta^{q}_{p} \sigma^{c}_{\alpha \beta}. \quad (88) $$

The set of superfield equations for super-$D$–3–brane \cite{83}–\cite{86} obtained above generalizes linearized equations for the $D$–3–brane studied in \cite{12} and is similar to equations proposed for the $D = 11$ super–5–brane in ref. \cite{13}.

Thus the generalized action proposed herein provides a bridge between the formulations of refs. \cite{1–4} and the superfield geometrical approach of refs. \cite{10,11,12,13}.

9 Conclusion

In conclusion we have proposed the Lorentz–harmonic formulation with irreducible $\kappa$–symmetry and the generalized action functional for Dirichlet super–p–branes in $D = 10$ type $II$ supergravity background. In this formulation not only the WZ term but the whole super–$D$–p–brane action is an integral of the differential $(p+1)$–form. From the generalized action we obtained the general form of the superfield equations of motion for all super–$D$–p–branes and specified them in more detail for $Type\ II B$ super–3–branes.

The superfield equations we obtained generalize linearized super–$D$–p–brane equations of ref. \cite{12} and have the form analogous to one proposed recently for $D = 11$ super–5–branes in Ref. \cite{13}. Thus we have established a relation between the “standard” approach to super–$D$–p–branes based on the DBI action \cite{1}–\cite{4} and the superfield approach to these objects \cite{12,13}.

A natural next step consists in studying the possibility of constructing a generalized action for the $D = 11$ super–5–brane of M–theory which should produce the superfield equations of ref. \cite{13}.

A covariant action for the $D = 11$ 5–brane proposed recently in \cite{35} as a generalization of results of \cite{36} provides a basis for this construction.

Acknowledgments. The authors are grateful to K. Lechner and P. Pasti for useful discussion.

Work of M.T. was supported by the European Commission TMR programme ERBFMRX–CT96–045 to which M.T. is associated. I.B. and D.S. acknowledge partial support from the grant N2.3/644 of the Ministry of Science and Technology of Ukraine and the INTAS Grants N 93–127, N 94–2317.

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