Dynamic modeling of the birefringence effects induced in semiconductor optical amplifier for all-optical telecommunication systems

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ABSTRACT

The semiconductor optical amplifiers (SOA) are all-optical multifunctional devices. The improvement of their performance will, therefore, be of great importance for modern optical telecommunication systems. We propose in this article to develop a dynamic model that enables us to simulate the dynamic behavior of SOA's birefringence effects. The determination of a numerical model is a multidisciplinary activity that needs engineering skills, optimization and physics. This numerical model enables to describe the propagation of a picosecond optical pulse passing through the SOA and takes into account its polarization and the phenomenon of energy coupling between the eigenmodes of SOA (TE mode and TM mode). In this paper, we will, first of all describe the numerical algorithm of our model, and then we will propose to make a dynamic characterization of the effect of the nonlinear polarization rotation in the SOA, which will allow us to study the all-optical logic gates as well as all the other digital components based on the nonlinear effect of birefringence in SOA.

Keywords: Birefringence effect, Dynamic Modeling, Numerical model, Semiconductor optical amplifier (SOA), Telecommunications systems

1. INTRODUCTION

Semiconductor Optical Amplifiers (SOAs) are expected to become key elements in optoelectronic integrated systems and fiber transmission systems [1-3]. There has been considerable progress in exploitation of optical nonlinearities in SOAs [4-7]. Much attention has been paid to the nonlinear effect of birefringence induced in SOA [4]. The effective refractive indices for transverse-electric (TE) mode and transverse-magnetic (TM) mode are different form each other due to intrinsic birefringence in SOA and induced birefringence in SOA, thus the polarization changes in SOA for TE and TM modes [4]. This change of nonlinear polarization in the SOA component will result in the appearance of optical logic gate characterized by high speed of response [8-12].

The design of a semiconductor optical amplifier (SOA) component must necessarily pass the modeling phase which allows studying a theoretical characterization of the SOA and anticipating its reactions on the basis of the operating conditions [1]. Mathematical models are required to aid in the design of SOAs and to predict their operational characteristics. The theoretical foundation of SOAs modeling was established in 1980s [1, 13]. Since then, major progress concerning SOA modeling start to focus on the material gain coefficient, spontaneous emission rate and the fraction of spontaneous emissions coupled with the guided
waves in an amplifier. Some numerical simulations which use the Finite Difference Method (FDM) have been studied intensively to solve carrier rate equation and photon traveling wave equations.

In this paper, we propose to make a dynamic characterization of the effect of the nonlinear polarization rotation in SOA by injecting into the component the pump and probe signals of different power and state of polarization at the entering of the SOA. Therefore, this paper is dedicated to the study of the modifications of our static numerical model to take into account the nonlinear effect of birefringence over time in the SOA [4]. Our detailed dynamic numerical model offers a set of equations and an algorithm that predict their behavior. The equations form a theoretical base from which we have coded our model in several files.cpp that the Language C++ executes. The model is used to predict the properties of optical logic gates and will be also beneficial for all other digital components design and optimization.

2. DYNAMIC NUMERICAL MODEL OF SOA

2.1. System equations of SOA

The polarization is a very important concept when studying the SOA component. This section is dedicated to the study of the modifications to be added to our static model [4] to take into account this polarization effect. The expressions of the coupled mode equations must be modified. For this, we have developed a more detailed model based on the coupled mode equations dependent on both time and polarization. And thus, we can study the nonlinear effect of the induced birefringence and the stability of SOA to use it well as a multifunctional component in optical telecommunications systems. This model treats separately the TE component and TM component of the optical field, introduces different confinement factors for the two polarization states and takes into account the phenomenon of the TE and TM modes energy coupling [4].

The developed model based on the assumption of quasi-stationary, can be seen in Figure 1.a is summed up in a several section division of the component gain region so as to take into account the non-uniform distribution of carrier density and refractive index. This allows describing the propagation of the field of the incident signal through the component until the output interface. The SOA of length \( L \) can be decomposed into \( N \) sections between its two facets.

Figure 1 also represents the configuration to analyze the dynamic characteristics of the short optical pulse amplification of wavelength \( \lambda_1 \) during its propagation through the SOA with the injection of a continuous wave (CW) of wavelength \( \lambda_2 \). This can be called an assist beam. The assist beam can be injected either in copropagative configuration or in counterpropagative configuration with the pulse of the entering optical signal.

![Figure 1. Schema of the optical signals transmission in a SOA (a) and the two polarization directions TE and TM (b)](image)

The pulse of the entering optical signal is a Gaussian pulse of half width FWHM of 10 ps. The field of the entering pulse is given by the following equation [14, 15]:

\[
E_{in}(\theta, t) = \sqrt{P_0} \times \exp \left[ -\frac{1}{2} \left( \frac{t}{\tau_0} \right)^2 \right] \times \exp(-j2\pi\theta_0 t) \quad (1)
\]

With:
- \( P_0 \) is the peak input optical power.
- \( \vartheta_0 \) is the peak frequency.
- \( t_p \) is the duration of the pulse linked with the half width FWHM by the relation: \( t_p \approx 1.665 \times t_0 \)

We thereafter present the mathematical equations that can describe the dynamic behavior of SOA. These differential equations describe the propagation of the optical signal, the amplified spontaneous emission ASE and the evolution of the carrier density and which depend on the time, the space and the two polarization modes (TE mode and TM mode).

To begin with, we decompose the incoming arbitrarily polarized electric field in a TE component and TM component as illustrated in Figure 1.b. These two polarization directions are along the principal axes \((x, y)\) that diagonalize the wave propagation in the SOA.

The total electric field is defined by [5]:

\[
\vec{E}(\vartheta, z, t) = E_{TE}(\vartheta, z, t)\vec{x} + E_{TM}(\vartheta, z, t)\vec{y}
\] (2)

With:

\[
\begin{align*}
E_{TE}(\vartheta, z, t) &= E_{in}(\vartheta, t) \times \cos(\vartheta) \times \exp(j\varphi_x) \\
E_{TM}(\vartheta, z, t) &= E_{in}(\vartheta, t) \times \sin(\vartheta) \times \exp(j\varphi_y)
\end{align*}
\]

Whereas \( \vartheta \) is the input polarization angle, \( \varphi_x \) and \( \varphi_y \) are initial phase of input signal for TE mode and TM mode, respectively. In our calculation, in left (input) and right (output) facets of SOA have power reflectivities \( R_1 \), and \( R_2 \), respectively. The signal wave gets partially transmitted and reflected from the two facets of the amplifier. Due to the input signal, the spatially varying component of the field in the SOA can be decomposed into forward and backward propagating \( E^+ \), \( E^- \), and the traveling-waving equation for TE mode and TM mode are [1, 7] and [14]:

\[
\frac{1}{v_g} \frac{d}{dz} E_{TM/TE}^\pm (z, \vartheta) = \left\{ \left[ -j \beta_{TM/TE}(\vartheta, n) + \frac{1}{2} g_{n, TM/TE}(\vartheta, n) \right] E_{TM/TE}^\pm (z, \vartheta) + C_{TM/TE}(\vartheta, n) E_{TM/TE}^\mp (z, \vartheta) \right\}
\]

(3)

Those spontaneous emission photon rates are observed in the following equation:

\[
\frac{dN_{TM/TE}(\vartheta, n)}{dz} = \pm \left\{ g_{n, TM/TE}(\vartheta, n) N_{TM/TE}^\pm (\vartheta, n) + R_{sp, TM/TE}(\vartheta, n) \right\}
\]

(4)

Whereas \( j = \sqrt{-1} \), \( C_{TE} \) and \( C_{TM} \) are the coefficients of coupling modes TE-TM and TM-TE respectively [4, 13].

\[
C_{TM/TE}(\vartheta, n) = \mp k_r \times \exp(\mp j \Delta \beta \times z)
\]

(5)

Whereas \( \Delta \beta = \beta_{TM}(\vartheta, n) - \beta_{TM}(\vartheta, n) \). \( k_r \) is the coupling constant and \( v_g \) is the group speed. The other coefficients are defined in [1, 4].

These amplified forward and backward signal waves should meet the following boundary conditions:

\[
E_{TM/TE}^+(z = 0, t) = (1 - r_2) E_{in, TM/TE}^-(t) + r_1 E_{TM/TE}^+(z = 0, t)
\]

(6)

\[
E_{TM/TE}^-(z = L, t) = r_2 E_{TM/TE}^+(z = L, t)
\]

(7)

Similarly, the output signal field to the right of the output facet is:

\[
E_{out, TM/TE}(t) = (1 - r_2) E_{TM/TE}^+(z = L, t)
\]

(8)

The initial conditions to solve the amplified spontaneous emission noise equations system.

\[
N_{TM/TE}^\pm (z = 0, t) = R_1 E_{TM/TE}^\mp (z = 0, t)
\]

(9)

The carrier density in section \( i \) inside the SOA obey the rate equation [1, 4, 12]:

\[
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\]
\[
\frac{dn(z,t)}{dt} = \frac{i}{edWL} - R(n) - \frac{f_{TE}}{dw} \left(f_{k,TE}(n) - 2f_{j,TE}(n)\right) - \frac{f_{TM}}{dw} \left(f_{k,TM}(n) - 2f_{j,TM}(n)\right)
\] (10)

Whereas \(I\) is the amplifier bias current, \(d\) and \(w\) are the active region thickness and width, respectively.

The recombination rate term \(R(n)\) is given by:
\[
R(n) = C_1 \times n + C_2 \times n^2 + C_3 \times n^3
\] (11)

Whereas \(C_1, C_2\) and \(C_3\) are recombination coefficients. \(f_{k,TE}, f_{r,TE}, f_{j,TE}\) and \(f_{j,TM}\) are defined by equations [4]:
\[
f_{k,TE/TM}(n) = \sum_{k=1}^{N_n} g_{m,TE/TM}(\theta_k, n) \left[|E_{TE/TM}^+|^2 + |E_{TE/TM}^-|^2\right]
\] (12)
\[
f_{j,TE/TM}(n) = \sum_{j=0}^{N_n-1} g_{m,TE/TM}(\theta_j, n) \left[N_{j,TM/H}^+ + N_{j,TE/TM}^-\right]
\] (13)

With, \(N_n\) numbers of signals injected with optical frequencies \(\theta\) \((k = 1 \text{ to } N_n)\) and \(N_{m_k}\) is an integer that determines the sampling step of the spontaneous emission frequencies. The physical meaning of each of the terms in equation (10) is shown in Table 1.

| Terms                          | Physical meaning                                      |
|--------------------------------|-------------------------------------------------------|
| \(dn(z,t)/dt\)                 | Variation of carrier density as a function of time and space. |
| \(i/edWL\)                    | Carrier density injected by the bias current.         |
| \(R(n)\)                      | Decreased carrier density caused by spontaneous recombination. |
| \(f_{k,TE}\)                  | Decreased carrier density caused by amplification of signals. |
| \(f_{j,TE/TM}\)               | Function describing the influence of ASE on carrier density. |

The systems of equations (3) and (4) are systems of five partial, nonlinear and coupled equations that do not have an analytic solution. A numerical resolution is then necessary. Partial derivatives appearing in these last equations are approximated by the finite differences in the first order and can be replaced by:

\[
\begin{aligned}
\frac{dE_{TE/TM}^+(x,t)}{dt} &= \pm \frac{E_{TE/TM}^+(x,t+\Delta t)-E_{TE/TM}^+(x,t)}{\Delta t} \\
\frac{dE_{TE/TM}^+(x)}{dx} &= \pm \frac{E_{TE/TM}^+(x+\Delta x)-E_{TE/TM}^+(x)}{\Delta x} \\
\frac{dN_{TE/TM}^-(x)}{dx} &= \pm \frac{N_{TE/TM}^-(x+\Delta x)-N_{TE/TM}^-(x)}{\Delta x}
\end{aligned}
\] (14)

Similarly, the evolution equation of carrier density \(\frac{dn(z,t)}{dt}\) is approximated by:
\[
\frac{dn(z,t)}{dt} \approx \frac{n(z,t+\Delta t)-n(z,t)}{\Delta t} \quad (15)
\]

### 2.2. Dynamic of numerical algorithm

In dynamic regime, we estimated the evolution of the carrier density \(\frac{dn(z,t)}{dt}\) by the method of FDM (15). The following algorithm, in Figure 2 summarizes the main steps used during the simulation:

**Step1:** Initialize to transparency, the initial conditions at \(t = 0\).

**Step2:** Begin iteration of time, then we estimate the signal fields and spontaneous emission photon rates using the method of FDM equations: (16), (17), (18) and (19).

**Step3:** The results found in the previous step for each iteration of the time are injected into the carrier evolution equation (20), and then the results found are saved for a complete study of the SOA component over time.
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Figure 3. Input optical pulse (a) and gain recovery dynamics with the injection of a TE polarized pulse (b)

Figure 4 illustrates the longitudinal distribution of carrier density and ASE photon rate at different times of the gain recovery dynamics of figure 3.b. It is noted that after the gain compression, the carrier density and the intensity of the photon rate ASE are weak in left and right facets of the component, which explains a slow gain recovery time but when the intensity of ASE is strong enough in the extremities sections of the SOA, the recombination of the stimulated emission becomes dominant compared to that of spontaneous emission. This makes it possible to accelerate the recovery of the carriers, in particular in the front end where the carrier density goes beyond its state of equilibrium (red curves).

Figure 4. Spatial distribution of carrier density and ASE photon rate for different gain recovery times

Figure 5 shows the gain recovery dynamics with co-propagating or counter-propagating injection of the different continuous power signals. The curves of Figure 5 show that gain recovery time of the SOA is clearly shortened with the increase of the power of assist beam for both configurations. Because, the more power of the assist beam is, the more important the amplification mechanism by stimulated emission. In all cases of figure, this gain recovery time is shorter in counter-propagating configuration. This efficiency is obtained by injecting the power of the assist beam at the output of the SOA, where the pulse signal needs many carriers to avoid gain saturation.
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Subsequently, we will study the effect on the gain recovery time when the input pulse, under different polarization angles of the assist beam.

One can notice for both polarizations of the incoming pulse (TE and TM), that the gain recovery time is remarkably shortened when the assist beam is polarized TE ($\theta_{cw} = 0^{\circ}$) shown in Figure 6 and the more the polarization angle of the assist beam is, the more important the gain recovery time. Add to this that the counter-propagating injection of the assist beam gives better results for the gain recovery time.

3.2. Time characterization of the outgoing optical pulse

In this subsection, we will analyze the temporal characteristics of the short optical pulse amplification in the SOA for different polarizations with and without the injection of a assist beam. We will show further that with the injection of an assist beam in the gain region, we can reduce the deformation of the outgoing optical pulse.

Figure 7 shows the evolution of the output power in the different sections of the SOA for the two polarization cases of the incoming optical pulse without injection of the assist signal. From this figure, it is found that peak power of the outgoing optical pulse becomes more important compared to that of the incoming pulse for the two polarization cases. But we can notice that the injection of the pulse in TE mode presents a great power of the outgoing optical pulse and a remarkable deformation compared to that of the TM mode injection.
Subsequently, we will study the case where the incoming optical pulse is assisted by the injection of an assist beam to reduce the deformation of the outgoing optical pulse of the SOA.

Figure 8 shows the time distribution of the outgoing optical pulse with the injection of the different powers of assist beam in co-propagating and counter-propagating configuration. The curves of this figure show a reduction in the deformation of the outgoing optical pulse and a decrease in peak power as the power of the assist beam is increased. We can notice that the reduction of deformation is clearly remarkable for the counter-propagating configuration. This is due to the decrease of the fast saturation of the carrier density the rising edge of the incoming pulse with the injection of a high power of the assist beam.

![Figure 8](image1)

(a): Copropagating configuration  
(b): Counterpropagating configuration

Figure 8. Time distribution of the outgoing optical pulse for different power of CW

Figure 9 illustrates the effect of polarization of an assist beam injected in co-propagating or counter-propagating configuration on the SOA component to which we also injected an optical pulse polarized in a TE. One can notice from figure 9 that the deformation of the outgoing optical pulse is dependent on the polarization of the assist beam. For the two cases of figure. 9, we observe that when the assist beam is polarized TE, the deformation of the outgoing optical pulse is well reduced compared to that polarized TM. This trend comes from the elastic constraints applied to SOA which largely favors TE transitions.

![Figure 9](image2)

(a): Copropagating configuration  
(b): Counterpropagating configuration

Figure 9. Time distribution of the normalized outgoing optical pulse for different polarization of CW

4. CONCLUSION

After explaining our numerical model in the time domain, we have listed the basic equations necessary to simulate the dynamic behavior of SOA by the FDM method. Subsequently, we have studied the dynamic characterization of the birefringence effects induced in the SOA component to overcome the slow gain recovery time and eventually obtain a deformation as reduced as possible of the outgoing optical pulse. The results of the simulations have shown that we can overcome these drawbacks by the injection of an assist beam in counterpropagation configuration, polarized TE and of a high power.

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