Nonlinear Modeling the Quadcopter Considering the Aerodynamic Interaction

JIANCHUAN YE, JIANG WANG, TAO SONG, ZELIANG WU, AND PAN TANG
School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China
Corresponding author: Tao Song (282233523@qq.com)

ABSTRACT This paper presents a nonlinear model of the quadcopter to simulate the actual motion. An aerodynamic model of the rotor is obtained through wind-tunnel tests, which consider the variations in the rotor’s angular velocity, incoming flow speed, and angle of attack of the rotor ($\alpha_p$). The quadcopter’s fuselage aerodynamics and mutual interference are simulated using the computational fluid dynamic (CFD) method. The mesh motion method is used to simulate the rotation of the rotor. It can be found that the interference has a significant impact on the aerodynamic load of the fuselage and the thrust and pitch moment of the rear rotor. The mathematical expression of the interference model is given by adopting the idea of the small disturbance hypothesis. The yaw dynamics model is also improved by considering the motor’s dynamics. The quadcopter’s nonlinear model can be obtained by combining the rotor model, fuselage model, motor model, and interference model. Then, flight experiments are done at hovering and forward flight states to verify the established model, and the experiment results are in good agreement with the nonlinear model. Finally, the quadcopter’s characteristic descriptive abilities of different accurate models are discussed to help designers to build a suitable quadcopter model according to the requirements.

INDEX TERMS Quadcopter, aerodynamic interaction, rotor model, wind-tunnel tests.

I. INTRODUCTION
Quadcopters have been increasingly applied in military and civilian fields due to their simpler mechanical structure and lower cost [1]. With the diversification of application requirements, new demands are put forward for the design and performance improvement of the quadcopter. In application scenarios such as target surveillance [2], film shooting, antimicro drones, and unmanned aerial vehicle (UAV) swarm [3], high position control accuracy is required. An efficient and robust controller algorithm could greatly improve the flight performance of the quadcopter. The synthesis of a robust flight control algorithm required high-order models, which can be used to describe the most out of the actual motions at several key flight statuses [4], [5]. When the flight controller designed based on a high-fidelity model is carried out in actual flight tests, the designed handing qualities could be better realized, and the risk of a crash can be greatly reduced.

Although there are many other previous studies on quadcopters [6]–[8], most of the outstanding works are only based on the research at hovering state or low speeds. Zhang et al. [9] and Chovancová et al. [10] summarize the model of the quadcopter in terms of aerodynamic, modeling methodology, and model identification. With the further increase in speed, the characteristics of the quadcopter model have undergone great changes, and those research results are difficult to apply in the forward flight mode. The modeling of the quadcopter in the forward flight states remains an underdeveloped area of research. Mellinger and Kumar [11] observed the trajectory tracking of the UAV through the optical motion capture system, and the results showed that the deviation between the flight trajectory of the UAV and the expected trajectory based on the hovering model design became larger as the speed increased. Amezquita-Brooks et al. [12] pointed out that the incoming airflow has a great influence on the propulsion system of the quadcopter. Kolaei et al. [13] and Theys et al. [14] tested the rotor in the wind tunnel with various inflow angles to simulate the rotor characteristic in forward flight. Ye et al. [15] analyzed the aerodynamic characteristics of the rotor in forward flight and pointed out that the thrust of the rotor increased with the increase of the incoming flow velocity at a small angle and decreased with the increase of the incoming flow velocity at a large angle. Luo et al. [16] proposed a quadcopter...
forward-flight mathematical model considering rotor wake mutual interference. However, the pitching moment is not reported and analyzed, which plays an important role in forward flight trim. Lei and Wang [17] studied the aerodynamic characteristics of a micro quadrotor considering the horizontal wind disturbance and indicated that in the presence of wind speed, the aerodynamic characteristics of the rotor had undergone major changes. Hwang et al. [18] and Misiorowski et al. [19] pointed out that the aerodynamic interference has a great influence on the aerodynamic model of the rear rotor during the forward flight in both cross configuration and plus configuration quadcopter. To summarize this, there are three main reasons for the variation in the forward flying model and the hovering model. 1) The aerodynamic characteristics of the rotor have changed. In addition to the thrust force and torque, it also produces hub force, pitching moment, and rolling moment; 2) The aerodynamic force on the fuselage becomes non-negligible due to the increase in dynamic pressure; 3) The mutual interference of each aerodynamic component has a strong influence on the aerodynamic and dynamic performance of the quadcopter [20]. In addition, the anti-torque generated by the motor-rotor plays an important role in the yaw channel. Therefore, motor dynamics must also be taken into consideration when designing the controller.

The accurate quadcopter models can realistically describe the dynamic characteristics of the quadcopter. This helps to verify the robustness of the controller and greatly reduces the cost and risk of the real flight test in the controller design process. To achieve more precise control of the quadcopter, this paper combines wind tunnel experiments and high-fidelity CFD simulations to give a model with high reliability. The isolated rotor is tested in a wind tunnel, and the model of the rotor is acquired by fitting the experimental results. CFD simulation analysis is performed on the fuselage, single rotor, and the whole quadcopter. By comparing the aerodynamic characteristics of the whole quadcopter and the individual components, the aerodynamic interference model in the quadcopter is obtained. And experiments are carried out in hovering and forward flight modes to verify the established model.

The main contributions of this paper are as follows:

1) This paper presents a novel rotor aerodynamic model with considering the incoming flow velocity and angle. Compared with previous models, the novel models include not only the thrust and torque, but also the hub force, pitching moment and rolling moment, which can be used to describe the rotor’s aerodynamic forces in hovering and forward states.

2) The study shows that the impact of aerodynamic interference on fuselage lift model and thrust and pitch moment of rear rotor needs to be considered, the rest interference effects can be neglected. The influential aerodynamic interaction forces are given.

3) Flight experiments show that the model of the quadcopter considering the novel rotor model and interference model can accurately describe motion of the UAV at different flight speeds. In the forward state, it is necessary to improve the accuracy of rotor aerodynamics. And the interference model can effectively improve the accuracy of the whole quadcopter’s model.

The paper is organized as follows: Chapter II introduces the basic dynamic model of the quadcopter, including the aerodynamic model of the rotor, the model of the fuselage, and the dynamic model of the motor. Chapter III discusses in detail the interference in the quadcopter and establishes the interference model. Chapter IV verifies the established nonlinear model through flight experiments. Chapter V discusses the descriptive ability of different accurate models. The conclusion is given in Chapter VI.

II. BASIC MODEL OF THE QUADCOPTER

The cross-configuration the quadcopters have better control performance during the forward flight than the plus-configuration flight because the maximum pitch and roll control torque of cross-configuration quadcopters are $\sqrt{2}$ times the plus-configuration quadcopters (when the length of arms are the same)[21]. Therefore, this paper focuses on the modeling of a cross-configuration quadcopters. The quadcopter modeling is divided into four parts, rotor modeling, fuselage modeling, motor modeling, and interference modeling. The rotor and fuselage models are suitable for all conventional platforms, and with an increase in the number of rotors, the effect of interference is much more pronounced and hence the quadcopter can be used as a standard to analyze this mutual interference.

The structure of the quadcopter model is shown in FIGURE 1. The control commands are for the four motion channels of pitch ($\delta_{\text{thm}}$), roll ($\delta_{\text{taw}}$), yaw ($\delta_{\text{ped}}$) and altitude ($\delta_{\text{col}}$). The Mixer Matrix converts control commands into control signals (Pulse width modulation, PWM) for each motor and controls the motors to drive the rotor to rotate. The model of the rotor is the function of the angular velocity and velocities ($u$, $v$, $w$) in the body system. The fuselage model is the function of the flight speed and attitude angles.

A. BASIC PARAMETERS OF THE QUADCOPTER

The quadcopter is cross-configuration, equipped with 20inch propellers, as shown in FIGURE 2-a. And the simplified model used for CFD is shown in FIGURE 2-b. The basic parameters of the studied quadcopter are shown in Table 1.

Several assumptions are used when modeling the quadcopter. 1) The ground is an inertial reference system, without considering the curvature of the earth, and the ground is a plane; 2) Quadcopter is a rigid body with constant mass; 3) The rotor blade is rigid, and there is no flapping motion; 4) The quadcopter is symmetrical in the longitudinal plane of the fuselage and to the transverse plane of the fuselage. (Product of inertia: $I_{xy} = I_{xz} = I_{yz} = 0$); 5) The center of mass is at the intersection point of the diagonal motor line and coincides with the plane of rotation of the rotors. Therefore, it is assumed that in any state of equilibrium, the front pair of motors have the same speed while the rear pair of motors have the same speed and the quadcopter lateral speed $v = 0$. 

!![](image-url)
With the assumptions above, the translational and rotational dynamics equations are given as follows [22]:

\[
\begin{align*}
\dot{v}_b &= gC(\Theta) + \frac{1}{m}(F_f + F_R) \\
J \cdot \dot{\omega}_b &= -\omega_b \times (J \cdot \omega_b) + (M_f + M_R)
\end{align*}
\]  

(1)

\(C(\Theta)\) is the transfer matrix of the body coordinate system to the inertial coordinate system. \(F_f\) represents the aerodynamic force acting on the fuselage. \(F_R\) represents the total external force generated by the four propellers. \(v_b\) is the velocity vector under the body coordinate system. \(M_f\) represents the moments produced by the fuselage. \(M_R\) represents the moments produced rotors. \(\omega_b\) is the angular velocity. \(J\) is the moment of inertia of the vehicle.

**B. ROTOR MODELING**

In the forward flight mode, the changes in the aerodynamic characteristics of the blade element of the rotor are very complicated. When the rotor rotates once, the angle of attack of the blade element changes dramatically, and there exists a recirculation area in the retreating sides of the rotor disk. It is difficult to accurately describe the aerodynamic characteristics of the rotor in the forward flight mode using traditional modeling methods (such as BEMT, etc.). In order
TABLE 2. Test conditions.

| Variables       | Range       |
|-----------------|-------------|
| Speed (m/s)     | 0, 10, 20, 30 |
| Angle (deg)     | 0, 10, 20, 30, 40 |
| Throttle (%)    | 40, 50, 60, 70, 80, 90 |

![FIGURE 4. Forces and moments acting on the rotor.](image)

to obtain an accurate rotor aerodynamic model, this paper tests the propeller through wind tunnel experiments, as shown in FIGURE 3. The forces and moments are measured by a 6-component high-resolution force/moment sensor (ME-K6D40). The RPM of the motor can be calculated according to the number of pole pairs of the motor and the alternating frequency of the phase current. The tested condition variables are incoming flow speed, incoming flow angle, and input throttle of the motor. The test conditions are shown in Table 2. The sampling frequency of all parameters is 1000Hz. In the specific test, first adjust the rotor rotation plane to the set angle, then operate the wind tunnel, and adjust the wind speed to the set speed, and finally control the motor rotation through the remote control at the designated throttle. Due to the periodically changing force in the rotor, the test of each specific test, first adjust the rotor rotation plane to the set angle, then operate the wind tunnel, and adjust the wind speed to the set speed, and finally control the motor rotation through the remote control at the designated throttle. Due to the periodically changing force in the rotor, the test of each working condition is maintained for more than 10 seconds after the rotor speed is stabilized, and then the average value of the data for these 10 seconds is taken as the truth value.

In the forward flight mode, the forces (thrust force ($T_p$), hub force ($H_{px}$) and side force ($H_{py}$)) and moments (torque ($Q_p$), pitch moment ($M_{px}$) and roll moment ($M_{py}$)) acting on the rotor are defined as shown in FIGURE 4. The incoming flow ($V_{flow}$) is broken down into velocity perpendicular to the rotor disc ($v_x$) and velocity parallel to the rotor disc ($v_y$). $\alpha_p$ is the angle of attack of the rotor.

According to the recorded data, the thrust coefficient ($C_T$), torque coefficient ($C_Q$), hub force coefficient ($C_H$), pitch moment coefficient ($m_y$) and roll moment coefficient ($m_x$) of the rotor can be written as (2)~(6), the side hub force is much small that can be ignored.

$$C_T = -0.0224\mu^2 - 0.0557\mu + 0.01097v + 0.02431$$

where $\mu$ is vertical advance ratio ($\mu = V_z/(\Omega R_r)$), $v$ is horizontal advance ratio ($v = V_h/(\Omega R_r)$), $\Omega$ is rotor’s RPM. The fitting results are shown in FIGURE 5. The goodness of fit ($R^2$) of all parameters is greater than 0.9, indicating that the models are well fitted. The rotor model is only valid within the test range in Table 2. Model beyond this range may have deficient accuracy. According to the rotor models, it can be found that the horizontal incoming flow can help increase the thrust of the rotor, while the vertical incoming flow can reduce the thrust. The horizontal flow also increases the pitch moment, roll moment, and hub force.

C. Fuselage Model

The aerodynamic loads on the fuselage can be separated into drag ($D_f$), lift ($L_f$), and pitch moment ($M_{sf}$). Normally the model of the fuselage are functions of the angle of attack ($\alpha$). The drag, lift, and pitch moment of the fuselage can be written as:

$$D_f = c_d(\alpha)\frac{1}{2}\rho V^2 S$$

$$L_f = c_l(\alpha)\frac{1}{2}\rho V^2 S$$

$$M_{sf} = c_m(\alpha)\frac{1}{2}\rho V^2 S$$

in which, $\rho$ is the local air density, $S$ is the planform area. The aerodynamic coefficients of the fuselage ($c_d$, $c_l$, $c_m$) can be obtained by using polynomials to fit the CFD simulation results. Aerodynamic force of the fuselage is more complicated when interference is considered. The specifics are discussed in detail in the next Section.

D. Motor Model and Correction of Torque Model

Motor is an extremely crucial part of the quadcopter. The dynamic of motor basically determines the response speed of commands and maneuverability of the quadcopter. And the anti-torque from the motor-rotor controls the yaw motion. The schematic diagram of the motor model system is shown in FIGURE 6.

$L_a$ is the motor equivalent inductance. $U_d$ is the input voltage. $r_a$ is the motor internal resistance. $K_T$ is motor torque coefficient. $B_V$ is coefficient of friction. $K_r$ is coefficient of back electromotive force. $J_{rr}$ is the moment of inertia of the propeller and motor outer rotor. $K_A$ is the equivalent torque
The coefficient of the rotor, can be written as:

$$K_A = 2\Omega_0 C_Q \rho R^5$$  \hspace{1cm} (10)

where $\Omega_0$ is the trim RPM of the motor. The transfer function of the RPM and input volt can be written as:

$$\frac{\Omega(s)}{U_d(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n \xi s + \omega_n^2} \frac{K_T}{s^2 + 2\omega_n \xi s + \omega_n^2}$$  \hspace{1cm} (11)

in which

$$\xi = \frac{(r_a J_{rr} + L_a (B_V + K_A))}{2\sqrt{L_a J_{rr} (B_V r_a + K_A r_a + K_T K_e)}}$$  \hspace{1cm} (12)

$$\omega_n^2 = \frac{B_V r_a + K_A r_a + K_T K_e}{L_a J_{rr}}$$  \hspace{1cm} (13)

The equivalent motor model is shown in FIGURE 7.

A step signal is given to the motor on the bench, and the counter torque of the motor-rotor is recorded through the torque sensor.

As shown in FIGURE 8, $Q_R$ is the reverse torque generated by the rotor, $T_L$ is the electromagnetic torque of the motor, and $Q_m$ is the anti-torque recorded by the torque sensor. It is obvious that the anti-torque acting on the fuselage should be the electromagnetic torque of the motor instead of the aerodynamic torque of the rotor. The yaw dynamics of quadcopter mainly rely on the anti-torque of the motor to balance and control the yaw motion of the UAV. The electromagnetic torque of the motor can also represent the anti-torque of the
rotor and the torque caused by the acceleration and deceleration of the rotor and outer rotor of the motor. The yaw moment produced by the rotor and motor can be written as:

\[ N_p = 4T_L = \sum (Q_{pi} + J_{m}\dot{\Omega}) \]  \hspace{1cm} (17)

### III. AERODYNAMIC INTERACTION OF THE QUADCOPTER

#### A. COMPUTATIONAL SET-UP

This study adopts the CFD to analyze the interactional aerodynamic phenomena with the commercial CFD solver ANSYS FLUENT. Additionally, the momentum, energy, turbulent kinetic energy, and specific dissipation rate equations are all in second-order upwind. According to the flight data, the trim states (such as attitude angle and rotor’s RPM) of the studied quadcopter at different forward flying speeds are shown in Table 3.

The Mach number is less than 0.4 in all flight states. Therefore, the flow motion is simulated using the three-dimensional, incompressible Reynolds-averaged Navier-Stokes equations with \( k - \omega \) sst turbulence model. Due to the left-right symmetry of the quadcopter, a half model is used to simulate and analyze the flow field of the quadcopter, as shown in FIGURE 10. The computational field is divided into four parts, the outer field, the dense area, and the rotation area of the two rotors. The front and up computational boundaries are more than 20 times radius \( R \) to the center of the quadcopter gravity (c.g). The side boundary is \( 32R \) to the c.g. The rear and bottom boundary are \( 40R \) to the c.g. And the radius of the rotation area is \( 1.2R \).

For mesh generation, unstructured tetrahedral and prism cells are adopted, all those areas’ interfaces are set as the couple interior wall. In the rotation areas, the maximum surface cell size is \( 1/3 \) of the mean chord \( \bar{C} \), and the minimum size is \( 1/100 \) of \( \bar{C} \). The boundary layer on all rotor surfaces was well designed, ensuring \( y^+ \leq 5 \). In the dense area, the volume cell size is \( 1/3 \bar{C} \). In the far-field, the volume cell size is \( 20 \bar{C} \), and the cell size on the interface with dense area keep the same as the dense area with a 1.2 growth rate.

A grid independence study is performed to check the computational domain with different numbers of cells. Table 4 lists the cell size of the mesh grid, the minimum size is set in the brackets, and the maximum size of the grid is in front of the brackets. FIGURE 9 shows the results of the drag coefficient \( (cd) \) of the fuselage and the rotor’s thrust coefficient \( (C_T) \).

In addition, the choice of time step is studied in this transient simulation using sliding grids. This simulation is performed for the hovering condition. The iteration steps of \( 0.5^\circ/\Delta t, 1^\circ/\Delta t, \) and \( 2^\circ/\Delta t \) are selected as shown in Table 5.
The rotor’s thrust coefficient and torque coefficient calculation results are shown in Table 5. The errors of the $0.5^\circ / \Delta t$ and $1^\circ / \Delta t$ calculation results are within 5%. The case of $10.35$ million grids is within the asymptotic range and exhibits a balance between accuracy and efficiency, and the time step of $1^\circ / \Delta t$ (the rotor rotates approximately 1 degree for each iteration step) are selected for the simulations. As a result, it is adopted for the following simulations.

Various combinations of individual front rotor, individual rear rotor, and individual fuselage are simulated; the combination conditions are shown in Table 6. Each case is analyzed according to the specific problem in the calculation Table 3. The mesh grid is as shown in FIGURE 11. The calculated flow field is a large rectangular parallelepiped area, and all the boundaries are at least ten rotor radii away from the gravity center of the quadcopter in all directions. The front, upper, and side planes away from the quadcopter are set as the velocity inlet, the bottom plane is set as the pressure outlet, and the symmetry plane is set as the symmetry. Boundary layer grids are divided on the surfaces of the rotors and the fuselage.

The meshing motion grid is used to simulate the rotation of the rotor. In order to improve the simulation accuracy, the grid is condensed at the interface between the rotating area (white and green areas) and the non-rotating area (gray area), as shown in FIGURE 12. To ensure the calculation accuracy of CFD, the rotor profile is scanned using a three-dimensional scanner, as shown in FIGURE 13.

As shown in FIGURE 14, the aerodynamics on the rotor are unsteady and changing periodically. A time-average value is adopted to be the valid aerodynamics of the rotor in the following discussion. The outcome of the validation is shown in Table 7 and the current solution is shown to have good agreement with the experimental results.

The Q criterion [23] of the whole quadcopter wake at different speeds is shown in FIGURE 15 and is colored by pressure. Vortex rollup can be observed clearly on both sides of the rotor disk. The vortex on the advancing side is stronger than the retreating side. As shown in FIGURE 15, when

---

**TABLE 5. Time step verification.**

| Iteration step | $C_T$  | $C_Q$  |
|----------------|--------|--------|
| $0.5^\circ / \Delta t$ | 0.0291 | 0.0035 |
| $1^\circ / \Delta t$ | 0.0283 | 0.0034 |
| $2^\circ / \Delta t$ | 0.0253 | 0.0031 |

**TABLE 6. Combination conditions.**

| Case No | Conditions          | Number of cells |
|---------|---------------------|-----------------|
| 1       | Individual fuselage | 1.36M           |
| 2       | Individual front rotor | 4.57M       |
| 3       | Individual rear rotor | 4.57M       |
| 4       | Front rotor + Rear rotor + fuselage | 10.37M   |

**FIGURE 11. Mesh grid of computational field.**

**FIGURE 12. Rotating area (white and green areas) and the non-rotating area (gray area).**

**FIGURE 13. Profile of the original rotor and scanning rotor.**

**FIGURE 14. Individual rear rotor CFD results of condition e.**

**FIGURE 15. Q criterion of the whole quadcopter wake at different speeds colored by pressure.**
hovering, the trails of the four propellers are all downwash vertically, and there is little interference with each other. At 5 m/s, the trailing tip vortex of the front propeller acts on the fuselage and the rear propeller, which interferes greatly with the fuselage and the rear propeller. At 10 m/s, both the advancing and retreating tip vortices of the front propeller act on the rear propeller and the fuselage, and the interference to the rear propeller is serious. At 15-20 m/s, due to the increase in the angular speed of the propellers, the wake vortices of the front propeller will be washed down more seriously, and the wake vortices are far below the rear propeller, which has little interference with the rear propeller. The wake of the fuselage interferes with the rear propeller.

**B. FRONT ROTOR**

According to the CFD results, the aerodynamics of the front rotor with and without interactional interference is shown in FIGURE 16. The rotor shows little differences (≤3%) in the thrust coefficient, torque coefficient, hub force coefficient, and pitch moment coefficient. The differences of the front rotor are small that can be negligible with respect to the accuracy of the simulation. The front rotor aerodynamic models keep the same as the individual rotor models. The lateral force is much smaller, which is not discussed.

**C. FUSELAGE**

Aerodynamic interactions on the fuselage are shown in Fig. FIGURE 17.a∼f. It can be seen that when the speed is below 5 m/s, the aerodynamic load of the fuselage is very small both with and without aerodynamic interaction. It can be seen from Fig. 16-a∼b that the interference has a great influence on the lift of the fuselage, which decreases with the increase of the pitch angle. It can be seen from FIGURE 17.c∼d that the interference has little effect on the drag force of the fuselage and can be ignored. As shown in FIGURE 17.e∼f, although the effect on the pitch moment of the fuselage is large, the pitch moment produced by the fuselage is much smaller than the pitch moment at the center of gravity produced by the rotor’s thrust (about 15.6 × 0.31 ≈ 5 N.m) when modeling and trimming the whole quadcopter, it is negligible in this paper.

**D. REAR ROTOR**

The comparison of the rear rotor with and without interactional interference is shown in FIGURE 18.a∼e. The aerodynamic interaction to hub force, roll moment, and torque are less than 5% that can be ignored for the limited loss of model accuracy in this study. The hub force, roll moment, and torque model of the rear rotor still adopt the isolated rotor model.

![FIGURE 15. CFD results of the quadcopter at each trim state.](image_url)
The lateral force of the rotor is very small that hardly affects the dynamic model of the quadcopter; it is not reported. The interference to thrust and pitch moment is large enough to affect the model accuracy that has to take into consideration when modeling the quadcopter.

Through the above analysis, it can be seen that the interference mainly exists at the flight speed of 5-15 m/s. When hovering, these interferences are very small and can be ignored. After 15 m/s, as the flight speed increases, these interferences gradually decrease. And only the interference on the rear rotors’ thrust and pitch moment and lift of the fuselage are significant to affect the dynamic model.

**E. AERODYNAMIC INTERFERENCE MODELING**

The interference in the quadcopter is very complicated. There is mutual interference between the fuselage and the rotors and between the front and rear rotors. When the flight speed of the quadcopter is determined, the flight pitch angle and the rotation speed of the front and rear rotors can be determined. In a word, when the quadcopter is stable flying at a certain speed, the interference to the rear rotor and fuselage can be determined. In this study, the aerodynamic interactional interference model of the rear rotor and fuselage can be simply expressed as a function of flying speed (V) and can be written as:

\[
C_E^F = C_E^F(V) = C_W^W(V) - C_I^F(V) \quad (18)
\]

\[
m_E^F = m_E^F(V) = m_W^W(V) - m_I^F(V) \quad (19)
\]

\[
C_I^E = C_I^E(V) = C_I^I(V) - C_I^F(V) \quad (20)
\]

Suppose \( \Lambda \) represents any interference item in the above equations. In which, \( \Lambda^E \) indicates interactional interference model, \( \Lambda^W \) indicates the CFD result of the whole quadcopter, \( \Lambda^I \) represents the isolate CFD simulation results.

In order to study the relationship between the interference model and the state varies (front rotor’s average RPM (\( \Omega_1 \)), rear rotor’s average RPM (\( \Omega_2 \)) and the velocity component in the body system (\( u \) and \( w \)), more cases are simulated near the trim state. This paper draws on the idea of local linearization, the interference model near the trim state can be written as:

\[
\Lambda^E = \chi^1_{\Lambda} (\Omega_1 - \Omega_1^0) + \chi^2_{\Lambda} (\Omega_2 - \Omega_2^0) + \chi^w_{\Lambda} (w - w^0) + \chi^u_{\Lambda} (u - u^0) + \Lambda_0^E \quad (21)
\]

**FIGURE 16. Forces and moments of the front rotor with and without aerodynamic interaction (AeroInter).**
For example, the two more front rotor’s RPM($\Omega_1 \pm \Delta\Omega_0$) are simulated at each trim point, the parameters about $\Omega_1$ in the interference model can be expressed as:

$$\chi^{\Omega_1}_r = \frac{\Lambda^E (\Omega_1 + \Delta\Omega_0) - \Lambda^E (\Omega_1 - \Delta\Omega_0)}{2\Delta\Omega_0}$$  \hspace{1cm} (22)

The expressions of the other interference models are basically similar and not discussed here. It can be seen from the CFD results that in the hovering mode, the interference of the front and rear rotors is very small, and the aerodynamic force act on the fuselage is also very small. The aerodynamic interference model in hovering can be ignored. The results of those interference parameters in forward flying states are shown in Table 8.

It is important that when calculating parameters about $\Omega_2$, $w$, and $u$, the individual rear rotor and the fuselage with new state varies also need to be calculated. As shown in Table 8, these coefficients respectively represent the influence of the corresponding state quantity on the interference model. For example, the $\chi^{\Omega_1}_{C_T}$ and $\chi^{\Omega_1}_{m_y}$ represent the front rotor’s RPM

**FIGURE 17.** Forces and moments of the fuselage with and without aerodynamic interaction.
variation on the interference thrust and pitch moment models, separately. All the $\chi_{C_1}$ and $\chi_{m_1}$ are less than 0, which means that the increase of $\Omega_1$ will increase the interference to the rear rotor, and the interference is to reduce the thrust and nose-up moment generated by the rear rotor. $\chi_{C_1}$ and $\chi_{m_1}$ are greater than 0, so the increases of $\Omega_1$ and $\Omega_2$ both help increase the lift of the fuselage; the main reason is that the downwash airflow in the rotor disc induces the upwash airflow outside the rotor disc to act on the fuselage, which has an upward force on the fuselage. Especially when flying at low speeds and small angles, the upwash induced by the rotors can change the lift on the fuselage from negative to positive. The significance of other parameters is similar, so they are not present repeatedly.

**IV. MODEL VERIFICATION**

**A. OVERVIEW OF THE MODEL**

With the interference model given in the last chapter, the accurate nonlinear model of the quadcopter is acquired. First, forces and moments (FM) acting on the propellers and
TABLE 8. Parameters of interference models.

| Coefficients | Trim state | 5 m/s | 10 m/s | 15 m/s | 20 m/s |
|--------------|------------|-------|--------|--------|--------|
| \(\chi_{C_2}^D\) | \(-6.71E-06\) | \(-1.44E-05\) | \(-1.23E-05\) | \(-5.45E-06\) |
| \(\chi_{C_2}^\omega\) | \(1.26E-06\) | \(-5.40E-06\) | \(-5.11E-06\) | \(-2.56E-07\) |
| \(C_{\tau}^{E}_f\) | \(1.13E-04\) | \(-3.66E-04\) | \(-5.70E-04\) | \(-1.01E-04\) |
| \(C_{\tau}^{E}_i\) | \(-2.63E-05\) | \(-1.44E-04\) | \(-3.18E-04\) | \(-2.23E-05\) |
| \(C_{\tau}^{E}_{T_0}\) | \(-2.41E-03\) | \(-3.48E-03\) | \(-2.45E-03\) | \(-1.62E-03\) |

TABLE 9. Comparison with other dynamic models.

| Performance analysis | Dynamic analysis |
|----------------------|------------------|
| hover | forward | hover | forward |
| Hwang[18] | Y | Y | N | N |
| Waqas[24] | Y | N | Y | N |
| Gutenberg[25] | Y | N | N | N |
| Díaz[26] | Y | Y | N | N |
| Jérémy[27] | Y | Y | N | N |
| Present model | Y | Y | Y | Y |

the fuselage are modified by the interference models as (23).

\[ FM = FM_i^0 + FM_i^f \]
\[ FR = FM_R^0 + FM_R^f \] (23)

in which, superscript \(^0\) indicates FM without interference; superscript \(^f\) indicates FM of the interference model; subscript \(_i\) indicates the rotor model; subscript \(_f\) indicates the fuselage model.

Then the FM are transferred to the body system and summed to obtain the resultant external force received by the quadcopter. When solving the yaw torque of the quadcopter, the electromagnetic torque model of the motor should be used. Those subsystem models are brought in (1) to get the full envelope dynamic model. The abilities of the acquired model are compared with the existing dynamic models, as shown in Table 9.

This model can both describe the performance and dynamic characteristics in the hovering and forward state despite the large cost.

**B. EXPERIMENT SET-UP**

Commercial Pixhawk PX4 flight control unit is utilized to conduct flight experiments and record experimental data. Attitude mode in PX4 controller is set to do the experiments. The closed-loop system of the UAV is shown in FIGURE 19, where the controller is a cascade PID structure, including an attitude PID controller and an angular velocity PID controller. The Mixer Matrix adopts the default value in PX4, and the same values are used in the nonlinear model built in this study. The flight experiment was carried out in hovering, forward flight at the pitch angle of 10 degrees (flight speed is about 10.5 m/s), and forward flight at the pitch angle of 15 degrees (flight speed is about 17 m/s).

In the experimental test, the quadcopter is excited to perform variable frequency sinusoidal oscillations at three attitude angles, respectively. The input of the excitation signal is inserted before the controller in the closed-loop system, as shown in FIGURE 19. The related motion information parameters of the quadcopter and the output signal of the controller (that is, the input signal (roll control command \(\delta_{lat}\)), pitch control command \(\delta_{lon}\), yaw control command \(\delta_{ped}\), and collective control command \(\delta_{col}\) of the model) are recorded, and the recording frequency is 100 Hz. The velocities \((u, v, w)\) under the body system are difficult to measure directly by the sensor, which are very important for model verification. The Extended Kalman Filter algorithm in [28] is adopted to reconstruct the recorded data to obtain the velocities and to ensure that the data used for verification has dynamic consistency.

**C. VERIFICATION METHOD**

Theoretically, the verification of the open-loop model does not require a controller. Directly input the experimentally recorded open-loop control commands into the nonlinear model, and compare the model output with the actual flight test data to know the accuracy of the model. The quadcopter is a system with very poor stability. When the flight data is brought into the nonlinear model, some of the actual flight disturbance (gust, sensor zero drift, etc.) is not considered. If an open-loop system is used for verification that the disturbance will cause rapid divergence of the model output. If the closed-loop model is used for model verification, the existence of the controller can easily cause the output of the model to be highly consistent with the actual flight data. The result of the model output at this time is the output of the controller and the quadcopter model. The verification result cannot explain the accuracy of the model. Therefore, this paper compares the input and output of the open-loop.
model in the closed-loop verification with the flight experiments. Only when the input and output comparison results are matched with the experiment data can the model be accurate.

In the closed-loop system simulation verification, the expected attitude angles of actual flight are used as the input of the system. The velocities obtained by EKF are used as the state in the simulation of the nonlinear model, and the Euler angle ($\phi, \theta, \psi$) and Euler angular rate ($p, q, r$) output by the model are used as the feedback of the controller.

**D. RESULTS**

The comparison results are shown in FIGURE 20~FIGURE 22. The attitude angle recorded by the experiment is in good agreement with the attitude angle output by the simulation in both hovering and forward flight. The median
FIGURE 22. Verification results at forward flight at a pitch angle of 15 degrees. (The blue line represents the experiment flight data, and the red line represents the input and output of the nonlinear model).

FIGURE 23. Trimming results.

value of the control command and the angular rate recorded in the experiment are basically consistent with the output of the simulation. The input and output of the nonlinear model are basically the same as the experiments, indicating that the established model can well describe the characteristic of the quadcopter in the hovering and forward flying state.

V. DISCUSSION
In this section, we mainly discuss the influence and necessity of the rotor forward flight model and the interference model on the overall model accuracy. Three different precision models are used for comparing, 1) Consider the interference model and the forward-flying rotor model (A-Model), 2) do not consider the interference model but use the forward-flying rotor model (B-Model), and 3) Use only the forward-flying rotor model (C-Model). The comparison results are shown in Figure 23. We can see that the A-Model and B-Model have a better agreement with the experimental data, especially in the low-speed region. This indicates that the interference model is necessary for improving the accuracy of the model. The C-Model is the worst of the three, which shows that the forward-flying rotor model alone is not sufficient for accurately describing the quadcopter's behavior.
rotor model (B-Model); 3) Do not consider the interference model and only uses the hovering rotor model which only the thrust and the torque are considered (C-Model).

These three models with different accuracy are trimmed to solve the performance of the quadcopter. It can be seen in FIGURE 23-a that the rotor’s RPM trim result of A-model is basically the same as the actual flight, and the result of B-model slightly deviates; the maximum difference is about 6.3% at 10m/s. When the flight speed is greater than 10m/s, the result of model C is completely different from the actual flight. It can also be seen that during forward flight, the speed of the rear rotor is greater than that of the front rotor. This is mainly because, in the forward flight mode, all rotors will generate head-up moments. In order to balance the head-up moment, the thrust generated by the rear rotor is greater than that of the front rotor.

It can be seen from FIGURE 23-b~d that the trim states (command, power, and pitch angle) of the C-model are very different from those of the A-model and B-model. In general, when describing the characteristics of the forward flight of the quadcopter, it is very necessary to fully consider the aerodynamic forces of the rotor during the forward flight.

Although A-model and B-model have similar trim pitch angles and which both are basically consistent with the flight experiment data, in the trim of the rotor speed, power, and control commands, the deviation between A-model and B-model can be up to 25%. This indicates that mutual aerodynamic interaction must be considered when evaluating the performance of the quadcopter.

In order to analyze the impact of different precision models on the accuracy of the quadrotor UAV dynamics, the trim results of the A-model are brought into different models and linearized at different speeds to obtain the eigenvalue of the quadcopter system. The quadcopter system can be divided into two subsystems: pitch-altitude subsystem (PAS) and roll-yaw subsystem (RYS). The eigenvalue perturbation of the two subsystems are shown in FIGURE 24.

It can be seen from FIGURE 24-a that the eigenvalues of the C-model are far from the A/B model in PAS. Both A-model and B-model have four eigenvalues, including two stable eigenvalues and a pair of unstable eigenvalues. The difference is that the unstable eigenvalues of the A-model are conjugate at all flight speed; the eigenvalues of the B-model are conjugate only when the flight speed is less than 15 m/s. As the flight speed increasing, the conjugate eigenvalues move to the real axis. This is because the changes in the state variables will cause a greater change in aerodynamic interference models at high speed than that at low speed, which causes the interference model to have a greater impact on the eigenvalues of the system as the speed increases.

It can be seen from FIGURE 24-b that all the eigenvalues of the C-model in RYS are at the origin, which is still very different from the eigenvalues of the A/B-model, but it can be seen that the eigenvalues in the A/B-model are basically the same, indicating that the aerodynamic interference has little effect on the dynamics in RYS.

In general, for the eigenvalue distributions of A and B models are relatively close at low flight speeds, the influence of aerodynamic interference on the model can be ignored when designing the controller; but as the flight speed increasing, the aerodynamic interference needs to be considered. Meanwhile, C-model cannot be used to describe the dynamic characteristics of the forward flight mode at all.

VI. CONCLUSION

In this study, a nonlinear model of quadcopter suitable for both hovering and forward flight states is established by wind tunnel tests and CFD.

Through the analysis of the established model, it is found that the aerodynamic characteristics of the rotor during the forward flight are very different from hovering. Therefore, it is very necessary to grasp the aerodynamic characteristics of the rotor when establishing the forward flight model. And
the motor’s dynamic has a great effect on the yaw motion of the quadcopter.

During the flight, there is also mutual aerodynamic interaction between different components. It can be seen that the loss of rear propeller thrust can reach 11%, and the pitching moment can be reduced by 42.7% at 10 m/s. Therefore, the influence of aerodynamic interaction needs to be considered when analyzing the performance of the quadcopter.

By comparing and analyzing the influence of interference on the dynamic characteristics of the model, in the range of 0–15 m/s, the interference on the system dynamics is small, but at the range of 15–20 m/s, interference on the system dynamics characteristic are large. Therefore, when the speed is below 15 m/s, the design of the controller may not consider the dynamic changes caused by aerodynamic interaction, but when the speed is greater than 15 m/s, the interference model needs to be considered.

In the future, some related works on optimizing the configuration of the quadcopter to reduce aerodynamic interaction can be carried out. In addition, some research related to nonlinear control can also be done.

REFERENCES

[1] A. S. Saeed, A. B. Younse, C. Cai, and G. Cai, “A survey of hybrid unmanned aerial vehicles,” *Prog. Aerosp. Sci.*, vol. 98, pp. 91–105, Apr. 2018.

[2] E. T. Efza, M. M. Mowlee, J.abin, I. Khan, and M. R. Islam, “Modeling of a high-speed and cost-effective FPV quadcopter for surveillance,” presented at the 23rd Int. Conf. Comput. Inf. Technol. (ICCIT), 2020.

[3] R. Rajaowana and P. Smithmaitre, “Mathematical modeling and validation of the aerial robot control system with the pixhawk flight controller,” *Int. J. Mech. Eng. Robot. Res.*, vol. 9, no. 7, pp. 1065–1071, 2020.

[4] L. Zivan and M. B. Tischler, “Development of a full flight envelope helicopter simulation using system identification,” *J. Amer. Helicopter Soc.*, vol. 55, no. 2, pp. 22003, Apr. 2010.

[5] M. A. Lotufo, L. Colangelo, and C. Novara, “Control design for UAV quadrotors via embedded model control,” *IEEE Trans. Control Syst. Technol.*, vol. 28, no. 5, pp. 1741–1756, Sep. 2020.

[6] S. I. Abdelmaksoud, M. Mailah, and A. M. Abdallah, “Practical real-time implementation of a disturbance rejection control scheme for a twin-rotor helicopter system using intelligent active force control,” *IEEE Access*, vol. 9, pp. 4886–4901, 2021.

[7] L. Amezquita-Brooks, E. Liceaga-Castro, M. Gonzalez-Sanchez, O. Garcia-Salazar, and D. Martinez-Vazquez, “Towards a standard design model for quad-rotors: A review of current models, their accuracy and a novel simplified model,” *Prog. Aerosp. Sci.*, vol. 95, pp. 1–23, Nov. 2017.

[8] Z. Benic, P. Piljek, and D. Kotarski, “Mathematical modelling of unmanned aerial vehicles with four rotors,” *Interdiscipl. Description Complex Syst.*, vol. 14, no. 1, pp. 88–100, 2016.

[9] X. Zhang, X. Li, K. Wang, and Y. Lu, “A survey of modelling and identification of quadrotor robot,” *Abstract Appl. Anal.*, vol. 2014, pp. 1–16, Oct. 2014.

[10] A. Chovancová, T. Fico, Ľ. Chovanec, and P. Hubinsk, “Mathematical modelling and parameter identification of quadrotor (a survey),” *Proc. Eng.*, vol. 96, pp. 172–181, Jan. 2014.

[11] D. Mellinger, N. Michael, and V. Kumar, “Trajectory generation and control for precise aggressive maneuvers with quadrotors,” *Auto. Robots*, vol. 33, nos. 1–2, pp. 143–156, 2012.

[12] L. Amezquita-Brooks, D. Hernandez-Alcantara, C. Santana-Delgado, R. Covarrubias-Fabelo, O. Garcia-Salazar, and A. M. E. Ramirez-Mendoza, “Improved model for micro-UV propulsion systems: Characterization and applications,” *IEEE Trans. Aeros. Electron. Syst.*, vol. 56, no. 3, pp. 2174–2197, Jun. 2020.

[13] A. Kolaei, D. Barcelos, and G. Bramesfeld, “Experimental analysis of a small-scale rotor at various inflow angles,” *Int. J. Aeros. Eng.*, vol. 2018, pp. 1–14, Oct. 2018.
TAO SONG received the Ph.D. degree in aircraft design from Beijing Institute of Technology Beijing, China, in 2014. He is currently an Assistant Researcher with the School of Aerospace Engineering, Beijing Institute of Technology. His main research interests include aircraft system modeling and identification.

ZELIANG WU received the B.S. degree from Beijing Institute of Technology, Beijing, China, in 2018, where he is currently pursuing the Ph.D. degree with the Institute of UAV Autonomous Control. His main research interests include aerodynamic analysis, modeling, and flight control of unmanned aircraft vehicles.

PAN TANG received the B.S. degree from Beijing Institute of Technology, Beijing, China, in 2016, where he is currently pursuing the Ph.D. degree with the Institute of UAV Autonomous Control. His main research interests include robust and fault-tolerant control of unmanned aircraft vehicles.