The Glueball Spectrum

D.V. Bugg$^a$, M. Peardon$^b$, B.S. Zou$^{a,1}$

$^a$Queen Mary and Westfield College, London E14NS, UK
$^b$Dept. of Physics, UCSD, La Jolla, California 92093-0319, USA

Abstract

Mass ratios of glueballs predicted by the latest Lattice QCD calculations in the quenched approximation agree well with four prime experimental candidates.

$^1$ Now at the Institute for High Energy Physics, Beijing 100039, China
There is an uncertainty of $\pm 10\%$ in the overall mass scale of present Lattice QCD predictions for glueballs, arising from normalisation to the mass of the $\rho$ meson and other benchmarks. All simulations agree that the lowest $0^+$ glueball is to be expected in the mass range 1500-1750 MeV (1, 2, 3). This has led to suggestions by many authors that the $0^+$ glueball is related to $f_0(1500)$ (4, 5), or the nearby broad state $f_0(1530)$ (3) or $f_J(1710)$ (3). The lowest $2^+$ and $0^-$ glueballs are predicted to lie close together in the mass range 2000-2400 MeV. A new feature of the latest calculations (1) is that a second $0^+$ glueball is also predicted in the latter mass range.

Mass ratios in the pure $SU(3)$ Yang-Mills theory are predicted with greater accuracy. Predictions from Ref. (1) are shown in the second column of Table 1. They agree remarkably well with the ratios between $f_0(1500)$, $f_0(2105)$, $\eta(2190)$ and $f_2(1980)$ which have exotic features, making them natural candidates for glueballs (1). The lattice calculations of (1, 2, 3) neglect the dynamics of the quark degrees of freedom and the lightest glueballs are stable, physical states of the resulting purely gluonic theory. In QCD, the glueball states can both decay to light mesons and mix strongly with nearby $q\bar{q}$ resonances. The limitations of current simulations mean that little is known about the glueballs in lattice QCD including light quarks; the latest studies (8, 9) suggest a lower value for the scalar mass. The mixing and decay of glueballs has been addressed in the quenched approximation by the IBM group (10, 11); they conclude that the scalar glueball is rather narrow, with a total decay width to pseudoscalar pairs of $\approx 100$ MeV and mixes strongly with the $s\bar{s}$ scalar meson. We first review the candidates presented in Table 1, beginning with $0^+$.

| Ratio                  | Prediction | Experiment |
|------------------------|------------|------------|
| $M(2^{++})/M(0^{++})$  | 1.39(4)    | 1.32(3)    |
| $M(0^{--})/M(0^{++})$  | 1.50(4)    | 1.46(3)    |
| $M(0^{++})/M(0^{++})$  | 1.54(11)   | 1.40(2)    |
| $M(0^{--})/M(2^{++})$  | 1.081(12)  | 1.043(36)  |

Table 1
Glueball mass ratios predicted by Ref. (1), compared with experimental candidates described in the text; errors are in parentheses.

A recent analysis of extensive data on $\bar{p}p \rightarrow \pi\pi, \eta\eta$ and $\eta\eta'$ has located many resonances in the mass range 1900–2400 MeV (12). A straight-line trajectory of $0^+$ states against the square of the mass $M$ may be constructed, as shown in Fig. 1(a), with the same slope as is observed for $2^+$ states, Fig. 1(b) and $4^+$. (It is presently uncertain whether $f_0(980)$ is predominantly $q\bar{q}$ or a ‘molecule’, but it does lie on the straight-line trajectory.) The $f_0(1370)$, $f_0(1770)$, $f_0(2020)$ and $f_0(2320)$ all appear to decay dominantly into non-strange states. One expects $s\bar{s}$ states $\sim 250$ MeV above non-strange, and $f_J(1710)$, which decays strongly to $K\bar{K}$, is a candidate for one of them; there is as yet no evidence for heavier
of data on $\bar{p}p$ analysis of $\bar{p}N$ have strong
in 1996 as $\theta$ angle
$\bar{p}p$ in independent analyses of Crystal Barrel data on $\bar{p}p$
the Particle Data Group (PDG) (13) lists as established $I = 0$
$\bar{p}N$ cannot be excluded. Next, the E765 collaboration (15) observed
striking peaks in $\eta$ in $\bar{p}p \rightarrow (\eta\eta)\pi^0$ at 1500, 1748 $\pm$ 10
and 2104 MeV, but without a determination of $J^P$. A similar series of
peaks is visible in data from Mark III (16) and DM2 (17). A re-analysis of
these data finds all three peaks to have $J^P = 0^+$, with decays mostly
to $\sigma\sigma$ (18). Data from the Crystal Barrel experiment on $\bar{p}p \rightarrow (\eta\eta)\pi^0$
provide a good determination of the mass
and width: $M = 1770 \pm 12$ MeV, $\Gamma = 220 \pm 40$ MeV (13). These values leave
little doubt that $f_0(1770)$ is distinct from the $\Theta$, $f_J(1710)$. Decay modes
are also distinct: $\eta\eta$ and $\sigma\sigma$ for $f_0(1770)$, but $K\bar{K}$
for $f_0(1710)$.

The $f_0(2105)$ was first identified in $J/\Psi \rightarrow \gamma(4\pi)$ (18). It has recently been
confirmed as a strong signal in $\bar{p}p \rightarrow \eta\eta$ (12). There, it has been found
$\Gamma(\pi^0\pi^0)/\Gamma(\eta\eta) = 0.71 \pm 0.17$, which is much too low for a normal non-strange
$q\bar{q}$ state; for the latter, the predicted ratio is $(1/0.8)^4 = 2.45$, since the non-
strange component of the $\eta$ wave function has coefficient 0.8. If the $f_0(2105)$
is treated as a mixed state $\cos \theta(u\bar{u} + d\bar{d})/\sqrt{2} + \sin \theta s\bar{s}$, this requires
a mixing
angle $\theta = (65 \pm 6)^\circ$. Its strong production from $\bar{p}p$ but dominant $\eta\eta$ decay
suggests exotic character. It is also produced strongly in $\bar{p}p \rightarrow (\eta\eta)\pi^0$ (20).
The PDG lists it under $f_2(2150)$. Two further $0^+$ states have been reported:
$f_0(2020)$ (21; 12) and $f_0(2320)$ (12).

The $f_0(1500)$ is too close in mass to $f_0(1370)$ for both to be explicable as $q\bar{q}$
states. The existence of $f_0(1370)$ is therefore crucial. It has been questioned
by Minkowski and Ochs (22). It is elusive because it decays weakly to $2\pi$
$K\bar{K}$ and $\eta\eta$, but dominantly into $4\pi$ . A recent comparison of $\bar{p}p \rightarrow \eta(\pi^0\pi^0)$
and $\bar{p}p \rightarrow \eta(\pi^+\pi^-\pi^+\pi^-)$ (23) establishes that $\Gamma(4\pi)/\Gamma(2\pi) \geq 5$
$\leq 5$ by Thoma (24). It has been reported in several sets
of data on $\bar{p}p \rightarrow 5\pi$ (13). In the $2\pi$ channel, it has been observed in two
independent analyses of Crystal Barrel data on $\bar{p}p \rightarrow 3\pi^0$ and $\eta\eta\pi^0$ (4; 25)
and also in $\bar{p}p \rightarrow (K\bar{K})\pi^0$ (20). Its $\pi\pi$ width is sufficiently small that it cannot
be identified definitively in CERN-Munich data on $\pi^+\pi^- \rightarrow \pi^+\pi^-$ (27; 28);
there, its peak cross section $\propto \Gamma_{2\pi}^2/\Gamma_{total}^2$. Nonetheless, it seems to be visible at
large $|t|$ in GAMS data on $\pi^+\pi^- \rightarrow \pi^0\pi^0$ (29). It has been reported recently
in central production of $2\pi$ (30) and $4\pi$ (31).

We turn now to $J^{PC} = 0^{-+}$. A very broad signal with these quantum numbers
is observed in $J/\Psi \rightarrow \gamma(\rho\rho)$ \cite{16,17}. Data on $J/\Psi$ radiative decays to $\rho\rho$, $\omega\omega$, $K^*\bar{K}^*$ and $\phi\phi$ may be fitted with a single broad resonance of mass $2190 \pm 40$ MeV having coupling constants which are flavour-blind within experimental errors of $\sim \pm 30\%$, i.e. in the ratio $3 : 1 : 4 : 1$ \cite{32}. This is the classic glueball signature.

Lastly, we review $2^+$ states. In Ref. \cite{12}, four $f_2$ were identified at 1920, 2020, 2210 and 2300 MeV. Fig. 1(b) shows parallel straight-line trajectories attributed to $\bar{q}q$ $^3P_2$ states $f_2(1270)$, $f_2(1565)$, $f_2(1920)$ and $f_2(2210)$ and $^3F_2$ states $f_2(2020)$ and $f_2(2300)$. The difference in mass between $^3F_2$ and $^3P_2$ may be interpreted as arising from the centrifugal barrier between $q$ and $\bar{q}$, which pushes $^3F_2$ up in mass. The $^3F_2$ states are almost degenerate with $^3F_1$ and $^3F_3$ \cite{33}; this is as expected, since the $L = 3$ centrifugal barrier shields the short-range $L.S$ and tensor interactions.

In addition to these $f_2$’s, a broad $2^+$ state has been observed with a mass of 1930–2000 MeV. It appeared first in the data on central production of $4\pi$ \cite{34}. It is also observed in $pp \rightarrow (\eta\eta)\pi^0$ as a broad $\eta\eta$ signal with $M = 1980 \pm 50$ MeV \cite{20}. There, one sees by eye a non-isotropic component in the decay angular distribution across the whole mass range 1650–2200 MeV; such a broad state cannot be explained by $f_2(1920)$. A broad $2^+$ signal of similar mass and width is also reported in $J/\Psi \rightarrow \gamma(K^*\bar{K}^*)$ \cite{35} and in $J/\Psi \rightarrow \gamma(4\pi)$ \cite{36}. The latest data on Central Production of $4\pi$ final states shows a clear broad peak with very little background \cite{34}; it has a mass of $1980 \pm 22$ MeV, with $\Gamma = 520 \pm 50$ MeV. Recent WA102 Central Production data for $\omega\omega$ \cite{37} show it to be distinct from $f_2(1920)$ of Fig. 1(b); the latter has a conventional width of $\sim 200$ MeV. Close, Kirk and Schuler \cite{38} show that $\phi$ distributions in Central Production data are similar for $f_0(1500)$ and $f_2(1980)$ but quite different to those for $f_0(1370)$ and $f_2(1270)$. We interpret the $f_2(1980)$ as the $2^+$ glueball, probably mixed with nearby $q\bar{q}$ states.

We now consider quantitative evidence concerning branching fractions for production of glueballs in $J/\Psi \rightarrow \gamma(gg)$. Close, Farrar and Li \cite{39} make predictions for branching fractions for individual spin states depending on their masses and widths. Using experimental values for these, Table 2 makes a comparison of their predictions with experiment. Their prediction is that $q\bar{q}$ states will be produced less strongly than glueballs by a factor 5–10.

The branching fraction of $f_0(1500)$ observed in the $4\pi$ final state \cite{18} is $(5.7 \pm 0.8) \times 10^{-4}$. Our present best estimate of decay widths of $f_0(1500)$, slightly updated from those of Ref. \cite{28}, are $\Gamma_{2\pi} : \Gamma_{K\bar{K}} : \Gamma_{\eta\eta} : \Gamma_{4\pi} = 48 : 6 : 5 : 72$ MeV. With these values, the total branching fraction of $f_0(1500)$ becomes $(1.03 \pm 0.14) \times 10^{-4}$. The result is close to prediction for a glueball and far above that predicted for $q\bar{q}$, as pointed out in Ref. \cite{38}. The production of $\eta(2190)$ in $\rho\rho$, $\omega\omega$, $\phi\phi$, $K^*\bar{K}^*$, $\eta\pi\pi$ and $K\bar{K}\pi$ \cite{18,32} is slightly above the
prediction. The $f_0(2105)$ has so far been observed in $J/\Psi$ radiative decays only to $\sigma\sigma$ (18). For this mode alone, the observed production is only 20% of the prediction; decays to $\eta\eta$ and $\pi\pi$ will also contribute, but their decay branching ratios relative to $4\pi$ are not presently known.

The prediction for the branching fraction of the $2^+$ glueball is large if the width is taken to be the 500 MeV fitted to $f_2(1980)$. Observed decays to $\sigma\sigma$ and $f_2(1270)\sigma$ account for $(10 \pm 0.7 \pm 3.6) \times 10^{-4}$ of $J/\Psi$ radiative decays (36) and $K^*\bar{K}^*$ decays a further $(7 \pm 1 \pm 2) \times 10^{-4}$ (35). If one assumes flavour-blindness for vector-vector final states, the vector-vector contribution increases to $(16 \pm 2 \pm 4.5) \times 10^{-4}$. The total of $2.6 \times 10^{-3}$ is still a factor 9 less than predicted for a glueball. This is presently a major flaw in identifying $f_2(1980)$ with the $2^+$ glueball.

It is possible that there are many decay modes as yet unobserved for the heavy $f_0(2105)$ and $f_2(1980)$. The total prediction for glueball production in $J/\Psi$ radiative decays is 4.2%, compared with $\sim 6\%$ for all radiative decays. As yet, less than half of the products of $J/\Psi$ radiative decays have been assigned to specific $J^P$. Data on radiative decays to $\eta\eta$ and $\eta\eta\pi\pi$ would be particularly valuable.

| State      | Prediction | Observation          |
|------------|------------|----------------------|
| $f_0(1500)$| $1.2 \times 10^{-3}$ | $(1.03 \pm 0.14) \times 10^{-3}$ |
| $\eta(2190)$| $1.4 \times 10^{-2}$ | $(1.9 \pm 0.3) \times 10^{-2}$ |
| $f_0(2105)$| $3.4 \times 10^{-3}$ | $(6.8 \pm 1.8) \times 10^{-4}$ |
| $f_2(1980)$| $2.3 \times 10^{-2}$ | $(2.6 \pm 0.6) \times 10^{-3}$ |

Table 2
Branching fractions of glueball candidates in $J/\Psi$ radiative decays, compared with predictions of Close, Farrar and Li (39).

We now discuss decays and mixing with $q\bar{q}$. Glueballs undoubtedly mix with neighbouring $q\bar{q}$ states. The decay branching ratios of $f_0(1500)$ and $f_0(2105)$ are certainly not flavour-blind (5). A feature of $f_0(1500)$, $f_0(2105)$ and $f_2(1980)$ is that they appear strongly in decays to $\eta\eta$ and $\sigma\sigma$. The $\eta\eta$ decay is natural for a glueball, as pointed out by Gershtein et al. (40). Strohmeier-Prescicek et al. (41) emphasize that the strong $\sigma\sigma$ decay points towards a large glueball component in $f_0(1500)$; analysing observed decays, they deduce a 0.75 coefficient for the glueball component in the wave function. A major challenge now is to understand this mixing quantitatively. In Ref. (6) and further references cited there, the mixing of the $0^+$ glueball with $q\bar{q}$ states has been fitted to an extensive range of data. Their conclusion is that $\sim 33\%$ of the glueball goes into $f_0(1500)$ and $\sim 53\%$ into a broad $0^+$ background, fitted as $f_0(1530)$ with $\Gamma = 1120 \pm 280$ MeV.
It is to be expected that glueballs will mix preferentially with the ninth (predominantly singlet) members of $q\bar{q}$ nonets. We interpret the $\eta(1440)$ as the first radial excitation of $\eta'(958)$. Mixing between the broad $\eta(2190)$ and $\eta(1440)$ explains naturally the strong production of $\eta(1440)$ in $J/\Psi$ radiative decays and in $\bar{p}p$ annihilation (32). The level repulsion between $\eta(2190)$ and $\eta(1440)$ can explain the low mass of $\eta(1440)$ compared with $\eta(1295)$, $\pi(1300)$ and $K_0(1460)$. There is presently controversy whether $f_J(1710)$ has $J = 0$ or 2, although recent publications by Dunwoodie (42) and WA102 (43) favour spin zero. Its strong decay to $K\bar{K}$ suggests it is the ninth member of a nonet having $J = 0$ or 2. Mixing with the $0^+$ or $2^+$ glueball can explain its strong production in $J/\Psi$ radiative decays.

Etkin et al. have observed anomalously strong production of $\phi\phi$ states with $J^P = 2^+$ in the mass range 2000-2340 MeV (44). We conjecture that these are predominantly states of $s\bar{s}$ states expected in this mass range, but enhanced strongly in $\pi\pi \rightarrow \phi\phi$ by mixing with the $2^+$ glueball, which couples to both initial and final states. An $s\bar{s}^3P_2$ state is expected $\sim 250$ MeV above $f_2(1920)$ and could be responsible for the S-wave $\phi\phi$ peak at 2150 MeV in Ref. (44) and the peak observed at the same mass in $K\bar{K}$ (43). A second $s\bar{s}^3F_2$ state is expected as partner to the $f_2(2020)$ and could explain the D-wave peaks observed in $\phi\phi$ at 2300–2340 MeV.

Many puzzles and questions remain. Why are the $\eta(2190)$ and $f_2(1980)$ so broad? A partial answer is that the glueball may mix with and spread over many neighbouring $q\bar{q}$ states. Nonetheless, the small width of $f_0(1500)$ is not understood. Further progress on the mixing process requires an understanding the mixing process is required.

On the experimental side, it is vital to confirm (or contradict) $f_2(1980)$ as the $2^+$ glueball. The glueball will couple as the SU(3) singlet $u\bar{u} + d\bar{d} + s\bar{s}$. The small $s\bar{s}$ content is hard to distinguish in environments rich in $u\bar{u} + d\bar{d}$, e.g. $\bar{p}p$ annihilation or pion-induced reactions. Hence it is probably best studied in $J/\Psi$ radiative decays and central production. It is important to test whether its decays to $\rho\rho$, $\omega\omega$, $\phi\phi$ and $K^*\bar{K}^*$ are flavour-blind. Present $J/\Psi$ data are statistically weak and contain too much background at high masses to allow a study of $\rho\rho$, $\omega\omega$ and $\phi\phi$ channels. It is also important to test flavour-blindness for decays of $0^+$ glueball candidates to $\pi\pi$, $\eta\eta$ and $K\bar{K}$. Presently, there are almost no data on $J/\Psi \rightarrow \gamma(\eta\eta)$.

In summary, although many puzzles remain, $f_0(1500)$, $f_0(2105)$, $\eta(2190)$ and $f_2(1980)$ display exotic characteristics and do not appear to fit naturally as $q\bar{q}$ states. If they are identified as glueballs, or states strongly mixed with nearby glueballs, mass ratios agree well with the latest predictions of Lattice Gauge calculations of the glueball spectrum.
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Fig. 1. Trajectories v. $M^2$ for (a) 0$^+$, (b) 2$^+$ states; numerical values give masses in MeV.