Hitting a BEC with a comb: Evolution of interference patterns inside a magnetic trap

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Abstract. We study the evolution inside a harmonic trap of Bose-Einstein condensates released from the periodic potential of an optical lattice. After a time-of-flight, harmonic motion of the interference peaks is observed as well as a breathing motion in the direction perpendicular to the optical lattice. We interpret these results in terms of a simple physical model and discuss the possibility of more detailed studies of such a system.

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1. Introduction

Optical lattices have been used in a number of experiments on Bose-Einstein condensates (BECs) in recent years. The periodic potential provided by these lattices has been exploited to study, amongst other things, dynamical phenomena such as Bloch oscillations [1, 2, 3], the Bogoliubov dispersion relation [1, 4] and Josephson oscillations [5] as well as phase properties as manifested in number squeezing [6], the Mott insulator transition [7] and coherent state revivals [8]. In most of these experiments, the interference pattern from the condensates trapped at the potential minima of the lattice was studied by switching off the optical lattice and magnetic trap simultaneously.

In [10], we have already demonstrated that an experimental protocol in which only the magnetic trap is switched off and the condensate is then allowed to expand freely inside the (one-dimensional) optical lattice can be used to infer the chemical potential of the condensate in the combined harmonic plus periodic potential. In the present work, we use a variation on this theme: instead of switching off the magnetic trap and leaving the optical lattice on, we suddenly switch off the optical lattice and allow the condensate to evolve inside the magnetic trap for a variable time. Thereafter, the magnetic trap is also switched off and the condensate is observed after a time-of-flight.

By suddenly switching off the optical lattice, we instantaneously release the condensate from a strongly deformed potential created locally by the individual wells of the optical lattice. In this way, one expects to excite the full spectrum of possible shape oscillations along the lattice direction and, through the non-linear coupling between the degrees of freedom, also in the other directions.

After the conclusion of our experiments, a theoretical paper discussing a scheme similar to ours was brought to our attention [11], and more recently a generalization to three dimensions was explored in [12].

This paper is organized as follows. In section 2, we outline our setup and the experimental protocol used. Section 3 presents the experimental results, followed by a discussion (Section 4). Finally, we present our conclusions and an outlook on further experiments in section 5.

2. Experimental setup and protocol

In a triaxial time-orbiting potential (TOP) trap with magnetic trapping frequencies in the ratio \( \omega_x : \omega_y : \omega_z \) of 2 : 1 : \( \sqrt{2 - \epsilon} \) (\( \epsilon \) goes to zero for large trapping frequencies and is typically of the order of 0.2 in the present experiment; \( z \) is the downward direction), we create BECs of around \( 1.5 \times 10^4 \) rubidium atoms as described in detail in [13]. Once the BEC has formed, the TOP trap is adiabatically relaxed from a mean trap frequency \( \omega_0 \) of about 100 Hz to \( \approx 30 \) Hz. Two linearly polarized laser beams with parallel polarizations intersecting at a (full) angle of 38 degrees at the position of the BEC provide a sinusoidal optical lattice potential of the form \( V(z) = V_0 \sin^2(\pi z/d) \) with period \( d = 1.2 \mu m \). This
lattice is superimposed onto the harmonic potential of the magnetic trap. The detuning of the lattice beams from the atomic resonance is \( \approx 30 \text{ GHz} \) and the intensity around \( 10 \text{ mW cm}^{-2} \) (for a beam waist of 1.8 mm), leading to a lattice depth \( V_0 \) of about \( 15 E_{\text{rec}} \), measured in units of the lattice recoil energy \( E_{\text{rec}} = \frac{\hbar^2 \pi^2}{(2md)^2} \) (where \( m \) is the mass of the rubidium atoms).

In an experimental cycle, after creating the BEC the optical lattice is switched on with a linear ramp of duration \( \tau = 1 \text{ ms} \) and then left at the final lattice depth for 1 ms. Thereafter, it is suddenly switched off and the BEC is allowed to evolve inside the magnetic trap for a duration \( t \). The switch-on time is chosen such that, in the Bloch band picture \[3\], the BEC remains in the ground state of the band structure. With respect to the magnetic trap frequencies \( \omega_i \), however, the switching-on procedure is non-adiabatic as \( 1/\tau \gg \omega_i \). In this way, we ”hit” the BEC with the comb-like lattice potential. After the evolution time \( t \), the magnetic trap is also switched off and the expanded BEC is observed after a time-of-flight of 21 ms. The experimental protocol is illustrated in Fig. 1.

3. Experimental results

When a BEC is released from an optical lattice (in the absence of a magnetic trap), after a time-of-flight \( t_{\text{TOF}} \) the periodic spatial pattern of the condensate density produces an interference pattern that is composed of a central peak and side-peaks at positions \( z_{\text{side}} = \pm 2nv_{\text{rec}}t_{\text{TOF}} \), where \( v_{\text{rec}} = \frac{\hbar}{2md} \) is the lattice recoil velocity and \( n \) is an integer \[1, 2, 6\]. For modest lattice depths \( (< 20 E_{\text{rec}}) \) only the side-peaks corresponding to \( n = 1 \) will be visible as the higher-order peaks are suppressed by the finite width of the condensates expanding from the individual wells. The population of the side-peaks relative to that of the central peak can be used to infer the widths of the locally trapped condensates and hence the lattice depth \[3\].

For the present experimental investigation with release from the optical lattice into the magnetic trap, the periodic spatial pattern created by the optical lattice produces the interference pattern described above, but that pattern is modified by the magnetic trap evolution. Figure 2 shows the interference pattern (monitored after the time of flight and integrated along the direction perpendicular to the lattice direction) for various evolution times inside the magnetic trap after the lattice is switched off. We observe two side-peaks whose positions vary in time as the released condensate evolves inside the magnetic trap. This behaviour is also evident in Fig. 3 (a). Here, we have plotted the position of the two side-peaks \[14\] relative to the central peak (in order to account for residual sloshing of the condensate) as a function of \( t \). The unequal heights of the two side-peaks in Fig. 2 are due to sloshing (and hence a non-zero initial velocity relative to the lattice) of the condensate inside the magnetic trap before the lattice is switched on.

In Fig. 3 (b), the widths \( \rho_\parallel \) and \( \rho_\perp \) of the central peak along and perpendicular to the lattice direction, respectively, are shown as a function of time. One clearly sees
oscillations of both the radial and the longitudinal widths.

4. Discussion

An optical lattice creates a regular spatial pattern of fragmented condensates, each of them being trapped by the local minimum of the lattice potential. When the optical lattice is switched off, the matter waves emitted from each individual condensate interfere. In the far field, the emission from a one-dimensional array of coherent sources produces a regular diffraction pattern with a central maximum and symmetric, equally spaced side-peaks. The separation between the central peak and the side-peaks is determined by the periodic spatial pattern of the condensate. The periodic evolution inside the magnetic trap of the condensate spatial pattern initially created by the optical lattice produces the observed oscillation of the side peaks.

The theoretical analysis of ref. [11] provides a simple picture for the interpretation of oscillatory motion of the side-peaks. When the optical lattice is removed, the interference of the condensate spatial pattern with initial period \( d \) inside the magnetic trap produces a periodic pattern with a central peak located at \( x = 0 \) and two side peaks. This fragmented structure of the condensate experiences the confinement of the harmonic magnetic trap potential. As a consequence, the two side-peaks execute an oscillatory motion. The three-peak structure experimentally observed after the time of flight is produced by the three-peak fragmentation of the condensate taking place inside the magnetic trap when the optical lattice is removed.

The observed motion of the two side-peaks can be explained by considering independently the evolution inside the magnetic trap of the three interference peaks created after switching off the optical lattice [11]. Since the lattice transfers momentum to the condensate in units of \( 2p_{\text{rec}} = 2mv_{\text{rec}} \), the side-peaks initially move with velocities \( \pm 2v_{\text{rec}} \) at \( t = 0 \) and then individually perform a harmonic motion of the form

\[
z(t) = \pm \frac{h}{m\omega_z d} \sin(\omega_z t),
\]

from which their expected positions after a time-of-flight can be easily calculated. The intuitive picture of the side-peaks moving like individual BECs is only valid once the condensates at the lattice sites have expanded sufficiently to overlap with each other and hence produce the three-peak interference pattern. For our experimental parameters, this time is less than 0.1 ms. In Fig. 3 (a), we have fitted the sinusoidal function of Eq. 1 for the expected position to the experimental points. The fitted values for \( \omega_z \) and \( d \) agree to within the experimental error with values obtained independently from measurements of the dipole (sloshing) frequency of a single condensate and the lattice spacing quoted above, respectively. The oscillation amplitude \( h/(m\omega_z d) \approx 20 \mu m \) is about twice as large as the Thomas-Fermi radius of the condensate inside the magnetic trap for the parameters of the present experiment.

Apart from the appearance of interference peaks along the lattice directions, one also expects to see shape oscillations of the condensate in the radial directions, as during
Hitting a BEC with a comb

the lattice phase the condensate gets broken up into individual condensates localized at the lattice sites (for $V_0 = 15E_{\text{rec}}$, the width of a wave-packet inside a lattice well is $\approx 0.17d$). When the lattice is suddenly switched off, the effective trapping frequency in the lattice direction jumps from $\approx 2600$ Hz (the harmonic frequency at a lattice site) to $30$ Hz. The condensate will, therefore, start expanding along the lattice direction and subsequently perform breathing oscillations inside the magnetic trap. As the degrees of freedom of the condensate are coupled through the non-linear interaction term in the Gross-Pitaevskii equation, the condensate will start breathing in the radial directions as well.

In practice, we expect to find frequency components corresponding to the breathing motions in the three directions of the trap in the evolution of all three in-trap widths [15]. After releasing the condensate from the trap, the in-trap width of the central interference peak will still be directly reflected in the radial direction (after taking into account the expansion), whereas the width in the lattice direction is not so easy to interpret because of interference effects. Fig. 3 shows a fit to $\rho_\perp$ using a fitting function with three sinusoidal oscillations with frequencies in the ratio of the three harmonic frequencies of our TOP-trap and independent amplitudes. This fit gives a frequency of $59.5$ Hz for the dominant contribution, corresponding to $\approx 2 \times \omega_z/(2\pi)$. A simple sinusoidal fit to the $\rho_\perp$-data gives a similar result for the frequency, but fits the experimental points considerably less well. The radial widths of the two side-peaks oscillate with the same frequency as the central peak (and with the same phase).

For the longitudinal width $\rho_{||}$, we have used a simple sinusoidal fit, giving a frequency of $49$ Hz $\approx 1.6 \times \omega_z/(2\pi)$. As mentioned above, $\rho_{||}$ does not simply reflect the in-trap longitudinal width of the individual condensates but is determined by the interference between all of them.

5. Conclusion and outlook

In this work, we have investigated the evolution inside a magnetic trap of a condensate "hit" by a periodic potential. The condensate evolution inside the magnetic trap produces a three peak fragmented structure of the condensate. Those peaks perform an oscillating motion at the magnetic trap frequency along the lattice direction, as viewed after the time-of-flight. Apart from the appearance of side-peaks, one can also study the breathing motion of the central interference peak. In this sense, the present work ties in with the investigations of collective modes of BECs [17, 18]. So far, both experimental and theoretical results have been obtained for low-lying modes of a condensate in a magnetic trap, including breathing modes [19], surface modes [20] and the scissors mode [21]. As in the present work, in these experiments the collective modes were typically excited by a sudden change in the trap frequency or geometry, and the frequency and damping rate of the subsequent oscillations were measured either in situ or after a time-of-flight.

In ref. [1], the effect of interactions on the evolution of the various peaks moving
inside the trap was discussed. The mean field interaction should produce a deviation for the motion of the side-peaks from the simple oscillating motion at frequency $\omega_z$. It would be interesting to investigate the effect of the mean field on the widths of the peaks after a time-of-flight and to compare these results to the non-interacting case. Furthermore, it should be investigated whether the mean field interaction could produce soliton-like features with the spatial pattern of the condensate and preserving its shape.

In future experiments, our method could be used to study in more detail the frequency spectrum of the radial breathing oscillations, requiring data for several oscillation cycles to give a reasonable resolution, as well as possible damping mechanisms both for the radial oscillations and the oscillations of the side-peaks. In our experiments, for $t > 15\text{ ms}$, corresponding to the first ”collision” of the side-peaks inside the trap, we see collisional haloes which hint at a possible damping mechanism.

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**Figure 1.** Experimental protocol. After creating a BEC in the magnetic trap (a), the optical lattice is switched on (b) for 1 ms. When the lattice is switched off again (c), the different momentum classes created by the lattice evolve freely inside the harmonic potential. This evolution is then monitored by switching off the magnetic trap and observing the BEC after a time-of-flight (d).
Figure 2. Evolution of the interference pattern (integrated perpendicular to the lattice direction) of a condensate released from the magnetic trap without (a) and after applying an optical lattice ((b)-(f)) for 1 ms. The evolution times inside the trap for (b)-(f) are 0.1, 6, 8, 10 and 18 ms, respectively.
Figure 3. Condensate evolution inside the magnetic trap after suddenly switching off the optical lattice. The harmonic motion of the sidepeaks is visible in (a). The solid and dashed sinusoidal curves with the smaller amplitude represent the positions of the sidepeaks inside the trap as calculated from the parameters extracted from the fit to the experimental points after a time-of-flight. The horizontal dotted and dashed lines indicate the width of the condensate inside the trap and after a time-of-flight of 21 ms, respectively. In (b), the widths perpendicular to (filled symbols) and along (open symbols) the lattice direction are shown along with the value measured without the optical lattice (dashed horizontal line). The solid and dashed sinusoidal curves are sine-fits to the experimental points; in (b), the solid line is a fit using three frequencies (see text).