Enhanced Hawking radiation in an out-of-equilibrium quantum fluid

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Spontaneous emission by the Hawking effect may be observed in quantum fluids. However, its detection is usually rendered challenging by the weak signal strength and the short propagation length of correlated waves on either side of the horizon. In quantum fluids of polaritons, out-of-equilibrium physics affects the dispersion relation, and hence the emission and propagation of waves. Here we explore the influence of the fluid properties on the Hawking effect and find that it may be strongly enhanced by supporting the phase and density of the fluid upstream of the horizon in the bistable regime. This brings spontaneous emission in out-of-equilibrium systems within experimental reach.

I. INTRODUCTION

Quantum fluctuations at the event horizon of black holes cause the emission of correlated waves by the Hawking effect [1]: while some waves (Hawking radiation) propagate away from the horizon to outer space, others (the partner radiation) fall inside the horizon. Since signalling from inside the horizon is impossible, only Hawking radiation may be detected and correlations between paired waves cannot be measured in astrophysics. The Hawking effect may also be observed in the laboratory thanks to analogue gravity setups [2–3]. These are media whose properties may be engineered such that waves within propagate on effectively curved spacetimes [4–5], as has been experimentally demonstrated in a variety of platforms ranging from optical waves in fibres to capillary waves in water tanks and sound waves in quantum fluids [6–18]. For example, there is a horizon for sound waves in a one-dimensional transsonic fluid (a fluid whose flow velocity goes from being sub- to super-sonic) where the flow velocity of the fluid equals the speed of sound. The Hawking effect at the sonic horizon yields the emission of correlated waves just like in astrophysics [4–5], with the notable difference that observation on both sides of the horizon is possible.

Experimental evidence for correlated emission at horizons was recently reported in analogue gravity setups based on classical [12–19] and quantum fluids [20]. While the thermal fluctuations of classical fluids overpower quantum fluctuations at the horizon such that spontaneous emission cannot be observed there, this can be done with quantum fluids. Spontaneous emission would yield a non-separable state at the output [21–30], whose degree of entanglement could be quantified from the density and correlation spectra [31–32].

Although most work on spontaneous emission has been dedicated to atomic Bose-Einstein condensate (BEC) analogues [33–41], correlated emission with comparable properties may also be observed in quantum fluids of microcavity polaritons where a sonic horizon has already been experimentally realised in one- and two-dimensional flows [11–12]. In both quantum fluids, the Hawking effect manifests itself as the emission of collective Bogoliubov excitations that propagate in opposite directions on either side of the horizon. However, to date, theoretical works on polariton analogues have predicted a signal that appears hardly measurable because of its low strength (correlations a $10^{-5}$ fraction of the fluid density) [43–45] and short propagation length (about 12 µm from the horizon for the experimental configuration of [11]). These limitations stem from the combination of various factors that ultimately influence the hydrodynamics of the fluid.

The main difference between quantum fluids of atoms and of polaritons is that the latter are intrinsically out of equilibrium. Radiative and nonradiative dissipative processes in microcavities must be compensated for by optical pumping, and so the non-equilibrium state is not determined by thermodynamic equation conditions [46]. The radiative decay of polaritons does not solely render real-time diagnosis of the fluid properties possible (a notable experimental simplification compared with Bose-Einstein condensates of atoms), it also is at the origin of a unique phenomenology in the collective dynamics. Specifically, in the regime of nonlinear interactions of interest to horizon physics, a gap opens between the branches of the dispersion relation [47–48]. Here we show how this affects the strength of the Hawking effect and how, in turn, two-point correlations can be used as a diagnostic for out-of-equilibrium effects.

In this paper, we explore the parameter space of quantum fluids of polaritons and find a regime favourable to the formation of correlations by the Hawking effect. The hydrodynamics of the fluid are controlled by its density and phase, which are in turn connected with the optical bistability of the system (the hysteresis cycle of its polariton-density-to-optical-power relationship) [49], and so we investigate spontaneous emission from this perspective. We study the influence of the regime of density on either side of the horizon on the properties of emission. In doing so, we explain how effects of out-of-equilibrium in the configurations considered in [11–14, 45] limit the emission of Bogoliubov excitations, and we show how to engineer the fluid such that the strength and spatial extension of the correlation signal become amenable to exper-
imental detection. We find that setting the density and phase of the fluid as close as possible to the turning point of the bistability aids the emission of correlated Bogoliubov excitations and enhances their propagation length upstream of the horizon. Thus, we obtain an order of magnitude increase in both the correlation strength and length when the fluid is kept at the sonic point of the bistability upstream of the horizon, and left free to evolve downstream. Fine control upon the working point provides us with a better understanding of the influence of the properties of the quantum fluid of polaritons on the propagation of Bogoliubov excitations therein as well as on emission by the Hawking effect in systems out of equilibrium (see companion paper [50]). Our results open the way to the experimental observation of spontaneous emission from the vacuum in polaritonic systems.

II. SONIC HORIZON IN A POLARITON FLUID

Our study is based on the consideration of the so-called waterfall geometry of the density of the quantum fluid in the laboratory frame. This waterfall is illustrated in Fig. 1 in laboratory frame coordinates \( x \) and \( t \). This flow profile is realised in a device called a wire, an elongated microcavity in which the polariton dynamics are effectively one-dimensional. We pump the microcavity with a continuous wave, coherent pump laser incident at a given angle with respect to the normal to form a stationary flow along the wire. We structure the light field with a step-like intensity profile (black line in Fig. 1(c)). In the region where the pump lies, the density and phase properties of the polariton fluid are set by those of the pump, while in the region where the pump intensity is zero, polaritons propagate ballistically [51]. As in [11, 45], we consider a cavity with an attractive defect (a localised 1 \( \mu \)m long broadening of the wire) placed after the region where the pump lies. The defect at \( x = 0 \) will create a dip in the fluid density and a spike in the flow velocity (in red in Fig. 1(e)) because of approximate conservation of the flow current. We show the speed of collective (or Bogoliubov) excitations \( c_B = \sqrt{\hbar g n/m^*} \) (\( h \) the reduced Planck constant, \( g \) the effective nonlinearity, \( n \) the fluid density, \( m^* \) the effective mass of polaritons) in blue in Fig. 1(c): we see that the speed is almost flat before the defect and decreases afterwards. The polariton dynamics are driven-dissipative: upon de-excitation (after their lifetime \( 1/\gamma \)), polaritons release photons that leak out of the cavity, enabling the direct monitoring of the density and phase of the fluid.

A. Modes of the system

In Appendix A we review the theory for polariton hydrodynamics in a homogeneous system as described by a modified Gross-Pitaevskii equation. There we show how the polaritons behave as a fluid and describe the dispersive properties of Bogoliubov excitations therein. Here we review the dispersive properties of the fluid in the waterfall configuration and the kinematics of Bogoliubov excitations therein: We find the modes of the homogeneous medium and construct the “global modes” (GMs) of the inhomogeneous system (including the waterfall). These are the modes in which scattering occurs.

The waterfall separates two regions of fluid density \( n \) and phase \( \theta \). To simplify the discussion, we consider regions of homogeneous properties whose laboratory-frame dispersion is modelled by

\[
\omega_{\pm}(k) = \pm \sqrt{\left( \Delta_p - \frac{\hbar \delta k^2}{2m^*} - 3gn \right) \left( \Delta_p - \frac{\hbar \delta k^2}{2m^*} - gn \right) + \nu\delta k - i\gamma/2},
\]

with \( \Delta_p = \omega_p - \omega_0 - \frac{\hbar k_p^2}{2m^*} \), the effective detuning between the pump energy \( \hbar \omega_p \) and that of polaritons \( \hbar \omega_0 \), where \( k_p \) is the wave-number of the pump field. \( v = \frac{\hbar}{m} \partial_x \theta \) is the flow velocity of the polariton fluid.

The laboratory frame dispersion relation (1) depends on both the flow velocity and density of the fluid. In Appendix A we show that there is a hysteresis relationship between \( gn \) and the pump intensity. We will henceforth refer to this relationship as the ‘bistability loop’. In the special case where \( \Delta_p = gn \), the dispersion curve (in the rest frame of the fluid) has a linear slope at low \( k \): \[ \omega_{\pm} = gn \quad \rightarrow \quad c_s k \], with \( c_s = c_B \). At large \( k \), the dispersion is that of free massive particles, \[ \omega_{\pm} = gn \quad \rightarrow \quad \hbar k^2/2m \]. There, \[ |\partial \omega_{\pm}/\partial k| > c_s \] — the gradient of the dispersion curve is larger than the speed of sound, so the dispersion is said to be ‘superluminal’ (in analogy with superluminal corrections to the dispersion in eg [52, 53]). Because of the linear, sound-like, dispersion of excitations at short \( k \), the case \( \Delta_p = gn \) is referred to as the “sonic point” of the bistability loop [54]. Operation at \( \Delta_p \neq gn \) is also possible, in which case the linear behaviour at short \( k \) disappears and the dispersion is always quadratic.

As we will show in Section II B this bears consequences on the emission and propagation of Bogoliubov excitations in the fluid. For now, we consider that \( \Delta_p = gn \).

In the configuration of Fig. 1(a), the fluid flow is transsonic: it goes from being sub- to supersonic with a sonic horizon \((v = c_s)\) at \( x = -2\mu \)m [55]. The region where the flow is subsonic is upstream of, or outside, the horizon. The region where the flow is supersonic is downstream of, or inside, the horizon. We plot the dispersion curve in the laboratory frame (the real part of Eq. (1)) of the supersonic fluid flow in Fig. 1(d), and of the supersonic fluid flow in Fig. 1(f): blue (orange) curves correspond to \( \omega_+ \) (\( \omega_- \)) solutions of Eq. (1). Note that in the rest frame of the fluid, these modes have strictly positive (negative) energies. Modes of the field are normalised with respect to a (Klein-Gordon) scalar product that is not positive definite [56]: modes with positive (negative) energies in the rest frame of the fluid have positive (negative) norm. However, in the laboratory frame, the Doppler effect modifies the shape of the dispersion relation. For subsonic fluid flows, the negative norm branch (blue) is at negative laboratory frame energies. For supersonic flows, part
of the negative norm branch (orange) is pulled up to positive laboratory frame energies (up to a maximum energy which we denote by \( \omega_{\text{max}} \)) while part of the positive norm branch (blue) is pulled down to negative laboratory frame energies.

Now that we have described the dispersive properties of the transsonic fluid, we consider the kinematics of Bogoliubov excitations therein. Because of the time invariance of the system, these are plane wave modes. Eq. (1) is a fourth-order polynomial, so there are four (positive laboratory-frame frequency) solutions to the equations of motion in each spatial region on either side of the interface. These solutions are found at the intersection point of an \( \omega = c_{\text{st}} \) line with the dispersion branches at positive energies in Fig. 1(d), (e) and (f). Although these solutions share the same \( \omega \) (which manifests energy conservation in the laboratory frame), they have distinct \( k \), i.e. they are local modes of the homogeneous system. For \( \omega > 0 \) in the upstream region there are two propagating modes of positive norm and two modes of complex \( \omega \) and \( k \), which are exponentially growing and decaying modes. For \( \omega < \omega_{\text{max}} \) in the downstream region, there are four propagating modes, two of which have positive norm while the other two have negative norm. For \( \omega > \omega_{\text{max}} \), there are two propagating modes of positive norm and two exponentially growing and decaying modes.

Local modes in a homogeneous region may be sorted by their respective group velocity \( v_g = \partial \omega \pm \partial k \): those which have positive group velocity propagate rightwards while those which have negative group velocity propagate leftwards. We proceed to construct modes of the transsonic fluid, the global modes (GMs) \([56, 57]\) — solutions to the equation of motion that are valid in both regions on either sides of the interface. GMs correspond to waves scattering at the interface, and they describe the conversion of an incoming field to scattered fields in both regions. The GMs are superpositions of the plane wave solutions in the two homogeneous regions on either side of the interface. We identify GMs via their ‘defining’ local mode. Specifically, in the upstream region, the unique local mode with positive group velocity defines an in GM \( u_{\text{in}} \), while the unique local mode with negative group velocity defines an out GM \( u_{\text{out}} \). In the downstream region, modes with negative group velocity define in GMs \( d_{1\text{in}} \) and \( d_{2\text{out}} \) and modes with positive group velocity define out GMs \( d_{1\text{out}} \) and \( d_{2\text{out}} \). GMs \( u_{\text{in}}, u_{\text{out}}, d_{1\text{in}}, \) and \( d_{1\text{out}} \) are positive-norm modes while GMs \( d_{2\text{in}} \) and \( d_{2\text{out}} \) are negative-norm modes. Each in GM describes the scattering of a plane wave to various outgoing plane waves. Conversely, each out GM describes a single plane wave resulting from the scattering of various incoming waves. The scattering can be described in the in as well as the out basis, and the transformation between the two bases defines the scattering matrix (see \([56, 57, 59]\) for an analytical derivation of the scattering matrix). Because the vacuum is basis dependent, spontaneous emission at the horizon will occur in correlated pairs \( u_{\text{out}} - d_{1\text{out}}, u_{\text{out}} - d_{2\text{out}} \) and \( d_{1\text{out}} - d_{2\text{out}} \) on top of the classical background formed by the polariton fluid (the mean-field).

### B. Effects of out-of-equilibrium physics

So far we have discussed the dispersive properties of the fluid when \( \Delta_p = gn \), when operating at the sonic point of the bistability loop. In the regime \( \Delta_p < gn \), the microcavity acts as an optical limiter \([47]\): as can be seen on Fig. 1(a), the growth of the speed of excitations \( c_B \) with the pump strength is sub-linear. While the fluid is stable in this regime, a gap opens between the positive- and negative-norm branches of the dispersion curve, see Fig. 1(e). We mark the bottom of the \( \omega_- \) curve as \( \omega_{\text{min}} \).

This behaviour is markedly different from that observed in systems close to thermal equilibrium like quantum fluids of atoms. There, the oscillation frequency of the condensate wavefunction corresponds to the chemical potential. Instead here it corresponds to \( \omega_p \) — the opening of the gap illustrates how tuning \( \Delta_p \) gives access to a unique phenomenology of collective dynamics. Specifically, the linear behaviour at short
\( k \) disappears as soon as the gap opens and the dispersion is always quadratic (in this case \( c_\beta \neq c_\alpha \) as there is no meaning to \( c_\alpha \)) so excitations are massive and elastic scattering is possible, meaning that the polariton ensemble cannot be superfluid even though its flow velocity is subsonic. As we will see in Section III A, this departure from superfluid propagation modifies the density of the fluid in the region \( x < 0 \) as a function of the pump strength and profile.

In Section III A, we have seen that the Hawking effect consists in the mixing of in GMs of opposite sign of norm at the horizon, \( u_{\text{in}} \) from \( x < 0 \) and \( d_{\text{in}} \) from \( x > 0 \). The downstream modes only exist over the limited interval \( 0 < \omega < \omega_{\text{max}} \). When the gap between \( \omega_0 \) and \( \omega_+ \) opens, \( \omega_{\text{in}} \) only exists for \( \omega > \omega_{\text{min}} \). When the frequency interval for scattering is reduced to \( \omega_{\text{min}} < \omega < \omega_{\text{max}} \). The curvature of the \( \omega \) curve also affects the condition of momentum conservation between \( \omega \) and \( \omega_{\text{in}} \). We will show in the simulations (cf Section III B) that the efficiency of the Hawking effect is thus decreased.

In brief, when \( \Delta_\rho < g_n \), out-of-equilibrium physics manifests itself in the opening of a gap between the branches of the dispersion relation and a modification of the shape of the dispersion to a purely quadratic form. This affects the generation of Bogoliubov excitations as well as their propagation in the fluid.

### III. EMISSION BY THE HAWKING EFFECT

We now perform calculations with the cavity parameters (11): \( \hbar \gamma = 0.047 \text{ meV}, \hbar g = 0.005 \text{ meV m} \mu, \quad m^* = 3 \cdot 10^{-5} m_e \). Importantly, \( \omega_p - \omega_0 = 0.49 \text{ meV} \) was kept constant throughout.

We study spontaneous emission via non-local correlations in the fluid density [33], which we quantify with the normalised spatial correlation function

\[
G^{(2)}(x, x') = \frac{G^{(2)}(x, x')}{G^{(1)}(x)G^{(1)}(x')}.
\]

\( G^{(2)}(x, x') \) and \( G^{(1)}(x) \) are the diagonal four-points and two-points correlation function of the field, respectively (cf Appendix B).

In Fig. 2 we plot the operation point on the bistability loop of the fluid on either side of the horizon (solid line, upstream, dashed line, downstream), the pump profile (black line) and ensuing properties of the inhomogeneous fluid – characterised by its velocity (red line) and the speed of excitations \( c_\beta \) (blue line) – as well as the resulting density-density correlations (2). We are interested in the fluid properties and their influence on correlated emission, so although we plot \( c_\beta \), since this is proportional to square root of the fluid density we will refer to the density in the discussion.

#### A. Fluid configurations

We consider various flow profiles on either side of the horizon. On the one hand, the bistability of the fluid on either side of the horizon may be tuned by controlling the wave-number of the fluid in either region by means of the pump \( k_{p,u} \) or \( k_{p,d} \) in the up- or downstream region, respectively, see Appendix A. On the other hand, the fluid density may be supported on the higher branch of the bistability loop by means of the pump intensity. There are, roughly speaking, 6 different working points along the bistability loop, meaning that in order to explore the full parameter space we have computed 36 correlation spectra. Not all combinations are interesting, though, so in Fig. 2 we present four configurations that give typical behaviours: row (a), the fluid density is set near (but not at) the sonic point in both regions \( k_{p,u} = 0.25 \mu m^{-1}, \quad k_{p,d} = 0.55 \mu m^{-1} \), the pump strength ramps down towards the horizon; row (b) the fluid density is set away from the sonic point on the upper branch of the bistability loop in both regions \( k_{p,u} = 0.25 \mu m^{-1}, \quad k_{p,d} = 0.58 \mu m^{-1} \); row (c), the fluid density is set away from the sonic point on the upper branch of the bistability loop in the upstream region \( k_{p,u} = 0.25 \mu m^{-1} \) and left free to evolve in the downstream region; row (d), the fluid density is set near (but not at) the sonic point in the upstream region \( k_{p,u} = 0.25 \mu m^{-1}, \quad k_{p,d} = 0.5 \mu m^{-1} \), the pump strength is set abruptly to zero at \( x = -7 \mu m \) so that the fluid is left free to evolve from that point on (across the defect into the downstream region).

In all configurations, the fluid builds up in the region \(-10 \mu m < x < x_d \): a relatively small amplitude bump in the density forms before the defect. This is yet another indication that the hydrodynamics in the upstream region are not superfluid, even when forcing operation near (but not at) the sonic point. On the other hand, while the density of the fluid is mostly flat in the configuration of Fig. 2(a), in that of Fig. 2(b) its amplitude undulates widely over 100 \mu m downstream of the horizon before flattening down. This illustrates how attempting to force the fluid properties to a working point away from the sonic point after it has propagated across an obstacle destabilises it. Meanwhile, the fluid density smoothly decreases when there is no pump in the downstream region.

Given the variety of fluid properties and the possible fast variations within, the description of the system as two homogeneous media adopted in Section III and amenable to analytical solutions is not always valid. Instead we must calculate the bistability and dispersion at all points. To this end, we use the truncated Wigner approximation (see Appendix B) to evolve the wave function and obtain the properties of the fluid at all points in the cavity as well as the dynamics of the Bogoliubov excitations therein [50]. This numerical method was first used in the context of analogue gravity with atomic quantum fluids [33] and was adapted to polaritonic quantum fluids in [44]. Here, it enables the study of spontaneous emission on highly varying backgrounds. All maps result from 100 000 Monte-Carlo realisations.

#### B. Correlation spectra

In all configurations, correlations may be sorted by the spatial region in which the involved modes propagate, which correspond to four quadrants in the plots. The South East quad-
Figure 2. **Correlated emission at the horizon.** Left column, bistability loop: solid black, upstream region; dashed grey, downstream region. **Middle column,** solid black, pump strength upstream; dashed black, pump strength downstream; blue, speed of excitations; red, fluid flow velocity \( k_{p,u} \). **Right column,** spatial correlation function \( g^{(2)}(x, x') \) (Eq. (2)), colour scale from \(-10^{-3}\) to \(10^{-3}\).

\( x < 0, x' < 0 \) corresponds to correlations in the upstream region; the South Wast and North East quadrants correspond to correlations across the horizon in the up- and downstream regions; the North West quadrant corresponds to correlations in the downstream region. All configurations have some common traces, which are most visible in Fig. 2(d): (i) anti-correlations along the \( x = x' \) diagonal that indicate anti-bunching under repulsive polariton interactions; (ii) a negative moustache-shaped trace in the upstream-downstream region that indicates correlations across the horizon between Hawking radiation and its partner radiation \( u_{out} - d_{2out} \) modes; (iii) strictly horizontal \( (x' = 0) \) and strictly vertical \( (x = 0) \) fringes. While traces (i) and (ii) are generic features of the Hawking effect in dispersive quantum fluids, see eg [33, 41, 44, 45, 56, 61], the strictly horizontal and vertical correlation lines (iii) are novel features that indicate correlations between the propagating modes \( u_{out} \) and \( d_{2out} \) and a mode bound to the horizon. In the companion paper [50], we explain that these are quasi-normal modes of the effective spacetime whose emission stems from vacuum-driven perturbations.

Configuration 2(a) leads to a wider diagonal in the downstream region than in the upstream region, and a Hawking moustache of amplitude \( 6 \cdot 10^{-4} \) about 40 \( \mu m \) and 50 \( \mu m \)-
long in the up- and down regions, respectively. The dispersive features of the diagonal stick to it in both regions, indicating a local behaviour. Configuration \(\text{2 (b)}\) also leads to a wider diagonal in the downstream region than in the upstream region. The Hawking moustache is of amplitude \(2 \cdot 10^{-4}\) and about \(10 \mu\text{m-}20 \mu\text{m}\)-long in the up- and down regions, respectively. A new feature emerges in the downstream region: an anti-correlation trace that begins at \(20 \mu\text{m}\), which is followed by a positive, local-correlations trace that starts at \(50 \mu\text{m}\). These features follow the amplitude undulations of the fluid density. In configurations \(\text{2 (c) and (d)}\), the diagonal width is constant in the upstream region and broadens in the downstream region. The Hawking moustache is of amplitude \(1.3 \cdot 10^{-4}\) and is about \(20 \mu\text{m}\)-long in both regions in configuration \(\text{2 (c)}\), while it is of amplitude \(7.5 \cdot 10^{-4}\) and is about \(35 \mu\text{m-}105 \mu\text{m}\)-long in the up- and down regions, respectively, in configuration \(\text{2 (d)}\).

In Refs \([11, 45]\) the spatial mode of the pump was Gaussian and an effective cavity formed between the edge of the pump and the attractive defect. Emission into \(u_{\text{out}}\) would form a standing wave in this cavity, thus effectively modulating emission at the horizon and reducing emission into \(d_{\text{in}}\) and hiding it. The physics at play here is different. In all four configurations, the set of fringes that surround the diagonal in the downstream region and the Hawking moustache hides correlated emission with the witness mode \(d_{\text{in}}\) which is weak. In configuration \(\text{2 (b)}\), the fringes that surround the diagonal are due to spontaneous emission on the spatially varying background, an effect that goes beyond this work. Besides this specific case, the fringes are due to the modulation of the Hawking effect by the quasi-normal modes of the effective spacetime \([50]\). Although the pump profile in \([44]\) (where correlated emission into \(d_{\text{in}}\) was visible) was flat as well, neither the effective cavity nor the modulation by quasi-normal modes was observed because the defect was repulsive (see supplemental information in \([50]\)).

C. Spontaneous emission and propagation of Bogoliubov excitations

In Fig. \(\text{2}\) we observe the influence of the regime of density of the fluid properties (working point on the bistability loop on either side of the horizon) on spontaneous emission at the horizon and propagation in either region thereafter. Emission occurs in all configurations. As the pump extends all the way to the defect in configurations \(\text{a, (b) and (c)}\), the dispersion is gaped in the upstream region. On the other hand, the pump stops before the defect in configuration \(\text{(d)}\) so the fluid propagates ballistically to the defect and across it and the dispersion is sound-like at short \(k\) in this case \([51,54]\). Likewise, the fluid is pumped in the downstream region in configurations \(\text{a and (b)}\) so the dispersion is gaped there as well, while it is sound-like there in configurations \(\text{c and (d)}\). Because the dispersion is sound-like in the upstream region in configuration \(\text{a}\), the frequency interval for emission at the horizon is wider in that configuration than in any other. This is why the amplitude of correlations in that configuration is the highest.

In configuration \(\text{a and (b)}\), the spatial extension of the Hawking moustache in the downstream region is short, meaning that the propagation of Bogoliubov excitations is limited there. Meanwhile, the spatial extension of the Hawking moustache on either side of the horizon is short when the fluid is pumped away from the sonic point and attains unprecedented lengths when pumping close to the sonic point.

In brief, We have established that operating such that the fluid is at the sonic point of the bistability loop upstream of the horizon and letting the fluid propagate ballistically downstream enhances the emission and propagation of Bogoliubov excitations. In that regard, operating with a flat pump profile whose spatial extension is well controlled is better than with a Gaussian profile.

D. Influence of the fluid properties near the horizon on spontaneous emission

Now that we have established the crucial role of the control of the fluid density in the vicinity of the horizon in the emission and propagation of Bogoliubov excitations, we further investigate the configuration of Fig. \(\text{2 (d)}\): the pump strength drops abruptly at a certain distance from the defect and the fluid accumulates in the region unpumped region thus created and propagates ballistically across the defect into the downstream region.

![Figure 3. Density of Bogoliubov excitations in the vicinity of the defect for various pump-horizon distances. \(k_{p,u} = 0.24 \mu\text{m}^{-1}\).](image)

The fluid density is supported in the upstream region such that it dips slightly where the pump strength drops, and then bumps back up. The shape of the peak in density of Bogoliubov excitations follows the shape of the fluid bump. In Fig. \(\text{3}\), we compute the density of Bogoliubov excitations on top of the fluid.

\[
\left\langle \delta \hat{\Psi}^\dagger(x) \delta \hat{\Psi}(x) \right\rangle = \left\langle \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \right\rangle - |\Psi(x)|^2 , \quad (3)
\]

for different distances between the pump and the horizon \(x_d - x_{\text{edge}}\), while keeping the pump strength constant. Because the amount of polaritons that ballistically flow into the
excitations are generated. So the Hawking effect is less efficient and less Bogoliubov heights. In other words, the horizon becomes shallower and speed of sound at the horizon decreases for increasing bump on top of the bump. As we see in Fig. 4, the slope of the x

We have verified that the shape and spatial extension of the up- and downstream increases from less than three-fold for $x_d - |x_{edge}| = 10 \mu m$ and drops afterwards. The ratio between the density of excitations density of excitations remains roughly constant in the downstream region because the dispersion is always sound-like there. On the one hand, the change in $M$ at the bump affects the generation of Bogoliubov excitations: we observe that, in the upstream region, the density of Bogoliubov excitations increases until $x_d - |x_{edge}| = 10 \mu m$ and drops afterwards. We have verified that the shape and spatial extension of the Hawking moustache is not significantly modified by changing $x_d - |x_{edge}|$.

We now we compute Eq. (3) as we vary the fluid density on top of the bump. As we see in Fig. 4 the slope of the speed of sound at the horizon decreases for increasing bump heights. In other words, the horizon becomes shallower and so the Hawking effect is less efficient and less Bogoliubov excitations are generated.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Density of Bogoliubov excitations in the vicinity of the defect for varying horizon steepness. $k_{p,u} = 0.24 \mu m^{-1}$.}
\end{figure}

In conclusion, Bogoliubov excitations are efficiently emitted in the upstream region as long as the fluid density is supported on the upper branch of the bistability loop in the vicinity of the horizon. We have shown that it is not necessary to operate at the sonic point of the bistability loop, although this does provide the largest enhancement of the Hawking effect.

IV. DISCUSSION

We showed how engineering the density of a quantum fluid of polaritons can enhance the emission and propagation of paired Bogoliubov excitations in a transsonic flow. Our work sheds light on the interplay between optical bistability and parametric amplification in fluids of light. The bistable behaviour of a system can thus be exploited to study field theoretic effects like the Hawking effect in the laboratory. Specifically, we have found that fine control over the fluid properties may be achieved with a step-like pump profile.

Optical bistability is ubiquitous in nonlinear optics and has been observed early on in semiconductor microcavities [49], where the density of the polariton fluid may be supported on the upper branch of the bistability loop. When this is done, it is possible to generate and thereafter enhance the (controlled) propagation of topological excitations of the quantum fluid [62-64]. Here we observed the generation and propagation of paired Bogoliubov excitations of the quantum fluid on either side of a sonic horizon when supporting the density of the fluid at various points in the bistable regime. Support of an inhomogeneous fluid density and velocity may be achieved by changing the wave-number of the pump. In an experiment, this is easily implemented with high spatial resolution (limited by diffraction) thanks to spatial light modulators [65]. We found that letting the fluid flow ballistically across a repulsive defect so as to form a horizon yields Hawking correlations of the order of $10^{-3}$ fraction of the fluid mean density over more than $100 \mu m$. These are a tenfold and a four- to tenfold enhancement, respectively, compared to previous results and render the observation of the Hawking effect realistic. Furthermore, we showed how moving away from this optimal configuration reduces the signal strength and length — two effects which we showed are directly linked to the kinematics of Bogoliubov excitations in the out-of-equilibrium fluid. In this way, our work demonstrates that the correlation traces are a diagnostic for the influence of out-of-equilibrium physics on mode conversion in inhomogeneous flows. Specifically, we have observed novel local correlation traces that stem from a dissipative quench of a mode bound to the horizon that couples to propagating modes, i.e. quasi-normal modes characteristic of relaxation of the effective spacetime formed by the fluid geometry. In the companion paper [50] we study the quantum-fluctuation-driven spacetime ring-down and its influence on Hawking radiation. Finally, our methods open the way to the theoretical and experimental study of the quantum statistics of the Hawking effect in driven-dissipative systems: for example, one could calculate (and observe) the Hawking correlations in reciprocal space [40], thus gaining frequency-resolved information on them [41] which could in turn be used to measure the entanglement content of the correlations [31].

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In the case where the pump’s intensity is zero in any spa-

tial region, the fluid will propagate ballistically there, as in eg

[11] [44] [45]. In this case, the dispersion in the unpumped

region will be 

\[ \omega_{p}^{2} = \pm \sqrt{\frac{\hbar (k - k_0)^2}{2m} + \frac{2\gamma n_0}} + \frac{\nu (k - k_0) - \gamma r}{2}, \]

where \( k_0 = m v / \hbar \) denotes the wave-number of the ballisitic fluid. It is the same sonic Bogoliubov dispersion relation as for \( \Delta p = n_0 \).

We remark that the ‘speed of sound’ is an ill-defined concept

in such highly dispersive media as our quantum fluid of polaritons.

However, it is generally accepted that the ‘local speed of sound’ \( c_s \) is given by the gradient of the tangent of the dispersion relation at \( \omega = 0 \) in the frame co-moving with the fluid. A sub-(super-)sonic flow is thus a flow for which \( \nu \) is

lower (larger) than this gradient. In that sense, it is possible to

define a horizon and to consider the region downstream from

it as its inner region although there may be waves that propa-

gate in both directions therein. There exist stricter definitions of

the curvature of the effective spacetime in analogue gravity, see eg [31], but the related considerations do not impact the conclusions we draw in the present work. Strictly speaking, the in-
terface at \( x = -\mu \) is a sonic horizon only at frequencies for

which there are two propagating local modes in the upstream

region and four propagating local modes (including negative-

norm modes) in the downstream region [36]; that is for \n
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two homogenous regions, although we shall eventually depart from this simplified picture. For now, we begin with the theoretical description of a homogeneous quantum fluid (of its phase and density) and of the propagation of quantum (i.e. small-amplitude density) fluctuations therein.

1. Polariton fluid and Bogoliubov excitations

Exciton-polaritons are quasi-particles resulting from the interaction of light with matter in a semiconductor microcavity. Photons emitted by a laser will be trapped in a cavity formed by two Bragg mirrors, wherein their dispersion is the usual Fabry-Perot dispersion. These trapped photons create excitons — bound electron-hole pairs — in the semiconductor microcavity. Strong coupling between the photons and excitons trapped in quantum wells gives rise to two eigenstates for the total Hamiltonian, known as the lower polariton (LP) and upper polariton (UP) branches. Furthermore, the Coulomb interaction between excitons results in an effective non-linearity for exciton-polaritons (polaritons). The dynamics of the mean-field are governed by a generalised Gross-Pitaevskii equation, which leads to Euler and continuity equations describing the system as a quantum fluid. Historically, polaritons have first been described as two-dimensional quasi-particles, although the theory may be reduced to one-dimensional cavities called ‘wires’, as in the present case.

In the majority of cases, all energies involved are small compared to the Rabi splitting so the exciton-polariton system can be described by the mean field approximation. At this level the system is described by a single scalar field, which can be described by the mean field approximation. The Coulomb interaction between excitons results in an effective non-linearity for exciton-polaritons (polaritons). The dynamics of the mean-field are governed by a generalised Gross-Pitaevskii equation, which leads to Euler and continuity equations describing the system as a quantum fluid. Historically, polaritons have first been described as two-dimensional quasi-particles, although the theory may be reduced to one-dimensional cavities called ‘wires’, as in the present case.

The first equation of (A3) is the Euler equation of atomic Bose-Einstein condensates (BECs) plus a term coming from the coherent pumping. The second equation of (A3) is the continuity of the fluid with a loss term and a term coming from the coherent pumping. We see that the properties of the fluid depend on two parameters, namely its density $n$ and phase $\theta$. The spatial variations of the phase are encapsulated in $v$, which we identify from (A3) as the flow velocity of the fluid.

Now that we have described the polariton fluid in terms of its density and phase, we consider the propagation of small amplitude fluctuations (such as quantum fluctuations) of the density in this fluid — the so-called ‘Bogoliubov excitations’. Bogoliubov excitations are mathematically obtained by linearising the GPE (A1) around a background: $\Psi \rightarrow \Psi + \delta \Psi$, and $\Psi \rightarrow \Psi^* + \delta \Psi^*$. $L$ is the ‘Bogoliubov matrix’ that describes the dynamics of the Bogoliubov excitations $(\delta \Psi, \delta \Psi^*)$: $i\hbar \partial_t \left( \begin{array}{c} \delta \Psi \\ \delta \Psi^* \end{array} \right) = L \left( \begin{array}{c} \delta \Psi \\ \delta \Psi^* \end{array} \right)$.

In the steady state, the GPE (A1) becomes

$$\omega_0 - \omega_p - \frac{\hbar}{2m^*} \partial_x^2 + V(x) + g|\Psi(x)|^2 \approx \frac{i\gamma}{2} \Psi(x) + F_p(x),$$

(A4)

where $\omega_p$ is the frequency of the pump. We first consider a configuration where the wire is pumped with a spatially homogeneous and monochromatic pump of incident wavevector $k_p$ (so there is no potential in Eq. (A1), $V(x) = 0$). The phase of the fluid is then set by, and equal to, $k_p x$, while its density is homogeneous. The steady-state GPE (A4) simplifies to

$$g|\Psi|^2 - \Delta_p - \frac{i\gamma}{2} \approx \frac{\hbar}{2m^*} + F_p = 0,$$

(A5)

where $\Delta_p$ is the ‘effective detuning’ defined as the difference between the pump energy and that of lower polaritons,

$$\Delta_p = \omega_p - \omega_0 - \frac{\hbar k_p^2}{2m^*}.$$  

(A6)

We go to the reference frame co-moving with the fluid via a Galilean transform $(x \rightarrow x - vt)$. In the special case of a homogeneous system where the interaction energy matches the detuning, $\gamma = \Delta_p$, the Bogoliubov matrix $L$ can be written in this frame as

$$L = \begin{pmatrix} gn + \frac{\hbar k_p^2}{2m^*} + i\gamma/2 & gn e^{-2i k_p x} \\ gn e^{2i k_p x} & gn + \frac{\hbar k_p^2}{2m^*} - i\gamma/2 \end{pmatrix}.$$  

(A7)

Upon diagonalization, we retrieve the Bogoliubov dispersion relation in this co-moving frame, which relates the wavenumber $k$ of Bogoliubov excitations to their frequency $\omega$ there:

$$\omega_{\pm} = gn \pm \sqrt{\frac{\hbar k_p^2}{2m^*} + 2gn} - i\gamma/2.$$  

(A8)
Figure 5. Dispersion curve of the polaritonic fluid in the rest frame of the fluid. Real part of the dispersion (A8) for a pump vector \( k_p \) = 0.25 \( \mu \text{m}^{-1} \). Blue, \( \omega_{\Delta_p = gn} \), positive-norm modes; orange, \( \omega_{\Delta_p = gn} \), negative norm modes. Black dashed lines show the speed of sound.

Figure 6. Bistability loop for an homogeneous polaritonic fluid. \( \omega_p - \omega_0 > 0 \) and \( k_p = 0 \). Black, stable points; dashed, unstable points. The system is bistable for \( F_2 < |F_p| < F_1 \) and follows the hysteresis cycle (1)-(4).

2. Optical bistability of the polariton fluid

Unlike the configuration considered in the previous paragraph, in the majority of cases the interaction energy does not match the effective detuning and the dispersion curve is thus modified. Furthermore, writing the density of the fluid as a function of the intensity of the laser yields several solutions. This degeneracy of fluid densities is due to optical bistability, which, as we will show, has tremendous influence on the physics at play may be investigated equivalently in the frame of the fluid. We begin by describing the relationship between the density of polaritons, \( n \), and the intensity of the pump laser, \( |F_p|^2 \) in the case where the energy of the laser is above that of the lower polaritons, \( \Delta_p > \gamma \sqrt{3}/2 \): we square Eq. (A5) and find

\[
(gn - \Delta_p)^2 + \frac{\gamma^2}{4} n = |F_p|^2 \quad (A9)
\]

or, equivalently,

\[
\left( \frac{m^* e^2}{\hbar} - \Delta_p \right)^2 + \frac{\gamma^2}{4} \frac{m^* e^2}{\hbar} g n = |F_p|^2. \quad (A10)
\]

The physics at play may be investigated equivalently in terms of the relationship between the speed of sound and the strength of the pump, as shown in Figure 6. At first, \( c_s \) increases slowly with \( |F_p| \) (arrow (1)), until \( |F_p| = F_1 \) where it increases abruptly (arrow (2)). For \( |F_p| > F_1 \), \( c_s \) increases slowly again. If the pump’s strength is decreased from \( |F_p| > F_1 \), \( c_s \) decreases slowly until \( |F_p| = F_2 \) (arrow (3)), where it falls abruptly (arrow (4)). Since \( F_1 > F_2 \), the \( c_s \) to \( |F_p| \) relationship presents a hysteresis cycle with two regimes of speed of sound: the linear regime when \( |F_p| < F_1 \) and \( c_s \) is low, and the non-linear regime when \( |F_p| > F_2 \) and \( c_s \) is high. This hysteresis cycle is the manifestation of optical bistability [49], so we will henceforth refer to it as the ‘bistability loop’. Note that the dashed line in Fig. 6 is unstable and the speed of sound will actually follow the hysteresis cycle schematised by arrows (1) - (4).

Now, in order to explicitly show the dependence of the Bogoliubov dispersion on the density of the fluid as well as the influence of optical bistability thereon, we generalise Eq. (A8): We diagonalise the Bogoliubov matrix \( L \) for a homogeneous system pumped with arbitrary strength and obtain

\[
\omega_{\pm}(k) = \pm \sqrt{ \left( \frac{\hbar k^2}{2m^*} + 2gn - \Delta_p \right)^2 - (gn)^2 - i\gamma/2}. \quad (A11)
\]

Eq. (1) is the Doppler shifted version of Eq. (A11). In Fig. 7 we show the dispersion curve for 5 different fluid densities along the bistability loop. As can be seen in Fig. 7(a) and (b), the shape of the dispersion does not change much in the linear regime; the two branches of the dispersion curve cross. When the fluid is bistable, in Fig. 7(b), we observe the appearance plateau characteristic of an unstable fluid at the crossing points. On the other hand, the shape of the dispersion curve changes significantly in the nonlinear regime depending on the position along the bistability loop: at high pump strength (Fig. 7(e)), the two branches are split in energy by a gap that increases with the pump strength. The sonic dispersion relation (A8) is recovered at point \( C \) (Fig. 7(c)), while for slightly lower pump strength (Fig. 7(d)), the plateau at low \( k \) is characteristic of an unstable fluid (similarly to Fig. 7(d)). Note that the dispersion curve has a linear slope at low \( k \) (and thus a sonic interpretation) at point \( C \) only, which is thus sometimes referred to as the ‘sonic point’ of the bistability. As Eq. (A11) is of order four in \( k \), the dispersion has four complex roots. The real part of these roots is non-zero in the linear regime (Fig. 7(a) and (b) as well as at points \( C \) and \( C' \) (Fig. 7(c) and (d)), but not at point \( D \) (Fig. 7(e)).
In this appendix, we have seen that the mean-field of a polariton system behaves as a fluid. We have studied the dispersion relation of Bogoliubov excitations in this fluid and seen that optical bistability of the fluid strongly influences the properties of this dispersion relation. These considerations may be generalised to a fluid whose density is not homogeneous.

Appendix B: Numerical method and correlation function

In this appendix we present the numerical method used to compute the correlation maps of the main text.

Our interest is in spontaneous emission, that is amplification of the quantum vacuum fluctuations at the horizon (the Hawking effect). The quantum description of spontaneous emission at the horizon is given by the normalised spatial correlation function

$$\langle \langle \rho \rangle \rangle = \langle \langle \rho(x) \rho(x') \rangle \rangle$$

where \( \langle \rangle \) denotes the statistical averaging over the realisations obtained with the TW A: the general rule for \( N \) arbitrary observables is

$$\langle O_1...O_N \rangle_W = \frac{1}{N!} \sum_{\text{permutations}} \langle P(\hat{O}_1, ..., \hat{O}_N) \rangle,$$  \hspace{1cm} (B2)

where \( \langle \rangle_W \) denotes the statistical averaging over the realisations. Note that the TWA is valid at the level of the Bogoliubov theory only.

Emission at the horizon by the Hawking effect is best detected via nonlocal correlations in the fluid density [33]. These may be quantified via the normalised spatial correlation function

$$g^{(2)}(x, x') = \frac{G^{(2)}(x, x')}{G^{(1)}(x) G^{(1)}(x')}, \hspace{1cm} (B3)$$

\( G^{(2)}(x, x') \) is the diagonal four-points correlation function of the field, which is calculated from \( \hat{H}^2 \) and normally ordered using Bose statistics:

$$G^{(2)}(x, x') = \langle \langle \hat{\Psi}^\dagger(x) \hat{\Psi}^\dagger(x') \hat{\Psi}(x') \hat{\Psi}(x) \rangle \rangle - (\langle \langle \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \rangle \rangle)^2,$$  \hspace{1cm} (B4)

while the diagonal two-points correlation function is

$$G^{(1)}(x) = \langle \langle \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \rangle \rangle - (\langle \langle \hat{\Psi}(x) \rangle \rangle)^2.$$  \hspace{1cm} (B5)

Appendix C: Constraints on the calculations

All configurations in Table. 2 have been realised with the cavity parameters of [11]. When exploring all possible configurations of fluid density on either side of the horizon, some constraints must be abode by. We present them in this appendix.

The first constraint is on the upstream pump wavevector \( k_{p,u} \) for a fluid near the sonic point. The fluid is at the sonic point for

$$c_u = \sqrt{\frac{\omega_p - \omega_0 - \hbar k_{p,u}^2 / 2m^*}{m^*}},$$  \hspace{1cm} (C1)

together with the upstream condition \( v_u < c_u \), this yields an upper bound for the upstream fluid flow velocity and thus for the wavevector of the pump:

$$\omega_p - \omega_0 > \frac{3}{2} m^* v_u^2.$$  \hspace{1cm} (C2)

For the value of detuning used throughout this paper the upper bound is around \( k_{p,u} = 0.28 \mu \text{m}^{-1} \). In most simulations, we used \( k_{p,u} = 0.25 \mu \text{m}^{-1} \) in order to be close to the bound while leaving a small interval for easier simulations.
Exploring all regimes of density in the downstream region comes with some constraints as well: For instance, placing the fluid in the upper part of the bistable regime as in configurations 2(b) is easier for a large bistable interval $F_1 - F_2$. Point $\beta$ in configuration 6 is obtained at

$$c_d = \sqrt{\frac{\omega_p - \omega_0 - \hbar k_{p,d}^2/2m^*}{2m^*}}.$$  \hspace{1cm} (C3)

and the width of the interval is then given by

$$\Delta I_{\text{bistable}} = |F_{p,max}|^2 - |F_{p,min}|^2 = \left[\frac{4}{9} \left(\omega_p - \omega_0 - \frac{hk_{p,d}^2}{2m^*}\right)^2 - \frac{1}{2}\right]^2 \times \frac{\omega_p - \omega_0 - \hbar k_{p,d}^2/2m^*}{3g}.$$  \hspace{1cm} (C4)

The larger downstream wavevector $k_{p,d}$ results in a larger interval, thus rendering simulation at a wanted point along the bistability loop easier. Nevertheless, a bistable regime exists only if $\omega_p - \omega_0 > \hbar k_{p,d}^2/2m^*$, hence an upper bound on $k_{p,d}$.

Furthermore, the speed of sound right after the defect, $c_{\text{def}}$ is fixed by the upstream parameters and the strength of the defect, $V_{\text{def}}$. Pumping in the upper branch of the bistability requires

$$c_d > \sqrt{\frac{\omega_p - \omega_0 - \hbar k_{p,d}^2/2m^*}{m^*}}.$$  \hspace{1cm} (C5)

This critical point needs to be below $c_{\text{def}}$ for the fluid density to be on the upper branch. (Note that it would also be possible to change the value of $c_{\text{def}}$, which can be achieved for a weaker defect potential — energy conservation before and just after the defect links $V_{\text{def}}$ and $c_{\text{def}}$). The choice of different $k_{p,d}$ in the simulations of Fig 2 is a consequence of all these constraints.