The Shape and Experimental Tests of the $Q^2$-Invariant Polarized Gluon Asymmetry

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Abstract. The absence of "valence-gluon" degrees of freedom combined with an examination of radiative QCD diagrams leads to an implication that the gluon spin asymmetry in a proton, defined as $A_G(x, Q^2) = \frac{\Delta G(x, Q^2)}{G(x, Q^2)}$, should be approximately $Q^2$ invariant. The condition for scale invariance completely determines the $x$-dependence of this asymmetry, which satisfies constituent counting rules and reproduces the basic results of the Bremsstrahlung model originated by Close and Sivers. This asymmetry can be combined with the measured unpolarized gluon density, $G(x, Q^2)$ to provide a prediction for $\Delta G(x, Q^2)$. Existing and proposed experiments can test both the prediction of scale-invariance for $A_G(x, Q^2)$ and the nature of $\Delta G$ itself.

I INTRODUCTION

The spin-weighted gluon density, $\Delta G(x, Q^2)$, is of fundamental importance in understanding the dynamics of hadron structure. Numerous experiments have been proposed [1–3] to determine this distribution experimentally. Measurements of the deep-inelastic scattering asymmetry $A_1(x, Q^2)$ for protons, neutrons and deuterons yield data from which polarized quark distributions may be inferred, but the shape and size of the polarized gluon density has not been determined. However, the constituent quark model provides a framework for predicting an essential feature of $\Delta G(x, Q^2)$. To understand this, we assume that the spin structure of proton does not have a significant component representing a valence or "constituent" gluon polarization. Hadronic spin observables at small $Q^2$ conform to the non-relativistic quark model in which spin degrees of freedom are associated with constituent quarks. This assumption does not imply that $\Delta G \to 0$ at low $Q^2$. In fact, when the spin structure of the constituent quarks is resolved by inelastic scattering measurements, this approach yields a variation of Close-Sivers Bremsstrahlung model [4] which displays a maximal gluon polarization at $x = 1$.

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In a positive helicity proton, we define the gluon polarization asymmetry as

\[ A_G(x, t) \equiv \Delta G(x, t)/G(x, t), \]  

where the evolution variable \( t \equiv \ln[\alpha_s(Q^2_0)/\alpha_s(Q^2)] \). It is assumed that the same factorization prescription is used to define all of the densities in equation (1). Since there are no overwhelming theoretical arguments favoring any single model for \( \Delta G \), we consider a more direct argument for its shape in terms of this asymmetry.

Our results follow from the observation that, in the absence of a “constituent” gluon, both \( G(x, t) \) and \( \Delta G(x, t) \) exhibit scaling violations which can be associated with measurements “resolving” radiative diagrams. The diagrams leading to positive and negative helicity gluons are the same. This implies that the relative probability of measuring a gluon of either helicity does not depend upon \( t \). Thus, the gluon polarization asymmetry is predicted to be scale invariant: \( \partial A_G(x, t)/\partial t = 0 \).

This would not be true if there were a valence gluon, since the shape of \( A_G \) would then depend upon the relative amount of valence and radiated gluons. It is reasonable to choose \( t = 0 \) to coincide with a typical hadronic scale, \( Q^2 = m^2_H \). The scale-invariance assumption provides the \( x \) dependence of \( A_G(x) \), which satisfies several important physical constraints:

- it obeys the constituent-counting rules,

- for large \( x \), where quark distributions dominate the gluon distribution, the predicted asymmetry coincides with the original QCD-Bremsstrahlung model of Close and Sivers [4]. At other values of \( x \), it corresponds to a natural extension of the QCD-Bremsstrahlung approach by allowing for radiation from both quarks and gluons, and

- for small \( x \), where the gluon distribution is expected to dominate the quark distributions, the scale-invariant asymmetry arises as a natural asymptotic limit, independent of the starting point.

Thus, the arguments originally presented in ref. [4] can be combined with existing parametrizations of polarized and unpolarized quark distributions to provide a quantitative estimate for \( \Delta G(x, Q^2_0) \) at any convenient reference scale.

\[ II \quad \text{THE SHAPE OF } A_G(X) \]

In 1977, Close and Sivers [4] proposed that the quark \textit{sea} should be polarized and that \textit{gluons} should exhibit a polarization of in the same direction as the proton. This was based on perturbative QCD and the theoretical understanding that valence quarks are polarized in the same sense as the proton. This happens since the \( \gamma^\mu \) coupling of the quark-gluon vertex conserves quark helicity when quark masses are neglected. Thus, when a gluon is radiated by a quark, its helicity has the same sign as the creating quark.
A phenomenological picture of the proton assumes that at low \( Q^2 \leq m_p^2 \), a proton consists of three “valence” quarks, surrounded by radiated gluons and \( q\bar{q} \) pairs. The Bremsstrahlung mechanism used in ref. [4] supplies a significant fraction of gluons in a proton found at low to medium values of \( Q^2 \). From a reference scale where the constituent quark picture is applicable, the QCD evolution equations can be used to generate a prediction for the quark and gluon distributions at higher \( Q^2 \).

The requirement that \( A_G(x, t) \) has no \( t \)-dependence implies that

\[
\frac{\partial A_G}{\partial t} = \frac{1}{G} \left[ \frac{\partial \Delta G}{\partial t} - A_G(x, t) \frac{\partial G}{\partial t} \right] = 0. \tag{2}
\]

The \( t \)-dependence of the gluon distributions is given by the corresponding DGLAP evolution equations. [5] Combining the DGLAP equations with equation (2) gives

\[
A_G = \frac{\partial \Delta G}{\partial t} = \frac{\left[ \Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes \Delta G \right]}{P_{Gq} \otimes q + P_{GG} \otimes G}. \tag{3}
\]

This follows since the diagrams which determine the \( t \) evolution of the distributions are the same as those which distribute the spin information to the gluons.

Since \( \Delta G \) has not been measured, equation (3) can be converted into a non-linear equation for \( A_G(x) \) by inserting \( \Delta G(x, t) = A_G(x) \cdot G(x, t) \) into the convolution,

\[
A_G = \frac{\left[ \Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes (A_G \cdot G) \right]}{P_{Gq} \otimes q + P_{GG} \otimes G}. \tag{4}
\]

An equation in this form can be solved iteratively. We first observe that for a given value of \( x \), the distributions in the DGLAP equations enter only in the range \([x, 1]\). Then, for a large enough \( x (x \geq 0.6) \), the gluon distributions on the right side of (4) can be neglected. Now, the polarized DIS data are consistent with the constituent counting rule result that \( \lim_{x \to 1} A_1(x, Q^2) \approx \lim_{x \to 1} \Delta u_v(x, Q^2)/u_v(x, Q^2) = 1 \). There exist parametrizations of the helicity-weighted quark distributions [6] which incorporate this result to reproduce all of the existing data. Thus, we make an initial approximation

\[
\lim_{x \to 1} A_G^0 = \left[ \frac{\Delta P_{Gq} \otimes \Delta u_v}{P_{Gq} \otimes u_v} \right]. \tag{5}
\]

in terms of the flavor non-singlet quark distributions, valid for large \( x \). We can then define the interactive approximation:

\[
A_G^{n+1} = \left[ \frac{\Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes (A_G^n \cdot G)}{P_{Gq} \otimes q + P_{GG} \otimes G} \right], \tag{6}
\]

which should converge for large enough \( n \). It is important to note that (6) determines the form of \( A_G(x) \) from the three distributions, \( \Delta q(x, t) \), \( q(x, t) \) and \( G(x, t) \),
extracted from data. The spin-weighted gluon asymmetry is then determined explicitly by
\[ \Delta G(x, t) = A_G(x) \cdot G(x, t). \]

At small-\(x\), we can also argue that \( \frac{\partial A_G}{\partial t} = 0 \). We can parametrize the asymmetry in the form \( A_G(x, t) = A_G(x) + \epsilon(x, t) \), where \( \epsilon(x, t) \) is a correction term which necessarily vanishes at large \( t \). Then,

\[
\frac{\partial A_G(x, t)}{\partial t} = \frac{\partial \epsilon(x, t)}{\partial t} = 1 + \frac{\partial \Delta G}{\partial t} - A_G(x, t) \frac{\partial G}{\partial t}.
\]

(7)

Now, insert the expression for \( A_G(x, t) \) in terms of \( A_G(x) \) and \( \epsilon \) into (7) to get

\[
\frac{\partial \epsilon(x, t)}{\partial t} = -\frac{\epsilon(x, t)}{G(x, t)} \cdot \frac{\partial G(x, t)}{\partial t}.
\]

(8)

This implies that \( \epsilon(x, t) \cdot G(x, t) \) is scale invariant. Thus, at small-\(x\), the growth in \( G \) predicted by the evolution equations ensures that \( \epsilon \to 0 \) at large \( t \) and that \( A_G \) maintains its \( x\)-dependent shape asymptotically in \( t \).

For the starting distributions in eq. (5) and the iterations of eq. (6), we use the polarized quark distributions outlined by GGR [6] and the CTEQ4M unpolarized distributions. [7] The evolution was performed in LO, since the NLO contributions to the splitting kernels, calculated in ref. [8], are most dominant at small-\(x\), where the asymmetry is the smallest. Work is in progress to ensure that the effects of NLO are not significant for the ratio \( \frac{\Delta G}{G} \). The iteration is relatively stable and converges within a couple of cycles. Some models of \( G(x) \) converge more uniformly than others. The resulting shape of \( A_G(x) \) is shown in Figure 1. This shape implies a larger polarized gluon distribution than the \( xG \) model of GGRA [6], so spin asymmetries which depend upon \( \Delta G \) are enhanced.

### III EXPERIMENTAL TESTS OF \( A_G(X) \) AND \( \Delta G \)

There are a number of existing and planned experiments are suitable for measuring either \( A_G(x) \) or a combination of \( \Delta G(x, Q^2) \) and \( G(x, Q^2) \). The HERMES experimental group at DESY has measured the longitudinal cross section asymmetry \( A_\| \) in high-\(p_T\) hadronic photoproduction. [1] From this and known values of \( \Delta q \) from DIS, a value for \( A_G(x_G) \) can be extracted. Here, \( x_G = \hat{s}/2M\nu \) is the nucleon momentum fraction carried by the gluon. Our corresponding value at \( x_G = 0.17 \) is within one \( \sigma \) of the quoted value of \( A_G = 0.41 \pm 0.18 \) (stat.) \( \pm 0.03 \) (syst.).

Both direct-\(\gamma\) production and jet production at RHIC provide the best means of extracting information about \( \Delta G(x, Q^2) \) and \( G(x, Q^2) \) separately. [2,9] The kinematic regions of STAR and PHENIX can determine \( A_G \) over a suitable range of \( x_{Bj} \) to test this model of the gluon asymmetry. Coupled with additional direct measurements of \( A_G(x_G) \) from HERMES, an appropriate cross check of \( \Delta G(x) \) and \( G(x) \) can be made. Since this model of \( \Delta G \) implies a larger polarized glue than the GGRA model used in ref. [9], all of the asymmetries for direct-\(\gamma\) and jet
production should be enhanced, making them easier to distinguish from the other parametrizations of $\Delta G$.

The COMPASS group at CERN. [3] plans to extract $A_G$ from the photon nucleon asymmetry, $A_{\gamma N}^{c\bar{c}}(x_G)$ in open charm muo-production, which is dominated by the photon-gluon fusion process. This experiment should be able to cover a wide kinematic range of $x_G$ as a further check of this model. The combination of these experiments will be a good test of the assumptions of our gluon asymmetry model and a consistency check on our knowledge of the gluon distribution in the nucleon and its polarization.

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