Optimization of Furnace Temperature Curve Based on GA

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Abstract. Aiming at the optimization problem of furnace temperature curve, this paper establishes a one-dimensional heat transfer model to determine the temperature field distribution of the reflow furnace; uses the law of thermodynamics to establish a temperature change model for the welding zone. Through the use of traversal algorithm and genetic algorithm to solve the optimal temperature of each temperature zone of the reflow furnace and the optimal speed of the conveyor belt passing the furnace, in order to achieve the optimal control of the furnace temperature curve. To solve the first problem, we assume that the temperature of each small temperature zone is stable, establish a one-dimensional alternating heat exchange model, determine the temperature distribution of each gap and the area before and after the furnace, and obtain the temperature distribution of the entire reflow furnace. According to Newton's cooling law, the temperature change model of the welding area is established. Substituting the passing speed of the conveyor belt and the temperature of each temperature zone in the transitional experiment into the model, and substituting the relevant data of the experiment, the thermal time constant of five types of different temperature zones is calculated by the genetic algorithm. The sequence is 59.75s, 45.55s, 45.79s, 45.48s, 23.91s. The obtained thermal time constant is replaced with the temperature change model of the welding area, and the furnace temperature curve is simulated. 0.02%, indicating that the required thermal time constant is better. In this law, the furnace temperature curve can be obtained by substituting the conveyor belt passing speed in problem 1 and the set value of each temperature zone temperature into the welding zone temperature change model. Among them, the temperature at the center of the small temperature zone 3, 6, 7 and the end of the small temperature zone 8 are 130.09°C, 167.33°C, 189.67°C, and 223.01°C. For the second problem, this task is a single-objective optimization problem, that is, based on the first task model, the temperature setting of each temperature zone is changed, the process limit is used as the constraint condition, and the maximum passing speed of the conveyor belt is the optimization goal. The traversal algorithm traverses the conveyor belt passing speed in the range of [65,100] with a step length of 0.017, and searches for the maximum passing speed that satisfies the constraints. The maximum speed obtained is: 92.478cm/min. For the third problem, this is a multi-decision variable single objective optimization problem established on the basis of problem one. It takes the temperature of each temperature zone of the reflow furnace and the furnace passing speed as decision variables, the process limit is the
constraint, and the furnace temperature curve exceeds 217°C to the minimum area covered by the peak temperature as the optimization goal. The optimal solution obtained by genetic algorithm is: the temperature of small temperature zone 1-5 is 181.2°C, the temperature of small temperature zone 6 is 198.5°C, the temperature of small temperature zone 7 is 226.6°C, and the temperature of small temperature zone 8-9 is 263.9°C. The temperature in the small temperature zone 10-11 is 17.9°C. The furnace passing speed of the conveyor belt is 95.2 cm/min. The minimum area is 447.8 and 217°C to be the smallest and the furnace temperature curve of the part exceeding 217°C is as symmetric as possible. The average deviation degree is defined to measure its symmetry. The smaller the value, the better the symmetry. The optimal solution obtained by genetic algorithm is: the temperature in the small temperature zone 1-5 is 183.4°C, and the temperature in the small temperature zone 6 is 188.9°C. The temperature in the temperature zone 7 is 226.7°C, and the temperature in the small temperature zone 8-9 is 261.2°C. The temperature in the small temperature zone 10-11 is 25.1°C. The furnace passing speed of the conveyor belt is 88.5 cm/min, the minimum area is 494.9°C, and the average deviation degree is 0.1749. Finally, the model established in this article is discussed and analyzed, the model established is comprehensively evaluated, and the direction of promotion is proposed.

Keywords: Target optimization model, heat transfer model, thermal time constant, genetic algorithm.

1. Introduction

1.1. Problem Background

With the continuous development and progress of surface mount technology (SMT), for manufacturers of electronic products such as integrated circuit boards, if they want to seize a certain share in the fiercely competitive market, excellent product quality is an important factor for success. In the production of such products, the multi-temperature zone reflow furnace is an indispensable link to realize the automatic soldering of electronic components to the printed circuit board (PCB). There is a heating circuit inside, which is attached to the PCB board by melting the solder realizes the firm fibrillation of the electronic components and the PCB board. Therefore, keeping each temperature zone in the reflow furnace at the temperature required by the process is very important to the quality and performance of the product. Otherwise, there may be phenomena such as virtual soldering, burning of components, and tin beads [1].

Nowadays, in China, regarding the setting of the temperature distribution of the reflow soldering temperature field, experimental testing methods are still used to formulate the required process temperature of each part of the reflow furnace, which has caused huge cost waste. Therefore, it is particularly important to establish a related mechanism model to study and analyze the temperature field distribution of the reflow furnace. This question has the following structure and related parameter settings:

(1) There are 11 small temperature zones in the reflow furnace and the front (rear) area of the furnace. The length of each small temperature zone is 30.5cm and there is a gap of 5cm between the small temperature zones. The length of the front (rear) area of the furnace is all 25cm.

(2) The temperature of the production workshop is maintained at 25°C.

(3) On the basis of the experiment temperature setting of this question, the temperature setting of the small temperature zone 1–9 can be adjusted within the range of ±10°C, and the temperature setting in
the small temperature zone 1~5, 8~9 is maintained Unanimous. In addition, the temperature in the small temperature zone 10-11 is kept at 25°C.

(4) The furnace passing speed of the conveyor belt can be adjusted within the range of 65~100cm/min.
(5) The thickness of the welding area is 0.15mm.

1.2. Restatement of the Problem

Based on the above background, we need to establish relevant mathematical models to solve the following problems:

Question 1: According to the relevant parameters and working principles of each part of the reflow furnace, comprehensively consider various heat transfer methods to analyze and explore the law of temperature changes in the welding area, and on the basis of the given conveyor belt passing speed and the set value of each temperature zone Above, find the temperature change over time in the center of the welding area.

Question 2: On the basis of question 1, change the set value of the temperature in each temperature zone, take the relevant process limit as the constraint condition, establish a single-decision variable single-objective optimization model with the maximum conveyor belt passing speed as the goal, and achieve the optimal design.

Question 3: Question three needs to consider the time when the center temperature of the welding zone exceeds the liquidus line (217°C) and the time when the peak temperature is reached. Taking the relevant process limits, the temperature setting value of the temperature zone and the adjustment range of the conveyor belt as constraints, a multi-decision variable single-objective optimization model is established with the goal of minimizing the area covered by the furnace temperature curve from the liquidus line to the peak (heating factor). The solution model gives the optimal furnace temperature curve, as well as the relevant setting parameters of each part of the reflow furnace, and gives the heating factor corresponding to the optimal furnace temperature curve.

Question 4: On the basis of question 3, adding the goal of symmetrical furnace temperature curves that exceed the liquidus line on both sides of the center line with the peak temperature as the goal, a multi-decision variable dual-objective optimization model is established. Solve the model and give the corresponding results.

2. Problem analysis

Question one: For the problem of heat transfer, the key to solving the problem is to correctly establish the heat transfer relationship between the measured object and the temperature field. In this problem, the temperature change in the center of the welding area needs to be obtained, but the physical parameters of the welding area are unknown, which makes the construction of the heat transfer model extremely difficult. Fortunately, the title provides a set of experimental data of the reflow furnace, so that the parameters of the welding area and the furnace environment can be inversely derived from the heat transfer model containing unknown parameters. Then, the calculated parameters are substituted into the heat transfer model with unknown parameters. At this point, the heat transfer relationship between the welding area and the temperature field can be established.

Question two: The second problem is to fix the specific temperature of each small temperature zone, and set the maximum transmission speed as the solution goal. It aims to increase the production capacity of the reflow furnace by increasing the conveying speed of the conveyor belt under the conditions of the clear production environment. When solving this problem, it must be established on the premise that the product quality is guaranteed to increase the furnace passing speed. Therefore, the second problem can be classified as a single-objective optimization problem. The guarantee of product quality is actually the process limit in the problem as a constraint, and the optimal target value can be solved by traversing the search algorithm.

Question three: Different from question 1 and question 2, question 3 does not give the specific temperature of each temperature zone. It hopes to get the temperature and conveyor belt passing speed under the ideal furnace temperature curve, and calculate the area covered by the furnace temperature
curve from the liquidus line to the peak. This is a multi-decision variable single-objective optimization problem, which can be solved by genetic algorithm. It should be noted that the focus of this question is the optimization of the furnace temperature curve, that is, when the process limit is met, the time and peak temperature of the curve exceeding 217°C are adjusted to achieve the minimum required area.

Question four: Question 4 is based on the combination of question 3. It is hoped that the furnace temperature curve exceeding 217°C should be centered on the peak temperature and try to maintain symmetry on both sides. So as to get a better furnace temperature control curve. This can add the curve symmetry requirement as an optimization goal to Problem 3, turn the problem into a multi-decision variable dual-objective optimization model, and then convert the dual-objective optimization model into a single-objective optimization model for solution.

3. General Assumption

For these problems we have the following assumptions:

Assumption I: When the reflow furnace is working, the temperature of the gap in the temperature range is stable, and the stored energy is constant

Assumption II: Each temperature zone of the reflow furnace has strong convection heat transfer ability, which can make the welding area obtain uniform heat transfer

Assumption III: Assuming that the soldering area and circuit board materials are all homogenous

Assumption IV: Assuming that the temperature of the soldering area and the circuit board does not affect the environment

4. Notations and Symbol Description

4.1. Notations

For the professional terms in the text, explain as follows:

(1) Preheating zone: In short, the preheating zone is an area where the ambient temperature is suitable before the welding components enter the high temperature range. Its significance is to slowly heat the welded components to a certain temperature to prevent the components from rising sharply when they enter the high temperature zone and causing damage such as cracking.

(2) Constant temperature zone: The constant temperature zone is between the preheating zone and the reflow zone. Its temperature is constant, which can make the temperature distribution of the soldered components even. In this way, the insufficiency of "bridging" and "cold welding" caused by uneven temperature distribution when entering the reflow zone can be avoided.

(3) Reflow area: The reflow area is the area where the original parts are soldered. It can melt the solder paste, and the flux contained in the solder paste can remove the oxide layer on the soldering surface, and finally form a liquid solder joint on the soldering position.

(4) Cooling zone: The cooling zone is the area where the liquid solder is cooled and solidified. When the soldered component enters the cooling zone from the reflow zone, the solder solidifies to form a bright solid point, forming a reliable connection between the component and the soldering point.

(5) Liquidus: generally, refers to the temperature of a material at the intersection of solid and liquid. The 217°C given in this question can be understood as liquidus

(6) Heating factor: defined as the area of the reflux curve on the liquidus line.
4.2. Symbol Description

| Symbol | Description |
|--------|-------------|
| $q_x$  | The rate of heat conduction in the forward direction of the conveyor belt |
| $\rho'$ | Heat flux |
| $h$    | Convection heat transfer coefficient |
| $Q$    | Convective heat transfer energy |
| $\rho$ | Welding area density |
| $c_p$  | Specific heat capacity of welding area at constant pressure |
| $V$    | Welding area volume |
| $\tau$ | Thermal time constant |
| $Q_H$  | Heating part of heating factor |

5. Models and Results

5.1. Question 1

5.1.1. Problem analysis. The problem requires us to find the temperature change in the center of the welding area under the given conveyor belt passing speed and the temperature conditions of each temperature zone. For this mechanism analysis problem, we know that once the PCB board enters the reflow furnace, it exchanges heat with the temperature field of the reflow furnace, and its temperature changes dynamically. If the temperature of each temperature zone of the reflow furnace and the furnace passing speed of the conveyor belt are determined, the furnace temperature curve at the center of the welding area can be obtained according to the heat transfer law between the welding area and the reflow furnace. Therefore, for the first question, this article will analyse and solve the problem from the following steps.

Step 1: Assuming that the temperature of each small temperature zone is stable, a one-dimensional steady-state heat conduction model is established to determine the temperature distribution of each gap and the area before and after the furnace, that is, the temperature field distribution of the reflow furnace is obtained.

Step 2: According to Newton’s law of cooling and the temperature field distribution of the reflow furnace, a temperature change model of the welding area is established.

Step 3: Substitute the passing speed of the conveyor belt and the temperature of each temperature zone into the model in the experiment given by the title. According to the data in Annex 1, the optimal parameters of the model are obtained based on genetic algorithm.

Step 4: Bring the conveyor speed through the furnace in question 1 and the set value of each temperature zone temperature into the model to get the furnace temperature curve.

5.1.2. Determination of gap temperature in one-dimensional steady-state temperature zone. From the question, in actual production, the temperature of each temperature zone can be determined by adjusting the settings, and the air in the furnace will reach the temperature in a short time after the reflow furnace is started, so we focus on the small temperature zone without special temperature control. The gap between the temperature distribution in the area before the furnace and the area behind the furnace. Since the gap temperature between the small temperature zones is affected by the adjacent temperature zones, the gap temperature needs to be analyzed and discussed. It can be seen from the
problem setting conditions that after the reflow furnace is started, the air temperature in the furnace will quickly stabilize, and there will be a certain temperature gradient in the forward direction of the conveyor belt. The temperature of each temperature zone in the direction of the vertical conveyor belt does not change, so the gap temperature change in the temperature zone can be regarded as a one-dimensional heat conduction problem [2]. The gap temperature distribution during the heating phase is shown in the figure below.

![Diagram of temperature profile](image)

**Figure 1.** Temperature profile

The meaning of the above figure is: the cold fluid with temperature $T_{\infty,1}$ reaches the gap with the limit temperature of $T_{s,1}$ through convection, and the temperature is conducted in the gap, from the gap limit with the temperature of $T_{s,2}$ to the hot fluid with the temperature of $T_{\infty,2}$. $q_x$ is the heat transfer rate (W) in the forward direction of the conveyor belt, and $h_1$, $h_2$ (W/(m$^2$·K)) is the convective heat transfer coefficient at different temperatures. The gap temperature distribution can be equivalent to a thermal resistance network, as follows:

![Diagram of equivalent thermal network](image)

**Figure 2.** Equivalent thermal network

The temperature distribution in the gap can be determined by solving the heat conduction equation with correct boundary conditions. Since the gap temperature is stable and there is no heat source, the corresponding heat conduction equation is:

$$ \frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0 $$

(1)
According to the heat conduction equation in the general rectangular coordinate system we can get:

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + q = \rho c_v \frac{\partial T}{\partial t}
\]  

(2)

Then we can get:

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dx = \rho'_s - \rho'_{s+dx}
\]  

(3)

This formula represents the net heat flux density entering the temperature zone gap in the x-coordinate direction, and \( \rho'_s \text{ (W/m}^2) \) is the heat flux density. And because the energy stored in the gap does not change, the heat flux density is a fixed constant and has nothing to do with the change of x.

In the case that the gap thermal conductivity is constant, the general solution of equation (1) can be obtained by integrating it twice:

\[
T(x) = C_1 x + C_2
\]  

(4)

Substituting the boundary conditions at \( x=0 \) and \( x=L \): \( T(0) = T_{s,1} \), \( T(L) = T_{s,2} \), it can be solved:

\[
\begin{cases}
C_2 = T_{s,1} \\
C_1 = \frac{T_{s,2} - T_{s,1}}{L}
\end{cases}
\]  

(5)

Finally, substituting \( C_1, C_2 \) into the general solution, the temperature distribution in the gap can be obtained:

\[
T(x) = (T_{s,2} - T_{s,1}) \frac{x}{L} + T_{s,1}
\]  

(6)

It can be seen from the results that the temperature change in the temperature zone gap with a one-dimensional, steady state, no internal heat source and a constant thermal conductivity is linear. In the same way, the temperature distribution of the temperature zone gap corresponding to the cooling stage is also linear. In addition, similar linear processing is done for the area in front of the furnace and the area behind the furnace.

Based on the above description of the temperature field distribution of the reflow furnace, we established the temperature distribution of the reflow furnace temperature field of an experiment in this question to determine the thermal time constant \( \tau \) corresponding to the following different temperature zones. The temperature distribution chart is as follows:
Figure 3. Temperature field distribution diagram of reflow furnace

In addition, the temperature distribution of the reflow furnace temperature field can also be established by referring to this rule in other cases where the temperature settings of the small temperature zones are different.

5.1.3. The establishment of the temperature change model of the welding area. The temperature distribution of the reflow furnace has been determined in the previous step, so here we can analyze the mechanism of the furnace temperature curve based on the known temperature distribution. For a certain heating zone of a specific reflow furnace, the heating model of the welding zone [3, 4] is shown in the figure below:

Figure 4. Model of convective heating welding area

After the reflow furnace is started, the air temperature in the furnace will stabilize in a short time. Therefore, it can be assumed that the heat transfer coefficient \( h(\text{W/m}^2\text{K}) \) between the hot gas and the welding area is uniform throughout the heating interval, and the heat transfer coefficient can be approximately regarded as one. Treating with a constant, the energy of convective heat transfer can be obtained, and the expression is as follows:

\[
Q = h \cdot A \cdot (T_{\text{air}} - T)
\]  

(7)
In the formula, $T_{\text{air}}$ is the temperature of the heated gas, $T$ is the surface temperature of the heated welding area, and $A$ is the area of the welding area, because the heat absorbed by the welding area per unit time is $dQ$, then:

$$dQ = hA(T_{\text{air}} - T)\,dt = \rho c_p V dT$$  \hspace{1cm} (8)

In the formula, $\rho$, $c_p$, and $V$ are the density of the welding area, the specific heat capacity at a constant pressure, and the volume respectively. Integrate both sides of equation (8) at the same time to obtain:

$$\int \frac{hA}{\rho c_p V} \,dt = -\int \frac{1}{T_{\text{air}} - T} \,d(T_{\text{air}} - T)$$  \hspace{1cm} (9)

$T(t = 0) = T_1$, $T(t = t) = T_2$. Substituting formula (9), we can get:

$$\int_0^t \frac{hA}{\rho c_p V} \,dt = -\int_{T_1}^{T_2} \frac{d(T_{\text{air}} - T)}{(T_{\text{air}} - T)}$$  \hspace{1cm} (10)

Calculated:

$$\frac{T_2 - T_1}{T_{\text{air}} - T_1} = 1 - e^{-hA/\rho c_p V} \Rightarrow T_2 = T_1 + (T_{\text{air}} - T_1) \cdot (1 - e^{-hA/\rho c_p V})$$  \hspace{1cm} (11)

Given the initial value, the change in the temperature of the center of the welding zone with time can be obtained according to the above formula, where $\frac{c_p \rho V}{hA}$ can be expressed by the thermal time constant $\tau$ which has a time dimension in engineering thermophysics [5]. It is a function of many factors such as $c_p$, $\rho$, $V$, $A$, $h$. It not only depends on the physical properties of the welding area itself, but also depends on the heat transfer coefficient between the air heat flow and the welding area, and reflects the speed of the temperature of the welding area to the air heat flow temperature. For the center of the soldering area on the PCB, heating in different temperature zones in the reflow furnace will correspond to a specific thermal time constant $\tau$. We assume that this reflow furnace is ideal and its heat transfer performance is good. The temperature at the center of the welding zone will approach the temperature setting value of the temperature zone. The temperature at the center of the welding zone will get a maximum heating rate when it just enters the heating zone. In the process of heating the end point of the zone, the heating rate gradually decreases, and then cyclically reciprocates between different temperature zones according to this law. This is the inherent essential law formed by the furnace temperature curve.

5.1.4. Solving the center temperature of welding zone based on finite difference.

1. Preparation before solution.

(1) Fit the measured furnace temperature curve

Due to the need to analyze and solve the error between the furnace temperature curve and the actual furnace temperature curve, the Fourier series is first used to fit the curve of the temperature at the center of the welding zone in Annex I with time, and its goodness of fit $R^2 = 0.999$. Expressed as follow:
According to the description of the temperature field in the model establishment part, the temperature zone of the reflow furnace, the gap between the temperature zones, and the one-dimensional heat transfer mathematical model of the front and back areas of the reflow furnace are proposed, and each area of the reflow furnace is established on the coordinate axis. The schematic is as follows:

\[ f_T(t) = -313.1 - 389.2 \cos(\omega t) + 792.6 \sin(\omega t) + 451.9 \cos(2\omega t) + 439.1 \sin(2\omega t) + 350.8 \cos(3\omega t) - 161.3 \sin(3\omega t) - 13.36 \cos(4\omega t) - 217.4 \sin(4\omega t) - 86.56 \cos(5\omega t) - 24.66 \sin(5\omega t) - 12.97 \cos(6\omega t) + 18.64 \sin(6\omega t) \]

(2) Establish temperature distribution and thermal time coefficient expression

According to the established coordinate axis, the temperature distribution expression of each part can be listed as follows:

\[
T_{\text{air}}(x) = \begin{cases} 
25 + x \cdot (T_{1-5} - 25)/25, & x_0 \leq x < x_1 \\
T_{1-5}, & x_1 \leq x < x_2 \\
T_{1-5} + x \cdot (T_6 - T_{1-5})/T_{1-5}, & x_2 \leq x < x_3 \\
T_6, & x_3 \leq x < x_4 \\
T_6 + x \cdot (T_7 - T_6)/T_6, & x_4 \leq x < x_5 \\
T_7, & x_5 \leq x < x_6 \\
T_7 + x \cdot (T_{8-9} - T_7)/T_7, & x_6 \leq x < x_7 \\
T_{8-9}, & x_7 \leq x < x_8 \\
T_{8-9} + x \cdot (T_{10-11} - T_{8-9})/T_{8-9}, & x_8 \leq x < x_9 \\
T_{10-11}, & x_9 \leq x < x_{10} \\
(T_{10-11} + x \cdot (25 - T_{10-11})/T_{10-11}, & x_{10} \leq x < x_{11} 
\end{cases}
\]  

Figure 5. Schematic diagram of the reflow furnace coordinate system

According to the established coordinate axis, the temperature distribution expression of each part can be listed as follows:

\[
T_{\text{air}}(x) = \begin{cases} 
25 + x \cdot (T_{1-5} - 25)/25, & x_0 \leq x < x_1 \\
T_{1-5}, & x_1 \leq x < x_2 \\
T_{1-5} + x \cdot (T_6 - T_{1-5})/T_{1-5}, & x_2 \leq x < x_3 \\
T_6, & x_3 \leq x < x_4 \\
T_6 + x \cdot (T_7 - T_6)/T_6, & x_4 \leq x < x_5 \\
T_7, & x_5 \leq x < x_6 \\
T_7 + x \cdot (T_{8-9} - T_7)/T_7, & x_6 \leq x < x_7 \\
T_{8-9}, & x_7 \leq x < x_8 \\
T_{8-9} + x \cdot (T_{10-11} - T_{8-9})/T_{8-9}, & x_8 \leq x < x_9 \\
T_{10-11}, & x_9 \leq x < x_{10} \\
(T_{10-11} + x \cdot (25 - T_{10-11})/T_{10-11}, & x_{10} \leq x < x_{11} 
\end{cases}
\]

In the formula, the number of temperature T subscript corresponds to the division of each small temperature zone.

The thermal time constant of the gap between the hypothetical temperature zones is regarded as equal to the previous temperature zone, so the thermal time constant expression is as follows:
Perform finite difference on equation (11)

Under the condition that the thermal time constant of each temperature zone is the same, the discretization idea can be used to solve the change of the center temperature of the welding zone with time. The basic idea is to discretize the continuous physical quantity in time and space on each node and use the finite difference method Numerically solve the physical quantity. Set the time step length to \( dt \), the displacement step length to \( dx = v \cdot dt \), and the temperature when initializing \( T_1 \) to \( t = dt \) is approximately the production workshop temperature of 25°C, according to formula (11):

\[
T_n + 1 = T_n + \left[T_{air}(n \cdot v \cdot dt) - T_n\right] \cdot \left[1 - e^{-dt/f(v \cdot dt)}\right]
\]

Where \( T_n \) is the temperature at the center of the welding area at \( t = ndt \). In this way, the temperature at the center of the welding area in each small discrete time period with a step length of \( dt \) can be obtained by recursion, which is the desired furnace temperature curve. The smaller the step length, the closer the desired temperature is to the true value.

2. Solve the thermal time constant \( \tau_1 \sim \tau_5 \) based on genetic algorithm

The genetic algorithm is a global random search method that imitates the biological evolution process. Its basic idea is: starting from a population of the optimization problem, according to the basic rules of survival of the fittest and survival of the fittest, a better and better population is evolved from generation to generation (A set of feasible solutions). This problem needs to solve the optimal solution of the thermal time constant of each temperature zone. The algorithm design is as follows:

Step1: Determine the code of the chromosome

The standard genetic algorithm adopts the binary coding method. Because the coding method causes a large amount of calculation in the algorithm execution process, this paper adopts real number coding.

Step2: population initialization

Refer to the literature [4] that the thermal time constant of each temperature zone of the reflow furnace is generally within 30~60s. To ensure the correctness of the result, the thermal time constant value range of each temperature zone is initialized here as \([20s, 80s]\). The initialized five thermal time constants constitute an individual and each individual is combined into a population.

Step3: Determine the fitness function

In the genetic algorithm, the larger the value of the individual fitness function \( F \), the closer the solution corresponding to the individual is to the optimal solution. This article sets the fitness function as:

\[
F = \frac{G}{1 + erro}
\]

\[
erro = \sum_{i=1}^{N1} |f_r(i dt) - T_i(\tau)|
\]

(13)
In the above formula, N1 is the total number of discrete points. When \( f_t(idt) \) is \( t = idt \), the temperature of the actual furnace temperature curve of Annex 1 in the experiment. That is, when the thermal time constant in the individual is \( \tau = (\tau_1, \ldots, \tau_5) \), the temperature at \( t = idt \) in the furnace temperature curve obtained by the solution, G is a positive real number, and we take 1000 in this article. The error value \( \text{erro} \) is the smallest, that is, the smaller the deviation between the calculated furnace temperature curve and the furnace temperature curve in the actual experiment, that is, the corresponding thermal time constant \( \tau = (\tau_1, \ldots, \tau_5) \) in the individual is better.

Step4: Select operation
Using the roulette method, based on the selection strategy of adaptive proportions, the probability of each individual's selection is:

\[
p_i = \frac{f_i}{\sum_{i=1}^{N} f_i}
\]  

\( f_i \) is the fitness \( F_i \) of the i-th individual in the group, and \( N \) is the number of individuals in the group.

According to the case of this article, the number of groups \( N \) is set to 1000.

Step5: Cross operation
Using the real number crossover method, the crossover operation method of the kth chromosome \( a_k \) and the lth chromosome \( a_l \) at the jth position is as follows:

\[
\begin{align*}
a_{kj} &= a_{kj} (1 - b) + a_{lj} b \\
a_{ij} &= a_{ij} (1 - b) + a_{kj} b
\end{align*}
\]

\( b \in [0, 1] \)

Step6: Mutation operation
Select the j-th gene of the i-th individual for mutation, which is expressed in mathematical language as follows:

\[
a_{ij} = \begin{cases} 
  a_{ij} + (a_{ij} - a_{\text{max}})^*f(g) & r > 0.5 \\
  a_{ij} + (a_{\text{min}} - a_{ij})^*f(g) & r \leq 0.5
\end{cases}, \quad f(g) = r_2 (1 - g/G_{\text{max}})^2
\]  

\( a_{\text{min}} \) is the lower bound of gene \( a_{ij} \), \( a_{\text{max}} \) is the upper bound of gene \( a_{ij} \); \( r \), \( r_2 \in [0, 1] \), is the generated random number. \( g \) is the current number of iterations, \( G_{\text{max}} \) is the maximum number of iterations set, this article sets the maximum number of iterations to 15 times.

Due to the large randomness of genetic algorithm, the termination condition based on heuristic rules is used here: if the algorithm is iterated for 15 times, the algorithm will terminate. The genetic algorithm program of this design is shown in the figure below:
Matlab is used to program to achieve the above process, where the genetic algorithm selects the cross-mutation part by calling the genetic algorithm toolbox of the University of North Carolina. The results of the calculation are as follows:

![Genetic algorithm iteration diagram](image)

**Figure 7. Genetic algorithm iteration diagram**

It can be seen from the above figure that when the number of iterations reaches 5, the optimal fitness in the population is equal to the average fitness in the population and remains unchanged thereafter. This shows that the fitness function value converges, and the solution sought can be considered as the best Excellent solution. Run to get the thermal time constant

$$\tau = (59.7508, 45.5472, 45.7920, 45.4779, 23.9056)$$

Substitute the calculated optimal thermal time constant into the temperature change model of the welding zone, and use the finite difference method to solve it, and obtain the furnace temperature curve distribution diagram, which is compared with the actual furnace temperature curve fitting diagram in Annex 1 as shown below:
According to the above figure, we can intuitively see that the fitted curve is close to the actual curve. By calculating the mean square percentage error MSPE: 0.02%. The MSPE is smaller, indicating that the fitting effect is better, and the parameter value $\tau$ is better.

3. Solution results of furnace temperature curve

Substitute the optimal thermal time constant $\tau$ obtained by the genetic algorithm into the finite difference model to determine the function $f_r(x)$. Determine the function $T_{air}(x)$ according to the furnace passing speed of the conveyor belt given in Question 1, and the set value of the temperature in each temperature zone. From equation (14), the determined temperature change model of the welding area can be obtained, and the furnace temperature curve can be obtained by writing code in matlab as follows:

Among them, the temperature at the midpoints of small temperature zones 3, 6, and 7 and the center of the welding area at the end of small temperature zone 8 are as follows: 130.09°C, 167.33°C, 189.67°C, 223.01°C.
5.2. **Question 2**

5.2.1. **Problem analysis.** The second question is based on the first question, changing the setting value of each temperature zone, requiring the determination of the maximum conveyor belt passing speed within the allowable conditions. The question is essentially a single-objective optimization problem, and the relevant process limits are considered as constraints. Conditions, a single decision variable single objective optimization model with the maximum conveyor belt passing speed as the goal is established, and the traversal idea is used to search for the conveyor belt’s maximum passing speed under the constraint conditions.

5.2.2. **Establishment of single objective optimization model.** It is known from the problem that the temperature of small temperature zone 1~5 is set to 182℃, the temperature of small temperature zone 6 is set to 203℃, the temperature of small temperature zone 7 is set to 237℃, and the temperature of small temperature zone 8~9 is set to 254℃, when the furnace temperature curve meets the process limit to ensure welding quality, while maximizing the conveyor belt passing speed can maximize the benefits. Therefore, the maximum conveyor belt passing speed design can be transformed into a single-objective optimization model.

**Objective function:**

\[
\text{max } v, \quad 65 \leq v \leq 100
\]

In the formula, \( v \) represents the furnace passing speed of the conveyor belt (cm/min).

**Restrictions:**

1. Considering that the temperature in the center of the soldering area rises or falls too fast, the components will be thermally shocked, causing component cracking and solder splash; too slow will cause insufficient solvent volatilization and affect the quality of soldering. Therefore, determine the range of furnace temperature curve slope change according to the process limit:

\[
\begin{align*}
0 & \leq k_u \leq 3 \\
-3 & \leq k_d \leq 0
\end{align*}
\]

In the formula, \( k_u \) represents the temperature rising slope, and \( k_d \) represents the temperature falling slope. Taking the absolute value of the temperature drop slope can get the variation range of the absolute value \( |k| \) of the furnace temperature curve slope:

\[
0 \leq |k| \leq 3
\]

2. Considering that the temperature is rising, the time between 150℃ and 190℃ should be appropriate. Refer to the process limit to obtain the value range:

\[
60 \leq t_{\text{between}} \leq 120
\]

In the formula, \( t_{\text{between}} \) (s) represents the time during which the temperature is rising between 150℃~190℃.

3. Considering that the time when the temperature is greater than 217℃ and the peak temperature will affect the change of the heating factor, thereby affecting the reliability of the solder joint, it is constrained within a reasonable range according to the process limit:

\[
\begin{align*}
40 & \leq t_{\text{exceed}} \leq 90 \\
240 & \leq T_{\text{max}} \leq 250
\end{align*}
\]

The formula \( t_{\text{exceed}} \) (s) represents the time when the temperature exceeds 217℃, and \( T_{\text{max}} \) (℃) represents the peak temperature.

So far, a single-objective optimization model for the optimized design of conveyor belt passing speed is established:

**Objective function:**
5.2.3. **Traverse search to solve the optimal speed.** Based on the single-objective optimization model established above, a certain scale of random speed can be generated first, and then solved by traversal search. The solution process is as follows:

1. Randomly generate 2000 speeds in the range of [60, 100], \( v_i \) represents the i-th speed, initialize \( i=0, v_{best} = 0 \).
2. Substitute \( v_i \) into the model to get the furnace temperature curve.
3. Calculate the maximum absolute slope of the furnace temperature curve, the time when the curve is at 150°C-190°C during the ascent process, the time when the temperature is greater than 217°C, and the peak temperature \( T_{max} \).
4. If the process limit is met, execute the next step, if not, execute the next iteration.
5. On the premise that the process conditions are met, if there is \( v_i > v_{best} \), the optimal speed \( v_{best} = v_i \) is updated; if \( v_i \leq v_{best} \), the optimal speed does not need to be updated, directly execute \( i = i + 1 \) and go to (2) for the next iteration.

The search continues until the randomly generated velocity iteration is completed. Finally, the maximum conveyor belt passing rate is 92.478 (cm/min).

5.3. **Question 3**

5.3.1. **Problem three analysis.** The third question is essentially a multi-decision variable single-objective optimization model based on the first question. Given the set temperature of each temperature zone and the furnace passing speed of the conveyor belt, the furnace temperature curve of the welding area can be obtained according to the finite difference model of Problem 1. According to the topic information, establish a single-objective optimization that takes the temperature of each temperature zone and the furnace passing speed of the conveyor belt as decision variables, and the minimum area covered by the furnace temperature curve exceeding 217°C to the peak temperature is established to meet the process limit as the constraint condition model.

5.3.2. **Establishment of single-objective optimization model.** Taking the solder in this question as an example, the furnace temperature curve diagram given is as follows:
Refer to related literature [7] that the area covered by the furnace temperature curve exceeding 217º C to the peak temperature is defined as the "heated part of the heating factor", which is recorded as $Q_H$. The control of the optimal range determines the control of the IMC thickness, which indirectly affects the reliability of the solder joints, and the closer its value is to the lower limit of the optimum range, the more reliable the quality of the solder joints, that is, the optimization target is the heating during the heating process. The factor $Q_H$ is the smallest. Let the function $T''(t, T_{1-5}, T_6, T_7, T_{8-9}, T_{10-11}, v)$ be the temperature distribution function of the center of the welding zone with time taking the temperature of each temperature zone and the furnace passing speed of the conveyor belt as independent variables, that is, the furnace temperature curve, then:

$$Q_H = \int_{t_0}^{t_1} (T''(t, T_{1-5}, T_6, T_7, T_{8-9}, T_{10-11}, v) - T_m) \, dt$$  \hspace{1cm} (22)$$

Where is the melting point temperature of the solder alloy (217ºC), is the starting time for the furnace temperature curve to reach, and is the time for the furnace temperature curve to reach the peak temperature. In view of the characteristics of the discretized data of this question, in the actual calculation process, the "heating factor" area is divided into countless approximate trapezoids by the step length $dt$, as shown in the figure below, solving for the area of each trapezoid and summing to get the heating factor heating part.

$$S_i = ((T''|t = jdt - 217) + (T''|t = (j-1)dt - 217)) \times dt \times 0.5$$  \hspace{1cm} (23)$$

$$Q_H = \sum_{i=1}^{n} S_i$$  \hspace{1cm} (24)$$
In the formula, $n$ is the number of discrete points in discrete time.

Based on the above analysis, it can be determined:

Optimization goal:

$$\min Q_H$$

Restrictions:

$$\begin{align*}
0 & \leq |k| \leq 3 \\
60 & \leq t_{between} \leq 120 \\
40 & \leq t_{exceed} \leq 90 \\
240 & \leq T_{max} \leq 250
\end{align*}$$

5.3.3. **Solving single-objective optimization model based on genetic algorithm.** For this multi-decision variable single-objective optimization problem, considering the inherent implicit parallelism of genetic algorithms and better global optimization capabilities; the use of probabilistic optimization methods can automatically obtain and guide the optimized search space, and adaptively adjust the search direction and find the global optimal solution without definite rules.

This paper still uses genetic algorithm to solve this problem.

The temperature of each temperature zone and the furnace passing speed of the conveyor belt set in the genetic algorithm initialization individual should meet:

$$\begin{align*}
160 & < T_{1-5} < 185 \\
185 & < T_6 < 205 \\
225 & < T_7 < 245 \\
245 & < T_{8-9} < 265 \\
15 & < T_{10-11} < 35 \\
65 & < v < 100
\end{align*}$$

The fitness function is $F = G_1 / (1 + S)$, $G_1$ is a positive real number, and we take 100 in this article. \[S = \begin{cases} Q_H, & \text{Furnace temperature curve } T' \text{ meets the process limit} \\ \inf, & \text{Furnace temperature curve } T'' \text{ does not meet the process limit} \end{cases}\] (25)

When an individual in the population does not meet the process limit, then the fitness function value of the individual is 0, and the individual will be automatically eliminated in the selection process. Individuals in the population that meet the process limits will be retained and continue to be inherited. The genes of individuals with a smaller $Q_H$ will be inherited with a greater probability.

Run through matlab code to get the individual optimal solution.

The following is an iterative diagram of genetic algorithm:
It can be seen from the above figure that when the number of iterations reaches 3 times, the population fitness value converges to 0.22 to reach the optimum. At this time, the temperature in the small temperature zone 1-5 is 181.2°C, the temperature in the small temperature zone 6 is 198.5°C, the temperature in the small temperature zone 7 is 226.6°C, and the temperature in the small temperature zone 8-9 is 263.9°C. The temperature in the small temperature zone 10-11 is 17.9°C. The furnace passing speed of the conveyor belt is 95.2 cm/min. In addition, the furnace temperature curve is shown in Figure 13. The area covered by the furnace temperature curve when the temperature exceeds 217°C to the peak temperature is \( 447.8 \, \text{°C} \).

5.4. Question 4

5.4.1. Problem four analysis. The fourth question is a multi-decision variable dual-objective optimization model based on the first question. Given the set temperature of each temperature zone and the furnace passing speed of the conveyor belt, the furnace temperature curve of the welding area can be obtained according to the finite difference model of Problem 1. According to the information of the
subject, the minimum area covered by the furnace temperature curve to the peak temperature and the part of the furnace temperature curve exceeding 217°C is called the target, and the two-objective optimization model is established to meet the process limit as the constraint condition, and then the two-objective optimization model is established. The model is transformed into a single-objective optimization model for solution.

5.4.2. The establishment of a two-objective optimization model. The function \( T''(t, T_{1-5}, T_6, T_7, T_{8-9}, T_{10-11}, v) \) is the temperature distribution function of the center of the welding zone over time with the temperature of each temperature zone and the furnace passing speed of the conveyor belt as independent variables, that is, the furnace temperature curve.

(1) Determination of optimization goals:

Known from problem three: the control of the optimal range of the area \( Q_H \) covered by the furnace temperature curve exceeding 217°C to the peak temperature determines the control of the IMC thickness, which indirectly affects the reliability of the solder joints, and the closer its value is to the optimal range, the more reliable the quality of the solder joints, that is, the optimization goal is to minimize the heating factor \( Q_H \) during the heating process.

\[
\min Q_H
\]

This question requires that on the basis of question three, consider that the furnace temperature curve that exceeds 217°C on both sides of the center line of the peak temperature should be as symmetrical as possible, that is, the furnace temperature curve must exceed 217°C and the peak temperature is the average of the center line. The degree of deviation \( \sigma \) is the smallest.

\[
\min \sigma
\]

![Figure 14. Schematic diagram of furnace temperature curve](image)

\[
\sigma = \sum_{i=1}^{n} (\Delta t_{i1} - \Delta t_{i2}) / n \tag{26}
\]

Discretize the temperature range between \( T_{\text{max}} \) and 217°C with \( \Delta T = 0.01 \) as the step size, and the degree of deviation of the i-th temperature value from the center line in time is \( \Delta t_{i1} - \Delta t_{i2} \). The smaller the average degree of deviation \( \sigma \) is, the closer the two sides of the curve are to symmetrical. When the average deviation degree \( \sigma = 0 \), the two sides of the curve are completely symmetrical.

(2) Constraints

Similar to the previous two questions, still need to meet the process boundary
The final two-objective optimization model

Objective function:
\[
\begin{align*}
\min Q_H \\
\min \sigma
\end{align*}
\]

Restrictions:
\[
\begin{align*}
0 \leq |k| \leq 3 \\
60 \leq t_{\text{between}} \leq 120 \\
40 \leq t_{\text{raised}} \leq 90 \\
240 \leq T_{\text{max}} \leq 250
\end{align*}
\]

In order to solve the dual-objective optimization problem, this paper uses the linear weighting method to transform the problem into a single-objective optimization problem: 
\[
\text{erro} = \omega Q_H + 1000\sigma (1 - \omega)
\]
and \(\omega\) are linearly weighted weights. Because the heating factor and the average deviation degree of the heating process are not in the same order of magnitude, in order to eliminate Dimensional influence, multiply \(\sigma\) by 1000. In this question, the importance of the heating factor and the average deviation degree is considered to be equal, so \(\omega = 0.5\). So, the optimization goal is transformed into the minimum erro.

5.4.3. Solving dual-objective optimization model based on genetic algorithm. The temperature of each temperature zone and the furnace passing speed of the conveyor belt set in the genetic algorithm initialization individual should meet:
\[
\begin{align*}
160 < T_{1-5} < 185 \\
185 < T_6 < 205 \\
225 < T_7 < 245 \\
245 < T_8-9 < 265 \\
15 < T_{10-11} < 35 \\
65 < u < 100
\end{align*}
\]

The fitness function is:
\[
F = G_1 / (1 + S) \quad (27)
\]

\(G_1\) is a positive real number, and we take 1000 in this article.

\[
S = \begin{cases}
\text{erro} & \text{Furnace temperature curve T' meets the process limit} \\
\text{inf} & \text{Furnace temperature curve T' does not meet the process limit}
\end{cases}
\]

When an individual in the population does not meet the process limit, then the fitness function value of the individual is 0, and the individual will be automatically eliminated in the selection process. Individuals in the population that meet the process limits will be retained and continue to be inherited. The genes of individuals with smaller \(Q_H\) and \(\sigma\), that is, with smaller erro, will be inherited with a greater probability.

Run through matlab to get the individual optimal solution.
The following is an iterative diagram of genetic algorithm:

![Genetic algorithm iteration diagram](image1.png)

**Figure 15.** Genetic algorithm iteration diagram.

It can be seen from the above figure that when the number of iterations reaches 10, the population fitness value converges to 0.09 to reach the optimum. At this time, the temperature in the small temperature zone 1-5 is 183.4°C, the temperature in the small temperature zone 6 is 188.9°C, the temperature in the small temperature zone 7 is 226.7°C, and the temperature in the small temperature zone 8-9 is 261.2°C. The temperature in the small temperature zone 10-11 is 25.1°C. The furnace passing speed of the conveyor belt is 88.5 cm/min.

The corresponding furnace temperature curve at this time is:

![Furnace temperature curve](image2.png)

**Figure 16.** Furnace temperature curve

The heating factor $Q_\text{H}$ of the heating process is $494.9308 \cdot ^\circ C \cdot s$, and the average degree of deviation $\sigma$ is 0.1749. It is considered that the curve has good symmetry.
6. Model Assessments and Promotion

This paper comprehensively considers the influence of various parameters on the change of the furnace temperature curve, establishes the temperature change model of the welding area, and applies it to the optimal control and design of the furnace temperature curve. The model and algorithm established in this paper have the following advantages and disadvantages.

6.1. Strengths

In the first question, considering that the temperature distribution of the temperature zone gap is one-dimensional, stable and without heat source, a temperature distribution model of the reflow furnace temperature field is further established. Comprehensively consider the influence of the welding zone constant pressure specific heat capacity, area, density, volume and convection heat transfer coefficient on the heating model, and based on the genetic algorithm and finite difference theory to calculate the furnace temperature curve of the given experiment in the question, and the actual measurement The relative error of the furnace temperature curve is found to be small, which verifies that the established model has good accuracy.

In the second question, a single-objective optimization model established with the maximum furnace passing speed as the goal. The maximum passing speed of the conveyor belt is solved by traversal search, which is less restricted by geometric conditions and can describe the physical process of the conveyor belt more realistically.

In the third question, considering that the heating factor of the heating part should be minimized as the optimization goal, a single-objective optimization model is established, which is solved based on the genetic algorithm, and the furnace temperature curve obtained is better.

In question four, based on the third question, the symmetry of the furnace temperature curve is further considered, a dual-objective optimization model is established, and the solution is based on genetic algorithm, which shows the temperature curve has good symmetry.

6.2. Weaknesses

The genetic algorithm used in the article has a certain instability. The error of the search algorithm in the second question has a certain probability.

6.3. Promotion

Reflow soldering is an important technology on the PCB production line. The solution results of the temperature change model of the soldering area established in this paper are more accurate than the measured data. By adjusting the parameters, it can be extended to other types of reflow furnaces and can be widely used.

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