Current-Controlled Majorana Bound States Using Magnetic Stripes

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A hybrid semiconductor-superconductor nanowire on the top of a magnetic film in the stripe phase experiences a magnetic texture from the underlying fringing fields. The Zeeman interaction with the highly inhomogeneous magnetic textures generates a large synthetic spin-orbit coupling. We show that this platform can support the formation of Majorana bound states (MBS) localized at the ends of the nanowire. The transition to the topological superconducting phase not only depends on the nanowire parameters and stripe size but also on the relative orientation of the stripes with respect to the nanowire axis. Therefore, by passing a charge current through the magnetic film to reorient the stripes, a topological phase transition with the corresponding emergence or destruction of MBS can be induced. The proposed platform removes the need for external magnetic fields and offers a non-invasive MBS tuning with the current flowing through the magnetic film, but not through the semiconductor.

Impressive experimental advances have made possible the realization and detection of Majorana bound states (MBS) in systems under certain conditions can be driven into the topological superconducting phase \cite{1–9}. Because of their non-Abelian statistics and topological protection, MBS are promising for the implementation of fault-tolerant topological quantum computing \cite{10, 11}. Such an implementation requires not only the creation but also the manipulations of MBS in order to realize braiding and fusion operations \cite{10–15}. The majority of proposals relies on systems with strong Rashba SOC and the use of an external magnetic field as a tuning tool \cite{16–24}. However, MBS can still be realized without the need for Rashba SOC nor external magnetic fields if appropriate magnetic textures are used. It is known that magnetic textures can generate both Zeeman and synthetic spin-orbit coupling \cite{25–28}, which together with superconductivity provide the basic ingredients for the formation of MBS. Therefore, there has been an increasing interest in the use of magnetic textures for the generation of MBS \cite{29–40}. In particular, the use of locally tunable magnetic fringing fields can enable the realization of both braiding and fusion operations \cite{41–43}. However, an experimental confirmation of the creation of MBS by using magnetic textures generated by fringing fields is still lacking \cite{44}. Here we show that in spite of its nonhelical character, the fringing field generated by experimentally created magnetic stripes in a Co/Pt multilayer thin film can support the formation of MBS in a semiconductor nanowire (NW) with proximity induced superconductivity. The controllability of the stripes orientation by means of a charge current allows for electrically tuning topological phase transitions in the NW.

The magnetic films consist of Co/Pt multilayers deposited on oxidized silicon wafers in an ultra-high vacuum physical vapor deposition system (Kurt Lesker PVD-12) by magnetron sputtering with the substrate at ambient temperature. The film consists of 20 Pt|Co repeats on a thin Ta seed layer capped with Pt, specifically, Ta(3nm)|[Pt(0.3nm)|Co(0.6nm)]x20|Pt(0.3nm). The domain images are obtained with a magnetic force microscope (MFM) at room temperature, shown in Fig. 1(a). An in-plane magnetic field of about 1 T is initially applied to align the stripes along the field direction, as shown in Ref. 45, (e.g. see Fig. 14 of [22]). The formed stripes are

![Magnetic force microscopy image](image-url)

**FIG. 1.** (a) Magnetic force microscopy image of the magnetic stripe phase formed in a Co/Pt multilayer thin film. (b) Schematic picture of the proposed set-up, consisting of an InAs-Al nanowire placed on the top of magnetic stripes. The nanowire is separated from the magnetic film by a thin insulating spacer (gray layer). (c) Magnetic fringing field pattern experienced by the carriers in the nanowire for a 5 nm thick insulating spacer and stripe domains of width (W) 160 nm, thickness (Lz) 40 nm, and domain-wall width (S) 20 nm. The color represents the out-of-plane component (Bz). The saturation magnetization (Ms) is 1.7 x 10^6 A/m. The nanowire is positioned at the middle, rectangular region (green) with dimensions 2 µm x 50 nm. (d) Variation of the three components of the fringing field with distance (x) along the length of the nanowire.
stable and survive for a long time after the external magnetic field is switched off. In this study, as detailed below, we explore the possibility of using the fringing fields generated by the stable stripes in the absence of an external magnetic field for inducing topological phase transitions in a semiconductor NW with proximity-induced superconductivity.

A schematic of the proposed set-up is shown in Fig. 1(b). Superconductivity is induced in the InAs NW by proximity to the Al half-covering. Although any semiconducting NW with high g-factor could be suitable, we consider InAs in which epitaxial growth and proximity-induced superconductivity have been demonstrated [46–48]. The NW is separated from the magnetic film by an insulator spacer (gray layer). We use realistic micromagnetic modeling of the magnetic textures using the finite-element method in COMSOL [49]. The magnetic fringing fields are simulated using stripe domains similar to the ones formed in experimentally realizable Co/Pt multilayer thin films. Due to the complexity of the problem, we consider a simplified, two-dimensional version of the actual NW. The z-component of the fringing field \( B_z \) in the plane of the NW is shown, as a function of position, in Fig. 1(c), where the green region represents the InAs NW with dimensions \( 2 \mu m \times 50 \) nm. For the numerical simulations we assumed a 5 nm thick insulating spacer and a film section of size \( 2 \mu m \times 100 \) nm and saturation magnetization \( M_s = 1.7 \times 10^6 \) A/m [50]. The stripes have a width \( W = 160 \) nm and are separated by domain-walls of thickness \( S = 20 \) nm. Changes in the fringing field components \( B_x, B_y, \) and \( B_z \) along the x axis are displayed in Fig. 1(d). The maximum amplitude of the fringing field lies below the critical field of about 1.9 T experimentally measured in InAs/Al NWs [51]. We note that although \( B_y \) is negligibly small and both \( B_x \) and \( B_z \) exhibit an oscillatory behavior, the z-component of the fringing field largely dominates. The resulting fringing field is therefore quite different from the helical-like and skyrmion-like textures considered in previous investigations of MBS [36, 37]. Therefore, it is somehow surprising that in spite of having a non-helical character, the textures illustrated in Fig. 1(d) can also support the formation of MBS, as shown below. This leads to the general question of which kind of magnetic textures, and therefore, which kind of synthetic SOC are compatible with the generation of MBS. An approximate topological condition as a function of slowly varying magnetic textures has been previously derived [41, 42] but a general answer to the question remains open.

The InAs NW is proximity coupled to superconducting Al and described by the following Hamiltonian,

\[
\mathcal{H} = -i \sum_{(i,j),\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + H.c.) - g^* \mu_B/2 \sum_i (\mathbf{B} \cdot \mathbf{\sigma})_{\sigma\sigma'} c^\dagger_{i\sigma} c_{i\sigma'} - \mu \sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma} + \Delta \sum_i (c^\dagger_{i1} c_{i1}^\dagger + H.c.),
\]

where \( c^\dagger_{i\sigma} (c_{i\sigma}) \) is the fermionic creation (annihilation) operator at site \( i \) with spin \( \sigma \), \( t = \hbar^2/(2m^*a^2) \) is the nearest-neighbor electron hopping energy, \( m^* \) is the effective mass of electrons and \( a \) is the unit lattice spacing of the underlying two-dimensional square lattice. The real-space information of the fringing field \( \mathbf{B} \) is contained in the Zeeman interaction (second term), where \( g^* \) is the effective g-factor of electrons, \( \mu_B \) is the Bohr magneton and \( \mathbf{\sigma} \) represents the Pauli spin matrices. The chemical potential measured from the bottom of the conduction band is represented by \( \mu \), and \( \Delta \) is the proximity induced superconducting gap. We use \( m^* = 0.026 m_0 \) (effective mass for InAs), \( a = 10 \) nm, \( \Delta = 0.2 \) meV, and unless otherwise specified, \( g^* = 15 \) [52].

The Hamiltonian (1) is solved by exact diagonalization after performing a transformation of the fermionic operators to the Bogoliubov-de Gennes (BdG) basis: \( c_{i\sigma} = \sum_n u^n_{\sigma} \gamma_n + v^n_{\sigma} \gamma_n^\dagger \), where \( u^n_{\sigma}, (v^n_{\sigma}) \) is the BdG quasiparticle (quasihole) amplitude, and \( \gamma_n^\dagger (\gamma_n) \) is a fermionic creation (annihilation) operator of a BdG quasiparticle or quasihole in the \( n \)th energy eigenstate.

As \( \mu \) is increased, the energy gap closes for certain range of \( \mu \) and zero-energy MBS emerge, as shown in Fig. 2(a), where the computed low-energy spectrum is displayed. Within this range (plotted in red), other energy levels are shifted away from zero energy, creating a minigap that protects the MBS. Near \( \mu = -0.08 \) meV, the lowest pair of energy levels come closer to zero energy but do not meet each other to produce robust MBS. To illustrate the effect of increasing the \( g^* \)-factor, the \( g^* \) dependence of the low-energy spectrum for \( g^* = 30 \) [i.e., twice the InAs \( g^* \)-factor used in Fig. 2(a)] is shown in Fig. 2(b). In practice, the effective \( g^* \)-factor can be enhanced in semiconducting quantum wells using alloys such as InAs\(_x\)Sb\(_{1-x}\) [53]. Magnetically doped semiconductors such as (In,Mn)As can reach \( g^* \) factors, the MBS are not only more robust but they appear within multiple ranges of \( \mu \). The existence of multiple topological regions originates from the mixing of different subbands when the \( g^* \)-factor is large enough. To visualize the real-space localization of the MBS, we plot the probability density \( |\Psi|^2 = \sum_n |u^n_{\sigma}\|^2 + |v^n_{\sigma}\|^2 \) corresponding to the lowest energy eigenstates at \( \mu = -0.4 \) meV (with existing MBS) and \( \mu = -0.7 \) meV (without any MBS) for \( g^* = 15 \), as shown in Fig. 2(c) and 2(d), respectively. The states are sharply localized near the ends of the NW for \( \mu = -0.4 \) meV, and inside the NW for \( \mu = -0.7 \) meV.
clearly distinguishing between the MBS and the non-MBS states. The probability densities of the two MBS at the two NW ends decay exponentially with distance and create a finite overlap, resulting in a finite splitting of the lowest energy pair when the length of the NW is small (not shown here). We also plot the local charge density \( \rho_i \), in units of e/nm\(^2\), corresponding to the lowest-energy state at (c) \( \mu=0.4 \) meV and (d) \( \mu=0.7 \) meV for \( g^*=15 \). Real-space profile of the charge density \( \rho_i \) for \( g^*=15 \).

In the case of a helical texture, the strength of the generated synthetic SOC is given by the helix wave number \( q = 2\pi/\lambda \) with \( \lambda \) denoting the period of the helix. In our platform the fringing field, although of non-helical character, is characterized by the stripes period \( \lambda = 2W + S \), as illustrated in Fig. 1(b). It is therefore relevant to investigate the topological phase transition of the proximitized NW as a function of \( \lambda \). The position dependence of the \( x \) and \( z \) components of the fringing fields generated by a stripe phase with periods \( \lambda = 200 \) nm and \( \lambda = 500 \) nm are shown in Figs. 3(a) and(b), while the corresponding low-energy spectrum as a function of \( \mu \) is displayed in Figs. 3(c) and(d), respectively. We note that both \( \lambda = 200 \) nm and \( \lambda = 500 \) nm leads to the formation of MBS for appropriate values of \( \mu \). However, the MBS for the longer period appear to be more robust, as seen from the fact that the minigap and the range of \( \mu \) within which the MBS emerge are larger for \( \lambda = 500 \) nm. Furthermore, for \( \lambda = 500 \) nm, multiple topological regions develop as the result of subbands mixing.

In order to better understand the effects of both \( \lambda \) and \( \mu \) on the realization of the topological superconducting phase, we first consider, for the sake of comparison, the case of a one-dimensional (1D) NW under a helical mag-

![FIG. 2. Energy spectrum, plotted as a function of the chemical potential (\( \mu \)), for stripes with period \( \lambda = 360 \) nm for (a) \( g^* = 15 \) and (b) \( g^* = 30 \). The red dots show the zero-energy MBS. Real-space profile of the probability density \( |\Psi_i|^2 \), in units of 1/nm\(^2\), corresponding to the lowest-energy state at (c) \( \mu=0.4 \) meV and (d) \( \mu=0.7 \) meV for \( g^*=15 \). Real-space profile of the charge density \( \rho_i \), in units of e/nm\(^2\), corresponding to the lowest-energy state at (e) \( \mu=0.4 \) meV and (f) \( \mu=0.7 \) meV for \( g^*=15 \).](image)

![FIG. 3. Two components (\( B_x \) and \( B_z \)) of the fringing field, plotted as a function of the distance (\( x \)) along the length of the nanowire for two different stripe periods: (a) \( \lambda = 200 \) nm and (b) \( \lambda = 500 \) nm. Low-energy spectrum as a function of the chemical potential (\( \mu \)) for (c) \( \lambda = 200 \) nm and (d) \( \lambda = 500 \) nm. (e) Phase diagram in the \( \mu - \lambda \) plane, showing the topological superconducting regime (in cyan), calculated for a one-dimensional (1D) nanowire in the presence of a helical texture with maximum field amplitude \( B = 1.3 \) T. The red lines obtained from numerically solving the BdG equation represent some of the \( \mu \) values for which MBS exist in the nanowire under the fringing field of magnetic stripes with different values of the period \( \lambda \). (f) Phase diagrams for a 1D nanowire in a helical texture with different values of field amplitudes: \( B = 0.5 \) T (blue), \( B = 0.7 \) T (green), \( B = 0.9 \) T (yellow), \( B = 1.1 \) T (red).](image)
netic texture. By defining \( g^* \mu_B B/2 - J \), where \( B \) is the amplitude of the helical magnetic field, the topological condition can be written as \( J^2 > (\mu - \eta)^2 + \Delta^2 \), where \( \eta = h^2 q^2/8m^* \) and \( q = 2\pi/\lambda \). This imposes boundaries on the values of \( \lambda \) for which the topological phase can be reached,

\[
\sqrt{\frac{\pi^2 t}{\mu + \sqrt{J^2 - \Delta^2}}} < \lambda < \sqrt{\frac{\pi^2 t}{\mu - \sqrt{J^2 - \Delta^2}}},
\]

where the upper bound applies only when \( \mu > \sqrt{J^2 - \Delta^2} \).

The corresponding phase diagram in the \( \mu-\lambda \) plane is shown in Fig. 3(e), where the cyan region represents the topological superconducting regime specified by Eq. (2) at \( B = 1.3 \) T. To compare this phase diagram with the solutions of Hamiltonian (1) for our platform, we compute the energy spectrum for several values of \( \lambda \), ranging from 150 nm to 520 nm, and plot the values of \( \mu \) at which MBS appear [red dots in Fig. 3(e)] in the same plot. Interestingly, in spite of the non-helical character of the magnetic texture generated by the stripes, many of the topological regions obtained for the set-up in Fig. 1(b) lie within the topological region of the helical texture on a 1D NW. Therefore, the phase diagram for the magnetic helix case still can be used for estimating the system parameters leading to the formation of MBS in the 2D-NW/stripes set-up, specially for \( \mu \lesssim 0.5 \) meV. For larger values of \( \mu \), the occupancy of multiple subbands in the 2D NW under the non-helical fringing field of the stripes leads to re-entrance into the topological regime and the emergence of MBS beyond the topological region of the 1D-NW/magnetic helix structure (note the multiple red dots outside the cyan region in Fig. 3(e)).

A phase diagram including the magnetic helix amplitude, in addition to \( \lambda \) and \( \mu \) is shown in Fig. 3(f) for the 1D-NW/magnetic helix structure. As explained above, we can use this phase diagram to draft some conclusions about the system parameters required to drive the proposed platform into the topological regime. Thus, Fig. 3(f) suggests that it is desirable to have larger fringing fields, wider stripes, and a smaller chemical potential to favor the formation of robust MBS.

We have also investigated the stability of MBS in the presence of random fluctuations of the fringing fields generated by the magnetic stripes. In particular, we found that the MBS are not only robust against this kind of disorder but, surprisingly, the topological region where MBS emerge can even be enhanced by increasing the strength of the magnetic field fluctuations [65, 66].

A particularly attractive functionality of the proposed platform for switching of topological phase transitions relies on the possibility of changing the magnetic stripes orientation by passing a current through the Co/Pt multilayer thin film [45, 67] and reorienting the stripes from being perpendicularly to being parallel to the NW. This modifies the form of the fringing fields experienced by the carriers in the NW and, eventually, affect the topological superconducting phase. Two relevant scenarios might occur when the stripes are parallel to the NW: (i) the NW is located on the top of the domain wall between two adjacent stripes. (b) same as in (a) but for a nanowire located on the top of a single stripe. (c) and (d) Low-energy spectrum as a function of the chemical potential, \( \mu \), for the cases (a) and (b), respectively. Parameters: \( \lambda = 360 \) nm, \( g^* = 15 \).

FIG. 4. (a) Spatial profile of the out-of-plane component, \( B_z \), of the fringing field generated by stripes parallel to a nanowire located on the top of the domain wall between two adjacent stripes. (b) same as in (a) but for a nanowire located on the top of a single stripe. (c) and (d) Low-energy spectrum as a function of the chemical potential, \( \mu \), for the cases (a) and (b), respectively. Parameters: \( \lambda = 360 \) nm, \( g^* = 15 \).
transition in the NW without the need for an external magnetic field. The driving current flows through Co/Pt multilayer film, which is isolated by the spacer from the NW and although electric contacts on the NW may be needed for MBS detection, the current-induced topological phase transition occurs in a non-invasive manner, i.e., with no charge transfer.

The detection of the MBS can be performed using tunneling spectroscopy by attaching metallic leads at the two ends of the NW and by looking for a zero-bias conductance peak (ZBCP) \[4, 58, 68-70\]. The current-induced switching of the stripes orientation offers an additional knob not only for investigating the nature of the ZBCP and its relation to the presence of MBS, but also for exploring their robustness as a function of tunable synthetic SOC. Furthermore, the proposed set-up could provide a proof-of-concept demonstration of the feasibility of using current-controlled synthetic SOC for the manipulation of MBS beyond recent experimental advances \[44\], paving the way to more complex platforms with extended tunability for the realization of braiding operations \[41, 42\].

This work is supported by DARPA Grant No. DP18AP900007, US ONR Grant No. N000141712793 (I. Ž. and A. M.-A.), and the UB Center for Computational Research. This work was performed in part at the Advanced Science Research Center NanoFabrication Facility of the Graduate Center at the City University of New York.

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