Torsion stability of a cylinder with circular and elliptical section under finite perturbations

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Abstract. In this paper, the stability of a cylinder with circular and elliptical section made of compressible and incompressible nonlinear elastic materials under finite perturbations is considered. A set of computational experiments was performed. The permissible boundaries of the region with respect to final and initial perturbations for given parameters of loading and structures are established. Finite sequences of bifurcation points are constructed, confirming, in contrast to stability at small perturbations, the existence of a hierarchy of stable equilibrium states. Classical linearized stability theories are evaluated. New phenomena and characteristic effects are established.

1. Introduction
In [1-3] the stability of the simplest problems of nonlinear elastic and nonlinear viscoelastic material under finite perturbations, when the subcritical state is homogeneous, is studied. In this paper, a solution to the problem of torsional stability of a cylinder with circular and elliptical section made of compressible and incompressible nonlinear elastic materials under finite perturbations is given. The problem of torsion stability of a cylinder with respect to small perturbations was considered in [4]. The analysis of the basic process of plate deformation is reduced to solving a nonlinear boundary value problem with respect to finite perturbations. Solutions for displacement perturbations are chosen in the form of eigenfunction series which are solutions of corresponding linearized problems and satisfy geometric boundary conditions [5]. After applying the principle of possible displacements, the question of the stability of the ground state of a nonlinear problem is reduced to the study of the stability of the zero solution of an infinite system of ordinary differential equations with constant coefficients, the number of terms in which is specified by the elastic potential. For the obtained system of equations, a function is constructed, which, under certain restrictions on the initial perturbations, is a the Lyapunov function [1-3].

2. Materials and methods
Let a round cylinder of radius \( R \) and length \( L \) be rigidly fixed at the end (plane) \( z=0 \), and a pair of forces is applied to the surface \( z=L \) with a moment \( M \). We assume that there is a simple torsion, i.e. the planes perpendicular to the axis of the cylinder rotate in their own planes by an angle \( \tau \). To simplify the solution, we consider a two-constant potential as in [1].
The field of displacements in the subcritical state is found in the form of a series of small parameter \( \delta \equiv \tau \) – twist, that is

\[
0 \quad u_i = \tau u_i + \tau^2 u_i + \tau^3 u_i + \ldots.
\]

The solution to this problem within the linear theory of elasticity was chosen as the first approximation. It has the form:

\[
0 \quad u_1 = -\frac{1}{2} i\xi z; \quad u_2 = \frac{1}{2} i\zeta z; \quad u_3 = 0.
\]

Here and further \( \xi_i \) are curvilinear coordinates:

\[
\xi_1 = x + iy = \zeta; \quad \xi_2 = x - iy = \overline{\zeta}; \quad \xi_3 = z,
\]

and \( x, y, z \) are Cartesian coordinates.

For the second and third approximations, the expressions for displacement components (1) are found as

\[
0 \quad u_1^{(2)} = -\frac{1}{2} \xi z; \quad u_2^{(2)} = \frac{1}{2} \zeta z;
\]

\[
u^{(2)}_3 = \frac{\mu (\lambda - \mu)}{3(\lambda + \mu)(\lambda + 2\mu)} z^3 - \frac{(\lambda + \mu)}{3(\lambda + 2\mu)} \frac{(\lambda - \mu)}{2(\lambda + \mu)} \xi z.
\]

\[
u^{(3)}_1 = -\frac{1}{2} i\xi z + \frac{3\mu + \lambda}{6(\lambda + \mu)} i\zeta z^3; \quad u_2^{(3)} = \frac{1}{2} i\zeta z - \frac{3\mu + \lambda}{6(\lambda + \mu)} i\xi z^3;
\]

\[
u^{(3)}_3 = \left[ \frac{2(\lambda + \mu)}{3(\lambda + 2\mu)} - \frac{2\mu (\lambda - \mu)}{3(\lambda + \mu)(\lambda + 2\mu)} \right] i\zeta^3 + \frac{\lambda - \mu}{\lambda + \mu} i\xi z.
\]

To determine the displacement field (1) of a round body in three approximations (2), (4), the equations of state, geometric relations in the form of Green’s formulas, equilibrium equations and boundary conditions were involved [2].

Composing the variational equation of the Bubnov-Galerkin method and searching [2] its solution in the form of series:

\[
u_i (\xi, t) = \sum_m f_m(t) \cdot \varphi_m (\xi); \quad (j = 1, 2, 3; \quad m = 1, 2, \ldots \infty),
\]

where \( f_j(t) \) are time-dependent undefined coefficients; \( \varphi_m^j(\xi) \) are functions satisfying geometric boundary conditions for \( u_m(\xi, t) \). Here, the repeated indices are summed from 1 to 3, if the dimension of the index is not specifically specified.

As variations we will accept expressions:

\[
\delta u_j = \sum_m \varphi_m^j (\xi) \cdot \delta f_m(t).
\]

Consideration of the convergence of series (5) presents great difficulties. Currently, there is no satisfactory proof of the convergence of a number of Bubnov-Galerkin method, which, however, does not prevent the use of this method in practical calculations. As a system of functions \( \varphi_m^j(\xi) \) we will
take the known forms corresponding to the linearized problem. A sufficient condition for the convergence of series (5) is the condition of completeness of the system of functions \( \varphi_i(\xi) \) [6].

From the series (5) corresponding to a nonlinear boundary value problem with a two-constant potential, we have a system of ordinary differential equations with constant coefficients with respect to \( f_k(t) \). This system of equations might be represented in the form

\[
A_k f_k + C_{np} f_p + D_{njk} f_j f_k + L_{nk} f_p f_j f_k = 0.
\]

(6)

The coefficients of the system are:

\[
A_k = \rho_0 \int \varphi^*_i \varphi_i dV; \quad C_{np} = \int \nabla \varphi^*_i \varphi_i dV;
\]

\[
D_{njk} = \int \nabla \varphi^*_i \varphi_i dV; \quad L_{nk} = \int \nabla \varphi^*_i \varphi_i dV.
\]

(7)

In the case of a two-contact potential \( T^{ijp} \), \( T^{ijpk} \), \( T^{ijpl} \) are

\[
T^{ijp} = g^{ijp} \nabla \varphi_i \varphi_p + \left( 2g^{ijp} \lambda \epsilon_i^0 + 2\mu g^{ijp} \epsilon_c^0 \right) \left( \nabla \varphi_i \varphi_p \right);
\]

(8)

\[
T^{ijpk} = \left( 2g^{ijp} \lambda \epsilon_i^0 + 2\mu g^{ijp} \epsilon_c^0 \right) \left( \nabla \varphi_i \varphi_p \right);
\]

\[
T^{ijpl} = \left( 2g^{ijp} \lambda \epsilon_i^0 + 2\mu g^{ijp} \epsilon_c^0 \right) \left( \nabla \varphi_i \varphi_p \right);
\]

where

\[
\epsilon_i^0 = \frac{1}{2} g^{ijp} f_i f_p \left( \nabla \varphi_i \varphi_j + \nabla \varphi_j \varphi_i + \nabla u^0 \varphi_i + \nabla \varphi_i \varphi_i + \nabla \varphi_i g^m \nabla u^0 \right) = f_i \epsilon_i^0;
\]

(9)

Here

\[
S^{ij} = S^{ij}(1) + S^{ij}(2),
\]

(10)

where

\[
S^{ij}(1) = 2g^{ijp} \lambda \epsilon_i^0(1) + 2\mu g^{ijp} \epsilon_c^0(1); \quad S^{ij}(2) = 2g^{ijp} \lambda \epsilon_i^0(2) + 2\mu g^{ijp} \epsilon_c^0(2).
\]

(11)

The Lyapunov function [3] is defined as

\[
\Pi = \frac{1}{2} C_{np} f_p f_n + \frac{1}{3} D_{njk} f_p f_j f_k + \frac{1}{4} L_{nk} f_p f_j f_k f_n - \sum_{n,p,k,l}^n.
\]

(12)

The stability condition will be the positivity of the Lyapunov function [3] for the values of initial perturbations found from the condition

\[
C_{np} f_p f_n + D_{njk} f_p f_j f_k + L_{nk} f_p f_j f_k f_n = 0, \quad \sum_{n,p,k,l}^n.
\]

(13)
The question of the sign-definiteness of the Lyapunov function (11) was investigated by numerical methods. The perturbation region was divided by a grid and for each parameter of the load (moment $M$) using the algebraic system of equations (12) a perturbation region $f$ in which the function $\Pi$ would be positive was determined. According to Lyapunov’s first stability theorem, the zero solution of a system of equations will be stable and, therefore, the ground (subcritical) state will be stable.

![Figure 1. Dependence of the torque on the dimensionless twist.](image1)

![Figure 2. Dependence of the permissible initial disturbance on the torque.](image2)

The results of the numerical experiment are presented in figure 1 and 2. In figure 1 we show a relationship between torque $M$ and dimensionless twist $\Psi$ in the case of stability studies in "small" [4]. In the case of stability studies in "large", the dependence of the permissible initial perturbation $|f|$ on the torque $M$ is shown in figure 2. The ratio of cylinder length to radius was assumed to be $\frac{L}{R} = 3$.

The $M_2$ value in figure 2 corresponds to the linearized stability theory and coincides at $\Psi=0.3$, with the point $M_2$ in figure 1. Numerical simulation was carried out for polystyrene [5].

It follows from the figures that a twisted cylinder of circular cross-section can lose stability before the critical moment obtained by the three-dimensional linearized theory, if only the initial perturbations exceed the absolute value of the permissible value.

Solid line on the plane $|f| - M$ (figure 2, $\Psi=0.3$) represents the upper bound of the stability domain, that is, the countable sequence of stable equilibrium states.

In the case of the study of the torsion stability of an elliptic body, for simplification, we take the Mooney potential [5], which describes the behavior of an incompressible material:

$$W = 2\lambda A_1 + 2\mu (2A_1 + A_2^2 - A_2),$$

where $A_1 = \varepsilon'_i; \ A_2 = \varepsilon'_i \cdot \varepsilon'_i$ are algebraic invariants.
The displacement field in the main state in the cylinder torsion task of the elliptical section was also found as a row (1) according to the small parameter \( \delta \), \( \delta \) – twist.

As the first approximation, the displacement field corresponding to the problem of linear elasticity theory was chosen, which has the form

\[
\begin{align*}
&u_1^{(0)} = -\frac{1}{2} i \xi z; \quad u_2^{(0)} = \frac{1}{2} i \xi z; \quad u_3^{(0)} = i \frac{R}{4} \left( \xi^2 - \zeta^2 \right), \quad p^{(0)} = 0, \\
&u_1^{(2)} = -\frac{1}{2} i \xi z - \frac{R}{12} \zeta z; \quad u_2^{(2)} = \frac{1}{2} i \xi z + \frac{R}{12} \zeta z; \quad u_3^{(2)} = -\frac{1}{3} \zeta z; \\
p^{(2)} = -\left( p_0 + 2C_{01} \right) \xi^2 - 3C_{01} \xi \zeta - C_{01} R^2 \xi \zeta + 2C_{01} R \zeta^2 + 2C_{01} R \xi \zeta^2.
\end{align*}
\]

where \( R = \frac{a^2 - b^2}{a^2 + b^2} \), \( a \) and \( b \) are large and small semi-axis of the ellipse.

Based on the basic relations for nonlinear elastic bodies [1], taking into account the conditions of rigid fixation \( u_{i3} = 0; \quad u_{23} = 0; \quad u_{33} = 0; \quad u_{13} = 0; \quad u_{22} = 0; \quad u_{33} + u_{32} = 0, \quad j = 1, 2, 3 \), the values standing at \( \tau^2 \) and \( \tau^3 \) in (1) are obtained. They are

\[
\begin{align*}
&u_1^{(3)} = -\frac{1}{2} i \xi z + \frac{C_{01}}{3 \left( p_0 + 2C_{01} \right)} i \xi \zeta + i \frac{R}{12} \zeta^3 + \frac{R}{6} \zeta^3 + \frac{C_{01}}{p_0 + 2C_{01}} i \xi \zeta^3; \\
&u_2^{(3)} = \frac{1}{2} i \xi z - \frac{C_{01}}{3 \left( p_0 + 2C_{01} \right)} i \xi \zeta^3 + i \frac{R}{12} \zeta^3 - \frac{R}{6} \zeta^3 - \frac{C_{01}}{p_0 + 2C_{01}} i \xi \zeta^3; \\
&u_3^{(3)} = \frac{2}{3} \zeta^3; \\
p^{(3)} = i \left[ 2 \left( p_0 + 2C_{01} \right) \xi^2 + 6C_{01} \xi \zeta - 2C_{01} R \zeta^2 - 2C_{01} R \zeta^2 z + 2C_{01} R \zeta^2 z + 2C_{01} R \zeta^2 z \right].
\end{align*}
\]

In this case the stress field defined up to the terms \( O(\tau^3) \) has the form

\[
T^{ij} = T^{ij(0)} + \tau^3 T^{ij(2)} + \tau^3 T^{ij(3)},
\]

where

\begin{align*}
T^{ij(0)} & = 2 g^{ij} C_{10} + 2 \left( 3 g^{ij} - g^{ii} g^{jj} g_{rs} \right) p_0 g^{ij}; \\
T^{ij(1)} & = 2C_{01} B^{ij(1)} + p_0 G^{ij(1)} + p^{(1)} G^{ij(0)}; \\
T^{ij(2)} & = 2C_{01} B^{ij(2)} + p_0 G^{ij(2)} + p^{(1)} G^{ij(1)} + p^{(2)} G^{ij(0)} + \left[ 2C_{01} B^{ij(0)} + p_0 G^{ij(0)} + p^{(1)} G^{ij(0)} \right] \nabla \dot{u}^{(1)}; \\
T^{ij(3)} & = 2C_{01} B^{ij(3)} + p_0 G^{ij(3)} + p^{(1)} G^{ij(2)} + p^{(2)} G^{ij(1)} + p^{(3)} G^{ij(0)} + \left[ 2C_{01} B^{ij(0)} + p_0 G^{ij(0)} + p^{(1)} G^{ij(0)} + p^{(2)} G^{ij(0)} \right] \nabla \dot{u}^{(2)} + \left[ 2C_{01} B^{ij(0)} + p_0 G^{ij(0)} + p^{(1)} G^{ij(0)} + p^{(2)} G^{ij(0)} \right] \nabla \dot{u}^{(3)}; \\
p_0 & = -(4 + 2C_{10}).
\end{align*}
Expressions for the metric tensor $G_{ij}$ of the deformable body, the component of the strain tensor $\varepsilon_{ij}$ [1, 2] and other quantities are determined by formulas of type (16).

In the cylindrical coordinate system, the basis functions of the solution corresponding to the nonlinear boundary value problem [1, 2] are taken in the form

$$
\phi_{1m} = -\frac{1}{2}\tau ir(\cos m\phi - i\sin m\phi)z - \frac{1}{2}\tau^2 \left[ r(\cos m\phi - i\sin m\phi)z + \frac{\kappa}{6} r^3 (\cos 3m\phi - i\sin 3m\phi) \right] + \\
+ \frac{\tau^3}{2} \left[ -r(\cos m\phi - i\sin m\phi)z + \frac{2C_{10}}{3(p_0 + 2C_{10})} r(\cos m\phi - i\sin m\phi)z^3 + \\
\frac{\kappa}{6} r^3 (\cos 3m\phi - i\sin 3m\phi) + \frac{\kappa}{3} r(\cos m\phi + i\sin m\phi)z^3 + \\
+ \frac{2C_{10}}{p_0 + 2C_{10}} r\kappa(\cos m\phi + i\sin m\phi)z^3 \right];
$$

$$
\phi_{2m} = -\frac{1}{2}\tau ir(\cos m\phi + i\sin m\phi)z + \frac{1}{2}\tau^2 \left[ r(\cos m\phi + i\sin m\phi)z + \frac{\kappa}{6} r^3 (\cos 3m\phi + i\sin 3m\phi) \right] + \\
+ \frac{\tau^3}{2} \left[ r(\cos m\phi + i\sin m\phi)z - \frac{2C_{10}}{3(p_0 + 2C_{10})} r(\cos m\phi + i\sin m\phi)z^3 + \\
\frac{\kappa}{6} r^3 (\cos 3m\phi - i\sin 3m\phi) - \frac{\kappa}{3} r(\cos m\phi - i\sin m\phi)z^3 - \\
- \frac{2C_{10}}{p_0 + 2C_{10}} r\kappa(\cos m\phi - i\sin m\phi)z^3 \right];
$$

$$
\phi_{3m} = -\frac{\kappa}{2} r^2 \sin 2m\phi - \frac{\tau^3}{3} z^3 + \frac{2\tau^2iz^3}{3}; \quad m = 1, 2, \ldots
$$

$$
\phi_{4m} = p_0 + \tau^2 \left[ -3C_{10}r^2 - (2C_{10} + p_0)z^3 - C_{10} \left( \frac{a^2 - b^2}{a^2 + b^2} \right)^2 r^2 + 4C_{10} \kappa r^2 \cos 2m\phi \right] + \\
+ \tau^3 \left[ 6C_{10}r^2 + 2(2C_{10} + p_0)z^3 - 4C_{10} \kappa r^2 \cos 2m\phi + 4C_{10} \kappa r^2 i \sin 2m\phi \right]. \quad (18)
$$

The system of equations with respect to generalized coordinates $f_a(t)$ has the form:

$$
A^p_n \ddot{f}_p + C^p_n f_p + D^p_{nk} f_k f_l + E^p_{nk} f_p f_k f_l + R^{pkl}_{nk} f_p f_k f_l f_d + \\
+ M^p_{nk} f_p f_k f_l f_d f_e = 0, \quad (n = 1, 2, \ldots; \quad p, k, l, \ldots, c = 1, 2, \ldots), \quad (19)
$$

where the coefficients are:

$$
A^p_n = \rho_0 \int_V \phi^p_{\mu\nu} \phi^p_{\mu\nu} dV; \quad C^p_n = \int_V T^{imp} \nabla \phi^p_{\mu\nu} dV; \quad D^p_{nk} = \int_V T^{imp} \nabla \phi^p_{\mu\nu} dV; \quad L^{pkl}_{nk} = \int_V T^{imp} \nabla \phi^p_{\mu\nu} dV;
$$

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\[ E_{n}^{pklr} = \int_{V} T^{\text{imp}klr} \nabla \phi_{mn} dV; \quad R_{n}^{pklrd} = \int_{V} T^{\text{imp}klrd} \nabla \phi_{mn} dV; \quad M_{n}^{pklrdc} = \int_{V} T^{\text{imp}klrdc} \nabla \phi_{mn} dV. \] (20)

Here as above \( T^{\text{imp}} \ldots T^{\text{imp}klrdc} \) are concretized by setting the shape of the elastic potential. In the case of the Mooney potential [5] they have the form

\[ T^{ij} = \left[ 2C_{01}B_{ij}^{\text{imp}} + p_{ij}G_{ij}^{\text{imp}} + u_{ij}^{G} \right] \nabla_{m} g_{ij} \phi_{ij}^{p} + \left[ 2C_{01}B_{ij}^{\text{imp}kl} + p_{ij}G_{ij}^{\text{imp}kl} + u_{ij}^{G} \right] \nabla_{m} u_{j}^{l} \theta_{m} \delta_{i}^{l}; \] (21)

\[ T^{ijkl} = \left[ 2C_{01}B_{ijkl}^{\text{imp}kl} + p_{ijkl}G_{ijkl}^{\text{imp}kl} + u_{ijkl}^{G} \right] \nabla_{m} g_{ijkl} \phi_{ijkl}^{p} + \left[ p_{ijkl}G_{ijkl}^{\text{imp}kl} + u_{ijkl}^{G} \right] \nabla_{m} u_{i}^{l} \theta_{m} \delta_{j}^{l}; \]

\[ T^{ijklc} = \left[ u_{ijkl}^{G} \right] \nabla_{m} g_{ijkl} \phi_{ijkl}^{c}, \]

where

\[ B^{ij} = \left[ 2g_{ij}^{m} \epsilon_{ij}^{m} - g_{ij}^{m} m_{ij}^{m} \epsilon_{ij}^{m} \right]; \]

\[ B^{ijkl} = \left[ 2g_{ijkl}^{m} \epsilon_{ijkl}^{m} - g_{ijkl}^{m} m_{ijkl}^{m} \epsilon_{ijkl}^{m} \right]; \] (22)

\[ \epsilon_{ij}^{m}(2) = \frac{1}{2} g_{ij}^{m} f_{ij} f_{m} \nabla \phi_{ij}^{p} \nabla \phi_{mn}^{mp}; \quad k, p = 1, 2, \ldots. \]

\[ G^{ij} = \epsilon_{mn}^{j} \left[ 2\delta_{1}^{i} \left( \delta_{m}^{j} + 2 \epsilon_{m}^{j} \right) \epsilon_{3}^{j} + 2\delta_{1}^{j} \left( \delta_{3}^{j} + 2 \epsilon_{3}^{j} \right) \epsilon_{3}^{j} - 2\delta_{1}^{i} \left( \delta_{3}^{j} + 2 \epsilon_{3}^{j} \right) \epsilon_{3}^{j} - 2\delta_{1}^{j} \left( \delta_{3}^{j} + 2 \epsilon_{3}^{j} \right) \epsilon_{3}^{j} \right]; \]

\[ G^{ijkl} = \epsilon_{mn}^{j} \left[ 2\delta_{1}^{i} \left( \delta_{m}^{j} + 2 \epsilon_{m}^{j} \right) \epsilon_{3}^{j} + 2\delta_{1}^{j} \left( \delta_{3}^{j} + 2 \epsilon_{3}^{j} \right) \epsilon_{3}^{j} - 2\delta_{1}^{i} \left( \delta_{3}^{j} + 2 \epsilon_{3}^{j} \right) \epsilon_{3}^{j} - 2\delta_{1}^{j} \left( \delta_{3}^{j} + 2 \epsilon_{3}^{j} \right) \epsilon_{3}^{j} \right]; \] (23)

\[ G^{ijkl} = \epsilon_{mn}^{j} \left[ 4\delta_{1}^{i} \left( \epsilon_{2}^{j} \epsilon_{3}^{j} + \epsilon_{2}^{j} \epsilon_{3}^{j} \epsilon_{3}^{j} \right) \right]; \]

\[ G^{ijklc} = \epsilon_{mn}^{j} \left[ 4\delta_{1}^{i} \epsilon_{2}^{j} \epsilon_{3}^{j} \epsilon_{3}^{j} \epsilon_{3}^{j} \right]. \]

A sufficient condition for the stability of the zero solution of the system (19) is the positivity of the function
\[ H = \frac{1}{2} C_p f_p f_n + \frac{1}{3} D_n f_p f_k f_n + \frac{1}{4} L_n f_p f_k f_n f_n + \frac{1}{5} E_n f_p f_k f_n f_n f_n + \frac{1}{6} R_n f_p f_k f_n f_n f_n f_n f_n + \]
\[ + \frac{1}{7} M_{pkld} f_p f_k f_n f_n f_n f_n f_n, \quad \left( \sum_{i=1}^{\infty} n, p, k, \ldots \right), \]  

(24)

for those values of coordinates and velocities that do not exceed the values found from the relations (19).

From this we obtain a nonlinear system of algebraic equations for determining the values of the initial perturbations at which the body can move to a new equilibrium position.

\[ C_n f_p + D_n f_p f_k f_n + E_n f_p f_k f_n f_n + R_n f_p f_k f_n f_n f_n + \]
\[ + M_{pkld} f_p f_k f_n f_n f_n f_n f_n = 0, \quad \left( n = 1, 2, \ldots \infty; \sum_{i=1}^{\infty} p, k, \ldots \right). \]

(25)

The question of the sign-definiteness of the Lyapunov function (24) was also investigated by numerical methods. The perturbation region was broken by a grid, and for each load parameter (torque \( M \)) from the relations (25) there was such a region of change of perturbations in which the function \( H \) would be positive. Following the Lyapunov's first stability theorem, in this domain the zero solution of the system of equations (19) will be stable.

3. Results and discussion

The results of the computational experiment are performed for the material rubber 2959. The dependence of the maximum perturbation modulus \( |f| \) on the torque \( M \) is shown in figure 3. In this case, the ratio of the geometric characteristics of the cylinder was chosen as \( \frac{L^2}{\sqrt{(a^2 + b^2)} ab} = 2 \), and \( \Psi = 0.3 \). The value \( M_* \) corresponds to the bifurcation point found by the linearized stability theory.

![Figure 3. The dependence of the maximum of the module of the perturbation on the torque.](image)

It follows from this plot that a twisted cylinder of elliptical section can lose stability before the critical moment determined by the linearized theory, if only the initial perturbations exceed the
absolute value of the permissible value. The curve in the $|f| - M$ plane (given geometric and physical characteristics) is the upper bound of the stability domain, that is, an infinite sequence of stable equilibrium states. Here we also note that in contrast to the classical theory of stability in the "small", which defines a single point of bifurcation, in the theory of stability in the "large" there is a region in which a hierarchy of stable states is observed. The calculation of the correlation dimension of the strange attractor [7–10] allows us to limit the phase space of the embedding for the set of initial values of perturbation amplitudes and their velocities, which allows us to reasonably limit the number of terms in the Bubnov-Galerkin series and, therefore, to solve the question of convergence of this series.

4. Conclusions
From the analysis of the presented figures it should be noted the following:
- reducing the size ratios of structures reduces the stability region with respect to both initial perturbations and elongations; replacing the compressible material with an incompressible one leads to the same results; for the compressible material, the Murnaghan potential gives an upper bound on the stability region compared to the two-constant one;
- for an incompressible material the Treloar potential gives a lower bound on the stability domain compared to the Mooney potential;
- the use of the stability criterion with respect to finite perturbations allows for a specific value of the load parameter to obtain a limited sequence of permissible values of initial perturbations in which the main deformation process will be stable and a hierarchy of stable equilibrium states is observed;
- setting the maximum permissible value of the initial perturbations leads to finding the area of change in the load parameter, in which the main deformation process will be stable.
Thus, the use of the stability criterion, consisting in the application of the second Lyapunov method with respect to finite perturbations, allows for a specific value of the load parameter to obtain a limited sequence of permissible values of initial perturbations, in which the main deformation process will be stable and a hierarchy of stable equilibrium States is observed, which determines the functioning of various complexes in the normal mode.

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