Spectral domain soliton-effect self-compression

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Abstract. On the basis of numerical studies, we demonstrate the nonlinear process of spectral self-compression in a fiber with anomalous dispersion, as a spectral analogue of soliton-effect self-compression (or soliton self-compression).

1. Introduction
The nonlinear process of spectral compression (SC) in a dispersive delay line (DDL) followed by a nonlinear fiber demonstrates promising applications to the signal analysis-synthesis problems in ultrafast optics [1]. In the SC system, the negative phase of pulse obtained in a DDL is compensating by the positive phase acquired by nonlinear self-phase modulation (SPM) in a fiber with normal dispersion [2, 3]. The impact of group velocity dispersion (GVD) on SC for sub-picosecond pulses in the range of normal dispersion, i.e. at wavelengths <1.3μm for standard silica fibers, is analyzed in [4] in view of shaping flattop pulses. In range of anomalous dispersion, i.e. at wavelengths >1.3μm for silica fibers, the combined impact of GVD and SPM leads to the formation of solitons [5, 6], when the contributions of GVD and SPM balance each other. The pulse self-compression phenomenon arises when the impact of SPM exceeds the GVD, and high-order solitons are shaped [7]. Recently, generation of sub-two-cycle pulses by soliton self-compression of 100-fs pulses from a Ti:sapphire laser at 85 MHz using a 4.85-mm long nonlinear photonic crystal fiber are demonstrated numerically [8] and experimentally [9]. The efficient soliton self-compression down to 5.07 fs is demonstrated also in As₂Se₃ and As₂S₃ chalcogenide photonic nanowires [10].

When the contribution of GVD exceeds the SPM impact in the fibers with anomalous dispersion, the spectral self-compression (self-SC), a spectral analogue of soliton self-compression, is anticipated. Recently, the soliton self-SC realization in a standard silica fiber with negative dispersion at the wavelength ≥ 1.3μm was predicted numerically [11] and observed experimentally [12]. The objective of our work is detailed numerical study of the soliton self-SC process revealing its nature, peculiarities and general regulations on the basis of physical pattern of the process.

2. Numerical studies
In our numerical experiments, we study the pulse and spectrum evolution during the propagation through a fiber with Kerr-nonlinearity and anomalous dispersion. We carry out simulations for initial Gaussian, secant-hyperbolic and super-Gaussian pulses.

The pulse propagation in a single-mode fiber is described by nonlinear Schrödinger equation [1]:

$$i \frac{\partial \psi}{\partial \zeta} = \frac{1}{2} \frac{\partial^2 \psi}{\partial \eta^2} + R|\psi|^2 \psi$$

(1)
where $\zeta = z/L_D$ is the dimensionless propagation distance normalized to the dispersive length $L_D = \tau_0^2/|k_2|$ (\(k_2\) is the coefficient of second-order dispersion); $\eta = (t - z/v)/\tau_0$ is the running time normalized to the initial pulse duration $\tau_0$. The nonlinearity parameter $R$ is given as $R = L_D/L_{NL}$, where $L_{NL} = (k_0 n_2 I_0)^{-1}$ is the nonlinearity length, $n_2$ is the Kerr index of silica, and $I_0$ is the peak intensity. The first and second terms of the right part of Eq. (1) describe the impact of GVD and nonlinearity, correspondingly. We use the split-step Fourier method for numerical solution of equation, with the fast Fourier transform algorithm on the dispersive step [13, 14].

We study soliton self-SC, which occurs when the dispersive length in the fiber is shorter than the nonlinear length ($L_D < L_{NL}$, i.e. $R < 1$). Therefore, at first the GVD stretches the pulse by acquiring a chirp. Afterwards, the accumulated impact of nonlinear SPM leads to the chirp compensation, and as a result, the spectrum becomes compressed. We study behavior of the pulse in a fiber with negative dispersion for different values of the nonlinearity parameter and fiber length.

The propagation mapping for Gaussian pulse and its spectrum for $R = 0.6$ are shown in Figures 1 (a, b), correspondingly.

![Figure 1. The 3D map of pulse self-interaction for pulse (a) and spectrum (b) ($R=0.6$, initially transform-limited Gaussian pulse).](image)

In initial propagation step the pulse is stretched and spectrum is compressed. Afterwards, the width of central peak of spectrum decreases and the main part of the pulse energy transfers to spectral satellites. At a certain fiber length the reverse process starts – the pulse self-compresses. The process can be explained in the following way: in the initial propagation stage the spectrum is compressed, which leads to decreasing of the GVD impact. As a result, the dispersive length increases, therefore, the nonlinearity parameter increases. If the condition $R > 1$ is satisfied ($L_D > L_{NL}$), the pulse is
compressed. Then, the spectrum stretches, which leads to the increase of GVD impact (decrease of $L_D$ and $R$). When the condition $R < 1$ ($L_D < L_{nl}$) is satisfied, the spectrum is compressed.

The above described process is of periodic character. However, for every next cycle the quality of the SC worsens as compared with the previous SC. In Figure 2, the peak value of spectrum and pulse is shown for initial Gaussian pulse. Our study clearly shows the periodic nature of the process for initial secant-hyperbolic pulse as well.

The behavior of spectrum in the propagation process is similar to the pulse behavior in the case of the soliton compression. First, the pulse is compressed on a distance equal to the period, and then it stretches. In our case, spectrum has similar behavior. However, because of incomplete chirp cancellation, the change in the spectrum does not exhibit strict periodicity. The study shows decrease of the process period with the reduction of nonlinearity parameter. For $R=0.25$, the SC ratio is ~ 4, without practical change of spectrum in the range of $\zeta \approx 11000-17000$ (Figure 3): the pulse large stretching (~6000$^5$) decrease the peak intensity of pulse which results in practical absence of the nonlinear self-interaction.

Figure 4 shows initial (dashed line) and compressed (solid line) spectra for initial transform-limited Gaussian ($A(t) = \exp(0.5 \cdot t / \tau_0)^2$) (a), secant-hyperbolic ($A(t) = \sec h(t / \tau_0)$) (b), and super-Gaussian ($I(\eta) \times 10^{-4}$).
Figures illustrate the case of maximal soliton self-SC. The SC ratio \( \Delta \Omega / \Delta \Omega_0 \) is \( \sim 6 \) for Gaussian pulse, obtained when the fiber length and nonlinearity parameter were \( \zeta \approx 50 \) and \( R = 0.6 \), correspondingly. For initial secant-hyperbolic and super-Gaussian pulses, the SC ratio is \( \Delta \Omega / \Delta \Omega_0 \approx 5 \) (\( \zeta \approx 81, R = 0.4 \)) and \( \Delta \Omega / \Delta \Omega_0 \approx 11 \) (\( \zeta \approx 80, R = 0.7 \)), correspondingly.

The corresponding pulses are shown in Figure 4 (d, e, f). The coefficient of soliton self-SC quality \( K \) (defined as the ratio of energy in the central part of pulse to the whole energy) equals to \( \sim 0.8 \) for Gaussian, secant-hyperbolic and super-Gaussian pulses.

We have studied also the self-SC for slightly chirped pulses. The studies are carried out for the Gaussian pulses dispersively stretched (1.2, 1.4, and 2.2) and chirped (with the chirp coefficients \( C = 0.83, 0.7 \) and \( 0.45 \)) in the +/- dispersive medium. The results of studies for \( C = 0.7 \) are shown in Figure 5. In this case the SC factor is \( \sim 24 \), for \( R = 0.5, \zeta \approx 700 \), and \( K \approx 0.3 \). The same SC factor we have for the \( C = 0.7 \) and also for transform limited pulses with the parameters for \( R = 0.46 \) (adequate to the 1.4 dispersed pulse with \( R = 0.5 \)), and \( \zeta \approx 600 \). This compassion shows that only the pulse peak
intensity and spectral bandwidth are important for the process, and initial phase of pulses (in the range of $C=+/-0.45-0.83$) does not impact to the process.

3. Conclusion
Our detailed study have shown the soliton spectral self-compression in the fiber "directly", without dispersive delay line, in the fiber with anomalous dispersion. We have demonstrated that there is an analogy between the processes of soliton self-compression and soliton spectral self-compression. The studies for the initially slowly +/- chirped pulses (with the chirp coefficients $C=-0.83$ to $+0.83$) show that only the pulse peak intensity and spectral bandwidth are important for the process, and initial phase of pulses does not impact to the process. Our studies curried out for the Gaussian, secant-hyperbolic and super-Gaussian pulses, in the range of parameters $R=0.25$ to 1 and $\zeta \approx 1$ to 20000, show up to $30^{th}$ SC.

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Figure 5. The pulse (a), spectrum (b), and chirp (c) for initially slightly chirped Gaussian pulses (stretching factor of the initial pulse is ~ 1.4).
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