Analysis of Connected Arrays and Capacitively Coupled Arrays

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ABSTRACT
The concept of connected and capacitively coupled dipoles in a planar array, which improves wideband performance, is investigated. A long wire with multiple feeds acts as an array in connected arrays, whereas a capacitively coupled array uses capacitors between array elements. Both approaches attempt to approximate the current sheet of Wheeler. Using spectral Green’s function, the array active impedance is calculated. The impedance mismatch of a connected/capacitively coupled dipole array is less than that of an array of unconnected dipoles of the same size. Both direct and capacitive connections can be used to design a wideband array antenna with a ground plane, although capacitive coupling performs better than a direct connection. This fact resulted from numerous analytical studies using infinite dipole array scan (active) impedance endorsed by full-wave simulations using CST MWS. The maximum mismatch losses over the frequency band for different source (reference) impedances are used to find the best solution at the broadside. The average reflected power over all scan angles is applied, for the scan performance, and then, the maximum average mismatch loss over the frequency band is investigated.

INDEX TERMS
Array active impedance, array scan impedance, connected array, tightly-coupled array, wideband array antenna.

I. INTRODUCTION
Extensive applications of wideband antennas have led to many efforts in recent years to design such antennas and improve their characteristics [1]–[4]. Arraying is one of the best solutions to increase antenna gain, scanning beams, and form desired radiation patterns. Array antenna bandwidth depends on antenna element and array structure, making it more challenging to increase array bandwidth than a simple antenna element.

Wideband array antennas have found many applications during the last decades [5]–[8]. Using a wideband array antenna, one can share a single aperture for multiple functions, resulting in cost and size reduction. Traditionally, array design starts from element design. A proper antenna element is designed to cover desired frequency band, then the array is formed in a lattice, and one tries to suppress mutual coupling and grating lobes. Usually, array bandwidth decreases relative to element bandwidth.

A new approach of array design, in contrast, tries to couple elements intentionally. This class of arrays, called tightly coupled arrays (TCAs), can achieve wideband performance. In this approach, coupling between array elements is done either capacitively or directly.

In capacitively coupled arrays (CCAs), the ends of the elements are connected by a capacitor together. The idea is based on extensive studies on frequency-selective surfaces [9], and numerous studies have been conducted to improve the array performance using this approach. However, another structure that can be said to generate a wide bandwidth is the connected array, in which the elements are connected directly to each other without capacitive reactance. Hansen [10] disapproved significant improvement in bandwidth using a capacitively coupled array. Since by connecting the elements, a continuous current distribution is achieved, Hansen used moment method simulations to prove the inefficiency of capacitive coupling. Around the center frequency, the antenna gain is significant and decreases sharply at frequencies away from the central frequency. In contrast, Hansen showed that array performance is significantly improved by directly connecting elements [11]. He also showed that using negative inductance works best. A review of articles and books demonstrates that much has been done with both approaches [12]–[21]. To properly evaluate this issue, which can be a good guide...
II. MATHEMATICAL FORMULATION

If we have an infinite array of dipole elements connected with impedance Z, as shown in Fig.1(a), writing the boundary condition of the field continuity on the array plane, we can obtain the current distribution in the array [27].

\[
i(x) = \frac{1}{\Delta x} \sum_{m_x = -\infty}^{\infty} \frac{-E(k_x)}{D(k_{xm})} e^{-jk_{xm}x};
\]

\[
D(k_{xm}) = \frac{1}{\Delta y} \sum_{m_y = -\infty}^{\infty} J_0 \left( k_{ym} \frac{d_x}{2} \right) G_{xx}(k_x, k_{ym})
\]

where \( k_{xm} = k_0 \sin \theta_x \cos \phi - \frac{2\pi m_x}{d_x} \) and \( k_{ym} = k_0 \sin \theta_y \sin \phi - \frac{2\pi m_y}{d_y} \) are wave numbers of the Floquet modes \( m_x, m_y \). \( G_{xx} \) represents the x-x component of the spectral Green’s function. \( J_0 \) is the Bessel function of the first kind and zero-order. A short derivation of \( G_{xx} \) for free space and backing reflector is presented in the Appendix.

\( E(k_x) \) is Fourier transform of the excitation field over a unit cell.

To model the connection impedance \( Z \) between elements, consider an array with two excitations in a unit cell [28], as shown in Fig. 1(b) so

\[
E(k_x) = \int_{-\Delta x/2}^{\Delta x/2} e^i x e^{j k_x x} dx
\]

\[
= \left( V_1 e^{j k_x d} + V_2 e^{-j k_x d} \right) \sin c \left( \frac{k_{xm} \delta_d}{2} \right)
\]

\[
(2)
\]

Here, \( E(k_x) \) is the 1-D Fourier transform of a unit cell’s impressed excitation field, \( e^i x \).

By averaging the current across the excitation gap and dividing it by the applied voltage, the input impedance can be obtained as:

\[
Y_{in,p} = \frac{1}{V_{p \Delta x}} \sum_{m_x = -\infty}^{\infty} \frac{-E(k_x) \sin c \left( \frac{k_{xm} \delta_d}{2} \right)}{D(k_{xm})} e^{-jk_{xm}x_p}
\]

(3)

where \( p \) represents the port number \( p \) in a unit cell and \( x_p \) indicates its location \((p = 1, 2)\).
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FIGURE 2. The active impedance of a connected array at broadside vs. number of modes in free space.

FIGURE 3. The active impedance of a connected array at broadside vs. number of modes with backing reflector.

III. CONNECTED AND CAP-COUPLED ARRAYS

In the scan impedance derived in equation (7), we assume a Z impedance between dipole elements of the array antenna. Using this general formula, we can investigate the impedance of ordinary, connected, and capacitively-coupled array dipoles. Setting Z to infinity, yields uncoupled elements, and we have an impedance of usual array dipoles. Setting Z to zero yields a direct connection between dipoles, and we have connected array dipoles. Finally, for evaluating capacitively-coupled array dipoles, we can set $Z = \frac{1}{j\omega C}$.

A. NUMBER OF MODES

The evaluation of impedances in equations (6) and (7) involves infinite series of Floquet modes. In practice, the truncation of the series to a finite number of modes needs. Before using these relations, the number of the modes that are sufficient to converge should determine. Fig. 2 shows the scan impedance of a connected array versus the number of modes in free space. Fig. 3 shows the same with the backing reflector.

Based on Fig. 2 and Fig. 3 minimum modes required to maintain error less than 5% is 20. So, we use 20 modes in all the following analyses and plots. Numerous simulations showed that this number of modes results in good agreement with full-wave simulation.

B. FULL-WAVE SIMULATION

Fig. 4 compares the analytic scan (active) impedance with the full-wave simulation using the CST MWS. A 5:1 frequency band (0.75–3.75 GHz) is used and is normalized to the upper frequency ($f_0$). The dipole is half-wave at the upper frequency and impedance calculated for 45° scan in the E-plane.

Fig. 4 shows a good agreement between analytical and full-wave scan impedances. So, analytical impedance can be used in the analysis and design of the array instead of a time-consuming full-wave approach.

C. IMPEDANCE MISMATCH LOSS

The performance of arrays was evaluated in terms of the impedance mismatch loss. The reflection coefficient ($\Gamma$) was calculated between the scan element impedance (active impedance) and source impedance (reference impedance). The mismatch loss was defined as $10 \log (1 - |\Gamma|^2)$. A 5:1 frequency band (133%) was used. Unit cell size is half-wave at the upper frequency, and dipole width is 0.05 of wavelength at this frequency. By changing the reference impedance, mismatch loss plotted versus frequency at broadside. Fig. 5, 6, and 7 show mismatch loss for uncoupled (ordinary) array, connected array, and cap-coupled array, respectively, without a backing reflector. It can be seen that the connected and cap-coupled arrays act better than an uncoupled array. The same result was obtained for the array with a backing reflector.

The mismatch loss for uncoupled array, connected array, and cap-coupled array with a backing reflector are plotted in Fig. 8, 9, and 10, respectively. Reflector distance to the array is a quarter-wave at the upper frequency. Coupling capacitance is 2.5 pF, so to compensate for the ground plane’s reactive impedance. In this section, the same value is used for free space. In the next section, we change coupling capacitance and backing reflector distance, to obtain optimum values for minimum mismatch loss over the frequency band and a range of scan angles.

The scan dependence of array impedance should be investigated. Scan(active) impedance can be plotted against the scan angle at different planes (for example, E, H, and D planes) for various frequencies. Instead, a proper evaluation can be used for the scanning performance of the array.
The average reflected power over the scan range, normalized to the unit input power, is a suitable measure for wide-angle scan performance [29]:

$$\left| \Gamma \right|_{avg}^2 = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\theta_{max}} |\Gamma(\theta, \phi, f)|^2 \sin \theta d\theta d\phi$$

(8)

$\theta$ and $\phi$ denote the scan direction in spherical coordinates, $|\Gamma|^2$ is the total reflected power, and $\theta_{max}$ the maximum scan angle.

Fig. 11 shows $|\Gamma|_{avg}^2$ for uncoupled, connected, and cap-coupled (2.5 pF) arrays in free space. Fig. 12 shows $|\Gamma|_{avg}^2$ with backing reflector. The reference impedance for calculating the reflection coefficient is 300 $\Omega$, and $\theta_{max} = 70^\circ$. It can be seen that the average reflected power for coupled arrays is less than that of an ordinary (uncoupled) array in both cases of free space and with a backing reflector. Changing backing reflector distance and coupling capacitance, better results can be achieved. This is done in the next section.

D. MINIMUM NUMBER OF ARRAY ELEMENTS

As stated in (II), the analytical impedance relations used here, developed assuming infinite array. So the question arises of how many elements are needed to use it. Fig. 13 shows scan element impedance for planar arrays with $N \times N$ elements. $N = 10$ is enough and over this number, impedance does not change significantly.
IV. CONNECTED OR CAP-COUPL ED ARRAY?

Both connected and cap-coupled arrays performed better than ordinary (uncoupled) arrays. However, choosing a suitable capacitor and backing reflector distance leads to better results than a connected one. In this section, the maximum mismatch loss over the frequency band is used as a measure to compare different cases.

A. BROADSIDE

Maximum mismatch loss over the frequency band at broadside ($\theta = 0^\circ$) is calculated for different reference impedances to find the best. It is done for uncoupled ($Z = \infty$), connected ($Z = 0$), and capacitively-coupled array. Coupling capacitance changes from 0.25 to 25 pF.

In the cases with backing reflector, the distance changed from 0.1 to 0.45 wavelengths at the upper frequency, and 0.4 showed the best results.

Fig. 14 shows maximum mismatch loss over the frequency band versus reference impedance in free space. It shows that cap-coupled with 5 pF and a reference impedance of 350 $\Omega$ performed the best. In this case, the maximum mismatch loss over the frequency band is 0.77 dB corresponding to VSWR $= 2.35$.

Fig. 15 shows maximum mismatch loss over the frequency band versus reference impedance with backing reflector placed at 0.4 wavelengths (at the upper frequency) behind the array. It shows that cap-coupled with 2.5 and 5 pF perform the best. Minimum mismatch loss occurs at the reference impedance 150 $\Omega$ for 2.5 pF and 200 $\Omega$ for 5 pF. Mismatch loss in both cases is 0.97 dB which corresponds to VSWR $= 2.6$. For the connected array, the best result occurs at 300 $\Omega$ with a mismatch loss of 2.35 dB.

B. AVERAGE OVER SCAN ANGLES

The average reflected power over scan angles is calculated using equation (8), and mismatch loss for this average is investigated ($\theta_{\text{max}} = 70^\circ$). Like the broadside, maximum mismatch loss over the frequency band, is calculated for different reference impedances to find the best. It is done for uncoupled ($Z = \infty$), connected ($Z = 0$), and capacitively-coupled arrays. Coupling capacitance changes from 0.25 to 25 pF.

In the cases with backing reflector, the distance changed from 0.1 to 0.48 wavelengths at the upper frequency, and 0.47 showed the best results.

Fig. 16 shows maximum average mismatch loss over the frequency band for free space. It shows that cap-coupled with 5 pF performs the best for free space. Minimum mismatch loss occurs at the reference impedance 350 $\Omega$ and is 0.83 dB which corresponds to VSWR $= 2.43$.

Fig. 17 shows maximum average mismatch loss over the frequency band versus reference impedance with backing reflector placed at 0.4 wavelengths (at the upper frequency) behind the array. It shows that cap-coupled with 2.5 and 5 pF perform the best. Minimum mismatch loss occurs at the reference impedance 150 $\Omega$ for 2.5 pF and 200 $\Omega$ for 5 pF. Mismatch loss in both cases is 1.12 dB which corresponds to VSWR $= 2.59$. For the connected array, the best result occurs at 300 $\Omega$ with a mismatch loss of 2.35 dB.
reflector placed at 0.47 wavelengths (at the upper frequency) behind the array. It shows that cap-coupled with 5 pF performs the best. Minimum mismatch loss occurs at the reference impedance of 150 Ω. Mismatch loss for this case is 1.66 dB which corresponds to VSWR = 3.6.

In free space, reducing bandwidth to 0.2 – 0.9 GHz, a 4.5:1 band, we have maximum mismatch loss of 0.56 dB for the connected array at 400 Ω reference impedance and 0.53 dB for the cap-coupled array with 25 pF capacitor at 350 Ω reference impedance. Corresponding VSWR is 2.07 and 2.03, respectively. Similarly, with the backing reflector, reducing the bandwidth to 0.3 – 1 GHz, a 3.3:1 band, we have maximum mismatch loss of 1.83 dB for connected array at 400 Ω reference impedance and 1.04 dB for a cap-coupled array with 2.5 pF capacitor at 200 Ω reference impedance. Corresponding VSWR is 3.84 and 2.71, respectively.

C. FINITE ARRAY FULL-WAVE SIMULATION

Fig. 19 shows realized gain of a finite dipole array designed and simulated using CST MWS with the best parameters obtained from the analytical study. A 20 x 20 array is used, and all ports are excited simultaneously. The maximum theoretical gain of the array aperture is plotted along with uncoupled, cap-coupled, and connected array antenna realized gain. Fig. 18 shows the model of a connected array in the CST
Co-pol and cross-pol radiation patterns for D-plane at 0.5 GHz are plotted in Fig. 22. Cross-pol pattern levels are less than $-30$ dBi.

Similar co-pol, and cross-pol radiation patterns for uncoupled, connected, and cap-coupled arrays show that the most crucial parameter in coupled array design is mismatch loss, as discussed in the previous sections.

V. CONCLUSION

An analytical investigation of tightly-coupled dipole arrays is accomplished in this paper. Scan (active) impedance of the infinite array, based on spectral Green’s function, is used for this investigation. Using an arbitrary coupling impedance in the formulation, we compare ordinary (uncoupled), connected, and capacitively-coupled dipole arrays.

A constant source impedance is assumed, then mismatch loss has been calculated and used as a measure to compare the wideband performance of the arrays, like choosing the best source impedance, coupling capacitor, and backing reflector distance. It is observed that connected and cap-coupled arrays performed better than an ordinary array. Both of them can be used to design the broadband arrays, but cap-coupled arrays, do the best. Capacitor coupling of the dipole elements compensates for the reactive contribution of the backing reflector and dipole reactance’s, resulting in a wideband performance.

The off-broadside scan performance of these dipole arrays has been evaluated using an average reflected power over all scan angles. Mismatch loss for this average reflected power has been used for a comprehensive wideband wide-angle performance comparison and design.

APPENDIX

A. SPECTRAL GREEN’S FUNCTION

The spectral Green’s function of a planar stratified media is needed for evaluating scan impedance. Full dyadic form of the spectral Green’s function and the method of derivation is described in appendix A of [27]. The xx-component of the dyadic spectral Green’s function of an electric source is stated as

$$G_{xx}(k_x, k_y, z) = -\frac{v_{TE} k_x^2 + v_{TM} k_y^2}{k_p^2}; \quad k_p = \sqrt{k_x^2 + k_y^2}$$  \hspace{1cm} (9)

$v_{TE/TM}$ is normalized voltage for equivalent transmission line fed by unit generator. For example, in the case of free space, the voltage in the transmission line is given by (parallel of the same impedance from top and down multiplied by unit current source)

$$v_{TE/TM} = \frac{Z_{0}^{TE/TM}}{2}$$  \hspace{1cm} (10)

TE/TM mode characteristic impedances are defined as

$$Z_0^{TE} = \frac{\eta k}{k_z}$$  \hspace{1cm} \hspace{1cm} $$Z_0^{TM} = \frac{\eta k}{k_p}$$  \hspace{1cm} \hspace{1cm} (11)

where $\eta$ is the intrinsic impedance of the medium, wavenumber $k = \omega \sqrt{\mu \varepsilon}$ and $k_z = \sqrt{k^2 - k_p^2}$. 

MWS. Coupling capacitor and backing reflector distance are set to the best values obtained in section IV-A. Bandwidth improvement of cap-coupled and connected array relative to uncoupled array is clear.

The aperture efficiencies of the arrays are shown in Fig. 20. Improved High aperture efficiency bandwidth with cap-coupled and connected array relative to ordinary (uncoupled) array is clear.

Array radiation pattern with uniform excitation is shown in Fig. 21 for 0.5 GHz. Pattern shapes are similar and SLL is about $-13.4$ dB due to uniform excitation. As we know from antenna array theory, better SLL can be achieved by tapering excitation.
By substituting (10) and (11) in (9)

\[
G^{bs}_{xx}(k_x, k_y, z = 0) = -\frac{\eta}{2k} \left( k^2 - k_z^2 \right) / k_z (1 - j cot (k_z))
\]  

(14)

In the case of a backing reflector placed at a distance \( h \) from the dipole, we have the same impedance from top and a short-circuited transmission line with length \( h \) from down

\[
v_{TE/TM} = \frac{J_0(k_z h)}{(1 + j \tan (k_z h))}
\]

(13)

By substituting (11) and (13) in (9)

\[
G^{bs}_{xx}(k_x, k_y, z = 0) = -\frac{\eta}{2k} \left( k^2 - k_z^2 \right) / k_z (1 - j cot (k_z))
\]

(14)

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