1. Introduction

I have been asked to review the theory of vector boson structure. Recent discussion about the possibility of composite vectors and the meaning of structure has been rather confusing, but I believe a consensus is beginning to emerge. Because the following talk by Dr. Miyamoto will review in some detail the phenomenological possibilities, I will concentrate on the theoretical underpinnings. In the context of the Next Linear Collider, I would say the main goal is to understand the extent to which studies of vector boson couplings can provide insights into new, unknown physics beyond the Standard Model, given the sensitivities that are likely to be achievable. The outline of my talk is as follows: In the next section, in order to introduce my nomenclature and to highlight the issues in a simpler context, I begin by considering the analogous situation for QED. In Section 3, I will generalize to the present-day situation. In Section 4, I review the now-standard parameterization of deviations of the triple-vector-boson couplings from the SM. In Section 5, I will expand on the case of the strongly interacting Higgs sector. In Section 6, I discuss the orders of magnitude to be expected on general theoretical grounds. In Section 7, I will try to indicate why gauge invariance is such a critical issue and why it would be so difficult to understand the present successes of the SM without it.

2. Before the Standard Model

One topic that has been debated is whether one must demand gauge invariance for the interactions of the vector bosons. There are many dimensions of this issue, and it is both pedagogically helpful and theoretically illuminating to begin with a somewhat simpler situation than we face today. Long before the Standard Model (SM) had been formulated, there was Quantum Electrodynamics (QED), which may be summarized by the Lagrangian:

\[ \mathcal{L}_{QED} = \mathcal{L}_g + \mathcal{L}_m, \]
\[ \mathcal{L}_g = -\frac{1}{4} F_{\mu\nu}^2 + \mathcal{L}_{g.f.} + \mathcal{L}_{F-P}, \]
\[ \mathcal{L}_m = \sum_n \overline{\psi_n} (i\partial - m_n) \psi_n \]

(1)

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu \psi_n = (\partial_\mu + ie_n A_\mu) \psi_n, \]

*Based in part on work done in collaboration with J. Wudka and C. Arzt.
†Some of the observations in this talk appeared originally in [1].
†Generally, the gauge-fixing term \( \mathcal{L}_{g.f.} \) is chosen so that the Faddeev-Popov ghosts in \( \mathcal{L}_{F-P} \) decouple, but with a view toward the non-Abelian case, we’ve included that term here.
where $A_\mu$ denotes the photon field and the $\psi_n$ denote the various matter fields. In ancient times, they were the electron and muon, nucleon and pion fields. (One may include the quarks and their interactions, now known to be described by QCD.) The action associated with this Lagrangian is gauge invariant under the $U_1^Q$ symmetry of electromagnetism. I want to recall two important things that are accomplished by this gauge symmetry:

1. To describe a massless photon, one wants only two physical degrees of freedom of the four-components of $A_\mu$. This is accomplished, first, because the time-derivative of $A_0$ does not occur, except through the gauge fixing term, so that $A_0$ is not a physical dynamical variable. This is important because this field would create ghosts, and gauge invariance insures that this feature is preserved by radiative corrections. Secondly, because of the gauge symmetry, of the remaining 3 dynamical space-components $A_i$, only two are physical, as can be seen by choosing a gauge (such as the Lorentz or Coulomb gauges) which imposes a constraint among them. I remind you of these well-known facts because tampering with gauge invariance by adding terms that explicitly break the gauge symmetry can be very dangerous and generally will imperil these important features.

2. A second important aspect of gauge invariance is universality. We are familiar with the fact that Lagrangian parameters are not directly observable, but gauge invariance insures that the ratio of physical charges equal the ratios of Lagrangian charges. This is why, if the electron and muon have the same charge in the Lagrangian, they have the same observed charges. This feature is preserved under the addition of strong interactions and protons (or quarks) are included with charge proportional to the electron charge. While it was not understood why the proton’s charge was equal and opposite to the electron’s charge, at least it was understood that if it were true for the Lagrangian parameters, it would be true for the physical charges as well. This relationship is extremely important, being ultimately responsible for the neutrality of atoms. Again, explicit breaking of gauge invariance would render it a great mystery and challenge for theory to explain these fundamental facts of nature.

There is a third point that is often times also cited in this connection. Frequently, it is said that gauge invariance forbids the occurrence of a photon mass term $m^2 A_\mu^2$, and keeps the photon massless. In this form it is true, but gauge invariance does not really prevent the photon from getting a mass, as Stückelberg pointed out in 1938. One may add to the theory a scalar field $\chi$ and a term to the Lagrangian of the form

$$\mathcal{L}_m \equiv (mA_\mu - \partial_\mu \chi)^2.$$  \hfill (2)

The $U_1^Q$ gauge symmetry is maintained by extending it to $\chi$:

$$A_\mu \rightarrow A_\mu + \partial_\mu \Theta, \quad \text{vector gauge field}$$

$$\chi \rightarrow \chi + m\Theta, \quad \text{scalar gauge field},$$  \hfill (3)

\footnote{Ghosts $\equiv$ states of negative norm.}

\footnote{In textbooks, this usually is associated with the equality of the charge and matter wave-function renormalization constants $Z_1 = Z_2$.}
The field $\chi$ is sometimes referred to as the Stückelberg field, but we might call it a “scalar” gauge field by analogy with $A_\mu$. While $\chi$ adds one degree of freedom to the theory, if this is the only place it enters, it may be regarded as a gauge artifact. In the “unitary gauge,” ($\chi = 0$) it disappears, and this term takes on the appearance of a mass term for the vector field. Indeed, this is the correct interpretation, showing that one can have a gauge-invariant, massive photon. I think it is fair to say that we still don’t really understand why the photon is massless, although the fact that it is may be a hint at grand unification.

Whereas the leptons were pointlike, nucleons had structure, indicated by the presence of form factors:

$$\langle p | J_\mu^{em} | p' \rangle = e u(p') \left[ \gamma_\mu F_1(q^2) + \frac{i \sigma_{\mu\nu} q^\nu}{2M} F_2(q^2) \right] u(p),$$

(4)

where $q \equiv p - p'$. The proton and neutron were found to have large magnetic moments:

$$F_2^p(0) = 1.8, \quad F_2^n(0) = -1.9.$$  

(5)

This was interpreted as the effects of the strong interactions, but, as the theme of this talk is the nature of structure, one might ask why one simply did not add Pauli-type interactions to the original Lagrangian

$$\frac{\alpha \psi F}{\Lambda} \psi \sigma_{\mu\nu} \psi F^{\mu\nu}$$

(6)

At the phenomenological level, the form factors for nucleons were soon found to be rapidly falling functions of $q^2$, but, at a more theoretical level, such an interaction term was objectionable because it was not renormalizable. Although it has not yet been experimentally verified, we now believe that, even after including a correct theory of strong interactions to account for the hadronic form factors, such local Pauli-interactions should be included in QED, even for leptons! They arise from weak vector boson effects, such as depicted in Fig. 1, and appear to be local interactions at scales far below the weak vector boson masses.

![Weak boson contributions to the anomalous magnetic moment.](Fig. 1)

\[This theory retains renormalizability. In the SM, one could do the same thing for the hypercharge field. As we shall see below, the similar trick in the non-Abelian case results in a non-renormalizable theory.\]
In fact, this illustrates what must be expected in any theory short of the ultimate theory. “Nonrenormalizable,” higher-dimensional interactions are generally present resulting from new physics at a higher energy scale and above which the present form of the local field theory breaks down, usually because new physical degrees of freedom come into play. For example, in Eq. (6), the scale \( \Lambda \) may be identified with the vector boson masses. If one regards \( \Lambda \) as a physical cutoff where the present theory breaks down, then these higher-dimensional operators do not spoil renormalizability in the following sense: For consistency, one must imagine that the Lagrangian includes all such higher dimensional operators allowed by the symmetries. Such a representation is called an effective field theory, and I am suggesting that any realistic theory (short of the ultimate) will be of this type. Then, although the inclusion of such “nonrenormalizable” vertices in loop corrections lead to divergences in arbitrary Green’s functions, all such divergences may be absorbed in relationships between Lagrangian parameters and observables.

This does imply, however, that, unlike a renormalizable theory, there are really an infinity of coupling constants to be fit to experiment, so you may well wonder how it is possible to predict anything. For example, given that there is a fundamental interaction of the Pauli type, how is it possible for QED to predict correctly the value of \( g − 2 \) for the electron and muon with such high precision? The answer is that it depends on the size of the nonrenormalizable vertex coming from the heavy sector as compared with the size of the radiative corrections coming from the light theory. The most important corrections to QED predictions actually come from hadronic effects, but, for pedagogical purposes, let me suppose that these can be precisely determined, as, in fact, they can be. It is illustrative for later comparison with corrections to the SM to estimate how large are the corrections coming from the weak loops in Fig. 1. If one expands the weak corrections in terms of the vector boson masses, there are logarithmic dependences that can be absorbed in the counterterm for the muon charge, but there are finite corrections to the magnetic moment Eq. (6) that may be estimated as follows: For example, the W-boson contribution is the product of three factors:

\[
\frac{\alpha \mu F}{\Lambda} = \left( \frac{e}{M_W} \right) \left( \frac{g^2}{16\pi^2} \right) \left( \frac{m_\mu}{M_W} \right).
\]

Each factor has significance: \( e/M_W \) represents the strength of photon coupling to \( A_\mu \) over scale of new physics; \( g^2/16\pi^2 \) represents a new interaction strength, \( g \), times a loop-factor of \( 1/(16\pi^2) \); a further suppression factor of \( m_\mu/M_W \) due to the chiral structure of electroweak interactions. This last suppression because of a global symmetry is fortunate for tests of QED, since it means that Pauli-term acts more like a dimension-six rather than a dimension-five operator.\( ^\sharp \) In the common parlance, this contribution is a correction to \( (g_\mu − 2)/2m_\mu \), even though the nominal scale of

\( ^\sharp \)This has been reviewed in Ref. [3], although there remain certain uncertainties due to the so-called hadronic light-by-light scattering corrections that may be as large as the weak corrections.

\( ^\sharp\sharp \)As small as this is, an experiment being constructed at BNL hopes to measure \( g_\mu − 2 \) with a design sensitivity about 5 times smaller than the size of the predicted weak corrections, so this discussion is quite relevant.
the contribution is not set by the muon mass but by the vector boson masses.

To sum up, we have argued that, because of physics at the weak scale and beyond, QED must be supplanted by new, “nonrenormalizable” vertices such as the one associated with $\alpha_{\mu F}$. Thus, the actual theory that we are dealing with takes the form

$$L_{\text{eff}} = L_{\text{QED}} + L_{\text{NR}},$$

$$L_{\text{NR}} = \frac{1}{\Lambda} \sum_i \alpha_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2} \sum_i \alpha_i^{(6)} O_i^{(6)} + \ldots \quad (8)$$

where the $O_i^{(N)}$ represent local operators of dimension $N$, corresponding to vertices represented by the coupling constants $\alpha_i^{(N)}$.

†† As if the new physics is associated with a single common new energy scale, appropriate factors of $\Lambda$ have been extracted so that each $\alpha_i^{(N)}$ are dimensionless. Of course, there may be a variety of scales of new physics, so that the inferred magnitudes or limits on the couplings must be interpreted in a specific context.

Schwinger’s famous prediction of $\alpha/2\pi$ for the one-loop contribution (Fig. 2) is cut-off independent by virtue of the renormalizability of QED. At the same time, from our present perspective, it should be regarded as uncertain by an amount of order $(m_\mu/\Lambda)^2$ since the physics at momentum scales above $\Lambda$ has not been correctly represented by the renormalizable terms we associated with QED alone. Stated otherwise, one must add to the loop corrections the “direct” contributions associated with $\alpha_{\mu F}$. It is only to the extent that Schwinger’s result is large compared to the size of this elementary vertex that QED radiative corrections may be tested.

Given that there is an elementary Pauli interaction illustrated by Eq. (6), with vertex proportional to $\alpha_{\mu F}$, should it be included in radiative corrections? Being “nonrenormalizable,” in the traditional sense of the term, the more times this vertex occurs, the greater the associated degree of divergence of the graph. However, familiar power-counting arguments can be used to show that such divergences only
contribute to local operators, so that their contributions simply renormalize other parameters of the effective field theory. Thus, for example, the self-energy diagram of Fig. 3 will produce a correction to the muon mass which, like the bare mass, is itself not directly observable.

![Anomalous vertex in a self-energy correction.](image)

Anomalous vertex in a self-energy correction.

Fig. 3

Indeed, the most convenient method of renormalizing the full effective Lagrangian Eq. (8) is to assume that the renormalized parameters, defined by minimal subtraction or another prescription, already include all divergent contributions, not from loops containing just renormalizable vertices but containing any vertices. Thus, an effective field theory is renormalizable, but an infinity of independent counterterms are required. As physical energy scales approach \( \Lambda \), increasingly many terms in \( \mathcal{L}_{NR} \) are required until the point is reached where the new physics can no longer be accurately represented in terms of local operators of light fields. Typically, new particles with masses of order \( \Lambda \) are produced whose propagation and interactions are not represented by \( \mathcal{L}_{eff} \).

In the preceding, we spoke only of leptons, but, of course, a similar calculation of the electromagnetic structure of the proton would break down at a scale of \( \Lambda_{QCD} \), even below the proton mass, revealing the modifications due to strong interactions. These too afflict leptonic calculations, but, fortunately (for us and for Schwinger,) they begin to contaminate QED predictions only at the two-loop level. This illustrates that the correlation between the sensitivity to new physics and its scale can be subtle, and we must be careful in interpreting our results.

3. The Standard Model and Beyond

Now let us focus on our main topic, the physics beyond the SM and its implications for vector boson couplings. Most of the lessons have been illustrated in the preceding discussion of QED, which is why we took such pains to explore it. Conceptually, we replace QED with the SM:

\[
\begin{align*}
U^Q_1 &\implies SU^L_2 \otimes U^Y_1, \\
\mathcal{L}_{QED} &\implies \mathcal{L}_{SM} \\
\mathcal{L}_{eff} &\implies \mathcal{L}_{SM} + \mathcal{L}_{NR}.
\end{align*}
\]

Now \( \mathcal{L}_{NR} \) represents the physics the lies beyond the SM. The assumption here is that there are no “light” particles below some scale \( \Lambda \) other than the ones we know and, possibly, the Higgs boson. This assumption could be wrong, e.g., there could be supersymmetric partners with masses not very different from these. In that case,
one would replace $\mathcal{L}_{SM}$ with the minimal SUSY model $\mathcal{L}_{MSSM}$ and begin again to seek the physics beyond expressed by $\mathcal{L}_{NR}$. But we shall suppose that the SM takes the form

$$\mathcal{L}_{SM} = \mathcal{L}_g + \mathcal{L}_m + \mathcal{L}_H,$$

$$\mathcal{L}_g = -\frac{1}{2} Tr W_{\mu\nu}^2 - \frac{1}{4} B_{\mu\nu}^2 + \mathcal{L}_{g.f.} + \mathcal{L}_{F-P},$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu + ig_2 [W_\mu, W_\nu],$$

$$W_\mu \equiv W^a_\mu \tau^a, \quad \tau^a = \text{Pauli matrices},$$

$$\mathcal{L}_m = \sum_n \overline{\psi}_{nL} \not{D} \psi_{nL} + \sum_n \overline{\psi}_{nR} \not{D} \psi_{nR} + \sum_{m,n} (g_{mn} \overline{\psi}_{mL} \phi \psi_{nR} + c.c.),$$

$$D_\mu \psi_{nL} \equiv (\partial_\mu + ig_2 W_\mu + ig_1 B_\mu Y_{nL}) \psi_{nL},$$

$$D_\mu \psi_{nR} \equiv (\partial_\mu + ig_1 B_\mu Y_{nR}) \psi_{nR}$$

The form of the Higgs Lagrangian $\mathcal{L}_H$ is unknown at present, but, in the “minimal” SM, it consists of a single Higgs doublet $\phi$ together with its requisite gauge interactions. I have chosen to include the Yukawa interactions of such a field, which give rise to the CKM mixing angles, in our “matter” Lagrangian $\mathcal{L}_m$, but that would be modified in nonminimal models. In fact, there is no compelling reason to restrict the number of Higgs doublets to one; there could be one doublet per generation or, as in supersymmetric models, two doublets, one giving mass to $T_3 = +1/2$ fermions and the other, to $T_3 = -1/2$ fermions. The Higgs could also be a composite field resulting from new strong interactions, as in technicolor models, in which case a nonlinear representation is often used, as will be reviewed subsequently. The immediate goal of electroweak experimentation is to unravel the nature of the Higgs sector and to uncover whatever can be deduced about the physics that lies beyond, embodied here in $\mathcal{L}_{NR}$.

As in QED, gauge invariance may not be explicitly broken without severe consequences for consistency of this whole framework. As previously, $SU_L^Y \otimes U_Y^I$ gauge symmetry provides both for the correct number of degrees of freedom carried by the gauge fields and for the universal character of the gauge interaction. Indeed, non-abelian universality is even stronger than the abelian case. While one may in principle vary the hypercharge coupling strength $g_1$ for each field, in the nonlinear character of the non-Abelian symmetry implies that there can be but a single non-Abelian coupling strength $g_2$ relating cubic and quartic self-couplings of the $W_\mu$ field as well as its coupling to fermions and scalars. Gauge invariance insures that this universality among Lagrangian parameters will be manifested by the physical couplings as well. It must be remembered that all these consequences are nontrivial, to say the least, and must be kept in mind if one begins to tinker with the dynamics of gauge fields. This is why we do not give masses to the weak vector bosons simply by adding mass terms

$$M_W^2 Tr W_{\mu\nu}^2 + M_B^2 B_{\mu\nu}^2$$

$^\dagger$These abelian couplings are constrained by anomaly cancellation, but, given the number of different fields, much arbitrariness remains.
The *raison d’etre* for $\mathcal{L}_H$ is to give masses in a gauge invariant manner, via spontaneous symmetry breaking and the Higgs mechanism. But there are two approaches to the description of $\mathcal{L}_H$:

1. The conventional approach involving one (or more) relatively light Higgs doublet ($m_H < 800$ GeV.) In the minimal model with a single doublet,

$$\mathcal{L}_H = (D_\mu \phi)^\dagger D^\mu \phi + m^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2,$$

(12)

$$D_\mu \phi = (\partial_\mu + ig_2 W_\mu - i \frac{g_1}{2} B_\mu) \phi$$

(13)

2. The second approach, although it arose historically by analogy with chiral perturbation theory for pions, may also be thought of as a way of generalizing Stückelberg’s trick to non-Abelian theories. The essence of his method is to replace the gauge fields by their gauge transforms, introducing in the process auxiliary scalar fields, now called would-be Goldstone bosons, as dynamical fields that are eaten to form the longitudinal components of the massive vector particles. That can be done via a nonlinear realization of the symmetry, introducing a $2 \times 2$ matrix $\Sigma$ according to

$$\Sigma \equiv \exp (i \vec{\tau} \cdot \vec{\omega} / v),$$

$$D_\mu \Sigma = \partial_\mu \Sigma + ig_2 W_\mu \Sigma - i \frac{g_1}{2} \Sigma B_\mu \tau_3$$

(13)

The $\vec{\omega}$ is composed of the three would-be Goldstone fields, and $v \approx 246$ GeV is the electroweak energy scale. By construction, $\Sigma$ transforms linearly under $SU_L^2 \otimes U_Y^1$

$$\Sigma \to e^{-i \vec{\omega}_L \cdot \tau / 2} \Sigma e^{(i \theta_R \tau_3 / 2)} \equiv U_L (\theta_L) \Sigma U_Y^1 (\theta_R)^\dagger$$

(14)

This approach includes the SM on scales below $m_H$. The corresponding matrix of Goldstone fields can be obtained from the SM Higgs field $\phi$ by the association

$$\bar{\phi} \equiv \frac{\sqrt{2}}{v} (\phi^\dagger \phi)$$

(15)

where $\bar{\phi} = i \tau_2 \phi^*$. Accordingly, this description in terms of the Goldstone bosons alone is more appropriate for the case of a very heavy or “strongly interacting” Higgs boson.

Defining $V_\mu \equiv \Sigma^\dagger (D_\mu \Sigma)$, the vector boson masses can be introduced in analogy with Stückelberg as

$$\mathcal{L}_2 \equiv \frac{v^2}{4} \text{Tr} (V_\mu V^\mu) + \frac{\beta_1}{8} \frac{v^2}{4} [\text{Tr} (\tau_3 V_\mu)]^2.$$

(16)

To see that these are simply gauge-invariant vector masses, go to the “unitary gauge” where $\Sigma = 1$, in which case,

$$\mathcal{L}_2 = \frac{g_2^2 v^2}{4} M_W^2 W_\mu^\dagger W^\mu + \frac{M_W^2}{2 \cos^2 \theta_w} (1 - \beta_1) Z_\mu^2,$$

(17)

where $W_\mu \equiv \frac{1}{\sqrt{2}} (W_\mu^1 - i W_\mu^2)$ is the $W^-$ field; $Z_\mu = \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu$, the $Z^0$ field, $M_W \equiv g_2 v / 2$, and $\theta_w$, the usual weak mixing angle defined by $\tan \theta_w \equiv g_1 / g_2$. In tree
approximation, obviously $M_W$ is the $W^\pm$ mass, and its relation to $M_Z$ is parameterized in terms of a parameter usually called $\rho$:

$$\rho \equiv \left( \frac{M_W}{M_Z \cos \theta_w} \right)^2.$$  \hfill (18)

Apparently, in tree approximation, we have $\rho = 1/(1 - \beta_1)$.

The preceding two descriptions are the only ways known to describe massive vector bosons in a gauge invariant manner. Experimentally, $\rho - 1$ agrees with SM predictions, including one loop corrections, to better than 0.5%,\footnote{Nevertheless, these corrections are proportional to the square of quark mass splittings, requiring $m_t < 180$ GeV. This custodial symmetry is not necessarily present when there are more than one Higgs doublet. It is, however, a natural consequence of chiral symmetry in technicolor models.} so it must be that $\beta_1$ is small, $\beta_1 \lesssim 1\%$. This would seem unnatural unless $\beta_1 = 0$ is associated with a higher symmetry. In fact, the minimal SM Lagrangian $\mathcal{L}_{SM}$ does possess an $SU_2^R$ global symmetry in the limit that hypercharge vanishes ($g_1 = 0$) and mass splittings within fermion doublets vanish. In the present context, that symmetry corresponds to embedding the $U_Y^7$ hypercharge transformation on $\Sigma$ in an $SU_2^R$ group under which

$$\Sigma \to \Sigma e^{(i\sigma_{R} \cdot \tau/2)}. \hfill (19)$$

In this limit, even after spontaneous symmetry breaking, there remains a vector-like isospin symmetry that preserves $M_W = M_Z$. Then, if the only explicit breakings are via hypercharge and Yukawa couplings, one can show that $\rho = 1$ plus loop corrections.\footnote{I'll return shortly to the non-linear realization.}

Possible physics “beyond the SM” is represented by $\mathcal{L}_{NR}$. As with $\mathcal{L}_H$, depending on which of the two frameworks one adopts, it takes on rather different forms with some different consequences. It is a bit easier to follow through the discussion in the linear case first before discussing the nonlinear representation, to which we shall return. In the case of a relatively light Higgs with a linearly realized representation of the symmetry, $\mathcal{L}_{NR}$ consists of writing down all higher dimensional, gauge-invariant operators involving the fields of the SM, as in Eq. (8). Assuming lepton and baryon conservation, there are no dimension 5 operators, so $\mathcal{L}_{NR}$ begins with dimension 6. The generic scale of new physics $\Lambda$ is unknown, but is certainly larger than the weak scale $v \approx 246$ GeV if this linear representation is to make sense. These have been catalogued several years ago;\footnote{I'll return shortly to the non-linear realization.} even with a single fermion generation, there are more than 80 independent dimension-6 operators. So you might think this parameterization would be overly general, but in fact, only a few affect any given process and one may classify these operators to some extent by the type of underlying physics that gives rise to them.

Since our interest here is in gauge boson structure, let us focus on those dimension-6 operators in $\mathcal{L}_{NR}$ that involve the vector fields and the Higgs field only. To simplify the discussion further, I will restrict my attention to the CP-conserving operators. Some of these operators correspond to vertices contributing directly to...
the vector boson vacuum polarization tensor. These are the ones that have been probed experimentally to some degree, so we’ll begin with them.

\[ O_\phi = |\phi^\dagger D_\mu \phi|^2, \]
\[ O_{BW} = g_1 g_2 (\phi^\dagger W_{\mu\nu} \phi) B^{\mu\nu}, \]
\[ O_{DB} = g_2^2 \frac{1}{2} \partial_\mu B_{\rho\sigma} \partial^\mu B^{\rho\sigma}, \]
\[ O_{DW} = g_2^2 \text{Tr}(D_\mu W_{\rho\sigma} D^\mu W^{\rho\sigma}) \]

Since these modify the Born amplitudes for $e^- e^+$ annihilation to fermion pairs at the $Z^0$, there already exist constraints on the magnitude of these operators as a result of the SLC/LEP-1 experiments. These are most easily quoted as if only one of these operators were present, and of course multiparameter fits will be weaker. Assuming that, and assuming that the scale of new physics is as low as possible for this expansion to make sense, $\Lambda = v$, then the typical constraint on the associated coupling constants $\alpha_i$ are

\[ |\alpha_\phi| \lesssim 0.3\% \quad |\alpha_{DW}| \lesssim 3\% \]
\[ |\alpha_{BW}| \lesssim 2\% \quad |\alpha_{DB}| \lesssim 10\% \]

As the operators in the first column explicitly break the custodial $SU_2^R$ symmetry, we would expect them to be small anyway. While these are useful numbers, at this level of accuracy, they give us no further insight into the physics beyond the SM.

There are other operators which are unobservable until Higgs boson interactions are observed. Such operators have been appropriately called “innocuous” and include

\[ O_{\tilde{\phi}\phi} = (\phi^\dagger \phi)(D_\mu \phi^\dagger D^\mu \phi), \]
\[ O_{BB} = \frac{g_2^2}{4} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}, \]
\[ O_{WW} = \frac{g_2^2}{4} (\phi^\dagger \phi) \text{Tr}(W_{\mu\nu} W^{\mu\nu}) \]

The point about these operators is that, if one goes to the unitary gauge and simply replaces $\phi$ by its vacuum expectation value, thereby suppressing the interactions of Higgs bosons, these operators simply become wave function renormalizations and are, therefore, unobservable.

A third class of operators are those whose vertices involve three or more vector bosons and, therefore, will contribute to vacuum polarization only through loops. These include

\[ O_{WWW} = g_2^2 \text{Tr}(W_{\mu\nu} W^{\mu\nu} W^\lambda W^\lambda) \]
\[ O_{B\phi} = i \frac{g_1}{2} (D_\mu \phi)^\dagger B^{\mu\nu} D_\nu \phi \]
\[ O_{W\phi} = g_2 (D_\mu \phi)^\dagger W^{\mu\nu} D_\nu \phi \]

One might think that, if the Higgs sector were strongly interacting, these effects might be enhanced, but, as we shall discuss later in this lecture, that is not likely to be the case, with technicolor models also suggesting a magnitude of no more than a few tenths of one percent. Rather than explore that branch of development immediately, it is pedagogically preferable to carry through within the context of the linear case.
Contributions of these operators to tree diagrams have not been experimentally tested, and, as we shall have more to say later, there are as yet no useful constraints from considering them in loops either.

4. Standard Parameterization of Deviations

Of primary interest for LEP-2 and the NLC is the nature of the triple-vector boson couplings. There are a variety of fields and tensor structures that make this discussion unavoidably tedious. In a now-standard reference, the most general trilinear vector couplings were displayed, assuming only Lorentz invariance. We will restrict our attention here to the CP-invariant terms only:

\[
\mathcal{L}_{WWV}/g_{WWV} = ig_1^V (W^l_{[\mu]}) W^{\mu}V^\nu - h.c. + i\kappa_V W^l_{[\mu]} W^\mu V^{\mu \nu} + ig_5^V \epsilon^{\mu \nu \sigma} (W^l_{[\mu]} \partial_\sigma W^\nu - \partial_\mu W^l_{[\nu]} W^\sigma),
\]

where \(W^l_{[\mu]}\) is the \(W^-\) field, \(V^\nu\) represents either the photon \(V = \gamma\) or the \(Z^0\)-boson \(V = Z\), \(V_{[\mu]} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu\) and \(W_{[\mu\nu]} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu\) are the “Abelian field strengths,” and the normalizations have been chosen to be \(g_{WW\gamma} \equiv e; g_{WWZ} \equiv e \cot \theta_w\). Requiring electromagnetic gauge invariance leads to \(g_1^\gamma = 1, \quad g_5^\gamma = 0\), and these are generally assumed true. The SM values for the rest are

\[
\begin{align*}
g_5^\gamma &= \kappa_\gamma = \kappa_Z = 1, & \lambda_\gamma &= \lambda_Z = g_5^Z = 0.
\end{align*}
\]

What is the relation between the previous effective Lagrangian formalism to these conventional parameters? By going to unitary gauge in Eqs. (20-23), one can extract a variety of results, such as:

\[
g_5^\gamma = 0, \quad \lambda_\gamma = \lambda_Z.
\]

These results are specifically the result of the truncation of \(\mathcal{L}_{NR}\) to dimension-6 operators and will be modified by higher dimensional operators. If experimental accuracy were sufficient to test these relations (about which we are rather skeptical,) they would provide an indication of the scale of new physics. The actual relation for the so-called “quadrupole moment” is

\[
\frac{\lambda_\gamma}{M_W^2} = \frac{3}{2} \frac{g_5^2}{\Lambda^2} \alpha_{WWW}
\]

This formula indicates that defining the dimensionless parameter \(\lambda\) by scaling out the mass \(M_W\) can be quite misleading, since the natural scale is set by \(\Lambda\), the scale of new physics. The relations for the other vertices are rather more complicated, involving many of the dimension-six operators. However, the degree to which

\[\text{The last term involving } g_5^V \text{ is separately C and P violating, and for some reason, is often dropped from these discussions despite the fact that the electroweak theory violates these symmetries maximally. The vertices in } \mathcal{L}_{WWV} \text{ were assumed to be for on-mass-shell vector bosons, which is sufficient for tree-level applications for which this formalism was originally developed.}
\[\text{In this respect, it is analogous to the traditional definition of } g_\mu - 2 \text{ by scaling out the muon mass, which is very misleading when it comes to discussing the magnitude of contributions from new physics at high scales.}\]
SLC/LEP-1 constrain those operators that contribute to the vacuum polarization tensor is so great as to render them beyond reach for LEP-2 experiments. So if one simply assumes that the operators that have not been constrained by existing experiments may be much larger than those that are, then one finds that the three deviations $\Delta \kappa_\gamma$, $\Delta \kappa_Z$, and $\Delta g_1^Z$ from their SM values may be expressed in terms of the two couplings $\alpha_{B\phi}$ and $\alpha_{W\phi}$. This leads to the non-trivial relation

$$\Delta g_1^Z = \Delta \kappa_Z + \tan^2 \theta_w \Delta \kappa_\gamma. \tag{28}$$

This relation also depends on the additional assumption that dimension-8 operators are negligible compared to the dimension-6 ones, which, as we shall see, is equivalent to assuming that the Higgs sector $L_H$ is not strongly interacting and can be represented linearly as in Eq. (12).

As a matter of fact, I know of no model that satisfies this assumption that the unconstrained dimension-6 operators are larger than those that have been constrained by existing experiments. Stated otherwise, I know of no model in which the underlying new physics contributes at one-loop order, for example, to the triple-vector boson vertices but does not also contribute at least as large a contribution to the vacuum polarization tensor. The assumption underlying Eq. (28) is also implausible since not all the dimension-6 operators that I have written down are linearly independent, that is, the choice of form of the higher order terms is somewhat arbitrary, because one may replace some operators by others without changing any physical consequences. Loosely speaking, it is the assumption that new physics does not prefer one basis set over another. However, the absence of “blind” directions is not a general principle about operators arising from new physics, since it is not the case that all models necessarily yield all interrelated dimension-6 operators of comparable magnitude. For example, depending on the underlying theory, four-fermion interactions may either be suppressed or enhanced relative to operators involving gauge bosons.

5. Strongly Interacting Higgs

We have indicated previously that, at scales well below the Higgs mass, or in models of dynamical symmetry breaking in which there may be no Higgs boson at all, in which case a nonlinear representation of $L_H$ is more appropriate than the linear prescription usually employed in the SM. Let us now turn to the question of how, in the nonlinear framework, the physics beyond the SM in $L_{NR}$ is to be represented. Dimensionality of the field no longer plays the key role that it did in the linear framework. However, Goldstone bosons are derivatively coupled, so their interactions are proportional to their momenta. Thus, one can perform a momentum

5This has been discussed at some length, for example, in Ref. [11]. For further discussion of the theoretical underpinnings, showing that this arbitrariness in the choice of operators may also be extended to their use in loop calculations, see Ref. [15].

4Loosely speaking, it is the assumption that new physics does not prefer one basis set over another. However, the absence of “blind” directions is not a general principle about operators arising from new physics, since it is not the case that all models necessarily yield all interrelated dimension-6 operators of comparable magnitude. For example, depending on the underlying theory, four-fermion interactions may either be suppressed or enhanced relative to operators involving gauge bosons.
expansion whose scale $\Lambda$ is set by the scale at which their interactions become strong. This has formed the basis for chiral perturbation theory for pions,[16, 17] where the expansion scale is set by the lowest-lying resonances, the vector mesons. On theoretical grounds, one may argue that, in general, $\Lambda$ is not expected to be larger than $4\pi v$, where $v$ is the scale of symmetry breaking. In this expansion, the leading terms, the mass terms of Eq. (17), were of "chiral-dimension" 2. In order to maintain manifest $SU_L^2 \otimes U_Y^1$ gauge-invariance, ordinary derivatives have been replaced by covariant derivatives so that the gauge field is counted as dimension one, as usual. There we inferred that, in such a theory, the mass terms that explicitly break the custodial $SU_R^2$ symmetry must necessarily be small, about 1% of those that do not. So we will expect that to be a characteristic of this description, in other words, for the operators that break the custodial symmetry, one may wish to associate a somewhat larger scale than $4\pi v$ or to regard the natural size of their couplings to be smaller than for the operators conserving $SU_R^2$. In any case, instead of $\mathcal{L}_{NR}$, we would similarly add

$$\mathcal{L}_{new} = \frac{1}{16\pi^2} \sum \alpha_k O_k, \quad (29)$$

where we have extracted a conventional[4] factor of $1/16\pi^2$ in the definition of the coupling constants, but, as indicated, it should really be a factor of $v^2/\Lambda^2$ where $\Lambda$ now represents the scale at which the longitudinal vector bosons become strongly interacting. The next terms in this expansion are chiral-dimension 4, having two more derivatives: it is a bit complicated to display a complete set of operators that are linearly independent under application of the classical equations of motion. Restricting ourselves to CP-conserving operators, ten were listed by Longhitano[18] a long time ago, and an eleven was recently pointed out in a recent paper by Appelquist and Wu.[19]Because of space limitations, I cannot share all of them with you, here, but three of the most important ones that, in the limit of zero hypercharge, conserve custodial symmetry, and can therefore expected to have the largest coefficients are, in the nomenclature of Ref. [19],

$$O_3 \equiv igTr(W_{\mu\nu}[V^\mu,V^\nu])$$

$$O_4 \equiv [Tr(V_\mu V_\nu)]^2$$

$$O_5 \equiv \alpha_5[Tr(V_\mu V^\mu)]^2, \quad (30)$$

where $V_\mu$ was defined above Eq. (16).

Whereas $O_3$ (and 5 other unspecified $SU_R^2$-breaking operators) contribute to a triple-vector boson vertex, the other two (plus 4 others that break the custodial symmetry) give quartic gauge-boson vertices and higher. These are the ones that involve interactions among the longitudinal vector bosons themselves and interact like the Goldstone bosons[7] so these are the ones that may be expected to be strongest. They are obviously important for discussing potentially strong $W W$-scattering processes, for example, at the NLC or SSC, or for linear colliders at extremely high

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1In the case of pions, the corresponding scale $v \equiv f_\pi \approx 95 MeV$ whereas $m_\rho = 770 MeV$.

2Longhitano's catalogue included 3 CP-violating operators, but Ref. [18] lists 5 additional ones.

3They survive even when you switch off the gauge fields.
energies, but it is $O_3$ that is most relevant to modifications of vector boson production at NLC. Now, you may note that, if the matrix $\Sigma$ were replaced by the doublet Higgs field $\phi$, $O_3$ would be of dimension 8. This illustrates a primary phenomenological distinction from the linear representation; operators that were of dimension-8 may, because of strong Higgs self-interactions, become as important as operators of dimension-6. Going to the unitary gauge, it is straightforward (but tedious) algebra to obtain the relations between these operators and the standard parameters.[19].

6. Orders of Magnitude

As indicated earlier, the experimental situation and potential will be summarized by Dr. Miyamoto in the next lecture. However, the sensitivity that seems to emerge for prospective facilities is

$$\begin{align*}
LEP(200) : & |\kappa - 1|, |\lambda| \lesssim 10\%, \\
NLC(500) : & |\kappa - 1|, |\lambda| \lesssim 1\%, \\
NLC(1000) : & |\kappa - 1|, |\lambda| \lesssim 0.1\%.
\end{align*}$$

(31)

Are there general theoretical or phenomenological arguments concerning the magnitude of the deviations to be expected from the SM predictions? Clearly, each gauge field brings in a power of a gauge coupling, but, more importantly, the triple vector boson operators are at least of one-loop order in any underlying theory conserving $SU_L^f \otimes U_Y^f$.[21] (This is not true for the four-point coupling, for example.) Therefore, it is natural to expect all the $\alpha_i$ that contribute directly to triple-vector-boson vertices to involve powers of the gauge couplings times at least a factor of $1/16\pi^2$ from the loop phase space factor. Thus, for example, we expect

$$\begin{align*}
\alpha_{WW} = \frac{g^3}{16\pi^2} x_W, \\
\alpha_{WB} = \frac{gg'}{16\pi^2} x_{WB},
\end{align*}$$

(32)

with the $x_i$ of order one. The next question is how small could $\Lambda$ possibly be? Certainly, in the usual, linear realization of $L_H$, the scale of new physics $\Lambda \sim v$. These observations imply that the natural values to be expected for deviations from the SM are no larger than a few tenths of one percent per loop:

$$\begin{align*}
|\kappa - 1| \lesssim 3 \times 10^{-3}, \\
|\lambda| \lesssim 2 \times 10^{-3}.
\end{align*}$$

(33)

Now it is reasonable to suppose that new physics involves several particles that might contribute constructively or that $\Lambda$ may be a bit smaller than $v$ so that the limits may be closer to 1%. Referring to the anticipated experimental accuracy, Eq. (31), we see that NLC(500) may just begin to provide interesting and meaningful constraints. LEP-2, on the other hand, would at best rule out rather bizarre models that have hundreds of particles adding up coherently.

*See the session at this conference on the strongly interacting Higgs sector for the phenomenological implications of these terms.
Now, you might think that one could generate larger effects in models in which there is a strongly interacting Higgs sector. But, in fact, their Goldstone character of the longitudinal modes means that, like pions, they couple weakly, until a scale considerably larger than the vector mass. Further, unitarity serves to limit their strength, so the effective scale parameter $\Lambda$ remains larger than the weak scale. In the final analysis, results on the overall strength are not expected to be very different from the linear case. This conclusion is supported by the recent general analysis of Ref. [1].

### 7. Gauge Invariance; Conclusions

In this lecture, I have emphasized that $SU^L_2 \otimes U^Y_1$ gauge symmetry strongly constrains the possibilities for new physics. This is actually a slightly controversial assertion that I will now address. A number of people [1, 8, 6] have remarked that the construction of the chiral Lagrangian involving the non-linear representation of the underlying symmetry be regarded as a non-Abelian generalization of the Stückelberg trick. Thus, any non-gauge invariant Lagrangian can be will-y-nilly assumed to be the unitary gauge expression of a gauge invariant theory. Therefore, some have emphasized (especially Ref. [8]) that gauge invariance is essentially without content. I will argue to the contrary. Is there really no difference between a weak vector boson and any other massive vector boson, like the $\rho$-meson, the deuteron [26], the $J/\psi$, etc.? This is a central issue—the scale of the structure of the particle. The essential difference is that a gauge particle is a vector particle that can be regarded as an elementary excitation of its field over a range of momentum that is large compared to its mass. Just because you can write an arbitrary vector interaction in gauge invariant form does not mean that is a useful description of nature beyond tree approximation or at large momentum scales. The only way that we know how to make a vector particle whose mass is small compared to the scale of its structure is via the Higgs mechanism. In the linear representation, it is manifest that the scale of new physics $\Lambda \gg v$. In fact, with an elementary scalar field, there are fine-tuning or naturalness issues [23] that suggest that there is a scale above which this description will break down. That scale cannot be larger than about $4\pi v$, not so very different from scale of compositeness in technicolor models. But the nonlinear realization is also only useful as an effective field theory if successive terms of higher chiral dimension are suppressed over some energy range large compared to the vector masses. This is a primary reason we are all so eager to probe the TeV region experimentally; the SM will change, at least through the addition of superpartners if not even more dramatically.

†Ref. [26] is extremely interesting in this context. However, that derivation of the “universal” character of the magnetic and quadrupole moments of a vector particle depends on assumptions about the number of subtractions required for convergence of their dispersion relations. This will ultimately depend on the short-distance structure of the theory, so I regard their conclusion as a consistency condition rather than a proof of universality.

‡This is almost a theorem [22].

§This is a different issue than the so-called triviality bound on the Higgs mass that shows that the local field theory becomes inconsistent for a Higgs mass larger than about 800 GeV. For a review, see Chpt. 9 of Ref. [24].
If the weak bosons were not gauge particles in the sense that I have described, then why are their couplings to fermions universal? Radiative corrections calculated within the SM involve loop integrals over a range of momenta up to the scale $\Lambda$. The particles in the loops are treated as structureless, point particles below that scale. Moreover, the dependences on this scale for different loops are constrained by gauge invariance. Outside of this context, the SM would not be a good first approximation insensitive to new physics at higher scales.

As SUSY models illustrate, this does not necessarily imply that all new particles lie at masses much larger than $M_W$ and $M_Z$, but it also does not mean that gauge invariance is without content. In particular, the structure of the gauge bosons are extremely sensitive to tampering. There have been many papers written showing this. For example, the electromagnetic self-energy of the $W^\pm$ would behave as $\Lambda^4$ if gauge-invariance were explicitly broken. Such dependences are not directly observable, since they simply multiply local operators and therefore renormalize Lagrangian parameters, but these contributions would change our present thinking considerably, because a theory that was not gauge invariant would involve many more coupling constants and mass parameters than are in the SM. For example, in that framework, there would be no reason for the precise relation between $M_W$ and $M_Z$, which would become independent parameters. The issue is one of naturalness; while fine-tuning is always possible, it is more likely that relations have reasons. To be a good first approximation at the scale $M_Z$, as they seem to be, the resulting corrections, like Schwinger’s old QED correction to $g^{-2}$, must be no smaller than the modifications coming from new physics. Thus, the scale $\Lambda$ of new physics ought to be larger than the vector masses.

This embedding of a non-gauge invariant Lagrangian in an underlying gauge theory, regarding it simply as the unitary gauge expression of a gauge invariant theory, does not dictate the underlying gauge group. We do not, for example, have to regard the general form in Eq. (24) as necessarily coming from an $SU_L^2 \otimes U_Y^1$ gauge invariant theory. We could choose another embedding having this as the unitary-gauge expression, e.g. $(\otimes U_1)^4$. Why does everyone choose to embed this in $SU_L^2 \otimes U_Y^1$? The statement that the underlying symmetry is an $SU_L^2 \otimes U_Y^1$ gauge symmetry has content.

Now I am aware that there is another point of view on these matters, one that has been extensively developed by members of the BMT collaboration. In this, one focuses exclusively on the vector boson interactions and attempts to build up relations among the observables $\kappa$ and $\lambda$ without assumptions about the underlying gauge-symmetry. This approach employs, for example, global symmetries or restrictions on the maximum power in the energy dependence of vector-boson scattering amplitudes in order to realize relations normally resulting from the effective Lagrangian approach that I have discussed. I simply have not been able to understand this alternate approach as a consistent field theory beyond tree level.

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*Universality for quarks is complicated by the CKM mixing angles (and the fact that quarks cannot be observed directly,) but the entire framework of phenomenology would be modified dramatically by strong interactions were it not for the underlying gauge symmetry.*
Gauge invariance is pervasive, and one may not simply assume that part of the Lagrangian explicitly violates it while the remainder respects it. The rules for calculating loop corrections become ambiguous, and the non-gauge invariance of one sector quickly spills over into the full theory. The challenge to proponents of this non-gauge-invariant alternative is to reproduce all that we know about electroweak interactions while modifying the interactions of the gauge bosons. I simply do not know how to do it other than as I have described within the framework of a gauge-invariant effective field theory.

I have already explained how, in the context of QED, the new divergences induced simply renormalize other couplings in the effective field theory. So while there had been some confusion in the past about how to use these nonrenormalizable operators in loop calculations, I think it is fair to say that the confusion has passed, and it is now well understood that an effective Lagrangian is in fact renormalizable in the sense that all divergences may be absorbed in renormalizations of the coefficients of one of its operators. So the degree of divergence of loop corrections or sensitivity to the cutoff $\Lambda$ can only be used for naturalness arguments and are not directly observable.

The gauge-invariant effective field theory approach that I have outlined in this lecture is the only way known that preserves the successes expressed by the SM Lagrangian while allowing for and including potential effects of new physics in a self-consistent manner. Unfortunately, nature has conspired to make new physics difficult to find, given our limited accuracy and energy, so that, if the threshold for new physics is at energies beyond our means, it is very difficult to find evidence of it through virtual effects. This is why the SM has withstood all tests to date. While the interactions among gauge bosons are of great interest, I am afraid the situation is unlikely to be different for the weak vector boson interactions, especially the triple-vector-boson couplings that can be probed at NLC. Although NLC can place interesting constraints on gauge boson interactions, given the limited accuracy that can be realistically anticipated, our best hope would seem to lie not in virtual effects, but in crossing the threshold for direct production of new particles.

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