D-branes of Covariant AdS Superstrings
– An Overview –

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ABSTRACT
We briefly review a covariant analysis of D-branes of type IIB superstring on the AdS$_5\times$S$^5$ background from the $\kappa$-invariance of the Green-Schwarz string action. The possible configurations of D-branes preserving half of supersymmetries are classified in both cases of AdS$_5\times$S$^5$ and the pp-wave background.

1. Introduction

D-brane is an important key ingredient in studies of non-perturbative aspects of superstring theories, and it is a recent interest to study D-branes on curved backgrounds. In particular, those on pp-wave backgrounds [1] have been well studied, since the Green-Schwarz strings on pp-waves are exactly solvable in light-cone gauge [2] and so one can study them directly by quantizing the theories [3-5].

Covariant studies of D-branes in type IIB and IIA strings on pp-waves were discussed in [7-8], respectively, by following the method of Lambert and West [9]. Motivated by these developments, we have carried out a covariant analysis of D-branes of type IIB string on the AdS$_5\times$S$^5$ background [10-11], by using the Green-Schwarz action obtained by Metsaev and Tseytlin [12]. The possible 1/2 supersymmetric (SUSY) D-brane configurations have been classified. This result is consistent to that of brane probe analysis in [5]. In addition, Penrose limits [13-14] of D-branes on the AdS$_5\times$S$^5$ give possible D-brane configurations in the type IIB pp-wave background.

On the other hand, by employing the methods of [16], the covariant analysis is also applicable to open supermembranes on the pp-wave [17-18] and AdS$_{4/7}\times$S$^{7/4}$ [19-20] backgrounds. These results are related via Penrose limit and are also consistent to the brane probe analysis in eleven dimensions [21].

We will briefly review the classification of D-branes on the AdS$_5\times$S$^5$ preserving half of supersymmetries, and discuss the Penrose limit of them.

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2. The action of type IIB string on the AdS$_5 \times S^5$

First of all, the action of AdS string we consider is written as [12]

\[ S = \int d^2 \sigma \left[ L_{\text{NG}} + L_{\text{WZ}} \right], \quad L_{\text{NG}} = -\sqrt{-g(X, \theta)} \cdot \quad (1) \]

The Nambu-Goto part of this Lagrangian is represented in terms of the induced metric $g_{ij}$, which is given by (For notation and convention, see [10])

\[ g_{ij} = E_i^M E_j^N G_{MN} = E_i^A E_j^B \eta_{AB}, \quad g = \det g_{ij}, \quad E_i^A = \partial_i Z^M E_M^A, \quad (2) \]

where $Z^M = (X^M, \theta^a)$ and $E_M^A$ are supervielbeins of the AdS$_5 \times S^5$ background. For D-strings, $g$ is replaced with $\det (g_{ij} + F_{ij})$ where $F$ is defined by $F = dA - B$ with the Born-Infeld $U(1)$ gauge field $A$ and the pull-back of the NS-NS two-form $B$. The Wess-Zumino term, which is needed for the $\kappa$-invariance and makes the theory consistent, is

\[ L_{\text{WZ}} = -2i \int_0^1 dt \tilde{E}^A \tilde{\theta} \Gamma_A \sigma \tilde{E}, \quad (3) \]

where $\tilde{E}^A \equiv E^A(t\theta)$ and $\tilde{E}^\alpha \equiv E^\alpha(t\theta)$. When we consider a fundamental string (F-string), the matrix $\sigma$ is given by $\sigma_3$. If we consider a D-string, then $\sigma$ is represented by $\sigma_1$. Since we would like to discuss boundary surfaces for both of F-string and D-string, we do not explicitly fix $\sigma$ in our consideration.

3. D-branes from $\kappa$-invariance

Let us consider D-branes on the AdS$_5 \times S^5$ by following the idea of Lambert and West [9]. They considered the Dp-branes from the $\kappa$-invariance of the Green-Schwarz type IIB string in flat space and obtained the standard fact that the value $p$ is odd. Such a constraint comes from the requirement that we should impose appropriate boundary conditions in order to delete the surface terms coming from the $\kappa$-variation and to ensure the consistency of the theory.

The idea of Lambert and West can be applicable to non-trivial backgrounds, including the AdS$_5 \times S^5$ and the pp-wave. In these cases, the boundary conditions restrict not only the value $p$ but also the configuration of a Dp-brane, and lead to the classification of possible D-branes [7,8,10,11,18,19,20].

3.1. The classification of 1/2 SUSY D-branes on the AdS$_5 \times S^5$

The classification of 1/2 SUSY D-branes on the AdS$_5 \times S^5$ [10] was given by considering the vanishing conditions of the $\kappa$-variation surface terms up to and including the fourth order in $\theta$. This result is still valid even at full order of $\theta$ [11]. The result is as follows: For the $d = 2 \ (\text{mod} \ 4)$ case, where $d$ is the number of Dirichlet directions, the possible configurations of D-branes need to satisfy the condition:
• The number of Dirichlet directions in the AdS\(_5\) coordinates \((X^0, \cdots, X^4)\) is even, and the same condition is also satisfied for the S\(_5\) coordinates \((X^5, \cdots, X^9)\).

For the \(d = 4 \text{ (mod 4)}\) case, D-branes satisfying the following condition are allowed:

• The number of Dirichlet directions in the AdS\(_5\) coordinates \((X^0, \cdots, X^4)\) is odd, and the same condition is also satisfied for the S\(_5\) coordinates \((X^5, \cdots, X^9)\).

For a D-brane on the AdS\(_5\)×S\(_5\), the directions to which the brane world-volume can extend are restricted. All the possible D-brane configurations at the origin are summarized in Tab.1. When we consider the D-branes sitting outside the origin, only a D-instanton is allowed as a 1/2 SUSY object.

Table 1: The possible 1/2 SUSY D-branes of F (D)-string on the AdS\(_5\)×S\(_5\), sitting at the origin.

| D-instanton | D (F)-string | D3-brane | D5 (NS5)-brane | D7 | D9 (NS9)-brane |
|-------------|--------------|----------|----------------|-----|----------------|
| (0,0)       | (0,2), (2,0) | (1,3), (3,1) | (2,4), (4,2) | (3,5), (5,3) | absent |

3.2. Penrose Limit of D-branes on the AdS\(_5\)×S\(_5\)

The Penrose limit \([13]\) of the AdS\(_5\)×S\(_5\) background leads to the maximally supersymmetric pp-wave background \([14]\). We may consider the Penrose limit of our classification result presented in the previous subsection. Then we can classify the possible D-branes on the pp-wave, including the well-known results in the light-cone analysis of the pp-wave string \([3,4,6]\) (For the detail, see our work \([10]\)). The result is summarized in Tab.2 which reveals the AdS origin of D-branes on the pp-wave. It is also consistent with the brane probe analysis \([5]\). Notably, we can see why 1/2 SUSY D-strings do not appear in the light-cone analysis.

Table 2: Penrose limit of D-branes on the AdS\(_5\)×S\(_5\).

| D7-brane | D5 (NS5)-brane |
|----------|----------------|
| \((3,5)\) | \((5,3)\) |
| \(D^2 \not\succ \ \\swarrow \ N^2\) | \(D^2 \not\succ \ \\swarrow \ N^2\) |
| \((+, -,; 2, 4)\) | \((+, -,; 2, 4)\) |

| D3-brane | D (F)-string |
|----------|--------------|
| \((1,3)\) | \((3,1)\) |
| \(D^2 \not\succ \ \\swarrow \ N^2\) | \(D^2 \not\succ \ \\swarrow \ N^2\) |
| \((1,3)\) | \((3,1)\) |

\(-\) : We cannot take this boundary condition.
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5. References

[1] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, JHEP 0201, 047 (2002); hep-th/0110242
[2] R. R. Metsaev, Nucl. Phys. B625, 70 (2002); hep-th/0112044  R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D65, 126004 (2002); hep-th/0202109
[3] M. Billo and I. Pesando, Phys. Lett. B536, 121 (2002); hep-th/0203028
[4] A. Dabholkar and S. Parvizi, Nucl. Phys. B641, 223 (2002); hep-th/0203231
[5] K. Skenderis and M. Taylor, JHEP 0206, 025 (2002); hep-th/0204054
[6] O. Bergman, M. R. Gaberdiel and M. B. Green, JHEP 0303, 002 (2003); hep-th/0205183
[7] P. Bain, K. Peeters and M. Zamaklar, Phys. Rev. D67, 066001 (2003); hep-th/0208038
[8] S. Hyun, J. Park and H. Shin, Phys. Lett. B559, 80 (2003); hep-th/0212343
[9] N. D. Lambert and P. C. West, Phys. Lett. B459, 515 (1999); hep-th/9905031
[10] M. Sakaguchi and K. Yoshida, Nucl. Phys. B684, 100 (2004); hep-th/0310228
[11] M. Sakaguchi and K. Yoshida, Phys. Lett. B591, 318 (2004); hep-th/0403243
[12] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B533, 109 (1998); hep-th/9805028
[13] R. Penrose, Any spacetime has a plane wave as a limit, Differential geometry and relativity, Reidel, Dordrecht, 1976, pp. 271-275. R. Gueven, Phys. Lett. B482, 255 (2000); hep-th/0005061
[14] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, Class. Quant. Grav. 19, L87 (2002); hep-th/0201081
[15] M. Hatsuda, K. Kamimura and M. Sakaguchi, Nucl. Phys. B632, 114 (2002); hep-th/0202190  Nucl. Phys. B637, 168 (2002); hep-th/0204002
[16] K. Ezawa, Y. Matsuo and K. Murakami, Phys. Rev. D57, 5118 (1998); hep-th/9707200  B. de Wit, K. Peeters and J. C. Plefka, Nucl. Phys. Proc. Suppl. 68, 206 (1998); hep-th/9710215
[17] K. Sugiyama and K. Yoshida, Nucl. Phys. B644, 113 (2002); hep-th/0206070
[18] M. Sakaguchi and K. Yoshida, Nucl. Phys. B676, 311 (2004); hep-th/0306213
[19] M. Sakaguchi and K. Yoshida, Nucl. Phys. B681, 137 (2004); hep-th/0310035
[20] M. Sakaguchi and K. Yoshida, hep-th/0405109
[21] N. Kim and J. T. Yee, Phys. Rev. D67, 046004 (2003); hep-th/0211029