Dynamic programming method in the tasks of optimal labor capital distribution programs

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Abstract. This article shows the application of the dynamic programming method to the tasks of the optimal distribution of labor resources by region. It discusses possible options for the formulation of this task, describes two models of the optimal distribution of labor resources, provides numerical examples.

1. Introduction
Several economic agents wish to co-organize the process of distributing labor capital in several regions. At the initial moment, the multitude of labor resources of economic agents were determined, distributed by region; the effectiveness of labor investment of economic agents in a particular region is also known. Each type of resource (capital) can be in one of a finite number of states that differ in the efficiency of its use. Each region of potential capital investment can be in one of a finite number of socio-economic conditions, characterized by the level of potential economic and financial development, the effectiveness of its investment in this region. At the same time, by the efficiency of investing labor capital of a given type in a given economic region, we mean its ability (possible time or probability) to transfer the investment object of a given region from a given financial and economic state to a state characterized by a higher (or lower) degree of economic development. It is necessary to distribute capital among economic regions in an optimal way, that is, in such a way that the total investment efficiency is maximum. By the beginning of the next period of time, depending on the decision made at the previous stage, and, possibly, for other reasons, many investment regions, many labor resources, as well as the effectiveness of a particular type of capital in a particular region may change. Consequently, a new situation arises in which it is necessary to solve a new problem on the optimal allocation of capital. For definiteness, let a finite set of such situations. Thus, a dynamic problem arises. It can be solved by the method of dynamic programming, and as strategies at each step, various solutions to the assignment problem are used [1-10].
2. Formal statement of the problem
We now turn to the formal statement of the problem. There are many labor resources (capital). We number them by index \( M_m, \ldots, M_1 \). There are many regions of possible investment. We number them by index \( L_l, \ldots, L_2, L_1 \). The process of investment and use of labor resources occurs during \( T \) time periods \( T_t, \ldots, T_2, T_1 \). We will evaluate the economic condition of the region \( l \) some number \( p(l) \) [11-13]. We introduce some system of numbers \( I_u \) \( u \in U = \{0, 2, 1, 0, \ldots\} \). We will say that the economic region is in a state (at the level of efficiency) \( a_k \), if the inequality

\[ u_k < p(l) \leq u_{k+1} \]

If before the start of the investment process, the region \( l \) was at the level of efficiency (in economic condition) \( a_{k_1} \), and after investing in a resource, he is in condition \( a_{k_2} \), then under the resource investment efficiency type \( m \) to the region \( l \) we will understand the value.

\[ E_{ml} = k_2 - k_1 \]

We denote the set of conditions in which economic regions may be located through \( A \) and number them by index \( a = 1, 2, \ldots, |A| \). Economic regions can be combined into a group, the state of which will be determined by the vector of economic conditions of individual regions. The set of conditions in which resources may exist (types of capital) is denoted by \( B \) and we index \( b = 1, 2, \ldots, |B| \). Types of labor resources can also be combined into a group, the state of which will be determined by the state vector of individual types of labor resources, that is, the state of the system is determined by

\[ s = \{ p_{m_1}, b_{m_2}^1, \ldots, b_{m|B|}^1, a_{l_1}^1, a_{l_2}^2, \ldots, a_{l|L|}^1 \} \]

Denote by \( S \) many states of the aggregated system and index them by index \( s = 1, 2, \ldots, |S| \). The number of system states is determined by the expression

\[ |S| = |A|^{|L|} \cdot |B|^{|M|}. \quad (1) \]

3. Options for solving the problem
Depending on how the system is transferred from one state to another, three options for the task can be considered. 1) Probabilistic option. If an investment resource (some type of capital) is in a state \( b \) nested in \( l \) economic region, the transition of this region from one state to another is determined by the probability matrix of the transition of the region \( p(m, b, l) \). Such matrices \( |M| \cdot |B| \cdot |L| \) pieces. If in \( l \) economic region investing in a state \( a \), invested resource type \( m \), then its transition from one state to another is determined by the probability matrix of the resource transition \( P(l, a, m) \) [16-19]. Such matrices \( L \cdot |A| \cdot |M| \) pieces. If at a point in time the system was in a state

\[ S_t = \{ p_{m_1}^{t_1}, b_{m_2}^{t_2}^1, \ldots, b_{m|B|}^{t|B|}, a_{l_1}^{t_1}^1, a_{l_2}^{t_2}^2, \ldots, a_{l|L|}^{t|L|}^1 \} \]

and at the moment \( t + 1 \) it’s located on status
then this transition occurs with probability

\[
\mathbf{\tilde{P}}_{x_{n+1}} = \prod_{m=1}^{M} P_{m^{n+1}}(l,a,m) \prod_{i=1}^{L} P_{q_{i}^{n+1}} a_{i}^{n+1}(m,b,l)
\]

We denote many strategies (that is, solutions to the problem of assigning labor resources of different types to various economic regions) by \(Q\) and number them by index \(q = 1,2,\ldots,|Q|\). Now we can say that a matrix is defined \(\mathbf{\tilde{P}}(q)\) probabilities of transition of a system from one state to another, depending on the chosen strategy

2) Deterministic option. In this case, both resources of different types and investment regions go into some specific states with a probability equal to one: that is, functions are defined \(F(m,b,l,a)\) transition region investment \(l\) from one state to another, if for some standard period of time in invested a certain type of investment resource \(m\), in condition \(b\), and functions \(\mathbf{\tilde{F}}(m,b,l,a)\) the transition of the invested resource from one state to another, if it was invested in the investment region \(l\), in condition \(a\). Then if at the moment \(t\) the system was in a state

\[
S_{t} = \left\{ b_{m_{1}}, b_{m_{2}}, \ldots, b_{m_{|M|}}, a_{l_{1}}, a_{l_{2}}, \ldots, a_{l_{|L|}} \right\}
\]

and at the moment \(t+1\) it’s located on status

\[
S_{t+1} = \left\{ \ldots, \mathbf{\tilde{F}}(l,a,m,b_{m_{i}}), \ldots, \ldots, \ldots, \right\},
\]

\[m = 1,2,\ldots,|M|,
\]

\[l = 1,2,\ldots,|L|.
\]

Using the transition functions

\[
F(m,b,l,a)
\]

and

\[
\mathbf{\tilde{F}}(m,b,l,a),
\]

can get function \(\mathbf{\tilde{F}}(q,s)\) system transition from one state to another [20-25].

3) Probabilistic-determinate option. This option is intermediate between the first and second, it stands out for the convenience of calculations. In this case, the transition of one of the groups (for example, a group of regions of investment) is carried out with certain probabilities in a particular state, its other group (for example, a group of invested labor resources) goes into a certain state depending on the chosen strategy. As in the probabilistic version, the matrix is defined \(\mathbf{\tilde{P}}(q)\) the probabilities of the transition of the system from one state to another, depending on the chosen strategy, however, in this embodiment, this matrix has a smaller dimension [26-28].

When the system transitions from state \(S_{t}\) in condition \(S_{t+1}\) there is an increase in the economic potential of the region \(r_{S_{t+1}}(q)\), which can be calculated in various ways, for example
4. Calculation of the optimal income in relation to the probabilistic version of the problem statement

To calculate the optimal income from the functioning of the system during \( T \) we will use the dynamic programming recurrence relations. Consider them in relation to the probabilistic version of the problem statement. Let be \( V^{T-t}(S_t) \) - maximum income from the functioning of the system during \( T-t \) time periods from state \( S_t \), Where \( S_t = 0,1,2,\ldots,T \) under optimal policy. The maximum income from the functioning of the system for one period of time is determined by the formula

\[
V^1(S_0) = \max_{q \in Q} \left\{ \sum_{S_1} P_{S_0S_1}(q) r_{S_0S_1}(q) \right\}.
\]

The maximum income from the functioning of the system for two periods of time is delivered by the expression

\[
V^2(S_0) = \max_{q \in Q} \left\{ \sum_{S_1} P_{S_0S_1}(q) \left[ r_{S_0S_1}(q) + V^1(S_1) \right] \right\},
\]

Where \( V^1(S_1) \) - maximum income from the functioning of the system in a one-step process. To calculate the optimal income, we have the following functional relation

\[
V^{T-t}(S_t) = \max_{q \in Q} \left\{ \sum_{S_{t+1}} \tilde{P}_{S_tS_{t+1}}(q) \left[ r_{S_tS_{t+1}}(q) + V^{T-t-1}(S_{t+1}) \right] \right\}.
\]

\[ V^0(S_t) \]

can be set equal to zero, which is natural.

5. Conclusion

Applying relation (5) sequentially for

\[ t = T - 1, T - 2, \ldots, 1, 0 \]

we calculate

\[ V^1(S_{T-1}), V^2(S_{T-2}), \ldots, V^{T-1}(S_1), V^T(S_0) \]

and we will be able to indicate the optimal distribution of labor resources by investment region at any given time. The described method for solving the problem can be implemented on a computer.

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References

[1] Badjin G M and Sychev S A 2013 Energy-economic house: Energy-Efficient construction technologies *Transmit World* 2(1)

[2] Badjin G M and Sychev S A 2015 Improving Technology of Constructing Pre-Fabricated Buildings in the Conditions of Northern Regions *Applied Mechanics and Materials* 725-6
[3] Bondarenko L A, Zubov A V, Orlov V B, Petrova V A and Ugegov N S 2016 Application in practice and optimization of industrial information systems Journal of Theoretical and Applied Information Technology 85(3)

[4] Bondarenko L A, Zubov A V, Zubova A F, Zubov S V and Orlov V B 2015 Stability of quasilinear dynamic systems with after effect Biosciences Biotechnology Research Asia 12(1)

[5] Dikusar V V, Zubov A V and Zubov N V 2010 Structural minimization of stationary control and observation systems Journal of Computer and Systems Sciences International 49(4)

[6] Malafeev O 1995 On the existence of nash equilibria in a noncooperative n-person game with measures as coefficients Communications in Applied Mathematics and Computational Science 5(4) 689-701

[7] Malafeev O and Nemnyugin S 1996 Generalized dynamic model of a system moving in an external field with stochastic components Theoretical and Mathematical Physics 107(3) 770

[8] Malafeyev O, Zaitseva I, Onishenko V, Zubov A, Bondarenko L, Orlov V, Petrova V and Kirjanen A 2019 Optimal location problem in the transportation network as an investment project: A numerical method AIP Conference Proceedings 2116 450058

[9] Murashko A Y, Orlov V B, Zubov A V, Bondarenko L A and Petrova V A 2019 Qualitative analysis of the behavior of one mechanical system International Journal of Innovative Technology and Exploring Engineering 8(7)

[10] Neverova E G, Malafeyev O A, Alferov G V and Smirnova T E 2015 Model of interaction between anticorruption authorities and corruption groups” International Conference on Stability and Control Processes in Memory of V.I. Zubov (SCP, Proceedings, SPb) pp 488-90

[11] Pichugin Y A, Malafeyov O A, Rylow D and Zaitseva I 2018 A statistical method for corrupt agents detection AIP Conference Proceedings

[12] Sychev S 2015 Technologies for fast economical construction of residential buildings ARPN Journal of Engineering and Applied Sciences 10(17)

[13] Sychev S and Badjin G An interactive construction project for method of statement based on BIM technologies for high-speed modular building Architecture and Engineering 1

[14] Sychev S and Sharipova D 2015 Monitoring and logistics of erection of prefabricated modular buildings Indian Journal of Science and Technology 8(29)

[15] Vlasov M A, Glebov V V, Malafeyev O A and Novichkov D N 1986 Experimental study of an electron beam in drift space Soviet journal of communications technology & electronics 31(3) 145-9

[16] Zaitseva I 2019 Numerical method of distribution of labor resources by game-theoretic model AIP Conference Proceedings 2116 450057

[17] Zaitseva I V, Ermakova A N, Shlaev D V, Shevchenko E A and Lugovskoy S I 2017 Workforce planning redistribution of the region’s results Research Journal of Pharmaceutical, Biological and Chemical Sciences 8(1) 1862-6

[18] Zaitseva I, Malafeyev O, Dolgopolova A, Zhukova V and Vorokhobina Y 2019 Numerical method for computing equilibria in economic system models with labor force AIP Conference Proceedings 2116 450060

[19] Zaitseva I, Malafeyev O, Kolesin I, Ermakova A and Shlaev D 2018 Modeling of the labour force redistribution in investment projects with account of their delay IEEE International Conference on Power, Control, Signals and Instrumentation Engineering, ICPCSI 2017

[20] Zaitseva I, Malafeyev O, Marenchuk Y, Kolesov D and Bogdanova S 2019 Competitive Mechanism for the Distribution of Labor Resources in the Transport Objective Journal of Physics: Conference Series

[21] Zaitseva I, Malafeyev O, Poddubnaya N, Vanina A and Novikova E 2019 Solving a dynamic assignment problem in the socio-economic system Journal of Physics

[22] Zaitseva I, Malafeyev O, Strekopytov S, Bondarenko G and Lovyanikov D 2018 Mathematic model of regional economy development by the final result of labor resources AIP Conference Proceedings
[23] Zaitseva I, Malafeyev O, Strekopytov S, Ermakova A and Shlaev D 2018 Game-theoretical model of labour force training Journal of Theoretical and Applied Information Technology
[24] Zubov A V, Dikusar V V and Zubov N V 2010 Controllability criterion for stationary systems Doklady Mathematics 81 (1)
[25] Zubov A V, Murashko A Y, Kolyada L G, Volkova E A and Zubova O A 2016 Fidelity issue of engineering analysis and computer aided calculations in sign models of dynamic systems Global Journal of Pure and Applied Mathematics 12(5)
[26] Zubov A V, Orlov V B, Petrova V A, Bondarenko L A and Pupysheva G I 2017 Engineering and Computer Aided Calculations upon Determination of Quality Characteristics of Dynamic Systems International Journal of Pure and Applied Mathematics 117(22)
[27] Zubov AV 2007 Stabilization of program motion and kinematic trajectories in dynamic systems in case of systems of direct and indirect control Automation and Remote Control 68(3)
[28] Zubov I V and Zubov A V 2009 The stability of motion of dynamic systems Doklady Mathematics 79(1)