Stress intensity factors of double edge cracks in large groove plates under mode I tension

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Abstract. The solutions of stress intensity factors (SIFs) of double edge cracks in large groove plates are investigated. The plate is assumed to fulfil the plain strain condition and it is axially stressed. According to the literature, the SIFs of the cracks in circular notches are available. However, the roles of cracks in large grooves are difficult to obtain. Therefore, different grooves and crack geometries are modelled numerically using the ANSYS finite element software. The present model is first validated using an existing model of double edge cracks without notches and it is found that both models are in close agreement. The large grooves are then introduced and the cracks are positioned in the mid-point. It is found numerically that the SIFs are strongly affected by the relative crack depth and the groove geometries. It is also found that the large groove is capable of reducing the SIFs in comparison with the circular notched due to lower stress concentration factors.

1. Introduction

In modern engineering, discontinuities in engineering components are unavoidable due to the design complexities. Under certain circumstances, cracks might appear on the surface of such discontinuities due to corrosion or fatigue [1]. Generally, stress intensity factor (SIF) approach is used successfully to characterize the cracks. On the other hand, other different methods are also available for the purpose of crack characterizations [2]. According to the recent literature survey, the solutions of SIFs for the circular holes or notches containing cracks are available [3-5]. The solutions of SIFs without notches are also vastly available [6-10]. However, the roles of cracks in large grooves are rarely investigated and not fully understood. Therefore, this present paper studies the effect of notches and large grooves containing different crack depths. In this study, four groove depths and eight crack lengths have been numerically modelled using ANSYS finite element software. The SIFs are based on the displacement extrapolation method and they are then normalized and plotted against the relative crack depth.

2. Theoretical background

The evaluation of J-integral around the crack tip is based on the domain integral method initially introduced by Shih \textit{et al.} [11]. This integral formulation uses area integration for 2-dimensional and volume integration for 3-dimensional problems which offer much better accuracy than contour integral and it is also much easier to implement numerically. Equation 1 represents the 2D domain J-integral
taking accounts the absence of thermal strain, path dependent plastic strain, body forces occur within
the integration area \[12\].

\[
J = \int A \left[ \sigma_{ij} \frac{\partial u_j}{\partial x_i} - w \delta_i^j \right] \frac{\partial q_j}{\partial x_i} \, dA
\]

(1)

where \(q_{ij}\) is the stress tensor, \(u_i\) is the displacement vector, \(w\) is the strain energy density, \(\delta_i^j\) is
the Kronecker delta, \(x_i\) is the coordinate axis and \(q\) is referred to as the crack extension vector.
The direction of \(q\) is similar with \(x\)-axis of the local coordinate specified at the crack tip and it is
normally chosen as zero at nodes along the contour \(\Gamma\). It is also a unit vector for all nodes inside \(\Gamma\) except
the mid-side nodes and known as virtual crack extension nodes. For 3-dimensional problems, the principal
is similar to 2-dimensional problems. However, domain integral representation of J-integral becomes
volume integration \[13\]. Two approaches for calculating stress intensity factor (SIF) are available in
ANSYS software: interaction integral and displacement extrapolation methods. The first method is
used because it is much easier to implement numerically, and it also offers better accuracy and fewer
mesh requirement. This method is similar to domain integral method for J-integral evaluation that is
described previously. Discussion on the domain integral methods can be found in \[13\]. The interaction
integral is defined as in Equation 2.

\[
I = \int \nabla \cdot \sigma_{ij} \cdot \delta_{j,i} - \sigma_{k,j}^\text{aux} \cdot \mu_{k,j} - \sigma_{k,j}^\text{aux} \cdot \mu_{k,j}^\text{aux} \, dv / \int \delta q_i \, ds
\]

(2)

where \(\sigma_{ij}, \varepsilon_{ij}\) and \(u_i\) are the stress, strain and displacement, and \(\sigma_{k,j}^\text{aux}, \varepsilon_{k,j}^\text{aux}\) and \(u_i^\text{aux}\) are the stress, strain and
displacement of the auxiliary field, and \(q_i\) is the crack extension vector.

\[
\sigma_{ij}^\text{aux} = \frac{K_{ij}^\text{aux}}{\sqrt{2\pi r}} f_{ij}^I (\theta) + \frac{K_{ij}^\text{aux}}{\sqrt{2\pi r}} f_{ij}^II (\theta) + \frac{K_{ij}^\text{aux}}{2\pi r} f_{ij}^III (\theta)
\]

(3)

\[
u_{ij}^\text{aux} = \frac{K_{ij}^\text{aux}}{2\mu} \sqrt{\frac{r}{2\pi}} g_{ij}^I (\theta, \nu) + \frac{K_{ij}^\text{aux}}{\mu} \sqrt{\frac{r}{2\pi}} g_{ij}^II (\theta, \nu) + \frac{2K_{ij}^\text{aux}}{\mu} \sqrt{\frac{r}{2\pi}} g_{ij}^III (\theta, \nu)
\]

(4)

\[
\varepsilon_{ij}^\text{aux} = \frac{1}{2} (u_{ij}^\text{aux} + \nu_{ij}^\text{aux})
\]

(5)

An expression for the energy release rate in terms of mixed-mode SIFs is defined as for plane strain
condition as in Equation 6, Equation 7 and Equation 8.

\[
J = \left( \frac{K_1^2 + K_2^2}{E} \right) (1 - \nu^2) + \frac{K_3^2}{E} (1 + \nu)
\]

(6)

\[
J = \left[ \left( K_1 + K_2^\text{aux} \right)^2 + (K_3 + K_3^\text{aux})^2 \right] \left( 1 - \nu^2 \right) + \left( \frac{K_3^2}{E} + \frac{K_3^\text{aux}}{E} \right) (1 + \nu)
\]

(7)

\[
J = J + J^\text{aux} + I
\]

(8)

The interaction integral can be associated with the SIFs as Equation 9.

\[
I = \frac{2(1 - \nu^2)}{E} \left( K_1 K_1^\text{aux} + K_2 K_2^\text{aux} \right) + \frac{1}{\mu} K_3 K_3^\text{aux}
\]

(9)

By setting \(K_2^\text{aux} = 1\) and \(K_1^\text{aux} = K_3^\text{aux} = 0\),

\[
K_1 = \frac{E}{2(1 - \nu^2)} I
\]

(10)
By setting $K_{II}^{aux} = 1$ and $K_{I}^{aux} = K_{III}^{aux} = 0$ and $K_{I}^{aux} = K_{II}^{aux} = 0$ leads to the relationship between modes II and III SIFs with $I$, respectively.

$$K_{II} = \frac{E}{2(1-\nu^2)} I$$

(11)

$$K_{III} = \mu I$$

(12)

where $K_i$ is a SIF with $i$ is a loading mode, $i = 1, 2$ and 3. J-integral can be represented as $J$ and $I$ is a interaction domain integral. Furthermore, $E$ and $\mu$ is a modulus of elasticity and modulus of rigidity, respectively.

3. Methodology

ANSYS finite element software is applied to model the cracks in large groove plates. The plate is assumed to fulfill the plain strain condition and it has two grooves at both sides and the cracks are located in the middle as shown in Figure 1. Due to the symmetrical geometry, only a quarter model is used and modelled numerically. A quarter finite element model is developed using ANSYS software and a special attention is given to the crack tip by employing 20-node iso-parametric elements. The square-root singularity of stresses and strains at the crack tip is modelled by shifting the mid-point nodes to the quarter-point location close to the crack tip, as shown in Figure 2. The bottom and right lines, except the crack face, are symmetrically restrained while the pressure is applied onto the left line. In order to calculate the stress intensity factors (SIFs), J-integral approach is used and it is then converted to SIFs according to Equation 13. The SIFs are normalized using Equation 14 and plotted against the relative crack depth, $a/d$.

$$K_i = \sqrt{\frac{J_i E}{1-\nu^2}}$$

(13)

$$F_{I,a} = \frac{K_i}{\sigma \sqrt{\pi a}}$$

(14)

where $K_i$ is a mode I stress intensity factor, $F_{I,a}$ is a normalized stress intensity factor, $J_i$ is an elastic J-integral, $E$ is a modulus of elasticity, $\nu$ is a Poisson’s ratio, $\sigma$ is a tension stress and $a$ is a crack length. Mode I is a condition where the crack is opened. Large groove is defined when $r \gg c$.

![Figure 1. Large groove plate containing double edge crack in the middle location](image)

![Figure 2. (a) Quarter finite element model and (b) Singular elements at the crack tip](image)
In this paper, the range of values of $a/d$ used is between 0.1 to 0.8. Meanwhile, the relative groove depth, $c/D$ are 0.125, 0.25, 0.375 and 0.50 where the plate width, $W = 2D$. In order to study the role of groove geometries, there are fifteen ratios of $c/r$ that are used depending on the radiuses and depth of the grooves. Before the model is further used, it is an important task to validate and compare the present model with an existing model. Since there is no cracks available in large grooves, then the crack without notch is used. It is found both models are in a good agreement as shown in Figure 3.

![Figure 3. Finite element validation](image)

4. Results and discussion
Figure 4 shows the effect of $a/d$ on the SIF for various groove geometries. The distributions of the SIFs are almost identical where the SIFs have reduced as longer cracks are used. However, when the crack length reached a certain crack depth depending on the groove geometries, the SIFs are increased. The turning point of the SIFs is approximated half of the plate width. This behaviour of crack is quite different with the cracks without notches where the SIFs increased as the crack length is increased [9, 10]. Figure 4 also reveals that the SIFs distributions can be divided into two regions. The first region shows that the SIFs are strongly depend on the crack depth and radius of the grooves. It is revealed that SIFs for circular notch are greater than the SIFs obtained from the large grooves. The dispersions of SIFs increased in magnitude if the groove depths are increased. In the second region where the cracks become deeper, all the SIFs converged into a single value indicating that varying the groove geometries are not affected the SIFs. Table 2 tabulates the regression statistics of the Equation 15.

$$F_{r,a} = 2.0324 - 0.5200 \left( \frac{c}{r} \right) - 1.491 \left( \frac{a}{L} \right) - 0.7810 \left( \frac{c}{r} \right)^2 + 0.2751 \left( \frac{r}{W} \right)^2 - 0.6060 \left( \frac{a}{L} \right)^2$$

$$- 0.0840 \left( \frac{c}{r} \right)^2 - 0.0920 \left( \frac{r}{W} \right)^3 + 2.3318 \left( \frac{a}{L} \right)^3 - 0.4130 \left( \frac{c}{r} \right)^3 + 0.1591 \left( \frac{r}{W} \right) \left( \frac{a}{L} \right)^2$$

$$+ 5.2377 \left( \frac{r}{W} \right) \left( \frac{c}{r} \right)^2 + 0.1247 \left( \frac{a}{L} \right) \left( \frac{r}{W} \right)^2 - 2.1930 \left( \frac{a}{L} \right) \left( \frac{c}{r} \right)^2 + 0.7629 \left( \frac{c}{r} \right) \left( \frac{r}{W} \right)^2$$

$$+ 0.7629 \left( \frac{c}{r} \right) \left( \frac{r}{W} \right)^2$$

The results conforms that by using Equation 15, the SIFs values can be estimated appropriately. In order to show the performance of the expression, a certain crack condition is selected for example for the case of $r/W = 0.6, r/c = 4.8$ and $a/L = 0.1$ to 0.8. The comparisons between two methods are shown in Figure 5 where both results are in good agreement.
Figure 4. The effect of relative crack depth, $a/d$ on the SIFs of various groove depth, (a) $c = 6.250$, (b) $c = 9.375$ and (c) $c = 12.500$. 
Table 1. Effect of $r/c$ on the stress intensity factors for different crack length

| $r/c$ | $a/L$ | 0.40 | 0.60 | 0.80 |
|-------|-------|------|------|------|
| 0.1   | 1.643 | 2.048| 2.463| 2.980|
| 0.2   | 1.451 | 1.797| 2.191| 2.703|
| 0.3   | 1.369 | 1.670| 2.042| 2.543|
| 0.4   | 1.347 | 1.619| 1.975| 2.470|
| 0.5   | 1.367 | 1.625| 1.974| 2.472|
| 0.6   | 1.430 | 1.687| 2.042| 2.556|
| 0.7   | 1.556 | 1.826| 2.202| 2.755|
| 0.8   | 1.803 | 2.109| 2.537| 3.172|

Table 2. Regression statistics of SIFs

| Regression Statistics | Value |
|-----------------------|-------|
| Multiple $R$          | 0.9969|
| $R$ Square            | 0.9937|
| Adjusted $R$ Square   | 0.9929|
| Standard Error        | 0.0396|
| Observations          | 128   |

Figure 5. Comparisons between numerical and closed form solutions

5. Conclusion
The use of double edge crack in the large grooved plates are successfully applied as a numerical model using ANSYS finite element software. Different groove and crack geometries are studied on the SIFs. Based on the numerical analysis, several conclusions can be drawn. First, the SIFs obtained from the circular notches are higher than the SIFs obtained using large groove due to the different in the effect of stress concentrations. Secondly, for relatively shallow crack but depending on the groove depths, the SIFs are strongly related with $c/r$. However, when $c/r$ ratio exceed a certain limit, the SIFs seem to have a single value. Last but not least, the developed closed form solution is capable to predict the SIFs well for wide ranges of groove geometries with high reliability.

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