City fire risk analysis is a very important basis for scientifically and effectively planning and deploying urban fire protection and formulating its related regulations. It also can guarantee and facilitate the coordinated development of city’s economic construction and social economy. Combining the city's development characteristic of China and the fire control safety management state, the thesis based on fire control safety engineering and systematic safety engineering theory and combine, has set up the fire risk fault tree of city area. Applying the method of the fuzzy important degree sorted the basic incident of the fault tree and did a fuzzy graduation on the factors of caused fire. The order and grade of fuzzy probability has shown that the main factors of limiting fire risk are the city area overall arrangement, the moved fire control strength and the personnel quality, which is in conformity with the main reason of the city fire of our country taken place over the years. For this reason, in order to limit the city fire risk fundamentally, we must manage the above three important factors strictly, which has a very positive significance for city fire safety management of China.

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Keywords: City fire risk, System safety engineering, Fault tree, Fuzzy probability, Triangle fuzzy number

1. Introduction

Fire safety assessment (also known as a fire risk assessment) is an important part of fire protection science
research. City safety assessment has its own characteristics. For the city's fire safety, its fire source is complicated and changeable and its disaster-pregnant environment has multiple hierarchies, complex structures. Moreover, there are also many uncertainty and fortuity. Therefore, the reasonable analysis method, adopted to analyze and study the urban fire safety and then accurately evaluate urban fire safety status, is an important guarantee for scientifically and effectively managing urban fire protection management, formulating corresponding fire protection regulation, rationally allocating urban fire fighting forces and promoting the coordinated development of city’s economic construction and social economy. In recent years, many scholars at home and abroad has done research work on urban fire risk. The fire risk analysis method studied by scholars at Abroad includes[1]: American Insurance Services Office (ISO) city fire gradation method based on community fire insurance rate, American Building Fire Safety Evaluation Method (BFSEM), Australian Risk Assessment Model (RAM), Japanese Building Comprehensive Fire Safety Design Method and Canadian FIRECAM Method, etc.. While the scholars in China, such as Xiao Guoqing, Lijie, Zhang Yixian etc., have also done a lot of research work on city fire risk [2]. However, there are certain defects in the research theory and results of many scholars above, which specifically embodied in these aspects: the overlook on the extent of the hazards after the catastrophe, the overlook on the possibility size of the catastrophe of hazard, the lack of scientific index system and the lack of the comprehensiveness and systematic in the urban fire risk assessment.

Fault tree analysis method is a very important analytical method in safety system analysis. In the fault tree triggering hazard (basic events) risk probability calculation, since the every incident which constitute the fault tree was expressed by the objective uncertainty factors, therefore, it is difficult for the traditional fault tree to use mathematical model or formula to analyze and calculate the uncertainty factors of the system, which has become the shortcoming that cannot be ignored in the traditional fault tree analysis.

The fuzzy theory is the best tool to deal with the problem above and also can solve the problem which can’t easily be solved by probability theory [3]. For those bottom incidents which are less than the accurate value of occurrence probability, the fuzzy mathematics theory can be applied. These bottom incidents occurrence probability can be considered as a fuzzy numbers, namely, fuzzy probability is used to depict the error behaviour of system and its constitution.

In the city fire risk analysis, since the statistical material of the basic incidents is hard to determine, is a typical fuzzy system and the non-quantitative system, therefore, expert scoring method be usually used to evaluate the uncertainty factors. Fault tree method and triangle fuzzy theory not only can analyze the basic incidents occurrence probability, but can analyze the main cause of limiting fire risk. It is a kind of multi-criteria and multi-objective analysis and evaluation model of complex issues, which can objectively describe people's subjective judgment and plays a very positive role in the city fire risk analysis.

**2. Triangular fuzzy numeral**

2.1. Basic Concepts

The fuzzy theory is proposed by L.A.Zadeh in 1965, which can be used to deal with the issue with imprecise and vague phenomenon [4]. Suppose \( U \) as a domain of discourse constituted by objects, then a fuzzy number \( \tilde{A} \) in domain of discourse \( U \) can be defined as a membership function:

\[
\mu_{\tilde{A}}(x) : U \rightarrow [0, 1] \quad x \in U
\]

The function record the real number, which is mapped to \([0, 1]\) from the elements in \( U \), as \( \tilde{A} = \int_{x \in U} \mu_{\tilde{A}}(x) \frac{dx}{x} \),

among which, \( \tilde{A}(x) \) is referred to as the degree that element \( x \) in domain of discourse \( U \) attaches to fuzzy sets \( \tilde{A} \), for short, \( x \) for \( A \) degree of membership. The greater \( \mu_{\tilde{A}}(x) \) is, the stronger the degree of \( x \) subordinate to \( A \) becomes. Sometimes membership function \( \mu_{\tilde{A}}(x) \) can be simply recorded as \( \tilde{A} \).

If a membership function of fuzzy number is composed by linear function, expressed as
\[ \mu_\tilde{A}(x) = \begin{cases} \frac{x-m+1}{l} & l \leq x \leq m \\ \frac{m+u-x}{u} & m \leq x \leq u \\ 0 & \text{others} \end{cases} \]

\( m \) is called the nucleus of \( \tilde{A} \) and \( u+l \) is called blind degrees. Its membership function is shown in figure 1. This kind of fuzzy number is known as triangular fuzzy number, which can be recorded as \( \tilde{A} \ (l, m, u) \).

\[
\begin{align*}
\tilde{q}_1 \oplus \tilde{q}_2 &= (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \\
\tilde{q}_1 \odot \tilde{q}_2 &= (l_1, m_1, u_1) \odot (l_2, m_2, u_2) = (l_1 \cdot l_2, m_1 \cdot m_2, u_1 \cdot u_2) \\
\tilde{q}_1 \oslash \tilde{q}_2 &= (l_1, m_1, u_1) \oslash (l_2, m_2, u_2) \\
\tilde{q}_1^2 &= (l_1^2, m_1^2, u_1^2) \\
C \otimes \tilde{q}_1 &= (Cl_1, Cm_1, Cu_1)
\end{align*}
\]

2.2. Triangular fuzzy numeral algorithm

According to the algorithm of fuzzy number, the following general algebraic operation formula of triangular fuzzy number can be derived.

1. “⊕” operation
\[ \tilde{q}_1 \oplus \tilde{q}_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \]

2. “⊙” operation
\[ \tilde{q}_1 \odot \tilde{q}_2 = (l_1, m_1, u_1) \odot (l_2, m_2, u_2) = (l_1 \cdot l_2, m_1 \cdot m_2, u_1 \cdot u_2) \]

3. “⊘” operation
\[ \tilde{q}_1 \oslash \tilde{q}_2 = (l_1, m_1, u_1) \oslash (l_2, m_2, u_2) \]
\[ \tilde{q}_1^2 = (l_1^2, m_1^2, u_1^2) \]
\[ C \otimes \tilde{q}_1 = (Cl_1, Cm_1, Cu_1) \]

2.3. Basic Incidents Fuzzy Probability Statistic

This thesis uses the method of 3σ to represent the basic incidents fuzzy probability [5]. Expert scoring method was adopted to determine the fuzzy probability of basic incidents which haven’t statistical information. The assessment work was operated by a panel of experts which has over 3 people. Every expert respectively give the estimate value of the every basic incidents probability; take the average value of the every estimated probability as \( m \) and the variance as \( \sigma \). Suppose a probability value obey normal statistical rule, according to the ruler of 3σ, the probability that its value falls in the interval \([u-3\sigma, u+3\sigma]\) is 99%. Therefore, suppose \( l=u=3\sigma \), the every probability value was fuzzily represent as \( (3\sigma, m, 3\sigma) \). This method was called 3σ characterization method in this thesis.

Suppose that \( n \) experts have evaluated the occurrence probability of some incident, its probability value can be set as:
\[ A_i = (a_{11}, a_{12}, \cdots, a_{1n}) \]
According to the knowledge of probability theory, for discrete random variables, the mathematical expectation $E(x)$, that is, its mean value $m$, can be $u = m = E(x) = (1/n) \bigotimes (a_{ij} + a_{ij} + \cdots + a_{in})$.

For discrete random variables, its mathematics covariance is:

\[
\sigma = \sqrt{\sum_{k=1}^{n} [x_k - E(x)]^2 P_k}
\]

In the function: $x_k$ is the probability value of the first $k$ items.

According to discrete probability theory, there are:

\[
P_k = P\{X = x_k\} \quad k=1, 2, \cdots, n
\]

For the occurrence probability of basic incidents which have statistical information, $P$ can be worked out, which can be processed as $\overline{q} = (p, p, p)$ with fuzzy method.

\section{Fault tree method and triangle fuzzy numeral theory coupling}

\subsection{Fault tree fuzzy processing operators}

The operators of logic and gate in traditional fault tree are:

\[
q_{\text{and}} = \prod_{i=1}^{n} q_i
\]

In the function: $q_i$ is the accurate occurrence probability value of incident $i$.

The fuzzy operators of and gate with fuzzy processing are:

\[
q_{\text{and}}^\text{and} = (l_{\text{and}}, m_{\text{and}}, u_{\text{and}}) = \prod_{i=1}^{n} q_i = \overline{q}_1 \otimes \overline{q}_2 \otimes \cdots \otimes \overline{q}_n = \left( \prod_{i=1}^{n} l_i, \prod_{i=1}^{n} m_i, \prod_{i=1}^{n} u_i \right)
\]

The operators of logic or gate in traditional fault tree are:

\[
q_{\text{or}} = 1 - \prod_{i=1}^{n} (1 - q_i)
\]

The fuzzy operators of or gates with fuzzy processing are:

\[
q_{\text{or}}^\text{or} = (l_{\text{or}}, m_{\text{or}}, u_{\text{or}}) = (1 \otimes \prod_{i=1}^{n} (1 \circ \overline{q}_i) =
\]

\[
((1 \circ \prod_{i=1}^{n} (1 \circ \overline{l}_i), (1 \circ \prod_{i=1}^{n} (1 \circ \overline{m}_i), (1 \circ \prod_{i=1}^{n} (1 \circ \overline{u}_i))
\]

\subsection{Fuzzy importance degree judgment data}

Definition 1: for figure 1, define $A_1 = \int_{x_1}^{x_n} \mu_m(x) \, dx$, $A_2 = \int_{x_1}^{x_m} \mu_m(x) \, dx$, $A = A_1 + A_2$. There are dot $Z$, which let the line passed this dot as dividing line. The area of left and right parts under the fuzzy number curve is equal, therefore, $Z$ is called the median of this fuzzy number.

Definition 2: suppose the structure function of fault tree as $I = \mu_m(x)$, and $x_i$ is the basic incident of fault tree. Record its possibility distribution of fuzzy numbers as $\tilde{x}_i$, therefore, the top incidents probability distribution as $\tilde{T} = \tilde{\phi}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) = (3\sigma, m, 3\sigma)$, and then record its median as $T_2$. 
\[ \tilde{T}_i = \tilde{\varphi}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{i-1}, 0, \tilde{x}_{i+1}, \ldots, \tilde{x}_n) = (3\sigma, m, 3\sigma), \text{ and then record its median as } T'_Z. \]

Fuzzy importance degree of basic incidents \( x_i \)

\[ S_i = T'_Z - T'_Z. \]  \( \text{(9)} \)

If \( S_i > S_j \), then consider that \( x_i \) is more important than \( x_j \), therefore, the effect to system caused by \( x_i \) is greater than \( x_j \).

4. City fire risk analysis

4.1. Index system establishment

City fire risk is the results of the comprehensive effects of the occurrence possibility of fire accident and the extent of damage caused by the fire accident. According to the relevant provisions in the Ministry of Public Security “Fire Fighting Management Regulation for Enterprises, Organizations and Institutions (Decree No.61)” and “Fire Control Law of the People's Republic of China” and in accordance with the importance degree and different properties of the various factors which lead to building fire, the urban fire risk fault tree model was established, as shown in figure 2[6, 7]. The name of all the incidents is the event-signature in Table 1.

4.2. Basic incidents occurrence probability processing

Taken some city of Hunan province fire risk current situation as example, we have done some work of statistics, summary and comparison on this city’s fire occurrence situation in recent 10 years. The basic time occurrence probability with readily available statistics can be obtained through the statistics and the quantification processing of the occurrence time, and then do fuzzy processing, namely, given fuzzy number \( \bar{\alpha} = (p, p, p) \). Because the statistics work of basic incident without statistics data is difficult, the occurrence probability can only be obtained
through experts scoring methods and giving language value, and then using the formula (4) and (5) to do fuzzy processing, the processed data as shown in table 1.

| Code | Incidents | $P_x$ | Code | Incidents | $P_x$ |
|------|-----------|-------|------|-----------|-------|
| $D$  | City fire risk | (0.054, 0.093, 0.054) | $X_{12}$ | Poor automatic fire extinguishing facilities | (0.015, 0.020, 0.020) |
| $A$  | Fire occurrence possibility | (0.120, 0.160, 0.120) | $X_{13}$ | Poor automatic alarm facilities | (0.015, 0.020, 0.020) |
| $A_1$ | Human error | (0.040, 0.040, 0.040) | $X_{14}$ | Poor smoke control facilities | (0.015, 0.020, 0.020) |
| $A_2$ | Material factors | (0.040, 0.060, 0.040) | $X_{15}$ | Less public fire protection training | (0.066, 0.086, 0.066) |
| $A_3$ | Poor environmental conditions | (0.001, 0.002, 0.001) | $X_{16}$ | Lack of public fire protection awareness | (0.010, 0.015, 0.010) |
| $A_4$ | Low technological level | (0.040, 0.040, 0.040) | $X_{17}$ | Defective fire protection regulation | (0.108, 0.200, 0.108) |
| $A_5$ | Lack of management factors | (0.060, 0.080, 0.060) | $X_{18}$ | Backward public fire control facilities | (0.252, 0.302, 0.252) |
| $B$  | Fire occurrence seriousness | (0.450, 0.580, 0.450) | $X_{19}$ | Non-standard fire protection design | (0.252, 0.302, 0.252) |
| $B_1$ | Inappropriate city layout | (0.060, 0.080, 0.060) | $X_{20}$ | Unreasonable city function distribution | (0.455, 0.565, 0.455) |
| $B_2$ | Complex building feature | (0.009, 0.009, 0.009) | $X_{21}$ | Unreasonable fire protection road design | (0.455, 0.565, 0.455) |
| $B_{21}$ | High building plot ratio | (0.085, 0.085, 0.085) | $X_{22}$ | Many leve-1 and level-2 refractory building | (0.107, 0.186, 0.107) |
| $B_3$ | High economic society indicator | (0.009, 0.068, 0.099) | $X_{23}$ | Large proportion of high-rise building | (0.107, 0.186, 0.107) |
| $B_4$ | Municipal fire water supply lack | (0.001, 0.001, 0.001) | $X_{24}$ | Low serviceability rate of fire control facilities | (0.040, 0.050, 0.040) |
| $B_5$ | Weak move fire-fighting power | (0.108, 0.108, 0.108) | $X_{25}$ | High per capita GDP | (0.010, 0.010, 0.010) |
| $B_{51}$ | Less effect of fire station | (0.040, 0.040, 0.040) | $X_{26}$ | Large population density | (0.105, 0.120, 0.105) |
| $B_{52}$ | Poor fire-fighting equipment | (0.283, 0.292, 0.292) | $X_{27}$ | Large proportion of above college population | (0.010, 0.015, 0.010) |
| $B_{53}$ | Less effect of fire man | (0.027, 0.027, 0.027) | $X_{28}$ | Large proportion of floating population | (0.001, 0.001, 0.001) |
| $X_1$  | Careless smoking | (0.005, 0.005, 0.005) | $X_{29}$ | Less fire protection water supply | (0.089, 0.094, 0.089) |
| $X_2$  | Illegal operations | (0.127, 0.127, 0.127) | $X_{30}$ | Poor Pipe water supply ability | (0.089, 0.094, 0.089) |
| $X_3$  | Intentional arson | (0.001, 0.001, 0.001) | $X_{31}$ | Less fire hydrant pipe | (0.089, 0.094, 0.089) |
| $X_4$  | Careless use of fire | (0.007, 0.007, 0.007) | $X_{32}$ | Large protection area | (0.100, 0.100, 0.100) |
| $X_5$  | Big power consumption | (0.045, 0.060, 0.045) | $X_{33}$ | Large building area | (0.100, 0.100, 0.100) |
| $X_6$  | More flammable items | (0.154, 0.286, 0.154) | $X_{34}$ | Long response time | (0.403, 0.455, 0.403) |
| $X_7$  | Low construction safety level | (0.038, 0.040, 0.040) | $X_{35}$ | Less serviceability rate of fire hydrant facilities | (0.042, 0.049, 0.042) |
| $X_8$  | Many gas pipeline network | (0.064, 0.080, 0.064) | $X_{36}$ | Poor equipment | (0.012, 0.015, 0.012) |
| $X_9$  | Low relative humidity | (0.001, 0.001, 0.001) | $X_{37}$ | Large numbers of fire-fighters protection | (0.033, 0.035, 0.033) |
| $X_{10}$ | Large wind speed | (0.001, 0.001, 0.001) | $X_{38}$ | Less limit working years of fire-fighters | (0.009, 0.010, 0.009) |
| $X_{11}$ | Poor automatic control facilities | (0.015, 0.020, 0.020) | $X_{39}$ | Poor fire extinguishing skills of fire-fighters | (0.086, 0.094, 0.086) |

### 4.3. Fire risk fuzzy evaluation computer-aided design

The self-made computer-aided fault tree analysis system has been used in the process of calculation. Based on VB and required windows2000/XP system as its running environment, this computer-aided system’s work will cover the fault tree graph drawing, the minimum path sets and minimum cut sets transformation and the solution of
the structure importance degree, the probability importance degree, the danger importance degree and the occurrence probability of the top incidents.

Using the “and” and “or” fuzzy operators of formula (7) and (8) determine the occurrence fuzzy probability value of the top events. The value is:

\[ q_D = (0.054, 0.093, 0.054) \]

The mean value of the occurrence probability of top incidents is 0.067.

Since the thesis uses the method of $3\sigma$ to process the triangular fuzzy number as shapes with the symmetrical left and right sides (conform to definition 1), combining function (9), the basic incident, based on the condition that the median $T_Z$ of the top incidents fuzzy number is definite value, can be ranked according to the size of $T_Z$ value (namely $m_i$). The sequence of bottom incidents (basic incidents) is as shown in Fig. 3.

4.4. City Fire Risk Evaluation Results

The analysis and calculation result in Fig. 3 shows that the key to limit this city’s fire risk level is the unreasonable urban layout, the city building fire protection features, the unsafe factors of human and the weak move fire-fighting power. In fact, these factors are in accordance with the cause of this city fire in recent years. In some major fire accident with many casualties and big economic losses, the most part of the cause of fire is the irregular operation, the careless use of fire and the overmuch flammable goods with the analysis of the statistical data provided by the fire safety department. However, the occurrence of disastrous accident after fire is due to the mixture of this city’s residential and commercial community, the narrow fire fighting access in the crowded places, the insufficient municipal fire water supply and the weak passive fire fighting force.

In the further research and analysis, during the evaluation of the top incidents state degree, using linguistic variables “safe”, “safer”, “more dangerous”, “dangerous” and “very dangerous” express the degree of its state, which can people have a general concept on the percept. The expert evaluation has also been used and the fuzzy probability processing with formula (6) – (9) has been done. The level of public awareness and the triangular fuzzy probability are as shown in Fig. 4 and Table 2. The fact that this city fire risk is in the level of “more dangerous” can be known when the method of $3\sigma$ be used to evaluate the occurrence degree of the top incidents (namely the city fire
risk occurrence probability is in a dangerous range).

![Fig.4. Triangle fuzzy numeral of public cognition degree.](image)

| Fuzzy Parameter | Safe | Safer | More dangerous | Dangerous | Very dangerous |
|-----------------|------|-------|----------------|-----------|----------------|
| \( m \)         | 0.000| 0.400 | 0.600          | 0.800     | 1.000          |
| \( 3 \sigma \)  | 0.200| 0.100 | 0.050          | 0.025     | 0.010          |

5. Conclusions

City fire risk evaluation is the basic link of urban disaster prevention. Only based on the foundation of correct evaluation of fire risk, the disaster prevention links like prediction and control will make sense; the validity and rationality of fire prevention and correct evaluation of fire risk complement each other.

(1) In the application, the insufficient data and the fuzziness of basic incidents restricted the analytical method of traditional fault tree. The solution is to introduce the fuzzy mathematics theory, to adopt the fault tree analysis methods based on triangular fuzzy number operation, to express the occurrence probability of bottom incidents with the language set and to quantify the expert evaluation set, which has solved these problems, including the selection of the fuzzy number unified form and unified handling. This method is more accord with the engineering practice and has higher engineering real value.

(2) On the basis of the full investigation, analysis and conclusion of the fire statistics at home and abroad, the thesis based on the building fire design manual and the related laws and regulations, put forward a three-level evaluation model of urban fire risk from the consideration of the occurrence possibility of fire and the seriousness of disaster after accident. Taken the fire situation in some city for example, it also deduced and verified this model through the analytical methods of coupling the triangle fuzzy number and fault tree, which has established a more perfect evaluation model for the city fire risk analysis and also provided a new guide basis for the city’s fire engineering.

(3) At present, China’s city building fire risk analysis and evaluation method is still in the incomplete stage. The author believes that the method of coupling the fault tree of city building fire risk and the triangular fuzzy theory coupling will continuously be perfect with the incessant development and improvement of fire safety engineering in China. These methods will play a more important role increasingly.
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