A spintronic analog of the Landauer residual resistivity dipole on the surface of a topological insulator containing a line defect

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Abstract
The Landauer ‘residual resistivity dipole’ is a well-known concept in electron transport through a disordered medium. It is formed when a defect/scatterer reflects an impinging electron causing negative charges to build up on one side of the scatterer and positive charges on the other. This charge imbalance results in the formation of a microscopic electric dipole that affects the electrical resistivity of the medium. Here, we show that an equivalent entity forms in spin polarized electron transport on the surface of a real topological insulator (TI) such as $\text{Bi}_2\text{Te}_3$ containing a line defect. When electrons reflect from such a scatterer, a local spin imbalance forms owing to spin accumulation on one side and depletion on the other side of the scatterer, resulting in a spin current that flows either in the same or in the opposite direction as the injected spin current, and hence, either decreases or increases the spin resistivity. Spatially varying local magnetic fields appear in the vicinity of the scatter, which will cause transiting spins to precess and emit electromagnetic waves. If the current injected into the TI is an alternating current, then the magnetic field’s polarity will oscillate in time with the frequency of the current and if the spins can follow quasi-statically, then they will radiate electromagnetic waves of the same frequency, thereby making the scatterer act as a miniature antenna.

Keywords: topological insulators, Landauer residual resistivity dipole, line defects, spin-momentum locking

(Some figures may appear in color only in the online journal)

1. Introduction

The Landauer residual resistivity dipole (LRRD) is a familiar concept in microscopic charge transport and has important consequences for electromigration [1, 2]. The basic idea behind the LRRD is illustrated in figure 1. A moving electron in a charge current (sometimes referred to as an ‘electron wind’) encounters a scatterer on the way and is reflected with some probability, causing negative charges to accumulate on the impinging side and deplete on the opposite side. This charge imbalance causes an electric dipole to form around the scatterer, which affects the medium’s electrical resistivity.

It is natural to ask if an equivalent magnetic entity can exist in spin polarized electron transport on the surface of a topological insulator (TI) containing scatterers that reflect
an impinging spin. The case of normal incidence is easy to visualize. If spin-momentum locking is not destroyed by the scatterer, then the reflected spin will be antiparallel to the incident spin since the wave vectors of the incident and reflected spins will be oppositely directed. Quantum interference between the incident and reflected electrons will make the spin polarization vary in space for some distance from the scatterer. On the other hand, if the reflection destroys spin-momentum locking, then the reflected and incident electrons need not have antiparallel spins. In this case, the building up of the electron population on the impinging side and depletion on the other side (in the manner of LRRD) will inevitably cause a spin imbalance around the scatterer. This is shown in figure 2. The case of oblique incidence is a little more difficult to visualize, but similar considerations hold. The local spin imbalance will cause a spin current which add to or subtract from the injected spin current, thereby decreasing or increasing the ‘spin resistivity’. In the case of LRRD, it was pointed out that the transport problem must be solved ‘self-consistently’ by solving the Boltzmann Transport Equation and the Poisson equation. Similar considerations will govern the problem of spin transport.

The local spin imbalance will also form local magnetic dipoles and local magnetic fields, which will cause transiting spins to precess and radiate electromagnetic waves as they traverse these regions, making the scatterer act as a source of radiation. Additionally, if the injected current polarity alternates in time (with not too high frequency), then the polarities of the magnetic fields and dipoles will also alternate in time with the same frequency as the current and radiate electromagnetic waves of the same frequency, which is the characteristic of an antenna.

In this paper, we analyze this phenomenon.

2. Model and formalism

For the sake of simplicity, we will consider a magnetic line defect on the surface of the TI (perpendicular to the direction of current flow) to be the scatterer. It can be created intentionally by implanting a row of magnetic impurities using shallow ion implantation or chemical vapor deposition through a mask. A magnetic impurity can reflect a spin with or without spin flip because it has an internal spin degree of freedom. Figure 3 shows such a system. This figure considers a spin incident from one of the two spin eigenstates of a TI. The reflected spin can be reflected into the same eigenstate with a reflection amplitude \( r \) and into the other eigenstate with a reflection amplitude \( r' \).

To make the mathematics simple, we will consider the line defect to have zero spatial width and hence act as a one-dimensional delta scatterer.

The Pauli equation for the spinor wave function \( \psi \) of an electron on the TI surface like Bi\(_2\)Te\(_3\) containing the line defect is [3, 4]

\[
\left[ \frac{\hbar^2}{2m} \left( \nabla^2 + k_x^2 + k_y^2 \right) + i \hbar \nu \left( k_x \sigma_y - k_y \sigma_x \right) + \Gamma \delta(x) + \Gamma' \sigma_y \delta(x) \right] \psi(x,y) = E \psi(x,y)
\]

(1)

where the scattering potential of the line defect has a spin-independent part of strength \( \Gamma \) and a spin-dependent part of strength \( \Gamma' \). Any band warping effect has been neglected in equation (1). The TI surface is assumed to be the \( x-y \) plane. The quantity \( \nu \) = \( \nu_0 \left( 1 + \alpha k^2 \right) \) [3, 4], but for simplicity we will assume \( \alpha = 0 \). Here, \( \nu_0 \) is the Dirac velocity and \( \sigma_f \) is the spin-flip matrix. We assume no other scattering, with or without spin flip. The ‘no-other-scattering’ (ballistic) approximation need not hold throughout the TI surface, but needs to hold in the vicinity of the line defect that we restrict our analysis to. The general form of the spin-flip matrix is [5]

\[
\sigma_f = e^{-i \sigma_x / 2} \begin{pmatrix} 1 & 0 \\ 0 & e^{i \gamma} \end{pmatrix} \begin{pmatrix} \sigma_y & 0 \\ 0 & e^{-i \gamma} \end{pmatrix},
\]

(2)
in between a real TI and the Rashba system have been discussed. TI system exhibit similar physics; for example, there is spin-
the TI, the opposite is true. The Rashba system and the real
a structural symmetry breaking electric field perpendicular to
Hamiltonian of a two-dimensional electron gas (2-DEG) with
energy dispersion relation

\[ E = \pm \hbar v_F k \] (6) where \( k = \sqrt{k_x^2 + k_y^2} \)
which are the familiar Dirac cones with perfect time reversal
symmetry. Real TIs, like Bi$_2$Te$_3$, however do not fit this bill
due to spin imbalance in the vicinity of the defect.

\[ \psi(x,y) = \Phi_\pm e^{(k^2 x + k_3 y)} \] (5)

Since there is no term in equation (1) that is a function of both x- and y-variables, the wave function is variable separable and hence we can write equation (1) using total derivatives:

\[ \left[ -\frac{\hbar^2}{2m^*} \frac{d^2}{dx^2} + \frac{\hbar^2}{2m^*} \frac{d^2}{dy^2} + \Gamma \delta(x) + \Gamma' \sigma_y \delta(x) \right] \psi(x,y) = E \psi(x,y). \]

Integrating both sides of the above equation from \(-\varepsilon\) to \(+\varepsilon\) and letting \( \varepsilon \to 0 \), we get [9]

\[ \frac{d\psi(x,y)}{dx} \bigg|_{x=0^+} - \frac{d\psi(x,y)}{dx} \bigg|_{x=0^-} = \frac{2m^*}{\hbar^2} \Gamma \left[ 1 + \left( \frac{\Gamma'}{\Gamma} \sigma_y \right) \right] \psi(x = 0, y). \] (4)

In deriving the above equation, we made use of the continuity of the wave function at \( x = 0 \).

Note that since the Hamiltonian in equation (1) is invariant in the coordinate y, the y-component of the wave vector \( k_y \) is a good quantum number. However, the x-components of the wave vectors in the two eigenspinors on the TI surface are different for any given energy [10] and these two wave vectors will be denoted as \( k_x^\pm \). They do not represent oppositely traveling states along the x-direction; rather, they are the x-components of wave vectors in the two spin eigenstates of a TI at any given energy.

We can write the wave functions in the two spin eigenstates on a pristine TI surface (without any defect) as

\[ \psi_\pm(x,y) = \Phi_\pm e^{(k^2 x + k_3 y)}, \]

where the eigenspinors are given by [6]

\[ \Phi_\pm = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_\pm \end{bmatrix}, \]

\[ \mu_\pm = \pm \left( \frac{k_x^+}{k_x^-} \right) \mp \left( \frac{k_y^+}{k_y^-} \right), \] (6)

and \( k^\pm = \sqrt{\left(k_x^\pm \right)^2 + k_y^2} \).

We will now consider the situation where an electron in the \( \Phi_+ \) eigenstate (one of the two eigenstates of the TI surface) is incident on the line defect at \( x = 0 \). The other case, when the incident electron is in the \( \Phi_- \) eigenstate, is similar and is omitted here for the sake of brevity. Let the reflection amplitude for reflecting into the first eigenstate be \( r \), the second eigenstate be \( r' \) and the transmission amplitudes into the two states be \( t \) and \( t' \), respectively.

The wave function to the left of the line defect is then

\[ \psi_L(x,y) = \Phi_+ e^{(k^2 x + k_3 y)} + r \Phi_+ e^{(-k^2 x + k_3 y)} + r' e^{(-k^2 x + k_3 y)} \Phi_- \]
while to the right, it is

\[ \psi_R(x,y) = t \Phi_+ e^{(k^2 x + k_3 y)} + t' \Phi_- e^{(k^2 x + k_3 y)} \] (see figure 3).

Enforcing the continuity of the wave function at \( x = 0 \), we get

\[ [1 + r] \Phi_+ + r' \Phi_- = rt \Phi_+ + t' \Phi_- . \] (7)

We also point out that the Hamiltonian in equation (1) is valid for the surface states near a Dirac cone in a three-dimensional TI. More complex effects that are associated with quantum spin Hall systems [6] or thin film structures of finite width or thickness [7] are ignored here, as is the effect of any external magnetic field [8].

In an ‘ideal’ TI with no defects, only the second term in the Hamiltonian in equation (1) will be present. That will make the energy dispersion relation \( E = \pm \hbar v_F k \) which are the familiar Dirac cones with perfect time reversal symmetry. Real TIs, like Bi$_2$Te$_3$, however do not fit this bill and the first term in the Hamiltonian will also be present, albeit it will be much smaller than the second term, except at very high wave vectors.

Curiously, with the first term present, the Hamiltonian in equation (1) looks identical to the two-dimensional Rashba Hamiltonian of a two-dimensional electron gas (2-DEG) with a structural symmetry breaking electric field perpendicular to the 2-DEG (if we replace \( \hbar v_F \) with the Rashba constant \( \alpha \)). The only difference is that in the Rashba system, the first term in the Hamiltonian is dominant over the second term, whereas in the TI, the opposite is true. The Rashba system and the real TI system exhibit similar physics; for example, there is spin-momentum locking in both. The similarities and differences between a real TI and the Rashba system have been discussed in [4].
Next, using equations (5) in (4), we obtain

\[
\begin{align*}
    ik^+_{\lambda} [1-r] \Phi_+ - ik^-_{\lambda} r' \Phi_- &= -\frac{2m^*}{\hbar^2} \Gamma'(1+r) \Phi_+ + r' \Phi_- - ik^-_{\lambda} r \Phi_+ - ik^+_{\lambda} r' \Phi_-
\end{align*}
\]

With the aid of equations (6) and (7) can be written as

\[
[A] = \begin{bmatrix} t & r \\ t' & r' \end{bmatrix} = [C],
\]

where

\[
[A] = \begin{bmatrix} \Phi_+ & \Phi_- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ \mu_+ & \mu_- \end{bmatrix},
\]

\[
[C] = \Phi_+ = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_+ \end{bmatrix}.
\]

Note that the matrix [A] is not unitary since \(k_x^+ \neq k_x^-\). From equation (9), we get

\[
\begin{bmatrix} t' & r' \end{bmatrix} = [A]^{-1} [C].
\]

Then from equation (8), we obtain

\[
\begin{align*}
    \left( ik^+_{\lambda} + \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_+ + \frac{2m^*}{\hbar^2} \Gamma' \Phi_- &= r \left( ik^+_{\lambda} - \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_+ - \frac{2m^*}{\hbar^2} \Gamma' \Phi_- \\
    &+ r' \left( ik^-_{\lambda} - \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_- - \frac{2m^*}{\hbar^2} \Gamma' \Phi_+ + t \left( ik^+_{\lambda} + t' ik^-_{\lambda} \right) \Phi_- \\
\end{align*}
\]

which can be written in matrix form as

\[
[B] \begin{bmatrix} r \\ r' \end{bmatrix} + [D] \begin{bmatrix} t \\ t' \end{bmatrix} = [K],
\]

where

\[
[B] = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix},
\]

\[
\begin{align*}
    b_1 &= \left( ik^+_{\lambda} - \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_+ - \frac{2m^*}{\hbar^2} \Gamma' \Phi_- \\
    b_2 &= \left( ik^-_{\lambda} - \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_- - \frac{2m^*}{\hbar^2} \Gamma' \Phi_+ \\
    b_3 &= \left( ik^+_{\lambda} - \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_+ + \frac{2m^*}{\hbar^2} \Gamma' \Phi_- \\
    b_4 &= \left( ik^-_{\lambda} - \frac{2m^*}{\hbar^2} \Gamma \right) \Phi_- + \frac{2m^*}{\hbar^2} \Gamma' \Phi_+ \\
\end{align*}
\]

Using equations (10) in (12), we obtain a solution for the reflection amplitudes into the two eigenstates of the TI as

\[
\begin{bmatrix} r \\ r' \end{bmatrix} = ([B] + [D])^{-1} [K] - \left\{ ([B] + [D])^{-1} [D] [A]^{-1} [C] \right\}.
\]

Finally, using equations (13) in (10), we get the solution for the transmission amplitudes:

\[
\begin{bmatrix} t \\ t' \end{bmatrix} = ([B] + [D])^{-1} [K] - \left\{ ([B] + [D])^{-1} [D] - [I] \right\} \times [A]^{-1} [C]
\]

where [I] is the 2 \times 2 identity matrix.

The wave function on the left of the line defect is (see figure 3)

\[
\psi_L = \Phi_+ e^{i(k^+_x x + k_y y)} + r \Phi_+ e^{-(k^+_x x + k_y y)} + r' \Phi_- e^{i(-k^-_x x + k_y y)} + \alpha_1(x, y)
\]

\[
\begin{align*}
    &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_+ \end{bmatrix} e^{i(k^+_x x + k_y y)} + \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_- \end{bmatrix} e^{-(k^-_x x + k_y y)} \\
    &+ r' \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_- \end{bmatrix} e^{i(-k^-_x x + k_y y)} \\
\end{align*}
\]

whereas on the right it is

\[
\psi_R = r \Phi_+ e^{i(k^+_x x + k_y y)} + t' \Phi_- e^{i(-k^-_x x + k_y y)}
\]

\[
\begin{align*}
    &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_+ \end{bmatrix} e^{i(k^+_x x + k_y y)} \\
    &+ t' \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \mu_- \end{bmatrix} e^{i(-k^-_x x + k_y y)} \\
\end{align*}
\]
Therefore, the x-, y- and z- components of the spin on the left of the line defect are
\[ S_1^x(x, y) = \hbar \text{Re} [\alpha_1^x(x, y) \alpha_2(x, y)] / M(x, y) \]
\[ S_1^y(x, y) = \hbar \text{Im} [\alpha_1^y(x, y) \alpha_2(x, y)] / M(x, y) \]
\[ S_1^z(x, y) = \frac{\hbar}{2} \left[ |\alpha_1|^2(x, y) - |\alpha_2|^2(x, y) \right] / M(x, y) \] (17)

and the same components on the right of the defect are
\[ S_2^x(x, y) = \hbar \text{Re} [\beta_1^x(x, y) \beta_2(x, y)] / M'(x, y) \]
\[ S_2^y(x, y) = \hbar \text{Im} [\beta_1^y(x, y) \beta_2(x, y)] / M'(x, y) \]
\[ S_2^z(x, y) = \frac{\hbar}{2} \left[ |\beta_1|^2(x, y) - |\beta_2|^2(x, y) \right] / M'(x, y) \] (18)

where \( M(x, y) = \sqrt{|\alpha_1|^2(x, y) + |\alpha_2|^2(x, y)} \) and \( M'(x, y) = \sqrt{|\beta_1|^2(x, y) + |\beta_2|^2(x, y)} \).

3. Results and discussion

To obtain representative results, we assume \( m^* = 0.1 m_0 \) (\( m_0 \) is the free electron mass), \( (1/2) m_0 \gamma_0^2 = 100 \text{ meV}, \) \( \Gamma = 6 \times 10^{-29} \text{ J} - \text{m} \) and \( \Gamma' = 4 \times 10^{-29} \text{ J} - \text{m} \). The energy dispersion relation on the surface of a TI (without any defect) can be obtained by diagonalizing the Hamiltonian in equation (1) after putting \( \Gamma = \Gamma' = 0 \), and is given by [10]
\[ E = \frac{\hbar^2 (k_x^2 + k_y^2)}{2m^*} \pm \hbar v_0 \sqrt{k_x^2 + k_y^2} \]. (19)

For the sake of simplicity, we will consider the case of normal incidence \( k_z = 0 \). All ensuing results are for normal incidence only since the case for oblique incidence \( k_z \neq 0 \) is mathematically more complicated. For normal incidence, the x-components of the wave vectors in the two spin eigenstates are related to the energy \( E \) as
\[ k_x^+ = \frac{m^* v_0}{\hbar} + \sqrt{\left( \frac{m^* v_0}{\hbar} \right)^2 + \frac{2m^* E}{\hbar^2}} \]
\[ k_x^- = \frac{-m^* v_0}{\hbar} + \sqrt{\left( \frac{m^* v_0}{\hbar} \right)^2 + \frac{2m^* E}{\hbar^2}} \] (20)

We have verified that the current continuity condition is always satisfied at every energy, i.e.
\[ |t(E)|^2 + |r(E)|^2 + \frac{k_x^-}{k_x^+}[t'(E)]^2 + \frac{k_x^+}{k_x^-}[r'(E)]^2 = 1 \]. (21)

The above equation is valid only for normal incidence \( k_z = 0 \). The current continuity equation for oblique incidence \( k_z \neq 0 \) is much more complicated and has been discussed in [10].

We use equation (20) in equations (13) and (14) to find the transmission and reflection probabilities as functions of the electron energy \( E \) for \( k_z = 0 \). They are plotted in figure 4.

\[ \text{Figure 4. Transmission and reflection probabilities associated} \]
\[ \text{with the line defect as functions of electron energy for} \]
\[ \Gamma = 6 \times 10^{-29} \text{ J} - \text{m and} \Gamma' = 4 \times 10^{-29} \text{ J} - \text{m and} \ k_z = 0. \]

For \( k_z = 0 \) (normal incidence), we find that \( |A|^{-1} |C| = \begin{bmatrix} 1 & 0 \end{bmatrix} \) (dagger represents Hermitian conjugate) and hence from equation (10), \( t = 1 + r \) and \( r' = r \).

Also for \( k_z = 0 \), \( \mu_z = \pm i \), and hence we get from equations (15) and (16):
\[ \alpha_1(x) = \frac{1}{\sqrt{2}} \left( e^{ik^+ x} + e^{-ik^+ x} + r'e^{-ik^- x} \right) \]
\[ \alpha_2(x) = \frac{i}{\sqrt{2}} \left( e^{ik^+ x} - e^{-ik^+ x} - r'e^{-ik^- x} \right) \]
\[ \beta_1(x) = \frac{1}{\sqrt{2}} \left( te^{ik^+ x} + r'e^{ik^- x} \right) \]
\[ \beta_2(x) = \frac{i}{\sqrt{2}} \left( te^{ik^+ x} - r'e^{ik^- x} \right) . \] (22)

These equations for normal incidence immediately show two interesting results: (a) if the transmitted spin does not couple into the second spin eigenstate, i.e. \( r' = 0 \), then the spin polarization on the right side of the scatterer would be completely y-polarized and spatially invariant \( \left[ \beta_1 \propto 1 / \sqrt{2} ; \beta_2 \propto i / \sqrt{2} \right] \), thus preserving spin-momentum locking; and (b) if the reflected spin does not couple into the second spin eigenstate, i.e. \( r' = 0 \), then the spin polarization on the left side of the scatterer will also be completely y-polarized and spatially invariant \( \left[ \alpha_1 \propto 1 / \sqrt{2} ; \alpha_2 \propto i / \sqrt{2} \right] \), again preserving spin-momentum locking. Only when the scatterer couples the spin into the second spin eigenstate that spin-momentum locking is broken.

We use the relations in equations (17) and (18) along with equation (20) to find the spin components \( S_x, S_y \) and \( S_z \) for normal incidence \( k_z = 0 \) as functions of the distances.
Figure 5. Spin components to the left and right of the magnetic line defect in units of $\hbar/2$, as functions of the distance from the magnetic line defect with $k_y = 0$.

from the magnetic line defect on both sides extending up to 25 nm for an electron energy $E = 100$ meV and plot them in figure 5. We assume that the distance of 25 nm is much shorter than the spin relaxation length, so we can neglect any spin relaxation.

Clearly, the spin polarization to the right of the scatterer is not 100% $y$-polarized and the $x$- and $z$-components are non-zero and oscillate in space. This is a consequence of the scatterer transmitting a spin incident from one eigenstate into the other eigenstate, and their spin polarizations are different.
Table 1. Spin components averaged over a fixed distance $W$ on the two sides of the magnetic line defect ($W = 25$ nm).

| $\langle S^x_t \rangle$ | $\langle S^y_t \rangle$ | $\langle S^z_t \rangle$ | $\langle S^x_r \rangle$ | $\langle S^y_r \rangle$ | $\langle S^z_r \rangle$ |
|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| -0.011                 | -0.003                 | 0.517                  | 0.459                  | 0.021                  | -0.027                 |

Figure 6. Angular separation between the spin polarizations at two locations on either side of the magnetic line defect as a function of the distance $|x|$ between those locations and the defect.

We define spatially averaged spin components on the two sides of the line defect as $\langle S^x_t \rangle = \int_{-W}^{0} S^x_t(x) \, dx \ [i = x, y, z]$, and $\langle S^y_r \rangle = \int_{0}^{W} S^y_r(x) \, dx \ [i = x, y, z]$, and then list them in table 1 in units of $\hbar/2$ for an arbitrary value of $W = 25$ nm for illustrative purposes.

Table 1 shows that there is a net spin imbalance between the two sides of the line defect up to some arbitrary small distance on both sides, causing the formation of a local magnetic dipole, local magnetization and an associated magnetic field. The spin imbalance will also cause a spin current to flow which can aid or oppose the injected spin current depending on the sign of the imbalance, thereby decreasing or increasing the spin resistivity. It is therefore a spintronic analog of the LRRD.

We also note from table 1 that the average value of the $z$-component of spin is non-zero. In a pristine TI surface, spin momentum locking would have ensured that the spin lies in the surface and hence the averaged $z$-component of the spin would have been zero. The fact that it is not, is a consequence of the breaking of spin-momentum locking. It is easy to see from equations (17) and (18) that $S_z \neq 0$ on the left of the scatterer only if $|\alpha_1(x,y)| \neq |\alpha_2(x,y)|$ and $S_z \neq 0$ on the right of the scatterer only if $|\beta_1(x,y)| \neq |\beta_2(x,y)|$. Equation (22) shows that the former can happen if neither $r$ nor $r'$ is zero, and the latter can happen if neither $t$ nor $t'$ is zero. As discussed earlier, these are the conditions for breaking of spin-momentum locking. Hence a non-zero out-of-plane spin component is a consequence of the breaking of spin-momentum locking caused by a scatterer coupling an electron incident from one spin eigenstate into both spin eigenstates.

In figure 6, we plot the angular separation between the net spin polarizations on the two sides of the magnetic line defect as a function of the distance from the defect. This quantity $\theta(x)$ is defined as

$$\cos \theta(x) = \frac{S^x_t(-x)S^y_r(x) + S^y_t(-x)S^z_r(x) + S^z_t(-x)S^x_r(x)}{\sqrt{[S^x_t(-x)]^2 + [S^y_t(-x)]^2 + [S^z_t(-x)]^2 \sqrt{[S^y_r(x)]^2 + [S^z_r(x)]^2}}$$

Because the spin components oscillate in space, the angular separation between the net spin polarizations on the two sides of the defect (at some fixed distance from the defect on either side), oscillates with that distance, as seen in figure 6.

4. Conclusions

In this work, we have shown the existence of a spin imbalance forming in the vicinity of a magnetic line defect on the surface of a current-carrying TI, reminiscent of the LRRD that causes
a charge imbalance. The spin imbalance causes the appearance of a magnetic dipole. Its existence can be verified experimentally with magnetic force microscopy or microscopy involving nitrogen vacancy (NV) centers in diamond.

The local magnetic fields resulting from the spin imbalances over microscopic distances near the scatterer will make spins transiting through these regions precess and emit electromagnetic waves \[11–13\], thereby making the defect act as an electromagnetic radiator, provided the damping is relatively small. The sample will be a broadband oscillator since the precession frequencies will be different at different sites. Finally, if instead of a static current, we use an alternating current, then the polarization of the impinging spins will alternate owing to spin momentum locking of impinging spins and hence the local magnetic fields will oscillate in time with the same frequency as the injected current. This will happen only if the frequency is low enough that the time variation of the field can keep up with the time variation of the current. The oscillating magnetic field too can radiate electromagnetic waves, making the line defect act as a miniature antenna \[14–17\]. These radiations can be detected with suitable detectors.

Data availability statement

No new data were created or analyzed in this study.

Conflict of interest

The authors declare no conflict of interest.

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