Optical levitation of a YIG nanoparticle and simulation of sympathetic cooling via coupling to a cold atomic gas

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We report on the first experimental demonstration of an optically levitated Yttrium Iron Garnet (YIG) nanoparticle in both air and vacuum as well as a proposal for ground state cooling of the translational motion. The theoretical cooling scheme involves the sympathetic cooling of a ferromagnetic YIG nanosphere with a spin-polarized atomic gas. Particle-atom cloud coupling is mediated through the magnetic dipole-dipole interaction. When the particle and atom oscillations are small compared to their separation, the interaction potential becomes dominantly linear which allows the particle to exchange energy with the \(N\) atoms. While the atoms are continuously Doppler cooled, energy is able to be removed from the nanoparticle’s motion as it exchanges energy with the atoms. The rate at which energy is removed from the nanoparticle’s motion was studied for three species of atoms (Dy,Cr,Rb) by simulating the full \(N + 1\) equations of motion and was found to depend on system parameters with scalings that are consistent with a simplified model.

I. INTRODUCTION

Cooling the motion of an optically levitated nanoparticle to the motional ground state has proven to be a formidable experimental challenge. Limiting the nanoparticle temperature is inefficient detection of scattered light, laser shot noise, and phase noise, among others. These limitations seen in conventional tweezer traps have sparked theorists and experimentalists alike to explore new and hybrid levitated systems that may offer alternative routes to the quantum regime. Passive/sympathetic cooling schemes involving coupling different degrees of freedom or nearby particles has been explored \[1–6\]. Cavity cooling has had success \[7–9\] where strong coupling rates have been achieved by coherent scattering through the addition of a tweezer trap \[10\]. All electrical or electro-optical hybrid systems utilizing electronic circuitry \[11–13\], even in its beginning stages are able to reach mK temperatures \[14,15\] with one particular experiment reaching the lowest reported occupation number of \(n \sim 16\) \((T = 100\mu\text{K})\) \[16\]. The field has also recently seen magnetic particles and traps being investigated \[17–20\], such as studying the dynamics of a ferromagnetic particle levitated above a superconductor \[21,22\].

In the spirit of promising new systems, we theoretically investigate a possible method of cooling the translational motion involving the coupling of an optically trapped ferromagnetic nanoparticle to a spin-polarized cold atomic gas. The coupling arises from the magnetic dipole-dipole interaction and allows significant energy exchange between the two systems. While the atom cloud is continuously Doppler cooled, energy is extracted from the nanoparticle through this energy exchange. The coupling of a nanoparticle to an atom cloud has been proposed previously with the coupling mediated by scattered light into a cavity \[6\]. The scheme proposed here does not require optical cavities and has the potential to cool to the quantum regime.

FIG. 1. Illustration of the proposed model. A ferromagnetic nanosphere is trapped at the focus of a Gaussian beam. The oscillation frequency for the nanoparticle in the \(y\) direction is \(\omega_p\). A cloud of atoms a distance \(y_0\) away are trapped in a separate, far red-detuned dipole trap oscillating at \(\omega_a\) in the \(y\) direction.

The theoretical cooling scheme proposed is best suited for, but not limited to, nanoparticle frequencies in the kHz range or larger. As it has yet to be shown in the literature, as proof of practice we report on the first demonstration of an optically trapped ferromagnetic yttrium iron garnet (YIG) nanoparticle which oscillates in the kHz range for each degree of freedom. While scanning electron microscope (SEM) images show a wide range of particle sizes, hydrodynamic fits to the power spectral densities suggest only particles with small diameters, \(\sim 55\) nm, are able to be trapped and are lost below
chamber pressures of 10 Torr without feedback cooling.

This article is organized as follows. In Sec. II the theoretical dynamics of a system of two linearly coupled harmonic oscillators is briefly explained. The theoretical proposal to couple a spin-polarized atomic gas to a ferromagnetic nanosphere through the magnetic dipole-dipole interaction is given in Sec. III with analogies drawn from Sec. II. In Sec. IV simulation results of the particle-atom cloud system with continuous atom Doppler cooling are given. Lastly, Sec. V presents the first demonstration of optical trapping of a YIG nanoparticle.

II. TOY MODEL

This toy model provides an understanding of the coupling mechanism used for the real proposed system found in Secs. III and IV. The toy system is comprised of two particles in one dimension with mass \( M_1 \) and \( M_2 \) individually trapped in their own harmonic traps with frequencies \( \omega_1 \) and \( \omega_2 \). The particles are linearly coupled via the potential energy \( U_{int} = c \sqrt{M_1 M_2} \omega_1 \omega_2 y_1 y_2 \) with \( y_i \) the position of particle \( i \) and \( c \) determining the interaction strength. The full potential is

\[
U = \frac{1}{2} M_1 \omega_1^2 y_1^2 + \frac{1}{2} M_2 \omega_2^2 y_2^2 + c \sqrt{M_1 M_2} \omega_1 \omega_2 y_1 y_2. \tag{1}
\]

The equations of motion

\[
y_1 = -\omega_1^2 y_1 - c \sqrt{M_2 \omega_1 \omega_2} y_2, \tag{2}
\]

\[
y_2 = -\omega_2^2 y_2 - c \sqrt{M_1 \omega_1 \omega_2} y_1, \tag{3}
\]

yield four normal mode eigenfrequencies

\[
\omega_{\pm}^2 = \frac{1}{2} \left( \omega_1^2 + \omega_2^2 \right) \pm \sqrt{\left( \omega_1^2 - \omega_2^2 \right)^2 + 4 \omega_1^2 \omega_2^2 c^2}, \tag{4}
\]

which lead to four formal solutions for \( y_1(t) \) and \( y_2(t) \). On resonance, \( \omega_1 = \omega_2 = \omega \), Eq. (4) simplifies to \( \omega_{\pm} = \pm \frac{\omega}{\sqrt{1 \pm c}} \approx \pm \omega (1 \pm c/2) \) in the weak coupling limit \( c \ll 1 \). To garner an idea of the dynamics, presuppose the initial conditions \( y_1(0) = A, \ y_1(0) = 0, \ y_2(0) = 0 \) and \( y_2(0) = 0 \) while on resonance in the weak coupling limit. The solutions of Eqs. (2) (3) are

\[
y_1(t) = \frac{A}{2} \left[ \cos \omega t + \cos \omega_+ t \right], \tag{5}
\]

\[
y_2(t) = \frac{A}{2} \sqrt{\frac{M_1}{M_2}} \left[ \cos \omega t - \cos \omega_+ t \right], \tag{6}
\]

which may be rewritten as

\[
y_1(t) = A \cos (\omega t) \cos \left( \frac{\omega_+ t}{2} \right), \tag{7}
\]

\[
y_2(t) = -A \sqrt{\frac{M_1}{M_2}} \sin (\omega t) \sin \left( \frac{\omega_+ t}{2} \right). \tag{8}
\]

Equations (4) and (8) show fast oscillations at frequency \( \omega \) enveloped in a slower beat frequency \( \omega c/2 \). The result describes energy exchange between the two oscillators at a rate four times that of the beat frequency \( f_{exch} = \frac{\omega c}{2 \pi} \) (see Fig. 2). If there is not one, but \( N \) non-interacting particles in the harmonic potential \( \omega_i \), each interacting with particle 1, the eigenfrequencies and therefore the rate of exchange increases by \( \sqrt{N} \) so that

\[
f_{exch} = \frac{\omega c \sqrt{N}}{\pi}. \tag{9}
\]

If the particles in trap 2 experience a continual damping force \( F_i = -\alpha v_i \) \( (i \neq 1) \) energy will be removed from particle 1’s motion through the exchange. In the overdamped regime, \( \alpha/M_2 \gg \omega c \), particle 1’s energy decreases according to

\[
E_1(t) = E_0 e^{-\gamma t}, \tag{10}
\]

where the cooling rate

\[
\gamma \propto f_{exch}^2 / \alpha M_2, \tag{11}
\]

determines the temperature reached by particle 1 in time \( t \) in the absence of heating and noise.

III. APPROXIMATE MODEL OF THE SYSTEM

The proposed physical system includes a ferromagnetic nanosphere of radius \( \hat{R} \) and mass \( M_p \) harmonically trapped in the focus of a laser beam traveling in the \( \hat{k} = \frac{2 \pi}{\lambda} \hat{z} \) direction. A ferromagnetic sphere with dipole moment \( \vec{m} \) produces a magnetic field \( \vec{B}_s \)

\[
\vec{B}_s = \left( \frac{\mu_0}{4\pi} \right) \left[ 3 (\vec{m} \cdot \hat{r}) \hat{r} - \frac{\vec{m}}{r^3} \right], \tag{12}
\]

where \( \hat{r} \) is directed outwards from the center of the sphere. The sphere’s moment will align along the \( y \)-axis if
a constant, uniform magnetic field \( \vec{B}_{\text{ext}} = B_0 \hat{y} \) is present, and a field distribution will surround the particle.

A distance \( y_0 \) above the focus of the nanoparticle trap, a single spin-polarized atom with dipole moment \( \vec{\mu}_a = -\mu_a \hat{y} \) and mass \( M_a \) is trapped in a far red-detuned dipole trap with oscillation frequency \( \omega_a \). The total particle-atom potential energy including the repulsive interaction \( U_{\text{int}} = -\vec{\mu}_a \cdot \vec{B}_a \) is

\[
U = \frac{1}{2} M_p \omega_p^2 y_p^2 + \frac{1}{2} M_a \omega_a^2 y_a^2 + U_{\text{int}}.
\]

where \( y_a \) (\( y_0 \)) is the atom (particle) position. If both the atom and the nanoparticle undergo small oscillations compared to the distance separating them, \( y_0 \), the interaction \( U_{\text{int}} = -\vec{\mu}_a \cdot \vec{B}_a \) is quasi-one-dimensional and may be expanded

\[
U_{\text{int}} = g f (y_a + y_0 - y_p)^3
\]

\[
\approx g \left[ \left( \frac{1}{y_0^3} \right) + 3 \left( \frac{y_p - y_a}{y_0^2} \right) + 6 \left( \frac{y_p^2}{y_0^3} \right) + 6 \left( \frac{y_a^2}{y_0^3} \right) - 12 \left( \frac{y_a y_p}{y_0^3} \right) + \ldots \right]
\]

where \( g = (2\mu_a|\vec{\mu}|\mu_a/4\pi) \) defines the interaction strength. Keeping only terms to second order in Eq. (15), the equations of motion for the two particles are

\[
\ddot{y}_p \approx - \left( \frac{3g}{M_p y_0^3} \right) \left[ \omega_p^2 + \frac{12g}{M_p y_0^3} \right] y_p + \left( \frac{12g}{M_p y_0^3} \right) y_a
\]

\[
= -a_p - \frac{\omega_p^2}{\omega_a^3} y_p + \left( \frac{\Omega_a^2}{\omega_a^3} \right) y_a
\]

\[
\ddot{y}_a \approx \left( \frac{3g}{M_a y_0^3} \right) \left[ \omega_a^2 + \frac{12g}{M_a y_0^3} \right] y_a + \left( \frac{12g}{M_a y_0^3} \right) y_p
\]

\[
= a_a - \frac{\omega_a^2}{\omega_a^3} y_a + \left( \frac{\Omega_a^2}{\omega_a^3} \right) y_p
\]

Comparing Eqs. (18) and (19) with Eqs. (2) and (3) in the previous section, the particle and atom will exchange energy with one another provided \( y_0 \gg (y_p, y_a) \) so that higher order terms in Eq. (15) do not contribute. The simple energy exchange mechanism provided by the linear coupling in Eqs. (18) (19) will break down when higher order terms emerge from Eq. (15). Retaining only lower order terms is only possible for nanoparticle temperatures much smaller than room temperature as the atoms’ positions will increase drastically as they exchange energy with the particle. The condition may also be satisfied if the motion of the atoms is continuously cooled via a cooling mechanism such as Doppler cooling which will be discussed in the next section.

Due to the frequency shifts \( \Omega_i^2 \) from \( U_{\text{int}} \), for coherent energy exchange the \( \omega_i \) need to be tuned to achieve resonance \( \omega_i = \omega_p = \omega_0 \). Assuming resonance has been reached, an energy exchange frequency can be extracted by comparing the potential energies in Eqs. (1) and (15) and using Eq. (9)

\[
f_{\text{exch}} = 12 \left( \frac{g}{y_0^3} \right) \left( \frac{\sqrt{N}}{\pi \omega_0} \right) \left( \frac{1}{\sqrt{M_p M_a}} \right)
\]

\[
= \frac{\Omega_p \Omega_a \sqrt{N}}{\pi \omega_0}.
\]

The equations describe two coupled harmonic oscillators that exchange energy with one another at a frequency \( f_{\text{exch}} \) provided \( y_0 \gg (y_p, y_a) \). Note that while the overall force due to the magnetic dipole-dipole interaction is repulsive, this scheme could work equally as well for an attractive interaction since the energy exchange affect is independent of the sign of the linear coupling term.

If the atom cloud is continuously Doppler cooled, motional energy can be extracted from the nanoparticle. Doppler cooled atoms experience a damping force \( F_D = -\alpha v \) with damp rate \( \alpha = \hbar k^2 / (I_0) \) tuned to reach the Doppler limit \( 24 \), where \( (I/I_0) \) is the saturation ratio. From Eq. (11), the rate at which energy is extracted from the nanoparticle in the absence of noise

\[
\gamma_{\text{cool}} = \frac{f_{\text{exch}}}{\alpha M_a} \propto \left( \frac{N}{\alpha M_p} \right) \left( \frac{g}{\omega_0 y_0} \right)^2
\]

\[
\propto \left( \frac{N q^2}{\alpha M_p} \right),
\]

depends on the number of atoms \( N \), the atom cooling rate \( \alpha \), and the magnetic interaction strength \( g = (2\mu_a |\vec{\mu}|\mu_a/4\pi) \propto M_p \). Equation (22) shows that slower atom cooling is beneficial for faster cooling of the nanoparticle. However, it is important that \( \alpha \) remain large enough so that the atoms remain in the regime \( y_0 \gg y_a \) and do not escape the trap as a result of heating.
IV. Simulations of the Full System

A. The system

To determine the extent to which energy can be extracted from a nanoparticle through the above scheme, several thousand simulations of the full \( N + 1 \) equations of motion were performed using the full non-linear one dimensional \( 1/y^3 \) potential in Eq. (14) while continuously Doppler cooling each atom. The nanosphere used for the simulations was composed of YIG with a radius \( R = 50 \) nm, density \( \rho = 5110 \) kg/m\(^3\), index of refraction \( n = 2.21 \) \cite{25}, and magnetic dipole moment \( |\vec{m}| = \frac{1}{5} N_p \mu_B = 4.05 \times 10^{-18} \) JT\(^{-1}\) where \( \mu_B \) is the Bohr magneton and \( N_p \) is the number of particles that make up the entire YIG nanosphere. This expression for the magnet moment is approximate, but is conservative compared to what is achievable experimentally \cite{26, 27}.

The nanoparticle was trapped in a \( \omega_p/2\pi = 100 \) kHz optical trap at the focus of laser beam linearly polarized along the lab frame \( x \)-direction and propagating in the \( z \)-direction with a wavelength \( \lambda = 1550 \) nm \( \gg R \), power 150 mW, and focused by a NA = 0.6 objective. The nanosphere was initially set with initial positions and velocities conforming to a Maxwell-Boltzmann distribution at a temperature of \( T = 1 \) K. Translational laser shot noise on the nanoparticle was included in the simulations as a Langevin process with heating rate \( 2E_T/k_B = 72.4 \) mK/s \cite{25}. Due to the frequency shifts \( \Omega^2 \) in Eq. (16), the nanoparticle’s frequency was shifted to match the frequency of the atom’s at the atom Doppler temperature.

Three species of atoms, dysprosium, chromium, and rubidium, were used for separate simulations. Relevant properties for these atoms may be seen in Table I. The atom cloud trap center was placed \( y_0 = \lambda/3 = 516 \) nm away from the nanoparticle trap’s center. The atom dipole trap frequency \( \omega_a/2\pi = 100 \) kHz remained unshifted. The atoms were initially set with velocities at their Doppler temperature and were continuously Doppler cooled (see the Appendix). Interactions between the atoms and atom loss from the trap were not included in the simulation.

From the parameters above, the beam waist of the particle trap is \( \sim 800 \) nm while the atom-particle separation distance is set at \( \sim 500 \) nm. While this distance is flexible, we envision the atom trap to be more tightly focused with a wavelength smaller than that of the nanoparticle’s. Further, the particle oscillation amplitude is \( \sim 10 \) nm at \( T = 1 \) K, an order of magnitude smaller than the separation distance. One could imagine a thin dielectric barrier placed between the particle and atoms if atom collisions with the nanoparticle would be of concern.

B. Results and discussion

For each atom species, the energy removal rate of the nanoparticle, \( \gamma \), was extracted for varying numbers of atoms in the trap, \( N \), as seen in Fig. 3. From several thousand averages, \( \gamma \) was obtained by fitting energy versus time plots to a decaying exponential, \( E(t) = A e^{-\gamma t} + C \), for a given \( N \) (see Fig. 3b). From Fig. 3a), the cooling rate depends linearly on the number of atoms in the trap as Eq. (22) predicts. As the particle exchanges energy with the atoms, each atom acquires a portion of the nanoparticle’s energy. When \( n \) more atoms are added to the trap, there are \( n \) more chances for removing that energy through Doppler cooling. As Fig. 3a) shows no deviation from a linear dependence for larger \( N \), the cooling rate may be extrapolated for larger \( N \) values. However, the nanoparticle energy is expected to be limited to the atom Doppler temperature. The average final temperatures reached for each atom species (Dy, Cr, Rb) was 794 \( \mu \)K, 406 \( \mu \)K, and 158 \( \mu \)K, respectively, for a cooling time of \( t = 100 \) ms at \( N = 10^4 \). Simulation time was the only constraint from observing the effects for \( N > 10^4 \).

Besides the number of atoms in the trap, Eq. (22) predicts that the cooling rate \( \gamma_{cool} \propto g^2/M_p \) depends on the square of the magnetic coupling strength \( g = (2\mu_a|\vec{m}|\mu_0/4\pi) \). Using the data in Fig. 3a) and the values for \( \mu_a \) in Table I \( g \propto \mu_a^2 \) is confirmed with a coefficient of determination \( r^2 = 0.997 \).

Note that \( \gamma \) in Fig. 3 includes the cooling rate \( \gamma_{cool} \) as well as the competing shot noise heating rate \( E_T \). The two parameters that these two quantities share are the density, which is predominantly fixed, and the size (radius \( R \)) of the nanoparticle. Approximating \( |\vec{m}| \propto R^3 \) we find \( \gamma_{cool} \propto R^2 \) while \( E_T \propto R^3 \) shares the same \( R \) dependence \cite{28}. However, the shot noise heating is linear in time while the cooling is exponential, indicating that larger particles may provide faster cooling, but will not influence the final temperature of the nanoparticle. The influence of shot noise heating may be further reduced by cooling the degree of freedom in the laser polariza-
TABLE I. Relevant properties of the three species of atoms and their Doppler parameters. From left to right: the atom mass, magnetic moment, decay rate, Doppler line, and Doppler temperature.

| Element | $M_a$ (a.u.) | $\mu_a/\mu_B$ | $\Gamma/2\pi$ (MHz) | $\lambda_{line}$ (nm) | $T_{min}$ ($\mu$K) |
|---------|--------------|----------------|---------------------|----------------------|-------------------|
| Dy      | 162.5        | 10             | 32.2                | 421                  | 760               |
| Cr      | 52           | 6              | 5.02                | 425                  | 124               |
| Rb      | 86.9         | 1              | 6.06                | 780                  | 146               |

The results above indicate that atom numbers of the order $N \sim 10^6$, which corresponds to $\gamma > 10^3$ Hz, would be sufficient for the nanoparticle to reach the atom’s Doppler temperature even for atom species with unit magnetic moment $\mu_a/\mu_B = 1$. The number of atoms that have been trapped experimentally is in the range $\sim 10^6$ to $10^8$ for chromium, rubidium, and others. Many of the commonly trapped atom species have unit magnetic moment, large $N$, and are able to reach the $1-10\mu$K regime. Comparing with the energy removal rate found for Rb in Fig. 3(a), these parameters are sufficient for motional ground state cooling of the nanoparticle. Further, Doppler cooling as the atom cooling method was chosen for simulation simplicity while retaining physicality. Other cooling methods offer lower temperatures such as sideband cooling, Sisyphus cooling, or using a spin-polarized Bose-Einstein condensate, which are able to reach nK temperatures.

V. EXPERIMENTAL TRAPPING OF A YIG NANOPARTICLE

For reasonable cooling times ($\sim$ ms), the above proposal is best suited for particle frequencies in the 100 kHz range. This section confirms the feasibility of that parameter regime by demonstrating optical levitation of a ferromagnetic YIG nanoparticle.

Two types of commercially available YIG samples were used for trapping. The first sample was a powder with a manufacturer size of $< 100$ nm. As can be seen in Fig. 4(d), SEM images of the sample show large variations in the particle size, from smaller than 100 nm to larger than 1 $\mu$m. The second sample used was a powder that was made by breaking a bulk YIG crystal with a mortar and a pestle.

In preparation of loading the sample, the YIG powder is diluted in water and is transferred to a nebulizer. The nebulizer forms droplets containing the YIG sample and the sample is sprayed towards the linearly polarized optical trap at atmospheric pressure. As the water droplets reach the trapping region, the water evaporates and the YIG is successfully trapped in the optical trap. A 532 nm laser and a CCD camera is used for the observation of the trapping procedure. The power of the 532 nm laser was much weaker than the trapping laser in order to minimize the problems of absorption and heating of the trapped YIG particle.

Inside a vacuum chamber, the trapping beam is formed with a 500 mW, 1550 nm laser traveling in the $\hat{z}$ direction that is tightly focused with an objective lens. Subsequently, the trapping laser is directed towards balanced detectors that are used to monitor the center of mass motion of the optically trapped particle. The $x$ and $y$ motion are observed from the change in direction of the laser beam (see Ref. [35]). Due to imperfect alignment, the $z$ motion can also be observed in the $x$ detector. The collected power spectral density (PSD) of the particle motion at two different pressures, 500 Torr and 15 Torr, are shown in Fig. 4(a) and (b). From the PSD’s, trapping frequencies at 104 kHz, 123 kHz, and 36 kHz are observed for the $x$, $y$, and $z$ directions, respectively.

From the measured PSD, the damping coefficient $\Gamma/2\pi$ is also obtained at different pressures. The measured linewidths of the trapped particles center of mass motion...
is shown in Fig. 4(c). As expected, the linewidth decreases as the pressure is lowered. The linewidth can be used for estimation of the hydrodynamic diameter, 2R, of the trapped YIG particle through Eq. (23)

$$\Gamma = \frac{6\pi \eta R}{\rho \left( \frac{4}{3} \pi R^3 \right)} C(K_n)$$  

(23)

where \( \eta \) is the viscosity of air, \( C(K_n) \) a non-linear function dependent on the Knudsen number \( K_n \), and \( \rho = 5110 \text{ kg/m}^3 \) as provided by the manufacturer. Using Eq. (23) with the results of Fig. 4(c), the average diameter for several trappings was calculated to be 56 ± 13 nm for the trapped YIG particle. As mentioned in Ref. [36], at lower pressures the trapped particle may heat due to absorption of light from the trapping laser in combination with less damping from the surrounding gas molecules. For this reason, Eq. (23) is applied at higher pressures for the calculation of the particle size. Particles from both samples were calculated to have similar radii.

VI. CONCLUSION

A theoretical proposal to sympathetically cool a levitated ferromagnetic nanoparticle via coupling to a spin-polarized atomic gas was studied and analyzed. While oscillating in their respective traps, the particle and atom cloud systems are coupled through the non-linear magnetic dipole-dipole interaction. For sufficiently large separation between the particle and the atom cloud relative to their displacements, the nanoparticle and atom cloud exchange energy with one another via the linear coupling term that is dominant in the magnetic force expansion. While the atoms are continuously Doppler cooled, energy is able to be removed from the particle’s motion. The cooling rate is proportional to the number of atoms in the trap as well as the square of the magnetic moment of the atom.

Simulations of the particle-atom cloud system were performed using the full, non-linear, magnetic dipole-dipole interaction for three species of atoms and varying numbers of atoms in the trap. The rate at which energy is removed from the particle motion is significant for \( 10^4 \) atoms in the trap when the atoms are continuously Doppler cooled. It is expected that the particle would reach the atom Doppler temperature as the number of atoms increases. This method of sympathetic cooling has potential to cool the nanoparticle to its motional ground state for atom species with lower Doppler temperatures. However, any atom cooling strategy that reaches low enough temperatures should allow for motional ground state cooling of the nanoparticle for large enough cooling rates.

As the proposed cooling scheme is best suited for nanoparticle frequencies in the 100 kHz regime, as proof of practice, the first experimental demonstration of optical levitation of a yttrium iron garnet nanoparticle was also presented. Fitting to a hydrodynamic model of gas damping, particles of diameter \( \sim 55 \text{ nm} \) were observed to be trapped.

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Appendix: Doppler Cooling

Doppler cooling and shot noise on the atoms was simulated by sampling the probability for absorption of a photon for each atom \( P = \mathcal{R} \) at each time step \( dt \), where

$$\mathcal{R} = \frac{\Gamma \Omega^2}{\left( \Delta + \vec{\nu} \cdot \vec{k} \right)^2 + \left( \Omega \right)^2 / 2 + \left( \Gamma \right)^2 / 4}$$

(A.1)

is the absorption rate. Here, the Rabi frequency \( \Omega = \Gamma \sqrt{\rho / 2} \) with \( \rho = I / I_{sat} = 0.1 \) the saturation intensity ratio, and the detuning \( \Delta = \Gamma / 2 \) were set to reach the Doppler limit. If a photon was absorbed the atom would experience a momentum kick \( h \vec{k} \) in the \( \vec{k} \) direction followed immediately by a kick in a random direction.

[1] S. Liu, T. Li, and Z. Qi Yin, J. Opt. Soc. Am. B 34, CS (2017)
[2] B. A. Stickler, S. Nimmrichter, L. Martinetz, S. Kuhn, M. Arndt, and K. Hornberger, Phys. Rev. A 94, 033818 (2016)
[3] Y. Arita, M. Mazilu, and K. Dholakia, Nat. Commun. 4, 2374 (2013)
[4] Y. Arita, E. M. Wright, and K. Dholakia, Optica 5, 910 (2018)
[5] L. Ge and N. Zhao, Phys. Rev. A 98, 043415 (2018)
[6] G. Ranjit, C. Montoya, and A. A. Geraci, Phys. Rev. A 91, 013416 (2015)
[7] N. Kiesel, F. Blaser, U. Delić, D. Grass, R. Kaltenbaek, and M. Aspelmeyer, Proceedings of the National Academy of Sciences 110, 14180 (2013), http://www.pnas.org/content/110/35/14180.full.pdf
[8] D. Windey, C. Gonzalez-Ballestero, P. Maurer, L. Novotny, O. Romero-Isart, and R. Reimann, arXiv e-prints , arXiv:1812.09176 (2018), arXiv:1812.09176 [quant-ph]
[9] C. Gonzalez-Ballestero, P. Maurer, D. Windey, L. Novotny, R. Reimann, and O. Romero-Isart, Phys. Rev. A 100, 013805 (2019).
[10] U. c. v. Deli´ c, M. Reisenbauer, D. Grass, N. Kiesel, V. Vuleti´ c, and M. Aspelmeyer, Phys. Rev. Lett. 122, 123602 (2019).
[11] D. Goldwater, B. A. Stickler, L. Martinetz, T. E. Northup, K. Hornberger, and J. Millen, Quantum Science and Technology 4, 024003 (2019).
[12] A. D. Rider, C. P. Blakemore, A. Kawasaki, N. Priel, S. Roy, and G. Gratta, Phys. Rev. A 99, 041802 (2019).
[13] C. Timberlake, M. Toro, D. Hempston, G. Winstone, M. Rashid, and H. Ulbricht, Appl. Phys. Lett. 114, 023104 (2019), https://doi.org/10.1063/1.5081045.
[14] M. Iwasaki, T. Yotsuya, T. Naruki, Y. Matsuda, M. Yoneda, and K. Aikawa, Phys. Rev. A 99, 051401 (2019).
[15] G. P. Conangla, F. Ricci, M. T. Cuairan, A. W. Schell, N. Meyer, and R. Quidant, Phys. Rev. Lett. 122, 233602 (2019).
[16] F. Tebbenjohanns, M. Frimmer, A. Militaru, V. Jain, and L. Novotny, Phys. Rev. Lett. 122, 223601 (2019).
[17] J.-F. Hsu, P. Ji, C. W. Lewandowski, and B. DuRso, Science Advances 2 (2016), 10.1126/sciadv.1501286, https://advances.sciencemag.org/content/2/3/eaaf1026.
[18] C. Gonzalez-Ballestero, J. Gieseler, and O. Romero-Isart, arXiv e-prints , arXiv:1907.04039 [quant-ph] (2019).
[19] T. Wang, S. Lourette, S. R. O’Kelley, M. Kayci, Y. Band, D. F. J. Kimball, A. O. Sushkov, and D. Budker, Phys. Rev. Applied 11, 044041 (2019).
[20] J. D. Jackson, Classical electrodynamics 3rd ed. (Wiley, New York, NY, 1999).
[21] C. J. Foot, Atomic physics, reprinted ed. (Oxford University Press, Oxford, 2011).
[22] S. H. Wemple, S. L. Blank, J. A. Seman, and W. A. Biolsi, Phys. Rev. B 9, 2134 (1974).
[23] M. A. Musa, R. S. Azis, N. H. Osman, J. Hassan, and T. Zangina, Results in Physics 7, 1135 (2017).
[24] V. Sharma, J. Saha, S. Patnaik, and B. K. Kuan, AIP Advances 7, 056405 (2017), https://doi.org/10.1063/1.4973199.
[25] T. Seberson and F. Robicheaux, arXiv e-prints , arXiv:1909.06469 (2019), arXiv:1909.06469 [physics.atom-ph].
[26] A. Griesmaier, J. Stuhler, and T. Pfau, Applied Physics B 82, 211 (2006).
[27] P. O. Schmidt, S. Hensler, J. Werner, T. Binhammer, A. G" orlitz, and T. Pfau, J. Opt. Soc. Am. B 20, 960 (2003).
[28] M. Lu, S. H. Youn, and B. L. Lev, Phys. Rev. A 83, 012510 (2011).
[29] L. Gabardos, S. Lepoutre, O. Gorceix, L. Vernac, and B. Laburthe-Tolra, Phys. Rev. A 99, 023607 (2019).
[30] M. Lu, N. Q. Burdick, S. H. Youn, and B. L. Lev, Phys. Rev. Lett. 107, 190401 (2011).
[31] B. Johnson and A. K. Walton, British Journal of Applied Physics 16, 475 (1965).
[32] J. Ahn, Z. Xu, J. Bang, Y.-H. Deng, T. M. Hoang, Q. Han, R.-M. Ma, and T. Li, Phys. Rev. Lett. 121, 033603 (2018).
[33] T. M. Hoang, J. Ahn, J. Bang, and T. Li, Nature Communications 7, 12250 (2016).
[34] S. A. Beresnev, V. G. Chernyak, and G. A. Fonyagin, Journal of Fluid Mechanics 219, 405421 (1990).