Brief Review

Black holes as particle accelerators: a brief review

Tomohiro Harada\textsuperscript{1} and Masashi Kimura\textsuperscript{2}

\textsuperscript{1}Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan
\textsuperscript{2}Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

E-mail: harada@rikkyo.ac.jp and M.Kimura@damtp.cam.ac.uk

Received 27 August 2014
Accepted for publication 24 September 2014
Published 28 November 2014

Abstract

Rapidly rotating Kerr black holes can accelerate particles to arbitrarily high energy if the angular momentum of the particle is fine-tuned to some critical value. This phenomenon is robust as it is founded on the basic properties of geodesic orbits around a near-extremal Kerr black hole. On the other hand, the maximum energy of the acceleration is subjected to several physical effects. There is convincing evidence that the particle acceleration to arbitrarily high energy is one of the universal properties of general near-extremal black holes. We also discuss gravitational particle acceleration in a more general context. This article is intended to provide a pedagogical introduction to and a brief overview of this topic for non-specialists.

Keywords: Kerr black hole, extremal black hole, particle collision
PACS numbers: 04.70.As, 04.70.Bw, 97.60.Lf

(Some figures may appear in colour only in the online journal)

1. Introduction

The existence of black holes is strongly suggested by astrophysical observations. However, it is not so clear whether the observed objects are really identical with what we know as black holes predicted in general relativity. It is necessary to understand the physics of black hole horizons for the direct observational confirmation of black holes. In this article, we will discuss the possibility of black holes being particle accelerators. What do black holes as particle accelerators mean?

Before asking this question, let us ask what it means that terrestrial particle accelerators, such as the Large Hadron Collider, accelerate particles. In most of terrestrial particle
accelerators, the kinetic energy of particles is increased through the work exerted on the particles by electromagnetic force. For the proton–proton collision in the Large Hadron Collider, the energy $E_{\text{cm}}$ measured by an observer who is at rest with respect to the centre of mass frame, which is called centre-of-mass (CM) energy and denoted as $E_{\text{cm}}$, becomes as high as 14 tera electron volts (TeV). This is 15,000 times the rest mass energy of proton approximately.

Black holes have gravitational force strong enough to trap light rays. The boundary behind which no light ray can escape to infinity is called an event horizon. Since gravitational force acts on both charged and neutral particles, black holes can accelerate not only charged particles but also neutral particles. From such a consideration, it can be regarded as natural that black holes accelerate particles. However, for the Schwarzschild black hole, which is a static spherically symmetric black hole, the CM energy $E_{\text{cm}}$ of two particles of equal rest mass $m$ which have been at rest at infinity can be $2\sqrt{5}mc^2$ at most, which corresponds to $\gamma = 9$ in terms of the relative velocity, and hence it cannot be regarded as a high energy particle accelerator.

In 2009, Bañados, Silk and West [1] found that $E_{\text{cm}}$ can be arbitrarily high for rotating black holes in the context of dark matter particle annihilation at the galactic centre. It should be noted that the unboundedly high $E_{\text{cm}}$ of particle collision had already been noticed in a different context in [2]. Because of the equivalence principle of general relativity, not only microscopic particles such as electrons, protons, neutrons, ions and molecules but also macroscopic objects such as black holes and compact stars can be accelerated by a rotating black hole with the same gamma factor if the size and mass of those objects are sufficiently small compared to the those of the central black hole.

This paper is organized as follows. In section 2, we elucidate that the CM energy of two colliding particles on the equatorial plane of a Kerr black hole can be arbitrarily high and discuss why this is possible in contrast to the case of a Schwarzschild black hole. In section 3, we review critical comments and basic questions on this scenario and respond to them from a physical point of view. In section 4, we discuss the possibility of particle acceleration in astrophysical black holes. In section 5, we briefly review the generalizations of the particle acceleration scenario. Section 6 is devoted to conclusion. In the following, we use the unit, where $G = c = 1$ unless explicitly noticed.

2. Kerr black holes as particle accelerators

2.1. Kerr black holes and geodesic particles

In general relativity, a stationary rotating vacuum black hole is uniquely described by a Kerr spacetime. The line element $\text{d}s^2 = g_{\mu\nu}\text{d}x^\mu\text{d}x^\nu$ in the Kerr spacetime is written in the Boyer–Lindquist coordinates in the following form [3, 4]

$$\text{d}s^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)\text{d}t^2 - \frac{4Mar}{\rho^2}\sin^2\theta\text{d}t\text{d}\phi + \frac{\rho^2}{\Delta}\text{d}r^2 + \rho^2\text{d}\theta^2 + \left(r^2 + a^2 + \frac{2Mar^2\sin^2\theta}{\rho^2}\right)\sin^2\theta\text{d}\phi^2,$$

(2.1)

where $\rho^2 = \rho^2(r, \theta) = r^2 + a^2\cos^2\theta$ and $\Delta = \Delta(r) = r^2 + a^2 - 2Mr$. Kerr black holes are parametrized by mass $M$ and spin $a$, which must satisfy $0 \leq |a| \leq M$ [5]. The spin parameter $a$ is related to the angular momentum with respect to the rotational axis of the black hole as
Black holes with the maximum value of the spin parameter are called maximally rotating black holes. We assume \( a \geq 0 \) without loss of generality. For later use, we define the non-dimensional spin parameter \( \alpha = a/M \).

The Kerr spacetime is stationary and axisymmetric with corresponding Killing vectors \( \partial/\partial t \) and \( \partial/\partial \phi \). The event horizon is located at \( r = r_H = M + \sqrt{M^2 - a^2} \), where \( \Delta(r) \) vanishes, and is rotating with the angular velocity \( \Omega_H = a/(r_H^2 + a^2) \). Note that the horizon is called extremal when \( r = r_H \) is a double root of \( \Delta(r) \), i.e. \( a = M \) \(^3\). The region given by \( \theta \ll r/M \), where the Killing vector \( \partial/\partial t \) of stationarity is spacelike, is called an ergoregion.

In general relativity, a free test particle moves along a geodesic of the spacetime. The energy \( E = -g_{\mu\nu} p^\mu \) and angular momentum \( L = g_{\rho\sigma} p^\rho \) of the particle with four-momentum \( p^\mu \) are conserved in accordance with the symmetries of the spacetime, where we can write \( p^\mu = \dot{x}^\mu \) with the dot being the derivative with respect to the affine parameter \( \lambda \). It should be noted that this energy \( E \) is with respect to an observer at rest at infinity and is distinct from the CM energy \( E_{\text{cm}} \). Additionally, it is known that there is another conserved quantity called the Carter constant, which is related to the total angular momentum. Because of the existence of the conserved quantities \( E, L \) and the Carter constant, geodesic equations are integrated to first-order differential equations.

For a particle which moves on the equatorial plane, for which the Carter constant vanishes, \( t \) and \( \phi \) can be written as

\[
\begin{align*}
    r^2 t &= \left( r^2 + a^2 \right) \frac{\left( E - L \right)^2 - aL}{\Delta} - a(E - L), \quad (2.2) \\
    r^2 \phi &= \frac{a}{\Delta} \left( E - L \right)(E - aL) - (a(E - L)), \quad (2.3)
\end{align*}
\]

respectively. Using \( g_{\mu\nu} p^\mu = -m^2 \), the geodesic equation for a particle on the equatorial plane is reduced to a simple one-dimensional potential problem given by

\[
    \frac{1}{2} \dot{r}^2 + V(r) = 0, \quad (2.4)
\]

with

\[
    V(r) = \frac{-\left( r^2 + a^2 \right) \left( E - aL \right)^2 - \Delta(r) \left[ m^2 r^2 + (L - aE)^2 \right]}{2r^4}
    = \frac{m^2 M}{r} + \frac{L^2 - a^2 \left( E^2 - m^2 \right)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{1}{2} \left( E^2 - m^2 \right), \quad (2.5)
\]

where \( m \) and \( V(r) \) are called the rest mass and the effective potential of the particle, respectively. For a massive particle, we have \( p^\mu = mu^\mu \), where \( u^\mu \) is the four-velocity satisfying \( u^\mu u_\mu = -1 \). The motion is possible only in the region, where \( V \leq 0 \).

Physical particles must satisfy the so-called forward-in-time condition which guarantees that the value of the time coordinate \( t \) increases along the trajectory of the particle’s motion.

\(^3\) For a general stationary black hole, an extremal horizon is defined as a horizon on which the surface gravity is zero [5].
Near the event horizon, equation (2.2) implies that this condition reduces to $E - \Omega L H \geq 0$. Here we call particles which satisfy $E - \Omega L H = 0$ critical particles.

2.2. Particle collision in the equatorial plane

If two particles 1 and 2 are at the same spacetime point, an observer at the CM frame is defined as the one whose four-velocity is parallel to the sum of the four-momenta $\mu p_1^\mu$ and $\mu p_2^\mu$ of particles 1 and 2, respectively. The CM energy $E_{cm}$ is defined as the energy measured by this observer and is given by

$$E_{cm}^2 = -g_{\mu\nu} \left( p_1^\mu + p_2^\nu \right) \left( p_1^\nu + p_2^\nu \right). \quad (2.6)$$

The CM energy is scalar invariant and physically observable in principle.

Let us first concentrate ourselves on two particles both of which move in the equatorial plane and collide with each other near the horizon. Figure 1 schematically illustrates the situation under consideration. In this case, the CM energy $E_{cm}$ for the collision in the vicinity of the horizon is calculated to give [6, 7]

$$E_{cm}^2 = \frac{m^2 r_H^2 \left( L_1 - a E_1 \right)^2}{r_H^2} \frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2) + \cdots, \quad (2.7)$$

where two labels 1 and 2 denote particles 1 and 2, respectively. ‘(1 $\leftrightarrow$ 2)’ denotes the term which is obtained by exchanging 1 and 2 in the first term, and ‘$\cdots$’ denotes the terms which are obviously finite. Therefore, $E_{cm}$ is divergent if either of particles 1 and 2 satisfies the critical condition $E - \Omega_H L = 0$. It should be noted that $E_{cm}$ is shown to be bounded if the collision point is not near the horizon.

Figure 1. The schematic figure of particle collision for which the CM energy can be very large. The arrow denotes the direction of the spin of the black hole. For arbitrarily high CM energy to be achieved, the critical particle orbits the black hole arbitrarily large number of times with arbitrarily long proper time.
For such a high energy collision to occur, the critical particle has to approach the horizon as a result of its motion. For a while, we concentrate ourselves on massive particles of rest mass $m$ in the equatorial plane. It is convenient to define non-dimensional specific energy $e = E/m$ and specific angular momentum $l = L/(mM)$. We introduce $l_c = E/(\Omega_H m M)$ as the critical value for $l$. The motion of geodesic particles on the equatorial plane can be fully analysed by the effective potential given by equation (2.5) and is extremely simple for a particle which is initially at rest at infinity, for which $e = 1$. In this case, we find

$$V(r_H) = - \frac{\left(r_H^2 + a^2\right)^2}{2r_H^2} \left(m - \Omega_H L\right)^2 \leq 0.$$  

(2.8)

If and only if $V(r) < 0$ for $r_H < r < \infty$, or equivalently, the quadratic equation

$$2r^2 - Ml^2 r + 2M^2 (l - a_s)^2 = 0$$

(2.9)

has no root in the region $0 < r < r_H$, such a particle approaches the horizon from infinity. This condition reduces to

$$-2\left(1 + \sqrt{1 + a_s}\right) = l_L < l < l_R = 2\left(1 + \sqrt{1 - a_s}\right).$$

Noting that $l_R \leq l_c$ and the equality holds only for $a_s = 1$, it turns out that the critical particle can reach the horizon for $a_s = 1$ but cannot for $a_s < 1$. Figure 2 shows the effective potentials of critical particles for $a_s = 0.9$, 0.99 and 1. We can see that the critical particle can reach the horizon only for $a_s = 1$ with infinitely long proper time. In the case of $a_s < 1$, if the spacetime is near-extremal $a_s \approx 1$, the critical particle reaches the radius

$$r \approx r_H + \frac{2\sqrt{2} \left(E^2 + m^2\right) M}{3E^2 - m^2} \sqrt{1 - a_s}$$

and is bounced back there.

Let us consider the collision between two particles of same rest mass $m$ which are initially at rest at infinity. If the two particles collide in the vicinity of the horizon for $a_s = 1$, equation (2.7) gives [1]

![Figure 2. The effective potential defined by equation (2.5) for marginally bound critical particles, where $r$ and $V$ are normalized by $M$ and $m^2$, respectively. $V$ must be non-positive in the allowed region of particle motion.](image)
\[
\frac{E_{\text{cm}}}{2m} = \sqrt{\frac{1}{2} \left( \frac{2 - l_1}{2 - l_2} + \frac{2 - l_2}{2 - l_1} \right)},
\tag{2.10}
\]

Therefore, if we fine-tune either \(l_1\) or \(l_2\) to the upper limit angular momentum \(l_R = 2(= l_c)\), \(E_{\text{cm}}\) can be arbitrarily large. If we choose \(l_1 = l_R\) and \(l_2 < l_2 < l_R\) and set the collision point at \(r_{\text{col}} \approx r_H = M\) for \(a_s = 1\), equation (2.6) gives [6]

\[
\frac{E_{\text{cm}}}{2m} \approx \sqrt{\frac{(2 - \sqrt{2})(2 - l_2)M}{2(r_{\text{col}} - M)}},
\tag{2.11}
\]

which again diverges in the limit \(r_{\text{col}} \to r_H = M\). On the other hand, if \(l_1 = l_R < l_c\) and \(l_2 < l_R\) for \(a_s < 1\), equation (2.7) gives [6, 8]

\[
\frac{E_{\text{cm}}}{2m} \approx \sqrt{\frac{(2 + \sqrt{2})(2 - l_2)}{2}} \frac{1}{\sqrt{1 - a_s^2}},
\tag{2.12}
\]

for the collision in the vicinity of the horizon, where \(a_s \approx 1\) is assumed. We find \(E_{\text{cm}} \to \infty\) as \(a_s \to 1\).

### 2.3. CM energy in finite acceleration time

It would be physically important how high the CM energy for the collision can be near the Kerr black hole if the acceleration continues for finite time. That is, we would like to estimate the maximum CM energy which can be achieved within finite time [9].

Let us consider the collision of two particles 1 and 2 of same rest mass \(m\) which are at rest at infinity near a maximally rotating black hole. We choose \(l_1 = l_R = 2\) and \(l_L < l_2 < l_R = 2\). Equations (2.2) and (2.4) implies

\[
\frac{d}{dr} = \frac{(r - M)^2\sqrt{2}M}{(r^2 + Mr + 2M^2)^{3/2}},
\tag{2.13}
\]

so that it takes infinite Killing time \(t\) for particle 1 to reach the horizon. Using equation (2.13), we can calculate the Killing time \(T\) needed for particle 1 to reach the collision point \(r = r_{\text{col}}\) from a distant location \(r = r_i\) around a maximally rotating Kerr black hole as

\[
T = - \int_{r_i}^{r_{\text{col}}} \frac{dr}{\sqrt{r^2 + Mr + 2M^2}} \approx 2\sqrt{2}M^2 \frac{r_{\text{col}} - M}{r_{\text{col}}},
\tag{2.14}
\]

where we have assumed \(M \ll r_i \ll M^2/(r_{\text{col}} - M)\) in the approximation on the right-hand side. This assumption is valid in reasonable astrophysical situations. From equations (2.11) and (2.14), we obtain

\[
\frac{E_{\text{cm}}}{2m} \approx \frac{1}{2} \sqrt{(\sqrt{2} - 1)(2 - l_2)} \frac{T}{M},
\tag{2.15}
\]

or

\[
E_{\text{cm}} \approx 2.5 \times 10^{20} \text{eV} \left( \frac{T}{10\text{ Gyr}} \right)^{1/2} \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \frac{m}{1\text{ GeV}} \right)^{1/2},
\tag{2.16}
\]

It is interesting to note that the maximum CM energy which can be achieved within the age of the universe is as high as ultra high energy cosmic rays.
2.4. Physical explanation of particle acceleration

We would like to propose an intuitive physical explanation for the particle acceleration. A black hole is defined by a region from which no light ray can escape. Since the event horizon is the boundary of the black hole region, it must be a null hypersurface on which the only possible causal curves are null geodesics. So we can interpret that the velocity of a critical particle, which asymptotes the event horizon in infinite proper time, also approaches the speed of light. In fact, we can show that the relative velocity of the critical particle with respect to a non-critical free-falling particle approaches the speed of light. This is the intuitive explanation why the CM energy of particle collision between critical and non-critical particles can be arbitrarily large near the horizon [8]. Figure 3 schematically shows the trajectories of the two colliding particles in the spacetime diagram.

We can interpret it in a slightly different way. No particle or no light ray can escape from behind the event horizon of the black hole. This can be understood that the velocity of a free-falling particle reaches the speed of light with respect to the distant observer. It should be noted, however, that the relative velocity of distant two bodies does not have any definite physical meaning in general relativity. So, if it is possible for a particle to stay at a constant radius $r$ near the horizon, which is analogous to a static observer in the $tr$ plane, the relative velocity of such a particle with respect to the non-critical free-falling particle approaches the speed of light in the limit of the spacetime point to the horizon. A similar discussion in terms of the velocity relative to the zero angular momentum observer is found in [10].

Here it would be helpful to study the difference in the behaviour of the particles orbiting near the horizon between a Schwarzschild black hole and a Kerr black hole and see how our explanation works. We focus on massive particles in the equatorial plane for clarity. For the Schwarzschild black hole, where the event horizon is located at $r_H = 2M$, any particle cannot stay in the vicinity of the horizon. This can be seen in terms of circular orbits, which can be located by $V(r) = V'(r) = 0$. Using equation (2.5) with $a = 0$, we find two roots
The energy of the particle can be determined by the condition \( V(r_\pm) = 0 \). \( r_+ \) and \( r_- \) correspond to stable and unstable circular orbits, respectively. We can see that \( 3M < r_+ \leq 6M \) and \( 6M \leq r_- < \infty \) for \( 2\sqrt{3}mM \leq |L| \), where the equalities \( r_+ = r_- = 6M \) hold only for \( |L| = 2\sqrt{3}mM \) and \( r = 6M \) is called the innermost stable circular orbit (ISCO). The reason why high energy particle collision does not occur in the Schwarzschild spacetime is that there is no circular orbit in the vicinity of the horizon, whether stable or not. Any two particles near the horizon plunge into the horizon with the velocity of light with respect to a static observer and the relative velocity between the two particles cannot be so large. It is the innermost circular orbit that determines the possibility of high energy particle collision in the vicinity of the horizon, although the ISCO is also very important on its own, which will be discussed later.

For the Kerr black hole, the explicit expression of the circular orbits is complicated but we can easily see whether there is the one in the vicinity of the horizon. Using equation (2.5), we find

\[
V(r_H) = - \frac{(r_H^2 + a^2)^2 (E - \Omega_H L)^2}{2r_H^4},
\]

\[
V'(r_H) = \frac{(r_H^2 + a^2)^2 (E - \Omega_H L)^2}{8r_H^6} - \frac{2r_H (r_H^2 + a^2) E (E - \Omega_H L) - (r_H - M) \left[ m^2 r_H^2 + (L - aE)^2 \right]}{r_H^4}.
\]

We should also note that the effective potential is analytic with respect to \( r \) at around \( r = r_H \). Then, we can easily see that there is no circular orbit in the vicinity of the horizon \( r = r_H \), whether it is stable or not, unless \( E = \Omega_H L \approx 0 \) and \( r_H \approx M \), i.e., the particle is at least nearly critical and the black hole is at least nearly maximally rotating.
For a nearly maximally rotating black hole, the radii of both the innermost unstable circular orbit and ISCO can be very close to that of the horizon. The situation is schematically shown in figure 4. As shown in [15], since all unstable circular orbits are located between the innermost unstable circular orbit and ISCO, the radii of unstable circular orbits can also be very close to that of the horizon. This implies that the geodesic particle with $E - \Omega_0 L \approx 0$ can keep its radius constant in the vicinity of the horizon. Such a particle approximately satisfies the critical condition. The relative velocity of a particle in the circular orbit near the horizon with respect to the generic particle plunging into the horizon has a definite physical meaning if these two particles share the same spacetime point. This relative velocity approaches the speed of light in the limit to the horizon radius or in the maximal rotation limit, leading to the arbitrarily large CM energy for the collision of these two particles.

For a maximally rotating black hole, if we release a critical particle at rest at infinity, it approaches the horizon in infinitely long proper time with infinitely many rotations because the horizon radius coincides with that of the maximum of the effective potential, although there is no circular orbit for a massive particle on the event horizon [6]. The relative velocity of the critical particle coasting and approaching the horizon with respect to the generic particle plunging into the horizon has a definite physical meaning if these two particles share the same spacetime point and approaches the speed of light in the limit of the collision point to the horizon.

3. Physical significance of particle acceleration

3.1. Criticisms and basic questions

Just after the rediscovery by Bañados et al [1], several critical comments were given in [8, 11]. Here we pick up the following relevant ones:

1. There exists an upper bound on the spin parameter $a \lesssim 0.998$ for astrophysical black holes, which is called Thorne’s bound [12]. Then, the maximum value of $E_{cm}/(2m)$ is $\sim 9.49$ for the collision of particles of mass $m$ which are initially at rest at infinity [6].

2. The backreaction effect due to the absorption of a pair of the colliding particles of mass $m$ by a maximally rotating black hole shifts the spin parameter $a_*$ from 1 to $1 - 2m/M$.

3. It needs arbitrarily long proper time for the critical particle to reach the horizon for the maximally rotating black hole.

4. The radiation reaction becomes so large for the critical particle that the particle acceleration may be suppressed.

We add the following questions:

5. Is it possible to fine-tune the angular momentum in nature?

6. Is the collision of high CM energy restricted on the equatorial plane?

7. How does the self-force of the particles affect the process?

Comment (3) has already been discussed in section 2.3. As for comment (1), Thorne’s bound is based on the standard accretion disk model and hence at least model-dependent. See, e.g. [13, 14] for the possible violation of this bound.

3.2. Fine-tuning problem and the ISCO

The analytic expression for the ISCO of the Kerr black hole is given in [15]. The ISCO is taken as the inner edge of the accretion disk in the standard accretion disk model. A compact object which adiabatically inspirals around the black hole undergoes a transition to a plunge
phase at the ISCO in the limit of extreme mass ratio. We can explicitly show that \( n_{ISCO} \rightarrow r_H \), \( E_{ISCO} \rightarrow m/\sqrt{3} \), \( L_{ISCO} \rightarrow 2mM/\sqrt{3} \) as \( a_0 \rightarrow 1 \). Noting \( \Omega_H \rightarrow 1/(2M) \) as \( a_0 \rightarrow 1 \), we find \( E_{ISCO} - \Omega_H L_{ISCO} \rightarrow 0 \). In other words, a particle orbiting at the ISCO approaches the horizon and asymptotically satisfies the critical condition in the limit of maximal rotation of the black hole. In fact, if particle 1 which orbits at the ISCO collides with particle 2 which is generic at the ISCO radius, we can find

\[
E_{cm} \approx \sqrt{2E_2 - l_2^2} \frac{1}{2^{1/4}3^{1/4}} \frac{1}{\sqrt{1 - a_0^2}}
\]  

for \( a_0 \approx 1 \), implying unboundedly high collision energy in the maximal rotation limit [6]. In view of the astrophysical significance of the ISCO, it turns out that the angular momentum of particles is naturally fine-tuned and \( E_{cm} \) can be very large for a rapidly rotating black hole.

### 3.3. Non-equatorial orbits and collisions

As for question (5), let us consider particles which are not restricted in the equatorial plane. If two general geodesic particles collide near the horizon, the CM energy is given by [7]

\[
E_{cm}^2 = \frac{m^2 r_H^2 + Q + (L_1 - aE_1)^2}{r_H^2 + a^2 \cos^2\theta} \frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2) + \cdots
\]  

where \( Q \) is the Carter constant. Therefore, we can see that \( E_{cm} \) is diverging if either of the two colliding particles satisfies the critical condition. As in the equatorial case, the critical particle can approach the horizon only if the black hole is maximally rotating. Furthermore, in this case, we can show that the polar angle \( \theta \) must be in some range so that the latitude from the equator must be between \( \pm 42.94^\circ \). Therefore, particle collision of arbitrarily high CM energy occurs only on the region of latitude lower than \( 42.94^\circ \) and does not on the region of higher latitude. See figure 5 for the schematic diagram of the high-velocity collision belt.

### 3.4. Effects of gravitational radiation reaction

A critical particle rotates around the black hole infinitely many times with infinitely long proper time before reaching the horizon of the rotating black hole. One might think that the
particle loses the considerable fraction of its energy and angular momentum through gravitational radiation and it results in the violation of the critical condition. However, the radiation power $E_{GW}$ of the particle orbiting at the ISCO is strongly suppressed if the black hole is rapidly rotating and the detailed numerical calculation suggests that it obeys the power law $E_{GW} \propto (1 - a_*)^4$ ($\lambda \approx 0.317$) for $a_* \approx 1$ [16, 17]. Assuming that this power law holds and that the radiated energy due to the circularly orbiting particle is extracted from the kinetic energy of the particle, we can follow the transition from the adiabatic inspiral phase to the plunge phase continuously and semi-analytically. Within this framework, if the mass ratio of the particle to the black hole is sufficiently small and the black hole is nearly maximally rotating, very high CM energy is still attainable [19].

3.5. Effects of self-gravity of the particles

As for comment (2), it should be noted that the black hole can be efficiently spun up by continuous mass accretion. On the other hand, comment (2) suggests that the gravitational field generated by the particles may affect the process. The rest mass as well as the energy of each particle are assumed to be sufficiently small compared to the gravitational mass of the central black hole. It is expected, however, that the gravitational field generated by the two colliding particles cannot be neglected if the CM energy of the collision is comparable with the gravitational energy of the black hole.

Since the treatment of this problem is very difficult because of lower symmetry of the system, we will try to learn a lesson from an analogous system with higher symmetry. For this reason, we replace the system of particles orbiting an axisymmetric rotating black hole with the system of electrically charged dynamical spherical shells around a static spherically symmetric electrically charged black hole, which is given by the Reissner–Nordström black hole. This is one of the examples where a very good analogy holds between the Kerr back hole and the Reissner–Nordström black hole. Although the latter is apparently very different from the original system, it is very analogous to the original system as the CM energy of the two spherical shells colliding in the vicinity of the horizon can be arbitrarily large if the gravitational and electromagnetic fields generated by each shell are neglected. See also [20] for the collision of two charged particles radially moving around a static spherically symmetric electrically charged black hole. The advantage of the electrically charged black hole–shell system is that we can fully exactly take the fields generated by the shells into account. It turns out that the CM energy $E_{cm}$ of the two colliding shells of equal proper mass in the vicinity but outside of the horizon is bounded as follows [21]

$$E_{cm} \lesssim 2^{1/4}M^{1/4}\mu^{3/4},$$

(3.3)

where $\mu$ is the proper mass of each shell. Although the ratio $E_{cm}/\mu$ can be very large if $M \gg \mu$, the boundedness of $E_{cm}$ for the finite values of $M$ and $\mu$ is important.

4. Towards astrophysical black holes

4.1. Observability of high-energy particles

The high energy collision of particles in the vicinity of the horizon can produce high energy and/or superheavy particles. Can these particles be observed by a distant observer? If such

---

4 Recently, a simple discussion has been proposed which suggests that no gravitational wave is emitted from a particle orbiting at the ISCO in the extremal limit of the Kerr black hole [18].
particles are to be emitted to infinity, extra energy gain is necessary as seen from simple energetics. In this context, it is known that the rotational energy of the black hole can be extracted. For this effect, the existence of the so-called ergoregion near the horizon of the rotating black hole, where the energy of the particle $E$ can be negative, plays an important role.

For a Penrose process [22], the most representative energy extraction process, particle 1 is released from infinity, disintegrates into particles 3 and 4 in the ergoregion and particle 3 escapes to infinity. If $E_4$ is negative, we have $E_3 = E_1 - E_4 > E_1$, i.e., net positive energy gain. Since we are interested in the collision of two particles, let us instead consider two incident particles 1 and 2. Also in this case, if $E_4$ is negative, we have $E_3 = E_1 + E_2 - E_4 > E_1 + E_2$, i.e. net positive energy gain. This process is called a collisional Penrose process [2]. In this process, we do not need to consider any artificial disintegration process in the ergoregion, which is necessary in the original Penrose process. See figure 6 for the schematic figures of both processes.

However, the requirement that particle 1 is a critical particle and particle 3 produced in the vicinity of the horizon escapes to infinity turns out to be a very strong restriction. It can be shown that the energy of the product particle 3 can be at most 218.6% of that of incident particle 1 [23, 24]. The energy extraction efficiency $E_3/(E_1 + E_2)$ is at most 137.2% for the inverse Compton scattering process, which is relatively more efficient among several important physical processes.

This does not necessarily mean that the high energy particle collision is unobservable in principle. In fact, the observational effects from high-energy collision of dark matter particles surrounding a rapidly rotating black hole have been calculated [25, 26]. It is suggested that there can appear signature in the spectrum of gamma ray emission of high energy collision of dark matter particles and subsequent pair annihilation around a black hole and this signature can be distinguished by the observation of the Fermi satellite [27]. However, consistently with the energy upper bound argument, it has been shown [28] that the flux directly emitted from the conventional Bañados–Silk–West process is unmeasurably small because of strong redshift as well as greatly diminished escape fraction. This point has been subsequently acknowledged and the potential indirect observability has been discussed by other authors [29].
4.2. Effects of magnetic fields

It is believed that there are strong magnetic fields around astrophysical black holes. The magnetic flux density is estimated to \( \sim 10^4 \) Gauss around supermassive black holes and to \( \sim 10^8 \) Gauss around stellar mass black holes. These magnetic fields are not so strong as to deform the black hole itself but strong enough to greatly affect the orbits of charged particles. The non-dimensional ratio \( b \) of the Lorentz force to the gravitational force is estimated as

\[
b = \frac{qBGM}{mc^3} \sim 10^{14}\left(\frac{m}{m_e}\right)^{-1}\left(\frac{B}{10^8 \text{ Gauss}}\right)\left(\frac{M}{10 M_\odot}\right).
\]  

(4.1)

where \( B, q, e, m_e \) are the magnetic flux density, the charge of the particle, the elementary charge and the electron mass, respectively, and \( G \) and \( c \) are restored. If the Lorentz force acts in the direction opposite to the gravitational force, the charged particle has a smaller ISCO radius and a higher ISCO velocity than the neutral particle. This can be viewed as an indirect acceleration of charged particles by the magnetic field, if we consider that a charged particle gradually shifts the radius of its circular orbit towards the ISCO for the charged particle by radiation reaction. For a Schwarzschild black hole immersed in a uniform magnetic field, the CM energy for the collision of particle 1 which is a charged particle orbiting at the ISCO against particle 2 which is a radially falling neutral particle is \[ E_{cm} \approx 1.74b^{1/4}m. \]  

(4.2)

For a Kerr black hole immersed in a uniform magnetic field, we can find \[ E_{cm}(b) \approx \frac{\sqrt{b}}{3^{1/4}}E_{cm}(0), \]  

(4.3)

where \( a_s \approx 1 \) and \( b \gg 1 \) are assumed and \( E_{cm}(0) \) is the CM energy in the absence of magnetic field, which is given by equation (3.1) for the collision of a particle orbiting at the ISCO. Therefore, for the Kerr black hole, the magnetic field is expected to enhance the acceleration of charged particles to \( \sim 10^4 \sim 10^5 \) times higher than the value for the absence of the magnetic field. It will be interesting to study the acceleration of charged particles around a Kerr black hole with more realistic configuration of the magnetic field.

5. Generalizations

So far, we have focused on the collision of geodesic particles around a Kerr black hole. In this section, we briefly review a variety of generalizations.

5.1. High energy particle collision with bounded physical quantities

In the process proposed by Bañados et al [1], it is important that the CM energy \( E_{cm} \) can be arbitrarily large in the limit to the horizon, even though the conserved quantities of the test particles with respect to a distant static observer are finite. We can separately discuss the condition for the divergence of \( E_{cm} \) leaving aside whether or not the colliding particles can reach the horizon by any physical process, e.g., geodesic motion from the distant region or continuous energy loss due to radiation. We should refer to several works from this point of view. Piran and Shaham [32] discuss that \( E_{cm} \) can be unbounded for the collision of ingoing particles and outgoing particles with finite conserved quantities in the vicinity of the horizon even in the non-extremal Kerr black hole, although such a collision is not physically well motivated. Grib and Pavlov [33] assume a near-critical particle in the vicinity of the horizon.
which is inside the barrier of the effective potential. Although such a particle cannot reach the vicinity of the horizon from a distant region through any geodesic motion, they invoke multiple scatterings. They showed that such a near-critical particle can collide with a non-critical falling particle with unbounded CM energy even around a non-extremal Kerr black hole. For these collisions, no physically realistic processes are known to give particles such special initial conditions.

5.2. High energy particle collision in non-Kerr black holes

As we have seen, Kerr black holes act as the accelerators of neutral particles. In fact, the Kerr-Newmann family of black holes \[34, 35\], accelerating and rotating black holes \[36\] and Sen black holes \[37\] are shown to accelerate neutral particles to arbitrarily high energy in the sense described in section 2. These are the examples of extremal rotating black holes which act as particle accelerators to unboundedly high energy if radiative reactions and self-force effects are neglected.

The Reissner–Nordström black holes are inefficient in accelerating neutral particles but can act as the accelerators of charged particles to unboundedly high energy \[20\]. It is also the case for general rotating and charged black holes \[38\]. These are the examples of extremal charged black holes as the accelerators of charged particles to unboundedly high energy.

Zaslavskii discussed high energy particle collision around ‘dirty’ black holes, which a certain class of stationary and axisymmetric black holes, including not only Kerr black holes but also black holes surrounded by matter distribution and black holes in alternative theories of gravity. He showed that the situation is similar to the case of the Kerr black holes. Irrespective of the explicit functional form of the metric, extremal dirty black holes are shown to accelerate neutral particles \[39\].

Recently, the particle acceleration scenario has been extended to higher dimensions. The Myers–Perry black holes, which are the higher-dimensional counterpart of the Kerr black holes, are shown to act as the accelerators of neutral particles \[40, 41\]. The universality of particle collision of unbounded CM energy in the vicinity of the horizon of extremal black holes suggests a tight link to the field instability universally seen on the horizon of the extremal black holes shown by Aretakis \[42–47\] and others \[48, 49\].

5.3. High energy particle collision in non-black hole spacetimes

At the event horizon of a black hole, the infalling velocity of the free-falling particle might be understood as the speed of light with respect to a distant static observer. We can interpret it as that the gravitational potential for such a particle is infinitely deep at the event horizon and hence photons emitted from such a particle are infinitely redshifted.

We should note that gravitational redshift can be very large even in non-black hole spacetimes. Photons emitted from a particle in a compact region to infinity can be strongly redshifted if the gravitational potential is very deep there. We can also expect that high energy particle collision occurs in such a region \[50, 51\]. In the absence of an event horizon, we can naturally consider collisions between ingoing and outgoing particles in such a high redshift region. In fact, it has been shown that the CM energy can be very high for such particle collisions in several non-black hole spacetimes \[50, 52–55\]. The efficiency and visibility of the high energy particle collisions around super-spinning near-extremal Kerr geometry are discussed in detail in \[56, 57\]. This is in contrast to the case of black holes, where the collision between ingoing and outgoing particles in the vicinity of the event horizon is not physically well motivated since the event horizon is a one-way membrane. The non-black
hole scenario has a few possible advantages over the black hole one. It has been shown that in the system of charged spherical shells the gamma factor between two colliding shells can be arbitrarily large even if the effect of self-gravity is fully taken into account in the context of the non-black hole scenario [51]. It has also been shown that the time necessary for the same CM energy to be achieved is much shorter than the corresponding black hole scenario [56].

6. Conclusion

We can expect that high energy particle collision occurs in the vicinity of the horizon of a rapidly rotating black hole as a result of the particle acceleration in rather general situations. This phenomenon is well founded on the properties of the geometry and the geodesic orbits of the extremal and near-extremal Kerr black holes. On the other hand, the acceleration to infinitely high energy is unphysical and there should exist an upper bound on the CM energy due to the finiteness of the acceleration time and probably due to the self-force of the colliding particles. Although we do not identify this particle acceleration mechanism with the direct acceleration mechanism of observed cosmic rays, it can imprint some indirectly observable signatures on the spectra and/or light curves of cosmic rays, electromagnetic waves, neutrinos and gravitational waves.

We have convincing evidence that the particle acceleration to arbitrarily high energy is one of the universal basic properties of extremal black holes not only in astrophysics but also in more general context. Here we have seen some of the simplest examples. Moreover, it should be noted that the particle acceleration is seen not only in the vicinity of the event horizon of black holes but also in the deep gravitational potential well in non-black hole spacetimes.

Acknowledgments

The authors thank V Frolov, T Igata, PS Joshi, T Kokubu, U Miyamoto, K-I Nakao, H Nemoto, M Patil, J Silk, H Tagoshi, N Tsukamoto and OB Zaslavskii for fruitful discussion. TH was partially supported by the Grant-in-Aid No. 26400282 for Scientific Research Fund of the Ministry of Education, Culture, Sports, Science and Technology, Japan. MK is supported by a grant for research abroad from JSPS.

References

[1] Bañados M, Silk J and West S M 2009 Kerr black holes as particle accelerators to arbitrarily high energy Phys. Rev. Lett. 103 111102
[2] Piran T, Shaham J and Katz J 1975 Astrophys. J. 196 107–8
[3] Wald R M 1984 General Relativity (Chicago, IL: University of Chicago Press)
[4] Kerr R P 1963 Gravitational field of a spinning mass as an example of algebraically special metrics Phys. Rev. Lett. 11 237
[5] Poisson E 2004 A Relativist’s Toolkit: The Mathematics of Black-Hole Mechanics (Cambridge: Cambridge University Press)
[6] Harada T and Kimura M 2011 Collision of an innermost stable circular orbit particle around a Kerr black hole Phys. Rev. D 83 024002
[7] Harada T and Kimura M 2011 Collision of two general geodesic particles around a Kerr black hole Phys. Rev. D 83 084041
[8] Jacobson T and Sotiriou T P 2010 Spinning black holes as particle accelerators Phys. Rev. Lett. 104 021101
[9] Patil M, Joshi P S, Nakao K-i, Kimura M and Harada T in preperation
[10] Zaslavskii O B 2011 Acceleration of particles by black holes: kinematic explanation Phys. Rev. D 84 024007

[11] Berti E, Cardoso V, Gualtieri L, Pretorius F and Sperhake U 2009 Comment on ‘Kerr black holes as particle accelerators to arbitrarily high energy’ Phys. Rev. Lett. 103 239001

[12] Thorne Kip S 1974 Disk accretion on to a black hole II. Evolution of the hole Astrophys. J. 191 507

[13] Abramowicz M A and Lasota J P 1980 Spin-up of black holes by thick accretion disks Acta Astron. 30 35

[14] Sadowski A, Bursa M, Abramowicz M, Kluzniak W, Lasota J P, Moderski R and Safarzadeh M 2011 Relativistic slim disks with vertical structure Astron. Astrophys. 532 A41

[15] Bardeen J M, Press W H and Teukolsky S A 1972 Rotating black holes: locally nonrotating frames, energy extraction, and scalar synchrotron radiation Astrophys. J. 178 347

[16] Kesden M 2011 Transition from adiabatic inspiral to plunge into a spinning black hole Phys. Rev. D 83 104011

[17] Chrzanowski P L 1976 Applications of metric perturbations of a rotating black hole: distortion of the event horizon Phys. Rev. D13 806

[18] Hadar S, Porfyriadis A P and Strominger A 2014 Gravity waves from extreme-mass-ratio plunges into Kerr black holes Phys. Rev. D 90 064045

[19] Harada T and Kimura M 2011 Collision of an object in the transition from adiabatic inspiral to plunge around a Kerr black hole Phys. Rev. D 84 124032

[20] Zaslavskii O B 2010 Acceleration of particles by nonrotating charged black holes Zh. Eksp. Teor. Fiz. 92 635

[21] Kimura M, Nakao K-i and Tagoshi H 2011 Acceleration of colliding shells around a black hole: validity of the test particle approximation in the Banados-Silk-West process Phys. Rev. D 83 044013

[22] Penrose R 1969 Gravitational collapse: the role of general relativity Rev. Nuovo Cimento 1 252

[23] Bejger M, Piran T, Abramowicz M and Håkanson F 2012 Collisional Penrose process near the horizon of extreme Kerr black holes Phys. Rev. Lett. 109 121101

[24] Harada T, Nemoto H and Miyamoto U 2012 Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole Phys. Rev. D 86 024027

[25] Bañados M, Hassanain B, Silk J and West S M 2011 Emergent Flux from Particle Collisions Near a Kerr Black Hole Phys. Rev. D 83 023004

[26] Williams A J 2011 Numerical estimation of the escaping flux of massless particles created in collisions around a Kerr black hole Phys. Rev. D 83 123004

[27] Cannoni M, Gómez M E, Pérez-García M A and Vergados J D 2012 New gamma ray signal from gravitationally boosted neutralinos at the galactic center Phys. Rev. D 85 115015

[28] McWilliams S T 2013 Black holes are neither particle accelerators nor dark matter probes Phys. Rev. Lett. 110 011102

[29] Gariel J, Santos N O and Silk J 2014 Unbound geodesics from the ergosphere and potential observability of debris from ultrahigh energy particle collisions Phys. Rev. D 90 063505

[30] Frolov V P 2012 Weakly magnetized black holes as particle accelerators Phys. Rev. D 85 024020

[31] Igata T, Harada T and Kimura M 2012 Effect of a weak electromagnetic field on particle acceleration by a rotating black hole Phys. Rev. D 85 104028

[32] Piran T and Shaham J 1977 Upper bounds on collisional Penrose processes near rotating black hole horizons Phys. Rev. D 16 1615–35

[33] Grib A A and Pavlov Y V 2011 On particles collisions near rotating black holes Gravit. Cosmol. 17 42–46

[34] Wei S W, Liu Y X, Guo H and Fu C E 2010 Charged spinning black holes as particle accelerators Phys. Rev. D 82 103005

[35] Liu C, Chen S and Jing J 2011 Collision of two general geodesic particles around a Kerr–Newman black hole (arXiv:1104.3225 [hep-th])

[36] Yao W, Chen S, Liu C and Jing J 2012 Effects of acceleration on the collision of particles in the rotating black hole spacetime Eur. Phys. J. C 72 1898

[37] Wei S W, Liu Y X, Li H T and Chen F W 2010 Particle collisions on stringy black hole background J. High Energy Phys. 1012 066
[38] Zhu Y, Wu S-F, Liu Y-X and Jiang Y 2011 General stationary charged black holes as charged particle accelerators Phys. Rev. D 84 043006
[39] Zaslavskii O B 2010 Acceleration of particles as universal property of rotating black holes Phys. Rev. D82 083004
[40] Abdurajbarov A, Dadhich N, Ahmedov B and Eskuvatov H 2013 Particle acceleration around a five-dimensional Kerr black hole Phys. Rev. D 88 084036
[41] Tsukamoto N, Kimura M and Harada T 2014 High energy collision of particles in the vicinity of extremal black holes in higher dimensions: Banados-Silk-West process as linear instability of extremal black holes Phys. Rev. D 89 024020
[42] Aretakis S 2011 Stability and instability of extreme Reissner–Nordström black hole spacetimes for linear scalar perturbations I Commun. Math. Phys. 307 17
[43] Aretakis S 2011 Stability and instability of extreme Reissner–Nordström black hole spacetimes for linear scalar perturbations II Ann. Henri Poincaré 12 1491
[44] Aretakis S 2012 Decay of axisymmetric solutions of the wave equation on extreme Kerr backgrounds J. Funct. Anal. 263 2770
[45] Aretakis S 2012 Horizon instability of extreme black holes (arXiv:1206.6598[gr-qc])
[46] Aretakis S 2013 A note on instabilities of extremal black holes under scalar perturbations from afar Class. Quantum Grav. 30 095010
[47] Aretakis S 2013 Nonlinear instability of scalar fields on extremal black holes Phys. Rev. D 87 084052
[48] Murata K, Reall H S and Tanahashi N 2013 What happens at the horizon(s) of an extreme black hole? Class. Quantum Grav. 30 235007
[49] Murata K 2013 Instability of higher dimensional extreme black holes Class. Quantum Grav. 30 075002
[50] Patil M and Joshi P S 2012 Ultra-high energy particle collisions in a regular spacetime without blackholes or naked singularities Phys. Rev. D 86 044040
[51] Nakao K-i, Kimura M, Patil M and Joshi P S 2013 Ultra high energy collision with neither black hole nor naked singularity Phys. Rev. D 87 104033
[52] Patil M and Joshi P S 2011 Kerr Naked Singularities as Particle Accelerators Class. Quantum Grav. 28 235012
[53] Patil M and Joshi P S 2011 High energy particle collisions in superspinning Kerr geometry Phys. Rev. D 84 104001
[54] Patil M and Joshi P S 2012 Acceleration of particles in Janis–Newman–Winicour singularities Phys. Rev. D 85 104014
[55] Patil M, Joshi P S, Kimura M and Nakao K i 2012 Acceleration of particles and shells by Reissner–Nordström naked singularities Phys. Rev. D 86 084023
[56] Stuchlík Z and Schee J 2012 Observational phenomena related to primordial Kerr superspinars Class Quantum Grav. 29 065002
[57] Stuchlík Z and Schee J 2013 Ultra-high-energy collisions in the superspinning Kerr geometry Class Quantum Grav. 30 075012