Giant thermopower in superconducting heterostructures with spin-active interfaces

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Abstract

We predict parametrically strong enhancement of the thermoelectric effect in metallic bilayers consisting of two superconductors separated by a spin-active interface. The physical mechanism for such an enhancement is directly related to electron-hole imbalance generated by spin-sensitive quasiparticle scattering at the interface between superconducting layers. We explicitly evaluate the thermoelectric currents flowing in the system and demonstrate that they can reach maximum values comparable to the critical ones for superconductors under consideration.

Keywords: Superconductivity, thermoelectric effect, spin-dependent electron scattering, electron-hole imbalance

1. Introduction

It is well known that application of a thermal gradient $\nabla T$ to a normal conductor along with electric field $E$ results in the electric current

$$j = \sigma_N E + \alpha_N \nabla T, \quad \alpha_N \sim (\sigma_N/e)(T/\varepsilon_F).$$

(1)

Here $\sigma_N$ defines Drude conductivity and $\varepsilon_F$ is the Fermi energy. Provided a metal is brought into a superconducting state, Eq. (1) is no longer correct since the electric field cannot penetrate into the bulk of a superconductor. Instead, one finds

$$j = j_s + \alpha_S \nabla T,$$

(2)

where $j_s$ is a supercurrent and $\alpha_S$ defines thermoelectric coefficient in a superconducting state. It turns out that by applying thermal gradient to a uniform superconductor it is not possible to induce and measure any current since thermal current would always be compensated by the supercurrent $j_s = -\alpha_S \nabla T$. The way out is to consider non-uniform superconducting structures in which case no such compensation generally occurs \cite{1,2} and the thermoelectric current can be detected experimentally. Making use of this idea the thermoelectric effect was indeed demonstrated in several experiments with bimetallic superconducting rings \cite{3,4,5}. Quite surprisingly, the magnitude of the effect was found to be several orders of magnitude bigger than predicted by theory \cite{6}. The authors of a very recent experimental work \cite{7} also observed a discrepancy between theory and their experimental data.

By now it is well understood that a small theoretical value of the thermoelectric coefficient in ordinary superconductors \cite{3,4} $\alpha_S \sim \alpha_N$ is directly linked to the assumption that electron-hole symmetry remains preserved in these structures. In this case contributions to the thermoelectric current provided by electron-like and hole-like excitations are of the opposite sign and almost cancel each other. Then, like in a normal metal, one inevitably finds that $\alpha_S$ is controlled by a parametrically small factor $T/\varepsilon_F \ll 1$.

The situation may change if for some reason the electron-hole symmetry gets violated. In this case – as it was demonstrated by a number of authors – a much stronger thermoelectric effect can be expected. The proposed mechanisms for the electron-hole symmetry violation and the related thermoelectric effect enhancement are diverse. In conventional superconductors doped by magnetic impurities, the presence of Andreev bound states formed near such impurities may yield an asymmetry between electron and hole scattering rates which in turn results in a drastic enhancement of the thermoelectric effect \cite{8}. Likewise, the formation of quasi-bound Andreev states near non-magnetic impurities in unconventional superconductors may lead to much larger values of $\alpha_S$ in such systems \cite{9}. Substantial enhancement of thermoelectric currents was also predicted in three terminal hybrid ferromagnet-superconductor-ferromagnet (FSF) \cite{10} as well as in FS junctions in the presence of a Zeeman spin-splitting field \cite{11}.

In a recent work \cite{12} we argued that the thermoelectric effect can be strongly enhanced also in metallic bilayers consisting of a superconductor and a normal metal (SN) provided these two metals are separated by a thin spin-active interface. By exactly solving the corresponding Bogolyubov-de-Gennes equations we evaluated the wave functions for electron-like and hole-like excitations in such systems demonstrating that spin-sensitive scattering at the SN interface can generate electron-hole imbalance and result in the presence of large thermoelectric currents in such systems.
systems. In this paper we will further extend our argu-
ments [13] to superconducting multilayers with spin-active
interfaces and demonstrate that thermoelectric properties
of such systems may drastically differ from those of
bulk superconductors. As a simple example of such sys-
tems below we will specifically consider a superconductor
with a thin ferromagnetic interlayer. We will show that
provided a temperature gradient is applied along this inter-
layer the system develops a thermoelectric current which
maximum values can be as high as the critical (depaire-
ing) current of a superconductor.

The structure of our paper is as follows. In Sec. 2
we will specify our model and outline our basic quasiclas-
sical formalism of Eilenberger equations to be employed
in our analysis of the thermoelectric effect. In Sec. 3 we
will present an efficient method enabling one to derive the
solution of these equations for the system under consider-
ation. With the aid of this solution we will then derive a
general expression for the thermoelectric current and also
briefly discuss our results in Sec. 4.

\section{The model and quasiclassical formalism}

In what follows we will consider an extended metallic
bilayer consisting of the two superconducting slabs \(S_1\) and
\(S_2\) as shown in Fig. 1. We will assume that both metals
are brought into direct contact with each other via a spin-
active interface that is located in the plane \(z = 0\). Such
an interface can be formed, e.g., by an ultrathin layer of
a ferromagnet. Our goal is to evaluate an electric cur-
rent response to a temperature gradient applied to the
system along the \(S_1S_2\) interface. This temperature grad-
ient is achieved by setting the temperature \(T\) at the left
\((x \to -\infty)\) and right \((x \to \infty)\) edges of the bilayer equal
respectively to \(T_a\) and \(T_b\), see Fig. 1. For the sake of sim-
plecity below we will assume that the temperature depends
only on \(x\) and does not vary along \(y\)- and \(z\)-directions.

Within the quasiclassical theory of superconductivity
[13], the current density \(j(r)\) in our system can be evalu-
ated by means of the standard formula

\[ j(r) = \frac{eN_0}{8} \int d\varepsilon \left\langle \nu_F \text{Sp}[\tilde{\tau}_3 \hat{\Delta}^K (p_F, r, \varepsilon)] \right\rangle, \quad (3) \]

where \(N_0\) is the density of state at the Fermi level, \(p_F =
\nu_F\) is the electron Fermi momentum vector, \(\tilde{\tau}_3\) is the
Pauli matrix in the Nambu space, the angular brackets
\(\langle \cdots \rangle\) denote averaging over the Fermi momentum
directions and \(\hat{\Delta}^K\) is the Keldysh block of the quasiclassical
Green-Eilenberger function matrix

\[ \hat{\gamma} = \begin{pmatrix} \hat{\gamma}^R & \hat{\gamma}^A \\ 0 & \hat{\gamma}^A \end{pmatrix}. \quad (4) \]

Here and below the “hat”-symbol denotes 4 \(\times\) 4 matrices in
the Nambu\(\otimes\)Spin space while the “check”-symbol labels
8 \(\times\) 8 matrices in the Keldysh\(\otimes\)Nambu\(\otimes\)Spin space.

The matrix function \(\hat{\gamma}\) obeys the transport-like Eilen-
berger equation [13]

\[ [\varepsilon \tilde{\tau}_3 - \hat{\Delta}(r), \hat{\gamma}] + i\nu_F \nabla \hat{\gamma}(p_F, r, \varepsilon) = 0 \quad (5) \]

as well as the normalization condition

\[ \hat{\gamma}^2 = 1. \quad (6) \]

The order parameter matrix \(\hat{\Delta}\) has only “retarded” and
“advanced” components,

\[ \hat{\Delta} = \begin{pmatrix} \hat{\Delta}^R & 0 \\ 0 & \hat{\Delta}^A \end{pmatrix}, \quad \hat{\Delta} = \begin{pmatrix} 0 & \Delta \sigma_0 \\ -\Delta^* \sigma_0 & 0 \end{pmatrix}, \quad (7) \]

where \(\sigma_0\) is the unity matrix in the spin space and \(\Delta\) is the
superconducting order parameter. As soon as we are
interested in the electronic transport along the interface
we set the phase difference between the two superconductors
\(S_1\) and \(S_2\) to zero. Under this assumption order parameter
can be made to be real everywhere in the system.

As usually, the quasiclassical equations (5) should be
supplemented by boundary conditions which describe elec-
tron transfer across the \(SFS\)-interface by matching the
Green function matrices \(\hat{\gamma}\) for incoming and outgoing
momentum directions at both sides of this interface, see Fig.
1. In the case of spin-active interfaces the corresponding
boundary conditions were derived in [14]. Here we will
employ an equivalent approach [15].

The simplest model of the spin-active interface is de-
scribed by three parameters, i.e. the transmission prob-
abilities for opposite spin directions \(D_{\uparrow}\) and \(D_{\downarrow}\) as well
as the so-called spin mixing angle \(\theta\). Previously we have
already made use of this model, e.g., while considering
crossed Andreev reflection in three-terminal FSF struc-
tures [16] or triplet pairing and dc Josephson effect in SFS
junctions [17]. For simplicity we assume that the above
three parameters do not depend on the sign of the quasi-
particle momentum along the interface, i.e. \(D_{\uparrow}(p_{\parallel}) =

\frac{38x-2656}{38x-2644} \text{ent is achieved by setting the temperature}
\text{system along the} \quad \frac{38x-2632}{38x-2632} \text{rent response to a temperature gradient applied to the}
\text{a ferromagnet. Our goal is to evaluate an electric cur-
\text{respectively to} \quad \frac{38x-2668}{38x-2668} T \quad \frac{38x-2262}{38x-2262} \text{active interface that is located in the plane}
\text{S} \quad \frac{38x-2144}{38x-2144} \text{are brought into direct contact with each other via a spin-
\text{S} \quad \frac{38x-2120}{38x-2120} \text{bilayer consisting of the two superconducting slabs}
\text{2. The model and quasiclassical formalism}
\text{electron Green functions for incoming and}
\text{going momentum values. These Green functions are matched at}
\text{the spin-active interface by means of the proper boundary conditions}
\text{as specified in the text.}

\begin{center}
Figure 1: The system under consideration consisting of two super-
conducting layers \(S_1\) and \(S_2\) separated by a spin-active interface. The
left and right edges of this superconducting bilayer are maintained
at temperatures \(T_a\) and \(T_b\), respectively. We also schematically indi-
cate the quasiclassical electron Green functions for incoming and
outgoing momentum values. These Green functions are matched at
the spin-active interface by means of the proper boundary conditions
as specified in the text.
\end{center}
$D_x(-p_f)$ and so on. Then the elements of the interface scattering matrices for electrons

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

(8)

and holes

$$\tilde{S} = \begin{pmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{21} & \tilde{S}_{22} \end{pmatrix}$$

(9)

take the form

$$S_{11} = S_{22} = \sqrt{R_\sigma} e^{i\theta_\sigma/2},$$

(10)

$$S_{12} = S_{21} = i\sqrt{D_\sigma} e^{-i\theta_\sigma/2},$$

(11)

$$\tilde{S}_{11} = \tilde{S}_{22} = \sqrt{R_{\sigma}} e^{-i\theta_\sigma/2},$$

(12)

$$\tilde{S}_{12} = \tilde{S}_{21} = i\sqrt{D_{\sigma}} e^{-i\theta_\sigma/2},$$

(13)

where $\theta_\sigma$ is a $2 \times 2$ diagonal matrix in the spin space defined as $\theta_\sigma = \theta_\sigma \gamma_3$. The matrices $D_{\pm \sigma}$ and $R_{\pm \sigma}$ are composed of transmission and reflection probabilities for opposite spin directions as

$$D_\sigma = \begin{pmatrix} D_{\uparrow} & 0 \\ 0 & D_{\downarrow} \end{pmatrix}, \quad D_{-\sigma} = \begin{pmatrix} D_{\downarrow} & 0 \\ 0 & D_{\uparrow} \end{pmatrix},$$

(14)

$$R_\sigma = \begin{pmatrix} R_{\uparrow} & 0 \\ 0 & R_{\downarrow} \end{pmatrix}, \quad R_{-\sigma} = \begin{pmatrix} R_{\downarrow} & 0 \\ 0 & R_{\uparrow} \end{pmatrix},$$

(15)

where we defined the spin-up and spin-down reflection coefficients respectively as $R_{\uparrow} = 1 - D_{\uparrow}$ and $R_{\downarrow} = 1 - D_{\downarrow}$.

3. Riccati parameterization

In order to proceed we will employ the so-called Riccati parameterization of the retarded and advanced Green functions\[13, 19, 20\],

$$\hat{g}^{R,A} = \pm \hat{N}^{R,A} \begin{pmatrix} 1 + \gamma^{R,A} \gamma^{R,A} & 2 \gamma^{R,A} \\ -2 \gamma^{R,A} & -1 - \gamma^{R,A} \gamma^{R,A} \end{pmatrix},$$

(16)

where $\hat{N}^{R,A}$ represent the following matrices

$$\hat{N}^{R,A} = \left( \begin{pmatrix} 1 - \gamma^{R,A} \gamma^{R,A} \end{pmatrix}^{-1} - 0 \\ 0 \right) \left( 1 - \gamma^{R,A} \gamma^{R,A} \right)^{-1}.$$

(17)

Here the Riccati amplitudes $\gamma^{R,A}$, $\tilde{\gamma}^{R,A}$ are $2 \times 2$ matrices in the spin space.

Parameterization of the Keldysh Green function involves two distribution functions\[12\], $x^K$, $\tilde{x}^K$ also being $2 \times 2$ matrices in the spin space, namely

$$\gamma^K = 2\hat{N}^R \begin{pmatrix} x^K & -\gamma^{R,A} \tilde{x}^A \\ -\tilde{x}^A & \tilde{x}^A - x^K \gamma^{R,A} \end{pmatrix} \hat{N}^A,$$

(18)

The amplitudes $\gamma^{R,A}$, $\tilde{\gamma}^{R,A}$ obey the Riccati equations

$$i\nu_F \nabla x^{R,A} = \begin{pmatrix} 1 & \gamma^{R,A} \end{pmatrix} \hat{h} \begin{pmatrix} -1 \\ \gamma^{R,A} \end{pmatrix},$$

(19)

$$i\nu_F \nabla \tilde{x}^{R,A} = \begin{pmatrix} \tilde{\gamma}^{R,A} & 1 \end{pmatrix} \hat{h} \begin{pmatrix} 1 \\ -\gamma^{R,A} \end{pmatrix},$$

(20)

while the distribution functions $x^K$ and $\tilde{x}^K$ satisfy the transport-like equations

$$i\nu_F \nabla x^K = x^K \begin{pmatrix} 1 & 0 \\ -\gamma^R & 1 \end{pmatrix} \hat{h} \begin{pmatrix} 1 \\ 0 \end{pmatrix} x^K,$$

(21)

and

$$i\nu_F \nabla \tilde{x}^K = \tilde{x}^K \begin{pmatrix} 0 & 1 \\ -\gamma^R & 1 \end{pmatrix} \hat{h} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tilde{x}^K,$$

(22)

where $\hat{h} = \hat{\varepsilon} \hat{\gamma}_3 - \hat{\Delta}(r)$.

Below it will be convenient for us to employ the parameterization of the Green function matrices in terms of the Riccati amplitudes $\gamma$, $\tilde{\gamma}$, $\Gamma$, $\tilde{\Gamma}$ as well as the distribution functions $x$, $\tilde{x}$, $X$, $\tilde{X}$ (all being $2 \times 2$ matrices in the spin space)\[13\],

$$\hat{g}_{in} = \hat{g}_{out}^{\pm\gamma} \hat{g}_{in}^{\pm\gamma} \begin{pmatrix} \gamma_1^{R,A} & \tilde{\gamma}_1^{R,A} \\ \tilde{\gamma}_1^{R,A} & \gamma_1^{R,A} \end{pmatrix} \hat{X}^{R,A},$$

(23)

$$\hat{g}_{out} = \hat{g}_{in}^{\pm\gamma} \hat{g}_{out}^{\pm\gamma} \begin{pmatrix} \gamma_1^{R,A} & \tilde{\gamma}_1^{R,A} \\ \tilde{\gamma}_1^{R,A} & \gamma_1^{R,A} \end{pmatrix} \hat{X}^{R,A},$$

(24)

Boundary conditions\[13\] allow to express the interface values of the capital functions $\Gamma$ and $\tilde{\Gamma}$ in terms of the lower-case functions $\gamma$ and $\tilde{\gamma}$. For Riccati amplitudes $\Gamma_1^{R,A}$ and $\tilde{\Gamma}_1^{R,A}$ at the interface we obtain

$$\Gamma_1^{R}(0) = \tilde{\Gamma}_1^{R}(0) = M_1^R \hat{e}^{-i\theta_\sigma} \begin{pmatrix} \gamma_1^{R}(0) & \tilde{\gamma}_1^{R}(0) \\ \tilde{\gamma}_1^{R}(0) & \gamma_1^{R}(0) \end{pmatrix} \sqrt{R_\uparrow R_\downarrow} \hat{R}_1^{R,A},$$

(25)

$$\Gamma_1^{A}(0) = \Gamma_1^{R}(0) = M_1^A \hat{e}^{-i\theta_\sigma} \begin{pmatrix} \gamma_1^{A}(0) & \tilde{\gamma}_1^{A}(0) \\ \tilde{\gamma}_1^{A}(0) & \gamma_1^{A}(0) \end{pmatrix} \sqrt{R_\uparrow R_\downarrow} \hat{R}_1^{R,A},$$

(26)

where

$$M_1^R = \left\{ \begin{array}{cc} 1 & \gamma_1^{R}(0) \hat{R}_1^{R,A} \\ -\gamma_1^{R}(0) & \gamma_1^{R}(0) \hat{R}_1^{R,A} \end{array} \right\}^{-1},$$

(27)

and

$$M_1^A = \left\{ \begin{array}{cc} 1 & \gamma_1^{A}(0) \hat{R}_1^{R,A} \\ -\gamma_1^{A}(0) & \gamma_1^{A}(0) \hat{R}_1^{R,A} \end{array} \right\}^{-1}.\quad (28)$$

The interface values of the Riccati amplitudes $\Gamma_1^{R,A}$ and $\tilde{\Gamma}_1^{R,A}$ can be obtained from Eqs.\[24-28\] by interchanging the indices $1 \leftrightarrow 2$. Here we used the fact that for the real order parameter one has $\gamma_1^{R,A} = \tilde{\gamma}_1^{R,A}$. From Eqs.\[24-28\] we observe that within the adopted model for the spin-active interface this equality also holds for the capital Riccati amplitudes $\Gamma_1^{R,A} = \tilde{\Gamma}_1^{R,A}$.\[3\]
In the same way we can express the interface values of the distribution functions. At the $S_1$-side of the interface we have

\[
X_1(0) = M_{\theta} R M_\Delta \left( x^K(0) \left\{ \sqrt{R_{\sigma}} - \left[ \gamma_2^R(0) \right]^2 \sqrt{R_{-\sigma} e^{i\theta}} \right\} \right.
\]

\[
\times \left\{ \sqrt{R_{-\sigma}} - \left[ \gamma_2^A(0) \right]^2 \sqrt{R_{\sigma} e^{-i\theta}} \right\}
\]

\[
+ x^K_2(0) \left\{ \sqrt{D_{\sigma}} - \gamma_1^R(0) \gamma_2^R(0) \sqrt{D_{-\sigma} e^{i\theta}} \right\}
\]

\[
\times \left\{ \sqrt{D_{-\sigma}} - \gamma_1^A(0) \gamma_2^A(0) \sqrt{D_{\sigma} e^{-i\theta}} \right\}
\]

\[
- \tilde{x}^K_2(0) \left\{ \gamma_1^R(0) \sqrt{R_{\sigma} D_{-\sigma}} - \gamma_2^R(0) \sqrt{R_{-\sigma} D_{\sigma}} \right\}
\]

\[
\times \left\{ \gamma_1^A(0) \sqrt{R_{\sigma} D_{-\sigma}} - \gamma_2^A(0) \sqrt{R_{-\sigma} D_{\sigma}} \right\},
\]

(29)

and

\[
\tilde{X}_1(0) = M_{\theta} R M_\Delta \left( \tilde{x}^K(0) \left\{ \sqrt{R_{\sigma}} - \left[ \gamma_2^R(0) \right]^2 \sqrt{R_{-\sigma} e^{i\theta}} \right\} \right.
\]

\[
\times \left\{ \sqrt{R_{-\sigma}} - \left[ \gamma_2^A(0) \right]^2 \sqrt{R_{\sigma} e^{-i\theta}} \right\}
\]

\[
+ \tilde{x}^K_2(0) \left\{ \sqrt{D_{\sigma}} - \gamma_1^R(0) \gamma_2^R(0) \sqrt{D_{-\sigma} e^{i\theta}} \right\}
\]

\[
\times \left\{ \sqrt{D_{-\sigma}} - \gamma_1^A(0) \gamma_2^A(0) \sqrt{D_{\sigma} e^{-i\theta}} \right\}
\]

\[
- x^K_2(0) \left\{ \gamma_1^R(0) \sqrt{R_{\sigma} D_{-\sigma}} - \gamma_2^R(0) \sqrt{R_{-\sigma} D_{\sigma}} \right\}
\]

\[
\times \left\{ \gamma_1^A(0) \sqrt{R_{\sigma} D_{-\sigma}} - \gamma_2^A(0) \sqrt{R_{-\sigma} D_{\sigma}} \right\}.
\]

(30)

The interface values for the distribution functions $X_2(0)$ and $\tilde{X}_2(0)$ can be recovered from Eqs. (29) and (30) simply by interchanging the indices 1 $\leftrightarrow$ 2. Within our simple model all Riccati amplitudes and distribution functions are diagonal in the spin space. The coordinate dependence of the distribution function can be easily found. We obtain

\[
x_i = \left[ 1 - \gamma_i^R \gamma_i^A \right] \times \left\{ h_a, \quad v_x > 0, \right\}
\]

\[
h_b, \quad v_x < 0,
\]

(31)

\[
\tilde{x}_i = - \left[ 1 - \gamma_i^R \gamma_i^A \right] \times \left\{ h_b, \quad v_x > 0, \right\}
\]

\[
h_a, \quad v_x < 0,
\]

(32)

\[
X_i = X_i(0) \left( \frac{1 - \Gamma^R \Gamma^A_i}{1 - \Gamma^R_i \Gamma^A} \right).
\]

(33)

\[
\tilde{X}_i = \tilde{X}_i(0) \left( \frac{1 - \Gamma^R \Gamma^A_i}{1 - \Gamma^R_i \Gamma^A} \right).
\]

(34)

where the functions $h_{a,b}$ are related to the equilibrium (Fermi) distribution function with temperatures $T_{a,b}$, i.e.

\[
h_{a,b} = \tanh \frac{\varepsilon}{2T_{a,b}}.
\]

(35)

4. Thermoelectric current

Let us apply the above quasiclassical formalism in order to derive the thermoelectric current flowing along the spin-active interface in the ballistic limit. According to Eq. (33) the component of the current density along the interface is expressed in terms of the combination $Sp(\tilde{\gamma}_3 \tilde{g}_{in}^K + \tilde{\gamma}_3 \tilde{g}_{out}^K)$. The above combination is evaluated by solving the Eilenberger equations (53, 61) supplemented by the proper boundary conditions at the $S_1 S_2$ interface. This task can be conveniently accomplished employing the Riccati parameterization of the Green functions [18, 19]. Making use of the results of the previous section we obtain

\[
Sp(\tilde{\gamma}_3 \tilde{g}_{in}^K + \tilde{\gamma}_3 \tilde{g}_{out}^K) = 2 Sp \left[ \frac{(x_i - \tilde{x}_i)(1 + \Gamma^R \Gamma^A)}{(1 - \gamma_i^R \Gamma^A_i)(1 - \gamma_i^A \Gamma^R_i)} \right], \quad i = 1, 2.
\]

(36)

With the aid of Eqs. (31)-(33) one can rewrite the above expression in the form

\[
Sp(\tilde{\gamma}_3 \tilde{g}_{in}^K + \tilde{\gamma}_3 \tilde{g}_{out}^K)
\]

\[
= 2 Sp \left[ \frac{(1 - \gamma_i^R \Gamma^A_i)(1 + \Gamma^R \Gamma^A_i)}{(1 - \gamma_i^R \Gamma^R)_i (1 - \gamma_i^A \Gamma^R_i)} \right] (h_a + h_b)
\]

\[
+ 2 Sp \left[ \frac{(1 - \Gamma^R \Gamma^A_i)(1 + \gamma_i^R \Gamma^A_i)}{(1 - \gamma_i^R \Gamma^R_i)(1 - \gamma_i^A \Gamma^A_i)} \right] X_i(0) - \tilde{X}_i(0).
\]

(37)

Eq. (37) can further be simplified if one observes that the four functions $\gamma_i^R, 1/\gamma_i^A, 1/\Gamma^R_i$ and $\Gamma^A_i$ obey the same Riccati equations implying that the combination

\[
\frac{1 - \Gamma^R \Gamma^A_i}{1 - \gamma_i^R \Gamma^R_i} \frac{1 - \gamma_i^A \Gamma^A_i}{1 - \gamma_i^R \Gamma^A_i}
\]

is spatially constant, i.e. it does not depend on the coordinates. Then one can rewrite Eq. (37) as

\[
Sp(\tilde{\gamma}_3 \tilde{g}_{in}^K + \tilde{\gamma}_3 \tilde{g}_{out}^K) = 2 Sp \left[ \frac{(1 - \gamma_i^R \Gamma^A_i)(1 + \Gamma^R \Gamma^A_i)}{(1 - \gamma_i^R \Gamma^R_i)(1 - \gamma_i^A \Gamma^A_i)} \right]
\]

\[
\times (h_a + h_b) + 2 \frac{1 + \gamma_i^R \Gamma^A_i}{1 - \gamma_i^R \Gamma^A_i}
\]

\[
\times Sp \left[ \frac{[1 - \gamma_i^A(0) \Gamma^R_i(0)]X_i(0) - \tilde{X}_i(0)}{[1 - \gamma_i^R(0) \Gamma^R_i(0)](1 - \gamma_i^A \Gamma^A_i)} \right].
\]

(38)

What remains is to find the difference of the distribution functions $X_i(0) - \tilde{X}_i(0)$ on both sides of the spin-active interface. Making use of Eqs. (29)-(30), we obtain

\[
X_i(0) - \tilde{X}_i(0) = M_{\theta}^R M_\Delta \left[ A_i (h_a - h_b) \sigma_3 \text{sgn} v_x + B_i (h_a + h_b) \right], \quad i = 1, 2.
\]

(39)
where

\[ A_1 = (R_1 - R_\downarrow) K_2 \left[ \gamma_1^R(0)\gamma_1^A(0) - \gamma_2^R(0)\gamma_2^A(0) \right], \quad (40) \]

\[ B_1 = K_2 \left[ 1 + \gamma_1^R(0)\gamma_1^A(0) \gamma_2^R(0)\gamma_2^A(0) \right] + (R_1 + R_\downarrow) K_2 \gamma_1^R(0)\gamma_2^A(0) \]

\[ - \sqrt{R_1 R_\downarrow} K_1 \left\{ \left[ \gamma_1^R(0)^2 \right] e^{i\theta_\sigma} + \left[ \gamma_2^A(0)^2 \right] e^{-i\theta_\sigma} \right\} \]

\[ - \sqrt{D_1 D_\downarrow} K_2 \left\{ \left[ \gamma_1^R(0)\gamma_1^A(0) e^{i\theta_\sigma} + \gamma_1^A(0)\gamma_2^R(0) e^{-i\theta_\sigma} \right] \right. \]

\[ - R_1 R_\downarrow K_2 \left[ \gamma_1^R(0)\gamma_1^A(0) + \gamma_2^R(0)\gamma_2^A(0) \right] \]

\[ - \sqrt{R_1 R_\downarrow} D_1 D_\downarrow K_2 \left[ \gamma_1^R(0)\gamma_2^A(0) + \gamma_2^R(0)\gamma_1^A(0) \right]. \quad (41) \]

Here we defined

\[ K_i = 1 - \gamma_i^R(0)\gamma_i^A(0), \quad i = 1, 2. \quad (42) \]

Analogous expressions can also be derived for \( A_2 \) and \( B_2 \).

Combining Eqs. (38)-(41) with the general expression (3) and observing that the terms containing the combination \( h_\uparrow + h_\downarrow \) do not contribute to the current and defining the unity vector in the \( x \)-direction \( e_x \), we arrive at the final result for the current in the \( S_1 \) superconductor

\[ j(z > 0) = e_x e^{i\theta_0} \int d\varepsilon \left[ \tanh \frac{\varepsilon}{2T_\downarrow} - \tanh \frac{\varepsilon}{2T_\uparrow} \right] \times \left\{ \sin(\theta) e_x \left( R_\uparrow - R_\downarrow \right) \right\} \]

\[ \left[ 1 - \gamma_1^A(0) \right] \left[ 1 - \gamma_1^R(0) \right] \left[ 1 + \gamma_1^R(0) \gamma_1^A(0) \right] \left[ 1 - \gamma_2^R(0) \gamma_2^A(0) \right] \times \left[ \gamma_1^R(0) \gamma_2^A(0) - \gamma_2^R(0) \gamma_1^A(0) \right] \langle \sigma_3 \mathcal{P} \rangle, \quad (43) \]

where

\[ \mathcal{P} = \left[ 1 - \gamma_1^R(0)^2 \right] \sqrt{R_1 R_\downarrow} e^{i\theta_\sigma} - \gamma_1^A(0)^2 \sqrt{R_1 R_\downarrow} e^{-i\theta_\sigma} \]

\[ - 2\gamma_2^R(0) \gamma_1^R(0) \sqrt{D_1 D_\downarrow} e^{i\theta_\sigma} + \gamma_2^A(0) \gamma_1^A(0) e^{2i\theta_\sigma} \].

\[ (44) \]

The current density in the second superconductor \( j(z < 0) \) is trivially derived from Eq. (43) by interchanging the indices \( 1 \leftrightarrow 2 \).

Note that at subgap energies we have \( \gamma_1^R(0)\gamma_2^A(0) \equiv 1 \) and, hence, the fraction \( [1 - \gamma_1^R(0)\gamma_1^A(0)]/[1 - \gamma_2^R(0)\gamma_2^A(0)] \) in Eq. (43) becomes indefinite. In this case with the aid of Eqs. (19) one can establish an equivalent representation for the above fraction

\[ 1 - \gamma_1^R(0)\gamma_1^A(0) \]

\[ 1 - \gamma_1^R(0)\gamma_1^A(0) \]

\[ = \exp \left( -i \text{sgn} \frac{z}{|v_\downarrow|} \int_0^z \Delta(z') \gamma_1^R(z') - \gamma_1^A(z') \right) dz', \quad (45) \]

which remains regular at subgap energies.

The above equations defining the thermoelectric current density \( j(z) \) flowing in each of the two superconductors \( S_1 \) and \( S_2 \) along the spin-active interface represent the main result of this work. It is easy to verify that provided the superconducting order parameter in one of the superconductors \( (\Delta(z > 0) \text{ or } \Delta(z < 0)) \) tends to zero, Eq. (43) reduces to the result for the thermoelectric current in an SN bilayer derived previously [12] within the framework of a different technique.

Let us briefly analyze the above results. To begin with we observe that the thermoelectric current (43) may differ from zero only in asymmetric structures consisting of different superconductors \( S_1 \) and \( S_2 \). Furthermore – similarly to [12] – the current (43) vanishes if at least one of the two conditions, \( D_\uparrow = D_\downarrow \) or \( \theta = 0 \), is fulfilled. On the other hand, provided both these conditions are simultaneously violated, the thermoelectric current differs from zero and its value can become large.

It is also worth pointing out that the thermoelectric currents (43), (44) flow in the opposite directions in the superconductors \( S_1 \) and \( S_2 \). In each of the superconductors the current density depends on the coordinate \( z \) in the vicinity of the interface and tends to some nonzero values far from it. The latter feature is specific for our ballistic model within which the elastic mean free path \( \ell \) tends to infinity and no electron momentum relaxation occurs. Assuming the mean free path to be finite (which is always the case in any real metal), one can demonstrate that the thermoelectric current density \( j(z) \) remains appreciable only in the vicinity of the spin-active interface \( |z| < \ell \) and decays exponentially into the superconducting bulk at distances from this interface exceeding the elastic mean free path. The corresponding analysis, however, goes beyond the frames of this work and will be made public elsewhere [20].

In order to accurately evaluate the general expression for the thermoelectric current defined in Eqs. (43), (44) one should selfconsistently determine the functions \( \gamma_i^A(0) \) as well as the order parameter \( \Delta(z) \) for any given values of the parameters \( D_\uparrow, D_\downarrow \) and \( \theta \). Technically this is a rather complicated task which can only be handled numerically. There is, however, no particular need in this calculation since the order-of-magnitude estimate of the current can easily be obtained directly from Eqs. (43), (44). The magnitude of the thermoelectric current density can roughly be estimated as

\[ |j(z)| \sim j_c (R_\uparrow - R_\downarrow) \sin \theta \frac{T_\downarrow - T_\uparrow}{T_c}, \quad (46) \]

where \( j_c \sim ev_F N_0 T_c \) is the critical current of a clean superconductor with the critical temperature \( T_c \). Hence, we observe that, in contrast to the standard situation [6], our result (46) does not contain the small factor \( T/\varepsilon_F \ll 1 \), i.e. the magnitude of the thermoelectric effect becomes really large in our case. For instance, by setting \( (R_\uparrow - R_\downarrow) \sin \theta \sim 1 \) and \( T_1 - T_2 \sim T_c \), one achieves
the thermoelectric current densities of the same order as the critical one \( j_c \).

To conclude, we demonstrated that quasiparticle scattering at spin-active interfaces generates electron-hole imbalance in superconductors which may yield an enhancement of thermoelectric currents in such structures up to values as high as the critical (depairing) current of a superconductor. The same effect is expected in more complicated (e.g., layered) superconducting structures containing spin-active interfaces. Such thermoelectric currents can easily be detected and investigated in modern experiments.

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