Equations of state and stability of color-superconducting quark matter cores in hybrid stars

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Abstract

The stable configurations of non-rotating and rotating hybrid stars composed of colour superconducting quark matter core are constructed using several equations of state (EOSs). We use a set of diverse EOSs for the nuclear matter which represents the low density phase. The EOSs at higher densities correspond to the quark matter in the colour superconducting phase and are computed within the NJL-like model for different values of the scalar diquark and vector current couplings strengths. The phase transition to the quark matter is computed by a Maxwell construction. We find that the stability of the hybrid stars are mainly governed by the behaviour of the EOSs for the colour superconducting quark matter. However the compositions of hybrid star are sensitive to the EOS of the nuclear matter. The value of the critical rotation frequency for the hybrid star depends strongly on the EOS of the nuclear matter as well as that for the colour superconducting quark matter. Our results indicate that the EOS for the colour superconducting quark matter can be obtained, by adjusting the parameters of the NJL model, to yield the stable configurations of the hybrid star having the maximum mass $\sim 1.5M_\odot$ in the non-rotating limit and the critical rotation frequency $\sim 1$ kHz.

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The present knowledge of quantum chromodynamics (QCD) suggests that quark matter might be in different color superconducting phases at high densities. Thus, one expects the core of the hybrid stars to be composed of color superconducting quark matter (CSQM) surrounded by a nuclear mantle. The possible CSQM phases are the two-flavor color superconductor (2SC) [1–3], the colour flavour locked (CFL) phase [4, 5], and crystalline color superconductor (CCS) [6, 7]. The speculation that the CSQM exists in the core of the hybrid stars has triggered many theoretical investigations both on the modeling of the equation of state (EOS) of quark matter and on the phenomenological signatures of the presence of quark matter in the compact stars [8].

The nuclear matter phase of the hybrid star is described by the various models which can be broadly grouped into (i) non-relativistic potential models [9], (ii) non-relativistic mean-field models [10–13], (iii) field theoretical based relativistic mean-field models [14–16] and (iv) Dirac-Brueckner-Hartree-Fock model [17–20]. The quark matter in the colour superconducting phases are usually described either within the MIT bag model or using a more realistic NJL-like model. The studies based on the MIT bag model indicate the existence of stable configurations of hybrid stars with the CFL quark matter core [21–24]. Further, the MIT bag model predicts the absence of the 2SC colour superconducting phase in the hybrid stars [25]. The scenario is some what different when NJL model is employed to study the hybrid stars with CSQM core. The stable configurations of hybrid stars with 2SC quark matter core are possible within the NJL model [26–29]. However, earlier investigations [30–32] based on the NJL model ruled out the possibility of CFL quark matter at the core of the hybrid stars, because, it rendered the hybrid star unstable. Only very recently [33, 34], it has been demonstrated that inclusion of the six-fermion interaction term together with large enough values of the scalar diquark coupling strength in the NJL model can yield stable configurations of the hybrid star containing 2SC or CFL quark matter core. The NJL model is also applied to study the possibility of existence of the CCS quark matter phase in the hybrid stars [35, 36].

The stability and the structure of the non-rotating hybrid stars are quite sensitive to the choice of the EOS of the nuclear matter and the quark matter [21, 37]. Further, one often finds that even though the stable configurations of the non-rotating hybrid star for a given EOS belong to the third family of compact stars, but, the maximum rotation frequency upto which these hybrid stars are stable is much lower than the corresponding mass-shedding (Keplerian) frequency [38, 39].
The EOSs for the quark matter in the unpaired or in the various colour superconducting phases employed in these investigations were obtained within the MIT bag model. Recently [40], a more realistic EOS for the unpaired quark matter computed within the NJL model is used to show that the maximum mass of the non-rotating hybrid stars depends sensitively on the choice of the EOS of the nuclear matter. It is necessary to construct stable configurations of the non-rotating and rotating hybrid stars using realistic EOSs for the nuclear matter and for the quark matter in the colour superconducting phases.

In the present work, we compute several EOSs and use them to study the properties of the non-rotating and rotating hybrid stars composed of CSQM core. The lower density part of these EOSs correspond to the nuclear matter and are based on the variational and mean-field approaches. Our set of EOSs for the nuclear matter around the saturation density ($\rho_0 = 0.16 \text{ fm}^{-3}$) is constrained by the bulk properties of the finite nuclei. But, their behaviour at densities, $\rho > \rho_0$, are significantly different. The EOSs for CSQM are calculated within the NJL model using different values for the scalar diquark and vector current coupling strengths. The EOS at intermediate densities are obtained using a Maxwell construction.

The paper is organized as follows, in Sec. II we describe, in brief, the models employed to construct the EOSs for nuclear matter and the CSQM. In Sec. III we present the results for the equilibrium sequences for non-rotating and rotating hybrid stars. In Sec. IV we state our conclusions.

II. EQUATIONS OF STATE

We compute the EOSs which correspond to the nuclear matter at lower densities and CSQM in the 2SC or CFL phases at higher densities. The EOS at intermediate densities are obtained using a Maxwell construction. For nuclear matter in the $\beta$ equilibrium, we employ a set of diverse EOSs which are obtained using various approaches, like, variational, non-relativistic mean field (NRMF) and relativistic mean field (RMF). In Fig. II we plot various nuclear matter EOSs. The low density behaviour these EOSs are very much similar as they are constrained by the bulk properties of the finite nuclei. But, their behaviour at higher densities are so different that the resulting neutron star properties are at variance. In Table II we list some key properties of the non-rotating neutron stars obtained using these nuclear matter EOSs. It can be seen from Table II that the values of the maximum neutron star masses are in the range of $2.0 - 2.8M_\odot$ and the radius $R_{1.4}$ at the canonical
neutron star mass vary between 11.3 – 14.8 km. It is interesting to note that the values of the maximum neutron star mass for both the APR and TM1 EOSs are equal, but, the radius at the canonical mass of the neutron star is reasonably smaller for the APR EOS. This is due to the fact that the APR EOS is softer relative to the TM1 at intermediate densities and it becomes stiffer at high densities as can be seen from Fig. 1. Similar is the case with SLY4 and BSR10 EOSs. We shall see in next section that these pairs of nuclear matter EOSs, for which the maximum neutron star masses are the same, yield significantly different structure for the hybrid stars.

The EOSs for the CSQM in the 2SC or CFL phase are obtained within the NJL model. The input variables of the NJL model are the chemical potentials for all the quark flavours and colours in the chemical equilibrium which is given by the matrix:

\[ \mu_{ab}^{\alpha\beta} = (\mu_3 T_3 + \mu_8 T_8) \delta_{ab} + [\mu_3 (T_3)_{ab} + \mu_8 (T_8)_{ab}] \delta^{\alpha\beta}, \]

where, \( \mu \) is the quark chemical potential, \( \mu_Q \) is the chemical potential of the electric charge equal to minus the electron chemical potential \( \mu_e \) and \( \mu_3 \) and \( \mu_8 \) are the colour chemical potentials associated with the two mutually commuting colour charges of the SU(3) \( c \) gauge group. The explicit form of the electric charge matrix \( Q_f = diag(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \), and for the colour charge matrices \( T_3 = diag(\frac{1}{2}, -\frac{1}{2}, 0) \), and \( \sqrt{3} T_8 = diag(\frac{1}{2}, \frac{1}{2}, -1) \). In the mean-field approximation, the pressure at vanishing temperature reads as,

\[ p = 4K \sigma_u \sigma_d \sigma_s - \frac{1}{4G_D} \sum_{c=1}^{3} |\Delta_c|^2 - 2G_s \sum_{a=1}^{3} \sigma_a^2 + \frac{\omega_0^2}{4GV} + \frac{1}{2\pi^2} \sum_{i=1}^{18} \int_0^\Lambda dk k^2 |\epsilon_i| + P_e - B \]

where, \( \sigma_{u,d,s} \) are the quark-antiquark condensates and \( \Delta_c \) are the three diquark condensates. The values of \( \sigma_i \) and \( \Delta_c \) are determined using,

\[ \frac{\partial p}{\partial \sigma_i} = 0 \]

\[ \frac{\partial p}{\partial \Delta_c} = 0. \]

In Eq. (2) \( \omega_0 \) is the mean field expectation value for isoscalar vector like meson \( \omega \) given as

\[ \omega_0 = 2GV(QM | \psi_u^\dagger \psi_u + \psi_d^\dagger \psi_d + \psi_s^\dagger \psi_s | QM) \].

This field modifies also the chemical potentials:

\[ \mu_{u,d,s} \rightarrow \mu_{u,d,s} - \omega_0 \].

The \( \epsilon_i \) are the dispersion relations computed by following the Ref. [3]. The \( \epsilon_i \) depend explicitly on the values of current quark masses, quark-antiquark and diquark condensates and various chemical potentials appearing in Eq. (1). The \( P_e = \mu_e^4/(12\pi^2) \) is the contribution to
the pressure from the electrons. The constant \( B \) is so determined that the pressure vanishes at zero density and temperature. In addition to the Eqs. (3) and (4), the pressure must satisfy,

\[ n_Q \equiv \frac{\partial p}{\partial \mu_Q} = 0, \]  
\[ n_3 \equiv \frac{\partial p}{\partial \mu_3} = 0, \]  
\[ n_8 \equiv \frac{\partial p}{\partial \mu_8} = 0, \]

so that local electric and colour charge neutrality conditions are met. Once, the pressure as a function of quark chemical potential is known, quark matter EOS can be easily computed.

The model parameters, the current quark masses \( m_u, m_d, m_s \), quark-antiquark coupling \( G_S \), the strength \( K \) of the six fermion or "t Hooft" interaction and the cutoff parameter \( \Lambda \) are taken to be \[ 41 \],

\[ m_u = m_d = 5.5 \text{ MeV}, \]  
\[ m_s = 140.7 \text{ MeV}, \]  
\[ G_S \Lambda^2 = 1.835, \]  
\[ K \Lambda^5 = 12.36, \]  
\[ \Lambda = 602.3 \text{ MeV}. \]

After fixing the masses of the up and down quarks, \( m_u = m_d = 5.5 \text{ MeV} \), the other four parameters are chosen to reproduce the following observables of vacuum QCD \[ 41 \]: \( m_\pi = 135.0 \text{ MeV}, m_K = 497.7 \text{ MeV}, m_{\eta'} = 957.8 \text{ MeV}, \) and \( f_\pi = 92.4 \text{ MeV} \). This parameter set gives \( m_\eta = 514.8 \text{ MeV} \).

The value of \( B \) for this set of parameters is \((425.4\text{MeV})^4 \). There are two more parameters, the diquark coupling strength \( G_D \) and the vector current coupling strength \( G_V \), which are not known. One expects that the diquark coupling has a similar strength as the quark-antiquark coupling. We construct quark matter EOS for \( G_D = 1.1 - 1.2G_S \) with \( G_V = 0 - 0.2G_S \).

In the 2SC phase, pairing occurs only between the \( u \) and \( d \) quarks and the \( s \) quarks remain unpaired leading to \( \Delta_1 = \Delta_2 = 0 \) and \( \Delta_3 \neq 0 \). On the other hand, in the CFL phase, \( \Delta_1 \neq 0, \Delta_2 \neq 0 \) and \( \Delta_3 \neq 0 \). In the left panels of Figs. 2 and 3, we plot the pressure as a function of the quark chemical potential for the nuclear matter and for the quark matter in the 2SC and CFL phases. The phase realized at a given chemical potential is the one having largest pressure. Thus, it is evident from the \( P - \mu \) curves that direct transition from nuclear matter to the CFL quark matter occurs
for the case of APR and SLY4 EOSs. For the TM1 and NL3 EOSs, transition from nuclear matter to the CFL quark matter proceeds via 2SC phase at intermediate densities. For the BSR10 EOS, nuclear matter to CFL quark matter phase transition proceeds via 2SC phase only for $G_D = 1.2G_S$ with $G_V = 0$. We see from these figures that the pressure, at which the transition from the nuclear to quark matter occurs, decreases with increasing $G_D$ or decreasing $G_V$. For instance, pressure at the phase transition reduces almost by a factor of two with increase in $G_D$ from $1.1G_S$ to $1.2G_S$. The solid circles on the various EOSs for the nuclear matter indicate the values of $P_{1.4}$ which is the pressure at the center of the neutron star with the canonical mass (1.4$M_\odot$). The values of transition pressure is close to that of $P_{1.4}$ for the cases plotted in the lower and the upper panels of Figs. 2 and 3, respectively. For the completeness, in the right panels of Figs. 2 and 3, we display the plots for the pressure as a function of the baryon density for the case of APR, BSR10 and NL3 EOSs. The phase transition to the quark matter is computed by a Maxwell construction.

III. HYBRID STARS WITH CSQM CORE

We construct the equilibrium sequences of the non-rotating and rotating compact stars using the EOSs obtained in the last section. These EOSs correspond to the nuclear matter at lower densities and the CSQM in the 2SC or CFL phase at higher densities as shown in Figs. 2 and 3. The nuclear matter EOSs are taken from the published literature as summarized in Table I. The EOSs for the CSQM are computed within the NJL model for different values of the scalar diquark coupling strength $G_D$ and the vector current coupling strength $G_V$. The other parameters of the model are determined by fit to some of the observables of the vacuum QCD. The various EOSs as obtained in the present work can be completely specified by (i) the source for the nuclear matter EOS as listed in Table I and (ii) the values of $G_D$ and $G_V$ used in computing the EOS of the CSQM within the NJL model. The properties of spherically symmetric non-rotating and axially symmetric rotating compact stars are obtained by solving the Einstein’s equations in 1D and 2D, respectively. The numerical computations are performed by using RNS code written by Stergioulas and Friedman [42].

The equilibrium sequence of compact stars for a given EOS is obtained by varying the central energy density $\epsilon_c$. For the stable configuration,

$$\frac{\partial M}{\partial \epsilon_c} \geq 0,$$

(13)

where, $M$ is the gravitational mass of the non-rotating compact star. The equilibrium sequences for the non-rotating compact stars resulting from our EOSs are plotted as mass versus radius in
Figs. 4 and 5. The central energy density increases as we move along these curves from the right hand side. The portion of the curves left to the solid circles represent the equilibrium sequences of hybrid stars with CFL quark matter core. The curves between the solid circle and triangle represent the sequences of hybrid star composed of 2SC quark matter core. It is clear from the lower panel of Fig. 4 and upper panel of Fig. 5 that the stable configurations of hybrid stars with CFL quark matter core belong to third family of compact stars. Further, irrespective of the choice of the EOS of the nuclear matter, the stable configurations of the non-rotating hybrid stars exist within the NJL model only when the EOSs for the CSQM are constructed for \( G_D = 1.2G_S \) with \( G_V \lesssim 0.1G_S \). These values of \( G_D \) and \( G_V \), for which the stable configurations of the hybrid star exist, are very much similar to the ones found in Ref. [33]. It appears that the stability of the hybrid stars with CSQM core depends solely on the choice of the EOS for the CSQM. However, the composition of the hybrid stars depend on the behaviour of the nuclear matter EOS. For instance, in case of the TM1 and the NL3 EOSs of the nuclear matter, the core of the hybrid stars are composed of CSQM which is either in 2SC phase or in the CFL phase. In the later case, CFL quark matter core is surrounded by a layer of 2SC quark matter with the outer layer composed of nuclear matter. The thickness of the 2SC quark matter at the maximum hybrid star mass is around 0.5 – 0.7km and its mass is \( \sim 0.1M_\odot \). On the other hand, no 2SC quark matter appears in the stable configurations of the hybrid star constructed using the EOSs for which the nuclear matter part correspond to the APR and SLY4.

The equilibrium sequences for the hybrid stars rotating with fixed rotation frequency \( f \) are constructed. As an illustration, in Fig. 6 we plot mass verses circumferential equatorial radius \( R_{eq} \) at fixed values of the rotational frequency obtained for two different EOSs. For the clarity, we mainly focus on the regions of the \( M – R_{eq} \) curves corresponding to the sequences of the hybrid stars which are relevant in the present context. We see that beyond certain frequency, so-called the critical rotation frequency \( f_{crit} \), the stable configuration for the rotating hybrid star does not exist. The solid black lines in Fig. 6 represent the result obtained at the \( f = f_{crit} \). In Fig. 7 we plot the values for the \( f_{crit} \) (left panel) calculated for the cases for which the stable configurations for the non-rotating hybrid star exist. It is evident that the values of \( f_{crit} \) are quite sensitive to the choice of the EOS for the nuclear matter as well as the CSQM. Depending on the EOSs, \( f_{crit} \) varies in the range of 350 – 1275 Hz. We also plot in Fig. 7 (right panel) the maximum mass \( M_{max} \) for the non-rotating hybrid stars with CSQM core. It is interesting to note from this Fig. that the values of \( G_D \) and \( G_V \) for a given nuclear matter EOS can be so adjusted that the resulting hybrid
star has (a) the maximum mass in the non-rotating limit larger than $1.44M_\odot$ which is the most accurately measured value for the maximum mass of a compact star \[43\] and (b) the maximum allowed rotation frequency is larger than the current observational limit of 716 Hz \[44\].

In Fig. 8 we show the correlations between the values of the $f_{\text{crit}}$ for the hybrid stars and the radius $R_{1.4}$ for the neutron star with the canonical mass. We see that $f_{\text{crit}}$ is large if value of $R_{1.4}$ is also large. Thus, hybrid star constructed for a given EOS for the CSQM can rotate faster if the EOS for the nuclear matter is stiffer. The existence of the correlations between the values of $f_{\text{crit}}$ and $R_{1.4}$ may be due to the fact that the pressure at which the nuclear to quark matter transition occurs is closer to the values of $P_{1.4}$ as can be seen from the lower and upper left panels of Figs. 2 and 3 respectively.

Finally, we would like to compare the present results with corresponding ones obtained within the MIT bag model \[37,45\]. The present results as obtained within the NJL model are significantly different with those for the MIT bag model. Within MIT bag model the EOS for the CSQM can be obtained by adjusting the value of the CFL gap parameter and the bag constant such that the resulting hybrid stars with CFL quark matter core are gravitationally stable up to the masses $\sim 2M_\odot$ in the static limit and the maximum allowed rotation frequency is much larger than 1 kHz. However, it can be seen from Fig. 7, stable configurations of the hybrid stars with CFL quark matter core obtained within the NJL model are having the maximum values for the mass and the rotational frequency appreciably lower than those obtained for the MIT bag model. The differences between the results for the MIT bag model and the NJL model can be attributed to the fact that the constituent quark masses, chiral condensates and the colour superconducting gaps in the later case are computed self-consistently as a function of baryon density.

IV. CONCLUSIONS

The stability of non-rotating and rotating hybrid stars, composed of the colour superconducting quark matter core surrounded by a nuclear mantle, is studied by using several EOSs. The EOSs for the nuclear matter, employed at lower densities, are based on the variational and the mean field approaches. We use a diverse set of nuclear matter EOSs such that the resulting maximum neutron star mass lie in the range of $2.2 - 2.8M_\odot$ and the radius at the canonical neutron star mass vary between 11.3 – 14.8 km. The EOSs at higher densities corresponding to the colour superconducting quark matter in the 2SC or the CFL phase, are calculated within the NJL model.
using different values of coupling strengths for the scalar diquark and isoscaler vector terms. The EOS at intermediate densities are obtained using a Maxwell construction.

We find that the stability of the non-rotating hybrid stars is very much sensitive to the EOSs for the colour superconducting quark matter and almost independent of the choice for the EOS for the nuclear matter. The stable configurations of the hybrid stars exist only for the large enough value for the scalar diquark coupling strength. Though, the stability of the hybrid stars are not sensitive to the choice of the EOSs for the nuclear matter, but, compositions of the hybrid stars are at variance for these EOSs. If the EOS for the nuclear matter is stiff, core of the hybrid star is composed of colour superconducting quark matter which is either in the 2SC or the CFL phase. In the later case, CFL quark matter core is surrounded by a thin layer of the 2SC quark matter and the outer layer composed of nuclear matter.

The stability of the rotating hybrid star is sensitive to the choice of the EOS for the nuclear matter as well as that for the colour superconducting quark matter. In particular, we find that the values of the critical rotation frequency vary from about 350 Hz to 1275 Hz depending upon the choice of the EOSs for the nuclear matter and the colour superconducting quark matter. Our results also indicate that the EOSs for the colour superconducting quark matter obtained within the NJL model may be adjusted for the various nuclear matter EOSs in such a way that it yields (a) the maximum mass in the non-rotating limit larger than $1.44M_\odot$ which is the most accurately measured value for the maximum mass of a compact star and (b) the maximum allowed rotation frequency is larger than the current observational limit of 716 Hz.

Finally, we would like to mention that our present study can be extended in several ways. The quark matter in the crystalline color superconducting phase, expected to appear at the intermediate densities, should also be considered. One might also include the contributions from the hyperons which would soften the hadronic EOS. The phase transition from hadron to the quark matter should proceed via mixed phase which can be constructed using the Gibbs conditions.
[1] M. G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B 422, 247 (1998).
[2] S. B. Rüster and D. H. Rischke, Phys. Rev. D 69, 0745011 (2004).
[3] S. B. Rüster, V. Werth, M. Buballa, I. A. Shovkovy, and D. H. Rischke, Phys. Rev. D 72, 034004 (2005).
[4] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. A 537, 443 (1999).
[5] K. Rajagopal and F. Wilczek, Phys. Rev. Lett. 86, 3492 (2001).
[6] M. G. Alford, J. Bowers, and K. Rajagopal, Phys. Rev. D 63, 074016 (2001).
[7] K. Rajagopal and R. Sharma, Phys. Rev. D 74, 094019 (2006).
[8] M. G. Alford, A. Schmitt, and T. Schfer, Rev. Mod. Phys. 80, 1455 (2008).
[9] V. R. Pandharipande and R. A. Smith, Nucl. Phys. A237, 507 (1975).
[10] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A627, 710 (1997).
[11] J. R. Stone, J. C. Miller, R. Konciewicz, P. D. Stevenson, and M. R. Strayer, Phys. Rev. C 68, 034324 (2003).
[12] L. Mornas, Eur. Phys. J. A24, 293 (2005).
[13] B. K. Agrawal, S. K. Dhiman, and R. Kumar, Phys. Rev. C 73, 034319 (2006).
[14] M. Prakash, J. R. Cooke, and J. M. Lattimer, Phys. Rev. D 52, 661 (1995).
[15] N. K. Glendenning and J. Schaffner-Bielich, Phys. Rev. C 60, 025803 (1999).
[16] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. Ellis, Phys. Rep. 411, 325 (2005).
[17] H. Mütther, M. Prakash, and T. L. Ainsworth, Phys. Lett. B199, 469 (1987).
[18] L. Engvik, M. Hjorth-Jensen, E. Osnes, G. Bao, and E. Østgaard, Phys. Rev. Lett. 73, 2650 (1994).
[19] L. Engvik, E. Osnes, M. Hjorth-Jensen, G. Bao, and E. Østgaard, Astrophys. J. 469, 794 (1996).
[20] H. J. Schulze, A. Polls, A. Ramos, and I. Vidana, Phys. Rev. C 73, 058801 (2006).
[21] M. Alford and S. Reddy, Phys. Rev. D 67, 074024 (2003).
[22] S. Banik and D. Bandyopadhyay, Phys. Rev. D 67, 123003 (2003).
[23] A. Drago, A. Lavagno, and G. Pagliara, Phys. Rev. D 69, 057505 (2004).
[24] I. Bombaci, G. Lugones, and I. Vidana, Astron. Astrophys. 462, 1017 (2007).
[25] M. Alford and K. Rajagopal, J. High Energy Phys. 06, 031 (2002).
[26] D. Blaschke1, S. Fredriksson, H. Grigorian, A. M. zta, and F. Sandin, Phys. Rev. D 72, 065020 (2005).
[27] D. N. Aguilera, D. Blaschke, and H. Grigorian, Astronomy and Astrophysics 416, 991 (2004).
[28] D. N. Aguilera, D. Blaschke, and H. Grigorian, Nucl. Phys. A757, 527 (2005).
[29] H. Grigorian, D. Blaschke, and D. N. Aguilera, Phys. Rev. C 69, 065802 (2004).
[30] M. Baldo, M. Buballa, F. Burgio, F. Neumann, M. Oertel, and H. Schulze, Phys. Lett. B 562, 153 (2003).
[31] M. Buballa, Phys. Rep. 407, 205 (2005).
[32] T. Klähn, D. Blaschke, F. Sandin, C. Fuchs, A. Faessler, H. Grigorian, G. Ropke, and J. Trümper, Phys. Lett. B 654, 170 (2007).
[33] G. Pagliara and J. Schaffner-Bielich, Phy. Rev. D 77, 063004 (2008).
[34] A. Drago, G. Pagliara, and J. Schaffner-Bielich, J. Phys. G 35, 014052 (2008).
[35] N. D. Ippolito, M. Nardulli, and M. Ruggieri, J. Heigh Energy Phys. 04, 036 (2007).
[36] N. D. Ippolito, M. Ruggieri, D. H. Rischke, A. Sedrakian, and F. Weber, Phys. Rev. D 77, 023004 (2008).
[37] M. Alford, M. Braby, M. Paris, and S. Reddy, Astrophys. J. 629, 969 (2005).
[38] S. Banik, M. Hanauske, and D. Bandyopadhyay, J. Phys. G 31, S841 (2005).
[39] A. Bhattacharyya, S. K. Ghosh, M. Hanauske, and S. Raha, Phys. Rev. C 71, 048801 (2005).
[40] F. Yang and H. Shen, Phys. Rev. C 77, 025801 (2008).
[41] P. Rehberg, S. P. Klevansky, and J. Hübner, Phys. Rev. C 53, 410 (1996).
[42] N. Stergioulas and J. L. Friedman, Astrophys. J. 444, 306 (1995).
[43] S. E. Thorsett and D. Chakrabarty, Astrophys. J 512, 288 (1999).
[44] J. W. T. Hessels, S. M. Ransom, I. H. Stairs, P. C. C. Freire, V. M. Kaspi, and F. Camilo, Science 311, 1901 (2006).
[45] B. K. Agrawal and S. K. Dhiman, Phys. Rev. D 79, 103006 (2009).
[46] A. Akmal, V. Pandharipande, and D. Ravenhall, Phys. Rev. C 58, 1804 (1998).
[47] A. Douchin and P. Haensel, Astron. Astrophys. 380, 151 (2001).
[48] S. K. Dhiman, R. Kumar, and B. K. Agrawal, Phys. Rev. C 76, 045801 (2007).
[49] K. Sumiyoshi, H. Kuwabara, and H. Toki, Nucl. Phys. A581, 725 (1995).
[50] G. A. Lalazissis, J. Konig, and P. Ring, Phys. Rev. C 55, 540 (1997).
TABLE I: Values of the maximum mass $M_{\text{max}}$ and corresponding central energy density $\epsilon_{\text{max}}$ and radius $R_{\text{max}}$ obtained for different EOSs of the nuclear matter. The radius $R_{1.4}$ for the neutron star with canonical mass ($1.4M_\odot$) are also given.

| EOS  | Approach | $\epsilon_{\text{max}}$ ($10^{15}$ g/cm$^3$) | $M_{\text{max}}$ ($M_\odot$) | $R_{\text{max}}$ (km) | $R_{1.4}$ (km) | Ref. |
|------|----------|---------------------------------------------|-------------------------------|------------------------|-----------------|------|
| APR  | Variational | 2.80                                         | 2.19                          | 9.9                    | 11.3            | [46] |
| SLY4 | NRMF     | 2.84                                         | 2.05                          | 10.0                   | 11.7            | [47] |
| BSR10$^a$ | RMF | 2.14                                         | 1.97                          | 11.6                   | 13.3            | [48] |
| TM1  | RMF      | 1.87                                         | 2.19                          | 12.4                   | 14.4            | [49] |
| NL3  | RMF      | 1.55                                         | 2.79                          | 13.3                   | 14.7            | [50] |

$^a$This EOS is obtained using one of the several parameter sets of the extended RMF model given in our earlier work [48]. Each of these parameterizations corresponds to different values of the strength $\zeta$ for the $\omega$-meson self-coupling term and neutron-skin thickness $\Delta r$ in $^{208}$Pb nucleus. The remaining parameters of the models were calibrated to yield reasonable fit to the bulk nuclear observables and nuclear matter incompressibility coefficient. In the present work we use the parameter set with $\zeta = 0.03$ and $\Delta r = 0.2$fm which will be referred henceforth as BSR10.
FIG. 1: (Color online) The nuclear matter EOSs plotted as pressure versus energy density. The energy density is normalized by $\varepsilon_0 = 150$ MeV/fm$^3$ which is the typical value of the energy density for the nuclear matter at the saturation density.
FIG. 2: (Color online) The pressure as a function of the quark chemical potential (left panel) and the baryon density (right panel) for the nuclear and quark matter. The EOSs for the quark matter in the 2SC and CFL phases are obtained within the NJL model using $G_V = 0$ with $G_D = 1.1G_S$ and $1.2G_S$. The solid circles on the various EOSs for the nuclear matter indicate the pressure at the center of the neutron star with the canonical mass ($1.4M_\odot$).
FIG. 3: (Color online) Same as Fig. 2 but, for $G_V = 0.1G_S$ and $0.2G_S$ with $G_D = 1.2G_S$. 
FIG. 4: (Color online) Plots for the mass-radius relationships for the equilibrium sequences of non-rotating compact stars obtained using various EOSs as shown in Fig. 2. The curves on the left of the solid circles represent the equilibrium sequences for the hybrid stars with core composed of the quark matter in the CFL phase. The curves between the solid circles and triangles represent the hybrid stars with 2SC quark matter core. The absence of solid triangle on a curve means that the hybrid stars contain quark matter only in the CFL phase.
FIG. 5: (Color online) Plots for the mass-radius relationships for the equilibrium sequences of the non-rotating compact stars obtained using various EOSs as shown in Fig. 3. The solid circles and triangles divide the curves according to the composition of the hybrid stars as described in Fig. 4.
FIG. 6: (Color online) Plots for the mass versus circumferential equatorial radius $R_{eq}$ at fixed values of the rotational frequency as indicated along each of the curves (in Hz). The black solid lines represent the results obtained at the critical frequencies $f_{crit}$. For $f > f_{crit}$, the stable configurations of hybrid star do not exist.
FIG. 7: (Color online) The values of the critical rotation frequency $f_{\text{crit}}$ (left panel) for the hybrid stars with the CSQM core and their maximum masses (right panel) in the non rotating limit as obtained using different EOSs for the nuclear matter and the CSQM.
FIG. 8: (Color online) Correlations between values of $f_{\text{crit}}$ for the hybrid stars and the radius $R_{1.4}$ for the neutron star with canonical mass as listed in Table I.