The Introduction of NPV and IRR
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ABSTRACT
This paper aims to present the initial concept of NPV and IRR by an example of a question to calculate and answer the question for extending contexts of NPV and IRR from the example. Besides, the paper uses the formula to build each question's process, which makes a systematic solution for leading to other concepts of NPV and IRR. The main goal of this study is to introduce concepts of NPV and IRR by solving the question, proposing a hypothesis to help a reader logically and precisely understand the concepts of NPV and IRR. Meanwhile, it will show examples of NPV and IRR to illustrate and come up with our hypothesis. An extra part makes NPV and IRR, and two different methods companies use to determine the investment by understanding the advantages and disadvantages of NPV and IRR to help you decide which is the best for your company's situation. By investigating NPV and IRR, the context of the paper will be as methods to help people gain financial value.

Keywords: NPV, IRR, Investment, risk-adjusted

1. INTRODUCTION
In the past years, the internal rate of return (IRR) has become a powerful method of investment evaluation. Many people prefer it to the NPV, probably because they can more easily comprehend a percentage measure. Ryan and Ryan found that 76% of the Fortune 1000 companies use IRR 75-100% of the time [1]. Earlier studies indicate that the preference for IRR over NPV. The general approach in determining the suitable decision for an investment project via investment appraisal techniques such as NPV and IRR is to treat the cash flows as known certainty. "When a manager needs to compare projects and decide which one to pursue, there are generally three options available: internal rate of return, payback method, and net present value said by Gallo [2]. In this paper, we propose to discuss the advantages and disadvantages of these two methods.

Net present value and internal rate of return are the most extensively used and accept discounted cash flow (DCF) indices to invest in business and project evaluation [3]. Despite the well-known shortcomings of the IRR as a criterion for evaluating the desirability of investment projects, it continues to be widely used in practice [4]. Finance textbooks (e.g., Brealey and Myers, 1991; Berk and DeMarzo, 2007), on the other hand, usually advise that selecting profitable investments amounts to undertaking projects with positive NPV. The most incredible conceptual difficulty for the practically important IRR criterion is its failure to generally provide a unique value against which to compare the company's hurdle rate to arrive at a decision that is consistent with the NPV rule.[5].

The organization of this paper is as follows. First, we review the introduction about the IRR and NPV rankings. Second, we combine past insights to deduce sufficient conditions for consistent IRR and NPV rankings. Third, we use examples to study IRR and NPV further and derive different methods for satisfying the adequate conditions for consistent IRR and NPV rankings. Finally, the last section of this paper offers some suggestions for choosing the appropriate size adjustment method and some shortcomings of this paper.

2. IRR CONCEPT
Over the past 60 years, the internal rate of return (IRR) has become a major, and it has always been used as a simple tool in investment evaluation. IRR is a discount rate that equates an investment to its cash flow. The definition by B.Graham showed: “An investment operation is one which, upon thorough analysis, promises safety of principal and an adequate return” in his cardinal
work The Intelligent Investor[6]. It will not be a suitable tool until some assumptions can be achieved. In the first place, a cost of capital or the hurdle rate should be available as a comparator. Furthermore, anything but the size as well as the timing of the free cash flow should be involved. And since the IRR is influenced by its cash flow and the investors reinvest cash flows, IRR will be misleading if the investors cannot find chances that result in the same IRR. So, there is an actually thoughtful matter, and it is unimportant to expect future investment opportunities to have the equal IRR as the project under consideration, especially with the high-risk investment demonstrating the high potential return. Here the discounted rate is computed such that PV of benefits equal the PV of costs. So, IRR is the “r” that can make the NPV zero.

\[
\text{IRR}= r \text{ such that } NPV = \sum \frac{B_r}{(1+r)^n} - \sum \frac{C_r}{(1+r)^n} = 0 \quad (1)
\]

(1). For independent projects, undertakes projects if the IRR is higher than the hurdle rate.

(2). For mutually exclusive projects, accept the one with the highest IRR greater than the hurdle rate.

(3). The IRR method assumes that the future cash flow will be reinvested by the investors to make them get a return equal to the IRR.

(4). IRR ignores the differences in the size as well as the scale of the projects.

(5). If there are some differences in the risk classes of the projects and capital rationing, it will cause ranking differences when just done based on IRR.

2.1. The Applicability of IRR

IRR is that discount rate that equates an investment to its resulting cash flow. It is not a measure of investment unless a cost of hurdle rate becomes available as a comparator. Though it is independent, the size and timing of the free cash flow need to be included. Since its cash flow determines the IRR and active investors typically reinvest cash flows, IRR will be unknown to lead if active investors cannot be able to find the best way or reasonable opportunities that occur at the same rate as IRR.

Ranking the investment project using IRR and rejecting marginal projects offers several preponderances over an NPV criterion with cash flow adjustments. First of all, IRR is less accusatory than adjusted by cash flow or discount rates. Moreover, when negotiating with divisions, boards of directors, or other stakeholders over capital requests, senior managers will generally prefer adjusting in less rather than more obtrusive ways [7]. Meanwhile, the IRR is the relatively simpler way when using it. We all know that the higher IRR is, the more heavily later cash flows are discounted. So, under an acceptable method that ranks projects using IRR and rejects marginal projects, any overstatement that improves the opportunities of project acceptance will decrease the weight on later cash flows, thus mitigating the effects of forecast bias.

3. NPV CONCEPT

NPV is represented as positive or negative future cashflows by cycling in investment. At the same time. It is calculated to find the difference between the project cost and cash outflows, or the income from the project and cash inflows. The first reference of npv in American economic literature which published in Irving Fisher’s book of “the rate of interest” in 1907. Fisher mentioned four principles of evaluating alternative investment proposals: maximum present value, return over cost, comparative advantages, and select “the differences of options by continuous gradations, which it forms its nearest rival to give a rate of return over cost equalizing as the rate of interest” [8]. “Net present value is the present value of cash flows at the required rate of return of your project compared to your initial investment” by Gallo [2]. NPV is more commonly used to analyze capital budgets, which it has equation to execute. The calculation of NPV is the present value of cash flows minus the present value of cash outflows, which means the present value will be defined as the future amount of money.

NPV is important tool for capital expenditure evaluation. However, NPV is more likely utilized in finance or the economy. What if we use NPV for other parts? Not only on business?

3.1. The Applicability of NPV

NPV can not only be used on commercial activity, but it can also be used on other significant parts. As we can see, we can assume CFT to cash flow generated by innovation project. First, NPV shows the risk cash tomorrow is less valuable than a particular cash today. The future cash flows are discounted by discount rate year by year. The discount rate represents the opportunity cost of the capital utilized, which increases to threat of innovation opportunity by estimated riskiness [9]. However, the more risker projects can create large amounts of high returns by controlling risk, which can be risk-adjusted. Second, NPV can be counted for all the future net cash flows for connecting to the innovation opportunity. “The NPV approach requires, on the one hand, the discounting and summing-up of all the future net cash-flows for which reasonable assumption can be made, and on the other hand to estimate and discount the final value of the remaining cash flows (the “final” value)” [9]. At the same time, the NPV can also be utilized by studying Environmental studies. Environmental studies have numerous contained economic perspectives [10]. The environment is a seriously crucial concept related to all people living on
this earth. Environmental studies can use the net present value to have valuable benefits and the cost of projects to let researchers decide the most advantageous plan for protecting the environment. From the research of NPV, we can extend the concept to generate Environmental modeling studies based on NPV to predict the future decision-making on Environmental protection. For example, various studies have used NPV to research specific prices paid for carbon storage /sequestration is a crucial way to award people to restrict carbon emissions [11]. Otherwise, some studies use NPV to optimize land-use strategies for economizing land to avoid unnecessary waste by human activities. For example, if the biodiversity conservation strategy does not influence the business strategy, we can assume the strategy is well acceptable for economic activities to decision-makers [11]. The NPV helps people calculate the utilization rate of land to maximize business benefits on the premise of protecting the environment.

4. THE RESEARCH OF CASE STUDY

In this section, we consider the relationship between IRR and NPV; in doing so, we develop the analytical technique, which we will apply to some empirical data to find out the result.

Table 1. The cash flows of two multiplied projects.

| Cash Flow ($) | Project A | Project B |
|---------------|-----------|-----------|
| CF₀           | -430      | -430      |
| CF₁           | 230       | 70        |
| CF₂           | 179       | 138       |
| CF₃           | 124       | 240       |
| CF₄           | 94        | 260       |

First of all, we will try to study the “what is the IRR for each of these projects? Using the IRR decision rule, which project should the company accept?”. And we use the IRR decision rule to solve it. We calculated that the IRR of project A is 20.44%, the IRR of project B is 18.84%.

Table 2. The NPV of each project for the discount rate of 0, 10, 20 and 30%.

| IRR   | Project A | NPV(0) | NPV(20%) | NPV(30%) |
|-------|-----------|--------|----------|----------|
| 20.44%| A         | 84.6   | 3.1      | -57.9    |

For this figure, we can conclude that if our decision is just based on the IRR criterion, the result will be unreasonable.

When the discount rate is between 0% and 15.295%, we may choose project B; When the discount rate is 15.29%, both project A and project B are optional; When the discount rate is between 15.29% and 20.44%, we may choose project A; And if the discount rate is above 20.44%, it is not wise for the investor to select these two projects.

5. DISCUSSION

5.1. Advantage and Disadvantage of IRR

Financial managers use IRR for business and IRR is formed as a percentage, which can help us easily compared to the required cost of capital. In the meantime, it provides you excellent guidance on the project’s value and the associated advantage of knowing the actual returns of money. However, IRR tells you about accepting the project or investment when IRR is greater than the weighted average cost of capital, but the discount rate changes every year, which is not stable, so it is difficult to compare with. Besides, if there are two or more mutually exclusive projects, IRR will not be effective under multiple situations.

5.2. Advantage and Disadvantage of NPV

NPV shows the importance of the Time Value of Money. The value of today is more than the value of tomorrow. It is the tool of which we often use for most financial analysts by considering the time value of money to translate future cash flows into today's dollars. Project profitability and risk factors are given high priority. Then, NPV helps you to maximize your wealth, and it shows your return greater than your real cost or not. NPV considers the before and after of cash flow over the life span of a project. However, NPV cannot give you an accurate decision when two or more projects have unequal conditions. At the same time, NPV will not show you how long projects or investments will get positive NPV due to the calculation. NPV suggests you accept the investment plan, but it won't show you the accurate
answer time about what time you will achieve positive NPV to achieve profits back. At last, it is difficult to calculate the appropriate discount rate for cash flows.

6. CONCLUSION

The paper illustrates comparing NPV and IRR. We used the example of a question to solve each part, in which we solved four questions. First, we list the concept and equation of IRR to work on how IRR is doing investment or commercial projects. Also, we figure out the disadvantages of IRR to make the hypothesis of expanding the concept of IRR. Second, we discuss NPV and still use the same process as the IRR part. Thirdly, we introduce the question and give the solution of calculating of NPV and IRR in the case study by using what we show on the foregoing paper of NPV and IRR. In the end, we list their relative merits, and it can be seen on paper for easy comparison to distinguish on what situation we will be able to use which method, IRR or NPV.

Among practitioners, the NPV approach is considered more reliable for wealth maximization. However, the ease with which IRRs can be interpreted has made the IRR approach the more popular of the two ranking methods (Graham and Harvey, 2001; Scott and Petty, 1984). Both ranking methods have their drawbacks. Investments may have multiple IRRs, and NPV rankings may be unstable, which will vary with changes in the discount rate.

Inconsistent IRR and NPV investment rankings have been attributed to differences in implied reinvestment rates, initial investment sizes, and terms (Cary and Dunn, 1997). Another reason IRR and NPV rankings can be inconsistent is that the unequal cash flows cause unequal periodic investments.

For the IRR decision method, there are some confusions over the meaning of IRR, and for example, it does not answer the question, "What is the return on this investment?"

IRR has several drawbacks --problems of reinvestment of cash flows, multiple IRRs, and investment ranking and timing. Finally, most investment opportunities exhibit different discount rates for their initial investment, cash flows, and terminal values. Varying discount rates make IRR analysis extremely complex. These drawbacks make IRR a not so suitable tool for most decision-making. In calculating IRR, which produces confusing results, the authors have been ignoring or "avoiding" the scale. And it is directly altering cause the wrong effect in obtaining unreasonable project.

But through the study of the instance, the authors have theoretically defined the operationally acceptable solution to the problem of choosing a reasonable decision method, so the aim of the paper has been achieved, but there are still some problems that haven't been solved. For example, if there exists the multiply IRR, how can we make a reasonable decision? So, the authors will carry out further studies to try to make a more comprehensive conclusion.

Although the paper show lots of concepts of IRR and NPV, we still think it is not insufficient. The paper shows the process of question, which is more likely ideal and fundamental. The question seems to not relate to real situations. Moreover, the paper lacks data to show the more logical process and the different operating IRR or NPV. The paper needs to expand more reasonable conditions to research more precisely about IRR and NPV.

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