Erratum

Ultimate quantum bounds on mass measurements with a nano-mechanical resonator

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A mistake is present in the application of the general formulae (5), (6) for the smallest measurable mass to the case of a coherent state: The $\alpha^4$ term in eq. (10) should be absent. This changes the scaling with the average number of excitations. More precisely, the paragraph starting with “Coherent state.” in the middle of the left column of page 3 should read as follows:

“For a coherent state $|\alpha\rangle = \exp\left(-|\alpha|^2/2\right) \sum_{n=0}^{\infty} (\alpha^n/\sqrt{n}) |n\rangle$, and $\alpha \in \mathbb{R}$, we have

$$f = -\left(\frac{1}{2} + \alpha^2\right) \sin^2 \tau - \alpha^2 \tau (\tau + \sin(2\tau)).$$

(10)

For fixed $\tau \gg 1$ and $\alpha^2 \gg 1$, we find $f \approx -\alpha^2 \tau^2$, and hence $\delta M_{\min}/M \approx 1/(\sqrt{N} \alpha \tau) = 1/(\sqrt{\langle n \rangle} \tau)$. Thus, for a coherent state, the sensitivity scales as the inverse square root of the average number of excitations in the oscillator, and one can, at least in principle, resolve arbitrarily small masses. We see again that $\delta M_{\min}$ reduces faster than $1/\sqrt{t}$ with measurement time. Compared to a Fock state with $n = \alpha^2$, there is a factor $\sqrt{\langle n \rangle}/2$ penalty in the scaling with $\langle n \rangle$ (but one gains a factor $1/\tau$). For $\alpha = 0$ we find $f = -\sin^2 \tau/2$ which leads back to the result for the Fock state $n = 0$.

The sentence “The analysis shows…” in the “Conclusions” section should read “The analysis shows that a coherent state allows one to achieve a $\delta M_{\min}/M$ that scales for fixed measurement time as the inverse square root of the average number of excitations.”

Fig. 1: (Colour on-line) Inverse minimal measurable mass $M/\delta M_{\min}$ (for $N = 1$) as a function of $\tau$ for selected pure states. Same symbols as in the original figure.

There was also a problem in the numerical optimization for small values of $\tau$. As a consequence, the sentence starting with “At $\tau = \pi/2$, the optimal pure state…” on the bottom right of page 3 should read: “At $\tau = \pi/2$, the optimal pure state with $L = 3$ allows still a reduction of $\delta M_{\min}/M$ by $\sim 3\%$ compared to $\psi_{ON}$, and by $\sim 18\%$ compared to the Fock state with the same $L$. The optimal state $\psi_{\text{opt}}$ quoted on the left top of page 4 should be
replaced by $|\psi_{\text{opt}}\rangle \simeq (-0.62057 + 0.0305i)|0\rangle - (0.00059 + 0.01198i)|2\rangle + (0.78252 - 0.038852i)|4\rangle$. Its Wigner function looks indistinguishable from the one that was plotted in fig. 2.

None of the quantitative results quoted on the left of page 4 or in the “Conclusions” section for realistic micromachined resonators or carbon nanotube resonators are changed, as these are all in the regime with $\tau \gg \alpha$, where the scaling with time dominated the smallest measurable mass.