A New User Grouping and Power Allocation Scheme for Downlink Non-orthogonal Multiple Access Systems

Yin Wang*, Xiaohui Li, Wenwu Wang, Gongquan Zhang and Hongwei Zhang

Key Laboratory of Intelligent Computing and Signal Processing of Ministry of Education, Anhui University, Hefei 230039, China

*Email: wangyin950203@163.com

Abstract. In this paper, we propose a new user grouping and power allocation scheme based on beamforming for downlink non-orthogonal multiple access systems. The proposed user grouping scheme can effectively reduce the interference from the other user and other beams as well, and can effectively improve the weak user rate especially when the SNR is not large. In addition, a power allocation scheme that can maximize the sum capacity while satisfying a certain fairness index is proposed. From the simulation results, the user grouping and power allocation method proposed in this paper can not only improve the overall system throughput performance, but also improve the fairness of weak users.

1. Introduction
Due to the increasing number of communication devices, there is an increasing demand for various applications of wireless networks. 5G requires higher spectral efficiency, faster rate and larger capacity. Non-orthogonal multiple access technology is widely studied in 5G because of its fast rate and higher spectrum efficiency [1]-[3]. Compared to existing techniques, the NOMA technology distinguished by power dimension. In a NOMA system, Superposed signals from multiple users are sent by one base station. The user with good channel condition allocates a smaller power, and the user with better channel condition allocates a larger power.

Combining NOMA with MIMO technology and adopting zero-forcing beam precoding is a hot topic in recent years. Since the receiver uses successive interference cancellation technology, it can support multiple users. The number of receive antennas in the NOMA downlink model is often larger than the number of transmit antennas, which may cause the target user to be interfered by other users. We can divide the total number of user antennas into several different groups. The interference generated between different groups can be eliminated by beamforming techniques. Each cluster contains multiple users, that is one beam supports multiple users, which inevitably creates interference. Therefore, in order to reduce interference and increase the achievable rate as much as possible, reasonable clustering and power allocation methods are indispensable.

In recent years, many researchers have done a lot of work on user grouping and power allocation. In [4], a method of joint user clustering and dynamic power allocation was proposed, the scheme maximize the energy efficiency of the MIMO-NOMA system downlink. In [5], a joint clustering and precoding algorithm based on the eigenspace of the channel matrix was proposed, which can eliminate interference between groups and enhance the total capacity of the MIMO-NOMA system. The authors in [6] propose a user grouping algorithm with large SNR, and a power allocation method to maximize the sum rate when the minimum target rate is met. The authors in [7] performed a proportional fair
based user grouping and power allocation method. Some of the above schemes do not consider the impact on user grouping when the value of SNR is very small, and some do not consider the fairness between users. In summary, this paper proposes a clustering method that selects strong users and weak users separately to improve sum capacity and to guarantee the fairness of the weak users.

The rest of this paper is organized as follows. The section II introduces our system model. The section III presents the user grouping and power allocation scheme. Numerical results and analyses are given in section IV. The section V is the summary of this paper. The symbol used are defined as follows, \( (\cdot)^T \), \( (\cdot)^* \), \( (\cdot)^\dagger \), \( E(\cdot) \) denotes matrix transpose, matrix conjugate transpose, matrix pseudo inverse, average value respectively.

2. System model
As shown in Figure 1, we considers a downlink multi-user MIMO-NOMA system. The base station is equipped with \( N \) antennas, and there are a total of \( K \) users in a cell. And each receiver have a single antenna. Different from the traditional MIMO-OMA technology, in order to increase the number of service users, each transmit antenna can support two or more users. In this case, up to \( N \) user groups are scheduled at the same time. For simplicity, we assume that there are two users in a cluster, and \( N \) beams can support \( 2N \) users. For two users in the same cluster, the user with large channel gain is called strong user, and the user with small channel gain is called weak user.

![Figure 1. The proposed multi-user MIMO-NOMA system model](image)

According to the above, the superposed signals of two users at the \( i \)-th cluster are expressed as

\[
x_i = \sqrt{\alpha_{i,1} P_{i,s,1}} + \sqrt{\alpha_{i,2} P_{i,s,2}}
\]

Where \( E\{[\mathbf{h} \mathbf{j}]^2\} = 1 \) for \( j = 1, 2 \). \( P_i \) is transmission power of the \( i \)-th cluster. \( \alpha_{i,1} \) and \( \alpha_{i,2} \) are power allocation coefficients assigned to strong users and weak users respectively, and satisfy \( \alpha_{i,1} + \alpha_{i,2} = 1 \). In the \( n \)-th cluster, \( y_{n,i} \) indicates the received signals of two users, given by

\[
y_{n,i} = h_{n,i} \sum_{k=1}^{N} w_k x_k + n_{n,i} \quad i = 1, 2
\]

Where \( h_{n,i} \) is the \( 1 \times N \) channel vector, which represents the channel coefficient of the \( i \)-th user of the \( n \)-th cluster. \( w_k \) is the \( N \times 1 \) precoding vector, and \( x_k \) is a superposed signal supported by a beam mentioned above, and \( n_{n,i} \) is the additive complex Gaussian white noise with zero mean and a variance of \( \sigma_n^2 \). In this paper, we assume that the channel is a Rayleigh fading channel with zero mean and unit variance, and the transmitter knows the full channel state information. In this paper, the ZFBF is used for the precoding scheme. The BF vector \( w_n \) is calculated by combining all the strong user channels \( h_m \) in the user set, and satisfies the following relationship:
\[ \frac{h_n}{|h_n|} W_n = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases} \]  

(3)

We set the channel matrix H to consist of strong user channel set, as follows:

\[ H = [h_{1,1}^T ... h_{N,1}^T]^T \]  

(4)

The precoding matrix W is obtained by

\[ W = \left[w_1 ... w_N \right] = \left(H \right)^T \left(\left(H\right)^T \left(H\right)^*\right)^{-1} \]  

(5)

Where \( w_n \) represents the precoding vector of the n-th cluster.

For a specific cluster, taking the n-th cluster as an example, the received signal of the strong user of the n-th cluster given by

\[ y_{n,1} = h_{n,1} W_n \sqrt{\alpha_{n,1} P_n} s_{n,1} + h_{n,1} W_n \sqrt{\alpha_{n,2} P_n} s_{n,2} + h_{n,1} \sum_{k=1, k \neq n}^N W_k x_k + n_{n,1} \]  

(6)

Where \( h_{n,1} \sqrt{\alpha_{n,1} P_n} s_{n,1} \) is the desired signal, \( h_{n,1} \sqrt{\alpha_{n,2} P_n} s_{n,2} \) is the interference from the other user, \( \sum_{k=1, k \neq n}^N W_k x_k \) is the interference from the other clusters to the n-th cluster. Since the SIC is used at the receiver, the interference of weak users to strong users can be eliminated by the SIC. We choose to generate BF vectors with strong user channels. According to the equation (3), it is known that \( h_{n,1} W_k = 0 \) for \( 1 < k < N \) and \( k \neq n \), so the inter-cluster interference can also be eliminated. The rate achieved by the n-th cluster of strong user given by

\[ R_{n,1} = \log_2 \left(1 + \frac{|h_{n,1} W_n|^2 \alpha_{n,1} P_n}{\sigma_n^2} \right) = \log_2 \left(1 + \frac{|h_{n,1}|^2 \alpha_{n,1} P_n}{\sigma_n^2} \right) \]  

(7)

The received signal of the weak user is expressed as follows:

\[ y_{n,2} = h_{n,2} \sqrt{\alpha_{n,2} P_n} s_{n,2} + h_{n,2} \sqrt{\alpha_{n,1} P_n} s_{n,1} + h_{n,2} \sum_{k=1, k \neq n}^N W_k x_k + n_{n,2} \]  

(8)

Because weak user do not satisfy equation (3) and cannot use SIC to eliminate the interference from the other user, weak user received interference from the other user and other beams. The achievable rate of the weak user is given by

\[ R_{n,2} = \log_2 \left(1 + \frac{|h_{n,2} W_n|^2 \alpha_{n,2} P_n}{|h_{n,2} W_n|^2 \alpha_{n,2} P_n + |h_{n,2} W_n|^2 \alpha_{n,1} P_n + \sum_{k=1, k \neq n}^N W_k x_k|^2 + \sigma_n^2} \right) \]  

(9)

We assume the transmit power of the base station transmitter to each cluster to be \( P_n \).

3. Proposed user clustering and power allocation

In this section, a user clustering and power allocation scheme will be proposed. The simulation results show that the scheme has good performance in the maximum sum capacity and the fairness of weak users.

3.1. User clustering

The proposed user clustering scheme aims to reduce interference from the other user and other beams to maximize sum rate. In this paper, we select N strong users from K total users by using the SUS algorithm in [8], and then assign a weak user to each of the N clusters from K-N remaining users. Select weak users to meet:
We select users whose channel correlation with strong user are greater than $\rho$ as weak users. When $\beta$ is not large, $\frac{1}{\beta |h_{n,2w_n}|^2}$ has a significant impact on the SINR of weak users. Therefore, the user with the largest $|h_{n,2w_n}|^2$ value is selected as the weak user of the cluster among the users whose correlation is greater than $\rho$. The specific user clustering scheme steps are as follows:

**Step 1** The set of total user $T = \{1,\ldots,K\}$, and $N$ strong users are selected into the set $S$ according to the SUS algorithm, $W$ represents a set of weak users, and $R$ represents a user remaining after picking out $N$ strong users from the total user set.

**Step 2** Initialize $U = \emptyset$, $i = 1$. For each user $k \in R$ in the set $R$, first calculate the correlation between all users in the $R$ set and corresponding strong users:

$$\text{Corr}_{(i,k)} = \left\{ \frac{|h_{i,k}h_k^H|}{\|h_{i,k}\|\|h_k\|} \right\}$$

When $\text{Corr}_{(i,k)} > \rho$, $U \leftarrow U \cup h_k$. If the user in the set $R$ has calculated and $U \neq \emptyset$, and then go to Step 3; if the user in the set $R$ has calculated and $U = \emptyset$, then

$$\pi(k) = \arg \max \left\{ \frac{|h_{i,k}h_k^H|}{\|h_{i,k}\|\|h_k\|} \right\}, W \leftarrow W \cup \pi(k), h_i, 2 = h_{\pi(k)}, i = i + 1$$

If $|W| < N$, repeat to Step 2, otherwise the user clustering scheme ends.

**Step 3** Let $j = 1$, $j \in U$.

$$\pi(j) = \arg \max \left\{ |h_{jw}|^2 \right\}$$

$W \leftarrow W \cup \{ \pi(j) \}$

$$h_{i,2} = h_{\pi(j)}$$

$i = i + 1$

If $|W| < N$, return to Step 2, otherwise the user clustering scheme ends.

### 3.2. Power allocation

After all users have completed the corresponding user grouping, the next step should be to allocate power to each user in the cluster, and maximize the sum rate of the cluster subject to that the fairness
index achieve $p$.

Since there are two users in a cluster, we define the fairness index as [9]:

$$F = \frac{\left( \sum_{i=1}^{2} R_i \right)^2}{2 \sum_{i=2}^{2} R_i^2}$$

which indicates the fairness of the system capacity of strong user and weak user in each cluster. The capacity for strong user and weak user gets close to each other when $F$ gets close to 1.

The proposed power allocation scheme aims to maximize the cluster sum capacity under a fairness constraint. The optimal allocation problem can be formulated as

$$\max_{\alpha_{n.1}, \alpha_{n.2}} (R_{n.1} + R_{n.2})$$

s.t. $F \geq p$, $\alpha_{n.1} + \alpha_{n.2} = 1$

$p$ is a predefined adaptive parameter. We obtain the power allocation coefficients for two users in a cluster through exhaustive search. If none of the power allocation coefficients meet the fairness constraint, then assign all of the power to strong user.

4. Simulation results and analysis
The simulation diagrams below in this paper all take $K=100$ users and the number of antennas $N=2$ as an example. The noise energy $\sigma^2 = 1$, and the channel is a Rayleigh fading channel with zero mean and unit variance. Figure 2 is a simulation comparison of the correlations in the proposed user clustering scheme. It can be seen from the simulation results that the sum capacity is the highest when the correlation value is 0.95, so we all simulated with the correlation value of 0.95 in the following simulation experiments.

![Figure 2](attachment:fig2.png)  
**Figure 2.** The relationship between the sum capacity and SNR of three different correlations at the fairness index $p = 0.8$  

![Figure 3](attachment:fig3.png)  
**Figure 3.** Capacity obtained by strong users and weak users with different fairness indexes

Figure 3 is a graph showing the relationship between weak users, strong users, and sum capacity and fairness index when SNR = 10 dB. It can be seen from the figure that the average achievable rate of weak users is almost zero when the fairness index is less than 0.5. When the fairness index is greater than 0.5, the average achievable rate of weak users increases with the increase of the fairness index and decreases with strong users. As the fairness index increases, the sum capacity decreases slightly.
The authors in [10] performed user clustering according to the correlation of the user channel and the channel difference, and proposes a power allocation method that maximizes total system capacity while keeping the capacity of weak users equal to or greater than the capacity of a traditional multi-user BF system. When the fairness index is set to 0.8, it can be seen from Figure 4 that the sum capacity of the proposed scheme is greater than that in [10], especially when the SNR is small. This is because the smaller the SNR, the greater the impact of $|h_{n, 2w}|^2$ on the weak user rate, and the proposed user clustering scheme can increase the weak user rate and thus increase the sum capacity. When the SNR is close to 20 dB, the scheme of the correlation of 0.75 in [10] is closer to the sum capacity of the proposed scheme. This is because the effect of $|h_{n, 2w}|^2$ on the weak user rate is negligible as the SNR increases. But overall, the proposed scheme is slightly better.

The relationship between the weak user capacity and SNR is shown in Figure 5. When the fairness index is set to 0.8, it can be seen from Figure 5 that the sum capacity of weak users in proposed scheme is significantly larger than that in [10], especially when the SNR is not large. In addition, when $\rho$ is high, the inter-cluster interference is reduced, thus enhancing the weak user’s capacity.

The relationship between the fairness index and SNR is shown in Figure 6. From Figure 6, the fairness index of the proposed scheme is 0.8, and the fairness index with the correlation of 0.95 in [10] is close to the maximum at 18 dB. The maximum value of the fairness index with the correlation of
0.75 in [10] is also less than 0.7 with the increase of SNR. It is obvious that the fairness index of the proposed scheme is better than that of the scheme in [10]. This means that the fairness of weak users is guaranteed by the proposed power allocation scheme.

In summary, it can be seen that through the comparison of the above three simulation diagrams, the proposed scheme is superior to the scheme proposed in [10] whether sum capacity or weak user rate and fairness.

5. Conclusion
This paper proposed a user clustering and power allocation scheme for downlink multi-user MIMO-NOMA system. The proposed user grouping scheme not only effectively reduces the intra-cluster interference and inter-cluster interference, but also takes into account the impact on the weak user capacity when the SNR is not large, and increases the capacity of the weak user. The proposed power allocation scheme is based on the optimal power coefficient allocation of the fairness index, which can maximize sum capacity while guaranteeing certain fairness. The simulation diagram proves that compared with the existing methods, the scheme improves system capacity, weak user capacity and fairness.

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