ACDM as image of open de Sitter, 
and how it predicts cosmological parameters and the H0 tension

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The de Sitter universe presents a cosmological model with many favorable properties. It follows as the unique cosmological solution from conservation of energy within the apparent horizon, with Minkowski spacetime as its local limit. Employing Painlevé-de Sitter coordinates, the freely falling observer in de Sitter observes flat space and gravitational curvature of time, instead of spatial curvature and flat time in FLRW-coordinates. Open de Sitter can be mapped onto a flat ACDM model by a coordinate transformation. This involves only two numbers: $\alpha = 2$ to represent the time dilation between the observer and the de Sitter horizon, and $\beta = 2/3$ in the transformation of curvature energy to matter energy (dust). Assuming identical time coordinates of the two models gives a unique solution with an exact density of dust $\Omega_m = [1 + \sinh^2(\frac{1}{3}\sinh(1))]^{-1} = 0.3178...$, which agrees with the Planck 2018 estimate $\Omega_{\text{Planck}} = 0.315 \pm 0.007$. The same mapping shows a ratio 0.8926 between the Hubble constants of both models, which agrees with the ratio $h_{\text{Planck}}/h_{\text{LOC}} = 0.9104 \pm 0.0247$ between the ACDM model dependent Planck 2018 estimate and the model independent local estimate by Riess et al., and so suggests a possible origin of this H0 tension. The local estimate of the Hubble constant is further shown to be in agreement with a decomposition of nonlocal energy in open de Sitter, so that $h_{\text{dS}}$ can be derived from the Planck estimate of the physical density $\Omega_h h^2$, yielding $h_{\text{dS}} = 0.7332 \pm 0.0016$, which matches $h_{\text{LOC}} = 0.7403 \pm 0.0142$. The dual character of the de Sitter universe (evolutionary vs. static) is considered in light of these results.

Since FLRW spacetimes are non-stationary, it is often argued that energy can not be conserved in the expanding Universe. While this indeed applies to most universe models, it is not generally the case. As Florides [1] showed, there are exactly six FLRW spacetimes (the ones with constant curvature) which have a static representation, namely: Minkowski, Milne, de Sitter (open, flat and closed), and anti-de Sitter. The common Friedmann form of these spacetimes is

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{\Lambda}{3} - Ka^{-2}. \quad (1)$$

Of these six, the flat de Sitter universe is cosmologically highly relevant in the early (inflating) and late universe, and vacuum energy within the de Sitter event horizon is indeed conserved. Thus conservation of global energy is not totally alien to cosmology. Moreover, these same six spacetimes show consistency between the three different forms of redshift [2], while this is still subject of controversy with FLRW spacetimes in general.

The spacetimes of Eq.(1), however, are all considered to be empty of matter, therefore may not seem viable cosmological models of the matter dominated era. We shall argue that this emptiness is only apparent; specifically, that open de Sitter can accommodate a flat ACDM model through transformation of curvature energy into (the form of) matter energy of dust. Hence, what normally is considered local energy of matter is instead identified with the nonlocal gravitational energy of the cosmic matter. Nonlocal energy is coordinate dependent, thus may appear in different forms and in different densities.

The apparent horizon is closely related to a notion of global energy conservation. Its radius is at the physical distance $R_a$ where inward directed photons momentarily are stationary relative to the observer at the origin. This means that sources presently beyond this distance are disconnected and can not gravitationally interact with a particle at the origin. The radius of gravitational interaction is therefore constrained by the apparent horizon, which satisfies the relation $(c = 1)$ [3],

$$H^2 = R_a^{-2} - Ka^{-2}. \quad (2)$$

This equation is quite similar to Eq.(1). If both hold, then it follows that $R_a^{-2} = \frac{\Lambda}{3}$, so that $R_a$ coincides with the constant radius $H^{-1}_\Lambda$ of the de Sitter event horizon. The Gaussian curvature of de Sitter therefore is $K = kR_a^{-2}$. Thus the equations combined give as only cosmological solution the de Sitter universe

$$H^2 = H^2_\Lambda(1 - ka^{-2}), \quad k = -1, 0, 1. \quad (3)$$

The de Sitter universe has many favorable properties. Apart from conservation of energy and consistent forms of redshift, the cosmological constant in de Sitter can be identified with the conserved total energy of recession within the horizon [4]. Also, open de Sitter has a diverging particle horizon at initial time, providing causal connection of all particles, thus preventing a horizon problem. It does not suffer from the coincidence problem either, since the equality of present densities, $\Omega_\Lambda = \Omega_m$, holds at any instance of present time, which can be related to the equality of recessional and peculiar energy within the apparent horizon, as discussed in section IV.

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I. GRAVITATIONAL TIME DILATION AT THE DE SITTER HORIZON

The open de Sitter model in Eq.(3) resembles the basic $\Lambda$CDM model, but is spatially curved, while the Universe appears spatially flat. Even so, gravitational energy is coordinate dependent, so that spatial curvature of open de Sitter in the constant time slices of comoving coordinates may appear differently in other coordinates. After all, one does not observe in constant time slices, but on the past light cone. While, at the large scale, the expanding Universe is considered homogeneous in constant time slices, one expects the matter density to be inhomogeneous on the light cone, which gives rise to curvature of space and/or of time, depending on coordinates chosen. This is illustrated by the static de Sitter metric in the Painlevé-de Sitter coordinates [5] of a freely falling observer,

$$ds^2 = (1 - \frac{\dot{R}^2}{R^2})dt^2 + 2\frac{\dot{R}}{R}dt dR - dR^2 - R^2 d\Omega^2, \quad (4)$$

where time is curved by gravitational time dilation, but space is flat in constant time slices. This is relevant since our cosmological observations are made in free fall, and essentially at a single point in time, like a snapshot. Thus, the comoving observer in de Sitter may conclude that space is flat and time is curved.

The metric shows that at $R = 0$ the cosmological constant $\frac{\dot{R}^2}{R^2}$ vanishes, and one obtains Minkowski as the local spacetime of the comoving observer, so that $t$ represents comoving time, and the speed of light is $c = \frac{dR}{dt}|_{R=0} = \pm 1$, as expected. According to the metric (4), due to curvature of time, one finds at the horizon a coordinate speed of light of outward directed photons of $c' = \frac{dR}{dt}|_{R=R_\Lambda} = 2$. In flat space this means that the comoving time at the horizon, as observed from $R = 0$, is gravitationally dilated and is represented by $dt' = \frac{1}{2} dt$, which agrees with a redshift $z = 1$ at the de Sitter horizon. The freely falling observer in de Sitter therefore finds a ratio of $\alpha = 2$ between local time $dt$ and observed horizon time $dt'$. Since this ratio is fixed in de Sitter, it follows that, at any time,

$$\alpha \equiv \frac{dt}{dt'} = \frac{t}{t'} = 2. \quad (5)$$

One therefore has $da/\dot{a} = 2da/dt$, so that (like the speed of light) the Hubble parameter at the horizon appears 2 times larger due to gravitational time dilation. This suggest that the cosmic energy density perceived by the observer, $\rho' = a^{-2}da^2/dt'^2$, is 4 times larger than the local energy density $\rho = a^{-2}da^2/dt^2$ in terms of comoving time $t$, i.e.,

$$\rho' = \alpha^2 \rho. \quad (6)$$

By the definition of the apparent horizon, the energy of inward directed null particles at the horizon is zero, while outward directed null particles travel at $c' = 2c$ in the frame of the observer, which is the sum of recessional and peculiar speed of the null particle at the horizon. This provides physical motivation of the factor $\alpha = 2$ for outward directed radiation. The observational relevance of outward directed photons can be understood in the sense that whatever radiates away from the horizon now is exactly the mirror image of what is being received at present by the observer from all directions, according to (infinitely many) comoving observers which are momentarily crossing the horizon in all directions. The factor $\alpha = 2$ is indirectly confirmed hereafter by observation, in two independent ways.

II. TRANSFORMATION OF OPEN DE SITTER TO $\Lambda$CDM

Application of the open de Sitter model to cosmological probes may get involved, as curvature of time rather disturbs the physics of standard cosmology. Another issue is that many probes assume a cosmology, usually $\Lambda$CDM, making certain estimates model dependent, which complicates comparison. The tension between direct and model dependent (CMB) estimates of $H_0$ may be an example of this [6, 7], as discussed in the next section.

An attractive alternative way to show observational validity of open de Sitter is the transformation of this model to the basic $\Lambda$CDM model with cosmological constant and dust term. This basically concerns substitution of the expansion coordinate, i.e., of the scale factor, so that $a^{-2} = \tilde{a}^{-3}$, which transforms the nonlocal curvature energy into (the form of) the local matter energy of dust, so that the two are mathematically equivalent. What makes the transformation particularly interesting is that open de Sitter has only one free parameter ($H_\Lambda$); the present densities are fixed, $\Omega_m = \Omega_k = \frac{1}{2}$. This means that transformation returns a specific value of the $\Lambda$CDM densities $\bar{\Omega}_m (1 - \tilde{\Omega}_\Lambda)$, which can be compared with reported estimates, as follows.

The Friedmann equation of the observational open de Sitter model Eq.(3), evaluated on the light cone at the horizon, where $t' = \alpha^{-1} t = \frac{1}{2} t$, can be written

$$H'^2 \equiv \frac{\dot{a}^2}{a^2 d\tau^2} = H_\Lambda^2 (1 + A' a^{-2}), \quad (7)$$

where the density ratio $A' \equiv \frac{\Omega_\Lambda}{\Omega_m} = 1$. It has the solution

$$a(t') = \sinh(H_\Lambda' t'). \quad (8)$$

The age $t'_0$ of this open de Sitter universe follows from the condition $a(t'_0) = 1$,

$$t'_0 = H_\Lambda'^{-1} \mathrm{asinh}(1). \quad (9)$$

The transformation of the open de Sitter model to $\Lambda$CDM
primarily regards a substitution of the scale factor, i.e.,
\begin{equation}
a \to \hat{a} = a^\beta,
\end{equation}
where $\beta = \frac{2}{3}$ (the hat denotes $\Lambda$CDM symbols). This substitution has some implications. It follows that
\begin{equation}
\frac{da}{a} = \beta^{-1} \frac{d\hat{a}}{\hat{a}}.
\end{equation}
so that the transformation $a \to \hat{a}$ causes the Hubble parameter $H'(t')$ to drop by a factor $\beta$ (when expressed in the same time coordinate). Since $H'_\Lambda$ is the asymptotic state of $H'$, scaling of $H'_\Lambda$ implies equal scaling of $H'_\Lambda$, i.e., $H'_\Lambda \to \hat{H}'_\Lambda = \beta H'_\Lambda$. At the same time, $H'_\Lambda = R^{-1} \frac{dR}{dt'} = R^{-1} \frac{dR}{dt} \mid_{R=1}$. Scaling of $H'_\Lambda$, therefore necessarily implies a scaling of the time coordinate, i.e., $t' \to \hat{t}' = \beta^{-1} t'$. Finally, for invariance of both the speed of light and total energy, scaling of the time coordinate demands in turn equal scaling of the radial coordinate, $R \to \hat{R} = \beta^{-1} R$. Although scaling of $R$ does not change the Friedmann equation, it does affect total energy. That is, $R \to \hat{R} = \beta^{-1} R$, so that total energy (per unit mass) $E' = H'^2 R'^2 = H'^2 R^2 = E$ is preserved.

This means, at least for the class of Friedmann-Lemaître models with cosmological constant and a single matter component, that different, but physically consistent representations of the same spacetime have their own set of (relative) coordinates. One is free to ignore this, but at the expense of arriving at seemingly incompatible results, like different total energy, different densities, and different ages, all of the same Universe. In contrast, consistency between open de Sitter and $\Lambda$CDM can be illustrated when coordinates transform according to the above as $(a, t', R) \to (\hat{a}, \hat{t}', \hat{R}) = (a^\beta, \beta^{-1} t', \beta^{-1} R)$, so that
\begin{equation}
\frac{d\hat{a}}{a} = \beta^{-2} \frac{da}{a} = \hat{H}'_\Lambda^2 (1 + \hat{A}' \hat{a}^{-3}),
\end{equation}
with solution
\begin{equation}
\hat{a}(\hat{t}') = \hat{A}'^{\frac{1}{3}} \sinh^{\frac{2}{3}}(\hat{H}'_\Lambda \hat{t}').
\end{equation}
The density ratio $\hat{A}'$ follows from the condition $\hat{a}(t'_0) = 1$. Since $H'_\Lambda t'_0 = H'_\Lambda t''_0$, the ratio is unchanged, $\hat{A}' = \hat{\Omega}_m / \hat{\Omega}_\Lambda = \sinh^{-2}[\beta \sinh(1)] = 1$, but now relates to dust. Total energy and the age of the universe $(\hat{t}'_0 = \frac{2}{3} t''_0)$ are preserved too. Thus by the above transformation one obtains equivalent models, yet with different coordinates.

The interesting question is what happens if the observer instead uses a single set of coordinates to probe models with different $\beta$. The effect of this is seen if one replaces $\hat{t}'$ by $t'$ in the $\Lambda$CDM model, so that solution Eq.(13) becomes
\begin{equation}
\hat{a}(t') = \hat{A}'^{\frac{1}{3}} \sinh^{\frac{2}{3}}(\hat{H}'_\Lambda t').
\end{equation}
Now the density ratio equals
\begin{equation}
\frac{\hat{\Omega}_m}{\frac{\hat{\Omega}_m}{\hat{\Omega}_\Lambda}} = \frac{\sinh^{-2}[\beta \sinh(1)]}{\beta^2} = 2.5849.
\end{equation}
The high value of $\hat{A}'$ is a reflection of the replacement of the time coordinate by a slower time coordinate $\hat{t}' \to t'$. The value itself, however, is not representative since the observer uses comoving time $t$ in the local coordinate system, so that representative values of the $\Lambda$CDM model are obtained after transformation of solution Eq.(14) to the local time coordinate, $t' \to t = \alpha t = 2t'$, i.e.,
\begin{equation}
\hat{a}(t) = \hat{A}'^{\frac{1}{3}} \sinh^{\frac{2}{3}}(\beta H'_\Lambda t) = \hat{A}'^{\frac{1}{3}} \sinh^{\frac{2}{3}}(\beta H'_\Lambda \alpha t'),
\end{equation}
where $\alpha = \frac{4}{3}$, so that the density ratio becomes
\begin{equation}
\hat{A} = \frac{\hat{\Omega}_m}{\hat{\Omega}_\Lambda} = \sinh^{-2}[\beta \sinh(1)] = 0.4659\ldots
\end{equation}
For $\hat{\Omega}_m = 1 - \hat{\Omega}_\Lambda$, this gives an analytical expression of the matter density
\begin{equation}
\hat{\Omega}_m = \frac{1}{1 + \sinh^{-2}[\beta \sinh(1)]} = 0.3178\ldots
\end{equation}
This accurately matches the Planck estimate $\Omega_m^{\text{Planck}} = 0.315 \pm 0.007$ [6], so indirectly confirms the observational validity of the open de Sitter model, including the factor $\alpha = 2$.

### III. H0 TENSION

The $\Lambda$CDM solution in Eq.(14) coincides with the open de Sitter solution
\begin{equation}
a(t') = \hat{a}'^{\frac{1}{3}}(t') = \hat{A}'^{\frac{1}{3}} \sinh^{\frac{2}{3}}(\frac{2}{3} H'_\Lambda t'),
\end{equation}
which gives a biased, $\Lambda$CDM model dependent value of the Hubble constant in an open de Sitter universe, i.e.,
\begin{equation}
H_0^{\text{ACDM}} = \frac{2}{3} H'_\Lambda \sqrt{1 + \hat{A}'} = H'_\Lambda 1.2623\ldots
\end{equation}
In contrast, local observation of the Hubble constant in the open de Sitter universe of Eq.(7) would return $H_0^{\text{ACDM}} = H'_\Lambda \sqrt{2}$, and hence
\begin{equation}
\frac{H_0^{\text{ACDM}}}{H_0^{\text{ACDM}}} = \frac{H'_\Lambda}{H'_\Lambda} = \frac{2}{3} \frac{1 + \hat{A}'}{\sqrt{2}} = 0.8926\ldots
\end{equation}
This analytical ratio matches within confidence limits the ratio of estimated Hubble constants from CMB anisotropy by the Planck collaboration [6], and from local HST observation by Riess et al. [7], i.e.,
\begin{equation}
\frac{\hbar_\text{Planck}}{\hbar_\text{LOC}} = \frac{0.674 \pm 0.005}{0.7403 \pm 0.0142} = 0.9104 \pm 0.0247.
\end{equation}
The mapping between open de Sitter and ΛCDM may therefore explain the origin and magnitude of the H0 tension. The result further shows that the model independent local H0 measurement may be associated with the open de Sitter universe.

IV. DECOMPOSITION OF ENERGY IN OPEN DE SITTER

There is another, simple way that leads to the same conclusion, based on [8], and briefly summarized as follows. The cosmic energy density ρ can be interpreted as density of the nonlocal kinetic energy associated with a particle of Newtonian mass m, at rest in the Hubble flow. Even while at rest, the particle is in relative motion with respect to all connected particles within the apparent horizon, which are in both recessional and peculiar motion. Within this relational framework it turns out that the total nonlocal kinetic energy associated with m, due to peculiar motion of connected particles in 3 directions, represents a nonlocal mass of 3m. Peculiar and recessional kinetic energy balance with the same potential energy (since due to the same connected particles), so that peculiar and recessional energy are equal, and both represent a nonlocal mass of 3m. Hence, the total nonlocal mass associated with m equals 6m, of which we locally experience only 1/6 part, i.e., the Newtonian mass in the peculiar motion of the particle in some (single) direction. This means that the Newtonian baryon density Ωb effectively represents a nonlocal mass density Ωb,eff = 6Ωb. If one also takes into account the factor α = 2 due to gravitational time dilation in de Sitter, which gives rise to an apparent density ρ′ = α2ρ, then this gives, in a purely baryonic Universe, an observed total density Ωtot = (6 × 4)Ωb = 1, that is, 24 times the Newtonian baryon density Ωb. Therefore

$$\Omega_b = \frac{1}{24} = 0.041666\ldots$$

(22)

This value can be validated using the physical density estimate Ωb,h^2 = 0.0224 ± 0.0001 from CMB anisotropy (Planck 2018 [6]), which is extremely accurate and agrees with independent BBN estimates, so can be considered fiducial. With the analytical value Ωb = 1/24, the Planck estimate Ωb,h^2 gives an equally tight estimate of the Hub-

V. DUALITY OF DE SITTER: STATIC VS. EVOLVING

The de Sitter cosmology shows several distinctive characteristics. Total recessional energy within the constant volume of the apparent horizon of de Sitter is conserved [4], thus appears as cosmological constant ΩΛ. Peculiar (i.e., curvature) energy density is more complicated: it vanishes as matter dilutes, which is the evolutionary picture of the expanding open de Sitter universe as observed on the light cone from a fixed point in time. In contrast, the present density of curvature energy equals Ω′k = Ω′k = 1/2 at any instance of present time, so is locally invariant in the comoving frame. These seemingly contradictory pictures present the duality of the de Sitter universe: evolving while being static. This must be attributed to the gravitational time dilation in de Sitter, which continuously renormalizes the perceived present energy level, so that locally it appears as if nothing changes, as one expects in Minkowski spacetime.

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