Probing the indefinite CP nature of the Higgs Boson through decay distributions in the process $e^+e^- \rightarrow t\bar{t}\Phi$

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The recently discovered scalar resonance at the LHC is now almost confirmed to be a Higgs Boson, whose CP properties are yet to be established. At the ILC with and without polarized beams, it may be possible to probe these properties at high precision. In this work, we study the possibility of probing departures from the pure CP-even case, by using the decay distributions in the process $e^+e^- \rightarrow t\bar{t}\Phi$, with $\Phi$ mainly decaying into a $b\bar{b}$ pair. We have compared the case of a minimal extension of the SM case (Model I) with an additional pseudoscalar degree of freedom, with a more realistic case namely the CP-violating Two-Higgs Doublet Model (Model II) that permits a more general description of the couplings. We have considered the ILC with $\sqrt{s} = 800$ GeV and integrated luminosity of 300 fb$^{-1}$. Our main findings are that even in the case of small departures from the CP-even case, the decay distributions are sensitive to the presence of a CP-odd component in Model II, while it is difficult to probe these departures in Model I unless the pseudoscalar component is very large. Noting that the proposed degrees of beam polarization increases the statistics, the process demonstrates the effective role of beam polarization in studies beyond the Standard Model. Further, our study shows that an indefinite CP Higgs would be a sensitive laboratory to physics beyond the SM.

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I. INTRODUCTION

Recently the Large Hadron Collider (LHC) discovered a new particle resonance that weighs about 125 GeV/c$^2$, see Refs. [1, 2], which may indeed be the long sought after Higgs Boson. While its presence is not in doubt, the properties of this particle are yet to be completely determined. What is clear is that it is electrically neutral, and that it is not a vector resonance, and almost certainly has spin-0. Whether or not it is the Standard Model (SM) Higgs Boson will be decided by the spin and parity properties of the particle, and also its couplings with other particles. The LHC being a hadronic machine may not be able to exhaustively study the properties of this particle. The proposed International Linear Collider (ILC) [3, 4], which is an $e^−−e^+$ collider, will carry out precision experiments on SM particles and establish their properties including that of the purported Higgs Boson. It has been pointed out, see Ref. [5] that beam polarization could significantly enhance the sensitivity of the machine to probe beyond the SM signals.

The Standard Higgs mechanism employed by the SM as a solution to the electroweak symmetry breaking (EWSB) introduces one Higgs state (doublet of SU(2)$_L$), resulting in a physical CP-even scalar particle. However, many extensions of the EWSB mechanism like the Supersymmetric (SUSY) extensions or the Two-Higgs Doublet Model (2HDM) predict more than one physical scalar particle. In CP-conserving models, these states are either CP-even or CP-odd. While it is too early to identify the CP nature of the new resonance at 125 GeV/c$^2$, it is almost certainly not a purely CP-even state. This follows from the fact that one of the discovery channels involve a $\sigma$ coupling, which would have been absent if $\Phi$ was purely CP-odd. This was also indicated from the recent data analysis results of both ATLAS [6] and CMS [7, 8], where they set an exclusion limit of around 3$\sigma$ for pure CP-odd ($J^P = 0^−$) state. However a CP-mixed state can fit in very well with the data.

CP-mixed states are possible in CP-violating versions of the SUSY models [11], as well as in the general 2HDM with CP violation in the Higgs sector [12]. Indeed, there had been many interesting studies on CP-violating Higgs sector within different versions of the SUSY models [13], and in other models. Some of the recent studies along these lines in the light of new LHC discovery may be found in Ref. [14] and references therein. Being heaviest among the SM particles, the top quark coupling to the Higgs Boson is the strongest, and therefore most promising to study the CP nature of the resonance. A recent study [15] has pointed out that the ILC is an ideal setting to probe the CP nature of the Higgs Boson in the process

\[ e^+e^- \rightarrow t\bar{t}\Phi. \]  

Here the deviation of the Higgs coupling to the top quark was parametrized by considering a CP-mixed Higgs state. The scalar and pseudo-scalar parts of such a CP-mixed Higgs Boson will couple differently to different polarization combinations of the top quark and top antiquark. A measurement of top quark polarization (and/or polarization
asymmetry) could therefore probe the CP-nature of the particle. More recently, in Ref. [16], it was shown that a combined use of total cross section and its energy dependence, the polarization asymmetry of the top quark and the up-down asymmetry of the antitop with respect to the top-electron plane can significantly help in determining the CP properties in the event of CP conservation and in that of mixing in the case of CP violation. The properties in the decay to \( \tau \) lepton pairs has also been considered, see Ref. [17], in the process \( e^+e^- \rightarrow Z\Phi \). Related papers are Refs. [18]–[19].

It is worth pointing out that, quite independently of the considerations of the CP properties of the Higgs Boson, there have been several studies of process in Eq. (1) in the context of the measurement of the top quark Yukawa coupling to the Higgs. For some early work, see, e.g. Ref. [20]–[21] and references therein. More recently the process has attracted renewed attention: in Refs. [22]–[23] the size of the signal and backgrounds when various decays are considered has been studied, while the issue of the process at a centre of mass energy of 500 GeV due to the QCD enhancement of the cross-section near \( tt \) threshold has been considered in some detail in Ref. [24]–[25], and finally the issue of a direct measurement using the semi-leptonic final state from the decays of the \( W \) arising from the decays of the top quarks is considered in Ref. [26].

Keeping in mind the above considerations, we now wish to study the possibility of fingerprinting the departure from the CP-even case [27] in decay distributions of the process in Eq. (1), which will necessarily require us to go beyond the analytical approach. In our study we commit ourselves to two definite scenarios which we denote as Model I and Model II. Model I corresponds to the minimal extension of the SM with one additional pseudo-scalar degree of freedom, which mixes with the SM scalar to form the physical Higgs Boson [15]. This model is characterized by one free parameter, which is denoted by \( b \). Model II is a more realistic case similar to the CP-violating 2HDM model which has some essential features that make it quite different from Model I. In particular, there is no theoretical constraint on parameters denoted by \( a \), \( b \) and \( c \) (as described in the Section II) and thus permits a more general discussion. However, we confine ourselves to some reasonable ranges for these parameters guided by the experimental indications that the resonance is close to a CP-even case.

In order to meet our objectives, considering that these are not amenable to analytical methods, and must necessarily involve numerical packages of great sophistication and complexity, we have used the integrated Monte-Carlo and event generation package WHIZARD [28] for our study. The SM, as well as some of its popular extensions are already implemented in this package. Further, any new model described through a Lagrangian can be incorporated into this package through an interface [29] generated using FeynRules [30]. In particular, in our work we introduce decays for the top-quark which can be implemented in WHIZARD. Our signal processes are

\[
e^+e^- \rightarrow t\bar{t}\Phi \rightarrow W^+W^-b\bar{b}\Phi, \ t\bar{t}b\bar{b},
\]

where the first final state is the result of \( tt \) pair decay, keeping \( \Phi \) fully reconstructed, while the second one considers the decay of \( \Phi \) into \( bb \) pair. In contrast to the existing studies of the \( tt\Phi \) production, we consider the effect of CP-violating Higgs Boson in the decay spectrum of both the top quark as well as the Higgs boson itself, noting that the decay distributions are the spin analysers of the parent particle.

The scheme of this paper is as follows. In Section II we first introduce and describe the basic structure of an indefinite CP Higgs sector in the two scenarios mentioned above. In Section III we describe the processes we consider. In Section IV we present the results of our analysis. In Section V we present a discussion and our conclusions.

II. FORMALISM AND MODELS

In this section we present the formalism we have adopted. In the Standard Higgs mechanism with one Higgs doublet acquiring vacuum expectation value (vev), there is only one CP-even physical scalar field. In the simplest extensions of this with an additional complex singlet or complex doublet, there are CP-odd states along with one or more CP-even states. If CP symmetry is violated in the Higgs sector, the physical Higgs states could be in a CP-mixed state. In this article we focus our attention on such a scenario, and its possible implications on the \( tt\Phi \) production at the ILC. A study of this process will thus give information on the CP nature of the Higgs Boson.

The process \( e^+e^- \rightarrow t\bar{t}\Phi \) goes through the channels shown in Fig. 1. The two channels with Higgs Boson radiating off the top quark or antiquark dominate the cross section, with about a few percent contribution from the third channel with the Higgs radiating off the \( Z \).

With CP-mixed Higgs Boson, both the \( tt\Phi \) as well as \( ZZ\Phi \) couplings take a form, which may be parametrized as follows [15]–[27]

\[
g_{\Phi\ell\ell} = -ie \frac{m_t}{s_W^2 2M_W^2} (a + ib\gamma_5)
g_{\Phi\mu\nu}^{\mu\nu} = -ic \frac{e M_Z}{s_W c_W} g^{\mu\nu},
\]

(3)
In the above, $s_W \equiv \sin \theta_W (= \sqrt{1 - c_W})$, where $\theta_W$ is the Weinberg angle. In the SM with only one scalar Higgs Boson ($h$), the parameters take values $a = 1$, $b = 0$ and $c = 1$.

A. Model I

In a minimal extension of the SM case, one imagines the presence of an additional pseudo-scalar degree of freedom $A$, which mixes with the scalar degree of freedom to produce a physical state:

$$\Phi = a \, h + b \, A.$$

We call this scenario as Model I in the rest of this article. The parameters $a$ and $b$ represent the mixing, and are related to each other by

$$a^2 + b^2 = 1.$$

Since the SM gauge Boson, $Z$ does not couple to the pseudo-scalar degree of freedom, we have $c = a$ in this scenario. The down-type quarks as well as the charged leptons will also have the same coupling structure as that of the up-type quarks, so that, for example, the $b$-quark couplings become

$$g_{\Phi bb} = -i \frac{e}{s_W} \frac{m_b}{2M_W} (a + ib\gamma_5).$$

B. Model II

While model I has the advantage in phenomenological analysis that there is only one free parameter, in most of the realistic cases of extensions beyond the SM the Higgs sector is more complex. For example, in the 2HDM and in the Minimal Supersymmetric Standard Model (MSSM) there are two Higgs doublet fields, leading to two neutral scalar Bosons and one pseudoscalar Boson in the physical spectrum in the CP-conserving case. Denoting the gauge eigenstates of the scalar fields as $\phi_1$ and $\phi_2$, and the mass eigenstates as $h$ and $H$, we can write down the relation between them in terms of the mixing matrix as follows.

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$

In the CP-violating case, all the three degrees of freedom mix to give CP-mixed physical mass eigenstates as below.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ A \end{pmatrix} = \mathcal{O}_{3 \times 3} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix},$$

where $A$ is the pseudo scalar gauge eigenstate $[11]$. This, in effect, removes the restricting relations between the parameters $a$, $b$ and $c$. For ready reference we take the example of MSSM case (or 2HDM) with and without CP.
violation in the Higgs sector, and list in Table I the couplings of the Higgs Boson with the fermions and the gauge Bosons, where tan $\beta$ is the ratio of the vev’s of the two Higgs fields. Comparing Table I with Eq. 3, we have

\begin{align}
\text{top quark} : & \quad a_u = O_{2i} / \sin \beta, \quad b_u = -O_{3i} \cot \beta \\
\text{bottom quark/} \tau - \text{lepton} : & \quad a_d = O_{1i} / \cos \beta, \quad b_d = -O_{3i} \tan \beta \\
\text{gauge Bosons} : & \quad c = O_{1i} \cos \beta + O_{2i} \sin \beta,
\end{align}

where we have introduced the subscripts $u$ and $d$ on the parameters $a$ and $b$ to denote the up-type and down-type quarks, respectively. The mixing matrix elements satisfy the normalization conditions:

\begin{equation}
O_{1i}^2 + O_{2i}^2 + O_{3i}^2 = 1.
\end{equation}

We call this scenario as Model II in the rest of this article. The lightest of the Higgs Bosons, $H_1$ will be assumed to be the discovered 125 GeV resonance (denoted as $\Phi$), while $H_2$ and $H_3$ are considered to be heavy enough to be out of LHC range investigated so far.

### C. Features of Models I and II

While the spin and parity measurements of the LHC resonance are not conclusive yet, LHC reports that the new resonance is consistent with a $J^P = 0^+$. It may be noted that these analyses are done with the either or hypothesis. The scope of a mixed CP-state need a more complex analysis, which may be beyond the capability of LHC at present. While in Model I the parameters $a$ and $b$ are directly proportional to the scalar and pseudoscalar components of the Higgs Boson, in Model II, even with small mixings, it is possible to have large changes in the couplings $a$ and $b$, owing to the relations expressed in Eq. 5. Taking the spin and parity measurements at LHC seriously, one may consider the mixing matrix elements $O_{3i}$ to be small. Further, considering the large contribution of top quark loop to both the diphoton decay process, as well as the gluon fusion production of the Higgs Boson, let us consider the case where the coupling $a_u$ is close to 1. In Table II we present a few possible sets of values of parameters along this line, corresponding to pseudo scalar component of 1% ($O_{31} = 0.10$) for two different $\tan \beta$ values of 2 and 20. For $a_u$ to be large, the scalar component should be mostly of $\phi_2$ type. We have considered the case with a small admixture (about 5%) of $\phi_1$ (corresponding to $O_{11} = 0.22$). On the other hand, the bottom Yukawa has a significant contribution of the pseudoscalar coupling, $b_d$. The effect of this should be visible in the $\Phi \rightarrow b \bar{b}$ and $\Phi \rightarrow \tau^+ \tau^-$ decays. With a more relaxed consideration of the spin and parity measurement of the resonance, we take the other two values of $O_{31} = 0.32$ and $O_{31} = 0.50$, corresponding to the pseudoscalar component of about 10% and 25%, respectively. Here, for small $\tan \beta$ the structure of the top quark Yuwaka couplings is modified, with large contribution from $b_u$, while for large $\tan \beta$, this parameter remains very small. In the former case, the CP-violating effect is significant in both the production as well as the decay.

Thus study using suitably chosen observables at the production level and decay level will be able to distinguish Model I from Model II, and possibly provide more information about the mixing in case of Model II.

| $g_{\Phi u}$ | $g_{\Phi b}$ | $g_{\Phi V}$ | CP-conserving | CP-violating |
|--------------|--------------|--------------|---------------|--------------|
| $-i \frac{e}{\sin \beta} \cos \alpha \frac{M_W}{2 m_W}$ | $-i \frac{e}{\sin \beta} \cos \alpha \frac{M_W}{2 m_W}$ | $-i g V M_V$ | $g_{\Phi u} \cos \alpha \frac{M_W}{2 m_W}$ | $g_{\Phi u} (-i) \cot \beta \gamma^5$ |
| $g_{\Phi b}$ | $-i \frac{e}{\sin \beta} \sin \alpha \frac{M_W}{2 m_W}$ | $g_{\Phi u} \sin \alpha \frac{M_W}{2 m_W}$ | $g_{\Phi b} \sin \alpha \frac{M_W}{2 m_W}$ | $g_{\Phi b} (-i) \tan \beta \gamma^5$ |
| $g_{\Phi V}$ | $-i g V M_V$ | $g_{\Phi V} \sin (\beta - \alpha)$ | $g_{\Phi V} \cos (\beta - \alpha)$ | $0$ | $g_{\Phi V} (O_{31} \cos \beta + O_{2i} \sin \beta)$ |

TABLE I: Couplings of the Higgs Bosons in the CP-conserving and CP-violating 2HDM. $V = W, Z$, with $g_W = e / \sin \theta_W$, $g_Z = e / (\sin \theta_W \cos \theta_W)$. 


TABLE II: Couplings of the Higgs Bosons ($H_1$) in the CP-violating 2HDM, as defined in Eq. (9), for different mixings given by $H_1 = \mathcal{O}_{11} \phi_1 + \mathcal{O}_{21} \phi_2 + \mathcal{O}_{31} \Phi$.

| Point | $\tan\beta$ | $\mathcal{O}_{11}$ | $\mathcal{O}_{21}$ | $\mathcal{O}_{31}$ | $Z$, $W$ | top | $b / \tau$ |
|-------|-------------|-----------------|------------------|-----------------|-----------|------|-----------|
|       |             | $c$             | $a_u$            | $b_u$           |           | $b$  | $a_d$     |
| P1    | 2           | 0.22 0.97       | 0.10 0.05       | -0.05          | 0.50 -0.20 |
| P2    | 2           | 0.22 0.92       | 0.32 0.05       | -0.05          | 0.50 -0.64 |
| P3    | 2           | 0.22 0.84       | 0.50 0.05       | -0.05          | 0.50 -1.00 |
| P4    | 20          | 0.22 0.97       | 0.10 0.05       | -0.05          | 4.48 -2.00 |
| P5    | 20          | 0.22 0.96       | 0.32 0.05       | -0.05          | 4.48 -6.40 |
| P6    | 20          | 0.22 0.84       | 0.50 0.05       | -0.05          | 4.48 -10.00 |

III. PROCESS

The process we consider is the associated production of Higgs Boson with $t\bar{t}$ pair in $e^+e^-$ collision. As per the Feynman diagrams in Fig. I, the process $e^+e^- \rightarrow t\bar{t}\Phi$ at leading order proceeds (i) through the $s$-channel production of $t\bar{t}$ pair and subsequent radiaton of Higgs Boson off either the top quark or the top antiquark, and (ii) through Higgsstrahlung process $e^+e^- \rightarrow Z^*\Phi$ followed by $t\bar{t}$ pair production from $Z^*$. However, the contribution of the latter channel is confined to few percent\cite{15} for $\sqrt{s} \leq 1$ TeV.

In our work, we consider an effective model deviating from the SM through modifications of $f\Phi$ and $VV\Phi$ couplings, where $f = t$, $b$, $\tau$ and $V = Z$, $W$. These changes can be implemented in the WHIZARD programme, by suitable modifications of the programme files of the SM case\footnote{We refer to \url{http://feynrules.irmp.ucl.ac.be/wiki/StandardModel} in this regard.} We have cross checked the correctness of our implementation by verifying the results of Ref. \cite{13} for the process being scrutinized.

It may be noted that the polarization of the top quarks produced here depend on the structure of the Yukawa coupling. Indeed, spin analysis of the top quark and/or the top antiquark produced in the process considered is a very useful tool to probe the Yukawa couplings, and thus the CP nature of the Higgs Boson. One approach is to reconstruct the top polarization, and study the polarization asymmetry. Such a study was carried out in Ref. \cite{15}, the result of which we reconfirm. One way to derive the spin information of the top quark is a study of the distributions of its decay products. The top quark decays nearly 100% through the electroweak interaction into a $W$ Boson and $b$ quark. The decay distribution of the $W$ will provide the information about the top quark polarization, and hence information about the top quark Yukawa interaction. We assume that the gauge structure is the same as that of the SM, and therefore, the $Ztt$ coupling remains the same as that in the SM. In our signal process in Eq. (2) restricting the $Wb$ invariant mass to be that of top quark mass guarantees that backgrounds are reduced to a satisfactory level, as will be demonstrated in Section IV.

In order to probe the other Yukawa couplings, we analyse the decay of the Higgs Boson. The main decay modes of the Higgs of mass around 125 GeV are $b\bar{b}$, $WW^*$ and $\tau^+\tau^-$ with branching fractions of 57.7%, 21.5% and 6.32% respectively. Since $W$’s couple only to the CP-even component of the Higgs Boson, the $WW$ channel can give only very limited information about CP mixing. Again, the branching fraction into $\tau\tau$ pair is very small, and therefore, we will mainly discuss the $b\bar{b}$ decay channel. In order to reduce the complexity, when $\Phi$ decay is considered, we assume that the top quark and top antiquark are fully reconstructed. This leaves us with a final state of four particles in our signal process, $e^+e^- \rightarrow t\bar{t}\Phi \rightarrow t\bar{t}b\bar{b}$. Here again, we will restrict the invariant mass of $b\bar{b}$ to be around 125 GeV to eliminate the background.

IV. RESULTS

We first consider Model I. As discussed in Section I, there is only one independent parameter, which is taken to be $b$, which can vary from 0 to 1. $b = 0$ corresponds to the purely CP-even Higgs Bosons, like that in the SM, while $b = 1$ corresponds to the purely CP-odd Higgs Boson.

The CP nature of the Higgs Boson affects the total production cross section, which is the first observable that we consider here. We plot this in Fig. 2 (left) for some values of $b$. The mass of the Higgs Boson is taken to be 125 GeV in
all our analyses. As we can see the total cross section is indeed sensitive to the parameter $b$, with maximum deviations from SM for non-zero $b$ values at centre of mass of 800 GeV. Notice that the deviations are not linear in $b$, with about 8% deviation for $b = 0.3$, which goes up to about 21% for $b = 0.5$. In Fig. 2 (right) we consider the cross section at $\sqrt{s} = 500, 800, 1000$ GeV plotted against the parameter $b$.

The top quark polarization in the process studied is decided by the CP properties of the Higgs Boson produced. A top polarization asymmetry measurement, as presented in [15] clearly shows the advantage of this observable in identifying the CP properties of the Higgs Boson. In the present work, we go beyond the production process to study the decay products of the top quark to analyse the $tt\Phi$ coupling, and thus the CP properties of the Higgs Boson.

It is important for us to point out the differences between the present work and that of Ref. [16]. While we have considered in detail the decay distributions in contrast to that work, we do not attempt to find, for example, the reach of the parameters $a$ and $b$ at the ILC, but rather we aim to illustrate the sensitivity of ILC within a realistic model, assuming specific values of the parameters considered in the light of the LHC discovery. It is well known that the decay distributions can be used as the top spin analysers. As the cross section peaks at around $\sqrt{s} = 800$ GeV, gaining in statistics, we will consider the distributions at this centre of mass energy. While considering the final state, $WWbb$ in this case, we need to worry about the background. In the following we call the signal along with the the background as the full process. In Fig. 3 we plot the angular and energy distributions of the $W^+$ of the signal process as well as the full process for the two extreme cases of purely scalar Higgs Boson ($b = 0$), like the SM...
FIG. 4: Model I: Energy distribution with a polar angle cut (left fig.) and $M_{bW}$ cut (right fig.) of $W^+$ at $\sqrt{s} = 800$ GeV with an integrated luminosity of 300 fb$^{-1}$ in the case of full process for extreme $b$ values.

Higgs Boson, and the purely pseudoscalar Higgs Boson ($b = 1$). As mentioned in Section II, it is very likely that the value of $b$ is close to zero. In such case the curves will stay closer to the SM curve. The angular distribution clearly shows a different forward-backward asymmetry in the case of signal process in comparison to the background process. The signal process is more peaked in the forward direction, indicating that an appropriate cut on $\cos \theta$ can increase the signal over the background. The distribution becomes more flat with increasing pseudo-scalar composition. A forward-backward asymmetry to be described later will use this feature to extract information on the reach of $b$ at ILC through the process considered. The energy distribution of the $W^+$ is also highly sensitive to the parameter $b$. The larger deviation in the central region indicates that a suitable kinematic cuts can enhance the sensitivity further.

It is clear from Fig. 3 that the background is very small, and even this small background can be controlled by an angular cut. Since in the signal process $W^+b$ is a product of the top quark decay, the invariant mass of this pair is expected to lie around top quark mass. Thus, a cut on this variable is an efficient way of controlling or even eliminating the background. We apply a cut of $170$ GeV $< M_{bW} < 175$ GeV to suppress the backgrounds in the full process. The resultant energy distributions of the $W^+$ with angular and $M_{bW}$ cut are given in Fig. 4. Thus, in the rest of the discussion we use only the signal process.

Taking cue from the angular distributions, we construct the forward-backward asymmetry and present this in Table III for different parameter values at the centre of mass of 800 GeV. The asymmetry is significant only for large values of $b$ with about 6% deviation from the SM case at $b = 0.7$ and about 40% deviation for $b = 1$. The beam polarization at ILC is expected to play an important role in studying the effects of new physics. This machine is supposed to provide high degree of polarization in longitudinal and transverse mode. To discuss the effects of initial longitudinal beam polarization we have generated similar distributions as we have for unpolarized case. For our study we have used a realistic 80% electron($e^-$) and 60% positron($e^+$) beam polarization. Since the final state particles do not have any common interaction vertex with the intial particles, the beam polarization is not expected to have any feature qualitatively different from that of the case of unpolarized beam. At the same time, in the present case the polarization helps improve the statistics. The forward-backward asymmetry of the $W^+$ is not affected by beam polarization, as can be see from Table III.

In Fig. 5 we present energy distributions of the $W^+$ and Higgs Boson for different values of the parameter $b$. We note that significant deviation is present only for values of $b$ is 0.5 and beyond. Apart from the reduction in the distribution for the whole range of the energy values, a shift in the maximum towards higher energy values for larger $b$ values is noted in the case of energy distribution of the Higgs Boson. This may give an additional handle to pinpoint the contribution of the pseudoscalar component in the Higgs Boson. In Fig. 6 we consider the energy distribution of the $W^+$ and the Higgs Boson in the presence of beam polarization. Firstly we note that the total number of events is almost doubled compared to the case of unpolarized beams. As a consequence, the sensitivity is improved, and it is possible to have more than 1σ deviation (considering only statistical uncertainty) for smaller value of $b$ compared to the case of unpolarized beams. While the reach of $b$ is certainly improved with beam polarization, we see that it
TABLE III: Forward-backward asymmetry of the $W^+$ and the total number of events in unpolarized (2nd-3rd column) and longitudinal polarized (4th-5th column) case for an integrated luminosity of 300 fb$^{-1}$ with different values of $b$ corresponding to Model I.

| Case | $N_{tot}^{unpol}$ | $A_{FB}^{unpol}$ | $N_{tot}^{pol}$ | $A_{FB}^{pol}$ |
|------|-------------------|------------------|----------------|----------------|
| P1   | 836               | 0.253            | 1669           | 0.212          |
| P2   | 762               | 0.252            | 1523           | 0.211          |
| P3   | 640               | 0.252            | 1278           | 0.210          |
| P4   | 679               | 0.253            | 1356           | 0.213          |
| P5   | 610               | 0.254            | 1219           | 0.212          |
| P6   | 509               | 0.253            | 1017           | 0.211          |

TABLE IV: Forward-backward asymmetry of the $W^+$ and the total number of events in unpolarized (2nd-3rd column) and longitudinal polarized (4th-5th column) case for an integrated luminosity of 300 fb$^{-1}$ with different cases corresponding to Model II are tabulated.

is still not really possible to probe pseudoscalar admixtures of a few percent or even up to 10-20%.

Next we come to the impact of CP violation in Higgs Boson decay. As mentioned in Section III we consider only the $\Phi \rightarrow b\bar{b}$ decay. In Model I, as described in Section II, the parameters $a$ and $b$ corresponding to bottom quark are taken to be the same as that corresponding to those for the top quark. The signal process we consider is $e^- e^+ \rightarrow t\bar{t}\Phi \rightarrow b\bar{b}W^+W^-\Phi$.

As in the previous case, we can contain the background by imposing a cut of $124 \leq M_{b\bar{b}} \leq 126$ on the invariant mass of $b\bar{b}$ ($M_{b\bar{b}}$). In Fig. 7 we present the angular, energy and $p_T$ distribution of the top quark for the signal process, as well as for the full process including the background. Comparing the two cases presented, viz the case with no kinematic cuts (the top row), and the case with cut on the invariant mass, $M_{b\bar{b}}$, it is evident that this almost eliminates the background. Fig. 8 shows different distributions of the top quark and the bottom quark with unpolarized beams.

Here again, we see that for large enough values of $b$ the presence of the pseudoscalar component can be identified.

FIG. 5: Model I: Energy distributions of $W^+$ and $\Phi$ in $e^- e^+ \rightarrow t\bar{t}\Phi \rightarrow b\bar{b}W^+W^-\Phi$ with unpolarized beams at $\sqrt{s} = 800$ GeV with an integrated luminosity of 300 fb$^{-1}$ for different values of $b$. 
FIG. 6: Model I: Energy distributions of $W^+$ and $\Phi$ in $e^+e^- \to tt\Phi \to b\bar{b}W^+W^-\Phi$ at $\sqrt{s} = 800$ GeV with an integrated luminosity of $300$ fb$^{-1}$ for different values of $b$. Beam polarizations of $80\%$ electron beam polarization and $60\%$ positron beam polarization are considered.

quite easily. As the case with beam polarization produce similar distributions, we do not displays them here. The effect of beam polarization is in an enhancement of the number of events, and thus increase the reach in probing the value of $b$, as in the case of the process $e^+e^- \to tt\Phi \to b\bar{b}W^+W^-\Phi$.

We next come to the Model II, which is a generic 2HDM with CP violation in the Higgs sector. As discussed in Section II, the parameters, $a$, $b$ and $c$ depend on the mixing matrix elements $O_{ij}$ and $\tan\beta$. For our analysis we consider only one light Higgs Boson $\Phi = H_1$ of mass around $125$ GeV/$c^2$, with the other two are heavy enough to be safely away from LHC bounds. Noting that for the top quark, the parameter $b$ is proportional to $\cot\beta$, we confine our studies to low $\tan\beta$ cases. Some illustrative values of mixing and the corresponding values of the couplings are given in Table II. Our analysis based on these couplings is given below.

In Fig. 2(middle) we plot the total production cross section for different mentioned cases. The cross section differs substantially from SM as now $a$ and $b$ parameters are bit free unlike Model I case where they are constrained by Eq. 5. Fig. 9 presents the angular, energy and $p_T$ distributions of the $W^+$, the invariant mass distribution of the $W$ pair, and the energy and $p_T$ distributions of the Higgs Boson in the process $e^+e^- \to tt\Phi \to b\bar{b}W^+W^-\Phi$. Remember that in case of P1 and P4 we have only $1\%$ of the pseudoscalar admixture in the Higgs Boson. Notice that unlike in the case of Model I, here, the parameters $a$ and $b$ can be larger than $1$, and vary with $\tan\beta$ for the same scalar - pseudoscalar composition of the Higgs Boson. In P1 case, the deviation from the SM is large, which could further be enhanced with the use of beam polarization. For larger $\tan\beta$ value, as indicated by P4, the effect is negligible for small CP-mixing. somewhat larger deviation. With $10\%$ and $25\%$ CP-mixing, deviations are significantly large to be observed in the case of $\tan\beta = 20$ (P5 and P6, respectively), while for $\tan\beta = 2$ (P2 and P3, respectively) the deviations are not so much. Recall that Model I required much larger mixing for the deviations to be significant. We also presented the forward-backward asymmetry values in Table IV for discussed sets. The asymmetry value show around $7\%$ deviation from SM in unpolarized case while for polarized case the deviation is almost negligible. Coming to the process $e^+e^- \to tl\Phi \to t\bar{tb}$, the energy, angle and $p_T$ distributions of the top quark, the invariant mass of the $tl$ pair, and the energy and $p_T$ distributions of the bottom quark are presented in Fig. 10. Here again the picture is similar to the previous case with possibility of large deviations from the SM case even in the case of small pseudoscalar admixtures.

Clearly, in the case of Model II, even for small pseudoscalar component of the Higgs Boson, there can be significant deviation in distributions from their SM values. While the analysis of the Higgs decay does not bring out any new features in the simple considerations of the distributions, it should be possible to construct specific observables which violate CP symmetry with the help of these final state particles, and could be the topic of a future study.
FIG. 7: Model I: Energy(left fig.), Polar angle(middle fig.) and $p_T$ distribution(right fig.) of top quark at $\sqrt{s} = 800$ GeV with an integrated luminosity of 300 fb$^{-1}$ for full and signal process using extremal $b$ values. The top row is without any kinematic cuts, while in the case of the second row a cut of $124 \text{ GeV} \leq M_{b\bar{b}} \leq 126 \text{ GeV}$ have been imposed.

V. SUMMARY AND CONCLUSIONS

In this work motivated by the observation of new scalar resonance[1][2] at LHC, we discussed the implications of indefinite CP properties of newly observed state at proposed linear collider ILC[3][4]. ILC is a next generation $e^-e^+$ collider machine which apart from searching beyond Standard Model Physics, promise to precisely determine the properties of various SM fields including Higgs at an unprecedented accuracy level which are beyond the realm of currently running LHC.

Here we thoroughly investigated the process $e^+e^- \rightarrow t\bar{t}\Phi$ with Higgs field $\Phi$ in an indefinite CP state. In general, CP properties of Higgs Yukawa couplings can be parameterized in terms of its CP-even (denoted by $'a'$) and CP-odd ($'b'$) components along with its gauge couplings parametrized by multiplicative factor $c$. These parameters obey certain constrained relations($a^2 + b^2 = 1$, $c = a$) in simplistic scenario, while they can vary much more freely in Models like MSSM or 2HDM for non supersymmetric case. In this work by categorizing these two scenarios as Model I and Model II for constrained and unconstrained cases, respectively, we studied their implications on various detector level observables. Here we incorporated the decays of the produced particles. Thus our observables are not only sensitive to indefinite CP parameters at production level but also to the ones which appears at subsequent decay vertex.

Compared to earlier model independent studies on this topic, we have considered the decay distributions of the top quark as well as the Higgs boson to analyse the couplings, and therefore the CP nature of the Higgs Boson under some realistic scenarios. Such analysis of the spin and parity of the decaying particles does not require a reconstruction of
FIG. 8: Model I: Various Distributions for Higgs decay case at $\sqrt{s} = 800$ GeV with an integrated luminosity of 300 fb$^{-1}$ for different $b$ values.

their polarizations. We have demonstrated that results based on the study of simplified case of Model I can differ drastically in a more realistic case like 2HDM of Model II. While the former require large pseudoscalar admixture in the Higgs Boson, and thus large CP violation in the Higgs sector, the latter can produce significant deviation from the SM case even with 10% or smaller fraction of pseudoscalar component in the Higgs Boson. Thus, we conclude that the indefinite CP properties of the Higgs as a window to physics beyond the SM can be probed effectively through the process considered here.
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FIG. 10: Model II: Various Distribution in the case of Higgs decay at $\sqrt{s} = 800$ GeV with an integrated luminosity of 300 fb$^{-1}$ for the signal process.

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