**Review on S-Shape Characteristics of Pump Turbine for Hydropower Generation**

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**Abstract:** This paper presents and discussed the results of an experimental investigation of S-shape characteristics of pump turbine. An experimental study of pump turbine has been carried out by many researchers to study its characteristics. By using the experimental results of tested pump of some previous researchers, The Numerical analysis and Experimental studies on the S-shaped characteristics of a pump turbine model, Impact of Parameters on S-shaped characteristics, mechanism of the internal flow of pump turbines as well as measurements of pressure fluctuations on reduced-scale model pump-turbines have been discussed.

**Keywords:** Pump Turbine, Pressure Fluctuations, S-Shaped Characteristics, Parameters  

**1. Introduction**  
The ability to store electricity is the main problem in electricity production to a large extent based on renewable energy. In reality, as We all know, these sources are sporadic and non-programmable, so it cannot meet the electricity needs of users. One of the most promising and useful means for power storage today is that of the pump storage plants characterized by a good dynamic (fastest way of altering from pump mode to turbine mode) with the provision of good energy. Reverse pump turbine (RPT) is the utmost communal mechanical device approved in the new generation of pumped-hydro energy storage it is usually preferred due to their profitability compare to other technical requirements Even though quite a lot of problems in the grid might be answer by Pumped Storage, pump-turbines also desires regular and quicker changes among pumping and generating modes outspreading the operation of the machine at off-design conditions. Nevertheless, pump turbine design comes from the outcome of a cooperation amongst the conflicting targets such as performance of pump and turbine, regulatory capacity, efficiency and cavitation behavior. The design criteria, which are more focused on the behavior of the pumping function, such as the increase in the depth of the flow field at the depth of the flow field can result in the operation of the unstable region (called the S-shaped region) which doesn’t have impact on Francis turbines functioning at the same conditions [1]. In view of the strong fluctuation of the torque caused by the occurrence of instability in the region, the RPT can be affected by a sudden change in the function (from turbine to pump and vice versa), along with substantial fluctuations of the head and flow rate through conceivable self-excited vibrations [2] otherwise water hammers emphasizing not the mechanical equipment alone but the entire power plant [3]. What brought about this S-Shaped characteristic appear to be in relation with the development of vortexes in the pump-turbine partly blocking the flow [4-6]. Established from Some studies indicate that the advancement of stall cells gyrating in the runner channels at 5070% of the rotational frequency of the runner [7-10].  

**1.1. Experimental Studies**  
The experimental study of the mechanism of the internal flow of pump turbines comprises of velocity fields measurements and visual observations as well as measurements of pressure fluctuations on reduced-scale model pump-turbines. Owing to the complication of the...
model, it is necessary to measure / monitor general special techniques. Experimental gadget for flow visualization and measurement of the pressure distribution of a model pump-turbine with S-shaped characteristics was developed by [11] particularly situations with no-flow. For transparent visualization of flow cells, and the use of laser Doppler velocimetry (LDV) to measure the convenience of transparent materials. During the experiment, no application of the rotor cover was done but adjusted the free surface of water directly beneath the upper surface of the rotor disk. Analysis was carried out on the impact of the direction of rotation and guide vanes on the radial distribution of static pressure. Observation was made on the flow patterns in the rotor and the draft tube. It was proposed that appropriate selection of the reaction rate of a pump-turbine, that is, the proportion of the total static head in the rotor to the overall head was required to stabilize the operating mode of the turbine. [12] Series of observations on the internal flows in a small-scale low-specific-speed model pump-turbine by means of S-shaped characteristics was performed by [13, 14, 15, 16, 17] The model pump turbine runner, guide vane and the draft tube becomes clear. There was a removal of the band of the runner for pressure measurements expediency. Addition of Aluminum flakes was made to show the internal flow patterns. Power station pumps are the most important in the development of free carbon dioxide in renewable energy and the safety of sustained growth in power supply components. In addition, the pump turbine is the principal element of the entire power station, which has significant impact on the network. As The performance of the turbine constantly display important steeper flow characteristics than the conventional high head Francis turbines having very similar specific speed in name of S-shaped profile for Q N (Figure 1) this is due to the fact that there is a cooperation between an optimal pump and an optimal turbine in the design of a pump turbine. For instance, the water hammer rises sharply due to the rapid changes in the flow rate as the turbines gets closer to the runaway speed. This can lead to serious problems frequency governing, particularly in idle speed [6].

![Figure 1. Four quadrant characteristics of the pump turbine model [6].](image)

Experimental research was conducted on the flow hydrodynamics of a low-specific-speed pump-turbine model under off-design operating conditions, including runaway and S-shaped turbine brake curves by [9, 18] they also performed high-speed flow visualizations in the vanless space amongst the runner and the guide vanes as well as Wall pressure measurements in the stator. It was pointed out that the pressure fluctuations in the guide vane channels stood very severe than in the spiral casing there was a rise of the pressure fluctuation around the S-shaped characteristics as well as the reverse pumping mode. The results are given away in Figure 2.
1.2. Experimental Analysis of S-Shape Characteristics

A lot has been dealt with on this phenomenon in many previous studies [18, 19]. And experimental studies been done [20] to measure the characteristics of flow with the use of PIV and image processing technology. It has been concluded that the instability can be attributed to the large-scale vortex structure at the leading edge of the runner vanes. This conclusion was made based on the analysis of the instability relationship between the external characteristics and the internal flow. Likewise, there was a study on the beginning and the development of flow instability [21]. [22] measured in S-shaped zone the thorough pressure fluctuations among wickets and concluded that the blocking flow could be caused by the stall in runner channels. They gave explanations for the S-shaped curve formation and could hypothetically be one or more of the following: a) Large ratio of diameter; b) heavy forward inclined vane; c) the design of leading edge in turbine mode. Nevertheless, the phenomenon of instability is still a major problem in engineering practice. With the aim of solving this problem meticulously, we must depend on the enhancement at the design stage. Therefore, a consistent numerical simulation method is required. Even though the traditional stoppage flow simulation based on the K-type turbulence model is the state of the art to turbomachinery [23-24], the flow on off-design conditions couldn’t still be predicted by it comparable to the S-shaped zones it still cannot predict the flow on off-design conditions like the S-shaped zones. For instance, the turbine characteristics of a reversible pump turbine was predicted [25], yet, the accuracy wasn’t very good nearby runaway point. The prediction of off-design conditions difficulty is as a result of the flow complexity for example the massive flow separations caused by large incident angle and the detached, reattached boundary layer flow and rotating stall also. In addition, the flow variable’s gradient could be so large to the extent that common algorithms are not very appropriate. So, more advanced turbulence model then algorithm should be discovered to solve this type of flow. Simulations of the stable and unstable flow on the runner based on the characteristics of S-shaped was carried out by. [26] They took the Time-changing in- and outflow into the vaneless space from the runner. there may perhaps be around 50% of the main flow to the turbine of the band of the fluctuations near runaway. The local vortices forming in the runner channels close to the leading edge possibly will be well thought-out as the source of the instability, also utmost of the energy dissipation for operating nearby runaway happened in the vaneless space flanked by guide vanes and the runner. As shown by the In-depth flow analysis results. [27] carried out numerical studies on the flows of two model pump-turbines and it was considered that zero torque to the shaft was achieved by the balance between the pumping and the power generation, at the speed no load condition. The contact amongst pumping, namely, the back flow at the runner inlet as a result of high centrifugal forces, also generating, that is, rendering of a complex local flow field occurred due to the comparatively high-velocity inflow outside the pumping region. The rainbow-colored back flow region at the runner inlet by a contour plot of radial velocity, in addition to the time-dependent variations in the amount then area of the back flow are displays in Figure 3. and Figure 4. methodically shows the basic characteristics of these flows based on SSTkω RANS turbulence model. Research was carried out by [28] on the flow patterns in a model pump-turbine runner on a reverse pump operating conditions in a diverse guide vane opening. There was an observation on the Vortex structures nearby guide vanes as well as stay vanes. Additional comparison was made among the internal flow features at turbine, no-load, then reverse pump operating points at the same guide vane opening. They recommended that under runaway condition, the speed no-load instability was caused by the dominant vortex nearby the runner inlet. [29] comparable analyses were made by. [30] and [31] with quite a lot of dissimilar guide vane openings. Confirmation was made on the Reduction in the strength of the stationary vortex structures in the vaneless space, along with increase of separation flows at the runner inlet, on runaway operating points with increasing guide vane opening.
2. Pump Turbine Specifications

According to Wang L Q, et a, the structured grid can be used for the entire domain and is separated into four parts namely; a spiral casing, vanes (stay vanes and guide vanes), runner and draft tube. All components are connected together through a fluid to a fluid interface. mesh refinement at the interfaces was performed to facilitate information upstream and downstream to be more precisely transferred. With the purpose of capturing the boundary flow separation, boundary refinement of guide vanes, stay vanes as well as runner are giving special attention. Figure 4 and figure 5 below shows a vertical view of the whole grid and the enlarged grid for the runner zone respectively [6].
Beneath the conventional control of guide vane opening, performance of transient of the pump turbine beneath load pressure distribution measurements, on a model pump turbine there was swapping of energy between the rotating mass and the runner and the guide vanes was measured using two-dimensional ensemble averaged PIV. PIV measurements in the vaneless space between the runner and the guide vanes through special linkages as well as two transparent guide vanes, and a window in the support ring of the guide vanes, the velocity in the vaneless space between the runner and the guide vanes was measured using two-dimensional ensemble averaged PIV. PIV measurements in the S-shaped characteristics region by [32, 33, 34, 35]. By means of engaging a spiral casing with an optical access, two different measurement planes, regions of recirculation and backflow may perhaps be partly noticed.

Impact of Parameters on S-Shaped Characteristics

Sequence of research was carried out by [36] and [37, 38] on the runner geometry effect on the S-shaped characteristics. They observed that there is a major influence on the steepness characteristics by the diameter and speed of rotation. A comprehensive parametric research on the impact of runner leading edge of a pump-turbine on the performance curve was accomplished by [38] in Identifying the complex flow patterns at the runner inlet on the S-shaped characteristic region by [37, 38, 39] projected a start-up plan for hydraulic turbines prone to S-shaped characteristic, based on a gain scheduling technique from finite horizon predictive control. Below are the summery of some parameters of pump-turbines which impact the S-shaped characteristics

1. Guide vane opening at speed no-load conditions, the slope of the \( Q_{11} \) \( n_{11} \) characteristics generally goes up with cumulative guide vane opening. Hence, the risk of unsteadiness increases with decreasing head [37].
2. Runner geometric parameters, table two below summarizes the study on the effect of the runner geometric parameters on the S-shaped characteristics.

### Table 1. The Parameters which influence S-shaped characteristics.

| References       | Pump-turbine                                                                 | geometric parameters examined with results                                                                 |
|------------------|------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| Zhu et al. [40]  | Model pump-turbine (corresponding prototype \( H_r \frac{1}{3} 259m, P_r \frac{1}{3} 255MW) | Negative blade lean lead to a weaker S-shaped characteristic                                               |
| Nielsen et al. [36] and Olimstad et al. [37,38] | (ns \( 3 \) 1 114) Low-specific-speed model pump-turbine | Higher values of diameter and speed of rotation gave steeper characteristics; increase the radius of curvature on the pressure side of the leading edge in turbine mode, decrease the inlet radius, increase inlet blade angle, or increase the length of the blade, lead to more stable runner |
| Yin et al. [41]  | Medium-specific-speed pump-turbine                                           | To some extent higher D 1 = D 2 then B=D 2 values required a stable performance curve                     |
| Liu et al. [42]  | Model for Xianju pumped storage power station (375 MW)                       | Number of runner blades, wrapping angle of the blade, diameter ratio between outlet and inlet, inlet and outlet blade angles, and the flow channels |

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3. Anti-S Characteristics Governor

An intelligent governor provided with an anti-S characteristics control was established for improving the performance of transient of the pump turbine beneath load rejection by Kawabata et al. [44, 45]. Analysis was made that beneath the conventional control of guide vane opening, there was swapping of energy between the rotating mass and the water column upstream then downstream the pump-turbine. As a result of these energy transfers, the pump-turbine should have recurrently tracked down and up the S-shaped characteristics producing extraordinary pressure along with discharge oscillations, until it settled to the speed-no-load condition at the end.

4. Misaligned Guide Vane (MGV)

This practice was initially implemented on COO II (Belgium) pump turbines, in reducing the S-shaped characteristics [46]. There was advancement in the start-up instability in providing for the opening of two individual guide vanes through special linkages as well as two additional small servomotors, to an accurately determined pre-opening angle. In china, quite a lot of pumped-storage stations have witness the efficiency in stabilizing the turbine start-up and load rejection processes demonstrated by this kind of device with misaligned guide vanes (MGV). for example, [47, 48, 58], [49], and many others. It has been
pointed out that attention must be paid to continuously monitoring the status of the pre-opened guide vanes to circumvent likely control errors. The use of MGV also made the controls system complex, increasing structural vibration, and quickening the decline of the seal ring of the inlet valve. Fig. 16 equates guide vane opening distribution among using MGV (with four pre-opened guide vanes) and synchronous guide vanes [50]. The experimental formula has been developed for runnery and zero- flow conditions. There was a confirmation in the usability of this fast prediction method in engineering practice in the site tests in Tianhuangping pumped storage plant. Liu et al. [51, 52] and [53] established through experimental and CFD calculations, that through the by utilization of four pre-opened guide vanes, great enhancement was observed in the S-shaped characteristics of a model pump-turbine. In the study of the effect of MGV on the pressure fluctuation in vanless space among the runner and the guide vanes It is suggested that the instability of the force in the radial force increases as the shaft runner simulates the collapse of the symmetrical flow structure compared to the case of synchronized guide vanes Xiao et al. a substantial reduction close to synchronous opening guide vanes through a numerical prediction was made by [54, 55].

There are reports from other researchers indicating that the pressure fluctuations close to the misaligned opening guide vanes augmented [56, 57].

![Figure 7. Openings of guide vanes (a) Synchronous guide vanes. (b) Misaligned guide vanes (MGV)]. [58].

### 5. Conclusion and Ways Forward

A comprehensive review study has been done on the Numerical Analysis and Experimental studies on the S-shaped characteristics of a pump turbine model. The design criteria, Experimental studies, Specifications of Pump turbine and Impact of Parameters on S-shaped characteristics have been explained, with conclusions on the results below.

1. It was pointed out that the pressure fluctuations in the guide vane channels stood very severe than in the spiral casing experimental results have showed that there was a rise of the pressure fluctuation around the S-shaped characteristics as well as the reverse pumping mode.
2. Guide vane opening at speed no-load conditions, the slope of the $Q_{n1}$, $n_{1}$ characteristics generally goes up with cumulative guide vane opening. Hence, the risk of unsteadiness increases with decreasing head.
3. An intelligent governor provided with an anti-S characteristics control has been recognized for improving the performance of transient of the pump turbine beneath load rejection. Analysis was made that beneath the conventional control of guide vane opening, there was swapping of energy between the rotating mass and the water column upstream then downstream the pump-turbine.
4. Operational conditions, such as guide vane opening and cavitation condition, along with a number of runner geometric parameters, particularly the leading-edge shapes of the runner, leading edge parameters, are recognized to have impacts on the S-shaped characteristics shape.

In spite of these outcomes more work is still needed since there are unstable integrally flows at off-design situations even though good results were obtained. Further studies should be focused on the characteristics of off-design conditions as well as Analysis of the Pump-turbine S Characteristics.

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