Amplification and Disorder Effects on the Coherent Backscattering in a Kronig-Penney Chain of Active Potentials

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Abstract

We report in this paper the analytical and numerical results on the effect of amplification on the transmission and reflection coefficient of a periodic one-dimensional Kronig-Penney lattice. A qualitative agreement is found with the tight-binding model where the transmission and reflection increase for small lengths before strongly oscillating with a maximum at a certain length. For larger lengths the transmission decays exponentially with the same rate as in the growing region while the reflection saturates at a high value. However, the maximum transmission (and reflection) moves to larger lengths and diverges in the limit of vanishing amplification instead of going to unity. In very large samples, it is anticipated that the presence of disorder and the associated length scale will limit this uninhibited growth in amplification. Also, there are other interesting competitive effects between disorder and localization giving rise to some nonmonotonic behavior in the peak of transmission.

Keywords: absorption, amplification, transmission, reflection, disorder.

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Introduction

Recently, there has been a lot of interest in non-hermitian hamiltonians and quantum phase transitions (typically localized to extended wavefunctions) in systems characterized by them. There are in general two classes of problems in this context: one in which the non-hermiticity is in the nonlocal part [1, 2] and the other in which it is in the local part [3-8]. In the first category, one considers an imaginary vector potential added to the momentum operator in the Schrodinger hamiltonian and this is shown to represent the physics of vortex lines pinned by columnar defects where the depinning is achieved [1] by a sufficiently high transverse magnetic field. In the case of a tight-binding hamiltonian, the non-hermiticity is introduced by a directed hopping in one of the directions (or more), and again in this case, it is intuitively clear that delocalization may be obtained in the preferred direction in the presence of randomness in the local potential even in 1D. In the second category (non-hermiticity in the local term), an imaginary term is introduced in the one-body potential. It is well-known from textbooks on quantum mechanics that depending on the sign of the imaginary term, this means the presence of a sink (absorber) or a source (amplifier) in the system. It may be noted that this second category does also have a counterpart in classical systems characterized by a Helmholtz (scalar) wave equation as well, where the practical application is in the studies of the effects of classical wave (light) localization due to backscattering in the presence of an amplifying (lasing) medium that has a complex dielectric constant with spatial disorder in its real part [3, 6]. There is a common thread binding both the problems though, namely that the spectrum for both becomes complex (the hamiltonian being non-hermitean or real non-symmetric), but can admit of real eigenvalues as well. The common property is that the real eigenvalues represent localized states and the eigenvalues off the real lines extended states. That it is so in the first category has been shown in the recent works starting with Hatano and Nelson and followed by others [1, 2]. For the second category with sources at each scatterer and in the absence of impurities, it seems counter-intuitive that there are localized solutions; but it has been shown in a simple way [8] that the real eigenvalues are always localized. At present, there is no unified analysis of non-hermiticity of both types present. In the rest of the
paper we would be concerned with non-hermitean hamiltonians of the second category only.

The interest in amplification effects of classical and quantum waves in disordered media has been strongly motivated by the recent experimental results on the amplification of light \cite{9}. The amplification was shown to strongly enhance the coherent backscattering and consequently increases the reflection \cite{1-3}. These results on the reflection naturally lead us also to predict an enhancement of the transmission in such amplifying systems (which has not been examined in previous works). However, recently Sen \cite{8} found for periodic systems that the transmission coefficient starts increasing exponentially up to a certain length scale where it reaches its maximum, then it oscillates strongly before decaying at larger length scales. The reflection seems to saturate to a constant value and becomes large asymptotically. In this paper, we study in details both analytically and numerically this scaling behavior of the reflection and transmission within the framework of the Kronig-Penney model which differs from the tight-binding one by the fact that it is a continuous multiband model where the bandwidth depends on the potential strength while the tight-binding (TB) framework is a discrete single band model where the bandwidth does not depend on the energy site. We compare the results with those obtained by Sen \cite{8} in the tight-binding model and study the evolution of this behavior with amplification. The competition effect between amplification and disorder is also examined.

**The Model**

We consider a non interacting electron moving in a periodic system of δ-peak potentials of a complex strength \( \lambda = \lambda_0 + i\eta \) where both \( \lambda_0 \) and \( \eta \) are constant numbers. By using the Poincare map \cite{10}, the Schrödinger equation of this system can be transformed to the following discrete second order equation \cite{11}

\[
\psi_{n+1} + \psi_{n-1} = \Omega \psi_n ,
\]

where \( \psi_n \) stands for the electron wavefunction at the site \( n \) and

\[
\Omega = 2\cos(\sqrt{E}) + \lambda \frac{\sin(\sqrt{E})}{\sqrt{E}} = 2\cos(k),
\]

where \( k \) is the wave-number. In the passive lattice (\( \lambda \) is real) the corresponding wave-number is
imaginary in the allowed band ($|\Omega| \leq 2$) and the wavefunction becomes Bloch like while in the band gap it becomes real and the wavefunction is evanescent. In the case of the active lattice ($\lambda$ is complex) the wave-number becomes complex ($k = k_s + i\gamma$) and Eq. (2) yields

$$2\cos(\sqrt{E}) + \lambda_0 \frac{\sin(\sqrt{E})}{\sqrt{E}} = (e^\gamma + e^{-\gamma})\cos(k_s),$$  \hspace{1cm} (3)$$

$$\eta \frac{\sin(\sqrt{E})}{\sqrt{E}} = (e^{-\gamma} - e^\gamma)\sin(k_s).$$  \hspace{1cm} (4)$$

The main difference between the tight-binding and this model is the direct dependence of the amplifying term $\gamma$ on the electronic energy. If we restrict ourselves to the first band ($0 < k_s < 2\pi$) we see from (4) that $\gamma$ is negative if $\eta$ is positive. Obviously, in successive bands the sign of $\eta$ must be changed alternatively to get the same sign of $\gamma$. We note also that since we choose in our model, for initial conditions of the discrete equation (1), an electron moving from the right side to the left side of the sample (see ref. [11]) the amplification should occur for negative values of $\gamma$. Therefore the imaginary part of the potential should be positive in the first allowed band of the corresponding passive system. Indeed, in the passive system the Hamiltonian is time reversal invariant but not in the active one, since the Hamiltonian is not hermitian. From Eq. (4) the transmission coefficient can be obtained as

$$T = \frac{4\sin^2(\sqrt{E})}{|ce^{ik_sL}e^{-\gamma L} - de^{-ik_sL}e^{\gamma L}|^2},$$  \hspace{1cm} (5)$$

and the reflection coefficient

$$R = \frac{|ae^{ik_sL}e^{-\gamma L} - be^{-ik_sL}e^{\gamma L}|^2}{|ce^{ik_sL}e^{-\gamma L} - de^{-ik_sL}e^{\gamma L}|^2},$$  \hspace{1cm} (6)$$

where

$$a = \left[ e^{i(k_s-\sqrt{E})}e^{-\gamma} - 1 \right] \left[ e^{i(k_s+\sqrt{E})}e^{-\gamma} - 1 \right],$$ \hspace{1cm} (7)$$

$$b = \left[ e^{-i(k_s+\sqrt{E})}e^\gamma - 1 \right] \left[ e^{-i(k_s-\sqrt{E})}e^\gamma - 1 \right],$$ \hspace{1cm} (8)$$

$$c = 2 - \left[ e^{i(k_s-\sqrt{E})}e^{-\gamma} + e^{-i(k_s-\sqrt{E})}e^\gamma \right],$$  \hspace{1cm} (9)$$

$$d = 2 - \left[ e^{i(k_s+\sqrt{E})}e^{-\gamma} + e^{-i(k_s+\sqrt{E})}e^\gamma \right].$$  \hspace{1cm} (10)$$

Since we are interested to scan the growing and decaying regions of the transmission coefficient (and also the reflection coefficient) it turns out to be more efficient to write the coefficients $c$ and
\[ c = e^{-i k_s \theta_c} e^{\gamma L_0}, \quad d = e^{i k_s \theta_d} e^{-\gamma L_1}, \]  \tag{11}

where

\[ L_0 = \frac{\ln 2 \left[ \cosh(\gamma) - \cos(k_s - \sqrt{E}) \right]}{\gamma}, \quad L_1 = -\frac{\ln 2 \left[ \cosh(\gamma) - \cos(k_s + \sqrt{E}) \right]}{\gamma}, \]  \tag{12}

and \( \theta_{c,d} \) are real phase parameters, which are expected to contribute to the oscillations of \( T \), and behave linearly in \( \gamma \) for vanishing amplification. The transmission then reads

\[ T = \frac{4 \sin^2(\sqrt{E}) \left| e^{i k_s e^{-\gamma} - e^{-i k_s e^\gamma}} \right|^2}{\left| e^{i(k_s L - \theta_c)} e^{-\gamma (L - L_0)} - e^{-i(k_s L - \theta_d)} e^{\gamma (L - L_1)} \right|^2}. \]  \tag{13}

**Results and Discussion**

From Eqs. (3 and 4) the amplification rate \( \gamma \) depends explicitly on the potential strength and the energy. However, since we are interested on the effect of \( \gamma \) on the transmission and reflection, we can, without loss of generality, fix the energy and the real part of the potential. The amplification will then depend on the imaginary part of the potential. In the rest of the text we take \( E = 1 \) and \( \lambda_0 = 0 \) except for the disordered case where \( \lambda_0 \) is taken to be uniformly distributed in the domain \([-W/2, W/2]\) where \( W \) is considered as the disorder strength. The decay of \( T \) for an absorbing chain is found from the above equations to be qualitatively similar to that for a disordered chain (with \( \eta = 0 \)). Thus, nothing particularly interesting takes place for absorbers. But, as we discuss below, in the amplifying chain there is an interesting competition between amplification and disorder in the small length scale regime. So our study below focusses on the amplification where \( \eta \) must be positive. For the numerical calculations, it is easier to use \( \eta \) instead of \( \gamma \). In the limit of small \( \gamma \) we have \( \eta = -2 \gamma \).

In figure 1, we show the transmission as a function of the sample length for two different amplifications. It is shown that the transmission grows exponentially up to an oscillatory region where it assumes a maximum value. For much larger lengths the transmission decays exponentially as in the case of an absorbing chain. A similar behavior is shown in figure 2 for the reflection.
coefficient where in contrast to the transmission, for large lengths the backscattering saturates (instead of decaying) at a high value of the reflection coefficient. This behavior is in a close agreement with that of the TB model [8] with a slight difference in the oscillatory region due to the different dependence of $\gamma$ on $\eta$. This means that this effect is globally model independent.

It is also shown from these figures that the maximum transmission and reflection increase by decreasing $\eta$ and shift to higher sample lengths. Indeed, from Eq.(13) we see that when $L < L_1$ the coefficient $d$ becomes dominant and then $T$ behaves as $\text{exp}(2|\gamma|L)$ while at asymptotically large lengths, the coefficient $c$ becomes dominant and the transmission decays as $\text{exp}(-2|\gamma|L)$. In the oscillatory region the two coefficients $c$ and $d$ are of the same order and the length of maximum transmission is

$$L_{\text{max}} = \frac{1}{\gamma} \ln \frac{\cosh(\gamma) - \cos(k_s - \sqrt{E})}{\cosh(\gamma) - \cos(k_s + \sqrt{E})}.$$ \hspace{1cm} (14)

It is clear from this equation that $L_{\text{max}}$ diverges for vanishing $\gamma$. However, since the maximum transmission must be naturally unity for a passive medium, $T_{\text{max}}$ should not diverge for $\gamma$ exactly equal to zero. Thus there is an infinite discontinuity at $\eta = 0$ which should turn towards a finite discontinuity at a finite disorder $W > 0$. In order to examine the limiting behavior as $\eta \to 0$, let us use a perturbative treatment for $\eta \ll 1$. In this limit $k_s$ tends to $\sqrt{E}$ as

$$k_s = \sqrt{E} + \frac{\gamma^2}{2\tan(1)},$$ \hspace{1cm} (15)

and from Eq.(12) the lengths $L_0$ and $L_1$ are given by

$$L_0 = \frac{\ln(\gamma^2)}{\gamma}, \quad L_1 = -\frac{\ln(4\sin^2k_s)}{\gamma}.$$ \hspace{1cm} (16)

It may be noted that for very small $\eta$, $T$ initially increases extremely slowly with $L$ until it comes quite close to $L_1$, and then it shoots up very fast to a very large value of peak transmission given by

$$T_{\text{max}} = \frac{1}{\gamma^2},$$ \hspace{1cm} (17)

and the length where this highest peak is obtained is given by

$$L_{\text{max}} = \frac{\ln(\gamma^2/\sin^2k_s)}{2\gamma},$$ \hspace{1cm} (18)
with the proviso that a negative value of $L_{\text{max}}$ indicates that the peak is only at $L_{\text{max}} = 0$. Obviously this divergence with a discontinuity is a somewhat unexpected behavior of the transmission. This is due to the fact that when $\eta \to 0^+$, $L_{\text{max}}$ diverges faster than the amplification length-scale $l_a = 1/\gamma$. Therefore $\gamma L_{\text{max}}$ will also diverge and whenever $\gamma$ is different from zero (positive), the current grows slowly up to a very large length scale and reaches very high values. One may note that the asymptotic reflection coefficient $R(L = \infty)$ also diverges as $\eta \to 0^+$ and has an infinite discontinuity at $\eta = 0$. Hence there is an extremely high amplification in the backscattered wave for a very small $\eta$. For example, for a chain with $\eta = 10^{-4}$, $E = 1.0$, the transmission peak occurs at $L_{\text{max}} = 2.07 \times 10^5$, and $T_{\text{max}} = 2.87 \times 10^{10}$, and the asymptotic $R(L = \infty) = 1.13 \times 10^9$ which occurs at $L > L_{\text{max}}$. It is also seen from Figs. 1 and 2 that the period of the oscillations increases when $\gamma$ decreases due to the increase of $k_s$. Before passing on we would like to mention that all the effects discussed above appears qualitatively similarly in the TB model as well. For simplicity, if we take the Fermi energy at the band-center ($E = 0$), then we find that the maximum peak for transmission occurs at an $L_{\text{max}} \simeq 1/\eta \ln(8\pi/\eta)$ which clearly diverges with $|\eta| \to 0$ and so does $T_{\text{max}}$.

However, the high amplitude of the largest peak in the transmission or the asymptotic value of the reflection coefficient even for very small amplification may not be observed experimentally since it occurs at very large sizes (see Eq. (18)) and the experimental realization of such perfect (disorder-free) systems is very difficult. Disorder, however small, would be present (in such a very large size system) and this may cut down strongly the divergences mentioned above. Now, as soon as one introduces disorder or, rather takes care of the disorder, however small, the question regarding whether we should average or not comes up. On the one hand it is clear that experimentalists work on a typical sample, and not on a hypothetical ‘average’ sample. On the other hand, it may not be easy to keep a sample in the same state for a long time due to different types of relaxation processes. Thus, the sample may change its characteristic with time if the characteristic under consideration is highly configuration dependent. Below we discuss both the non-averaged and averaged transmission properties.
First, we discuss the properties for a particular configuration. For this part, we keep the disorder strength constant at $W = 1$. In Fig.3 we show the effect of disorder on the transmission for different imaginary potentials. We see clearly that the disorder destroys the amplification at larger scales and shifts the maximum transmission to smaller lengths. The transmission fluctuations appearing in Fig.3 increase with the amplification ($\eta$). As is well known, disorder introduces an exponential decay of the transmission with a rate $\gamma_{\text{dis}} = W^2/96E$ [12] where $E$ is the energy of the incoming electron and $\gamma_{\text{dis}}$ is the Lyapunov exponent due to the disorder. Stated differently, disorder introduces the localization length $\xi_{\text{dis}} = 1/\gamma_{\text{dis}}$ into the problem. For a small $\eta$, the length $L_{\text{max}}$ up to which the exponential growth occurs in pure systems may be much larger than $\xi_{\text{dis}}$. So, in general, the transmission starts decaying due to disorder effects before it gets the maximal amplification due to a non-zero $\eta$. Therefore, the divergence in $T_{\text{max}}$ observed in periodic systems disappears with the included disorder as shown in the Fig.4. For very small $\eta$, $T_{\text{max}}$ tends to the trivial constant value of unity with $L_{\text{max}} = 0$. But we have to remember that for $\xi_{\text{dis}} < L < L_{\text{max}}$ (for pure systems), there is a fine competition between the amplification-dependent growth and disorder-dependent decay which affects the transmission sensitively in this regime. As given by the above formula, for $W = 1$, $\xi_{\text{dis}} \approx 100$. Yet, indeed, there is a non-monotonic behavior at much larger lengths corresponding to some compensation between disorder and amplification. For $\eta \approx 10^{-3}$, the transmission in general decays for $L > \xi_{\text{dis}}$ but only to pick up again at a still larger $L$, and one observes a peak of $T_{\text{max}}$ (for the particular disorder configuration in Figs. 3 and 4)) at $L \approx 260$. This transmission peak seems to correspond to one of the Azbel resonances that becomes sensitively amplified by a tuned value of $\eta \approx 10^{-3}$. We have actually checked that this resonance peak $T_{\text{max}}$ occurs at the same $L_{\text{max}}$ but becomes weaker both by increasing or by decreasing $\eta$ around 0.001 as shown in Fig.4 and thus $T_{\text{max}}$ has a peak close to this special value of 0.001 for this particular configuration. In particular, if we decrease $\eta \to 10^{-6}$, the peak remains at $L_{\text{max}} \approx 260$ while $T_{\text{max}} \to 1$ continuously. For $\eta < 10^{-6}$, the (local) peak transmission at $L \approx 260$ becomes less than unity and hence the global $T_{\text{max}} = 1$ (trivial constant) and $L_{\text{max}}$ jumps back to the trivial value of zero discontinuously (see the insert of Fig.4). Further, as expected, we found
that in other configurations, the peak in $T_{max}$ at the special value of $\eta \simeq 10^{-3}$ as shown in Fig.4 does not exist.

Next we discuss the characteristics of averaged samples. The question of what quantity to average becomes crucial now. In Fig.5, we choose $E = 1$, $W = 0.01$ and $\eta = 0.1$ and show the transmission as a function of $L$ (in semi-log plot) by averaging in (a) the quantity $T$ itself, and in (b) the quantity $\ln T$. For comparison we have also shown the case without disorder by dashed lines. The full line is the result of averaging with 100 configurations and the dotted line is the same for 10000 configurations in both the cases. Whereas in Fig.5(a) the average with 10000 configurations lies higher than that with 100 configurations (both of them larger than the pure case as well!), the logarithmic average shown in Fig.5(b) is much more well-behaved in every respect. The results shown here are consistent with the fact that all the moments of the transmission and reflection diverge in the amplifying case [4, 5]. So, we restrict ourselves to logarithmic averaging.

In the Fig.6, we show such averaged $T_{max}$ for two different $\eta$’s (0.01 with open squares, and 0.1 with crosses). To show both the cases with very different $T_{max}$’s we have normalized both of them by their values for the pure case. Now, one expects that the nonmonotonic behavior as seen above should disappear since the Azbel resonances disappear on averaging. But, interestingly enough the fine tuning between disorder and amplification is still at work, and some nonmonotonic effects still survive. We have shown in the inset of Fig.6 the magnified view of the $y$-axis around 1. Now we find that for the case of $\eta = 0.1$, there are some values of disorder $W$ around 0.01 where the $T_{max}$ is somewhat larger than its value for the pure case. Further, we could not find such an interesting non-monotonic behavior for the case of $\eta = 0.01$ after a lot of search, which means that even if it is there it is probably very weak or lies in an extremely narrow region for this case. In any case, Fig.6 shows amply that the fine tuning between disorder and amplification may lead to quite interesting and unexpected results.

**Conclusion**

We have studied in this paper, within the framework of the Kronig-Penney model, the effect of amplification on the transmission and reflection of a periodic system. The behavior
shown is in a close agreement with that shown in the tight-binding model \[8\]. Therefore, this effect seems to be model independent. However, this result means that diverging transmission will be obtained at very large sample sizes for vanishing amplification while in a passive system the transmission and reflection do not exceed one. This limiting effect is due to the divergence of the maximum transmission length faster than \(1/\gamma\). This effect is probably experimentally irrealsizable since large periodic samples cannot be growth without disorder which can destroy this divergence. Indeed, we found that the maximum transmission decreases when amplification decreases and tends to sature at one for vanishing \(\gamma\). A peak at \(\eta = 0.001\) appears and seems to correspond to the compensation between disorder and amplification. However, the decay of the transmission is slower for smaller amplification leading to the delocalization of the electronic states. However, the compensation effect leading to some of the non-monotonic behavior in Fig.4 persist by averaging in Fig.6 and remain not well understood. Therefore they should be extensively examined. Also the generalization of this study to different electron energies and non-zero real parts of the potential \(\lambda_0\) is necessary since the bandwidth depends on the scattering potential in this model. On the other hand, for a further understanding of the surprising amplification effect on the periodic system, it is interesting to study the amplification effect on the resonant tunnelling in a simple system of a double barrier which can give us a basis for the periodic system.

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Figure Captions

**Fig.1** Transmission coefficient versus the sample size $L$ for $\eta = 0.05$ (solid curve) and 0.1 (dashed curve).

**Fig.2** Reflection coefficient versus the sample size $L$ for the same parameters as in Fig.1.

**Fig.3** $T$ versus $L$ for a disordered lattice of $\lambda_0$ uniformly distributed between -1/2 and 1/2 ($W = 1$) and $\eta = 0.1$ (solid curve) 0.01 (dashed curve) and $10^{-7}$ (dotted curve).

**Fig.4** $T_{\text{max}}$ versus $(\eta)$ for the same configuration of the random real potential as in Fig.3. The insert shows the corresponding length at the maximum transmission ($L_{\text{max}}$) as a function of $(\eta)$. The dashed curve is only a guide for the eyes.

**fig.5** Transmission versus length for $E = 1$, $W = 0.01$ and $\eta = 0.1$ for an averaging over 100 samples (solid curve), over 10000 samples (dotted curve) and without disorder (dashed curve). (a) averaging the quantity $T$ itself, (b) averaging ln($T$).

**fig.6** The normalized-averaged maximum Transmission $T_{\text{max}}$ versus disorder for $\eta = 0.1$ (cross ‘+’) and $\eta = 0.01$ (open squares). The inset shows a blown up y-axis region between 0.99 and 1.01.
FIGURE 2
FIGURE 3

Transmission coefficient vs. Sample size for different values of $\eta$: $\eta=10^{-7}$, $\eta=0.01$, $\eta=0.1$. The graph shows a decrease in the transmission coefficient as the sample size increases for each value of $\eta$. The y-axis represents the transmission coefficient, ranging from 0.1 to 10, while the x-axis represents the sample size, ranging from 200 to 1000.
FIGURE 4
FIGURE 5
FIGURE 6