Central MONDian spike in spherically symmetric systems

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ABSTRACT

Under a MONDian view, astrophysical systems are expected to follow Newtonian dynamics whenever the local acceleration is above the critical $a_0 = 1.2 \times 10^{-10}\text{m s}^{-2}$, and enter a modified regime for accelerations below this critical value. Indeed, the dark matter phenomenology on galactic and subgalactic scales appears always, and only, at low accelerations. It is standard to find the $a < a_0$ regime towards the low density outskirts of astronomical systems, where under a Newtonian interpretation, dark matter becomes conspicuous. Thus, it is standard to find, and to think, of the dense central regions of observed systems as purely Newtonian. However, under spherical symmetry in the MONDian as in the Newtonian case, the local acceleration will tend to zero as one approaches the very centre of a mass distribution. It is clear that for spherically symmetric systems, an inner $a < a_0$ region will necessarily appear interior to a critical radius which will depend on the details of the density profile in question. Here we calculate analytically such a critical radius for a constant density core, and numerically for a cored isothermal profile. Under a Newtonian interpretation, such a central MONDian region will be interpreted as extra mass, analogous to the controversial black holes sometimes inferred to lie at the centres of globular clusters, despite an absence of nuclear activity detected to date. We calculate this effect and give predictions for the “central black hole” mass to be expected under Newtonian interpretations of low density Galactic globular clusters.

Key words: gravitation — stars: kinematics and dynamics — galaxies: star clusters: general

1 INTRODUCTION

The continual null detection of any dark matter particles, in particular over the last year, LHC results eliminating simple super-symmetric candidates (CMS collaboration 2016), the astrophysical searches for dark matter annihilation signals being fully consistent with zero dark matter signal (e.g. Fermi-LAT and DES collaborations 2016 searching for such a signal in local dwarf galaxies reporting results consistent with expected backgrounds), and various recent direct detection experiments returning only ever stricter exclusion limits (e.g. Yang et al. 2016 reporting no dark matter signal from the PANDAX-II experiment, ruling out previous claims, and Szydagis et al. 2016 for the LUX and LZ collaborations reporting also no dark matter signal), encourage the sustained exploration of alternative explanations for the gravitational anomalies appearing in the low acceleration regime, and generally ascribed to the presence of dark matter.

Further, recent theoretical developments have shown novel possible fundamental physical origins for a change in regime for gravity when reaching the low acceleration region of MOND, e.g. the emergent gravity ideas of Verlinde (2016) (but see also Lelli et al. 2017, Hees et al. 2017, Turtora et al. 2017), or the covariant models including torsion of Barrientos & Mendoza (2016). Empirically, Durazo et al. (2017) analysing pressure supported systems from globular clusters to elliptical galaxies, and McGaugh et al. (2016) and Desmond (2017) studying spiral rotation curves, all find the ratio between inferred dark matter content and observed baryonic mass to be consistent with MONDian expectations across a wide range of galaxy types and masses.

In light of the above, it appears reasonable to continue the study of MONDian gravity and its predictions. In this paper we present a novel effect, the expectation of a modified gravity region towards the centre of astrophysical systems, and perform a first exploration of its consequences. Under MONDian gravity the usual Newtonian physics are recovered in the high acceleration regime, while whenever local acceleration falls below the critical $a_0 = 1.2 \times 10^{-10}\text{m s}^{-2}$ value, the gravitational force shifts towards $(GM/a_0)^{1/2}/R$, for test particles orbiting at a radius $R$ from a point mass $M$. This low acceleration regime typically occurs towards the outskirts of astrophysical systems, where lower matter densities naturally lead to low accelerations. It becomes clear however, that given the validity of Newton’s theorems for
spherical mass distributions in both Newtonian and MONDian gravity (e.g. Mendoza et al. 2011), the local acceleration will tend to zero for \( R \to 0 \). Necessarily, this will lead to an inner MONDian region on crossing inwards of a threshold radius interior to which \( a < a_0 \), as \( a \to 0 \) in going to the actual centre of an astrophysical system.

We present a first development of this effect for constant stellar density cores through a simple analytic development to show the physical scalings and orders of magnitude expected for the size of this inner MONDian region, as well as its expected force amplitude, as a function of the density of such a region. We show that if dynamical tracers within this enhanced gravity region are interpreted under Newtonian assumptions, an extra mass component would be inferred, perhaps akin to the intermediate mass black holes reported by some authors (e.g. Ibata et al. 2009, Feldmeier et al. 2013, Kamann et al. 2016) in the centres of Galactic globular clusters, and which have failed to appear under direct searches (e.g. Maccarone & Servillat 2010, Lu & Kong 2011 and Strader et al. 2012 report only upper limits on GC black holes through accretion activity diagnostics). A numerical treatment for cored isothermal stellar distributions, intended merely as accurate phenomenological descriptions of the observed density profiles of globular clusters, is also performed, with parameters representative of Galactic globular clusters, in order to explore and give some predictions regarding the gravitational anomaly expected under MONDian gravity towards the centres of these systems. Such region should range from somewhat over the inner 10 pc to less than 1 pc, in going from systems with central densities spanning from a few tens to a few thousands of \( M_⊙ \) pc\(^{-3}\).

In section 2 we derive analytically the details of this effect for constant density cores, showing the physical scalings which result. In section 3 we extend the results numerically for cored isothermal stellar profiles, as representative models of the actual stellar density profiles of observed globular clusters. In this section we also compare with recent studies addressing the controversy of intermediate mass black holes in globular clusters, and give predictions for such an effect in low density globular clusters. Our concluding remarks appear in section 4.

2 CENTRAL MONDIAN SPIKES

Within a MONDian gravity scheme one expects the force felt by a test particle orbiting a total mass \( M \) to be well described by the Newtonian expression of \( F_N(R) = -GM/R^2 \) whenever the acceleration is above \( a_0 \), and to follow the MONDian expression of \( F_M(R) = -(GMa_0)^{1/2}/R \) in the low acceleration regime. The transition between regimes appears at the point where \( F_N(R) = F_M(R) \), yielding a characteristic MOND radius \( R_M = (GMa_0)^{1/2} \) e.g. Milgrom (1984), Mendoza et al. (2011). The condition \( a < a_0 \) for the modified regime yielding directly \( R > R_M \), the MOND region appears at distances larger than the characteristic \( R_M \).

Given the validity of Newton’s theorems for spherically symmetric mass distributions also under the MONDian regime (e.g. Mendoza et al. 2011), the above expressions can be generalised directly by substituting \( M \to M(R) \). If we now assume a constant density core, as appropriate for a wide range of astrophysical systems, e.g. the central regions of the King halos generally fitted to observed surface brightness profiles of Galactic globular clusters, we can take \( M(R) = 4\pi \rho_0 R^3/3 \) to yield:

\[
R_M = \left( \frac{4\pi G \rho_0 R_0^3}{3a_0} \right)^{1/2},
\]

where \( \rho_0 \) is the density of the core region in question. As the condition \( a < a_0 \) remains \( R > R_M \), and as \( R_M \) scales with \( R_{\odot}^3 \), it is clear that \( R_M \) will fall to zero towards the centre at a faster rate than the radial coordinate, yielding necessarily an inner critical radius interior to which a central MONDian region is to be expected. This critical radius, \( R_c \), can be found by equating the radial coordinate to \( R_M \), which gives:

\[
R_c = \frac{3a_0}{4\pi G \rho_0}
\]

the previous relation in astronomical units reads:

\[
R_c = 211.3 \left( \frac{M_{\odot} \text{ pc}^{-3}}{\rho_0} \right) \text{ pc}.
\]

For a constant density core, interior to the above radius dynamics will show a transition towards a MONDian regime. We see that in a globular cluster with a typical central density of \( 100 M_\odot \text{ pc}^{-3} \) only the central 2 pc will lie within the \( a < a_0 \), \( R > R_M \) inner modified regime. While in going to high density globular clusters, with central densities an order of magnitude higher, the central MONDian region will be limited to a tiny 0.2 pc region. For low density systems, the modified regime could easily extend to the central 10 pc or more. The development leading to equation (1) can be repeated for a stellar density profile having a central scaling \( \rho(R) \propto R^n \) where the condition for \( R_M \) to fall to zero towards \( R \to 0 \) faster than the radial coordinate becomes \( n > -1 \). Thus, we see that for all stellar profiles having a central cusp shallower than \( R^{-1} \), an internal low acceleration \( a < a_0 \) region will exist. A clear precedent for this effect can be found in the study by Ciotti et al. (2006), where in their figure (1) a “dark matter” spike appears for the cases of central baryonic density distributions shallower than \( R^{-1} \), as shown above.

In the above we have ignored the details of any MOND transition function and assumed an abrupt shift from the Newtonian to the MONDian regimes. Such details, in MOND as such, are not well constrained beyond the requirement of a fairly abrupt transition e.g. Milgrom (2014). In Hernandez & Jimenez (2012) we showed that under a MONDian modelling of Galactic globular cluster structure, that when considering MONDian force laws at the Newtonian level, the transition function is required to be very sudden, as also implied in order to avoid any conflict with solar system dynamics.

We can now calculate what extra central mass will be inferred under Newtonian dynamics from dynamical tracers within \( R_c \) by equating the actual force at a given radius, to the corresponding Newtonian expression, to which we add a hypothetically central potential:

\[
\frac{GM(R)a_0}{R} = \frac{GM(R)}{R^2} + \frac{GM_{BH}}{R^2}.
\]

If we now go to the constant density core of a baryonic
mass distribution, e.g. the central regions of Galactic globular clusters, $M(R) = 4\pi R^3/3$ and we can solve for $M_{BH}$ from the above expression to yield:

$$M_{BH} = \left(\frac{4\pi \rho_0 R^3}{3}\right)^{1/2} \left[\left(\frac{a_0}{G}\right)^{1/2} \left(\frac{4\pi \rho_0 R}{3}\right)^{1/2}\right]$$  \hspace{0.5cm} (5)

We see that for large densities and radii the above equation becomes negative, whenever $R < R_M$ c.f. equation (2), the Newtonian term dominates and the equation is not valid, as no MONDian regime appears. Also, notice that the inferred black hole mass under a Newtonian interpretation is a function of the radial distance at which the tracers being used for the inference are found, i.e., consistent, unique inferred black hole mass will appear when treating tracers found over a range of radial distances. Of course, if tracers within the central MONDian spike are analysed under a Newtonian assumption, some extra mass will necessarily be required, and if one then fits a model with such a black hole mass as a free parameter, statistically, some preferred value will appear.

We estimate the maximum black hole mass which under a Newtonian interpretation could appear as a result of the central MONDian region, by setting the radial derivative of the previous equation equal to zero and finding the radius at which a maximum black hole mass appears, $R_{MX}$ given by:

$$R_{MX} = \left(\frac{25}{36}\right) R_c,$$ \hspace{0.5cm} (6)

i.e., slightly inside of the outer edge of the MONDian central region one finds the radius at which tracers will, under a Newtonian interpretation, require a maximal black hole mass to fit dynamics. This maximum black hole mass will now be given by:

$$M_{BH}(R_{MX}) = \left(\frac{a_0}{G\rho_0}\right)^3 \left(\frac{4\pi}{3}\right)^{1/2} \left(\frac{25}{48\pi}\right)^{5/2} \left[1 - \left(\frac{25\pi}{36}\right)^{1/2}\right].$$ \hspace{0.5cm} (7)

which going to astronomical units yields:

$$M_{BH}(R_{MX}) = 3.82 \times 10^{-3} \left(\frac{a_0}{G\rho_0}\right) M_\odot.$$ \hspace{0.5cm} (8)

Hence, for central galactic cluster densities of below $51.3 M_\odot/pc^3$, in the range of what is inferred for some well studied Galactic globular clusters (e.g. Harris 1996 and updates thereof), central black hole masses of more than $1000 M_\odot$ will be required to understand the dynamics of stellar tracers within a constant density region, under Newtonian assumptions. For higher central galactic cluster densities, the maximum required black hole mass drops as shown above, with the square of the core density.

3 PREDICTIONS FOR GALACTIC GLOBULAR CLUSTERS

In the previous section we have given analytic estimates for the effect being presented, under the assumption of a core region of rigorously constant stellar density. In reality, even within the core region of King halos, as commonly used to describe observed globular clusters, a slight radial drop in density occurs. Since we are clearly dealing with an effect which is very centrally located, and since all King halos tend strongly within their core regions to a cored isothermal profile (see e.g. fig 4-9 in Binney & Tremaine 1987), we can accurately estimate the effect of the slight curvature in the density profile within the core region of Galactic globular clusters, without reference to particular central potential values, by modelling the situation through a cored isothermal stellar density profile, intended solely as an empirical description of the density profile of observed globular clusters.

We begin by calculating cored isothermal profiles, by solving numerically the equation:

$$\sigma^2 \frac{dp}{dR} = -\rho \frac{GM(R)}{R^2},$$ \hspace{0.5cm} (10)

under the conditions $\rho(0) = \rho_0$, $dp/dR = 0$ at the origin and taking a fixed velocity dispersion of $\sigma = 5 \text{km/s}$, a value representative of what is observed in Galactic globular clusters, e.g. Harris (1996), Scarpa et al. (2011). The central stellar densities of these profiles were chosen to span a range of values bracketing that which is inferred for Galactic globular clusters, we used 50, 100, 300, 1000 and 3000 $M_\odot/pc^3$.

Once the full density profiles and corresponding mass profiles $M(R)$ were calculated, we evaluated the ratio between the radial coordinate and the local MOND radius of $(GM(R)/a_0)^{1/2}$. The result is shown in figure (1), where said ratio appears as a function of the logarithm of the radial distance for the cored isothermal halos mentioned above, top to bottom for decreasing values of the central density. The horizontal line in the figure gives $R = R_M$, with the central MONDian region appearing interior to the first crossing of the corresponding curves and this line. These crossing points.

**Figure 1.** The figure gives the ratio of the radial coordinate to the local MOND radius, $R/R_M$, as a function of the logarithm of the radial distance in pc, for cored isothermal halos characterised by a velocity dispersion of $\sigma = 5 \text{km/s}$ and central density values of 50, 100, 300, 1000 and 3000 $M_\odot/pc^3$, top to bottom respectively. The horizontal line gives $R = R_M$, with values above indicative of a central MONDian spike.
and corresponding radii are within a few percent of the results of equation (2). are always within a few percent of the analytic estimates for rigorously constant stellar density cores of equation (2).

We now solve for the Newtonian central black hole mass required to reproduce the actual MONDian radial force within the internal $R > R_M$ region, when added to the Newtonian force produced by the actual stellar density profile present, i.e., we solve for $M_{BH}$ from equation (4), using this time the full cored isothermal $M(R)$ profiles described above. The results are shown in figure (2), where in a log-log plot we give the central black hole mass which would be inferred under a Newtonian modelling of dynamical tracers observed within the halos described above. It is interesting that the actual maximum values for these masses, and the radii at which these occur, are never more than a few percent off from the rigorously constant stellar density core results of the previous section. As expected from the analytic results, the maximum black hole mass appears just inside the onset of the central MONDian spike, the $M_{BH}(R)$ functions in figure (2) drop very abruptly shortly after reaching their maxima.

Going back to figure (1), we see that the inner MONDian region extends outwards as the central density assumed is reduced, in consistency with equation (3). In going to progressively lower densities, the inner $a < a_0$ region extends to cover a growing percentage of the cluster extent, until a density threshold is reached at which the inner MONDian region extends into the typical $a < a_0$ outer low acceleration one, and the globular cluster appears entirely MONDian. Indeed, such is the case of NGC 288, a very low density system fully within the low acceleration regime. As expected under MONDian gravity, Hernandez et al. (2017) recently found that this cluster shows no drop in its velocity dispersion profile, being fully isothermal and fully consistent in both velocity dispersion and projected surface density profiles with MONDian dynamical models.

As mentioned previously, in the preceding development we have assumed that the force law which applies is the one which corresponds to the $a << a_0$ limit of the deep MOND regime, that is, the l.h.s. of equation (4) considers a purely MOND-limit force law. This would be accurate in the case of an abrupt transition between the Newtonian and the modified regime, for example, if this transition proved to be of quantum origin. If however, a more gradual transition between the two limit regimes applies, as is generally assumed through the $\mu(a/a_0)$ transition functions of MOND (e.g. Famaey & McGaugh 2012), the l.h.s. of equation (4) would be modified to include a more complicated expression accommodating a gradual transition to the standard Newtonian expression in the $a >> a_0$ limit. As can be seen in figure (1), the $a_0$ transition point is approached quite gradually for the cored isothermal density distribution profiles considered, the case for constant density cores is quite similar. This last means that for gradual transition functions, the resulting MONDian spike could be quite different than for the limit case shown in figures (1) and (2).

In Bekenstein (2004) it is shown that for the relativistic extension of TeVeS, the function which mediates the transition between the MOND and the Newtonian regimes for a MOND description of $a_\mu(X) = GM/R^2$, where $X = a/a_0$, is:

$$\mu X = \left(\frac{1 + 4X^{1/2}}{1 + 4X^{1/2} + 1}\right)^{1/2} - 1,$$

the so called 'simple' MOND transition function is:

$$\mu(X) = \frac{X}{X + 1},$$

while the 'n-family' of transition functions is:

$$\mu X = \frac{X}{(X^n + 1)^{1/n}}.$$

Taking $n = 2$ in the last expression yields what is generally referred to as the standard $\mu$ function. All of the above clearly have the required limits of $\mu(X) \to 1$ for $X >> 1$ and $\mu(X) \to X$ for $X << 1$. Figure (3) is analogous to figure (2), but shows in all cases the results for a central density value of $300 M_\odot pc^{-3}$, for different assumed $\mu(X)$ functions, top to bottom, the Bekenstein transition function, the simple MOND $\mu(X)$, the 'standard' MOND function, followed by $n = 4, 8, 16$ and 32 of the 'n-family'. The lowermost curve is the $a << a_0$ limit shown in figure (2).

We see that for the cases of very soft transition functions, the MONDian modification of the mildly Newtonian regime for accelerations slightly above $a_0$, is significant and results in extremely large gravitational anomalies which from the point of view of Newtonian dynamics would be interpreted as extra matter components. In fact, these soft transition functions are already ruled out by solar system consistency checks, where the absence of measurable discrepancies with regards to Newtonian physics eliminates $\mu(X)$ functions which introduce even small modified gravity effects at solar system acceleration scales, e.g. Mendoza et al. (2011). Even low $n$-family transition functions are hard to accommodate, e.g., Hernandez & Jimenez (2012) showed...
that dynamical models for Galactic globular clusters constrained to fit both surface brightness and velocity dispersion profiles required very abrupt $\mu(X)$ functions. As progressively higher $n$ values are taken, it is clear that the transition function becomes more and more abrupt, with the results tending to the limiting MOND case of equation (4) and shown in figure (2). Thus, we propose that detailed studies of the radial range over which this central MONDian spike can be detected (or alternatively intermediate mass black holes in the absence of any high energy activity signature under Newtonian assumptions), can serve to constrain the abruptness of the MOND transition function.

As discussed above, the systems under consideration hover close to the transition $a = a_0$ point, so that a return to a scaled Newtonian gravity in the deep $a << a_0$ regime, as happens when an $\epsilon_0$ term is included in the transition function (Famaey & McGaugh 2012) would have negligible consequences.

A further caveat to what we present here is that under MOND as such, some Galactic globular clusters would be expected to behave as essentially Newtonian systems throughout, even if their internal accelerations are lower than $a_0$. This a consequence of the so called external field effect, where if the external gravitational acceleration produced by a larger system - in this case the Milky Way - is actually larger than $a_0$, the internal dynamics would revert to a scaled Newtonian character e.g. Famaey & McGaugh (2012). It is not known if this effect appears at all under the various covariant plausible theoretical origins of the MOND phenomenology proposed, e.g. Barrientos & Mendoza (2016) or Verlinde (2016) and indeed, systems showing evidence of MONDian phenomenology in cases where the EFE would predict essentially a return to Newtonian dynamics have been reported, e.g. kinematics of wide binaries in the solar neighbourhood (Hernandez et al. (2012b) or Galactic globular clusters asymptotically following the deep MOND expected "Tully-Fisher" scaling between their total baryonic content and their velocity dispersion profiles (Hernandez et al. (2013)). Still, if the external field effect of MOND as such proves to be a reality, the analysis presented here would hold only for the more distant Galactic globular clusters which lie at locations where their orbital accelerations about the Milky Way are actually lower than $a_0$, e.g. Sanders (2012) for NGC 2419. Indeed, recent proper motion determinations by Massari et al. (2017) have determined this particular cluster has an orbit which results in a Galactocentric radial variation of between 53 and 98 kpc.

Figure 3. This figure is analogous to figure (2), but shows in all cases the results for a central density value of $300 M_\odot pc^{-3}$, for different assumed $\mu(X)$ functions, top to bottom, the Bekenstein transition function, the simple MOND $\mu(X)$, the 'standard' MOND function, followed by $n = 4, 8, 16$ and 32. The lowermost curve is the $a << a_0$ limit shown in figure (2)

Over the past years, a number of authors have claimed inferences of black holes at the centres of Galactic globular clusters, e.g. Ibata et al. (2009), Feldmeier et al. (2013) and Kamann et al. (2016). Black hole claims in globular clusters remain controversial, as direct searches looking for any central nuclear activity tend to return only upper limits on the putative black hole masses, e.g. Maccarone & Servillat (2010), Lu & Kong (2011) and Strader et al. (2012). Indeed, Baumgardt (2017) through an extensive Newtonian grid of N-body simulations calibrated to specific globular clusters where black hole masses have been claimed, finds that no such black holes are needed, or are even compatible with the observed joint surface density and velocity dispersion observations, above about $1000 M_\odot$, much below the typical claimed black hole masses. The only exception to the above is the case of $\omega$ Cen, where Baumgardt (2017) does find evidence for a $\sim 40,000 M_\odot$ central black hole.

In the context of our study, given the central densities of the globular clusters for which central black holes have been inferred, of order $10^4 M_\odot pc^{-3}$ and above (Harris 1996), the recent results of Baumgardt (2017) are consistent, as we would expect only very small “black hole” masses to be required under Newtonian modelling of the central MONDian region of these clusters of less than $1 M_\odot$. C.f. equation (9) under abrupt $\mu(X)$ transition functions. It is of course not impossible that some of these globular clusters, due to dynamical evolution processes and their high stellar densities, might actually contain central black holes, as might probably be the case of $\omega$ Cen. Indeed, the added central potential produced by the inner MONDian region could play a part in enhancing the formation efficiency of central black holes in globular clusters.

As changes in the transition function away from the abrupt limit of equation (4) yield always increasing “black hole masses”, the effect presented here is falsifiable, the lack of any inferred black hole (under Newtonian assumptions) at the level of the predictions of equation (9) for suitably located dynamical tracers, would clearly invalidate the hypothesis of the analysis presented, and challenge the MONDian view point. Thus, a prediction remains for gravitational anomalies in the centres of lower central density globular clusters. A more solid understanding of the effect of this central MONDian region in the context of dynamical evolution, mass segregation and N-body relaxation of globular clusters lies beyond the scope of this first presentation of the effect, but would certainly be desirable to more fully assess its consequences.
4 FINAL REMARKS

We have shown that for stellar density profiles having a central cusp shallower than $\rho \propto R^{-1}$, an inner low acceleration region will exist where $a < a_0$. For the particular case of a constant density core of density $\rho_0$, the extent of this inner region will be of $211.3(\rho_0/M_\odot pc^{-3})pc$.

Under MONDian gravity, this implies that within this inner region, dynamical tracers will behave as if under the gravitational influence of an extra mass component, when interpreted under Newtonian assumptions.

If this extra potential under Newtonian interpretations is ascribed to a central black hole, its mass will depend on the radial range over which dynamical tracers are used, with a maximum inferred black hole mass of $M_{BM}(R_{MX}) = 2.65 \times 10^6 (\rho_0/M_\odot pc^{-3})^2 M_\odot$.

The previous expression becomes hence a prediction for Newtonian inferences from volume resolved dynamics of $\rho_0 \sim 100 M_\odot pc^{-3}$ Galactic globular clusters.

As the $a_0$ threshold is approached gradually, the effect described is very sensitive to the details of the MONDian transition function used, this presents the possibility of tightly constraining this transition function through detailed measurements of the extra central “dark matter” in galactic globular clusters.

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