Note on Evaluation of Hierarchical Modular Systems

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This survey note describes a brief systemic view to approaches for evaluation of hierarchical composite (modular) systems. The list of considered issues involves the following: (i) basic assessment scales (quantitative scale, ordinal scale, multicriteria description, two kinds of poset-like scales), (ii) basic types of scale transformation problems, (iii) basic types of scale integration methods. Evaluation of the modular systems is considered as assessment of system components (and their compatibility) and integration of the obtained local estimates into the total system estimate(s). This process is based on the above-mentioned problems (i.e., scale transformation and integration). Illustrations of the assessment problems and evaluation approaches are presented (including numerical examples).

Keywords: modular systems, system design, system evaluation, multicriteria analysis, quantitative scales, ordinal scales, poset-like scales, interval multiset estimates, frameworks, heuristics

1. Introduction

In recent decades, the significance of modular (multi-component) systems has been increased (e.g., [2],[5],[8],[14],[17],[20]). This survey note describes a brief systemic view to approaches for evaluation of hierarchical composite (modular) systems. The list of considered issues involves the following: (i) basic assessment scales (quantitative scale, ordinal scale, multicriteria description, two kinds of poset-like scales), (ii) basic types of scale transformation problems (i.e., mapping 1: initial scale ⇒ resultant scale), (iii) basic types of scale integration approaches. (i.e., mapping 2: initial scales ⇒ resultant integrated scale). It is assumed that the above-mentioned mappings are monotone (or anti-monotone). Here, data envelopment analysis is not considered (e.g., [29]). Our evaluation of composite (modular) systems is examined as assessment of system components (and their compatibility) and integration of the obtained local estimates into the total system estimate(s) (e.g., [12],[13],[14],[18]). Mainly, integration of component estimates is considered (estimates of system component compatibility can be examined as additional system components). Thus, the described system evaluation approach considered as a combination of the above-mentioned problems (i.e., transformation of scales and integration of scales). Now, it is reasonable to point out the following:

(a) composite (modular) system (e.g., two-layer hierarchy) \( S = S_1 \star \ldots \star S_i \star \ldots \star S_m \) (where \( S_1, \ldots, S_i, \ldots, S_m \) are the system components/parts) (Fig. 1) (e.g., [12],[13],[14],[17],[18]),

(b) local domains (e.g., scales, sets of estimates) to evaluate the quality (excellence, “utility”) of the system components \( \{S_1, \ldots, S_i, \ldots, S_m\} \) (and/or their design alternatives DAs: \( \{X_{i,1}, \ldots, X_{i,q}\mid i = 1, m\} \)) and a total domain (scale, set of estimates) to evaluate the whole system \( S \) (Fig. 2) (e.g., [12],[13],[14],[17],[18]).

Generally, two basic situations can be examined (e.g., [12],[13],[14]):

**Situation 1.** Evaluation of a whole system to directly obtaining the total system estimate (e.g., expert judgment procedures, system testing procedures, statistical data processing, collection and processing of data from databases, technical measurement procedures, hybrid procedures).

**Situation 2.** Two-stage framework: 2.1. Evaluation (assessment) of system components. 2.2. Integration of the components estimates into the total system estimate (this stage can be executed several times hierarchically).

Usually, the following basic approaches are used:

1. Expert judgment (e.g., domain experts).
2. Measurement procedures: (a) technical measurement (i.e., physical system testing), (b) statistical measurement and data processing, (c) expert judgment, (d) assessment based on data bases, and (e)
composite (hybrid) procedure.

3. Computer simulation.

\[ S = S_1 \star \ldots \star S_i \star \ldots \star S_m \]

Fig. 1. Composite (modular) system

In this paper, the following evaluation problems are examined: (1) assessment of DAs for leaf nodes of the system model (i.e., system components) (e.g., quantitative scale, ordinal scale, multicriteria description, poset-like scales); (2) integration of the obtained estimates for DAs to obtain the integrated (total) estimate for the composite final system (or its versions). An illustration of the evaluation procedure for two-layer system is presented in Fig. 2.

Fig. 2. Evaluation scheme for two-layer system

An example of three-layer system structure is presented in Fig. 3.

Fig. 3. Three-layer composite (modular) system

Here, the following evaluation problems are considered: (1) assessment of DAs for leaf nodes of the system model (i.e., system components) (e.g., quantitative scale, ordinal scale, multicriteria description, poset-like scales); (2) integration of the obtained estimates for DAs to obtain integrated estimates for the composite system nodes (i.e., system parts, at the higher system hierarchy); (3) integration of the obtained estimates for system parts to obtain integrated (total) estimates for the final system (or its versions). An illustration of the evaluation procedure for three-layer system is presented in Fig. 4.
Fig. 4. Evaluation scheme for three-layer system

2. Considered Types of Assessment Scales

Table 1 contains the considered types of assessment scales (for system parts/components, for final system): quantitative scale, ordinal scale, multicriteria description, poset-like scales (e.g., $[6,9,12,13,14,18,22,24,28,30]$).

| Types of scales (descriptions)                                      | Sources     |
|-------------------------------------------------------------------|-------------|
| 1. Quantitative scale                                             | $[6,9,28]$  |
| 2. Ordinal scale                                                  | $[4,11,12,22,24,30]$ |
| 3. Multicriteria description (vector-like estimate based on quantitative and/or ordinal estimates) | $[9,22,23,24,28]$ |
| 4. Poset-like scale based on ordinal estimates                    | $[12,13,14,17]$ |
| 5. Poset-like scale based on interval multiset estimates          | $[18]$      |

Let us consider illustrations for the above-mentioned basic assessment scales.

First, Fig. 5 depicts illustrations for quantitative scale, qualitative ordinal scale, and multicriteria description:

(a) quantitative scale, e.g., interval $(\beta, \alpha)$, $\alpha$ corresponds to the best point, $\beta$ corresponds to the worst point (Fig. 5a);

(b) qualitative (ordinal) scale: $[1, 2, ..., \kappa]$, 1 corresponds to the best point, i.e., point $i$ dominates point $i + 1$ (Fig. 5b); and

(c) multicriteria description (i.e., vector-like estimates) (Fig. 5c).

Note, domination binary relations for the points, which belong to the scales in cases (a) and (b), are evident. In the case (c), domination is illustrated in Fig. 5: $\alpha_2 \succ \beta_2$, $\alpha_2 \succ \beta_3$, $\alpha_2 \succ \beta_4$. In the case of domination by Pareto-rule (e.g., $[22,23]$), the basic domination binary relation is extended by cases as $\alpha_2 \succ^P \beta_1$. Here, the following ordered layers of quality can be considered (as a special ordinal scale $D$, by illustration in Fig. 5c):

(i) the ideal point (the best point) $\alpha^I$,  


(ii) a layer of Pareto-efficient points (e.g., points: \( \{ \alpha_1, \alpha_2, \alpha_3, \alpha_4 \} \)),

(iii) near Pareto-efficient points (the points are close to the Pareto-layer, e.g., points: \( \{ \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \} \)),

(iv) a next layer of quality (i.e., between near Pareto-efficient points and the worst point, e.g., points: \( \{ \gamma_1, \gamma_2 \} \)), and

(v) the worst point.

Fig. 5. Quantitative scale, ordinal scale, multicriteria description

The description of poset-like scales (or lattices) for quality of composite (modular) systems (based on ordinal estimates of DAs and their compatibility) was suggested within framework of HMMD approach (e.g., [12][13][14][17]). Here, two cases have to be examined: (1) scale for system quality based on system components ordinal estimates \( \psi = \sum_{k=1}^{n} \nu_{r} = m \) and by component compatibility. Fig. 6 depicts the scale-poset and estimates for assessment problem \( S = X \times Y \times Z \) is considered (Fig. 1). The following ordinal scales are used: (a) ordinal scale for elements (priorities) is \( [1, 2, 3, 4] \), (b) ordinal scale for compatibility is \( [0, 1, 2, 3, 4] \). For this case, Fig. 6a depicts the poset of system quality by components and Fig. 6b depicts an integrated poset with compatibility (each triangle corresponds to the poset from Fig. 6a).

Generally, the following layers of system excellence can be considered (Fig. 6b, this corresponds to the resultant system scale \( D \) in Fig. 5b):

1. The ideal point \( N(S^I) \) (\( S^I \) is the ideal system solution).

2. A layer of Pareto-efficient solutions: \( \{ S^p_1, S^p_2, S^p_3 \} \); estimates are: \( N(S^p_1) = (2; 3, 0, 0) \), \( N(S^p_2) = (3; 1, 1, 1) \), and \( N(S^p_3) = (4; 0, 2, 1) \).

3. A next layer of quality (e.g., neighborhood of Pareto-efficient solutions layer): \( \{ S'_1, S'_2, S'_3 \} \); estimates are: \( N(S'_1) = (1; 3, 0, 0) \), \( N(S'_2) = (2; 1, 1, 1) \), and \( N(S'_3) = (3; 0, 2, 1) \).

4. A composite solution of this set can be transformed into a Pareto-efficient solution on the basis of a simple improvement action(s) (e.g., as modification of the only one element).

5. The worst point \( S_0 \); estimate is: \( N(S_0) = (1; 0, 3, 0) \).

Note, the compatibility component of vector \( N(S) \) can be considered on the basis of a poset-like scale too (as \( n(S) \) ) [13][14]. In this case, the discrete space of system excellence will be an analogical lattice.

The poset-like scales based on interval multiset estimates have been suggested in [18]. Analogically, two cases have to be considered: (i) system estimate by components, (ii) system estimate by components and by component compatibility. Fig. 7 depicts the scale-poset and estimates for assessment problem \( P^{3,4} \) (assessment over scale \( [1, 3] \) with four elements; estimates \( (2, 0, 2), (3, 0, 1) \), and \( (1, 0, 3) \) are not used) [18]. Evidently, the above-mentioned resultant system ordinal scale \( D \) can used here as well.
Fig. 6. Poset-like scale for composite system based on ordinal estimates

(a) Poset-like scale by elements $n(S)\prec 3,0,0$ Ideal point

(b) Poset-like scale by elements and by compatibility $N(S)$

Fig. 7. Poset-like scale ($P^{3.4}$) [18]
An example of four-component system composition is presented in Fig. 8. It is assumed, interval multiset estimates (scale from Fig. 7) are used for assessment of DAs. For evaluation of the final system consisting of four components, it is necessary to take into account estimates of compatibility (e.g., \([0, 1, 2, 3]\)). The corresponding integrated poset-like scale is depicted in Fig. 9 (median-like integral system estimates are assumed \([3, 4, 0, 0]\)).

\[
S = X \ast Y \ast Z \ast V \\
S_1 = X_3 \ast Y_5 \ast Z_3 \ast V_2 \\
S_2 = X_3 \ast Y_5 \ast Z_3 \ast V_2 \\
S_3 = X_3 \ast Y_3 \ast Z_2 \ast V_2
\]

Fig. 8. Example of four-component system

compatibility \(w = 1\)

\[
\begin{align*}
\epsilon_{1}\cdot{} & (1; 4, 0, 0) \\
\epsilon_{2}\cdot{} & (1; 3, 1, 0) (S_1^p) \\
\epsilon_{3}\cdot{} & (1; 2, 2, 0) \\
\epsilon_{4}\cdot{} & (1; 1, 3, 0) (S_1^p) \\
\epsilon_{5}\cdot{} & (1; 0, 4, 0) \\
\epsilon_{6}\cdot{} & (1; 0, 3, 1) \\
\epsilon_{7}\cdot{} & (1; 1, 2) \\
\epsilon_{8}\cdot{} & (1; 0, 2, 2) \\
\epsilon_{9}\cdot{} & (1; 0, 1, 3) (S''') \\
\epsilon_{10}\cdot{} & (1; 0, 0, 4) (S_0)
\end{align*}
\]

compatibility \(w = 2\)

\[
\begin{align*}
\epsilon_{1}\cdot{} & (2; 4, 0, 0) \\
\epsilon_{2}\cdot{} & (2; 3, 1, 0) \\
\epsilon_{3}\cdot{} & (2; 2, 2, 0) (S_2^p) \\
\epsilon_{4}\cdot{} & (2; 1, 3, 0) \\
\epsilon_{5}\cdot{} & (2; 0, 4, 0) \\
\epsilon_{6}\cdot{} & (2; 0, 3, 1) \\
\epsilon_{7}\cdot{} & (2; 1, 1, 2) \\
\epsilon_{8}\cdot{} & (2; 0, 2, 2) \\
\epsilon_{9}\cdot{} & (2; 0, 1, 3) (S_2'') \\
\epsilon_{10}\cdot{} & (2; 0, 0, 4)
\end{align*}
\]

compatibility \(w = 3\)

\[
\begin{align*}
\epsilon_{1}\cdot{} & (3; 4, 0, 0) (S^I) \\
\epsilon_{2}\cdot{} & (3; 3, 1, 0) \\
\epsilon_{3}\cdot{} & (3; 2, 2, 0) \\
\epsilon_{4}\cdot{} & (3; 1, 3, 0) \\
\epsilon_{5}\cdot{} & (3; 0, 4, 0) \\
\epsilon_{6}\cdot{} & (3; 0, 3, 1) \\
\epsilon_{7}\cdot{} & (3; 1, 1, 2) \\
\epsilon_{8}\cdot{} & (3; 0, 2, 2) (S_3^p) \\
\epsilon_{9}\cdot{} & (3; 0, 1, 3) \\
\epsilon_{10}\cdot{} & (3; 0, 0, 4)
\end{align*}
\]

Fig. 9. Integrated poset (assessment problem \(P^{3,4}\)), compatibility scale \([1, 2, 3]\)

Further, an illustration of the resultant system ordinal scale \(D\) is the following (Fig. 9):

1. The ideal solution \(S^I\), estimate is: \(e(S^I) = (3; 4, 0, 0)\).
2. A layer of Pareto-efficient solutions: \(\{S^p_1, S^p_2, S^p_3\}\); estimates (points) are: \(e(S^p_1) = (1; 3, 1, 0)\), \(e(S^p_2) = (2; 2, 2, 0)\), and \(e(S^p_3) = (3; 0, 2, 2)\).

3. A next layer of quality (e.g., neighborhood of Pareto-efficient solutions layer): \(\{S'_1, S'_2\}\); estimates (points) are: \(e(S'_1) = (1; 1, 3, 0)\), and \(e(S'_2) = (2; 0, 1, 3)\).

4. A next layer of quality \(S''\); estimate is: \(e(S'') = (1; 0, 1, 3)\).

5. The worst solution \(S_0\); estimate is: \(e(S_0) = (1; 0, 0, 3)\).

3. Transformation of Scales

Generally, main transformation problems for basic assessment scales are shown in Table 2 (note, resultant ordinal scale corresponds often to final solutions).

| Initial scale | Resultant scale |
|---------------|----------------|
|               | Quantitative scale | Ordinal scale | Multicriteria description | Poset-like scale based on ordinal estimates | Poset-like scale based on interval multiset estimates |
| 1. Quantitative scale | ⋆ | ⋆ | − | − | − |
| 2. Ordinal scale | − | ⋆ | − | − | − |
| 3. Multicriteria description (based on ordinal and/or quantitative estimates) | ⋆ | ⋆ | ⋆ | ⋆ | ⋆ |
| 4. Poset-like scale based on ordinal estimates | − | ⋆ | − | − | − |
| 5. Poset-like scale based on interval multiset estimates | − | ⋆ | − | − | ⋆ |

Here, the following basic scale transformation problems are considered:

1. Quantitative scale ⇒ Quantitative scale.
2. Quantitative scale ⇒ Ordinal scale.
3. Ordinal scale ⇒ Ordinal scale.
4. Multicriteria description ⇒ Ordinal scale. This is multicriteria ranking or sorting problem (e.g., [12,21,22,24,30]).
5. Poset-like scale ⇒ Ordinal scale (e.g., [12,14,18]).
6. Multicriteria description ⇒ Quantitative scale. This is decision making based on utility function analysis (e.g., [6,9,28]).
7. Multicriteria description (ordinal scales) ⇒ Poset-like scale, based on ordinal estimates. Here, the same ordinal scales are assumed (i.e., for each system part/component). This scale transformation type is described in (e.g., [12,14,15,17]).
8. Multicriteria description (ordinal scales) ⇒ Poset-like scale, based on interval multiset estimates. Here, the same ordinal scales are assumed (i.e., for each system part/component). This scale transformation type is described in (e.g., [18]).
9. Poset-like scale, based on interval multiset estimates $\Rightarrow$ Poset-like scale, based on interval multiset estimates. This scale transformation type is briefly described in (e.g., [18]).

10. Multicriteria description $\Rightarrow$ Multicriteria description. (some simple mappings, multidimensional scaling, etc., e.g., [3]).

Note, the above-mentioned types 4, 6, 7, and 8 correspond to the scale integration problem.

The first type of transformation (i.e., quantitative scale $\Rightarrow$ quantitative scale) (Fig. 10) can be based on a linear function ($y = ax + b$).

![Fig. 10. Quantitative scale $\Rightarrow$ quantitative scale](image)

The second type of transformation (i.e., quantitative scale $\Rightarrow$ ordinal scale) is illustrated in Fig. 11. Here, the quantitative scale (or considered value interval ($\beta, \alpha$)) is divided into a set of intervals, and each interval corresponds to a level of the resultant ordinal scale. The dividing procedure (i.e., definition of the thresholds) may be based on various approaches (e.g., computing scheme, expert judgment, usage of reference points) (e.g., [1,11,21,26]).

![Fig. 11. Quantitative scale $\Rightarrow$ ordinal scale](image)

The third type is the following. Two typical cases for transformation (i.e., mapping) ordinal scale $\Rightarrow$ ordinal scale are depicted in Fig. 12. The mapping can be based on expert judgment (i.e., professional knowledge of domain experts).

For the fourth type of the above-mentioned transformation (i.e., multicriteria description $\Rightarrow$ ordinal scale), the following main approaches are used:

1. two-stage method: vector-estimates $\Rightarrow$ utility function $\Rightarrow$ resultant ordinal estimate,
2. series detection of Pareto-layers,
3. series detection of maximal points,
4. usage of dividing curves of equal quality, i.e., curves of equal quality or subdomains of equal quality: here, expert judgment procedures or logical methods can be used (e.g., [21]) (Fig. 13),
5. frameworks based on analysis and usage of reference solutions,
6. outranking techniques (ELECTRE, PROMETHEE, etc.) (e.g., [12,24]),
7. special interactive procedures based on logical methods (e.g., [1,21,26]).
8 usage of an ordinal scale $D$ (e.g., Fig. 5c, Fig. 6b): (i) the ideal solution, (ii) Pareto-efficient points, (iii) near Pareto-efficient points (the points are close to the Pareto-layer), (iv) some other points, (v) the worst point.

In the fifth third case (poset-like scale $\Rightarrow$ ordinal scale), analogical methods (as for the transformation type 2) can be used, for example: series detection of Pareto-layers, etc.

For the case eight, Fig. 14 depicts the layers of quality (an ordinal scale $D$ as in Fig. 6b): (i) the ideal solution $e(S^I)$, (ii) Pareto-efficient points (i.e., $\{e(S_{1}^p), e(S_{2}^p), e(S_{3}^p)\}$), (iii) points of the next layer of quality, (i.e., $\{e(S_{1}'), e(S_{2}'), e(S_{3}')\}$), (iv) another point (the next layer of quality) (i.e, $e(S'')$, and (v) the worst point. Here, $I, S_{i}^p (i = 1, 4), S_{j}' (j = 1, 3), S''$ correspond to system versions.

Analogically (case nine), the total ordinal scale for system quality is depicted for poset-like scale based on interval multiset estimates in Fig. 9: (i) the ideal solution, (ii) Pareto-efficient points; (iii) points of the next layer of quality; (iv) another point (the next layer of quality); and (v) the worst point.

\begin{figure}[h]
\begin{center}
\includegraphics[width=\textwidth]{ordinal_scale}
\end{center}
\caption{Ordinal scale $\Rightarrow$ ordinal scale}  
\end{figure}

\begin{figure}[h]
\begin{center}
\includegraphics[width=\textwidth]{curves_equal_quality}
\end{center}
\caption{Curves of equal quality}  
\end{figure}

\begin{figure}[h]
\begin{center}
\includegraphics[width=\textwidth]{layers_poset_like_scale}
\end{center}
\caption{Layers at poset-like scale}  
\end{figure}

4. Integration of Scales and System Quality

Some approaches to integration of system component/compatibility estimates into a total system estimate (i.e., system evaluation) are the following (Table 3): 1. Quantitative estimates $\Rightarrow$ integrated quantitative estimate: (1.1) utility function approaches (e.g., [59]), (1.2) AHP and its modifications (e.g., [25]), (1.3) TOPSIS-like methods (TOPSIS: technique for order performance by similarity to ideal solution) (e.g., [10,27]), (1.4) frameworks based on analysis and usage of reference solutions, and (1.5) hybrid methods.

2. Quantitative estimates and ordinal estimates $\Rightarrow$ integrated ordinal estimates (or sorting problems) (e.g., [21,134]): (2.1) usage of ordinal scale $D$ (e.g., [12,15]), (2.2) series detection of Pareto-efficient points (as Pareto-layers) (e.g., [22,23]), (2.3) series detection of maximal points, (2.4) outranking techniques (e.g., [424]), (2.5) frameworks based on analysis and usage of reference solutions, and (2.6) hybrid/composite methods (e.g., [19,21]).

3. Ordinal estimates $\Rightarrow$ integrated ordinal estimates (or sorting problems) (e.g., [21,134]): (3.1) integration tables (e.g., [7,14]), (3.2) man-machine interactive procedures (expert judgment) to design
the class bounds at the total system quality domain (i.e., ordinal scale for system quality) (Fig. 15) (e.g., [11,21]), (3.3) man-machine interactive procedures (expert and logical methods) to design the class bounds at the total system quality domain (i.e., ordinal scale for system quality) (Fig. 15) (e.g., [1,21,26]), (3.4) frameworks based on analysis and usage of reference solutions, and (3.5) hybrid methods (e.g., [19,21]).

4. **Ordinal estimates** ⇒ integrated poset-like estimate (e.g., [12,14,17]): (4.1) computing the integrated poset-like estimates, (4.2) usage of expert judgment to get the integrated poset-like estimates.

5. **Poset-like estimates** ⇒ integrated poset-like estimate (e.g., [18]): (5.1) integrated estimate, (5.2) median-like estimate, (5.3) usage of expert judgment.

6. **Vector-like estimates** ⇒ integrated vector-like estimate: (6.1) unification of the initial multicriteria (i.e., multidimensional) domains, (6.2) simple integration of the initial multicriteria (i.e., multidimensional) domains (e.g., summarization by components), (6.3) special mappings.

| Methods | Scales for system components | Scale for total system quality | Type of integration | Some sources |
|---------|-----------------------------|--------------------------------|---------------------|-------------|
| 1. Utility analysis, TOPSIS, AHP | Quantitative | Quantitative | Utility function, TOPSIS, AHP | [6,9,10], [25,27] |
| 2. Integration tables | Ordinal | Ordinal | Hierarchical integration tables | [7,14] |
| 3. Pareto-approach | Quantitative, ordinal | Ordinal | Detection of Pareto-layer | [22,23] |
| 4. Outranking methods (ELECTRE, PROMETHEE) | Quantitative, ordinal | Ordinal | Detection of dominating points | [4,24] |
| 5. Layer of maximal (minimal) elements | Quantitative, ordinal | Ordinal | Detection of maximal (or/and minimal) elements | |
| 6. Man-machine procedure (expert judgment) | Ordinal | Ordinal | Dividing class bounds for multicriteria domain | [11] |
| 7. Interactive procedure (expert and logical methods) | Ordinal | Ordinal | Dividing class bounds for multicriteria domain | [11,21,26] |
| 8. Unification of measurement domains | Multicriteria description | Multicriteria description | Integration of domains (unification, consensus) | [12,14,18] |
| 9. HMMD with ordinal estimates | Ordinal estimates | Poset based on ordinal estimates | Detection of Pareto layer | [12,15,16], [17] |
| 10. HMMD with interval multiset estimates | Interval multiset estimates | Poset based on interval multiset estimates | (a) Integrated or median-like estimate, (b) Pareto layer | [18] |

From the engineering viewpoint (i.e., experience of domain experts), it may be reasonable to illustrate two methods: (a) integrated tables (Fig. 16 and Fig. 17; numerical examples of system, integrated of
tables, and system evaluation), and (b) TOPSIS (Fig. 18; an illustration of an extended version for several ideal points).

Fig. 15. Class bounds for ordinal system quality

Best point \((1, 1, \ldots, 1)\)

Bound 1

Solution class 1

Solution class 2

Bound 2

Local scales:

- \([1, 2, \ldots, k_1]\)
- \([1, 2, \ldots, k_2]\)
- \([1, 2, \ldots, k_l]\)

Bound \((r - 1)\)

Solution class \(r\)

Worst point \((k_1, k_2, \ldots, k_l)\)

Fig. 16. Example of system structure, scales for components

\[ S = A \ast B = (X \ast Y) \ast (E \ast H \ast G) \]

(scale \([1, 2, 3, 4, 5]\))

\[ A = X \ast Y \] (scale \([1, 2, 3, 4]\))

\[ B = E \ast H \ast G \] (scale \([1, 2, 3, 4]\))

\[ X \] (scale \([1, 2, 3, 4]\))

\[ Y \] (scale \([1, 2, 3]\))

\[ E \] (scale \([1, 2, 3]\))

\[ H \] (scale \([1, 2]\))

\[ G \] (scale \([1, 2]\))

Fig. 17. Integration of scales by tables
In the basic versions of TOPSIS-like methods, transformation of multicriteria description of alternatives into a final ordinal scale is based on a simplification of the problem by consideration of proximity of the alternatives to the best solution. Generally, the alternatives are ordered by the vector \( \rho = (\rho^-, \rho^+) \) where \( \rho^+ \) corresponds to proximity to the best point(s) (e.g., the ideal point(s)), \( \rho^- \) corresponds to proximity to the worst point(s).

5. Numerical Examples

Here, simple numerical examples for four-component student team is described (Fig. 19) (system component compatibility is not examined). Table 4 contains initial estimates of team elements (i.e., alternatives for system components DAs) for four types of scales:

(i) quantitative estimates (scale \((1, 3)\), \(1\) corresponds to the best level);
(ii) vector-like (two-element) ordinal estimates (scale \((x, y)\), \((1, 1)\) corresponds to the best level, e.g., \(x\) corresponds to “Mathematics”, \(y\) corresponds to “Physics”);
(iii) ordinal estimates (scale \([1, 2, 3]\), \(1\) corresponds to the best level); and
(iv) interval multiset estimates (assessment problem \(P^{3, 4}\), Fig. 7).

![Fig. 19. Example of four-component team](image)

| DA   | Quantitative estimates (scale \((1, 3)\)) | Vector-like estimates \((x, y)\) | Ordinal estimates (scale \([1, 2, 3]\)) | Interval multiset estimates (assessment problem \(P^{3, 4}\)) |
|------|-----------------------------------------|-------------------------------|-----------------------------------------|-----------------------------------------------------------|
| \(L_1\) | 1.5                                      | \((2, 1)\)                  | 1                                       | \((3, 1, 0)\)                                             |
| \(L_2\) | 1.8                                      | \((2, 2)\)                  | 2                                       | \((0, 4, 0)\)                                             |
| \(Q_1\) | 1.1                                      | \((1, 1)\)                  | 1                                       | \((4, 0, 0)\)                                             |
| \(Q_2\) | 2.7                                      | \((2, 3)\)                  | 3                                       | \((0, 3, 1)\)                                             |
| \(G_1\) | 1.2                                      | \((1, 1)\)                  | 1                                       | \((3, 1, 0)\)                                             |
| \(G_2\) | 2.4                                      | \((3, 2)\)                  | 2                                       | \((1, 2, 1)\)                                             |
| \(H_1\) | 1.4                                      | \((1, 2)\)                  | 1                                       | \((2, 2, 0)\)                                             |
| \(H_2\) | 3.1                                      | \((3, 3)\)                  | 3                                       | \((0, 2, 2)\)                                             |
The following numerical examples are presented:

**Example 1.** Quantitative estimates of DAs are integrated by the simplest additive (i.e., utility) function (Fig. 20): \( f(T_1) = 1.5 + 1.1 + 1.2 + 1.4 = 5.2 \) (the best solution), \( f(T_2) = 1.8 + 1.1 + 2.4 + 3.1 = 8.4 \), \( f(T_3) = 1.5 + 1.1 + 2.4 + 3.1 = 8.1 \), and \( f(T_4) = 1.5 + 2.7 + 1.2 + 3.1 = 8.5 \); the corresponding preference relation is: \( T_1 \succ T_3 \succ T_2 \succ T_4 \).

![Fig. 20. Resultant quantitative scale for modular solutions](image)

**Example 2.** Ordinal estimates of DAs are integrated into the resultant ordinal estimates for modular solutions (via method of integration tables, Fig. 21): \( \{e(T_1) = 1\}, \{e(T_2) = 4\}, \{e(T_3) = 3\}, \) and \( \{e(T_4) = 3\} \).

![Fig. 21. Integration of by tables for example 2](image)

**Example 3.** Vector-like (two-element) estimates are integrated into an ordinal scale for modular solutions: (1) summarization (by vector-estimate components) for each modular solution (i.e., \( T_1,T_2,T_3,T_4 \)), (2) selection of Pareto-efficient solutions) (Fig. 22):

(a) vector-like estimates: \( e(T_1) = (5,5), e(T_2) = (9,8), e(T_3) = (9,7), \) and \( e(T_4) = (8,8) \);

(b) domination (preferences): \( T_1 \succ T_2, T_1 \succ T_3, T_1 \succ T_4, T_3 \succ T_2, \) and \( T_4 \succ T_2 \).

(c) the resultant ordinal scale (type \( D \)): the layer of Pareto-efficient solution (layer 1): \( \{T_1\} \); the next layer (layer 2): \( \{T_3,T_4\} \); the next layer (layer 3): \( \{T_2\} \).

Thus, the resultant priorities are obtained: \( r(T_1) = 1, r(T_2) = 3, r(T_3) = 2, \) and \( r(T_4) = 2 \).

**Example 4.** Ordinal estimates of DAs are transformed into poset-like estimate for modular solutions (Fig. 23), selection of Pareto-efficient solutions:

(a) poset-like estimates: \( n(T_1) = (4,0,0), n(T_2) = (1,2,1), n(T_3) = (2,1,1), \) and \( n(T_4) = (2,1,1) \);
(b) domination (preferences): \( T_1 \succ T_2, T_1 \succ T_3, T_1 \succ T_4, T_3 \succ T_2, T_4 \succ T_2; \)
(c) the resultant ordinal scale (type \( D \)): the layer of Paret-efficient solutions (layer 1): \( \{T_1\} \), the next layer (layer 2): \( \{T_3, T_4\} \), the next layer (layer 3): \( \{T_2\} \).
Thus, the resultant priorities are obtained: \( r(T_1) = 1, r(T_2) = 3, r(T_3) = 2, \) and \( r(T_4) = 2. \)

\[ \begin{align*}
\begin{array}{c}
\text{Criterion 1} \\
\text{("Mathematics")}
\end{array} & \quad \begin{array}{c}
\text{Criterion 2} \\
\text{("Physics")}
\end{array} \\
4 & \quad \begin{array}{c}
\text{Best point} \\
(0, 0)
\end{array} \\
8 & \quad \begin{array}{c}
T_2 \\
\bullet
\end{array} \\
12 & \quad \begin{array}{c}
T_3 \\
\bullet
\end{array} \\
\text{Worst point} & \quad \begin{array}{c}
\text{(12, 12)}
\end{array}
\end{align*} \]

Fig. 22. Multicriteria description

\[ \begin{align*}
\begin{array}{c}
\text{The ideal point} \\
\text{n}(T_1)
\end{array} & \quad \begin{array}{c}
\text{The worst point} \\
\text{n}(T_2)
\end{array} \\
\text{n}(T_3), n(T_4) & \quad \begin{array}{c}
\text{n}(T_2)
\end{array}
\end{align*} \]

Fig. 23. Poset \( n(T) = (\eta_1, \eta_2, \eta_3) \)

\begin{itemize}
\item[(a)] interval multiset estimates: \( n(T_1) = (3, 1, 0), n(T_2) = (0, 4, 0), n(T_3) = (1, 3, 0), \) and \( n(T_4) = (1, 3, 0); \)
\item[(b)] domination (preferences): \( T_1 \succ T_2, T_1 \succ T_3, T_1 \succ T_4, T_3 \succ T_2, T_4 \succ T_2; \)
\item[(c)] the resultant ordinal scale (type \( D \)): the layer of Paret-efficient solution (layer 1): \( \{T_1\} \), the next layer (layer 2): \( \{T_3, T_4\} \), the next layer (layer 3): \( \{T_2\} \).
\end{itemize}

Thus, the resultant priorities are obtained: \( r(T_1) = 1, r(T_2) = 3, r(T_3) = 2, \) and \( r(T_4) = 2. \)

6. Conclusion

This survey paper briefly described approaches to evaluation of composite (modular) systems. In the future, it may be reasonable to consider the following research directions: (1) study of other scale transformation problems (e.g., \( \text{poset } \lambda \Rightarrow \text{poset } \mu \)), (2) study of multi-stage scale transformation procedures (frameworks), (3) examination of various real-world applications, (e.g., usage of stochastic models, fuzzy sets), (4) analysis and usage of reference solutions; (5) taking into account uncertainty, (6) special analysis of the correspondence between considered system evaluation problems, scale transformation problems, and traditional decision making problems, (7) additional attention to issues of system component compatibility assessment and integration of the corresponding estimates into the total system estimates, (8) design of a special software tool for scale transformation/integration (e.g., library of various scales, visualization support, automatic and interactive procedures), and (9) usage of the described system evaluation approaches in education (computer science, engineering, applied mathematics, management).
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