Numerical analysis of thin ring foundations under different loading conditions

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Abstract. The problem of ring foundation under uniform load or different point loads are a very common one in practice. The resulting independent degrees of freedom for thin depth ring members are the deflection and the twisting angle at any section in the ring. The governing equations for bending of ring members resting on Winkler foundation which derived by the previous researcher are solved in this research using finite differences. Also, finite element methods are used to model the problem by using two-dimensional and three-dimensional elements and spring elements to model the soil. The behaviour under point and distributed loads are investigated and compared with previous analytical results and good agreements were obtained. This study revealed that the maximum deflection decreased by 86% when the loading changed from point to uniformly distributed. Also, when the subgrade reaction was increased from (1.7 to 6 MPa), the maximum deflection decreased by 72% and the maximum moment decreased by 31%.

1. Introduction
For any building or structure, the foundation is must be sufficient to resist applied loads or to support the building. Sufficient and efficient foundations depend on the selection of foundation type and the properties of the soil beneath it. The ring shape footing is the most suitable foundation for heavy structures and it may be considered to be economical for elevated tanks, silos, cooling towers, and chimneys. The foundation with ring shape will enable the construction engineer to reduce the used materials and construction cost. Several researches were implemented on ring members placed on soil by various researchers. In 2008, Mehrjardi [1] made a study to evaluate the bearing capacity and deflection of ring foundations. The response of ring footing and circular footing was compared and presented. The research made a conclusion that the ring foundations are more appropriate and economical than circular ones especially in shoring axisymmetric constructions.
In 2009, Algin [2] studied theoretically the rectangular footing elastic settlement. The surface footing was subjected to eccentric loads and placed on sand deposits. The solution was obtained in closed-form. The resulting influence factors were given in order to calculate the elastic settlement for footing under eccentric load. The expressions were also expanded for the analytical problems of infinitely deep incompressible soil layer. This research is an attempt to understand the elastic and differential settlement of footing under eccentric load.
In 2012, Zhou et. al. [3] used the curved flexural member or beam theory to derive the differential equations for the flexural deformation of ring footings. Different boundary conditions were investigated. The elastic deflection of ring foundation subjected to point load is obtained mathematically through
solving the problem and substituting the boundary conditions. A parameter relating soil stiffness to the flexural member stiffness (non-dimensional) is suggested by the researchers. In that study, the parameter is recognized to govern the ring foundation response results. The mathematical model is verified using finite elements with different variables and the study revealed that such non-dimensional parameter gives a brief indication and guidance in the ring foundation design of special structures.

In 2013, Atalar et.al [4] studied the soil bearing capacity for shallow footing subjected to inclined eccentric load. The researchers conducted small scale laboratory tests in order to obtain the ultimate bearing capacity of a strip footing. The foundation was placed on sand and under inclined eccentric load. The embedment ratio of footing in sand was varying from zero to one. The laboratory outcomes were compared with the empirical relationships and the difference between them was 25%.

In 2015, Naseri and Hosseinionia [5] carried out finite difference analysis study on the deflection of ring footing on soil. The aim of the study was to obtain solution in closed-form in order to calculate ring footing elastic deflection. This calculation is made through using the theory of elasticity and defining displacement influence factors. These factors consider the non-homogeneity of soil, the ring dimensions, ring stiffness, and the embedment of ring footing. The obtained results were compared with the previous analytical outcomes and good agreement was obtained.

In 2016, Nayyeri et. al. [6] derived the governing equations of ring on Winkler foundation considering the bending and torsion effects. The soil was simulated as elastic material in the derivation assumptions. The load on foundation was assumed to be a normal concentrated load. Estimation of the deflection and twist has been obtained for a range of practical field applications. The response of rings under several point loads can be calculated using the developed closed-form equations using superposition principle. The finite elements were used to check the validity of the proposed equations in calculating the settlement and angular twist for three ring foundations with different radii and cross sections.

In 2017, Keshavarz A., Kumar J. [7] used the (SCM) stress characteristics method to estimate the footing bearing capacity. The considered problem was ring foundation with smooth and rough bases. Different failure mechanisms were considered for ring footings. Two mechanisms were selected for smooth base case and four mechanisms for rough base case. A curved non-plastic wedge was utilized for a rough base footing. The stress singularities were considered in the analysis at the inner and outer edges of foundations. The bearing capacity coefficients, Nc, Nq and Nc were given in the form of a function of inner to outer radii ratio of footing and internal friction angle of soil. The analysis results were compared with the reported computational and experimental data and good correlation between them were obtained.

Based on previous researches the differential equation derived by Nayyeri et. al. [6] is selected in the present study. The finite difference method is selected to solve the differential equations numerically as the closed-form solution developed by Nayyeri et. al. [6] is limited to simple loading and boundary conditions. The Nayyeri et. al. [6] solution focused on obtaining expressions for deflection and twist only. The 2D and 3D finite elements through ABAQUS software are used in the present study to check the finite difference developed model.

2. Finite difference method

The differential equations for bending of thin ring member on Winkler foundation was derived by Nayyeri et. al. [6]. The derivation was established on basis of small strain theory and linear Hooke’s law.

\[
\frac{d^2}{ds^2}\left(-EI\left(\frac{d^2w}{ds^2} + \frac{d^2w}{R ds^2}\right)\right) - \frac{d}{ds}\left(GJ\left(-\frac{1}{R} \frac{dw}{ds} + \frac{d^2\phi}{ds^2}\right)\right) = q - kw = 0 \tag{1}
\]

\[
\frac{d}{ds}\left(GJ\left(-\frac{1}{R} \frac{dw}{ds} + \frac{d^2\phi}{ds^2}\right)\right) - \frac{EI}{R} \left(\frac{d^2w}{ds^2}\right) + T - k'\phi = 0 \tag{2}
\]

where s is the distance along the centerline of the ring,

w=w(s) is the centerline ring deflection,

\(\phi = \phi(s)\) is angular twist,
G is the ring shear modulus,
J is the polar moment of inertia of ring section,
k is the vertical subgrade modulus,
k’ is the rotational subgrade modulus,
EI is the ring bending rigidity,
q is the distributed load,
M is the bending moment,
V is the shear force,
T is the distributed torque
and R is the radius of ring centerline.

The slab cross section is shown in Figure 1.
The bending and twisting moments can be calculated using the equations following:

\[ M = -EI \left( \frac{\theta}{R} + \frac{d^2w}{ds^2} \right) \]  
(3)

\[ T = GJ \left( -\frac{1}{R} \frac{dw}{ds} + \frac{d\theta}{ds} \right) \]  
(4)

*Figure 1. General representation of the prismatic ring member [6]*

The finite differences are used to solve the governing differential equation using polar coordinates. At each node, the differential equations of ring foundation are commuted by difference expressions to simulate the ring. The ring peripheral length in the (s) direction is divided into length intervals (\(\Delta s\) or \(R\Delta\theta\)) (Figure 2).
3. Finite element method
The numerical modelling is made using ABAQUS software. The 8 nodes quadrilateral element is used to model the problem in two dimensions and 8 node brick element to model the problem in three dimensions. The soil is modelled using Winkler spring or spring element. Figure 3 shows the numerical model assumed for the ring foundation under point load. The finite element analysis is made carefully through selecting the suitable elements number for modelling the ring problem. A mesh sensitivity analysis is carried out such that more mesh refinement did not affect the solution results.

4. Verification of the numerical method
The problem of the ring resting on springs was modelled using two numerical methods. Namely, the finite differences and finite elements are used in order to obtain a brief analysis results. Both numerical solutions are based on discretizing the differential equations or the continuum into a member or element of finite size. To study the response of the ring footing under a single point load, three different loading examples solved by Nayyeri et. al. [6] are selected. The finite elements simulation of the ring on springs under a point load was implemented using ABAQUS software. Comparisons of the settlements and angular twists of the ring were made between the analytical Nayyeri et. al. [6] solution and the present study numerical solution. Three different footings examples, case 1, case 2 and case 3 were analysed with different radii of 3050, 6100 and 9150 mm, respectively. The values of width, depths and radii of the ring examples are shown in Figure 4. The ring is under the action of a 10 kN point load with different locations. The ring material elasticity modulus was 20.7 MPa and Poisson’s ratio was 0.35 in the selected examples. The subgrade reaction of soil (k) (multiplied by ring width) is taken to be 1.7MPa for the settlement and $k' = k \left( \frac{b^3}{12} \right)$ for twist.
Figure 4. Case previously studied by Nayyeri et al. [6]

Figure 5 shows the deflection or settlement under the ring footing for the first case using finite difference and finite element methods (brick elements) with a radial position (the 10 kN point load at θ=0). Also, Figure 6 shows the other two cases and good agreement is obtained with the previous analytical study of Nayyeri et al. [6] with a maximum tolerance of 3% and 5% for finite differences and finite elements respectively. The maximum deflection of the ring footing occurs at load application position (θ = 0). The deflection reduces with enlarging the value of θ. The effect of the point load on the deflection for θ= π/2 and beyond can be neglected for the studied cases. Figure 7 show for case 1 the variation of the angle of twist with radial positions. Small differences between the previous analytical and the present study numerical results were obtained.

| Case number | Radius (cm) | Ring width (b) (cm) | Ring depth (d) (cm) |
|-------------|-------------|---------------------|---------------------|
| 1           | 305 (120)   | 61.24               | 30.5 (12)           |
| 2           | 610 (240)   | 91.5 (36)           | 45.75 (18)          |
| 3           | 915 (360)   | 122.8 (48)          | 61 (24)             |
5. Parametric study
The influence of subgrade reaction, ring depth and load type on the response of ring foundations were investigated. A case 1 ring foundation under equivalent uniform load (with value equal point load divided by the ring area) is considered in the parametric study.

5.1 Effect of subgrade reaction
The maximum ring settlement is found to be reduced as the soil subgrade reaction was enlarged as shown in Figure 8. When the soil subgrade reaction (multiplied by ring width) was enlarged from (1.7 to 6 MPa), the maximum settlement or deflection reduced by 72%. Figure 9 illustrates the variation of soil subgrade reaction with ring maximum bending moment. It is recognized that the maximum moment reduced with enlarging the subgrade modulus for the same range by 31%.

5.2 Effect of ring depth
Figure 10 shows the variation of ring depth with ring maximum deflection. The maximum settlement or deflection had a lesser value when the ring depth was enlarged. If the ring depth enlarged from (305 to 915 mm), the maximum deflection decreased by 1% because the footing self-weight is neglected in the analysis. Figure 11 shows the variation of ring depth with ring maximum bending moment. It is recognized that the maximum moment increased with increasing ring depth by 52% and this is may be due to the increase in the flexural rigidity of the footing.
5.3 Effect of loading type
The effect of load type on ring maximum deflection was investigated. The maximum deflection had a reduction from (2.2 mm to 0.308 mm) or by 86% when the loading changed from point to uniformly distributed.

6. Conclusions
Numerical simulation for the problem ring footings resting on Winkler foundation was developed herein using finite difference and finite element techniques as the closed-form solutions are limited to very simple applications. These techniques extended the application for analysing ring foundation with complicated geometry and loading.

The results of the previous analytical solution were compared with present numerical solutions and for several foundation sizes and varying positions for verification purposes. The numerical solution results are shown to be comparable with previous analytical solution. The maximum and average relative errors of 3 and 5% were obtained for finite differences and finite elements respectively.

A parametric study was made to study the behaviours of ring foundations under different loading types. The maximum deflection had a reduced value by 86% when the loading changed from point to uniformly distributed with equal resultant force. When the soil stiffness increased or the subgrade reaction increased from (1.7 to 6 MPa), the maximum deflection decreased by 72% and the maximum moment decreased with increasing subgrade by 31%. When the ring depth was increased from (305 to 915 mm), the maximum deflection decreased by 1% because the footing self-weight is neglected in the analysis. While the maximum moment increased with increasing ring depth by 52% and this is may be due to the increase in the flexural rigidity of the footing.

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