Magnetic Circuit Model combined with Play Model Obtained from Landau-Lifshitz-Gilbert Equation

H Tanaka¹, K Nakamura¹ and O Ichinokura¹
¹Graduate School of Engineering, Tohoku University, 6-6-05 Aoba Aramaki Aoba-ku, Sendai 980-8579, Japan
E-mail: hideaki.tanaka.s7@dc.tohoku.ac.jp

Abstract. This paper presents a novel magnetic circuit model considering magnetic hysteresis behavior, in which dc hysteresis is expressed by a play model while classical and anomalous eddy current losses are calculated by magnetic circuit elements. We describe an efficient method for obtaining the play model from the minimum measured $B$-$H$ loops. It is proved that the proposed magnetic circuit model can calculate hysteresis loops under Pulse Width Modulation (PWM) excitation with high accuracy in a short time.

1. Introduction
In recent years, establishment of a practical and simple method for calculating iron loss including magnetic hysteresis behavior is required for developing high-efficient electric machines with short-term. A magnetic circuit model is one of the potential candidates because it is simple model with fast computation time. In our previous paper [1], a magnetic circuit model considering the magnetic hysteresis was proposed. In this model [1], dc hysteresis is expressed by a simplified micromagnetic model using Landau-Lifshitz-Gilbert (LLG) equation [2], while classical and anomalous eddy current losses are calculated by magnetic circuit elements. Although the simplified micromagnetic model is derived from a few measured dc hysteresis loops, the magnetic circuit model [1] is able to simulate dc hysteresis loops in various maximum flux densities with high accuracy. However, this model takes relatively long time for calculation since the LLG equation requires convergence calculation. Meanwhile, a Play model [3] is a practical method from a viewpoint of the calculation time. However, a large amount of measured hysteresis loops is required to obtain the play model. To overcome these problems, we present an efficient method for obtaining the play model from the simplified micromagnetic model which can be derived from the minimum measured dc hysteresis loops.

2. Play model obtained from simplified micromagnetic model
The motion of the magnetizations in a magnetic substance can be represented by the following LLG equation:

$$\frac{d\mathbf{m}_i}{dt} = -\frac{1}{1+\alpha^2}\left\{\gamma \left(\mathbf{m}_i \times \mathbf{H}_{\text{eff}}\right) + \alpha \left(\mathbf{m}_i \times (\mathbf{m}_i \times \mathbf{H}_{\text{eff}})\right)\right\}$$

where the magnetization vector is $\mathbf{M}_i = M_s \mathbf{m}_i$, the spontaneous magnetization is $M_s$, the normalized magnetization vector is $\mathbf{m}_i$, the gyromagnetic ratio of electron is $\gamma$, the damping constant is $\alpha$, and the effective field is $\mathbf{H}_{\text{eff}}$, respectively.
The simplified micromagnetic model [2] is derived based on several assumptions: each crystal grain has single domain particle, and the domain wall motion is neglected, etc. Thereby, the effective field $H_{\text{eff}}$ in (1) is given by

$$H_{\text{eff}} = H_{\text{app}} + H_{\text{ani}} + H_{\text{ela}}$$

$$= H_{\text{app}} - \frac{1}{M_s} \frac{\partial}{\partial m} (E_{\text{ani}} + E_{\text{ela}}),$$

where the applied field is $H_{\text{app}}$, the anisotropy field is $H_{\text{ani}}$, the field generated by magnetoelastic energy is $H_{\text{ela}}$, the magnetic anisotropy energy is $E_{\text{ani}}$, and the magnetoelastic energy is $E_{\text{ela}}$, respectively. The magnetic anisotropy energy $E_{\text{ani}}$ and the magnetoelastic energy $E_{\text{ela}}$ are defined as

$$E_{\text{ani}} = \frac{h_{\text{ani}} M_s}{2} (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2),$$

$$E_{\text{ela}} = b_2 x^2 + b_4 x^4 + b_6 x^6,$$

where the direction cosines of magnetization vectors with respect to $x$, $y$, and $z$ axes (easy axes) of each grains are $a_1$, $a_2$, and $a_3$, the coefficient of magnetic anisotropy is $h_{\text{ani}}$, and the parameters of Taylor expansion are $b_2$, $b_4$, and $b_6$, mean magnetization is $\bar{m}$, respectively. It is considered that the magnetoelastic energy is symmetric with respect to the direction of the mean magnetization. Therefore, it is assumed that the magnetoelastic energy is expressed by Taylor expansion. In addition, although the LLG equation is generally used for expressing the dynamic hysteresis behavior, only steady-state value of the LLG equation is used for expressing dc hysteresis. Figure 1 shows dc hysteresis loops calculated by the simplified micromagnetic model. All the parameters of the simplified micromagnetic model are determined to fit the dc hysteresis loops of the maximum flux density $B_m = 1.2$T in Figure 1. As shown in the figure, the model can calculate dc hysteresis loops in various flux densities with high accuracy using the same parameters. However, the calculation time of the simplified micromagnetic model is relatively long because the LLG equation requires convergence calculation.

Meanwhile, a play model [3], which needs no convergence calculation, is efficient for simulating hysteresis behavior from a viewpoint of calculation time. However, the play model requires a large amount of measured symmetric dc hysteresis loops. To overcome these problems, we present an efficient method for obtaining the play model from the simplified micromagnetic model. Figure 2 indicates the symmetric dc hysteresis loops in various maximum flux densities at intervals of 0.04 T which is calculated by the simplified micromagnetic model. By using the simplified micromagnetic model, the play model can be obtained without a large amount of measurement.

3. Proposed magnetic circuit model and simulation results
To prove the usefulness of the proposed method, the dynamic analysis of a transformer shown in Figure 3 was conducted. As shown in the proposed magnetic circuit model, dc hysteresis is expressed by the play model which is determined by the simplified micromagnetic model [2], while classical and anomalous eddy current losses are calculated by magnetic circuit elements. Figure 4 shows the measured and calculated hysteresis loops under Pulse Width Modulation excitation. It is clear that the proposed model is able to calculate the hysteresis loops under PWM excitation with high accuracy including minor loops. Table 1 indicates the comparison of the calculation time between previous [1] and proposed models. It is understood that the proposed magnetic circuit model reduced remarkably the calculation time.

4. Conclusion
This paper presented the novel magnetic circuit model which is combined with the play model. The play model can be obtained from the simplified micromagnetic model using LLG equation without a
large amount of measurement. The proposed model can calculate the hysteresis loops under PWM excitation with high accuracy and remarkably reduced the calculation time in comparison with the previous model.

This work was supported by Grant-in Aid for JSPS Fellows (26-5193).

**Figure 1.** Measured and calculated dc hysteresis loops of non-oriented silicon steel at the maximum flux density $B_m = 0.4$ T, 0.8 T, 1.2 T.

**Figure 2.** Calculated dc hysteresis loops from $B_m = 0.04$ T to 1.2 T at intervals of 0.04 T.

**Figure 3.** Experimental circuit and the proposed magnetic circuit model.

**Figure 4.** Measured and calculated hysteresis loops under PWM excitation.

| Table 1. Comparison of the calculation time between previous [1] and proposed models. |
|-----------------------------------------------------------
| Number of steps per 1 cycle | Previous model [1] | Proposed model |
|-----------------------------|--------------------|---------------|
|                             | 400                | 400           |
| Calculation time of 1 cycles| 85 min.            | 0.15 min.     |

**References**

[1] Tanaka H, Nakamura K, and Ichinokura O 2015 *J. Magn. Soc. Jpn.* **39** 65-70

[2] Oshima H, Uehara Y, Shimizu K, Inagaki K, Furuya A, Fujisaki J, Suzuki M, Kawano K, Mifune T, Matsuo T, Watanabe K, and Igarashi H 2014 *J. Jpn. Soc. Powder Metallurgy* **61** S238-S241

[3] Bobbio S, Milano G, Serpico C, and Visone C 1997 *IEEE Trans. Magn.* **33** 4417-4426