Superalgebras in Many Types of M-Brane Backgrounds and Various Supersymmetric Brane Configurations

TAKESHI SATO

Institute for Cosmic Ray Research, University of Tokyo, 3-2-1 Midori-cho, Tanashi, Tokyo 188-8502 Japan

Abstract

We derive superalgebras in many types of supersymmetric M-brane backgrounds. The backgrounds examined here include the cases of the M-wave and the M-Kaluza-Klein monopole. On the basis of the obtained algebras, we deduce all the supersymmetric non-orthogonal intersections of the M-Kaluza-Klein monopole and the M-5-brane at angles. In addition, we present a 1/4 supersymmetric worldvolume 3-brane soliton on the M-5-brane in the M-5-brane background as an extended solution of the 3-brane solitons of the M-5-brane by Howe, Lambert and West. This soliton can be interpreted as a certain intersection of three M-5-branes.
1 Introduction

The M-theory is currently a most hopeful candidate for a unified theory of particle interactions\[1\][2] and is extensively studied from various points of view\[3\][4][5]. Among them, the analysis via superalgebra is one of the most powerful approaches to investigate its various properties\[5\][6]. Since there are, of course, two kinds of supersymmetries, two kinds of algebras have been discussed so far: spacetime superalgebra and worldvolume ones. The former was first constructed as the most general modification of the standard D=11 supersymmetry algebra\[8\][9], and then deduced explicitly from M-brane actions in the flat background via Noether method\[10\](see also ref.\[11\]). The latter, defined on the flat (p+1)-dimensional worldvolumes of p-branes, were constructed as the maximal extensions of the (p+1)-dimensional supertranslation algebras\[12\][13]. Various possible supersymmetric brane intersections were deduced from both of the above algebras. The same analyses were also applied to the cases of string theories\[11\][13][14][15], although there are some subtleties in the worldvolume cases. In this way the discussions of superalgebras had been based only on the flat cases until recently.

In the previous paper\[16\], however, we have proposed the method of deriving spacetime superalgebras in supersymmetric brane backgrounds, i.e. “non-flat” cases, in terms of M-theory.† The first motivation for this extension to non-flat cases has been to get the superalgebras of the 10-dimensional massive IIA theory\[17\][18][19], which does not admit the flat background owing to the existence of the cosmological constant\[20\][21]. (We have applied the method to this case in ref.\[22\].) The idea presented in ref.\[16\] is as follows: let us consider a “test” brane, the action of which is invariant under local supertransformation. First, suppose we take the background of the test brane to be flat, as done in ref.\[11\][10]. Then, the flat background has supertranslation symmetry, of course, and the brane action is proved to have the same supertranslation symmetry. In other words, the symmetry of the brane action in a fixed background is equal to the (unbroken) symmetry of the fixed background. So, we can define the corresponding Noether supercharge and obtain the superalgebra. Next, suppose the test brane to be in a brane background which have some portions of supersymmetry.‡ (This means that we take its background to be the brane solution.) Then, by analogy with the above case, the test brane action can be expected to have the same portions of supersymmetry as the background. If the action does have the supersymmetry, it should be possible to define

† The possibility of this computation has been pointed out in the earlier paper ref.\[11\] for a different purpose (related to nontrivial topologies), although it was not shown explicitly in it.
‡ Here, we assume that this background actually consists of such a large number of coincident M-branes that the setting of “test brane” is justified, as done in ref.\[23\].
the corresponding Noether supercharge and obtain the superalgebra in the same way. (We call it “the superalgebra via the brane probe” because we “probe” the supersymmetry of the background via the test brane.) Since the anti-commutator of the supercharge is written in terms of an embedding of the test brane in the brane background, the consistency of this method should be confirmed by deducing from the superalgebra the previously obtained supersymmetric configurations of two M-branes, as the corresponding supersymmetric embeddings. In the paper we have examined the above idea explicitly in the four cases: a test M-2-brane and a test M-5-brane in the M-2-brane background and the M-5-brane background, and we have confirmed their consistency by deducing from the algebras all the 1/4-supersymmetric orthogonal intersections of the four combinations of two M-branes known before.

It is not evident, however, that the above discussions hold true in cases of backgrounds and probes other than the M-2- and the M-5-branes. So, the first purpose of this paper is to clarify how generically the method is applicable. In section 2, we investigate all the cases of 1/2-supersymmetric “basic” M-brane backgrounds and probes possible to discuss: the M-wave, the M-2-brane, the M-5-brane and the M-Kaluza-Klein monopole as backgrounds, and the M-wave, the M-2-brane and the M-5-brane as probes (i.e. eight extra cases in addition to the previous four ones). The concrete procedures are as follows: First, we substitute one of the M-brane solutions for the background of each test brane action as was done in ref., and prove the invariance of the action under the unbroken supersymmetry transformation. Next, we derive the representation of the supercharge in terms of the worldvolume fields of the test brane and their conjugate momenta, compute its anti-commutator to obtain the superalgebra, Finally, we confirm their consistency by deducing all the previously known supersymmetric orthogonal intersections of any combinations of the two M-branes among the above. We note that we cannot discuss the cases of the other 1/2-supersymmetric “basic” M-branes: the M-9-brane background, the M-Kaluza-Klein monopole probe and the M-9-brane probe. This is because full \( \kappa \)-symmetric actions have not been constructed yet in these cases (see ref.).

Another purpose of this paper, inspired by the above extension, is to investigate supersymmetric configurations in this set-up, i.e. supersymmetric embeddings of test branes in brane backgrounds. The supersymmetric configurations examined here include not only non-orthogonal intersections of two M-branes at angles but also a nontrivial worldvolume soliton on a brane in a brane background. As to the former cases, non-orthogonal intersections at angles have been investigated so far by using constraints of Killing spinors.

§ In this case the test brane corresponds to one of the two M-branes and the background corresponds to the other.
of the branes, as in ref.[30][31][32]. However, it should be possible to deduce them from
the obtained superalgebras, since all the intersecting configurations of two branes should
be expressed as the corresponding embeddings of probes in brane backgrounds in this
method. What we should do first is to find the supersymmetric embeddings that corre-
spond to the non-orthogonally intersecting two M-branes at angles, and the next is to
examine the preserved supersymmetry for each value of the angles. In subsection 3.1
we investigate the cases of the M-5-brane in the M-Kalza-Klein monopole background,
and deduce all the supersymmetric intersections of the two M-branes at angles from the
superalgebras, most of which have been previously unexamined.

In subsection 3.2 we present the latter: a worldvolume 3-brane soliton on the M-5-
brane in the M-5-brane background, that is, this is a solution of the equations of motions
of the brane action in the nontrivial (brane) background, while the usual worldvolume
solitons are constructed as the solutions of the equations of motions of the brane action in
the flat backgrounds[33][34][35][12]. The soliton we present here is the extended version
of the 3-brane soliton of the M-5-brane presented by Howe, Lambert and West in ref.[12].
The soliton is interesting not only in that it can be interpreted as a certain intersection
of three M-5-branes, but also in that each of the three branes is expressed in a way
different from the other two: a solution of supergravity, a(n) (embedded) source, and a
worldvolume soliton. We can easily prove by using the superalgebra that the soliton has
1/4 supersymmetry.

Throughout this paper, the invariance of the test brane actions are proved to the full
order in $\theta$, while the explicit computations are performed only up to the low orders which
might contribute to the central charges at zeroth order in fermionic coordinates $\theta$. (It
is very difficult to derive superalgebras to the full order in $\theta$.) The obtained algebras,
however, is useful enough since we can discuss all the supersymmetric configurations only
on the basis of the bosonic terms of the superalgebras. The important fact in the compu-
tations is that we can reduce the superspace in a brane background with supercoordinates
$(x, \theta)$ to that with the coordinates $(x, \theta^+)$, where the index $+$ of $\theta^+$ implies that $\theta^+$ has a
definite worldvolume chirality of the background. The reason is the following: since half
of supersymmetry is already not the symmetry of the system owing to the existence of
the background brane, the corresponding parameter $\theta^-$ must not be transformed. So, the
conjugate momentum of $\theta^-$ does not appear in the supercharge $Q^+$, which means that
the terms including $\theta^-$ cannot contribute to the central charges at zeroth order in $\theta$. So,
we ignore the terms including $\theta^-$ and set $\theta^- = 0$ from the beginning.

This paper is organized as follows: in section 2 we show the invariance of various test
M-brane actions in different M-brane backgrounds stated above under the supertransfor-
mation corresponding to the symmetry of the backgrounds, and derive superalgebras in the backgrounds, respectively. In subsection 2.1 we begin with the M-2-brane background, and we deal with the M-5-brane background in subsection 2.2. We discuss the M-wave background in subsection 2.3, and the M-Kaluza-Klein monopole background in subsection 2.4. In subsection 3.1 we deduce all the non-orthogonal intersections of the M-5-brane with the M-Kaluza-Klein monopole at angles from the superalgebra. In subsection 3.2 we present a worldvolume 3-brane soliton on the M-5-brane in the M-5-brane background. Finally, in section 4 we give short summary.

Before starting discussions we present the notations in this paper. We use “mostly plus” metrics for both worldvolume and spacetime. We use capital Latin letters (M, N, ..) for superspace indices, small Latin letters (m, n, ..) for spacetime vectors and early small Greek letters (α, β, ..) for spinors. Furthermore, we use late Greek letters (µ, ν, ..) for spacetime vectors parallel to the background branes and early Latin letters (a, b, ..) for spacetime vectors transverse to them. We use hatted letters (\hat{M}, \hat{m}, \hat{a}, \hat{α}..) for all the inertial frame indices and middle Latin letters (i, j, ..) for worldvolume vectors. And we use Majorana (32 × 32) representation for Gamma matrices Γ\hat{m} which are all real and satisfy \{Γ\hat{m}, Γ\hat{n}\} = 2η\hat{m}\hat{n}. Γ\hat{0} is antisymmetric and others symmetric. Charge Conjugation is C = Γ\hat{0}. We denote the eleventh Gamma matrix Γ\hat{10} as Γ\hat{♯}, as used in ref.[7]. Finally, we choose Γ\hat{0}..\hat{9} = 1.

2 Spacetime superalgebras in M-brane backgrounds

In this section we discuss spacetime superalgebras in terms of various 1/2-supersymmetric M-brane backgrounds and probes. In the following subsections we discuss the M-2-brane, the M-5-brane, the M-wave and the M-Kaluza-Klein monopole background in turn, and probe each background via the M-2-brane, the M-5-brane and the M-wave in this order. This order is so arranged as to begin with the easiest case to deal with and discuss more difficult ones later. To be more concrete, we first present the M-brane background solutions and construct superfields and their supertransformations in the backgrounds. Then, we discuss superalgebras via each probe, separately. We prove the invariance of the test M-brane action under the supertransformations which correspond to the symmetries of the backgrounds, derive spacetime superalgebras in the M-brane backgrounds and deduce from them all the orthogonal intersections the two M-branes to confirm the consistency of this method.

* Though some of the combinations have been discussed in ref.[16], we present them again in order for this paper to be complete.
2.1 In the M-2-brane background

First of all, we give some preliminaries about the M-2-brane background, the superfields and their supertransformation in the background.

The M-2-brane background solution is

\[ ds^2_{11} = H^{-2/3} \eta_{\mu \nu} dx^\mu dx^\nu + H^{1/3} dy^a dy^b \delta_{ab} \]

\[ C_{012} = H^{-1} \quad \text{the others} = 0, \quad (2.1) \]

where \( \eta_{\mu \nu} \) is the 3-dimensional Minkovski metric with coordinates \( x^\mu \) and \( H \) is a harmonic function on the transverse 8-space with coordinates \( y^a \), that is, \( H = 1 + \frac{q_2}{y^6} \) where \( y = \sqrt{y^a y^b \delta_{ab}} \) and \( q_2 \) is a constant. This background admits a Killing spinor \( \varepsilon \) which satisfies

\[ \delta \psi_m = (\partial_m + \frac{1}{4} \omega_m^{ij} \Gamma_{ij} + T_{m/n_1 n_2 n_3 n_4} F_{n_1 n_2 n_3 n_4}) \varepsilon = 0 \quad (2.2) \]

where \( T_{m/n_1 n_2 n_3 n_4} = -\frac{1}{288} (\Gamma_{m/n_1 n_2 n_3 n_4} + 8 \Gamma^{[n_1 n_2 n_3} \delta_{n_4]}_{m}) \). Then the Killing spinor has the form \( \varepsilon = H^{-1/6} \varepsilon_0 \) where \( \varepsilon_0 \) has the positive worldvolume chirality: \( \bar{\Gamma}_0 \varepsilon_0 \equiv \Gamma_{012} \varepsilon_0 = +\varepsilon_0 \).

Since \( \bar{\Gamma} \) satisfies \( \bar{\Gamma}^T = \bar{\Gamma} \) and \( \bar{\Gamma}^2 = 1 \), both \( \frac{1+\bar{\Gamma}}{2} \) and \( \frac{1-\bar{\Gamma}}{2} \) are projection operators. So, if we denote \( \frac{1+\bar{\Gamma}}{2} \zeta \) as \( \zeta^\pm \) for a spinor \( \zeta \), the background is invariant under the transformation generated by the supercharge \( Q^+ \), and each test brane action in this background is also expected to be invariant. On the other hand, the background and each brane action are not invariant under the transformation by \( Q^- \), which means that we should set the corresponding transformation parameter \( \varepsilon^- \) to be zero. Then, the conjugate momentum \( \Pi^- \) of \( \theta^- \) does not appear in the Noether charge \( Q^+ \). So, the terms including \( \theta^- \) never contribute to the central charges at zeroth order in \( \theta \). Thus, we can ignore the terms and set \( \theta^- = 0 \) from the beginning. From now on, we will use these in all the cases we treat in this paper. Related with this, we exhibit the properties of \( \bar{\Gamma} \):

\[ [\bar{\Gamma}, \Gamma_\bar{\mu}] = [\bar{\Gamma}, C] = \{\bar{\Gamma}, \Gamma_\bar{a}\} = 0. \quad (2.3) \]

Now, we have prepared to get the explicit representations of the superfields and their supertransformations in terms of superspace coordinates to low orders in \( \theta \). By substituting \( (2.1) \) and \( \theta^- = 0 \) to the usual expressions\([37]\) (and using \( (2.3) \)), we see that only the \( E_{\bar{\alpha}}^\bar{A} \) has the nontrivial contribution from the background. From the results the superspace 1-form on the inertial frame \( E_{\bar{A}}^\bar{M} dZ^\bar{M} \) is given by \( \dagger \)

\[ E^\bar{\mu} = dx^\nu H^{-1/3} \delta_{\nu}^{\bar{\mu}} - i \bar{\theta}^+ \Gamma^\nu d\theta^+ + O(\theta^4) \]

\( \dagger \)In fact we need to know the (vanishing of the) contribution from \( E_m^n \) at order \( \theta^2 \). We can infer its vanishing in this specific simple background, but the expression of \( E_m^n \) at order \( \theta^2 \) in general background was obtained \([38]\), by which our inference is confirmed.
\[ E^\hat{a} = d y^b H^{1/6} \delta^\hat{a}_b + \mathcal{O}(\theta^4) \]
\[ E^{\hat{a}} = d \theta^{\hat{a}+} + \frac{1}{6} H^{-1} d H \theta^{\hat{a}+} + \mathcal{O}(\theta^3). \] (2.4)

Since the 1-form \( E^\hat{A} \) has no superspace (curved) indices, \( E^\hat{A} \) is invariant under the local supertransformation[37] \( \delta Z^M = \Xi^M \) in this background given by
\[ \Xi^\mu = i \tilde{\varepsilon}^+ \Gamma^\mu \theta^+ + \mathcal{O}(\theta^3) \]
\[ \Xi^a = 0 + \mathcal{O}(\theta^3) \]
\[ \Xi^{\hat{a}} = \varepsilon^{\hat{a}+} + \mathcal{O}(\theta^2). \] (2.5)

We can easily check the invariance of \( E^\hat{A} \) explicitly up to second order in \( \theta \). We note that the coordinates \( y^a \) transverse to the background brane are not transformed (at least up to the second order in \( \theta \)). Namely, this is the supertranslation parallel to the background brane. (So, the Noether supercharge we will define later corresponds to this.) It is also to be noted that \( \Gamma^\mu = H^{1/3} \Gamma^\nu \delta^\mu_\nu \), i.e. the gamma matrices with the spacetime indices depend on the harmonic function.

The remaining fields are superspace gauge potentials: 3-form \( C^{(3)} \) and 6-form \( C^{(6)} \). The former is introduced by the gauge invariant 4-form field strength[37][39]
\[ R^{(4)} \equiv dC^{(3)} = \frac{i}{2} E^{\hat{a}} E^{\hat{b}} E^{\hat{c}} (\Gamma_{\hat{a}\hat{b}\hat{c}})_{\hat{d}\hat{e}} E^{\hat{d}} + \frac{1}{4!} E^{\hat{m}_1} E^{\hat{m}_2} E^{\hat{m}_3} E^{\hat{m}_4} F_{\hat{m}_4 \hat{m}_3 \hat{m}_2 \hat{m}_1}, \] (2.6)
where \( F_{\hat{m}_4 \hat{m}_3 \hat{m}_2 \hat{m}_1} \) is the bosonic field strength which is in this case associated with the electric M-2-brane background.

Here, we assume that all the fermionic (but not bosonic) cocycles in the superspaces are trivial. (We assume this in all the cases in this paper.) Then, the invariance of \( R^{(4)} \) under the super-transformation (2.5) means that \( \delta C^{(3)} \) can be written as a d-exact form to full order in \( \theta \).

From (2.6) we can get the explicit expression of \( C^{(3)} \) as
\[ C^{(3)} = \frac{1}{3!} H^{-1} (-\epsilon_{\mu \nu \rho}) dx^\mu dx^\nu dx^\rho - \frac{i}{2} H^{-2/3} dx^\rho \delta^\hat{a}_\rho dx^\sigma \delta^\hat{b}_\sigma \theta^+ \Gamma_{\hat{a}\hat{b}} d\theta^+ \]
\[- \frac{i}{2} H^{1/3} dx^c \delta^\hat{c}_c dx^d \delta^\hat{d}_d \theta^+ \Gamma_{\hat{a}\hat{b}} d\theta^+ + \mathcal{O}(\theta^4) \] (\( \epsilon_{012} = -1 \)), (2.7)
and hence the supertransformation of \( C^{(3)} \):
\[ \delta C^{(3)} \equiv d(\tilde{\varepsilon}^+ \Delta_2) = d(-\frac{i}{2} H^{1/3} dy^c \delta^\hat{c}_c dy^d \delta^\hat{d}_d \theta^+ + \mathcal{O}(\theta^3)). \] (2.8)

\footnote{Although the \( \hat{\alpha} \) of \( \theta^{\hat{\alpha}+} \) is the index of the inertial frame, \( \theta^{\hat{\alpha}} = \theta^+ \delta^\hat{\alpha}_\beta + \mathcal{O}(\theta^3) \). So, we need not distinguish the two indices in this paper.}
The latter superspace 6-form $C^{(6)}$ is introduced by the 7-form field strength which takes the form \[ R^{(7)} \equiv dC^{(6)} - \frac{1}{2} C^{(3)} R^{(4)} \]

\[ = \frac{i}{5!} E^{m_1} \ldots E^{m_5} E^\alpha (\Gamma_{m_5 \ldots m_1})^\alpha^\beta E^\beta + \frac{1}{7!} E^{m_1} \ldots E^{m_7} F^{(7)}_{m_7 \ldots m_1} \]

(2.9)

where the 7-form $F^{(7)}$ is the Hodge dual of the bosonic 4-form field strength. We note that $C^{(6)}$ cannot be expressed globally in this case because it has a part of magnetic potential which originates from the existence of the M-2-brane. (We denote it by $C^{(6)}_{\text{mag}}$ formally.) Then, in the same way as $\delta C^{(3)}$, the invariance of $R^{(7)}$ under (2.5) means that $\delta C^{(6)} + \frac{1}{2} \delta C^{(3)} C^{(3)}$ can be written as a d-exact form.\(^\S\) From (2.9) we get

\[ \delta C^{(6)} + \frac{1}{2} \delta C^{(3)} C^{(3)} \equiv d(\bar{\varepsilon}^+ \Delta_5) \]

\[ = d(-\frac{i}{4!} H^{1/3} dx^\nu \delta_\nu^a \delta_{d_1}^a \ldots \delta_{d_4}^a (\Gamma_{d_1 \ldots d_4} \theta^+)^\alpha + \mathcal{O}(\theta^3)). \]

(2.10)

Now, we have finished preliminaries about the background, the superfield and the supertransformation. So, we will discuss each of the probes separately in the next.

(2.1a) via the M-2-brane probe

At first we review the case of a test M-2-brane floating in the M-2-brane background discussed in ref.\(^{[16]}\). The M-2-brane action in a D=11 supergravity background is

\[ S_{M2} = S^{(0)} + S_{WZ} = -\int d^3 \xi \sqrt{-\det \tilde{g}_{ij}} + \int d^3 \xi \frac{1}{3!} \varepsilon^{ijk} \tilde{C}^{(3)}_{ijk} \]

(2.11)

where $\tilde{g}_{ij} = E^{m_i} E^{m_j} \eta_{\tilde{m} \tilde{n}}$ is the induced worldvolume metric and $C^{(3)}_{ijk}$ is the worldvolume 3-form induced by the superspace 3-form gauge potential. $E^A_i = \partial_i Z^M E^M_A$ where $E^M_A$ is the supervielbein. Note that the action is invariant under local super-transformation at this moment. Let’s fix the background to the M-2-brane solution (2.1). Since it holds $\delta E^A_i = 0$ under the supertransformation (2.3), $\tilde{g}_{ij}$ and hence $S^{(0)}$ are also invariant under (2.3). On the other hand, from (2.8) we get $\delta \mathcal{L}_{WZ} = d(\bar{\varepsilon}^+ \Delta_2)$. So, the action (2.11) in the M-2-brane background (2.1) is invariant up to total derivative under the supertransformation (2.3). (In the cases of the other backgrounds, we can prove the invariance of the test M-2-brane action under the supertransformations corresponding to the symmetries of the backgrounds in the same way.)

So, we can define the corresponding Noether supercharge $Q^+_{\alpha}$ in the Hamiltonian formulation as an integral over the test brane at fixed time $M_2$, given by

\[ Q^+_{\alpha} \equiv Q^+_{\alpha}^{(0)} - i \int_{M_2} (\mathcal{C} \Delta_2)_\alpha \]

\(^\S\)We note that $C^{(6)}_{\text{mag}}$ is invariant under the super-transformation (2.3) owing to the inertness of the transverse coordinates $y^a$ (see, (2.3)).
\[ \int_{\mathcal{M}_2} d^2\xi (i\Pi^+_\alpha - \Pi_\mu (\mathcal{C}^\mu \theta^+)_{\alpha}) - \frac{1}{2} \int_{\mathcal{M}_2} dy^a dy^b (\mathcal{C}_{ab} \theta^+)_{\alpha} + \mathcal{O}(\theta^3) \]  

(2.12)

where \( \Pi_\mu \) and \( \Pi^+_\alpha \) are the conjugate momenta of \( x^\mu \) and \( \theta^+ \), respectively. \( Q^+_{\alpha} \) is the momentum part, the form of which is almost common to all the branes. Then, we get the superalgebra of \( Q^+_\alpha \)

\[ \{ Q^+_\alpha, Q^+_\beta \} = 2 \int_{\mathcal{M}_2} d^2\xi \Pi_\mu (\mathcal{C}^\mu)_{\alpha\beta} + 2 \int_{\mathcal{M}_2} dy^a dy^b (\mathcal{C}_{ab})_{\alpha\beta} + \mathcal{O}(\theta^2). \]  

(2.13)

Before discussing this result, we give the explicit expression of \( \Pi_\mu \):

\[ \Pi_\mu = \frac{\delta L^{(0)}}{\delta \dot{x}^\mu} + \frac{1}{2} \varepsilon^{0ij} \partial_i x^\nu \partial_j x^\rho C^{(3)}_{\mu\nu\rho} + \mathcal{O}(\theta^2) \equiv \Pi^{(0)}_\mu + \Pi^{WZ}_\mu \]  

(2.14)

where \( L^{(0)} \) is the Nambu-Goto Lagrangian.

The implications of this algebra are as follows: since we are interested only in static configurations, we choose the static gauge:

\[ \partial_0 x^\mu = \delta^\mu_0, \partial_i x^0 = \delta^i_0. \]  

(2.15)

Then, if the test brane is oriented parallel to the background brane, the term like a central charge arises from the \( \Pi^{WZ}_\mu \), although the original central charge vanishes. \( \Pi^{(0)}_\mu \) and \( \Pi^{WZ}_\mu \) are obtained respectively as

\[ \int_{\mathcal{M}_2} d^2\xi \Pi^{(0)}_\mu = | \int_{\mathcal{M}_2} dx^1 dx^2 H^{-1} | \delta^0_\mu + \mathcal{O}(\theta^2) \]  

(2.16)

\[ \int_{\mathcal{M}_2} d^2\xi \Pi^{WZ}_\mu = \int_{\mathcal{M}_2} dx^1 dx^2 H^{-1} \delta^0_\mu + \mathcal{O}(\theta^2). \]  

(2.17)

Thus, we conclude that the parallel configuration (for example, \( x^1 = \xi^1, x^2 = \xi^2 \)) with a certain orientation of the test brane has 1/2 spacetime supersymmetry and the one with the other orientation breaks all the supersymmetry, which is consistent with the previous result [36][42]. We note that (2.16) and (2.17) are invariant under the 12-plane rotation and hence the discussion above also holds, as it should be.

On the other hand, if the test brane is oriented orthogonally to the background brane (i.e. zero-brane intersection), the central charge does have the nonzero value. In the static gauge with the test brane to be fixed, for example, to 34-plane, the algebra becomes

\[ \{ Q^+_\alpha, Q^+_\beta \} = 2 \int_{\mathcal{M}_2} d^2\xi H^{1/3} (1 - \Gamma_{34})_{\alpha\beta}, \]  

(2.18)

which means that 1/4 spacetime supersymmetry is preserved in this configuration (0|M2,M2).

We can easily see from the algebra (2.13) that this is the only orthogonal intersection preserving supersymmetry, which is also consistent with the previous result given in ref. [24][25][26][13].
\( (2.1b) \) via the M-5-brane probe

The M-5-brane action is [13]

\[
S_{M5} = S^{(0)} + S_{WZ} = -\int d^6\xi \sqrt{-\det(g_{ij} + \bar{\mathcal{H}}_{ij}) + \frac{\sqrt{-g}}{4(\partial a)^2}(\partial a)(\mathcal{H})^{ij}k_2(\partial a)(\partial a)] + \int (C^{(6)} + \frac{1}{2}H^i}\),
\]

(2.19)

where \( \mathcal{H} \) is the “modified” field strength of the worldvolume self-dual 2-form \( A_2 \) given by

\[
\mathcal{H} = dA_2 - C^{(3)}.
\]

(2.20)

\((\mathcal{H})^{ijk} \) and \( \bar{\mathcal{H}}^{ij} \) are defined as \( (\mathcal{H})^{ijk} = \frac{1}{3!\sqrt{-g}}\varepsilon^{ijk}j_{k'}\mathcal{H}_{ij}^k, \) \( \bar{\mathcal{H}}^{ij} = \frac{1}{\sqrt{-\partial^2}}(\mathcal{H})^{ijk}\partial_ka \), respectively. \( a \) is an auxiliary worldvolume scalar field. The super-transformation of \( A_2 \) is determined by the requirement of the invariance of the “modified” field strength \( \mathcal{H} \) as in ref. [10]. The transformation in this M-2-brane background is

\[
\delta A_2 = \bar{\varepsilon}^{+}\Delta_2 = -\frac{i}{2}H^{1/3}dy^\phi d\bar{\phi}d\bar{\phi}d\varepsilon^+\Gamma_{ab}\theta^+ + O(\theta^3),
\]

where \( \Delta_2 \) is defined in (2.8). Since \( g_{ij} \) and \( \mathcal{H} \) are invariant, the kinetic action \( S^{(0)} \) is also invariant under the transformation (2.4). On the other hand, \( \delta L_{WZ} \) is shown to be the following d-exact form:

\[
\delta L_{WZ} = \delta C^{(6)} - \frac{1}{2}\delta C^{(3)}\mathcal{H} = d(\bar{\varepsilon}^{+}\Delta_5 - \frac{1}{2}dA_2\varepsilon^+\Delta_2) \equiv d(\bar{\varepsilon}^{+}\Delta)
\]

(2.22)

where \( \bar{\varepsilon}^{+}\Delta_5 \) is defined in (2.10). So, the M-5-brane action (2.14) in the M-2-brane background is invariant up to total derivative under the supertransformation (2.3), and the supercharge is given as before by \( Q^{+}_\alpha \equiv Q^{+}_\alpha^{(0)} - i\int_{M5}(\mathcal{C}\Delta)_\alpha \) where \( Q^{+}_\alpha^{(0)} \) takes the form [11]

\[
Q^{+}_\alpha^{(0)} = \int_{M5} d^5\bar{\xi}[(i\Pi^{\alpha}_\beta - \Pi(\Gamma^\alpha_\beta)^{+}) + \frac{i}{2}\mathcal{P}^{\bar{\alpha}}_\beta(\mathcal{C}\Delta_2)_{\bar{\alpha}}^{\beta}],
\]

(2.23)

where \( \bar{\alpha} \) is the space index of the test M-5-brane worldvolume and \( \Pi_\mu, \Pi^{\alpha}_+ \) and \( \mathcal{P}^{\bar{\alpha}}_\beta \) are the conjugate momenta of \( x^\mu, \theta^+ \) and \( A_{ij} \), respectively. Then, the superalgebra is obtained as

\[
\{Q^{\alpha}_+, Q^{\beta}_+\} = 2\int_{M5} d^5\bar{\xi}[\Pi(\Gamma^\alpha_\beta)_{\alpha\beta} - \frac{1}{2}\mathcal{P}^{\bar{\alpha}}_\beta\partial y^\alpha\partial y^\beta(\Gamma^\alpha_\beta)_{\alpha\beta}] + \frac{1}{4}\int_{M5} dx^\mu dy^\alpha \ldots dy^\alpha(\Gamma^\mu_{\alpha\ldots\beta})(\partial y^\alpha)_{\alpha\beta} - \frac{1}{4}\int_{M5} dA_2 dy^\alpha dy^\beta(\Gamma^\alpha_\beta)_{\alpha\beta} + O(\theta^2).
\]

(2.24)

In the static gauge the third term in (2.24) means that only the string intersection with the M-2-brane background leads to preservation of 1/4 supersymmetry, which is again consistent with ref. [25][26][12][13].
The M-wave is a massless superparticle running at the speed of light. The action is
\[ S_{MW} = \int d\tau e(\tau) E_{\tau}^\alpha E_{\tau}^\beta \eta_{\alpha\beta}, \]
where \( \tau \) is the time on its worldline and \( e(\tau) \) is an einbein. Since \( E_{\tau}^\alpha = \partial_\tau Z^N E_N^\alpha \) and \( e(\tau) \) are invariant, the action (2.25) in the M-2-brane background is also invariant under the supertransformation (2.3). We note that the action is exactly invariant because it has no Wess-Zumino term. Let us choose the gauge \( e(\tau) = \frac{1}{2} \). The supercharge is written as
\[ Q_\alpha^+ = i\Pi_\alpha^+ - \Pi_\mu (\nabla^\mu)_{\alpha}, \]
and the superalgebra is obtained as
\[ \{Q_\alpha^+, Q_\beta^+\} = 2\Pi_\mu (\nabla^\mu)_{\alpha\beta}. \]
We note that this form is exact to the full order in \( \theta \).

Since \( \Pi_\mu \) is the momentum parallel to the background M-2-brane, supersymmetry is preserved only if the absolute value of the parallel momentum is equal to the energy \( \Pi^0 \). For example, when we fix the gauge (i.e. the embedding) to \( x^0 = x^1 = \tau \), it holds \( \Pi^0 = \Pi^1 \), and the algebra is written as
\[ \{Q_\alpha^+, Q_\beta^+\} = 2\Pi^0(1 - \Gamma_{01}), \]
which means that this embedding lead to preservation of 1/4 supersymmetry. Since this embedding can be interpreted as \((1|MW,M2)\) with 1/4 supersymmetry given in ref. [26] [45] [13], the superalgebra (2.27) is also consistent.

### 2.2 In the M-5-brane background
The M-5-brane background solution is given by
\[ ds_{11}^2 = H^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu + H^{2/3} dy^a dy^b \delta_{ab} \]
\[ F_{abcd} = -\varepsilon_{abcd} \partial_\alpha H, \]
where \( \mu = 0, 1, \ldots, 5 \) and \( a = 6, \ldots, 9, \hat{5} \). The Killing spinor \( \varepsilon \) has the form \( \varepsilon = H^{-1/12}\varepsilon_0 \) where \( \varepsilon_0 \) has the positive chirality of the worldvolume of the background: \( \Gamma^\prime \varepsilon_0 \equiv \Gamma_{012345}\varepsilon_0 = +\varepsilon_0 \). Since \( \frac{1+\Gamma^\prime}{2} \) are again projection operators, we denote here \( \frac{1+\Gamma^\prime}{2} \) as \( \psi \) for a spinor \( \psi \). Then, for the same reason stated in the case of the M-2-brane background, only \( Q^+ \) is
expected to be the symmetry of the system and we set $\varepsilon^- = 0$ and hence $\theta^- = 0$. We note that $\hat{\Gamma}$ satisfies the (anti-)commutators \( \{ \hat{\Gamma}^\prime, C \} = \{ \hat{\Gamma}^\prime, \hat{\Gamma}_\mu \} = [\hat{\Gamma}^\prime, \hat{\Gamma}_a] = 0 \). By using this relations and the formula presented in ref. [37], the superspace 1-form on the inertial frame is given by

\[
E^\hat{\mu} = dx^\nu H^{-1/6} \delta^\mu_{\hat{\mu}} - i \bar{\theta}^+ \Gamma^\hat{\mu} d\bar{\theta}^+ + \mathcal{O}(\theta^4)
\]

\[
E^\hat{a} = dy^b H^{1/3} \delta^a_{\hat{a}} + \mathcal{O}(\theta^4)
\]

\[
E^\hat{\alpha} = d\theta^\hat{\alpha} + \frac{1}{12} H^{-1} dH \theta^\hat{\alpha} + \mathcal{O}(\theta^3).
\]

The supertransformations of the supercoordinates are the same forms as those in the M-2-brane background (2.5) except for the ranges of $\mu$ and $a$. The superspace 3-form $C^{(3)}$ and the 6-form $C^{(6)}$ are introduced by (2.6) and (2.9), in the same way as the case of the M-2-brane background. Note that $C^{(3)}$ cannot be expressed globally in this background because the 3-form has a magnetic part $C^{(3)}_{mag}$, which originate from the existence of the M-5-brane. However, $C^{(3)}_{mag}$ is invariant under the supertransformation at least up to second order in $\theta$, owing to the inertness of the transverse coordinates $y^a$ under the supertransformation. As a result,

\[
\delta C^{(3)} \equiv d(\hat{\varepsilon}^+ \Delta_2) = d(-i H^{1/6} dx^\nu \delta^\mu_{\hat{\mu}} dy^b \delta^a_{\hat{a}} \bar{\theta}^+ \Gamma_{\hat{\mu} \hat{a}} + \mathcal{O}(\theta^3))
\]

\[
\delta C^{(6)} + \frac{1}{2} \delta C^{(3)} C^{(3)} \equiv d(\hat{\varepsilon}^+ \Delta_3)
\]

\[
= d\left(-i \frac{1}{12} H^{1/3} dx^\nu \delta^\mu_{\hat{\mu}} \ldots dx^\nu_3 \delta^\mu_3 dy^b_1 \delta^a_{\hat{a}_1} \ldots dy^b_3 \delta^a_{\hat{a}_3} (\Gamma_{\hat{\mu}_1 \ldots \hat{\mu}_3 \hat{a}_1 \hat{a}_2 \hat{a}_3} \theta^+) \right)
\]

\[
- \frac{i}{4!} H^{1/3} dx^\nu \delta^\mu_{\hat{\mu}} \ldots dy^b dy^b dy^b (\Gamma_{\hat{\mu}_1 \ldots \hat{\mu}_3 a_1 a_2 a_3} \theta^+)
\]

\[
- \frac{i}{2} C^{(3)}_{mag} dx^\nu \delta^a_\hat{\alpha} \Gamma_{\hat{\alpha} \hat{a}} \theta^+ \mathcal{O}(\theta^3). \quad (2.32)
\]

Since we have finished preliminaries about the M-5-brane background, we will discuss each of the probes, respectively, in the same way as the M-2-brane background.

(2.2a) via the M-2-brane probe

The test M-2-brane action is the same as (2.11) while the background is fixed to the M-5-brane solution (2.25). $g_{ij}$ and hence $S^{(0)}$ are also invariant under the supertransformation for the same reason, and the whole action (2.11) in the M-5-brane background (2.25) is invariant up to total derivative because of (2.31). So, we can define the corresponding Noether supercharge and obtain the superalgebra as

\[
\{ Q^+_\alpha, Q^+_\beta \} = 2 \int_{M_2} d^2 \xi \Pi_\mu (C \Gamma^\mu)_{\alpha \beta} + 2 \int_{M_2} dx^\nu dy^a (C \Gamma_{\mu a})_{\alpha \beta} + \mathcal{O}(\theta^2). \quad (2.33)
\]

The second term implies that the string intersection of the test brane with the background is the only 1/4-supersymmetric configuration permitted in this background, which is consistent with the previous results [25, 26, 12, 13] and (3.1b).
The test M-5-brane action is again (2.19) while the background is fixed as (2.29). The transformation of $A_2$ in the M-5-brane background is determined by the invariance of $H$, just the same as the case of the M-2-brane background:

$$\delta A_2 = \varepsilon^+ \Delta_2' = -iH^{1/6} dx^\nu \delta^\nu_\mu dy^\nu \delta^\nu_\nu \varepsilon^+ \Gamma_{\mu \nu} \theta^+ + O(\theta^3). \quad (2.34)$$

The proof of the invariance of the action in the M-5-brane background is almost the same as that in the M-2-brane background, except that $\Delta_5$ and $\Delta_2$ in (2.22) are replaced with $\Delta_5'$ and $\Delta_2'$ defined in (2.32) and (2.31). So, in the same way, the superalgebra in the M-5-brane background is

$$\{Q_\alpha^+, Q_\beta^+\} = 2 \int_{M_5} d^5 \xi \left[ \Pi_\mu (\mathcal{C} \Gamma^\mu)_{\alpha \beta} - \frac{1}{2} \mathcal{P}^{ij} \partial_i x^\mu \partial_j y^a (\mathcal{C} \Gamma_{\mu a})_{\alpha \beta} \right]$$

$$+ \frac{1}{12} \int_{M_5} dx^{\mu_1} ... dx^{\mu_3} dy^{a_1} dy^{a_2} (\mathcal{C} \Gamma_{\mu_1 ... \mu_3 a_1 a_2})_{\alpha \beta} + \frac{2}{4!} \int_{M_5} dx^\mu dy^{a_1} ... dy^{a_4} (\mathcal{C} \Gamma_{\mu a_1 ... a_4})_{\alpha \beta}$$

$$- \frac{2}{2} \int_{M_5} (dA_2 - C_{mag}^{(3)}) dx^\mu dy^a (\mathcal{C} \Gamma_{\mu a})_{\alpha \beta} + O(\theta^2), \quad (2.35)$$

where $\Pi_\mu$ is

$$\Pi_\mu = \frac{\delta \mathcal{L}^{(0)}}{\delta \dot{x}^\mu} + \frac{1}{2} \varepsilon^{0i_1 ... i_5} \partial_{i_1} x^{i_1} ... \partial_{i_5} x^{i_5} C^{(6)}_{\mu_1 ... \nu_5} + O(\theta^2), \quad (2.36)$$

where $\mathcal{L}^{(0)}$ is the kinetic term of the M-5-brane Lagrangian.

The implications of the algebra are as follows: in the static gauge (2.15) the form of the momentum $\Pi_\mu$ is similar to that in the case of the test M-2-brane in the M-2-brane background. So, a parallel configuration with a certain orientation leads to the preservation of 1/2 supersymmetry and the other orientation breaks all the supersymmetry, which is consistent with the previous result (24) (12). The third term implies that 1/4 supersymmetry is preserved in the case of any three brane intersections. Finally, we can prove that any string intersections lead to preservation of 1/4 supersymmetry. This proof is a bit more complex than the others because in addition to the fourth term, the last term including the magnetic 3-form $C_{mag}^{(3)}$ do not vanish in this case. We will show that in the next.

Suppose $A_2 = 0, \theta = 0$ and that the test brane is fixed as $x^1 = \xi^1, y^7 = \xi^2, y^8 = \xi^3, y^9 = \xi^4, y^5 = \xi^5$. Choosing the gauge $a = \xi^0$, we have

$$\{Q_\alpha^+, Q_\beta^+\} = 2 \int_{M_5} d^5 \xi \left[ \Pi^0 \cdot H^{-1/6} \delta_{\alpha \beta} - H^{7/6} \{\mathcal{C} \Gamma_{17895}^{(3)}$$

$$+ H^{-1}(\mathcal{C} \Gamma_{17}^{(3)} C_{mag895}^{(3)} + \mathcal{C} \Gamma_{18}^{(3)} C_{mag795}^{(3)} + \mathcal{C} \Gamma_{19}^{(3)} C_{mag785}^{(3)} + \mathcal{C} \Gamma_{15}^{(3)} C_{mag789}^{(3)})_{\alpha \beta} \right] \quad (2.37)$$
where the momentum is given by

\[ \Pi^0 = H^{1/3} \sqrt{1 + H^{-2} [(C_{mag789})^2 + (C_{mag782})^2 + (C_{mag792})^2 + (C_{mag892})^2]} \]  

(2.38)

which originates from the kinetic term \( \mathcal{L}^{(0)} \). Then, since the last five matrices in (2.37) anti-commute with each other, they can be gathered into a traceless matrix \( \tilde{\Gamma} \) multiplied by their “norm” such as

\[
C_{\Gamma\hat{1}\hat{7}\hat{8}\hat{9}} + H^{-1} [C_{\Gamma\hat{1}\hat{7}C_{mag789}} + C_{\Gamma\hat{1}\hat{8}C_{mag782}} + C_{\Gamma\hat{1}\hat{9}C_{mag792}} + C_{\Gamma\hat{1}\hat{9}C_{mag892}}]
= H^{-4/3} \Pi^0 \tilde{\Gamma},
\]

(2.39)

where \( (\tilde{\Gamma})^2 = 1 \). So, 1/4 supersymmetry is preserved in this case, too. We can see from the algebra (2.35) that these are the only orthogonal intersections preserving supersymmetry. All of the above are consistent with the result of ref.\[25\]\[26\]\[27\]\[45\]\[13\].

(2.2c) via the M-wave probe

The action is (2.25) while the background is fixed to the M-5-brane solution. The proof of the invariance of the action in this background is again the same as the one in the M-2-brane case. So, we only show the results. The superalgebra is obtained as

\[
\{Q^+_\alpha, Q^+_{\beta}\} = 2\Pi_\mu (C\Gamma^\mu)_{\alpha\beta}.
\]

(2.40)

As is the case with the M-2-brane background, 1/4 supersymmetry is preserved only in the embeddings in which the absolute value of the momentum parallel to the background brane is equal to the energy \( \Pi^0 \), which is also consistent with ref.\[26\]\[15\]\[13\].

2. 3 In the M-wave background

In this subsection we discuss the M-wave background, which is given by \[46\]

\[ ds^2 = (K - 1)dt^2 - 2Kdtdx^1 + (1 + K)(dx^1)^2 + dy^a dy^b \delta_{ab}, \text{(the others = 0)} \]

(2.41)

where \( K \) is a harmonic function in the variables \(-t + x^1\) and \( y^a\) \((a = 2, 3, ..., \hat{\zeta})\). The Killing spinor in this background is constant and satisfies \( \Gamma^\alpha \varepsilon = \Gamma_{\alpha\hat{1}} \varepsilon = + \varepsilon \). Since \( \frac{1+\Gamma^\nu(\tau)}{2} \) are again projection operators, we denote here \( \frac{1+\Gamma^\nu}{2} \zeta \) as \( \zeta^\pm \) for a spinor \( \zeta \). Then, for the same reason as the case of the previous backgrounds, only \( Q^+ \) is the symmetry of the system and we set \( \varepsilon^- = 0 \) and hence \( \theta^- = 0 \). We note that \( \tilde{\Gamma}^\nu \) satisfies the (anti-)commutators \( \{\tilde{\Gamma}^\nu, C\} = \{\tilde{\Gamma}^\nu, \Gamma_\mu\} = [\tilde{\Gamma}^\nu, \Gamma_\alpha] = 0 \). By using these relations and the formula presented in ref.\[37\], the superspace 1-form on the inertial frame \( E^M \equiv dZ^N E^M_N \) is given by

\[
E^\hat{0} = \left(1 - \frac{K}{2}\right)dx^0 + \frac{K}{2}dx^1 - i\theta^+ \Gamma^\hat{0} d\theta^+
\]
\[ E^i = -\frac{K}{2} dx^0 + (1 + \frac{K}{2}) dx^1 - i \bar{\theta}^i \Gamma^i d\theta^+ \]
\[ E^\hat{a} = dy^\hat{a} \]
\[ E^{\hat{a}} = d\theta^{+\hat{a}}. \]  
(2.42)

The supertransformations of the supercoordinates are the same forms as those in the M-2-brane background (2.3) except that \( \mu = 0, 1 \) and \( a = 2, 3, \ldots, 9, \hat{a} \). The superspace 3-form \( C^{(3)} \) and the 6-form \( C^{(6)} \) are introduced by (2.6) and (2.9), in the same way as the case of the M-2-brane background. Their (combinations of) supertransformations can also be written as d-exact forms by the same proofs, given by

\[ \delta C^{(3)} \equiv d(\bar{\epsilon}^+ \Delta_2^\mu) = d(-idx^\mu dy^a \bar{\epsilon}^+ \Gamma_{\hat{\mu}a} \theta^+ + \mathcal{O}(\theta^3)) \quad (2.43) \]
\[ \delta C^{(6)} + \frac{1}{2} \delta C^{(3)} C^{(3)} \equiv d(\bar{\epsilon}^+ \Delta_5^\mu) = d(-\frac{i}{4!} dx^\mu dx^a_1 \cdots dx^a_4 \bar{\epsilon}^+ \Gamma_{\hat{\mu}a_1 \cdots a_4} \theta^+ + \mathcal{O}(\theta^3)). \quad (2.44) \]

We note that \( \frac{1}{2} \delta C^{(3)} C^{(3)} \) is \( \mathcal{O}(\theta^3) \) in this case.

Next we discuss each of the probes, respectively. The original actions are the same as the previous cases and the proofs of the invariance of the test brane actions under the supertransformation are also the same, while the background is replaced by (2.41). So, we present only the results and their implications.

(2.3a) via the M-2-brane probe

The superalgebra is
\[ \{Q_\alpha^+, Q_\beta^+\} = 2 \int_{M_2} d^2 \xi \Pi_\mu (\Sigma^\mu)_{\alpha\beta} + 2 \int_{M_2} dx^\mu dy^a (\Sigma \Gamma_{\mu a})_{\alpha\beta} + \mathcal{O}(\theta^2). \]  
(2.45)

The second term implies that in the static gauge only the string intersection leads to the preservation of 1/4 supersymmetry, which is also consistent with ref.\[26\] [13].

(2.3b) via the M-5-brane probe

In this case the supertransformation of \( A_2 \) is \( \delta A_2 = \bar{\epsilon}^+ \Delta_2^\mu \) where \( \Delta_2^\mu \) is defined in (2.43). The superalgebra is

\[ \{Q_\alpha^+, Q_\beta^+\} = 2 \int_{M_5} d^5 \xi [\Pi_\mu (\Sigma^\mu)_{\alpha\beta} - \frac{1}{2} \Sigma \Gamma_{\mu a} \partial_\mu \partial_\nu \partial_\rho \partial_\sigma (\Sigma \Gamma_{\nu \rho \sigma})_{\alpha\beta}] \]
\[ + \frac{2}{4!} \int_{M_5} dx^\mu dy^a_1 \cdots dy^a_4 (\Sigma \Gamma_{\mu a_1 \cdots a_4})_{\alpha\beta} - \frac{2}{2} \int_{M_5} dA_2 dx^\mu dy^a (\Sigma \Gamma_{\mu a})_{\alpha\beta} + \mathcal{O}(\theta^2) \]  
(2.46)

The third term implies that only string intersection leads to the preservation of 1/4 supersymmetry, which is also consistent with ref.\[26\] [13].

(2.3c) via the M-wave probe

In the gauge \( e(\tau) = \frac{1}{2} \) the momentum is given by

\[ \Pi_0 = -(1 - K) \partial_\tau x^0 - K \partial_\tau x^1 + \mathcal{O}(\theta^2) \]
\[ \Pi_1 = -K \partial_\tau x^0 + (1 + K) \partial_\tau x^1 + \mathcal{O}(\theta^2) \]
\[ \Pi_a = \partial_\tau y^a. \]  
(2.47)
Then, the superalgebra becomes

\[
\{Q_\alpha^+, Q_\beta^+\} = 2\Pi_\mu (C\Gamma^\mu)_{\alpha\beta} = 2(\partial_\tau x^0 - \partial_\tau x^1), \tag{2.48}
\]

which means that only such an embedding as \(x^0 = x^1 = k\tau\) for a positive constant \(k\) leads to preservation of 1/2 supersymmetry. This is the configuration \((1|\text{MW,MW})\) with 1/2 supersymmetry, which is also consistent with ref. [13].

### 2. 4 In the M-Kaluza-Klein monopole background

The M-Kaluza-Klein monopole background solution is \([1]\)

\[
ds_{11}^2 = \eta_{\mu\nu}dx^\mu dx^\nu + Vdy_ady_b\delta^{ab} + V^{-1}(dy^2 - A_a dy^a)^2 \tag{2.49}
\]

where \(\mu, \nu = 0, 1..6\) and \(a, b = 7, 8, 9\). \(\eta_{\mu\nu}\) is the 7-dimensional Minkovski metric with coordinates \(x^\mu\). \(A_a\) is a magnetic potential of a monopole on the transverse 3-space with coordinates \(y^a\) and \(V\) is a harmonic function on the same 3-space satisfying the equation:

\[
\partial_a V = \varepsilon^{abc}\partial_b A_c.
\]

This background admits a constant Killing spinor \(\varepsilon\) which satisfies \(\hat{\Gamma}^{m}\varepsilon_0 \equiv \Gamma_{\hat{\theta}_1,\hat{\theta}_6}\varepsilon_0 = +\varepsilon_0\). For the same reason, if we denote \(\frac{1+i\Gamma^{m}}{2}\varepsilon\) as \(\zeta^\pm\) for a spinor \(\zeta\), \(Q^+\) corresponds to the symmetry of the system, while \(Q^-\) is not, and we set \(\varepsilon^- = 0\) and \(\theta^- = 0\) from the beginning. \(\hat{\Gamma}^{m}\) satisfies the relations \([\hat{\Gamma}^{m},\mathcal{C}] = [\hat{\Gamma}^{m}, \Gamma_\hat{a}] = \{\hat{\Gamma}^{m}, \Gamma_\hat{a}\} = \{\hat{\Gamma}^{m}, \Gamma_\hat{a}\} = 0\).

The superspace 1-form on the inertial frame is given by

\[
E^\mu = dx^\nu \delta^\nu_\mu - i\hat{\theta}^+\hat{\Gamma}^\mu d\theta^+ + \mathcal{O}(\theta^4)
\]

\[
E^a = V^{1/2}dy^b\delta^a_b + \mathcal{O}(\theta^4)
\]

\[
E^\hat{a} = V^{-1/2}dy^2 - V^{-1/2}A_a dy^a + \mathcal{O}(\theta^4)
\]

\[
E^\hat{a} = d\theta^\hat{a} + \mathcal{O}(\theta^3). \tag{2.50}
\]

The supertransformations of the supercoordinates are again the same forms as those in the M-2-brane background \((2.3)\) except that \(\mu = 0, 1,..,6\) and \(a = 7, 9, .., \hat{a}\). The (combinations of) supertransformations of the superspace 3-form \(C^{(3)}\) and the 6-form \(C^{(6)}\) in this background are proved to be d-exact forms given respectively by

\[
\delta C^{(3)} \equiv d(\varepsilon^+\Delta^m_2) = d(-\frac{i}{2}dx^{\mu_1}\delta^\mu_1_{\mu_2}\delta^{\mu_2}_{\mu_3}\varepsilon^+\hat{\Gamma}_{\hat{\nu}_1\hat{\nu}_2}\theta^+ \\
\quad - \frac{i}{2}Vdy^{a_1}\delta^{a_1}_{a_2}\delta^{a_2}_{a_3}\varepsilon^+\hat{\Gamma}_{\hat{b}_1\hat{b}_2}\theta^+ \\
\quad - idy^a\delta^a_b dy^b\varepsilon^+\hat{\Gamma}_{\hat{c}_1\hat{c}_2}\theta^+ - iA_a dy^a dy^b\delta^b_c\varepsilon^+\hat{\Gamma}_{\hat{c}_3}\theta^+ + \mathcal{O}(\theta^3)) \tag{2.51}
\]

\[
\delta C^{(6)} + \frac{1}{2}\delta C^{(3)} C^{(3)} \equiv d(\varepsilon^+\Delta^m_3) = d(-\frac{i}{5!}dx^1 \cdots dx^5 \varepsilon^+\hat{\Gamma}_{\hat{\mu}_1\cdots\hat{\mu}_5}\theta^+)
\]

16
Next, we will discuss each of the probes, respectively. Each original action is the same as the previous cases while the background is chosen to be \((2.49)\). Since the proofs of the invariance of the test brane actions under the supertransformation are also the same, we present only the results and their implications again.

\((2.4a)\) via the M-2-brane probe

The superalgebra in the M-Kaluza-Klein monopole background via the M-2-brane probe is

\[
\{Q_\alpha^+, Q_\beta^+\} = 2 \int_{M_2} d^2 \xi \; \Pi^\mu (C \Gamma^\mu)_{\alpha\beta} + \frac{2}{2} \int_{M_2} dx^\mu dx'^\nu (C \Gamma_{\mu\nu})_{\alpha\beta} + \frac{2}{2} \int_{M_2} V dy^a dy^b (C \Gamma_{\hat{a}\hat{b}})_{\alpha\beta} + 2 \int_{M_2} dy^a dy^b (C \Gamma_{\hat{a}\hat{b}})_{\alpha\beta} + 2 \int_{M_2} A_a dy^a dy^b (C \Gamma_{\hat{a}\hat{b}})_{\alpha\beta} + O(\theta^2). \tag{2.53}
\]

In the static gauge the second term implies that 1/4 supersymmetry is preserved in the case of 2-brane intersection. The last three terms imply that 1/4 supersymmetry is also preserved in 0-brane intersection. The proof of this preservation is essentially similar to the proof in the case of string intersection of the two M-5-branes in \((2.2b)\). If the test brane is fixed as \(y^a = \xi^1\), \(y^2 = \xi^2\), only the fourth term in addition to the first term does not vanish. So, we can easily see that 1/4 supersymmetry is preserved. But, if the test brane is fixed as, for example, \(y^7 = \xi^1\), \(y^8 = \xi^2\), it is not so simple because the last term do contribute to the r.h.s. of the algebra, which becomes

\[
\{Q_\alpha^+, Q_\beta^+\} = 2 \int_{M_2} d^2 \xi \; \sqrt{V^2 + (A_7)^2 + (A_8)^2} \cdot 1 + VC \Gamma_{\hat{7}\hat{8}} + A_7 \Gamma_{\hat{5}\hat{8}} \hat{8} \Gamma_{\hat{5}} - A_8 \Gamma_{\hat{5}\hat{7}} \hat{7} \Gamma_{\hat{5}} |_{\alpha\beta}. \tag{2.54}
\]

Then, since the last three matrices in \((2.54)\) anti-commute with each other, they can be gathered into a traceless matrix \(\tilde{\Gamma}'((\tilde{\Gamma}')^2 = 1)\) multiplied by their “norm”, which is equal to the energy \(\Pi^0 = \sqrt{V^2 + (A_7)^2 + (A_8)^2}\). So, 1/4 supersymmetry is preserved in this embedding. We can see again from the algebra \((2.53)\) that these are the only orthogonal intersections preserving supersymmetry. All of the above are consistent with the result of ref. \([26, 45, 13]\).

\((2.4b)\) via the M-5-brane probe

The superalgebra in the M-Kaluza-Klein monopole background via the M-5-brane
probe is

\[ \{ Q^+_\alpha, Q^+_\beta \} = 2 \int_{M_5} d^5 \xi \left[ \Pi_\mu (C \Gamma^\mu)_{\alpha\beta} - \frac{1}{2} \partial^\mu \partial_\nu (C \Gamma_{\mu\nu})_{\alpha\beta} \right] \]

\[ + \frac{2}{5!} \int_{M_5} dx^{\mu_1} \ldots dx^{\mu_5} (C \Gamma_{\mu_1 \ldots \mu_5})_{\alpha\beta} + \frac{2}{12} \int_{M_5} V \left. dx^{\mu_1} \ldots dx^{\mu_3} dy^{\alpha_1} dy^{\alpha_2} (C \Gamma_{\mu_1 \ldots \mu_3 \alpha_1 \alpha_2})_{\alpha\beta} \right|_{\alpha=\beta=1} \]

\[ + \frac{2}{3!} \int_{M_5} dx^{\mu_1} \ldots dx^{\mu_3} dy^{a} dy^{b} (C \Gamma_{\mu_1 \ldots \mu_3 \alpha \beta})_{\alpha\beta} + \frac{2}{3!} \int_{M_5} A_a dy^{a} dy^{b} (C \Gamma_{\mu_1 \ldots \mu_3 \alpha \beta})_{\alpha\beta} \]

\[ + \frac{2}{3!} \int_{M_5} V dy^{a} dy^{b} (C \Gamma_{\alpha \beta})_{\alpha\beta} + dy^{a} dy^{b} (C \Gamma_{\alpha \beta})_{\alpha\beta} + A_a dy^{a} dy^{b} (C \Gamma_{\alpha \beta})_{\alpha\beta} + O(\theta^2) \].

(2.55)

The third term implies that 5-brane intersection leads to preservation of 1/4 supersymmetry. The fourth, the fifth and the sixth terms implies that 3-brane intersection also leads to preservation of 1/4 supersymmetry. We can prove the latter by the same procedure as the case of the zero-brane intersection of the test M-2-brane with this M-KK background in (2.4c). The seventh term means that 1/4 supersymmetry is also preserved in the string intersection. We can see from the algebra that there are no other orthogonal intersections with supersymmetry. All of the above are again consistent with the result of ref. [26][45][13].

(2.4c) via the M-wave probe

The superalgebra in the M-Kaluza-Klein monopole background via the M-wave probe is

\[ \{ Q^+_\alpha, Q^+_\beta \} = 2 \Pi_\mu (C \Gamma^\mu)_{\alpha\beta}. \]

(2.56)

Since there are no central charges and the momentum \( \Pi^\mu \) is parallel to the background, (1|MW,MKK) is the only embeddings to preserve (1/4) supersymmetry, which is also consistent with ref. [26][45][13].

3 Various supersymmetric brane configurations from the superalgebras

3.1 Supersymmetric intersections of the M-5-brane with the M-Kaluza-Klein monopole at angles from the superalgebra

In this subsection we discuss non-orthogonal supersymmetric intersections of two M-branes at angles. In the same way as the paper ref. [30][31][32], we start from the configurations of test branes “maximally parallel” to background branes and rotate the test branes. When either the background or the probe is the M-2-branes or the M-waves, at
most only two angles are needed to parametrize generic rotation from “maximally parallel” configurations. Then, the combinations have supersymmetry only in the cases of orthogonal intersections, all of which are already known and has been reproduced in the previous section. So, the only combinations of two M-branes discussed above to permit the existence of non-orthogonal intersections with supersymmetry are the cases of (3.2b) and (3.4b): the test M-5-brane in the M-5-brane background and the test M-5-brane in the M-Kaluza-Klein monopole background. (The former is investigated in detail in ref. [32].) When the two branes intersect orthogonally, we can prove straightforwardly the preservation of 1/4 supersymmetry in both cases. When they intersect non-orthogonally at angles, however, the right hand side of the superalgebra (2.35) in the former case (3.2b) becomes very complicated, especially owing to the existence of the magnetic 3-form gauge potential $C^{(3)}_{mag}$, and it is difficult to see how much supersymmetry is preserved. So, in this paper we discuss non-orthogonal intersections in the latter case (3.4b). Although the superalgebra (2.55) also appears to include the magnetic 1-form $A_a$, the algebra can be written such that $A_a$ does not appear in it, in the expression of only vector (i.e. completely “hatless”) indices, as

$$\{Q^+_\alpha, Q^+_\beta\} = 2 \int_{M_5} d^5 \xi \Pi^\mu (C^\mu)^{\alpha \beta} + P^\parallel \{-\frac{1}{2} \partial^\mu x^\nu \partial_\beta x^\nu (C_{\mu \nu})^{\alpha \beta}\}$$

$$+ \frac{2}{5!} \int_{M_5} dx^{\mu_1} ... dx^{\mu_5} (C_{\mu_1 ... \mu_5})^{\alpha \beta} + \frac{2}{12} \int_{M_5} dx^{\mu_1} ... dx^{\mu_3} dy^{a_1} dy^{a_2} (C_{\mu_1 ... \mu_3 a_1 a_2})^{\alpha \beta}$$

$$+ \frac{2}{3!} \int_{M_5} dx^{\mu_1} ... dx^{\mu_3} dy^a dy^b (C_{\mu_1 ... \mu_3 a b})^{\alpha \beta} + \frac{2}{3!} \int_{M_5} dx^\mu dy^{a_1} ... dy^{a_3} dy^5 (C_{\mu a_1 ... a_3})^{\alpha \beta}$$

$$- \frac{2}{2} \int_{M_5} dA_2 [dx^\mu dx^\nu (C_{\mu \nu})^{\alpha \beta} + dy^a dy^b (C_{ab})^{\alpha \beta} + dy^a dy^5 (C_{a 5})^{\alpha \beta}] + \mathcal{O}(\theta^2). \quad (3.1)$$

We investigate the preserved supersymmetry on the basis of this expression.

Let us find the embeddings corresponding to the intersection of the M-5-brane with the M-Kaluza-Klein monopole at angles. Since the M-Kaluza-Klein monopole is essentially a 6-brane, the generic rotation is parametrized by four independent angles $\theta_i$ ($i = 1, ..., 4$). (Namely, they always intersect at least on a string.) A “naive” embedding is expected without loss of generality as $x^0 = \xi^0, x^i = \xi^i \cos \theta_i, y^{i+6} = \xi^i \sin \theta_i$ ($i = 1, 2, 3, 4$), $x^5 = \xi^5$ and $x^6 = \text{const}$, where $\theta_i$ is the “angle” of the rotation of the i-(i+6)-plane for $i = 1, 2, 3, 4$, respectively. This embedding, however, does not preserve any supersymmetry in the cases of non-orthogonal intersections. Instead, the supersymmetry is preserved by the embedding:

$$\frac{\partial x^\mu}{\partial \xi^i} = \cos \theta_i e_i^\mu = \cos \theta_i \delta_i^\mu \quad \text{(for } i = 1, ..., 4),$$

$$\frac{\partial y^a}{\partial \xi^i} = \sin \theta_i e_{i+6}^a \quad \text{(for } i = 1, ..., 3),$$
where $e_m^a$ is the inverse of a vielbein $e_a^m$ in the M-Kaluza-Klein monopole background (2.49). This is so constructed as to satisfy the relations:

$$\frac{\partial y^a}{\partial \xi^4} = \sin \theta_4 e_4^a, \quad \frac{\partial x^\mu}{\partial \xi^5} = e_5^\mu = \delta_1^\mu, \quad \frac{\partial x^6}{\partial \xi^i} = 0 \quad (3.2)$$

where $e^n_m$ is the inverse of a vielbein $e_m^n$ in the M-Kaluza-Klein monopole background (2.49). This is so constructed as to satisfy the relations:

$$\frac{\partial y^a}{\partial \xi^4} = \sin \theta_4 e_4^a, \quad \frac{\partial x^\mu}{\partial \xi^5} = e_5^\mu = \delta_1^\mu, \quad \frac{\partial x^6}{\partial \xi^i} = 0 \quad (3.2)$$

for the following five matrices to be such a basis as all of them are equal to the identity. And since all of them are traceless, we can arrange by the same technique as that in ref.[32]. Their eigenvalues are all $$\pm \frac{\sqrt{64}}{2}$$

Since the gamma matrix products in (3.4) commute with each other and $\Gamma_{0123456}$, all these matrices can be simultaneously diagonalized. So, we can analyse the above consequence by the same technique as that in ref.[32]. Their eigenvalues are all $\pm 1$ because the square of them are all equal to the identity. And since all of them are traceless, we can arrange for the following five matrices to be such a basis as

$$\Gamma_{0123456} = \bar{\Gamma}_A \equiv \text{diag} (1, \cdots, 1, -1, \cdots, -1), \quad 16$$

$$\Gamma_{012345} = \bar{\Gamma}_B \equiv \text{diag} (1, \cdots, 1, -1, \cdots, -1, \cdots), \quad 8$$

$$\Gamma_{078345} = \bar{\Gamma}_C \equiv \text{diag} (1, \cdots, 1, -1, \cdots, 1, -1, \cdots, -1, \cdots), \quad 4$$

$$\Gamma_{072845} = \bar{\Gamma}_D \equiv \text{diag} (1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, -1, -1, -1, \cdots), \quad 16$$

$$\Gamma_{07235} = \bar{\Gamma}_E \equiv \text{diag} (1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, \cdots). \quad (3.5)$$

The representations of the rest of the matrices appearing in (3.4) are determined because each of the rest is the product of the above five. (We note that it is sufficient for us to know

* Although this embedding is not written explicitly but written in terms of differential equations, the embedding can be determined by integrating them from points at infinity.

† We omit hats of the vector indices of Gamma matrices from now on.
only the first 16 components of the matrices because \( Q^+ \) is the supercharge projected by the matrix \( \frac{1 + \Gamma}{2} \). Then, we can derive the following expression:

\[
\{Q^+_\alpha, Q^-_\beta\} = 4 \int_{\mathcal{M}_5} d\xi^5 \cdot \text{diag.}(\sin^2 \frac{\theta_1 - \theta_2 - \theta_3 - \theta_4}{2}, \sin^2 \frac{\theta_1 - \theta_2 - \theta_3 + \theta_4}{2}, \\
\sin^2 \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{2}, \sin^2 \frac{\theta_1 + \theta_2 - \theta_3 + \theta_4}{2}, \\
\sin^2 \frac{\theta_1 + \theta_2 - \theta_3 - \theta_4}{2}, \sin^2 \frac{\theta_1 + \theta_2 + \theta_3 - \theta_4}{2}, \\
\cos^2 \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{2}, \cos^2 \frac{\theta_1 + \theta_2 + \theta_3 - \theta_4}{2}, \\
\cos^2 \frac{\theta_1 + \theta_2 - \theta_3 + \theta_4}{2}, \cos^2 \frac{\theta_1 + \theta_2 - \theta_3 - \theta_4}{2} \ldots). \quad (3.6)
\]

We use this result to provide a systematic analysis of preserved supersymmetry. Before analyzing the result, we clarify the ranges of \( \theta_i \). As opposed to the cases of two M-branes of the same kind, there are no differences between parallel \((\theta = 0)\) and "anti-parallel" \((\theta = \pi)\) configurations as to the combinations of two M-branes of different kinds. So, we can set \(-\frac{\pi}{2} \leq \theta_i \leq \frac{\pi}{2}\) without the loss of generality.

(3a) one angle

To begin with, we deal with the simplest case of a rotation by single angle \( \theta_1 \), that is, we set the other angles to zero. Then, denoting \( I_n \) as the \( n \times n \) identity matrices, we get

\[
\{Q^+_\alpha, Q^-_\beta\} = 4 \int_{\mathcal{M}_5} d\xi^5 \cdot \text{diag.}(\sin^2 \frac{\theta_1}{2} I_8, \cos^2 \frac{\theta_1}{2} I_8, \ldots), \quad (3.7)
\]

which means that all the supersymmetry is broken unless \( \theta_1 = 0 \) (or \( \theta_1 = \pi \)). When this condition is satisfied (i.e. \((5|\text{M5,MKK})\) given in ref. [26]), 1/4 supersymmetry is preserved.

(3b) two angles

Now, we get

\[
\{Q^+_\alpha, Q^-_\beta\} = 4 \int_{\mathcal{M}_5} d\xi^5 \cdot \text{diag.}(\sin^2 \frac{\theta_1 - \theta_2}{2} I_4, \sin^2 \frac{\theta_1 + \theta_2}{2} I_4, \\
\cos^2 \frac{\theta_1 + \theta_2}{2} I_4, \cos^2 \frac{\theta_1 - \theta_2}{2} I_4, \ldots), \quad (3.8)
\]

which means that all the supersymmetry is broken unless \( \theta_1 \pm \theta_2 = 0, \pm \pi \). When one of these conditions is satisfied, 1/8 supersymmetry is preserved. When two of them are satisfied, 1/4 supersymmetry is preserved, which are the cases of \( \theta_1 = \pm \theta_2 = \pm \frac{\pi}{2} \) (i.e. \((3|\text{M5,MKK})\)).

(3c) three angles
We have
\[
\{Q^+_\alpha, Q^+_\beta\} = 4 \int_{M_5} d\xi^5 \text{diag} \left( \sin^2 \frac{\theta_1 - \theta_2 - \theta_3}{2} 1_2, \sin^2 \frac{\theta_1 - \theta_2 + \theta_3}{2} 1_2, \sin^2 \frac{\theta_1 + \theta_2 - \theta_3}{2} 1_2, \sin^2 \frac{\theta_1 + \theta_2 + \theta_3}{2} 1_2, \cos^2 \frac{\theta_1 + \theta_2 + \theta_3}{2} 1_2, \cos^2 \frac{\theta_1 + \theta_2 - \theta_3}{2} 1_2, \cdots \right),
\]
which means that all the supersymmetry is broken unless \( \theta_1 \pm \theta_2 \pm \theta_3 = 0, \pm \pi \). When one of these conditions is satisfied, 1/16 supersymmetry is preserved. In special cases the supersymmetry is enhanced. When one of \( \theta_i \) is \( \pm \frac{\pi}{2} \) in addition to the condition, 1/8 supersymmetry is preserved. If another \( \theta_i \) is \( \pm \frac{\pi}{2} \), 1/4 supersymmetry is preserved, to be sure, but in this case it holds the other angle is equal to zero or \( \pm \pi \). So, these should be classified in “two angles”.

(3d) four angles

The superalgebra is given in (3.6). Supersymmetry is completely broken unless \( \theta_1 \pm \theta_2 \pm \theta_3 \pm \theta_4 = 0, \pm \pi \). When one of these conditions is satisfied, 1/32 supersymmetry is preserved. Furthermore, the supersymmetry is enhanced in the following cases: suppose none of the indices i,j,k,l are the same. Then, 1/16 supersymmetry is preserved in the three cases:

\[
\begin{align*}
\theta^i &= \pm \theta^j \\
\theta^i \pm \theta^j &= \pm \frac{\pi}{2} \\
\theta^i \pm \frac{\pi}{2} \\
\theta^k &= \pm \theta^l
\end{align*}
\]

3/32 supersymmetry is preserved in the two cases:

\[
\begin{align*}
\theta^i &= \pm \theta^j \\
\theta^i &\pm \theta^j = \pm \frac{\pi}{2} \\
\theta^i &\pm \theta^j = \pm \frac{\pi}{2}
\end{align*}
\]

1/8 supersymmetry is preserved in the two cases:

\[
\begin{align*}
\theta^i &= \pm \theta^j \\
\theta^i &\pm \theta^j = \pm \frac{\pi}{2} \\
\theta^i &\pm \theta^j = \pm \frac{\pi}{2}
\end{align*}
\]

Finally, 1/4 supersymmetry is preserved in the cases of \( \theta^i = \pm \theta^j = \pm \theta^k = \pm \theta^l = \pm \frac{\pi}{2} \), which is (1|M5,MKK) given in ref.[45][13].

3. 2 A worldvolume soliton on the M-5-brane in the M-5-brane background

We have discussed so far supersymmetric configurations of test branes in (nontrivial) brane backgrounds so far. In fact these configurations can be regarded as trivial examples
of worldvolume solitons on test branes in brane backgrounds because they are solutions of the equations of motion of the test branes in the brane backgrounds. In this subsection, related to this interpretation, we present a (nontrivial) supersymmetric worldvolume 3-brane soliton on the (test) M-5-brane in a non-flat (M-5-brane) background. Namely, this is a solution of the equations of motion of the M-5-branes in the M-5-brane background, while usual worldvolume solitons are constructed as the solutions of branewave equations of branes in flat backgrounds\([33][34][35][12]\). The soliton we present here is an extended solution of 3-brane solitons on the M-5-brane in the flat spacetime given by Howe, Lambert and West in ref.\([12]\). So, before presenting our original result, we give a short review of the 3-brane solitons given in ref.\([12]\).

Suppose the M-5-brane lies in D=11 flat background with coordinates \(X^m\) (for \(m=0,1,\ldots,9,\bar{z}\)). We denote the 6-dimensional worldvolume coordinates of it by \(\xi^i\) (\(i=0,1,\ldots,5\)), and suppose the 3-brane soliton lies in the hyperplane \(\xi^0,\xi^1,\xi^2,\xi^3\), which means that the coordinates transverse to the 3-brane is \(\xi^4,\xi^5\), which we denote as \(\xi^{i'}\). Here, we assume that all fields depend only on the transverse coordinates \(\xi^{i'}\) just like the cases of brane solutions of spacetimes. Then, the 3-brane soliton in ref.\([12]\) (in our notation) is written as

\[
X^0 = \xi^0, X^1 = \xi^1, X^2 = \xi^2, X^3 = \xi^3, X^4 = \xi^4, X^5 = \xi^5, \\
X^6 = X^6(\xi^4,\xi^5), X^7 = q \ln \sqrt{(\xi^4)^2 + (\xi^5)^2} \\
X^8, X^9, X^{\bar{z}} = (\text{const.}), A_2 = 0,
\]

(3.13)

where \(X^6\) and \(X^7\) satisfy the relations:

\[
\epsilon_{i'j'} \partial^{i'} X^6 = \pm \partial_{i'} X^7
\]

(3.14)

(where \(\epsilon_{i'j'}\) is the volume element on the transverse space). The scalar \(X^6\) is interpreted as the worldvolume dual of a 5-form field strength \(G_5\) of a 4-form gauge field \(B_4\) defined as

\[
G_{i_1\ldots i_5} \equiv dB_4 = \epsilon_{i_1\ldots i_6} \partial^{i_6} X^6.
\]

(3.15)

So, \(q\) is an electric charge in terms of the 4-form \(B_4\) while it is a magnetic charge in terms of \(X^1\). So, \(X^1\) cannot be expressed globally, though the solution does exist. This is a worldvolume 3-brane soliton on the M-5-brane with 1/2 worldvolume (i.e. 1/4 spacetime) supersymmetry. In terms of spacetime this soliton is interpreted as the configuration of two M-5-branes intersecting on the 3-brane. The 3-brane is considered to be an intersecting subspace of another M-5-brane, and the two nontrivial scalars \(X^1, X^2\) pointing to the two directions transverse to the original M-5-brane, are considered to be the rest subspace of the another M-5-brane.
Now, we present our original result. On the analogy of the above 3-brane soliton, we construct by rule of thumb a 3-brane soliton on the M-5-brane in the M-5-brane background (2.29) given by

\[
\begin{align*}
  x^0 &= \xi^0, x^1 = \xi^1, x^2 = \xi^2, x^3 = \xi^3, y^6 = \xi^4, y^7 = \xi^5, \\
  x^4 &= x^4(\xi^4, \xi^5), x^5 = q \ln (\xi^4)^2 + (\xi^5)^2, \\
  y^8, y^9, \tilde{y} \ &= \text{constant}, A_2 = 0,
\end{align*}
\]

where \(x^4\) is interpreted as the dual scalar of a worldvolume five-form field strength \(G_5\) defined as \(G_5 = (\partial x^4)^*\), and \(q\) is a magnetic charge of \(x^4\). \(x^4\) and \(x^5\) satisfy the equation:

\[
\epsilon_{i'j'} x^{i'} = \partial_i x^5, \quad \text{(for } i', j' = 4, 5) \quad (3.17)
\]

where \(\epsilon_{i'j'}\) is the volume element on the space \(\xi^4, \xi^5\).

Let us first explain the interpretation of this soliton. From upper part of (3.16) the test brane is considered to be embedded in the 12367-hyperplane, which means that this is the 3-brane intersection of the test M-5-brane with the background M-5-brane. Besides, from the worldvolume point of view this can be interpreted as a 3-brane soliton in the plane \(\xi^0, \xi^1, \xi^2, \xi^3\), while the coordinates transverse to the 3-brane are \(\xi^4, \xi^5\). And the two worldvolume scalars point to \(x^4, x^5\). So, on the basis of the interpretation presented above, this soliton can be interpreted as the following three M-5-brane intersection:

\[
\begin{array}{cccccccccc}
\text{directions} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \text{#} \\
\text{the background M-5-brane} & \circ & \circ & \circ & \circ & \circ & - & - & - & - & - & - \\
\text{the test M-5-brane} & \circ & \circ & \circ & - & - & \circ & \circ & - & - & - & - \\
\text{the M-5-brane as the soliton} & \circ & \circ & \circ & \circ & \circ & \circ & \circ & - & - & - & - \\
\end{array}
\]

where the underbarred circles in the table mean that they are scalars transverse to the test M-5-brane.

Next, we show that the embedding (3.16) is a worldvolume soliton, i.e. a solution of the equations of motion of the M-5-branes in the M-5-brane background (2.29). To see this we present the expressions of induced worldvolume fields in this case: the metric \(\tilde{g}_{ij}\), the 3-form \(\tilde{C}^{(3)}\), and the 6-form \(\tilde{C}^{(6)}\). By using the relations (3.17), the induced worldvolume \((6 \times 6)\) metric \(\tilde{g}_{ij}\) is

\[
\tilde{g}_{ij} = 
\begin{pmatrix}
-H^{-1/3} & 0 & 0 & 0 & 0 & 0 \\
0 & H^{-1/3} \cdot \text{1}_{3 \times 3} & 0 & 0 & 0 & 0 \\
0 & 0 & H^{2/3} \cdot \delta_{i'j'} + H^{-1/3}(\partial_i x^4 \partial_{j'} x^4 + \partial_i x^5 \partial_{j'} x^5) & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
= H^{-1/3} \text{diag.}(-1, 1, 1, 1, H + \delta_{i'j'} \partial_i x^4 \partial_{j'} x^4 + \partial_i x^5 \partial_{j'} x^5, H + \delta_{i'j'} \partial_i x^5 \partial_{j'} x^5).
\]

(3.18)

We note that \(H|_{\theta=0} = -\tilde{C}^{(3)}|_{\theta=0} = -\tilde{C}_{\text{mag}}^{3} = 0\) in the cases of three brane intersection of the probe with the background, because \(\tilde{C}_{\text{mag}}^{3} \neq 0\) only if the three indices are all
transverse to the background M-5-brane. On the other hand, $\tilde{C}^{(6)}|_{\theta=0} = \tilde{C}_{\text{ele}}^{(6)}$ has a only nonzero component

$$
\tilde{C}_{\text{ele}}^{(6)}_{012345} = H^{-1}(\partial_4 x^4 \partial_5 x^5 - \partial_4 x^5 \partial_5 x^4) = H^{-1} \delta^{i'j'} \partial_{i'} x^5 \partial_{j'} x^5.
$$

Then, we can see that this soliton (3.16) solves the equations of motion of the M-5-branes (2.19) in the nontrivial (M-5-brane) background (2.29) given in this case by

$$
\frac{\delta L}{\delta x^m} = \partial_i [\delta L / \delta \partial_i x^m] = \partial_i [\sqrt{-\det \tilde{g}} \tilde{g}^{ij} \partial_j x^n g_{nm} + \frac{1}{5!} \epsilon^{i i_1 \cdots i_5} \epsilon_{m_1 \cdots m_5} \partial_{i_1} x^{m_1} \cdots \partial_{i_5} x^{m_5} H^{-1}],
$$

where the last term is the contribution of $C_{\text{ele}}^{(6)}$ in the Wess-Zumino-like term of the M-5-brane action (2.19). Thus, we can call it a worldvolume soliton.

The final issue we have to discuss is the preserved supersymmetry of the soliton. This configuration is expected to have 1/4 supersymmetry because it can be interpreted as the above three M-5-brane intersection. We can easily confirm that 1/4 supersymmetry is preserved by using the superalgebra (2.35). Substituting the solution (3.16) for (2.35) we have

$$
\{Q^+_\alpha, Q^+_\beta\} = 2 \int_{M_5} d^5 \xi \delta_{\alpha \beta} + 2 \int_{M_5} d^5 \xi (C \Gamma_{12367})_{\alpha \beta}.
$$

So, 1/4 supersymmetry is preserved in this solution.

Though it seems possible to construct other solutions of this type, we do not discuss them here.

4 Summary

We have derived superalgebras in many type of M-brane backgrounds via various probes, and checked their consistency by deducing from the algebras all the previously known supersymmetric orthogonal intersections (and parallel configurations) of various combinations of two M-branes. In addition, on the basis of the superalgebras, we have derived all the non-orthogonal supersymmetric intersections of the M-5-brane and the M-Kaluza-Klein monopole at angles, most of which were previously unexamined. Finally, we have presented the 1/4-supersymmetric worldvolume 3-brane soliton on the M-5-brane in the M-5-brane background, which can be interpreted as the intersection of three M-5-branes.

We note that the setup of this method is inappropriate just on the background branes because of the singular behavior of their metrics. Generally speaking, however, each of various approaches has its merits and demerits, or good “regions” and bad ones to
describe something (in this case, branes). So, thinking of the results presented above, we can conclude that our method is useful in some aspects to investigate M-theory and String theory.

**Acknowledgement**

I would like to thank Prof. J. Arafune for careful reading of the manuscript and useful comments. I am grateful to Akira Matsuda for many useful discussions and encouragement. I would also like to thank Taro Tani for stimulating discussions and encouragement. I am obliged to Tsunehide Kuroki for useful comments.

**References**

[1] P. K. Townsend, Phys. Lett. **B350** (1995) 184.

[2] E. Witten, Nucl. Phys. **B443** (1995) 85.

[3] For a review, P. K. Townsend, Four Lectures on M-theory, hep-th/9612121 and references therein.

[4] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, M Theory As A Matrix Model: A Conjecture, Phys. Rev. **D55** (1997) 5112.

[5] For a review, D. Bigatti, L. Susskind, Review of Matrix Theory, hep-th/9712072 and references therein.

[6] C. M. Hull, Nucl. Phys. **B509** (1998) 216.

[7] For a review, P. K. Townsend, M-theory from its superalgebra, hep-th/9712004 and references therein.

[8] J. W. van Holten and A. Van Proeyen, J. Phys. A:Math Gen. **15** (1982) 3763.

[9] P. K. Townsend, p-brane democracy, hep-th/9507048.

[10] D. Sorokin and P. K. Townsend, Phys. Lett. **412** (1997) 265.

[11] J. A. de Azcarraga, J. P. Gauntlett, J. M. Izquierdo and P. K. Townsend, Phys. Rev. Lett. **63** (1989) 2443.

‡ I would like to thank Taro Tani for pointing out that.
[12] P. S. Howe, N. D. Lambert and P. C. West, Phys. Lett. B419 (1998) 79.
[13] E. Bergshoeff, J. Gomis and P.K. Townsend, Phys. Lett. B421 (1998) 109.
[14] H. Hammer, Nucl. Phys. B521 (1998) 503.
[15] K. Kamimura and M. Hatsuda, Nucl. Phys. B527 (1998) 381; Nucl. Phys. B535 (1998) 499.
[16] T. Sato, The spacetime superalgebras from M-branes in M-brane backgrounds, Phys. Lett. 439 (1998) 12; [hep-th/9804202].
[17] E. Bergshoeff and M. de Roo, Phys. Lett. B380 (1996) 265.
[18] M. B. Green, C. M. Hull and P.K. Townsend, Phys. Lett. B382 (1996) 65.
[19] E. Bergshoeff and P.K. Townsend, Nucl. Phys. B490 (1997) 145.
[20] L. Romans, Phys. Lett. 169B (1986) 374.
[21] E. Bergshoeff, M. de Roo, M. B. Green, G. Papadopoulos and P. K. Townsend, Nucl. Phys. B470 (1996) 113.
[22] T. Sato, The spacetime superalgebras in a massive IIA background via brane probes Phys. Lett. 441 (1998) 105; [hep-th/9805203].
[23] J. Gomis, D. Mateos, J. Simon and P. K. Townsend, Phys. Lett. 430 (1998) 231.
[24] R. Güven Phys. Lett. 276B (1992) 49.
[25] G. Papadopoulos and P. K. Townsend, Phys. Lett. B380 (1996) 273.
[26] A. A. Tseytlin, Nucl. Phys. B475 (1996) 149; I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B475 (1996) 179.
[27] J. P. Gauntlett, D. A. Kastor and J. Traschen, Nucl. Phys. 478 (1996) 544.
[28] E. Bergshoeff, E. Eyras and Y. Lozano, Phys. Lett. 430 (1998) 77.
[29] E. Bergshoeff and J. P. van der Schaar, On M-9-branes, [hep-th/9806069].
[30] M. Berkooz, M. R. Douglas and R. G. leigh, Nucl. Phys. B480 (1996) 265.
[31] E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, Nucl. Phys. B502 (1997) 149.
[32] N. Ohta and P. K. Townsend, Phys. Lett.  B418 (1998) 77.

[33] C. G. Callan and J. M. Maldacena, Nucl. Phys.  B513 (1998) 198.

[34] P. S. Howe, N. D. Lambert and P. C. West, Nucl. Phys.  B515 (1998) 203.

[35] G. W. Gibbons, Nucl. Phys. B514 (1998) 603.

[36] M. J. Duff and K. S. Stelle, Phys. Lett. 350B (1991) 113.

[37] E. Cremmer and S. Ferrara, Phys. Lett. 91B (1980) 61.

[38] B. de Wit, K. Peeters and J. Plefka, Nucl. Phys. 532 (1998) 99.

[39] L. Brink and P. S. Howe, Phys. Lett. 91B (1980) 384.

[40] A. Candiello and K. Lechner, Nucl. Phys. B412 (1994) 479.

[41] E. Bergshoeff, E. Sezgin and P. K. Townsend, Phys. Lett. B189 (1987) 75; Ann.Phys.(NY) 185 (1988) 330.

[42] A. A. Tseytlin, Nucl. Phys. 487 (1997) 141.

[43] P. Pasti, D. Sorokin and M. Tonin, Phys. Lett. 398B (1997) 41; I.Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, Phys. Rev. Lett. 78 (1997) 4332.

[44] L. Brink and J. H. Schwarz, Phys. Lett. 100B (1981) 310.

[45] E. Bergshoeff, M. de Roo, E. Eyras, B. Janssen and J. P. van der Schaar, Class. Quant. Grav. 14 (1997) 2757.

[46] C. Hull, Phys. Lett. 139B (1984) 39.

[47] P. K. Townsend, Brane surgery, Nucl. Phys. Proc. Suppl. 58 (1997) 163.