Internal Report ITESRE 198/1997

A PRELIMINARY STUDY ON DESTRIPING TECHNIQUES OF PLANCK LFI MEASUREMENTS VERSUS OBSERVATIONAL STRATEGY

C. Burigana¹, M. Malaspina¹, N. Mandolesi¹,
L. Danese², D. Maino²,
M. Bersanelli³,¹ & M. Maltoni⁴,¹

¹Istituto TESRE, CNR, Bologna, Italy
²SISSA – International School for Advanced Studies, Trieste, Italy
³Istituto di Fisica Cosmica, CNR, Milano, Italy
⁴Dipartimento di Fisica, Università di Ferrara, Italy

November 1997
A PRELIMINARY STUDY ON DESTRIPING TECHNIQUES OF PLANCK LFI MEASUREMENTS VERSUS OBSERVATIONAL STRATEGY

C. Burigana\textsuperscript{1}, M. Malaspina\textsuperscript{1}, N. Mandolesi\textsuperscript{1}, L. Danese\textsuperscript{2}, D. Maino\textsuperscript{2}, M. Bersanelli\textsuperscript{3} & M. Maltoni\textsuperscript{4,1}

\textsuperscript{1}Istituto TESRE, CNR, Bologna, Italy
\textsuperscript{2}SISSA – International School for Advanced Studies, Trieste, Italy
\textsuperscript{3}Istituto di Fisica Cosmica, CNR, Milano, Italy
\textsuperscript{4}Dipartimento di Fisica, Università di Ferrara, Italy

SUMMARY – We present here the basic issues of our simulations of Planck observations, focusing on the study of the stripes generated by $1/f$-type noise due to amplifiers noise temperature variation of LFI radiometers generated by gain fluctuations. Our simulations include a realistic estimate of $1/f$ knee frequency, based on a recent analytical study of the properties of the Planck LFI radiometers and of their systematic effects, as well as realistic choices of load, amplifier noise and payload environment temperatures. By comparing simulated observed maps of CMB anisotropies obtained by including or not the instrumental noises we are able to quantify the magnitude of the striping effect. The “standard” destriping method presented in the report on the Phase A study has been adapted to our purposes. We quantify the efficiency of this destriping technique for different scanning strategies, for typical off-axis LFI beams. Our results are compared with those of other previous analyses.

1 Introduction

The Planck Surveyor is an European Space Agency (ESA) satellite mission to map spatial anisotropy in the Cosmic Microwave Background (CMB) over a wide range of frequencies with an unprecedented combination of sensitivity, angular resolution, and sky coverage (Bersanelli et al. 1996). The data gathered by this mission will revolutionize modern cosmology by shedding light on fundamental cosmological questions such as the age and present expansion rate of the universe, the average density of the universe, the amount and the kind of dark matter, and other questions. As with any CMB experiment, achieving the desired performance requires careful attention to the control of systematic effects. $1/f$-type noise in the
radiometer output is one of the most critical systematic effect pertaining to the Low Frequency Instrument (LFI) radiometers because it may lead to striping in the final sky maps and increase the noise level. In general, a value of the $1/f$ knee frequency, $f_k$, significantly greater than the spacecraft rotation frequency, $f_s$, will lead to some degradation in sensitivity (Janssen et al. 1996). In this paper we examine the impact of the $1/f$-type noise by adopting a realistic estimate of $1/f$ knee frequency as a function of the load, amplifier noise and payload environment temperatures, of the radiometer bandwidth and of the level of fluctuations of each stage of the radiometer amplifiers, based on recent analytical studies of systematic effects in Planck LFI radiometers (Seiffert et al. 1997).

In section 2 we summarize the basic concepts relevant for understanding the basic properties of instrumental noises.

In section 3 we present the mathematical formalism of our numerical code for the simulation of the Planck observations, including instrumental noises, and the data stream generation; we typically refer here to Planck like scanning strategies, but our code is versatile enough to allow the study of other observational schemes.

In section 4 we describe how we have converted our simulated data streams in simulated observed maps.

In section 5 we discuss in detail the mathematical formalism of the proposed destriping technique, including some considerations about the numerical efficiency.

The “standard” estimators that quantify the magnitude of the striping effect and the efficiency of destriping techniques are presented in section 6.

Some preliminary results are presented in section 7.

Finally, in section 8 we draw out the main conclusions of our analysis, compare our results with those of previous works and draw out a brief guideline for a future work. We discuss there the main implications of our study, focusing on their impact for the optimization of the Planck observational strategy.

2 Sources of instrumental noises

Planck LFI radiometers are modified Blum correlation receivers (Blum 1959, Colvin 1961). The modification is that the temperature of the reference load is quite different from the sky temperature (Bersanelli et al. 1995). To compensate, differing DC gains are applied after the two detector diodes. Adjusting the ratio of DC gains, $r$, allows one to null the output signal, minimize sensitivity to RF gain fluctuations, and achieve the lowest white noise in the output. Although it may not be immediately apparent, the fact that the reference load is not at the same temperature as the sky does not increase the white noise level compared to a standard correlation receiver.

The ideal sensitivity of our radiometer for a single observation with an integration time $\tau$ is

$$\Delta T_{\text{white}} = \sqrt{\frac{2}{\beta\tau}} \left( T_n + T_x \right),$$  \hspace{1cm}(1)$$

where $\beta$ is the effective bandwidth, $T_x$ is the noise temperature of the signal entering one of the two horns and $T_n$ is the amplifier noise temperature.

In order to null the average output signal of the radiometer, the DC gains ratio after the two detector diodes, $r$, must be adjusted to the proper value. In the simple case that the gains of the two amplifiers entering the two horns and their noise temperatures can be considered equal, $r$ must be set to the value:
\[ r = \frac{T_x + T_n}{T_y + T_n}, \tag{2} \]

where \( T_y \) is the reference load temperature entering the other horn. The above temperatures are antenna temperatures; \( T_x \) is due properly to the sum of the sky temperature (essentially the CMB monopole antenna temperature, related to the CMB thermodynamic temperature \( T_0 \approx 2.726 \text{K} \), plus “minor” contributions from CMB dipole and anisotropies, galactic and extragalactic foregrounds, bright sources, Zodiacal light ...) and of the “environment” temperature (of about 1 K) due to the satellite emission.

There are several potential concerns for the current radiometer scheme. Amplifiers noise temperature variations, that derive from gain fluctuations, could be confused with the sky signal variations, that we are interested in measuring, by introducing a change in the observed signal, \( \Delta T_{\text{equiv}} \), which can mimic a true sky fluctuation. The amplifiers noise temperature variations have the characteristics of \( 1/f \) noise and this leads to \( 1/f \)-type noise in the radiometer output.

We have recently estimated the \( 1/f \) knee frequency of our radiometer (Burigana et al. 1997a, Mandolesi et al. 1997a, Seiffert et al. 1997) on the simplifying assumption that there is no \( 1/f \) contribution from the detector diodes and neglecting the effects of the phase shifting that is designed to control this contribution; the diodes are assumed to be perfect square law detectors and we have assumed that the bandpass of the signal is the same in both legs of the radiometer. Here we briefly summarize the main arguments relevant for the expected magnitude of gain and noise temperature fluctuations and the main concepts relevant for the estimate of the \( 1/f \) knee frequency.

The contribution to the \( 1/f \) noise directly due to amplifier gain fluctuations results to be zero at first order. Under quite reasonable assumptions, the noise contributions due to reference load fluctuations and fluctuations in the ratio of DC gains are much less than that due to the amplifier noise temperature fluctuations term, which then results to be the dominant source of \( 1/f \) noise in the radiometer output. Imperfect isolation does not significantly modify this conclusion. Further, the sensitivity of our radiometer to differences between the gains and noise temperatures of the two amplifiers is not critical. As a consequence, all these complications cannot significantly change the knee frequency respect to the results we draw out here below.

Cryogenic HEMT amplifiers have noise temperature fluctuations with a \( 1/f \)-type spectrum because induced by the \( 1/f \)-type gain fluctuations of amplifiers (Wollack 1995, Jarosik 1996, Seiffert et al. 1996). The magnitude of noise temperature fluctuations can be computed from the following argument. Assuming that each stage of the amplifier has the same level of fluctuation, we can conclude that the transconductance of an individual HEMT device also fluctuates according to:

\[ \frac{\Delta g_m}{g_m} = \frac{1}{2\sqrt{N_s}} \frac{\Delta G}{G}, \tag{3} \]

where \( N_s \) is the number of stages of the amplifier, typically \( \sim 5 \). An optimal low noise amplifier design will have equal noise contributions from the gate and drain of the HEMT, which means the changes in \( g_m \) will lead to changes in \( T_n \) (Pospieszalsky 1989). This can be expressed as

\[ \frac{\Delta T_n}{T_n} \sim \frac{\Delta g_m}{g_m}. \tag{4} \]
We can write the $1/f$ spectrum of the gain fluctuations as:

$$\frac{\Delta G}{G} = \frac{C}{\sqrt{f}}. \quad (5)$$

Putting this together we get:

$$\frac{\Delta T_n}{T_n} \approx \frac{1}{2\sqrt{N_s}} \frac{C}{\sqrt{f}}. \quad (6)$$

We can therefore write the amplifier noise temperature fluctuations as

$$\frac{\Delta T_n}{T_n} = \frac{A}{\sqrt{f}}, \quad (7)$$

with $A = C/(2\sqrt{N_s})$; a normalization of $A \simeq 1.8 \times 10^{-5}$ (relying on the references above) is appropriate for the Planck radiometers at 30 and 45 GHz. Throughout, we will use units of $K/\sqrt{Hz}$ for $\Delta T$ so that we will not need to refer to the sampling frequency of the radiometer. In these units then, $\Delta T/T$ has units of $Hz^{-1/2}$ and $A$ is dimensionless. We also note that the value of $A$ will generally depend on the physical temperature of the amplifier. The values for $A$ given here should be regarded as estimates rather than precise values. For the radiometers at higher frequencies, it will be necessary to use HEMT devices with a smaller gate width to achieve the lowest amplifier noise figure. We expect that the gate widths will be roughly 1/2 that of the devices used for the lower frequency radiometers and this will lead to $g_m$ fluctuations that are roughly a factor of $\sqrt{2}$ higher (Gaier 1997, Weinreb 1997). We will therefore adopt a normalization of $A = 2.5 \times 10^{-5}$ for the 70 and 100 GHz radiometers.

Starting from the expression of the average output of the differential radiometer, one derives the change in the output signal for the above small change in the noise temperature of one of the amplifier; and by multiplying it by a factor $\sqrt{2}$ because both amplifiers (which have uncorrelated noise) can contribute to this effect, we have the change in the observed signal, $\Delta T_{\text{equiv}}$, given by:

$$\Delta T_{\text{equiv}} = \sqrt{2}\Delta T_n \left[1 - \frac{r}{2}\right]. \quad (8)$$

We define the “knee” frequency as the post-detection frequency, $f_k$ at which the $1/f$ contribution and the ideal white noise contribution are equal, i.e. $\Delta T_{\text{equiv}} = \Delta T_{\text{white}}$. For the computation of the knee frequency we use an integration time $\tau = 1/(2\Delta f)$ and $\Delta f = 1$ Hz, according to the choosen units for $A$ and $\Delta T_n$. The knee frequency is then given by:

$$f_k = \frac{A^2 \beta}{8}(1 - r)^2 \left(\frac{T_n}{T_n + T_x}\right)^2. \quad (9)$$

This expression shows as the knee frequency depends upon several factors including the radiometer bandwidth, reference load temperature, and the intrinsic level of fluctuation in the HEMT devices; values of $\sim$ few $\times$ 0.1 Hz, according to the frequency, should be reached with only passive cooling of the radiometer (to about 50 K), whereas active cooling (to about 20K or less) can further reduces the knee frequency. As examples, assuming a 20% bandwidth for our frequency channels and an antenna temperature $T_x = 3K$, the knee frequency is 0.046 Hz and 0.11 Hz respectively at 30 GHz (assuming $T_y = 20K$ and $T_n = 10K$) and at 100 GHz (assuming $T_y = 20K$ and $T_n = 40K$).
For a more realistic evaluation we have to accurately repeat the analytical considerations by Seiffert et al. (1997) to distinguish between the amplifier noise temperature, $T_n$, and the system temperature, $T_{sys}$. In reality, it would be more appropriate to insert $T_{sys}$ in eq. (1) and to carefully consider when the two temperatures enter in the determination of the knee frequency. On the other hand, the difference between these two temperature is estimated to be of few K while the instrumental situation is still partially unclear; then a careful distinction between them is presently not necessary from practical point of view, although interesting. We will address this point in a future work.

The knee frequency must be compared to the spin frequency $f_s$; for the Planck observational strategy proposed for the Phase A study $f_s = 1\text{ r.p.m.}$, i.e. 0.017 Hz.

As a comparison, for a total power radiometer, laboratory measurements have found knee frequencies between 10 and 100 Hz; the modified correlation radiometer scheme reduces the knee frequency by more than two order magnitudes.

3 Simulation of the mission

We have written a code that simulates the basic properties of Planck observations in order to study the stripping effect due to $1/f$-type noise on the measured sky temperatures. For simplicity our sky map includes only CMB fluctuations, generated in equi-cylindrical pixelisation (ECP) by using the method of Muciaccia et al. (1997); then we have projected it in COBE-cube pixelisation in order to have quasi equal area pixels. For our tests we have considered CMB fluctuations in a typical CDM scenario ($\Omega_b = 0.05$, with COBE/DMR normalization). We have used here an input map $M_{in}$ at a resolution of $19.4'$ [i.e. at COBE-cube resolution 9] that we have derived from an ECP map with 1024 grid points along each parallel computed by using the multipoles up $l = 512$. The typical dimension of a pixel of an input map at resolution 9 is comparable with the beam FWHM of 30' of the channels at 30GHz, that we have considered for the present tests. Our input map is shown in Figure 1.

3.1 The simulation of the sky observation

The instrument design for Planck mission calls for multi-frequency focal plane arrays placed at the focus of off-axis optical systems, in order to achieve proper angular resolution, sensitivity, and spectral coverage. As a consequence, not all the feedhorns can be located very close to the centre of the focal plane; the study of the implications of the related beam optical distortions on temperature measurements has been presented in other works (Burigana et al. 1997b,c). For the present purpose we do not convolve the sky map with the beam response, but we simply read the map temperature corresponding to the pixel identified by the central direction of a given beam during the sky scanning.

Figure 2 shows the schematic representation of the observational geometry.

Let $i$ be the angle between a unit vector $\vec{s}$, along the satellite spin axis (outward the Sun direction), and the normal to the ecliptic plane, and $\vec{p}$ the unit vector of the direction of the optical axis of the telescope, at an angle $\alpha$ from the spin axis ($i = 90^\circ$ and $\alpha = 70^\circ$ for the Phase A study (Bersanelli et al. 1996). We choose two coordinates $x$ and $y$ on the plane tangent to the celestial sphere in the telescope optical axis direction, with unit vector $\vec{u}$ and $\vec{v}$ respectively; we choose the $x$ axis according to the condition that the unit vector $\vec{u}$ points always toward the satellite spin axis; indeed, for standard Planck observational strategy, this condition is preserved as the telescope scans different sky regions. With this choice of reference frame, we have that $\vec{v} = \vec{p} \wedge \vec{s} / |\vec{p} \wedge \vec{s}|$ and $\vec{u} = \vec{v} \wedge \vec{p} / |\vec{v} \wedge \vec{p}|$ (here $\wedge$ indicates the vector product). In general the coordinates $(x_0,y_0)$ of the beam centre will be identified by two angles; we use here the colatitude $\theta_B$ and the longitude $\phi_B$ in the $\vec{u},\vec{v},\vec{p}$ reference
frame (see the Appendix A for details on geometrical transformations). For the present test we assume a typical off-axis location of the considered beam: $\theta_B = 2.8^\circ$, $\phi_B = 45^\circ$. We note that our choice of $\theta_B$ is representative of typical LFI beam position for a telescope with a primary mirror of 1.5m aperture in the new Planck optical configuration (see Mandolesi et al. 1997b). Our choice of $\phi_B$ corresponds to a case of intermediate efficiency respect to the destriping technique; the cases $\phi_B = 0^\circ$ (beam located along the $\vec{u}$ direction in the TICRA U-V plane) and $\phi_B = 90^\circ$ (beam located along the $\vec{v}$ direction in the U-V plane) are equivalent respectively to an on-axis beam and to a beam that suitably distributes the crossings between different circles in two regions, close to the ecliptic poles, of maximum size. The off-axis choice for $\theta_B$, when $\phi_B$ significantly differs from 0, also assures that, even in the particular case of an angle of 90$^\circ$ between the spin axis and the telescope direction, the scanning circles do not cross always exactly the ecliptic poles but two somewhat larger regions around them also for scanning circles corresponding to significantly different spin axis positions.

We remember that in the Planck scanning strategy of the Phase A study the sampling time of each receiver is chosen in order to have three samplings when the telescope axis describes in the celestial sphere an angle with a length equal to the beam FWHM ($\sim$ three samplings per beam). For any value of the angle $\alpha$, this condition determines the number of samplings, $n_p$, per scan circle.

We will study in detail the effect introduced by the telescope motion in a future work. For the present purpose, we simply assign the proper beam directions to each sampling, extract the corresponding pixel and read its temperature in the simulated input map; given the assumed FWHM of 30$'$ and the pixel dimension 19.4$'$ we have typically 2–3 different pixels explored in an integration time corresponding to 3 samplings.

We remember that, in order to be able to reduce the white noise in a simple way, we want to “close” the scanning circle, i.e. we need to modify just a little the integration time by requiring that the telescope points always at the same set of directions when it repeats the (120 for the Phase A study) number of cycles with the same spin axis direction. Finally we have applied a further reassessment of the integration time to make $n_p$ multiple of 12, in order to be able of applying in the future the destriping technique not only with one but also with 2, 3 or 4 level constants for circle (see sections 5 and 7.3).

Our code let us free to implement arbitrary scanning strategies (see also Appendix A); we consider here cases with the spin axis always on the ecliptic plane and with an angle $\alpha$ between the spin axis and the telescope direction of 80$^\circ$, 85$^\circ$, 90$^\circ$; lower values of $\alpha$, like that of 70$^\circ$ assumed for the Phase A study, will require wide oscillations ($15^\circ$–20$^\circ$) of the spin axis on the ecliptic plane (with relevant problems for the thermal stability) in order to observe the regions close to ecliptic poles, that are indeed very informative, being not significantly contaminated by the galactic emission. We have considered also a case with $\alpha = 90^\circ$ and with ten $10^\circ$ sinusoidal oscillations of the spin axis on the ecliptic plane. For all the cases we have considered a 360 days mission in order to compare results obtained with the same mission duration.

3.2 The generation of instrumental noise series

We generate white noise and 1/f noise using a random number generator code (see Press et al. 1992). For the white noise this is very simple: we have simply to rescale a gaussian random noise distribution, by taking into account the radiometer rms white noise [see eq. 4]. For including the 1/f-type noise we have adapted in FORTRAN an original IDL code provided us by M. Seiffert, based on the power spectrum expansion in the Fourier space of the noise components. We generate together white and 1/f noise. Firstly a random gaussian distribution
of (white) noises is generated; then we calculate its power spectrum using an FFT code (see Press et al. 1992); the amplitude of this power spectrum is then multiplied by \((1 + f_k/f)^{1/2}\) for including the \(1/f\) contribution. Finally we have the time series with both the noises by computing the (inverse) FFT of the power spectrum. When we take into account the real properties of the 30 GHz receivers, eq. (9) (Seiffert et al. 1997) gives the \(1/f\) knee frequency of our radiometers. We adopt as reference for the present simulations \(f_k = 0.05\text{Hz}\) a value adequate to a cooling efficiency that allows to keep a load temperature \(T_y \simeq 20\text{K}\) (we will use here \(T_n = 9\text{K}, \beta = 6\text{GHz}\)). Our noise series consist of about \(2 \times 10^6\) evaluations and a single series covers the integrations for 8 different contiguous spin axis directions (16 hours). We use a single series for white noise and \(1/f\) noise together, but we generate also a noise series that includes white noise only; this increases the computation time of only few percent (see above) but it is useful for comparison and for the quantification of the stripes magnitude and of the destriping efficiency (see sections 6 ÷ 10). In our code the generation of noise series is coupled with the sky observation; more precisely, because of the noises dependence on the antenna temperature \(T_x\), we include the exact local sky temperature in the noise magnitudes. This kind of flexibility may be useful also in the future for simulations that will include Galaxy emission and thermal drifts too. Then, the only simplification is in the estimate of the knee frequency for the \(1/f\) noise series, which is assumed to be constant (i.e. with constant \(T_x\), not allowing for spatial or time variations); the adopted value is consistent with the CMB monopole antenna temperature and a typical environment temperature of 1 K.

### 3.3 The data stream generation and recording

We have recorded our data streams in 4 matrices; any row of these matrices refers to the data obtained from a given spin axis direction; the number of columns is equal to the number of samplings \(n_p \sim 2100\), depending on \(\alpha\); the number of rows is equal to the number, \(n_s\), of different spin axis directions (4320 for a shift of 5’ in the spin axis direction). We have recorded the following data:

- the matrix \(N\) which contains the pixel numbers – 4 bytes per pixel – (in COBE-cube pixelisation) corresponding to the different integrations;

- the matrix \(T\) which contains the "global" temperatures observed by the receiver in the above sky directions, directly averaged over the number of cycles per scan circle (120 in the Phase A study scheme) in order to avoid the use of a large useless amount of memory space. \(T\) includes: the "input" sky temperature fluctuation as observed in the adopted geometrical scheme, which obviously can be computed a single time for any given spin axis direction; the white noise and the \(1/f\) noise averaged over the number of cycles for each spin axis direction;

- the matrix \(W\), generated in the same way as the matrix \(T\), but containing the temperatures that will be observed in presence of the white noise only (see section 3.2);

- the matrix \(G\) contains the temperatures that will be observed in absence of instrumental noise and will be useful to check the goodness of the geometrical part of our flight simulation code.

Of course the matrices \(W\) and \(G\) do not have the corresponding ones in a real observation. In principle, if we simulate the mission for a time long enough that the spin axis return on the same direction after a certain period (360 days, as a reference allowing for the case \(\alpha = 90^\circ\)) we can average the data of the second period with the corresponding rows of the first period, and take memory that their will be affect by a (statistical) error \(\sqrt{2}\) times
smaller than that corresponding to rows that are observed for a single period. For a more realistic simulation, we need to record a further matrix $E$ with the statistical sensitivities corresponding to the pixels of matrix $N$, which takes into account this fact as well as a possible degradation in sensitivity for some elements of data streams due for example to cosmic rays, spurious effects ... . We neglect these kinds of complications for the present analysis, which is equivalent to say that all the elements of $E$ assume a constant value. Nevertheless, in section we draw out our formulas by including this possible effect, for sake of generality.

For implementing the destriping techniques (see section 5) we need to recognize when the pointing directions for different spin axis directions are substantially identical. The sky pointing direction is stored in this scheme only through the corresponding pixel number. Then, from a statistical point of view, two pointing directions are considered identical provided that their distance is smaller than the pixel size. The resulting number of pixels in common is then related to the assumed pixel size. We have then recorded:

- additional matrices $N_H$ (H=1,2,...,$n_R$) contains the pixel number corresponding to the different integrations for a certain number, $n_R$, of resolutions higher than that used for the input/output maps (for example at resolution 10 or 11 for input map at resolution 9). By exploiting these matrices we will able to test the adopted destriping technique under more stringent conditions on the average distance between their pointing directions in the research of the pixels in common.

4 From data streams to observed simulated maps

Given the above simulated data streams, it is quite simple to obtain the following simulated observed maps (see also section 5.3), that can be easily compared one to each other.

We compute the sensitivity map, $M_S$, with which any map pixel is observed, by recognizing how many times a given pixel is observed from the analysis of the whole matrix $N$ (and $E$ if this is the case).

From the matrices $N$ and $T$, we average the temperatures corresponding to the same pixel in different matrix positions to have the observed temperature map $M_T$ (including noises).

In similar way, from the matrices $N$ and $W$ (or $G$) we obtain the observed temperature map $M_W$ (or $M_G$) computed in presence of white noise only (or in absence of instrumental noises). We have verified that the map $M_G$ is identical to the “input” map for all the observed pixels, so confirming the validity of the geometrical part of our flight simulation code.

5 Destriping techniques

We have developed a technique in order to eliminate the effects of gain drifts in Planck signal due to the $1/f$ noise effect. The method is derived from that proposed for the Phase A Study and re-analyzed by Delabrouille (1997). On the other hand, our treatment of the Planck observation simulation, although simplified, is general enough to be close to the “real” Planck observations; so we draw out here below the destriping mathematical formalism, in a way directly applicable to our simulated data streams.

5.1 Mathematical formalism

In this section we discuss how to eliminate the effects of gain drifts on timescales greater than that for which the spin axis points at a given direction (2 hours for the Phase A scheme, or 1 hour or 40 minutes for possible new observational strategies in which the spin axis shift of
2.5’ or 1.6’ in 1 hour or 40 minutes respectively), i.e. the satellite scans a given circle in the sky.

After we removed the drifts within any given scan circle by averaging the observations over the corresponding cycles (see section 3.1), each set of observations at a given spin axis direction, denoted by the index \(i\), is characterized by an additive level \(A_j\) which is related to the “mean” \(1/f\) noise level during the observation in that scan circle. These levels \(A_i\) are different for different circles, due to gain fluctuations. Our goal is to obtain a reduced set of observations of different scan circles by removing the contamination that affects any circle. So we will subtract to all sets of observations on a given scan circle their own characteristic level \(A_i\). As a variance, we can attribute more constant levels, say \(n_l\) levels, per single scan circle.

From the computational point of view this is exactly equivalent to rearrange all the matrices in section 3.3 by dividing their rows in \(n_l\) parts that have to be appropriately relocated to construct new matrices with \(n_s \times n_l\) rows and \(n_p/n_l\) columns that can be then analysed exactly as in the case of a single constant per circle.

For estimating all these levels we use a computation scheme able to simultaneously find the pixels in common between different scan circles and generate a linear system whose solution gives the unknowns \(A_i\).

The observations of different directions in the sky explored by the satellite have been recorded in three matrices \(\mathbf{N}, \mathbf{T}, \mathbf{E}\) of \(n_s\) rows and \(n_p\) columns, where \(n_s\) is the number of different spin axis directions, and \(n_p\) is the number of samplings at different horn pointing directions in a given circle. Here \(N_{il}\) \((i = 1, \ldots, n_s, l = 1, \ldots, n_p)\) contains the pixel number corresponding to the observed direction in the sky, and \(T_{il}\) and \(E_{il}\) are respectively the corresponding observed temperature (full signal due to the sky plus noises) and the estimates of the \(\text{rms}\) noise, essentially due to the white noise. We observe that \(E_{il}\) is properly related to the amplifier noise temperature and to the observed antenna temperature which depends in a real case on the local thermal conditions and on the true sky temperature but it may depend also on possible “spurious effects”. We have in reality only “first-order” informations about all these quantities at this level; on the other hand its accurate knowledge is not crucial, being the amplifier noise temperature typically higher than the observed temperature. Anyway we hope to have more accurate informations from accurate thermal models and from iterating our data reduction scheme to have a good determination of sky temperature, which is of course our goal.

In the following the first index will denote the row index and the second the column index.

We must check for all the possible crossing points for two different circles for the whole ensemble of \(n_s\) circles (see also section 3.3).

Let \(\pi\) be an index that identifies a generic couple of different observations corresponding to the same pixel in the sky, i.e. a pixel in common between two scan circles: \(\pi\) ranges from 1 to \(n_c\), where \(n_c\) is the total number of couples found. Therefore the index \(\pi\) is related to two elements in the matrix \(\mathbf{N}\): \(\pi \rightarrow (il, jm)\). Here \(i\) and \(j\) identify the two circles for which we found common pixels, \(l\) and \(m\) are the common pixels positions on the circle \(i\) and on the circle \(j\) respectively. So we have \(N_{il} = N_{jm}\). As a variance, we can replace the matrix \(\mathbf{N}\) with one of the matrices \(\mathbf{N_H}\) (see section 3.3 and 7.4) to search for the pixels in common, according to the adopted averaged maximum distance for recognizing two pixels in common.

We want to minimize the quantity:

\[
S = \sum_{\text{all couples}} \left[ \frac{[(A_i - A_j) - (T_{il} - T_{jm})]^2}{E_{il}^2 + E_{jm}^2} \right] = \sum_{\pi=1}^{n_c} \left[ \frac{[(A_i - A_j) - (T_{il} - T_{jm})]^2}{E_{il}^2 + E_{jm}^2} \right]_{\pi} \tag{10}
\]

respect to the set of the unknown levels \(A_i\); the index \(\pi\) in the right hand side of this equation remembers that each set \((il, jm)\) derive from a given pixel \(\pi\). \(S\) is quadratic in all the unknown
on the other hand, only the differences between the levels $A_i$ enter in this expression, so that the solution will be indeterminate, i.e. the levels are determined apart from an arbitrary additional constant (with no physical meaning, as obvious for anisotropy measurements).

To remove this indetermination, we add a constraint to the $A_i$ quantities:

$$\sum_{h=1}^{n_s} A_h = 0 .$$

(11)

This is equivalent to minimize the quantity:

$$S' = S + \left( \sum_{h=1}^{n_s} A_h \right)^2$$

(12)

Now let’s go into some algebra. We perform the derivate of the previous equation, and finally we have:

$$\frac{1}{2} \frac{\partial S'}{\partial A_k} = \sum_{\pi=1}^{n_c} \left[ \frac{[A_i - A_j] - (T_{il} - T_{jm}) \cdot [\delta_{ik} - \delta_{jk}]}{E^2_{il} + E^2_{jm}} \right]_\pi + \sum_{h=1}^{n_s} A_h = 0$$

(13)

for all $k = 1, \ldots, n_s$ (here the $\delta$ are the usual Kronecker symbols). So we have a set of $n_s$ equations:

$$\sum_{t=1}^{n_s} C_{kt} A_t = B_k , \quad k = 1, \ldots, n_s .$$

(14)

We denote with $C$ and $B$, respectively, the matrix of the coefficients $C_{kt}$ and the vector of the coefficients $B_k$.

To be concrete, we show here as $C$ and $B$ are formed as we extract the pixels in common between the different rows. First of all, we set $B = 0$ and $C_{kt} = 1 \forall k, t$ (setting all $C_{kt}$ to 1 takes into account the second term of eq. 13). Then for each couple $\pi$ of pixels in common between two scan circle we define:

$$\chi_\pi = \left[ \frac{1}{E^2_{il} + E^2_{jm}} \right]_\pi$$

(15)

and

$$\tau_\pi = \left[ \frac{T_{il} - T_{jm}}{E^2_{il} + E^2_{jm}} \right]_\pi$$

(16)

From the above equation and the definition of Kronecker symbol, we easily have that a given couple $\pi$ contributes only to two equations of our linear system, those for $k = i$ or $k = j$, where as usual $i$ and $j$ corresponds to two different observations of the same pixel. If we iteratively increment the coefficients of $C$ and $B$ as we find a new couple, explicitely we have [remember $\pi \rightarrow (il, jm)$]:

$$C_{ii} \rightarrow C_{ii} + \chi_\pi$$

(17)

$$C_{ij} \rightarrow C_{ij} - \chi_\pi$$

(18)

$$C_{ji} \rightarrow C_{ji} - \chi_\pi$$

(19)

$$C_{jj} \rightarrow C_{jj} + \chi_\pi$$

(20)

$$B_i \rightarrow B_i + \tau_\pi$$

(21)

$$B_j \rightarrow B_j - \tau_\pi$$

(22)
Summing up, we have that each couple $\pi$ contributes to only six terms, and the resulting system shows a complete symmetry with respect to the exchange of the indexes $i$ and $j$.

The linear system defined by eq. (14) has some interesting properties which considerably simplify the numerical computation of its solution. In particular, the matrix $C$:

- is symmetric, so we can hold in memory only half of the matrix (say the upper-right part) and solve the system and speed-up the code by computing only half of the matrix coefficients. This is possible because the Gauss reduction algorithm preserves at each step the symmetry of remaining part of the matrix;
- is positive defined, so we never find a null pivot when reducing a non-singular matrix (Strang 1976). This allow us to solve the system without having to exchange rows or columns, so preserving the symmetry;
- is not singular (provided that there are enough intersections between scan circles), because the only fundamental indetermination has been removed by imposing the constraint (11).

Anyway, after the system has been solved, it is our care to replace the solution into the original $C$ matrix, to verify its correctness and check for rounding errors and/or accidental degenerations.

### 5.2 Some remarks on numerical efficiency, RAM requirements and off-sets

In order to speed the construction of the matrix $C$ and of the vector $B$, we have found that it is very advantageous to firstly order (we use the quick sort algorithm) all the elements of the matrix $N$, i.e. the observed pixels (4 bytes integers), in the first column of a new “matrix” $U$ (of $n_p \times n_s$ rows and 3 “columns”) by keeping memory of their locations (2×2 bytes integers), in the original matrix $N$ in the other two “columns” of $U$. In this way we simply extract once for all each pixel in common between two scan circles for all scan circles, being the same pixel located in contiguous rows in the matrix $U$, by considering all the possible pairs of rows of $U$ with the same element in the first column, with the simple caution that the elements of the second column of $U$, i.e. the original rows in the matrix $N$, are different. In this way the “scanning” of the matrix $N$ and the construction of $C$ and $B$ according to the rules of section 5.1 turns to be very fast. It is immediate to use the matrices $N_H$, containing the pixel numbers at higher resolutions, in the construction of $C$ and $B$ if one want to adopt more stringent conditions on the distance between pixels in common. In addition, working with the additional matrix $U$ optimize the construction of the simulated maps from the simulated data streams (see section 4), being immediate to recognize in the matrix $U$ when the same pixel has been observed.

For the solution of the linear system (14) we have found that the Gauss elimination method works very well. We prefer to construct and solve the system by using double precision accuracy, to have high numerical accuracy and to be sure of avoiding artificial numerical singularity; also, due to matrix symmetry and positive definiteness, we do not need pivot.

To build up the linear system, we have to keep the memory space for the system matrix, the system known terms, the observed temperature matrix and the auxiliary (integer) matrix $U$; by taking advantage of this symmetry, and by considering in general $n_l$ constant levels per scan circle the memory requirement is: 8bytes $\times \left[ n_l (n_l n_s + 1)/2 + n_l n_s + n_s n_p \right] + 4\text{bytes} \times 2 \times n_s n_p$. For example, at 30 GHz (FHWM $\simeq 30'$, $n_p \simeq 2100$), for the case of 5' shift of the spin axis ($n_s = 4320$) we need about 220 (440) Mbytes by working with $n_l = 1$ ($n_l = 2$); for a 2.5' shift of the spin axis, $n_s = 8640$ and we need about 590 (1500) Mbytes by working with $n_l = 1$ ($n_l = 2$). For sake of illustration, if we have a beam of $\simeq 10'$ (like the nominal
100 GHz beams) and we want to record 4 samplings per beam we will have $n_p \approx 8400$; for a 2.5' shift of the spin axis and by working with $n_l = 1$ ($n_l = 2$) the memory requirement is of about 1500 (2400) Mbytes. This memory problem can be solved by taking advantage of disk buffers; we discuss our solution in the Appendix B.

For solving the system we only need to keep in memory the system matrix and known terms, and the memory requirement is: $8 \text{bytes} \times [n_ln_s(n_sn_l + 1)/2 + n_sn_l]$. At 30 GHz and with 3 samplings per beam, we need about 75 Mbytes if $n_s = 4320$ and $n_l = 1$, 300 Mbytes if $n_s = 4320$ and $n_l = 2$ or $n_s = 8640$ and $n_l = 1$ and 1200 Mbytes if $n_s = 8640$ and $n_l = 2$. This problem maybe crucial depending on the available amount of RAM, especially because the Gauss elimination continuously changes the system components. Also this memory problem can be solved by taking advantage of disk buffers (Appendix B).

After the solution of the linear system, we obtain a “destriped” matrix $\tilde{D}$ by subtracting the level $A_i$ to the $i$–th row $(i = 1, n_s)$ of the matrix $T$. Then we apply to this matrix the same treatment of section \[\text{and we have the observed destriped temperature map } M_D.\]

We observe that, contrary to the case of pure white noise, the average of a $1/f$-type noise series can be significantly different from zero. Then, the map $M_T$ (as well as the map $M_D$, but in general with a somewhat different value) may present an off-set with respect to the map $M_G$; on the contrary the off-set between the map $M_W$ and the map $M_G$ is negligible. As a typical example, for the simulation with $\alpha = 90^\circ$ (see Table 1) we find that these off-sets are $\approx 4.8\mu K$; for comparison, the off-set we find between the maps $M_W$ and $M_G$ is much smaller, $\approx 0.075\mu K$. The off-set between $M_T$ ($M_D$, $M_W$) and $M_G$ is of course not relevant for anisotropy measurements, nor we are able to subtract it in a real case (on the other hand we must pay attention to the fact that off-sets may be present between maps produced by different receivers at the same frequency). The off-set of the map $M_T$ ($M_D$, $M_W$) must be removed by subtracting the difference between the average of the map $M_T$ ($M_D$, $M_W$) and of the map $M_G$: this is necessary for a correct quantitative analysis of stripes magnitude and destriping efficiency.

We indicate with $\tilde{M}_T$, $\tilde{M}_D$ (and $\tilde{M}_W$, but it is not relevant in practice for this matrix) the above maps, when this kind of off-set has been removed.

### 6 Estimators of the destriping efficiency

In the previous sections we have described our simulations of the Planck observations and the basic treatment to convert observational data streams into sky maps, including destriping techniques. Here we analyse the efficiency of the adopted destriping technique, by considering well known estimators.

#### 6.1 Ratio between the $\chi_i^2$’s

We expect that the average of the squares of differences between the elements of the maps $\tilde{M}_W$ and $M_G$ divided by the observation sensitivity [essentially the estimator $\chi_i^2$; we will call it in this case $\langle \chi_i^2 \rangle_W$] is very close to 1, because only the white noise is present in this case. We can compute the same estimator for $\tilde{M}_T - M_G$ ($\langle \chi_i^2 \rangle_T$, undestriped case) and $\tilde{M}_D - M_G$ ($\langle \chi_i^2 \rangle_D$ destriped case). (For the above consideration, by using observed maps without removing the off-sets one will find a meaningless amplification of the $\chi_i^2$).

On the other hand, we find that the exact value of $\langle \chi_i^2 \rangle_W$ may be just a little different from 1 depending indeed on the assumed sensitivity; for example it is just a little different is we divide $\tilde{M}_W - M_G$ by the map $M_s$ (i.e. by using the sensitivity proper of any pixel – we will use this definition in our tables) or the average of the sensitivities in the map $M_S$ or the estimate of the average sensitivity obtained on the basis the global mission time, the observed
number of pixels (393216 for our maps at COBE-cube resolution 9) and the properties of considered receiver. Then we prefer to use the ratio between the \( (\chi^2_{r})_{T} - (\chi^2_{r})_{D} \) and \( (\chi^2_{r})_{W} \) as estimator. This “renormalized” estimator (that we will denote by \( \chi^2_{r,n,T} \) and \( \chi^2_{r,n,D} \) respectively for the undestripped and destripped cases) results independent of the choice of the sensitivity adopted for the \( \chi^2 \) calculation, so allowing a better understanding of the magnitude of striping effect and of destriping efficiency. We will quantify the destriping efficiency by using the relative decrease of the renormalized \( \chi^2_{r} \), i.e. with the quantity \[ \left( (\chi^2_{r})_{D} - (\chi^2_{r})_{T} \right) / ((\chi^2_{r})_{T} - 1) \].

6.2 Magnitude of stripes temperature

From the values of \( \chi^2_{r,n,T} \) and \( \chi^2_{r,n,D} \) defined above and from the average \( \text{rms} \) white noise, \( \text{rms}_{W} \), for the observed pixels derived from the sensitivity map \( M_{S} \) we can easily give an estimate of the \( \text{rms} \) temperature of the stripes before, \( \text{rms}_T \), and after destriping, \( \text{rms}_D \). Under the hypothesis that the error introduced by the noises in each pixel may be thought as a sum of two uncorrelated contributions from white noise and \( 1/f \) noise, we have \[ \text{rms}_T = \text{rms}_{W} \sqrt{\chi^2_{r,n,T} - 1} \] and \[ \text{rms}_D = \text{rms}_{W} \sqrt{\chi^2_{r,n,D} - 1} \] [Method (a)].

Another estimate [Method (b)] of the \( \text{rms} \) stripes temperature can be obtained by directly evaluating the global \( \text{rms} \) difference before, \( \text{rms}_{tot,T} \), or after, \( \text{rms}_{tot,D} \), the destriping from the comparison with the maps \( M_{G} \) and by assuming that they are given by the sum in quadrature of \( \text{rms}_{W} \) and \( \text{rms}_T \) or \( \text{rms}_D \). From the values of \( \text{rms}_{W}, \text{rms}_{tot,T} \) and \( \text{rms}_{tot,D} \) we can calculate \( \text{rms}_T \) or \( \text{rms}_D \).

Finally [Method (c)], we can treat the \( 1/f \) contribution to the total noise like a systematic (and not statistical) error and therefore to assume that \( \text{rms}_{tot,T} \) or \( \text{rms}_{tot,D} \) are simply given by the sum of \( \text{rms}_W \) and \( \text{rms}_T \) or \( \text{rms}_D \).

For estimating the destriping efficiency in terms of residual stripes temperature we will by using the relative decrease of the stripes temperature, i.e. the quantity \( (\text{rms}_D - \text{rms}_T) / \text{rms}_T \).

7 Results

From the visual inspection of the simulated maps the effect of the noises is of course not clearly evident; we only recognize a somewhat degradation of the map details. The stripes become more evident when we plot the noises map only (see Figure 3); they can be obtained by subtracting the CMB fluctuation map (\( M_{G} \)) to the observed map. The stripes figures well reproduce the adopted scanning strategy.

7.1 Destriping versus scanning strategy

The visual inspection of our “stripes” maps do not allow to quantify the striping magnitude and its reduction obtained from the destriping procedure (see Figures 3 and 4).

The statistical analysis of the maps allows a much better understanding of the destriping procedure efficiency. We present here (see Tables 1 ÷ 4) the results of our simulations (for a typical channel at 30 GHz) in terms of reduced \( \chi^2_{r,n} \) and of stripes temperature for the three chosen values of the (constant) angle \( \alpha \) between the spin axis and the telescope direction and for the considered case with \( \alpha = 90^\circ \) and spin axis oscillations (see also the Appendix A). We report also in the tables some informations on relevant quantities: the ideal white noise for a single sampling time which weakly increases with \( \alpha \) for geometrical reasons if we want to have 3 samplings per beam; the (single receiver) average \( \text{rms} \) white noise, \( \text{rms}_{W} \), (expressed in mK) for the observed pixels for a 360 days mission; the square of the ratio \( \mathcal{R} \) between these white noises, normalized to the intermediate case of \( \alpha = 85^\circ \); the percentage of sky which results to be observed by the considered single off-axis beam, which also increases
with $\alpha$; the number of couples in common find in the destriping procedure and the number of constant levels per scan circle used in the destriping code; the map resolution and the resolution used for searching the pixels in common. We remember the adopted value, 0.05 Hz, of $1/f$ knee frequency and of the bandwidth, 6 GHz, and that the sampling time is of about 0.03 sec, the exact value depending on the chosen value of $\alpha$. In the tables we report our values of $\chi^2_{r,n}$ before and after the destriping procedure and the relative (%) decrease without the multiplicative factor $R^2$ and by taking it into account. Indeed the (white noise) sensitivity per pixel is different for different scanning strategies; then, by including the factor $R^2$ we “renormalize” the values of $\chi^2_{r,n}$ at the same (white noise) sensitivity level, so making the them essentially independent of the $\alpha$–dependent sensitivity.

**Table 1:** destriping results; $\alpha = 90^\circ$.

| Some global parameters | Before destriping | After destriping | % improvement | Method |
|------------------------|------------------|-----------------|--------------|--------|
| $\Delta T_W = 1.322$mK | $r m s_W = 27.18\mu K$ | $R^2 = 1.00666$ | $% sky = 99.98$ | $n_l = 1$ |
| Map Res. = 9 | Res. common pix. = 9 | | | pix. in common = $2.70 \times 10^8$ |
| $\chi^2_{r,n,T} = 1.1770$ | $\chi^2_{r,n,D} = 1.0216$ | | 87.8 % | |
| $\chi^2_{r,n,T}R^2 = 1.1848$ | $\chi^2_{r,n,D}R^2 = 1.0284$ | | 84.6 % | |
| $r m s_T = 11.4\mu K$ | $r m s_D = 3.99\mu K$ | | 65.1 % | (a) |
| $r m s_T = 17.3\mu K$ | $r m s_D = 6.93\mu K$ | | 59.9 % | (b) |
| $r m s_T = 5.03\mu K$ | $r m s_D = 0.87\mu K$ | | 82.7 % | (c) |

**Table 2:** destriping results; $\alpha = 85^\circ$.

| Some global parameters | Before destriping | After destriping | % improvement | Method |
|------------------------|------------------|-----------------|--------------|--------|
| $\Delta T_W = 1.318$mK | $r m s_W = 27.09\mu K$ | $R^2 = 1$ | $% sky = 99.43$ | $n_l = 1$ |
| Map Res. = 9 | Res. common pix. = 9 | | | pix. in common = $2.34 \times 10^8$ |
| $\chi^2_{r,n,T} = 1.2709$ | $\chi^2_{r,n,D} = 1.0142$ | | 94.7 % | |
| $\chi^2_{r,n,T}R^2 = 1.2709$ | $\chi^2_{r,n,D}R^2 = 1.0142$ | | 94.7 % | |
| $r m s_T = 14.1\mu K$ | $r m s_D = 3.23\mu K$ | | 77.1 % | (a) |
| $r m s_T = 21.6\mu K$ | $r m s_D = 7.31\mu K$ | | 66.2 % | (b) |
| $r m s_T = 7.98\mu K$ | $r m s_D = 0.969\mu K$ | | 87.9 % | (c) |
Table 3: destriping results; $\alpha = 80^\circ$.

Some global parameters

| $\Delta T_W$ = 1.311mK | $rms_W = 26.90\mu K$ | $R^2 = 0.98602$ | $%\ sky = 98.12$ |
|-----------------------|----------------------|-----------------|-----------------|
| Map Res. = 9          | Res. common pix. = 9 | $n_I = 1$       | pix. in common = $2.20 \times 10^8$ |

| Before destriping | After destriping | $%$ improvement | Method |
|-------------------|------------------|----------------|--------|
| $\chi^2_{r,n,T}$ = 1.0396 | $\chi^2_{r,n,D}$ = 1.0292 | 26.19 $%$ | |
| $\chi^2_{r,n,T}R^2 = 1.0251$ | $\chi^2_{r,n,D}R^2 = 1.0148$ | 41.0 $%$ | |
| $rms_T = 5.35\mu K$ | $rms_D = 4.60\mu K$ | 14.1 $%$ | (a) |
| $rms_T = 8.05\mu K$ | $rms_D = 6.94\mu K$ | 13.8 $%$ | (b) |
| $rms_T = 1.17\mu K$ | $rms_D = 0.88\mu K$ | 24.8 $%$ | (c) |

Table 4: destriping results; $\alpha = 90^\circ \pm 10^\circ$ (10 sinusoidal oscillations).

Some global parameters

| $\Delta T_W$ = 1.322mK | $rms_W = 29.07\mu K$ | $R^2 = 1.15152$ | $%\ sky = 100$ |
|-----------------------|----------------------|-----------------|----------------|
| Map Res. = 9          | Res. common pix. = 9 | $n_I = 1$       | pix. in common = $2.36 \times 10^8$ |

| Before destriping | After destriping | $%$ improvement | Method |
|-------------------|------------------|----------------|--------|
| $\chi^2_{r,n,T}$ = 1.1165 | $\chi^2_{r,n,D}$ = 1.0153 | 86.9 $%$ | |
| $\chi^2_{r,n,T}R^2 = 1.2857$ | $\chi^2_{r,n,D}R^2 = 1.1691$ | 40.8 $%$ | |
| $rms_T = 9.92\mu K$ | $rms_D = 3.60\mu K$ | 63.8 $%$ | (a) |
| $rms_T = 15.2\mu K$ | $rms_D = 8.62\mu K$ | 43.3 $%$ | (b) |
| $rms_T = 3.73\mu K$ | $rms_D = 1.25\mu K$ | 66.5 $%$ | (c) |

7.2 Destriping versus $1/f$ knee frequency

For the interesting case $\alpha = 90^\circ$ with no oscillations, we carried out other a simulation with a much larger value of the $1/f$ knee frequency, $f_k = 10$Hz, of order of that expected for total power radiometers.

It is interesting to study the stripes effect and destriping performance under this very pessimistic condition.

Tables 5 shows our results that have to be compared with those of Table 1, based on the theoretical estimate of $f_k$ of our kind of radiometers.
Table 5: destriping results; $\alpha = 90^\circ$; $f_k = 10$Hz.

Some global parameters

| $\Delta T_W = 1.322$mK | $rms_W = 27.18\mu$K | $R^2 = 1.00666$ | $n_l = 1$ | $\%$ sky = 99.98 |
|-------------------------|---------------------|-----------------|----------|-------------------|
| Map Res. = 9            | Res. common pix. = 9|                 |          | pix. in common = $2.70 \times 10^8$ |

Before destriping | After destriping | % improvement | Method  |
|------------------|------------------|---------------|---------|
| $\chi^2_{r,n,T}$ = 9.2846 | $\chi^2_{r,n,D}$ = 2.6769 | 79.8 $\%$ |         |
| $\chi^2_{r,n,T}R^2$ = 9.3464 | $\chi^2_{r,n,D}R^2$ = 2.6947 | 79.7 $\%$ | (a)     |
| $rms_T = 78.2\mu$K | $rms_D = 35.4\mu$K | 54.8 $\%$ | (b)     |
| $rms_T = 258\mu$K | $rms_D = 67.7\mu$K | 73.7 $\%$ | (b)     |
| $rms_T = 233\mu$K | $rms_D = 45.8\mu$K | 80.3 $\%$ | (c)     |

7.3 Destriping with more than one constant per scan circle

For the reference case $\alpha = 90^\circ$, both with the theoretical prediction for $f_k$ and for the case with $f_k$ representative of total power radiometers, we have applied our destriping code by using two constants per scan circle. The results are shown in Tables 6 and 7 that must be compared with Tables 1 and 5 respectively. We find that the use of more constant per circle does not help the destriping technique.

Table 6: destriping results; $\alpha = 90^\circ$.

Some global parameters

| $\Delta T_W = 1.322$mK | $rms_W = 27.18\mu$K | $R^2 = 1.00666$ | $n_l = 2$ | $\%$ sky = 99.98 |
|-------------------------|---------------------|-----------------|----------|-------------------|
| Map Res. = 9            | Res. common pix. = 9|                 |          | pix. in common = $2.70 \times 10^8$ |

Before destriping | After destriping | % improvement | Method  |
|------------------|------------------|---------------|---------|
| $\chi^2_{r,n,T}$ = 1.1770 | $\chi^2_{r,n,D}$ = 1.0280 | 84.2 $\%$ |         |
| $\chi^2_{r,n,T}R^2$ = 1.1848 | $\chi^2_{r,n,D}R^2$ = 1.0348 | 81.2 $\%$ | (a)     |
| $rms_T = 11.4\mu$K | $rms_D = 4.55\mu$K | 60.1 $\%$ | (a)     |
| $rms_T = 17.3\mu$K | $rms_D = 7.62\mu$K | 56.0 $\%$ | (b)     |
| $rms_T = 5.03\mu$K | $rms_D = 1.05\mu$K | 79.2 $\%$ | (c)     |
### Table 7: destriping results; $\alpha = 90^\circ$; $f_k = 10$Hz.

Some global parameters

| $\Delta T_W$ | $rms_W$ | $R^2$ | $n_l$ | % sky |
|--------------|---------|-------|-------|-------|
| 1.322mK      | 27.18$\mu$K | 1.00666 | 2     | 99.98 |
| Map Res. = 9 | Res. common pix. = 9 | Pix. in common = $2.70 \times 10^8$ |

| Before destriping | After destriping | % improvement | Method |
|-------------------|------------------|---------------|--------|
| $\chi^2_{r,n,T}$  | $\chi^2_{r,n,D}$ | 78.8 %        |        |
| 9.2846            | 2.7584           |               |        |
| $\chi^2_{r,n,T}R^2$ | $\chi^2_{r,n,D}R^2$ | 78.7 %        |        |
| 9.3464            | 2.7768           |               |        |
| $rms_T$ = 78.2$\mu$K | $rms_D$ = 36.2$\mu$K | 53.7 %        | (a)    |
| $rms_T$ = 258$\mu$K | $rms_D$ = 70.0$\mu$K | 72.9 %        | (b)    |
| $rms_T$ = 233$\mu$K | $rms_D$ = 47.9$\mu$K | 79.4 %        | (c)    |

#### 7.4 Destriping versus distance conditions

For the reference case $\alpha = 90^\circ$, both with the theoretical prediction for $f_k$ and for the case with $f_k$ representative of total power radiometers, we have applied our destriping code by using the map pixels at higher resolution to search for pixels in common. The results are shown in Tables 8 and 9 that must be compared with Tables 1 and 5 respectively. We conclude that the use of a more stringent condition to find the coincidences of the pointing directions in different scan circles does not help the destriping technique.

### Table 8: destriping results; $\alpha = 90^\circ$.

Some global parameters

| $\Delta T_W$ | $rms_W$ | $R^2$ | $n_l$ | % sky |
|--------------|---------|-------|-------|-------|
| 1.322mK      | 27.18$\mu$K | 1.00666 | 1     | 99.98 |
| Map Res. = 9 | Res. common pix. = 10 | Pix. in common = $6.97 \times 10^7$ |

| Before destriping | After destriping | % improvement | Method |
|-------------------|------------------|---------------|--------|
| $\chi^2_{r,n,T}$  | $\chi^2_{r,n,D}$ | 86.7 %        |        |
| 1.1770            | 1.0235           |               |        |
| $\chi^2_{r,n,T}R^2$ | $\chi^2_{r,n,D}R^2$ | 83.6 %        |        |
| 1.1848            | 1.0303           |               |        |
| $rms_T$ = 11.4$\mu$K | $rms_D$ = 4.73$\mu$K | 58.5 %        | (a)    |
| $rms_T$ = 17.3$\mu$K | $rms_D$ = 7.16$\mu$K | 58.6 %        | (b)    |
| $rms_T$ = 5.03$\mu$K | $rms_D$ = 0.928$\mu$K | 81.6 %        | (c)    |
Table 8': destriping results; $\alpha = 90^\circ$.

Some global parameters

| $\Delta T_W$ | $rms_W$ | $R^2$ | % sky |
|--------------|---------|-------|-------|
| 1.322mK      | 27.18µK | 1.00666 | 99.98 |

Before destriping | After destriping | % improvement | Method |
|-------------------|------------------|---------------|--------|
| $\chi^2_{r,n,T}$ | 1.1770           | 81.1 %        |        |
| $\chi^2_{r,n,T}R^2$ | 1.1848         | 78.2 %        |        |
| $rms_T$          | 11.4µK          | 52.1 %        | (a)    |
| $rms_T$          | 17.3µK          | 53.2 %        | (b)    |
| $rms_T$          | 5.03µK          | 76.5 %        | (c)    |

Table 9: destriping results; $\alpha = 90^\circ$; $f_k = 10$Hz.

Some global parameters

| $\Delta T_W$ | $rms_W$ | $R^2$ | % sky |
|--------------|---------|-------|-------|
| 1.322mK      | 27.18µK | 1.00666 | 99.98 |

Before destriping | After destriping | % improvement | Method |
|-------------------|------------------|---------------|--------|
| $\chi^2_{r,n,T}$ | 9.2846           | 79.5 %        |        |
| $\chi^2_{r,n,T}R^2$ | 9.3464         | 79.5 %        |        |
| $rms_T$          | 78.2µK          | 70.6 %        | (a)    |
| $rms_T$          | 258µK           | 73.5 %        | (b)    |
| $rms_T$          | 233µK           | 80.1 %        | (c)    |
Table 9': destriping results; $\alpha = 90^\circ$; $f_k = 10$Hz.

Some global parameters

\[
\begin{array}{l}
\Delta T_W = 1.322\,\text{mK} & rms_W = 27.18\,\mu\text{K} & R^2 = 1.00666 & \%\,\text{sky} = 99.98 \\
\text{Map Res.} = 9 & \text{Res. common pix.} = 11 & n_l = 1 & \text{pix. in common} = 1.62 \times 10^7
\end{array}
\]

| Before destriping | After destriping | % improvement | Method |
|-------------------|------------------|---------------|--------|
| $\chi^2_{r,n,T}$ = 9.2846 | $\chi^2_{r,n,D}$ = 2.7752 | 78.6 % | (a) |
| $\chi^2_{r,n,T}R^2$ = 9.3464 | $\chi^2_{r,n,D}R^2$ = 2.7937 | 78.5 % | |
| $rms_T = 78.2\,\mu\text{K}$ | $rms_D = 24.2\,\mu\text{K}$ | 69.0 % | |
| $rms_T = 258\,\mu\text{K}$ | $rms_D = 70.5\,\mu\text{K}$ | 72.7 % | (b) |
| $rms_T = 233\,\mu\text{K}$ | $rms_D = 48.4\,\mu\text{K}$ | 79.2 % | (c) |

8 Discussion and conclusions

An analytical estimate of the maximum excess noise factor, $F$, due to the stripes related to $1/f$ effect has been given by Janssen et al. (1996); they found $F \simeq 1 + \tau f_k (2\ln n_p + 0.743)^{1/2}$. By thinking $F^2$ as equal to $1 + (\Delta rms rms_W)^2$, we have a fractional additional $rms$ noise respect to the $rms$ noise, $rms_W$, obtained in the case of pure white noise given by $(\Delta rms rms_W)^2 = \tau f_k (2\ln n_p + 0.743)$. For example for our simulations we have $n_p \simeq 2100$, a sampling time $\tau \simeq 28\text{msec}$, corresponding to an angle of $\simeq 10'$ in the sky, and $f_k = 0.05\text{Hz}$ (or 10 Hz). With these number we get $(\Delta rms rms_W)^2 \simeq 0.022$ (or 4.5) for a pixel of $10' \times 10'$. Our map pixel is $19.4' \times 19.4'$ (COBE-cube resolution 9); therefore $rms^2_W$ reduces by a factor $\simeq 4$ and we expect to have an additional $(\Delta rms rms_W)^2$ per pixel, i.e. an additional reduced $\chi^2$, of about 0.1 (or 18). The results shown in our tables are in quite good agreement (always within a factor 2) with these analytical estimates. Somewhat larger values may be expected from the larger observational time toward high ecliptic latitudes, the consequent reduction of the white noise and the increasing of the relative weight of the $1/f$ noise. We stress here that, contrary to the case of pure white noise, even for the same scanning strategy the final effect of $1/f$ noise may be quite different for different simulations: larger or smaller effects can be obtained according to the (simulated) behaviour of $1/f$ gain fluctuations. Only from a very large set of simulations for each considered scanning strategy we can derive a robust evaluation of the "averaged" final effect of $1/f$ noise (see below). For this reason, the results shown in our tables have to be considered as first order estimates of the final $1/f$ noise effect expected in a given scanning strategy rather than detailed predictions.

Without applying the destriping procedure, the typical amount of stripes temperatures (i.e. the $rms$ values) per resolution element (of $\simeq 19.4' \times 19.4'$) ranges from few $\mu\text{K}$ to one or two tens of $\mu\text{K}$ (according to the method adopted for estimating them) and is always significantly less than the corresponding single beam sensitivity of about $27 \div 30\mu\text{K}$.

The residual stripes that remain after the reduction through our destriping code show typical $rms$ temperatures of few $\mu\text{K}$, roughly independently of the adopted scanning strategy. The efficiency of the destriping algorithm is quite good ($\sim 20\% \div 90\%$, according to the
adopted estimator and depending in part on the scanning strategy). We stress that in any case, for our kind of radiometers, the reduced additional noise is of few percent in terms of increased reduced $\chi^2$ and of few $\mu$K in terms of stripes temperature. This is not particularly critical. Nevertheless it must be compared with the final Planck sensitivity as it results by combining the data from all the receivers at the same frequency (see below).

In the case of much larger $1/f$ contaminations like those expected for higher values of the knee frequency in the case of total power radiometers, the final impact on observed sky maps is much higher. The destriping code allows to reduce a large fraction of the added noise (see Table 5), nevertheless a significant increase of the final noise still remains. Then, the importance of the reduction via “hardware” of the $1/f$ noise is strongly recommended.

About the dependence of the striping effect and of the destriping efficiency on the scanning strategy, our preliminary results suggest the following conclusions.

- The magnitude of the sensitivity degradation due to the stripes is only weakly dependent on $\alpha$ for undestripped maps as well as for destripped maps (see Tables 1 ÷ 3).

- The final residual added noise is roughly independent of the scanning strategy and on the unreduced added noise, almost not relevant unreduced contaminations (see Tables 1 ÷ 4).

- Oscillations of the spin axis do not improve significantly the destriping efficiency (see Table 4). On the contrary it is well known that oscillations of the spin axis may introduce further systematic effects related to variations of the illumination by the sun and of shielding performances.

- Our results are of course related to the beam position in the sky field of view. For on-axis beams and for beams located at $\phi_B \sim 0^\circ$ the case at $\alpha = 90^\circ$ is expected to give the worst results, whereas even in this case the destriping results are expected to improve for off-axis beams located at $\phi_B \sim 90^\circ$.

- We find that, in spite of the larger computing time, the efficiency of the destriping technique does not improve by using two level constants per scan circle (see Tables 6 and 7). Indeed the number of “physical” conditions (the number of pixels in common) do not depend on the chosen number of level constants adopted in the destriping procedure. Then, by searching (in the solution of the linear system, see section 5.1, obtained by the condition of minimization of $S$) for a number of unknown larger by a factor two, by using the same number of informations, we expect that the uncertainty of each unknown will be larger. We infer that the advantage of using more constant per scan circle found by Delabrouille (1997) for a similar knee frequency but for the case of thermal drifts (noise spectrum proportional to $1/f^2$) has to be related to the different kind of noise spectrum. The noise fluctuations on long timescales will be higher in the case of $1/f^2$ noise spectrum than in the case of $1/f$ noise spectrum; in the former case the use of more constants per circle may allow a more appropriate subtraction of the gain fluctuations in each circle, whereas in our case this advantage is balanced by the increased uncertainty in the determination of the levels and the global effect results in to a small decreasing of destriping efficiency.

- The use of more stringent conditions for identifying the pixels in common does not improve (see Tables 8 and 9) the destriping results. This option allows to reach a more accurate superposition of the pixels used in the destriping; nevertheless the number of couples decreases about by a factor 4 (or 16) by using pixels 4 (or 16) times smaller, and so a number of conditions significantly smaller than that found before enters in the minimization of $S$ (see section 5.1).
Given all these results we believe that a simple scanning strategy with constant $\alpha$ in the range $85^\circ \div 90^\circ$ can offer the advantages of a large (practically full) sky coverage, quite good destriping performances together with the minimization of all the systematic effects related to the variations of thermal conditions.

We draw here below a brief guide-line for the future simulation work on this topic.

- We intend to test different sampling strategies: for example with a smaller shift ($2.5'$) of the spin axis direction and a corresponding smaller observation time for spin axis direction (1 hour). Such a kind of sampling, possibly with the same ($\sim 8700$, corresponding to 4 samplings per beam at 100 GHz – FWHM $\sim 10'$) number of samplings per scan circle independently of the frequency, may be the final LFI/Planck sampling. A strategy of this kind allows to have a larger number of pixels in common in the destriping procedure; on the other hand the sensitivity of the temperature measurements for the different scan circles degrades. It may be interesting to investigate pro and contro of a variance of this kind from the point of view of the $1/f$ noise reduction.

- We may be also interested in an accurate verification of the validity of the chosen telescope rotation velocity around the spin axis.

- It may be interesting to apply our codes to higher ratios between the beam FWHM and the map pixel size, for example by working, at 30 GHz, with maps at COBE-cube resolution 10. Indeed, considering input maps at a higher resolution roughly tests the importance of assuming a better efficiency in the data streams deconvolution, for example by fully exploiting the beam oversampling.

- As better approximation, it is interesting to implement the convolution with the beam for a moving telescope in the observation simulation code and to search for robust and fast criteria for establishing in this situation when it is possible to consider that integrations in different scan circles can be really referred to the same direction in the sky. Of course this problem is correlated to the technique adopted for deconvolving the data streams to obtain observed maps; on the other hand, the stripes magnitude must be small enough to not significantly alter the deconvolution procedure.

- We have found that the noise added by the $1/f$ effect is not particularly critical, compared to the single beam (white noise) sensitivity. Nevertheless it must be compared with the final Planck sensitivity as it results by combining the data from all the receivers at the same frequency. Indeed, we do not expect that the $1/f$ noise magnitude decreases as the square root of the number of receivers, as white noise does: by carrying out several simulation with the same set of physical parameters for the same scanning strategy and averaging the corresponding maps, we can address this topic.

- We intend to apply the methods of inversion of CMB maps (Muciaccia et al. 1997) for deriving the angular power spectrum of the observed maps. By comparing the CMB angular power spectrum obtained in presence of $1/f$ noise contamination with that derived in the case of pure white noise (and of course with that of the input map) it is possible to estimate the $1/f$ noise impact on the extraction on the key cosmological informations, almost in the case of gaussian fluctuations like those expected in inflationary scenarios. Particular attention has to be attempted for evaluating the impact on our science in the context of topological defects, like cosmic strings for example, which introduce non gaussian features in the power spectrum. The characteristic geometrical pattern of $1/f$ noise stripes (related to the scanning strategy) have to be used for disantangle between instrumental and cosmological deviations from the gaussianity.
• Of course, we intend to extend in the next future our analysis to Planck measurements at higher frequencies.

The full success of missions like Planck and MAP require a good control of all the relevant sources of systematic effects. Discrete sources above the detection limit must be carefully removed and accurate models for foregrounds radiation and anisotropies (Brandt et al. 1994, Danese et al. 1996, Bouchet et al. 1997, Toffolatti et al. 1995, 1997) are required to keep the sensitivity degradation in the knowledge of CMB anisotropies below few tens percent (Dodelson 1997). Optical distortions, which produce a non-symmetric beam response for feed-horns located away from the centre of the focal plane, introduce other systematic effects; they must be minimized by optimizing the telescope and the focal plane assembly design (Mandolesi et al. 1997b). Thermal drifts (Bersanelli et al. 1996), which couple to the 1/f-type noise here discussed, can also generate stripes in the observed maps; efficient shield is required together with accurate reduction of sidelobe effects and optimization of the thermal conditions during the mission.

All in all maximum efforts should be addressed to optimize the cooling efficiency and the observational strategy and to improve the methods for the data analysis in order to reduce the magnitude of the striping effect and of the other instrumental systematic effects.

Acknowledgements – We warmly thank M. Seiffert for useful discussions on Planck LFI receivers and for having provided us its original IDL code for the generation on 1/f-type noise, J. Delabrouille and K. Gorski for useful discussions on simulations and destriping techniques during their visits in Bologna and P. Natoli and N. Vittorio for having provided us their code for the generation of CMB anisotropy maps.

Appendix A: Geometrical transformations between coordinate systems

Let \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) the standard unit vectors in ecliptic coordinates and \( \mathbf{s} \) a unit vector along the satellite spin axis outward the Sun direction. Let \( i \) the angle between \( \mathbf{s} \) the and \( \mathbf{k} \) (i.e. the ecliptic colatitude of \( \mathbf{s} \)) and \( \phi \) the angle between \( \mathbf{i} \) and \( \mathbf{s} \) (i.e. the ecliptic longitude of \( \mathbf{s} \)). For general scanning strategies \( i \) will be described by a, possibly not constant, function \( i = i(\phi) \). Then \( \mathbf{s} = \sin i \cos \phi \mathbf{i} + \sin i \sin \phi \mathbf{j} + \cos i \mathbf{k} \). Let \( \mathbf{i}' = \mathbf{s} \) and \( \mathbf{k}' \) a unit vector horizontal to \( \mathbf{s} \) on the plane identified by the vectors \( \mathbf{k} \) and \( \mathbf{s} \), namely \( \mathbf{k}' = -\cos i \cos \phi \mathbf{i} - \cos i \sin \phi \mathbf{j} + \sin i \mathbf{k} \). Let \( \mathbf{j}' = \mathbf{k}' \wedge \mathbf{i}' \) (here \( \wedge \) indicates the vector product). Let \( \mathbf{p} \) the unit vector that identifies the pointing direction of the telescope optical axis. In the reference \( \mathbf{i}', \mathbf{j}', \mathbf{k}' \) the vector \( \mathbf{p} \) can be defined by two angles: the angle \( \alpha \) from \( \mathbf{s} \) (\( \alpha = 70^\circ \) for the Phase A study, Bersanelli et al. 1996) and the angle, \( \psi \), between its projection on the plane identified by \( \mathbf{j}', \mathbf{k}' \) and \( \mathbf{i}' \), with the convention \( \mathbf{p} = \cos \alpha \mathbf{i} + \sin \alpha \sin \psi \mathbf{j}' + \sin \alpha \cos \psi \mathbf{k}' \). Given \( \mathbf{i}', \mathbf{j}', \mathbf{k}' \) in terms of \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) it is easily to derive \( \mathbf{p} \) in the same basis. We choose two coordinates \( x \) and \( y \) on the plane tangent to the celestial sphere in the telescope optical axis direction, \( \mathbf{p} \), with unit vector \( \mathbf{u} \) and \( \mathbf{v} \) respectively; we choose the \( x \) axis according to the condition that the unit vector \( \mathbf{u} \) points always toward the satellite spin axis; indeed, for standard Planck observational strategy, this condition is preserved as the telescope scans different sky regions. With this choice of reference frame, we have that \( \mathbf{v} = \mathbf{p} \wedge \mathbf{s}/|\mathbf{p} \wedge \mathbf{s}| \) and \( \mathbf{u} = \mathbf{v} \wedge \mathbf{p}/|\mathbf{v} \wedge \mathbf{p}| \). In general, the coordinates \( (x_0, y_0) \) of the beam centre in a ("satellite") reference \( x_T, y_T, z_T \), corresponding to the unit vectors \( \mathbf{u}, \mathbf{v}, \mathbf{p} \), can be identified by two angles; we use here the colatitude \( \theta_B \) and
the longitude $\phi_B$ in this reference. Finally, the pointing direction of this generic (on-axis or off-axis) beam is given by the unit vector $\vec{B} = \cos \theta_B \vec{p} + \cos \phi_B \sin \theta_B \vec{u} + \sin \phi_B \sin \theta_B \vec{v}$.

For sake of illustration, we consider here briefly three different kinds of scanning strategies (the first two options have been considered in the present simulations).

- **Spin axis always on the ecliptic plane.**
  
  In this simple case we have $i = 90^\circ$ and $\vec{s} = \cos \phi \vec{i} + \sin \phi \vec{j}$.
  
  Given the angle $\alpha$, we need to give the time dependences, $\phi = \phi(t)$ and $\psi = \psi(t)$, of the spin axis longitude and of the telescope projection to fully determine the scanning strategy. The reference case is to change $\phi$ of a certain angle $\Delta \phi$ (5', for example) after a given time interval (2 hours, for example) and to choose a given spin frequency (1 r.p.m., for example) of the continuous rotation of $\psi$.

- **Sinusoidal oscillations of the spin axis.**
  
  In this case we need also to define the amplitude of the oscillations, $\delta$ (10°, for example), and the number of complete oscillations, $n_{osc}$ (10 oscillations for 360 days, for example), per a complete rotation of the spin axis over the ecliptic (in 360 days, for example). Then, by choosing $i = 0$ when $\phi = 0$, we have simply $i = \delta \sin(n_{osc} \phi)$.

- **Precession of the spin axis.**
  
  This case is just a little more complicated. We can consider a further unit vector $\vec{f}$ which moves always on the ecliptic plane: its ecliptic longitude, $\eta = \eta(t)$ defines it: $\vec{f} = \cos \eta \vec{i} + \sin \eta \vec{j}$. Let $\vec{j}'' = -\sin \eta \vec{i} + \cos \eta \vec{j}$. The satellite spin axis $\vec{s}$ precesses around $\vec{f}$ (we can consider for example again 10 precessions for 360 days, per a complete rotation of the axis $\vec{f}$ over the ecliptic plane in 360 days). Let $\xi$ the angle between its projection on the plane identified by $\vec{j}''$ and $\vec{k}$ and the vector $\vec{k}$ and $\delta$ the angle between $\vec{f}$ and $\vec{s}$ (10°, for example). Then, the relation $\vec{s} = \cos \delta \vec{f} + \sin \delta \cos \xi \vec{k} + \sin \delta \sin \xi \vec{j}''$ easily gives the colatitude $i$ and the longitude $\phi$ of spin axis.

**Appendix B: System creation and solution with low memory usage**

As discussed in section 5.2, the creation of the linear system requires a large amount of memory, usually much more than the available RAM. This problem can be avoided by taking advantage of disk buffers, essentially by splitting a large matrix into smaller blocks and creating it a block at a time. Our strategy to do this is very simple:

1. a memory buffer is created, large enough to keep $L$ lines;

2. the algorithm described in section 5.1 is performed: for each couple $\pi$ of pixel in common between two scan circles $i$ and $j$, the quantities $\chi_\pi$ and $\tau_\pi$ are evaluated;

3. if $i$ is in the range $[0, \ldots, L - 1]$, then equations (17), (18) and (21) are applied; if $j$ is in the range $[0, \ldots, L - 1]$, then equations (19), (20) and (22) are applied;

4. after all the couples have been evaluated, the memory buffer is saved in a file; steps 2–4 are then repeated for $i$ and $j$ in the range $[L, \ldots, 2L - 1]$, and then for $i$ and $j$ in $[2L, \ldots, 3L - 1]$, and so on until all the $N$ matrix lines have been created.
At the end of this loop, the linear system has been created and stored in a file in binary format; the coefficients are organized in a matrix of \( N \) rows and \( N + 1 \) columns, where the last column contains the known terms.

It is easy to see that this strategy saves memory space but increases considerably CPU time, because time required to create the whole matrix is directly proportional to the number of pieces \( N/L \) into which the linear system is divided. Due to the fact that program execution time is essentially dominated by the routines which solve the system, while creation time is negligible for the present practical purpose, we didn’t bother to improve our code.

A very different situation occurs for system solution, because both a large amount of memory and a lot of computation time are usually required. While time efficiency can’t be usually improved (except in a few particular cases, when the matrix which defines the system has special symmetry properties and a lot of zero coefficients), memory requirements can be considerably reduced by taking advantages of disk swapping. However, a special care is required when choosing the strategy for disk operations, because disk time can easily blow up and overtake CPU time. Our algorithm behaves as follows:

1. as for system creation, a memory buffer is allocated, large enough to keep \( L \) lines;
2. then the first \( L \) lines \((0,\ldots,L-1)\) are loaded in memory, and complete Gauss elimination is performed on them: each line is reduced by the preceding lines and is used to reduce the following lines;
3. each of the remaining \( N-L \) lines is sequentially load into memory, and reduced by each of the \( L \) lines stored in the memory buffer. In this way, we perform \( L \) steps of the Gauss elimination algorithm with a single disk operation;
4. the memory buffer is flushed, and the next \( L \) lines \((L,\ldots,2L-1)\) are loaded into memory, reduced by one another and used to sequentially reduce all the remaining lines of the system. Steps 3-4 are repeated until the original matrix has been completely reduced.

As can be seen, our algorithm differs from the “standard” version only in the order in which the elimination is performed; in this way, we can limit memory requirements without increasing total solution time. It is easy to see that the total time spent for disk operation is:

\[
t_{\text{disk}} = \beta \frac{N}{2} \left( \frac{N}{L} + 1 \right) (N + 1) \approx \beta_{\text{disk}} \frac{N^3}{L} \tag{23}
\]

CPU time scales as the cube of the linear size of the system:

\[
t_{\text{cpu}} = \beta_{\text{cpu}} N^3 \tag{24}
\]

So we conclude that the ratio \( \eta \) between disk time and CPU time is independent from the size \( N \) of the linear system, and only depend on the total number of lines \( L \) then we can hold simultaneously in memory:

\[
\eta = \frac{t_{\text{disk}}}{t_{\text{cpu}}} = \frac{\beta_{\text{disk}}}{\beta_{\text{cpu}} \cdot L} \tag{25}
\]

Of course, the memory necessary to hold a line is proportional to the line length, so we need a larger amount of memory to solve a larger system; however, the size of the buffer is linear in system size \( N \), while the memory required to hold all the coefficients simultaneously is quadratic in \( N \).
REFERENCES

Bersanelli M., Mandolesi N., Weinreb S., Ambrosini R. & Smoot G.F., 1995, Int. Rep. ITESRE 177/1995 – COBRAS memo n.5
Bersanelli M. et al., 1996. ESA, COBRAS/SAMBA Report on the Phase A Study, D/SCI(96)3
Blum E.J., 1959, Annales d’Astrophysique, 22-2, 140
Bouchet F. et al., 1997, in proceedings of The XVIth Moriond Astrophysics Meeting, Les Arcs, Savoie, France, 16-23 March 1996.
Brandt W.N. et al., 1994, ApJ 424, 1
Burigana C. et al., 1997a, Int. Rep. ITesRE/CNR 186/1997
Burigana C. et al., 1997b, in proceedings of Particle Physics and Early Universe Conference, Cambridge 7-11 April 1997, [http://www.mrao.cam.ac.uk/ppeuc/proceedings/]
Burigana C. et al., 1997c, A&A, submitted
Colvin R.S., 1961, Ph.D. thesis, Stanford University
Danese L., Toffolatti L., Franceschini A., Bersanelli M. & Mandolesi N. 1996, Astroph. Lett & comm, 33, 257.
Delabrouille J., 1997, A&A, submitted
Dodelson S., 1997, ApJ 482, 577
Gaier T., 1997, private communication
Janssen M.A. et al., 1996, Astrophys. J. Lett., submitted
Jarosik N.C., 1997, ApJ 482, 577
Mandolesi N. et al., 1997a, in proceedings of Particle Physics and Early Universe Conference, Cambridge 7-11 April 1997, [http://www.mrao.cam.ac.uk/ppeuc/proceedings/]
Mandolesi N. et al., 1997b, Int. Rep. TTeSRE/CNR 198/1997
Muciaccia P.F. et al., 1997, preprint astro-ph/9703084
Pospieszalsky, 1989, MTT Sep, p. 1340
Press W.H. et al., 1992, “Numerical Recipes in Fortran”, Cambridge University Press
Seiffert M. et al., 1996, Rev. Sci. Instrum., submitted
Strang G., 1976, “Linear Algebra and Its Applications”, Academic Press, Inc.
Toffolatti L. et al., 1995, Astro. Lett. & Comm. 32, 125
Toffolatti L. et al., 1997, MNRAS, submitted
Weinreb S., 1997, private communication
Wollack E.J., 1995, Rev. Sci. Instrum., 66, 4305
**FIGURE CAPTIONS**

**Figure 1:** The simulated (input) map of CMB anisotropies (CDM model, the dipole term is neglected). Galactic coordinates have been used for the plot.

**Figure 2:** Schematic representation of the observational geometry.

**Figure 3:** The unreduced simulated noise map (white plus $1/f$ noise) for the simulation with $\alpha = 85^\circ$. Note the two small circular regions close to the ecliptic poles that are not observed by the considered off-axis beam; for graphic purposes we have filled them with a random noise distribution with variance given by the noise variance of the observed pixels. Note also the elongated sky region with noise significantly larger than the average, which corresponds to the sky regions that are observed a single time only, due to the choosen value of $\alpha$. (Galactic coordinates have been used for the plot).

**Figure 4:** The reduced simulated noise map (white plus “reduced” $1/f$ noise) for the simulation with $\alpha = 85^\circ$. Note again the two small circular regions close to the ecliptic poles filled with a random noise distribution. We note that the the elongated sky region with noise significantly larger than the average disappears as result of the destriping procedure. Also the stripes in the sky became much less evident. (Galactic coordinates have been used for the plot).
