Local time evolution in Personal Communication Service model

Liliia Ziganurova\textsuperscript{1,2} and Lev Shchur\textsuperscript{1,3}

\textsuperscript{1}National Research University Higher School of Economics, 101000 Moscow, Russia
\textsuperscript{2}Science Center in Chernogolovka, 142432 Chernogolovka, Russia
\textsuperscript{3}Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

Abstract. We investigate the local time evolution in the Personal Communication Service (PCS) model simulated with the parallel discrete event simulation method's optimistic algorithm. We propose a model for the optimistic local virtual time evolution (OLVT) in PCS, which is reminiscent of statistical physics's surface growth. We use Rensselaer's optimistic simulation system with the Time Warp implementation. We compare the results of the simulations of both PCS and OLVT models and found good agreement. We discuss the highlights of our approach in the analysis of scalability and synchronization using the OLVT model.

1. Introduction

Parallel Discrete Event Simulation (PDES) [1] is a large-scale simulation technique used in a wide range of fields, from physics and engineering to biology and social sciences. The discrete-event model contains a description of the system's status and the events that can occur in it. Events occur at random time moments and change the system's state (for example, in a magnetic system, an event is a spin-flip; and in the logistics, the event is a change of the status of delivery).

Today, many software platforms (simulators) for PDES facilitate using the method for solving applied problems. Among them, the most famous are ROSS [2], WARPED2 [3], Simian [4], SPEEDS [5], PDEVS [6], and NS-3 [7, 8]. The main functionality of PDES simulators is the event planner, i.e., organizing the execution of events by logical processes and receiving and transmitting messages containing the event information, and providing the other details of the simulation process. The successful application of the optimistic PDES simulator was demonstrated on almost two million cores [9].

Simulators’ critical features are the scalability of parallel simulations and the synchronization of the processes [1, 10, 11]. Analysis of modeling profiles is successfully used to optimize modeling tools and algorithms.

In the paper, we present the model of the model, i.e., the model of local virtual time evolution (OVLT) of the optimistic simulator running the particular model of interest. The OVLT model can capture the essential properties of the real simulation model and provide information on the — i) possible lockdown; ii) on the scalability, and iii) on the desynchronization of parallel processing units depending on the parameters of simulations. We should stress that information is valid for a particular model and a wide range of parameters. In contrast to the widely used modeling profiles for each particular model and particular values of parameters, the proposed
approach provides information on the particular model’s behavior in a wide range of parameters. We do not pretend on a very detailed analysis of the algorithm performance and provide additional information on the simulator’s general behavior running the particular model, which can be informative for the general understanding performance. In other words, we do not pretend that our approach is superior to any other approaches or can substitute them. Instead, our point is to present the new and complementary way of algorithm analysis.

We present the preliminary results of applying our approach to the Personal Communication Service (PCS) model. The results of the PCS simulations are in good agreement with the OLVT model. Both models undergo a roughening transition.

Our approach is based on the previous analysis of the models of conservative algorithm [12] and optimistic algorithm [13] and models on the small-world networks [14, 15, 16]. In contrast to these studies, we build the OLVT model of optimistic simulation of a particular model — the PCS model. The new ingredient is the simulation of the model for the specific pair of two parameters in the model, which is correlated with the PCS model.

2. PCS model
PCS (personal communication service) model simulates a wireless communication network that provides communication services for mobile units [17, 18, 19]. The service area of the network is partitioned into checkerboard areas (cells). The simulation model consists of cell objects and portable objects associated with mobile devices. Each cell object represents a cellular receiver/transmitter with a fixed number of channels. The portable object represents a mobile unit that resides within the cell for some time and can move during the call to one of the four neighboring cells. The portable object’s events are call arrival, call completion, and move to a neighboring cell. When a portable device moves to the neighboring cell, the currently allocated channel must be freed, and the destination cell object must allocate a channel to the portable object. It happens practically instantaneously.

The model has several customizable parameters, and typically the number of portables is much larger than the number of cells. The most important are the parameters that specify the mean values of the distributions:

- **MOVE_CALL_MEAN** - the average time between mobile devices moving from one cell to another,
- **NEXT_CALL_MEAN** - the next call average,
- **CALL_TIME_MEAN** - the average call length.

We run the PCS model using the ROSS simulator (Rensselaer’s optimistic simulation systems) [2]. The ROSS simulator is a parallel discrete event simulation system based on the Time Warp synchronization algorithm [20] and is widely used in the scientific community [21, 22, 23, 24, 25].

The ROSS simulator’s output contains the total number of events processed, the total number of rollbacks, the number of events processed per second (event rate), the number of events with the portable moves to the neighboring cell. In our case, it is essential to get in the output the local virtual times of processes. However, it can be done when calculating the global virtual time (GVT), which happens after 256 events processed by the default setting.

3. OLVT model
In this section, we introduce the model of the local virtual (LVT) time evolution in PCS model simulated with an optimistic algorithm - the OLVT model. It is based on the ideas of the models of local time evolution in the conservative PDES algorithm [12], FaS algorithm [26], and optimistic algorithm [13].
In the OLVT model, we associate each cell object with the logical process \( \text{LP}_i \) and initiate the flat profile of the local virtual time \( \tau_i \) of each \( \text{LP}_i \) \((i = 1, 2, \ldots, N)\).

The simulation of the LVT with an optimistic algorithm consists of two essential parts. The first part is the evolution of the system for some time, assuming there is no causality violation – it is the forward computations. The second part is the check of causality violation with rolling back those events, which violates the time order – it is the rollback computations.

To model the forward part in OLVT we use the same approach as in Refs. [12, 26, 13] computing the increment of LVT with the exponentially distributed random variable \( \eta_i \) with the mean equal to unity

\[
\tau_i(t + 1) = \tau_i(t) + \eta_i, \quad \text{where } i = 1, 2, \ldots, N.
\]

The wall time \( t \) in the OLVT model is measured in the units of the Time Warp window [2].

To model the rollback part in OLVT, we choose randomly from the exponential distribution with the mean \( b \) the rollback depth value \( k \) and choose \( kN \) LPs randomly. For each chosen LP, check the rollback condition with the following law.

With probability \( p \) the LVT \( \tau_j(t + 1) \) of the chosen \( \text{LP}_j \) will remain the same with probability \((1 - p)\) and will be updated with probability \( p \)

\[
\tau_j(t + 1) = \begin{cases} 
\tau_r(t + 1), & \text{if } \tau_j(t + 1) > \tau_r(t + 1) \\
\tau_j(t + 1), & \text{otherwise}
\end{cases}
\]

where \( \tau_r(t + 1) \) is LVT of the one of the nearest neighbors chosen randomly. The LVT update corresponds to the event of the active mobile device moving from the cell \( r \) to the cell \( j \).

We should stress that the rollback law in the OLVT model is different from the one in the optimistic LVT model [13], and use some ideas of the LVT model defined on the small-world network in Ref. [16].

4. Parameters and observables

4.1. Parameters in OLVT model

The parameter of OLVT model defined in line with the optimistic LVT model [13] as the ratio of the forward increment to the sum of the forward and rollback increments

\[
q = \frac{1}{1 + b},
\]

which takes the limiting values 1 and 0 while \( b \) varies from 0 to \( \infty \).

Analyzing the simple LVT evolution model in the paper [13] we found that at a low enough value of parameter \( q \), the LVT profile becomes flat and does not increase the mean value. For a large enough value of the parameter \( q \), the LVT profile becomes rough and propagates with the wall time \( t \), and the mean value of the LVT profile becomes proportional to the wall time \( t \).

There is some critical value \( q_c \) separating the two regimes, the value of the roughening transitions known in statistical physics [27] (for details, we refer the reader to reviews [28, 29]). The critical feature of the roughening transition is that the LVT profile behavior near the critical value \( q_c \) is universal, giving the same law of the mean LVT profile dependence from the parameter \( q_c \), and the same law of the mean LVT profile width dependence from the number of cells \( N \).

In the language of computer science, the first dependence is connected to the efficiency of simulations (the loading of the processing elements), and the second dependence is connected to the synchronization of the processing elements.

In addition to the parameter \( q \), we already defined the parameter \( p \) for the OLVT model as the probability of the event describing the active mobile device moving to the nearest cell.
4.2. Parameters in PCS model
Let us introduce the parameter $q'$

$$q' = 1 - \frac{\# \text{ rollbacks}}{\# \text{ events}} \quad (4)$$

which defined through the relative number of rollbacks ($\# \text{ rollbacks}$) to the number of the processed events ($\# \text{ events}$).

As we will see, the parameter $q'$ plays the same role as the parameter $q$ in the OLVT model.

In the PCS model, the parameter $p'$ has the same meaning as the parameter $p$ in the OLVT model. We keep different notations for $p, p'$ and $q, q'$ because in PCS model parameters $p'$ and $q'$ is not set before simulations as in OLVT model and rather computed in PCS simulation. The parameters $p'$ and $q'$ depend on the value of ROSS parameter MOVE_CALL_MEAN, which is the average time between calls that leave the tower’s coverage area.

4.3. Observables
We calculate time-dependent averages at each time step $t$ and after simulations average them over $M = 10$ realizations of the process in the PCS model and $M = 1000$ in the OLVT model using the following definitions

(i) The mean LVT profile

$$\langle \tau(t) \rangle = \frac{1}{M} \sum_{j=1}^{M} \left( \frac{1}{N} \sum_{i=1}^{N} \tau_i(t) \right)_j \quad , \quad (5)$$

(ii) The mean velocity

$$\langle v(t) \rangle = \frac{1}{M} \sum_{j=1}^{M} (\tau(t+1) - \tau(t))_j \quad , \quad (6)$$

(iii) The mean LVT profile square width

$$\langle w^2(t) \rangle = \frac{1}{M} \sum_{j=1}^{M} \left( \frac{1}{N} \sum_{i=1}^{N} [\tau_i(t) - \tau(t)]^2 \right)_j \quad . \quad (7)$$

5. Simulations
5.1. The profile evolution
The PCS model was simulated on 64 cores of the computing node R2D26 computer. The number of logical processes and the number of kernel processes was $N = 256$. The parameter MOVE_CALL_MEAN sets the average time between calls moving from one network coverage area to another.

We use values of $p', q'$ calculated in the PCS model simulations as the values of parameters $p, q$ in OLVT simulations. The data presented in Table 1. Below, we use the notations $v(p, q)$ for the profile velocity in the OLVT model and similar notations for the profiles’ square width.

The OLVT model was simulated on the R2D26 computer. We run each model with several logical processes $N = 256$ and different parameters $p$ and $q$ independently on the available cores to average over random realizations.

Mean OLVT profile velocity $\langle v(t) \rangle$ saturated after some transition time, forgetting the initial flat state. We name this velocity the steady-state velocity $v(p, q)$. The steady-state velocity does depend on the value of parameter $q$ defined by expression 3) and is shown in Figure 1-a.

In the figures, each point supplied the error bars, computed with the number of realizations. The size of error bars is typically less than the symbols. The velocity vanishes at some value
Table 1. Parameters of the simulations.

| MOVE_CALL_MEAN | $p'$ | $q'$ |
|---------------|------|------|
| 300           | 0.1821 | 0.24 |
| 450           | 0.1368 | 0.30 |
| 600           | 0.1078 | 0.36 |
| 800           | 0.0828 | 0.43 |
| 1000          | 0.0666 | 0.50 |
| 1250          | 0.0531 | 0.57 |
| 1500          | 0.0439 | 0.64 |
| 1750          | 0.0373 | 0.68 |
| 2000          | 0.0323 | 0.71 |
| 3000          | 0.0209 | 0.76 |
| 4000          | 0.0154 | 0.7759 |
| 4500          | 0.0135 | 0.7761 |

a) Steady-state velocity in OLVT as function of parameter $q$.

b) Normalized event-rate in PCS model as function of parameter $q'$.

Figure 1. Critical behaviour of the a) steady-state velocity $v(p, q)$ in OLVT model and b) normalized event-rate in PCS model.

$q_c$, which is the critical point of the roughening transition [28, 29]). The approximation of the $v$ near the $q_c$ for the values of $q < 0.33$ gives the following expression

$$v \sim (q - q_c)^\nu,$$

with $q_c \approx 0.132(5)$ (the number in parentheses is the statistical error of the fitting procedure) and $\nu \approx 1.50(1)$.

The normalized event-rate in the PCS model can be estimated as the number of events in the unity time divided by the number of maximum event rates. In a qualitative analogy with the steady-state velocity in the OLVT model, the normalized event-rate in the PCS model demonstrates the roughening transition. The fit to the data in Figure 1-b for the values of $q' < 0.33$ give the following approximation of the event-rate behavior near the roughening point

$$\text{Event rate} \sim (q' - q'_c)^{\nu'},$$

with $q'_c \approx 0.101(2)$ and $\nu' \approx 1.54(3)$. 
The qualitative analogy of the steady-state velocity in the OLVT model and the event rate become even better while plotting depends on them as functions of \( p \) and \( p' \), correspondingly. In Figure 2 both the left and right curves drops done about five times in the same variation range of the parameters \( p \) and \( p' \).

![Graph](image)  

**Figure 2.** Behaviour of the a) steady-state velocity \( v \) in OLVT model and b) event-rate in PCS model.

5.2. The width of the profile evolution

The width of the profile can be associated with the level of processing elements synchronization. Figure 3 shows the dependence of the square width of the OLVT profile and the square width of the PCS profile. The name roughening of the transitions comes from the fact that below \( q_c \) (\( q'_c \)), both models have the flat profile \( \langle w^2(\infty) \rangle \) after saturation, and for larger values of parameter \( q_c \) (\( q'_c \)) the width is finite, reflecting the rough random profile in both models.

![Graph](image)  

**Figure 3.** The mean square width of LVT profiles in OLVT and PCS models.

We should note the similar qualitative behavior in the left and right panel of Figure 3.
6. Discussions

We introduce the model of the local virtual time (LVT) profile evolution in the PCS model, the OLVT model. The model reflects the essentials of the LVT evolution. The comparison of the direct simulation of the PCS model with the ROSS simulator and OLVT model demonstrates the qualitative analogy in the LVT profile evolution. Moreover, both models have the roughening transition, and the behavior of models near the transition belongs to the same universality class of the transition.

The paper’s main results are 1) introduction of the OLVT model and 2) validation of the model by comparing simulation results for the LVT profile of both models.

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