Single and double changes of Entanglement

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Abstract

Entanglement behavior for different classes of two qubit systems passing through a generalized amplitude damping channel is discussed. The phenomena of sudden single, double changes and the sudden death of entanglement are reported for correlated and non-correlated noise. It is shown that, for less entangled states, these phenomena appear for small values of channel strength. The effect of the channel can be frozen for these classes as one increases the channel strength. Maximum entangled states are more fragile than partial entangled states, where the entanglement decays very fast. However, one can not freeze the effect of the noise channel for systems initially prepared in maximum entangled states. The decay rate of entanglement for systems affected by non-correlated noise is much larger than that affected by correlated noise.

Keyword: Entanglement, Qubits, Discord, noise channels

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1 Introduction

Entanglement represents an invaluable recourse for most of quantum information tasks [1]. It is possible to generate maximum entangled states but due to its interaction with environment, the entanglement decreases and consequently the efficiency of using it to perform some quantum information and computations decreases. Sometimes, the entanglement decays during sending the states from the lab to the users. However, there are some cases where the travelling states are forced to pass through a noise channel [2]. Therefore it is important to investigate and quantify the decay rate of entanglement due to this undesired interactions.

There are some common noise channels that have been considered as unavoidable noise during performing quantum information tasks. For example, Yu and Eberly [3] showed that, the abrupt and asymptotically gradual decay of entanglement predicted in amplitude and phase damping channels. However for some systems the entanglement is completely vanishes in a finite time. This type of entanglement decay is called entanglement sudden-death, ESD [4] and has been reported extensively in many studies [5].

It has been shown that, there are some classes of noise states whose teleportation fidelity can be enhanced if one of the two qubits subject to dissipative interaction with environment via amplitude damping channel [6]. The possibility of recovering entanglement in the presence of amplitude damping channel is discussed by Sun, et.al [7, 8]. Recently, Montealegre et. al [9], have investigated the effect of different types of noise channels on the on-norm geometric discord.

This motivated us to study the entanglement behavior of different classes of maximum and partial entangled states. The current manuscript is different from the previous manipulation [9], where it is assumed that both qubit interact with the noise channel. The noise could be correlated or non-correlated. The behavior of entanglement shows that there are different types of noise channels. The decay rate of entanglement is investigated for both types.

The decay of entanglement is performed using computer simulations. The main features of the decay are illustrated in figures 1 and 2. The entanglement decreases more for non-correlated noise. The decay rate of entanglement is shown in figure 3. The decay rate of entanglement is shown in figure 3. The decay rate of entanglement is shown in figure 3. The decay rate of entanglement is shown in figure 3. The decay rate of entanglement is shown in figure 3. The decay rate of entanglement is shown in figure 3.

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However, the effect of the noise channel can be frozen by a judicious choice of the initial states and channel parameter.

The paper is organized as follows: In Sec.II, we describe the model of a two qubit-system. The initial degree of entanglement is quantified for different classes via negativity. The generalized amplitude damping and its effect on the two qubit system is discussed in Sec.III, where we consider that the noise could be correlated or non-correlated. In Sec. IV, we conclude our results.

2 The suggested system

Let us assume that there are two users, Alice and Bob, that share a two-qubit state. This state can be described by 15 parameters: 3 parameters, represent the Bloch vector for each qubit and the 9 remainder parameters represent the correlation between the two qubits [10]. In these parameters the two qubit state can be written as,

\[
\rho_{ab} = \frac{1}{4} (1 + s \cdot \vec{\sigma}^{(1)} + t \cdot \vec{\sigma}^{(2)} + \vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) \quad (1)
\]

where \(s = (s_x, s_y, s_z)\) and \(t = (t_x, t_y, t_z)\) are the Bloch vectors for the first and the second qubit respectively. The vectors \(\vec{\sigma}^{(i)} = (\sigma_x^{(i)}, \sigma_y^{(i)}, \sigma_z^{(i)})\) and \(\vec{\sigma}^{(2)} = (\sigma_x^{(2)}, \sigma_y^{(2)}, \sigma_z^{(2)})\) are the Pauli-vectors for each qubit, respectively. The dyadic, \(\vec{C}\) is a 3 \times 3 matrix represents the correlation between the two qubits, where \(c_{ij} = \text{tr}\{\rho \sigma_{kl}^{(i)} \sigma_{kl}^{(j)}\}, i, j = 1, 2\) and \(k, l = x, y, z\).

The state (1) is a general form of a two qubit systems, where we can construct several common classes in the context of quantum information and computation. However, if we set \(s = t = 0\), while the elements of the cross dyadic \(\vec{C}\) are \(c_{ij} = 0\) for \(i \neq j\) and \(c_{ij} \neq 0\) for \(i = j\), one gets what is called X-state [13]. The Werner state [14] can be obtained from X-state class if we set \(c_{11} = c_{22} = c_{33} = x\). The four Bell maximum entangled states [15] \(\rho_{\phi^+}, \rho_{\phi^-}, \rho_{\psi^+}\) and \(\rho_{\psi^-}\) can be obtained for different values of the parameters’ \(c_i\). For example the singlet maximum entangled state \(\rho_{\psi^-}\) can be obtained if we set \(c_{11} = c_{22} = c_{33} = -1\), where \(\rho_{\psi^-} = |\psi^\text{--}\rangle \langle \psi^\text{--}|\) and \(|\psi^\text{--}\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)\).

In our consideration, we assume that the initial system is prepared in X-state, Werner, or maximum entangled state. Since the main aim of this contribution is investigating the behavior of the amount of entanglement contained in these states after they passing through a noisy channel, it is important to quantify the initial amount of entanglement in these classes. For this purpose, we use a measure of entanglement called negativity [16]. This measure represents one of the most common measures of entangled two qubit states. This measure based on positive partial transpose criterion [17]. Due to its operational and calculations, negativity has been recently quantified experimentally by Sliva et. al., [18]. This measure state that if \(\lambda_i\) represent the eigenvalues of \(\rho_{T_2}^{ab}\), then the negativity is given by [16]

\[
\mathcal{N}(\rho_{ab}) = \sum_{k=1}^{4} |\lambda_k| - 1, \quad (2)
\]

where \(T_2\) represents a partial transposition for the second qubit. As a function of the \(c_i\) parameters, the negativity can be written as

\[
\mathcal{N}(\rho_{ab}) = -\frac{1}{2} + \frac{1}{2} \text{tr}\left\{\vec{C}^T \cdot \vec{C}\right\}. \quad (3)
\]
For example the degree of entanglement for Werner state is given by $N(\rho_{\text{Werner}}) = -\frac{1}{2} + \frac{3|\alpha|^2}{2}$ and $N(\rho_{\text{Bell}}) = 1$. In Fig.(1), we display the behavior of the negativity $N(\rho^{(f)})$ for different classes of initial states, where we set $c_{xx} = -1$ and $c_{yy}, c_{zz} \in [-1,0]$. However, classes which are described by $|c_{yy}| < 1$ or $|c_{zz}| < 1$, represent partial entangled states, PES, where the degree of entanglement is smaller than one, i.e., $0 \leq N(\rho_{ab}) < 1$.

3 Noise channel

The generalized amplitude damping channel, GAD represents one of the most important channel in quantum information context. It corresponds to the interaction of a two-qubit system with a squeezed thermal bath via a dissipative interaction [11]. Srikanth and Banerjee [11] showed that, the amplitude damping channel preserves the non-classical phenomenon from incoherence. This motivates us to investigate the behavior of the initial entangled states passing through this channel. The evolution of the state $\rho^{(f)}_{ab}$ under the effect of the generalized amplitude damped channel[9] is given by,

$$\rho^{(f)}_{ab} = \sum_{i=0}^{3} \left\{ U_a^{(i)} U_b^{(i)} \rho_{ab} U_b^{\dagger (i)} U_a^{\dagger (i)} \right\},$$

(4)

where $U_a$ and $U_b$ are the operators of the generalized amplitude damping channel for qubits ”$a$” and ”$b$”, respectively. In the computational basis, ”$0$” and ”$1$” these operators can be written as,

$$U_i^{(0)} = \sqrt{p} \left( |0\rangle \langle 0| + \sqrt{1-\gamma} |1\rangle \langle 1| \right), \quad U_i^{(1)} = \sqrt{p} \left| 0\right\rangle \langle 1\right|$$

$$U_i^{(2)} = \sqrt{1-p} \left( \sqrt{1-\gamma} |0\rangle \langle 0| + |1\rangle \langle 1| \right), \quad U_i^{(3)} = \sqrt{1-p} \sqrt{\gamma} |1\rangle \langle 0|,$$

(5)

where $i = a, b, \gamma = 1 - Exp[-\gamma_0 t]$ and $\gamma_0$ is the decay rate. It is clear that, if we set $p = 1$ or 0, then the generalized amplitude channel reduces to the usual amplitude channel. The final state $\rho^{(f)}_{ab}$ depends on the initial state between the two users. In what follows we consider different classes prepared initially with different degrees of entanglement.
3.1 Correlated Noise

In this case, we assume that each qubit is affected by the same noise at the same time. For example, if the operator $U_{a}^{(0)}$ is operators on the first qubit, then the second qubit will be affected by the operator $U_{b}^{(0)}$. Explicitly, the final state evolves as,

$$\rho_{ab}^{(f_{c})} = U_{a}^{(0)}U_{b}^{(0)}\rho_{ab}U_{a}^{(i(0))}U_{b}^{(i(0))} + U_{a}^{(1)}U_{b}^{(1)}\rho_{ab}U_{a}^{(i(1))}U_{b}^{(i(1))} + U_{a}^{(2)}U_{b}^{(2)}\rho_{ab}U_{a}^{(i(2))}U_{b}^{(i(2))} + U_{a}^{(3)}U_{b}^{(3)}\rho_{ab}U_{a}^{(i(3))}U_{b}^{(i(3))}, \quad (6)$$

where $\rho_{ab}^{(f_{c})}$ is the final state in the presence of correlated noise. In this case the final state (4) is described by

$$\rho_{ab}^{(f_{c})} = \frac{1}{4}(1 + c_{1}\sigma_{x}^{(1)}\sigma_{x}^{(2)} + c_{2}\sigma_{y}^{(1)}\sigma_{y}^{(2)} + c_{3}\sigma_{z}^{(1)}\sigma_{z}^{(2)}), \quad (7)$$

where,

$$c_{1} = B_{2} + B_{3} + B_{7} + B_{8}, \quad c_{2} = -c_{1}, \quad c_{3} = (B_{1} + B_{6}) - (B_{4} + B_{5}), \quad (8)$$

with

$$\begin{align*}
B_{1} & = \frac{1 + c_{zz}}{4}\left[p^{2} + (1 - p)^{2}(1 - \gamma)^{2}\right], \\
B_{2} & = \frac{c_{xx} - c_{yy}}{4}\left[(1 - \gamma)(1 - p)^{2} + p^{2}(2 - \gamma)\right], \\
B_{3} & = \frac{c_{xx} - c_{yy}}{4}(1 - \gamma)\left[p^{2} + (1 - p)^{2}\right] + \gamma(1 - p), \\
B_{4} & = \frac{1 - c_{zz}}{4}(1 - \gamma)\left[p^{2} + (1 - p)^{2}\right], \\
B_{5} & = B_{4}, \quad B_{6} = \frac{1 + c_{zz}}{4}\left[p^{2}(1 - \gamma)^{2} + (1 - p)^{2}\right], \\
B_{7} & = B_{8} = \frac{c_{xx} + c_{yy}}{4}\left[p^{2}(1 - \gamma)^{2} + (1 - \gamma)(1 - p)^{2}\right]. \quad (9)
\end{align*}$$

The behavior of $N(\rho^{(f_{c})})$ for a class of state described by $c_{xx} = -0.1, c_{yy} = -0.2$ and $c_{zz} = -0.7$ is described in Fig.(2a). From this figure three different phenomena can be predicated. Firstly, there is a sudden decay of entanglement as one increases the parameters $\gamma$ or $p$. Secondly, there is a range of $\gamma$ and $p$, where the effect of the noisy channel is completely frozen. Thirdly, a sudden death of entanglement happens at $\gamma = 1$. In Fig. (2b), we investigate the effect of a particular value of the channel strength $\gamma$ on the same system. This figure describes clearly the three previous phenomena. The upper bounds of entanglement decrease as $p$ increases. As soon as $\gamma$ increases, the entanglement decays sharply to reach its minimum bounds which depend on the values of $p$. However, for larger values of $\gamma$ the effect of the channel is frozen, namely, the entanglement is immutabl. The frozen interval increases with $p$: the larger value of $p$ is, the larger frozen interval. The last remark which appears in this behavior is the sudden entanglement death as $\gamma$ further increases.

In Fig.(3), we consider a class of Werner state defined by $c_{xx} = c_{yy} = c_{zz} = -0.03$. It is shown that, for small values of $p$, the entanglement decays hastily to vanish completely at
Figure 2: The negativity $N(f_\gamma)$ against the channel parameters $p$ and $\gamma$ for a state initially described by $c_{xx} = -0.1$, $c_{yy} = -0.2$ and $c_{zz} = -0.7$ and (b) the same as (a) but for particular values of $p$. The solid, dot and dash curves for $p = 0.1, 0.2$ and 0.3, respectively.

$\gamma = 1$. However for larger values of $p$, the entanglement is almost constant for $\gamma \in [0, 0.25]$, i.e., the effect of the generalized amplitude channel is frozen. As $\gamma$ increases further, the entanglement suddenly decays to vanish completely at $\gamma = 1$. The behavior of $N$ shows that the frozen interval of the channel increases as $p$ increases. The effect of larger values of $p$ on the negativity of this class is depicted in Fig.(3b), where we set $p = 0.7, 0.8$ and 0.9. It is clear that, the upper bounds of entanglement is smaller than those depicted in Fig.(3a). Although the negativity decreases in general, it increases for larger values of $p$. The behavior of $N(\rho(f_\gamma))$ shows that the phenomena of the sudden single, double changes and channel frozen appear for larger values of $p$.

It is important to investigate the behavior of entanglement for systems which are initially prepared in maximum entangled states. For this aim, we consider a class defined by $c_{xx} = c_{yy} = c_{zz} = -1$, where the entanglement $N = 1$. Fig.(4a) shows that the entanglement decays as $\gamma$ increases and completely dies at $\gamma = 1$. On the other hand, for larger values of $p$, the upper bounds decrease.

Fig.(4b) displays the entanglement behavior of a class of $X-$ states, where we set $c_{xx} = -0.1, c_{yy} = -0.2$ and $c_{zz} = -0.3$. It is clear that, the entanglement slightly decays, then suddenly decays to death completely at $\gamma = 1$. The sudden changes occurs faster for smaller values of $p$.

From the previous figures one can conclude that, it is impossible to freeze the noisy channel for systems prepared initially in maximum entangled states. For less entangled state the phenomena of sudden changes and sudden death appear for larger values of the channel strength. However, the possibility of the frozen noisy channel increases for less entangled state and larger values of the channel’s strength.
Figure 3: (a) The negativity $\mathcal{N}(f_c)$ for class initially prepared in Werner state with $c_{xx} = c_{yy} = c_{zz} = -0.03$. The solid, dot and dash curves for, $p = 0.01, 0.1, 0.2$, respectively and (b) the same as (a) but $p = 0.7, 0.8, 0.9$ for the solid, dot and dash curves, respectively.

Figure 4: The same as Fig.(3) but the system is initially prepared in (a) maximum entangled state i.e., $c_{xx} = c_{yy} = c_{zz} = -1$(b) in $X-$ state with $c_{xx} = -0.1, c_{yy} = -0.2$ and $c_{zz} = -0.3$.

### 3.2 Non- correlated Noise

In this subsection, we investigate the effect of the generalized amplitude damping channel for non correlated noise. In this case the final state is given by,

$$\rho_{ab}^{(f_{nc})} = \sum_{i=0}^{3} \sum_{j=0}^{3} \left\{ U_a^{(i)} U_b^{(j)} \rho_{ab} U_b^{(j)} U_a^{(i)} \right\}, \quad (10)$$

where $\rho_{ab}^{(f_{nc})}$ is the final state for the initial state (1) subject to non-correlated noise. This state can be written explicitly in the form (7) but with different coefficients $B_i, i = 1, ... 8$.
Figure 5: The same as Fig. (2), but it is assumed that the noise is non-correlated.

given by,

\[
\begin{align*}
\tilde{B}_1 &= \frac{1 + c_{zz}}{4} \left( p + (1 - p)(1 - \gamma) \right)^2 \\
\tilde{B}_2 &= \frac{1 - c_{zz}}{4} \left[ (1 - \gamma)(p^2 + (1 - p)^2) + p(1 - p)(1 + (1 - \gamma)^2) \right], \quad \tilde{B}_3 = \tilde{B}_2, \\
\tilde{B}_4 &= \frac{1 + c_{zz}}{4} \left( p^2(1 - \gamma)^2 + (1 - p)^2 + p(1 - p)(1 - \gamma) \right) + \frac{c_{xx} - c_{yy}}{4} \gamma^2 (1 - p)^2, \\
\tilde{B}_5 &= \frac{c_{xx} - c_{yy}}{4} \sqrt{1 - \gamma}(\gamma + p(1 - \gamma) + \frac{c_{xx} + c_{yy}}{4} (1 - \gamma) \left[ (2p - 1) + p(1 - p)(2 + \gamma) \right]} \\
\tilde{B}_6 &= \frac{c_{xx} - c_{yy}}{4} \sqrt{1 - \gamma} \left( p^2 + \gamma(1 - p)^2 + p(1 - p)(1 + \gamma) \right) \\
&\quad + \frac{c_{xx} + c_{yy}}{4} \left[ (1 - \gamma) + \gamma p(1 - p) \right], \\
\tilde{B}_7 &= \frac{c_{xx} - c_{yy}}{4} (1 - \gamma)(1 + p^2) + \frac{c_1 + c_2}{2} p \sqrt{1 - \gamma} \\
\tilde{B}_8 &= \frac{c_{xx} - c_{yy}}{4} (1 - \gamma)(p^2 + p - 1) + \frac{c_1 + c_2}{2} \gamma \sqrt{1 - \gamma}(1 - p)
\end{align*}
\]

In Fig. (5a), we investigate the effect of the non-correlated noise on the same class that is described in Fig. (2) (correlated noise). It is clear that, the upper bounds of entanglement are smaller than those depicted for correlated noise (see Fig. (2a)). As \( \gamma \) increases the negativity \( \mathcal{N} \) decreases to vanish completely at \( \gamma = 1 \). At \( \gamma = 0 \), the negativity slightly decreases in the interval \( p \in [0, 0.75] \). However for further values of \( p \), there is a remarkable increase in the negativity.

To describe the previous behavior of the negativity, we consider different values of \( p \) as shown in Fig. (5b). It is clear that, initially (\( \gamma = 0 \)) the negativity decreases as \( p \) increases. For larger values of \( \gamma \in [0, 0.70] \), the negativity decreases sharply. However for larger values of \( \gamma \), the negativity increases as \( p \) increases.

Fig. (6) shows the behavior of entanglement for a class is initially prepared in Werner state defined by \( c_{xx} = c_{yy} = c_{zz} = -0.03 \) subjects to non-correlated noise. In this case the
entanglement decays faster than that shown for correlated noise (see Fig. 3). The phenomena of sudden decay and sudden change appear clearly as \( \gamma \) increases. However as \( p \) increases the degree of entanglement increases and doesn’t vanish even at \( \gamma = 1 \). The behavior of the negativity \( N \) for larger values of \( p \) is described in Fig. (6b), where we set \( p = 0.7, 0.8, 0.9 \), respectively.

It is clear that, the rate of entanglement decay is much smaller than that shown in Fig. (6a).

4 conclusion

We have studied the effect of the generalized amplitude damping channel on different classes of two qubit systems. Particulary, we considerd maximum entangled class, partial entangled class of \( X \)-states and a class of Werner states. The degree of entanglement is quantified for different classes of initial state setting before passing through the generalized amplitude damping channel.

In this investigation, it is assumed that the noise could be correlated or non-correlated. The general behavior of entanglement shows that, as the channel parameters increase the entanglement decays. The decay rate depends on the initial state setting and the noise type (correlated or non-correlated). Our results shows that the decay rate of entanglement is larger in the presence of the non-correlated noise.

The phenomena of single, double changes and sudden death of entanglement, as well as the frozen channel appear according to the initial state setting, the type of the noise and the values of the channel strength. For correlated noise, if we start from maximum entangled states, then the entanglement decays very fast and the decay’s rate increases as the channel strength increases. The phenomena of single changes and sudden death of entanglement appear for systems prepared initially in Werner and \( X \)-states with small degree of entanglement. However, the sudden double change of entanglement and the frozen channel phenomena appear for systems prepared initially with small entanglement and larger values of the channel strength. Although the entanglement decreases in the presence of
non-correlated noise, the upper bounds increase for smaller values of the channel strength. However, for larger values of the channel strength the entanglement decays smoothly to non-zero value.

In conclusion: important phenomena might appear for two qubit systems passing through correlated or non-correlated noise: single, double changes, sudden death of entanglement and the frozen noisy channel. One can increase the range of the frozen channel interval by controlling the channel parameters. The maximum entangled states are fragile while the partial entangled states are more robust.

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