Reconstructing interacting new agegraphic polytropic gas model in non-flat FRW universe

K. Karami · A. Abdolmaleki

Abstract We study the correspondence between the interacting new agegraphic dark energy and the polytropic gas model of dark energy in the non-flat FRW universe. This correspondence allows us to reconstruct the potential and the dynamics for the scalar field of the polytropic model, which describe accelerated expansion of the universe.

Keywords Dark energy theory · Polytropic model

1 Introduction

Type Ia supernovae observational data suggest that the universe is dominated by two dark components containing dark matter and dark energy (Riess et al. 1998; Perlmutter et al. 1999, 2003; de Bernardis et al. 2000). Dark matter (DM), a matter without pressure, is mainly used to explain galactic curves and large-scale structure formation, while dark energy (DE), an exotic energy with negative pressure, is used to explain the present cosmic accelerating expansion. However, the nature of DE is still unknown, and people have proposed some candidates to describe it (for a good review see Karami et al. 2009).

Recently, the original agegraphic dark energy (OADE) and new agegraphic dark energy (NADE) models were proposed by Cai (2007) and Wei and Cai (2008a), respectively.

Cai (2007) proposed the OADE model to explain the accelerated expansion of the universe, based on the uncertainty relation of quantum mechanics as well as the gravitational effect in general relativity. The OADE model had some difficulties. In particular, it cannot justify the matter-dominated era (Cai 2007). This motivated Wei and Cai (2008a) to propose the NADE model, while the time scale is chosen to be the conformal time instead of the age of the universe. The evolution behavior of the NADE is very different from that of the OADE. Instead the evolution behavior of the NADE is similar to that of the holographic DE (Cohen et al. 1999; Horava and Minic 2000; Thomas 2002; Li 2004; Zhang and Wu 2005, 2007; Zhang 2006; Li et al. 2009a, 2009b; Sheykhi 2009b; Gao et al. 2009; Karami 2010). But some essential differences exist between them. In particular, the NADE model is free of the drawback concerning the causality problem which exists in the holographic DE model. The ADE models assume that the observed DE comes from the spacetime and matter field fluctuations in the universe (Wei and Cai 2008a, 2009). The ADE models have been studied in ample detail by Kim et al. (2008a, 2008b), Wu et al. (2008), Zhang et al. (2008), Wei and Cai (2008b), Neupane (2009), and Sheykhi (2009a, 2010b).

Karami et al. (2009) introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. An example of a polytropic gas is a gas where the pressure is dominated by degenerate electrons in white dwarfs or degenerate neutrons in neutron stars. Another example is the case where pressure and density are related adiabatically in main sequence stars (Karami et al. 2009).

Reconstructing the holographic and agegraphic scalar field models of DE is one of the interesting issues which have been investigated in the literature. For instance, we have holographic quintom (Zhang 2006), holographic
quintessence (Zhang 2007), holographic tachyon (Zhang et al. 2007), holographic Ricci quintom (Zhang 2009), new holographic quintessence, tachyon, K-essence and dilaton (Granda and Oliveros 2009; Karami and Fehri 2010), interacting new agegraphic tachyon, K-essence and dilaton (Karami et al. 2010), and interacting agegraphic tachyon (Sheykhi 2010a).

All mentioned above motivates us to investigate the correspondence between the interacting NADE and the polytropic gas model of DE in the non-flat FRW universe. This paper is organized as follows. In Sect. 2, we study the interacting NADE with the cold DM in the non-flat FRW universe. In Sect. 3, we investigate the polytropic gas model of DE. In Sect. 4, we suggest a correspondence between the interacting NADE and the polytropic gas model of DE. We reconstruct the potential and the dynamics for the scalar field of the polytropic model, which describe accelerated expansion. Section 5 is devoted to conclusions.

2 Interacting NADE model in non-flat FRW universe

We consider the Friedmann–Robertson–Walker (FRW) metric for the non-flat universe,

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \]

where \( a \) is the cosmic scale factor and \( k = 0, 1, -1 \) represent a flat, closed and open FRW universe, respectively. Observational evidence supports the existence of a closed universe with a small positive curvature (\( \Omega_k \sim 0.02 \)) (Bennett et al. 2003; Spergel 2003; Tegmark et al. 2004; Seljak et al. 2006; Spergel et al. 2007). Besides, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large (Huang and Li 2004). It is still possible that there is a contribution to the Friedmann equation from the spatial curvature when studying the late universe, though much smaller than other energy components according to observations.

For the non-flat FRW universe containing the DE and DM, the first Friedmann equation has the following form:

\[ H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} \left( \rho_\Lambda + \rho_m \right), \]

where \( \rho_\Lambda \) and \( \rho_m \) are the energy density of DE and DM, respectively. Let us define the dimensionless energy densities as

\[ \Omega_m = \frac{\rho_m}{\rho_{ct}} = \frac{\rho_m}{3M_p^2 H^2}, \]

\[ \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{ct}} = \frac{\rho_\Lambda}{3M_p^2 H^2}, \]

\[ \Omega_k = \frac{k}{a^2 H^2}, \]

then the first Friedmann equation yields

\[ \Omega_m + \Omega_\Lambda = 1 + \Omega_k. \]  

Following Sheykhi (2009a), the energy density of the NADE is given by

\[ \rho_\Lambda = \frac{3n^2 M_p^2}{\eta^2}, \]

where the numerical factor \( 3n^2 \) is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime (since the energy density is derived for Minkowski spacetime), and so on. The astronomical data for the NADE gives the best-fit value (with 1σ uncertainty) \( n = 2.716^{+0.111}_{-0.099} \) (Wei and Cai 2008b).

It was found that the coincidence problem could be solved naturally in the NADE model provided that the single model parameter \( n \) is of order unity (Wei and Cai 2008b). Also \( \eta \) is the conformal time of the FRW universe, and it is given by

\[ \eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha}. \]

Note that in the energy density of the OADE model, the age of the universe appears in (5) instead of \( \eta \). This causes some difficulties. In particular, it fails to describe the matter-dominated epoch properly (Cai 2007). The DE density (5) has the same form as the holographic DE, but the conformal time stands instead of the future event horizon distance of the universe. Thus the causality problem in the holographic DE is avoided. This is because the existence of the future event horizon requires an eternal accelerated expansion of the universe (Wei and Cai 2008a).

From the definition \( \rho_\Lambda = 3M_p^2 H^2 \Omega_\Lambda \), we get

\[ \eta = \frac{n}{H \sqrt{\Omega_\Lambda}}. \]

We consider a universe containing an interacting NADE density \( \rho_\Lambda \) and the cold dark matter (CDM), with \( \omega_m = 0 \). The energy equations for NADE and CDM are

\[ \dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \]

\[ \dot{\rho}_m + 3H\rho_m = Q, \]

where following Kim et al. (2006), we choose \( Q = \Gamma \rho_\Lambda \) as an interaction term and \( \Gamma = 3b^2 H (1 + \Omega_\Lambda \rho_\Lambda / \rho_m) \) is the decay rate of the NADE component into CDM with a coupling constant \( b \). Although this expression for the interaction term may look purely phenomenological, different Lagrangians have been proposed in support of it (Tsujikawa and Sami 2004). The choice of the interaction between both components was to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of DE and DM becomes a constant (Hu and Ling 2006). Note that choosing \( H \) in the \( Q \)-term is motivated purely by mathematical simplicity. Because from the