The analysis of the high-precision light rangefinder CD-1200 optical path

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Abstract. The scheme of a high-precision laser rangefinder with a crystal light modulator, which has the ability to work both on the compensation and on the two-phase mode of the linear measurements modulating method, is considered. The two-phase method is implemented by introducing another electro-optical crystal EOC in the receiving path, rotated 90º relative to the first one around the optical Z axis. In this case, a second signal is formed in the optical channel of the light rangefinder, shifted relative to the first signal by 180º. The measurement of the residual phase cycle occurs when the periodically received signals are equal. A theoretical analysis of the proposed scheme is carried out and in this case it is shown that, the receiving points fixing sensitivity increases, leading to an increase in the accuracy of linear measurements in 3-4 times, estimated by the phase determination error value of $m\phi=0.02-0.03$ mm.

1. Introduction
At the present stage, the role of engineering-construction and geodetic works on such special objects as Comparators, reference lines on tectonic plate faults, large space antennas, storage of charged particles, etc. is increasing.

Performing the high-precision linear measurements using the high-precision laser rangefinders and refractometers is one of the main tasks of modern engineering geodesy. Consequently, the development and production of the high-precision laser rangefinders and refractometers with a relative measurement error of no more than $3\times10^{-7}$ becomes an urgent scientific and technical problem [1,2].

In foreign developments of the high-precision light rangefinders based on the compensation method of electro-optical light modulation, an increase in the measurement accuracy is realized by introducing a shift of the demodulated light minimum position and by a small deviation of the modulation frequency in the Mekometer ME-5000 and Geomensor GR-204 [3,4] at 500 MHz modulation frequencies, the error of determining the residual part of the phase cycle equal to $m\phi=0.25$ mm is significantly reduced.

The use of these light rangefinders in various linear measurements and in the study of crustal deformation [5,6] and the studies’ results of DVCD-1200 by the specialists of Dresden Technical University [7] and Central Research Institute of Geodesy and Cartography [8] confirm that the compensation method at a modulation frequency of 500 MHz by fixing the position of the light minimum provided the measurement accuracy of $m\phi = 0.4-0.5$ mm, and on 1200 MHz $m\phi=0.2-0.25$ mm.
The presented work considers the possibility of constructing a light rangefinder based on the real results of measurements carried out in the accelerator using the CD-1200 model developed in the problem laboratory of geodetic measurements of National University of Architecture and Construction of Armenia, when the error in determining the phase was \( m_\phi = 0.035 \text{ mm} \) [9]. The basis of the new light rangefinder is based on identical modulation-demodulation of laser radiation in an electro-optical modulator on EOC crystals and on the two-phase method of the phase measurements, and periodically incoming signals are formed in the optical channel with a phase shift of 180° by introducing an additional crystal into the receiving path, rotated relative to the first one by 90° around the optical Z axis.

2. Methodology

Let us consider a block diagram of a light rangefinder with the light crystal modulator-demodulator (modem) that can work both by the compensation method of extremum and by the two-phase method.

![Block diagram of a light rangefinder](image)

**Figure 1.** The light meter block diagram with the ability to work with compensating and two-phase method

In the microwave range, the two-phase method can be implemented, for example, by introducing another crystal in the receiving path, rotated relative to the first one by 90° around the Z axis. The block diagram of a light rangefinder with a crystal oriented \( \theta' = 90^\circ \) in the receiving path is shown in Figure 1 and works as follows. Linear-polarized laser radiation from LGN-207A passes through a crystal with an orientation \( \theta' = 0^\circ \), modulated by polarization and, reflecting from the reflector, passes through two crystals A and B. The crystal B is rotated by 90° relative to A and they are glued with nontransparent glue. At the same polarity of the electric field on crystals A and B, the intensity of the rays falling on the photomultiplier tube (PMT) is different. The intensity of these rays after the analyzer changes depending on the position of the reflector according to the law are shown in the graph Figure 2. As can be seen from the graph, the same values of the intensities of the rays M and N falling on the PMT correspond to the maximum steepness of the curves M and N, and the equal intensities periodicity is \( \lambda/4 \).
This paper considers a mathematical solution to this problem. To study a light rangefinder with the light crystal modulator-demodulator constructed according to the functional scheme shown in Fig. 1, consider two cases:

1. Direct and reflected light rays are passed through the same Z-cut crystal located in the microwave field of the resonator. The power lines of the microwave field are parallel to the Z axis of the crystal and the light flux direction, Figure 3.

2. Direct and reflected light rays are passed through the EOC crystals, one of which is rotated 90° relative to the other Z-axis, Figure 4.

In the diagram in Figure 3 in order to avoid the influence of side reflections on the receiving path, the direct and reflected rays are spatially separated, which is almost easily carried out in one crystal due to the use of a laser beam with a diameter of 1-2 mm.

To the positive features of the considered scheme (Figure 3) it should be attributed to its simplicity and the presence of almost complete identity of conditions for both direct (modulation channel) and reflected (demodulation channel) rays. The scheme is designed for the measurements by the extreme compensation method, when the minimum light flux is fixed at the output of the analyzer, the polarization plane of which is perpendicular to the direction of the laser beam polarization.
Figure 4. Direct and reflected light rays pass through the EOC crystals that are rotated 90º degrees relative to each other.

The arrangement of the crystals in Fig. 4, leads to the fact that in the diagram Fig.4 compared to the diagram in Fig.3 the minimum intensity of the beam at the output of the analyzer is shifted by the value $\lambda/4$. This circumstance allows using both measurement schemes simultaneously to implement the two-phase method for measurements with rangefinders and improve the accuracy of phase measurements.

We will conduct a theoretical analysis of the above schemes.

The coordinate system X, Y is chosen so that the X axis is 45º with the direction of laser polarization. Induced by a modulating electric field applied along the Z-axis, the X'-axis and Y'-axis of the EOC crystal coincide with the accepted X, Y-axes of the coordinate system, and the main X$_{cr}$ and Y$_{cr}$ axes make up an angle of 45º with them. The maximum transmission axis of the analyzer is also 45º with the coordinate system axes, Fig.5.

If the intensity of a plane-polarized ray that falls on a crystal is equal to $I_0$, then the components of the electric field vector of the light wave in the directions X and Y will be respectively

$$E_X = \sqrt{I_0}/2, \quad E_Y = -\sqrt{I_0}/2.$$  \hspace{1cm} (1)

The calculation of the light wave incident on the photodetector after passing through the various elements of the light rangefinder optical system can be performed using the Jones matrix method \cite{10}. The resulting matrices for the schemes Fig. 3 and Fig. 4 are written as \cite{11}

$$M_{ip} = M_a \cdot M_{cr} \cdot M_{ref} \cdot M_{cr} \cdot M_l;$$  \hspace{1cm} (2)

$$M_{ip} = M_a \cdot M'_{cr} \cdot M_{ot} \cdot M_{cr} \cdot M_c,$$  \hspace{1cm} (3)

where $M_{ref}$ - denotes the Jones matrix for the reflector.

Figure 5. The functional scheme for the external modulation implementation.
\[ M_{\text{cr}} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix}, \]  

(4)

Where “a” indicates the ratio of the light intensity at the crystal output to the light intensity returning to the crystal after passing the measured distance D.

To obtain the effective modulation, the laser beam polarization must be parallel to the X_{cr} or Y_{cr} axes, i.e. \( \theta' = 0 \). The axis of the analyzer with the coordinate axes should be 45º \( (\theta = 90^\circ) \). In this case, the \( M_a \), \( M_{\text{cr}} \), and \( M'_{\text{cr}} \) matrices are simplified and written as

\[ M_{\text{cr}} = \begin{bmatrix} e^{iG_{1/2}} & 0 \\ 0 & e^{-iG_{1/2}} \end{bmatrix}, \quad M'_{\text{cr}} = \begin{bmatrix} e^{-iG_{1/2}} & 0 \\ 0 & e^{iG_{1/2}} \end{bmatrix}, \quad M_a = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \]  

(5)

Taking into account the optical component matrices, the expressions (2) and (3) can be written as follows:

\[ M_{0^\circ} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{i\Gamma_{1/2}} & 0 \\ 0 & e^{-i\Gamma_{1/2}} \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} e^{iG_{1/2}} & 0 \\ 0 & e^{-iG_{1/2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix}, \]  

(6)

\[ M_{90^\circ} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-iG_{1/2}} & 0 \\ 0 & e^{iG_{1/2}} \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} e^{iG_{1/2}} & 0 \\ 0 & e^{-iG_{1/2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix}. \]  

(7)

After the calculations for the components \( E_x \) and \( E_y \) of the output light, we get the expressions that are the same in amplitude, but differ in phase

\[ \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{0^\circ} = a \frac{1}{2} \sqrt{\frac{1}{2}} \begin{bmatrix} 2 \cos \frac{G_2 + G_1}{2} & 0 \\ 2 \cos \frac{G_2 - G_1}{2} & 0 \end{bmatrix}. \]  

(8)

\[ \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{90^\circ} = a \frac{1}{2} \sqrt{\frac{1}{2}} \begin{bmatrix} 2 \sin \frac{G_2 + G_1}{2} & 0 \\ 2 \sin \frac{G_2 - G_1}{2} & 0 \end{bmatrix}. \]  

(9)

The light intensity falling on the PMT is equal to the absolute value of the angles \( E_x \) and \( E_y \) sum.

\[ I_{\theta' = 0^\circ} = |E_x + E_y| = 2E^2 = a^2 I_0 \sin^2 \frac{G_2 + G_1}{2}, \]  

(10)

\[ I_{\theta' = 90^\circ} = |E_x^2 + E_y^2| = 2E^2 = a^2 I_0 \sin^2 \frac{G_2 - G_1}{2}. \]  

(11)

If the modulating voltage is the harmonic time function of \( U = U_0 \sin \omega_\text{t} \) then for \( G_1 \) and \( G_2 \) we receive

\[ G_1 = \pi \frac{U_0}{\omega_\text{t}} \sin \omega_\text{t} t, \quad G_2 = \pi \frac{U_0}{\omega_\text{t}} \sin (\omega_\text{t} t + \varphi), \]  

(12)

where \( \varphi = \frac{4\pi D}{\lambda_\text{m}} \), \( D \) is the measured distance.

Substituting the values of \( G_1 \) and \( G_2 \) in the expressions (10) and (11) and performing the transformations, we get

\[ I_{\theta' = 0^\circ} = a^2 I_0 \sin^2 \left[ \pi \frac{U_0}{\omega_\text{t}} \sin \left( \omega_\text{t} t + \frac{\varphi}{2} \right) \cos \frac{\varphi}{2} \right], \]  

(13)

\[ I_{\theta' = 90^\circ} = a^2 I_0 \sin^2 \left[ \pi \frac{U_0}{\omega_\text{t}} \cos \left( \omega_\text{t} t + \frac{\varphi}{2} \right) \sin \frac{\varphi}{2} \right]. \]  

(14)

In the expressions (13) and (14), the light intensity falling on the photodetector changes with time, the average intensity value is determined by the integrating expressions (13) and (14) over the modulation period

\[ I_{\theta' = 0^\circ} = \frac{a^2 I_0}{\pi} \int_0^\pi \sin^2 \left[ \pi \frac{U_0}{\omega_\text{t}} \sin \left( \omega_\text{t} t + \frac{\varphi}{2} \right) \cos \frac{\varphi}{2} \right] dt, \]  

(15)
The following expressions are used for the integration:

\[ \cos(A \sin \varphi) = J_0(A) + 2 \sum_{n=1}^{\infty} J_{2n}(A) \cos 2n \varphi, \quad \cos(A \cos \varphi) = J_0(A) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(A) \cos 2n \varphi, \]

where \( J_n \) denotes the Bessel function of the first kind of \( n \)-th order.

For the diagrams in Fig.3 and Fig.4, the integration results are different:

\[ I_{\theta'=0^\circ} = \frac{a^2 i_0}{2} \left[ 1 - J_0 \left( 2 \pi \frac{U}{U_{\pi}} \cdot \cos \frac{\varphi}{2} \right) \right], \quad (17) \]

\[ I_{\theta'=90^\circ} = \frac{a^2 i_0}{2} \left[ 1 - J_0 \left( 2 \pi \frac{U}{U_{\pi}} \cdot \sin \frac{\varphi}{2} \right) \right]. \quad (18) \]

The resulting expressions have minimums that depend on the distance \( D \), since \( \frac{\varphi}{2} = \frac{2\pi D}{\lambda_m} \). The Bessel function \( J_0(A) \) is maximal at \( A=0 \), and \( J_0 \) decreases with increasing \( A \). The minimum value for \( I_{\theta=0^\circ} \) will occur when \( \cos \varphi/2=0 \), at the time when

\[ 2\pi D / \lambda_m = (N + 1/2)\pi; \quad D_{\theta=0^\circ} = N \lambda_m / 2 + \lambda_m / 4. \quad (19) \]

Similarly, we get

\[ D_{\theta=90^\circ} = N \lambda_m / 2, \quad (20) \]

Where \( N=1,2...n \) is the number of the modulation waves half-lengths in the measured distance.

3. Results

The resulting expressions allow us to conclude:

1. The Minimum output currents of the PMT for the circuit in Fig.4 are separated by a value of \( \lambda_m / 4 \) as a result of the rotation of one of the crystals relative to the other by a value of \( \theta' = \pi / 2; \)

2. At low modulation levels (\( U < U_z \)), the output light intensity changes sinusoidal with the change of \( D \), as the modulating voltage increases, the minima become more acute, and the maxima are widening or double (Fig. 6).

3. If there is an additional phase shift \( \rho \) caused by, for example, the natural anisotropy of the crystal and other factors, the light intensity at the output path decreases by the law:

\[ I_{\theta'=\rho^\circ} = \frac{a^2 i_0}{2} \left[ 1 - \cos \rho J_0 \left( 2 \pi \frac{U}{U_{\pi}} \cdot \cos \frac{2\pi D}{2} \right) \right], \quad (21) \]

Where \( a \) is the light intensity loss in the measured distance.

The presence of an additional phase shift \( \rho \) that occurs along the path of light does not shift the position of the minimum, but only expands it, Figure 7.
Figure 6. Dependence of $I/I_0$ on the distance for different power modes of the light

![Figure 6](image)

Figure 7. Demodulation characteristic in the presence of depolarization $p$

If we combine the diagrams in Fig. 4 and Fig. 5 in one scheme so that on the PMT is fed rays from both a crystal with an azimuth of 0º and a crystal rotated by 90º, we get a rangefinder scheme that works both by the extreme method and the two-phase method. As it can be seen from the expressions (13) and (14), the equality of amplitudes, i.e. the light intensities signals coming to the PMT, occurs under the condition $\cos 2\pi D/\lambda_m = \sin 2\pi D/\lambda_m$, which take place when

$$2\pi D/\lambda_m = (N + 1/2)\frac{\pi}{2}$$

(22)

From the condition (22), we obtain a formula for determining the distance for a rangefinder operating in two-phase mode

$$D_{\theta=0^\circ} = N \lambda_m/4 + \lambda_m/8.$$  

(23)

4. Summary

The above expression (23) shows that the periodicity of the points with the same amplitudes occurs at a distance of $\lambda_m/4$ instead of $\lambda_m/2$. This feature makes it possible to reduce the travel length of the optical delay line by half.

The arrangement of the crystals in Fig. 4, leads to the fact that in the diagram Fig.4 compared to the diagram in Fig.3 the minimum intensity of the beam at the output of the analyzer is shifted by the value $\lambda/4$. This circumstance allows using both the measurement schemes simultaneously to implement the two-phase method of the rangefinders measurements and increase the phase measurements accuracy by 3-4 times.

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