We study the coupled $\Lambda\Lambda nn - \Xi^- pnn$ system to check whether the inclusion of channel coupling is able to bind the $\Lambda\Lambda nn$ system. We use a separable potential three-body model of the coupled $\Lambda\Lambda nn - \Xi^- pnn$ system as well as a variational four-body calculation with realistic interactions. Our results exclude the possibility of a $\Lambda\Lambda nn$ bound state by a large margin. However, we have found a $\Xi^- t$ quasibound state above the $\Lambda\Lambda nn$ threshold.

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I. INTRODUCTION

Bound states of two neutrons and two Λ hyperons are a controversial subject. Recently, Bleser et al. [1] have offered a new interpretation of the results of the BNL AGS-E906 experiment to produce and study double hypernuclei through a \((K^-, K^+)\) reaction on \(^9\text{Be}\) [2]. Following a suggestion made by Avraham Gal, they explored the conjecture that decays of a \(^4\Lambda\Lambda\) double hypernucleus may be responsible for some of the observed structures in the correlated \(\pi^-\pi^-\) momenta. However, in a recent calculation using the stochastic variational method in a pionless effective field theory approach [3], it has been concluded that the \(\Lambda\Lambda nn\) system is unbound by a large margin. We had previously come to the identical conclusion [4] in a study of the uncoupled \(\Lambda\Lambda nn\) system using local central Yukawa-type Malfliet-Tjon interactions reproducing the low-energy parameters and phase shifts of the \(nn\) system and the latest updates of the \(n\Lambda\) and \(\Lambda\Lambda\) Nijmegen ESC08c potentials. It is important to notice that in order to create a \(\Lambda\Lambda nn\) bound state the four particles must coincide simultaneously since the system does not contain two- or three-body subsystem bound states, so that the probability of the event occurring is rather small.

In this work we take the calculation one step further by considering the coupled \(\Lambda\Lambda nn - \Xi^-pn\) system to check if the inclusion of channel coupling is able to bind the \(\Lambda\Lambda nn\) system. If this were not the case, we will study whether there could be a \(\Xi^-pn\) sharp resonance or quasibound state above the \(\Lambda\Lambda nn\) threshold. In the \(\Lambda\Lambda nn - \Xi^-pn\) system, the effect of channel coupling arises from the process \(\Lambda\Lambda \rightarrow \Xi N\) in the two-body channel \((i, j) = (0, 0)\). The channel \(\Xi N\) can be realized in two ways, \(\Xi^0n\) or \(\Xi^-p\); however, if one restricts the calculation to \(S\) waves, the subchannel \(\Xi^0n\) can not contribute since one can not have three nucleons with a symmetric space wave function. Thus, only the subchannel \(\Xi^-p\) will contribute.\(^1\)

We present two different approaches. First, we address a three-body model \(\Lambda\Lambda(nn) - \Xi^-pn\)
Ξ−p(nn), where the dineutron (nn) is treated as a particle of isospin 1 and spin 0, and all
the two-body interactions are assumed to be simple Yamaguchi separable potentials. This
allows us to search for solutions both in the real axis, bound states, and in the complex plane,
resonances and quasibound states. Later, we perform a variational four-body calculation
with realistic local two-body interactions which necessarily will be restricted to energies in
the real axis.

II. THE ΛΛ(nn) − Ξ−p(nn) THREE-BODY MODEL

In this model we treat the dineutron (nn) as an elementary particle with mass \(m_{nn} = 2m_n\), isospin 1, and spin 0 with two-body interactions given by Yamaguchi separable potentials \[5\]. It is based on the model proposed in Ref. \[6\] to search for resonances of the
\(\Lambda\Lambda N − \Xi NN\) system. If one of the nucleons in the lower and upper channels is replaced
by a dineutron, \(N \rightarrow (nn)\), the equations of Ref. \[6\] are similar to those of this work. The
differences originate from the fact that in the \(\Lambda\Lambda N − \Xi NN\) system two of the three particles
in the upper channel are identical while in the \(\Lambda\Lambda(nn) − \Xi−p(nn)\) system the three particles
in the upper channel are different.

A. Three-body equations

We take the dineutron (nn) as particle 1. In the lower channel the two \(\Lambda\)'s are particles 2
and 3 while in the upper channel particles 2 and 3 are the \(\Xi−\) and \(p\), respectively. Following
the graphical method of Ref. \[7\] the equations of the \(\Lambda\Lambda(nn) − \Xi−p(nn)\) system are,

\[
\langle 1|T_1 \rangle = 2\langle 1|t^{\Lambda\Lambda}_1|1\rangle\langle 1|3\rangle G_0(3)\langle 3|T_3 \rangle + \langle 1|t^{\Lambda\Lambda−\Xi−p}_1|1\rangle\langle 1|2\rangle G_0(2)\langle 2|U_2 \rangle + \langle 1|t^{\Lambda\Lambda−\Xi−p}_1|1\rangle\langle 1|3\rangle G_0(3)\langle 3|U_3 \rangle,
\]

\[
\langle 3|T_3 \rangle = -(3|t^{(nn)\Lambda}_3|3\rangle\langle 2|3\rangle G_0(3)\langle 3|T_3 \rangle + \langle 3|t^{(nn)\Lambda}_3|3\rangle\langle 3|1\rangle G_0(1)\langle 1|T_1 \rangle,
\]

\[
\langle 1|U_1 \rangle = \langle 1|t^{\Xi−p}_1|1\rangle\langle 1|2\rangle G_0(2)\langle 2|U_2 \rangle + \langle 1|t^{\Xi−p}_1|1\rangle\langle 1|3\rangle G_0(3)\langle 3|U_3 \rangle + 2\langle 1|t^{\Xi−p−\Lambda\Lambda}_1|1\rangle\langle 1|3\rangle G_0(3)\langle 3|T_3 \rangle,
\]

\[
\langle 2|U_2 \rangle = \langle 2|t^{(nn)p}_2|2\rangle\langle 2|3\rangle G_0(3)\langle 3|U_3 \rangle + \langle 2|t^{(nn)p}_2|2\rangle\langle 2|1\rangle G_0(1)\langle 1|U_1 \rangle,
\]

\[
\langle 3|U_3 \rangle = \langle 3|t^{(nn)\Xi−}_3|3\rangle\langle 3|2\rangle G_0(2)\langle 2|U_2 \rangle + \langle 3|t^{(nn)\Xi−}_3|3\rangle\langle 3|1\rangle G_0(1)\langle 1|U_1 \rangle. \quad (1)
\]
For all the uncoupled interactions we assume separable potentials of the form,
\[ V^\rho_i = g^\rho_i \rangle \lambda^\rho_i \langle g^\rho_i , \]  
(2)
such that the two-body \( t \)-matrices are,
\[ t^\rho_i = g^\rho_i \rangle \tau^\rho_i \langle g^\rho_i , \]  
(3)
with
\[ \tau^\rho_i = \frac{1}{(\lambda^\rho_i)^{-1} - \langle g^\rho_i | G_0(i) | g^\rho_i \rangle} . \]  
(4)
In the case of the two-body channel responsible for the channel coupling, \((i, j) = (0, 0)\), we use a separable interaction of the form,
\[ V^{\rho\sigma}_1 = g^{\rho\sigma}_1 \rangle \lambda^{\rho\sigma}_1 \langle g^{\rho\sigma}_1 , \]  
(5)
such that
\[ t^{\rho\sigma}_1 = g^{\rho\sigma}_1 \rangle \tau^{\rho\sigma}_1 \langle g^{\rho\sigma}_1 , \]  
(6)
with
\[ \tau^{\rho\sigma}_1 = g^{\rho\sigma}_1 \rangle \tau^{\rho\sigma}_1 \langle g^{\rho\sigma}_1 , \]  
(7)
and
\[ G^{\Lambda\Lambda} = \langle g^{\Lambda\Lambda}_1 | G_0 | g^{\Lambda\Lambda}_1 \rangle , \]  
\[ G^{\Xi^- p} = \langle g^{\Xi^- p}_1 | G_0 | g^{\Xi^- p}_1 \rangle , \]  
(8)
where for simplicity we have redefined \( \tau^{\Lambda\Lambda}_1 \equiv \tau^{\Lambda\Lambda-\Xi^- p}_1 , \lambda^{\Lambda\Lambda}_1 \equiv \lambda^{\Lambda\Lambda-\Lambda\Lambda}_1 , \) etc.

Using Eqs. (3) and (6) into the integral equations (1) and introducing the transformations \( \langle i | T_i \rangle = \langle i | g^{\alpha i}_i \rangle \langle i | X_i \rangle \) and \( \langle i | U_i \rangle = \langle i | g^{\beta i}_i \rangle \langle i | Y_i \rangle \), one obtains the one-dimensional integral
Eqs. (9) can be extended into the complex energy plane following the method of Ref. [8]. Thus, for each uncoupled two-body channel we have to fit the two parameters equations.

\[ \langle 1|X_1 \rangle = 2\tau_1^{\Lambda\Lambda} \langle g_1^{\Lambda\Lambda}|1\rangle \langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)\Lambda}\rangle \langle 3|X_3 \rangle 
\]
\[ + \tau_1^{\Lambda\Lambda-E^-p} \langle g_1^{E^-p}|1\rangle \langle 1|2\rangle G_0(2)\langle 2|g_2^{(nn)p}\rangle \langle 2|Y_2 \rangle 
\]
\[ + \tau_1^{\Lambda\Lambda-E^-p} \langle g_1^{E^-p}|1\rangle \langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)E^-}\rangle \langle 3|Y_3 \rangle , \]
\[ \langle 3|X_3 \rangle = -\tau_3^{(nn)\Lambda} \langle g_3^{(nn)\Lambda}|3\rangle \langle 2|3\rangle G_0(3)\langle 3|g_3^{(nn)\Lambda}\rangle \langle 3|X_3 \rangle 
\]
\[ + \tau_3^{(nn)\Lambda} \langle g_3^{(nn)\Lambda}|3\rangle \langle 3|1\rangle G_0(1)\langle 1|g_1^{\Lambda\Lambda}\rangle \langle 1|X_1 \rangle , \]
\[ \langle 1|Y_1 \rangle = \tau_1^{E^-p} \langle g_1^{E^-p}|1\rangle \langle 1|2\rangle G_0(2)\langle 2|g_2^{(nn)p}\rangle \langle 2|Y_2 \rangle 
\]
\[ + \tau_1^{E^-p} \langle g_1^{E^-p}|1\rangle \langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)E^-}\rangle \langle 3|Y_3 \rangle 
\]
\[ + 2\tau_1^{E^-p-\Lambda\Lambda} \langle g_1^{\Lambda\Lambda}|1\rangle \langle 1|3\rangle G_0(3)\langle 3|g_3^{(nn)\Lambda}\rangle \langle 3|X_3 \rangle , \]
\[ \langle 2|Y_2 \rangle = \tau_2^{(nn)p} \langle g_2^{(nn)p}|2\rangle \langle 2|3\rangle G_0(3)\langle 3|g_3^{(nn)E^-}\rangle \langle 3|Y_3 \rangle 
\]
\[ + \tau_2^{(nn)p} \langle g_2^{(nn)p}|2\rangle \langle 2|1\rangle G_0(1)\langle 1|g_1^{E^-p}\rangle \langle 1|Y_1 \rangle , \]
\[ \langle 3|Y_3 \rangle = \tau_3^{(nn)E^-} \langle g_3^{(nn)E^-}|3\rangle \langle 3|2\rangle G_0(2)\langle 2|g_2^{(nn)p}\rangle \langle 2|Y_2 \rangle 
\]
\[ + \tau_3^{(nn)E^-} \langle g_3^{(nn)E^-}|3\rangle \langle 3|1\rangle G_0(1)\langle 1|g_1^{E^-p}\rangle \langle 1|Y_1 \rangle . \]

\[ (9) \]

Eqs. (9) can be extended into the complex energy plane following the method of Ref. [8].

### B. Two-body inputs

The \( \Xi^- t \rightarrow \Lambda nn \) process occurs with quantum numbers \( (I, J) = (1, 0) \) so that, since we restrict our calculation to \( S \) waves, the contributing two-body channels in our three-body model are: the \( (nn)p \) channel \( (i, j) = (1/2, 1/2) \), the \( (nn)\Lambda \) channel \( (i, j) = (1, 1/2) \), the \( (nn)\Xi^- \) channel \( (i, j) = (3/2, 1/2) \), and the \( \Lambda\Lambda - \Xi^- p \) channel \( (i, j) = (0, 0) \).

We use Yamaguchi form factors for the separable potentials of Eqs. (2) and (5), i.e.,

\[ g(p) = \frac{1}{\alpha^2 + p^2}. \]

\[ (10) \]

Thus, for each uncoupled two-body channel we have to fit the two parameters \( \alpha \) and \( \lambda \).

In the case of the \( (nn)p \) subsystem with quantum numbers \( (i, j) = (1/2, 1/2) \), the tritium channel, for a given value of the range \( \alpha \) the tritium binding energy (8.48 MeV) determines the strength \( \lambda \) through Eq. (11) as,

\[ \lambda = \frac{1}{\langle g|G_0(E_B)|g \rangle} . \]

\[ (11) \]
while the value of $\alpha$ is determined from the binding energy of $^4\text{He}$ (28.2 MeV) through the solution of the three-body system ($nnpp$). The parameters of this model are given in Table I.

In the case of the ($nnp$) subsystem with quantum numbers $(i,j) = (1/2,1/2)$, we fit the two parameters of the interaction to the ground state and spin-excitation energies of the $^4\Lambda\text{H}$ hypernucleus. It is considered as a three-body system ($nnp\Lambda$) with quantum numbers $(I,J) = (1/2,0)$. For the ($nnp$) subsystem we use the interaction previously described and for the $p\Lambda$ the separable potentials for $j = 0$ and $j = 1$ constructed in Ref. [6]. Thus, for a given value of the range $\alpha$, we fit the strength $\lambda$ to the binding energy of $^4\Lambda\text{H}$ (10.52 MeV) [9]. In order to obtain the range $\alpha$ we calculate the binding energy of the excited state $(I,J) = (1/2,1)$ (9.43 MeV) [9], obtaining for $\alpha = 1$, 2, and 3 fm$^{-1}$ the values 9.93, 9.81 and 9.77 MeV, respectively, which are labeled as models 1, 2, and 3 in Table I. As it is well known, the $^4\Lambda\text{H}$ spin excitation is difficult to fit since it depends strongly on the tensor force arising from the transition $\Lambda N - \Sigma N$ [9–12]. Therefore, we did not consider larger values of $\alpha$.

In the case of the ($nn\Xi^-$) subsystem with quantum numbers $(i,j) = (3/2,1/2)$, we do not have any experimental information available to calibrate our separable potential model. However, in a couple of recent calculations [13, 14] based in the strangeness $-2$ Nijmegen ESC08c potential [15] a bound state is predicted with a binding energy of 2.89 MeV below the $\Xi NN$ threshold. Thus, we have used this result to obtain the strength $\lambda$ of the separable

**TABLE I: Parameters of the different separable potential models for the uncoupled partial waves:** $\alpha$ (in fm$^{-1}$) and $\lambda$ (in fm$^{-2}$).

| Model | Subsystem | $(i,j)$      | $\alpha$ | $\lambda$  |
|-------|-----------|--------------|----------|------------|
| 1     | ($nn)p$   | (1/2,1/2)   | 1.07     | -0.5444    |
|       | ($nn)\Lambda$ | (1,1/2)     | 1.0      | -0.1655    |
|       | ($nn)\Xi^-$ | (3/2,1/2)   | 1.0      | -0.2904    |
| 2     | ($nn)\Lambda$ | (1,1/2)     | 2.0      | -1.1560    |
|       | ($nn)\Xi^-$ | (3/2,1/2)   | 2.0      | -1.7719    |
| 3     | ($nn)\Xi^-$ | (3/2,1/2)   | 3.0      | -5.4162    |
TABLE II: Parameters of the two separable potential models for the coupled partial wave \((i, j) = (0, 0)\): \(\alpha_1^{AA}, \alpha_1^{\Xi N}\) (in fm\(^{-1}\)), \(\lambda_1^{AA}, \lambda_1^{\Xi N}\), and \(\lambda_1^{AA-\Xi N}\) (in fm\(^{-2}\)).

| Model | \(\alpha_1^{AA}\) | \(\lambda_1^{AA}\) | \(\alpha_1^{\Xi N}\) | \(\lambda_1^{\Xi N}\) | \(\lambda_1^{AA-\Xi N}\) |
|-------|-------------------|-------------------|-------------------|-------------------|-------------------|
| A     | 1.3465            | -0.1390           | 1.1460            | -0.3867           | 0.0977            |
| B     | 1.25              | -0.0959           | 4.287             | 1.302             | 1.243             |

potential using Eq. (11) and taking the range \(\alpha\) equal to that of the \((nn)\Lambda\) subsystem. We give in Table III the parameters corresponding to the different models 1, 2, and 3.

In the case of the coupled \(\Lambda\Lambda - \Xi^- p\) subsystem first we use a recent lattice QCD study by the HAL QCD Collaboration \[16\] with almost physical quark masses \((m_\pi = 146\) MeV and \(m_K = 525\) MeV). In this model the \(H\) dibaryon was calculated through the coupled channel \(\Lambda\Lambda - \Xi N\) system, appearing as a very sharp resonance just below the \(\Xi N\) threshold \[16, 17\].

We have constructed a model, labeled as A, giving similar \(\Lambda\Lambda\) and \(\Xi N\) phase shifts as those of Ref. \[16\]. The parameters of this model are given in Table II. Besides, we have also considered the separable potential model of the \(\Lambda\Lambda - \Xi N\) system constructed in Ref. \[6\] which is based in the Nijmegen ESC08c potential \[15\]. This model is shown in Table III as model B. Of course, in the \(\Lambda\Lambda(nn) - \Xi^- p(nn)\) calculation we use the parameters \(\lambda_1^{\Lambda\Lambda-\Xi^- p} = \lambda_1^{\Lambda\Lambda-\Xi N}/\sqrt{2}\) and \(\lambda_1^{\Xi^- p} = \lambda_1^{\Xi N}/2\).

C. Results

We show in Table III the energy eigenvalue of the two models A–B of the coupled \(\Lambda\Lambda - \Xi N\) system and the three models 1–3 of the \((nn)\Lambda\) and \((nn)\Xi^-\) systems. We also give in parentheses the energy of the uncoupled \(\Xi^- t\) system. As one can see from this table,

| Model | 1 | 2 | 3 |
|-------|---|---|---|
| A     | \(-12.80 - i 0.05\) \((-12.73)\) | \(-13.46 - i 0.04\) \((-13.37)\) | \(-13.52 - i 0.04\) \((-13.43)\) |
| B     | \(-10.99 - i 0.06\) \((-10.92)\) | \(-11.04 - i 0.07\) \((-10.93)\) | \(-10.89 - i 0.07\) \((-10.77)\) |
the real part of the energy eigenvalue is slightly below the energy of the uncoupled \( \Xi^{-}t \) system and the imaginary part of the energy eigenvalue is roughly the difference between the uncoupled energy and the real part of the energy eigenvalue. Thus, this state appears as a narrow \( \Xi^{-}t \) quasibound state decaying to \( \Lambda\Lambda nn \). The reason for the narrow width of the \( \Xi^{-}t \) state stems from the weakness of the \( \Lambda\Lambda - \Xi N \) transition potential [15, 16], that on the other hand is also responsible for the \( H \) dibaryon appearing as a very sharp resonance just below the \( \Xi N \) threshold [17].

Finally, we give in Table IV the corresponding values of the \( \Xi^{-}t \) scattering lengths of the two models A−B which may be of use in the calculation of the energy shift of the atomic levels of the \( \Xi^{-}t \) atom.

### III. THE \( \Lambda\Lambda nn \) AND \( \Xi^{-}pnn \) FOUR-BODY PROBLEMS

#### A. Four-body calculation

The four-body problem has been addressed by means of a Generalized Gaussian Variational (GGV) method [18, 19]. The nonrelativistic Hamiltonian is given by,

\[
H = \sum_{i=1}^{4} \frac{\vec{p}_i^2}{2m_i} + \sum_{i<j=1}^{4} V_{ij}(\vec{r}_{ij}),
\]

where \( V(\vec{r}_{ij}) \) is a local central two-body potential.

The four-body wave function is taken to be a sum over all allowed channels with well-defined symmetry properties:

\[
\psi(\vec{x}, \vec{y}, \vec{z}) = \sum_{\kappa=1}^{s} \chi_{\kappa}^{SI} R_{\kappa}(\vec{x}, \vec{y}, \vec{z}),
\]

where \( s \) is the number of channels allowed by the Pauli principle. \( \vec{x} = \vec{r}_1 - \vec{r}_2, \, \vec{y} = \vec{r}_3 - \vec{r}_4, \) and \( \vec{z} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1 + m_2) - (m_3\vec{r}_3 + m_4\vec{r}_4)/(m_3 + m_4) \) are the Jacobi coordinates. \( \chi_{\kappa}^{SI} \) are
orthonormalized spin-isospin vectors and $R_\kappa(\vec{x}, \vec{y}, \vec{z})$ is the radial part of the wave function of the $\kappa^{th}$ channel. In order to get the appropriate symmetry properties in configuration space, $R_\kappa(\vec{x}, \vec{y}, \vec{z})$ is expressed as the sum of four components,

$$R_\kappa(\vec{x}, \vec{y}, \vec{z}) = \sum_{n=1}^{4} w_n^\kappa R_n^\kappa(\vec{x}, \vec{y}, \vec{z}),$$  \hspace{1cm} (14)$$

where $w_n^\kappa = \pm 1$. Finally, each $R_n^\kappa(\vec{x}, \vec{y}, \vec{z})$ is expanded in terms of $N$ generalized Gaussians

$$R_n^\kappa(\vec{x}, \vec{y}, \vec{z}) = \sum_{i=1}^{N} \alpha_i^\kappa \exp \left[ -a_i^\kappa \vec{x}^2 - b_i^\kappa \vec{y}^2 - c_i^\kappa \vec{z}^2 - d_i^\kappa s_1^\kappa \vec{x} \cdot \vec{y} - e_i^\kappa s_2^\kappa \vec{x} \cdot \vec{z} - f_i^\kappa s_3^\kappa \vec{y} \cdot \vec{z} \right],$$  \hspace{1cm} (15)$$

where $s_i^\kappa$ are equal to $\pm 1$ to guarantee the symmetry properties of the radial wave function and $\alpha_i^\kappa, a_i^\kappa, \cdots, f_i^\kappa$ are the variational parameters. The latter are determined by minimizing the intrinsic energy of the four-body system. We follow closely the developments of Refs. [18, 19], where further technical details can be found about the wave function and the minimization procedure.

The numerical method described in this section has been tested in different few-body calculations in comparison to the hyperspherical harmonic formalism, see for example Refs. [19, 20], or the stochastic variational approach of Ref. [21] for some of the results presented in Ref. [22]. As a benchmark calculation to show the capability of the method we have studied the $^4$He, a $nnpp$ system with $(I, J) = (0, 0)$, using the spin-averaged Malfliet-Tjon (MT-V) potential of Ref. [23]. Results for the $(I, J) = (0, 0)$ four-nucleon problem can be found in Table 11.2 of Ref. [21]. It was solved with different numerical methods getting a full converged binding energy of 31.3 MeV.

We have studied the $(I, J) = (0, 0) nnpp$ state with the GGV method using the MT-V potential of Ref. [23],

$$V_{ij}(r) = -A e^{-\mu A r} + B e^{-\mu B r},$$  \hspace{1cm} (16)$$

**TABLE V:** $S$ wave two-body channels contributing to the $nnpp$ system with $(I, J) = (0, 0)$.

| $V_{12}$ | $V_{34}$ | $V_{13}$ | $V_{24}$ |
|----------|----------|----------|----------|
| $nn$ $(i, j) = (1, 0)$ | $pp$ $(i, j) = (1, 0)$ | $np$ $(i, j) = (0, 1)$ | $np$ $(i, j) = (0, 1)$ |
| $np$ $(i, j) = (1, 0)$ | $np$ $(i, j) = (1, 0)$ | $np$ $(i, j) = (1, 0)$ |
with parameters: $A = 578.09$ MeV, $\mu_A = 1.55$ fm$^{-1}$, $B = 1458.05$ MeV, $\mu_B = 3.11$ fm$^{-1}$. As in Ref. [21] we have used $\hbar^2/m_N = 41.47$ MeV fm$^2$. Being a pure $S$ wave calculation, the different two-body channels contributing to the $(I, J) = (0, 0)$ $nnpp$ state are shown in Table VI. With $N = 25$ generalized Gaussians in Eq. (15) we have obtained a binding energy of 31.2 MeV, which shows the capability of our method and gives confidence in the results. Let us note that the spin-averaged MT-V potential reproduces reasonably well the tritium binding energy, giving a result of 8.25 MeV.

### B. The $\Lambda\Lambda nn$ system

The uncoupled $\Lambda\Lambda nn$ system with $(I, J) = (1, 0)$ was examined in detail in Ref. [4] using local central Yukawa-type Malfliet-Tjon interactions. We summed up in Table VI the different two-body channels contributing to the $(I, J) = (1, 0)$ $\Lambda\Lambda nn$ state. The parameters of the $\Lambda N$ and $\Lambda\Lambda$ two-body channels were obtained by fitting the low-energy data and the phase-shifts of each channel as given in the most recent update of the strangeness $-1$ [24].

| Ref. | $(i, j)$ | $A$(MeV fm) | $\mu_A$(fm$^{-1}$) | $B$(MeV fm) | $\mu_B$(fm$^{-1}$) | $a$(fm) | $r_0$(fm) |
|------|----------|--------------|------------------|-------------|------------------|---------|----------|
| NN   | (1,0)    | 513.968      | 1.55             | 1438.72     | 3.11             | -23.56  | 2.88     |
|      | (1/2,0)  | 416          | 1.77             | 1098        | 3.33             | -2.62   | 3.17     |
|      | (1/2,1)  | 339          | 1.87             | 968         | 3.73             | -1.72   | 3.50     |
| AN   | (0,0)    | 121          | 1.74             | 926         | 6.04             | -0.85   | 5.13     |
|      | (0,0)    | 207.44       | 1.87             | 627.6       | 3.63             | -0.62   | 7.32     |
and $-2 \pm 1$ Nijmegen ESC08c potential. The low-energy data and the parameters of these models, together with those of the $NN$ interaction from Ref. [23], are given in Table VII. As can be seen in Fig. 2 of Ref. [4] there is no $\Lambda\Lambda nn$ bound state.

The system hardly gets bound for a reasonable increase of the strength of the $\Lambda\Lambda$ interaction. Although one cannot exclude that the genuine $\Lambda\Lambda$ interaction in dilute states as the one studied here could be slightly stronger that the one reported in Ref. [15], however, one needs a multiplicative factor in the attractive term of Eq. (16) $g_{\Lambda\Lambda} \geq 1.8$ to get a bound state. Such modification would destroy the agreement with the Nijmegen ESC08c $\Lambda\Lambda$ phase shifts. Note also that this is a very sensitive parameter for the study of double-$\Lambda$ hypernuclei [26] and this modification would produce an almost $\Lambda\Lambda$ bound state in free space, in particular it would give rise to $a_{i_{\Lambda\Lambda}^{S_{0}}} = -29.15 \text{ fm}$ and $r_{0i_{\Lambda\Lambda}^{S_{0}}} = 1.90 \text{ fm}$. The four-body system would also become bound by taking a multiplicative factor 1.2 in the $NN$ interaction. However, such a change would make the $^{1}S_{0}$ $NN$ potential as strong as the $^{3}S_{1}$ [23] and thus the singlet $S$ wave would develop a dineutron bound state, $a_{i_{NN}^{S_{0}}} = 6.07 \text{ fm}$ and $r_{0i_{NN}^{S_{0}}} = 1.96 \text{ fm}$. The situation is slightly different when dealing with the $\Lambda N$ interaction. We have used a common factor $g_{NA}$ for the attractive part of the two $\Lambda N$ partial waves, $^{1}S_{0}$ and $^{3}S_{1}$. The four-body system develops a bound state for $g_{NA} = 1.1$, giving rise to the $\Lambda N$ low-energy parameters: $a_{i_{\Lambda N}^{S_{0}}} = -5.60 \text{ fm}$, $r_{0i_{\Lambda N}^{S_{0}}} = 2.88 \text{ fm}$, $a_{i_{\Lambda N}^{S_{1}}} = -2.91 \text{ fm}$, and $r_{0i_{\Lambda N}^{S_{1}}} = 2.99 \text{ fm}$, far from the values constrained by the existing experimental data. In particular, these scattering lengths point to the unbound nature of the $\Lambda\Lambda nn$ system based on the hyperon-nucleon interactions derived from chiral effective field theory in Ref. [27], because it is less attractive: $a_{i_{\Lambda N}^{1S_{0}}} \in [-2.90, -2.91] \text{ fm}$ and $a_{i_{\Lambda N}^{3S_{1}}} \in [-1.40, -1.61] \text{ fm}$ (see Table 1 of Ref. [27]).

It is also worth mentioning that Ref. [28] tackled the same problem by fitting low-energy parameters of older versions of the Nijmegen-RIKEN potential [29, 30] or chiral effective field theory [31, 32], by means of a single Yukawa attractive term or a Morse parametrization. The method used to solve the four-body problem is similar to the one we have used in our calculation, thus the results might be directly comparable. Our improved description of the two- and three-body subsystems and the introduction of the repulsive barrier for the $^{1}S_{0}$ $NN$ partial wave, relevant for the study of the triton binding energy (see Table II of Ref. [33]), leads to a four-body state above threshold, that cannot get bound by a reliable modification of the two-body subsystem interactions. As clearly explained in Ref. [28], the window of Borromean binding is more and more reduced for potentials with harder inner
For the sake of consistency with Sec. II we have repeated the calculation using the latest \( \Lambda \Lambda \) interaction derived by the HAL QCD Collaboration \cite{16}. The parameters of the \( \Lambda \Lambda \) HAL QCD potential are given in the last line of Table VII. Although the \( \Lambda \Lambda \) interaction of Ref. \cite{16} is slightly more attractive than that of the Nijmegen ESC08c potential \cite{15}, the \( \Lambda \Lambda nn \) state remains unbound. The more attractive character of the HAL QCD \( \Lambda \Lambda \) interaction can be easily tested by trying to generate a \( \Lambda \Lambda nn \) bound state with the multiplicative factor in the attractive term of Eq. (16) of the \( \Lambda \Lambda \) interaction. While with the model of Ref. \cite{15} a multiplicative factor \( g_{\Lambda \Lambda} = 1.8 \) is necessary to get a bound state, with that of Ref. \cite{16} the bound state is developed for \( g_{\Lambda \Lambda} = 1.6 \).

We have also studied the coupled \( \Lambda \Lambda nn - \Xi^- pnn \) system, to check if the coupling to the upper channel \( \Xi^- pnn \) could help to generate a \( \Lambda \Lambda nn \) bound state. For this purpose one needs a parametrization of the \( \Lambda \Lambda - \Xi N \) transition potential. As has been explained in Sec. II B the HAL QCD Collaboration has recently derived a \( \Lambda \Lambda - \Xi N \) transition potential \cite{16} with almost physical quark masses. In their results, the \( H \) dibaryon appears as a very sharp resonance just below the \( \Xi N \) threshold, what points to a rather weak \( \Lambda \Lambda - \Xi N \) transition potential. We have parametrized this interaction by means of a Malflie-Tjon interaction as in Eq. (16) with parameters: \( A = 61.66 \text{ MeV}, \mu_A = 1.79 \text{ fm}^{-1}, B = 227.01 \text{ MeV}, \mu_B = 3.25 \text{ fm}^{-1} \). The details of the \( \Xi N \) interaction are discussed in the next subsection. The coupled \( \Lambda \Lambda nn - \Xi^- pnn \) system is clearly unbound. Thus, it only remains to study the possible existence of a \( \Xi^- pnn \) bound state that may decay to \( \Lambda \Lambda nn \).

C. The \( \Xi^- pnn \) system

We now study the uncoupled \( \Xi^- pnn \) system with quantum numbers \((I, J) = (1, 0)\), to look for a possible bound state. This system contains several bound states made of subsets of two- and three-body particles. It contains the deuteron, the tritium, the \((i, j) = (1, 1)\) \( \Xi N \) bound state predicted by the Nijmegen potential \cite{15} with a binding energy of 1.56 MeV, and the \((i, j) = (3/2, 1/2)\) \( \Xi NN \) bound state with a binding energy of 2.89 MeV discussed in Sec. II B. If there is a \( \Xi^- pnn \) bound state, it would not be stable unless its binding energy exceeds \( m_{\Xi^- p} - m_{\Lambda \Lambda} = 28.6 \text{ MeV} \). Otherwise it would decay to \( \Lambda \Lambda nn \). If its binding energy would be larger than that of the tritium, it would appear as a \( \Xi^- t \) resonance or quasibound
TABLE VIII: S wave two-body channels contributing to the $\Xi^−pnn$ system with $(I, J) = (1, 0)$.

| $nn\ (i, j) = (1, 0) - p\Xi^−\ (i, j) = (0, 0)$ | $np\ (i, j) = (1, 0) - n\Xi^−\ (i, j) = (1, 0)$ |
| $nn\ (i, j) = (1, 0) - p\Xi^−\ (i, j) = (1, 0)$ | $np\ (i, j) = (0, 1) - n\Xi^−\ (i, j) = (1, 1)$ |

state decaying to $ΛΛnn$.

To perform this study we need the $ΞN$ in three different partial waves. We show in Table VIII the different two-body channels contributing to the $(I, J) = (1, 0) \Xi^−pnn$ state. Firstly, we use the full set of $ΞN$ interactions of the Nijmegen group [15]. As in the case of the two-body channels in Sec. [311] we have constructed the two-body amplitudes for all subsystems entering the four-body problem studied by solving the Lippmann–Schwinger equation of each $(i, j)$ channel,

$$t_{ij}(p, p'; e) = V_{ij}(p, p') + \int_{0}^{\infty} p''^2 dp''V_{ij}(p, p'') \frac{1}{e - p''^2/2\mu}t_{ij}(p'', p'; e),$$  \hspace{1cm} (17)$$

where

$$V_{ij}(p, p') = \frac{2}{\pi} \int_{0}^{\infty} r^2 dr j_0(pr)j_0(p'r)V_{ij}(r),$$  \hspace{1cm} (18)$$

and the two-body potentials consist of an attractive and a repulsive Yukawa term as in Eq. [16]. The parameters of the $ΞN$ channels were obtained by fitting the low-energy data as given in the most recent update of the strangeness $−2$ Nijmegen ESC08c potential [15]. Besides, as mentioned above, the HAL QCD Collaboration [16] has recently derived a potential for the $(i, j) = (0, 0) \LambdaΛ − ΞN$ channel with almost physical quark masses. Thus, we

TABLE IX: Low-energy parameters and parameters of the local central Yukawa-type potentials given by Eq. (16) for the $ΞN$ system contributing to the $(I, J) = (1, 0) \Xi^−pnn$ state.

| Ref. | $(i, j)$ | $A$(MeV fm) | $\mu_A$(fm$^{-1}$) | $B$(MeV fm) | $\mu_B$(fm$^{-1}$) | $a$(fm) | $r_0$(fm) |
|------|---------|-------------|-----------------|-------------|-----------------|--------|---------|
| [16] | (0, 0)  | 161.38      | 1.17            | 197.5       | 2.18            | -      | -       |
| [15] | (0, 0)  | 120         | 1.30            | 510         | 2.30            | -      | -       |
| [15] | (1, 0)  | 290         | 3.05            | 155         | 1.60            | 0.58   | -2.52   |
| [15] | (1, 1)  | 568         | 4.56            | 425         | 6.73            | 4.91   | 0.53    |
have performed the calculation with both models for the \((i, j) = (0, 0)\) \(\Lambda\Lambda - \Xi N\) channel, Nijmegen ESC08c \[15\] and HAL QCD \[16\]. The low-energy data and the parameters of the different \(\Xi N\) interactions are given in Table \[VIII\].

With \(N = 15\) generalized Gaussians in Eq. \((15)\) we have obtained a \(\Xi^- pnn\) bound state of 14.43 MeV with the \((i, j) = (0, 0)\) HAL QCD interactions and 10.78 MeV with the Nijmegen potentials\(^2\). In both cases, the \((I, J) = (1, 0)\) \(\Xi^- pnn\) state lies below the lowest two-body threshold, \(\Xi^- t\). Such state would decay to the \(\Lambda\Lambda nn\) channel with a very small width as shown in Sec. \[II C\] and Ref. \[34\]. The results are in close agreement with those obtained with the separable potential three-body model shown in Table \[III\]. In all models the binding is larger than that of the tritium and a slightly deeper bound state is obtained when using the HAL QCD interactions for the two-body coupled channel \((i, j) = (0, 0)\). By including the Coulomb \(\Xi^- p\) potential the binding energies are increased roughly by 0.75 MeV with the HAL QCD interaction and 0.53 MeV with the Nijmegen potentials, driving to final binding energies of 15.18 MeV and 11.31 MeV, respectively.

**IV. OUTLOOK**

It has been suggested in Ref. \[1\] that some of the structures observed in the correlated \(\pi^- - \pi^-\) momenta by the BNL AGS-E906 experiment \[2\], aiming to produce and study double hypernuclei through a \((K^- , K^+)\) reaction on \(^9\)Be, could result from the decays of a \(^4\)\(\Lambda\Lambda n\) double hypernucleus. We have studied the coupled \(\Lambda\Lambda nn - \Xi^- pnn\) system to check if the inclusion of channel coupling is able to bind the \(\Lambda\Lambda nn\) system. We have used two different approaches. The first one is a separable potential three-body model of the coupled \(\Lambda\Lambda nn - \Xi^- pnn\) system tuned to the known experimental data that allows us to evaluate the \(\Xi^- t\) binding energy and its decay width to \(\Lambda\Lambda nn\). The second one is a generalized Gaussian variational method based on realistic two-body interactions tuned in the known two-, three- and four-body systems experimental data.

With the available two-body interactions that are adjusted to describe what is known about the two- and three-baryon subsystems, neither a \(\Lambda\Lambda nn\) bound state nor a resonance is obtained. However, we have found a \(\Xi^- t\) quasibound state with quantum numbers \((I, J) = \)

\(^2\) Note that the mass of \(^4\)He changes by 0.24 MeV from \(N = 15\) to \(N = 25\), so the result is fully converged.
(1, 0) above the ΛΛnn threshold. The stability of the state is increased by considering the Coulomb potential. The different approaches to the ΛΛ − ΞN interaction drive to similar results, the weakness of the ΛΛ − ΞN transition potential explaining the narrow width of the Ξ−t quasibound state. Finally, we have calculated the Ξ−t scattering length, which may be useful in the calculation of the energy shift of the atomic levels of the Ξ−t atom.

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