Computational Schemes for the Propagation of Ultra High Energy Cosmic Rays

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Abstract

We discuss the problem of ultra high energy particles propagation in astrophysical backgrounds. We present two different computational schemes based on both kinetic and Monte Carlo approaches. The kinetic approach is an analytical computation scheme based on the hypothesis of continuous energy losses while the Monte Carlo scheme takes into account also the stochastic nature of particle interactions. These schemes, that give quite reliable results, enable the computation of fluxes keeping track of the different primary and secondary components, providing a fast and useful workbench to study Ultra High Energy Cosmic Rays.

Keywords: Particles Astrophysics, Ultra High Energy Cosmic Rays, Astrophysical Backgrounds.

1 Introduction and Conclusions

Ultra High Energy Cosmic Rays (UHECR) are the most energetic particles observed in nature, with energies up to few $\times 10^{20}$ eV. Nowadays the experimental study of UHECR is conducted by three different experiments: Auger in Argentina, HiRes and Telescope Array in the USA.

The propagation of UHECR from the source to the observer is conditioned by their interactions with astrophysical backgrounds: the Cosmic Microwave Background (CMB) and the Extragalactic Background Light (EBL). Understanding the key features of the propagation is of paramount importance to interpret experimental observations paving the way for the discovery of the astrophysical origin of these fascinating particles.

Several features of the observed spectrum can be directly linked to the chemical composition of UHECR and to their sources (Greisen 1966, Zatsepin & Kuzmin 1966, Berezinsky et al. 2006a, Aloisio et al. 2007, Aloisio & Boncioli 2011). Among such features particularly important is the Greisin, Zatsepin and Kuzmin (GZK) suppression of the flux, an abrupt depletion of the observed proton spectrum, arising at energies $E \approx 5 \times 10^{19}$ eV, due to the interaction of UHE protons with the CMB radiation field (Greisen 1966, Zatsepin & Kuzmin 1966). The GZK suppression, as follows from the original papers, is referred to protons and it is due to the photon production process on the CMB radiation field ($p + \gamma_{\text{CMB}} \rightarrow \pi + p$). In the case of nuclei the expected flux also shows a suppression at the highest energies that, depending on the nuclei specie, is due to the photo-disintegration process on the CMB and EBL radiation fields ($A + \gamma_{\text{CMB,EBL}} \rightarrow (A - n N) + n N$) (Aloisio et al. 2012a). Another important feature in the spectrum that can be directly linked with the nature of the primary particles and their origin (galactic/extra-galactic) is the pair-production dip (Berezinsky et al. 2006a, Aloisio et al. 2007). This feature is present only in the spectrum of UHE extragalactic protons and, as the GZK, is a direct consequence of the proton interaction with the CMB radiation field, in particular the dip brings a direct imprint of the pair production process $p + \gamma_{\text{CMB}} \rightarrow p + e^+ + e^-$ suffered by protons.

From the experimental point of view the situation is far from being clear with different experiments claiming contradictory results. The HiRes experiment, not anymore taking data, showed a proton dominated spectrum till the highest energies (HiRes collaboration 2008, 2010) while the Auger observations show an heavy mass composition at energies $E > 4 \times 10^{18}$ eV (Auger collaboration 2010).

This puzzling situation, with different experiments favoring different scenarios, shows once more the importance of a systematic study of UHECR propagation in astrophysical backgrounds. In the present paper we will review the main points of two alternative computation schemes which enable the determination of the fluxes expected on earth fixing the injection spectrum and the distribution of sources. These two schemes are based on different
approaches to model particle interactions with backgrounds: the continuum energy losses (CEL) approximation, which is the base for the kinetic approach, and the Monte Carlo (MC) technique.

As we will discuss in the following these two different schemes give reliable results that, in the framework of the different assumptions, agree each other offering a suitable theoretical framework to study experimental results unveiling the intimate nature of UHECR.

2 Kinetic Equations

The main assumption under which the kinetic theory is build is the CEL approximation (Berezinsky et al 1990), through which particle interactions are treated as a continuum process that continuously depletes the particles energy.

UHECR propagating through astrophysical backgrounds suffer different interaction processes

- **protons** - UHE protons interact only with the CMB radiation field giving rise to the two processes of pair production and photo-pion production. Both of these reactions can be treated in the CEL hypothesis.

- **nuclei** - UHE nuclei interact with the CMB and EBL radiation fields, suffering the process of pair production, for which only CEL is relevant, and photo-disintegration, that involves both backgrounds CMB and EBL. While the first process can be treated in the CEL hypothesis, being the nuclei specie conserved, the second cannot be, producing a change in the nucleus specie. Following Aloisio et. al. 2012a, in the framework of the kinetic approach, we will treat the photo-disintegration process as a "decaying" process that simply depletes the flux of the propagating nucleus.

Taking into account all energy losses processes we can describe the propagation of protons and nuclei through kinetic equations of the type:

\[
\frac{1}{\tau_A} = \frac{c}{2\Gamma^2} \int_{e_{\text{cut}}(A)}^{\infty} \frac{d\epsilon}{\epsilon} \sigma(\epsilon, A) \nu(\epsilon) \epsilon \int_{e_{\text{cut}}/(2\Gamma)}^{\infty} \frac{d\epsilon}{\epsilon^2} \frac{n_{p\text{bg}}(\epsilon)}{2} \quad (3)
\]

where \(\sigma(\epsilon, A)\) is the photo-disintegration cross-section and \(\nu(\epsilon)\) is the molteplicity associated with this process, namely the average number of nucleons extracted from the nucleus by a single interaction and \(n_{p\text{bg}} = n_{\text{CMB}} + n_{\text{EBL}}\). The dependence on red-shift of \(\tau_A\) directly follows from the evolution with red-shift of the background photon densities \(n_{\text{CMB}}\) and \(n_{\text{EBL}}\). In the case of CMB this dependence is known analytically while for the EBL one should refer to evolution models (in our computations we have used the model by Stecker et. al. 2006).

One important feature of the photo-disintegration process is that it starts to contribute to the propagation of nuclei at a Lorentz factor that is almost independent of the nuclei specie \(\Gamma_{\text{cut}} \simeq 2 \times 10^9\) (Aloisio et al. 2012a). This is an important general characteristic of nuclei photo-disintegration process from which we can immediately deduce the dependence on the nucleus specie of the energy corresponding to the photo-disintegration suppression of the flux: \(E_{\text{cut}}^{A} = A m_{N} \Gamma_{\text{cut}}\) being \(A\) the atomic mass number of the nucleus and \(m_{N}\) the proton mass. From this expression for \(E_{\text{cut}}^{A}\) it is evident how the flux behavior could bring informations on the chemical composition of the UHECR, in the case of Helium (\(A = 4\)) the suppression is expected around energies \(E \simeq 10^{19}\) eV while in the case of Iron (\(A = 56\)) the suppression is expected at higher energies \(E \simeq 10^{20}\) eV.

Let us discuss now the generation function \(Q_{A}(t)\) in the right hand side of Eq. (2). One should distinguish among primary nuclei, i.e. nuclei accelerated at the source and injected in the intergalactic space, and secondary nuclei and nucleons, i.e. particles produced as secondaries in the photo-disintegration chain. In the case of primaries the injection function is an assumption of the source model, while the injection of secondaries should be modeled taking into account the characteristics of the photo-disintegration process. The dominant process of photo-disintegration is the one nucleon (\(N\)) emis-
sion, namely the process \((A + 1) + \gamma_{\text{bkg}} \rightarrow A + N\), this follows directly from the behavior of the photo-disintegration cross-section (see Aloisio et al. 2012a and references therein) that shows the giant dipole resonance corresponding to one nucleon emission. Moreover, at the typical energies of UHECR \((E > 10^{17} \text{ eV})\) one can safely neglect the nucleus recoil so that photo-disintegration will conserve the Lorentz factor of the particles. Therefore the production rate of secondary \(A\)–nucleus and \(A\)–associated nucleons will be given by

\[
Q_A(\Gamma, z) = Q_p^A(\Gamma, z) = \frac{n_{A+1}(\Gamma, z)}{\tau_{A+1}(\Gamma, z)} \quad (4)
\]

where \(\tau_{A+1}\) is the photo-disintegration life-time of the nucleus father \((A+1)\) and \(n_{A+1}\) is its equilibrium distribution, solution of the kinetic equation (2).

Using equation (4) we can build a system of coupled differential equations that starting from primary injected nuclei \((A_0)\) follows the complete photo-disintegration chain for all secondary nuclei \((A < A_0)\) and nucleons. Clearly secondary proton\(^1\) propagation will be described by the proper kinetic equation (1) with an injection term given by (4). The solution of the kinetic equation for protons and nuclei can be worked out analytically. In the case of protons:

\[
n_p(\Gamma, z) = \int_z^{z_{\text{max}}} \frac{dz'}{(1 + z') H(z')} Q_p(\Gamma', z) \frac{d\Gamma'}{d\Gamma} \quad (5)
\]

being \(Q_p\) the injection of primary protons or secondary protons (equation (3)) and \(\Gamma' = \Gamma'(\Gamma, z)\) is the characteristic function of the kinetic equation (Aloisio et al. 2012a). In the case of nuclei:

\[
n_A(\Gamma, z) = \int_z^{z_{\text{max}}} \frac{dz'}{(1 + z') H(z')} Q_A(\Gamma', z) \frac{d\Gamma'}{d\Gamma} e^{-\eta_A(\Gamma', z')}. \quad (6)
\]

being, again, \(Q_A\) the injection of primary or secondary nuclei. The exponential term in Eq. (6) represents the survival probability during the propagation time \(t' - t\) for a nucleus with fixed \(A\) and can be computed according to Aloisio et al. 2012a. The derivative term \(d\Gamma'/d\Gamma\) present in both solutions (5) and (6) is analytically given (Aloisio et al. 2012a).

### 3 Monte Carlo

The kinetic approach outlined above neglects interactions fluctuations considering an (average) continuum loss of energy suffered by particles. This approximation in the case of protons, has a limited effect on the flux computation only at the highest energies \((E > 100 \text{ EeV})\) (Berezinsky et al. 2006a, Berezinsky et al. 2006b, Aloisio et al. 2007).

\(^1\)Neutrons decay very fast into protons, so we will always refer to secondary protons.

"Figure 1: Flux of iron and secondary nuclei \((A = 50, 40, 30, 20, 10)\) at \(z = 0\) in the case of pure iron injection at the source with a power law injection index \(\gamma = 2.2\). Full squares correspond to the SimProp result (Aloisio et al. 2012b) while continuous lines correspond to the solution of the nuclei kinetic equation of Aloisio et al. 2012a.

In order to evaluate the effects of fluctuations on the expected nuclei flux we have build a computation scheme alternative to the kinetic one, that uses the MC technique to simulate nuclei interactions. First of all, let us remark that fluctuations could be relevant only in the case of nuclei photo-disintegration, this follows from the fact that the pair-production process involving nuclei can be considered as an interaction process of the inside nucleon, therefore being fluctuations in the protons pair-production irrelevant (Berezinsky et al. 2006b) the same holds for nuclei. The MC simulation scheme we have developed SimProp (Aloisio et al. 2012b) is mono-dimensional: it does not take into account spatial distributions tagging sources only through their distance from the observer (red-shift). The MC simulation propagates particles in steps of red-shift following the injected nuclei, secondary nuclei and protons produced at each photo-disintegration interaction and calculating their losses up to the observer, placed at red shift zero. The nuclear model on which SimProp is based is the same used for the kinetic approach (see Aloisio et al. 2012a, 2012b and references therein). The stochastic nature of the nuclei photo-disintegration process is modeled through the survival probability of a nucleus of atomic mass number \(A\) and Lorentz factor \(\Gamma\)

\[
P(\Gamma, z) = \exp \left( -\int_z^{z^*} \frac{1}{\tau_A(\Gamma, z')} \frac{dt'}{dz'} \right) \quad (7)
\]

where \(z\) and \(z^*\) are the values of the redshift of the current step (from \(z^*\) to \(z\)).

The SimProp code is designed in such a way that any red-shift distribution of sources and any injection spectrum can be simulated. This is achieved drawing events from a flat distribution in the red-shift
of the sources and of the logarithm of the injection energy. Once the event is recorded at \( z = 0 \) the actual source/energy distribution is recovered through a proper weight attributed to the event (Aloisio et al. 2012b). We will compare now the spectra obtained using SimProp (Aloisio et al. 2012b) with those calculated solving the kinetic equation associated to the propagation of nuclei (Aloisio et al. 2012a). To pursue such comparison, a pure iron injection with a power law injection of the type \( \propto E^{-\gamma} \) with \( \gamma = 2.2 \) have been assumed. The sources have been assumed to be homogeneously distributed in the red-shift range \( 0 < z < 3 \). In figure 1 the fluxes expected at \( z = 0 \) are shown for iron and secondary nuclei produced in the photo-disintegration chain suffered by primary injected irons. The points refer to the SimProp results while the continuous lines to the fluxes computed in the kinetic approach. A good agreement between the two schemes is clearly visible in figure 1. At the highest energies the path-length of iron nuclei is very short. Therefore, to achieve a good sampling in the MC simulation, higher statistics is needed; this is the reason for larger errors bars in the SimProp results at the highest energies. Let us conclude discussing why it is useful to go beyond the kinetic approach. The kinetic approach has the important feature of being analytical: fluxes are computed mathematically solving a first principles equation (Aloisio et al. 2012a). This means that the flux of primaries and secondaries is expressed in terms of several integrals that can be computed numerically, once the injection spectrum and the sources distribution are specified. In particular, the flux of secondary nuclei and nucleons produced in the photo-disintegration chain of a primary \( A_0 \) is obtained by the numerical computation of \( A_0 \) nested integrals and this computation should be repeated each time the hypothesis on sources (injection and distribution) are changed. This computation, while it is always feasible numerically, takes some time that can be substantially reduced using a MC computation scheme. This follows by the fact that, as discussed above, within the SimProp approach it is possible to simulate different source distributions and injection spectra without repeating the overall propagation of particles. In this sense the MC approach presented here, which is the minimal stochastic extension of the kinetic approach, provides a faster computation scheme.

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DISCUSSION

CARLO GUSTAVINO: The difference between Auger and the other experiments can be due to the fact they are looking from different hemispheres?

ROBERTO ALOISIO This is an hypothesis that was recently put forward. I personally do not believe in such explanation because of the simple reason that at energies around \( 2 \div 3 \times 10^{19} \) eV, where already the difference between Auger and HiRes starts, the universe visible in UHECR has a huge scale of the order of Gpc. Therefore it is very unlikely to have differences between observations carried out from the southern and northern hemispheres.