Gamma-ray emission from primordial black hole-neutron star interaction

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ABSTRACT
The interaction of an asteroid-mass primordial black hole (PBH) with a slowly-rotating neutron star (NS) can lead to detectable gamma-ray emission via modern observatories like Fermi-LAT or ASTROGRAM. Depending on the specific PBH relativistic orbit in the NS Schwarzschild spacetime and the relative orientation of this binary system with respect to Earth, the PBH Hawking radiation will show a characteristic temperature profile over time. Essentially, a moderate heating behaviour (or even a constant temperature value) is found for the majority of the event, followed by a sudden and dramatic cool-down at the end of the burst. Our theoretical model might provide a means of identification of such hypothetical PBH-NS interactions, based on the distinctive temperature evolution of thermal-like gamma ray bursts (GRBs) described in this article.

Key words: gamma-ray bursts – dark matter – black hole - neutron star mergers

1 INTRODUCTION
Primordial black holes constitute an hypothetical type of black holes (BHs) formed in the early Universe through gravitational collapse of extremely dense regions (Zeldovich 1967; Hawking 1971; Carr 1974). The study of such ultracompact objects of primitive origin is a subject of continuous research since it is thought that part or the totality of the dark matter (DM) might be formed by PBHs of different masses (Inomata 2017; Bartolo 2019; Belotsky 2019; deLuca 2021). In this regard, the currently allowed PBH masses range in the $10^{13} - 10^{14}$ kg, $10^{17} - 10^{21}$ kg and $10 - 10^{10} M_\odot$ window (Carr 2020, 2022), significantly lower than the already evidenced supermassive black holes (Akiyama 2019, 2022). Larger PBHs in the solar-mass range have recently been associated with gravitational wave signals (Kohri 2021) while PBHs smaller than $10^{12}$ kg would have presently evaporated due to its Hawking emission (Ackermann 2018). Current observations reported plausible PBH detection via their Hawking radiation in the mass range $8 \times 10^{11} - 10^{13}$ kg (Wang 2021). Moreover, ultra-light PBHs with masses less than $10^6$ kg (which would have evaporated before big-bang nucleosynthesis) might have been predominant in the early Universe (Papanikolaou 2021).

Among all of these different mass windows, the so-called asteroid-mass PBHs have drawn increasing attention. These light BHs within the mass interval $7 \times 10^{13} - 8 \times 10^{18}$ kg (Montero-Camacho 2019) have lifetimes ranging from hundreds to several millions times the age of the Universe and constitute a serious candidate for the cosmological dark matter (Coogan 2021). New constraints on this asteroid-mass window have been lately reported (Smyth 2020; Miller 2022) and they are likely to be discovered via the future gamma ray telescopes such as the upcoming AMEGO (Ray 2021).

Recently, a plausible PBH origin for thermal GRBs was stated, where an atomic-sized PBH described a radial fall onto a massive black hole (Barco 2021, 2022). As a result, the numerical calculations for the PBH Hawking emission were consistent with thermal-like gamma ray bursts (GRBs) in the MeV domain. However, this study considered a quite restrictive interaction between both black holes, that is, a plunging orbit where the PBH is being captured by its massive companion. No other possible relativistic orbits were analyzed in this paper and a detailed discussion of the rate of occurrence of such events was not provided.

In the present article, the above-mentioned astrophysical scenario and the related theoretical model are both updated to take into account a more general PBH interaction event. Indeed, an asteroid-mass PBH experiences a close approach to a slowly-rotating neutron star (Yazadjiev 2016; Motahar 2017; Bounaza 2021) well approximated by a Schwarzschild spacetime. Two different relativistic orbits are studied in our model, that is, a plunge orbit (where the PBH is being captured by the NS) and a scattering orbit around the NS, both with characteristic and recognizable Hawking emission profiles. Moreover, a complete analysis of the capture rate of an asteroid-mass PBH by a NS (following Capela (2013) formalism) is carried out, being consistent with the observed ratio of GRBs.

The paper is organized as follows. In section 2 we describe our astrophysical scenario and develop the theoretical framework to calculate the PBH relativistic orbits and its fluence spectrum $\nu S_\nu$ during the infalling event. A detailed numerical study concerning the PBH plunge and scattering orbits and its associated Hawking temperature evolution are performed in section 3. A moderate heating behaviour (or even a constant temperature value) is found for the majority of the event, followed by a sudden and dramatic cool-down at the end of the burst. The gamma-ray emission due to the PBH Hawking radiation might be detected via modern observatories like Fermi-LAT or ASTROGRAM. For completeness, the rate of occurrence of such PBH-NS interaction is fully analyzed in section 4. Finally, we summarize our results in section 5.

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2 THEORETICAL BASIS

Let us consider the astrophysical scenario depicted in Fig. 1. Here, an asteroid-mass PBH describes a relativistic orbit around an isolated slowly-rotating NS of radius $R_{\text{NS}}$, at distance $d_c$ from Earth. It is also assumed that our NS does not accrete matter from any stellar material, with such a low temperature so that its proper thermal emission is practically negligible (Pearson 2018, 2019; Fantina 2020).

According to Fig. 1, when the PBH is located at position (1) (within the so-called non-detection area), its Hawking emission lies below the typical sensitivity of modern observatories and, consequently, it cannot be initially detected. Once the PBH approaches the NS and its velocity component is pointing directly towards the Earth (as in position (2)), its Hawking radiation is significantly Lorentz-boosted and might be measured in the MeV range. The parameter $\theta_e$ corresponds to the angle between the radial coordinate $\hat{r}$ and the photon emission direction (supposed above the Earth’s line of sight, ELS). As the PBH goes around the NS (in the case of a scattering orbit), the velocity component is now opposite to the ELS so a deboosting effect occurs (please, see position (3) in Fig. 1). In this situation, the Hawking emission drops below the gamma-ray observatories sensitivity and enters again the non-detection area.

Under these premises, the NS spacetime can be properly modelled by the Schwarzschild line element

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

where $r$ indicates the radial coordinate of the PBH (our emitter), $f(r) = 1 - 2M_{\text{NS}}/r$ and $M_{\text{NS}}$ stands for the mass of our NS. Trivially, for a distant observer placed at $r_{\text{obs}}$ (for example, an observatory on Earth), the parameter $f(r_{\text{obs}}) \approx 1$.

Throughout the article, the geometrized system of units ($c = G = 1$) is adopted, except for radiometric magnitudes where the S.I. system of units is considered. We also assume that the PBH motion is confined at the equatorial plane $\theta = \pi/2$ with conserved specific energy $E_s$ and specific angular momentum $L_s$. Both parameters will determine the specific PBH orbit, as will be briefly discussed.

On the other hand, the kinematic component of the time dilation $\gamma$ can be expressed as (Hartle 2003)

$$\gamma = \left(1 - \beta^2\right)^{-1/2} = \frac{E_s}{\gamma G},$$

where $\beta$ is the velocity of the PBH and $\gamma_G = \sqrt{f(r)/f(r_{\text{obs}})} \approx \sqrt{f(r)}$. We can also express the coordinate time $t$ (for a distant observer) and the azimuthal angle $\phi$ as a function of the PBH radial coordinate $r$ via the following differential equations (Hartle 2003)

$$dt = \frac{E_s}{f(r)} \left[ E_s^2 - f(r) \left( 1 + \frac{L_s^2}{r^2} \right) \right]^{-1/2} dr,$$

and

$$d\phi = \left[ \frac{1}{b^2} - f(r) \left( \frac{1}{L_s^2} + \frac{1}{r^2} \right) \right]^{-1/2} dr,$$

where $b = L_s/E_s$. After numerical resolution of both expressions (3) and (4), the event duration (as measured by an observer on Earth) and the PBH specific orbit will be respectively derived.

Furthermore, the PBH Hawking temperature as measured by the emitter ($T_P$) and a distant observer ($T_P$) are related via the next equation

$$T_P = \alpha T_P,$$

where the redshift (blueshift) factor $\alpha$ is given by (McMaken 2022)

$$\alpha = \gamma_G \left( \frac{E_s}{f(r)} + \cos \theta_e \sqrt{\frac{E_s^2}{f(r)} - 1 - \frac{L_s^2}{r^2} - \frac{L_{\text{eff}}}{r} \sin \theta_e} \right)^{-1}.$$ (6)

It should be noted that the PBH Hawking temperature $T_P$ depends on its mass (that is, lighter PBHs would be at higher temperature), the constants of motion $E_s$ and $L_s$ (i.e., the specific PBH orbit in the NS spacetime) and the photon emission direction $\theta_e$ (or, equivalently, the position of the PBH in its relativistic orbit).

The PBH emission associated with its Hawking radiation can be well approximated by a Planckian distribution with temperature $T_P$ (Hawking 1974, 1975)

$$T_P = \left( 6.169 \times 10^{-9} \right) \frac{M_{\odot}}{M_p},$$

where $M_p$ corresponds to the PBH mass and the temperature is expressed in Kelvins. Considering a slowly-rotating PBH, it is thus appropriate to model the Hawking emission as a blackbody radiator (Murata 2006; Barco 2021), so the fluence spectrum $\nu S_{\nu}$ for a static observer on Earth can be written as (S.I. units)

$$\nu S_{\nu} = \frac{2\pi h\nu^4}{c^2} \left( \exp \left( \frac{h\nu}{kT_P} \right) - 1 \right)^{-1},$$

and $\Omega = \pi r_0^2 / d_L^2$ is the solid angle subtended by a point source for a detector of radius $r_D$ (which, in turn, is proportional to the effective area of our gamma-ray telescope). As discussed in the next section, our astrophysical scenario predicts a fluence spectrum $\nu S_{\nu}$ of finite duration which lies within the MeV range, a result compatible with the already evidenced thermal-like GRBs.

3 NUMERICAL RESULTS

In this section, a complete numerical analysis of the PBH relativistic orbits during its infall onto the NS and the associated Hawking emission (in connection with thermal-like GRBs) is carried out.

Firstly, let us analyze the different PBH relativistic orbits allowed in our model within the theoretical framework of the NS Schwarzschild spacetime. Without loss of generality, the mass of our PBH will be $10^{12}$ kg throughout the article (that is, in the range of the aforementioned asteroid-mass interval). As already stated, the specific PBH orbit depends on the conserved magnitudes $E_s$ and $L_s$ via the effective potential $V_{\text{eff}}$ (Hartle 2003)

$$V_{\text{eff}}(r) = \frac{1}{2} \left[ \frac{E_s}{f(r)} \right] \left( 1 + \frac{L_s^2}{r^2} \right) - 1.$$ (9)

When $(L_s/M_{\text{NS}}) > \sqrt{12}$, the effective potential has one maximum and one minimum: this will be the situation considered in our model. Moreover, the specific orbit depends on the relationship between the parameter $\varepsilon = (E_s^2 - 1)/2$ and $V_{\text{eff}}$. If $\varepsilon > V_{\text{eff}}^{(\text{max})}$ the PBH describes a plunge orbit onto the NS (and, eventually, it might be captured by the latter, as remarked in section 4). When $0 < \varepsilon < V_{\text{eff}}^{(\text{max})}$ a scattering orbit occurs where the PBH comes in from infinity, orbits the NS and moves out to infinity again (please, see again Fig. 1).

The effective potential $V_{\text{eff}}$ is calculated via equation (9) and represented in Fig. 2 for a typical neutron star of mass $M_{\text{NS}} = 1.4M_\odot$ and radius $R_{\text{NS}} = 1.2 \times 10^3$ m. The selected value for the PBH specific angular momentum was $L_s = 9 \times 10^3$ m. With this choice, the NS effective potential presents one maximum and one minimum, though this prerequisite is not essential for our model (as in the case when $(L_s/M_{\text{NS}}) < \sqrt{12}$ where only plunge orbits are allowed).
Two different values of the parameter $\epsilon$ are shown in Fig. 2, each representing a characteristic PBH orbit: a plunge orbit for $\epsilon_p = 0.105$, corresponding to a PBH specific energy $E_p = 1.1$ and a velocity at infinity of 0.41c, and a scattering orbit for $\epsilon_s = 0.0$, where the PBH exhibits a free fall from infinity. In the previous case, the intercept of $\epsilon$ with the effective potential curve gives us the minimum approach of the PBH to the NS (that is, $r_{\text{min}} = 1.38 \times 10^4$ m).

The specific shape of each PBH orbit (obtained via numerical resolution of equation (4)) is represented in Fig. 3, where the radial $r$ and azimuthal $\phi$ coordinates (last one in degrees) are plotted as polar diagrams in the equatorial plane $\theta = \pi/2$. Top panel depicts two identical asteroid-mass PBHs describing different plunging orbits (in relation to their relative orientation with respect to the ELS and denoted as PBH(1) and PBH(2)), while bottom panel shows two scattering orbits for the same astrophysical scenario. In both cases, the same parameters as those considered in Fig. 2 have been chosen. In connection with Fig. 1, the infalling PBH approaching the NS from infinity is not initially detected (due to its faint Hawking emission). Depending on its particular orbit, the PBH Hawking radiation will be rather different and recognizable, as briefly discussed.

In Fig. 4, the PBH Hawking temperature $T_P$ is illustrated as a function of the radial coordinate $r$ for each particular orbit in Fig. 3. All our numerical calculations have been performed via equations (5) and (6). For both plunge and scattering orbits, there exists a minimum value of the PBH Hawking temperature $T_p^{(\text{min})}$ (i.e., the dashed horizontal lines in top and bottom panels in Fig. 4) below which the fluence spectrum $S_{\nu}$ is dim enough to be detected by modern observatories. It should be also pointed out that this temperature threshold strongly depends on the PBH mass, its distance to the Earth (or the solid angle subtended by such point source) and the sensitivity of the gamma-ray telescopes.

Once our PBH is close enough to the NS, its Hawking temperature exceeds $T_p^{(\text{min})}$ and the gamma-ray emission is likely to be measured: this is the beginning of the thermal GRB of finite duration (and calculated after numerical resolution of the differential equation (3)).

This prompt gamma-ray emission is concluded when $T_P$ drops below the threshold temperature $T_p^{(\text{min})}$ again, a process which is highly dependent on the PBH relativistic orbit. Indeed, the PBH(2) orbit in Fig. 4(a) will produce a detectable GRB of 0.9 s, while the plunge orbit for PBH(1) will remain unnoticed for current gamma-ray obser-
vatories. On the other hand, both scattering orbits in Fig. 4(b) will be experienced as thermal-like GRBs of specific duration (about 35 s for PBH(2)).

It can be noticed that, in both cases, the PBH Hawking temperature presents an initial heating behaviour (due to the Lorentz-boosting of $T_P$ when the PBH velocity component is directed towards the ELS), followed by a dramatic cool-down when the PBH velocity component is opposite to the Earth’s line of sight (and a Lorentz-deboosting process occurs). This heating evolution is more pronounced (and eventually reaching a well-defined maximum temperature) for relativistic orbits where the PBH velocity component is mainly maintained towards us during practically the whole event (please, see again the PBH(2) cases in Fig. 4). In the opposite scenarios (as in the PBH(1) events), there exists a moderate or null heating process (such as the scattering orbit in bottom panel) or a progressive and constant cooling, as in the plunge case in Fig. 4(a). The latter situation is more consistent with already reported cooling behaviour of thermal GRBs (Ryde 1999, 2004; Barco 2021, 2022).

In relation to the fluence spectrum $\nu_S$, plausibly measurable by modern gamma-ray stations, Fig. 5 shows this parameter for the PBH(2) events calculated via equation (8) (please, see again Figs. 3 and 4). It can be observed a typical Planckian-like distribution for the fluence spectrum, according to the PBH thermal emission of its Hawking radiation.

For the plunge orbit case (top panel), the fluence spectrum corresponding to $T_P^{(\text{min})} = 16.5$ MeV practically reaches the sensitivity of the Fermi-LAT observatory (deAngelis 2018). For this calculation, an estimated distance to Earth of 4.2 Gparsec and an effective area of $10^4$ cm$^2$ for the Fermi-LAT (Coogan 2021) has been considered. Furthermore, the PBH fluence spectrum $\nu_S$ for the maximum Hawking temperature 17.7 MeV (i.e., the red dot shown in Fig. 4(a)) is also represented, where now $\nu_S$ is well above the Fermi-LAT sensitivity.

In Fig. 5(b), the scattering orbit event is depicted for a PBH-NS system at 2.3 Gparsec from Earth. One notices that the minimum fluence spectrum (associated with the threshold PBH Hawking temperature $T_P^{(\text{min})} = 10.7$ MeV, please see again Fig. 4(b)) is of the order of ASTROGRAM observatory sensitivity (deAngelis 2018), with an estimated effective area of $2 \times 10^3$ cm$^2$ in the MeV domain (Coogan 2021). The maximum value for $\nu_S$ (corresponding to a PBH temperature of $T_P = 14.5$ MeV, as illustrated in Fig. 4(b) with a red dot) peaks above the Fermi-LAT sensitivity, being highly probable its detection at the final stage of the burst via such modern observatories.

At the same time, it should be recalled that our slowly-rotating
Figure 5. Fluence spectrum $\nu S_\nu$ for the minimum and maximum PBH Hawking temperatures (same parameters as in Figs. 3 and 4) for the PBH;2; in falling events. Our numerical results were calculated via equation (8) for (a) plunge and (b) scattering orbits. As an indicative matter, the sensitivities of the ASTROGRAM and Fermi-LAT telescopes are shown.

NS does not possess an accretion disk (Pearson 2018, 2019; Fantina 2020), unlike other younger and more active NSs. The presence of such structure in our binary astrophysical scenario might modify our previous fluence calculations due to the thermal emission of this accretion disk. Additionally, as previously commented, our isolated NS has cooled down sufficiently so that its proper thermal emission is negligible.

To complete this section, it is worth asking how frequent are such PBH-NS interactions in order to be detectable by modern gamma-ray observatories. We will try to give a satisfactory response on this topic in the next section.

4 RATE OF OCCURRENCE ANALYSIS

Let us now examine the rate of occurrence of such PBH-NS scenarios and its plausible relation to the observed ratio of GRBs. Such astrophysical events have been earlier studied in the literature, where a PBH is captured by a neutron star (Capela 2013; Pani 2014).

Following the formalism by Kouvaris (2008) and Capela (2013), the capture rate $\mathcal{F}$ of a PBH by a NS is derived by integrating a Maxwellian dark matter distribution, with velocity dispersion $\bar{v}$, over the $E_\nu$ and $L_\nu$ space with well-defined integration limits. Within this theoretical framework, a PBH becomes gravitationally bound when it loses its initial energy due to the accretion of star’s material. Despite this constitutes such a restrictive situation (that is, we do not need in our model that the PBH is confined within the NS), it will provide a good basis to describe the probability of occurrence of these PBH-NS events.

The capture rate $\mathcal{F}$ of a PBH by a NS of radius $R_{NS}$ can be written as (Capela 2013)

$$\mathcal{F} = \frac{\Omega_{PBH} \rho_{DM}}{\Omega_{DM}} \frac{R_{NS} R_{s}}{M_{P}} \frac{1}{\bar{v}(R_{NS})} \left[ 1 - \exp \left( \frac{3E_{\text{loss}}}{M_{P} \bar{v}} \right) \right],$$

(10)

where $\Omega_{PBH}/\Omega_{DM}$ is the fraction of PBHs to dark matter, $\rho_{DM}$ the local DM density, $R_{s} = 2M_{NS}$ the Schwarschild radius of the NS and $E_{\text{loss}}$ the PBH energy loss during the collision

$$\frac{E_{\text{loss}}}{M_{P}} = 6.3 \times 10^{-12} \left( \frac{M_{P}}{10^{22}g} \right).$$

(11)

Assuming that the currently estimated DM density ranges from low density regions (where $\rho_{DM} \approx 0.5$ GeV cm$^{-3}$ at the edge of a galaxy), to high values around 840 GeV cm$^{-3}$ at the center of a standard galaxy (Read 2014; Sofue 2020; Chang 2021), a velocity dispersion $\bar{v} = 7000$ m s$^{-1}$ and typical NS parameters (i.e., $R_{NS} = 12$ km and $M_{NS} = 1.4M_{\odot}$), the capture rate $\mathcal{F}$ for an asteroid-mass PBH of $10^{13}$ kg should be of $4 \times 10^{13}$ yr$^{-1}$ (according to equations (10) and (11)). Considering a fraction of dark matter in the form of asteroid-mass PBHs as $\Omega_{PBH}/\Omega_{DM} \approx 10^{-5}$ (Bartolo 2019; deLuca 2021), the capture rate for a single capture event corresponds to $\mathcal{F} = 4 \times 10^{-18}$ yr$^{-1}$.

Taking into account an estimated number of $10^{8}$ NSs in a galaxy similar to the Milky Way (Sartore 2010; Cordes 2016) and $10^{11}$ galaxies in the observable universe (Conselice 2016; Lauer 2021), the number of such asteroid-mass PBH captures should be roughly 38 events per year (assuming the lowest value of DM density in our calculations). It is also worth recalling that this value must be smaller due to the differences in PBH concentration and DM densities in the Universe, excluding young NSs in the galaxies. Even in this scenario, the capture rate $\mathcal{F}$ here derived is consistent with the GRBs rate for gamma-ray missions like BATSE or Fermi-LAT (Sacahui 2016).

Our estimations show that the astrophysical model reported in this article is in accordance with the observed ratio of GRBs. Nevertheless, the number of measured thermal-like GRBs (Ryde 1999, 2004) is substantially lower than the predicted by our theory. In the next section we will discuss this issue in more detail.

5 CONCLUSIONS

Summarizing, an astrophysical scenario where an asteroid-mass PBH undergoes a close approach to a slowly-rotating galaxy can lead to detectable gamma-ray emission, via modern observatories like Fermi-LAT or ASTROGRAM. As widely studied in the previous sections, the PBH Hawking radiation over time shows a well-defined and characteristic temperature profile, depending on the specific PBH orbit in the NS Schwarzschild spacetime. Related to this, the PBH stable circular orbit (that is, when the parameter $e$ equals the minimum value of $V_{eff}$, please see again Fig. 2) has not been discussed in our model. The main reason is that this scenario would produce periodic thermal GRBs of very short duration, an aspect that (to our knowledge) has not yet been reported.

No afterglows are expected for the scattering case, where our PBH goes around the NS and moves away again to infinity (that is, a dark GRB would occur (Fynbo 2001; Jakobsson 2004; Melandri 2012)). Nonetheless, subsequent thermal emission at longer wavelengths (i.e., afterglows within the X-ray or ultraviolet domain) might happen for a plunge event due to the accretion of heated stellar material, once the PBH is trapped by the NS. This accretion mechanism should be more efficient for more massive incoming primordial black holes (such as planetary-mass PBHs) where their event horizon is much greater than the size of an atom: let us recall that the size of an asteroid-mass PBH (like the one considered in our work) is of the order of an atomic nucleus, roughly $10^{-15}$ m.

As commented in section 4, the capture rate of such asteroid-mass PBHs by NSs in the observable Universe (under the restrictive conditions already described) should be of about 38 events per year, a result which is in accordance with the observed ratio of GRBs.
However, the number of measured thermal-like GRBs is significantly lower than those estimated by our model. A possible explanation is that the capture events leading to detectable thermal-like GRBs occur at cosmological distances (that is, when such asteroid-mass PBHs were more abundant than presently), constraining thus the number of PBH-NS plausible scenarios. It can also be expected that future gamma-ray observatories with improved sensitivities might be able to detect such GRBs of primordial black hole origin.

On the other hand, similar results to those detailed in this article are expected if we consider a stellar-mass black hole as the massive central object, instead of a NS. In this context, the asteroid-mass PBH could get through the BH photon sphere and, as a consequence, the cool-down process might be more pronounced and accelerated (Barco 2022).

In conclusion, our theoretical model might provide a way of identifying such feasible PBH-NS interactions, based on the specific temperature profile of the before-mentioned thermal-like GRBs.

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DATA AVAILABILITY

Provided the theoretical nature of this paper, all data and numerical results generated or analysed during this study are included in this article (and based on the references therein). All the numerical calculations have been carried out on the basis of a Fortran 90 compiler.

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