Far- and Near-Field Heat Transfer in Transdimensional Plasmonic Film Systems

Svend-Age Biehs and Igor V. Bondarev*

A confinement-induced nonlocal electromagnetic response model is applied to study radiative heat transfer processes in transdimensional plasmonic film systems. The results are compared to the standard local Drude model routinely used in plasmonics. The former predicts greater Woltersdorff length in the far-field and larger film thicknesses at which heat transfer is dominated by surface plasmons in the near-field, than the latter. The analysis performed suggests that the theoretical treatment and experimental data interpretation for thin and ultrathin metallic film systems must incorporate the confinement-induced nonlocal effect in order to provide reliable results in radiative heat transfer studies. The fact that the enhanced far- and near-field radiative heat transfer occurs for much thicker films than the standard Drude model predicts is crucial for thermal management applications with thin and ultrathin metallic films and coatings.

1. Introduction

Present-day nanofabrication techniques make it possible to produce ultrathin films of precisely controlled thickness down to a few monolayers.[1–10] Often referred to as transdimensional (TD) quantum materials,[10–14] such films offer high tailorable properties of their electronic and optical properties not only by altering their chemical and electronic composition (stoichiometry, doping) but also by varying their thickness (the number of monolayers).[14–20] Plasmonic TD materials (ultrathin metallic films) are irreplaceable for studies of the fundamental properties of the light-matter interaction as it evolves from a single 2D atomic layer to a larger number of layers approaching the 3D bulk material properties.[11,16] They offer controlled light confinement and large tailorbility of their optical properties due to their thickness-dependent localized surface plasmon (SP) modes.[12–20] The strong vertical quantum confinement makes these modes distinct from those of conventional thin films commonly described either by 2D or by 3D material properties with boundary conditions on their top and bottom interfaces.[21–29] Their properties can be understood in terms of the confinement-induced nonlocal Drude electromagnetic (EM) response theory proposed[15] and verified both experimentally[20,30] and computationally[3,31] recently. The EM response nonlocality was earlier reported experimentally to be a remarkable intrinsic property of quantum-confined metallic nanostructures.[12,33] It is this nonlocality that enables a variety of new quantum phenomena in ultrathin TD plasmonic film systems, including the thickness-controlled plasma frequency red shift,[10,15] the low-temperature plasma frequency dropoff,[16] the SP mode degeneracy lifting,[14,15] a series of quantum-optical[13] and nonlocal magneto-optical effects,[16] as well as quantum electronic transitions that are normally forbidden.[12,15,36]

The confinement-induced nonlocal Drude EM response theory is built on the Keldysh–Rytova (KR) pairwise electron interaction potential[15] (and so referred to as the KR model in what follows for brevity). The KR interaction potential takes into account the vertical electron confinement due to the presence of substrate and superstrate materials with dielectric permittivities much less than that of the film.[37,38] For ultrathin films the KR potential is much stronger than the Coulomb interaction potential.[37] It turns into the Coulomb potential with film thickness increase, suggesting the film thickness as a parameter to control the nonlocal optical response of TD materials. The nonlocal KR model is unique in that it covers the entire thickness range from atomically thin films to conventional films of the order of a few optical wavelengths in thickness.[30,36] We perform a comparative study of the far-field and near-field heat transfer processes in the metallic TD film systems using the nonlocal KR model and the standard local Drude EM response model (a ‘workhorse’ routinely used in plasmonics). We show that the nonlocal KR model results in the greater Woltersdorff length (the film thickness at which its thermal emission is maximal) in the far-field regime and predicts larger film thicknesses at which the near-field heat transfer starts being dominated by surface plasmons, as compared to those resulted from...
Equation 1 can be seen to gradually approach $\omega_p^0$, the bulk material screened plasma frequency, and Equation 2 takes the local Drude form. This is the essence of the nonlocal KR model of the plasmonic TD film EM response.

The degree of the confinement-induced nonlocality associated with $k$-dependence in Equation 2 can be evaluated by effecting the inverse Fourier transformation from the reciprocal 2D space to the direct coordinate 2D space. This gives the coordinate/frequency-dependent specific permittivity (per unit area) in the form [16]

$$\frac{\varepsilon(\rho, \omega)}{\varepsilon_k} = \frac{\omega_p^{10}}{\omega(\omega + i\Gamma_0)} \delta(\rho) - \frac{(\omega_p^{10})^2}{2\pi\omega(\omega + i\Gamma_0)^2} \int_0^\infty dk \frac{kJ_0(kp)}{1 + 1/(\varepsilon kd)}$$

where $k_c$ and $k_{\text{in}}(= 2\pi/L)$ with $L$ being the film lateral size) stand for the plasmon upper and lower cut-off wave vectors, respectively. The latter indicates that the largest possible plasma wavelength cannot exceed the lateral size of the film. For thick films with $\varepsilon kd \gg 1$, Equation 3 gives the local response of the form

$$\frac{\varepsilon(\rho, \omega)}{\varepsilon_k} = 1 - \frac{(\omega_p^{10})^2}{\omega(\omega + i\Gamma_0)} \delta(\rho)$$

For ultrathin films with $\varepsilon kd \ll 1$, one obtains ($L \to \infty$)

$$\frac{\varepsilon(\rho, \omega)}{\varepsilon_k} = 1 - \frac{\varepsilon k_d(\omega_p^{10})^2}{\omega(\omega + i\Gamma_0)} \left[ \frac{\cos(k\rho + 3\pi/4)}{\sqrt{2}\pi k_c} \rho^{5/2} \right]$$

under extra condition $k_c\rho > 1$ to specify the distances at which the electrostatic repulsion of electrons is screened and their motion in the form of collective plasma oscillations is well defined. The ultrathin film permittivity can now be seen to decay with distance as $\rho^{-3/2}$ due to the confinement-induced EM response nonlocality of the KR model we use.

The damping constant $\Gamma_0$ for the ultrathin films can be expressed as [10]

$$\Gamma_0 = \Gamma_{\text{bulk}} + \frac{\hbar\nu_F}{l_{\text{eff}}} + A/d$$

with the bulk damping $\Gamma_{\text{bulk}}$, the surface roughness correction $A$, and the surface scattering determined by the effective mean free path

$$l_{\text{eff}} = d \left[ 1 + \ln \left( \frac{l_{\text{bulk}}}{d} \right) \right]$$

with the bulk mean free path $l_{\text{bulk}}$ and the Fermi velocity

$$\nu_F = \frac{\varepsilon_0 k_c}{\hbar}$$

introducing the thermally averaged plasma frequency of Equation 1 in the TD regime [10, 14]

$$\bar{\omega}_p = \frac{2C^2d^2\omega_p^{10}}{(1 + 2Cd)\sqrt{C\pi(1 + Cd) + \sinh^{-1}(\sqrt{Cd})}}$$

2. Confinement-Induced Nonlocality

If the environment has a lower dielectric constant than that of the film, such as the case of the finite-thickness metallic slab shown in Figure 1a, where $\varepsilon_{1,2} < \varepsilon_b$ are, respectively, the static permittivities of the substrate, the superstrate and the slab (contributed by the positive ion background and interband electronic transitions), then the increased ‘outside’ contribution makes the Coulomb interaction of electrons vertically confined inside the slab stronger than that in a homogeneous medium with the same permittivity [17]. The interaction potential takes the KR form to give the in-plane plasma oscillation frequency as follows [15]

$$\omega_p = \omega_p(k) = \frac{\omega_p^{10}}{\sqrt{1 + 1/(\varepsilon kd)}}$$

(1)

Here, $\varepsilon = \varepsilon_b/\varepsilon_c$ is the relative dielectric constant of the film, $d$ is its thickness, and $k$ is the in-plane momentum absolute value. With this plasma frequency, the low-energy (well below the first interband transition energy) in-plane EM response function of the plasmonic film takes the form

$$\frac{\varepsilon(k, \omega)}{\varepsilon_k} = 1 - \frac{\omega_p^{10}(k)}{\omega(\omega + i\Gamma_0)}$$

(2)

which is $k$-dependent and so spatially dispersive, or nonlocal. Here, the damping constant $\Gamma_D$ stands to (phenomenologically) include the inelastic electron scattering rate. It can be seen that as $d$ decreases, $\omega_p(k)$ shifts to the red and Equation 1 acquires the $\sqrt{k}$-type nonlocal spatial dispersion of 2D materials. In the TD regime, for ultrathin plasmonic films of finite thickness, the low-frequency response is controlled by Equations 1 and 2, with $\varepsilon_b = 10$ contributed by both positive ion background and interband transitions [10, 14]. As $d$ increases and gets large,
with $C = \hat{E} k_s$. The local Drude model can be retrieved from the above expressions in the limit $d \to \infty$, which leads to $\omega_p^{(0)}$ for both $\omega_p(k)$ and $\omega_p$.

In our comparative study below we use TiN, Au, and Ag parameters taken from experimental data and listed in Table 1 for convenience.\textsuperscript{(10,39,40)} Transition metal nitrides have emerged as a new class of materials with great promise to substitute noble metals such as gold and silver,\textsuperscript{(41)} which have exceptional plasmonic properties but relatively low melting temperatures making them incompatible with semiconductor fabrication technologies. Titanium nitride has low-loss plasmonic response and high melting point, making it structurally stable TD material capable of forming stochiometrically perfect ultrathin films of controlled thickness down to one nanometer at room temperature. For the sake of convenience as surface roughness correction $A$ we use throughout the manuscript the value $A = 0.12$ eV nm as experimentally found for nanometer-thick TD TiN films in Ref.\textsuperscript{(10)} This value of the surface roughness is of course highly dependent on the specific sample, but also the Drude parameters show some variance for different samples\textsuperscript{(39,40)} and in particular for thin films.\textsuperscript{(42)}

### Table 1. Materials studied.

| Material | $n$ | $\omega_p$ | $k_s$ | $\ell_{bulk}$ | $\Gamma_{bulk}$ |
|----------|----|--------|------|--------------|---------------|
| Au       | 1.39 | 9.5 | 2.9 | 3.3 | 42 | 0.02 |
| Ag       | 1.39 | 4.2 | 4.5 | 5.2 | 56 | 0.016 |
| TiN      | 0.67 | 9.1 | 2.5 | 6.0 | 45 | 0.2 |

3. Far-Field Thermal Radiation

We consider a thin film as used in transmission electron microscopy or soft X-ray spectral purity filter applications,\textsuperscript{(43)} see Figure 1a. The HF $\Phi_{df}$ of a single film at temperature $T_f$, radiating in its surrounding at temperature $T_b$, is given by\textsuperscript{(44)}

$$\Phi_{df} = \int_0^{\infty} \frac{d\omega}{2\pi} \left[ \Theta(\omega, T_f) - \Theta(\omega, T_b) \right] \int_0^{\infty} \frac{dk}{2\pi} k^2 \sum_{r,s,p} c_i$$

(10)

with $k$ and $k_0 = \omega/c$ being the in-plane and vacuum wave vector absolute values, respectively. The thermal emission is driven by the temperature difference of a harmonic oscillator

$$\Theta(\omega, T) = \frac{h \omega}{e^{\frac{h \omega}{k_b T}} - 1}$$

(11)

For a given polarization $i = s, p$, the film emissivity is $c_i(\omega, k)$, $1 - |R_i|^2 - |T_i|^2$, where $R_i$ and $T_i$ are the Fresnel reflection and transmission coefficients at a given frequency $\omega$ and the incidence angle (represented here by $k$). They are given by

$$R_i = \frac{r_i (1 - e^{i2\pi d})}{1 - r_i e^{i2\pi d}}$$

(12)

$$T_i = \frac{(1 - r_i^2) e^{i2\pi d}}{1 - r_i e^{i2\pi d}}$$

(13)

where $k_s = \sqrt{k_s^2 - \ell^2}$ and $r_i$ are the interface Fresnel reflection coefficients defined as

$$r_i = \frac{k_0 - k_s}{k_s + k_0}$$

(14)

$$r_p = \frac{k_0 e(\omega, k) - k_s}{k_0 e(\omega, k) + k_s}$$

(15)

with $k = \sqrt{k_s^2 - \ell^2}$. If, for example, $|R_i = s, p|^2 = |T_i = s, p|^2 = 0$ for all $\omega$ and $k$, then

$$\Phi_{bs} = \sigma (T_f^4 - T_b^4)$$

(16)

with $\sigma = 5.6705 \times 10^{-8}$ W m$^{-2}$K$^{-4}$ being the Stefan–Boltzmann constant, to give the blackbody HF $\Phi_{bs} = 64.4$ W m$^{-2}$ for $T_f = 310$ and $T_b = 300$ K.

Figure 2 compares the film thickness dependence of the HF from Equation 10 for TiN, Au, and Ag films using the local Drude model and the non-local KR model. For large $d$ the HF converges to a constant bulk sample value. This value highly depends on the damping, the bulk damping $\Gamma_{bulk}$ in this case, which determines the absorptivity and therewith the emissivity, so that the higher is $\Gamma_{bulk}$ the stronger is the thermal emission. Nonetheless, in all three cases considered the radiative HF is one to three orders of magnitude less than the blackbody value, confirming that metals are poor thermal emitters due to their high reflectivity. With film thickness reduction the HF starts increasing, reaches a maximum, and then declines. This effect is known for a long time.\textsuperscript{(45)} It was discussed within the framework of fluctuational electrodynamics in Ref.\textsuperscript{(44)} and was measured for Ru and Pt thin films previously (see Refs.\textsuperscript{(43, 46)}). The maximum is known to take place at a length scale called the Woltersdorff length $d_{max}$ which for metals is typically very small and within the local Drude model in the limit $\omega \ll \Gamma_{bulk} \ll \omega_p$ can be written as Ref.\textsuperscript{(44)}

$$d_{max} = \frac{2 \Gamma_{bulk}}{\omega_p^2}$$

(17)

It can be seen in Figure 2 that $d_{max}$ of the local model is far below 10 nm for all three materials, whereas in the non-local model it shifts towards thicknesses greater than 10 nm. For TiN, in particular, it shifts from $d_{max} = 1.8$ nm (local model) to $d_{max} = 38$ nm (non-local model), whereby the nonlocal KR model predicts over an order of magnitude Woltersdorff length increase as compared to the conventional local Drude model. This can be understood from Equation 17, where $\omega_p \propto \omega_p^0 \sqrt{kd}$ with $kd \ll 1$ and the damping is dominated by $\Gamma_D \propto A \ell$ which is the case for the ultrathin TD films described by the non-local KR model as given by Equations 1 and 6, respectively. Clearly, $d_{max}$ has the tendency to increase for thinner films then. As a consequence, for $d > 10$ nm larger HF can be found in the non-local than in local model. For 20 nm thick Au and Ag films the HF predicted by the KR model is one order of magnitude greater than that of the local model. As the Woltersdorff length is in the range of 10–100 nm in the non-local model, the predicted HF enhancement effect might be used to test this model experimentally with well-characterized ultrathin film samples.
4. Near-Field Thermal Radiation

Here we aim to study the radiative heat exchange between the two thin films separated by distances smaller than the room-temperature thermal wavelength $\lambda_{th} = 10\mu m$; see Figure 1b. In this near-field regime, as opposed to the far-field regime discussed above, the blackbody HF value is no longer a limit due to the dominant contribution of evanescent waves unaccounted for by the Stefan–Boltzmann law.\[47\] The near-field thermal radiation enhancement was previously studied in a great number of experiments measured the HF between two planar samples.\[48–56\] For thin films supporting surface phonon polaritons, an enhancement factor of 700 times over the blackbody value was found for separations under 30 nm.\[57\] For metals there is also a near-field enhancement due to the s-polarized (magnetic) contribution of eddy currents, first reported by Polder and van Hove,\[47\] in contrast to the p-polarized (electric) surface phonon-polariton or SP contributions widely discussed for thin films.\[44,58,60\] For ultrathin TD metallic films the low-frequency SP contribution can also show an enhancement effect.\[14,44,60\] Similar effects for phonon-polaritonic films are predicted\[61,62\] and experimentally verified\[63\] as well.

The HF between the two identical thin films at temperatures $T_1$ and $T_2$ is given by  
\[
\Phi = \int_0^\infty \frac{d\omega}{2\pi} \left[ \Theta(\omega, T_i) - \Theta(\omega, T_j) \right] \frac{dk}{2\pi} k \left( \sum_{i=x,p} T_i \right) 
\]
with transmission coefficients of different form for the propagating ($k < k_0$) and evanescent ($k > k_0$) waves as follows ($i = s, p$)

\[
T_i = \begin{cases} 
  \left(1 - \frac{R_i}{|1 - R_i e^{i\omega d}|^2} \right)^2, & k < k_0 \\
  \frac{4\text{Im}(R_i)^2 e^{-2\omega l}}{|1 - R_i e^{i\omega d}|^2}, & k > k_0
\end{cases}
\]

Figure 2. The behavior of $\Phi_{BB}$ for the local (dashed lines) and nonlocal (solid lines) case of a free standing TiN, Au, Ag layer as function of the film thickness $d$ with $T_i = 310$ and $T_b = 300 K$.

Note that the KR model is valid for $k = 2\pi/\lambda > 2\pi/L$, and so our results here hold for samples with $L > \lambda_{th}$, where $\lambda_{th}$ is the dominant thermal wavelength determined by the Wien’s law.

Figure 3. Heat flux enhancements $\Phi_{nonloc}/\Phi_{BB}$ (solid lines) and $\Phi_{loc}/\Phi_{BB}$ (dashed lines) between two free standing same-thickness TiN films as functions of interfilm separation distance $l$ for films of thicknesses $d = 1, 5,$ and $10 nm$ at $T_1 = 310$ and $T_2 = 300 K$.

Figure 3 compares $\Phi_{loc}/\Phi_{BB}$ and $\Phi_{nonloc}/\Phi_{BB}$ computed from these equations for two TiN films of fixed $d$ within the local and non-local model, respectively, as functions of the interfilm distance $l$. The near-field enhancement $\Phi_{nonloc}/\Phi_{BB}$ can be seen very high for TD film systems with $l \leq 10 nm$ but is extremely reduced in the far-field for $l \geq 10\mu m$ in agreement with Figure 2. For ultrathin TD films separated by distances $l \leq 100 nm$ this effect should be easily accessible for measurements with today’s experimental methods. Since $k \geq 2\pi/L$ in the non-local KR model, $\exp(-2k,l) = \exp(-2kl)$ in the near-field quasi-static regime ($k \gg k_0$) in $T_3$; in Equation 19, whereby the $k$-integrand in Equation 18 is dominated by $k = 1/l$. Hence, $k \geq 2\pi/L$ translates into $L \geq 2\pi l$ which is typically the case in near-field experiments. In general, for $l < h_{bulk}$ the Lindhard–Mermin nonlocality might play a role as well.\[58,59\] In the most important low-frequency domain, where $k \leq k_s$ and the interelectron interaction is screened with collective plasma oscillations well defined, this is $k^2$-infinitesimal order nonlocality similar to that due to the pressure term of hydrodynamical Drude models, which is much smaller than the confinement-induced $k^2$-infinitesimal order nonlocality of the KR model we discuss.\[15\] A full treatment of such nonlocalities is out of the scope of the present work.

Figure 4a compares $\Phi_{loc}/\Phi_{BB}$ and $\Phi_{nonloc}/\Phi_{BB}$ as functions of $d$ for pairs of same-thickness TiN, Au, and Ag films separated by $l = 100 nm$. Depending on $d$ the KR model gives either larger or smaller HFs than the local model. For ultrathin TD films of $d < 10 nm$, in particular, the nonlocal model gives a much larger near-field enhancement than the local model. This enhancement is particularly strong for Au and Ag films. For the local model a similar enhancement would occur at distances less than 1 nm where the macroscopic approach is no longer valid.

The HF behavior in Figure 4a can be explained by the transition from a thick-film regime where the $s$-polarization is dominant to the TD regime where the $p$-polarized evanescent waves prevail.\[14,44,60\] This is shown in Figure 4b separately for $s$- and $p$-polarized contributions in a pair of same-thickness TiN films.
With \( d \) decreasing, the s-polarized wave contribution first dominates the HF but then drops down abruptly, making the HF decline. The sharply increasing p-polarized wave contribution of SPs picks up and starts playing the dominant role instead. This makes the HF increase again, with much greater enhancement predicted by the nonlocal KR model than that of the local Drude model for the ultrathin TD films.

The dominance of the SPs in the HF of ultrathin TD films can be directly observed in the transmission coefficients \( T_i \) (\( i = s, p \)) as well. Figure 5 shows \( T_i \) (\( i = s, p \)) in the \( \omega-k \) space for propagating (s) and evanescent (p) waves in a pair of TiN films with \( d = l = 5 \) nm. In the local model \( T_s \) has a significant contribution from the s-polarized eddy currents, whereas for the nonlocal case this contribution does not exist for such thin layers. Contrary to the local model, in the KR model the SP modes of \( T_p \) come out red-shifted\(^{10,15} \) toward the thermally accessible frequency range \( \approx 2 \times 10^{14} \text{rads}^{-1} \) (\( \lambda_{\text{th}} \approx 10 \mu m \)), explaining the greater HF enhancement predictions by the nonlocal KR model compared to the local Drude model.
5. Impact of a Substrate

Finally, we want to discuss the impact of a substrate on the far- and near-field thermal emission of the TD films. We focus on TiN films deposited on semi-infinite MgO and Si substrates studied recently in spectroscopic experiments at room and cryogenic temperatures. For their dielectric permittivities we use $\varepsilon_{\text{MgO}} = 3$ as reported in Ref. [10] and $\varepsilon_{\text{Si}} = 11.7$ as tabulated in Ref. [64].

To incorporate substrate material in the theory, one can still use the expressions in Equations 10 and 18 but with slightly different reflection coefficients as follows ($i = s, p$)

$$R_i = \frac{r_{sub} - k_{d} e^{i k_{d} d}}{1 - r_{sub} e^{i k_{d} d}}$$

(20)

with

$$r_{sub,i} = \frac{k_{sub} - k_{d}}{k_{sub} + k_{d}}$$

(21)

$$r_{sub,p} = \frac{k_{sub} e^{i (\omega, k)} - k_{d} e_{1}}{k_{sub} e^{i (\omega, k)} + k_{d} e_{1}}$$

(22)

Here, $k_{sub} = \sqrt{k_{d}^{2} e_{1} - k_{d}^{2}}$ with $e_{1}$ representing the substrate dielectric constant, which also enters Equations 1 and 2 of the nonlocal KR model. The latter is now taken to be equal to $\varepsilon_{\text{MgO}}$ and $\varepsilon_{\text{Si}}$, respectively. There are no transmission coefficients in the equations above because a semi-infinite substrate has zero transmission, whereby our results in what follows are valid for optically thick substrates only.

Figure 6a shows the single-film normalized far-field heat flux calculated for finite-thickness TiN films on MgO and Si substrates in comparison with the free-standing TiN film of the same thickness (cf. Figure 2). It can be seen that with substrate present the heat flux maximum is no longer there due to the dominant contribution of the substrate itself, whereby in the limit of $d \rightarrow 0$ almost entire flux $\Phi_{sf}$ comes from the substrate alone. This is true for both local and nonlocal model. However, for TiN films thinner than 100 nm the nonlocal model is significantly more sensitive to the substrate material than the local one. For instance, the local model gives $\Phi_{sf} \approx 0.25 \Phi_{BB}$ for 10 nm-thick films on all substrates, whereas the nonlocal model predicts $\Phi_{sf} \approx 0.57 \Phi_{BB}$ and $\Phi_{sf} \approx 0.68 \Phi_{BB}$ for MgO and Si substrate, respectively. Figure 6b shows our near-field regime results for two identical finite-thickness TiN films separated by distance $l = 100$ nm with and with no substrate. In this case the substrate impact exhibited by the non-local model is even more crucial as opposed to the local model predicting no substrate material sensitivity for films thicker than 3 nm. For ultrathin TD films the nonlocal KR model gives the heat flux in the range $10 \Phi_{BB} < \Phi < 90 \Phi_{BB}$ depending on the substrate material type. Note that the substrate permittivities considered here are such that $\varepsilon_{\text{MgO}} < \varepsilon_{\text{Si}}$ in the infrared. Higher permittivity substrates suppress the surface plasmon stronger which can be seen in our case for TD films thinner than 100 nm. Therefore, to observe experimentally an increased near-field radiative heat transfer due to surface plasmons in TD films, substrates with permittivities close that of vacuum are preferable.

6. Conclusion

In this work, we show that in the far-field regime the nonlocal KR electromagnetic response model predicts a significant increase for film thickness values at which the thermal emission of thin metallic films has a maximum, i.e. the Woltersdorff length is increased compared to the local Drude model. In the near-field regime the thickness of the metallic films at which the near-field heat transfer becomes SP dominated is also shifted to larger thickness values when using the nonlocal KR model. This leads to strongly enhanced near-field HFs between two thin metallic films or materials with thin metal coatings. Thus, the nonlocal KR model makes a significant impact on the understanding of thermal radiation transfer in TD materials both in the near-field and in the far-field regime. The nonlocal KR model can be tested in far- or near-field measurements...
with existing setups and well-characterized plasmonic films. Our analysis suggests that the theoretical treatment and experimental data interpretation for thin and ultrathin metallic films must incorporate the nonlocal effect as described by the KR model in order to provide reliable results in radiative heat transfer studies. Finally, the fact that the enhanced far- and near-field emission is achievable in the nonlocal KR model with much thicker films than the standard local Drude model erroneously predicts is crucial for thermal management applications with thin metallic films and coatings.

Acknowledgements
S.-A.B. acknowledges support from Heisenberg Programme of the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation), project no. 461632548. I.V.B. was supported by the U.S. National Science Foundation under Condensed Matter Theory Program Award No. DMR-1830874. Both authors gratefully acknowledge support from the Kavli Institute for Theoretical Physics (KITP), UC Santa Barbara, under U.S. National Science Foundation grant no. PHY-1748958, where this collaborative work was started. I.V.B. acknowledges KITP hospitality during his invited visit as a KITP Fellow 2022–23 made possible by the Heising-Simons Foundation.

Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords
non-local electromagnetic response, radiative heat transfer, transdimensional plasmonic materials, Woltersdorff length

Received: November 14, 2022
Revised: February 6, 2023
Published online: March 24, 2023

[1] A. Kossoy, V. Merk, D. Simakov, K. Leossion, S. Këna-Cohen, S. A. Maier, Adv. Opt. Mater. 2015, 3, 71.
[2] D. Shah, H. Reddy, N. Kinsey, V. M. Shalaev, A. Boltasseva, Adv. Optical Mater. 2017, 5, 1700065.
[3] G. M. Dubrovkin, B. Qiang, H. N. S. Krishnamoorthy, N. I. Zheludev, Q. J. Wang, Nat. Commun. 2018, 9, 1762.
[4] A. Rodríguez Echarri, J. D. Cox, F. J. García de Abajo, Optica 2019, 6, 630.
[5] Z. M. Abd El-Fattah, V. Mkhitaryan, J. Brede, L. Fernández, C. Li, Q. Guo, A. Ghosh, A. Rodríguez Echarri, D. Naveh, F. Xia, J. Enrique Ortega, F. J. García de Abajo, ACS Nano 2019, 13, 7771.
[6] K. P. Kelley, T. G. Folland, J. Ryan Nolen, N. Engheta, J. D. Caldwell, J.-P. Maria, Nano Lett. 2019, 19, 948.
[7] R. A. Manjivra, D. Rodrigo, R. Yu, J. Canet-Ferrer, D. S. Ghosh, R. Yongsumthon, D. E. Baker, A. Rezikyan, F. J. García de Abajo, V. Pruneri, Nat. Photon. 2019, 13, 328.
[8] Z. Sun, J. Beaumariage, Q. Wan, H. Alnatah, N. Houglund, J. Chisholm, Q. Cao, K. Watanabe, T. Taniguchi, B. M. Hunt, I. V. Bondarev, D. Sneke, Nano Lett. 2021, 21, 7669.
[9] D. Shah, M. Yang, Z. Kudyshev, X. Xu, V. M. Shalaev, I. V. Bondarev, A. Boltasseva, Nano Lett. 2022, 22, 4622.
[10] I. V. Bondarev, V. M. Shalaev, ACS Photon. 2019, 6, 1.
[11] I. V. Bondarev, C. M. Adhikari, Phys. Rev. Applied 2021, 15, 034001.
[12] I. V. Bondarev, Ann. Phys. (Berlin) 2022, https://doi.org/10.1002/andp.202200331.
[13] I. V. Bondarev, H. Mousavi, V. M. Shalaev, Phys. Rev. Research 2020, 2, 013070.
[14] I. V. Bondarev, V. M. Shalaev, Optical Mater. Express 2017, 7, 3731.
[15] I. V. Bondarev, H. Mousavi, V. M. Shalaev, MRS Commun. 2018, 8, 1092.
[16] I. V. Bondarev, Optical Mater. Express 2019, 9, 285.
[17] I. V. Bondarev, O. L. Berman, R. Y. Kezerashvili, Y. E. Lozovik, Commun. Phys. 2021, 4, 134.
[18] J. Pelliciari, S. Lee, K. Gilmore, J. Li, Y. Gu, A. Barbour, I. Laurige, C. H. Ahn, F. J. Walker, V. Bisogni, Nat. Mater. 2021, 20, 188.
[19] L. Zundel, P. Gieri, S. Sanders, A. Manjavacas, Adv. Optical Mater. 2022, 10, 2102550.
[20] R. H. Ritchie, Phys. Rev. 1957, 106, 874.
[21] E. N. Economou, Phys. Rev. 1969, 182, 539.
[22] D. A. Dahl, L. J. Sham, Phys. Rev. B 1977, 16, 651.
[23] T. N. Theis, Surf. Sci. 1980, 98, 515.
[24] T. Ando, A. B. Fowler, F. Stern, Rev. Mod. Phys. 1982, 54, 437.
[25] A. V. Chaplik, Surf. Sci. Rep. 1985, 5, 289.
[26] Z. L. Wang, Micron 1996, 27, 265.
[27] J. M. Pitarke, V. M. Silkin, E. V. Chulkov, P. M. Echenique, Rep. Prog. Phys. 2007, 70, 1.
[28] A. Poltano, G. Chiarello, Front. Mater. 2014, 1, 9.
[29] L. Vertchenko, L. Leandro, E. Shkondin, O. Takayama, I. V. Bondarev, N. Akopian, A. V. Lavrinenko, Optical Mater. Express 2019, 9, 2117.
[30] L. J. Mendoza-Herrera, M. C. Tebaldi, L. B. Scaffardia, D. C. Schinca, Phys. Chem. Chem. Phys. 2022, 24, 28019.
[31] X. Liu, J.-H. Kang, H. Yuan, J. Park, S. J. Kim, Y. Cui, H. Y. Hwang, M. L. Brongersma, Nat. Nanotechn. 2017, 12, 866.
[32] D. A. Iranzo, S. Nanot, E. J. C. Dias, I. Epstein, C. Peng, D. K. Efetov, M. B. Lundeberg, R. Parret, J. Osmond, J.-Y. Hong, J. Kong, D. R. Englund, M. N. R. Peres, F. H. L. Koppens, Science 2018, 360, 291.
[33] S. Campione, I. Brener, F. Marquier, Phys. Rev. B 2015, 91, 121408(R).
[34] N. Rivera, I. Kaminer, B. Zhen, J. D. Joannopoulos, M. Soljačić, Science 2016, 353, 263.
[35] C. M. Adhikari, I. V. Bondarev, J. Appl. Phys. 2021, 129, 015301.
[36] L. V. Keldysh, JETP Lett. 1980, 29, 658.
[37] N. S. Rytova, Moscow University Physics Bulletin 1967, 3, 30.
[38] R. L. Olmon, B. Slovick, T. W. Johnson, D. Shelton, S.-H. Oh, G. D. Boreman, M. B. Raschke, Phys. Rev. B 2012, 86, 235417.
[39] H. U. Yang, J. D’Archangel, M. L. Sundheimer, E. Tucker, G. D. Boreman, M. B. Raschke, Phys. Rev. B 2015, 91, 235337.
[40] A. Boltasseva, H. A. Atwater, Science 2011, 331, 290.
[41] S. Lang, H. S. Lee, A. Y. Petrov, M. Sturmer, M. Ritter, M. Eich, Appl. Phys. Lett. 2013, 103, 021905.
[42] P. J. van Zwol, D. F. Vles, W. P. Voortuuijzen, M. Peter, H. Vermeulen, W. J. van der Zande, J. M. Sturm, R. W. E. van de Kruis, F. J. Bijkert, J. Appl. Phys. 2015, 118, 213107.
[43] S.-A. Biels, D. Reddig, M. Holthaus, Eur. Phys. J. B 2007, 55, 237.
[44] W. Woltersdorff, Z. Phys. 1934, 91, 230.
[46] G. D. Mahan, D. T. F. Marple, Appl. Phys. Lett. 1983, 42, 219.
[47] D. Polder, M. van Hove, Phys. Rev. B 1971, 4, 3303.
[48] L. Hu, A. Narayanaswamy, X. Chen, G. Chen, Appl. Phys. Lett. 2008, 92, 133106.
[49] R. S. Ottens, V. Quetschke, S. Wise, A. A. Alemi, R. Lundock, G. Mueller, D. H. Reitze, D. B. Tanner, B. F. Whiting, Phys. Rev. Lett. 2011, 107, 014301.
[50] T. Kralik, P. Hanzelka, M. Zobac, V. Musilova, T. Fort, M. Horak, Phys. Rev. Lett. 2012, 109, 224302.
[51] M. Lim, S. S. Lee, B. J. Lee, Phys. Rev. B 2015, 91, 195136.
[52] J. I. Watjen, B. Zhao, Z. M. Zhang, Appl. Phys. Lett. 2016, 109, 203112.
[53] M. P. Bernardi, D. Milovich, M. Francoeur, Nat. Commun. 2016, 7, 12900.
[54] B. Song, D. Thompson, A. Fiorino, Y. Ganjeh, P. Reddy, E. Meyhofer, Nat. Nanotechn. 2016, 11, 509.
[55] T. Kralik, V. Musilova, T. Fort, A. Srnka, Phys. Rev. B 2017, 95, 060503 (R).
[56] A. Fiorino, D. Thompson, L. Zhu, R. Mittapally, S.-A. Biehs, O. Bezenecnet, N. El-Bondry, S. Bansropun, P. Ben-Abdallah, E. Meyhofer, P. Reddy, ACS Nano 2018, 12, 5774.
[57] A. Fiorino, D. Thompson, L. Zhu, B. Song, P. Reddy, E. Meyhofer, Nano Lett. 2018, 18, 3711.
[58] A. I. Volokitin, B. N. Persson, Phys.-Usp. 2007, 50, 879.
[59] P.-O. Chapuis, S. Volz, C. Henkel, K. Joulain, J.-J. Greffet, Phys. Rev. B 2008, 77, 035431.
[60] S.-A. Biehs, Eur. Phys. J. B 2007, 58, 423.
[61] M. Francoeur, M. P. Menguc, R. Vaillon, Appl. Phys. Lett. 2008, 93, 043109.
[62] P. Ben-Abdallah, K. Joulain, J. Drevillon, G. Domingues, J. Appl. Phys. 2009, 106, 044306.
[63] B. Song, Y. Ganjeh, S. Sadat, D. Thompson, A. Fiorino, V. Fernández-Hurtado, J. Feist, F. J. Garcia-Vidal, J. C. Cuevas, P. Reddy, E. Meyhofer, Nat. Nanotechn. 2015, 10, 253.
[64] E. D. Palik, Handbook of Optical Constants of Solids, Academic Press, Boston 1991.