Overdamped large-eddy simulations of turbulent pipe flow up to $Re_\tau = 1500$

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Abstract. We present results from large-eddy simulations (LES) of turbulent pipe flow in a computational domain of 42 radii in length. Wide ranges of shear the Reynolds number and Smagorinsky model parameter are covered, $180 \leq Re_\tau \leq 1500$ and $0.05 \leq C_s \leq 1.2$, respectively. The aim is to assess the effect of $C_s$ on the resolved flow field and turbulence statistics as well as to test whether very large scale motions (VLSM) in pipe flow can be isolated from the near-wall cycle by enhancing the dissipative character of the static Smagorinsky model with elevated $C_s$ values. We found that the optimal $C_s$ to achieve best agreement with reference data varies with $Re_\tau$ and further depends on the wall normal location and the quantity of interest. Furthermore, for increasing $Re_\tau$, the optimal $C_s$ for pipe flow LES seems to approach the theoretically optimal value for LES of isotropic turbulence. In agreement with previous studies, we found that for increasing $C_s$ small-scale streaks in simple flow field visualisations are gradually quenched and replaced by much larger smooth streaks. Our analysis of low-order turbulence statistics suggests, that these structures originate from an effective reduction of the Reynolds number and thus represent modified low-Reynolds number near-wall streaks rather than VLSM. We argue that overdamped LES with the static Smagorinsky model cannot be used to unambiguously determine the origin and the dynamics of VLSM in pipe flow. The approach might be salvaged by e.g. using more sophisticated LES models accounting for energy flux towards large scales or explicit anisotropic filter kernels.

1. Introduction
One of the most conspicuous features of wall-bounded turbulent flows is the prevalence of near-wall streaks, which consist of elongated neighbouring regions of high- and low speed fluid alternating in the spanwise direction [1]. It is well established, that their typical streamwise length and spanwise spacing in viscous units is independent of the Reynolds number ($Re_\tau$) and is about $O(10^3)$ and $O(10^2)$, respectively. For the example of streaks in pipe flow, characteristic streamwise and azimuthal (spanwise) wavelengths in the energy spectra are $\lambda^+ \approx 800$ and $\lambda^\theta \approx 130$, respectively, e.g. [2], and two-point correlations indicate a radial thickness of roughly 80 viscous units, e.g. [3].

For sufficiently high $Re_\tau \geq O(10^3)$, there is growing experimental evidence of the rise of very large-scale motions (VLSM) of several outer units in extent. In pipe flow, very long meandering structures carrying a substantial portion of the kinetic energy in streamwise wavelengths exceeding $12R$, where $R$ is the pipe radius, have been reported [4, 5, 6, 7]. In boundary layer flow, Hutchins & Marusic [8, 9] detected superstructures commonly exceeding $20\delta$ in streamwise length, where $\delta$ is the boundary layer thickness. Their spanwise extent is $O(1)$ in outer units,
and so as $Re$ increases there is an increasing scale separation between superstructures and near-wall structures. While glimpses of VLSM have been observed [10] already at $Re_{\tau}$ as low as 380, a distinct outer peak in the azimuthal spectra only emerges for $Re_{\tau} \geq 1500$ [2]. A distinct outer peak in the streamwise spectra develops at much larger $Re_{\tau}$ [11, 12]. In addition to these smoothly growing outer peaks in the energy spectra, a sudden increase of the streamwise turbulence intensity in the outer region occurs at $Re_{\tau} \approx 1500$ and can be traced back to VLSM-relevant length scales [13, 14, 2]. Because there is no universal definition available yet, we hereinafter use the terms superstructures and VLSM interchangeably.

Uncovering the onset and origin of superstructures, analysing their dynamics, and determining their contribution to the global momentum transport are challenging tasks because of the large Reynolds numbers at which they can be unambiguously observed. Numerical studies of wall-bounded turbulence at the highest Reynolds numbers attained to date (see e.g. [15, 16] for channel flow at $Re_{\tau} = 4200$ and 5200 or [2, 17] for pipe flow at $Re_{\tau} = 1500$ and 3008) barely reach the onset of superstructures. This limitation is because of the enormous computational costs of resolving all spatial and temporal scales with direct numerical simulation (DNS) of the Navier–Stokes equations. Note that not only $Re_{\tau}$ must be large, but additionally the computational domain must also be large enough in terms of outer units (e.g. $42R$ in pipe flow) to capture the energetically most relevant length scales associated with VLSM [2].

Large-eddy simulations (LES) [18, 19] with the static Smagorinsky model [20] have been extensively used by Cossu, Hwang and co-workers in an attempt to isolate superstructures from the near wall cycle in turbulent channel [21, 22, 23] and plane Couette [24] flows. Their approach — termed overdamped LES (ODL) in [25] — is based on the idea that by increasing the Smagorinsky parameter $C_s$, an increasing range of small-scale structures is damped out (i.e. accounted for by the eddy-viscosity model), whereas large structures may be maintained without losing valuable resolution [25]. For two reasons, ODL is a very appealing approach to investigate the physical mechanisms underlying superstructures. First, compared to DNS, LES allows for a Reynolds number roughly one order of magnitude larger for a given amount of computational resources. Second, and even more important, the use of ODL reduces the complexity of the resolved flow field substantially because of the absence of small rapidly varying structures. This makes high $Re$ flows, and thus superstructures, accessible to dynamical-systems approaches, which have proven very useful in understanding low Reynolds number (transitional) flows [26, 27, 28].

We here apply the ODL approach proposed by Cossu, Hwang and co-workers to pipe flow. We perform LES with the Smagorinsky model in a computational domain of length $42R$ for $180 \leq Re_{\tau} \leq 1500$ and $0.05 \leq C_s \leq 1.2$. The paper is structured as follows. In §2, we briefly introduce the details of our numerical approach, before comparing our results to DNS reference data sets, as well as former ODL of channel and Couette flows in §3. Finally, in §4 we discuss the effect of $C_s$ on the resolved flow field and turbulence statistics and assess the feasibility of studying superstructures with ODL.

2. Numerical approach
We consider an incompressible Newtonian fluid with constant properties confined by a cylindrical wall of diameter $D = 2R$ and length $L$. The fluid is driven in the axial direction with a constant bulk velocity $u_b$. The flow is described through the Navier–Stokes equations, where $t$ denotes the time, $p$ the pressure, and $u$ the velocity vector. Its components $u_r$, $u_\theta$, and $u_z$ point along the cylindrical coordinates in the radial ($r$), azimuthal ($\varphi$), and axial ($z$) directions.

The pressure gradient $\nabla p = \nabla p' + P$ is separated into a fluctuating part $\nabla p'$ and a constant part $P$ driving the flow, such that periodic boundary conditions (BC) can be employed for $p'$ in $\theta$ and in $z$. For $u$, the governing equations are complemented with periodic BC in $\theta$ and $z$ as well as with no-slip and impermeability BC at $r = R$ to model a smooth and rigid pipe wall.
The only physical control parameter is the bulk Reynolds number

$$Re = \frac{Du_b}{\nu}$$

(1)

where $\nu$ is the kinematic viscosity of the fluid.

We use the pseudo-spectral simulation code openpipeflow.org [29] to solve the filtered Navier–Stokes equations in the form

$$\nabla \cdot \mathbf{u} = 0 \quad \text{and} \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \frac{1}{Re} \Delta \mathbf{u} - \nabla \cdot \mathbf{\tau} = \mathbf{P}$$

(2)

in combination with a standard eddy viscosity closure model. The driving pressure gradient $\mathbf{P}$ is adapted every time step such that a constant mass flux through the pipe domain is maintained. Filtered quantities are denoted by an overbar and $\mathbf{\tau}$ is the unknown residual stress tensor. To integrate eq. (2) in time, openpipeflow.org employs a second-order predictor-corrector scheme with dynamic time-step control. For the spatial discretisation, Fourier expansions are used in the two homogeneous directions $\theta$ and $z$ and finite differences with a seven point stencil are used in $r$. Grid points are distributed in $r$ according to a hyperbolic tangent function in such a way that high wall-normal velocity gradients in the viscous sublayer can be resolved. Additionally, the first few points for $r \approx 0$ are also clustered to preserve the high order of the finite difference scheme across the pipe axis.

The simple but robust linear eddy viscosity model of Smagorinsky [20] is used to relate the unknown residual stresses to the filtered rate of strain tensor ($\mathbf{S}$) via

$$\mathbf{\tau} = -2\nu_{r} \mathbf{S}$$

with

$$\nu_{r} = D_{s}^{2} \left(2S_{ij} S_{ij}\right)^{1/2}$$

and

$$l_{s} = C_{s} \Delta,$$

where

$$\Delta = \left(\Delta z \cdot r \Delta \theta \cdot \Delta \tau\right)^{1/3},$$

are the components of $\mathbf{S}$. Through the model parameter $C_{s}$, the Smagorinsky (mixing) length scale $l_{s}$ is taken to be proportional to the volumetric mean grid filter width

$$D(y^{+}) = \left(1 - e^{-\left(y^{+}/26\right)^{2}}\right)^{2}$$

(5)

to enforce the correct behaviour of $l_{s}$ close to the wall, see e.g. [31, 32, 33, 19] for commonly used formulations. Here, $y^{+} = (R - r)Re_{r}$ is the distance from the pipe wall in viscous (wall) units denoted by plus. The relevant viscous length scale is, as in [21, 22, 24], adaptively calculated in the course of the simulation based on the instantaneous shear velocity $\langle u_{r}\rangle_{\theta,z}$, where angle brackets denote averaging with respect to the noted subscripts. Note that as for all eddy viscosity approaches with $\nu_{r} > 0$, the Smagorinsky model does not allow for backscatter, i.e. no energy can be transferred from unresolved (small) to resolved (large) scales.

A summary of the spatial resolutions used in our LES is provided in tab. 1. For all $Re_{r}$, the
resolution is chosen to be roughly the same when scaled in wall units. When $C_s$ is increased, according to the ODL approach proposed by Cossu and co-workers [25] the grid resolution is kept constant, so that $C_s$ is the only free parameter to change the mixing length scale $l_s$ and thus enhance the dissipative character of the closure model. With the resolution listed in tab. 1, $\Delta z^+$ evaluates to $O(\Delta z^+) \leq 5$ depending on $r$. As a consequence, $l_s$, which is a uniform length scale representative of the smallest resolved eddies [34], reaches at most values of $O(4.2) \leq l_s^+ \leq O(42)$ (depending on $r$) for the highest $C_s$ considered here, see tab. 2. A typical size in $r$, $\theta$, and $z$ of the target structures to be damped out by the ODL approach — i.e. typical near-wall streaks — is roughly $80 \times 130 \times 800$ viscous units in pipe flow, e.g. [2, 3].

For the lowest $Re_\tau \approx 180$ considered here, a statistically steady flow field was generated in a well-resolved DNS and used as initial condition for the LES. For successively higher Reynolds numbers a statistically steady flow field from the LES of the previous $Re_\tau$ was used. For the LES at successively higher $C_s$, a statistically steady flow field from the previous $C_s$ was used. In all cases, turbulence statistics were sampled over at least $375^D/\nu_b$ after a transient phase of at least the same length.

### 3. Large eddy simulation results

#### 3.1. Instantaneous flow field

We visualise streaks by plotting iso-surfaces of the streamwise velocity fluctuations $u'_z = u_z - \langle u_z \rangle_{t,\theta,z}$ and focus on low-velocity streaks — i.e. connected regions of fluid moving slower than the local mean velocity $\langle u_z \rangle_{t,\theta,z}$ — defined by a negative threshold of $u'_z = -2.5u_\tau$. Although this choice is to some extent arbitrary, thresholds in the range of $-0.5u_\tau$ to $-2.5u_\tau$ are commonly used in the literature [10, 35, 36, 22]. In what follows, velocities are scaled in units of the actual $u_\tau \equiv \langle u_\tau \rangle_{t,\theta,z}$ resulting from each simulation and not with the nominal friction velocity according to Blasius’ law.

The effect of the Smagorinsky parameter $C_s$ on the structure and organisation of low velocity streaks is shown in fig. 1. The top panels show typical turbulent flow fields for two selected nominal $Re_\tau \in \{360,1500\}$ from LES with a commonly used moderate value of $C_s = 0.10$. They reveal the characteristic elongated streak structures, which decrease in size (when scaled in bulk units) and hence increase in number with growing $Re$. These qualitative observations for the regular $C_s$ values are in excellent agreement with flow field visualisations from pipe flow.
Reτ ≈ 360 (Re = 11 700)  

Reτ ≈ 1500 (Re = 59 600)

Figure 1. Visualisation of low-speed streaks in instantaneous flow field realisations from different LES. Shown are iso-surfaces of \( u_z' = -2.5u_\tau \) for nominal Reτ ≈ 360 (left) and Reτ ≈ 1500 (right) and increasing values of the Smagorinsky parameter Cs (top to bottom). The flow is directed from the top left to the bottom right, and out of the plane, respectively.

DNS data, see e.g. [37, 38, 14, 2] amongst many others. In addition to the near-wall streaks, the incipient formation of clusters with a large wavelength modulation can be observed for the highest Reτ ≈ 1500 considered here. This modulated clusters provide qualitative evidence for the rise of very large scale coherent motions [14], which can be more clearly discerned at even higher Reτ [8, 11, 12].

As Cs increases, the smallest visible length scales are gradually removed from the picture, while very large smooth streaks appear instead. The resulting three- and fourfold symmetries correspond to azimuthal streak spacings of approximately one to two pipe radii, while their radial extent is slightly less than \( R \). This trend — exemplarily shown for the two selected Reynolds
Table 2. Measured mean friction Reynolds number $\langle Re_{\tau} \rangle$ and its relative error $\epsilon_{\text{rel}}$ with respect to the nominal $Re_{\tau}$ from Blasius’ law as a function of the Smagorinsky model parameter $C_s$ at four different bulk Reynolds numbers ($Re$).

| $Re$ (contr. param.) | 5300 | 11 700 | 25 800 | 59 600 |
|----------------------|------|--------|--------|--------|
| $Re_{\tau}$ (Blasius) | 180.4| 360.7  | 720.6  | 1499.3 |
| $\langle Re_{\tau} \rangle$ | 195.3 | 398.6 | 784.8 | 1626.2 |
| $\epsilon_{\text{rel}}$ | 0.083 | 0.105 | 0.089 | 0.085 |
| $\langle Re_{\tau} \rangle$ | 165.6 | 366.2 | 741.8 | 1586.1 |
| $\epsilon_{\text{rel}}$ | -0.082 | 0.015 | 0.029 | 0.058 |
| $\langle Re_{\tau} \rangle$ | 160.9 | -0.108 |
| $\epsilon_{\text{rel}}$ | |

For $Re_{\tau} = 1500$ — the lowest Reynolds number for which we expect VLSM-related effects [2] — the Smagorinsky parameter has to be increased to $C_s = 1.2$ to observe a degree of small-scale quenching comparable to that observed here for $Re_{\tau} \approx 360$ or in [21, 22, 24], where $C_s \approx 0.3$ is sufficient. For only moderately increased values of $C_s \lesssim 0.4$, there is almost no reduction of complexity (removal of small scales) at $Re_{\tau} \approx 1500$. However, the observed clustering of smaller structures has been completely suppressed already at $C_s = 0.4$. Thus, increasing $C_s$ is comparable to reducing the Reynolds number, since the observed clustering emerges first for $Re_{\tau} \gtrsim O(1500)$ [14].

3.2. Non-dimensional shear stress

The effect of $C_s$ on the integral shear stress in the turbulent flow is quantified here by comparing the actual shear Reynolds number

$$\langle Re_{\tau} \rangle = \frac{R \langle u_{\tau} \rangle_{1,\theta,z}}{\nu}$$

extracted from each LES to the nominal $Re_{\tau} = f(Re)$ given by Blasius’ law at a fixed $Re$. Note that as $R$ and $\nu$ are fixed via the control parameter $Re$, changes in $\langle Re_{\tau} \rangle$ are due to changes in the wall shear stress $\tau_w \sim u_{\tau}^2$, and hence of the total energy input. The relative errors $\epsilon_{\text{rel}} = \langle Re_{\tau} \rangle/Re_{\tau} - 1$ for four different nominal $Re_{\tau}$ are given in fig. 2b and tab. 2, where positive/negative values correspond to over-/underestimated shear stress. With increasing $C_s$ the measured shear stress decreases and reaches a minimum (which depends on $Re_{\tau}$). For the two lowest $Re_{\tau}$, the flow laminarises completely around this minimum when $C_s$ is further
increased. The shear stress corresponding to this laminar flow state is indicated by full symbols and is much higher compared to the shear stress of Hagen-Poiseuille (HP) flow at the same $Re$, which is further discussed in §3.3. This demonstrates, that a large $\langle Re_\tau \rangle$ does not necessarily imply dynamical/kinematic similarity with a truly high $Re_\tau$ turbulent pipe flow. For the two highest $Re_\tau$, the measured $\langle Re_\tau \rangle$ increases again when $C_s$ is further increased, before ultimate laminarisation takes place. The dependence of the laminarisation threshold $C_{s,\text{lam}}$ on $Re_\tau$ is illustrated in fig. 2a.

Minimizing $|\epsilon_{\text{rel}}|$ by linear interpolation yields an estimate of the optimal $C_{s,\text{LES}}$ for a standard LES in the sense that the measured shear stress is closest to the nominal value from Blasius’ law for a fully-developed healthy turbulent flow. Of course this choice does not necessarily optimise one-point statistics, energy spectra, etc. as discussed in the following sections. Interestingly, the value of $C_{s,\text{LES}}$ increases from about 0.075 to 0.151 as $Re_\tau$ is increased (see squares in fig. 2a). To some extent, $C_{s,\text{lam}}$ quantifies the proposed/observed reduction of the effective Reynolds number with increasing Smagorinsky parameter: a two or roughly three times higher $C_s$ is needed to laminarise the flow for a two or three times higher $Re_\tau$, respectively.

### 3.3. Low-order-moment turbulence statistics

As a next step, we analyse the effect of an elevated Smagorinsky parameter on low-order turbulence statistics. Figure 3 shows the mean and root-mean-square (RMS) velocity profiles for $Re_\tau \approx 360$ scaled in inner (a, c, e, g) as well as in outer (b, d, f, h) units. Independent DNS data sets [38, 2] are shown as a reference (with excellent agreement). The best matches to the DNS data are obtained for Smagorinsky parameters roughly between $C_s = 0.05$ and 0.10, depending on the quantity and the wall normal location. Thus, the optimal $C_s$ value estimated by matching first- and second-order turbulence statistics are slightly smaller than that from the (integral) mean shear stress ($C_{s,\text{LES}} = 0.114$).
\( u_z = y \)
\( u_z = 5/2 \log \left( y^+ \right) + 5 \)
\( Re \tau \approx 360 \)

Figure 3. Mean (a, b) and RMS (c-h) velocity statistics calculated from LES at \( Re \tau \approx 360 \) with varying \( C_s \) and fixed spatial discretisation. The data is presented in inner (a, c, e, g) and outer (b, d, f, h) units. Independent DNS data [38, 2] as well as the HP solution (b), the linear law (a), and the logarithmic law (a) are shown as references. Red and green lines localise best matches to the DNS data. Unlabelled grey lines correspond to intermediate \( C_s \) as listed in tab. 2. Additionally, an instantaneous velocity profile for the stationary flow state after laminarisation is presented in panel b). DNS data for the next lower \( Re \tau \approx 180 \) is included in panel h).
By further increasing $C_s$, the mean velocity profile departs almost everywhere from the reference profile. Figure 3a shows that the mean velocity develops a large wake component already for moderately increased $C_s$. Note that large wake components are characteristic of low Reynolds number (transitional) pipe flow. Figure 3b shows that for $C_s \rightarrow C_s,\text{lam}$, the HP solution is quickly approached but never reached. The wall-normal mean velocity gradient, and thus the measured $\langle \tau \rangle$, is still much higher despite the flow being laminar ($u_r' = u_\theta' = u_z' = 0$). This is because the implementation of the Smagorinsky model used here and also in [21, 22, 24] leads, incorrectly, to $\tau = C_s \langle \nabla \bar{u} \rangle$ for a laminar flow with $C_s \neq 0$, due to the non-vanishing components of $\mathbf{S}_{ij}$ related to the mean velocity profile, see pp. 597 in [19]. Alternative implementations based on $\mathbf{S}_{ij} - \langle \mathbf{S}_{ij} \rangle$ (e.g. [39, 31]), however, suffer from the disadvantage that an appropriate averaging operation has to be performed during the simulation.

Figures 3c–f reveal that with increasing $C_s$ both cross-stream components of the RMS velocity fluctuations are reduced almost everywhere in the radial direction except for a small region close to the centreline, where the level of turbulence seems nearly unaffected by $C_s$. The RMS of the streamwise velocity fluctuations shown in fig. 3g–h exhibit a progressive outward shift of the high intensity near-wall peak as $C_s$ increases. As a consequence of the shift, the streamwise turbulence intensity in the outer region increases substantially. While the peak amplitude remains roughly unchanged in wall units, it is gradually reduced for increasing $C_s$ when scaled in outer units. Additionally, fig. 3h includes DNS reference data for the next lower $Re \approx 180$. This shows that increasing the Smagorinsky parameter for $Re \approx 360$ by a factor of two (from $C_s = 0.1$ to $C_s = 0.2$), perfectly matches the RMS level (for over 80% of the radial direction) and the RMS peak position of DNS reference data for $Re \approx 180$. These features are common to all simulations performed here. Figure 4 shows the variation (in inner and outer units) of the near-wall peak (level and location) and of the centreline value of $\text{RMS}(u_z)$ as a function of $Re$ and $C_s$. The observed trends suggest a smooth outward displacement of the near-wall cycle as $C_s$ is increased. Hence, the effect of increasing $C_s$ is actually qualitatively similar to reducing the effective Reynolds number of the flow, even though the bulk Reynolds number (i.e. the control parameter) is unchanged and the measured shear stress is roughly similar (see fig. 2b). This suggests, that the measured $\langle \tau \rangle$ cannot be used to infer dynamical similarity for a given $Re$. $Re$ implies similarity in the near wall region and only has general/global meaning for fully developed, genuinely turbulent pipe flow. It does not guarantee global kinematic similarity in other situations. For example, in oscillatory pipe flow, the instantaneous $\langle \tau \rangle$ can be very high,
In the following, we examine the effect of $C_s$ on the energy distribution across scales at different wall-normal locations. Since VLSM are mostly recognised as long and wide regions of purely positive or negative $u_z$, we focus on streamwise and azimuthal energy spectra of $u_z$, denoted as $E_{zz} (\kappa_z)$ and $E_{\theta z} (\kappa_\theta)$, respectively. All spectra are scaled in outer units, since we are mainly concerned with superstructures, which scale in outer units. Figures 5a–d show streamwise spectra at $Re_\tau \approx 360$ for increasing distance from the wall and different values of $C_s$. Additionally, fig. 6a–d show azimuthal spectra for the same parameters. Comparison with the reference DNS data reveals again best matches for $0.05 < C_s < 0.15$, which agrees with the optimal $C_{s, LES} = 0.114$ obtained by matching the measured dimensionless shear stress with the nominal value from Blasius law (see §3.2). The optimal $C_s$ for the spectra depends slightly
on the wavenumber range and also on the wall-normal location. In other words, $C_s$ changes the slope of the spectrum differently strong at different wall-normal locations. At $y^+ = 15$, a moderate value of $C_s = 0.10$ gives best results for all $\kappa_z$. At $r'/R = 0.8$, on the other hand, best matches change gradually from $C_s = 0.10$ to 0.15 with increasing $\kappa_z$, while at $r'/R = 0.4$ a uniform $C_s = 0.15$ leads to best agreement for all $\kappa_z$. For the entire range of $C_s$, the energy in the lower wavenumbers increases by factors of $8$ to $10$ in the outer region ($r < 0.8R$). In the buffer region, the energy level also increases with $C_s$, but only by factors of $6$ to $8$ in the lower wavenumbers, see fig. 5 and 6. In contrast to the lowest $Re_z \approx 180$ (not shown), for all $Re_z \geq 360$ the highest energy increase does not occur for the strongest possible overdamping (i.e. $C_s = 0.3$ in this case), but for moderately elevated $0.12 < C_s < 0.3$. This effect is mainly restricted to the buffer region (black arrows in fig. 5a–b) and absent in the outer region (black arrows in fig. 5d). The azimuthal spectra presented in fig. 6 have similar features and similar effects are observed at all other $Re$ considered (hence not shown). The observed trends are consistent with the fact, that the LES filter is local in physical space and thus non-local in Fourier space (the energy spectra change slope everywhere).

3.5. Pre-multiplied energy spectra

We now turn to pre-multiplied energy spectra, which — in contrast to the energy spectra — allow a direct comparison of the energetic contribution of different ranges of wavenumbers. To compensate for the varying amount of energy input, which is proportional to $u_w^2 \sim \tau_w$ for fixed $u_b$, the energy spectra are scaled in inner units based on the measured ($Re_z$). As the effect of $C_s$ on the pre-multiplied spectra is similar at all $Re_z$, we here focus on $Re_z \approx 1500$, at which incipient VLSM are present. Figures 7 and 8 present streamwise ($\kappa_z E_{zz}(\lambda_z)$) and azimuthal ($\kappa_\theta E_{zz}(\lambda_\theta)$) pre-multiplied energy spectra of the streamwise velocity as a function of the wavelength $\lambda$.

Figures 7a–c demonstrate that at wall-normal locations characteristic of the near-wall cycle ($y^+ \approx 15$) the energy in the range of wavelengths likewise characteristic of the near-wall cycle ($\lambda_z^+ \approx 800$) is strongly damped as $C_s$ increases beyond $C_{s, LES}$. However, in the outer region (fig. 7d–f) the energy in the range of wavelengths associated with VLSM ($\lambda_z \approx 20R$) substantially grows far beyond the level of energy observed for DNS or standard LES ($C_{s, LES} \approx 0.15$), as soon as $C_s$ is elevated to values from which on a clear damping of small scales is obvious and only smooth long streaks remain (fig. 1). For the same high $C_s = 1.2$, closer to the wall (fig. 7b–c), the level of energy at $\lambda_z \approx 20R$ drops far below the values observed for standard LES. For only moderate, but still elevated values (e.g. $C_s = 0.4$) on the other hand, the artificial increase of energy in the relevant large and intermediate $\lambda$ can be even larger than for the highest $C_s = 1.2$, e.g. fig. 7b–d.

Similar trends are observed in the azimuthal energy spectra presented in fig. 8. For sufficiently high $C_s$ in the sense of a clear reduction of small scales (fig. 1), energy in wavelengths associated with the azimuthal spacing of near-wall streaks ($\lambda_\theta^+ = 130$) is substantially damped out, whereas it is strongly increased at larger wavelengths associated with the azimuthal spacing of VLSM ($\lambda_\theta = 1R$). In the buffer region, energy is first strongly increased and than damped out as $C_s$ is smoothly elevated, as can be seen from fig. 8b–c. Looking at the series of spectra for smoothly increased $C_s$, this can be explained as follows. The near-wall peak, which is known to appear at $\lambda_\theta^+ \approx 130$ around $y^+ \approx 15$, is well reproduced at $C_s \in [0.10, 0.15]$. As the Smagorinsky parameter is increased by a factor of roughly three ($C_s = 0.4$), the near wall peak is shifted to a roughly three times larger wall distance ($y^+ = 50$) and a roughly six times larger wavelength (fig. 8c). Its near-wall-low-wavelengths flank replaces the near-wall peak at its original location (fig. 8b), while its far-wall-high-wavelength flank traverses the location of the VLSM (fig. 8d), i.e around $r'/R = 0.8$ and $\lambda_\theta = R$ [2]. At a fixed location and wavelength this seemingly appears as an energy amplification followed by damping as the modified near-wall peak passes by. This means, that the original near-wall peak is modified in physical as well as in wavenumber space
Figure 7. Comparison of streamwise pre-multiplied energy spectra for $u_z$ from different LES at $Re_x \approx 1500$ with varying $C_s$. Shown are spectra extracted at several wall normal locations from the edge of the viscous sublayer ($y^+ = 5$) up to the outer region ($r \leq 0.8R$). Unlabelled grey lines correspond to further intermediate $C_s$ as listed in tab. 2.

when $C_s$ is elevated, until, eventually, its dynamics are constrained by the pipe wall. In order to verify this claim, an examination of the dynamics of the structures for different $C_s$ should be conducted, as in [42], where various quantitative measures are used to categorise eddies.

4. Discussion
We performed an extensive LES study of turbulent pipe flow driven at a constant bulk velocity by using the classic static Smagorinsky model [20]. The computational domain length was fixed to $42R$, allowing the formation of VLSM [2], whereas wide ranges of the Reynolds number and the Smagorinsky model parameter were covered, $180 \leq Re_x \leq 1500$ and $0.05 \leq C_s \leq 1.2$, respectively. Overall, our analysis of wall shear stresses, mean velocity profiles, RMS velocity fluctuations, and energy spectra confirm the high quality of our LES of turbulent pipe flow for reasonable parameter ranges of $C_s$. The optimal Smagorinsky parameter to achieve best agreement with DNS reference data varies with $Re_x$ and further depends on the wall normal location, as well as the quantity of interest.

We selected an optimal value of the Smagorinsky constant ($C_{s,LES}$) by minimising the difference between the nominal friction according to Blasius’ law and the friction obtained from the LES. For the two lower $Re_x \approx 180$ and 360, we found $C_{s,LES} \approx 0.075$ and 0.114 respectively, which is very similar to those values reported in the literature ($0.065 \leq C_{s,LES} \leq 0.125$) to adequately reproduce reference data in turbulent pipe [37, 43, 30] and channel [31, 44, 45] flow LES at similar $Re_x$. For increasing $Re_x$, we found that increasing values of the Smagorinsky parameter produced better results. For the highest $Re_x \approx 1500$ investigated, $C_{s,LES} = 0.151$ approaches values normally used in LES of isotropic turbulence, as in the seminal work of Smagorinsky [20] or in the theoretical derivation by Lilly [46], with $C_s = 0.2$, and 0.17
respectively. This is in agreement with the fact that for increasing Reτ the flow gradually becomes more isotropic in a growing portion of the pipe domain. Hence it would be interesting to perform LES at yet larger Reτ than here to determine the value at which Cs,LES eventually saturates.

For the highest Reτ ≈ 1500 considered here, the incipient formation of long clusters of streaks leading to VLSM could be observed — in good agreement with recent pipe flow DNS, e.g. [17, 14], and experiments, e.g. [6, 10]. Our long term goal is to understand and model VLSM with dynamical systems approaches, i.e. via exact coherent solutions (ECS) of the governing equations. This requires however to substantially reduce the complexity of high Reynolds number flow fields by isolating VLSM from smaller-scale turbulent motions. We wanted to take this challenge by using overdamped LES following the work of Cossu [25] and co-workers. Their idea is that increasing the effective filter width (ls) of the Smagorinsky model via Cs, passivates an increasing range of small-scale structures without losing their dissipative effect on the next larger scales. We found in good qualitative agreement with these studies that small-scale streaks in simple flow field visualisations are gradually quenched and replaced by much larger smooth streaks as Cs increases. Surprisingly, for all parameter values reached here, the uniform length scale at which the Smagorinsky model is acting (i.e \( t_s^+ = C_s X_s^+ \leq C_s 35 \leq 42 \)), remains always much smaller than the size of the inhomogeneous/anisotropic target near-wall structures (i.e. \( 80 \times 130 \times 800 \) viscous units). However, our analysis of mean velocities, RMS velocity profiles and energy spectra suggest, that these structures originate from an effective reduction of the Reynolds number and thus represent enlarged/modified near-wall streaks rather than superstructures isolated from the original near-wall cycle. As the Cs is increased, the wall cycle progressively moves away from the wall and the associated streaks become longer and wider.

Figure 8. Comparison of azimuthal pre-multiplied energy spectra for \( u_z \) from different LES at Reτ ≈ 1500 with varying Cs. Shown are spectra extracted at several wall normal locations from the edge of the viscous sublayer (\( y^+ = 5 \)) up to the outer region (\( r \leq 0.8R \)). Unlabelled grey lines correspond to further intermediate Cs as listed in tab. 2.
when scaled in inner and outer units. Our streak visualisations in §3.1 reveal that for all \(Re_s\) the remaining structures reach an azimuthal and radial size similar to those of usual near-wall streaks in a turbulent low-Reynolds number (\(Re_s \approx 90\)) pipe flow, i.e. \(\Delta \theta = \Delta \theta^+ / Re_s = 130/90 = 1.4 R\) and \(\Delta r = \Delta r^+ / Re_s = 80/90 = 0.9 R\). Just before the flow is forced to relaminarise, connected streaks end up filling our 42R-long pipe domain. Furthermore, the energy contained in VLSM-related wavelengths is artificially increased by up to factors of seven (inner units) and ten (outer units) for elevated \(C_s\), likely because the modified higher-intensity near-wall cycle shifts to outer-scale locations/wavelengths where VLSM are expected to live. The method changes the flow similarly at all investigated \(Re_s\) regardless of the clear presence of VLSM, which in DNS may only be discerned at the highest Reynolds number investigated here (\(Re_s \approx 1500\)) \[2, 14\]. These observations are consistent with other LES studies, where \(C_s\) is varied more moderately \[47, 30, 43\], and can be explained by the purely dissipative nature of the Smagorinsky model, whose dissipation is strongly enhanced by increasing \(C_s\). The Smagorinsky model exclusively acts as an energy sink at a uniform length scale and, therefore, can neither mimic structural features of the unresolved motions nor account for feedback (e.g. backscatter, cluster formation etc.) to the resolved scales.

In view of these findings, we here suggest that the overdamped LES approach based on the static Smagorinsky model is not suitable to unambiguously determine the origin and the dynamics of superstructures in pipe flow. We expect that ECS computed from the ODL equations would not truly represent the nature of superstructures in turbulent pipe flow. Nevertheless, we believe that the proposal of Cossu \[25\] and co-workers — to eliminate the wall equations would not truly represent the nature of superstructures in turbulent pipe flow. We expect that ECS computed from the ODL approach might be salvaged by e.g. using a more sophisticated LES model accounting for backscatter \[48\] or maybe by applying explicit anisotropic filtering tailored to match the size and orientation of the target structures. Moreover, ODL appears to be a valuable numerical tool to find ECS using \(C_s\) as a third continuation parameter (besides \(Re\) and \(L\)), as already described in e.g. \[49\]. However, eventually \(C_s\) has to be reduced back to zero or at least to \(C_s\) LES to converge the intermediate ECS to solutions of the Navier-Stokes or the standard LES equations, so that they can actually be interpreted as exact coherent superstructures.

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References
[1] Kline S J, Reynolds W C, Schraub F A and Runstadler P W 1967 J. Fluid Mech. 30 741–773
[2] Feldmann D, Bauer C and Wagner C 2018 J. Turbul. in print 1–22
[3] Feldmann D and Wagner C 2012 J. Turbul. 13 1–28
[4] Kim K C and Adrian R J 1999 Phys. Fluids 11 417–422
[5] Guala M, Homemaa S E and Adrian R J 2006 J. Fluid Mech. 554 521–542
[6] Monty J P, Stewart J A, Williams R C and Chong M S 2007 J. Fluid Mech. 589 147–156
[7] Monty J P, Hutchins N, NG H C H, Marusic I and Chong M S 2009 J. Fluid Mech. 632 431–442
[8] Hutchins N and Marusic I 2007 Philos. T. Roy. Soc. A 365 647–664
[9] Hutchins N and Marusic I 2007 J. Fluid Mech. 579 1–28
[10] Hellström L H O, Sinha A and Smits A J 2011 Phys. Fluids 23 011703
[11] Mathis R, Hutchins N and Marusic I 2009 J. Fluid Mech. 628 311–337
[12] Rosenberg B J, Hultmark M, Vallikivi M, Bailey S C C and Smits A J 2013 J. Fluid Mech. 731 46–63
[13] Jiménez J 2007 Rev. R. Acad. Cien. Serie A. Mat. 101 187–203
[14] Bauer C, Feldmann D and Wagner C 2017 Phys. Fluids 29 125105
[15] Lozano-Durán A and Jiménez J 2014 *Phys. Fluids* **26** 011702
[16] Lee M and Moser R D 2015 *J. Fluid Mech.* **774** 395–415
[17] Ahn J, Lee J H, Kang J h and Sung H J 2015 *Phys. Fluids* **27** 065110
[18] Sagaut P 2006 *Large Eddy Simulation for Incompressible Flows: An Introduction* 3rd ed (Springer)
[19] Pope S B 2000 *Turbulent Flows* 7th ed (Cambridge U. Press)
[20] Smagorinsky J 1963 *Month. Weather Rev.* **91** 99–164
[21] Hwang Y and Cossu C 2010 *Phys. Rev. Lett.* **105** 044505
[22] Hwang Y and Cossu C 2011 *Phys. Fluids* **23** 061702
[23] Hwang Y, Willis A P and Cossu C 2016 *J. Fluid Mech.* **802** R1
[24] Rawat S, Cossu C, Hwang Y and Rincon F 2015 *J. Fluid Mech.* **782** 515–540
[25] Cossu C 2017 Exact invariant solutions for coherent turbulent motions in Couette and Poiseuille flows (KITP Talk)
[26] Eckhardt B, Schneider T M, Hof B and Westerweel J 2007 *Ann. Rev. Fluid Mech.* **39** 447–468
[27] Kawahara G, Uhlmann M and van Veen L 2012 *Ann. Rev. Fluid Mech.* **44** 203–225
[28] Cvitanović P 2013 *J. Fluid Mech.* **726** 1–4
[29] Willis A P 2017 *SoftwareX* **6** 124–127
[30] Schmidt S, McIver D M, Blackburn H M, Rudman M and Nathan G J 2001 *14th Australasian Fluid Mech. Conf.* pp 917–920
[31] Moin P and Kim J 1982 *J. Fluid Mech.* **118** 341–376
[32] Piomelli U, Ferziger J H and Moin P 1987 Models for large eddy simulations of turbulent channel flows including transpiration Tech. Rep. TF-32 Stanford University
[33] Kim W W and Menon S 1999 *Int. J. Numer. Meth. Fluids* **31** 983–1017
[34] Piomelli U, Moin P and Ferziger J H 1988 *Phys. Fluids* **31** 1884
[35] Lozano-Durán A, Flores O and Jiménez J 2012 *J. Fluid Mech.* **694** 100–130
[36] Lee J, Ahn J and Sung H J 2015 *Phys. Fluids* **27** 025105
[37] Unger F 1994 *Numerische Simulation turbulenter Rohrströmungen* Dissertation Technische Universität München
[38] El Khoury G K, Schlatter P, Noorani A, Fischer P F, Brethouwer G and Johansson A V 2013 *Flow Turbal. Combust.* **91** 475–495
[39] Schumann U 1975 *J. Comput. Phys.* **18** 376–404
[40] Feldmann D and Wagner C 2016 *Int. J. Heat Fluid Flow* **61** 229–244
[41] Kühnen J, Song B, Scarselli D, Budanur N B, Riedl M, Willis A P, Avila M and Hof B 2018 *Nature Physics*
[42] Jimenez J 2018 *J. Fluid Mech.* **842** P1–100
[43] Rudman M and Blackburn H M 1999 *Second International Conference on CFD in the Minerals and Process Industries* (Melbourne)
[44] Schmitt L, Richter K and Friedrich R 1986 *Direct and Large Eddy Simulation of Turbulence* (Notes on Numerical Fluid Mechanics vol 3) ed Schumann U and Friedrich R (Wiesbaden: Vieweg+Teubner Verlag) pp 161–176
[45] Arnal M and Friedrich R 1993 *Turbulent Shear Flows* **8** 1987 (Berlin, Heidelberg: Springer Berlin Heidelberg) pp 169–187
[46] Lilly D K 1967 *IBM Scientific Computing Symposium on environmental sciences* pp 195–210
[47] Prinz S, Boeck T and Schumacher J 2018 *Eur. J. Mech. B/Fluids* submitted
[48] Cimarelli A and De Angelis E 2014 *Phys. Fluids* **26**
[49] Rawat S, Cossu C and Rincon F 2017 *Procedia IUTAM* **20** 94–98