Output control of a single-airscrew helicopter’s longitudinal motion spectrum

Abstract

The physical principle of a single-airscrew helicopter’s spatial motion provides the opportunity to categorize it as a complex multidimensional dynamic system (control object). At the same time, it is possible to mark out the following problems in less detail, which can be attributed to the control of helicopters as multidimensional dynamic systems.1,2

a. The problem of stabilization, or the problem of stabilizing control law synthesis, i.e. determination of a feedback (of a controller), which provides stability for a disturbed motion. In searching for this kind of problem solution, as a rule, modal control methods or pole control methods are used.3–6

b. The closed-loop system decoupling problem, i.e. determination of a controller’s coefficients that provide decoupling of the control object’s subsystems. The group of methods used is based on the analysis and usage of the reference system operators’ kernels.7,8

c. The problem of making a set of stabilizing control laws, i.e. obtaining all, or almost all, control laws (algorithms), which provide stability for the closed-loop system, if the solution to the first problem, from those listed above, is not the only one. As a rule, the methods used are based on the solutions to Diophantine equations and the subsequent parameterization (for instance, the Youla-Kučera parametrization).7,9–12

Introduction

It is possible to find other, not so commonly used, statements and solutions to the problem. For instance, there is a statement and a solution to the problem when stability and decoupling of the closed-loop controlled system, and also specified placement of the invariant system’s zeros, i.e. complex frequencies, on which the closed-loop system “locks” completely or partially, can be provided simultaneously. The strengthened type of decoupling, when it is necessary to provide not only the block-diagonal form of the closed-loop system’s operator, but also specific placement of the invariant system’s zeros on the complex plane, belongs to it. Many of the practically important solutions to the problems listed above are not accompanied by published methods on how to obtain them. The analytical solutions have to fill this gap. In this paper for the first time the analytical solution to the problem of a single-airscrew helicopter’s stabilization in the vertical plane with the specified placement of poles and the incomplete measurement vector of motion parameters is presented. We use the linearized model of longitudinal motion of a single-airscrew helicopter (SH) having the following form:13

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
\dot{V}_x \\
\dot{V}_y \\
\dot{V}_z
\end{bmatrix}
= \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66}
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y \\
V_z \\
v_x \\
v_y \\
v_z
\end{bmatrix}
+ \begin{bmatrix}
V_x \\
V_y \\
V_z \\
V_x \\
V_y \\
V_z
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y \\
u_z
\end{bmatrix}
\tag{1}
\]

Here \( V_x \) – deviation from specified value of the longitudinal speed; \( V_y \) – deviation from specified value of the vertical speed; \( V_z \) – deviation from specified value of the pitch angular velocity; \( v_x \) – deviation from specified value of the angle of pitch; \( u_x \) – deviation angle of a main rotor’s cone in the longitudinal direction; \( a_{ij} \) – general pitch of a main rotor; \( a_{ij} \) – angular velocity of a main rotor; \( a_{ij} \) – linearization coefficients.1,4–14

We use the following notation

\[
x = (\Delta V_x, \Delta V_y, \Delta V_z, \Delta u_x, \Delta u_y, \Delta u_z)^T,
\]

and also

\[
a_1=a_{11}, a_2=a_{12}, a_3=a_{13}, a_4=a_{14}, a_5=a_{15}, a_6=a_{16},
\]

\[
a_7=a_{21}, a_8=a_{22}, a_9=a_{23}, a_{10}=a_{24}, a_{11}=a_{25}, a_{12}=a_{26},
\]

\[
a_13=a_{31}, a_14=a_{32}, a_15=a_{33}, a_{16}=a_{34}, a_{17}=a_{35}, a_{18}=a_{36},
\]

\[
b_1=b_{11}, b_2=b_{12}, b_3=b_{13}, b_4=b_{14}, b_5=b_{15}, b_6=b_{16},
\]

\[
b_7=b_{21}, b_8=b_{22}, b_9=b_{23}, b_{10}=b_{24}, b_{11}=b_{25}, b_{12}=b_{26},
\]

\[
b_13=b_{31}, b_14=b_{32}, b_15=b_{33}, b_{16}=b_{34}, b_{17}=b_{35}, b_{18}=b_{36}.
\]
Then instead of expression (1) we will obtain the following linearized model of the SH longitudinal motion:

$$\dot{x} = A x + B u, \quad y = C x. \quad (3)$$

Where $x \in \mathbb{R}^n$, $n=4$ is the state vector; $u \in \mathbb{R}^r$ is the input vector (control vector), where $r=2$; $y\in\mathbb{R}^m$ is the output vector (measurement vector), where $m=3$. Here $\mathbb{R}$ is the set of real numbers.

In the equations (3) the corresponding matrices can be written as

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (4)$$

If, as a control law (3), to suggest the expression of the following form:

$$u = y^T = FCx,$$  

Where $F \in \mathbb{R}^{r\times m}$ is a matrix of the output controller, then, in accordance with the system under consideration (3) – (5) a case of the dynamic MIMO-system output control will take place.

Here after we assume that matrix $B \in \mathbb{R}^{4\times 2}$ (4) has a full rank (rank $B=2$ in this case), or that its equivalent matrix $B^T \tilde{B}$ is invertible, i.e. $\det(B^T \tilde{B}) \neq 0$. Let us now consider the matrix spectrum $A \in \mathbb{R}^{4\times 4}$ (5). It will be understood as a set of matrices $A$ eigen values (poles), $\text{eig}(A) = \{\lambda_i \in \mathbb{C} : \det(\lambda_i I - A) = 0\}, \quad i \in [1,4]$. Here $I$ – identity matrix of size $4 \times 4$, $\mathbb{C}$ – the set of complex numbers. Let $A$ be the given spectrum of the matrix $A+BFC$ of the corresponding close-loop controlled system, i.e.

$$A = \begin{pmatrix} \lambda_1, & \lambda_2, & \lambda_3, & \lambda_4 \end{pmatrix}. \quad (6)$$

It is required to determine (i.e. synthesize) explicitly the controller matrix $F \in \mathbb{R}^{2\times 3}$, such that the equality $A = \text{eig}(A+BFC)$ should be satisfied exactly. The complexity of this problem is a necessity for obtaining a solution in explicit analytical form, since, in general, not necessarily the equality $B_i^e B_i^e = I_m$, is implemented.

**Theorem1.** Let $m > r$, and the following matrices exist and are pair wise completely controllable:

$$G_i = B_i A_i C_i, \quad H_i = B_i C_i, \quad i \in [0,M],$$

Then, there exists a nonempty set of matrices $K = \{0, M\}$, such that

$$\Phi_i = G_i + K_i H_i, \quad \Phi_i = \begin{pmatrix} B_i & A_i C_i & H_i \end{pmatrix}, \quad i \in [0, M],$$

satisfy the equalities of spectra

$$\text{eig}(A_i + B_i F_i C_i) = \bigcup_{i = k}^{M} \text{eig}(\Phi_i), \quad k \in [0, M],$$

Moreover, the following matrices exist and are pair wise completely controllable:

$$G_k = \begin{pmatrix} B_k C_k \end{pmatrix}, \quad H_k = \begin{pmatrix} B_k C_k \end{pmatrix}, \quad k \in [0, N].$$

Then, there exists a nonempty set of matrices $L_i, \quad i \in [0, N]$, such that

$$F_i = B_i^e \Psi_i + H_i L_i, \quad \Psi_i = \begin{pmatrix} B_i^e C_i \end{pmatrix}, \quad \Psi_i = \begin{pmatrix} B_i^e A_i C_i + (B_i^e C_i) \end{pmatrix}, \quad i \in [0, M],$$

and, for $F = B^e \Psi + A^e C^e \Psi_i^T F = B^e \Psi_i F_i, \quad \Psi_i = A_i \Psi_i, \quad k \in [0, N]$. 

**Decomposition of a dynamic system**

As a first step of the given problem solution we will consider the multilevel decomposition of the SH model suggested.\textsuperscript{15}–\textsuperscript{17} Since in this case the inequality $m > r$ (i.e. the number of system’s outputs is greater than the number of its inputs) is implemented, then, in general, not taking into consideration specific numerical values for $m$ and $r$, we consider the multilevel decomposition of system (3) – (5) of the following form: -- zero decomposition level

$$\dot{A}_0 = A_0, \quad B_0 = B, \quad C_0 = C \quad (7)$$

$– k$th decomposition level ($k = [0, M]$), where $M = \text{ceil}(n/r)$, \textsuperscript{*} is the operation of rounding the number($* \quad \text{upwards}$)

$$A_0 = B_0 A_0, \quad B_0 = B_0 A_0, \quad B_0 = B_0 A_0, \quad C_0 = C_0 A_0, \quad B_0^e A_0$$

Equations (7), (8) for a set of indices $k = 0, M$ involve the matrices with the following properties:

$$B_k^e | B_k^e, \quad B_k^e B_k^e = 0, \quad B_k B_k = I_m,$$

$$C_k^e = [C_k^e]^{-1}, \quad C_k C_k^e = 0, \quad C_k C_k^e = I_m,$$

Where the superscript «⊥» denotes orthogonal annihilators (divisors of zero), and the superscript «+» denotes the Moore-Penrose pseudo inverse matrices.\textsuperscript{18}–\textsuperscript{19} Also, we consider the recurrence formulae of controllers for the spectrum control on the corresponding decomposition levels, written down in reverse order:

$$- M \quad \text{-th decomposition level}, \quad F_M = [\Phi_M B_M - B_M A_M] C_M,$$

$$- k \quad \text{-th decomposition level}, \quad k = 0, M-1$$

$$F_k = [\Phi_k B_k - B_k A_k] C_k, \quad B_k = B_k - F_{k+1} C_k B_k,$$

The multilevel decomposition procedure considered is then implemented.

**III. Algorithm for synthesis of the MIMO-system output control**

The following statement is true that has been proven\textsuperscript{19} and given here with an allowance for replacement of matrix annihilators, which are not possessed of a property of orthogonality, i.e., replacement with matrices, for which, in general, not necessarily the equality $B_k^e B_k^e = I_m$, is implemented.

**Theorem2.** Let $m > r$, and the following matrices exist and are pair wise completely controllable:

$$G_i = B_i A_i C_i, \quad H_i = B_i C_i, \quad i \in [0,M],$$

Then, there exists a nonempty set of matrices $K = \{0, M\}$, such that

$$\Phi_i = G_i + K_i H_i, \quad \Phi_i = \begin{pmatrix} B_i A_i C_i & B_i C_i \end{pmatrix}, \quad i \in [0, M],$$

satisfy the equalities of spectra

$$\text{eig}(A_i + B_i F_i C_i) = \bigcup_{i = k}^{M} \text{eig}(\Phi_i), \quad k \in [0, M],$$

Moreover, the following matrices exist and are pair wise completely controllable:

$$G_k = \begin{pmatrix} B_k C_k \end{pmatrix}, \quad H_k = \begin{pmatrix} B_k C_k \end{pmatrix}, \quad k \in [0, N].$$

Then, there exists a nonempty set of matrices $L_i, \quad i \in [0, N]$, such that

$$\Psi_i = G_i + H_i L_i, \quad \Psi_i = \begin{pmatrix} B_i C_i \end{pmatrix}, \quad \Psi_i = \begin{pmatrix} B_i A_i C_i + (B_i C_i) \end{pmatrix}, \quad i \in [0, M],$$

and, for $F = B^e \Psi + A^e C^e \Psi_i^T F = B^e \Psi_i F_i, \quad \Psi_i = A_i \Psi_i, \quad k \in [0, N]$.
where, taking in view the following notation \( \Gamma = b_1 b_22 - b_2 b_12 \), the matricesˈ elements in (12), (13) is equal to:

\[
l_1 = b_{11} b_{23} - b_{21} b_{13}, \quad l_2 = -(b_{11} b_{23} - b_{21} b_{13}), \quad l = l_1 + l_2 + 1,
\]

and provide for the close-loop system «HS + control system» of a specified earlier spectrum(6). We perform for the system (2) with matrices(3), (4) – the multilevel decomposition described in Section 1, which has in this case two \( \text{eig}(A_0 + B_0 E_0 C_0) = \bigcup_{i=0}^{M} \text{eig}(\Psi_i) = \Lambda \). The condition \( m_{SR} \geq 2 \) in Theorem 1 is not restrictive; it is introduced to indicate that, in the present case \( F \), matrix from (5) is conventionally considered as a matrix of controller (i.e. the number of inputs is less than the number of outputs). For the case \( m_{SR} = 1 \), Theorem 1 has a dual formulation, and matrix \( F \) is replaced with the observer matrix \( L \) (the number of inputs is greater than the number of outputs).

\[
C_0^T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} b_{11}^0 & b_{12}^0 & b_{13}^0 & 0 \end{bmatrix}, \quad \text{in which, in}
\]

presence of the introduced parameter

\[
b_{11}^0 = b_{11}^2 + 2b_{12}^2 + b_{13}^2 - 2b_{12} b_{13}, \quad b_{12}^0 = b_{11}^2 - b_{13}^2, \quad b_{13}^0 = b_{11}^2 - b_{12}^2 + 2b_{13},
\]

Corresponding elements can be expressed as

\[
b_{11}^0 = b_{12} b_{13} - b_{13} b_{12}, \quad b_{12}^0 = b_{13} b_{13} - b_{12} b_{12}, \quad b_{13}^0 = b_{13} b_{13} - b_{12} b_{12},
\]

Besides, the following equations take place:

\[
H_0 = \begin{bmatrix} C_0^T \end{bmatrix}^T \quad \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad G_0 = \begin{bmatrix} B_0 C_0^T \end{bmatrix}^T \quad \begin{bmatrix} b_{11}^0 & b_{12}^0 & b_{13}^0 \end{bmatrix}^T
\]

\[
= \begin{bmatrix} (b_{12}^2 + b_{13}^2 + b_{13} b_{12}) & (b_{12}^2 + b_{13}^2 + b_{13} b_{12}) & (b_{12}^2 + b_{13}^2 + b_{13} b_{12}) \end{bmatrix}
\]

\[
\]
The following notation is used:

\( C_i^T = 0, \quad B_i = \begin{bmatrix} b_{i1}^+ \ b_{i2}^+ \\ b_{i1}^- \ b_{i2}^- \end{bmatrix} \)

\( H_i = (B_i^* C_i^{-1})^T = I_2, \quad H_i' = (H_i)^T = I_2 \) \hspace{1cm} (15)

\( G_i = [B_i^* C_i^{-1}]^{-1} [B_i^* A_i C_i^{-1}]^T = 0 \)

Here we denote linear combinations:

\( b_{11}^0 = b_{11}^+ - b_{11}^-, \quad b_{12}^0 = b_{12}^+ - b_{12}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

According to Theorem 1, we complete the system of the zero decomposition level using (7). For this purpose we use equation (14) and, applying new notation, we obtain

\( G_0 = \begin{bmatrix} a_{01}^+ & a_{02}^- \end{bmatrix} \)

Where we denote

\( a_{01}^+ = b_{01}^0 + d_{11}^0 / d, \quad a_{02}^- = b_{02}^0 + d_{22}^0 / d \),

\( d = (b_{12}^0)^2 + (b_{22}^0)^2 \),

\( d_1 = a_{01}^+ b_{11}^- + a_{02}^- b_{11}^+ + a_{02}^- b_{12}^- + a_{01}^+ b_{12}^+ \),

\( d_2 = a_{01}^+ b_{21}^- + a_{02}^- b_{21}^+ + a_{02}^- b_{22}^- + a_{01}^+ b_{22}^+ \),

Now, we should determine \( \hat{O} \) for the zero decomposition level. For this purpose, we decompose the matrices \( H_0, G_0 \) of the zero level into two sublevels and calculate the corresponding matrices. We obtain

\( H_0 = H_0_1 \quad G_0 = G_0_1 \quad (H_0_1)^{-1} = \begin{bmatrix} b_{11}^0 / b_{01}^0 \ 0 \end{bmatrix} \)

\( H_0 = \begin{bmatrix} b_{11}^0 / b_{01}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ b_{22}^0 / b_{02}^0 \ \end{bmatrix} \)

Here we denote linear combinations:

\( b_{11}^0 = b_{11}^0 - b_{11}^-, \quad b_{12}^0 = b_{12}^+ - b_{12}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

\( b_{21}^0 = b_{21}^+ - b_{21}^- \quad b_{22}^0 = b_{22}^+ - b_{22}^- \)

To calculate matrix \( B_i \) that is needed for determining the zero level controller, we use the second formula in (10). As a result, we obtain the expression

\( B^- = B^+ - F C B^+ \begin{bmatrix} b_{11}^0 \ b_{12}^0 \ b_{21}^0 \ b_{22}^0 \ \end{bmatrix} \begin{bmatrix} b_{11}^0 \ b_{12}^0 \ b_{21}^0 \ b_{22}^0 \ \end{bmatrix} \)

\( B^- = b_{11}^0 ((a_{11}^- - \hat{\lambda}) / b_{11}^- + b_{22}^0 / b_{12}^- / b_{22}^-) \)

Next, we calculate the matrix

\( B^- = b_{11}^0 b_{12}^0 ((a_{11}^- - \hat{\lambda}) / b_{11}^- + b_{22}^- / b_{12}^- / b_{22}^-) \)

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Where
\[ h_{11}^0 = p_{11}^0 (a_{11}^{aw} - \lambda_{11}^0) / h_{11}^{aw} + h_{12}^{aw} / h^{aw}, \]
\[ h_{12}^0 = p_{12}^0 (a_{11}^{aw} - \lambda_{11}^0) + (h_{12}^{aw})^2 / h^{aw}, \]
and specify again the matrix of Eigen values of the zero sublevel of the zero decomposition level by \( \Phi_0 = \lambda \).

Finally, we find matrix \( K_0 \) by the rule
\[ K_0 = (P_0 G_0 - (H_0 G_0)_0 = (K_{11} + K_{12}) \),

Where in the case under examination
\[ K_{11} = a_{11}^{aw} (h_{12}^{aw} / h^{aw}) - b_{p1,12} (a_{11}^{aw} - \lambda_{11}^0) / h_{12}^{aw} - a_{21}^{aw} (a_{11}^{aw} - \lambda_{11}^0) / h_{12}^{aw} \]
\[ + (h_{12}^{aw})^2 / h^{aw} - \lambda_{12} (h_{12}^{aw} / h^{aw}) - b_{p1,12} (a_{11}^{aw} - \lambda_{11}^0) / h_{12}^{aw} - a_{21}^{aw} (a_{11}^{aw} - \lambda_{11}^0) + \]
\[ + (h_{12}^{aw})^2 / h^{aw} + a_{12}^{aw} (h_{12}^{aw} / h^{aw} - b_{p1,12} (a_{11}^{aw} - \lambda_{11}^0) / h_{12}^{aw} ) \]

As a result, we obtain using equation (11), the matrix \( \Phi_0 \), whose eigen values \( \lambda_\), \( \lambda_4 \) are ensured by the output controller, namely,
\[ \Phi_0 = \begin{pmatrix}
\lambda_1 - h_{11}^{aw} / h_{11}^{aw} & a_{12}^0 + K_{11} \\
\lambda_2 - h_{12}^{aw} / h_{12}^{aw} & a_{22}^0 + K_{12} 
\end{pmatrix}.
\]

Further calculations, which were described, for instance, finally yield the following formula for the SH output controller vector:
\[ F = \begin{pmatrix}
f_{11} & f_{12} & f_{13} \\
f_{21} & f_{22} & f_{23} 
\end{pmatrix}, \] (17)

Here elements of the following notation:
\[ d_{11} = a_{11}^{aw} h_{12}^{aw} / h_{11}^{aw}, \quad d_{22} = a_{22}^0 + K_{12}, \quad d_{21} = a_{21}^{aw} h_{12}^{aw} / h_{11}^{aw}, \quad d_{22} = a_{22}^0 + K_{12}, \]
Can be expressed as
\[ f_{11} = b_{11} d_{11} - b_{12} d_{21} - b_{13} d_{31} - b_{14} d_{41} + b_{15} d_{51}, \]
\[ f_{12} = b_{11} d_{12} - b_{13} d_{32} - b_{14} d_{42} + b_{15} d_{52}, \]
\[ f_{13} = b_{11} d_{13} - b_{14} d_{43} + b_{15} d_{53}, \]
\[ f_{21} = b_{21} d_{11} - b_{22} d_{21} - b_{23} d_{31} + b_{24} d_{41} + b_{25} d_{51}, \]
\[ f_{22} = b_{21} d_{12} - b_{23} d_{32} - b_{24} d_{42} + b_{25} d_{52}, \]
\[ f_{23} = b_{21} d_{13} - b_{24} d_{43} + b_{25} d_{53}, \]
\[ f_{24} = b_{21} d_{14} - b_{25} d_{54}, \]
\[ f_{25} = b_{21} d_{15} - b_{24} d_{45} + b_{25} d_{55}. \]

The synthesized controller (the SH control system laws) ensures exactly the specified spectrum (6) for controlled longitudinal motion. This assertion can be directly checked with the help of appropriate analytical calculations. For this purpose it is sufficient to make use of the package Symbolic Toolbox Matlab; namely, one can use the \( \text{eig} \) instruction to calculate the eigen values of the \( A+BFC \) matrix, where matrices can be expressed as(4), and the \( F \) matrix is determined by (17).

**Numerical analysis**

Suppose that we want the closed-loop system “SH + control system” to have the following multiple real spectrum

\[ \Lambda = \{-1.5, -1.5, -1.5, -1.5\} . \] (18)

Let use for the hypothetical SH the \( A, B \) and \( C \) (4) matrices having the following numerical values:
\[ A = \begin{pmatrix}
-0.0598 & -0.0233 & 0.1095 & -0.1703 \\
-0.0268 & -0.0889 & -1.2091 & -0.0135 \\
1.5158 & -2.3207 & -2.3464 & 0 \\
-0.5050 & -0.0228 
\end{pmatrix},
\]
\[ B = \begin{pmatrix}
-1.2817 & 2.0890 \\
28.178 & 22.7255 \\
0 & 0 
\end{pmatrix},
\]

With use of equation (17) for the controller we obtain the numerical value for the matrix \( F \). As a result we have
\[ F = \begin{pmatrix}
3.6163 & -1.8622 & -3.4996 \\
-7.9736 & 2.2309 & 3.6237 
\end{pmatrix}.
\]

The computation of Eigen values of the matrix \( A+BFC \) yields the set
\[ \text{eig}(A+BFC) = \{-1.5, -1.5, -1.4924 \pm 0.0131i\} \].

**Conclusion**

The problem of a hypothetical single-airscrew helicopter’s longitudinal motion stabilization for lack of information about the vertical speed of its motion has been analytically solved. The solution is based on the method of the output signal control synthesis that provides a specified spectrum of the MIMO-system’s motion, presented 13-16-19.

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**Conflict of interest**

The author declares there is no conflict of interest.

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