Harrison-Zel’dovich spectrum gets back?

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The Data Release 4 of the Atacama Cosmology Telescope (ACT) shows an agreement with an Harrison-Zel’dovich primordial spectrum \(n_s = 1.009 \pm 0.015\), introducing a tension with a significance of 99.3% CL with the results from the Planck satellite. The discrepancy on the value of the scalar spectral index is neither alleviated with the addition of large scale structure information nor with the low multipole polarization data. We discuss possible avenues to alleviate the tension relying on either neglecting polarization measurements from ACT or in extending the inflationary sector of the theory.

Keywords: Cosmological Tensions; Inflation

I. INTRODUCTION

Inflation provides the most successful cosmological scenario able to generate the initial conditions of our Universe and simultaneously solving the standard cosmological problems. However, and despite this remarkable success, the inflationary paradigm is still lacking firm observational confirmation.

A "smoking-gun" evidence for inflation would be the detection of primordial B-modes in the Cosmic Microwave Background (CMB) power spectrum produced by primordial gravitational waves. In the most typical inflationary models, the amplitude of tensor perturbations is expected to be proportional to the quantity \(|n_s - 1|^2\) with \(n_s\) the scalar spectral index of the primordial scalar spectrum: the larger the departure of \(n_s\) from unity, the more likely tensor modes would be within observational reach. Therefore determining how much the former index deviates from one dictates the theoretical, phenomenological, also experimental perspectives of the field. For instance, a cosmological model with \(n_s = 1\) — that will corresponds to the phenomenological model proposed by Harrison, Zel’ dovich, and Peebles [1–3] — will imply a major theoretical breakthrough, as it would imply that the origin of cosmic perturbations may lie in some unknown fundamental theory different from the standard inflationary picture or in extensions of the latter [4–13].

In this regard, the latest observations of the CMB temperature and polarization anisotropies, echoes of the Big Bang, provided by the Planck satellite have reached subpercent accuracy on the extraction of the majority of the cosmological parameters [14, 15], resulting in a \(\sim 8\sigma\) evidence for \(n_s \neq 1\) and establishing inflation as the most accredited theory of the early Universe. However, this is both a blessing and a curse, since, as high precision parameter extraction becomes a reality, the possible discrepancies among different data sets may grow in significance.

Currently, there are several anomalies that can not be fully understood in the minimal cosmological constant plus cold dark matter (ΛCDM) scenario [16, 17]. The most significant 5σ disagreements is related to the value of the Hubble constant \(H_0\) extracted from local distances and redshifts in the nearby Universe and that inferred from CMB observations [18–21]. Other less significant disagreements concern the parameter \(S_8\), whose values differ for CMB and weak lensing estimates [22–24], and the so-called lensing anomaly [14, 25], related to the fact that the Planck CMB data show a preference for additional lensing. Interestingly, while inflation predicts a perfectly flat Universe, this excess of lensing in the damping tail produces an indication for a closed Universe at level of 3.4 standard deviations [14, 26–29] that, if confirmed, would be very hard to explain in the simplest models of inflation.

In this letter we exhaustively explore yet a new potential challenge for inflationary cosmology arising from Planck-independent observations of the cosmic microwave background, namely the \(\sim 2.7\sigma\) discrepancy in the value of the scalar spectral index \(n_s\) measured by Planck \((n_s = 0.9649 \pm 0.0044)\) [14] and by the Atacama Cosmology Telescope (ACT) \((n_s = 1.008 \pm 0.015)\) [30] (see also the recent [31] and [32]). As with the other tensions mentioned above, this \(n_s\) discrepancy could result from a statistical fluctuation, to a (yet unknown)
systematic effect in the ACT or Planck data, or a departure from the theoretical ΛCDM framework by assuming canonical inflation as the dominant mechanism for producing the perturbations in the early Universe. In this regard, recent analyses [12, 33–35] suggest a potential prominent role of ns in solving the aforementioned cosmological tensions, motivating the need of a systematic investigation on the nature of this discrepancy to evaluate its robustness.

### II. ANALYSIS

We therefore scrutinize the emergent tension on canonical inflationary scenarios, exploring different cosmological observations at distinct epochs in the cosmic evolution. Our main datasets consist of the observations of the Cosmic Microwave Background provided by the Planck 2018 temperature and polarization likelihood [14, 15, 36], Planck (TT TE EE), including low multipole data (ℓ < 30), the Atacama Cosmology Telescope DR4 likelihood [37], ACT (TT TE EE), and South Pole Telescope polarization measurements [38], SPT (TE EE). From the large scale structure perspective, we shall make use of the Baryon Acoustic Oscillations (BAO) and Redshift Space Distortions (RSD) measurements from BOSS DR12 [39], (BAO DR12) and eBOSS [40], (BAO DR16) cosmological observations, and also the shear-shear, galaxy-galaxy, and galaxy-shear correlation functions from the first year of the Dark Energy Survey [41] (DES). For different combinations of these datasets, Monte Carlo Markov Chain (MCMC) analyses are performed using the publicly available package COBAYA [42], which computes the observables by means of the Boltzmann code CAMB [43, 44].

| Dataset                        | Scalar Spectral Index \((n_s)\) |
|-------------------------------|---------------------------------|
| ACT                           | 1.009 ± 0.015                   |
| ACT+BAO (DR12)                | 1.006 ± 0.013                   |
| ACT+BAO (DR16)                | 1.006 ± 0.014                   |
| ACT+DESy1                     | 1.007 ± 0.013                   |
| ACT+SPT+BAO (DR12)            | 0.996 ± 0.012                   |
| Planck                        | 0.9649 ± 0.0044                 |
| Planck+BAO (DR12)             | 0.9668 ± 0.0038                 |
| Planck+BAO (DR16)             | 0.9677 ± 0.0037                 |
| Planck (2 ≤ ℓ ≤ 650)          | 0.9655 ± 0.0043                 |
| Planck (ℓ > 650)              | 0.9634 ± 0.0085                 |

Table I: The marginalized 1σ bounds for the scalar spectral index for various data combination obtained assuming a standard cosmological model.

All our results are summarized in Table I. As already mentioned, considering CMB data alone, the measurements of ns from Planck \((n_s = 0.9649 ± 0.0044)\) and from ACT \((n_s = 1.006 ± 0.015)\) differ by \(\sim 2.7\sigma\). This can be clearly observed in Figure 1, where in the two-dimensional plane it can be definitely noted that the direction of the \(\Omega_b h^2-n_s\) degeneracy is opposite for ACT and Planck, and the disagreement here is significantly exceeding \(\sim 3\sigma\). In the absence of low-ℓ CMB data, as in the case of ACT measurements, there is a strong degeneracy between the baryon energy density \(\Omega_b h^2\) and the scalar spectral index \(n_s\): a lower value of the former (increasing the damping of the low ℓ acoustic peaks) can be always mimicked by a larger value of the latter, tilting the spectrum in the opposite direction. ACT measurements of the small scale CMB spectra favor a cosmology with a lower value of \(\Omega_b h^2\) and a higher spectral index \(n_s\) and, consequently, a lower amplitude of the first acoustic peak in the TT power spectrum than that preferred by both the Wilkinson Microwave Anisotropy Probe (WMAP) [45] or Planck CMB observations. Indeed, in Ref. [30] the mis-

![Figure 1: One-dimensional posterior distributions and the two-dimensional joint marginalized contours at 68% and 95% CL inferred for the baryon energy-density \((\Omega_b h^2)\), the optical depth at reionization \((\tau)\) and the spectral index of inflationary perturbations \((n_s)\) by analyzing the different data-sets and cosmological models indicated in the legend.](image)

\footnote{For instance, when fitting the ACT data, fixing the value of the spectral index to the Planck measured value \(n_s = 0.9649\) would give a larger \(\chi^2 = 286.6\) than fixing \(n_s = 1\) (\(\chi^2 = 279.0\)).}
match in the values of $n_s$ was interpreted as a consequence of the lack of information concerning the first acoustic peak of the temperature power spectrum. To verify this origin of the discrepancy in the CMB values of $n_s$, we have performed two separate analyses of the Planck observations, splitting the likelihood into low ($2 \leq \ell \leq 650$) and high ($\ell > 650$) multipoles. We find that the discrepancy still persists at the level of $3\sigma$ ($2\sigma$) for low (high) multipole temperature data. Our results therefore cast doubts on the claim that the mismatch in $n_s$ between ACT and Planck is due to the lack of information on the first acoustic peak in ACT data. In fact, Planck data still prefers a value of the scalar spectral index smaller than unity at $\sim 4.3\sigma$ when the information on the first acoustic peak is removed. In addition, by focusing only on the high-$\ell$ region of the Planck spectra, the disagreement with ACT is actually reduced at the level of $2\sigma$, but this is due to a loss of constraining power rather than a true shift of the mean value of $n_s$, see also Figure 2. Therefore this discrepancy, although minor, should be seriously taken into account, as one would expect a reasonable agreement between two experiments measuring an overlapping range of multipoles. Conversely the low-$\ell$ end of the Planck data is in strong disagreement with ACT: even with larger error bars, the low-$\ell$ data exhibits a tension higher than when the full Planck multipole range is used.

Figure 1 illustrates that there is also a degeneracy between the reionization optical depth $\tau_{\text{reio}}$ and $n_s$. The reionization optical depth is defined as:

$$\tau_{\text{reio}}(z) = \int_z^\infty dz' \frac{e}{z'} (n_e'(z') - n_{e,0}'(z')) \sigma_T ,$$

where $n_e(z) = n_{H}(0)(1+z)^3 x_e(z)$ and $n_{e,0}(z) = n_{H}(0)(1+z)^3 x_{e,0}(z)$, being $n_{H}(0)$ the number density of hydrogen at present, $x_e(z)$ the free electron fraction and $x_{e,0}(z)$ the free electron fraction leftover from the recombination epoch. It is well-known that the statement that $n_s = 1$ is observationally excluded no longer applies if one treats reionization in a general manner [46]. One could therefore consider to add an additional prior on the reionization optical depth to the ACT constraints on the cosmological parameters. If a prior on $\tau_{\text{reio}} = 0.065 \pm 0.015$ [30] ($\tau_{\text{reio}} = 0.0544 \pm 0.0070$ [14]) is applied, we have $n_s = 1.009 \pm 0.015$ (1.007 \pm 0.015), barely changing the $\sim 3\sigma$ discrepancy with the Planck results. The same conclusion is reached if ACT data is directly combined with the low-$\ell$ polarization Planck (lowE) data.

The next logical step is to investigate the effect of complementary (i.e. non-CMB) data: the addition of BAO measurements normally alleviates tensions and restores the parameter values to those corresponding to $\Lambda$CDM. One well-known example is that of the curvature $\Omega_k$: the addition of BAO measurements to Planck observations is indeed very consistent with a flat cosmology ($\Omega_k = 0.0007 \pm 0.0019$ at 68% CL). Unfortunately, this is not the case here: an inspection into Figure 1 clearly shows that neither the addition of BAO DR12 nor that of BAO DR16 alleviates the tension in the measured value of $n_s$. The combination of both BAO measurements and a prior on the reionization optical depth leads to a tension even more significant, as the mean value of $n_s$ remains unchanged but their error bars are reduced. Although not present in Figure 1, we have also considered the combination of ACT with DES galaxy clustering and cosmic shear observations, obtaining a tension of $3.1\sigma$. Yet another possibility usually explored when finding anomalies in the cosmological parameter values when combining different data sets is to extend the minimal cosmological model. Relaxing the dark energy sector physics alleviates some discrepancies, such as the Hubble constant one (see Refs. [16, 19–21, 47] and references therein). We have therefore allowed the dark energy equation of state $w$ to be a free parameter, finding values of $n_s = 1.010 \pm 0.014$ ($n_s = 0.9654 \pm 0.0042$) for ACT with a prior of $\tau_{\text{reio}} = 0.065 \pm 0.015$ + BAO DR12 (Planck + BAO DR12). The tension is enhanced to the $3.2\sigma$ level. Leaving the assumption of flatness provides a solution to the aforementioned lensing anomaly [27]. Motivated by this near result, we have explored here the possibility of having a non-zero curvature parameter $\Omega_k$. While the Planck (TT TE EE) data show a definite preference for a closed Universe at more than 99% CL [32], ACT is perfectly consistent with the inflationary prediction for a flat Universe, i.e. with $\Omega_k = 0$.

Figure 2: One-dimensional posterior distributions and the two-dimensional joint marginalized contours inferred by combining the Atacama Cosmology Telescope measurements of the CMB temperature anisotropies and the South Pole Telescope measurements of the CMB polarization anisotropies and splitting the Planck likelihood in low ($2 \leq \ell \leq 650$) and high ($\ell > 650$) multipoles.
we find $\Omega_k = -0.0011^{+0.014}_{-0.0093}$ for the case of ACT plus a prior on $\tau_{\text{reio}} = 0.0544 \pm 0.0070$ [14]. A very similar conclusion is achieved if the reionization prior considered is $\tau_{\text{reio}} = 0.065 \pm 0.015$ [30] or low multipole polarization data from Planck is used. Last, but not least, ACT data is perfectly compatible with $A_{\text{lens}} = 1$ [31], as $A_{\text{lens}} = 0.984_{-0.094}^{+0.082}$ for the combination of ACT plus a prior $\tau_{\text{reio}} = 0.0544 \pm 0.0070$ [14].

III. DISCUSSION

From all our analyses and consistency checks detailed above the lesson we learned is that there is a clear discrepancy between the extraction of the scalar spectral index of primordial perturbations from Planck and ACT data. Since future strategies in B modes searches in the CMB polarization pattern depend crucially on the amplitude of these fluctuations, it is mandatory to understand where this anomaly comes from and what we can conclude about the precise value of $n_s$ from present cosmological observations [35].

In order to understand the nature of this anomaly and recover the measured value of the scalar spectral index $n_s$ by Planck consistent with the predictions from most of the canonical inflationary scenarios, one could follow two distinct avenues. One possible logical first step is to identify which of the data sets could be responsible for the $n_s$ discrepancy, and discard it in the cosmological parameter inference analyses. We have made a number of tests along this line. Figure 2 shows the results of the combination of ACT temperature anisotropies (TT) with SPT polarization data (TE EE), that is, we neglect any information arising from ACT polarization measurements (TE EE). In this case the disagreement with Planck is reduced below 2σ, but with the ACT and SPT data still preferring a value of $n_s$ around unity. However the tension is only mitigated by the larger error bars and once BAO are combined with the ACT and SPT data the disagreement with Planck in fact grows again to the statistical level of $\sim 2.4 \sigma$ ($n_s = 0.996 \pm 0.012$). Nonetheless, the combination of the ACT temperature and the SPT polarization produces a significant shift in the plane ($n_s$, $\Omega_b h^2$) resulting in a value of the baryon energy density $\Omega_b h^2 = 0.02237 \pm 0.00030$ that is now in perfect agreement with the Planck result. So the degeneracy between these two parameters seems to play only a partial role in this potential tension: restoring the agreement for $\Omega_b h^2$ is not enough to reconcile the $n_s$ discrepancy.

A second possible different approach is to extend the inflationary scenario, accounting also for the possibility of a running in the scalar spectral index $\alpha_s = dn_s/d \log k$. Figure 1 depicts the impact of considering a running of the scalar spectral index in the parameter allowed regions for the cases of Planck and ACT data either alone or combined with BAO DR12 measurements.

In the case of CMB data alone, $n_s = 0.950 \pm 0.011$ ($n_s = 0.982 \pm 0.020$) for Planck (ACT) data, leading to a very mild 1.6σ discrepancy. If the BAO DR12 dataset is also considered, the disagreement gets diluted, as it barely reaches the 1σ significance. However, the tension in $n_s$ maps into a controversy in the values of $\alpha_s$: while Planck alone prefers a negative value of $\alpha_s = -0.0119 \pm 0.0079$, ACT measurements seem to favor a positive one $\alpha_s = 0.058 \pm 0.028$, leading to a 2.5σ tension, see also Ref. [48] where such an indication for a positive tilted spectrum is shown to persist combining ACT with WMAP 9-year observations, as well. The addition of BAO DR12 barely modifies this result. The preference for a positive $\alpha_s$ from small scale CMB observations from the ACT experiment challenges canonical inflationary scenarios, as the predictions from all these models provide a negative value of $\alpha_s$, see e.g. [49–51].

A third avenue is to relax the reionization scenario [46]. In all our previous results, we have restricted ourselves to parameterize the reionization history in terms of the optical depth to reionization, see Equation 1. To study the impact of a more general reionization scenario on the tension on $n_s$, we have explored, as a first attempt, the so-called redshift-symmetric parameterization, which assumes that the free electron fraction follows a step-like function, taking the recombination leftover value at high redshifts and becoming close to one at low redshifts, and being described by the hyperbolic tangent function [52]

$$x_c \tanh(z) = \frac{1 + f_{\text{He}}}{2} \left[ 1 + \tanh \left( \frac{y(z_{re}) - y(z)}{\Delta y} \right) \right],$$  \hspace{1cm} (2)

where $f_{\text{He}} = n_{\text{He}}/n_{\text{H}}$ is the Helium fraction, $y(z) = (1 + z)^{3/2}$, $\Delta y = 3/2(1 + z_{re})^{1/2} \Delta z$, and $\Delta z$ is the width of the transition. Therefore, the free parameters in this simple approach are the reionization redshift $z_{re}$ and $\Delta z$. However, this reionization scenario renders the very same results, and the $\sim 3 \sigma$ tension on $n_s$ still persists. More general reionization schemes, such as a Principal Component Analysis (PCA) approach of Refs. [53–60] or non-parametric forms for the free electron fraction $x_c(z)$, which is instead described using the function values $x_c(z_i)$ in a number $n$ of fixed redshift points $z_1, \ldots, z_n$, are promising and viable phenomenological alternatives that will be further explored in future work.

In conclusion, both Planck and ACT show intriguing anomalies that seem to challenge the typical predictions of inflationary theories: the first data-set seems to disfavor the inflationary prediction for a flat background geometry at more than 99.9 % CL while the second, albeit in perfect agreement with spatial flatness, shows a preference for a larger spectral index consistent with a Harrison-Zel’dovich scale-invariant spectrum ($n_s = 1$) of primordial density perturbations, introducing a tension with a significance of 99.3% CL with the results from the
Planck satellite. Our analysis proves that this preference remains robust with both the addition of large scale structure information and the inclusion of low multipole polarization data, suggesting either the presence of important observational systematics in one or both data-sets or a departure from the theoretical framework. Possible phenomenological avenues to settle this issue include a possible running of the scalar spectral index $\alpha_s$ (albeit in this latter case the tension is translated into a discrepancy on $\alpha_t$) or alternative reionization scenarios. From the theoretical, model-building approach, non-standard inflation theories may also provide a solution, while being testable by near future CMB B-mode experiments.

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