Preliminary study of feasibility of an experiment looking for excited state double beta transitions in Tin

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Abstract

A first attempt to study the feasibility of an experiment to search for double beta decay in $^{124}$Sn and $^{112}$Sn was carried out by using ultra-low background HPGe detector (244 cm$^3$) inside the Gran Sasso National Laboratory (LNGS) of the INFN (Italy). A small sample of natural Sn was examined for 2367.5 h. The radioactive contamination of the sample has been estimated. The data has also been considered to calculate the present sensitivity for the proposed search: half-life limits $\sim 10^{17} - 10^{18}$ years for $\beta^+EC$ and EC-EC processes in $^{112}$Sn and $\sim 10^{18}$ years for $\beta^-\beta^-$ transition in $^{124}$Sn were measured. In the last section of the paper the enhancement of the sensitivity for a proposed experiment with larger mass to reach theoretically estimated values of half-lives is discussed.

Keywords: Double beta decay, ultra low background, HPGe $\gamma$ detector

1. Introduction

The existence of non-zero mass of neutrino has been established by neutrino flavor-oscillation experiments. However, the nature of the neutrino, either Dirac or Majorana, can only be tested through observation of neutrino-less

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double beta decay, which is a rare second order transition involving two isobars. Moreover, the double beta decay experiments have the potential to establish the absolute scale of the neutrino mass, to prove the hierarchy of neutrino mass, to test the existence of right-handed admixtures in the weak interaction, and to test some other effects beyond the Standard Model. The $0(2)\nu\beta\beta$ decay can occur as:

\begin{align}
(A, Z) \rightarrow (A, Z + 2) + 2e^- + (2\nu_e) & \quad (1) \\
(A, Z) \rightarrow (A, Z - 2) + 2e^+ + (2\nu_e) & \quad (2) \\
e^- + (A, Z) \rightarrow (A, Z - 2) + e^+ + (2\nu_e) & \quad (3) \\
2e^- + (A, Z) \rightarrow (A, Z - 2) + (2\nu_e) & \quad (4)
\end{align}

Triggered by the important implication of neutrino mass, a new generation of experiments are aimed to observe $0\nu\beta\beta$ decay in various isotopes and with different experimental techniques [4]. Among the main experimental activities, we remind that recently three $\beta\beta$ experiments have published new experimental data: GERDA [5], EXO-200 [6] and KamLAND-Zen [7]. GERDA is searching for $0\nu\beta\beta$ decay in $^{76}\text{Ge}$ while EXO-200 and KamLAND-Zen are looking for the decay in $^{136}\text{Xe}$. Several other experiments are going through their R&D phase. These include CUORE ($^{130}\text{Te}$) [8], SuperNEMO ($^{82}\text{Se}$) [9], MAJORANA ($^{76}\text{Ge}$) [10], SNO+ ($^{150}\text{Nd}$) [11], NEXT ($^{136}\text{Xe}$) [12] etc.. Present experiments are sensitive to half-lives of the order of $10^{25}$ years and a corresponding effective mass of the neutrino $\langle m_{\beta\beta} \rangle \sim 100$ meV [13]. Constrained by the uncertainties in the calculation of nuclear transition matrix elements [14] and by the $g_A$ value [15] that are used to determine $\langle m_{\beta\beta} \rangle$ as well as by the different background for different isotopes, it is essential to measure the half-life for $0\nu\beta\beta$ decay in several isotopes.

Although Sn was considered one of the potential candidates since the 1980s [16], there have been only a few studies on Sn. It can be mentioned that one of the earliest attempts for the experimental study of double beta decay was done by Fireman [17] in 1949 using $^{124}\text{Sn}$.
Natural tin contains three isotopes which can decay via double beta transition: $^{122,124}$Sn through two electron mode and $^{112}$Sn through $\beta^+\text{EC}$ and EC-EC processes, as given in the following:

$$^{122}\text{Sn} \rightarrow ^{122}\text{Te} + 2e^- + (2\bar{\nu}_e) \quad (5)$$

$$^{124}\text{Sn} \rightarrow ^{124}\text{Te} + 2e^- + (2\bar{\nu}_e) \quad (6)$$

$$e^- + ^{112}\text{Sn} \rightarrow ^{112}\text{Cd} + e^+ + (2\nu_e) \quad (7)$$

$$2e^- + ^{112}\text{Sn} \rightarrow ^{112}\text{Cd} + (2\nu_e) \quad (8)$$

The Q-values for the decay transitions in $^{112}$Sn, $^{122}$Sn and $^{124}$Sn are 1919.82$\pm$0.16 keV, 372.9$\pm$2.7 keV and 2291.1$\pm$1.5 keV [18], respectively. The natural isotopic abundances for these isotopes are 0.97(1)$\%$, 4.63(3)$\%$ and 5.79(5)$\%$ [19], respectively. The gamma rays from $^{122}$Sn decays are at low energy where the background is high and makes it more difficult to study this decay modes in an external source experiment. Therefore, we have not considered this nucleus for the present study.

Recently, investigations were carried out using natural tin samples for $\beta^-\beta^-$ decay of $^{124}$Sn to the excited states of the daughter nucleus $^{124}$Te and $\beta^+\text{EC}$ and EC-EC processes in $^{112}$Sn by Dawson \textit{et al.} [20, 21], Kim \textit{et al.} [22] and Barabash \textit{et al.} [23]. Half-life limits of the order of $10^{18} - 10^{21}$ years were obtained in those experiments. Searches for $\beta^+\text{EC}$ and EC-EC processes in $^{112}$Sn were also carried out by Kidd \textit{et al.} [24] and Barabash \textit{et al.} [25, 26] using enriched material obtaining the best half-life limit of $10^{21}$ years. The Kims collaboration [27] gave a half-life limit $2.0\times10^{20}$ years for neutrino-less double beta decay of $^{124}$Sn using tin-loaded liquid scintillator.

A first attempt to study the feasibility of an experiment to search for double beta decay in $^{124}$Sn and $^{112}$Sn was carried out using an ultra-low background HPGe detector. The gamma rays produced by the de-excitations of the excited levels of either $^{112}\text{Cd}$ or $^{124}\text{Te}$ can be detected by the HPGe detector. The aim of the present study was to investigate the decay processes in $^{124}$Sn to excited states and to the ground state as well as the decay of $^{112}$Sn to the excited states
using $\gamma$-ray spectrometry. We will discuss in the last section also a possible enhancement of the sensitivity with a proposed experiment with larger mass to reach the half-life values estimated by theory.

2. Experimental set-up and measurements

A coaxial closed-end n-type ultra low background HPGe (GeBer) $\gamma$ detector ($244 \text{ cm}^3$) was used for the measurement. The detector is in place in the Gran Sasso National Laboratory (LNGS) of the INFN, Italy, located underground at $\approx 3600$ meters of water equivalent. The schematic diagram of the experimental setup is shown in Fig.1. The physical dimensions of the detector are given in Table 1. More details about the detector and the ultra low background setup can be found in [28].

Monte-Carlo simulation of HPGe detector has been performed using the GEANT4 software library [29]. To validate the results of the simulation, we have compared the simulated efficiencies with known experimental results. The full-energy-peak (FEP) efficiency of the HPGe detector was measured for different
Figure 2: Energy spectrum with 13.3 g of natural tin sample (Sn) for 2367.5 h of measurement in comparison with background spectrum (Bg) of ultra low-background HPGe detector measured for 6109.4 h. The spectra are normalised in time of measurement of tin sample. The energy of the γ lines are in keV.

γ ray energies using $^{241}$Am, $^{133}$Ba, $^{137}$Cs and $^{60}$Co point sources. The sources were placed along the z-axis at a distance of 26.5 cm from the end cap of the detector. The energy resolution (FWHM) of the detector was measured as 2.0 keV at 1332.5 keV.

A natural tin (purity 99.997%) sample of 13.3 g and with a thickness of 4.5 mm was placed on the end-cap, and measured with the HPGe detector in order to study the double beta decay processes in $^{112}$Sn and $^{124}$Sn nuclei. Data were accumulated for 2367.5 h (sample) and for 6109.4 h (background). The energy spectra of sample and background, normalized to the time of measurement of sample, are shown in Fig. 2.

3. Results and Discussion

3.1. Efficiency of the detector

The FEP efficiency of the detector was simulated for the calibration point sources using Monte-Carlo cases based on the software libraries GEANT4 [29].

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Table 1: The geometrical parameters of the detector.

| Detector parameter         | Nominal value (mm) |
|---------------------------|--------------------|
| Ge crystal radius         | 30.85              |
| Ge crystal length         | 81.70              |
| Carbon (C) window thickness | 0.76              |
| Ge crystal-C window distance | 4.00              |
| Hole radius               | 4.95               |
| Hole length               | 73.40              |
| Ge side dead layer        | 0.0                |
| Ge front dead layer       | 0.0                |
| Hole inner dead layer     | 0.3                |

Table 2: Comparison of experimental efficiency with simulated efficiency computed with two different Monte-Carlo codes: EGS4 (E4) and GEANT4 (G4). Efficiencies related to MC1 and MC2 are calculated considering two different thickness of the inner dead-layer: 1.5 mm and 5.0 mm, respectively.

| Source | Energy (keV) | Exp % | MC1 (E4)% | MC2 (E4) % | MC1 (G4) % | MC2 (G4) % |
|--------|--------------|-------|-----------|------------|------------|------------|
| 241Am  | 26.3         | 0.0048| 0.0064    | 75%        | 0.0064     | 73%        |
|        | 59.5         | 0.0938| 0.1089    | 86%        | 0.1088     | 83%        |
|        | 133Ba        | 81.0+79.6| 0.0913    | 0.1073     | 85%        | 0.1044     |
|        | 276.4        | 0.0111| 0.0131    | 85%        | 0.0114     | 84%        |
|        | 302.9        | 0.0270| 0.0305    | 89%        | 0.0271     | 86%        |
|        | 356.0        | 0.0798| 0.0926    | 86%        | 0.0799     | 84%        |
|        | 383.8        | 0.0110| 0.0126    | 87%        | 0.0107     | 88%        |
|        | 137Cs        | 661.7 | 0.0668    | 0.0818     | 82%        | 0.0679     |
|        | 60Co         | 1173.2| 0.0527    | 0.0652     | 81%        | 0.0531     |
|        | 1332.5       | 0.0478| 0.0600    | 80%        | 0.0476     | 75%        |
and EGS [30]. Generally, the results of the Monte Carlo simulations deviate significantly (> 10%) from the experimental results [31]. The difference can be due to two reasons: the uncertainties associated with the detector shape parameters provided by the manufacturer or incomplete charge collection in the crystal during the measurement process. Therefore, to match the experimental efficiency values, the detector parameters and the dead layer thickness can be varied [31]. In the first step, the detector parameters provided by the manufacturer was used in the simulation (see 2nd column of Table 1). A fine adjustment was then made by varying only the inner dead layer thickness in a systematic way to match the experimental efficiency values. In order to show the dependence of the calculated efficiencies on the inner hole dead layer, the results obtained by considering two different thicknesses (1.5 mm and 5.0 mm) are compared with the experimental values in Table 2 and in Fig. 3. In the latter case the FEP efficiencies have been divided by the branching ratios of the corresponding gammas. From Table 2 we can see that for the optimized dead layer of the inner hole, thickness of 5.0 mm, the simulated efficiencies (for both

Figure 3: Comparison of the experimental efficiencies with those obtained from Monte-Carlo simulation using two different codes EGS4 and GEANT4. Here the FEP efficiencies have been divided by the branching ratios of the corresponding gammas.
EGS4 and GEANT4) are in very good agreement with the experimental results except for the very low energy region. The uncertainty is within 4%, which is quite reasonable considering the statistical fluctuations. However, at very low energy, difference is quite large. This could be due to the very small mean free path of \( \gamma \) rays at this energy scale. Therefore, some \( \gamma \) rays are absorbed before reaching the detector. The optimized parameters were then used to calculate the FEP efficiency for a volume source of dimensions equal to the tin sample used in the present work.

3.2. Radioactive contamination in the sample

The sources of background radiations measured in an underground laboratory can be classified into several categories: environmental radioactivity including radon, gamma rays, neutrons from natural fission and from the \( (\alpha, n) \) reaction, radioactive impurities in the detector and shielding materials and cosmic rays with relevant contributions from muons and neutrons \[32, 33\]. In the measured spectra, background components can be identified by their characteristic \( \gamma \)-emission peaks. The specific activities of the radioactive nuclei present in the natural tin sample were calculated with the formula: \[28\]

\[
A = \frac{(S_s/t_s - S_b/t_b)}{m \eta i},
\]

where \( S_s \) and \( S_b \) denote the area under a peak in the sample and background spectra, respectively. The measurement time of the sample and background spectra are denoted by \( t_s \) and \( t_b \), respectively. The mass of the sample is represented by \( m \), \( \eta \) is the efficiency of the full-energy peak detection and \( i \) is the decay fraction of the \( \gamma \) ray which was obtained from National Nuclear Data Center (NNDC) \[34\]. Activities of the natural radioactivity sources present in the tin sample are listed in Table 3. Activity limits are given at 90 % C. L. according to the Feldman-Cousins method \[35\].

3.3. Double beta decay study of \(^{112,124}\)Sn

In the accumulated spectra with the tin sample, no peaks are observed which could be unambiguously attributed to the double beta decay processes of
Table 3: The radioactive contamination present in the Tin sample measured with HPGe detector. Limits are given at 90% C. L.

| Source | Energy (keV) | Count rate from Signal (mBq) | Count rate from background (mBq) | \( \eta \times \text{i} \) (\%) | Activity in (mBq/Kg) |
|--------|-------------|-------------------------------|--------------------------------|--------------------------|----------------------|
| \(^{228}\text{Ac}\) | 911.2       | 0.014±0.002                   | 0.012±0.001                   | 1.16                     | \( \leq 30 \)        |
| \(^{212}\text{Pb}\) | 238.6       | 0.053±0.005                   | 0.048±0.003                   | 6.94                     | \( \leq 15 \)        |
| \(^{208}\text{Tl}\) | 583.2       | 0.021±0.002                   | 0.015±0.001                   | 5.83                     | 7±3                  |
| \(^{214}\text{Bi}\) | 727.3       | 0.004±0.001                   | 0.006±0.001                   | 0.37                     | \( \leq 34 \)        |
| \(^{214}\text{Bi}\) | 1764.5      | 0.098±0.001                   | 0.006±0.001                   | 0.41                     | 42±22                |
| \(^{214}\text{Pb}\) | 351.9       | 0.048±0.004                   | 0.033±0.002                   | 3.81                     | 30±9                 |
| \(^{210}\text{Pb}\) | 46.5        | 0.335±0.008                   | 0.196±0.004                   | 1.30                     | 801±51               |
| \(^{234}\text{Th}\) | 63.3        | 0.095±0.005                   | 0.103±0.004                   | 1.11                     | \( \leq 28 \)        |
| \(^{226}\text{Ra}\) | 186.2       | 0.056±0.006                   | 0.060±0.004                   | 0.72                     | \( \leq 79 \)        |
| \(^{234}\text{mPa}\) | 1001.0      | 0.004±0.001                   | 0.003±0.001                   | 0.03                     | \( \leq 739 \)       |
| \(^{137}\text{Cs}\) | 661.7       | 0.030±0.002                   | 0.003±0.001                   | 5.03                     | 39±4                 |
| \(^{60}\text{Co}\) | 1173.2      | 0.020±0.002                   | 0.012±0.001                   | 3.63                     | 17±4                 |
| \(^{40}\text{K}\) | 1460.7      | 0.094±0.003                   | 0.087±0.002                   | 0.34                     | 155±79               |

Therefore, only half-life limits are calculated using the formula

\[
T_{1/2} \geq \frac{(\ln 2)N \eta t}{limS},
\]

where \( N \) is the number of \( \beta \beta \)-active nuclei, \( t \) is the measurement time in years and \( \eta \) is the FEP detection efficiency. The expression \( limS \) is the number of events of the effect searched for which can be excluded at a given confidence level (C. L.). All the limits reported in the present study are given at 90% C. L.. The values of \( limS \) were calculated using the Feldman-Cousins procedure \[35\]. The detection efficiencies of the double beta processes in the tin isotopes were calculated using the GEANT4 software library \[29\]. The DECAY0 \[36\] event generator has been used to generate the initial kinematics of the particles.
Taking into account the isotopic composition of natural tin, the sample contained $6.55 \times 10^{20}$ nuclei of $^{112}$Sn and $3.90 \times 10^{21}$ nuclei of $^{124}$Sn. The measured limits on half-lives for these two isotopes along with today’s best experimental limits and theoretical estimates are reported in Tables 4 and 5.

![Partial decay scheme of $^{112}$Cd](image)

Figure 4: Partial decay scheme of $^{112}$Cd. The energies of the excited levels and the emitted $\gamma$ quanta are in keV with relative intensities of $\gamma$ quanta are given in parenthesis.

3.3.1. EC-EC decay of $^{112}$Sn

In the $2\nu$ EC-EC decay of $^{112}$Sn to the ground state of $^{112}$Cd, all the energy release is carried away by the neutrinos except for a very small amount emitted as X-rays. These X-rays lie below the energy threshold of the measurement apparatus. In case of the neutrino-less mode, bremsstrahlung $\gamma$ quanta are emitted with an energy equal to

$$E_\gamma = Q_{2B} - \epsilon_1 - \epsilon_2 - E_{exe},$$

where $\epsilon_i$ are electron binding energies of daughter nuclide, and $E_{exe}$ is the populated level energy of $^{112}$Cd. The partial decay scheme of $^{112}$Sn is shown in Fig. 4. For the transition to the excited state, the bremsstrahlung $\gamma$ quanta
are accompanied by γ rays emitted from nuclear deexcitation. We did not observe any peak with the expected energies for the EC-EC decay of $^{112}$Sn. Only lower limits of half-lives are obtained using the Feldman-Cousins prescription [35]. The $T_{1/2}$ limits along with the γ energies and the corresponding detection efficiencies are shown in Table 4. The results of the fit for different γ rays are shown in Fig. 6.

3.3.2. $\beta^+ EC$ processes in $^{112}$Sn

The $(0\nu + 2\nu)\beta^+EC$ transition to the ground state of daughter nuclei is accompanied by two annihilation γ quanta with energy 511.0 keV. Moreover, the detector surroundings and the sample contains the radioactive element $^{208}$Tl which emits γ quanta of energy 510.8 keV. The measurements with and without the sample give count rates of $(0.0549 \pm 0.0030)$ and $(0.0424 \pm 0.0062) \text{ h}^{-1}$, respectively for the peak at 510.8 keV. So, this peak can be attributed to the background. Taking into account the FEP efficiency ($\eta$) of 7.58%, the lower limit of the $(0\nu + 2\nu)\beta^+EC$ transition to the ground state is $1.61 \times 10^{17}$ y.

In the case of the $(0\nu + 2\nu)\beta^+EC$ transition to the excited $2^+_1$ (617.5 keV) states of $^{112}$Cd, no peak has been observed. The limit on the half-life is reported in Table 4. The theoretical $T_{1/2}$ is estimated to be very high because of the low phase space available for this transition.

3.3.3. The $\beta^- \beta^-$ decay of $^{124}$Sn

For $\beta^- \beta^-$ decay of $^{124}$Sn, only transitions to excited states of $^{124}$Te can be studied as we are measuring gamma quanta. The partial decay scheme of $^{124}$Te is shown in Fig. 5 [39]. No peak has been observed at 602.7 keV. Therefore, we only report the lower limits of the half-lives in Table 5. The fits of the various energy windows together with the expected γ peaks are shown in Fig. 7.

The overall result is that due to the small mass of the tin sample (13.3 g), the obtained limits are rather poor compared to the existing experimental data. Therefore, it is certainly worthwhile to repeat the measurement with a larger
Figure 5: Partial decay scheme of $^{124}$Te. The energies of the excited levels and of the $\gamma$ quanta are in keV with relative intensities of $\gamma$ quanta are given in parenthesis.

sample, may be even enriched in the isotopes of interest, in order to obtain more stringent limits.

4. Double beta decay study with enhanced sensitivity

The sensitivity of a future experiment can be enhanced by either increasing the mass of the sample and the efficiency of the detector or by reducing the background. The efficiency of the detector depends on the position and the shape of the sample. The $\gamma$ rays can undergo either self-attenuation loose in the sample itself or absorption in other interposed media. The intensity of the gamma ray decreases to half to its initial value passing through a thickness of a half-value layer (HVL). The HVL depends on the atomic number of the element and also on the energy of the $\gamma$ ray. HVL for tin material at different $\gamma$-ray energies were calculated from the mass attenuation coefficients ($\mu/\rho$) values at
Figure 6: Energy spectra with 13.3 g of natural tin for 2367.5 h of measurement by ultra-low background HPGe detector. The fits are shown by solid lines. The arrows show position of the expected peaks due to double beta transition of $^{112}$Sn to the excited states of $^{112}$Cd.
Figure 7: Energy spectra with 13.3 g of natural tin for 2367.5 h of measurement by ultra-low background HPGe detector. The fits are shown by solid lines. The arrows show position of the expected peaks due to double beta transition of $^{124}$Sn to the excited states of $^{124}$Te.

different energies taken from NIST data [41].

Efficiencies of the detector were simulated for various sample thicknesses
and different sample-detector distances. At a fixed $\gamma$ ray energy, the efficiency decreases with an increasing thickness of the sample, and the number of double beta nuclei is getting bigger augmenting the mass of the sample. However, the product of the efficiency and the number of nuclei making the sample thicker.

Considering the geometry of the present HPGe detector, strips of tin of mass 1080 g can be placed on the sample holder. A tin sample, enriched in $^{112}$Sn up-to 90%, will contain $5.23 \times 10^{24}$ nuclei of this isotopes. Enriching in $^{124}$Sn instead by the same factor, the sample will contain $4.72 \times 10^{24}$ nuclei of it. For a measurement period of two years with the efficiencies 3.49 % and 4.56 % for $(0\nu)$ and $(2\nu)$EC-EC processes, respectively for $^{112}$Sn measurement, half-lives of up to $10^{22}$ years could be reached. This result would be better than the previously measured best half-life limit\[23\]. For the double-beta decay transition of $^{124}$Sn to the excited states of $^{124}$Te, with 90% enriched $^{124}$Sn and an efficiency of 4.53 %, the sensitivity would be the same as for $^{112}$Sn, up to $10^{22}$ years for the half-life. Both the results are comparable to the theoretical prediction made in different model calculations \[40, 42\].
Table 4: The experimental limits for the \((0\nu + 2\nu)\) \(\beta^+\) EC and \((0\nu)\) and \((2\nu)\) EC-EC processes and theoretical predictions for \((2\nu)\) \(\beta^+\) EC and \((2\nu)\) EC-EC processes in \(^{112}\text{Sn}\).

| Transition | \(E_\gamma\) (keV) | \(\eta\) (%) | LimS (90% C. L.) | \(T^{\text{exp}}_{1/2}\) (90% C. L.) \((10^{17})y\) | \(T^{\text{th,exp}}_{1/2}(2\nu)\) \(y\) \([\text{ES}]\) | \(T^{\text{th,exp}}_{1/2}(2\nu)\) \((10^{20})y\) |
|------------|-------------------|--------------|------------------|---------------------------------|--------------------------------|---------------------------------|
| \(\beta^+\) EC \((0\nu + 2\nu)\): g.s. | 511.0 | 7.58 | 58.0 | 1.61 | 0.97 | \(3.8 \times 10^{24}\)
| \(\beta^+\) EC \((0\nu + 2\nu)\); \(2^+_1\) | 617.5 | 5.95 | 6.0 | 12.17 | 7.02 | \(2.3 \times 10^{32}\)
| EC-EC \((0\nu)\); \(K^1L^2\): g.s. | 1889.1 | 2.16 | 1.6 | 16.56 | 8.15 | 4.0 \times 10^{34} |
| EC-EC \((0\nu)\); \(2^+_1\) | 617.5 | 4.59 | 6.0 | 9.40 | 9.72 | \[26\]
| EC-EC \((0\nu)\); \(0^+_1\) | 606.9 | 3.32 | 40.1 | 1.02 | 12.86 | \[26\]
| | 617.5 | 3.22 | 6.0 | 6.54 | 6.54 | \[26\]
| EC-EC \((0\nu)\); \(2^+_2\) | 617.5 | 2.35 | 6.0 | 4.81 | 8.89 | \[26\]
| | 694.9 | 2.11 | 35.9 | 0.72 | 0.72 | \[26\]
| | 1312.3 | 0.86 | 18.0 | 0.59 | 0.59 | \[26\]
| EC-EC \((0\nu)\); \(0^+_2\) | 617.5 | 2.85 | 6.0 | 5.83 | 6.86 | \[26\]
| | 815.8 | 1.01 | 28.3 | 0.44 | 0.44 | \[26\]
| EC-EC \((0\nu)\); \(2^+_3\) | 617.5 | 2.06 | 6.0 | 4.21 | 6.46 | \[26\]
| | 851.1 | 1.59 | 5.2 | 3.75 | 3.75 | \[26\]
| | 1468.8 | 1.05 | 4.2 | 3.07 | 3.07 | \[26\]
| EC-EC \((0\nu)\); \(0^+_3\) | 617.5 | 4.76 | 6.0 | 9.73 | 13.43 | \[26\]
| | 1253.3 | 2.72 | 24.4 | 1.37 | 1.37 | \[26\]
| EC-EC \((2\nu)\); \(2^+_1\) | 617.5 | 6.06 | 6.0 | 12.39 | 11.94 | \(1.9 \times 10^{32}\)
| EC-EC \((2\nu)\); \(0^+_1\) | 606.9 | 4.26 | 40.1 | 1.30 | 16.25 | \(7.4 \times 10^{24}\)
| | 617.5 | 4.51 | 6.0 | 9.22 | 9.22 | \[26\]
| EC-EC \((2\nu)\); \(2^+_2\) | 617.5 | 3.26 | 6.0 | 6.67 | 11.24 | \(1.9 \times 10^{32}\)
| | 694.9 | 2.96 | 35.9 | 1.01 | 1.01 | \[26\]
| | 1312.3 | 1.25 | 18.0 | 0.85 | 0.85 | \[26\]
| EC-EC \((2\nu)\); \(0^+_2\) | 617.5 | 3.90 | 6.0 | 7.97 | 8.64 | \[26\]
| | 815.8 | 1.39 | 28.3 | 0.61 | 0.61 | \[26\]
| EC-EC \((2\nu)\); \(2^+_3\) | 617.5 | 2.80 | 6.0 | 5.73 | 8.19 | \(6.2 \times 10^{31}\)
| | 851.1 | 2.14 | 5.2 | 5.07 | 5.07 | \[26\]
| | 1468.8 | 1.42 | 4.2 | 4.15 | 4.15 | \[26\]
| EC-EC \((2\nu)\); \(0^+_3\) | 617.5 | 4.76 | 6.0 | 9.73 | 13.43 | \(5.4 \times 10^{34}\)
| | 1253.3 | 2.72 | 24.4 | 1.37 | 1.37 | \[26\]
Table 5: The experimental results for \((0\nu + 2\nu + 0\nu\chi^0)\) \(\beta^-\beta^-\) decay and theoretical predictions of \((2\nu)\beta^-\beta^-\) decay of \(^{124}\text{Sn}\) to the excited states of \(^{124}\text{Te}\).

| Excited state (keV) | \(E_\gamma\) (keV) | \(\eta\) (%) | LimS | \(T_{1/2}^{exp}(90\%\text{ C. L.})\) (10\(^{18}\)y) | \(T_{1/2}^{theo}(2\nu)\) (10\(^{21}\)y) |
|---------------------|-------------------|---------|------|---------------------------------|-----------------|
| \(2^+_1\) (602.7)  | 602.7             | 6.05    | 29.0 | 1.53                            | 4.8 \times 10^{21} |
| \(2^+_2\) (1325.5) | 602.7             | 3.94    | 29.0 | 1.01                            | 2.5 \times 10^{27} |
|                    | 722.8             | 3.40    | 27.9 | 0.89                            |                 |
| \(0^+_1\) (1657.3) | 602.7             | 4.59    | 29.0 | 1.16                            | 1.2             |
|                    | 1054.5            | 3.21    | 8.7  | 2.70                            |                 |
| \(0^+_2\) (1882.9) | 557.4             | 3.70    | 6.3  | 4.30                            | 1.2             |
|                    | 602.7             | 2.86    | 29.0 | 0.72                            |                 |
|                    | 722.8             | 2.51    | 27.9 | 0.66                            |                 |
| \(2^+_3\) (2039.4) | 602.7             | 3.07    | 29.0 | 0.78                            | 0.86            |
|                    | 1436.7            | 1.57    | 9.2  | 1.25                            |                 |
| \(2^+_4\) (2091.6) | 602.7             | 4.60    | 29.0 | 1.16                            | 0.96            |
|                    | 1488.9            | 2.31    | 0.3  | 56.39                           |                 |
| \(0^+_3\) (2153.3) | 602.7             | 3.19    | 29.0 | 0.82                            | 0.95            |
|                    | 722.8             | 2.04    | 27.9 | 0.54                            |                 |
|                    | 827.8             | 2.26    | 26.3 | 0.63                            |                 |
|                    | 1550.4            | 0.56    | 5.4  | 0.76                            |                 |
| \(2^+_5\) (2182.4) | 602.7             | 4.06    | 29.0 | 1.03                            |                 |
|                    | 1579.7            | 1.86    | 4.7  | 2.90                            |                 |
5. Conclusion

In our preliminary study for the search of excited state transitions concerning possible double beta decays of tin isotopes with a HPGe detector, we measured half-life limits of $10^{17} - 10^{18}$ years for $\beta^+\text{EC}$ and EC-EC processes in $^{112}\text{Sn}$ and $10^{18}$ years for $\beta^-\beta^-$ transition in $^{124}\text{Sn}$. For the EC-EC decay mode of $^{112}\text{Sn}$ to the ground state of its daughter nucleide we obtained a half-life limit of $1.66 \times 10^{18}$ years. We showed that an experiment with larger mass, and material possibility enriched to high levels of either $^{112}\text{Sn}$ or $^{124}\text{Sn}$, could reach with the same HPGe detector half-life limits for both tin isotopes on the order of $10^{22}$ years for the decays to the excited levels. In India, an effort has been started to build up a bolometric Sn detector for experimental study of $0\nu\beta\beta$ decay in $^{124}\text{Sn}$ [43]. The experimental setup will be housed at upcoming India-based Neutrino Observatory site.

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