QUARK-LEPTON SYMMETRY

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ABSTRACT

Quark-lepton symmetric models are a class of gauge theories motivated by the similarities between the quarks and leptons. In these models the gauge group of the standard model is extended to include a “color” group for the leptons. Consequently, the quarks and leptons can then be related by a \(Z_2\) discrete quark-lepton symmetry which is spontaneously broken by the vacuum. Models utilizing quark-lepton symmetry with acceptable and interesting collider phenomenology have been constructed. The cosmological consequences of these models are also discussed.

1. The minimal quark-lepton symmetric model

Historically, there has been a rough correspondence between the hadrons and leptons which is now translated into a quark-lepton “symmetry”. This rough correspondence can be seen in the similarities of the weak interactions for the quarks and leptons. Therefore it maybe interesting to postulate an exact symmetry between quarks and leptons in the context of gauge theories. In the Minimal Standard Model (MSM), the quarks carry color whereas the leptons do not and it is also assumed that there are no right-handed neutrinos. To implement quark-lepton symmetry (hereafter referred to as q-\(\ell\) symmetry), equal numbers of quark and lepton degrees of freedom are needed. To achieve this we will introduce (i) the right-handed neutrino, \(\nu_R\), and (ii) a “color” group for the leptons. This then necessitates extending the MSM gauge group, \(G_{SM}\), to \(G_{q\ell} = SU(3)_\ell \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_X\) supplemented by a \(Z_2\) discrete symmetry between the quarks and leptons. Here \(SU(3)_q\) is the usual color group and \(SU(3)_\ell\) is its leptonic partner. The expanded fermionic generation is defined by the transformation laws

\[Q_L \sim (1, 3, 2)(1/3), \quad u_R \sim (1, 3, 1)(4/3), \quad d_R \sim (1, 3, 1)(-2/3),\]
\[F_L \sim (3, 1, 2)(-1/3), \quad E_R \sim (3, 1, 1)(-4/3), \quad N_R \sim (3, 1, 1)(2/3).\]

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The $Z_2$ discrete symmetry: $F_L \leftrightarrow Q_L$, $E_R \leftrightarrow u_R$, $N_R \leftrightarrow d_R$, $G_{q,\ell}^u \leftrightarrow G_{q,\ell}^d$, $C^u \leftrightarrow -C^u$ can now be defined [where $G_{q,\ell}^u$ are the gauge bosons of SU(3)$_q$ and $C^u$ is the gauge boson of U(1)$_X$]. Standard hypercharge is given by $Y = X + \frac{1}{2}T$, where $T = \text{diag}(-2,1,1)$ is a generator of SU(3)$_\ell$. Standard leptons are identified with the leptonic color triplets, while the $T = 1$ components are the exotic charge $\pm 1/2$ leptons, called liptons.

The MSM works very well for energies up to about 100 GeV. In order to spontaneously break SU(3)$_\ell$ and the quark-lepton discrete symmetry, as well as giving mass to the liptons, the Higgs bosons $\chi_1 \sim (3,1,1)(-2/3)$ and $\chi_2 \sim (1,3,1)(2/3)$ are introduced with $\chi_1 \leftrightarrow \chi_2$. The $T = 2$ component of $\chi_1$ develops a nonzero vacuum expectation value (VEV), while the VEV of $\chi_2$ is completely zero. Electroweak symmetry breaking is achieved through the Higgs doublet, $\phi \sim (1,1,2)(1)$, with $\phi \leftrightarrow \phi^\dagger$ (charge conjugate field) under q-$\ell$ symmetry. The overall symmetry breaking pattern can be summarised as follows:

\[
G_{q,l} \xrightarrow{(\chi_1)} SU(2)' \otimes G_{SM} \xrightarrow{(\phi)} SU(2)' \otimes SU(3)_q \otimes U(1)_Q.
\]

This minimal Higgs sector results in the tree-level mass relations, $M_u = M_e$ and $M_d = M_{\nu}^{\text{Dirac}}$, where $M_{u,e,d,\nu}$ refer to the $3 \times 3$ fermion mass matrices. These mass relations arise as a consequence of (i) the assumption that q-$\ell$ symmetry is a symmetry of the Yukawa Lagrangian and (ii) using only one Higgs doublet. If the minimal model is extended to contain two Higgs doublets, then the abovementioned mass relations can be avoided at tree-level but at the expense of predictivity. Alternatively, a certain q-$\ell$ symmetric model with a non-minimal gauge group has been shown to contain radiative corrections which can yield correct but unpredictable fermion masses.

2. Phenomenological Implications

2.1. Gauge Bosons

The low energy gauge group of the q-$\ell$ model is $SU(2)' \otimes SU(3)_q \otimes U(1)_Q$. As a result of the larger symmetry there will be additional gauge bosons, both massive and massless, to that of the MSM. The decomposition of the SU(3)$_\ell$ gauge bosons under the low energy gauge group is: $8 \rightarrow (1,1)(0) \oplus (2,1)(-\frac{1}{2}) \oplus (2,1)(\frac{1}{2}) \oplus (3,1)(0)$. The neutral component, the $Z'$ boson, can mix with the standard Z boson. The SU(2)' doublets of charge $\pm \frac{1}{2}$ are the massive SU(3)$_\ell$/SU(2)' bosons which can contribute

\[
V = \lambda_1 \left[\chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2 - u^2\right]^2 + \lambda_3 \left(\phi^\dagger \phi - u^2\right)^2 + \lambda_2 \chi_1^\dagger \chi_1 \chi_2^\dagger \chi_2 + \lambda_4 \left[\phi^\dagger \phi - u^2 + \chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2 - v^2\right]^2.
\]

To show that the required pattern of symmetry breaking for the minimal q-$\ell$ model can be realised, consider the Higgs potential

$\lambda_1 \chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2 - v^2 \Rightarrow \lambda_3 \left(\phi^\dagger \phi - u^2\right) \Rightarrow \lambda_4 \left[\phi^\dagger \phi - u^2 + \chi_1^\dagger \chi_1 + \chi_2^\dagger \chi_2 - v^2\right]^2.$

The mass relation involving the neutrinos can be avoided if Majorana masses are given to the right-handed neutrinos. This can be achieved through the Higgs multiplets $\Delta_1 \sim (6,1,1)(4/3)$ and $\Delta_2 \sim (1,6,1)(-4/3)$ with $\Delta_1 \leftrightarrow \Delta_2$. It is assumed that the $T = -4$ component of $\Delta_1$ develops a nonzero VEV while the VEV of $\Delta_2$ remains zero.
to FCNC processes at 1-loop (e.g. $\mu \to e + \gamma$).

The remaining triplet contains the massless $SU(2)'$ gauge bosons. By fitting the model parameters to the low energy electroweak data, the lower bound on the mass of the $Z'$ boson was found to be around 700 GeV. If the number of fermions is not too large then $SU(2)'$ is expected to be asymptotically free and by analogy with QCD we assume that it confines all $SU(2)'$ colored states. Thus we expect $SU(2)'$ glueballs to exist with a mass of the order of the $SU(2)'$ confinement scale, $\Lambda_{SU(2)'}$.

2.2. Liptons

The charged $\pm \frac{1}{2}$ liptons transform as doublets under $SU(2)'$ and are expected to be confined into unstable integrally charged bound states. These bound states are non-relativistic since the mass of the lipton ($> O(100 \text{ GeV})$) is expected to be much larger than $\Lambda_{SU(2)'}$ ($\leq \Lambda_{QCD}$). The liptons also participate in electroweak interactions so they could be produced at colliders. Likewise the liptonic bound states can decay into ordinary particles via electroweak interactions.

3. Cosmological Implications

3.1. Domain Walls

Spontaneously broken discrete symmetries can lead to cosmologically unacceptable domain walls. The domain wall problem in the q-\ell model can be avoided as follows: (i) Embed the discrete q-\ell symmetry into a continuous one. The resulting chain of symmetry breakings will produce a string-wall network in which the walls become unstable. (ii) There exists a parameter space for the Higgs potential such that discrete q-\ell symmetry restoration cannot occur at high temperatures. This allows one to consistently arrange, as an initial condition of the Big Bang, for all the vacua of causally disconnected regions to be the same. (iii) It could be possible that an inflationary period occurs after the discrete q-\ell symmetry phase transition during the course of the evolution of the early universe.

3.2. $SU(2)'$ Glueballs

If the $SU(2)'$ glueball has a mass around 1 GeV then its lifetime is constrained to be less than 1 sec for compatibility with standard Big Bang nucleosynthesis. If the mass of the glueball is around 1 keV then its existence will not interfere with nucleosynthesis and is a possible candidate for Dark Matter.

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4 This will put a bound on the mass of the coset space gauge bosons in terms of mixing angles and lipton masses. There will also be contributions from the Higgs sector which we have not considered.
4. References

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