\[ K \to \pi\pi, \ K_L-K_S \text{ mass difference} \]
and the Dalitz decays of \( K_L^* \)

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Abstract

\( K \to \pi\pi, \ K_L-K_S \) mass difference, \( K_L \to \gamma\gamma \) and the Dalitz decays of \( K_L^* \) are studied systematically by assuming that their amplitude can be described in terms of a sum of short distance and long distance contributions. Dominance of the short distance effect on the \( K_L-K_S \) mass difference will be checked by the Dalitz decays of \( K_L \) in a way consistent with the \( K \to \pi\pi \) and \( K_L \to \gamma\gamma \) decays.

I. INTRODUCTION

Our starting point to study nonleptonic weak processes is to assume that their amplitude can be decomposed into a sum of short distance and long distance terms and that the long distance amplitude is dominated by dynamical contributions of various hadrons \([1]\).

It has been known that a short distance contribution is small in the \( K_L \to \gamma\gamma \) decay \([2]\) and a naively factorized \( \Delta I = 1/2 \) amplitude for the \( K \to \pi\pi \) decays which is now classified into the short distance one is also much smaller than the observed one \([3]\). However, importance of long distance contribution to the \( K_L-K_S \) mass difference, \( \Delta m_K = m_{K_L} - m_{K_S} \), is still in controversy although the mass difference has been used to test theories within or beyond the standard model by assuming explicitly or implicitly dominance of short distance contribution. Therefore, it will be meaningful to study a role of the long distance contribution in \( \Delta m_K \) and test it in some other processes. To this, we will study the following two extreme cases since theoretical and experimental ambiguities are still large: (i) the short distance contribution vanishes, \( (\Delta m_K)_{SD} = 0 \), and (ii) the long distance contribution vanishes, \( (\Delta m_K)_{LD} = 0 \). Then we investigate responses of the Dalitz decays of \( K_L^* \) in the above two extreme cases.

Before we study amplitudes for the weak processes mentioned above, we review briefly the effective weak Hamiltonian which is usually written in the form \([4,5]\).

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\[ H_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \left\{ c_1 O_1 + c_2 O_2 + (\text{penguin}) \right\} + h.c., \]  
(1)

or equivalently,

\[ H_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \left\{ c_- O_- + c_+ O_+ + (\text{penguin}) \right\} + h.c. \]  
(2)

with \( O_\pm = O_1 \pm O_2 \), where the four quark operators \( O_1 \) and \( O_2 \) are given by

\[ O_1 =: (\bar{u}s)_{V-A}(\bar{d}u)_{V-A} : \quad \text{and} \quad O_2 =: (\bar{u}u)_{V-A}(\bar{d}s)_{V-A} :. \]  
(3)

\( O_\pm \) transform like \( 8_a \) and \( 27 \) of the flavor \( SU_f(3) \) and are responsible for the \( \Delta I = 1/2 \) and \( 3/2 \) amplitudes for the \( K \to \pi \pi \) decays, respectively. \( V_{ij} \) denotes a CKM matrix element which is taken to be real since CP invariance is always assumed in this paper. When we apply the factorization prescription to the \( K \to \pi \pi \) amplitudes later, we use the so-called BSW Hamiltonian

\[ H^{BSW}_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \left\{ a_1 O_1^H + a_2 O_2^H + (\text{penguin})^H \right\} + h.c. \]  
(4)

which can be obtained from Eq.(1) by using the Fierz reordering. The operators with the superscript \( H \), i.e., \( O_1^H \), \( O_2^H \) and \( (\text{penguin})^H \), should no longer be Fierz reordered. The coefficients \( a_1 \) and \( a_2 \) are given by

\[ a_1 = c_1 + \frac{c_2}{N_c} \simeq 1.14, \quad a_2 = c_2 + \frac{c_1}{N_c} \simeq -0.209, \]  
(5)

where \( N_c \) is the color degree of freedom. Numerical values are obtained by using the values of \( c_1 \) and \( c_2 \) with the leading order QCD corrections.

In the next section, we will study two photon decays, \( K_L \to \gamma \gamma^{(*)} \), and the Dalitz decays of \( K_L \), where \( \gamma^{(*)} \) denotes an (off-mass-shell) photon. The amplitude will be given by two independent matrix elements of \( H_w \) taken between pseudo scalar meson states and between helicity \( \lambda = \pm 1 \) vector meson states. In 3, the \( K_L-K_S \) mass difference will be investigated. Its short distance term \( (\Delta m_{K^0})_{\text{SD}} \) is proportional to \( \langle K^0|O_{\Delta S=2}|\bar{K}^0 \rangle \) arising from the box diagrams and the long distance one \( (\Delta m_{K})_{\text{LD}} \) is dominated by a sum of contributions of pseudo scalar and vector meson poles and of \( (\pi \pi) \) intermediate states. In 4, the \( K \to \pi \pi \) decays will be investigated. The short distance amplitude is estimated by using the naive factorization prescription. The long distance amplitude is assumed to be dominated by dynamical contributions of various hadron states and is estimated by using a hard pion approximation. Since the naively factorized (short distance) amplitude does not satisfy the \( \Delta I = 1/2 \) rule and its \( \Delta I = 1/2 \) part is much smaller than the observed one, the (long distance) hard pion amplitude of \( \Delta I = 1/2 \) should be much larger than the factorized one and satisfy the \( \Delta I = 1/2 \) rule to reproduce the observation. To realize this, asymptotic matrix elements of \( H_w \) taken between \( \langle \pi | \) and \( | K \rangle \) (or \( | K^{(*)} \rangle \)) must satisfy the rule, which will be demonstrated by using a simple quark counting in 5. Asymptotic matrix elements of \( H_w \) taken between the ground-state-meson states will be parameterized in the same section. Inserting the parameterization into the long distance amplitudes, we will compare our result with experimental data on \( K \to \pi \pi \), \( \Delta m_K \), \( K_L \to \gamma \gamma \) and the Dalitz decays of \( K_L \) in 6. In the final section, we will provide a brief summary.
II. TWO PHOTON DECAYS OF $K_L$

Now we study the $K_L \rightarrow \gamma \gamma$ and $\gamma \gamma^*$ decays. We consider Lorentz invariant amplitudes in the infinite momentum frame (IMF) for later convenience. As mentioned before, it is known \[1\] that short distance contribution to the $K_L \rightarrow \gamma \gamma$ is small. Therefore, we neglect it and consider only long distance effects which will be dominated by pole amplitudes since contributions of two and more pion intermediate states are suppressed because of the approximate CP invariance and small phase space volume, respectively. However a sum of pseudo scalar meson ($P = \pi^0$, $\eta$, $\eta'$) pole amplitudes \[13\]

$$A_P(K_L \rightarrow \gamma \gamma) = \sum_{P_i} \frac{\langle K_L|H_w|P_i\rangle A(P_i \rightarrow \gamma \gamma)}{(m_{P_i}^2 - m_K^2)}$$  

(6)

with the usual $\eta$-$\eta'$ mixing angle, $\theta_P \simeq -20^\circ$ \[14\], is not sufficient \[13\] to reproduce the observed rate \[14\], $\Gamma(K_L \rightarrow \gamma \gamma)_{\text{expt}} = (7.26 \pm 0.35) \times 10^{-12}$ eV. Therefore we have to take into account some other contributions. Although a possible role of the pseudo scalar glue-ball ($\iota$) through the penguin effect has been considered in Ref. \[13\], it will be not very important in the present perspective because of its high mass and small rate $B(\iota \rightarrow \gamma \gamma)_{\text{expt}} < 1.2$ keV. Another possible contribution to the $K_L \rightarrow \gamma \gamma$ will be the $K^*$ meson pole with the vector meson dominance (VMD) \[10\]. However there have been some arguments against it \[17\]. These arguments are based on the field algebra \[18\] and their weak Hamiltonian consists of symmetric products of left-handed currents and transforms like $\mathbf{8_0}$ of $SU_f(3)$. It is much different from the standard model presented in the previous section. Therefore we should not be restricted by such arguments. Since the VMD in the electro-magnetic interactions of hadrons can be derived \[19\] independently of the field algebra, we now can be free from the above arguments in Ref. \[17\] even if we use the VMD. In this way, we can safely take into account the $K^*$ pole contribution in the $K_L \rightarrow \gamma \gamma$ decay \[20\]. Its off-mass-shell amplitude is given by

$$A_{K^*}(K_L \rightarrow \gamma \gamma^*(k^2)) = \sum_{V_i} \sum_{V_j} \sqrt{2} X_{V_i} X_{V_j} G_{K^0 K^0} \langle K^0|H_w|V_j\rangle \frac{1}{(m_{V_i}^2 - k^2)(m_{V_j}^2 - k^2)}$$

$$\times \left\{ \frac{1}{(m_{V_i}^2 - k^2)(m_{V_j}^2 - k^2)} + \frac{1}{(m_{V_i}^2 - k^2)m_{V_j}^2} \right\}$$  

(7)

with $V_i = \rho^0$, $\omega$ and $\phi$. $X_{V_i} = e m_{V_i}^2 / f_{V_i}$ is the photon-vector meson coupling strength and $f_{V_i}$ is the usual photon-vector meson transition moment. The subscript $\lambda = \pm 1$ of the matrix element $\langle K^0|H_w|V_j\rangle_{\lambda = \pm 1}$ denotes the helicity of the vector meson states which sandwich $H_w$. The $K^*$ pole amplitude for the $K_L \rightarrow \gamma \gamma$ decay is simply obtained by putting $k^2 = 0$ in the above off-mass-shell amplitude, Eq. (7).

The pseudo scalar meson pole amplitude, Eq. (6), can be extrapolated into the off-mass-shell region approximately in the form,

$$A_P(K_L \rightarrow \gamma \gamma^*(k^2)) = \sum_{P_i} \frac{\langle K_L|H_w|P_i\rangle A(P_i \rightarrow \gamma \gamma)}{(m_{P_i}^2 - m_K^2)(1 - k^2/\Lambda_P^2)}$$  

(8)

since the observed form factors for the $\pi^0$, $\eta$ and $\eta' \rightarrow \gamma \gamma^*$ decays are approximately described in the form \[21\], $\sim (1 - k^2/\Lambda_P^2)^{-1}$ with $\Lambda_P \simeq m_{\rho}$. For more precise arguments, however, we
may have to use a more improved result from recent measurements \[22\], \(\Lambda_{\pi} = 776 \pm 10 \pm 12 \pm 16 \text{ MeV}, \ \Lambda_{\eta} = 774 \pm 11 \pm 16 \pm 22 \text{ MeV}\) and \(\Lambda_{\eta'} = 859 \pm 9 \pm 18 \pm 20 \text{ MeV}\). In this way, the amplitude and the form factor for the \(K_L \to \gamma \gamma^*\) are approximately given by

\[
A(K_L \to \gamma \gamma^*(k^2)) \simeq A_P(K_L \to \gamma \gamma^*(k^2)) + A_{K^*}(K_L \to \gamma \gamma^*(k^2))
\]

and

\[
f(k^2) = \frac{A(K_L \to \gamma \gamma^*(k^2))}{A(K_L \to \gamma \gamma)},
\]

respectively, where

\[
A(K_L \to \gamma \gamma) = A(K_L \to \gamma \gamma^*(k^2 = 0)).
\]

As seen in Eqs.(9) and (10) with Eqs.(6), (7) and (8), the amplitude \(A(K_L \to \gamma \gamma^*)\) and the form factor \(f(k^2)\) for the \(K_L \to \gamma \gamma^*\) have been written in terms of asymptotic ground-state-meson matrix elements of \(H_w\) (matrix elements of \(H_w\) taken between the ground-state-meson states with infinite momentum).

Since the Dalitz decay of \(K_L\) proceeds dominantly as \(K_L \to \gamma \gamma^* \to \gamma \ell^+ \ell^-\), its branching fraction is given by the following formula \[23\],

\[
R_{\ell^+ \ell^-} = \frac{\Gamma(K_L \to \gamma \ell^+ \ell^-)}{\Gamma(K_L \to \gamma \gamma)} = [\Gamma(K_L \to \gamma \gamma)]^{-1} \int_{x_{\text{min}}}^1 dx \left[ \frac{d\Gamma(K_L \to \gamma \ell^+ \ell^-)}{dx} \right],
\]

with \(x_{\text{min}} = (2m_\ell/m_K)^2\), where

\[
[\Gamma(K_L \to \gamma \gamma)]^{-1} \frac{d\Gamma(K_L \to \gamma \ell^+ \ell^-)}{dx} = \left( \frac{2\alpha}{3\pi} \right) \left( \frac{1 - x}{x} \right)^3 \left[ 1 + 2 \left( \frac{m_\ell}{m_K} \right)^2 \frac{1}{x} \right] \left[ 1 - 4 \left( \frac{m_\ell}{m_K} \right)^2 \frac{1}{x} \right]^{1/2} |f(x)|^2.
\]

In the previous analyses \[23,24\] which were restricted by the arguments in Ref. \[17\], the \(K^*\) pole had to vanish in the \(K_L \to \gamma \gamma\) while it survived in the Dalitz decays of \(K_L\). In this case, however, it will be hard to reproduce the observed \(\Gamma(K_L \to \gamma \gamma)\) in consistency with the \(K \to \pi \pi\) decays if the usual \(\eta-\eta'\) mixing angle \(\theta_P \simeq -20^\circ\) is taken, as mentioned before.

### III. \(K_L-K_S\) MASS DIFFERENCE

Now we study the \(K_L-K_S\) mass difference \(\Delta m_K\) by decomposing it into a sum of short distance and long distance contributions \[25\],

\[
\Delta m_K = (\Delta m_K)_{SD} + (\Delta m_K)_{LD}.
\]

The short distance contribution \((\Delta m_K)_{SD}\) is proportional to the matrix element of the \(\Delta S = 2\) box operator \[3\] taken between \(\langle K^0 \rangle\) and \(\langle \bar{K}^0 \rangle\), i.e.,

\[
(\Delta m_K)_{SD} \propto \langle K^0 \rangle |O_{\Delta S=2}| \langle \bar{K}^0 \rangle.
\]
The right-hand side of the above equation can be related to the matrix element of the \( \Delta I = 3/2 \) operator \( O_{\Delta I=3/2} \) in the effective weak Hamiltonian,

\[
\langle K^0 | O_{\Delta S=2} | \bar{K}^0 \rangle = \sqrt{2} \langle \pi^0 | O_{\Delta I=3/2} | \bar{K}^0 \rangle,
\]

in the \( SU_f(3) \) symmetry limit \(^{23}\) or by using the asymptotic \( SU_f(3) \) symmetry \(^{24}\) which implies a flavor \( SU_f(3) \) symmetry of matrix elements of operators (like charges, currents, etc.) taken between single hadron states with 1-8 mixing in the IMF \(^{25}\).

As will be seen later, the short distance amplitudes for the \( K \to \pi\pi \) decays do not satisfy the well-known \( \Delta I = 1/2 \) rule and their \( \Delta I = 1/2 \) part is much smaller than the observed one. However the long distance amplitudes are given approximately by asymptotic ground-state-meson matrix elements of \( H_w \) in the present perspective and are expected to be much larger than the short distance ones to reproduce the observation. Therefore the \( \Delta I = 1/2 \) rule in the \( K \to \pi\pi \) decays will be understood rather easily if \( (\Delta m_K)_{SD} \) is suppressed. On the contrary, if \( (\Delta m_K)_{SD} \) dominates \( \Delta m_K \), it will be hard to explain the observed \( \Delta I = 1/2 \) rule in the \( K \to \pi\pi \) decays in a simple way.

The long distance contribution \( (\Delta m_K)_{LD} \) can be written in an elegant form in the IMF \(^{10}\),

\[
(\Delta m_K)_{LD} = \int \frac{dm_n^2}{2m_K(m_n^2 - m_{\pi^0}^2)} \left\{ \left[ \langle n | H_w | K_L \rangle \right]^2 - \left[ \langle n | H_w | K_S \rangle \right]^2 \right\}.
\]

It will be dominated by pole contributions of pseudo scalar and vector mesons and by \((\pi\pi)\) continuum contributions,

\[
(\Delta m_K)_{LD} \simeq (\Delta m_K)_{pole} + (\Delta m_K)_{\pi\pi}.
\]

As the \( \pi\pi \) continuum contribution, we here take the following value \(^{29}\),

\[
\frac{(\Delta m_K)_{\pi\pi}}{\Gamma_{K_S}} = 0.22 \pm 0.03,
\]

which has been obtained by using Omnes-Mushkevili equation and the measured \( \pi\pi \) phase shifts, where \( \Gamma_{K_S} \) denotes the full width of \( K_S \). Therefore we hereafter can concentrate on the pole contribution which is approximated by

\[
(\Delta m_K)_{pole} \simeq \left\{ \sum_{P_i} \frac{|\langle K_L | H_w | P_i \rangle|^2}{2m_K(m_n^2 - m_{P_i}^2)} - \sum_{V_i} \frac{|\langle K_S | H_w | V_i \rangle|^2}{2m_K(m_n^2 - m_{V_i}^2)} \right\},
\]

in the IMF, where \( P_i = \pi^0, \eta \) and \( \eta' \) and \( V_i = \rho^0, \omega \) and \( \phi \). (The \( \iota \) contribution has been neglected since it is expected to be not very important because of its high mass.) As seen in Eq. (20), the pole contribution to the \( K_L-K_S \) mass difference \( (\Delta m_K)_{pole} \) has been described in terms of asymptotic ground-state-meson matrix elements of \( H_w \).

### IV. TWO PION DECAYS OF K MESONS

Amplitudes for two pion decays of \( K \) mesons are again classified into short distance and long distance ones. The former will be estimated by using the naive factorization below
Table I. Naively factorized amplitudes for the $K \to \pi\pi$ decays where terms proportional to $f_-$ are neglected.

| Decay                  | $M_{SD}$                           |
|------------------------|------------------------------------|
| $K_S \to \pi^+\pi^-$   | $-iV_{ud}V_{us}(G_F/\sqrt{2})a_1f_\pi(m_K^2 - m_\pi^2)f_+^{\pi K}(m_\pi^2)$ |
| $\bar{K}_S \to \pi^0\pi^0$ | 0                                  |
| $K^+ \to \pi^+\pi^0$    | $iV_{ud}V_{us}(G_F/2)(a_1 + a_2)f_\pi(m_K^2 - m_\pi^2)f_+^{\pi K}(m_\pi^2)$ |

while the latter is assumed to be dominated by dynamical contributions of various hadron states as in the previous sections and is estimated later by using a hard pion approximation in the IMF.

The short distance amplitudes for the $K \to \pi\pi$ decays are estimated by using the naive factorization in the BSW scheme $\text{(a)}$. As an example, we consider the amplitude for the $K^+ \to \pi^+\pi^0$ decay which is given by

$$M_{SD}(K^+(p) \to \pi^0(p')\pi^+(q)) = \frac{G_F}{\sqrt{2}}V_{us}V_{ud}\{a_1\langle \pi^+(q)|(|\bar{u}d|_{V-A}|0\rangle\langle \pi^0(p')|(\bar{s}u)_{V-A}|K^+(p)\rangle + a_2\langle \pi^0(p')|(\bar{u}u)_{V-A}|0\rangle\langle \pi^+(q)|(|\bar{s}d)_{V-A}|K^+(p)\rangle\}. \quad (21)$$

Factorizable amplitudes for the other $K \to \pi\pi$ decays also can be calculated in the same way. To evaluate these amplitudes, we use the following parameterization of matrix elements of currents,

$$\langle \pi(q)|A_\mu^{(\pi)}|0\rangle = -iq_\mu g_\mu, \quad \text{etc.}, \quad (22)$$

$$\langle \pi(p')|V_\mu^{(\pi K)}|K(p)\rangle = (p + p')_\mu f_+^{(\pi K)}(q^2) + q_\mu f_-^{(\pi K)}(q^2), \quad \text{etc.}, \quad (23)$$

where $q = p-p'$. Using these expressions of current matrix elements, we obtain the factorized amplitudes listed in Table I, where terms proportional to $f_-(q^2)$ have been neglected since their coefficients are small in the spectator decays and, in possible annihilation decays, they are proportional to $a_2$. The penguin contribution has also been neglected since, recently, it is considered to be very small $\text{(b)}$ in contrast with the old expectation $\text{(c)}$. If the values of $a_1$ and $a_2$ with the leading order QCD corrections $\text{(d)}$ are taken, it will be seen, since $|a_1| \gg |a_2|$, that the factorized amplitude for the $K^0 \to \pi^0\pi^0$ decay which is described by the color mismatched diagram, $\bar{s} \to \bar{d} + (u\bar{u})_1$, is proportional to $a_2$ and therefore is much smaller (the color suppression) than those for the spectator decays and that the short distance amplitude for the $K^+ \to \pi^+\pi^0$ decay is considerably larger than the observed one. For the same reason, the size of the $\Delta I = 1/2$ amplitude is not much larger than the $\Delta I = 3/2$ part. Therefore it is hard to reproduce the well-known approximate $\Delta I = 1/2$ rule by the short distance amplitudes for the $K \to \pi\pi$ decays.

Next we study long distance amplitudes for these decays by using a hard pion approximation in the IMF, $i.e.$, we evaluate the amplitudes at a slightly unphysical point $q \to 0$ in the IMF $\text{(e)}$, $\text{(f)}$. The hard pion amplitude as the long distance one is written in the form,
\[ M_{LD}(K \rightarrow \pi_1\pi_2) \simeq M_{ETC}(K \rightarrow \pi_1\pi_2) + M_S(K \rightarrow \pi_1\pi_2), \]

where \( M_{ETC} \) and \( M_S \) are given by

\[ M_{ETC}(K \rightarrow \pi_1\pi_2) = \frac{i}{\sqrt{2f_\pi}} \langle \pi_2 | [V_{\pi_1}, H_w] | K \rangle + (\pi_2 \leftrightarrow \pi_1) \]

and

\[ M_S(K \rightarrow \pi_1\pi_2) = \frac{i}{\sqrt{2f_\pi}} \left\{ \sum_n \left( \frac{m^2_\pi - m^2_{\pi_1}}{m^2_n - m^2_{\pi_1}} \right) \langle \pi_2 | A_{\pi_1} | n \rangle \langle n | H_w | K \rangle + \sum_\ell \left( \frac{m^2_\pi - m^2_{\pi_1}}{m^2_\ell - m^2_{\pi_1}} \right) \langle \pi_2 | H_w | \ell \rangle \langle \ell | A_{\pi_1} | K \rangle \right\} + (\pi_2 \leftrightarrow \pi_1), \]

respectively, where \([V_{\pi} + A_{\pi}, H_w] = 0\) has been used. (See Refs. [11] and [12] for notations.) \( M_{ETC} \) has the same form as the one in the old soft pion approximation but now has to be evaluated in the IMF. The surface term has been given by a sum of all possible pole amplitudes, \( i.e., n \) and \( \ell \) run over all possible single meson states, not only ordinary \( \{q\bar{q}\} \), but also hybrid \( \{q\bar{q}g\} \), four-quark \( \{qq\bar{q}\} \), glue-balls, etc. However, since values of wave functions of orbitally excited \( \{q\bar{q}\}_{L \neq 0} \) states at the origin are expected to vanish in the non-relativistic quark model, and more generally, wave function overlappings between the ground-state \( \{q\bar{q}\}_0 \) and excited-state-meson states are expected to be small, we neglect contributions of excited-state mesons to the amplitudes except for the \( K^+ \rightarrow \pi^+\pi^0 \) in which the ground-state-meson contributions can be strongly suppressed because of the (approximate) \( \Delta I = 1/2 \) rule in the asymptotic ground-state-meson matrix elements of \( H_w \) as will be seen later.

Asymptotic matrix elements of isospin \( V_{\pi} \) and its axial counterpart \( A_{\pi} \) involved in the amplitudes can be well parameterized by using (asymptotic) \( SU_f(3) \) symmetry. Therefore the hard pion amplitude in Eq. (24) with Eqs. (25) and (26) as the non-factorizable long distance contribution is approximately described in terms of asymptotic ground-state-meson matrix elements of \( H_w \).

Amplitudes for dynamical hadronic processes, in general, can be described in the form, \( (\text{continuum contribution}) + (\text{Born term}) \). In the present case, \( M_S \) is given by a sum of pole amplitudes so that \( M_{ETC} \) corresponds to the continuum contribution [34] which can develop a phase relative to the Born term. Therefore, using isospin eigen amplitudes \( M^{(f)}_{ETC} \)'s and their phases \( \delta_f \)'s, we here parameterize the ETC terms as

\[ M_{ETC}(K^0_{\pi} \rightarrow \pi^+\pi^-) = \frac{2}{3} M^{(2)}_{ETC}(K \rightarrow \pi\pi)e^{i\delta_2} + \frac{1}{3} M^{(0)}_{ETC}(K \rightarrow \pi\pi)e^{i\delta_0}, \]

\[ M_{ETC}(K^0_{\pi} \rightarrow \pi^0\pi^0) = -\frac{2\sqrt{2}}{3} M^{(2)}_{ETC}(K \rightarrow \pi\pi)e^{i\delta_2} + \sqrt{\frac{2}{3}} M^{(0)}_{ETC}(K \rightarrow \pi\pi)e^{i\delta_0}, \]

\[ M_{ETC}(K^+ \rightarrow \pi^+\pi^0) = \overline{M^{(2)}_{ETC}(K \rightarrow \pi\pi)e^{i\delta_2}}, \]

since the \( S \)-wave \( \pi\pi \) final states can have isospin \( I = 0 \) and 2. Therefore the so-called final state interactions are now included in the long distance amplitudes. It is much more natural than the usual case in which phase factors are multiplied to the factorized amplitudes by hand.
In this way, we see that the long distance amplitudes for the $K \to \pi\pi$ decays will satisfy well the $\Delta I = 1/2$ rule if the asymptotic ground-state-meson matrix elements of $H_w$ satisfy the same rule. Inversely, to reproduce the observed approximate $\Delta I = 1/2$ rule in the $K \to \pi\pi$ amplitudes, the long distance contributions dominate these decays and the ground-state-meson matrix elements of $H_w$ should satisfy the same rule. Therefore we here assume that the hard pion amplitude as the long distance contribution dominates the $\Delta I = 1/2$ amplitude in the $K \to \pi\pi$ decays since the short distance $\Delta I = 1/2$ amplitude is much smaller than the observed one as discussed before and that the asymptotic ground-state-meson matrix elements of $H_w$ satisfy the $\Delta I = 1/2$ rule. (We will demonstrate in the next section, using a simple quark counting, that they are obliged to satisfy the $\Delta I = 1/2$ rule.)

By neglecting small contributions of excited states and seemingly small $\Delta I = 3/2$ asymptotic ground-state-meson matrix elements of $H_w$, the long distance amplitudes for the $K \to \pi\pi$ decays can be summarized as follows,

$$M_{LD}(K_S^0 \to \pi^+\pi^-) \simeq -\frac{i}{f_{\pi}}\langle \pi^+ | H_w | K^+ \rangle \left\{ e^{i\delta_0} + \sqrt{\frac{1}{2}} \langle \pi^+ | H_w | K^+ \rangle \right\},$$

$$M_{LD}(K_S^0 \to \pi^0\pi^0) \simeq -\sqrt{\frac{1}{2}}M_{LD}(K_S \to \pi^+\pi^-),$$

$$M_{LD}(K^+ \to \pi^+\pi^0) \simeq 0,$$

where the size of $h = \langle \pi^- | A_{\pi^-} | \rho^0 \rangle$ is estimated to be $|h| \simeq 1.0$ from the observed decay rate \cite{14}, $\Gamma(\rho \to \pi\pi)_{\text{expt}} \simeq 150$ MeV, by using PCAC. In this way, the $K \to \pi\pi$ amplitudes have been described approximately by the asymptotic ground-state-meson matrix elements of $H_w$ and the iso-scalar $S$-wave $\pi\pi$ phase shift $\delta_0$. In Eq.\(\ref{eq:32}\), the right hand side is vanishing since the excited-state meson contributions and the $\Delta I = 3/2$ asymptotic ground-state-meson matrix elements of $H_w$ have been neglected. As will be seen later, the $\Delta I = 3/2$ part of the long distance amplitude can be supplied through four-quark meson pole amplitudes even if the asymptotic ground-state-meson matrix elements of $H_w$ satisfy the $\Delta I = 1/2$ rule. They can interfere destructively with the too big $\Delta I = 3/2$ part of the factorized amplitudes in Table I.

V. Parameterization of Asymptotic Matrix Elements of $H_w$

We have described approximately the long distance amplitudes for the $K \to \pi\pi$ decays, the $K^0\bar{K}^0$ mixing and the $K_L \to \gamma\gamma^{(*)}$ using the asymptotic ground-state-meson matrix elements of $H_w$ and have seen that the $\Delta I = 1/2$ rule in the $K \to \pi\pi$ decays is mainly controlled by the same selection rule in the asymptotic ground-state-meson matrix elements of $H_w$. We have also seen that $(\Delta m_K)_{\text{SD}}$ is related to $\langle \pi^0 | O_{\Delta I=3/2} | \bar{K}^0 \rangle$ and hence the former should vanish if the ground-state-meson matrix elements of $H_w$ satisfy the $\Delta I = 1/2$ rule.

Before we parameterize asymptotic matrix elements of $H_w$, we study constraints on them using a simple quark counting \cite{31}. The effective weak Hamiltonian $H_w$ has been given by a sum of four quark operators $O_\pm$ (and the penguin operator which always satisfies the $\Delta I = 1/2$ rule) in Eq.\(\ref{eq:3}\). The normal ordered four-quark operators $O_\pm$ can be expanded into a sum of products of (a) two creation operators to the left and two annihilation operators
to the right, (b) three creation operators to the left and one annihilation operator to the right, (c) one creation operator to the left and three annihilation operators to the right, and (d) all (four) creation operators or annihilation operators of quarks and anti-quarks. We associate (a)–(d) with quark-line diagrams describing different types of matrix elements of $O_{\pm}$. For (a), we utilize the two creation and annihilation operators to create and annihilate, respectively, the quarks and anti-quarks belonging to the meson states $\{q\bar{q}\}$ and $\{\bar{q}q\}$ in the asymptotic matrix elements of $O_{\pm}$. For (b) and (c), we need to add a spectator quark or anti-quark to reach physical processes, $\langle\{q\bar{q}\}\rangle$ and $\langle\{q\bar{q}\}\rangle$, where $\{q\bar{q}\}$ denotes four-quark mesons $^{[24]}$. They can be classified into the following four types, $\{q\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q}) \oplus \{(qq)(\bar{q}\bar{q}) \pm (qq)[\bar{q}\bar{q}]\}$, where ( ) and [ ] denote symmetry and antisymmetry, respectively, under the exchange of flavors between them. We here consider contributions only of the first two since the others do not have $J^{P(C)} = 0^{+(+)}$.

While we count all possible connected quark-line diagrams, we forget color degree of freedom of quarks since they will be compensated by a sea of soft gluons carried by light mesons and have to be careful with the order of the quark(s) and anti-quark(s) in $O_{\pm}$ since symmetry (or antisymmetry) property of wave functions of meson states sandwiching $O_{\pm}$ under exchanges of quark and anti-quark plays an important role. Noting that the wave function of the ground-state $\{q\bar{q}\}$ meson is antisymmetric under the exchange of its quark and anti-quark $^{[33]}$, we obtain the following constraints on asymptotic matrix elements of $O_{\pm}$ $^{[31]}$,

\[
\langle\{q\bar{q}\}\rangle_{O_{\pm}}\langle\{q\bar{q}\}\rangle = 0, \tag{33}
\]

\[
\langle\{qq\rangle[\bar{q}\bar{q}]\rangle_{O_{+}}\langle\{q\bar{q}\}\rangle = \langle\{q\bar{q}\}\rangle_{O_{+}}[\{qq\rangle[\bar{q}\bar{q}]\rangle] = 0, \tag{34}
\]

\[
\langle\{qq\rangle(\bar{q}\bar{q})\rangle_{O_{-}}\langle\{qq\}\rangle_{O_{+}}\langle\{qq\rangle(\bar{q}\bar{q})\rangle = 0. \tag{35}
\]

The above equations imply that the asymptotic ground-state-meson matrix elements of $H_w$ satisfy the $\Delta I = 1/2$ rule and its violation in the long distance amplitudes for the $K \to \pi \pi$ decays can be supplied through the four-quark $\langle qq\rangle(\bar{q}\bar{q})$ meson contributions which can interfere destructively with the too big $\Delta I = 3/2$ part of the factorized amplitude in Table I. However, since our purpose in this paper is not to discuss the $\Delta I = 1/2$ rule and its violation, we do not consider them any more. The same quark counting leads directly to $\langle K_{\pm} | O_{\Delta S=2} | \bar{K}_{\mp} \rangle = 0$ $^{[20][27]}$ which is compatible with Eq. (33) as discussed before.

Now we are ready to parameterize the asymptotic ground-state-meson matrix elements of $H_w$. To reproduce the observed $\Delta I = 1/2$ rule in the $K \to \pi \pi$ decays, we need the $\Delta I = 1/2$ rule for the ground-state-meson matrix elements of $H_w$ with a sufficient precision. It is all right if one accepts the above quark counting. (If not, one has to assume the $\Delta I = 1/2$ rule for the ground-state-meson matrix elements of $H_w$.) Anyway, neglecting seemingly small (or zero in the above quark counting) $\Delta I = 3/2$ contributions, we parameterize the ground-state-meson matrix elements of $H_w$ as follows, (A) helicity $\lambda = 0$ matrix elements:

\[
\langle \pi^- | H_w | K^- \rangle = \langle \pi^- | H_w | K^+ \rangle = \langle \rho^- | H_w | K^- \rangle = (1 + r_0) H_0, \tag{36}
\]

\[
\langle \pi^0 | H_w | K^0 \rangle = \langle \pi^0 | H_w | K^*0 \rangle = \langle \rho^0 | H_w | K^0 \rangle = -\sqrt{\frac{1}{2}} (1 + r_0) H_0, \tag{37}
\]

\[
\langle \eta_0 | H_w | K^0 \rangle = \langle \eta_0 | H_w | K^{*0} \rangle = \langle \omega | H_w | K^0 \rangle = -\sqrt{\frac{1}{2}} (1 - r_0) H_0, \tag{38}
\]

\[\text{9}\]
\begin{equation}
\langle \eta_s | H_w | \bar{K}^0 \rangle = \langle \eta_s | H_w | \bar{K}^{*0} \rangle = \langle \phi | H_w | \bar{K}^0 \rangle = r_0 H_0, \tag{39}
\end{equation}

(B) helicity \( \lambda = \pm 1 \) matrix elements:

\begin{align*}
\langle \rho^0 | H_w | \bar{K}^{*0} \rangle_{\pm 1} &= -\sqrt{\frac{1}{2}}(1 + r_1)H_1, \\
\langle \omega | H_w | \bar{K}^{*0} \rangle_{\pm 1} &= -\sqrt{\frac{1}{2}}(1 - r_1)H_1, \\
\langle \phi | H_w | \bar{K}^{*0} \rangle_{\pm 1} &= r_1 H_1,
\end{align*}

where iso-singlet pseudo scalar mesons \( \eta \) and \( \eta' \) are written as

\begin{align*}
\eta &= \left( \sqrt{\frac{1}{3}} \cos \theta_P - \sqrt{\frac{2}{3}} \sin \theta_P \right) \eta_0 - \left( \sqrt{\frac{2}{3}} \cos \theta_P + \sqrt{\frac{1}{3}} \sin \theta_P \right) \eta_s, \\
\eta' &= \left( \sqrt{\frac{1}{3}} \sin \theta_P + \sqrt{\frac{2}{3}} \cos \theta_P \right) \eta_0 + \left( -\sqrt{\frac{2}{3}} \sin \theta_P + \sqrt{\frac{1}{3}} \cos \theta_P \right) \eta_s,
\end{align*}

in terms of their components \( \eta_0 \sim (u\bar{u} + d\bar{d})/\sqrt{2} \) and \( \eta_s \sim (s\bar{s}) \). The mixing angle is usually taken to be \( \theta_P \approx -20^\circ \) [14]. The \( \omega-\phi \) mixing has been assumed to be ideal. The parameters \( r_0 \) and \( r_1 \) denote contributions of the penguin relative to \( O_\perp \) in the helicity \( \lambda = 0 \) and \( \lambda = \pm 1 \) matrix elements of \( H_0 \), respectively. \( H_0 \) and \( H_1 \) provide their normalizations. We have parameterized the asymptotic matrix elements of \( H_w \) between pseudo scalar and vector meson states in the same manner. It can be justified by a simple algebraic procedure [11,12] (spins are not very important in the IMF). In (B), the helicity \( \lambda = \pm 1 \) matrix elements, \( r_1 \), will be neglected hereafter since it is expected to be small because of the small coefficient of the penguin and because of a helicity consideration.

VI. COMPARISON WITH EXPERIMENTS

Inserting the above parameterization into the amplitudes given in the previous sections, we can compare our result with experiments. Since our result contains many parameters, however, we here estimate them by using various experimental data.

Sizes of the amplitudes \( A(P_i \rightarrow \gamma\gamma)'s \) in Eq.(3) can be estimated from the measured values of the decay rates [14], \( \Gamma(\pi^0 \rightarrow \gamma\gamma)_{\text{expt}} = (7.7 \pm 0.6) \text{ eV} \), \( \Gamma(\eta \rightarrow \gamma\gamma)_{\text{expt}} = (0.46 \pm 0.04) \text{ keV} \) and \( \Gamma(\eta' \rightarrow \gamma\gamma)_{\text{expt}} = (4.26 \pm 0.19) \text{ keV} \). Their relative signs are taken to be compatible with the quark model. The \( V-V' \)-P, \( V, V' = K, \rho, \omega \) and \( \phi; P = K, \pi, \eta \) and \( \eta' \), coupling constants can be estimated from the observed rates for the radiative decays of \( K^* \) by using \( SU_f(3) \) symmetry and the VMD with the \( \gamma-V \) coupling strengths [14], \( X_\rho(0) = 0.033 \pm 0.003 \text{ (GeV)}^2 \), \( X_\omega(0) = 0.011 \pm 0.001 \text{ (GeV)}^2 \) and \( X_\phi(0) = -0.018 \pm 0.001 \text{ (GeV)}^2 \), estimated from experiments on photoproductions of vector mesons. Although these coupling strength can have momentum square \( (k^2) \) dependence, we neglect it in this paper since they are mild in the region \( k^2 < m_{K^*}^2 \). From \( \Gamma(K^{*0} \rightarrow K^0\gamma)_{\text{expt}} = (0.115 \pm 0.012) \text{ MeV} [14] \), we obtain \( |G_{K^{*0}K^{0}\rho}| \approx 0.856 \text{ (GeV)}^{-1} \) and then \( G_{\omega\rho}^{K^{*0}K^{0}\rho} = -2G_{K^{*0}K^{0}\rho} \) using \( SU_f(3) \). In this way, we can reproduce well the observed rate, \( \Gamma(\pi^0 \rightarrow \gamma\gamma)_{\text{expt}} \). The value of the matrix elements of axial charges can be estimated to be \( |h| = |\langle \pi^- | A_\pi^- | \rho^0 \rangle| \) \( (= \sqrt{2} |\langle K^+ | A_{K^+} | K^{*0} \rangle|) \approx 1.0 \)
from the observed rate $[14]$, $\Gamma(\rho \to \pi\pi)_{\text{expt}} \simeq 150$ MeV, by using PCAC and (asymptotic) $SU_f(3)$. The above value of $|h|$ can reproduce considerably well $\Gamma(K^* \to K\pi)_{\text{expt}}$. The size of $\langle \pi|H_w|K\rangle$ can be estimated from the observed rate for the $K_S \to \pi^+\pi^-$ decay by using Eq.$(33)$ with $\langle \pi|H_w|K\rangle = \langle \pi|H_w|K^*\rangle$ and the $S$-wave $\pi\pi$ phase shift $\delta_0 \simeq (50 - 60)^\circ$ at $m_K$ $[35]$ and by taking into account the small contribution of the factorized amplitude for the same decay in Table I:

$$\langle \pi^+|H_w|K^+\rangle = \langle \pi^+|H_w|K^{*-}\rangle \simeq 1.69 \times 10^{-7}m_K^2. \tag{45}$$

The parameters which are included in the amplitudes given in the previous sections but still have not been estimated are $\alpha_{K^*} = \langle \rho^+|H_w|K^{*-}\rangle_{\pm1}/\langle \pi^+|H_w|K^{*-}\rangle$ and $r_0$ (describing the contribution of the penguin relative to $O_-$ in the asymptotic ground-state-meson matrix elements of $H_w$ with the helicity $\lambda = 0$). We now search for values of these parameters to reproduce $(\Delta m_K)_{\text{expt}}$ and $\Gamma(K_L \to \pi\pi)_{\text{expt}}$ mentioned before. [We have already used $\Gamma(K_S \to \pi^+\pi^-)_{\text{expt}}$ to estimate the size of $\langle \pi^+|H_w|K^+\rangle$.] For the $\Delta m_K$, relative importance between $(\Delta m_K)_{\text{SD}}$ and $(\Delta m_K)_{\text{LD}}$ is still not known. However, if we accept the result from the quark counting presented in the previous section, we have $(\Delta m_K)_{\text{SD}} = 0$ and we can rather easily understand the observed $\Delta I = 1/2$ rule in the $K \to \pi\pi$ decays. In this case (i), the pole contribution which we have calculated in $[3]$ should be compared with

$$\frac{(\Delta m_K)_{\text{pole}}}{\Gamma_{K_S}} \simeq \frac{(\Delta m_K)_{\text{expt}}}{\Gamma_{K_S}} - \frac{(\Delta m_K)_{\pi\pi}}{\Gamma_{K_S}} \simeq (0.477 \pm 0.002) - (0.22 \pm 0.03), \tag{46}$$

where the value of $(\Delta m_K)_{\pi\pi}/\Gamma_{K_S}$ has been given in Ref. $[23]$ as mentioned before. Inserting the parameterization of the asymptotic ground-state-meson matrix elements of $H_w$ in the previous section into $(\Delta m_K)_{\text{pole}}$ in Eq.$(20)$ and $A(K_L \to \gamma\gamma)$ in Eq.$(11)$, we find two possible solutions,

(a) $0.31 < r_0 < 0.35$ and $1.05 < \alpha_{K^*} < 1.20$, \tag{47}

(b) $0.31 < r_0 < 0.35$ and $3.40 < \alpha_{K^*} < 3.55$, \tag{48}

which can reproduce the value of $(\Delta m_K)_{\text{pole}}/\Gamma_{K_S}$ in Eq.$(46)$ and $\Gamma(K_L \to \pi\pi)_{\text{expt}}$, where $|A_P(K_L \to \gamma\gamma)| > |A_{K^*}(K_L \to \gamma\gamma)|$ for (a) and $|A_P(K_L \to \gamma\gamma)| < |A_{K^*}(K_L \to \gamma\gamma)|$ for (b), respectively.

Next, we consider the case (ii) in which $(\Delta m_K)_{\text{SD}}$ dominates the $K_L$-$K_S$ mass difference, i.e., $(\Delta m_K)_{\text{LD}} = 0$. In this case, it is not very easy to understand the $\Delta I = 1/2$ rule in the $K \to \pi\pi$ decays since $\langle \pi|O_{\Delta I=3/2}|K\rangle \neq 0$. Using Eq.$(10)$ and the observed value of $\Delta m_K$, we obtain, $\langle \pi^0|H_w^{\Delta I=3/2}|K^0\rangle \simeq 0.22 \times 10^{-7}m_K^2$, which is considerably smaller than the size of $\langle \pi^+|H_w|K^+\rangle$ estimated phenomenologically in Eq.$(33)$ by neglecting the $\Delta I = 3/2$ part of asymptotic ground-state-meson matrix elements of $H_w$, i.e.,

$$\left| \sqrt{\frac{1}{2}} \langle \pi^+|H_w^{\Delta I=1/2}|K^+\rangle \langle \pi^0|H_w^{\Delta I=3/2}|K^0\rangle \right| \simeq 5.4 \tag{49}$$

while the ratio of the coefficient $c_-$ to $c_+$ is $c_-/c_+ \simeq 4.3$. Then the long distance amplitude $M_{\text{LD}}(K^+ \to \pi^+\pi^0)$ which will be proportional to $\langle \pi|H_w^{\Delta I=3/2}|K\rangle$ if contributions of excited-meson-states are neglected can interfere destructively with too big $M_{\text{SD}}(K^+ \to \pi^+\pi^0)$. A
Table II. Branching fractions for the Dalitz decays of $K_L$. The values of the parameters in (a), (b) and (c) are taken from the corresponding solutions in the text. Data values with (*), (†) and (‡) are taken from Refs. [14], [38] and [40], respectively.

|                      | $R_{\gamma\mu^+\mu^-}(\times10^{-4})$ | $R_{\gamma e^+e^-}(\times10^{-2})$ |
|----------------------|--------------------------------------|---------------------------------|
| (i) $(\Delta m_K)_{SD} = 0$ | (a) $r_0 = 0.330, \alpha_{K^*} = 1.13$ | 1.6 | 5.6 |
|                      | (b) $r_0 = 0.330, \alpha_{K^*} = 3.47$ | 1.7 | 6.6 |
| (ii) $(\Delta m_K)_{LD} = 0$ | (c) $r_0 = 0.139, \alpha_{K^*} = 4.32$ | 1.7 | 6.8 |
| Experiments          | (1.6 ± 0.1)(*)                        | (5.6 ± 0.8)(†)                  |
|                      | (5.9 ± 1.8)(‡)                        |                                 |

The sum of these two amplitudes leads to $\Gamma(K^+ \rightarrow \pi^+\pi^0) \approx 0.14 \times 10^8 \text{ s}^{-1}$ which should be compared with $\Gamma(K^+ \rightarrow \pi^+\pi^0)_{\text{expt}} \approx 0.17 \times 10^8 \text{ s}^{-1}$. Anyway, we neglect the small $\Delta I = 3/2$ part in the asymptotic ground-state-meson matrix elements of $H_{\text{ex}}$ and then we can use the parameterization of them in the previous section and the value of $|\langle \pi^+|H_{\text{ex}}|K^+\rangle|$ in Eq.(15) as its approximate value. Since $(\Delta m_K)_{\text{pole}}/\Gamma_{K_S}$ should be cancelled by $(\Delta m_K)_{\pi\pi}/\Gamma_{K_S}$ in this case (ii), we put

$$\frac{(\Delta m_K)_{\text{pole}}}{\Gamma_{K_S}} = -\frac{(\Delta m_K)_{\pi\pi}}{\Gamma_{K_S}} = -(0.22 \pm 0.03).$$

Then we find another possible solution which can reproduce the value of $(\Delta m_K)_{\text{pole}}/\Gamma_{K_S}$ in the above equation and $\Gamma(K_L \rightarrow \gamma\gamma)_{\text{expt}},$

$$(c) \ 0.13 < r_0 < 0.15 \quad \text{and} \quad 4.25 < \alpha_{K^*} < 4.39.$$  

Inserting the above sets (a) – (c) of values of $r_0$ and $\alpha_{K^*}$ into Eq.(10), we obtain three different results on the form factor for the Dalitz decays of $K_L$. For experimental data on the form factor, there exist three different data, i.e., two of them are from the $\gamma e^+e^-$ final states [36,37] and the other is from the $\gamma\mu^+\mu^-$ [38]. The existing data from different types of the final states are not consistent with each other near the $\gamma\mu^+\mu^-$ threshold. Our results from the solutions (b) and (c) are not very far from the data from the $\gamma e^+e^-$ final states but not consistent with the data from the $\gamma\mu^+\mu^-$ final states while the one from the solution (a) is close to the data from the $\gamma\mu^+\mu^-$ final states near the threshold of the $K_L \rightarrow \gamma\mu^+\mu^-$. At higher $x = k^2/m^2_{K_L} (> 0.4)$, all the three results are consistent with almost all the data within their large errors.

Substituting the above results on the form factor for the Dalitz decays of $K_L$ into the formula Eq.(12) with Eq.(13), we can calculate their branching fractions [39] as listed in Table II. The rate for the Dalitz decay of $K_L$ is mainly determined by the values of the form factor near the threshold. Therefore, the rate $\Gamma(K_L \rightarrow \gamma e^+e^-)$ is not very useful to discriminate different theories since its threshold is close to $x = 0$ where the form factor is usually normalized to be $f(0) = 1$. However the threshold of the $K_L \rightarrow \gamma\mu^+\mu^-$ decay is considerably distant from the normalization point $x = 0$. Therefore, we may discriminate
the above three different solutions using this decay since they give considerably different values of the form factor around the $\gamma \mu^+ \mu^-$ threshold. In Table II, it is seen that the data from the $\gamma \mu^+ \mu^-$ final state seem to favor the solution (a). However, at the present stage where theoretical and experimental ambiguities are still large, it is hard to say definitely what solution is the best.

VII. SUMMARY

We have investigated $\Delta m_K$, $K \to \pi\pi$, $K_L \to \gamma\gamma$ and the Dalitz decays of $K_L$ systematically. For $\Delta m_K$, we have considered two extreme cases, i.e., (i) $(\Delta m_K)_{SD} = 0$ and (ii) $(\Delta m_K)_{LD} = 0$, since we do not know relative weight between $(\Delta m_K)_{SD}$ and $(\Delta m_K)_{LD}$ in $\Delta m_K$. We have searched possible solutions to reproduce the data on $\Delta m_K$ and $\Gamma(K_L \to \gamma\gamma)$ simultaneously and found two possible solutions, (a) and (b), in the case (i) and a possible solution, (c), in the case (ii). Then, using these solutions, we have calculated the form factor for the Dalitz decays of $K_L$ and their decay rates. Our predictions have been compared with the existing experimental data. However, the existing data on the form factor from the $K_L \to \gamma e^+ e^-$ decay [36,37] and the $K_L \to \gamma \mu^+ \mu^-$ decay [38] are not consistent with each other near the $\gamma \mu^+ \mu^-$ threshold. The results from two of our solutions, (b) and (c), are almost consistent with the data from the $\gamma e^+ e^-$ final state but considerably higher than the data from the $\gamma \mu^+ \mu^-$ final state around its threshold. The form factor predicted by the solution (a) is close to the data from the $\gamma \mu^+ \mu^-$ final states. The rate $\Gamma(K_L \to \gamma \mu^+ \mu^-)$ will be useful to discriminate these three solutions in contrast with $\Gamma(K_L \to \gamma e^+ e^-)$. The data from E799 [38] seems to favor the solution (a) and hence a suppression of $(\Delta m_K)_{SD}$ which can explain rather easily the $\Delta I = 1/2$ rule in the $K \to \pi\pi$ amplitudes and can be derived by a simple quark counting. However, it is hard to conclude definitely the above statement since theoretical and experimental ambiguities are still too large. Therefore more theoretical and experimental investigations of the Dalitz decays of $K_L$ will be needed.

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