Financial Storage Rights in Electric Power Networks

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Abstract—The decreasing cost of energy storage technologies coupled with their potential to bring significant benefits to electric power networks have kindled research efforts to design both market and regulatory frameworks to facilitate the efficient integration of such technologies. The primary challenge resides in designing market systems that provide the correct incentives to deploy and operate storage systems efficiently in both the short and long-run. In the following paper, we propose an open access approach to the integration of storage in which storage is treated as a communal asset centrally operated by the System Operator (SO) to maximize social welfare; not unlike the operation of the transmission network today. Concomitantly, we propose a novel electricity derivative, which we refer to as financial storage rights (FSRs), to enable the redistribution of the additional merchandising surplus (attributable to storage) collected by the SO. FSRs do not interfere with the socially optimal operation of storage, and their definition as a sequence of nodal power injections facilitates their use by market participants to mitigate the cost and/or risk of meeting contractual commitments. Moreover, the revenue collected by the SO through the sale of FSRs can be used to remunerate capital expenditures in storage.

Index Terms—Energy storage, electricity markets, financial derivatives.

I. INTRODUCTION

The increased penetration of supply derived from variable renewable energy resources, coupled with the recent decline in the cost of electric energy storage technologies, has brought about an opportunity to significantly reduce the cost of managing the electric power system through careful planning, deployment, and operation of storage resources [2]. Broadly, the short-run value of an energy storage network derives from its ability to arbitrage energy across space and forward in time, enabling both the absorption of power imbalances on short time scales and the more substantial reshaping of supply and demand profiles over longer periods of time. The extent to which the deployment of an energy storage network might benefit the system depends critically, however, on the collective sizing, placement, and operation of said devices [3]. The challenge resides in the design and implementation of electricity markets and instruments that induce strategic expansion and operation of storage in a manner that is consistent with the maximization of social welfare over both the long and short run, respectively.

The coordinated optimal dispatch of a collection of distributed energy storage resources clearly offers the possibility of sizable reduction in the cost of servicing demand through implicit reshaping of the nodal demand profiles to alleviate both transmission congestion and the reliance on peak power generation [4]. Of interest then is the characterization of mechanisms for integration of storage capable of realizing an efficient dispatch. And of critical importance to this effort is the resolution of the question: who commands the storage? Among the variety of possible answers to this question, there are two extremes – differing in terms of the degree of government intervention – which we naturally refer to as regulated and competitive. Each implies a distinct mechanism for both the operation of the physical storage assets and the remuneration of the services provided.

Broadly, the regulated integration of storage entails a centralized operating paradigm in which storage is treated as a communal asset and is centrally dispatched by the system operator (SO) to maximize social welfare; not unlike the operation of the transmission network today. The socially optimal dispatch of storage, in concert with conventional generation and transmission, naturally improves upon the welfare of the system. Thus, in order to incentivize investment in storage capacity, such a regulated approach to integration necessarily requires the establishment of a value proposition to storage owners. Namely, the delineation of a mechanism to extract and redistribute the gain in social welfare (attributable to storage) back to the owners of the responsible storage assets.

Conversely, the competitive or market-based integration of storage entails a decentralized operating paradigm in which storage owners pursue their own rational (profit maximizing) interests in various energy and ancillary service markets. The collective behavior of strategic market participants may, however, deviate from perfect competition – resulting in substantial efficiency loss relative to the socially optimal outcome. In fact, Sioshansi [5] shows that, in the setting of a simplified energy market, the operating strategies of storage owner-operators at a Nash equilibrium can result in significant efficiency loss relative to the socially optimal operation of storage. Moreover, a potential hindrance of the market-based approach to storage integration derives from uncertainty in the revenue that storage owner-operators can obtain from the market. Energy storage is a capital intensive technology. And the risk of incomplete capital cost recovery inherent to the market-based approach may serve to inhibit initial investment in storage assets. A More nuanced discussion surrounding this issue can be found in [6], [7].

A. An Open Access Approach to Storage Integration

There has been a recent move in academia and industry to identify alternative paradigms to support the efficient integration of storage into system operations [4], [7]. One stream of literature centers on an open access approach to integration.
of storage; or more simply, open access storage (OAS) [8], [9]. Loosely, we refer to OAS as a regulatory framework in which energy storage systems are treated as communal assets accessible to both regulated and non-regulated firms in the wholesale market.

To the best of our knowledge, only two concrete approaches to OAS have been proposed. He et al. [8] proposes a market framework where storage owners sell physically binding rights to their storage capacity through sequential auctions coordinated by the SO. The collection of physical rights, which are defined as a sequence of nodal power injections within a specified time horizon, determine the actual operation of storage, except perhaps for any use of the residual storage capacity the SO may be allowed to perform for reliability or economic purposes.\(^1\) As such, the physical rights associated with a particular storage device must be collectively feasible with respect to the corresponding physical device constraints. While such physical rights might be used by market participants to execute price arbitrage or mitigate the cost of honoring existing contractual energy commitments, there are several important limitations. Namely, the extent to which a market participant might leverage on a physical storage right is dependent on her physical location within the network and the potential interference of transmission congestion. More importantly, the eventual physical dispatch of storage is determined by a sequence of auctions; the outcome of which is likely to deviate from the socially optimal dispatch, because of strategic interactions between parties bidding for physical dispatch rights.

On the other hand, Taylor [9] suggests an approach to OAS that centers on a paradigm where storage owners sell financially binding rights to their storage capacity through a (unspecified) market mechanism coordinated by the SO. The SO is charged with the task of centrally operating storage in a socially efficient manner – not unlike its non-discriminatory operation of the transmission network. As financial rights, they do not interfere with the optimal operation of storage, but rather, they represent entitlements to portions of the merchandising surplus collected by the SO. A central component to the proposal in [9] is the definition of the financial rights in terms of shadow prices (i.e. Lagrange multipliers) on the various physical constraints on the storage devices. This is analogous to the definition of Flow-Gate Rights (FGRs) \([10], [11], [12]\) in the context of transmission access. And, as a result, such a definition of financial storage rights is naturally endowed with advantages and disadvantages comparable to those of FGRs in the context of transmission. We refer the reader to Remark 6 and [13], [14] for a more detailed discussion.

B. Main Contribution

In this paper, we propose a novel market and regulatory framework to enable open access storage, in which storage devices are treated as communal assets accessible by all regulated and non-regulated agents within the system. Broadly, our framework centers on the concept of financial storage rights (FSRs).\(^2\) We show that the proposed FSRs, in combination with financial transmission rights (FTRs), enable the full redistribution of the merchandising surplus among the right holders, thus allowing the SO to operate storage in a socially optimal manner without jeopardizing its independence. Specifically, we define FSRs as a sequence of nodal power injections that entitle the holder to the associated profit at the corresponding sequence of nodal prices. Such a definition represents a financial analog to the physical storage rights proposed by He et al. [8], and is in contrast to the financial constraint-based rights proposed in [9].

The formulation of financial storage rights as a sequence of injections is natural, as it enables market participants to directly leverage on such rights to hedge against price volatility in the short-run and mitigate the cost of meeting side energy commitments [8]. Moreover, by virtue of their definition, FSRs can be auctioned at nodes without physical storage – a feature which genuinely democratizes access to storage. In addition, the revenue collected by the system operator from auctions for FSRs can be used to directly remunerate those entities responsible for the initial investment in the physical storage assets. And, as these auctions could be cleared anywhere from one day to several years in advance of delivery, their revenue would serve as a less risky alternative for a storage owner, as compared to the variable revenue stream a storage owner-operator might derive through direct participation in the energy market. Clearly, such a framework has the potential to substantially mitigate the risk of incomplete capital cost recovery that is currently suppressing the expansion of storage assets in the electric power system.

C. Organization

The paper is organized as follows. In Section II, we present a formulation of multi-period economic dispatch with storage and characterize the optimality conditions. This is followed by a precise formulation of financial storage rights and analysis of their properties in Section III. We close with a summary and discuss directions for future research in Section IV.

II. MODELS AND FORMULATION

A. Notation

Let \(\mathbb{R}\) denote the set of real numbers and \(\mathbb{R}_+\) the non-negative real numbers. Denote the transpose of a vector \(x \in \mathbb{R}^n\) by \(x^\top\). And let \(x_i\) denote the \(i\)th entry of a vector \(x \in \mathbb{R}^n\). We define by \(1\) a column vector of all ones of dimension appropriate to the context. For two matrices \(A, B \in \mathbb{R}^{m \times n}\) of equivalent dimension, we denote their Hadamard product by \(A \circ B\). Given a matrix \(A \in \mathbb{R}^{m \times n}\), we write \(A = 0\) to denote entrywise equivalence to zero.

\(^1\)Such operation of the residual storage capacity available, however, constitutes the same fundamental problem of having the SO centrally operating storage without a method to redistribute the additional surplus collected with the difference that storage constraints would be now modified by the aggregated physical rights.

\(^2\)Taylor [9] offers an alternative (capacity-based) definition of financial storage rights. See Remark 6 for a detailed comparison between the two approaches.
B. Network Model

Consider a transmission network defined on a set of \( n \) nodes (buses) connected by \( m \) edges (transmission lines). The associated graph of the network is assumed connected. The nodes are labeled \( 1, 2, \ldots, n \). We operate under the assumption of a linear model of steady state power flow defined by the so-called DC power flow approximation, where the vector of nodal power injections \( \mathbf{v} \) is linearly mapped to a vector of (directional) power flows along the \( m \) transmission lines through the mapping \( H \in \mathbb{R}^{2m \times n} \), commonly referred to as the shift-factor matrix. Let \( \mathbf{c} \in \mathbb{R}^{2m} \) denote the corresponding vector of transmission line capacities. It follows that the set of feasible power injections is described by the polytope \( \mathcal{P} \subset \mathbb{R}^n \),

\[
\mathcal{P} = \left\{ \mathbf{v} \in \mathbb{R}^n \mid H\mathbf{v} \leq \mathbf{c}, \ 1^\top\mathbf{v} = 0 \right\}. \tag{1}
\]

One can readily verify the compactness of \( \mathcal{P} \), as \( \text{rank}(H) = n - 1 \) and \( 1^\top \) is linearly independent from the rows of \( H \).

C. Cost Model

At the core of the formulation considered in this paper is the problem of multi-period economic dispatch over \( N \) discrete time periods, which we index by \( k = 0, \ldots, N - 1 \). We appraise the cost and benefit of the net injection \( \mathbf{v}(k) \in \mathbb{R}^n \) at time \( k \) according to

\[
C(\mathbf{v}(k), k) = \sum_{i=1}^{n} C_i(v_i(k), k),
\]

where each constituent function \( C_i(v, k) \) is assumed to be increasing, convex, and differentiable in \( v \) over \( \mathbb{R} \). Moreover, each function is assumed to satisfy \( C_i(0, k) = 0 \), \( C_i(v, k) > 0 \) for \( v > 0 \), and \( C_i(v, k) < 0 \) for \( v < 0 \). This implies that \( C_i(v, k) \) represents the convex cost of generation for \( v > 0 \) at node \( i \) and time \( k \). Conversely, \( -C_i(v, k) \) represents the concave benefit of consumption for \( v < 0 \) at node \( i \) and time \( k \). A more detailed explanation of said model can be found in [15]. Finally, we have allowed the functions \( \{C_i(\cdot, k)\} \) to vary with time in order to capture the potential variation in the nodal demand preferences over time.

D. Energy Storage Model

We consider an arbitrary collection of \( n \) perfectly efficient energy storage devices built into the transmission network, where we associate with each node \( i \) a storage device with energy capacity \( b_i \in \mathbb{R}_+ \). And we denote by \( \mathbf{b} = [b_1, \ldots, b_n]^\top \) the vector of nodal energy storage capacities. The collective storage dynamics are naturally modeled as a linear difference equation

\[
z(k+1) = z(k) - \mathbf{u}(k) \tag{2}
\]

for \( k = 0, 1, \ldots, N - 1 \), where the vector \( z(k) \in \mathbb{R}^n \) denotes the vector of energy storage states just preceding time period \( k \) and the input \( \mathbf{u}(k) \in \mathbb{R}^n \) denotes the vector of energy extractions or injections during period \( k \). The notational convention is such that \( u_i(k) > 0 \) (\( u_i(k) < 0 \)) represents a net energy extraction from (injection into) the storage device at node \( i \) during time period \( k \). Without loss of generality, we assume a zero initial condition \( z(0) = 0 \) for the remainder of the paper. The capacity limitation of energy storage requires that \( 0 \leq z(k) \leq \mathbf{b} \) for all \( k \). Iterating the linear difference equation (2) back to its initial condition, one can express the storage capacity constraint as

\[
0 \leq -\sum_{\ell=0}^{k-1} \mathbf{u}(\ell) \leq \mathbf{b} \tag{3}
\]

for \( k = 1, \ldots, N \). As a matter of notational convenience, we consider an equivalent characterization of the energy storage capacity constraints (3), which enables a structural separation of the constraints across nodes. More specifically, letting \( \mathbf{u}_i = [u_i(0), \ldots, u_i(N-1)]^\top \) denote the entire sequence of injections and extractions from the storage device at node \( i \), one can recast (3) as

\[
\mathbf{u}_i \in \mathcal{U}_i = \left\{ \mathbf{u} \in \mathbb{R}^N \mid 0 \leq L\mathbf{u} \leq b_i \mathbf{1} \right\} \tag{4}
\]

for \( i = 1, \ldots, n \), where \( L \in \mathbb{R}^{N \times N} \) is a lower triangular matrix with entries \( L_{k\ell} = -1 \) for all \( k \geq \ell \) and zero otherwise. It is immediate to see that \( \mathcal{U}_i \) is a compact polytope containing the origin for each \( i = 1, \ldots, n \).

Remark 1. While the model of storage considered is quite stylized, much of the analysis and subsequent conclusions derived can be easily generalized to accommodate nonidealities such as constraints on allowable rates of charging and discharging, inefficiencies, and dissipative losses.

E. Multi-Period Economic Dispatch

Working within the idealized setting considered, we now formulate the problem of multi-period economic dispatch with storage. Broadly, the objective of the SO is to select a vector of nodal prices for energy that sustain competitive supply and demand at a feasible system operating point that maximizes social welfare, a so-called economic dispatch. Formally, the multi-period economic dispatch problem is stated as:

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{N-1} C(\mathbf{v}(k), k) \\
\text{subject to} & \quad \mathbf{v}(k) + \mathbf{u}(k) \in \mathcal{P}, \quad k = 0, \ldots, N - 1 \tag{6} \\
& \quad \mathbf{u}_i \in \mathcal{U}_i, \quad i = 1, \ldots, n \tag{7}
\end{align*}
\]

where the minimization is taken with respect to the variables \( \mathbf{v}(k) \in \mathbb{R}^n \) and \( \mathbf{u}(k) \in \mathbb{R}^n \) for \( k = 0, \ldots, N - 1 \). We will occasionally denote the decision variables more compactly as \( V = [\mathbf{v}(0), \ldots, \mathbf{v}(N-1)] \) and \( U = [\mathbf{u}(0), \ldots, \mathbf{u}(N-1)] \).

F. Optimality Conditions

Definition 1. A pair \( (V, U) \) is a feasible dispatch if it satisfies constraints (6)-(7). A pair \( (V, U) \) is an (optimal) economic dispatch if it solves problem (5)-(7).

The multi-period economic dispatch problem (5) - (7) is a convex optimization problem with linear constraints. As such, an economic dispatch \( (V, U) \) is characterized by the existence of Lagrange multipliers such that the Karush-Kuhn-Tucker (KKT) conditions (6) - (13) hold, which we now introduce.
More specifically, we associate Lagrange multipliers $\gamma(k) \in \mathbb{R}$ and $\mu(k) \in \mathbb{R}_{+}^{2m}$ with the power balance and line flow capacity constraints (6) at time $k$, respectively. Similarly, we define $\mathcal{L}_{i} \in \mathbb{R}_{+}^{m}$ and $\mathcal{T}_{i} \in \mathbb{R}_{+}^{m}$ as the Lagrange multipliers associated with the energy capacity constraints (7) of the storage device at node $i$. In specifying the KKT conditions, it will be convenient to define as $\lambda(k) \in \mathbb{R}^{n}$ a particular linear combination of Lagrange multipliers given by:

$$\lambda(k) = \gamma(k)1 - H^{T}\mu(k)$$

for each time $k = 0, \ldots, N - 1$. The stationarity condition is given by:

$$\nabla C(v(k), k) = \lambda(k), \quad k = 0, \ldots, N - 1$$

$$L^{T}(\mathcal{T}_{i} - \mathcal{L}_{i}) = \lambda_{i}, \quad i = 1, \ldots, n$$

where we have defined $\lambda_{i} = [\lambda_{i}(0), \ldots, \lambda_{i}(N - 1)]^{T}$. The complementary slackness condition is given by:

$$\mu(k)\circ(H(v(k) + u(k)) - c) = 0, \quad k = 0, \ldots, N - 1$$

$$\mathcal{L}_{i} \circ L_{u} = 0, \quad i = 1, \ldots, n$$

$$\mathcal{T}_{i} \circ (b_{i}1 - L_{u}) = 0, \quad i = 1, \ldots, n$$

III. FINANCIAL STORAGE RIGHTS

We now introduce several basic features of competitive nodal pricing that will prove useful in developing our concept of financial storage rights. For the remainder of the paper we refer to $\lambda(k) \in \mathbb{R}^{n}$ as the vector of nodal prices at time $k$. More specifically, $\lambda_{i}(k)$ denotes the price at which energy is transacted at node $i$ and time $k$. We denote by $\Lambda = [\lambda(0), \ldots, \lambda(N - 1)]$ the corresponding sequence of nodal prices from time $k = 0$ to $N - 1$. We have the following definition according to [15].

**Definition 2.** The triple $(V, U, \Lambda)$ constitutes a market equilibrium if it satisfies (6), (7) and (9). The triple $(V, U, \Lambda)$ is deemed efficient if $(V, U)$ is also an economic dispatch.

The requirement that $(V, U)$ satisfy (6) and (7) can be interpreted as a market clearing condition, as it requires that supply equal demand at each time period $k$, while ensuring that the line flow and storage capacity constraints are met. Condition (9) is tantamount to requiring consumer and supplier equilibrium at every node $i$ and time $k$. Namely, relation (9) requires that the marginal cost of supply (benefit of demand) equal the nodal price $\lambda_{i}(k)$ for all nodes $i$ and times $k$. Consequently, at equilibrium, there is no opportunity for profitable arbitrage of energy across nodes or time.

It is important to note that there may exist multiple market equilibria – all of which need not constitute an economic dispatch. In other words, the system operating point at a market equilibrium may not maximize social welfare. One can, however, implement an economic dispatch $(V, U)$ at a market equilibrium $(V, U, \Lambda)$, if the nodal prices $\Lambda$ are set according to (8) with the Lagrange multipliers derived from the corresponding economic dispatch. This is commonly referred to as locational marginal pricing (LMP) [16].

A. Merchandizing Surplus

In selecting and implementing a market equilibrium $(V, U, \Lambda)$, the SO is responsible for collecting monies from the demanders and remunerating the suppliers according to their respective operating points and nodal prices. In doing so, the SO may collect a nonzero surplus. We refer to this excess as the merchandising surplus (MS). Indeed, it is a straightforward generalization of [15] to show that the MS can be either positive or negative at a market equilibrium. The latter outcome is undesirable, as it requires the SO to make whole those transactions with a negative MS. In the following, we briefly discuss the effects of dispatch efficiency and congestion, in both transmission and storage, on the MS. First, we have a definition.

**Definition 3.** The merchandising surplus (MS) at a market equilibrium $(V, U, \Lambda)$ is defined as

$$\text{MS} = - \sum_{k=0}^{N-1} \lambda(k)^{T}v(k).$$

One can further refine the structure of the MS in order to reveal the effects of both transmission and storage congestion on its value. In order to do so, we first require a specification of the line flows induced by the net injection profile for each period of time. More formally, let $(V, U)$ be an arbitrary feasible dispatch. And denote by $p_{ij}(k)$ the corresponding power flow over the line from node $i$ to $j$ at time $k$. We adopt a sign convention such that $p_{ij}(k) > 0$ if power flows from node $i$ to $j$. It follows from Kirchhoff’s Current Law that $v_{i}(k) + u_{i}(k) = \sum_{j=1}^{N} p_{ij}(k)$ for all $i = 1, \ldots, n$. A direct substitution of the prior equation into (14), in combination with simple algebraic manipulations, yields an equivalent decomposition of the merchandising surplus as

$$\text{MS} = \text{TCS} + \text{SCS},$$

where the first term is commonly referred to as the transmission congestion surplus (TCS), and the second term we refer to as the storage congestion surplus (SCS). They satisfy:

$$\text{TCS} = \frac{1}{2} \sum_{k=0}^{N-1} \sum_{i,j=1}^{n} (\lambda_{j}(k) - \lambda_{i}(k)) p_{ij}(k),$$

$$\text{SCS} = \sum_{k=0}^{N-1} \sum_{i=1}^{n} \lambda_{i}(k)u_{i}(k).$$

**Lemma 1.** The MS, TCS, and SCS derived at a market equilibrium $(V, U, \Lambda)$ are nonnegative quantities, if $(V, U)$ is also an economic dispatch.

While we omit the proof of Lemma 1, it follows readily from the feasibility of the dispatch $(V, U) = (0, 0)$ – a property that also holds for more complex models of power flow or storage.

**Remark 2.** Lemma 1 reveals an important property. Namely, at an efficient market equilibrium, the collective transactions

\footnote{According to the formulation of DC power flow considered in Section II-B, the line flow $p_{ij}(k)$ corresponds to a single entry of the vector $H(v(k)) + u(k)$. And if there is no line connecting nodes $i$ and $j$, then $p_{ij}(k) = -p_{ji}(k) = 0$ necessarily.}
between supply and demand are guaranteed to be revenue adequate (i.e. MS ≥ 0). Moreover, the reformulation of the MS in Equation (15) reveals a decomposition of the effects due to congestion in both transmission and storage on the excess money collected by the SO. That is to say that if the transmission network remains to be uncongested at an economic dispatch (i.e. μ(k) = 0 for all k), then the TCS = 0 regardless of the congestion in storage. Similarly, if the collection of storage devices remain uncongested (i.e. Vi = 0 for all i), then the SCS = 0 irrespective of the congestion pattern in transmission. This is immediate to see upon simple algebraic manipulations of the KKT conditions (6)-(13).

B. Financial Transmission Rights

In the event that there is transmission congestion at an economic dispatch and the SO does indeed collect a positive merchandising surplus (MS), it is common practice in electricity markets to reallocate the MS in the form of financial transmission rights (FTRs) [17]. Essentially, a FTR is a financial instrument that entitles the owner the right to receive a fraction of the transmission congestion surplus (TCS) derived at a market equilibrium (traditionally day-ahead). In particular, a FTR is defined in terms of (i) a quantity of power T ∈ R+, (ii) a point of delivery i, and (iii) a point of receipt j. Given a vector of nodal prices \( \lambda \in \mathbb{R}^n \), said FTR yields the holder the rent (or liability) \( (\lambda_j - \lambda_i)T \). FTRs have become an important component of LMP-based electricity markets, in part, because of their ability to provide market participants with a perfect hedge against nodal price differentials across the transmission network. See [18], [19] for a recent survey.

In the following, we explore the limitations of FTRs in terms of their ability to fully recover the MS in the presence of congested storage. We first provide a formal definition of a FTR extended to the multi-period setting.

**Assumption 1.** For the remainder of the paper, we let \((V, U, \Lambda)\) denote an efficient market equilibrium, unless otherwise specified.

**Definition 4.** A financial transmission right (FTR) is any triple \((i, j, T_{ij})\) with \( T_{ij} \in \mathbb{R}_{+}^N \). Said FTR yields the holder a rent (or liability) of \( (\lambda_j - \lambda_i)T_{ij} \). We refer to the FTR more compactly as \( T_{ij} \).

**Remark 3.** We have implicitly required injection/extraction symmetry in our definition of FTRs, as we have considered a lossless model of power flow. See [20] for a more general characterization of FTRs that accommodates lossy power networks.

We refer to an arbitrary collection of FTRs as the set

\[ \mathcal{T} = \{ T_{ij} \mid i, j = 1, \ldots, n \} \]

where each element \( T_{ij} \) sums all FTRs of the same type \((i, j)\).

**Definition 5.** We define the rent associated with a collection of FTRs \( \mathcal{T} \) as

\[ \Phi(\mathcal{T}) = \sum_{i,j=1}^{n} (\lambda_j - \lambda_i)^\top T_{ij}. \]

In general, the MS collected by the SO will not equal the rent \( \Phi(\mathcal{T}) \) due to a collection of FTRs \( \mathcal{T} \). Thus, it is crucial to restrict the allocation of FTRs in such a manner as to guarantee that the SO retains a nonnegative balance, i.e. \( MS - \Phi(\mathcal{T}) \geq 0 \). A well known requirement (in the absence of storage) is that of simultaneous feasibility [15], [17], [20]. We now extend this notion to the multi-period setting to incorporate the additional flexibility in power flow afforded by the availability of storage in the network.

**Definition 6.** Let \( \mathcal{T} \) be a collection of FTRs. And let \( P \in \mathbb{R}^{n \times N} \) denote the sequence of net power injections implied by \( \mathcal{T} \), whose \( i \)th row is determined by:

\[ p_i^\top = \sum_{j=1}^{n} (T_{ij} - T_{ji})^\top. \]  

The collection of FTRs \( \mathcal{T} \) is deemed simultaneously feasible if there exists a sequence of storage injections \( Q \in \mathbb{R}^{n \times N} \) such that \((P, Q)\) is a feasible dispatch according to Def. 1.

We have the following result bounding the maximum rent achievable by any simultaneously feasible collection of FTRs.

**Lemma 2.** If \( \mathcal{T} \) is a simultaneously feasible collection of FTRs, then the associated rent satisfies

\[ \Phi(\mathcal{T}) \leq TCS. \]  

This inequality is tight, in the sense that there exists a collection of simultaneously feasible FTRs with an associated rent equal to the TCS.

**Remark 4.** We remark that FTRs were originally developed within the mathematical framework of single-period economic dispatch, thus obviating the possibility of accommodating the intertemporal congestion effects of storage in their design. Lemma 2 reveals such a limitation. Namely, FTRs alone cannot yield a rent beyond that of the transmission congestion surplus (TCS), thus preventing their recovery of the full MS in the event of storage congestion. This is undesirable, as it may allow the SO to retain a substantial fraction of the MS in the event that the storage congestion surplus (SCS) is large. This may create an incentive to the SO to inflate said revenue by operating the system suboptimally [15]. This motivates our definition of financial storage rights (FSRs) – a novel financial instrument that, in combination with FTRs, is capable of fully recovering the MS, among other favorable characteristics.

C. Financial Storage Rights

We now present our definition of financial storage rights.

**Definition 7.** A financial storage right (FSR) is any double \((i, S_i)\) with \( S_i \in \mathbb{R}^N \). Said FSR yields the holder a rent (or liability) of \( \chi_i^\top S_i \). We refer to the FSR more compactly as \( S_i \). We call an FSR balanced if \( 1^\top S_i = 0 \).

Before embarking upon a formal analysis of FSRs and their properties, we provide a brief qualitative discussion surrounding the structure of FSRs and their potential uses. First, FSRs offer market participants the ability to perfectly hedge price volatility at any node in the network. Such price volatility is a
natural consequence at an economic dispatch, because of the inherent time variation in supply cost, demand preferences, and congestion in storage. The intertemporal hedging capability of FSRs represents a natural complement to FTRs and their ability to hedge spatial price differentials across the network.

Second, the revenue collected by the SO through the sale of FSRs can be used to (partially) remunerate those parties responsible for the initial investment in storage — not unlike the role of FTRs in supporting remuneration of transmission investment.

**Remark 5.** (The Balancedness Requirement). A FSR \( S_i \) can be alternatively interpreted as the profit a market participant would derive from a sale of a power profile \( S_i \) at node \( i \) over the time horizon \( k = 0,\ldots,N-1 \). Moreover, the requirement of balancedness ensures the usage of FSRs purely for intertemporal price arbitrage at a particular node in the network. We require balancedness of a FSR in a manner which is analogous to the requirement of symmetry of FTRs.

Such a formulation derives from our assumption of perfectly efficient storage. Accommodating storage nonidealities, such as dissipative losses, will naturally require a relaxation of the balancedness requirement.

**Remark 6.** (A Comparison with Capacity-Based FSRs). Taylor [9] offers an alternative definition of FSRs in terms of the capacity of individual storage devices, priced according to their corresponding Lagrange multipliers derived at an economic dispatch. We refer to Taylor’s alternative form as capacity-based financial storage rights (C-FSRs). This is in contrast to our injection profile-based definition of FSRs (cf. Definition 7).

While the two forms are mathematically equivalent in terms of their ability to yield a perfect hedge against intertemporal price variation, there are important differences in terms of practical market implementation considerations. We elaborate on several such differences.

One point of distinction pertains to market liquidity. First, note that a market for C-FSRs (as defined in [9]) would entail the definition of as many financial rights as there are physical storage devices in the power system; which may grow to be large in the evolving power system. In such event, the corresponding market for C-FSRs risks insufficient liquidity stemming from a possible dearth of interested buyers for each and every right. In contrast, a market for injection profile-based FSRs entails at most \( n \) financial rights, where \( n \) is the number of buses in the transmission network. However, the requirement that injection profile-based FSRs be simultaneously feasible may serve to limit their liquidity.

Another point of distinction stems from the system information required to perfectly hedge bilateral transactions against intertemporal price variation. It is straightforward to show that a multi-period bilateral transaction can be perfectly hedged with a single injection profile-based FSR associated with the bus at which power is being traded. In contrast, the capacity-based definition of financial storage rights may require the bundling of multiple C-FSRs to produce an equivalent hedge. The difficulty in implementation derives from the information required to construct such C-FSR bundles. Namely, the buyer is required to have knowledge of physical storage models used to solve the economic dispatch problem. In addition, any variation in the underlying storage models (e.g., due to maintenance, degradation, or outage of a storage device) would jeopardize the efficacy of such C-FSR hedges. This is analogous to the effect of variation in the shift factor matrix (e.g., due to transmission outages) on the hedging capability of flowgate rights in the context of transmission congestion pricing.

A final point of contrast between C-FSRs and FSRs derives from the financial risk they impose on the holder. The rent associated with a C-FSR is guaranteed to be nonnegative, while a FSR may yield a negative rent leaving the holder liable for payment.

We now formally characterize the maximum rent achievable with any combination of FTRs and FSRs. First, we refer to an arbitrary collection of FSRs as the set

\[
\mathcal{S} = \{ S_i \mid i = 1,\ldots,n \},
\]

where each element \( S_i \) sums all FSRs of the same type \( i \).

**Definition 8.** We define the rent associated with a collection of FSRs \( \mathcal{S} \) as

\[
\Sigma(\mathcal{S}) = \sum_{i=1}^{n} \lambda^T_i S_i.
\]

We now extend Definition 6 of simultaneous feasibility to accommodate a combination of FSRs \( \mathcal{S} \) and FTRs \( \mathcal{T} \).

**Definition 9.** Let \((\mathcal{T},\mathcal{S})\) be a collection consisting of both FTRs and FSRs. And let \( P \in \mathbb{R}^{n \times N} \) denote the sequence of net power injections implied by \((\mathcal{T},\mathcal{S})\), whose \( i^{th} \) row is determined by:

\[
p_i^T = -S_i^T + \sum_{j=1}^{n} (T_{ij} - T_{ji})^T.
\]

The collection \((\mathcal{T},\mathcal{S})\) is deemed simultaneously feasible if there exists a sequence of storage injections \( Q \in \mathbb{R}^{n \times N} \) such that \((P,Q)\) is a feasible dispatch according to Def. 1.

**Remark 7.** (Accommodating Storage Inefficiency). While we have heretofore operated under the assumption of an ideal storage model, this definition of simultaneous feasibility can be easily extended to accommodate inefficiencies in storage by refining the underlying storage constraints on which it is based.

Employing Definition 9 of simultaneous feasibility, we have the following result which characterizes the maximum rent attainable with any collection of (jointly) simultaneously feasible FTRs and FSRs.

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4Indeed, said contrast is analogous to the distinction between flowgate rights (FGRs) and point-to-point financial transmission rights (FTRs) in the context of transmission congestion pricing.

5Similar issues arise in debating the viability of FGRs versus FTRs. See for example, [13], [14], [21], [22].

6To the context of transmission congestion pricing, the requirement that FTRs pass a simultaneous feasibility test may have limited their liquidity in secondary markets [21].
Theorem 3. If \((\mathcal{T}, \mathcal{F})\) is a simultaneously feasible collection of financial rights, then the associated rent satisfies
\[
\Phi(\mathcal{T}) + \Sigma(\mathcal{F}) \leq MS. \quad (19)
\]

And, moreover, this inequality is tight, in the sense that there exists a collection of simultaneously feasible rights \((\mathcal{T}, \mathcal{F})\) with an associated rent equal to the MS.

Theorem 3 is reassuring. Whereas in Lemma 2 we have exposed the impossibility of constructing simultaneously feasible FTRs whose associated rent is guaranteed to fully recover the MS, Theorem 3 reveals that there does indeed exist a collection of FTRs and FSRs \((\mathcal{T}, \mathcal{F})\) whose joint rent is guaranteed to recover the entire MS, i.e. \(\Phi(\mathcal{T}) + \Sigma(\mathcal{F}) = MS\). Moreover, there does not exist a collection of simultaneously feasible rights \((\mathcal{T}, \mathcal{F})\) whose joint rent exceeds the MS. This guarantees revenue adequacy on behalf of the SO when issuing such rights in a manner that is simultaneously feasible.

IV. Conclusion and Future Work

In this paper, we have proposed a general regulatory and market framework to enable the open access integration of storage, in which storage is treated as a communal asset accessible to all agents within the system. Such an approach is a substantial departure from the more standard storage integration paradigm in which storage owner-operators pursue their individual profit maximizing interests within conventional market settings. Central to our proposal is the concept of financial storage rights (FSRs), which are defined as a sequence of nodal power injections that entitle the holder to the associated profit at the corresponding sequence of nodal prices. Qualitatively, FSRs represent a financial right to the flexibility offered to the system by storage. This is in sharp contrast to the physical rights proposed in [8]. We establish through a sequence of results (Lemma 2 and Theorem 3) that FSRs are essential to a well-functioning electricity market (in which storage is centrally dispatched), as their allocation is necessary to enable the full redistribution of the merchandising surplus collected by the system operator (SO) under locational marginal pricing. An essential advantage of FSRs and the modus operandi they entail is that their allocation does not interfere with the socially optimal operation of storage or the independence of the SO. Moreover, the proposed FSRs are naturally defined products that market participants can easily incorporate into their short and long-term decision making process to hedge against intertemporal price variation.

More broadly, we envision storage owners trading such FSRs with other market participants through (short and long term) forward auctions and secondary markets centrally coordinated by the SO; not unlike markets for financial transmission rights today. And, by selling financial rights to a fraction of their energy storage capacity (and holding on to the remainder), storage owners can more finely manage their exposure to revenue risk. An advantage of the proposed market paradigm is the potential to substantially mitigate the risk of incomplete capital cost recovery that is currently suppressing the expansion of storage assets in the electric power system [6], [7].

The study of financial storage rights presented in this paper represents but only an initial point of analysis. Many interesting questions remain open. First, our definition of (joint) simultaneous feasibility for FTRs and FSRs would require the coordination of both the FSR and FTR markets. This might be too cumbersome to be practical. As such, it would be of interest to explore the development of more restrictive definitions of simultaneous feasibility to enable the decoupling of FTR and FSR auctions. Second, the potential value that energy storage offers to the power system goes well beyond the application of intertemporal arbitrage considered in this paper [7]. For example, certain storage technologies possess the capability of providing voltage or frequency regulation services. A natural question then, is how might one expand the concept of FSR to incorporate these value streams as well? Third, it would be of interest to generalize the market framework considered to accommodate a broader family of technologies capable of shifting energy in time (e.g. flexible demand-side resources).

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REFERENCES

[1] D. Muñoz-Álvarez and E. Bitar, “Financial storage rights: definition and basic properties,” in North American Power Symposium (NAPS), 2014. IEEE, 2014.
[2] E. Hinds and J. Boyer-Dry, “The emergence of an electric energy storage market,” The Electricity Journal, vol. 27, no. 2, pp. 6 – 13, 2014.
[3] E. Bitar and S. Bose, “Zero crossings and the locational marginal value of energy storage,” in Conference on Decision and Control (CDC), 2014. IEEE, 2014.
[4] PJM, “Energy storage as a transmission asset,” PJM, Technical report, 2012. [Online]. Available: http://www.pjm.com
[5] R. Sioshansi, “Welfare impacts of electricity storage and the implications of ownership structure.” Energy Journal, vol. 31, no. 2, 2010.
[6] U.S. Department of Energy, “Grid energy storage,” Report, December 2013. [Online]. Available: http://energy.gov/oe/downloads/grid-energy-storage-december-2013
[7] R. Sioshansi, P. Denholm, and T. Jenkin, “Market and policy barriers to deployment of energy storage,” Economics of Energy and Environmental Policy Journal, vol. 1, no. 2, p. 47, 2012.
[8] X. He, E. Delarue, W. D’haezeleer, and J.-M. Glachant, “A novel business model for aggregating the values of electricity storage,” Energy Policy, vol. 39, no. 3, pp. 1575 – 1585, 2011.
[9] J. Taylor, “Financial storage rights,” Power Systems, IEEE Transactions on, vol. PP, no. 99, pp. 1–9, 2014.
[10] H.-P. Chao and S. Peck, “A market mechanism for electric power transmission,” Journal of regulatory economics, vol. 10, no. 1, pp. 25–59, 1996.
[11] ———, “An institutional design for an electricity contract market with central dispatch,” The Energy Journal, vol. 18, no. 1, pp. 85–110, 1997.
[12] H.-P. Chao, S. Peck, S. Oren, and R. Wilson, “Flow-based transmission rights and congestion management,” The Electricity Journal, vol. 13, no. 8, pp. 38–58, 2000.
[13] W. W. Hogan, “Flowgate rights and wrongs,” John F. Kennedy School of Government, Harvard University, 2000. [Online]. Available: www.hks.harvard.edu/fs/whogan/flow0800cr.pdf
[14] S. S. Oren, P. T. Spiller, P. Varaiya, and F. Wu, “Nodal prices and transmission rights: A critical appraisal,” The Electricity Journal, vol. 8, no. 3, pp. 24–35, 1995.
where (a) and (b) follow from a direct substitution of the stationarity condition (10) and complementary slackness conditions (12)-(13), respectively. And the SCS is nonnegative, as both b and \( \mathbf{1} \mathbf{v}_i \) are nonnegative vectors for all \( i \).

It follows that \( MS = TCS + SCS \geq 0 \), completing the proof.

**PROOF OF LEMMA 2**

Let \( (V, U, \Lambda) \) be an efficient market equilibrium (cf. Def. 2). And let \( \mathcal{F} \) be a simultaneously feasible collection of FTRs. Define \( P \in \mathbb{R}^{n \times N} \) and \( Q \in \mathbb{R}^{n \times N} \) as the sequence of net power injections and net storage injections, respectively, implied by the rights \( \mathcal{F} \). For their precise specification, see Def. 6. We now establish a sequence of results that will prove essential in establishing the desired result.

**Proposition 3.** \( \Phi(\mathcal{F}) = -\sum_{k=0}^{N-1} \lambda(k)^\top p(k) \).

**Proof:** Combining identity (16) with the Def. 5 of rent, one obtains the following string of equalities:

\[
\Phi(\mathcal{F}) = \sum_{i=1}^{n} \sum_{j=1}^{n} (\lambda_j - \lambda_i)^\top T_{ij} = \sum_{i=1}^{n} \lambda_i^\top T_{ij} - \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i^\top T_{ij}
\]

\[
= \sum_{i=1}^{n} \lambda_i^\top \left[ \sum_{j=1}^{n} (T_{ij} - T_{ij}) \right] = -\sum_{i=1}^{n} \lambda_i^\top p_i,
\]

which yields the desired result, as \( \sum_{i=1}^{n} \lambda_i^\top p_i = \sum_{k=0}^{N-1} \lambda(k)^\top p(k) \).

**Proposition 4.** For any simultaneously feasible collection of transmission rights \( \mathcal{F} \), the implied injections \((P, Q)\) satisfy:

(i) \( 1^\top p(k) = 0 \) for all \( k = 0, \ldots, N - 1 \)

(ii) \( Q = 0 \).

**Proof:** The proof of part (i) is immediate, as one can show

\[
1^\top p(k) = \sum_{i=1}^{n} p_i(k) = \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij}(k) - T_{ji}(k)
\]

\[
= -\sum_{j=1}^{n} p_j(k) = -1^\top p(k).
\]

As for part (ii), the simultaneous feasibility of \( \mathcal{F} \) requires that \((P, Q)\) satisfy

\[
1^\top (p(k) + q(k)) = 0, \ k = 0, \ldots, N - 1,
\]

It follows from part (i) and (22) that \( 1^\top q(k) = 0 \) for all \( k \). Moreover, given our assumption of zero initial stored energy (i.e. \( z(0) = 0 \)), one can readily prove by induction that \( q(k) \leq 0 \) for all \( k \); from which it follows that \( q(k) = 0 \) for all \( k \), proving that \( Q = 0 \).
We have the following result, which we will use to relate the rent $\Phi(\mathcal{T})$ due to a collection of simultaneously feasible transmission rights $\mathcal{T}$ with the transmission congestion surplus, TCS.

**Proposition 5.** For any collection of vectors $x(k) \in \mathbb{R}^n$ satisfying $x(k) \in \mathcal{P}$ for $k = 0, \ldots, N - 1$, it holds that

$$- \sum_{k=0}^{N-1} \lambda(k)^{\top} x(k) \leq \text{TCS}.$$  

**Proof:** Let $(\gamma(k), \mu(k), \nu_k, \lambda_k)$ denote the optimal Lagrange multipliers satisfying the KKT conditions (6)-(13) for all $k$ and $i$. It trivially holds that

$$\gamma(k)^{\top} x(k) = 0,$$  

$$\mu(k)^{\top} (Hx(k) - c) \leq 0,$$  

as $x(k) \in \mathcal{P}$ for all $k$. Subtracting (23) from (24) and summing over $k$ yields

$$- \sum_{k=0}^{N-1} (\gamma(k)^{\top} - H^{\top} \mu(k))^{\top} x(k) \leq \sum_{k=0}^{N-1} \mu(k)^{\top} c.$$  

It follows from Prop. 1 that right-hand side of the inequality above equals the TCS. And the desired result follows, as locational marginal prices are defined as $\lambda(k) = \gamma(k)^{\top} - H^{\top} \mu(k)$ for all $k$.

It follows from Propositions 3 - 5 that

$$\Phi(\mathcal{T}) = - \sum_{k=0}^{N-1} \lambda(k)^{\top} p(k) \leq \text{TCS},$$  

as Prop. 4 implies that $p(k) \in \mathcal{P}$ for all $k$.

It now remains to prove tightness of the inequality (25); namely, that there exists a collection of simultaneously feasible transmission rights whose associated rent achieves the TCS. We construct such a collection of rights from an optimal dispatch of a system without storage. More specifically, let $b = 0$. And let $(V, U)$ be the optimal dispatch solving the corresponding problem (5) - (7). Clearly, $U = 0$. It follows that

$$\text{MS} = - \sum_{k=0}^{N-1} \lambda(k)^{\top} v(k) = \text{TCS},$$  

as the SCS $= 0$ necessarily. It follows that any collection of transmission rights $\mathcal{T} = \{T_{ij} \mid i,j = 1, \ldots, n\}$ satisfying

$$v_i = \sum_{j=1}^{n} (T_{ij} - T_{ji})$$  

is both simultaneously feasible and yields a rent $\Phi(\mathcal{T}) = \text{TCS}$. There are, in general, infinitely many such rights. ■

**Proof of Theorem 3**

Let $(V, U, \Lambda)$ be an efficient market equilibrium (cf. Def. 2). And let $(\mathcal{T}, \mathcal{S})$ be a simultaneously feasible collection of FTRs and FSRs, respectively. Define $P \in \mathbb{R}^{n \times N}$ and $Q \in \mathbb{R}^{n \times N}$ as the sequence of net power injections and net storage injections, respectively, implied by the collection $(\mathcal{T}, \mathcal{S})$. For their precise specification, see Def. 9. We now establish a sequence of results that will prove essential in establishing the desired result.

**Proposition 6.** $\Phi(\mathcal{T}) + \Sigma(\mathcal{S}) = - \sum_{k=0}^{N-1} \lambda(k)^{\top} p(k)$.

**Proof:** The proof is analogous to that of Prop. 3. Using Equation (18), which relates the collection of financial transmission and storage rights to their implied net power injections $P$, one can show that

$$\Phi(\mathcal{T}) = - \sum_{i=1}^{n} \lambda_i^\top (p_i + S_i).$$  

It follows that

$$\Phi(\mathcal{T}) + \Sigma(\mathcal{S}) = - \sum_{i=1}^{n} \lambda_i^\top (p_i + S_i) + \sum_{i=1}^{n} \lambda_i^\top S_i,$$

yielding the desired result. ■

**Proposition 7.** For any collection of vectors $x_i \in \mathbb{R}^N$ satisfying $x_i \in \mathcal{U}_i$ for $i = 1, \ldots, n$, it holds that

$$\sum_{i=1}^{n} \lambda_i^\top x_i \leq \text{SCS}.$$

**Proof:** Let $(\gamma(k), \mu(k), \nu_k, \lambda_k)$ denote the optimal Lagrange multipliers satisfying the KKT conditions (6)-(13) for all $k$ and $i$. It follows from nonnegativity of the Lagrange multipliers $(\nu_k, \lambda_k)$ and the assumption that $x_i \in \mathcal{U}_i$ for all $i$ that

$$\nu_k^\top L x_i \geq 0,$$  

$$\nu_k^\top (1 b_i - L x_i) \geq 0.$$  

Adding (26) and (27) and summing over $i$ yields

$$- \sum_{i=1}^{n} (\nu_i - \nu_j)^\top L x_i \leq \sum_{i=1}^{n} (\nu_i^\top 1)b_i.$$  

It follows from Prop. 2 that the right-hand side of the inequality above equals the SCS. The result follows as the locational marginal prices $\Lambda$ satisfy the stationarity condition $\lambda_i = L^\top (\nu_i - \nu_j)$ for all $i$.

Using Prop. 6, one can rewrite the total rent as

$$\Phi(\mathcal{T}) + \Sigma(\mathcal{S}) = - \sum_{k=0}^{N-1} \lambda(k)^\top (p(k) + q(k)) + \sum_{i=1}^{n} \lambda_i^\top q_i \leq \text{TCS} + \text{SCS} = \text{MS},$$

where the inequality is a consequence of Propositions 5 and 7, as the pair $(P, Q)$ is a feasible dispatch according to Def. 1. This proves revenue adequacy.

It remains to prove that the inequality in Theorem 3 is tight, in sense that there exists a collection of financial transmission and storage rights whose collective rent fully recovers the MS. Recall that $(V, U)$ is an optimal dispatch solving problem (5) - (7). Define the collection of FSRs, $\mathcal{S} = \{S_i \mid i = 1, \ldots, n\},$
to satisfy $S_i = u_i$ for all $i$. And let the FTRs be any collection, $\mathcal{F} = \{ T_{ij} \mid i, j = 1, \ldots, n \}$, satisfying

$$v_i = -u_i + \sum_{j=1}^{n} (T_{ij} - T_{ji}).$$

There are, in general, infinitely many such rights. The resulting collection of rights $(\mathcal{F}, \mathcal{S})$ is by nature of its construction simultaneously feasible. It follows from Prop. 6 that

$$\Phi(\mathcal{F}) + \Sigma(\mathcal{S}) = -\sum_{k=0}^{N-1} \lambda(k)^\top v(k),$$

completing the proof, as the right-hand side is equal to the MS by Def. 3. ■