“Light from chaos” in two dimensions

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We perform a Monte-Carlo study of the lattice two-dimensional gauged XY-model. Our results confirm the strong-coupling expansion arguments that for sufficiently small values of the spin-spin coupling the “gauge symmetry breaking” terms decouple and the long-distance physics is that of the unbroken pure gauge theory. We find no evidence for the existence, conjectured earlier, of massless states near a critical value of the spin-spin coupling. We comment on recent remarks in the literature on the use of gauged XY-models in proposed constructions of chiral lattice gauge theories.

I. MOTIVATION AND SUMMARY

The gauged two-dimensional lattice XY-model has been studied for quite some time. Its simplicity allows for analytic studies via lattice dualities and strong-coupling expansions [1] as well as for numerical analysis, in either a Hamiltonian [2] or Euclidean (see, e.g., [3]) formulation. The model exhibits many phenomena found in more complicated higher-dimensional theories, notably the presence of both confining and Higgs phases [1, 2, 3, 4, 5].

The motivation for this short study stems from our interest in using the gauged XY-model, as well as its non-abelian higher-dimensional analogues [4, 6, 7], in lattice constructions of chiral gauge theories. The lattice formulation of chiral gauge theories is an outstanding problem with no practical solution yet, despite much recent progress; for reviews, see [5]. While not our topic here, we note that the recent constructions of [9, 10] have some attractive features (for an earlier proposal of similar flavor, see [11]). Most importantly, they may offer a way around the difficult and unsolved problem of the explicit nonperturbative construction of the fermion measure for general chiral lattice gauge theories. Ref. [9] aims to achieve this by combining older ideas [12] to use non-gauge strong dynamics to decouple the mirror fermions in a vectorlike theory with the recently discovered exact lattice chirality of the Neuberger-Dirac operator.

A concrete two-dimensional realization of the proposal [9] has the gauged XY-model as an essential part and was studied in various limits in [10]. Recently, [13] considered the perturbative spectrum of the models of [9] at nonzero gauge coupling. It was claimed there that the gauge boson is always massive and, therefore, the construction of [9, 10] was argued to be irrelevant to the study of unbroken chiral gauge theories, on account of this fact alone.

This brings us to the main topic of this paper. The arguments of [13] do not invoke either the fermions or the strong mirror dynamics (admittedly, complicated and not yet completely understood), but concern only the spectrum of the gauged XY-model, arguing that it only

has a phase with a massive gauge boson. This conclusion contradicts strong-coupling expansion arguments for the decoupling of the “gauge-breaking” terms for small values of the spin-spin coupling. As these arguments have been made many times in the past [2, 4, 6], we will not repeat them at length here.

Instead, given that confusion around the issue appears to still persist, we use Monte-Carlo simulations to argue the same point. These methods allow us to also study regions beyond the reach of strong-coupling expansions or perturbation theory and to observe in detail how the infrared physics changes smoothly as the XY-model spin-spin coupling $\kappa$ is varied. We show that for $\kappa < \kappa_c$, with $\kappa_c = O(1)$, the long-distance physics of the gauged XY-model is that of the unbroken pure gauge theory. We numerically demonstrate that, in accord with the strong-coupling expansion, for subcritical $\kappa$, the leading effect of the spin-spin coupling can be incorporated into a shift of the gauge coupling.

We also study the spectrum in coupling regimes inaccessible by controlled analytic methods. The lattice Hamiltonian of the gauged XY-model was studied long ago via a strong-coupling expansion in the gauge coupling, valid for arbitrary values of $\kappa$, with a subsequent Padé resummation to small gauge coupling [2]. A dip in the spectrum of charged particle pairs near a critical value of $\kappa$ of order unity was found, and it was conjectured that it may indicate the presence of massless states. We look for such states by measuring the susceptibilities of the relevant operators. We find a slight increase of the susceptibility around similar values of $\kappa$, consistent with the results of the Padé resummation of the strong-coupling series. However, we find no finite-size scaling of the susceptibility and hence no evidence for massless states in the intermediate “critical” regime.

Finally, as far as lattice formulations of chiral gauge theories are concerned, our results address the objection of [13] to the proposal [9]. Much remains to be done to see whether constructions along the lines of [11, 9] fulfill their designed goal—to give rise to long-distance gauge theories with chiral fermions in complex representations. While we hope to report more on at least some of the outstanding issues in the future, our main point here is that the reason for their failure would have to be more subtle.
II. THE MODEL

The Euclidean action of the gauged XY-model is:

$$-S = \sum_x \left( \frac{\beta}{2} \prod_{\text{plaq}} U + \frac{\kappa}{2} \sum_\mu \phi_x^* U_{x,\mu} \phi_{x+\hat{\mu}} \right) + \text{h.c.}, \quad (1)$$

where $U_{x,\mu} = e^{i A_{x,\mu}}$ is the link group element, $\prod_{\text{plaq}} U$ is the usual plaquette Wilson action, and $\phi_x = e^{i \eta_x}$ is a unitary Higgs field. $x$ denotes lattice sites on a two-dimensional square lattice and $\hat{\mu}$ is a unit lattice vector in the $\mu = 1, 2$ direction. Here $A_{x,\mu}$ and $\eta_x$ are angular variables defined on links and sites, respectively; throughout the paper the lattice spacing is set to unity.

The partition function of the model is defined via a lattice path integral over all angular variables. The action can be more succinctly expressed (and the simulation simplified) if one notes that upon a change of the link variables the dependence of the action on $\eta_x$ can be completely eliminated, leaving us with a simpler theory to study:

$$-S = \sum_x \left( \beta \cos F_x + \kappa \sum_\mu \cos A_{x,\mu} \right), \quad (2)$$

where $F_x = A_{x+1,2} - A_{x,2} - A_{x+2,1} + A_{x,1}$ is the lattice gauge field strength. In the naive continuum limit, taking $\beta = \frac{1}{g^2}$, the action (2) becomes:

$$-S_{\text{naive}} = \sum_x -\frac{1}{2g^2} F_x^2 - \frac{\kappa}{2} (A_{x,1}^2 + A_{x,2}^2) + \kappa \mathcal{O}(A^4), \quad (3)$$

and describes a massive gauge boson of mass $m_W^2 = g^2 \kappa$—a conclusion which holds for sufficiently large $\kappa$. For small $\kappa$, however, the perturbative expansion around the naive continuum limit is not a good guide to the spectrum of the theory, because the naive continuum limit misses two important points: the angular nature of the variables and the fact that at small $\kappa$ the quantum fluctuations of the longitudinal component of the gauge field $A_{x,\mu}$ are not suppressed. For $\kappa \ll 1$, the $\mathcal{O}(A^4)$ terms lead to strong interactions between the longitudinal component of $A_{\mu}$—the fluctuations of these modes are not suppressed by the gauge invariant kinetic term and for small $\kappa$ the spin-spin (or “mass”) term does not suppress them either. In contrast, for $\kappa \gg 1$, these modes’ fluctuations are suppressed by the “mass” term in (3) and the $(A_{\mu})^4$ interactions are irrelevant.

For small $\kappa$, therefore, a strong coupling expansion in $\kappa$ is more appropriate than the perturbative expansion—the longitudinal components of the gauge field strongly fluctuate on the scale of the lattice spacing, are thus heavy, and can be integrated out. The small-$\kappa$ expansion is most conveniently performed in the original gauge invariant formulation in terms of the variables $A_{x,\mu}$ and $\eta_x$ of (1). To leading order in $\kappa$, the effect on the infrared physics of the gauge field of the rapid fluctuations of the Higgs field $\phi_x$ is to renormalize the gauge coupling $g$.

The long-distance—compared to the correlation length of the Higgs field excitations, which is smaller than the lattice spacing for $\kappa < \kappa_c$—effective action is of the form:

$$-S_{\kappa<\kappa_c} \simeq \sum_x \left( \beta + \frac{\kappa^4}{8} \right) \cos F_x + \ldots \quad (4)$$

where dots denote higher-dimensional gauge invariant terms. Computing the coefficient of $\kappa^4$ in (4) can be read off the results of our simulation (clearly, this is numerically possible only if $\beta$ is not too large). The Monte-Carlo methods also allow us to see how the infrared physics changes as a function of $\kappa$ and to study the spectrum of various operators in regimes where neither the strong-coupling expansion nor perturbation theory apply.

III. THE MONTE-CARLO STUDY

Our Monte-Carlo study focuses on the evaluation of several quantities: the Wilson loop, the Polyakov loop, and several zero-momentum correlators, or susceptibilities. We study their variation with $\kappa$, $\beta$, and the lattice size $N$, our primary interest being the $\kappa$ dependence for small gauge coupling.

This choice of observables is motivated by the nature of two-dimensional physics: we remind the reader that the two-dimensional pure $U(1)$ gauge theory (which we argue describes the infrared of the gauged XY-model for subcritical $\kappa$) has no local propagating degrees of freedom. Nonetheless, it exhibits nontrivial dynamics manifesting itself in a nonzero expectation value of the Wilson loop, with an area law behavior for all values of the gauge coupling. Similarly, the Polyakov loop expectation value vanishes in the pure gauge theory for any coupling and temperature. On the other hand, the gauged XY-model with nonzero sufficiently large $\kappa$ has propagating local degrees of freedom, and information on their spectrum can be inferred from correlators of local operators.

All our simulations are performed via the Metropolis algorithm, using the partition function in “unitary gauge” defined by the action (2). The autocorrelation time for the various observables is monitored. The largest autocorrelation time—that of the Polyakov loop, varying between about ten and a few hundred of lattice updates, depending on $\beta$ and $\kappa$—is used to estimate the errors.
A. The Wilson and Polyakov loops

We denote by $W_{ij}$ the expectation value of the Wilson loop operator for a rectangular loop of size $i \times j$. In the pure gauge theory, $\kappa = 0$, the Wilson loop expectation value can be exactly calculated. The string tension $\sigma$ of the pure $U(1)$ gauge theory, which confines for all values of the gauge coupling, see e.g. [14], is easily shown to be, in the infinite volume limit:

$$\sigma(\beta) = -\frac{\ln W_{ij}}{i \times j} = -\ln \frac{I_1(\beta)}{I_0(\beta)}, \quad (5)$$

where $I_1, I_0$ are Bessel functions. At strong coupling, the string tension is $\sigma = \ln 4g^2 + \mathcal{O}(1/g^2)$, while at weak coupling one instead obtains $\sigma \simeq g^2$.

Finite-volume corrections to the string tension will sometimes be significant in our simulations. In order to disentangle these effects, we have computed the Wilson loop expectation value in the pure gauge theory in finite volume, using the dual representation (see, e.g., the second reference in [11]) of the partition function. We thus obtain for $W_{ij}$ in finite volume:

$$W_{ij} = \frac{\sum_n I_n(\beta)N^2 - A I_{n-1}(\beta)^A}{\sum_m I_m(\beta)N^2}, \quad (6)$$

where $A = ij$ is the area of the loop, $N^2$ is the lattice volume, and the sums are over all integers. It is easy to check a few simple limits: as $N \to \infty$ [6] reproduces the result [5] for the string tension; also, it correctly yields $W_{NN} = 1$. We use [6] to show that for subcritical $\kappa$ our measured Wilson loop expectation values reduce to the ones of the pure gauge theory also in cases where finite-size effects are numerically significant.

Turning on $\kappa \neq 0$, for large $\kappa$ we expect from Eqn. [3] to obtain a perimeter law for the Wilson loop. For small $\kappa$, from Eqn. [4] we should expect an area law instead. To avoid confusion, we stress that the theory does have charged matter excitations—the quanta of the Higgs field $\phi(x)$ which can screen external charges and lead to perimeter law for the Wilson loop. In the small-$\kappa$ limit, however, the charged excitations are heavier than the UV cutoff. For any charged fields’ mass a sufficiently long “string” will eventually find it energetically favourable to break, we can imagine taking the continuum and large-volume limits so that the breaking does not occur even for the longest possible string, i.e., $\sigma N/2 < m_S$, where $N$ is the lattice size and $m_S$ is the lowest energy of a pair of charged particles, much higher than the UV cutoff ($m_S \sim \kappa^{-1}$ in the strong-coupling expansion [2]).

We begin the discussion of our Monte-Carlo results by showing the dependence of the “string tension” $\sigma_{ij} = -\ln W_{ij}/(ij)$ on $\kappa$ for Wilson loops with $i=j=1,2,3,4$. On Fig. 1, we show $\sigma$ for an $8^2$ lattice and $\beta = 4$, while on Fig. 2, we show $\sigma$ on a $16^2$ lattice for the same $\beta = 4$. For these loop sizes and values of $\beta$ the finite volume effects are small. For small $\kappa < \mathcal{O}(1)$, we see that in both cases, the pure gauge theory value of the string tension $\sigma(4) \approx .147$, as given by Eqn. [5], is reproduced. For large values of $\kappa \gg 1$, on the other hand, we see a gradual transition to a perimeter law for the Wilson loop, with the “string tensions” for $\kappa = 4,5,8$ scaling as inverse powers of $\sqrt{A}$ ($\kappa = 8$ comes closest to the pure perimeter law).

The strong-coupling expansion Eqn. [4], predicts that for sufficiently small $\kappa$ the Wilson loop expectation value should be given by the same equation as in the pure gauge theory, Eqn. [5], but with $\beta \to \beta + \kappa^4/8$, so long as the loop is larger than the correlation length of the massive charged excitations. The small effect due to the $\mathcal{O}(\kappa^4)$
shift of the bare gauge coupling can not be seen for the
relatively large values of $\beta$ on Figs. 1, 2. To test this
prediction and see for what values of $\kappa$ it breaks down,
we have measured $W_{11}$, $W_{22}$, and $W_{33}$ as a function of
$\beta$, near $\beta \simeq 1.6$, for a number of values of $\kappa$, ranging
from $-1$ to $1$ with spacing $\delta \kappa = 0.1$. Then, we determine
the function $\Delta \beta(\kappa)$ such that $W_{ii}$ is constant. On Fig. 3
we plot $\Delta \beta = \beta(\kappa) - \beta$ vs. the small-$\kappa$ prediction of
the strong-coupling expansion, $\Delta \beta_{str.coupl.} \simeq \kappa^4/8$. The
agreement for $\kappa \leq 0.5$ is quite persuasive.

We present one more plot of the Wilson loop expectation
value, this time for the larger value of $\beta = 20$ and an $8^2$
lattice, on Fig. 4 where we show $W_{ii}$ of sizes
$1^2$, $2^2$, $3^2$, and $4^2$, as function of $\kappa$. The prediction for
$W_{ii}$ of the pure gauge theory, including finite-size effects, Eqs. 6, is given in the caption of Fig. 4 for $\kappa < 0.7$, the agreement with the numerical results is evident.

Finally, we study the expectation value of the Polyakov loop operator around the $\mu$-th compact direction, $P_\mu$; we recall that its vanishing signals the onset of a confining phase where the free energy of an isolated quark is infinite. On Fig. 5 we show $P_\mu$ (the loops in the two directions are identical, up to numerical errors) measured on a $16^2$
lattice, for a range of values of $\beta$, as a function of $\kappa$. As with the Wilson loop, the fact that the Polyakov loop achieves the value appropriate to the pure gauge theory (zero) at $\kappa < 1$ is clearly visible on the figure.

B. Zero-momentum Green’s functions

In this Section, we study the following zero-momentum correlators (susceptibilities):

$$\chi^S_{\mu \nu} = \sum_x \langle \cos A_{x, \mu} \cos A_{0, \nu} \rangle^C,$$

$$\chi^V_{\mu \nu} = \sum_x \langle \sin A_{x, \mu} \sin A_{0, \nu} \rangle^C, \quad (7)$$

where $\langle \ldots \rangle^C$ denotes a connected correlator; in this Section, we take the indices to be $\mu, \nu = 0, 1$.

We use the susceptibilities to look for massless states in the relevant scalar ($S$) and vector ($V$) channel via a large-volume scaling. In a Hamiltonian formulation, the operator $\text{Re}(\phi^* U_{x,1} \phi_{x+1})$, equal to $\cos A_{x,1}$ in unitary gauge, creates pairs of oppositely charged particles, while $\text{Im}(\phi^* U_{x,1} \phi_{x+1}) = \sin A_{x,1}$ creates (massive) vector states. The spectrum of the lattice gauged XY-model Hamiltonian was studied in ref. [2] via an expansion in
The measurements of the scalar susceptibilities, \( \langle \cos A_\mu \cos A_\nu \rangle \), are shown on Fig. 7. The states created by \( \cos A_{x,0} \) and \( \cos A_{x,1} \) mix, as the \( Z_4 \) symmetry does not forbid them to (in the Hamiltonian formulation, the mixing is between states created by the operators \( \text{Re}(\phi_x U_{2\pi/k} \phi_{x+1}) \) and \( \cos \pi x \)). We show the two eigenvalues of the susceptibility matrix on Fig. 7. Clearly, while an increase of the susceptibility (and hence the correlation length) around the critical value of \( \kappa \) is visible on Fig. 7—consistent with the Padé resummation of the 1/\( g^2 \) expansion—we do not show these data as they are not particularly illuminating; moreover, even if one takes the value of the zero-momentum Green’s functions for \( \kappa \leq O(1) \) as indicative of the square of the Compton wavelength of the state (this, in fact, is a gross overestimate, as the true energies of the states go to infinity as \( \kappa \to 0 \)), a quick look at Figs. 7, 8 shows that the maximum Compton wavelength is about \( \sqrt{0.7} \approx 0.84 \) in units of the lattice spacing. This is consistent with the strong-coupling expansion in \( \kappa \), as described at the end of Section II, as well as with the results of ref. [2], which showed that both \( V \) and \( S \) states decouple at \( \kappa < \kappa_c \).

While our main interest in this Section was the susceptibility (the zero-momentum Green’s function) and its large-volume scaling, we have also measured position-dependent Green’s functions. The results reveal that, for large \( \kappa \), valid for large \( \kappa \), is also indicated.

1/\( g^2 \), valid for all values of \( \kappa \). Subsequently, a Padé resummation was used to continue the series to \( g^2 \to 0 \). The [4, 4] Padé approximant of the scalar correlator computed there showed a tendency for scalar (S) states to become light for \( \kappa \) near \( \kappa_c \) (see Figs. 8, 9 of ref. [2]). While ref. [2] admitted that it is not clear that the procedure used to arrive at this result is to be trusted, it is conjectured that there may be massless states near \( \kappa_c \).

We first show our results for the vector, \( \langle \sin A_\mu \sin A_\nu \rangle \), susceptibilities [7] on Fig. 8 for lattices of various sizes and for \( \beta = 4 \). A few remarks are in order. First, the off-diagonal (\( \mu \nu = 01 \)) components of the vector correlators vanish due to the \( A_0 \to A_1, A_1 \to -A_0 \) symmetry. Second, the 00 and 11 components are identical due to the same \( Z_4 \) rotational symmetry of the Euclidean action; hence, we only show the 00 component. Third, it is clear from the figure that there is no large-volume scaling of the vector susceptibility for any value of \( \kappa \), and hence no indication that massless states are present; we have also checked that this lack of scaling persists for larger values of \( \beta \). Finally, as explained in Section II, for \( \kappa \to \infty \) perturbation theory is valid and the susceptibility [7] can be approximated by:

\[
\chi^{V}_{00} \approx \sum_x \langle A_{x,0} A_{x,0} \rangle^C \simeq g^2 \lim_{\mu \to 0} \frac{g_{00} - \frac{p_p p_0}{p^2 + g^2 \kappa}}{p^2 + g^2 \kappa} = \frac{1}{\kappa}, \tag{8}
\]

where we used the perturbative expansion of the action, Eqn. [5], to calculate the zero-momentum two-point function of the massive gauge boson (the overall factor of \( g^2 \) is due to the field normalization and the metric is Euclidean). Comparing the results of the simulation with the 1/\( \kappa \) curve, also drawn on Fig. 6, we see that the expected perturbative behavior [8] is well reproduced by our Monte-Carlo results for large \( \kappa \), and that the Green’s function is drastically different for small values of \( \kappa \).

1/\( \kappa \) behavior expected in perturbation theory, see Eqn. [5], valid for large \( \kappa \), is also indicated.

FIG. 6: The values of the vector susceptibilities \( \chi^V \) (upper set of curves) and \( \chi^0 \) (vanishing lower curves) as functions of \( \kappa \) for lattices of size \( 4^2, 8^2, 16^2 \). The 1/\( \kappa \) behavior expected in perturbation theory, see Eqn. [5], valid for large \( \kappa \), is also indicated.

FIG. 7: The two eigenvalues of the scalar susceptibility matrix \( \chi^S_{\mu \nu} \) as functions of \( \kappa \) for lattices of size \( 4^2, 8^2, 16^2 \). Clearly, no large-N scaling is observed near the transition.
IV. CONCLUSIONS FROM THE MONTE-CARLO STUDY

We performed a detailed study of a number of observables in the gauged XY-model, for various values of $\beta$ and $\kappa$. Here is a summary of the results of this paper:

1. We showed that for $\kappa < \kappa_c$, the string tension precisely agrees with the exactly known pure gauge theory expression, Eqn. (5), for a number of lattice sizes and values of $\beta$, see Figs. 1 2 4. This even includes observable finite size effects in Fig. 4, which correctly reproduce the finite-volume pure gauge theory result of Eqn. (4).

2. In Fig. 3 we observed even the small $O(\kappa^4)$ correction to the bare gauge coupling of Eqn. (4) from the leading-order strong-coupling (small-$\kappa$) expansion.

3. We showed that the Polyakov loop expectation value vanishes for $\kappa$ in the same range, $\kappa < \kappa_c$, where the string tension is that of the pure gauge theory, see Fig. 5.

4. Our results for the scalar, Fig. 1 and vector, Fig. 6, susceptibilities revealed no evidence for massless states in the spectrum in the intermediate regime of $\kappa \sim \kappa_c$.

5. Furthermore, the vector susceptibility was shown, see Fig. 6, to match that expected of a massive gauge boson for large values of $\kappa \gg 1$, precisely where perturbation theory is expected to be valid, see Section III. The same zero-momentum propagator is, however, drastically different for $\kappa < \kappa_c$, indicating that the correspondent state is heavier than the inverse lattice spacing and decouples from the infrared physics.

These results show that we have achieved our main objective: to provide numerical evidence for the fact that at small $\kappa < \kappa_c = O(1)$ the “gauge symmetry breaking” decouples and that the infrared physics of the theory is that of the pure gauge theory, with a small shift of the bare gauge coupling.

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