Experimental evidence of stochastic resonance without tuning due to non Gaussian noises

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In order to test theoretical predictions, we have studied the phenomenon of stochastic resonance in an electronic experimental system driven by white non Gaussian noise. In agreement with the theoretical predictions our main findings are: an enhancement of the sensibility of the system together with a remarkable widening of the response (robustness). This implies that even a single resonant unit can reach a marked reduction in the need of noise tuning.

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The phenomenon of Stochastic resonance (SR) has attracted enormous interest due to both its potential technological applications for optimizing the response to weak external signals in nonlinear dynamical systems and its connection with some biological mechanisms. A recent review shows the state of the art [1]. There, a large number of applications are shown in science and technology, ranging from paleoclimatology [2], to electronic circuits [3, 4], lasers [5], chemical systems [6], and the connection with some situations of biological interest (noise-induced information flow in sensory neurons in living systems, influence in ion-channel gating or in visual perception) [6]. Recent works have shown a tendency pointing towards achieving an enhancement of the system response (by means of the coupling of several SR units in what conforms an extended medium [6, 7, 8]), or analyzing the possibility of making the system response less dependent on a fine tuning of the noise intensity [1], as well as different ways to control the phenomenon [2].

It is worth remarking here that a majority of such studies on SR, with very few exceptions [13], have been done assuming that the noises were Gaussian. However, some experimental results in sensory systems, particularly for one kind of crayfish [13] as well as recent results for rat skin [13], offer strong indications that the noise source in these systems could be non Gaussian. Some recent studies in neural networks also point in this direction [16].

In recent works [7, 13], a nice way of generating a non Gaussian noise with a “fine” control on the degree of non Gaussianity was introduced and some interesting results concerning the effect of using non Gaussian noises in the study of SR have been obtained. The results of some analytical approximations and of numerical simulations indicate a certain degree of enhancement of the system response when it departs from Gaussian behavior. However, a most remarkable finding was that the system shows a marked “robustness” against noise tuning. Such robustness means that the maximum of the signal-to-noise ratio (SNR) curve can flatten when departing from Gaussian behavior, implying that the system does not require a fine tuning of the noise intensity in order to maximize its response to a weak external signal.

In this letter we analyze the case of SR when the noise source is non Gaussian but from an experimental point of view. We have studied an experimental set up similar to the one used in [3] but now using a non Gaussian noise source which was built exploiting the form of noise introduced in [17, 18], but for the particular case of white noise. In [7, 13] a particular class of Langevin equations was studied, having non Gaussian stationary distribution functions [19]. The work in [19] is based on the generalized thermostatistics proposed by Tsallis [20] which has been successfully applied to a wide variety of physical systems [21]. Those equations were

\[ \dot{x} = f(x, t) + g(x)\eta(t) \]
\[ \dot{\eta} = -\frac{1}{\tau} \frac{d}{d\eta} V_q(\eta) + \frac{1}{\tau} \xi(t) \]

where \( \xi(t) \) is a Gaussian white noise of zero mean and correlation \(< \xi(t)\xi(t') >= 2D\delta(t-t') \), and \( V_q(\eta) \) is given by [19]

\[ V_q(\eta) = \frac{1}{\beta(q-1)} \ln[1 + \beta(q-1)\eta^2], \]

with \( \beta = \frac{\tau}{\tau_0} \). The function \( f(x, t) \) was derived from a potential \( U(x, t) \), consisting of a double well potential and a linear term modulated by \( S(t) \sim F \cos(\omega t) \). The expression adopts (for \( \omega = 0 \)) to the case of diffusion in a potential \( U_0(x) \), induced by \( \eta \), a colored non-Gaussian noise. Clearly, when \( q \rightarrow 1 \) we recover the limit of \( \eta \) being a Gaussian colored noise (Ornstein-Uhlenbeck process). The stationary probability distribution for the random variable \( \eta \) and \( q > 1 \) is given by

\[ P_q(\eta|\eta) = \frac{\tau_0^{q-1}}{\tau^{q+1}} [1 + \beta(q-1)\eta^2/2]^{-\frac{q+1}{2}}, \]

with \( \eta \in (-\infty, \infty) \), and \( Z_q \) the normalization factor. When \( q < 1 \) the expression adopts the form:

\[ P_q(\eta) = \begin{cases} \frac{\tau_0^{q-1}}{Z_q} [1 + \beta(q-1)\eta^2/2]^{-\frac{q+1}{2}} & \text{if } |\eta| < w \\ 0 & \text{otherwise} \end{cases} \]

with \( w^{-2} = (1-q)\beta \), and with noise intensity \( D =\)
2\beta^{-1}/(5 - 3q).

By applying a path-integral formalism to the Langevin equations given by Eqs. (1), (2), and making an adiabatic-like elimination procedure \(22\), it was possible to arrive at an effective Markovian approximation. The specific details, not relevant here, have been shown elsewhere \(17\). In the present work we have used a stationary probability distribution for the noise with the same form indicated above, and for \(q < 1\). To obtain a number with the mentioned probability distribution we generate two uniformly distributed random numbers \(x \in (-w, w)\) and \(y \in (0, P_q(0))\). \(x\) is accepted if \(y < P_q(x)\).

To make the experiment we have used an electronic (analogical) circuit. We have used the well known Schmitt trigger, in an arrangement similar to \(3\). Analogical approaches have been extensively used in order not only to study SR (see Section II C of \(1\)) but also to analyze the propagation of freezing fronts, spatiotemporal SR, irreversibility of classical fluctuations, SR in nonlinear electronic systems, resonant propagation, etc. \(23\).

The experimental setup was implemented with a LM741 integrated opamp with positive feedback and variable resistors in a voltage divider configuration that could be used to set appropriately the threshold level of the circuit. The input and output signals were driven by a 12 bit data acquisition board (Advantech PCL-818L) connected to a PC. The frequencies of the periodic signal provided to the circuit was chosen as 100 Hz or 250 Hz, with an amplitude that could be varied between 0 and 3.536 \(V_{\text{rms}}\). The threshold level of the circuit was set as 3.018 \(V_{\text{rms}}\). The optimal temporal resolution achieved with the system was 70 \(\mu s\). The noises were obtained using a numerical generator as indicated above.

The set of parameters controlled during the experiments were the amplitude \((V_s)\) and frequency \((\nu)\) of the periodic signal, the noise intensity \(D\) and the parameter \(q\) of the noise distribution. For each election of the parameter values we performed measurements comprising 300000 points with a time lapse of 70 \(\mu s\) between two consecutive acquisitions. The phenomena of SR was characterized by the residence time distribution \(1\). In particular, we considered the area \((A_{\nu})\) and the height \((H_{\nu})\) of the first peak of the distribution, centered at \(t = 1/(2\nu)\). In Figs. 1 and 2 we display curves showing the behavior of \(A_{\nu}\) as a function of \(D\), for several situations.

Figure 1 shows the effect of varying the \(q\) parameter for \(\nu = 100\) Hz and \(V_s = 1.414V_{\text{rms}}\). We can see that for each value of \(q\), there is a clear resonant noise intensity.

As \(q\) decreases we find that the resonant noise intensity shifts to higher \(D\) values and the region where \(A_{\nu}\) remains high is much wider (robustness). At the same time, the value of the maximum of each curve is higher. These results are in correspondence with previously reported ones obtained from numerical simulations \(18\). In Fig. 2 we show the effect of changing the amplitude \(V_s\) and the frequency \(\nu\) of the periodic signal. We can see that the resonant range for \(D\) turns wider and the sensitivity of the system increases for growing \(V_s\). On the other hand, the effect of increasing the frequency is to shift the resonant peak towards higher values of \(D\).

As mentioned earlier, \(A_{\nu}\) corresponds to the area under the first peak of the residence time distribution. This quantity produces neat curves that allow us to show more clearly the SR effect. However, if we want to compare the effect of \(q\) on the sensitivity of the system it is more convenient to plot the height of the peak, \(H_{\nu}\), for a chosen value of \(V_s\) and \(\nu\), and several \(q\) values. The latter is shown in Fig. 3 where we can see that there is a clear enhancement of the sensitivity of the system with decreasing \(q\).

Summarizing, motivated by some experimental results in sensory systems \(14, 15\), as well as some recent theoretical results \(7, 18\), we have experimentally analyzed the problem of SR when the noise source is non Gaussian. We have chosen a white non Gaussian noise source with a probability distribution based on the generalized thermostatistics \(24\).

Our experimental results indicate that

- The resonance range increases significantly with the non-gaussianity of the noise. That shows that tuning the noise intensity is not determinant in order to increment the signal perception. This effect is what we call increased robustness of the system.

- At the same time there is a clear enhancement of the sensitivity of the system (rising of \(H_{\nu}\)) with decreasing \(q\).

- For larger amplitude \((V_s)\) of the periodic signal, the “resonant” range for \(D\) becomes wider and the height of the response increases.

- The effect of increasing the frequency \(\nu\) is to shift the resonance peak towards lower values of \(D\).

As we depart from Gaussian behavior (with \(q < 1\)), the SNR shows two main aspects: firstly it becomes less dependent on the precise value of the noise intensity, and secondly its maximum as a function of the noise intensity increases. Both aspects are of great relevance for technological applications \(1\). Moreover, as was indicated in \(13\), non Gaussian noises could be an intrinsic characteristic in biological systems, particularly in sensory systems \(7, 14, 17\). In addition to the increase in the response (SNR), the reduction in the need of tuning a precise value of the noise intensity is of particular relevance both in technology and in order to understand how a biological system can exploit this phenomenon. It is worth remarking here that such effects have been obtained considering a single resonant
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Figure 1: \( A_\nu \) vs. \( D \) for several \( q \) values. \( \nu = 100 \text{Hz} \) and \( V_s = 1.414 \text{ V}_\text{rms} \); (dashed) \( q = 0.5 \); (dotted) \( q = 0.25 \); (dash-dotted) \( q = 0.1 \).

Figure 2: \( A_\nu \) vs. \( D \) for \( q = 0.1 \) and varying \( V_s \) and \( \nu : \) (dotted) \( \nu = 100 \text{Hz} \) and \( V_s = 2.121 \text{ V}_\text{rms} \); (dashed) \( \nu = 250 \text{Hz} \) and \( V_s = 1.414 \text{ V}_\text{rms} \); (solid) \( \nu = 100 \text{Hz} \) and \( V_s = 1.414 \text{ V}_\text{rms} \).

Figure 3: \( H_\nu \) vs. \( q \), for \( \nu = 100 \text{Hz} \) and \( V_s = 1.414 \text{ V}_\text{rms} \).