RAIL: Risk-Averse Imitation Learning

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Abstract

Imitation learning algorithms learn viable policies by imitating an expert’s behavior when reward signals are not available. Generative Adversarial Imitation Learning (GAIL) is a state-of-the-art algorithm for learning policies when the expert’s behavior is available as a fixed set of trajectories. We evaluate in terms of the expert’s cost function and observe that the distribution of trajectory-costs is often more heavy-tailed for GAIL-agents than the expert at a number of benchmark continuous-control tasks. Thus, high-cost trajectories, corresponding to tail-end events of catastrophic failure, are more likely to be encountered by the GAIL-agents than the expert. This makes the reliability of GAIL-agents questionable when it comes to deployment in risk-sensitive applications like robotic surgery and autonomous driving. In this work, we aim to minimize the occurrence of tail-end events by minimizing tail risk within the GAIL framework. We quantify tail risk by the Conditional-Value-at-Risk ($CVaR$) of trajectories and develop the Risk-Averse Imitation Learning (RAIL) algorithm. We observe that the policies learned with RAIL show lower tail-end risk than those of vanilla GAIL. Thus the proposed RAIL algorithm appears as a potent alternative to GAIL for improved reliability in risk-sensitive applications.

1 Introduction

Reinforcement learning (RL) Sutton and Barto (1998) is used to learn an effective policy of choosing actions in order to achieve a specified goal in an environment. The goal is communicated to the agent through a scalar cost and the agent learns a policy that minimizes the expected total cost incurred over a trajectory. RL algorithms, along with efficient function approximators like deep neural networks, have achieved human-level or beyond human-level performance at many challenging planning tasks like continuous-control Lillicrap et al., Schmidhuber et al. (2015, 2015) and game-playing Silver et al., Mnih et al. (2015, 2016). In classical RL, the cost function is handcrafted based on heuristic assumptions about the goal and the environment. This is challenging in most real-world applications and also prone to subjectivity induced bias. Imitation learning or Learning from Demonstration (LfD) Argall et al., Schaal, Atkeson and Schaal, Abbeel and Ng, Abbeel and Ng, Ng, Russell, and others (1997, 1997, 2000, 2004, 2009, 2011) addresses this challenge by providing methods of learning policies through imitation of an expert’s behavior without the need of a handcrafted cost function. In this paper we study the reliability of existing imitation learning algorithms when it comes to learning solely from a fixed set of trajectories demonstrated by an expert with no interaction between the agent and the expert during training.

Imitation learning algorithms fall into two broad categories. The first category, known as Behavioral Cloning Pomerleau, Bojarski et al., Bojarski et al. (1989, 2016, 2017), uses supervised learning to fit a policy function to the state-action pairs from expert-demonstrated trajectories. Despite its simplicity, Behavioral Cloning fails to work well when only a limited amount of data is available. These algorithms assume that observations are i.i.d. and learn to fit single time-step decisions. Whereas, in sequential decision making problems where predicted actions affect the future observations (e.g. driving), the i.i.d. assumption is violated. As a result, these algorithms suffer from the problem of compounding error due to covariate shift Ross and Bagnell, Ross, Gordon, and Bagnell (2010, 2011). Approaches to ameliorate the issue of compounding error like SMILe Ross and Bagnell (2010), SEARN Daumé, Langford, and Marcu (2009), CPI Kakade and Langford (2002) suffer from instability in practical applications Ross, Gordon, and Bagnell (2011) while DAGGER Ross, Gordon, and Bagnell (2011) and AGGREVATE Ross and Bagnell (2014) require the agent to query the expert during training which is not allowed in our setting of learning from a fixed set of expert demonstrations. Another drawback of Behavioral Cloning is that it does not allow the agent to explore alternate policies for achieving the same objective that might be efficient in some sense other than what the expert cared for.

The second category of algorithms, known as Inverse Reinforcement Learning (IRL) Russell, Ng, Russell, and others (1998, 2000, 2011), attempt to uncover the expert’s reward function from the demonstrated trajectories. This reward function succinctly encodes the expert’s behavior and can be used by an agent to learn a policy through an RL algorithm. The method of learning policies through RL after IRL is known as Apprenticeship...
Learning Abbeel and Ng (2004). IRL algorithms find reward functions that prioritize entire trajectories over others. Unlike behavioral cloning, they do not fit single time-step decisions, and hence they do not suffer from the issue of compounding error. However, IRL algorithms are indirect because they learn a reward function that explains expert behavior but do not tell the learner how to act directly Ho and Ermon (2016). That job is left to an RL algorithm. Moreover, IRL algorithms are computationally expensive and have scalability issues in large environments Finn, Levine, and Abbeel Levine and Koltun (2012, 2016).

The recently proposed Generative Adversarial Imitation Learning (GAIL) algorithm Ho and Ermon (2016) presents a novel mathematical framework in which the agent learns to act by directly extracting a policy from expert-demonstrated trajectories, as if it were obtained by RL following IRL. The authors show that unlike Behavioral Cloning, this method is not prone to the issue of compounding error and it is also scalable to large environments. Currently, GAIL provides state-of-the-art performance at several benchmark control tasks, including those in Table 1.

Risk sensitivity is integral to human learning Nagengast, Braun, and Wolpert Niv et al. (2010, 2012), and risk-sensitive decision-making problems, in the context of MDPs, have been investigated in various fields, e.g., in finance Ruszczyński (2010), operations research Howard and Matheson Borkar (1972, 2002), machine learning Heger Mihatsch and Neuneier (1994, 2002) and robotics Shalev-Shwartz, Shammah, and Shashua Shalev-Shwartz, Shammah, and Shashua, Abbeel et al. (2007, 2016, 2017). Garcia and Fernández (2015) give a comprehensive overview of different risk-sensitive RL algorithms. They fall in two broad categories. The first category includes methods that constrain the agent to safe states during exploration while the second modifies the optimality criterion of the agent to embed a term for minimizing risk. Studies on risk-minimization are rather scarce in the imitation learning literature. Majumdar et al. (2017) take inspiration from studies like Glimcher and Fehr, Shen et al. [and Hsu et al.] (2005, 2013, 2014) on modeling risk in human decision-making and conservatively approximate the expert’s risk preferences by finding an outer approximation of the risk envelope. Much of the literature on imitation learning has been developed with average-case performance at the center, overlooking tail-end events. In

Figure 1: Histograms of the costs of 250 trajectories generated by the expert and GAIL agents at high-dimensional continuous control tasks, Hopper-v1 and Humanoid-v1, from OpenAI Gym. The inset diagrams show zoomed-in views of the tails of these distributions (the region beyond 2σ of the mean). We observe that the GAIL agents produce tails heavier than the expert, indicating that GAIL is more prone to generating high-cost trajectories.
this work, we aim to take an inclusive and direct approach to minimizing tail risk of GAIL-learned policies at test time irrespective of the expert’s risk preferences.

In order to evaluate the worst-case risk of deploying GAIL-learned policies, we studied the distributions (see Figure 1) of trajectory-costs (according to the expert’s cost function for the GAIL agents and experts at different control tasks (see Table 1). We observed that the distributions for GAIL are more heavy-tailed than the expert, where the tail corresponds to occurrences of high trajectory-costs. In order to quantify tail risk, we use Conditional-Value-at-Risk (CVaR) [Rockafellar and Uryasev (2000)]. CVaR is defined as the expected cost above a given level of confidence and is a popular and coherent tail risk measure. We observe that the value of CVaR is much higher for GAIL than the experts at most of the tasks (see Table 1) which again suggests that the GAIL agents encounter high-cost trajectories more often than the experts. Since high trajectory-costs may correspond to events of catastrophic failure, GAIL agents are not reliable in risk-sensitive applications. In this work, we aim to explicitly minimize expected worst-case risk for a given confidence bound (quantified by CVaR) along with the GAIL objective, such that the learned policies are more reliable than GAIL, when deployed, while still preserving the average performance of GAIL. Chow and Ghavamzadeh (2014) developed policy gradient and actor-critic algorithms for mean-CVaR optimization for learning policies in the classic RL setting. However these algorithms are not directly applicable in our setting of learning a policy from a set of expert-demonstrated trajectories. We take inspiration from this work and make the following contributions:

1. We formulate the Risk-Averse Imitation Learning (RAIL) algorithm which optimizes CVaR in addition to the original GAIL objective.
2. We evaluate RAIL at a number of benchmark control tasks and demonstrate that it obtains policies with lesser tail risk at test time than GAIL.

The rest of the paper is organized as follows. Section 2 builds the mathematical foundation of the paper by introducing essential concepts of imitation learning. Section 3 defines relevant risk-measures and describes the proposed Risk-Averse Imitation Learning algorithm. Section 4 specifies our experimental setup and section 5 outlines the evaluation metrics. Finally, section 6 presents the results of our experiments comparing RAIL with GAIL followed by a discussion of the same. Section 7 concludes the paper with scope of future work.

## 2 Mathematical Background

Let us consider a Markov Decision Process (MDP), $\mathcal{M} = (\mathcal{S}, \mathcal{A}, T, c, p_0, \gamma)$, where $\mathcal{S}$ denotes the set of all possible states, $\mathcal{A}$ denotes the set of all possible actions that the agent can take, $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$ is the state transition function such that, $T(s'|s, a)$ is a probability distribution over next states, $s' \in \mathcal{S}$ given current state $s \in \mathcal{S}$ and action $a \in \mathcal{A}$, $c : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is the cost function which generates a real number as feedback for every state-action pair, $p_0 : \mathcal{S} \rightarrow [0, 1]$ gives the initial state distribution, and $\gamma$ is a temporal discount factor.

A policy $\pi : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$ is a function such that $\pi(a|s)$ gives a probability distribution over actions, $a \in \mathcal{A}$ in a given state, $s \in \mathcal{S}$. Let $\xi = (s_0, a_0, s_1, \ldots, s_L)$ denote a trajectory of length $L$, obtained by following a policy $\pi$. We define expectation of a function $f(\cdot, \cdot)$ defined on $\mathcal{S} \times \mathcal{A}$ with respect to a policy $\pi$ as follows:

$$E_\pi[f(s, a)] = \mathbb{E}_{\xi \sim \pi} \left[ \sum_{t=0}^{L-1} \gamma^t f(s_t, a_t) \right]$$

### Generative Adversarial Imitation Learning

Apprenticeship learning or Apprenticeship Learning via Inverse Reinforcement Learning algorithms [Abbeel and Ng (2004)] first estimate the expert’s reward function using IRL and then find the optimal policy for the recovered reward function using RL. Mathematically, this problem can be described as:

$$\text{RL} \circ \text{IRL} (\pi_E) = \underset{\pi \in \Pi}{\text{argmin}} \max_{c \in \mathcal{C}} \mathbb{E}_\pi[c(s, a)] - \mathbb{E}_{\pi_E}[c(s, a)]$$

$$-H(\pi)$$

where, $\pi_E$ denotes the expert-policy, $c(\cdot, \cdot)$ denotes the cost function. $\Pi$ and $\mathcal{C}$ denote the hypothesis classes for policy and cost functions. $H(\pi)$ denotes entropy of policy $\pi$. The term $-H(\pi)$ provides causal-entropy regularization [Ziebart et al. (2008, 2010)] which helps in making the policy optimization algorithm unbiased to factors other than the expected reward.

[Ho and Ermon (2016)] proposed Generative Adversarial Imitation Learning (GAIL) which packs the two step process of $\text{RL} \circ \text{IRL}_\omega (\pi_E)$ into a single optimization problem with special considerations for scalability in large environments. The name is due to the fact that this objective function can be optimized using the Generative Adversarial Network (GAN) [Goodfellow et al. (2014)] framework. The following is objective function of GAIL:

$$\underset{\pi \in \Pi}{\text{argmin}} \max_{D \in \{0, 1\}^{S \times A}} \mathbb{E}_\pi[\log(D(s, a))]$$

$$+ \mathbb{E}_{\pi_E}[\log(1 - D(s, a))] - H(\pi)$$

Here, the agent’s policy, $\pi$, acts as a generator of state-action pairs. $D$ is a discriminative binary classifier of the form $D : \mathcal{S} \times \mathcal{A} \rightarrow (0, 1)$, known as discriminator, which given a state-action pair $(s, a)$, predicts the likelihood of it being generated by the generator. A two-player adversarial game is started, wherein the generator tries to generate $(s, a)$ pairs that closely match the expert, while the discriminator tries to correctly classify the $(s, a)$ pairs of the expert and the agent. At convergence, the agent’s actions resemble those of the expert in any given state.
We use CVaR where Value at Risk (VaR) variable is defined as the minimum value of returns is not normal. Let Dalleh (2011) and unlike other measures like Variance and investment will move beyond three standard deviations. because there is a probability, which may be small, that an events that have a small probability of occurring. When the investment moving more than three standard deviations away from the mean is greater than what is shown by a normal distribution Investopedia (2017). Tail risk corresponds to events that have a small probability of occurring. When the distribution of market returns is heavy-tailed, tail risk is high because there is a probability, which may be small, that an investment will move beyond three standard deviations.

Conditional-Value-at-Risk (CVaR) Rockafellar and Uryasev (2000) is the most conservative measure of tail risk Dalaleh (2011) and unlike other measures like Variance and Value at Risk (VaR), it can be applied when the distribution of returns is not normal. Let Z be a random variable. Let \( \alpha \in [0, 1] \) denote a probability value. The Value-at-Risk of Z with respect to confidence level \( \alpha \), denoted by \( VaR_\alpha(Z) \), is defined as the minimum value \( z \in \mathbb{R} \) such that with probability \( \alpha \), Z will not exceed \( z \).

\[
VaR_\alpha(Z) = \min(z \mid P(Z \leq z) \geq \alpha)
\]

Next, we formulate the optimization problem to optimize CVaR of \( \mathcal{R}^\pi(\xi|c(D)) \) as:

\[
\min_{\pi} \max_{c} CVaR_\alpha(\mathcal{R}^\pi(\xi|c(D))) = \min_{\pi, \nu} \max_{c} H_\alpha(\mathcal{R}^\pi(\xi|c(D)), \nu)
\]

Integrating this with the GAIL objective of equation (3) we have the following:

\[
\min_{\pi, \nu} \max_{D \in (0,1)^S \times A} J = \max_{\pi, \nu} \max_{D \in (0,1)^S \times A} -H(\pi)
+ E_\pi [\log(D(s, a))] + E_{\pi_E} [\log(1 - D(s, a))]
+ \lambda_{CVaR} H_\alpha(\mathcal{R}^\pi(\xi|c(D)), \nu)
\]

Note that as \( c(\cdot) \) is order-preserving, the maximization with respect to \( \nu \) in equation (8) is equivalent to maximization with respect to \( D \) in equation (9) \( \lambda_{CVaR} \) is a constant that controls the amount of weightage given to CVaR optimization relative to the original GAIL objective. Equation (9) comprises the objective function of the proposed Risk-Averse Imitation Learning (RAIL) algorithm. Algorithm 1 gives the pseudo-code. Please refer to the supplementary material for the expressions of gradients of the CVaR term, \( H_\alpha(\mathcal{R}^\pi(\xi|c(D)), \nu) \) with respect to \( \pi, D \) and \( \nu \) and their derivations. We use Adam algorithm Diederik Kingma (2015) for gradient ascent in the discriminator and Trust Region Policy Optimization (TRPO) Schulman et al. (2015) for policy gradient descent in the generator. The CVaR term \( \alpha \) is trained by batch gradient descent Haykin (1998).

Algorithm 1 Risk-Averse Imitation learning (RAIL)

**Input:** Expert trajectories \( \xi_E \sim \pi_E \), hyper-parameters \( \alpha, \beta, \lambda_{CVaR} \)

**Output:** Optimized learner’s policy \( \pi \)

1: Initialization: \( \theta \leftarrow \theta_0, w \leftarrow w_0, \nu \leftarrow \nu_0, \lambda \leftarrow \lambda_{CVaR} \)
2: repeat
3: Sample trajectories \( \xi_i \sim \pi_{\theta_0} \)
4: Estimate \( \hat{H}_\alpha(D^\pi(\xi|c(D)), \nu) \)

\[
= \nu + \frac{1}{1 - \alpha} \mathbb{E}_{\xi_i} [(D^\pi(\xi|c(D))) - \nu]^+]; \; (x)^+ = \max(x, 0)
\]

5: Gradient ascent on discriminator parameters using:

\[
\nabla_{\theta_i} \mathcal{J} = \mathbb{E}_{\xi_i} [\nabla_{\theta_i} \log(D(s, a))] - \nabla_{\theta_i} H(\pi_\theta)
+ \lambda_{CVaR} \nabla_{\theta_i} \hat{H}_\alpha(\mathcal{R}^\pi(\xi|c(D)), \nu)
\]

6: KL-constrained natural gradient descent step (TRPO) on policy parameters using:

\[
\nabla_{\nu_i} \mathcal{J} = \mathbb{E}_{(s, a) \sim \xi_i} [\nabla_{\nu_i} \log(p_\theta(a|s))Q(s, a)]
+ \lambda_{CVaR} \nabla_{\nu_i} \hat{H}_\alpha(\mathcal{R}^\pi(\xi|c(D)), \nu)
\]

7: Gradient descent on CVaR parameters:

\[
\nabla_{\nu_i} \mathcal{J} = \nabla_{\nu_i} \hat{H}_\alpha(\mathcal{R}^\pi(\xi|c(D)), \nu)
\]

8: until \( i = \max_{\text{iter}} \)
We compare the tail risk of policies learned by GAIL and RAIL for five continuous control tasks listed in Table 1. All these environments, were simulated using MuJoCo Physics Simulator [Todorov, Erez, and Tassa 2012]. Each of these environments come packed with a “true” reward function in OpenAI Gym [Brockman et al. 2016]. Ho and Ermon [2016] trained neural network policies using Trust Region Policy Optimization (TRPO) [Schulman et al. 2015] on these reward functions to achieve state-of-the-art performance and have made the pre-trained models publicly available for all these environments as a part of their repository OpenAI-GAIL [2017]. They used these policies to generate the expert trajectories in their work on GAIL [Ho and Ermon 2016]. For a fair comparison, we use the same policies to generate expert trajectories in our experiments. Table 1 gives the number of expert trajectories sampled for each environment. These numbers correspond to the best results reported in Ho and Ermon (2016).

Again, following Ho and Ermon (2016), we model the generator (policy), discriminator and value function network (used for advantage estimation Sutton and Barto (1998) for the discriminator) with multi-layer perceptrons of the following architecture: observationDim = fc_100 – tanh – fc_100 – tanh – outDim, where fc_100 means fully connected layer with 100 nodes, tanh represents the hyperbolic-tangent activation function of the hidden layers, observationDim stands for the dimensionality of the observed feature space, outDim is equal to 1 for the discriminator and value function networks and equal to the twice of the dimensionality of the action space (for mean and standard deviation of the Gaussian from which the action should be sampled) for the policy network. For example, in case of Humanoid-v1, observationDim = 376 and outDim = 34 in the policy network. The value of the CVaR coefficient $\lambda_{CVaR}$ is set as given by Table 1 after a coarse hyper-parameter search. All other hyperparameters corresponding to the GAIL component of the algorithm are set identical to those used in Ho and Ermon (2016) and their repository OpenAI-GAIL (2017) for all the experiments. The value of $\alpha$ in the CVaR term is set to 0.9 and its lone parameter, $\nu$, is trained by batch gradient descent with learning rate 0.01.

### Table 1: Hyperparameters for the RAIL experiments on various continuous control tasks from OpenAI Gym.

| Task           | #training iterations | #expert trajectories | $\lambda_{CVaR}$ |
|----------------|----------------------|----------------------|------------------|
| Reacher-v1     | 200                  | 18                   | 0.25             |
| HalfCheetah-v1 | 500                  | 25                   | 0.5              |
| Hopper-v1      | 500                  | 25                   | 0.5              |
| Walker-v1      | 500                  | 25                   | 0.25             |
| Humanoid-v1    | 1500                 | 240                  | 0.75             |

#### 4 Experimental Setup

In this section we define the metrics we use to evaluate the efficacy of RAIL at reducing the tail risk of GAIL learned policies. Given an agent $A$’s policy $\pi_A$, we roll out $N$ trajectories $T = \{\xi_i\}_{i=1}^N$ from it and estimate VaR$_\alpha$ and CVaR$_\alpha$ as defined in Section 3. VaR$_\alpha$ denotes the value under which the trajectory-cost remains with probability $\alpha$ and CVaR$_\alpha$ gives the expected value of cost above VaR$_\alpha$. Intuitively, CVaR$_\alpha$ gives the average value of cost of the worst cases that have a total probability no more than $(1 - \alpha)$. The lower the value of both these metrics, the lower is the tail risk.

In order to compare tail risk of an agent with respect to the expert, $E$, we define percentage relative-VaR$_\alpha$ as follows:

$$VaR_\alpha(A|E) = 100 \times \frac{VaR_\alpha(E) - VaR_\alpha(A)}{|VaR_\alpha(E)|} \%$$

Similarly, we define percentage relative-CVaR$_\alpha$ as:

$$CVaR_\alpha(A|E) = 100 \times \frac{CVaR_\alpha(E) - CVaR_\alpha(A)}{|CVaR_\alpha(E)|} \%$$

The higher these numbers, the lesser is the tail risk of agent $A$. We define Gain in Reliability (GR) as the difference in percentage relative tail risk between RAIL and GAIL agents.

$$GR-VaR = VaR_\alpha(RAIL|E) - VaR_\alpha(GAIL|E)$$

$$GR-CVaR = CVaR_\alpha(RAIL|E) - CVaR_\alpha(GAIL|E)$$

#### 5 Evaluation Metrics

#### 6 Experimental Results and Discussion

In this section, we present and discuss the results of comparing between GAIL and RAIL. The expert’s performance is used as a benchmark. Tables 2 and 3 present the values of our evaluation metrics for different continuous-control tasks. We set $\alpha = 0.9$ for VaR$_\alpha$ and CVaR$_\alpha$ and estimate all metrics with $N = 50$ sampled trajectories (as followed by Ho and Ermon 2016). The following are some interesting observations that we make:

- RAIL obtains superior performance than GAIL at both tail risk measures – $VaR_{0.9}$ and $CVaR_{0.9}$, across a wide range of continuous-control tasks. This shows that RAIL is a superior choice than GAIL for imitation learning in risk-sensitive applications.

- The applicability of RAIL is not limited to environments in which the distribution of trajectory-cost is heavy-tailed for GAIL. Rockafellar and Uryasev (2000) showed that if the distribution of the risk variable $Z$ be normal, $CVaR_\alpha(Z) = \mu_Z + a(\alpha)\sigma_Z$, where $a(\alpha)$ is a constant for a given $\alpha$, $\mu_Z$ and $\sigma_Z$ are the mean and standard deviation of $Z$. Thus, in the absence of a heavy tail, minimization of $CVaR_\alpha$ of the trajectory cost aids in learning
Figure 2: Convergence of mean trajectory-cost during training. The faded curves correspond to the original value of mean trajectory-cost which varies highly between successive iterations. The data is smoothened with a moving average filter of window size 21 to demonstrate the prevalent behavior and plotted with solid curves. RAIL converges at least as fast as GAIL at all the five continuous-control tasks, and at times, even faster.

Table 2: Comparison of expert, GAIL, and RAIL in terms of the tail risk metrics - $\text{VaR}_{0.9}$ and $\text{CVaR}_{0.9}$. All the scores are calculated on samples of 50 trajectories. With smaller values of $\text{VaR}$ and $\text{CVaR}$, our method outperforms GAIL in all the 5 continuous control tasks and also outperforms the expert in many cases.

| Environment    | Dimensionality | VaR Expert | VaR GAIL | VaR Ours | CVaR Expert | CVaR GAIL | CVaR Ours |
|----------------|----------------|------------|----------|----------|-------------|----------|----------|
| Reacher-v1     | 11 Observation  | 5.88       | 9.55     | 7.28     | 6.34        | 13.25    | 9.41     |
| Hopper-v1      | 11 Observation  | -3754.71   | -1758.19 | -3745.90 | -2674.65   | -1347.60 | -3727.94 |
| HalfCheetah-v1 | 17 Observation  | -3431.59   | -2688.34 | -3150.31 | -2310.54   | -3359.29 | -3939.99 |
| Walker-v1      | 17 Observation  | -5402.52   | -3314.05 | -5404.00 | -2220.64   | -2945.76 |
| Humanoid-v1    | 376 Observation | -9839.79   | -2641.14 | -9252.29 | -491.43    | -1298.80 | -4640.42 |

Table 3: Values of percentage relative tail risk measures and gains in reliability on using RAIL over GAIL for different continuous control tasks.

| Environment     | $\text{VaR}_{0.9}(A|E)$ (%) GAIL | $\text{VaR}_{0.9}(A|E)$ (%) RAIL | GR-$\text{VaR}$ (%) | $\text{CVaR}_{0.9}(A|E)$ (%) GAIL | $\text{CVaR}_{0.9}(A|E)$ (%) RAIL | GR-$\text{CVaR}$ (%) |
|-----------------|-----------------------------------|----------------------------------|----------------------|-----------------------------------|-----------------------------------|----------------------|
| Reacher-v1      | -62.41                            | -23.81                           | 38.61                | -108.99                           | -48.42                           | 60.57                |
| Hopper-v1       | -53.17                            | -0.23                            | 52.94                | -49.62                            | 39.38                            | 89.00                |
| HalfCheetah-v1  | -21.66                            | -8.20                            | 13.46                | -33.84                            | -12.24                           | 21.60                |
| Walker-v1       | -1.64                             | 0.03                             | 1.66                 | 45.39                             | 70.52                            | 25.13                |
| Humanoid-v1     | -73.16                            | -5.97                            | 67.19                | -71.71                            | 1.07                             | 72.78                |

better policies by contributing to the minimization of the mean and standard deviation of trajectory cost. The results on Reacher-v1 corroborate our claims. Although the histogram does not show a heavy tail (Figure 3), the mean converges fine (Figure 2) and tail risk scores are improved (Table 2) which in this case indicates the distribution of trajectory-costs is more condensed around the mean than GAIL. Thus we can use RAIL instead of GAIL, no matter whether the distribution of trajectory costs is heavy-tailed for GAIL or not.

- Figure 2 shows the variation of mean trajectory cost over training iterations for GAIL and RAIL. We observe that RAIL converges at least as fast as GAIL at all the continuous-control tasks in discussion, and at times, even faster.
The success of RAIL in learning a viable policy for Humanoid-v1 suggests that RAIL is scalable to large environments. Scalability is one of the salient features of GAIL. RAIL preserves the scalability of GAIL while showing lower tail risk.

RAIL agents show lesser tail risk than GAIL agents after training has been completed. However it still requires the agent to act in the real world and sample trajectories (line 3 in Algorithm 1) during training. One way to rule out environmental interaction during training is to make the agent act in a simulator while learning from the expert’s real-world demonstrations. The setting changes to that of third person imitation learning Stadie, Abbeel, and Sutskever (2017). The RAIL formulation can be easily ported to the formulation of Studie, Abbeel, and Sutskever (2017) but we do not evaluate that in this paper.

7 Conclusion

This paper presents the RAIL algorithm which incorporates CVaR optimization within the original GAIL algorithm to minimize tail risk and thus improve reliability of learned policies. We report significant improvement over GAIL at a number of evaluation metrics on five continuous-control tasks. Thus the proposed algorithm is a viable step in the direction of learning low-risk policies by imitation learning in complex environments, especially in risk-sensitive applications like robotic surgery and autonomous driving. We plan to test RAIL on fielded robotic applications in the future.

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