The thinking process of the teachers with pseudo contingent when giving scaffolding in the teaching and learning of mathematics

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Abstract. This study aims to describe thinking processes of teachers with pseudo contingent when they give scaffolding in the teaching and learning of mathematics. The participants of this study were two teachers who teach mathematics at high schools in Banda Aceh and Aceh Besar. To collect the data, the researchers recorded the teachers’ classroom practices by using videotape. Later on, the researchers interviewed the teachers by using a method namely stimulated recall. Furthermore, the researchers explored the teachers’ thinking processes based on three criteria consisted of generating ideas, clarifying ideas and assessing ideas. Findings showed that the participants only considered learning outcome as a component which has to be mastered by the students rapidly. In other words, the teachers only assisted the students to understand the subject without considering the value of their assistance. Moreover, the participants were not eager to understand the mathematical concepts. Therefore, they had misconceptions about contents of mathematics given and were not able make decisions when they gave the scaffolding to the students.

1. Introduction
In an effective teaching and learning mathematics, some students usually ask about the topics. The students might ask questions to represent individuals or groups to have an explanation about what students have not understood. Furthermore, the teacher seeks to assist in the form of words, directions, explanations or gestures in the form of dialogue to develop students' thinking so that students are more independent in their learning (see [1] and [2]). This assistance is known as scaffolding. As stated by [3], the scaffolding refers to support that is tailored by teacher to fulfill students' needs. However, the teacher's thinking process when providing scaffolding has not been widely revealed by researchers yet. Exploring the thinking process of the teacher when providing scaffolding is necessary to gain knowledge about how to provide a good scaffolding. Therefore, efforts to uncover the teacher's thinking process when scaffolding is given is very important.

Several studies of scaffolding have been suggested and carried out by [4], [5]-[8], and [9]. [4] - [6] have identified two scaffolding characteristics namely contingent and non-contingent. Contingently, this means that there is a match between scaffolding given by the teacher and the response (level of understanding) of the student. If this is not the case then it is called non-contingent scaffolding. Meanwhile, [7] and [8] have identified three types of scaffolding given by teachers in teaching and learning mathematics when students ask questions, namely contingent dominant, non-contingent dominant, and pseudo contingent. The thinking process of teachers with contingent dominant
characteristics can occur intuitively and analytically. [9] and [10] suggested that the need to examine teachers' scaffolding in mathematics teaching. Meanwhile, the thinking process of teachers with other characteristics, namely non-contingent dominant and pseudo contingent, has not been revealed. Considering the length of time, the study was conducted, this study only examined the thinking processes of teachers who were pseudo contingent.

From 32 teachers observed by [8] in Malang and Banda Aceh, it was revealed that 15,625% belong to the pseudo contingent type. Specifically, in Aceh, the number of teachers included in that type was 25%. This fact occurs in a balanced manner both at the junior high school level and at senior high school. This fact also shows the concern of all mathematics educators that teachers still teach mathematical concepts incorrectly, unsure or hesitant in conveying material, even though the teacher is able to provide scaffolding well. However, some of these studies have only identified scaffolding contingency problems in learning but have not revealed the teacher's thinking process when scaffolding occurs. Therefore, further efforts to uncover this problem need to be done by exploring the teacher's thinking process in order to find the ideal scaffolding theory in mathematics learning.

The results of this study are needed to find the characteristics of teacher thinking when providing appropriate scaffolding between actions and what teachers think. As suggested by [11] that the teacher's thinking process is an important thing to study besides the teacher's actions. The results of this study are also very necessary to assist the government in order to detect or map the thinking processes of mathematics teachers in providing scaffolding during learning, for example in the implementation of the Teacher Professional Education Program and the Field Experience Practice Program for prospective teacher students.

2. Scaffolding as a result of the teacher's thinking process

What the teacher gives as scaffolding is, of course, the teacher's decision after going through the thinking process. According to [12], the thinking process is an idea of someone about what he did after experiencing an event. Based on the observation of researchers in teaching and learning mathematics in Aceh and Malang, in general, teachers help students naturally [8]. Some teachers help students by giving direct answers and some are indirect. Directly means the teacher himself solve the problem given so that students know the solution. In this case, the teacher has more ambitious processes of answering rather than involving the activeness of students to construct their knowledge. Indirectly means the teacher uses certain strategies to construct student knowledge, for example through asking about what is known and how students complete it. The teacher might ask students to compare what is known with other students' answers so they think. Such thinking processes can certainly differ from one teacher to another teacher. Moreover, the provision of scaffolding is very dependent on the time spent during scaffolding is given [3]

3. Contingency of scaffolding in teaching and learning mathematics

As the main characteristics of scaffolding [4][5][6], the compatibility between the assistance provided by the teacher and the level of understanding of students (contingency) is the need for every learning. Moreover, [4], [5] and [6] have examined the fragments of conversations between teachers and students to find contingencies in the provision of small group scaffolding on social science material. The results of the research by [4],[5] and [6] shown that there are two contingency patterns of interaction that the teacher makes when providing scaffolding to students, namely contingent and non-contingent. These findings can be different for other abstract material such as mathematics. [7], [8] and [9] have reviewed teacher scaffolding based on the number of fragments of a contingent or non-contingent scaffolding that occur when learners ask questions. If more interaction fragments are contingent, the researchers call them dominant contingents. If the interaction fragments are more non-contingent, the researchers call them non-contingent dominant. It may also be that an interaction fragment leads to a dominant contingent but the concepts conveyed by the teacher are incorrect or doubtful. Such interaction fragments are called pseudo contingents.

In this study, researchers adopted a contingency-based learning phase conducted by [4][5][6], as a
result, adapted from the assessment [13]. There are three strategies that teachers can do to check the pattern of the scaffolding of a teacher's learning, namely diagnostic strategy, checking of diagnosis, and intervention strategy. A teacher acts contingent depending on the initial strategy applied by the teacher, the diagnostic strategy [4][5][6]. In this strategy, the teacher seeks to know what students know as capital for the students' knowledge. The teacher collects all information about concepts that are known to students or strategies that are owned by students, the teacher can later decide or use appropriate methods to deal with problems based on students' responses. For example, the teacher asks the parts that are not yet understood by students. When students can reflect their understanding well, students will be better asked to explain their understanding to the teacher. This is useful for tracking the extent to which students understand the material to be learned.

According to [4], [5] and [6], there are two ways that teachers can track students' knowledge, namely: 1) reading written by students or 2) asking questions that are diagnostic (knowing the ability level of students without teacher assistance). In practice, the teacher may combine these two ways. The checking of diagnosis strategies is defined as the activity of verifying whether the teacher understands students correctly or not. This depends on the diagnostic strategy used. Through this strategy, teachers will get more information about students' knowledge. The teacher explores what students say and ask whether they have understood the teacher's assistance correctly and the interrelationships between the materials.

In the intervention strategy, the teachers adjust his actual assistance with information obtained by the teacher in the first and second stages. The intervention is done by the teachers to follow up with the students' understanding, not to take over the work of the students. According to [4], [5] and [6], diagnostic and intervention strategies are needed for contingent learning to take place. Intervention strategies can be done based on students' understanding that it is seen when the diagnostic strategy is carried out. Intervention (emphasis) on students is given so that students know or be directed towards what they are doing. This emphasis seems to be in the form of questions or orders that are forced. In this way, learners are told to continue the next activity. Included in the intervention strategy include: providing feedback, instructions, orders, models or asking questions.

4. Examples of scaffolding fragments of pseudo contingent types in teaching and learning mathematics

The following are the fragments of teacher scaffolding when facilitating an open sentence and closed sentence material. Both fragments are taken from [7] and [8].

Fragment 1
Students: Ms. open sentences …
Teacher: If the open sentence was a sentence that can be declared true or false or not both. Well, if closed means it ... what?
Students: Oh ...

Fragment 2
Teacher: So in conclusion ... what is a closed sentence? The closed sentence is the example of this ... this ... up to here ... Continue to be grouped into this group ... this group ... this group ... this group ... this group ... this group ... What does this closed sentence mean?
Student: [speechless]
Teacher: Sentences that are ... that cannot be ...
Students: Declared true.
Teacher: Or ...?
Student: Declared wrong.
Teacher: Please write it in your own words!

Noting Fragments 1 and 2, students' understanding is unclear. This is because the teacher incorrectly mentions the definition of open sentences and closed sentences. The results of the analysis of video recordings show that teachers often misteach material and seem hesitant. Among the definitions
conveyed by the teacher are: "Closed sentences are sentences that cannot be declared true or false or both. Open sentences are sentences that cannot be stated correctly or incorrectly or both. Such statements are the same as what was revealed by the teacher during the interview. Therefore, such teacher interaction is coded as pseudo contingent.

5. Thinking Process

Experts vary in defining thinking. Some define thinking processes as "mental activities" [14], or as "logic and reasons" for drawing conclusions [15]. Some equate thinking with reason [16]. However, in general, when a person thinks, the person is mentally required to doubt, ascertain, design, estimate, measure, differentiate, connect, or see all the possibilities that will occur so that the decision taken is the right and correct decision. Therefore, thinking is a very important activity in human life.

[17] divides thinking into two types, namely good thinking and bad thinking. To distinguish the two types of thinking, [17] relate it to intelligence (intelligence) This is because people's thinking skills are determined by how intelligence is used. For people who have high intelligence often defend their ideas when looking at a subject. Because the person defends his argument very well, the person does not need to dig deep into the subject or listen to other alternative views. According to [17], thinking like that is bad thinking. However, when someone views something and continues to explore his ideas in depth and consider various things, the process is called good thinking.

Many teachers do not think about what they are teaching [18]. Teachers prioritize the delivery of teaching materials to students but the teacher forgets to think of the best way to guarantee students learn [19]. One important activity by the teacher is to think of ways to facilitate subject matter effectively and efficiently so that it can bring students' understanding correctly. Therefore, mathematics learning facilitated by teachers should not only prioritize the results or answers of students but also how to facilitate students to think about the process.

6. Thinking Process Based on Swartz Framework

[19] divides the teacher's thinking activities in thinking about learning with three characteristics, namely generating ideas, clarifying ideas, and assessing the reasonableness of the ideas. Generating ideas is someone's creative imagination to produce something new. This is done after synthesizing existing thinking. According to [19] a person's ability to generate ideas comes from two things, namely 1) experience and 2) ability to mix experience. In that case, the role of imagination is very large. Generating ideas will help us gain understanding to overcome problems. When we have many ideas, we have many possibilities to solve the problem.

In this research, in the category of generating ideas, the teacher needs to come up with various alternative ideas that might be needed to encourage the success of learning starting from the learning planning stage. In providing scaffolding, teachers need to think about the things needed to help students related to facts, concepts, principles, or procedures needed, the form of questions, forms of intervention, forms of diagnostics, gestures, etc.

Ideas that are imagined by someone need to be clarified so that others know the reason for the idea. According to [19], teacher activities at this stage aim to increase understanding and the ability to use existing information. Therefore, the reason why a teacher does a certain way of providing scaffolding to students needs to be clarified.

At the clarifying idea stage, the teacher needs to explain the reasons for selecting ideas that have been done at the idea generation stage. For example, the teacher has set a topic for learning. The teacher must be able to explain the reason for scaffolding when learning interactions occur between the teacher and students. The reason is the analysis of ideas or arguments made by the teacher. Some ways that can be done is to analyze the ideas or arguments that are built by comparing, contrasting, defining or classifying, taking part or all, and sorting out the ideas.

At the stage of assessing the reasonableness of the ideas is the stage of assessing what the teacher will do to make a good decision. In other words, at this stage, someone takes the best decision based on the analysis of information that has been built at the clarifying idea stage. According to researchers, in
this third stage, someone assesses whether an idea raised by someone is reasonable or unnatural to do. In the category of assessing the reasonableness of ideas, the teacher assesses all considerations related to good initial information about scaffolding that will be given at the time of the learning interaction. This is done by predicting the accuracy of the information in the mind of the teacher, whether the value is positive, and can be accounted for, to produce the right decision to be implemented.

7. Method
The participants of this study were two teachers who were selected based on [8]. In this study, four out of 16 mathematics teachers in Banda Aceh and Aceh Besar were identified as the pseudo contingent type of scaffolding. They were two teachers at the junior high school and two teachers at the senior high school level. However, in this study, only two participants were selected from the junior high schools because both of them had more scaffolding fragments than the two pseudo contingent teachers at the high school level. Also, the two participants were willing to provide the data needed to reveal their thinking processes through interviews. Both of these subjects had initial ZL and ER. ZL is a 10-year experienced male teacher who teaches mathematics at SMP level in Banda Aceh with part-time status. ER is a female teacher who has worked for 10 years at the MTs level in Aceh Besar. Both have not been certified yet.

The purposive sampling method was used to select two participants [20]. As a qualitative study, the phenomenon under study is unique from the pseudo contingent type of teacher thinking processes in providing scaffolding to mathematics learning, obtained from interviews. Researchers want to understand the unique things of the learning process carried out by the teacher type pseudo contingent. After the interview results are transcribed, then the data are analyzed qualitatively based on the stages of thinking by [19] through data reduction procedures, data presentation, and conclusions. Finally, to ensure the validity of the data obtained, researchers conducted interviews twice at different periods.

8. Results and Discussion
The thinking processes of the two teachers in this study when providing scaffolding are summarized in Table 1. Based on Table 1, the idea that was raised in the thinking process by ZL for the first time when providing scaffolding to all students was to ask students to give examples of questions based on a material known in previous learning. Other ideas such as student conditions, books, and formulas are not considered by ZL when providing scaffolding. The conditions are similar to scaffolding given by teachers with intuitive contingent dominant types [9]. At this stage, according to [1], ZL’s scaffolding is at level 1.

Even though students only wrote the incomplete sequence from the example they made, it was enough as scaffolding by ZL. Next, ZL intervenes by adding a few commands to the example made by the student by giving a few extra words. For example, ZL adds the word "specify the term of ..." from a known sequence, even though the addition of the word seems unclear commands. ZL provides the intervention without realizing the meaning. As a result, the addition of the word seems not so important to make the student understand the given problem. According to [17], the way of such thinking is classified as bad thinking and it justifies what is said by [18].

The process of appointing students who were asked to give examples on the board was also not based on clear considerations ZL. ZL did not consider whether the requested student is a capable student or not. The important thing for him was that the student can continue the assignment until the results are obtained based on the formula that has been given by using the modified questions rapidly as answers. As a decision, ZL conducts scaffolding activities to help work quickly, focusing on the acquisition of results. The thinking process when giving scaffolding is almost the same as done by ZL for all of his teaching and learning. Besides, because ZL thinks that textbooks are rarely wrong and are often used by teachers, ZL is less daring to go out of context in the book, including using the formula Un arithmetic sequence like in the book, $Un = b \times n + (U1 + b)$. In fact, the formula is the same as $Un = a + (n-1) b$. 


**Table 1. Comparison of the pseudo contingent teachers’ thinking processes when providing scaffolding in teaching and learning mathematics**

| Teacher’s Initial | Generating Idea                                                                 | Clarifying Idea                                                                 | Assessing Idea                                                                 |
|-------------------|----------------------------------------------------------------------------------|----------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| **ZL**            | Consider the examples, materials, students, textbooks, and formulas              | Students have studied the material before                                        | Helping students work quickly rather than slowly                               |
|                   |                                                                                  | Examples of questions asked from students                                        | Focus on the results of the settlement                                           |
|                   |                                                                                  | It’s not very important who the students are giving the example                  | Often it depends on textbooks                                                   |
|                   |                                                                                  | Textbooks are rarely wrong so they can reinforce understood material            | Do not dare to go out of context in the textbook                                |
|                   |                                                                                  | The formula is a reference to work                                              | Use formulas like those in the textbook and don’t dare to change them           |
| **ER**            | Consider the media, example problems, students, textbooks, and rules (formulas)  | Students have studied the material before                                        | Help students work quickly                                                       |
|                   |                                                                                  | Examples of problems given by the teacher                                        | Focus on the results of the settlement                                           |
|                   |                                                                                  | Teacher do not so concerned with students who appear presentations              | Because textbooks are rarely wrong, they are often used                        |
|                   |                                                                                  | Textbooks as a practice guide of understood material                           | Do not dare to go out of context in the textbook                                |
|                   |                                                                                  | The media is used to help the learning process                                  | Using homemade media                                                            |
|                   |                                                                                  |                                                                                  | Try to understand for himself                                                    |
|                   |                                                                                  |                                                                                  | the concepts given and the results are wrong                                    |

ZL only considered learning outcomes as something that students must master in a short time. ZL pays less attention to student learning conditions. If there are students who have not been able to do some basic things such as multiplying two numbers, 6 times 7 = ..., then ZL provides scaffolding by providing inductive intervention. For example, ZL started asking the more basic questions that are 1 times 7, 2 times 7, and so on. That is, ZL does not consider the learning process as a form of interaction between teachers and students (although this method includes scaffolding level 2 according to [1] because the direct interaction between teachers and students is specifically related to mathematics being considered). The appointment of students to present subject matter is carried out randomly only for equal opportunity. Also, ZL is less critical in understanding mathematical concepts. As a result, ZL tends to facilitate misconceptions to students or hesitate in conveying material. The decision-making process in providing scaffolding is carried out quickly to achieve results, for example through inductive scaffolding. Referring to [21] and [22], ZL's thinking process is type 2 of his thinking activity and the given scaffolding is at level three [1].

In contrast to ZL, the first idea raised by ER was to intervene by asking students to answer questions made by ER based on the material that had been facilitated in the previous week. Other ideas such as student conditions, textbooks, and formulas did not pay much attention to the teachers in providing scaffolding. Furthermore, ER conducts a checking strategy or intervention by providing alternative problem-solving techniques using colored stone media. This is done when students answer one of the questions given. As a result, it appears that the instructions given by the ER did not provide opportunities for students to think but instead went directly to the stones that had been prepared. This of course only has an impact on students' momentary understanding.

The thinking process by ER can be said to be better than ZL. ER already involves generating ideas by facilitating media as learning tools. Unfortunately, the existence of such media is not very helpful for the teacher if the teacher has the misunderstanding. For example, when ER explains the division of
integers by 0, the stone media used by ER cannot help students find the right answer. For example, ER said that -96: 0 = -96 (can be seen in Figure 1) or 2: 0 = 2 (can be observed in Interview 1) ER has given students the wrong concept. Such understanding causes fatal errors in mathematics learning even though scaffolding facilitated by ER can be said to be contingent dominant. This observation shows that there has been tremendous confusion for students when ER answered the problem of dividing the integer by 0. However, ER easily answers that when what is divided "does not exist" then "what remains" is the answer. This fact is referred to by [7] – [9] as something "pseudo". Referring to [21] and [22], the ER thinking process is a type 1 of her thinking activity. Although ER has entered level 3 of scaffolding according to [1], the way of the thinking is classified as bad thinking [17]. As a result, the teaching did not become meaningful and it justified what is said by [19] that teachers are focused on the delivery of material but they forget to think of the best way to guarantee students understand.

ER is a teacher who is very enthusiastic about learning mathematics. This is evident from his preparation in learning, for example, ER provides learning media that he thinks of herself, namely black and white stones. This is not an imitation of another person. ER never received the process of using the media from other people or from training. ER beliefs like that turned out to be incompatible with the actual concepts in mathematics. Some facts about student answers that the teacher justifies when answering the problem of a number divided by zero can be noted in Fragment 3 and Fragment 4. Fragment 3 and Fragment 4 are two ER dialogues with students when giving scaffolding for a -96 problem divided by 0.

**Figure 1. The answers of the student who has written that -96: 0 = -96**

**Fragment 3**

Student : Ma'am divided by 0?
Teacher : -96 divided by 0 .... 1 divided by 0 ... is 0 there exist?
Student: no
Teacher: It doesn't mean the result ... 
Student : -96 [students write their answers based on analogy 1: 0 = 1]
Teacher: [The teacher smiles and leaves the student]

**Fragment 4**

Student: Ma'am ... [student asks division by 0]
Teacher: 1 divided by 0 ... is 0 exist?
Student: no
Teacher: No one means anything ... to whom? ’
Student: [smile]
Teacher: I have 2 mangoes ... 2 divided by 0 ... for me [while giving gestures by pointing his hands at the teacher] because 0 has no one]
Student: [students write their answers]

Based on Fragment 3 and Fragment 4, ER is very sure if what conveys is true. Based on interviews, the results of such thinking arise from a spontaneous understanding of themselves. The aim is that students simply understand what is being facilitated. Some ER thinking processes like that need to be addressed so that no more mistakes occur in the future. The teacher needs to realize that what he is thinking and doing is wrong. Therefore, in the future, it should be sought to help teachers to provide scaffolding correctly and the material being facilitated is also correct.
In interview 1, it was revealed that the ER tried to convince students to answer \(-96: 0\) by giving a positive intervention even though it was wrong by giving a reversal question which is "2: 0 ... is there 0?" In this way, ER tries to ensure that students will receive the remaining undivided (numerator) as the answer.

**Interview 1**

**Researcher:** When there are questions from students about -96: 0 ... do you answer by asking a feedback question that is 1: 0 ... 0 is there? What does it mean?

**Teacher:** So like this ... He asked me -96: 0 ... then I asked him back ... 1: 0 ... 0 is there? Well, there's nothing ... how much is the rest ... 1 ... so if -96: 0 ... 0 is there ... no so what is the result ... -96 ...

**Researcher:** I see?

**Teacher:** Because I imagine with him that there are 0 or not?

**Researcher:** Now that happened several times ... the same ... there were some children ... [while showing some facts in the video]. After seeing this video someone thought of me ... is that something right? 2 divided by 0 ... 0 there are no people means 2: 0 ...

**Teacher:** Two

**Researcher:** Is that something right ... 2: 0 ... two?

**Teacher:** Because like this ... I imagine 0 is taka da people ... we have manga 2 ...

**Researcher:** yes

**Teacher:** I want to divide ... to 0 ... 0 there is no one ... meaning who is the manga for me ... for me ... so how many mangoes ... yes ... two

Seeing such understanding as from Interview 1, the researcher felt the need to provide scaffolding such as interview 2.

**Interview 2** [Researcher's conversation with the teacher discusses the division by 0]

**Researcher:** Now ... for whom is it shared, Mrs?

**Teacher:** [pause]

**Researcher:** Say this ... suppose I have 6 mangoes ... want to be divided by 2 ... there are two people ... then how many people have each?

**Teacher:** Got three

**Researcher:** For whom does the quotient mean?

**Teacher:** For people ... that's why there are people ... but if it's zero ... there's no one

**Researcher:** Well, what does the result mean for us? Does that mean the review is for people or for us?

**Teacher:** Because there is no one ... yes for us ....

**Researcher:** Well ... what happened?

**Teacher:** No ...

**Researcher:** [If that's the case] Where do you get such understanding from?

**Teacher:** Alone

**Researcher:** When you were in college?

**Teacher:** No ... that's probably my language is wrong sir ...

**Researcher:** Have you been doing this for a long time?

**Teacher:** Already sir

Based on interviews 1 and 2, ER can find out that what he did was wrong. But, ER was happy to know that division of integers with 0 cannot be done because there is no process to work on (in more general terms known as undefined). ER has considered the learning process as a form of interaction between teacher and student, namely by implementing a checking strategy. However, the concepts conveyed are the result of her understanding as a misunderstanding. The appointment of students to present subject matter in front of the class is also done randomly just to equalize opportunities for students. ER is less important in understanding mathematical concepts. As a result, ER gives the wrong concept to students or is hesitant in delivering material. The decision-making process when providing scaffolding is done quickly aimed at the results to be achieved, even though inductively. [26] suggested
that teachers like ER should be given training or included to attend upgrading about scaffolding in mathematics learning.

The results of this study have shown that becoming a professional teacher is not easy. Although the teacher has experienced in teaching and learning mathematics for about 10 years, this does not guarantee that the teacher has good scaffolding in helping students who ask questions. Teachers still tend to depend on their daily experiences or experiences when attending lectures. The assistance provided is something wrong. According to [23], mathematics teachers can have had pseudo thinking and this has happened in various materials in learning mathematics such as algebra, geometry, and calculus. Therefore, according to [23] and [24], to overcome this, teachers must be given upgrading or training. [25] has mentioned it as a pseudo reasoning thinking process, that is, someone seems to think following the reasoning process, but the thought process of the person has not yet followed the correct reasoning process so that the results are artificial or fake. This is certainly dangerous in the development of student knowledge because students will bring something wrong. Of course, the problem must be fixed immediately. One way to fix this is to practice giving good scaffolding [26]. For example, prospective teachers need to be implanted about that knowledge since fieldwork programs at schools or when participating in the teacher professional programs. Teachers need to be introduced this type of scaffolding as often as possible in order to help them teaching mathematics. As a result, As s [1] argued that scaffolding can improve mathematics learning outcomes. Therefore, a professional teacher needs to prepare early for his scaffolding knowledge.

9. Summaries
The pseudo contingent type of teachers’ thinking process in providing scaffolding in mathematics learning is more dominant considering the facilitated subject matter. The teachers’ consideration in providing scaffolding is more focused on the results of problem-solving rather than considering the scaffolding process itself. The teacher's consideration comes from the textbook and what is obtained during lectures. However, the teacher's decision to provide scaffolding is based on wrong interpretations and is different from those in books or what is learned from lectures.

The results showed that the teacher only considered learning outcomes as something students had to master quickly. Subjects pay less attention to the condition of students in learning. Therefore, the appointment of students to present the subject matter is made randomly only for equal opportunity. The learning process is considered by participants, not as a form of interaction between teachers and students. Participants are also less critical in understanding mathematical concepts so that the subject gives wrong concepts to students or hesitates in the presentation of the material and the decision-making process when giving scaffolding is done quickly.

To support research on the teachers' thinking process of pseudo contingent types, it is necessary to do some kind of reinforcement in the form of teacher training with these types. Further, future research needs to be done by taking some teachers who are scattered in Indonesia based on this type. Although not to generalize findings, these results can be input for the government in making decisions, for example, in implementing teacher education programs or when implementing peer teaching.

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