Investigating the symmetry energy of nuclear equation of state with heavy-ion reactions.

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Abstract. Heavy-ion collisions provide a unique opportunity to examine nuclear matter at temperatures, densities, and neutron-to-proton (N/Z) ratios away from that of ground state nuclei. A wide range of observables from heavy-ion collisions has been used to study and constrain the nuclear equation-of-state (EOS). Recently much work has been focused on the symmetry energy of the EOS. The transverse collective flow of light charged particles and intermediate mass fragments is sensitive to the fragmentation mechanism and – in particular – the density dependence of the nuclear symmetry energy. The present work demonstrates the impact of the N/Z of the system on the transverse flow of LCPs and IMFs from the reactions 70Zn + 70Zn, 64Zn +64Zn, and 64Ni + 64Ni at 35 Mev/nucleon. The IMF flow is consistent with a “stiff” equation of state as implemented in the AMD model.

1. Introduction

Heavy-ion collisions provide a unique opportunity to examine nuclear matter at temperatures, densities, and neutron-to-proton (N/Z) ratios away from that of ground state nuclei. A wide range of observables from heavy-ion collisions have been used to study and constrain the nuclear equation-of-state (EOS) [1, 2]. While the EOS for symmetric nuclear matter (N=Z) is relatively well constrained [3–5], predictions for the density dependence of the symmetry energy, E_{sym}(\rho), can still vary widely [2, 5]. The collective transverse flow of light charged particles (LCPs) has been predicted to be a useful probe for applying constraints on the asymmetric part of the EOS at both high and low densities [2, 6]. The transverse collective flow has been shown to depend on both the mass and N/Z of the colliding system[7-12]. The examination of the balance energy demonstrated that the transverse flow was strongly dependent on the mass, A, of the colliding system [7]. Pak et al. have shown that the transverse collective flow for Z=1-3 particles increases with an increasing neutron-to-proton ratio of the system (N/Z)_{sys} [9,10]. The isospin dependence of the transverse flow and balance energy was attributed to the isospin dependent potential and in-medium NN-cross sections [11,12]. Along with the mass-dependent mechanisms (mean-field and NN-collisions), theoretical simulations have also demonstrated the importance of the Coulomb potential in describing the transverse flow [13-15].
While LCPs have typically been used in studying the transverse flow, we also investigated IMFs to explore the dependences of the transverse flow on the mass and charge of the colliding systems. Additionally, the IMF flow is shown to be a viable probe to the EOS, particularly the density dependence of the symmetry energy.

2. Experiment
The reactions were measured using the NIMROD-ISiS detector array [16] at the Texas A&M Cyclotron Institute. Beams of $^{70}$Zn, $^{64}$Zn, and $^{64}$Ni were accelerated by the K500 superconducting cyclotron to 35 MeV/nucleon. These beams were impinged on $^{70}$Zn, $^{64}$Zn, and $^{64}$Ni self-supporting targets, respectively. NIMROD-ISiS consists of 14 concentric rings providing coverage from 3.6 to 167 in lab as shown in figure 1. The first 8 rings, ranging from 3.6 to 45.0, had the same geometry as the INDRA detector [17] and the final 6 rings were of the ISiS geometry [18]. Isotopic resolution was achieved, in the forward angles, for Z=1-17 particles and elemental identification was obtained up through the charge of the beam. In the backward angles detector thresholds allowed only for identification of Z=1-2 particles. The entire charged particle array was surrounded by the 4π TAMU Neutron Ball which provided an average event-by-event neutron multiplicity.

3. Impact parameter determination
The impact parameter for the experimental data was determined using minimum bias 2-D distributions of the raw neutron multiplicity plotted against the charged particle multiplicity for each system. Five bins (0-4) were created from the 2-D distributions such that each bin would represent a $b/b_{max}$, or $b_{rad}$, width of 0.2 if one assumes a corresponding triangular impact parameter distribution. Molecular dynamics simulations [19,20], filtered using a software replica of the NIMROD-ISiS array, showed that the 80% (20%) of the events in the most peripheral (central) bin were correctly identified. The impact parameter determination for the more central bins was less certain. Thus, Bin 0 does not necessarily contain the most central events but rather the most violent events, while the bins representing the peripheral collisions do provide a relatively accurate impact parameter estimation.

4. Transverse Flow of LCP
In order to calculate the in-plane transverse momentum for the fragments, the reaction plane for each event was reconstructed using the azimuthal correlation method [21]. The azimuthal correlation method does not differentiate the forward, quasi-projectile, and backward, quasi-target, sides of the flow. Therefore, the forward flow side of the reaction plane was determined using the transverse momentum analysis method [22]. The particle of interest (POI) was removed from the calculation of the reaction plane in order to avoid autocorrelations [21-23]. Thus, the reaction plane was calculated for each particle in an event rather than once for the whole event. In order to ensure that only quasi-complete events were used in the analysis an event criterion was imposed such that the total detected
charge for an event must be greater than 40% of the total charge in the colliding system. The transverse flow is often quantified as the slope of the average in-plane momentum, \( <P_x/A> \), at mid-rapidity as shown in figure 2 for protons. In the following analysis, the flow was defined as the slope of the \( <P_x/A> \) over a reduced rapidity region \((Y_r)\) of -0.45 to 0.45.

![Figure 2: Average in plane momentum of protons as a function of rapidity from the \(^{64}\text{Ni}+^{64}\text{Ni}\) reaction. The slope of the line at mid-rapidity represents the measured flow.](image)

Figure 3 demonstrates the effect of both the isospin of the reaction system as well as the mass of both the particle of interest and the reaction system. The flow of \( Z=1 \) and \( Z=2 \) particles are shown as a function of the N/Z of the reaction system. As one can see the flow is larger for the \(^{64}\text{Ni}+^{64}\text{Ni}\) system than it is for the \(^{64}\text{Zn}+^{64}\text{Zn}\) system. This increase in flow with neutron richness, which is shown by the blue line, is consistent with earlier data by Pak.[9,10]. Based only on the N/Z one would expect that the \(^{70}\text{Zn}+^{70}\text{Zn}\) would have yet a larger flow. However the flow measured from the \(^{70}\text{Zn}+^{70}\text{Zn}\) system is lower than that of the \(^{64}\text{Ni}+^{64}\text{Ni}\) system because it is a larger mass system. This can be understood through the mass dependence of the transverse flow which is related to the balance energy relationship derived by Westfall et al. [7]. Thus, one would expect the \(^{70}\text{Zn}\) system to exhibit a decreased flow in comparison to the \( A=64 \) systems since it has a lower balance energy due to the increased repulsive NN-collisions relative to the attractive mean-field potential. As expected the flow for the \( Z=2 \) particles is larger than for the \( Z=1 \) particles.

The effect of the isospin of the particle of interest is shown in figure 4. The observation that the flow increases for more neutron-rich systems made from the nonisotopically resolved data (shown in figure 3) holds true here when one breaks it down into the individual isotopes. For each of the isotopes shown in figure 4 the \(^{64}\text{Zn}+^{64}\text{Zn}\) system has a lower flow than the \(^{64}\text{Ni}+^{64}\text{Ni}\) system.
Figure 3. Transverse collective flow of $Z=1,2$ particles as a function of the N/Z of the reaction. The lines are drawn to guide the eye. The blue line represents the expected change in the flow for a constant mass $A_{proj}=A_{tgt}=64$ system. The red line represents the decrease in flow due to the mass for the $A_{proj}=A_{tgt}=70$ system based upon the blue line for the $A=64$ system.

Figure 4. Transverse collective flow per nucleon as a function of the charge times the mass of the measured particle for the $^{70}\text{Zn} + ^{70}\text{Zn}$, $^{64}\text{Zn} + ^{64}\text{Zn}$, and $^{64}\text{Ni} + ^{64}\text{Ni}$ reactions at 35 Mev/nucleon.
Figure 4 plots the flow per nucleon as a function of the Z times the A of the particle. Using the Flow/nucleon allows us to remove the difference in flow due to particle size and focus on more interesting differences. Within a given Z the flow always decreases as the N/Z of the fragment increases. The triton to $^3\text{He}$ comparison demonstrated the effect of the neutron richness of the fragment where there is not difference in size.

5. IMF flow

In the NIMROD-ISiS array thresholds produced incomplete detection of intermediate mass fragments (IMFs) at negative reduced rapidities ($Y_r=\frac{Y_{cm}}{Y_{cm,proj}}$). Therefore, the transverse flow was quantified by calculating the average in-plane transverse momentum from $0.0<Y_r<0.45$ [24–26]. The flow is extracted only from the positive rapidity fragments and is designated as $\langle P_x \rangle$. Details of this method can be found in Ref. [27].

The transverse flow $\langle P_x \rangle$, for Z=1-9 particles is shown in Figure 5 for the five centrality bins, ranging from Bin 0 (most violent collisions) to bin 4 (most peripheral collisions). The expected increase in the transverse flow with the increasing charge of the fragments is clear [9, 28]. In bin 0 the $\langle P_x \rangle$ of the IMFs from the $^{64}\text{Ni}$ and $^{64}\text{Zn}$ systems are nearly equivalent and larger than the $^{70}\text{Zn}$ system. This is consistent with previous work by Westfall on the mass dependence of the balance energy. [7] In the peripheral reactions, Bins 3 and 4, the $\langle P_x \rangle$ of the IMFs from the Zn systems become similar and decreased with respect to the $^{64}\text{Ni}$ system. This represents a clear dependence of the charge of the system on the IMF flow. The larger repulsive Coulomb force in the Zn (Z=30) systems causes a decreased flow in comparison to the $^{64}\text{Ni}$ (Z=28) system. The increased effect of the charge-dependent forces, relative to the mass-dependent forces, in the peripheral collisions may be due to the decreased interaction volume. For example, the number of NN-collisions would be greatly diminished in the peripheral reactions. A separation of the IMF flow between all 3 systems occurs in the mid-peripheral reactions, bin 2, in which the IMF’s $\langle P_x \rangle$ from the $^{64}\text{Zn}$ system is less than that from the $^{64}\text{Ni}$ system yet larger than the $^{70}\text{Zn}$ flow, exhibiting a behavior between the extremes of the mass (bin 0) and charge (bin 4) dependent flow. The difference between the IMF flow in the $^{64}\text{Ni}$ and $^{64}\text{Zn}$ systems is similar to the (N/Z)$_{sys}$ dependence observed by Pak et al. for LCPs in $\Lambda=58$ systems [9]. However, in context with the results from the IMF flow of the $^{70}\text{Zn}$ system, which has a similar (N/Z)$_{sys}$ to the $^{64}\text{Ni}$ system, the difference between the A=64 IMF flow appears to be due to a balancing between the mass and charge dependent mechanisms.
Figure 5. Transverse collective flow of fragments from Z=1-9 as a function of particle Z for five bins in impact parameter. Bin 0 is the most central or most violent while Bin 4 represented the most peripheral collisions.

Figure 6. $R_{\text{Flow}}$ for Z=6-9 (colored lines) as a function of impact parameter in $b_{\text{red}}$ bins. The black dashed line represents a linear transition from $R_{\text{Flow}}=1$ at bin 0 to $R_{\text{Flow}}=0$ at bin 4 (not shown due to statistical uncertainties in bin 4).
In order to examine this trend more quantitatively the ratio

\[ R_{\text{Flow}} = \frac{\langle P_x \rangle ^{64\text{Zn}} - \langle P_x \rangle ^{70\text{Zn}}}{\langle P_x \rangle ^{64\text{Ni}} - \langle P_x \rangle ^{70\text{Zn}}} \]

can be used to define the magnitude of the flow from the $^{64\text{Zn}}$ system in comparison to the $^{64\text{Ni}}$ and $^{70\text{Zn}}$ systems. Thus, when $R_{\text{Flow}} = 1$ the IMF flow of the $^{64\text{Zn}}$ system equals that of the $^{64\text{Ni}}$ system and when $R_{\text{Flow}} = 0$ the $^{64\text{Zn}}$ and $^{70\text{Zn}}$ systems have equivalent values of flow. In Figure 6, the individual $R_{\text{Flow}}$ values of the $Z=6-9$ fragments and the average $R_{\text{Flow}}$ value of $Z=4-9$ fragments are plotted as a function of the centrality bin number. The ratio values exhibit a systematic trend from $R_{\text{Flow}} = 1$ for the most violent collisions to $R_{\text{Flow}} = 0$ for the most peripheral reactions. This trend, observed in Figures 5 and 6, shows a transition from the IMF’s $\langle P_x \rangle$ being strongly dependent on the mass of the system to a dependence on the charge of the system.

The observed mass to charge dependence of the IMF transverse flow should be sensitive to the density dependence of the symmetry energy since there is a mean-field component to the flow. Scalone et al. predicted that the difference in the transverse flow of LCPs from two systems with the same mass and differing $(N/Z)_{\text{sys}}$ would be sensitive to $E_{\text{sym}}(\rho)$ [29]. Therefore, changing the isospin-dependent part of the mean-field should affect the balance between the mass and charge dependent forces. The antisymmetrized molecular dynamics with wave packet Diffusion and Shrinking (AMD-DS) model [20] was used to investigate the sensitivity of the IMF flow to $E_{\text{sym}}(\rho)$. The dynamics of the reaction were simulated up to a time of 300 fm/c, after which the GEMINI code [30] was used to statistically de-excite the hot fragments. The excitation energy and angular momentum of the hot fragments, produced by AMD, were used as inputs into the GEMINI code. Additional details about the coupling of dynamical AMD and statistical GEMINI codes can be found in Ref. [20]. The momentum dependent Gogny and Gogny-AS effective interactions provided an incompressibility of symmetric nuclear matter of $\text{K}=228$ MeV while allowing for the density dependence of the symmetry energy to be varied [20]. The Gogny and Gogny-AS interactions produce a soft and stiff density dependence of the symmetry energy, respectively [31].

![Figure 7](image-url)

**Figure 7.** The ratio of the difference in transverse collective flow between the $^{64\text{Zn}}$ and $^{64\text{Ni}}$ compared to that of the $^{70\text{Zn}}$ system for $Z=4-9$ fragments as a function of the reduced impact parameter (left). The corresponding symmetry energy as a function of density for the interactions in the AMD results (right).
In Figure 7 the average $R_{\text{flow}}$ value for $Z=4-9$ fragments is shown as a function of $b_{\text{red}}$ from the AMD-Gemini simulations in comparison to the experimental data. The experimental results are equivalent to those presented in Figure 6 except that $R_{\text{flow}}$ is shown as a function of the average bred of each centrality bin. The average $b_{\text{red}}$ was determined using the filtered molecular dynamics simulations to provide an estimate of the impact parameter range selected in each centrality bin. The impact parameter for each event of the AMD-Gemini simulation was known and, therefore, the average $b_{\text{red}}$ values shown in Figure 7 are exact. While the same experimental procedure discussed above was used to extract the IMF flow, the AMD-Gemini results (Figure 7) were not filtered due to statistical limitations. The results of Figure 7 demonstrate that the differences in the IMF flow between systems have a strong sensitivity to the density dependence of the symmetry energy. The Gogny-AS interaction, or stiff $E_{\text{sym}}(\rho)$, clearly demonstrates the best agreement with the experimental data, showing a decreasing $\langle R_{\text{flow}} \rangle_{Z=4-9}$ value with increasing $b_{\text{red}}$. In comparison, the soft symmetry energy parameterization, or Gogny interaction, is unable to reproduce the experimental trend. In the Gogny calculation the $^{64}\text{Zn}$ flow increases relative to the $^{64}\text{Ni}$ flow, eventually becoming larger ($R_{\text{flow}} > 1$). This is related to the larger symmetry energy at low-density for the Gogny interaction, which is more repulsive for the more neutron-rich $^{64}\text{Ni}$ system relative to the $^{64}\text{Zn}$ system. The isospin-dependent part of the Gogny-AS interaction is less repulsive at low-density and therefore, the $^{64}\text{Ni}$ flow remains larger than the $^{64}\text{Zn}$ flow producing agreement with the experimental data. It is clear that the isospin-dependent part of the interaction is an important component in describing the observed transition from a mass to charge dependence of the IMF transverse flow.

6. Conclusions
The transverse flow of the IMFs has been shown to be sensitive to both the mass and charge of the colliding system. The strong dependence of the IMF flow on the charge of the system in the peripheral collisions provides new insight into the mechanisms responsible for the transverse flow. The AMD-Gemini simulation demonstrated that the differences in the IMF flow between the systems was sensitive to the isospin-dependent part of the nucleon-nucleon interaction. Comparison between the experimental and AMD-Gemini provided strong evidence supporting a stiff $E_{\text{sym}}(\rho)$. Future research examining the IMF flow should allow for additional constraints on the nuclear Equation of State.

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