On energy estimation of high energy muon events in KM3 detectors based on a more exact range fluctuations of high energy muons

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Abstract. The analyses of muon events above 1 TeV in high energy neutrino astrophysics are under progress in IceCube, ANTARES and Baikal-GVD. In these analyses, the verification of validity of the utilized procedures is particularly important. In present report, an analysis procedure for the KM3 detectors is proposed. The procedures thus utilized are composed of two parts, namely, (1) the exact treatment towards the parent muons and (2) the exact treatment of Cherenkov light due to both the parent muon and their dissipated energies. As for the former, we check the validity, based on the investigation of range fluctuations of high energy muon, and, for the latter, we check the verification on three dimensional cascade showers which are the origin of the main part of the Cherenkov light. Every process due to the muon and accompanying cascade shower are exactly investigated in stochastic manner. Thus, by using the established procedure, we analyse Fully Contained Event in the KM3 detector exclusively. The comparisons are made of the relation between the generated Cherenkov Yields and the observed ones.

1. Introduction
In the analyses of the neutrino events above 1 TeV in high energy neutrino astrophysics, such as IceCube, ANTARES, Baikal-GVD, frequencies for the muon neutrino events is far higher than those of electron neutrino events due to their big difference in effective lengths in high energy neutrino events.

Therefore, the reliable energy estimation of the muon neutrino events has become particularly important among those of high energy neutrino astrophysics. Originally, neutrino events in higher energies are rare events. From the energy estimation of such rare events, we are forced to construct the physical images due neutrinos. This is the reason why we should pursue the reliable energy estimation as exactly as possible.

2. Calculation procedures
This paper is a natural extension of the preceding paper [1]. The higher energy neutrino events are generated mainly due to the deep inelastic scattering of the neutrinos, such as

\[ \nu_\mu + N \rightarrow \mu + X \] (1)
For muon neutrino interactions, the energies of hadronic parts (hadronic showers) are smaller than those of the generated muons and the traversed lengths of muons are far longer than those of the hadronic parts. The muons thus produced traverse in water (ice) 1 kilometer or more, dissipating their energies due to either direct pair production, or bremsstrahlung or nuclear interaction. These processes are in nature of stochastic characters. The behaviors of these muons are investigated under the concept of the range fluctuations of the muons.

In previous paper [1], we treat the range fluctuations of muons as exactly as possible and examine the application limit of Stanev and Lipari theory [2] and similar theories [3] which are the basement of the muon analysis in the present high energy astrophysics, such as IceCube, Antares and Baikal-GVD.

2.1. The validities of the Monte Carlo methods concerned

Nowadays, the range fluctuations of high energy muons are investigated exclusively by Monte Carlo method. However, it is essentially difficult to verify the validity of the Monte Carlo method concerned due to inherent nature involved in the method.

As far as the validity of our Monte Carlo method on the range fluctuations of muons is concerned, its validity is confirmed by the comparison of our method with the analytical theory, methodologically independent one of the Monte Carlo procedure, and the agreement between them is confirmed [1, 4].

In the present analysis of the muon events, their dissipated energies due to stochastic characters, namely, due to direct pair production, bremsstrahlung and nuclear interaction are treated as ensemble of three-dimensional electromagnetic cascade showers - hereafter, simply, cascade shower - whose incident particles are different in different interactions. Also, these cascade showers are treated by an exact Monte Carlo method.

The dissipated energies due to respective interactions are converted into the cascade showers. The validity of our Monte Carlo method in the calculation of three-dimensional cascade showers is verified by the comparison of the mean square lateral and angular spread of shower particles obtained by our Monte Carlo method with corresponding quantities obtained by the analytical theory [5] based on Nishimura-Kamata theory [6], the methodologically independent of the Monte Carlo method. We confirm the agreement between them. The details of the certification of the validities will be described in detail in a full paper.

2.2. The Cherenkov lights due to the segments of electrons in the cascade showers

The shower particles in a cascade shower are pursued exactly in stochastic manner including the ionization loss three-dimensionally up to 1 MeV under Approximation B in the traditional cascade shower theory [6, 7]. The cascade showers due to the dissipated energies from the parent muons develop three-dimensionally, taking into account of the multiple scattering of shower particles along the direction of the muons whose multiple scatterings are neglected.

![Figure 1. The geometrical relation between cosine direction of dt, an electron segment and those of the Cherenkov lights.](image)

The Cherenkov lights due to $dt$, a segment in shower electrons are emitted with the direction cosine $(l' \ m' \ n')$ for the $dt$, whose direction cosine $(l \ m \ n)$ is shown in figure 1.

Here, $dt$ denotes an electron segment in a cascade shower with the direct cosine $(l \ m \ n)$ at the depth $t$ in a three-dimensional Monte Carlo electromagnetic cascade shower. The Cherenkov lights
due to the $dt$ radiate the direction with $\theta$ the emitted angle of the Cherenkov light, and $\varphi$, azimuthal angle, with the cosine direction ($l', m', n'$) as shown in figure 1.

Furthermore, the relation between two kinds of the direct cosine is given as

$$
(l' \quad m' \quad n') = \begin{pmatrix}
\ln(l'^2 + m'^2)^{-1/2} & -m(l'^2 + m'^2)^{-1/2} & l \\
mln(l'^2 + m'^2)^{-1/2} & l(l'^2 + m'^2)^{-1/2} & m \\
-(l'^2 + m'^2)^{1/2} & 0 & n
\end{pmatrix}
\begin{pmatrix}
\sin\theta \cos\varphi \\
\sin\theta \sin\varphi \\
\cos\theta
\end{pmatrix}
$$

(2)

The Cherenkov lights radiated from $dt$, an electron segment, which are absorbed at the most adjacent detector. Let us the absorption coefficient of the Cherenkov light be $\lambda$, then the Cherenkov light absorbed at the detector is given as

$$N = N_0 \times \exp\left(-\frac{t}{\lambda}\right)$$

(3)

where $N_0$ denotes the generated Cherenkov lights due to the $dt$ and $N$ denotes the observed Cherenkov light at the detector and $t$ denotes the distance from the $dt$ to the place for detection for Cherenkov lights. Here, $\lambda$ is taken as 40 meter.

In our calculation, we examine two cases of the total Cherenkov lights, namely the generated Cherenkov lights and the observed ones.

Related to the mutual relations between [generated points] and [terminated points] among the muon events, these events are classified in the four types, namely, [Fully Contained Event], [Partially Contained Events], [Stopping Event] and [Passing through Events].

In present paper, we examine Fully Contained Events only.

3. The analyses of Fully Contained Events

3.1. Do the Fully Contained Events guarantee the reliable energy estimations?

It is needless to say that only Fully Contained Events may give the most reliable energy estimations of the muon events concerned compared with other three types of events, from the point of view of energy conservation.

In present calculations, we could examine the Fully Contained Events whose muon energies are below 1 TeV, while in higher energies above ~1 TeV, we could hardly observe Fully Contained Events.

Here, let us examine characteristics of the three Fully Contained Events.

**Table 1.** The generated Cherenkov lights for three Fully Contained Events. B, N, D, B+N+D denote the kinds of interactions, namely, bremsstrahlung, nuclear interaction, direct production, and the total number of interactions, respectively. M (muon), E (shower), M+E denote generated Cherenkov lights due muon, electron shower, muon + electron shower (generated total Cherenkov lights), respectively.

| Traversed depth | Number of Interactions | Generated Cherenkov |
|----------------|------------------------|---------------------|
|                | B  | N  | D  | B+N+D | M: muon | E: shower | M+E       |
| #FC-4        | 491.8 | 1 | 0 | 46 | 47 | 1.23×10^7 | 7.27×10^7 | 8.50×10^7 |
| #FC-390      | 603.8 | 1 | 3 | 34 | 38 | 1.51×10^7 | 6.63×10^7 | 8.14×10^7 |
| #FC-401      | 975.6 | 3 | 1 | 64 | 68 | 2.44×10^7 | 6.10×10^7 | 8.53×10^7 |

In table 1, we summarize the characteristics of three Fully Contained Events. In spite of the big difference of the traversed lengths (491.8m, 603.8m and 975.6m) among three events, it should be noticed that totally generated Cherenkov lights is nearly the same, as they must be, because total
Cherenkov lights exactly correspond to the primary energies. Total Cherenkov light generated exactly corresponds to the total track lengths for electrons segments in the cascade showers and the parent muon. However, in real measurements, it should be noticed that the generated Cherenkov lights are never observed, because the some part of generated Cherenkov lights are lost due to their absorption (see equation (3)).

In table 2, we give the ratios of the observed Cherenkov lights to the generated Cherenkov ones in the case of $\lambda = 40$ m for the different unit for the detection of Cherenkov lights for the event [FC-4].

Here, for examples, [60 c.u.] denotes that each measurement is made in the unit of 60 c.u. over the distance from the beginning to the end. Namely, we measure Cherenkov light at 60 c.u., 120 c.u., 240 c.u. and so on, until the muon events concerned terminate and the ratio of the observed Cherenkov lights to corresponding generated ones are obtained at each measurement point. The ratio may be a good measure for estimation of the reliability on energies of the muon events concerned. It is easily understood from the table that reliability of the energy estimation decreases as the distance for the measurements increase even for Fully Contained Events.

| Observed Yield | Total       | $\Sigma E_{\text{shower}}$ | $\Sigma \mu$ |
|----------------|-------------|-----------------------------|--------------|
| 60 c.u.        | $6.92 \times 10^7$ | $6.05 \times 10^7$          | $8.69 \times 10^6$ |
| Ratio          | $8.15 \times 10^{-1}$ | $8.33 \times 10^{-1}$      | $7.07 \times 10^{-1}$ |
| 120 c.u.       | $6.68 \times 10^7$ | $6.03 \times 10^7$          | $6.44 \times 10^6$ |
| Ratio          | $7.86 \times 10^{-1}$ | $8.30 \times 10^{-1}$      | $5.24 \times 10^{-1}$ |
| 240 c.u.       | $1.74 \times 10^7$ | $1.35 \times 10^7$          | $3.85 \times 10^6$ |
| Ratio          | $2.04 \times 10^{-1}$ | $1.48 \times 10^{-1}$      | $3.13 \times 10^{-1}$ |
| 480 c.u.       | $1.51 \times 10^7$ | $1.33 \times 10^7$          | $1.83 \times 10^6$ |
| Ratio          | $1.78 \times 10^{-1}$ | $1.83 \times 10^{-1}$      | $1.49 \times 10^{-1}$ |

### 3.2. The behaviours of a Fully Contained Event: #FC-4: Longitudinal development for Cherenkov Lights

Here, we show several different graphs for different physical quantities for one Fully Contained Event (#FC-4) to grasp the physical image of the event concerned as a whole. The characteristics of this event are as follows: The total traversed length is 491.8 meter, total number of interactions ever experienced is 47. The details are 46 direct pairs, 1 bremsstrahlung and 0 nuclear interaction. The biggest energy loss by the muon is due to the bremsstrahlung whose energy is $8.41 \times 10^{11}$ eV and whose generation point is $2.94 \times 10^4$ g/cm$^2$ from the starting point of the incident muon.

In figure 2, we give two cascade curves for Cherenkov light yields as the function of the depth. One is the generated Cherenkov light (dotted line) and the other is the observed one (continuous line) in the unit of 60 c.u. It is clear from both curves that the both peaks from both curves correspond to the catastrophic energy loss due to bremsstrahlung mentioned above.

### 3.3. The Lateral distribution for Cherenkov light yields from both the muon and cascade shower particles integrated by the depth

In figure 3, the lateral distribution for the generated Cherenkov light yields integrated by the depth is given. The contribution from both cascade shower electrons and the incident muon are shown in the figure. It is clear from the figure that the contribution from the incident muon is far small compared with those from electrons, which is also shown in table 1. It is estimated from figure 3 that the average spread of the 1 TeV muon event (#FC-4) is of 100 c.u. (36 meters). The spreads of the muon events come mainly from cascade shower particles due to their multiple scattering and they decrease as their primary energies increase.
Figure 2. Transition curves for Cherenkov light in the cases of both generated Cherenkov lights and observed ones (λ = 40 m).

Figure 3. The lateral distribution for generated Cherenkov Lights integrated by depth.

3.4. The cosθ distribution for both the muon and cascade shower particles integrated by depth

In figure 4, we give the cosθ distribution for total Cherenkov lights with regard to the direction of the parent muon, for the parent muon and electrons from cascade showers in the case of generated Cherenkov light. In our calculation, we adopt the cosθ of the muon is of cosθ=0.752 (=1/n, n=1.4, refraction index in water). If we assume the Cherenkov lights due to the dissipated energies from the incident muon are generated along the same direction as the Cherenkov lights by the muon, then we should expect the delta function type distribution for around cosθ=0.752. However, the real distribution for figure is quite different from the delta function type distribution. This is due to the multiple scattering of cascade electrons as shown in figure 1. It is also noticed that the contributions from the incident muon are far smaller than those from cascade electrons.

Figure 4. cosθ distribution of the generated Cherenkov lights for the parent muon and cascade shower electrons integrated by depth.

Figure 5. A part of the arrival time distribution of the generated Cherenkov light. The Cherenkov light yields due to bremsstrahlung are clearly registered. The time is measured from the generation points for the parent muon. The time resolution is one nanosecond.
3.5. The arrival time distribution for the Cherenkov light yields from the both the muon and cascade shower particles integrated by the depth

Finally, we give the arrival time distribution for the total Cherenkov light whose time resolution is 1 nanosecond. The peak at around 1020 nanosecond corresponds to the peak of the Cherenkov light in figure 2. It is clear from figure 2 and figure 5 that we could find easily the phenomena whose dissipated energies are so high. The higher dissipated energy comes mainly from the catastrophic energy loss due to bremsstrahlung.

4. The range distribution for muons with different primary energies

In figure 6, we show the range distributions for muons whose primary energies cover from 1 TeV to 1 PeV. Sampling number for each primary muon is 1000. It is clear from the figure that fluctuations with regard to the depth increase rapidly as primary energies increase. It is easily concluded from the figure the following. Namely, it is almost impossible for us to find Fully Contained Events whose primary energies exceed 10 TeV within the interior of the KM3 detector. Even if one is interested in the muon events with ~1 TeV, it should be noticed that the majority is occupied by Partially Contained Events. Only several percent of muon events with 1 TeV are classified as Fully Contained Events.

![Figure 6. Range distributions for the incident muons with incident energies $10^{12}$ eV to $10^{15}$ eV. The minimum observation energies are taken as $10^9$ eV. Each sampling number is 1000.](image)

5. Conclusion

For the moment, we restrict our theme to the Fully Contained Events with 1 TeV only. In the present paper, we have clarified that it is not so easy to estimate energy of the muon events even for Fully Contained Events with 1 TeV. In the next paper, we analyse the Partially Contained Events whose primary energies exceed 10 TeV from the view point of reliable energy estimation.

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