Stokes efficiency of temporally rocked ratchets

Raishma Krishnan, Jim Chacko, Mamata Sahoo and A M Jayannavar

Institute of Physics, Sachivalaya Marg, Bhubaneswar 751005, India
E-mail: raishmakrishnan2000@yahoo.co.in, chacko@iopb.res.in, mamatas@iopb.res.in and jayan@iopb.res.in

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Abstract. We study the generalized efficiency of an adiabatically rocked ratchet with both spatial and temporal asymmetry. We obtain an analytical expression for the generalized efficiency in the deterministic case. Generalized efficiency of the order of 50% is obtained by fine tuning of the parameter range. This is unlike the case of thermodynamic efficiency, where we could readily get an enhanced efficiency of up to 90%. The observed higher values of generalized efficiency is attributed to be due to the suppression of backward current. We have also discussed briefly the differences between thermodynamic, rectification or generalized efficiency and Stokes efficiency. The temperature is found to optimize the generalized efficiency over a wide range of parameter space, unlike in the case of thermodynamic efficiency.

Keywords: Brownian motion, stochastic processes (theory), molecular motors (theory)
1. Introduction

Nonequilibrium fluctuations can induce directed transport along periodic extended structures without the application of a net external bias. Diverse studies exist in the literature which centre on this phenomena of noise-induced transport [1]–[4]. The extraction of useful work by the rectification of thermal fluctuations inherent in the medium at the expense of an overall increase in the entropy of the system plus the environment [5]–[7] has become a major area of research in nonequilibrium statistical mechanics. The key criterion for the possibility of such transport is the presence of unbiased nonequilibrium perturbations and a broken spatial or temporal symmetry. With the increase in prominence of the study of nano-size particles, the concurrent thermal agitations can no longer be ignored. The perceptivity of the basic mechanism of ratchet operation has been disclosed through various models such as flashing ratchets, rocking ratchets, time-asymmetric ratchets, frictional ratchets, etc [1]–[4].

Extensive studies have been performed to understand the nature of currents, their possible reversals and also the efficiency of energy transduction. These results are of immense use in the development of appropriate models that efficiently separate particles of micro- and nano-sizes and also, in turn, in the development of machines at nanoscales [8]. Processes in which the chemical energy stored in a nonequilibrium bath is transformed into useful work are believed to be the basis of molecular motors and are of great importance in active biological processes.

With the development of a separate subfield called stochastic energetics [5, 9], the reaction force exerted by the stochastic system on the bath is identified with the heat discarded by the system to the bath. With this definition, it has become possible to establish the compatibility between the Langevin or Fokker–Planck formalism with the laws of thermodynamics. This framework helps us to calculate various physical quantities such as the efficiency of energy transduction [10], energy dissipation (hysteresis loss), entropy production [11] etc, thereby rendering a new tool to study systems far from equilibrium.

In the present work we consider a time-asymmetric ratchet [12]–[14] where the ratchet potential is rocked adiabatically in time in such a way that a large force field $F(t)$ acts for a short time interval in the forward direction compared to a smaller force field for
a longer time interval in the opposite direction. The intervals are chosen so that the net external force or bias acting on the particle over a time period is zero. With such time-asymmetric forcing, one can generate enhanced unidirectional currents even in the presence of a spatially symmetric periodic potential [13].

The schematic figure of the ratchet potential, $V_0(q)$, chosen for our present work and the time-asymmetric forcing, $F(t)$, are shown in figure 1. Time-asymmetric forcing can also be generated by applying a biharmonic force or harmonic mixing [15]. Theoretically, time-asymmetric ratchets have been considered in earlier literature under different physical contexts [16,17]. Several experimental studies have also been explored, such as the generation of photo-currents in semiconductors [18], transport in binary mixtures [16], and the realization of Brownian motors using cold atoms in a dissipative optical lattice [19] etc.

One of the key concepts in the study of the performance characteristics of Brownian engines/ratchets is the notion of the efficiency of energy transduction from the fluctuations [20]. The primary need for efficient motors arises either to decrease the energy consumption rate and/or to decrease the heat dissipation in the process of operation, and it is the latter concept which is of more importance in the present world of miniaturization of components [5]. As the ratchet operates in a nonequilibrium state, there is always an unavoidable and irreversible transfer of heat via fluctuations (in coordinate and accompanying velocity) thereby making it less efficient as a motor. Any irreversibility or finite entropy production will reduce the efficiency. For instance, the attained value of thermodynamic efficiency in flashing and rocking ratchets are below the sub-percentage regime ($<0.01$). However, it has been shown that, at very low temperatures, fine tuning of parameters could easily lead to a larger efficiency, the regime of parameters being very narrow [21]. Protocols to optimize the efficiency in a saw-tooth ratchet potential in the

Figure 1. Plot of saw-tooth potential as a function of coordinate $q$ and the time-asymmetric forcing, $F(t)$, as a function of $t$. 
presence of spatial symmetry and symmetric temporal rocking have been worked out in detail in [21, 22].

By construction of a special type of flashing ratchet with two asymmetric double-well periodic-potential states displaced by half a period [23], a high efficiency of an order of magnitude higher than in earlier models [5, 9, 10, 24] was obtained. The basic essence here was that, even for diffusive Brownian motion, the choice of an appropriate potential profile ensures the suppression of backward motion, leading to a reduction in the accompanying dissipation. Similarly to the case of flashing ratchets [23], we had earlier studied the motion of a particle in a rocking ratchet by applying a temporally asymmetric but unbiased periodic forcing in the presence of a sinusoidal [12] and saw-tooth potential [25]. The efficiency obtained was very high, far above the sub-percentage level, about ∼30–40% without fine tuning for the sinusoidal case and ∼90% for the saw-tooth case in the presence of temporal asymmetry.

It is to be pointed that, in all ratchet models, the particles move in a periodic potential system and hence it ends up with the same potential energy, even after crossing over to the adjacent potential minimum. There is no extra energy stored in the particle that can be usefully expended when needed. Hence, to have an engine out of a ratchet, it is necessary to use its systematic motion to store potential energy, which in turn is achieved if a ratchet lifts a load [5, 26]. Thus a load force $L$ is applied in a direction opposite to the direction of current in the ratchet. With this definition, the thermodynamic efficiency assumes a zero value when no load force is acting [5, 27].

However, as not all motors are designed to pull loads, alternate proposals for efficiency have arisen, depending on the task that the motor has been proposed to perform, without taking recourse to the application of a load force. Some motors may have to achieve high velocity against a frictional drag. This consequently implies that the objective of the motor considered is to move a certain distance in a given time interval with minimal fluctuations in its velocity and its position. In such a case, one defines the generalized efficiency [28] or rectification efficiency [29] which, in the absence of load, is sometimes called Stokes efficiency [30], given by the expression

$$\eta_s = \frac{E_{\text{min}}}{E_{\text{in}}},$$

(Eq. 1)

$E_{\text{min}} = \gamma \langle v \rangle^2$ is the minimum average power necessary to maintain the motion of the motor with an average velocity $\langle v \rangle$ against an opposing frictional force. $E_{\text{in}}$ is the average input power. In the presence of load, the generalized efficiency or rectification efficiency is defined as [28, 29, 31]

$$\eta_{g-r} = \frac{L \langle v \rangle + \gamma \langle v \rangle^2}{E_{\text{in}}}.$$  

(Eq. 2)

The numerator in the above equation is the sum of the average power necessary to move against the external load and against the frictional drag with velocity $\langle v \rangle$, respectively. The thermodynamic efficiency of energy transduction is given by

$$\eta_t = \frac{L \langle v \rangle}{E_{\text{in}}}.$$  

(Eq. 3)

This definition can be used for the overdamped [29] as well as the underdamped case [27]. For the case of an underdamped Brownian motor, there is an added advantage that the
input power can be written in terms of experimentally observable quantities, namely $\langle v \rangle$ and its fluctuations [27,29]. This is independent of the model of the ratchet chosen.

In the present work, we mainly analyse the nature of generalized efficiency in the absence of load, namely Stokes efficiency. The behaviour in the presence of a load is also briefly discussed. We obtain values of Stokes efficiency of the order of 50% by fine tuning the parameters. In the generic parameter space, we obtain efficiencies far above the sub-percentage regime. The earlier model for the case of a flashing ratchet [29] gave a generalized efficiency of the order of 0.2. In a recent study [32], it has been shown that Stokes efficiency exhibits a high value of around 0.75, when the motor operates in an inertial regime and at very low temperatures. However, these inertial motors do not exhibit high thermodynamic efficiency. We also show that, unlike thermodynamic efficiency, the generalized efficiency is aided or optimized by temperature.

Our paper is organized as follows. We first describe our model in section 2. Results and discussions are given in section 3, which is followed by the conclusions in section 4.

2. The model

Our model consists of an overdamped Brownian particle with co-ordinate $q(t)$ in a spatially asymmetric potential $V(q)$ subjected to a temporally asymmetric rocking. The stochastic differential equation or the Langevin equation for such a particle is given by [33]

$$\dot{q} = -\frac{(V'(q) - F(t))}{\gamma} + \xi(t),$$

(4)

with $\xi(t)$ being the randomly fluctuating Gaussian thermal noise having zero mean and correlation, and $\langle \xi(t)\xi(t') \rangle = (2k_B T/\gamma)\delta(t - t')$ with $\gamma$ being the friction coefficient. In the present work, we consider a piecewise linear ratchet potential, as in the case of Magnasco [34] with periodicity $\lambda = \lambda_1 + \lambda_2$ set equal to unity (figure 1). This also corresponds to the spacing between the wells. Later on, we scale all the lengths with respect to $\lambda$:

$$V(q) = \begin{cases} 
\frac{Q}{\lambda_1} q, & q \leq \lambda_1 \\
\frac{Q}{\lambda_2} (1-q), & \lambda_1 < q \leq \lambda 
\end{cases}.$$

(5)

$F(t)$, which is the externally applied time-asymmetric force with zero average over the period, is also shown in figure 1. The forces in the gentler and steeper side of the potential are, respectively, $f^+ = -Q/\lambda_1$ and $f^- = Q/\lambda_2$, and $Q$ is the height of the potential.

We are interested in the adiabatic rocking regime where the forcing $F(t)$ is assumed to change very slowly, i.e. its frequency is smaller than any other frequency related to the relaxation rate in the problem, such that the system is in a steady state at each instant of time.

Following Stratonovich’s interpretation [35], the corresponding Fokker–Planck equation [36] is given by

$$\frac{\partial P(q,t)}{\partial t} = \frac{\partial}{\partial q} \left[ k_B T \frac{\partial P(q,t)}{\partial q} + [V'(q) - F(t) + L] P(q,t) \right].$$

(6)
The probability current density $j$ for the case of constant force (or static tilt) $F$ is given by

$$j(F_0) = \frac{P_2^2 \sinh\{\lambda[F_0 - L]/2k_B T\}}{k_B T(\lambda/Q)^2 P_3 - (\lambda/Q) P_1 P_2 \sinh\{\lambda[F_0 - L]/2k_B T\}}$$

(7)

where

$$P_1 = \Delta + \frac{\lambda^2 - \Delta^2 F_0 - L}{4Q}$$

(8)

$$P_2 = \left(1 - \frac{\Delta[F_0 - L]}{2Q}\right)^2 - \left(\frac{\lambda[F_0 - L]}{2Q}\right)^2$$

(9)

$$P_3 = \cosh\{(Q - \Delta/[F_0 - L])/k_B T\} - \cosh\{\lambda[F_0 - L]/2k_B T\}$$

(10)

where $\Delta = \lambda_1 - \lambda_2$ is the spatial asymmetry factor. In the above expression we have also included the presence of an external load $L$, which is essential for defining thermodynamic efficiency. The current in the stationary adiabatic regime averaged over the period $\tau$ of the driving force $F(t)$ is given by

$$\langle j \rangle = \frac{1}{\tau} \int_0^\tau j(F(t)) \, dt.$$ 

(11)

The form of the time-asymmetric ratchets with a zero mean periodic driving force that we have chosen \[12\]–\[14\] is given by

$$F(t) = \begin{cases} 
1 + \frac{\epsilon}{1 - \epsilon} F, & (n\tau \leq t < n\tau + \frac{1}{2}\tau(1 - \epsilon)), \\
-\frac{1}{1 - \epsilon} F, & (n\tau + \frac{1}{2}\tau(1 - \epsilon) < t \leq (n + 1)\tau).
\end{cases}$$

(12)

Here, the parameter $\epsilon$ signifies the temporal asymmetry in the periodic forcing, $\tau$ is the period of the driving force $F(t)$, and $n = 0, 1, 2, \ldots$ is an integer. For this forcing, in the adiabatic limit the expression for the time-averaged current is \[10,13\]

$$\langle j \rangle = j^+ + j^-,$$

(13)

with

$$j^+ = \frac{1}{2}(1 - \epsilon) j \left(\frac{1 + \epsilon}{1 - \epsilon} F\right),$$

$$j^- = \frac{1}{2}(1 + \epsilon) j(-F),$$

(14)

where $j^+$ is the current fraction in the positive direction over a fraction of time period $(1 - \epsilon)/2$ of $\tau$ when the external driving force field is $((1 + \epsilon)/(1 - \epsilon))F$ and $j^-$ is the current fraction over the time period $(1 + \epsilon)/2$ of $\tau$ when the external driving force field is $-F$. The input energy $E_{in}$ per unit time is given by \[10,12\]

$$E_{in} = F \left[\left(\frac{1 + \epsilon}{1 - \epsilon}\right) j^+ - j^-\right].$$

(15)

In order for the system to do useful work, a load force $L$ is applied in a direction opposite to the direction of current in the ratchet. The overall potential is then

$$V(x) = -[V_0(x) - xL].$$

As long as the load is less than the stopping force $L_s$, current...
flows against the load and the ratchet does work. Beyond the stopping force, the current flows in the same direction as the load and hence no useful work is done. Thus, in the operating range of the load, \(0 < L < L_s\), the Brownian particles move in the direction opposite to the load and the ratchet does work \[26\]. The average rate of work done over a period is given by \[10\]

\[
E_{\text{out}} = L[j^+ + j^-].
\]  

(16)

The thermodynamic efficiency of energy transduction is \[5,9\]

\[
\eta_t = \frac{L[j^+ + j^-]}{F\left[\left((1 + \epsilon)/(1 - \epsilon)\right)j^+ - j^-\right]}.
\]  

(17)

At very low temperatures or in the deterministic limit and also in the absence of applied load, the barriers in the forward direction disappear when \(\left((1 + \epsilon)/(1 - \epsilon)\right)F > Q/\lambda_1\) or \(F > Q(1 - \epsilon)/\lambda_1(1 + \epsilon)\), and a finite current starts to flow in the forward direction. When \(F > Q/\lambda_2\), the barriers in the backward direction also disappear and hence we now have a current in the backward direction as well leading to a decrease in the average current. In between the above two values of \(F\), the current increases monotonically and peaks around \(Q/\lambda_2\). In this range, a high efficiency is expected \[11,12,21,25\]. In the limit when there is only forward current in the ratchet, i.e. \(j^+ \gg j^-\) and \(L = 0\), generalized efficiency reduces to Stokes efficiency and is given by

\[
\eta_S = \frac{(1 - \epsilon)j^+}{(1 + \epsilon)F}.
\]  

(18)

In the present work, we mainly focus on the case when the load \(L = 0\).

For the case of adiabatic rocking, the ratchet can be considered as a rectifier \[21\], and in the deterministic limit of operation and with zero applied load when \(F\) is in the range \(Q/\lambda_2 > F > Q(1 - \epsilon)/\lambda_1(1 + \epsilon)\), finite forward current alone exists and the analytic expression for current is given by

\[
j^+ = \frac{1}{2}\left[\frac{\lambda_1^2}{(1 + \epsilon)F\lambda_1 - Q(1 - \epsilon)} + \frac{\lambda_2^2}{(1 + \epsilon)F\lambda_2 + Q(1 - \epsilon)}\right]^{-1}.
\]  

(19)

Thus, equations (18) and (19) give an analytical expression for the Stokes efficiency in the deterministic limit. We take all physical quantities in dimensionless units. The energies are scaled with respect to the height of the ratchet potential, \(Q\); all lengths are scaled with respect to the period of the potential, \(\lambda\), which is taken to be unity, and we also set \(\gamma = 1\). In the following section, we present our results followed by discussions \[12,25,33\].

3. Results and discussions

In figures 2 and 3, we plot the generalized efficiency in the absence of load or Stokes efficiency as a function of \(F\) for different values of \(\epsilon\) at \(T = 0.01\) for symmetric potential \(\Delta = 0.0\) and asymmetric potential \(\Delta = 0.9\), respectively. As we increase \(F\) in the interval from zero to \(F_{\text{min}} = Q(1 - \epsilon)/\lambda_1(1 + \epsilon)\), the current is almost zero, since barriers to motion exist in both forward (right) and backward (left) directions. This critical value of \(F\) will decrease as we increase \(\epsilon\), as seen from figures 2 and 3. For \(F > F_{\text{min}}\), barriers to the
right disappear and, as a consequence, the current (inset) increases as a function of $F$ till $F_{\text{max}} = Q/\lambda_2$, beyond which the current also starts flowing in the backward direction. The behaviour of Stokes efficiency reflects the nature of current (cf equation (18)). Note that the value of $F_{\text{max}}$ does not depend on the time asymmetry parameter $\epsilon$, as is clear from figures 2 and 3. Beyond $F_{\text{max}}$, barriers to motion in both directions disappear and current, as well as generalized efficiency, starts decreasing beyond $F_{\text{max}}$. We have seen that the input energy increases monotonically with $F$ for all the parameters. Hence the qualitative behaviour of current is reflected in the nature of generalized efficiency. From the plot, we see that the dependence of the generalized efficiency on $\epsilon$ is not in a chronological manner. A high $\epsilon$ value need not correspond to high generalized efficiency. For a given $\epsilon$, the current and Stokes efficiency exhibit a peak around $F_{\text{max}}$.

We see from equation (19) that, for $\lambda_1 \gg \lambda_2$ (i.e., large spatial asymmetry) and $F_{\text{min}} < F < F_{\text{max}}$, the analytical results from equation (19) for the forward current fraction are simply given by $j_+ = (1 + \epsilon)F/2\lambda_1$, while the Stokes efficiency becomes $\eta_S = (1 - \epsilon)/2\lambda_1$. It is obvious from figure 3 that, in this domain, $j_+$ is a linear function of $F$ (inset) while $\eta_S$ exhibits a plateau in this regime. This plateau regime is clearly observable for $\epsilon = 0.9$ and $\Delta = 0.9$, as in figure 3. For these parameters, the ranges between $F_{\text{min}}$ and $F_{\text{max}}$ is large and, moreover, $\lambda_1 \gg \lambda_2$. The value of $\eta_S$ at the plateau is 0.05, which is consistent with the analytical result. In contrast to the nature of $\eta_S$, we see that the average current, for a given $F$ and $\Delta$, always increases as $\epsilon$ is increased; see the insets of figures 2 and 3. However, we see from equation (18) that $\eta_S$ also depends on $\epsilon$ through the factor $(1 - \epsilon)/(1 + \epsilon)$, which is a decreasing function of $\epsilon$, and hence the existence of an optimal value of $\epsilon$ for $\eta_S$ is understandable. For a large spatial asymmetry, $\Delta = 0.9$, in the ratchet potential the magnitudes of the average currents are quite large, even for a given $\epsilon$, compared to the case when $\Delta$ is small.
From figure 2, we notice that the optimum value of generalized efficiency obtained is around 20%. This is the case of a symmetric potential driven by temporally asymmetric force. From figure 3, we notice that the inclusion of spatial asymmetry in the potential helps in enhancing the generalized efficiency, and we can obtain an optimal value of nearly 50% for efficiency in a particular parameter space.

In figure 4, we plot the generalized efficiency with zero load (or Stokes efficiency $\eta_S$) as a function of $\epsilon$ for different values of $F$ and symmetric potentials. We have taken $T = 0.01$ so as to be closer to the deterministic limit. The inset shows the same plot with asymmetric potential. We observe that, for a given value of $F$, only those $\epsilon$ values contribute to $\eta_S$ for which $F > F_{\text{min}}$. The minimum value of $\epsilon$ is given by $\epsilon_{\text{min}} = (Q - \lambda_1 F)/(Q + \lambda_1 F)$. For larger $F$, $\epsilon_{\text{min}}$ shifts to a smaller value, as can easily be seen from the figure. Moreover, from equation (18) we can see that, as $\epsilon$ approaches 1, the $\eta_S$ approaches zero (even though, strictly speaking, the $\epsilon \to 1$ limit is pathological). Thus, for the chosen parameter values, the $\eta_S$ exhibits a peaking behaviour. Note that the current vanishes due to the spatial symmetry of the potential in the limit $\epsilon \to 0$.

We now study the case when there is a spatial asymmetry, which is shown in the inset of figure 4. Here, a finite current can arise even when $\epsilon = 0$, provided that the force $F > Q/\lambda_1$. Thus $\eta_S$ in this regime can have a finite value at $\epsilon = 0$ and can show a peaking behaviour. For $F \gg Q/\lambda_1$, efficiency shows a monotonically decreasing behaviour as a function of $\epsilon$. This clearly brings out the fact that, in certain parameter ranges, time-asymmetric driving need not help in enhancing $\eta_S$ in the presence of a spatially asymmetric potential. In the range $F < Q/\lambda_1$, currents are zero at $\epsilon = 0$; thus $\eta_S$ exhibits a peaking behaviour with a value of zero for $\epsilon = 0$ and $\epsilon = 1$, in accordance with equation (18). These results show that $\eta_S$ is not a monotonically increasing function of $\epsilon$.

We next discuss the behaviour of $\eta_S$ with temperature. In figure 5, we plot $\eta_S$ as a function of temperature for a fixed $F = 0.1$ and temporal asymmetry $\epsilon = 0.9$ but with...
varying potential asymmetry $\Delta$. In most of the generic parameter space, we observe that temperature (or noise) facilitates $\eta_S$, which is quite opposite to the generically observed behaviour of thermodynamic efficiency [25]. For example, if we take a particular curve, say $\Delta = 0.0$, $F = 0.1$, $\epsilon = 0.9$, we can see that the value of efficiency is zero at $T = 0$. This is because of the presence of the barriers in either direction during rocking. Thus, when $F < F_{\text{min}}$, the efficiency (current) is zero and, as temperature is increased, current starts to build up, since Brownian particles can readily overcome the barriers to the right in the adiabatic limit. Beyond a certain $T$, the current or efficiency will start to subside again, as too much noise will help the particle to overcome the barriers in both directions, thereby reducing the ratchet effect. Hence, both current and generalized efficiency will fall. Thus, for $F < F_{\text{min}}$, temperature always facilitates $\eta_S$. With an increase in spatial asymmetry, we see a finite current even when the temperature is zero. This is because of the disappearance of the barriers to motion in the forward direction. However, the Stokes efficiency increases and shows a peaking behaviour as a function of temperature even in this range. In figure 5, note that the peak value of current (inset) shifts to the right as we increase $\Delta$. With an increase in $\Delta$, barriers to the left increase and hence, to overcome these larger barriers, a higher temperature is required. Only above these temperatures does current in the backward direction begin to flow, causing a decrease in the average current. Hence it is understandable that the peak in average current shifts to higher $T$ with an increase in $\Delta$.

When $F > F_{\text{max}}$, the barriers in both directions disappear. We have separately verified that, in this case, both $\eta_S$ and the net current decrease monotonically as a function of temperature.

In the end, we discuss briefly the differences between the nature of thermodynamic efficiency ($\eta_t$), generalized efficiency ($\eta_{g-r}$) and Stokes efficiency ($\eta_S$). For the sake of
comparison, we apply a load to the system. In figure 6, we plot the $\eta_t$, net current $\langle j \rangle$, and input ($E_{\text{in}}$) and output ($E_{\text{out}}$) powers as a function of load for $\epsilon = 0.9$, $\Delta = 0.9$, $F = 0.1$ and $T = 0.01$. In the inset, we plot $\eta_{g-r}$, $\eta_S$ and $\eta_t$ as a function of load for the same set of parameters, so as to have a comparative idea of the behaviour of the different definitions of efficiency.

We notice that the thermodynamic efficiency increases with load from zero and exhibits a high value (90%) just before the stopping force or critical load, the range within which $\eta_t$ is defined. In contrast, $\eta_S$ (shown in the inset) has a finite value, even when the load force is zero and then decreases monotonically with load. $\eta_S$ is almost zero, and so is the velocity and current in the range where $\eta_t$ is very high. The magnitudes of current, $\eta_S$, $E_{\text{in}}$ and $E_{\text{out}}$ are very small near the stopping force, and hence are not observable on figure 6 due to the scale used. Both $\eta_{g-r}$ or $\eta_S$ starts with a finite value when load $L = 0$, and it differs from $\eta_t$ in the low load limit. When the load value increases, $\eta_{g-r}$ also increases, and at larger values of $L$ (near stopping force) it coincides with $\eta_t$. The main contribution to $\eta_{g-r}$ comes essentially from the work done against the load, since the velocity of the particle is almost negligible and thus the average power needed to move against the frictional drag becomes very small.

Another observation is that the average work done exhibits a peaking behaviour where the thermodynamic efficiency is small, and it is vanishingly small where the latter peaks. The average input power and current decreases monotonically with the load. The figure clearly indicates that high thermodynamic efficiency does not lead to higher currents/work/Stokes efficiency. These results clearly bring out glaring differences between different definitions of efficiency, as they are based on physically different criteria of motor performance [28, 29, 32, 37].
4. Conclusions

We have studied the generalized efficiency in an adiabatically rocked system in the presence of spatial and temporal asymmetry. The Stokes efficiency exhibits a value of 50% by fine tuning the parameters. Moreover, in a wide range of parameters, this efficiency is much above the subpercentage regime. We have shown that, in a wider parameter space, temporal asymmetry may or may not facilitate the generalized efficiency whereas, generically, temperature facilitates it. In the regime of parameter space where the current is zero in the deterministic limit, temperature always facilitates Stokes efficiency. In contrast, if the current is nonzero in the deterministic regime, depending on the parameters, it may happen that Stokes efficiency decreases monotonically with temperature. The obtained high values for both the thermodynamic and generalized efficiency is attributed to the effect of suppression of current in the backward direction. Recently, it has also been shown that the same effect in these ratchet systems leads to enhanced coherency or reliability in transport [38]. In conclusion, in suitable parameter ranges, our system exhibits high values for all the performance characteristics, namely Stokes efficiency and thermodynamic efficiency, along with a pronounced transport coherency.

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