Simulation of the Influence of Internal Pressure on Free Vibrated Aluminum Conveying Fluid Pipe

Ali Abbas W Altai and Ghanim Sh Sadiq

Department of Mechanical Engineering, Engineering College, Al-Nahrain University, Baghdad, Iraq.

E-mail: ghanim.s.sadiq@ced.nahrainuniv.edu.iq

Abstract. High temperature and pressure conditions lead to expanding the pipelines used to transport different types of fluids. A compressive axial force arises if this expansion is prevented. The present work aims to analyze pipelines' instability due to internal pressure influence by making a simulation using ANSYS - Mechanical APDL 2019 R3. The study presents comparisons between increasing internal pressure from (0 to 10 MPa) on some pipe parameters like a natural frequency and instability for two pinned–pinned pipe thicknesses, which are 0.5 and 1 mm. It is concluded that the increase in internal pressure will decrease the natural frequency and stability of the vibrated system but will increase the modal damping ratio and total deformation of the tested pipe.

Keywords. Internal pressure, Simulation analysis by ANSYS, Pipeline.

1. Introduction

It is observed that a lot of previous researchers neglected the internal fluid pressure influence and axial tension in the pipe despite its importance in many industrial applications. Firstly, Fung et al. (1957) [1] was studied the vibration modes and frequency spectra of circular cylinders manufactured of thin-walls due to internal pressure. They demonstrated that the internal pressure for very thin wall cylinders has a considerable influence on natural vibration characteristics. Also, they showed that the fundamental frequency increases sharply as the internal pressure increase. Paidoussis and Issid (1974) [2] re-investigated the fluid conveying pipe dynamics with different boundary conditions. Also, they studied the internal pressure influences. They proved that even pipes with both ends supported are subject to buckling and coupled-mode flutter instabilities. Ilgamov et al. (1994) [3] supplied a model that physically based for the characterization of lateral internal forces in a vertical conveying fluid pipe, and they concluded that the internal pressure plays a serious rule in minimizing the critical value of the flutter velocity of the system for even a small value of the perturbation in the configuration of a straight beam. Olav Fyrileiv (2010) [4] showed that, for the long-span pipe, the natural frequency increase as the internal pressure increase (direct relationship) and concluded that pipe arc shows a significant effect on the natural frequency. Also, increasing the internal pressure from a short to moderate span reduces the natural frequency values. Khoruzhiy and Taranenko (2019) [5] analyzed the internal pressure influence and internal volume variations on the natural frequencies. The studied validity and the correctness of bending vibration determination for a straight pipe and the natural frequency of the fluid by comparing three types of computational methods. The ANSYS (Mechanical) package was utilized to calculate the bent pipe natural frequency.
based on the finite element method. The internal pressure effect and the fluid density was considered as well. Xiao et al. (2019) [6] showed the influence of some parameters like axial and bending loads on the pipelines internal pressure limit value, they carried out large numbers of (FE) calculations, and they demonstrated that in actually buried pipelines that happened during the examination of the pipe, the maximum existed value of the bending strain was (0.4%), so they take it as the reference moment. They concluded that both axial and bending moments could decrease the internal pressure limit value compared to that corresponding, which only carries internal pressure. As shown in the literature and according to the authors’ experience, there is a scarcity in the approaches based on the analytical solutions. Moreover, there is no comprehensive study for the behavior of pipe parameters under the influence of internal pressure, so that the authors will study the influence of the internal pressure on some pined – pined pipe parameters utilizing ANSYS - Mechanical APDL 2019 R3 simulation.

2. Theoretical discussion

2.1. The Mathematical model

The beam-like models for estimating the dynamical behavior are convincing for the most practical applications, as many researchers have shown in references [7,8, and 9], Figure (1).

![Figure 1. Beam model.](image)

In this assumption, the model derived by Paidoussis and Li (1993) [8] is used because it considers the influence of most of the pipe parameters. Also, it is acceptable for steady and unsteady fluid velocities. Starting with the study, two elements of length δx in the axis, as shown in Figure (2), the first one is the fluid element, and the second is the pipe element. Firstly, Supposing the deformation angle as θ (smaller value), and by neglecting the influences from high order trace, and based on appropriate equation 

\[\cos \theta = 1, \sin \theta = \theta = \frac{\partial y}{\partial x}\].

The force balance in the x and y directions for the fluid element of Figure (2) yields:

\[F \frac{\partial y}{\partial x} - A \frac{\partial P}{\partial x} - qS = 0\]  

\[F + A \frac{\partial}{\partial x} \left(P \frac{\partial y}{\partial x}\right) + qS \frac{\partial y}{\partial x} + m_f \left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial x}\right)^2 y = 0\]  

Where; \(m_f\): fluid mass, \(V\): fluid velocity, \(P\): internal pressure, \(A\): flow cross-section area, \(q\): shear stress developed on the internal surface of the pipe, \(S\): pipe inner section circumference, and \(F\): per unit length transverse force between fluid and wall of the pipe. Similarly, the pipe element is shown in Figure (2) force balance yields:

\[\frac{\partial T}{\partial x} + qS - F \frac{\partial y}{\partial x} = 0\]  

\[\frac{\partial Q}{\partial x} + F + \frac{\partial}{\partial x} \left(T \frac{\partial y}{\partial x}\right) + qS \frac{\partial y}{\partial x} - m_p \frac{\partial^2 y}{\partial t^2} = 0\]  

Where; \(m_p\): The mass of pipe, \(T\): longitudinal tension, \(Q\): transverse shear force in the pipe. The relation between the bending moment (M) and transverse shear force (Q) is as follows:

\[M = E I \frac{\partial^2 y}{\partial x^2}\] and \[Q = - \frac{\partial M}{\partial x}\]

So,
Now, if we collecting equations (2), (4), and (5), will get:

\[ EI \frac{\partial^{4}y}{\partial x^{4}} - \frac{\partial}{\partial x} \left[ (T - PA) \frac{\partial y}{\partial x} \right] + m_f \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right)^{2} y + m_p \frac{\partial^{2}y}{\partial t^{2}} = 0 \] (6)

Equation (6) takes the following form:

\[ EI \frac{\partial^{4}y}{\partial x^{4}} + (m_f V^{2} + P_l A) \frac{\partial^{2}y}{\partial x^{2}} + 2m_f V \frac{\partial y}{\partial x} + m_f \frac{\partial y}{\partial t} \frac{\partial y}{\partial x} + \left( m_f + m_p \right) \frac{\partial^{2}y}{\partial t^{2}} = 0 \] (7)

For simplicity of calculations, the dimensionless form is as below:

\[ \eta''' + \left( U^2 + \gamma \right) \eta'' + 2\beta U \eta' + \dot{U} \eta' + \ddot{\eta} = 0 \] (8)

Which is rearranged as below due to the fluid velocity must be assumed steady in analyzing vibrations, so that Equation (12) is reduced to the following form:

\[ \eta''' + \left( U^2 + \gamma \right) \eta'' + 2\beta U \eta' + \ddot{\eta} = 0 \] (9)

Where,

\[ \eta = \frac{y}{l} , \xi = \frac{x}{l} , V = \sqrt{\frac{m_f}{EI}} , \gamma = \frac{p_l A L^2}{EI} , \beta = \sqrt{\frac{m_f}{m_f + m_p}} \text{ and } \tau = \frac{t}{l^2} \sqrt{\frac{EI}{m_f + m_p}} \]

Equation (9) can be solved by trying the following trial solution [7]:

\[ \eta(\xi, \tau) = \sum_{j=1}^{4} \Omega e^{i\beta \xi} e^{i\beta \tau} \] (10)

Where,

\[ \Omega = \omega L^2 \sqrt{\frac{m_f + m_p}{EI}} \]

By substituting Equation (10) in Equation (9) will get the following polynomial equations for \(\lambda\)'s:

\[ \lambda^4 - \left( U^2 + \gamma \right) \lambda^2 - 2\beta U \Omega \lambda - \Omega^2 = 0 \] (11)

The fundamental forms of the roots of Equation (11) are given as in the below form because the four roots of such a polynomial should consist of two real and two complex conjugates [6]:

\[ \lambda_{1,2} = -1/2 \sqrt{\alpha} \pm i/2 \left( \frac{4\beta U \Omega / \sqrt{\alpha}}{\alpha - 2\kappa} \right)^{1/2} \]

\[ \lambda_{3,4} = 1/2 \sqrt{\alpha} \pm i/2 \left( \frac{4\beta U \Omega / \sqrt{\alpha}}{\alpha + 2\kappa} \right)^{1/2} \] (12)
Where;
\[ \kappa = U^2 + \gamma = 2\kappa/3 + 0.42 S_1/S_2 + 0.265 S_2, \]
\[ S_1 = \kappa^2 - 12 \Omega^2, \]
\[ S_2 = \left( S + \sqrt{S^2 - 4 S_1^2} \right)^{1/3}, \text{ and } S = 108 (\beta^2 U \Omega)^2 - 72 \kappa \Omega^2 - 2\kappa^3 \]  \hspace{1cm} (13)

So, rearranging Equation (12) and writing it in the complex forms as below:
\[ \lambda_{1,2} = -a \pm ib_1, \]
\[ \lambda_{3,4} = a \pm b_2 \]  \hspace{1cm} (14)

Where; \( a = 1/2 \sqrt{\alpha} \), \( b_1 = 1/2 \left[ (4\beta U \Omega/\sqrt{\alpha}) + \alpha - 2\kappa \right]^{1/2}, \)
\[ \text{and } b_2 = 1/2 \left[ (4\beta U \Omega/\sqrt{\alpha} - \alpha + 2\kappa \right]^{1/2} \]  \hspace{1cm} (15)

Consequently, the general solution for the vibration equation of conservative conveying fluid pipes can be concluded by inserting \( \lambda \)'s presented by Equation (14) into the solution presented by Equation (10) and making some algebraic and geometrical simplifications which can be written as below:
\[ \eta(\xi, \tau) = e^{i(\Omega \tau - a\xi)} [A \sinh b_1 \xi + B \cosh b_1 \xi] + e^{i(\Omega \tau + a\xi)} [D \sin b_2 \xi + E \cos b_2 \xi] \]  \hspace{1cm} (16)

Where; The constants A, B, D, and E are related to \( C_1, C_2, C_3, \) and \( C_4 \)

2.2. Natural frequencies

Subsequently, the natural frequency equations of the three types of conservative conveying fluid pipes will be estimated by applying the boundary conditions, which are as below:
\[ \eta(0, \tau) = 0, \eta''(0, \tau) = 0, \eta(1, \tau) = 0, \eta''(1, \tau) = 0 \]  \hspace{1cm} (17)

After application of the boundary conditions given in Equation (17) on the general solution given in Equation (16) gives to the following set of algebraic equations:
\[ B + D = 0, \]
\[ (b_1^2 - a^2)B - 2iab_1A - (b_2^2 - a^2)E + 2iab_2D = 0, \]
\[ e^{-ia}[A \sinh b_1 + B \cosh b_1] + e^{ia}[D \sin b_2 + E \cos b_2] = 0, \]
\[ e^{-ia}[(b_1^2 - a^2)A - 2iab_1B] \sinh b_1 + [(b_2^2 - a^2)B - 2iab_1A] \cosh b_1] + e^{ia}[-(b_2^2 + a^2)D - 2iab_2E] \sin b_2 + [-(b_2^2 + a^2)E + 2iab_2D] \cos b_2] = 0 \]  \hspace{1cm} (18)

By substituting the first of Equation (18) into the other equations yields three synchronous equations in terms of A, B, and E. For a non-trivial solution, the determinant of the coefficient of A, B, and C must equal to zero, so:
\[ \begin{vmatrix}
-2iab_1 & b_1^2 + b_2^2 & 2iab_2 \\
e^{-ia} \sinh b_1 & e^{-ia} \sinh b_1 & e^{ia} \sin b_2 \\
e^{-ia} (b_2^2 - a^2) \sinh b_1 - 2iab_1 \cosh b_1 & D_{32} & e^{ia} [- (b_2^2 + a^2) \sin b_2 + 2iab_2 \cos b_2]
\end{vmatrix} = 0 \]  \hspace{1cm} (19)

Where;
\[ D_{32} = -2iab_1 e^{-ia} \sinh b_1 + (b_1^2 - a^2) e^{-ia} \cosh b_1 + 2iab_2 e^{ia} \sin b_2 + (b_2^2 + a^2) e^{ia} \cos b_2 \]

The frequency equation for pinned-pinned pipes conveying fluid produces from the expansion of Equation (19) determinant. Despite Equation (19) produces a complex expression; nevertheless, the successive algebraic and geometrical simplifications result in real expression because the imaginary terms are canceled out.

Finally, the pinned-pinned pipe conveying fluid resulting frequency equation will be as the following form [10]:

\[ \text{...} \]
The pinned-pinned pipes conveying fluid, natural frequencies can have produced from solving Equation (20) together with Equation (15).

3. Numerical analysis by ansys
Using ANSYS - Mechanical APDL 2019 R3, the simulation can be utilized to compare with the theoretical and experimental results. The two-pipe models were presented as described in Table (1) below:

Table 1. The test samples descriptions.

| Sample No. | Material | D (mm) | t (mm) |
|------------|----------|--------|--------|
| 1          | Aluminum | 10     | 1      |
| 2          | Aluminum | 10     | 0.5    |

The below sections show the obtained results from ANSYS - MECHANICAL, which is free vibration, where the data are listed in Tables (2) and (3), and its shapes are shown in Figures (3) to (14). Tables (2) and (3) below show the obtained natural frequency, stability, and modal damping ratio data from ANSYS - MECHANICAL for two pipe samples at different internal pressure values.

Table 2. Data for free vibration of aluminum pipe (0.5 mm thickness) at different pressure values.

| Pressure (MPa) | Mode No. | Natural Frequency [Hz] | Stability [Hz] | Modal Damping Ratio |
|---------------|----------|------------------------|----------------|---------------------|
| 0             | 1        | 60.53                  | -0.60524       | 0.009999            |
|               | 2        | 166.55                 | -1.6654        | 0.009999            |
|               | 3        | 325.73                 | -3.257         | 0.009999            |
|               | 1        | 53.297                 | -1.4397        | 0.027003            |
| 2             | 2        | 157.07                 | -2.7092        | 0.017246            |
|               | 3        | 315.47                 | -4.3614        | 0.013824            |
|               | 1        | 49.0635                | -2.0368        | 0.042827            |
| 5             | 2        | 152.005                | -3.3118        | 0.021939            |
|               | 3        | 310.17                 | -4.9546        | 0.016009            |
|               | 1        | 44.83                  | -2.6339        | 0.058651            |
| 10            | 2        | 146.94                 | -3.9144        | 0.026631            |
|               | 3        | 304.87                 | -5.5478        | 0.018194            |

Table 3. Data for free vibration of aluminum pipe (1 mm thickness) at different pressure values.

| Pressure (MPa) | Mode No. | Natural Frequency [Hz] | Stability [Hz] | Modal Damping Ratio |
|---------------|----------|------------------------|----------------|---------------------|
| 0             | 1        | 57.596                 | -0.5759        | 1.00E-02            |
|               | 2        | 158.51                 | -1.585         | 1.00E-02            |
|               | 3        | 310.1                  | -3.1007        | 1.00E-02            |
|               | 1        | 54.529                 | -0.9123        | 0.016728            |
| 2             | 2        | 154.41                 | -2.0259        | 0.013119            |
|               | 3        | 305.62                 | -3.5755        | 0.011698            |
|               | 1        | 52.8965                | -1.1051        | 0.021019            |
| 5             | 2        | 152.3                  | -2.26025       | 0.014864            |
|               | 3        | 303.35                 | -3.8206        | 0.0126              |
|               | 1        | 51.264                 | -1.2979        | 0.02531             |
| 10            | 2        | 150.19                 | -2.4946        | 0.016608            |
|               | 3        | 301.08                 | -4.0657        | 0.013502            |
Figure 3. Pressure effect on natural frequency of (p-p) Aluminum pipe for two different pipe thickness.

Figure 4. Pressure effect on stability of (p-p) Aluminum pipe for two different pipe thickness.

Figure 5. Pressure effect on modal damping ratio of (p-p) Aluminum pipe for two different pipe thickness.

Figure 6. Pressure effect on logarithmic decrement of (p-p) Aluminum pipe for two different pipe thickness.
The figures below show the mid-span behavior of the pipe samples under free vibration at different internal pressure values.

**Figure 7.** Total deformation of free vibrated Aluminum pipe conveying water.

**Figure 8.** Total deformation of free vibrated Aluminum pipe conveying water.

**Figure 9.** Total deformation of free vibrated Aluminum pipe conveying water.

**Figure 10.** Total deformation of free vibrated Aluminum pipe conveying water.
Figure 11. Total deformation of free vibrated Aluminum pipe conveying water.

Figure 12. Total deformation of free vibrated Aluminum pipe conveying water.

Figure 13. Total deformation of free vibrated Aluminum pipe conveying water.

Figure 14. Total deformation of free vibrated Aluminum pipe conveying water.
4. Conclusions
According to the obtained results, the authors conclude the following. The increase in pressure increases the velocity of the fluid flow and reduces the damping influence, as it clearly noted from the data listed in Tables (2 and 3), which show increasing the stability values on the opposing side. It is noted from the simulation by ANSYS that the internal pressure has a considerable influence on the characteristics of the natural vibration. Also, the fundamental frequency decreases dramatically as the internal pressure increases for all three modes of vibration; it also decreases with increasing pipe thickness. The increasing in internal pressure will increase the total deformation of the tested pipe. Finally, the modal damping ratio of the vibrated system will be increased with increasing of the internal pressure and decreases with decreasing the wall pipe thickness.

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