The Integrated Sachs-Wolfe Effect in the Bulk Viscous Dark Energy Model

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ABSTRACT

We examine linear perturbation theory to evaluate the contribution of viscosity coefficient in the growing of dark matter perturbations in the context of the bulk viscous dark energy model inspired by thermodynamical dissipative phenomena proposed by Mostaghel et al. (2017). As the cosmological implementations, we investigate the Integrated Sachs-Wolfe (ISW) auto-power spectrum, the ISW-galaxy cross-power spectrum and derive limits on $f\sigma_8$. The dimensionless bulk viscosity coefficient ($\gamma$) in the absence of interaction between dark sectors, modifies the Hubble parameter and the growth function, while the Poisson equation remains unchanged. Increasing $\gamma$ reduces the dark matter growing mode at the early epoch while a considerable enhancement will be achieved at the late time. This behavior imposes non-monotonic variation in the time evolution of gravitational potential generating a unique signature on the CMB photons. The bulk viscous dark energy model leads to almost a decreasing in ISW source function at the late time. Implementation of the Redshift Space Distortion (RSD) observations based on "Gold-2017" catalogue, shows $\Omega^0_m = 0.303^{+0.044}_{-0.038}$, $\gamma = 0.033^{+0.098}_{-0.013}$ and $\sigma_8 = 0.769^{+0.080}_{-0.088}$ at 1$\sigma$ level of confidence. Finally, tension in the $\sigma_8$ is alleviated in our viscous dark energy model.

Key words: methods: analytical–methods: data analysis – cosmic background radiation – dark energy – large-scale structure of Universe.

1 INTRODUCTION

The agent of the late time accelerating expansion of the Universe confirmed by many observational data sets ranging from background evolution to perturbations dynamics is mysterious not only for theoretical cosmology but also in observations (Riess et al. 1998; Perlmutter et al. 1999).

The standard cosmological model (ΛCDM) containing six free parameters is a trivial prescription to explain dynamics of our Universe. This scenario has been confirmed by various observations such as Supernova Type Ia (SNIa), Cosmic Microwave Background radiation (CMB), Baryonic Acoustic Oscillations (BAO) (Ade et al. 2016a,b), the ISW effect (Boughn & Crittenden 2004, 2005a; Ade et al. 2016c) and weak lensing (Peel et al. 2017; Heymans et al. 2013; Lewis & Challinor 2006; Contaldi et al. 2003). Despite the outstanding consistencies between ΛCDM and the observational cosmic data sets, the cosmological constant has some fundamental problems and concordance model has tensions remained unresolved (Ade et al. 2016a; Riess et al. 2016; Bernal et al. 2016). Recent observations indicated a deficiency in amplitude of ISW cross-correlation with astronomical objects based on concordance model (Granett et al. 2008; Kovács et al. 2013; Flender et al. 2013; Ferraro et al. 2015). Taking into account the late-Universe measurements of the dark matter growth rate which is proportional to the $\sigma_8$ implied an orientation to low value with respect to that of computed by CMB in the context of ΛCDM (Ade et al. 2016c; Heymans et al. 2012; Erben et al. 2013). Subsequently, there is some room for alternative scenarios mainly classified into the following categories: Dynamical dark energy including the field theory orientation, phenomenological dark fluids, modification of the general relativity and thermodynamics point of view (Copeland et al. 2006; Amendola et al. 2013; Horndeski 1974; Kônig et al. 2016; Bento et al. 2002; Zlatev et al. 1999; Caldwell 2002; Amendola 1993; Germani & Kehagias 2010; Huterer & Shafer 2018; Mostaghel et al. 2017, and references therein). In an interesting approach, recently, N. Khosravi suggested a new proposal to modify the standard cosmology based on the idea of taking ensemble average over the various gravitational models (Khosravi et al. 2010a, 2010b, 2013).
The $\Lambda(t)$ cosmology, considering a typical form of dark energy and/or interaction between dark sectors in the Universe are some proposals to reduce such discrepancies (Wang et al. 2010a; Velten et al. 2015; Kunz et al. 2015; Barbosa et al. 2017; Fan et al. 2016; Mainini & Mota 2011).

Another proposal for dark energy is an exotic fluid with some thermodynamical features such as bulk and/or shear viscosities. Meanwhile, finding proper observational measures which can precisely probe the influence of dark energy component and distinguishing between various scenarios has received extensive attention. Geometrical and topological properties of cosmological random fields (Ling et al. 2015; Fang et al. 2017) and considering the primary and the secondary probes have been discovered and applied for evaluation and discrimination of dark energy models (Huterer & Shafer 2018, and references therein).

Dark energy can affect on the various elements of the Universe. A trivial contribution of dark energy can be realized in the rate of background expansion. Hence all quantities containing the Hubble parameter are affected by dark energy density. At the background level, due to changing the expansion history of the Universe, the distance to the last scattering surface is modified changing the so-called acoustic signatures (Hu & White 1996). Dark energy perturbations can also alter the lensing potential (Acquaviva & Baccigalupi 2006; Carbone et al. 2013) and consequently can modify the lensing $B$-mode (Amendola et al. 2014). Dark energy modifies the primordial gravitational wave and therefore it changes the amplitude of primordial $B$-mode (Antolini et al. 2013; Raveri et al. 2015; Amendola et al. 2014). The matter perturbations growth is also affected by dark energy density (Peebles 1984; Barrow & Saich 1993) causing to have some discrepancies between amplitude of fluctuations computed by late-time observations and CMB map (Durrer et al. 1999; Baldi & Pettorino 2011; Ade et al. 2016b).

Secondary CMB anisotropies are widely experiencing different epoch of the Universe, therefore, we expect that the CMB stochastic field is a proper measure to explore dark energy. A relevant probe of dark energy in the context of CMB observations, is ISW effect which is a secondary anisotropy (Sachs & Wolfe 1967; Rees & Sciama 1968; Kolman & Starobinski 1985; Hu & Sugiyama 1994; Crittenden & Turok 1996; Cooray 2002; Afshordi 2004; Schäfer & Bartelmann 2006; Schäfer 2008b; Ade et al. 2016c). ISW effect is related to the frequency changes in the CMB photons when they encounter with the time evolving gravitational potential. Since dark energy is dominating at the low redshift (1), the primordial CMB fluctuations alone cannot provide a considerable and precise probe. However, the secondary anisotropies in the CMB is more sensitive to the dark energy dynamics (Schäfer 2008b; Kovács et al. 2013; Huterer & Shafer 2018). The cosmic variance for almost those multipoles that ISW has signature on the CMB power spectrum washouts the importance of ISW alone to explore dark energy models (Song et al. 2007).

Practically, the cross-correlation of ISW with the tracers of the large scale structures magnifies the ISW signal and it would be distinguishable from primordial processes (Afshordi et al. 2004; Afshordi 2004; Ho et al. 2008; Oliveses et al. 2008; Giannantonio et al. 2008; Douspis et al. 2008; Wang et al. 2010a; Ferraro et al. 2015; Lesgourgues 2013).

The ISW signal has been computed, in order to examine the $A$ contribution in concordance model relying on rare superstructures identified in the SDSS Luminous Red Galaxy catalogue (Nadathur et al. 2012; Flender et al. 2013). The ISW effect and its cross-correlation with large scale structures have been investigated in the context of alternative to LCDM models such as the extended quintessence model in both the metric and Palatini formalisms (Fan et al. 2016), quintessence cosmological model (Wang et al. 2010b), particular form of interaction between dark sectors (Schäfer 2008b; Schäfer 2008a; Schäfer 2009), non-ideal fluid dark energy with anisotropic stress component (Majerotto et al. 2015), clustering of dark energy (Khosravi et al. 2016) and also other more general cosmological models (Scranton et al. 2003; Ferraro et al. 2015; Velten et al. 2015; Fosalba et al. 2003; Dent et al. 2009; Sapon et al. 2009; Padmanabhan et al. 2005; Dent et al. 2009).

More recently, inspired by thermodynamical dissipative phenomena and taking into account the isotropy of the Universe at the background level, we proposed a bulk viscous dark energy (BVDE) model (Mostaghel et al. 2017). In this model, the old cosmological objects can be accommodated and the tension in the Hubble parameter was alleviated (Mostaghel et al. 2017). Relying on dissipative process in a realistic fluid, Israel et al. proposed a causal dissipative theory for assessing the irreversible processes (Israel & Stewart 1979). Accordingly, bulk and shear viscous terms are most relevant parts for a feasible relativistic fluid. The Eckart’s theory including the first-order dissipative relativistic fluid, is acasual and has instabilities (Eckart 1940; Hiscock & Lindblom 1985; Hiscock & Salmonson 1991) (see also (Landau & Lifshitz 1987; Jou et al. 2009)). There are many approaches to construct causal and stable theory of relativistic viscous fluid for a certain range of relevant quantities and proper conditions (Hiscock & Lindblom 1983, 1988; Donzelli 2014). However, examining the problem of causality and stability of relativistic theories is under debate (Rezzolla & Zanotti 2013).

For cosmological implementation, it has been demonstrated that Israel approach converges to the Eckart’s theory (Hiscock & Salmonson 1991). Since, the collision time scale in the transport equation of our proposed fluid is zero, consequently, our bulk viscous model is necessarily acasual and unstable (Maartens 1996). For making a causal bulk viscous dark energy, in principle, one should take into account full Israel-Stewart transport equation and keeping the collision time scale (Israel & Stewart 1979). On the other hand, the functional form of viscosity in our model leads to crossing phantom divide barrier which has been observationally confirmed (Ade et al. 2016b). In order to resolve mentioned instability and regularize underlying model, in some cases, it turns out that various dark energy cosmological models can provide different range of redshifts depending on corresponding natures.
one can take into account interaction between dark components of the Universe (Amendola et al. 2013). The effective dark energy in a suitable interacting model can possibly have phantom crossing without divergences and it is fundamentally related to the collision time scale in the corresponding transport equation. It is worth noting that in a simple dark energy model with constant equation of state accompanying a typical interaction term leads to another instability (Välikivi et al. 2008). One approach to reduce the effect of instability may possibly assume a proper rate of interaction between viscous dark energy and dark matter. Another way for regularization the instability is suppressing any initial perturbations of dark energy by taking proper initial conditions.

In this paper, we are interested in discussing the consequence of BVDE model incorporating the background expansion on the large scale structures. To this end, we will take into account the linear perturbation of dark matter in the presence of our viscous dark energy model to study its contributions on the rate of structure formations and ISW effect 2.

We will focus on the ISW auto-power spectrum and cross-correlation between CMB fluctuations mainly considered by ISW part with large scale structure to explore the contribution of bulk viscosity in the dark energy budget. To make our discussion more complete, we will use the Redshift Space Distortion (RSD) data set to constrain model free parameters. The contribution of viscosity coefficient represents a non-monotonic behavior of ISW auto- and cross-power spectrums leading to low clustering compared to ΛCDM model and also it can reduce the tension in σs. It is worth reviewing that the BVDE cosmological model could alleviate H0 tension (Mostaghel et al. 2017).

The layout of the rest of this paper is as follows: In section 2, for the sake of clarity, we give a brief introduction of our bulk viscous dark energy (BVDE) model. Background dynamics of the Universe will be explained in this section. Section 3 is devoted to the linear perturbation of the dark matter when bulk viscous dark energy is considered. Growth function, growth rate and bias independent parameter, namely fσs for BVDE model are derived in section 3. Angular auto-power spectrum of ISW and ISW-galaxy power spectrum in the flat sky approximation for BVDE model will be derived in section 4. We will check the observational consistency using RSD data set in section 5. Summary and concluding remarks are given in section 6.

### 2 COSMIC EVOLUTION IN THE BULK VISCOUS DARK ENERGY FRAMEWORK

A proposal for dark energy model inspired by dissipative fluid dynamics is considering bulk and shear viscous terms for the energy-momentum tensor. Recently, we consider an exotic dissipative fluid playing a responsible for the late time acceleration (Mostaghel et al. 2017). The pressure for dark energy (DE) component in the BVDE model is defined by:

\[ P_{DE} = -\rho_{DE} - \zeta \Theta(t), \]

where \( \zeta = \omega_{DE} \) and \( \Theta(t) = \nabla^\mu u_\mu \) are viscosity and expansion scalar, respectively. In the FLRW space-time, expansion scalar is \( \Theta(t) = 3H(t) = 3a/\dot{a} \). In addition, we have an assumption (Mostaghel et al. 2017):

\[ \zeta (\rho_{DE}, H) = \xi \frac{\sqrt{\rho_{DE}}}{H}, \]

where \( \xi \) is positive constant with mass square dimension. Other functional form for \( \zeta \) can be found in (Li & Barrow 2009; Hiscock & Salmonson 1991; Barbosa et al. 2017, and references therein). By solving the continuity equation, the evolution of dark energy component becomes:

\[ \rho_{DE}(a) = \rho_{DE}^0 \left( 1 + \frac{9\gamma}{2\sqrt{\rho_{DE}^0}} \ln a \right)^2. \]

The superscript "0" represents the value of DE component at present time. Using the flat FLRW metric in the Einstein’s fields equations, we find the Friedman equations as:

\[ H^2 = \frac{8\pi G N}{3} \rho_{\text{tot}}, \]

\[ \frac{\dot{a}}{a} = -\frac{4\pi G N}{3} (\rho_{\text{tot}} + 3p_{\text{tot}}), \]

where \( \rho_{\text{tot}} = \rho_s + \rho_r + \rho_m + \rho_{DE} \) and \( p_{\text{tot}} = p_s + p_r + p_m + p_{DE} \). The G_N is Newton’s constant. The indices "s", "r" and "m" correspond to radiation, baryonic matter and cold dark matter (CDM), respectively. Finally, the Hubble parameter without any interaction between dark sectors reads:

\[ H^2 = H_0^2 \left[ \Omega_{DE}^0 a^{-4} + \Omega_m^0 a^{-3} + \Omega_{DE}^0 \left( 1 + \frac{9\gamma}{2\sqrt{\rho_{DE}^0}} \ln a \right)^2 \right] + H_0^2 (1 - \Omega_{\text{tot}}^0) a^{-2}, \]

where \( \Omega_{\text{tot}}^0 = \Omega_s^0 + \Omega_r^0 + \Omega_m^0 + \Omega_{DE}^0 \) and throughout this paper, we consider flat Universe (\( \Omega_{\text{tot}} = 1 \)). In Eq. (5), we define:

\[ \Omega_i^0 = \frac{8\pi G N}{3H_0^2} \rho_i^0, \quad \Omega_m^0 = \frac{8\pi G N}{3H_0^2} \rho_m^0, \quad \Omega_{DE}^0 = \frac{8\pi G N}{3H_0^2} \rho_{DE}^0, \]

and the dimensionless viscosity coefficient, \( \gamma \) is:

\[ \gamma \equiv \sqrt{\frac{8\pi G N}{3H_0^2}} \xi. \]

It turns out that for \( \gamma = 0 \), the standard ΛCDM model is recovered. Due to viscosity term, we have non-trivial behavior for DE in BVDE model compared to cosmological constant. In Fig. 1, we indicate the evolution of \( \Omega_{DE}(a) \) as a function of scale factor. As indicated in the inset plot of this figure, around \( a \lesssim 0.06 \) the contribution of DE in the BVDE model is more than Λ. While for the interval \( 0.06 \lesssim a \lesssim 0.9 \) the value of \( \Omega_{DE} \) is less than ΛCDM model. At very early epoch, again the \( \Omega_{DE} \) is similar to Λ. Subsequently, this kind of behavior has non-trivial impact on the ISW as well as other structure formation phenomena. In the next section, we will study the structure formation in the BVDE framework.
energy momentum tensor becomes \( \delta T^\mu_\nu = \rho_m [\Delta_m u^\mu u^\nu + u^\nu \delta u^\mu + u^\mu \delta u^\nu] \) and density contrast is represented by \( \Delta_m \equiv \delta \rho_m / \rho_m \). By applying Fourier Transform of the perturbation equations, we find the following perturbed Einstein equations:

\[
\begin{align}
&k^2 \ddot{\Phi} + 3H(\dot{\Phi} - H\Psi) = 4\pi G N a^2 \Delta_m \rho_m, \\
&\dot{\Psi} = \ddot{\Phi}, \\
&\ddot{\Psi}'' + 2H \dot{\Psi} - H^2 \Psi = (H^2 + 2H') \Psi = 0.
\end{align}
\]

The sign \( - \) corresponds to the Fourier mode. The velocity divergence in the Eqs. (11) is \( \delta \equiv \text{ik}_j \dot{v}_j \). Combining Eqs. (11) and continuity equation, the evolution equation for the CDM density contrast at the linear regime on the sub-horizon scale is as follows:

\[
d\frac{\Delta_{m0}}{dN^2} + \left( \frac{d \ln H}{dN} + 1 \right) \frac{d\Delta_{m0}}{dN} - \frac{3\Omega_m}{2} = 0,
\]

where \( N \equiv \ln a \) is the number of e-foldings. One can derive a set of differential equations representing the perturbations in the BVDE component. However, we consider suitable initial conditions to suppress the dark energy perturbations and finally, the effect of instability is resolved.

Since Eq. (12) is independent from scale of structure, therefore, one can define \( \Delta_m(k, a) \equiv \delta_m(a) \Delta_m(k) \) with two independent modes which are called decaying (\( \delta_m \)) and growing (\( \delta_m^+ \)) modes. The linear growth rate of the density contrast, \( f \), which is related to the peculiar velocity in the linear theory is defined by (Peebles 1993):

\[
f(a) \equiv \frac{d \ln D^+_{m0}(a)}{d \ln a}
\]

where \( D^+_{m0}(a) \equiv \delta_m^+(a)/\delta_m(a = 1) \) which is known as growth function. The growth rate measurements are characterized by the peculiar velocities obtained from the Redshift Space Distortion (RSD) observations (Kaiser 1987). To achieve proper observational quantity comparable with the growth rate computed from the linear perturbation theory, one can compare transverse and line of sight anisotropies influenced by the peculiar motion in the redshift space clustering of galaxies. Weak lensing and/or RSD (Song & Percival 2009; Nesseris et al. 2017) yield a robust combination, namely \( f\sigma(z) \equiv f(z)\sigma(z) \) which is bias independent observable quantity. Here the variance of the linear density contrast on scale \( R_8 = 8h^{-1} \text{Mpc} \) is \( \sigma(z) \) which is bias independent observable quantity. Here the equation for the linear density contrast on scale \( R_8 = 8h^{-1} \text{Mpc} \) is given by:

\[
\sigma^2(R_s, z) = D^{+}_{m0}(z) \left[ \int_{0}^{\infty} \frac{dk}{2\pi} k^2 P_{m}(k) W^2(kR_s) \right]^{1/2}
\]

where \( W(kR) = \frac{\delta_{m0}(k\sqrt{H_0^2 - k^2} + kR)}{\delta_{m0}(kR)} \) and \( R = (3M/4\pi \rho_m)^{1/3} \). The matter power spectrum is introduced by \( \langle \tilde{\Delta}_m(k) \tilde{\Delta}_m(k') \rangle = (2\pi)^3 \delta_D(k - k') P_m(k) \). The theoretical formula for the present matter power spectrum is \( P_m(k) = P_0 k^{n_s} T^2(k) \) where \( P_0 \) is normalization constant and \( n_s = 0.968 \pm 0.006 \) according to the recent reanalysis of the Planck data (Ade et al. 2016a). \( T(k) \) is transfer function. Since, at the very high redshift, our BVDE model is almost similar to that of supposed by \( \Lambda \text{CDM} \), therefore, we use the BBKS transfer function model (Bardeen et al. 1986):

\[
T(k) = C_T [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4},
\]
Mostaghel et al. 2017, and references therein) (e.g. to assess Smith et al. 2003 indicates the ratio of BVDE growing mode to the ΛCDM growing mode ratio in comparison of ΛCDM.

\[
\frac{\delta_{+;\text{BVDE}}}{\delta_{+;\text{ΛCDM}}} = 0.295, \gamma = 0.01570 \\
\frac{\delta_{+;\text{BVDE}}}{\delta_{+;\text{ΛCDM}}} = 0.304, \gamma = 0.01404 \\
\frac{\delta_{+;\text{BVDE}}}{\delta_{+;\text{ΛCDM}}} = 0.316, \gamma = 0.03200
\]

For all values of free parameters, we present the evolution of CDM growth function as a function of scale factor for various values of free parameters. These values have been set according to background estimations represented in the Ref. Mostaghel et al. (2017).

\[
\Gamma = \Omega_m^2 h^2 \exp(-\Omega_m^0 h^2 - \sqrt{2\Omega_m^0 h^2}/\Omega_m^0).
\]

where \(C_q \equiv \ln(1 + 2.34q)/2.34q\) and \(q \equiv k/\Gamma\). Here \(\Gamma\) is the shape parameter, given by:

\[
D_+(a) = \frac{\delta_{+;\text{BVDE}} / \delta_{+;\text{ΛCDM}}}{m / \delta_{+;\text{ΛCDM}}} + \frac{\delta_{+;\text{BVDE}} / \delta_{+;\text{ΛCDM}}}{m / \delta_{+;\text{ΛCDM}}}.
\]

It is worth noting that in the general case, one should recalculate the transfer function to find robust results (Wang et al. 2010b, and references therein) (e.g. to assess non-linear evolution see (Smith et al. 2003; McDonald et al. 2006; Lewis & Challinor 2006)).

\[
\frac{\delta_{+;\text{BVDE}} / \delta_{+;\text{ΛCDM}}}{m / \delta_{+;\text{ΛCDM}}} = 0.295, \gamma = 0.01570 \\
\frac{\delta_{+;\text{BVDE}} / \delta_{+;\text{ΛCDM}}}{m / \delta_{+;\text{ΛCDM}}} = 0.304, \gamma = 0.01404 \\
\frac{\delta_{+;\text{BVDE}} / \delta_{+;\text{ΛCDM}}}{m / \delta_{+;\text{ΛCDM}}} = 0.316, \gamma = 0.03200
\]

4 INTEGRATED SACHS-WOLFE EFFECT

One of the sensitive and feasible tracers of the dark energy density is ISW effects. This effect is due to the interaction of CMB photons with time-varying gravitational potential. This term contains all processes due to the non-static metric fluctuations. During radiation and dark energy dominated epochs, the gravitational potential, \(\Phi\), has dynamics leading to non-vanishing contribution on the CMB fluctuations. Here, we ignore the non-significant early ISW effect associated with the radiation dominated Uni-
verse. The quantitative formula for computing the ISW contribution on the CMB fluctuation seen in direction \(\hat{n}\) reads as (Sachs & Wolfe 1967; Seljak & Zaldarriaga 1996; Gordon & Hu 2004; Alshordi et al. 2004; Ho et al. 2008; Olivares et al. 2008; Ade et al. 2016c):

\[
\left( \frac{\Delta T}{T_{CMB}} \right) (\hat{n}) = -2 \int_0^{\chi_{CMB}} d\chi a^2 H(a) \frac{\partial \Phi(\hat{n}, a)}{\partial a},
\]

(17)

here \(\chi_{CMB}\) is the comoving distance to the last scattering surface. Using the Eq. (11a), the Poisson equation in the Fourier space for the scales smaller than the Hubble radius, \(k \gg H\), becomes:

\[
k^2 \Phi = 4\pi G_N a^2 \rho_m \delta_m^G(a) \Delta_m(k)
\]

(18)

Finally, we get the gravitational potential as:

\[
\Phi = 3H_0^2 a^2 E^2(\Omega_m(a) \delta_m^G(a) \Delta_m(k)),
\]

(19)

where \(E^2(\alpha) \equiv H^2(\alpha)/H_0^2\). Now we define the function \(Q(a)\) by (Schäfer 2008b):

\[
Q(a) \equiv a^2 E^2(\alpha) \Omega_m(a) D_m^2(\alpha) \delta_m^G(a = 1),
\]

(20)

Finally, the Eq. (17) equates to:

\[
\left( \frac{\Delta T}{T_{CMB}} \right) (\hat{n}) = -3H_0^2 \int_0^{\chi_{CMB}} d\chi a^2 H(a) \frac{dQ(a)}{da} \int \frac{dk}{(2\pi)^{3/2}} k^2 e^{-ik \cdot \Delta_m(k)}
\]

(21)

According to this quantitative equation, the \(Q(a)\) function and its derivative are the sources of ISW effect (Schäfer 2008b; Wang et al. 2010a,b). In our BVDE model, the ISW is altered by modified Hubble parameter and growth function, while the Poisson equation remains unchanged due to the absence of interaction between the BVDE and cold dark components. As indicated in the upper part of Fig. 5, the \(Q(a)\) function in the BVDE model is lower than the \(\Lambda CDM\) model. The \(dQ(a)/da\) behaves as non-monotonic function versus scale factor leading to non-trivial contribution in the ISW effect. Actually taking into account the \(\gamma\) (dimensionless viscosity coefficient), the magnitude of \(dQ(a)/da\) as a source of ISW, becomes smaller than \(\Lambda CDM\). This behavior can be justified by considering the contribution of \(\Omega_{DE}\) indicated in Fig. 1 and also by Fig. 3. In addition, the behavior of \(H(a)\) in BVDE model is different compared to \(\Lambda CDM\) (Mostaghel et al. 2017). As discussed before, for \(\gamma \gtrsim \gamma_x\), we expect to have opposite contribution of BVDE resulting in the higher value for the source of ISW.

Till now we explained the physical model for ISW of CMB fluctuations, but due to stochastic nature of CMB field, the practical observable measures are inferred from probabilistic framework in the context of n-point autocorrelation and cross-correlation approaches. We turn to the two-point correlation function of CMB temperature fluctuation expanded in terms of spherical harmonic basis functions: \(C_\ell(\theta) = \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) C_\ell^T W_\ell^2 P_\ell(\cos \theta)/4\pi\). Here \(C_\ell^T\) is temperature angular power spectrum, \(P_\ell\) is the Legendre polynomial and \(W_\ell\) is a smoothing function\(^3\).

\(^3\)For a Gaussian kernel function, we have \(W_\ell = \exp\left(-\theta_{beam}^2((\ell + 1)/2)\right)\) and \(\theta_{beam} \equiv \theta_{FWHM}/\sqrt{8\ln 2}\).

\[c_{ISW}^{1\ell \ell} = \int_0^{\chi_{CMB}} d\chi W_\ell^2 H_0^2 \frac{W_\ell^2(\chi)}{1} \frac{\Delta(\ell + 1/2)}{(\ell + 1/2)}\chi^2,\]

(22)

where \(k = (\ell + 1/2)/\chi\) (Ho et al. 2008) and the weighting function including the evolution of gravitational potential is \(W_\ell(\chi) \equiv a^2 H(a) dQ(a)/da\). Fig. 6 indicates \(W_\ell^2\) as a function of scale factor for the BVDE and \(\Lambda CDM\) models. For \(\gamma \lesssim \gamma_x\), the contribution of bulk viscosity decreases the amount of \(W_\ell^2\) comparing to \(\Lambda CDM\). This is also justified by increasing the contribution of cold dark matter in the BVDE model. Non-monotonic behavior with respect to viscosity coefficient for \(\gamma \gtrsim \gamma_x\) is a consequence of previous results shown in Fig. 5.

\(^4\)Actually, the integration over various values of \(k\) is replaced by the most dominant contribution term.
Fig. 7 illustrates the effect of bulk viscosity on the ISW power spectrum. The ISW power spectrum for BVDE model is lower than ΛCDM. Increasing γ leads to decrease $C_{TT}\ell$ for $\gamma \lesssim \gamma_x$. This behavior is no longer valid for $\gamma \gtrsim \gamma_x$ due to the non-monotonic contribution of viscosity effect in the BVDE model.

Physical interpretation for this behavior is clarified by looking at the source terms in Eq. (22). For $\gamma \lesssim \gamma_x$ the ratio of $\Omega_m/\Omega_{DE}$ is higher than that of for ΛCDM during long period of evolution. Therefore, the amount of variation in the gravitational potential is less than ΛCDM expressed by $dQ(a)/da$ (a representative for ˆΦ) in Fig. 5 remaining in the lower value of late ISW. For $\gamma \gtrsim \gamma_x$, we expect to have more (less) contribution of viscous dark energy at the early (late) time. This manner leads to having a turning point for $\gamma$ dependency of ISW phenomenon alone (see Fig. 6) restricted to the non-monotonic contribution of viscosity effect in the BVDE model.

The physical processes governing on the CMB fluctuations can be classified into the primary and secondary classes. Therefore, the observed CMB map is a superposition of mentioned processes having almost different contributions in the various scales with almost same frequency dependent. Practically, it is very hard to distinguish the corresponding footprints. The early ISW effect which is due to the metric perturbation just after photon decoupling epoch on the sub-sound-horizon scales, has mainly contribution on the lower value of late ISW. For $\gamma \gtrsim \gamma_x$ the scale independent density contrast of observable galaxies is:

$$\delta_g(\hat{n}) = \int b(z) \frac{dN}{dz} \Delta_m(\hat{n}, z) dz,$$

where $b(z)$ is the bias between galaxies and dark matter density perturbations. Also, $dN/dz$ is the selection function of the survey. Similar to the previous strategy, the line of sight integral for density contrast of visible galaxies reads as:

$$\delta_g(\hat{n}) = \int_0^{\chi_{CMB}} d\chi b(a) a^2 \frac{dN}{dz} H(a) D_m^+(a) \delta_m^+(a = 1)$$

$$\times \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} e^{-i \mathbf{k} \cdot \mathbf{n}} \Delta_m(k).$$

We define the weight function for observable galaxy density contrast as: $W_g(\chi) \equiv b(a) a^2 (dN/da) H(a) D_m^+(a) \delta_m^+(a = 1)$. The angular cross-correlation of CMB fluctuations and visible galaxy density contrast considering only ISW contribu-
tion in the flat-sky approximation is given by:

\[ C_{T_{\nu},T}^{\text{ISW}}(\ell) = \int_0^{X_{\text{CMB}}} d\chi H_0^2 W_{\nu}(\chi) W_{\nu}(\chi) \frac{P_{\Delta}(\ell + 1/2)/\chi^2}{[\ell(\ell + 1)/\chi^2]^2}. \]  

(25)

In order to compute cross-correlation power spectrum, we use the NRAO VLA Sky Survey (NVSS) sample (Condon et al. 1998) which has:

\[ \left( \frac{dN}{dz} \right)_{\text{NVSS}} = b_{\text{eff}} \alpha_{\gamma} \zeta_{\gamma} \Gamma(\alpha) e^{-\alpha_{\gamma} \zeta_{\gamma}} \]  

(26)

where \( b_{\text{eff}} = 1.98, z_{\gamma} = 0.79 \) and \( \alpha = 1.18 \) (Ho et al. 2008).

The NVSS catalog includes the North sky of \(-40^\circ\) declination in 1.4 GHz continuum band (Condon et al. 1998). For other observed samples released by different surveys, we should use proper redshift distribution functions as reported e.g. in (McDonald et al. 2005; Giannantonio et al. 2008; Douspis et al. 2008; Ho et al. 2008; Xia et al. 2009; Wang et al. 2010a; Ferraro et al. 2015; Ade et al. 2016).

The ISW cross-power spectrum is shown in Fig. 8. From observational point of view, the value of \( C_{T_{\nu},T}^{\text{ISW}} \) shows a considerable magnification compared to \( C_{T_{\nu},T}^{\text{F}} \). In the upper panel of Fig. 8, we show the effect of bulk viscosity on the ISW-cross power spectrum. By increasing \( \gamma \), the value of the ISW-cross power spectrum goes down and we have a turning point in which the behavior of the ISW-cross power spectrum changes in opposite way but not for all multipoles as revealed by the mentioned figure. For \( \gamma \sim \gamma_{\gamma} \), at the late time, we have a deficiency in \( dQ/da \) (Fig. 5), while for almost long interval of scale factor, it gets higher value compared to other cases yielding non-monotonic behavior for ISW cross-power spectrum with galaxies. This behavior provides an opportunity to get more consistent theoretical predictions with recent observations. The lower panel of Fig. 8 illustrates the \( C_{T_{\nu},T}^{\text{ISW}} \) in \( \mu K \) scale compared to the NVSS (Ho et al. 2008).

In the next section, we will examine the observational consistency of BVDE model based on observational quantities derived in the perturbations approached.

5 OBSERVATIONAL CONSISTENCY

In our previous paper, we mainly concentrated on the background evolution and derived the best fit values for model free parameters. To make more complete present discussion on the contribution of bulk viscous dark energy model in the dark matter perturbations evolution, we focus on an observable quantity coming from perturbation formalism, namely \( f\sigma_8(z) \). Table 1 contains the observed value of \( f\sigma_8(z) \) with associated 1\( \sigma \) uncertainty according to the "Gold-2017" catalogue (Nesseris et al. 2017, see references therein). Our relevant model free parameters are \( \gamma \) and \( \Omega_{m0}^* \) with constant priors. The Hubble parameter at present time is mainly constrained by SNIa+BAO+Planck reported by Mostaghel et al. (2017).

In order to compare the observational data set with that of predicted by our model, we utilize likelihood function with the following \( \chi^2 \):

\[ \chi^2_{\text{RSD}} \equiv \Delta f\sigma_8^\text{obs} - \Delta f\sigma_8 \]  

(27)

Table 1. The current observational value of the \( f\sigma_8(z) \) according to the "Gold-2017" catalogue (Nesseris et al. 2017).

| Index | Redshift | \( f\sigma_8,\text{obs} \) |
|-------|----------|-----------------|
| 1     | 0.02     | 0.428 ± 0.0465 |
| 2     | 0.02     | 0.398 ± 0.065  |
| 3     | 0.02     | 0.314 ± 0.048  |
| 4     | 0.10     | 0.370 ± 0.130  |
| 5     | 0.15     | 0.490 ± 0.145  |
| 6     | 0.17     | 0.510 ± 0.060  |
| 7     | 0.18     | 0.360 ± 0.090  |
| 8     | 0.25     | 0.3512 ± 0.0583|
| 9     | 0.32     | 0.384 ± 0.095  |
| 10    | 0.37     | 0.4602 ± 0.0378|
| 11    | 0.38     | 0.440 ± 0.060  |
| 12    | 0.44     | 0.413 ± 0.080  |
| 13    | 0.59     | 0.488 ± 0.060  |
| 14    | 0.60     | 0.550 ± 0.120  |
| 15    | 0.60     | 0.390 ± 0.063  |
| 16    | 0.73     | 0.637 ± 0.072  |
| 17    | 0.86     | 0.400 ± 0.110  |
| 18    | 1.40     | 0.482 ± 0.116  |

Table 2. Best fit values for BVDE model using RSD data at 68\% and 95\% confidence intervals.

| Parameter | RSD |
|-----------|-----|
| \( \Omega_{m0}^* \) | 0.30(0.044±0.003)–0.038–0.070 |
| \( \gamma \) | 0.033(0.098±0.182)–0.033–0.033 |
| \( \sigma_8 \) | 0.769(0.080±0.181)–0.080–0.154 |

where \( \Delta f\sigma_8 \equiv f\sigma_8^\text{obs}(z) - f\sigma_8^\text{th}(z; \Omega_{m0}^*, \gamma) \) and \( C \) is the covariance matrix of RSD data set. The best fit values for BVDE free parameters based on RSD observation are reported in Table 2. Fig. 9 indicates the behavior of \( f\sigma_8 \) as a function of redshift for various values of model free parameters. The symbols correspond to the most new catalog including observational values. The marginalized likelihood function for \( \sigma_8, \Omega_{m0}^* \) and \( \gamma \) determined by RSD observations are depicted in the upper panel of Fig. 10. The lower panel of Fig. 10 indicates the contour plots illustrating the marginalized confidence regions at 68\% and 95\% levels. Taking into account the bulk viscosity for dark energy component, manipulates the growing of the dark matter in the Universe decreasing tension in the present fluctuation spectrum, \( \sigma_8 \), which has been mentioned in (Ade et al. 2016a; Heymans et al. 2012; Erben et al. 2013).

6 SUMMARY AND CONCLUSION

In this paper, following our previous paper on proposing a new dynamical dark energy model inspired by thermodynamical dissipative phenomena, we examined the linear perturbation theory of the dark matter in the presence of viscous dark energy model. In order to probe the dark energy properties, the clustering of large scale structure and ISW have been elucidated.

Taking into account viscosity coefficient for our BVDE model, suppresses the delta\( \Omega_{m0}^* \) comparing to the same quantity

\[ \text{http://www.cv.nrao.edu/nvss/} \]
computed for ΛCDM at early epoch. While dark matter growing mode is boosted at the late time. For higher value of the viscous coefficient, we obtained higher value of scale factor for which the δ_m becomes higher than ΛCDM (see Fig. 2). As illustrated in Fig. 2, the dark matter growing mode is not monotonic function versus scale factor and structure formation experiences a delay in the presence of viscous dark energy.

Fig. 3 indicated the value of growth function (D_m(z)). By increasing the bulk viscosity coefficient in the BVDE model, we found a deviation from the standard ΛCDM model. Therefore we expect to have a manipulation in the σ_m introduced by Eq. (14). The linear growth rate in Fig. 4 indicated that for almost $\gamma \lesssim \gamma_\times$ (Mostaghel et al. 2017), the early dark energy contribution can not compensate the higher role of cold dark matter at the late time. Consequently, the growth rate is higher than ΛCDM. Such behavior is no longer valid for almost $\gamma \gtrsim \gamma_\times$. We found a bump in $f(z)$ around $z \sim 0.8$ for higher $\gamma$.

ISW power spectrum for our BVDE model has been illustrated in Fig. 7 confirming non-monotonic contribution of viscosity effect in the BVDE model. To give a physical interpretation for this behavior, we should look at the contribution of $\Omega_D$, $H$, $D_m^+(a)$, $Q(a)$ and $dQ(a)/da$. The bulk viscosity parameter can extensively modify the rate of Hubble expansion and consequently, it mainly affects the evolution of the dark matter density parameter and $D_m^+(a)$. In another word, incorporating bulk viscosity for our dark energy fluid, reduces $\Omega_D/\Omega_m$ for long period of cosmic evolution. According to Fig. 2, the growing mode of matter is higher than ΛCDM. The source term in ISW is diminished considerably leading to have more static gravitational potential and therefore less ISW. This behavior is not monotonic with respect to $\gamma$. For almost $\gamma \gtrsim \gamma_\times$, at intermediate range of scale factor, the BVDE model effectively reduces the structure formation giving higher ISW term. It is worth mentioning that, such magnification can never compensate the sharp reduction in $\Omega^2_T$ and finally we get the lower ISW value comparing to the ΛCDM model (Figs. 5, 6 and 7).

Indeed, incorporating viscosity for $\gamma \lesssim \gamma_\times$ damps the growing mode of the newtonian potential slower than ΛCDM model.

To magnify the amount of ISW signal in one hand and on the other hand to reduce the degeneracies in determining the source of fluctuations in the power spectrum, we computed the ISW cross-correlation with visible galaxy density contrast. Such cross-correlation can magnify the value of $C_{T\ell}$ compared to $C_{TT\ell}$. In BVDE model, the value of the ISW-cross power spectrum decreases by increasing $\gamma$ and we found a turning point confirming non-monotonic behavior of ISW-cross power spectrum. Fig. 8 illustrates the $C_{TSW}$ in $\mu$K scale compared to the NVSS.

To examine the observational consistency, we have utilized "Gold-2017" RSD observations. The value of $f\sigma_8$ as a function of redshift has been indicated in Fig. 9. Our posterior analysis indicated that $\Omega_m^0 = 0.303 \pm 0.044$, $\Omega_k^0 = 0.038 \pm 0.035$, $\gamma = 0.033 \pm 0.058 \pm 0.035$ and $\sigma_8 = 0.789 \pm 0.098 \pm 0.141$ at 1σ and 2σ level of confidences, respectively. It seems that our model could reduce the tension in $\sigma_8$ (see Fig. 10).

Finally, considering the contribution of coupling between dark sectors of the Universe incorporating viscosity for the dark energy could be interesting and utilizing structure formation with background observations would be able to distinguish between such families of models (Schäfer 2008b). Taking into account such interaction may provide an opportunity to examine the stability of underlying viscous dark energy model. To resolve acausal problem for the bulk viscous dark energy fluid, one can keep the collision time scale in the transport equation. We will address them in the future work.

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