Dark Matter in Minimal Trinification*

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ABSTRACT

We study an example of Grand Unified Theory (GUT), known as trinification, which was first introduced in 1984 by S. Glashow. This model has the GUT gauge group as $[SU(3)]^3$ with a discrete $\mathbb{Z}_3$ to ensure the couplings are unified at the GUT scale. In this letter we consider this trinification model in its minimal formulation and investigate its robustness in the context of cosmology. In particular we show that for a large set of the parameter space the model doesn’t seem to provide a Dark Matter candidate compatible with cosmological data.

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1 Introduction

In this letter we would like to present the preliminary and suggestive results of a more ambitious and extensive research project. We study an example of Grand Unified Theory (GUT) in the context of certain requirements dictated by cosmology. In other words we require the model in examination to address questions like “Is there a Dark Matter (DM) candidate? How abundant is this at present?” or “Can we find successful mechanisms for Baryogenesis and Reheating?”. These questions arise from the more general program of using present cosmological data to constrain the enormous proliferation of phenomenological works describing physics beyond the Standard Model (SM).

We are going to concentrate on one of such models, known as “trinification”, introduced for the first time in 1984 by S. Glashow [1], successively studied in detail by Babu et al. [2] and more recently by Willenbrock et al. [3] [4]. In particular we consider the minimal formulation of the model (respect to how the SM is embedded in it), and focus on the question of Dark Matter. We show that the model does not have a stable DM candidate compatible with $\Omega_{DM} \leq 0.24$. We find that if we adjust the parameters such that we have a stable candidate, there is far too much DM. On the other hand, making the candidate unstable conflicts with Big Bang Nucleosynthesis (BBN) constraints. In section 2 we are going to briefly present the model, list its salient features and focus on its advantages and downsides. We present our results in section 3. We conclude in section 4 discussing possible improvements of the model that would circumvent its difficulty in providing a viable DM candidate.

2 Trinification in a nutshell

The name “trinification” comes from its gauge symmetry: a triple replica of $SU(3)$ conventionally written as $SU_C(3) \times SU_L(3) \times SU_R(3) \times \mathbb{Z}_3$. The discrete group $\mathbb{Z}_3$ guarantees the gauge couplings of the single $SU(3)$ factors are the same at the GUT scale. The SM embedding is obtained by identifying the $SU_C(3)$ with the QCD gauge symmetry while the electroweak gauge group emerges as a result of breaking the other two $SU(3)$ factors. Each SM fermion generation is embedded in a $27 = (1, 3, 3) \oplus \bar{(3}, 1, 3) \oplus (3, \bar{3}, 1)$ representation of the gauge group. In order to better understand the field content we can
re-express this representation in terms of SM quantum numbers

\[
E \equiv (1, 2, -\frac{1}{2}) \quad N_1 \equiv (1, 1, 0) \quad E^c \equiv (1, 2, \frac{1}{2})
\]

\[
L \equiv (1, 2, -\frac{1}{2}) \quad N_2 \equiv (1, 1, 0) \quad e^c \equiv (1, 1, 1)
\]

\[
Q \equiv (3, 2, \frac{1}{6}) \quad u^c \equiv (3, 1, \frac{2}{3}) \quad d^c \equiv (3, 1, -\frac{1}{3})
\]

\[
B \equiv (3, 1, -\frac{1}{3}) \quad B^c \equiv (3, 1, -\frac{1}{3})
\]

We immediately notice that each generation includes the usual lepton and quark doublets and singlets plus some additional fields: we get two additional lepton doublets \(E\) and \(E^c\), two neutral singlets \(N_1\) and \(N_2\) and two quark singlets \(B\) and \(B^c\). We will see that the doublets under the breaking of 
\([SU(3)]^3\) will acquire a heavy mass \((\sim O(M_{GUT}))\) at tree level and become Dirac fermions. The singlets, instead, remain light and require a more refined adjustment involving radiative corrections.

In order to have unification of the SM couplings at the GUT scale we also need two copies of scalars both in the same \(\Phi^a(27) \equiv \Phi^a_\ell (1, 3, \bar{3}) \oplus \Phi^a_q(3, 1, 3)\) or \(\Phi^a_q(3, \bar{3}, 1)\) representation the fermions are in. Let’s concentrate only on the \(\Phi^a_\ell\) and write its field content in terms of SM quantum numbers

\[
\phi^a_{1\ell} \equiv (1, 2, -\frac{1}{2}) \quad \phi^a_{2\ell} \equiv (1, 2, \frac{1}{2}) \quad \phi^a_{3\ell} \equiv (1, 2, \frac{1}{2})
\]

\[
S^a_{1\ell} \equiv (1, 1, 0) \quad S^a_{2\ell} \equiv (1, 1, 1) \quad S^a_{3\ell} \equiv (1, 1, 0)
\]

The breaking 
\([SU(3)]^3 \rightarrow SU_C(3) \times SU_L(2) \times U_Y(1)\) is obtained giving vevs to some of the singlets \(S^a_{3\ell}\). The most general choice being \((S^a_{2\ell}\) are electrically charged so they cannot assume a vev)

\[
< S^1_{3\ell} >= v_1 \quad < S^2_{1\ell} >= v_2 \quad < S^2_{3\ell} >= v_3, \quad (1)
\]

with \(v’s \sim O(M_{GUT} = 10^{14} \text{GeV})\) [3]. In the same fashion the electroweak symmetry is broken giving vevs to the electrically neutral components of the doublets \(\phi^a_{\ell}\) charged under the \(SU_L(2)\). A very general choice is

\[
< (\phi^2_{1\ell})^0 >= n_1 \quad < (\phi^2_{2\ell})^0 >= n_2 \quad < (\phi^2_{3\ell})^0 >= n_3
\]

\[
< (\phi^1_{1\ell})^0 >= u_1 \quad < (\phi^1_{2\ell})^0 >= u_2, \quad (2)
\]
with $u_i, n_i \sim O(M_{EW})$. The other scalar fields $\Phi_i^a$ and $\Phi_i^\nu$ do not acquire any vev since they carry color charge and would break $SU_C(3)$. They are generically very heavy due to radiative corrections to their mass terms and do not show up in the low energy spectrum of the theory. From now on we will assume their masses to be of the order of $M_{GUT}$.

2.1 A simple case

In a simplified version of the model, we set $n_1 = n_2 = n_3 = v_3 \equiv 0$ but keep all other vevs $(u_1, u_2, v_1, v_2)$ non-zero. The qualitative results will be exactly equal to the more general case and suffice to illustrate our point. A linear combination of the $\phi_{i\ell}^a$ and four of the $S_{i\ell}^a$ are eaten by twelve of the gauge bosons\footnote{The gauge bosons are in the $24 = (1,1,8) \oplus (1,8,1) \oplus (8,1,1)$ adjoint representation of $[SU(3)]^3$} that become heavy with masses proportional to the $v_i$. Fine-tuning the quartic couplings, it is possible to obtain at most 5 light ($\sim M_{EW}$) Higgs doublets. At the same time Yukawa terms give masses for the fermions at the tree level. In general, such terms can be built pairing two fermion doublets and a scalar singlet or one scalar doublet contracting indices with one fermion doublet plus one one fermion singlet. In the first case we end up with a heavy (mass $\sim M_{GUT}$) Dirac fermion meanwhile in the second case the fermions are light (mass $\sim M_{EW}$). Limiting our analysis to one fermion generation we obtain at tree level

$$
\begin{align*}
    m_B &\simeq \sqrt{g_1^2 v_1^2 + g_2^2 v_2^2} \\
    m_u &= g_1 u_2 \\
    m_d &\simeq g_1 u_1 \\
    m_e &\simeq h_1 u_1 \\
    m_\nu &= h_1 u_2 \\
    m_N &= h_1 u_2 \\
    m_N &\simeq \frac{h_1^2 u_1 u_2}{m_E}.
\end{align*}
$$

(3)

where $h_a$ and $g_a$ are couplings associated with the Yukawa terms proportional to $\phi_{i\ell}^a$. This spectrum has some positive qualitative features and a negative one. As we have already mentioned $B, B^c$ and $E, E^c$ pair up to become very heavy Dirac fermions. The up and down quarks as well as the electron get different light masses. Unfortunately $N_2, N_1$ and $\nu$ are also light, with the additional inconvenience that the last two seem to pair up to form a light Dirac fermion. This is highly undesirable and can be fixed invoking radiative correction induced by cubic scalar couplings [3] [4]. Calculating the one loop
contribution to mass terms for light fermions and using a seesaw mechanism
the spectrum for these light neutral fields become

\[ m_{N_{1,2}} \sim g_q^2 F \quad m_{\nu} \sim \frac{h_1^2 u_2^2}{g_q^2 F} \]  

(4)

where \( F \leq M_{GUT} \) is a factor of pure one-loop origin. In the presence of other
two fermion generations there are additional \( N_i \)'s and they can mix up and
appear in a sort of hierarchy where the lightest of all can be pushed as far
down as \( 10^5 \) GeV.

2.2 Light and darkness of Trinification

We end this section by briefly reviewing the positive and negative features
of this GUT model. Thanks to the two sets of scalar fields (six weak dou-
blets \( \phi^{a}_{u,d} \)) this model achieves unifications of the SM gauge couplings at a
scale around \( 10^{14} \) GeV. The mechanism is similar to that of generic SUSY
SU(5) GUT (although this unifies at \( 10^{16} \) GeV): the six Higgs contribution
to the \( \beta \)-function is equivalent to the contribution of the two SUSY Higgs
doublets and their fermion superpartners. The only difference resides in the
fact that in Trinification there aren’t gauge boson superpartners that are
ultimately responsible of moving the GUT scale upwards.

Usually, in other non-SUSY unification models there are mass degene-
racies between quarks and leptons since they come in the same represen-
tation of the gauge group. In Trinification this is avoided since quarks and leptons
are in different representations in which the \( 27 \) is decomposed. In conclusion
SM masses for fermions can be arbitrarily adjusted through Yukawa cou-
plings to fit the experimental values. From this point of view Trinification is
not any more predictive than the SM itself. Masses for the scalars are not
protected and receive one loop quadratic corrections requiring fine tuning for
at least one Higgs light doublet in \( \Phi^7 \). Trinification does not provide any
mechanism to solve the so called hierarchy problem.

Unlike SU(5), in trinification gauge bosons conserve Baryon number and
so do not mediate proton decay: the proton can only decay through Yukawa
interactions. An acceptable value for its lifetime is recovered without the
need of fine-tuning the involved Yukawa couplings.

Due to abundant number of scalar fields, it is possible that baryogenesis
is achieved at GUT scales [6] or at electroweak scales through a first order
phase transition [7]. Heavy scalars may be important in some Inflationary scenario. Moreover light neutral singlets may play the role of sterile neutrinos and be used to invoke a Leptogenesis mechanism [5]. The punch line being that this model offers a wide variety of possible developments in the context of Cosmology. In the next section we will address one of these issues. We investigate the possibility that the lightest of the neutral singlets can function as a viable candidate for Dark Matter and try to give an indicative answer.

3 A Dark Matter Candidate

Let us indicate the lightest neutral singlet as $N_{\chi}$. There are two main requirements that $N_{\chi}$ needs to satisfy in order to be a possible Dark Matter candidate. First, its relative abundance $\Omega_{N_{\chi}}$ has to be less than or equal to 0.24. Second, its decay time needs to be much longer than the age of the Universe.

3.1 Relative abundance

The relative abundance at present (assuming the particle is stable) is calculated using the Boltzmann equation. The form of its solution depends on the regime (relativistic or not) at which it is approximated. To check if $N_{\chi}$ was non-relativistic at freeze out we need to verify under which condition the inequality $x_f = \frac{m}{T} \geq 3$ is valid. If we assume that $N_{\chi}$ is non-relativistic the value of $x_f$ depends logarithmically on the annihilation cross section and the mass of $N_{\chi}$ [8].

$$x_f \simeq \ln \left( \frac{0.038}{\sqrt{g_*}} \frac{g}{m_{N_{\chi}}} m_{N_{\chi}} \langle \sigma v \rangle \right)$$

Since we are interested only in estimating this expression we consider the dominant contribution to the annihilation channel. This is given by the following tree level Feynman graph$^2$

$^2$The contribution coming from two $N_{\chi}$ singlets annihilating in a virtual heavy $X$ boson is even more suppressed since it is proportional to $\frac{\alpha^2}{m_X^2}$ and $m_X^2 \sim v_g^2 g_{gauge}$. 


Assuming $\phi_{i\ell}$ and $\phi^*_{i\ell}$ are light Higgs we are lead to the s-wave expression for the cross-section

$$\langle \sigma v \rangle \sim \frac{h^4}{(4\pi)^2m_E^2}.$$  \hspace{1cm} (7)

Here $h$ is a generic Yukawa coupling. Plugging this expression in (6) and assuming the reasonable range $10^5 \text{ GeV} \leq m_{N_\chi} \leq 10^{10} \text{ GeV}$ we arrive at the conclusion that $\frac{v_i}{\chi} \leq 10^9 \text{ GeV}$ in order to have $x_f \geq 3$. Since $v_i \sim M_{\text{GUT}}$ the inequality cannot be satisfied implying that $N_\chi$ is highly relativistic when it freezes out.

We then equate the annihilation rate of $N_\chi$ to the Hubble rate $H$ to obtain the freeze-out temperature

$$T_f \approx 9.1 \frac{\sqrt{g_*(T_f)}}{M_{\text{Pl}}(\langle \sigma v \rangle)} \sim 10^{12} \sqrt{g_s} h^{-2} \text{ GeV}.$$  \hspace{1cm} (8)

This gives a present-day abundance that is rather insensitive to $\langle \sigma v \rangle$ and much too large, namely

$$\Omega_{N_\chi} \sim 2 \times 10^8 g_{*}^{-1}(T_f) \frac{m_{N_\chi}}{\text{GeV}}.$$  \hspace{1cm} (9)

### 3.2 Decay time

The previous estimate is based on the assumption that $N_\chi$ is stable over the course of the universe requiring that $\tau_{N_\chi} \gg 10^{10} \text{ yr}$. However, this is not correct for most reasonable choices of parameters in this model. We find that it decays not too long after it freezes out, and tends to destroy Big Bang Nucleosynthesis (BBN) unless we adjust some parameters appropriately. In order to estimate the lifetime of $N_\chi$ we will consider only the dominant contribution, as we did earlier for the cross section. The most favorable decay channel is given by the following
In this graph $\phi_{i q}$ is one of the scalars carrying color charge with mass of the order of $M_{\text{GUT}}$. In calculating the decay rate it is a very good approximation to consider the decay products massless since they are highly relativistic. In such an approximation the decay rate is

$$\Gamma_{N_x} \sim \frac{128g_q^4m_{N_x}^5}{(2\pi)^2 m_{\phi_{i q}}^4} \tag{11}$$

with $g_q$ being the Yukawa coupling associated with the heavy colored scalar. For $m_{\phi_{i q}} \sim 10^{14}$ GeV and $m_{N_x} \sim 10^5$ GeV we get the following estimate for the decay time

$$\tau_{N_x} \sim 10^{30}g_q^{-4} \text{GeV}^{-1} \sim 10^6 g_q^{-4} \text{s}. \tag{12}$$

For a typical value of $g_q = 0.1$ the decay time is about a 600 years making $N_x$ rather unstable and completely absent at present time in the universe. Moreover its decay products are so highly relativistic and get produced in such abundance that they destroy any product of BBN. We can reverse the reasoning and put an upper bound on the Yukawa coupling that stabilizes $N_x$

$$\tau_{N_x} = 6.3 \times 10^{-2} g_q^{-4} \text{yr} \gg 10^{10} \text{yr} \quad \Rightarrow \quad g_q \ll 2 \times 10^{-3}. \tag{13}$$

This fine-tuning of $g_q$ is not so unreasonable, but it may backfire on the one-loop corrections to the mass of the neutral singlets compromising the efficiency of the radiative seesaw mechanism.

## 4 Conclusion and discussion

We have seen that the lightest of the neutral singlets in the minimal trinification model is greatly overabundant at freeze out with the consequence that it is completely ruled out by cosmological data. This may be overcome
if the particle is not very stable but then it ruins BBN and creates another big problem. As it stands this GUT model does not seem to be a complete model of particle physics since it does not withstand one of the most needed cosmological requirements: the existence of Dark Matter.

There is room for improvement though. On one side we may introduce a $Z_2$ symmetry like the one introduced in SUSY models and make $N_\chi$ more stable without the need of adjusting the Yukawas opportunely. Such a discrete symmetry would forbid the Yukawa terms responsible for the dominant decay channel and stabilize the singlet. On the other hand we have to admit that our calculation of the cross-section for the relative abundance is a little “naive” since it doesn’t take into account possible mixing of $N_i$’s with the other neutral Weyl fermions of the model. For example within just one generations of leptons we have $E_0 \in E$ and $E_0^c \in E^c$ plus the two neutral singlets, $N_1$ and $N_2$, and the neutrino $\nu$. The mass matrix of these five neutral fields is far from diagonal and can provide some mixing between them. The net result is that the lightest among them may have some non-zero coupling with the $Z^0$ boson greatly enhancing the cross section amplitude. This mechanism may help in reducing the abundance to more accepted values.

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