Probing the sheath electric field using thermophoresis in dusty plasma.
Part II: Experimental measurements

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Abstract—A two-dimensional dust crystal levitated in the sheath of a modified Gaseous Electronics Conference (GEC) reference cell is manipulated by heating or cooling the lower electrode. The dust charge is obtained from top-view pictures of the crystal using a previously developed analytical model. By assuming a simple force balance, and measuring the radial confining force, the vertical electric field profile in the sheath is reconstructed. The dust crystal is shown to levitate on the plasma side of the Bohm point. Finally, it is shown that the ion drag plays an important role in the vertical force balance, even for large dust grains.

Index Terms—Dust crystal, dusty plasma, ion drag, modified GEC cell, sheath electric field, thermophoresis.

I. INTRODUCTION

Dust particles immersed in laboratory plasma collect electrons and ions and become negatively charged. The charge on a single particle is typically estimated to be between 1 and 5 electron charges per nanometer radius [1], depending on the background pressure [2]. This means that micrometer-sized particles used in dusty plasma experiments can carry a negative charge of thousands of electron charges. Because of this, the dust particles are levitated against gravity in the strong electric fields present in the sheath. The electrostatic interaction between the particles is shielded by the plasma, but depending on the discharge parameters, strongly coupled systems can be formed [3]. By applying a radial confinement, these so-called dust crystals can be used to study many solid state physics phenomena on a scale accessible to ordinary optical diagnostic techniques.

In order to understand the behavior of these systems, knowledge about the screening length and the dust charge is of paramount importance. Furthermore, once the charge on the dust particles is known, the vertical force balance between gravity and the electric field can provide insight into the structure of the electric field in the sheath. This is important for both dusty plasma experiments and many plasma enhanced industrial processes, where ions can be accelerated by the sheath electric field from the plasma bulk to a processing substrate [4].

A method for determining the Debye length and dust particle charge for a dust crystal suspended above a spherical electrode in the lower electrode of a RF discharge, by measuring observable quantities in top-view pictures of dust crystals, was developed in [5]. This method was recently investigated computationally and the results presented elsewhere in this issue [6].

In this paper, we present experimental results of the determination of the Debye length and the dust charge for two-dimensional dust crystals levitated in a modified GEC cell. By using thermophoresis to move the dust crystal through the sheath, the electric field was reconstructed based on the measured dust charge and the assumed balance between gravity and the electrostatic force in the sheath. It is shown that this method is consistent with our computational results, that the ion drag most likely plays an important role in the vertical force balance, and finally that the dust crystal is levitated on the plasma side of the classical Bohm point.

II. EXPERIMENTAL SETUP

A GEC cell was used [7] to create a standard 13.56 MHz radio-frequency (RF) discharge in argon. The cell was modified in order to conduct dusty plasma experiments. Cover plates can be attached to top of the powered bottom electrode, which have cylindrically shaped depressions milled in them, with different diameters. These cutouts provide an electric field that confines the dust particles radially. The upper electrode is a hollow grounded cylinder, which provides optical access to the dust crystal. Above this electrode dust containers are installed, which, when tapped, release dust particles which fall through the hollow electrode into the plasma, until they reach their equilibrium levitation height.

A camera mounted above the hollow upper electrode is used to provide top-view pictures of the horizontal dust crystal structure. A second camera installed in front of a side-port takes side-view pictures of the vertical structure of the dust crystal. Two red diode-lasers with a wavelength of 640 nm and 686 nm are equipped with cylindrical lenses that shape the laser beams into thin laser sheaths. One sheath is oriented vertically, the other horizontally. The light scattered by the dust particles is captured by either the top-view or side-view camera. Filters can be used with the cameras to only allow light at the laser wavelength to be observed, rejecting the plasma glow. Typically, only one camera-laser combination was used at a time. The cameras and lasers can be moved in three directions; however, the focus of the cameras is fixed, so that the resolution did not change between pictures. The...
resolution of the top-camera and side-camera was measured to be 26 and 14 \( \mu \text{m} \) per pixel, respectively.

The lower electrode is constructed such that it can be cooled or heated by running liquid through tubes inserted in a circular channel drilled in the electrode. For heating and cooling the liquid, we used a MGW Lauda Brinkman RM3T chiller/heater, connected to a Ranco ETC-111000-000 thermostat. The temperature was measured employing thermocouples attached to the tube near the thermostat, as well as on the bottom of the electrode. Due to the hysteresis of the thermostat/chiller-heater system, the temperature fluctuated approximately 2\(^\circ\)C above and below the set temperature. A rapid rise to temperatures above the set temperature was followed by a slow decrease to a point 2 degrees below the set-point. By measuring the electrode-temperature variation, we determined that the heater/thermostat rapidly heats the system, but then overshoots by a few degrees. After reaching the maximum temperature, the electrode slowly cools down, giving a relatively long time-window where the temperature may be considered constant. This was long enough to take the necessary top- and side-view pictures, since the cameras operated at 60 and 30 Hz, respectively. Figure 1 shows the change in temperature of the lower electrode measured by a thermocouple attached to the bottom of the electrode, together with the ambient temperature. To determine the electric field profile, the set temperature was varied between 0\(^\circ\)C and 60\(^\circ\)C in steps of 10\(^\circ\)C.

For the experiments presented here, 8.89 \( \mu \text{m} \) diameter melamine-formaldehyde (MF) particles were used, in an argon discharge run at 2 W of input power and 200 mTorr of background pressure. The diameter of the cutout in the top-plate attached to the lower electrode was 1 inch (25.4 mm) with the depth of the cutout roughly half a millimeter.

![Fig. 1. The room-temperature and the temperature of the powered electrode measured with thermocouples. The room-temperature was roughly constant at 23\(^\circ\)C. The temperature on the electrode was set at 20\(^\circ\)C. It can be seen that the heater/thermostat rapidly heats the system, then overshoots by a few degrees. After reaching the maximum temperature, the electrode slowly cools down, giving a relatively long time-window where the temperature may be considered fairly constant.](image)

### III. Observables

A complete description of the analytical method used to determine the Debye length and dust charge was developed in [5] and discussed in [6]. We here restrict ourselves to mentioning only the observables required to determine the Debye length and the dust charge, together with the equations relating these to the observables.

Given a single-layer dust crystal consisting of \( N \) particles, the radius of the crystal \( R_M \), the inter-particle spacing at the edge of the crystal, \( \Delta_M \), and the inter-particle spacing at the center of the crystal \( \Delta_0 \) were measured. The theoretical edge of the crystal, \( R_\infty = R_M + \Delta_M \sqrt{3}/2 \) was then calculated, so that the Debye length could be determined from

\[
\lambda_D = \Delta_0 \left[ \frac{A - S}{3S - A} \right],
\]

where we have defined the total crystal surface area as \( A = \pi R_\infty^2 \) and the total surface area covered by \( N \) Wigner-Seitz cells measured at the center to be \( S = \sqrt{3} \Delta_0^2 N/2 \).

After determining the Debye length, the dust charge can be calculated from

\[
q_D = \sqrt{\frac{4\pi \varepsilon_0 \Delta_0 \kappa k R_\infty^2}{3 \left( 3 + \frac{2\pi^2}{\kappa^2} \right) \exp\left( \frac{A - S}{3S - A} \right)}}
\]

where \( k \) is the constant of the radial restoring force. It is assumed that the radial potential well is parabolic, so that \( V(r) \propto cr^2 \). In this case, \( k \) depends on the steepness of the potential well, which depends on the geometry of the cutout. Also, \( k \) changes with the applied temperature, and hence with any change in the thermophoretic force \( F_{th} \):

\[
k = 2c \left[ m_D g - F_{th} \right].
\]

We see that when the thermophoretic force acts downward (\( F_{th} < 0 \)), \( k \) increases and the dust crystal is radially compressed. When the thermophoretic force acts upward, \( k \) decreases and the dust crystal expands radially.

In [5] spherical cutouts were used, and the curvature of the potential well, \( c \), was related to the radius of curvature of the cutout, \( R_C \): \( c = 1/(2R_C) \). We will assume a similar relation for our cylindrical cutout, so that we have

\[
k = \left[ m_D g - F_{th} \right] / R_C.
\]

### A. Measuring \( k \) without thermophoresis

The common assumption concerning the radial confinement resulting from cutouts or other adaptations to the lower electrode is that it can be described by a potential well parabolic, so that \( V(r) \propto cr^2 \). In this case, \( k \) is a priori clear for our case.

In order to determine the radial restoring force, we ran the discharge at high pressure (1 Torr in this case) in order to minimize any oscillatory motion of the dust particles, without heating or cooling of the lower electrode, so that \( F_{th} = 0 \), and dropped the dust particles just outside of the cutout. We then monitored the particles while they moved inwards into the potential well of the cutout. Assuming that in this case the forces acting on the particles are the electrostatic force (determined by the potential well) and the gas drag, the equation of motion for these particles can be written as:

\[
m_D \ddot{x} = -\alpha \dot{x} - k_0 x,
\]

where the first term on the right hand side represents the neutral drag, with \( \alpha \) the Epstein friction coefficient [9], and the...
second term the restoring force. Note that here we presume the force is due to a harmonic potential, hence the use of Hooke’s law. We assume that the particles are monodisperse, so that they all have the same mass, \( m_D \). A solution to equation 3 can be found by substituting \( x(t) = a \exp(-bt) \) for overdamped motion (for high neutral drag at 1 Torr), which gives us the value of \( b \) directly: \( b = -\dot{x}/x = (k_0/\alpha) \).

Figure 2 shows three selected particle trajectories. Plotted is the natural logarithm of the distance from the final position in the last frame. For the first 0.3 seconds, the trajectories form straight lines. The dashed lines are linear fits to the natural logarithm of the distance, all with \( R^2 > 0.996 \). From these trajectories the value of \( k_0 \) is determined.

IV. DETERMINATION OF THE DEBYE LENGTH, CHARGE, AND ELECTRIC FIELD

Now that we have determined \( k_0 \) for the 1-inch diameter cutout in our modified GEC cell, we can take pictures of the dust crystals formed with different applied temperature gradients. From side-view pictures the levitation height was determined, whereas using graphics software [10], the number of particles, crystal radius and central inter-particle distance were determined from top-view pictures.

Figure 3 shows a top-view image of a typical crystal levitated in the discharge. This crystal was formed while the lower electrode was cooled to 0\(^\circ\) C. It contains roughly 1000 particles in the first layer, has a radius of \( R_\infty = 8.2 \) mm, and the central inter-particle distance was measured to be \( \Delta_0 = 386 \) micron. This yields \( A = 8.2 \times 10^{-4} \) m\(^2\) and \( S = 1.29 \times 10^{-4} \) m\(^2\). From figure 5, the levitation height of the upper crystal-layer was determined to be 5.83 mm.

Repeat these calculations for different applied lower electrode temperatures, the Debye length, dust charge, and electric field were obtained as a function of the temperature. A plot of the dust levitation height versus applied temperature, as determined from side-view pictures, is shown in figure 5. It can be well fitted to an exponential, which is consistent with the dependence of the levitation height on temperature as shown in [6]. The Debye length, dust charge, and electric field are shown in figure 6.

As shown, the Debye length varies between 200 and 360 microns, having its smallest values higher in the plasma.
seems reasonable, since the plasma density is expected to increase farther away from the electrode, and $\lambda_D \propto \sqrt{T/n}$. The values obtained for the Debye length are in reasonable agreement with values reported in the same GEC cell (roughly 250 $\mu$m for a discharge at 200 mTorr and 5 W power), measured using a different technique [11], and also with the observed inter-particle distances, which in these crystals typically are a few times the Debye length [3].

The dust charge becomes continuously less negative from -120,000e to -50,000e with increasing height above the lower electrode. These charges seem to be too large by a factor of two to three, since 9-micron diameter particles are expected to have a charge of less than 50,000 electron charges.

The electric field decreases approximately linearly for heights above 6.5 mm. The obtained vertical electric field in the sheath is low (due to the high reconstructed dust charge), and varies from -400 V/m (closer to the electrode) to -120 V/m (farther up in the plasma).

However, even if we multiply the electric field by three, the drift velocity of argon ions would correspond to an energy of roughly 0.3 eV. This is much lower than the electron temperature, which is roughly 3-5 eV, so that the dust is levitated on the plasma side of the Bohm point, consistent with [5].

V. DISCUSSION AND CONCLUSIONS

Our results show that the application of thermophoresis provides an interesting tool to probe electric field structures in the sheath. Debye lengths obtained in this manner appear to be in agreement with values obtained by others. The dust charge appears to be overestimated however, roughly by a factor of two to three, resulting in unrealistically low values for the electric field. Since the Debye length and inter-particle distance seem to be in agreement with other measurements, the unrealistically high negative charge implies that the value obtained for $k$ is too high, even though the value determined from the particle trajectories seems to be rather precise.

Since we did not include the ion drag in the vertical force balance, the radius of curvature of the cutout in the cover-plate on top of the powered electrode determined from the value of $k_0$ obtained without additional heating is given by

$$R_C = \frac{m_D g}{k_0}.$$  \hspace{1cm} (4)

However, if the ion drag does play a significant role, the radius of curvature would be given by

$$R_C = \left(\frac{m_D g + F_{id}}{k_0}\right),$$  \hspace{1cm} (5)

resulting in a larger radius of curvature, since the ion drag acts downwards and adds to gravity. The resulting change in $k$ due to the thermophoretic force $F_{th}$ is then given by

$$\Delta k = k_{\Delta T} - k_0 = -\frac{F_{th}}{R_C},$$  \hspace{1cm} (6)

which would be smaller due to the larger value of $R_C$. This would result in smaller charges for a calculated $\lambda_D$, which follows from equation[2]. In order to have a significant change in the charge, the ion drag would need at a minimum to be of the same order as gravity, which is not expected for particles of this size [5]. Nonetheless, the ion drag seems to play an important role in the vertical force balance.

Despite ignoring the ion drag in the analysis, our measurements show that the variation in the electric field can be probed on the sub-millimeter scale, and that the dust crystal is levitated on the plasma side of the Bohm point.

Of the three observables used in this determination, the number of particles turned out to be the hardest to obtain. This is due to a few factors. First, in order to obtain high enough resolution to accurately measure the central inter-particle distance, the field of view of the top-mounted camera had to be reduced, so that the crystal did not fit completely in the frame. However, it was still possible to estimate what fraction of the crystal was missing and correct the number of particles. Secondly, in some cases the horizontal laser sheath also illuminated particles below the top-layer, resulting in an overcount of the total number of particles. Finally, and most importantly, despite all our efforts, more and more dust particles were lost from the top layer when we increased the temperature on the lower electrode. At this point, the initially single-layer crystal seemed to split into additional layers. This
was also observed with the side-view camera. As a result, the equation of state, which strongly depends on the single-layer structure of the crystal, may no longer have been valid.

An interesting observation, which we could not find in the literature, was that a measurable change in the natural DC bias on the lower electrode occurred. This bias became less negative for electrode temperatures above room-temperature, changing from \(-13\) V when the electrode was at room-temperature to roughly \(-10.5\) V at \(+60^\circ\) C. When the electrode was cooled below room-temperature, we did not observe the opposite effect. Since a decrease in the negative DC bias causes dust particles to levitate closer to the lower electrode, negating the effect of the applied temperature, we maintained the DC bias at \(-13\) V for all temperatures using an external power supply. This insured that no changes in the DC bias could occur, so that the DC bias would not play a role in the levitation of the dust. Even so, very small changes in the bias can not be excluded, and the question arises whether or not this might also have played a role in past experiments employing thermophoresis.

![Graph](image)

**Figure 7.** The levitation height shown together with the height of the lower boundary of the plasma glow, measured from side-view images taken at various temperatures. The glow appears to be moving together with the dust for higher temperatures.

Figure 7 shows the obtained crystal levitation height together with the lower boundary of the plasma glow for different bottom electrode temperatures, estimated using images obtained with the side-view camera. We see that at higher temperatures, the plasma glow seems to move up together with the dust crystal. Even though dust particles have been shown to have a significant effect on plasma emission in a dusty argon plasma [12], we cannot at this point say whether this is due to the influence of the dust crystal on the plasma, or due to the observed changes in the DC bias. This effect should be more carefully investigated in future work.

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