HETEROTIC INSTANTONS AND SOLITONS
IN ANOMALY-FREE SUPERGRAVITY

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Abstract

We extend the classical heterotic instanton solutions to all orders in $\alpha'$ using the equations of anomaly-free supergravity, and discuss the relation between these equations and the string theory $\beta$-functions.
1 Introduction

During the past few years the study of classical (low energy) equations for 10-dimensional superstrings has yielded a number of interesting solutions, see for instance \[1, 2, 3, 4, 5, 6, 7\]. The standard method of finding these is to solve the equations of motion of ordinary supersymmetric Einstein-Yang-Mills theory (augmented by the Lorentz Chern-Simons term in the definition of the antisymmetric tensor), which equal the string \(\beta\)-functions to the lowest order. To show that a solution obtained this way is also a solution to string theory, one then tries to construct the corresponding superconformal sigma-model.

However, from the space-time point of view it might appear somewhat unsatisfactory to look for supersymmetric solutions using a set of manifestly non-supersymmetric equations. A supersymmetrization of the coupled Einstein-Yang-Mills theory including the Lorentz Chern-Simons term would also naturally extend the equations to higher orders in \(\alpha'\), which is interesting in itself. Such a supersymmetrization has actually already been performed, both in the normal case (with a 3-form \(H\)) \[8, 9, 10\] using an important observation by Bonora, Pasti and Tonin \[11\], and in the dual case \[12\]. The equations of this model, which we call the Anomaly-Free Supergravity (AFS), give \(\alpha'\) corrections to the standard equations, leading to implicit equations for the physical fields. In the general case one must then expand to all orders in \(\alpha'\). AFS is hence a classically consistent, all orders in \(\alpha'\) (on-shell) supersymmetric theory which incorporates the Green-Schwarz condition for anomaly cancellations \[13\].

Fortunately, for the purpose of generalizing the classical solutions of \[2, 3\], it will turn out to be sufficient to use the original implicit equations. This is what is done in this paper. We find a solution to the AFS equations in a closed form containing only \(O(\alpha'^0)\) and \(O(\alpha'^1)\) terms. This solution, which is really a family of solutions containing the ones by Callan, Harvey, and Strominger referred to above, consists of a non-linear differential equation for the field appearing in the metric, which, in general, might have to be solved as an expansion in \(\alpha'\). Also the relation between the dilaton and the metric contains derivatives.

It should perhaps be pointed out here that although AFS agrees with the effective string theory to lowest order and contains terms to all orders, it is extremely unlikely that it will turn out to be equivalent to the massless effective string theory. It does not incorporate the \(\zeta(3)\) terms in an obvious fashion, and it has been shown that AFS can at least in principle be extended to a non-minimal version containing extra representations which can accommodate such terms \[14\]. We believe, however, that the minimal AFS provides a better approximation to string theory than the one normally used, and, as is argued at the end of this paper, it might even provide a necessary condition for a solution to be a solution of string theory.
2 The instanton solution

We will here study the generalization to AFS of the heterotic five-brane solution by Strominger \[2\]. We follow his calculation closely, only making a slightly more general ansatz. As in the lowest order case, the solution also turns out to incorporate the wormhole solution \[3, 4\] related to the solution with a five-brane source à la Duff and Lu \[7\]. Since AFS corrections are rather complicated, we find it most convenient to work directly in the variables of \[10\], instead of performing the field redefinitions \[15, 2\] to obtain the \(\sigma\)-model variables and a flat five-brane metric. We use mainly the conventions of \[9, 10\]. For instance, 

\[g_{MN} = (+, -, ..., -),\]

\[\{\gamma_M, \gamma_N\} = 2g_{MN}\]

and a \(p\)-form is defined as 

\[\omega(p) = \omega_{M_1...M_p} dz^{M_1} \wedge ... \wedge dz^{M_p}.\]

However, to make comparison easier, we use the index conventions of \[2\], that is, 

\[M, N, P, ...\]

are 10-dimensional space-time indices, and 

\[A, B, C, ...\]

are the corresponding tangent space indices.

We now want to find a maximally symmetric, supersymmetric solution, that is, we want a solution to the AFS equations with all spinorial fields equal to zero, and with a (non-zero) Majorana-Weyl spinor \(\epsilon\) satisfying \[9, 10\]

\[\delta \psi_M = D_M \epsilon + \frac{1}{36} \gamma_M \gamma_{A_1A_2A_3} \epsilon \ T^{A_1A_2A_3} = 0,\] \hspace{1cm} (1a)

\[\delta \lambda = -2i \gamma_A \epsilon \partial_A \phi + i \gamma_{A_1A_2A_3} \epsilon \ Z^{A_1A_2A_3} = 0,\] \hspace{1cm} (1b)

\[\delta \chi = -\frac{1}{4} \gamma_{A_1A_2} \epsilon \ F^{A_1A_2} = 0.\] \hspace{1cm} (1c)

To the lowest order, \(T^{A_1A_2A_3}\) and \(Z^{A_1A_2A_3}\) in the gravitino and dilatino transformation equations are both proportional to \(H^{A_1A_2A_3}\). This is no longer the case in AFS. Here instead, we have the torsion \[4\]

\[T_{A_1A_2A_3} = -3 e^{-\frac{4}{3\phi}} H_{A_1A_2A_3} - 2\gamma_1 e^{-\frac{4}{3\phi}} W_{A_1A_2A_3}\] \hspace{1cm} (2)

where \(\gamma_1 \sim \alpha'\) and

\[W_{A_1A_2A_3} = \frac{1}{2} \Box T_{A_1A_2A_3} + 3 T_{B_1B_2[A_1} R_{B_2]A_2A_3} + 3 T_{B_1[A_1A_2} R_{B_2A_3]} + 4 T_{B_1B_2[A_1} T_{B_2}^{B_3} T_{A_3]} B_3 - \left(\frac{2}{27} + h_1\right) T_{A_1A_2A_3} T^2\] \hspace{1cm} (3)

with \(h_1\) a free parameter reabsorbable in a redefinition of the dilaton:

\[e^{\frac{4}{3\phi}'} = e^{\frac{4}{3\phi}} - 2 h_1 \gamma_1 T^2\] \hspace{1cm} (4)

\[\text{Here, as in the following, we drop all fermionic terms.}\]
It is this torsion which turns up in covariant derivatives and the curvature tensor; 
\( D = D(\Omega), \ R = R(\Omega), \) and \( \Omega = \omega + T, \) and which also occurs in the gravitino transformation law, while in the gaugino transformation law we have

\[
Z_{A_1 A_2 A_3} = \frac{1}{6} T_{A_1 A_2 A_3} + 6\gamma_1 e^{-\frac{4\phi}{A_1 A_2 A_3}}
\]

with

\[
W^{(5)}_{A_1 A_2 A_3} = \frac{1}{36} \Box T_{A_1 A_2 A_3} - \left(\frac{1}{9} + \frac{3}{2} h_1\right) T_{B_1 B_2 [A_1 R_{B_1 B_2 A_2 A_3}] + \left(\frac{2}{9} + 3h_1\right) T_{B_1 [A_1 A_2 R_{B_1} A_3]}
\]

\[
+ \frac{1}{36} D_{B_1} T_{B_2 [A_1 A_2 T_{B_1 A_3} R_{B_1 B_2}} + \left(\frac{1}{4} + 3h_1\right) T_{B_1 B_2 B_3} [A_1 D_{A_2} T_{A_3} B_1 B_2 B_3}
\]

\[- \left(\frac{5}{54} + \frac{5}{3} h_1\right) \frac{1}{5!} \epsilon_{A_1 A_2 A_3 B_1 B_2 B_3 C_1 C_2 C_3 C_4 D_{C_1} T_{C_2 C_3 C_4}
\]

\[- \left(\frac{5}{18} + \frac{35}{6} h_1\right) \frac{1}{5!} \epsilon_{A_1 A_2 A_3 B_1 B_2 B_3 C_1 C_2 C_3 C_4 D_{C_1} T_{C_2 C_3 C_4}
\]

\[+ \left(\frac{5}{9} + 5h_1\right) T_{B_1 B_2 [A_1 T_{B_2 B_3 A_1} T_{B_3 A_3} B_1 + \left(\frac{5}{18} + \frac{5}{2} h_1\right) T_{B_1 [A_1 A_2 T_{A_3} B_2 B_3 B_1 B_2 B_3}
\]

\[- \left(\frac{1}{108} + \frac{7}{36} h_1\right) T_{A_1 A_2 A_3 T^2} \right].
\]

Obviously the field redefinitions in for instance [15] would lead to a rather long calculation which is not needed for the present purpose. If we further demand that the solution fulfil

\[
D_{[M} H_{NPQ]} = -4 \text{ Tr}(F_{[MN} F_{PQ]}) - \gamma_1 \text{ Tr}(R_{[MN} R_{PQ]}),
\]

with the traces defined just as the sum over the group indices, we will also automatically satisfy the bosonic equations of motion [14]. In order to find a five-brane solution we split up space-time into

\[
z^M \rightarrow (y^a, x^\mu); \quad a = 0, 1, ..5; \; \mu = 6, ..9
\]

and assume a metric of the form

\[
g_{MN} = \begin{pmatrix}
 e^{2A} & -e^{2A} & \cdots & \cdots & \cdots & \cdots \\
 0 & e^{2A} & -e^{2B} & \cdots & \cdots & \cdots \\
 0 & 0 & e^{2B} & \cdots & \cdots & \cdots \\
 0 & 0 & 0 & \cdots & \cdots & \cdots \\
 0 & 0 & 0 & 0 & \cdots & \cdots \\
 0 & 0 & 0 & 0 & 0 & \cdots
\end{pmatrix}.
\]
Here $A = A(r)$ and $B = B(r)$ are arbitrary scalar fields which, as well as all the fields in the following, depend only on $r = (\delta_{\mu\nu}x^\mu x^\nu)^{1/2}$. Our strategy will now be to solve (1a)-(1c) with $Z_{A_1 A_2 A_3}$ and $T_{A_1 A_2 A_3}$ regarded as independent fields and only put the solution into (5) afterwards.

We start by studying the dilatino equation (1b). Just like in [2] it can be solved by defining chiral spinors

$$\frac{1}{\sqrt{g_6}} \epsilon_{a_1 \ldots a_6} \gamma^{a_1 \ldots a_6} \epsilon_{\pm} = \pm 6! \epsilon_{\pm}$$

and

$$\frac{1}{\sqrt{g_4}} \epsilon_{\mu_1 \ldots \mu_4} \gamma^{\mu_1 \ldots \mu_4} \epsilon_{\pm} = \pm 4! \epsilon_{\pm}$$

with $g_6 = -det(g_{ab}) = e^{12A}$, $g_4 = det(g_{\mu\nu}) = e^{8B}$, and by putting

$$Z_{\mu_1 \mu_2 \mu_3} \sim \epsilon_{\mu_1 \mu_2 \mu_3} \nabla_{\nu} \phi(r) e^{C(r)},$$

and the rest of the components to zero. We then immediately find

$$Z_{\mu_1 \mu_2 \mu_3}^{\pm} = \pm \frac{1}{3} \epsilon_{\mu_1 \mu_2 \mu_3} \nabla_{\nu} \phi(r) e^{-4B}.$$  

(10)

Note that the factor $e^{-4B} = \frac{1}{\sqrt{g_4}}$ is exactly what is needed to make $Z_{\mu_1 \mu_2 \mu_3}^{\pm}$ a tensor in $x$-space. Proceeding to the gravitino equation, we make a similar, but independent ansatz:

$$T_{\mu_1 \mu_2 \mu_3} = \epsilon_{\mu_1 \mu_2 \mu_3} \nabla_{\nu} D(r) e^{E(r)}.$$  

(11)

The $M = a$ component of (1c) is then

$$0 = \partial_a \epsilon_{\pm} + \frac{1}{2} \gamma_a \gamma^\mu \epsilon_{\pm} \partial_\mu A + \frac{1}{6} \gamma_a \gamma^\mu \epsilon_{\pm} \partial_\mu D e^{E+4B}$$

and can be solved by making $\epsilon$ independent of $y^a$, $D^{\pm} = \pm 3A^{\pm}$ (+constant), and $E = -4B$. For $M = \mu$ we get

$$0 = \partial_\mu \epsilon_{\pm} + \frac{1}{6} \epsilon_{\pm} \partial_\mu D + \frac{1}{2} \gamma_{\mu\nu} \epsilon_{\pm} \partial^\nu (B \pm \frac{2}{3} D).$$

(12)

A solution is

$$\epsilon_{\pm} = e^{A/2} \eta_{\pm}$$

with $\eta_{\pm}$ constant chiral and antichiral spinors,

$$B = -2A + \text{constant},$$

(17)

\footnote{The $\epsilon_{\ldots}$'s are here defined as tensor densities; $\frac{1}{\sqrt{g}} \epsilon_{\ldots}$ are the proper tensors in respective space.}
and hence

$$T_{\mu_1 \mu_2 \mu_3} = \pm \epsilon_{\mu_1 \mu_2 \mu_3}^{\nu} \partial_\nu A e^{8A}. \quad (18)$$

The constant in (17) can be absorbed in a constant rescaling of the coordinates and is dropped below. The gaugino equation (1c) is now directly solved by the (anti)instanton configuration

$$F_{\mu \nu} = \pm \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} F_{\rho \sigma} e^{8A}. \quad (19)$$

In the $\gamma_1 = 0$ case we would have had $Z_{\mu \nu \rho} \sim T_{\mu \nu \rho}$ and hence directly $A \sim \phi$. For $\gamma_1 \neq 0$ we have to insert our solution into (5). After a straightforward, but cumbersome calculation, we find that only

$$W_{\mu_1 \mu_2 \mu_3}^{(5)} = \pm \epsilon_{\mu_1 \mu_2 \mu_3}^{\nu} e^{8A} \left[ \frac{1}{12} \partial^\rho \partial_\rho \partial_\nu A + \frac{1}{2} \partial^\rho \partial_\rho A \partial_\nu A ight. \\
- \left. \left( \frac{1}{2} + 9h_1 \right) \partial^\rho A \partial_\rho \partial_\nu A - \left( \frac{3}{2} + 27h_1 \right) \partial^\rho A \partial_\rho A \partial_\nu A, \right] \quad (20)$$

is different from zero. Using (20) and contracting with $\epsilon_{\mu_1 \mu_2 \mu_3}^{\nu} e^{8A}$ we get

$$\partial_\mu \phi = -\frac{3}{2} \partial_\mu A - 18\gamma_1 e^{-\frac{4}{3}A} \left[ \frac{1}{12} \partial^\rho \partial_\rho \partial_\mu A + \frac{1}{2} \partial^\rho \partial_\rho A \partial_\mu A \\
- \left( \frac{1}{2} + 9h_1 \right) \partial^\rho A \partial_\rho \partial_\mu A - \left( \frac{3}{2} + 27h_1 \right) \partial^\rho A \partial_\rho A \partial_\mu A \right] \quad (21)$$

which can directly be integrated to

$$e^{-\frac{4}{3}A} = k - 2\gamma_1 e^{2A} (\partial^\nu \partial_\nu A - 3(1 + 18h_1) \partial^\mu A \partial_\mu A) \quad (22)$$

with $k$ constant. For $\gamma_1 = 0$ we have to choose $k > 0$, and it can be put equal to one by once more rescaling the coordinates. However, in the AFS case, there are also solutions for $k \leq 0$ as we shall see below, so we choose to keep $k$ as a free parameter. Finally, our solution has to satisfy (7). Again we find that only the $[\mu \nu \rho \sigma]$ component is different from zero, and it yields

$$\partial^\mu \left\{ \partial_\mu A e^{-\frac{4}{3}A + 2A} + \gamma_1 e^{-A(\partial^\nu \partial_\nu A - 3(1 + 18h_1) \partial^\mu A \partial_\mu A)} \\
- 6(1 + 18h_1) \partial^\nu A \partial_\nu A \partial_\mu A \right\} = -\frac{4}{3} e^{-4A} Tr(F_{\mu \nu} F_{\mu \nu}) \quad (23)$$

We insert (22), and use

$$\partial^\mu = -e^{A} \partial_\mu$$

$$F_{\mu \nu} = e^{8A} F_{\mu \nu} \quad (24)$$

and obtain
\[
\n\nabla^2[k e^{-6A} + 6 \gamma_1 \nabla^2 A] = -8 \text{ Tr} F^2
\]

Here \( \nabla^2 = \frac{1}{r^3} \frac{\partial}{\partial r} r^3 \frac{\partial}{\partial r} \) is the Laplacian in four-dimensional Euclidean space.

Comparing the expression we use to those of other authors, see for instance [16, 17] and also [15, 2], we find agreement for \( \alpha' = k_2^2 g_2^2 \) and \( \gamma_1 = -2 \alpha' \) and, in particular,

\[
\text{Tr}(F_{\mu \nu} F^{\mu \nu}) = \frac{1}{8 \cdot 30} \alpha' \text{ Tr}(F_{\mu \nu} F^{\mu \nu})_{\text{Strominger}}
\]

Hence, our solutions are exactly the same (as they should be) for \( \gamma_1 = 0 \) (\( \phi = -\frac{1}{2} \phi_S \)). The general solution given our ansatz is then any \( A(r) \) and \( \phi(r) \) satisfying

\[
\begin{align*}
    k e^{-6A} + 6 \gamma_1 \nabla^2 A &= k e^{-6A_0} + k' \frac{r^2}{r^2 + 8 \alpha' \frac{r^2 + 2 \rho^2}{(r^2 + \rho^2)^2}} \quad (27a) \\
    e^{\frac{4}{3} \phi + 2A} &= k - 2 \gamma_1 e^{2A} (\nabla^2 A - 3(1 + 18 h_1) (\nabla A)^2) \quad (27b)
\end{align*}
\]
together with (9), (12), (17), and (18). As a consistency check, as well of [10] as of the calculations above, the equations of motions were also studied, and found to be linear combinations of (derivatives of) the equations (27).

3 Analysis of the solution and discussion.

In equations (27) we have three, so far arbitrary, integration constants. However, the solution is not physical, and might not even exist for all values of \( k, k', \) and \( A_0 \). For instance, the constants have to be chosen so that \( A \) and \( \phi \) are real. We will here first restrict ourselves to a discussion of a few cases already mentioned in the literature. All these solutions can, of course, be extended to multi-instanton or multi-wormhole solutions in the standard fashion. Afterwards, we will give examples of other exact, but mainly unphysical, solutions. The instanton solution [2] (gauge solution in [3, 4]) has \( k' = 0 \). If we assume that \( A \) can be written as a power series in \( \frac{1}{r} \) at infinity, we find

\[
A = A_0 + \frac{A_1}{r^2} + O\left(\frac{1}{r^4}\right), \quad r \to \infty.
\]

The \( \gamma_1 \) part gives then no contribution at infinity and the calculation of mass, axion charge, and the Bogomoln’yi bound still give the same result as in the paper by Strominger. Furthermore, since we still have the same continuous symmetries, the zero-modes should remain unchanged, and a solution of this form giving a real \( \phi \) is then indeed an extension of [2] to all orders in \( \alpha' \).

If we instead let \( \rho \to 0 \) we obtain the generalization of the neutral solution [3], related to [4]. Some care must be taken in this case since it is not obvious
that taking the limit \( \rho \to 0 \) in (27) and then solving for \( A \) and \( \phi \) will give the same result as the other way round.

Perhaps the most important example, however, is that of the symmetric solution of [3, 4], which is argued to have no higher order corrections. In order to be able to define a Lorentz connection which equals the Yang Mills potential we use the "original" version of the instanton potential, see e.g. [18]. In analogy with the authors quoted above we thus put

\[
A_\mu = \sum_{\mu\nu} \partial_{\nu} \log \left( 1 + \frac{\rho^2}{r^2} \right) = -6 \sum_{\mu\nu} \partial_{\nu} A \tag{29}
\]

with \( \rho^2 = n \alpha' e^{6A_0} \). That is

\[
A = A_0 - \frac{1}{6} \log \left( 1 + \frac{\rho^2}{r^2} \right). \tag{30}
\]

Inserting this into (27a) we get

\[
k e^{-6A_0} \left( 1 + \frac{\rho^2}{r^2} \right) - 8 \alpha' \frac{\rho^4}{r^2(r^2 + \rho^2)^2} = k e^{-6A_0} + \frac{k'}{r^2} + 8 \alpha' \frac{r^2 + 2\rho^2}{(r^2 + \rho^2)^2}, \tag{31}
\]

which is satisfied if we choose

\[
k' = (nk - 8)\alpha'. \tag{32}
\]

Since \( A \) also satisfies

\[
\nabla^2 A = 6 (\nabla A)^2, \tag{33}
\]

we can eliminate the correction term in (27b) if we choose the parameter of AFS

\[
h_1 = \frac{1}{18}. \tag{34}
\]

We obtain

\[
\phi = -\frac{3}{2} A + \text{constant} \tag{35}
\]

The symmetric solution is hence a solution also to AFS for the choice of \( h_1 \) in (34). Since a particular choice of \( h_1 \) just corresponds to a field redefinition (4), this value of \( h_1 \) must give the same choice of \( \phi \) as in the references above.

So far, the symmetric solution is the only one we have given explicitly, only assuming that there exist well-behaved solutions of (27) of the neutral and gauge type too, albeit not in a closed form. The symmetric solution is, however, not the only example of a simple solution of (27), although the others we have found do not, in general, have an immediate physical interpretation. Both for the
symmetric solution and for these new ones, we have cancellations between $\text{tr} \ F^2$ and $\text{tr} \ R^2$ so they do not have a proper limit as $\gamma_1 \to 0$. They also have $k \leq 0$.

In (27a) we have already implicitly assumed that $A$ has a well-defined value, $A_0$, as $r \to \infty$, so that the metric is Minkowski at infinity, and that $k \neq 0$. We now relax these constraints and put $k \ e^{-6A_0} = k''$. For $k'' = 0$ we find the solution

$$A = A_0 + \frac{1}{3} \log \left(1 + \frac{r^2}{\rho^2}\right) + \frac{1}{3} \log \left(\frac{r}{\rho}\right),$$

with $k < 0$ and $e^\phi < 0$ if we assume (34). Putting $k = 0$ we can also add non-logarithmic terms to $A$, and we find another solution

$$A = A_0 + \frac{A_2}{r^2} - \frac{1}{6} \log \left(1 + \frac{r^2}{\rho^2}\right) - \frac{k'}{24\alpha'} \log \left(\frac{r}{\rho}\right) - \frac{k''}{96\alpha'} r^2,$$  

which has two free integration constants, $A_0$ and $A_2$, and is hence the general solution for $k = 0$. In order to remove the essential singularities, we must choose $A_2$ and $k''$ as zero, and certain values of $k'$ might then yield interesting solutions.

The effective Lagrangian of string theory should also contain higher order terms multiplied by the transcendental coefficient $\zeta(3)$ [19]. It is very hard to imagine how these could occur within the framework of AFS, although it has been suggested that they might depend on the boundary conditions chosen when solving (3) to construct an effective Lagrangian [1]. Another, perhaps more likely, source is to note [14] that AFS as used here is a minimal supersymmetric extension of the anomaly-free Einstein-Yang-Mills theory, and that it is possible to extend it, by relaxing the constraints used in solving the Bianchi identities. One can then accommodate precisely the superfield needed [20] for the $\zeta(4) \ R^4$-terms. They might then act as counterterms, the precise value of the coefficients being decided from cancellation of divergences, as suggested in [21].

To correspond to the string $\beta$-functions, all equations of AFS should then be augmented by $\zeta(3)$ terms which, for simple non-transcendental solutions such as the ones given above, have to be satisfied separately. Since it is argued, using the $\sigma$-model approach, that the symmetric solution is really a solution to the string [3, 4], we can assume that the $\zeta(3)$ equations are indeed satisfied separately in this case. Hence, a necessary condition for all “normal” classical solutions to string theory should be that they satisfy the AFS equations. It would be interesting to study other solutions like the black five-brane one [1], and also to search for new exotic compactifications, in this framework. It would of course also be interesting to derive the full, non-minimal AFS, and introduce the right $\zeta(3)$ coefficients, but judging from the derivation of the equations of motion for AFS [11] this might require an unrealistic amount of computing power.
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References

[1] A. Dabholkar, G. Gibbons, J. A. Harvey and F. R. Ruiz, Nucl. Phys. B340 (1990) 33.

[2] A. Strominger, Nucl. Phys. B343 (1990) 167.

[3] C. G. Callan, J. A. Harvey and A. Strominger, Nucl. Phys. B359 (1991) 611; PUPT-1244, EFI-91-12.

[4] C. G. Callan, Instantons and Solitons in Heterotic String Theory, PUPT-1278 (1991).

[5] J. A. Harvey and A. Strominger, Phys. Rev. Lett. 66 (1991) 549.

[6] G. T. Horowitz and A. Strominger, Nucl. Phys. B360 (1991) 197; S. B. Giddings and A. Strominger, Exact Black Fivebranes in Critical Superstring Theory, UCSBTH-91-35. (1991).

[7] M. J. Duff and J. X. Lu, Nucl. Phys. B354 (1991) 141; Phys. Rev. Lett. 66 (1991) 1402.

[8] L. Bonora, M. Bregola, K. Lechner, P. Pasti and M. Tonin, Nucl. Phys. B296 (1988) 877.

[9] R. D’Auria, P. Frè, M. Raciti and F. Riva, J. Mod. Phys. A3 (1988) 953.

[10] I. Pesando, The equations of motion of anomaly free supergravity, DFTT 1/91 (1991) to appear in Phys. Lett. B; Completion of the anomaly free supergravity programme: the field equations, DFTT 9/91 (1991) to appear in Class. Quantum Grav.

[11] L. Bonora, P. Pasti and M. Tonin, Phys. Lett. B188 (1987) 335.

[12] H. Nishino, Phys. Lett. B258 (1991) 104.

[13] M. B. Green and J. H. Schwarz, Phys. Lett. 149B (1984) 117.

[14] K. Lechner, P. Pasti, and M. Tonin, Mod. Phys. Lett. A2 (1987) 929; K. Lechner and P. Pasti, Mod. Phys. Lett. A4 (1989) 1721.

[15] A. Strominger, Nucl. Phys. B274 (1986) 253.

[16] L. Castellani and R. D’Auria and P. Frè, Supergravity and superstrings, World Scientific, 1991.
[17] M. B. Green, J. H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, 1987.

[18] R. Rajaraman, *Solitons and instantons*, North Holland, 1982.

[19] M. T. Grisaru, A. E. M. Van de Ven and D. Zanon, Phys. Lett. B173 (1986) 423; Nucl. Phys. B277 (1986) 388, 409; D. J. Gross and E. Witten, Nucl. Phys. B277 (1986) 1; D. J. Gross and J. H. Sloan, Nucl. Phys. B291 (1987) 41.

[20] R. Kallosh, Physica Scripta T15 (1987) 118; B. E. W. Nilsson and A. K. Tollstén, Phys. Lett. B181 (1986) 63.

[21] E. A. Bergshoeff and M. de Roo, Nucl. Phys. B238 (1989) 439.