Interacting holographic generalized Chaplygin gas model

M.R. Setare *
Department of Science, Payame Noor University. Bijar, Iran

Abstract

In this paper we consider a correspondence between the holographic dark energy density and interacting generalized Chaplygin gas energy density in FRW universe. Then we reconstruct the potential of the scalar field which describe the generalized Chaplygin cosmology.
1 Introduction

The accelerated expansion that based on recent astrophysical data [1], our universe is experiencing is today’s most important problem of cosmology. Missing energy density - with negative pressure - responsible for this expansion has been dubbed Dark Energy (DE). Wide range of scenarios have been proposed to explain this acceleration while most of them can not explain all the features of universe or they have so many parameters that makes them difficult to fit. The models which have been discussed widely in literature are those which consider vacuum energy (cosmological constant) [2] as DE, introduce fifth elements and dub it quintessence [3] or scenarios named phantom [4] with \( w < -1 \), where \( w \) is parameter of state.

An approach to the problem of DE arises from holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. It was shown by 'tHooft and Susskind [5] that effective local quantum field theories greatly overcount degrees of freedom because the entropy scales extensively for an effective quantum field theory in a box of size \( L \) with UV cut-off \( \Lambda \). As pointed out by [6], attempting to solve this problem, Cohen et al. showed [7] that in quantum field theory, short distance cut-off \( \Lambda \) is related to long distance cut-off \( L \) due to the limit set by forming a black hole. In other words the total energy of the system with size \( L \) should not exceed the mass of the same size black hole i.e. \( L^3 \rho_\Lambda \leq L M_p^2 \) where \( \rho_\Lambda \) is the quantum zero-point energy density caused by UV cutoff \( \Lambda \) and \( M_p \) denotes Planck mass \( (M_p^2 = 1/G) \). The largest \( L \) is required to saturate this inequality. Then its holographic energy density is given by \( \rho_\Lambda = 3 c^2 M_p^2 / L^2 \) in which \( c \) is free dimensionless parameter and coefficient 3 is for convenience. Based on cosmological state of holographic principle, proposed by Fischler and Susskind [8], the Holographic model of Dark Energy (HDE) has been proposed and studied widely in the literature [9, 10].

Some experimental data has implied that our universe is not a perfectly flat universe and recent papers have favored the universe with spatial curvature [11, 12]. As a matter of fact, we want to remark that although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [13]. Defining the appropriate distance, for the case of non-flat universe has another story. Some aspects of the problem has been discussed in [13, 14]. In this case, the event horizon can not be considered as the system’s IR cut-off, because for instance, when the dark energy is dominated and \( c = 1 \), where \( c \) is a positive constant, \( \Omega_\Lambda = 1 + \Omega_k \), we find \( \dot{R}_h < 0 \), while we know that in this situation we must be in de Sitter space with constant EoS. To solve this problem, another distance is considered- radial size of the event horizon measured on the sphere of the horizon, denoted by \( L \) - and the evolution of holographic model of dark energy in non-flat universe is investigated.

It is fair to claim that simplicity and reasonability of HDE provides more reliable frame to investigate the problem of DE rather than other models proposed in the literature[2, 3, 4]. For instance the coincidence or ”why now” problem is easily solved in some models of HDE based on this fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [15]. In fact a suitable evolution of the Universe is obtained when, in addition to the holographic dark energy, an interaction (decay of dark energy to matter) is assumed. In a very interesting paper Kamenshchik, Moschella, and Pasquier [16] have studied a
homogeneous model based on a single fluid obeying the Chaplygin gas equation of state

\[ P = \frac{-A}{\rho} \]

where \( P \) and \( \rho \) are respectively pressure and energy density in comoving reference frame, with \( \rho > 0; A \) is a positive constant. This equation of state has raised a certain interest [17] because of its many interesting and, in some sense, intriguingly unique features. Some possible motivations for this model from the field theory points of view are investigated in [18]. The Chaplygin gas emerges as an effective fluid associated with d-branes [19] and can also be obtained from the Born-Infeld action [20].

Inserting the equation of state (1) into the relativistic energy conservation equation, leads to a density evolving as

\[ \rho_\Lambda = \sqrt{A + \frac{B}{a^6}} \]

where \( B \) is an integration constant.

In present paper, using the generalized Chaplygin gas model of dark energy, we obtain equation of state for interacting Chaplygin gas energy density in non-flat universe. The current available observational data imply that the dark energy behaves as phantom-type dark energy, i.e. the equation-of-state of dark energy crosses the cosmological-constant boundary \( w = -1 \) during the evolution history. We show this phantomic description of the interacting generalized Chaplygin gas dark energy in non-flat universe, and reconstruct the potential of the phantom scalar field. Finally we introduce the squared speeds of sound for generalized Chaplygin gas and interacting holographic fluid. We find that the squared speed for generalized Chaplygin gas is negative when, \( \alpha < 0, A > 0 \) or \( \alpha > 0, A < 0 \). Also we show that the squared speed for interacting holographic fluid is negative when choosing \( c = 0.84 \), and taking \( \Omega_\Lambda = 0.73, \Omega_k = 0.01 \) for the present time.

## 2 Interacting generalized Chaplygin gas model

In this section we obtain the equation of state for the generalized Chaplygin gas when there is an interaction between generalized Chaplygin gas energy density \( \rho_\Lambda \) and a Cold Dark Matter(CDM) with \( w_m = 0 \). The continuity equations for dark energy and CDM are

\[ \dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q, \]

\[ \dot{\rho}_m + 3H\rho_m = Q. \]

The interaction is given by the quantity \( Q = \Gamma \rho_\Lambda \). This is a decaying of the generalized Chaplygin gas component into CDM with the decay rate \( \Gamma \). Taking a ratio of two energy densities as \( r = \rho_m/\rho_\Lambda \), the above equations lead to

\[ \dot{r} = 3Hr \left[ w_\Lambda + \frac{1 + r}{r} \frac{\Gamma}{3H} \right] \]

Following Ref.[21], if we define

\[ w_\Lambda^{\text{eff}} = w_\Lambda + \frac{\Gamma}{3H}, \quad w_m^{\text{eff}} = -\frac{1}{r} \frac{\Gamma}{3H}. \]
Then, the continuity equations can be written in their standard form

\[ \dot{\rho}_\Lambda + 3H(1 + w^\text{eff}_\Lambda)\rho_\Lambda = 0, \]  
\[ \dot{\rho}_m + 3H(1 + w^\text{eff}_m)\rho_m = 0 \]

We consider the non-flat Friedmann-Robertson-Walker universe with line element

\[ ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right). \]

where \( k \) denotes the curvature of space \( k=0,1,-1 \) for flat, closed and open universe respectively. A closed universe with a small positive curvature \( (\Omega_k \sim 0.01) \) is compatible with observations \([11, 12]\). We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

\[ H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}\left[\rho_\Lambda + \rho_m\right]. \]  

Define as usual

\[ \Omega_m = \rho_m / \rho_{cr} = \rho_m / 3M_p^2H^2, \quad \Omega_\Lambda = \rho_\Lambda / \rho_{cr} = \rho_\Lambda / 3M_p^2H^2, \quad \Omega_k = k / a^2H^2 \]

Now we can rewrite the first Friedmann equation as

\[ \Omega_m + \Omega_\Lambda = 1 + \Omega_k. \]

Using Eqs.(11,12) we obtain following relation for ratio of energy densities \( r \) as

\[ r = \frac{1 + \Omega_k - \Omega_\Lambda}{\Omega_\Lambda} \]

In the generalized Chaplygin gas approach \([20]\), the equation of state to (1) is generalized to

\[ P_\Lambda = \frac{-A}{\rho_\Lambda^\alpha}. \]

The above equation of state leads to a density evolution as

\[ \rho_\Lambda = [A + \frac{B}{a^{3(1+\alpha)}}]^{\frac{1}{1+\alpha}} \]

By considering the above equations, one can find

\[ w_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = \frac{-A}{a^{3(1+\alpha)}[A + Ba^{-3(1+\alpha)}]}. \]

From Eqs.(6, 16), we have the effective equation of state as

\[ w^\text{eff}_\Lambda = \frac{-A}{a^{3(1+\alpha)}[A + Ba^{-3(1+\alpha)}]} + \frac{\Gamma}{3H}. \]

Here as in Ref.[22], we choose the following relation for decay rate

\[ \Gamma = 3b^2(1 + r)H \]
with the coupling constant $b^2$. Now using the definition of generalized Chaplygin gas energy density $\rho_\Lambda$, and using $\Omega_\Lambda$, we can rewrite Eq.(17) as

$$w_\Lambda^{\text{eff}} = -\frac{A}{(3M_p^2H^2\Omega_\Lambda)^{1+\alpha}} + \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda}$$  \hspace{1cm} (19)$$

Now we suggest a correspondence between the holographic dark energy scenario and the generalized Chaplygin gas dark energy model.

In non-flat universe, our choice for holographic dark energy density is

$$\rho_\Lambda = 3c^2M_p^2L^{-2}.$$  \hspace{1cm} (20)$$

$L$ is defined as the following form[13]:

$$L = ar(t),$$  \hspace{1cm} (21)$$

here, $a$, is scale factor and $r(t)$ is relevant to the future event horizon of the universe.

Given the fact that

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \frac{1}{\sqrt{|k|}}\sin^{-1}(\sqrt{|k|}r_1)$$

$$= \begin{cases} 
\sin^{-1}(\sqrt{|k|}r_1)/\sqrt{|k|}, & k = 1, \\
r_1, & k = 0, \\
\sinh^{-1}(\sqrt{|k|}r_1)/\sqrt{|k|}, & k = -1,
\end{cases}$$  \hspace{1cm} (22)$$

one can easily derive

$$L = \frac{a(t)\sin[\sqrt{|k|}R_h(t)/a(t)]}{\sqrt{|k|}},$$  \hspace{1cm} (23)$$

where $R_h$ is the future event horizon given by

$$R_h = a\int_t^{\infty} \frac{dt}{a} = a\int_0^{\infty} \frac{da}{Ha^2}$$  \hspace{1cm} (24)$$

By considering the definition of holographic energy density $\rho_\Lambda$, one can find [23, 24]:

$$w_\Lambda = -\left[\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{1}{\sqrt{|k|}} \cos(\sqrt{|k|}R_h/a) + \frac{\Gamma}{3H}\right].$$  \hspace{1cm} (25)$$

where

$$\frac{1}{\sqrt{|k|}}\cos(\sqrt{|k|}x) = \begin{cases} 
\cos(x), & k = 1, \\
1, & k = 0, \\
\cosh(x), & k = -1.
\end{cases}$$  \hspace{1cm} (26)$$

Substituting Eq.(18) into Eq.(16), and using Eq.(6) one can find

$$w_\Lambda^{\text{eff}} = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda} - c^2\Omega_k}{3c}.$$  \hspace{1cm} (27)$$
If we establish the correspondence between the holographic dark energy and generalized Chaplygin gas energy density, then using Eqs. (15, 20) we have

\[ A = (3c^2M_p^2L^{-2})^{1+\alpha} - \frac{B}{a^{3(1+\alpha)}} \]  

(28)

Using definitions \( \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} \) and \( \rho_{cr} = 3M_p^2H^2 \), we get

\[ H L = \frac{c}{\sqrt{\Omega_\Lambda}} \]  

(29)

Now, by comparing the effective equation of states (19, 27) we obtain

\[ A = (3M_p^2H^2\Omega_\Lambda)^{1+\alpha}(\frac{b^2(1+\Omega_k)}{\Omega_\Lambda} + \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} + \frac{1}{3}) \]  

(31)

Substituting the above relation into Eq. (28) we have

\[ B = (3M_p^2H^2\Omega_\Lambda a^3)^{1+\alpha}[1 - (\frac{b^2(1+\Omega_k)}{\Omega_\Lambda} + \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} + \frac{1}{3})] \]  

(32)

3 The correspondence between interacting generalized Chaplygin gas and holographic phantom

For the non-flat universe, the authors of [27] used the data coming from the SN and CMB to constrain the holographic dark energy model, and got the 1 \( \sigma \) fit results: \( c = 0.84^{+0.16}_{-0.03} \). If we take \( c = 0.84 \), and taking \( \Omega_\Lambda = 0.73, \Omega_k = 0.01 \) for the present time, using Eq. (27) we obtain \( w_\Lambda^{eff} = -1.007 \). Also for the flat case, the X-ray gas mass fraction of rich clusters, as a function of redshift, has also been used to constrain the holographic dark energy model. The main results, i.e. the 1 \( \sigma \) fit values for \( c \) is: \( c = 0.61^{+0.45}_{-0.21} \), in this case also we obtain \( w_\Lambda^{eff} < -1 \). This implies that one can generate phantom-like equation of state from an interacting holographic dark energy model in flat and non-flat universe only if \( c \leq 0.84 \). It must be pointed out that the choice of \( c \leq 0.84 \), on theoretical level, will bring some troubles. The Gibbons-Hawking entropy will thus decrease since the event horizon shrinks, which violates the second law of thermodynamics as well. However, the current observational data indicate that the parameter \( c \) in the holographic model seems smaller than 1. Now we reconstruct the phantom potential and the dynamics of the scalar field in light of the holographic dark energy with \( c \leq 0.84 \). According to the following forms of phantom energy density and pressure

\[ \rho_\Lambda = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \]  

(33)

\[ P_\Lambda = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \]  

(34)

\footnote{As one can see in this case the \( A \) and \( B \) can change with time. Similar situation can arise when the cosmological constant has dynamic, see for example eq. (12) of [16], (see also [25]), according to this equation

\[ A = \Lambda(\Lambda + \rho_m) \]  

(30)

therefore, if \( \Lambda \) vary with time [26], \( A \) does not remain constant.}
One can easily derive the scalar potential and kinetic energy term as

$$V(\phi) = \frac{1}{2}(1 - w_\Lambda)\rho_\Lambda$$

(35)

$$\dot{\phi}^2 = -(1 + w_\Lambda)\rho_\Lambda$$

(36)

Differenating Eq.(10) with respect to the cosmic time $t$, one find

$$\dot{H} = \frac{\dot{\rho}}{6HM_p^2} + \frac{k}{a^2}$$

(37)

where $\rho = \rho_m + \rho_\Lambda$ is the total energy density, now using Eqs.(3, 4)

$$\dot{\rho} = -3H(1 + w)\rho$$

(38)

where

$$w = \frac{w_\Lambda \rho_\Lambda}{\rho} = \frac{\Omega_\Lambda w_\Lambda}{1 + \frac{k}{a^2}H^2}$$

(39)

Substituting $\dot{\rho}$ into Eq.(37), we obtain

$$w = \frac{2/3(\frac{k}{a^2} - \dot{H})}{H^2 + \frac{k}{a^2}} - 1$$

(40)

Using Eqs.(39, 40), one can rewrite the holographic energy equation of state as

$$w_\Lambda = -\frac{1}{3\Omega_\Lambda H^2(2\dot{H} + 3H^2 + \frac{k}{a^2})}$$

(41)

Substituting the above $w_\Lambda$ into Eqs.(35, 36), we obtain

$$V(\phi) = \frac{M_p^2}{2}[2\dot{H} + 3H^2(1 + \Omega_\Lambda) + \frac{k}{a^2}]$$

(42)

$$\dot{\phi}^2 = M_p^2[2\dot{H} + 3H^2(1 - \Omega_\Lambda) + \frac{k}{a^2}]$$

(43)

In similar to the [28, 31], we can define $\dot{\phi}^2$ and $V(\phi)$ in terms of single function $f(\phi)$ as

$$V(\phi) = \frac{M_p^2}{2}[2f'(\phi) + 3f^2(\phi)(1 + \Omega_\Lambda) + \frac{k}{a^2}]$$

(44)

$$1 = M_p^2[2f'(\phi) + 3f^2(\phi)(1 - \Omega_\Lambda) + \frac{k}{a^2}]$$

(45)

In the spatially flat case the Eqs.(44, 45) solved only in case of presence of two scalar potentials $V(\phi)$, and $\omega(\phi)$. Here we have claimed that in the presence of curvature term $\frac{k}{a^2}$, Eqs.(44, 45) may be solved with potential $V(\phi)$. Hence, the following solution are obtained

$$\phi = t, \quad H = f(t)$$

(46)

From Eq.(45) we get

$$\frac{k}{a^2} = 3f^2(\phi)(\Omega_\Lambda - 1) - 2f'(\phi) + \frac{1}{M_p^2}$$

(47)
Substituting the above $\frac{k}{\sigma^2}$ into Eq.(44), we obtain the scalar potential as

$$V(\phi) = 3M_p^2\Omega_\Lambda f^2(\phi) + \frac{1}{2}$$  \hspace{1cm} (48)

One can check that the solution (46) satisfies the following scalar field equation

$$-\ddot{\phi} - 3H\dot{\phi} + V'(\phi) = 0$$  \hspace{1cm} (49)

Therefore by the above condition, $f(\phi)$ in our model must satisfy following relation

$$3f(\phi) = V'(\phi)$$  \hspace{1cm} (50)

In the other hand, using Eqs.(20, 16) we have

$$V(\phi) = \frac{3H^2\Omega_\Lambda}{16\pi G}\left[\frac{4}{3} + \frac{2\sqrt{\Omega_\Lambda - c^2\Omega_k}}{3c} + \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda}\right]$$ \hspace{1cm} (51)

$$\dot{\phi} = \frac{H}{2\sqrt{\pi G}}\left[-1 + \frac{\sqrt{\Omega_\Lambda - c^2\Omega_k}}{c} + \frac{3b^2(1 + \Omega_k)}{2\Omega_\Lambda}\right]^{1/2}$$  \hspace{1cm} (52)

Using Eq.(52), we can rewrite Eq.(51) as

$$V(\phi) = 3M_p^2\Omega_\Lambda H^2\left(1 + \frac{\dot{\phi}^2}{6M_p^2H^2\Omega_\Lambda}\right),$$  \hspace{1cm} (53)

or in another form as following

$$V(\phi) = 3M_p^2\Omega_\Lambda [f^2(\phi) + \frac{1}{6M_p^2\Omega_\Lambda}] = 3M_p^2\Omega_\Lambda f^2(\phi) + \frac{1}{2}$$  \hspace{1cm} (54)

which is exactly the result (48).

From Eq.(45) for the flat case we have

$$2f'(\phi) = 3f^2(\phi)(\Omega_\Lambda - 1) + \frac{1}{M_p^2}$$  \hspace{1cm} (55)

By derivative of the above equation respect to $\phi$ we obtain

$$2f''(\phi) = 6ff'(\phi)(\Omega_\Lambda - 1) + 3f^2\Omega_\Lambda'$$ \hspace{1cm} (56)

then

$$\Omega_\Lambda' = \frac{2f''}{3f} + \frac{2f'}{f}(1 - \Omega_\Lambda)$$ \hspace{1cm} (57)

Now using Eqs. (50), (54) we have

$$2ff' + f\Omega_\Lambda' = \frac{1}{M_p^2}$$ \hspace{1cm} (58)

Substituting $\Omega_\Lambda'$ from Eq.(57) into the above equation we obtain

$$2f'' + 6ff' - \frac{3f}{M_p^2} = 0$$ \hspace{1cm} (59)
Therefore, \( f(\phi) \) must satisfy the above equation in flat case. Using Maple software one can obtain following relation

\[
\int \frac{f(\phi)}{W(c_1 e^{-(3x^2+1)}) + 1} = \phi + c_2
\]

where \( W \) is the Lambert \( W \)-function.\(^2\)

4 Squared speed for generalized Chaplygin gas and interacting holographic dark energy

Here we introduce the squared speed of generalized Chaplygin gas as

\[
v_g^2 = \frac{dP_\Lambda}{d\rho_\Lambda}
\]

Using Eq.(14), we have

\[
v_g^2 = \frac{A\alpha}{\rho^{\alpha+1}}
\]

For \( \alpha < 0, A > 0 \) or \( \alpha > 0, A < 0 \) (see recent paper [33]) ; \( v_g^2 < 0 \), in this cases the generalized Chaplygin gas model is instable (see also [34]). The squared speed of interacting holographic dark energy fluid is as

\[
v_\Lambda^2 = \frac{dP_\Lambda}{d\rho_\Lambda} = \frac{\dot{P}_\Lambda}{\rho_\Lambda}
\]

where

\[
\dot{P}_\Lambda = \dot{w}_{\Lambda}^{\text{eff}} \rho_\Lambda + w_{\Lambda}^{\text{eff}} \rho_\Lambda
\]

with

\[
\dot{w}_{\Lambda}^{\text{eff}} = H \frac{dw_{\Lambda}^{\text{eff}}}{dx}
\]

\(^2\)Consideration of Lambert \( W \) function can be traced back to J. Lambert around 1758, and later, it was considered by L. Euler but it was recently established as a special function of mathematics on its own[29]. The Lambert \( W \) function is defined to be the function satisfying

\[
W[z] e^{W[z]} = z
\]

It is a multivalued function defined in general for \( z \) complex and assuming values \( W[z] \) complex. If \( z \) is real and \( z < -1/e \), then \( W[z] \) is multivalued complex. If \( z \) is real and \( -1/e \leq z \leq 0 \), there are two possible real values of \( W[z] \). The one real value of \( W[z] \) is the branch satisfying \( \leq -1W[z] \), denoted by \( W_0[z] \), and it is called the principal branch of the \( W \) function. The other branch is \( W[z] \leq -1 \) and is denoted by \( W_{-1}[z] \). If \( z \) is real and \( z \geq 0 \), there is a single real value for \( W[z] \) which also belongs to the principal branch \( W_0[z] \). Special values of the principal branch of the Lambert \( W \) function are \( W_0[0] = 0 \) and \( W_0[-1/e] = -1 \). The Taylor series of \( W_0[z] \) about \( z = 0 \) can be found using the Lagrange inversion theorem and is given by [30]

\[
W[z] = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^{-2}}{(n-1)!} z^n = z - z^2 + \frac{3}{2} z^3 - \frac{8}{3} z^4 + \frac{125}{24} z^5 - \frac{54}{5} z^6 + \ldots
\]

The ratio test establishes that this series converges if \( |z| < 1/e \).
where $x = Lna$. Using Eq.(27) and following equation

$$\frac{d\Omega_A}{dx} = \frac{\dot{\Omega}_A}{H} = 3\Omega_A(1 + \Omega_k - \Omega_\Lambda)[\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c}\frac{1}{\sqrt{|k|}}\cos(n\sqrt{|k| R_h/a})]$$

(68)

We can write

$$\dot{w}_A^{eff} = \frac{-H}{c\sqrt{\Omega_\Lambda} - c^2\Omega_k}\Omega_A(1 + \Omega_k - \Omega_\Lambda)[\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c}\frac{1}{\sqrt{|k|}}\cos(n\sqrt{|k| R_h/a})]$$

(69)

Where we have assumed $\frac{d\Omega_\Lambda}{dx} = 0$. Substituting the above equation and Eq.(38) into Eq.(65) we obtain (to see the non-interacting case relation refer to [34])

$$v_\Lambda^2 = w_\Lambda^{eff} - \frac{\dot{w}_A^{eff}}{3H(1 + w)}$$

(70)

From Eq.(69) one can see that $\dot{w}_A^{eff} < 0$, also as we have mentioned in section 3 if we take $c = 0.84$, and taking $\Omega_\Lambda = 0.73$, $\Omega_k = 0.01$ for the present time, we obtain $w_\Lambda^{eff} = -1.007$. One can see from Eq.(40) that in the phantom phase where $\dot{H} > 0$, $w + 1 < 0$, hence we obtain a negative value for squared speed of interacting holographic fluid. Due to this the interacting holographic fluid similar to generalized Chaplygin gas is instable. In a recent paper Myung [34] has shown that the perfect fluid for holographic dark energy is classically unstable, our result show interacting fluid of holographic dark energy is also unstable. However, in contrast to the Chaplygin gas fluid where the squared speed is always non-negative, for the generalized Chaplygin gas may be one can obtain negative value for the squared speed. Hence the holographic interpretation for generalized Chaplygin gas in contrast with Chaplygin gas is not problematic.

5 Conclusions

In order to solve cosmological problems and because the lack of our knowledge, for instance to determine what could be the best candidate for DE to explain the accelerated expansion of universe, the cosmologists try to approach to best results as precise as they can by considering all the possibilities they have. Within the different candidates to play the role of the dark energy, the Chaplygin gas, has emerged as a possible unification of dark matter and dark energy, since its cosmological evolution is similar to an initial dust like matter and a cosmological constant for late times. Inspired by the fact that the Chaplygin gas possesses a negative pressure, people [32] have undertaken the simple task of studying a FRW cosmology of a universe filled with this type of fluid.

In this paper, by considering an interaction between generalized Chaplygin gas energy density and CDM, we have obtained the equation of state for the interacting generalized Chaplygin gas energy density in the non-flat universe. Then we have considered a correspondence between the holographic dark energy density and interacting generalized Chaplygin gas energy density in FRW universe. Then we have reconstructed the potential of the scalar field which describe the generalized Chaplygin cosmology. Also we calculated the squared speeds of sound for generalized Chaplygin gas and interacting holographic fluid, then we have shown that interacting holographic fluid similar to the generalized Chaplygin gas is instable.
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