Open charm effects in $e^+e^- \rightarrow J/\psi \eta$, $J/\psi \pi^0$ and $\phi \eta_c$.  

Qian Wang$^1$, Xiao-Hai Liu$^1$, and Qiang Zhao$^{1,2}$  

1) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P.R. China  
2) Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China  

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We propose to study the open charm effects in $e^+e^- \rightarrow J/\psi \eta$, $J/\psi \pi^0$ and $\phi \eta_c$. We show that the exclusive cross section lineshapes of these processes would be strongly affected by the open charm effects. Since the final state light meson productions are through soft gluon radiations, we assume a recognition of this soft process via charmed meson loops at hadronic level. A unique feature among these three reactions is that the $DD^* + c.c.$ open channel is located in a relatively isolated energy, i.e. $\sim 3.876$ GeV, which is sufficiently far away from the known charmonia $\psi(3770)$ and $\psi(4040)$. Therefore, the cross section lineshapes of these reactions may provide an opportunity for singling out the open charm effects with relatively well-defined charmonium contributions. In particular, we find that reaction $e^+e^- \rightarrow J/\psi \pi^0$ is sensitive to the open charm $DD^* + c.c.$ Due to the dominance of the isospin-0 component at the charmonium energy region, we predict an enhanced model-independent cusp effect between the thresholds of $D^0 \bar{D}^0 + c.c.$ and $D^+ D^{-} + c.c.$ This study can also help us to understand the $X(3900)$ enhancement recently observed by Belle Collaboration in $e^+e^- \rightarrow D \bar{D} + c.c.$  

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I. INTRODUCTION  

During the past years the progress in experiment in the study of hadron spectroscopy has brought a lot of surprises. In the charmonium sector, a number of new resonance-like signals have been observed by the B-factories [1]. These observations have not only initiated tremendous interests in their nature, but also revived the efforts on the search for exotics in both experiment and theory (e.g. see Refs. [2, 3] for a recent review on these issues.  

Although various theoretical prescriptions have been proposed in order to understand the underlying dynamics for the production and decay of these new “resonances”, such as hybrid charmonium, tetraquark, baryonium, and hadronic molecule, an interesting feature with those observed resonance-like signals is that most of them are close to open charm thresholds. An example is the $X(3872)$ which is located in the vicinity of $D^0 \bar{D}^0$. Because of this, a molecular prescription has been broadly investigated in the literature. An alternative view is its indication of the underlying non-perturbative mechanisms arising from open charm thresholds.  

Phenomenologically, the open channel effects may have more general dynamical implications. They would allow different partial waves to contribute in exclusive processes. In contrast, the hadronic molecule scenario would require a relative $S$ wave between the interacting hadrons. On the other hand, an additional $q\bar{q}$ pair creation near the open channel is highly non-perturbative. Therefore, this non-perturbative mechanism would play an important role near the open heavy-flavor threshold to shift the hadrons’ masses nearby, and change their wavefunctions and decay properties. It is realized that a better understanding of the open channel effects would be a prerequisite for our ultimate understanding of the hadron spectroscopies.  

In this work we shall use an effective Lagrangian approach based on the heavy quark symmetry and chiral symmetry [6, 12] to study the open charm effects in $e^+e^- \rightarrow J/\psi \eta$, $J/\psi \pi^0$ and $\phi \eta_c$. Our motivation is based on the following points: i) In $e^+e^- \rightarrow D \bar{D} + c.c.$, it is observed by Belle Collaboration [13] that an enhancement around 3.9 GeV, i.e. $X(3900)$, of which the nature is unclear. As we know, the vector charmonium ($J^{PC} = 1^{--}$) spectrum has been better established since the charmonium states can be directly produced via the time-like virtual photon in $e^+e^-$ annihilations. Therefore, the observation of the enhancement $X(3900)$ provides an ideal place to investigate the underlying dynamics beyond the known charmonium spectrum. ii) In Ref. [8], the $DD^* + c.c.$ open charm effects are investigated and seem to provide a natural explanation for the $X(3900)$ enhancement without introducing any exotic components. In order to confirm the nature of the $X(3900)$, one should investigate other possible reflections of such a mechanism. iii) Note that the position of the $X(3900)$ is located between the known $\psi(3770)$ and $\psi(4040)$, its coupling to $J/\psi \eta$ and isospin-violating $J/\psi \pi^0$ would receive relatively small interferences from the nearby resonances.  

Apart from this anticipation, we also consider the $\phi \eta_c$ channel, of which the threshold is very close to the $DD^* + c.c.$ Therefore, peculiar threshold effects due to the open $DD^* + c.c.$ might be detectable in $e^+e^- \rightarrow J/\psi \eta$, $J/\psi \pi^0$ and $\phi \eta_c$. Although the experimental measurement of these three exclusive channels are not available, the CLEO Collaboration recently provide an upper limit of the cross sections for $e^+e^- \rightarrow J/\psi \eta$ and $J/\psi \pi^0$ [14], which would be a guidance for us to examine the open charm effects in the vector charmonium excitations.  

This paper is organized as below. In Sec. II we present the effective Lagrangian approach with formulae. The parameters are fitted in Sec. III. Section IV is devoted to numerical results and discussions. The summary is given.
in the last section.

II. FORMULAE

The effective Lagrangian approach has been successfully applied to various charmonium decay processes as one of the most important non-perturbative mechanisms in order to explain some of the long-standing puzzles in the charmonium energy region. For instance, it was shown that the open charm coupled-channel effects would lead to sizeable non-$D\bar{D}$ decay branching ratios for $\psi(3770)$ \cite{3}, and account for the large breaking of the helicity selection rule in charmonium decays \cite{3,10,12}. They are written as follows:

$$\mathcal{L}_2 = ig_2Tr[R_{c\bar{c}}\delta_{i\alpha}H_{ci}] + H.c.,$$

where the $S$-wave charmonium states are expressed as

$$R_{c\bar{c}} = \left(\frac{1 + \gamma_5}{2}\right) (\psi^{\mu\gamma_\mu} - \eta_c) \left(\frac{1 - \gamma_5}{2}\right),$$

and the charmed and anti-charmed meson triplet are

$$H_{ci} = \left(\frac{1 + \gamma_5}{2}\right) [D_i^{\mu*}\gamma_\mu - D_i]\gamma_5],$$

$$H_{2ci} = [D_i^{\mu*}\gamma_\mu - D_i]\gamma_5],$$

where $D$ and $D^*$ are the pseudoscalar charmed mesons ($(D_0^0, D_{+}^+, D_{s}^{*+})$) and vector charmed mesons ($(D^{*0}, D^{*+}, D_{s}^{*+})$), respectively. Additionally, the $J/\psi$ and $\eta_c$, and $D^*$ and $D$, can be considered as doublet states based on the heavy quark spin symmetry. The Lagrangian describing the interactions between light meson and charmed mesons reads

$$\mathcal{L} = iTr[H_\mu v^\mu D_{\mu\nu}\bar{H}_{\nu}] + igTr[H_\mu v^\mu v_{\mu\nu}(V_{\mu} - \rho_{\mu})_{ij}\bar{H}_{\nu} + i\beta Tr[H_i v^\mu (V_{\mu} - \rho_{\mu})_{ij}\bar{H}_{\nu} + i\lambda Tr[H_i\sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ij}\bar{H}_{\nu}]],$$

where the operator $A_{\mu} = \frac{1}{2}(\xi^\mu \partial^\mu \xi - \xi \partial^\mu \xi^\mu)$ with $\xi = \sqrt{\Sigma} = e^{\mu\nu} \Sigma$, and $F_{\mu\nu}(\rho) = \partial_{\mu}\rho_{\nu} - \partial_{\nu}\rho_{\mu} + [\rho_{\mu}, \rho_{\nu}]$. $M$ and $\rho$ denote the light pseudoscalar octet and vector nonet, respectively \cite{15,16}:

$$M = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^+}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\
-\frac{\pi^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{K^0}{\sqrt{2}} \end{pmatrix},
\rho = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \rho^+ & K^{*+} \\
-\frac{\rho^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 & \phi \end{pmatrix}.$$ (5)

To keep the same convention as Eq. (3), the superfield $H$ is defined as below \cite{17}:

$$H_i = \left(\frac{1 + \gamma_5}{2}\right) [D_i^{\mu*}\gamma_\mu - D_i]\gamma_5],$$

$$\bar{H}_i = [D_i^{\mu*}\gamma_\mu + D_i\gamma_5]\left(\frac{1 - \gamma_5}{2}\right).$$ (6)

Substituting Eq. (3) to Eq. (6), it is easy to obtain the detailed form of the interactions to the leading order \cite{15}:

$$\mathcal{L} = -g_{D^*D^*P} (D^\dagger \partial^\mu P_{ij}D^\mu + D^\dagger \partial^\mu P_{ij}D^\mu) + \frac{1}{2}g_{D^*D^*P} \epsilon_{\mu\nu\rho\sigma} D_i^{\mu*} \partial^\rho P_{ij}^\nu \partial^\sigma D_j^{\dagger *},$$

$$-ig_{D^*D^*V} \partial_{\mu} D_j^{\nu*} (V_{\mu}^i)^{\dagger} - 2if_{D^*D^*V} \epsilon_{\mu\nu\rho\sigma} (\partial^\rho V^i)^{\dagger} (D^\dagger \partial^\nu D_j^{\nu*} + D^j^{\nu*} \partial^\rho D_i^{\dagger})$$

$$+ ig_{D^*D^*V} D_i^{\nu*} \partial_{\mu} D_j^{\nu*} (V_{\mu}^i)^{\dagger} + 4if_{D^*D^*V} \epsilon_{\mu\nu\rho\sigma} (\partial^\rho V^i)^{\dagger} \partial^\nu V^i D_j^{\nu*},$$ (7)
where $\epsilon_{\alpha\beta\mu\nu}$ is the Levi-Civita tensor and $D$ meson field destroys a $D$ meson.

The kinematics for a typical transition of Fig. 1 are defined as: $e^+(k_2)e^-(k_1) \to D(p_1)\bar{D}(p_2)[D(p_3)] \to J/\psi(k)\eta(q)$ ($J/\psi(k)\pi^+(q)$, $\phi(k)\eta(q)$), with $k_1$, $k_2$, $k$, and $q$, the four-vector momenta of the corresponding particles, respectively. Consequently, the transition amplitude for an intermediate vector charmonium $\psi$ can be expressed as:

$$M = \bar{v}(k_2)e\gamma^\mu u'(k_1)\frac{-g_{\mu\nu}v_m^2\epsilon^{\nu\nu}}{s-m_\psi^2} + \frac{\epsilon^{\nu\nu}}{m_\psi^2}\sum_{i=0}^f M_{i\alpha},$$

where $M_{i\alpha}$ could be a complex number serving as a vertex function for the intermediate $\psi$ coupling to final state $VP$ via $D$ meson loops. For the processes of Fig. 1, $M_{i\alpha}$ has the following expressions:

$$M_{i\alpha} = -\int\frac{d^4p_3}{(2\pi)^4}2g_{J/\psi\bar{D}D}\bar{g}_\nu\bar{g}_\rho\bar{g}_\sigma\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\nu}g_{\mu\nu}p_{1\alpha}^\lambda p_{2\nu}^\rho F(p_3),$$

$$M_{b\alpha} = -\int\frac{d^4p_3}{(2\pi)^4}2g_{J/\psi\bar{D}D}\bar{g}_\nu\bar{g}_\rho\bar{g}_\sigma\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\nu}g_{\mu\nu}p_{1\alpha}^\lambda p_{2\nu}^\rho F(p_3),$$

$$M_{e\alpha} = -\int\frac{d^4p_3}{(2\pi)^4}g_{J/\psi\bar{D}D}\bar{g}_\nu\bar{g}_\rho\bar{g}_\sigma\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\nu}g_{\mu\nu}p_{1\alpha}^\lambda p_{2\nu}^\rho F(p_3),$$

$$M_{d\alpha} = -\int\frac{d^4p_3}{(2\pi)^4}g_{J/\psi\bar{D}D}\bar{g}_\nu\bar{g}_\rho\bar{g}_\sigma\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\nu}g_{\mu\nu}p_{1\alpha}^\lambda p_{2\nu}^\rho F(p_3),$$

$$M_{f\alpha} = \int\frac{d^4p_3}{(2\pi)^4}g_{J/\psi\bar{D}D}\bar{g}_\nu\bar{g}_\rho\bar{g}_\sigma\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\nu}g_{\mu\nu}p_{1\alpha}^\lambda p_{2\nu}^\rho F(p_3),$$

with $a_1 \equiv p_1^2 - m_1^2$, $a_2 \equiv p_2^2 - m_2^2$ and $a_3 \equiv p_3^2 - m_3^2$. As we know, the meson loop integrals have ultra-violet divergence. To cut off the unphysical contributions in the high momentum transfers, we introduce a form factor as broadly applied in the literature. A typical dipole form factor for the integrals is as follows:

$$F(p_3^2) = \left(\frac{\Lambda^2 - m_3^2}{\Lambda^2 - p_3^2}\right)^2,$$

where $\Lambda$ is the cutoff energy and can be parameterized as $\Lambda = m + \alpha\Lambda_{QCD}$ with $m$ the mass of the exchanged particle and $\Lambda_{QCD} = 220$ MeV.

In this study, we include five resonances, i.e. $J/\psi$, $\psi(3686)$, $\psi(3770)$, $\psi(4040)$, and $\psi(4160)$. Thus, the total transition amplitude can be expressed as

$$M = M_{J/\psi} + M_{\psi(3686)} + e^{i\theta}M_{\psi(3770)} + e^{i\beta}M_{\psi(4040)} + e^{i\phi}M_{\psi(4160)},$$

where $\theta$, $\beta$, and $\phi$ are the relative phase angles which can be determined by experimental data.

In Eq. (8), the dimensionless vector charmonia couplings to the virtual photon, $e/f_V$, can be determined by the VMD model in $V \to e^+e^-$:

$$\frac{e}{f_V} = \left[\frac{3\Gamma_{Y\to e^+e^-}}{2\alpha_e|p_e|}\right]^{1/2},$$
where $\Gamma_{\psi \rightarrow e^+e^-}$ is the vector meson partial decay width to $e^+e^-$, and $p_\psi$ is the three-vector momentum of the final state electron in the vector meson rest frame. With the partial decay widths from the Particle Data Group [18], the couplings $e/f_V$ for those low-lying charmonia are listed in Table I.

![Diagram](image)

**FIG. 1**: Schematic diagrams for $e^+e^- \rightarrow J/\psi \pi^0$ via charmed $D (D^*)$ meson loops. The diagrams for the $\phi \eta_c$ mode are similar.

| $V \rightarrow e^+e^-$ | Partial decay widths (keV) | $e/f_V$ |
|------------------------|----------------------------|---------|
| $J/\psi \rightarrow e^+e^-$ | 5.55 | $2.71 \times 10^{-2}$ |
| $\psi(3686) \rightarrow e^+e^-$ | 2.38 | $1.63 \times 10^{-2}$ |
| $\psi(3770) \rightarrow e^+e^-$ | 0.26 | $5.4 \times 10^{-3}$ |
| $\psi(4040) \rightarrow e^+e^-$ | 0.86 | $9.35 \times 10^{-3}$ |
| $\psi(4160) \rightarrow e^+e^-$ | 0.83 | $9.06 \times 10^{-3}$ |

It should be noted that the $V D^*\bar{D}^*$ coupling consists of two terms with the relative angular momentum $L = 1$ between $D^*$ and $\bar{D}^*$, i.e. $\epsilon_V \cdot \epsilon_{D^*} \cdot (k-q) + \epsilon_{D^*} \cdot \epsilon_V \cdot (k-q)$. The coefficients of these two terms are universal for $J/\psi D^*\bar{D}^*$, i.e. $g_{J/\psi D^*\bar{D}^*}$, while for the $\phi D^*\bar{D}^*$, the coupling structure is $4f_{D^*\bar{D}^* V}(\epsilon_V \cdot \epsilon_{D^*} \cdot (k-q) + \epsilon_{D^*} \cdot \epsilon_V \cdot (k-q)) - g_{D^*\bar{D}^* V}\epsilon_{D^*} \cdot \epsilon_{D^*} \epsilon_V \cdot (k-q)$. Here $k$ and $q$ are the incoming four momentum of $D^*$ and $\bar{D}^*$, respectively. The total spin of $D^*\bar{D}^*$ system in the first term is $S = 2$, but $S = 0$ in the second term. These two couplings, $f_{D^*\bar{D}^* V}$ and $g_{D^*\bar{D}^* V}$, are equal to each other due to the heavy quark spin symmetry for a quarkonium coupling to the charmed mesons.

### III. PARAMETERS

The charmonium couplings to the charmed mesons are extracted under the SU(3) flavor symmetry and heavy quark symmetry [12, 13]:

$$
\begin{align*}
    g_{\psi D^* D^*} &= \frac{g_{\psi D D}}{M_D}, \quad g_{\psi D^* D^*} = g_{\psi D D} \sqrt{\frac{m_{D^*}}{m_D} m_{D^*}}, \quad M_D = \sqrt{m_D m_{D^*}}, \\
    g_{\eta_c D^* D^*} &= \frac{g_{\eta_c D D}}{m_{\eta_c}} \sqrt{\frac{m_{D^*}}{m_{D^*} m_{D^*}}}, \quad g_2 = \frac{\sqrt{m_{\psi}}}{2 m_D f_\psi},
\end{align*}
$$

(13)
where $m_\psi$ and $f_\psi$ are the mass and decay constant of $J/\psi$ with $f_\psi = 405$ MeV. Since $J/\psi$ and $\psi(3686)$ are below the $D\bar{D}$ threshold, their couplings to $D\bar{D}$ cannot be directly measured by experiment. We adopt $g_{J/\psi DD} = 7.44$ which is from the VMD model \cite{13, 20}. The couplings of $\psi'(\psi(3770))$ to $D(\bar{D})D(\bar{D})$ have been extracted from the cross section lineshape of $e^+e^- \rightarrow D\bar{D}$ in Ref. \cite{3}, where quite significant isospin violation effects are found with the couplings, i.e. $g_{\psi'\psi p\bar{p}} = 9.05 \pm 2.34$, $g_{\psi'\bar{D}-\bar{D}^+} = 7.72 \pm 1.02$, $g_{\psi'(3770)\bar{D}p\bar{p}} = 13.58 \pm 1.07$, $g_{\psi'(3770)\bar{D}+\bar{D}^-} = 10.71 \pm 1.75$. Meanwhile, one notices that these extracted values still possess large uncertainties due to the relatively poor status of the experimental data \cite{13, 21, 22}.

In general, such isospin breaking contributions will bring model-dependence to the predictions for the open charm effects. Since we are still lacking experimental observables to constrain these parameters, we assume that to leading order the couplings between $\psi'(\psi(3770))$ and the charged and neutral charmed mesons are the same. Namely, we take the average values for these couplings, i.e. $g_{\psi'DD} = (9.05 + 7.72)/2 = 8.4$, and $g_{\psi'(3770)DD} = (13.58 + 10.71)/2 = 12.1$. By requiring that the cross sections of $e^+e^- \rightarrow \psi' \rightarrow J/\psi \eta$ agree with the experimental data, we can determine the form factor parameter $\alpha$, which can be then fixed in the predictions for $e^+e^- \rightarrow J/\psi \pi^0$ and $\phi \eta_c$. We also take the average value of charged and neutral ones for strange-charmed mesons. The other couplings between charmonium and charmed mesons can be obtained from Eq. \cite{15}.

To evaluate the couplings of the $\psi(4040)$ to the charmed mesons, we assume that the phase space allowed $D^*\bar{D}\bar{D}^*$ and $D^*\bar{D}\bar{D}^*$ modes account for the total width of $\psi(4040)$. BaBar Collaboration measured the branching ratio fractions of these channels \cite{24}, i.e. $Br(\psi(4040) \rightarrow D^*\bar{D})/Br(\psi(4040) \rightarrow D\bar{D}) = 0.24 \pm 0.05 \pm 0.12$ and $Br(\psi(4040) \rightarrow D\bar{D}^*)/Br(\psi(4040) \rightarrow D^*\bar{D}) = 0.18 \pm 0.14 \pm 0.03$. The center values are used here to extract the coupling $g_{\psi(4040)DD} = 2.02$, $g_{\psi(4040)D^*\bar{D}} = 4.24$ and $g_{\psi(4040)D\bar{D}^*} = 1.6$ GeV$^{-1}$. The couplings of the $\psi(4160)$ to the charmed mesons can be obtained in the same way using the data, $Br(\psi(4160) \rightarrow D^*\bar{D})/Br(\psi(4160) \rightarrow D\bar{D}^*) = 0.02$ and $Br(\psi(4160) \rightarrow D\bar{D}^*)/Br(\psi(4160) \rightarrow D^*\bar{D}) = 0.34$ from Ref. \cite{24}. It gives $g_{\psi(4160)DD} = 0.53$, $g_{\psi(4160)D^*\bar{D}} = 0.71$ GeV$^{-1}$ and $g_{\psi(4160)D\bar{D}^*} = 3.08$. This can be regarded as a reasonable way to extract the couplings of $\psi(4040)$ and $\psi(4160)$.

For the light meson couplings to the charmed mesons, they are determined as those in Refs. \cite{8, 15}:

\begin{align}
    g_{D^*\bar{D}V} &= \frac{2g}{f_\pi} \sqrt{m_{D^*}m_{\bar{D}}}, \\
    g_{D\bar{D}^*V} &= \frac{g_{D\bar{D}^*}}{\sqrt{m_{D^*}m_{\bar{D}}}}, \\
    g_{DDV} &= \frac{g_{DDV}}{\sqrt{2}}, \\
    g_{D^*\bar{D}\bar{D}^*} &= \frac{g_{D^*\bar{D}\bar{D}^*}}{m_{D^*}} = \frac{\lambda g_{D^*\bar{D}V}}{\sqrt{2}}, \\
    g_{D\bar{D}^*\bar{D}} &= \frac{g_{D\bar{D}^*\bar{D}}}{m_{D^*}} = \frac{\lambda g_{D\bar{D}^*V}}{\sqrt{2}}.
\end{align}

where $g = 0.59$, $\beta = 0.9$, $\lambda = 0.56$ GeV$^{-1}$ and $f_\pi = 132$ MeV. Since the SU(3) flavor symmetry works well at leading order in this energy scale, we adopt the following relations: $g_{D^*\bar{D}^*(uu)} = g_{D^*\bar{D}^*-(dd)} = g_{D^*\bar{D}^*(ss)}$, and $g_{D\bar{D}^*(ss)} = g_{D^*\bar{D}^*(qq)} = 0$, where $qq$ stands for a non-strange light quark-antiquark pair. For the pion coupling, $g_{\pi DD} = \sqrt{2}g_{D^*\bar{D}^*(qq)(0\gamma)}$ is employed.

The flavor wavefunctions of $\eta$ and $\eta'$ are as below,

\begin{align}
    \eta &= \cos \alpha_p |n\bar{n}i\rangle - \sin \alpha_p |s\bar{s}\rangle, \\
    \eta' &= \sin \alpha_p |n\bar{n}i\rangle + \cos \alpha_p |s\bar{s}\rangle,
\end{align}

where $|n\bar{n}i\rangle \equiv |u\bar{u} + d\bar{d}|/\sqrt{2}$ and $\alpha_p \equiv \theta_p + \arctan \sqrt{2}$ with $\theta_p = -19.1^\circ$ \cite{8}.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this Section we present the calculated cross sections for $e^+e^- \rightarrow J/\psi \eta$, $J/\psi \pi^0$, and $\phi \eta_c$ in terms of the c.m. energy $W$. Five charmonium states are included, i.e. $J/\psi$, $\psi(3686)$, $\psi(3770)$, $\psi(4040)$ and $\psi(4160)$, which are the main resonance contributions in the energy region that we are interested in. As pointed out in the Introduction, these processes are highly non-perturbative near threshold, which gives rise to the contributions from the vector charmonia via the charmed meson loops as a natural mechanism to evade the OZI rule.

Since there are no data available to constrain the relative phases among the resonance transition amplitudes, we shall examine several phase combinations to test the sensitivities of the cross sections to the relative phases. Since the contribution from $J/\psi$ is negligibly small, we simply take it in phase with the $\psi(3686)$. The other resonance amplitudes can shift phases in respect of the $\psi(3686)$. Apart from the phase angles, the only parameter left is the form factor parameter $\alpha = 1.57$, which is fixed by the cross section $\sigma(e^+e^- \rightarrow \psi' \rightarrow J/\psi \eta) = 8351.5 \times 3.28\% = 274$ nb at the mass of $\psi'$, with the $\psi'$ production cross section $\sigma(e^+e^- \rightarrow \psi') = 8351.5$ nb, and $BR(\psi' \rightarrow J/\psi \eta) = 3.28\%$ \cite{15}. As broadly applied in the literature, the form factor should cut off the unphysical contributions in the region sufficiently
far away from the singularity. We shall discuss later that the introduction of form factors may cause unphysical thresholds which should be distinguished from the physical ones. With the other coupling parameters fixed in the previous Section, the calculated cross sections in the first scheme are presented in Figs. 2 and 3 for $e^+e^- \to J/\psi\eta, J/\psi\pi^0$ and $\phi\eta_c$, respectively.

Our main results are summarized as follows:

i) The dominant contributions are from the $\psi(3686)$ in $e^+e^- \to J/\psi\eta$ and $J/\psi\pi^0$. Although the $\psi(3686)$ is below the $\phi\eta_c$ threshold, it still plays an important role in $e^+e^- \to \phi\eta_c$. It is because the mass of the $\psi(3686)$ is close to the energy region considered here, and the coupling constant of $\psi(3686)$ to the virtual photon is two times larger than that of the $\psi(3770)$. In contrast, although the coupling constants of $\psi(4040)$ and $\psi(4160)$ to the virtual photon are compatible with that of the $\psi(3770)$, their couplings to the charmed mesons are rather small. Thus, they give relatively small contributions to the cross sections.

ii) As shown by Figs. 2 and 3, the lineshape of the $\psi(3770)$ is shifted significantly by the $D\bar{D}^*$ threshold. It is an evidence that the open charm coupled-channel effects would shift the lineshape of the particle nearby. The same phenomenon appears in $e^+e^- \to \phi\eta_c$ process at the energy about 4.08 GeV due to the $D_1^+ D_1^-$ threshold as illustrated in Fig. 4. It can be understood, when amplitudes of different meson loops are added together, it would make the vector charmonium contributions non-trivial. In particular, since masses of the thresholds of the open charmons are different, the open charm effects that distort the Breit-Wigner would then be highlighted.

iii) In Figs. 2 and 3 the open $D^{(s)}\bar{D}^{(s)}$ thresholds are explicitly denoted. As mentioned earlier, the introduction of form factors may cause unphysical thresholds in the cross section lineshape. Thus, it is necessary to clarify this in order to correctly understand the calculated results.

It shows that the dipole form factor of Eq. (10) should be more suitable for the study of the cross section lineshape and would not introduce additional thresholds apart from $m_1 + m_2$. This can be seen from the regularization of the propagators in association with the dipole form factor:

$$\frac{1}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \left( \frac{\Lambda_2^2 - m_2^2}{\Lambda_2^2 - p_2^2} \right)$$

$$\sim C(s, m_1^2, m_2^2, m_3^2) - C(s, m_4^2, m_5^2, m_6^2, \Lambda_3^2)$$

$$+ \frac{\Lambda_3^2 - m_3^2}{\varepsilon} \left[ C(s, m_4^2, m_5^2, m_6^2, \Lambda_3^2 + \varepsilon) - C(s, m_4^2, m_5^2, m_6^2, \Lambda_3^2) \right],$$

where function $C$ is the three-point function, and $\varepsilon$ is a small quantity.

As a comparison, a tri-monopole form factor will cause unphysical thresholds, namely,

$$F(p_i^2) \equiv \prod_{i=1}^{3} \left( \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - p_i^2} \right),$$

where $m_i$ ($p_i$) is the mass (four momentum) of the exchanged particle, and $\Lambda_i \equiv m_1 + \alpha \Lambda_{QCD}$. In this case, the regularization leads to unphysical thresholds, $m_1 + \Lambda_2, \Lambda_1 + m_2$ and $\Lambda_1 + \Lambda_2$, in the cross section. This reflects the model-dependent feature arising from the form factors. In particular, we point out that the cusp effects caused by these unphysical thresholds would be amplified in $e^+e^- \to J/\psi\pi^0$, although their effects are negligibly small in $e^+e^- \to J/\psi\eta$ and $\phi\eta_c$.

iv) The above analysis helps us to identify model-independent features produced by open charm thresholds. We stress that the isospin violating transitions in $e^+e^- \to J/\psi\pi^0$ would provide a great opportunity for disentangling the open charm effects. Comparing the results of Figs. 2 and 3 we can see that the predicted cross sections for $J/\psi\pi^0$ are greatly suppressed. For $e^+e^- \to J/\psi\pi^0$, since the contributing intermediate vector charmonia are mainly from $\psi$ resonances with isospin 0, the cross sections would have vanished if isospin symmetry were conserved. In Fig. 3 the non-vanishing cross sections are produced by the mass differences (as a result of isospin violation) between the charge and charge-neutral $D (D^*)$ mesons in the intermediate meson loops. Namely, the charged and charge-neutral meson loop amplitudes cannot cancel out completely. As a consequence, a peak (cusp) appears between the thresholds of $D^0\bar{D}^{*0} + c.c.$ and $D^+\bar{D}^{-*} + c.c.$ which stands like a resonance, i.e. so-called $X(3900)$ around 3.876 GeV.

Although the cross sections for both $e^+e^- \to \psi \to J/\psi\eta$ and $J/\psi\pi^0$ are rather sensitive to the relative phases introduced among the transition amplitudes, the peak structure $X(3900)$ has a model-independent feature and can be searched in experiment. More importantly, since the thresholds of $D^0\bar{D}^{*0} + c.c.$ and $D^+\bar{D}^{-*} + c.c.$ are isolated from the known $\psi(3770)$ and $\psi(4040)$, the enhancement here would be a clear evidence for non-resonant peaks in $e^+e^-$ annihilations. In contrast, although the $DD^* + c.c.$ loops have relatively large contributions to the cross sections in $e^+e^- \to J/\psi\eta$, their contributions are submerged by other amplitudes and cannot be indisputably identified in the cross section lineshape. In this sense, the observation of the $X(3900)$ by the Belle Collaboration [3] may have
TABLE II: Comparison of the cross section of $e^+e^- \rightarrow J/\psi\eta$ between the experiment data from CLEO [14] and our results with phase $(\theta, \beta, \phi) = (0, 0, 0)$ and $(\pi, 0, 0)$. The form factor parameter $\alpha = 1.57$ is adopted in the calculation.

| Phase | CLEO [14] | Results with $(0, 0, 0)$ | Results with $(\pi, 0, 0)$ |
|-------|-----------|-------------------------|--------------------------|
|       | $\sigma(e^+e^- \rightarrow J/\psi\eta)$ | $< 29 \text{ pb}$ | $15^{+7}_{-3} \pm 8 \text{ pb}$ | $< 32 \text{ pb}$ |
|       | $3.97 \sim 4.06 \text{ GeV}$ | $3.8 \sim 39 \text{ pb}$ | $28.9 \sim 42.9 \text{ pb}$ | $27.6 \text{ pb}$ |
|       | $4.12 \sim 4.2 \text{ GeV}$ | $0.57 \sim 22.2 \text{ pb}$ | $14.9 \sim 24.2 \text{ pb}$ | $14.9 \text{ pb}$ |
|       | $4.26 \text{ GeV}$ | | | |

TABLE III: Comparison of the cross section of $e^+e^- \rightarrow J/\psi\pi$ between the experiment data from CLEO [14] and our results with phase $(\theta, \beta, \phi) = (0, 0, 0)$ and $(\pi, 0, 0)$. The form factor parameter $\alpha = 1.57$ is adopted.

| Phase | CLEO [14] | Results with $(0, 0, 0)$ | Results with $(\pi, 0, 0)$ |
|-------|-----------|-------------------------|--------------------------|
|       | $\sigma(e^+e^- \rightarrow J/\psi\pi^0)$ | $< 10 \text{ pb}$ | $< 3 \text{ pb}$ | $< 12 \text{ pb}$ |
|       | $3.97 \sim 4.06 \text{ GeV}$ | $(6.67 \sim 28.3) \times 10^{-3} \text{ pb}$ | $(16.4 \sim 20.0) \times 10^{-3} \text{ pb}$ | $1.2 \times 10^{-2} \text{ pb}$ |
|       | $4.12 \sim 4.2 \text{ GeV}$ | $(0.45 \sim 17) \times 10^{-3} \text{ pb}$ | $(6.92 \sim 9.1) \times 10^{-3} \text{ pb}$ | $5.07 \times 10^{-3} \text{ pb}$ |
|       | $4.26 \text{ GeV}$ | | | |

suggested a hint of the open $DD^* + c.c.$ effects in $e^+e^- \rightarrow D\bar{D}$, but should be further investigated in the $J/\psi\pi^0$ channel.

We also note that the $\psi(3686)$ has a predominant contribution to $e^+e^- \rightarrow \psi \rightarrow J/\psi\pi^0$ due to its strong isospin violation couplings via the $D$ meson loops [23, 24]. Such a resonance enhancement should be detectable of which the cross section measurement will provide a calibration for the $X(3900)$ structure.

v) It should be pointed out that this structure as the open charm effect is a collective one from the $DD^* + c.c.$ loops to which all the vector charmonia have contributions. That is why such a $P$ wave configuration between $DD^* + c.c.$ can produce the significant enhancement in $e^+e^- \rightarrow J/\psi\pi^0$. This mechanism is much likely to be different from the $X(3872)$, which has been broadly investigated in the literature as a dynamically generated $DD^* + c.c.$ bound state in a relative $S$ wave.

vi) It is interesting to see the model predictions for $e^+e^- \rightarrow \phi\eta_c$ in Fig. 4. In this case, the physical open charm threshold is $D_{s+}^+D_{s}^- + c.c.$ which causes rather significant cusp effects in the cross section. Since the cusp is close to the $\psi(4040)$ mass, interferences between the open $D_{s+}^+D_{s}^- + c.c.$ and $\psi(4040)$ can be investigated. In addition, although the cross sections exhibit obvious dependence on the relative phases, we can still see some systematic trends in terms of the c.m. $W$.

vii) We emphasize again that the relative phases among the amplitudes would lead to very different predictions for the cross section lineshapes as illustrated by those curves in Figs. 2, 3 and 4. Because of this, it is important to have experimental constraints for the model parameters. As mentioned earlier, there are measurements of the cross sections from the CLEO Collaboration at several energies [14]. Although only the upper limits of the cross sections are provided, it can still give a rough guidance for the parameter ranges adopted in the calculations. As an example, we compare the experimental upper limits with the predicted cross sections with phases $(\theta, \beta, \phi) = (0, 0, 0)$ and $(\pi, 0, 0)$ in Tables I and II for $e^+e^- \rightarrow J/\psi\eta_c$, $J/\psi\pi^0$, respectively. It shows that the predicted cross sections with the adopted parameters are consistent with the so-far available experimental information, and indeed give the correct orders of magnitude of the cross sections.

V. SUMMARY

In summary, we have proposed to study the coupled channel effects in $e^+e^-$ annihilating to $J/\psi\eta_c$, $J/\psi\pi^0$ and $\phi\eta_c$. In particular, we show that the reaction $e^+e^- \rightarrow J/\psi\pi^0$ will be extremely interesting for disentangling the resonance contributions and open charm effects taking the advantage that the open $DD^*$ threshold is relatively isolated from the nearby known charmonia $\psi(3770)$ and $\psi(4040)$. Although we also find that the predicted cross sections are rather sensitive to the model parameters adopted, we clarify that the open charm effects from the $DD^* + c.c.$ channel are rather model-independent. Therefore, it is extremely interesting to search for the predicted enhancement around 3.876 GeV (i.e. $X(3900)$) in experiment. Confirmation of this prediction would allow us to learn a lot about the nature of non-pQCD in the charmonium energy region.
FIG. 2: The predicted cross section for $e^+e^- \rightarrow J/\psi \eta$ in terms of the c.m. energy $W$ with the cutoff parameter $\alpha = 1.57$. The cross sections with different phases, i.e. $(\theta, \beta, \phi) = (0, 0, 0)$, $(0, 0, \pi)$, $(0, \pi, 0)$, $(\phi, 0, \pi)$, $(\pi, \pi, \pi)$, $(\pi, \pi, \pi)$, are presented and denoted by different curves. The vertical lines label the open charm thresholds.

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FIG. 3: The predicted cross section for $e^+e^- \rightarrow J/\psi \pi^0$ in terms of the c.m. energy $W$. The notations are similar to Fig. 2.

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FIG. 4: The predicted cross section for $e^+e^- \rightarrow \phi \eta_c$ in terms of the c.m. energy $W$. The notations are similar to Fig. 2.