Empirical Equations and Poisson’s Ratios for Initial Load-Extension Properties of Some Complex Glass and Aramid Technical Weft Knitted Structures

Bazı Karmaşık Cam ve Aramid Teknik Atkı Örme Kumaşların Başlangıç Yük-Uzama Özellikleri için Ampirik Denklemler ve Poisson Oranları

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Online Erişime Açıldığı Tarih (Available online): 31 Aralık 2019 (31 December 2019)

Bu makaleye atıf yapmak için (To cite this article):
Arif KURBAK, Sinem ÖZTÜRK (2019). Empirical Equations and Poisson’s Ratios for Initial Load-Extension Properties of Some Complex Glass and Aramid Technical Weft Knitted Structures, Tekstil ve Mühendis, 26: 116, 381-405.

For online version of the article: https://doi.org/10.7216/1300759920192611611

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EMPIRICAL EQUATIONS AND POISSON’S RATIOS FOR INITIAL LOAD-EXTENSION PROPERTIES OF SOME COMPLEX GLASS AND ARAMID TECHNICAL WEFT KNITTED STRUCTURES

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Gönderilme Tarihi / Received: 25.04.2019
Kabul Tarihi / Accepted: 02.12.2019

ABSTRACT: This work builds on former research carried out concerning load-extension properties of plain knitted glass technical fabrics. In related former research, it was determined that a load-extension or a load-contraction curve could be considered in three stages of extension or contraction. These stages were a) the extension or contraction of the fabric (the first stage), b) the extension of the yarn along with the change of the shape of the sample (the second stage), and, c) the extension or contraction of the fibres (the third stage). In the same works, theoretical analyses were then provided to explain the first stage of extension or contraction, and thus some simple equations between load and extension and between load and contraction together with Poisson’s ratio were obtained. In obtaining the extension of the loop head curve, the equation of extension of a circular ring had been applied. This formulation enabled the emergence of a method that separates the frictional restraints and/or fabric jamming forces from the experimentally obtained quadratic curve fittings for plain knitted fabrics. Building on those former studies, for the present work, similar experimental studies for the first stages of extensions or contractions, were carried out for some more complex technical weft knitted structures that use E-glass and Aramid yarns. The curve fitting equations are obtained and some empirical equations are given, assuming that the same method of separating frictional restraints and/or fabric jamming forces from quadratic curve fittings also applies for complex structures. Extension and contraction rates are also calculated and discussed further. These empirical equations can, of course, be used in related engineering software.

Keywords: Weft knitted fabric, load-extension, contraction, empirical equations, glass fiber, aramid fiber, initial extension rates, initial contraction rates.

BAZI KARMAŞIK CAM VE ARAMİD TEKNİK AKTI ÖRME KUMAŞLARIN BAŞLANGIÇ YÜK-UZAMA ÖZELLİKLERİ İÇİN AMPİRİK DENKLEMLER VE POİSSON ORANLARI

ÖZET: Bu çalışma, daha önce cam düz örme kumaşların yük-uzama özellikleri üzerine yapılan bir çalışmanın devamını niteliğindedir. Daha önceki çalışmada cam düz örme kumaşlarının yük-uzama ve yük-yandan daralma eğrilerinin üç aşamada gerçekleştigi gösterilmiştir. Bular; a) kumaş uzaması ve yandan daralması (1. aşama), b) iplik uzaması, incelmesi ve deney numunesinin şekillenmesi (2. aşama) ve c) lif uzaması ve incelmesidir (3. aşama). Ayrıca çalışmada, 1. aşamının açıklanması için teorik çalışma yapılmış, yük-uzama, yük-yandan daralma eğrileri için bazı formüller bulunmaktadır ve Poisson oranları hesaplanabilir. Teorik çalışmada ilmek bağı uzaması, ilk defa, dairesel elastik halkanın uzaması problemi kullanılarak hesaplanmıştır. Bu formülasyon, düz örme kumaşlarında deneySEL sonuçlara parabolik regresyon denklemi ve yandan daralma eğrileri için lineer regresyon kullanılarak elde edilmiştir. Bu durum, düz örme kumaşları genel olarak deneySEL sonuçlar parabolik regresyon denklemi ve yandan daralma eğrileri için lineer regresyon kullanılarak elde edilmiştir. Elde edilen parabolik regresyon denklemi daha önceki çalışmada düz örgüler için elde edilen denklemi esas almaktadır, buna göre çalışmada bulunan ‘sürtünme ve iç kuvvetler ile dış kuvvetlerin ayırılması’ metodu kumaşık örgüler için de geçerli olduğu kabul edilmiştir. Bu ampirik formüller kullanılarak yük-uzama oranları ve yük-yandan daralma oranlarının hesaplanabildiği yeni bir metod geliştirilmiş ve ayrıntılı olarak incelenmiştir. Bu yeni çıkarılan ampirik formüller, tabi ki, ilgili mühendislik bilgisayar paket programlarında kullanılabilecektir.

Anahtar Kelimeler: Atkı örme kumaş, yük-uzama eğrileri, enden daralma, ampirik denklem, cam lifleri, aramid lifleri, başlangıç uzama oranları, başlangıç enden daralma oranları

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DOI: 10.7216/1300759920192611611, www.tekstilvemuhendis.org.tr
1. INTRODUCTION

Technical applications of knitted fabrics are increasing day by day. Some technical functions of knitted fabrics depend on the geometry of the employed structure as well as on potential changes in its shape under applied loads. In relation to this, Hepworth [1], MacRory et al [2,3], Popper [4], Shanahan and Postle [5], Hong et al [6] have worked on load-extension properties of plain knitted fabrics in course-wise direction. Hepworth [1], MacRory et al [2,3], Popper [4], Shanahan and Postle [7], Hong et al [6], moreover, have worked on load-extension properties of plain knitted fabrics in wale-wise direction. Most of the aforementioned research, however, applied the non-inflectional elastica theory in modelling the extension of the loop head curve. The lack of those models was that discontinuities occurred between the loop head curve and the rest of the loop curves, hence the models did not completely match the experimental results. Moreover, frictional resistances were not properly considered in those early models. Later on, further research on the geometrical and mechanical properties of technical glass plain knitted fabrics in their dry relaxed [8,9] and loaded [10,11] conditions was conducted by Kurbak. Those works [8,9] pointed out that the dry relaxed technical slack plain knitted fabrics were kept their stable shapes by the effect of frictional restraints. When the fabrics were loaded in the wale-wise direction, the contraction rate of Some Complex Glass and Aramid Technical Weft Knitted Structures

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Figure 1: Plain knitted slack fabric model (Kurbak [11]).

\[
\begin{align*}
\varepsilon_{1W} &= \frac{S_{W10d}}{(c_0-S_{W10})} \frac{T_w}{2R} \\
\varepsilon_{2W} &= \frac{1}{(w_0+B_w)} \left[ -D_w - v(w_0 + 2d) \frac{\sqrt{0.956/W}}{9.672/2} \right] \\
\varepsilon_{1C} &= \frac{1}{(w_0+B_c)} \left[ -D_c + (w_0 + 2d) \frac{\sqrt{1.076/W}}{9.672/2} \right] \\
\varepsilon_{2C} &= -\frac{c_{20d}d}{(c_0-c_{10d})} \frac{T_c}{R} 
\end{align*}
\]

where \(\varepsilon_{1W}\) is the extension rate in the wale-wise loaded samples. \(\varepsilon_{2W}\) is the contraction rate in the course-wise direction under load applied in the wale-wise direction. \(\varepsilon_{1C}\) is the extension rate in the course-wise loaded samples. \(\varepsilon_{2C}\) is the contraction rate in the wale-wise direction under load applied in the course-wise direction.

\[D_w = v(w_0 + 2d) \frac{\sqrt{0.956/W}}{9.672/2}\]

\[F_{SW} = 0.405 \, g\]

\[S_{W10} = \mu [\sin \alpha_{1av} - \cos \alpha_{10}] = 0.08705\]

\[S_{W20} = [1 + \mu \sin \alpha_{1av}] = 1.1954\]

\[D_c = (w_0 + 2d) \frac{\sqrt{1.076/W}}{9.672/2}\]

\[F_{SC} = 0.695 \, g\]

\[S_{C10} = \mu [\cos \alpha_{10} + v(\sin \alpha_{1av} - \cos \alpha_{10})] = 0.1516\]

\[S_{C20} = [v + \mu \sin \alpha_{1av}] = 0.6874\]

Where \(w_0\) and \(c_0\) are the wale-spacing and course-spacing of fully relaxed fabric (global minimum without friction). \(F_{SW}, F_{SC}, S_{W20d}\) and \(S_{C20d}\) are the frictional forces in Equations (2 - 5) which are applied in dry relaxed state. \(D_w, D_c, S_{W10d}\) and \(S_{C10d}\) are the differences between the dry relaxed fabric parameters (\(c, w\)) and fully relaxed fabric parameters (\(c_0, w_0\)). The indices in Equations (2 - 5) are used as such that \(c\) and \(w\) are used for the load
In the present work, building on Kurbak’s [8-11] works that are described above, the first stages of load-extension and/or load contraction properties of some complex weft knitted structures are studied experimentally. The aim here is to investigate whether empirical equations based on the Equations (2-4) can be found for such complex knitted fabrics as well, based on the assumption that the $y_0$ values in Regression Equations (7) being frictional restrains and/or fabric jamming forces also applies for complex fabrics. The outstanding equations are expected to be useful for engineering software that are generally used for technical applications of textiles, such as composite reinforcements, stretch sensors, air permeabilities etc. These kinds of applications usually start from dry relaxed fabric conditions; therefore the study particularly focuses on this state. The work, furthermore, is primarily conducted for composite reinforcement applications, therefore high modulus yarns such as E-glass and para-aramid are chosen for the study.

2. EXPERIMENTAL PROCEDURE

- The knitted structures chosen for this study were 1x1 Rib, Milano Rib, Half Cardigan and Full Cardigan Derivative. Knit notations of these fabrics are given in Figures 2a, 2b, 2c and 2d. It should be noted here that due to some spiralities that occurred in the fabrics with Full Cardigan structure itself, the Full Cardigan Derivative structure that is given in Figure 2d was chosen for this work.

- Samples were knitted on a 7 gauge (7 needles per inch) V-bed hand-knitting machine. In order to study the effect of tightness, the samples were knitted at different cam settings of the machine, namely, three different cam settings for 1x1 Rib (R6, R8, and R10), and two different cam settings for Milano Rib (M8 and M10), Half Cardigan (Y8 and Y10) and Full Cardigan Derivative (S8 and S10).

- Two types of yarns were used to knit the samples, namely, 136 tex E-glass yarn (EC9 68 1x2 twisted E-glass yarn) and 168 tex Aramid yarn (Twaron [12], 2012).

### Table 1. Reference state parameters of glass plain knitted fabric (Kurbak [11]).

| Ref. State parameters | Reaction Force |
|-----------------------|----------------|
|                       | $w_0/d$ $c_0/d$ $l/d$ $R[g]$ |
| Wale-wise loaded samples | 6.751 4.410 22.987 3.840 |
| Course-wise loaded samples | 6.763 4.455 23.361 4.328 |

The values given in Table 1 were calculated theoretically through the geometrical and physical parameters at the relaxed state.

In obtaining Equations (3 and 4): (a) The loop head curves were modelled by using the equation of the extension of a circular ring (see Kurbak [10]) therefore no discontinuities occurred between the loop head curve and the rest of the loop curves during the extension. (b) It was shown that when the experimental quadratic curve fitting equations $y = a_1 x^2 + b_1 x + c_1$ (6)

were written in the form of $y = a_1 (x - x_0)^2 + y_0$ (7)

the $y_0$ values were the frictional restrains or/and inter fabric jamming forces. Therefore the equations were written in the form of $x - x_0 = \sqrt{\frac{y - y_0}{a_1}}$ (8)

and thus the Equations (3 and 4) were obtained.

Where $y$ is the total load applied in a direction (wale-wise or course-wise), $x$ is the sample length ($h$) for extension, or the minimum fabric width ($b_{min}$) for contraction.
Estimations of the yarn diameters: Kurbak [11] estimated the diameter of 136 tex E-glass yarn as
\[ d_g = 1.18 \times d_{ap} = 0.354 \text{ mm} \quad (9) \]

where \( d_{ap} \) was the yarn diameter in the yarn cross-section calculated by using the open fibre packaging model by Hearle, Grosberg and Backer [13]. It is also known that the yarn diameter can be related to the square root of the yarn tex. When Equation (9) is given in terms of the yarn tex, the equation
\[ d_g = 0.03035 \times \sqrt{\text{tex}_{\text{glass}}} \quad (10) \]
is obtained. The yarn diameter of 168 tex aramid is estimated here by using the equivalent yarn tex of the aramid in glass as
\[ d_a = 0.03035 \times \sqrt{\frac{168 + 2.55}{1.44}} = 0.5236 \text{ mm} \quad (11) \]
where
\[ \rho_g = 2.55 \text{ g/cm}^3 \] is the density of the glass fibres and \( \rho_a = 1.44 \text{ g/cm}^3 \) is the average density of the aramid fibres.

Estimations of bending rigidities of yarns used: Kurbak [11] used the same 136 tex E-glass yarn and found the bending rigidity \( B_g \) for 136 tex E-glass yarn as
\[ B_g = 2.87 \text{ g mm}^2 \quad (12) \]
by using an equation given by Lomov and Verpoest [14] for glass yarns as
\[ B_g = 1.886 \times 10^{-5} \text{ tex}^2 + 1.937 \times 10^{-2} \text{ tex} \times [g \text{ mm}^2] \quad (13) \]
On the other hand, Kurbak [15], using a different measuring system, measured the values 2.66 g mm\(^2\) and 18 g mm\(^2\) respectively for the bending rigidities of 136 tex E-glass and 168 tex aramid yarns. Assuming that the ratio \( \frac{B_g}{B_a} \) obtained between Lomov and Verpoest’s system of measurements and Kurbak’s system of measurements for glass yarns is also valid for aramid yarns, the bending rigidity of 168 tex aramid yarn can be estimated as
\[ B_a = \frac{2.87}{2.66} \times 18 = 19.3483 \text{ g mm}^2 \quad (14) \]

In summary, for the 136 tex E-glass and 168 tex aramid yarns that are used in this work, the estimated yarn diameters that are given in Equations (9) and (11) together with the estimated bending rigidities given in Equations (12) and (14) are used throughout the work.

- Two sets of samples were prepared for each structure, each chosen tightness points, and each yarn. One of the sets was prepared for wale-wise loadings and the other for loadings in the course-wise direction.

The samples were prepared in the following manner:

i) Wale-wise loading samples: The intention was to obtain a \( hxb=20\times10 \text{ cm}^2 \) main area as the main measuring area of the samples. Therefore, the sample lengths \( (b) \) at dry relaxed fabric condition were chosen in order to fulfil the minimum of 20 cm main sample length plus additional fabric parts above and below the main sample to be used in fixing the jaws of any measurement tester.

Because of the differences in structure and tightness, it was challenging to adjust the sample widths \( (b) \) to exact 10 cm in their dry relaxed states. Therefore, a constant knitting zone composed of 35 needles from each bed (from the back and front beds) was set. The sample widths \( (b) \), accordingly, were obtained as 10 cm in average.

Because of the high bending rigidities of aramid and glass technical yarns, fabric samples unravel easily. To prevent the unravelling, ten rows of cotton borders were knitted at the beginning and end of each sample. Some examples of E-glass and aramid wale-wise load-extension measuring samples can be seen in Figure 3a. The average sample widths \( (b) \), which are obtained by measuring the samples in dry relaxed state at their minimum places , is given in Table 2. A total of five wale-wise loading samples were knitted for each case.

ii) Course-wise loading samples: Again, the abovementioned \( hxb=20\times10 \text{ cm}^2 \) main sample area was intended for the course-wise loading samples as well. In this case, a knitting zone constitutes the sample lengths \( (h) \) of the course-wise loading samples. Therefore a constant knitting zone of 110 needles on each bed (front and back) was set. Through this setting, it was possible to obtain the minimum sample length of 20 cm as well as the additional fabric parts at the lower and upper sides necessary for fixing the samples in the jaws of any measuring tester. Again, it was challenging to keep the sample widths \( (b) \) at exactly 10 cm.

Based on the initial tests that were conducted, sufficient numbers of rows of fabric were knitted as the widths \( (b) \) of the course-wise loading samples and the intended average sample widths \( (b) \) around 10 cm was achieved.

A total of 5 samples per case were knitted to measure the course-wise extension of the fabrics. The averages of the 5 sample widths \( (b) \), which were measured at their dry relaxed conditions, are given in Table 2.

To prevent unraveling, 10 rows of cotton borders were knitted at the beginning and at the end of each sample. It was assumed that these cotton borders had negligible minimal effect on the load-extension measurements.

Some examples of course-wise loading samples are given in Figure 3b.
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Table 2. The average sample widths (b) in cm.

| Knitted Structures | Tightness | Loop Lenght (l) [mm] | Course per cm (cpc) | Wales per cm (wpc) | Mass per unit area g/m² |
|--------------------|-----------|----------------------|---------------------|--------------------|------------------------|
| 1x1 Rib            | R6        | 5.923                | 9.52                | 4.69               | 783.409                |
|                    | R8        | 7.330                | 7.65                | 3.54               | 575.587                |
|                    | R10       | 9.185                | 5.37                | 3.02               | 460.176                |
| Milano Rib         | M8        | Rib/Plain            | 7.815 / 5.807       | 8.06               | 718.239                |
|                    | M10       | Rib/Plain            | 9.698 / 7.307       | 6.63               | 475.402                |
| Half Cardigan      | Y8        | Rib/Tuck             | 7.295 / 7.56        | 6.32               | 842.921                |
|                    | Y10       | Rib/Tuck             | 9.243 / 9.842       | 4.8                | 678.354                |
| Full Cardigan      | S8        | Rib/Tuck             | 7.463 / 7.739       | 4.1                | 842.755                |
|                    | S10       | Rib/Tuck             | 8.860 / 9.378       | 4                  | 659.663                |

- For each experimental case, one extra sample was prepared to measure the samples' dimensional properties.
- In total, 216 samples were prepared:

4 types of structure x (2 different tightness value +1 extra tightness value for 1x1 rib) x 2 types of yarn x 2 directions (wale-wise and course-wise) x (5 samples from each case to measure the load-extension properties + 1 extra sample for measuring dimensional properties of the samples) = 216 samples.

- The measurement methods of dimensional properties at dry relaxed fabric condition:

i) Courses per cm (cpc) and wales per cm (wpc):

After two weeks of dry relaxation on a smooth surface, courses per cm (cpc) and wales per cm (wpc) were counted with a two-inch square magnifying glass at three different places of the wale-wise loaded samples. The same was done for the course-wise loaded samples. Then, the average value was obtained and divided by 2x2.54 as (cpc) and (wpc).

In the half cardigan structure, the number of courses on the front side is twice the number of courses on the backside. Thus, in this work, the number of courses on the backside was taken into consideration. To obtain the number of courses on the front side, the number of courses obtained here should be multiplied by two.

ii) Loop length (l):

After two weeks of dry relaxation on a smooth surface, areas in the size of 10x5 cm² were pointed out at three different regions of each sample and these areas were cut out. From each piece, 5 rows were separated and their course lengths (L) were measured by hanging 10 g weights below them. With 10 g weight, the curled yarns were observed to be straightened. The average of 3 parts x 5 rows= 15 course lengths (L) was divided by the average number of loops (at the back and in the front sides) in a row in order to obtain the loop length (l).

Dönmez and Kurbak [16] and also Kurbak and Alpyıldız [17], 2009b showed that the yarn length of a tuck stitch was slightly bigger than the stitch length (l) of a normal loop. In the present work, however, if a row is constructed as a combination of tuck stitches and normal stitches, the yarn length of tuck stitches and the yarn length of normal stitches are assumed to be equal. Thus, the average loop length (l) was found by dividing the average course length (L) by the total number of normal stitches plus tuck stitches in a particular row.

If a structure unit, like the Half Cardigan, has rib rows and rows that contain tuck stitches, the stitch lengths of the two types of rows were given separately.

If a structure unit, like the Milano Rib, is a combination of rib rows and plain knit rows, their stitch lengths were, again, given separately.

The obtained dimensional properties of dry relaxed samples are given in Table 3a for E-glass samples and in Table 3b for Aramid samples.

Table 3a. Dimensional properties of dry relaxed Glass fabric samples

| Knitted Structures | Tightness | Loop Lenght (l) [mm] | Course per cm (cpc) | Wales per cm (wpc) | Mass per unit area g/m² |
|--------------------|-----------|----------------------|---------------------|--------------------|------------------------|
| 1x1 Rib            | R6        | 5.923                | 9.52                | 4.69               | 783.409                |
|                    | R8        | 7.330                | 7.65                | 3.54               | 575.587                |
|                    | R10       | 9.185                | 5.37                | 3.02               | 460.176                |
| Milano Rib         | M8        | Rib/Plain            | 7.815 / 5.807       | 8.06               | 718.239                |
|                    | M10       | Rib/Plain            | 9.698 / 7.307       | 6.63               | 475.402                |
| Half Cardigan      | Y8        | Rib/Tuck             | 7.295 / 7.56        | 6.32               | 842.921                |
|                    | Y10       | Rib/Tuck             | 9.243 / 9.842       | 4.8                | 678.354                |
| Full Cardigan      | S8        | Rib/Tuck             | 7.463 / 7.739       | 4.1                | 842.755                |
|                    | S10       | Rib/Tuck             | 8.860 / 9.378       | 4                  | 659.663                |
Table 3b. Dimensional properties of dry relaxed Aramid fabric samples

| Knitted Structures | Tightness | Loop Length(mm) | Course per cm (cpc) | Wales per cm (wpc) | Mass per unit area g/m² |
|--------------------|-----------|-----------------|---------------------|-------------------|-----------------------|
| 1x1 Rib            | R6        | 6.620           | 7.66                | 3.98              | 749.002               |
|                    | R8        | 7.358           | 7.49                | 3.37              | 691.027               |
|                    | R10       | 8.813           | 5.79                | 3.31              | 677.532               |
| Milano Rib         | M8        | Rib / Plain     | 7.835/ 6.775        | 8.03              | 4.13                  | 866.605               |
|                    | M10       | Rib / Plain     | 9.769/ 8.805        | 5.72              | 3.3                   | 658.282               |
| Half Cardigan      | Y8        | Rib / Tuck      | 7.524/ 7.961        | 5.18              | 2.88                  | 833.477               |
|                    | Y10       | Rib / Tuck      | 8.917/ 9.498        | 3.36              | 2.81                  | 682.489               |
| Full Cardigan      | S8        | Rib / Tuck      | 6.844/ 7.210        | 6.8               | 2.9                   | 815.225               |
|                    | S10       | Rib / Tuck      | 9.007/ 9.804        | 3.5               | 2.5                   | 651.702               |

It should be remembered that the inverse of \((cpc)\) and the inverse of \((wpc)\) in Tables 3a and 3b are equal to the course-spacing \((c)\) and the wale-spacing \((w)\) respectively as

\[
\begin{align*}
c &= \frac{10}{(cpc)} \text{[mm]} \quad (15) \\
w &= \frac{10}{(wpc)} \text{[mm]} \quad (16)
\end{align*}
\]

- Methods of load-extension and load-contraction measurements:

The load-extension and load-contraction properties of the samples were measured by a special apparatus (Elmalı [18]) seen in Figure 4, since standard methods would not have allowed exploring the load-contraction behaviours of the fabrics in question.

Since technical yarns are too brittle and often break when being fixed in the jaws of load-extension measurement testers, a special friction system was designed to be used at the upper and lower ends of the samples in order to prevent yarn breakages. Load-extension and load-contraction behaviours of the samples were measured by hanging dead weights at the lower friction system given in Figure 4. The dead weights were gradually increased and, at each load level, the length \((h)\) and the width \((b_{min})\) at the mid points of the samples were measured with a ruler.

According to the initial plan that was set for this study, the first experiments were in fact devoted to measure all three extension and contraction stages (the first, second, and third stage). Therefore, initially, 12 constant dead weights were arranged as given in Table 4.

![Figure 4](attachment:image.png)

**Figure 4.** Special apparatus to measure the load-extension properties in this work (Elmalı [18]).

During the initial experiments, it was observed that some extension in the fabric parts placed in the inner side of the special friction system shown in Figure 4 inevitably occurred at about the eighth load level. Therefore, despite the original plan, it was later decided that, while continuing to measure all stages of the load-extension and load-contraction curves with 12 constant dead weights, the present work would be devoted only to the first stage of extensions or contractions. The rest of the measurements were solely used for a) predicting the exact load point at which the first stage of extension ended b) figuring out whether all three stages of extensions or contractions can be distinguished in complex structures as in plain knitted structure previously given by Kurbak [11].

![Figure 5](attachment:image.png)

**Figure 5.** The shape of a sample under loading, a) sample shape for the first stage of extension, b) sample shape for the second and third stages of extension (Kurbak [11]).
Table 4. The dead weights used to measure load-extension and load-contraction properties.

| Load Levels | 1  | 2  | 3  | 4  | 5  | 6  |
|--------------|----|----|----|----|----|----|
| Amount of weight (g) | 0  | 238.4 | 480 | 800 | 1050 | 1563.33 |
| Load Levels | 7  | 8  | 9  | 10 | 11 | 12 |
| Amount of weight (g) | 2050.5 | 4050.5 | 6001.5 | 8360 | 10700 | 12570.5 |

As a result, load-extension and load contraction curves of all relevant cases were drawn to scale for all 12 dead weights as given in Table 4.

It was observed that:

i) All three stages of extensions and contractions were distinguishable in complex structures as they were in plain knitted fabrics as formerly proven by Kurbak [11]. Some examples of the obtained load extension and load-contraction curves are given in Figures 6a and 6b.

ii) The end point of the first stage is determined to be the 5th load level given in Table 4 for all the cases in question in this study.

iii) Due to the use of a special friction system, no slippages of fabrics occurred in the friction system for the first stages of extensions and contractions.

iv) Since the aim of the present work is focused on the exploration of the first stages of load-extension and load-contraction properties of the cases in question, hereafter, only the first 5 load levels given in Table 4 will be used.

Figure 6. Examples of load-extension and load-contraction results for all three stages (see ref [19]), a) load-extension, b) load-contraction.

3. EXPERIMENTAL RESULTS

Experimentally obtained h and b_{min} values with changing load level T are given at tables in APPENDIX A together with example drawings of load-extension and load-contraction curves for each table.

Regression analyses are applied to all cases in question and given in the form of Equation (6). The constants of Regression Equations (6) are given in Table 5, together with their correlation coefficients R^2:

As seen in Table 5, the correlation coefficients are very high; therefore, Regression Equations (6) can be used.

For comparison purposes, Regression Equations (6) and their constants given in Table 5 are written in terms of unit structure parameters as follows:

a) For the wale-wise loaded samples

\[ T_w = \frac{y}{b \text{ (wpc)}} \] (17)

is defined, where b (wpc) is the average number of loops on a face of sample width (b). For the case (i) of contraction equations of the wale-wise loaded samples, the equation

\[ w_{min} = \frac{x}{b \text{ (wpc)}} \] (18)

is used, whereas for the case (ii) of extension equations of the wale-wise loaded samples (see Figure 5), the equation

\[ c_{av} = \frac{x}{h \text{ (cpc)}} \] (19)

is used.

It should be noted here that the variable y is the total load applied on a sample, the variable x is the measured length or width of a sample under loading condition. the parameter b and h are the sample width and length respectively at dry relaxed fabric conditions. cpc and wpc are the course per cm and wales per cm of the samples at dry relaxed conditions. \( T_w, w_{min} \) and \( c_{av} \) values are the calculated wale-wise applied load, minimum wale-spacing (see Figure 5) and average course-spacing of a loop under wale-wise loading conditions.

If the general regression equation is

\[ y = a_1 x^2 + b_1 x + c_1 \] (20)

for the contraction equations of the wale-wise loaded samples

\[ T_w = \frac{a_1(b \text{ wpc})^2}{100} w_{min}^2 - \frac{b_1}{10} w_{min} + \frac{c_1}{(b \text{ wpc})} \] (21)

change of variables are applied, whereas for the extension equations of the wale-wise loaded samples

\[ T_w = \frac{a_1(b \text{ cpc})^2}{100 (b \text{ wpc})} c_{av}^2 - \frac{b_1(b \text{ cpc})}{100 (b \text{ wpc})} c_{av} + \frac{c_1}{(b \text{ wpc})} \] (22)

change of variables are applied, where cpc and wpc are given in Tables 3a and 3b while b parameters are given in Table 2.
It should be noted here that the $w_{min}$ and $c_{ave}$ parameters are given in mm, therefore $a_1$ and $b_1$ constants in Equations (21) and (22) are also divided by 100 and 10 respectively.

b) On the other hand, for the course-wise loaded samples, the equation

$$T_c = \frac{y}{b_{(cpc)}}$$

(23)

is valid. In this category

$$c_{min} = \frac{x}{b_{(cpc)}}$$

(24)

for (i) contraction equations and

$$w_{av} = \frac{x}{h_{(wpc)}}$$

for (ii) extension equations should also be considered. Using Equations (23, 24 and 25), the change of variable equations

$$T_c = \frac{a_1(b_{cpc})^2}{100}c_{min} - \frac{b_1}{10}r_{min} + \frac{c_1}{(b_{cpc})}$$

(26)

$$T_c = \frac{a_1(h_{wpc})^2}{100(b_{cpc})}w_{av} - \frac{b_1(h_{wpc})}{10(b_{cpc})}w_{av} + \frac{c_1}{(b_{cpc})}$$

(27)
are obtained to be applied for the contraction equations \((i)\) and for the extension equations \((ii)\) of the course-wise loaded samples respectively.

Where \(T_c\), \(c_{\text{min}}\) and \(w_{\text{av}}\) values are the calculated course-wise applied load, minimum course-spacing and average wale-spacing of a loop under the course-wise applied loading conditions.

After applying Equations (21, 22, 26 and 27) in related Regression Equations (6 or 20) through Table 5, the following forms of equations are assumed and applied in accordance with the obtaining of Equation (8) as given in the work of Kurbak [11] for plain knitted glass fabrics.

a) For the wale-wise loaded samples

\[ w_{\text{av}} = w_0 \pm \frac{\sqrt{T_w + m}}{a_2} \]  
\[ c_{\text{av}} = c_0 \pm \frac{\sqrt{T_w + m}}{a_2} \]  

b) For the course-wise loaded samples

\[ w_{\text{av}} = w_0 \pm \frac{\sqrt{T_c + m}}{a_2} \]  
\[ c_{\text{av}} = c_0 \pm \frac{\sqrt{T_c + m}}{a_2} \]

Here, it should be noted that the \(m\) values in Equations (28-31) are assumed to be as frictional restrains and/or fabric jamming forces as were the \(y_0\) values given in Equation (8).

Assuming that the side edges of loaded samples constitute parabolic curves (see Figure 5a) for the first stages of extensions, Kurbak [11] found the following equations to obtain \(w_{\text{av}}\) and \(c_{\text{av}}\) in places of \(w_{\text{min}}\) and \(c_{\text{min}}\) in Equations (28 and 30), thus:

\[ w_{\text{av}} = \frac{1}{3} w_0 + \frac{2}{3} w_{\text{min}} \]  
\[ c_{\text{av}} = \frac{1}{3} c_0 + \frac{2}{3} c_{\text{min}} \]  

When Equations (32) and (33) are replaced in Equations (28) and (30), the following forms are obtained

\[ w_{\text{av}} = \frac{2w_0 + w'_{0}}{3} \pm \frac{\sqrt{w_0 + m}}{1.5a_2} \]  
\[ c_{\text{av}} = \frac{2c_0 + c'_{0}}{3} \pm \frac{\sqrt{c_0 + m}}{1.5a_2} \]  

These changes of variables are also applied to all of the related Regressions Equations (6) through Table 5, by equalizing the \(w\) and \(c\) values obtained from Tables 3a and 3b through the Equations (15) and (16) to the \(w'_0\) and \(c'_0\) in Equations (34) and (35) respectively.

With these final changes, the final forms of Equations (28-31) become as follows:

a) For the wale-wise loaded samples

\[ w_{\text{av}} = w_0 \pm \frac{\sqrt{T_w + m}}{a_3} \]  
\[ c_{\text{av}} = c_0 \pm \frac{\sqrt{T_w + m}}{a_3} \]

b) For the course-wise loaded samples

\[ w_{\text{av}} = w_0 \pm \frac{\sqrt{T_c + m}}{a_3} \]  
\[ c_{\text{av}} = c_0 \pm \frac{\sqrt{T_c + m}}{a_3} \]

where

\[ w_0 = \frac{2w_0 + w'_{0}}{3} \]  
\[ c_0 = \frac{2c_0 + c'_{0}}{3} \]  

are taken for Equations (36) and (38) and also

\[ a_3 = 1.5 a_2 \]  

is taken for the same Equations (36) and (38) while \(a_2\) do not change in Equations (37) and (39) as

\[ a_3 = a_2 \]  

Similar equations as Equations (17-43) are also valid for the cases in which the linear regression equations are applied. The only differences are that the \(c_{\text{min}}^2, w_{\text{min}}^2, w_0^2\) and \(c_{\text{av}}^2\) parameters should be equalized to zero and the phrases \(\sqrt{T_w + m}\) and \(\sqrt{T_c + m}\) should be replaced by \(T_w\) and \(T_c\) all through the Equations (17-43).

The parameters \(w_0, c_0, m\) and \(a_3\) of Equations (36-39) are given in Table 6 for all cases considered.

When \(T\) is equalized to zero in Equations (36-39), two roots are obtained for each case if \(m\) is positive. For these cases, the relaxed
fabrics can either be in root 1 condition or in root 2 condition, which are also given in Table 6. If the $m$ parameter is negative, there is no root for $T$ is equal to zero. Therefore, for those cases, 0 (zero) are placed in Table 10. If a regression equation is linear, there is no $m$ value and thus there is only one root. For these cases ‘–’ (dash) are put in places of $m$ and root in Table 6. All these situations are explained further in the next section.

Since it is not useful in practice to have too many equations and also to be able to cover the intermediate tightnesses, some empirical equations are obtained in the next section.

It is seen from Table 6 that the $m$ values are small enough to be considered as frictional restrains and/or fabric jamming forces. Therefore, considering Equation (8) and assuming $v_0$ values in Equation (8) or the $m$ values in Table 6 as frictional restrains and/or fabric jamming forces is reasonable.

### Table 6. The calculated parameters of Equations (36-39) for all the present experimental cases

| Alai | $w_0$ | $c_0, w_0$ | $m$ | $\alpha_1$ | root |
|------|-------|------------|-----|-------------|------|
| 1    |       |            |     |             |      |
| Alaii| $c_0$ | R6         | 2.0661 | -5.4683     | 9.2703 | 0 |
|      |       | R8         | 2.8076 | -0.2393     | 5.4510 | 0 |
|      |       | R10        | 3.1287 | 0.4250      | 4.5693 | 2 |
| Albi | $c_0$ | R6         | 1.0952 | -5.4757     | 7.672 | 0 |
|      |       | R8         | 1.4018 | 0.1784      | 6.5731 | 1 |
|      |       | R10        | 2.1100 | 0.8543      | 4.2566 | 1 |
| Albii| $w_0$ | R6         | 1.4534 | 2.2529      | 5.1559 | 1 |
|      |       | R8         | 1.2322 | 0.1623      | 6.8245 | 2 |
|      |       | R10        | 1.5405 | 6.1105      | 7.7338 | 2 |
| Allai| $w_0$ | M8         | 2.6770 | -6.0390     | 6.7073 | 0 |
|      |       | M10        | 1.5241 | -0.7999     | 3.9805 | 0 |
| Allaii| $c_0$ | M8        | 1.2176 | -6.6286     | 7.7173 | 0 |
|      |       | M10        | 1.5316 | -0.5462     | 4.6992 | 0 |
| Allbi | $c_0$ | M8        | 1.9541 | 6.4716      | 4.1885 | 1 |
|      |       | M10        | 1.6784 | 0.2625      | 4.7693 | 1 |
| Allbii| $w_0$ | M8 | 2.0741 | -7.695 | 2.0020 | 0 |
|      |       | M10        | 3.4023 | -0.2083     | 1.5174 | 0 |
| AllIai| $w_0$ | Y8 | 3.2385 | 0.0085 | 8.9118 | 2 |
|      |       | Y10        | 3.6573 | 2.3877      | 9.1478 | 2 |
| AllIaia| $c_0$ | Y8 | 1.9204 | 2.0085 | 4.3222 | 1 |
|      |       | Y10        | 2.5908 | 3.1642      | 3.5485 | 1 |
| AllIbi | $c_0$ | Y8 | 1.7003 | 0.9413 | 8.6265 | 1 |
|      |       | Y10        | 2.0311 | 0.5120      | 8.0103 | 2 |
| AllIlbi | $w_0$ | Y8 | 4.3315 | 0.9230 | 0.9632 | 1 |
|      |       | Y10        | 6.3169 | 5.7310      | 0.9677 | 1 |
| AllIVai | $w_0$ | S8 | 3.5294 | 0.5082 | 4.9052 | 1 |
|      |       | S10        | 4.2268 | -0.3499     | 3.5729 | 0 |
| AllIVaii | $c_0$ | S8 | 2.4229 | -0.7712 | 1.9462 | 0 |
|      |       | S10        | 3.1094 | 1.2093      | 2.0126 | 1 |
| AllIVbi | $c_0$ | S8 | 2.6127 | 1.4298 | 7.4070 | 1 |
|      |       | S10        | 2.3966 | 0.3816      | 6.8799 | 2 |
| AllIVbii | $w_0$ | S8 | 4.4746 | 1.9387 | 1.2611 | 1 |
|      |       | S10        | 6.8353 | 11.1244     | 1.3427 | 1 |

| Blai | $w_0$ | $c_0$ | $m$ | $\alpha_1$ | root |
|------|-------|-------|-----|-------------|------|
| 1    |       |       |     |             |      |
| Blaii| $c_0$ | R6 | 1.3054 | - | 273.64 | - |
|      |       | R8 | 1.3300 | - | 63.558 | - |
|      |       | R10 | 1.6961 | - | 6.21023 | 0 |
| Blbi | $c_0$ | R6 | 1.3056 | - | 359.5424 | - |
|      |       | R8 | 1.3472 | - | 58.9942 | - |
|      |       | R10 | 1.7298 | - | 3.1813 | 0 |
| Blbii| $w_0$ | R6 | 2.4762 | - | 193.785 | - |
|      |       | R8 | 2.9450 | - | 101.0805 | - |
|      |       | R10 | 3.0211 | - | 33.738 | - |
4. SUGGESTIONS OF SOME EMPIRICAL EQUATIONS, POISSON’S RATIOS AND CALCULATIONS OF EXTENSION RATES

During the obtaining of empirical equations, the following points were considered.

1. All of the regression equations for glass yarn gave similar equations to Equations (3) and (4), which were obtained for plain knitted fabrics by Kurbak [11]. Therefore, for glass yarn, the following empirical forms of equations should be written:

   a) for wale-wise loaded samples

   i) contraction

   \[ w_{av} = w_0 - v_1 \frac{f(x) - d}{x_2 + m} \]  

   ii) extension

   \[ w_{av} = w_0 + \frac{f(x) - d}{x_2 + m} \]  

   b) for course-wise loaded samples

   i) contraction

   \[ c_{av} = c_0 - v_2 \frac{f(x) - d}{x_2 + m} \]  

   ii) extension

   \[ c_{av} = c_0 + \frac{f(x) - d}{x_2 + m} \]

2. The Poisson’s ratio \( u_1 \) in Equation (44) should be equal to the \( u_2 \) in Equation (46) as

\[ u_1 = u_2 = \frac{b_2}{b_3} \]  

During equalizing Poisson’s ratios in Equations (44 and 46), \( f'' \) functions were so changed that equalities in Equations (44 and 46) were remained the same. 

3. The constant \( a_4 \) in Equation (44) should be equal to \( a_7 \) in Equation (47), while the same equality between \( a_5 \) and \( a_6 \) in Equations (45) and (46) should also be valid:

\[ a_4 = a_7 = b_2 \]  

\[ a_5 = a_6 = b_3 \]  

Again during equalizing \( a_4 \) and \( a_7 \) and also equalizing \( a_5 \) and \( a_6 \) in Equations (44-47), \( f'' \) functions were so changed that equalities in Equations (44-47) were remained the same. 

\[ f = \frac{b_2}{a_5} f' \]  

\[ f = \frac{b_2}{a_6} f' \]  

Table 6. The calculated parameters of Equations (36-39) for all the present experimental cases

| BI | w₀ | M8 | M10 | M8 | M10 | M8 | M10 | M8 | M10 | M8 | M10 |
|----|----|----|-----|----|-----|----|-----|----|-----|----|-----|
| BI 1a | 2.4244 | 2.4244 | 3.0344 | 3.0344 | 294.84 | - | 68.2995 | - |
| BI 1b | 1.2358 | 1.2358 | 1.7476 | 1.7476 | 290.4571 | - | 55.4983 | - |
| BI 2b | 1.2483 | 1.2483 | 1.7618 | 1.7618 | 298.725 | - | 83.6595 | - |
| BI 2b | 2.4054 | 2.4054 | 3.0145 | 3.0145 | 59.6303 | - | 16.4531 | - |
| BI 1a | 2.4054 | 2.4054 | 3.0145 | 3.0145 | 294.84 | - | 68.2995 | - |
| BI 1b | 1.2358 | 1.2358 | 1.7476 | 1.7476 | 290.4571 | - | 55.4983 | - |
| BI 2b | 1.2483 | 1.2483 | 1.7618 | 1.7618 | 298.725 | - | 83.6595 | - |
| BI 2b | 2.4054 | 2.4054 | 3.0145 | 3.0145 | 59.6303 | - | 16.4531 | - |

*a) A) Glass, B) Aramid, I) 1x1 Rib, II) Milano Rib, III) Half Cardigan, IV) Full Cardigan Derivative, a) Wale-wise loaded samples, b) Course-wise loaded samples, i) Contraction, ii) Extension
4. The phrase \((w_0 + 2d)\) in Equations (3) and (4) was created by Kurbak [11] because this phrase was proportional with the loop head radius, \(a_0\), of the relaxed plain knitted fabrics. Since, in most of the rib structures, the radius of the loop heads, \(a_0\), proportional with \((w_0 - d)\), assumes that wale jamming conditions occurred, for the extension or contraction in the course’s direction, function \(f\) in Equations (44) and (47) should depend on \((w_0 - d)\). In this work
\[
f(w_0 - d) = (w_0 - d)^n \tag{51}
\]
is taken.

On the other hand, there is no such knowledge for wale-wise extensions or contractions, but the function \(f\) for Equations (45) and (46) should depend on \(c_0\) because the length \(c_0\) extends at two ends of a curved part of a loop. Therefore, for \(f\) function of Equations (45) and (46), the form
\[
f(c_0) = c_0^n \tag{52}
\]
is taken.

The \(f\) functions given in Equations (51) and (52) are used here for various purposes as such that

i) the upper indices \(n\) is used for estimating the intermediate tightnesses,

ii) if any jamming occurs in the fabric in a direction, this can be easily considered by changing the values of upper indices \(n\) etc.,

The final empirical parabolic equations to be used in this work are as

a) for wale-wise loaded samples

i) contraction
\[
w_{av} = w_0 - v \frac{(w_0-d)^n/\sqrt{T_w/2+m/2}}{b_2 \sqrt{B}} \tag{53}
\]
ii) extension
\[
c_{av} = c_0 + \frac{(c_0)^n/\sqrt{T_c/2+m/2}}{b_3 \sqrt{B}} \tag{54}
\]

b) for course-wise loaded samples

i) contraction
\[
c_{av} = c_0 - v \frac{(c_0)^n/\sqrt{T_c+m}}{b_3 \sqrt{B}} \tag{55}
\]
ii) extension
\[
w_{av} = w_0 + \frac{(w_0-d)^n/\sqrt{T_c+m}}{b_2 \sqrt{B}} \tag{56}
\]

5. For aramid fabrics, most of the regression equations are linear in forms as
\[
w_{av} = w_0 \pm \frac{T}{b_2} \tag{57}
\]
\[
c_{av} = c_0 \pm \frac{T}{b_2} \tag{58}
\]
The form of Equations (57) and (58) obtained for aramid fabrics can best be explained by using Equations (55) of Kurbak’s work [11] for plain knitted fabrics as such that Equation (55) in Kurbak’s work [11] was as
\[
T_c + F_{sc} = \frac{2B}{3a_0} 8.770 (x - 1)^2 \tag{59}
\]
where
\[
x = \frac{a}{a_0} = \frac{w_{av}+2d}{w_0+2d} \tag{60}
\]
When Equation (59) is opened up, the equation
\[
T_c + F_{sc} = \frac{2B}{3a_0} 8.770 (x^2 - 2x + 1) \tag{61}
\]
is obtained. Since the bending rigidity, \(B\), of aramid is high as given in Equation (14), the inclination angles of parabolic curves between \(T\) and \(w_{av}\) are very high; thus, the arms of the parabolic curves obtained for this yarn are extended along which are near the vertical line as given in Figure 7. Because of these higher inclination angles of parabolic curves, the values of \(w_{av}\) and \(w_0\) becomes too close to each other, and, thus, \(x^2\) in Equations (61) becomes as
\[
x^2 = \frac{a^2}{a_0^2} = \frac{(w_{av}+2d)^2}{(w_0+2d)^2} \approx 1 \tag{62}
\]
When Equations (62) is replaced in Equation (61), an equation is obtained as
\[
T_c + F_{sc} = -\frac{2B}{3a_0} 8.770 (2x - 2)
\]
and thus
\[
T_c + F_{sc} = -b’2B (x - 1) \tag{63}
\]

Figure 7. Comparison of parabolic curve fittings of load-extension properties of aramid and glass samples.

It is thought that this must be one of the reasons for obtaining linear regression equations for aramid fabrics. One more reason for obtaining linear regression equations for aramid fabrics will be given later in this section.

As a result of the above discussion, the following forms of empirical equations are used for linear equations of aramid fabrics:
Empirical Equations and Poisson’s Ratios for Initial Load-Extension Properties of Some Complex Glass and Aramid Technical Weft Knitted Structures

Arif KURBAK
Sinem ÖZTÜRK

For wale-wise loaded samples

i) contraction

\[ w_{av} = w_0 - b \frac{(w_0 - d)^n}{b_2(2B)} T_0 \]  
\[ (64) \]

ii) extension

\[ c_{av} = c_0 + \frac{c_0^m}{b_2(2B)} T_0 \]  
\[ (65) \]

For course-wise loaded samples

i) contraction

\[ c_{av} = c_0 - b \frac{c_0^m}{b_2(2B)} T_c \]  
\[ (66) \]

ii) extension

\[ w_{av} = w_0 + b \frac{(w_0 - d)^n}{b_2(2B)} T_c \]  
\[ (67) \]

According to the above points, the empirical equations were obtained by the method of trial and error, and they are given in APPENDIX B.

It should be noted here that Empirical Equations in APPENDIX B include one more parameter called ‘a’. This parameter is included to equalize two different Poisson’s ratios in one structure. The parameter, \( a \), is put wherever it is necessary in the equations.

To be able to apply Equations in APPENDIX B, the parameter, \( m \), in the equations should also be estimated and discussed further as given below:

i) if the parameter, \( m \), is negative

If the parameter, \( m \), in an equation is negative and when the external load \( T \) is set to zero, the square root in the equation becomes irrational. It is thought that these kind of structures are jammed structures, and some of the external loads are spent to overcome these jamming forces. The amounts of inner fabric forces are equal to \( m \); therefore the equation

\[ T_0 = m \]  
\[ (68) \]

should be valid where \( T_0 \) is the external load to overcome the inner fabric force.

In this situation, an equation in the form

\[ w_{av} = w_0 \pm b \frac{(w_0 - d)^n}{b_2(2B)} \sqrt{T_1 + m} \]  
\[ (69a) \]

\[ c_{av} = c_0 \pm c_0^m \sqrt{T_1 + m} \]  
\[ (69b) \]

has no root when \( T_1 \) is equal to zero as seen in Figure 8.

It is not difficult to conclude that, if an extension equation has a minus \( m \) value, the load-extension curve would follow the route which is given in Figure 9a. Again, if a contraction equation has minus \( m \) value, it should take the route as given in Figure 9b.

For these kind of structures, when a load-extension or a load-contraction equation is given in the form as

\[ (w_{av} \text{ or } c_{av}) = (w_0 \text{ or } c_0) \pm D_1(T_i + m) \]  
\[ (70) \]

practically obtained \( w_0 \) or \( c_0 \) values should be equal to \( w_0 \) and \( c_0 \) values in Equation (70). The rate of extensions or contractions can be given for the negative \( m \) case in the form as

\[ \varepsilon = \frac{(w_{av} \text{ or } c_{av}) - (w_0 \text{ or } c_0)}{(w_0 \text{ or } c_0)} = \pm D_1(T_i + m) \quad \text{for } T_i \geq |m| \]  
\[ (71) \]

The cases AIa \( \rightarrow \) (R6, R8), AIaii \( \rightarrow \) (R6), Albi \( \rightarrow \) (R6), Allia \( \rightarrow \) (M8, M10), Allia \( \rightarrow \) (M8, M10), Albi \( \rightarrow \) (M8, M10), AlVai \( \rightarrow \) (S10), AlVaii \( \rightarrow \) (S10) in E-glass fabrics have these kinds of load-extension or load-contraction curves (see Figure 10a).

On the other hand, in spite of the linear regression equations that are applied, these kinds of load-extension or load-contraction curves can be distinguished in some of the aramid fabrics such as Biai \( \rightarrow \) (R8, R10), Blaii \( \rightarrow \) (R8, R10), Biai \( \rightarrow \) (R6, R8), Blia \( \rightarrow \) (M8, M10), Bliai \( \rightarrow \) (M8, M10), Blibi \( \rightarrow \) (M8, M10), Blibi \( \rightarrow \) (M8), Bliai \( \rightarrow \) (Y8, Y10), Blibi \( \rightarrow \) (Y8), BlVai \( \rightarrow \) (S8), BlVaii \( \rightarrow \) (S8), BIVbi \( \rightarrow \) (S8) (see Figure 10b).
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Sinem ÖZTÜRK

The diameter of the aramid yarn was \( d = 0.5236 \text{ mm} \) while the diameter of the E-glass yarn was \( d = 0.354 \text{ mm} \). Therefore, the aramid fabric samples obtained here were tighter than the E-glass samples. Since the jamming forces occur in tight fabrics, most of the load-extension or load-contraction curves of the aramid fabric samples would have minus \( m \) values if they could be written in parabolic form. It is thought that this is the other reason to obtain linear regression equations in load-extension or load-contraction curves of the aramid fabrics in addition to the reason given earlier in this section. It can be understood that, although the linear regression equations are applied, some of the aramid load-extension or load-contraction curves for the first stage are actually combinations of two different regions, one of which is the \( T_0 \) region and the other is the parabolic region, as seen in Figure 10b:

\[
T_{\text{w}}(h) \quad \text{and} \quad T_{\text{c}}(h)
\]

**Figure 10.** a) The load-extension results of wale-wise extended glass 1x1 rib structure knitted with 6 cam setting and parabolic curve fitting, b) the load-contraction results of wale-wise extended aramid full cardigan derivative sample knitted with an 8 cam setting and parabolic curve fitting.

For further explanation, regression equations to obtain these roots for a fabric sample are written in the form as

**a) wale-wise loaded samples**

\[ w_{1,2} = w_0 \pm D_{2w}(m/2) \]  
\[ c_{1,2} = c_0 \pm D_{1w}(m/2) \]

**b) course-wise loaded samples**

\[ w_{1,2} = w_0 \pm D_{2c}(m) \]  
\[ c_{1,2} = c_0 \pm D_{1c}(m) \]

It should be noted here that the minus signs in Equations (73-76) stand for root 1 while plus signs stand for root 2 where

\[ D_{2w}(m/2) = v \frac{(w_0-d)^2 \sqrt{m/2}}{b_2 \sqrt{B}} \]  
\[ D_{1w}(m/2) = \frac{c_0 \sqrt{m/2}}{b_2 \sqrt{B}} \]  
\[ D_{2c}(m) = v \frac{c_0 \sqrt{m}}{b_2 \sqrt{B}} \]  
\[ D_{1c}(m) = \frac{(w_0-d)^2 \sqrt{m}}{b_2 \sqrt{B}} \]

can be given for the terms in Equations (73-76). Equations (77-80), in turn, should be evaluated from Empirical Equations in APPENDIX B for the case considered.

Some more definitions are given here to be used in the rest of the work as

\[ D_{2w}(T_{\text{w}}/2 + m/2) = v \frac{(w_0-d)^2 \sqrt{T_{\text{w}}/2+m/2}}{b_2 \sqrt{B}} \]  
\[ D_{1w}(T_{\text{w}}/2 + m/2) = \frac{c_0 \sqrt{T_{\text{w}}/2+m/2}}{b_2 \sqrt{B}} \]  
\[ D_{2c}(T_{\text{c}} + m) = v \frac{c_0 \sqrt{T_{\text{c}}+m}}{b_2 \sqrt{B}} \]
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\[ D_{1c}(T_c + m) = \frac{(w_0 - d)^2 \sqrt{1 + m}}{w_0 \sqrt{B}} \]  

(84)

where Equations (81-84) are used for wale-wise loaded contraction, wale-wise loaded extension, course-wise loaded contraction and course-wise loaded extension respectively.

The evaluations of Equations (81-84) again should be done by using Empirical Equations in APPENDIX B for the case considered.

As analysed in the work of Kurbak [11] the main reason for obtaining the two roots is because of friction resistances which are applied in the dry relaxed fabrics. In the actual situation, the fabric in a dry relaxed state can either be in root 1 \((w_1 \text{ or } c_1)\) condition or in root 2 \((w_2 \text{ or } c_2)\) condition.

As a result of the above discussion, to be able to use Empirical Equations in APPENDIX B, \(w_0\) and \(c_0\) values should be known. These values, in turn, can be obtained by using Equations (73-76). The unknown parameters \(m\) in Equations (73-76) should be estimated first for obtaining \(w_0\) and \(c_0\) values for the intermediate tightness points along with which root should be taken as the starting point of extension or contraction (root 1 or root 2).

The parameters, \(m\), are estimated and the pattern of the starting root of each curve for E-glass fabric samples were searched assuming that this present pattern is repeatable and thus the same pattern could be obtained in future works. This work is given as in the following:

- The tightness, \(\frac{l}{d}\), for each case is calculated first using Table 3a and Equation (9). During these calculations, the plain knit row of the unit cell of Milano Rib, the rib row of the unit cell of Half Cardigan and the rib row of the unit cell of Full Cardigan Derivative Structure are used.
- The parameters, \(m\), given in Table 6 and \(\frac{l}{d}\) are drawn in the graphical forms as seen in Figures 12a and 12b for 1x1 rib; in Figures 13a and 13b for Milano Rib; in Figures 14a and 14b for Half Cardigan; and in Figures 15a and 15b for Full Cardigan Derivative structures.
- Assuming linear relations between \(m\) values and tightness \(\frac{l}{d}\) values, a pattern of \(m\) values for each structure was obtained and is given in the related figures (Figures 12-15).
- The starting roots of extensions and contractions are also shown on these graphical representations by putting (–) for root 1, (+) for root 2, and (.) for negative \(m\) values on top of each case.

When Figures (12-15) are searched, it is seen that the negative \(m\) values are seen to be obtained mainly for tight fabrics.

It should be added that the glass milano rib structure samples have mostly negative \(m\) values. The reason may be the tighter plain knit rows of the unit cell of the milano rib. The rib row and the plain knit rows of the unit cell of the milano rib have been knitted in the same cam setting which is against the experience that the normal tightness of plain knit requires a higher cam setting than the normal cam settings of the 1x1 Rib.

- The relaxed fabrics which are being in root 1 or root 2, is mostly effective on the extension rates and the contraction rates. Since the rate of extension and the rate of contraction are preferred in places of the extension and the contraction themselves in practice, calculations of the extension rate and contraction rate are explained briefly as follows:

![Figure 12](image1.png)

Figure 12. The parameter \(m\) vs. \(\frac{l}{d}\) for 1x1 Rib, a) Wale-wise loaded samples, b) Course-wise loaded samples.

![Figure 13](image2.png)

Figure 13. The parameter \(m\) vs. \(\frac{l}{d}\) for milano rib, a) Wale-wise loaded samples, b) Course-wise loaded samples.
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Figure 14. The parameter $m$ vs. $l/d$ for half cardigan, a) Wale-wise loaded samples, b) Course-wise loaded samples.

Figure 15. The parameter $m$ vs. $l/d$ for full cardigan derivative, a) Wale-wise loaded samples, b) Course-wise loaded samples.

Figure 16: Load-extension and load-contraction curves when the fabric is in root 1 condition, a) load-extension curve, b) load-contraction curve.

i) If the relaxed fabric state is in root 1, for example, for the contraction of the wale-wise loaded samples, the contraction rate would be as

$$
\varepsilon_{2w} = \frac{[w_0-(w_0-D_{2w}(m/2))] - D_{2w}(m/2) + \pm m/2}{w_0-D_{2w}(m/2)}
$$

If the relaxed fabric state is in root 1, for example, for the extension of the wale-wise loaded samples, the extension rate would be as

$$
\varepsilon_{1w} = \frac{[w_0-(w_0-D_{1w}(m/2))] + D_{1w}(m/2) + \pm m/2}{w_0-D_{1w}(m/2)}
$$

The relations between $w_{2w}$ and $\varepsilon_{1w}$ and between $w_{1w}$ and $\varepsilon_{2w}$ would be as in Figures 16a and 16b, respectively.

ii) If the relaxed fabric state is in root 2 condition, for the contraction of the same wale-wise loaded sample, the contraction rate would be as

$$
\varepsilon_{2w} = \frac{w_0-(w_0+D_{2w}(m/2)) - D_{2w}(m/2) - \pm m/2}{w_0+D_{2w}(m/2)}
$$

If the relaxed fabric state is in root 2 condition, for the extension of the same wale-wise loaded sample, the extension rate would be as

$$
\varepsilon_{1w} = \frac{w_0+(w_0+D_{1w}(m/2)) - D_{1w}(m/2) - \pm m/2}{w_0+D_{1w}(m/2)}
$$

The relations between $w_{2w}$ and $\varepsilon_{1w}$ and between $w_{1w}$ and $\varepsilon_{2w}$ would be as in Figures 17a and 17b.

Very similar equations as Equations (85-88) and very similar curves as in Figures (16-17) are obtained for the course-wise loaded samples, as well.

It can be seen from the above discussion that the extension rates and/or the contraction rates are different according to the relaxed fabrics being in root 1 or root 2. Using the present experimental results, the equations of $m$ values in terms of tightness $(l/d)$ (assuming linear relations), the starting roots of the relaxed fabrics, and the contraction or the extension rates for glass fabrics are given in APPENDIX C.
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To complete the work, the calculated load-extension and load-contraction rates are compared with the experimentally obtained load-extension and load-contraction rates in Figures 18-26 for all the structures, tightness, and types of yarns used in this work.

During obtaining the theoretical load-extension curves in Figures 18-26, the \( m, c_0, \) and \( w_0 \) values are taken from Table 6. According to the roots given in Table 6, similar equations as in Equation (71, 85-88) are calculated to obtain the extension or contraction rate values. As noted above, example calculations of extension and contraction rates are also given in APPENDIX C for the glass samples.

During obtaining the experimental extension or contraction rates, Tables in APPENDIX A are used as

\[
\varepsilon_2 = \frac{b_{\min}(i) - b_i}{1.5 b_0}; \quad (i = 1 \text{ to } 5)
\]

\[
\varepsilon_1 = \frac{b_i - b_0}{b_0}; \quad (i = 1 \text{ to } 5)
\]

It should be noted here that the contractions are measured in the middle of the samples as \( b_{\min} \). In order to obtain \( b_{\min} \) values, the side edges of the samples are assumed to be as in Figure 5a in this work, and similar equations as Equations (34-35) are used for obtaining \( b_{\min} \) values. Therefore, a dividing factor 1.5 exists in Equation (89).

The load applied, \( T \), are calculated by dividing the total loads given at the Tables in APPENDIX A by the average number of loops on a face of the cross-section of fabric that carries out the total loads as

\[
T_w = \frac{\text{Total wale-wise Loads (see Tables 5 and 7)}}{b \ (\text{wpc})}
\]

\[
T_c = \frac{\text{Total course-wise Loads (see Table 6 and 8)}}{b \ (\text{cpc})}
\]

The applied loads \( T_w \) and \( T_c \) given in Equations (91-92) are used for obtaining the theoretical calculations of extension and contraction rates as well as for obtaining the experimental load-extension and load-contraction results.

It should be said further that \( b \ (\text{wpc}) \) and \( b \ (\text{cpc}) \) in Equations (91-92) are the average number of loops on one face of the samples. The chosen faces were explained during obtaining (wpc) and (cpc) parameters by using Equations (15-16). These definitions of applied loads makes easier to obtain the carried load by a unit cross-sections of fabrics at any structures by having \( T_w \) (wpc) [g/cm] and \( T_c \) (cpc) [g/cm] and so on.

In some of the cases, \( c_0 \) and \( w_0 \) values given in Table 6 do not give the calculated extension and/or contraction rates correctly, so the drawn curves do not follow the experimental points. For these cases some suitable \( c_0 \) and/or \( w_0 \) values are calculated to fit the experimental points. These calculated new \( c_0 \) and/or \( w_0 \) values are given in Table 6 in brackets. They may be due to five experimental points which are less for obtaining the correct \( c_0 \) and/or \( w_0 \) values.

In some of the tight fabric cases, the calculated curves do not follow the experimental points. These discrepancies can be explained as follows; for example, for the glass 6 cam setting tight fabrics, the first point was put out of the regression equation. It was later seen that the second point in the 6 cam setting and the first point in the 8 cam setting of the 1x1 rib glass fabric should also be put out of the regression equations as shown in Figure 10. This kind of discrepancy is also the result of fewer experimental points since, if the second experimental point in the 6 cam setting of the glass fabrics were also left out of the regression equations, only three point would be left for a quadratic curve fitting.

In spite of some discrepancies mentioned above, it is seen that Empirical Equations in APPENDIX B can be used to estimate the extension and the contraction rates for a given load value for any kind of complex knitted structure at any tightness.

Figure 17: Load-extension and load-contraction curves when the fabric is in root 2 condition, a) load-extension curve, b) load-contraction curve.
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Figure 18: Glass 1x1 rib load-extension results and empirical equations.

Figure 19: Glass miliano rib load-extension results and empirical equations.
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Figure 20: Glass half cardigan load-extension results and empirical equations.

Figure 21: Glass full cardigan load-extension results and empirical equations.
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Figure 22: Aramid 1x1 rib load-extension results and empirical equations.

Figure 23: Aramid milano load-extension results and empirical equations.
5. RESULTS AND DISCUSSION

Results of the initial load-extension and load-contraction properties of complex weft knitted fabrics will be given in comparison with the same properties of plain knitted fabric which was given by Kurbak [11] such that:

i) For the extension and contraction in fabric length direction of plain knitted glass fabric, linear regression equations with higher correlation coefficients could be applied while they were quadratic curve fittings for the fabric width directions. For the extension and contraction in both directions for complex weft knitted glass fabrics, however, quadratic curve fittings could be applied with higher correlation coefficients. It may be because of the loop arms which are nearly two dimensional curves for plain knitted fabrics while they are three dimensional for complex weft knitted structures.

ii) Mainly three kinds of quadratic curve fittings are obtained for complex weft knitted fabrics as given in schematic Figures 26a, 26b and 26c.
Calculations of extension and contraction rates in relation to the tightness of fabrics are given in APPENDIX C for glass samples. These rates are also given in Equations (71, 85-88). Apart from these, example calculations of extension and contraction rates are provided in Figures 12-15 during the present experimental work using a measuring tester.

The red dotted line given in Figure 26c along with the blue line is the actual load-extension curve which can be obtained by applying, load-extension rate results are obtained as in Figure 26b. This can occur because there are jamming forces in tight fabrics, and slack fabrics in Figures 26b and 26c. It is thought that two roots exist for complex weft knitted fabrics. This can happen because there are jamming forces in tight fabrics, and slack fabrics in Figures 26b and 26c. It is thought that two roots exist for complex weft knitted fabrics.

Equation (69b) is negative, the situation in Figure 26a is obtained. If the load and extension rate relationship is uncertain as in Figure 26c, it cannot be used for cyclic loading, for example, for “stretch sensor” applications (see ref. [20]).

It is thought that if the parameter \( m \) in Equation (69a) or in Equation (69b) is negative, the situation in Figure 26a is obtained. In this situation, no root is obtained when the load is equal to zero. This can occur because there are jamming forces in tight fabrics, and slack fabrics in Figures 26b and 26c. It is thought that two roots exist for complex weft knitted fabrics.

In most of the cases, the calculated load-extension curves fit with the given Empirical Equations in APPENDIX B. However, when the parameter \( m \) is positive, the two cases are obtained as in Figures 26b and 26c. It is thought that two roots exist for Equations (73-76), namely root 1 and root 2, and slack fabrics in a dry relaxed state can be in root 1 or root 2 conditions, according to their relaxation from loaded knitting conditions. Fabrics in these conditions can be in a friction induced equilibrium state. If a relaxed fabric is in root 2 condition and the extension load is applied, load-extension rate results are obtained as in Figure 26b. Again, if a fabric is in root 1 condition and the extension load is applied, load-extension rate results are obtained as in Figure 26c. The red dotted line given in Figure 26c along with the blue line is the actual load-extension curve which can be obtained by using a measuring tester.

The negative \( m \) case, the root 1 case, and the root 2 case obtained during the present experimental work are given in Figures 12-15 for glass samples. Calculations of extension and contraction rates are also given in Equations (71, 85-88). Apart from these, example calculations of extension and contraction rates in relation to the tightness of fabrics are given in APPENDIX C for glass samples.

When Kurbak’s work [11] is examined, it can be seen that root 1 and root 2 conditions also exist in plain knitted glass fabrics.

In complex knitted fabrics, the other curve fittings obtained are the linear regression equation which is given in Figure 26d. While linear regression equations were obtained for the wale-wise load-extension properties in plain knitted fabric because of the yarn arms of plain knitted fabrics which were nearly two dimensional, the linear regression equations obtained for three dimensional complex weft knitted fabrics are because of higher bending rigidities along with higher tightness. Linear regression equations are obtained in tight aramid fabrics. It is thought that these tight complex weft knitted fabrics with higher bending rigidities also have parabolic load-extension or load-contraction curves, but their extensions or contractions are too small to be approximated by linear curve fittings.

The estimations of Poisson’s ratios are reasonable since they decrease according to the complexity of the structure, as they should be.

More research, however, must be conducted by having more load levels in the region during experiments at any tightness point and also to increase the tightness points to cover intermediate tightness. It is thought that with this future research, the parameters \( c_0 \), \( \omega_0 \), and \( m \) in Table 6 can be formulated in relation to the tightness, \( l/d \), and, thus, the above procedure will become a complete method.

Finally, some suggestions can be made on some of the technical applications of knitted fabrics according to the present results.

- If the load and extension rate relationship is uncertain as in Figures 26b and 26c, it cannot be used for cyclic loading, for example, for “stretch sensor” applications. \( \text{Figure 26. Schematic drawings of curve fittings obtained for load against extension rates (T vs. %E) of complex weft knitted structures a) quadratic curve fitting with no root, b) quadratic curve fitting with root 2, c) quadratic curve fitting with root 1 and d) the linear load-extension curve.} \)
• In composite reinforcement application of knitted fabrics; Araujo et al. [21] suggested that if a fabric is extended at a certain rate, the low load region of the load-extension curves can be omitted and the breaking strengths of composites can be increased. They used this method with a 20% extension rate.

According to the present work, this suggestion is not suitable for every tightness of the fabric since the fixed amount of extension rate could be bigger than the maximum extension of tight fabrics. Apart from this, fabrics can be in one of the load-extension curves of every tightness of the fabric since the fixed amount of extension region will not be completely omitted. It is thought that, instead of using a certain extension rate, the certain amount of load applied on a loop should be more suitable to use.

In addition to the above argument that the wale-wise breaking strengths of the composites reinforced with the plain knitted fabrics and the milano rib fabrics are obtained to be higher than the other weft knitted structures in practice, it is interesting to see that the wale-wise extensions of plain-knitted fabrics and the milano rib fabrics start from their relaxed conditions given in Figures 26a, 26b and 26d as seen in Figure 19a, Figure 23a, and Figure 6 of the reference paper by Kurbak [11].

• Lastly, the mechanical properties of fabrics are calculated by using their geometrical models nowadays (see refs. [22, 23]). Before starting of these kinds of calculations, the initial load-extension properties of the subjected fabric should first be experimentally investigated. Otherwise, some discrepancies may occur between the theoretically calculated values and the measured experimental results. There is one exception: if a fabric has a special tightness at which the course and/or wale jamming has just started to occur, on this special tightness point, the m value becomes zero. For this special tightness point, theoretical calculations can be carried out without the need for the initial experimental results.

6. CONCLUSION

In this work, an experimental investigation on the initial load-extension and load-contraction properties of some complex weft knitted technical fabrics was carried out based on the work of Kurbak [11] in plain knitted glass fabrics. The work is summarized and the conclusion given below:

a) Two types of yarn were used namely 136 tex E-glass and 168 tex aramid. Four different structures were chosen: 1x1 rib, milano rib, half cardigan, and full cardigan derivative. Two types of samples were prepared for the wale-wise direction and the course-wise direction. For the 1x1 rib, three tightness points were taken: the 6 cam setting, the 8 cam setting, and the 10 cam setting of the 7 gauge V-bed hand knitting machine. For the milano rib, the half cardigan, and full cardigan derivative, two tightness points were taken, the 8 cam setting and the 10 cam setting. Six samples for each experiment were prepared. A total of 216 samples were prepared.

Samples were adjusted so that the usable length (b) and width (w) were about 20x10 cm² plus enough length to hold them by a load-extension measuring apparatus.

One sample for each experimental point was used to measure the initial dimensional properties of the fabric: the loop length (l), the course-per-cm (cpe), and wales-per-cm (wpc).

The load-extension and load-contraction properties of the 20x10 cm² samples were then measured on a special apparatus (see Figure 4) using 5 level dead weights. The load levels were increased from zero to the maximum load of the first region. For every load level, the lengths (l) and the middle widths (bmin) of the samples were measured. Five samples for each experimental point were measured, and their averages were calculated.

b) Regression analyses were carried out on the relation between the loads (T) and lengths (l) and also between the loads (T) and widths (wmin). Mostly quadratic curve fittings with higher correlation coefficients were obtained between the load-extension and the load-contraction results except that, for some, tight aramid fabrics linear regression equations could be applied to the experimental results with higher correlation coefficients.

c) For comparison purposes, the regression equations (Equations 6 and Table 5) are written in terms of the loop parameters $c_{ey}$, $c_{ey}$, $w_{ey}$, $w_{ey}$, $T_e$, and $T_n$ using Equations (21-22, 26-27).

d) The last obtained equations were then turned into the forms of equations as given in Equations (28-31) according to the assumption made about the separating method of the frictional restraints and/or fabric jamming forces from quadratic curve fittings. In the form of Equations (28-31) the m values were considered as the frictional restraints or/fabric jamming forces for the unit structures of fabrics.

e) For the first stages of extensions and contractions, Kurbak [11] gave some equations as Equations (32-35) to replace $c_{min}$ and $w_{min}$ values with the $c_{ey}$ and $w_{ey}$ values by assuming that the side edges of the samples follow parabolic curves under loading conditions (see Figure 5a). Equations (32-35) are applied to all the $c_{min}$ and $w_{min}$ values in the regression equations, and they are turned into the forms given in Equations (36-39). The parameters of all the obtained regression equations in the form of Equations (36-39) are given in Table 6. It is seen in Table 6 that the m values are small enough to be considered as frictional restraints and/or fabric jamming forces. Therefore, the assumption made about the separating method of the m values from quadratic curve fittings is the reasonable one.

f) When all the parabolic curve fittings were examined, three kinds of situations were distinguished:

i) When parameter m is negative (in this case, when external load $T_e$ or $T_c$ is equal to zero), the square root in Equations...
(53-56) become imaginary; therefore, no root is obtained. This situation was primarily obtained for tight fabrics.

ii) When parameter \( m \) is positive; there are two roots, root 1 and root 2. The fabric can be in each of the roots when the external load is equal to zero. These cases— the negative \( m \) case, the root 1 case, and the root 2 case—are also given in Table 6 as 0, 1, and 2 respectively. One more case is that when the linear regression equation is applied. For the linear equation case, a “dash” (-) is used in Table 6.

g) The first region (initial region) is the most geometrically uncertain region of the load-extension and load-contraction curves; thus, if one wants to obtain the mechanical properties of knitted fabrics by using their geometrical models, this region may create some problems. Therefore, the attempt was made to find out if some empirical equations can be given for this region that can be used in related engineering software. The empirical equations are given as in APPENDIX B. While obtaining the empirical equations, Poisson’s ratios of the structures were also estimated. The estimated Poisson’s ratios are reasonable since they reduce with the complexities of the fabrics, as they should be.

h) In Empirical Equations given in APPENDIX B, the parameters \( c_0, w_0 \), and \( m \) values are taken from Table 6. It is already known that the parameters \( c_0 \) and \( w_0 \) are related to the tightness \( l/d \). It is attempted to obtain some relationship between the \( m \) parameters and the tightness \( l/d \), as given in Figures 12-15. It was thought that, to obtain the exact relations between the parameter \( m \) and the tightness \( l/d \), some more tightness points should be taken from each fabric structure during the experiments.

i) Using Table 6 and the Empirical Equations in APPENDIX B, the load-extension rates and load-contraction rates are obtained through some equations which are similar to Equations (71, 85-88) for all the cases considered here. The results obtained were compared with the experimentally obtained extension and contraction rates as given in Figures 18-25.

Figures 18-25 show that the Empirical Equations in APPENDIX B correctly estimated the extension and contraction rates. Because only 5 load levels are taken for the first extension or contraction region in this work, for 7 points out of 70 points, however, the \( c_0 \) or \( w_0 \) values were obtained, which are different from the given points in Table 6. The new points are also written in brackets in Table 6.

Finally, it can be concluded that three different parabolic curve fittings of the load-extension and load-contraction rates were obtained as given schematically in Figures 26a, 26b and 26c. The linear curve fittings were also obtained for tighter aramid fabrics. The three kinds of curve fittings in Figures 26a, 26b and 26c could be calculated by Empirical Equation in APPENDIX B through some equations similar to Equations (71, 85-88); thus, this can be considered as a new method. It is thought that, when this method is improved by future experiments, it will be useful for obtaining the load-extension or load-contraction properties of such a geometrically uncertain region, namely the initial load-extension or load-contraction region.

ACKNOWLEDGEMENT

Experimental section of this work was partly supported by TUBITAK (Scientific and Technological Research Council of Turkey) through the project No.1002/114M022. The authors would like to thank to TUBITAK for this support. The authors would like to thank Assoc. Prof. Dr. Tuba Alpaydiz for her contributions during carrying out the experimental section of the work. The authors would like to thank also Dr. Gonca Balci Kilic, Ph D Students; Mehmet Korkmaz and Berrak Bukan Avc, MSc Student Gökköker Devrim, BSc Student Mehmet Çağatay Bahçı and MSc Arch. Darioush Bashiri for their contributions during preparation of this paper.

The research received no specific grant any funding agency in the public, commercial, or not-for-profit sectors.

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APPENDIX A

The experimentally obtained $h$ and $b_{min}$ values with changing load levels $T$ are given in Tables A1a and A1b for wale-wise loaded glass samples, in Table A2a and A2b for course-wise loaded glass samples, in Table A3a and A3b for wale-wise loaded aramid samples and in Table A4a and A4b for course-wise loaded aramid samples.

Example drawings of load-extension and load contraction curves for each table are also given in Figures A1a, A1b, A2a, A2b, A3a, A3b, A4a and A4b.

Table A1: Measured initial (the first stage) load-extension and load contraction results for wale-wise loaded glass samples.

| Load [g] | R6 | R8 | R10 | M8 | M10 | Y8 | Y10 | S8 | S10 |
|----------|----|----|-----|----|-----|----|-----|----|-----|
| 0        | 20 | 20 | 20  | 20 | 20  | 20 | 20  | 20 | 20  |
| 238.4    | 20.94 | 27.1 | 28.88 | 21.32 | 26.1 | 32.58 | 33.46 | 28.34 | 35.02 |
| 480      | 26.24 | 30.48 | 33  | 25.22 | 31.66 | 36.62 | 36.82 | 36.48 | 42.14 |
| 800      | 30.12 | 33.06 | 35.36 | 28.58 | 34.26 | 39.62 | 40.02 | 40.54 | 45.68 |
| 1050     | 30.92 | 33.92 | 36.24 | 29.88 | 35.48 | 40.9 | 40.98 | 41.98 | 46.8 |

Figure A1: Example drawings of load-extension and load-contraction results given in Table A1; a) load-extension, b) load-contraction.
Table A2: Measured initial (the first stage) load-extension and load-contraction results for course-wise loaded glass samples.

| Glass course-wise extension | Results | 1x1 Rib | Milano Rib | Half Cardigan | Full Cardigan | Derivative |
|----------------------------|---------|---------|------------|---------------|---------------|------------|
| a) extension (h) [cm]      |         |         |            |               |               |            |
| Load [g]                   | R6      | R8      | R10        | M8            | M10           | Y8         | Y10        | S8          | S10          |
| 0                          | 20      | 20      | 20         | 20            | 20            | 20         | 20         | 20          | 20           |
| 238.4                      | 23.22   | 40.82   | 57.26      | 22.04         | 25.66         | 39.3       | 50.2       | 40.04       | 45.52        |
| 480                        | 30.72   | 49.86   | 66.26      | 24.9          | 30.08         | 46.6       | 56.1       | 46.66       | 49.08        |
| 800                        | 38.48   | 55.58   | 70.32      | 28.3          | 33.26         | 51.34      | 60.34      | 51.74       | 52.08        |
| 1050                       | 41.88   | 57.76   | 72.08      | 29.88         | 34.02         | 53         | 61.66      | 53.04       | 52.9         |
| b) contraction (b) [cm]    |         |         |            |               |               |            |
| Load [g]                   | R6      | R8      | R10        | M8            | M10           | Y8         | Y10        | S8          | S10          |
| 0                          | 10.1    | 6.7     | 8.6        | 9.2           | 9.7           | 9.3        | 9.0        | 8.3         | 8.6          |
| 238.4                      | 9.7     | 4.5     | 3.6        | 8.8           | 7.8           | 8.1        | 6.8        | 7.3         | 6.2          |
| 480                        | 8.6     | 3.1     | 2.4        | 7.7           | 5.5           | 7.2        | 5.8        | 6.5         | 5.2          |
| 800                        | 6.6     | 2.5     | 2.0        | 6.0           | 3.8           | 6.4        | 5.1        | 5.7         | 4.3          |
| 1050                       | 5.9     | 2.3     | 1.8        | 5.1           | 3.4           | 5.9        | 4.7        | 5.4         | 4.0          |

Figure A2: Example drawings of load-extension and load-contraction results given in Table A2; a) load-extension, b) load-contraction.
Table A3: Measured initial (the first stage) load-extension and load-contraction results for wale-wise loaded aramid samples.

### Aramid wale-wise extension

#### Results

**a) extension \((h)\) [cm]**

| Load [g] | R6 | R8 | R10 | M8 | M10 | Y8 | Y10 | S8 | S10 |
|----------|----|----|-----|----|-----|----|-----|----|-----|
| 0        | 20 | 20 | 20  | 20 | 20  | 20 | 20  | 20 | 20  |
| 238.4    | 20.36 | 20.42 | 24.96 | 20.12 | 21.26 | 22.02 | 23.28 | 21.64 | 25.4 |
| 480      | 20.62 | 22.76 | 33.06 | 20.46 | 23.36 | 23.78 | 25.98 | 24.6 | 27.92 |
| 800      | 21.1 | 25.72 | 36  | 21.08 | 25.44 | 25.04 | 27.92 | 27.2 | 29.52 |
| 1050     | 21.46 | 26.9 | 36.82 | 21.46 | 26.24 | 25.62 | 28.68 | 28.04 | 30.1 |

**b) contraction \((b)\) [cm]**

| Load [g] | R6 | R8 | R10 | M8 | M10 | Y8 | Y10 | S8 | S10 |
|----------|----|----|-----|----|-----|----|-----|----|-----|
| 0        | 7.8 | 10.1 | 12.1 | 8.9 | 9.8  | 11.7 | 12.2 | 11.3 | 13.2 |
| 238.4    | 7.6 | 9.9  | 10.1 | 8.8 | 9.4  | 11.4 | 11.6 | 11.2 | 12.4 |
| 480      | 7.5 | 9.2  | 8.1  | 8.7 | 8.7  | 11.0 | 11.1 | 10.9 | 11.5 |
| 800      | 7.3 | 8.3  | 7.4  | 8.5 | 8.0  | 10.6 | 10.6 | 10.4 | 10.6 |
| 1050     | 7.2 | 7.9  | 7.2  | 8.3 | 7.6  | 10.5 | 10.4 | 10.1 | 10.2 |

Figure A3: Example drawings of load-extension and load-contraction results given in Table A3; a) load-extension, b) load-contraction.
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Sinem OZTÜRK

Table A4: Measured initial (the first stage) load-extension and load-contraction results for course-wise loaded aramid samples.

Aramid course-wise extension

| Results          | 1x1 Rib | Milano Rib | Half Cardigan | Full Cardigan | Derivative |
|------------------|---------|------------|---------------|---------------|------------|
| Load [g]         | R6      | R8         | R10           | M8            | M10        | Y8         | Y10        | S8         | S10        |
| 0                | 20      | 20         | 20            | 20            | 20         | 20         | 20         | 20         | 20         |
| 238.4            | 21.02   | 21.94      | 38.08         | 20.22         | 21.42      | 25.36      | 40.74      | 27.18      | 42.94      |
| 480              | 22.92   | 25.1       | 45.08         | 20.62         | 23.14      | 31.02      | 47.84      | 33.98      | 49.06      |
| 800              | 25.56   | 28.64      | 52.48         | 21.32         | 25.54      | 35.4       | 52.54      | 38.8       | 53         |
| 1050             | 26.8    | 30.5       | 57.76         | 21.78         | 27.08      | 37.16      | 54.2       | 40.52      | 55.04      |

b) contraction (b) [cm]

| Load [g] | R6 | R8 | R10 | M8 | M10 | Y8 | Y10 | S8 | S10 |
|----------|----|----|-----|----|-----|----|-----|----|-----|
| 0        | 9.1| 9.3| 8.5 | 9.6| 10.2| 9.5| 8.3 | 8.1| 7.1 |
| 238.4    | 9.0| 9.1| 7.4 | 9.6| 9.9 | 9.3| 7.5 | 7.9| 6.3 |
| 480      | 8.8| 8.8| 6.6 | 9.5| 9.5 | 8.8| 6.5 | 7.6| 5.4 |
| 800      | 8.5| 8.2| 5.0 | 9.3| 8.8 | 8.3| 5.7 | 7.1| 4.7 |
| 1050     | 8.3| 7.8| 3.8 | 9.1| 8.4 | 8.0| 5.3 | 6.8| 4.3 |

Figure A4: Example drawings of load-extension and load-contraction results given in Table A4; a) load-extension, b) load-contraction.
APPENDIX B

EMPIRICAL EQUATIONS

A) GLASS

I) 1x1 RIB STRUCTURE

a) for wale-wise loaded samples

i) contraction

\[ w_{av} = w_0 - v \left( w_0 - d \right)^{1.3826} \frac{\sqrt{7w}}{3.4231 \sqrt{B}} \]  
(B1)

ii) extension

\[ c_{av} = c_0 + \frac{c_{av}}{3.7080 \sqrt{B}} \]  
(B2)

b) for course-wise loaded samples

i) contraction

\[ c_{av} = c_0 - \frac{v_0 \sqrt{7w}}{3.7080 \sqrt{B}} \]  
(B3)

ii) extension

\[ w_{av} = w_0 + \left( w_0 - d \right)^{1.3826} \frac{\sqrt{7w}}{3.4231 \sqrt{B}} \]  
(B4)

where

\[ v = \frac{1}{2.3319} = 0.4288 \]  
(B5)

\[ n_1 = 2.6582 - 2.1194 \left( \frac{c_0}{d} - 3.7932 \right) \]  
(B6)

\[ n_2 = 2.34071 \left( \frac{w_0}{d} - 5.5671 \right)^{-1/3.25} \]  
(B7)

A) GLASS

II) MILANO RIB STRUCTURE

a) for wale-wise loaded samples

i) contraction

\[ w_{av} = w_0 - v \left( w_0 - d \right)^{1.675} \frac{\sqrt{7w}}{4.4476 \sqrt{B}} \]  
(B8)

ii) extension

\[ c_{av} = c_0 + \frac{c_{av}}{4.9330 \sqrt{B}} \]  
(B9)

b) for course-wise loaded samples

i) contraction

\[ c_{av} = c_0 - v \left( \frac{c_0}{d} - 5.52 \right) \]  
(B10)

ii) extension

\[ w_{av} = w_0 + \left( w_0 - d \right)^{1.675} \frac{\sqrt{7w}}{4.4476 \sqrt{B}} \]  
(B11)

where

\[ v = \frac{1}{2.5816} = 0.3874 \]  
(B12)

\[ n_1 = 2.4441 - \frac{\left( \frac{c_0}{d} - 5.52 \right)}{1.6674 \frac{c_0}{d} - 5.8590} \]  
(B13)

\[ n_2 = 2.44034 - \frac{\left( \frac{w_0}{d} - 5.8590 \right)}{3.7364} \]  
(B14)

A) GLASS

III) HALF CARDIGAN STRUCTURE

a) for wale-wise loaded samples

i) contraction

\[ w_{av} = w_0 - v \left( w_0 - d \right)^{a_1} \frac{\sqrt{7w}}{1.2504 \sqrt{B}} \]  
(B15)
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\( c_{av} = c_0 + c_0^{\frac{0.66}{a} \sqrt{\frac{d}{T_w/2+m/2}} \sqrt{B}} \) \hspace{1cm} (B16)

**b) for course-wise loaded samples**

\( c_{av} = c_0 - v \frac{c_0^{0.02(1/a)^{3/2}}}{\sqrt{B}} \)

\( w_{av} = w_0 + \frac{(e_0-d)^{0.1(1/a)} \sqrt{C+m}}{2.2346} \) \hspace{1cm} (B18)

where Poisson’s ratio and a constant, \( a \), are as

\[ v = \frac{1}{5.0994} = 0.1996 \] \hspace{1cm} (B19)

\[ a = 1.2891 \] \hspace{1cm} (B20)

\[ n_1 = 0.2506 - \frac{w_0}{d} - 9.1484 \] \hspace{1cm} (B21)

\[ n_2 = 2.8312 - (c_0/d - 4.8031)/1.5423 \] \hspace{1cm} (B22)

\[ n_3 = 0.7535 - (w_0/d - 12.2359)/32.4436 \] \hspace{1cm} (B23)

**A) GLASS**

**IV) FULL CARDIGAN DERIVATIVE**

**a) for wale-wise loaded samples**

\( c_{av} = c_0 + c_0^{n_1 \frac{a \sqrt{d/2+m/2}}{0.08065 \sqrt{B}}} \) \hspace{1cm} (B25)

\( w_{av} = w_0 - v \frac{(e_0-d)^{1.517} \sqrt{d/2+m/2}}{2.2267} \) \hspace{1cm} (B24)

\( w_{av} = w_0 - \frac{(e_0-d)^{1.10} \sqrt{d/2+m/2}}{2.2267} \) \hspace{1cm} (B27)

where

\[ v = \frac{1}{5.1790} = 0.1930 \] \hspace{1cm} (B28)

\[ a = 0.9762 \] \hspace{1cm} (B29)

\[ n_1 = -0.037338 - \frac{c_0/d - 6.8443}{30.9019} \] \hspace{1cm} (B30)

\[ n_2 = 0.7555 - \frac{w_0/d - 12.6400}{30.7872} \] \hspace{1cm} (B31)

**B) ARAMID**

**I) 1x1 RIB STRUCTURE**

**a) for wale-wise loaded samples**

\( c_{av} = c_0 + c_0^{n_1 \frac{a \sqrt{d/2+m/2}}{0.08065 \sqrt{B}}} \) \hspace{1cm} (B26)

\( w_{av} = w_0 + v \frac{(e_0-d)^{1.10} \sqrt{d/2+m/2}}{2.2267} \) \hspace{1cm} (B27)

\( c_{av} = c_0 - v \frac{(e_0-d)^{1.10} \sqrt{d/2+m/2}}{2.2267} \) \hspace{1cm} (B26)

where

\[ v = \frac{1}{5.4590} = 0.1990 \] \hspace{1cm} (B28)

\[ a = 0.9762 \] \hspace{1cm} (B29)

\[ n_1 = -0.037338 - \frac{c_0/d - 6.8443}{30.9019} \] \hspace{1cm} (B30)

\[ n_2 = 0.7555 - \frac{w_0/d - 12.6400}{30.7872} \] \hspace{1cm} (B31)
\[
R10 \quad c_{av} = c_0 + \frac{a n_2}{3.1709 \sqrt{B}}
\]

\[b) \text{ for course-wise loaded samples} \]

i) contraction
\[
c_{av} = c_0 - v \frac{a n_3 (1/a)}{3.1709 (2B) T_c}
\]

ii) extension
\[
w_{av} = w_0 + \frac{(w_0 - d)n_4 (1/a)}{0.5662 (2B)} T_c + m
\]

where Poisson’s ratio
\[
v = \frac{1}{2.3319} = 0.4288
\]
\[
a = 1.0500
\]
\[
n_1 = \begin{cases}
-12.4049 + 2.2727 \frac{w_0}{d}; \text{for} \frac{w_0}{d} \leq 5.6671 \\
 0.2520; \text{for} \frac{w_0}{d} \geq 5.6671
\end{cases}
\]
\[
n_2 = \begin{cases}
-343.9483 + 137.3074 \frac{c_0}{d}; \text{for} \frac{c_0}{d} \leq 2.5401 \\
 3.3608; \text{for} \frac{c_0}{d} \geq 2.5401
\end{cases}
\]
\[
n_3 = \begin{cases}
3.6553 + 5.6671 (c_0/d - 2.5730); \text{for} \frac{c_0}{d} \leq 2.5730 \\
3.6553 + (c_0/d - 2.5730)/2.1812; \text{for} \frac{c_0}{d} \geq 2.5730
\end{cases}
\]
\[
n_4 = \begin{cases}
-1.9575 + \frac{w_0}{1.9235}; \text{for} \frac{w_0}{d} \leq 5.6671 \\
 1.2957; \text{for} \frac{w_0}{d} \geq 5.6671
\end{cases}
\]

B) ARAMID

II) MILANO RIB STRUCTURE

a) for wale-wise loaded samples

i) contraction
\[
w_{av} = w_0 - v \frac{(w_0 - d)^{5.2548} (1/a)}{34.6900 (2B)} T_c
\]

ii) extension
\[
c_{av} = c_0 + \frac{a^{0.7778} (1/a)}{8.3467 (2B) T_c}
\]

b) for course-wise loaded samples

i) contraction
\[
c_{av} = c_0 - v \frac{a^{3.6933} (1/a)}{8.3467 (2B) T_c}
\]

ii) extension
\[
w_{av} = w_0 + \frac{(w_0 - d)^{4.5893} a}{34.6900 (2B) T_c}
\]

where
\[
v = \frac{1}{2.5949} = 0.3857
\]
\[
a = 1.2369
\]

B) ARAMID

III) HALF CARDIGAN STRUCTURE

a) for wale-wise loaded samples

i) contraction
\[
w_{av} = w_0 - v \frac{(w_0 - d)^{n_1} \sqrt{T_w/2 + m/2}}{0.5016 \sqrt{B}}
\]

ii) extension
\[ c_{av} = c_0 + \frac{c_0^{n_2} \sqrt{T_w/2 + m/2}}{0.7295 \sqrt{B}} \]  
(B52)

**b) for course-wise loaded samples**

1. **contraction**
   \[ \begin{align*}
   Y8) & \quad c_{av} = c_0 - v \frac{c_0^{n_3}}{0.7295 (2B)} T_c \\
   Y10) & \quad c_{av} = c_0 - v \frac{c_0^{n_4}}{0.7295 \sqrt{T_c + m}}
   \end{align*} \]  
(B53)  
(B54)

2. **extension**
   \[ \begin{align*}
   w_{av} = w_0 + \frac{(w_0 - d)^{n_5} \sqrt{T_c + m}}{0.5016 \sqrt{B}}
   \end{align*} \]  
(B55)

where
\[ \begin{align*}
   v &= \frac{1}{5.0994} = 0.1996 \quad \text{(B56)} \\
   n_1 &= 0.1514 - 2.4037 (w_0 / d - 7.07861) \quad \text{(B57)} \\
   n_2 &= -1.0356 + \frac{c_0 / d - 3.5470}{1.616} \quad \text{(B58)} \\
   n_3 &= -0.2381 + \frac{4.7227}{w_0^2 - 5.0750} \quad \text{(B59)} \\
   n_4 &= 0.7911 - \frac{d}{9.9688} \quad \text{(B60)}
\end{align*} \]

**B) ARAMID**

**IV) FULL CARDIGAN DERIVATIVE**

1. **a) for wale-wise loaded samples**
   
   1. **contraction**
      \[ \begin{align*}
      S8) & \quad w_{av} = w_0 - v \frac{(w_0 - d)^{n_1} \sqrt{T_w}}{0.7275 (2B)} \\
      S10) & \quad w_{av} = w_0 - v \frac{(w_0 - d)^{n_2} \sqrt{T_w}}{0.7275 \sqrt{T_c + m}}
      \end{align*} \]  
(B61)  
(B62)

   2. **extension**
      \[ \begin{align*}
      c_{av} = c_0 + \frac{c_0^{n_3}}{0.7692 \sqrt{B}} \frac{\sqrt{T_w/2 + m/2}}{0.7275 \sqrt{B}}
      \end{align*} \]  
(B63)

2. **b) for course-wise loaded samples**
   
   1. **contraction**
      \[ \begin{align*}
      S8) & \quad c_{av} = c_0 - v \frac{c_0^{n_3}}{0.7692 (2B)} T_c \\
      S10) & \quad c_{av} = c_0 - v \frac{c_0^{n_4}}{0.7692 \sqrt{T_c + m}}
      \end{align*} \]  
(B64)  
(B65)

   2. **extension**
      \[ \begin{align*}
      w_{av} = w_0 + \frac{(w_0 - d)^{n_5} \sqrt{T_c + m}}{0.7275 \sqrt{B}}
      \end{align*} \]  
(B66)

where
\[ \begin{align*}
   v &= \frac{1}{5.1790} = 0.1930 \quad \text{(B67)} \\
   n_1 &= -3.2302 + \frac{w_0 / d}{1.6392} \quad \text{(B68)} \\
   n_2 &= 7.2022 - 12.190 c_0 / d \quad \text{(B69)} \\
   n_3 &= 0.4086 + \frac{c_0 / d}{0.87654} \quad \text{(B70)} \\
   n_4 &= 0.7410 - \frac{w_0 / d - 9.5493}{7.7947} \quad \text{(B71)}
\end{align*} \]
APPENDIX C

EXAMPLE CALCULATIONS OF THE PARAMETER \( m \), THE RATES OF EXTENSIONS \( (\varepsilon_{1w}, \varepsilon_{1c}) \), AND THE RATES OF CONTRACTIONS \( (\varepsilon_{2w}, \varepsilon_{2c}) \) FOR GLASS SAMPLES

The following route and notations should be known in order to understand the calculations:

i) \( c_0 \) and \( w_0 \) values should be taken from Table 6 for related cases

ii) The linear relations between tightness points \( l/d \) and the parameter \( m \) of each fabric structure are assumed to occur as in Figures 12-15.

iii) In the equations in the Appendix, notations as in Equations (77-84) are used. Evaluations of the right-hand side of Equations (77-84), in turn, should be made by using the related case equations given in Empirical Equations at APPENDIX B.

Example calculations are given below.

A) GLASS

I) 1x1 RIB STRUCTURE

a) Wale-wise loaded samples

i) Contraction

\[
\varepsilon_{2w} = \begin{cases} 
    -T_0 = 1.452 \left( \frac{l}{d} - 20.5 \right); & \text{for } \frac{l}{d} \leq 20.5; m < 0 \\
    \frac{l}{d - 20.5} \leq 20.5; & \text{for } \frac{l}{d} \geq 20.5; \text{root 2} \\
    \frac{D_{2w}(T_w/2 - T_0/2)}{w_0} \leq 20.5; & \text{for } \frac{l}{d} \leq 20.5; T_w/2 \geq T_0/2 \\
    \frac{D_{2w}(m_2/2) - D_{2w}(T_w/2 + m_2/2)}{w_0 + D_{2w}(m_2/2)} \leq 20.5; & \text{for } \frac{l}{d} \geq 20.5
\end{cases}
\]  

(ii) Extension

\[
\varepsilon_{1w} = \begin{cases} 
    -T_0 = 1.452 \left( \frac{l}{d} - 20.5 \right); & \text{for } \frac{l}{d} \leq 20.5; m < 0 \\
    \frac{l}{d - 20.5} \leq 20.5; & \text{for } \frac{l}{d} \geq 20.5; \text{root 1} \\
    \frac{D_{1w}(T_w/2 - T_0/2)}{c_0} \leq 20.5; & \text{for } \frac{l}{d} \leq 20.5; T_w/2 \geq T_0/2 \\
    \frac{D_{1w}(m_1/2) + D_{1w}(T_w/2 + m_1/2)}{c_0 - D_{1w}(m_1/2)} \leq 20.5; & \text{for } \frac{l}{d} \geq 20.5
\end{cases}
\]  

b) Course-wise loaded samples

i) Contraction

\[
\varepsilon_{2c} = \begin{cases} 
    -D_{2c}(m) - D_{2c}(T_c + m) \leq 20.5 \text{ for } \frac{l}{d} \leq 20.5; c_0 - D_{2c}(m) \leq 20.5 \text{ for } \frac{l}{d} \geq 20.5
\end{cases}
\]  

(ii) Extension

\[
\varepsilon_{1c} = \begin{cases} 
    -D_{1c}(m) - D_{1c}(T_c + m) \leq 20.5 \text{ for } \frac{l}{d} \leq 20.5; \text{root 1} \\
    \frac{D_{1c}(T_c - T_0)}{w_0} \leq 20.5; \text{ for } \frac{l}{d} \geq 20.5; T_c \geq T_0
\end{cases}
\]
II) MILANO RIB

a) Wale-wise loaded samples
   i) Contraction
   \[ m = \frac{1}{2.316 \cdot \sqrt{d - 21.1425}} \text{ for } \frac{1}{d} \leq 21.1425; \frac{m}{d} < 0 \]  
   \[ \varepsilon_{2w} = -\frac{D_{2w}(T_w^2/2 - T_0/2)}{w_0} \text{ for } \frac{1}{d} \leq 21.1425; \frac{T_w}{2} \geq \frac{T_0}{2} \]  
   \[ \frac{c_0}{c_0} \text{ C9 } \]
   ii) Extension
   \[ m = \frac{1}{6.559 \cdot \sqrt{d - 21.1425}} \text{ for } \frac{1}{d} \leq 21.1425; \frac{m}{d} < 0 \]  
   \[ \varepsilon_{1w} = \frac{D_{1w}(T_c - T_0)}{w_0} \text{ for } \frac{1}{d} \leq 21.1425; \frac{T_c}{2} \geq \frac{T_0}{2} \]  
   \[ \frac{c_0}{c_0} \text{ C10 } \]

b) Course-wise loaded samples
   i) Contraction
   \[ m = \frac{1}{2.284 \cdot \sqrt{d - 22.75}} \text{ for } \frac{1}{d} \geq 22.75; \frac{m}{d} > 0 \]  
   \[ \varepsilon_{2c} = \frac{D_{2c}(m - D_{2c}(T_c + m))}{\sqrt{20.58}} \text{ for } \frac{1}{d} \geq 22.75 \]  
   \[ \frac{c_0}{c_0} \text{ C11 } \]
   ii) Extension
   \[ m = \frac{1}{1.0661 \cdot \sqrt{d - 20}} \text{ for } \frac{1}{d} \geq 22.75; \frac{m}{d} > 0 \]  
   \[ \varepsilon_{1c} = \frac{D_{1c}(T_c - m)}{w_0} \text{ for } \frac{1}{d} \geq 22.75 \]  
   \[ \frac{c_0}{c_0} \text{ C12 } \]

III) HALF CARDIGAN

a) Wale-wise loaded samples
   i) Contraction
   \[ m = \frac{1}{2.284 \cdot \sqrt{d - 22.75}} \text{ for } \frac{1}{d} \leq 22.75; \frac{m}{d} < 0 \]  
   \[ \varepsilon_{2w} = \frac{-D_{2w}(m/2 - D_{2w}(T_w/2 + m/2))}{w_0 + D_{2w}(m/2)} \text{ for } \frac{1}{d} \leq 22.75 \]  
   \[ \frac{c_0}{c_0} \text{ C13 } \]
   ii) Extension
   \[ m = \frac{1}{0.966 \cdot \sqrt{d - 20}} \text{ for } \frac{1}{d} \geq 22.75; \frac{m}{d} > 0 \]  
   \[ \varepsilon_{1w} = \frac{D_{1w}(T_w/2 + m/2)}{w_0} \text{ for } \frac{1}{d} \geq 22.75 \]  
   \[ \frac{c_0}{c_0} \text{ C14 } \]

b) Course-wise loaded samples
   i) Contraction
   \[ m = \frac{1}{2.284 \cdot \sqrt{d - 22.75}} \text{ for } \frac{1}{d} \geq 22.75; \frac{m}{d} > 0 \]  
   \[ \varepsilon_{2c} = \frac{-D_{2c}(m - D_{2c}(T_c + m))}{\sqrt{20}} \text{ for } \frac{1}{d} \geq 22.75 \]  
   \[ \frac{c_0}{c_0} \text{ C15 } \]
   ii) Extension
   \[ m = \frac{1}{1.0661 \cdot \sqrt{d - 20}} \text{ for } \frac{1}{d} \geq 22.75; \frac{m}{d} > 0 \]  
   \[ \varepsilon_{1c} = \frac{D_{1c}(m)}{w_0} \text{ for } \frac{1}{d} \geq 22.75 \]  
   \[ \frac{c_0}{c_0} \text{ C16 } \]
IV) FULL CARDIGAN DERIVATIVE

a) Wale-wise loaded samples

i) Contraction

\[ m = \begin{cases} \frac{-(l/d-22.75)}{2.3863} &; \text{for } l/d \leq 22.75; \text{root 1} \\ -T_0 = \frac{-(l/d-22.75)}{6.5095} &; \text{for } l/d \geq 22.75; m < 0 \end{cases} \]  

\[ \varepsilon_{2w} = \begin{cases} \frac{D_{2w}^{(m/2)}-D_{2w}^{(T/2+m/2)}}{w_0-D_{2w}^{(m/2)}} &; \text{for } l/d \leq 22.75 \\ -\frac{D_{2w}^{(T/2)-T_0/2}}{w_0} &; \text{for } l/d \geq 22.75; T_w/2 \geq T_0/2 \end{cases} \]  

(C25)

(C26)

ii) Extension

\[ m = \begin{cases} \frac{-(l/d-22.75)}{2.1654} &; \text{for } l/d \leq 22.75; m < 0 \\ \frac{-(l/d-22.75)}{1.8837} &; \text{for } l/d \geq 22.75; \text{root 1} \end{cases} \]  

\[ \varepsilon_{1w} = \begin{cases} \frac{D_{1w}^{(m/2)}+D_{1w}^{(T/2+m/2)}}{c_0-D_{1w}^{(m/2)}} &; \text{for } l/d \leq 22.75 \\ \frac{D_{1w}^{(T/2)-T_0/2}}{c_0} &; \text{for } l/d \geq 22.75 \end{cases} \]  

(C27)

(C28)

b) Course-wise loaded samples

i) Contraction

\[ m = \begin{cases} \frac{-(l/d-22.75)}{1.1680} &; \text{for } l/d \leq 22.75; \text{root 1} \\ \frac{-(l/d-22.75)}{5.9700} &; \text{for } l/d \geq 22.75; \text{root 2} \end{cases} \]  

\[ \varepsilon_{2c} = \begin{cases} \frac{D_{2c}^{(m)}-D_{2c}^{(T_c+m)}}{c_0-D_{2c}^{(m)}} &; \text{for } l/d \leq 22.75 \\ \frac{-D_{2c}^{(T_c)-T_0/2}}{c_0+D_{2c}^{(m)}} &; \text{for } l/d \geq 22.75 \end{cases} \]  

(C29)

(C30)

ii) Extension

\[ m = 2.2125 \left( \frac{l/d-20}{l/d-20} \right) ; \text{for } l/d \geq 20; \text{root 1} \]  

\[ \varepsilon_{1c} = \frac{D_{2c}^{(m)}+D_{2c}^{(T_c+m)}}{w_0-D_{2c}^{(m)}} ; \text{for } l/d \geq 20 \]  

(C31)

(C32)