Enhancement Effects of Transition and Vavilov-Cherenkov Radiation Mechanisms Under Grazing Interaction of Fast Electrons With a Thick Substrate Applied by Thin Layer

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Abstract. The paper presents the results of a theoretical study and a mathematical model of radiation processes occurred during the grazing interaction of fast electrons with semi-infinite targets applied on a thin amorphous layer. The developed model considers Vavilov-Cherenkov and transition radiation mechanisms and predicts the possibility to enhance the angular radiation density under grazing incidence of fast electrons on the layer. The characteristics of possible extreme vacuum ultraviolet and soft X-ray sources are estimated.

1. Introduction

An important spectral range of electromagnetic radiation are the extreme vacuum ultraviolet (EUV) and soft X-ray regions which are used in various fields of science and technology [1]. Nowadays many EUV sources have been developed, the most powerful of which are synchrotrons and free electron lasers. The main drawback of these sources is the high cost of their production and operation which limits their availability. A promising direction for the development of EUV sources lies in the use of radiation mechanisms occurred during the interaction of fast charged particles with matter [2–4].

The coherent mechanisms of X-ray generation by relativistic electrons in periodic structures like a multilayer mirror [5,6] or a crystal [7-9] can be used to obtain high radiation fluxes for different applications. But the existence of a strong radiation absorption in the target limits the use of the mentioned mechanisms to develop EUV and soft X-rays sources. It is important to mention also that the coherent mechanism yield is substantially decreased when the energy of the emitting particle decreases [10,11].

The photoabsorption problem can be solved when radiation is generated during contactless interaction of the charged particle with the structured target, for example the Smith-Purcell radiation. In addition, multiple passes of the emitting particles through targets mounted inside cyclical accelerators can be used to decrease the influence of photoabsorption on the radiation yield in EUV and soft X-rays regions [12-14]. Among the drawbacks of this mechanism one can underline the necessity of a small value of the impact parameter of the interaction between the incident particle and
the target structured surface and a small size of the transverse beam to obtain a high efficiency of the source. In this context, transition radiation (TR) and Vavilov-Cherenkov (VC) mechanisms are convenient and can be used to develop EUV and soft X-ray sources. An important feature of TR and VC is the significant yield in the soft spectral range for relativistic electrons. The possibility to use TR and VC as a radiation source was shown in [15,16]. Nevertheless, the yield of TR decreases substantially when the energy of the emitting particle decreases and the VC mechanism is limited by photoabsorption. The possibilities to enhance the yield of these mechanisms are relevant task. An approach to solve this task was proposed in [17,18], where the grazing interaction of fast electrons with a flat semi-infinite amorphous target is considered. The main result of these works is the predicted possibility to increase the angular radiation density by more than than times in comparison to the usual case.

This paper presents the calculation results of TR and VC characteristics obtained during the interaction between fast electrons and a target which consists of a thin amorphous layer applied on the surface of a semi-infinite amorphous substrate (the thickness of the substrate layer is much more than the absorption length of the considered photons). Such structure of the target can be easily produced to develop EUV and soft X-ray radiators. The high quality of the surface is an indispensable condition for the realization of the considered effects. The relativistic system of units used for the calculations presented below is $\hbar = c = 1$.

2. Theory

Let us consider the radiation produced by a relativistic electron moving with a constant velocity $\vec{V}$ in a target consisting of a thin amorphous layer with thickness $L$ applied on a substrate with thickness much more than the photoabsorption length of the considered radiation. The semi-infinite model for the substrate can be used as it is presented in Fig.1 where $\vec{n}$ is propagation direction of a radiated photon, $\chi^1(\omega)$ and $\chi^2(\omega)$ are the dielectric susceptibility of the layer and substrate respectively ($\omega$ is the energy of the radiated photon). The presented geometry of the radiation process represents real targets, which can be produced with different methods of film coating.

The radiation characteristics can be calculated from Maxwell equations and ordinary boundary conditions for the electromagnetic field. Using the Fourier-transform of electric and magnetic fields one can obtain the following equations for the Fourier-components of the electric field in vacuum – I, the layer – II and the substrate – III.
\[
\begin{align*}
E_{a,k}^t &= \frac{i\omega e}{2\pi^2} \delta\left(\omega - \tilde{k} \tilde{V} \right) \left(\tilde{V} - \tilde{k} \tilde{V}\right) + \tilde{a}_k \delta\left(k_\perp - \sqrt{\omega^2 - k_\parallel^2}\right), \\
E_{a,k}^{II} &= \frac{i\omega e}{2\pi^2} \delta\left(\omega - \tilde{k} \tilde{V} \right) \left(\tilde{V} - \tilde{k} \tilde{V}\right) + \tilde{b}_k \delta\left(k_\perp - \sqrt{\omega^2 e_1 - k_\parallel^2}\right) + \tilde{c}_k \delta\left(k_\perp + \sqrt{\omega^2 e_1 - k_\parallel^2}\right), \\
E_{a,k}^{III} &= \frac{i\omega e}{2\pi^2} \delta\left(\omega - \tilde{k} \tilde{V} \right) \left(\tilde{V} - \tilde{k} \tilde{V}\right) + \tilde{d}_k \delta\left(k_\perp + \sqrt{\omega^2 e_2 - k_\parallel^2}\right). 
\end{align*}
\]

The vectors \(\tilde{a}_k\), \(\tilde{b}_k\), \(\tilde{c}_k\) and \(\tilde{d}_k\) determine the radiation in the layer and inside the substrate. Calculating the vector \(\tilde{a}_k\) is of great interest because it represents the radiation from the target in vacuum. It is convenient to introduce the basis of orthogonal unit vectors

\[
\vec{e}_1, \quad \vec{e}_\parallel = \frac{\vec{k}_1}{k_1}, \quad \vec{e}' = \frac{\vec{k}_1}{k_1}.
\]

The unit vectors \(\vec{e}_1\) and \(\vec{e}'\) lie in the surface of the target. The unknown coefficients for the electric fields in (2) can be determined using ordinary boundary conditions. The coefficient \(\tilde{a}_k\) determines the radiation field in vacuum and it can be represented in the following form

\[
\tilde{a}_k = \left(\vec{e}_1 - \vec{e}_\parallel k_\parallel / k_1\right) a_1 + \vec{e}' a'.
\]

Calculating the Fourier-integral in wave-zone by the stationary phase method, one can obtain the following expression for the emission amplitude

\[
\bar{E}_{a,o}^{\text{Rad}} = \int \hat{d}^3 k \hat{a}_k e^{i\hat{V} \hat{k}} \to -2\pi i \omega n_{\perp} \hat{a}_{\perp, a,o} \frac{e^{i\omega r}}{r} = \bar{A}_{a,o} e^{i\omega r},
\]

\[
\omega \frac{dN}{d\omega d\Omega} = |\bar{A}|^2 = |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2 = 4\pi^2 \omega^2 n_\parallel^2 \left(\frac{\bar{a}_\perp^2}{n_\perp^2} + \bar{a}_\parallel^2\right).
\]

The resultant expression for the calculation of the spectral-angular distribution of radiation in considered conditions can be represented in the next form

\[
|\bar{A}_\perp|^2 = \frac{\epsilon^2}{\pi^2} \frac{n_\parallel^2}{V_\perp n_\parallel^2} \left(\epsilon_1 \gamma + \epsilon_2 \beta\right) \left(\beta + \epsilon_1 n_\perp\right) e^{-i\omega l(k+\beta - p)} + \left(\epsilon_1 \gamma - \epsilon_2 \beta\right) \left(\beta - \epsilon_1 n_\perp\right) e^{i\omega l(k+\beta - p)}
\]

\[
= \frac{2\beta (\chi_1 - \chi_2) n_\parallel^2 (\epsilon_1 V_\perp - p) - \gamma (\epsilon_1 n_\parallel V_\perp - n_\parallel^2)}{p - \gamma}
\]

\[
+ \frac{\chi_1}{p^2 - n_\parallel^2} \left(\epsilon_1 \gamma + \epsilon_2 \beta\right) \left(\beta (n_\parallel V_\perp - n_\parallel^2) + n_\parallel^2 \left(p - V_\perp\right)\right) e^{-i\omega l(k+\beta - p)} +
\]

\[
+ \frac{\chi_1}{p^2 - n_\parallel^2} \left(\epsilon_1 \gamma - \epsilon_2 \beta\right) \left(\beta (n_\parallel V_\perp - n_\parallel^2) - n_\parallel^2 \left(p - V_\perp\right)\right) e^{i\omega l(k+\beta - p)}
\]
\[
\left| A \right|^2 = \frac{e^2}{\pi^2} \frac{V^2 n^2 \gamma^2}{V^2 \gamma} \frac{1}{(\gamma + \beta)(\beta + n) e^{-i\omega(\beta + p)}} \left( \frac{2\beta X_1 - X_2}{p} \frac{1}{p^2 - \beta^2} \right) - \frac{X_1}{p^2 - \beta^2} \left( \frac{\gamma + \beta e^{-i\omega(\beta + p)}}{p - \beta e^{-i\omega(\beta + p)}} - \frac{\gamma - \beta e^{-i\omega(\beta + p)}}{p + \beta e^{-i\omega(\beta + p)}} \right)^2,
\]

where \( p = \left(1 - \frac{\tilde{V}_\parallel}{V_\perp}\right) / V_\perp \), \( \beta = \sqrt{n^2 + \chi' + i\chi''} \), \( \gamma = \sqrt{n^2 + \chi' + i\chi''} \), \( \chi'' > 0 \).

The components of vectors \( \tilde{n} \) and \( \tilde{V} \) can be presented in a convenient form for the calculation of spectral-angular distribution of emitted photons

\[
V_\perp = V \sin(\xi), \quad V_\parallel = V_x = V \cos(\xi), \quad n_\perp = \cos(\Theta), \quad n_x = \sin(\Theta) \cos(\varphi), \quad n_y = \sin(\Theta) \sin(\varphi).
\]

The presented expressions (6a) and (6b) can be used for the calculation of radiation spectral-angular characteristics for both cases when emitting electron flight in the target (\( V_\perp < 0 \)) and flight out from one (\( V_\perp > 0 \)).

3. Calculations

An important case for calculations is the radiation produced in conditions of grazing interaction of fast electrons with the target surface because of the prediction of radiation effects enhancement for TR [17] and VC [18]. These effects were predicted for the radiation generated during the flight out of the radiating electron from the semi-infinite amorphous target. The semi-infinite target is not convenient because of strong multiple scattering of the electron inside the target under grazing incidence conditions. The expressions (6) allow calculating the possibility of the enhancement realization as in [17,18] when the electrons cross the layer moving from vacuum. In accordance with [17] the angular density of TR can increase in ten times. This effect is explained by the combined action of two mechanisms of the formation of maxima in the angular distribution of TR. The first one corresponds to the case of total external reflection, when the maximum of the angular distribution is observed near the angle of total external reflection. The second mechanism is occurs due to the reconstruction of the Coulomb field of the charged particle when the velocity increases. The angular density can increase substantially when both mechanisms take place. The results of TR angular distribution calculations are presented in Fig.2 for the Si substrate and C layer. The real and imaginary parts of the dielectric susceptibilities for the layer and the substrate were calculated according to [19]. One can see the growing of TR angular density at the angle of incidence \( \xi = \pi / 8 \) (curve 3). The curve 3 illustrates the manifestation of TR enhancement as described in [17].
Figure 2. The effect of TR angular density enhancement for 50 eV photons under grazing interaction of 300 keV electrons with the target consisting of C layer 300 nm thickness on Si substrate. Figures “a” and “b” are the radiation under incidence of electrons from layer and vacuum sides respectively. Curves: 1 - $\xi = \pi / 2$; 2 - $\xi = \pi / 4$; 3 - $\xi = \pi / 8$; 4 - $\xi = \pi / 18$

A second case of interest for calculations is the VC radiation produced under conditions when the emitting electrons interact with the target provided that the incidence angle is less than the VC cone angle. The radiation angular density can grow under that condition in ten times [18]. The theoretical results of VC angular distribution are presented in Fig.3. One can see the angular density growing and the cone structure transformation. The oscillations manifested near the peaks are explained by the interference effects occurred the photons reflection at the layer surfaces.

Figure 3. The effect of VC angular density enhancement for 284.1 eV photons under grazing interaction of 10 MeV electrons with the target consisting of C layer 300 nm thickness on Si substrate. Figures “a” and “b” are the radiation under incidence of electrons from layer and vacuum sides respectively. Curves: 1 - $\xi = \pi / 15$; 2 - $\xi = \pi / 60$

4. Conclusions
The model of TR and VC radiation is developed for the grazing incidence geometry of the interaction between fast electrons and a target consisting of thin amorphous layer applied on a thick amorphous substrate. The performed calculations predict the possibility to enhance the angular density of TR and
VC radiation under the grazing incidence of electrons on the layer from the vacuum side. The mechanism of radiation enhancement is the same as described in [18].

It is important to highlight the simple target construction and the radiation process geometry which facilitates the application of the mentioned effect. The EUV and soft X-ray radiation can be generated by electrons with an energy of the order of 100 keV and 10 MeV for the TR and VC mechanisms respectively. The TR mechanism allows to produce wideband radiation in a wide spectral range, VC mechanism allows to produce a quasimonohromatic radiation in contrast with TR. The typical radiation yield for TR generated under grazing interaction of electrons with the target is about of $10^{-6}$ photons per (electron eV sr.) and for the VC is about of 0.1 photons per (electron eV sr.). The difference for the realization of the presented characteristics consists in the energy of the incidence electrons – about 10 MeV for VC and 0.1 MeV for TR.

The results of the work can be used to develop laboratory radiation sources. A compact setup as described in [20] can be used to develop the VC source. The TR source can be built using electron guns with an energy order of 100 keV.

Acknowledgments

The work was supported by a program of the ministry of education and science of The Russian Federation for higher education establishments, project №14.578.21.0063 (RFMEFI57814X0063).

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