Non-linear supersymmetry and intersecting D-branes

I. Antoniadis$^1$, M. Tuckmantel$^{1,2}$

$^1$Department of Physics, CERN - Theory Division
CH–1211 Geneva 23, Switzerland

$^2$Institut für Theoretische Physik, ETH Hönggerberg
CH–8093 Zürich, Switzerland

Abstract

We study the non-linear realization of supersymmetry. We classify all lower dimensional operators, describing effective interactions of the Goldstino with Standard Model fields. Besides a universal coupling to the energy momentum tensor of dimension eight, there are additional model dependent operators whose strength is not determined by non-linear supersymmetry, within the effective field theory. Their dimensionality can be lower than eight, starting with dimension six, leading in general to dominant effects at low energies. We compute their coefficients in string models with D-branes at angles. We find that the Goldstino decay constant is given by the total brane tension, while the various dimensionless couplings are independent from the values of the intersection angles.

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1 Introduction

In this work, we study the non-linear realization of supersymmetry, present in a class of D-brane models with non-supersymmetric spectra. In fact generic D-brane configurations in type II closed superstring theories [1], either combined with orientifolds [2] or at angles [3, 4, 5], break all bulk supersymmetries which are however still realized on the D-branes world-volume in a non-linear way [6, 7]. A consequence of non-linear realization is the existence of a (tree-level) massless Goldstino(s) which is a brane field [7, 8, 9]. In the compact case, it is expected to acquire a small mass by radiative corrections, suppressed by the compactification volume [9, 10]. Our analysis is of particular interest for models where the string scale is in the TeV region and supersymmetric bulk [11], or even in models with light Goldstino and all superparticles heavier than the electroweak scale [12, 13, 14, 15].

Our aim is to determine the effective action describing the lower dimensional interactions of the Goldstino with all kinds of fields: gauge bosons, scalars and (chiral) fermions. It is known that non-linear supersymmetry implies a universal coupling between the Goldstino and matter stress-energy tensors of dimension eight, whose strength is fixed by the Goldstino decay constant, in analogy to low-energy theorems for spontaneously broken global symmetries. However, it was noticed that besides this coupling there may exist other supersymmetric interactions whose strength is left undetermined within the effective field theory [13, 14, 15]. For instance, a general analysis of Goldstino to fermions interactions, which are described to lowest order by dimension eight four-fermion operators, revealed the existence of a free parameter, associated to the coefficient of a second operator allowed by non-linear supersymmetry, besides the one corresponding to the product of the two stress-tensors. This parameter can be computed in principle in string theory by considering the low energy expansion of appropriate four-fermion amplitudes [16, 9].

In this work, we extend the general analysis of Goldstino interactions to gauge and scalar fields and compute the leading coefficients in string theory. At the four-point level, we find for instance additional operators of dimension eight, involving two scalars and two Goldstinos, similar to the four-fermion operator described above. However, now there are more interactions of lower dimensionality. In particular, there are two operators of dimension six that contain a single Goldstino coupled to a matter fermion and a gauge or scalar field. The presence of these operators complicates the extraction of the Goldstino effective action from four-point string amplitudes because they generate reducible contact terms that have to be subtracted. Fortunately, such terms are absent for generic brane intersection angles, allowing to compute the
coefficients of all lower dimensional four- and three-point vertices in the effective action. Thus, although four-point contact terms appear to depend on intersection angles, the coefficients of irreducible effective operators turn out to be model independent constants.

More precisely, we study the non-linear supersymmetry present locally on the intersection of two sets of coincident D6-branes. The intersection is point-like in the six-dimensional internal (compact) manifold and extends in our three space non-compact coordinates. The Goldstino is a linear combination of the two gauge singlet fermions localized, respectively, on the two stacks. Moreover, its decay constant is given by the effective total brane tension on the intersection. We then determine its leading interactions with the massless gauge and matter fields living on the two sets, as well as with the chiral fermions and scalars localized at the intersection.

Our paper is organized as follows. The next three sections (2-4) are devoted to the effective field theory. In Section 2, we recall the main properties of non-linear supersymmetry, such as field transformations and invariant actions. In Section 3, we describe the superfield formalism, while in Section 4, we derive all supersymmetric Goldstino couplings to gauge fields, scalars and fermions, up to dimension eight. In the following three sections (5-7), we determine all three- and four-point couplings among brane fields. In Section 5, we compute all four-point functions on a disc world-sheet, involving two Goldstinos and two matter fields from a single set of coincident D-branes. In Section 6, we generalize this computation for matter fields living on brane intersections. For particular values of intersection angles, there are additional massless gauge or scalar fields that have 3-point interactions with Goldstinos leading to reducible contributions in the 4-point functions. By studying the various degeneration limits, we extract the 3-point interactions. In Section 7, we combine the previous results and determine the strength of all effective operators describing the lowest dimensional Goldstino interactions, up to four-point level. Finally, Section 8 contains our conclusions. For convenience of the reader, there are also two appendices with a summary of our results. Appendix A contains the Goldstino couplings from the general analysis of non-linear supersymmetry, while Appendix B contains the low-energy effective action derived from the string computations.

2 Non-linear supersymmetry

In the standard realization of non-linear supersymmetry, the Goldstino field $\lambda$ transforms according to
\[ \delta \lambda_\alpha = \frac{\xi_\alpha}{\kappa} - i \kappa (\lambda \sigma^\mu \xi - \xi \sigma^\mu \lambda) \partial_\mu \lambda_\alpha \]
\[ \delta \bar{\lambda}^\dot{\alpha} = \frac{\bar{\xi}^\dot{\alpha}}{\kappa} - i \kappa (\lambda \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\lambda}) \partial_\mu \bar{\lambda}^\dot{\alpha} \]  

(2.1)

Here \( \xi_\alpha, \bar{\xi}^\dot{\alpha} \) are the (Grassmann) parameters of the transformation and \( \kappa \) is some constant with units of \( \text{length}^2 \) which parametrizes the supersymmetry breaking scale. It is the Goldstino decay constant and plays a role similar to \( f_\pi \) in soft-pion dynamics.

We wish to construct a Lagrangian for an effective low-energy description of the Goldstino and its interactions with Standard Model fields. We first consider the part of the effective action which contains only self-couplings of the Goldstino. This must contain the standard kinetic term for a Weyl spinor with some additional terms necessary in order to make the action invariant under the standard non-linear realization (2.1). An action which satisfies these criteria has been constructed by Akulov and Volkov [6]. Indeed, we define the quantity

\[ A^\nu_\mu = \delta^\nu_\mu + i \kappa^2 \lambda \leftrightarrow \partial_\mu \sigma^\nu \bar{\lambda} \]  

(2.2)

from which we can construct the Akulov-Volkov action

\[ S_{AV} = \int d^4x \ L_{AV} = -\frac{1}{2\kappa^2} \int d^4x \ det A. \]  

(2.3)

Using (2.1) it can be shown that \( L_{AV} \) transforms as a total divergence:

\[ \delta (det A) = -i \kappa \partial_\mu [ (\lambda \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\lambda}) det A ] \equiv \kappa \partial_\mu (\Lambda^\mu det A) , \]  

(2.4)

where we have introduced a useful short-hand notation

\[ \Lambda^\mu = -i (\lambda \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\lambda}) . \]  

(2.5)

This shows that the above action is supersymmetric. Expanding in powers of \( \kappa \) we obtain

\[ L_{AV} = -\frac{1}{2\kappa^2} - \frac{i}{2} \lambda \sigma^\mu \partial_\mu \bar{\lambda} + \ldots \]  

(2.6)

where the dots denote self-couplings proportional to the second or higher powers of \( \kappa \). The first term plays the role of a cosmological constant. As will be shown later, in string theory it is given by the total effective tension of the D-branes. The remaining part of the Lagrangian is just the non-linear supersymmetric extension of the kinetic energy of a Weyl spinor.
The standard realization can be extended to matter (and gauge) fields \([12, 15]\). Let \(\phi_i\) denote some generic field with \(i\) an index in some representation of the Lorentz group or of an internal symmetry group. Then we define

\[
\delta \phi_i = \kappa \Lambda^\mu \partial_\mu \phi_i .
\]  
(2.7)

It can be checked that this indeed provides a representation of supersymmetry. Note that (2.7) has the same form as the transformation of the Goldstino (2.1) except for the absence of the inhomogeneous term \(\xi_\alpha\).

As a special case, we can consider a gauge field \(B_\mu\). By gauge invariance, such a field can enter the Lagrangian only through the field-strength tensor and gauge-covariant derivatives. Both of these contain ordinary derivatives and therefore will not transform covariantly according to the standard realization even though \(B_\mu\) does. This can be remedied by defining a modified field strength-tensor:

\[
F^a_{\mu\nu} \equiv (A^{-1})_\mu^\sigma (A^{-1})_\nu^\rho F^a_{\sigma\rho} ,
\]  
(2.8)

where \(F^a_{\mu\nu}\) is the ordinary field-strength and \((A^{-1})_\mu^\nu\) is the inverse of the matrix defined in (2.2). If we expand the right-hand side in powers of \(\kappa\), the first term will be \(F^a_{\mu\nu}\), followed by appropriate couplings to the Goldstino field. As a result, the quantity \(F^a_{\mu\nu}\) transforms covariantly according to the standard realization. The same procedure also works for the covariant derivative. Starting from the ordinary gauge-covariant derivative, we define a supersymmetry-covariant derivative according to

\[
D_\mu \phi_i \equiv (A^{-1})_\mu^\nu D_\nu \phi_i .
\]  
(2.9)

Thus, if \(\phi_i\) is a field transforming in the standard realization, then so is \(D_\mu \phi_i\).

It is now a simple task to construct an invariant effective action. The Standard Model Lagrangian has the form

\[
\mathcal{L}_{SM} = \mathcal{L}_{SM}(\phi_i, D_\mu \phi_i , F^a_{\mu\nu}) .
\]  
(2.10)

By replacing all quantities with their SUSY-covariant counterparts (that is \(F^a_{\mu\nu} \to F^a_{\mu\nu}\) and \(D_\mu \to D_\mu\)), the resulting Lagrangian transforms as a field in the standard realization:

\[
\delta \mathcal{L}_{SM}(\phi_i, D_\mu \phi_i , F^a_{\mu\nu}) = \kappa \Lambda^\sigma \partial_\sigma \mathcal{L}_{SM}(\phi_i, D_\mu \phi_i , F^a_{\mu\nu}) .
\]  
(2.11)

Multiplying with \(det A\) we obtain the invariant action

\[
S_{eff} = \int d^4x \mathcal{L}_{eff} = \int d^4x \ det A \ \mathcal{L}_{SM}(\phi_i, D_\mu \phi_i , F^a_{\mu\nu}) .
\]  
(2.12)
Indeed, using the transformation of $\text{det} A$ given in (2.4) we see that this action is supersymmetric. Furthermore, expanding the effective Lagrangian $\mathcal{L}_{\text{eff}}$ in powers of $\kappa$, the lowest ($\kappa$-independent) term is the Standard Model (SM) Lagrangian itself. The additional terms are required to make the action supersymmetric and describe appropriate interactions of the SM-fields to the Goldstino. Explicitly:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}(\phi_i, D_\mu \phi_i, F_{\mu\nu}^a) + (i\kappa^2 \lambda \bar{\sigma}^\nu \lambda) T_{\mu\nu} + \ldots$$

(2.13)

where the dots denote higher powers of $\kappa$ that can be neglected in the low-energy limit. $T_{\mu\nu}$ is the manifestly gauge-invariant energy-momentum tensor

$$T_{\mu\nu} = \eta_{\mu\nu} \mathcal{L}_{\text{SM}} - \frac{\partial \mathcal{L}_{\text{SM}}}{\partial (D_\mu \phi_i)} D^\nu \phi_i + 2 \frac{\partial \mathcal{L}_{\text{SM}}}{\partial (F_{\mu\lambda}^a)} F_{\nu}^{\lambda}.$$ 

(2.14)

Notice that once we have fixed the normalization of the Goldstino and SM-fields, this coupling is completely determined and model-independent. This is the low-energy theorem for supersymmetry (SUSY). However, it turns out that the above procedure does not lead to the most general effective action invariant under non-linear supersymmetry. It is possible to find additional supersymmetric terms which could in principle be added to the above effective Lagrangian. Since these terms would need to be supersymmetric by themselves, their overall normalization is not determined within the effective field theory, but depends on the underlying fundamental theory. If we restrict ourselves to terms suppressed by at most two powers of $\kappa$, then supersymmetry allows only a small number of such terms. In the next section, we use the superfield formalism of non-linear supersymmetry [17] to find the complete list.

### 3 The superfield formalism

Starting with a generic field $\phi_i$ in some representation of supersymmetry, we can systematically construct a superfield $\Phi_i$ with $\phi_i$ being its lowest component [18]:

$$\Phi_i(x, \theta, \bar{\theta}) = \exp(\theta Q + \bar{\theta} \bar{Q}) \times \phi_i(x),$$

(3.1)

where the multiplication symbol $\times$ means that the supercharges $Q$ and $\bar{Q}$ operate on $\phi_i$ in the appropriate representation. If we apply this prescription to the case of a field transforming in the standard realization, we obtain

$$\Phi_i(x, \theta, \bar{\theta}) = \phi_i(x) - i\kappa(\lambda \sigma^\mu \bar{\theta} - \bar{\theta} \sigma^\mu \bar{\lambda}) \partial_\mu \phi_i(x) + \ldots$$

$$= \phi_i(x) + \kappa \Lambda^\mu \partial_\mu \phi_i(x) + \ldots$$

(3.2)
which contains ordinary derivatives of \( \phi_i \). Therefore, if \( \phi_i \) transforms covariantly in some representation of the gauge group, then this representation does not carry over “nicely” to the corresponding superfield. However, it is possible to rewrite the superfield without explicitly using derivatives. To this end, we define a new variable

\[
\tilde{x}^\mu \equiv x^\mu + \Lambda^\mu(\tilde{x}).
\] (3.3)

The claim is then that, starting with some field in the standard realization, we can obtain the corresponding superfield simply by making the replacement \( x \to \tilde{x} \). In other words

\[
\Phi_i(x, \theta, \bar{\theta}) \equiv \phi_i(\tilde{x}).
\] (3.4)

It is understood here that the right-hand side needs to be expanded in a formal power-series around \( x \). To check this claim, we first note that \( \Lambda^\mu(x) \) transforms in the standard realization:

\[
\delta \Lambda^\mu(x) = \kappa \Lambda^\nu(x) \partial_\nu \Lambda^\mu(x).
\] (3.5)

Then we expand \( \phi_i(\tilde{x}) \) in a power series. A straightforward calculation using (2.7) and (3.5) shows that this can be written as

\[
\phi_i(\tilde{x}) = \phi_i(x) + \delta \phi_i(x) + \frac{1}{2!} \delta^2 \phi_i(x) + \frac{1}{3!} \delta^3 \phi_i(x) + \frac{1}{4!} \delta^4 \phi_i(x)
= \exp(\theta Q + \bar{\theta} \bar{Q}) \times \phi_i(x).
\] (3.6)

Thus, the claim follows by comparing with (3.1). In the same way, it can be shown that the Goldstino superfield can be written as

\[
G_\alpha(x, \theta, \bar{\theta}) = \frac{\theta_\alpha}{\kappa} + \lambda_\alpha(\tilde{x}).
\] (3.7)

When expressed in this way, the superfield \( \Phi \) does not involve any explicit derivatives. Suppose \( \phi_i(x) \) transforms in some representation of the gauge group,

\[
\phi_i'(x) = \exp(i \Omega^A(x)t^A)_{ij} \phi_j(x).
\] (3.8)

Since gauge transformations correspond to a symmetry of the theory it is natural to require that \( \phi_i' \) should also transform in the standard realization (this in turn implies that the parameter \( \Omega^A(x) \) must transform in the standard realization as well). Following the discussion above, the transformation of the corresponding superfields is then obtained by substituting \( x \to \tilde{x} \):

\[
\Phi_i'(x, \theta, \bar{\theta}) = \exp(i \Omega^A(\tilde{x})t^A)_{ij} \Phi_j(x, \theta, \bar{\theta}).
\] (3.9)
The superfield transforms just like its lowest component field except that the parameter of the transformation is now itself a superfield. Using the fact that \( \tilde{x} \) is real, we also have
\[
\Phi^\dagger_i(x, \theta, \bar{\theta}) = \Phi^\dagger_j(x, \theta, \bar{\theta}) \exp(-i\Omega^A(x)\lambda^A_{ji}).
\] (3.10)

This procedure works for any field in the standard realization and in particular for SUSY-covariant derivatives of such fields as defined in (2.9). In this way, we can construct derivatives of superfields:
\[
\mathcal{D}_\mu \Phi_i(x, \theta, \bar{\theta}) \equiv ((A^{-1})^\nu_\mu D_\nu \phi_i)(\tilde{x}).
\] (3.11)

Under gauge transformations, \( \mathcal{D}_\mu \Phi_i \) transforms just like \( \Phi_i \). With these ingredients it is a straightforward task to construct gauge-invariant quantities out of superfields and their derivatives. Any function of \( \Phi_i, \Phi^\dagger_i \) and their covariant derivatives (as defined in (3.11)) that is invariant under global gauge transformations is also invariant under local gauge transformations.

## 4 Supersymmetric Goldstino couplings

We shall at first consider only scalars and gauge fields. Starting from fields in the standard realization we obtain the corresponding superfields by substituting \( x \rightarrow \tilde{x} \). The relevant superfields are the Goldstino, the field-strength tensor, the scalar field and possibly their derivatives:
\[
\kappa G_\alpha(x, \theta, \bar{\theta}) = \theta_\alpha + \kappa \lambda_\alpha(\tilde{x})
\] (4.1)
\[
\mathcal{F}^a_{\mu\nu}(x, \theta, \bar{\theta}) = ((A^{-1})^\sigma_\mu(A^{-1})^\rho_\nu F^a_{\sigma\rho})(\tilde{x})
\] (4.2)
\[
\Phi_i(x, \theta, \bar{\theta}) = \phi_i(\tilde{x})
\] (4.3)

The field \( \kappa G_\alpha \) has mass dimension \(-\frac{1}{2}\) while \( \mathcal{F}^a_{\mu\nu} \) and \( \Phi_i \) have dimensions 2 and 1, respectively. Notice that when expanding the above superfields, every insertion of the Goldstino field \( \lambda_\alpha \) comes with exactly one power of \( \kappa \).

Out of these “elementary” superfields we can construct gauge-invariant superfield Lagrangians \( \mathcal{L}^{SF} \). The corresponding action is obtained as usual by a superspace integration:
\[
S = \int d^4xd^2\theta d^2\bar{\theta} \mathcal{L}^{SF}.
\] (4.4)

Consider specifically an operator \( \mathcal{O} \) built out of the above superfields. The corresponding supersymmetric field operator is given by
\[
O = \int d^2\theta d^2\bar{\theta} \mathcal{O} = O^{(0)} + \kappa^2 O^{(2)} + \kappa^4 O^{(4)} + \ldots
\] (4.5)
where the right-hand side denotes an expansion in powers of $\kappa$. Since there is one power of $\kappa$ for each $\lambda$, the operator $O^{(i)}$ contains $i$ Goldstino fields. Interactions between the Goldstino, scalars, gauge fields and an even number of matter fermions must contain an even number of $\lambda$, so the expansion (4.5) contains in this case only even powers of $\kappa$. However, as we will see below, there are also interactions involving an odd number of Goldstinos, and thus, odd powers of $\kappa$.

If $\mathcal{O}$ has mass dimension $D \geq 0$ then $O$ has dimension $d = D + 2$ and $O^{(i)}$ has dimension $d_i = D + 2i + 2$. Thus, on dimensional grounds, the operator $O$ gives an effective Lagrangian term of the form

$$\mathcal{L}_O = g\kappa^{D-1}O = g\kappa^{D-1}O^{(0)} + g\kappa^{D+1}O^{(2)} + g\kappa^{D+3}O^{(4)} + \ldots$$

(4.6)

where $g$ is some numerical constant. We shall only be interested in low-dimensional interactions suppressed by at most two powers of $\kappa$. In that case it is easy to see that it is enough to consider superfields $\mathcal{O}$ with $D \leq 2$ and we only need to keep $O^{(0)}$ and $O^{(2)}$ in the expansion (4.6). Of course $O^{(0)}$ does not involve any Goldstinos and exists already in the effective action independently of non-linear supersymmetry, while $O^{(2)}$ for $D > 2$ has dimension bigger than 8 and we drop.

To find all possible couplings of the Goldstino to Standard Model fields consistent with non-linear supersymmetry, we shall proceed in two ways. Usually, it will be simpler to write down all gauge invariant superfields $\mathcal{O}$ with $D \leq 2$ and then perform the superspace integration to obtain the corresponding contribution to the effective action. However, in a few cases it will be more convenient to work in the other way around: first find all viable candidates for $O^{(2)}$ and then show that these can indeed be realized in terms of superfields. This will be trivial whenever all Goldstino fields are acted upon by derivatives. In this case, we simply have to replace all fields by the corresponding superfields and multiply the whole expression by $\kappa^4G^2\bar{G}^2$. This is then manifestly supersymmetric and after carrying out the integration over the fermionic coordinates, we recover $O^{(2)}$ as the first term in the expansion in powers of $\kappa$. In fact this is equivalent to the “supersymmetrization” procedure introduced in ref. [12].

### 4.1 Couplings to gauge fields

We first assume for simplicity that there is no $U(1)$ factor in the gauge group. In that case, gauge invariance requires that the field-strength enters

\footnote{We shall always use capital $D$ to denote the mass dimension of superfields and lowercase $d$ for the dimension of ordinary fields.}
$\mathcal{L}^{SF}$ quadratically, which brings already a factor of dimension 4. Because the Goldstino superfields are Grassmann variables, any superfield can contain at most four of them if no derivatives are involved. Since they have dimension $-1/2$, no terms of dimension $D < 2$ are possible while $D = 2$ operators necessarily involve four Goldstino superfields. From the identities

$$G_\alpha G_\beta = \frac{1}{2} G^2 \epsilon_{\alpha\beta}$$

$$\bar{G}_\dot{\alpha} \bar{G}_\dot{\beta} = -\frac{1}{2} G^2 \epsilon_{\dot{\alpha}\dot{\beta}}$$

it follows that there is only one possibility:

$$\mathcal{O}_1 = -\frac{1}{4} \kappa^4 F^a_{\mu\nu} F^{a\mu\nu} G^2 \bar{G}^2.$$  \hspace{1cm} (4.8)

If $S(x, \theta, \bar{\theta}) \equiv s(\tilde{x})$ is a superfield with $s(x)$ its lowest component, then:

$$(\kappa^4 S G^2 \bar{G}^2)_{\mu_1 \mu_2} = s(x) + i\kappa^2 \lambda \sigma^\mu \tilde{\partial}_\mu \tilde{\lambda} s(x) + O(\kappa^4).$$ \hspace{1cm} (4.9)

With the help of this identity and the definition (2.8) we obtain

$$\mathcal{O}_1 = -\frac{1}{4} \kappa^4 F^a_{\mu\nu} F^{a\mu\nu} + i\kappa^2 (\lambda \sigma^\mu \tilde{\partial}_\mu \tilde{\lambda}) (-F^a_{\nu\alpha} F^{a\nu\alpha} - \frac{\delta^\mu_{\nu}}{4} F^a_{\alpha\beta} F^{a\alpha\beta}) + \cdots$$ \hspace{1cm} (4.10)

The quantity in parenthesis in the second term is just the contribution of the field-strength to the energy-momentum tensor and so $\mathcal{O}_1$ simply reproduces the coupling required by the low-energy theorem.

In principle we could also have a term

$$\mathcal{O}_2 = -\frac{\theta}{64\pi^2} \kappa^4 \epsilon^{\mu\rho\sigma\nu} F^{a}_{\mu\nu} F^a_{\rho\sigma} G^2 \bar{G}^2.$$ \hspace{1cm} (4.11)

This would again reproduce the result of the low-energy theorem, but since $\text{Tr}(F\bar{F})$ is a total derivative, its contribution to the energy-momentum tensor vanishes.

If the gauge group contains $U(1)$ factors, then we should also consider terms linear in $F_{\mu\nu}$. Using (4.7), it is easy to see that no such terms exist at dimension $D = 0$. To construct $D = 1$ operators we need to use derivatives of the Goldstino superfield. If we start with terms containing a single superderivative, then there are two possible operators:

$$\mathcal{O}_3 = \kappa^4 F^a_{\mu\nu} G^2 \mathcal{D}^\mu G\sigma^\nu \tilde{G}$$

$$\mathcal{O}_4 = \kappa^4 F^a_{\mu\nu} \epsilon^{\mu\nu\alpha\beta} G^2 \mathcal{D}_\alpha G\sigma_\beta \tilde{G}$$ \hspace{1cm} (4.12)
If we take \( S_{\mu\nu}(x, \theta, \bar{\theta}) \) to be some superfield with lowest component \( s_{\mu\nu}(x) \), then up to total derivatives we have the identity

\[
(\kappa^4 S_{\mu\nu} G^2 \partial^\mu G \sigma^\nu \bar{G})_{\theta^2 \bar{\theta}^2} = \frac{i}{2} \kappa^2 s_{\mu\nu} \partial_\alpha \lambda \sigma^\alpha \sigma^\nu \partial^\mu \lambda + O(\kappa^4).
\]  

(4.13)

This is proportional to the equations of motion and therefore \( O_3 \) and \( O_4 \) do not contribute at order \( \kappa^2 \).

If we add an extra derivative then there are additional possibilities. These are obtained by considering all possible contractions of indices in the following superfield:

\[
O_5 = \kappa^4 G^2 \partial_\alpha G J_{(1/2,0)}^{\beta\gamma} D_\beta G,
\]  

(4.14)

where \( J_{(1/2,0)}^{\mu\nu} \) are the generators of the \((1/2,0)\) representation of the Lorentz group:

\[
J_{(1/2,0)}^{\mu\nu} = \frac{i}{4} (\sigma^\mu \sigma^\nu - \sigma^\nu \sigma^\mu).
\]  

(4.15)

Carrying out the fermionic integration and using the equations of motion for the Goldstino it is easy to see that these terms again do not contribute at order \( \kappa^2 \). This exhausts all possibilities for \( D = 1 \).

We still need to check the case \( D = 2 \). Instead of working with superfields it turns out to be more convenient to find all dimension \( d = 8 \) field operators containing two Goldstinos and being linear in \( F_{\mu\nu} \). The prototype for these operators is \( F_{\mu\nu} \partial_\alpha \lambda \sigma_\beta \partial_\gamma \partial_\delta \bar{\lambda} \). Looking for all possible ways of contracting the indices with the metric or \( \epsilon^{\mu\nu\sigma\rho} \), it is easy to show that all these terms except one either vanish or are proportional to the equations of motion. The remaining term is\(^2\)

\[
O_5^{(2)} = \kappa^4 \partial^\alpha \lambda \sigma^\mu \sigma^\nu \bar{\lambda} \partial_\alpha F_{\mu\nu}.
\]  

(4.16)

Since both Goldstino fields are coming with derivatives, it immediately follows from the discussion at the end of last subsection that this term is supersymmetric. It was already found in ref. [15].

### 4.2 Couplings to scalar fields

We shall proceed systematically beginning with interactions which are quartic in the scalar fields.

\(^2\)The analogous term containing \( \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\sigma\rho} F^{\sigma\rho} \) brings nothing new since up to terms proportional to the equations of motion we have \( \partial^\alpha \lambda \sigma^\mu \partial^\nu \bar{\lambda} \partial_\alpha \tilde{F}_{\mu\nu} = -i (\partial^\alpha \lambda \sigma^\mu \partial^\nu \bar{\lambda}) \partial_\alpha F_{\mu\nu} \).
1. **Quartic interactions**

No $D = 0, 1$ terms are possible since these would require respectively 8 and 6 Goldstino superfields. There is a single $D = 2$ operator:

$$\mathcal{O} = \frac{\kappa^4}{4!} M_{ijkl} \Phi_i^{(1)} \Phi_j^{(2)} \Phi_k^{(3)} \Phi_l^{(4)} G^2 \bar{G}^2. \quad (4.17)$$

The superscripts on the scalar fields indicate that they might transform in different representations of the gauge group which are coupled through the Clebsch-Gordan coefficients $M_{ijkl}$. Using the identity (4.9), this results in a $\phi^4$ term together with the coupling of the Goldstino to the corresponding contribution to the energy-momentum tensor. In fact this coupling is proportional to the equations of motion and can be omitted.

2. **Cubic interactions**

Again there can be no $D = 0$ terms. For $D = 1$, we can have a term similar to (4.17) but cubic in $\Phi$. This again reproduces the low-energy coupling to the stress-energy tensor and in any case it can be omitted since it is proportional to the equations of motion. For $D = 2$, restricting first to couplings containing at most one superderivative, we find two terms:

$$\mathcal{O}_1 = \frac{\kappa^2}{3!} \Phi_i^{(1)} \Phi_j^{(2)} \Phi_k^{(3)} M_{ijk} G^2 \quad (4.18)$$

$$\mathcal{O}_2 = \frac{\kappa^4}{3!} \Phi_i^{(1)} \Phi_j^{(2)} \Phi_k^{(3)} M_{ijk} G^2 \mathcal{D}_\mu G\sigma^\mu \bar{G} \quad (4.19)$$

Since $\mathcal{D}_\mu (\kappa G_\alpha) = (\kappa \partial_\mu \lambda_\alpha)(\bar{x}) + O(\kappa^3)$, the operator $\mathcal{O}_2$, up to higher order terms, is proportional to the equations of motion and can be omitted. $\mathcal{O}_1$ on the other hand gives something new. Carrying out the superspace integration, we obtain the supersymmetric action:

$$S = i\kappa^2 \int d^4x \phi_i^{(1)} \phi_j^{(2)} \phi_k^{(3)} M_{ijk} (\partial_{\mu} \lambda J_\mu^{(2)} (\frac{4}{3}, 0) \partial_{\nu} \lambda). \quad (4.20)$$

Adding a second superderivative yields additional terms but all of these reproduce (4.20). Terms with more than two superderivatives generate interactions with $d > 8$, which we omit.

3. **Quadratic interactions**

Here again it is more convenient to work directly with ordinary fields and first look for possible candidates for $O^{(2)}$. These operators should
contain two Goldstinos together with some derivatives, one of which at least must act on the Goldstinos since the coupling must vanish for constant $\lambda$. At mass dimension $d = 6$ we find

$$O_1^{(2)} = i\lambda \sigma^\mu \partial_\mu \overline{\phi}_i^{(1)} \phi_j^{(2)} M_{ij}, \quad (4.21)$$

which is proportional to the equations of motion. At $d = 7$ we have four possibilities:

$$O_1^{(2)} = (D_\mu \phi_i^{(1)} \phi_j^{(2)} M_{ij} - \phi_i^{(1)} D_\mu \phi_j^{(2)} M_{ij}) \lambda \partial \mu \lambda$$

$$O_2^{(2)} = (D_\mu \phi_i^{(1)} \phi_j^{(2)} M_{ij} - \phi_i^{(1)} D_\mu \phi_j^{(2)} M_{ij}) \lambda J^{\mu \nu}_{(\frac{1}{2},0)} \partial_\nu \lambda$$

$$O_3^{(2)} = \phi_i^{(1)} \phi_j^{(2)} M_{ij} \partial_\alpha \lambda \partial^\alpha \lambda$$

$$O_4^{(2)} = \phi_i^{(1)} \phi_j^{(2)} M_{ij} \partial_\mu \lambda J^{\mu \nu}_{(\frac{1}{2},0)} \partial_\nu \lambda$$

Up to total derivatives, $O_1^{(2)}$ can be rewritten as

$$\frac{1}{2} (D^2 \phi_i \phi_j M_{ij} - \phi_i D^2 \phi_j M_{ij}) \lambda^2, \quad (4.22)$$

which is proportional to the equations of motion of the scalar fields. Also, up to terms proportional to the equations of motion of the Goldstino, $O_1^{(2)}$ and $O_2^{(2)}$ are equivalent. The same is true for $O_3^{(2)}$ and $O_4^{(2)}$. This leaves us with only $O_4^{(2)}$ which is trivially supersymmetric. According to (4.6) with $D = 1$, this operator then contributes to the action a term

$$S = \kappa \frac{1}{2} \int d^4 x \phi_i^{(1)} \phi_j^{(2)} M_{ij} \partial_\mu \lambda J^{\mu \nu}_{(\frac{1}{2},0)} \partial_\nu \lambda. \quad (4.23)$$

Finally, we need to consider $d = 8$ operators. There are three possibilities

$$O_1^{(2)} = (D_\alpha \phi_i^{(1)} \phi_j^{(2)} M_{ij} - \phi_i^{(1)} D_\alpha \phi_j^{(2)} M_{ij}) \lambda \sigma^\mu \partial^\mu \overline{\lambda}$$

$$O_2^{(2)} = (D_\alpha \phi_i^{(1)} \phi_j^{(2)} M_{ij} - \phi_i^{(1)} D_\alpha \phi_j^{(2)} M_{ij}) \lambda J^{\mu \nu}_{(\frac{1}{2},0)} \partial_\mu \lambda$$

$$O_3^{(2)} = D_\alpha \phi_i^{(1)} D_\beta \phi_j^{(2)} M_{ij} \lambda \sigma^\mu \partial^\mu \overline{\lambda}$$

Since $D_\alpha \phi_i^{(1)} D_\beta \phi_j^{(2)} M_{ij}$ is the contribution to the energy-momentum tensor coming from the kinetic term of the scalar fields, $O_3^{(2)}$ corresponds to the interaction of the low-energy theorem. On the other hand, $O_1^{(2)}$ and $O_2^{(2)}$ are equivalent due to the on-shell identity

$$\partial_\alpha \lambda \sigma^\mu \partial^\mu \overline{\lambda} = -ie^{\mu \alpha \beta} \partial_\nu \lambda \sigma_\alpha \partial_\beta \overline{\lambda}. \quad (4.24)$$

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Thus, we are left with the operator $O_1^{(2)}$, which is supersymmetric and whose contribution to the action is given by (1.6):

$$S = \kappa^2 \int d^4x \left( D_\mu \phi_1^{(1)} \phi_2^{(2)} M_{ij} - \phi_1^{(1)} D_\mu \phi_2^{(2)} M_{ij} \right) \partial_\alpha \lambda \sigma^\mu \partial_\alpha \bar{\lambda}.$$  (4.25)

This exhausts all possible couplings to scalar fields.

### 4.3 Couplings to scalars and gauge field-strengths

Operators with two scalars should contain for dimensional reasons only one power of $F_{\mu\nu}$, together with four Goldstino superfields. From the identity (4.7) it then follows that the Goldstinos must appear in the form $G^2 \bar{G}^2$ and this implies that the spacetime indices of $F_{\mu\nu}$ must be contracted among themselves, giving a vanishing result. We conclude that these couplings can only contain a single scalar field which must be in the adjoint representation. Generically they have the form:

$$O = \Phi^a O'^a (F_{\mu\nu}^a, G_\alpha, \bar{G}_\beta),$$  (4.26)

where $O'$ has dimension $D \leq 1$. From the analysis of Section 4.1 it then follows that none of these couplings can contribute at order $\kappa^2$.

### 4.4 Couplings to fermions

If we assume that lepton and baryon number are conserved, any such interaction must involve an equal number of matter fermions and anti-fermions. This case has already been considered in ref. [13] and here we just quote the result. It was found that up to order $\kappa^2$ there is a single possible term besides the standard coupling to the energy-momentum tensor:

$$S = \kappa^2 \int d^4x (f \partial^\mu \lambda)(\bar{f} \partial_\mu \bar{\lambda}).$$  (4.27)

However, since lepton and baryon number conservations result from accidental global symmetries of the Standard Model that may be broken by non-renormalizable interactions, it is natural to consider also terms which do not preserve these symmetries. The above four-fermion coupling then generalizes to

$$S_1 = \kappa^2 \int d^4x M_{ij} \left( f_i^{(1)} \partial^\mu \lambda \right) \left( \bar{f}_j^{(2)} \partial_\mu \bar{\lambda} \right).$$  (4.28)
Furthermore, there is an additional four-fermion interaction\(^3\):

\[
S_2 = \int d^4 x M_{ij} (f^{(1)}_i \partial_\mu \lambda)(f^{(2)}_j \partial^\mu \lambda) 
\]

(4.29)

We can also consider terms which are linear in \(\lambda\), allowing for several dimension 6 operators:

\[
\begin{align*}
O_1 &= \kappa (f^a \sigma_\alpha \partial_\beta \lambda) F^{a \alpha \beta}_\mu \partial^\mu \\
O_2 &= \kappa (f^a \sigma_\alpha \bar{\partial}^\beta \lambda) F^{a}_{\alpha \beta} \\
O_3 &= \kappa M_{ij} (f^i \partial_\alpha \lambda) D^{\alpha} \phi^j
\end{align*}
\]

(4.30)

All these operators are supersymmetric. Actually, using the equations of motion for \(\lambda\), \(O_1\) is proportional to \(O_2\). Moreover, \(O_2\) cannot appear in the Standard Model because it requires fermions transforming in the adjoint representation. One could however use a gauge singlet fermion, such as a right-handed neutrino, coupled to the hypercharge field-strength. On the other hand, \(O_3\) is allowed provided that \(f^i\) is a lepton doublet and \(\phi^j\) is the Higgs field. There can be additional terms linear in \(\lambda\) of order \(\kappa^2\) but we shall not attempt to construct these systematically here, since they are too many.

This exhausts all possibilities for coupling the Goldstino to Standard Model fields consistent with non-linear supersymmetry. The new couplings we found are of the same, or even lower, order in \(\kappa\) as the coupling to the energy-momentum tensor and must therefore be taken into account in the low-energy theory. Their (dimensionless) coefficients are model-dependent and can be determined by the underlying fundamental theory. In the next section we shall determine these values in string theory with D-branes. For future reference, we summarize the full list of operators in Appendix A.

Before proceeding, there is an important technical point that needs to be mentioned. In this section, we have found two interactions of order \(\kappa\): (A.3) and (A.4). They both give rise to three-point functions involving massless particles, which obviously vanish on-shell because of the derivatives. Since in string theory we can only do on-shell calculations, these operators cannot be determined “directly” by computing three-particle interaction amplitudes. A way to compute the coefficients \(C_1\) and \(C_2\) is to look instead at four-point tree-level amplitudes, obtained by combining two order \(\kappa\) vertices. These are in fact contact terms, because the propagator of the intermediate massless state gets canceled. Furthermore, these reducible contact terms must

---

\(^3\)The action \(S_3 = \int d^4 x M_{ij} (f^{(1)}_i \partial_\mu \lambda)(f^{(2)}_j \partial^\mu \lambda)\), which is also supersymmetric, is related to (4.29) by a Fierz rearrangement. We thank A. Brignole for pointing this out to us.
be (non-linear) supersymmetric, because they arise by combining two super-symmetric vertices. Thus, they are indistinguishable from other irreducible contributions to the amplitudes, obtained by order $\kappa^2$ terms in the list of operators. This makes it difficult to disentangle the two contributions.

As we shall see, it is possible to obtain irreducible amplitudes by considering generic string interactions on intersecting stacks of D-branes where the intermediate states generating reducible contributions are massive. Assuming that all effective couplings depend continuously on the intersection angles, it is then possible to obtain their values also for single stacks of D-branes.

A similar problem occurs with the 3-point interaction (A.8). However, in this case the contact term generated at four-point level is of order $\kappa^4$ which we do not compute. Thus, we do not determine in this work the coefficient $C_6$. We also leave undetermined the coefficient $C_8$ which requires the computation of a 5-point function.

In Sections 5 and 6, we compute all the relevant string amplitudes. In Section 7 we combine the results to extract the values of the coefficients $C_i$ and of the Goldstino decay constant $\kappa$.

5 Single stack of Dp-branes

Our first task is to identify the Goldstino in the spectrum. We consider space-time to be the product of (3+1)-dimensional Minkowski space $M_4$ with some internal six-dimensional compact manifold $M$ which we assume to preserve all 32 supersymmetries of the type II superstring theory. We shall first consider a single stack of $N$ D$p$-branes with $p \geq 3$. We identify the first 4 dimensions of D-brane world-volume with $M_4$ while the remaining $p - 3$ dimensions are wrapped around a cycle of the internal manifold. The presence of branes breaks half of the supercharges spontaneously and from the point of view of (3+1)-dimensions we have $\mathcal{N} = 4$ supersymmetry. The massless open string spectrum consists of an $\mathcal{N} = 4$ vector multiplet transforming in the adjoint representation of $U(N)$ \footnote{In the general case, one could add orientifolds and break (part of) the supersymmetry, but their presence does not modify our analysis.}. For each of the four broken supersymmetries there is a pair of massless Goldstone fermions (Goldstinos) forming a two-component Weyl spinor in four dimensions. These carry the same quantum numbers as the broken supercharges. Since the supercharges commute with the generators of the gauge group, the Goldstinos should be gauge singlets and must therefore be identified with the gauginos in the $U(1)$ vector multiplet which is part of $U(N)$. We shall pick one of these Goldstinos and its CPT conjugate and calculate its effective interactions with the particles in...
the $SU(N)$ vector multiplet. This will allow to extract an expression for the supersymmetry breaking scale $\kappa$ in terms of the string scale $M_s$ and to determine the coefficients of the (model-dependent) supersymmetric operators obtained in the previous section.

### 5.1 Interactions with gauge bosons

As is well-known, the tree-level interaction amplitude involving open string states is obtained by evaluating correlation functions of vertex operators on the boundary of the disc. By a suitable conformal transformation, the disc can be mapped onto the upper half-plane with the vertex operators located on the real axis. After integrating over the positions of the vertices we must divide by the volume of $PSL(2,R)$, the conformal group on the disc. Alternatively, we can fix the location of three vertex operators to some arbitrary positions which we shall choose to be $x = 0, 1, \infty$. Using the conformal symmetry of the disc, the range of integration of the fourth operator can be mapped to the interval $[0,1]$. The amplitude has then the form

$$A(1, 2, 3, 4) = A(1, 2, 3, 4) \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4 + \lambda^1 \lambda^3 \lambda^2 \lambda^1)$$

$$+ A(4, 2, 3, 1) \text{Tr}(\lambda^4 \lambda^2 \lambda^3 \lambda^1 + \lambda^4 \lambda^3 \lambda^2 \lambda^4)$$

$$+ A(1, 2, 4, 3) \text{Tr}(\lambda^1 \lambda^2 \lambda^4 \lambda^3 + \lambda^3 \lambda^4 \lambda^2 \lambda^1)$$

(5.1)

with

$$A(1, 2, 3, 4) = iC_D \int_0^1 dx \langle (e\nu_{q_1})(0)\nu_{q_2}(x)(e\nu_{q_3})(1)(e\nu_{q_4})(\infty) \rangle.$$  

(5.2)

Here $\lambda^i$ is the Chan-Paton matrix that comes with the vertex operator $\nu_{q_i}(x)$. The subscripts $q_i$ denote the ghost numbers which for the case of the disc must add up to $-2$ in order to cancel the superconformal background charge $[19]$. The constant $C_D$ depends only on the topology of the world-sheet and can be determined by unitarity. It is given by

$$C_D = \frac{1}{2\alpha'^2 g_{YM}^2 V_c},$$

(5.3)

where $g_{YM}$ is the four-dimensional Yang-Mills coupling$^5$ and $V_c$ is the compactification volume along the Dp-brane world-volume. Finally, the gauge-fixing procedure requires that each “fixed” vertex operator is accompanied by a c-ghost insertion.

$^5$Here we use a somewhat non-standard convention where the generators of the gauge group are normalized according to $\text{Tr}(T^a T^b) = \delta_{ab}$. This implies in particular that the canonically normalized kinetic term for the gauge bosons is given by $-\frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu})$. 

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It will be convenient to express all amplitudes in terms of the Mandelstam variables:

\[ s = -(k_1 + k_2)^2 \quad t = -(k_1 + k_3)^2 \quad u = -(k_1 + k_4)^2, \]  

(5.4)

where \( k_i \) is the spacetime momentum of the state \( i \).

We shall first evaluate the interaction of two Goldstinos and two gauge bosons. As stated above, the spectrum contains eight Goldstinos characterized by their helicities in the internal directions. We shall conventionally choose the Goldstino to be the abelian gaugino with internal helicities given by \{+ + +\}, which by virtue of the GSO projection will have positive spacetime chirality. Its CPT conjugate will then have internal helicities \{- - -\} and negative spacetime chirality. The corresponding vertex operators in the \((-\frac{1}{2})\)-ghost picture are

\[ \nu_{-\frac{1}{2}}^G(x, k, u_L) = (4\alpha'^3)^{\frac{1}{2}} gYM u_L \Theta^a e^{-\frac{\phi}{2}} e^{-i(H_1+H_2+H_3)} e^{ikX} \]

\[ \bar{\nu}_{-\frac{1}{2}}^G(x, k, u_R) = (4\alpha'^3)^{\frac{1}{2}} gYM u_R \Theta^a e^{-\frac{\phi}{2}} e^{i(H_1+H_2+H_3)} e^{ikX} \]  

(5.5)

where \( \phi \) is the bosonized superconformal ghost and \( H_i \) are the usual bosonized fermionic coordinates in the internal and transverse directions. The \( \Theta^a \) are four-dimensional spin fields and \( u_{L,R} \) are left- and right-handed Weyl spinors (in four-component notation), respectively. For gauge bosons \( (B^\mu) \), we shall need the vertex operators in both the \((0)-\) and \((-1)-\)ghost picture:

\[ \nu_{-1}^B(x, k, \epsilon) = \sqrt{2\alpha' gYM} e^{-\phi(\epsilon\psi)} e^{ikX} \]

\[ \nu_0^B(x, k, \epsilon) = 2gYM \left( i(\epsilon\partial X) + \alpha'(k\psi)(\epsilon\psi) \right) e^{ikX} \]  

(5.6)

where \( \epsilon \) is the polarization vector. The normalizations of the vertices can be obtained by computing three-point functions in the point-particle limit and comparing with the corresponding result in Yang-Mills theory.

The correlation function we need to evaluate is:

\[ <\nu_{-\frac{1}{2}}^G(x_1, k_1, u_{L1}) \bar{\nu}_{-\frac{1}{2}}^G(x_2, k_2, u_{R2}) \nu_{-1}^B(x_3, k_3, \epsilon_3) \nu_0^B(x_4, k_4, \epsilon_4) > . \]  

(5.7)

Inserting the vertex operators, this factorizes into a product of well-known

\footnote{The c-ghost insertions are implicit here. They will always contribute a factor \( x_{13}x_{14}x_{34} \) to the correlator.}
correlators \(^{19}\) and the result is
\[
< \nu^G_\mathfrak{g_1} \nu^G_{-\mathfrak{g_2}} \nu^B_{-1} \nu^B_0 > = 4\sqrt{2} \alpha^3 \sqrt{g_{YM}} \frac{\sum x_{ij} x_{14} x_{34}}{x_{12} x_{13} x_{23}} (2\pi)^4 \delta^{(4)}(\sum_i k_i) V_c \prod_{i<j} x_{ij}^2 x_{ij}^2
\]
\[
+ \left( \frac{1}{\sqrt{2}} u^T_{L1} C \epsilon \delta u_{R2} \left( \frac{x_{13}}{x_{14}} + \frac{x_{23}}{x_{24}} \right) + \frac{1}{\sqrt{2}} u^T_{L1} C \epsilon \delta u_{R2} \left( \frac{x_{13}}{x_{34}} - \frac{1}{\sqrt{2}} (\epsilon_3 \epsilon_4) u^T_{L1} C \epsilon \delta u_{R2} \frac{x_{13}}{x_{34}} - \frac{x_{12}}{x_{34}} \right) \right)
\]
where \(x_{ij} = x_i - x_j\) and \(C\) is the charge-conjugation matrix. The volume factor \(V_c\) in the numerator comes from the correlator of exponentials:
\[
< \prod_i e^{ik_i X} > = (2\pi)^4 \delta^{(4)}(\sum_i k_i) V_c \prod_{i<j} x_{ij}^2 x_{ij}^2.
\](5.8)

The integration over the zero modes of \(X^{0,1,2,3}\) gives the delta function, while integration over the zero modes of \(X^{4,\ldots,p}\) gives the compactification volume along the D-brane.

To obtain the amplitude \(A(1, 2, 3, 4)\), we set \((x_1, x_2, x_3, x_4) \rightarrow (0, x, 1, \infty)\) and then perform the integration in (5.24). The other two permutations in (5.21) are obtained by simply permuting the positions of the vertex operators. For example \(A(4, 2, 3, 1)\) corresponds to setting \((x_1, x_2, x_3, x_4) \rightarrow (\infty, x, 1, 0)\) before performing the integration. Putting everything together, we obtain the total amplitude:
\[
A(\lambda_L \lambda_R \rightarrow BB) = 2i\alpha^2 g_{YM}^2 (2\pi)^4 \delta^{(4)}(\sum_i k_i) K_{GB}(1, 2, 3, 4)
\]
\[
\left( B(s, u) \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4 + \lambda^1 \lambda^2 \lambda^3 \lambda^4) + B(t, u) \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4 + \lambda^1 \lambda^2 \lambda^3 \lambda^4) \right.
\]
\[
+ B(s, t) \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4 + \lambda^1 \lambda^2 \lambda^3 \lambda^4) \right)
\](5.9)

where \(K_{GB}(1, 2, 3, 4)\) is the kinematic factor
\[
K_{GB}(1, 2, 3, 4) = -uu^T_{L1} C \epsilon \delta u_{R2} (k_2 \epsilon_4) - \frac{s}{2} uu^T_{L1} C \epsilon \delta u_{R2} \epsilon_4 (k_2) + t uu^T_{L1} C \epsilon \delta u_{R2} \epsilon (k_2 \epsilon_4) + uu^T_{L1} C \epsilon \delta u_{R2} (k_2 \epsilon_4)
\]
\[
- uu^T_{L1} C \epsilon \delta u_{R2} (k_2 \epsilon_4)
\](5.10)
and \(B(x, y)\) is given in terms of gamma functions as
\[
B(x, y) = \frac{\Gamma(-\alpha' x)\Gamma(-\alpha' y)}{\Gamma(1 - \alpha' x - \alpha' y)} = \frac{1}{\alpha'^2 xy}(1 - \frac{\pi^2}{6}\alpha'^2 xy + \ldots). \tag{5.11}
\]

One can easily check that the amplitude \((5.9)\) reproduces correctly, in the low-energy limit, the corresponding quantum field theory (QFT) result. Taking \(\alpha' \to 0\) and using the expansion \((5.11)\) in \((5.9)\) we obtain for the s-channel pole
\[
A^{(0)}(\lambda_L \lambda_R \to BB) \to -2 i g_{YM}^2 (2\pi)^4 \delta^{(4)}(\sum k_i) \frac{1}{s} \text{Tr}(\lambda^1, \lambda^2)[\lambda^3, \lambda^4])
\[
\left( u^T_{L1} C \epsilon_4 u_{R2} (k_4 \epsilon_3) - u^T_{L1} C \epsilon_5 u_{R2} (k_3 \epsilon_4) - u^T_{L1} C \epsilon_4 u_{R2} (\epsilon_3 \epsilon_4) \right).
\]

This agrees with the s-channel contribution of the corresponding QFT amplitude with fermions in the adjoint representation of \(U(N)\). The Chan-Paton matrices coincide with the generators of the gauge group. A similar check can be performed in the t- and u-channels. This confirms that we have correctly chosen the phases of the six permutations that contribute to the amplitude.

We can now specialize to the case where the fermions are the \(U(1)\) gauginos by using the appropriate Chan-Paton matrices:
\[
\lambda^1 = \lambda^2 = \frac{1}{\sqrt{N}} \mathbb{I}_N. \tag{5.12}
\]

The point-particle limit now vanishes and the interaction results from purely “stringy” effects with massive string modes as intermediate states. Inserting \((5.12)\) in \((5.9)\) and using again the expansion \((5.11)\), we obtain the first correction to the QFT amplitude:
\[
A^{(2)}(\lambda_L \lambda_R \to BB) = -\frac{2 i \pi^2 \alpha'^2 g_{YM}^2}{N} (2\pi)^4 \delta^{(4)}(\sum k_i) \text{Tr}(\lambda^3 \lambda^4) K_{GB}(1, 2, 3, 4). \tag{5.13}
\]

### 5.2 Interactions with scalars

An analogous calculation can be done for the interaction between the scalars and Goldstinos. The scalars are just the components of the ten-dimensional gauge bosons in the transverse and internal directions (in the latter case we need to take the zero-modes of the Kaluza-Klein expansion). The vertex operators can be obtained from \((5.6)\) simply by choosing the gauge bosons
to be polarized in these directions. For example, we can obtain one such complex scalar \( \phi^{(1)} \) by choosing the gauge boson to be polarized in the 45-direction:

\[
\epsilon = \frac{1}{\sqrt{2}}(0, 0, 0, 1, -i, 0, 0, 0).
\]

Using this polarization in (5.6) and bosonizing the fermions, we obtain

\[
\nu^{(1)}_{-1}(x,k) = \sqrt{2}\alpha' g_{YM} e^{-iH_1} e^{ikX}
\]

The CPT conjugate scalar has then polarization vector \( \epsilon^* \). Its vertex operator is obtained by reversing the internal helicity \( e^{-iH_1} \rightarrow e^{iH_1} \) and taking the complex conjugation \( iX^5 \rightarrow -iX^5 \). Similarly, the scalars \( \phi^{(2)} \) and \( \phi^{(3)} \) correspond to gauge bosons polarized in the 67- and 89-planes, respectively, and their vertices \( \nu^{(2)}_{-1} \) and \( \nu^{(3)}_{-1} \) are obtained just like (5.15).

The calculation now proceeds in exactly the same way as for the gauge bosons and the result is the same as in (5.9) except that we need to replace the kinematic factor

\[
K_{GB}(1, 2, 3, 4) \rightarrow K_S(1, 2, 3, 4) = u_L^T C h_4 u_R.
\]

Inserting the Chan-Paton factors (5.12), we can expand in powers of \( \alpha' \), retaining only the first stringy correction. This yields

\[
\mathcal{A}^{(2)}(\lambda_L \lambda_R \rightarrow \phi^{(i)} \bar{\phi}^{(j)}) = -\frac{i}{2\pi}(\frac{2\alpha'^2 g_{YM}^2}{N})^4 \delta^{(i)}(\sum k_i) \times \text{Tr}(\lambda^3 \lambda^4) \delta^{ij} K_S(1, 2, 3, 4).
\]

### 5.3 Interactions with fermions

The interactions of two fermions with two Goldstinos of opposite helicity were studied in ref. [9]. It was found that there are two inequivalent cases, depending on the internal helicities of the fermions. In the next section we shall recover these results by computing the corresponding interaction on intersecting D-branes and taking the limit of coincident branes. Here, we simply quote the result:

\[
\mathcal{A}^{(2)}(\lambda_L \lambda_R \rightarrow f_L \bar{f}_R) = -2i\frac{2\pi \alpha'^2 g_{YM}^2}{N} \times (2\pi)^4 \delta^{(i)}(\sum k_i) \text{Tr}(\lambda^3 \lambda^4) K_F(1, 2, 3, 4),
\]
where the kinematic factor is
\[
K_F(1, 2, 3, 4) = \begin{cases} 
\frac{tu+sv}{2s} & \text{case I} \\
\frac{tu}{2s} & \text{case II}
\end{cases}
\]
(5.19)

Case I corresponds to the amplitude where the left-handed fermion \( f_L \) has internal helicities \((-,-,-)\), whereas in case II it has mixed internal helicities \((+,-,-),(+,+,-)\) or \((-,+,+).\)

In order to study the effective operator \( [A.6] \) in string theory, we also need to compute analogous four-fermion amplitudes involving two Goldstinos with the same helicity. If we take the (incoming) Goldstinos to be left-handed, then conservation of internal helicity requires the outgoing fermions to be left-handed as well, with internal helicities \((-,-,-)\). This corresponds to case I. The computation is straightforward. The vertex operators for both the Goldstino and the fermions are given by (5.5) together with appropriate Chan-Paton factors. The final result is:

\[
\mathcal{A}^{(2)}(\lambda_L \lambda_L \rightarrow f_L f_L) = -2i(\frac{2\alpha' \alpha'^2 g_{YM}^2}{N})(2\pi)^4 \delta^4(\sum_i k_i) \text{Tr}(\lambda^3 \lambda^4)K_F'(1, 2, 3, 4)
\]
(5.20)

where
\[
K_F'(1, 2, 3, 4) = \begin{cases} 
\frac{8}{2} & \text{case I} \\
0 & \text{case II}
\end{cases}
\]
(5.21)

6 Intersecting D-branes

Intersecting D-branes have been used in recent years to construct semi-realistic models with particle spectra and gauge groups close to the Standard Model (see for example ref. [20] and references therein). It is therefore interesting to examine the dynamics of Goldstinos in this framework.

We consider the case of two stacks of D-branes intersecting in some \(d\)-dimensional world-volume. The two stacks will be denoted by \(Dp\) and \(Dp'\) with \(N, N'\) being the respective number of branes in each stack. The gauge group is then \(U(N) \times U(N')\) and the massless spectrum is divided into several sectors transforming in different representations of the gauge group:

- Open strings with both endpoints located on the same stack. The corresponding excitations will (upon compactification) fill out vector multiplets of \(\mathcal{N} = 4\) supersymmetry. Strings with both ends on \(Dp\) will transform in the adjoint representation of \(U(N)\) and in the trivial representation of \(U(N')\) while those with both ends on \(Dp'\) will be in the trivial of \(U(N)\) and in the adjoint of \(U(N')\).
Open strings stretched between the two stacks. The corresponding states are localized at the intersection and transform in bifundamental representations of the gauge group. Specifically, an open string with its $\sigma = 0$ end on the D$p$ and its $\sigma = \pi$ end on the D$p'$ ($pp'$ string) transforms in $(N, \bar{N'})$. Here $\sigma$ is the world-sheet coordinate along the open string. By interchanging the endpoints we obtain a $p'p$ string transforming in $(\bar{N}, N')$. If the two stacks are localized at the same position in the transverse directions, then the intersection will carry massless fermions. For special values of the intersection angles corresponding to unbroken supersymmetry (locally), the intersection will also carry massless scalars which combine with the fermions into supermultiplets. For generic angles, however, the scalars are massive or tachyonic and the full massless spectrum is non-supersymmetric.

6.1 Interactions with chiral fermions

We will be interested in the interactions between Goldstinos and the fermions located on the intersection. The study of non-linear supersymmetry in Section 4.4 has shown that, besides the coupling to the energy-momentum tensor, there are some possible additional four-fermion interactions at order $\kappa^2$. We wish to determine whether these coupling do appear in string theory for generic D-brane intersection angles.

For simplicity, we shall specialize to the case of two stacks of D6-branes intersecting in a (3+1)-dimensional world-volume. In such a configuration, there can be no common transverse directions and the spectrum will generically contain massless fermions. As noted in ref. [3], the GSO projection requires these fermions to be chiral. Furthermore, by choosing the intersection angles appropriately, it is possible to preserve at most one supersymmetry\footnote{Of course, by setting some intersection angle(s) to zero, it is possible to preserve more supersymmetry. However in this case, from the four-dimensional point of view, the fermions on the intersection will not be chiral anymore. We therefore take all intersection angles to be non-vanishing.}. In this case, there will be two (CPT conjugate) massless scalars on the intersection (see subsection 6.2) which combine with the fermions into chiral multiplets.

We start with two parallel stacks of D6-branes which we take to be oriented in the 0123468 directions. We then rotate the $D6'$ stack in the 45, 67 and 89 planes by angles $\phi_1$, $\phi_2$ and $\phi_3$, respectively. The two stacks then intersect in the 0123 directions. It is convenient to introduce complex
coordinates:
\[
Z^1 = X^4 + iX^5 \quad Z^2 = X^6 + iX^7 \quad Z^3 = X^8 + iX^9 \quad (6.1)
\]
Consider now a 66' string. In terms of the coordinates \( Z^k \), it will satisfy the following boundary conditions:
\[
\text{Re}(\partial_\sigma Z^k) = 0 \quad \text{Im}(Z^k) = 0 \quad \text{at} \quad \sigma = 0
\]
(6.2)
\[
\text{Re}(e^{-i\phi_k \partial_\sigma} Z^k) = 0 \quad \text{Im}(e^{-i\phi_k} Z^k) = 0 \quad \text{at} \quad \sigma = \pi
\]
They imply that the mode expansions of \( \partial Z^k \) and \( \partial \bar{Z}^k \) have mode numbers shifted by \( \frac{\phi_k}{\pi} \):
\[
\partial Z^k(e^{2i\pi z}) = e^{2i\phi_k} \partial Z^k(z) \quad \partial \bar{Z}^k(e^{2i\pi z}) = e^{-2i\phi_k} \partial \bar{Z}^k(z) \quad (6.3)
\]
where \( z \) is the complex world-sheet coordinate.

To construct the vertex operators which create the 66' states we can proceed in analogy with strings on orbifolds. Indeed, bosonic fields satisfying the periodicity condition (6.3) can be thought of as belonging to the twisted sector of an orbifold theory. Let us first assume that the rotation angle \( \phi_k \) is positive. Following ref. [21], the correct boundary condition near the insertion of the vertex operator is implemented by means of a twist field \( \sigma_+ \) which has an appropriate operator product expansion with \( \partial Z^k \) and \( \partial \bar{Z}^k \):
\[
\partial Z^k(z)\sigma_+^k(0) \sim z^{-(1 - \frac{\phi_k}{\pi})} \tau_+^k(0) \quad \partial \bar{Z}^k(z)\sigma_+^k(0) \sim z^{\frac{\phi_k}{\pi}} \tau_+^k(0) \quad (6.4)
\]
where \( \tau_+ \) and \( \tau_+' \) are “excited” twist fields. The conformal dimension of \( \sigma_+^k \) is \( h_{\sigma} = \frac{1}{2} \left( 1 - \frac{\phi_k}{\pi} \right) \).

By superconformal symmetry, twisting the bosonic fields requires a similar twisting of their world-sheet superpartners. Let \( \psi^k \) and \( \bar{\psi}^k \) be the complexified fermions which are the superpartners of \( \partial Z^k \) and \( \partial \bar{Z}^k \). World-sheet supersymmetry and condition (6.3) require that, in the Ramond sector, they satisfy
\[
\psi^k(e^{2i\pi z}) = -e^{2i\phi_k} \psi^k(z) \quad \bar{\psi}^k(e^{2i\pi z}) = -e^{-2i\phi_k} \bar{\psi}^k(z) \quad (6.5)
\]
which insures that the world-sheet supercurrent is anti-periodic on the plane (or periodic on the cylinder). We therefore need to insert along with \( \sigma_+^k \) a fermionic twist field \( S_+^k \) such that
\[
\psi^k(z)S_+^k(0) \sim z^{\frac{\phi_k}{\pi} - \frac{1}{2}} \tau_+^k(0) \quad \bar{\psi}^k(z)S_+^k(0) \sim z^{\frac{1}{2} - \frac{\phi_k}{\pi}} \tau_+^k(0) \quad (6.6)
\]
In terms of the bosonized fermions \( H_k \), the fermionic twist fields can be expressed as \( S^k_+ = e^{i\left(\frac{\phi_k}{\pi} - \frac{1}{2}\right)H_k} \). Their conformal dimension is \( h_S = \frac{1}{2}(\frac{\phi_k}{\pi} - \frac{1}{2})^2 \).
and therefore $h_\sigma + h_S = \frac{1}{8}$ independent of the rotation angle. Moreover, 

$$t'^k_+ = e^{i(\frac{2}{3} + \frac{1}{2})H_k} \quad \text{and} \quad t^k_+ = e^{i(\frac{2}{3} - \frac{1}{2})H_k}.$$  

In the context of open strings on intersecting D-branes, the twist fields can be thought of as implementing discrete changes of boundary conditions. As one moves along the boundary of the world-sheet, crossing a twist field amounts to moving (in the target space) across an intersection from one stack to another.

The case of a negative rotation angle is similar. Instead of (6.3) and (6.5) we now have similar expressions with $\phi_k \rightarrow -\phi_k^8$. This boundary condition is implemented by means of anti-twist fields $\sigma^k_- \quad \text{and} \quad S^k_-$. The corresponding expressions for the operator product expansions and conformal weights are obtained simply by making the replacement $\frac{\phi_k}{\pi} \rightarrow (1 - \frac{\phi_k}{\pi})$. In particular, the conformal weights of twist and anti-twist fields are the same.

The vertex operators for the massless fermions on the intersection can be obtained from (5.5) by replacing the bosonized spin fields in the 45, 67 and 89 planes by the appropriate twist and anti-twist fields. Consider first the case of a 66' string; we obtain:

$$\nu^{66'}_{-\frac{1}{2}} = g_0 \lambda^{66'} e^{-\frac{\phi}{\pi}} u_\alpha \Theta_\alpha^3 \prod_{k=1}^3 (S^k_\pm \sigma^k_\pm) e^{i p X}.$$  

The Chan-Paton factor $\lambda^{66'}$ is a $(N + N') \times (N + N')$ matrix with entries only in the upper off-diagonal block in accordance with the fact that a 66' string transforms in the $(N, N')$ representation of $U(N) \times U(N')$. The sign of the twist $\sigma^k_\pm S^k_\pm$ is given by the sign of the rotation in the corresponding plane. These signs determine via the GSO projection the chirality of the Weyl spinor $u_\alpha$.

The 6'6 string is obtained by interchanging the endpoints of the 66' string. This amounts to flipping the signs of all twists and replacing the Chan-Paton matrix $\lambda^{66'}$ by one with non-vanishing entries only in the lower off-diagonal block:

$$\nu^{6'6}_{-\frac{1}{2}} = g_0 \nu^{6'6}_{\frac{1}{2}} e^{\frac{\phi}{\pi}} v_\alpha \Theta_\alpha^\gamma \prod_{k=1}^3 (S^k_\pm \sigma^k_\pm) e^{i p X}.$$  

This string transforms in the conjugate representation $(\bar{N}, N')$. In this sense, the 6'6 string can be viewed as the CPT conjugate of the 66' string. Furthermore, the GSO projection requires $v_\alpha$ to be a Weyl spinor of chirality opposite to $u_\alpha$.

---

8From now on, we shall always take $\phi_k$ to denote the magnitude of the rotation angle and $\epsilon_k = \pm$ to denote its sign.
We now have to distinguish between two types of intersections. Let $\epsilon_{1,2,3}$ denote the sign of the rotation in the 45, 67 and 89 plane, respectively. We shall call the product $\epsilon_1 \times \epsilon_2 \times \epsilon_3$ the chirality of the intersection. We then have:

1. Positive intersections
   For this type of intersection, the GSO projection requires (conventionally) $u_\alpha$ to be left-handed and $v_\alpha$ to be right-handed. This implies that, in our convention, positive intersections carry only negative helicity particles ($66'$ strings) and positive helicity anti-particles ($6'6$ strings).

2. Negative intersections
   This is precisely the opposite situation. The GSO projection requires $u_\alpha$ and $v_\alpha$ to be right- and left-handed, respectively. Therefore negative intersections carry only positive helicity particles and negative helicity anti-particles.

For our purposes it will be sufficient to specialize to the case of positive intersections. The vertex operators are then given by

$$\nu^-_{\vec{\nu}^{66'}} = g_0 \lambda^{66'} e^{-\frac{\phi}{2}} u_{L\alpha} \Theta^\alpha \prod_{k=1}^{3} (S_{+\epsilon_k}^k \sigma_{\epsilon_k}^k) e^{ipX}$$

$$\nu^\nu_{\vec{\nu}^{6}} = g_0 \lambda^{6'} e^{-\frac{\phi}{2}} u_{R\alpha} \Theta^\alpha \prod_{k=1}^{3} (S_{-\epsilon_k}^k \sigma_{-\epsilon_k}^k) e^{ipX} \quad (6.7)$$

where $\epsilon_1 \epsilon_2 \epsilon_3 = +1$ and

$$\lambda^{66'} = \begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix} \quad \lambda^{6'} = \begin{pmatrix} 0 & 0 \\ t' & 0 \end{pmatrix}. \quad (6.8)$$

The constant $g_0$ is a normalization which can be expressed in terms of the Yang-Mills couplings $g_{YM}$ and $g_{YM}'$ by comparing the point-particle limit of string amplitudes with the corresponding field theory calculation. We can for example compute the three-point function of two fermions and a gauge boson of $U(N)$. This is given by

$$\mathcal{A}(1, 2, 3) = A(1, 2, 3) \text{ Tr}(\lambda^1 \lambda^2 \lambda^3) + (2 \leftrightarrow 3) \quad (6.9)$$

with

$$A(1, 2, 3) = i C_D < c\nu^{66'}_{\frac{1}{2}}(0, p_1, u_{L1}) c\nu^{\nu 6}_{\frac{1}{2}}(1, p_2, u_{R2}) c\nu^{-1}_{\infty}(\infty, p_3, \epsilon) >. \quad (6.10)$$

---

9Positive (negative) chirality intersections make positive (negative) contributions to the intersection number $I_{66'}$ as defined in ref. [20].
Inserting the vertex operators (6.7) and (5.6) and using (5.3) for the constant $C_D$, we obtain

$$A(1, 2, 3) = \frac{g_0^2}{2\alpha'^2 g_{YM} V_c} (u_{1L}^C e'/u_{2R})(2\pi)^4 \delta^4 \left( \sum_i p_i \right) \text{Tr}(\lambda^1, \lambda^2, \lambda^3).$$

(6.11)

Here we have used the correlator of twist fields which is completely determined (up to a normalization which we fix to unity) by conformal invariance and the conformal weights $h_{\sigma}, h_{S}$:

$$< (S^k \sigma^k)_+ (x_1) (S^k \sigma^k)_- (x_2) > = x_{12}^{-\frac{1}{3}}.$$  (6.12)

Also, the correlator of exponentials is given by

$$< \prod_i e^{ip_i X} > = (2\pi)^4 \delta^4 \left( \sum_i p_i \right) \prod_{i<j} x_{ij}^{2\alpha' p_i p_j}.$$  (6.13)

Note that the factor $V_c$ appearing in (5.8) is now missing. This is because the fields $Z^k$ and $\bar{Z}^k$ do not have any zero modes over which we have to integrate since the strings are localized on the intersection. It follows that there is an extra factor of $V_c^{\frac{1}{2}}$ in the normalization constant $g_0$. Indeed, comparing (6.11) with the corresponding vertex in Yang-Mills theory we obtain

$$g_0 = (4\alpha'^3)^{\frac{1}{2}} g_{YM} V_c^{\frac{1}{2}}.$$  (6.14)

If instead we had chosen a gauge boson of $U(N')$ we would have obtained the same result with $g_{YM} \rightarrow g'_{YM}$ and $V_c \rightarrow V_c'$. In particular, this implies the relation

$$\frac{g_{YM}}{g'_{YM}} = \left( \frac{V_c'}{V_c} \right)^{\frac{1}{2}},$$  (6.15)

which follows from the fact that both Yang-Mills couplings are related to the string coupling $g_s$ by dimensional reduction.

We need again to identify the Goldstinos in the spectrum. In contrast to the case of a single stack of D-branes, there is now an additional complication from the fact that the amount of broken supersymmetry, and therefore the number of Goldstinos, depends on the intersection angles. As mentioned in the previous section, the Goldstinos should be gauge singlets and therefore must be identified with the gauginos of the $U(1)$ factors in $U(N)$ and $U(N')$. This would result in a total of 16 Goldstinos which is in agreement with Goldstone’s theorem in the case of non-supersymmetric configurations. However, if some unbroken supersymmetry remains, then not all $U(1)$ gauginos can be identified with Goldstinos. Instead, it turns out that in this case the physical
Goldstinos correspond to certain linear combinations of \(U(1)\) gauginos. The coefficients appearing in these linear combinations can be determined by the condition that the Goldstinos, being gauge neutral, do not interact with the chiral fermions in the QFT limit \(\alpha' \to 0\).

The vertex operator for the D6 gaugino is given in (5.5). It needs to be supplemented by the appropriate Chan-Paton factor \(\lambda\) which in the present context is an \((N + N') \times (N + N')\) matrix with the identity in the upper diagonal block and all other entries vanishing:

\[
\lambda = \frac{1}{\sqrt{N}} \begin{pmatrix}
  1 & 0 \\
  0 & 0
\end{pmatrix} .
\]

To obtain the vertices for the D6' gauginos, we substitute \(g_{YM} \to g'_{YM}\) and \(\lambda \to \lambda'\) where \(\lambda'\) has the identity in the lower diagonal block. Also, we have to replace \(H_k\) by \(H'_k = H_k + \epsilon_k \phi_k\). This shift comes from the rotation of the D6' stack relative to the D6. Indeed, \(e^{\pm i H_k}\) is the bosonization of \(\psi^{2k+2+2i\psi^{2k+3}}\), while \(e^{\pm i H'_k}\) is the bosonization of \(\psi^{2k+2+2i\psi^{2k+3}}\). The latter is obtained from the former by applying a rotation by an angle \(\phi_k\) in a positive (\(\epsilon_k = +1\)) or negative (\(\epsilon_k = -1\)) direction:

\[
\psi^{2k+2+2i\psi^{2k+3}} = e^{\pm i \epsilon_k \phi_k} \psi^{2k+2+2i\psi^{2k+3}} .
\]

In terms of the fields \(H_k\), this rotation corresponds to a shift by \(\epsilon_k \phi_k\). Putting everything together, the vertex operators for the D6' gauginos become

\[
\nu^{G}_{-\frac{1}{2}}(x, k, u_L) = (4\alpha'^3)^{\frac{1}{2}} g'_{YM} e^{i\Phi} \epsilon_1 \Theta^a u_L \Theta^a e^{-\frac{i}{2}(H_1 + H_2 + H_3)} e^{ipX}.
\]

\[
\bar{\nu}^{G}_{-\frac{1}{2}}(x, k, u_R) = (4\alpha'^3)^{\frac{1}{2}} g'_{YM} e^{i\Phi} \epsilon_2 \Theta^a u_R \Theta^a e^{-\frac{i}{2}(H_1 + H_2 + H_3)} e^{ipX} .
\]

where we have defined \(\Phi\) as the sum of rotation angles

\[
\Phi = \epsilon_1 \phi_1 + \epsilon_2 \phi_2 + \epsilon_3 \phi_3
\]

and

\[
\lambda' = \frac{1}{\sqrt{N'}} \begin{pmatrix}
  1 & 0 \\
  0 & 0
\end{pmatrix} .
\]

In (6.19) we take into account the respective signs of the rotations.

It turns out that the physical Goldstino with internal helicities \(- - -\) and its CPT conjugate are given by the following linear combinations of gauginos:

\[
\nu^{G_{phys}}_{-\frac{1}{2}} = \frac{1}{\sqrt{NV_c + N'V'_c}} (\sqrt{N} \nu^G_{-\frac{1}{2}} + \sqrt{N'} \nu^G_{-\frac{1}{2}}) .
\]

\[
\bar{\nu}^{G_{phys}}_{-\frac{1}{2}} = \frac{1}{\sqrt{NV_c + N'V'_c}} (\sqrt{N} \bar{\nu}^{G}_{-\frac{1}{2}} + \sqrt{N'} \bar{\nu}^{G}_{-\frac{1}{2}}) .
\]
As we shall see, in the QFT limit, the physical Goldstino does not interact with the massless “matter” and gauge fields. In addition, in the limit $\phi_k \to 0$ (5.21) reduces to (5.5).

The scattering amplitude $A(\lambda_L \lambda_R \to f_L \bar{f}_R)$ is obtained from (5.1) and (5.2) and has four contributions:

$$ A = \frac{NV_c A^{66} + NV'_c A^{66'} + \sqrt{NN'cV'c} A^{66'} + \sqrt{NN'cV'c} A^{66}}{NV_c + NV'_c}. \quad (6.22) $$

The superscripts of the amplitudes on the right-hand side indicate which gauginos are involved. For instance, according to (5.1) and (5.2), to evaluate the superscripts of the amplitudes on the right-hand side indicate which gauginos are involved. For instance, according to (5.1) and (5.2), to evaluate $A^{66}$ we need the correlation function on the disc:

$$ <\nu_{\frac{1}{2}}^{66} \nu_{\frac{1}{2}}^{66} \nu_{\frac{1}{2}}^{66} \nu_{\frac{1}{2}}^{66} >. \quad (6.23) $$

Inserting the vertex operators, this factorizes into a product of simple correlators and we obtain

$$ <\nu_{\frac{1}{2}}^{66} \nu_{\frac{1}{2}}^{66} \nu_{\frac{1}{2}}^{66} \nu_{\frac{1}{2}}^{66} > = (4\alpha^3)^4 g_{YM}^4 V_c(2\pi)^4 \delta^4 \left( \sum_i p_i \right) (u^T_{1L} Cu_{4L})(u^T_{2R} Cu_{3R}) $$

$$ \times \prod_{i<j} x_{ij}^{2\epsilon_p p_j} \left\{ x_{12}^{1 \frac{\Phi_3}{2\pi} - \frac{\epsilon-1}{2}} x_{14}^{\frac{\Phi_3}{2\pi} + \frac{\epsilon+1}{2}} x_{23}^{\frac{\Phi_3}{2\pi} + \frac{\epsilon-1}{2}} x_{24}^{\frac{\Phi_3}{2\pi} - \frac{\epsilon+1}{2}} \right\}, \quad (6.24) $$

where we have defined the quantity

$$ \epsilon = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3 - 1}{2}, \quad (6.25) $$

which equals either $+1$ or $-1$ for positive intersections. Using this correlator in (5.2), one finds the function $A(1,2,3,4)$. Including also the other two permutations of vertex operators according to (5.1), we end up with

$$ A^{66}(1,2,3,4) = 2i\alpha' g_{YM}^2 (2\pi)^4 \delta^4 \left( \sum_i p_i \right) (u^T_{1L} Cu_{4L})(u^T_{2R} Cu_{3R}) $$

$$ \left( \frac{\Gamma(-\alpha's)\Gamma(-\alpha'u - \frac{\Phi}{2\pi} + \frac{\epsilon+1}{2})}{\Gamma(-\alpha's - \alpha'u - \frac{\Phi}{2\pi} + \frac{\epsilon+1}{2})} \right) \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4 + \lambda^4 \lambda^3 \lambda^2 \lambda^1) $$

$$ \pm \frac{\Gamma(-\alpha't + \frac{\Phi}{2\pi} - \frac{\epsilon-1}{2})\Gamma(-\alpha'u - \frac{\Phi}{2\pi} + \frac{\epsilon+1}{2})}{\Gamma(-\alpha't - \alpha'u + 1)} \text{Tr}(\lambda^1 \lambda^3 \lambda^2 \lambda^4 + \lambda^4 \lambda^2 \lambda^3 \lambda^1) $$

$$ - \frac{\Gamma(-\alpha's)\Gamma(-\alpha't + \frac{\Phi}{2\pi} - \frac{\epsilon-1}{2})}{\Gamma(-\alpha's - \alpha't + \frac{\Phi}{2\pi} - \frac{\epsilon-1}{2})} \text{Tr}(\lambda^1 \lambda^2 \lambda^4 \lambda^3 + \lambda^3 \lambda^4 \lambda^2 \lambda^1) \right). \quad (6.26) $$
Note that, generically, there are no massless poles in either the $t$- or $u$-channels. This is expected since the intermediate states in these channels are twisted and correspond to scalars ($u$-channel) or vectors ($t$-channel) on the intersection, both of which are massive or tachyonic for generic values of the intersection angles. On the other hand, the massless intermediate state in the $s$-channel is a gauge boson. Therefore, there must be a pole except if the external fermions are neutral. In this case, the $s$-channel poles coming from the first and third term in (6.26) must cancel and this fixes the negative sign of the third term. To fix the sign of the second term we go to the limit $\phi_k \to 0$ where the two stacks become coincident. If $\epsilon = 1$, there is a massless pole in the $t$-channel corresponding to an intermediate gauge boson. Since the fermions are then in the adjoint representation, this pole must be proportional to $\text{Tr}([\lambda^1, \lambda^3][\lambda^2, \lambda^4])$, fixing the sign of the second term to be positive. By a similar reasoning, it can be easily seen that for the case $\epsilon = -1$, the sign of the second term must be negative.

The appropriate Chan-Paton matrices are $\lambda^1 = \lambda^2 = \lambda$, $\lambda^3 = \lambda'^6$ and $\lambda^4 = \lambda'^6$. Using the explicit expressions (6.8) and (6.16), we see that only the first and third term contribute. The second term corresponds to orderings of the vertex operators where the chiral fermions lie between the gaugino vertices on the world-sheet boundary, implying that the gauginos must be located on different stacks.

The other gaugino amplitudes can be obtained directly from (6.26). Thus, to obtain ($A^{66'}, A^{66'}, A^{66'}$) we have to substitute

$$g^2_{YM} \rightarrow (g^{f^2}_{YM}, e^{i\frac{\phi}{2}}g_{YM}g'_{YM}, e^{-i\frac{\phi}{2}}g_{YM}g'_{YM})$$

and use the appropriate Chan-Paton matrices for the gauginos:

$$(\lambda^1, \lambda^2) = (\lambda', \lambda'), (\lambda, \lambda'), (\lambda', \lambda), \quad (6.27)$$

respectively. The phase factors in $A^{66'}$ and $A^{66'}$ are crucial in order to cancel any dimension 6 operators in the low-energy effective action.

Putting everything together in (6.22) and using the relation (6.15), we finally obtain the full Goldstino-fermion scattering amplitude:

$$A(\lambda_L \lambda_R \rightarrow f_L \bar{f}_R) = -2i \left( \frac{2\pi^2 \alpha'^2 g_{YM}g'_{YM}V_c\bar{V}_{c'}}{NV_c + N'V_{c'}} \right)$$

$$(2\pi)^4 \delta^4(\sum_i p_i) \text{Tr}(tt')K_F(1, 2, 3, 4), \quad (6.28)$$

where $K_F(1, 2, 3, 4)$ is the kinematic factor

$$K_F(1, 2, 3, 4) = -\frac{1}{\alpha' \pi^2} (u^T_{1L}Cu_{4L})(u^T_{2R}Cu_{3R})f_c(s, t, u) \quad (6.29)$$

30
with
\[
 f_\epsilon(s, t, u) = \frac{\Gamma(-\alpha's)\Gamma(-\alpha'u - \frac{\Phi}{2\pi} + \frac{\epsilon+1}{2})}{\Gamma(-\alpha's - \alpha'u - \frac{\Phi}{2\pi} + \frac{\epsilon+1}{2})} - \frac{\Gamma(-\alpha's)\Gamma(-\alpha't + \frac{\Phi}{2\pi} - \frac{\epsilon-1}{2})}{\Gamma(-\alpha's + \alpha't + \frac{\Phi}{2\pi} - \frac{\epsilon-1}{2})} \\
+\epsilon\cos\left(\frac{\Phi}{2}\right)\frac{\Gamma(-\alpha't + \frac{\Phi}{2\pi} - \frac{\epsilon-1}{2})\Gamma(-\alpha'u - \frac{\Phi}{2\pi} + \frac{\epsilon+1}{2})}{\Gamma(-\alpha't - \alpha'u + 1)}.
\]

(6.30)

Below we give the leading term in the expansion of this function for the two cases \(\epsilon = +1\) and \(\epsilon = -1\).

I. All three rotations are positive (\(\epsilon = 1\)):
\[
f_1(s, t, u) = \begin{cases} 
\alpha'u^2 \pi^2 & \text{if } \Phi = 0, \\
-\alpha't^2 - \alpha's^2 \pi^2 & \text{if } \alpha's, \alpha't, \alpha'u << |\Phi|
\end{cases}
\]

(6.31)

II. One rotation is positive and two are negative (\(\epsilon = -1\)):
\[
f_{-1}(s, t, u) = \begin{cases} 
-\alpha't^2 \pi^2 & \text{if } \Phi = 0, \\
-\alpha't^2 + \alpha's^2 \pi^2 & \text{if } \alpha's, \alpha't, \alpha'u << |\Phi|
\end{cases}
\]

(6.32)

Combining the above expansions with the explicit expressions for the Weyl spinors we obtain for the kinematic factor:

I. \(K_F(1, 2, 3, 4) = \begin{cases} 
su + tu & \text{if } \Phi = 0, \\
\frac{su + tu}{2s} & \text{if } \Phi \neq 0
\end{cases}\)

(6.33)

II. \(K_F(1, 2, 3, 4) = \begin{cases} 
tu & \text{if } \Phi = 0, \\
\frac{tu}{2s} & \text{if } \Phi \neq 0
\end{cases}\)

(6.34)

As promised, the quantum field theory limit \(\alpha' \to 0\) vanishes and the first stringy correction corresponds to an effective dimension 8 operator. Furthermore, we see that the case \(\Phi = 0\) reproduces the results of ref. [9] for a single stack of D-branes that were given in (5.19).\(^{10}\) Starting with the vertex operators (6.7) and taking the limit of coincident stacks, we see that in case I the fermions have internal helicities \((+, +, +)\) and \((-,-,-)\) while in case II they have mixed internal helicities.

\(^{10}\)Notice however the difference of our result from the result of ref. [9] in the \(\mathcal{N} = 2\) supersymmetric case of orthogonal branes, where one rotation angle vanishes and the other two equal \(\pm \pi/2\).
6.2 Interaction with twisted scalars

As stated previously, generically the intersection does not carry massless scalars. A notable exception is of course the case of supersymmetric configurations. In the Neveu-Schwarz (NS) sector, the zero-point energy for twisted states is

\[ E_0 = \sum_k \frac{\phi_k}{2\pi} - \frac{1}{2} ; \quad 0 \leq \phi_k \leq \frac{\pi}{2}. \]  

(6.35)

The lowest state, which we denote as \(|0; p>\), has squared-mass \(m^2 = E_0\) and is typically tachyonic. However, it is expected to be projected out by an appropriate GSO projection. The lowest lying scalars in the physical spectrum are then:

\[ |j \equiv \bar{\psi}_{-\frac{1}{2} + \frac{\phi_j}{\pi}}|0; p> j = 1, 2, 3 \]  

(6.36)

where \(\bar{\psi}_{-\frac{1}{2} + \frac{\phi_j}{\pi}}\) are the creation operators appearing in the mode expansion of \(\bar{\psi}^j\) in the NS sector. Here we have assumed that all rotations defining the intersection are positive, i.e. \(\epsilon_1 = \epsilon_2 = \epsilon_3 = 1\). If some \(\epsilon_k = -1\), then we need to replace \(\bar{\psi}_{-\frac{1}{2} + \frac{\phi_k}{\pi}} \rightarrow \psi_{-\frac{1}{2} + \frac{\phi_k}{\pi}}\).

The masses of the states (6.36) are

\[ m_1^2 = \frac{1}{2\pi\alpha'}(-\phi_1 + \phi_2 + \phi_3) \]
\[ m_2^2 = \frac{1}{2\pi\alpha'}(\phi_1 - \phi_2 + \phi_3) \]
\[ m_3^2 = \frac{1}{2\pi\alpha'}(\phi_1 + \phi_2 - \phi_3) \]  

(6.37)

If all angles are non-vanishing, then at most one of these scalars can be made massless and this happens precisely when one supersymmetry is restored. One can then consider interactions of this scalar with the Goldstinos in the low-energy effective action.

By conservation of the internal helicities, it is clear that the operator (A.9) cannot appear in the effective action. The remaining operators involving scalars are (A.4) and (A.7), both of which will contribute to the amplitude \(A(\lambda_L\lambda_R \rightarrow \phi\bar{\phi})\). Since the computation of this amplitude in string theory is independent of which of the scalars is massless, we consider for definiteness a configuration where the massless one is \(|j = 3>\).

To construct the vertex operator for this state, we start from the tachyon vertex and act with \(\bar{\psi}_{-\frac{1}{2} + \frac{\phi_j}{\pi}}^3\). The construction of the tachyon vertex follows closely the method discussed in the previous subsection for the fermion vertices, except of course that the tachyon is in the NS sector. To implement
the boundary condition (6.3) we insert a (bosonic) twist field \( \sigma^+_k \). By worldsheet supersymmetry, this field must be accompanied by a fermionic twist field \( S^+_k = e^{i \frac{\phi_k}{\pi} H_k} \). The corresponding operator product expansion (OPE) with the fields \( \psi^k, \bar{\psi}^k \) is

\[
\psi^k(z) S^+_k(0) \sim z^{\frac{\phi_k}{\pi}} t'^+_k(0) \quad \bar{\psi}^k(z) S^+_k(0) \sim z^{-\frac{\phi_k}{\pi}} t^+_k(0)
\]

where

\[
t'^+_k = e^{i \frac{\phi_k}{\pi} - 1} H_k \\
t^+_k = e^{i \frac{\phi_k}{\pi} + 1} H_k
\]

This makes the supersymmetry current periodic on the plane (or anti-periodic on the cylinder), as is appropriate for the NS sector. The full vertex operator for the 66' tachyon is then

\[
\nu^0_{-1} = g_0 e^{-\phi} \lambda^{66'} \prod_{k=1}^{3} \left( \sigma^k_+ S^k_+ \right) e^{ipX}.
\]

where \( g_0 \) is some normalization constant.

The vertex for the massless scalar in the \((-1)\)-ghost picture is then given by

\[
\nu^{(3)}_{-1}(0) = \overline{\psi}^{3}_{-1+\frac{\phi}{\pi}} \cdot \nu^0_{-1}(0) = \oint dz \ z^{\frac{\phi_k}{\pi} - 1} \overline{\psi}^3(z) \nu^0_{-1}(0).
\]

Using the OPEs (6.38), we see that the appropriate vertex operator is obtained from (6.39) by replacing \( S^3_+ \to t^3_+ \):

\[
\nu^{(3)}_{-1}(z) = g_0 e^{-\phi} \lambda^{66'} (\sigma^1_+ S^1_+) (\sigma^2_+ S^2_+) (\sigma^3_+ t^3_+) e^{ipX}.
\]

Conformal invariance requires this operator to have conformal weight 1, from which we recover the mass-shell condition (6.37).

In order to compute the 4-point amplitude, we also need the scalar vertex operator in the 0-picture. This is obtained by operating with the supercharge \( G_{-\frac{1}{2}} \) on \( \nu^3_{-1} \) after removing the superconformal ghost \( e^{-\phi} \) [19]:

\[
\nu^{(3)}_0(0) = Q_{BRST} \cdot \nu^{(3)}_{-1}(0) = \int dz e^{\phi} T_F(z) \nu^{(3)}_{-1}(0).
\]

From the OPEs (6.4) and (6.38) we get

\[
G_{-\frac{1}{2}} \cdot \left( \sigma^k_+ S^k_+ \right) = i \sqrt{\frac{2}{\alpha'}} \tau^k_+ t^k_+ \\
G_{-\frac{1}{2}} \cdot \left( \sigma^k_+ t^k_+ \right) = i \sqrt{\frac{2}{\alpha'}} \tau^k_+ S^k_+ \\
G_{-\frac{1}{2}} \cdot e^{ipX} = \sqrt{\frac{2}{\alpha'}} \alpha' (p \cdot \psi) e^{ipX}.
\]
which yields

\[
\nu^{(3)}_0 = \sqrt{\frac{2}{\alpha'}} g_0 \lambda^{66'} \left( i (\tau^1 \tau^1_+ \sigma^2_+ S^2_+ \sigma^3_+ t^3_+ + \sigma^1_+ S^1_+ \tau^2_+ \sigma^3_+ t^3_+ 
+ \sigma^1_+ S^1_+ \sigma^2_+ S^2_+ \tau^3_+ S^3_+ ) + \prod_{k=1}^2 (\sigma^k_+ S^k_+) \sigma^3_+ t^3_+ \alpha' (p \cdot \psi) \right) e^{ipX}. \tag{6.44}
\]

The first three terms do not contribute to the amplitude, since they result in correlators of the form

\[
< \sigma_+ (z) \tau_+ (z') > < \sigma_+ (z) \tau'_+ (z') >
\]

both of which vanish by conformal invariance. Effectively, the scalar vertex then becomes

\[
\nu^{(3)}_0 \rightarrow \sqrt{\frac{2}{\alpha'}} g_0 \lambda^{66'} \prod_{k=1}^2 (\sigma^k_+ S^k_+) \sigma^3_+ t^3_+ \alpha' (p \cdot \psi) e^{ipX}. \tag{6.46}
\]

This construction easily generalizes to arbitrary twists with the result:

\[
\nu^{(3)}_0 = g_0 e^{-\phi} \lambda^{66'} \prod_{k=1}^2 (\sigma^k_+ S^k_+) \sigma^3_+ t^3_+ \alpha' (p \cdot \psi) e^{ipX}. \tag{6.47}
\]

\[
\nu^{(3)}_{-1} (z) = g_0 e^{-\phi} \lambda^{66'} \prod_{k=1}^2 (\sigma^k_+ S^k_+) \sigma^3_+ t^3_+ \alpha' (p \cdot \psi) e^{ipX}. \tag{6.48}
\]

The vertex for the CPT conjugate state is obtained as usual by flipping the signs of all twists and replacing the Chan-Paton matrix \( \lambda^{66'} \rightarrow \lambda^{66} \). Also, the normalization constant \( g_0 \) can be deduced by the same method as for the chiral fermions with the result \( g_0 = \sqrt{2\alpha' g_{YM} V_c^2} \). The calculation then proceeds in exactly the same way as for the fermions. The final result is

\[
\mathcal{A}(\lambda_L \lambda_R \rightarrow \phi^3 \phi^3) = -i \left( \frac{2\pi^2 \alpha'^2 g_{YM} g_{YM}' V_c^2 V_c'^2}{NV_c + N'V_c'} \right) \times (2\pi)^4 \delta^4 \left( \sum_k p_k \right) \text{Tr}(tt') K_S(1, 2, 3, 4) \tag{6.49}
\]

where the kinematic factor \( K_S(1, 2, 3, 4) \) is given by

\[
K_S(1, 2, 3, 4) = \frac{2}{\alpha' \pi^2} u^T L C g_4 u R_2 f_{\epsilon_3} (s, t, u). \tag{6.50}
\]

Using the expansions for the function \( f_\epsilon (s, t, u) \) given in (6.31) and (6.32), we obtain the first non-vanishing term in the expansion of \( K_S \):
I. $\epsilon_3 = 1$ and the kinematic factor becomes

$$K_S(1, 2, 3, 4) = u_L^T C \gamma_4 u_{2R} (u - t) \quad (\Phi \neq 0) \quad (6.51)$$

II. Using (6.19) and the fact that the scalar is massless (i.e. $\phi_3 = \phi_1 + \phi_2$), it is easy to see that $\Phi = 0$ requires $\epsilon_3 = +1$ while $\Phi \neq 0$ requires $\epsilon_3 = -1$. We then obtain:

$$K_S(1, 2, 3, 4) = u_L^T C \gamma_4 u_{2R} \begin{cases} u & \text{if } \Phi = 0 \\ u - t & \text{if } \Phi \neq 0 \end{cases} \quad (6.52)$$

The same result applies for any other of the scalars in (6.36), as long as the angles are chosen to make this particular scalar massless.

6.3 Interactions with strings located on single D-brane stacks

In Section 5, we computed interaction amplitudes involving gauginos, gauge bosons and adjoint fermions for the case of a single stack of $N$ D-branes. The Goldstino was identified as the gaugino of the $U(1)$ supermultiplet. These results can be immediately translated into corresponding results for the case of two intersecting stacks of D-branes. Indeed, in (6.21) we identified the Goldstino as a linear combination of the $U(1)$ gauginos of the two stacks.

Consider for example interactions of the Goldstino with gauge bosons of the D6 stack. The amplitude $A(\lambda_L \lambda_R \rightarrow BB)$ has four contributions just like the four-fermion amplitude (6.22) computed in the previous subsection. However, due to the vanishing of the corresponding traces of Chan-Paton matrices, only one of these contributions, $A^{66}$, survives. As expected, the $U(N)$ gauge bosons interact only with the $U(1)$ gauginos of the D6 stack.

The first non-vanishing term in an expansion of the amplitude in powers of $\alpha'$ is then:

$$A^{(2)}(\lambda_L \lambda_R \rightarrow BB) = \frac{NV_e}{NV_e + N'V'_e} A^{66} \quad (6.53)$$

where $A^{66}$ is given by (5.13) (remember that in (5.13) the Goldstino is just the $U(1)$ gaugino). Using the relation (6.15) we obtain finally:

$$A^{(2)}(\lambda_L \lambda_R \rightarrow BB) = -i \left( \frac{2\pi^2 \alpha' g_{YM} g'_{YM} V_e^{1/2} V'_e^{1/2}}{NV_e + N'V'_e} \right) \times (2\pi)^4 \delta^{(4)} \left( \sum_i k_i \right) \text{Tr}(\lambda^3 \lambda^4) K_{GB}(1, 2, 3, 4) \quad (6.54)$$
Here the Chan-Paton matrices $\lambda^3$ and $\lambda^4$ are in the Lie-Algebra of $U(N)$ and the kinematic factor $K_{\text{GB}}(1, 2, 3, 4)$ is given by (5.10). The result is unchanged if we replace $B$ by $B'$, the gauge boson of the $D6'$ stack, except of course that the Chan-Paton matrices then lie in the Lie-Algebra of $U(N')$.

In the same way we can obtain interaction amplitudes of the Goldstino with adjoint scalars and fermions of $U(N)$. The Goldstino again “appears” as a gaugino with a non-canonical normalization and the amplitudes $A^{(2)}(\lambda_L \lambda_R \rightarrow \phi^{(i)} \bar{\phi}^{(j)})$, $A^{(2)}(\lambda_L \lambda_R \rightarrow f_L \bar{f}_R)$ and $A^{(2)}(\lambda_L \lambda_L \rightarrow f_L f_L)$ are obtained by multiplying respectively (5.17), (5.18) and (5.20) with the factor $\frac{N V_c}{N V_c + N' V'_c}$:

$$A^{(2)}(\lambda_L \lambda_R \rightarrow \phi^{(i)} \bar{\phi}^{(j)}) = -i \left( \frac{2 \pi^2 \alpha'^2 g_{\text{YM}} g_{YM}' V_c^2 V_c'^2}{N V_c + N' V'_c} \right) \times (2 \pi)^4 \delta^{(4)} \left( \sum_i k_i \right) \text{Tr}(\lambda^3 \lambda^4) \delta^{ij} K_S(1, 2, 3, 4)$$ (6.55)

$$A^{(2)}(\lambda_L \lambda_R \rightarrow f_L \bar{f}_R) = -2i \left( \frac{2 \pi^2 \alpha'^2 g_{\text{YM}} g_{YM}' V_c^2 V_c'^2}{N V_c + N' V'_c} \right) \times (2 \pi)^4 \delta^{(4)} \left( \sum_i k_i \right) \text{Tr}(\lambda^3 \lambda^4) K_F(1, 2, 3, 4)$$ (6.56)

$$A^{(2)}(\lambda_L \lambda_L \rightarrow f_L f_L) = -2i \left( \frac{2 \pi^2 \alpha'^2 g_{\text{YM}} g_{YM}' V_c^2 V_c'^2}{N V_c + N' V'_c} \right) \times (2 \pi)^4 \delta^{(4)} \left( \sum_i k_i \right) \text{Tr}(\lambda^3 \lambda^4) K_F'(1, 2, 3, 4)$$ (6.57)

The kinematic factors $K_S(1, 2, 3, 4)$, $K_F(1, 2, 3, 4)$ and $K_F'(1, 2, 3, 4)$ are given respectively by (5.16), (5.19) and (5.21). Again these results are valid for adjoint scalars and fermions of both the $D6$ and $D6'$ stacks, provided that the Chan-Paton factors lie, respectively, in the Lie-Algebra of $U(N)$ and $U(N')$. Notice also that in the limit $\phi_k \rightarrow 0$ where the two stacks become coincident, we recover the results of section 5.

7 The low-energy effective action

We are now ready to compare the string computations with the effective low-energy quantum field theory. We first concentrate on the twisted sector that corresponds to strings localized on brane intersections and then study the untwisted sector corresponding to strings ending on a single D-brane stack.
7.1 States on D-brane intersections

As discussed before, on the intersection there is a massless left-handed fermion transforming in the \((N, \bar{N}')\) representation of \(U(N) \times U(N')\) (the 66' string) and its CPT conjugate right-handed (anti-)fermion transforming in \((\bar{N}, N')\) (the 6'6 string). The vertex operators for these states are given in (6.7).

Let us define \(f_{iJ}\) the quantum field which absorbs the left-handed fermion and creates the right-handed anti-fermion. This is a left-handed Weyl spinor transforming in \((N, \bar{N}')\):

\[i\] is an index in the fundamental of \(U(N)\) and \(J\) an index in the anti-fundamental of \(U(N')\).

There is also a potentially massless scalar transforming in \((N, \bar{N}')\) and its CPT (complex) conjugate. Accordingly, let us define \(\phi_{iJ}\) the quantum field which annihilates this scalar and creates its CPT conjugate. Of course, this scalar will appear in the effective theory only if the choice of intersection angles makes it massless, i.e. if the D-brane configuration is supersymmetric.

As mentioned before, we leave the coefficients of \(S_6\) and \(S_8\) (see appendix A) undetermined. The latter does not contribute to four-point functions and therefore will not affect the following discussion. On the other hand, \(S_6\) can not be discarded so easily. Indeed, the gauge bosons \(A_\mu\) and \(A'_\mu\) of \(U(1)\) and \(U(1)'\), respectively, couple to fermions on the intersection via the usual Yang-Mills (YM) coupling:

\[
L_I = \frac{g_{YM}}{\sqrt{N}} f_{iJ} \sigma^\mu \bar{f}_{iJ} A_\mu - \frac{g'_{YM}}{\sqrt{N'}} f_{iJ} \sigma^\mu \bar{f}_{iJ} A'_\mu .
\] (7.1)

Combining such a YM vertex with \(S_6\) yields a contact term which has just the form of \(S_3\). A similar problem occurs for the scalars on the intersection: \(S_6\) together with the YM coupling results in a contact term indistinguishable from \(S_5\). It might therefore seem that without determining \(C_6\) we cannot obtain a definite prediction for the coefficients \(C_3\) and \(C_5\) in string theory. Fortunately, a more detailed analysis shows that these reducible contributions actually vanish. This is due to the fact that strings located on the intersection couple to the gauge bosons of \(U(1)\) and \(U(1)\)' with opposite charges, as can be seen from the relative minus sign in the two terms of (7.1). As a result, the previously mentioned contact terms receive in fact two contributions which are equal and opposite and they cancel out.

This cancellation is easiest to see in the “string frame” with Chan-Paton matrices normalized to \(N (N')\) for the \(D6 (D6')\) stack. The tree-level low-energy effective Lagrangian for the \(U(1)\) and \(U(1)'\) vector multiplets is then:

\[
L = \frac{NV_c M^3}{g_s} L^{D6} + \frac{N'V'_c M^3}{g_s} L^{D6'}
\] (7.2)
with
\[ \mathcal{L}^{D6} = -\frac{i}{2} g_{\mu}^{\tau} \sigma_{\mu} \bar{\gamma} - \frac{1}{4} F^2 + \frac{C}{M_s^4} \partial^\alpha g_{\mu}^{\tau} \partial^\mu \bar{\gamma} \partial_\alpha F_{\mu\nu} + \cdots \] (7.3)
\[ \mathcal{L}^{D6'} = -\frac{i}{2} g'_{\mu}^{\tau} \sigma_{\mu} \bar{\gamma}' - \frac{1}{4} F'^2 + \frac{C}{M_s^4} \partial^\alpha g'_{\mu}^{\tau} \partial^\mu \bar{\gamma}' \partial_\alpha F'_{\mu\nu} + \cdots \] (7.4)

Here \( g \) and \( g' \) are the gauginos of \( U(1) \) and \( U(1)' \), respectively, and the dots indicate further terms that are not relevant for our discussion. Notice that \( \mathcal{L}^{D6} \) and \( \mathcal{L}^{D6'} \) have identical forms: all “stack-dependant” normalizations have been absorbed in the fields and appear as overall factors of the two terms in (7.2); \( g_s^{-1} \) is the contribution of the disc to the genus expansion, while \( V_c \) and \( V'_c \) result from the compactification of the internal dimensions of the two stacks. Also, all string amplitudes are weighted by traces of products of Chan-Paton matrices. For strings belonging to the \( U(1) \) (\( U(1)' \)) supermultiplet and with our present normalization convention, these Chan-Paton factors are just unit matrices and their trace contributes an overall factor \( N (N') \).

We can now re-express (7.2) in terms of canonically normalized fields by rescaling:
\[ A_\mu \rightarrow \left( \frac{g_s}{NV_c M_s^4} \right)^{\frac{1}{2}} A_\mu \quad g \rightarrow \left( \frac{g_s}{NV_c M_s^4} \right)^{\frac{1}{2}} g \] (7.5)
and similarly for \( A'_\mu \) and \( g' \). Also, we have to use (6.21) to express the Goldstino as a linear combination of \( g \) and \( g' \). The Lagrangian then becomes
\[ \mathcal{L} = -\frac{i}{2} \lambda_{\mu}^{\tau} \sigma_{\mu} \bar{\lambda} - \frac{1}{4} F^2 - \frac{1}{4} F'^2 \\
+ \frac{1}{M_s^4} \frac{g_s}{NV_c + NV'_c} \mathcal{G}_Y \sqrt{\mathcal{G}} (\partial^\alpha \lambda_{\mu}^{\tau} \partial^\mu \bar{\lambda}) \partial_\alpha F_{\mu\nu} \\
+ \frac{1}{M_s^4} \frac{g_s}{NV_c + NV'_c} \mathcal{G}_Y' \sqrt{\mathcal{G}} (\partial^\alpha \lambda'_{\mu}^{\tau} \partial^\mu \bar{\lambda}') \partial_\alpha F'_{\mu\nu} + \cdots \] (7.6)
where we have used that \( g_s^2 M_s^4 = g_{Y_c}^2 V_c = g_{Y'_c}^2 V'_c \). The last two terms will contribute to the action two copies of \( S_6 \), one for each abelian gauge boson. Combining these with (7.1) to build four-fermion interactions of the same type as those generated by \( S_3 \), we obtain two diagrams which contribute with opposite sign and cancel out. The same reasoning applies if we replace the chiral fermions by their scalar superpartners on the intersection (if they are present). We conclude that \( S_6 \) will in fact never affect the determination of \( C_3 \) and \( C_5 \) and thus for our purposes its coefficient \( C_6 \) is irrelevant.

In the twisted case, we can exclude the term \( S_1 \) from the effective action since there are is no adjoint matter on the intersection. Gauge invariance
also excludes the term $S_4$. Finally, as explained in Section 6.2, conservation of internal helicity forbids the interaction $S_7$. The full effective action has then the form \[ S = S_0 + S_2 + S_3 + S_5, \] (7.7)
where $S_0$ is the model-independent coupling
\[ S_0 = \int d^4x (i\kappa^2 \lambda \partial^\mu \sigma^\nu \bar{\lambda}) T_{\mu\nu}. \] (7.8)
In the present context, the relevant part of the energy-momentum tensor is
\[ T_{\mu\nu} \equiv T_{\mu\nu}^F + T_{\mu\nu}^S, \]
\[ = -\frac{i}{2} (f_{ij} \sigma_\mu \bar{D}_\nu f_{ij}) + (D_\mu \phi)_{ij}^\dagger D_\nu \phi_{ij} + (D_\nu \phi)_{ij}^\dagger D_\mu \phi_{ij} \] (7.9)
where it is understood that the scalar part $T_{\mu\nu}^S$ must be included only if the scalar is massless.

The general form of the couplings $S_i$ is given in appendix A. Using that the fields transform in bifundamentals, we obtain:
\[ S_2 = C_2 \kappa \int d^4x (f_{ij} \partial_\alpha \lambda) D^\alpha \phi_{ij}^\dagger + h.c. \] (7.10)
\[ S_3 = C_3 2\kappa^2 \int d^4x (f_{ij} \partial_\mu \bar{\lambda}) (f_{ij} \partial_\mu \lambda) \] (7.11)
\[ S_5 = C_5 \kappa^2 \int d^4x (\phi_{ij}^\dagger D_\mu \phi_{ij} - (D_\mu \phi_{ij})^\dagger \phi_{ij}) i\partial_\alpha \lambda \sigma_\mu \partial^\alpha \bar{\lambda} \] (7.12)

Here $C_3$ and $C_5$ are real while $C_2$ is in general complex. Again $S_2$ and $S_5$ are included only if there is a massless scalar in the spectrum. However, as we show below, the interaction $S_2$ may be absent even when a massless scalar is present.

A necessary condition for $S_2$ to be present in the effective action is that the vertex operator of the massless scalar appears in the fusion product of the vertex operators of the chiral fermion and the Goldstino. We therefore start by considering OPEs of the vertex operators involved in (7.10). The OPE of the 66' string fermion vertex, $\nu_{66'}^{-12}$, with that of the gaugino (5.5) gives:
\[ \nu_{-\frac{1}{2}}^G(x_1) \nu_{-\frac{1}{2}}^{66'}(x_2) = \frac{x_{12}^{2\alpha'}}{2} \nu_{-\frac{1}{2}}^{S}(x_2) + \ldots \] (7.13)
\[ \text{Since } S_6 \text{ and } S_8 \text{ are irrelevant for the following discussion, we have not included them in (7.7).} \]
where
\[ \nu_{-1}^S = e^{-\phi} \prod_{k=1}^{3} \sigma_k^k e^{i(\phi_k + \frac{\phi}{2} - \frac{\pi}{2}) H_k e^{i(p_1 + p_2)X}}. \] (7.14)

The string state created by the vertex \( \nu_{-1}^S \) is a spacetime scalar located on the intersection of the D-branes. Its mass can be obtained directly by counting conformal weights:
\[ \alpha' m^2 = \frac{\epsilon + 1}{2} - \frac{\Phi}{2\pi}, \] (7.15)

where \( \Phi \) is given by (6.19). This is the lightest state that can appear by "fusing" a left-handed Goldstino with a massless fermion excitation of a 66' string. The dots on the right-hand side of (7.13) denote scalars of higher mass. In a simplified notation, (7.13) can be written as:
\[ \nu_{-1}^G \times \nu_{-1}^{66'} = \nu_{-1}^S + \ldots \] (7.16)

In the same way, one can compute the fusion of a right-handed Goldstino and a massless fermion excitation of 66' string. Now the emerging states are spacetime vectors and we shall only need the lightest one:
\[ \nu_{-1}^G \times \nu_{-1}^{66'} = \nu_{-1}^V + \ldots \] (7.17)

where
\[ \nu_{-1}^V = e^{-\phi}(\zeta \psi) \prod_{k=1}^{3} \sigma_k^k e^{i(\phi_k + \frac{\phi}{2} + \frac{\pi}{2}) H_k e^{i(p_1 + p_2)X}}. \] (7.18)

Here, \( \zeta \) is a polarization vector which is given in terms of the spinors appearing in \( \nu_{-1}^G \) and \( \nu_{-1}^{66'} \) by \( \zeta^\mu = u^T \gamma^\mu u_L \). The string state corresponding to this vertex has a mass
\[ \alpha' m^2 = \frac{1 - \epsilon}{2} + \frac{\Phi}{2\pi}. \] (7.19)

Since the masses of the lightest resulting states (7.15) and (7.19) depend on the signs of the rotations, we must consider separately the cases \( \epsilon = 1 \) (case I) and \( \epsilon = -1 \) (case II):

I. The vertex operator \( \nu_{-1}^S \) corresponds to the state
\[ | \bar{\psi}_{-1/2 + \phi_1} \bar{\psi}_{-1/2 + \phi_2} \bar{\psi}_{-1/2 + \phi_3} |0; p > \] (7.20)
which is massive for all angles \( 0 \leq \phi_k \leq \frac{\pi}{2} \), as can be seen from (7.15).

We conclude that \( S_2 \) does not appear in the effective action and therefore \( C_2 = 0 \). To determine \( C_3, C_5 \) and \( \kappa^2 \) we should evaluate the amplitudes \( \mathcal{A}(\lambda_L \lambda_R \rightarrow f_L \bar{f}_R) \) and \( \mathcal{A}(\lambda_L \lambda_R \rightarrow \phi \bar{\phi}) \) in the effective quantum
field theory and compare them with the corresponding amplitudes in string theory. Starting from the effective action (7.7) with $C_2$ set to zero, a short and straightforward computation gives the following QFT amplitudes:

$$A^{\text{QFT}}(\lambda_L \lambda_R \rightarrow f_L \bar{f}_R) = -2i\kappa^2(2\pi)^4\delta^4(\sum p_i)\delta_{IJ}\delta_{ij} \frac{2tu + C_3su}{2s}$$

$$A^{\text{QFT}}(\lambda_L \lambda_R \rightarrow \phi \bar{\phi}) = -i\kappa^2(2\pi)^4\delta^4(\sum p_i)\delta_{IJ}\delta_{ij} \times (u_1L \alpha_4u_{2R})(u - t + C_5s)$$

These can be compared with the corresponding results of the string calculations in the previous section, (6.28), (6.33) and (6.49), (6.51). We thus obtain:

$$\kappa^2 = \frac{2\pi^2\alpha'^2g_{YM}g'_{YM}V_c^2V'_c}{NV_c + NV'_c} \quad C_3 = 1 \quad C_5 = 0$$

The supersymmetry breaking scale is proportional to the square of the string length and depends on the intersection angles only through the four-dimensional Yang-Mills couplings. More precisely, it is given by the effective tensions $T_3$ and $T'_3$ of the 3-brane stacks obtained upon compactification of the internal directions along the D6-branes [9]:

$$\frac{1}{2\kappa^2} = NT_3 + N'T'_3 \quad T_3 = \frac{1}{4\pi^2\alpha'^2g_{YM}^2} \quad T'_3 = \frac{1}{4\pi^2\alpha'^2g'_{YM}^2}$$

As a result, in case I, the leading effective interactions of the Goldstino with massless fields on brane intersections are given simply by the coupling to the energy-momentum tensor and the four-fermion interaction (7.11) with coefficient $C_3 = 1$. Note that the interaction $S_2$ is absent even in the presence of massless scalars (6.37) at the intersection, when some combination of angles $\phi_i + \phi_j - \phi_k = 0$ for $i \neq j \neq k \neq i$.

II. From the fusion rule (7.16) we now obtain:

$$\nu^{-G}_{-1} \times \nu^{G}_{-1} = \nu_{-1} + \ldots$$

where $\nu_{-1}$ corresponds to one of the scalars in (6.36). In fact, it is the scalar which becomes massless precisely when $\Phi = 0$. For example, if $\vec{e} = (-,-, +)$ then $\nu_{-1} \equiv \nu^{(3)}_{-1}$ (see (6.47)) and the scalar is just

\[\text{integral constant} \]
\( j = 3 > \text{ in (6.36)} \). This is the reason for the apparent discontinuity in the kinematic factors (6.34) and (6.52) when \( \Phi \to 0 \). Indeed, as long as \( \Phi \neq 0 \), the scalar appearing on the right-hand side of (7.25) is massive and just as in case I there is no operator \( S_2 \) in the effective action. We then have again

\[
\frac{1}{2\kappa^2} = NT_3 + N'T_{3}' \quad C_3 = 1 \quad C_5 = 0 \quad (7.26)
\]

By continuity we expect that (7.26) also holds when \( \Phi \to 0 \). But in this limit the scalar becomes massless and \( S_2 \) will contribute to the QFT amplitudes \( A(\lambda_L\lambda_R \to f_Lf_R) \) and \( A(\lambda_L\lambda_R \to \phi\bar{\phi}) \). This extra contribution is responsible for the discontinuities. The QFT amplitudes then become:

\[
A^{QFT}(\lambda_L\lambda_R \to f_Lf_R) = -2i\kappa^2(2\pi)^4\delta^4(\sum p_i)\delta_{IJ}\delta_{ij}
\]

\[
\frac{1}{2s} \left( tu(2 - \frac{|C_2|^2}{4}) + su(C_3 - \frac{|C_2|^2}{4}) \right) \quad (7.27)
\]

\[
A^{QFT}(\lambda_L\lambda_R \to \phi\bar{\phi}) = -i\kappa^2(2\pi)^4\delta^4(\sum p_i)\delta_{IJ}\delta_{ij}
\]

\[
(u_1LC\bar{p}_4u_2) (u - t + C_5s + \frac{|C_2|^2}{4}t) \quad (7.28)
\]

Comparing these expressions with the results of the string calculations (6.28), (6.34) and (6.49), (6.52) in the case \( \Phi = 0 \) we obtain

\[
\frac{1}{2\kappa^2} = NT_3 + N'T_{3}' \quad |C_2|^2 = 4 \quad C_3 = 1 \quad C_5 = 0 \quad (7.29)
\]

Thus, in case II, the effective action involving fields on brane intersections is the same as in case I when \( \Phi \neq 0 \). However, when \( \Phi = 0 \), there is a massless scalar in the spectrum which couples to the Goldstino via \( S_0 \) and the additional 3-point interaction \( S_2 \). Note that in this case the intersection preserves locally \( \mathcal{N} = 1 \) supersymmetry, which pairs the massless scalar with the fermion in a chiral supermultiplet.

### 7.2 States on single D-brane stacks

We now consider untwisted fields corresponding to excitations of open strings with both ends on the same stack of D-branes. The spectrum consists of a \( U(N) \) gauge boson, four helicity \( \frac{1}{2} \) adjoint fermions and their CPT conjugates and three complex adjoint scalars.
The fermions are labeled by their internal helicities. In our conventions, the left-handed (right-handed) fermions have negative (positive) internal chirality. The vertex operators for two CPT conjugate fermions are

\[
\nu_{-\frac{1}{2}}^{(j)} = e^{-\frac{\phi}{2}} \lambda^a u_{L\alpha} \Theta^\alpha \hat{e} e^{-\frac{i}{2} \epsilon_1 H_1 + \epsilon_2 H_2 + \epsilon_3 H_3} e^{ikX}
\]

\[
\bar{\nu}_{-\frac{1}{2}}^{(j)} = e^{-\frac{\phi}{2}} \lambda^a u_{R\alpha} \Theta^\alpha \hat{e} e^{\frac{i}{2} \epsilon_1 H_1 + \epsilon_2 H_2 + \epsilon_3 H_3} e^{ikX}
\]

(7.30)

Here, the parameters \(\epsilon_k = \pm 1\) must be chosen such that \(\epsilon_1 \epsilon_2 \epsilon_3 = 1\). The handedness of the spinors is then imposed by the GSO projection. The index \(j\) labels the internal helicities in the following way:

\[
j = \begin{cases} 
0 & \text{if } \vec{\epsilon} = (+, +, +) \\
1 & \text{if } \vec{\epsilon} = (+, -, -) \\
2 & \text{if } \vec{\epsilon} = (-, +, -) \\
3 & \text{if } \vec{\epsilon} = (-, -, +) 
\end{cases}
\]

(7.31)

We define \(f^a_j\) the (left-handed) quantum field which annihilates the left-handed fermion corresponding to \(\nu_{-\frac{1}{2}}^{(j)}\) and creates its CPT conjugate antifermion \(\bar{\nu}_{-\frac{1}{2}}^{(j)}\). The gauge index \(a\) is in the adjoint representation.

In the same way, we label the massless scalars. The vertex operators are

\[
\nu_{-1}^{(j)} = e^{-\phi} \lambda^a e^{-i H_j} e^{ikX}
\]

\[
\bar{\nu}_{-1}^{(j)} = e^{-\phi} \lambda^a e^{i H_j} e^{ikX}
\]

(7.32)

where \(j = 1, 2, 3\). We define \(\phi^a_j\) the complex scalar field which annihilates \(\nu_{-1}^{(j)}\) and creates \(\bar{\nu}_{-1}^{(j)}\).

The leading effective action takes the form

\[
S = S_0 + S_1 + S_2 + S_3 + S_4 + S_5
\]

(7.33)

and the energy-momentum tensor is given by

\[
T_{\mu\nu} = T_{\mu\nu}^{GB} + T_{\mu\nu}^{F} + T_{\mu\nu}^{S}
\]

(7.34)

where

\[
T_{\mu\nu}^{GB} = -\text{Tr}(F_{\nu\sigma} F^\sigma_{\mu} + \frac{\eta_{\mu\nu}}{4} F_{\alpha\beta} F^{\alpha\beta})
\]

(7.35)

\[
T_{\mu\nu}^{F} = -\frac{i}{2} \text{Tr}(f_j \sigma_{\mu} D_{\nu} \bar{f}_j)
\]

(7.36)

\[
T_{\mu\nu}^{S} = \text{Tr}((D_\mu \phi_j)^\dagger D_\nu \phi_j + (D_\nu \phi_j)^\dagger D_\mu \phi_j)
\]

(7.37)
Besides $S_0$ involving $T_{\mu\nu}$, the additional interactions are obtained from Appendix A in the special case where the fields transform in the adjoint of $U(N)$:

\[
S_1 = \kappa \sum_{j=0}^{3} C_1^j \int d^4 x \ i F^{a}_{\mu\nu} f_j^a \sigma^\mu \partial^\nu \bar{\lambda} + h.c. \quad (7.38)
\]

\[
S_2 = \kappa \sum_{j=1}^{3} C_2^j \int d^4 x (f_j^a \partial_\alpha \lambda) \bar{\phi}_j^{a\dagger} + h.c. \quad (7.39)
\]

\[
S_3 = \sum_{j=0}^{3} C_3^j 2\kappa^2 \int d^4 x (\bar{f}_j^a \partial^\mu \bar{\lambda}) (f_j^a \partial_\mu \lambda) \quad (7.40)
\]

\[
S_4 = \sum_{j=0}^{3} C_4^j \kappa^2 \int d^4 x (\bar{f}_j^a \bar{f}_j^a) (\partial_\mu \lambda \partial^\mu \lambda) + h.c. \quad (7.41)
\]

\[
S_5 = \kappa^2 \sum_{j=1}^{3} C_5^j \int d^4 x (\phi_j^{a\dagger} D_\mu \phi_j^a - (D_\mu \phi_j^a)^\dagger \phi_j^{a\dagger}) i \partial_\alpha \lambda \sigma^\mu \partial^\alpha \bar{\lambda} \quad (7.42)
\]

The interaction $S_4$ can only appear if the fermions have internal helicities opposite to the Goldstinos, as we have anticipated in (7.41).

All untwisted vertex operators can be obtained from the corresponding twisted expressions by taking the limit $\phi_k \to 0$. Starting with the vertex operators $\nu_{-\frac{1}{2}}^{66'}$ and $\bar{\nu}_{-\frac{1}{2}}^{66}$ and taking the limit $\phi_k \to 0$ (using that the bosonic twist $\sigma^k$ at vanishing angle is the identity operator), we obtain, respectively, the fermion vertices $\nu_{-\frac{1}{2}}^{(j)}$ and $\bar{\nu}_{-\frac{1}{2}}^{(j)}$.

More precisely, if we start from case I we obtain $j = 0$, i.e. fermions with internal helicities $(+, +, +)$ and $(-, -, -)$. In this limit, the fusion rule (7.16) and the mass formula (7.15) imply that the intermediate scalars are massive and therefore the interaction $S_2$ is absent. On the other hand, (7.17) and (7.19) show that a left-handed fermion can combine with a right-handed Goldstino to yield a gauge boson. This implies that a new interaction $S_1$ may be present and should be taken into account.

In case II, starting from the vertex $\nu_{\frac{1}{2}}^{66'}$ and taking the limit $\phi_k \to 0$ we obtain $\nu_{\frac{1}{2}}^{(j)}$ with $j = 1, 2, 3$. According to the fusion rules, these fermions can couple with Goldstinos to yield massless scalars but not gauge bosons. Moreover, the fermion field $f_j^a$ can only couple to the scalar $\phi_j^{a\dagger}$, as we have anticipated in (7.39).

We conclude that

\[
C_1^j = 0 \quad \text{for} \quad j=1,2,3 \quad \text{and} \quad C_2^0 = 0 \quad . \quad (7.43)
\]
The remaining coefficients can be obtained by the same amplitudes studied in the previous subsection. Using the action (7.33), a straightforward calculation yields:

\[ \mathcal{A}^{QFT}(\lambda_L \lambda_R \rightarrow f^i_L \bar{f}^j_R) = -2i\kappa^2(2\pi)^4\delta^4(\sum_i k_i)\delta^{ab} \]
\[ \times \frac{1}{2s} \left( tu(2 - \frac{|C_1^j|^2}{2} - \frac{|C_2^j|^2}{4}) + su(C_3^j - \frac{|C_3^j|^2}{4}) \right) \]

(7.44)

\[ \mathcal{A}^{QFT}(\lambda_L \lambda_R \rightarrow \phi^j \bar{\phi}^j) = -i\kappa^2(2\pi)^4\delta^4(\sum_i k_i)\delta^{ab} \]
\[ \times (u_1^L C_{j4} u_2^R)(u - t + C_3^j s + \frac{|C_2^j|^2}{4} t) \]

(7.45)

Since the above amplitudes involve Goldstinos of opposite helicity, the interaction \( S_4 \) does not contribute. Note also that in (7.44) \( j = 0, \ldots, 3 \) while in (7.45) \( j = 1, 2, 3 \). Comparing these expressions with the string results (6.55) and (6.56) and using (7.24) for the SUSY-breaking scale, we obtain

\[ |C_0^1|^2 = 2 \quad |C_2^{1,2,3}|^2 = 4 \quad C_3^{0,1,2,3} = 1 \quad C_4^j = 0 \]

(7.46)

As an additional check we can use the effective action to compute the QFT amplitude involving two Goldstinos and two gauge fields, \( \mathcal{A}(\lambda_L \lambda_R \rightarrow BB) \), and show that it agrees with the string result (5.10) and (6.54). Indeed, we obtain

\[ \mathcal{A}^{QFT}(\lambda_L \lambda_R \rightarrow BB) = -i\kappa^2(2\pi)^4\delta^4(\sum_i k_i)\delta^{ab} K^{QFT}_{GB} \]

(7.47)

with

\[ K^{QFT}_{GB} = K_{GB}(1, 2, 3, 4) + (1 - \sum_{j=0}^{3} \frac{|C_1^j|^2}{2}) K'(1, 2, 3, 4) \]

(7.48)

where \( K_{GB}(1, 2, 3, 4) \) is the kinematic factor (5.10) and \( K'(1, 2, 3, 4) \) is given by

\[ K'(1, 2, 3, 4) = u(\epsilon_2\epsilon_4) u_1^T L C_{j4} u_2^R - s(k_1\epsilon_3) u_1^T L C_{j4} u_2^R + \frac{s}{2} u_1^T L C_{j4} \epsilon_4 C_3^j u_2^R \]
\[ + 2u_1^T L C_{j4} u_2^R((k_3\epsilon_4)(k_2\epsilon_3) - (k_4\epsilon_3)(k_2\epsilon_4)) \]

(7.49)

Using that \( |C_0^1|^2 = 2 \) and \( C_1^{1,2,3} = 0 \) one finds perfect agreement with the string computation.
To conclude this section, we still have to determine $C^j_i$. This can be done by computing the amplitude $\mathcal{A}(\lambda_L \lambda_L \rightarrow f_L f_L)$, which is non-vanishing only if $j = 0$ in (7.31). We conclude:

$$C^1_{4,2,3} = 0.$$  \hfill (7.50)

A simple computation in QFT using the effective action (7.33) (actually only $S_1$ and $S_4$ contribute) then yields the amplitude:

$$\mathcal{A}^{QFT}(\lambda_L \lambda_L \rightarrow f^0_L f^0_L) = -2i\kappa^2 (2\pi)^4 \delta^4(\sum_i k_i) \delta^{ab} \frac{s}{2} \left(\frac{(C^0_1)^2}{2} + 2C^0_4\right),$$  \hfill (7.51)

where $C^0_1$ is the complex conjugate of $C^0_1$. This result is not enough to determine $C^0_4$ because $C^0_1$ has in principle a phase ambiguity. However, if we assume that $C^0_1$ is real, then we obtain simply that $C^0_4 = 0$ as can be seen by comparing with (6.57).

There are two reasons which motivate the assumption that $C^0_1$ (and $C^j_2$) are real. First, it makes the interactions $S_1$ and $S_2$ invariant under spacetime parity transformations. Second, it allows to write all order $\kappa$ interactions linear in the Goldstino as a coupling to a supercurrent $J^\mu$ of the form

$$S = k \int d^4x \kappa(\partial^\mu \lambda J^\mu + \partial^\mu \bar{\lambda} \bar{J}^\mu),$$  \hfill (7.52)

where $k$ is a real constant.

In order to see this property, we consider first a supersymmetric D-brane configuration. The $D6$ stack preserves the following supercharges:

$$Q = Q + \beta_\perp \bar{Q},$$  \hfill (7.53)

where $Q$ and $\bar{Q}$ are the bulk supercharges of the type II theory and $\beta_\perp = (\Gamma^5 \Gamma)(\Gamma^7 \Gamma)(\Gamma^9 \Gamma)$ implements a reflection in the directions transverse to the D-branes which results from successive T-duality transformations. Here $\Gamma^M$ are the ten-dimensional gamma-matrices and $\Gamma \equiv \Gamma^{11}$ is the chirality operator. Similarly, the $D6'$ stack of D-branes preserves the supercharges

$$Q' = Q + \beta'^\perp \bar{Q},$$  \hfill (7.54)

where $\beta'^\perp$ corresponds now to a reflection in the directions transverse to the $D6'$ stack. Consider the supercharges

$$(Q_{\alpha,--}, Q^\alpha_{--,++})$$  \hfill (7.55)

and

$$(Q'_{\alpha,--}, Q'^\alpha_{--,++}),$$  \hfill (7.56)
where $\alpha, \dot{\alpha}$ are indices in the $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}\right)$ representations of the four-dimensional Lorentz group, respectively. These supercharges form Majorana spinors in four dimensions. The internal helicities are chosen to coincide with those of the Goldstino and its CPT conjugate.

Generically, the supercharges (7.55) and (7.56) are distinct, in which case neither is preserved by the D-brane intersection. However, if we choose the intersection angles such that
\[
(\beta \perp \bar{Q})_{\alpha,---} = (\beta' \perp \bar{Q})_{\alpha,---} \quad \text{and} \quad (\beta \perp \bar{Q})_{\dot{\alpha},+++} = (\beta' \perp \bar{Q})_{\dot{\alpha},+++}
\]
(7.57)
then the two supercharges coincide and the full spectrum is supersymmetric. This happens precisely in case II (where not all angles have the same sign) when the angles satisfy $\Phi = 0$. The associated conserved supercurrent is given by
\[
J^\mu = J^\mu_{\text{untwisted}} + J^\mu_{\text{untwisted}}' + J^\mu_{\text{twisted}} + \ldots
\]
(7.58)
where
\[
J^\mu_{\text{untwisted}} = \frac{i}{4} F^a_{\alpha\beta}[\sigma^\alpha, \bar{\sigma}^\beta]\bar{f}^a_0 - \sqrt{2} \sum_{j=1}^{3} D_\nu \phi_j^\dagger \sigma^\nu \tilde{f}_j^a
\]
\[
J^\mu_{\text{twisted}} = -\sqrt{2} D_\nu \phi_{iJ}^\dagger \sigma^\nu \bar{f}_{iJ}
\]
(7.59)
and $J^\mu_{\text{untwisted}}$ has exactly the same form as $J^\mu_{\text{untwisted}}'$ with all fields replaced by primed fields corresponding to states on the $D6'$-branes. The dots in (7.58) denote terms that contain massive fields, which are excluded from the effective action. Also, in (7.59), we have omitted terms that become proportional to the equations of motion of the Goldstino when inserted in (7.52). Inserting (7.58) and (7.59) in (7.52) we obtain
\[
S = k \int d^4x \kappa \left(2\sqrt{2} \sum_{j=1}^{3} (f^a_j \partial_\mu \lambda) D^\mu \phi_j^\dagger - 2i F^a_{\mu\nu} \partial^\nu \lambda \sigma^\mu \bar{f}^a_0 + 2\sqrt{2}(f_{iJ} \partial_\mu \lambda) D^\mu \phi_{iJ}^\dagger + h.c.\right).
\]
(7.60)
This reproduces the order $\kappa$ couplings $S_1$ and $S_2$ obtained in this section, provided that we set $k^2 = \frac{1}{2}$.

If we vary the angles, then typically the intersection breaks the supersymmetries (7.55) and (7.56). The scalar superpartner of the chiral fermion becomes massive and the coupling of the Goldstino to $J^\mu_{\text{twisted}}$ is removed from the low-energy effective action. However, the spectrum on either D-brane stack is still supersymmetric locally. On the $D6$-branes, the Goldstino
couples to the supercurrent \( J^\mu_{\text{untwisted}} \) whose supercharge (7.55) is preserved by the stack. Similarly, the \( D6' \)-branes preserve the supercharge (7.56) and the Goldstino couples to the associated supercurrent \( J^\mu_{\text{untwisted}}' \).

To conclude, it is suggestive to postulate that the Goldstino couples linearly to the supercurrent which has the same internal helicities as itself. In our case, where the Goldstino and its CPT conjugate were chosen with internal helicities \((- - -)\) and \((+ + +)\), this yields the interaction (7.60). This implies in particular that the coefficients \( C_2 \), \( C_0 \), and \( C_2' \) as defined in (7.10), (7.38), and (7.39) are real. As a final remark, we note that this coupling requires extended supersymmetry and is different from the well-known coupling of the Goldstino to the “spontaneously broken” supercurrent (under which the Goldstino transforms non-linearly). Indeed, this supercurrent would involve bosons and fermions of the same supermultiplet and the Goldstino coupling would be proportional to the corresponding mass-splitting. However, in our case, these mass-splittings are strictly speaking infinite since there are no superpartners under the non-linear supersymmetry present on the branes.

Assuming the reality of all coefficients, the complete effective action is given in Appendix B. It depends on the angles only through the supersymmetry breaking scale. All coefficients are independent of the angles. Of course, these results are only valid in the limit \( \alpha' s, \alpha' t, \alpha' u << \Phi \). In this regime the energies are too small to “resolve” the intersection and all interactions become insensitive to the precise values of the angles. In order to probe the “structure” of the intersection, we expect that it is necessary to consider energies (in string units) of the same order or higher than \( \Phi \).

8 Concluding remarks

In this work, we classified all lower dimensional effective operators describing interactions of the Goldstino with gauge fields, scalars and chiral fermions, listed in Appendix A. Their strength is set by appropriate powers of the supersymmetry breaking scale (or equivalently, the Goldstino decay constant) times dimensionless coefficients, which we have computed in string theory with intersecting D-branes. The Goldstino decay constant is given by the total effective 3-brane tension, while all couplings turn out to be universal constants, independent from the values of the brane intersection angles. They are summarized in Appendix B.

In the framework of low scale string theories with fundamental scale in the TeV region and supersymmetric bulk, our analysis provides all possible couplings of the Goldstino to Standard Model fields. Two of them correspond to
dimension six operators and can lead in principle to the most dominant effects at low energy. They generate three-point interactions of a single Goldstino with a chiral fermion and a gauge field (A.3) or a scalar (A.4). In the absence of extra states, such as adjoint fermions, the former involves in principle the hypercharge and a fermion singlet such as the right-handed neutrino, while (A.4) can exist for the Higgs and lepton doublets. Both couplings seem to violate lepton number. A study of their effects is currently under way [22].

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A Goldstino couplings

In this appendix, we summarize the full list of Goldstino couplings consistent with non-linear supersymmetry up to order $\kappa^2$. The model-independent coupling reads

$$ S_0 = \int d^4 x i(\kappa^2 \lambda \bar{\sigma}^\nu \lambda) T_{\mu\nu}, $$

(A.1)

where $T_{\mu\nu}$ is the energy-momentum tensor\(^{13}\)

$$ T_{\mu\nu} = \eta_{\mu\nu} \mathcal{L}_SM - \frac{\partial \mathcal{L}_SM}{\partial \left(D_\mu \phi_i\right)} D_\nu \phi_i + 2 \frac{\partial \mathcal{L}_SM}{\partial \left(F_{\mu\lambda}^a\right)} F_{\lambda}^{a\mu}. $$

(A.2)

\(^{13}\)In the definition of the energy-momentum tensor, the fields $\phi_i$ denote “matter” fields. Otherwise, the symbol $\phi$ is used only for scalar fields.
In addition, we have found eight model-dependent couplings:

\[ S_1 = C_1 \int d^4 x \, i \kappa F^a_{\mu
u} f^a \sigma^\mu \sigma^\nu \tilde{\lambda} + h.c. \]  
(A.3)

\[ S_2 = C_2 \int d^4 x \, \kappa M_{ij} (f^i \partial_\alpha \lambda) D^\alpha \phi^j + h.c. \]  
(A.4)

\[ S_3 = C_3 \int d^4 x \, \kappa^2 M_{ij} (\partial^\mu \lambda f^{(1)}_i (\partial_\mu \tilde{f}^{(2)}_j) + h.c. \]  
(A.5)

\[ S_4 = C_4 \int d^4 x \, \kappa^2 M_{ij} (f^{(1)}_i f^{(2)}_j) (\partial_\mu \lambda \partial^\mu \lambda) + h.c. \]  
(A.6)

\[ S_5 = C_5 \int d^4 x \, \kappa^2 M_{ij} (D_\mu \phi^{(1)}_i \phi^{(2)}_j - \phi^{(1)}_i D_\mu \phi^{(2)}_j) i \partial_\alpha \lambda \sigma^\mu \partial^\alpha \tilde{\lambda} + h.c. \]  
(A.7)

\[ S_6 = C_6 \int d^4 x \, \kappa \partial^\alpha \lambda \sigma^\mu \partial^\alpha \tilde{\lambda} \partial_\nu F_{\mu\nu} + h.c. \]  
(A.8)

\[ S_7 = C_7 \int d^4 x \, \kappa^3 \phi^{(1)}_i \phi^{(2)}_j M_{ij} \partial_\mu \lambda J^{\mu\nu}_{\frac{1}{2},0} \partial_\nu \lambda + h.c. \]  
(A.9)

\[ S_8 = C_8 \int d^4 x \, i \kappa^2 M_{ijk} \phi^{(1)}_i \phi^{(2)}_j \phi^{(3)}_k (\partial_\mu \lambda J^{\mu\nu}_{\frac{1}{2},0} \partial_\nu \lambda) + h.c. \]  
(A.10)

In this list, we omitted operators linear in \( \lambda \) and suppressed by more than one power of \( \kappa \). The coefficients \( C_i \) are dimensionless (in general complex) numbers, in principle of order one, that depend on the underlying fundamental theory. We have determined them for the case of string theory on D-branes. The result is summarized in Appendix B.

## B Low-energy effective action

In this appendix we give the low-energy effective action of string theory describing Goldstino interactions on D6-branes. This action has the form

\[ S_{\text{eff}} = S_0 + S_{\text{twisted}} + S_{\text{untwisted}} + S'_{\text{untwisted}} \]  
(B.11)

\( S_0 \) is the Goldstino interaction with the energy-momentum tensor:

\[ S_0 = \int d^4 x (i \kappa^2 \lambda \partial^\mu \sigma^\nu \tilde{\lambda}) T_{\mu\nu} \]  
(B.12)

where

\[ T_{\mu\nu} = \eta_{\mu\nu} \mathcal{L}_{\text{SM}} - \frac{\partial \mathcal{L}_{\text{SM}}}{\partial (D_\mu \phi_i)} D^\nu \phi_i + 2 \frac{\partial \mathcal{L}_{\text{SM}}}{\partial (F^a_{\mu\lambda})} F^a_{\nu\lambda} \]  
(B.13)

and \( \kappa^2 \) is given in terms of the effective 3-brane tensions as

\[ \frac{1}{2\kappa^2} = NT_3 + N'T_3' \]  
(B.14)
$S_{\text{twisted}}$ contains the interactions of the Goldstino with the matter localized on the intersection of two $D6$-brane stacks. It is given by

$$S_{\text{twisted}} = 2\kappa^2 \int d^4x (\tilde{f}_{i,j} \partial_\mu \tilde{\lambda})(f_{i,j} \partial^\mu \lambda) + 2\kappa (\int d^4x (f_{i,j} \partial_\alpha \lambda) D^\alpha \phi_{i,j}^\dagger + \text{h.c.})$$ (B.15)

Here $f_{i,j}$ is the chiral fermion on the intersection and $\phi_{i,j}^\dagger$ is its superpartner with respect to the supercharge (7.55), (7.56). The precise definitions of the quantum fields are given in Section 7.1. It is understood that the second term should be included only if the scalar involved in the interaction is massless, i.e. whenever the sum of the relative rotation angles $\Phi = 0$ in (6.19) which is the condition for the supersymmetry (7.55), (7.56) to be preserved by the intersection. Only the four-fermion interaction is generically present.

The remaining two contributions to $S_{\text{eff}}$ in (B.11) contain the interactions with matter on the $D6$ and $D6'$ stacks, respectively:

$$S_{\text{untwisted}} = \sqrt{2}i\kappa \int d^4x F_{\mu\nu} f^a_0 \sigma^\mu \partial^\nu \bar{\lambda} + 2\kappa \sum_{j=1}^3 \int d^4x (f^a_j \partial_\mu \lambda) D^\mu \phi^a_j \tag{B.16}$$

The action $S'_{\text{untwisted}}$ has exactly the same form as $S_{\text{untwisted}}$. For the precise definitions of the quantum fields see Section 7.2.

In our analysis, we left undetermined the coefficient of the five-point function $C_8$, as well as $C_6$ which may exist only for abelian gauge fields. All other couplings that do not appear in (B.15) and suntwist), such as $C_4$, $C_5$ and $C_7$, are vanishing. We have also argued that $C_1$ and $C_2$ are real. In this case, the terms linear in the Goldstino correspond to its coupling to the supercurrent with the same internal helicities as itself. This coupling is given by

$$S_{\text{linear}} = \frac{\kappa}{\sqrt{2}} \int d^4x (\partial_\mu \lambda J^\mu + \partial_\mu \bar{\lambda} \bar{J}^\mu)$$ (B.17)

with $J^\mu$ and $\bar{J}^\mu$ the left- and right-handed part of the supercurrent, respectively. In our conventions, where the Goldstino and its CPT conjugate were chosen to have internal helicities $(- - -)$ and $(+ + +)$, the massless part of the supercurrent is given in (7.58) and (7.59).
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