Duality-Symmetric Three-Brane and its Coupling to Type IIB Supergravity

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Abstract

Starting from the bosonic sector of the M-theory super-five-brane we obtain the action for duality-symmetric three-brane and construct the consistent coupling of the proposed action with the bosonic sector of type IIB supergravity.

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1 Introduction

S-duality is conjectured to be an exact symmetry of the type IIB superstring theory, whose BPS spectrum includes in particular the IIB three-brane, being invariant under the action of S-duality symmetry group.

The special role of the three-brane in the type IIB superstring theory has been noted in literature (see, for example, [1] – [9]) time and again in connection with conjecture about possible reformulation of the $D = 10$ IIB superstring theory as a theory in twelve dimensions [10], [11]. In frames of this conjecture the type IIB superstring theory could be interpreted as a theory of fundamental supersymmetric three-branes [2], [3], whose $SL(2, \mathbb{Z})$ symmetry would be a consequence of the ‘electro-magnetic’ worldvolume duality, with S-duality invariant effective action in type IIB theory background. However, one of the obstacles in construction of this effective action is the field content of type IIB supergravity [12], whose bosonic sector includes a self-dual antisymmetric gauge field.

The problem of a compatibility of the Lagrangian description of self-dual fields and space-time covariance has attracted great deal of attention during a long period of time [14]. The progress in construction of the Lagrangians for self-dual antisymmetric gauge fields was achieved by Schwarz and Sen [15]. Their results was the generalization of early work by Floreanini and Jackiw [16] on $d = 2$ chiral scalars and by Henneaux and Teitelboim [17] on $d = 2(p + 1)$ chiral p-forms. However, the approach of [15], being manifestly duality invariant, is not manifestly space-time covariant. But it is desirable to have a covariant formulation, especially when one is interested in consideration of models coupled to gravity or supergravity. This requires the use of auxiliary fields, from one to infinity in dependence on the approach [18], [19], [20]. In what follows we will be concentrated on the approach proposed by Pasti, Sorokin and Tonin (PST) [20], [21] with single auxiliary field ensuring the covariance of the model, but entering to the action in a non-polynomial way. Recently this approach was successfully applied to a different kind of models with self-dual or duality-symmetric antisymmetric gauge fields including the M-Theory super-five-brane [22], [23]; the bosonic sector of $D = 10$ type IIB supergravity [24] and duality-symmetric $D = 11$ supergravity [26]. In this paper we apply the PST approach to obtain the effective action of the type IIB superstring theory with three-brane source.

As a preliminary step in Section 2 we obtain the action for three-brane with duality-symmetric worldvolume gauge fields by worldvolume dimensional reduction of the action for bosonic five-brane [27]. This special structure of the three-brane action is dictated by the structure of the bosonic sector of IIB supergravity [24], which contains a self-dual forth rank antisymmetric gauge field naturally coupled to three-brane, having the special symmetries being characteristic of theories with self-dual gauge fields [20], [21]. This symmetries have to be preserved under the coupling of three-brane with the type IIB supergravity that automatically demands the duality-symmetric structure of three-brane as we will see later on. In Section 3 we review the PST approach to the bosonic sector of $D = 10$ IIB supergravity [24], entering to the effective action of the type IIB superstrings, and obtain the action describing consistent coupling of three-brane with the bosonic sector of $D = 10$ IIB supergravity. Discussion of the obtained results and some concluding remarks are collected in the last section.
2 Three-brane from five-brane

Our starting point is the action for bosonic five-brane [27]:

$$ S = - \int d^6 \xi \sqrt{- \det (g_{\mu \nu} + i \tilde{H}_{\mu \nu})} + \sqrt{-g} \frac{1}{4(\partial a)^2} \tilde{H}^{\mu \nu} H_{\mu \rho \nu} \partial^\rho a $$

with

$$ \tilde{H}_{\mu \nu} \equiv \frac{1}{\sqrt{-(\partial a)^2}} H^*_{\mu \nu} \partial^a; \quad H^{\mu \nu \lambda} = \frac{1}{3!} \sqrt{-g} \epsilon^{\mu \nu \lambda \rho \sigma \delta} H_{\rho \sigma \delta} $$

where $H_{\mu \nu \lambda}$ is the field strength of the worldvolume antisymmetric tensor field $B_{\mu \nu}(\xi)$ and $g_{\mu \nu}$ is the metric in six dimensions.

To obtain the action describing duality-symmetric three-brane we perform a dimensional reduction by splitting the six-dimensional indices $\mu$ onto the set of $(m, \alpha)$ [29], where $m$ is the four-dimensional index and $\alpha$ is the index in compact dimensions. We will assume $a \neq a(y^\alpha)$, where $y^\alpha$ are the coordinates of a compact space; $B_{mn} = 0$, $B_{pp} \neq B_{pp}(y^\alpha)$ and $B_{\alpha \beta} = 0$; and will choose the standard form of vielbeins [28], [29] that allows us to write the six-dimensional Levi-Civita symbol $\epsilon_{\mu \nu \rho \sigma \delta}$ as the direct product of the four-dimensional Levi-Civita symbol $\epsilon_{mnrq}$ and the unit antisymmetric tensor $\epsilon_{\alpha \beta}$ in compact dimensions.

Using this ansatz, after some algebra we obtain the following action for three-brane with duality-symmetric worldvolume gauge fields:

$$ S = - V \int d^4 \xi \sqrt{-G} \left( 1 + \tilde{F}_{\alpha m} \tilde{F}^{\alpha m} + \frac{1}{4} \epsilon^{\alpha \beta \gamma \delta} \tilde{F}_{\beta \delta} \frac{\partial a}{\sqrt{-(\partial a)^2}} \right)^2 $$

$$ - \frac{1}{2} \tilde{F}_{\alpha m} \epsilon^{\alpha \beta} F^{\beta m} \frac{\partial a}{\sqrt{-(\partial a)^2}} $$

where

$$ \tilde{F}_{\alpha m} = \frac{\partial^a a}{\sqrt{-(\partial a)^2}} F^*_{\alpha m}; \quad F^{\beta m} = 2 \partial_{[m} B_{n]}^\beta; \quad \sqrt{-G} = \sqrt{- \det G_{mn}}, $$

the metric tensor for the internal space is $g_{\alpha \beta} = \delta_{\alpha \beta}$, and $V$ is the volume of a compact space.

The action (3), written in manifestly covariant form, is invariant under

- worldvolume diffeomorphisms,
- usual gauge invariance
  $$ \delta B^\alpha_m = \partial_m \phi^\alpha, $$
- local transformations of the form
  $$ \delta B^\alpha_m = \partial_m a(\xi) \varphi^\alpha(\xi); \quad \delta a = 0, $$
- and additional local symmetry
  $$ \delta a(\xi) = \Phi(\xi), \quad \delta B^\alpha_m = \frac{\Phi(\xi)}{(\partial a)^2} \epsilon^{\alpha \beta} F^{\beta m}; \quad \hat{F}^\alpha_m = V^\alpha_m - \epsilon^{\alpha \beta} F_{\beta m} \partial^a a $$
where
\[ \mathcal{V}_m^{\alpha} = \sqrt{-(\partial a)^2} \frac{\delta}{\delta F_0^m} \sqrt{1 + \partial^2 a} \left( \partial \partial a \right) \left( \partial \partial a \right)^2 \] (8)

All these symmetries become obvious if we consider a variation of the action (3) under \( B_0^\alpha \) and \( a(\xi) \), written in differential forms as
\[ \delta S = -\mathcal{V} \int_{\mathcal{M}^4} \left( \delta B^{(1)\alpha} - \frac{\delta a}{(\partial a)^2} \right) \epsilon^{\alpha\beta} \mathcal{F}_{(1)\beta} \wedge d\left( \frac{1}{\sqrt{-(\partial a)^2}} \partial a \wedge \mathcal{F}_{(1)}^\alpha \right). \]

The presence of the symmetry (7) is crucial for establishing a connection with non-covariant formalism of [30], [31]. Effectively, the symmetry (7) allows one to fix a gauge in such a way that, say,
\[ \partial_m a(\xi) = \delta_m^0, \] (9)

or
\[ \partial_m a(\xi) = \delta_m^3, \] (10)

and the local symmetry (7) ensures the auxiliary role of the field \( a(\xi) \). Thus, fixing the gauge (9) or (10), we can eliminate \( a(\xi) \) from the action without losing dynamical information and obtain the action (8) similar to that of [30] and [31].

The symmetry (6) allows one to reduce the general solution for the equations of motion of \( B_0^\alpha \) fields to the form
\[ \mathcal{V}_m^{\alpha} - \epsilon^{\alpha\beta} F_{mn,\beta} \partial^0 a = 0. \] (11)

Eq. (11) is a generalization of the self–duality condition to the case of self-interacting gauge fields [32]. In the linearized limit, where we take the flat metric and insert in (3) the term \(-\epsilon^0_{\alpha} \mathcal{F}_{0m} \) instead of the square root, the condition (11) becomes the self-duality condition for duality-symmetric Maxwell electrodynamics [20]
\[ \mathcal{F}_{mn}^\alpha = 0 \]

with
\[ \mathcal{F}_{mn}^\alpha = \epsilon^{\alpha\beta} F_{\beta mn} - F_{m}^{*\alpha}. \]

Thus, the action (3) describes a duality-symmetric three-brane. It involves a pair of a gauge fields \( B_0^\alpha \) related to each other by virtue of the self-duality condition (11). After elimination of one of the Abelian fields we recover the usual Born-Infeld action as it was shown in [30], [31]. The presence of an additional field \( a(\xi) \) allows one to covariantize the approach of [30], [31] and to make electric-magnetic duality manifest at the level of action.

To obtain the three-brane source term of the effective action for type IIB theory we have to consider three-brane coupling with IIB \( D = 10 \) background fields, including the dilaton \( \phi \), the second rank antisymmetric tensor \( C_{mn} \) and the metric \( g_{mn} \) in NS-NS sector and the axion \( \chi \), the second rank antisymmetric tensor \( \tilde{C}_{mn} \) together with the self-dual fourth rank antisymmetric tensor \( A_{mnpq} \) in RR sector \( (m, n, p, q = 0, \ldots , 9) \).

\textsuperscript{3}The choice (8) corresponds to the action (15) of [31] and as it has noted in [30], the choice (10) leads to the action with a pair of electric fields.
First consider the action (3) in dilaton-axion background, describing by the $2 \times 2$ matrix valued scalar field

$$M = \frac{1}{\lambda_2(x)} \begin{pmatrix} 1 & \lambda_1(x) \\ \lambda_1(x) & \lambda_1^2 + \lambda_2^2 \end{pmatrix},$$  

(12)

$$\lambda = \lambda_1 + i\lambda_2 \equiv \chi + i e^{-\phi}, \quad \phi = \phi(x(\xi)), \quad \chi = \chi(x(\xi)),$$

(13)
satisfying the following conditions:

$$M^T = M; \quad M\epsilon M^T = \epsilon.$$  

(14)

$M$, $\epsilon$ and $B_m^\alpha$ transform under the global $SL(2, R)$ transformations $\omega$ as follows:

$$M \longrightarrow \omega^T M \omega, \quad \omega \epsilon \omega^T = \epsilon, \quad B_m^\alpha = (\omega^T B_m^\alpha),$$  

(15)

$$\omega = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad AD - BC = 1.$$

Then the $SL(2, R)$ invariant action has the following form [4]:

$$S = -V \int d^4\xi \sqrt{-G} \left( 1 + \tilde{F}_m^\alpha (e^T M \epsilon)_{\alpha\beta} \tilde{F}_m^\beta + \frac{1}{4} (e^{abcd} \tilde{F}_a e^{\alpha\beta} \tilde{F}_d e^{\gamma \delta} \frac{\partial f}{\sqrt{-(\partial a)^2}})^2 - \frac{1}{2} \tilde{F}_m^\alpha e^{\alpha\beta} F_{mn} e^{\beta\gamma} \frac{\partial^\alpha a}{\sqrt{-(\partial a)^2}} \right).$$  

(16)

To find the action (16) in a background of antisymmetric gauge fields of IIB $D = 10$ supergravity, we have to replace the field strength $F_m^\alpha$ with

$$H_m^\alpha = F_m^\alpha - C_m^\alpha,$$

(17)

and add to the action (16) a Wess-Zumino term (see [4], [9], [27]). The resulting action becomes

$$S = -V \int d^4\xi \sqrt{-G} \left( 1 + \tilde{H}_m^\alpha (e^T M \epsilon)_{\alpha\beta} \tilde{H}_m^\beta + \frac{1}{4} (e^{abcd} \tilde{H}_a e^{\alpha\beta} \tilde{H}_d e^{\gamma \delta} \frac{\partial f}{\sqrt{-(\partial a)^2}})^2 - \frac{1}{2} \tilde{H}_m^\alpha e^{\alpha\beta} H_{mn} e^{\beta\gamma} \frac{\partial^\alpha a}{\sqrt{-(\partial a)^2}} \right) + \int_{M^4} A(4) + \frac{1}{2} e^{\alpha\beta} F_\alpha^\beta \wedge C_\beta^\alpha,$$

(18)

where $A(4)$ and $C(2)$ are the pullbacks of the corresponding D=10 forms onto worldvolume $M^4$.

The structure of the Wess-Zumino term is governed by the requirement of invariance of (16) under the modified symmetries (4) – (7)

$$\delta a = 0; \quad \delta B_m^\alpha = \partial_m \phi^\alpha + \partial_m a \phi^\alpha,$$

$$\delta a(\xi) = \Phi(\xi), \quad \delta B_m^\alpha = \Phi(\xi) (\partial m)^2 e^{\alpha\beta} H_m^\beta, \quad H_m^\alpha = \tilde{V}_m^\alpha - \varepsilon^{\alpha\gamma} H_{mn}^\gamma \partial^\alpha a$$  

(19)

To check this invariance, the transformation law $\tilde{F}_m^\alpha = (\tilde{F}_m^\alpha)^\alpha$ has to be used.
with
\[ \tilde{V}_m^{\alpha} = \sqrt{-(\partial a)^2} \delta \sqrt{1 + \tilde{H}_m^{\alpha}(\epsilon^T M \epsilon)_{\alpha\beta} \tilde{H}^{\beta m} + \frac{1}{4}(\epsilon^{abdf} \tilde{H}_{ba} \epsilon^{\alpha\beta} \tilde{H}_{d\beta} \frac{\partial f_a}{\sqrt{-(\partial a)^2}})^2} \delta \tilde{H}_m^{\alpha}, \]
that is, the Wess–Zumino term is required to preserve local symmetries of the action when three–brane couples to the antisymmetric fields (see [27], where this fact was pointed out for the first time).

To complete a description, let us note that the action (18) is \( SL(2, \mathbb{R}) \) invariant under the following \( SL(2, \mathbb{R}) \) transformation of the background fields:
\[ C_{mn}^{\alpha} = \left( \begin{array}{c} C_{mn} \\ \tilde{C}_{mn} \end{array} \right) \rightarrow (\omega^T C)^{\alpha}_{mn}. \] (20)

3 D=10 IIB supergravity and its coupling to three-brane

Although the Lorentz covariant equations of motion for \( D = 10 \) IIB supergravity are well known for a long time [12], the problem of construction of the complete \( D = 10 \) IIB supergravity action functional [13] remains open. However, from the modern point of view the main obstacle in construction of this action connected with the presence of a self-dual antisymmetric gauge field in the bosonic sector of Type IIB supergravity has been avoided in the work by Dall’Agata, Lechner and Sorokin [24] where the PST approach [20, 21] was applied. In this section we review the approach of [24] and obtain the effective action for the Type IIB superstring theory with the three-brane source.

The bosonic sector of the type IIB supergravity [12] is described by the following action [24]:
\[
S = \int d^{10}x \sqrt{-g} \left[ R - 2 \partial_m \phi \partial^m \phi - 2 e^{2\phi} \partial_m \chi \partial^m \chi - \frac{1}{3} e^{-\phi} H_{imn} H^{imn} \right.
\]
\[
- \frac{1}{3} e^{\phi}(\tilde{H}_{imn} - \chi H_{imn})(\tilde{H}^{imn} - \chi H^{imn}] \right]
\[
- \frac{1}{6} \int d^{10}x \sqrt{-g} \partial_a(x) M_{m_1 \ldots m_5}^{* \alpha_1 \ldots \alpha_5} \tilde{M}^{m_1 \ldots m_5} \partial_a(x) - 4 \int_{\mathcal{M}_{10}} A^{(4)} \wedge H \wedge \tilde{H}, \] (21)
where \( R(x) \) is a \( D = 10 \) scalar curvature, \( H^{(2)} = dC^{(2)} \) and \( \tilde{H}^{(2)} = d\tilde{C}^{(2)} \) are, respectively, the field strength of the NS-NS and R-R two-forms;
\[
M^{(5)} = dA^{(4)} + \frac{1}{2} C^{(2)} \wedge d\tilde{C}^{(2)} - \frac{1}{2} \tilde{C}^{(2)} \wedge dC^{(2)} \] (22)
is the five-form field strength of the R-R self-dual gauge field \( A^{(4)}(x), \mathcal{M}^{(5)} = M^{(5)} - M^{* (5)} \) is the anti-self-dual part of \( M^{(5)}(x), \) and the auxiliary scalar field \( a(x) \) ensures the manifest \( D = 10 \) covariance of the \( A^{(4)} \) part of the action.

The action (21) possesses the following symmetries:

- \( D = 10 \) general covariance;
usual gauge invariance
\[ \delta C^{(2)} = d\alpha^{(1)}, \quad \delta \tilde{C}^{(2)} = d\tilde{\alpha}^{(1)}, \]
\[ \delta A^{(4)} = d\phi^{(3)} - \frac{1}{2} d\alpha^{(1)} \wedge \tilde{C}^{(2)} + \frac{1}{2} d\tilde{\alpha}^{(1)} \wedge C^{(2)}; \] (23)

- global \( SL(2, R) \) symmetry mixing of \( \phi \) and \( \chi \) and of \( C^{(2)} \) and \( \tilde{C}^{(2)} \)
- and additional local symmetries:
\[ \delta A^{(4)} = da \wedge \varphi^{(3)}, \quad \delta a = 0, \]
\[ \delta a = \Phi(x), \quad \delta A^{(4)} \mid_{mnpq} = \frac{\delta a}{(\partial a)^2} M^{(5)} \mid_{mnpq} \partial^2 a. \] (24)

Apparently, the kinetic terms involving \( H^{(3)} \) and \( \tilde{H}^{(3)} \) are the gauge invariant. The invariance under the symmetries corresponding to the transformations of \( A^{(4)} \) (23) and (24) can be established by the variation of the last two terms of the action (21) having the following form in the differential forms notations:
\[ \delta S_{M+CS} = -4 \int_{M^{10}} \frac{\delta a}{\sqrt{-(\partial a)^2}} i_v M^{(5)} + \delta A^{(4)} + \frac{1}{2} C^{(2)} \wedge \delta \tilde{C}^{(2)} - \frac{1}{2} \tilde{C}^{(2)} \wedge \delta C^{(2)} \wedge d(v \wedge i_v M^{(5)}) \]
\[ + 2(d\tilde{C}^{(2)} \wedge \delta C^{(2)} - dC^{(2)} \wedge \delta \tilde{C}^{(2)}) \wedge (v \wedge i_v M^{(5)} - \frac{1}{2} M^{(5)}) \]
\[ + dM^{(5)} \wedge (\delta A^{(4)} + \frac{1}{2} C^{(2)} \wedge \delta \tilde{C}^{(2)} - \frac{1}{2} \tilde{C}^{(2)} \wedge \delta C^{(2)}) \]
\[ + \delta A^{(4)} \wedge dC^{(2)} \wedge d\tilde{C}^{(2)} - \delta A^{(4)} \wedge d\tilde{C}^{(2)} \wedge \delta C^{(2)} + dA^{(4)} \wedge dC^{(2)} \wedge \delta \tilde{C}^{(2)}), \] (25)

where \( v = \frac{da}{\sqrt{-(\partial a)^2}} \) and the contraction \( i_v M^{(5)} \) is defined as
\[ i_v M^{(5)} = \frac{1}{4!} dx^{m_1} \wedge \ldots \wedge dx^{m_4} v^2 M_{pm4\ldots m}. \] (26)

The first line of the (23) makes the gauge invariance obvious since \( \delta_{gauge} M^{(5)} = 0 \), and \( \delta_{gauge} S_{CS} = 0 \) up to the total derivative. Concerning the invariance under the transformations (24), it should be noted that the form of the Chern-Simons term is completely determined by the requirement of keeping the symmetries (24) and by the form of \( M^{(5)} \). On the other hand, the form of the Chern-Simons term is fixed by the requirement of supersymmetry. Thus, the symmetry (24) is the criterion of selfconsistency for the bosonic theories with the self-dual fields playing the same role as supersymmetry in superfield theories or as kappa-symmetry for super-p-branes.

As for the global \( SL(2, R) \), the presence of this symmetry becomes clear if we write down the action (21) using the matrix (12) with \( \phi = \phi(x) \) and \( \chi = \chi(x) \):
\[ S = \int d^{10} x \sqrt{-g} [R - g_{mn} tr(\partial \alpha \mathcal{M} \partial \alpha \mathcal{M}) - \frac{1}{3} H_{lmn}^\alpha (\epsilon^T \mathcal{M} \epsilon)_{\alpha \beta} H^{3lmn}] \]
\[-\frac{1}{6} \int d^{10}x \sqrt{-g} \partial a(x) \mathcal{M}^{(5)}_{\alpha \beta} \partial a(x) + 2 \int_{\mathcal{M}^{10}} A^{(4)} \wedge \mathcal{H}^{\alpha} \epsilon_{\alpha \beta} \wedge \mathcal{H}^{\beta}, \tag{27}\]

where
\[
\mathcal{H}^{\alpha} = \left( \frac{dC}{dC} \right), \quad \epsilon_{\alpha \beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{28}\]

and taking into account the transformation laws for \(\mathcal{M}^{(4)}\) and for \(C^{(2)}\).

Equation of motion for \(A^{(4)}\) field is reduced by (24) to the self-duality condition
\[
\mathcal{M}^{(5)} = M^{(5)} - M^{* (5)} = 0, \tag{29}\]

provided a fulfillment of the equation of motion
\[
dM^{(5)} = -dC^{(2)} \wedge d \tilde{C}^{(2)}, \tag{30}\]

derived early in [12].

To construct a consistent coupling of three-brane to type IIB supergravity let us firstly analyze the equations (29) and (30) following from the action (21). The self-duality of \(M^{(5)}\) provides the symmetry between equation of motion for \(M^{(5)}\) and its Bianchi identities:
\[
dM^{(5)} = -dC^{(2)} \wedge d \tilde{C}^{(2)}, \quad dM^{(5)} = -dC^{(2)} \wedge d \tilde{C}^{(2)}, \tag{31}\]

hence, if one would like to introduce sources into these equations [25], one should follow the prescription used by Dirac for describing magnetic monopoles coupled to D=4 Maxwell fields. Following Dirac we can provide a fulfillment of the Bianchi identities
\[
d\hat{M} = 0, \tag{32}\]

replacing \(M^{(5)}\) with \(\hat{M}^{(5)} = M^{(5)} - G^{(5)}\), where
\[
dG^{(5)} = \frac{1}{4} J^{(4)}, \tag{33}\]

and consider the generalization of the Dirac string to a Dirac four-brane stemmed from the three-brane.

Following [26] we extend the action for three-brane to an integral over \(D = 10\) space-time by inserting a \(\delta\)-function closed six-form
\[
J^{(4)} = \frac{1}{4!6!} dx^{m_1} \wedge \ldots \wedge dx^{m_6} \epsilon_{m_1 \ldots m_6} \int_{\mathcal{M}^4} d\tilde{x}^{n_1} \wedge \ldots \wedge d\tilde{x}^{n_4} \delta(x - \tilde{x}(\xi)) \tag{34}\]

dual to the current \(J^{(4)}\) minimally coupled to \(A^{(4)}(x)\). Then the action describing consistent coupling is
\[
S = \int d^{10}x \sqrt{-g} \left[ R - g_{mntr} \partial^{(m} \mathcal{M}^{n} \epsilon \partial^{t} \mathcal{M}^{r} \right] - \frac{1}{3} \mathcal{H}^{\alpha}_{mn} (\epsilon T \mathcal{M}^{\alpha} \epsilon) \mathcal{H}^{\beta mn} - \frac{1}{6} \int d^{10}x \sqrt{-g} \partial a(x) \mathcal{M}^{(5)}_{\alpha \beta} \partial a(x) + 2 \int_{\mathcal{M}^{10}} A^{(4)} \wedge \mathcal{H}^{(2)} \epsilon_{\alpha \beta} \wedge \mathcal{H}^{(2)\beta} - \sqrt{-G} \left[ 1 + \mathcal{H}^{(4)} (\epsilon T \mathcal{M}^{\alpha} \epsilon) \mathcal{H}^{\beta mn} \right] + \frac{1}{4} (\epsilon^{abcd} \tilde{H}_{ab} \epsilon^{\alpha \beta} \tilde{H}_{d3} \frac{\partial f a}{\sqrt{-G}})^2 \tag{35}\]

\[
- \mathcal{V} \int d^{4}\xi \sqrt{-G} \left( 1 + \tilde{H}_{m}^{\alpha} (\epsilon T \mathcal{M}^{\alpha} \epsilon) \tilde{H}_{mn} + \frac{1}{4} (\epsilon^{abcd} \tilde{H}_{ab} \epsilon^{\alpha \beta} \tilde{H}_{d3} \frac{\partial f a}{\sqrt{-G}})^2 \right) \tag{36}\]

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\[ -\frac{1}{2} \hat{H}_a^m \epsilon^{\alpha\beta} H_{mn\beta} \frac{\partial^a a}{\sqrt{-(\partial a)^2}} + \int_{\mathcal{M}^4} A^{(4)} - 2 \int_{\mathcal{M}^{10}} \epsilon^{\alpha\beta} H^{(2)}_a \wedge dC^{(2)}_\beta \wedge G^{(5)}, \quad (35) \]

where \( \hat{M}^{(5)} = M^{(5)} - G^{(5)} \) and \( \hat{M}^{(5)} = \hat{M}^{(5)} - \hat{M}^{(5)} \).

The requirement of consistency demands preservation of the local symmetries (19) and (24) as well as global \( SL(2, \mathbb{R}) \). This puts the restriction to the additional scalar field \( a(x) \) to be the image onto the worldvolume of three-brane, i.e. \( a = a(x(\xi)) \) (see [26] for details).

Thus, we have constructed the effective action for type IIB supergravity with the three-brane source.

### 4 Discussion

To summarize, we start from the action for the bosonic sector of the M-theory super-five-brane and obtain the action for three-brane with duality-symmetric worldvolume gauge fields. This fields are related to each other by virtue of the self-duality condition and elimination of one of them yields the usual Born-Infeld action. In the presence of the IIB \( D = 10 \) background fields the proposed action possesses an \( SL(2, \mathbb{R}) \) symmetry and the requirement of keeping the additional local symmetries characteristic of the PST approach to the theories with chiral gauge fields completely restricts the structure of the Wess-Zumino term.

The duality-symmetric structure of the three-brane action allows to construct the consistent coupling to the type IIB supergravity preserving all of the local and global symmetries of the effective action. It leads to appearance of minimal term coupled to \( A^{(4)} \) as well as to the new types of non-minimal terms such as \( \epsilon^{\alpha\beta} H^{(2)}_a \wedge dC^{(2)}_\beta \wedge G^{(5)} \) that reflects the presence of electric and magnetic charges carrying by the worldvolume of three-brane.

The straightforward developments of the obtained results are the investigation of the equations of motion of the effective action, the anomaly analysis [33] and the construction of the supersymmetric generalization of the proposed action for three-brane in a spirit of [34].

The investigation of these and other questions we postpone to future papers.

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