Simulation of multi-agent interaction between mobile operators and users

I V Zaitseva1,2, O A Malafeyev1, I F Kefeli3, N V Poddubnaya2, A V Shuvaev2

1Saint-Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia
2Stavropol State Agrarian University, Zootekhnichesky Lane, 12, Stavropol, 355017, Russia
3Center for geopolitical expertise and publishing projects of the North-Western Institute of management Russian, Academy of national economy and state service under the President of the Russian Federation, Saint-Petersburg, Russia, 4 Pesochnaya Embankment, Saint-Petersburg, 197376, Russia

E-mail irina.zaitseva.stv@yandex.ru

Abstract. Game-theoretic models for two classes of players are considered in this paper. The first class will consist of mobile operators, who, based on information about competitors' actions, will plan changes in prices for services. The second class will consist of dedicated user cohorts who, depending on the current situation, will plan their actions, namely, change or not change the operator. The presented models solve the problem of maximizing the income of mobile operators and minimizing the cost of communication by users. A turn-based game is considered where a possible strategy for a first-class player will be considered a non-zero price vector for the provided communication services. A possible strategy for a second-class player will be considered the sequence of choosing a mobile operator for each of the services. The payoff function of both classes players will be built at each stage. According to the winnings, there is a compromise solution for different classes of players.

Keywords: Game-theoretic model, multi-agent interaction, operator, mobile communication, user, game tree

1. Introduction

There are companies on the market that provide various communication services for users, let's call them mobile operators. Each mobile operator is characterized by a price vector for various communication services.

The following types of communication are understood as communication services in the model: voice communication in the “home region”, long-distance communication, international communication, SMS, mobile Internet. The “home” region is understood as the subscriber connection region.

If a user of mobile communications services uses different mobile operators for different services, he incurs costs compared with the user who has one operator for all mobile communication services.
If the user changes the mobile operator to one of the services that the other mobile operator previously provided to him, he incurs slightly different costs. These costs can vary from small values, for those users who, for example, come for a short time to another city and want to save on roaming, so they temporarily change the operator. To large amounts, when, for example, the company incurred large costs for advertising a particular subscriber number and cannot stop using it, so as not to lose customers. Also, these costs may not be material, and their monetary equivalent will be taken into account in the model. Such costs, for example, may arise if the user of communication services has not changed the operator for a long time and, changing the number, his friends who were not informed, may lose contact with him.

2. Informal statement of the problem

Since the number of users of communication services has already exceeded the number of residents in Russia, and in order not to consider these users, they are divided into cohorts (groups) by the number and type of consumed services, by the value of the operator change cost parameter. Thus, the dimension of the problem can be significantly reduced.

In this model, we will consider two classes of players. The first class will consist of mobile operators, who based on information about competitors’ actions, will plan changes in prices for services. The second class will consist of dedicated user groups, which depending on the current situation will plan their actions (change or not change the operator). In this model, in order for both classes of players to have a maximization problem, suppose that second-class players have a constant income that exceeds communication costs, and they maximize the difference between income and communication costs [1-3].

Incomes of mobile operators are calculated by multiplying the total number of users who have chosen this particular operator for this communication service by the price per unit of service. The goal of players of both classes is to maximize their income function. We will consider a turn-based game, i.e. first-class players alternately change prices for the provided communication services, after each step of the first, second-class players either change the operator to certain types of communications, or decide to keep it. A possible strategy for a first-class player will be considered a non-zero price vector for the provided communication services. A possible strategy for a second-class player will be considered the sequence of choosing a mobile operator for each of the services.

After each of the mobile operators makes one move, the final wins of the players of both classes are calculated. At each stage, the payoff function of players of both classes is built, by which we find a compromise solution for different classes of players.

3. Problem formalization of choosing a vector of prices by mobile operators

The following task is considered: there are n mobile operators T_1, T_2, T_3, ..., T_n, ... T_n, i = 1, n, which provide k different services U_1, U_2, U_3, ..., U_k, where j = 1, k. Each mobile operator T_i, i = 1, n, is defined by a vector of dimension (k+2), T_i = (t_{i1}, t_{i2}, ..., t_{ij}, ..., t_{ik}, p_j, q_j) in which t_{ij}, i = 1, n, j = 1, k, are the prices of the mobile operator T_i, j = 1, k, for the service U_j, p_j, i = 1, n, is the cost of the operator T_i, i = 1, n, for advertising, and q_j, i = 1, n, is the cost of the operator T_i, i = 1, n, for the development of mobile communications network [4-5].

Each user of communication services is set by the vector of their needs I = (i_1, i_2, i_3, ..., i_k) and also by the parameter of the cost of changing the operator s = s(t_i, f(I)), i ≠ j where t_i, whose operator is changed, t_j, the operator to which they change, the function f(I) takes different values, depending on the type of communication services user [6].

Since the number of users of communication services significantly exceeds the number of mobile operators and the services they provide, we group them. Denote by S the set of values of the parameter s. We divide this set exactly on R disjoint subsets and enumerate them.
Next, we will group communication service users into cohorts by the parameter of operator change cost, so that if for some group of users this parameter is in the subset \( s_i \), we will say that this communication service user is in the \( i \)-th cohort. Moreover, the entire cohort can be considered as one user of communication services, whose needs are equal to the sum of the needs of the cohort participants, and the cost of changing an operator is equal to the average for the cohort. Thus, each cohort is denoted by \( Q_l \), \( l = 1, r \) and is a vector \( Q_l = (q_{l1}, q_{l2}, ..., q_{ln}, s_l) \) which contains the need for communication services of this cohort as well as the cost of changing the operator for this cohort.

At the initial moment of time, a matrix is defined, \( W = W_{k^n} \), in which the element \( w_{ij} \) denotes the price of the \( i \)-th mobile operator \( I_i \), \( i = \overline{1, n} \) for the \( j \)-th service \( U_j \), \( j = \overline{1, k} \).

By the payoff function \( H_i \), \( i = \overline{1, n} \) of each mobile operator, we consider the income received from users of communication services who have chosen for their service \( U_j \), \( j = \overline{1, k} \), operator \( T_i \), \( i = \overline{1, n} \).

For uniformity, we assume that each cohort of users receives a certain stable income \( B_{l=\overline{1, r}} \), with \( B >> \max(q_{lj}) \times \max(t_{lj}) \), \( l = \overline{1, r} \), \( i = \overline{1, n} \), \( j = \overline{1, k} \). The cohort gain function will be the difference between \( B \) and the cost of communication services and a possible change of operators.

By the top of the tree we mean the value of the matrix \( W = W^{(k+2)\times n} \), an indication of which of the operators will go next, as well as the entire history of the change in the matrix \( W^{[7,9]} \).

Consider the actions of mobile operators. For the many possible strategies of the operator \( I_i \), \( i = \overline{1, n} \) we take the set of price vectors for communication services that this operator can offer.

### 4. Dimension analysis of the practically solved multi-agent interaction problem

Let us consider an example of the interaction of 4 mobile communication operators providing 4 different communication services: voice communication, long-distance communication, SMS and mobile Internet. As an example, take a multi-step game with full information on a tree-like finite graph. By the top of the graph in this model we mean the price matrix of mobile operators for communication services, \( W = W^{4\times4} \), in which the element \( w_{ij} \) denotes the price of the \( i \)-th mobile operator, \( I_i \), \( i = \overline{1, n} \) for the \( j \)-th service, \( U_j \), \( j = \overline{1, k} \). In the initial period of time, the matrix \( W_0 \) is determined. Further, the first operator, Tele2, offers 2 different strategies, in which it indicates 2 different tariffs for communication services and in each cases its costs for advertising and network development. After that, Megafon makes a move, after which MTS move and Beeline completes.

As a result, we get a game tree in which for each vertex, as well as for the set of final vertices, we write down the profit of each operator, taken from a quarterly report published by mobile operators. The winning function for each player will be equal to the profit earned for the reporting period.

In the practical implementation of this example, cohorts of users can choose their own operator for each of the services in an arbitrary way. In the interaction of 4 mobile operators providing 4 services, in the first step we get \( 4^4 = 64 \) the choice for each of the cohorts, in the second step the cohort of communication service users can change the operator at random, which will give an option \( 64 \times 64 \times 2 = 8192 \). For all the terminal vertices of the game tree, we get \( 64^5 = 1073741824 \) the options for getting to this terminal vertex. It is also necessary to take into account that there are only 16 such vertices, which increases the dimension of the problem by one more order.

To reduce the dimension of the problem, in the examples considered, a model with two mobile operators providing two services and a model with three mobile operators providing three services will be used. In the latter case, we get 27 options for choosing a mobile operator to meet demand at one vertex. And 531441 choices for each of the 8 end vertices.
5. Examples of user choices for a mobile operator

Let us consider the number of different choices for a mobile operator by the user of communication services. With the interaction of 2 operators providing 2 services, we get 4 options: ((1,1), (1,2), (2,1), (2,2)). The first option means that for voice communication in the home region the user of communication services chooses the first mobile operator for himself, for using mobile Internet the user of communication services chooses the first mobile operator for himself. The second option means that for voice communication in the home region the user of communication services chooses the first mobile operator for himself, for using mobile Internet the user of communication services chooses the second mobile operator for himself. The third option means that for voice communication in the home region the user of communication services chooses a second mobile operator for himself, for using mobile Internet the user of communication services chooses his first mobile operator. The fourth option means that for voice communication in the home region the user of communication services chooses a second mobile operator, for using mobile Internet the user of communication services chooses a second mobile operator.

5.1. An example of a positional game for 2 mobile operators

In this example, we will only consider price changes by the mobile operator and we will not consider the costs of advertising and network development.

The initial price matrix for communication services $W_0$ of mobile operators looks like $W_0 = \begin{pmatrix} 10 & 15 \\ 7 & 8 \end{pmatrix}$. The first operator offers two different price vectors for communication services. Thus, we obtain the matrix $W_1$ and the matrix $W_2$: $W_1 = \begin{pmatrix} 6.9 & 17.5 \\ 7 & 18 \end{pmatrix}$, $W_2 = \begin{pmatrix} 6 & 16 \\ 7 & 18 \end{pmatrix}$.

After that, the user of communication services has 4 choice strategies ((1,1), (1,2), (2,1), (2,2)), i.e. he can leave everything as it is, can change one operator for one service, can exchange one operator for another service and can exchange both operators for both services.

There is a second operator and offers two price vectors for communication services. After that, the user of communication services is again redistributed between operators:

$W_3 = \begin{pmatrix} 6.9 & 17.5 \\ 6 & 17 \end{pmatrix}$, $W_4 = \begin{pmatrix} 6.9 & 17.5 \\ 6.5 & 15 \end{pmatrix}$, $W_5 = \begin{pmatrix} 6 & 16 \\ 5 & 15.5 \end{pmatrix}$, $W_6 = \begin{pmatrix} 6 & 16 \\ 5.9 & 15.9 \end{pmatrix}$.

For a mobile operator, the game tree will look like this (Figure 1).

![Figure 1. Game tree](image)

In this case, the income in each of the terminal vertices will depend on the sequence in which and how the user of communication services will change (or not change) the mobile operator.
Consider the following user of communication services: voice communication, the first type of service he needs 3 units, mobile Internet, the second type of service he needs 2 units. The cost of changing the operator for him will be 1 unit. We will write the winnings of all the players in each of the terminal vertices, subject to a different choice of the user of communication services, the mobile operator. We denote by $H_1$ the gain of the communication services user, $H_2$ is the gain of the first mobile operator, $H_3$ is the gain of the second mobile operator.

| Vertex $W_3$ | $H_1$ | $H_2$ | $H_3$ | User way |
|-------------|-------|-------|-------|----------|
| 164,4       | 141,4 | 21    |       | $(2,1) \rightarrow (1,1) \rightarrow (1,1)$ |
| 164,4       | 106,4 | 55    |       | $(2,2) \rightarrow (1,1) \rightarrow (1,2)$ |
| 161         | 100   | 60    |       | $(2,2) \rightarrow (2,1) \rightarrow (2,1)$ |
| 161         | 65    | 94    |       | $(2,1) \rightarrow (2,1) \rightarrow (2,2)$ |
| 166         | 0     | 166   |       | $(2,2) \rightarrow (2,2) \rightarrow (2,2)$ |
| 171,4       | 171,4 | 0     |       | $(1,1) \rightarrow (1,1) \rightarrow (1,1)$ |

| Vertex $W_4$ | $H_1$ | $H_2$ | $H_3$ | User way |
|-------------|-------|-------|-------|----------|
| 164,4       | 141,4 | 21    |       | $(2,1) \rightarrow (1,1) \rightarrow (1,1)$ |
| 160,4       | 106,4 | 51    |       | $(2,1) \rightarrow (1,1) \rightarrow (1,2)$ |
| 162,5       | 100   | 61,5  |       | $(2,1) \rightarrow (1,2) \rightarrow (1,2)$ |
| 158,5       | 65    | 91,5  |       | $(2,1) \rightarrow (2,1) \rightarrow (2,2)$ |
| 163,5       | 0     | 163,5 |       | $(2,2) \rightarrow (2,2) \rightarrow (2,2)$ |
| 171,4       | 171,4 | 0     |       | $(1,1) \rightarrow (1,1) \rightarrow (1,1)$ |

| Vertex $W_5$ | $H_1$ | $H_2$ | $H_3$ | User way |
|-------------|-------|-------|-------|----------|
| 153         | 130   | 21    |       | $(2,1) \rightarrow (1,1) \rightarrow (1,1)$ |
| 153         | 98    | 52    |       | $(2,1) \rightarrow (1,1) \rightarrow (1,2)$ |
| 151         | 112   | 36    |       | $(2,1) \rightarrow (1,1) \rightarrow (2,1)$ |
| 151         | 80    | 67    |       | $(2,1) \rightarrow (1,1) \rightarrow (2,2)$ |
| 160         | 0     | 160   |       | $(2,2) \rightarrow (2,2) \rightarrow (2,2)$ |
| 160         | 160   | 0     |       | $(1,1) \rightarrow (1,1) \rightarrow (1,1)$ |

| Vertex $W_6$ | $H_1$ | $H_2$ | $H_3$ | User way |
|-------------|-------|-------|-------|----------|
| 153         | 130   | 21    |       | $(2,1) \rightarrow (1,1) \rightarrow (1,1)$ |
| 153,8       | 98    | 52,8  |       | $(2,1) \rightarrow (1,1) \rightarrow (1,2)$ |
| 153,7       | 112   | 38,7  |       | $(2,1) \rightarrow (1,1) \rightarrow (2,1)$ |
| 154,5       | 80    | 70,5  |       | $(2,1) \rightarrow (1,1) \rightarrow (2,2)$ |
| 163,5       | 0     | 163,5 |       | $(2,2) \rightarrow (2,2) \rightarrow (2,2)$ |
| 160         | 160   | 0     |       | $(1,1) \rightarrow (1,1) \rightarrow (1,1)$ |

Thus, we obtain a solution for the cohort minimum cost of communication services users (Figure 2). This will be the vertex of $W_5$, and here for the cohort there are two strategies with the same gain: $(2,1) \rightarrow (1,1) \rightarrow (2,1)$; $(2,1) \rightarrow (1,1) \rightarrow (2,2)$.

It is in this option that you can see how even insignificant costs for advertising or network development will help the first mobile operator to attract this particular cohort, because her costs will increase by only 2 units, while the profit of the first operator will increase by 50 units. The second mobile operator at the vertex of $W_6$ has a similar opportunity, the costs of the cohort will grow by only 1.5 units, and the operator’s profit will grow by 49.9 units.
In this example, the compromise solution for mobile operators coincides with the solution that would have been learned if the first operator chooses a cautious strategy. So at the first step, evaluating your minimum profit, the first operator chooses the $W_2$ strategy, where this minimum profit is greater. In the second step, the second operator chooses a strategy that maximizes his profit, and this is the $W_5$ strategy, which coincides with the compromise solution of this game tree.

5.2. An example of a positional game for 2 mobile operators providing 2 services and 6 different user cohorts

In the second example, we consider the interaction of 2 mobile operators and 6 user cohorts of communications services with different needs for voice and Internet traffic. We write down each of the cohorts needs in the following table 1.

| Cohort number | Voice Requirements | Mobile Internet Needs | Operator Change Cost |
|---------------|--------------------|-----------------------|----------------------|
| 1             | 4                  | 5                     | 2                    |
| 2             | 5                  | 2                     | 2                    |
| 3             | 6                  | 3                     | 5                    |
| 4             | 7                  | 4                     | 5                    |
| 5             | 5                  | 10                    | 30                   |
| 6             | 10                 | 5                     | 30                   |

Now we describe the payoff functions each of the cohorts and mobile operators at each of the end vertices $W_3$, $W_4$, $W_5$, $W_6$. $H_1$ - $H_6$ payoffs of the cohorts from the first to the sixth, $H_7$ and $H_8$ are the payoffs of mobile operators. And also for each of the cohorts we will write the path by which she came to the current vertex.

| For the vertex $W_3$ | $H_7$ | $H_8$ | Way                  |
|----------------------|-------|-------|----------------------|
| $H_1$                | 331,5 | 162,5 | (2,1) $\rightarrow$ (2,1) $\rightarrow$ (2,2) |
| $H_2$                | 202   | 100   | (2,1) $\rightarrow$ (2,1) $\rightarrow$ (2,1) |
| $H_3$                | 275   | 150   | (2,1) $\rightarrow$ (2,1) $\rightarrow$ (2,1) |
| $H_4$                | 345   | 200   | (2,1) $\rightarrow$ (2,1) $\rightarrow$ (2,1) |
| $H_5$                | 619   | 619   | (1,1) $\rightarrow$ (1,1) $\rightarrow$ (1,1) |
| $H_6$                | 465   | 0     | 465 $\rightarrow$ 465 $\rightarrow$ 465 |
| $H_7$ and $H_8$      | 1231,5| 990   | (2,2) $\rightarrow$ (2,2) $\rightarrow$ (2,2) |
Based on the payoffs of the mobile operators, we will find a compromise solution. To do this, we define at each vertex of the operator that made the least profit, and find the vertex at which this smaller value will be the largest. Such a vertex $W_3$, in which the first mobile operator makes a profit of 1231.5 units, and the second mobile operator receives 990 units. Moreover, for each of the cohorts, the maximum profit is brought by the vertex $W_5$. Unfortunately, user cohorts of communication services can only indirectly influence mobile operators, therefore, when choosing their move, mobile operators are more focused on the moves of competitors than on the actions of user cohorts of communication services.

Figure 3. Compromise solution
With such a game tree (Figure 3), a compromise solution will not be reached, because predicting its profit in the first move, the first mobile operator will choose the strategy W2. During the course of the second mobile operator, he will also strive to maximize his profit and, when the first operator chooses the strategy W1, the second operator will choose W4 to maximize his profit. The first mobile operator in this case will receive its minimum profit. Fearing just this, the first operator will choose the W2 strategy in its first move, and the second mobile operator will choose the W5 strategy to maximize its profit. At the same time, the user cohorts will receive the maximum of their profit.

6. Conclusion
Dividing into cohorts (groups) (by the number and type of consumed services, by the value of the operator change cost parameter) has significantly reduced the dimension of the multi-agent interaction model between mobile operators and users. Two classes of players are distinguished in the work: mobile operators and dedicated user cohorts. In this model, the goal of both class players was to maximize the profit function. The presented turn-based game contains strategies for players of the first and second classes. The payoff function of both class players built at every stage of the game made it possible to find a compromise solution for different classes of players.

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