MEASURING THE INITIAL MASS FUNCTION OF LOW MASS STARS AND BROWN DWARFS

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Abstract. I review efforts to determine the form and any lower limit to the initial mass function in the Galactic disk, using observations of low-mass stars and brown dwarfs in the field, young clusters and star forming regions. I focus on the methodologies that have been used and the uncertainties that exist due to observational limitations and to systematic uncertainties in calibrations and theoretical models. I conclude that whilst it is possible that the low-mass IMFs deduced from the field and most young clusters are similar, there are too many problems to be sure; there are examples of low-mass cluster IMFs that appear to be very discrepant and the IMFs for brown dwarfs in the field and young clusters have yet to be reconciled convincingly.

1 Introduction

The initial mass function (IMF) is the name given to the distribution of masses with which stars and brown dwarfs are born. Early in our astrophysical careers we learn some form of the Russell-Vogt theorem: that the mass and, to a lesser extent, composition of a star determine its radius, luminosity and its subsequent evolution in the Hertzsprung-Russell (HR) diagram. If we could determine the IMF within a population that has broadly the same composition, then in principle we would know almost everything about that stellar population and its evolution.

The IMF is perhaps the most fundamental output and diagnostic of the star formation process. As such, it acts as an external constraint that must be satisfied by any star formation theory. We can search for features or variations in the IMF as a function of environment or metallicity, thus identifying the factors that are important in shaping it (e.g. see reviews by Chabrier 2003; Bastian et al. 2010).

The low-mass IMF may be especially important as a star formation diagnostic. There is abundant evidence that low-mass stars ($m < 0.3 M_\odot$) and brown dwarfs

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DOI: (will be inserted later)
(BDs, $m < 0.075 M_\odot$) form in a similar way to stars like the sun (e.g. Luhman et al. 2007; Whitworth et al. 2007, 2010), but in many cases they must have masses well below any average thermal Jeans mass of the cloud where they were born. So what produces them and what stops them from accreting more mass? Many ideas have been put forward (ejection, turbulent fragmentation, photoevaporation) and are discussed elsewhere in this volume, but the important point is that these processes will put their imprint on the IMF. Some excellent examples of this can be found in a series of papers where smoothed particle hydrodynamic simulations are used to investigate how the inclusion of different pieces of physics, or differing intial conditions, can affect the IMF (Bate & Bonnell 2005; Bate 2005, 2009a, 2009b).

The IMF also provides a strong link between stellar physics and the physics of galaxies. Bastian et al. (2010) introduce (tongue-in-cheek) Davé’s theorem; that all problems in extragalactic astrophysics can be solved with an appropriate choice of IMF! It is certainly true that a knowledge of the IMF is vital in understanding the observable properties and mass budget of whole stellar systems and unresolved populations. For typical assumptions for the form of the IMF, more than 60 per cent of the stellar mass is in relatively dim stars with $m < 1 M_\odot$. Some of the highest profile results in extragalactic astronomy, such as the Schmidt-Kennicutt law relating galactic star formation rates to the surface density of gas (Schmidt 1959; Kennicutt 1998), or the “Madau diagram” plotting the star-forming rate in the universe as a function of redshift (Madau et al. 1996, 1998; Gonzalez et al. 2010), rely to some extent on an assumed IMF. The observational tracers of star formation activity ($H\alpha$, rest-frame UV continuum etc.) generally trace luminous high-mass populations that only make up a small fraction of the total stellar mass. Mass-to-light ratios and star formation rates, in solar masses per year, are inferred from synthesis models (e.g. Bruzual & Charlot 2003) combined with assumptions about the IMF.

In this article I summarise a series of lectures that were given as part of the 2011 Evry-Schatzman school on “Low-mass stars and the transition from stars to brown dwarfs”. The objective of these lectures was to review progress towards measuring the IMF of the Galactic disk based on observations of low-mass stars and brown dwarfs in the field and in young clusters. There is an emphasis on examining the methodologies used in these scenarios and I focus on weaknesses in techniques or uncertainties in physical models that render some results more reliable than others.

In section 2 I start with a historical perspective and examine how (sub)stellar masses are deduced from observational indicators and how mass-luminosity relations are calibrated. In section 3 I examine attempts to determine the low-mass stellar IMF from a census of local field stars, and this is extended to a consideration of the substellar field IMF in section 4 where different techniques need to be used. Section 5 is devoted to measuring the IMF in open clusters and star forming regions and the problems that must be overcome to yield reliable results. Section 6 reviews attempts to find any lower limit to the IMF in clusters and the field. Section 7 provides a summary of the major points, discusses areas where
progress needs to be made and perspectives for future research.

2 Luminosity Functions to Mass Functions

2.1 A historical perspective

The history of IMF and MF determinations begins with studies of the luminosity functions (LFs) of Galactic field populations conducted in the early part of the 20th century (read the excellent review of Reid & Hawley 2000). Perhaps the main motivation at that time for determining $\Phi(M)$, the number of stars per unit absolute magnitude ($M$) interval per cubic parsec, was to interpret star count data in an effort to understand the structure of our own Galaxy.

Early attempts to determine $\Phi(M)$ were of course hampered by a lack of accurate trigonometric parallaxes. A number of clever statistical techniques – the method of mean parallaxes or the use of relationships between absolute magnitude and “reduced proper motion” – were devised and exploited by pioneers such as Kapteyn and Luyten (see for example Kapteyn 1902; Luyten 1923). There were also significant problems with incompleteness for early samples selected from their large proper-motions. Later studies such as those of Kuiper (1942) attempted to construct volume-limited samples of nearby stars using trigonometric parallaxes, supplemented by stars with distances estimated from a relationship between absolute magnitude and colour – a photometric parallax. There were (and still are) significant uncertainties associated with such relationships for cool, low-mass stars. This was partly because the colour commonly used at that time was $B - V$, which becomes degenerate for $T_{\text{eff}} \leq 3600\,\text{K}$, but also because few stars at these temperatures were bright enough to have measured parallaxes that could be used for calibration. As a consequence, very little could be said for certain about the LF for $M_V > 11$ (spectral types cooler than about M2).

The situation has of course improved a great deal in the last few decades and will be discussed in more detail in section [x], but two clear points emerge from these early studies. (i) That incompleteness in samples of nearby stars is often a major problem when trying to study the IMF at low-masses and (ii) that even should the determination of $\Phi(M)$ be exquisitely accurate, we still need a way of converting luminosities to masses before the MF or even the IMF can be estimated.

2.1.1 Some definitions and formalism

Much of the formalism adopted in IMF studies is due to Salpeter (1955) and Miller & Scalo (1979), who looked at the LF of field stars in the Galactic disk and made attempts to determine the IMF from it. First there are the definitions of the luminosity and mass functions.

$$\Phi(M) = \frac{dN}{dM} \quad (2.1)$$
is the number of stars per unit magnitude ($M$) interval, per cubic parsec.

$$\phi(\log m) = \frac{dN}{d\log m} \quad (2.2)$$

is the number of stars per unit logarithmic mass ($m$) interval, per cubic parsec. Sometimes a linear mass function ($\phi(m) = dN/dm$) is preferred, but given that the IMF covers 4 orders of magnitude (from high mass stars to planets) and there are some arguments that the IMF may be log-normal in form (see below), then the logarithmic representation is commonly adopted.

There are a number of ways that the MF or IMF can be parameterised. The most common are a power law form with index $\alpha$

$$\phi(m) \propto m^{-\alpha} \quad \text{or} \quad \phi(\log m) \propto m^{-\alpha+1}, \quad (2.3)$$

or the log-normal form

$$\phi(\log m) \propto \exp \left( -\frac{(\log m - \log m_c)^2}{2\sigma^2} \right), \quad (2.4)$$

that has a width parameter $\sigma$ and a characteristic mass $m_c$ (Chabrier 2002, 2003, 2005). Other more exotic forms have been proposed, for example a smoothed two-power law

$$\phi(m) \propto m^{-\Gamma}(1 - \exp[-(m/m_p)^\gamma+\Gamma]) \quad (2.5)$$

that has power law indices $\Gamma$ at high masses, $\gamma$ at low-masses and a transition mass $m_p$ (de Marchi & Paresce 2001; Parravano et al. 2011). There is also the simple expedient of modelling the MF as a series of connected power law relationships with characteristic transition masses (e.g. Kroupa 2002).

If an attempt is made to measure the MF from field stars, this is not the IMF, because massive stars with short lives may well have completed their evolution and become white dwarfs, neutron stars or black holes, and will hence be under-represented. Some fraction (that increases with mass) of all stars with $m \geq 0.9 M_\odot$ will have evolved from the main-sequence given an age for the Galactic disk $T_G \approx 10$ Gyr. We refer to the MF measured in this way as the present day mass function (PDMF, $\phi_{PD}(\log m)$). Miller & Scalo (1979) defined a “creation function” $C(\log m, t)$, which is the rate at which stars form per unit (logarithmic) mass interval per cubic parsec. If we assume that the IMF is constant in time, then the mass dependence and time dependence are separable and allow the PDMF to be written as

$$\phi_{PD}(\log m) = \phi(\log m) \int_{T_G-T_{MS}}^{T_G} f(t) \, dt \quad \text{for } T_{MS} < T_G, \quad (2.6)$$

$$\phi_{PD}(\log m) = \phi(\log m) \quad \text{for } T_{MS} \geq T_G, \quad (2.7)$$

where $T_{MS}$ is the main sequence lifetime for a star of mass $m$, and $f(t)$ is an arbitrary function of time that encapsulates the star-forming history of the Galactic
disk. $f(t)$ is normalised such that its integral over time from zero to $T_G$ is unity. Thus we see that for stars with $M > 0.9 \, M_\odot$, some estimate of the star forming history is required to transform the PDMF to the IMF. For less massive stars the PDMF is the IMF, but we shall see that there is an additional complication that means that $f(t)$ becomes important once more for stars and BDs with $m \leq 0.1 \, M_\odot$.

2.1.2 Early attempts to obtain the IMF

The basic observational experiment is to determine the LF in the form $dN/dM$ and then multiply this by the slope of a mass-magnitude relationship, $dM/d \log m$, to get $\phi_{PD}$. An assumed star forming history is then used to correct $\phi_{PD}$ and obtain the IMF. Taking an LF determined by van Rhijn (1936) and Luyten (1941), a crude mass-magnitude relationship and assuming a uniform star forming history, Salpeter (1955) was able to propose that $\phi(m) \propto m^{-\alpha}$, with $\alpha = 2.35$, for stars with $0.4 < m/M_\odot < 10$. Miller & Scalo (1979) investigated various star forming histories and updated the field LF using what were the best proper-motion and parallax surveys of the time (e.g. Luyten 1968; Wielen 1974). Miller & Scalo found their IMF was best represented by a semi log-normal distribution with a characteristic mass of $0.1 \, M_\odot$ and width of $0.7$ dex for stars with $0.1 < m/M_\odot < 50$. This parameterisation roughly agrees with Salpeter over the mass range common to both, but corresponds to a larger value of $\alpha$ at $m > 10 \, M_\odot$ and smaller $\alpha$ for $m < 0.4 \, M_\odot$.

Both of these highly cited early attempts to measure the IMF are entirely reliant on the fidelity and completeness of the low-mass star surveys and an accurate relationship between mass and magnitude. The former will be discussed in section 3, the latter is discussed here.

2.2 Mass-Magnitude relationships for low-mass stars

The conversion of a LF to a MF relies on a mass-magnitude relationship. This can be taken from evolutionary models of stars, especially for pre-main sequence (PMS) or very low-mass objects (see sections 4 and 5), but can be determined empirically for low-mass main sequence stars. The requirements for this calibration are stars with a range of masses that have an accurate distance and photometry, in order to determine their absolute magnitude, and of course a means of measuring their mass. The former is most usually found through a trigonometric parallax, but the latter is more difficult to obtain.

2.2.1 Stellar masses

The masses of stars come chiefly from measurements of stars in binary systems – either resolved, astrometric binaries or close, eclipsing binaries.

In an astrometric binary, the relative orbit is an ellipse with the primary star at one focus. An inclined orbital plane means that the projected orbit on the plane of the sky is also an ellipse, but the primary is not at the focus. The displacement
of the primary from the focus yields the orbital inclination and then the semi-major axis and eccentricity can be deduced. The semi-major axis and orbital period give the total system mass from Kepler’s third law, but the semi-major axis needs to be in physical, rather than angular units, so the distance to the binary is required (see Kraus et al. 2009 for an example of this approach). If one has absolute orbits (i.e. the RA and Dec of each component as a function of time, rather than just separation and position angle), then the ratio of the apparent orbital displacements gives the inverse mass-ratio. Combined with the system mass this gives the masses of the individual components. Likewise, the mass-ratio can be constrained by measurements of the radial velocity (RV) curves of the two components (e.g. see Andersen et al. 1991; Ségransan et al. 2000 for applications to low-mass stars).

In an eclipsing binary, the mass-ratio again comes directly from the amplitude ratio of the RV curves and gives one constraint on the individual masses. The system inclination (and the stellar radii) can be obtained by modelling the eclipses and then a second constraint comes from the dynamical mass function

$$\frac{m_2 \sin^3 i}{(m_1 + m_2)^2} = \frac{PK_1^3}{2\pi G},$$

(2.8)

where $P$ is the binary orbital period and $K_1$ is the measured RV amplitude of the primary. This leads to component masses without the need for an accurate parallax (e.g. see Morales et al. 2009; Torres et al. 2010).

2.2.2 The empirical and theoretical mass-magnitude relationships

After measuring masses, the photometry and parallaxes are combined to give an empirical mass-magnitude relationship. This can be used straightforwardly to derive mass functions from luminosity functions. A comparison with models is valuable though, because we do need to understand some details of the structure and atmospheres of low-mass objects in order to identify possible sources of systematic error in the relationships. It is also a pre-requisite that models match the data we have before we can have any confidence in extrapolating these models to ages and masses that are not yet well-calibrated by observational data.

The best-known relationships for low-mass stars are those presented in Delfosse et al. (2000, see their Fig.3; but see also Henry & McCarthy 1993; Xia et al. 2008). The main conclusion that can be drawn from this study is that the tightest relationships are found between near-infrared absolute magnitudes and mass. These are in good agreement with the main-sequence properties predicted by evolutionary models such as those of Baraffe et al. (1998) and Siess et al. (2000) over the range $0.1 < m/M_\odot < 0.8$. In contrast there is much more scatter (about one magnitude, or about 30 per cent in mass) in the $V$-band relationship, and the models show significant discrepancies with data at the lowest masses.

The likely explanation is that the $V$-band magnitudes of cool, low-mass stars are highly dependent on some uncertain atmospheric opacities, that themselves are quite metallicity dependent (see elsewhere in this volume and Fig. 3). A field
population of low-mass M-dwarfs will have a range of metallicity, so the scatter in the mass-magnitude relation is not unexpected, but the overall discrepancies probably indicate remaining problems in calculations of atmospheric opacities at low temperatures. The near-infrared magnitudes are much less uncertain and much less dependent on metallicity. The lesson is clear – the most reliable results will come from near-infrared assessments of field luminosity functions.

A further issue to consider is any age dependence of the mass-magnitude relation. At $m > 0.8 \, M_\odot$ the oldest field stars will have started to increase in luminosity as they evolve on the main sequence (see Fig 2) and this must be accounted for in any detailed estimate of the MF. At younger ages, low-mass objects are still descending towards the ZAMS (which they will never reach if $m < 0.07 \, M_\odot$). The PMS timescale is roughly equivalent to the Kelvin-Helmholtz time, which is approximately proportional to $m^{-1}$. For a uniform star formation rate over 10 Gyr, about 10 per cent of stars with $m = 0.1 \, M_\odot$ will still be on the PMS, thus
estimates of the field MF could safely use a main sequence mass-magnitude relationship for $0.1 < m/M_\odot < 0.8$ (see Fig. 2), but stars outside of this range would be more luminous (on average) than a main sequence relationship would predict. Of course this time-dependence can be important at all masses when dealing with young clusters (see section 5).

### 2.3 Mass-Magnitude Relationships for Brown Dwarfs

For $m \geq 0.08 M_\odot$ the core temperature eventually becomes high enough to initiate hydrogen fusion, but the cores of objects with $m < 0.07 M_\odot$ never attain this status; such objects reach a minimum radius and then simply cool and fade as degenerate BDs (see Chabrier & Baraffe 1997; Burrows et al. 1997). A further complication is that the brief phase of deuterium burning in the early PMS is more extended in the lowest mass objects, perhaps 100 Myr at $m = 0.015 M_\odot$, but at even lower masses, even deuterium burning does not occur.

This complicated luminosity evolution means that BDs are never stationary in luminosity or the HR diagram. This offers problems and opportunities. In principle, like PMS stars, position in the HR diagram could determine both mass and age (if the models were correct!). In practice, a typical $T_{\text{eff}}$ measurement uncertainty (for instance from a spectral type) of $\pm 150$ K, which may also have systematic errors, encapsulates BD “cooling tracks” with a mass range of factors of 2–3 (see Fig. 11 in Burrows et al. 1997). Furthermore, although the various available models are in reasonable agreement on the mass-luminosity relation as a
function of time, they do not agree on the $T_{\text{eff}}$ scale, so that different models would lead to very large differences in masses deduced from the HR diagram (Konopacky et al. 2010).

Progress can be made by testing the theoretical mass-luminosity relationships for very low mass objects. If the model predictions could be confirmed for a few benchmark objects then we could be more confident in extrapolation or interpolation to other masses and ages.

Dynamical masses for BDs can be estimated in similar ways to those described earlier for low-mass stars. However, Kepler’s third law tells us that $(m_1 + m_2) \propto a^3/P^2$, where $a$ is the semi-major axis. To measure orbital motion on decadal timescales requires separations of less than a few au. BD binaries, even at 10 pc, would still be very faint and have angular component separations less than a few tenths of an arcsecond. Separating these requires high spatial resolution that until recently could only be achieved for a few nearby objects with adaptive optics (Close et al. 2003) or from space (Bouy et al. 2003).

The 2MASS, SDSS and UKIDSS surveys have resulted in an explosion of BD discoveries, approximately 10 per cent of which are in binary systems. Most of these are not bright enough to act as their own natural guide stars, so it is only with the advent of laser guide star technology that large programs monitoring many potential binary systems have become possible (Dupuy et al. 2009; Konopacky et al. 2010).

One of the first BD masses to be confirmed was GL 569Bab with a system mass of 0.123 M$_\odot$ (Lane et al. 2001). This 890-day binary has now been followed for more than a decade, and RV measurements give component mass of 0.073 M$_\odot$ and 0.053 M$_\odot$ (Konopacky et al. 2010). Compared with the theoretical HR diagram and the appropriate cooling tracks it is found that whatever its age and whichever models are used, the GL569Bab system is either too luminous or the temperatures determined from matching synthetic models to near infrared spectra are too cool. To put it another way, the mass deduced from the HR diagram would be far too low. Konopacky et al. (2010) have conducted this kind of analysis for a dozen low mass L-dwarf binaries and found a consistent picture of dynamical masses that are higher than expected. This could be a problem either with the evolutionary models or perhaps it is a problem with the atmospheres used to model the effective temperature. It is also of interest that the one T-dwarf binary studied by Konopacky et al. has a lower dynamical mass than expected.

A different kind of test and one which is more incisive for IMF studies is to investigate the theoretical mass-luminosity relationship for binaries with known age. Unfortunately, BD binaries in the nearest open clusters are currently beyond reach, but more indirect methods of estimating age can be used. Close et al. (2005) used adaptive optics to measure the orbit and estimate the mass of AB Dor C, a faint companion to a young star which is the prototype member of a proposed coeval kinematic group in the solar neighbourhood. For a mass of 0.09 M$_\odot$, and an assumed age for the AB Dor association of 50 Myr, Close et al. concluded that AB Dor C is a factor of 2–3 underluminous compared with model predictions. However, the ages of these kinematic groups are not beyond dispute. Luhman et
al. (2005) reassessed the age of the AB Dor association, finding an age of 100–
125 Myr. At this older age, the models predict the correct luminosity for AB Dor
C.

Another recent example is HD 130948BC, a close-to-equal luminosity (and
hence mass) L-dwarf binary around a field G-star with a known rotation period
and metallicity. Dupuy et al. (2009) obtained a relative orbit and system mass of
0.109 M⊙. The rotation of the G-star suggests an age of 800 Myr. Like other BD
binaries (see above), the components of HD 130948BC are either too luminous or
too cool compared with the theoretical HR diagram cooling tracks. The relatively
precise age determination (± ≃ 200 Myr) shows that using the measured luminos-
ity and current evolutionary models would over-estimate the component masses
by 30 per cent.

A contrasting picture is found for the recently discovered system ǫ Indi Bab.
This has great potential to constrain models, because its components are very cool.
It is a T1+T6 close binary in a wide orbit around a K4.5V star at 3.6 pc. The 12
year T-dwarf binary orbit gives a dynamical system mass of 121 M_Jup (Cardoso
et al. 2010), which given their luminosities indicates that both components are
likely to be BDs. The positions of the components in colour-magnitude diagrams
are not well described by current models (King et al. 2010). This is not a new
result (see later), but what is more alarming is that while the magnetic activity
of the K4.5V primary star indicates an age of < 2 Gyr, the measured luminosity
and dynamical mass demand an age of 4 ± 0.3 Gyr. In other words, just using
the measured luminosity and the age from magnetic activity would result in the
deduction of a mass that was at least 25 per cent too low.

In summary, whilst the mass-magnitude relationships for low mass stars with
m > 0.1 M⊙ are reasonably secure, there remain significant uncertainties when
converting luminosities to masses in the case of objects with m < 0.1 M⊙ and a hint
of discrepancies that may vary with the age or surface temperature of the object.
In addition to the physical uncertainties introduced by an age dependence in the
mass-luminosity relation and the difficulty of estimating the ages unless objects
are in a coeval cluster, it seems quite likely that there are some quite important
systematic errors in the evolutionary models and/or the model atmospheres that
remain to be resolved.

3 The Initial Mass Function from Low Mass Field Stars

In this section I discuss measurements of the Galactic disk IMF using observations
of low-mass field stars. The PDMF is roughly equivalent to the IMF for stars
with 0.1 < m/M⊙ < 0.8, in the sense that the effects of stellar evolution on the
mass-magnitude relations are minimal (section 2.1.1). Of course the PDMF of
an arbitrary star sample might still not be the IMF if there were any significant
variation of the IMF in time or space.

The basic technique consists of three steps: (i) Measure the luminosity or
absolute magnitude of stars that lie within some defined volume; i.e. compute
the LF. (ii) Convert the luminosities or absolute magnitude to masses using the
relationships discussed in the last section. (iii) Correct the derived MF (or the intermediate LF) for incompleteness, biases, binarity, Galactic structure and stellar evolution. It is also possible to use a more inductive approach. This would assume a form for the IMF to predict the star counts or apparent magnitude distribution of a sample by folding it through a mass-magnitude relation, a Galactic structure model and then filter the result according to observational sensitivity and uncertainties. However, the former three-point procedure is pedagogically more sensible to describe here and within that scheme there are broadly two routes.

The first uses star counts in the solar neighbourhood to build up a volume limited sample. An implicit assumption is that the gradual dispersion of Galactic orbits means that the solar neighbourhood offers a representative sample of stars from the disk, even if modest corrections to the IMF are needed to account for the Sun’s present position. The advantages of this technique arise from the proximity of the stars. Even very low-mass stars are reasonably bright if nearby; distances can be measured with potentially very precise trigonometric parallaxes and it is possible to resolve a large fraction of any binary population. On the other hand the sample may suffer from uncertain levels of incompleteness because of the inhomogeneous methods used to identify nearby stars. Samples may also have small number statistics if selected from a very small local volume, and could be afflicted by Lutz-Kelker bias (see section 3.1.2).

A second method can be characterised as a photometric field star survey. These may be wide and shallow or very deep pencil-beam surveys. The advantages here are that one can obtain very large samples and the level of incompleteness can be well controlled. Against this must be set the disadvantages of relying on photometric parallaxes for distance measurements, Malmquist bias, unresolved binarity and a much greater reliance on Galactic structure models in the interpretation of deep surveys.

3.1 The IMF from the local field population

3.1.1 A census of the solar neighbourhood

Since the beginning of the 20th century proper motion measurements have been used to identify nearby stars. Typical tangential velocities combined with proximity leads to high proper motion (e.g. Luyten 1924; van Maanen 1937). The early work culminated in classic works collating high proper motion stars – the Luyten Two-Tenths catalogue (LTT – Luyten 1957) the Luyten Half Second catalogue (LHS – Luyten 1979a) and the New Luyten Two Tenths catalogue (NLTT - Luyten 1979b). These surveys, which subsequently formed the basis for most work on nearby stars, are strongly biased against the inclusion of southern hemisphere stars, and to this day there are still possibilities to find new, relatively bright, nearby stars in the south (e.g. Hambly et al. 2004; Lépine 2005).

Proper motions were used by Gliese (and later Jahreiss) to compile the first, second and third catalogues of nearby stars, within 20 pc, 22 pc and 25 pc respectively (Gliese 1957; Gliese 1969; Gliese & Jahreiss 1991 – CNS3). About half of the
stars in this latter catalogue had trigonometric parallaxes; the rest were included on the basis of photometric and spectroscopic parallaxes (many have since proved to be erroneously included). In the 1990s the Palomar Michigan State University (PMSU) survey conducted extensive spectroscopic investigations of CNS3, refining it and defining complete samples to magnitude-dependent distance limits (Reid et al. 1995; Hawley et al. 1996; Gizis et al. 2002). Throughout the last decade efforts have focused on (i) pushing these limits further outward for faint low-mass stars using the NLTT/2MASS catalogues and spectroscopy (Reid et al. 2004; Cruz et al. 2007); (ii) using reanalyses of older plate material to increase completeness (Lépine et al. 2003; Lépine 2005); and (iii) establishing better trigonometric parallax data for the very nearest objects (Henry et al. 2006). It is now likely that for stars, the sample is complete to 8 pc in the northern hemisphere, with further work to do in the south; is rapidly becoming complete to 10 pc; and is about 80 per cent complete to 20 pc (Reid et al. 2007).

3.1.2 Lutz-Kelker bias

How is a nearby sample constructed? Unfortunately, even if trigonometric parallaxes are available, there are significant problems in using these unless they are precise. Lutz & Kelker (1973) showed that if stars are uniformly distributed in space then more are scattered into a sample from a larger true parallax than are scattered out. This means that if the true parallax is $\pi_0$, the observed parallax is $\pi$ and the uncertainty in the measurement is $\sigma_\pi$, then the mean value of $\pi_0/\pi$ is less than unity. The estimated absolute magnitude of such a sample will be biased such that $M_{\text{true}} = M_{\text{obs}} + \Delta M$. The bias $\Delta M$ is negative and a rapidly increasing function of $\sigma_\pi/\pi$, such that $\Delta M = -0.11$ mag for $\sigma_\pi/\pi = 0.1$, $\Delta M = -0.28$ mag for $\sigma_\pi/\pi = 0.15$ and essentially no meaningful correction can be made for $\sigma_\pi/\pi \geq 0.2$. In other words, contamination of the sample with objects that are at up to an infinite distance becomes important! In practice, the corrections may be complicated by the fact that the parent sample is not uniform, for example there could be growing incompleteness at larger distances (Hanson 1979).

The Lutz-Kelker effect was amply demonstrated when some of the earlier nearby star catalogues were compared with much more accurate parallax data (for bright stars) from the Hipparcos satellite (Jahreiss & Wielen 1997). About one third of the original CNS3 sample was excluded, and this fraction would have been higher if many stars had not already been excluded from CNS3 using additional photometric/spectroscopic parallax information.

A further important application that may be susceptible to this bias is when samples with trigonometric parallax are used to calibrate mass versus absolute magnitude or absolute magnitude versus colour relationships. Such relationships should be treated with extreme caution if the samples used have parallax uncertainties any larger than 10 per cent.
A representative (and pedagogic) example of the nearby star approach is provided by Reid et al. (2002). These authors constructed a volume limited sample for $8 < M_V < 16$ (roughly spectral types M0V and cooler), confined to declinations $>-30^\circ$ (558 stars in 448 systems). This was supplemented with Hipparcos data for $M_V < 8$ (1028 stars in 764 systems), which after filtering out giants and white dwarfs, should be complete to 25 pc. About 90 per cent of this sample have precise trigonometric parallaxes.

A number of steps were taken to assess the completeness of the sample. There is a trivial spatial incompleteness in the faint sample that is easily dealt with. Another factor considered and rejected was whether there was a bias away from finding high proper motion stars in crowded fields close to the Galactic plane, though there was no investigation of this as a function of magnitude. Differential completeness among fainter stars was assessed in absolute magnitude bins by plotting stellar density as a function of distance, under the assumption that it should be uniform. This resulted in a completeness limit of 20–22 pc for stars with $M_V \simeq 9$, but only 5 pc at $M_V = 15.5$.

Two further important issues were investigated. The first was whether selection by proper-motion leads to incompleteness. As the nearby stars were predominantly identified on the basis of their large proper motions it is possible that objects with low proper motions, which are more likely to be at larger distances, have not been identified. For instance, detection in the NLTT catalogue at a proper motion of $>0.2$ arcseconds/yr corresponds to a tangential velocity of $>20$ km s$^{-1}$ at 20 pc, which some stars may not possess. This was tested by comparing the transverse motions with those of a sample selected only by their photometric parallaxes. No significant differences were found, but there remains the likelihood that some low proper motion objects remain undiscovered. The second issue was binarity. Two LFs can be constructed; one with the luminosity of “systems”, the second with the luminosity of the components of those systems. Binarity has been subjected to extensive investigation in the PMSU survey, but less attention has been given to the brighter Hipparcos sample! Reid et al. coped with this by assuming that the known binary population in the brighter stars is representative of the whole sample and simply assigning these stars double weight in the analysis to deal with an assumed factor of two incompleteness in the census. Despite this additional assumption, knowledge of the binary fraction and companion properties is a key advantage of any local population survey.

The nearby star LF is shown in Fig. 3, where the “system” and total LF are shown separately. Points to note are the significant peak in the LF at $M_V \simeq 12$ (spectral type M3, $\simeq 0.25$ $M_\odot$). There is also a hint of the “Wielen dip” (Wielen 1974) at $M_V \simeq 7.5$, which is now known to be caused by a kink in the mass-magnitude relation caused by the details of changing opacities in the outer envelope (Kroupa & Tout 1997). Finally, note the large Poissonian error bars that increase for $M_V > 10$ (due to the small volume sampled at these magnitudes) and that systematic uncertainties due to unresolved or undiscovered binaries are also likely
Reid et al. convert this LF into a PDMF using two different mass-magnitude relations (Kroupa et al. 1993; Delfosse et al. 2000), with little difference in the results. It is worth noting that they used the absolute $V$ band magnitude in a population that must have a wide spread of metallicities. From section 2.2.2 we understand that a near infrared mass-magnitude relationship would be less metallicity sensitive and shows less scatter, but if the calibrating binary population has a similar metallicity distribution to the field population then the results should be reasonably robust.

To get from this observed PDMF to an IMF requires corrections for (i) stellar evolution, (ii) the Galactic density distribution and (iii) the mix of stellar populations that are included in the sample. Reid et al. assume that the creation function $C(\log m, t)$ is time-independent in which case the correction factor at any mass is $\tau_G/\tau_{MS}$ (if $\tau_{MS} < \tau_G$). The second point arises because lower mass stars are (on average) older, have a larger velocity dispersion and hence a large scale height with respect to the Galactic plane. For a density of the form $\rho = \rho_0 \exp(-z/z_0)$, then the surface density of the disk is $\propto \rho_0 z_0$. Reid et al. model this with a steep rise in scale height from $z_0 = 100 \, \text{pc}$ for $M_V < 3$, to $z_0 = 250 \, \text{pc}$ for $M_V > 4$. Hence this is a very significant correction and potentially there may yet be some mass dependence among the lower mass stars. Finally, Reid et al. make a minor 10 per cent downward correction for $M_V > 4$, assuming that this fraction of the census consists of very old, extended thick disk stars that make no contribution to the IMF of the Galactic disk.

The Reid et al. (2002) IMF of the Galactic disk is reasonably well modelled

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**Fig. 3.** The luminosity function of nearby stars from Reid et al. (2002). The contribution of systems and all the components of systems are shown separately.
between $0.1 < m/M_\odot < 3$ with a pair of power laws of the form $\phi(m) \propto m^{-\alpha}$, with $\alpha \simeq 2.8$ for $m > 1.1M_\odot$ and $\alpha \simeq 1.3$ below this. The IMF shows no significant evidence for a peak in the considered mass range (see Fig. 4) and is less well represented by a log-normal parameterisation (equation 2.4).

3.2 Photometric field star surveys

3.2.1 Wide field and pencil beam surveys

The alternative approach is to select a sample of low-mass stars from a photometric survey; estimate their distances from a photometric parallax; and then construct a LF that must be corrected for issues such as binarity, Galactic structure and Malmquist bias (see below). These surveys can arbitrarily be divided between those that cover relatively large areas on the sky, but reach comparatively shallow apparent magnitude limits (e.g. Tinney et al. 1993; Covey et al. 2008; Bochanski et al. 2010), and narrow, “pencil-beam” surveys going to great depths (e.g. Gould et al. 1997; Martini & Osmer 1998; Zheng et al. 2001; Schultheis et al. 2006).

The earlier studies were focused on whether the $\alpha \simeq 2.35$ “Salpeter law” extended to very low masses, because this would clearly have had a bearing on the question of baryonic dark matter. In the event, these studies appear to show that the slope of the IMF must fall to $\alpha \simeq 1.0 \pm 0.5$ somewhere between $0.5 M_\odot$ and $1.0 M_\odot$, in agreement with the nearby star IMF, and so low-mass stars are not a significant contributor to dark matter.

3.2.2 A case study

The recent work by Bochanski et al. (2010) makes an excellent case study. This consisted of magnitude- and colour-selected samples from 8400 square degrees of the SDSS. After cleaning out bad photometry and galaxies, the sample consists of 20 million late K- and M-type dwarfs. As the depth limit of $r' < 22$ is sufficient to probe well above the Galactic plane, the survey is both deep enough and wide enough to encompass problems that affect both general types of photometric field star survey.

Bochanski et al. use several colour-magnitude relationships in order to estimate distances to their sample stars. These relationships were calibrated in the SDSS photometric bands using stars with very precise ($\sigma_\pi/\pi < 0.1$) trigonometric parallaxes. The typical scatter is 0.4 mag, which is important when considering Malmquist bias (see below). For each half-magnitude interval, the stellar density is calculated as a function of position with respect to the Sun. These densities were matched to a 2-D Galactic model; the free parameters consisted of vertical and radial scale heights for both thick and thin disk components, $f$, the fraction of the sample belonging to the thick disk population, and a normalisation equal to the local density at the solar position. The values of local density are the raw measurement of the LF that then needs correcting for systematic effects.

The colour-magnitude relationships have a poorly calibrated metallicity dependence and this combined with the expected metallicity gradient as a function
of height above the Galactic plane lead to systematic differences in the LF – the scale heights become smaller and the local densities a little larger. For plausible metallicity gradients this effect is small.

Extinction is modelled using a Galactic dust model. The dominant effect is the reddening of stars rather than attenuation, which causes absolute magnitudes to be underestimated. Most of the stars in the sample probably have very small extinction \( (A_r < 0.4) \) and so even if extinction is ignored entirely it only increases the LF by \( \sim 10 \) per cent among the brighter stars.

### 3.2.3 Malmquist Bias

The luminosity function of a magnitude-limited sample, or one where distances have been estimated from photometric/spectroscopic parallax, needs to be corrected for Malmquist bias (Malmquist 1936). This arises because distant stars with brighter absolute magnitudes (at a given colour or spectral type) scatter into the survey volume either because of the intrinsic dispersion in the colour-magnitude relation or because of observational uncertainties. A classical expression for the Malmquist bias, assuming a Gaussian dispersion \( \sigma \) in the colour-magnitude relation, is

\[
M = M_{\text{true}} - \frac{\sigma^2}{N(\text{mag})} \frac{dN(\text{mag})}{d\text{mag}},
\]

where \( M \) is the mean absolute magnitude estimate, \( M_{\text{true}} \) is the true mean absolute magnitude, and \( N(\text{mag}) \) is the number of stars per unit apparent magnitude. For a uniform spatial distribution of stars this leads to the well known result that \( M = M_{\text{true}} - 1.38 \sigma^2 \) (e.g. Butkevich et al. 2005). In turn, this correction affects the observed LF because the sampled volume is effectively increased and also, if \( \Phi(M) \) has a steep slope, the LF is being sampled at a value \( M < M_{\text{true}} \).

In non-uniform samples the Malmquist bias is more complex. Bochanski et al. (2010) deal with it using Monte Carlo simulations to inject the appropriate scatter into the absolute magnitudes and colours and then recalculating the LFs. The result is a significant (10–40 per cent), luminosity-dependent decrease in the LF (and hence MF). It is worth noting that if a survey is sensitive enough to reach out to, and beyond, the edge of a spatial distribution – for example a very deep pencil beam survey out of the Galactic plane – then the Malmquist bias may be negligible (e.g. Gould et al. 1997).

### 3.2.4 Binarity

Binarity is another thorny issue in photometric surveys because usually the sample stars are so distant that no sensible attempt to resolve binary components is possible, and no systematic survey for RV variations will have been carried out. This is certainly the case for the SDSS sample studies by Bochanski et al. (2010). Unresolved binarity causes two main problems in LF and MF determinations. First, the actual number of stars present in the survey is greater than first thought and the masses of those stars will differ from a simple estimate based on the system
luminosity. Second, in a magnitude-limited survey, the observed LF and MF are enhanced because binaries are brighter than single stars of the same colour and can be seen to greater distances. A simple example would be a survey containing only equal-mass binaries. There would be twice as many stars as there are measured stars in the survey, but they will gathered from $2\sqrt{2}$ times the volume that is estimated from the colour-magnitude relationship for single stars.

Bochanski et al. chose to construct two separate LFs, one corresponding to whole systems and the other to components of those systems (including single stars). To calculate the component LF, a fraction of binary systems is added to the original stellar catalogue. These have secondary component luminosities (and corresponding colours) drawn randomly from the system LF, but constrained to be fainter than the primary. The flux and combined colour of each pair is then calculated. This new stellar catalogue is passed through the analysis pipeline again to determine a new system LF and this is compared with the initial system LF. The initial system LF is then “tweaked” and the process iterated until the artificially generated system LF matches that which was observed.

Dealing with binarity will always be a weakness of unresolved field star surveys because the binary properties of the lowest mass stars are rather poorly known. The binary frequency is likely to decrease from $\sim 50$ per cent at a solar mass to about 10–20 per cent at the lowest stellar masses (see Duquennoy & Mayor 1991; Fischer & Marcy 1992; Allen 2007 and Artigau’s contribution in this volume). Bochanski et al. (2010) tried a variety of binary prescriptions, with both fixed binary frequencies of 30–50 per cent and also a frequency that declined with decreasing luminosity between these two limits. These differing assumptions play a significant role in estimating the LF of the component stars, especially at the lowest luminosities (and masses, $m < 0.3 M_\odot$), because many stars will have unresolved companions at these masses (see Fig. 21 in Bochanski et al. 2010).

3.2.5 Results and parameterisations

Bochanski et al. (2010) convert their LFs to MFs using the J-band LF combined with the mass-magnitude relationship from Delfosse et al. (2000). The results for single stars plus the components of binary systems are reasonably comparable with the IMF from local field stars (Reid & Gizis 1997; Reid et al. 2002) and are best fitted with a log-normal mass function (see equation 2.4) with $m_c = 0.18 M_\odot$ and $\sigma = 0.34$ dex, though a broken power law with $\alpha = 2.38$ for $m > 0.32 M_\odot$ and $\alpha = 0.35$ for $0.1 < m/M_\odot < 0.32$ is almost as good (see Fig. 4). The system IMF is fit with $m_c = 0.25 M_\odot$ and $\sigma = 0.28$ dex. In fact, the evidence for any peak in the IMF is very weak on the basis of this sample alone; the peak of the fitted function appears to coincide with a slight (but insignificant) dip in the IMF found from both local and extended samples. Note that whilst the statistical uncertainties in the Bochanski IMF are extremely small, the contribution of the systematic uncertainties (extinction, metallicity dependence, binarity, the Galactic model, Malmquist bias) means that the final estimated uncertainties are as large as the Poissonian uncertainties in the IMF determined from small local volumes,
The initial mass function for stars (single stars plus components of binary systems) based on a local sample (from Reid et al. 2002) and derived from a deep photometric survey (Bochanski et al. 2010). The IMFs have been corrected for stellar evolution (for \( m > 1 \, M_\odot \)) and are based on the local density of stars. The solid line is a log-normal fit to the Bochanski et al. IMF, with \( m_c = 0.18 \, M_\odot \) and \( \sigma = 0.34 \, \text{dex} \) (see equation 2.4).

Compared to the IMF from nearby stars there is an excess of stars in the Bochanski IMF for for \( 0.15 < m/ M_\odot < 0.4 \) and a deficit for \( m > 0.6 \, M_\odot \). A suggested explanation for the deficit is neglecting to include lower mass binary companions to stars with \( m > 0.9 \, M_\odot \) that are not included in the survey (e.g. Parravano et al. 2011). Certainly, the width of the fitted log-normal functions are considerably narrower than the \( \sigma = 0.55 \, \text{dex} \) suggested by Chabrier (2005) for the field IMF, although the peak mass is similar.

### 3.3 Summary of the Field Star IMF

Assuming that the PDMF is almost equal to the IMF readily allows its determination for stars with \( 0.1 < m/ M_\odot < 0.8 \) in the general field population. This can be done using volume-limited local surveys or larger, magnitude-limited photometric surveys. The uncertainties in local surveys are dominated by small number statistics at low masses, whilst large scale photometric surveys are dominated by systematic uncertainties that include the metallicity dependence of photometric parallax relations, extinction, Malmquist bias and corrections for binarity.

The IMF from both types of survey is converging to a uniform result. The IMF can be represented in terms of more than one power law (Kroupa 2002; Parravano et al. 2011) or a log-normal function with characteristic mass of 0.15–0.25 \( M_\odot \).
Fig. 5. A comparison of the different functional forms that have been proposed to describe the IMF of the Galactic disk. The functions have been normalised to be similar at $0.4 \, M_\odot$ where the observational data are most constraining. Above $1 \, M_\odot$ the IMF is Salpeter-like, but significant differences between the different parameterisations arise for $m < 0.2 \, M_\odot$.

(Chabrier 2005). The key point that seems almost universally agreed is that the IMF flattens significantly from a Salpeter-like slope at masses of $0.5-1 \, M_\odot$. That there is little agreement on which parameterisations is best is hardly surprising. When plotted such that they are normalised to be equal at $\simeq 0.4 \, M_\odot$, there is little difference between them in the range $0.2 < m/ M_\odot < 0.8$ where the data are most secure (see Fig. 5). There are however clear differences at lower masses and especially in the brown dwarf regime and it is to the IMF of BDs that we turn next.

4 The Mass Function for Brown Dwarfs

The IMF of low-mass stars is estimated by constructing a LF and then converting this to a MF using mass-magnitude (or mass-luminosity) relationships that are (i) quite well calibrated and (ii) not very age dependent. This section moves to lower masses ($m < 0.1 \, M_\odot$), where neither of these properties can be assumed. The age-dependent mass-luminosity relationship demands a different, inductive approach. Despite these difficulties, enormous efforts have been expended on this task because there is hope that the form of the IMF at low-masses will not only solve the problem of which parameterisation best represents it (see Fig. 5), but more importantly will actually yield physical insight into which mechanisms control the form of the IMF, determine it’s characteristic mass and what might be responsible for IMF
variations (e.g. Padoan & Nordlund 2004; Hennebelle & Chabrier 2008). In this section I explain the methodologies adopted and discuss the results.

4.1 The mass-luminosity relation for brown dwarfs

BDs are very low-luminosity objects with no nuclear source of heat (bar a brief deuterium-burning phase if $m > 0.015 \, M_\odot$). As field objects they were first identified as companions to low-mass stars (Nakajima et al. 1995), and even in the near-IR are $> 7$ mag fainter than solar-type stars. The brightness contrast is less severe for young brown dwarfs and another fruitful place to find them has proved to be young clusters like the Pleiades and the Orion Nebula cluster (see section 5; Rebolo et al. 1995; Lucas & Roche 2000). Searching for isolated objects in the field with $m < 0.1 \, M_\odot$ poses significant problems that have only recently been addressed.

The mass-luminosity relation of BDs was discussed in section 2.3, but it is worth re-iterating the main points. First, the mass-luminosity relation of BDs is age-dependent. That is, even were the evolutionary models absolutely accurate, then the age of a BD is required before one can estimate a mass from its luminosity. Second, the accuracy of the models cannot be assumed. There is already a lot of evidence that the theoretical evolutionary models and the model atmospheres do not correctly predict luminosities of BDs of a given mass and age, and they do not correctly reproduce the positions of BDs in the Hertzsprung-Russell diagram. To quote King et al. (2010) from their work on the BD binary $\epsilon$ Indi Bab – “Given the preliminary dynamical system mass it therefore appears that with current theoretical models and spectroscopically derived effective temperatures, one cannot obtain reliable mass estimates for T-dwarfs such as these, even when precise luminosity constraints are available.” With that depressing assessment, we turn to methods of deducing the IMF for BDs using these very models!

4.2 Simulating field brown dwarf luminosity functions

The age-dependent mass-luminosity relationship means that determining the IMF from an LF is inextricably linked to the age distribution of the population in question and hence to the star (BD) formation history. Several authors have suggested an inductive approach that predicts what the LF (or the distribution of effective temperatures/spectral types) of a low-mass field population will be given assumed functional forms for the IMF and star forming history. The advantages of this approach (in contrast to work on young clusters discussed in section 5) are that the field BDs will be close and more amenable to determining their binary characteristics, and it is hoped that, despite the limitations discussed above, the evolutionary models may be more accurate for older BDs. Burgasser (2004) gives a very clear description of the general approach; the conclusions it reaches and the limitations of such work are similar to work by Reid et al. (1999), Allen et al. (2005), Deacon & Hambly (2006) and Burgasser (2007).

In Burgasser (2004) IMFs of the form $\phi(m) \propto m^{-\alpha}$ are considered along with
the log-normal form suggested by Chabrier (2003). These IMFs cover objects with \( m < 0.1 \, M_\odot \); the normalisation was set to match local field stars (both single and in multiple systems) with \( 0.09 < m/\, M_\odot < 0.1 \), noting that this scaling factor is still uncertain by 30 per cent. Considered star forming histories include a uniform distribution, an exponentially decaying function and various “burst” models. It is assumed, as usual, that the IMF is time-independent. Monte Carlo simulations, select objects at random from the age distribution and from the IMF with masses between \( 0.1 \, M_\odot \) and a number of possible low-mass cut-offs. The present day properties of the objects were derived using the models of Burrows et al. (1997) and Baraffe et al. (2003) and these were binned to form luminosity and \( T_{\text{eff}} \) distributions.

The main limitations of this approach are the absolute reliance on evolutionary and atmospheric models to give the correct properties for BDs of a given mass and age. Presumably the local field population of BDs spans a range of metallicities, but the metallicities of field L- and T-dwarfs are not well known and the evolutionary models available all assumed solar metallicity.

The key results of these simulations, which are confirmed by the later works, are (see Figs. 9–12 in Burgasser 2004):

1. The LFs and \( T_{\text{eff}} \) distributions are morphologically similar, because most BDs with ages of 1–10 Gyr have radii close to \( 1 \, R_{\text{Jup}} \). There is a peak among late M-dwarfs caused by long-lived \( 0.08-0.1 \, M_\odot \) stars; a trough through the L-dwarf regime \( (1400 < T_{\text{eff}} < 2300 \, \text{K}) \) because the evolutionary models show that BDs with \( m < 0.075 \, M_\odot \) cool rapidly at these temperatures; there is a “pile-up” of cooling T-dwarfs, rising to a peak in late T-dwarfs because the cooling time of such objects reaches 10 Gyr (see Fig. 6).

2. The LFs and \( T_{\text{eff}} \) distributions become increasingly sensitive to variations in the adopted IMF at lower luminosities and cooler temperatures. There are factors of a few variations in the late T-dwarf LF for a range of plausible IMFs (see Fig. 6). If \( \alpha = 1 \), there should be \( \sim 60 \) T6–T8 dwarfs within 10 pc of the sun, but only \( \sim 25 \) if \( \alpha = 0 \).

3. There is little dependence on adopted evolutionary model for spectral types cooler than late-L. This must be tempered by the fact that the evolutionary models do have some common assumptions and presumably weaknesses. There is some model-dependence (at the level of factors of two) for late M and early L-dwarfs.

4. There is only a modest dependence on the birthrate model. Any dependence is confined to L-dwarfs. In particular, the space density of \( \geq \)T6 dwarfs is insensitive to the star forming history of the Galactic disk.

5. Only the space density of the very coolest T-dwarfs (T8–Y0?) is sensitive to any cut-off or minimum mass in the BD IMF if that cut-off is \(< 0.015 \, M_\odot \).

In summary, the space density of late T-dwarfs is most sensitive to the detailed shape of the BD IMF, but is almost insensitive to the star forming history. The
present day density of L dwarfs may offer some sensitivity to the BD birthrate history, but it will be difficult to disentangle this from systematic differences of similar size between the predictions of alternative evolutionary models.

A final complication is the influence of binarity on these simulations. The binary properties of BDs are quite uncertain. Resolved BD binaries among the nearby field population amount to about 15–30 per cent of the observed population (see Table 1 in Burgasser 2007), with evidence of higher binarity among early L8–T3 dwarfs, which may be consistent with an underlying (resolved) binary frequency of $11_{-3}^{+6}$ per cent. Burgasser (2007) has conducted a further series of simulations incorporating the binary population, predicting what would be observed among volume-limited or magnitude-limited surveys with no binary resolution. The effect is small in volume-limited surveys, but if, as is more usual, a survey is magnitude-limited then there can be significant corrections because of the additional volume from which binaries can contribute.

4.3 Measuring field brown dwarf luminosity functions

Armed with predictions from simulations, it is merely(!) a case of conducting a census of local BDs, grouping them into luminosity or temperature bins and comparing them with the models. The challenges involve (i) finding cool BDs and defining a clean sample, (ii) assessing completeness, (iii) assessing contamination, (iv) determining distances to calculate a space density and (v) correcting these densities for Malmquist bias and the presence of binary systems. A number of major surveys have been brought to bear on this topic. Key literature includes Allen et al. (2005), Cruz et al. (2007), Metchev et al. (2008), Reylé et al. (2010) and Burningham et al. (2010), that have used a variety of survey data and photometric colours to select samples of cool BDs (see Table I).

It can prove difficult to separate very low-mass objects from the higher mass M-dwarfs that dominate at any magnitude. The optical spectrum of M-dwarfs is dominated by TiO bands, but these become progressively weaker and disappear by spectral type L3. At the same time a number of atomic lines (rubidium, caesium) become broad and prominent, while the potassium 7665/7699Å line becomes a broad trough in the spectrum at L5. In the near infrared, water absorption appears in M-dwarfs and grows in strength. At around 1400 K, methane absorption becomes prominent (defining the T-dwarf class) and significantly alters the near infrared spectrum. The effects of these spectral changes on commonly used photometric colours is well illustrated in Fig. 4 of Chiu et al. (2006). L dwarfs are a little redder than M-dwarfs in all colours and hence many candidates are found in 2MASS and SDSS data. However, the separation in colour from M-dwarfs is insufficient to prevent photometrically selected samples being heavily contaminated by the much more common M-dwarfs, because both intrinsic colour dispersion and photometric uncertainties scatter them into the selection region. The situation is more favourable for T-dwarfs, because while being redder than L-dwarfs in $i - z$ and $z - J$, they are significantly bluer at $J - H$ and $H - K$. Hence whilst late L- and T-dwarf candidates can be selected in (for instance) $i - z$ vs $z - J$ diagrams
Table 1. A summary of surveys for the local field BD population. Columns list the sample source (see text), magnitude limit, size of sample, the deduced power-law index of the IMF (equation 2.3) and the sampled mass range.

| Study          | Survey         | Method              | Sample         | $\alpha$     | Mass range   |
|----------------|----------------|---------------------|----------------|--------------|--------------|
| Allen et al. 2005 | 2MASS 14800 deg$^2$ | vol-lim(ML)         | 180 ML         | +0.3 ± 0.6   | 0.04–0.10 M$_\odot$ |
|                | $J < 16.5$     | mag-lim(T)          | 18 T5-T8       |              |              |
| Cruz et al. 2007 | 2MASS 14800 deg$^2$ | vol-lim             | 45 L           | $\leq 1.5$   | $\leq 0.075$ M$_\odot$ |
|                | $J < 16.5$     | $JHK+$spec.         |                |              |              |
| Metchev et al. 2008 | SDSS/2MASS 2099 deg$^2$ | mag-lim              | 15 T0-T8       | $\leq 0$     | $\leq 0.075$ M$_\odot$ |
|                | $z < 21$       | $izJHK+$spec.       |                |              |              |
| Reylé et al. 2010 | CF BDS 444 deg$^2$ | mag-lim             | 102 L5-T9      | $\leq 0$     | $\leq 0.075$ M$_\odot$ |
|                | $z < 22.5$     | $izJ+$spec.         |                |              |              |
| Burningham et al. 2010 | UKIDSS 980 deg$^2$ | mag-lim             | 47 T6-T9       | $-0.5 \pm 0.5$ | 0.02–0.04 M$_\odot$ |
|                | $J < 18.8$     | $zYJH+$spec.        |                |              |              |

(e.g. Metchev et al. 2008; Reylé et al. 2010), the addition of $H$ or $K$ data appears to offer much more discrimination for T-dwarfs (Burningham et al. 2010).

Attempts to determine spectral types from photometry are almost impossible. The intrinsic dispersion of colours at a given spectral type can reach 0.5 mag. The reasons for this are poorly understood but probably include the presence of unresolved binary systems (especially around the L/T transition), differences in metallicity, patchy clouds/condensates and a range of surface gravities because objects at a given $T_{\text{eff}}$ have a range of ages and masses (e.g. see Knapp et al. 2004; Liu et al. 2006; Burgasser 2007; Marley et al. 2010). This means that detailed comparisons with simulations demand spectroscopic spectral type determination.

Both completeness and contamination are issues to be dealt with in these studies. Contamination arises because numerous unwanted objects are present or are scattered into the photometric selection box. This can be dealt with either by detailed simulations (Reylé et al. 2010) or spectroscopic follow-up to eliminate contaminants. The simulation technique requires an accurate knowledge of the (possibly non-Gaussian) tail of the photometric uncertainty distribution. Completeness corrections are required to deal with the sensitivity of the observations, but also to account for any possibility that photometric dispersion can also scatter
genuine candidates out of the selection box. Different approaches to this include making a correction based on the recovery of previously known objects (Cruz et al. 2007), or simply ignoring spectral type ranges where the photometric selection box may exclude genuine BDs (Burningham et al. 2010).

After settling on a census then distances are estimated. Parallaxes are rarely available so relationships between colour, or preferably spectral type, and absolute magnitude are used, calibrated using some objects with known (and precise – see section 3.1.2) parallax (e.g. Cruz et al. 2003 for late M- and early L-dwarfs or Vrba et al. 2004 for L- and T-dwarfs). Like the spectral type versus colour relationships, there is a dispersion, possibly caused by gravity and metallicity differences. There is also a pronounced kink or plateau for early T dwarfs, which may partly be explained by binarity (Liu et al. 2006). Different authors use different forms of these relationships that can lead to factors of two systematic differences in their estimates of space densities. These space densities then have to be corrected for the significant Malmquist bias due to dispersion in the spectroscopic parallax.

Studies also differ in how they treat binarity. The effect of binarity on volume-limited LFs is small because the binary frequency of BDs is low. Nevertheless, a significant correction to a LF derived from a magnitude-limited survey is still required because the volume sampled by binary systems can be up to \(2\sqrt{2}\) larger (for a mass ratio of 1) than for single BDs. Corrections have been calculated using analytical approximations (e.g. see Burgasser 2004; Burningham et al. 2010), by Monte Carlo simulation (Metchev et al. 2008), reducing the LFs by up to a factor of two, or have been neglected (Reylé et al. 2010).

4.4 The IMF of field brown dwarfs

The final LFs are compared with simulated LFs predicted according to different assumptions about the IMF and birthrate history. Before doing so it is worth pausing to recall the systematic uncertainties present. The most important are: (i) The veracity or otherwise of the evolutionary models and their predictions of luminosity and \(T_{\text{eff}}\) as a function of mass and age. (ii) A relationship between \(T_{\text{eff}}\) and spectral type is needed to make an observational comparison (e.g. Golimowski et al. 2004). Systematic uncertainties in this will redistribute objects between spectral types. (iii) Systematic uncertainties in spectroscopic parallaxes could change space densities by factors of 1.5–2. (iv) The model normalisation at 0.1 \(M_\odot\) is uncertain by about 30 per cent. (v) Corrections for binarity are uncertain at a similar level.

A comparison between measured and predicted LFs is shown in Fig. 6. The model loci are from Burgasser (2007) and assume a uniform star forming rate, a local field density normalisation of 0.0037 pc\(^{-3}\) for stars with 0.09 < \(m/M_\odot\) < 0.1 (Reid et al. 1999), the evolutionary models of Burrows et al. (2001) and Baraffe et al. (2003) and the “COND” models of Allard et al. (2001). A variety of IMFs of the form \(dN/dm \propto m^{-\alpha}\) were explored by Burgasser, and also a log-normal IMF with parameters taken from Chabrier (2002). Subject to all the caveats listed above, it seems that the current observational data favours a value of \(\alpha \leq 0\). The log-
Fig. 6. The luminosity function of field BDs as a function of their spectral type. Results are shown for a number of recent surveys (see text) Also shown are model loci taken from Burgasser (2007). These predict the observed LF based on a set of evolutionary models and atmospheres together with an assumed value for the IMF \((dN/dm \propto m^{-\alpha})\) and a uniform formation rate. Also shown is a locus derived using a log-normal IMF. Overall it appears that values of \(\alpha < 0\) are favoured, though a log-normal IMF with a smaller dispersion parameter may also be possible (see text).

normal model of Chabrier (2002) does not describe the observations well but the parameters of this distribution, \(m_c = 0.1 M_\odot\) and \(\sigma = 0.627 \text{dex} \) (see equation 2.4) were updated to \(m_c = 0.2 M_\odot\) and \(\sigma = 0.55 \text{dex}\) by Chabrier (2005) and may describe the observations significantly better.

5 The initial mass function from young clusters

5.1 Advantages and disadvantages

An alternative to finding low mass stars and BDs in the field is to look for them in star clusters. The primary scientific advantage of determining the IMF from clusters is that whilst the local field IMF is some sort of average over all star forming environments, clusters offer the opportunity to probe differing star forming conditions (e.g. Jeans mass, density, radiation environment). Open clusters contain approximately coeval stars with similar initial composition. In principle the ages of clusters are known, either from the main sequence turn-off at high masses or
from other techniques, so the difficulties of a time-dependent mass-luminosity relationship are resolved. Distances can also be determined by main sequence fitting or other means, and even though clusters are usually more distant than field BDs identified in wide photometric surveys, their youth means that low mass objects in clusters are hotter and intrinsically more luminous. The net result is that despite their distance, BDs in clusters can be brighter than field objects of similar mass (see Fig. 7).

In the debit column, very young clusters (< 10 Myr) are afflicted by considerable and variable extinction that makes luminosities hard to estimate. Very young objects are often surrounded by circumstellar accretion disks which alter their spectral energy distributions making an estimate of the underlying luminosity (and hence mass) problematic (Hillenbrand 1997; Da Rio et al. 2010). The masses must be estimated using theoretical models but these have not been well-tested at at younger ages and are likely to be affected by significant uncertainties (e.g. Baraffe et al. 2002, 2009). At very young ages a small age spread could make a significant difference to the mass-luminosity relation. The reality or not of significant age spreads is currently disputed (e.g. Huff & Stahler 2006; Jeffries et al. 2011).
In older clusters coevality must be a good approximation, there is also little variable extinction and no accretion. But the measured MF may no longer be the IMF. Mass segregation effects that are either primordial (therefore affecting young clusters too) or which develop dynamically as clusters gets older can alter the MF significantly. Care must be taken that a census of cluster members is not biased by any radial dependence of the MF. Some fraction of the lower mass objects may be preferentially evaporated from the cluster resulting in the PDMF underestimating their contribution (Allison et al. 2009; de Marchi et al. 2010). As most cluster MFs make no attempt to resolve binary systems, a more subtle effect could be any change in binary frequency with age or environment (e.g. Kroupa & Bouvier 2003). In these circumstances there may be little alternative but to appeal to simulations in an effort to predict what the IMF was.

Whichever type of cluster is being observed, to perform an accurate census there are two issues that must always be considered; contamination and completeness. Contamination of samples, whether they be selected on the basis of colours, proper-motions or radial velocities will usually be a problem. Ideally one would like to work with samples that have been cleaned of significant contamination (e.g. background/foreground field stars, background galaxies and quasars) by applying multiple membership criteria or through independent membership criteria. The same selection criteria may also lead to incompleteness in the sample if they are too stringent. In addition, photometric surveys are usually incomplete at some level due to magnitude or colour limits, variable extinctions and obscuration by brighter stars or nebulosity.

5.2 Cluster Membership

5.2.1 Photometry

The first (and sometimes only) line of attack in cluster MF determinations is to examine colour-magnitude or colour-colour diagrams to select cluster members and exclude non-members. In the absence of age spreads, coeval cluster members of uniform chemical composition are expected to follow well-defined loci in such diagrams. Sequential selection in a number of diagrams can be made but there is almost always some contamination that remains (e.g. Lodieu et al. 2009; Béjar et al. 2011). The selection boundaries must be broad enough to include all possible cluster members (including binary or possibly even higher multiple systems) or the selection criteria must be well enough understood that the mass-dependence of any incompleteness can be accounted for at a later stage. There is a tension between ensuring completeness and minimising contamination. The amount of contamination depends on the depth of the survey, the photometric bands being used and the Galactic longitude and latitude of the cluster. A variety of techniques can be used to estimate contamination based solely on the photometric information or by using semi-empirical models for the foreground and background Galactic populations (e.g. see Jeffries et al. 2004; Oliveira et al. 2009).

The selection region for cluster members is often defined with reference to
theoretical isochrones if the cluster age is well-known. However, this approach is susceptible to uncertainties in the theoretical models and atmospheres, a particular hazard for very low-mass stars and brown dwarfs. Guidance is often taken from the known location of cluster members that have been confirmed using alternative techniques (RVs, proper motions, Li abundances etc.).

In general, photometric selection of cluster members works better in older clusters where there is a negligible spread in luminosity caused by any plausible age spread, no accreting objects with peculiar luminosities, and often a uniform extinction. For selecting very low-mass stars and brown dwarfs it is almost mandatory to have data at infrared wavelengths, although selection in $I, I - Z$ or $I, R - I$ diagrams can be effective.

5.2.2 Kinematic selection

Cluster members are often assumed to share a common space velocity. Measurements of the proper motions (PMs) and RVs of clean samples of members in older well-populated clusters suggest velocity dispersions of about 1 km s$^{-1}$ for most open clusters (e.g. Geller et al. 2010; Jackson & Jeffries 2010; Bonatto & Bica 2011). If clusters are virialised then equipartition would give a larger velocity dispersion to lower mass stars. Individual clusters show signs of both sub- and super-virial motions, reflecting details of their formation or dissolution processes (e.g. de Grijs et al. 2008; Proszkow et al. 2009). Some formation models for very low-mass stars and BDs predict much higher velocity dispersions for these objects (Reipurth & Clarke 2001; Umbreit et al. 2005). With these caveats in mind, kinematic observations provide a useful, though rarely decisive indication of cluster membership.

The utility of PM measurements is a function of their precision, the distance to the cluster, the peculiar tangential motion of the cluster relative to the field population and the density of foreground and background sources. Older, all-sky PM catalogues based on photographic plates with long observation baselines have precisions of a few mas/year, that are just capable of resolving cluster velocity dispersions at $\sim 100$ pc, but are limited to $V < 20$ which, whilst useful in tracing the low-mass population struggles to probe very low-mass stars and BDs in clusters any further than this. Frequently, PMs are combined with photometric constraints to provide a cleaner sample (e.g. Hambly et al. 1999; Deacon & Hambly 2004). The advent of new digital far-red and infrared surveys such as 2MASS, SDSS and UKIDSS have improved the situation considerably in nearby clusters (e.g. Adams et al. 2002; Kraus & Hillenbrand 2007; Lodieu et al. 2011). Nevertheless, unless the cluster population dominates the field population in some limited area (e.g. NGC 3603, Rochau et al. 2010), PMs can only be used to reject cluster non-members in more distant clusters (e.g. Caballero 2010).

Different considerations apply to RV measurements. The distance to the cluster is only relevant in that precise RV measurements require at least intermediate resolution ($R \geq 5000$) spectroscopy. The specificity of RV selection depends on the cluster RV compared to that of the field population. For clusters with distances
of a few hundred pc or more, greater membership discrimination can be achieved with RVs than with PMs, and are facilitated by wide-field fibre spectrographs (e.g. FRinchaboy & Majewski 2008), although challenging measurements in the far-red or near-infrared are needed to reach BDs (e.g. Kenyon et al. 2005; Sacco et al. 2008) and it is difficult to achieve high levels of completeness. Nevertheless, the precision of such surveys often yield unexpected results such as revealing that some clusters consist of multiple populations with different ages and at different distances (e.g. Jeffries et al. 2006), with obvious implications for IMF determinations!

5.2.3 Youth indicators

A variety of youth indicators are widely used to identify members of young clusters – which can be useful in empirically defining where in CMDs the cluster members lie. All of these methods have their problems and are unlikely to provide the sole means of completing a cluster census.

Magnetic activity is ubiquitous among young, low-mass stars and is associated with their convective envelopes and rapid rotation. The rapid rotation of young stars in clusters can be established either by multi-epoch photometric monitoring, which determines rotation periods from modulation by cool, magnetic starspots (e.g. Irwin et al. 2007; Hartman et al. 2010) or by large-scale, high resolution spectroscopic surveys that measure rotational broadening (e.g. Jackson & Jeffries 2010). Unfortunately, only a fraction of young stars exhibit rotational modulation in any survey and rotational broadening can be masked by small rotation axis inclinations. An additional problem is that whilst rapidly rotating field stars are rare among field G- and K-dwarfs, the spin-down timescales become longer in M-dwarfs, and rapid-rotators become common in field dwarfs cooler than M4 and in BDs (e.g. Reiners & Basri 2010; Reiners et al. 2012), so that incompleteness and contamination of samples are problematic.

X-rays are emitted from the magnetically confined coronae of young, low-mass stars (Guedel 2004). X-ray imaging surveys can be used to identify candidate cluster members and trace their spatial extent. However, limited sensitivity means that these surveys rarely reach the lowest masses, and BDs are probably weak X-ray sources except at very young ages (Grosso et al. 2007; Berger et al. 2010). IMFs derived solely from X-ray selected samples would have uncertain levels of incompleteness, especially in star forming regions where variable extinction and accretion also serve to increase the scatter in observed X-ray luminosity at a given mass (Flaccomio et al. 2010).

Chromospheric Hα emission can also be used as a magnetic activity indicator and is strong in the active K- and M-dwarfs of young clusters. The completeness levels of any survey can be well-controlled and the observation of chromospheric activity is a strong membership indicator in young K-stars but becomes weaker in cooler stars because a large fraction of field M-dwarfs are also magnetically active. Emission persists until spectral types of at least M8, but suffers a sharp decline at cooler temperatures (e.g. Mohanty & Basri 2003).
Indicators of circumstellar material or accretion are expected from a fraction of very young low-mass stars. Examples include strong, broad H$\alpha$ emission and infrared excesses radiated from circumstellar dust. The former can be found either in wide-field spectroscopic surveys, but can be investigated more economically via photometric means (see Valdivielso et al. 2009 for an application to very low mass stars and BDs). Secure identification of infrared excesses was revolutionised with 3-8$\mu$m observations by the Spitzer satellite. Most stars have detectable circumstellar dust at ages of 1 Myr, but this declines to a negligible fraction over the course of 10 Myr (see Hernández et al. 2008 and references therein). Any sample constructed from stars selected solely on the basis of an infrared excess will be incomplete, although contamination levels will be extremely low. An additional concern is that the level of incompleteness may be mass-dependent. Although young BDs are also found with discs and accretion, it is possible that their timescales for disc retention are longer (e.g. Bouy et al. 2007).

5.2.4 Spectral types and gravity-sensitive features

In star forming regions it is important to establish spectral types as a means of estimating extinction, separating photospheric emission from accretion and hence establishing accurate luminosities. Extinction values can be used to select cluster members and it is often useful to define an “extinction-limited” sample (i.e. a complete subset of the full sample, with extinction less than some value) with which to investigate the IMF; under the assumption that extinction and mass are independent (e.g. Luhman 2007).

Low-mass PMS stars are larger than their main-sequence counterparts and have lower surface gravity. At a given age, the difference is largest for lower mass stars and leads to observable consequences in their spectra. A prime example is the neutral alkali lines (K$\lambda$, Na$\lambda$) in the far-red spectrum. These are weaker in cluster members than in main-sequence dwarfs of similar colour or spectral type, but stronger than in giants (see for example Luhman 2004; Lodieu et al. 2011). As a method of identifying cluster members this is most effective in M-dwarfs and can be done with relatively low-resolution spectra ($R > 1000$).

5.2.5 Lithium depletion

Lithium is burned in the cores of fully convective PMS stars as they contract and heat up. Because convective mixing is effective, very rapid and total Li depletion results except in higher mass stars ($m > 0.6 M_\odot$) where the genesis and expansion of a radiative core occurs early enough to preserve some of the original Li. The rate of PMS contraction is age-dependent so the amount of Li depletion depends on both age and mass in a complex way (see Jeffries 2006), but Li is undepleted for all low-mass stars ($m < 0.25 M_\odot$) younger than 30 Myr and for BDs of any age with $m < 0.06 M_\odot$. In principle this is a powerful membership indicator in young clusters, even for BDs because they can be separated from older Li-rich BDs by their much higher temperatures (e.g. Rebolo et al. 1996; Zapatero-Osorio et al.)
Fig. 8. The MFs for young clusters with ages 30–120 Myr (offset for clarity). The solid line indicates the best-fitting log-normal description of the Pleiades MF \( m_c = 0.25 \, M_\odot \) and \( \sigma = 0.52 \, \text{dex} \). Figure constructed by Bouvier & Moraux (private communication).

However, the technique requires precise measurements of the Li\( ^{6} \) feature using spectroscopy with \( R > 3000 \).

5.3 Results for Older Open Clusters

Once a census has been taken then the masses of the stars and BDs must be determined. The adopted process differs between open clusters with age > 10 Myr, where accretion, discs, variable extinction and age spreads can be ignored, and the younger clusters and star forming regions where they cannot. As a representative of the former, I take the Pleiades as an example; a nearby cluster with a precisely known age \((125 \pm 10 \, \text{Myr}, \text{Stauffer et al. 1998})\) and distance \((135 \pm 4 \, \text{pc}, \text{An et al. 2007})\), with a well-established upper main sequence and mean proper motion.

The boundary between stars and BDs occurs at \( I = 17.8 \), so relatively deep far-red or infrared photometry is needed to locate its very low mass members. Moraux et al. (2003) used an \( IZ \) survey with “optical” CCD cameras in order to
cover large areas (6.4 deg$^2$) of the cluster. Selection in the $I$, $I-Z$ CMD gave 40 candidates down to an assumed completeness limit of $I \simeq 22$, $m \simeq 0.03 M_{\odot}$. Contamination was estimated to be about 30 per cent based on the LF and colours of nearby stars. Masses were estimated directly from absolute $I$ magnitudes using evolutionary models, with no attempt to correct for unresolved binarity. The resulting MF was well represented either by a power law with $\alpha = 0.60 \pm 0.11$ for $0.03 \leq m/M_{\odot} \leq 0.45$ or a log-normal form with $m_c = 0.25$ and $\sigma = 0.52$ dex applicable for a wider range of $0.03 \leq m/M_{\odot} \leq 2$ (see Fig. 8). Lodieu et al. (2007) used “optical” $I Z$, 2MASS infrared data and deep infrared photometry from the UKIDSS survey over 12 deg$^2$ to select 456 stars and BDs using multiple CMDs and PMs. Membership probabilities are summed to form an LF and this is converted to an MF using evolutionary models. The results are in good agreement with Moraux et al. although the best-fitting dispersion of the log-normal is narrower at $\sigma = 0.34$ dex.

A number of prior and subsequent studies of the Pleiades using similar methods are consistent with these results. Very similar work has now been carried out in a number of open clusters with ages of 30–150 Myr (Alpha Per, Barrado y Navascúes et al. 2002; NGC 2547, Jeffries et al. 2004; IC 4665, de Wit et al. 2006; Blanco 1, Moraux et al. 2007). In common with the Pleiades studies these clusters are either too old or too distant for systematic spectroscopic studies of the very low mass and BD populations and rely mostly on selection in multiple CMDs and in some cases PMs and a handful of spectra. The results for these clusters also agree with those for the Pleiades; the IMF below $0.3 M_{\odot}$ could be represented with a log-normal function with $m_c \simeq 0.25 M_{\odot}$ and $\sigma \simeq 0.5$ dex, or a power law with $\alpha \simeq 0.5$ (and power laws with larger $\alpha$ at higher masses), with no evidence for significant variations from cluster to cluster.

The agreement between these clusters is encouraging, but even in these relatively young clusters there are clear signs of mass segregation (Jeffries et al. 2004; Moraux et al. 2007). As these surveys tend to be centred on cluster cores, there is a risk that low-mass objects are under-represented. This is readily apparent in older clusters that have been analysed in a similar way, where 90 per cent of BDs may have been evaporated (e.g. the Hyades, age 600 Myr, Bouvier et al. 2008). On the contrary, other older clusters still agree quite well with the Pleiades IMF (e.g. Praesepe, age 600 Myr, Boudreault et al. 2010). The reasons for these discrepancies are unclear. They may reflect differing IMFs or they may arise from differing dynamical histories. This uncertainty may lead to systematic underestimate of the very low-mass IMFs of clusters.

### 5.4 Results for younger clusters and star forming regions

The possibilities of dynamical mass segregation drive us to consider measuring IMFs in very young clusters and star forming regions, with the additional advantage that very low-mass stars and BDs are much brighter at younger ages (see Fig. 7). The presence of extinction, discs and possible age spreads requires a different approach to measuring IMFs. A summary of earlier observational work is
Fig. 9. The IMF determined in a number of young (< 10 Myr) clusters and star forming regions (offset for clarity). The solid lines show the log-normal model that best fits the Pleiades (see Fig. 8). The MFs may be generally consistent with that of the Pleiades but the MF of Upper Sco is quite different. Figure constructed by Bouvier & Moraux.

provided by Luhman et al. (2007) and I consider the determination of the IMF in Chamaeleon I by Luhman (2007) as a representative example.

Chamaeleon I has an age of ≃ 3 Myr and a distance of 170 pc. An initial selection of candidate members is made using a number of generous cuts in far-red and near-infrared CMDs, using the positions of previously identified members (through Hα emission, X-ray emission and infrared excesses) as a guide. The depths of these surveys (approximately I < 21, H, K < 14 for a wide area and a smaller region covered to I < 26) gave access to the IMF over the range 0.01 < \( m/\text{M}_\odot < 3.5 \). Luhman obtained flux-calibrated, low resolution (600 < R < 2000) far-red and near-infrared spectra of the candidates, using RVs, alkali line strengths, Li absorption and infrared excesses to confirm membership, determine spectral types and estimate extinction. Cluster members were placed on an HR diagram using spectral types to estimate temperatures and bolometric corrections to the absolute J magnitudes. Evolutionary tracks are then used to estimate both the
masses and ages of each object. This differs from the procedure for older clusters because the apparent spread of age in Chamaeleon I of from < 1 to ∼ 20 Myr means that a single mass-luminosity or mass-magnitude relationship could not apply to all objects. Complete, extinction-limited samples are then defined for each of the photometric surveys and the masses define IMFs for a wider and an inner region of the cluster respectively.

The two IMFs determined by Luhman (2007) for Chamaeleon I are consistent with each other. They are also consistent with an IMF determined in a similar way for IC 348 (age ∼ 3 Myr) by Luhman et al. (2003), and could be represented with a log-normal IMF that peaks at $m_c \simeq 0.15 M_\odot$. In fact the IMF of a number of young clusters and star forming regions are remarkably similar (see Fig. 9), both to each other and to the IMFs of the Pleiades and the older clusters shown in Fig. 9 (Barrado y Navascués et al. 2005; Oliveira et al. 2009; Caballero et al. 2009). Nevertheless, some clusters do appear to have a different IMF; Upper Sco has a significant excess of BDs (Lodieu et al. 2007b) compared to other clusters and the IMF of the Taurus-Auriga association clearly peaks at higher masses (see Luhman 2007).

An alternative metric for comparing the IMFs is the ratio of stars (0.08–1 M$_\odot$) to BDs (0.03–0.08 M$_\odot$) (Andersen et al. 2008; Oliveira et al. 2009). This ratio varies from $3.3^{+0.2}_{-0.7}$ in the Orion Nebula cluster (ONC) to $8.3^{+3.3}_{-2.6}$ in IC 348. There are no clear trends with cluster size, density or the presence of strongly ionising O-stars. Andersen et al. (2008) present an analysis that suggests the variations seen in this ratio are entirely consistent with all the clusters considered (Taurus, Pleiades, ONC, Mon R2, Chamaeleon I, NGC 2024 and IC 348, but they did not include Upper Sco) being drawn from the same log-normal IMF proposed by Chabrier (2005).

5.5 Systematic theoretical uncertainties

Aside from the problems of dynamical evolution and either ongoing or primordial mass segregation or ejection, there are a number of theoretical uncertainties which may have more of an impact on attempts to determine the IMF in young clusters than in field stars.

The mass-luminosity (or mass-magnitude) relationships are highly dependent upon the assumed ages of PMS stars and young BDs. But there is considerable suspicion about the age spreads, or even the absolute ages, deduced for very young clusters from HR diagrams and model isochrones (e.g. Naylor 2009). Even the originators of some of the evolutionary models caution against their reliability for ages < 10 Myr (e.g. Baraffe et al. 2002), which is of course the regime where the majority of the very young clusters are. Unresolved issues include the treatment of convection and an appropriate representation of the atmospheres for cool stars where molecules become important. It is well known that the use of different sets of evolutionary models will lead to different IMFs in the young clusters, not changing the overall shape, but significantly changing the fitted parameters of log-normal parameterisations or power-law exponents (e.g. Da Rio et al. 2012).
Fig. 10. Components of pre-main sequence eclipsing binary systems placed in the HR diagram (see text). Lines join components of the same (and presumably coeval) binary system. The objects are labelled with their dynamical masses (in $M_\odot$) and compared with evolutionary tracks (also labelled in $M_\odot$) from Baraffe et al. (2002). Isochrones are also shown at ages of 1, 3, 10 and 100 Myr. The dynamical masses are determined with precisions of 1–8 per cent and in several cases show significant disagreement with the evolutionary tracks. The objects highlighted in blue are Par 1802AB and 2M0535-05AB, which are discussed in section 5.5.

Beyond these difficulties, most models also neglect physical effects that may give rise to significant errors in the determinations of mass from evolutionary tracks. There is a fierce debate about whether ongoing or earlier episodes of heavy accretion might lead to a drastic modification of the luminosity of objects with ages $< 10$ Myr (e.g. Baraffe et al. 2009; Hosokawa et al. 2011). This would systematically change both the deduced ages and masses of stars and BDs from the HR diagram, but worse, the effect would depend on the history of the object rather than its currently observed properties! At older ages it is likely that the star “forgets” its past and relaxes back towards the standard evolutionary tracks. However, even for low-mass stars in older clusters at $\sim 100$ Myr there are strong suggestions that their strong magnetic fields and starspots could modify their luminosity and radii, leading to erroneous mass determinations based on current models that neglect these effects (e.g. Jackson et al. 2009; Mohanty et al. 2010).
There has been much less work testing the validity of mass determinations in PMS stars and BDs than for older field objects. The main problem is a lack of suitable eclipsing binary systems and the difficulty of resolving astrometric binary systems at the distances of the nearest clusters. Those measurements that exist indicate a large scatter (∼ 50 per cent) when comparing dynamical masses and masses from evolutionary tracks for astrometric binaries with $0.4 < m/M_\odot < 1.5$ (Hillenbrand & White 2004). Among PMS eclipsing binaries the situation is worse! Figure 10 shows the HR diagram for the components of PMS binaries in Orion, taken from Covino et al. (2001), Stassun et al. (2004) and Stempels et al. (2008). The stars are labelled with their derived dynamical masses and compared to the evolutionary tracks of Baraffe et al. (2002). The positions of some of the binary components are in agreement with their measured dynamical mass, but others are very different. Highlighted in blue are two recent awkward examples from Stassun et al. (2006, 2008). Par 1802AB have equal masses of $(0.41 \pm 0.02) M_\odot$ yet masses deduced from the HR diagram would be unequal and twice as large in the case of one component. In the BD binary system 2M0535-05AB, the more massive component is twice the mass suggested by evolutionary tracks and cooler than the less massive component.

5.6 Summary of the IMF from young clusters

Clusters offer significant advantages (coeval, same composition, known age and distance, brighter BDs) over field star studies, but also suffer from the disadvantages of difficult luminosity estimates and the binary properties of samples cannot be investigated easily. Whilst significant efforts are required to exclude contamination and prove cluster membership in complete samples there is reasonable evidence that most clusters have IMFs that are consistent with a log-normal representation (equation 2.4), with $m_c \simeq 0.2 M_\odot$ and $\sigma \sim 0.3$ dex. An equally valid representation of the IMF for $0.3 M_\odot < m < 0.5 M_\odot$ would be a power-law with $\alpha \simeq 0.5$. Some older clusters show clear signs of mass segregation and evaporation of low-mass objects and there are at least two well-studied star forming regions that show IMFs that are clearly inconsistent with the general picture. There is ample evidence from current observations of PMS binaries that the HR diagram does not yield reliable masses. Systematic uncertainties in the IMF due to choice of evolutionary models or deficiencies in evolutionary models are unlikely to affect the cluster-to-cluster comparisons (where IMFs have been determined using the same models), but may affect any comparison with the low mass IMF determined from the field.

6 The bottom of the initial mass function?

The smallest mass with which a BD can form is an important constraint for formation theories (see Hennebelle’s contribution to this volume). It may be set by the thermodynamics of the primary fragmentation phase of a collapsing cloud or circumstellar disc – “the opacity limit”. The lowest possible masses may be in the
region of 0.005–0.01 M⊙ (e.g. Low & Lynden-Bell 1976), but lower values have been suggested (0.001–0.004 M⊙, Whitworth & Stamatellos 2006).

6.1 Measurements in the field

Evolutionary models suggest that field BDs with spectral types of T8-T9 have 500 < Teff < 700 K and masses of 0.01 < m/M⊙ < 0.03 for typical ages of 1–10 Gyr. To look for lower mass objects requires the identification of cooler, less-luminous objects, known as Y-dwarfs. Some examples of such very cool objects have been reported, thanks largely to 3–5µm imaging from the Wide-field Infrared Survey Explorer satellite (WISE, Cushing et al. 2011). These objects are redder than T8 dwarfs in their [3.6]-[4.6] micron colours, have a broad spread of J−H colours, and present spectra with evidence for ammonia absorption. A preliminary parallax for one of these Y-dwarfs suggests it is 6–7 magnitudes fainter than a T8 dwarf in the J and H bands (Kirkpatrick et al. 2011).

Unfortunately, without an age estimate, the precise masses of these objects remain uncertain; model-dependent spectroscopic estimates of masses and radii via Teff and log g suggest 0.005–0.03 M⊙ (Cushing et al. 2011). Yet, following the same approach described in section 4 it may be possible to distinguish between different combinations of IMF and minimum BD mass, through their effects on the local space density of Y-dwarfs. Kirkpatrick et al. (2011) use a preliminary census from the WISE data to suggest that they can already rule out IMFs with α < −1 or with a minimum BD mass of > 0.01 M⊙.

6.2 Measurements in clusters

The prospects for finding very low-mass objects in young clusters are promising. The known age and distance of clusters means that a (model-dependent) mass can be readily estimated from a luminosity. Conversely, there is perhaps less confidence in the models to accurately predict a mass at young ages. The models of Burrows et al. (1997) predict that any T-dwarf in a cluster with age < 30 Myr will have m < 0.01 M⊙. Very low-mass BDs in young clusters can also be brighter (see Fig. 7). At 5 Gyr, a 0.005 M⊙ field object at 5 pc has H ≃ 27, Teff ≃ 200 K, whereas the same object in a 5 Myr old cluster at 400 pc will have H ≃ 20, Teff ≃ 1400 K (Baraffe et al. 2003).

The first claims for the discovery of very low-mass BDs (also called isolated planetary mass objects or “planemos”) were made by Lucas & Roche (2000) and Zapatero-Osorio et al. (2000) in the ONC and around σ Ori respectively. They used near-infrared CMDs to select objects and used their absolute magnitudes to infer masses at or below 0.01 M⊙. Of course the possibility of contamination by background or foreground objects must be ruled out. Spectroscopy on such faint objects (H > 19) is very difficult, but Zapater-Osorio et al. (2000) and Lucas et al. (2006) compared some low-resolution near infrared spectra with those of nearby field BDs and concluded that their candidates had spectral types cooler than M9. The spectra suggest a low-gravity but it is unclear at present what the spectra of
very low-gravity L- and T-dwarfs look like.

Another technique aimed specifically at finding T-dwarfs is “methane-imaging” (Tinney et al. 2005). Narrow-band photometric filters, approximately 0.12 μm wide, are centred at and just bluerward of the deep methane absorption band seen in T-dwarfs at 1.6–1.8 μm. The flux ratio (or colour) defined by these bandpasses becomes very blue when strong methane absorption is established in mid T-dwarfs and is only weakly affected by reddening.

Attempts to exploit this technique are reported by Burgess et al. (2009) in IC 348 (∼ 3 Myr), Haisch et al. (2010) around ρ Oph (age ∼ 1 Myr) and Peña Ramírez et al. (2011) around σ Ori (age ∼ 3 Myr). Burgess et al. find 3 candidate T-dwarf members of IC 348. Estimating their spectral types is complicated by extinction but also because calibration of the methane narrow band indices must be done either with older field T-dwarfs or using models that suggest that young T-dwarfs have a lower $T_{\text{eff}}$ for the same methane index. However two of these candidates are rejected as T-dwarfs by virtue of being too blue in $z - J$. Burgess et al. show that foreground or background contamination by field T-dwarfs or extragalactic objects is unlikely. The nature of these contaminants is therefore unclear. The remaining candidate could have a mass in the range 0.001–0.005 $M_{\odot}$. The presence of one object in this mass range is consistent with an extrapolation of the log-normal IMF that fits the low-mass stars of the cluster. In contrast, an extrapolation of an $\alpha = 0.6$ power law IMF would predict an order of magnitude more T-dwarfs.

Similar work by Peña Ramírez et al. (2011) finds 2 T-dwarf candidates near σ Ori. Both have proper motions inconsistent with cluster membership, yet the probability of contamination by foreground field T-dwarfs is claimed to be very low. Haisch et al. (2010) find 22 candidate T-dwarfs in ρ Oph and again, calculate that foreground and background contamination should be negligible. This number would be almost 10 per cent of the total cluster population determined by Alves de Oliveira et al. (2012) and would seem more consistent with a power-law IMF extrapolation than a log-normal IMF. None of these candidates have so far been confirmed as cluster members with spectroscopy or PMs, but Alves de Oliveira et al. do spectroscopically confirm 5 L-dwarfs with inferred masses of $\simeq 0.01 M_{\odot}$.

Hence observations in the field and in young clusters suggest that the bottom of the IMF is not reached until at least 0.01 $M_{\odot}$ and there are a number of candidate objects, which if confirmed, would be of lower mass. However a problem common to all these studies is their complete reliance on the fidelity of theoretical models in mass and age regimes almost untested by empirical data.

7 A Summary and Future Perspectives

In this review I have attempted to describe a range of approaches for determining the shape of the local stellar and substellar IMF and to highlight their assumptions and limitations.

The IMF of the Galactic disk stellar population is now reasonably well known and there is broad agreement between determinations based on a census of the
local population and deeper surveys of field stars over more limited solid angles. The IMF is Salpeter-like above a solar mass but becomes flat (when expressed as $\phi(\log m)$) for lower masses down to $0.1 \, M_\odot$. The IMF for $0.1 < m/M_\odot < 2$ could be described in terms of two power laws with a break at $\sim 0.5 \, M_\odot$ or in terms of a log-normal with a broad peak at $\sim 0.25 \, M_\odot$. Improvements in the IMF of low-mass stars will certainly come about with the launch of the Gaia satellite (in 2013?), which will provide accurate parallaxes for $V \leq 19$ (i.e. for objects with an absolute magnitude of 19 at 10 pc). This will readily define much larger volume-limited samples and remove much of the statistical uncertainty that currently exists below $0.3 \, M_\odot$. However, much work will still be required to assess the contribution of binary systems and separately determine the “system” and single-star IMFs. To reduce systematic uncertainties, it would be much better to determine masses using $K$-band mass-luminosity relationships and to better calibrate these by finding and monitoring many more low-mass astrometric binary systems and by quantitatively assessing the variations introduced by metallicity dispersions in the Galactic disk.

The substellar IMF of field objects is much more uncertain. It could be described by a power law ($dN/dm \propto m^{-\alpha}$) with $\alpha \leq 0$ or possibly with a log-normal IMF that peaks at $m_c \sim 0.2$ with a reasonably narrow dispersion ($\sigma \leq 0.5$ dex), though simulations of the latter have not yet appeared in the peer-reviewed literature. On the observational side the T-dwarf (and now Y-dwarf) census is still quite small, but this will be vastly improved by ongoing ground-based wide field surveys and the extra sensitivity afforded by WISE (e.g. Kirkpatrick et al. 2012). Systematic uncertainties are due to limited knowledge of the binary properties of field BDs and differences in adopted relationships between absolute magnitude and spectral types. The former can be attacked with new instruments like SPIROU (see Artigau in this volume) and the spatial resolution afforded by the upcoming E-ELT (about 6 mas at 1$\mu$m). The latter requires a significant effort to measure precise trigonometric parallaxes for very faint objects beyond the reach of Gaia and to understand the effects of gravity and metallicity on their spectra.

A more serious problem is that much of the available data on BDs in binary systems suggests that their masses are not well-predicted by the luminosities given by the current generation of evolutionary models. There is an urgent need to obtain masses for benchmark BDs in binaries with known age. These could be members of nearby kinematic moving groups or perhaps companions to white dwarfs. An alternative would be to find BD binaries in the nearest open clusters, but these will need the resolution of the E-ELT to measure their orbits.

The MFs of stars and BDs in young clusters already show remarkable self-consistency and observations of young clusters may be the best method of finding any lower limit to the IMF. Deeper, wider censuses will monitor the effects of mass segregation and evaporation; longer baseline PMs based on digital infrared surveys should prove effective in membership determination. Confirmation from PMs or spectroscopy becomes more and more important at lower masses, where the declining MF means there may be relatively few cluster members among a large number of contaminants. The status of the evolutionary models at young ages is
even more precarious than for older BDs. Existing measurements clearly show large discrepancies between the dynamical masses and masses deduced from HR diagrams for both PMS stars and young BDs. More astrometric and particularly eclipsing binaries must be identified in clusters at a range of ages to in order to adequately test and refine the models.

It is appealing to agree with Chabrier (2005) that almost all IMF estimates in the field and clusters are now converging on a universal log-normal form, with \( m_c \simeq 0.25 \, M_\odot \) and \( \sigma \simeq 0.5 \) dex. However, agreement of the system IMFs might imply a disparity in the single/component IMFs, given the age-dependence of mass-luminosity relationships and the likely evolution of the binary population between young clusters and the field. A single log-normal form does represent most young cluster IMFs well, though there are clear exceptions such as the Taurus and Upper Sco associations. It may be that these exceptions lead us to important insights on the star and cluster formation process, but no clear patterns are yet established. The field IMF may also have this form, but this is yet to be demonstrated convincingly. The currently published simulations favour a power law IMF with \( \alpha \leq 0 \), an index that appears to be significantly smaller than the \( \alpha \simeq 0.5 \) that could represent very low-mass stars and BDs in clusters. There is an urgent need to update these simulations to include the log-normal IMF parameters suggested above, but also to update the evolutionary models and atmospheres they use and to make predictions about the gravity distribution of field BDs of a given spectral type that could be tested by observations. Given the systematic problems identified with models and observations in both the the young clusters and field IMF determinations, it would be remarkable indeed if any close agreement were found at this stage.

Acknowledgements

I would like to thank Corinne Charbonnel, Céline Reylé, Mathias Schultheis and the rest of the local organising and scientific committees for arranging an exceptional meeting and for inviting me to be a lecturer. Thanks are due to the Programme National de Physique Stellaire and the CNRS for their financial support. I would like to acknowledge the helpful discussions I have had with Jérôme Bouvier, Adam Burgasser, Ben Burningham and Kelle Cruz whilst preparing this material.

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