Quantum interference of particles and resonances

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Though the phenomenon of quantum-mechanical interference has been known for many years, it still has many open questions. The present review discusses specifically how the interference of resonances may and does work. We collect data on the search for rare decay modes of well-known resonances that demonstrate a wide variety of possible different manifestations of interference. Some special kinds of resonance interference, not yet sufficiently studied and understood, are also briefly considered. The interference may give useful experimental procedures to search for new resonances with arbitrary quantum numbers, even with exotic ones, and to investigate their properties.

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I. INTRODUCTION

In one of his Physics Lectures, Feynman discussed problems of scientific imagination. In particular, he asked [1] ‘whether it will ever be possible to imagine beauty that we can’t see. It is an interesting question. When we look at a rainbow, it looks beautiful to us. Everybody says, ‘Ooh, a rainbow.’ ... But how would we describe a rainbow if we were blind? ... Do we have enough imagination to see in the spectral curves the same beauty we see when we look directly at the rainbow?’

Similar problems arise with respect to quantum interference phenomena. Everybody is sure that the quantum interference does exist. But one cannot see it directly, only by means of measuring devices. And it is not always easy to imagine how the interference will manifest itself in a particular case. As a result, when one looks at a spectral curve containing interference contributions, their presence is frequently not recognized (or may be even refused).

Meanwhile, the interference of resonances has not only academic interest. Even now it has applications, e.g., to search for and to study rare decay modes of well-known resonances. Investigation of CP violation, especially for B-mesons, also uses interference phenomena. The area of their applications may become wider in future.

In this review, we collect and discuss well-established examples of interference of resonances. Our aim here is to evolve experience and intuition of how the interference may work.

II. GENERAL NOTES: TIME OSCILLATIONS OF PARTICLES

Everybody knows today that quantum physics, and quantum mechanics in particular, is probabilistic. However, this appears to be not its most specific feature. Classical physics may be probabilistic as well. Even few-body classical systems can reveal a chaotic time evolution, where the probabilistic description arises quite naturally. The probabilistic character of statistical physics, being applied to many-particle systems, is also widely known.

Thus, the basic difference between quantum and classical physics lies not in probabilities themselves, but in the way the probabilities should be described for various situations. In the quantum case, one begins with a wavefunction, its squared absolute value providing the respective probability. There can be two (or more) coherent configurations, such that their wavefunctions may be linearly combined. Then, the resulting probability contains not only the sum of probabilities for the two separate configurations, but also an additional term, describing interference of these configurations. The interference may be absent by some reason, e.g., if the interfering configurations are orthogonal. Then the quantum case looks indistinguishable from the classical one. In the absence of interference, a physical system could be described in some classical manner. It is, however, impossible to eliminate all interference contributions from quantum physics, and just this impossibility enables the Bell inequalities [2] to discriminate between a true quantum case and a hidden-variable (classical in essence) situation.

One of the most unfamiliar results of quantum physics is the possibility for some particles to oscillate in time, changing their characteristics. This may emerge if the corresponding particles can be transformed to each other, and their wavefunctions may be coherent.

Of course, the possibility for particles in microworld to oscillate is a direct manifestation of the particle-wave duality for quantum objects. The classical notion of a (point) particle admits the possibility of oscillating motion, but does not admit oscillation of any internal properties (say, of mass, or some other characteristic). On the other hand, wave description of a quantum particle opens different possibilities for classical modeling. Classical propagation of continuous waves, e.g., sound or light, is always directly related to some kind of oscillations; interference of the waves is a well-known and familiar phenomenon. Note, however, that classical mechanical systems (not a point particle!), with oscillating motion, may also model quantum interference effects. A simple example may be given by the system of two (or more) pendula [3]. If they are coupled, say, by a spring, they move in a correlated manner, which is reminiscent of the
Historically, the first example of particle oscillations was provided by strangeness mixing and oscillations in the neutral kaon decays, as suggested by Gell-Mann and Pais [3]. Let us briefly recall this case (for a more detailed description see Ref. [3]; for a brief historical review of the kaon, as well as neutrino oscillations and quark mixing, see the more recent paper by Cabibbo [5]; more references, for both experimental data and their theoretical interpretation, may be found in the Review of Particle Physics [6]). Strong and/or electromagnetic interactions produce neutral kaon states \(K^0\) and/or \(\overline{K}^0\), with a definite value of flavor (here it is the strangeness). However, because of strangeness violation in weak interactions, definite (and slightly different) eigenvalues of mass and lifetime belong to other states, \(K_S\) and \(K_L\), which are linear combinations of \(K^0\) and \(\overline{K}^0\). The strangeness may be tagged by the sign of the electric charge for a lepton, generated in semi-leptonic decays: \(K^0 \rightarrow l^+\), while \(\overline{K}^0 \rightarrow l^-\).

If we have initially pure \(K^0\) and trace time dependence of the semi-leptonic decays, then initially we can observe only \(l^+\). However, \(K^0\) is a definite coherent combination of \(K_S\) and \(K_L\). Since \(K_S\) decays more rapidly, the survived combination of \(K_S\) and \(K_L\) in the later moments of time will be different from initial. In terms of flavor, it will inevitably contain both \(K^0\) and \(\overline{K}^0\). Therefore, with time, we should discover generation of both \(l^+\) and \(l^-\). The ratio of leptonic yields \(l^-/l^+\) changes in time just from the beginning: it oscillates, tending to a constant (\(\approx 1\)) at large times. The two lepton yields become here nearly the same, with the exponential time dependence of pure \(K_L\)-decays.

Another picture of time evolution is seen if we trace the same neutral kaon state through pion pairs produced in its decays. The decay amplitude of \(K_L \rightarrow 2\pi\) is suppressed by the \(CP\)-parity (and is not completely vanishing only due to the \(CP\)-violation). Therefore, the \(K_L\) contribution is initially very small, and time dependence of the two-pion yield at small times is almost purely exponential, corresponding to \(K_S\)-decays. At later times, however, the \(K_S\)-content diminishes, while \(K_L\) is dying much slower. After some time the contributions of \(K_S\) and \(K_L\) to the \(2\pi\)-yield become comparable. Here, they strongly interfere and provide time oscillations. Even later, the \(K_S\)-content and its contribution to the \(2\pi\)-yield become negligible, so we again see the pure exponential time dependence, but now characteristic of \(K_L\)-decays. Contrary to semi-leptonic decays, the two-pion channel shows clear non-pure-exponential behavior (and oscillations) only at some intermediate times, not at early times.

These two examples demonstrate an essential difference between time dependences of decays for non-interfering and interfering particles. In a familiar (non-interfering) case, the decay time dependence is universal, it is the same exponential function for any possible decay mode. In contrast, oscillations for the interfering unstable particles are not universal: different decay channels may have different time dependences.

It is worth also emphasizing that the \(K_L\) and \(K_S\) have strongly different mean lifetimes (the ratio \(\tau_L/\tau_S \approx 500\)). At first sight, this could mean that they decay at very different moments of time and are not able to interfere. Nevertheless, experiments clearly demonstrate that \(K_L\) and \(K_S\) can (and do) interfere in decays! The reason is that the lifetime \(\tau\) in quantum physics is only an average quantity, so the kaon may split into its decay products at any moment, without waiting its lifetime. The decay process begins immediately at the moment of production and continues till the last kaon dies.

Similar flavor oscillations of beauty have been discovered in decays of neutral \(B\)-mesons (for both kinds of them, \(B_d\) and \(B_s\) [4]). Coherent oscillations of two (or more) different flavors are also possible.

Mixing, related to the flavor oscillations, is now known to exist for neutral \(D\)-mesons as well [4]. However, oscillations themselves cannot be seen in the neutral \(D\)-meson decays, since their lifetime is less than 1% of the oscillation period.

Neutrinos seem to oscillate as well [4]. More exactly, only neutrino flavor disappearance has been observed, together with constancy of the neutrino flux summed over flavors. However, explicit appearance of a changed flavor, definitely known for the neutral \(K\), \(B\), and \(D\) mesons, has not been found yet in experiments with neutrinos.

Thus, in spite of some uncertainties, existence of the quantum phenomenon of particle mixing and oscillations is firmly established. Moreover, such an effect is not unique.

Let us briefly consider the space picture of the particle oscillations. Again, we begin with neutral kaons. The short-lived kaons have the mean lifetime \(\tau_S = 0.9 \times 10^{-10} \text{ s}\), which gives \(c\tau_S = 2.7 \text{ cm}\). Therefore, for realistic energies, the kaon oscillations take place most probably at distances of some centimeters or meters from the production point. For the \(B\)-mesons, with \(c\tau_B = 0.46 \text{ mm}\), the oscillations may be seen from some millimeters up to some centimeters. Contrary to these, oscillation effects for neutrinos are seen at much larger distances: about 10 km for atmospheric neutrinos, some hundreds kilometers for reactor antineutrinos and accelerator (anti)neutrinos, and even astronomical distances for solar neutrinos. Thus, the microscopic phenomenon of quantum interference may generate quite macroscopic manifestations!

## III. INTERFERENCE OF RESONANCES

The known hadron resonances have principally the same structure as the stable hadrons (it is more correct to call them ‘stable’, since most of them are not really stable, they decay through weak or electromagnetic interactions). Therefore, interference of resonances could
be considered on the same ground as, say, interference of neutral kaons. For example, when studying time evolution of decays into two pions for a coherent mixture of the meson resonances \( \rho^0 \) and \( \omega \), we should see a time dependence qualitatively similar to \( 2\pi \) decays of a coherent mixture of \( K_S \) and \( K_L \). Just as \( K_L, \omega \) is the longer-lived component \( (\tau_\omega/\tau_\rho = \Gamma_\rho/\Gamma_\omega \approx 18) \). And, again in similarity with the \( (K_L, K_S) \) case, the amplitude for \( \omega \to 2\pi \) is much smaller than for \( \rho^0 \to 2\pi \) (because of isospin violation, while \( K_L \to 2\pi \) is suppressed because of \( CP \)-violation). Therefore, the time dependence for \( 2\pi \) decays of the \( (\rho^0, \omega) \)-mixture should reveal three regions:

- Exponential behavior with characteristic time \( \tau_\rho \), at small times.
- Manifestations of interference, at some intermediate times.
- Exponential behavior with characteristic time \( \tau_\omega \), at large times.

Regrettably, such a picture is absolutely unobservable. Indeed, all hadronic resonances decay with very short lifetimes \( \tau < 10^{-20} \) s, \( c\tau < 3 \cdot 10^{-10} \) cm. Therefore, if a resonance has been produced off a nucleus, the whole space picture of its decay (including oscillations) sits totally inside the surrounding atom. Of course, it cannot be seen (in any sense), and time oscillations for resonance decays, with such short intervals, cannot be traced.

The situation is, however, not hopeless. While the interference of resonances cannot be seen as time oscillations, one can use the complementary variable, the energy (or the mass, in the rest frame).

It is a frequent opinion that in the energy representation a resonance reveals itself in the energy distributions only as a Breit-Wigner (BW) peak of the form

\[
\left| \frac{a}{E - E_0 + i\Gamma/2} \right|^2 = \frac{|a|^2}{(E - E_0)^2 + (\Gamma/4)^2}. \tag{1}
\]

However, interference, in the energy representation, may violate such expectations very essentially.

It is interesting to note that the time and energy representations are not only complementary, but even, with respect to observability of particle oscillations, appear to be inconsistent. Indeed, as has just been explained, interference of hadronic resonances is absolutely invisible in time, but can be studied in the energy representation (see below). On the contrary, for the neutral kaons, time oscillations are clearly seen. But if one tried to study energy distribution for the two pions produced in the kaon decays, one should see two slightly separated BW peaks, corresponding to the unstable particles \( K_L \) and \( K_S \). However, proper widths of the two peaks and the distance between them are so tiny \( (< 10^{-5} \) eV\) that could not be measured with any realistic experimental resolution. For resonances, both peak widths and distances between peaks are larger, and the peaks may be separated experimentally. Thus, the mixing and interference of particles and/or resonances can be studied either in time or in energy representation, but not in both of them.

Let us return to the interference of resonances. If the energy dependence of an amplitude contains not only a resonance BW term, but also some additional contributions, which provide a background \( B \) with respect to the resonance, equation \( 1 \) for the energy distribution changes and takes the form

\[
\left| B + \frac{a}{E - E_0 + i\Gamma/2} \right|^2 = |B|^2 + \frac{|a|^2}{(E - E_0)^2 + (\Gamma/4)^2} + \frac{2|Ba|\cos \varphi \cdot (E - E_0) + |Ba|\sin \varphi \cdot \Gamma}{(E - E_0)^2 + (\Gamma/4)^2}, \tag{2}
\]

where \( \varphi \) is the relative phase between \( a \) and \( B \). On the right-hand side of equation \( 2 \), the first two terms provide the non-coherent sum of the background and BW contributions, while the third term describes just their interference. Let us consider properties of the interference in more detail.

- The interference contribution is linear in both \( |a| \) and \( |B| \). Its relative role depends on \( |a/B| \). At small \( |a/B| \) the interference may appear more important than the BW contribution itself. This can be considered as amplification of a small resonance signal by interference with large background.
- The interference contribution depends on the relative phase \( \varphi \) between \( B \) and \( a \).
- The interference may be either positive (constructive) or negative (destructive).
- In comparison with the BW contribution, the interference may have an additional energy dependence and may decrease with energy slower than the proper BW contribution.
- Because of the factor \( (E - E_0) \), the interference contribution may change its sign. Generally, any BW term gives rise to both constructive and destructive interference (in different energy regions).
- The background itself, \( |B| \) and \( \varphi \), also may depend on energy. As a result, the background may appear different in regions of constructive and destructive interference, and the relative role of these regions may be very different, up to full vanishing of one of them. Thus, the presence of interference may provide either bump, or dip, or both. Positions of the bump and/or dip are, generally, shifted from the true position of the resonance.
- The same resonance can interfere differently in different decay channels, at least due to different properties of the corresponding backgrounds.
Let us consider now, how the known resonances interfere in relatively simple cases when two (or more) resonances generate the same decay products, thus producing the same final states. Such resonances can still have different quantum numbers, as, e.g., $\rho^0 \rightarrow \pi^+\pi^-$ ($J^P = 1^-$) and $f_0 \rightarrow \pi^+\pi^-$ ($J^P = 0^+$). At first sight, they cannot interfere, because of different partial waves of the final pions ($P$- and $S$-waves, respectively). This is true indeed, but only after integration over all directions of the relative momentum of the pions. If one takes a particular direction or a restricted set of directions, the interference is possible. It can manifest itself observationally by an asymmetry of the pions flight direction, in the pair rest-frame, with respect to a chosen direction (say, to the direction of the laboratory momentum of the pair). Interference of such a kind seems to be well understood theoretically. Experimentally, the angular asymmetry has been recently used to separate the resonances $\rho^0$ and $f_0(980)$ in photoproduction of the $\pi^+\pi^-$-pair off the proton [8]. Note, however, that the $(S, P)$-wave interference in the $(\pi^+\pi^-)$-pair, but also by difference in final-state interactions of $\pi^+p$ and $\pi^-p$.

A clearer case is the interference of direct channel resonances, e.g., in the annihilation $e^+e^- \rightarrow$ hadrons. It may be called the direct interference of resonances.

**A. Resonances and their direct interference in $e^+e^-$ annihilation**

Production of hadrons in $e^+e^-$ annihilation, up to the lowest order in electroweak interactions, goes through the virtual photon (or Z-boson). Therefore, all direct channel resonances, seen in the annihilation, have the same quantum numbers $J^{PC} = 1^{--}$ as the photon. Of course, this simplifies theoretical description of their interference. In addition, experiments on $e^+e^-$ annihilation have reached very high precision of measurements for some final states.

If an isolated resonance decays into the final state $X$ (see figure 1), its integrated contribution to the process $e^+e^- \rightarrow X$ is proportional to $(\Gamma_X - \Gamma_X)/\Gamma$, where $\Gamma$ is the total width of the resonance, $\Gamma_X$ and $\Gamma_X$ are its partial widths for decays into $e^+e^-$ and $X$, respectively. The energy dependence of the cross section for an isolated resonance is determined by the BW expression.

If two (or more) resonances in $e^+e^-$-annihilation, with the same $J^{PC}$, have in addition the same decay products $X$, they are always coherent and, therefore, can (and, moreover, should) interfere. Of course, with background determined by another resonance, equation (2) shows that the interference becomes weaker (decreases) when the mass difference of the two resonances increases. Nevertheless, its manifestation may still be quite essential.

At first sight, two resonances could interfere only if their BW peaks overlap, i.e., if distance between their masses is not greater than the sum of their widths. In reality, however, this is not necessary, since every BW amplitude has long tails, with not very rapid decrease. Examples below demonstrate that such a tail may provide sufficient background to interfere with another resonance.

The situation recalls, to some extent, the time representation picture of interfering particles with strongly different lifetimes. For example, in decays of neutral kaons, the $(K_S, K_L)$-interference becomes to be clearly seen only after several $\tau_S$, though the $K_S$-component decreases exponentially. In energy representation, BW tails decrease essentially slower. As a result (and we will see it below), they provide the possibility for interference of resonances even if the mass difference of the resonances is noticeably larger than the sum of their widths.

Now we are ready to discuss particular cases of direct interference.

1. **Final state $\pi^+\pi^-\pi^0$**

Experimental data on the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0$, as measured by the SND group, are collected in [9]. Figure 2 shows the cross section as a function of energy (mass). Its most evident feature is the clear BW peak of the $\omega$-resonance, which has the mass $m_\omega = 783$ MeV, the rather narrow total width $\Gamma_\omega = 8.5$ MeV, and the large branching ratio for the $3\pi$ decay (the partial width $\Gamma(\omega \rightarrow \pi^+\pi^-\pi^0) = 7.6$ MeV). All numerical values here and below correspond to the latest Review of Particle Physics [8].

The BW tail of $\omega$, together with BW tails of higher resonances $\omega'$ and $\omega''$, also having essential decays to the $3\pi$-channel, provides almost constant background in the $\phi$-meson region, near its tabulated mass $m_\phi = 1019.5$ MeV [6]. But the $\phi$-meson itself, the well-known and firmly established resonance, does not evolve here a standard BW-peak. Instead figure 2 demonstrates nearly ideal interference curve: the cross section shows the bump, then rapidly drops to dip. After that the cross section increases again, though slower, up to the value close (but not equal) to the pre-bump one. The distance between masses of the maximum and minimum is of order $\Gamma_\phi = 4.3$ MeV, and the $\phi$-meson mass lies just between the values of the maximum and minimum. Such behavior qualitatively appears to correspond to the case of equation (2) with constant background $\tilde{B}$ and small $|a/B|$ ratio: the interference term is comparable or even

![Feynman diagram](image-url)
larger than the proper BW term. Indeed, fit to the data leads to the small partial width
\[ \Gamma(\phi \rightarrow \pi^+\pi^-\pi^0) = 0.65 \text{ MeV}. \]
This is essentially smaller than \( \Gamma(\omega \rightarrow \pi^+\pi^-\pi^0) \), in agreement with the Zweig rule suppression.

The role of the \( \rho' \)-meson is worth special consideration. Masses of \( \rho^0 \) and \( \omega \) are nearly equal (775 MeV and 783 MeV, correspondingly), but the decay \( \rho^0 \rightarrow \pi^+\pi^-\pi^0 \) is expected to be strongly suppressed, due to the isospin (or, equivalently, G-parity) violation. Therefore, at first sight, the \( \rho^0 \) contribution should always be negligible in comparison with the \( \omega \) one. This is true, indeed, near the vertex of the BW peak, but may not be so for the BW tails, because of very different total widths. The large \( \rho^0 \)-width, \( \Gamma_{\rho^0} = 149.4 \text{ MeV} \), is about 18 times larger than the \( \omega \)-width, suggests slower decrease of the \( \rho^0 \) BW tails as compared to the \( \omega \)-ones. As a result, the \( (\rho^0, \omega) \)-interference in \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) is quite negligible near the vertex of the BW-peak, but may be noticeable for BW-tails (below we will see such a situation explicitly, for the \( \pi^0\gamma \) final state). Fit to experimental data leads indeed to the very small value \[ \Gamma(\rho^0 \rightarrow \pi^+\pi^-\pi^0) = 0.015 \text{ MeV}. \]

2. Final state \( \eta \gamma \)

Now we consider the reaction \( e^+e^- \rightarrow \eta \gamma \). Its experimental cross section, also measured by the SND group, is shown in figure 3 [10]. At first sight, it looks quite natural and demonstrates two BW peaks, in the \( (\rho^0, \omega) \) and \( \phi \) regions, without any interference. However, let us consider the situation in more detail.

We begin with the \( (\rho^0, \omega) \) peak. Both \( \rho^0 \) and \( \omega \) may contribute to the peak, since they both are produced in \( e^+e^- \) annihilation and may decay to \( \eta \gamma \). We can estimate the expected role of the two resonances, since all the necessary parameters can be determined independently of the reaction under discussion.

To the present best knowledge [6],
\[ \Gamma(\rho^0 \rightarrow e^+e^-) = 7.04 \text{ keV}, \quad \Gamma(\rho^0 \rightarrow \eta \gamma) = 44.9 \text{ keV}; \]
\[ \Gamma(\omega \rightarrow e^+e^-) = 0.60 \text{ keV}, \quad \Gamma(\omega \rightarrow \eta \gamma) = 4.1 \text{ keV}. \]

Note that the evident smallness of widths (4) as compared to widths (3) is a result of interference inside the resonances, at the quark level. In the framework of the quark-antiquark picture for mesons, both \( \rho^0 \) and \( \omega \) are superpositions of \( u\bar{u} \) and \( d\bar{d} \) pairs. For widths (3), the contributions of \( u \) and \( d \) quarks interfere constructively and increase the widths, while for widths (4) their interference is destructive and suppresses the widths.

The values (3) and (4) suggest that the \( \rho^0 \)-contribution to the reaction \( e^+e^- \rightarrow \eta \gamma \) should dominate over the \( \omega \)-contribution. Nevertheless, the left peak in figure 3 has its maximum just near the \( \omega \)-mass, while near the \( \rho^0 \)-mass one can see only a hint of break. Such a structure cannot be explained by the sum of two BW peaks, but may emerge due to constructive interference of the two resonance contributions. We will discuss this point in more detail below, in connection with the \( 2\pi \) final state.

The right peak in figure 3 related to the \( \phi \)-meson, also gives evidence for the presence of interference: the peak

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FIG. 2: The \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) cross section measured by the SND group and collected in [8]. The curve is the fit with the \( \omega, \phi, \rho^0, \omega' \) and \( \omega'' \) resonances. Used with permission of the SND group.

FIG. 3: The \( e^+e^- \rightarrow \eta \gamma \) cross section measured by the SND group; the curve is the best fit [10]. Used with permission of the SND group.
is not symmetrical, as would be expected for the pure BW term. The right side of the peak is sharper than the left one. The necessity of the interference becomes even more evident if one considers the cross section for $e^+e^- \rightarrow \eta\gamma$ in a wider energy interval. Figure 4 shows measurements of detectors SND and CMD-2, both below and above the $\phi$-meson [9]. The cross sections just below and just above the narrow $\phi$-peak are noticeably different. This becomes possible due to the contribution of the interference term, which decreases, with moving off the resonance mass, slower than the proper BW contribution, and has different signs below and above the resonance.

3. Final state $\pi^+\pi^-$

Interference of $\rho^0$ and $\omega$ in decays to $2\pi$ is one of the earliest and most famous examples of interference of two resonances. Moreover, it was the first case where the interference phenomenon allowed study of a very rare decay, which could hardly be discovered without interference.

Indeed, the decay $\omega \rightarrow \pi^+\pi^-$ is suppressed because of isospin symmetry violation. Interference contribution of this decay is also suppressed, but weaker. In addition, contrary to the proper BW contribution of $\omega$, its interference with the $\rho^0$-meson is amplified due to the intense $\rho^0$-production and the nearly 100% branching ratio for the decay $\rho^0 \rightarrow \pi^+\pi^-$. The presence of the $(\rho^0, \omega)$-interference results in distortion of the canonical form of the $\rho^0$-meson BW peak. For the reaction $e^+e^- \rightarrow \pi^+\pi^-$, this is well seen in figure 5 [9], which shows the cross section of this reaction, and/or in figure 6 [11] for the charged pion form factor, closely related to the $e^+e^-$ annihilation into $\pi^+\pi^-$. The behavior of the form factor may be discussed in more detail on the basis of measurements by the CMD-2 group [12]. Their high-precision results (see figure 7) allow us to consider the inset with details of the interference region.

The left side of the $\rho^0$ peak (below the $m_{\rho^0}$) looks to be undistorted. But slightly above $m_{\rho^0}$ the form fac-
tor rapidly falls due to destructive interference. If the \( \rho^0 \) and \( \omega \) contributions to the \( e^+e^- \rightarrow \pi^+\pi^- \) amplitude were real with respect to each other, then, according to equation (2), the interference would change its sign just at \( m_\omega \). However, earlier phenomenological analysis [13] presented some evidence for complexity of the \( (\rho^0, \omega) \) mixing. Now we see that the interference changes its sign somewhat above \( m_\omega \), thus confirming the presence of small, but non-vanishing complexity between the \( \rho^0 \) and \( \omega \)-resonance contributions.

Parameterization of the fit shown in figure 7 and the emerging values of the parameters are given in [12]. With corrections for the current \( \rho^0 \)-properties [8], they lead to the partial width

\[
\Gamma(\omega \rightarrow \pi^+\pi^-) = 0.13 \text{ MeV} \quad (5)
\]

(compare with \( \Gamma(\rho^0 \rightarrow \pi^+\pi^-) = 148 \text{ MeV} \) [8]). It is worth emphasizing that the partial width (5) is known only due to the \( (\rho^0, \omega) \) interference.

Note that even change of the interference sign, above \( m_\omega \), does not lead to the growing behavior of the form factor anywhere in the interference region. This can be understood as due to both the mentioned complexity and the decrease of the interfering background. Indeed, background for the \( \omega \) BW contribution is the \( \rho^0 \) BW contribution; in the constructive interference area it is noticeably lower than in the destructive one. Therefore, the constructive interference of \( \rho^0 \) and \( \omega \) is amplified much weaker than the destructive one, and is hardly seen. Above 800 MeV, the interference dies out, and the form factor is, again, determined mainly by the pure \( \rho^0 \)-contribution.

It is interesting to discuss how the picture would look if the relative sign of the \( \rho^0 \) and \( \omega \)-contributions were just opposite to the existing one. In such a case, we would need to reverse the above description. Slightly above \( m_{\rho^0} \) the interference would be constructive, and the form factor would rise, instead of fall. It would look like some break near \( m_{\rho^0} \). Then, about \( m_\omega \), the form factor would begin to decrease, partly due to the change of the interference sign and partly due to the decreasing \( \rho^0 \) amplitude together with the dying out \( \omega \) amplitude. We can note that, qualitatively, such a hypothetical picture just corresponds to the structure of the \( (\rho^0, \omega) \) peak in the reaction \( e^+e^- \rightarrow \eta\gamma \) (see figure 8 and its discussion above).

4. Final state \( \pi^0\gamma \)

The process \( e^+e^- \rightarrow \pi^0\gamma \) also demonstrates direct interference of resonances \( \rho^0, \omega, \phi \). Its cross section, measured by the SND group [11, 14], is shown in figure 8. The energy dependence looks here similar to the case of 3\( \pi \) annihilation (figure 2). It has a clear BW-like peak in the \( (\rho^0, \omega) \) region and a bump-dip structure in the \( \phi \) region. Again, as in the 3\( \pi \) case, for the relative role of the \( \rho^0 \) and \( \omega \) resonances, we can give qualitative evaluations, based on other processes and independent of data on the reaction under discussion.

The decay \( \omega \rightarrow \pi^0\gamma \) is rather intensive and may be studied in many reactions. After all, it has the partial width (recall that the numerical values correspond to the summary tables of [6])

\[
\Gamma(\omega \rightarrow \pi^0\gamma) = 0.76 \text{ MeV} \quad (6)
\]

FIG. 7: The pion form factor squared in the region of the \( \rho^0 \) meson peak as measured by the detector CMD-2 [12]; the curve is the best fit. The inset shows details of the interference region. Used with permission of the CMD-2 group.

FIG. 8: The cross section of the process \( e^+e^- \rightarrow \pi^0\gamma \). Data are measurements of the SND group [14]. two fitting curves correspond to models with \( \rho^0 + \omega + \phi \) (solid) or \( \omega + \phi \) (dashed) intermediate states. Used with permission of the SND group.
The $\rho^0$ radiative width

$$\Gamma(\rho^0 \to \pi^0\gamma) = 0.09 \text{ MeV}$$  \hspace{1cm} (7)$$

is reliably known today just from the $e^+e^-$ annihilation which we are discussing now. But as its rough estimate one could use the $\rho^\pm$ radiative width $\Gamma(\rho^\pm \to \pi^\pm\gamma) = 0.067 \text{ MeV}$, measured in different ways, e.g., by the Coulomb excitation $\pi^\pm \to \rho^\pm$ (the difference of the two radiative widths, for $\rho^0$ and $\rho^\pm$, may be explained by the $(\rho^0, \omega)$ interference, absent for $\rho^\pm$; see, e.g., Ref. [13]).

Note that the essential difference of the radiative widths (6) and (7) is due to quark-level interference, just as for the widths (3) and (4). The interference of widths (6) and (7) is due to quark-level interference, just without interference manifestations. Second, one of the decay products, $\omega$, is itself a resonance and, in its turn, can be studied only through its (several) decay modes. Thus, we need to deal here with a cascade decay, which may affect the interference picture (in the above examples, only the $\eta$-meson has similar properties, but it is much narrower than $\omega$).

The $\omega$-meson has two frequent decay modes, $\omega \to \pi^0\gamma$ (9%) and $\omega \to \pi^+\pi^-\pi^0$ (89%). To study decay (5) in $e^+e^-$ annihilation, one may respectively use processes

$$e^+e^-\to \omega\pi^0, \quad e^+e^-\to \omega^0\pi^0\gamma.$$  \hspace{1cm} (8)$$

Both reactions were investigated experimentally in several measurements of the SND group; their results for the decay (5) have been used for the tables of the Review of Particle Physics [15] (the corresponding references are also given there).

Recently, the KLOE Collaboration presented new data for the two reactions [13]. They are measured in the same experiment and with better precision than before. The obtained cross sections are shown in figure 9 together with the resulting fits. For each of the cases, the cross section reveals a dip in the $\phi$-meson region. However, its detailed form is different for the two cascade branches.

In the bottom panel (decay channel $\omega \to \pi^0\gamma$), the lowest point of the dip is at the standard mass value $m_{\phi} = 1019.5 \text{ MeV}$, the dip width corresponds to the standard $\phi$-meson width $\Gamma_{\phi} = 4.3 \text{ MeV}$. Fits to background before and after the dip continue each other. The curve in the $\phi$-meson region looks like the BW peak with
the reversed sign. In terms of equation (2), such form corresponds to \( \cos \varphi = 0 \), \( \sin \varphi = -1 \) (i.e., \( \varphi = -\pi/2 \)), and \( |Ba| \Gamma > |a|^2 \). Indeed, the accurate fit of the KLOE Collaboration \( \text{[13]} \) gives the resonance versus background relative phase near \(-\pi/2\).

The top panel (decay channel \( \omega \to \pi^+\pi^-\pi^0 \)) shows different behavior of the cross section. The lowest point of the dip is reached below \( m_\omega \). The background after the dip is higher than a simple continuation of the background before the dip. Such properties mean that \( \cos\varphi > 0 \) and \( \sin\varphi < 0 \). These inequalities for the four-pion branch of the decay cascade are satisfied indeed by the KLOE fit \( \text{[15]} \), which gives \( \varphi \approx -\pi/4 \).

Thus, cascade decays of a resonance may provide different interference pictures in different branches of the cascade, even with the same first-step decay. It is not amusing, of course, because the different branches of the cascade generate different final states, which interfere with different backgrounds.

The KLOE analysis extracts the amplitude for the decay \( \text{[8]} \) that corresponds to the branching ratio

\[
\text{Br}(\phi \to \omega\pi^0) = (4.4 \pm 0.6) \cdot 10^{-5}.
\]

It is consistent with the earlier value given in tables \( \text{[6]} \), but is somewhat lower and has twice smaller uncertainty. It leads to the very small partial width

\[
\Gamma(\phi \to \omega\pi^0) \approx 0.19 \text{ keV}.
\]

Note that this strong-interaction partial width is smaller indeed than any of the partial widths considered above, including even radiative widths and widths of decays into \( e^+e^- \). Such is a price of the double suppression (Zweig rule + isospin symmetry) for the decay \( \text{[8]} \).

**B. Rescattering interference of resonances**

Up to now, we have considered examples of direct interference of two (or more) resonances. All decay products could come from decays of any of the two interfering resonances. However, this is not the only possible form of interference. Resonances can interfere even if only one of the final particles may be among decay products of both resonances. Moreover, experimenters encountered such a kind of interference long ago, in 1960s, when studying the \( \rho \)-resonance at the dawn of the era of hadron resonances.

Indeed, the \( \rho \)-meson may be produced, e.g., in the reaction

\[
\pi^+ p \to \pi^+\pi^0 p, \tag{9}
\]

by the subprocess

\[
\pi^+ p \to \rho^+ p \to \pi^+\pi^0 p. \tag{10}
\]

However, reaction \( \text{[9]} \) may result also from other subprocesses, e.g.,

\[
\pi^+ p \to \pi^0\Delta^{++} \to \pi^+\pi^0 p, \tag{11}
\]

(Feynman diagrams for the subprocesses \( \text{[10]}, \text{[11]} \) and \( \text{[12]} \) are shown in figure \( \text{[10]} \). All the subprocesses generate the same set of final particles and, therefore, can interfere. Contribution to the cross section, provided by such two-resonance interference (figure \( \text{[11]} \), has a structure similar to rescattering diagrams for three-particle interactions, where a particle interacts first with one partner, and then proceeds to interact with another one (figure \( \text{[12]} \). That is why interference of such a kind may be called rescattering interference. Note that the rescattering plays the leading role in widely used description of three-particle quantum systems by the Faddeev equations \( \text{[10]} \).

This kind of interference may also be called the rearrangement interference, since the observed final particles in this process rearrange in different ways to reveal different resonances.

The phenomenon of rescattering interference has various analogies in quantum physics. For instance, in the case of \( (\Delta^{++}, \rho^+) \) rescattering interference, one cannot discriminate whether the \( \pi^+ \) was produced from \( \Delta^{++} \) or from \( \rho^+ \). This is similar to the case of the two-slit quan-
tum interference, where one cannot discriminate which of two slits was traversed by the quantum particle.

However, the cases of direct and rescattering interferences have essential and interesting differences. Two directly interfering resonances should possess strictly related quantum numbers: they both should be either mesons or baryons, they should carry the same charge and the same flavor - strangeness, beauty, and so on (we have seen that isospin of two directly interfering resonances may be different, as for \( \rho^0 \) and \( \omega \); this is possible since the isospin symmetry is not exact, even in the framework of strong interactions). In contrast, two resonances providing rescattering interference may have totally unrelated quantum numbers. They may be, e.g., a meson and a baryon, as in reactions (10)–(12), they may even have different charges and/or flavors. All such differences do not exclude the possibility to interfere.

Instead, the rescattering interference imposes restrictions of another kind. To be coherent, final states of different processes (different cascade branches) should, of course, have the same particle content. But this condition is insufficient. The final states should also be kinematically consistent. Such consistency is much more restrictive for rescattering interference than for the direct one. As a result, the direct interference may be studied as a function of only one essential parameter, the total energy (total mass). It is just such consideration that was used above for the discussion of \( e^+e^- \) annihilation. In difference, the rescattering interference depends on several parameters. For three-particle production, such parameters are, first of all, the pair masses. The total energy, at first glance, should not be essential. However, the kinematical consistency implies that the rescattering interference of two particular resonances may be noticeable only in a limited interval of the total energy. Momentum transfers (only two of the possible six are kinematically independent for transitions of 2 \( \rightarrow \) 3 particles) can also affect the rescattering interference picture.

Existence of the rescattering-type interference, and the necessity of accounting for it, was clearly demonstrated, e.g., by Michael [17]. He fitted the \( \rho^+ \)-resonance peak in reaction (10) at 2.67 GeV/c. The form and parameters of the peak had been found to vary as a function of position in the Dalitz plot. The major variations were explained by interference of the subprocess (10) with other subprocesses. The dominant effect came from the subprocess (11). A smaller contribution was attributed to the production of \( \pi N^* \), where \( N^* \) meant nucleon-like \( N \pi \) resonances with masses in the interval 1500 – 1700 MeV. It is interesting that the model description [17] has needed also to use interference with the diffractively produced final states \( \pi^+(p\pi^0) \).

Note that the rescattering interference may emerge not only in particle collisions, but also in particle decays into several particles. This effect is well known for \( B^- \) and \( D^- \)-meson decays. If the direct interference has become an instrument to study rare decays of various known meson resonances (see above), the rescattering-type interference has become an instrument to study parameters of the \( B^- \) and \( D^- \)-meson decays, or relative phases of final state strong interactions (for a recent example, see [18]). In the following subsection, we will discuss interference effects for decays of heavy hadrons in some more detail.

Since rescattering interference of two (or more) resonances (in similarity with direct one) deforms the resonance peaks, it became a standard approach, when studying resonances, to eliminate interference as much as possible. For such a purpose, intensively produced resonances (say, \( \Delta^{++} \) in reaction (9) ) are usually cut out. This may be the reason why the existing literature does not provide any study of the general structure and properties for the rescattering interference, though there are many model-dependent considerations which fit data on particular reactions through accounting for rescattering-type interferences of various resonances.

Elimination of interference is an appropriate method for extracting a resonance with sufficiently high apparent cross section of its production, in comparison with non-resonant contributions. The situation may be different, however, for rare decay modes, or if there exist resonances with relatively low production cross section.

Possible existence of unusual hadrons, having suppressed couplings to the familiar hadrons, as a result of specific internal structure, was suggested long ago [19]. With such states, interference of some kind could be really helpful to search for their manifestations.

Indeed, the exotic baryon \( \Theta^+ \) (initially, \( Z^+ \)) was later predicted [20] on the basis of the quark-soliton model. It should have evidently non-canonical quark structure and, by prediction [20], is expected to have small width (and small decay coupling). Its production may represent a new kind of hard processes [21], thus implying relatively small production cross section. In any case, \( \Theta^+ \) has not been seen in many experiments. If \( \Theta^+ \) does, nevertheless, exist, its production is strictly limited.

In particular, rather low upper boundary for the \( \Theta^+ \)-photoproduction on the proton was given by the CLAS Collaboration [22]. To amplify a possible signal of the \( \Theta^+ \), it was suggested [23] to look for the interference of final states for two subprocesses (see figure 13)

\[ \gamma p \rightarrow \phi p \rightarrow K_S K_{LP}, \quad \gamma p \rightarrow K_S \Theta^+ \rightarrow K_S K_{LP}. \] 

(13)

This is just a rescattering-type interference, similar to the interference of \( \rho \) and \( \Delta \) discussed above. But this time, a possible weak (and unobserved) proper \( \Theta^+ \)-signal may be enhanced by the strong signal of the \( \phi \)-resonance. Note that contribution of the \( \phi \)-photoproduction is cut out in the published analysis [22], and any potential interference with \( \phi \) has been discarded.
Subprocesses leading to multiparticle final states with more than three particles also can (and should) interfere. Such processes depend on even larger number of physical parameters (pair masses and momentum transfers), which may affect the possible interference picture. It is, therefore, essentially more complicated type of rescattering interference than the three-particle cases discussed up to now in this subsection. Nevertheless, it can also be helpful to search for new resonances and investigate them. For instance, for $\Theta^+$, interference of the subprocesses
\[
\gamma p \rightarrow \phi \Delta^+ \rightarrow K^- K^+ \pi^+ n,
\]
\[
\gamma p \rightarrow K^* 0 \Theta^+ \rightarrow K^- K^+ K^0 n,
\]
shown in figure 14, is also suggested to be investigated \[23\]. Here, the two good resonances, $\phi$ and $\Delta$, may enhance manifestation of $\Theta^+$.

Note an interesting difference between processes \[13\] and \[14\]. The process \[13\] may provide a peak in the system $K_L p$ (or $K_S p$), and one could not discriminate between $\Sigma$-like state (with $S = -1$) or $\Theta$-like state (with $S = +1$). In contrast, a peak in the system $K^+ n$, expected in the process \[14\], has the tagged strangeness $S = +1$.

C. Interference phenomena in decays

As was mentioned in the previous subsection, various kinds of interference may be seen also in decays of heavy hadrons. They may be similar to the direct interference, or the rescattering one, or be of more complicated type (as will be seen below). For example, an essential part of decays $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ goes through the intermediate two-body channels $\rho^0 \pi^0$, $\rho^+ \pi^-$, $\rho^- \pi^+ \pi^0$ (see the corresponding branching ratios in the tables \[6\]). Of course, all these channels interfere with each other (just as in rescattering interference) and, thus, affect the distribution of events over the Dalitz plot. Therefore, to accurately extract the coupling constant between $J/\psi$ and $\rho\pi$, one should account for the interference. Such necessity becomes even more important for decays of even heavier hadrons.

Collaborations BaBar and Belle, working at $B$-factories, have collected great sets of data on multiparticle decays of $B$, $D$-mesons, and some other heavy hadrons. A large set of data on $D$-meson decays has been gathered also by the CLEO Collaboration.

Those decays provide many examples of various kinds of interference. They are worth a special review paper, and we will not consider here all details of interference in the decays. Instead, we will mainly be concerned with similarities and/or differences with respect to the interference manifestations described above. Nevertheless, we will briefly discuss also some particular examples.

The interference picture in collisions, as in $e^+ e^-$ annihilation or, e.g., in reaction \[9\], depends on the total energy. It essentially changes (or even disappears) when the total energy changes. For decays, in contrast, the total energy is fixed by the mass of the decaying particle. Moreover, collisions generally produce states with various values of parity and total angular momentum. In contrast to this, decays produce only final states with the $J^P$ value of the initial particle. In this respect, the situation is similar to the $e^+ e^-$ annihilation where hadrons are produced through the virtual photon with fixed $J^P = 1^-$. At first sight, these two points should simplify the interference picture in decays. However, decay properties may complicate the situation. For example, the strong-interaction decays $J/\psi \rightarrow \rho \pi$ go (up to smaller electromagnetic contributions) with isospin conservation and are described by one coupling constant. On the other side, weak decays $B^0 \rightarrow \rho \pi$ (quark decay $b \rightarrow u d d$) violate isospin, and all three couplings of $B^0$ to the three channels $\rho^0 \pi^0$, $\rho^+ \pi^-$ and $\rho^- \pi^+$ may be independent. Experimentally, the branching ratio for $\rho^0 \pi^0$ is several times smaller than for $\rho^\pm \pi^\mp$ \[24\]. Difference of $CP$-violating parameters for the decay channels $\rho^+ \pi^-$ and $\rho^- \pi^+$ \[24\] supports difference of the corresponding decay amplitudes. Thus, all the charge channels look unrelated indeed.

In addition, instead of the total energy, decays have another variable, the time between production and decay of the hadron. For neutral mesons with open flavor, the
interference picture may change with changing this time (see below).

Generally, decays of heavy hadrons provide a rich source of resonances which are seen in multiparticle final states. If, e.g., we consider the three-particle decay $B^+ \rightarrow \pi^+ \pi^- \pi^0$, its essential part comes from the quasi-two-body decay $\rho^0(770)\pi^\pm \pi^\mp$. But, in addition, there are also other sub-decays: $f_0(980)\pi^\pm$, $f_2(1270)\pi^\pm$, and $\rho^0(1450)\pi^\pm$. This example demonstrates, that heavy hadron decays may allow us to investigate various resonances insufficiently studied up to now, including radial excitations. On the other side, the presence of many resonances complicates the problem of their accurate separation, because of numerous interference contributions. Especially important (and difficult) is accounting for interference between states with the same $J^P$-values, such as, e.g., $\rho(770)$ and $\rho(1450)$ or other radial excitations, since their interference cannot be suppressed by angular integration of the produced pion pair (just as in the case since their interference cannot be suppressed by angular integration of the produced pion pair (just as in the case of direct interference). Separation of such states may be done today only in a model-dependent way. This is just what is done for $B^0$-meson decays [24].

Sure, the above notes are qualitatively applicable to various decays of $B$-mesons, as well as other heavy hadrons, e.g., of $D$-mesons, or charmed baryons. Interference is very interesting also for final states with strange or even charmed hadrons. And, of course, interference becomes even more essential for decays into final states with four or more hadrons.

A specific feature of weak decays of hadrons is the possibility of $CP$-violation. Manifestations of this phenomenon for neutral flavored mesons, $K^0(\overline{K^0})$, $B^0(\overline{B^0})$, $B_s(\overline{B_s})$, and, possibly, $D^0(\overline{D^0})$, are closely related to interference between amplitudes of meson and anti-meson decays. Such interference generates oscillatory time behavior for particular decays of those mesons. The most famous example is demonstrated by oscillations in the decay $K^0 \rightarrow 2\pi$. But there exist less familiar manifestations of interference also related to the $CP$-violation. Here we briefly discuss two such effects.

An interesting problem is a discrete ambiguity in measuring $CP$-violating parameters. Its origin may be traced [26] to a phase factor. Mathematically, this may be illustrated by a simple example. Recall that any measurable value in quantum physics is related to the absolute value squared of some amplitude. Now, if one knows $|a|, |b|$, and

$$|c|^2 = |a + b \cdot e^{i\alpha}|^2,$$

one can determine the phase factor $\exp(i\alpha)$ only with the two-fold ambiguity, up to the sign of its imaginary part. This ambiguity will be eliminated, if one can also find

$$|c|^2 = |a + b \cdot e^{i(\alpha + \theta)}|^2,$$

where $\theta$ is a known function of some parameter and has a definite (!) sign.

As an example, let us consider a particular weak decay $B^0(\overline{B^0}) \rightarrow J/\psi K^0(\overline{K^0})$, with the quark-level decay $b \rightarrow c\bar{c}s$ or charge conjugate. The $CP$-violation in this channel, as compared to any other $B$-decay, has been measured most precisely (see recent overview [27]). To resolve ambiguity in this decay mode, it was suggested [26] to study the whole decay sequence, including the secondary kaon decay, in dependence on both $t_B$ (time of the $B$-decay) and $t_K$ (time of the $K$-decay).

Then the coherent beauty-strangeness oscillations provide the additional phase factor, which comes from the kaon time evolution. It is related to the $(K_S,K_L)$ oscillations, and the sign of its phase is determined by the known sign of the mass difference $m_S - m_L$. Regrettably, the Monte Carlo simulations [26] show that such an approach needs very high statistics, as yet unavailable.

More realistic has appeared another method, similar to that suggested earlier [26] for the $B \rightarrow \rho\tau$ decays. The BaBar Collaboration [26] investigated the decay

$$B \rightarrow J/\psi K \pi,$$

also with the quark-level decay $b \rightarrow c\bar{c}s$. Here the reference sign for the $CP$-violating phase comes from the interference of amplitudes for the $(K\pi)$-pair produced in the $S$- and $P$-wave states. When the $(K\pi)$ mass goes through the band of the resonance $K^*(890)$, the $S$-wave phase stays nearly constant, while the $P$-wave phase strongly changes (increases), according to the Breit-Wigner formula. This known behavior of the phases (and of their difference in the $S-P$ interference) has allowed experimentalists to eliminate the sign ambiguity in the $CP$-violating phase factor as well [26].

Another interesting (and somewhat unexpected) effect arises in decays with secondary neutral kaons. It was first discovered theoretically for decays $D \rightarrow K\pi$ [30]. The neutral kaons are usually registered by their two-pion decay. It appears that $CP$-violating $(K_S,K_L)$ interference in this secondary decay may imitate small $CP$-violation for the $D$-meson branchings, even if it was totally absent at the first stage of the decay. Such an effect was later rediscovered, also theoretically, for the $\tau$-lepton decays $\tau^+ \rightarrow \nu\pi^\pm K_S$ [31]. Of course, this ‘secondary violation’ is totally determined by the neutral kaon properties and, by itself, gives no new information. It should be present in any decay with secondary neutral kaons, but, e.g., for the decay $B^0(\overline{B^0}) \rightarrow J/\psi K_S$ it is practically unimportant due to large $CP$-violation at the first step of the process. But for $D$-meson or $\tau$-decays, with expected small $CP$-violation, it may provide a useful reference point. Experimentally, this effect has not yet been observed.

Even the few effects, briefly described here, demonstrate how diverse may be interference manifestations in decays. They may be very useful and important for studies of both spectroscopy and properties of new resonances, as well as for more detailed investigation of the known resonances.
IV. DISCUSSION AND CONCLUSIONS

Many examples of the interference of resonances, discussed in this paper, are rather simple. They, nevertheless, allow us to demonstrate various features, inherent also in more general and complicated cases. That is why we are now able to formulate a number of sufficiently general conclusions.

- Interference of resonances has the same quantum nature as oscillations of particles, though they are observed in complementary variables - energy (mass) for the former, or time for the latter.

- Two resonances can interfere even if they do not overlap, i.e., if their mass difference is large, larger than the sum of their widths. For instance, $\phi$ and $\omega$ apparently interfere in several decay modes, though $M_\phi - M_\omega \approx 240$ MeV, while $\Gamma_\phi + \Gamma_\omega \approx 13$ MeV. Similarly, particle decays can reveal interference (and oscillations) even if lifetimes of the particles are essentially different. For instance, $K_S$ and $K_L$ mesons demonstrate well-known oscillations, though $\tau_L/\tau_S \approx 500$.

- If a resonance produces only a feeble signal (due to a rare decay mode, or due to mild production cross section), the contribution of its interference with a large background may appear more essential than the proper resonance signal. The corresponding background may be non-resonant, but may also come from another resonance having a profound signal. The both cases can be considered as amplification of the feeble resonance by the large background, be it resonance or non-resonance.

- Interference of a resonance and a background may be either positive (constructive) or negative (destructive), depending on the relative phase between the resonance and the background. Moreover, the interference contribution usually has an additional energy dependence, in comparison with the familiar Breit-Wigner form, even if the background is energy-independent (it is more so for a resonant or any other energy-dependent background). Generally, the interference term changes its sign at some energy near the resonance position.

- Generally, the interference reveals both bump and dip, with their positions shifted from the true position of the resonance. The relative intensity of the bump and the dip may be very different, essentially depending on the energy behavior of the background. Some cases may show only one kind of structure, either bump, or dip.

- The same resonance may produce different interference pictures even in the same reaction when being observed in different decay modes. The situation is similar for particle oscillations: e.g., oscillations of neutral kaons look differently in semileptonic and two-pion decay channels.

- Resonances can interfere in various ways. The simplest case is direct interference, when the resonances generate the same decay products. Evidently, this is possible only if (at least) some quantum numbers (such as flavors, baryon numbers, and so on) are the same for the interfering resonances. However, there can also be rescattering (or rearrangement) interference, when only some of the final particles may emerge in decays of both resonances. Such case of the resonance interference is more complicated. It needs correlated kinematics for products of the interfering resonances, but does not impose any restrictions on the resonance quantum numbers. Note that for the rescattering-type interference, the position of the interference bump (or dip) may, and even should, move when changing some parameters, e.g., momentum transfers.

- Decays of heavy hadrons may demonstrate combinations of various kinds of interference. Account for these effects is necessary, and has been used, to separate different decay sub-channels, with different secondary resonances produced, and to extract related parameters. Regrettfully, the structure of both the rescattering interference and different interference effects in decays is not yet clearly understood. That is why fits to experimental data are still very model-dependent in many cases.

Concluding this brief discussion, one should emphasize that direct interference has become a useful instrument for searching and studying rare decays of well-established resonances. However, its possibilities are limited by restrictions for the resonance quantum numbers. Rescattering interference is not limited by such requirements and, therefore, may provide effective methods to search and study new resonances with arbitrary quantum numbers. Data on multihadron decays of heavy particles also present a new rapidly expanding area for applications of different kinds of interference both to study spectroscopy of resonances and to establish their characteristics.

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