SENSIBLE QUANTUM MECHANICS:
ARE ONLY PERCEPTIONS
PROBABILISTIC? *

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Abstract
Quantum mechanics may be formulated as Sensible Quantum Mechanics (SQM) so that it contains nothing probabilistic, except, in a certain frequency sense, conscious perceptions. Sets of these perceptions can be deterministically realized with measures given by expectation values of positive-operator-valued awareness operators in a quantum state of the universe which never jumps or collapses. Ratios of the measures for these sets of perceptions can be interpreted as frequency-type probabilities for many actually existing sets rather than as propensities for potentialities to be actualized, so there is nothing indeterministic in SQM. These frequency-type probabilities generally cannot be given by the ordinary quantum “probabilities” for a single set of alternatives. Probabilism, or ascribing probabilities to unconscious aspects of the world, may be seen to be an aesthemamorphic myth.

No fundamental correlation or equivalence is postulated between different perceptions (each being the entirety of a single conscious experience and thus not in direct contact with any other), so SQM, a variant of Everett’s “many-worlds” framework, is a “many-perceptions” framework but not a “many-minds” framework. Different detailed SQM theories may be tested against experienced perceptions by the typicalities (defined herein) they predict for these perceptions. One may adopt the Conditional Aesthetic Principle: among the set of all conscious perceptions, our perceptions are likely to be typical.

An experimental test is proposed to compare SQM with a variant, SQMn.
1 Basic Ideas of Sensible Quantum Mechanics

Probabilities can seem rather mysterious in any theory or description that is supposed to be complete. If probabilities are interpreted as indicating fundamental uncertainties, then any theory that describes things probabilistically appears to be uncertain and incapable of being as complete as some alternative theory that describes more precisely what happens.

In many cases the incompleteness of a theory is overbalanced in our evaluation by the relative simplicity it has in comparison with a more complete theory. Thus in classical physics, for example, we may consider a statistical theory better than a more complete alternative which gives the precise trajectory in phase space, because the statistical theory may be much simpler. However, in such cases we readily agree that the statistical theory is incomplete and usually believe that a more complete description exists in principle.

Someone may despair of ever knowing a complete description of a certain system (e.g., the complete history of the universe), and it may indeed be true that he will never succeed in finding it, but that personal despair should not be misinterpreted as evidence that no such complete description exists in principle. Furthermore, one can argue that a complete description certainly exists, namely, the system itself. (One might prefer that a complete theory appear simpler than the system it describes, but that is a separate question from that of the existence of a complete theory.) So in this paper I shall assume that a complete theory of the universe does exist in principle, and that it is a goal of physics to search for one or at least to try to get closer to one.

In the absence of a better alternative, it seems worthwhile to consider whether quantum mechanics is a suitable framework for a complete theory of the universe. But then one runs into the problem that quantum mechanics is usually interpreted probabilistically, which seems to indicate that it cannot be complete. For example, if the quantum probabilities are interpreted as propensities for several possible sequences of events to be actualized, and if only one sequence actually occurs, then the theory is incomplete in not describing which particular sequence does occur.

Here, expanding upon previous work [1], I argue that quantum mechanics is a complete framework for describing the unconscious aspects of reality (here called the quantum world), because those aspects should not be described probabilistically. Instead, I claim it is consistent to assume that they are completely described by the quantum amplitudes (e.g., by the amplitudes in a path integral in a sum-over-histories approach, or by an algebra of operators and a quantum state giving the expectation value of these operators).

(More modestly, I am really merely claiming that quantum mechanics may be a complete framework if it is absolutely correct. In view of the progression of science, that latter assumption may be thought to be highly dubious, but in the spirit of what Feynman called Wheeler’s “radical conservatism,” I want to push as far as
possible our present principles, in this case the assumption that quantum mechanics is correct, which I shall make throughout the remainder of this paper. I wish to transcend Wald’s insightful remark on page 309 of [3], “If you believe in quantum mechanics, then you can’t take it seriously.” In particular, I am arguing that the objections that can be raised against a probabilistic theory need not be objections against quantum mechanics in the version I am proposing.

However, our universe also includes conscious perceptions or sensations or phenomenal first-person experiences (in what is here called the conscious world to distinguish it from the unconscious quantum world, though both together can be taken to be the physical world if one accepts that terminology; this physical world includes both the “mental world” and the “physical world” of Fig. 8.1 of Penrose’s Shadows of the Mind [3]). These perceptions seem to have certain classical aspects that are not captured merely by the full set of amplitudes for the quantum world, so I propose that quantum mechanics be augmented to give real measures for sets of conscious perceptions. In the particular augmentation I am proposing, which I call Sensible Quantum Mechanics, or SQM, these measures are given by the expectation values of particular awareness operators in the state of the quantum world. (This strong connection between the conscious world and the quantum world in a thus-unified physical world means that Sensible Quantum Mechanics is not fundamentally dualistic in any negative sense, any more than the quantum world is dualistic for having the distinct elements of paths and amplitudes, or of operators and a quantum state. However, it is dualistic in the sense of Chalmer’s The Conscious Mind [4], with which I agree virtually completely for the first half of the book, except for some minor issues of terminology, and for the final chapter, on the interpretation of quantum mechanics, even though I had not seen this excellent book when I wrote the present paper.)

Ratios of the measures of appropriate sets of conscious perceptions can be interpreted as the classical conditional probabilities for these sets. In this way Sensible Quantum Mechanics gives something like the usual probabilistic interpretation applied to ordinary quantum mechanics, but I am proposing that in the most fundamental sense, probabilities are entirely restricted to conscious perceptions or sensations. One might summarize this proposal by the slogan, “No nonsensical probabilities!”

Thus I am proposing a framework or viewpoint in which probabilism, or interpreting the unconscious quantum world itself probabilistically, is an aesthemamorphism (from the Greek αἰσθησις: perception, sense, sensation). It is a myth of attributing a fundamental property of conscious perceptions to the quantum world, rather analogous to the myth of animism that ascribes living properties to inanimate objects. Of course, probabilism may be a convenient myth, just as animism is a convenient myth when we say such things as, “A charged particle feels an electromagnetic field,” or when Feynman [5] wrote, “It isn’t that a particle takes the path of least action but
that it smells all the paths in the neighborhood . . . ,” but it would give us a better understanding of the world if we recognized it as a myth. My claim that probabilism is a myth is also analogous to the claim that classical physics is a myth, since it is only an approximation to an underlying quantum reality. (Of course, it may be that quantum mechanics itself is a myth, but, as discussed above, in this paper I am making the radically conservative assumption that it is absolutely correct. My claim is that I have good reasons for identifying probabilism as a myth, whereas I do not see any good evidence yet that quantum mechanics itself is a myth.)

Thus, by eliminating probabilities from the quantum world, quantum mechanics is permitted to be a complete framework for that world. On the other hand, someone may object that Sensible Quantum Mechanics reintroduces probabilities for the conscious world and so cannot possibly be a complete theory for the entire physical world of both the quantum world and the conscious world.

This objection would indeed be valid if the probabilities for sets of perceptions in the conscious world were merely propensities for the existence of these perceptions, so that the particular set of perceptions which are actualized is not uniquely determined by the theory. However, Sensible Quantum Mechanics instead gives the picture of all sets of perceptions with nonzero measure as actually occurring. In this way the theory is really not fundamentally probabilistic in the propensity sense even for the conscious world. Instead, it is a many-perceptions theory, with the probabilities to be interpreted almost in the frequency sense as the ratios of numbers of perceptions that actually exist in the various sets. (I say “almost,” because the probabilities need not be rational ratios, as the “numbers” of perceptions in the sets really refer to the measured continua of perceptions in the sets.)

Thus Sensible Quantum Mechanics is very closely related to the “many-worlds” interpretation [6, 7], in which probabilities also have the “frequency” interpretation of being the ratios of measures. In both the quantum state never collapses as a result of a measurement or perception. The main difference is that in Sensible Quantum Mechanics, the “many” applies to conscious perceptions rather than to anything in the unconscious quantum world.

2 Why I Claim Probabilism Is a Myth

Because the many-worlds interpretation is not fundamentally probabilistic in the propensity sense, it seems to give a complete theory for the quantum world itself that is probabilistic in the frequency sense. So why should I claim that such probabilism is a myth? My argument is that it seems to be both ugly and unnecessary to ascribe probabilities (even in merely the “frequency” sense) to the unconscious quantum world.

The ugliness of applying probabilities to the quantum world lies in the arbitrariness of the choice of which set of possibilities is to be assigned probabilities. This is
the uncertainty of which set of amplitudes should be squared to get probabilities.

For example, one viewpoint is that it is the amplitude for each macroscopically
distinct outcome of a “measurement” that should be squared to get a probability.
(More precisely, this view is that one takes the expectation values of a complete set
of orthogonal projection operators, each representing one of the macroscopically-
distinct outcomes of the measurement process.) This viewpoint has the difficulty or
ugliness of requiring the specification of precisely what a “measurement” is supposed
to be, and precisely which projection operators are supposed to be measured by it.

A more inclusive viewpoint is that the expectation value of any projection op-
erator is a probability for the corresponding “event.” An even broader viewpoint
is that one can square the amplitude given by projecting the wavefunction not just
by one, but by a whole sequence of (possibly noncommuting) projection operators
representing a “history” or sequence of “events.” (For the resulting probabilities
to obey the sum rules under a coarse-graining of the projection operators, the
sequences must obey certain consistency conditions \[8, 9\].) One can extend this
viewpoint, of assigning probabilities to “consistent histories,” yet further to the
viewpoint that one can project the wavefunction by sums of sequences of projection
operators that represent coarser-grained histories. (Then one needs “decoherence”
conditions for the resulting probabilities to obey the sum rules for “decohering histo-
ries” \[10, 11, 12, 13, 14, 15, 16, 17, 18\].) An even further extension is the viewpoint
that probabilities are the real parts of the expectation values of sums of sequences
of projection operators, whenever these obey a “linear positivity” condition of being
nonnegative, giving probabilities for “linearly positive histories” that automatically
obey the sum rules \[19\].

In each of these broader viewpoints there is a family of many different allowed
sets of possibilities (e.g., the family of different complete sets of orthogonal pro-
jection operators in the first viewpoint of the previous paragraph, or the family of
consistent sequences of projection operators in the second viewpoint, etc.). To get
a set of frequency-type probabilities that sum to unity, there must apparently be a
mysterious choice of a unique set of possibilities out of the family of all such sets of
possibilities. In the absence of any definite simple principle for selecting this set, its
choice, and the resulting many-worlds (or perhaps “many-histories”) theory for the
unconscious quantum world itself, seems ugly.

One conceivable proposal for allowing probabilism without requiring quite so
much ugliness is the idea that all possibilities in all sets of possibilities in one of these
families actually occur, with measures given by the quantum expectation values in
one of the ways discussed above. Since, for a normalized quantum state, these
expectation values are designed to add up to unity for a single set of possibilities,
the measures will sum to more than unity when one adds up all the sets in a family.
In fact, typically the number of allowed sets of possibilities is infinite. For example,
even for a two-state spin-half system, there is a complete set of two orthogonal rank-
one projection operators for each direction in space, and hence an infinite number of such sets in the family of all complete sets of rank-one projection operators. This means that the sum of the measures for all sets of possibilities is unnormalizable, which is problematic.

This problem may be circumventable for certain choices of the family of sets of possibilities. For example, suppose that the family is given by the set of all decompositions of the identity operator into an ordered set of $m$ orthogonal projection operators $P_i$ of respective ranks $r_i, i = 1, \ldots, m$, for a quantum system with a Hilbert space of dimension $n = \sum_{i=1}^{m} r_i$. This family forms the proper flag manifold $U(n)/\prod_{i=1}^{m} U(r_i)$, a compact homogeneous manifold of real dimension $n^2 - \sum_{i=1}^{m} r_i^2$. For each point on this flag manifold, the expectation values of the corresponding projectors give a set of $m$ probabilities that sum to unity. Of course, just as in the $n = 2$ case discussed above, the sum of the expectation values over the continuous infinity of points of the proper flag manifold diverges. However, the proper flag manifold can be given a natural homogeneous volume element that integrates to unit volume over the entire manifold. Then the expectation value of the $i$th projector $P_i$ can be reinterpreted as a probability density for the positive outcome of that projector. If one integrates this probability density over a finite volume of the proper flag manifold, one can interpret the result as the joint probability that the set of possibilities is within that region of the flag manifold and that the $i$th projector has a positive outcome.

For a family of consistent or decohering histories, defined by sets of class operators $C_\alpha(x^i)$ that depend upon some parameters or coordinates $x^i$ that locally label each such set of histories in the family, one could introduce the Riemannian metric

$$g_{ij} dx^i dx^j = \sum_\alpha \text{Re} \text{Tr} \{[C_\alpha^\dagger(x^i + dx^i) - C_\alpha^\dagger(x^i)][C_\alpha(x^j + dx^j) - C_\alpha(x^j)]\}$$

and then take the volume element of this metric (normalized by the total volume over the entire family). Then one could integrate the expectation value of $C_\alpha^\dagger(x^i)C_\alpha(x^i)$ (the probability assigned by the consistent or decohering histories approach to the particular outcome $\alpha$ in the family labeled by the particular coordinates $x^i$, now to be reinterpreted as a probability density over families of histories) over this normalized volume element for the families in some set of ranges of the $x^i$'s to get the joint probability that the set of histories is within that region and that the outcome $\alpha$ is positive.

In this way one could get a “many-many-worlds” interpretation with frequency-type probabilities for the unconscious quantum world in which not only are all possibilities (with nonzero expectation value) within one set of possibilities interpreted as actually occurring, but also all sets of possibilities within an appropriate family are interpreted as actually occurring. However, one might object that even this construction leaves it ambiguous which family of sets of possibilities should be chosen. For example, I might prefer to choose the family given by the set of all decomposi-
tions of the identity operator into an ordered set of orthogonal rank-one projection
operators, whereas someone else might prefer the family of all decohering histories.
One might also object that the constructions outlined above for these two particular
classes of families (those decomposing the identity operator into orthogonal projec-
tion operators $P_i$ and those decomposing it into other suitable operators $C_\alpha$) are
rather cumbersome and hence do not completely avoid the problem of ugliness.

These attempts to ascribe probabilities to the unconscious quantum world ap-
pear not only to be ugly but also to be *unnecessary*. Probabilities from an accepted
theory can help us to predict what types of experiences we may expect, and the
probabilities assigned to our experiences by an uncertain theory may be used to
test that theory, but those predictions and tests strictly apply to experiences as
consciously perceived. We really have no way of assigning a meaning to probabili-
ties for things that are not consciously experienced, and neither can we test such
probabilities. Probabilities can be arbitrarily assigned to things in the unconscious
quantum world, as in the “many-many-worlds” interpretation outlined above, but
without relating them to conscious perceptions, their meaning is ambiguous. Such
probabilities are fundamentally unnecessary for selecting and using a theory for our
experiences.

This diatribe against taking probabilism as a fundamental truth is not meant
to be a denial of the current historical circumstance that for heuristic purposes it
may often be convenient to use the myth of probabilism to assign “probabilities”
to things in the unconscious quantum world as a substitute or approximation for
what I am arguing are the more fundamental probabilities, in the “frequency” sense,
for sets of conscious perceptions. (It is similarly often extremely convenient to use
the myth of classical physics.) If such “probabilities” in the quantum world are
remembered to be purely mythical, than one can often simply use the ordinary
many-worlds interpretation with some arbitrary choice of the set of possibilities,
without worrying about the arbitrariness of this choice. A different choice would
merely give a different set of mythical probabilities, useful perhaps for approximating
the true probabilities for some different sets of conscious perceptions. Only if the
probabilities are to be interpreted as something fundamental need one worry about
the ambiguity or ugliness in the choice in the set of possibilities.

After writing this section, I heard Coleman’s beautiful lecture, “Quantum Me-
chanics in Your Face” [20], a rerun of his 1993 Dirac Memorial Lecture with a more
censored title [21], which argues for “NO special measurement process, NO reduc-
tion of the wavefunction, NO indeterminacy, NOTHING probabilistic in quantum
mechanics.” Coleman further states that a interaction of an infinite set of spins with
a measuring device can lead to “a definite deterministic state, definitely a random
sequence.” (See the forthcoming [22] for a proof of this.) He argues that it is a mis-
take to call the Everett interpretation “many worlds” and prefers that it be called
the “unitarian interpretation” of quantum mechanics.
The present paper agrees with this viewpoint for the unconscious quantum world, though Albert’s summary [23] (p. 124) of Newman’s argument against “the bare theory” persuades me that we need something like probabilities (e.g., in the “frequency” sense) for perceptions in the conscious world to explain what is typical about our experience. For example, if I imagine a perception of remembering having thrown two million fair coins and knowing how many came up heads, I would expect a typical such perception to have a memory of getting within a few thousand of one million heads rather than of getting merely a few thousand heads total, even though I would expect there to be nonzero amplitudes for both sets of corresponding brain states. Unfortunately, showing that tossing an infinite number of coins may under suitable circumstances (fair coins, etc.) definitely lead to a random infinite sequence of heads or tails [22] does not seem to help explain the experience we have with finite sequences, since any finite sequence can be the beginning of an infinite random sequence.

One might accept that we do need something beyond bare quantum mechanics to predict that our actual finite measurement results are typical but object to appealing to consciousness to do that. This might be a valid objection if all we are trying to explain can be described purely without reference to consciousness. But the idea of Sensible Quantum Mechanics is not that one needs to consider consciousness in order to provide a suitable interpretation of the unconscious quantum world (which I believe is adequately described by the bare “unitarian” theory that Coleman advocates), but rather that one needs to consider consciousness precisely when one is seeking to explain properties of conscious experiences themselves.

3 Axioms of Sensible Quantum Mechanics

Sensible Quantum Mechanics (SQM) is given by the following three basic postulates or axioms:

Quantum World Axiom: The unconscious “quantum world” \( Q \) is completely described by an appropriate algebra of operators and by a suitable state \( \sigma \) (a positive linear functional of the operators) giving the expectation value \( \langle O \rangle \equiv \sigma [O] \) of each operator \( O \).

Conscious World Axiom: The “conscious world” \( M \), the set of all perceptions \( p \), has a fundamental measure \( \mu(S) \) for each subset \( S \) of \( M \).

Quantum-Consciousness Connection: The measure \( \mu(S) \) for each set \( S \) of conscious perceptions is given by the expectation value of a corresponding “awareness operator” \( A(S) \), a positive-operator-valued (POV) measure [24], in the state \( \sigma \) of the quantum world:

\[
\mu(S) = \langle A(S) \rangle \equiv \sigma [A(S)].
\] (2)

The Quantum World Axiom is here deliberately vague as to the precise nature of the algebra of operators and of the state, because as the details of various quantum
theories of the universe are being developed, I do not want the general framework of Sensible Quantum Mechanics at this time to be made too restrictive. For example, SQM is not designed necessarily to exclude the possibility that the operators may be the pairs of multi-time (or even more general) class operators (or perhaps even arbitrary linear combinations of them) in the decohering histories approach [10, 11, 12, 13, 14, 15, 16, 17, 18, 19], with the expectation values being given by decoherence functionals, even though SQM does reject as mythical the usual probability interpretation given these functionals for the unconscious quantum world.

In the Conscious World Axiom and elsewhere in this paper, a perception is taken to mean all that one is consciously aware of or consciously experiencing at one moment (or, more strictly, the entirety of a single conscious experience). Lockwood, in a book expressing remarkably similar ideas to those that I initially arrived at independently [25], calls a “phenomenal perspective” or “maximal experience.” It could also be expressed as a total “raw feel” that one has at once. It can include components such as a visual sensation, an auditory sensation, a pain, a conscious memory, a conscious impression of a thought or belief, etc., but it does not include a sequence of more than one immediate perception that in other proposals might be considered to be strung together to form a stream of consciousness of an individual mind.

I should emphasize that by a perception, I mean the phenomenal, first-person, subjective experience, and not the processes in the brain (which I call quantum, even if they are accurately described classically) that accompany these subjective phenomena. In his first chapter, Chalmers [4] gives an excellent discussion of the distinction between the former, which he calls the phenomenal concept of mind, and the latter, which he calls the psychological concept of mind. In his language, what I mean by a perception (and by other approximate synonyms that I might use, such as sensation or awareness) is the phenomenal concept, and not the psychological one.

I should perhaps also emphasize that each perception has a unique character given by its content (including its qualia), so, by definition, there are no pairs of different perceptions with precisely the same character. In this way perceptions are different from the interpretation of points of a connected manifold in general relativity, since there the points are all equivalent (e.g., under active diffeomorphisms) until one lays down suitably inhomogeneous fields on the manifold that can then be used to distinguish the points. In contrast, for the set of perceptions, the individual perceptions are assumed to be distinguished entirely by their content before any other structures are laid down, such as the measure for each subset of the set . The appropriate description of the distinguishing features of each perception appears to be a difficult problem that I am generally leaving aside from my discussion of the framework of Sensible Quantum Mechanics, so in this way my discussion is definitely incomplete. For my purposes here, I am merely assuming that such
a distinction between all different perceptions $p$ can in principle be given (though perhaps not in practice by any conscious being within our universe).

The Quantum-Consciousness Connection Axiom states my assumption of the structure of what might be called the ‘psychophysical laws,’ the laws that presumably give the ‘neural correlates of consciousness.’ This axiom, when combined with the other two, gives what to me seems to be the simplest and most conservative framework for “bridging principles that link the physical facts with consciousness” and for stating “the connection at the level of ‘Brain state X produces conscious state Y’ for a vast collection of complex physical states and associated experiences” [4] in language that is consistent with a quantum theory having “NO special measurement process, NO reduction of the wavefunction, NO indeterminacy” [21, 20] (in particular, with a many-perceptions variant of Everett’s quantum theory, in which measures for sets of conscious perceptions are added to the bare unitary quantum theory that Coleman advocates).

Of course, the Quantum-Consciousness Connection Axiom, like the Quantum world Axiom, is here also deliberately vague, because I do not have a detailed theory of consciousness, but only a framework for fitting it with quantum theory. My suggestion is that any theory of consciousness that is not inconsistent with bare quantum theory should be formulated within this framework (unless a better framework can be found, of course). I am also suspicious of any present detailed theory that purports to say precisely under what conditions in the quantum world would consciousness occur, since it seems that we simply don’t know yet. I feel that present detailed theories may be analogous to the cargo cults of the South Pacific after World War II, in which an incorrect theory was adopted, that aircraft with goods would land simply if airfields and towers were built.

Since all sets $S$ of perceptions with $\mu(S) > 0$ really occur in the framework of Sensible Quantum Mechanics, it is completely deterministic if the quantum state and the $A(S)$ are determined: there are no random or truly probabilistic elements in SQM. Nevertheless, because the framework has measures for sets of perceptions, one can readily use them to calculate quantities that can be interpreted as conditional probabilities. One can consider sets of perceptions $S_1, S_2$, etc., defined in terms of properties of the perceptions. For example, $S_1$ might be the set of perceptions in which there is a conscious memory of having tossed a coin one hundred times, and $S_2$ might be the set of perceptions in which there is a conscious memory of getting more than seventy heads. Then one can interpret

$$P(S_2|S_1) \equiv \frac{\mu(S_1 \cap S_2)}{\mu(S_1)}$$

as the conditional probability that the perception is in the set $S_2$, given that it is in the set $S_1$. In our example, this would be the conditional probability that a perception included a conscious memory of getting more than seventy heads, given that it included a conscious memory of having tossed a coin one hundred times.
An analogue of this conditional “probability” is the conditional probability that a person in early 1997 is the Queen of England. If we consider a model of all the five to six billion people, including the Queen, that we agree to consider as living humans on Earth in 1997, then at the basic level of this model the Queen certainly exists in it; there is nothing random or probabilistic about her existence. But if the model weights each of the five to six billion people equally, then one can in a manner of speaking say that the conditional probability that one of these persons is the Queen is somewhat less than $2 \times 10^{-10}$. I am proposing that it is in the same manner of speaking that one can assign conditional probabilities to sets of perceptions, even though there is nothing truly random about them at the basic level.

Another analogue one could give for the meaning of the measures of perceptions postulated in Sensible Quantum Mechanics (particularly when they are incommensurate) is the following picture, not to be taken literally, but to be taken as an aid for conceptualizing the measure: Assume for simplicity that the total measure for all perceptions is finite, and assume that the number of perceptions is countable. Then imagine that God moves His finger across each perception, staying at each one so that it occurs for a “time” that is proportional to the measure. Of course this “time” should not be confused with any physical time within our universe, or with any conscious awareness of time within any of the perceptions, but it should merely used as a picture for a continuous variable that can illustrate the measure. The picture is then that the measure for perceptions may be viewed as somewhat analogous to the measure of time used for calculating time averages in dynamical systems, for example.

As it is defined by the three basic axioms above, Sensible Quantum Mechanics is a framework and not a complete theory for the universe, since it would need to be completed by giving the detailed algebra of operators and state of the quantum world, the set of all possible perceptions of the conscious world, and the awareness operators $A(S)$ for the subsets of possible perceptions, whose quantum expectation values are the measures for these subsets.

Furthermore, even if such a complete theory were found, it would not necessarily be the final theory of the universe, since one would like to systematize the connection between the elements given above. As Chalmers eloquently puts it on pages 214-15 of his book [4], “An ultimate theory will not leave the connection at the level of ‘Brain state $X$ produces conscious state $Y$’ for a vast collection of complex physical states and associated experiences. Instead, it will systematize this connection via an underlying explanatory framework, specifying simple underlying laws in virtue of which the connection holds. Physics does not content itself with being a mere mass of observations about the positions, velocities, and charges of various objects at various times; it systematizes these observations and shows how they are consequences of underlying laws, where the underlying laws are as simple and as powerful as possible. The same should hold of a theory of consciousness. We should seek to explain the
supervenience of consciousness upon the physical in terms of the simplest possible set of laws.

“Ultimately, we will wish for a set of fundamental laws. Physicists seek a set of basic laws simple enough that one might write them on the front of a T-shirt; in a theory of consciousness, we should expect the same thing. In both cases, we are questing for the basic structure of the universe, and we have good reason to believe that the basic structure has a remarkable simplicity. The discovery of fundamental laws may be a distant goal, however. . . .

“When we finally have fundamental theories of physics and consciousness in hand, we may have what truly counts as a theory of everything. The fundamental physical laws will explain the character of physical processes; the psychophysical laws will explain the conscious experiences that are associated; and everything else will be a consequence.”

Returning to the elements above of a postulated completed, but not necessarily final, Sensible Quantum Mechanics theory, it is presently premature to try to give these elements precisely, particularly the awareness operators that have generally been left out of physics discussions. However, it might be helpful to hypothesize certain simplifying forms that these awareness operators might have. Before doing even that, it is useful to consider the structure of the set of all possible perceptions and to postulate a prior measure for that set.

4 Hypotheses for a Prior Measure

I shall hypothesize that the set $M$ of all possible conscious perceptions $p$ is a suitable topological space with a prior measure that I shall denote as

$$\mu_0(S) = \int_S d\mu_0(p).$$  \hspace{1cm} (4)

Then, because of the linearity of positive-valued-operator measures over sets, one can write each awareness operator as

$$A(S) = \int_S E(p)d\mu_0(p),$$  \hspace{1cm} (5)

a generalized sum or integral of “experience operators” or “perception operators” $E(p)$ for the individual perceptions $p$ in the set $S$. Similarly, one can write the measure on a set of perceptions $S$ as

$$\mu(S) = \langle A(S) \rangle = \int_S d\mu(p) = \int_S m(p)d\mu_0(p),$$  \hspace{1cm} (6)

in terms of a measure density $m(p)$ that is the quantum expectation value of the experience operator $E(p)$ for the same perception $p$:

$$m(p) = \langle E(p) \rangle \equiv \sigma[E(p)].$$  \hspace{1cm} (7)
Strictly speaking, the prior measure $\mu_0(S)$ is not an essential part of a complete Sensible Quantum Mechanics theory, since once the algebra of operators, the quantum state, the set of perceptions, and the awareness operators are specified, the theory is complete. However, since we do not yet have any precise knowledge of these elements, the prior measure is a very helpful tool to use while postulating and testing various hypotheses about these elements.

Perhaps the simplest hypothesis that one can make about the set $M$ of all possible perceptions is that it is countably discrete. Such a class of Sensible Quantum Mechanics theories with discrete perceptions may be labeled SQMD. This leads to a natural prior measure

$$\mu_0D(S) = N(S) = \text{number of perceptions in } S. \tag{8}$$

Then the “integrals” in Eqs. (4)-(6) are simply sums over the discrete perceptions $p$ that make up the set $S$.

In the alternative hypothesis of SQMC in which the set of all possible perceptions forms a continuum, there is not such an obvious natural prior measure. If the awareness operators are of trace class, one might choose

$$\mu_0CT(S) = Tr[A(S)], \tag{9}$$

which is equivalent to requiring $Tr[E(p)] = 1$. This would be reasonable if the set of possible quantum states formed a finite-dimensional Hilbert space, or perhaps if each experience operator depended nontrivially upon all of the quantum system except for a finite-dimensional subsystem, but these possibilities seems unduly restrictive, so I am doubtful that $\mu_0CT(S)$ and the resulting SQMCT is a realistic choice except in toy models.

Another hypothesis that one might make in the case in which $M$ is a continuum is to assume that there is a preferred prior state $\sigma_0$ and then to take the prior measure to be the measure given by the expectation values of the awareness operators in this state (rather than in the actual state $\sigma$, which gives the true measure):

$$\mu_0P(S) = \sigma_0[A(S)]. \tag{10}$$

Such a subset of the SQMC theories might be labeled SQMC$\sigma_0$. If the set of possible quantum states forms a Hilbert space of finite dimension $n$, one might choose $\sigma_0$ to be the maximally mixed state, the one giving $\sigma_0[O] = Tr[O]/n$ for any operator $O$. Then $\mu_0P(S)$ would simply be $\mu_0CT(S)/n$. However, $\mu_0P(S)$ could be defined in more general situations in which $\mu_0CT(S)$ is not definable, though in such situations there might not be such a natural choice for $\sigma_0$. It may be hard enough to find a simple state $\sigma$ for the quantum world of our universe that fits observations, without the added difficulty of requiring also the specification of a prior state $\sigma_0$. (In quantum field theory in Minkowski spacetime, the Minkowski vacuum would appear to be a
natural choice for $\sigma_0$, but finding a state $\sigma$ in that framework consistent with our observations seems nontrivial. When one includes gravitation, the situation seems reversed in that there may be a certain natural choices for $\sigma$ [20, 27, 28, 29, 30], but then one seems to lose the Minkowski vacuum as a natural choice for $\sigma_0$.

Yet another hypothesis one might make for the prior measure $\sigma_0$ in the continuous case is that it is given by the Riemannian volume element

$$\mu_0^R(S) = \int_S g^{1/2}(p) d^n p$$

(11)
of a Riemannian metric on the set of set $M$ of all perceptions. If one took the set of experience operators $E(p)$ as basic rather than the awareness operators $A(S)$ to be derived from them by Eq. (3), one might define the Riemannian metric to be

$$g_{ij} dp^i dp^j = Re Tr \{[E(p^i + dp^i) - E(p^i)] [E(p^j + dp^j) - E(p^j)]\}$$

(12)

if this is both finite and nondegenerate. In such a case both the prior measure $\mu_0^R(S)$ and the awareness operators $A(S)$ would follow from a single suitable set of experience operators $E(p)$. One might label such a family of Sensible Quantum Mechanics theories SQMCR, the final $R$ denoting the Riemannian metric (12) and the corresponding prior volume element $\mu_0^R(S)$.

5 Hypotheses for Experience Operators

After one has chosen a suitable prior volume element $\mu_0(S)$ so that one then can derive the awareness operators $A(S)$ from the experience operators $E(p)$, one can formulate various hypotheses for the latter. Clearly, one could get any measure density $m(p)$ one wanted from Eq. (7) applied to any arbitrary quantum state $\sigma$ simply by choosing $E(p) = m(p)I$, where $I$ is the identity operator (so long as the state $\sigma$ satisfies the normalization requirement $\sigma[I] \equiv \langle I \rangle = 1$). However, this would put no burden of the explanation of the measure of perceptions onto the quantum state. I strongly suspect that this would not lead to the simplest possible choice of $E(p)$’s giving the correct measure density $m(p)$, since presumably the actual quantum state is a crucial element in the simplest complete description of even just the conscious world. (Otherwise, since one’s perception in the conscious world is all one can be aware of, why should one even postulate the existence of the quantum world? Surely the justification we have for the postulate of the existence of the quantum world is that such a postulate simplifies the explanation of the conscious world that we directly experience. In this way the whole physical world, comprising both the conscious world and the quantum world, related by the Quantum-Consciousness Connection axiom, can be simpler than just the conscious world considered by itself. Whether the whole physical world is also simpler than just the quantum world considered by itself is a question posed for the future; the fact that it does not seem
to be so within our present level of understanding seems to me to be one of the main reasons why consciousness has so far been largely banished from physics.)

One very natural requirement that we may wish to put on the experience operators is the following:

**Pairwise Independence Hypothesis**: $E(p)$ and $E(p')$ are linearly independent for two different perceptions, $p \neq p'$.

This would be sufficient to rule out the ad hoc proposal $E(p) = m(p)I$ above in which the quantum state would have no effect on the measure for perceptions. Of course, it is much stronger than necessary for merely that purpose. It implies that no two different perceptions have the same experience operator (or even the same up to normalization). The Quantum-Consciousness Connection and the assumption of a prior measure that leads to Eq. (5) says that there is a unique map from the set of perceptions $M$ in the conscious world to a subset of operators in the quantum world that I am calling experience operators $E(p)$; the Pairwise Independence Hypothesis says that there is a unique inverse map from this subset of operators back to the set of perceptions. These assumptions together provide a particular type of mind-body unity that seems rather plausible (or at least relatively simple), though it certainly is not logically necessary, and I doubt that there could even be any direct observational evidence for it apart from appeals to simplicity.

Unless explicitly stated otherwise, in the following I shall generally assume the Pairwise Independence Hypothesis. Where necessary, Sensible Quantum Mechanics with this additional assumption may be denoted SQMI (or SQMDI in the discrete case, etc., skipping the P for “pairwise” to avoid confusion with the P below for “projection”), though I shall generally just assume the Pairwise Independence Hypothesis implicitly and not bother listing the I for it.

A related but much stronger hypothesis than the Pairwise Independence Hypothesis is the following:

**Linear Independence Hypothesis**: The set of all the $E(p)$’s is a linearly independent set. Such SQM theories may be labeled SQMLI.

If the $E(p)$’s are positive operators in a Hilbert space of finite complex dimension $N$, then there are at most $N^2$ linearly independent such operators, so that would be the limit on the number of perceptions $p$ in SQMLI in a finite-dimensional Hilbert space. This may be a plausible restriction, but since it does not seem to be necessary, and since I do not see too much other motivation for the Linear Independence Hypothesis (except as a possible consequence of other hypotheses that I may wish to consider), I shall not generally assume it but shall only implicitly assume the Pairwise Independence Hypothesis below when I speak in an unqualified way about Sensible Quantum Mechanics.

Now one can turn to more particular assumptions about the structure of the experience operators. One of the simplest hypotheses one can make about them is that they are projection operators:
**Projection Hypothesis:** $E(p) = P(p)$, a projection operator that depends on the perception $p$.

Such forms of Sensible Quantum Mechanics can be denoted by attaching the letter $P$ to the abbreviation. Thus the general form of SQM with the Projection Hypothesis added is SQMP (or SMQIP if it is necessary to show explicitly that the projection operators are all assumed to be different for different perceptions); the form with a discrete set of perceptions (each with its own projection operator) would be SQMDP (or SQMDIP); the form with a continuum of perceptions with the prior measure given by the volume element of the Riemannian metric would be SQMCRP (or SQMCRIP); etc.

The Projection Hypothesis appears to be a specific mathematical realization of part of Lockwood’s proposal [25] (p. 215), that “a phenomenal perspective [what I have here been calling simply a perception $p$] may be equated with a shared eigenstate of some preferred (by consciousness) set of compatible brain observables.” Here I have expressed the “equating” by Eqs. (2) and (5), and presumably the “shared eigenstate” can be expressed by a corresponding projection operator $P(p)$.

I should also emphasize that if the same conscious perception is produced by several different orthogonal “eigenstates of consciousness” (e.g., different states of a brain and surroundings that give rise to the same perception $p$), then in the Projection Hypothesis the projection operator $P(p)$ would be a sum of the corresponding rank-one projection operators and so would be a projection operator of rank higher than unity. This is what I would expect, since surely the surroundings could be different and yet the appropriate part of the brain, if unchanged, would lead to the same perception. On the other hand, if $E(p)$ were a sum of noncommuting projection operators corresponding to nonorthogonal states, or a weighted sum of projection operators with weights different from unity, then generically $E(p)$ would not be a projection operator $P(p)$ as assumed in the Projection Hypothesis.

If one has a constrained system, such as a closed universe in general relativity, the quantum state may obey certain constraint equations, such as the Wheeler-DeWitt equations. The projection operators $P(p)$ of perceptions in the Projection Hypothesis may not commute with these constraints, in which case they may give technically ‘unphysical’ states when applied to the quantum state. But so long as their expectation values can be calculated and are nonnegative real numbers, that is sufficient for giving the perception measure density $m(p)$. What it means is that in the Projection Hypothesis, the perception operators should be considered as projection operators in the space of unconstrained states, even though the actual physical state does obey the constraints.

Alternatively, if one wishes to write the perception operators $E(p)$ as operators within the space of constrained states, the Projection Hypothesis could be modified to the following assumption to give perception operators $E(p)$ that commute with the constraints and so keep the state ‘physical’:
**Constrained Projection Hypothesis:** \( E(p) = P_C P(p) P_C \), where \( P_C \) is a projection operator within the space of unconstrained states that takes any state to a corresponding constrained state, and \( P(p) \) is a projection operator in the space of unconstrained states that depends on the perception \( p \).

One might label such theories SQMP(C), SQMDP(C), SQMCRP(C), etc., where the C for “constrained” is put in parentheses to indicate that it modifies the P and to distinguish it from a possible earlier C for “continuum” or a later one for “commuting” (see below).

One can also get something like the Constrained Projection Hypothesis, say the **Symmetrized Projection Hypothesis**, even for unconstrained systems if they have symmetries (e.g., the Poincaré symmetries of quantum field theory in a classical Minkowski spacetime, though one would not expect these symmetries to survive when one includes gravity), since one might then expect that \( E(p) \) should be invariant under the symmetry group with elements \( g \). Then if one starts with a projection operator \( P(p) \) that is not invariant under the action of each group element, say \( P(p) \neq g P(p) g^{-1} \), then one might expect \( E(p) \) to be proportional to the sum or integral of \( g P(p) g^{-1} \) over the group elements \( g \), so that \( E(p) = g E(p) g^{-1} \). Unless all these different \( g P(p) g^{-1} \)’s are orthogonal (which does not appear possible for a continuous symmetry group), the resulting \( E(p) \) will generically not be a projection operator, but it can be said to have arisen from one. Such theories might be labeled SQMP(S), SQMDP(S), SQMCRP(S), etc.

One might prefer to make an even more restrictive assumption and strengthen the Projection Hypothesis to the Linearly Independent Projection Hypothesis, the Commuting Projection Hypothesis or, stronger yet, the Orthogonal Projection Hypothesis:

**Linearly Independent Projection Hypothesis:** \( E(p) = P(p) \), with the set of all these \( P(p) \)’s being a linearly independent set. (This combination of the Projection Hypothesis with the Linear Independence Hypothesis is much stronger than the combination of the Projection Hypothesis with the now-implicit Pairwise Independence Hypothesis given earlier above, which merely requires that each pair of different \( E(p) \)’s be linearly independent.) Such SQM theories may be labeled SQMLIP.

**Commuting Projection Hypothesis:** \( E(p) = P(p) \), with \( [P(p), P(p')] = 0 \) for all pairs of perceptions \( p \) and \( p' \). Such SQM theories may be labeled SQMPC, putting the C for “commuting” after the P for “projection.”

**Orthogonal Projection Hypothesis:** \( E(p) = P(p) \), with \( P(p) P(p') = 0 \) for all pairs of different perceptions \( p \neq p' \). Such SQM theories (a subset of the SQMPC theories) may analogously be labeled SQMPO. Unlike the Projection Hypothesis and the Commuting Projection Hypothesis by themselves, the Orthogonal Projection Hypothesis obviously implies the Pairwise Independence Hypothesis and the Linear Independence Hypothesis.
The Commuting Projection Hypothesis and the Orthogonal Projection Hypothesis allow one to define measures on pairs of perceptions that have nice properties. However, such joint measures are not fundamental to SQM and may be viewed as analogous to yet another case of mythical probabilities. Therefore, the possibility of their definition in SQMPC and SQMPO theories does not seem to me to be a strong argument in favor of such restriction of SQMP theories.

Another direction one can go in hypothesizing properties of experience operators is to make the following assumption about the structure of each perception (which, as defined above, is all that one is aware of at once, or all of a single conscious experience):

**Assumption of Perception Components**: Each perception \( p \) itself consists of a set of discrete components \( c_i(p) \) contained within the perception, say \( p = \{c_i(p)\} \). Different perceptions \( p \) may share various components in common, but at least some components must differ in order that the perceptions themselves differ.

Because of the apparent unity of perception, the Assumption of Perception Components seems likely to be more of an approximation for certain aspects of perceptions than a general truth. In other words, there may not be any fundamental decomposition of perceptions into components. However, there are cases in which it appears to be a reasonably good approximation, such as for a perception with one component being a conscious memory of having tossed a coin one hundred times, and with another component being a conscious memory of getting more than seventy heads.

If one does make the Assumption of Perception Components, perhaps just as an approximation for certain aspects of a perception that are easy to describe, it may be natural to make the following extension of the Projection Hypothesis:

**Product Projection Hypothesis**: \( E(p) = \prod_i P[c_i(p)] \), where each \( P[c_i(p)] \) is a projection operator that depends on the perception component \( c_i(p) \), with all the \( P[c_i(p)] \)'s commuting for the same \( p \). The corresponding theories, subsets of those obeying the Projection Hypothesis, can be labeled by a double PP replacing the single P.

One can also strengthen the Product Projection Hypothesis in ways analogous to those in which the Projection Hypothesis were strengthened above:

**Commuting Product Projection Hypothesis**: \( E(p) = \prod_i P[c_i(p)] \), where each \( P[c_i(p)] \) is a projection operator that depends on the perception component \( c_i(p) \), with all the \( P[c_i(p)] \)'s commuting (i.e., for all different \( p \)'s, as well as for all \( c_i(p) \)'s with the same \( p \) as in the Product Projection Hypothesis itself). The resulting theories may be labeled SQMPPC.

**Orthogonal Product Projection Hypothesis**: \( E(p) = P(p) = \prod_i P[c_i(p)] \), with all the \( P[c_i(p)] \)'s commuting, and with \( P(p)P(p') = 0 \) for all pairs of different perceptions \( p \neq p' \). In other words, each pair of different perceptions has at least one corresponding pair of different components whose corresponding projection op-
erators are orthogonal. Like the Orthogonal Projection Hypothesis, the Orthogonal Product Projection Hypothesis implies both the Pairwise Independence Hypothesis and the Linear Independence Hypothesis. Such theories may be labeled SQMPPO.

After discussing all of these stronger hypotheses that one might add to the Projection Hypothesis (or to one of its slight variants, such as the Constrained Projection Hypothesis or the Symmetrized Projection Hypothesis), I should say that one might prefer to stop at an even weaker hypothesis, such as the following:

**Sequence of Projections Hypothesis**: \( E(p) = C^\dagger(p)C(p) \), where \( C(p) = P(p, n)P(p, n-1) \cdots P(p, 2)P(p, 1) \) is a product of a sequence of (possibly noncommuting) projection operators (or a “homogeneous history”), with the integer \( n \) and the projection operators \( P(p, i) \) all depending on the perception \( p \). Such theories can be labeled by adding \( S \) to the previous abbreviation (as \( P \) is added for theories obeying the Projection Hypothesis). Note that when \( n = 1 \), SQMS reduces to SQMP.

For \( n > 1 \), one might prefer to restrict the sequences to those which obey the conditions of “consistent histories” \([8, 9]\). Thus one might make the following restriction:

**Consistent Sequence of Projections Hypothesis**: The Sequence of Projections Hypothesis holds, and the set of all experience operators \( E(p) \) forms a subset of a single set of consistent histories, that is, a set of histories with the property that, for each pair of distinct sequences \( C(p) \) and \( C(p') \) such that \( \hat{C}(p, p') = C(p) + C(p') \) is also a homogeneous history (a product of a sequence of projection operators), \( \sigma[\hat{C}(p, p')^\dagger\hat{C}(p, p')] = \sigma[C^\dagger(p)C(p)] + \sigma[C^\dagger(p')C(p')] \). Such a theory can be labeled SQMSC.

An example obeying the Sequence of Projections Hypothesis would be the case in which the individual projection operators are those of the components \( c_i(p) \) of the perception \( p \). However, if these projection operators are actually noncommuting, contrary to the Product Projection Hypothesis above, it may not be clear what fixes the order of the sequence. For example, it would not seem natural to try to impose any time ordering, since the perception is all that one is aware of “at once” and so might be interpreted as happening at one time. One might try to order the components of a perception that are interpreted as memories by the order in the time at which the events remembered are felt to have happened, but this seems very imprecise in many cases and certainly would not seem to apply to the components of a perception that are felt to be present sensations of concurrent events.

**Histories Hypothesis**: \( E(p) = C(p)^\dagger C(p) \), where each \( C(p) \) is a (possibly one-term) sum (with unit coefficients) of products of sequences of projection operators. Such theories can be labeled by adding \( H \) instead of \( P \) or \( S \) to the previous abbreviation. SQMH is a further generalization of SQMP than SQMS is; SQMP is a subset of SQMS, which is itself a subset of SQMH.

Just as one might like to restrict the sequences of projection operators in SQMS
to consistent histories, so one might analogously want to restrict the $C$’s in SQMH to one or another of the “decoherent histories” requirements [16, 17, 18, 19]:

**Individually Weak Decoherent Histories Hypothesis:** The Histories Hypothesis holds, with each $C(p)$ being a member of a weak decoherent set of histories consisting of $C(p)$ and $I - C(p)$. That is, $\sigma[C(p)\dagger C(p)] = Re \sigma[C(p)]$. Such theories may be called SQMHWD.

**Individually Medium Decoherent Histories Hypothesis:** The Histories Hypothesis holds, with each $C(p)$ being a member of a medium decoherent set of histories consisting of $C(p)$ and $I - C(p)$. That is, $\sigma[C(p)\dagger C(p)] = \sigma[C(p)]$. This version of SQMH may be called SQMHIMD.

**Individually Strong Decoherent Histories Hypothesis:** The Histories Hypothesis holds, with each $C(p)$ being a member of a strong decoherent set of histories consisting of $C(p)$ and $I - C(p)$. That is, there exists a projection operator $P(p)$ such that $\sigma[OC(p)] = \sigma[OP(p)]$ for any operator $O$. This is SQMHISD.

**Weak Decoherent Histories Hypothesis:** The Histories Hypothesis holds, with the $C(p)$’s forming part of a weak decoherent set of histories. That is, $Re \sigma[C(p)\dagger C(p')] = 0$ for $p \neq p'$. This would be SQMHWD.

**Medium Decoherent Histories Hypothesis:** The Histories Hypothesis holds, with the $C(p)$’s forming part of a medium decoherent set of histories. That is, $\sigma[C(p)\dagger C(p')] = 0$ for $p \neq p'$. This can be labeled SQMHMD.

**Strong Decoherent Histories Hypothesis:** The Histories Hypothesis holds, with the $C(p)$’s forming part of a strong decoherent set of histories. That is, there exists a set of orthogonal projection operators $P(p)$, a distinct one for each distinct perception $p$, such that $\sigma[OC(p)] = \sigma[OP(p)]$ for any operator $O$. This may be denoted SQMHSD.

In addition to these various forms of the Histories Hypothesis, one can also consider the extension to Linearly Positive Histories [19]:

**Linearly Positive Histories Hypothesis:** $E(p) = Re C(p)$, where each $C(p)$ is a sum of one or more products of sequences of projection operators such that $\sigma[Re C(p)] \geq 0$. Such theories, SQMLPH, unlike the previous versions of the Histories Hypothesis, are not necessarily subsets of SQMH. However, they give the same measure densities $m(p)$ as SQMHWD or its subsets SQMHIMD, SQMHSD, SQMHWD, SQMHMD, or SQMHSD when the $C(p)$’s obey the corresponding more restrictive conditions of those theories [19].

It is certainly logically possible that perceptions might depend on histories (characterized by $C$’s that are sums of products of sequences of generically noncommuting projection operators) rather than on events (characterized by individual projection operators) that one could consider localized on hypersurfaces of constant time if the quantum world has such a time. However, as a previous advocate of the “marvelous moment” approach to quantum mechanics in which only quantities on one
such hypersurface can be tested [31], I find it more believable to assume that perceptions are caused by brain states which could be at one moment of time if there are such things in the physical world. The generalization of this hypothesis to the case in which there may not be a well-defined physical time leads me personally to prefer adopting the Projection Hypothesis (or perhaps the Constrained Projection Hypothesis for constrained systems) rather than stopping at the Sequence of Projections Hypothesis or the Histories Hypothesis, though of course the general framework of Sensible Quantum Mechanics is broad enough to encompass any of these more specific hypotheses.

On the other hand, I am not myself convinced that the evidence strongly suggests going beyond the Projection Hypothesis to the Commuting Projection Hypothesis, the Orthogonal Projection Hypothesis, the Product Projection Hypothesis, or any of the restrictions of the latter, though it would certainly be worth investigating these more specific hypotheses to see whether one of them might indeed lead to a simple complete theory that is consistent with our experience. Some of the properties implied by these various hypotheses will be discussed in more detail in the section below on toy models.

While I am considering hypotheses weaker than the Projection Hypothesis, it might be worth listing a few other very weak requirements, restricting only the normalization of the experience operators $E(p)$, that nevertheless would generally be sufficient to exclude the ad hoc proposal $E(p) = m(p)I$:

**Constant-Maximum-Normalization Hypothesis:** The expectation value of each $E(p)$ has a constant maximum value, say unity, in the set of all states $\sigma$ that are normalized to give $\sigma[I] = 1$. This weaker hypothesis would be a consequence of the Projection Hypothesis but of course does not imply the Projection Hypothesis. Such theories could be called SQMNCM, putting the initial for “normalization” before that of its modifiers.

**Unit-Normalization Hypothesis:** $Tr[E(p)] = 1$ for each experience operator. This is a consequence of Eq. (9) for SQMCT or SQMDT, but it is not consistent with the Projection Hypothesis unless all the projection operators $P(p)$ are of rank one, which seems extremely implausible. These theories would be SQMNNU.

**Projection-Normalization Hypothesis:** Each experience operator is normalized so that $Tr[E(p)] = Tr[E(p)E(p)]$. This is a consequence (at least if $E(p)$ is of trace class so that the left side of this normalization condition is defined) of the Projection Hypothesis, since then $E(p) = E(p)E(p)$. Such theories might be labeled SQMNP. However, the trace-class condition on $E(p)$ that seems necessary for imposing the Projection-Normalization Hypothesis seems unrealistic to impose if the quantum system has an infinite number of states.

Finally, for comparison’s sake, let me describe some hypotheses that would take one outside Sensible Quantum Mechanics itself, as I have defined it by the three axioms above, by violating the particular Quantum-Consciousness Connection ax-
These broader alternative hypotheses would give the measure \( \mu(S) \) on sets \( S \) of perceptions as \textit{nonlinear} functionals of the quantum state. For example, one might set

\[
\mu(S) = \int_S f[p, m(p)]d\mu_0(p) \equiv \int_S f(p, \langle E(p) \rangle)d\mu_0(p) \equiv \int_S f(p, \sigma[E(p)])d\mu_0(p),
\]

where \( f \) is some arbitrary nonnegative finite function, depending possibly upon the perception \( p \) itself, of its other argument, the nonnegative number \( m(p) \). The simplest class would be those in which the function is purely of \( m(p) \), i.e., \( f[p, m(p)] = f[m(p)] \). A simple set of examples would be to set \( f[m(p)] = m(p)^n \) for some positive constant \( n \). Such extensions of Sensible Quantum Mechanics might be labeled SQMf, or SQMn in the case in which \( f \) is the power-law function. Of course, SQM1, the case in which the power is \( n = 1 \), is ordinary Sensible Quantum Mechanics, which is what I shall henceforth assume unless explicitly stated otherwise.

One can see that there are many possible characteristics for the general form of the experience operators in Sensible Quantum Mechanics or its extensions SQMf. From one viewpoint this merely illustrates the incompleteness of the bare framework of SQM, but in a slightly different way of looking at it, it shows part of the enormous gap of our knowledge about consciousness, even within this one framework, which itself is merely a proposal.

6 Sensible Classical Mechanics

For illustrative purposes, it may be helpful to note that one could have a rather similar relation between consciousness and mechanics even if mechanics were classical. Then one might propose a theory of \textit{Sensible Classical Mechanics} (SCM) with the following three axioms analogous to those proposed above for Sensible Quantum Mechanics:

**Classical World Axiom:** The unconscious “classical world” \( C \) is completely described by an appropriate set of classical histories and by a particular history \( h \) within that set.

**Conscious World Axiom:** The “conscious world” \( M \), the set of all perceptions \( p \), has a fundamental measure \( \mu(S) \) for each subset \( S \) of \( M \).

**Classical-Consciousness Connection:** The measure \( \mu(S) \) for each set \( S \) of conscious perceptions is given by the value of a corresponding “classical awareness functional” \( A_C(S) \), a positive functional, linear in the set \( S \) of histories, evaluated for the specific history \( h \) of the classical world:

\[
\mu(S) = A_C(S)[h].
\]
set
\[ A_C(S) = \int_S E_C(p) d\mu_0(p) , \] (15)
a generalized sum or integral of “classical experience operators” or “classical perception operators” \( E_C(p) \) for the individual perceptions \( p \). Then one could use the relevant part of Eq. (12) to define the measure density \( m(p) \) as
\[ m(p) = E_C(p)[h] , \] (16)
the analogue of Eq. (7) above.

In some forms of Sensible Classical Mechanics, one might take the set of classical histories to be time-parametrized trajectories, obeying some set of classical equations of motion, in some phase space. Then one might take the value of a classical awareness functional \( A_C(S)[h] \) to be a time integral, along the trajectory of the actual history \( h \) in the phase space, of some positive function of the phase space that is linear in \( S \). For example, \( E_C(p)[h] \) could be the time integral, along the history \( h \), of a suitable delta-function of the phase space, so that each perception has a particular corresponding point of the phase space and so that the measure density \( m(p) \) is, say, unity if the trajectory of the actual history \( h \) goes through the point in the phase space corresponding to the perception \( p \) and is zero otherwise.

Then if the set of all perceptions were homeomorphic to some subset of the phase space, and if the correspondence between each perception and its corresponding point of this subset of the phase space were a homeomorphism, then an actual classical history passing through the subset of the phase space would give a corresponding one-parameter trajectory though the set of all perceptions. This is indeed what one might naively expect in a classical model for a single conscious being having a continuous temporal sequence of perceptions, but even within SCM one could certainly have many different variants of this simple example. (For example, one would probably want to allow for more than one conscious being, so that appropriate points of the phase space would correspond to several conscious perceptions, say one for each conscious being that existed at that point of the phase space.) However, I shall not bother listing various detailed possibilities as I did above for the more realistic possibility of Sensible Quantum Mechanics.

Sensible Classical Mechanics seems rather moot, since the unconscious “mechanical” part of the physical world appears to be quantum rather classical. However, I give it mainly to show that the connection I am proposing between the mechanical and the conscious aspects of the world is not strongly dependent upon the quantum nature of mechanics, though the detailed mathematical form of the connection does depend upon the mathematical form of the description of the mechanics. (Contrast [32] for an opposing viewpoint in which the form of the mechanics is considered crucial.) In other words, if it turns out that a classical approximation is adequate for the mechanics of the brain that leads to consciousness, then there would not seem to be any fundamental problem with using a particular SCM as an approximation to
the appropriate SQM. For example, if SQM is correct and the experience operators \( E(p) \) are approximately projection operators onto certain brain configurations that can be described classically to a good approximation, then a SCM with perceptions corresponding to the regions of phase space with those brain configurations would presumably be a good approximation to at least that part of the SQM in which the quantum state behaves according to the classical approximation for it.

Another use for Sensible Classical Mechanics might be in attaching a Sensible theory of conscious perceptions to the de Broglie-Bohm theory of quantum mechanics [33, 34, 35, 36, 23, 38, 39, 40, 1] in which there is not only the algebra of operators and the quantum state, but also a classical history or de Broglie-Bohm trajectory (albeit not one obeying the same equations of motion as that of the classical approximation to the quantum theory). If SQM is applied to this theory according to the rules above, then the measure for conscious perceptions would be completely unaffected by the de Broglie-Bohm trajectory, so that this trajectory would have absolutely no influence upon what is consciously experienced. (Thus my advocacy of SQM leaves me with no personal motivation for augmenting quantum mechanics with what would then be a totally unobservable de Broglie-Bohm trajectory.)

However, someone else who does believe in the existence of a de Broglie-Bohm classical trajectory and who does believe it has an observable effect might thus choose not to adopt SQM but might prefer to adopt some form of SCM with the history given by the de Broglie-Bohm trajectory (giving, say, an SCMBB theory). Although I myself would not advocate doing that, it seems that one must have something similar in mind if one believes that a de Broglie-Bohm version of quantum mechanics is in principle observationally different from quantum mechanics without the de Broglie-Bohm trajectories. Nevertheless, the statistical predictions for perceptions in this SCMBB theory could be similar to those of certain SQM theories in which the experience operators are projection operators in configuration space, at least if one averages over a suitable prior distribution for the unknown precise de Broglie-Bohm trajectory of the former. Thus the observational difference that would be manifest if one had access to the entire conscious world could very well not be sufficient to make the two theories distinguishable by any single typical perception. Indeed, many advocates of a de Broglie-Bohm version of quantum mechanics would say that it is observationally indistinguishable from ordinary quantum mechanics. (See the third paper of [1] for a further discussion of these points.)

After this interlude on how relating consciousness to classical mechanics need not be all that different from relating it to quantum mechanics, I shall return to the assumption that bare quantum mechanics is the correct framework for the mechanical aspects of the physical world and that Sensible Quantum Mechanics is the correct framework for the combination of the mechanical and the conscious aspects.
7 Perceptions rather than Minds

Another point I should emphasize is that in Sensible Quantum Mechanics, the set $M$ of perceptions is fundamental, but not any higher power of this set. Thus there is a linear measure on subsets $S$ of perceptions, which can be expressed as the "integral" (6) (a discrete sum when the set $M$ is discrete) of a measure density $m(p)$ times a prior measure element $d\mu_0(p)$, but there is no nontrivial fundamental measure density $m(p_1, p_2, \ldots, p_n)$ on $n$-tuples of perceptions. Thus, for example, there is no fundamental notion of a correlation between individual perceptions given by any measure.

(On the other hand, if a perception can be broken up into component parts, say $c_1$ and $c_2$, there can be a correlation between the parts, in the sense that the measure $\mu(S_1 \cap S_2)$ for all perceptions in the set $S_1$ containing the component $c_1$ and in the set $S_2$ containing the component $c_2$ need not be proportional to $\mu(S_1)\mu(S_2)$, the measure for all perceptions containing $c_1$ times the measure for all perceptions containing $c_2$. The enormous structure in a single perception seems to suggest that such correlations within perceptions are highly nontrivial, but I see no evidence for ascribing any fundamental meaning to a nontrivial correlation between complete perceptions $p$, since no two different complete perceptions can be perceived together.)

In saying that SQM posits no fundamental correlation between complete perceptions, I do not mean that it is impossible to define such correlations from the mathematics, but only that I do not see any fundamental physical meaning for such mathematically-defined correlations. As an example of how such a correlation might be defined, consider that if a perception operator $E(p)$ is a projection operator, and the quantum state of the universe is represented by the pure state $|\psi\rangle$, one can ascribe to the perception $p$ the pure Everett "relative state"

$$|p\rangle = \frac{E(p)|\psi\rangle}{\|E(p)|\psi\rangle\|} = \frac{E(p)|\psi\rangle}{\langle\psi|E(p)E(p)|\psi\rangle^{1/2}}.$$  

(17)

Alternatively, if the quantum state of the universe is represented by the density matrix $\rho$, one can associate the perception with a relative density matrix

$$\rho_p = \frac{E(p)\rho E(p)}{Tr[E(p)\rho E(p)]}.$$  

(18)

Either of these formulas can be applied when the perception operator is not a projection operator, but then the meaning is not necessarily so clear.

Then if one is willing to say that $m(p) = Tr[E(p)\rho]$ is the absolute probability for the perception $p$ (which might seem natural at least when $E(p)$ is a projection operator, though I am certainly not advocating this naïve interpretation), one might also naïvely interpret $Tr[E(p')\rho_p]$ as the conditional probability of the perception $p'$ given the perception $p$. 

25
Another thing one can do with two perceptions \( p \) and \( p' \) is to calculate an “overlap fraction” between them as

\[
f(p, p') = \frac{\langle E(p)E(p') \rangle \langle E(p')E(p) \rangle}{\langle E(p)E(p) \rangle \langle E(p')E(p') \rangle}.
\] (19)

If the quantum state of the universe is pure, this is the same as the overlap probability between the two Everett relative states corresponding to the perceptions:

\[ f(p, p') = |\langle p|p' \rangle|^2. \]

Thus one might in some sense say that if \( f(p, p') \) is near unity, the two perceptions are in nearly the same one of the Everett “many worlds,” but if \( f(p, p') \) is near zero, the two perceptions are in nearly orthogonal different worlds. However, this is just a manner of speaking, since I do not wish to say that the quantum state of the universe is really divided up into many different worlds. In a slightly different way of putting it, one might also propose that \( f(p, p') \), instead of \( \text{Tr}[E(p')\rho_p] \), be interpreted as the conditional probability of the perception \( p' \) given the perception \( p \). Still, I do not see any evidence that \( f(p, p') \) should be interpreted as a fundamental element of Sensible Quantum Mechanics. In any case, one can be conscious only of a single perception at once, so there is no way in principle that one can test any properties of joint perceptions such as \( f(p, p') \).

An amusing property of both of these “conditional probabilities” for one perception given another is that they would both always be zero if the Orthogonal Projection Hypothesis were true. Even though the resulting SQMPO theory would generally be a “many-perceptions” theory, it could be interpreted as being rather solipsistic in the sense that in the relative density matrix \( \rho_p \) corresponding to my present perception, no other perceptions would occur in it with nonzero measure! This has the appearance of being somewhat unpalatable, and might be taken to be an argument against adopting the Orthogonal Projection Hypothesis (and hence perhaps for stopping at the Commuting Projection Hypothesis, or perhaps the Commuting Product Projection Hypothesis if one adopts the Assumption of Perception Components, as the strongest reasonable hypothesis), but it is not clear to me that this is actually strong evidence against the Orthogonal Projection Hypothesis.

In addition to the fact that Sensible Quantum Mechanics postulates no fundamental notion of any correlation between individual perceptions, it also postulates no fundamental equivalence relation on the set of perceptions. For example, the measure gives no way of classifying different perceptions as to whether they belong to the same conscious being (e.g., at different times) or to different conscious beings. The most reasonable such classification would seem to be by the content (including the qualia) of the perceptions themselves, which distinguish the perceptions, so that no two different perceptions, \( p \neq p' \), have the same content. Based upon my own present perception, I find it natural to suppose that perceptions that could be put into the classification of being alert human perceptions have such enormous structure that they could easily distinguish between all of the \( 10^{11} \) or so persons that are typically assigned to our history of the human race. In other words, in practice,
different people can presumably be distinguished by their conscious feelings.

Another classification of perceptions might be given by classifying the perceptions operators $E(p)$ rather than the content of the perceptions themselves. This would be more analogous to classifying people by the quantum nature of their bodies (in particular, presumably by the characteristics of their brains). However, I doubt that in a fundamental sense there is any absolute classification that uniquely distinguishes each person in all circumstances. (Of course, one could presumably raise this criticism about the classification of any physical object, such as a “chair” or even a “proton”: precisely what projection operators correspond to the existence of a “chair” or of a “proton”?) Therefore, in the present framework perceptions are fundamental, but persons (or individual minds), like other physical objects, are not, although they certainly do seem to be very good approximate entities (perhaps as good as chairs or even protons) that I do not wish to deny. Even if there is no absolute definition of persons in the framework of Sensible Quantum Mechanics itself, the concept of persons and minds does occur in some sense as part of the content of my present perception, just the concepts of chairs and of protons do (in what are perhaps slightly different “present perceptions,” since I am not quite sure that I can be consciously aware of all three concepts at once, though I seem to be aware that I have been thinking of three concepts).

In this way the framework of Sensible Quantum Mechanics proposed here is a particular manifestation of Hume’s ideas [41], that “what we call a mind, is nothing but a heap or collection of different perceptions, united together by certain relations, and suppos’d, tho’ falsely, to be endow’d with a perfect simplicity and identity” (p. 207), and that the self is “nothing but a bundle or collection of different perceptions” (p. 252). As he explains in the Appendix (p. 634), “When I turn my reflexion on myself, I never can perceive this self without some one or more perceptions; nor can I ever perceive any thing but the perceptions. ’Tis the composition of these, therefore, which forms the self.” (Here I should note that what Hume calls a perception may be only one component of the “phenomenal perspective” or “maximal experience” that I have been calling a perception $p$, so one $p$ can include “one or more perceptions” $c_i(p)$ in Hume’s sense.)

Furthermore, each experience or perception operator $E(p)$ need not have any precise location in either space or time associated with it, so there need be no fundamental place or time connected with each perception. Indeed, Sensible Quantum Mechanics can easily survive a replacement of spacetime with some other structure (e.g., superstrings) as more basic in the quantum world. Of course, the contents of a perception can include a sense or impression of the time of the perception, just as my present perception when I perceive that I am writing this includes a feeling that it is now A.D. 1995, so the set of perceptions $p$ must include perceptions with such beliefs, but there need not be any precise time in the physical world associated with a perception. That is, perceptions are ‘outside’ physical spacetime (even if
spacetime is a fundamental element of the physical world, which I doubt.

As a consequence of these considerations, there are no unique time-sequences of perceptions to form an individual mind or self in Sensible Quantum Mechanics. In this way the present framework appears to differ from those proposed by Squires [12], Albert and Loewer [13, 23], and Stapp [32]. (Stapp’s also differs in having the wavefunction collapse at each “Heisenberg actual event,” whereas the other two agree with mine in having a fixed quantum state, in the Heisenberg picture, which never collapses.) Lockwood’s proposal [25] seems to be more similar to mine, though he also proposes (p. 232) “a continuous infinity of parallel such streams” of consciousness, “differentiating over time,” whereas Sensible Quantum Mechanics has no such stream as fundamental. On the other hand, later Lockwood [44] does explicitly repudiate the Albert-Loewer many-minds interpretation, so there seems to me to be little disagreement between Lockwood’s view and Sensible Quantum Mechanics except for the detailed formalism and manner of presentation. Thus one might label Sensible Quantum Mechanics as the Hume-Everett-Lockwood-Page (HELP) interpretation, though I do not wish to imply that these other three scholars, on whose work my proposal is heavily based, would necessarily agree with my present formulation.

Of course, the perceptions themselves can include components that seem to be memories of past perceptions or events. In this way it can be a very good approximation to give an approximate order for perceptions whose content include memories that are correlated with the contents of other perceptions. It might indeed be that the measure density \( m(p) \) for perceptions including detailed memories is rather heavily peaked around approximate sequences constructed in this way. But I would doubt that the contents of the perceptions \( p \), the perception operators \( E(p) \), or the measure densities \( m(p) \) for the set of perceptions would give unique sequences of perceptions that one could rigorously identify with individual minds.

Because the physical state of our universe seems to obey the second law of thermodynamics, with growing correlations in some sense, I suspect that the measure density \( m(p) \) may have rather a smeared peak (or better, ridge) along approximately tree-like structures of branching sequences of perceptions, with perceptions further out along the branches having contents that includes memories that are correlated with the present-sensation components of perceptions further back toward the trunks of the trees. This is different from what one might expect from a classical model with a discrete number of conscious beings, each of which might be expected to have a unique sharp sequence or non-branching trajectory of perceptions. In the quantum case, I would expect that what are crudely viewed as quantum choices would cause smeared-out trajectories to branch into larger numbers of smeared-out trajectories with the progression of what we call time. If each smeared-out trajectory is viewed as a different individual mind, we do get roughly a “many-minds” picture that is analogous to the “many-worlds” interpretation [6, 7], but in my framework of
Sensible Quantum Mechanics, the “many minds” are only approximate and are not fundamental as they are in the proposal of Albert and Loewer \cite{13}. Instead, Sensible Quantum Mechanics is a “many-perceptions” or “many-sensations” interpretation. One might thus label it philosophically as Mindless Sensationalism.

Even in a classical model, if there is one perception for each conscious being at each moment of time in which the being is conscious, the fact that there may be many conscious beings, and many conscious moments, can be said to lead to a “many-perceptions” interpretation. However, in Sensible Quantum Mechanics, there may be vastly more perceptions, since they are not limited to a discrete set of one-parameter sharp sequences of perceptions, but occur for all perceptions $p$ for which $m(p)$ is positive. In this way a quantum model may be said to be even “more sensible” (or is it “more sensational”?) than a classical model. One might distinguish SQM from a classical model like SCM with many perceptions by calling SQM a “very-many-perceptions” framework, meaning that almost all (say as defined by the prior measure) possible perceptions actually occur with nonzero measure density. (Thus SQM might, in a narrowly literal sense, almost be a version of panpsychism, but the enormous range possible for the logarithm of the measure density means that it is really quite far from the usual connotations ascribed to panpsychism. This is perhaps comparable to noting that there may be a nonzero amplitude that almost any system, such as a star, has a PC in it, and then calling the resulting many-worlds theory pancomputerism.)

One might fear that the present attack on the assumption of any definite notion of a precise identity for persons or minds as sequences of perceptions would threaten human dignity. Although I would not deny that I feel that it might, I can point out that on the other hand, the acceptance of the viewpoint of Sensible Quantum Mechanics might increase one’s sense of identity with all other humans and other conscious beings. Furthermore, it might tend to undercut the motivations toward selfishness that I perceive in myself if I could realize in a deeply psychological way that what I normally anticipate as my own future perceptions are in no fundamental way picked out from the set of all perceptions. (Of course, what I normally think of as my own future perceptions are presumably those that contain memory components that are correlated with the content of my present perception, but I do not see logically why I should be much more concerned about trying to make such perceptions happy than about trying to make perceptions happy that do not have such memories: better to do unto others as I would wish they would do unto me.) One can find that Parfit \cite{15} had earlier drawn similar, but much more sophisticated, conclusions from a view in which a unique personal identity is not fundamental.
8 Quantum Field Theory Model

Although Sensible Quantum Mechanics transcends quantum theories in which space and time are fundamental, and although I believe that such theories will need to be transcended to give a good theory of our universe, it might help to get a better feel for the spacetime properties of perceptions by considering the context of quantum field theory in an unquantized curved globally-hyperbolic background spacetime in which spacetime points are unambiguously distinguished by the spacetime geometry (so that the Poincaré symmetries are entirely broken and one need not worry about integrating over $gP(p)g^{-1}$s to satisfy superselection rules for energy, momentum, and/or angular momentum [46]). This simplified model might in some sense be a good approximation for part of the entire quantum state of the universe in a correct theory if there is one that does fit into the framework of Sensible Quantum Mechanics and does give a suitable classical spacetime approximation.

In the Heisenberg picture used in this paper, the quantum state is independent of time (i.e., of a choice of Cauchy hypersurface in the spacetime), but the Heisenberg equations of evolution for the fundamental fields and their conjugate momenta can be used to express the operators $E(p)$ in terms of the fields and momenta on any Cauchy hypersurface. The arbitrariness of the hypersurface means that even in this quantum field theory with a well-defined classical spacetime, and even with a definite foliation of the spacetime by a one-parameter (time) sequence of Cauchy hypersurfaces, there is no unique physical time that one can assign to any of the perceptions $p$; they are ‘outside’ time as well as space.

Furthermore, the operators $E(p)$ in this simplified model are all likely to be highly nonlocal in terms of local field operators on any Cauchy hypersurface, since quantum field theories that we presently know do not seem to have enough local operators to describe the complexities of an individual perception, unless one considers high spatial derivatives of the field and conjugate momentum operators. However, for a given one-parameter (time) sequence of Cauchy hypersurfaces, one might rather arbitrarily choose to define a preferred time for each perception $p$ as the time giving the Cauchy hypersurface on which the corresponding $E(p)$, if expressed in terms of fields and momenta on that hypersurface, has in some sense the smallest spatial spread at that time.

For example, to give a tediously explicit *ad hoc* prescription, on a Cauchy hypersurface labeled by the time $t$ one might choose a point $P$ and a ball that is the set of all points within a certain geodesic radius $r$ of the point. Then one can define an operator $E'(p; t, P, r)$ that is obtained from $E(p)$ written in terms of the fields and conjugate momenta at points on the hypersurface by throwing away all contributions that have any fields or conjugate momenta at points outside the ball of radius...
from the point $P$. Now define the overlap fraction

$$F(p; t, P, r) = \frac{\langle E(p)E'(p; t, P, r) \rangle \langle E'(p; t, P, r)E(p) \rangle}{\langle E(p)E(p) \rangle \langle E'(p; t, P, r)E'(p; t, P, r) \rangle}. \quad (20)$$

(If both $E(p)$ and $E'(p; t, P, r)$ were projection operators, and the actual quantum state were a pure state, then $F$ would be the overlap probability between the states obtained by projecting the actual quantum state by these projectors and normalizing.) If $E(p)$ is nonlocal, this fraction $F$ will be small if the radius $r$ is small but will be nearly unity if the radius $r$ is large enough for the ball to encompass almost all of the Cauchy hypersurface. For each perception $p$, time $t$, and point $P$, one can find the smallest $r$ that gives $F = 1/2$, say, and call that value of the radius $r(p; t, P)$. Then one can find the point $P = P(p; t)$ on the hypersurface that gives the smallest $r(p; t, P)$ on that hypersurface for the fixed perception $p$ and call the resulting radius $r(p; t)$. Finally, define the preferred time $t_p$ as the time $t$ for which $r(p; t)$ is the smallest, and label that smallest value of $r(p; t)$ for the fixed perception $p$ as $r_p$.

If the perception operator $E(p)$ for some human conscious perception is not unduly nonlocal in the simplified model under present consideration, and if the quantum state of the fields in the spacetime has macroscopic structures that at the time $t_p$ of the perception are fairly well localized (e.g., with quantum uncertainties less than a millimeter, say, which would certainly not be a generic state, even among states which give a significant $m(p)$ for the perception in question), one might expect that at this time the ball within radius $r_p$ of the point $P(p; t_p)$ on the hypersurface labeled by $t_p$ would be inside a human brain. It would be interesting if one could learn where the point $P(p; t_p)$ is in a human brain, and what the radius $r_p$ is, for various human perceptions, and how the location and size of this ball depends on the perception $p$.

9 Testing and Comparing Sensible Quantum Mechanics Theories

Any proposed theory should be tested against experience before being accepted. If one has a theory in which only a small subset of the set of all possible perceptions is predicted to occur (e.g., a classical theory in which there are a finite number, one for each conscious being, of time-sequences of perceptions that are determined by the trajectory in the phase space of the system), one can simply check whether an experienced perception is in that subset. If it is not, that is clear evidence against the theory.

The situation is unfortunately more complicated in very-many-perceptions theories, such as Sensible Quantum Mechanics, in which almost all perceptions are predicted to occur with nonzero measure density $m(p)$. Unless one experienced a
perception in the set, say $S_0$, for which the particular SQM theory under investigation predicts $m(p) = 0$, one could not absolutely rule out that theory. For a typical SQM theory, the set $S_0$ is of measure zero, either using the prior measure, or even using the measure of an alternative typical SQM theory, such as, presumably, the (unknown) correct theory. I.e., it is likely that $\mu'(S_0) = 0$ from the measure $\mu'(S)$ for almost any other theory, such as the unknown correct one. Thus one is not at all likely to have a perception that would absolutely rule out almost any specific SQM theory.

The best one can hope for with a very-many-perceptions theory is to find likelihood evidence for or against it, where the likelihood is the probability that the theory assigns or predicts for an experienced outcome. Even this cannot be done directly for a particular experienced perception $p$ in SQM theories, since they merely assign a measure density $m(p)$ to the perception and not a probability to it. One somehow needs to get an assigned probability for the perception or some aspect of it from the theory.

In the case of SQMD with a countably discrete set of perceptions, and in the case in which the total measure $\mu(M)$ of the set $M$ of all perceptions is finite, one could assign the normalized probability $P_r(p) = m(p)/\mu(M)$ to each individual perception $p$. However, even in this highly restricted case, a very low $P_r(p)$ assigned to the experienced perception would not necessarily be strong evidence against the particular SQMD theory that made this assignment, for it might simply be that there are a huge number of possible perceptions in the theory, each of which is assigned a similarly low $P_r(p)$.

If there were only a finite total number $N$ of possible perceptions in some SQMD theory, one could say that a typical perception $p$ should have $P_r(p)$ not too much lower than $P_{r0}(p) \equiv \mu_0(p)/\mu_0(M) = 1/N$, the probability that one gets from the prior measure that weights each discrete perception equally. Thus an experience of a perception $p$ for which the theory predicts $NP_r(p) \ll 1$ would be evidence against that theory.

However, the apparent complexity of my present perception suggests to me that if one restricted attention to theories having only a finite number $N$ of possible perceptions, the simplest of these theories giving my perception among the $N$ would have $N$ very large. (One could logically have a theory in which only my present perception existed, an utterly extreme form of solipsism, but I strongly doubt that such a theory could be so simple as a theory with many possible perceptions. This is analogous to saying that although logically there exists a theory which gives the single positive integer $8568193572865287529475652899568765824569287623819923752927591010$, this theory is not so simple in some sense as the simplest theory which gives all the first $9^{99}$ positive integers.) Furthermore, if the simplest such theory with a finite set of possible perceptions, including mine, gave $N$ very large, I would suspect that
an even simpler theory existed which includes my perception and in which the total
number of possible perceptions is not restricted to be finite. (This is analogous to
saying that the infinite set of all positive integers is simpler than the large but finite
set of the first $9^{99}$ integers.)

Because of such examples showing how an infinite set can easily be simpler than
a finite set, I suspect that the total number of possible perceptions is infinite. (An
extension of this reasoning further suggests to me that it may be simpler to have
the set of possible perceptions continuous rather than discrete, though in this case
it is less compelling. If one continued accepting a sequence of such arguments, one
would apparently be led to a set with infinite cardinality, which seems as if it might
in the end be more complex, but that might just be the appearance to our limited
way of thinking.)

In any case, it would be nice to have a way of calculating the likelihood for one’s
perception in SQM theories more general than those with a finite number of possible
perceptions. To do this, it seems that one needs to go beyond the probability for the
mere existence of the perception (which in the existential sense is unity in SQM for
all perceptions with positive measure density, and which in the frequency sense is
useful only if $\mu(p)/\mu(M)$ is nonzero, which requires that the perceptions be discrete
so that the measure $\mu(p)$ for a single perception be positive, and furthermore requires
that the total measure $\mu(M)$ for the set $M$ of all perceptions be finite).

One probability assigned by a particular SQM theory to a perception that may
be calculated simply from the measure and the measure density is the probability
that a perception is as “typical” as it is, where the (ordinary) typicality $T(p)$ of one’s
perception $p$ may be defined in the following way if the total measure $\mu(M)$ for all
perceptions is finite: Let $S_{\leq}(p)$ be the set of perceptions $p’$ with $m(p’) \leq m(p)$. Then

$$T(p) \equiv \frac{\mu(S_{\leq}(p))}{\mu(M)}.$$  \hspace{1cm} (21)

For $p$ fixed and $\tilde{p}$ chosen randomly with the infinitesimal measure $d\mu(\tilde{p})$, the probability that $T(\tilde{p})$ is less than or equal to $T(p)$ is

$$P_T(p) \equiv P(T(\tilde{p}) \leq T(p)) = T(p).$$  \hspace{1cm} (22)

In the case in which $m(p)$ varies continuously in such a way that $T(p)$ also varies
continuously, this typicality $T(p)$ has a uniform probability distribution between 0
and 1, but if there is a nonzero measure of perceptions with the same value of $m(p)$,
then $T(p)$ has discrete jumps. (In the extreme case in which $m(p)$ is one constant
value over all perceptions, $T(p)$ is unity for each $p$.)

Using this particular criterion of typicality and assuming that one’s perception
$p$ is indeed typical in this regard, one might say that agreement with observation
requires that the prediction, by the theory in question, of $P_T(p) = T(p)$ for one’s
observation $p$ be not too much smaller than unity.
Once one defines a typicality, such as by Eq. (21), one can use a Bayesian approach and assign prior probabilities $P(H_i)$ to individual hypotheses $H_i$. Suppose that each such hypothesis gives a particular SQM theory in detail, and hence its predictions for a measure density $m_i(p)$ over all perceptions, from which one can assign a particular typicality $T_i(p)$ to the perception one experiences. Then the probability $P_{T_i}(p)$ that the theory predicts that a random perception would have a typicality no greater than $T_i(p)$, which by Eq. (22) is $T_i(p)$ itself, may be taken to be the likelihood of $H_i$ given $p$. By Bayes’ rule, the posterior conditional probability that one should then rationally assign to the hypothesis $H_i$, if one followed this prescription of interpreting the typicality as the conditional probability (given the hypothesis $H_i$) for one’s particular perception $p$, would be

$$P(H_i|p) = \frac{P(H_i)T_i(p)}{\sum_j P(H_j)T_j(p)}.$$  \hspace{1cm} (23)

The main new difficulty in this Bayesian approach, even if it is assumed that one can indeed calculate the typicality for the perception given each hypothesis $H_i$, is the assignment of the prior probabilities $P(H_i)$. These probabilities are certainly not the frequency-type probabilities that occur within one SQM theory, nor are they even “probabilities” assigned to the unconscious quantum world in what I am claiming are some sort of mythical idealization or approximation for what I am proposing are the true frequency-type probabilities for the conscious world. Instead, these prior probabilities would be purely subjective probabilities (in a way that I am claiming that the ratios of measures in the conscious world are not; the latter are supposed to be entirely objective frequency-type probabilities, though their precise values would be inaccessible to us unless we were given the correct precise SQM theory for our universe). The prior probabilities are more like propensities in that they might be interpreted as our guesses for the propensities for God to have created a universe according to the particular SQM theories in question. (It is conceivable that they could be interpreted as frequencies for an ensemble of universes described by the various different SQM theories, but this would require a meta-theory for such an ensemble, which definitely goes beyond the ensemble of worlds in the Everett many-worlds interpretation of one single closed quantum system such as the universe, or the ensemble of conscious perceptions within one single SQM theory and corresponding physical world.)

Since the prior probabilities assigned to particular SQM hypotheses $H_i$ are subjective, they may be assigned rather arbitrarily. Based upon the goal of getting a simple complete theory for the universe, one might prefer to choose them so that simpler theories would be given higher prior probabilities. For example, one simple choice for a countably infinite set of hypotheses is the set of prior probabilities

$$P(H_i) = 2^{-n_i},$$  \hspace{1cm} (24)

where $n_i$ is the rank of $H_i$ in order of increasing complexity.
Unfortunately, even to make this simple assignment, one would need to assume some particular background knowledge with respect to which one might define “complexity.” In the cases in which one simply wants to compare the complexity of very complex items, the background knowledge, if sufficiently small compared with the information in the items, is not too important. But for the goal of finding a complete theory of the universe which may not have a large amount of information in it, the background knowledge is relatively important and seems to thwart an attempt to use the simple formula Eq. (24). However, since Eq. (24) is a subjective (if apparently simple) choice anyway, one could simply use it with a subjective choice of the background knowledge with respect to which the rank of $H_i$ in order of increasing complexity is made. The only difficulty is that if a different choice were made, then even for the same perception $p$, the same set of hypotheses $H_i$, and the same calculations of the typicalities $T_i(p)$, Eq. (23) would give a different assignment of the posterior probabilities for the hypotheses. It is then an open question whether the theory that is thus assigned the highest posterior probability would then the same in both cases, though that could be true if the typicalities assigned by the various theories varied so much that the posterior probabilities (23) are then relatively stable with respect to the changes in the prior probabilities (24). (This is indeed roughly what seems to occur for many well-established theories within present physics, such as Maxwell’s electromagnetism, which most physicists accept within a certain domain of applicability, since they would assign very low prior probabilities to more complicated alternatives that fit the data, even if they did not agree precisely how low to set such prior probabilities.)

There is also the potential technical problem that one might assign nonzero prior probabilities to hypotheses $H_i$ in which the total measure $\mu(M)$ for all perceptions is not finite, so that the right side of Eq. (21) may have both numerator and denominator infinite, which makes the typicality $T_i(p)$ inherently ambiguous. To avoid this problem, one might use, instead of $T_i(p)$ in Eq. (23), rather

$$T_i(p; S) = \mu_i(S \leq p \cap S) / \mu_i(S)$$  \hspace{1cm} (25)

for some set of perceptions $S$ containing $p$ that has $\mu_i(S)$ finite for each hypothesis $H_i$. This is related to a practical limitation anyway, since one could presumably only hope to be able to compare the measure densities $m(p)$ for some small set of perceptions rather similar to one’s own, though it is not clear in quantum cosmological theories that allow an infinite amount of inflation how to get a finite measure even for a small set of perceptions. Unfortunately, even if one can get a finite measure by suitably restricting the set $S$, this makes the resulting $P(H_i|p; S)$ depend on this chosen $S$ as well as on the other postulated quantities such as $P(H_i)$.

Instead of using the particular typicality defined by Eq. (21) above, one could of course instead use any other property of perceptions which places them into an ordered set to define a corresponding “typicality.” For example, I might be
tempted to order them according to their complexity, if that could be well defined. Thinking about this alternative “typicality” leaves me surprised that my own present perception seems to be highly complicated but apparently not infinitely so. What simple complete theory could make a typical perception have a high but not infinite complexity?

However, the “typicality” defined by Eq. (21) has the merit of being defined purely from the prior and fundamental measures, with no added concepts such as complexity that would need to be defined. The necessity of being able to rank perceptions, say by their measure density, in order to calculate a typicality, is indeed one of my main motivations for postulating a prior measure Eq. (4).

Nevertheless, there are alternative typicalities that one can define purely from the prior and fundamental measures. For example, one might define a reversed typicality $T_r(p)$ in the following way (again assuming that the total measure $\mu(M)$ for all perceptions is finite): Let $S_{\geq}(p)$ be the set of perceptions $p'$ with $m(p') \geq m(p)$. Then

$$T_r(p) \equiv \mu(S_{\geq}(p))/\mu(M).$$

For $p$ fixed and $\tilde{p}$ chosen randomly with the infinitesimal measure $d\mu(\tilde{p})$, the probability that $T_r(\tilde{p})$ is less than $T_r(p)$ is

$$P_{T_r}(p) \equiv P(T_r(\tilde{p}) \leq T_r(p)) = T_r(p),$$

the analogue of Eq. (22) for the ordinary typicality.

In the generic continuum case in which $m(p)$ varies continuously in such a way that there is only an infinitesimally small measure of perceptions whose $m(p)$ are infinitesimally near any fixed value, the reversed typicality $T_r(p)$ is simply one minus the ordinary typicality, i.e., $1 - T(p)$, and also has a uniform probability distribution between 0 and 1. Its use arises from the fact that just as a perception with very low ordinary typicality $T(p) \ll 1$ could be considered unusual, so a perception with an ordinary typicality too near one (and hence a reversed typicality too near zero, $T_r(p) \ll 1$) could also be considered unusual, “too good to be true.”

Perhaps one might like to combine the ordinary typicality with the reversed typicality to say that a perception giving either typicality too near zero would be evidence against the theory. For example, one might define the dual typicality $T_d(p)$ as the probability that a random perception $\tilde{p}$ has the lesser of its ordinary and its reversed typicalities less than or equal to that of the perception under consideration:

$$T_d(p) \equiv P(\min[T(\tilde{p}), T_r(\tilde{p})] \leq \min[T(p), T_r(p)]) \equiv \mu(S_d(p))/\mu(M),$$

where $S_d(p)$ is the set of all perceptions $\tilde{p}$ with the minimum of its ordinary and reversed typicalities less than or equal to that of the perception $p$, i.e., the set with $\min[T(\tilde{p}), T_r(\tilde{p})] \leq \min[T(p), T_r(p)]$. In the case in which $T(p)$, and hence also $T_r(p)$, varies continuously from 0 to 1,

$$T_d(p) = 1 - |1 - 2T(p)|.$$
Then the dual typicality $T_d(p)$ would be very small if the ordinary typicality $T(p)$ were very near either 0 or 1.

Of course, one could go on with an indefinitely long sequence of typicalities, say making a perception “atypical” if $T(p)$ were very near any number of particular values at or between the endpoint values 0 and 1. But these endpoint values are the only ones that seem especially relevant, and so it would seem rather *ad hoc* to define “typicalities” based on any other values. Since $T_d(p)$ is symmetrically defined in terms of both endpoints (or, more precisely, in terms of both the $\leq$ and the $\geq$ relations for $m(p')$ in comparison with $m(p)$), in some sense it seems the most natural one to use. Obviously, one could use it, or its modification along the lines of Eq. (25), instead of $T(p)$ in the Bayesian Eq. (23).

To illustrate how one might use these typicalities in a Bayesian analysis, consider the SQM$_n$ alteration of SQM given by Eq. (13) with $f[m(p)] = m(p)^n$. Suppose that the exponent $n$ is postulated to be uncertain (unlike in pure SQM, where it is postulated to be precisely 1), say with a prior probability distribution $P(n)dn$ simply equal to $dn$ and hence uniform over all $n$. This prior distribution is obviously not normalizable, but the normalization or lack thereof will cancel out in Eq. (23) in the use I am giving it.

Now consider a simple toy model in which the perceptions $p$ form a continuum of one dimension (labeled by a single real number, which for simplicity I shall also call $p$), with the uniform prior measure $d\mu_0(p) = dp$, and with $m(p)$, the expectation value of $E(p)$, having a gaussian distribution in $p$, say $m(p) = e^{-p^2/2}$ with the origin and scale of the numbers $p$ adjusted so that the mean of the distribution is at $p = 0$ and the standard deviation is 1. Now suppose that a perception of a particular value $p$ occurs.

Since $f[m(p)] = m(p)^n = e^{-np^2/2}$ is also a gaussian in $p$ centered at zero, but with standard deviation $1/\sqrt{n}$, one can readily calculate that for positive $n$ the typicality of the perception $p$ in the theory labeled by the exponent $n$ is

$$T(p) = \text{erfc}(\sqrt{np^2/2}) \equiv 1 - \text{erf}(\sqrt{np^2/2}) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{np^2/2}} e^{-x^2} dx, \quad (30)$$

the reversed typicality is

$$T_r(p) = 1 - T(p) = \text{erf}(\sqrt{np^2/2}), \quad (31)$$

and the dual typicality is

$$T_d(p) = 1 - \left| 1 - 2T(p) \right| = 1 - \left| 1 - 2T_r(p) \right|$$

$$= 1 - \left| 1 - 2\text{erfc}(\sqrt{np^2/2}) \right| = 1 - \left| 1 - 2\text{erf}(\sqrt{np^2/2}) \right|. \quad (32)$$

For negative $n$ the measure density $f[m(p)] = m(p)^n$ diverges for large $p$, so the typicality and dual typicalities are both zero in that case, whereas the reversed typicality is unity.
Now we can insert these typicalities and the \textit{ad hoc} prior measure $P(n)dn = dn$ into the Bayesian Eq. (23) to calculate the posterior probability distribution for the exponent $n$ given a particular perception $p$. Using the ordinary typicality $T(p)$, the sum in the denominator of Eq. (23) becomes an integral over $n$, which can be restricted to positive $n$, since $T(p)$ vanishes for negative $n$, and which gives the value $1/p^2$. Thus the corresponding posterior probability distribution becomes

$$P(n|p)dn = p^2 \text{erfc}(\sqrt{p^2n/2})dn$$

(33)

for positive $n$, and 0 for negative $n$.

This probability density $P(n|p)$ is monotonically decreasing with $n$, with the $m$th moment of $n$ being

$$\langle n^m \rangle = \frac{(2m + 1)!!}{(m + 1)p^{2m}}.$$  

(34)

(Here and in the remainder of this Section the angular brackets $\langle \rangle$ denote the expectation value in the probability distribution for $n$ given $p$, not the quantum expectation value in the state $\sigma$ that the angular brackets denote in other parts of this paper.) Thus the mean posterior value for $n$ is $3/(2p^2) = 1.5/p^2$, and its standard deviation is $\sqrt{11}/(2p^2) \approx 1.658312/p^2$. (The mean and standard deviation are both larger than 1 even if $p = 1$, essentially because we started with the uniform prior distribution $P(n)dn = dn$ which has an infinite mean and standard deviation, at least if it is restricted to $n \geq 0$ where the typicality of a finite $p$ is not 0. One would get a mean closer to unity if one had instead started with a prior distribution such as $P(n)dn = [\pi(1 + n^2)]^{-1}dn$, which is invariant under $n \rightarrow 1/n$, but I shall not further consider here such a more complicated prior distribution.)

For $n \gg 1/p^2$ the probability distribution of Eq. (33) has an exponentially decreasing asymptotic form

$$P(n|p)dn \sim \sqrt{\frac{2}{\pi p^2 n}}e^{-\frac{1}{2}p^2n}.$$  

(35)

Thus a perception of, say, $p \sim 1$, which is roughly what one would expect if the exponent $n$ were indeed 1 as SQM would give, would by Eq. (23), with the ordinary typicality used there as the likelihood of the perception $p$ given the hypothesis of a particular value of $n$, lead one to a very small posterior probability that $n \gg 1$ even if one started with the unnormalized uniform prior distribution $P(n)dn = dn$ that is almost entirely weighted at arbitrarily large values of $n$.

If we average the posterior probability distribution Eq. (33) over the gaussian distribution $e^{-p^2/2}$ that would be given for $p$ if indeed SQM and its value of $n = 1$ were correct, then one would get the following averaged posterior distribution for $n$:

$$\bar{P}(n)dn = 2(\arctan \frac{1}{\sqrt{n}} - \frac{\sqrt{n}}{n + 1})dn.$$  

(36)

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This distribution is normalized, unlike the prior distribution for \( n \) that was adopted, but since for large \( n \), \( \hat{P}(n) \sim (4/3)n^{-3/2} \), the mean and higher moments for \( n \) are infinite. Therefore, if all equal ranges for \( n \) are a priori assumed to be equally likely, then the average of all observations of \( p \), if \( n = 1 \) were really unknowingly correct, would damp the posterior probability distribution for \( n \) at large \( n \) so that it would become normalizable, but it would be damped so weakly that the mean would still be undefined. Nevertheless, any single observation of a \( p \neq 0 \) would give a posterior distribution Eq. (33) exponentially damped for sufficiently large \( n \), and hence with a finite mean and rms value, even though these finite values from individual observations would average out to divergent values when averaged over the distribution of \( p \) that would result if indeed \( n = 1 \) (or indeed if \( n \) were any other precise positive value). In any case, with or without the averaging over the values of \( p \), the posterior probability would be very small that \( n \) would have a value that is sufficiently large, giving evidence against such large values of \( n \).

On the other hand, the posterior probability distribution \( P(n|p)dn \) of Eq. (33) does not provide significant evidence against a value of \( n \) much smaller than \( 1/p^2 \), since it is even relatively larger at very small values of \( n \) than was the uniform \( P(n)dn \) from which it was derived. This illustrates the limitations of using merely the ordinary typicality to deduce posterior probabilities, since it provides no penalty for results “too good to be true,” namely ordinary typicalities very near unity. In this example, if \( n \) were very small, a \( p \) near one would be much closer to its mean of zero than the standard deviation \( 1/\sqrt{n} \) for \( p \) in the distribution \( f(m(p)) = m(p)^n = e^{-np^2/2} \). Intuitively, we ought be be surprised if we get a result much closer to the peak of a gaussian probability distribution than one standard deviation, but using only the ordinary typicality does not capture this intuition, since it says we should only be surprised if we get a result too many standard deviations from the mean.

Using purely the reversed typicality \( T_r(p) \) instead would not be any good here, since it would not give any penalty for getting a result with very low ordinary typicality. In fact, the denominator of Eq. (23) would then simply diverge from the integration over negative \( n \), which would give a reversed typicality of unity. If one put a cutoff at large negative \( n \), did the calculation, and then let the cutoff tend to negative infinity, one would find that the resulting \( P_r(n|p) \) would have almost all its contribution from arbitrarily negative \( n \).

The best of the three typicalities to use in Eq. (23) would thus appear to be the dual typicality \( T_d(p) \). Inserting this and the uniform prior distribution for \( n \) into Eq. (23) gives

\[
P_d(n|p)dn = Np^2 \min \{ \text{erfc}(\sqrt{p^2n/2}), \text{erf}(\sqrt{p^2n/2}) \} dn,
\]

where

\[
N^{-1} = 1 - 2x_1^2 + 8 \int_0^{x_1} dx \text{erfc}(x) \approx 0.857348 \approx (1.166387)^{-1}.
\]
with \( x_1 \approx 0.476936 \) being the positive value of \( x \) for which \( \text{erf}(x) = \text{erfc}(x) = 1/2 \). This posterior probability distribution \( P_d(n|p)dn \) is not only damped at large \( p^2n \), which would give \( p \) a low ordinary typicality, but also at small \( p^2n \), which would give \( p \) a low reversed typicality (i.e., an ordinary typicality unusually near one). \( P_d(n|p)dn \) gives a mean value for \( n \) of about \( 1.727468/p^2 \), and a standard deviation of about \( 1.686141/p^2 \), which are both even higher than those for \( P(n|p)dn \), essentially because \( P_d(n|p)dn \) damps the contribution at small \( n \) and gives a higher weight to the contribution for larger \( n \).

Thus we may note that the damping at small \( p^2n \) is so weak, going only as the square root of \( n \), that a perception \( p \), even if it is reasonably close to one standard deviation from the mean, does not put very tight limits on \( n \) in SQMn with a uniform prior distribution for \( n \). In this way it seems, at least for a gaussian distribution of expectation values for experience or perception operators \( E(p) \) (or for a discrete distribution that is approximately gaussian, such as a binomial distribution for a large number of possibilities), that if one allows the measures for the perceptions to be an arbitrary power \( n \) of the expectation values with a broad prior distribution for \( n \), then no observation (i.e., perception) can give very tight limits on \( n \). Thus it may be that there is actually very little evidence (except for the simplicity that leads me to propose \( n = 1 \) in Sensible Quantum Mechanics itself) that probabilities in quantum mechanics (which I have argued apply only to conscious perceptions) are proportional to the squares of the absolute values of appropriate amplitudes (i.e., to the first power of the expectation values of positive perception operators). It would be interesting to see whether there are any highly nongaussian distributions for perceptions that would be suitable for putting stringent limits on the power \( n \) of the expectation value that enters in SQMn theories.

One idea for testing the exponent \( n \) more stringently in SQMn theories is the following: Consider a set of perceptions that may be divided up into a large number \( N \) of subsets, and for which one has some control on the relative expectation values of the experience operators \( E(p) \). For example, consider the set \( S \) of perceptions that include a conscious awareness of all the digits of a nonnegative decimal integer with no more than \( k \) digits. (It seems hard for me, by looking at numbers on a computer terminal, to convince myself that I can be consciously aware, in a single simultaneous perception, of the values of more than about \( k = 8 \) digits, and I am not sure I can really be conscious of all those at once, but \( k = 8 \) is my rough subjective estimate of the limit for me. Note that this is not the number of digits I can memorize, for I need not have all the digits that I can remember ever simultaneously be in any single conscious perception.)

This set of perceptions can now be divided up into the \( N = 10^k \) subsets of perceptions that each include a component of being consciously aware of the values of the decimal digits of a particular nonnegative integer of \( k \) digits or less. Group these \( N = 10^k \) sets into three sets, the first (\( S_1 \)) containing the perceptions of a
particular subset of \( N_1 \) of the integers, the second \((S_2)\) containing the perceptions of a second subset of \( N_2 \) integers (not overlapping with the first), and the third \((S_3)\) containing the the perceptions of the remaining \( N_3 = N - N_1 - N_2 \) integers.

Now employ some quantum decision-making device (such as a nonalgorithmic random number generator that invokes quantum measurements and is nondeterministic in the usual sense, even if it is deterministic in my global view when one considers its effects within the entire wavefunction, i.e., across all the Everett many worlds in that crude way of describing things, but not within a single randomly chosen Everett world, where it will appear random). Use this device to produce a quantum expectation value for each of the integers to appear, by itself, on a computer terminal or printout where it can be read, with the expectation value being roughly independent of the particular integer within each of the three sets described above, but depending on which set the integer is in.

There is then the process of reading the integer and transferring the information to whatever part of the brain that produces the conscious experience that includes the awareness of all the digits of the integer (somewhat more accurately, the part of the brain that has the relevant structure for the experience operator \( E(p) \) for each of the possible perceptions of the entire integer). If this process can be idealized as making \( m(p) = \langle E(p) \rangle \equiv \sigma[E(p)] \) proportional to the quantum expectation value for the corresponding integer to appear on a terminal or printout (an expectation value that, up to a constant of proportionality, can be controlled by the experimenter), then one can choose the relative values of the \( m(p) \)'s, say \( m_1, m_2, \) and \( m_3 \) respectively, for each of the three sets of integers.

Now in ordinary Sensible Quantum Mechanics, the measure density for each perception \( p \) is simply the corresponding \( m(p) \), but in the SQM\(_n\) alteration of SQM given by Eq. (13), the measure density is \( f[m(p)] = m(p)^n \), where the exponent \( n \) may be different from the value of 1 that it is postulated to have in pure SQM. Therefore, in SQM\(_n\), the conditional probability that a perception is in the set \( S_2 \), say, given that it is in the set \( S = S_1 \cup S_2 \cup S_3 \), is, under the idealizations above,

\[
P_n(S_2|S) \equiv \frac{\mu(S_2)}{\mu(S)} = \frac{m_2^n N_2}{m_1^n N_1 + m_2^n N_2 + m_3^n N_3}, \tag{39}
\]

which depends on \( n \) and the ratios of the \( m \)'s, though not on the overall normalization of the \( m \)'s, which is arbitrary.

For example, let \( N_1 = 1, N_2 = 10^{k/2} \), so that then (assuming \( 10^k \gg 1 \), as it is for, say, \( k = 8 \)) \( N_3 = 10^k - 10^{k/2} - 1 \approx 10^k = N \). Then if one selects the quantum decision-making device so that, up to an arbitrary constant overall normalization factor, the \( m \)'s are \( m_1 = 1/N_1 = 1, m_2 = 1/N_2 = 10^{-k/2}, \) and \( m_3 = 1/N_3 \approx 10^{-k} \), one gets

\[
P_n(S_2|S) \approx \frac{1}{10^{(n-1)k/2} + 1 + 10^{-(n-1)k/2}}. \tag{40}
\]
One can now see from this that if one has ordinary SQM with \( n = 1 \), there is roughly one chance in three that a perception in the set \( S \) will be in the particular subset \( S_2 \), so such a perception would not be atypical if it occurred. But if such a perception occurred and one assumed that an SQM\(_n\) theory applied with \( n \) significantly different from 1, then the probability that a perception within \( S \) is also within \( S_2 \) is only roughly \( 10^{-|n-1|k/2} \), e.g., only \( 10^{-4} \) if \( k = 8 \) and either \( n = 0 \) or \( n = 2 \). Therefore, such a perception would be highly atypical within such a theory, and one would then have strong statistical evidence for rejecting such a theory, unless one had assigned it a prior probability \( P(H_i) \) that were very much nearer unity than the prior probability assigned to ordinary SQM with \( n = 1 \). In another way of stating it, at the 99% confidence level (i.e., rejecting hypotheses that predict that the conditional probability of the observed perception within \( S_2 \) is less than 1%), the value of the exponent \( n \) would obey \( |n - 1| \lesssim 4/k \).

It is somewhat discouraging that this bound, if it indeed can be found to be true, is not very tight, say if \( k \) is roughly 8, but at least it is sufficient to rule out at the 99% confidence level what might be seen as the simplest alternative to SQM with \( n = 1 \), namely SQM\(_n\) with \( n = 2 \). (The theory with \( n = 0 \) could probably be ruled out by other considerations, such as not predicting that it is any more typical to perceive getting a million heads than to get zero after throwing a fair coin two million times, if one is not perceiving the order of the million heads, and if one can avoid the Attention Effect that would no doubt amplify the measure for perceptions of zero heads.)

It may also be somewhat discouraging to note that with the quantum expectation values, the \( m \)'s, chosen as above, there is a two-thirds conditional probability even within SQM that the perception will not be within the set \( S_2 \). If so, this test would tell one virtually nothing, since then the perception (or at least the fact that it is in \( S \) but not in \( S_2 \)) would be typical for any SQM\(_n\) theory. One might want to select the quantum decision-making device to make \( m_2 \) relatively larger than the choice above, so that then if SQM is correct the perception within \( S \) will almost certainly also be within \( S_2 \), but then the conditional probability that it would be within \( S_2 \) even if SQM\(_n\) were true with a different value of \( n \) would also be higher, so the test would be less sensitive in ruling out different values of \( n \).

Of course, if one wanted instead to try to show that \( n \) is not equal to 1, one might make \( m_2 \) sufficiently large that a perception within \( S \) would almost certainly also be within \( S_2 \) if \( n = 1 \), and then if it were found that such a perception were not within \( S_2 \), this would be statistical evidence that \( n \) is not one. Given the high prior probability that one might tend to assign to \( n = 1 \), it might take a perception outside \( S_2 \) with \( m_2 \) very high in order to make much of a case for concluding that actually \( n \) is not equal to unity.

It is important to note that, given the irreducibly first-person nature of conscious experience, it would not be sufficient evidence for anyone to be told of the result

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of such an experiment to test the value of $n$, since in SQM$^n$ one’s consciousness of being told of the result would have a measure that is proportional to the $n$th power of the expectation value of the experience operator $E(p)$ for the consciousness of being told, not of the $E(p)$ for the consciousness of the digits of the decimal integer. Anyone who wished to experience the result of the experiment would need to be directly conscious of the digits themselves (which could of course be printed in the report of the experiment, so that each reader could in a sense do the consciousness part of the experiment for himself or herself). It would also not even be sufficient for an experimenter simply to remember having done the experiment and having, say, gotten a result supporting $n$ equal to or close to 1, if he or she were not in the same perception consciously aware of the digits of the number itself, because what would matter for that conscious memory would be the $n$th power of the expectation value of the experience operator $E(p)$ for the consciousness of the memory, not of the $E(p)$ for the consciousness of the digits of the decimal integer itself. This is another aspect that makes it apparently very difficult to get much strong evidence about the value of the exponent $n$ in the generalization of Sensible Quantum Mechanics to SQM$^n$ theories.

10 Toy Models for Sensible Quantum Mechanics

To illustrate some of the structures and hypotheses discussed above for SQM itself, let us first consider a simple toy system for which the quantum world has a Hilbert-space dimension of two, i.e., the spin states of a spin-half system with basis states $|1\rangle = |\sigma_z = 1\rangle = |\uparrow\rangle$ and $|2\rangle = |\sigma_z = -1\rangle = |\downarrow\rangle$. A general positive operator for this system has the matrix form

$$
\begin{pmatrix}
t + z & x + iy \\
x - iy & t - z
\end{pmatrix}
$$

for

$$
t \geq \sqrt{x^2 + y^2 + z^2}.
$$

(42)

Hence any awareness operator $A(S)$ for any set of perceptions must have this form for this two-state quantum system. The set of parameters $\{t, x, y, z\}$ with the inequality (42) forms a four-dimensional manifold with boundary, which one can readily visualize in this example as the interior of the future light cone, and its boundary, in a (fictitious) four-dimensional Minkowski spacetime. For the $A(S)$ to be a positive-operator-valued measure as the Quantum-Consciousness Connection assumption demands, the parameters $\{t, x, y, z\}$ must be linear functions of the sets $S$, so that the parameters for the union of disjoint subsets are the sums of the corresponding parameters for the subsets themselves.

First let us consider SQMC theories in which the set of perceptions is continuous. If this set forms a manifold, one can assign a volume element $d\mu_0(p)$ to it to give
the prior measure (4), from which by Eq. (5) one can define an experience operator \( E(p) \) for each perception \( p \), also necessarily of the form (41) as each \( A(S) \) is.

If the dimensionality of the set \( M \) of perceptions in this model is larger than three, and if the map from this set to the set of (necessarily positive) experience operators \( E(p) \) is smooth, then necessarily two different perceptions must have experience operators proportional to each other. Thus the Pairwise Independence Hypothesis will not hold in this case.

In order to require that the Pairwise Independence Hypothesis hold, I shall henceforth assume that the dimensionality of the set \( M \) of perceptions in this toy model is no larger than three (i.e., one less than the square of the dimension of the quantum Hilbert space being considered). First, consider the case in which the dimensionality is exactly three, and for simplicity assume that perceptions in the set are parametrized by the triplet \( \{u, v, w\} \) of real numbers obeying the inequality

\[
   r(u, v, w) \equiv \sqrt{u^2 + v^2 + w^2} \leq 1. \tag{43}
\]

Then one simple choice for the experience operators is

\[
   E(p) \equiv E(u, v, w) = \begin{pmatrix} t(1 + w) & t(u + iv) \\ t(u - iv) & t(1 - w) \end{pmatrix}, \tag{44}
\]

where \( t = t(u, v, w) \) is some nonnegative weight function over the set of perceptions. One can also write the prior measure volume element as

\[
   d\mu_0(p) = m_0(u, v, w)dudvdw \tag{45}
\]

for some nonnegative weight function \( m_0(u, v, w) \). Then Eq. (5) gives

\[
   A(S) = \int_S t(u, v, w)m_0(u, v, w)dudvdw \begin{pmatrix} 1 + w & u + iv \\ u - iv & 1 - w \end{pmatrix}. \tag{46}
\]

One sees that the awareness operators do not depend separately upon the weight functions \( t(u, v, w) \) and \( m_0(u, v, w) \), but only upon their product. Of course, the prior measure

\[
   \mu_0(S) \equiv \int_S d\mu_0(p) = \int_S m_0(u, v, w)dudvdw \tag{47}
\]

does depend upon \( m_0(u, v, w) \) alone, so it is logically an independent degree of freedom. However, in this model, one could adopt one of the normalization hypotheses for the experience operators to fix \( t(u, v, w) \). For example, the Constant-Maximum-Normalization Hypothesis leads to \( t = 1/[1 + r(u, v, w)] \); the Unit-Normalization Hypothesis, which is equivalent to the hypothesis (I) of SQMCT, leads to \( t(u, v, w) = 1/2 \); and the Projection-Normalization Hypothesis leads to \( t = 1/[1 + r(u, v, w)^2] = 1/(1 + u^2 + v^2 + w^2) \). Furthermore, one could use Eqs. (11) and (12) of SQMCR to get \( m_0(u, v, w) \) in terms of \( t(u, v, w) \) and its first derivatives (in a slightly complicated formula not worth copying here for the general \( t(u, v, w) \)). For example,
combining Eqs. (11) and (12) with the Projection-Normalization Hypothesis leads to
\[ m_0(u, v, w) = \sqrt{8/(1 + u^2 + v^2 + w^2)^3}, \]
which is what I shall take for concreteness in the following discussion of this example.

Now if one takes the quantum state to have the pure state \[ \rho = |1\rangle\langle 1|, \]
then \[ m(p) = \langle E(p) \rangle = t(1 + w) = (1 + w)/(1 + u^2 + v^2 + w^2). \]
One can insert this into Eq. (6) to get the measure for any set of perceptions, with the set \( S \) here being given by some region of the three-dimensional space with coordinates \( (u, v, w) \):
\[ \mu(S) = \int_S m(u, v, w)m_0(u, v, w)dudvdw = \int_S \sqrt{8(1 + w)/(1 + u^2 + v^2 + w^2)^4}dudvdw. \] (48)

One could then take appropriate ratios of such measures, such as in Eq. (3), as giving conditional probabilities for various sets of perceptions. For example, one could calculate the probability \[ \langle 22 \rangle \] that the typicality of a random perception in this measure is less than that of a particular perception \( p \) labeled by \( (u, v, w) \). For a generic perception in this present example, this calculation appears to be too messy to be worth doing here, but the point is that, given the assumptions made above, it can in principle be done (perhaps numerically if it is not possible to give explicit elementary formulas for the result).

Another example within SQMC that one might consider is the case in which the set \( M \) of perceptions forms a manifold of dimension one, say the circle \( S^1 \) parametrized by the angle \( \phi \) that runs from zero to \( 2\pi \) and then repeats. (One might regard this angle as denoting a cyclic time as perceived, but of course this would depend on what is the precise content of the perception \( p \) parametrized by the angle \( \phi \). The contents of the perceptions are features that are not captured merely by the topology of the set \( M \), the prior measure, and the experience operators.) In this case one can adopt the Projection Hypothesis as well as the Pairwise Independence Hypothesis, giving SQMP (or, to state more explicitly the assumption of the Pairwise Independence Hypothesis, SQMIP). For example, the experience operators can take the form Eq. (44) with \[ t = 1/2, \ u = \cos \phi, \ v = \sin \phi, \] and \( w = 0 \), giving the projection operators
\[ E(p) \equiv E(\phi) = P(\phi) = \frac{1}{2} \left( \begin{array}{c} 1 \\ e^{-i\phi} \\ e^{i\phi} \\ 1 \end{array} \right). \] (49)

If we write the prior measure volume element as
\[ d\mu_0(p) = m_0(\phi)d\phi, \] (50)
for some nonnegative weight function \( m_0(\phi) \), then Eq. (5) gives
\[ A(S) = \int_S m_0(\phi)d\phi \frac{1}{2} \left( \begin{array}{c} 1 \\ e^{-i\phi} \\ e^{i\phi} \\ 1 \end{array} \right). \] (51)

In this case of a one-dimensional set of perceptions and the experience projection operators of Eq. (49), the only natural choice for the prior measure density \( m_0(\phi) \)
is a constant, which is what will be assumed here. For example, Eqs. (11) and (12) of SQMCR give $m_{0}(\phi) = 1/\sqrt{2}$, but the value of the constant is not relevant to testable conditional probabilities or typicalities.

If the quantum state is the pure state

$$\rho = (\cos \frac{1}{2} \theta |1\rangle + \sin \frac{1}{2} \theta |2\rangle)(\cos \frac{1}{2} \theta \langle 1| + \sin \frac{1}{2} \theta \langle 2|),$$

then

$$m(p) = \langle E(p) \rangle = \text{Tr}(E(p)\rho) = \frac{1}{2}(1 + \sin \theta \cos \phi).$$

Assuming for simplicity that $\sin \theta > 0$, this leads to the following expressions for the ordinary, reversed, and dual typicalities:

$$T(p) = 1 - |\phi + \sin \theta \sin \phi|/\pi,$$

$$T_r(p) = |\phi + \sin \theta \sin \phi|/\pi,$$

$$T(p) = 1 - |1 - 2(|\phi + \sin \theta \sin \phi|/\pi)|.$$

For example, if it were hypothesized that $\theta = \pi/2 = 90^\circ$, and one observed $\phi = 5\pi/6 = 150^\circ$, then $T(p) = (\pi - 3)/(6\pi) \approx 0.007512$, so by the criterion that the typicality should not be too small, one could rule out this hypothesis for $\theta$ at the 99% confidence level.

Although we have been using the Heisenberg picture, in which the state is fixed, it might be helpful to think of the present example in the Schrödinger picture in which the experience operator $E(p)$ is held fixed as a function of the “perceived time” $\phi$, and instead the state $\rho$ changes with this “time.” In this example, one can then think of the state as representing the direction of the spin of a spin-half particle which precesses around the equatorial plane perpendicular to an axis at an angle $\theta$ from the direction corresponding to the experience operator. When the spin direction is closer to that of the experience operator, one gets a larger measure density $m(p)$ for the corresponding perception. This example illustrates how the measure density need not be constant as a function of the “perceived time,” as Sensible Quantum Mechanics has no requirement of any “unitarity” in the sense of conservation with “time” of any probability or measure or measure density.

One could still maintain the Projection Hypothesis and yet extend the example to one in which the perceptions form a two-dimensional space. For example, the experience operators could take the form Eq. (44) with $t = 1/2$, $u = \sin \vartheta \cos \varphi$, $v = \sin \vartheta \sin \varphi$, and $w = \cos \vartheta$, giving the projection operators

$$E(p) \equiv E(\vartheta, \varphi) = P(\vartheta, \varphi) = \frac{1}{2} \begin{pmatrix} 1 + \cos \vartheta & \sin \vartheta e^{i\varphi} \\ \sin \vartheta e^{-i\varphi} & 1 - \cos \vartheta \end{pmatrix}.$$ 

Here $(\vartheta, \varphi)$ are polar coordinates for a two-sphere. If one restricts $\vartheta$ not to include the values of 0 and $\pi$ that it would take at the poles of the sphere, one could take
φ to be a “perceived time,” as ϕ was in the previous example. Then one can take 
θ to be an a second component of the perception, e.g., a “perceived temperature.”
One might even divide up the space of perceptions so that those with $0 < \vartheta < \pi/2$
are defined to be those of a “cold” individual mind, and those with $\pi/2 < \vartheta < \pi$
are defined to be those of a “hot” individual. (The “lukewarm” mind with $\vartheta = \pi/2$
forms a set of perceptions of measure zero and will be henceforth ignored.) However,
as mentioned above, this division of perceptions into individual “minds” is \textit{ad hoc}
and not fundamental to Sensible Quantum Mechanics.

If one takes the rotationally-invariant prior measure volume element as

$$d\mu_0(\vartheta) = \sin \vartheta d\vartheta d\varphi,$$

which is proportional to what Eqs. (11) and (12) of SQMCR would give, then Eq. (11)
gives

$$A(S) = \int_S \sin \vartheta d\vartheta d\varphi \frac{1}{2} \left( \frac{1 + \cos \vartheta}{\sin \vartheta} \sin \varphi e^{i\varphi} + \sin \vartheta \cos \varphi \right).$$

Again taking the quantum state to be given by the (pure) density matrix of
Eq. (52), the measure density for each perception is

$$m(p) \equiv m(\vartheta, \varphi) = \langle E(p) \rangle = Tr(E(p)\rho)$$
$$= \frac{1}{2} (1 + \cos \theta \cos \vartheta + \sin \theta \sin \vartheta \cos \varphi) = \cos^2 \frac{1}{2} \psi,$$

where $\psi$ is the angle between the spin direction corresponding to the state $\rho$, at polar
coordinates ($\theta, \phi = 0$), and that corresponding to the experience operator $E(p)$, at
polar coordinates ($\vartheta, \varphi$). Then the measure for any set $S$ of perceptions, here being
given by some region of the two-sphere, is

$$\mu(S) = \int_S m(p)d\mu_0(p) = \int_S m(\vartheta, \varphi) \sin \vartheta d\vartheta d\varphi$$
$$= \int_S \frac{1}{2} (1 + \cos \theta \cos \vartheta + \sin \theta \sin \vartheta \cos \varphi) \sin \vartheta d\vartheta d\varphi.$$ 

From this measure, one can readily calculate the various typicalities of each
perception as

$$T(p) = \cos^4 \frac{1}{2} \psi,$$

$$T_r(p) = 1 - \cos^4 \frac{1}{2} \psi,$$

$$T(p) = 1 - |1 - 2 \cos^4 \frac{1}{2} \psi|.$$ 

One can also calculate various conditional probabilities, such as the conditional
probability distribution for the “perceived temperature” $\vartheta$ at a fixed range (perhaps
infinitesimal) of the “perceived time” $\varphi$, or even the inverse conditional probability distribution for the “time” at a fixed “temperature.” (One easily sees there need be no preference for the perceived time component of a perception to be used as a condition, rather than for any other component or aspect of a perception to be thus used. That is, Sensible Quantum Mechanics not only does not need a preferred time variable in the description of the quantum state and operators, but also it does not need to single out a preferred temporal aspect of perceptions on which to base all conditional probabilities.)

There are also other probabilities that one can calculate, such as the probability that a perception belongs to a “cold” individual, which, using the ad hoc division of the perceptions into those of “cold” and “hot” individuals above, comes out to be $(2 + \cos \theta)/4$. In a similar way, in a complete Sensible Quantum Mechanics theory, and with a precisely-defined ad hoc division of the possible perceptions into those of, say, humans, dogs, insects, electronic computers, etc., one (with this one admittedly being probably only a sufficiently intelligent being outside our universe) should in principle be able to calculate the relative (frequency-type) probabilities that a random perception fits into one of these categories. It would certainly be interesting to know what these relative probabilities are (for some reasonable choice of the classification). Since my own perception is human (or at least I presently perceive it to be), and since I see no reason why it should not be typical, I personally would suspect (even though one can readily see that this suspicion does not directly follow merely from the assumption that the typicality of my perception is not small) that the probability of a human perception would be greater than, or at least of a comparable magnitude to, that of dogs, insects, or electronic computers, even including those with perceptions that they are far to the future of what we perceive the present epoch to be.

Next, let us consider SQMD theories in which the set of perceptions is discrete, but still for the moment continue to use the simple toy system for which the quantum world has a Hilbert-space dimension of two. If we adopt the Pairwise Independence Hypothesis and the Projection Hypothesis, thus getting SMQIP theories, but do not also adopt the Linear Independence Hypothesis to get the Linearly Independent Projection Hypothesis and the resulting SQMLIP theories, then the experience operators $E(p)$ can be projection operators of the form given by Eq. (57) corresponding to each of any discrete set of directions in three-space or points on a two-sphere (e.g., points each labeled by a pair of polar coordinates $(\vartheta, \varphi)$). (One could also have one of the experience operators being equal to the identity operator, the only rank-two projection operator for the present toy system.) There can be an arbitrary large number (even a countably infinite number, or even an uncountably infinite number) of such pairwise independent discrete experience projection operators, so without further hypotheses they are not limited by the dimension of the quantum Hilbert space. As a result, there can be an arbitrarily large number of measure “densities”
m(p) even in the discrete case.

If the set of these distinct experience projection operators is not fixed, then for any pure quantum state such as the one given by the density matrix of Eq. (44), the ratios of all the m(p)’s can be arbitrary. (If the state is not pure, then the ratios are bounded by the ratio of the larger to the smaller eigenvalue of the density matrix.) On the other hand, if the set of E(p)’s is fixed, and there is only the freedom of what the state is (including how impure it may be), then since there are at most four linearly independent positive operators (or projection operators) in the Hilbert space of dimension two, there are at most four independent m(p)’s and three independent ratios of them. In general, even if the quantum world is described by a Hilbert space of finite dimension N, with N × N positive, hermitian density matrices as states, and even if there is an arbitrarily large but discrete number of possible perceptions p and corresponding experience operators E(p), the corresponding measures m(p) for these perceptions can have arbitrary ratios for any pure state and hence do not determine the state if the experience operators themselves are not known. On the other hand, if the experience operators are known, then there are at most N^2 linearly independent m(p)’s (or N^2 − 1 if the density matrix is assumed to be normalized), so for a generic set of known E(p)’s, N^2 − 1 ratios of m(p)’s would uniquely determine the state up to normalization.

Of course, in reality, any conscious being within the system has access to only one perception and does not even have access to its measure (except that he may typically assume that the measure is high enough that it is not too atypical), so he can only use something like the Bayesian reasoning of Eq. (23) with some assignment of prior probabilities (say based on simplicity) in order to make an effectively probabilistic guess of the correct theory (e.g., of the correct experience operators and the correct quantum state, if the framework of Sensible Quantum Mechanics is assumed).

(Perhaps it would be more nearly true to say that if SQM is true, the correct experience operators and the correct quantum state simply directly determine the perceptions, and their measures, of beliefs in various theories, but in my perception they seem to be determining that I come up with some sort of rational idealization of how this choice might be made by an agent that is truly free to assign probabilities to theories based on Bayesian reasoning. It is a deeper mystery how such a free agent may be an idealized approximation to us when we choose theories if our thought processes are not really free to choose to follow logical reasoning, say starting with certain Bayesian assumptions, including a set of prior probabilities for various hypotheses.)

Because there are only N^2 − 1 independent ratios of m(p)’s for a fixed set of experience operators in a Hilbert space dimension of dimension N, one might prefer (though I myself do not see a strong reason for this preference) to restrict the number of experience operators to be no greater than N^2 and to require that they
obey the Linear Independence Hypothesis, which, once the Projection Hypothesis is also assumed, leads to SQMLIP theories obeying the Linearly Independent Projection Hypothesis. For the simple two-dimensional Hilbert-space model we have been considering, one can have four rank-one projection operators of the form given by Eq. (57), with the corresponding four directions in three-space not all being coplanar, or, equivalently, with the corresponding points on the two-sphere not all being on the same great circle. If one has instead the identity operator and three rank-one projection operators, then the corresponding three directions in three-space must not be coplanar, or the corresponding points on the two-sphere must not all be on the same great circle.

If one makes the Commuting Projection Hypothesis instead of the Linearly Independent Projection Hypothesis, then for the two-dimensional Hilbert-space model one can have at most three experience operators, the identity operator and two orthogonal rank-one projection operators (i.e., two corresponding to two opposite directions in three-space, or to antipodal points on the two-sphere). These three are of course not linearly independent, since the two orthogonal rank-one projection operators add up to the identity operators, so this particular SQMPC theory does not obey the Linearly Independent Projection Hypothesis. If one added the latter, one could have at most two experience operators, either the identity operator and one (arbitrary) rank-one projection operator, or else two orthogonal rank-one projection operators.

As a final example obeying the Projection Hypothesis, one may adopt the Orthogonal Projection Hypothesis, which is the strongest of the possible hypotheses listed above for the experience operators and which implies all the others (except the Unit-Normalization Hypothesis, which is inconsistent with the Projection Hypothesis unless all the projection operators are of rank one). Then one could either have up to two orthogonal rank-one projection operators, or the one identity operator, for the set of experience operators. Only in the former case does the Unit-Normalization Hypothesis also hold.

If in the discrete SQMD case we weaken the Projection Hypothesis to the Sequence of Projections Hypothesis, then without further restrictions we can have an arbitrarily large number of experience operators, even if we continue to assume the Pairwise Independence Hypothesis. Of course, if we also make the Linear Independence Hypothesis, then we are again limited to at most $N^2$ experience operators (e.g., four for the two-dimensional Hilbert-space model), but in the following I shall not assume this.

Suppose we consider the case in which each sequence has two rank-one projection operators, of which the first can be, in matrix notation, either

$$Q = \frac{1}{2} \begin{pmatrix} 1 + \cos \theta_1 & \sin \theta_1 e^{i\phi_1} \\ \sin \theta_1 e^{-i\phi_1} & 1 - \cos \theta_1 \end{pmatrix}$$

(65)
or $I - Q$, and of which the second can be either
\[
R = \frac{1}{2} \begin{pmatrix} 1 + \cos \theta_2 & \sin \theta_2 e^{i\phi_2} \\ \sin \theta_2 e^{-i\phi_2} & 1 - \cos \theta_2 \end{pmatrix}
\] (66)
or $I - R$. The resulting set of four sequences are thus
\[
C(1) = R Q,
\]
(67)
\[
C(2) = R (I - Q) = R - R Q,
\]
(68)
\[
C(3) = (I - R) Q = Q - R Q.
\]
(69)
\[
C(4) = (I - R)(I - Q) = I - Q - R + R Q,
\]
(70)
and by the Sequence of Projections Hypothesis the experience operators are then
\[
E(1) = C(1) \dagger C(1) = QRQ,
\]
(71)
\[
E(2) = (I - Q) R (I - Q) = R - QR - R Q + QRQ,
\]
(72)
\[
E(3) = Q (I - R) Q = Q - QRQ,
\]
(73)
\[
E(4) = (I - Q) (I - R) (I - Q) = I - Q - R + QR + R Q - QRQ.
\]
(74)

Now assume that the state is given by the pure-state density matrix (i.e., another rank-one projection operator)
\[
\rho = \frac{1}{2} \begin{pmatrix} 1 + \cos \theta_0 & \sin \theta_0 e^{i\phi_0} \\ \sin \theta_0 e^{-i\phi_0} & 1 - \cos \theta_0 \end{pmatrix},
\]
(75)
Then in this case the Consistent Sequence of Projections Hypothesis, the Individually Weak Decoherent Histories Hypothesis, and the Weak Decoherent Histories Hypothesis all give the same single real equation
\[
\sigma [QR + RQ - 2QRQ] \equiv 2Re Tr[(QR - QRQ)\rho] = 0.
\]
(76)
Similarly, the Individually Medium Decoherent Histories Hypothesis, the Medium Decoherent Histories Hypothesis, and the Individually Strong Decoherent Histories Hypothesis all give the one complex equation
\[
\sigma [QR - QRQ] \equiv Tr[(QR - QRQ)\rho] = 0.
\]
(77)
In the present example, the Strong Decoherent Histories Hypothesis is impossible to satisfy, since it would require four orthogonal projection operators (one for each sequence) in the two-dimensional Hilbert space. Finally, the Linearly Positive Histories Hypothesis gives the inequality
\[
\max \{0, \sigma [Q + R - I]\} \leq Re \sigma [QR] \equiv Re Tr(Q R\rho) \leq \min (\sigma [Q], \sigma [R]).
\]
(78)
One can express these conditions (76)-(78) geometrically in the following manner: On the unit two-sphere representing the spin directions corresponding to the projection operators and state above in the two-dimensional Hilbert space, draw three great circles, one through \((\theta_0, \phi_0)\) and \((\theta_1, \phi_1)\), one through \((\theta_0, \phi_0)\) and \((\theta_2, \phi_2)\), and one through \((\theta_1, \phi_1)\) and \((\theta_2, \phi_2)\). These will generically divide the two-sphere into eight spherical triangles, four of which are parity reverses of the antipodal four. Now the conditions (76)-(78) can be represented by geometric properties of these triangles.

In particular, the condition given by the real Eq. (76) is equivalent to the condition that the two great circles through the point \((\theta_1, \phi_1)\) (which represents \(Q\)) intersect orthogonally there, so that each of the eight triangles are right spherical triangles (or the degenerate limit in which \((\theta_1, \phi_1)\) or the antipodal point to that coincides with one of the two vertices representing \(\rho\) and \(R\)). This condition is satisfied by a three-parameter subset (of measure zero) of the four-parameter set of directions \((\theta_1, \phi_1)\) and \((\theta_2, \phi_2)\), assuming that the direction \((\theta_0, \phi_0)\) representing the state is kept fixed.

Similarly, the condition given by the complex Eq. (77) is only satisfied in the degenerate case in which \((\theta_1, \phi_1)\) or its antipode coincides with either \((\theta_0, \phi_0)\) or \((\theta_2, \phi_2)\); i.e., when \(Q\) or \(I - Q\) coincides with either \(\rho\) or \(R\). This condition is satisfied by a discrete family of two-parameter subsets (also of measure zero) of the four-parameter set of directions \((\theta_1, \phi_1)\) and \((\theta_2, \phi_2)\).

Finally, the Linearly Positive Histories condition, given by the inequality (78), is equivalent to the condition that none of the eight triangles have area or solid angle greater than \(\pi\) (which is twice the average). Unlike the other conditions, which give sets of measure zero, this inequality condition has a positive measure, \((\sqrt{128} - 9)/15\), or about 0.154247, of the total measure for all possible choices of the two directions \((\theta_1, \phi_1)\) and \((\theta_2, \phi_2)\) that define the sequence of projection operators \(C(p)\), assuming a measure density for these directions that is uniform over the two-sphere.

One point of all these examples is to show that the additional structure of the conscious world and the corresponding awareness and experience operators in the quantum world lead to probabilities that need not be merely proportional to the ordinary quantum “probabilities” in any single set of possibilities in the same Hilbert space (the “probabilities” that I am claiming are merely fictitious). Only in the cases of very strong hypotheses, such as the Commuting Projection Hypothesis, the Orthogonal Projection Hypothesis, and some of the various Histories Hypotheses, can one get such proportionals, since in more general cases one simply has more perceptions than possibilities in any single set. Unless one starts from a broader set of probabilities than those for a single set of possibilities (whether for events or for histories) in the same Hilbert space, one simply does not get anything proportional to the true (frequency-type) probabilities for perceptions in Sensible Quantum Mechanics unless one makes restrictive assumptions about the additional structure of
the expectation values of the awareness or experience operators.

One way to circumvent this conclusion is to extend the Hilbert space to a tensor product of copies of the original Hilbert space and consider ordinary quantum “probabilities” in this larger Hilbert space. For example, suppose one has the sequence of \( n \) projection operators \( C(h) = \{P(h, 1), P(h, 2), \ldots, P(h, n)\} \) representing a homogeneous history \( h \). (Note that, unlike in the Sequence of Projections Hypothesis, this \( C \) is not the product of \( n \) projection operators in the original Hilbert space, but an ordered sequence of \( n \) projection operators.) This sequence can now be regarded as a projection operator on the tensor product of \( n \) copies of the original Hilbert space \([13, 15, 16, 18]\) with the corresponding state \( \sigma^n \) which is the tensor product of \( n \) copies of \( \sigma \). Then one can define a decoherence functional on pairs of atomic histories as

\[
D(h, h') = \sigma^n [C(h') \dagger C(h)] \\
= \sigma[P(h', 1)P(h, 1)]\sigma[P(h', 2)P(h, 2)] \cdots \sigma[P(h', n)P(h, n)].
\]

One can now extend this definition of \( C(h) \) for a homogeneous history \( h \) to one for an inhomogeneous history \( h' \) (a sum of sequences that cannot be written as a single sequence) by linearity and then define the corresponding decoherence functional \( D(h, h') \) from Eq. (79) by requiring it to be bilinear in both \( C(h) \) and \( C(h') \). This decoherence functional is then not the standard one that is an expectation value in the state \( \sigma \) in the original Hilbert space, but it is an expectation value in the product state \( \sigma^n \) in the product Hilbert space. It obviously obeys all the decoherence conditions of hermiticity, positivity, additivity, and normalization \([10, 11, 13, 15, 16, 18]\) for any normalized positive state \( \sigma \), thus illustrating how Eq. (79) and its linear extension to inhomogeneous histories gives a decoherence functional that decoheres for all pairs of histories.

Now, for simplicity, consider the case in which there is only a finite number \( n \) of possible perceptions in the set \( M \), each with its corresponding positive experience operator \( E(p) \). (The case in which there is an infinite number of possible perceptions would necessitate going to an infinite tensor product of the original Hilbert space, which will not be done here.) Assume, as is the case in which the state is given by a density matrix in a finite-dimensional Hilbert space, that each \( E(p) \) can be written as a sum, with positive coefficients, of a complete set of orthogonal projection operators \( P(h_p) \):

\[
E(p) = \sum_{h_p} \lambda_{h_p} P(h_p).
\]

Here we can regard \( p \) as an integer between 1 and \( n \), inclusive, that labels the \( n \) perceptions in the set \( M \). Now we can regard a basic homogeneous history \( h \) as a particular sequence \( h_1, h_2, \ldots, h_n \) of the \( n \) labels, giving a particular sequence of projection operators, \( C(h) = \{P(h_1), P(h_2), \ldots, P(h_n)\} \). Then from the decoherence functional \( D(h, h') \) given by Eq. (79), whose diagonal element \( D(h, h) \) can be
considered to be the quantum “probability”

\[ P_r(h) \equiv P_r(h_1, h_2, \ldots, h_n) \equiv D(h, h) = \sigma[P(h_1)]\sigma[P(h_2)] \cdots \sigma[P(h_n)], \tag{81} \]

one can indeed form linear combinations for the measure \( m(p) = \sigma[E(p)] \) for each perception:

\[ m(p) = \sum_{h_1, \ldots, h_n} \lambda_{h_p} P_r(h_1, \ldots, h_n). \tag{82} \]

However, if the number \( n \) of perceptions is larger than the square of the dimension \( N \) of the original Hilbert space, then in general one cannot write the measure for each perception as a linear combination (using coefficients that depend only on the experience operators and that are independent of the state itself) of a set of “probabilities” that are expectation values for a single set of possibilities (whose “probabilities” add up to one) in a state in the original Hilbert space.

Of course, if one adopts a “many-many-worlds” interpretation for the quantum world and assumes the reality of the “probabilities” for all sets of possibilities for some appropriate family defining all of these sets, then this broader set of “probabilities” in suitable cases can give quantities that are proportional to the expectation values of the awareness or experience operators. Thus one could say that the effect of a quantum state upon the measures in the conscious world where our experiences lie is proportional to an appropriate set of “probabilities” that can be ascribed to that state in the quantum world (assuming that the awareness or experience operators are known), but it can easily be within Sensible Quantum Mechanics that the measures for the conscious world is not directly given by any single set of “probabilities” (adding up to unity) in the same Hilbert space of the quantum world. Unless Sensible Quantum Mechanics takes on a very restricted form, or unless one extends the Hilbert space sufficiently, the ordinary probabilism applied to the quantum world is simply inadequate to give directly the measure for all experiences.

11 EPR and Schrödinger’s Cat

It may be of interest to give a brief analysis of the Einstein-Podolsky-Rosen (EPR) “paradox” [17] combined with that of Schrödinger’s cat [18]. I shall use a variant of Bohm’s modification [19] of the EPR experimental setup to two spin-half atoms in a singlet state.

Suppose that the two atoms are moved far apart (while their spins remain undisturbed in their perfectly anticorrelated singlet state of total angular momentum zero), and then interactions are made with the atoms’ spins in two spacelike-separated regions of flat Minkowski spacetime, say A and B. Suppose that in region A, a perfect nondemolition measurement interaction is made of the spin of the atom there in the \( z \)-direction, and suppose that the measuring device is further coupled
to a conscious being so that there results a set of conscious perceptions $S_{↑}$ corresponding to perceiving that the experiment has been done, that one is in region A, that the spin direction is now known by the being, and that the atom was measured to have spin up, and another set of conscious perceptions $S_{↓}$ corresponding to perceiving that the experiment has been done, that one is in region A, that the spin direction is now known by the being, and that the atom was measured to have spin down. Here I shall assume an SQMPC theory with the Commuting Projection Hypothesis (and later the restriction of this to a SQMPPC theory with the Commuting Product Projection Hypothesis when I discuss a conscious being in region B with different discrete components to his or her perceptions), so that the awareness operators $A(S_{↑})$ and $A(S_{↓})$ for the two sets of perceptions described above are commuting projection operators which also commute with awareness operators I shall describe momentarily for certain sets of perceptions of being in region B.

I shall further assume for simplicity the idealization that the quantum state with the experimental setup in region A, including the coupling to the conscious being, is such that $A(S_{↑})$ and $A(S_{↓})$ have the same expectation values as commuting projection operators $P_{↑}$ and $P_{↓}$ for the spin of the atom in region A, each multiplied by a common commuting projection operator $P_{\text{other}}$ for other common factors that lead to the perception that the experiment has been done, that one is in region A, and that the spin direction is now known by the being. Thus I am assuming a perfect correlation between the spin of the atom and whatever (e.g., some state in the brain) it is that directly causes the perception that the spin is up or down. Furthermore, the idealization I am assuming implies that whether the spin is up or down has no effect on the expectation value of the awareness operator (i.e., the measure) for the set of perceptions that the experiment has been done, that one is in region A, and that the spin direction is now known by the being. (For example, I am assuming that no anesthetic is administered to the being if and only if the atom has spin up, which certainly could affect the relative measures of the two sets of conscious perceptions by reducing the measure for $A(S_{↑})$, though in principle SQM is capable of handling such effects by a more complicated analysis.)

Now since the pair of atoms was in the singlet state (further assuming that the perception that the experiment has been done does indeed imply that, though again SQM could in principle handle the more realistic case in which such perceptions may be mistaken), the expectation values for $P_{↑}$ and $P_{↓}$ are identical, and so by our idealization the measures $\mu(S_{↑})$ and $\mu(S_{↓})$ are also identical, being the expectation values of $A(S_{↑})$ and $A(S_{↓})$ that are the same as those of the commuting projection operators $P_{↑}$ and $P_{↓}$ multiplied by the common commuting projection operator $P_{\text{other}}$. Thus in region A there is an equal relative probability to perceive the atom there having spin up or down, completely independent of what may be happening in region B. In other words, under the assumption of local quantum field theory (which may be only approximately valid if the fundamental theory is something different,
such as string theory), and under the assumption that the awareness operators for the perceptions that one is in region A are operators that can be confined to that region (which might be only approximately valid as well, even under the assumption of local field theory for the quantum world), there is no superluminal propagation of anything from region B that can have any effect on anything observed or perceived in the spacelike-separated region A \[50\].

Now let us suppose that in region B there is a device to make a nondemolition measurement interaction with the spin of the atom there in a direction at an angle $\theta$ from the $z$-direction (which I assume is parallelly propagated across the flat spacetime from the $z$-direction in region A). Suppose that further there is a “diabolical device” \[48\] to poison a cat if and only if the spin is measured to be down (i.e., if the measuring device, whose record of the measurement interaction is assumed to have become perfectly correlated with the atom spin in the $\theta$-direction, has a record indicating that the spin is down). For the purposes of the following discussion, divide the cat into “head” and “body” (conceptually, not physically; I do not mean to behead the cat!). Assume that if the cat is poisoned, that both the head and the body are dead, but that if the cat is not poisoned, both the head and body are alive.

If the projection operator for the atom spin in region B to be up in the $\theta$-direction is $P_{\text{up}}$ and to be down is $P_{\text{down}}$, if the projection operator for the cat head to be alive is $P_{\text{head alive}}$ and to be dead is $P_{\text{head dead}}$, and if the projection operator for the cat body to be alive is $P_{\text{body alive}}$ and to be dead is $P_{\text{body dead}}$ (all of which are assumed to commute), then the density operator for the atom spin and the cat’s property of having its head and body alive or dead (multiplied by the unit operator for the rest of the total system) is, under the idealization that the pair of atoms started in the singlet state and that the coupling to the measuring apparatus and diabolical device were perfect, proportional to

$$P_{\text{up}}P_{\text{head alive}}P_{\text{body alive}} + P_{\text{down}}P_{\text{head dead}}P_{\text{body dead}}. \quad (83)$$

Thus there is assumed to be a perfect correlation between the spin of the atom in region B and the “liveliness” of the cat’s head and body.

Now suppose we add a conscious being that perceives that the experiment has been done as stated (which I shall for simplicity assume excludes the conscious perceptions of the cat, even if it is still alive; if the cat could perceive that the experiment has been done and could correctly perceive the spin direction, presumably it could only perceive that the spin were up, since if the spin were down, the cat would be dead and presumably would have no perceptions) and that the being is in region B, and, as for the being perceived to be in region A, assume the idealization that these perceptions are indeed perfectly correlated with the state of affairs but are unaffected by whether the atom spin is up or down. (For example, the poison is not supposed to kill this conscious being or otherwise affect the measure of these perceptions of him or her.)
Furthermore assume that this conscious being interacts with (e.g., sees) the liveliness of the cat’s head and body. I now explicitly assume that we have an SQMPPC theory with the Assumption of Perception Components, one for the perceived state of the head of the cat and one for the state of the body.

We now must face the question of what it is about the head and body of the cat that can lead to different perception components. The most realistic idealization seems to be that the SQMPPC theory is such that if the head and body are alive, they will be perceived to be alive, and if they are dead, they will be perceived to be dead. Then different conceivable perceptions could be in the set \( S_{(\text{head alive, body alive})} \) with the components \( \{c_{\text{head alive}}, c_{\text{body alive}}, \ldots\} \) if both the head and body were perceived to be alive, with the corresponding awareness operator having an expectation value equal to that of \( P_{\text{head alive}}P_{\text{body alive}} \) and hence being nonzero. Or, they could be in the set \( S_{(\text{head dead, body dead})} \) with components \( \{c_{\text{head dead}}, c_{\text{body dead}}, \ldots\} \) if both the head and body were perceived to be dead, with the corresponding awareness operator having an expectation value equal to that of \( P_{\text{head dead}}P_{\text{body dead}} \) and hence being the same nonzero value, in the idealization of perfect coupling and with the pair of atoms starting out in the singlet spin state. Thus the relative probability of perceiving that the cat is alive is the same as that of perceiving that the cat is dead, assuming SQMPPC and the various idealizations above, independent of what may be happening in region A.

Other conceivable perceptions of the conscious being in region B that correctly perceives that the experiment has been done as stated could be in the set \( S_{(\text{head alive, body dead})} \) with the components \( \{c_{\text{head alive}}, c_{\text{body dead}}, \ldots\} \) if the head were perceived to be alive but the body were perceived to be dead, with the corresponding awareness operator having an expectation value equal to that of \( P_{\text{head alive}}P_{\text{body dead}} \), or in the set \( S_{(\text{head dead, body alive})} \) with components \( \{c_{\text{head dead}}, c_{\text{body alive}}, \ldots\} \) if the head were perceived to be dead but the body were perceived to be alive, with the corresponding awareness operator having an expectation value equal to that of \( P_{\text{head dead}}P_{\text{body alive}} \). However, both of these sets of perceptions have zero expectation value in the idealized state being assumed, so there will be no perception of a disagreement between the liveliness of the head and body of the cat with the idealizations being made. This seems to agree fairly well with my perception of what has been reported to me, though in my sheltered life as a theoretical physicist I cannot presently recall any memories of actually having seen a dead cat myself.

However, one could imagine an alternative SQMPPC theory to the original one just described, in which the components of perceptions are not directly coupled to the eigenstates of the liveliness of the cat’s head and body, but, say, to equal linear combinations of these eigenstates (for either the head or the body liveliness subsystem as may be the case),

\[
|\text{head}+\rangle = (|\text{head alive}\rangle + |\text{head dead}\rangle)/\sqrt{2}, \tag{84}
\]

\[
|\text{head}−\rangle = (|\text{head alive}\rangle − |\text{head dead}\rangle)/\sqrt{2}, \tag{85}
\]
\[
|\text{body+}\rangle = \frac{(|\text{body alive}\rangle + |\text{body dead}\rangle)}{\sqrt{2}},
\]
(86)
\[
|\text{body−}\rangle = \frac{(|\text{body alive}\rangle - |\text{body dead}\rangle)}{\sqrt{2}},
\]
(87)

or, more precisely, to the corresponding projection operators
\[
P_{\text{head+}} = |\text{head+}\rangle \langle \text{head+}| \otimes I_{\text{all but head}},
\]
(88)
\[
P_{\text{head−}} = |\text{head−}\rangle \langle \text{head−}| \otimes I_{\text{all but head}},
\]
(89)
\[
P_{\text{body+}} = |\text{body+}\rangle \langle \text{body+}| \otimes I_{\text{all but body}},
\]
(90)
\[
P_{\text{body−}} = |\text{body−}\rangle \langle \text{body−}| \otimes I_{\text{all but body}},
\]
(91)

where \(I_{\text{all but head}}\) is the identity operator for all of the system except for the head liveliness subsystem with its states \(|\text{head+}\rangle\) and \(|\text{head−}\rangle\), and similarly \(I_{\text{all but body}}\) is the identity operator for all of the system except for the body liveliness subsystem.

Then different conceivable perceptions could be in the set \(S_{\text{(head+, body+)}}\) with the components \(\{c_{\text{head+}}, c_{\text{body+}}, \ldots\}\) if both the head and body were perceived to be in their + states, with the corresponding awareness operator having an expectation value equal to that of \(P_{\text{head+}}P_{\text{body+}}\); in the set \(S_{\text{(head−, body−)}}\) with components \(\{c_{\text{head−}}, c_{\text{body−}}, \ldots\}\) if both the head and body were perceived to be in their − states, with the corresponding awareness operator having an expectation value equal to that of \(P_{\text{head−}}P_{\text{body−}}\); in the set \(S_{\text{(head+, body−)}}\) with the components \(\{c_{\text{head+}}, c_{\text{body−}}, \ldots\}\) if the head were perceived to be in the + state but the body were perceived to be in the − state, with the corresponding awareness operator having an expectation value equal to that of \(P_{\text{head+}}P_{\text{body−}}\); or in the set \(S_{\text{(head−, body+)}}\) with components \(\{c_{\text{head−}}, c_{\text{body+}}, \ldots\}\) if the head were perceived to be in its − state but the body were perceived to be in its + state, with the corresponding awareness operator having an expectation value equal to that of \(P_{\text{head−}}P_{\text{body+}}\). All of these awareness operators would have the same nonzero expectation value in the idealizations of perfect coupling and of the pair of atoms starting out in the singlet spin state.

Thus in this alternative SQMPPC theory and in the idealized experiment being described, there would be no correlation between the perception of the states of the cat’s head and body, when one averages over all sets of perceptions, weighted by their measures, in which the experiment is perceived to have occurred (assuming that this component of the perception is indeed perfectly correlated with whether or not the experiment occurred as stated). Presumably we might describe the conscious being as being confused if he or she perceives a disagreement between the states of the cat’s head and body. In the original SQMPPC theory described above, the measure was zero (under the idealized assumptions) for such “confused” perceptions, so all perceptions with positive measure were “unconfused,” but in the alternative SQMPPC the weighted fraction of unconfused perceptions was only one half (and would have been only \(2^{-n}\) if one had conceptually divided the cat into \(n\) parts that were all assigned + and − states to which the conscious perceptions were perfectly correlated).
The comparison of these two conceivable SQMPPC theories shows that if it is desired that perceptions $p$ be all unconfused in idealized cases, their components $c_i(p)$ should have their corresponding projection operators having expectation values equal to those of $P[c_i(p)]$ coupled to things (e.g., states of parts of the cat) that are entirely correlated (e.g., the “liveliness” property of being alive or dead, rather than the $+$ or $-$ properties) in these cases. These preferred projection operators for unconfused conscious perceptions are similar to the Information Basis of States for quantum measurements [51], except that no claim is made in SQM that the operators associated with perceptions form a complete basis.

However, it still is somewhat confusing to me why in idealized cases our perceptions actually seem to be rather unconfused, why the original rather than the alternative SQMPPC theory seems more accurate (or more likely to make our unconfused perceptions typical). One might argue that if they were not unconfused, then we could not act coherently and so would not survive. This would seem to be a good argument only if our perceptions really do affect our actions in the quantum world and are not just epiphenomena that are determined by the quantum world without having any effect back on it. Another argument similarly suggesting that explanations might be simpler if conscious perceptions acted back on the quantum world will be mentioned briefly in the Conclusions. But on the other hand, it is not obvious how perceptions could affect the quantum world in a relatively simple way in detail (though it is easy to speculate on general ways in which there might be some effect; see [1] and the Conclusions below). So although it appears to be unexplained, it conceivably could be that conscious perceptions do not affect the quantum world but are determined by it in just such a way that in most cases they are not too confused. To mimic Einstein, I might say, “The most confusing thing about perceptions is that they are generally unconfused.”

As an aside, I should say that although epiphenomenalism seems to leave it mysterious why typical perceptions are unconfused, I do not think it leaves it mysterious that perceptions occur, despite a naïve expectation that the latter is also mysterious. The naïve argument is that if the conscious world has no effect on the quantum world (usually called the physical world [3, 4], in contrast to my use of that term to include both the quantum world and the conscious world), and if the development of life in the quantum world occurs by natural selection, the development of consciousness would have no effect on this natural selection and so could not be explained by it.

Nevertheless, one can give an answer analogous to that I have heard was given by the former Fermilab Director Robert Wilson when he was was asked by a Congressional committee what Fermilab contributed to the defense of the nation: ”Nothing. But it helps make the nation worth defending.” Similarly, if epiphenomenalism is correct, consciousness may contribute nothing to the survival of the species, but it may help make certain species worth surviving. More accurately, it may not contribute to the evolution of complexity, but it may select us (probably not uniquely)
as complex organisms which have typical perceptions. Then our consciousness would not be surprising, because we are selected simply as typical conscious beings.

This selection as typical conscious beings might also help explain why we can do highly abstract theoretical mathematics and physics that does not seem to help us much with our survival as a species. If we are selected by the measure of our consciousness, and if that is positively correlated with a certain kind of complexity that is itself correlated with the ability to do theoretical mathematics and physics, then it would not be surprising that we can do this better than the average hominid that survives as well as we do (say averaging over all the Everett many worlds).

Returning to a consideration of the EPR experiment, the perceptions by conscious beings in the spacelike-separated regions A and B occur independently in SQM (under the idealizations of local quantum field theory and of the assumption that the relevant awareness operators are confined to either region A or B) and do not show any of the EPR correlations. To perceive the EPR correlations between regions A and B, one needs perceptions with awareness operators whose expectation values are affected by what is happening in both regions. One obvious way is to have these awareness operators in a region C which is to the causal future of both regions A and B. For example, a signal could be sent from each region, a signal that is perfectly correlated with the result of the spin measurement in that region (another idealization made for simplicity). Then one can imagine conscious perceptions of a being in C (meaning that the corresponding awareness operators can be localized to that region) with one component of each determined by the signal from A and the other component determined by the signal from B.

For example, there could be the set $S_{\uparrow\uparrow}$ of perceptions in which both signals indicate that the corresponding spin is up in the measured directions (which differed by the angle $\theta$), the set $S_{\uparrow\downarrow}$ of perceptions that the spin in A is up and that the spin in B is down, the set $S_{\downarrow\uparrow}$ of perceptions that the spin in A is down and that the spin in B is up, and the set $S_{\downarrow\downarrow}$ of perceptions that the spin in both A and B is down. The awareness operators for these four sets of perceptions could have expectation values, ideally, the same as those of $P_{\uparrow}P_{\text{up}}$, $P_{\uparrow}P_{\text{down}}$, $P_{\downarrow}P_{\text{up}}$, and $P_{\downarrow}P_{\text{down}}$, respectively, each multiplied by a common commuting projection operator $P_{\text{otherC}}$ for other common factors that lead to the perception that the experiment has been done, that one is in region C, and that both spin directions are now known as a result of receiving signals from both regions A and B.

Now one can readily calculate that if the pair of atoms starts in the singlet spin state, then under the idealizations above, the sets of perceptions $S_{\uparrow\uparrow}$ and $S_{\downarrow\downarrow}$ each have the same measure, which is $\tan^2 \theta/2$ of the measure for each of the sets $S_{\uparrow\downarrow}$ and $S_{\downarrow\uparrow}$. Thus if $\theta = 0$, there will be no measure for the first two sets of perceptions, and the conscious being in C will necessarily perceive that the two spins are opposite, the perfect EPR anticorrelation, even though the measure for each of the last two sets of perceptions is equal so that one cannot uniquely predict whether it will be
the spin in A or the spin in B that is perceived to be up in all perceptions with nonzero measure.

However, since one sees that the EPR correlations between regions A and B can be perceived by conscious beings (assuming that their awareness operators can be fairly well localized to where we can thus define these beings to be) only if they are to the causal future of both A and B (e.g., in region C), we do not have any superluminal propagation of anything in SQM if it is based on local quantum field theory. (Of course, the ultimate replacement of quantum field theory, e.g., by string theory, may eliminate this locality property of the quantum world, except as some sort of approximation in suitable circumstances.) Furthermore, just as one may conclude concerning quantum mechanics in the Everett interpretation, so too the Einstein-Podolsky-Rosen physical reality is completely described by Sensible Quantum Mechanics, contrary to the claim that Einstein, Podolsky, and Rosen made about quantum mechanics when all they had was the Copenhagen interpretation.

12 Questions and Speculations

One can use the framework of Sensible Quantum Mechanics to ask questions and make speculations that would be difficult without such a framework. I shall here give some examples, without intending to imply that Sensible Quantum Mechanics itself, even if true, would guarantee that these questions and speculations make sense, but it does seem to allow circumstances in which they might.

First, in the model of quantum field theory on a classical spacetime with no symmetries, and with a quantum state having well-localized human brains on some Cauchy hypersurface labeled by time $t$, one might ask whether it is possible to have two quite different perceptions, say $p$ and $p'$, in nearly the same Everett world in the sense of having the $f(p, p')$ of Eq. (19) near unity, and giving $E(p)$ and $E(p')$ both with the same preferred time $t_p = t$ and both localized (by the rather ad hoc prescription of Section 8) in balls in the same brain. In other words, can one brain have two different (maximal) perceptions in the same world at the same time, each not aware of the other? Unless we are solipsists (or unless we adopt the Orthogonal Projection Hypothesis, in which case we say that different perceptions all occur in different Everett worlds), we generally believe this is possible for two separate brains, but would one brain be sufficient? Furthermore, if it is possible, can the two balls (corresponding to $p$ and $p'$ respectively) be overlapping spatially, or need they be separate regions in the brain?

Second, one might ask whether and how the sum (or integral) of the measures (or measure densities) $m(p)$ associated with an individual brain region at the time $t$ depends on the brain characteristics. One might speculate that it might be greater for brains that are in some sense more intelligent, so that in a crude sense brighter brains have more perceptions. This could explain why you do not perceive yourself
to be an insect, for example, even though there are far more insects than humans.

Third, one might conjecture that an appropriate measure on perceptions might give a possible explanation of why most of us perceive ourselves to be living on the same planet on which our species developed. This observation might seem surprising when one considers that we may be technologically near the point at which we could leave Earth and colonize large regions of the Galaxy [52], presumably greatly increasing the number of humans beyond the roughly $10^{11}$ that are believed to have lived on Earth. If so, why don’t we have the perceptions of one of the vast numbers of humans that may be born away from Earth? One answer is that some sort of doom is likely to prevent this vast colonization of the Galaxy from happening [53, 54, 55, 56], though these arguments are not conclusive [57]. Although I would not be surprised if such a doom were likely, I would naively expect it to be not so overwhelmingly probable that the probability of vast colonization would be so small as is the presumably very small ratio of the total number of humans who could ever live on Earth to those who could live throughout the Galaxy if the colonization occurs. Then, even though the colonization may be unlikely, it may still produce a higher measure for conscious perceptions of humans living off Earth than on it.

However, another possibility is that colonization of the Galaxy is not too improbable, but that it is mostly done by self-replicating computers or machines who do not tolerate many humans going along, so that the number of actual human colonizers is not nearly so large as the total number who could live throughout the Galaxy if the computers or machines did not dominate the colonization. If the number of these computers or machines dominate humans as “intelligent” beings (in the sense of having certain information-processing capabilities), one might still have the question of why we perceive ourselves as being humans rather than as being one of the vastly greater numbers of such machines. But the explanation might simply be that the weight of conscious perceptions (the sum or integral of the $m(p)$’s corresponding to the type of perceptions under consideration) is dominated by human perceptions, even if the number of “intelligent” beings is not. In other words, human brains may be much more efficient in producing conscious perceptions than the kinds of self-replicating computers or machines which may be likely to dominate the colonization of the Galaxy. If such machines are more “intelligent” than humans in terms of information-processing capabilities and yet are less efficient in producing conscious perceptions, our perceptions of being human would suggest that the measure of perceptions is not merely correlated with “intelligence.” (On the other hand, if the measure of perceptions is indeed strongly correlated with “intelligence” in the sense of information-processing capabilities, perhaps it might be the case that Galactic colonization is most efficiently done by self-replicating computers or machines that are not so “intelligent” as humans. After all, insects and even bacteria have been more efficient in colonizing a larger fraction of Earth than have humans.)

It might be tempting to take the observations that these speculations might
explain (our perceptions of ourselves as human rather than as insect, and our perceptions of ourselves as humans on our home planet) as evidence tending to support the speculations. One could summarize such reasoning as a generalization of the Weak Anthropic Principle [58, 59, 60, 61, 62, 63, 64] that might be called the Conditional Aesthetic Principle (CAP): given that we are conscious beings, our conscious perceptions are likely to be typical perceptions in the conscious world with its measure.

If one uses the dual typicality $T_d(p)$ defined by Eq. (28) as an indication of how “likely” a perception is, one can say that there is a 99% likelihood that $T_d(p) \geq 0.01$. For example, if one restricts oneself to the perception of a continuous variable for which the measure density has a gaussian distribution, then at the 99% likelihood level, the variable should be between about 0.0062666117 and about 2.8070337863 standard deviations from the mean. Values closer to the mean are “too good to be likely,” and values further from the mean are “too bad to be likely,” at least at the 99% likelihood level.

In addition to the typicality $T(p)$ defined by Eq. (21) and the dual typicality $T_d(p)$ defined by Eq. (28) as indications of how “likely” a perception is, the discussion above suggests the usefulness of other measures of the typicality or likelihood of a perception. Unfortunately, there is the apparent arbitrariness of their definition. If one makes a rather ad hoc definition of typicality and then finds that one’s perception is atypical with respect to this definition, one may have grounds to be sceptical that it really is evidence against the detailed Sensible Quantum Mechanics theory that predicts the low typicality-thus-defined of one’s perception. For example, I have the perception of having been one of the last $10^{-7}$ or so of those born in 1948 (since I was born about 90 minutes before midnight on December 31 according to the local time, ten time zones west of Greenwich, in the lowly-populated Territory of Alaska, fairly near the point $B$ of Figure 8.3 on page 212 of my Ph.D. advisor’s textbook [65], with only Hawaii providing a significant population in that or any other time zone further west), so I am atypical in that regard, but it is probably not so surprising that after living over 40 years I have finally found some particular detail about myself that by itself might appear unusual.

I also noticed recently that the fourth (and, I would guess, last) in the sequence of at least four Mersenne primes given by the (non-Mersenne) prime seed $N_0 = 2$ and the recursion relation $N_{n+1} = 2^{N_n} - 1$ is within one-half of one percent of the inverse gravitational fine structure constant for the proton (the square of the ratio of the Planck mass to the proton mass). (The logarithms of these two large numbers agree to about one part in 19000 ± 500.) If the inverse of the gravitational fine structure constant is an environmental constant that varies from component to component of the quantum state of the universe, then I would expect it to be rather atypical to perceive it to be so close to $2^{127} - 1$, but since there are presumably so many other unusual things that I could have found instead (and which indeed Eddington [66]
did find and claim to have found a theory for), I would think that surely it is just
another numerical coincidence and thus that the apparent resulting “atypicality” is
just an artifact of an ad hoc definition for it.
I could also cite various other perceptions that naively might seem atypical, such
as my frequent remembrance that in 1992 the U.S., Canada, and the new Russian
Republic had ages in years that were all perfect cubes (216, 125, and 1 respectively),
my noticing that an integer often considered unlucky in Western culture is the only
positive integer fourth root of the sum of two successive positive square integers, and
my fascination with the fact that there is a mystery word (for which I have a long
standing offer of $10 for the first person who can find it) whose modern definition in
the British Oxford English Dictionary gives a certain quantity, but whose definition
in the American Webster’s Third International Dictionary gives a different quantity,
larger by the ratio of about 1.00655197916815342586087238773730720409883464864441784178751363.

Part of the explanation for such ‘unusual’ perceptions is what I have called the
Attention Effect (see the last paper in [1]). This is the fact that unusual events
attract our attention, so that we tend to focus on them and spend a longer time
being conscious of them. The measure for a set of perceptions presumably increases
with the time (when that approximate concept is applicable) spent having them, so
events that attract our attention for longer periods of time will presumably lead to
sets of conscious perceptions having greater measure and hence being less atypical
than we might have thought.

Therefore, one should be very careful in using the Conditional Aesthetic Principle,
even though it might be useful in explaining certain features of our perceptions
that might more naively be thought to be surprising.

13 Conclusions

In conclusion, I am proposing that Sensible Quantum Mechanics is the best
framework we have at the present level for understanding conscious perceptions
and the interpretation of quantum mechanics. Of course, the framework would only
become a complete theory once one had the set $M$ of all perceptions $p$, the awareness
operators $A(S)$, and the quantum state $\sigma$ of the universe (and preferably also the
prior measure $\mu_0(S)$ in order to test the theory and compare it with others).

Even such a complete theory of the quantum world and the conscious world
affected by it need not be the ultimate simplest complete theory of the combined
physical world. There might be a simpler set of unifying principles from which one
could in principle deduce the perceptions, awareness operators, and quantum state,
or perhaps some simpler entities that replaced them. For example, although in
the present framework of Sensible Quantum Mechanics, the quantum world (i.e., its
state), along with the awareness operators, determines the measure for perceptions in
the conscious world, there might be a reverse effect of the conscious world affecting the quantum world to give a simpler explanation than we have at present of the coherence of our perceptions (as pondered in Section 11) and of the correlation between will and action (why my desire to do something I feel am capable of doing is correlated with my perception of actually doing it, i.e., why I “do as I please”). If the quantum state is partially determined by an action functional, can desires in the conscious world affect that functional (say in a coordinate-invariant way that therefore does not violate energy-momentum conservation)? Such considerations may call for a more unified framework than Sensible Quantum Mechanics, which one might call Sensational Quantum Mechanics [1]. Such a more unified framework need not violate the limited assumptions of Sensible Quantum Mechanics, though it might do that as well and perhaps reduce to Sensible Quantum Mechanics only in a certain approximate sense.

To explain these frameworks in terms of an analogy, consider a classical model of spinless massive point charged particles and an electromagnetic field in Minkowski spacetime. Let the charged particles be analogous to the quantum world (or the quantum state part of it), and the electromagnetic field be analogous to the conscious world (the set of perceptions with its measure $\mu(S)$). At the level of a simplistic materialist mind-body philosophy, one might merely say that the electromagnetic field is part of, or perhaps a property of, the material particles. At the level of Sensible Quantum Mechanics, the charged particle worldlines are the analogue of the quantum state, the retarded electromagnetic field propagator (Coulomb’s law in the nonrelativistic approximation) is the analogue of the awareness operators, and the electromagnetic field determined by the worldlines of the charged particles and by the retarded propagator is the analogue of the conscious world. (Here one can see that this analogue of Sensible Quantum Mechanics is valid only if there is no free incoming electromagnetic radiation.) At the level of Sensational Quantum Mechanics, at which the conscious world may affect the quantum world, the charged particle worldlines are partially determined by the electromagnetic field through the change in the action it causes. (This more unified framework better explains the previous level but does not violate its description, which simply had the particle worldlines given.) At a yet higher level, there is the possibility of incoming free electromagnetic waves, which would violate the previous frameworks that assumed the electromagnetic field was uniquely determined by the charged particle worldlines. (An analogous suggestion for intrinsic degrees of freedom for consciousness has been made by Linde [57].) Finally, at a still higher level, there might be an even more unifying framework in which both charged particles and the electromagnetic field are seen as modes of a single entity (e.g., to take a popular current speculation, a superstring).

Therefore, although it is doubtful that Sensible Quantum Mechanics is the correct framework for the final unifying theory (if one does indeed exist), it seems to
me to be a move in that direction that is consistent with what we presently know about consciousness and the physical world. At least it seems to be an augmentation of ordinary quantum mechanics (without the collapse postulate) that cannot be criticized as being incomplete for not predicting (when the framework is fleshed out into a complete SQM theory) precisely what happens during observations.

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