Abstract

The vulnerability of deep neural networks (DNNs) to adversarial examples has attracted great attention in the machine learning community. The problem is related to local non-smoothness and steepness of normally obtained loss landscapes. Training augmented with adversarial examples (a.k.a., adversarial training) is considered as an effective remedy. In this paper, we highlight that some collaborative examples, nearly perceptually indistinguishable from both adversarial and benign examples yet show extremely lower prediction loss, can be utilized to enhance adversarial training. A novel method called collaborative adversarial training (CoAT) is thus proposed to achieve new state-of-the-arts. Code will be made publicly available.

1 Introduction

Adversarial examples [37, 4] crafted by adding imperceptible perturbations to benign examples are capable of fooling DNNs to make incorrect predictions. The widely existence of such adversarial examples has raised many security concerns and attracted great recent attention.

Much endeavour has been devoted to improve the adversarial robustness of DNNs over the past few years. As one of the most effective methods, adversarial training [25, 47] introduces powerful and adaptive adversarial examples during the model training process, and encourages the model to classify them correctly.

In this paper, to gain a deeper understanding of DNNs, robust or not, we examine the valley of their loss landscapes and explore the existence of collaborative examples in the $\epsilon$-bounded neighborhood of benign examples, which demonstrate extremely lower prediction loss in comparison to that of their neighbors. Somewhat unsurprisingly, the existence of such examples can be related to the adversarial robustness of DNNs. More specifically, if given a model which was trained to be adversarially more robust, then it is less likely to discover a powerful collaborative example. Furthermore, incorporating such collaborative examples into model training seemingly also improves the adversarial robustness. On this point, we propose collaborative adversarial training (CoAT), in which adversarial example and collaborative example of each benign example are optimized jointly, such that their maximum possible prediction discrepancy is directly constrained, with a formulation partially inspired by that of TRADES [47]. Extensive experimental results demonstrate that our CoAT outperforms state-of-the-arts on several benchmark datasets significantly, and it can also be readily combined with a variety of recent efforts, e.g., RST [6] and AWP [42], to further improve the performance.

2 Background and Related Work

2.1 Adversarial Examples

We first introduce symbols and notations in this paper. Let $x_i$ and $y_i$ denote a benign example (e.g., a natural image) and its label from a dataset $S = \{(x_i, y_i)\}_{i=1}^n$, respectively, where $x_i \in \mathcal{X}$ and $y_i \in \mathcal{Y}$.
where $y_i \in \mathcal{Y} = \{0, \ldots, C - 1\}$. We use $\mathcal{B}_\epsilon[x_i] = \{x' \mid \|x' - x_i\|_\infty \leq \epsilon\}$ to represent the $\epsilon$-bounded $l_\infty$ neighborhood of the natural image $x_i$. The DNN model parameterized by a set of parameters $\Theta$ can be defined as a function $f_\Theta(\cdot) : \mathcal{X} \to \mathbb{R}^C$. Without ambiguity, we shall drop the subscript $\Theta$ in $f_\Theta(\cdot)$ and write it as $f(\cdot)$ in the sequel of this paper.

In general, adversarial examples are almost perceptually indistinguishable to benign examples, yet lead to arbitrarily incorrect predictions on the victim models. One typical formulation for generating an adversarial example is to maximize the prediction loss in a constrained local neighborhood of a benign example.

Projected gradient descent (PGD) [25] (or the iterative fast gradient sign method, i.e., I-FGSM [22]) is commonly chosen for achieving the aim. It seeks possible adversarial examples by leveraging the gradient of $g = \ell \circ f$ w.r.t. its inputs, where $\ell$ is a loss function (e.g., the cross-entropy loss $\text{CE}(\cdot, y)$). Given a starting point $x_0$, an iterative update is performed with:

$$x^{t+1} = \Pi_{B_\epsilon[x]}(x^t + \alpha \cdot \text{sign}(\nabla_{x^t} \text{CE}(f(x^t), y_i))),$$

where $x^t$ is a temporary result obtained at the $t$-th step and function $\Pi_{B_\epsilon[x]}(\cdot)$ projects its input onto the $\epsilon$-bounded neighborhood of the benign example. The starting point can be the benign example (for I-FGSM) or its randomly neighbor (for PGD).

Besides I-FGSM and PGD, the single-step FGSM [13], C&W’s attack [5], DeepFool [27], and the momentum iterative FGSM [12] are also popular and effective for generating adversarial examples. Some work also investigates the way of generating adversarial examples without any knowledge of the victim model, which are known as black-box attacks [29, 7, 19, 8, 45, 15] and no-box attacks [29, 24]. Recently, the ensemble of a variety of attacks becomes popular for performing adversarial attack and evaluating adversarial robustness. Such a strong adversarial benchmark, called AutoAttack (AA) [10], consists of three white-box attacks, i.e., APGD-CE, APGD-DLR, and FAB [9], and one black-box attack, i.e., the Square Attack [1]. We adopt it in experimental evaluations.

In this paper, we explore valley of the loss landscape of DNNs and study the benefit of incorporating collaborative examples into adversarial training. In an independent paper [38], hypocritical examples were explored for concealing mistakes of a model, as an attack. These examples also lied in the valley. Yet, due to the difference in aim, studies of hypocritical examples in [38] were mainly performed based on mis-classified benign examples according to their formal definition, while our work concerns local landscapes around all benign examples. Other related work include unadversarial examples [34] and assistive signals [31] that designed 3D textures to customize objects for better classifying them.

### 2.2 Adversarial Training (AT)

Among the numerous methods for defending against adversarial examples, adversarial training that incorporates such examples into model training is probably one of the most effective ones. We will revisit some representative adversarial training methods in this subsection.

**Vanilla AT** [25] formulates the training objective as a simple min-max game. Adversarial examples are first generated for instance PGD to maximize some loss (e.g., the cross-entropy loss) in the objective, and then the model parameters are optimized to minimize the same loss with the obtained adversarial examples:

$$\min_{\Theta} \max_{x_i' \in \mathcal{B}_\epsilon[x_i]} \text{CE}(f(x_i'), y_i).$$

Although effective in improving adversarial robustness, the vanilla AT method inevitably leads to decrease in the prediction accuracy of benign examples, therefore several follow-up methods discuss improved and more principled ways to better trade off clean and robust accuracy [47, 20, 40, 42]. Such methods advocate regularizing the output of benign example and its adversarial neighbors. With remarkable empirical performance, they are regarded as strong baselines, and we will introduce a representative one, i.e., TRADES [47].

**TRADES** [47] advocates a learning objective comprising two loss terms. Its first term penalizes the cross-entropy loss of benign training samples, and the second term regularizes the difference between benign output and the output of possibly malicious data points. Specifically, the worst-case Kullback-Leibler (KL) divergence between the output of each benign example and that of any suspicious data point in its $\epsilon$-bounded $l_\infty$ neighborhood is minimized in the regularization term:

$$\min_{\Theta} \sum_i (\text{CE}(f(x_i), y_i) + \beta \max_{x_i' \in \mathcal{B}_\epsilon[x_i]} \text{KL}(f(x_i'), f(x_i))).$$

2
Other efforts have also been devoted in the family of adversarial training research, e.g., MART [40], robust self training (RST) [6], and adversarial weight perturbation (AWP) [42]. More specifically, after investigating the influence of mis-classified samples on model robustness, MART advocates giving specific focus to these samples for robustness. AWP identifies that flatter loss changing with respect to parameter perturbation leads to improved generalization of adversarial training, and provides a novel double perturbation mechanism. RST proposes to boost adversarial training by using unlabeled data and incorporating semi-supervised learning. Rebuffi et al. [32] focus on data augmentation and study the performance of using generative models. There are also insightful work that focuses on model architectures [18, 41, 3, 26, 30], batch normalization [43], and activation functions [44, 11]. Distilling from adversarially trained models has also been widely studied [49, 35, 2].

Our CoAT for improving adversarial robustness is partially inspired by TRADES, and we will discuss and compare them in Section 4.2, 5.1, and 5.2. Besides, our method can be naturally combined with a variety of other prior efforts introduced in this section, to achieve further improvements, as will be demonstrated in Section 5.

3 Collaborative Examples

With the surge of interest in adversarial examples, we have achieved some understandings of plateau regions on the loss landscape of DNNs. However, valleys of the landscape seem less explored. In this section, we examine the valleys and explore the existence of collaborative examples, i.e., data points are capable of achieving extremely lower classification loss, in the $\epsilon$-bounded neighborhood of benign examples. In particular, we discuss how adversarial robustness of DNNs and the collaborative examples affect each other, by providing several intriguing observations.

3.1 Valley of The Loss Landscape

Unlike adversarial examples that are data points with higher or even maximal prediction loss, we pay attention to local minimum around benign examples in this subsection.

To achieve this, we here simply adapt the PGD method to instead minimize the prediction loss with:

$$x^{t+1} = \Pi_{\mathbb{S}^d_{\epsilon}}(x^t - \alpha \cdot \text{sign}(\nabla_{x^t} \text{CE}(f(x^t), y))).$$

(4)

Comparing Eq. (4) to (1), it can be seen that their main difference is that, in Eq. (4), gradient descent is performed rather than gradient ascent. Similar to the I-FGSM and PGD attack, we clip the result in Eq. (4) after each update iteration to guarantee that the perturbation is within a presumed budget, e.g., $4/255, 8/255, 16/255, \text{ or } 32/255$. We perform such an update with a step size of $\alpha = 1/255$. ResNet [17] and wide ResNet [46] models trained on CIFAR-10 and CIFAR-100 are tested.

Figure 1: The average cross-entropy loss value of benign examples (blue) and collaborative examples (red) on (a) ResNet-18 trained using CIFAR-10, (b) wide ResNet-34-10 trained using CIFAR-10, (c) ResNet-18 trained on CIFAR-100, and (d) wide ResNet-34-10 trained on CIFAR-100. Shaded areas indicate scaled standard deviations. The collaborative examples are crafted with a fixed step size of $\alpha = 1/255$ and various perturbation budgets.
Figure 2: Visualization of benign example (left), collaborative example crafted on a normally trained ResNet-18 model (middle), and collaborative example crafted on a robust ResNet-18 model (right).

After multiple update steps (preciously 100 steps for this experiment), we evaluate the cross-entropy loss of the obtained examples. We compare it to the benign loss in the left most panels in Figure 1. It can be seen that, though benign data shows relatively low cross-entropy loss (i.e., ~ 0.2 on average for ResNet-18 on the CIFAR-10 test set) already, there always exists neighboring data that easily achieve extremely lower loss values (i.e., almost 0 on average). Such data points that show considerably lower prediction loss are collaborative examples of our interest. See Figure 2 for visualization of the collaborative examples generated on the basis of a randomly chosen CIFAR-100 image.

The existence of collaborative examples implies large local Lipschitz constants or non-smoothness of $g = \ell \circ f$, from a somehow different perspective against the conventional adversarial phenomenon. To shed more light on this, we further test with DNN models that were trained to be robust to adversarial examples, using the vanilla AT [25], TRADES [47], and MART [40] $^2$. The results can be found in the right panels in Figure 1. We see it is more difficult to achieve zero prediction loss with these models, probably owing to smoother and flatter loss landscapes [23]. See Figure 3, in which the perturbation direction $u$ is obtained utilizing Eq. (4), and $v$ is random chosen in a hyperplane orthogonal to $u$.

We analyze the angle between collaborative perturbations and PGD adversarial perturbations. After a bunch of update steps, we observe that it lies in a limited range around $90^\circ$, which is unsurprising in the high dimensional input space. However, for more robust models, we see that the angle deviate more from $90^\circ$, indicating that powerful collaborative and adversarial perturbations become more correlated on the robust landscapes. See Section B in our supplementary material for more details.

3.2 How Can Collaborative Examples Aid?

Given the results that the collaborative examples are less “destructive” on a more adversarially robust model, we raise a question:

**How can collaborative examples in return benefit adversarial robustness?**

Towards answering the question, one may first try to incorporate the collaborative examples into the training phase to see whether adversarial robustness of the obtained DNN model can be improved. To this end, we resort to the following learning objective:

$$
\min_{\Theta} \sum_i (CE(f(x_i), y_i) + \beta \cdot KL(f(x_i^{col}), f(x_i))),
$$

(5)

where $x_i^{col}$ is a collaborative example generated using the method introduced in Section 3.1. Eq. (5) minimizes the output discrepancy between the collaborative examples and their corresponding benign examples, in addition to the loss term that encourages correct prediction on benign examples.

This simple and straightforward method has been similarly adopted in Tao et al.’s work [38] for resisting hypocritical examples. A quick experiment is performed here to test its benefit to adversarial robustness in our settings. The inner update for obtaining collaborative examples is performed over 10 steps, with a step size of $\alpha = 2/255$ and a perturbation budget of $\epsilon = 8/255$. We evaluate prediction accuracy on the PGD adversarial examples and benign examples for comparison. Figure 4 shows the test-set performance of ResNet-18 trained as normal and trained using Eq. (5) on CIFAR-10 and

$^2$All models trained via adversarial training show quite good robust accuracy under adversarial attacks on CIFAR-10 and CIFAR-100. Clean and robust accuracy of these models are provided in Section A in our supplementary material.
Figure 4: Changes in clean and robust accuracy during training ResNet-18 on (a) CIFAR-10 and (b) CIFAR-100, using Eq. (5). Best viewed in color.

CIFAR-100, respectively. It can be seen that the robust accuracy is improved remarkably by solely incorporating collaborative examples into training. In the meanwhile, those normally trained models consistently show \( \sim 0\% \) robust accuracy.

However, there still exists a considerable gap between the obtained performance of Eq. (5) and that of TRADES (see Figure 8 in our supplementary material). Robust overfitting \cite{nguyen2018simple} can be observed on the red curves in Figure 4 (especially after the 80-th epoch), even though training with collaborative examples and testing with adversarial examples, and it seems more severe in comparison to that occurs with existing adversarial training methods, e.g., TRADES. We also test other DNN architectures, e.g., VGG \cite{simonyan2014very} and wide ResNet \cite{zagoruyko2016wide}, and the results are similar.

4 Collaborative Adversarial Training (CoAT)

In the above section, we have experimentally shown that collaborative examples exist and they can be used to improve the adversarial robustness of DNNs, by simply enforcing their output probabilities to be close to the output of their corresponding benign neighbors. In this section, we consider utilizing collaborative examples and adversarial examples jointly during training, aiming at regularizing non-smooth regions of the whole loss landscape altogether.

4.1 Method

Considering that the adversarial and collaborative examples control the upper and lower bound of the prediction loss, respectively, we penalize the maximum possible output discrepancy of any two data points within the \( \epsilon \)-bounded neighborhood of each benign example. Inspired by the formulation of TRADES \cite{dong2018boosting}, we adopt a benign prediction loss term in combination with a regularization in which possible adversarial examples and possible collaborative examples are jointly regularized. By contrast, the benign example itself is not necessarily involved in error accumulation from the regularization. That is, the output of a benign example is neither explicitly encouraged to be “adversarial” nor to be “collaborative”. We name this novel method as collaborative adversarial training (CoAT), since it jointly regularizes these two sorts of non-smooth and steep regions of the loss landscape. The learning objective of our CoAT is:

\[
\begin{align*}
\min_\Theta & \sum_i (\text{CE}(f(x_i), y_i) + \beta \max_{x' \in B_\epsilon(x_i)} \ell_{\text{reg}}(f(x'_i), f(x_i'))) \\
\text{s.t.} & \quad p_f(y_i | x'_i) \geq p_f(y_i | x_i) \geq p_f(y_i | x''_i), \quad \forall i,
\end{align*}
\]

where \( \ell_{\text{reg}}(\cdot) \) is a regularization function which evaluates the discrepancy between two probability vectors, and \( \beta \) is a scaling factor which balances clean and robust errors. The constraint in Eq. (6) is introduced to ensure that the two examples obtained in the inner optimization include one adversarial example and one collaborative example, considering that it makes little sense to minimize the gap between two adversarial examples or between two collaborative examples. There are several different choices for the regularization function \( \ell_{\text{reg}} \), e.g., the Jensen–Shannon (JS) divergence, the squared \( l_2 \) distance, and the symmetric KL divergence, which are formulated as follows:

1. JS:

\[
\ell_{\text{reg}} = \frac{1}{2} \left( \text{KL}\left(\frac{f(x') + f(x'')}{2}, f(x')\right) + \text{KL}\left(\frac{f(x') + f(x'')}{2}, f(x'')\right) \right),
\]

where \( \text{KL}(\cdot, \cdot) \) is the Kullback-Leibler divergence.
(2) (Squared) $l_2$:
$$\ell_{reg} = \|f(x') - f(x'')\|^2_2,$$
(3) Symmetric KL:
$$\ell_{reg} = \frac{1}{2} (KL(f(x'), f(x'')) + KL(f(x''), f(x'))).$$

Among these choices, the (squared) $l_2$ and JS divergence are already symmetric, and we also adapt the original KL divergence to make it satisfy the symmetry axiom of desirable metrics, such that the adversarial examples and collaborative examples are treated equally.

With the formulation in Eq. (6), pairs of collaborative and adversarial examples are obtained simultaneously, and the inner optimization are thus different from Eq. (1) and (4). The implementation is shown in Algorithm 1, with modified PGD for achieving temporary results and consistent comparisons between prediction loss of the temporary results for guaranteeing chained inequality.

**Algorithm 1 Collaborative Adversarial Training (CoAT)**

**Input**: A set of benign example and their labels $S$, number of training iterations $T$, learning rate $\eta$, number of inner optimization steps $K$, perturbation budget $\epsilon$, step size $\alpha$, and a choice of regularization function $\ell_{reg}$.

**Initialization**: Perform random initialization for $f$.

**for** $t = 1, \ldots, T$ **do**

Sample a mini-batch of training data $\{(x_i, y_i)\}_{i=1}^m$.

**for** $i = 1, \ldots, m$ (in parallel) **do**

$x_i' \leftarrow x_i + 0.001 \cdot \mathcal{N}(0, I)$.

$x_i'' \leftarrow x_i + 0.001 \cdot \mathcal{N}(0, I)$.

**while** $K \geq 0$ **do**

$x_i^{adv} \leftarrow \arg \max_{x_i \in \epsilon^x} \text{CE}(f(x_i), y_i)$.

$x_i^{col} \leftarrow \arg \min_{x_i \in \epsilon^x} \text{CE}(f(x_i), y_i)$.

$g_{inner} = \ell_{reg}(f(x_i^{adv}), f(x_i^{col}))$.

$x_i' \leftarrow \Pi_{\mathcal{B}_i, |x_i|} (x_i^{adv} + \alpha \cdot \text{sign}(\nabla_{x_i^{adv}} g_{inner}))$.

$x_i'' \leftarrow \Pi_{\mathcal{B}_i, |x_i|} (x_i^{col} + \alpha \cdot \text{sign}(\nabla_{x_i^{col}} g_{inner}))$.

$K \leftarrow K - 1$.

end while

$g_i \leftarrow \text{CE}(f(x_i), y_i) + \beta \cdot \ell_{reg}(f(x_i^{adv}), f(x_i^{col}))$.

end for

$$\Theta \leftarrow \Theta - \eta \sum_{i=1}^m \nabla \Theta g_i$$

end for

**Output**: A robust classifier $f$ parameterized by $\Theta$.

4.2 Discussions and Comparisons with TRADES

As has been revisited in Section 2, TRADES regularizes the KL divergence between benign examples and their neighboring data points. While, a neighboring data point that shows maximal KL divergence from the benign output, can be an adversarial example or a collaborative example actually.

In Figure 5, we demonstrate the ratio of collaborative examples and the prediction loss of different examples along with the training using TRADES proceeds. It can be observed that the ratio of collaborative examples used for model optimization decreases consistently, and, even at the very beginning, it is less than 50%. Our CoAT aims to use 100% of the collaborative and adversarial examples, if possible.

Comparing with TRADES, the $\ell_{reg}$ term in our CoAT imposes a stricter regularization, since the maximum possible output discrepancy between

![Figure 5: Changes in average cross-entropy loss of benign examples, adversarial examples, and collaborative examples during training using TRADES (upper), and the changes in the ratio of collaborative examples utilized (lower). The experiment is performed with ResNet-18 on CIFAR-10. Shaded areas represent scaled standard deviations. Best viewed in color.](image-url)
any two data points within the $\epsilon$-bounded neighborhood is penalized, which bounds the TRADES regularization from above. In this regard, the worst-case outputs (i.e., adversarial outputs) and the best-case outputs (collaborative outputs) are expected to be optimized jointly and equally.

Moreover, as has been mentioned, since CoAT does not explicitly enforce the benign output probabilities to match the output of adversarial examples as in TRADES, we may expect improved trade-off between robust and clean accuracy. Extensive experimental comparison with TRADES in Section 5 will verify that our CoAT indeed outperforms TRADES significantly, probably benefits from these merits. With improved adversarial robustness, we expect little negative social impact of this work.

5 Experiments

In this section, we compare the proposed CoAT to several state-of-the-art methods, including the vanilla AT [25], TRADES [47], and MART [40], on popular benchmark datasets, including CIFAR-10, CIFAR-100 [21], and SVHN [28]. Table 1 summarizes our main results, with ResNet [17]. We also test with wide ResNet [46] to show that our method works as well on large scale classification models, whose results can be found in Table 3 and 4.

It is worth noting that our CoAT can be readily combined with many recent advances, e.g., AWP [42] and RST [6]. AWP utilizes weight perturbation in addition to input perturbation. We can combine CoAT with it by replacing its learning objective with ours. RST uses additional unlabeled data to boost the performance of adversarial training. It first produces pseudo labels for unlabeled data, and then minimizes a regularization loss on both labeled data and unlabeled data.

**Training settings.** In most experiments in this section, we perform adversarial training with a perturbation budget of $\epsilon = 8/255$ and an inner step size $\alpha = 2/255$, except for the SVHN dataset, where we use $\alpha = 1/255$. In the training phase, we always use an SGD optimizer with a momentum of 0.9, a weight decay of 0.0005, and a batch size of 128. All images are re-scaled to the numerical range of $[0.0, 1.0]$. Random crop and random horizontal flip are used as data augmentation strategies for CIFAR-10 and CIFAR-100. We train ResNet-18 [16] for 120 epochs on CIFAR-10 and CIFAR-100, and we adopt an initial learning rate of 0.1 and cut it by $10\times$ at the 80-th and 100-th epoch. For SVHN, we train ResNet-18 for 80 epochs with an initial learning rate of 0.01, and we cut by $10\times$ at the 50-th and 65-th epoch. We adopt $\beta = 6$ for TRADES and $\beta = 5$ for MART by following their original papers. The final choice for the regularization function $\ell_{reg}$ and the scaling factor $\beta$ in our CoAT will be given in Section 5.1. All models are trained on an NVIDIA Tesla-V100 GPU.

**Evaluation details.** We evaluate the performance of adversarially trained models by computing their clean and robust accuracy. For robust accuracy, we perform various white-box attack methods including FGSM [13], PGD [25], C&W's attack [5], and AutoAttack (AA) [10]. Specifically, we perform PGD-20, PGD-100, and C&W$_\infty$ (i.e., the $l_\infty$ version of C&W’s loss optimized using PGD-100) under $\epsilon = 8/255$ and $\alpha = 2/255$, following [42]. Since adversarial training generally shows overfitting [33], we select the model with the best PGD-20 performance from all checkpoints, as suggested in many recent papers [48, 42, 40, 14].
Table 1: Clean and robust accuracies of robust ResNet-18 models trained using different adversarial training methods on CIFAR-10, CIFAR-100, and SVHN. The robust accuracy is evaluated under a $\ell_\infty$ threat model with $\epsilon = 8/255$. We perform seven runs and report the average performance with 95% confidence intervals.

| Dataset   | Method     | Clean | FGSM | PGD-20 | PGD-100 | C&W  | AA   |
|-----------|------------|-------|------|--------|---------|------|------|
| CIFAR-10  | Vanilla AT | 82.78%| 56.94%| 51.30% | 50.88%  | 49.72%| 47.63%|
|           | TRADES     | 82.41%| 58.47%| 52.76% | 52.47%  | 50.43%| 49.37%|
|           | MART       | 80.70%| 58.91%| 54.02% | 53.58%  | 49.35%| 47.49%|
|           | CoAT (ours)| 83.10%| 59.51%| 54.62% | 54.39%  | 51.43%| 50.50%|
|           | TRADES+AWP | 81.16%| 57.86%| 54.56% | 54.45%  | 50.95%| 50.31%|
|           | CoAT (ours)+AWP | 82.53%| 59.73%| 55.56% | 55.47%  | 52.05%| 51.23%|
| CIFAR-100 | Vanilla AT | 57.27%| 31.81%| 28.66% | 28.49%  | 26.89%| 24.60%|
|           | TRADES     | 57.94%| 32.37%| 29.25% | 29.10%  | 25.88%| 24.71%|
|           | MART       | 55.03%| 33.12%| 30.32% | 30.20%  | 26.60%| 25.13%|
|           | CoAT (ours)| 58.44%| 33.35%| 30.53% | 30.39%  | 26.70%| 25.61%|
|           | TRADES+AWP | 58.76%| 33.82%| 31.53% | 31.42%  | 27.03%| 26.06%|
|           | CoAT (ours)+AWP | 59.06%| 34.50%| 32.22% | 32.16%  | 27.83%| 26.86%|
| SVHN      | Vanilla AT | 89.21%| 59.81%| 51.18% | 50.35%  | 48.39%| 45.96%|
|           | TRADES     | 90.20%| 66.40%| 54.49% | 54.18%  | 52.09%| 49.51%|
|           | MART       | 88.70%| 64.16%| 54.70% | 54.13%  | 46.95%| 44.98%|
|           | CoAT (ours)| 90.68%| 66.68%| 56.35% | 56.00%  | 52.57%| 50.54%|
|           | TRADES+AWP | 89.80%| 66.30%| 59.01% | 58.63%  | 54.72%| 52.54%|
|           | CoAT (ours)+AWP | 90.77%| 67.77%| 59.95% | 59.76%  | 55.26%| 53.37%|

5.1 Comparison of Different Discrepancy Metrics

To get started, we compare the three choices of discrepancy metric for $\ell_{\text{reg}}$, e.g., the JS divergence, the squared $l_2$ distance, and the symmetric KL divergence. In Figure 6, we summarize the performance of CoAT with these choices. We vary the scaling factor $\beta$ to demonstrate the trade-off between clean and robust accuracy, and the robust accuracy is evaluated using AutoAttack [10] which provides reliable evaluations. For a fair comparison, we also evaluate “TRADES” with the original KL divergence being replaced with these newly introduced discrepancy functions and illustrate the results in the same plots (i.e., Figure 6(a), 6(b), and 6(c)) correspondingly. The performance curve of the original TRADES is shown in every sub-figure (in grey). See Table 2 for all $\beta$ values in the figure.

From Figure 6(a) to 6(c), one can see that considerably improved trade-off between the clean and robust accuracy is achieved by using our CoAT, in comparison to TRADES using the same discrepancy metric for measuring the gap between probability vectors. Moreover, in Figure 6(d), it can be seen that our CoAT with different choices for the discrepancy functions always outperforms the original TRADES by a large margin, and using the symmetric KL divergence for $\ell_{\text{reg}}$ leads to the best performance overall. We will stick with the symmetric KL divergence for CoAT in our following comparison, and we use $\beta = 6$ for CIFAR-10, $\beta = 4$ for CIFAR-100, and $\beta = 8$ for SVHN.

5.2 Comparison to State-of-the-arts

Table 1 reports the performance of our adversarial training method CoAT and its competitors. Intensive results demonstrate that CoAT outperforms the vanilla AT [25], TRADES [47], and MART [40]...
significantly, gaining consistently higher clean and robust accuracy on CIFAR-10, CIFAR-100, and SVHN. In other words, our CoAT significantly enhances adversarial robustness with less degradation of clean accuracy, indicating better trade-off between clean and robust performance. Specifically, on CIFAR-10, TRADES shows classification accuracy of 82.41% and 49.37% on the clean and adversarial test sets, respectively, while our CoAT with symmetric KL achieves 83.10% and 50.50%. Combining with AWP [42], we further gain an absolute improvement of 1.37% and 0.92% in clean and robust accuracy, respectively, compared to TRADES+AWP, on CIFAR-10. Similar observations can be made on CIFAR-100 and SVHN. Complexity analyses of our method is deferred to Section C in our supplementary material, and training curves are given in Figure 8 to demonstrate less overfitting than that in Figure 4.

In addition to the experiments on ResNet-18, we also employ larger-scale DNNs, i.e., wide ResNet (WRN) [46]. We train robust WRN models, including robust WRN-34-5 and robust WRN-34-10 on CIFAR-10, and we report their results in Table 3. Obviously, the WRN models lead to higher clean and robust accuracy than that of the ResNet-18 models. Importantly, our CoAT still outperforms TRADES on these networks, showing that the effectiveness of our method holds when the size of DNN scales. Another WRN, i.e., WRN-28-10, is also considered and the same observations can be made. We will test with it carefully in Section 5.3, in which additional unlabeled data is utilized during training.

5.3 Training with Unlabeled Data

RST [6] is a recent work that confirms unlabeled data could also be properly incorporated into training for enhancing adversarial robustness. Here we consider a simple and direct combination with it. Recall that, in the RST paper, it extracted 500K unlabeled data from 80 Million Tiny Images [39]. To utilize these unlabeled images, it first generates pseudo labels for them and then performs TRADES on the whole training set including all CIFAR-10 training images and the 500K originally unlabeled data. Our CoAT can easily be incorporated after obtaining the pseudo labels.

We implement RST and our combination with it (called CoAT-RST) by strictly following the training details in the original paper of RST. Table 4 shows that our CoAT-RST gains a significant improvement not only in adversarial robustness but also in clean accuracy, showing better trade-off. We further compare with two recent work, i.e., GAIR-RST [48] and AWP-RST [42]. The evaluation results are collected directly from their papers. One can see that, despite the improvement in robustness, their performance is inferior to ours. Especially for AWP-RST, it improves 0.50% robust accuracy by 1.41% sacrifice on natural accuracy, while CoAT-RST achieves 0.74% and 1.21% improvements on natural and robust accuracy respectively.

6 Conclusion

In this paper, we have studied the loss landscape of DNN models (robust or not), by paying more attention to the valley region of the landscapes where collaborative examples widely exist. We have verified that collaborative examples can be utilized to benefit adversarial robustness. In particular, we have proposed CoAT, a collaborative adversarial training method, to take both adversarial examples and collaborative examples into accounts, jointly and equally, for regularizing the loss landscape during DNN training, forming a novel regularization regime. Extensive experiments have shown that our CoAT outperforms current state-of-the-arts across different benchmark datasets and network architectures, and it can be combined with recent advances (including RST and AWP) to gain further progress in improving adversarial robustness.
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A Performance of Models in Figure 1

Table 4: The clean and robust accuracy of models used in Figure 1, and the robust accuracy is evaluated by AutoAttack.

|                  | CIFAR-10, ResNet-18 | CIFAR-10, WRN-34-10 | CIFAR-100, ResNet-18 | CIFAR-100, WRN-34-10 |
|------------------|----------------------|----------------------|----------------------|----------------------|
|                  | Clean | AA   | Clean | AA   | Clean | AA   | Clean | AA   | Clean | AA   |
| Normal           | 95.09%| 0.00%| 95.28%| 0.00%| 76.33%| 0.00%| 79.42%| 0.00%|
| Vanilla AT       | 82.66%| 47.62%| 86.90%| 48.31%| 57.72%| 24.67%| 61.82%| 25.39%|
| TRADES           | 82.51%| 49.18%| 84.80%| 52.94%| 57.99%| 24.58%| 57.10%| 26.76%|
| MART             | 81.15%| 47.28%| 83.62%| 50.93%| 55.28%| 25.15%| 57.99%| 27.12%|

B Collaborative and Adversarial Directions

Figure 6: The distributions of angles between the PGD adversarial perturbations and collaborative perturbations. At the first update iteration, an collaborative example and its corresponding adversarial examples show opposite directions. However, after more and more update steps, since the gradient is computed w.r.t. different inputs, the adversarial and collaborative directions become less and less correlated and their angle finally lies in a range around $90^\circ$. Interestingly, on a more robust model (TRADES), their correlated is more obvious and less concentrated around $90^\circ$.

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C  Computational Complexity of CoAT

Since the adversarial examples and collaborative examples are both required in CoAT, the computational complexity in its inner optimization increases. Yet, we note that the two sorts of examples can be computed in parallel, thus the run time of our CoAT can be similar or only slightly higher than that of the baseline. Furthermore, the performance of previous state-of-the-arts does not in fact improve with higher computational capacity (e.g., more inner optimization steps). For instance, TRADES show slightly better robust accuracy (AA: 49.76% ± 0.09%) but decreased clean accuracy (81.57% ± 0.14%) on CIFAR-10 with 2× more inner steps, which are both not better than our CoAT (AA: 50.50% ± 0.07%, and clean accuracy: 83.10% ± 0.10%).

D  Training Curves of CoAT

Figure 7: How the average cross-entropy loss changes during training using our CoAT. The experiment is performed with ResNet-18 on CIFAR-10. Shaded areas represent scaled standard deviations. It can be seen that the gap between the collaborative examples and the benign examples and that between the adversarial examples and the benign examples are both obviously reduced, comparing to Figure 5. Best viewed in color.

Figure 8: Changes in clean and robust accuracy during training ResNet-18 on (a) CIFAR-10 and (b) CIFAR-100, using TRADES and our CoAT. The robust accuracy is evaluated using PGD adversarial examples generated over 20 steps with $\epsilon = 8/255$ and a step size of $\alpha = 2/255$. Best viewed in color.

Figure 9: Changes in clean and robust accuracy during training WRN-28-10 on CIFAR-10, using RST and CoAT-RST. By following the original paper of RST, the robust accuracy is evaluated using PGD adversarial examples generated over 20 steps with $\epsilon = 0.031$ and a step size of $\alpha = 0.007$ on the first 500 images in the test set. Although we only train 200 epochs as suggested in the RST paper for comparison in Table 4 in our main paper, it can be seen that more training epochs can still be beneficial to our method. Best viewed in color.
E To Make “Adversarial” and “Collaborative” Clear

In this paper, we generalize the definition of adversarial examples to include all data points (in the bounded neighborhood of benign examples) showing considerable higher prediction loss than that of the benign loss, in contrast to the definition in a narrower sense saying that different label prediction ought to be made. The collaborative examples are similarly “defined”, somewhat non-rigorously. Throughout the paper, we use the word “adversarial” and “collaborative” to indicate perturbed data points with considerable higher and lower loss than the benign loss, respectively.