Simulation of Multiple Scattering of THz Wave Propagation in Sandstorm

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ABSTRACT. Terahertz wave has important application prospects in communication, but its transmission in the atmosphere will be affected by the atmospheric environment. Based on Mie scattering theory and Monte Carlo method, the attenuation characteristics of THz wave multiple scattering caused by dust storms in Tengger desert are studied in this paper. The relationship between the attenuation rate and visibility and frequency of THz wave are obtained by calculation. The results show that the multiple scattering of THz wave in sandstorm is very significant. There is a big error in the calculation of attenuation rate based on single scattering. The difference between the two methods decreases with the increase of visibility. The attenuation rate of THz wave varies with the frequency in the sandstorm. When the frequency of THz wave is less than 2.4 THz, its attenuation rate increases with the increase of frequency, and when the frequency is more than 2.8 THz, its attenuation rate decreases with the increase of frequency.

1. INTRODUCTION

THz wave refers to the electromagnetic wave whose frequency is in the 0.1-10 THz band, which lies between microwave and infrared in the electromagnetic spectrum and in the transitional region from electronics to photonics. In terms of communication transmission, THz wave has the characteristics of high resolution, large capacity, good direction, confidentiality and anti-jamming ability [1-2]. This makes THz wave more advantageous than traditional microwave and light wave, and determines that THz wave has a broad application in secure communication and space communication. The research on the atmospheric transmission characteristics of THz wave is the basis of the application of THz wave space communication, which has important academic significance and practical value.

Sandstorm refers to the weather phenomenon in which strong winds sweep a large amount of dust and sand into the air, making the air particularly turbid and the horizontal visibility less than 1 km. The absorption and scattering of electromagnetic waves by suspended sand particles in the air caused by dust storms will not only cause serious attenuation of signals, but also easily lead to signal phase fluctuation, waveform distortion, depolarization and cross-polarization. Therefore, dust storms have a
great impact on electromagnetic wave propagation. Single scattering method is commonly used to calculate the propagation attenuation of electromagnetic radiation in random media. Although this method is simple, sometimes the calculation results have large errors. It has been pointed out that when the visibility is high, the signal attenuation can be calculated based on single scattering, but for the sandstorm weather with low visibility, the number density of sand particles is large and the phenomenon of multiple scattering is significant, so the influence on the attenuation results can’t be ignored\cite{3}. For the radiation transfer problem with multiple scattering, many research methods have been proposed, such as successive iteration method, discrete coordinate method and Monte Carlo statistical method. Monte Carlo method\cite{4-6} is based on probability model, and it uses random number and probability statistics to study the propagation of light in random medium. Monte Carlo method can better reveal the phenomenon of multiple scattering of particles, especially when the concentration of particles is relatively high, the effect of multiple scattering is more prominent. Using Monte Carlo method, as long as the number of photons is enough, the problem of multiple scattering of particles can be simulated accurately.

In this paper, based on Mie scattering theory and Monte Carlo method, the attenuation characteristics of THz wave caused by multiple scattering in sandstorms are calculated and analyzed.

2. SINGLE SCATTERING OF SAND PARTICLES

2.1 Efficiency Factor of Single Scattering

Mie scattering theory is the exact solution of plane electromagnetic wave scattering by an isotropic homogeneous sphere. When studying the scattering characteristics of electromagnetic wave signals by dust storms, the sand particles can be regarded as spherical particles and analyzed and calculated by Mie theory. According to Mie scattering theory, the extinction efficiency factor, scattering efficiency factor and absorption efficiency factor of a single particle for incident wave are respectively:

\[
Q_e = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(2n+1)}{(2n+1)} |\text{Re}(a_n + b_n)|
\]

\[
Q_s = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(2n+1)}{(2n+1)} (|a_n|^2 + |b_n|^2)
\]

\[
Q_a = Q_e - Q_s
\]

Where, \(x=2\pi r/\lambda\), \(r\) is the radius of the particle, \(\lambda\) is the wavelength, and \(a_n, b_n\) are the Mie scattering coefficients.

The calculation of efficiency factor involves the complex refractive index of sand grains, which is determined by the complex dielectric coefficient. The complex dielectric coefficient of sand varies with water content and incident wave frequency. In fact, the complex dielectric coefficient of dry sand is independent of frequency (\(\varepsilon'_s=3\)), and the relationship between imaginary part and electromagnetic wave frequency is\cite{7}:

\[
\varepsilon'_s = \begin{cases} 
1.8 \times 10^{12} f^{-2} & \text{if} \quad 0.8 \text{GHz} \leq f \leq 80 \text{GHz} \\
18.256 / f & \text{if} \quad f \geq 80 \text{GHz}
\end{cases}
\]

So, the complex refractive index of dry sand is:

\[
m = (3 + i \varepsilon'_s)^{1/2}
\]

Taking THz wave frequency \(f = 1\) THz, the relationship between efficiency factor and particle size \(D\) is calculated as shown in Figure 1. It can be seen that the scattering effect of sand particles on THz wave is remarkable, the absorption effect is weak, and the extinction is mainly caused by scattering. When the diameter of sand particles is small (less than 5\(\mu\)m), the scattering of THz wave is weak. With the increase of sand particle diameter, the scattering effect increases obviously.
Figure 1. Relation between the efficiency factor and sand particle diameter

Figure 2. Relation between extinction efficiency factors of different THz waves and sand particle diameter

Figure 2 is the relationship between extinction efficiency factor of THz wave with different frequencies and sand particle diameter. It can be seen that the higher the frequency of THz wave is, the faster the extinction of THz wave increases with the size of sand grains, and the change is oscillatory. The extinction of 0.5 THz with lower frequency increases most slowly with the increase of sand size, and there is no oscillating change.

2.2 Attenuation Due to Single Scattering

According to Lamber-Beer law and Mie scattering theory, the attenuation rate of electromagnetic radiation per unit distance in discrete random medium is [8-9]:

\[
A = 4.434 \times 10^3 \int \pi r^2 Q_e(r)N(r)dr
\]

Where, \(N(r)\) is the particle size distribution function. For dust particles, lognormal distribution function is often used to describe their scale distribution [10]:

\[
N(r) = \frac{N_0}{2r\sigma\sqrt{2\pi}} \exp\left(-\frac{[\ln(2r) - m_0]^2}{2\sigma^2}\right)
\]

Where, \(m_0\) and \(\sigma\) are the mean and standard deviation of ln(2r) respectively, and \(N_0\) is the density of particles. Because \(N_0\) is a difficult physical quantity to measure, scholars usually use optical visibility \(V_b\) to express the particle number density of sandstorms. The relationship is:

\[
N_0 = \frac{15}{4.343 \times 10^3 V_b \int 2\pi^2 p(r)dr}
\]
Where, \( p(r) = N(r)/N_r \) is the probability density function of particle size distribution. By substituting formulas (7) and (8) into formulas (6), the attenuation rates of electromagnetic waves in sandstorms can be obtained as follows:

\[
A = \frac{15\pi}{2\pi} \int_0^2 r^2 p(r) dr
\]

The attenuation rate formula (9) is based on the single scattering theory, which can describe the scattering attenuation characteristics of THz signals under certain conditions. However, in sandstorm weather conditions, the dust particles in the atmosphere are relatively dense and visibility is low. In this case, single scattering is difficult to truly reflect the actual scattering attenuation, so it is necessary to consider the effect of multiple scattering.

3. ATTENUATION DUE TO MULTIPLE SCATTERING OF SAND PARTICLES

The transmission of THz waves in sandstorms inevitably involves multiple scattering. The Monte Carlo method has unique advantages in calculating the multiple scattering of electromagnetic radiation in random media. The basic idea of Monte Carlo method is to treat electromagnetic radiation as a photon beam composed of many photons, and the transmission of electromagnetic radiation is transformed into the transmission of photons. The transmission process of each photon in a random medium is determined by the random number generated by the computer. By sampling the scattering direction and traveling path of the photon, the photon is tracked until the photon weight is less than the threshold, or the photon leaves the medium or reaches the receiving interface. By tracking a large number of photons, and finally counting the photons arriving at the receiving interface, we can obtain stable transmittance estimation.

Let terahertz wave shoot vertically into dust storm with thickness \( H \) along \( z \) direction. The random step of photon motion between two adjacent collisions is determined by the cumulative probability distribution of mean free path.

\[
L = -\frac{1}{\mu_\varepsilon} \ln \xi
\]

Where \( \xi \) denotes the random number uniformly distributed on (0,1), \( \mu_\varepsilon \) is the attenuation coefficient determined by the following formula:

\[
\mu_\varepsilon = \int_0^\infty \pi r^2 Q_s(r) N(r) dr
\]

After the collision of photons with sand particles at a certain point, the new direction of motion is determined by the H-G scattering phase function. According to the H-G scattering phase function, the sampling value of scattering angle \( \theta \) can be obtained.

\[
\theta = \cos^{-1}\left[\frac{\frac{1}{2g} - (1 + g)^2}{\frac{1}{2g} - (1 - g/2g)^2}\right]
\]

Where, \( g \) is the average asymmetric factor:

\[
g = \frac{\int_0^\infty \pi r^2 Q_s(r) N(r) g(r) dr}{\int_0^\infty \pi r^2 Q_s(r) N(r) dr}
\]

The azimuth \( \varphi \) of photon scattering can be considered to be uniformly distributed in (0, \( 2\pi \)), and its sampling value is:

\[
\varphi = 2\pi \xi
\]

Let \( (\mu_\varepsilon, \mu_r, \mu_\varphi) \) be the direction cosine before the collision between photons and sand particles. The new direction cosine of photons after collision can be obtained by coordinate transformation.
\[
\mu' = \frac{\sin \theta}{\sqrt{1-\mu^2}} (\mu_x \cos \varphi - \mu_y \sin \varphi) + \mu_x \cos \theta \\
\mu' = \frac{\sin \theta}{\sqrt{1-\mu^2}} (\mu_x \cos \varphi + \mu_y \sin \varphi) + \mu_x \cos \theta \\
\mu' = -\sin \theta \cos \varphi \sqrt{1-\mu^2} + \mu_x \cos \theta
\]

According to the idea of weighted Monte Carlo, the initial weight \( W \) of a photon is assigned to 1. Every time the photon collides with a sand particle, the energy of the photon will be absorbed, and its weight will be reduced and updated to:

\[
W' = W \cdot (1 - \frac{\mu' x}{\mu_x})
\]

Where, \( \mu_x \) is the absorption coefficient and its value is determined by the frequency and the size distribution function of sand particles. The photon travels a long \( L \) in the dust storm and reaches the next collision point. Its coordinates are as follows:

\[
x' = x + \mu_x L_x, \quad y' = y + \mu_y L_y, \quad z' = z + \mu_z L_z
\]

If \( z' < 0 \) or \( z' > L \), it means that the photon escapes from the dust storm and the tracking process ends. If the weight factor of the photon is less than the set threshold \( 10^{-4} \), it means that the photon is absorbed and the tracking process ends. If the photon is neither absorbed nor escaped, repeat the above steps until the photon reaches the receiving interface.

Tracking \( N \) photons \( (N=10^6) \), if \( n \) photons arrive at the receiving interface, the average transmittance can be estimated by counting the weight of the \( n \) photons.

\[
T = \frac{1}{N} \sum_{i=1}^{n} W_i
\]

The attenuation rate of THz wave transmission unit distance in sandstorm is (dB/km):

\[
A = -\frac{10}{H} \log T
\]

4. COMPUTATIONAL RESULTS

There are differences in the causes of sand occurrence and the scale distribution of sand grains in different areas. This paper takes the sandstorm in Tengger sandy area as an example to calculate. The parameters in formula (7) are \( m_0 = -2.31, \sigma = 0.296 \) \cite{10}, and the frequencies of THz waves are 0.5, 1, 3 and 5 THz, respectively. The attenuation rates with visibility are calculated based on formula (9) and (19), respectively. The results are shown in Figure 3.
Figure 3. Relation between attenuation rate and visibility of THz wave in Sandstorm

From Figure 3, it can be seen that the attenuation rate of THz wave in sandstorms decreases with the increase of visibility whether calculated by single scattering or multiple scattering based on MC method. When the visibility is the same, the attenuation rate varies with the frequency of THz wave. In addition, the attenuation of single scattering and multiple scattering of THz wave caused by sandstorms is obviously different. The attenuation of multiple scattering calculated by MC method is less than that caused by single scattering, which indicates that the phenomenon of multiple scattering of THz wave in sandstorms is very significant, and there is a big error in the calculation of attenuation rate only based on single scattering. The difference between the two methods decreases with the
increase of visibility, because the larger the visibility, the smaller the sand concentration, the less obvious the multiple scattering phenomenon.

Figure 4 is a comparison of attenuation rates of four frequency THz waves in sandstorms based on MC method. Under the same visibility, the attenuation of 0.5THz wave is the smallest, and that of 3THz wave is the largest.

Figure 5 is the relationship between attenuation rate and frequency of THz wave propagation in dust storms of Tengger area based on MC method. It can be seen that when the frequency of THz wave is less than 2.4 THz, the attenuation rate of Sandstorm to THz wave increases with the increase of frequency, and the attenuation rate of Sandstorm to THz wave between 2.4 THz and 2.8 THz is the strongest; when the frequency of THz wave is more than 2.8 THz, the attenuation rate decreases gradually with the increase of frequency.

Figure 4. Comparison of transmission attenuation rates of different THz waves in sandstorms

Figure 5. Relation between transmission attenuation rate and frequency of THz wave in Sandstorm

5. CONCLUSION
The scattering of THz waves by suspended dust particles in the air caused by dust storms can cause serious attenuation of signals. When dust storms are formed, the visibility is relatively low (less than 1 km), and the density of dust particles in the air is relatively high. The THz wave signal will produce multiple scattering phenomenon when it is transmitted in dust storms. The influence of multiple scattering phenomenon should be taken into account when calculating signal attenuation. Based on Monte Carlo method, the attenuation rate of THz wave caused by multiple scattering is analyzed and calculated in this paper. The results show that the attenuation of THz wave in Sandstorm decreases with the increase of visibility. The attenuation rate of multiple scattering calculated by MC method is less than that caused by single scattering. Attenuation rate is also closely related to the frequency of THz wave signal. Sandstorms in Tengger area attenuate the THz wave with the frequency between 2.4 THz-2.8 THz most. The attenuation of THz wave by sandstorms in different areas may be different.
due to the difference of the causes of sanding and the scale distribution of sand grains in different areas.

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