Quark Propagation in a Quark-Gluon Plasma with Gluon Condensate

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Abstract

We present a calculation of the thermal quark propagator taking the gluon condensate above the critical temperature into account. The quark dispersion relation following from this propagator, describing two massive modes, is discussed.

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Finite temperature QCD is used to describe the properties of a quark-gluon plasma (QGP), which is supposed to exist in the early stage of the Universe and in the fireball of a relativistic heavy ion collision. Besides lattice QCD also perturbation theory has been applied to determine phenomenologically relevant properties of this state. In contrast to lattice calculations the latter method is able to deal also with dynamical properties, a finite baryon density and non-equilibrium situations. As an important example the dispersion relations for quarks and gluons in a quark-gluon plasma have been determined in this way [1–3]. However, using only bare propagators gauge dependent and infrared divergent results have been found in many cases. In order to avoid these problems (at least partially), Hard Thermal Loop (HTL) resummed propagators and vertices have to be used within an effective perturbation theory [4,5]. However, these perturbative methods could be reliable at best for temperatures far above the critical temperature, where the temperature dependent running coupling constant becomes small.

In the present letter we investigate the consequences of a non-perturbative quantity, namely the gluon condensate measured in lattice QCD above the critical temperature \( T_c \) [6], on the quark propagator and the quark dispersion relation. Our approach is similar to the QCD sumrule approach to hadron physics at zero temperature. We will combine purely non-perturbative input from lattice QCD as parametrized by temperature-dependent condensates and purely perturbative results as obtained e.g. in HTL calculations. To do this consistently, i.e. to avoid double counting, we have to subtract all perturbative parts in our calculation involving the condensates.

The gluon condensate above \( T_c \) is given by the difference of the zero temperature condensate and the interaction measure \( \Delta = (\epsilon - 3p)/T^4 \), where \( \epsilon \) is the energy density and \( p \) the pressure of the QGP [7,8].

\[
\langle G^2 \rangle_T = \langle G^2 \rangle_{T=0} - \Delta T^4, \tag{1}
\]

where \( \langle G^2 \rangle_T = (11\alpha_s)/(8\pi) \langle G_{\mu\nu}^a \rangle^2 \) and \( \langle G^2 \rangle_{T=0} = (2.5 \pm 1.0) T_c^4 \) [8].

To lowest order the interaction of a quark with the gluon condensate is given by the
self energy diagram shown in Fig.1. For massless bare quarks it reads at finite temperature using the imaginary time formalism and the notation $P \equiv (p_0, \mathbf{p})$, $p \equiv |\mathbf{p}|$

$$\Sigma(P) = \frac{4}{3} i g^2 \int_{k_0 = 2\pi/nT} d^3k \frac{d^3k}{(2\pi)^3} \tilde{D}_{\mu\nu}(K) \gamma^\nu \frac{P - K}{(P - K)^2} \gamma^\mu,$$  \hspace{1cm} (2)

where $\tilde{D}_{\mu\nu} = D_{\mu\nu}^{full} - D_{\mu\nu}^{pert}$ is the non-perturbative gluon propagator containing the gluon condensate. The perturbative gluon propagator has been subtracted since we are not interested in the contribution to the quark self energy coming from the bare gluon propagator, which is not related to the gluon condensate but is contained in the HTL part. The diagram of Fig.1 together with the subtracted gluon propagator $\tilde{D}_{\mu\nu}$ has been used already in a zero temperature calculation [9]. Owing to the subtraction of the perturbative propagator, $\tilde{D}_{\mu\nu}$ is gauge independent.

The most general expression for the fermion self energy (in the case of a vanishing bare fermion mass) in the rest frame of the heat bath is given by [3]

$$\Sigma(P) = -a(p_0, \mathbf{p}) P - b(p_0, \mathbf{p}) \gamma_0$$  \hspace{1cm} (3)

with the scalar functions

$$a = \frac{1}{4p^2} \left[ tr(P\Sigma) - p_0 tr(\gamma_0 \Sigma) \right],$$

$$b = \frac{1}{4p^2} \left[ p^2 tr(\gamma_0 \Sigma) - p_0 tr(P\Sigma) \right].$$  \hspace{1cm} (4)

The most general ansatz for the non-perturbative gluon propagator, reads [11]

$$\tilde{D}_{\mu\nu}(K) = \tilde{D}_L(k_0, k) P_{\mu\nu}^L + \tilde{D}_T(k_0, k) P_{\mu\nu}^T,$$  \hspace{1cm} (5)

where the longitudinal and transverse projectors are given by

$$P_{\mu\nu}^L = \frac{K_\mu K_\nu}{K^2} - g_{\mu\nu} - P_{\mu\nu}^T,$$

$$P_{\mu\nu}^T = 0, \quad P_{\mu\nu}^T = \delta_{ij} \frac{k_i k_j}{k^2}.$$  \hspace{1cm} (6)

In order to relate the propagator (5) to the gluon condensate we follow the zero temperature calculation [8] and expand the quark propagator in (2) for small loop momenta $K$.  

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Keeping only terms which are bilinear in $K$, we can relate the gluon condensate to moments of the gluon propagator \[1\]. This approach, known as plane wave method, has been widely used in QCD sum rules calculations \[11\]. At zero temperature it reproduces the well known results for the quark self energy as discussed in \[9\]. Here we assume that it can also be applied to the finite temperature case.

At finite temperature, where $k_0$ and $k$ have to be treated separately, new contributions containing bilinear combinations of $k_0$ and $k$ appear, which cannot be related to the gluon condensate. However, since $|k_0| \ll p$ as a consequence of the plane wave method and since $T > T_c$, it should be a good approximation to limit ourselves to the lowest Matsubara mode $k_0 = 2\pi i n T = 0$ as long as $p$ is not much larger than $T_c$. This approximation is a crucial step as it is necessary to relate at finite temperature the non-perturbative gluon propagator (5) to the condensates (8).

Then expanding the quark propagator for small three momenta $k \ll p$ leads to

\[
\begin{align*}
    a &= -\frac{4}{3} g^2 \frac{1}{p_0} T \int \frac{d^3 k}{(2\pi)^3} \left[ \left( \frac{1}{3} p^2 - \frac{5}{3} p_0^2 \right) k^2 \tilde{D}_L(0, k) + \left( \frac{2}{5} p^2 - 2p_0^2 \right) k^2 \tilde{D}_T(0, k) \right], \\
    b &= -\frac{4}{3} g^2 \frac{p_0}{p^6} T \int \frac{d^3 k}{(2\pi)^3} \left[ \frac{8}{3} p_0^2 k^2 \tilde{D}_L(0, k) - \frac{16}{15} p_0^2 k^2 \tilde{D}_T(0, k) \right].
\end{align*}
\]  

(7)

The moments of the longitudinal and transverse gluon propagator in (7) can be expressed by the chromoelectric and chromomagnetic condensates similarly as in \[9\]:

\[
\begin{align*}
    \langle E^2 \rangle_T &= 8T \int \frac{d^3 k}{(2\pi)^3} k^2 \tilde{D}_L(0, k) + O(g), \\
    \langle B^2 \rangle_T &= -16T \int \frac{d^3 k}{(2\pi)^3} k^2 \tilde{D}_T(0, k) + O(g).
\end{align*}
\]  

(8)

Keeping terms proportional to $k_0^2$ in (9), taking the zero temperature limit, and replacing $iT \int d^3 k/(2\pi)^3 \rightarrow \int d^4 K/(2\pi)^4$ and $\tilde{D}_L(k_0, k) = \tilde{D}_T(k_0, k) = -\tilde{D}(K^2)$, (7) and (8) reproduce the zero temperature results \[9\]: $a = -g^2 \langle G_{\mu \nu}^a \rangle_{T=0}/(36 P^4)$ and $b = 0$.

The condensates defined in Minkowski space are related to the space and timelike plaquette expectation values, $\Delta_\sigma$ and $\Delta_\tau$, measured on a lattice in the case of a pure SU(3) gauge theory \[8\], by
\[
\frac{\alpha_s}{\pi} \langle E^2 \rangle_T = \frac{4}{11} T^4 \Delta_\tau - \frac{2}{11} \langle G^2 \rangle_{T=0}, \]
\[
\frac{\alpha_s}{\pi} \langle B^2 \rangle_T = -\frac{4}{11} T^4 \Delta_\sigma + \frac{2}{11} \langle G^2 \rangle_{T=0}.
\]

The plaquette expectation values are related to the interaction measure \( \Delta \) of (1) by \( \Delta = \Delta_\sigma + \Delta_\tau \).

Using the results found for \( \Delta_{\sigma,\tau} \) in Ref. [8] we find that the electric condensate \((\alpha_s/\pi) \langle E^2 \rangle_T\) increases like about \( T^{3.5} \) between \( T=1.1T_c \) and \( 4T_c \), whereas the magnetic condensate \((\alpha_s/\pi) \langle B^2 \rangle_T\) is close to zero up to \( T=2T_c \) and increases strongly afterwards. These results differ from previous lattice calculations, where approximately temperature independent electric and magnetic condensates of equal size have been found above and close to \( T_c \) [12].

In the perturbative regime \((g \to 0)\) \( \Delta_\tau = -\Delta_\sigma = 11g^2/30 + O(g^4) \) holds [8]. This result corresponds to the Stefan-Boltzmann limit: \( \epsilon_{SB} = (\langle E^2 \rangle_{SB} + \langle B^2 \rangle_{SB})/2 = (2\pi/11\alpha_s)(\Delta_\tau - \Delta_\sigma)T^4 = (8\pi^2/15)T^4 \). In order to obtain the condensate in the perturbative regime, the Stefan-Boltzmann contribution has to be subtracted from (9) in the limit \( g \to 0 \) [12]. Then the condensates are of order \( g^4T^4 \), describing perturbative corrections to the ideal gas limit. It should be noted that the subtraction of the Stefan-Boltzmann contribution, defined in the limit \( g \to 0 \), does not change the results for the condensates near \( T_c \) since there the plaquette expectation values are non-perturbative and cannot be written as being proportional to \( g^2 \) plus higher orders. Subtracting the Stefan-Boltzmann contribution of \( \Delta_{\sigma,\tau} \) at a finite value of \( g \) would lead to an explicit \( g \)-dependence of the condensates in (9).

The quark propagator containing the gluon condensate follows from the self energy as \( S(P) = 1/(P - \Sigma) \). The analogous calculation for \( T = 0 \), which was performed in [4], leads to a gauge dependent quark propagator. The gauge dependent term was found to be proportional to the quark condensate. As the latter vanishes in our case \((T > T_c)\) due to chiral symmetry restoration we end up with a gauge independent result.

Decomposing this propagator according to its helicity eigenstates it can be written as [13]...
\[ S(P) = \frac{\gamma_0 - \hat{P} \cdot \gamma}{2D_+(P)} + \frac{\gamma_0 + \hat{P} \cdot \gamma}{2D_-(P)}, \quad (10) \]

where

\[ D_\pm(P) = (-p_0 \pm p)(1 + a) - b. \quad (11) \]

The dispersion relation of a quark interacting with the thermal gluon condensate in a QGP is given by the roots of \( D_\pm(P) \). Combining (7), (8), and (9) the dispersion relations shown in Fig.2 for \( T = 1.1T_\text{c} \), for \( T = 2T_\text{c} \), and for \( T = 4T_\text{c} \) are found. It should be noted that these dispersion relations follow from lattice results in the case of a pure gauge theory. Unfortunately lattice calculations with dynamical quarks for the thermal gluon condensate are not reliable so far [14]. However, our results for the dispersion relations are insensitive to small variations in the values of the condensates.

As shown in Fig.2 there are two real positive solutions of \( D_\pm(P) = 0 \), where the upper curve \( p_0^+ \) corresponds to solutions of \( D_+(P) = 0 \) and the lower curve \( p_0^- \) to \( D_-(P) = 0 \). Similar as in the case of the dispersion relation following from the HTL resummed quark propagator [13] the dispersion relation \( p_0^- \) describes the propagation of a quark mode with a negative helicity to chirality ratio (plasmino) which is absent in the vacuum. As in the HTL case the plasmino branch shows a minimum leading to Van Hove singularities in the soft dilepton production rate [13].

The plasmino mode rapidly approaches the free dispersion for increasing momenta, indicating that it is a purely collective long wave-length mode as in the case of the HTL dispersion [13]. (The residue of this pole being proportional to \( (p_0^2 - p^2)^3 \) for large momenta, vanishes for \( p \gg T \).) The \( p_0^+ \)-mode, on the other hand, is given by \( p_0^+ = p + c_1 \) for large \( p \gg T \), where \( c_1 = [(2\pi/9)\alpha_s(\langle E^2 \rangle_T + \langle B^2 \rangle_T/5)]^{1/4} \).

Both branches are situated above the free dispersion relation \( p_0 = p \) and start from a common effective quark mass

\[ m_{\text{eff}} = \left[ \frac{2\pi^2}{3} \frac{\alpha_s}{\pi} \left(\langle E^2 \rangle_T + \langle B^2 \rangle_T/5\right) \right]^{1/4}. \quad (12) \]
This effective mass is given by $m_{\text{eff}} = 1.15T$ between $T = 1.1T_c$ and $4T_c$. Thus it is independent of $g$. For small momenta $p \to 0$ the dispersion relation behaves like $p^\pm = m_{\text{eff}} \pm c_2 p$, where $c_2 = (3/4) \langle E^2 \rangle_T / (\langle E^2 \rangle_T + \langle B^2 \rangle_T)$.

For high temperatures the effective mass is proportional to $gT$ since the condensates in $(9)$ are of order $g^4T^4$ once that the Stefan-Boltzmann contribution has been subtracted. Therefore it is of the same order as the HTL quark mass $m_{\text{HTL}} = gT/\sqrt{6}$. Although this contribution to the effective quark mass is a perturbative correction, we do not expect it to be identical to the HTL result. For the loop momentum of the quark self energy using the HTL approximations is hard ($k \gg p$), whereas it is soft ($k \ll p$) in our case.

Let us note here that the HTL mass has been used in related studies considering quarks and gluons in the QGP as quasiparticles in order to explain the lattice data for the interaction measure $\Delta$.\cite{16}

As a possible application the quark dispersion and the effective quark propagator containing the gluon condensate might be used to determine the dilepton and photon production from a QGP. To lowest order the dilepton production rate is derived from the Born term (quark-antiquark annihilation), where the quark dispersion could be used for the quark-antiquark pair. The dilepton production taking into account a finite temperature gluon condensate above $T_c$ has been discussed recently in Ref.\cite{17}. In contrast to the HTL propagator our effective propagator has no imaginary part below the light cone. Therefore there is no contribution of the photon self energy to the photon production rate at the one-loop level using an effective quark propagator as it is the case within the HTL resummation scheme\cite{18}. However, the effective propagator could be used in the tree level calculation of the photon rate, i.e., for the Compton scattering and quark-antiquark annihilation (with gluon emission) diagrams, together with the dispersion relation for the external quarks.

Finally we want to comment on the possibility of deriving a gluon propagator containing a gluon condensate at finite temperature. This would be of great interest as the non-perturbative gluon propagator might show a static magnetic screening, which is not present in perturbative calculations leading to infrared divergences even when HTL resummed prop-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Example figure caption.}
\end{figure}
agators are used (see e.g. [4]). Unfortunately, the gluon propagator containing a gluon condensate is a very complicated object already at zero temperature [19]. In order to preserve the Slavnov-Taylor identities, i.e. the transversality of the gluon self energy containing the gluon condensate, higher order condensates have to be included. These condensates, however, are not known at finite temperature. Furthermore this gluon propagator exhibits a dependence on the gauge parameter rendering a physical interpretation of its poles doubtful.

Summarizing, we have shown that the presence of a gluon condensate above $T_c$, as suggested by lattice calculations, leads to an interesting quark dispersion relation, which exhibits a large similarity to the dispersion resulting from the HTL quark propagator. In both cases two massive branches are found, where the plasmino mode shows a minimum at a finite value of the momentum. For large momenta all branches approach the free dispersion relation. However, our effective quark mass differs from the HTL mass, which is proportional to $gT$. For temperatures close to $T_c$ it is proportional to $T$, while it is proportional to $gT$ in the perturbative regime.

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FIG. 1. Quark self energy containing a gluon condensate.
FIG. 2. Quark dispersion relations at \( T = 1.1 T_c \) (a), \( T = 2T_c \) (b), \( T = 4T_c \) (c) and dispersion relation of a non-interacting massless quark (dashed lines).