Formation Control for Multiple Connected and Automated Vehicles on Multi-lane Roads

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Abstract—Coordinated decision making and control can improve traffic efficiency while guaranteeing driving safety. This paper proposes a formation control method for multiple Connected and Automated Vehicles (CAVs) on multi-lane roads. A bi-level planning framework is proposed to smoothly and effectively switch the structure of the formation in different scenarios. The relative coordinate system is established and the conflict-free relative paths are planned in the upper level. Multi-stage trajectory planning and tracking are performed in the lower level. Case study is conducted to verify the function of the proposed method and simulation in the lane-drop bottleneck scenario is carried out under different traffic volume. Numerical results indicate that the proposed method can improve traffic efficiency at high traffic volume.

Index Terms—Formation Control, Connected and Automated Vehicles, Multi-lane Roads

I. INTRODUCTION

Coordinated control for multiple Connected and Automated Vehicles (CAVs) has been proved to be able to improve driving safety of single vehicles and traffic efficiency of the whole traffic system than single-vehicle automated control in multiple traffic scenarios [1]–[3]. Most of the existing multi-vehicle coordinated control methods focus on single-lane traffic scenarios where only the longitudinal behavior of vehicles is considered [4], [5]. Less research looks into the coordination of multiple vehicles on multi-lane traffic scenarios.

Lane assignment, on-ramp merging and lane-bottleneck merging are the three main research topics in the field of multi-lane coordination. Lane assignment methods aim to assign proper lanes to vehicles based on the lane capacity and occupancy of the highway system [6], [7]. It helps distribute vehicles spatially on the drivable lanes to prevent or reduce traffic congestion. These methods provide optimal lane occupation suggestion for vehicles but not actually control them, which limits the improvement of traffic efficiency. On-ramp and lane-bottleneck merging control methods aim to resolve conflict for vehicles driving on different lanes. The difference of these two topics is that vehicles are allowed to change lanes at any collision-free points in lane-bottleneck scenarios but have to wait for passing the merging point on-ramps.

In the existing research, priority is usually designed or calculated and vehicles are controlled to drive according to the passing sequence [8], [9]. The aforementioned methods focus on solving the problem of traffic congestion within local scenarios. However, there still lacks a multi-vehicle coordination framework which can provide driving strategies for vehicles to adapt to multiple traffic scenarios effectively and smoothly.

Formation control (FC), also called as convoy control, considers multiple CAVs as a group and plans the movement of vehicles from the overall perspective. The main difference between FC methods and the above on-ramp merging and lane-bottleneck coordination methods is that the FC methods focus on the global coordination of vehicles. Vehicles drive as a whole and switch the formation structure according to the changing driving environment, which enables the FC methods to be applied to multiple scenarios. The idea of multi-lane formation of vehicles is firstly proposed in [10]. The spatial distribution of vehicles in the formation is designed and the formation is able to coordinatedly change shape to pass lane-bottlenecks smoothly. The distributed graph-based convoy control method for group of vehicles to maintain the formed formation is proposed in [11], where distributed communication topology of vehicles is defined and the formation controller is designed. The idea of combining formation control and task assignment for CAVs is firstly proposed in [12]. Vehicles are assigned to generated targets in real time when the formation structure is changed according to the driving environment. This enables the formation to dynamically change its structure and improve its scenario adaptability, e.g., the FC method is proved to be able to improve traffic efficiency in multi-lane intersections [13]. The shortcomings of the existing FC methods include that: (1) the formation switching process is predefined and the explanation of how the formation reacts to different scenarios is lacked; (2) the collisionlessness of vehicles in formation switching process has not been considered and explained in detail.

This paper proposes a multi-lane FC method for multiple CAVs to improve the overall traffic efficiency and guarantee the smoothness of formation switching process between different scenarios. The main contributions of this paper are as follows:

- A bi-level motion planning framework is proposed to guarantee safety and smoothness of FC for CAVs. In the upper level, conflict-free relative path planning for vehicles is performed in relative coordinate system. In the lower level, vehicles are controlled to track the tra-
Section VI gives the conclusion of this paper.

and Section IV provide the upper-level motion planning establishes the formation relative coordinate system. Section III defines the bi-level motion planning framework and establishes the formation relative coordinate system. The upper-level motion planning framework is proposed in this paper. The upper-level planner performs trajectory planning for the vehicle to travel through tracked trajectories to pass the key points from the upper level.

A conflict-free relative path planning algorithm is proposed in relative coordinate system, where motion conflicts of vehicles can be described more clearly. The conflict types of vehicles moving in formation are defined and potential collision is avoided by iterative updating assignment result.

The rest of this paper is organized as follows. Section II defines the bi-level motion planning framework and establishes the formation relative coordinate system. Section III and Section IV provide the upper-level motion planning and lower-level trajectory planning methods respectively. Section V carries out the simulation and provides the results. Section VI gives the conclusion of this paper.

II. BI-LEVEL MOTION PLANNING FRAMEWORK AND THE RELATIVE COORDINATE SYSTEM

A. Bi-level Motion Planning Framework

In order to guarantee collisionlessness of vehicles and formation switching smoothness, a bi-level motion planning framework is proposed in this paper. The upper-level planner firstly calculates collision-free paths for vehicles. The paths consist of several key points which can guide vehicles to avoid collision with others and switch the formation to the desired structure. In the lower level, vehicles generate and track trajectories to pass the key points from the upper level. Fig. 1 shows the working process of the bi-level motion planning framework.

The conflicts and potential collisions of vehicles in the Geodetic Coordinate System (GCS) is hard to handle because of the nonlinear vehicle model. However, if we change the perspective from GCS to the moving vehicles, their relative state and motion can be described much more easily. Thus, we establish the Relative Coordinate System (RCS) to describe and regulate vehicles’ relative motion and the vehicles can be treated as free-moving robots. The established RCS is shown in the ‘Relative state’ in Fig. 1.

B. Setup of RCS

First of all, the perspective is transformed from GCS to the moving vehicles. According to the established relative axes, the most forward vehicle has the smallest X coordinate. The Y axis is set to pass through the center of rear axle of the most forward vehicle. The X coordinate \( x^r \) of a vehicle represents the relative longitudinal distance \( d^r_{i,0} \) between the vehicle and the most forward vehicle. The relative X coordinate of vehicle \( i \), noted as \( x^r_i \), can be expressed as:

\[
x^r_i = d^r_{i,0} = \frac{d^0_{i,0}}{d_g},
\]

where \( d^0_{i,0} \) represents the longitudinal Euclidean distance between vehicle \( i \) and the most forward vehicle in GCS, and \( d_g \) represents the safe following gap between the heads of two adjacent vehicles in the same lane.

The relative Y coordinate \( y^r \) of a vehicle in RCS represents the ID of the lane that the vehicle occupies. The ordered pair \((x^r, y^r)\) represents the relative position of a vehicle in a formation, which is called as the Formation Relative Coordinate (FRC). The points in RCS whose X coordinate \( x^r \) and Y coordinate \( y^r \) are both integers are the key relative points \((P_{rc,s})\). Since RCS is a dynamic coordinate system that moves with vehicles in formations, the \( P_{rc,s} \) in FCRS are one-to-one matched with key road points \((P_{ro,s})\) in GCS at certain time.

C. Motion Rules in RCS

In order to guarantee collisionlessness while vehicles move simultaneously in formation, some rules for relative motion in RCS are made.

**Rule 1:** The time and space are both discretized for relative motion in RCS. The time is discretized with constant interval \( T \), so that available time set in RCS is \( T = \{t_i | t_i = iT, i = 0, 1, 2, 3, \ldots \} \), where \( t_0 \) is the initial time point. The space is discretized to \( P_{rc,s} \). For each \( t_i \) in \( T \) where \( i > 0 \), the FRC of a vehicle must be at one of the \( P_{rc,s} \).

**Rule 2:** For each two adjacent time points \( t_i \) and \( t_j \) in \( T \) where \( i > 0 \) and \( j = i + 1 \), the RCS of a vehicle \((x^r_i, y^r_i)\) and \((x^r_j, y^r_j)\) satisfy the following constraints:

\[
x^r_j - x^r_i, y^r_j - y^r_i \in \{-1, 0, 1\}.
\]

The relationship of \( P_{rc} \) and \( P_{ro} \) is shown in Fig. 1. The green vehicle temp to join the four forward vehicles to form a standard five-lane interlaced formation. As shown in the relative state, the initial \( P_{rc} \) is \((3, 2)\) and the final \( P_{rc} \) is \((2, 3)\). The relative motion that the vehicle should take is shown as ‘Relative motion’. For each relative state in RCS, there is a corresponding real-world state \( P_{ro} \) in GCS at the same fixed time point. The formation is supposed to move with a constant velocity \( v_F \), so the longitudinal distance between two \( P_{ro,s} \) corresponding to the same \( P_{rc} \) at two adjacent time point is \( v_F \times T \). The lower-level then planner performs trajectory planning for the vehicle to travel through the sequence of \( P_{rc,s} \).
III. Upper-level Motion Planning

A. Formation Geometric Structure

The geometric structure of vehicular formations on multi-lane roads determines the relative position between vehicles. There are two common structures in the existing research: the interlaced structure (also known as diamond structure) and the parallel structure. For the interlaced structure, vehicles leave some vacant places in the formation to make lane changing and formation switching more convenient. For the parallel structure, vehicles tend to occupy every available place and form tight formation. Existing research revealed that although the parallel structure has higher vehicle density, the interlaced structure is more suitable for multi-lane vehicle coordination considering lane-changing efficiency and driving safety [13]. In the rest of this paper, the interlaced structure is considered as the standard formation geometric structure. Additionally, vehicles should occupy more forward positions in RCS prior to the other positions.

B. Vehicle-target Optimal Assignment

Targets are generated according to the geometric structure. The number of targets is equal to the number of vehicles that are going to form a formation. Then, an assignment needs to be done to one-to-one match vehicles and targets.

1) Setup of the Optimal Assignment Problem: An assignment builds the one-to-one matching relationship between vehicles and targets. A match between a vehicle and a target has a corresponding cost for the vehicle to travel to the target. The assignment with the minimum total matching cost of all the vehicles is the optimal assignment. The setup process is explained in our previous paper [12], [13] and we omit here.

2) The Conflict-free Assignment Algorithm: The Hungarian Algorithm (HA), which is commonly used for multi-agent task assignment, is able to solve the assignment problem provided in [13], and returns the assignment with minimum total cost [14]. The A* algorithm is then used to calculate the optimal path with the smallest relative travelled distance for vehicles to drive to their targets [15]. The travelled distance of the path calculated by the A* algorithm is also equal to the FRD between the two points. The A* algorithm calculates the relative paths and the sets of $P_{res}$ that need to be passed orderly for vehicles in the formation. For any two vehicles whose relative paths overlap, a collision may potentially happen. HA doesn’t consider the potential collision during the formation switching process. In order to resolve the collision, we need to define the types of conflict firstly.

Definition 1: Define the types of conflict between two vehicles during the formation switching process: for vehicle $i$ and vehicle $j$, the targets assigned to them are target $k_i$ and target $k_j$, respectively. The conflict relationship can be categorized into three types:

Conflict type 1: if target $k_i$ locates on the path for vehicle $j$ to travel to target $k_j$, and the steps for vehicle $i$ to travel to target $k_i$ is fewer than the steps for vehicle $j$ to travel to target $k_i$, then vehicle $i$ and vehicle $j$ are in conflict type 1, shown as vehicle 3 and 4 in Fig. 2.

Conflict type 2: if target $k_i$ locates on the path for vehicle $j$ to travel to target $k_j$, and the steps for vehicle $i$ to travel to target $k_i$ is equal to or more than the steps for vehicle $j$ to travel to target $k_i$, then vehicle $i$ and vehicle $j$ are in conflict type 2, shown as vehicle 1 and 2 in Fig. 2.

Conflict type 3: if vehicle $i$ and vehicle $j$ are not in Conflict Type 1 and Conflict Type 2, and part of their paths towards their assigned targets overlap, then they are in Conflict Type 3, shown as vehicle 4 and 5 in Fig. 2.

For any two vehicles where potential collision may happen, there must be some common $P_{res}$ of their relative paths $P^r$. However, not all the conflicts will cause collision. Among the three aforementioned types of conflict, only the first type will cause deadlock and can not be resolved by that one vehicle relatively stops and waits for the other to go. However, the classical HA cannot resolve the conflict because the two possible assignments have the same total cost. In order to resolve the potential collision caused by the conflict, a novel algorithm, the conflict-free assignment algorithm, is proposed based on HA in Algorithm 1 for the proposed conflict-free assignment algorithm, the following theorems hold.

Theorem 1: Step 3 of Algorithm 1 will end in finite number of steps.

Theorem 2: Exchanging the targets of two vehicles can resolve the existing Conflict Type 1 and will not cause new Conflict Type 1.

Proof: We keep to consider the scenario in the proof of Theorem 2. Vehicle $i$ will block the path of vehicle $j$ because $L_i < L_{j,1}$ and vehicle $i$ will arrive at the conflict point earlier than vehicle $j$. After exchanging the targets, the scenario changes to that the target of vehicle $j$ locates on the path of vehicle $i$ and vehicle $j$ will arrive at the point later than or at the same time with vehicle $i$, so the Conflict Type 1 is resolved and a new Conflict Type 2 is generated.

Since Conflict Type 2 will not cause endless loop because the vehicle which is closer to its target can wait for a step to let the other vehicle pass. The exchange of the two targets doesn’t change the position of all the targets and the paths of other vehicles, so no new Conflict Type 1 will be generated. This completes the proof.

Theorem 3: Algorithm 1 returns the optimal assignment, which has the same total cost with the result of HA. Theorem 1 guarantees that Algorithm 1 will return a solv-
Algorithm 1: The conflict-free assignment algorithm

**Input:** \( N^V \): the number of vehicles
\[ \{ (x^i_{r,V}, y^i_{r,V}) | i = 1, 2, 3, ..., N^V \} \]: FRC of vehicles.
\[ \{ (x^i_{r,t}, y^i_{r,t}) | i = 1, 2, 3, ..., N^V \} \]: FRC of targets.
\( C \in \mathbb{R}^{N^V \times N^V} \): the cost matrix.

**Output:** \( A^H \in \mathbb{R}^{N^V \times 1} \): the optimal assignment result.
\[ \{ P^i_r | i = 1, 2, 3, ..., N^V \} \]: the relative paths for vehicles in RCS.

1. **Step 1** calculating the initial assignment \( A^H \): apply HA to \( N^V \), \( \{ (x^i_{r,V}, y^i_{r,V}) \} \), \( \{ (x^i_{r,t}, y^i_{r,t}) \} \), and \( C \) to get \( A^H \).

2. **Step 2** relative path planning:
3. for \( i \leftarrow 1 \) to \( N^V \) do
4. set \( k_i = A^H(i) \).
5. apply A* algorithm to \( (x^i_{r,V}, y^i_{r,V}) \) and \( (x^i_{r,t}, y^i_{r,t}) \) to get \( P^i_r \).
6. end

7. **Step 3** conflict resolution:
8. while find any vehicles pairs \( i \) and \( j \) that are in Conflict Type 1 do
9. exchange the targets of vehicle \( i \) and \( j \) and update \( A^H \).
10. end

Theorem 2 indicates that the assignment result will not cause unavoidable collision. Theorem 3 guarantees the optimality of Algorithm 1. Due to page limit, the proves of Theorem 1 and Theorem 3 will be presented in an extended version.

IV. LOWER-LEVEL TRAJECTORY PLANNING AND TRACKING

Motion planning in RCS calculates the collision-free relative paths for vehicles to travel to the assigned targets. The output of the relative motion planning is sequences of \( P_{r,s} \).

In this section, the real-world trajectories which pass the \( P_{r,s} \) corresponding to the \( P_{r,s} \) are generated for vehicles. Then, the multi-stage motion control method is proposed to reduce the tracking error.

A. Trajectory Planning Using Bézier Curves

Bézier curves are widely used for trajectory planning for CAVs. In this paper, the cubic Bézier curve is used for trajectory generating. Since there are possibly more than two points for the vehicles to pass, the whole trajectory consists of several Bézier curves and vehicles perform multi-stage motion control.

B. Multi-stage Motion Control

As given in Rule 1 and Rule 2, vehicles arrive at the \( P_{r,s} \) at each \( t_i \) in \( T \) = \{ \( t_i \) | \( t_i = iT \), \( i = 0, 1, 2, 3, \ldots \) \} in RCS. Bézier curves are generated for the vehicles to pass through the corresponding \( P_{r,s} \) in GCS. The key is to calculate longitudinal and lateral control inputs for the vehicles to arrive at the desired position at desired time. The trajectory in GCS passes through a sequence of \( P_{r,s} \) and the inaccuracy during the trajectory following process may accumulate by time. To prevent severe following error which may lead to collision, a multi-stage motion control framework is proposed and vehicles will replan their control inputs after desired time intervals.

1. **Vehicle Model:** The vehicle model used in this paper is the bicycle model. The center of the rear axle is chosen to represent the position of the vehicle, and the coordinate in GCS is \( (x^v, y^v) \). The yaw angle and the steer angle are represented as \( \theta \) and \( \delta \) respectively. \( L \) represents the wheelbase of the vehicle. The state variable \( z^v \) of the vehicle contains \( x^v, y^v, v \) and \( \theta \), where \( v \) represents the velocity of the vehicle. The control input for the vehicle model contains the acceleration \( a \) and the steer angle \( \delta \), and the state is calculated as:

\[
\dot{z}^v = \begin{bmatrix} v \sin(\theta) \\ v \cos(\theta) \\ a \end{bmatrix}, \quad z^v = \begin{bmatrix} x^v \\ y^v \\ \frac{v}{a} \tan(\delta) \\ \theta \end{bmatrix}.
\]

2. **Lateral Control:** For the lateral control, a linear feedback controller is designed. The vehicle looks ahead to the preview point \( P^l \) and the closest point \( P^c \) on the trajectory. The controller calculates the angle between the vehicle velocity and the desired velocity on \( P^l \), and the distance between the vehicle and \( P^c \) as feedbacks.

3. **Optimal Longitudinal Control With Path Constraints:**

The discretized longitudinal state equation is given as:

\[
\begin{align*}
    s(k + 1) &= s(k) + v(k)\Delta t, \\
    v(k + 1) &= v(k) + a(k)\Delta t,
\end{align*}
\]

where \( s(k) \), \( s(k) \) and \( s(k) \) represent the longitudinal position, speed and acceleration respectively. The time is discretized by sample interval \( \Delta t \). The distance that the vehicle will travel from the starting point to the final point is denoted as \( S_t \).

Since the vehicle moving in RCS should pass a sequence of \( P_{r,s} \), \( S_t \) consists of series of segments, whose length are denoted as \( S_1, S_2, ..., S_{N^v} \), where \( N^l \) is the number of segments. The time interval for the vehicle to cover each segment is \( T \). Taking the control energy of the whole control process as the cost, the discretized longitudinal control can be described as:

\[
\min_{a^2(k)} \sum_{k=0}^{N^lN^k-1} a^2(k),
\]

s.t. \( s(0) = 0 \),

\[
\begin{align*}
    s(k) &= \sum_{n=1}^{i} S_n, \quad k_i = iN^k, \quad i = 1, 2, 3, ..., N^l, \\
    v(0) &= v(N^lN^k) = v^l, \\
    v^{\min} \leq v &\leq v^{\max}, \\
    a^{\min} \leq a &\leq a^{\max},
\end{align*}
\]
where \( N^k = \frac{T}{k} \) is the steps in one segment, \( a(k) \) is the longitudinal control input (acceleration) of the vehicle at step \( k \), \( v(k) \) is the longitudinal speed, \( v^F \) is the desired longitudinal speed of the formation, and \( v_{\text{min}}, v_{\text{max}}, a_{\text{min}} \) and \( a_{\text{max}} \) are the bounds of speed and acceleration.

The optimal control problem in (5) is solved and the sequence of control input is calculated to guide the vehicle to travel desired distance at desired time. Since the linear feedback preview controller is designed for lateral control, the inaccuracy of lateral motion may cause deviation for longitudinal motion. In order to resolve the accumulated inaccuracy, the optimal problem is reformed and solved every time the vehicle completes the following of one segment at time \( t = iT (t = 1, 2, 3, ..., N^i) \).

V. SIMULATION AND RESULTS

In this section, the simulation is conducted and the results are analyzed. The simulation is implemented with MATLAB 2017b and SUMO 0.32.0 [16] on a personal computer with CPU Intel CORE i7-8700@3.2GHz. The average travel time of vehicles is analyzed to evaluate the performance. The parameters chosen for this simulation is presented in Table I.

| TABLE I | SIMULATION PARAMETERS |
|---------|-----------------------|
| Safe one-lane following gap | \( d_g \) | 15 m |
| Formation switching cycle | \( T \) | 5 s |
| Desired speed in formation | \( v_p \) | 28.8 m/s |
| Minimum speed of vehicle | \( v_{\text{min}} \) | 0 m/s |
| Maximum speed of vehicle | \( v_{\text{max}} \) | 33.3 m/s |
| Minimum acceleration of vehicle | \( a_{\text{min}} \) | \(-10 \) m/s² |
| Maximum acceleration of vehicle | \( a_{\text{max}} \) | 5 m/s² |
| Minimum steering angle of vehicle | \( \delta_{\text{min}} \) | \(-40^\circ \) |
| Maximum steering angle of vehicle | \( \delta_{\text{max}} \) | \(40^\circ \) |

Case study is firstly conducted to verify the function of the proposed method. Three scenarios are chosen for case study: (1) the number of lanes changes from three to one; (2) the number of lanes changes from three to two; (3) the number of lanes changes from three to four. The formation switching process is shown in Fig. 3 where the initial interlaced formations switch their structure to adapt to the changed scenarios. It indicates that the proposed method is able to handle different scenarios and can switch the formation smoothly with continuous and smooth trajectories.

The lane-drop bottleneck from three lanes to two lanes are chosen as the scenario of the comparative simulation, as shown in Fig. 4. The road segment is divided into three parts: the upstream multi-lane driving part \( S_1 \), the formation switching part \( S_2 \), and the downstream multi-lane driving part \( S_3 \). The length of them are \( l_1 + l_2 = 1000 \) m and \( l_3 = 200 \) m. The beginning point of \( S_2 \) is determined according to the result of motion planning of each single formation. The number of lanes of \( S_1 \) and \( S_2 \) are three, and that of \( S_3 \) is two. Vehicles are generated at the beginning of \( S_1 \) (the green points) and try to form standard formation when driving on \( S_1 \). The formation calculates the time cycle it needs to switch to the two-lane structure and thus get the length of \( S_2 \) (\( l_2 \)). Formation switching is performed on \( S_2 \) and the third lane (on the top) is cleared before the most forward vehicle of the formation arrives at the beginning of \( S_2 \). Vehicles leave the road segment at the end of \( S_3 \) (the red points).

The default driving model of SUMO is adopted as reference. The longitudinal car following model is Intelligent Driving Model (IDM) and the lateral lane change model is the default strategic lane changing [17]. The simulation is conducted under different input traffic volume changing from 250 veh/(hour · lane) to 2000 veh/(hour · lane). The simulation time under each traffic volume is 600 s. The average travel time of vehicles is used to evaluate the efficiency improvement of the proposed method.

Heatmaps are presented in Fig. 5 and Fig. 6 to show the speed distribution among vehicles on time and space dimensions under different traffic input volume. When vehicles driving without FC, congestion may happen and spread upstream from the lane-bottleneck point (1000 m). When FC is conducted, the congestion is significantly improved and vehicles tend to maintain a constant formation speed (28.8 m/s).

The snapshots of the simulation process are provided in Fig. 7. The figures are taken at the fixed simulation time 600 s under each input traffic volume. The color of the vehicles represents their speed. From the snapshots we can see that the proposed FC method reasonably distributes vehicles on the lanes and guides them to drive with the expected speed (28.8 m/s) and avoids congestion at the lane-drop bottleneck. Although some vehicles can achieve higher speed when not using FC, the performance at the bottleneck is poor and severe congestion is formed at high traffic volume.

Numerical results are shown in Fig. 8. It indicates that vehicles pass the whole road segment faster when
not using FC with the input traffic volume lower than 1000 veh./(hour · lane). It’s because there is no or slight congestion near the bottleneck and the expected speed of vehicles is higher when using the reference method. When the input traffic volume get higher, the congestion becomes severer, the average speed of vehicles becomes lower, and the average travel time gets higher. It is noticeable that the travel time of FC method keeps almost unchanged under different traffic volume, because the input traffic volume hasn’t reach the theoretical maximum congestion-free volume, which is around 2500 veh./(hour · lane) and can be calculated by the following gap and desired formation speed.

VI. CONCLUSIONS

This paper proposes a formation control method for multiple vehicles driving on multi-lane roads. The bi-level planning framework is proposed for formation switching in different scenarios. Conflict-free relative motion planning and multi-stage trajectory planning are conducted on the upper-level and lower-level planners respectively. Case study and the result of the simulation verifies the ability of the proposed method to smoothly switch the structure of formations in different scenarios and indicates that the proposed method can improve traffic efficiency at high traffic volume.

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