TDGL Simulation on Angular Dependence of Critical Current Density in Superconductors with Columnar Defects

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Abstract. We numerically solved the three dimensional time-dependent Ginzburg-Landau (TDGL) equations to visualize the motion of the quantized magnetic flux lines in the superconductor under the transverse magnetic field B. We investigated the angular dependence of the critical current density $J_c$ in the columnar pins using TDGL equations. As a result of simulations, it was confirmed that $J_c$ decreases as increasing the angle $\theta$ between the columnar pins and the quantized magnetic flux lines. In particular at $B = 0.4$, $J_c$ sharply decreased at $\theta$ between 20° and 30°. This result was compared with the calculation result of the overlapped volume between the quantized magnetic flux and the pins. The calculated result by TDGL equations was in good agreement with the calculated result by the overlapped volume. In addition, $J_c$ was calculated using various pins such as the sphere, the columnar and the plane pins. It was found that plane pins are more effective than pins of other shapes. In addition, although it is experimentally difficult to fabricate, $J_c$ of the star-shaped pin was obtained by the TDGL simulations. It was found that the $J_c$ of the star-shaped pin shows the highest performance among all the kinds of configurations for the columnar pins in high magnetic fields.

1. Introduction

It is well known that the critical current density $J_c$ varies depending on various conditions for pins trapping quantized magnetic flux lines in the type-II superconductors under the transverse magnetic fields [1–3]. The angular dependencies of $J_c$ have been observed by experiments where the columnar pins were introduced by the heavy ion beams injected along the c-axis of oxide superconductors [4]. However, few cases of these dependencies were theoretically described. According to Moore's Law, computer performance has developed rapidly and has become very high, the computer numerical calculation becomes very important in addition to recent theories and experiments. It was reported that the two dimensional numerical calculation based on the time-dependent Ginzburg-Landau (TDGL) equations with a simple point pin [5, 6] in about 30 years ago. Recently, there are some reports on the pinning simulations based on the three dimensional TDGL equations [7, 8]. Further investigations using the TDGL equations with various kinds of shape and structure of pins are needed to understand the
pinning effect on the quantized magnetic flux lines. Here, we focused on the pin shape and the angular dependence. Numerical simulations are expected to provide effective methods to characterize the pinning effect concerned. Instead of executing complicated and time-consuming experiments, we adopted to use the numerical simulation because it is desired to obtain some useful scheme efficiently to improve the superconducting properties for $J_c$.

In this study, we numerically solved the TDGL equations to visualize the movement of the quantized magnetic flux lines of the superconductor under the transverse magnetic field and to find angular dependence of $J_c$ by the columnar pins. In addition, we investigated various kinds of pins of a sphere, a columnar, a plane and a star-shaped defects to obtain the $J_c$ characteristics dependent on various electromagnetic and materials parameters.

2. Methods

The critical current densities $J_c$ in various conditions of the pinning in a small superconducting cube exposed to a transport current and a transverse magnetic field were investigated. The TDGL equations for the superconducting cube was numerically solved by using the Euler method. The dimensions of the cubic material were set to be smaller than the London penetration depth [7]. In this case, the vector potential $A$ depends only on the external magnetic field $B$. This approximation is known as the thin wire approximation, where the GL parameter $\kappa$ is expected to be sufficiently high. We show the three-dimensional dynamics of the quantized magnetic flux lines by plotting the contour surfaces of the superconducting electron density $|\Psi|^2$, where $\Psi$ is the order parameter.

In this study, the parameters using in the original TDGL equations were normalized using the coherence length $\xi$ and the upper critical field $B_{c2}$ and so on for reducing the number of the constants in the TDGL equations. Hence, the calculation of the order parameter $\Psi$ and scalar potential $V$ are easier compared with the original TDGL equations, which are obtained by solving a set of two differential equations. The reduced TDGL equations are given as follows [8]:

$$\frac{d\Psi}{dt} + iV\Psi + (i\nabla + A)^2\Psi - \Psi + |\Psi|^2\Psi = 0 \tag{1}$$

$$\sigma \nabla^2 V = \frac{1}{2i} (\Psi^\ast \nabla^2 \Psi - \Psi \nabla^2 \Psi^\ast) - \nabla \cdot (|\Psi|^2 A) \tag{2}$$

where, $\sigma$ is the normal conductivity.

We considered a superconducting cube of which side length is $10\xi$ in the vacuum. In addition, 4 columnar pins of diameter $\xi$ were introduced with the distance $d$ of pins as shown in Figure 1. Here, we define the order parameter $\Psi$ as 0 inside of the pins [5]. This approximation $\Psi = 0$ seems to be too strong setting for the pinning. However, the calculation result of $J_c$ was not so large compared with that taken into account of the proximity effect in the normal conducting region.

![Figure 1. Geometry of the superconducting cube and the columnar pins.](image-url)
The boundary condition for the electric current density $\mathbf{j}$ is that its normal component is zero at the surfaces of the cube. The current density and the magnetic field are applied to the $y$-axis and the $z$-axis, respectively. That is (0, $J$, 0) and (0, 0, $B$), respectively. Hence, the vector potential can be given by $(A_x, A_y, A_z) = (0, xB, 0)$ for the transverse magnetic field. $J$ and $B$ at each time are kept constant at a normalized value. Figure 2 shows the example of the quantized magnetic flux lines each of which is trapped by the 4 columnar pins, where pixels of $|\Psi|^2 < 0.1$ is colored. The hue of the color is corresponding to the phase of the order parameter. It is to be noted that almost all parameters such as $B$, $J$ and $\Psi$ are normalized. The magnetic field is normalized by upper critical field, $B_c(2)$. The normalized current density of 0.385 means the pair breaking current density. And the order parameter is normalized by the order parameter of equilibrium point.

Figure 3 shows the superconducting cube with different angle of the columnar pins. It is assumed that $\theta = 0^\circ$ represents for the case when the pin and the magnetic field are in parallel, as shown in Figures 3(b), (c), and (d). Instead of rotating the magnetic field, the pin is rotated in this study to suppress the surface effects. Simulations were made with the external magnetic field $B = 0.1, 0.2, \cdots, 0.5$, the electric current density $J = 0.01, 0.02, \cdots, 0.30$, and angle $\theta = 0, 10, \cdots, 90^\circ$.

In addition, we calculate the overlapped volume between the quantized magnetic flux and the pins. As shown in Figure 4, the quantized magnetic flux and the pin are assumed to be the quadrangular prisms. That is, the quantized magnetic flux has a cross-sectional area of $2\xi \times 2\xi$ and the pin has a cross-sectional area of $2r \times 2r$. And, we start to consider the case where the length of superconducting region along $z$-axis, $L_z = \infty$. Since the bold line is $r/\sin \theta$, the area of the red parallelogram is $4r\xi/\sin \theta$. In the case of $r < \xi$, the overlapped volume between the quantized magnetic flux is $8r^2 \xi/\sin \theta$. Then, the case where $L_z$ is finite is also analytically considered. Since, the detail of the calculation is complicated, equations are omitted for the case of the finite length of the superconducting region.

In order to investigate the pinning effects on $J_c$ for different shapes of the pins, we investigated for a sphere pin as a point pin, and a plane pin. Figure 5 shows the superconducting cube with sphere pins. Figures 5(b), (c), and (d) show the number of pins for 1, 4 and 8, respectively. Figure 6 shows the superconducting cube with a plane pin. Figures 6(a) and (b) show the thickness of the plane pin for the thickness $r = 0.6, 1.0$, respectively. The pin thickness $r$ is normalized with $\xi$. Figure 7 shows the superconducting cube with a star-shaped pin. Although it is experimentally difficult to fabricate the shape of the star-shaped pin, it is relatively easy to execute simulations with that kind of pins. This is one of the advantages of the TDGL numerical study. A number of simulations were made with various external magnetic fields $B = 0.05, 0.10, \cdots, 0.60$, the electric current densities $J = 0.01, 0.02, \cdots, 0.30$.

**Figure 2.** Simulation example: the flux lines are trapped by the columnar pins. Color represents the phase of the order parameter.

**Figure 3.** Superconducting cube with different angle of the columnar pins: (b) $\theta = 0^\circ$, (c) $\theta = 45^\circ$, (d) $\theta = 90^\circ$. 

31st International Symposium on Superconductivity (ISS2018) 
IOP Conf. Series: Journal of Physics: Conf. Series 1293 (2019) 012018 doi:10.1088/1742-6596/1293/1/012018
Figure 4. Overlapped volume between a quantized magnetic flux and a pin for the finite length of the superconducting region.

Figure 5. Superconducting cube (a) with the sphere pins: (b) 1 pin, (c) 4 pins, (d) 8 pins.

Figure 6. Superconducting cube with the plane pin: (a) $r = 0.6$, (b) $r = 1.0$.

Figure 7. Superconducting cube with the star-shaped pins.
3. Results and discussion

Figure 8 shows the numerical results of the angular dependence of \( J_c \) at high magnetic fields: \( B = 0.3, 0.4, 0.5 \) for the columnar pins. The critical current density \( J_c \) decreases as increasing the angle. \( J_c \) at \( B = 0.4 \) abruptly decreases when \( \theta \) is between 20° and 30°. Although the quantized magnetic flux lines could be trapped by the columnar pins for \( \theta = 0 - 20° \), the quantized magnetic flux lines could not be trapped and slip through at high magnetic fields. We also examined the case of low magnetic fields: \( B = 0.1, 0.2 \), then it was found that the result that \( J_c \) increases as increasing the angle. These results of high and low cases seem to be a contradiction, which can be solved by the following explanation. At low magnetic fields, the space between the quantized magnetic flux lines is too large and only one flux line appeared in the small superconducting cube. Then the probability that the quantized magnetic flux lines slip through at \( \theta = 0° \) was high. On the other hand, the quantized magnetic flux lines is likely to be trapped at \( \theta = 90° \), since the pin and the quantized magnetic flux lines always cross each other. Therefore, we found that \( J_c \) increases as increasing the angle at low magnetic fields. This result is due to the limitation of the present calculation with the small superconducting cube.

Figure 9 shows the numerical results of the angular dependence of \( J_c \) at \( B = 0.4 \) and the calculation result of the overlapped volume between the quantized magnetic flux and the pins. The calculation result of the overlapped volume sharply decreases at \( 10° \leq \theta \leq 30° \) and then gently decreases. Thus, the simulated result based on the TDGL equations is in good agreement with the analytically calculated result by the overlapped volume. Therefore, it is concluded that the angular dependence of \( J_c \) is determined by the overlapped volume between the quantized magnetic flux and the pins.

Figure 10 shows the \( J_c-B \) property for the sphere pins. In the range of \( 0.35 \leq B \leq 0.6 \), it is found that \( J_c \) is relatively large in the case of 8 sphere pins. This can be due to the fact that as increasing \( B \), the number of the quantized magnetic flux lines increases by entering the superconducting region. In addition, \( J_c \) of the model with 4 sphere pins became the largest at \( B = 0.3 \), because the number of the quantized magnetic flux lines is 2 and the flux line lattice spacing is closed to the distance between the sphere pins, and the matching effect occurs. This effect is known as the matching effect with periodic modulations of the pins [9]. Thus, the \( J_c \) shows the peak around \( B = 0.3 \).

Figure 11 shows the \( J_c-B \) property for the plane pin. It is found both results \( r = 0.6 \) and 1.0 are the same. Therefore, the pinning works effectively both at \( r = 0.6 \) and \( r = 1.0 \).

Those results of \( J_c-B \) property are reasonable as expected by theories and experiments. The advantage for using numerical calculation is easy to change the shape of pins compared with the theories and experiments. Figure 12 shows the \( J_c-B \) property for the star-shaped pin, the sphere pins and the plane pin. It is found that the sphere pin shows the highest \( J_c \) at low magnetic fields, and the plane pin shows the highest \( J_c \) at middle magnetic fields. At high magnetic fields, the star-shaped pin shows the highest \( J_c \). This is because the volume of the star-shaped pin is lower than that of the plane pin, and the degradation of superconductivity of the star-shaped pin is less than that in the case of the plane pin. Therefore, it is concluded that the numerical calculation by using TDGL has high efficiency to find a new type of shape or configuration for pins to design high \( J_c \) superconducting materials.

Figure 13 shows the angular dependence of the critical current density at \( B = 0.4 \). It is confirmed that the angular dependence is almost flat. And the critical current density slightly negative drops at \( \theta = 30, 90° \). This is due to the 6-fold rotational symmetry of the star-shaped pin. It was reported that Gd-based coated superconductor with the crossed columnar defects produced by heavy-ion irradiation shows better angular dependence of the critical current density [4]. Therefore, if the number of crossed columnar defects increases, the angular dependence of the critical current density becomes more flat according to the present work.

On the other hand, it is also true that there is a limit to the present calculation. For example, in Figs. 10 to 12, the value of the normalized upper critical field is about 0.7 – 0.8, which is not unity. The main reason is the limitation of calculation space as \( 10\% \times 10\% \times 10\% \) due to the calculation time and cost. Hence, the surface effect is remarkable in the present calculation. In addition, it is difficult to discuss
quantitatively with the present numerical calculation. Therefore, it is desired to obtain large calculation space by using such as a parallel calculation method and so on.

**Figure 8.** Numerical results of the angular dependence of $J_c$ at $B = 0.3, 0.4, 0.5$ of columnar pins.

**Figure 9.** Numerical results of the angular dependence of $J_c$ at $B = 0.4$ and the calculation result of overlapped volume between the quantized magnetic flux and the columnar pins.

**Figure 10.** Magnetic field dependence of critical current density in the case of the sphere pin.

**Figure 11.** Magnetic field dependence of critical current density in the case of the plane pin.
4. Conclusion

We investigated the angular dependence of $J_c$ in the columnar pins using simulations based on the three dimensional time-dependent Ginzburg-Landau (TDGL) equations. The critical current density $J_c$ decreases as increasing the angle. Especially $J_c$ at $B = 0.4$ abruptly decreases in the range of $\theta = 20^\circ - 30^\circ$. This numerical result was compared with the calculation result of the overlapped volume between the quantized magnetic flux and the pins. And, the simulation result by TDGL agrees well with the calculated result by the overlapped volume. In this way, experimentally observed angular dependence of $J_c$ was reproduced by the simulations in this study.

The numerical simulations based on the TDGL equations were also applied for the sphere pins and the plane pin. In the sphere pins, it is found that $J_c$ in the model of 8 sphere pins is relatively large in the range of $0.35 \leq B \leq 0.6$. This can be due to the fact that as increasing $B$, the number of the quantized magnetic flux lines increases by entering the superconducting region. In addition, when $B = 0.3$, $J_c$ of the model of 4 sphere pins became the largest. Furthermore, $J_c$ in the model of 4 sphere pins became the largest at $B = 0.3$, because the number of the quantized magnetic flux lines is 2, and the flux line lattice spacing is closed to the distance between the sphere pins, and the matching effect occurs. Thus, the $J_c$ shows the peak around $B = 0.3$. In the plane pin, both $r = 0.6$ and $r = 1.0$ show similar characteristics. Therefore, the pinning works effectively both at $r = 0.6$ and $r = 1.0$. At high magnetic fields, the star-shaped shape pin shows the highest $J_c$. The TDGL simulation was shown to have considerable advantages to find a new type of pin shape to design engineering superconducting materials of high $J_c$ with good properties with respect to various kinds of parameter dependence. That is, we predicted the $J_c$-$B$ characteristics with the defects that have been neither fabricated nor tested by real experimental measurements. On the other hand, because of computational limitation that the computation space is small, unnatural results were obtained. Therefore, it is necessary to extend the computation space using parallel computation and so on.

![Figure 12](image1.png)  
**Figure 12.** Magnetic field dependence of critical current density in the case of the star-shaped pin, the sphere pins and the plane pin.

![Figure 13](image2.png)  
**Figure 13.** Angular dependence of critical current density at $B = 0.4$. 

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