Robust PID controller design for unstable processes with parametric uncertainty

J. Zavacka, M. Bakosova, K. Matejickova a*

*Faculty of Chemical and Food Technology, Slovak University of Technology in Bratislava, Radlinského 9, 812 37 Bratislava, Slovak Republic

Abstract

The paper presents a method for robust PI controller design for system affected by parametric uncertainty. The proposed method is based on plotting the stability boundary locus in the plane of controller parameters that is called \((k_p, k_i)\) - plane. The design approach is verified using simulations of control of the continuous stirred tank reactor (CSTR) with hydrolysis of propylene oxide to propylene glycol. The reactor which has three uncertain parameters is controlled into its unstable steady-state.

Keywords: Robust PI controller; parametric uncertainty; chemical reactor; unstable steady-state

1. Introduction

The CSTRs belong to the most important plants in chemical and food technology. Their operation is corrupted with various uncertainties. Some of them arise from varying or not exactly known parameters. In others cases, operating point of reactor vary or reactor dynamics is affected by various changes of parameters of inlet stream. All these uncertainties can cause poor performance or even instability of closed-loop control systems. Application of robust control is one of the ways to overcome these problems, [2], [4]. In case when the unstable steady-state coincides with the point that yields the maximum reaction

* Corresponding author. Tel.: +421-259-325-366; fax: +421-259-325-340.
E-mail address: jana.zavacka@stuba.sk.
rate at a prescribed temperature, it is necessary to control CSTRs into the prescribed open-loop unstable steady-state [1], [2], [6].

For design of robust PI controllers, which stabilize nonlinear system of CSTR with uncertain parameters [5] in unstable steady-state, is simple and fast method presented [7]. The method is based on plotting the stability boundary locus in the \((k_p, k_i)\)–plane. The parameters of robust stabilizing PI controller are determined from the stability region [10]. Robust PI controllers stabilize a controlled system with parametric uncertainties, if the stability region is found for sufficient number of Khartitonov plants [3].

2. Robust PI controller design

Consider a single-input single-output uncertain control system in the form

\[
G(s) = \frac{N(s)}{D(s)}
\]  

(1)

where \(N(s)\) is uncertain polynomial of system numerator and \(D(s)\) is uncertain polynomial of denominator. \(C(s)\) is a PI controller in the form

\[
C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s}
\]  

(2)

The problem is to find the parameters of the PI controller (2). Decomposing the numerator and the denominator polynomials in (1) [8] into their even and odd parts, and substituting \(s = j\omega\), where \(\omega\) is the frequency, gives

\[
G(j\omega) = \frac{N_e(-\omega^2) + j\omega N_o(-\omega^2)}{D_e(-\omega^2) + j\omega D_o(-\omega^2)}
\]  

(3)

Using (3) the closed loop characteristic equation \(\Delta(j\omega)\) can be written as

\[
\Delta(j\omega) = [k_i N_e(-\omega^2) - k_p \omega^2 N_o(-\omega^2) - \omega^2 D_o(-\omega^2)] + \]

\[
+j[k_p \omega N_e(-\omega^2) + k_i \omega N_o(-\omega^2) + \omega D_e(-\omega^2)] = 0
\]  

(4)

Then, equating the real and the imaginary parts of \(\Delta(j\omega)\) to zero, one obtains [7], [9]

\[
k_p \left( -\omega^2 N_o(-\omega^2) \right) + k_i \left( N_e(-\omega^2) \right) = \omega^2 D_o(-\omega^2)
\]  

(5)

and

\[
k_p \left( N_e(-\omega^2) \right) + k_i \left( N_o(-\omega^2) \right) = -D_e(-\omega^2)
\]  

(6)

Solving the equations (5), (6) simultaneously for \(\omega \geq 0\), the set of parameters \(k_p\) and \(k_i\) is obtained. Then, it is possible to plot the dependence of \(k_i\) on \(k_p\), and the stability boundary locus \(l(k_p, k_i, \omega)\) in the \((k_p, k_i)\)-plane is obtained. The stability boundary divides the parameter plane into stable and unstable regions.

The described robust PI controller design is fast and effective. The controller problem consists in finding a proper interval of frequency \(\omega\). Hence is for this method necessary to find real values of \(\omega\) that satisfy condition

\[
\text{Im}[G(j\omega)] = 0
\]  

(7)
3. Description of the controlled processes

As a controlled process the continuous stirred tank reactor for hydrolysis of propylene oxide to propylene glycol was chosen, see e.g. [5]. The reaction is described according to the scheme \( \text{C}_3\text{H}_6\text{O} + \text{H}_2\text{O} \rightarrow \text{C}_3\text{H}_8\text{O}_2 \), where \( \text{C}_3\text{H}_6\text{O} \) and \( \text{H}_2\text{O} \) represent source substances propylene and water, respectively and \( \text{C}_3\text{H}_8\text{O}_2 \) is product propylene glycol. Chemical reaction run in a reaction vessel and reaction heat generated by exothermic reaction is removed from the reactor by coolant in a reactor jacket. The excess of water provides higher selectivity to propylene glycol and eliminates consecutive reactions of propylene oxide as a key component. The mathematical model of CSTR consist of Arrhenius equation which describe the exponential dependency of reactant concentrations on the temperature of the reaction mixture, the mass balance for any species in the system, the simplified enthalpy balance of the reacting mixture and the simplified enthalpy balance of the cooling medium, see e.g. [2].

The values of constant parameters and steady-state inputs of the reactor are summarized in Tab. 1.

| Variable | Description | Value | Unit |
|----------|-------------|-------|------|
| \( V_r \) | volume of the reaction mixture | 2.407 | m³ |
| \( V_c \) | volume of coolant | 2.000 | m³ |
| \( \rho_r \) | density of the reaction mixture | 974.19 | kg m⁻³ |
| \( \rho_c \) | density of the coolant | 998.00 | kg m⁻³ |
| \( c_{pr} \) | specific heat capacity of the reaction mixture | 3.7187 | kJ kg⁻¹ K⁻¹ |
| \( c_{pc} \) | specific heat capacity of the coolant | 4.182 | kJ kg⁻¹ K⁻¹ |
| \( A \) | heat exchange surface area | 8.695 | kJ min⁻¹ K⁻¹ |
| \( g=(E/R) \) | activation energy divided by gas constant | 10183 | K |
| \( q_r \) | volumetric flow rates of reaction mixture | 0.072 | m³ min⁻¹ |
| \( q_c \) | volumetric flow rates of coolant | 0.6307 | m³ min⁻¹ |
| \( T_{rf} \) | inlet temperature of the reaction mixture | 299.05 | K |
| \( T_{cf} \) | inlet temperature of the coolant | 288.15 | K |
| \( c_{r,A} \) | inlet concentration of the \( \text{C}_3\text{H}_6\text{O} \) | 0.0824 | kmol m⁻³ |
| \( c_{r,B} \) | inlet concentration of the \( \text{C}_3\text{H}_8\text{O}_2 \) | 0 | kmol m⁻³ |

The model of the CSTR contains two inputs: volumetric flow rates of reaction mixture \( q_r \) and coolant \( q_c \) and three outputs: concentration of the \( \text{C}_3\text{H}_6\text{O} \), the reaction mixture temperature in the reaction vessel \( T_r \) and the coolant temperature in the jacket \( T_c \). Model uncertainties of the reactor follow from the fact that are three physical parameters in this reactor: the reaction enthalpy \( \Delta_r H \), the pre-exponential factor \( k_\infty \) and the overall heat transfer coefficient \( \alpha \). The boundaries values of which vary within certain intervals are in Tab. 2.
Table 2. Uncertain parameters in the CSTR

| Variable | Description           | Minimal Value      | Maximal value     | Unit       |
|----------|-----------------------|--------------------|-------------------|------------|
| $\Delta H$ | reaction enthalpy     | $-5.508 \times 10^6$ | $-5.412 \times 10^6$ | kJ mol$^{-1}$ |
| $k_\infty$ | pre-exponential factor | $2.5867 \times 10^{11}$ | $3.0667 \times 10^{11}$ | min$^{-1}$ |
| $\alpha$   | heat transfer coefficient | 13.0            | 14.6             | kJ min$^{-1}$ K$^{-1}$ |

The steady-state behavior of the model CSTR was studied. It can be stated the CSTR has three steady-states, two of them are stable and one is unstable. From the viewpoint of safety operation or in the case of unstable steady-state coinciding with the point that yields the maximum reaction rate at a prescribed temperature, it is necessary to control CSTRs near the prescribed open-loop unstable steady-state [2]. Therefore the main operating point is described by unstable steady-state values of state variables. The heat analysis the nominal model is shown in Fig. 2, where $Q_{GEN}$ is the heat generated by chemical reactions and $Q_{OUT}$ is the heat removed by the jacket and the product stream. The operating point of the nominal system is described using $[c_A, T_s, T_c] = [0.0371 \text{ kmol m}^{-3}, 343.0992 \text{ K}, 290.5456 \text{ K}]$.

Fig. 1. Steady-states of nominal model of CSTR obtained as intersections of $Q_{GEN}$ and $Q_{OUT}$

4. Control of the CSTR

Robust controller has been designed to achieve control of the reaction mixture temperature in the reaction vessel. Control input is volumetric flow rate of the reaction mixture $q_r$ and control output is temperature of the reaction mixture $T_r$. For controller design, the mathematical model of the continuous stirred tank reactor with three uncertain parameters (Tab. 2) is obtained in the form of a transfer function

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

where the values of the uncertain parameters are shown in Tab. 3. Using sixteen Kharitonov plants [3] are generated for the CSTR controlled system (8) to design robust PI controller.
Table 3. Uncertain parameters

| Parameter | Minimal value | Maximal value | Parameter | Minimal value | Maximal value |
|-----------|---------------|---------------|-----------|---------------|---------------|
| $b_2$     | -20.0485      | -16.8807      | $a_2$     | 0.26          | 0.2672        |
| $b_1$     | -6.6816       | -5.7959       | $a_1$     | -0.0259       | -0.0228       |
| $b_0$     | -0.0802       | -0.0180       | $a_0$     | $-0.8117 \times 10^{-3}$ | $-0.7658 \times 10^{-3}$ |

Using the (4), (5) to obtain the PI controller tuning rules in the form (9), (10)

\[
q_i = \frac{b_2 a_3 \omega^6 + (a_2 b_1 - a_1 b_2 - a_3 b_0) \omega^4 + (-a_0 b_1 + a_1 b_0) \omega^2}{b_2^2 \omega^4 + (2 b_0 b_2 + b_1^2) \omega^2 + b_0^2}
\]  

\[
q_p = \frac{a_3 \omega^4 + (-a_1 - b_2 k_i) \omega^2 + b_0 k_i}{b_1 \omega^2}
\]  

After a suitable choice of $\omega \in (0; 0.3289)$ (7), the stability boundary locus as the dependence of $q_i = f(\omega)$ on $q_p = f(\omega)$ is plotted. In Fig. 2 are shown the stability regions of sixteen Kharitonov plants, where intersection of these regions represents the stable region.

![Stable region for sixteen Kharitonov plants](image)

Each Kharitonov system divided the $(k_p, k_i)$ – plane into two parts, stable and unstableregion which are shown in the Fig. 3. Robust stability region is obtained as a intersection of all particular stability regions generated by 16 Kharitonov plants. The required robust PI controller is tuned using the parameters from the stable region. The robust controller parameters $k_p$ and $k_i$ which stabilizes all sixteen Kharitonov systems are chosen from the stable region and design PI controller is described for example by (11)

\[
C(s) = \frac{-0.03s - 0.004}{s}
\]  

(11)
5. Simulation results

The designed PI controller (11) for control of nonlinear uncertain model of unstable CSTR by simulations in MATLAB-Simulink environment has been verified.

Fig. 4 presents simulation results obtained with the designed controller (11) and nonlinear models of the CSTR - nominal and two systems obtained with minimal and maximal values of uncertain parameters. The control output is the reaction mixture temperature $T_r$. The setpoint changes at time 0 min from initial value 345.0992 K to unstable working point 345.0992 K, then at time 150 min to new value of temperature 347.0992 K, then at time 450 min to 344.0992 K and at time 600 min again to unstable working point 345.0992 K. At time 300 min the disturbance has been generated which is represented by change of the inlet temperature $T_r$ from 299.05 K to 300.05 K. In Figures 4 are shown control inputs represented by volumetric flow rates of reaction mixture $q_r$ for nominal system and two systems obtained with minimal and maximal values of uncertain parameters.

![Fig. 3. Control responses for controller $C_1$.](image1)

![Fig. 4. Control inputs generated by controller $C_1$.](image2)
Robust PI controller (11) which has been designed using the robust stability region of all stabilizing parameters $k_p$ and $k_i$ was able stabilize nonlinear model of CSTR with uncertainties into unstable working point and his surroundings. The controller also has been able to remove the disturbance which was generated by change of the inlet temperature $T_{in}$.

6. Conclusion

In this paper, robust PI controller for control uncertain system in unstable steady-state has been designed. For calculation of robust PI controller was used method which is based on the plotting the stability boundary locus in the ($k_p$, $k_i$)-plane. Design controllers were used for control of nonlinear model of CSTR with three uncertain parameters. The aim was control of the reactor in unstable state for example from the viewpoint of safety operation. Therefore, the reactor was in the unstable steady-state stabilized which coincides with the point that yields the maximum reaction rate at a prescribed temperature.

Presented simulation results shows that designed robust PI controllers is able to stabilize the uncertain CSTR in unstable working point and his surroundings.

References

[1] Bakošová M, Oravec J, Kačur M, Závacká J. Stabilization of chemical reactors using robust and optimal controllers. In: Markoš J, editors. Proceeding of the 38th International Conference of Slovak Society of Chemical Engineering; 2011, p. 988–997.
[2] Bakošová M, Puna D, Dostál P, Závacká J. Robust stabilization of a chemical reactor. Chem Pap 2009;65:527–536.
[3] Barmish B. New Tools for Robustness of Linear Systems. New York: Macmillan; 1994.
[4] Ding B. Constrained robust model predictive control via parameter-dependent dynamic output feedback. Automatica 2010;46:1517–1523.
[5] Molnár A, Markoš J, Jelemenský Š. Accuracy of mathematical model with regard to safety analysis of chemical reactors. Chem Pap 2002;56:357–361.
[6] Saulen NPG, Secchi AR, Trierweiler JO, Neumann GA. Multivariable control strategy based on bifurcation analysis of an industrial gas-phase polymerization reactor. In: International Symposium on Advanced Control of Chemical Processes, 2006, p. 166.pdf.
[7] Tan N, Kaya I. Computation of stabilizing PI controllers for interval systems. In: Mediterranean Conference on Control and Automation, Rhodes, Greece; 2003.
[8] Tan N, Kaya I, Yeroglu C, Artherton DP. Computation of stabilizing PI and PID controllers using the stability boundary locus. In: Energy Conv Manag 2006;47:3045–3058.
[9] Závacká J, Bakošová M, Vaneková K. Control of laboratory chemical reactor using robust PI controller. AT&P Journal Plu2 2009;2:84–88.
[10] Závacká J, Bakošová M, Vaneková K. Riadenie laboratórneho chemického reaktora robustným PI regulátorom. Automatizace 2009;52:362–365.