Parameter discordance in Planck CMB and low-redshift measurements: projection in the primordial power spectrum

Dhiraj Kumar Hazra,$^{a,b}$ Arman Shafieloo$^{c,d}$ and Tarun Souradeep$^e$

$^a$Istituto Nazionale Di Fisica Nucleare, Sezione di Bologna,
Viale Berti Pichat, 6/2, I-40127 Bologna, Italy
$^b$Osservatorio di Astrofisica e Scienza dello Spazio di Bologna,
Istituto Nazionale di Astrofisica,
via Gobetti 101, I-40129 Bologna, Italy
$^c$Korea Astronomy and Space Science Institute,
Daejeon 34055, Korea
$^d$University of Science and Technology,
Daejeon 34113, Korea
$^e$Inter-University Centre for Astronomy and Astrophysics,
Post Bag 4, Ganeshkhind, Pune 411 007, India
E-mail: hazra@bo.infn.it, shafieloo@kasi.re.kr, tarun@iucaa.in

Received November 14, 2018
Revised March 13, 2019
Accepted April 8, 2019
Published April 23, 2019

Abstract. We discuss the discordance between the estimated values of the cosmological parameters from Planck assuming the concordance ΛCDM model and low-redshift measurements. In particular, we consider the Hubble constant mismatch between Planck temperature constraint for the ΛCDM model and the Riess et al. [1] local measurements as well as the discordance between the estimated value of $S_8$ from Planck and some weak lensing surveys such as Kilo Degree Survey (KiDS-450) and Dark Energy Survey (DES) observations. The discordance can come from a wide range of non-standard cosmological or astrophysical processes as well as from some particular systematics of the observations. In this paper, without considering any particular astrophysical process or extension to the standard model at the background level, we seek solely to project the effect of these differences in the values of the key cosmological parameters on to the shape of the primordial power spectrum (PPS). In order to realise this goal, we uncover the shape of the PPS by implementing the Modified Richardson-Lucy algorithm (MRL) that fits the Planck temperature data as acceptably as
the case of the standard model of cosmology, but with a Hubble constant consistent with local measurements as well as improving the consistency between the derived $S_8$ and $\sigma_8$ parameters with estimations of the weak lensing surveys.

**Keywords:** cosmological parameters from CMBR, CMBR theory, cosmological parameters from LSS, physics of the early universe

**ArXiv ePrint:** 1810.08101
An intriguing discrepancy between the estimated values of the cosmological parameters from Planck CMB measurements and some local observations has persisted over the last couple of years. While measurements of the Hubble constant from the local Universe is converging at $H_0 \sim 73 \text{ km/s/Mpc}$ with progressively narrower errorbars [1–4], Planck CMB measurements consistently estimate $H_0 = 67.8 \pm 0.9 \text{ km/s/Mpc}$ for the case of the concordance ΛCDM model [5, 6]. On the other hand we are also confronted with a discrepancy between growth of structure measurements from weak lensing surveys such as KiDS [7], CFHT [8], DES [9], HSC [10] and that deducted from Planck CMB measurements as reflected in the estimated values of $S_8 = \sigma_8 \sqrt{\Omega_{m0}/0.3}$. While most of these constraints on the cosmological parameters from different surveys are in fact model dependent estimations and, in particular, assume the concordance ΛCDM model of the Universe, the persistence of the discrepancy has made it imperative to investigate and reveal the nature of these disagreements. In fact, if these tensions are real and become more statistically significant with future observations (and not attributable to different systematics), we may be forced to seek physics beyond the concordance model of cosmology that effectively require exciting extensions or modifications beyond the current model [11–13]. In this work we seek a modified form of the primordial spectrum deviating from its conventional minimal power-law form within the context of the concordance model that satisfies different astrophysical and cosmological observations and also alleviates the tensions in the different estimations of key cosmological parameters. We show that it is indeed possible to project the discrepancies onto the form of the primordial power spectrum (PPS) and the reconstructed form of the PPS might potentially hint toward some specific physics in the early Universe. Nevertheless, the shape of the reconstructed form of the PPS might prove to be a guide toward the identification of some systematics in the pipelines of the data reduction and analysis of Planck CMB data as well. While the discrepancies can result from various systematics in weak lensing surveys [14] or Planck CMB data [15, 16], we also point to the consideration of the baryonic feedback in weak lensing analysis that has been suggested in literature to help reducing the tensions [17].

The paper is organized as follows: first we describe the formalism used for our analysis and reconstruction, then we present our results and state our conclusion at the end. Note that throughout the paper, we make use of solely publicly released Planck data from 2015.
2 Formalism

The primary goal behind this work is to reconstruct a form of the primordial power spectrum that results in the same CMB observables of the best fit standard ΛCDM model while also being consistent with substantially different combinations of the background cosmological parameters to accommodate some discrepancies observed. To achieve that we invoke strong priors on the key background parameters such as Hubble constant $H_0$, matter density $\Omega_m$ and $\sigma_8$ consistent with low redshift observations and then reconstruct the form of the PPS that results in the same CMB temperature observables corresponding to the best fit ΛCDM model. For the purpose of reconstruction, we use the modified Richardson-Lucy (MRL) formalism that has been introduced in [18] and has been subsequently used in [19, 20] for parameter estimation and consistency check besides aiming at finding the features in the primordial power spectrum. For early works on the Richardson-Lucy algorithm and its variants, see [21–25]. In this work we have used the following eq. (2.1). Here PPS at $i$+1th iteration, $P_{k}^{(i+1)}$, is given as a modification to the PPS at $i$th iteration, $P_{k}^{(i)}$ as has been provided in eq. (2.1). $\tilde{G}_{\ell k}$ is the transport kernel, normalized in each $\ell$. $C_{\ell}^{T(i)}$ is the theoretical angular power spectrum corresponding to the PPS at $i$th iteration, $P_{k}^{(i)}$.

$$P_{k}^{(i+1)} - P_{k}^{(i)} = P_{k}^{(i)} \times \left[ \sum_{\ell=\ell_{\text{min}}}^{\ell_{\text{max}}} \tilde{G}_{\ell k} \left\{ \left( \frac{C_{\ell}^{D} - C_{\ell}^{T(i)}}{C_{\ell}^{T(i)}} \right) \right\} \tanh^2 \left[ Q_{\ell}(C_{\ell}^{D} - C_{\ell}^{T(i)}) \right] \right]$$

(2.1)

$C_{\ell}^{D}$ is the angular power spectrum (i.e. the data) and $Q_{\ell}$ is the error covariance matrix, but following [20] and as an approximation we use just the diagonal elements.

Planck baseline is defined using standard ΛCDM model with power law form of primordial power spectrum. In addition the sum of neutrino mass is fixed to be 0.06eV and the effective number of relativistic species is fixed to be 3.046. We use Planck baseline best-fit power spectrum to Planck 2015 TT and lowT [26] data and we reconstruct the form of primordial power spectra that matches the Planck best fit power spectrum while accommodating different and specifically chosen set of background cosmological parameters. Therefore, we use the smooth best fit angular power spectrum (ΛCDM model to Planck TT data) as an input data ($C_{\ell}^{D}$ from eq. (2.1)) in our algorithm. As the input power spectrum is always positive, we do not use the binned reconstruction part even when the signal-to-noise ratio falls [18].

In our analysis we assume that the signatures of non-standard physics that may give rise to the discordance in estimated parameters between Planck CMB and other observations can be substantially captured by the features in the reconstructed PPS. This assumption is motivated from some of our previous analyses. In [19, 25] we demonstrated that allowing a free form of the PPS increases the degeneracy in the background cosmological parameters substantially. In [20], we showed, when effects of CMB lensing is not included in the analysis, MRL projects its partial imprints in the PPS as some specific form of features.

We have used a scale invariant power spectrum as an initial guess for the reconstruction but earlier studies ([23]) have shown that this algorithm converges to a power spectrum that is practically independent of the initial guess. Since we are using theoretical best fit and not the data, we do not have the disadvantage of fitting noise. In particular, for Planck 2013 data combined from all frequency channels, we had demonstrated in [20] that with higher iterations, CMB lensing effect can be significantly mimicked by the reconstructed features.
To start with and to calculate the radiative transport kernel we fix $H_0 = 73.48$ km/s/Mpc [1] (Note the form of the Kernel in figure 1 of [20]). To be consistent with results from weak lensing surveys we consider the matter density to be lower than the derived values for the case of the standard ΛCDM model fitting the Planck data. We find that using the Planck TT + lowT best fit for $\omega_b = \Omega_b h^2$ and $\omega_{CDM} = \Omega_{CDM} h^2$ provides a value of $\Omega_m = 0.259$ if $H_0 = 73.48$ km/s/Mpc is used which is suitable for our purpose. While heights in the CMB acoustic peaks and dips are sensitive to $\omega_b$ and $\omega_{CDM}$ [27–29], using the same values for our kernel results in minimal features in the form of the primordial spectrum at large scales.

Once we have the reconstructed form of the PPS from our MRL algorithm, we include scope for some fine tuning (in order to maximize the likelihood) by varying $A_{\text{scale}}$ (between 0.6 and 1.3) the overall amplitude of the reconstructed PPS, allowing a lateral shift of the feature positions using $\Delta \ln k$ (between -0.2 and 0.16) and also performing a Gaussian smoothing of the features with a smoothing width of $\Delta_{\text{smooth}}$ (between -0.7 and 1.4). Next we use the reconstructed PPS for cosmological background parameter estimation. While in the case of the standard ΛCDM model one assumes a power-law form of the PPS to perform parameter estimation, here we fix the form of the PPS to be what we have reconstructed and then establish how much cosmological parameters can vary while satisfying Planck temperature observations. In particular we are interested to see how much $\Omega_m$ and $H_0$ can vary around their fixed assumed values used to reconstruct the primordial power spectrum.

For background parameters we use the conventional parameters $\Omega_b h^2$, $\Omega_{CDM} h^2$, $100 \theta_{\text{MC}}$ and $\tau$ that denotes baryon and cold dark matter densities, the ratio of the sound horizon to the angular diameter distance at decoupling and the Thomson scattering optical depth respectively. The priors used are same as Planck baseline analysis (see, table 1 of [30]). We use CAMB [31] and CosmoMC [32] for parameter estimation. We calculate the angular power spectra at all multipoles without interpolation. We use publicly available Planck 2015 data and likelihood codes [5, 33]. In particular we use Planck TT + lowT data for comparison. We do not use the polarization data at this stage to follow a conservative approach.

One important issue here to note is the CMB lensing. In our reconstruction procedure and the likelihood analysis we considered the lensing effect for different forms of the PPS and the choice of the background parameters.

In MRL formalism the angular power spectrum data is used in unlensed format ($C_{\ell}^{\text{unlensed}}$) since the effect of lensing needs to be subtracted in order to connect the primordial spectrum to angular power spectrum via our kernel which is a function of the background parameters.

$$C_{\ell}^{\text{lensed}} = C_{\ell}^{\text{unlensed}} + C_{\ell}^{\text{lensing template}}$$

The theoretical $C_{\ell}^{\text{lensing template}}$ is obtained from the differences between the lensed and unlensed CMB angular power spectra at all multipoles. In our work we aim to have the same CMB temperature angular power spectrum observables of the best fit ΛCDM model for the case of our reconstruction. Hence to consider the lensing effect in our analysis, we should have the equality relation,

$$C_{\ell}^{\text{unlensed}}[\text{Planck best fit}] + C_{\ell}^{\text{lensing template}}[\text{Planck best fit}] = C_{\ell}^{\text{unlensed}}[\text{assumed model}] + C_{\ell}^{\text{lensing template}}[\text{assumed model}]$$

Note that in our analysis the $C_{\ell}^{\text{unlensed}}[\text{assumed model}]$ is used for the reconstruction and our assumed model described earlier has the combination of $\Omega_m = 0.259$ and $H_0 = 73.48$ km/s/Mpc where $\omega_b$, $\omega_{CDM}$ and $\tau$ are the same values of the best fit ΛCDM model.
Figure 1. The shape of the reconstructed primordial power spectra assuming $H_0 = 73.48 \text{ km/s/Mpc}$ and $\Omega_m = 0.259$. While for this combination $\omega_b, \omega_{\text{CDM}}$ have the same values as the case of the best fit $\Lambda$CDM model, these parameters can satisfy local $H_0$ measurement as well as the constraints from weak lensing surveys. Different power spectra are drawn from the samples that are consistent with the data after lateral shifts and smoothing to the original reconstructed spectrum.

3 Results

Figure 1 shows the reconstructed forms of the PPS assuming $H_0 = 73.48 \text{ km/s/Mpc}$ and $\Omega_m = 0.259$ where we also allow some lateral shifts or smoothing of the features after the reconstruction to maximize the likelihood. Results obtained overlap with each other to generate a very narrow band. The reconstructed form of the PPS show some prominent features at different scales. While we see a suppression of power at large scales, we notice sharp fluctuations in the form of the PPS at wavenumbers larger than $0.02 \text{ Mpc}^{-1}$.

We present a comparison of the parameter constraints between the standard $\Lambda$CDM model with power law PPS and the case of our study in figures 2 and 3. The comparison of the marginalized probabilities of these three parameters are provided in figure 4. The power law case is marked with Planck2015. Using the reconstructed PPS leads to high $H_0$ and low matter density around the assumed initial values (where we fixed the kernel for reconstruction) as expected. Using the reconstructed form of the PPS reduces the size of the confidence ball considerably because of the fine tuned features and no flexibility in the tilt of the reconstructed PPS. The constraints obtained on the background parameters are as follows: $\Omega_b h^2 = 0.0224 \pm 0.0002$, $\Omega_{\text{CDM}} h^2 = 0.116 \pm 0.0012$, $100\theta_{\text{MC}} = 1.054^{+0.0095}_{-0.0094}$ and $\tau = 0.095^{+0.05}_{-0.03}$. We have three parameters to tune the reconstructed power spectrum. The obtained bounds are: $A_{\text{scale}} = 0.94 \pm 0.07$, $\Delta \ln k = 0.0035 \pm 0.004$ and $\Delta_{\text{smooth}} = -6.1^{+0.3}_{-\text{unbounded}}$.

Constraints on $\sigma_8$ however remains similar, and even slightly larger than the case of the standard model with power-law form of the PPS as we allow the overall amplitude of the power spectrum to vary and $\sigma_8$ mainly depends on this variation. We also plot the marginalized constraints on $\Omega_m$ and $\sigma_8$ in figure 3 from the KiDS observation [34] in the background. We do not allow the tilt of the power spectrum to vary for the case of our reconstruction and for the case of the power law model, spectral tilt is a free parameter that is correlated with matter density. In absence of variation in tilt, matter density is tightly constrained around the value we have used to reconstruct. The reduced error in figure 2 reflects and
Figure 2. Marginalized 68% and 95% confidence contours in the $\Omega_m$ and $H_0$ plane. Planck 2015 constraints using power law as PPS (in red) and using reconstructed PPS (in blue) are shown. Note that the reconstruction prefers a higher value of $H_0$ that matches with the local Hubble parameter measurement.

Figure 3. Marginalized 68% and 95% confidence contours of $\Omega_m$ and $\sigma_8$ normalization. Planck 2015 constraints using power law PPS (in red) and using reconstructed PPS (in blue) are shown while KiDS-450 weak lensing constraints are shown in grey. Note that the discrepancy is completely removed in the case of our reconstruction where there is an overlap between the 1σ regions.

confirms the above reasoning. Here $\sigma_8$ here is primarily correlated with the overall normalization $A_{\text{scale}}$ used in the analysis and hence the degeneracy with matter density is largely lifted. It is evident that the 3σ tension can be removed by our specific form of PPS from reconstruction. In fact the marginalized estimated value of $S_8 = 0.811 \pm 0.03$ from our reconstruction fitting Planck CMB data is clearly more consistent with $S_8 = 0.745 \pm 0.038$ from
Figure 4. Marginalized probability distribution of $H_0$ [left], $\Omega_m$ [centre], $\sigma_8$ [right] are plotted. The Hubble parameter constraints are shifted to higher values that agrees with the local measurement of $H_0$ [1]. The mean value of matter density is substantially lower in the case of our reconstruction compared to the baseline case. Note that in both the cases the constraints are tighter in the case of reconstruction as its form has been fixed while there is a flexibility in the tilt of the power-law PPS. The $\sigma_8$ normalization constraints are comparable in both cases as we allow an overall amplitude shift of the PPS which is directly connected to the amplitude of the $\sigma_8$.

Figure 5. Constraints on $S_8$ from different surveys and from Planck using the reconstructed PPS. A shift to the lower value is evident with the reconstructed PPS using Planck only data.

KiDS, $S_8 = 0.783^{+0.025}_{-0.021}$ from DES and $S_8 = 0.737 \pm 0.038$ from CFHTLenS (in comparison with the case of the power law PPS). In figure 5 we plot the constraints on $S_8$ from different studies. It is intriguing that our reconstructed form of the PPS combined with the choice of the parameters we assumed, can indeed satisfy Planck CMB data as well as the local Hubble measurement and also resolves the tension between the Planck measured values of matter density and matter power normalization $\sigma_8$ with that inferred using the lensing data from KiDS-450, DES and other weak lensing surveys.
4 Discussion

In this paper we project the inconsistencies in the estimated values of the key cosmological parameters assuming the concordance cosmological model from different surveys, to the form of the primordial spectrum. In other words, we reconstruct a form of the primordial spectrum that for a particular set of cosmological parameters (that are consistent with different low redshift observations), can result identically as the case of the concordance ΛCDM model with power-law form of the PPS fitting Planck CMB temperature data. In the result section we show that this is possible. The particular form of the reconstructed PPS derived by enforcing the higher values of $H_0$ from local measurements also satisfies the lower value of $S_8$ estimated by various weak lensing surveys. While the main purpose of this paper has been to demonstrate this possibility, there are few important issues to note:

1. The features at high $k$ values are very similar to the features we reconstructed previously in [20] when we did not consider CMB lensing (trying to project the effect on the form of the PPS). While in our analysis we have considered the lensing effect for each point in the parameter space, our results appear to suggest that the CMB lensing templates prefer a background cosmology very close to the concordance model with its best fit parameters. It is not so straightforward to interpret this observation however, it might be worthy of further investigation. Our results indirectly suggest that the lensing templates which we use to analyze CMB data can have substantial effect on the constraints on the form of the PPS and background cosmological parameters [35]. This might not be evident when we use a power-law form of the PPS, but the issue brought to light more clearly when features in the form of the primordial spectrum are allowed.

2. At face value it appears to be unnatural to generate the complex form of the reconstructed PPS within an inflationary scenario without extreme fine tuning. However, we do not provide any conclusive reason to close the possibility of a physical early Universe explanation. In fact the reconstructed form of the PPS at large scales might provide a hint for a specific phenomenological model while at the small scales we might have to consider other issues such as the lensing effects indicated in the earlier point.

3. Using polarization data it should be possible to validate further the possibility of the reconstructed form of the PPS. Likewise, using polarization data we might be able to look for a more optimized form of the PPS to remove tensions from different observations. This is deferred to future study.

4. A wider exploration of the underlying parameter space of the cosmological model would be essential to reveal potential routes to ameliorate the disagreements in cosmological parameters inferred. In this work we fixed two of the key cosmological parameters, $H_0$ and $\Omega_m$ for reconstruction. An interesting possible extension of this work where the parameters are held fixed for the MRL reconstruction, is then to allow them to vary within a particular range of interest. This is planned for future works. In the MRL algorithm, we plan to make several reconstruction of the PPS corresponding to different values of $H_0$ and $\Omega_m$ consistent with low-redshift measurements. Note that we performed MRL reconstruction by varying the background parameters and thereby estimated the cosmological parameters with free-form primordial power spectrum in [19].

5. We considered a combination of key cosmological parameters in order to satisfy the local $H_0$ measurement as well as estimations of the $\sigma_8$ and $S_8$ from weak lensing surveys.
However, these constraints from weak lensing surveys are generally based on assumption of the concordance model itself where power-law form of the primordial spectrum is assumed. While we have a different form of the PPS from our reconstruction, observational constraints of $\sigma_8$ and $S_8$ might become slightly different from what they have been reported! In fact a comprehensive analysis requires an iterative approach considering all the theoretical effects as well as the effects on observational constraints. This is also beyond the scope of this work which requires a more generalized pipeline to estimate cosmological parameters.

6. Though we did not discuss in the paper, our result may also alleviate the tension between the cosmological constant dark energy and the Ly-\(\alpha\) forest BAO measurement at $z = 2.33$ [36]. The tension has been reported as a serious problem for the cosmological constant and a hint for an evolving dark energy [11, 12]. By allowing a lower matter density (with respect to the standard model cosmology fitting Planck data), the expansion history $h(z)$ in the case of $\Lambda$ dark energy can have substantially lower values at redshifts higher than two that may help to make this model more consistent with the Ly-\(\alpha\) BAO observations.

Acknowledgments

The authors would like to thank Shahab Joudaki for discussions regarding the KiDS results. The authors would like to acknowledge the use of APC cluster (https://www.apc.univ-paris7.fr/FACeWiki/pmwiki.php?n=Apc-cluster.Apc-cluster). A.S. would like to acknowledge the support of the National Research Foundation of Korea (NRF-2016R1C1B2016478). A.S. would like to acknowledge the support of the Korea Institute for Advanced Study (KIAS) grant funded by the Korea government.

References

[1] A.G. Riess et al., New Parallaxes of Galactic Cepheids from Spatially Scanning the Hubble Space Telescope: Implications for the Hubble Constant, Astrophys. J. 855 (2018) 136 [arXiv:1801.01120] [inSPIRE].

[2] HST collaboration, Final results from the Hubble Space Telescope key project to measure the Hubble constant, Astrophys. J. 553 (2001) 47 [astro-ph/0012376] [inSPIRE].

[3] A.G. Riess et al., A 2.4\% Determination of the Local Value of the Hubble Constant, Astrophys. J. 826 (2016) 56 [arXiv:1604.01424] [inSPIRE].

[4] A.G. Riess et al., Milky Way Cepheid Standards for Measuring Cosmic Distances and Application to Gaia DR2: Implications for the Hubble Constant, Astrophys. J. 861 (2018) 126 [arXiv:1804.10655] [inSPIRE].

[5] Planck collaboration, Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. 594 (2016) A13 [arXiv:1502.01589] [inSPIRE].

[6] Planck collaboration, Planck 2018 results. VI. Cosmological parameters, arXiv:1807.06209 [inSPIRE].

[7] H. Hildebrandt et al., KiDS-450: Cosmological parameter constraints from tomographic weak gravitational lensing, Mon. Not. Roy. Astron. Soc. 465 (2017) 1454 [arXiv:1606.05338] [inSPIRE].
[8] S. Joudaki et al., *CFHTLenS revisited: assessing concordance with Planck including astrophysical systematics*, Mon. Not. Roy. Astron. Soc. **465** (2017) 2033 [arXiv:1601.05786] [SPIRE].

[9] DES collaboration, *Dark Energy Survey year 1 results: Cosmological constraints from galaxy clustering and weak lensing*, Phys. Rev. D **98** (2018) 043526 [arXiv:1708.01530] [SPIRE].

[10] HSC collaboration, *Cosmology from cosmic shear power spectra with Subaru Hyper Suprime-Cam first-year data*, arXiv:1809.09148 [SPIRE].

[11] V. Sahni, A. Shafieloo and A.A. Starobinsky, *Model independent evidence for dark energy evolution from Baryon Acoustic Oscillations*, Astrophys. J. **793** (2014) L40 [arXiv:1406.2209] [SPIRE].

[12] G.-B. Zhao et al., *Dynamical dark energy in light of the latest observations*, Nat. Astron. **1** (2017) 627 [arXiv:1701.08165] [SPIRE].

[13] A. Shafieloo, B. L’Huillier and A.A. Starobinsky, *Falsifying ΛCDM: Model-independent tests of the concordance model with eBOSS DR14Q and Pantheon*, Phys. Rev. D **98** (2018) 083526 [arXiv:1804.04320] [SPIRE].

[14] S. Das, R. de Putter, E.V. Linder and R. Nakajima, *Weak lensing cosmology beyond Lambda CDM*, JCAP **11** (2012) 011 [arXiv:1102.5090] [SPIRE].

[15] Planck collaboration, *Planck 2013 results. III. LFI systematic uncertainties*, Astron. Astrophys. **571** (2014) A3 [arXiv:1303.5064] [SPIRE].

[16] Planck collaboration, *Planck intermediate results. XLVI. Reduction of large-scale systematic effects in HFI polarization maps and estimation of the reionization optical depth*, Astron. Astrophys. **596** (2016) A107 [arXiv:1605.02985] [SPIRE].

[17] M. Yoon, M.J. Jee, J.A. Tyson, S. Schmidt, D. Wittman and A. Choi, *Constraints on Cosmology and Baryonic Feedback with the Deep Lens Survey Using Galaxy-Galaxy and Galaxy-Mass Power Spectra*, Astrophys. J. **870** (2019) 111 [arXiv:1807.09195] [SPIRE].

[18] D.K. Hazra, A. Shafieloo and T. Souradeep, *Primordial power spectrum: a complete analysis with the WMAP nine-year data*, JCAP **07** (2013) 031 [arXiv:1303.4143] [SPIRE].

[19] D.K. Hazra, A. Shafieloo and T. Souradeep, *Cosmological parameter estimation with free-form primordial power spectrum*, Phys. Rev. D **87** (2013) 123528 [arXiv:1303.5336] [SPIRE].

[20] D.K. Hazra, A. Shafieloo and T. Souradeep, *Primordial power spectrum from Planck*, JCAP **11** (2014) 011 [arXiv:1406.4827] [SPIRE].

[21] W.H. Richardson, *Bayesian-Based Iterative Method of Image Restoration*, J. Opt. Soc. Am. **62** (1972) 55.

[22] L.B. Lucy, *An iterative technique for the rectification of observed distributions*, Astron. J. **79** (1974) 745 [SPIRE].

[23] A. Shafieloo and T. Souradeep, *Primordial power spectrum from WMAP*, Phys. Rev. D **70** (2004) 043523 [astro-ph/0312174] [SPIRE].

[24] A. Shafieloo, T. Souradeep, P. Manimaran, P.K. Panigrahi and R. Rangarajan, *Features in the Primordial Spectrum from WMAP: A Wavelet Analysis*, Phys. Rev. D **75** (2007) 123502 [astro-ph/0611352] [SPIRE].

[25] A. Shafieloo and T. Souradeep, *Estimation of Primordial Spectrum with post-WMAP 3 year data*, Phys. Rev. D **78** (2008) 023511 [arXiv:0709.1944] [SPIRE].

[26] Planck legacy archive, http://pla.esac.esa.int/pla/#home.

[27] W. Hu, M. Fukugita, M. Zaldarriaga and M. Tegmark, *CMB observables and their cosmological implications*, Astrophys. J. **549** (2001) 669 [astro-ph/0006436] [SPIRE].
[28] A. Aghamousa, M. Arjunwadkar and T. Souradeep, \textit{Model-independent forecasts of CMB angular power spectra for the Planck mission}, \textit{Phys. Rev. D} \textbf{89} (2014) 023509 [arXiv:1303.5143] [inSPIRE].

[29] A. Aghamousa, A. Shafieloo, M. Arjunwadkar and T. Souradeep, \textit{Unveiling acoustic physics of the CMB using nonparametric estimation of the temperature angular power spectrum for Planck}, \textit{JCAP} \textbf{02} (2015) 007 [arXiv:1412.3552] [inSPIRE].

[30] PLANCK collaboration, \textit{Planck 2013 results. XVI. Cosmological parameters}, \textit{Astron. Astrophys.} \textbf{571} (2014) A16 [arXiv:1303.5076] [inSPIRE].

[31] A. Lewis, A. Challinor and A. Lasenby, \textit{Efficient computation of CMB anisotropies in closed FRW models}, \textit{Astrophys. J.} \textbf{538} (2000) 473 [astro-ph/9911177] [inSPIRE].

[32] A. Lewis and S. Bridle, \textit{Cosmological parameters from CMB and other data: A Monte Carlo approach}, \textit{Phys. Rev. D} \textbf{66} (2002) 103511 [astro-ph/0205436] [inSPIRE].

[33] PLANCK collaboration, \textit{Planck 2015 results. XI. CMB power spectra, likelihoods and robustness of parameters}, \textit{Astron. Astrophys.} \textbf{594} (2016) A11 [arXiv:1507.02704] [inSPIRE].

[34] S. Joudaki et al., \textit{KiDS-450: Testing extensions to the standard cosmological model}, \textit{Mon. Not. Roy. Astron. Soc.} \textbf{471} (2017) 1259 [arXiv:1610.04606] [inSPIRE].

[35] J.A. Kable, G.E. Addison and C.L. Bennett, \textit{Quantifying the CMB Degeneracy Between the Matter Density and Hubble Constant in Current Experiments}, \textit{Astrophys. J.} \textbf{871} (2019) 77 [arXiv:1809.03983] [inSPIRE].

[36] J.E. Bautista et al., \textit{Measurement of baryon acoustic oscillation correlations at $z = 2.3$ with SDSS DR12 Ly-$\alpha$-Forests}, \textit{Astron. Astrophys.} \textbf{603} (2017) A12 [arXiv:1702.00176] [inSPIRE].