Hadron Mass Scaling in Regge Phenomenology

L. Burakovsky†

Theoretical Division, T-8
Los Alamos National Laboratory
Los Alamos NM 87545, USA

Abstract

We show that Regge phenomenology is consistent with the only universal scaling law for hadron masses, \( M^*/M = (\alpha'/\alpha'^*)^{1/2} \), where asterisk indicates a finite-temperature quantity. Phenomenological models further suggest the following expression of the above scaling in terms of the temperature-dependent gluon condensate: \( M^*/M = (\alpha'/\alpha'^*)^{1/2} = (\langle G_{\mu\nu}^a \rangle^*/\langle G_{\mu\nu}^a \rangle)^{1/4} \).

Key words: Regge phenomenology, hadron masses, finite temperature
PACS: 11.10.Wx, 12.40.Nn, 12.40.Yx, 12.90.+b

It is known that the properties of hadrons undergo modifications at finite temperature or in nuclear medium \[1\]. Recently considerable amount of work has been devoted to the question of the reduction of the vector meson masses at finite temperature and/or density from both theoretical and experimental viewpoints. It has been suggested that the quenching of the longitudinal response in the quasi-elastic electron-nucleus scattering \[2\] and the enhancement of the \( K^+N \) scattering cross-section \[3\] can be understood as possible consequences of the reduction of the vector meson mass in nuclear medium.

*Presented at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT ’98, Houston, Texas, Feb 9-11, 1998
†E-mail: BURAKOV@QMC.LANL.GOV
Recent experiments in heavy ion collisions which measure the dilepton mass spectrum seem to support this possibility. A detailed theoretical calculation of the dilepton mass spectrum which incorporates various dynamical aspects, including the reduction of the vector meson masses in medium, was done in [6].

Also, the question of a universal scaling behavior for all hadron masses at finite temperature and/or density has been drawn much attention in the literature. Using the effective classical Lagrangian which incorporates approximate scale and chiral invariance [7], and assuming that the ground state properties are dominated by the dilatation field which connects the chiral symmetry breaking scale with the gluon condensate via the QCD trace anomaly, Brown and Rho [8] have suggested the following universal scaling,

$$\frac{M^*(\rho)}{M(\rho)} \approx \frac{M^*(\sigma)}{M(\sigma)} \approx \frac{M^*(N)}{M(N)} \approx \frac{f_\pi^*}{f_\pi} \approx \left( \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle} \right)^{1/3} \approx \left( \frac{\langle G^a_{\mu\nu}G^a_{\mu\nu} \rangle}{\langle G^a_{\mu\nu}G^a_{\mu\nu} \rangle} \right)^{1/4},$$

(1)

where $\langle \bar{q}q \rangle$ and $\langle G^a_{\mu\nu}G^a_{\mu\nu} \rangle$ are the quark and gluon condensates, respectively, $f_\pi$ is the pion decay constant, and asterisk stands for a temperature- and/or density-dependent quantity. Physical implications of this scaling have been studied for nuclear physics [9], the equation of state for neutron stars [10], and the equation of state for hot hadronic matter [11]. It was, however, argued in [12] that the universal scaling behavior (1) is not realized in the Nambu–Jona-Lasinio model with the inclusion of a dilatation field for reasonable parameter ranges.

The purpose of the present paper is to consider the question of the universal scaling behavior for all hadron masses from the viewpoint of Regge phenomenology.

It is well known that the hadrons composed of light $(n(= u, d, s))$ quarks populate linear Regge trajectories; i.e., the square of the mass of a state with orbital momentum $\ell$ is proportional to $\ell / \alpha' + \text{const}$, where the slope $\alpha'$ only very weakly depends on the flavor content of the states lying on the corresponding trajectory: $\alpha'_{n\bar{n}} = 0.88$ GeV$^{-2}$, $\alpha'_{s\bar{n}} = 0.85$ GeV$^{-2}$, $\alpha'_{s\bar{s}} = 0.81$ GeV$^{-2}$; it therefore may be taken as a universal slope in the light quark sector, $\alpha' \approx 0.85$ GeV$^{-2}$. In this respect, the hadron masses as populating collinear trajectories do exhibit a universal behavior. Hence, it is quite natural to address the question of the scaling of the hadron masses in the framework of Regge phenomenology.

Let us therefore consider the case of finite temperature (which does not differ technically from the case of finite chemical potential), and assume a universal scaling behavior for all hadron masses which populate collinear trajectories (at zero $T$):

$$\frac{M^*}{M} = \gamma = \gamma(T),$$

(2)

where $\gamma$ is the same function of temperature for every hadronic state. It then follows that the pattern of initial collinear trajectories constrain the form of $\gamma$ in Eq. (2).

Consider three states with $\ell = 1, 3, 5$ which belong to a common trajectory at finite $T$:

$$1 = \alpha' M^2(\ell = 1) + \alpha(0), \quad 3 = \alpha' M^2(\ell = 3) + \alpha(0), \quad 5 = \alpha' M^2(\ell = 5) + \alpha(0).$$

(3)
Therefore,
\[ M^2(\ell = 5) - M^2(\ell = 3) = M^2(\ell = 3) - M^2(\ell = 1) = \frac{2}{\alpha}. \quad (4) \]

At finite \( T \), it follows from (2),(4) that
\[
M^2(\ell = 5) - M^2(\ell = 3) = M^2(\ell = 3) - M^2(\ell = 1) = 2 \alpha'.
\]
(5)

Since similar considerations are applied to three states of every trajectory, in view of (2), one easily concludes that at finite \( T \) hadrons still populate linear Regge trajectories, albeit with different slope: (since spin does not change with temperature)
\[
1 = \alpha' M^2(\ell = 1) + \alpha^*(0), \quad 3 = \alpha' M^2(\ell = 3) + \alpha^*(0), \quad 5 = \alpha' M^2(\ell = 5) + \alpha^*(0),
\]
(6)

Thus, the slopes of collinear hadron trajectories must satisfy a scaling law, as well as the hadron masses, and
\[
\frac{M^*}{M} = \left( \frac{\alpha'}{\alpha^*} \right)^{1/2}.
\]
(7)

It is now easy to show that trajectory intercepts do not change with \( T \): as follows from \( \ell = \alpha' M^2(\ell) + \alpha(0) = \alpha' M^2(\ell) + \alpha^*(0) \) and Eq. (7),
\[
\alpha^*(0) = \alpha(0).
\]
(8)

Hence, a universal scaling law (2) for all hadron states which populate collinear trajectories implies a similar scaling law for the slopes of these trajectories, Eq. (6), and the constancy of their intercepts.

Accordingly, the question about the explicit form of this scaling law in terms of temperature-dependent quark and/or gluon condensates (e.g., Eq. (1)) is directly related to the corresponding dependence of the Regge slope on these condensates.

Although an exact answer to this question is not known to us yet, a preliminary consideration of this point can be given.

In the MIT bag model, the Regge slope can be related to the bag constant \( B \) [13]:
\[
\alpha' = \frac{1}{16 \pi^{3/2}} \left( \frac{3}{2} \right)^{1/2} \frac{1}{\sqrt{\alpha_s \sqrt{B}}},
\]
(9)

which for the phenomenological values \( \alpha_s = 0.55 \) and \( B^{1/4} = 0.146 \) MeV [13] gives
\[
\alpha' \approx 0.87 \text{ GeV}^{-2},
\]
(10)
in good agreement with data. The bag pressure can be interpreted as the difference between the energy density of the physical (nonpreturbative) vacuum outside the bag.
and perturbative vacuum inside the bag (it is easily seen from the bag equation of state $p = p_{\text{free}} - B$, $\rho = \rho_{\text{free}} + B$, and therefore $B = 1/4 (\rho - 3p) = 1/4 \, \text{Tr} \, T_{\mu\nu}$). Since the energy density of the physical vacuum is related to the gluon condensate, via the trace anomaly, it is clear that $B \propto \langle G_{\mu\nu}^a G^{a\mu\nu} \rangle$, and therefore, in view of (9), $1/\alpha'^2 \propto \langle G_{\mu\nu}^a G^{a\mu\nu} \rangle$.

Also, Nambu [14] derived a string-like equation for the path-ordered phase factor $U[\sigma]$, \[ \left( \frac{\delta}{\delta \sigma_{n\tau}} \frac{\delta}{\delta \sigma_{n\tau}} + C \right) U[\sigma] = 0, \quad U[\sigma] = P \exp \left( i \int_\sigma A_\mu dz^\mu \right), \quad A_\mu = \frac{1}{2} g \Sigma A^a_\mu \lambda_a, \] (11) with \[ C = G_{n\tau} G^{n\tau} \] (12) is the gluon field tensor in the normal, $n$, and tangential, $\tau$, directions along the string. Identifying Eq. (11) with the string equations from the Nambu-Goto action leads further to \[ C = - \left( \frac{1}{2\pi \alpha'} \right)^2. \] (13) Hence, two above examples suggest that \[ \frac{1}{\alpha'^2} \propto \langle G_{\mu\nu}^a G^{a\mu\nu} \rangle. \] (14)

Although an explicit relation of $\alpha'$ to the condensates is not known to us yet, Eq. (14) may be considered as a first approximation to the actual relation. In view of (14), Eq. (7) may be rewritten as \[ \frac{M^*}{M} = \left( \frac{\alpha'}{\alpha^*} \right)^{1/2} = \left( \frac{\langle G_{\mu\nu}^a G^{a\mu\nu} \rangle^*}{\langle G_{\mu\nu}^a G^{a\mu\nu} \rangle} \right)^{1/4}. \] (15)

Following arguments of ref. [15], one can show that the scaling law (7) can be generalized to the pion decay constant, via the relation \[ \alpha' f_\pi^2 = 8\pi \beta \approx \text{const}, \] where $\beta$ is a coupling constant in front of the $\pi^+\pi^- \to \pi^+\pi^-$ Veneziano scattering amplitude. An explicit expression of this scaling law in terms of the quark condensate (or both the gluon and quark condensates) is still an open question.

Finally, we remark that, in view of the scaling law (7), it seems quite natural to express the hadron masses in terms of the Regge slope $\alpha'$. Such relations for the hadron masses were derived in [16] for mesons and baryons (below we present relations for the ground states only), \[ M^2(\rho) = \frac{1}{2\alpha'}, \quad M^2(K^*) = \frac{11}{16\alpha'}, \quad M^2(\phi) = \frac{7}{8\alpha'}, \] (16) \[ M^2(N) = \frac{3}{4\alpha'}, \quad M^2(\Sigma^*) = \frac{9}{8\alpha'}, \quad M^2(\Xi) = \frac{3}{2\alpha'}, \] (17) \[ M^2(\Delta) = \frac{5}{4\alpha'}, \quad M^2(\Sigma^*) = \frac{13}{8\alpha'}, \quad M^2(\Xi^*) = \frac{2}{\alpha'}, \quad M^2(\Omega) = \frac{19}{8\alpha'}. \] (18)
(in 17) \( M^2(\Sigma') \equiv (M^2(\Lambda) + M^2(\Sigma))/2 \), and in [17] for glueballs,

\[
M^2(0^{++}) = \frac{9}{4\alpha'}, \quad M^2(2^{++}) = \frac{9}{2\alpha'},
\]

from collinear Regge trajectories for ordinary hadrons and glueballs, and the cubic mass spectra associated with them. Eqs. (16)-(18) are in excellent agreement with experiment, and Eq. (19) with lattice QCD simulations, for \( \alpha' \approx 0.85 \text{ GeV}^{-2} \).

**Concluding remarks**

We have shown that Regge phenomenology is consistent with the only scaling behavior for all hadron masses which is given in Eq. (7), which implies that at finite temperature hadrons keep populating collinear trajectories the slope of which depends on temperature but the intercepts do not. Some phenomenological models further specify the expression of the scaling law (7) in terms of the temperature-dependent gluon condensate, as in (15). The answer to the question about whether such a universal scaling is realized in the real world, as well as its relation to the temperature-dependent quark condensate (as, e.g., in (1)), requires further theoretical and experimental study.

**Acknowledgements**

The author wishes to thank T. Goldman and L.P. Horwitz for very valuable discussions during the preparation of the present work.

**References**

[1] C. Adami and G.E. Brown, Phys. Rep. 234 (1993) 1
C.M. Ko and G.Q. Li, J. Phys. G 22 (1996) 1673

[2] G.E. Brown and M. Rho, Phys. Lett. B 222 (1989) 324

[3] G.E. Brown, C.B. Dover, P.B. Siegel and W. Weise, Phys. Rev. Lett. 60 (1988) 2723
C.M. Chew and D.J. Ernst, Phys. Rev. C 45 (1992) 2019

[4] HELIOS-3 Coll. (M. Masera et al.), Nucl. Phys. A 590 (1995) 93c
CERES Coll. (P. Wurm et al.), ibid. 103c

[5] I. Tserruya, Nucl. Phys. A 590 (1995) 127c

[6] G.Q. Li, C.M. Ko and G.E. Brown, Nucl. Phys. A 606 (1996) 568
C.M. Ko, G.Q. Li, G.E. Brown and H. Sorge, Nucl. Phys. A 610 (1996) 342c
[7] B.A. Campbell, J. Ellis and K.A. Olive, Phys. Lett. B 235 (1990) 325; Nucl. Phys. B 345 (1990) 57

[8] G.E. Brown and M. Rho, Phys. Rev. Lett. 99 (1991) 2720

[9] A. Hosaka and H. Toki, Nucl. Phys. A 529 (1991) 429
K. Kubodera and M. Rho, Phys. Rev. Lett. 67 (1991) 3479

[10] J. Ellis, J.I. Kapusta and K.A. Olive, Nucl. Phys. B 348 (1991) 345

[11] G.E. Brown, H.A. Bethe and P.M. Pizzochero, Phys. Lett. B 263 (1991) 337
G.E. Brown, A.D. Jackson, H.A. Bethe and P.M. Pizzochero, Nucl. Phys. A 560 (1993) 1035
L. Burakovsky and L.P. Horwitz, Nucl. Phys. A 614 (1997) 373

[12] K. Kusaka and W. Weise, Phys. Lett. B 288 (1992) 6

[13] K. Johnson and C.B. Thorn, Phys. Rev. D 13 (1976) 1934

[14] Y. Nambu, Phys. Lett. B 80 (1979) 372

[15] J. Pasupathy, Mod. Phys. Lett. A 12 (1997) 1943

[16] L. Burakovsky, Relativistic Statistical Mechanics and Particle Spectroscopy, Talk at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT ’98, Houston, Texas, USA, Feb 9-11, 1998, LANL preprint LA-UR-98-1694, and Found, Phys., in press

[17] L. Burakovsky, Glueball Spectroscopy in Regge Phenomenology, Talk at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT ’98, Houston, Texas, USA, Feb 9-11, 1998, LANL preprint LA-UR-98-1695 [hep-ph/9804465], and Found, Phys., in press