Measurement of $xF_3$ and $F_2$ Structure Functions in Low $Q^2$ Region with the İHEP-JINR Neutrino Detector

A. V. Sidorov$^a$, V. B. Anykeyev$^b$, Y. A. Batusov$^a$, S. A. Bunyatov$^a$, A. A. Borisov$^b$, N. I. Bozhko$^b$, S. K. Chernichenko$^b$, G. L. Chukin$^b$, O. Y. Denisov$^b$, R. M. Fachrutdinov$^b$, V. N. Goryachev$^b$, M. Y. Kazarinov$^a$, M. M. Kirsanov$^b$, O. L. Klimov$^a$, A. I. Kononov$^b$, A. S. Kozhin$^b$, A. V. Krasnoperov$^a$, V. I. Kravtsov$^b$, V. E. Kuznetsov$^a$, V. V. Lipajev$^b$, A. I. Mukhin$^b$, Y. A. Nefedov$^a$, B. A. Popov$^a$, S. N. Prakhov$^a$, Y. I. Salomatín$^b$, V. I. Snyatkov$^a$, Y. M. Sviridov$^b$, V. V. Tereshchenko$^a$, V. L. Tumakov$^b$, V. Y. Valuev$^a$, A. S. Vovenko$^b$

$^a$ JINR, 141980 Dubna Moscow Region Russia

$^b$ İHEP, 142284 Protvino Moscow Region Russia

Abstract

The isoscalar structure functions $xF_3$ and $F_2$ are measured as functions of $x$ averaged over all $Q^2$ permissible for the range of 6 to 28 GeV of incident neutrino (anti-neutrino) energy at the İHEP-JINR Neutrino Detector. The QCD analysis of $xF_3$ structure function provides $\Lambda_{\overline{MS}}^{(4)} = (411 \pm 200)$ MeV under the assumption of QCD validity in the region of low $Q^2$. The corresponding value of the strong interaction constant $\alpha_S(M_Z) = 0.123^{+0.010}_{-0.013}$ agrees with the recent result of the CCFR collaboration and with the combined LEP/SLC result.
1 Introduction

The data on deep-inelastic neutrino and anti-neutrino scattering in a wide region of momentum transfer provide a reliable basis for precise verification of QCD predictions \([1]\). In this paper we present the measurements of the \(x F_3\) and \(F_2\) structure functions (SF) and the QCD analysis of \(x F_3\) in the kinematic region of relatively small momentum transfer \(0.55 < Q^2 < 4.0\) GeV\(^2\). The value of the strong interaction constant \(\alpha_S(M_Z)\) is also evaluated and compared with the results of other experiments.

2 Data Samples

The analysis is based on data collected with three independent exposures of the IHEP-JINR Neutrino Detector \([2]\) to the wide-band neutrino and anti-neutrino beams \([3]\) of the Serpukhov U-70 accelerator. The exposure to the anti-neutrino beam (\(\bar{\nu}_\mu\)-exposure) was performed at the proton beam energy \(E_p = 70\) GeV, whereas the two \(\nu_\mu\)-exposures were carried out at \(E_p = 70\) GeV and at \(E_p = 67\) GeV. The energy of the resulting \(\nu_\mu\) (\(\bar{\nu}_\mu\)) was in the range of \(6 < E_{\nu_\mu(\bar{\nu}_\mu)} < 28\) GeV.

The experimental set-up and the selection criteria of charged current (CC) neutrino and anti-neutrino interactions are discussed in \([4]\). We restricted the range of measurements to \(W^2 > 1.7\) GeV\(^2\) in order to reject quasi-elastic and resonance events and select mainly deep-inelastic neutrino and anti-neutrino interactions. The number of protons on target (p.o.t.) for each exposure, the selected number of \(\nu_\mu\) CC and \(\bar{\nu}_\mu\) CC events and the mean values of \(Q^2\) for the three data samples are given in Table 1.

| Beam | \(\bar{\nu}_\mu\) | \(\nu_\mu\) | \(\nu_\mu\) |
|------|----------------|-------------|-------------|
| \(E_p,\) GeV | 70 | 70 | 67 |
| \(N_{p.o.t.} \times 10^{17}\) | 2.86 | 1.05 | 2.11 |
| Final statistics | 741 | 2 139 | 3 848 |
| \(\langle Q^2 \rangle,\) GeV\(^2\) | 1.2 | 2.3 |
3 Data Analysis

The SF were measured as functions of \( x \) averaged over all \( Q^2 \) permissible for the energy range \( 6 < E_\nu(\overline{\nu}) < 28 \) GeV. The events were binned in intervals of \( x \), and the values of \( x F_3 \) and \( F_2 \) were calculated in these intervals.

The number of \( \nu_\mu \) interactions, \( n^{\nu} \), and \( \overline{\nu}_\mu \) interactions, \( n^{\overline{\nu}} \), in a given bin of \( x \) is a linear combination of the average values \( \{F_2\} \) and \( \{xF_3\} \) of the respective SF in this bin (we assume invariance under the charge conjugation):

\[
\begin{align*}
n^{\nu} &= a^{\nu} \cdot \{F_2\} - b^{\nu} \cdot \{xF_3\}, \\
n^{\overline{\nu}_{1,2}} &= a^{\overline{\nu}_{1,2}} \cdot \{F_2\} + b^{\overline{\nu}_{1,2}} \cdot \{xF_3\}.
\end{align*}
\]

The subscripts 1 and 2 correspond to the \( \nu_\mu \)-exposures at \( E_p = 70 \) GeV and \( E_p = 67 \) GeV respectively. The quantities \( a^{\nu(\overline{\nu})} \) and \( b^{\nu(\overline{\nu})} \) are the integrals ("flux integrals") of products of the differential neutrino (anti-neutrino) flux \( \phi^{\nu(\overline{\nu})}(E) \) and the known factors depending on the scaling variables \( x \) and \( y \), as given by the standard form of the differential cross-section for deep-inelastic \( \nu_\mu(\overline{\nu}_\mu)\)-scattering off an isoscalar target:

\[
\begin{align*}
a^{\nu(\overline{\nu})} &= N G^2 \frac{M}{\pi} \times \\
&\int \left(1 - y - \frac{Mxy}{2E} + \frac{y^2}{2(R + 1)}\right) E \phi^{\nu(\overline{\nu})}(E) \, dx \, dy \, dE, \\
b^{\nu(\overline{\nu})} &= N G^2 \frac{M}{\pi} \int y \left(1 - \frac{y}{2}\right) E \phi^{\nu(\overline{\nu})}(E) \, dx \, dy \, dE.
\end{align*}
\]

Here \( N \) is the number of nucleons in the fiducial volume of the detector and the parameter \( R = (F_2 - 2xF_1)/2xF_1 \) measures the violation of the Callan-Gross relation \([3]\).

The number \( n^{\nu(\overline{\nu})} \) of neutrino (anti-neutrino) interactions in a given bin of \( x \) was obtained from the measured number of neutrino (anti-neutrino) events in this bin corrected for acceptance, smearing effects arising from Fermi motion and measurement uncertainties, radiative effects (following the prescription given by De Rújula et al. \([7]\)) and target non-isoscalarity (assuming \( d_\nu/u_\nu = 0.5 \)) \([8]\). To determine appropriate correction factors, the Monte Carlo simulation of the experimental set-up has been carried out using the CATAS program \([9]\). We used the Buras and Gaemers (BEBC) parameterization \([10]\) for quark distributions. The charm quark content of the nucleon was assumed to be zero. The kinematic suppression of \( d \to c \) and
$s \rightarrow c$ transitions was taken into account assuming slow rescaling \[10\] and the charm and strange quark masses of $m_c = 1.25$ GeV and $m_s = 0.25$ GeV respectively. The Fermi motion of nucleons was simulated according to \[11\]. The details of the Monte Carlo simulation are described in \[4, 12\].

The number of interactions in a given bin of $x$ is subjected to kinematic constraints imposed by the cuts on the muon momentum ($p_\mu > 1$ GeV$/c$ \[4\]), on the neutrino (anti-neutrino) energy ($6 < E_\nu(\bar{\nu}) < 28$ GeV) and on the invariant mass square of the hadronic system ($W^2 > 1.7$ GeV$^2$). These constraints were taken into account in the calculation of the flux integrals by appropriate modification of the volume of integration.

The measured values of $F_2$ and $xF_3$ structure functions are given in Table 2 and in Fig. 1.

Table 2: The isoscalar structure functions $F_2$ and $xF_3$ obtained with the assumption of $R = 0$. The difference $\Delta F_2$ between the values of $F_2$ obtained with $R = 0.1$ and with $R = 0$ is also presented. The bin edges are at $x = 0.02, 0.1, 0.2, 0.3, 0.4, 0.5$ and $0.65$. The shown systematic errors do not include the normalization error of 4% for $F_2$ and 11% for $xF_3$ originating from the uncertainties in the $\nu_\mu$ and $\overline{\nu}_\mu$ flux prediction \[13\].

| $\langle x \rangle$ | $\langle Q^2 \rangle$, GeV$^2$ | $F_2$ | stat | syst | $\Delta F_2$ | $xF_3$ | stat | syst |
|----------------------|-------------------------------|-------|-------|-------|-------------|-------|-------|-------|
| 0.052                | 0.55                          | 1.169 | .039  | 0.047 | 0.023       | 0.445 | .458  | 0.062 |
| 0.148                | 1.4                           | 1.097 | .036  | 0.022 | 0.022       | 0.583 | .087  | 0.017 |
| 0.248                | 2.2                           | 0.894 | .032  | 0.018 | 0.019       | 0.622 | .075  | 0.019 |
| 0.346                | 2.9                           | 0.576 | .028  | 0.017 | 0.013       | 0.556 | .109  | 0.011 |
| 0.447                | 3.4                           | 0.390 | .025  | 0.012 | 0.009       | 0.336 | .070  | 0.007 |
| 0.563                | 4.0                           | 0.182 | .017  | 0.004 | 0.004       | 0.177 | .117  | 0.005 |

The systematic errors presented in Table 2 come from the uncertainties in the knowledge of neutrino flux and cross-sections \[4\], imperfect detector calibration and uncertainties in the correction factors due to the choice of the input quark distributions. The overall normalization error, originating from the uncertainties in the $\nu_\mu$ and $\overline{\nu}_\mu$ flux prediction \[13\], was estimated to be 4% for $F_2$ and 11% for $xF_3$. The correction factor uncertainties were evaluated by repeating the calculation of the SF using the Field-Feynman \[14\] and GRV \[15\] parameterizations of quark distributions.
The obtained experimental data on the $xF_3$ were then compared with the QCD prediction for $Q^2$-evolution by the Jacobi polynomials method in the next-to-leading order (NLO) QCD approximation [16, 17, 18]. Performing the QCD analysis of the $xF_3$ SF, we do not discuss here the problem of validity of application of perturbative QCD predictions for the kinematic region of low $Q^2$ and do not take into account nuclear effects, heavy quarks threshold effects and higher order QCD corrections.

In order to take into account the target mass corrections, the Nachtmann moments [19] of $F_3$ are expanded in powers of $M^2_{\text{nuc}}/Q^2$. Retaining only the terms of the order of $M^2_{\text{nuc}}/Q^2$, one could obtain:

$$M_3(N, Q^2) = M_3^{QCD}(N, Q^2) + \frac{N(N+1)}{N+2} \frac{M^2_{\text{nuc}}}{Q^2} M_3^{QCD}(N+2, Q^2).$$

Here $M_3^{QCD}(N, Q^2)$ are the Mellin moments of $xF_3$:

$$M_3^{QCD}(N, Q^2) = \int_0^1 x^{N-2} xF_3(x, Q^2) dx, \quad N = 2, 3, \ldots$$

The $Q^2$-evolution of Mellin moments is defined by QCD [20, 21] and is presented here for the non-singlet case for simplicity:

$$M_3^{QCD}(N, Q^2) = \left[ \frac{\alpha_s(Q^2_0)}{\alpha_s(Q^2)} \right]^{dN} H_N(Q^2_0, Q^2) \times M_3^{QCD}(N, Q^2_0), \quad N = 2, 3, \ldots$$

$$d_N = \gamma^{(0)NS}_N / 2\beta_0,$$

$$\beta_0 = 11 - \frac{2}{3} n_f.$$

Here $\alpha_s(Q^2)$ is the strong interaction constant, $\gamma^{(0)NS}_N$ are the non-singlet leading order anomalous dimensions and $n_f$ is the number of flavours. The factor $H_N(Q^2_0, Q^2)$ contains all next-to-leading order QCD corrections [18, 21, 22].
The unknown coefficients $M_3^{QCD}(N, Q_0^2)$ in (3) could be parameterized as the Mellin moments of some function:

$$M_3^{QCD}(N, Q_0^2) = \int_0^1 x^{N-2} A x^b (1-x)^c \, dx,$$

where the constants $A$, $b$ and $c$ should be determined from the fit to the data.

Having defined the moments (1) – (4) and following the method discussed in [16, 17], we can write the $x F_3$ SF in the form:

$$x F_3^{QCD}(x, Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta^{\alpha,\beta}_n(x) \times \sum_{j=0}^{n} c^n_j(\alpha, \beta) M_3(j+2, Q^2),$$

where $\Theta^{\alpha,\beta}_n(x)$ are the Jacobi polynomials and $c^n_j(\alpha, \beta)$ are the coefficients of the expansion of $\Theta^{\alpha,\beta}_n(x)$ in powers of $x$:

$$\Theta^{\alpha,\beta}_n(x) = \sum_{j=0}^{n} c^n_j(\alpha, \beta) x^j.$$

The accuracy of the SF approximation better than 1% is achieved for $N_{\text{max}} = 9$ in a wide region of the parameters $\alpha$ and $\beta$ [17].

The higher-twist (HT) contribution is also taken into account:

$$x F_3(x, Q^2) = x F_3^{QCD}(x, Q^2) + \frac{h(x)}{Q^2},$$

where $h(x) = 0.166 - 3.746 \cdot x + 9.922 \cdot x^2 - 6.730 \cdot x^3$ is chosen by an interpolation of the NLO result for the HT contribution from [23]. This shape of $h(x)$ is in a good agreement with the theoretical prediction of [24] and with the result of [25, 26] obtained for a higher $Q^2$ kinematic region.

Using nine Mellin moments and taking into account target mass corrections, we have determined four parameters – $A$, $b$, $c$ and the QCD parameter $\Lambda_{\overline{MS}}$ (Table 3). In order to decrease the number of free parameters we have fixed the value of parameter $A$ using the Gross-Llewellyn Smith sum rule $Q^2$ - dependence: $S_{GLS} = 3 \left( 1 - \frac{\alpha_s(Q_0^2)}{\pi} \right)$ [27]. The fit was performed using the MINUIT program [28]. Three sources of errors – statistical, systematic and...
Table 3: The results of the NLO QCD fit to the $xF_3$ SF data for $n_f = 4$, $Q_0^2 = 3 \text{ GeV}^2$, $N_{max} = 9$, $\alpha = 0.7$, $\beta = 3.0$.

| Parameter | Value |
|-----------|-------|
| $\chi^2$  | 0.22  |
| $A$       | 10.4 (fixed) |
| $b$       | $0.86 \pm 0.14$ |
| $c$       | $3.83 \pm 0.61$ |
| $\Lambda_{MS}^{(4)}$ | $(411 \pm 200) \text{ MeV}$ |
| $\alpha(S(M_Z))$ | $0.123^{+0.010}_{-0.013}$ |

normalization – were summed up in quadrature. The errors for the free parameters corresponding to the 70% confidence level were obtained using the procedure described in [29]. A relatively good accuracy of the measurements of $\Lambda_{MS}^{(4)}$ was achieved due to a high sensitivity of the QCD evolution equations to the variations of $\Lambda_{MS}^{(4)}$ in the low $Q^2$ region ($0.55 < Q^2 < 4.0 \text{ GeV}^2$ in our case).

The value of $\alpha_S(M_Z)$ corresponding to the measured value of $\Lambda_{MS}^{(4)}$ was calculated from the so-called “matching relation” [30] and found to be $\alpha_S(M_Z) = 0.123^{+0.010}_{-0.013}$.

4 Discussion of the results

We have compared our results with the measurements performed by other experiments. The comparison led to the following comments:

- The parameter $\Lambda_{MS}^{(4)} = (411 \pm 200) \text{ MeV}$ of the fit to the $xF_3$ SF data is in agreement with the NLO analyses with HT contribution of the CCFR $xF_3$ data: $\Lambda_{MS}^{(4)} = (381 \pm 53 \text{ (stat)} \pm 17 \text{ (HT)}) \text{ MeV}$ [32] and $\Lambda_{MS}^{(4)} = (428 \pm 158 \text{ (exp)}) \text{ MeV}$ [26].

- The value of $\Lambda_{MS}^{(4)}$ obtained from the NLO analysis of the $xF_3$ SF provides the value of the strong interaction constant at the point of $Z$ boson mass of $\alpha_S(M_Z) = 0.123^{+0.010}_{-0.013}$ which is in agreement with the result of the analysis of the CCFR data $\alpha_s(M_Z) = 0.119 \pm 0.002 \text{ (exp)} \pm 0.004 \text{ (theory)}$ [32] and with the combined LEP/SLC
result $\alpha_s(M_Z) = 0.124 \pm 0.0043$. Our current measurement is higher by one standard deviation than the value $\alpha_s(M_Z) = 0.113 \pm 0.003 (exp) \pm 0.004 (theory)$, obtained in the $F_2$ structure function analysis of the BCDMS and SLAC data on $\mu N$ and $e N$ deep-inelastic scattering.

5 Conclusion

We have presented the measurements of the structure functions $F_2$ and $xF_3$ in the kinematic range $0.02 < x < 0.65$ and $0.55 < Q^2 < 4.0 \text{ GeV}^2$, obtained from the inclusive deep-inelastic $\nu\mu$ and $\overline{\nu}\mu$ scattering data collected at the IHEP-JINR Neutrino Detector. The NLO QCD analysis with HT contributions of $xF_3$ under the assumption of QCD validity in the region of low $Q^2$ provides $\Lambda^{(d)}_{\overline{\text{MS}}} = (411 \pm 200) \text{ MeV}$; the corresponding value of the strong interaction constant is $\alpha_s(M_Z) = 0.123^{+0.010}_{-0.013}$.

Acknowledgments

This work has been supported by the Russian Foundation for Basic Research under grants 96-02-17608, 96-02-18562 and 99-01-00091.

References

[1] G. Altarelli, in Proceedings of the “QCD – 20 Years Later” Conference, Aachen, 1992, edited by P.M. Zerwas and H.A. Kastrup (World Scientific, 1993), v.1., p.172.

[2] S.A. Bunyatov et al., in Proceedings of the International Conference on Neutrino Physics, Balatonfured, 1982, v.2, p.249.

[3] D.G. Baratov et al., Sov. J. of Technical Phys. 47 (1977) 991; A.P. Bugorsky et al., Nucl. Instr. and Meth. 146 (1977) 367.

[4] V.B. Anikeev et al., Z. Phys. C70 (1996) 39.

[5] C.G. Callan and D.J. Gross, Phys. Rev. Lett. 22 (1969) 156.

[6] A. De Rújula et al., Nucl. Phys. B154 (1979) 394.

[7] H. Abramowicz et al., Z. Phys. C17 (1983) 283.
[8] A.S. Vovenko et al., Nucl. Instr. and Meth. 212 (1983) 155.

[9] A.J. Buras and K. J.F. Gaemers, Nucl. Phys. B132 (1978) 249; K. Varvell et al., Z. Phys. C36 (1987) 1.

[10] H. Georgi and H.D. Politzer, Phys. Rev. D14 (1976) 1829; J. Kaplan and F. Martin, Nucl. Phys. B115 (1976) 333; R. Brock, Phys. Rev. Lett. 44 (1980) 1027.

[11] A. Bodek and J.L. Ritchie, Phys. Rev. D23 (1981) 1070.

[12] J. Blumlein, in Materials of the VIII Workshop on the IHEP-JINR Neutrino Detector, Dubna, 1988, p.115.

[13] Y.M. Sapunov and Y.M. Sviridov, in Materials of the XVII Workshop on the IHEP-JINR Neutrino Detector, Dubna, 1995, p.81.

[14] R. Field and R.P. Feynman, Phys. Rev. D15 (1977) 2590.

[15] M. Glück, E. Reya and A. Vogt, Z. Phys. C48 (1990) 471.

[16] G. Parisi, N. Sourlas, Nucl. Phys. B151 (1979) 421; I.S. Barker, C.B. Langensiepen and G. Shaw, Nucl. Phys. B186 (1981) 61; V.G. Krivokhizhin et al., Z. Phys. C36 (1987) 51; A. Benvenuti et al. Phys. Lett. 195B (1987) 97; 223B (1989) 490.

[17] V.G. Krivokhizhin et al., Z. Phys. C48 (1990) 347.

[18] A.L. Kataev and A.V. Sidorov, Phys. Lett. B331 (1994) 179.

[19] O. Nachtmann, Nucl. Phys. B63 (1973) 237; H. Georgi, H.D. Politzer, Phys. Rev. D14 (1976) 1829; S. Wandzura, Nucl. Phys. B122 (1977) 412.

[20] F.J. Yndurain, Quantum Chromodynamics (An Introduction to the Theory of Quarks and Gluons) (Springer-Verlag, Berlin, 1983), 117.

[21] A. Buras, Rev. Mod. Phys. 52 (1980) 199.

[22] A.L. Kataev et al., Phys. Lett. B388 (1996) 179.

[23] A.V. Sidorov, JINR Rapid Comm. 80 (1996) 11.

[24] M. Dasgupta and B.R. Webber, Phys. Lett. B382 (1996) 273.
[25] A.V. Sidorov, Phys. Lett. B389 (1996) 379.

[26] A.L. Kataev et al., Phys. Lett. B417 (1998) 374; INR P0989/98; JINR E2-98-265 (hep-ph/9809500).

[27] D.J. Gross, C.H. Llewellyn-Smith, Nucl. Phys. B14 (1969) 337. S.G. Gorishny, S.A. Larin, Phys. Lett. B172 (1986) 109. E.B. Zijstra, W.L. van Neerven, Phys. Lett. B297 (1992) 377.

[28] MINUIT, CERN Program Library Long Writeup D506 (1992).

[29] F. James, Interpretation of the Errors on Parameters as Given by MINUIT, Supplement to Long Writeup D506 (1978).

[30] W. Marciano, Phys. Rev. D29 (1984) 580.

[31] J.H. Kim et al., Phys. Rev. Lett. 81 (1998) 3595.

[32] W.G. Seligman et al., Phys. Rev. Lett. 79 (1997) 1213.

[33] Particle Data Group, C. Caso et al., Eur. Phys. J. C3 (1998) 85.

[34] M. Virchaux and A. Milsztajn, Phys. Lett. B274 (1992) 221.
Figure 1: The measured $x$-dependence of the isoscalar structure functions $F_2(x)$ and $xF_3(x)$. The statistical and systematic errors are added in quadrature (the normalization errors of 4% for $F_2$ and 11% for $xF_3$ are not shown).