Cosmic Censorship: the Role of Quantum Physics

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(August 24, 2021)

The cosmic censorship hypothesis introduced by Penrose thirty years ago is still one of the most important open questions in classical general relativity. The main goal of this paper is to put forward the idea that cosmic censorship is intrinsically a quantum phenomena. We construct a gedanken experiment which seems to violate the cosmic censorship principle within the purely classical framework of general relativity. We prove, however, that quantum physics restores the validity of the conjecture. It is therefore suggested that cosmic censorship might be enforced by a quantum theory of gravity.

I. MOTIVATION AND OBJECTIVES

It is well-known that atoms are unstable objects if viewed in the framework of the purely classical laws. The stability of atomic systems, and therefore our very existence depends in an essential manner on a quantum effect, namely the Heisenberg quantum uncertainty principle.

In some respects the black hole plays the same role in gravitation that the atom played in the early development of quantum mechanics. The quantum nature of black holes manifests itself most vividly in the form of a discrete eigenvalue spectrum for the black-hole horizon area. Thus, it is not at all surprising that the stability of the black-hole horizon might also be a quantum phenomena, basically not different from the stability of the atom. Attempts to establish a general proof for the stability of the black-hole event horizon in the framework of the purely classical laws may therefore seem quite presumptuous.

The main goal of this paper is to put forward the idea that the stability of the black-hole horizon, and therefore the cosmic censorship principle are intrinsically quantum phenomena. To that end, we shall construct a gedanken experiment which seems to violate the cosmic censorship hypothesis within the purely classical framework of general relativity. We will prove, however, that in accordance with our conjecture quantum effects restore the cosmic censorship principle to its proper status.

II. A BRIEF REVIEW OF FORMER GEDANKEN EXPERIMENTS

Spacetime singularities arising from realistic gravitational collapse scenarios are always hidden inside of black holes, and are therefore invisible to distant observers. This is the essence of the (weak) cosmic censorship hypothesis, put forward by Penrose thirty years ago. The conjecture is widely believed to be true and has become one of the corner stones of classical general relativity. However, despite the flurry of activity over the years, we are still lack a general proof of this conjecture.

When a rigorous mathematical proof seems to be beyond our present reach, it may seems quite tempting to obtain a convincing counterexample and show that the conjecture is false. If the conjecture fails, then it is quite possible that the formation of a black hole would be a non-generic outcome of gravitational collapse. If so, one might expect to find some evidence for an instability of the black-hole event horizon in physical processes which seem to have a chance of exposing the singularity hidden inside a black hole. For the advocates of the cosmic censorship principle the task remains to find out how such candidate processes eventually fail to remove the horizon.

We shall not attempt to review the numerous works that have been written addressing the question of whether or not the cosmic censorship hypothesis holds (for some of the recent reviews and lists of references, see e.g. [5, 6]). Rather, we will briefly describe those works which are directly related to the present paper.

One of the earliest attempts to eliminate the horizon of a black hole is due to Wald [7]. As is well-known, the Reissner-Nordström metric with $M < Q$ (where $M$ and $Q$ are the mass and charge of the configuration) does not contain an event horizon, and therefore describes a naked singularity. One may start with an extremal black hole (characterized by $Q = M$), and try to “supersaturate” the extremality condition by dropping in a test charged particle whose charge-to-mass ratio is greater than unity. Wald showed, however, that such an attempt would fail because of the Coulomb potential barrier surrounding the black hole.

Hiscock [8], and independently Bekenstein and Rosenzweig [9] attempted to overcharge a black hole in a different version of the gedanken experiment: suppose there exist two different types of local charge, namely type-$q \in U(1)$ and
type $k \in U'(1)$, e.g., electric and magnetic charge. The black hole is assumed to be an extremal Reissner-Nordström black hole, possessing a $U'(1)$ charge, but no $U(1)$ charge. Thus, the black hole is not endowed with a $U(1)$ gauge field, and an infalling charge $q \in U(1)$ seems to encounter no repulsive electrostatic potential barrier.

Bekenstein and Rosenzweig \cite{11} considered the specific case of a charged particle which starts falling from spatial infinity (thus, the particle’s energy-at-infinity is larger than its rest mass). It was shown in \cite{9} that such an attempt to overcharge a black hole would fail because the required classical radius of the charged body (the analogous of the well-known classical radius of the electron) is larger than the black-hole size.

The natural question immediately arises: what physical mechanism insures the stability of the horizon if the charged particle is slowly lowered towards the black hole? In this case, the energy delivered to the black hole can be red-shifted by letting the assimilation point approach the black-hole horizon. At first sight, therefore, the particle is not hindered from entering the black hole and removing its horizon, thus violating cosmic censorship. The final outcome of this ‘dangerous’ gedanken experiment has remained unclear for almost two decades. Recently, Hod \cite{11} has reexamined this old question and showed that this process actually fails to remove the horizon; the black hole preserves its integrity thanks to two factors not considered in former gedanken experiments: the effect of the spacetime curvature on the electrostatic self-interaction of the charged body (the black-hole polarization), and the finite size imposed on a charged body which respects the weak (positive) energy condition.

Quinn and Wald \cite{11} have independently suggested to use the self-energy correction in the context of the gedanken experiment recently proposed by Hubeny \cite{12}. Perhaps somewhat surprisingly, it was shown in \cite{9} that the test particle approximation actually allows a near extremal black hole to “jump over” extremality by capturing a charged particle which starts falling from spatial infinity. The (classical) effect of the self-energy turns over this conclusion.

In this paper we inquire into the physical mechanism which protects the black-hole horizon from being eliminated by the assimilation of a charged object which is slowly lowered into a (near extremal) black hole (this is the more ‘dangerous’ version of the original gedanken experiment \cite{9} [12]). We will prove that purely classical effects are actually helpless against the exposure of a naked singularity in this gedanken experiment. However, we shall propose a resolution out of this ‘embarrassing’ situation which involves the quantum properties of the vacuum.

III. THE GEDANKEN EXPERIMENT

We consider a charged body of rest mass $\mu$, charge $q$, and proper radius $b$, which is slowly descent into a (near extremal) black hole. The total energy $E$ of the body in a black-hole spacetime is made up of three contributions: 1) $E_0$, the energy associated with the body’s mass (red-shifted by the gravitational field); 2) $E_{elec}$, the electrostatic interaction of the charged body with the external electric field; and 3) $E_{self}$, the gravitationally induced self-energy of the charged body.

The first two contribution, $E_0 + E_{elec}$, are given by Carter’s \cite{13} integrals of the Lorentz equations of motion for a charged particle moving in a charged black-hole background \cite{14}:

$$E_0 + E_{elec} = \frac{\mu \ell (r_+ - r_-)}{2r_+^2} [1 + O(\ell^2/r_+^2)] + \frac{qQ}{r_+} - \frac{qQ^2 (r_+ - r_-)}{4r_+^2} [1 + O(\ell^2/r_+^2)] ,$$

where $r_\pm = M \pm (M^2 - Q^2)^{1/2}$ are the locations of the black-hole (event and inner) horizons (we use gravitational units in which $G = c = 1$), and $\ell$ is the proper distance from the horizon. Namely,

$$\ell = \ell (r) = \int_{r_+}^r \sqrt{g_{rr}} dr ,$$

with $g_{rr} = r^2/(r - r_+)(r - r_-)$.

The third contribution, $E_{self}$, reflects the effect of the spacetime curvature on the particle’s electrostatic self-interaction. The physical origin of this force is the distortion of the charge’s long-range Coulomb field by the spacetime curvature. This can also be interpreted as being due to the image charge induced inside the (polarized) black hole \cite{14} [15]. The self-interaction of a charged particle in the black-hole background results with a repulsive (i.e., directed away from the black hole) self-force. A variety of techniques have been used to demonstrate this effect in black-hole spacetimes \cite{14} [24]. In particular, the contribution of this effect to the particle’s (self) energy in the Reissner-Nordström background is $E_{self} = M q^2 / 2r_+^2$ \cite{14} [24], which implies $E_{self} = M q^2 / 2r_+^2$ to leading order in $(\ell/r_+)^2$.

We thus obtain

$$E(\ell) = \frac{\mu \ell (r_+ - r_-)}{2r_+^2} + \frac{qQ}{r_+} - \frac{qQ^2 (r_+ - r_-)}{4r_+^2} + \frac{M q^2}{2r_+^2} .$$
This expression is actually the effective potential governing the motion of a charged body in the black-hole background. Provided \( qQ > 0 \), it has a maximum located at \( \ell = \ell^* (\mu, q; M, Q) = \mu r_+^2 / qQ \). The charged body has to be over this potential barrier in order to be captured by the black hole.

The gradual approach to the black hole must stop when the proper distance from the body’s center of mass to the black-hole horizon equals \( b \), the body’s radius. For bodies which satisfy the restriction \( b \leq \ell^* \) one should therefore evaluate \( \mathcal{E} \) at the point \( \ell = b \). An assimilation of the charged object results with a change \( \Delta M = \mathcal{E} \) in the black-hole mass and a change \( \Delta Q = q \) in its charge. The condition for the black hole to preserve its integrity after the assimilation of the body is therefore

\[
q + Q \leq M + \mathcal{E} .
\]

Substituting \( \mathcal{E} = \mathcal{E}_0 + \mathcal{E}_{elec} + \mathcal{E}_{self} \) from Eq. (3) we find a necessary and sufficient condition for removal of the black-hole horizon:

\[
(q - \varepsilon)^2 + \frac{2\varepsilon}{M} \left( \mu b - q^2 - \frac{q b^2}{2M} \right) + \frac{q \varepsilon^2}{M} < 0 ,
\]

where \( r_\pm \equiv M \pm \varepsilon \). This condition is accurate to order \( O(\varepsilon^2) \). The expression on the l.h.s. of Eq. (5) is minimized for \( q = \varepsilon + O(\varepsilon^2/M) \), thus yielding

\[
2\mu b - q^2 - q b^2 / M < 0 ,
\]

as a necessary and sufficient condition for removal of the black-hole horizon.

The total mass of the charged body is given by \( \mu = \mu_0 + f q^2 / b \), where \( \mu_0 \) is the mechanical (nonelectromagnetic) mass, and \( f \) is a numerical factor of order unity which depends on how the charge is distributed inside the body. The Coulomb energy attains its minimum, \( q^2 / 2b \), when the charge is uniformly spread on a thin shell of radius \( b \), which implies \( f \geq 1 / 2 \) (an homogeneous charged sphere, for instance, has \( f = 3 / 5 \)). Therefore, any charged body which respects the weak (positive) energy condition must be larger than \( r_c \equiv q^2 / 2\mu \).

In deriving the lower bound on particle’s size, \( r_c \), one neglects the mechanical mass of the body. In fact, large stresses may be placed inside the charged body and the charge distribution must have forces of nonelectromagnetic character holding it stable. Therefore, a purely classical electromagnetic model has little relevance to the real world. Nonelectromagnetic forces imply a large contribution \( \mu_0 \) to the mass of the body from such forces. The large nonelectromagnetic contribution will prevent us from getting close to the minimal size limit \( r_c \); Atomic nuclei, for instance, are bounded by strong forces, which are often much stronger than the force exerted by the surface electric field. In fact, even atomic nuclei, which are the densest charged objects (with negligible self-gravity) in nature, satisfy the relation \( b/r_c \sim 10^2 - 10^3 \) and are therefore far larger than \( r_c \) ! Black holes with their extreme gravitational binding character are in fact the only objects in nature whose size can come close to the limit \( r_c \): An extremal Reissner-Nordström black hole, in particular, satisfies the relation \( b/r_c = 2 \) (other black holes satisfy \( b/r_c > 2 \)). Therefore, even the gravitational interaction in its extremal form as displayed in black holes cannot allow a charged object to be as small as \( r_c \).

Thus, one may safely conjecture that a charged body which respects the weak (positive) energy condition must satisfy the restriction \( b/r_c \geq 2 \) (where the equality is only saturated by the extremal Reissner-Nordström black hole). This result, combined with the inequality \( b \leq \ell^* \) (i.e., \( \mu b \geq q b^2 / M \)) implies that \( 2\mu b - q^2 - q b^2 / M \geq 0 \). We therefore conclude that the black-hole horizon cannot be removed by an assimilation of such a charged body – cosmic censorship is upheld!

For an elementary charge which is subjected to Heisenberg’s quantum uncertainty principle with \( b \sim h/\mu, r_c \) is not the measure of particle size. In fact, for \( U(1) \) charges found free in nature (weak coupling constant \( q^2 \ll h \), e.g., an electron), the classical radius \( r_c \) is far smaller than the Compton length. This is incompatible with the necessary condition Eq. (4), and we therefore recover our previous conclusion that the black-hole horizon cannot be removed.

Charged bodies which satisfy the relation \( b > \ell^* \) must have a minimal energy of \( \mathcal{E}_{min} = \mathcal{E}(\ell^*) \) in order to overcome the potential barrier, and to be captured by the black hole (this is also true for any charged object which is released to fall in from \( \ell > \ell^* \)). Taking cognizance of Eq. (5) we find that a necessary and sufficient condition for removal of the black-hole horizon is \( 2\mu \ell^* - q^2 - q \ell^{*2} / M < 0 \), or equivalently,

\[
\mu^2 / q^3 < E ,
\]

where \( E = Q/r_+^2 = M^{-1} + O(\varepsilon/M^2) \) is the black-hole electric field in the vicinity of its horizon.

Is there a physical mechanism which restrains the black-hole electric field from growing beyond the ‘dangerous’ value given in Eq. (5)? The answer seems to be negative within the framework of the purely classical laws. Note that the two series of inequalities \( q^2 / \mu \leq b \leq r_+ \leq E^{-1} \leq q^2 / \mu^2 \) and \( \mu / qE \equiv \ell^* \leq b \leq r_+ \leq E^{-1} \) are easily satisfied.
by charged objects with $\mu \ll q$. Thus, without any obvious physical mechanism which bounds the black-hole electric field, the assimilation of a charged object by a charged black hole which satisfies the condition Eq. (7) is expected to violate the cosmic censorship conjecture.

This physical picture changes, however, in the framework of the quantum laws because now there is a physical mechanism which bounds the electric-field strength – Vacuum polarization effects (Schwinger discharge of the black hole) do set an upper bound to the black-hole electric field!

A variety of techniques have been used to calculate the rate of particle production by a constant electric field in flat [26] and curved [27] spacetimes. The critical electric field, $E_c(\mu, q)$, for pair-production of particles with rest mass $\mu$ and charge $q$ is found to be $E_c(\mu, q) = \pi \mu^2/q \bar{\hbar}$. This order of magnitude can easily be understood on physical grounds: In a quantum theory the vacuum is continuously undergoing fluctuations, where a pair of “virtual” particles is created and then annihilated. The electric field tends to separate the charges. If the field is strong enough, the particles tunnel through the quantum barrier and materialize as real particles. Schwinger-type pair-production is therefore exponentially suppressed unless the work done by the electric field on the virtual pair of (charged) particles in separating them by a Compton wavelength is (at least) of the same order of magnitude of the particle’s mass.

In practice, the electric field of a black hole is bounded quantum mechanically by pair-production of the lightest charged particles in nature. This implies $E \leq E_c \equiv \pi m_e^2/|e|\bar{\hbar}$, where $m_e$ and $e$ are the rest mass and charge of the electron, respectively. We thus conclude that a necessary and sufficient condition for a violation of the cosmic censorship conjecture within the framework of a quantum theory is the existence in nature of a charged object which satisfies the inequality

$$q^3 E_c/\mu^2 > 1.$$  \hspace{1cm} (8)

Obviously, the most dangerous threat to the integrity of the black hole is imposed by the electron, which has the largest charge-to-mass ratio in nature. However, even the electron itself satisfies the relation $q^3 E_c/\mu^2 = \pi \alpha < 1$ (where $\alpha = e^2/\hbar \approx 1/137$ is the fine structure constant), and thus it cannot remove the black-hole horizon. Atomic nuclei, the densest composite charged objects in nature satisfy the relation $q^3 E_c/\mu^2 < \sim 10^{-7}$ and are therefore absolutely harmless to the black hole. Thus, we conclude that the mechanism of vacuum polarization (Schwinger discharge of the black hole) insures the integrity of the black hole. This quantum phenomena therefore prevents an exposure of a naked singularity.

IV. CONCLUSIONS

Motivated by the plausible analogy between black holes and atoms we have conjectured that the stability of the black-hole event horizon, like the stability of the atom, is intrinsically a quantum phenomena. To prove this conjecture we have constructed a gedanken experiment which may be regarded as the most ‘dangerous’ one among a large family of gedanken experiments [7–10,12] that have been designed to challenge the validity of the cosmic censorship hypothesis. It was shown that purely classical effects are actually helpless against the exposure of a naked singularity when a charged object is assimilated by a black hole. We have demonstrated explicitly, however, that quantum effects help save cosmic censorship.

Although the question of whether cosmic censorship holds remains very far from being settled, the physical picture that now arises can be summarized by the following two statements:

• The black-hole event horizon may be classically unstable while absorbing charged objects. This suggests that the purely classical laws of general relativity do not enforce cosmic censorship.

• The stability of the black-hole event horizon depends in an essential manner on quantum effects. This suggests that cosmic censorship might be enforced by a quantum theory of gravity.

We find it most intriguing that quantum effects must be invoked in order to insure the stability of the black-hole event horizon, and hence to restore the validity of the cosmic censorship principle [28]. We thus conclude that the cosmic censor must be cognizant of quantum physics.

ACKNOWLEDGMENTS

It is a pleasure to thank Jacob D. Bekenstein, Tsvi Piran, and Avraham Gal for stimulating discussions. I also wish to thank Veronika E. Hubeny for a correspondence. This research was supported by a grant from the Israel Science Foundation.
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[3] The idea to use quantum effects in order to insulate the stability of the black-hole event horizon was first suggested in the seminal work of Bekenstein and Rosenzweig [4], in which an elementary $U(1)$ charged particle with $q^2 > h$ is thrown into a Reissner-Nordström black hole with a different type of $U(1)$ charge. There are, however, two major shortfalls in this gedanken experiment which make this example not very convincing (although it is surely indicative): The discussion given in [4] is only qualitative because the analysis of a strongly coupled $U(1)$ theory is not yet feasible. More important, an elementary charged particle with $q^2 > h$ actually violates the positive energy condition, which is a classical condition. Therefore, the very existence of such particles is a quantum phenomena, and thus it is a priori obvious that quantum physics must be used in order to protect the black-hole horizon against such particles. In this paper we shall prove, perhaps somewhat surprisingly, that quantum effects must be invoked in order to preserve the black-hole integrity while absorbing absolutely ‘innocent’ objects, i.e., charged bodies which do respect the classical energy condition.

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[28] An analogous conclusion had been considered to hold true in the context of the instability of Cauchy horizons inside charged black holes embedded in de Sitter spacetimes. It was believed that these inner horizons may be classically stable, thus violating strong cosmic censorship (see [24] and references therein for additional details). Quantum effects have been suggested in order to insuire the instability of these Cauchy horizons, and thus to enforce cosmic censorship [24]. However, Brady et. al. [21] have recently shown that these Cauchy horizons are actually always classically unstable. This restores the full validity of strong cosmic censorship within the purely classical framework of general relativity, and makes it unnecessary to invoke quantum effects.

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