Supporting Information for
Friction induces anisotropic propulsion in sliding magnetic microtriangles

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Section S1: Experimental methods

Preparation of the magnetic microtriangles. PDMS molds holding microwells with triangular cross sections were fabricated using standard soft-lithographical techniques and templated by an SU8 resin as detailed in a previous work.1

To prepare the colloidal precursor dispersion we use superparamagnetic particles made of hematite with a silica shell (GE Healthcare, Serasil-Mag, diameter = 400nm) in a liquid monomer ethoxylated trimethylolpropane triacrylate (ETPTA, Sigma, Mn ~ 428). This suspension was used as a precursor formulation to fill the microwells and make the triangles. To enhance the stability of the magnetic particles within ETPTA, the magnetic particles were first treated at room temperature at a concentration of 0.1% v/v within a 5:1 methanol:ammonia(aq) (10%wt) solution with 0.5%v/v 3-(tremethoxysilyl)propyl methacrylate for two days. The now treated magnetic particles were then cleaned by five cycles of centrifugation and supernatant removal with methanol before finally transferring to ETPTA at 33%v/v with 4%v/v of the photoinitiator 2-hydroxy-2-methyl-1-phenyl-propan-1-one (Sigma) added to the final mixture.
PDMS microwells were filled with the magnetic colloidal dispersion in ETPTA by sliding a 20μL droplet of it over the PDMS surface by tilting the mold to 45°. After filling, the dispersion was reticulated in the wells overnight under a 254 nm hand-held UV lamp (NU4 KL, Benda Laborgeraete).

**Experimental setup.** The dynamics of the propelling particles are observed using an upright light microscope (Eclipse Ni, Nikon) equipped with a Charge-Coupled Device Camera (Scout scA640-74f, Basler) and different oil immersion objective (100× and 60×), depending on the degree of magnification required. We mount on the microscope stage a set of custom made magnetic coils arranged to apply time dependent magnetic fields. The coils that generate rotating field are driven by a power amplifier (IMG STA-800, stage line) which is controlled via a wave generator (Aim- TTI TGA1244). We obtain a rotating magnetic field in a given plane by passing through two perpendicular coils two sinusoidal current with 90° phase shift. Further, a static field is obtained by using a DC power supplier (TTi El 302).

**Section S2: Numerical simulation**

We provide here a detailed account of the terms of Eq. ?? which determine the dynamics of the microtriangle. \( F^g_i = -mg\hat{z} \) is the gravitational force. \( F^{LJ}_i(z_i) \) accounts for the steric interaction of a bead located a distance \( z_i \) above the bounding planar solid wall. The steric potential is the Weeks-Chandler-Andersen (WCA) potential, which consists in the repulsive part of the Lennard-Jones potential. To calculate \( F^m_i \) we assume that the magnetic torque is applied to the centre of mass of the triangle at position \( r_{CM} \).

\[
\mathbf{\tau} = \sum_{i=1}^{3} (r_i - r_{CM}) \times F^m_i \tag{1}
\]
As the torque does not induce any net force on the triangle, we add the extra constraint $\sum F_i^m = 0$, which allows to rewrite Eq 1 as

$$\tau = (r_1 - r_3) \times F_1^m + (r_2 - r_3) \times F_2^m$$

(2)

We also impose that the torque does not produce any local tensions along the sides of the triangle. At each step, we solve numerically the system of equations by performing a lower-upper (LU) decomposition combined with a backward and a forward substitution algorithm. Additionally, we ensure the separation between particles remains fixed by means of the MILC SHAKE algorithm.

$F_i^H$ accounts for the hydrodynamic interactions. In the vicinity of a stationary bounding wall, a bead moving in a viscous fluid experiences a flow generated by its own image, but also by the motion of all the other beads and their images. For Stokes flow, the fluid velocity at the position of bead $i$ can be expressed as

$$v_{iH} = \Delta \nu_i F_i^t + \sum_{j \neq i} G(r_i, r_j) F_j^t$$

(3)

in terms of the total force each bead is subject to, $F_i^t = F_i^m + F_i^m + F_i^{LJ}$. The first term in the right side of Eq. 3 corresponds to the self interaction contribution of the particle with its own image on the stationary bounding wall. The tensor $\Delta \nu$ captures this interaction $\Delta \nu = \nu(1 - \hat{z} \hat{z}) + 2\nu \hat{z} \hat{z}$, where $\nu = -\frac{3}{16} \frac{a}{z_i}$, $a$ is the hydrodynamic radius of the bead, and $z_i$ its $z$ coordinate distance to the stationary bounding wall. The second term provides the cross hydrodynamic interactions between different beads: $G(r_i, r_j) \equiv G_{ij}$ which takes into account the hydrodynamic flux contribution between a bead $j$, its image, and bead $i$. Due to this velocity field, particles experience an hydrodynamic drag that can be calculated as $F_H^i = -\gamma (\dot{r}_i - v_{iH})$, so that the final expression for the force is

$$F_H^i = -\gamma \left( \hat{n} \frac{\gamma_0}{\gamma_0} \hat{p} + \hat{p} \right) \left[ \dot{r}_i - \Delta \nu_i F_i^t \right] - \frac{1}{8\pi \eta} \sum_j F_j^t G_{ij}$$

(4)

Here we have considered an asymmetric friction $\dot{\gamma}$, which takes into account the difference in
friction of beads when they move parallel to the plane of the triangle, or perpendicular to it. The terms $\hat{n}\hat{n}$ and $\hat{p}\hat{p}$ are tensors that determine the hydrodynamic friction normal ($\hat{n}$) or perpendicular ($\hat{p}$) to the triangle plane. The scalars $\gamma_0^\perp$, $\gamma_0^\parallel$ denote the bead friction perpendicular and parallel to the triangle plane. This difference in friction accounts for the planar geometry of the triangle.

The tensor $\hat{p}\hat{p}$ defines the plane of the triangle, $\hat{p}\hat{p} = 1 - \hat{n}\hat{n}$. In the limit $\gamma_0^\perp = \gamma_0^\parallel$, Eq. 4 reduces to the scalar form of the friction tensor for spherical beads. We consider the far field hydrodynamic coupling between beads and consider the Blake-Green expression for $G_{ij}$, which takes into account the hydrodynamic interaction between beads in the presence of a stationary plane at $z = 0$.

Using the characteristic length of the triangle, $r_c$, and the characteristic relaxation time $\tau = \gamma_0^\parallel r_c^2 / |m||B|$, one can express Eq. ?? in dimensionless form as

$$\frac{t_a}{\tau} \ddot{\hat{r}}_i = -\dot{\gamma} \hat{r}_i + (\hat{F}_i^m + \hat{F}_i^g + \hat{F}_i^{LJ})(1 + \dot{\gamma} \Delta \nu_i) + \frac{3}{4} \frac{a}{r_c} \sum_j \hat{F}_j^t \hat{G}_{ij}$$

(5)

where $t_a = \frac{m}{\gamma}$ is the inertial time, $\hat{r}_i = r_i / r_c$, and $\hat{F}_i^m \equiv ||F_i^m|| |r| / |m||B|$, and the factor $a/r_c$ compares the thickness to the size of the triangle.

The force $\hat{F}_i^g = -\xi \hat{e}_z$, with $\xi \equiv r_c mg / |m||B|$ accounts for the relative magnitude of the gravitational field compared with the applied magnetic torque. A large torque compared to the triangle weight, induced by $|\vec{B}||\vec{m}|/r_c \gg mg$ implies a negligible $\xi$. Experimentally this parameter can only be controlled through the applied magnetic field. $\hat{F}_i^{LJ}$ accounts for the interaction of each bead with the bounding wall, and reads $\hat{F}_i^{LJ} = (r_c u_0 / |m|B)|\hat{e}_z| |\hat{r}_z|^{-13}$, with $u_0$ being the strength of the steric repulsion.

Therefore, the relevant parameters of the model are $a/r_c$, $\frac{\gamma_0^\perp}{\gamma_0}$, $B_y/|\vec{B}_{rot}|$, $\xi$, $f$. We tune in simulation the first three parameters, while obtain the other from the experimental system. As characteristic values, we take the hydrodynamic radius to be close to triangle thickness, $a \simeq 0.15$, $\gamma_0^\parallel = 1$ and $\frac{\gamma_0^\perp}{\gamma_0} = 2$. The rest of the parameters, $B_y/|\vec{B}_{rot}|$, $\xi$, $f\tau$ will be varied to characterise the different dynamic regimes in simulations.
We integrate Eq. 5 using an implicit, two step Velocity-Verlet algorithm in matrix notation to deal with the coupling introduced by the tensorial friction.

Section S3: Supporting video files

With the article there are 74 videoclips as support of Figures and Main text.

- **VideoS1**(AVI): This videoclip illustrates the reorientation dynamics of two magnetic micro-triangles initially aligned by a static field of amplitude 1mT along the vertical (y) direction which is subsequently switched along the horizontal (x) direction.

- **VideoS2**(AVI): This videoclip illustrates the wheel motion of a magnetic microtriangle which is driven first towards top and later towards bottom by inverting the chirality of the precessing field. The precessing field has frequency $f = 10$Hz, and amplitudes $B_x = B_z = 1.6$mT, $B_y = 0$. The video corresponds to the sequence of images at the top of Figure 2(c) of the article.

- **VideoS3**(AVI): Tumbling motion of a magnetic microtriangle which is driven first towards top and later towards bottom by inverting the chirality of the precessing field. The applied field has frequency $f = 10$Hz, and amplitudes $B_x = B_z = 1.6$mT and $B_y = 0.32$mT. The video corresponds to the sequence of images in the middle of Figure 2(c) of the article.

- **VideoS4**(AVI): Video showing the surfing like propulsion of a magnetic microtriangle which is driven first towards top and later towards bottom by inverting the chirality of the precessing field. The precessing field has frequency $f = 10$Hz, and amplitudes $B_x = B_z = 1.6$mT and $B_y = 0.32$. The video corresponds to the sequence of images at the bottom of Figure 2(c) of the article.

- **VideoS5**(AVI): This videoclip shows how a microtriangle performs a close trajectory by acquiring a transversal speed due to friction anisotropy. The center of mass position is superimposed to the image. The video has been speed-up 10×. The precessing magnetic field
has frequency \( f = 10 \text{Hz} \), and amplitudes \( B_x = B_z = 1.6 \text{mT} \) and \( B_y = 0.32 \). The video corresponds to the sequence of images in Figure 3(a) of the article.

- **VideoS6(.AVI)**: Video showing the position of the three tips of a microtriangle in the sliding mode obtained from numerical simulation. The used parameters are \( f = 0.5/\tau \) being \( \tau \) the reduced time (see text), \( B_y/B_{\text{rot}} = 1.25 \) where \( B_{\text{rot}} = |B_x + B_z| \), and \( B_x = 1.0, B_z = 1.0 \) where all field amplitudes have been made adimensional.

- **VideoS7(.AVI)**: This videoclip shows the collective transport of 6 microtriangle initially assembled to form a chain. The precessing magnetic field has frequency \( f = 10 \text{Hz} \), and amplitudes \( B_x = B_z = 1.6 \text{mT} \) and \( B_y = 1.22 \text{mT} \). The video corresponds to the sequence of images in Figure 5(a) of the article.

**References**

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