Evasive Maneuvers in Route Collision With Space Debris Cloud

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Abstract. Collisions between operational vehicles and space debris can completely derail the continuity of space missions, especially if there is chain collisions between debris, which generate even smaller fragments. In this paper, we investigate the dynamics on between an operational vehicle and space debris that form a cloud, considering the possibility of collisions between debris during an evasive maneuver the vehicle. For a radius of 3 km celestial sphere, we find possibilities of collision between debris up to 10 m, while the vehicle performs an evasive maneuver in time 3,000 s range. These results depend on the time collision, the angular positions of the collisional objects and the amount of debris that form the cloud.

1. Introduction

Debris clouds potentially represent the biggest nightmare in space missions in the regions where they occur. In many regions these clouds are generated mainly due to secondary collisions (with debris) and also fragmentation of the rockets and satellites bodies. The satellite fragmentation total is 37.7 % of cataloged objects and due to collisions are only 0.6 % (NASA - JSC, 2004). With the space activities, the distribution of these objects in the operating regions increased, even with mitigation measures and the natural reduction in the atmospheric drag of objects in LEO. Collisions with debris larger than 10 cm are considered catastrophic and still trigger a collision cascade process. This type of collision produces a long term critical density, without perspective of reduction, unless the quantity of large objects is sufficiently reduced. Control techniques should be adopted, in view of the growing demands of space campaign perpetuating along with the growth of debris in the operating regions (Kessler et al, 1978 Kessler, 1991). Simulations indicate that in a few decades the debris from fragments of the collisions will be dominant, at least in altitudes 800-1400 km (ESA, 2013). Operate in LEO regions, MEO and GEO necessarily imposes concerns about the safety of space activities due to the distribution of space debris in these altitudes. In MEO focus navigation constellations (GPS, GLONASS, etc.) and about 16,000 debris with diameters larger than 1 cm are predicted to cross the orbits in the region. Most of them have non-zero eccentricities, leading them to achieve GPS altitude at apogee. The energy produced in a collision (10⁴ J) with such objects would cause severe damage to the spacecraft (Klinkrad 2006; Rossi 2005; Rossi and Valsecchi, 2006; Smirnov, 2002). Disturbances occurring in GEO, unlike what happens in LEO due to atmospheric drag, does not reduce the amount of debris (Milani et al., 1987). The growth of GEO debris without removal...
increases in 30 debris 30 to year (Valk et al., 2009). This paper studies evasive maneuvers in debris cloud environment with the possibility of collision between them.

2. Mathematical model

The relative dynamic equations between the collisional bodies in Cartesian coordinates are:

\[
\ddot{x} - 2\omega \dot{y} - 3\omega^2 x = -v_{ex} \frac{d}{dt} \ln(M(t)),
\]

\[
\ddot{y} + 2\omega \dot{x} = -v_{ey} \frac{d}{dt} \ln(M(t)),
\]

\[
\ddot{z} + \omega^2 z = -v_{ez} \frac{d}{dt} \ln(M(t)).
\]

These questions are Hills Equations (Hill, 1878) adapted in this paper for the relative dynamics between the operational vehicle and space debris, subject to two forces: the Earth’s gravitational and the vehicle propellant (non-gravitational force). The vehicle orbits the Earth with circular \( \vec{\omega} = \omega \hat{k} \), and the y-axis is taken in the direction of its radius, as shown in Figure 1. The coordinates \( \phi \) and \( \theta \) are spherical angles to locate the debris in a system centered on the satellite with a radius \( r \). The propulsion force is required to implement the evasive maneuvers in the orbital regions (LEO, MEO, GEO) in which the angular speed assumes specific values.

This system describes the motion and is fixed on the vehicle such that all technical control evasive maneuver is performed from it. The dynamic equations are valid in that the relative distance between objects \( (r = |\vec{r}|) \) is very small compared to the distance from the vehicle to the center of the Earth \( (R = |\vec{R}|) \), that is, \( (r/R << 1) \). \( M(t) \) is the total mass of space vehicle at time \( t \). For this dynamic, we adopted exponential variation of the vehicle mass in time, such
that, 
\[ M(t) = M_o + m(t), \]  
(4) 
\[ \chi = \frac{M_o}{m_o}, \]  
(5) 
\[ M(t) = m_o(\chi + \exp(-\gamma t)), \gamma > 0. \]  
(6) 
The exhaust velocity \((v_{ex}, v_{ey}, v_{ez})\), the motor power factor of the engine \(\gamma > 0\) and the mass factor \((\chi > 1)\), i.e., the ratio of the mass of the spacecraft mass \((M_o)\) and the initial mass of the propellant \((m_o)\), call the technological parameters. For this model, the components of the propulsion force are:
\[ F_x = \gamma v_{ex}m_o e^{-\gamma t}, \]  
(7) 
\[ F_y = \gamma v_{ey}m_o e^{-\gamma t}, \]  
(8) 
\[ F_z = \gamma v_{ez}m_o e^{-\gamma t}. \]  
(9) These equations show that the propulsion force on the exponential model mass is proportional to the power factor. This factor can be controlled such that the evasive maneuver can be implemented in a given time interval, maintaining the circular orbit vehicle or operating nearly circular.

2.1. Strategy for evasive maneuvers
The evasive maneuvers are performed by acting propellants under appropriate technological strategies for each mission. The internal computer data requires the implementation of software that will use the equations of dynamics on between the vehicle and the debris. Initially, the homogeneous solution of equations (1) to (3), provide the relatives initial velocity and position on favorable to the collision. The coordinates of the relative position of these objects are obtained by scanning the spherical angles in space that contains them. The solution \(r(t)\) of the non-homogeneous equations (1) to (3) is a function of technological parameters that characterize the evasive maneuvers for our exponential mass model, Equation (4). The relative position vector Cartesian components are, therefore,
\[ x(t) = 2A \sin(nt) - 2B \cos(nt) + Et + \sum_{n=1}^{\infty} F_n e^{-n\gamma t} + G, \]  
(10) 
\[ y(t) = A \cos(nt) + B \sin(nt) - \sum_{n=1}^{\infty} C_n e^{-n\gamma t} + D, \]  
(11) 
\[ z(t) = H \cos(nt) + I \sin(nt) - \sum_{n=1}^{\infty} J_n e^{-n\gamma t}. \]  
(12) This solution was obtained before by Jesus et al (2012). All these equations coefficients depend on the initial conditions and the technological parameters which in general can be written as \(L = L(\vec{r}_o, \vec{v}_o, \vec{v}_e, \gamma, \chi, n)\). Thus, the relative final position between the collisional objects can be controlled from the adjustment of these coefficients through the suitable technological parameters to implement of evasive maneuvers. These maneuvers are possible whenever the relative final distance (on time to collision) is compared with the size of collisional objects.
So, initially without the propulsion, find the favorable initial conditions to and collision time. This can be done with a radar on board the vehicle operational. During the collision time the propulsion is burned to perform an evasive maneuver. The technological parameters must be such that the evasive maneuver can be implemented in the collision interval. We chose a "nominal solution" in the set of possible collision and the collision time. This solution is characterized by initial velocities of interest for technological applications and for a time allowing the computer implementation of the evasive maneuver. Specifically, we have chosen the initial relative velocity equal to \((7.76 \text{ km/s}, 4.25 \text{ km/s}, 1.01 \text{ km/s})\) to the regions LEO, MEO and GEO, respectively, in non-planar maneuvers. The time interval is of the order 45 minutes.

3. Study of collision with debris cloud
The study of evasive maneuvers in debris cloud environment is not trivial, because if we suppose that a spacecraft escaped from a cloud of debris, not imply that it will escape from others. In addition, the debris can themselves collide during the time of the evasive maneuver, generating smaller debris, increasing the cloud and the difficulty to complete the space mission. Depending on the sizes of objects, the first collision may occur by more than one at a time. Our results show situations in which the vehicle escapes from debris colliding with each other and further showing the angular regions that are favorable to collisions between them. We found the distribution of the approximations of positions between the debris during the collision time or evasive maneuvers time. The initial relative position between the debris and the spacecraft is 3 km and they are in different angular positions, all on a collision course with the vehicle. The initial relative velocities (extracted from histogram initial conditions) are such that spatial objects are on a collision course. Figure 2 shows the approach and avoidance trajectories between two debris and an operational vehicle.

In the Figure 2, the curves in red and blue represent the course of a collision of the debris with the space vehicle, that is, the collision of the debris 1 with the satellite and the debris 2 with the satellite, respectively. The curves in black and green are escape routes of the vehicle in front of the collision debris possibility. A cross between black and green paths are approximations...
between the space debris and, depending on their sizes and angular positions, represent collisions between them, while spacecraft escapes of both. In the histogram of Figure 3, show the amount of 90 approaches between two debris with sizes up to 50 meters. These approaches would be collisions if the debris sizes within this range. For example, in the range between 0 and 10 meters, exist 18 approaches that would be collisions if the debris is smaller than or equal to 10 meters. These results were obtained for both debris located on two pairs of fixed angle ($\phi = a$ and $\theta = b$) and a relative initial distance of 3 km with the spacecraft.

It is important to be said also that these collisions are dangerous because debris can fragment and generate a cloud of debris that further damage the evasive maneuvers of the spacecraft. These results were obtained for two pairs of fixed angles. In the Figure 4, shows a histogram in which there is a distribution of debris sizes (between 0 and 50 cm) in an angular region where the angles between 0 and 90 degrees (I Octant Figure 1).

In this graph, we see that many collisions and/or approaches (more than 10^4) occur often and the highest concentration of debris of them occurs for up to 1 cm in size.

### 3.1. Dynamic with debris cloud

In this case, we simulated collision dynamics for 5 debris. The Figure 5 shows results for the distribution of debris collisions between 5 and 10 cm in size. The angular coordinates of the debris are: $a = 0^\circ$ for all the debris and $b = 1, ..., 5^\circ$ to 5 debris, respectively. The graph shows collisions between these 5 debris without discriminating which of them collide. The statistics were performed to register any collisions between two debris and even between 5 debris. For this configuration of the angular distribution were more than 160 collisions of debris sizes between 1 and 10 cm, while an evasive maneuver is implemented by the spacecraft.

### 4. Conclusions

Our results show that occur approaches debris, while the operational vehicle performs an evasive maneuver. These approaches may be collisions between debris, depending on their sizes. We found 90 approaches between two debris up to 50 meters for fixed spherical angles for both and initial relative distance between them and the vehicle equal to 3 km. Setting the spherical angles
of the debris and varying angles in the first octant of the second debris encountered over 104 approaches for size distribution up to 50 cm and an increased incidence of collision of debris to 1 cm. For a cloud of debris with 5 fixed angles in the plane and out of plane ranging between 1 and 5 degrees, found 160 approaches and/or collisions between debris of the 5 and 10 cm sizes. The debris cloud environment is confirmed to be highly dangerous for space operations, even if they are evasive maneuvers of spacecraft, especially if they happen in octant where debris are localized initially.

5. References

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