Isolated horizons in higher dimensional Einstein–Gauss–Bonnet gravity

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Abstract

The isolated horizon framework was introduced in order to provide a local description of black holes that are in equilibrium with their (possibly dynamic) environment. Over the past several years, the framework has been extended to include matter fields (dilaton, Yang–Mills etc) in $D = 4$ dimensions and cosmological constant in $D \geq 3$ dimensions. In this paper, we present a further extension of the framework that includes black holes in higher dimensional Einstein–Gauss–Bonnet (EGB) gravity. In particular, we construct a covariant phase space for EGB gravity in arbitrary dimensions which allows us to derive the first law. We find that the entropy of a weakly isolated and non-rotating horizon is given by

$$S = \frac{1}{4G_D} \oint_{S^{D-2}} \tilde{\epsilon}(1 + 2\alpha R).$$

In this expression $S^{D-2}$ is the $(D - 2)$-dimensional cross section of the horizon with an area form $\tilde{\epsilon}$ and the Ricci scalar $R$, $G_D$ is the $D$-dimensional Newton constant and $\alpha$ is the Gauss–Bonnet parameter. This expression for the horizon entropy is in agreement with those predicted by the Euclidean and Noether charge methods. Thus we extend the isolated horizon framework beyond Einstein gravity.

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1. Introduction

The isolated horizon framework [1–4] provides a very elegant mathematical description of the mechanics of black holes by replacing the event horizon with an inner boundary contained in the spacetime manifold. There are several reasons to use this quasilocal description of black
holes in favour of the old one developed in the seventies [5–8]. Among the more significant are the following. (i) The conventional definition of a black hole is of a non-local nature because the entire future of the spacetime must be known before the event horizon can be located. (ii) If a black hole is assumed to be in equilibrium, then the surrounding spacetime must also be in equilibrium. This situation is clearly not realistic, as radiation and other forms of matter outside the black hole may be dynamical, while only the hole itself is in equilibrium. (iii) The conventional definitions of energy and angular momentum for a black hole are defined in terms of asymptotic infinity; the first law, for instance, relates quantities that are defined at spatial infinity to quantities that are defined at the horizon. Clearly, then, a more local notion of black holes must be introduced to take these issues into account.

Isolated horizons provide such a description, by ‘imitating’ the existence of a Killing vector that becomes null at the horizon. It turns out that the existence of an expansion-free null normal at the horizon is sufficient for the zeroth and first laws of black-hole mechanics to be satisfied. This is the only physical assumption in the boundary conditions. In particular, the zeroth law follows from basic differential geometry, the energy conditions and the Raychaudhuri equation. The first law then follows as a necessary and sufficient condition in the Hamiltonian evolution upon choosing an appropriate time translation vector field that points in the direction of the null normal. However, unlike its predecessor, the first law for an isolated horizon relates quantities that are all defined on the horizon. For example, the first law for rotating isolated horizons in Einstein–Maxwell (EM) theory states that variations of the mass $M$, surface area $a$, angular momentum $J$ and charge $Q$ are related via

$$\frac{\delta M}{\delta t} = \kappa \frac{\delta a}{\delta t} + \Omega \frac{\delta J}{\delta t} + \Phi \frac{\delta Q}{\delta t},$$

where the parameters $\kappa$, $\Omega$, $\Phi$ are, respectively, the surface gravity, angular velocity and electric potential. This is the equilibrium form of the first law which relates the changes between two nearby equilibrium states within the space of all solutions.

Isolated horizons have been extensively studied in Einstein gravity. In particular, the canonical phase space and covariant phase space were constructed first in terms of complex self-dual connections and $SL(2, \mathbb{C})$ soldering forms [9–11]. Shortly afterwards followed a detailed study of dilaton couplings and Yang–Mills fields [12, 13]. The formalism was then refined and polished by re-expressing the covariant phase space in terms of real Lorentz connections and tetrads [14], which paved the way for extensions to include e.g. rotation [15] and non-minimally coupled scalar fields [16]. Geometrical issues were extensively studied in [17–19]. The framework was extended to higher dimensional spacetimes in [20–22]. Important questions that need to be addressed are the following: can the isolated horizon framework be extended beyond Einstein gravity? And, if so, can the resulting framework be extended to include matter couplings? The aim of the present work is to answer the first of these questions in the affirmative, by extending the framework to Einstein–Gauss–Bonnet (EGB) gravity in arbitrary dimensions.

2. Gauss–Bonnet term in the second-order formulation

The appearance of curvature-squared terms in the effective action for gravity from superstring theory is well known [23]. This alone is enough justification for studying the effects of these extra terms on gravitational objects in higher dimensions. In addition, there are now other physical models of unification that employ a large extra dimension, including for example braneworld cosmology [24] and induced-matter theory [25].

In four dimensions, there is a unique combination of higher curvature terms containing at most second derivatives of the metric $g_{ab}$ $(a, b, \ldots \in \{0, \ldots, 3\})$ that can be added to the
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Einstein–Hilbert action, such that the equations of motion are the (vacuum) Einstein field equations. This is the Gauss–Bonnet (GB) term \[26, 27\]

\[ \mathcal{L}_{\text{GB}} = R^2 - 4 R_{ab} R^{ab} + R_{abcd} R^{abcd}, \]  

(2)

where \( R_{abcd} \) is the Riemann curvature tensor, \( R_{ab} = R^c_{acb} \) is the Ricci tensor and \( R = g^{ab} R_{ab} \) is the Ricci scalar. In this paper we employ the convention of Wald \[28\] for the Riemann tensor; the definition is given by equation (A.6) in the appendix. The complete action on a four-dimensional manifold \((\mathcal{M}, g_{ab})\) (assumed for the moment to have no boundaries) with a cosmological constant \(\Lambda_1\) is then given by

\[ S = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4 x \sqrt{-g} \left( R - 2 \Lambda + \alpha \mathcal{L}_{\text{GB}} \right). \]  

(3)

Here, \(\alpha\) is the GB parameter. In four dimensions, the GB term is a topological invariant of \(\mathcal{M}\) known as the Euler characteristic \(\chi(\mathcal{M})\), and (up to surface terms) does not contribute to the equations of motion. In \(D \geq 5\) dimensions, however, the GB term is no longer a topological invariant of \(\mathcal{M}\) (see section 3 below), and gives non-trivial modifications to the dynamics of gravity. This is precisely what happens with the Einstein–Hilbert action: it is the Euler characteristic of a two-dimensional manifold, but in \(D \geq 3\) dimensions describes the dynamics of spacetime! Therefore the GB term cannot be excluded from the action principle in dimensions \(D \geq 5\). Moreover, from the superstring theory point of view, the GB term is the only combination of curvature-squared interactions for which the low-energy effective action is ghost-free \[29\]. Therefore we consider here the gravitational action

\[ S = \frac{1}{2k_D} \int_{\mathcal{M}} d^D x \sqrt{-g} \left( R - 2 \Lambda + \alpha \mathcal{L}_{\text{GB}} \right), \]  

(4)

where now the indices run \(a, b, \ldots \in \{0, \ldots, D-1\}\). In this work, we use the standard convention for the coupling constant such that \(k_D = 8\pi G_D\) with \(G_D\) being the \(D\)-dimensional Newton constant \[30\]. The cosmological constant is given by

\[ \Lambda = \frac{\epsilon}{2l^2} (D-1)(D-2), \]  

(5)

where \(\epsilon \in \{-1, 1\}\) and \(l\) is the de Sitter radius \[31\]. In braneworld models one usually only considers the case for which \(\epsilon = -1\), i.e. asymptotically AdS spacetime.

The equations of motion are given by \(\delta S = 0\), where \(\delta\) is the first variation, i.e. the stationary points of the action. Varying action (4) with respect to the metric gives the EGB field equations

\[ G_{ab} = -\Lambda g_{ab} + \alpha \left[ \frac{1}{2} \mathcal{L}_{\text{GB}} g_{ab} - 2 R R_{ab} + 4 R_{abc} R^c_b + 4 R_{abcd} R^{cd} - 2 R_{acde} R_{b}^{\ cde} \right] \]  

(6)

\[ G_{ab} = \frac{1}{2} R g_{ab}. \]

When \(\alpha = 0\) these equations reduce to the Einstein field equations \(G_{ab} = -\Lambda g_{ab}\). The EGB equations admit the following class of (static) black hole solutions \[33\]:

\[ ds^2 = -h(r) dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_{(k)(D-2)}^2 \]  

(7)

\[ h(r) = k + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - \frac{8\alpha \Lambda}{(D-1)(D-2)} + \frac{8k_D \alpha M}{(D-2)\mathcal{V}_{(k)(D-2)}D^{-1}}} \right). \]

Here, \(\mathcal{V}_{(k)(N-1)} = \pi^{N/2}/\Gamma(N/2 + 1)\) is the volume of an \((N - 1)\)-dimensional space \(S^{N-1}\) of constant curvature with metric \(d\Omega_{(k)(N-1)}^2\). \(k\) is the curvature index with \(k = 1\) corresponding to positive constant curvature, \(k = -1\) corresponding to negative...
constant curvature, and $k = 0$ corresponding to zero curvature. $M$ is the mass of the black hole, and $\tilde{\alpha}$ is related to the GB parameter via

$$\tilde{\alpha} = (D - 3)(D - 4)\alpha.$$  \hfill (8)

The singular surfaces with radii $r_*$ are given by the roots to the equation $h(r = r_*) = 0$. We denote the event horizon by $r_*$. The location of this surface depends on the sign of the cosmological constant: if $\Lambda \leq 0$ then the largest root $r_*$ is the event horizon, and if $\Lambda > 0$ then the largest root is the cosmological horizon and therefore the second largest root is the event horizon.

The thermodynamics of the black hole is determined in the usual way [32]. In particular, the average energy $\langle E \rangle$ and entropy $S$ are given by

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \left( \ln Z \right) \quad \text{and} \quad S = \beta \langle E \rangle + \ln Z,$$  \hfill (9)

where $\ln Z$ is the (zero-loop) partition function and $\beta$ is the inverse temperature. The partition function is determined via $\ln Z = -\tilde{I}[g]$ by evaluating the Euclidean action $\tilde{I}[g]$ (in the stationary phase approximation where $g$ are solutions to the equations of motion $\delta \int \tilde{I} = 0$), and the inverse temperature is determined by requiring that the Euclidean manifold does not contain any conical singularities at $r_*$ where the manifold closes up. For the black hole solution (7) one finds that [33]

$$\langle E \rangle = M \quad \text{and} \quad S = \frac{A_{D-2} r_*^{D-2}}{4 G_D} \left[ 1 + \left( \frac{D - 2}{D - 4} \right) \frac{2\tilde{\alpha} k}{r_*^2} \right].$$  \hfill (10)

Here, $A_{N-1} = 2\pi^{N/2} / \Gamma(N/2)$ is the surface area of a unit $(N - 1)$ sphere. This shows that the entropy acquires a correction due to the presence of the GB term. A more geometrical expression for the entropy can be obtained by using the Noether charge formalism [34–36]. For EGB gravity, one finds that the entropy is [37]

$$S = \frac{1}{4 G_D} \int_{S^{D-2}} d^{D-2}x \sqrt{h(1 + 2\alpha R)},$$  \hfill (11)

where $R = R_{ij} h^{ij}$ ($i, j, \ldots \in \{0, \ldots, D - 2\}$) is the Ricci scalar determined by the metric $h_{ij} = r_*^2 \Omega_{(D-2)}^2$ on the surface $S^{D-2}$. Note, however, that this surface need not be a space of constant curvature. The assumption in the Noether charge approach is stationarity; the existence of a globally defined Killing vector field is required. One purpose of the isolated horizon framework is to relax this assumption, and to derive the zeroth and first laws of black-hole mechanics with minimal conditions assumed about the spacetimes in question. In this sense the isolated horizon framework generalizes the notion of a Killing horizon to include situations where fields outside the horizon may be dynamical.

Let us now proceed to the connection formulation of EGB gravity, which will pave the way for the construction of the corresponding covariant phase space.

3. Gauss–Bonnet term in the first-order formulation

In the connection formulation of general relativity, the configuration space consists of the pair $(e^I, A^I_j)$, where the co-frame $e^I_a = e^I_a dx^a$ determines the metric

$$g_{ab} = \eta_{IJ} e^I_a \otimes e^K_b,$$  \hfill (12)

and the connection $A^I_j = A^I_a_j dx^a$ determines the curvature 2-form

$$\Omega^I_j = dA^I_j + A^I_K \wedge A^K_j.$$  \hfill (13)
Internal indices $I, J, \ldots \in \{0, \ldots, D - 1\}$ are raised and lowered using the Minkowski metric $\eta_{IJ} = \text{diag}(-1, 1, \ldots, 1)$. The curvature defines the Riemann tensor $R^I_{JKL}$ via

$$R^I_{JKL} = \frac{1}{2} R^I_{JKL} e^K \wedge e^L.$$  \hfill (14)

The Ricci tensor is then $R_{IJ} = R^K_{IKJ}$, and the Ricci scalar is $R = \eta^{IJ} R_{IJ}$. The gauge covariant derivative $\mathcal{D}$ acts on generic fields $\Psi^I_{IJ}$ such that

$$\mathcal{D} \Psi^I_{IJ} = d \Psi^I_{IJ} + A^I_{J} \wedge \Psi^K_{I} - A^K_{J} \wedge \Psi^I_{K}.$$ \hfill (15)

Finally, the co-frame defines the $(D - m)$-form

$$\Sigma_{I_1 \ldots I_m} = \frac{1}{(D - m)!} \epsilon_{I_1 \ldots I_m I_{m+1} \ldots I_D} e^{I_{m+1}} \wedge \cdots \wedge e^{I_D},$$ \hfill (16)

where the totally antisymmetric Levi-Civita tensor $\epsilon_{I_1 \ldots I_D}$ is related to the spacetime volume element by

$$\epsilon_{a_1 \ldots a_D} = \epsilon_{I_1 \ldots I_D} e^{I_1}_{a_1} \cdots e^{I_D}_{a_D}. \hfill (17)$$

In this configuration space the action for EGB gravity becomes [26, 27]

$$S = \frac{1}{2k_D} \int_M \Sigma_{IJK} \wedge \Omega^{IJK} - 2 \Lambda \epsilon + \alpha \Sigma_{IJKLM} \wedge \Omega^{IJK} \wedge \Omega^{JKLM},$$ \hfill (18)

where $\epsilon = e^0 \wedge \cdots \wedge e^{D-1}$ is the spacetime volume element. Here the equations of motion are derived from independently varying the action with respect to the connection and co-frame. The equation of motion for the connection is

$$\mathcal{D}(\Sigma_{IJK} + 2\alpha \Sigma_{IJKLM} \wedge \Omega^{JKLM}) = 0.$$ \hfill (19)

This equation says that, in general, there exists a non-vanishing torsion $T^I = \mathcal{D}e^I$. To see what constraints are imposed on $T$, we can use the Bianchi identity $\mathcal{D}\Omega^{IJ} = 0$ together with the identity

$$\mathcal{D} \Sigma_{I_1 \ldots I_m} = \mathcal{D} e^M \wedge \Sigma_{I_1 \ldots I_m M}.$$ \hfill (20)

Substituting these into equation (19) gives

$$T^I \wedge (\Sigma_{IJK} + 2\alpha \Sigma_{IJKLM} \wedge \Omega^{LM}) = 0.$$ \hfill (21)

In analogy with Einstein gravity, we assume directly that the torsion in (21) vanishes. (The torsion in Einstein gravity is zero, but this is not an assumption. The condition follows directly from the equation of motion for the connection.) To get the equation of motion for the co-frame we note that the variation of $\Sigma$ is given by

$$\delta \Sigma_{I_1 \ldots I_m} = \delta e^M \wedge \Sigma_{I_1 \ldots I_m M}.$$ \hfill (22)

This leads to

$$\Sigma_{IJK} \wedge \Omega^{JK} - 2 \Lambda \Sigma_{I} + \alpha \Sigma_{IJKLM} \wedge \Omega^{JK} \wedge \Omega^{LM} = 0.$$ \hfill (23)

Equations (19) and (23) for the connection and co-frame are equivalent to equations (6) in the metric formulation.
4. Boundary conditions

The reasons to consider a quasilocal description of black holes were outlined in section 1. Therefore we proceed directly to the main definitions for the existence of an isolated horizon, adapted here to EGB gravity in arbitrary dimensions.

We consider a $D$-dimensional spacetime manifold $M$ with topology $\mathbb{R} \times M$ containing a $(D-1)$-dimensional null surface $\Delta$ as inner boundary (representing the event horizon), and is bounded by $(D-1)$-dimensional manifolds $M^\pm$ that intersect $\Delta$ in $(D-2)$ spaces $S^\pm$ and extend to the boundary at infinity $\mathcal{B}$. The manifold $M$ is said to be globally hyperbolic if it can be foliated by a one-parameter family of spacelike hypersurfaces $M_t$. It follows that $M_t$ are (partial) Cauchy surfaces. Then, any wave equation with solutions restricted to $M$ will have a well defined initial-value formulation (see e.g. [38]). The outer boundary $\mathcal{B}$ is some arbitrary $(D-1)$-dimensional surface, and is loosely referred to as the ‘boundary at infinity’. In other words, we consider the purely quasilocal case and neglect any subtleties that are associated with the outer boundary (see figure 1).

**Definition 1.** A non-expanding horizon $(\Delta, q_{ab}, \ell^a)$ is a $(D-1)$-dimensional null hypersurface $\Delta$ (with topology $\mathbb{R} \times S^{D-2}$) together with a degenerate metric $q_{ab}$ of signature $0^+\cdots+^+$ (with $D-2$ non-degenerate spatial directions) and a null normal $\ell^a$ such that (a) the expansion $\theta(\ell)$ of $\ell^a$ vanishes on $\Delta$, (b) the field equations hold on $\Delta$, and (c) the Ricci tensor is such that $-R_{ab}\ell^a\ell^b$ is a future-directed causal vector.

Condition (c) is analogous to the dominant energy condition imposed on any matter fields that may be present in the neighbourhood of the horizon; in Einstein gravity the condition is imposed on the stress–energy tensor, but here the condition must be imposed directly on the Ricci tensor because $G_{ab} \neq k_D T_{ab}$. Conditions (a) and (c) hold for any null normal regardless of the normalization of $\ell$. Condition (a) implies that the surface $\Delta$ is ‘time independent’ in the sense that all of its cross sections have the same area. Condition (a) also implies that $\Delta$ is a congruence of null geodesics, which in turn implies (by the Frobenius theorem) that the rotation tensor is zero. The Raychaudhuri equation then implies that $R_{ab} \ell^a\ell^b = -\sigma_{ab}\sigma^{ab}$, where $\sigma_{ab}$ is the shear tensor. From condition (c) it follows that $\sigma_{ab} = 0$ and $R_{ab}\ell^a\ell^b = 0$.

The vanishing of the expansion, rotation and shear implies that $\nabla_a \ell_b \approx \omega_a \ell_b$. (We are using the convention from constrained Hamiltonian systems whereby ‘$\approx$’ denotes equality restricted to a submanifold—in the present context the restriction is to $\Delta \subset M$. The underarrow indicates pull-back to $\Delta$.) Thus the 1-form $\omega$ is the natural connection (in the normal bundle) induced on the horizon. The ‘time independence’ of $\omega$ on $\Delta$ ensures the weak isolation of a non-expanding horizon.
**Definition 2.** A weakly isolated horizon \((\Delta, q_{ab}, [\ell])\) is a non-expanding horizon \(\Delta\) together with an equivalence class of null normals \([\ell]\) such that \(\mathcal{L}_\ell \omega_a = 0\) for all \(\ell \in [\ell]\) (where \(\ell' \sim \ell\) if \(\ell' = c\ell\) for some constant \(c\)).

The above condition is a restriction on the rescaling freedom of \(\ell\). Now, for any vector \(t^a\) tangent to \(\Delta\) we have that

\[
t^a \nabla_a \ell^b = t^a \omega_a \ell^b.
\]

In particular, because \(\ell^a\) is tangent to \(\Delta\) we have that

\[
\ell^a \nabla_a \ell^b = \ell^a \omega_a \ell^b,
\]

which means that \(\ell^a\) is geodesic. This defines the surface gravity \(\kappa(\ell) = \ell^a \omega_a\). It is important to keep in mind, however, that \(\kappa(\ell)\) is an intrinsic property not of the horizon but of the null normal; the rescaling freedom of \(\ell\) means that if \(\ell' = f \ell\) for some function \(f\), then \(\omega_a' = \omega_a + \nabla_a f\) and \(\kappa(\ell') = f \kappa(\ell) + \mathcal{L}_\ell f\). Note that under this rescaling \(\omega\) transforms as a connection. This suggests that \(\kappa(\ell)\) may not be constant on \(\Delta\). It turns out that \(\mathcal{L}_\ell \omega \approx 0\) is sufficient to obtain \(d(\ell^a \omega_a) = 0\) (see [14]). The zeroth law therefore follows from the boundary conditions and is independent of the functional content of the Lagrangian.

In this paper, for simplicity, we will restrict our attention to non-rotating weakly isolated horizons. That is, we will assume that \(\omega_a = -\kappa(\ell) n_a\). Such horizons include, but are not restricted to, those with spherical symmetry. The name arises from the fact that the non-\(n\) components of \(\omega_a\) are associated with the angular momentum of a horizon. Specifically, given a foliation of \(\Delta\) into spacelike \((D-2)\) surfaces \(S_v\) and a rotational vector field \(\phi^a\) parallel to those surfaces, the angular momentum of the horizon associated with \(\phi^a\) on a given slice is

\[
J_{\phi} = \oint_{S_v} \tilde{\epsilon} \phi^a \omega_a,
\]

where \(\tilde{\epsilon}\) is the area form on the surface. Thus, for a non-rotating horizon, \(J_{\phi} = 0\) for all rotational vector fields. For further discussion of rotational vectors and angular momentum, see, e.g. [15, 39] or one of the review articles [2–4].

### 5. Variation of the boundary term

We have seen that the boundary conditions for an isolated horizon need to be modified for EGB gravity by imposing the analogue of the dominant energy condition directly on the Ricci tensor. In the action principle, the main modification to the formalism is the appearance of an additional surface term. Let us therefore reconsider action (18) but for a region of the manifold \(M\) that is bounded by a null surface \(\Delta\) and spacelike surfaces \(M^\pm\) which extend to the (arbitrary) boundary \(\mathcal{B}\) (figure 1).

Denoting the pair \((e, A)\) collectively as a generic field variable \(\Psi\), the first variation gives

\[
\delta S = \frac{1}{2k_D} \int_M E[\Psi] \delta \Psi + \frac{1}{2k_D} \int_{\partial M} J[\Psi, \delta \Psi].
\]

Here \(E[\Psi] = 0\) symbolically denotes the equations of motion and

\[
J[\Psi, \delta \Psi] = \bar{\Sigma}_{IJ} \wedge \delta A^{IJ}
\]

is the surface term, with \(\bar{\Sigma} = (-1)^{(D-2)} \equiv (-1)^D\) and \((D-2)\)-form

\[
\bar{\Sigma}_{IJ} = \Sigma_{IJ} + 2\alpha \Sigma_{IKL} \wedge \Omega^{KL}.
\]

If the integral of \(J\) on the boundary \(\partial M\) vanishes then the action principle is said to be differentiable. We must show that this is the case. Because the fields are held fixed at \(M^\pm\)
and at $\mathcal{B}$, $J$ vanishes there. So we only need to show that $J$ vanishes at the inner boundary $\Delta$. To show that this is true we need to find an expression for $J$ in terms of $A$ and $\Sigma$ pulled back to $\Delta$. This is accomplished by fixing an internal basis consisting of the (null) pair $(\ell, n)$ and $D - 2$ spacelike vectors $\vartheta_{(i)}$ $(i \in \{2, \ldots, D - 1\})$ such that

$$e_0 = \ell, \quad e_1 = n, \quad e_i = \vartheta_{(i)},$$

(30)

together with the conditions

$$\ell \cdot n = -1, \quad \ell \cdot \ell = n \cdot n = n \cdot \vartheta_{(i)} = n \cdot \vartheta_{(i)} = 0, \quad \vartheta_{(i)} \cdot \vartheta_{(j)} = \delta_{ij}.$$

(31)

In the following we also apply the summation convention over repeated spacelike indices $(i, j, k$ etc). As these are Euclidean indices their position (up or down) will be adjusted according to the dictates of notational convenience. Thus, we employ a higher dimensional analogue of the Newman–Penrose (NP) formalism [40, 41].

To find the pull-back of $A$ we first note that

$$\nabla_a^\epsilon \ell_I \approx \nabla_a^\epsilon \ell_I + \nabla_a^\epsilon \ell_I \approx \nabla_a^\epsilon \omega_a \ell_I \approx \omega_a \ell_I,$$

(32)

where we used $\nabla_a^\epsilon \ell_I = 0$ in going from the second to the third line (a consequence of the metric compatibility of the connection). Now, taking the covariant derivative of $\ell$ acting on internal indices gives

$$\nabla_a \ell_I \approx \partial_a \ell_I + A_a^I J \ell_J,$$

(33)

where $\partial$ is a flat derivative operator that is compatible with the internal co-frame on $\Delta$. Thus $\partial_a \ell_I \approx 0$ and $\nabla_a \ell_I \approx A_a^I J \ell_J$. Putting this together with (32) we have that

$$A_a^I J \ell_J \approx \omega_a \ell_I,$$

(34)

where the $a_i$ and $b_{ij}$ are 1-forms in the cotangent space $T^*(\Delta)$. It follows that the variation of (34) is

$$\delta A_a^I J \ell_J \approx -2\ell(I \cdot n) \delta \omega_a + a_i^I \ell(I \vartheta_{(i)}) + b_{ij}^I \vartheta_{(i)} \vartheta_{(j)}.$$

(35)

Then, either by direct calculation from (34) or from the considerations of the appendix, it can be shown that on any weakly isolated and non-rotating horizon the pull-back of the associated curvature is

$$\Omega_a^b_{ij} \approx \vartheta_a^I \vartheta_b^I \mathcal{R}_{kl}^{ij} \vartheta_{(i)} \vartheta_{(j)} + 2\ell(I \vartheta_{(i)}) \Omega_a^b_{ij} \vartheta_{(i)} \vartheta_{(j)},$$

(36)

where $\mathcal{R}^{ij}_{kl}$ is the Riemann tensor associated with the $(D - 2)$ metric $\tilde{g}_{ab} = g_{ab} + \ell_a n_b + n_a \ell_b$. That is, given a foliation of $\Delta$ into spacelike $(D - 2)$ surfaces, the spacelike $\vartheta_a^{(i)}$ give an orthonormal basis on those surfaces and $\mathcal{R}^{ij}_{kl}$ is the corresponding curvature tensor; for a non-expanding horizon, these quantities are independent of both the slice of the foliation and the particular foliation itself.

To find the pull-back to $\Delta$ of $\overline{\Sigma}$, we use the decomposition

$$
\vartheta_a^I \approx -\ell(I \vartheta_a) + \vartheta_a^{(i)} \vartheta_a^{(i)},
$$

(37)
whence the \((D - 2)\)-form
\[
\sum_{IJ} \approx -\frac{1}{(D - 3)!} \epsilon_{IJ A_1 \ldots A_{D-2}} \epsilon^{A_1} (\partial_{(i_1}) \cdots \partial_{(i_{D-3})}^A (n \wedge \vartheta^{(i_1)} \wedge \cdots \wedge \vartheta^{(i_{D-3})}) \\
+ \frac{1}{(D - 2)!} \epsilon_{IJ A_1 \ldots A_{D-2}} \partial_{(i_1)}^{A_1} \cdots \partial_{(i_{D-2})}^{A_{D-2}} (\vartheta^{(i_1)} \wedge \cdots \wedge \vartheta^{(i_{D-2})}),
\]
and in \(D \geq 5\) dimensions, the \((D - 4)\)-form
\[
\sum_{IJKL} \approx -\frac{1}{(D - 5)!} \epsilon_{IJKL A_1 \ldots A_{D-4}} \epsilon^{A_1} \cdots \partial_{(i_1)}^{A_1} \cdots \partial_{(i_{D-4})}^{A_{D-4}} (n \wedge \vartheta^{(i_1)} \wedge \cdots \wedge \vartheta^{(i_{D-4})}) \\
+ \frac{1}{(D - 4)!} \epsilon_{IJKL A_1 \ldots A_{D-4}} \partial_{(i_1)}^{A_1} \cdots \partial_{(i_{D-4})}^{A_{D-4}} (\vartheta^{(i_1)} \wedge \cdots \wedge \vartheta^{(i_{D-4})}).
\]
In four dimensions \(\sum_{IJKL} = \epsilon_{IJKL}\).

These expressions are somewhat formidable but on combining them to find \(\sum_{IJ} \wedge \delta A^{IJ}\) there is significant simplification. The key is to note that each term includes a total contraction of \(\epsilon_{i_1 \ldots i_{D-4}}\). This contraction must include one copy of each of \(\ell^I, n^I\) and the \(\vartheta_{(i_1})\)—else that term will be zero. Similarly the resulting \((D - 1)\)-form must be proportional to \(n \wedge \vartheta^{(2)} \wedge \cdots \wedge \vartheta^{(D-1)}\).

Then (28) becomes
\[
J[\Psi, \delta \Psi] \approx \tilde{\varepsilon} \wedge \delta \vartheta + \frac{2\alpha}{(D - 4)!} (\epsilon_{IJKL A_1 \ldots A_{D-4}} \epsilon^{A_1} n^I \vartheta_{(i_1)} \vartheta_{(i_2)} \vartheta_{(i_3)} \cdots \vartheta_{(i_{D-4})} \\
\times \mathcal{R}_{mn}^{ij} \delta \vartheta_{(i_1)} \wedge \cdots \wedge \vartheta^{(i_{D-4})} \wedge \vartheta^{(m)} \vartheta^{(n)} \wedge \delta \vartheta).
\]

The first and second terms respectively come from the \(\sum_{eIJ}\) and \(\sum_{eIJKL}\) parts of \(\sum_{eIJ}\) while
\[
\tilde{\varepsilon} = \vartheta^{(1)} \wedge \cdots \wedge \vartheta^{(D-2)}
\]
is an area element and we keep in mind that the horizon is non-rotating so that \(\vartheta_u = -\kappa_{(e)} n_u\).

The second term therefore also simplifies. Given that there are only \((D - 4)\) elements in the spacelike basis it is reasonably easy to see that this term sums over cases where \((m, n)\) and \((i, j)\) are the same set of indices. That is (up to a numerical factor) the second term amounts to contracting \(m\) with \(i\) and \(n\) with \(j\) so that the full surface term reduces to
\[
J[\Psi, \delta \Psi] \approx \tilde{\varepsilon} (1 + 2\alpha R) \wedge \delta \vartheta.
\]

The final step is to note that \(\delta \varepsilon \propto \varepsilon\) for some \(\varepsilon\) fixed in \([\ell]\), and this together with \(\varepsilon_{\ell} \delta \vartheta = 0\) implies that \(\varepsilon_{\ell} \delta \vartheta = 0\) for \(\delta \vartheta = 0\) on the initial and final cross sections of \(\Delta\) (i.e. on \(M^- \cap \Delta\) and on \(M^+ \cap \Delta\), and because \(\delta \vartheta\) is Lie dragged on \(\Delta\) it follows that \(J \approx 0\). Therefore the surface term \(J|_{\partial M} = 0\) for EGB gravity, and we conclude that the equations of motion \(E[\Psi] = 0\) follow from the action principle \(\delta S = 0\).

### 6. Covariant phase space and the first law

In order to derive the first law we need to find the symplectic structure on the covariant phase space \(\Gamma\) consisting of solutions \((e, A)\) to the EGB field equations on \(M\). Generally, the antisymmetrized second variation of the surface term gives the symplectic current, and integrating over a partial Cauchy surface \(M\) gives the symplectic structure (the choice of \(M\) being arbitrary). Following [14], we find that the second variation of the EGB surface term (28) gives
\[
J[\Psi, \delta_1 \Psi, \delta_2 \Psi] = D[\delta_1 \tilde{\Sigma}_{IJ} \wedge \delta_2 A^{IJ} - \delta_2 \tilde{\Sigma}_{IJ} \wedge \delta_1 A^{IJ}].
\]
Integrating over $M$ defines the *bulk* symplectic structure
\[
\Omega_{\text{bulk}}(\delta_1, \delta_2) = \frac{D}{2k_D} \int_M \left[ \delta_1 \tilde{\Sigma}_{IJ} \wedge \delta_2 A^{IJ} - \delta_2 \tilde{\Sigma}_{IJ} \wedge \delta_1 A^{IJ} \right].
\] (44)

In addition, we need to find the pull-back of $J$ to $\Delta$ and add the integral of this term to $\Omega_{\text{bulk}}$ so that the resulting symplectic structure on $\Gamma$ is conserved. From (42) we find that
\[
\Omega_{\text{surface}} \approx \frac{D}{k_D} \int_{\Delta} \left[ \delta_1 [\tilde{\epsilon}(1 + 2\alpha \mathcal{R})] \wedge \delta_2 \omega - \delta_2 [\tilde{\epsilon}(1 + 2\alpha \mathcal{R})] \wedge \delta_1 \omega \right].
\] (45)

It turns out that this term is a total derivative. To see this we define a potential $\psi$ for the surface gravity such that
\[
\mathcal{L}_{\ell} \psi = \kappa(\ell).
\] (46)

Taking this into account and using the Stokes theorem, the total derivative over $\Delta$ becomes an integral over $S^{D-2}$. The full symplectic structure for EGB gravity is therefore
\[
\Omega(\delta_1, \delta_2) = \frac{1}{2k_D} \int_M \left[ \delta_1 \tilde{\Sigma}_{IJ} \wedge \delta_2 A^{IJ} - \delta_2 \tilde{\Sigma}_{IJ} \wedge \delta_1 A^{IJ} \right]
+ \frac{1}{k_D} \oint_{S^{D-2}} \left[ \delta_1 [\tilde{\epsilon}(1 + 2\alpha \mathcal{R})] \wedge \delta_2 \psi - \delta_2 [\tilde{\epsilon}(1 + 2\alpha \mathcal{R})] \wedge \delta_1 \psi \right],
\] (47)

where we have absorbed the overall (irrelevant) factor of $D$.

We can now proceed to derive the first law. To do so we need to specify a time evolution vector field $t^a$. Just as for Killing horizons, this vector field is required to approach an asymptotic time translation at infinity, and at the horizon must be a symmetry. Therefore we can restrict this vector field to the equivalence class $[\ell]$ of null vectors on the horizon. (For a rotating horizon we would also add a rotational vector $\Omega R^a$ with $\Omega$ being the angular velocity of the horizon.) The system is said to be Hamiltonian iff there exists a function $H_t$ such that
\[
\Omega(\delta, \delta_t) = \delta H_t.
\] (48)

Evaluating the symplectic structure (47) with $(\delta, \delta_t)$ gives two surface terms, one at infinity (which is identified with the ADM energy), and one at the horizon. At the horizon, we find that
\[
\Omega|_{\Delta}(\delta, \delta_t) = \delta H_t.
\] (49)

Here, we used $\kappa(\ell) = \mathcal{L}_{\ell} \psi = t \cdot \omega$. The right-hand side will be a total variation if the normalization of $t^a$ is chosen such that the functional dependence of the surface gravity is $\kappa(\ell) = \kappa(\ell) (\oint_{S^{D-2}} \tilde{\epsilon}(1 + 2\alpha \mathcal{R})).$. The vector fields with this type of normalization are commonly referred to as ‘live’ vector fields. For details see for example [14] for non-rotating horizons and [15] for rotating horizons. With this choice made, the right-hand side in the above expression is a total variation, i.e. there exists a function $E/\Delta$ such that $\Omega|_{\Delta}(\delta, \delta_t) = \delta E/\Delta$. We conclude that
\[
\delta E/\Delta = \frac{\kappa(\ell)}{k_D} \oint_{S^{D-2}} \tilde{\epsilon}(1 + 2\alpha \mathcal{R}),
\] (50)

which is the first law for the isolated horizon with energy $E_{\Delta}$. In its standard form, the first law of thermodynamics (for a quasi-static process) is $\delta E = T \delta S + \text{(work terms)}$. Here, the temperature is $T = \kappa(\ell)/2\pi$. This identifies the entropy of the isolated horizon
\[
S = \frac{1}{4G_D} \oint_{S^{D-2}} \tilde{\epsilon}(1 + 2\alpha \mathcal{R}).
\] (51)

This expression is in exact agreement with the Noether charge expression (11). As in that approach, no assumptions about the cross sections $S^{D-2}$ of the horizon need to be made. An
important difference, however, is that we did not assume the existence of a globally defined Killing vector. Instead we had to specify the existence of a time translation vector field which mimics the properties of a Killing vector but is not defined for the entire spacetime.

For the black hole solution (7) with $\Lambda = 0$ and $k = 1$, the Ricci scalar is $R = (D - 2)(D - 3)/r_+^2$ (the Ricci scalar of a $(D - 2)$-sphere with radius $r_+$), and (51) reduces to (10). Our entropy expression is therefore in agreement with the Euclidean expression as well. In our derivation, however, the entropy (51) automatically satisfies the first law (50). Note that it is possible to have black holes with negative entropies for negative constant curvature horizons when $2\alpha R < 1$. This was first discovered by Cvetič et al in [42] and later confirmed by Clunan et al [37]. For non-rotating horizons, the first law (50) implies that the energy is also negative; this is not surprising, as negative energy solutions are possible when $\Lambda < 0$ [43].

### 7. Discussion

We have shown that the isolated horizon framework can be extended beyond Einstein gravity. By constructing a covariant phase space for EGB gravity in arbitrary dimensions, we derived an expression for the entropy of the corresponding isolated horizons. This derivation is classical. The next step is to study the quantum geometry of the horizons using the state-counting arguments that were developed in [44–46], specifically for the five-dimensional solution (7). This should lead to some interesting physics. In fact, inclusion of the GB term has physical effects in four dimensions as well, because the variation $\delta L_{GB|\mathcal{M}_4} = \delta \chi (\mathcal{M}_4)$ gives a surface term that cannot be excluded if $\mathcal{M}_4$ has boundaries [47].

In this paper we considered vacuum gravity. An obvious question is whether the first law holds for cases where gravity is coupled to matter. This has been studied extensively for Einstein gravity in four dimensions [12, 13, 14, 16]. The situation is different in higher dimensions. For instance, the only Lagrangian for gravity coupled to electromagnetism in four dimensions is the EM Lagrangian

$$ S_{EM} = \frac{1}{16\pi G} \int_M \Sigma_{IJ} \wedge \Omega^{IJ} - \frac{1}{4} F \wedge * F, $$

where $F = dA$ is the curvature of the potential 1-form $A$ and $*$ denotes the Hodge dual. In $D \geq 5$ dimensions, however, one can add to the action a Chern–Simons (CS) term $A \wedge F^n$ ($n = D/2 - 1$) for the Maxwell fields. Of particular interest is the action in five dimensions, which describes minimal ($N = 1$) supergravity and is known to admit black hole solutions with non-vanishing Killing spinors [48].

One of the main assumptions that we made in our calculations was that the horizons are non-rotating. Extension of the phase space of solutions to include rotation by relaxing the condition $\tilde{\omega} = 0$ would be of interest, which can be done by using the framework that was developed in [15].

The formalism presented here can be further extended by including torsion. Recall that in section 3 we assumed $T^t = 0$ directly, which became crucial when we derived the pull-back to $\Delta$ of the connection. However, as the equation of motion for $A$ indicates, the torsion-free condition is not imposed in $D \geq 5$ dimensions; in four dimensions $\Sigma_{IJKL} = \epsilon_{IJKL}$ so that equation (19) reduces to $\mathcal{D} e = 0$ by virtue of the Bianchi identity $\mathcal{D} \Omega = 0$. If the torsion is non-zero in $D \geq 5$ dimensions then the pull-back to $\Delta$ of $A$ is not given by (34). In order to derive the modified pull-back of $A$ in the presence of torsion we would need to find $\nabla_a e^b_i$ explicitly. In addition, the Raychaudhuri equation would be different as well, and so the boundary conditions would require a more careful analysis. The effects of torsion on isolated horizons should therefore lead to some interesting consequences. This would be a particularly
interesting project to work out in five dimensions, for which a constant curvature black hole is known to have an entropy that is proportional to the surface area of the inner horizon rather than the event horizon \[49\]. To study this curiosity within the isolated horizon framework would require a modification of the boundary conditions from horizon topology \(R \times S^3\) to \(R^3 \times S^1\), which is more or less a dimensional continuation of the three-dimensional isolated horizons that was developed in \[11\].

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Appendix. Restrictions to the Riemann tensor on \(\Delta\)

In this appendix we show that for a weakly isolated and non-rotating horizon

\[
\Omega_{ab}^{1J} \approx q_a^{(k)}q_b^{(l)}R_{ij}^{(l)}\bar{\partial}_j^{(i)} + 2\bar{\Omega}_a^{(k)}\bar{\Omega}_b^{(l)}\kappa_{KL}N_{KL},
\]

as stated in equation (36).

First we establish some notation. The pull-back operator onto \(\Delta\) may be written as

\[
q_a^b = g_a^b + \ell_a\eta^b,
\]

while the pull-back operator into the tangent subspace spanned by the spacelike \(\bar{\theta}_a(i)\) is

\[
\tilde{q}_a^b = g_a^b + \ell_a\eta^b + n_a\ell^b = q_a^b + n_a\ell^b,
\]

and \(\tilde{q}_{ab}\) is also the metric on this subspace. We also define the angular momentum density

\[
\tilde{\omega}_a \equiv \tilde{q}_b^a \omega_b.
\]

It is clear that

\[
\omega_a = -\kappa(\ell)n_a + \tilde{\omega}_a,
\]

and so a horizon is non-rotating if and only if \(\tilde{\omega}_a\) vanishes.

In thinking about these quantities it is useful to keep in mind the case where \(\Delta\) is foliated into spacelike \((D-2)\) surfaces \(S_v\) which are labelled by a parameter \(v\) and \(n\) is chosen to be \(-dv\). Then \(\ell^a\) evolves the foliation surfaces while \(\ell_a\) and \(n_a\) together span the normal bundle \(T^\perp(S_v)\) on which \(\tilde{\omega}_a\) is the connection. Furthermore, the \(\bar{\theta}_a(i)\) span the tangent bundle \(T(S_v)\) and \(\tilde{q}_{ab}\) is the metric tensor for the \(S_v\).

We now turn to the Riemann tensor with the first two indices pulled back to \(\Delta\). By definition,

\[
R^{c}_{ab}d^e = -q_a^e q_b^f (\nabla_c \bar{\partial}_f - \nabla_f \bar{\partial}_c)\ell^e,
\]

and with the horizon identity \(\nabla_a \ell^b = \omega_a \ell^b\) along with the decomposition \((A.5)\), a few lines of algebra gives

\[
R^{c}_{ab}d^e = (\bar{\partial}_a \kappa(\ell)) - 2\bar{q}_a^e \bar{q}_b^f (\bar{\partial}_c \bar{\partial}_f - 2n_a \bar{q}_b^f \partial_c \bar{\partial}_f))\ell^e,
\]

where \(\bar{d}_a\) is the covariant derivative that is compatible with the metric \(\tilde{q}_{ab}\). For a weakly isolated horizon the zeroth law ensures that \(\bar{d}_a \kappa(\ell) = 0\) and if the horizon is non-rotating then \(\hat{\omega} = 0\) also, whence

\[
R^{c}_{ab}d^e = 0.
\]
Finally, using this result and (A.3) it is straightforward to see that
\[
R_{abcd}^{(i)} \gamma^{a}_{d} \gamma^{d}_{(j)} = \tilde{q}_{a}^{(i)} \tilde{q}_{b}^{f} \gamma^{f}_{c} \nabla_{d} \left( \tilde{q}_{e}^{g} \tilde{q}_{j}^{h} \gamma^{(i)}_{h} \right),
\]
(A.9)
from here one can use the fact that
\[
\tilde{d}_{a} \tilde{d}_{b} \gamma^{(i)}_{c} = \tilde{q}_{a}^{(i)} \tilde{q}_{b}^{f} \gamma^{f}_{c} \nabla_{d} \left( \tilde{q}_{e}^{g} \tilde{q}_{j}^{h} \gamma^{(i)}_{h} \right),
\]
(A.10)
and the identity for the Riemann tensor \( R_{abcd} \) associated with \( \tilde{q}_{ab} \)
\[
R_{abcd}^{(i)} \gamma^{a}_{d} = \left( \tilde{d}_{a} \tilde{d}_{b} - \tilde{d}_{a} \tilde{d}_{b} \right) \gamma^{(i)}_{a},
\]
(A.11)
along with (A.3) to show the Gauss relation
\[
\tilde{q}_{a}^{(i)} \tilde{q}_{b}^{f} \gamma^{f}_{e} \nabla_{d} \gamma_{(i)}^{d} = R_{abcd} + \left( k_{abc}^{(i)} k_{bd}^{(n)} + k_{ac}^{(i)} k_{bd}^{(n)} \right) - \left( k_{ac}^{(i)} k_{bd}^{(n)} + k_{bc}^{(n)} k_{ad}^{(i)} \right). \]
(A.12)
Here \( k_{ab}^{(i)} = \tilde{q}_{a}^{i} \tilde{q}_{e}^{d} \nabla_{e} \tilde{\ell}_{d} \) and \( k_{ab}^{(n)} = \tilde{q}_{a}^{i} \tilde{q}_{e}^{d} \nabla_{e} n_{d} \) are the extrinsic curvatures associated with \( \tilde{\ell}_{d} \) and \( n_{d} \). However, \( k_{ab}^{(i)} \) is \((1/\tilde{\theta})\partial_{(i)} \tilde{q}_{ab} + \sigma_{ab} \), and on a non-expanding horizon both the expansion and shear vanish. Thus for the cases in which we are interested
\[
\tilde{\gamma}_{a}^{(i)} \tilde{\gamma}_{b}^{f} \gamma_{c}^{f} \nabla_{d} \gamma_{d}^{(i)} = R_{abcd}.
\]
(A.13)
Then equation (36) directly follows on expanding the frame indices of \( \Omega_{ab}^{IJ} \) in terms of the \( \ell^{I} \), \( n^{I} \) and \( \theta_{(i)}^{I} \), and applying (A.8) and (A.13).

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