Brane plus Bulk Supersymmetry in Ten Dimensions

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ABSTRACT

We discuss a generalized form of IIA/IIB supergravity depending on all R-R potentials $C^{(p)}$ ($p = 0, 1, \ldots, 9$) as the effective field theory of Type IIA/IIB superstring theory. For the IIA case we explicitly break this R-R democracy to either $p \leq 3$ or $p \geq 5$ which allows us to write a new bulk action that can be coupled to $N = 1$ supersymmetric brane actions.

The case of 8-branes is studied in detail using the new bulk & brane action. The supersymmetric negative tension branes without matter excitations can be viewed as orientifolds in the effective action. These D8-branes and O8-planes are fundamental in Type I' string theory. A BPS 8-brane solution is given which satisfies the jump conditions on the wall. As an application of our results we derive a quantization of the mass parameter and the cosmological constant in string units.
MOTIVATION

Our purpose is to construct supersymmetric domain walls of string theory in $D = 10$ which may shed some light on the stringy origin of the brane world scenarios. In the process of pursuing this goal we have realized that all descriptions of the effective field theory of Type IIA/B string theory available in the literature are inefficient for our purpose. This has led us to introduce new versions of the effective supergravities corresponding to Type IIA/B string theory.

The standard IIA massless supergravity includes the $C^{(1)}$ and $C^{(3)}$ R-R potentials and the corresponding $G^{(2)}$ and $G^{(4)}$ gauge-invariant R-R forms. Type IIB supergravity includes the $C^{(0)}$, $C^{(2)}$ and $C^{(4)}$ R-R potentials and the corresponding $G^{(1)}$, $G^{(3)}$ and (self-dual) $G^{(5)}$ gauge-invariant R-R forms. On the other hand, string theory has all $D^p$-branes, odd and even, including the exotic ones, like 8-branes in IIA and 7-branes in IIB theory. These branes, of co-dimension 1 and 2, are special objects which are different in many respects from the other BPS-extended objects like the $p$-branes with $0 \leq p \leq 6$, which have co-dimension greater than or equal to 3. The basic difference is in the behavior of the form fields at large distance, $G^{(p+2)} \sim r^{p-8}$. For example the $G^{(10)}$ R-R form of the 8-brane does not fall off at infinity but takes a constant value there. It is believed that such extended objects can not exist independently but only in connection with orientifold planes [1]. However, the realization of the total system in supergravity is rather obscure.

It has been realized a while ago [2] that massive IIA supergravity, discovered by Romans [3], was the key to understand the spacetime picture of the 8-branes, which are domain walls in $D = 10$. A significant progress towards the understanding of the 8-brane solutions was made in [4, 5], where the bulk supergravity solution was found. Also, in [5], the description of the cosmological constant via a 9-form potential, based upon the work of [6, 7], was discussed. In [8] a standard 8-brane action coupling to this 9-form potential has been shown to be the appropriate source for the second Randall–Sundrum scenario [9]. Solutions for the coupled bulk & brane action system automatically satisfy the jump conditions and so they are consistent, at least from this point of view. A major unsolved problem was to find an explicitly supersymmetric description of coupled bulk & brane systems like it was done in [10]. Such a description should allow us to find out some important properties of the domain walls like the distance between the planes, the status of unbroken supersymmetry in the bulk and on the brane etc. We expect that realizing such a bulk & brane construction will lead to a better insight into the fundamental nature of extended objects of string theory.

The string backgrounds that we want to describe using an explicitly supersymmetric bulk & brane action are one-dimensional orbifolds obtained by modding out the circle $S^1$ by a reflection $Z_2$. The orbifold direction is the transverse direction of the branes that fill the rest of the spacetime. Now, the orbifold $S^1/Z_2$ being a compact space, we cannot place a single charged object (a D8-brane, say) in it, but we have to have at least two oppositely charged objects. However, this kind of system cannot be in supersymmetric equilibrium unless their tensions also have opposite signs. We are going to identify these negative-tension objects with O8-planes and we will propose an O8-plane action to be coupled to the bulk supergravity action. O8-planes can only sit at orbifold points because they require the spacetime to be mirror symmetric in their transverse direction and, thus, they can sit in any of the two
endpoints of the segment $S^1/Z_2$. We are going to place the other (positive tension, opposite R-R charge) brane at the other endpoint. Clearly we can, from the effective action point of view, identify the positive tension brane as a combination of O8-planes and D8-branes with positive total tension and the negative tension brane as a combination of O8-planes and D8-branes with negative total tension.

Our strategy will be to generalize the 5-dimensional construction of the supersymmetric bulk & brane action, proposed in [10]. The construction of [10] allowed to find a supersymmetric realization of the brane-world scenario of Randall and Sundrum [9]. We will repeat the construction of [10] in $D = 10$ with the aim to get a better understanding of branes and planes in string theory.

To solve the discrepancy between the bulk actions with limited field content (lower-rank R-R forms) and the wide range of brane actions that involve all the possible R-R forms, we have constructed a new formulation of IIA/IIB supergravity up to quartic order in fermions. In particular, the new formulation gives an easy control over the exotic $G^{(0)}$ and $G^{(10)}$ R-R forms associated with the mass and cosmological constant of the $D = 10$ supergravity. This in turn allows a clear study of the D8–O8 system describing a pair of supersymmetric domain walls which are fundamental objects of the Type I’ string theory. The quantization of the mass parameter and cosmological constant in stringy units are simple consequences of the theory. Apart from being a tool to understand the supersymmetric domain walls we were interested in, it can be expected that the new effective theories of $D = 10$ supersymmetry will have more general applications in the future.

In this talk we will summarize the results of [11].

A NEW DUAL FORMULATION OF D=10 SUPERGRAVITY

The standard formulation of $D = 10$ IIA (massless [12, 13, 14] and massive [3]) and IIB [15, 16] supergravity has the following field content

\begin{align}
\text{IIA} & : \left\{ g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}, \psi_{\mu}, \lambda \right\}, \\
\text{IIB} & : \left\{ g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}_{\mu}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\nu\rho\sigma}, \psi_{\mu}, \lambda \right\}.
\end{align}

In the IIA case, the massive theory contains an additional mass parameter $G^{(0)} = m$. In the IIB case, an extra self-duality condition is imposed on the field strength of the four-form. It turns out that one can realize the N=2 supersymmetry on the R-R gauge fields of higher rank as well. These are usually incorporated via duality relations. To treat the R-R potentials democratically we propose a new formulation based upon a pseudo-action. This democratic formulation describes the dynamics of the bulk supergravity in the most elegant way. However, it turns out that this formulation is not well suited for our purposes. For the IIA case, we therefore give a different formulation where the constant mass parameter has been replaced by a field.
The Democratic Formulation

To explicitly introduce the democracy among the R-R potentials we propose a pseudo-action whose equations of motion are supplemented by duality constraints (see below). Of course this enlarges the number of degrees of freedom. Since a $p$- and an $(8-p)$-form potential carry the same number of degrees of freedom, the introduction of the dual potentials doubles the R-R sector. Including the highest potential $C^{(9)}$ in IIA does not alter this, since it carries no degrees of freedom. This 9-form potential can be seen as the potential dual to the constant mass parameter $G^{(0)} = m$. The doubling of number of degrees of freedom will be taken care of by a constraint, relating the lower- and higher-rank potentials. This new formulation of supersymmetry is inspired by the bosonic construction of [17], and, in the case of IIB supergravity, is related to the pseudo-action construction of [18].

A pseudo-action [18] can be used as a mnemonic to derive the equations of motion. It differs from a usual action in the sense that not all equations of motion follow from varying the fields in the pseudo-action. To obtain the complete set of equations of motion, an additional constraint has to be substituted by hand into the set of equations of motion that follow from the pseudo-action. The constraint itself does not follow from the pseudo-action. The construction we present here generalizes the pseudo-action construction of [17, 18] in the sense that our construction (i) treats the IIA and IIB case in a unified way, introducing all R-R potentials in the pseudo-action, and (ii) describes also the massive IIA case via a 9-form potential $C^{(9)}$ and a constant mass parameter $G^{(0)} = m$.

Our pseudo-action has the extended field content

\[
\text{IIA} : \quad \left\{ g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}_{\mu}, C^{(3)}_{\mu\nu\rho}, C^{(5)}_{\mu\nu\rho\sigma}, C^{(7)}_{\mu\nu\rho\sigma\tau}, C^{(9)}_{\mu\nu\rho\sigma\tau\upsilon}, \psi_{\mu}, \lambda \right\},
\]

\[
\text{IIB} : \quad \left\{ g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(0)}_{\mu}, C^{(2)}_{\mu\nu}, C^{(4)}_{\mu\nu\rho}, C^{(6)}_{\mu\nu\rho\sigma}, C^{(8)}_{\mu\nu\rho\sigma\tau}, \psi_{\mu}, \lambda \right\}. \tag{2}
\]

It is understood that in the IIA case the fermions contain both chiralities, while in the IIB case they satisfy

\[
\Gamma_{11}\psi_{\mu} = \psi_{\mu}, \quad \Gamma_{11}\lambda = -\lambda, \quad \text{(IIB)}. \tag{3}
\]

In that case they are doublets, and we suppress the corresponding index. The explicit form of the pseudo-action is given by

\[
S_{\text{pseudo}} = -\frac{1}{2\kappa^{10}_0} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ R(e) - 4(\partial\phi)^2 + \frac{1}{2} H \cdot H + 2\partial^\mu \phi \chi^{(1)}_{\mu} + H \cdot \chi^{(3)} + 2\psi_{\mu} \Gamma^{\mu\nu\rho} \nabla_\nu \psi_\rho - 2\lambda \Gamma^{\mu\nu} \nabla_\nu \lambda + 4\lambda \Gamma^{\mu\nu} \nabla_\mu \psi_\nu \right] + \sum_{n=0,1/2}^{5,9/2} \frac{1}{4} G^{(2n)} \cdot G^{(2n)} + \frac{1}{2} G^{(2n)} \cdot \Psi^{(2n)} \right\} + \text{quartic fermionic terms}. \tag{4}
\]

It is understood that the summation in the above pseudo-action is over integers ($n = 0, 1, \ldots, 5$) in the IIA case and over half-integers ($n = 1/2, 3/2, \ldots, 9/2$) in the IIB case.

\footnote{We use the notation and conventions of [11].}
In the summation range we will always first indicate the lowest value for the IIA case, before the one for the IIB case. Furthermore,

\[ \frac{1}{2\kappa_{10}^2} = \frac{g^2}{2\kappa^2} = \frac{2\pi}{(2\pi\ell_s)^8}, \]

where \( \kappa^2 \) is the physical gravitational coupling, \( g \) is the string coupling constant and \( \ell_s = \sqrt{\alpha'} \) is the string length. For notational convenience we group all potentials and field strengths in the formal sums

\[ G = \sum_{n=0,1/2}^{5,9/2} G^{(2n)}, \quad C = \sum_{n=1,1/2}^{5,9/2} C^{(2n-1)}. \]

The bosonic field strengths are given by

\[ H = dB, \quad G = dC - dB \wedge C + G^{(0)} e^B, \]

where it is understood that each equation involves only one term from the formal sums (only the relevant combinations are extracted). The corresponding Bianchi identities then read

\[ dH = 0, \quad dG - H \wedge G = 0. \]

In this subsection \( G^{(0)} = m \) indicates the constant mass parameter of IIA supergravity. In the IIB theory all equations should be read with vanishing \( G^{(0)} \). The spin connection in the covariant derivative \( \nabla_\mu \) is given by its zehnbein part:

\[ \omega_{ab}^{\mu} = \omega_{ab}^{\mu} (e). \]

The bosonic fields couple to the fermions via the bilinears \( \chi^{(1,3)} \) and \( \Psi^{(2n)} \), which read

\[ \chi^{(1)} = -2\bar{\psi}_{\alpha} \Gamma^{\nu} \psi_{\mu} - 2\bar{\lambda} \Gamma^{\nu} \Gamma_{\mu} \psi_{\nu}, \]

\[ \chi^{(3)}_{\mu\nu\rho} = \frac{1}{2} \bar{\psi}_{\alpha} \Gamma^{[\alpha} \Gamma_{\mu\nu\rho]} \psi_{\beta} + \bar{\lambda} \Gamma_{\mu\rho} \beta \Psi_{\beta} - \frac{1}{2} \bar{\lambda} \Gamma_{\mu\rho} \lambda, \]

\[ \Psi^{(2n)}_{\mu_1 \cdots \mu_{2n}} = \frac{1}{2} e^{-\phi} \bar{\psi}_{\alpha} \Gamma_{\mu_1 \cdots \mu_{2n}} \Gamma^{\beta} \Psi_{n} + \frac{1}{2} e^{-\phi} \bar{\lambda} \Gamma_{\mu_1 \cdots \mu_{2n}} \Gamma^{\beta} \Psi_{n} + \frac{1}{4} e^{-\phi} \bar{\lambda} \Gamma_{\mu_1 \cdots \mu_{2n} - 1} \mathcal{P} \Gamma_{\mu_{2n}} \lambda. \]

We have used the following definitions:

\[ \mathcal{P} = \Gamma_{11} \quad \text{(IIA)} \quad \text{or} \quad -\sigma^3 \quad \text{(IIB)}, \]

\[ \mathcal{P}_n = (\Gamma_{11})^n \quad \text{(IIA)} \quad \text{or} \quad \sigma^1 (n + 1/2 \text{ even}), \quad i\sigma^2 (n + 1/2 \text{ odd}) \quad \text{(IIB)}. \]

Note that the fermions satisfy

\[ \Psi^{(2n)} = (-)^{\text{Int}[n]+1} \Psi^{(10-2n)}. \]

Due to the appearance of all R-R potentials, the number of degrees of freedom in the R-R sector has been doubled. Each R-R potential leads to a corresponding equation of motion:

\[ d \star (G^{(2n)} + \Psi^{(2n)}) + H \wedge \star (G^{(2n+2)} + \Psi^{(2n+2)}) = 0. \]
Now, one must relate the different potentials to get the correct number of degrees of freedom. We therefore by hand impose the following duality relations

\[ G^{(2n)} + \psi^{(2n)} = (-)^{\text{Int}[n]} \star G^{(10-2n)}, \]  

(13)

in the equations of motion that follow from the pseudo-action \( (4) \). It is in this sense that the action \( (4) \) cannot be considered as a true action. Instead, it should be considered as a mnemonic to obtain the full equations of motion of the theory. As usual, the Bianchi identities and equations of motions of the dual potentials correspond to each other when employing the duality relation. For the above reason the democratic formulation can be viewed as self-dual, since \( (13) \) places constraints relating the field content \( (2) \).

The pseudo-action \( (4) \) is invariant under supersymmetry provided we impose the duality relations \( (13) \) after varying the action. The supersymmetry rules read (here given modulo cubic fermion terms):

\[ \delta_e e^\mu_a = e \Gamma^a \psi^\mu, \]

\[ \delta_e \psi^\mu = \left( \partial^\mu + \frac{1}{2} \phi^\mu + \frac{1}{8} \mathcal{H}_\mu \right) \epsilon + \frac{1}{16} e^\phi \sum_{n=0,1/2}^{5,9/2} \frac{1}{(2n)!} G^{(2n)} \Gamma_\mu \mathcal{P}_n \epsilon, \]

\[ \delta_e B^\mu = -2 e \Gamma_\mu [\mathcal{P} \psi], \]

\[ \delta_e C_{\mu_1 \ldots \mu_{2n-1}}^{(2n-1)} = -e^{-\phi} \epsilon \Gamma_{[\mu_1 \ldots \mu_{2n-2}} \mathcal{P}_{n} \left( (2n-1) \psi_{\mu_{2n-1]} - \frac{1}{2} \Gamma_{\mu_{2n-1}] \lambda} \right) + \]

\[ + (n-1)(2n-1) C_n^{(2n-3)} \Gamma_{[\mu_1 \ldots \mu_{2n-3}} \delta_\lambda \mathcal{P}_{\mu_{2n-2} \ldots \mu_{2n-1}]}, \]

\[ \delta_e \lambda = \left( \partial \phi + \frac{1}{12} \mathcal{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^\phi \sum_{n=0,1/2}^{5,9/2} \frac{1}{(2n)!} G^{(2n)} \mathcal{P}_n \epsilon, \]

\[ \delta_e \phi = \frac{1}{2} e \lambda, \]  

(14)

where \( \epsilon \) is a spinor similar to \( \psi^\mu \), i.e. in IIB: \( \Gamma_{11} \epsilon = \epsilon \). Note that for \( n \) half-integer (the IIB case) these supersymmetry rules exactly reproduce the rules given in eq. (1.1) of [19].

Secondly, the pseudo-action \( (4) \) is also invariant under the usual bosonic NS-NS and R-R gauge symmetries with parameters \( \Lambda \) and \( \Lambda^{(2n)} \) respectively:

\[ \delta_\Lambda B = d\Lambda, \quad \delta_\Lambda C = (dL - G^{(0)} \Lambda) \cdot e^B, \quad \text{with} \quad L = \sum_{n=0,1/2}^{4,7/2} \Lambda^{(2n)}. \]  

(15)

Finally, there is a number of \( \mathbb{Z}_2 \)-symmetries. However, in the IIA case these \( \mathbb{Z}_2 \)-symmetries are only valid for \( G^{(0)} = m = 0 \). Below we show how these symmetries of the action act on supergravity fields. For both massless IIA and IIB there is a fermion number symmetry \( (-)^F \) given by

\[ \{ \phi, g_{\mu\nu}, B_{\mu\nu} \} \rightarrow \{ \phi, g_{\mu\nu}, B_{\mu\nu} \}, \]

\[ \{ C_{\mu_1 \ldots \mu_{2n-1}}^{(2n-1)} \} \rightarrow -\{ C_{\mu_1 \ldots \mu_{2n-1}}^{(2n-1)} \}, \]

\[ \{ \psi^\mu, \lambda, \epsilon \} \rightarrow +\mathcal{P} \{ \psi^\mu, -\lambda, \epsilon \}, \quad (\text{IIA}), \]

\[ \{ \psi^\mu, \lambda, \epsilon \} \rightarrow +\mathcal{P} \{ \psi^\mu, \lambda, \epsilon \}, \quad (\text{IIB}). \]  

(16)
In the IIB case there is an additional worldsheet parity symmetry $\Omega$ given by

\[ \{ \phi, g_{\mu\nu}, B_{\mu\nu} \} \rightarrow \{ \phi, g_{\mu\nu}, -B_{\mu\nu} \}, \]
\[ \{ C^{(2n-1)}_{\mu_1...\mu_{2n-1}} \} \rightarrow (-)^{n+1/2} \{ C^{(2n-1)}_{\mu_1...\mu_{2n-1}} \}, \]
\[ \{ \psi_{\mu}, \lambda, \epsilon \} \rightarrow \sigma^{1/2} \{ \psi_{\mu}, \lambda, \epsilon \}. \] (17)

In the massless IIA case there is a similar $I_9\Omega$-symmetry involving an additional parity transformation in the 9-direction. Writing $\mu = (\mu, 9)$, the rules are given by

\[ x^9 \rightarrow -x^9, \]
\[ \{ \phi, g_{\mu\nu}, B_{\mu\nu} \} \rightarrow \{ \phi, g_{\mu\nu}, -B_{\mu\nu} \}, \]
\[ \{ C^{(2n-1)}_{\mu_1...\mu_{2n-1}} \} \rightarrow (-)^{n+1} \{ C^{(2n-1)}_{\mu_1...\mu_{2n-1}} \}, \]
\[ \{ \psi_{\mu}, \lambda, \epsilon \} \rightarrow +\Gamma^9 \{ \psi_{\mu}, -\lambda, \epsilon \}. \] (18)

The parity of the fields with one or more indices in the 9-direction is given by the rule that every index in the 9-direction gives an extra minus sign compared to the above rules.

In both IIA and IIB there is also the obvious symmetry of interchanging all fermions by minus the fermions, leaving the bosons invariant.

The $\mathbb{Z}_2$-symmetries are used for the construction of superstring theories with sixteen supercharges, see [20]. $(-)^F_L$ gives a projection to the $E_8 \times E_8$ heterotic superstring (IIA) or the SO(32) heterotic superstring theory (IIB). $\Omega$ is used to reduce the IIB theory to the SO(32) Type I superstring, while the $I_9\Omega$-symmetry reduces the IIA theory to the Type I′ SO(16) × SO(16) superstring theory.

One might wonder at the advantages of the generalized pseudo-action (4) above the standard supergravity formulation. At the cost of an extra duality relation we were able to realize the R-R democracy in the action. Note that only kinetic terms are present; by allowing for a larger field content the Chern–Simons term is eliminated. Under T-duality all kinetic terms are easily seen to transform into each other [21]. The same goes for the duality constraints. This formulation is elegant and comprises all potentials. However, it is impossible to construct a proper action in this formulation due to the doubling of the degrees of freedom. Therefore, to add brane actions to the bulk system, the democratic formulation is not suitable. This is due to two reasons. First, the $I_9\Omega$ symmetry is only valid for $G^{(0)} = 0$, but we will need this symmetry in our construction of the bulk & 8-brane system. Secondly, to describe a charged domain wall, we would like to have opposite values for $G^{(0)}$ at the two sides of the domain wall, i.e. we want to allow for a mass parameter that is only piecewise constant. The R-R democracy has to be broken to accommodate for an action and this will be discussed in the next subsection.

The Dual Formulation

We will present here the new dual formulation with action, available for the IIA case only. A proper action will be constructed in this formulation. It is this formulation that we will apply in our construction of the bulk & brane system. We will call this the dual formulation.
The independent fields in this formulation are
\[
\left\{ e_\mu^a, B_{\mu\nu}, \phi, G^{(0)}, G^{(2)}_{\mu\nu}, G^{(4)}_{\mu_1...\mu_4}, A^{(5)}_{\mu_1...\mu_5}, A^{(7)}_{\mu_1...\mu_7}, A^{(9)}_{\mu_1...\mu_9}, \bar{\psi}_\mu, \lambda \right\}.
\tag{19}
\]

The bulk action reads
\[
S_{\text{bulk}} = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ R(\omega(e)) - 4(\partial \phi)^2 + \frac{1}{2} H \cdot H - 2\partial^\mu \phi \chi^{(1)}_\mu + H \cdot \chi^{(3)} + 2\bar{\psi}_\mu \Gamma^{\mu\rho\nu} \nabla_\rho \psi_\nu - 2\lambda \Gamma^{\mu} \nabla_\mu \lambda + 4\lambda \Gamma^{\mu\nu} \nabla_\nu \psi_\mu \right] + \sum_{n=0,1,2} \frac{1}{2} G^{(2n)} \cdot G^{(2n)} + G^{(2n)} \cdot \psi^{(2n)} + \left[ \frac{1}{2} G^{(4)} G^{(4)} B - \frac{1}{2} G^{(2)} G^{(2)} B^2 + \frac{1}{6} G^{(2)} B^3 + \frac{1}{6} G^{(0)} G^{(4)} B^3 - \frac{1}{8} G^{(0)} G^{(2)} B^4 + \frac{1}{40} G^{(0)} B^5 + e^{-B} G d(A^{(5)} - A^{(7)} + A^{(9)}) \right] + \text{quartic fermionic terms},
\right\}
\tag{20}
\]
where all $\wedge$'s have been omitted in the last two lines. In the last term a projection on the 10-form is understood. Here $G$ is defined as in (6) but where $G^{(0)}, G^{(2)}$ and $G^{(4)}$ are now independent fields (which we will call black boxes) and are no longer given by (7). Note that their Bianchi identities are imposed by the Lagrange multipliers $A^{(9)}, A^{(7)}$ and $A^{(5)}$. The NS-NS three-form field strength is given by (7). Note that the standard action for IIA supergravity can be obtained by integrating out the dual potentials in (20).

The symmetries of the action are similar to those of the democratic formulation with some small changes. In the supersymmetry transformations of gravitino and gaugino, the sums now extend only over $n = 0, 1, 2$:
\[
\delta e_\mu^a = \epsilon \Gamma^a \psi_\mu,
\]
\[
\delta e \psi_\mu = \left( \partial_\mu + \frac{1}{2} e_\mu^a \partial_a + \frac{1}{8} \Gamma_{11} H_\mu \right) \epsilon + \frac{1}{8} \epsilon \phi \sum_{n=0,1,2} \frac{1}{(2n)!} G^{(2n)} \Gamma_\mu (\Gamma_{11})^n \epsilon,
\]
\[
\delta e B_{\mu\nu} = -2 \epsilon \Gamma_{[\mu} \Gamma_{11] \nu] \epsilon,
\]
\[
\delta e \lambda = \left( \partial \phi + \frac{1}{12} \Gamma_{11} H \right) \epsilon + \frac{1}{4} \epsilon \phi \sum_{n=0,1,2} \frac{5 - 2n}{(2n)!} G^{(2n)} (\Gamma_{11})^n \epsilon,
\]
\[
\delta e \phi = \frac{1}{2} \epsilon \lambda,
\]
\[
\delta e A = e^{-B} E,
\]
\[
\delta e G = d E + G \wedge \delta e B - H \wedge E,
\]
with $E^{(2n-1)}_{\mu_1...\mu_{2n-1}} = -e^{-\phi} \epsilon \Gamma_{[\mu_1...\mu_{2n-2}} (\Gamma_{11})^n \left( (2n - 1) \psi_{\mu_{2n-1}} - \frac{1}{2} \Gamma_{\mu_{2n-1}} \right) \lambda$.
\tag{21}

The transformation of the black boxes $G$ follow from the requirement that $e^{-B} G$ transforms in a total derivative. Here the formal sums
\[
A = \sum_{n=1}^{5} A^{(2n-1)}, \quad E = \sum_{n=1}^{5} E^{(2n-1)}, \quad G = \sum_{n=0}^{5} G^{(2n)},
\tag{22}
\]
have been used. Note that the first formal sum in (22) contains fields, $A^{(1)}$ and $A^{(3)}$, that do not occur in the action. The same applies to $G$, which contains the extra fields $G^{(6)}, G^{(8)}$ and $G^{(10)}$. Although these fields do not occur in the action, one can nevertheless show that
the supersymmetry algebra is realized on them. To do so one must use the supersymmetry rules of (21) and the equations of motion that follow from the action (20).

The gauge symmetries with parameters $\Lambda$ and $\Lambda^{(2n)}$ are

$$\delta_{\Lambda} B = d\Lambda, \quad \delta_{\Lambda} A = dL - G^{(0)} \Lambda - d\Lambda \wedge A,$$

$$\delta_{\Lambda} G = d\Lambda \wedge (G - e^{B \wedge (dA + G^{(0)})}) + e^B \Lambda \wedge dG^{(0)}.$$

(23)

Note that, with respect to the R-R gauge symmetry, the $A$ potentials transform as a total derivative while the black boxes are invariant.

Finally, there are $\mathbb{Z}_2$-symmetries, $(-)^F L$ and $I_9 \Omega$, which leave the action invariant. In contrast to the democratic formulation these two $\mathbb{Z}_2$-symmetries are valid symmetries even for $G^{(0)} \neq 0$. The $(-)^F L$-symmetry is given by

$$\{ \phi, g_{\mu \nu}, B_{\mu \nu} \} \rightarrow \{ \phi, g_{\mu \nu}, B_{\mu \nu} \},$$

$$\{ G^{(2n)}_{\mu_1 \cdots \mu_{2n}}, A^{(2n-1)}_{\mu_1 \cdots \mu_{2n-1}} \} \rightarrow \{-G^{(2n)}_{\mu_1 \cdots \mu_{2n}}, A^{(2n-1)}_{\mu_1 \cdots \mu_{2n-1}} \},$$

$$\{ \psi_\mu, \lambda, \epsilon \} \rightarrow +\Gamma_{11} \{ \psi_\mu, -\lambda, \epsilon \},$$

while the second $I_9 \Omega$-symmetry reads

$$x^9 \rightarrow -x^9,$$

$$\{ \phi, g_{\mu \nu}, B_{\mu \nu} \} \rightarrow \{ \phi, g_{\mu \nu}, -B_{\mu \nu} \},$$

$$\{ G^{(2n)}_{\mu_1 \cdots \mu_{2n}}, A^{(2n-1)}_{\mu_1 \cdots \mu_{2n-1}} \} \rightarrow (-)^{n+1} \{ G^{(2n)}_{\mu_1 \cdots \mu_{2n}}, A^{(2n-1)}_{\mu_1 \cdots \mu_{2n-1}} \},$$

$$\{ \psi_\mu, \lambda, \epsilon \} \rightarrow +\Gamma^9 \{ \psi_\mu, -\lambda, \epsilon \}.$$  

(24)

(25)

**ADDING THE BRANE ACTIONS**

Having established supersymmetry in the bulk, we now turn to supersymmetry on the brane. As mentioned in the introduction, our main interest is in one-dimensional orbifold constructions with 8-branes at the orbifold points. Using the techniques of the three-brane on the orbifold in five dimensions \[\mathbb{O}\], we want to construct an orientifold using a $\mathbb{Z}_2$-symmetry of the bulk action. On the fixed points we insert brane actions, which will turn out to be invariant under the reduced ($N=1$) supersymmetry. For the moment we will not restrict to domain walls (in this case eight-branes) since our brane analysis is similar for orientifolds of lower dimension. In the previous section we have seen that our bulk action possesses a number of symmetries, among which a parity operation. To construct an orientifold, the relevant $\mathbb{Z}_2$-symmetry must contain parity operations in the transverse directions. Furthermore, in order to construct a charged domain wall, we want for a $p$-brane the $(p+1)$-form R-R potential to be even. For the 8-brane the $I_9 \Omega$ symmetry satisfies the desired properties. For the other $p$-branes, it would seem natural to use the $\mathbb{Z}_2$-symmetry

$$I_{9,8,\ldots,p+1} \Omega \equiv (I_9 \Omega)(I_8 \Omega) \cdots (I_{p+1} \Omega).$$

(26)
where $I_q\Omega$ is the transformation (18) with 9 replaced by $q$, and $I_q$ and $\Omega$ commute. However, for some $p$-branes ($p = 2, 3, 6, 7$) the corresponding $C^{(p+1)}$ R-R-potential is odd under this $Z_2$-symmetry. To obtain the correct parity one must include an extra $(-)^{F_L}$ transformation in these cases, which also follows from T-duality [24]. This leads for each $p$-brane to the $Z_2$-symmetry indicated in Table 1.

| $p$ | IIB    | IIA    |
|-----|--------|--------|
| 9   | $\Omega$ | -      |
| 8   | -      | $I_9\Omega$ |
| 7   | $(-)^{F_L}I_{9,8}\Omega$ | -      |
| 6   | -      | $(-)^{F_L}I_{9,8,7}\Omega$ |
| 5   | $I_{9,8,...,6}\Omega$ | -      |
| 4   | -      | $I_{9,8,...,5}\Omega$ |
| 3   | $(-)^{F_L}I_{9,8,...,4}\Omega$ | -      |
| 2   | -      | $(-)^{F_L}I_{9,8,...,3}\Omega$ |
| 1   | $I_{9,8,...,2}\Omega$ | -      |
| 0   | -      | $I_{9,8,...,1}\Omega$ |

Table 1: The $Z_2$-symmetries used in the orientifold construction of an $O_p$-plane. The T-duality transformation from IIA to IIB in the lower dimension induces each time a $(-)^{F_L}$. Thus the correct $Z_2$-symmetry for a general IIA $O_p$-plane is given by

$$((-)^{F_L})^{p/2}I_{9,8,...,p+1}\Omega.$$  \hspace{1cm} (27)

The effect of this $Z_2$-symmetry on the bulk fields reads (the underlined indices refer to the worldvolume directions, i.e. $\mu = (\mu, p + 1, \ldots, 9)$

$$\{x^{p+1}, \ldots, x^9\} \rightarrow -\{x^{p+1}, \ldots, x^9\},$$

$$\{\phi, g_{\mu\nu}, B_{\mu\nu}\} \rightarrow \{\phi, g_{\mu\nu}, -B_{\mu\nu}\},$$

$$\{A^{(5)}_{\mu_1...\mu_5}, A^{(9)}_{\mu_1...\mu_9}, G^{(2)}_{\mu_1...\mu_2}\} \rightarrow (-)^{\frac{p(p+1)}{2}} \{A^{(5)}_{\mu_1...\mu_5}, A^{(9)}_{\mu_1...\mu_9}, G^{(2)}_{\mu_1...\mu_2}\},$$

$$\{A^{(7)}_{\mu_1...\mu_7}, G^{(0)}_{\mu_1...\mu_4}, G^{(4)}_{\mu_1...\mu_4}\} \rightarrow (-)^{\frac{p(p+1)}{2} + 1}\{A^{(7)}_{\mu_1...\mu_7}, G^{(0)}_{\mu_1...\mu_4}, G^{(4)}_{\mu_1...\mu_4}\},$$

$$\{\psi_{\mu, \epsilon}\} \rightarrow -\alpha \Gamma^{p+1...9}(-\Gamma_{11})^{\frac{p}{2}} \{\psi_{\mu, \epsilon}\},$$

$$\{\lambda\} \rightarrow +\alpha \Gamma^{p+1...9}(+\Gamma_{11})^{\frac{p}{2}} \{\lambda\},$$

and for fields with other indices there is an extra minus sign for each replacement of a worldvolume index $\mu$ by an index in a transverse direction. We have left open the possibility of combining the symmetry with the sign change of all fermions. This possibility introduces a number $\alpha = \pm 1$ in the above rules. This symmetry will be used for the orientifold construction.
For this purpose we choose spacetime to be $M^{p+1} \times T^{9-p}$ with radii $R^\vec{p}$ of the torus that may depend on the world-volume coordinates. All fields satisfy

$$\Phi(x^\vec{p}) = \Phi(x^\vec{p} + 2\pi R^\vec{p}),$$

(29)

with $\vec{p} = (p + 1, \ldots, 9)$. The parity symmetry (27) relates the fields in the bulk at $x^\vec{p}$ and $-x^\vec{p}$. At the fixed point of the orientifolds, however, this relation is local and projects out half the fields. This means that we are left with only $N = 1$ supersymmetry on the fixed points, where the branes will be inserted. Consider for example a nine-dimensional orientifold. The projection truncates our bulk $N = 2$ supersymmetry to $N = 1$ on the brane; only half of the 32 components of $\epsilon$ are even under (25). The original field content, a $D = 10, (128 + 128)$, $N = 2$ supergravity multiplet, gets truncated on the brane to a reducible $D = 9, (64 + 64)$, $N = 1$ theory consisting of a supergravity plus a vector multiplet. One may further restrict to a constant torus. This particular choice of spacetime then projects out a $N = 1$ supersymmetry on the fixed points, however, this relation is local and projects out half of the 32 components of $\epsilon$.

We propose the $p$-brane action $(p = 0, 2, 4, 6, 8)$ to be proportional to

$$L_p = -e^{-\phi} \sqrt{-g_{(p+1)}} - \alpha \epsilon^{(p+1)} \varepsilon^{(p+1)} C(p+1), \quad \text{with } \varepsilon^{(p+1)} C(p+1) \equiv \varepsilon^{(p+1)} \mu_0 \cdots \mu_p C(p+1) \mu_0 \cdots \mu_p,$$

(30)

with $\varepsilon^{(p+1)} \mu_0 \cdots \mu_p = \varepsilon^{(10)} \mu_0 \cdots \mu_p \cdots \mu_9$, which follows from $\epsilon^{(1)} = 0$ (being odd). Here the underlined indices are $(p + 1)$-dimensional and refer to the world-volume. The parameter $\alpha$ is the same that appears in (28) and takes the values $\alpha = +1$ for branes, which are defined to have tension and charge with the same sign in our conventions, and $\alpha = -1$ for anti-branes, which are defined to have tension and charge of opposite signs. Note that due to the vanishing of $B$ on the brane the potentials $C(p+1)$ and $A^{(p+1)}$ are equal. The $p$-brane action can easily be shown to be invariant under the appropriate $N = 1$ supersymmetry:

$$\delta, L_p = -e^{-\phi} \sqrt{-g_{(p+1)}} \epsilon(1 - \alpha \Gamma_{p+1-p} \cdot (\Gamma_{11}) \frac{\hat{\mathbf{p}}}{2}) \Gamma_\mu (\psi_\mu - \frac{1}{16} \Gamma_\mu \lambda).$$

(31)

The above variation vanishes due to the projection under (28) that selects branes or anti-branes depending on the sign of $\alpha$ ($+1$ or $-1$ respectively). In the following discussions we will assume $\alpha = 1$ but the other case just amounts to replacing branes by anti-branes.

By truncating our theory we are able to construct a brane action that only consists of bosons and yet is separately supersymmetric. Having these at our disposal, we can introduce source terms for the various potentials. In general there are $2^{9-p}$ fixed points. The compactness of the transverse space implies that the total charge must vanish. Thus the total action will read (we take the special case that all branes are equally distributed over all $2^{9-p}$ fixed points)

$$\mathcal{L} = \mathcal{L}_{\text{bulk}} + k_p L_p \Delta_p,$$

with $\Delta_p \equiv (\delta(x^{p+1}) - \delta(x^{p+1} - \pi R^{p+1})) \cdots (\delta(x^9) - \delta(x^9 - \pi R^9))$

(32)

where the branes at all fixed points have a tension and a charge proportional to $\pm k_p$, a parameter of dimension $1/[\text{length}]^{p+1}$. Since anti-branes do not satisfy the supersymmetry
condition (31), we need both positive and negative tension branes to accomplish vanishing total charge. As explained in the introduction we are going to interpret the negative tension branes as O-planes.

The equations of motion following from (32) induce a $\delta$-function in the Bianchi identity of the $8-p$-form field strength. In general, an elegant solution is difficult to find, but in the eight-brane case the situation simplifies.

**QUANTIZATION OF MASS AND COSMOLOGICAL CONSTANT**

Consider the eight-brane case only. The equation of motion of the nine-form is modified by the brane & plane actions such that the solution for $G^{(0)}$ is given by

$$G^{(0)} = \alpha \frac{n - 8}{2\pi\ell_s} \varepsilon(x^9).$$

Thus we may identify the mass parameter of Type IIA supergravity as follows:

$$m = \begin{cases} 
\frac{\alpha(n - 8)}{2\pi\ell_s}, & x^9 > 0, \\
-\frac{\alpha(n - 8)}{2\pi\ell_s}, & x^9 < 0.
\end{cases}$$

The mass is quantized in string units and it is proportional to $n - 8$ where there are $2n$ and $2(16 - n)$ D8-branes at each O8-plane. The mass vanishes only in the special case $n = 8$ when the contribution from the D8-branes cancels exactly the contribution from the O8-planes. In general, the mass takes only the restricted values

$$2\pi\ell_s|m| = 0, 1, 2, 3, 4, 5, 6, 7, 8.$$

This is a quantization of our mass parameter, and for the cosmological constant it follows that

$$m^2 = (G^{(0)})^2 = \left(\frac{n - 8}{2\pi\ell_s}\right)^2.$$

Thus the mass parameter and the cosmological constant are quantized in the units of the string length in terms of the integers $n - 8$.

The quantization of the mass and of the cosmological constant in $D = 10$ was discussed before in [2, 4, 22] as well as in [5, 23]. In the latter two references, two independent derivations of the quantization condition were given. In [5], the T-duality between a 7-brane & 8-brane solution was investigated. Here it was pointed out that, in the presence of a cosmological constant, the relation between the $D = 10$ IIB R-R scalar $C^{(0)}$ and the one reduced to $D = 9$, $c^{(0)}$, is given via a generalized Scherk–Schwarz prescription:

$$C^{(0)} = c^{(0)}(x^9) + mx^8.$$
Here \((x^8, x^9)\) parametrize the 2-dimensional space transverse to the 7-brane. \(x^9\) is a radial coordinate whereas \(x^8\) is periodically identified (it corresponds to a U(1) Killing vector field):

\[
x^8 \sim x^8 + 1.
\]

Furthermore, due to the \(SL(2, \mathbb{Z})\) U-duality, the R-R scalar \(C^{(0)}\) is also periodically identified:

\[
C^{(0)} \sim C^{(0)} + 1.
\]

Combining the two identifications with the reduction rule for \(C^{(0)}\) leads to a quantization condition for \(m\) of the form

\[
m \sim \frac{n}{\ell_s}, \quad n \text{ integer}.
\]

The same result was obtained by a different method in [23].

We are able to give a new, and independent, derivation of the quantization condition for the mass and cosmological constant. The conditions given in (34), (36) follow straightforwardly from our construction of the bulk & brane & plane action.

Note that the Scherk–Schwarz reduction in (37) and the quantization of \(SL(2, \mathbb{R})\) were essential in deriving the quantization of \(m\). In the new dual formulation we can derive a similar T-duality relation between the 7-brane and the 8-brane, including the source terms. However, in this case the T-duality relation does not imply a quantization condition for \(m\) since we do not know how to realize the \(SL(2, \mathbb{R})\) symmetry in the dual formulation. Another noteworthy feature is that the derivation of the T-duality rules in the dual formulation does not require a Scherk-Schwarz reduction. This is possible due to the fact that the R-R scalar only appears after solving the equations of motion.

**CONCLUSIONS**

We have constructed new formulations of Type II \(D = 10\) supergravity. For both Type IIA and IIB theories, we constructed democratic bulk theories with a unified treatment of all R-R potentials. Due to the doubling of R-R degrees of freedom one had to impose extra duality constraints and thus a proper action was not possible. A so-called pseudo-action, containing kinetic terms for all R-R potentials but without Chern-Simons terms, was discussed. Furthermore, we have broken the self-duality explicitly in the IIA case, allowing for a proper action. Instead of all R-R potentials only half of the \(C^{(p)}\)'s occur in these theories. Both the standard \((p = 1, 3)\) as well as the dual \((p = 5, 7, 9)\) formulations were discussed. Using these actions all bulk & brane systems can be described.

A notable difference of our scenario from the HW [25, 26] scenario is that the walls are the O8 and D8 objects which exist in string theory. The main goal of the HW theory was to present a scenario for appearance of chiral fermions starting with \(D = 11\) supersymmetric theory with non-chiral fermions. Our O8-D8 construction may reach this precise goal in an interesting and controllable way due to stringy nature of this construction and due to the complete control over supersymmetries in the bulk & on the walls. We remark that the strong coupling limit of Type I' string theory is equal to the HW theory. Using the results of this paper, it would be interesting to investigate whether and how in this limit the O8-D8 objects can be related to the HW branes.
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