Weak Matrix Elements and $|V_{cb}|$
in New Formulation of Heavy Quark Effective Field Theory

W.Y. Wang, Y.L. Wu and Y.A. Yan
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China
(ylwu@itp.ac.cn)

Abstract

The heavy quark effective field theory (HQEFT) containing both effective ‘quark fields’ and ‘antiquark fields’ is investigated in detail. By integrating out (but not neglecting) the effective antiquark fields, we present a new formulation of effective theory which differs from the usual heavy quark effective theory (HQET) and exhibits valuable features because of the inclusion of the contributions from the antiquark fields. Matrix elements of vector and axial vector heavy quark currents between pseudoscalar and vector mesons containing a heavy quark (b or c) are then evaluated systematically up to the order of $1/m_Q^2$ and parameterized by a set of universal form factors. With a consistent normalization condition between the effective heavy hadron states, the form factors at zero recoil are related to the ground state meson masses, which enables us to estimate the values of form factors at zero recoil. In particular, the Luke’s theorem comes out automatically in the new formulation of HQEFT without the need of imposing the equation of motion $iv \cdot DQ^+ = 0$. Consequently, the differential decay rates of both $B \to D^* l\nu$ and $B \to D l\nu$ do not receive $1/m_Q$ order corrections at zero recoil, which is not the case in the usual HQET. Thus we quote that the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ can nicely be extracted from either of these two exclusive semileptonic decays at the order of $1/m_Q^2$. Our estimates for $|V_{cb}|$ are presented.

PACS numbers: 11.30.Hv 12.39.Hg 13.20.He 13.20.Fc
I. INTRODUCTION

It is well known that, because of the heavy quark spin-flavor symmetry \[1\]–\[3\] in the infinite quark mass limit revealed by QCD, the dynamics of hadrons consisting of one heavy quark and any number of light quarks simplifies greatly. Consequently, the effective theories for heavy quarks \[4\]–\[21\] have developed rapidly since the observation of spin-flavor symmetry, and achieved great success. The effective theories manifestly exhibit the spin-flavor symmetry of hadrons containing a single heavy quark at the leading order. This enables one to separate the long distance dynamics from the short one in a reliable way. In particular, the Luke’s theorem \[12\] has shown that the order \(1/m_Q^2\) corrections to the transition matrix elements of heavy meson decays vanish at zero recoil. These features make weak transitions such as \(B \to D^* l \bar{\nu}\) rather suitable for the determination of the Cabibbo-Kobayashi-Maskawa matrix element \(|V_{cb}|\) \[20\].

In the usual framework of heavy quark effective theory (HQET), one generally decouples the ‘quark fields’ and ‘antiquark fields’ and treats only one of them independently. Strictly speaking, in quantum field theory particle and antiparticle decouple completely only in the infinite heavy quark mass limit \(m_Q \to \infty\). To consider the finite quark mass correction, it is necessary to include the contributions from the components of the antiquark fields. For that, one can simply extend the usual HQET to a heavy quark effective field theory with keeping both effective quark and antiquark fields (which may be called for simplicity as HQEFT so as to distinguish with the usual HQET). This was actually first pointed out by one of us \[19\], where a new formulation of heavy quark effective Lagrangian was derived from full QCD. Its form permits an expansion in powers of the heavy quark momentum characterizing its off-shellness divided by its mass. In this paper we will provide a more detailed study on the new formulation of the HQEFT \[19\] and apply it to evaluate the weak transition matrix elements between the heavy hadrons containing a single heavy quark.

Our paper is organized as follows: In section II, based on the construction of a new formation of HQEFT in ref. \[19\], and using the equations of motion, we derive an effective Lagrangian with integrating out antiparticles. And the \(1/m_Q^2\) expansion for an arbitrary heavy quark current is also obtained. Alternatively, these can be achieved through the approach of the functional integral method, which is briefly presented in appendix A. The functional integral method has also been adopted in Ref. \[16\]. But the HQET in \[16\] only considered the components of heavy ‘particle fields’ without including ‘antiparticle fields’. Though the new effective Lagrangian and current derived in section II of this paper are in terms of only effective quark fields, they include additional contributions from antiquark components, so that they differ from the usual HQET counterparts. In section III, within the new framework, a consistent normalization condition between two heavy hadrons is introduced to reflect spin-flavor symmetry. Transition matrix elements of heavy hadrons are generally investigated up to the order of \(1/m_Q^2\). In section IV, a set of universal form factors are introduced and the transition matrix elements of the vector and axial vector currents between pseudoscalar as well as vector ground state mesons are evaluated in detail up to the order of \(1/m_Q^2\). In particular, it is explicitly shown that the Luke’s theorem automatically holds within the framework of the new formulation of HQEFT. With the new normalization condition, it is found that the form factors at zero recoil are related to the meson masses. The decomposition of some Lorentz tensors used in this section are given in appendix B.
And appendix C contains the general results for meson form factors. In section V, as the differential decay rates of both $B \to D^* l\nu$ and $B \to D l\nu$ have no $1/m_Q$ order corrections at zero recoil in the new formulation of HQEFT, and the most important relevant form factors at zero recoil can be fitted from the ground state meson masses, the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ are extracted from these two exclusive semileptonic decays at the order of $1/m_Q^2$. A brief summary is presented in section VI. In this paper we restrict our discussions to the tree level.

II. HQEFT WITH KEEPING BOTH PARTICLES AND ANTIPARTICLES

We now briefly describe the new formulation of heavy quark effective field Lagrangian that contains both effective quark and antiquark fields [19]. Firstly, denote the heavy quark field as

$$ Q = Q^+ + Q^- $$

(2.1)

with $Q^+$ and $Q^-$ the components of ‘quark field’ and ‘antiquark field’ respectively. More strictly speaking, they are corresponding to the two solutions of the Dirac equation.

Defining

$$ \hat{Q}_v^\pm \equiv \frac{1\pm \not{v}}{2} Q^\pm, \quad R_v^\pm \equiv \frac{1\mp \not{v}}{2} Q^\pm $$

(2.2)

with $v^\mu$ an arbitrary four-vector satisfying $v^2 = 1$, the original fields $Q$ and $\bar{Q}$ can be expressed by the new field variables $\hat{Q}_v$ and $\hat{Q}_{\bar{v}}$,

$$ \begin{cases} 
Q = [1 + (1 - \frac{iD_v^\parallel + m_Q}{2m_Q})^{-1} \frac{iD_v^\perp}{2m_Q}] \hat{Q}_v \equiv \hat{Q}_v \hat{\omega} \\
\bar{Q} = \hat{\bar{Q}}_v [1 - \frac{iD_{\bar{v}}^\perp}{2m_Q} (1 - \frac{iD_{\bar{v}}^\parallel + m_Q}{2m_Q})^{-1}] \equiv \hat{\bar{Q}}_v \hat{\bar{\omega}}.
\end{cases} $$

(2.3)

The QCD Lagrangian becomes

$$ L_{QCD} = L_{\text{light}} + L_{Q,v}, $$

(2.4)

where $L_{\text{light}}$ represents the part of Lagrangian containing no heavy quarks, and

$$ L_{Q,v} = L_{Q,v}^{(++)} + L_{Q,v}^{(- -)} + L_{Q,v}^{(+-)} + L_{Q,v}^{(-+)} $$

(2.5)

with

$$ \begin{align*}
L_{Q,v}^{(\pm \pm)} &= \bar{Q}_v^\pm [iD^\parallel - m_Q + \frac{1}{2m_Q} iD^\perp (1 - \frac{iD^\parallel + m_Q}{2m_Q})^{-1} iD^\perp] \hat{Q}_v^\pm \equiv \bar{Q}_v^\pm \hat{A} \hat{Q}_v^\pm \\
L_{Q,v}^{(\mp \mp)} &= \bar{Q}_v^\mp [\frac{1}{2m_Q} iD^\perp (1 - \frac{iD^\parallel + m_Q}{2m_Q})^{-1} iD^\perp - \frac{1}{4m_Q^2} (-i D^\perp) \\
&\times (1 - \frac{iD^\parallel + m_Q}{2m_Q})^{-1} iD^\perp (1 - \frac{iD^\parallel + m_Q}{2m_Q})^{-1} iD^\perp] \hat{Q}_v^\mp \\
&\equiv \bar{Q}_v^\mp \hat{B} \hat{Q}_v^\mp.
\end{align*} $$

(2.6)
\( \vec{D}^\mu, \, \mathcal{D}_\parallel, \) and \( \mathcal{D}_\perp \) are defined as
\[
\begin{align*}
\int \kappa \, \vec{D}^\mu \varphi & = - \int \kappa D^\mu \varphi, \\
\mathcal{D}_\parallel & = \psi (v \cdot D), \\
\mathcal{D}_\perp & = \bar{\psi} (v \cdot D).
\end{align*}
\]  

When quark fields and antiquark fields decouple completely, it is reasonable to deal with only section \( L^{(++,+)}_{Q,v} \) or \( L^{(-,-)}_{Q,v} \) independently. This is just the case considered in the framework of the usual HQET. In this paper, we will consider the complete effective lagrangian and investigate its possible new effects on the physical observables.

Note that \( L_{Q,v} \) in eq. \((2.3)\) accounts for only one flavor of heavy quarks moving with velocity \( v^\mu \). If there are \( N_f \) flavors of heavy quarks and they move in different velocities, the heavy quark Lagrangian should generalize to \( \sum Q L_{Q,v} \). But in this paper we use \( L_{Q,v} \) for simplicity. It should also be mentioned that the effective Lagrangian eq. \((2.3)\) as shown in ref. \([19]\) is automatically velocity reparametrization invariant and Lorentz invariant.

The Lagrangian given by eqs. \((2.5)\) and \((2.6)\) contains both quark fields and antiquark fields manifestly. But in many processes it is reasonable and more convenient to deal with either quarks or antiquarks since the initial and final states contain no antiquarks or quarks. For this reason, we will derive an effective Lagrangian which on one hand contains only heavy quark fields, and on the other hand, takes the contributions of antiquarks into account. Two methods can achieve this. One is to replace the antiquark fields by quark fields with the help of the equations of motion, and the other is to apply the functional integral method to ‘integrate out’ the antiquark fields.

It is much simpler to do that through the equations of motion. The following equations of motion can be easily yielded from eqs. \((2.5)\) and \((2.6)\),
\[
\hat{A} \tilde{Q}_v^- + \hat{B} \tilde{Q}_v^+ = 0 \quad \tilde{Q}^-_v \hat{A} + \tilde{Q}^+_v \hat{B} = 0,
\]
or
\[
\tilde{Q}^-_v = - (\hat{A})^{-1} \hat{B} \tilde{Q}^+_v \quad \tilde{Q}^+_v = - (\hat{A})^{-1} \hat{B} \tilde{Q}^-_v.
\]  

where \( \hat{A} \) and \( \hat{B} \) are just \( \hat{A} \) and \( \hat{B} \) but with replacing \( D^\mu \) by \( - \hat{D}^\mu \). By inserting \( \tilde{Q}^-_v \) and \( \tilde{Q}^+_v \) in eq. \((2.8)\) into eq. \((2.6)\), we then arrive at
\[
L_{eff}^{(++)} = \tilde{Q}^+_v (\hat{A} - \hat{B} \hat{A}^{-1} \hat{B}) \tilde{Q}^+_v
\]
\[
= \tilde{Q}^+_v (i \mathcal{D}_\parallel - m_Q + \frac{1}{2m_Q} i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel + m_Q}{2m_Q})^{-1} i \mathcal{D}_\perp)
\]
\[
\times \frac{-i}{4m_Q^2} \left[ (i \mathcal{D}_\parallel - m_Q + \frac{1}{2m_Q} i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel + m_Q}{2m_Q})^{-1} i \mathcal{D}_\perp) \right. \\
\times \left. (i \mathcal{D}_\parallel - m_Q + \frac{1}{2m_Q} i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel + m_Q}{2m_Q})^{-1} i \mathcal{D}_\perp) \right]
\]
\[
	imes \left[ (i \mathcal{D}_\parallel - m_Q + \frac{1}{2m_Q} i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel + m_Q}{2m_Q})^{-1} i \mathcal{D}_\perp) \right]
\]  

One can make a further transformation to eliminate the mass terms in eq. \((2.9)\) through introducing new field variables \( Q_v \) and \( \tilde{Q}_v \):
\[ Q_v = e^{i\hat{n}m_Q v \cdot x} \hat{Q}_v, \quad \bar{Q}_v = \hat{Q}_v e^{-i\hat{n}m_Q v \cdot x}. \]  

Since eq.\((2.10)\) is nothing more than a field variable redefinition, \(\hat{m}_Q\) may be any parameter with mass dimension. Here we write for convenience

\[ \hat{m}_Q \equiv m_Q + \Lambda \]  

It will be found to be useful for taking \(\Lambda\) as a binding energy \(\bar{\Lambda}\) which is independent of the heavy flavors and reflects the contributions of all light degrees of freedom in the hadron.

Noticing the fact that \(\slashed{v}\) commutes with \(\slashed{D}_\parallel\) but anticommutes with \(\slashed{D}_\perp\), it is not difficult to construct a new form of Lagrangian in terms of \(Q^+_v\) and \(\bar{Q}^+_v\). When the mass of a heavy quark is much larger than the QCD scale \(\Lambda_{QCD}\), this effective Lagrangian can be expanded in inverse powers of the quark mass and be straightforwardly written as follows

\[ L_{\text{eff}}^{[++]} = \bar{Q}^+_v \{ i\slashed{D}_\parallel + \Lambda \} Q^+_v + \sum_{n=1}^{\infty} \frac{1}{(2m_Q)^n} i\slashed{D}_\perp (i\slashed{D}_\parallel - \Lambda)^{n-1} i\slashed{D}_\perp \} \frac{1}{i\slashed{D}_\parallel + \Lambda} \{ i\slashed{D}_\parallel + \Lambda \}
+ \sum_{n=1}^{\infty} \frac{1}{(2m_Q)^n} i\slashed{D}_\perp (i\slashed{D}_\parallel - \Lambda)^{n-1} i\slashed{D}_\perp \} Q^+_v \equiv L_{\text{eff}}^{(0)} + L_{\text{eff}}^{(1/m_Q)} \]  

with

\[ L_{\text{eff}}^{(0)} \equiv \bar{Q}^+_v (i\slashed{D}_\parallel + \Lambda) Q^+_v, \]  

\[ L_{\text{eff}}^{(1/m_Q)} \equiv \frac{1}{m_Q} \bar{Q}^+_v (i\slashed{D}_\perp)^2 Q^+_v + \frac{1}{2m_Q^2} \bar{Q}^+_v i\slashed{D}_\perp (i\slashed{D}_\parallel - \Lambda) i\slashed{D}_\perp Q^+_v
+ \frac{1}{4m_Q^2} \bar{Q}^+_v (i\slashed{D}_\perp)^2 \frac{1}{i\slashed{D}_\parallel + \Lambda} (i\slashed{D}_\perp)^2 Q^+_v + O\left(\frac{1}{m_Q^3}\right), \]

where \(L_{\text{eff}}^{(0)}\) possesses spin-flavor symmetry, whereas \(L_{\text{eff}}^{(1/m_Q)}\) contains symmetry breaking terms and suppressed by the \(1/m_Q\). Note that \(L_{\text{eff}}^{[++]}\) also automatically preserves the velocity reparametrization invariance as well as Lorentz invariance without the need of summing over the velocity.

As the resulting improved effective Lagrangian eq.\((2.12)\) describe interactions concerning ‘quark fields’, it is convenient to be applied to physical processes in which the initial and final states contain only quarks. As quark and antiquark appear simultaneously in any full quantum field theory, a similar effective Lagrangian \(L_{\text{eff}}^{(--)}\) for antiquark fields can be simply obtained by replacing the effective quark field variables \(Q^+_v\) and \(\bar{Q}^+_v\) in eq.\((2.12)\) with the effective antiquark field variables \(Q^-_v\) and \(\bar{Q}^-_v\).

The effective heavy quark currents can also be derived easily via the \(1/m_Q\) expansion approach. The general form of a heavy quark current may be written as

\[ J(x) = \bar{Q}'(x) \Gamma Q(x) \]  

with \(\Gamma\) an arbitrary Dirac matrix. Using eq.\((2.1)\) this current can be decomposed into four parts,

\[ J = \bar{Q}'^+ \Gamma Q^+ + \bar{Q}'^+ \Gamma Q^- + \bar{Q}'^- \Gamma Q^+ + \bar{Q}'^- \Gamma Q^-, \]
where $Q^\pm$ are related to $\hat{Q}^\pm_v$ by eq.(2.3), whereas $\hat{Q}^+_v$ and $\hat{Q}^+_{v'}$ are related to new variables $Q^+_v$ and $Q^+_{v'}$ by eq.(2.10).

The equations of motion in eq.(2.8) can be used to eliminate the field variables $\hat{Q}^+_v$ and $\bar{Q}^+_v$. The four parts in eq.(2.10) then get the following form in the framework of the new formulation of HQEFT

\[
\begin{align*}
< \hat{Q}^+Q^+ > & = \bar{Q}^+_v \Gamma \hat{\omega} \hat{Q}^+_v, \\
< \hat{Q}^+Q^- > & = \bar{Q}^+_v \bar{\omega} \Gamma \hat{\omega} \hat{Q}^-_v = \bar{Q}^+_v \bar{\omega} \Gamma \hat{\omega} (-\hat{A}^{-1}\hat{B}) \hat{Q}^+_v, \\
< \hat{Q}^-Q^+ > & = \bar{Q}^+_v \bar{\omega} \Gamma \hat{\omega} \hat{Q}^+_v = \bar{Q}^+_v (-\hat{B} (\hat{A})^{-1}) \bar{\omega} \Gamma \hat{\omega} \hat{Q}^+_v, \\
< \hat{Q}^-Q^- > & = \bar{Q}^+_v \bar{\omega} \Gamma \hat{\omega} \hat{Q}^-_v = \bar{Q}^+_v (-\hat{B} (\hat{A})^{-1}) \bar{\omega} \Gamma \hat{\omega} (-\hat{A}^{-1}\hat{B}) \hat{Q}^+_v. \quad (2.17)
\end{align*}
\]

The operators $\hat{\omega}, \bar{\omega}$ and $\hat{A}, \hat{B}$ have been defined in eqs.(2.3) and (2.6). Here we have used the symbol $< \hat{j} >$ to represent the effective operator of $\hat{j}$ within the framework of the new formulation of HQEFT.

Expanding the formulae in eq.(2.17) in powers of $1/m_Q$ and summing them up, one then obtains the effective heavy quark current

\[
J^{(++)}_{\text{eff}} \equiv < \bar{Q} \Gamma Q > = e^{i(m_Q v' - \bar{m}_Q v)x} \{ \bar{Q}^+_v \Gamma Q^+_v + \frac{1}{2m_Q} \bar{Q}^+_v \Gamma \frac{1}{i\not{D}} + \Lambda (i\not{D}_\perp)^2 Q^+_v \\
+ \frac{1}{2m_Q'} \bar{Q}^+_v (-i \not{D}_\perp)^2 \frac{1}{-i \not{D}_\parallel + \Lambda} \frac{1}{\not{D}_\perp} \Gamma Q^+_v + \frac{1}{4m_Q^2} \bar{Q}^+_v \Gamma \frac{1}{i\not{D}_\parallel + \Lambda} (i\not{D}_\perp - \Lambda) \}
\times i\not{D}_\perp Q^+_v + \frac{1}{4m_Q^2} \bar{Q}^+_v (-i \not{D}_\perp)(-i \not{D}_\parallel - \Lambda)(-i \not{D}_\perp) \frac{1}{\not{D}_\parallel + \Lambda} \Gamma Q^+_v \\
+ \frac{1}{4m_Q^2 \bar{m}_Q} \bar{Q}^+_v (-i \not{D}_\perp)^2 \frac{1}{\not{D}_\parallel + \Lambda} \frac{1}{\not{D}_\perp} \Gamma (i\not{D}_\perp)^2 Q^+_v + O\left(\frac{1}{m_Q^3}\right) \}
\equiv J^{(0)}_{\text{eff}} + J^{(1/m_Q)}_{\text{eff}} \quad (2.18)
\]

with $J^{(0)}_{\text{eff}}$ the leading term $J^{(0)}_{\text{eff}} = e^{i(m_Q v' - \bar{m}_Q v)x} \bar{Q}^+_v \Gamma Q^+_v$ and $J^{(1/m_Q)}_{\text{eff}}$ the remaining terms in $J^{(++)}_{\text{eff}}$. In appendix A we also present the derivation of $L^{(++)}_{\text{eff}}$ and $J^{(++)}_{\text{eff}}$ from the functional integral method. We see that the approaches of the functional integral and the equations of motion are indeed equivalent.

One can see from eqs.(2.3), (2.6) and (2.10) that, if we neglect the antiquark contributions and choose $\Lambda = 0$, the Lagrangian becomes

\[
L_{\text{eff}} = \bar{Q}(i\not{D} - m_Q)Q = Q_v i\sigma \cdot DQ_v + Q_v i\not{D}_\perp \frac{1}{2m_Q + i\not{v} \cdot D} i\not{D}_\perp Q_v \\
= \bar{Q}_v i\sigma \cdot DQ_v + \frac{1}{2m_Q} \bar{Q}_v (i\not{D}_\perp)^2 Q_v - \frac{1}{4m_Q^2} \bar{Q}_v i\not{D}_\perp i\not{v} \cdot D i\not{D}_\perp Q_v + O\left(\frac{1}{m_Q^3}\right). \quad (2.19)
\]

which is just the Lagrangian used in the usual HQEFT, whose starting point is separating the quark field $Q$ into ‘large’ and ‘small’ component fields $Q_v$ and $\chi_v$ as follows,
\[
Q(x) = e^{-im_Q v \cdot x}[Q_v(x) + \chi_v(x)],
\]
where
\[
Q_v(x) = e^{im_Q v \cdot x} \frac{1 + \frac{\gamma^\mu}{2} Q(x)}{2}, \quad \chi_v(x) = e^{im_Q v \cdot x} \frac{1 - \frac{\gamma^\mu}{2} Q(x)}{2}.
\]

In this case, the arbitrary current \( \bar{Q}' \Gamma Q \) turns into the form in the usual HQET, i.e.,
\[
J = \bar{Q}' \Gamma Q = e^{i(m_{Q'} - m_Q)v \cdot x}\{\bar{Q}_v' \Gamma Q_v + \frac{1}{2m_Q} \bar{Q}_v' \Gamma i \not{D}_Q Q_v + \frac{1}{2m_{Q'}} \bar{Q}_v' (-i \not{D}_Q) \Gamma Q_v
\]
\[ - \frac{1}{4m_Q} Q_v' \Gamma (iv \cdot D) i \not{D}_Q Q_v - \frac{1}{4m_{Q'}} Q_v' (-i \not{D}_Q) (-iv' \cdot D) \Gamma Q_v
\]
\[ + \frac{1}{4m_Q m_{Q'}} Q_v' (-i \not{D}_Q) \Gamma i \not{D}_Q Q_v + O\left( \frac{1}{m_{Q'}^3} \right) \}.
\]

III. WEAK TRANSITION MATRIX ELEMENTS

In the framework of the effective theory, the hadronic matrix element in full QCD theory can be expanded into a series of matrix elements in terms of \( 1/m_Q \). We will study in this section the hadronic matrix element up to the order of \( 1/m_Q^2 \).

Using the following conventional relativistic normalization
\[
< H(p') | H(p) > = 2p^0 (2\pi)^3 \delta^3 (\vec{p}' - \vec{p}),
\]
the conservation of the vector current \( \bar{Q} \gamma^\mu Q \) leads to
\[
< H(p) | \bar{Q} \gamma^\mu Q | H(p) > = 2p^\mu = 2m_H v^\mu,
\]
where $|H>$ denotes a hadron state in QCD and $p^\mu = m_H v^\mu$ is the momentum of the heavy hadron $H$.

In the effective theory, it is better to introduce an effective heavy hadron state $|H_v>$ which exhibits a manifest spin-flavor symmetry. Thus, the hadron state $|H_v>$ should be related to the state $|H>$ by the following relation

$$\frac{1}{\sqrt{m_H m_{H'}}} < H'|Q\Gamma Q|H> = \frac{1}{\sqrt{\bar{\Lambda}_H \bar{\Lambda}_{H'}}} < H'|J^{(++)}_{e f f} e^{i \int d^4x L_{e f f}^{(1/m_Q)}} |H_v>.$$  \hspace{1cm} (3.3)

Here $\bar{\Lambda}_H$ and $\bar{\Lambda}_{H'}$ are binding energies defined as

$$\bar{\Lambda}_H \equiv m_H - m_Q, \quad \bar{\Lambda}_{H'} \equiv m_{H'} - m_{Q'}.$$  \hspace{1cm} (3.4)

The flavor-dependent factor $\frac{1}{\sqrt{\Lambda_H \Lambda_{H'}}}$ appears due to the different normalizations of the two hadron states $|H>$ and $|H_v>$. Here the normalization of $|H_v>$ is taken so as to exhibit a manifest spin-flavor symmetry, i.e.,

$$< H_v|\bar{Q}_v^+ \gamma^\mu Q_v^+ |H_v> = 2 \bar{\Lambda} v^\mu$$  \hspace{1cm} (3.5)

with

$$\bar{\Lambda} = \lim_{m_Q \to \infty} \bar{\Lambda}_H$$

being a heavy flavor-independent binding energy that reflects the effects of the light degrees of freedom in the heavy hadron.

In the heavy quark expansion, $1/m_Q$ corrections to the hadronic matrix elements can be classified into three parts [17,18]: (I). corrections from purely effective current $J^{(1/m_Q)}_{e f f}$; (II). corrections from purely effective Lagrangian $L^{(1/m_Q)}_{e f f}$; and (III). mixed corrections from both $J^{(1/m_Q)}_{e f f}$ and $L^{(1/m_Q)}_{e f f}$.

The leading matrix element is simply given by

$$A^{(0)} \equiv < H_v'|J^{(0)}_{e f f}|H_v> = < H_v'|\bar{Q}_v^+ \Gamma Q_v^+ |H_v>.$$  \hspace{1cm} (3.6)

The three types of corrections are found to be

$$A^{(I)} \equiv < H_v'|J^{(1/m_Q)}_{e f f}|H_v> = \frac{1}{2m_Q} < H_v'|\bar{Q}_v^+ O_1(\Gamma) Q_v^+ |H_v>$$

$$+ \frac{1}{2m_Q} < H_v'|\bar{Q}_v^+ O'_1(\Gamma) Q_v^+ |H_v> + \frac{1}{4m_Q^2} < H_v'|\bar{Q}_v^+ O_2(\Gamma) Q_v^+ |H_v>$$

$$+ \frac{1}{4m_Q^2} < H_v'|\bar{Q}_v^+ O'_2(\Gamma) Q_v^+ |H_v> + \frac{1}{4m_Q m_Q} < H_v'|\bar{Q}_v^+ O_4(\Gamma) Q_v^+ |H_v> + O\left(\frac{1}{m_Q}\right),$$  \hspace{1cm} (3.7)

$$A^{(II)} \equiv < H_v'|J^{(0)}_{e f f} e^{i \int d^4x L^{(1/m_Q)}_{e f f}} |H_v> = - \frac{1}{m_Q} < H_v'|\bar{Q}_v^+ O_1(\Gamma) Q_v^+ |H_v>$$

$$+ \frac{1}{4m_Q} < H_v'|\bar{Q}_v^+ O_2(\Gamma) Q_v^+ |H_v> + \frac{1}{4m_Q^2} < H_v'|\bar{Q}_v^+ O_4(\Gamma) Q_v^+ |H_v> + O\left(\frac{1}{m_Q}\right),$$  \hspace{1cm} (3.8)

$$A^{(III)} \equiv < H_v'|J^{(0)}_{e f f} e^{i \int d^4x L^{(1/m_Q)}_{e f f}} |H_v> = - \frac{1}{m_Q} < H_v'|\bar{Q}_v^+ O_1(\Gamma) Q_v^+ |H_v>$$

$$+ \frac{1}{4m_Q} < H_v'|\bar{Q}_v^+ O_2(\Gamma) Q_v^+ |H_v> + \frac{1}{4m_Q^2} < H_v'|\bar{Q}_v^+ O_4(\Gamma) Q_v^+ |H_v> + O\left(\frac{1}{m_Q}\right).$$  \hspace{1cm} (3.9)
\[-\frac{1}{m_Q'} < H_{\nu'} | \bar{Q}^+_{\nu'} O'_1 (\Gamma) Q^+_v | H_v > - \frac{1}{2m_Q^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O_2 (\Gamma) Q^+_v | H_v >\]

\[-\frac{1}{2m_{Q'}^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O_2 (\Gamma) Q^+_v | H_v > + \frac{1}{4m_Q^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O_3 (\Gamma) Q^+_v | H_v >\]

\[+ \frac{1}{4m_{Q'}^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O_3 (\Gamma) Q^+_v | H_v > + \frac{1}{m_Q' m_Q} < H_{\nu'} | \bar{Q}^+_{\nu'} O_4 (\Gamma) Q^+_v | H_v > + O \left( \frac{1}{m_Q^{3 \epsilon}} \right), \quad (3.8)\]

\[\mathcal{A}^{(III)} \equiv < H_{\nu'} | J_{eff}^{(1/m_Q)} e^{i \int d^4 x L_{eff}^{(1/m_Q)}} | H_v > = - \frac{1}{2m_Q^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O_3 (\Gamma) Q^+_v | H_v >\]

\[-\frac{1}{2m_{Q'}^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O_3 (\Gamma) Q^+_v | H_v > - \frac{1}{m_Q' m_Q} < H_{\nu'} | \bar{Q}^+_{\nu'} O_4 (\Gamma) Q^+_v | H_v > + O \left( \frac{1}{m_Q^{3 \epsilon}} \right), \quad (3.9)\]

where the operators \( O_1 (\Gamma) \) and \( O'_1 (\Gamma) \) are defined as follows:

\[O_1 (\Gamma) = \Gamma \left( \frac{1}{i \not{D}^\parallel + \Lambda} (i \not{\partial}^\perp)^2 \right),\]

\[O'_1 (\Gamma) = (-i \not{\partial}^\perp)^2 \left( \frac{1}{-i \not{D}^\parallel + \Lambda} \right) \Gamma,\]

\[O_2 (\Gamma) = \Gamma \left( \frac{1}{i \not{D}^\parallel + \Lambda} (i \not{\partial}^\perp)^2 (i \not{D}^\parallel - \Lambda) i \not{\partial}^\perp \right),\]

\[O'_2 (\Gamma) = (-i \not{\partial}^\perp)^2 \left( -i \not{D}^\parallel - \Lambda \right) (-i \not{\partial}^\perp) \left( \frac{1}{-i \not{D}^\parallel + \Lambda} \right) \Gamma,\]

\[O_3 (\Gamma) = \Gamma \left( \frac{1}{i \not{D}^\parallel + \Lambda} (i \not{\partial}^\perp)^2 \frac{1}{i \not{D}^\parallel + \Lambda} (i \not{\partial}^\perp)^2 \right),\]

\[O'_3 (\Gamma) = (-i \not{\partial}^\perp)^2 \left( \frac{1}{-i \not{D}^\parallel + \Lambda} (-i \not{\partial}^\perp)^2 \right) \left( \frac{1}{-i \not{D}^\parallel + \Lambda} \right) \Gamma,\]

\[O_4 (\Gamma) = (-i \not{\partial}^\perp)^2 \left( \frac{1}{-i \not{D}^\parallel + \Lambda} \right) \frac{1}{i \not{D}^\parallel + \Lambda} (i \not{\partial}^\perp)^2. \quad (3.10)\]

The first type of corrections \( \mathcal{A}^{(I)} \) can be read from eq. (2.18). In obtaining the second and the third types of corrections \( \mathcal{A}^{(II)} \) and \( \mathcal{A}^{(III)} \), we have contracted the heavy quark pair and used the heavy quark propagator \( i/(i \not{D}^\parallel + \Lambda) \).

The hadronic matrix element is then given by:

\[\mathcal{A} \equiv < H_{\nu'} | J_{eff}^{(\ldots)} e^{i \int d^4 x L_{eff}^{(1/m_Q)}} | H_v > = \mathcal{A}^{(0)} + \mathcal{A}^{(I)} + \mathcal{A}^{(II)} + \mathcal{A}^{(III)}\]

\[= < H_{\nu'} | \bar{Q}^+_{\nu'} \Gamma Q^+_v | H_v > - \frac{1}{2m_Q} < H_{\nu'} | \bar{Q}^+_{\nu'} O_1 (\Gamma) Q^+_v | H_v >\]

\[-\frac{1}{2m_{Q'}^2} < H_{\nu'} | \bar{Q}^+_{\nu'} O'_1 (\Gamma) Q^+_v | H_v > - \frac{1}{m_Q' m_Q} < H_{\nu'} | \bar{Q}^+_{\nu'} O_2 (\Gamma) Q^+_v | H_v >\]
\[-\frac{1}{4m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_2^\ast (\Gamma) Q_{\nu}^+ | H_{\nu} > - \frac{1}{4m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_3^\ast (\Gamma) Q_{\nu}^+ | H_{\nu} > \]
\[-\frac{1}{4m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_4^\ast (\Gamma) Q_{\nu}^+ | H_{\nu} > + \frac{1}{4m_Q m_Q} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_4 (\Gamma) Q_{\nu}^+ | H_{\nu} > \]
\[+ O \left( \frac{1}{m_Q^3} \right). \tag{3.11} \]

From eqs. (3.3) and (3.11) as well as the hadron state normalization conditions in eqs. (3.2) and (3.5), one can extract the hadron mass by setting \( \nu' = \nu \). Explicitly, one has

\[2m_H v^\mu = < H | Q^\gamma \mu Q | H > = \frac{m_H}{\Lambda_H} \left\{ 2 \Lambda v^\mu - \frac{1}{m_Q} < H_{\nu}^\prime | Q_{\nu}^+ O_4 (\gamma^\mu) Q_{\nu}^+ | H_{\nu} > \right. \]
\[\left. - \frac{1}{2m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ (O_2 (\gamma^\mu) + O_3 (\gamma^\mu)) Q_{\nu}^+ | H_{\nu} > \right. \]
\[\left. + \frac{1}{4m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_4 (\gamma^\mu) Q_{\nu}^+ | H_{\nu} > + O \left( \frac{1}{m_Q^3} \right) \right\}, \tag{3.12}\]

which leads to the following relation

\[\tilde{\Lambda}_H = \tilde{\Lambda} - \frac{1}{2m_Q} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_4 (\gamma^\mu) Q_{\nu}^+ | H_{\nu} > - \frac{1}{4m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ (O_2 (\gamma^\mu) + O_3 (\gamma^\mu)) Q_{\nu}^+ | H_{\nu} > \]
\[+ \frac{1}{8m_Q^2} < H_{\nu}^\prime | \tilde{Q}_{\nu}^+ O_4 (\gamma^\mu) Q_{\nu}^+ | H_{\nu} > + O \left( \frac{1}{m_Q^3} \right). \tag{3.13}\]

The heavy hadron mass is then given by

\[m_H = m_Q + \tilde{\Lambda}_H = m_Q + \tilde{\Lambda} + O(1/m_Q), \tag{3.14}\]

which coincides with the fact that the mass of a hadron equals the sum of the heavy quark mass \( m_Q \), the binding energy \( \tilde{\Lambda} \) due to light degrees of freedom and terms suppressed by \( 1/m_Q \).

Let us consider the case that both the initial and final states are pseudoscalar mesons (or both vector mesons as well). An interesting example is the \( B \to D \) transition matrix element of vector current. From eqs. (3.3) and (3.11), we obtain

\[< D | \bar{c} \gamma^\mu b | B > = \sqrt{\frac{m_{DmB}}{\Lambda_D \Lambda_B}} \left\{ < D_{\nu}^\prime | \bar{c}_{\nu}^\gamma \gamma^\mu b_{\nu}^+ | B_{\nu} > - \frac{1}{2m_b} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_4 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > \right. \]
\[\left. - \frac{1}{2m_c} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_1 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > - \frac{1}{4m_b^2} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_2 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > \right. \]
\[\left. - \frac{1}{4m_c^2} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_2 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > - \frac{1}{4m_b^2} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_3 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > \right. \]
\[\left. - \frac{1}{4m_c^2} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_3 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > + \frac{1}{4m_c m_b} < D_{\nu}^\prime | \bar{c}_{\nu}^+ O_4 (\gamma^\mu) b_{\nu}^+ | B_{\nu} > \right. \]
\[\left. + O \left( \frac{1}{m_b^3} \right) \right\}. \tag{3.15}\]
From spin-flavor symmetry, one can use the following relations for operator $O_i$

$$< B_v | \bar{b}_i^\nu O_i b_v^\nu | B_v > = < D_v | \bar{c}^\nu O_i c_v^\nu | D_v > = < P_v | \bar{Q}_v^\nu O_i Q_v^\nu | P_v > .$$  \hspace{1cm} (3.16)

With these relations and eqs.\{3.13\}, eq.\{3.15\} can be simplified to the following form at zero recoil $v' = v$,

$$< D | \bar{c}^\nu b > |_{q^2 = q^2_{max}} = 2 \sqrt{m_B m_D} v | \{ 1 + \frac{1}{32 \Lambda^2} \left( \frac{1}{m_b} - \frac{1}{m_c} \right)^2 \} < P_v | \bar{Q}_v^\nu O_1(\tilde{\psi}) Q_v^\nu | P_v >^2$$

$$- \frac{1}{16 \Lambda} \left( \frac{1}{m_b} - \frac{1}{m_c} \right)^2 < P_v | \bar{Q}_v^\nu O_4(\tilde{\psi}) Q_v^\nu | P_v > + O \left( \frac{1}{m_{3b(c)}^2} \right) ,$$  \hspace{1cm} (3.17)

which explicitly shows that when working with the normalizations in eqs.\{3.2\}, \{3.3\} and \{3.3\}, the transition matrix elements of heavy quark vector current between two pseudoscalar states with different spins, it is not so manifest and one needs to analyze the concrete Lorentz structures of currents and meson states. This is the object of the next section.

IV. MESON FORM FACTORS AND LUKE'S THEOREM

The HQET has been applied fruitfully to study heavy meson decays. In particular, the Luke’s theorem \[12\] plays an important role in the determination of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ from exclusive $B \rightarrow D^* l \bar{\nu}$ decay. This is because the order $1/m_Q$ corrections to the $b \rightarrow c$ transition matrix element of weak current are absent at zero recoil. As the Luke’s theorem was deduced from the heavy quark effective Lagrangian and currents in the usual HQET, it is interesting to check whether the Luke’s theorem remains valid within the framework of the new formulation of HQEFT. At the same time, the order $1/m_Q^2$ corrections, though expected to be small, are worthy to be considered since they can provide us information about the convergence of the $1/m_Q$ expansion. And as we will illustrate, some useful features can be observed because the new formulation of the heavy quark effective Lagrangian (i.e. eq.\{2.12\}) and currents (eq.\{2.18\}) are different from the usual ones due to new contributions from integrating out antiparticles.

Generally, matrix elements of vector and axial vector currents between pseudoscalar and vector ground meson states are described by 18 meson form factors defined as follows

$$< D'(v') | \bar{c}^\nu b | B(v) >= \sqrt{m_D m_B} [ h_1(\omega)(v + v')^\mu + h_2(\omega)(v - v')^\mu ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu b | B(v) >= i \sqrt{m_D m_B} [ h_1(\omega) \epsilon^\nu \nu^\mu \epsilon_{\alpha \beta} \nu_\alpha v_\beta ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu c^\alpha b | B(v) >= \sqrt{m_D m_B} [ h_1(\omega)(1 + \omega) \epsilon^\nu \nu^\mu - h_2(\omega)(\epsilon^\nu \nu^\mu) ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu c^\alpha c^\beta b | B(v) > = \sqrt{m_D m_B} [ h_1(\omega)(1 + \omega) \epsilon^\nu \nu^\mu - h_2(\omega)(\epsilon^\nu \nu^\mu) ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu c^\alpha c^\beta b | B^*(v, \epsilon) >= i \sqrt{m_D m_B} [ \epsilon^\nu \nu^\mu \epsilon_{\alpha \beta} \nu_\alpha v_\beta ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu c^\alpha c^\beta b | B^*(v, \epsilon) >= i \sqrt{m_D m_B} [ \epsilon^\nu \nu^\mu \epsilon_{\alpha \beta} \nu_\alpha v_\beta ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu c^\alpha c^\beta b | B^*(v, \epsilon) >= i \sqrt{m_D m_B} [ \epsilon^\nu \nu^\mu \epsilon_{\alpha \beta} \nu_\alpha v_\beta ] ,$$

$$< D^*(v', \epsilon') | \bar{c}^\nu c^\alpha c^\beta b | B^*(v, \epsilon) >= i \sqrt{m_D m_B} [ \epsilon^\nu \nu^\mu \epsilon_{\alpha \beta} \nu_\alpha v_\beta ] .$$  \hspace{1cm} (4.1)
In order to relate explicitly the form factors $h_i(\omega)$ to matrix elements of operators, eq. (3.11) can be rewritten as

$$
\mathcal{A} \equiv < H_{v'}^{\prime} | J_{\text{eff}}^{(++)} e^{i \int d^4x L_{\text{eff}}^{(1/\alpha_q)}} | H_v > = \mathcal{A}^{(0)} + \mathcal{A}^{(I)} + \mathcal{A}^{(II)} + \mathcal{A}^{(III)}
$$

$$
= < H_{v'}^{\prime} | Q^+_{v'} \Gamma Q^+_v | H_v > - \frac{1}{2m_Q} < H_{v'}^{\prime} | Q^+_{v'} \Gamma \frac{1}{\Lambda + i v \cdot D} P_+ [D_+^2 + i 2 \sigma_{\alpha \beta} F^{\alpha \beta}] Q^+_v | H_v >
$$

$$
- \frac{1}{2m_Q} < H_{v'}^{\prime} | Q^+_{v'} \Gamma \frac{1}{\Lambda + i v \cdot D} P_+ [D_+^2 + i 2 \sigma_{\alpha \beta} F^{\alpha \beta}] Q^+_v | H_v >
$$

$$
- \frac{1}{4m_Q^2} < H_{v'}^{\prime} | Q^+_{v'} \Gamma \frac{1}{\Lambda + iv \cdot D} P_+ [\Lambda D_+^2 + i D_+^2 (v \cdot D) - i D_+^2 v_\alpha F^{\alpha \beta} + \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta}] | H_v >
$$

$$
+ \Lambda - i \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta} - \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta} (v \cdot D) - \sigma^{\alpha \beta} v_\beta F^{\alpha \beta} + \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta} P_+ \frac{1}{\Lambda + i v \cdot D} P - \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta} | Q^+_v | H_v >
$$

$$
- \frac{1}{4m_Q^2} < H_{v'}^{\prime} | Q^+_{v'} \Gamma [\Lambda D_+^2 + D_+^2 (v \cdot D) - i D_+^2 v_\alpha F^{\alpha \beta} + \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta}] | H_v >
$$

$$
+ \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta} - \frac{1}{2} (v \cdot D) \sigma_{\alpha \beta} F^{\alpha \beta} - F^{\alpha \beta} D_\alpha v_\beta \sigma^{\alpha \beta} + \frac{i}{2} \sigma_{\alpha \beta} F^{\alpha \beta} P_+ \frac{1}{\Lambda + i v \cdot D} P - \frac{1}{2} \sigma_{\alpha \beta} F^{\alpha \beta} | Q^+_v | H_v >
$$

with $F^{\alpha \beta} = [D_\alpha, D_\beta]$ the field strength of gluons tensor, $\sigma^{\alpha \beta} = \frac{i}{2} [\gamma^\alpha, \gamma^\beta]$, $P_+ = \frac{1+i \epsilon^\mu}{2}$ and $P'_+ = \frac{1+i \epsilon' \cdot D}{2}$.

A simple way to evaluate the hadronic matrix elements is to associate the spin wave functions adopted in refs. [17, 18, 20]. In the framework of new formulation of HQEFT with the normalization condition eq. (3.3), the spin wave functions have the following form

$$
\mathcal{M}(v) = \sqrt{\Lambda} P_+ \left\{ \begin{array}{ll}
-\gamma^5, & \text{pseudoscalar meson} \\
\epsilon_\mu, & \text{vector meson}
\end{array} \right\}
$$

with meson states $|M_v>$ defined in the HQEFT. Here $\epsilon^\mu$ is the polarization vector of the vector meson.

Lorentz invariance enables us to parameterized the relevant matrix elements by the following trace formulre

$$
< M_{v'}^{\prime} | Q^+_{v'} \Gamma Q^+_v | M_v > = -\xi(\omega) Tr[\mathcal{M} \mathcal{M}],
$$

$$
< M_{v'}^{\prime} | Q^+_{v'} \Gamma -\frac{1}{\Lambda + i v \cdot D} P_+ D_+^2 Q^+_v | M_v > = -\kappa_1(\omega) \frac{1}{\Lambda} Tr[\mathcal{M} \mathcal{M}],
$$

12
\[
< M'_{\nu} \bar{Q}^+_{\nu} \Gamma \frac{-1}{\Lambda + iv \cdot D} P + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_\nu | M_{\nu} > = \frac{1}{\Lambda} Tr[\kappa_{\alpha\beta}(v, v') \tilde{\mathcal{M}} \Gamma P \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]
\[
< M'_{\nu} |Q^+_{\nu} \Gamma \frac{1}{\Lambda + iv \cdot D} P [iD^2 (v \cdot D) - iD^\alpha v^\beta F_{\alpha\beta}] Q^+_\nu | M_{\nu} > = -\frac{\eta_1}{\Lambda} Tr[\tilde{\mathcal{M}} \Gamma \mathcal{M}],
\]
\[
< M'_{\nu} |\bar{Q}^+_{\nu} \Gamma \frac{1}{\Lambda + iv \cdot D} P [\frac{-1}{2} \sigma_{\alpha\beta} F^{\alpha\beta}(v \cdot D) - \sigma^{\alpha\gamma} v^\beta D_{\alpha\beta} Q^+_\nu | M_{\nu} >
= \frac{1}{\Lambda} Tr[\eta_{\alpha\beta}(v, v') \tilde{\mathcal{M}} \Gamma TP \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]
\[
< M'_{\nu} |\bar{Q}^+_{\nu} \Gamma \frac{1}{\Lambda + iv \cdot D} P [D^2 \frac{1}{\Lambda + iv \cdot D} P + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_\nu | M_{\nu} >
= \frac{1}{\Lambda} Tr[\chi_{\alpha\beta}(v, v') \tilde{\mathcal{M}} \Gamma TP \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]
\[
< M'_{\nu} |\bar{Q}^+_{\nu} \Gamma \frac{1}{\Lambda + iv \cdot D} P [\frac{-1}{2} \sigma_{\alpha\beta} F^{\alpha\beta}(v \cdot D) + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_\nu | M_{\nu} >
= -\frac{1}{\Lambda} Tr[\chi_{\alpha\beta}(v, v') \tilde{\mathcal{M}} \Gamma TP \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]
\[
< M'_{\nu} |\bar{Q}^+_{\nu} \Gamma \frac{-1}{\Lambda - iv \cdot D} P + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_\nu | M_{\nu} > = \frac{1}{\Lambda} Tr[\eta_{\alpha\beta}(v, v') \tilde{\mathcal{M}} \Gamma TP \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]
\[
< M'_{\nu} |\bar{Q}^+_{\nu} \Gamma \frac{1}{\Lambda - iv \cdot D} P + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_\nu | M_{\nu} > = \frac{1}{\Lambda} Tr[\eta_{\alpha\beta}(v, v') \tilde{\mathcal{M}} \Gamma TP \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]
\[
< M'_{\nu} |\bar{Q}^+_{\nu} \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} P' \frac{-1}{\Lambda - iv \cdot D} \Gamma Q^+_\nu | M_{\nu} > = \frac{1}{\Lambda} Tr[\tilde{\kappa}_{\alpha\beta}(v', v) \tilde{\mathcal{M}} \Gamma P' \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]

(4.4)

Note that all other matrix elements in eq.(4.4) can also be represented by the form factors introduced in eq.(4.4). For example, one can conjugate the second equality in eq.(4.4) to get

\[
< M'_{\nu} |\bar{Q}^+_{\nu} \Gamma \frac{1}{\Lambda + iv \cdot D} P + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_\nu | M_{\nu} > = \frac{1}{\Lambda} Tr[\tilde{\kappa}_{\alpha\beta}(v', v) \tilde{\mathcal{M}} \Gamma P' \frac{i}{2} \sigma_{\alpha\beta} \mathcal{M}],
\]

(4.5)

where \( \omega = v \cdot v' \), \( \tilde{\mathcal{M}} = \gamma^0 \tilde{\mathcal{M}} \gamma^0 \) and \( \tilde{\kappa}_{\alpha\beta}, \tilde{\eta}_{\alpha\beta}, \tilde{\chi}_{\alpha\beta} \) are defined similarly. The decomposition of the Lorentz tensors \( \kappa_{\alpha\beta}(v, v'), \eta_{\alpha\beta}(v, v'), \chi_{\alpha\beta}(v, v'), \tilde{\eta}_{\alpha\beta}(v, v'), \tilde{\chi}_{\alpha\beta}(v, v') \) and \( \eta_{\alpha\beta\gamma\delta}(v, v') \) are presented in the appendix [5, 7, 11]. \( \xi(1) \) is the Isgur-Wise function that normalizes to unity at the point of zero recoil \( \omega = 1 \), i.e., \( \xi(1) = 1 \) [12, 20, 21].

When taking the \( \Lambda \) as the heavy flavor-independent binding energy \( \tilde{\Lambda} \), the momentum \( \tilde{k} = P_Q - \hat{m}_Q v \) carried by the effective field \( Q^+_\nu \) within the heavy hadron is expected to be much smaller than the binding energy \( \tilde{\Lambda} \). So we can perform following expansion

\[
\frac{1}{\Lambda + iv \cdot D} \rightarrow \frac{1}{\Lambda + iv \cdot D} = \frac{1}{\tilde{\Lambda}} \left( 1 + O\left( \frac{iv \cdot D}{\tilde{\Lambda}} \right) \right) \sim \frac{1}{\tilde{\Lambda}},
\]

(4.6)
Here $\bar{\Lambda}$ characterizes the effects of the light degrees of freedom in the heavy hadron. This may be understood as follows: In general a heavy quark within a hadron cannot truly be on-shell due to strong interactions among heavy quark and light quark as well as soft gluons. The off-shellness of the heavy quark in the heavy hadron is characterized by a residual momentum $k = \bar{\Lambda}v + \bar{k}$. The total momentum $P_Q$ of the heavy quark in a hadron may be written as: $P_Q = m_Qv + k = \hat{m}_Qv + \hat{k}$. Thus the residual momentum $k = \bar{\Lambda}v + \bar{k}$ of the heavy quark within a hadron is assumed to comprise the main contributions of the light degrees of freedom. Where $\bar{k}$ is the part which depends on heavy flavor and is suppressed by $1/m_Q$. With this picture the heavy quark may be regarded as a ‘dressed heavy quark’, and the heavy hadron containing a single heavy quark is more reliable to be considered as a dualized particle of a ‘dressed heavy quark’. This differs from the picture in the usual HQET, where one mainly deals with the heavy quark and treats the light quark as a spectator. For this reason, the form factors defined in the ways of eq.(4.4) should have a very weak dependence on the light constituents of heavy hadrons. Thus it is useful in the new formulation of HQEFT to define the ‘dressed heavy quark’ mass as

$$\hat{m}_Q \equiv \lim_{m_Q \to \infty} m_H = m_Q + \bar{\Lambda}. \quad (4.7)$$

One can complete the trace calculation and write the matrix elements of vector and axial vector currents between pseudoscalar and vector mesons in terms of Lorentz scalar factors $\kappa_i, \varrho_i, \chi_i$ and $\eta_i$. At the zero recoil point, we obtain

$$h_+(1) = \sqrt{\frac{1}{\Lambda_D \Lambda_B}} \bar{\Lambda} \{ \xi - \left( \frac{1}{2m_b \Lambda} + \frac{1}{2m_c \Lambda} - \frac{1}{4m_b^2} - \frac{1}{4m_c^2} \right) (\kappa_1 + 3\kappa_2) - \frac{1}{4\Lambda^2} \left( \frac{1}{m_b^2} + \frac{1}{m_c^2} \right) (\varrho_1 \bar{\Lambda} + 3\varrho_2 \bar{\Lambda} + \chi_1 + 3\chi_2 - 3\chi_4 - 9\chi_5 - 6\chi_6) + \frac{1}{4m_b m_c \Lambda^2} (\eta_1 + 6\eta_3 - 3\eta_4 - 9\eta_5 - 6\eta_6) \},$$

$$h_{A_1}(1) = \sqrt{\frac{1}{\Lambda_D \Lambda_B^*}} \bar{\Lambda} \{ \xi - \left( \frac{1}{2m_b \Lambda} - \frac{1}{4m_b^2} \right) (\kappa_1 + 3\kappa_2) - \frac{1}{4\Lambda^2} \left( \frac{1}{m_b^2} + \frac{1}{m_c^2} \right) (\varrho_1 \bar{\Lambda} + 3\varrho_2 \bar{\Lambda} + \chi_1 + 3\chi_2 - 3\chi_4 - 9\chi_5 - 6\chi_6) - \frac{1}{4m_b^2 \Lambda^2} (\varrho_1 \bar{\Lambda} - \varrho_2 \bar{\Lambda} + \chi_1 - \chi_2 - 3\chi_4 - \chi_5 + 5\chi_6) + \frac{1}{4m_b m_c \Lambda^2} (\eta_1 + 2\eta_2 + \eta_4 + 3\eta_5 + 2\eta_6) \},$$

$$h_1(1) = \sqrt{\frac{1}{\Lambda_D \Lambda_B^*}} \bar{\Lambda} \{ \xi - \left( \frac{1}{2m_b \Lambda} + \frac{1}{2m_c \Lambda} - \frac{1}{4m_b^2} - \frac{1}{4m_c^2} \right) (\kappa_1 - \kappa_2) - \frac{1}{4\Lambda^2} \left( \frac{1}{m_b^2} + \frac{1}{m_c^2} \right) (\varrho_1 \bar{\Lambda} - \varrho_2 \bar{\Lambda} + \chi_1 - \chi_2 - 3\chi_4 + \chi_5 + 2\chi_6) + \frac{1}{4m_b m_c \Lambda^2} (\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) \},$$

$$h_{\gamma}(1) = -\sqrt{\frac{1}{\Lambda_D \Lambda_B^*}} \bar{\Lambda} \{ \xi - \left( \frac{1}{2m_b \Lambda} + \frac{1}{2m_c \Lambda} - \frac{1}{4m_b^2} - \frac{1}{4m_c^2} \right) (\kappa_1 - \kappa_2) - \frac{1}{4\Lambda^2} \left( \frac{1}{m_b^2} + \frac{1}{m_c^2} \right) (\varrho_1 \bar{\Lambda} - \varrho_2 \bar{\Lambda} + \chi_1 - \chi_2 - 3\chi_4 + \chi_5 + 2\chi_6) + \frac{1}{4m_b m_c \Lambda^2} (\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) \},$$
where $\xi(1)$ has been written as $\xi$ for simplicity, and similarly for other form factors.

From the normalization condition eq. (3.2), the first two equalities of eq. (4.8) and the definition of eq. (4.11), one arrives at the following relations

$$\bar{\Lambda}_{D(B)} = \bar{\Lambda} - \left( \frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2} \right) (\kappa_1 + 3\kappa_2) - \frac{1}{2m_{c(b)}^2}(\varrho_1 \bar{\Lambda} + 3\varrho_2 \bar{\Lambda} + \chi_1 + 3\chi_2 - 3\chi_4 - 9\chi_5 - 6\chi_6)
$$

$$-9\chi_5 - 6\chi_6 + \frac{1}{4m_{c(b)}^2}(\eta_1 + 6\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6) + O\left( \frac{1}{m_{c(b)}^3} \right),$$

$$\bar{\Lambda}_{D^*(B^*)} = \bar{\Lambda} - \left( \frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2} \right) (\kappa_1 - \kappa_2) - \frac{1}{2m_{c(b)}^2}(\varrho_1 \bar{\Lambda} - \varrho_2 \bar{\Lambda} + \chi_1 - \chi_2)$$

$$-3\chi_4 - \chi_5 + 2\chi_6 + \frac{1}{4m_{c(b)}^2}(\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) + O\left( \frac{1}{m_{c(b)}^3} \right),$$

where the normalization of $\xi(\omega)$ at zero recoil: $\xi(\omega = 1) = 1$ has been used.

Putting the above results for $\bar{\Lambda}_{D(B)}$ and $\bar{\Lambda}_{D^*(B^*)}$ back into eq. (4.8), we get directly:

$$h_+(1) = 1 + \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\kappa_1 + 3\kappa_2)^2 - \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\eta_1 + 3\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6),$$

$$h_{A_1}(1) = 1 + \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\kappa_1 + 3\kappa_2)^2 - \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\eta_1 + 3\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6)$$

$$- \frac{1}{8m_b^2\Lambda^2}(\eta_1 - \eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) + \frac{1}{4m_b^2m_c\Lambda^2}(\eta_1 + \eta_2 + \eta_4 + 3\eta_5 + 2\eta_6),$$

$$h_1(1) = 1 + \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\kappa_1 - \kappa_2)^2 - \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\eta_1 - \eta_2 - 3\eta_4 - \eta_5 + 2\eta_6),$$

$$h_7(1) = -\{1 + \frac{1}{8\Lambda^2}(\frac{1}{m_b} - \frac{1}{m_c})^2(\kappa_1 - \kappa_2)^2 - \frac{1}{8\Lambda^2}(\frac{1}{m_b}^2 + \frac{1}{m_c}^2)(\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6)$$

$$+ \frac{1}{4m_b^2m_c\Lambda^2}(\eta_1 - 2\eta_2 + \eta_4 - \eta_5 - 2\eta_6)\}.$$ (4.11)

Here we only give the form factors for the case at zero recoil point. The most general results at non-zero recoil are quite lengthy and we present them in the appendix. Unlike the framework of the usual HQET, we observe that $h_-(\omega) = h_2(\omega) = 0$ in the new formulation of HQEFT. Such an interesting feature results from the fact that in the new framework of HQEFT, the operators in the effective Lagrangian and effective current contain only terms with even powers of $\vec{p}_{\perp}^2$.

Generally, as shown in the appendix, mesonic matrix elements up to second order power corrections can be described by a set of 29 form factors, which are universal functions of the kinematic variable $\omega = v \cdot v'$. Such a number is less than the one introduced in the usual HQET, where 34 form factors are needed. This is also due to the new structures of the effective Lagrangian eq. (2.12) and the effective current eq. (2.18).
At zero recoil point, some of the form factors are kinematically suppressed, only 15 universal form factors are needed to describe the mesonic matrix elements up to order $1/m_Q^2$. Where $\kappa_1$ and $\kappa_2$ characterize the contributions of the order $1/m_Q$ operators at zero recoil. As shown in eq. (4.9), the first order corrections to the meson mass arise from these two form factors. They play the same roles as the parameters $\lambda_1$ and $\lambda_2$ defined in the framework of the usual HQET \cite{18} \cite{21} - \cite{26} though the corresponding operator forms are different.

It is seen that the order $1/m_Q$ corrections in the meson transition matrix elements of weak currents are absent at zero recoil, though the forms of the effective Lagrangian and currents in the new formulation of HQEFT are obviously different from the ones in the usual HQET. This means that Luke’s theorem remains valid within the framework of the new formulation of HQEFT.

V. EXTRACTION OF FORM FACTORS AND $|V_{cb}|$

Since the discovery of spin-flavor symmetry in the heavy quark limit, great efforts have been made to extract the CKM matrix element $|V_{cb}|$ from the exclusive semileptonic decay modes $B \to D^* l \nu$ and $B \to D l \nu$ in the usual framework of HQET \cite{18} \cite{20} \cite{23} - \cite{25} \cite{27} - \cite{28}.

The differential decay rates of $B \to D^* l \nu$ and $B \to D l \nu$ decays can be simply formulated as follows \cite{24} \cite{27}

$$\frac{d\Gamma(B \to D^* l \nu)}{d\omega} = \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 m_D^3 \sqrt{\omega^2 - 1} (\omega + 1)^2 \times [1 + \frac{4\omega}{\omega + 1} \frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}] |V_{cb}|^2 F^2(\omega),$$

(5.1)

and

$$\frac{d\Gamma(B \to D l \nu)}{d\omega} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} |V_{cb}|^2 G^2(\omega)$$

(5.2)

with

$$F(\omega) = \eta_A h_{A_1}(\omega),$$

(5.3)

$$G(\omega) = \eta_V [h_+(\omega) - \frac{m_B - m_D}{m_B + m_D} h_-(\omega)],$$

(5.4)

where the coefficients $\eta_A$ and $\eta_V$ characterize short distance QCD corrections, whereas the functions $h_{A_1}(\omega)$, $h_+(\omega)$ and $h_-(\omega)$ contain long distance dynamics.

Based on experimental measurements of the differential decay rate, one can extract $|V_{cb}| F(\omega)$ and $|V_{cb}| G(\omega)$ and then extrapolate them to $\omega = 1$ to obtain the following quantities

$$|V_{cb}| F(1) = |V_{cb}| \eta_A h_{A_1}(1) = |V_{cb}| \eta_A (1 + \delta^*),$$

(5.5)

and

$$|V_{cb}| G(1) = |V_{cb}| \eta_V [h_+(1) - \frac{m_B - m_D}{m_B + m_D} h_-(1)] = |V_{cb}| \eta_V (1 + \delta).$$

(5.6)
In the framework of the usual HQET, it was shown that, from the theoretical point of view, the $B \to D^*\ell\nu$ decay is more favorable for the extraction of $|V_{cb}|$ because its decay rate at zero recoil is strictly protected by Luke’s theorem against first-order power corrections in $1/m_Q$. For decay channel $B \to D\ell\nu$, though the transition matrix element is protected by Luke’s theorem, the decay rate do receive order $1/m_Q$ corrections from the form factor $h_-(\omega)$, as can be seen form eq.(5.6).

In the new framework of HQEFT, however, we have seen in last section that $h_-(\omega) = 0$. This means that both the differential decay rates of channels $B \to D\ell\nu$ and $B \to D^*\ell\nu$ receive no order $1/m_Q$ corrections.

Since all quantities in eqs.(5.1) and (5.2) are the same as in the usual HQET except for the form factors $h_{A_1}(\omega)$, $h_+(\omega)$ and $h_-(\omega)$, we follow the same strategy and use

$$|V_{cb}|F(1) = 0.0352 \pm 0.0026;$$
$$|V_{cb}|G(1) = 0.0386 \pm 0.0041,$$

which are average results of recent experimental data used in Ref. [35]. $\eta_A$ and $\eta_V$ have been calculated by many authors [25]–[33]. Here we use [33]

$$\eta_A = 0.960 \pm 0.007,$$
$$\eta_V = 1.022 \pm 0.004.\quad (5.9)$$

The form factors $h_{A_1}(1)$ and $h_+(1)$ in eqs.(5.5) and (5.6) are given in terms of 14 unknown scalar form factors: $\kappa_1$, $\kappa_2$, $\varrho_1$, $\varrho_2$, $\chi_1$, $\chi_i$ and $\eta_i$ ($i = 1, 2, 4, 5, 6$). These form factors contain information of long-distance interaction in the heavy hadron. While little is known about the long-distance dynamics, evaluating these forms factors and extracting their values at zero recoil are great challenges. Until now their values can only be estimated by using either certain models or QCD sum rules. Within the framework of new formulation of HQEFT, the ground state meson masses which have been measured to a relatively high accuracy are correlated with the form factors via eq.(4.9). This allows us to figure out the most important form factors and to extract the important CKM matrix element $|V_{cb}|$.

As commented in last section, the number of form factors contributing at zero recoil increases quickly as the power of $1/m_Q$ expansion becomes higher. At the $1/m_Q^2$ order, there are already 15 form factors that contribute to hadronic matrix elements at zero recoil so that some simplifications must be made before extracting $|V_{cb}|$.

Firstly, we note that operators $O_3(\Gamma)$, $O_4(\Gamma)$ and $O_4(\Gamma)$ appear at order $1/m_Q^2$ and have similar forms. Their contributions to hadronic matrix elements should not be significant. At the zero recoil point $v = v'$, supposing that residual momenta of the heavy quarks are approximately equal, i.e., $k_1^2 \approx k_2^2 \approx \Lambda$, then the left and right actions of the derivative operators on the heavy effective fields almost leads to the same results, i.e.,

$$-\hat{D}_\mu \sim D_\mu, \quad \frac{1}{i\not{D}_\parallel + \Lambda} \sim \frac{1}{-i\not{D}_\parallel + \Lambda} \sim \frac{1}{\Lambda}.$$  

(5.10)

Under these considerations, we have $O_3(\gamma^\mu) = O_3'(\gamma^\mu) = O_4(\gamma^\mu)$, which implies the following relations among the form factors,

$$\chi_1 = \eta_1; \quad \chi_2 = 2\eta_2; \quad \chi_i = \eta_i \quad (i = 4, 5, 6).$$

(5.11)
As argued in Ref. [18], the $1/m_Q$ corrections can be well described by neglecting the form factors arising from the chromomagnetic moment operator. They corresponds to the fictitious limit of vanishing field strength, $F^{\alpha\beta} \to 0$. Now we will use a similar treatment, but only neglect the contributions arising from operators with two gluon field strength tensors and still keep the contributions of operators containing one gluon field strength tensor. This implies that the form factors $\chi_j$ and $\eta_j (j = 4, 5, 6)$ can be dropped out.

From above considerations, there are only six form factors left. eq.(4.9) is simplified to be

$$\bar{\Lambda}_{D(B)} = \bar{\Lambda} - (\frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2})(\kappa_1 + 3\kappa_2) - \frac{1}{4m_{c(b)}^2}(F_1 + 3F_2) + O(\frac{1}{m_{c(b)}^3}),$$

$$\bar{\Lambda}_{D^*(B^*)} = \bar{\Lambda} - (\frac{1}{m_{c(b)}} - \frac{\bar{\Lambda}}{2m_{c(b)}^2})(\kappa_1 - \kappa_2) - \frac{1}{4m_{c(b)}^2}(F_1 - F_2) + O(\frac{1}{m_{c(b)}^3}),$$

where $F_1$ and $F_2$ are defined as

$$F_1 = \chi_1 + 2\bar{\Lambda}q_1, \quad F_2 = \chi_2 + 2\bar{\Lambda}q_2.$$

Thus the $h_{A_1}(1)$ and $h_+(1)$ in eq.(4.11) turn into

$$h_{A_1}(1) = 1 + \frac{1}{8\Delta^2}[\frac{1}{m_b}(\kappa_1 + 3\kappa_2) - \frac{1}{m_c}(\kappa_1 - \kappa_2)]^2 - \frac{1}{8m_b^2\Delta^2}(F_1 + 3F_2 - 2\bar{\Lambda}q_1 - 6\bar{\Lambda}q_2) - \frac{1}{8m_c^2\Delta^2}(F_1 - F_2 - 2\bar{\Lambda}q_1 + 2\bar{\Lambda}q_2) + \frac{1}{4m_bm_c\Delta^2}(F_1 + F_2 - 2\bar{\Lambda}q_1 - 2\bar{\Lambda}q_2),$$

$$h_+(1) = 1 + \frac{1}{8\Delta^2}[\frac{1}{m_b} - \frac{1}{m_c}]^2[(\kappa_1 + 3\kappa_2)^2 - (F_1 + 3F_2) + 2\bar{\Lambda}(q_1 + 3q_2)].$$

We will take the heavy quark masses $m_b$ and $m_c$ as well as the ‘dressed heavy quark’ mass $\hat{m}_b = m_b + \bar{\Lambda}$ ( or the heavy flavor-independent binding energy $\bar{\Lambda}$ ) as three basic parameters of the theory. Usually, one fixes the value of the mass difference $m_b - m_c$, to extract $|V_{cb}|$ as functions of mass $m_b$ ( or $m_c$ ) from either exclusive or inclusive B decays [18][23]- [28]. In our present considerations, we permit the value of $m_b - m_c$ change between 3.32GeV and 3.41GeV.

Based on eq.(5.12) and using ground state meson masses [34] as input, we can extract the four form factors $\kappa_1, \kappa_2, F_1$ and $F_2$ as functions of $m_b, m_b - m_c$ and $m_b + \bar{\Lambda}$. Thus it allows us to calculate the parameters $\delta^*$ and $\delta$ up to the $1/m_Q^2$ order corrections in the $B \to D^{*}l\nu$ and $B \to Dl\nu$ transition matrix elements at zero recoil.

The form factors $\kappa_1, \kappa_2, F_1$ and $F_2$ as functions of $m_b, m_b - m_c$ and $m_b + \bar{\Lambda}$ are plotted in Fig.1. One sees from Figs.1a-1b that $\kappa_1$ is insensitive to $m_b$ and $m_b - m_c$, but it is quite sensitive to $m_b + \bar{\Lambda}$ as explicitly shown in Fig.1c. From Figs.1d-1e, one sees that $\kappa_2$ only slightly changes against $m_b$ and $m_b - m_c$ and is independent of $m_b + \bar{\Lambda}$ as shown from the straight lines in Fig.1f. The combined form factors $F_1$ and $F_2$ have only a slight dependence on $m_b$ as shown in Figs.1g and 1j, but both of them heavily depends on $m_b - m_c$ and decrease
as $m_b - m_c$ increases, which have been shown in Figs.1h and 1k. They are sensitive to $m_b + \bar{\Lambda}$ as seen from Figs.1j and 1l, but their variations against $m_b + \bar{\Lambda}$ go to opposite direction, where $F_1$ decreases and $F_2$ increases as $m_b + \bar{\Lambda}$ goes to be large. As can be seen from these figures and eqs.5.14, the values of $F_2$ are very small and its contributions to $h_{A_1}(1)$ and $h_{+}(1)$ are negligible. Thus the main contributions to the form factors $h_{A_1}(1)$ and $h_{+}(1)$ arise from the form factors $\kappa_1$ and $\kappa_2$ at the order $1/m_Q$ as well as the form factor $F_1$ at the order $1/m_Q^2$.

To show typical values for the form factors $\kappa_1$, $\kappa_2$, $F_1$ and $F_2$ at zero recoil, we take the central values of $m_b$ and $m_b - m_c$, namely, $m_b = 4.7$GeV and $m_b - m_c = 3.36$GeV, and $m_b + \bar{\Lambda} = 5.21$GeV. The reliability of taking $m_b + \bar{\Lambda} = 5.21$GeV will be seen explicitly below. With these data, one can straightforwardly read from Figs.1a-1l the following reasonable values for the form factors

$$\kappa_1 \sim -0.615$GeV$^2$, \quad \kappa_2 \sim 0.056$GeV$^2$, \quad F_1 \sim 0.917$GeV$^4$, \quad F_2 \sim 0.004$GeV$^4$. \quad (5.16)$$

With these typical values, $\delta^*$ and $\delta$ can be simply represented as functions of $\varrho_1$ and $\varrho_2$

$$\delta^* \approx -0.045 + 0.14 \varrho_1 - 0.362 \varrho_2,$$

$$\delta \approx -0.1 + 0.14(\varrho_1 + 3\varrho_2). \quad (5.17)$$

So $\delta^*$ and $\delta$ strongly depend on $\varrho_2$ and change in opposite direction. $\delta^*$ decreases whereas $\delta$ increases as $\varrho_2$ goes up. Their dependence on $\varrho_1$ is relatively weak and has a similar behaviour.

Up to now, the two form factors $\varrho_1$ and $\varrho_2$ have not yet been determined. But their contributions should be such that the values of $|V_{cb}|$ estimated from the two channels $B \to D^*l\nu$ and $B \to Dl\nu$ approximately equal each other. From this consideration, we find that it is reliable to take

$$\varrho_1 \approx 0.3$GeV$^3$, \quad \varrho_2 \approx 0.11$GeV$^3. \quad (5.18)$$

This can be explicitly seen from eqs.(5.3) and (5.6) and Fig.2. $|V_{cb}|$ can be extracted from either of the two exclusive B decay channels. As shown in Fig.2a, the $|V_{cb}|$ extracted from $B \to D^*l\nu$ channel becomes large as $\varrho_2$ increases, whereas the $|V_{cb}|$ extracted from $B \to Dl\nu$ channel decreases as $\varrho_2$ increases as shown in Fig.2b. Fig.2c shows $|V_{cb}|$ extracted from both of the channels as function of $\varrho_2$. When taking $\varrho_2 = 0.08$GeV$^3$, the values of $|V_{cb}|$ from the two channels differ obviously. While for $\varrho_2 = 0.11$GeV$^3$, the two curves of $|V_{cb}|$ almost coincide with each other. Their little difference may also be seen from Fig.2d.

With the values of $\varrho_1$ and $\varrho_2$ given in eq.(5.18), we obtain a reliable $|V_{cb}|$. The numerical results are listed in Table. 1-3.

We also show in Figs.3a-3b the parameter $\delta^*$ as function of $m_b + \bar{\Lambda}$ for $m_b - m_c = 3.41$GeV, 3.36GeV, 3.32GeV, respectively. The three lines in each figure correspond to $m_b = 4.6$GeV, $m_b = 4.7$GeV and $m_b = 4.8$GeV. Clearly, on each curve there is a minimum around which the curve becomes relatively flat. This means that the correction $\delta^*$ is not very sensitive to the value of $m_b + \bar{\Lambda}$ around that minimal point. As a consequence, the resulting values for $|V_{cb}|$ also becomes more stable around that minimal point of $m_b + \bar{\Lambda}$. One can also see from Table. 1-3 or those figures in Fig.3 that the minimal value is near the point $m_b + \bar{\Lambda} = 5.2$GeV, which has been taken as a reliable value in getting the estimates in eq.(5.16).
It can be read from Fig.3 that within $4.6 \, GeV \leq m_b \leq 4.8 \, GeV$ and $3.32 \, GeV \leq m_b - m_c \leq 3.41 \, GeV$, the values of $\delta^*$ at the minimal point on a curve range from $-0.07$ to $-0.01$. This gives an optimistic estimate:

$$\delta^* = -0.04 \pm 0.03.$$  \hspace{1cm} (5.19)

To be more conservative, one may take a range of $m_b + \bar{\Lambda}$ around the minimal point. For the region $5.175 \, GeV \leq m_b + \bar{\Lambda} \leq 5.275 \, GeV$, the values of $\delta^*$ are found to be

$$-0.07 < \delta^* < 0.01.$$  \hspace{1cm} (5.20)

The two figures in Fig.4 exhibit the $m_b$ and $m_b - m_c$ dependences of $\delta^*$. It can be seen from Fig.4a that the variation of $m_b$ (or $m_c$) only very slightly influences $\delta^*$ when $m_b - m_c$ is fixed. On the contrary, Fig.4b shows that $\delta^*$ is rather sensitive to the mass difference $m_b - m_c$ between the b and c quarks. Fig.5 shows the resulting $|V_{cb}|$ as function of $m_b$, $m_b - m_c$ and $m_b + \bar{\Lambda}$.

By using the values of $\delta^*$ given in eqs.(5.19) and (5.20), we then obtain from eq.(5.5) the important CKM matrix element $|V_{cb}|$ within the framework of the new formulation of HQEFT. Here we would like to present the numerical result for $|V_{cb}|$ from the eq.(5.20)

$$|V_{cb}| = 0.0378 \pm 0.0028\exp \pm 0.0018_{th}.$$  \hspace{1cm} (5.21)

Now turn to the case of $B \rightarrow Dl\nu$ decay. Following analogous analyses, we obtain

$$\delta = -0.03 \pm 0.05 \text{ (optimistic estimate)},$$  \hspace{1cm} (5.22)

$$-0.08 < \delta < 0.06 \text{ (conservative estimate)},$$  \hspace{1cm} (5.23)

and

$$|V_{cb}| = 0.0382 \pm 0.0041\exp \pm 0.0028_{th}.$$  \hspace{1cm} (5.24)

**VI. SUMMARY**

We have derived an effective Lagrangian in terms of effective heavy quark (or antiquark) fields by integrating out heavy antiquark (or quark) fields based on the framework of new formulation of HQEFT [19]. The resulting heavy quark effective Lagrangian differs from the one in the usual HQET because of the additional contributions from the antiquark fields. An arbitrary heavy quark current is expanded into a power series of $1/m_Q$. Week matrix elements between heavy hadron states have been investigated in detail and explicitly evaluated up to the order of $1/m_Q^2$. The resulting special structures of the heavy quark effective Lagrangian and currents within the framework of new formulation of HQEFT results in some interesting features: Firstly, it has been explicitly shown that Luke’s theorem comes out automatically without the need of imposing the equation of motion $iv \cdot DQ^+ = 0$. Secondly, the consistent normalization condition between two heavy hadrons naturally reflects spin-flavor symmetry, and the form factors at zero recoil are found to be related to the meson masses, so that the most important relevant form factors at zero recoil have been
fitted from the ground state meson masses. Thirdly, we find \( h_-(\omega) = 0 \), so the differential decay rates of both \( B \to D^*l\nu \) and \( B \to Dl\nu \) have no \( 1/m_Q \) order corrections at zero recoil. This enables one to extract the CKM matrix element \( |V_{cb}| \) from either of the two exclusive semileptonic decays at the order of \( 1/m_Q^2 \). Finally, a set of 29 universal form factors up to the order of \( 1/m_Q^2 \), which is less than the one in the usual HQET, has been introduced to describe transition matrix elements of weak currents between ground state pseudoscalar and vector mesons. Note that at zero recoil point, some of the form factors are kinematically suppressed and only 15 form factors contribute to the weak matrix elements. By reasonable considerations, the zero recoil heavy meson transition matrix elements can be approximately described by only 6 form factors. These form factors are hadronic parameters concerning long distance dynamics and are hardly evaluated, four of them have been estimated from ground state meson masses. Following the conventional strategy, we have extracted \( |V_{cb}| \) from the exclusive semileptonic decay channel \( B \to D^*l\nu \) with the value

\[
|V_{cb}| = 0.0378 \pm 0.0028_{\text{exp}} \pm 0.0018_{\text{th}},
\]

and from the exclusive semileptonic decay channel \( B \to Dl\nu \) with the value

\[
|V_{cb}| = 0.0382 \pm 0.0041_{\text{exp}} \pm 0.0028_{\text{th}}.
\]

In this paper, we have shown some interesting features in applying the new formulation of HQEFT to the exclusive semileptonic decays of heavy hadrons. More interesting features can be found in applying the new formulation of HQEFT to the inclusive decays of heavy hadrons \([36]\), such as: the new formulation of HQEFT allows us to simply clarify the well known ambiguity of using the quark mass or hadron mass in the inclusive heavy hadron decays, as a consequence, the CKM matrix element \( |V_{cb}| \) can also be well determined from the inclusive semileptonic decay rate, and the result is nicely consistent with the above one; the resulting lifetime differences between bottom mesons and baryons also agree well with the experimental data.

ACKNOWLEDGMENTS

We would like to thank professor Y.B. Dai for useful discussions. This work was supported in part by the NSF of China under the grant No. 19625514.

APPENDIX A: FUNCTIONAL INTEGRAL METHOD

Corresponding to eq.(2.3), the generating functional relevant to heavy quarks is as follows,

\[
\tilde{W}[\eta_v^+, \eta_v^-, \eta_v^-, \eta_v^-] = N \int D\tilde{Q}_v^+ D\dot{Q}_v^+ D\tilde{Q}_v^- D\dot{Q}_v^- \exp\{i \int d^4x [L_{Q,v} + \eta_v^+ \dot{Q}_v^+ \\
+ \tilde{Q}_v^+ \eta_v^- + \eta_v^- \dot{Q}_v^- + \tilde{Q}_v^- \eta_v^-] \}
= N \int D\tilde{Q}_v^+ D\dot{Q}_v^+ D\tilde{Q}_v^- D\dot{Q}_v^- \exp\{i \int d^4x [\tilde{Q}_v^+ \dot{A} \dot{Q}_v^+ + \tilde{Q}_v^- \dot{A} \dot{Q}_v^- \\
+ \tilde{Q}_v^+ \dot{B} \dot{Q}_v^- + \tilde{Q}_v^- \dot{B} \dot{Q}_v^- + \eta_v^+ \dot{Q}_v^- + \tilde{Q}_v^+ \eta_v^- + \eta_v^- \dot{Q}_v^- + \tilde{Q}_v^- \eta_v^-] \},
\]

(A1)
where $\eta^+_v$, $\eta^+_v$ and $\eta^-_v$, $\eta^-_v$ are external sources of quark and antiquark fields, respectively. $N$ is a normalization constant. The operators $\hat{A}$ and $\hat{B}$ have been defined in eq.\((2.6)\).

Integrating over $\tilde{Q}^-_v$ and $\tilde{Q}^+_v$, eq.\((A1)\) becomes

$$W[\eta^+_v, \eta^+_v, \eta^-_v, \eta^-_v] = N \det(-i\hat{A}) \int D\tilde{Q}^+_v D\tilde{Q}^-_v \exp\{i \int d^4x (\tilde{Q}^+_v (\hat{A} - \hat{B} \hat{A}^{-1} \hat{B}) \tilde{Q}^-_v$$

$$+ (\eta^+_v - \eta^-_v \hat{A}^{-1} \hat{B}) \tilde{Q}^+_v + \tilde{Q}^+_v (\eta^+_v - \hat{B} \hat{A}^{-1} \eta^-_v) - \eta^+_v \hat{A}^{-1} \eta^-_v)]\}.$$  \((A2)\)

And then setting the sources $\eta^-_v$ and $\eta^-_v$ to be zero, we get

$$W[\eta^+_v, \eta^+_v] = N \det(-i\hat{A}) \int D\tilde{Q}^+_v D\tilde{Q}^+_v e^{i \int d^4x (\tilde{Q}^+_v (\hat{A} - \hat{B} \hat{A}^{-1} \hat{B}) \tilde{Q}^+_v + \tilde{Q}^+_v \eta^+_v + \eta^+_v \tilde{Q}^+_v)}. \quad (A3)$$

The factor $N \det(-i\hat{A})$ contains no quark field and can be seen as a new normalization factor. Then the effective Lagrangian eq.\((2.3)\) can be read from eq.\((A3)\).

To get the effective current, one can express $\tilde{Q}^-_v$ and $\tilde{Q}^+_v$ in eq.\((2.16)\) by the variationals of the generating functional eq.\((A1)\) over the corresponding external sources,

$$\tilde{Q}^-_v = \left(\frac{\delta}{i \delta \tilde{Q}^-_v} W[\eta^+_v, \eta^+_v, \eta^-_v, \eta^-_v]\right)_{\eta^-_v = \eta^-_v = 0},$$

$$\tilde{Q}^+_v = \left(\frac{\delta}{i \delta \tilde{Q}^+_v} W[\eta^+_v, \eta^+_v, \eta^-_v, \eta^-_v]\right)_{\eta^-_v = \eta^-_v = 0}.$$  \((A4)\)

Combining eqs.\((2.3)\), \((A2)\) and \((A4)\), eq.\((2.17)\) is obtained once more.

**APPENDIX B: DECOMPOSITION**

Using the identity $v_\alpha P_+ \sigma^{\alpha \beta} \mathcal{M} = 0$, the Lorentz tensors $\kappa_{\alpha \beta}(v, v')$, $\eta_{\alpha \beta}(v, v')$, $\eta_{\alpha \beta}(v, v')$, $\chi_{\alpha \beta}(v, v')$, and $\eta_{\alpha \beta \gamma}(v, v')$ can be decomposed into the following general forms in terms of Lorentz scalar factors

$$\kappa_{\alpha \beta}(v, v') = i \kappa_2(\omega) (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha),$$

$$\eta_{\alpha \beta}(v, v') = i \eta_2(\omega) v_\alpha \gamma_\beta - v_\beta \gamma_\alpha),$$

$$\chi_{\alpha \beta}(v, v') = i \chi_2(\omega) \sigma_{\alpha \beta} + \chi_3(\omega) (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha),$$

$$\eta_{\alpha \beta \gamma}(v, v') = i \eta_{\alpha \beta}(v, v') + \chi_6(\omega) (v_\beta \gamma_\alpha - v_\alpha \gamma_\beta),$$

$$\chi_{\alpha \beta \gamma}(v, v') = \chi_4(\omega) (g_{\alpha \beta} g_{\gamma \delta} g_{\gamma' \delta'} - g_{\beta \gamma} g_{\gamma' \delta} g_{\gamma \delta'}),$$

$$\eta_{\alpha \beta \gamma}(v, v') = \eta_4(\omega) (g_{\alpha \beta} g_{\gamma \delta} g_{\gamma' \delta'} - g_{\beta \gamma} g_{\gamma' \delta} g_{\gamma \delta'}),$$

$$\eta_{\alpha \beta \gamma}(v, v') = \eta_6(\omega) (g_{\alpha \beta} g_{\gamma \delta} g_{\gamma' \delta'} - g_{\beta \gamma} g_{\gamma' \delta} g_{\gamma \delta'}).$$  \((B1)\)
APPENDIX C: MESON FORM FACTORS

Since all form factors are functions of $\omega$, we will neglect the variable $\omega$ in following formulæ for simplicity. To second order in the new framework of HQEFT, the meson form factors defined in eq. (1.1) are given by

$$h_+ = \xi + (\xi - 1)\frac{1}{2\Lambda}(\frac{1}{m_b} + \frac{1}{m_c})(\kappa_1 + 3\kappa_2) + \frac{1}{4\Lambda}(\frac{m_b^2}{m_b} + \frac{m_c^2}{m_c})(c_1 - 3c_2 - \frac{1}{\Lambda}c_4)$$

$$+\xi[\frac{1}{8\Lambda^2}c_6 + \frac{1}{8\Lambda^2}(\frac{3}{m_b^2} + \frac{3}{m_c^2} + \frac{2}{m_bm_c})(\kappa_1 + 3\kappa_2)^2] + \frac{1}{\Lambda}(1 - \omega)$$

$$\times[\frac{1}{m_b}(\frac{1}{m_c})\kappa_3 - (\frac{1}{2m_b^2} + \frac{1}{2m_c^2})c_3] + \frac{1}{4\Lambda^2}(\frac{1}{m_b^2} + \frac{1}{m_c^2})[(4\chi_7 + 2\chi_8)(1 - \omega^2)$$

$$-4(\chi_9 + \chi_{12})(1 - \omega)] + \frac{1}{4m_b m_c \Lambda^2}[\eta_1 + 6\eta_2 - 4\eta_3(1 - \omega) - \eta_4(1 + 2\omega)]$$

$$-9\eta_5 - 6\eta_6 + (4\eta_7 + 2\eta_8)(1 - \omega^2) - 8(\eta_9 + 2\eta_{10})(1 - \omega)]$$

$$-\frac{1}{4\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})^2(\kappa_1 + 3\kappa_2)^2 + \frac{1}{4\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})^2\kappa_3(\kappa_1 + 3\kappa_2)(1 - \omega), \quad (C1)$$

$$h_- = 0, \quad (C2)$$

$$h_1 = \xi + (\xi - 1)\frac{1}{2\Lambda}(\frac{1}{m_b} + \frac{1}{m_c})(\kappa_1 - \kappa_2) + \frac{1}{4\Lambda}(\frac{m_b^2}{m_b} + \frac{m_c^2}{m_c})(c_1 + c_2 - \frac{1}{\Lambda}c_5)$$

$$+\xi[\frac{1}{8\Lambda^2}(\frac{1}{m_b^2} + \frac{1}{m_c^2})c_7 + \frac{1}{8\Lambda^2}(\frac{3}{m_b^2} + \frac{3}{m_c^2} + \frac{2}{m_bm_c})(\kappa_1 - \kappa_2)^2]$$

$$+\frac{1}{4m_b^2\Lambda^2}[2\chi_8(1 + \omega^2) - (2\chi_9 + 2\chi_{10} - 6\chi_{11} - 2\chi_{12})(1 - \omega)]$$

$$+\frac{1}{4m_c^2\Lambda^2}[2\chi_8(1 + \omega^2) - (2\chi_{10} - 6\chi_{11} - 2\chi_{12})(1 - \omega)] + \frac{1}{4m_b m_c \Lambda^2}[\eta_1$$

$$-2\eta_2 - \eta_4(1 + 2\omega) - \eta_5 - 2\eta_6(1 - 2\omega) + 2\eta_8(1 - \omega^2) - 4(\eta_9 - \eta_{10})(1 - \omega)]$$

$$-\frac{1}{4\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})^2(\kappa_1 - \kappa_2)^2, \quad (C3)$$

$$h_2 = 0, \quad (C4)$$

$$h_3 = \xi + (\xi - 1)\frac{1}{2\Lambda}(\frac{1}{m_b} + \frac{1}{m_c})(\kappa_1 - \kappa_2) + \frac{1}{4\Lambda}(\frac{m_b^2}{m_b} + \frac{m_c^2}{m_c})(c_1 + c_2 - \frac{1}{\Lambda}c_5)$$

$$+\xi[\frac{1}{8\Lambda^2}(\frac{1}{m_b^2} + \frac{1}{m_c^2})c_7 + \frac{1}{8\Lambda^2}(\frac{3}{m_b^2} + \frac{3}{m_c^2} + \frac{2}{m_bm_c})(\kappa_1 - \kappa_2)^2]$$

$$-\frac{1}{m_c\Lambda}\kappa_3(1 - \omega) + \frac{1}{2m_c^2\Lambda}(1 - \omega)c_3 + \frac{1}{4m_b^2\Lambda^2}[2\chi_8(1 + \omega^2)$$

$$-2(\chi_{10} + 2\chi_{12})(1 - \omega)] + \frac{1}{4m_c^2\Lambda^2}[(4\chi_7 + 2\chi_8)(1 - \omega^2)$$

$$-(4\chi_{10} + 8\chi_{12})(1 - \omega)] + \frac{1}{4m_b m_c \Lambda^2}[\eta_1 - 2\eta_2 + 2\eta_3(1 - \omega)$$

$$+\eta_4 - \eta_5 - 2\eta_6 - 2(\eta_9 + \eta_{10})(1 - \omega)] - \frac{1}{4\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})^2(\kappa_1 - \kappa_2)^2$$

$$-\frac{1}{2m_c\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})\kappa_3(\kappa_1 - \kappa_2)(1 - \omega), \quad (C5)$$

23
\[ h_5 = \xi + (\xi - 1)\left[ \frac{1}{2\Lambda b} + \frac{1}{m_c} \right] \kappa_1 - \kappa_2 \left( \kappa_1 - \kappa_2 \right) + \frac{1}{4\Lambda} \left( \frac{1}{m_b} + \frac{1}{m_c} \right) \left( c_1 + c_2 - \frac{1}{\Lambda c_5} \right) \]
\[ + \xi \left[ - \frac{1}{8\Lambda} \left( \frac{1}{m_b} + \frac{1}{m_c} \right) c_7 + \frac{1}{8\Lambda^2} \left( \frac{3}{m_b} + \frac{3}{m_c} + \frac{2}{m_b m_c} \right) \left( \kappa_1 - \kappa_2 \right)^2 \right] \]
\[ - \frac{1}{m_b \kappa_3} \left( 1 - \omega \right) - \frac{1}{2m_b \kappa_3} \left( 1 - \omega \right) + \frac{1}{4m_b \kappa_3} \left( \left( 4\chi_7 + 2\chi_8 \right) \left( 1 - \omega^2 \right) \right) \]
\[ - \left( 4\chi_{10} + 4\chi_{11} + 8\chi_{12} \right) \left( 1 - \omega \right) \right] + \frac{1}{4m_b \kappa_3} \left( 2\chi_8 \left( 1 - \omega^2 \right) - 2\chi_{10} - 6\chi_{11} \right) \]
\[ - 2\chi_{12} \left( 1 - \omega \right) \right] + \frac{1}{4m_b \kappa_3} \left[ \eta_1 - 2\eta_2 + 2\eta_3 \left( 1 - \omega \right) + \eta_4 - \eta_5 - 2\eta_6 \right] \]
\[ - 2 \left( \eta_9 + \eta_{10} \right) \left( 1 - \omega \right) \right] - \frac{1}{4\Lambda} \left( \frac{1}{b m_b} + \frac{1}{m_c} \right)^2 \left( \kappa_1 - \kappa_2 \right)^2 \]
\[ - \frac{1}{2m_b \kappa_3} \left( \frac{1}{b m_b} + \frac{1}{m_c} \right) \left( \kappa_1 - \kappa_2 \right) \left( 1 - \omega \right), \] (C6)

\[ h_6 = \frac{1}{m_b \kappa_3} - \frac{1}{2m_b \kappa_3} \left( \frac{1}{b m_b} + \frac{1}{m_c} \right) \left( \kappa_1 - \kappa_2 \right) \left( 1 - \omega \right) \]
\[ - \frac{1}{4m_b \kappa_3} \left( \left( 4\chi_7 + 2\chi_9 - 2\chi_{10} - 10\chi_{11} - 10\chi_{12} \right) \right) \]
\[ + \frac{1}{2m_b \kappa_3} \left( \frac{1}{b m_b} + \frac{1}{m_c} \right) \left( \kappa_1 - \kappa_2 \right) \], (C7)

\[ h_7 = \xi + (\xi - 1)\left[ \frac{1}{2\Lambda b} + \frac{1}{m_c} \right] \kappa_1 + \frac{1}{4\Lambda} \left( \frac{1}{m_b} + \frac{1}{m_c} \right) c_1 + \frac{1}{4m_b \kappa_3} \left( c_2 - \frac{1}{\Lambda c_5} \right) \]
\[ + \xi \left[ - \frac{1}{2\Lambda} \left( \frac{1}{m_b} + \frac{1}{m_c} \right) \kappa_2 + \frac{1}{4m_b \kappa_3} \left( c_2 - \frac{1}{\Lambda c_5} \right) - \frac{1}{8\Lambda^2} \left( \frac{1}{m_b} + \frac{1}{m_c} \right) c_7 \right] \]
\[ + \frac{1}{8\Lambda^2} \left( \kappa_1 - \kappa_2 \right)^2 \left[ \frac{3}{m_b} + \frac{3}{m_c} + \frac{2}{m_b m_c} \right] + \frac{1}{2\Lambda} \left[ \frac{1}{m_b} - \frac{1}{m_c} \right] \left( 1 - 2\omega \right) \] κ_2 \]
\[ + \frac{1}{4m_b \kappa_3} c_2 \left( 1 - 2\omega \right) + \frac{1}{4m_b \kappa_3} \left[ 3\chi_4 + \chi_5 \left( 5 - 4\omega \right) + 2\chi_6 \left( 1 - 2\omega \right) \right] \]
\[ + 2 \left( \chi_7 + \chi_8 \right) \left( 1 - \omega^2 \right) - \chi_9 \left( 3 - \omega - 2\omega^2 \right) - \chi_{10} \left( 1 + \omega - 2\omega^2 \right) \]
\[ + \chi_{11} \left( 3 - 5\omega + 2\omega^2 \right) - \chi_{12} \left( 1 + 5\omega - 6\omega^2 \right) + \frac{1}{4m_b \kappa_3} \left[ 2\chi_8 \left( 1 - \omega^2 \right) \right] \]
\[ - \left( 2\chi_9 + 2\chi_{10} - 6\chi_{11} - 2\chi_{12} \right) \left( 1 - \omega \right) \right] + \frac{1}{4m_b \kappa_3} \left[ \eta_1 - 2\omega \eta_2 + \omega \eta_4 \right] \]
\[ + \left( \eta_5 + 2\eta_6 \right) \left( 1 - 2\omega \right) - \eta_8 \left( 1 - \omega^2 \right) + \left( 3\eta_9 - \eta_{10} \right) \left( 1 - \omega \right) \right] - \frac{1}{4\Lambda} \left( \frac{1}{m_b} + \frac{1}{m_c} \right)^2 \]
\[ \times \kappa_1 \left( \kappa_1 - \kappa_2 \right) + \frac{1}{4\Lambda} \left( \frac{1}{m_b} + \frac{1}{m_c} \right) \left[ \frac{1}{m_b} - \frac{1}{m_c} \right] \left( 2\chi_8 \left( 1 - 2\omega \right) \right) \kappa_2 \left( \kappa_1 - \kappa_2 \right), \] (C9)
\[ h_8 = -\frac{1}{m_b\Lambda}(1 + \omega)(\kappa_2 - \kappa_3) + \frac{1}{2m_b^2\Lambda}(1 + \omega)(c_2 - c_3) + \frac{1}{4m_b^2\Lambda^2}[4(\chi_5 + \chi_6)(1 + \omega) + 2\chi_7(1 + 2\omega + \omega^2) - \chi_9(1 + 3\omega + 2\omega^2) + \chi_{10}(1 - \omega - 2\omega^2) - \chi_{11}(3 + 5\omega + 2\omega^2) - 3\chi_{12}(1 + 3\omega + 2\omega^2)] + \frac{1}{4m_bmc_\Lambda^2}(2\eta_2 - 2\eta_3 + \eta_4 + 2\eta_5 - \eta_9 + 3\eta_9)(1 + \omega) + \eta_8(1 + 2\omega + \omega^2)] - \frac{1}{2m_b\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})(1 + \omega)(\kappa_1 - \kappa_2)(\kappa_2 - \kappa_3), \] (C10)

\[ h_9 = -\frac{2}{m_b\Lambda}\kappa_2 - \frac{1}{m_b^2\Lambda}c_2 - \frac{1}{\Lambda}(\frac{1}{m_b} + \frac{1}{m_c})\kappa_3 + \frac{1}{2\Lambda}(\frac{1}{m_b^2} + \frac{1}{m_c^2})c_3 - \frac{1}{4m_b^2\Lambda^2}[8\chi_5 + 8\chi_6 + 4\chi_7(1 + \omega) - 2\chi_9(1 + 2\omega) + 2\chi_{10}(1 - \omega) - \chi_{11}(6 + 4\omega) - 6\chi_{12}(1 + 2\omega)] - \frac{1}{4m_c^2\Lambda^2}[-4\chi_7(1 + \omega) - 2\chi_9 + 2\chi_{10} + 10\chi_{11} + 10\chi_{12}] - \frac{1}{m_bmc_c\Lambda^2}[\eta_2 - \eta_3 + \eta_5 + \eta_6 + \eta_9 + \eta_{10}] + \frac{1}{m_b\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})\kappa_2(\kappa_1 - \kappa_2) - \frac{1}{2\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})^2 \times \kappa_3(\kappa_1 - \kappa_2), \] (C11)

\[ h_{10} = \frac{1}{m_b\Lambda^2}[4\chi_7(1 + \omega) + 2\chi_9 - 2\chi_{10} - 10\chi_{11} - 10\chi_{12}] + \frac{1}{4m_bmc_\Lambda^2}[2\eta_4 - 4\eta_6 + 2\eta_8(1 + \omega) - 6\eta_9 + 2\eta_{10}], \] (C12)

\[ h_{11} = \frac{1}{m_b\Lambda}(-2\kappa_2 + \kappa_3) + \frac{1}{m_b^2\Lambda}c_2 - \frac{1}{2\Lambda}(\frac{1}{m_b} + \frac{1}{m_c})\kappa_3 + \frac{1}{4m_b^2\Lambda^2}[8\chi_5 + 8\chi_6 + 4\chi_7(1 + \omega) - 2\chi_9(1 + 2\omega) + 2\chi_{10}(1 - \omega) - \chi_{11}(6 + 4\omega) - 6\chi_{12}(1 + 2\omega)] + \frac{1}{4m_bmc_c\Lambda^2}[4\eta_2 - 2\eta_3 + 2\eta_4 + 4\eta_5 + 2\eta_6(1 + \omega) - 2\eta_9 + 6\eta_{10}] + \frac{1}{2m_b\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})(-2\kappa_2 + \kappa_3)(\kappa_1 - \kappa_2), \] (C13)

\[ h_{12} = \frac{1}{m_b\Lambda}\kappa_3 - \frac{1}{2m_b^2\Lambda}c_3 - \frac{1}{2m_bmc_\Lambda^2}\eta_3 + \frac{1}{2m_b\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})\kappa_3(\kappa_1 - \kappa_2), \] (C14)

\[ h_V = -\xi - (\xi - 1)[\frac{1}{2m_b\Lambda}(\kappa_1 + 3\kappa_2) + \frac{1}{2m_b\Lambda}(\kappa_1 - \kappa_2) + \frac{1}{4\Lambda}(\frac{1}{m_b} + \frac{1}{m_c})c_1 - \frac{1}{4\Lambda}(\frac{3}{m_b} - \frac{1}{m_c})c_2 - \frac{1}{4\Lambda^2}\kappa_3 + \frac{1}{8\Lambda^2}(\kappa_1 + 3\kappa_2)^2 + \kappa_1 + \kappa_2(\kappa_1 - \kappa_2)) + \frac{1}{2m_b^2\Lambda^2}c_3(1 - \omega) - \frac{1}{m_b\Lambda}\kappa_3(1 - \omega) - \frac{1}{4m_b^2\Lambda^2}[4\chi_7 + 2\chi_{12}](1 - \omega^2) - 4(\chi_9 + \chi_{12})(1 - \omega)] - \frac{1}{4m_c^2\Lambda^2}[2\chi_8(1 - \omega^2) - (2\chi_9 + 2\chi_{10} - 6\chi_{11})] - 2\chi_{12}(1 - \omega) - \frac{1}{4m_bmc_\Lambda^2}[\eta_1 + 2\eta_2 - 2\eta_3(1 - \omega) + \eta_4 + 3\eta_5 + 2\eta_6 + 2(\eta_9 + \eta_{10})(1 - \omega)] + \frac{1}{4\Lambda^2}(\frac{1}{m_b} + \frac{1}{m_c})(\kappa_1 + 3\kappa_2 + \frac{1}{m_c}(\kappa_1 - \kappa_2))^2 - \frac{1}{2m_b\Lambda^2}\kappa_3(1 - \omega)(\frac{1}{m_b}(\kappa_1 + 3\kappa_2) + \frac{1}{m_c}(\kappa_1 - \kappa_2)), \] (C15)
\[ h_{A_1} = \xi + (\xi - 1)[\frac{1}{2m_b \Lambda} (\kappa_1 + 3\kappa_2) + \frac{1}{2m_c \Lambda} (\kappa_1 - \kappa_2) + \frac{1}{4\Lambda} (\frac{1}{m_b^2} + \frac{1}{m_c^2})c_1 \\
- \frac{1}{4\Lambda} (\frac{3}{m_b^2} - \frac{1}{m_c^2})c_2 - \frac{1}{4m_b^2 \Lambda^2} c_4 - \frac{1}{4m_c^2 \Lambda^2} c_5] + \xi[-\frac{1}{8m_b^3 \Lambda^2} c_6 - \frac{1}{8m_c^3 \Lambda^2} c_7 \\
+ \frac{1}{8\Lambda^2} (\frac{3}{m_b^2} (\kappa_1 + 3\kappa_2)^2 + \frac{3}{m_c^2} (\kappa_1 - \kappa_2)^2 + \frac{2}{m_b m_c} (\kappa_1 + 3\kappa_2)(\kappa_1 - \kappa_2))] \\
+ \frac{1}{m_b \Lambda} \kappa_3 (1 - \omega) - \frac{1}{2m_b^2 \Lambda} c_3 (1 - \omega) + \frac{1}{4m_b^2 \Lambda^2} [(4\chi_7 + 2\chi_8)(1 - \omega^2) \\
- 4(\chi_9 + \chi_12)(1 - \omega)] + \frac{1}{4m_b m_c \Lambda^2} [\eta_1 + 2\eta_2 - 2\eta_3 (1 - \omega) + \eta_4 + 3\eta_5 + 2\eta_6 \\
+ 2(\eta_9 + \eta_{10})(1 - \omega)] - \frac{1}{4\Lambda^2} [\frac{1}{m_b} (\kappa_1 + 3\kappa_2) + \frac{1}{m_c} (\kappa_1 - \kappa_2))^2 \\
+ \frac{1}{2m_b \Lambda^2} \kappa_3 (1 - \omega) [\frac{1}{m_b} (\kappa_1 + 3\kappa_2) + \frac{1}{m_c} (\kappa_1 - \kappa_2)], \tag{C16} \]

\[ h_{A_2} = \frac{1}{m_b \Lambda} \kappa_3 - \frac{1}{2m_b^2 \Lambda} c_3 - \frac{1}{4m_b^2 \Lambda^2} [4\chi_7(1 + \omega) + 2\chi_9 - 2\chi_{10} - 10\chi_{11} - 10\chi_{12}] \\
+ \frac{1}{4m_b m_c \Lambda^2} [-2\eta_3 + 2\eta_4 + (4\eta_7 + 2\eta_8)(1 + \omega) - 2\eta_9 - 10\eta_{10} \\
- \frac{1}{2m_c \Lambda} \kappa_3 [\frac{1}{m_b} (\kappa_1 + 3\kappa_2) + \frac{1}{m_c} (\kappa_1 - \kappa_2)], \tag{C17} \]

\[ h_{A_3} = \xi + (\xi - 1)[\frac{1}{2m_b \Lambda} (\kappa_1 + 3\kappa_2) + \frac{1}{2m_c \Lambda} (\kappa_1 - \kappa_2) + \frac{1}{4\Lambda} (\frac{1}{m_b^2} + \frac{1}{m_c^2})c_1 \\
- \frac{1}{4\Lambda} (\frac{3}{m_b^2} - \frac{1}{m_c^2})c_2 - \frac{1}{4m_b^2 \Lambda^2} c_4 - \frac{1}{4m_c^2 \Lambda^2} c_5] + \xi[-\frac{1}{8m_b^3 \Lambda^2} c_6 - \frac{1}{8m_c^3 \Lambda^2} c_7 \\
+ \frac{1}{8\Lambda^2} (\frac{3}{m_b^2} (\kappa_1 + 3\kappa_2)^2 + \frac{3}{m_c^2} (\kappa_1 - \kappa_2)^2 + \frac{2}{m_b m_c} (\kappa_1 + 3\kappa_2)(\kappa_1 - \kappa_2))] \\
+ \frac{1}{m_b \Lambda} \kappa_3 (1 - \omega) - \frac{1}{2\Lambda} [\frac{1}{m_b^2} (1 - \omega) - \frac{1}{m_c^2} c_3 - \frac{1}{m_b \Lambda} \kappa_3 + \frac{1}{4m_b^2 \Lambda^2} [(4\chi_7 + 2\chi_8) \\
\times (1 - \omega^2) - 4(\chi_9 + \chi_12)(1 - \omega)] + \frac{1}{4m_c^2 \Lambda^2} [4\chi_7(1 + \omega) + 2\chi_8(1 - \omega^2) \\
+ 2\chi_9 - 2\chi_{10} (2 - \omega) - 2\chi_{11} (2 + 3\omega) - 2\chi_{12} (4 + \omega)] + \frac{1}{4m_b m_c \Lambda^2} [\eta_1 \\
+ 2\eta_2 + 2\eta_3 \omega - \eta_4 + 3\eta_5 + 2\eta_6 - (4\eta_7 + 2\eta_8)(1 + \omega) + 2\eta_9 (2 - \omega) \\
+ 2\eta_{10} (6 - \omega)] - \frac{1}{4\Lambda^2} [\frac{1}{m_b} (\kappa_1 + 3\kappa_2) + \frac{1}{m_c} (\kappa_1 - \kappa_2))^2 \\
+ \frac{1}{2\Lambda^2} [\frac{1}{m_b} (1 - \omega) - \frac{1}{m_c} \kappa_3 [\frac{1}{m_b} (\kappa_1 + 3\kappa_2) + \frac{1}{m_c} (\kappa_1 - \kappa_2)], \tag{C18} \]

where

\[ c_1 = -\tilde{\Lambda} \kappa_1 + q_1 + \frac{1}{\bar{\Lambda}} \chi_1, \tag{C19} \]
\[ c_2 = \bar{\Lambda}\kappa_2 - \varrho_2 - \frac{1}{\bar{\Lambda}}\chi_2, \quad \text{(C20)} \]
\[ c_3 = \bar{\Lambda}\kappa_3 - \varrho_3 - \frac{1}{\bar{\Lambda}}\chi_3, \quad \text{(C21)} \]
\[ c_4 = 3\chi_4 + 9\chi_5 + 6\chi_6, \quad \text{(C22)} \]
\[ c_5 = 3\chi_4 + \chi_5 - 2\chi_6, \quad \text{(C23)} \]
\[ c_6 = \eta_1 + 6\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6, \quad \text{(C24)} \]
\[ c_7 = \eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6. \quad \text{(C25)} \]
REFERENCES

[1] E.V. Shuryak, Phys. Lett. B 93, 134 (1980); Nucl. Phys. B 198 83 (1982).
[2] S. Nussinov and W. Wetzel, Phys. Rev. D 36, 130 (1987).
[3] N. Isgur and M.B. Wise, Phys. Lett. B 232, 113 (1989); B 237, 527 (1990).
[4] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 45, 292(1987); 47, 199 (1988).
[5] E. Eichten, Nucl. Phys. (Proc. Suppl.) B 4, 170 (1988).
[6] E. Eichten and B. Hill, Phys. Lett. B 234, 511 (1990).
[7] H. Georgi, Phys. Lett. B 240, 447 (1990).
[8] B. Grinstein, Nucl. Phys. B 339, 253 (1990).
[9] A. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B 343, 1 (1990).
[10] T. Mannel and Z. Ryzak, Phys. Lett. B 247, 2388 (1990).
[11] F. Hussain, J.G. Körner, K.Schilcher, G. Thompson and Y.L. Wu, Phys. Lett. B 249, 295 (1990).
[12] M.E. Luke, Phys. Lett. B 252, 447 (1990).
[13] A.F. Falk, B. Grinstein and M.E. Luke, Nucl. Phys. B 357, 185 (1991).
[14] Y.L. Wu, Mod. Phys. Lett. A 6, 1277 (1991).
[15] J.G. Körner and G. Thompson, Phys. Lett. B 264, 185 (1991).
[16] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B 368, 204 (1992).
[17] B. Grinstein, ‘Light-Quark, Heavy-Quark System’, SSCL-Preprint-34, 1992.
[18] A.F. Falk and M. Neubert, Phys. Rev. D 47, 2965 (1993).
[19] Y.L. Wu, Modern Phys. Lett. A 8, 819 (1993).
[20] M. Neubert, Phys. Lett. B338(1994)2031;
       Int. J. Mod. Phys. A11(1996)4173; Phys. Rept. 245(1994)259, and reference therein.
[21] M.B. Wise, ‘Heavy Quark Physics’, CALT-68-2172, hep-ph/9805468
[22] M. Neubert, Phys. Rev. D 46, 1076 (1992).
[23] T. Mannel, Phys. Rev. D 50, 428 (1994).
[24] M. Neubert, Phys. Lett. B 338, 84 (1994).
[25] M. Neubert, CERN-TH/95-107, hep-ph/9505238.
[26] K. K. Jeong and C. S. Kim, hep-ph/9811473.
[27] M. Neubert, Phys. Lett. B 264, 455 (1991).
[28] M. Shifman, N. G. Uraltsev and A. Vainshtein, Phys. Rev. D 51, 2217 (1995).
[29] J. E. Paschalis and G. J. Gounaris, Nucl. Phys. B 222, 473 (1983).
[30] A. F. Falk and B. Grinstein, Phys. Lett. B 247, 406 (1990).
[31] X. Ji and M. J. Musolf, Phys. Lett. B 257, 409 (1991).
[32] M. Neubert, Nucl. Phys. B 371, 149 (1992).
[33] A. Czarnecki, Phys. Rev. Lett. 22, 4124 (1996).
[34] Particle Data Group, Eur. Phys. J. C 3, 1998.
[35] M. Neubert, CERN-TH/98-2, hep-ph/9801269.
[36] Y.A. Yan, Y.L. Wu and W.Y. Wang, AS-ITP-99-07, 1999.
FIGURES
Fig. 1 $\kappa_1$, $\kappa_2$, $F_1$ and $F_2$ as functions of $m_b$, $m_b - m_c$ and $m_b + \bar{\Lambda}$ for (a), (d), (g), (j): $m_b - m_c = 3.36\text{GeV}$; (b), (e), (h), (k): $m_b = 4.7\text{GeV}$; and (c), (f), (i), (l): $m_b - m_c = 3.36\text{GeV}$.
Fig. 2 $|V_{cb}|$ extracted from the two channels $B \to D^* l \nu$ and $B \to D l \nu$ as function of $\rho_1$, $\rho_2$ and $m_b + \Lambda$. (a)–(c): $m_b = 4.7 \text{GeV}$, $m_b - m_c = 3.36 \text{GeV}$ and $m_b + \Lambda = 5.21 \text{GeV}$; (d): $m_b = 4.7 \text{GeV}$, $m_b - m_c = 3.36 \text{GeV}$, $\rho_1 = 0.3 \text{GeV}^3$ and $\rho_2 = 0.11 \text{GeV}^3$. 
Fig. 3 $\delta^*$ as function of $m_b + \Lambda$ for (a): $m_b - m_c = 3.41\text{GeV}$; (b): $m_b - m_c = 3.36\text{GeV}$; (c): $m_b - m_c = 3.32\text{GeV}$. 
Fig. 4 $\delta^*$ as function of $m_b$ and $m_b - m_c$ for (a): $m_b - m_c = 3.36$ GeV; (b): $m_b = 4.7$ GeV.
Fig. 5 $|V_{ub}|$ extracted from $B \to D^* \ell \nu$ decay as function of $m_b$, $m_b - m_c$ and $m_b + \bar{\Lambda}$ for (a), (c): $m_b - m_c = 3.36$ GeV; (b): $m_b = 4.7$ GeV.
Table.1 Some results extracted when \( m_b = 4.6 \text{GeV} \), \( g_1 = 0.3 \text{GeV}^3 \) and \( g_2 = 0.11 \text{GeV}^3 \). Here \( |V_{cb}|_{BD} \) (\( |V_{cb}|_{BD} \)) refers to the value of \( |V_{cb}| \) extracted from \( B \to D^*\ell\nu \) (\( B \to D\ell\nu \)) decay.

| \( m_b - m_c \) | \( m_b + \Lambda \) | \( \kappa_1 \) | \( \kappa_2 \) | \( \bar{F}_1 \) | \( \bar{F}_2 \) | \( \delta^* \) | \( \delta \) | \( |V_{cb}|_{BD} \) |
|---|---|---|---|---|---|---|---|---|
| 3.32 | 5.18 | -0.7954 | 0.0558 | 1.4042 | 0.0070 | -0.0573 | -0.0354 | 0.0389 | 0.0392 |
| | 5.20 | -0.6778 | 0.0558 | 1.2356 | 0.0085 | -0.0572 | -0.0270 | 0.0389 | 0.0388 |
| | 5.21 | -0.6190 | 0.0558 | 1.1480 | 0.0094 | -0.0556 | -0.0217 | 0.0388 | 0.0386 |
| | 5.24 | -0.4426 | 0.0558 | 0.8730 | 0.0120 | -0.0461 | -0.0026 | 0.0384 | 0.0379 |
| | 5.27 | -0.2662 | 0.0558 | 0.5811 | 0.0148 | -0.0310 | 0.0202 | 0.0378 | 0.0370 |
| 3.36 | 5.18 | -0.7709 | 0.0562 | 1.1598 | 0.0029 | -0.0377 | -0.0097 | 0.0381 | 0.0381 |
| | 5.20 | -0.6541 | 0.0562 | 0.9916 | 0.0044 | -0.0384 | -0.0016 | 0.0381 | 0.0378 |
| | 5.21 | -0.5957 | 0.0562 | 0.9045 | 0.0051 | -0.0371 | 0.0036 | 0.0381 | 0.0376 |
| | 5.24 | -0.4205 | 0.0562 | 0.6315 | 0.0076 | -0.0280 | 0.0228 | 0.0377 | 0.0369 |
| | 5.27 | -0.2453 | 0.0562 | 0.3429 | 0.0102 | -0.0128 | 0.0461 | 0.0371 | 0.0361 |
| 3.41 | 5.18 | -0.7442 | 0.0562 | 0.8931 | -0.0015 | -0.0116 | 0.0248 | 0.0371 | 0.0369 |
| | 5.20 | -0.6284 | 0.0562 | 0.7266 | -0.0002 | -0.0134 | 0.0324 | 0.0372 | 0.0366 |
| | 5.21 | -0.5765 | 0.0562 | 0.6406 | 0.0005 | -0.0125 | 0.0374 | 0.0371 | 0.0364 |
| | 5.24 | -0.3968 | 0.0562 | 0.3721 | 0.0027 | -0.0042 | 0.0566 | 0.0368 | 0.0357 |
| | 5.27 | -0.2231 | 0.0562 | 0.0894 | 0.0051 | 0.0109 | 0.0802 | 0.0363 | 0.0350 |

Table.2 Some results extracted when \( m_b = 4.7 \text{GeV} \), \( g_1 = 0.3 \text{GeV}^3 \) and \( g_2 = 0.11 \text{GeV}^3 \).

| \( m_b - m_c \) | \( m_b + \Lambda \) | \( \kappa_1 \) | \( \kappa_2 \) | \( \bar{F}_1 \) | \( \bar{F}_2 \) | \( \delta^* \) | \( \delta \) | \( |V_{cb}|_{BD} \) |
|---|---|---|---|---|---|---|---|---|
| 3.32 | 5.18 | -0.8238 | 0.0561 | 1.3967 | 0.0070 | -0.0603 | -0.0545 | 0.0390 | 0.0399 |
| | 5.20 | -0.7022 | 0.0561 | 1.2398 | 0.0084 | -0.0635 | -0.0459 | 0.0392 | 0.0396 |
| | 5.21 | -0.6414 | 0.0561 | 1.1567 | 0.0092 | -0.0629 | -0.0400 | 0.0391 | 0.0393 |
| | 5.24 | -0.4590 | 0.0561 | 0.8901 | 0.0115 | -0.0541 | -0.0179 | 0.0388 | 0.0385 |
| | 5.27 | -0.2766 | 0.0561 | 0.5987 | 0.0141 | -0.0374 | 0.0090 | 0.0381 | 0.0374 |
| 3.36 | 5.18 | -0.7962 | 0.0566 | 1.1605 | 0.0029 | -0.0385 | -0.0266 | 0.0381 | 0.0388 |
| | 5.20 | -0.6754 | 0.0566 | 1.0014 | 0.0041 | -0.0427 | -0.0182 | 0.0383 | 0.0385 |
| | 5.21 | -0.6150 | 0.0566 | 0.9175 | 0.0048 | -0.0423 | -0.0125 | 0.0383 | 0.0382 |
| | 5.24 | -0.4338 | 0.0566 | 0.6491 | 0.0069 | -0.0339 | 0.0098 | 0.0380 | 0.0374 |
| | 5.27 | -0.2526 | 0.0566 | 0.3572 | 0.0092 | -0.0172 | 0.0372 | 0.0373 | 0.0364 |
| 3.41 | 5.18 | -0.7660 | 0.0572 | 0.9015 | -0.0017 | -0.0096 | 0.0106 | 0.0370 | 0.0374 |
| | 5.20 | -0.6462 | 0.0572 | 0.7411 | -0.0007 | -0.0151 | 0.0184 | 0.0372 | 0.0371 |
| | 5.21 | -0.5863 | 0.0572 | 0.6568 | -0.0001 | -0.0152 | 0.0240 | 0.0372 | 0.0369 |
| | 5.24 | -0.4066 | 0.0572 | 0.3884 | 0.0018 | -0.0076 | 0.0464 | 0.0369 | 0.0361 |
| | 5.27 | -0.2269 | 0.0572 | 0.0981 | 0.0038 | 0.0092 | 0.0745 | 0.0363 | 0.0352 |
Table 3: Some results extracted when $m_b = 4.8\text{GeV}$, $\varrho_1 = 0.3\text{GeV}^3$ and $\varrho_2 = 0.11\text{GeV}^3$.

| $m_b - m_c$ | $m_b + \Lambda$ | $\kappa_1$ | $\kappa_2$ | $F_1$ | $F_2$ | $\delta^*$ | $\delta$ | $|V_{cb}|_{BD}$ | $|V_{ub}|_{BD}$ |
|-----------------|-----------------|----------|----------|--------|--------|----------|--------|----------------|----------------|
| 3.32            | 5.18            | -0.8524  | 0.0564   | 1.3013 | 0.0077 | -0.0502  | -0.0716| 0.0386         | 0.0407         |
|                 | 5.20            | -0.7268  | 0.0564   | 1.1690 | 0.0091 | -0.0628  | -0.0650| 0.0391         | 0.0404         |
|                 | 5.21            | -0.6640  | 0.0564   | 1.0969 | 0.0097 | -0.0651  | -0.0592| 0.0392         | 0.0401         |
|                 | 5.24            | -0.4756  | 0.0564   | 0.8574 | 0.0119 | -0.0606  | -0.0345| 0.0390         | 0.0391         |
|                 | 5.27            | -0.2872  | 0.0564   | 0.5851 | 0.0144 | -0.0437  | -0.0027| 0.0383         | 0.0379         |
| 3.36            | 5.18            | -0.8214  | 0.0569   | 1.0843 | 0.0038 | -0.0246  | -0.0396| 0.0376         | 0.0393         |
|                 | 5.20            | -0.6960  | 0.0569   | 0.9469 | 0.0049 | -0.0334  | -0.0384| 0.0381         | 0.0391         |
|                 | 5.21            | -0.6342  | 0.0569   | 0.8725 | 0.0055 | -0.0412  | -0.0277| 0.0382         | 0.0388         |
|                 | 5.24            | -0.4470  | 0.0569   | 0.6271 | 0.0074 | -0.0372  | -0.0028| 0.0381         | 0.0379         |
|                 | 5.27            | -0.2598  | 0.0569   | 0.3502 | 0.0095 | -0.0202  | 0.0296 | 0.0374         | 0.0367         |
| 3.41            | 5.18            | -0.7873  | 0.0576   | 0.8453 | -0.0007| 0.0096   | 0.0030 | 0.0363         | 0.0377         |
|                 | 5.20            | -0.6635  | 0.0576   | 0.7033 | 0.0002 | -0.0061  | 0.0084 | 0.0369         | 0.0375         |
|                 | 5.21            | -0.6016  | 0.0576   | 0.6268 | 0.0007 | -0.0096  | 0.0139 | 0.0370         | 0.0373         |
|                 | 5.24            | -0.4159  | 0.0576   | 0.3764 | 0.0022 | -0.0066  | 0.0389 | 0.0369         | 0.0364         |
|                 | 5.27            | -0.2302  | 0.0576   | 0.0961 | 0.0040 | 0.0104   | 0.0720 | 0.0363         | 0.0352         |