SIVERS EFFECT IN SEMI-INCLUSIVE DEEPLY INELASTIC SCATTERING AND DRELL-YAN

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The Sivers function is extracted from HERMES data on single spin asymmetries in semi-inclusive deeply inelastic scattering. The result is used for making predictions for the Sivers effect in the Drell-Yan process.

1. Introduction

The recent HERMES and COMPASS data\textsuperscript{1,2,3,4} on transverse target single spin asymmetries (SSA) in semi-inclusive deeply inelastic scattering (SIDIS) can be described\textsuperscript{5} — on the basis of generalized factorization theorems\textsuperscript{6,7,8} — in terms of the Sivers\textsuperscript{9,10,11,12} and Collins\textsuperscript{13} effects. These effects may also contribute to SSA in hadron-hadron-collisions\textsuperscript{14} and longitudinal SSA in SIDIS\textsuperscript{15,16,17,18}, but the status of factorization for these SSA is not clear. All that is clear is that the longitudinal target SSA in SIDIS are dominated by subleading-twist effects\textsuperscript{19} and more difficult to interpret\textsuperscript{20,21}.

The Collins and Sivers effects were subject to intensive phenomenological studies in hadron-hadron-collisions\textsuperscript{22–25} and in SIDIS\textsuperscript{26–33}. In this note we will concentrate on the Sivers effect and review our recent work\textsuperscript{30,33,34}. Studies of the equally interesting Collins effect are reported elsewhere\textsuperscript{32,35}.

The Sivers function belongs to the class of so-called “naively time-reversal-odd” distributions, which have been predicted to obey an unusual “universality property”, namely to appear with opposite sign in SIDIS and in the Drell-Yan (DY) process\textsuperscript{11}. We show that an experimental test of this prediction, which is among the most important issues for the future spin physics, is feasible in the running or planned experiments at RHIC, COMPASS and GSI.
Sivers function from preliminary $P_{h\perp}$-weighted data \cite{1}

In SIDIS the Sivers effect gives rise to a transverse target SSA with a specific angular distribution of the produced hadrons \( \propto \sin(\phi - \phi_S) \), where \( \phi (\phi_S) \) is the azimuthal angle of the produced hadron (target polarization vector) with respect to the axis defined by the hard virtual photon\(^5\).

Weighting the events entering the spin asymmetry with \( \sin(\phi - \phi_S) P_{h\perp} \) (\( P_{h\perp} \) = transverse momentum of the produced hadron) yields the Sivers SSA given\(^5\) (if we neglect soft factors\(^6,7,8\)) by

\[
A^{\sin(\phi - \phi_S) P_{h\perp}}_{UT/MN} = (-2)^{e_a} \frac{\sum_a e_a^2 x_{f_a}^{(1)a}(x) z D_1^{a/\pi}(z)}{\sum_a e_a^2 x_{f_a}^{(1)a}(x) D_1^{a/\pi}(z)},
\]

where the transverse moment of the Sivers function is defined as

\[
\int d^2 p_T \frac{p_T^2}{2 M_N^2} f_{UT}^{(1)a}(x, p_T^2).
\]

Preliminary data analyzed in this way are available\(^1\). Neglecting \( f_{UT}^{a\bar{q}} \) and taking the ansatz motivated by predictions from the large-\( N_c \) limit\(^3\)

\[
f_{UT}^{(1)a}(x) \xrightarrow{large N_c} -f_{UT}^{(1)d}(x) \xrightarrow{ansatz} A x^b (1 - x)^5,
\]

the Sivers function was extracted from these data\(^3\). The fit result (\( \chi^2 \) per degree of freedom \( \equiv \chi^2_{ndf} \sim 0.3 \)) refers to a scale of about 2.5 GeV\(^2\) and is shown in Fig. 1. For \( f_1^a \) and \( D_1^a \) the parameterizations\(^37,38\) were used.

Transverse parton momenta and the Gaussian ansatz

However, the currently available published data\(^2,3\) were analyzed without a \( P_{h\perp} \)-weight, and can only be interpreted by resorting to some model for the distribution of the transverse parton momenta in the “unintegrated” distribution and fragmentation functions\(^39\). Here (for other models see\(^31,32\)) we assume the distributions of transverse parton momenta to be Gaussian:

\[
f_1^a(x, p_T^2) = f_1^a(x) \frac{\exp(-p_T^2/p_{unp}^2)}{\pi p_{unp}^2},
\]

\[
f_{UT}^{(1)a}(x, p_T^2) = f_{UT}^{(1)a}(x) \frac{\exp(-p_T^2/p_{Sin}^2)}{\pi p_{Sin}^2},
\]

\[
D_1^a(z, K_T^2) = D_1^a(z) \frac{\exp(-K_T^2/K_{D1}^2)}{\pi K_{D1}^2},
\]

and take the Gaussian widths to be flavour and \( x- \) or \( z- \)independent. This model describes well the distributions of low (with respect to the relevant
hard scale) transverse hadron momenta in various hard reactions\textsuperscript{24} — which is the case at HERMES\textsuperscript{2}, where \( \langle P_{h \perp} \rangle \sim 0.4 \text{ GeV} \ll \langle Q^2 \rangle^{1/2} \sim 1.5 \text{ GeV} \).

In order to test the ansatz (4) for \( f^T_1 \) and \( D^T_1 \) in SIDIS we consider the HERMES data\textsuperscript{17} on the average transverse momentum of the produced hadrons given by \( \langle P_{h \perp}(z) \rangle = \frac{\sqrt{\pi} M_N}{2 \sqrt{p^2_{\text{Siv}} + K^2_{D_1}/z^2}} \) in the Gaussian ansatz\textsuperscript{33}.

With the (fitted) parameters \( p^2_{\text{unp}} = 0.33 \text{ GeV}^2 \) and \( K^2_{D_1} = 0.16 \text{ GeV}^2 \) the Gaussian ansatz provides a good description of the data\textsuperscript{17} — Fig. 1. For comparison we also show the description one obtains using the parameters obtained from an analysis\textsuperscript{31} of EMC data\textsuperscript{40} on the Cahn effect\textsuperscript{41}, which is equally satisfactory. The good agreement observed in Fig. 1 indicates that the mechanisms generating the Cahn effect in the EMC data\textsuperscript{40} and transverse hadron momenta at HERMES\textsuperscript{17} could be compatible\textsuperscript{31,33}.

4. Sivers function from final (published) HERMES data [2]

In the Gaussian model the expression for the Sivers SSA weighted without a power of transverse hadron momentum is given by\textsuperscript{28}

\[
A^{\text{SIV}}_{UT}(\phi - \phi_S) = a_{\text{Gauss}}^\text{SIDIS} A^{\text{SIV}}_{UT}(\phi - \phi_S)/M_N \quad ; \quad a_{\text{Gauss}}^\text{SIDIS} = \frac{\sqrt{\pi} M_N}{2 \sqrt{p^2_{\text{Siv}} + K^2_{D_1}/z^2}}.
\]

Positivity\textsuperscript{42} constrains \( p^2_{\text{Siv}} \) to be in the range\textsuperscript{33} \( 0 < p^2_{\text{Siv}} < 0.33 \text{ GeV}^2 \). Though vague, this information is sufficient for the extraction of the transverse moment of the Sivers function.

Using the same assumptions (large-\( N_c \), neglect of \( \bar{q} \), etc.) as in Sec. 2 the fit (\( \chi^2_{\text{dof}} \sim 0.3 \)) to the final data\textsuperscript{2} was obtained shown in Fig. 1. The fit\textsuperscript{33} to the final data\textsuperscript{2} obtained assuming the Gaussian model is compatible — see Fig. 1 — with the “model-independent” fit\textsuperscript{30} to the preliminary data. This observation is a valuable test of the Gaussian ansatz for the Sivers function.

5. Sivers effect from deuteron at COMPASS [3]

At COMPASS the deuteron Sivers effect was found consistent with zero within error bars\textsuperscript{3}. Notice that the Sivers SSA from deuteron is sensitive solely to \( (f^\perp_{1u} + f^\perp_{1d}) \) which is subleading in the large-\( N_c \) limit. Thus, the deuteron Sivers SSA is \( \sim O(N_c^{-1}) \), while the proton Sivers SSA is \( \sim O(N_c^0) \).

This suppression naturally explains\textsuperscript{30,33} the compatibility of the HERMES and COMPASS results\textsuperscript{2,3}, within errors. The COMPASS data\textsuperscript{3} confirm the utility of the constraint (3) at the present stage.
6. Sivers function from most recent preliminary data [4]

Increasing precision of data will, sooner or later, require to relax the strict large-$N_c$ constraint (3). The preliminary HERMES data released recently are considerably more precise compared to the final (published) data.

Could these data already constrain $1/N_c$-corrections? In order to answer this question, we repeat here the procedure of [33] described in Sec. 4 with the preliminary HERMES data. The resulting fit is compatible with the fit obtained from the published data — Fig. 1.

The $\chi^2_{\text{dof}} \sim 2$ of this fit is larger than previously, see Sec. 4, which indicates that the description of the preliminary data could be improved, e.g., by considering $1/N_c$ corrections. However, it could be equally sufficient to introduce more parameters in the large-$N_c$ ansatz (3). Thus, our large-$N_c$ ansatz is still useful to describe the most recent preliminary data.

7. Sivers effect in the Drell-Yan process

On the basis of the first study of the preliminary $P_{h\perp}$-weighted data it was found that the Sivers effect can give rise to SSA in DY large enough to be measured in the planned COMPASS and PAX experiments. This conclusion is now solidified by the study of the published HERMES data. Thus, the predicted sign reversal of the quark Sivers function in SIDIS and DY can be tested at COMPASS (PAX) in $p^+\pi^- (p^-\bar{p})$ collisions.

At RHIC in DY from $p^+p$-collisions, however, antiquark Sivers distribu-
tions are of importance, which are not constrained by the present SIDIS data\textsuperscript{33}. In order to see, what one can learn from RHIC about the Sivers SSA, let us assume the $\bar{q}$-Sivers distributions are given by

$$f_{1T}^{(1)\bar{q}}(x) = f_{1T}^{(1)q}(x) \times \begin{cases} 0.25 = \text{const} \quad \text{(model I)} \\ \\
\frac{(f_u^a + f_d^a)(x)}{(f_u^a + f_d^a)(x)} \quad \text{(model II)}, \end{cases} \quad (6)$$

with $f_{1T}^{(1)\bar{q}}(x)$ from the fit to the published HERMES data\textsuperscript{2} — Sec. 4. The models I, II are consistent\textsuperscript{33} with theoretical constraints, and with SIDIS data\textsuperscript{1,2,3}. This makes them well suited to visualize possible effects of $\bar{q}$-Sivers distributions at RHIC.

The Sivers SSA in DY is defined similarly to that in SIDIS. Here $(\phi - \phi_S)$ is the azimuthal angle between the virtual photon and the polarization vector (the polarized proton moves into the positive $z$-direction). We again neglect soft factors and assume the Gaussian model, so that the SSA is

$$A_{\sin(\phi - \phi_S)} = 2 a_{\text{Gauss}}^{DY} \frac{\sum_a e^2_a x_1 f_{1T}^{(1)a}(x_1) x_2 f_{1T}^{(2)}(x_2)}{\sum_b e^2_b x_1 f_{1T}^{(1)b}(x_1) x_2 f_{1T}^{(2)}(x_2)}, \quad (7)$$

where $x_{1,2} = (Q^2/s)^{1/2} e^{\pm y}$ with $s = (p_1 + p_2)^2$, $Q^2 = (k_1 + k_2)^2$, and $y = \frac{1}{2} \ln \frac{p_1 \cdot (k_1 + k_2)}{p_2 \cdot (k_1 + k_2)}$. The momenta of the incoming proton (outgoing lepton) pair are denoted by $p_{1/2}$ ($k_{1/2}$). The Gaussian factor reads

$$a_{\text{Gauss}}^{DY} = \frac{\sqrt{\pi}}{2} \frac{M_N}{\sqrt{p_{\text{Siv}}^2 + p_{\text{unp}}^2}}. \quad (8)$$

Considering the sign reversal\textsuperscript{11} we obtain the results shown in Fig. 2. In our estimate we assume the ratio $\sum_a e^2_a f_{1T}^{(1)a}/\sum_b e^2_b f_{1T}^{(1)b}$ to be weakly scale-dependent, and roughly simulate Sudakov dilution by assuming that $p_{\text{unp/Siv}}^2$ increase by a factor of two from HERMES to RHIC energies. Notice that SSA weighted appropriately with the transverse dilepton momentum\textsuperscript{30} were argued to be less sensitive to Sudakov suppression\textsuperscript{16}.

In the region of positive rapidities $1 \leq y \leq 2$ the Sivers SSA at RHIC is well constrained by the SIDIS data\textsuperscript{2} and shows little sensitivity to the unknown $\bar{q}$-Sivers distributions. Thus, in this region STAR and PHENIX can also test the sign reversal\textsuperscript{11} of the quark Sivers function.

For negative $y$ the Sivers SSA is strongly sensitive to the antiquark Sivers function — with the effect being more pronounced at larger dilepton masses $Q$.\textsuperscript{33} This reveals the unique feature of RHIC, which — in contrast to COMPASS and PAX — can also provide information on the antiquark Sivers distribution.
Figure 2. The Sivers SSA $A_{UT}^{\sin(\phi - \phi_S)}$ in $p^+p \rightarrow l^+l^-X$ as a function of $y$ for the kinematics of the RHIC experiment with $\sqrt{s} = 200$ GeV, and $Q^2 = (4 \text{GeV})^2$. The inner error band (thick lines) shows the $1\sigma$ uncertainty of the fit$^{33}$. The $x$-region explored at HERMES is included to indicate where the SIDIS data constrain the prediction. The outer error band (thin lines) arises from assuming Sivers $\bar{q}$-distribution functions according to model I (left) and model II (right) in Eq. (6). Also shown are the regions in which PHENIX and STAR can detect $\mu^+\mu^-$ or $e^+e^-$ pairs.

8. Conclusions

We reviewed our studies$^{30,33}$ of the HERMES and COMPASS data on the Sivers effect in SIDIS$^{1,2,3,4}$. The data from various targets are compatible with each other and can be well described assuming a Gaussian distribution of parton transverse momenta in the distribution and fragmentation functions. The parameters in the Gaussian ansatz were constrained by HERMES data and are compatible with results obtained from studies of the Cahn effect$^{31}$. The data$^{1,2,3,4}$ confirm the predictions from the large-$N_c$ limit on the flavour dependence of the Sivers function$^{36}$. The sign of the extracted quark Sivers functions is in agreement with the intuitive picture discussed in [47]. Results by other groups$^{31,32}$ confirm these findings — see the detailed comparison in Ref. [48].

The information on the quark Sivers distributions extracted from SIDIS is required for reliable estimates of the Sivers effect in DY for current or planned experiments. We estimated that the Sivers SSA in DY are sizeable enough to be observed at RHIC, COMPASS and PAX$^{30,33,34}$ allowing one to test the QCD prediction$^{31}$ that the Sivers function should appear with opposite signs in SIDIS and in DY. In addition, RHIC can provide information on the antiquark Sivers distributions$^{34}$. 
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