Pseudospin-dependent Acoustic Topological Insulator by Sonic Crystals With Same Hexagonal Rods

Ding Jia, Shuai Gu, Shuai Jiang, Yong Ge, Shou-qi Yuan and Hong-xiang Sun*

1Research Center of Fluid Machinery Engineering and Technology, School of Physics and Electronic Engineering, Jiangsu University, Zhenjiang, China, 2School of Computer Science and Communications Engineering, Jiangsu University, Zhenjiang, China, 3State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing, China

We report the experimental and numerical realization of a pseudospin-dependent acoustic topological insulator based on two sonic crystals constructed by the same regular hexagonal rods. Based on the zone folding mechanism, we obtain double Dirac cones with a four-fold deterministic degeneracy in the sonic crystal, and realize a band inversion and topological phase transition by rotating the rods. We observe the topologically protected one-way sound propagation of pseudospin-dependent edge states in a designed topological insulator composed of two selected sonic crystals with different rotation angles of the rods. Furthermore, we experimentally demonstrate the robustness of topological sound propagation against two types of defects, in which the edge states are almost immune to backscattering, and remain pseudospin-dependent characteristics. Our work provides a diverse route for designing tunable topological functional sound devices.

Keywords: sonic crystal, acoustic topological insulator, pseudospin, dirac cone, acoustics

INTRODUCTION

In the past few years, acoustic topological insulators (ATIs) have become a hot topic in the area of acoustics for its great potential applications, such as acoustic-noise reduction [1], acoustic beam splitter [2] and acoustic communications [3]. However, the intrinsic difference between electrons and acoustics poses a challenge to realize a spin-like degree of freedom for acoustics due to the longitudinal polarization characteristics of sound. To overcome it, four types of ATIs have been proposed successively. To mimic the quantum Hall effect and break time-reversal symmetry, topologically protected edge states were observed in acoustic waveguides by introducing circular flowing air [4–9]. Later, analogous Floquet ATIs were proposed based on coupled acoustic trimers or strong coupling ring resonator waveguides [10–13]. Moreover, to realize acoustic valley Hall (AVH) states, the nonzero Berry curvature located near the valley K/K' was obtained by breaking mirror or inversion symmetry, and valley-projected topological edge states were observed at the interface between two sonic crystals (SCs) with distinct AVH phases [14–28]. Furthermore, to mimic quantum spin Hall states in acoustics, the SCs with $C_6$ crystal symmetry were introduced to realize artificial pseudospins-1/2 states by hybridizing degenerate modes [29–39]. Generally, the observed double Dirac cones are accidental degeneracy in these pseudospin-dependent ATIs, and the sizes or shapes of rods in the SCs are different, which is still a challenge for practical applications.

Recently, by introducing the zone folding mechanism, a type of pseudospin-dependent ATI without depending on a fixed filling ratio was proposed [30, 31], in which the observed double Dirac cones were a deterministic degeneracy. These types of pseudospin-dependent ATIs were designed by fictitious soft metamaterial rods [30] and three circular rods with different radius [31], and the
complex structures greatly limit their applications. Based on the SCs composed of same rods, the experimental realization of the pseudospin-dependent ATIs with a deterministic degeneracy is still a challenge.

In this work, we propose a pseudospin-dependent ATI constructed by two airborne SCs which consist of the same rods. Based on the zone folding mechanism, we obtain double Dirac cones with the deterministic degeneracy, and realize a band inversion by simply rotating the rods in the SCs. Moreover, we design an ATI composed of two SCs with different rotation angles of the rods, and observe a one-way sound propagation of topological edge states. Finally, we experimentally demonstrate the robustness of the topologically protected sound propagation against two types of defects of a disorder and a bend.

RESULTS AND DISCUSSIONS

Figure 1A shows an airborne SC composed of regular hexagonal rods with the side length $l = 14.4$ mm. The proposed SC can be described by the unit A or B, in which the lattice constants of the units A and B are $a_0 = 30$ mm and $a = 30\sqrt{3}$ mm, respectively. By simply rotating these rods, we obtain another type of SC composed of the unit B (shown in Figure 1B), in which the rotation angles of a central rod and six surrounding rods are $\theta_0$ and $\theta$, respectively. Figure 1C shows the folding mechanism of the Brillouin zones (BZs) for the units A and B, corresponding to the black and red open regular hexagons, respectively.

![Figure 1 | (A) Schematic of an airborne SC composed of regular hexagonal rods. Black and red open regular hexagons refer to the units A and B with the lattice constants $a_0$ and $a$, respectively. (B) Schematic of the unit B with two types of configurations. Red regular hexagons exhibit its rotation state, and the parameters $\theta_0$ and $\theta$ are the rotation angles of a central rod and six surrounding rods, respectively. Blue regular hexagons represent its initial configuration ($\theta_0 = \theta = 0^\circ$). (C) Schematic of the folding mechanism of the BZs for the units A and B, corresponding to the black and red open regular hexagons, respectively.](image-url)

Figures 2A, B show the dispersion relations of the units A and B in the SC-I ($\theta_0 = 0^\circ$ and $\theta = 0^\circ$), respectively. As shown in Figure 2A, there exists a Dirac cone with the two-fold degeneracy at the point $K_0$ for the unit A, which is attributed to its $C_{6v}$ symmetry. However, for the unit B, we observe double Dirac cones with a four-fold deterministic degeneracy at the point $\Gamma$ (Figure 2B). The dispersion characteristics of both units agree well with the folding of the BZ.

Figures 2C,D show the dispersion relations of the unit B in the SC-II ($\theta_0 = 30^\circ$ and $\theta = 0^\circ$) and SC-III ($\theta_0 = 0^\circ$ and $\theta = 30^\circ$), respectively, in which the four-fold degeneracy is broken, and is divided into a double two-fold degeneracy ($p$ and $d$ types) for both cases. We can see that the bands of the $d$ type are above the $p$
type in the SC-II (Figure 2C), while the bands of the $p$ type are above the $d$ type in the SC-III. The phenomenon of band inversion arises from the increasing coupling strength between the hexagonal rods by changing the values of $\theta_0$ and $\theta$. Additionally, the pressure eigenfunctions of the dipolar ($p_x$ and $p_y$) and quadrupolar ($d_{xy}$ and $d_{x^2-y^2}$) modes which

**FIGURE 2** | Dispersion relations of the units (A) A and (B) B in the SC-I ($\theta_0 = 0^\circ$ and $\theta = 0^\circ$), and of the unit B in the (C) SC-II ($\theta_0 = 30^\circ$ and $\theta = 0^\circ$) and (D) SC-III ($\theta_0 = 0^\circ$ and $\theta = 30^\circ$). Black dots in Figures 2A,B represent the Dirac points. Eight insets in Figures 2C,D are the simulated pressure eigenfunctions of the pseudospin dipolar ($p_x$ and $p_y$) and quadrupolar modes ($d_{xy}$ and $d_{x^2-y^2}$) at the BZ center (red and blue dots) of the SC-II and SC-III, respectively.

**FIGURE 3** | (A) Dispersion relations of the ATI composed of the SC-II and SC-III. (B) Simulated pressure eigenfunctions ($M_\uparrow$ and $M_\downarrow$ modes) of the supercell at the point A ($k_z = -0.4$). (C) Simulated acoustic energy flow of the pseudospin-dependent edge states at the points A and B ($k_z = 0.4$). Red dashed lines refer to the domain wall of the ATI.
correspond to the blue and red dots are displayed in eight insets of Figures 2C,D, further showing a typical characteristic of the band inversion. Additionally, we also calculated the Chern numbers of SC-II and SC-III based on the Wilson-loop method (shown in the supplementary material), which further verify that both SCs have different topological phases. Here, it is worth mentioning that the pseudospin-1/2 modes of sound are the basis for imitating quantum spin Hall effect in acoustics, which can be realized by hybridizing the modes $M_1$ and $M_2$ as the forms of $M_1+iM_2$ and $M_1-iM_2$, respectively. To further exhibit the pseudospin-dependent characteristics, we simulate the distributions of energy flow around the domain wall of a supercell at the points A and B (shown in Figure 3C). We find that there exist energy vortices around the domain wall, and the chirality of the vortex is clockwise at the point A, but is anticlockwise at the point B, which corresponds to the pseudospin− (red circular arrow) and pseudospin+ (blue circular arrow), respectively.

Next, we simulate the pseudospin-dependent one-way propagation of sound in the ATI composed of the SC-II and SC-III, in which the intensity distributions excited by two types of chiral sources (green stars) with the opposite chirality at 16.0 kHz are shown in Figure 4. The chiral sources are realized by six point sound sources arranged in a circular array with different initial phase delays, which are placed at the center of the domain wall. As shown in Figure 4A, when the chirality of the source is anticlockwise, the edge state propagates toward the left side along the domain wall with almost negligible backscattering. In addition, it is found from the zoom of R1 that, the chirality of the energy vortex remains the same during its propagation along the domain wall, which further demonstrates the one-way propagation of the pseudospin + edge state. However, when the chirality of the source is clockwise, the pseudospin edge state is excited and propagates toward the right side. Therefore, we demonstrate the one-way sound propagation of the pseudospin-dependent edge states.

To demonstrate the robustness feature of the pseudospin-dependent edge states, we separately introduce two kinds of defects (a lattice disorder and a bend) around the domain wall of the ATI. Figure 5A shows the photograph of the sample without the defects. The description of experimental set-up is displayed in the supplementary material. Figures 5B–D show the simulated intensity distributions in the ATIs without defects and with the defects of a disorder and a bend, respectively, in which a point source is placed at the left side of the domain wall. Meanwhile, the measured intensity distributions in the scanning region (green open rectangle) are shown at the right side. We find that the smooth propagations of the pseudospin edge states still exist with both types of defects (Figures 5C,D), and the corresponding intensity distributions are almost the same as those in Figure 5B. The experimental results agree well with the simulations. Beyond that, in the zooms of R3–R5 (Figures 5B–D), the chirality of edge states is not affected by both defects, showing high robustness of the pseudospin-dependent edge states.

Finally, to demonstrate the topologically protected sound propagation, we also experimentally measure the transmission spectra for the three types of ATIs in Figures 5B–D, which is shown in Figure 6. Note that the transmittance spectra for the three cases are almost the same, especially in the overlapped bulk band gap (black shaded region). Therefore, we further experimentally demonstrate the robustness of the pseudospin-dependent edge states.
CONCLUSIONS

In conclusion, we have demonstrated a pseudospin-dependent ATI composed of two SCs with the same regular hexagonal rods. The results show that, by using the zone folding mechanism, the double Dirac cones with a four-fold deterministic degeneracy are observed in the SC, and the band inversion is realized by rotating the rods. Moreover, the ATI composed of two SCs with different rotation angles of the rods is designed, and the one-way sound propagation of the pseudospin-dependent edge states excited by the chiral sound sources in the ATI is observed. Furthermore, by measuring the transmission spectra in the ATIs with and without defects, the robustness of sound propagation against two types of defects (a lattice disorder and a bend) is demonstrated. The simulated and measured results agree well with each other. Our work opens a new venue to design tunable pseudospin-dependent ATIs with various functionalities and applications.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.
AUTHOR CONTRIBUTIONS
All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

FUNDING
This work was supported by the National Natural Science Foundation of China (Grant Nos. 11774137, 51779107 and 12174159), the China Postdoctoral Science Foundation (Grant No. 2020M671351), and the Postdoctoral Research Funding Program of Jiangsu Province (Grant No. 2021K567C).

SUPPLEMENTARY MATERIAL
The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/phy2.2021.762567/full#supplementary-material
Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher’s Note: All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

Copyright © 2021 Jia, Gu, Jiang, Ge, Yuan and Sun. This is an open-access article distributed under the terms of the Creative Commons Attribution License (CC BY). The use, distribution or reproduction in other forums is permitted, provided the original author(s) and the copyright owner(s) are credited and that the original publication in this journal is cited, in accordance with accepted academic practice. No use, distribution or reproduction is permitted which does not comply with these terms.