Equivalent model of a dually-fed machine for electric drive control systems

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Abstract. The article shows that the mathematical model of a dually-fed machine is complicated because of the presence of a controlled voltage source in the rotor circuit. As a method of obtaining a mathematical model, the method of a generalized two-phase electric machine is applied and a rotating orthogonal coordinate system is chosen that is associated with the representing vector of a stator current. In the chosen coordinate system in the operator form the differential equations of electric equilibrium for the windings of the generalized machine (the Kirchhoff equation) are written together with the expression for the moment, which determines the electromechanical energy transformation in the machine. Equations are transformed so that they connect the currents of the windings, that determine the moment of the machine, and the voltages on these windings. The structural diagram of the machine is assigned to the written equations. Based on the written equations and accepted assumptions, expressions were obtained for the balancing the EMF of windings, and on the basis of these expressions an equivalent mathematical model of a dually-fed machine is proposed, convenient for use in electric drive control systems.

1. Introduction
A dually-fed machine (DFM) is generally referred to as a connection circuit of asynchronous motor with a phase rotor in which a voltage from the external source is applied to the rotor, with the possibility of adjusting the amplitude, frequency, and phase of this voltage [1]. The use of a DFM has significant advantages over the use of an asynchronous motor with a phase rotor and a rotor station, since it allows the flow of slip energy to be controlled and high energy efficiency to be achieved. To control the DFM in the electric drive control systems, it is necessary to build its mathematical model. The task of this paper is to obtain a simple DFM model suitable for the above purposes.

Mathematical description of the dually-fed machine is very complicated because the machine moment is a non-linear function of the winding currents of the stator and rotor, so its identification is difficult. Besides, due to the presence of additional voltage source in the rotor, methods of mathematical modeling that are well suited to describe the asynchronous motor with a voltage source only in the stator are inconvenient for describing the DFM.

2. Statement of the problem
In [1] and [2], a DFM simulation using the method of an orthogonal coordinate system oriented along the representing vectors of machine coordinate was proposed. To simplify the mathematical model the authors chose a coordinate system associated with the representing vector of the stator current. In such a coordinate system, the model of electrical and electromechanical transformations in the DFM is written as a system of equations (1).
\[
\begin{align*}
    i_{Sg} &= \frac{1}{R_g} \left( u_{Sg} + pL_\mu i_{Rg} + \omega_S L_{\mu0} i_{Rg} \right), \\
    i_{Rg} &= \frac{1}{R_g} \left( u_{Rg} - pL_\mu i_{Sg} + \omega_S L_{\mu0} i_{Sg} \right), \\
    i_{Ri} &= \frac{1}{R_g} \left( u_{Ri} + \omega_S sL_\mu i_{Rg} - \omega_S sL_{\mu0} i_{Sg} \right), \\
    M &= \frac{3}{2} z_p L_\mu i_{Sg} i_{Ri}, \\
    M - M_c &= J_z \frac{d\omega}{dt},
\end{align*}
\]

where

- \(i_{Sg}\) – projection of the representing vector of stator current on the axis \(g\) of the rotating orthogonal coordinate system \(g-i\),
- \(i_{Rg}, i_{Ri}\) – projections of the representing vector of rotor current on the axis \(g\) and \(i\) of the rotating orthogonal coordinate system \(g-i\), respectively;
- \(u_{Sg}\) – projection of the representing vector of stator voltage on the axis \(g\) of the rotating orthogonal coordinate system \(g-i\), respectively;
- \(u_{Rg}, u_{Ri}\) – projections of the representing vector of stator voltage on the axis \(g\) and \(i\) of the rotating orthogonal coordinate system \(g-i\), respectively;
- \(T_S, T_R\) – electromagnetic constants of stator and rotor time, respectively;
- \(R_S, R_R\) – active resistance of the stator and rotor, respectively;
- \(L_\mu\) – magnetizing inductance of the machine;
- \(z_p\) – number of pole pairs of the machine;
- \(M, M_c\) – electromagnetic moment of the machine and the moment of loading, respectively;
- \(J_z\) – total moment of inertia of the working mechanism brought to the rotor of the machine;
- \(\omega\) – angular speed of rotation of the machine rotor;
- \(\omega_g\) – angular velocity of rotation of the coordinate system, for the selected system determined by

\[
    \arccos \left( \frac{u_{Sg}}{u_{S_{\text{max}}}} \right) = f_c \cdot \arccos \left( \frac{u_{Sg}}{u_{S_{\text{max}}}} \right) = f_{\omega_g} \left( u_{Sg} \right),
\]

the expression \(\omega_g = \frac{\arccos \left( \frac{u_{Sg}}{u_{S_{\text{max}}}} \right)}{T}\) = \(f_c \cdot \arccos \left( \frac{u_{Sg}}{u_{S_{\text{max}}}} \right) = f_{\omega_g} \left( u_{Sg} \right)\), where \(f_c\) – the voltage frequency on the stator, \(u_{S_{\text{max}}}\) – the voltage amplitude on the stator;

- \(s\) – relative frequency of the emf of the rotor, determined by the expression

\[
    s = \frac{\omega_S - \omega}{\omega_S} = \frac{E_S - E_R}{E_S},
\]

where \(E_S = k_E \omega_S\) is the emf of the stator, \(E_R = k_E \omega - \text{emf of the rotor},\)

- \(k_E\) – coefficient of internal feedback on the motor emf;
- \(p\) – differentiation operator.

The system of equations (1), taking into account the indicated notations, can be associated with the DFM structural diagram, shown in figure 1.
Figure 1. Structural diagram of the dually-fed machine in asynchronous mode in the orthogonal coordinate system associated with the stator current representing vector.

The DFM structural diagram shown in figure 1, although has simple transfer functions for windings, has a drawback – the presence of cross feedbacks, the corresponding emf, induced by the windings of opposite phases of the stator and rotor. In [1] it is proved that the equations of electric equilibrium of induction motor along the axes of coordinate system can be written in the operator form in the following form:

\[ u = \frac{1}{k_e} \left( T_e p + 1 \right) \cdot y + f_c \left( \tilde{i}_s, \tilde{i}_r, \tilde{\psi}_s, \tilde{\psi}_r, f_r \right), \]  

(2)

where \( u \) – projection of the generalized vector of stator or rotor voltage on the corresponding axis of orthogonal coordinate system,

\( T_e \) – equivalent time constant of the machine;

\( k_e \) – equivalent rate of gain of the machine;

\( i \) – projection of the generalized stator or rotor current vector on the corresponding axis of the orthogonal coordinate system;

\( f_c \left( \tilde{i}_s, \tilde{i}_r, \tilde{\psi}_s, \tilde{\psi}_r, f_r \right) \) – compensation function of mutual influence of phase windings of the stator and rotor, depending on the selected coordinate system.
Provided that the stator and rotor phases of the machine are symmetrical and the currents and voltages in the windings are sinusoidal, we can assume that these emfs are small in magnitude compared to the emfs induced by the windings stator phases in the windings of the same phases of the rotor and vice versa, and are balanced by the emf of self-induction, appearing in the respective windings. The compensation function in this case is determined by balancing emfs, which makes it possible to simplify the proposed mathematical model of the DFM and to obtain a simpler equivalent model of the machine.

Balancing emf in the chosen coordinate system in accordance with the above considerations, the equations of the system (1) and the scheme shown in figure 1, is determined by the expressions (3):

$$\Sigma e_{RgSg} = L_\mu \frac{di_{Rg}}{dt}, \Sigma e_{RiSg} = L_\mu \omega_g i_{Ri}, \Sigma e_{SgSg} = L_\mu \frac{di_{Sg}}{dt},$$
$$\Sigma e_{RiRi} = \omega_g sL_R i_{Ri}, \Sigma e_{RgRi} = \omega_g sL_{pRg}, \Sigma e_{SgRi} = \omega_g sL_i i_{Sg},$$

where $\Sigma e$ is the balancing emf;
index $S$ – stator winding;
index $R$ – winding of the rotor;
index $g$ – winding on the axis $g$ of the two-phase DFM model;
index $i$ – winding on the $i$-axis of the two-phase DFM model.

The DFM structural scheme, taking into account the balancing emf (3), is shown in figure 2.

![DFM structural diagram with the balancing of emf](image_url)
Taking into account the accepted assumptions, expressions (3) and signs of emf, the following relations are valid for the balancing ones:

\[
\begin{align*}
\Sigma e_{RgSg} \approx -e_{RgSg}, \quad \Sigma e_{RiSg} \approx -e_{RiSg}, \quad \Sigma e_{SgRg} \approx -e_{SgRg}, \\
\Sigma e_{RiRs} \approx -e_{RiRs}, \quad \Sigma e_{RgRs} \approx -e_{RgRs}, \quad \Sigma e_{SgRs} \approx -e_{SgRs}.
\end{align*}
\] (4)

In the structure diagrams shown in figures 1 and 2, the controlling signals (forming the moment of the machine) are \(u_{Sg}\) and \(u_{Ri}\). In the DFM, as a rule, the voltage at the stator \(u_S\), and, consequently, its projection \(u_{Sg}\) are constant [1]. Let us consider the diagram shown in figure 2 with respect to the control action \(u_{Ri}\) under constant control action \(u_{Sg}\). The DFM structure diagram, shown in figure 2, with the constant voltage at the stator and observing condition (4) takes the form shown in figure 3.

![Figure 3. Equivalent structure diagram of a dually-fed machine.](image)

In accordance with the diagram shown in figure 3 the transfer function for the DFM speed in the coordinate system associated with the stator representing vector at balancing emf and the constant voltage on the stator has the form:

\[
\frac{\omega(p)}{u_{Ri}(p)} = \frac{3U_{Sg}z_{p}L_{\mu}}{2JR_{R}R_{S}p^{2} + 2JR_{R}R_{S}p + 3\frac{\omega_{S}}{R_{S}\omega_{0}}U_{Sg}^{2}z_{p}L_{\mu}^{2}} = \frac{1}{2JR_{R}R_{S}T_{B}} \left( \frac{U_{Sg}L_{\mu}\omega_{S}}{R_{S}\omega_{0}} \right) p^{2} + \frac{2JR_{R}R_{S}}{3\frac{\omega_{S}}{R_{S}\omega_{0}}U_{Sg}^{2}z_{p}L_{\mu}^{2}} p + 1.
\]

By introducing the DFM mechanical time constant \(T_{M} = \frac{2JR_{R}R_{S}}{3\frac{\omega_{S}}{R_{S}\omega_{0}}U_{Sg}^{2}z_{p}L_{\mu}^{2}}\),
and the equivalent DFM transfer factor 
\[ k_E = \frac{1}{\left( \frac{U_s L_2 \omega_s}{R_s \omega_0} \right)} \], we get

\[ \frac{\omega(p)}{u_R(p)} = \frac{k_e}{T_M T_R p^2 + T_M p + 1}. \]  \hspace{1cm} (5)

The transfer function of the moment with respect to the rotor voltage, taking into account the above notations, has the form:

\[ \frac{M(p)}{u_R(p)} = \frac{T_M / J (T_R p + 1)}{T_M T_R p^2 + T_M p + 1}. \]  \hspace{1cm} (6)

It follows from expressions (5) and (6) that, with the assumptions made, the transfer functions for the velocity and electromagnetic moment of the machine have a simple form convenient for use in the electric drive control systems.

3. Conclusion

Thus, the application of the rotating coordinate system associated with one of the machine coordinates, as well as the balancing of the mutual influence of the windings emf of the motor rotor and stator like phases, make it possible to simplify considerably the mathematical model of the dually-fed machine and bring it closer to the mathematical model of the DC motor with an independent excitation. This allows the operation of the machine to be analyzed and its active rotor current and electromagnetic moment to be easily identified, which is important for practical purposes.

References

[1] Onischenko G B and Lokteva I L 1979 Asynchronous Valve Cascades and Dually fed Motors (Moscow: Energia) p 200

[2] Ostrovlyanchik V Yu and Popolzin I Yu 2016 Proc. of Int. Sci. Conf. in Knowledge-based Technologies of Development and Use of Mineral Resources (Novokuznetsk: SibSIU) vol 2 pp 303–309