Toward Understanding the Origin of Asteroid Geometries: Variety in Shapes Produced by Equal-Mass Impacts

K. Sugiura, H. Kobayashi, and S. Inutsuka

Department of Physics, Nagoya University, Aichi 464-8602, Japan e-mail: sugiura.keisuke@nagoya-u.ac.jp

1. Introduction

Planets are formed in protoplanetary disks around protostars through collisional coalescence of planetesimals [Safronov 1969, Hayashi et al. 1985]. The growth mode of planetesimals is considered as “runaway”, that is, larger planetesimals grow more rapidly than smaller ones (e.g., Greenberg et al. 1978, Wetherill & Stewart 1989, Kokubo & Ida 1996). The runaway growth produces a bimodal mass distribution of bodies composed of proto-planets and remnant planetesimals of mass around the onset of runaway growth (Kobayashi et al. 2016). Main-belt asteroids located between the orbits of Mars and Jupiter may be remnants of planetesimals (e.g., Petit et al. 2001, Bottke et al. 2005). A large number of asteroids (more than one hundred thousand asteroids for those with diameters > 1 km) allows statistical discussion to reveal the history of the solar system.

Asteroids have variety of shapes. Recent in-situ observations by spacecraft and light curve observations by ground-based telescopes reveal shapes of about 1,000 asteroids, which are summarized in Database of Asteroid Models from Inversion Techniques (DAMIT; Durouch et al. 2010). According to the database and other observations, shapes of many asteroids smaller than 100 km are distinctly different from planet shapes, which are almost spheres (Fujisawa et al. 2006, Durech et al. 2010, Marchis et al. 2014, Cibulkova et al. 2016). Some asteroids with diameters larger than 100 km also have irregular shapes. For example, (624) Hektor has a very elongated shape with the intermediate axis length of 195 km and the major axis length of 370 km (Storrs et al. 1999).

Irregular shapes of asteroids may be formed through collisional destruction of planetesimals. Irregular-shape formation of rubble piles through collisional destruction of planetesimals and gravitational reaccumulation is investigated using Smoothed Particle Hydrodynamics (SPH) method or N-body code with models of material strength. Some impact simulations reproduce formation of elongated shapes like (25143) Itokawa or bilobed shapes like (67P) Churyumov-Gerasimenko (Michel & Richardson 2013, Jutzi & Asphaug 2015, Jutzi & Benz 2017).

Shapes of objects formed through collisional destruction or coalescence depend on impact conditions (e.g., Jutzi & Asphaug 2015). For example, collisions with equal-mass and low-velocity (50 – 400 m/s) impacts. We clarify a range of the impact angle and velocity to form each shape. Our results indicate that irregular shapes, especially flat shapes, of asteroids with diameters larger than 10 km are likely to be formed through similar-mass and low-velocity impacts, which are likely to occur in primordial environment prior to the formation of Jupiter.
2. Method

2.1. SPH method

To investigate planetesimal collisions, we use SPH method for elastic dynamics (Lubersky & Petschek [1991]). SPH method is a computational fluid dynamics method utilizing Lagrangian particles (Monaghan [1992]). In a framework of SPH method, we represent continuum material such as rock using a swarm of particles. Motion of each particle is described by the equation of motion. Each particle has physical quantities such as density and internal energy, and these physical quantities are calculated from time evolution equations such as the equation of energy.

In order to treat elastic bodies by the SPH method we use following forms of basic equation:

\[
\begin{align*}
\frac{dp_i}{dt} &= -\sum_j \frac{m_j}{\rho_j} (v^j_i - v_i) \frac{\partial}{\partial x^i} W(|x_i - x_j|, h), \\
\frac{du_i^\alpha}{dt} &= \sum_j \frac{1}{2} \left[ \frac{m_j}{\rho_j} (v^j_i - v_i)^2 + \Pi_j \delta^\alpha_\beta \right] \frac{\partial}{\partial x^i} W(|x_i - x_j|, h) + \sum_j g^j_i, \\
\frac{du_i^\beta}{dt} &= -\frac{1}{2} \sum_j \frac{m_j}{\rho_j} (v^j_i - v_i)^2 \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} \left[ (v^j_i - v^\gamma_i) \frac{\partial}{\partial x^\gamma} + (v^\gamma_i - v^j_i) \frac{\partial}{\partial x^\gamma} \right] W(|x_i - x_j|, h).
\end{align*}
\]

Here, \( m_i \) is mass of the \( i \)-th SPH particle, \( \rho_i \) is its density, \( v_i \) is its velocity vector, \( x_i \) is its position vector, \( h \) is a smoothing length, \( \sigma^\alpha_\beta \) is its stress tensor, \( u_i \) is its specific internal energy, \( p_i \) is its pressure, \( S^\alpha_\beta \) is its deviatoric stress tensor, \( \delta^\alpha_\beta \) is the Kronecker delta, \( \Pi_j \) is a artificial viscosity, and \( g_j \) is the gravity between the \( i \)-th and \( j \)-th particles. A superscript of Greek letter means a direction or component of a vector or tensor, and a subscript of Roman letter means the particle number. We also assume a summation rule over repeated indices of Greek letter. Using the pressure \( p_i \) and the deviatoric stress tensor \( S^\alpha_\beta \), the stress tensor \( \sigma^\alpha_\beta \) is represented as

\[
\sigma^\alpha_\beta = -p \delta^\alpha_\beta + S^\alpha_\beta.
\]

For the kernel function, we use a Gaussian kernel given by

\[
W(r, h) = \left[ \frac{1}{h} \frac{1}{\sqrt{2\pi}} \right]^3 \exp \left( -\frac{r^2}{h^2} \right).
\]

We set the smoothing length to be constant because of insignificant density variation. The smoothing length is determined by

\[
\Pi_{ij} = \begin{cases} 
\frac{-\alpha_{vis}(C_i^\alpha + C_j^\beta)/2 \beta_{vis}}{\rho_i \rho_j} & (v_i - v_j) \cdot (x_i - x_j) < 0, \\
0 & (v_i - v_j) \cdot (x_i - x_j) > 0.
\end{cases}
\]

Here, \( \alpha_{vis} \) and \( \beta_{vis} \) are parameters for the artificial viscosity. We adopt \( \alpha_{vis} = 1.0 \) and \( \beta_{vis} = 2.0 \). According to the kernel function, the gravity between the \( i \)-th and \( j \)-th particles is calculated as

\[
g_{ij} = -G \frac{x_i - x_j}{|x_i - x_j|^3}.
\]

Initial average particle spacing. The artificial viscosity is represented as

\[
\frac{\partial \epsilon_{\alpha\beta}^0}{\partial t} = 2\mu (\epsilon_{\alpha\beta}^0 - \frac{1}{3} f_{ij} \delta_{\alpha\beta}) + S_{ij} R_{ij} + S_{ij} R_{ij},
\]

where \( \mu \) is the shear modulus, \( \epsilon_{\alpha\beta}^0 \) and \( R_{ij} \) are a strain rate tensor and a rotational rate tensor respectively, and represented as

\[
\epsilon_{\alpha\beta}^0 = \frac{1}{2} \left( \frac{\partial u^\alpha_i}{\partial x^\beta_j} + \frac{\partial u^\beta_j}{\partial x^\alpha_i} \right),
\]

\[
R_{ij} = \frac{1}{2} \left( \frac{\partial u^\alpha_i}{\partial x^\beta_j} - \frac{\partial u^\beta_j}{\partial x^\alpha_i} \right).
\]

Note that \( \epsilon_{\alpha\beta}^0 \) and \( R_{ij} \) are described by sums of velocity gradients. To treat rigid body rotation correctly, we adopt equations of velocity gradients with the correction matrix \( \Pi_i \), developed by Bonet & Lok ([1999]).

For time integration, we use a leapfrog method with a kick-drift-kick scheme. Here, we use leapfrog equations with a form where the position and other physical quantities such as the velocity are both updated at the end of each step (e.g., Hubber et al. [2013]). Detailed description for the time integration scheme is given in Appendix A.

For fast calculation of the time evolution equations (1), (2) and (3), we parallelize our simulation code using Framework for Developing Particle Simulator (FDPS; Iwasawa et al. [2015, 2016]).
2.2. Models for the fracture and friction

To treat the collisional destruction of rocky material, we apply appropriate models for the fracture of rock and the friction between completely damaged material. 

Benz & Asphaug (1995) introduce a fracture model based on the model for brittle solid (Grady & Kipp 1980) to SPH method. In this model, we introduce a damage parameter $D$. Each SPH particle has this state variable $D$. SPH particles with $D = 0$ represent intact rock, and those with $D = 1$ represent completely damaged rock, which means that these SPH particles do not feel any tensile stress. The damage parameter increases according to the function modeled by Benz & Asphaug (1999) if local strain exceeds flaw’s activation threshold. Flaw’s activation threshold is determined by material dependent parameters and total volume of rock. For these parameters we also use values for basalt described in Benz & Asphaug (1999).

According to the fracture model, we modify the pressure and use damage relieved pressure $p_{di}$:

$$p_{di} = \begin{cases} (1-D)p_i & p_i < 0 \\ p_i & p_i > 0 \end{cases}$$

for Eqs. (2) and (3).

We treat the friction of damaged rock ($D > 0$) according to Jutzi (2015). For collisions of our interest, the energy dissipation by the friction of partially damaged rock ($0 < D < 1$) is much smaller than that of completely damaged rock ($D = 1$). Therefore, we only explain the treatment for the friction of completely damaged rock.

To represent the friction of granular materials, we set yielding strength $Y_{dj}$ as:

$$Y_{dj} = \mu_d p_{di},$$

where $\mu_d$ is the friction coefficient. Here we assume $\mu_d = \tan(40^\circ) = 0.839$, which corresponds to a material with the angle of repose of $40^\circ$. Note that the angle of repose of lunar sand is estimated to be $30^\circ - 50^\circ$ (e.g., Heiken et al. 1991). Using the yielding strength of Eq. (13), we modify the deviatoric stress tensor as:

$$S_{ij}^{\alpha\beta} \rightarrow f_i S_{ij}^{\alpha\beta},$$

$$f_i = \min[Y_{dj}, \sqrt{J_{2j}}, 1],$$

$$J_{2j} = \frac{1}{2} S_{ij} S_{ij}^{\alpha\beta}.$$  \hspace{1cm} (14)

Owing to this friction model, the formation of irregular shapes of rubble piles are reproduced.

3. Initial conditions of impacts and analysis of results

3.1. Initial conditions of impacts

For simplicity, we use a sphere of basalt with zero rotation as an initial planetesimal. The radius of planetesimals is set to $R_t = 50$ km, and we focus on collisions between two equal-mass planetesimals with mass of $M_{\text{target}} = 4\pi\rho_0 R_t^3/3$, where $\rho_0$ is the uncompressed density of basalt. To enable detailed investigation of the dependence of results on two impact parameters, the total number of SPH particles for a simulation is set to 100,000, i.e., relatively rough resolution. Validity of this number of SPH particles is discussed in Section 4.1.

For a basaltic planetesimal with the radius of 50 km, under the hydrostatic equilibrium condition central density is almost the same as uncompressed density. Thus we use a uniform sphere with the mean density of basalt as initial planetesimals. An isotropic SPH particle distribution is more preferable, for example, than particles placed on cubic lattices, so that we prepare a particle distribution with uniform disposition from a random distribution. Detailed procedures to produce the uniform and isotropic particle distribution are as follows: Firstly we put SPH particles within a cubic domain with periodic boundary conditions randomly so that desired resolution and desired mean density are achieved. Secondly we let the particles move under forces that make the particle distribution uniform until the standard deviation of density becomes less than 0.1% of the mean density. These forces are anti-parallel to density gradients, and become 0 for uniform particle distributions. Finally, we remove particles outside of a shell with the radius of 50 km, and then a uniform and isotropic sphere is obtained.

![Fig. 1. Impact geometry and the definition of the impact velocity $v_{\text{imp}}$ and angle $\theta_{\text{imp}}$.](image)

We define the impact velocity $v_{\text{imp}}$ as the relative velocity between two planetesimals at the time of impact, and the impact angle $\theta_{\text{imp}}$ as the angle between the line joining centers of two planetesimals and the relative velocity vector at the time of impact. Thus the impact angle of $0^\circ$ means a head-on collision, and that of $90^\circ$ means a grazing collision. This definition is also adopted by Genda et al. (2012). Fig. 1 schematically shows the definition of the impact velocity and angle. At the beginning of numerical simulations, centers of two planetesimals are apart at a distance of $4R_t$.

3.2. Analysis of results

We conduct numerical simulations of impacts and subsequent gravitational reaccumulation over a period of $1.0 \times 10^5$ s. The typical timescale of reaccumulation is estimated as $t_{\text{acc}} = 2R_t/v_{\text{esc}}$, where $v_{\text{esc}}$ is the two-body escape velocity of planetesimals. The value of $t_{\text{acc}}$ is calculated as:

$$t_{\text{acc}} = \frac{2R_t}{v_{\text{esc}}} = \frac{2R_t}{\sqrt{2GM_{\text{target}}/R_t}} = \sqrt{\frac{3}{2\pi G\rho_0}} \approx 1600 \text{ s}.$$  \hspace{1cm} (15)
Thus 1.0 × 10^5 s is about 100 times longer than the typical timescale of reaccumulation, and we also confirmed that after 1.0 × 10^6 s gravitational reaccumulation is sufficiently finished.

After collisional simulations, we identify the largest remnants using a friends-of-friends algorithm. We find a swarm of SPH particles with spacing less than 1.5 times the largest remnant. Then we measure shapes of the largest remnants. To do so, we quantitatively measure axis lengths of the largest remnants using the inertia moment tensor. We approximate the largest remnant as an ellipsoid that has the same inertia moment tensor, and then we identify axis lengths of that ellipsoid with those of the largest remnant.

The inertia moment tensor of the largest remnant composed of k SPH particles is calculated as

\[ \sum_k m_k \left[ (x_k^y - x_{\text{CoM}}^y) (x_k^y - x_{\text{CoM}}^y) \delta^{\phi \phi} - (x_k^\alpha - x_{\text{CoM}}^\alpha) (x_k^\alpha - x_{\text{CoM}}^\alpha) \right], \]

**(16)**

where \( x_{\text{CoM}} \) is the position vector at the center of mass of the largest remnant. Then, three principal moments of inertia \( I_1, I_2, \) and \( I_3 \) are obtained from \( I^{\phi \phi} \). Here, \( I_1 > I_2 > I_3 \). For a uniform ellipsoid with the length of major axis \( a \), intermediate axis \( b \), and minor axis \( c \), the three principal moments of inertia are represented as

\[ I_1 = \frac{1}{20} (a^2 + b^2) M_k, \]

\[ I_2 = \frac{1}{20} (a^2 + c^2) M_k, \]

\[ I_3 = \frac{1}{20} (b^2 + c^2) M_k, \]

**(17)**

where \( M_k \) is the mass of the largest remnant. Eq. [17] is rewritten as

\[ a = \sqrt{\frac{10 (I_1 + I_2 - I_3)}{M_k}}, \]

\[ b = \sqrt{\frac{10 (I_1 - I_2 + I_3)}{M_k}}, \]

\[ c = \sqrt{\frac{10 (-I_1 + I_2 + I_3)}{M_k}}. \]

**(18)**

Therefore, we obtain \( I_1, I_2, I_3, \) and \( M_k \) of the largest remnant through a simulation, and then derive its \( a, b, \) and \( c \) from Eq. [18]. Shapes of objects are characterized by ratios between the major, intermediate, and minor axis lengths, i.e., \( b/a \) and \( c/a \) (\( b/a \) and \( c/a \) are 0 – 1 and \( b/a > c/a \) by definition). Bodies with \( c/a \approx 1 \) have almost spherical shapes. For \( b/a < 1 \) and \( c/a < 1 \), bodies have flat shapes. Bodies with \( b/a \ll 1 \) have elongated shapes.

**4. Results**

**4.1. Resolution dependence on the resultant shape**

Figure 2 represents snapshots of the SPH simulation with the impact angle \( \theta_{\text{imp}} \) of 15° and the impact velocity \( v_{\text{imp}} \) of 200 m/s. In Fig. 2, the collision induces shattering of planetesimals. Then two planetesimals are stretched in the direction perpendicular to the line joining centers of two contacting planetesimals and fragments are ejected (Fig. 2). Ejected materials are mainly reaccumulated from the direction of the long axis of the largest remnant (Fig. 2). Finally a very elongated shape with the ratio \( b/a \) of about 0.2 is formed (Fig. 2). The accretion on the largest body is mostly done within \( t \approx 5.0 \times 10^4 \) s.

**Figure 2.** Snapshots of the impact simulation with the impact angle \( \theta_{\text{imp}} \) of 15°, the impact velocity \( v_{\text{imp}} \) of 200 m/s and the total number of SPH particles \( N_{\text{total}} = 1 \times 10^4 \) (a, \( 2.0 \times 10^4 \) s(b), \( 6.0 \times 10^4 \) s(c), \( 2.0 \times 10^4 \) s(d), \( 3.0 \times 10^4 \) s(e), and \( 5.0 \times 10^4 \) s(f). Scale on Panel (a) is also valid for all Panels (b)-(f).

**Figure 3.** Shapes of the largest remnants at \( 5.0 \times 10^4 \) s for the impact simulations with \( \theta_{\text{imp}} = 15°, \) \( v_{\text{imp}} = 200 \) m/s, and \( N_{\text{total}} = 5 \times 10^3 \) (a), \( 2 \times 10^3 \) (b), and \( 8 \times 10^3 \) (c), respectively.

Figure 3 shows shapes of the largest remnants at \( 5.0 \times 10^4 \) s with three different resolutions (the total number of SPH particles \( N_{\text{total}} \) of \( 5 \times 10^3 \) (a), \( 2 \times 10^3 \) (b), and \( 8 \times 10^3 \) (c)). Even if \( N_{\text{total}} \) becomes ten times larger, the characteristic of elongated shape does not change. Figure 4 shows the dependence of the mass and axis ratios of the largest remnants on the number of SPH particles \( N_{\text{total}} \). The mass of the largest remnants slightly decreases with increasing \( N_{\text{total}} \), because numerical dissipation by the artificial viscosity decreases for higher resolution. This tendency is the same as the result of Genda et al. (2015). The axis ratios slightly increase with increasing \( N_{\text{total}} \), and the difference of \( b/a \) between \( N_{\text{total}} \) of \( 5 \times 10^4 \) and \( 8 \times 10^5 \) is about 0.03. Difference of axis ratios less than 0.1 is unimportant for the analysis of asteroidal shapes because difference of axis measurements also causes such minor errors. Therefore, the number of SPH particles of \( 10^4 \) is sufficient to capture at least the feature of shapes.

**4.2. Mass of the largest remnants**

Hereafter, we use \( 10^4 \) SPH particles for a simulation, and we measure the mass and axis ratios of the largest remnants at \( 1.0 \times 10^5 \) s after impacts.
Dependence of the mass and axis ratios of the largest remnants on the number of SPH particles \( N_{\text{total}} \) for the impact with \( \theta_{\text{imp}} = 15^\circ \) and \( v_{\text{imp}} = 200 \text{ m/s} \). Red solid line shows the ratio \( b/a \), green dashed line shows the ratio \( c/a \), and blue dotted line shows the mass of the largest remnants \( M_L \) normalized by the mass of an initial planetesimal \( M_{\text{target}} \). Left vertical axis shows the axis ratios, and right vertical axis shows the mass of the largest remnant.

![Dependence of the mass and axis ratios of the largest remnants on the number of SPH particles](image)

**Fig. 4.** Dependence of the mass and axis ratios of the largest remnants on the number of SPH particles \( N_{\text{total}} \) for the impact with \( \theta_{\text{imp}} = 15^\circ \) and \( v_{\text{imp}} = 200 \text{ m/s} \). Red solid line shows the ratio \( b/a \), green dashed line shows the ratio \( c/a \), and blue dotted line shows the mass of the largest remnants \( M_L \) normalized by the mass of an initial planetesimal \( M_{\text{target}} \). Left vertical axis shows the axis ratios, and right vertical axis shows the mass of the largest remnant.

For \( v_{\text{imp}} > 300 \text{ m/s} \), \( M_L \) has a minimum value at \( \theta_{\text{imp}} \approx 15^\circ \) (see Fig. 5). For head-on collisions, the collision results in an overall destruction, and the most of the impact energy is dissipated and not transformed to the ejection processes. For slightly higher \( \theta_{\text{imp}} \), the impact energy is more effectively used for ejection, and thus the mass of the largest remnant becomes smaller. However, for much higher \( \theta_{\text{imp}} \), the velocity component normal to colliding bodies is small, so that the impact energy is not effectively used for destruction. Therefore, an intermediate \( \theta_{\text{imp}} \) yields smallest \( M_L \).

We note that collisions with \( v_{\text{imp}} > 400 \text{ m/s} \) and low \( \theta_{\text{imp}} \) result in \( M_L \approx 0.1M_{\text{target}} \). The largest remnants resulting from such impacts are composed of less than about 5000 SPH particles, and resolved by less than 20 SPH particles along each axis direction. Thus axis ratios obtained from such a small number of SPH particles are not measured accurately. For \( \theta_{\text{imp}} > 45^\circ \), only edges of planetesimals are destroyed by collisions rather than overall deformation, so that the investigation of such impact angles is not interesting. Therefore, we investigate the collisions with \( 50 \text{ m/s} \leq v_{\text{imp}} \leq 400 \text{ m/s} \) and \( 5^\circ \leq \theta_{\text{imp}} \leq 45^\circ \), because in this parameter range the resolution of the largest remnants is mainly sufficient and significant shape deformation occurs.

**4.3. Characteristic shapes formed by collisions**

As a result of impact simulations with \( 50 \text{ m/s} \leq v_{\text{imp}} \leq 400 \text{ m/s} \) and \( 5^\circ \leq \theta_{\text{imp}} \leq 45^\circ \), we find that resultant shapes of the largest remnants are roughly classified into five categories. In this subsection, we introduce the results of typical impacts to form five different characteristic shapes and catastrophic collisions.

**4.3.1. Bilobed shapes**

If the impact velocity is very small, the initial spherical shapes of colliding bodies are preserved and collisional merging forms bilobed shape. Fig. 6 shows impact snapshots with \( v_{\text{imp}} = 50 \text{ m/s} \) and \( \theta_{\text{imp}} = 30^\circ \). The impact forms a bilobed shape (Fig. 6b). The two-body escape velocity \( v_{\text{esc}} \) is about 60 m/s, which is slightly larger than \( v_{\text{imp}} \). For \( v_{\text{imp}} < v_{\text{esc}} \), the impact energy is too small to largely deform the initial spherical shapes, and colliding bodies are gravitationally bound. Thus the bilobed shapes resulting from such low velocity impacts are independent of \( \theta_{\text{imp}} \).

**4.3.2. Spherical shapes**

The initial spherical shape is sufficiently deformed with \( v_{\text{imp}} \sim 100 \text{ m/s} \), which results in a single sphere due to merging of two planetesimals. Fig. 7 shows an impact producing a spherical shape with \( v_{\text{imp}} = 100 \text{ m/s} \) and \( \theta_{\text{imp}} = 10^\circ \).

It should be noted that a relatively low speed collision with \( \theta_{\text{imp}} \geq 40^\circ \) results in local destruction due to hit-and-run, whose outcome is also close to two spheres.

**4.3.3. Flat shapes**

Figure 8 shows impact snapshots with \( v_{\text{imp}} = 200 \text{ m/s} \) and \( \theta_{\text{imp}} = 5^\circ \). The initial spherical shapes are completely deformed (Fig. 8b,c), and the resultant shape is flat (Fig. 8d-f).
4.3.4. Elongated shapes

A collision forming extremely elongated shape is shown in Fig. 2. The collision results in $M_{\text{lr}} \sim 2M_{\text{target}}$; collisional merging mainly occurs.

Some hit-and-run collisions also produce elongated shapes. Fig. 9 shows snapshots of the impact with $v_{\text{imp}} = 250 \text{ m/s}$ and $\theta_{\text{imp}} = 20^\circ$, and Fig. 10 shows a zoom out view of Fig. 9. The impact results in significant destruction and deformation (Fig. 9b,c). Although two planetesimals do not merge (Fig. 10), the reaccretion of surrounding fragments produces two elongated shapes (Fig. 9d-f and Fig. 10). Note that the largest and second largest objects in hit-and-run collisions have almost the same shape (Fig. 10).

4.3.5. Hemispherical shapes

In Fig. 11 we show an impact forming a hemispherical shape. Significant destruction occurs around the impact point and a large amount of fragments is ejected straightforwardly.
This collisional truncation results in a hemispherical shape (Fig. 11f).

![Fig. 11. Snapshots of the impact simulation with $\theta_{imp} = 45^\circ$ and $v_{imp} = 350 \text{ m/s}$ at $0 \text{ s (a), } 2 \times 10^3 \text{ s (b), } 1.0 \times 10^4 \text{ s (c), } 2.0 \times 10^4 \text{ s (d), and } 1.0 \times 10^5 \text{ s (e).}]

4.3.6. Catastrophic destruction

![Fig. 12. Snapshots of the impact simulation with $\theta_{imp} = 5^\circ$ and $v_{imp} = 400 \text{ m/s}$ at $0 \text{ s (a), } 2 \times 10^3 \text{ s (b), } 2.6 \times 10^4 \text{ s (c), } 5.0 \times 10^4 \text{ s (d), } 8.0 \times 10^4 \text{ s (e), and } 1.0 \times 10^5 \text{ s (f).}]

Figure 12 represents the result of the impact simulation with $v_{imp} = 400 \text{ m/s}$ and $\theta_{imp} = 5^\circ$. The impact of very high $v_{imp}$ produces a large curtain of ejected fragments (Fig. 12b), and the gravitational fragmentation of the curtain forms many clumps (Fig. 12c). Then the largest remnant is formed through the coalescence of clumps (Fig. 12f).

In catastrophic destructions with $M_{fr} < 0.4M_{target}$, the largest bodies are formed through significant reaccretion of ejecta. Even small difference of initial conditions produces significant difference of the distribution of ejecta, which lead to variety of shapes. Therefore, high-resolution simulations are required. We will conduct simulations with much higher number of SPH particles in our future work. In this paper, we just call impacts with $M_{fr} < 0.4M_{target}$ catastrophic destruction, and do not classify shapes for such destructive impacts.

4.4. Summary of shapes formed by collisions

![Fig. 13. Dependence of the ratios $b/a$ and $c/a$ of the largest remnants on $v_{imp}$ and $\theta_{imp}$. Color represents (a) the ratio $b/a$, (b) the ratio $c/a$, respectively. For impacts in hatched region, we do not measure the axis ratios of the largest remnants, because the mass of the largest remnants is too small (smaller than 0.15 $M_{target}$). The meaning of the shaded regions is the same as in Fig. 5. Parameters surrounded by green boxes represent impacts with the second collision as shown in Appendix B.]

Table 1. Threshold for the categorization of shapes.

| Shape                | $b/a$ | $c/a$ | $M_{fr}/M_{target}$ |
|----------------------|-------|-------|---------------------|
| Bilobed              | < 0.7 | > 0.7 | 2.0                 |
| Spherical            | > 0.7 | > 0.7 | ···                 |
| Flat                 | > 0.7 | < 0.7 | 1.0                 |
| Elongated            | < 0.7 | < 0.7 | < 2.0               |
| Hemispherical        | > 0.7 | < 0.7 | 1.0                 |
| Catastrophic destruction | ···   | ···   | < 0.4               |

We categorize shapes of collisional outcomes into bilobed, spherical, flat, elongated, hemispherical, and catastrophic destruction according to Table 1. The result of classification by the threshold listed in Table 1 is in good agreement with that by eye. Fig. 13 shows the result of classification of shapes. From Fig. 13, impact parameters of collisions producing each shape are mainly as follows: $v_{imp} \sim 50 \text{ m/s}$, or $v_{imp} \sim 100 \text{ m/s}$ and $\theta_{imp} > 25^\circ$ (bilobed shapes), $v_{imp} \sim 100 \text{ m/s}$ and $\theta_{imp} < 25^\circ$ (spherical shapes), $v_{imp} > 100 \text{ m/s}$ and $\theta_{imp} < 30^\circ$ (flat shapes), $v_{imp} > 100 \text{ m/s}$ and $\sin \theta_{imp} > 30^\circ$ (hemispherical shapes), and $\theta_{imp} < 30^\circ$ (elongated shapes).
5. Discussion

5.1. Three conditions for elongated shapes

The threshold of $v_{\text{imp}} > 100 \text{ m/s}$ is required for significant deformation. We estimate necessary impact velocity to deform planetesimals as follows. Frictional force of $\mu \rho v^2$ acts on the area of $\pi R^2$ and the energy dissipation occurs due to frictional deformation on the length scale of $\sim 4R$. The dissipated energy $E_{\text{dis}}$ is estimated as

$$E_{\text{dis}} = 4\pi R^3 \mu \rho p.$$  \hspace{1cm} (21)

The timescale for deformation of whole bodies is estimated to be $2R/v_{\text{imp}}$, which is much longer than the shock passing time $\sim 2R/C_s$, where $C_s \approx 3 \text{ km/s}$ is the sound speed. High pressure states caused by shocks are already relaxed before the end of the deformation, and shocks do not contribute to the pressure for frictional force given in Eq. (21). Therefore, the pressure is mainly determined by the self-gravity and estimated to be central pressure of a planetesimal with the radius of $R$, and uniform density of $\rho_0$, given by

$$p = \frac{2}{3} G \rho_0^2 R^2.$$ \hspace{1cm} (22)

Equating the total initial kinetic energy for two equal-mass bodies $(1/4)M_{\text{target}}v_{\text{imp}}^2$ to $E_{\text{dis}}$, we obtain the critical deformation velocity as

$$v_{\text{imp, crit}} = \sqrt{\frac{4E_{\text{dis}}}{M_{\text{target}}}} = \sqrt{\frac{3}{2}} \mu_\rho v_{\text{esc}}$$
$$= 97.4 \left( \frac{R}{50 \text{ km}} \right) \left( \frac{\mu_\rho}{\tan(40^\circ)} \right)^{1/2} \left( \frac{\rho_0}{2.7 \text{ g/cm}^3} \right)^{1/2} \text{ m/s},$$ \hspace{1cm} (23)

where $2.7 \text{ g/cm}^3$ is the mean density of basalt described in Benz & Asphaug (1999). The impact velocity obtained in Eq. (23) well agrees with $v_{\text{imp}} \approx 100 \text{ m/s}$ in Eq. (19) in spite of rough estimation of the dissipated energy $E_{\text{dis}}$.

The condition of $\theta_{\text{imp}} < 30^\circ$ in Eq. (19) is needed for the avoidance of hemispherical shapes caused by hit-and-run collisions. For $\theta_{\text{imp}} \geq 30^\circ$ half or smaller of a target is directly interacted by an impactor, resulting in hit-and-run collisions (Asphaug 2010; Leinhardt & Stewart 2012). To form elongated shapes, it is necessary for almost whole volume of two planetesimals to be deformed. For $\theta_{\text{imp}} < 30^\circ$ most of two planetesimals are directly affected by collisions, which lead to deformation to be elongated shapes.

Collisional elongation requires large shear velocity $v_{\text{imp}} \sin \theta_{\text{imp}}$, which determines the condition of $v_{\text{imp}} \sin \theta_{\text{imp}} > 30 \text{ m/s}$ in Eq. (19). Impacts with $v_{\text{imp}} \sin \theta_{\text{imp}} < 30 \text{ m/s}$ produce flat shapes, while those with $v_{\text{imp}} \sin \theta_{\text{imp}} > 30 \text{ m/s}$ result in elongated shapes. Elongated shapes are formed through stretch of planetesimals in the direction of shear velocity (see Figs. 2, 19). However, self-gravity prevents deformation, which occurs if $v_{\text{imp}} \sin \theta_{\text{imp}} < v_{\text{esc}}$. We find that elongation needs $v_{\text{imp}} \sin \theta_{\text{imp}} > v_{\text{esc}}/2 \approx 30 \text{ m/s}$.

Distribution of pressure is determined by the self-gravity unless the impact velocity is comparable or larger than the sound speed. Since frictional force is proportional to the pressure, the friction is also determined by the self-gravity. Thus, unless the material strength is dominant, force on bodies (right hand side of Eq. (2)) is solely determined by the self-gravity, so that results of impacts are characterized by dimensionless velocity $v_{\text{imp}}/v_{\text{esc}}$ regardless of the scale or size of planetesimals. For rocky planetesimals with $R_s \geq 0.5 \text{ km}$, the friction is dominant rather than the material strength. For $R_s \leq 200 \text{ km}$, $v_{\text{esc}}$ is smaller than $0.1C_s$. Therefore, the conditions to form elongated shapes of Eq. (20) and the shape classification of Fig. 14 with upper horizontal axis are also valid for equal-mass impacts with the angle of repose of $40^\circ$ and $10^\circ km \leq R_s \leq 10^2 \text{ km}$. 

---

**Fig. 14.** Summary of resultant shapes and the conditions of Eq. (19). Blue squares represent the impact parameters, $v_{\text{imp}}$ and $\theta_{\text{imp}}$, producing bilobed shapes, gray triangles represent those of spherical shapes, brown inverted triangles represent those of flat shapes, red circles represent those of elongated shapes, green diamonds represent those of spherical shapes, and black crosses represent those for catastrophic destruction. Dotted line shows $v_{\text{imp}} = 100 \text{ m/s}$, dashed line shows $\theta_{\text{imp}} = 30^\circ$, chain curve shows $v_{\text{imp}} \sin \theta_{\text{imp}} = 30 \text{ m/s}$, and solid curve shows $M_\text{t}/M_{\text{target}} = 0.4$. The meanings of the shaded region and green boxes are the same as in Fig. 4 or Fig. 13.
5.2. Applications

We analyze shapes of 139 asteroids with diameters \( D > 80 \) km, which are obtained using DAMIT database (http://astro.troja.mff.cuni.cz/projects/asteroids3D/web.php).

We derive axis ratios of asteroids in DAMIT database according to experimental method (Fujisawa et al.1978). We find three elongated asteroids that have \( b/a < 0.6, D > 80 \) km; (63) Ausonia, (216) Kleopatra, and (624) Hektor. We also find two flat asteroids that have \( b/a > 0.9, c/a < 0.5, \) and \( D > 80 \) km; (419) Aurelia and (471) Papagena. Therefore, \( \sim 3\% \) of asteroids with \( D > 80 \) km have irregular shapes.

(624) Hektor is a Jupiter trojan, and the others are main-belt asteroids. In the main belt, the keplerian velocity is \( \sim 20 \) km/s and mean impact velocity is roughly estimated to be \( v_{\text{imp}} \approx \sqrt{v_{\text{esc}}^2 + v_{\text{esc} / K}} \approx 4 \) km/s with mean orbital eccentricity of \( e_{\text{esc}} = 0.15 \) and inclination of \( i_{\text{esc}} = 0.13 \) (Ueda et al.2017). This mean impact velocity is much higher than that treated in our simulations \( (v_{\text{imp}} < 400 \) m/s). We estimate distribution of impact velocities between main-belt asteroids using orbital parameters obtained from the JPL small-body Database Search Engine (https://sdss.jpl.nasa.gov/sbdb_query.cgi/sx) and the method to obtain the relative velocity at the orbital crossing according to Kobayashi & Ida (2001). This gives the mean collisional velocity of 5 km/s. The probability that impact velocities become less than 400 m/s is about 0.15%. Therefore, expected production of irregular shaped asteroids is too low to reproduce the current fraction of the asteroids.

Similar-mass impacts with the mean impact velocity in the main belt result in catastrophic destruction. Elongated, or bilobed shapes are also formed through largely destructive impacts. Many remnants are formed in large curtain of ejected fragments as shown in Fig. 12 and then these remnants may again collide with each other with \( v_{\text{imp}} \sim v_{\text{esc}} \), which leads to bilobed shapes. We additionally conduct a higher resolution simulation of a largely destructive impact, which shows that bilobed asteroids are formed. However, flat shapes are not formed in the destructive impact. Moreover, collisional lifetimes of asteroids with \( D > 10 \) km are longer than the age of the solar system, so that catastrophic collisions of those asteroids are unlikely to occur in the present main belt (O’Brien & Greenberg 2005).

For large-mass-ratio impacts, deformation occurs only in the scale of impactors much smaller than that of targets, so that overall deformation does not occur via a single collision. Although such impacts are frequent, isotropic impacts to almost spherical asteroids do not form irregular shapes.

Therefore, irregular shapes, especially flat shapes, are likely to be formed through similar-mass and low-velocity impacts. Relative velocities between planetesimals are increased by gravitational interaction with planets, especially Jupiter (e.g., Kobayashi et al.2010). Thus, collisional velocities in the main belt may be much lower prior to Jupiter formation. Jupiter formation may significantly deplete asteroids (Bottke et al.2005), and similar-mass impacts may be frequent prior to Jupiter formation. Irregular shaped asteroids are possibly formed in such environments. Therefore, irregular shaped asteroids with \( D ≥ 10 \) km may be formed in primordial environment and remain the same until today.

6. Summary

Asteroids are believed to be remnants of planetesimals formed in planet formation era and may retain information of the history of the solar system. Irregular shapes of asteroids are possible to be formed through collisional destruction and coalescence of planetesimals. Thus clarifying impact conditions to form specific shapes of asteroids leads to constrain epoch or collisional environment forming those asteroids.

Our simulations show the relationship between impact conditions and resultant shapes of planetesimals. We carry out simulations of collisions between planetesimals using SPH method for elastic dynamics with the self-gravity and the models for fracture and friction. We consider collisions between two planetesimals with the radius of 50 km, because significant shape deformation occurs in equal-mass impacts. We vary the impact velocity \( v_{\text{imp}} \) from 50 m/s to 400 m/s and the impact angle \( \theta_{\text{imp}} \) from 5 to 45°. Then we measure shape of the largest remnant formed in each impact simulation.

We confirm that various shapes are formed by equal-mass impacts. We classify shapes of the largest remnants into 5 categories if the mass of the largest remnant \( M_{\text{t}} \) is larger than 0.4 of that of an initial planetesimal \( M_{\text{t, ini}} \). The result of the shape classification is as follows:

- For \( v_{\text{imp}} < 50 \) m/s, or \( v_{\text{imp}} > 100 \) m/s and \( \theta_{\text{imp}} > 25° \), bilobed shapes are formed because of merging of planetesimals with preserving the initial spherical shapes (see Fig.9).
- For \( v_{\text{imp}} > 100 \) m/s and \( \theta_{\text{imp}} < 20° \), spherical shapes are formed because a part of planetesimal is crushed (see Fig.7).
- For \( v_{\text{imp}} > 100 \) m/s and \( v_{\text{imp}} \sin \theta_{\text{imp}} > 30 \) m/s, flat shapes are formed because of nearly head-on collisions and large deformation (see Fig.8).
- For \( v_{\text{imp}} > 100 \) m/s and \( v_{\text{imp}} \sin \theta_{\text{imp}} > 30 \) m/s, and \( \theta_{\text{imp}} > 30° \), elongated shapes are formed because planetesimals are stretched to the direction perpendicular to the line joining centers of two contacting planetesimals (see Fig.2).
- For \( v_{\text{imp}} > 100 \) m/s and \( \theta_{\text{imp}} > 30° \), hemispherical shapes are formed because of excavation of edges of planetesimals (see Fig.11).

As a result of the shape classification, we find four conditions to form elongated shapes with the ratio \( b/a \) smaller than 0.7. Those four conditions and the meaning of each condition are as follows:

- \( v_{\text{imp}} > 1.6v_{\text{esc}} \): Overall deformation of planetesimals requires large impact velocity.
- \( \theta_{\text{imp}} < 30° \): Impacts with large impact angles result in erosion of only edges of planetesimals.
- \( v_{\text{imp}} \sin \theta_{\text{imp}} > 0.5v_{\text{esc}} \): Elongated shapes are formed through stretch of planetesimals to the direction of shear velocity \( v_{\text{imp}} \sin \theta_{\text{imp}} \), so that large shear velocity is also required.
- \( M_{\text{t}} > 0.4M_{\text{t, ini}} \): In largely destructive impacts the largest remnants are formed through rigorous reaccumulation of fragments and resultant shapes tend to be spherical.

These conditions are also valid for equal-mass impacts with the angle of repose of 40° and 10° km ≤ \( R_s \) ≤ 10° km.

According to our simulations, various irregular shapes are formed through impacts with two equal-mass planetesimals and low impact velocities < 400 m/s. Impacts with the average relative velocity in the main belt = 5 km/s result in catastrophic destruction for similar-mass impacts or moderate destruction for high-mass-ratio impacts. However, both catastrophic destruction and impacts with high mass ratio are unlikely to produce flat shapes. Asteroids with diameters ≥ 10 km have longer collisional lifetime than the age of the solar system. Therefore, we suggest that irregular shapes, especially flat shapes, are likely to
be formed through similar-mass and low-velocity impacts in primordial environment prior to the formation of Jupiter, at least for asteroids with diameters $\geq 10$ km.

We only consider collisions between two rocky planetesimals with the radius of 50 km and limited impact velocity range. Thus investigations of following impacts are our future work: impacts with different radii of planetesimals, high mass ratios of colliding two planetesimals, and higher impact velocities, which form smaller fragments. Clarifying more detailed relationship between impact conditions and resultant shapes of planetesimals may allow us to extract more detailed information of the history of the solar system from shapes of asteroids.

Acknowledgements. The authors thank Hidekazu Tanaka and Ryuki Hyodo for the useful discussions and comments. We also thank Martin Jutzi and Natsuki Hosono for giving us the useful information about the numerical simulation methods. KS is supported by JSPS KAKENHI Grant Number JP 17H01703. HK and SI are supported by Grant-in-Aid for Scientific Research (17K05632, 17H01105, 17H01103, 23244027, 16H02160). Numerical simulations in this work were carried out on Cray XC30 at Center for Computational Astrophysics, National Astronomical Observatory of Japan.

References

Agnor, C. & Asphaug, E. 2004, ApJ, 613, 157
Asphaug, E. 2010, Chem. Erde/Geochem., 70, 199
Benz, W. & Asphaug, E. 1995, Comput. Phys. Commun., 87, 253
Benz, W. & Asphaug, E. 1999, Icarus, 142, 5
Bonet, J. & Lok, T.-S. L. 1999, Comput. Methods Appl. Mech. Engrg., 180, 97
Bottke, Jr., W. F., Durda, D. D., Nesvorný, D., et al. 2005, Icarus, 175, 111
Cibulková, H., Šurech, J., Vokrouhlický, D., & Oszkiewicz, D. A. 2016, A&A, 596, A57
Fujiwara, A., Kamimoto, G., & Tsukamoto, A. 1978, Nature, 272, 602
Genda, H., Fujita, T., Kobayashi, H., Tanaka, H., & Abe, Y. 2015, Icarus, 262, 58
Grady, D. E. & Kipp, M. E. 1980, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 17, 147
Hayashi, C., Nakazawa, K., & Nakagawa, Y. 1985, in Protostars and Planets II, ed. D. C. Black & M. S. Matthews, 1100–1153
Heikin, G. H., Vaniman, D. T., & French, B. M. 1991, Lunar sourcebook - A user’s guide to the moon
Hubber, D. A., Allison, R. J., Smith, R., & Goodwin, S. P. 2013, MNRAS, 430, 1599
Itoh, S. 2002, J. Comput. Phys., 179, 238
Iwasawa, M., Tanikawa, A., Hosono, N., et al. 2015, in Proceedings of the 5th International Workshop on Domain-Specific Languages and High-Level Frameworks for High Performance Computing, WOLFHPC ’15 (New York, NY, USA: ACM), 1:1–1:10
Iwata, M., Tanikawa, A., Hosono, N., et al. 2016, PASJ, 68 (4), 54 (1)
Jutzi, M. 2015, Planet. Space. Sci., 107, 3
Jutzi, M. & Asphaug, E. 2015, Science, 348, 1355
Jutzi, M. & Benz, W. 2017, A&A, 597, A62
Kobayashi, H. & Iida, S. 2001, Icarus, 153, 416
Kobayashi, H., Tanaka, H., Kirov, A. V., & Inaba, S. 2010, Icarus, 209, 836
Kobayashi, H., Tanaka, H., & Okuzumi, S. 2016, ApJ, 817, 105
Kokubo, E. & Iida, S. 1996, Icarus, 123, 180
Leinhardt, Z. M. & Stewart, S. T. 2012, ApJ, 745, 79
Libersky, L. D. & Petschek, A. G. 1991, in Lecture Notes in Physics, Berlin Springer Verlag, Vol. 395, Advances in the Free-Lagrange Method Including Contributions on Adaptive Gridding and the Smooth Particle Hydrodynamics Method, ed. H. E. Trease, M. F. Frits, & W. P. Crowley, 248–257
Michel, P. & Richardson, D. C. 2013, A&A, 554, L1
Monaghan, J. J. 1992, ARA&A, 30, 543
O’Brien, D. F. & Greenberg, R. 2005, Icarus, 178, 179
Petit, J.-M., Morbidelli, A., & Chambers, J. 2001, Icarus, 153, 338
Safronov, V. S. 1969, Evolution of the protoplanetary cloud and formation of the Earth and the planets (Moscow: Nauka)
Shirr, A., Weiss, B., Zellner, B., et al. 1999, Icarus, 137, 260
Sugrue, K. & Inutsuka, S. 2016, J. Comput. Phys., 308, 171
Sugrue, K., & Inutsuka, S. 2017, J. Comput. Phys., 333, 78
Tillotson, J. S. 1962, General Atomic Rept., GA-3216, 1
Ueda, T., Kobayashi, H., Takeuchi, T., et al. 2017, AJ, 153, 232
Wetherill, G. W. & Stewart, G. R. 1989, Icarus, 77, 330

Article number, page 10 of 11
Appendix A: Time development method

Here, we describe detailed equation of the leapfrog integrator used in this study. At the \( n \)-th step we update the position of the \( i \)-th particle \( x_i^n \) and other quantities of the \( i \)-th particle \( q_i^n \) as

\[
\begin{align*}
x_i^{n+1} &= x_i^n + v_i^n \Delta t + \frac{1}{2} \left( \frac{dv_i}{dt} \right)^n \Delta t^2, \\
q_i^{n+1} &= q_i^n + \frac{1}{2} \left[ \left( \frac{dq_i}{dt} \right)^n + \left( \frac{dq_i}{dt} \right)^{n+1} \right] \Delta t,
\end{align*}
\]  
(A1)

where \( \Delta t \) is the timestep.

In Eq. (A1), for example \( v_i^{n+1} \) is determined by \( (dv_i/dt)^{n+1} \). However, to calculate \( (dv_i/dt)^{n+1} \) with the equation of motion (2) we need \( v_i^{n+1} \), so that we cannot directly derive \( v_i^{n+1} \). Thus we update physical quantities as following procedure: Firstly, we predict the quantities of the \((n+1)\)-th step only using the quantities of the \( n \)-th step as

\[
q_i^n = q_i^n + \left( \frac{dq_i}{dt} \right)^n \Delta t. \tag{A2}
\]

At the same time we update the positions as Eq. (A1). Then we calculate \( (dq_i/dt)^{n+1} \) using \( q_i^n \) and \( x_i^{n+1} \), and we obtain \( q_i^{n+1} \) from Eq. (A1). We can reuse \( (dq_i/dt)^{n+1} \) at the next step, so that we calculate derivatives of variables only once at every step. Moreover, if all variables vary linearly in time, this procedure does not produce any integration error. Therefore, this integration scheme has second-order accuracy in time.

The timestep \( \Delta t \) is determined by considering the Courant condition as

\[
\Delta t = \min_i \frac{h_i}{C_{s,i} \text{ CFL}}, \tag{A3}
\]

where \( C_{s,i} \) is local bulk sound speed at the position of the \( i \)-th particle. \( C_{s,i} \) is calculated from the equation of state, the density, the specific internal energy, and the pressure of the \( i \)-th particle. We adopt the value of \( \text{CFL} = 0.5 \).

Appendix B: Elongated-shape-forming impacts with the second collisions

Figure B.1 represents a collision with \( \theta_{\text{imp}} = 20^\circ \) and \( v_{\text{imp}} = 175 \) m/s, which results in the second collision of two elongated objects. As in Fig. 9, the first collision produces two elongated objects (Fig. B.1b,c). However, the energy dissipation by the collision makes colliding bodies gravitationally bounded (Fig. B.1d), and the resultant body formed by the merging is no longer elongated object (Fig. B.1e). Although the resultant body is not elongated, this impact should be also classified to elongated-shape-forming collision.

Fig. B.1. Snapshots of the impact simulation with \( \theta_{\text{imp}} = 20^\circ \) and \( v_{\text{imp}} = 175 \) m/s at 0.0 s(a), 4.0 \times 10^3 s(b), 3.0 \times 10^4 s(c), 7.0 \times 10^4 s(d), and 1.0 \times 10^5 s(e).