Quantum Entanglement on Cosmological Scale

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Abstract

It has been suggested that relational logic, a form of logic developed by C. S. Peirce, is the common inner syntax of quantum mechanics and string theory. A relation may be represented by a spinor and the Cartan-Penrose connection of spinor to geometry, allows to abstract geometry from a calculus of relations-spinors, reviving Wheeler’s pregeometry. With a single spinor related to the null cone of Minkowski space-time, we search for the geometry emerging when we entangle a left-handed spinor and a right-handed spinor. We find that the quantum entanglement generates an extra dimension and the distance in the extra dimension is measured by the amount of entanglement. The emerging geometry corresponds to a Milne space-time, with two branes coexisting in the extra dimension. One brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles. A distinct phenomenology accompanies our proposal. During the brane collision all points are causally connected, making less pressing the inflationary scenario. The left-right symmetry is achieved with having two “mirror” branes and the neutrino appears as the ideal mediator between the branes. We may revisit also the dark matter, dark energy issues, with everything on the other brane and in the bulk appearing “dark” to us. If biohomochirality is due to parity violation, we anticipate that the life-forms in the other brane demonstrate opposite chiralities. Our scheme brings closer logic - quantum theory - string theory – geometry – cosmology and space-time, rather than an abstract and a priori construction, appears as the outcome of a quantum logical act.
Introduction

Everything is under evolution. The entire universe appears as an ever-changing entity, with distinct stages in evolution. Ordinary matter and radiation, stars, galaxies, clusters of galaxies emerge as parts of an unfolding cosmic evolution. Next to the cosmological evolution, the biological evolution follows and finally the human language and culture appears. Do we dispose the necessary tools to understand the pivotal process of universal evolution? Can we detect and study the different modes of evolution? What kind of theory will bind very different processes together and will reproduce the observed time scales? Should we revisit the fundamental notions of space and time and seek for a dynamical emergence of geometry and time?

In another direction, there is the unification drive in physics, the effort of putting together within a unified framework distinct theoretical approaches. Along the unification of terrestrial and celestial Mechanics (Newton in the 17th century), of optics with the theories of electricity and magnetism (Maxwell in the 19th century), of spacetime geometry and gravity (Einstein in 1916), the next step in this process is the unification of quantum mechanics and general relativity. String theory, in this respect, appears as a promising example of unification. Not unrelated is the quest for the foundations and the internal architecture of a theory. Different theories may share the same internal architecture or syntax, thus reaching a deeper unified comprehension.

It has been suggested [1] that relational logic, a form of logic developed by C. S. Peirce during the years 1870-1880 [2, 3], may offer new insights into these difficult issues. Algebraic logic attempts to express the laws of thought in the form of mathematical equations, and Peirce introduced relation as the irreducible primary datum. All other terms or objects are defined in terms of relations, transformations, arrows, morphisms. A relation $R_{ij}$ may receive multiple interpretations: as the proof of the logical proposition $i$ starting from the logical premise $j$, as a transition from the $j$ state to the $i$ state, as a measurement process that rejects all impinging systems except those in the state $j$ and permits only systems in the state $i$ to emerge from the apparatus. At the core of the Peircean logical system is the composition of relations. Two relations of the form $R_{ij}, R_{jk}$ may be composed giving rise to a third transitive relation $R_{ik}$. It has been indicated that the logical structure developed by Peirce bears great resemblance to category theory, a remarkably rich branch of mathematics developed by Eilenberg and Maclane in 1945 [4]. A consistent effort of categorification of physics has been developed already [5-7].

The symmetries and the dynamics of the Peircean relations lead to the essential laws of quantum mechanics (the probability rule, the commutation rules), indicating relational logic as the conceptual foundation of quantum mechanics [1]. In another direction, when we adopt a double line representation for relations, the composition rule for relations appears as string joining or string splitting [1]. Repetition of the “cubic-string” interaction leads to geometrical patterns reminiscent of Regge’s discrete version of Riemannian geometry [8], of Penrose’s spin networks [9], of simplicial quantum gravity [10], of string foam models [11]. A relation standing for a logical proof, may be represented by a spinor. Consider for example the simplified case of two propositions $(i, j = 1, 2)$. The emerging algebra is $SU(2)$, thus the underlying dynamics is similar to a “spin 1/2 particle”. A proposed logical proof (proposition $j$ entails proposition $i$)
receives as an answer a “yes” or a “no” statement, resembling a spinor, which under measurement is revealed as “spin up” or “spin down” [1, 12]. We understand that a spin network may represent the proof of an entire theorem in logic. Our approach goes in parallel and vindicates Wheeler’s vision. Wheeler pointed out that the genuine explanations about the nature of something do not come about by explicating a concept in terms of similar ones, but by reducing it to a different, more basic kind of object. In this spirit Wheeler considered that geometry is preceded by pregeometry, which is based on the calculus of propositions [13, 14]. We suggest that this calculus of propositions is governed by the rules of quantum relational logic. It should be added that relations have a dynamical character and relational logic may provide the appropriate framework to study and analyze evolutionary dynamics.

The profound connection between spinors and geometry was established a hundred years ago by Cartan, who introduced spinors as representation of the rotation group [15]. Penrose used spinor as the building block of discrete space-time and as a powerful tool to study physics issues [16]. Along this line we have used spinors to represent a string worldsheet or an AdS$_3$ space [12]. In the present work we further extend the Cartan-Penrose argument. If a single spinor, represented by a point on a Riemann sphere, is connected to the Minkowski null cone, we may search for the geometry emerging when two propositions-spinors are combined.

A spinor may be represented by a Majorana field or a Dirac field. We find that the composition of two left-handed fields, following Majorana’s instructions, leads to an entangled state respecting the well known entanglement rules. On the other hand the composition of a left-handed spinor and a right-handed spinor, in Dirac’s way, generates an extra dimension and the emerging geometry corresponds to a Milne universe. In the third section we examine the phenomenology of the proposed Milne universe, a universe made out of two branes coexisting in the extra dimension. We point out the similarity with the ekpyrotic scenario [17-19] and the distinct difference, since within our model a brane (ours) hosts left-handed fields, while the other brane hosts right-handed fields. We address the issue of uniform behavior across the brane and we find that the need for an inflationary scenario, to secure homogeneity and isotropy, is less pressing. We study also the symmetries of the Milne universe (especially the left-right symmetry), the mediation between the two branes and we reconsider the subject of dark matter, dark energy within the new framework. Tentatively we examine the handedness inherent in our model and the homochirality observed in biology.

Overall we suggest a new scheme which brings together logic, quantum mechanics, string theory, geometry. Space-time is created out of a quantum entanglement process and quantum logic governs not only subatomic physics but cosmos itself. In the conclusions we summarize our findings and indicate directions for future research.

**Entanglement and geometry**

Consider the proposition-spinor $|u⟩ = \begin{pmatrix} \xi \\ \eta \end{pmatrix}$. The relationship $R$ is defined by

$$R = |u⟩ ⟨u| = \begin{pmatrix} \xi \xi^* & \xi \eta^* \\ \eta \xi^* & \eta \eta^* \end{pmatrix}$$

(1)
\( \mathbf{R} \) receives the decomposition

\[
\mathbf{R} = \frac{1}{2} \sum_{\mu=0}^{3} X_{\mu} \sigma^\mu
\]  

with \( \sigma^0 = 1 \) and \( \sigma^i \) (\( i = 1, 2, 3 \)) the Pauli matrices. We deduce that

\[
X_{\mu} = \langle \mathbf{u} | \sigma_{\mu} | \mathbf{u} \rangle.
\]  

(3)

\( X_{\mu} \) satisfies identically the equation

\[
X_{1}^2 + X_{2}^2 + X_{3}^2 - X_{0}^2 = 0
\]  

(4)

Eq. (4) receives a double interpretation. It may represent a null vector belonging to Minkowski spacetime \([15, 16]\), indicating the logic-algebraic origin of Minkowski spacetime. On the other hand, with \( X_{0} = 1 \), it represents the spinor Riemann-Bloch sphere. Inversely, given eq. (4), we may search for its spinorial representations. We obtain the Cartan-Weyl equations

\[
\left( \mathbf{\bar{X}} \cdot \sigma - X_{0} \right) |u_L\rangle = 0
\]  

(5)

\[
\left( \mathbf{\bar{X}} \cdot \sigma + X_{0} \right) |u_R\rangle = 0
\]  

(6)

with \( |u_L\rangle \) and \( |u_R\rangle \) the left-handed and right-handed Weyl spinors.

With a single spinor related to Minkowski spacetime, we may ask what kind of geometry emerges when we entangle two spinors. Let’s consider first the Majorana type entanglement. Given a left-handed spinor \( |\psi_L\rangle \), following Majorana’s recipe \([20, 21]\), we may construct a right-handed spinor \( |\chi_R\rangle \) by

\[
|\chi_R\rangle = \sigma_2 |\psi_L\rangle^*.
\]  

(7)

Starting with two left-handed Weyl spinors

\[
|\chi_L\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad |\psi_L\rangle = \begin{pmatrix} \gamma \\ \delta \end{pmatrix}
\]  

(8)

we define the four-component Majorana spinor

\[
|\Psi_M\rangle = \begin{pmatrix} |\chi_L\rangle \\ \sigma_2 |\psi_L\rangle^* \end{pmatrix}.
\]  

(9)

With \( \langle \Psi_M | = |\Psi_M\rangle^\dagger \gamma_0 \) we find

\[
X_1 = \langle \Psi_M | \gamma_1 | \Psi_M \rangle = a^*b + b^*a - cd^* - dc^*
\]  

(10)

\[
X_2 = \langle \Psi_M | \gamma_2 | \Psi_M \rangle = i (-a^*b + b^*a - cd^* + dc^*)
\]  

(11)

\[
X_3 = \langle \Psi_M | \gamma_3 | \Psi_M \rangle = a^*a - b^*b - cc^* + dd^*
\]  

(12)

\[
X_0 = \langle \Psi_M | \gamma_0 | \Psi_M \rangle = -a^*a - b^*b + cc^* + dd^*
\]  

(13)

where we used

\[
\gamma_{\mu} : \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma^i & 0 \end{pmatrix}, \quad \gamma_0 = \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix}.
\]  

(14)
The quantity $X_1^2 + X_2^2 + X_3^2 - X_0^2$ is not anymore zero. We find

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 \equiv M_M^2 = 4 \left| (ad - cb) \right|^2 \quad (15)$$

Considering

$$X_4 = i \langle \Psi_M \mid \Psi_M \rangle = -2i \text{Im} (ad - cb)$$
$$X_5 = \langle \Psi_M \mid \gamma_5 \mid \Psi_M \rangle = 2i \text{Re} (ad - cb)$$

with

$$\gamma_5 = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3 = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}, \quad (16)$$

we obtain

$$M_M^2 = - \left( X_4^2 + X_5^2 \right). \quad (17)$$

The requirement that the Riemann-Bloch sphere remains intact, equ.(4), implies the condition

$$ad - cb = 0 \quad (18)$$

On the other hand a generic two-qubit state is written as

$$|\Psi\rangle = a |00\rangle + b |01\rangle + c |10\rangle + d |11\rangle \quad (19)$$

The state $|\Psi\rangle$ is “separable”, i.e. it can be written as a simple product of individual kets, when the same condition (18) is satisfied. Thus our Majorana entanglement reproduces the ordinary two-qubit entanglement, usually obtained by making appeal to the quaternion formalism [22, 23].

For the Dirac entanglement we select a left-handed Weyl spinor and a right-handed Weyl spinor. Writing

$$|\chi_L\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \quad |\psi_R\rangle = \begin{pmatrix} c \\ d \end{pmatrix} \quad (20)$$

and

$$|\Psi_D\rangle = \begin{pmatrix} |\chi_L\rangle \\ |\psi_R\rangle \end{pmatrix} \quad (21)$$

we obtain

$$X_1 = \langle \Psi_D \mid \gamma_1 \mid \Psi_D \rangle = a^*b + b^*a - c^*d - d^*c \quad (22)$$
$$X_2 = \langle \Psi_D \mid \gamma_2 \mid \Psi_D \rangle = i (-a^*b + b^*a + c^*d - d^*c) \quad (23)$$
$$X_3 = \langle \Psi_D \mid \gamma_3 \mid \Psi_D \rangle = |a|^2 - |b|^2 - |c|^2 + |d|^2 \quad (24)$$
$$X_0 = \langle \Psi_D \mid \gamma_0 \mid \Psi_D \rangle = - \left( |a|^2 + |b|^2 \right) - \left( |c|^2 + |d|^2 \right) \quad (25)$$

The quantity $X_1^2 + X_2^2 + X_3^2 - X_0^2$ is not anymore zero. We obtain

$$X_1^2 + X_2^2 + X_3^2 - X_0^2 \equiv -M_D^2 = -4 \left| (a^*c + b^*d) \right|^2 \quad (26)$$

Considering

$$X_4 = i \langle \Psi_D \mid \Psi_D \rangle = -2 \text{Im} (a^*c + b^*d)$$
$$X_5 = \langle \Psi_D \mid \gamma_5 \mid \Psi_D \rangle = -2 \text{Re} (a^*c + b^*d)$$
we find
\[ M_D^2 = \left( X_4^2 + X_5^2 \right) \] (27)

The absence of entanglement and the re-establishment of the Riemann-Bloch sphere is obtained, when the condition
\[ a^*c + b^*d = 0 \] (28)
is fulfilled.

Let us define \( T = X_0, t = M_D \). The Dirac entanglement, equ.(26), takes then the form of a space-like hyperboloid
\[ T^2 - \sum_{i=1}^{3} X_i^2 = t^2 \] (29)

The above constraint is automatically satisfied by
\[ T = t \cosh \rho \quad X_i = n_i t \sinh \rho \quad \sum_{i=1}^{3} n_i^2 = 1 \] (30)

The amount of quantum entanglement is determined by \( t \). The topological equivalence between the Riemann-Bloch sphere and the null cone of Minkowski space is obtained by letting the radius of the Riemann-Bloch sphere becoming the continuous time \( X_0 \). In a similar fashion we consider \( t \) as a continuous variable. The induced metric becomes
\[ ds^2 = dT^2 - \sum_{i=1}^{3} dX_i^2 \]
\[ = dt^2 - t^2 (d\rho)^2 - t^2 \sinh^2 \rho d\Omega_2^2 \]

This represents a Milne universe. Within our approach, geometry and space-time, rather than abstract and a priori mathematical constructions, emerge as the outcome of a quantum act, the act of quantum entanglement. In the next section we will examine the phenomenological implications of a Milne universe.

**Phenomenology of a Milne Universe**

Our starting point is logic, relational logic as a foundation of quantum mechanics and string theory. The composition rule of relations builds up both geometry and quantum mechanics. This development gives further credit to the notion that a quantum system may encode a logical proof [24]. Space-time itself is sought as an emergent property of a deeper theory or, to use Wheeler’s terminology, a pregeometry [14]. The close connection we found between a relation and a spinor, led us to follow the Cartan-Penrose argumentation [9, 15, 16] and use the relation-spinor as the building block of space-time. If a single spinor is associated to Minkowski space-time, our study indicates that the entanglement of two spinors gives rise to a more complex space-time. At first quantum entanglement generates an extra dimension, indicating that a quantum system in \( d \) dimensions is analogous to a classical system in \( d + 1 \) dimensions [25]. Further the emerging metric corresponds to a Milne space-time, with two 3-dimensional branes coexisting in the extra dimension. The outcome is very similar to the ekpyrotic model.
According to the ekpyrotic scenario a violent collision between the two branes results to a conflagration, resembling the conventional “big bang”. It should be noted though that the ekpyrotic model is derived within the heterotic M-theory [26], while the Milne universe we suggest emerges out of a quantum logical process. Furthermore the two branes are not identical in our case. By construction one brane hosts left-handed particles (our brane), while the other brane hosts right-handed particles.

The Lagrangian for a particle in a Milne universe is

\[ L = \frac{1}{2} \left[ \dot{t}^2 - t^2 \dot{\varrho}^2 - t^2 \sinh^2 \varrho \left( \dot{n}_1^2 + \dot{n}_2^2 + \dot{n}_3^2 \right) \right] \]  

The equations of motion are easily solved in the case of one-dimensional branes. We find

\[ \varrho \simeq -\log t \]  

As \( t \) goes to zero we approach the time of brane-collision (the ekpyrosis moment) and the distance \( \varrho \) across the brane becomes infinite. We gather that at the collision time the correlation along the brane is infinite and all the points in the brane are causally connected [27]. The apparent homogeneity and isotropy of our universe can be accounted therefore, making less pressing the need for an inflatiorary scenario [28].

The conventional way to restore left-right symmetry is to introduce an extra \( SU(2)_R \) gauge group in the energy desert above the scale of the standard \( SU(2)_L \) interactions. The right-handed gauge bosons are more massive compared to the left-handed gauge bosons, leading to parity violation at low energies [29, 30]. Within our approach the left-right symmetry is achieved with the extra dimension hosting two “mirror” branes, a left-handed brane and a right-handed brane. Higgs scalars, denoted by \( \phi_L \) and \( \phi_R \), live in the corresponding branes, though having different vacuum expectation values. An interaction term \( \lambda \phi_L^2 \phi_R^2 \) may induce a mixing of the two Higgs scalars, serving also as a mediator between the two branes. At the ekpyrotic moment \( (t = 0) \), the full conformal \( (L + R) \) symmetry is achieved. From a phenomenological point of view the particles living in the “other” brane behave as mirror-duplicate of the particles in our visible world [31, 32].

The most prominent candidate for mediation between the two branes is the neutrino particle. The left-handed neutrino, an essential ingredient of the standard model, resides in our brane, while its counterpart, the right-handed neutrino, resides in the other brane. The two braneworlds are equivalent to a two-sheeted spacetime \( M_4 \times Z_2 \) [32, 33], with \( M_4 \) standing for a four-dimensional continuous manifold and the fifth dimension reduced to two discrete points separated by a distance \( 2t \) (the amount of entanglement). In a five dimensional \( Z_2 \)-Dirac equation, the neutrino mass will appear as a term connecting the two branes and neutrino oscillations will acquire the novel form of swapping between the branes. Our scheme offers a natural explanation for the neutrino masses. The neutrino mass is at the same time the entanglement between two spinors and the distance between the two branes. The tiny neutrino masses reflect the small distance separating the branes. Notice also the evolving nature of the neutrino mass. Close to ekpyrosis the Dirac neutrino is massless, affecting in a distinct way cosmology.

In our universe we register three main components. Visible matter accounts for about 4%, dark matter for 21% and dark energy for the remainder. In the usual
approach we consider that dark matter consists of particles (classified as cold, warm, hot), while the dynamics of dark energy is assumed by the cosmological constant $\Lambda$. In our case we explain dark matter and dark energy by invoking geometry. Everything that is localized in the other brane and in the extra dimension appears dark to us. If we assume that in our observed brane, matter is equally shared between visible and invisible matter, with the same analogy holding in the other brane, then we obtain naturally that our visible matter is $\sim \frac{1}{5}$ of the entire matter content, in agreement with the observations. Similar arguments have been developed in models where the brane is folded many times inside the extra dimension [34]. Since we have located most of the matter outside our brane, we may easily imagine that the local distribution of visible matter is not identical to the local distribution of the entire matter. In an imaginative journey, as we leave the center of our galaxy, moving a fraction of the millimeter across the extra dimension to reach the “other” brane, we are not going to land at the center of the “other” galaxy. The local matter distribution in the “mirror” brane is not the same with the matter distribution in our brane, and the net effect is a displacement of the dark matter density with respect the visible matter density. Such an effect has been recently observed. The peak of the local dark matter density differs from the center of our galaxy by several hundred parsec [35].

Gravity is not confined to the brane and matter fields on the brane can emit gravitational waves into the bulk. The brane energy-momentum tensor is not conserved therefore and it is essential to include this energy flow into realistic cosmological models. The leakage of gravitational energy into extra dimension affects the cosmological expansion [36], inducing an accelerating expansion for the three dimensional space [37]. Thus an alternative to the non-zero cosmological constant scenario is offered.

It is well known that life manifests a strong biohomochirality. In the living organisms we encounter only left-handed aminoacids and right-handed sugars. There are many hypotheses advanced regarding the origin of biohomochirality. We would like to point out the hypothesis linking the bio-asymmetry to the fundamental parity violation of the weak interactions. The weak neutral currents (mediated by $Z^0$) stabilize preferentially the L-aminoacids and D-sugars over the D-aminoacids and L-sugars. The difference in energy between the two enantiomers is small. A phenomenon of autocatalysis in a far from equilibrium state, amplifies the small difference and in a period of $10^4$ years leads to a unique chirality [38]. If this working hypothesis is a valid one, then in the “mirror” brane the corresponding weak neutral currents will operate in the opposite direction. In that case the life-forms of the other brane will have exclusively D-aminoacids and L-sugars.

**In lieu of conclusions**

We are used first to wonder about particles or states and then about their interactions. First to ask about “what is it” and afterwards “how is it”. On the other hand, quantum mechanics and string theory display a highly relational nature. We are led to reorient our thinking and consider that things have no meaning in themselves, and that only the correlations between them are “real” [39]. We adopted the Peircean relational logic as a consistent framework to prime correlations and gain new insights into these
Our representation of a relation by a spinor allowed us to connect logic with geometry and space-time. We found that the entanglement of two spinors generates an extra dimension and the distance in the extra dimension is measured by the amount of entanglement. Finally two brane-geometries are entangled together: one brane hosting left-handed particles and another brane hosting right-handed particles. We would like to draw also a parallel with the CFT/AdS duality [40, 41]. A spinor may be viewed as the space-time orientation of a pixel on a holographic screen [42]. Through the pixel on the light-cone a null ray passes. What the quantum entanglement offers is the exploration of the internal space of the light-cone (the bulk). On very general grounds it has been shown the correspondence between quantum entanglement, classical renormalization group and holographic gauge/gravity duality [43-46]. The stringy degrees of freedom operating in the CFT/AdS duality are assumed in our case by the entanglement degrees of freedom and quantum phenomena are encoded in classical geometry.

The emerging geometry, a Milne space-time with two chirally oriented branes, offers a rich phenomenology: a novel way to approach left-right symmetry on cosmological scale, a link of dark matter and dark energy to the dynamics of extra space, particles and forces as mediators between the branes, a connection between the left-right symmetry breaking and biohomochirality. Within our approach time acquires a new role. The beginning of time is not set at the “big bang”, 13.7 billions years ago, but somehow is lost in the previous aeons. The time allocated to universe’s evolution is longer than 13.7 billion years, involving also the time periods between successive “ekpyrotic” moments. During these periods-aeons part of the creation of the universe was carried out. Thus the difficulty in accommodating the different distinct phenomena within a single history, exemplified by the account of the “anthropic principle” [47], is significantly reduced. Searching for the ruins from a previous aeon, or the archeology of the universe, is the most intricate and complicated task. Yet, it has been already suggested that in the CMB there are traces from an activity preceding the “big-bang” [48].

Altogether the new syntax we have introduced brings closer logic – quantum mechanics – string theory - cosmology and allows addressing foundational questions regarding the evolution of the universe. A relevant phenomenology may serve as the testing ground of these ideas.

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