Abstract

We show that the availability of longitudinally polarized electron beams at a 500 GeV Linear Collider would allow, from an analysis of the reaction $e^+e^- \rightarrow W^+W^-$, to set stringent bounds on the couplings of a $Z'$ of the most general type. In addition, to some extent, it would be possible to disentangle observable effects of the $Z'$ from analogous ones due to competitor models with anomalous trilinear gauge couplings.
1 Introduction

It has been recently suggested [1] that theoretical models with one extra $Z \equiv Z'$ whose couplings to quarks and leptons are not of the ‘conventional’ type would be perfectly consistent with all the available experimental information from either LEP1 [2] and SLD [3] or CDF [4] data. Starting from this observation, a detailed analysis has been performed of the detectability in the final two-fermion channels at LEP2 of a $Z'$ whose fermion couplings are arbitrary (but still family independent) [5]. Also, in [5] the problem of distinguishing this model from competitor ones (in particular, from a model with anomalous gauge couplings) has been studied.

The final two-fermion channel is not the only one where virtual effects generated by a $Z'$ can manifest themselves. The usefulness of the final $W^+W^-$ channel in $e^+e^-$ annihilation to obtain improved information on some theoretical properties of such models, has already been stressed in previous papers in the specific case of longitudinally polarized beams for models of ‘conventional’ type (e.g., $E_6$, LR, etc.), showing that the role of polarization in these cases would be essential [6].

The effects of a $Z'$ of ‘unconventional’ type in the $W^+W^-$ channel have also been considered, and compared with those of models with anomalous gauge couplings in Ref. [7]. In particular, in [7] it was shown that the benchmark of the model with a $Z'$ would be the existence of a peculiar connection between certain effects observed in the $W^+W^-$ channel and other effects observed in the final lepton-antilepton channel.

The aim of this paper is that of considering whether the search for indirect effects of a ‘unconventional’ $Z'$ in the $W^+W^-$ channel would benefit from the availability of longitudinal polarization of initial beams, as it is the case for the ‘conventional’ situation. We shall show in the next Sect. 2 that this is indeed the case, i.e., that in the parameter space the expected experimental sensitivity in the polarized processes is by far better than in the unpolarized case. For what concerns the differentiation from other sources of nonstandard effects, in particular those with anomalous gauge couplings, we shall also show in Sect. 3 that the characteristic feature of such a $Z'$ would be the existence of certain peculiar properties of different observables, all pertaining to the final $W^+W^-$ channel.

All our discussions assume that longitudinal lepton polarization will be available in the considered examples. In practice, this would be feasible at the future planned...
500 GeV linear Collider (LC). Our conclusions for the specific cases that we consider, as summarized in Sect. 4, will therefore be strongly in favour of polarization. An Appendix will be devoted to the derivation of several expressions for the relevant experimental observables.

2 Derivation of the constraints for general \(Z'\) parameters

The starting point of our analysis will be the expression of the invariant amplitude for the process

\[ e^+ + e^- \rightarrow W^+ + W^- . \]  

(1)

In Born approximation, this can be written as a sum of a \(t\)-channel and of an \(s\)-channel component. In the Standard Model (SM) case, the latter will be written as follows:

\[ M_s^{(\lambda)} = \left( -\frac{1}{s} + \cot \theta_W \left( v - 2\lambda a \right) \right) \times G^{(\lambda)}(s, \theta), \]

(2)

where \(s\) and \(\theta\) are the total c.m. squared energy and \(W^-\) production angle; \(v = (T_{3,e} - 2Q_e s_W^2)/2s_W c_W\) and \(a = T_{3,e}/2s_W c_W\) with \(T_{3,e} = -1/2\) and \(s_W = \sin \theta_W\), \(c_W = \cos \theta_W\) (\(\theta_W\) is the conventional electroweak mixing angle); \(\lambda\) denotes the electron helicity (\(\lambda = \pm 1/2\) for right/left-handed electrons); finally, \(G^{(\lambda)}(s, \theta)\) is a kinematical coefficient, depending also on the final \(W\)’s helicities. For simplicity we omit its explicit form, which is not essential for our discussion here, and can be either found in the literature [8] or easily derived from the entries of Tab. 1 in the subsequent Sect. 3, which also shows the form of the \(t\)-channel neutrino exchange. Note that, at this stage, we are writing an ‘effective’ Born approximation that contains both the physical \(Z\) couplings and the physical \(Z\) mass. We shall systematically ignore extra contributions at one loop. In fact, our purpose is that of evaluating deviations from the SM expressions due to one extra \(Z\). In this spirit, we shall also consider the \(Z'\) contribution using an ‘effective’ Born approximation with physical \(Z'\) couplings and mass. The more rigorous one-loop treatment would require also the calculation of the, potentially dangerous, QED radiation effects whose study has not yet been performed, to our knowledge, for polarized beams at the LC. We shall assume in the sequel that the results of a rigorous treatment reproduce those of an effective approximation without QED after a suitable, apparatus dependent, calculation as...
it is the case for the unpolarized case. Then, for the evaluation of the deviations due to the \( Z' \), the residual purely electroweak one-loop contributions will be safely neglected.

Working in this framework, the effective expression of the invariant amplitude after addition of one extra \( Z \) will be written as:

\[
M^{(\lambda)}_s = \left( -\frac{1}{s} + \frac{g_{WWZ_1}(v_1 - 2\lambda a_1)}{s - M_{Z_1}^2} + \frac{g_{WWZ_2}(v_2 - 2\lambda a_2)}{s - M_{Z_2}^2} \right) \times G^{(\lambda)}(s, \theta). \tag{3}
\]

In Eq. (3), we have retained two possible sources of effects from, respectively, the ‘light’ and the ‘heavy’ neutral gauge bosons \( Z_1 \) and \( Z_2 \). In general, in models with two neutral gauge bosons, the ‘light’ \( Z \) is formally not identical to the SM \( Z \) and, within the accuracy of experimental data relevant to measurements of the standard \( Z \) parameters, it potentially can have mass and couplings different from the SM prediction. Such modifications of the \( Z \) couplings, reflecting the presence of the additional extra \( Z' \), can be induced, e.g., through the mechanism of \( Z - Z' \) mixing. To account for this fact, the ‘light’ \( Z \) is now denoted as \( Z_1 \), and the same convention applies to its couplings \( v_1, a_1 \) and \( g_{WWZ_1} \). The second effect is due to the actual extra heavy \( Z \) exchange diagram, and will be treated by denoting the physical heavy \( Z \) as \( Z_2 \) and its physical couplings by analogous notations.

It turns out that it is convenient to rewrite Eq. (3) in the following form:

\[
M^{(\lambda)}_s = \left( -\frac{g_{WW\gamma}}{s} + \frac{g_{WWZ}(v - 2\lambda a)}{s - M_Z^2} \right) \times G^{(\lambda)}(s, \theta), \tag{4}
\]

where the ‘effective’ gauge boson couplings \( g_{WW\gamma} \) and \( g_{WWZ} \) are defined as:

\[
g_{WW\gamma} = 1 + \Delta_\gamma = 1 + \Delta_\gamma(Z_1) + \Delta_\gamma(Z_2), \tag{5}
\]

\[
g_{WWZ} = \cot \theta_W + \Delta_Z = 1 + \Delta_Z(Z_1) + \Delta_Z(Z_2), \tag{6}
\]

with

\[
\Delta_\gamma(Z_1) = v \cot \theta_W \left( \frac{\Delta a}{a} - \frac{\Delta \chi}{\chi} \right) \left( 1 + \Delta \chi \right) \chi; \quad \Delta_\gamma(Z_2) = v g_{WWZ_2} \left( \frac{a_2}{a} - \frac{v_2}{v} \right) \chi_2.
\]

\[
\Delta_Z(Z_1) = \Delta g_{WWZ} + \cot \theta_W \left( \frac{\Delta a}{a} + \Delta \chi \right); \quad \Delta_Z(Z_2) = g_{WWZ_2} \frac{a_2}{a} \chi_2. \tag{7}
\]

\[^3\text{In Eq. (3), the couplings to } W^+W^- \text{ of both } Z_1 \text{ and } Z_2 \text{ have been tacitly assumed of the usual Yang-Mills form.} \]
In Eqs. (7) and (8) we have introduced the deviations of the fermionic and trilinear bosonic couplings \( \Delta v = v_1 - v, \Delta a = a_1 - a \) and \( \Delta g_{WWZ} = g_{WWZ_1} - \cot \theta_W \), and the neutral vector boson propagators (neglecting their widths):

\[
\chi(s) = \frac{s}{s - M_Z^2}; \quad \chi_2(s) = \frac{s}{s - M_{Z_2}^2}; \quad \Delta \chi(s) = -\frac{2M_Z \Delta M}{s - M_Z^2},
\]

where \( \Delta M = M_Z - M_{Z_1} \) is the \( Z-Z_1 \) mass-shift.

It should be stressed that, not referring to specific models, the parametrization (4)-(6) is both general and useful for phenomenological purposes, in particular to compare different sources of nonstandard effects contributing finite deviations (7) and (8) to the SM predictions.

Concerning the \( Z_1 \) couplings to electrons, present constraints from experimental data indicate that their values should be rather close to the SM values \( v \) and \( a \) listed above. Indeed, results for the effective leptonic vector and axial-vector couplings derived from the combined LEP data give \( v/v_{SM} = 1.0008 \pm 0.0235 \) and \( a/a_{SM} = 0.9902 \pm 0.0006 \) [9], and the analysis of the \( \rho \)-parameter suggests an upper limit on \( \Delta M \) of the order of \( 150 - 200 \text{MeV} \) [10, 11]. Thus, the deviations \( \Delta v \) and \( \Delta a \) should be small numbers, to be treated as a perturbation to the SM results, and the same is true for the mass-shift \( \Delta M/M_Z \), with \( \Delta M > 0 \) if this is due to \( Z-Z' \) mixing.

As we are interested here in the sensitivity of process (1) to general features of virtual, ‘indirect’, \( Z' \) effects and in their comparison with analogous effects of anomalous gauge boson couplings, we do not consider modifications of the \( t \)-channel amplitude. In principle, such effects can arise in specific models due to the presence of new heavy fermions, and their inclusion in the general parametrization of the amplitude deviations from the SM would require a much more complicated analysis, taking into account three different sources of nonstandard effects at the same time (\( Z' \), lepton mixing and anomalous couplings). Although perhaps possible, this is beyond the purpose of this paper.\(^4\)

We now focus on the effects of the heavy \( Z \) on polarized observables. Although this is not necessarily a unique choice, we mostly consider for a first investigation the case of only polarized electron beams. The general expression for the cross section of process (1) with longitudinally polarized electron and positron beams can be

\(^4\)For an attempt of separating heavy-lepton mixing effects from \( Z' \) ones see, e.g., Ref. [12].
expressed as
\[
\frac{d\sigma}{d\cos\theta} = \frac{1}{4} \left[ (1 + P_L)(1 - \bar{P}_L) \frac{d\sigma^+}{d\cos\theta} + (1 - P_L)(1 + \bar{P}_L) \frac{d\sigma^-}{d\cos\theta} \right],
\]
where \( P_L \) and \( \bar{P}_L \) are the actual degrees of electron and positron longitudinal polarization, respectively, and \( \sigma^\pm \) are the cross sections for purely right-handed and left-handed electrons. From Eq. (10), the cross section for polarized electrons and unpolarized positrons corresponds to \( \bar{P}_L = 0 \). The polarized cross sections can be generally decomposed as follows:
\[
\frac{d\sigma^\pm}{d\cos\theta} = \frac{\pi\alpha_{em}^2\beta_W^2}{2s} \sum_i F_i^\pm \mathcal{O}_i(s, \cos\theta),
\]
where: \( \beta_W = \sqrt{1 - 4M_W^2/s} = 2p/\sqrt{s} \), with \( p = |\vec{p}| \) the CM momentum of the \( W \); \( F_i^\pm \) are combinations of couplings involving in particular the deviations from the SM couplings, e.g., of the kind previously introduced; \( \mathcal{O}_i \) are functions of the kinematical variables. To make the paper self-contained, we list the explicit expressions of the relevant \( F_i^\pm \) and \( \mathcal{O}_i \) in the Appendix.

In practice, we shall denote by \( \sigma^L \) and \( \sigma^R \) the cross sections corresponding, in Eq. (10), to the values \( P_L = -0.9 \) and \( P_L = 0.9 \), respectively. Such degrees of longitudinal polarization should be realistically obtainable at the LC [13].

Our analysis proceeds in this way. Following the suggestions of previous dedicated searches [14], the sensitivity of \( \sigma^L \) and \( \sigma^R \) to \( \Delta_\gamma \) and \( \Delta_Z \) is assessed numerically by dividing the angular range \( |\cos\theta| \leq 0.98 \) into 10 equal ‘bins’, and defining a \( \chi^2 \) function in terms of the expected number of events \( N(i) \) in each bin:
\[
\chi^2 = \sum_i^{\text{bins}} \left[ \frac{N_{SM}(i) - N(i)}{\delta N_{SM}(i)} \right]^2,
\]
where the uncertainty on the number of events \( \delta N_{SM}(i) \) combines both statistical and systematic errors as
\[
\delta N_{SM}(i) = \sqrt{N_{SM}(i) + (\delta_{\text{syst}} N_{SM}(i))^2},
\]
(we assume \( \delta_{\text{syst}} = 2\% \)). In Eq. (12), \( N(i) = L_{\text{int}}\sigma_i \varepsilon_W \) with \( L_{\text{int}} \) the time-integrated luminosity, and \( (z = \cos\theta) \):
\[
\sigma_i = \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left( \frac{d\sigma}{dz} \right) dz.
\]
Finally, \( \varepsilon_W \) is the efficiency for \( W^+W^- \) reconstruction, for which we take the channel of lepton pairs \((e\nu + \mu\nu)\) plus two hadronic jets, giving \( \varepsilon_W \simeq 0.3 \) from the relevant branching ratios. An analogous procedure is followed to evaluate \( N_{SM}(i) \).

As a criterion to derive the constraints on the coupling constants in the case where no deviations from the SM were observed, we impose that \( \chi^2 \leq \chi^2_{\text{crit}} \), where \( \chi^2_{\text{crit}} \) is a number that specifies the chosen confidence level. With two independent parameters in Eqs. (3) and (4), the 95% CL is obtained by choosing \( \chi^2_{\text{crit}} = 6 \).

From the numerical procedure outlined above, we obtain the allowed bands for \( \Delta\gamma \) and \( \Delta Z \) determined by the polarized cross sections \( \sigma^R \) and \( \sigma^L \) (as well as \( \sigma^{\text{unpol}} \)) depicted in Fig. 1, where \( L_{\text{int}} = 50 \text{ fb}^{-1} \) has been assumed.

One can see from inspection of Fig. 1 that the role of polarization is essential in order to set meaningful finite bounds. Indeed, contrary to the unpolarized case, which evidently by itself does not provide any finite region for \( \Delta\gamma \) and \( \Delta Z \) (unless one of the two parameters is fixed by some further assumption), from the combined and intersecting bands relative to \( \sigma^L \) and \( \sigma^R \) one can derive the following 95% CL allowed ranges

\[
-0.002 < \Delta\gamma < 0.002 \\
-0.004 < \Delta Z < 0.004.
\] (15)

Reflecting the generality of the parametrization (4)-(6), Eq. (15) represents the most general constraint that would be derivable at the LC for a ‘unconventional’ \( Z' \) with polarized electron beams. It should be stressed that the constraint is completely model independent. To have a feeling of how polarization works in more specific cases, we consider as an application the familiar situation of an extra \( Z \) of extended gauge origin, in particular generated by a previous \( E_6 \) symmetry [16]. Denoting by \( \phi \) the \( Z-Z' \) mixing angle defined by following the conventional prescriptions, Eqs. (7) and (8) would read now:

\[
\Delta\gamma = v \cot \theta_W \phi \left( \frac{a'}{a} - \frac{v'}{v} \right) \left( 1 - \frac{\chi^2}{\chi} + \Delta \chi \right) \chi, \tag{16}
\]

\[
\Delta Z = \cot \theta_W \left[ \frac{\phi a'}{a} \right. \left( 1 - \frac{\chi^2}{\chi} \right) + \Delta \chi], \tag{17}
\]

with \( v' \) and \( a' \) fixed by the specific model and, in general

\[
\tan^2 \phi = \frac{M_Z^2 - M_{Z'}^2}{M_{Z'}^2 - M_Z^2} \approx \frac{2M_Z\Delta M}{M_{Z'}^2}.
\] (18)
Consequently, in the $(\Delta \gamma, \Delta Z)$ plane of Fig. 1 each model is now represented, in
linear approximation in $\phi$, by a line of equation:

$$\Delta Z = \Delta \gamma \frac{1}{v \chi} \frac{(a'/a)}{(a'/a) - (v'/v)}.$$  \hspace{1cm} (19)

Such relation is rather unique, and does not depend on either $\phi$ or $M_{Z_2}$, but only on
ratios of the fermionic couplings. In the case considered here, $v'$ and $a'$ are explicitly
parametrized in terms of a (generally unconstrained) angle $\beta$ which characterizes the
direction of the $Z'$-related extra $U(1)$ generator in the $E_6$ group space, and reflects
the pattern of symmetry breaking to $SU(2)_L \times U(1)_Y$ [16, 17]:

$$v' = \cos \beta \frac{1}{c_W \sqrt{6}}; \hspace{1cm} a' = \frac{1}{2c_W \sqrt{6}} \left( \cos \beta + \sqrt{\frac{5}{3}} \sin \beta \right).$$  \hspace{1cm} (20)

In Fig. 2 we depict, as an illustration, the cases corresponding to the models cur-
cently called $\chi$, $\psi$ and $\eta$, corresponding to the choices $\beta = 0; \pi/2; \pi - \arctan \sqrt{5/3} \approx
128^\circ$, respectively. From this figure, two main conclusions can be drawn: i) polarization
systematically reduces the allowed range for the model parameters, in some cases
($\psi$, $\eta$ models) more spectacularly than in other ones ($\chi$ model), and ii) depending on
the considered model, different polarization values are relevant, i.e., for the $\eta$ and $\psi$
cases $\sigma^R$ is essential while for the $\chi$ model $\sigma^L$ provides the main constraint.

As a simple quantitative illustration of these features, we can consider in more
detail the case of the $\eta$ model. As it can be read from Fig. 2, for this case the bounds
obtainable from $\sigma^{\text{unpol}}$ only are the following:

$$-0.005 < \Delta \gamma < 0.005$$
$$-0.003 < \Delta Z < 0.003.$$  \hspace{1cm} (21)

The use of $\sigma^R$ allows the improvement of Eq. (21) to the more stringent bounds:

$$-0.002 < \Delta \gamma < 0.002$$
$$-0.001 < \Delta Z < 0.001.$$  \hspace{1cm} (22)

The ranges of $\Delta \gamma$ and $\Delta Z$ allowed to the specific models in Fig. 2 can be translated
into limits on the mixing angle $\phi$ and the heavier gauge boson mass $M_{Z_2}$, using
Eqs. (16)-(19). Continuing our illustrative example of the $\eta$ model, in this case the
resulting allowed region (at the 95% CL) in the $(\phi, M_{Z_2})$ plane is limited by the

\[\text{footnote text}\]
thick solid line in Fig. 3. We have chosen for $\Delta M$ in Eq. (9) an upper limit of about $150 - 200 \text{MeV}$, although the limiting curves do not appreciably depend on the specific value of this quantity. Also, the indicative current lower bound on $M_{Z_2}$ from direct searches [15], as well as the bound obtainable from full exploitation of the $e^+e^-$ annihilation into lepton-antilepton pairs with polarized electrons [18, 19], are reported in Fig. 3. For a comparison, the dashed line in Fig. 3 represents the maximal region allowed to $\phi$ by the model-and process-independent relation (18), with the same upper bound on $\Delta M$. Finally, the thin solid lines exemplify the allowed region for a particular $E_6$ ‘superstring inspired’ model with all Higgses belonging to the 27-plet representation, such that the mixing angle can be related to the masses of $Z_1$ and $Z_2$ as [16]:

$$\phi \simeq C \frac{M^2_{Z_1}}{M^2_{Z_2}}.$$  (23)

In this case, with $\sigma$ the ratio of appropriate Higgs vacuum expectation values squared:

$$C = 4s_W \left( \frac{\cos \beta}{2\sqrt{6}} - \frac{\sigma - 1}{\sigma + 1} \sqrt{10} \sin \beta \right).$$  (24)

As one can conclude from Fig. 3, the $W^+W^-$ channel with polarized electron beams represents a quite sensitive, and independent, source of information on deviations from the SM model due to the extra $Z$, which can be combined with that provided by the final leptonic channel and nicely complements it.

One may observe that, in principle, the deviations $\Delta_\gamma$ and $\Delta_Z$ in Eqs. (16) and (17) are energy-dependent, reflecting the energy dependence of (7) and (8) through the neutral vector boson propagators. Numerically, however, for the considered values of the energy $\sqrt{s} = 0.5\text{TeV}$ and $M_{Z'}$ such that $M_Z \ll \sqrt{s} \ll M_{Z'}$, one expects $\Delta_\gamma$ and $\Delta_Z$ to be dominated by the contributions $\Delta_\gamma(Z_1)$ and $\Delta_Z(Z_1)$. Indeed, $\Delta_\gamma(Z_2)$ and $\Delta_Z(Z_2)$ should be suppressed relatively to $\Delta_\gamma(Z_1)$ and $\Delta_Z(Z_1)$ by the ratio $|\chi_2/\chi|$, which is of the order of $3 \times 10^{-1}$ and $3 \times 10^{-2}$ for $M_{Z_2} = 1\text{TeV}$ and $3\text{TeV}$, respectively. For these values of $\sqrt{s}$ and $M_{Z'}$, $\Delta_\gamma(Z_1)$ and $\Delta_Z(Z_1)$ are almost energy-independent and therefore so are $\Delta_\gamma$ and $\Delta_Z$. Correspondingly, in this case, if we define the sensitivity of process (1) to the deviations from the SM by the statistical significance

$$S = \frac{\sigma - \sigma^{SM}}{\delta \sigma} = \frac{\Delta \sigma}{\sqrt{\sigma^{SM}}} \sqrt{L_{\text{int}}},$$  (25)

where $\delta \sigma$ is the statistical uncertainty and $L_{\text{int}}$ the time-integrated luminosity, such $S$ is determined only by the explicit $s$-dependence of Eq. (4) and is found to behave,
with $\sqrt{s}$, as $S \propto \sqrt{L_{\text{int}} s}$ at fixed $L_{\text{int}}$. Conversely, for lighter $Z'$, the $Z_2$ contribution can become more significant and somewhat modify the $s$-behaviour of the sensitivity.

The previous discussion should have shown, hopefully in a clear and simple way, the advantages of longitudinal electron polarization at the LC in order to study the effects of a general model with an extra $Z$ in the reaction (1) at a linear electron-positron collider. Actually, our analysis has focused on the derivation of bounds, starting from the (negative) assumption that no effects, i.e., no deviations from the SM predictions are observed within the expected accuracy on the cross section. In the next section we shall take, instead, the (positive) attitude of assuming that certain deviations from the SM are observed in $\sigma^L$ and/or $\sigma^R$. In such a case, it might be possible to identify, to some extent, the relevant source of the observed deviation.

3 Comparison with a model with anomalous gauge couplings

It has already been pointed out [7, 14] that a model with one extra $Z$ would produce virtual manifestations in the final $W^+W^-$ channel at the LC that in principle could mimic those of a model (of completely different origin) with anomalous trilinear gauge boson couplings. As shown by Eqs. (4)-(8), this is due to the fact that the effects of the extra $Z$ can be reabsorbed into a redefinition of the $V_{WW}$ couplings ($V = \gamma, Z$). Therefore, the identification of such an effect, if observed at the LC, becomes a relevant problem.

Using the notations of, e.g., Ref. [8], the relevant trilinear $WWV$ interaction which conserves $U(1)_{\text{em}}, C$ and $P$, can be written as ($e = \sqrt{4\pi\alpha_{\text{em}}}$):

$$L_{\text{eff}} = -ie(1 + \delta_\gamma) \left[ A_\mu \left( W^{\mu\nu}W^{\nu+} - W^{\mu+}W^{\nu-} \right) + F_{\mu\nu}W^{\mu+}W^{\nu-} \right]$$

$$- ie \left( \cot \theta_W + \delta_Z \right) \left[ Z_\mu \left( W^{-\mu\nu}W^{\nu+} - W^{\mu+}W^{\nu-} \right) + Z_{\mu\nu}W^{\mu+}W^{\nu-} \right]$$

$$- ie x_\gamma F_{\mu\nu}W^{\mu+}W^{\nu-} - ie x_Z Z_{\mu\nu}W^{\mu+}W^{\nu-}$$

$$+ ie \frac{y_\gamma}{M_W^2} F^{\nu\lambda}_{\mu\nu}W^{\mu+}_{\nu\lambda} + ie \frac{y_Z}{M_W^2} Z^{\nu\lambda}_{\mu\nu}W^{\mu+}_{\nu\lambda},$$

(26)

where $W^{\pm}_{\mu\nu} = \partial_\mu W^\pm_{\nu} - \partial_\nu W^\pm_{\mu}$ and $Z_{\mu\nu} = \partial_\mu Z_{\nu} - \partial_\nu Z_{\mu}$. In the SM at the tree-level, the anomalous couplings in (26) vanish: $\delta_\gamma = \delta_Z = x_\gamma = x_Z = y_\gamma = y_Z = 0$.

Corresponding to (26), the helicity amplitudes $M^{(\lambda)}$ have the structure shown in Tab. 1 [20], where $\tau$ and $\tau'$ indicate the various possible $W^+$ and $W^-$ polarizations.
Here, for practical purposes, the Yang-Mills parts and their deviations proportional to \( \delta_\gamma \) and \( \delta_Z = g_{WWZ} - \cot \theta_W \) are reported separately from the anomalous ‘magnetic’ and ‘quadrupole’ terms conventionally denoted as, respectively

\[
\Delta k_\gamma = k_\gamma - 1 = x_\gamma; \quad \Delta k_Z g_{WWZ} = (k_Z - 1) g_{WWZ} = x_Z, \tag{27}
\]

and

\[
\lambda_\gamma = y_\gamma; \quad \lambda_Z \cot \theta_W = y_Z. \tag{28}
\]

The deviations \( \Delta_\gamma \) and \( \Delta_Z \) needed in Eqs. (4)-(8) are easily derived from the relevant entries in Tab. 1. Differently from the previous case of the \( Z' \), where \( \Delta_\gamma \) and \( \Delta_Z \) have an explicit (although numerically not quite significant) \( s \)-dependence through Eqs. (6) and (8), the anomalous trilinear gauge boson couplings are taken here as effective \textit{constants}. As a consequence, one must assume \( \delta_\gamma \equiv 0 \) to ensure \( U(1)_{\text{e.m.}} \) gauge invariance. As a matter of fact, for maximum generality one might allow for some \( s \)-dependence of the anomalous couplings in (26) \textit{via} form factors (in particular, the one relevant to \( \delta_\gamma \) must vanish at \( s = 0 \)). Clearly, that would complicate a

| Table 1: Helicity amplitudes for \( e^+ e^- \to W^+ W^- \) |
|-----------------|---|---|
| \( e^-_\lambda e^\lambda_\gamma \to W^-_L W^L_\gamma \) | \( \tau = \tau' = 0 \) | \( -\frac{\epsilon^3 S_\lambda}{2\lambda} \sin \theta \) |
| \( -\frac{2(1+\delta_\gamma)}{S} + \frac{2(\cot \theta_W + \delta_Z)}{M_Z^2} (v - 2a\lambda) \) | \( \frac{S}{2M_W} [\cos \theta - \beta_W (1 + \frac{2M_\gamma}{S})] \) | \( -\beta_W (1 + \frac{S}{2M_\gamma}) \) |
| \(-\frac{x_\gamma}{S} + \frac{y_\gamma}{M_Z^2} (v - 2a\lambda) \) | \( -\beta_W \frac{S}{M_W} \) | \( 0 \) |
| \( e^+_\lambda e^\lambda_\gamma \to W^+_T W^-_T \) | \( \tau = \tau' = \pm 1 \) | \( -\frac{\epsilon^3 S_\lambda}{2\lambda} \sin \theta \) |
| \( -\frac{2(1+\delta_\gamma)}{S} + \frac{2(\cot \theta_W + \delta_Z)}{M_Z^2} (v - 2a\lambda) \) | \( \cos \theta - \beta_W \) | \( -\cos \theta - 2\beta_\lambda \) |
| \(-\frac{x_\gamma}{S} + \frac{y_\gamma}{M_Z^2} (v - 2a\lambda) \) | \(-\beta_W \) | \( 0 \) |
| \( e^-_\lambda e^\lambda_\gamma \to W^-_T W^+_T \) | \( \tau = 0, \tau' = \pm 1 \) | \( \frac{\epsilon^3 S_\lambda}{2\lambda} (\tau' \cos \theta - 2\lambda) \) |
| \( \frac{2(1+\delta_\gamma)}{S} + \frac{2(\cot \theta_W + \delta_Z)}{M_Z^2} (v - 2a\lambda) \) | \( \frac{\sqrt{S}}{2M_W} [\cos \theta (1 + \beta_Z^2) - 2\beta_W] - \frac{2M_\gamma}{\sqrt{S}} \tau' \sin^2 \theta \) | \( \frac{\sqrt{S}}{2M_W} [\cos \theta (1 + \beta_Z^2) - 2\beta_W] - \frac{2M_\gamma}{\sqrt{S}} \sin \theta \tan \theta \) |
| \(-\frac{x_\gamma+y_\gamma}{S} + \frac{x_\gamma+y_\gamma}{M_Z^2} (v - 2a\lambda) \) | \(-\beta_W \frac{S}{M_W} \) | \(-\beta_W \frac{S}{M_W} \) |
true model-independent analysis by the introduction of an overwhelming number of independent parameters.

According to the present viewpoint, anomalous gauge couplings are understood to parametrize some New Physics defined at a scale \( \Lambda \) much greater than the Fermi scale, \( \Lambda \gg v = (\sqrt{2}G_F)^{-1/2} \approx 250 \text{GeV} \), and involving new, very heavy, particles. After integration of the heavy degrees of freedom, a ‘residual’ low-energy interaction among the standard ‘light’ degrees of freedom should remain. It is natural to assume that also the new physics respects \( SU(2) \times U(1) \) gauge symmetry spontaneously broken by the Higgs vacuum expectation value. The weak interaction is then described by an effective Lagrangian, representing a ‘low-energy’ expansion in powers of the small ratio \( v^2/\Lambda^2 \) (and \( s/\Lambda^2 \)) \(^{21} \):

\[
L_W = L_{SM} + \sum_{d \geq 6} \sum_k f_k^{(d)} \frac{\Lambda^d}{\Lambda^4} O_k^{(d)}. \tag{29}
\]

The second term in the RHS of Eq. (29) contains the anomalous trilinear gauge boson couplings, and is given by \( SU(2) \times U(1) \) gauge invariant operators \( O_k^{(d)} \) made of \( \gamma, W, Z \) and Higgs fields, with dimension \( d \). The values of the corresponding coupling constants \( f_k^{(d)} \) are not fixed by the symmetry and therefore must be considered as \textit{a priori} arbitrary constants, to be determined from experimental data (for a detailed analysis and the explicit expressions of the relevant operators we refer to \(^{22} \)).

Clearly, in this framework, the lower dimension operators in (29) are expected to be the leading ones, the higher ones being suppressed by inverse powers of the large scale \( \Lambda \). Therefore, while Eq. (29) potentially includes all the anomalous \textit{constants} of Eq. (26) as well as their possible slopes in \( s \), in practice such slopes are generated by the higher dimension \( (d \geq 8) \) operators and, due to the suppression, are assumed to give a negligible effect in the numerical analysis. Specifically, the slope \( \delta' \) of the \( W^+W^-\gamma \) Yang-Mills coupling, \( \delta_\gamma = s \delta' \), is generated by a \( d = 8 \) operator, while the other slopes involve \( d \geq 10 \) operators \(^{22} \). Furthermore, it can be shown that the assumption of ‘custodial’ global \( SU(2) \) symmetry of the New Physics, which naturally accounts for the smallness of the \( \Delta \rho \) parameter, would imply \( \delta' = 0 \) at the \( d = 8 \) level, because the relevant operator would not respect this symmetry.

\(^6\)One can notice that in Eq. (26) \( \delta V \) and \( xV \) multiply dimension 4 operators, while \( yV \) multiply dimension 6 operators. Thus, more consistently with \(^{24} \), the latter ones should be scaled to \( \Lambda^2 \) rather than \( M_W^2 \) as it is usually done.
It might be useful to recall that the truncation of the sum in Eq. (29) to the lowest significant dimension $d = 6$ would allow the reduction of the number of independent operators, and the corresponding anomalous coupling constants, to three (and all slopes identically vanishing) [21, 22]. In this case, by choosing, e.g., as independent couplings $\delta_{Z}$, $x_{\gamma}$ and $y_{\gamma}$, one has the relation

$$x_{Z} = -x_{\gamma}\tan\theta_{W}; \quad y_{Z} = y_{\gamma}\cot\theta_{W}.$$  

(30)

Additional assumptions, or specific dynamical models, allow to further reduce the number of independent anomalous constants (see, e.g., [8]).

As far as the present information on the five anomalous couplings in (26) is concerned, indirect constraints on $WW\gamma$ and $WWZ$ vertices have been obtained by comparing low-energy data ($\sqrt{s} < 2M_{W}$) with SM predictions for observables that can involve such vertices at the loop level, therefore could affect the electroweak corrections. The results are [23]: $\delta_{Z} = -0.059 \pm 0.056$, $\Delta k_{\gamma} = 0.056 \pm 0.056$, $\Delta k_{Z} = -0.0019 \pm 0.0440$, $\lambda_{\gamma} = -0.036 \pm 0.034$ and $\lambda_{Z} = 0.049 \pm 0.045$. These constraints are obtained from a global analysis of the data by taking the trilinear couplings independently one by one, and fixing the remaining ones at the SM values. However, allowing the simultaneous presence of all five trilinear anomalous couplings in a multiparameter fit, due to the possibility of cancellation and/or correlations, the limits obtained from such analysis would considerably weaken to about $O(0.1 - 1)$ or so.

For our analysis, we would have now to account for the deviations from the SM induced by the various anomalous couplings on polarized observables, considering that the general model in Eq. (26) introduces five independent parameters. Thus, regardless of the attempt to distinguish this case from the simpler one of an extra $Z$ (where only two parameters are involved), the determination of suitable experimental observables, depending on reduced subsets of anomalous gauge boson couplings, would represent an important issue by itself which deserves a separate treatment.

To illustrate a ‘minimal’ identification program, as anticipated in the previous section, we assume that a virtual signal has been detected to a given, conventionally fixed confidence level, in either $\sigma^{L}$ or $\sigma^{R}$, or both. In our notations, that would be expressed as:

$$\frac{\Delta\sigma^{L,R}}{\delta\sigma^{L,R}} = \frac{\sigma^{L,R}_{\text{exp}} - \sigma^{L,R}_{\text{SM}}}{\delta\sigma^{L,R}_{\text{SM}}} \geq \kappa,$$

(31)
where \( \delta \sigma \) is the expected statistical uncertainty on the cross section and the value of \( \kappa \) corresponds to an assigned number of standard deviations.

As the next step, we try to define an observable which is ‘orthogonal’ to the \( Z' \) model, in the sense that such variable should depend only on those four couplings \((x_V, y_V)\) that are specific of the Lagrangian [23], but not on \( \delta Z \) which would induce an effect in common with the \( Z' \) model of Sect. 2.

An illustrative, simple, example of such quantity can be worked out by introducing the following polarized observables, along the lines proposed in [20]:

\[
\sigma_{F_B} = \int \frac{d\sigma^-}{d\cos\theta} d\cos\theta - \int \frac{d\sigma^-}{d\cos\theta} d\cos\theta, \tag{32}
\]

and

\[
\sigma_{CE}(z^*) = \int \frac{d\sigma^\pm}{d\cos\theta} d\cos\theta - \left[ \int \frac{d\sigma^\pm}{d\cos\theta} d\cos\theta + \int \frac{d\sigma^\pm}{d\cos\theta} d\cos\theta \right]. \tag{33}
\]

Similar to (11), one can expand (32) and (33) as

\[
\sigma_{F_B} = \frac{\pi^2 \alpha_{em}^2 \beta_W^2}{2s} \left[ F_0^- \mathcal{O}_{0,F_B} + F_2^- \mathcal{O}_{2,F_B} + (F_6^- + F_7^-) \mathcal{O}_{6,F_B} \right], \tag{34}
\]

\[
\sigma_{CE}(z^*) = \frac{\pi \alpha_{em} \beta_W}{2s} \sum_i F_i^\pm \mathcal{O}_{i,CE}(z^*), \tag{35}
\]

where the explicit expressions of couplings and corresponding kinematical coefficients can be obtained from the Appendix. One can notice, also, that \( \sigma_{F_B} = 0 \). The \( \delta Z \) contribution in Eqs. (34) and (35) is contained in \( F_1^\pm \) and \( F_2^- \) (recall that \( F_2^+ = 0 \)). Therefore, the simplest observable ‘orthogonal’ to \( \delta Z \) is represented, at \( \sqrt{s} = 500 \text{ GeV} \), by the quantity

\[
\sigma_{CE}(z^* \simeq 0.4). \tag{36}
\]

Indeed, numerical inspection of the formulae in the Appendix shows that, at \( z^* \simeq 0.4 \) (and \( \sqrt{s} = 500 \text{ GeV} \)) the coefficient \( \mathcal{O}_{1,CE} \) vanishes, leaving a pure dependence of (36) from \((x_V^+, y_V^+)\) but not from \( \delta Z \). Clearly, the position of this zero is entirely determined by \( M_W \) and the CM energy \( \sqrt{s} \). The possibility to measure (36) strongly depends on the angular resolution for the \( W \) and its decay products.
Another example, which would eliminate both $F_1^-$ and $F_2^-$ in Eqs. (34) and (35), should be the specific combination:

$$Q^- = \sigma A^-_{FB} - \frac{O_{2,FB}}{O_{2,CE}(0.4)} \left( \sigma A^-_{CE}(0.4) \right) \simeq \sigma A^-_{FB} + 0.029 \cdot \sigma A^-_{CE}(0.4).$$  \hfill (37)

Still another possibility could be represented, in principle, by the combination

$$P^+(z^*) = \sigma^+ - \frac{O_1}{O_{1,CE}(z^*)} \left( \sigma A^+_{CE}(z^*) \right),$$  \hfill (38)

with $\sigma^+$ the total cross section for right-handed electrons and arbitrary $z^* \neq 0.4$. The coefficients $O_1$, $O_{1,CE}$ and $O_{2,CE}$ can be easily calculated from the formulae given in the Appendix. Concerning the dependence on the remaining anomalous couplings, in the linear approximation to the $F_i^\pm$ (see Eq. (A4)) the combination (37) depends on $(x_V, y_V)$, while (38) is determined by $(x_V^+, y_V^+)$ similar to (36), but with different parametrical dependence hence with different sensitivities on the anomalous couplings.

In Figs. 4-6 we depict the statistical significance of the variables (36)-(38) as a function of the relevant anomalous couplings. As anticipated in the previous section, for any observable $O$ such significance is defined as the ratio $S = \Delta O / (\delta O)_{stat}$ where $\Delta O = O(x_V, y_V) - O^{SM}$ and $(\delta O)_{stat}$ is the statistical uncertainty attainable on $O$. The different sensitivities to the various couplings $x_V$ and $y_V$ can be directly read from these figures and, as one can see, in certain cases they can be substantial. Figs. 4-6 are obtained by varying one of the parameters at a time and setting all the other ones at the SM values.

By definition, the observables (36)-(38) require 100% longitudinal electron polarization, a situation that will not be fully obtained in practice. However, the presently planned degree of electron polarization at the NLC, $|P_L| \simeq 0.90 - 0.95$ \cite{13, 24} is high enough that Eq. (10) can represent a satisfactory approximation to evaluate such observables. Clearly, such approximation would be substantially improved if also positron polarization were available, e.g., $|\bar{P}_L| \simeq 0.6$ \cite{24}, because in that case in Eq. (10) the coefficient of $\sigma^+$ could be emphasized over that of $\sigma^-$ by a large factor, or viceversa. Moreover, we can remark that the independence of $Q$ in Eq. (37) from $\delta Z$ holds for both $\pm$ cases and, consequently, also for the case of unpolarized beams.

Concerning a possible discrimination between the $Z'$ model of Sect. 2 and the model considered in this section, a strategy could be the following. If a signal is
observed in either $\sigma^L$ and/or $\sigma^R$ and also in at least one of the ‘orthogonal’ observables defined above, we can conclude that it is due to the model with anomalous gauge couplings, and we can try to derive the values of some of them by properly analyzing the observed effects [23, 24]. If, conversely, only $\sigma^L$ and/or $\sigma^R$ show an effect, we are left with the possibility that both models are responsible for such deviations. In this situation, we still have a simple tool to try to distinguish among the two models, which uses the observation that, under the assumption that only $\delta_V$ and $\Delta_V$ are effective, the expressions of the consequent deviations of the integrated cross sections $\sigma^L$ and $\sigma^R$ are, respectively:

$$\Delta \sigma^{R,L} \simeq \Delta \sigma^\pm \propto \delta \gamma - \delta Z g^{R,L}_e \chi, \quad (39)$$

and

$$\Delta \sigma^{R,L} \simeq \Delta \sigma^\pm \propto \Delta \gamma - \Delta Z g^{R,L}_e \chi. \quad (40)$$

Here, both $\Delta_V$ and $\delta_V$ have been taken nonvanishing, and $g^{L,R}_e = v \pm a$ are the left- and right-handed electron couplings, respectively. However, recalling that $\delta_{\gamma} = 0$ in the case of anomalous trilinear gauge boson couplings, using the experimental value of $s^2_W \simeq 0.23$, one has for such a model the very characteristic feature

$$\Delta \sigma^L \simeq \left(1 - \frac{1}{2s^2_W}\right) \Delta \sigma^R = -1.17\Delta \sigma^R, \quad (41)$$

where the explicit expressions of $g^L_e$ and $g^R_e$ have been used. If, on the contrary, the effect is due to a model with a $Z'$, no a priori relationship exists between $\Delta \sigma^L$ and $\Delta \sigma^R$. Accordingly, from inspection of these two quantities, if they are found not to be related by Eq. (41) to a given confidence level, one would conclude that the observed effect should be due to the general extra $Z$ discussed in Sect. 2. Then, depending on the actual values of the experimental deviations, a determination of the two parameters $\Delta_{\gamma}$ and $\Delta_Z$ might be carried on.

Actually, if the deviations of $\sigma^{L,R}$ satisfy the correlation Eq. (41), a small residual ambiguity would remain. Although the possibility that in a model with both $\Delta_{\gamma}$ and $\Delta_Z$ nonvanishing the correlation Eq. (41) is satisfied just by chance seems rather unlikely, one cannot exclude it a priori. Should this be the real situation, further analysis, e.g., in the different final fermion-antifermion channel would be required. The discussion of this essentially unlikely case can be performed, but is beyond the purpose of this paper.
4 Concluding remarks

We have shown in this paper that the availability of longitudinal electron beam polarization at the LC would be very useful for the study of the most general model with one extra $Z$ from an analysis of the final $W^+W^-$ channel. In principle, it would also be possible to discriminate this model from a rather ‘natural’ competitor one where anomalous gauge boson couplings are present. This could be done by analyzing suitable experimental variables, all defined in the same $W^+W^-$ final channel.

The interesting property of polarized observables in the $W^+W^-$ channel should be joined to analogous interesting features that are characteristic of polarization asymmetries in the final two-fermion channel, whose general discussion has been presented recently [27].

All these facts allow us to conclude that polarization at the LC would be, least to say, a highly desirable opportunity.

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Appendix

Limiting to CP conserving couplings, to generally describe the deviations from the SM of the cross section for process (1) of interest here, we have to account for the effect of six parameters, i.e., the five anomalous gauge couplings ($\delta_Z$, $x_V$ and $y_V$) of Eq. (26) plus the deviation $\delta_\gamma$ possible in the Z' model. Moreover, in this case the index $i$ in Eq. (11) runs from 1 to 11.

Referring to the expression of polarized differential cross section $d\sigma^\pm$ in Eq. (11), one can easily realize that the $F_i$ there are conveniently expressed in terms of the combinations of anomalous coupling constants defined as follows:

$$\delta_V^\pm = \delta_\gamma - \delta_Z g_e^\pm \chi; \quad x_V^\pm = x_\gamma - x_Z g_e^\pm \chi; \quad y_V^\pm = y_\gamma - y_Z g_e^\pm \chi,$$

where

$$g_e^+ = v - a = \tan \theta_W; \quad g_e^- = v + a = g_e^+ \left(1 - \frac{1}{2s_W^2}\right),$$

and $v$, $a$ and the $Z$ propagator $\chi$ have been previously defined in Sect. 2 with regard to Eqs. (2) and (3). Introducing the combination

$$g_s^\pm = 1 - \cot \theta_W g_e^\pm \chi,$$

we have:

$$F_0^- = \frac{1}{16s_W^4}; \quad F_1^\pm = \left(g_s^\pm + \delta_V^\pm\right)^2; \quad F_2^- = -\frac{1}{2s_W^2} \left(g_s^- + \delta_V^+\right),$$

$$F_3^+ = \left(g_s^+ + \delta_V^+\right) x_V^+ \simeq g_s^+ x_V^+; \quad F_4^+ = \left(g_s^+ + \delta_V^+\right) y_V^+ \simeq g_s^+ y_V^+,$$

$$F_6^- = -\frac{1}{4s_W^2} x_V^-; \quad F_7^- = -\frac{1}{4s_W^2} y_V^-,$$

$$F_9^\pm = \frac{1}{2} \left(x_V^\pm\right)^2; \quad F_{10}^\pm = \frac{1}{2} \left(y_V^\pm\right)^2; \quad F_{11}^\pm = \frac{1}{2} \left(x_V^\pm y_V^\mp\right).$$

All other $F$’s vanish. Clearly, the polarized cross sections will depend on on the anomalous parameters as $\sigma^+ = \sigma^+(\delta_V^+, x_V^+, y_V^+)$ and $\sigma^- = \sigma^-(\delta_V^-, x_V^-, y_V^-)$.

With $t = M_W^2 - \frac{s}{2} (1 - \beta_W \cos \theta)$, and $\beta_W$ defined previously, the corresponding kinematical coefficients $O_i(s, \cos \theta)$ that appear in Eq. (11) are [20]:

$$O_0 = 8 \left[\frac{2s}{M_W^2} + \frac{\beta_W^2}{2} \left(\frac{s^2}{t^2} + \frac{s^2}{4M_W^4}\right) \sin^2 \theta\right],$$

7 The general expansion including also CP violating couplings can be found, e.g., in [20].
\[ \mathcal{O}_1 = \frac{\beta_W^2}{8} \left[ \frac{16s}{M_W^2} + \left( \frac{s^2}{M_W^2} - \frac{4s}{M_W^2} + 12 \right) \sin^2 \theta \right], \]

\[ \mathcal{O}_2 = 16 \left( 1 + \frac{M_W^2}{t} \right) + 8\beta_W^2 \left[ \frac{s}{M_W^2} + \frac{1}{16} \left( \frac{s^2}{M_W^4} - \frac{2s}{M_W^2} - \frac{4s}{t} \right) \sin^2 \theta \right], \]

\[ \mathcal{O}_3 = \frac{\beta_W^2 s^2}{2M_W^4} \left[ 1 + \frac{6M_W^2}{s} - \left( 1 - \frac{2M_W^2}{s} \right) \cos^2 \theta \right]; \]

\[ \mathcal{O}_4 = \frac{4\beta_W^2 s}{M_W^2}, \]

\[ \mathcal{O}_6 = \frac{\beta_W s^3}{2tM_W^4} \left[ -\beta_W \left( 1 + \frac{6M_W^2}{s} \right) + \left( 1 + \frac{4M_W^2}{s} - \frac{16M_W^4}{s^2} \right) \cos \theta \right. \]

\[ \left. + \beta_W \left( 1 - \frac{2M_W^2}{s} \right) \cos^2 \theta - \beta_W^2 \cos^3 \theta \right], \]

\[ \mathcal{O}_7 = \frac{4\beta_W s^2}{tM_W^4} \left[ -\beta_W + \left( 1 - \frac{2M_W^2}{s} \right) \cos \theta \right], \]

\[ \mathcal{O}_9 = \frac{\beta_W^2 s^2}{2M_W^4} \left[ 1 + \frac{2M_W^2}{s} - \left( 1 - \frac{2M_W^2}{s} \right) \cos^2 \theta \right], \]

\[ \mathcal{O}_{10} = \frac{\beta_W^2 s^2}{M_W^4} \left[ 1 + \frac{M_W^2}{s} - \left( 1 - \frac{M_W^2}{s} \right) \cos^2 \theta \right]; \]

\[ \mathcal{O}_{11} = \frac{2\beta_W^2 s}{M_W^2} \left( 1 + \cos^2 \theta \right). \quad (A5) \]

Then, defining

\[ C = \frac{2M_W^2 - s}{2\sqrt{s}} = -\frac{1 + \beta_W^2}{2\beta_W}; \quad L_{FB} = \ln \frac{C^2 - 1}{C^2}, \]

defining the coefficients \( \mathcal{O}_{i,FB} \) in Eq. (A4) can be expressed as:

\[ \mathcal{O}_{0,FB} = 32 \left[ \frac{1}{C} + CL_{FB} \right]; \quad \mathcal{O}_{2,FB} = 4 \left[ \beta_W + \frac{4M_W^2}{s} \left( 2 + \frac{M_W^2}{s} \right) L_{FB} \right], \]

\[ \mathcal{O}_{6,FB} = \mathcal{O}_{7,FB} = 32 \frac{M_W^2}{s\beta_W} L_{FB}. \quad (A6) \]

Finally, defining

\[ L_{CE}(z^*) = \ln \frac{C + 1}{C - 1} - 2\ln \frac{C + z^*}{C - z^*}, \]

defining the coefficients \( \mathcal{O}_{i,CE}(z^*) \) in Eq. (A5) have the following expressions:

\[ \mathcal{O}_{0,CE} = 32 \left[ (2z^* - 1) \left( \frac{s}{M_W^2} - 1 \right) + \left( z^* - \frac{z^*}{3} - \frac{1}{3} \right) \frac{\beta_W^2 s^2}{8M_W^4} \right. \]

\[ \left. - 2z^* \frac{C^2 - 1}{C^2 - z^{*2}} + 1 - CL_{CE}(z^*) \right], \]
\[ O_{1,CE} = \beta_W^2 \left[ 4 \left( 2z^* - 1 \right) \frac{s}{M_W^4} + \frac{1}{2} \left( z^* - \frac{z^{*3}}{3} - \frac{1}{3} \right) \left( \frac{s^2}{M_W^4} - \frac{4s}{M_W^2} + 12 \right) \right], \]
\[ O_{2,CE} = 2 \left( 2z^* - 1 \right) \left( 20 + \frac{8 \beta_W^2 s}{M_W^4} - \frac{8 M_W^2}{s} \right) + \frac{2 \beta_W^2 s^2}{M_W^4} \left( 1 - \frac{2 M_W^2}{s} \right) \left( z^* - \frac{z^{*3}}{3} - \frac{1}{3} \right) \]
\[ -16 \frac{M_W^2}{s} \left( 2 + \frac{M_W^2}{s} \right) \frac{1}{\beta_W} L_{CE}(z^*), \]
\[ O_{3,CE} = \frac{\beta_W^2 s^2}{M_W^4} \left[ (2z^* - 1) \left( 1 + \frac{6 M_W^2}{s} \right) + \frac{1}{3} \left( 1 - 2z^{*3} \right) \left( 1 - \frac{2 M_W^2}{s} \right) \right], \]
\[ O_{4,CE} = \frac{8 \beta_W^2 s}{M_W^4} (2z^* - 1), \]
\[ O_{6,CE} = \frac{2}{3} \left( 1 - 2z^{*3} \right) \frac{\beta_W^2 s^2}{M_W^4} + 2 \left( 2z^* - 1 \right) \frac{s^2}{M_W^4} \left( 1 + \frac{4 M_W^2}{s} - \frac{16 M_W^4}{s^2} \right) \]
\[ -\frac{32 M_W^2}{s} \frac{1}{\beta_W} L_{CE}(z^*), \]
\[ O_{7,CE} = \frac{16 s}{M_W^4} \left[ (2z^* - 1) \left( 1 - \frac{2 M_W^2}{s} \right) - \frac{2 M_W^4}{s^2} \frac{1}{\beta_W} L_{CE}(z^*) \right], \]
\[ O_{9,CE} = \frac{\beta_W^2 s^2}{M_W^4} \left[ (2z^* - 1) \left( 1 + \frac{2 M_W^2}{s} \right) + \frac{1}{3} \left( 1 - 2z^{*3} \right) \left( 1 - \frac{2 M_W^2}{s} \right) \right], \]
\[ O_{10,CE} = \frac{2 \beta_W^2 s^2}{M_W^4} \left[ (2z^* - 1) \left( 1 + \frac{M_W^2}{s} \right) + \frac{1}{3} \left( 1 - 2z^{*3} \right) \left( 1 - \frac{M_W^2}{s} \right) \right], \]
\[ O_{11,CE} = \frac{8 \beta_W^2 s}{M_W^4} \left( z^* + \frac{z^{*3}}{3} - \frac{2}{3} \right). \] (A7)
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**Figure captions**

**Fig. 1** Allowed bands for $\Delta \gamma$ and $\Delta Z$ (95% CL) from $\sigma^L$ and $\sigma^R$ at $\sqrt{s} = 500 GeV$ and $L_{int} = 50 fb^{-1}$, and combined allowed domain. Also the domain determined by $\sigma^{unpol}$ alone is reported.

**Fig. 2** Same as Fig. 1, with the straight lines (19) for the $\eta$, $\chi$ and $\psi$ models superimposed.

**Fig. 3** Thick solid contour: allowed domain (95% CL) in the $(\phi, M_{Z\prime})$ plane for the $\eta$ model from process (1) at $\sqrt{s} = 0.5 TeV$. Dotted lines: constraints from the $Z - Z'$ mass-matrix relation (18) with the upper limit $\Delta M = 200 MeV$. Thin solid contours: constraints for the particular case of Eq. (23) with $\sigma = 0$ and $\sigma = \infty$ in Eq. (24). The current limit on $M_2$ from direct searches and that expected from $e^+e^- \rightarrow l^+l^-$ at $\sqrt{s} = 0.5 TeV$ with polarized electrons are also indicated.

**Fig. 4** Statistical significance in $x_V^+$ of the observables $\sigma A_{CE}^+$ of Eq. (36) and $P^+$ of Eq. (38) with $z^* = 0.5$.

**Fig. 5** Statistical significance in $y_V^+$ of the observables $\sigma A_{CE}^+$ of Eq. (36) and $P^+$ of Eq. (38) with $z^* = 0.5$.

**Fig. 6** Statistical significance in $x_V^-, y_V^-$ of the variable $Q^-$ of Eq. (37).
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