Drops in the wind: their dispersion and COVID-19 implications

Mario Sandoval and Omar Vergara
Department of Physics, Universidad Autónoma Metropolitana-Iztapalapa, Mexico City 09340, Mexico.

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Most of the works on the dispersion of droplets and their COVID-19 (Coronavirus disease) implications address droplets’ dynamics in quiescent environments. As most droplets in a common situation are immersed in external flows (such as ambient flows), we consider the effect of canonical flow profiles namely, shear flow, Poiseuille flow, and unsteady shear flow on the transport of spherical droplets of radius ranging from 5 µm to 100 µm, which are characteristic lengths in human talking, coughing or sneezing processes. The dynamics we employ satisfies the Maxey-Riley (M-R) equation. An order-of-magnitude estimate allows us to solve the M-R equation to leading order analytically, and to higher order (accounting for the Boussinesq-Basset memory term) numerically. Discarding evaporation, our results to leading order indicate that the maximum travelled distance for small droplets (5 µm radius) under a shear/Poiseuille external flow with a maximum flow speed of 1 m/s may easily reach more than 250 meters, since those droplets remain in the air for around 600 seconds. The maximum travelled distance was also calculated to leading and higher orders, and it is observed that there is a small difference between the leading and higher order results, and that it depends on the strength of the flow. For example, this difference for droplets of radius 5 µm in a shear flow, and with a maximum wind speed of 5 m/s, is seen to be around 2 m. In general, higher order terms are observed to slightly enhance droplets’ dispersion and their flying time.

I. INTRODUCTION

So far, COVID pandemic is still growing, as of 18:34pm Central European Time (CET), February 6 2021, there have been 104, 956, 439 corroborated cases of COVID-19, encompassing 2, 290, 488 deaths, reported to the World Health Organization (WHO). It is believed that airborne transmission is one of the main mechanisms for COVID spreading. The dynamics that droplets are breathing, sneezing, coughing or simply talking. Unfortunately, most of the reported literature neglects the effect of ambient flows on droplets transport. These flows are often present in daily activities such a wind flows, ventilation generated currents in offices, homes, malls, among other public places.

Recent studies suggest that talking may be among the most dangerous mechanisms for generating infected droplets. According to Tan and Bourouiba, speech induced plumes can travel 1.3 m in 2 s or even 8 m, which is a distance way longer than the 2 m social distancing. Abkarian and Stone using high-speed imaging showed that pronouncing consonants (typical to many languages) such as ‘Pa’, ‘Ba’, and ‘Ka’ are potent aerolization mechanisms. Abkarian et al using theory, experiments, and simulations documented the flow generated after speaking and breathing, which is in fact the responsible for droplets’ transport. Notice that the 2 m social distancing, only considered quiescent flow. This situation is not always satisfied in daily conditions, where wind is frequently present either naturally or induced by air condition in buildings or even by simple motion of people. Further evidence of airborne transmission of COVID-19 disease possible enhanced by air currents are: A reported infection of 96 people out of 216 working in a eleventh floor in a call center of South Korea, a singing rehearsal in Washington, where 53 singers were infected even they were located in a volleyball court but ventilation air currents were present. A study precisely on the effectiveness of air ventilation and COVID implication suggests that only certain type of ventilation called ‘displacement ventilation’ properly designed to extract contaminated hot air, could be the most effective air condition mechanism to reduce the risk of infection. Related works where droplets produced by breathing, sneezing, coughing, or talking, and immersed in external flows are few. Some examples are Cummins et al who studied micrometric spherical droplets in the presence of a source-sink flow, which simulated a scenario where droplets are produced and subjected to an extraction mechanism (air condition). Incorporating evaporation, humidity, and an uniform external wind, Dbnouk and Drikakis presented a conjugated heat and flow transfer problem (occurring in a saliva droplet) and coupled to a CFD analysis. They proposed a transient Nusselt number and varied relative humidity (RH), temperature, and speed of flow. Contrary to past studies, they conclude that evaporation of droplets is enhanced at low RH and high temperatures. As an example, they results indicate that in a cloud of droplets in an environment at RH=50%, temperature $T < 30 ^\circ C$, and under an external flow of 4 km/h, there will not be evaporation. Their simulation however, only reached five seconds, hence the full dispersion of droplets could not be reached. B. Blocken et al also performed a CFD analysis for people exhaling while walking or running, and emanating from them possible infected droplets. However, their simulation only considered droplets of radius 20 µm and beyond. As it has been observed that there is a small difference between the leading and higher order results, and that it depends on the strength of the flow. For example, this difference for droplets of radius 5 µm in a shear flow, and with a maximum wind speed of 5 m/s, is seen to be around 2 m. In general, higher order terms are observed to slightly enhance droplets’ dispersion and their flying time.

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a) Electronic mail: sem@xanum.uam.mx
recently been observed, the smallest the size of a droplet, the more dangerous it may be.\(^{13}\) Feng et al.\(^ {13}\) located two virtual humans 3.05m apart and let one human to eject droplets while coughing or sneezing. Using a computational particle fluid dynamics model that considered evaporation, external wind and condensation, and even Brownian motion, they found that at this distance, potentially infected droplets easily reach hair and face of the other human. They also performed a study on the filtration efficiency of several masks. In conclusion, most of the previous works suggest that the safe distancing is not enough under wind conditions.

In this work, we consider micrometrical spherical droplets emanating from a person while talking either normally (exit speed of droplets around 1 m/s) or strongly (exit speed of droplets around 5 m/s),\(^ {10}\) and under the presence of an external flow \(\mathbf{u}(r,t)\). FIG. 1. Schematic of the problem studied. A person talking either normally (exit speed of droplets around 1 m/s) or strongly (exit speed of droplets around 5 m/s),\(^ {10}\) and under the presence of an external flow \(\mathbf{u}(r,t)\).

II. PHYSICAL MODEL

Let us analyze the motion of noninteracting spherical droplets of mass \(m\), radius \(a\), in an environment with air density \(\rho_a\) and air viscosity \(\mu_a\), under gravity \(\mathbf{g}\), and subject to an external flow of the form \(\mathbf{u}(r,t)\), with a characteristic speed \(U_{\text{max}}\), where \(r(t) = (x,y,z)\) is the droplet’s position, here \(t\) represents time. In this study we will be considering three flows namely shear, Poiseuille, and unsteady shear flow. Droplets immersed in these profiles will have a characteristic speed \(v(t)\) which decreases as the droplets fall due to gravity. With the latter physical quantities a Reynolds number (\(Re\)) can be defined as \(Re = \rho_a(U_{\text{max}} - v)\alpha/\mu_a\) which satisfies \(Re < 1\). Therefore, these droplets individual dynamics described by their translational velocity, \(\mathbf{v}(t) = (v_x, v_y, v_z)\), follows the Maxey-Riley equation\(^ {13}\)

\[
m\frac{d\mathbf{v}}{dt} = (m - m_a)\mathbf{g} + m_a\frac{D\mathbf{u}}{Dt}\bigg|_r - \frac{m_a}{2}\frac{d}{dt}\left(\mathbf{v}_T + \frac{a^2}{15}\nabla^2\mathbf{u}\right)_r - R_T\mathbf{v}_T - R_T a\frac{\mathbf{v}_T(0)}{\sqrt{\pi}a}\]

\[
- R_T a\int_0^t d\tau \frac{d\mathbf{v}_T}{d\tau} \frac{d\tau}{\sqrt{\pi}a(t-\tau)},
\]

where \(m_a\) is the mass of air displaced by the sphere, \(v_a = \mu_a/\rho_a\) represents the kinematic viscosity, \(R_T = 6\pi a\mu\) is the resistance coefficient; while \(\rho\) indicates the droplet’s density. In addition, the following definitions are included:

\[
\mathbf{v}_T = \mathbf{v} - \mathbf{u}_r - \frac{a^2}{6}\nabla^2\mathbf{u}\bigg|_r
\]

\[
\frac{D\mathbf{u}}{Dt} = \frac{\partial\mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u},
\]

\[
\frac{d\mathbf{u}}{d\tau} = \frac{\partial\mathbf{u}}{\partial \tau} + \mathbf{v} \cdot \nabla \mathbf{u}.
\]

The forces on the right hand side of Eq. (1) are respectively, the droplet’s weight; buoyancy force: forces in the undisturbed flow due to local pressure gradients; the added or virtual mass force; drag force; and the last two terms represent the Boussinesq-Basset memory force. To find the order-of-magnitude of each term in the Maxey-Riley Eq. (1), one can introduce the following dimensionless quantities \(\mathbf{r} = r/a, \mathbf{v} = \mathbf{v}/U, \mathbf{t} = (U/a)t\), where the characteristic speed is defined as \(U = mg/R_T\), which is the terminal velocity of a falling droplet. After applying these dimensionless variables to Eq. (1), one arrives at

\[
Re \frac{d\mathbf{v}}{dt} = -\frac{9\lambda}{2}(1 - \lambda)\mathbf{k} + \lambda Re \frac{D\mathbf{u}}{Dt}\bigg|_r - \frac{\lambda Re}{2}\frac{d}{d\tau}\left(\nabla^2\mathbf{u}\right)_r - \frac{9\lambda}{2}\mathbf{v}_T - \frac{9\lambda\sqrt{Re}}{2\sqrt{\pi}}\mathbf{v}_T(0)
\]

\[
- \frac{9\lambda\sqrt{Re}}{2\sqrt{\pi}}\int_0^\mathbf{t} d\mathbf{v}_T \frac{d\tau}{\sqrt{(\mathbf{t} - \tau)}}.
\]
The latter result belongs to the so-called overdamped approximation, where inertia does not play a role, and particles immediately reach the external flow speed. Alternatively, by keeping inertia and neglecting terms of order $\sqrt{\text{Re}}$ and higher, the Maxey-Riley equation can be rewritten as

$$\frac{d\mathbf{v}}{dt} = -\mathbf{g} (\mathbf{k} + \mathbf{v}_T).$$

This equation allows us to observe the dynamics of droplets at short times.

### A. Shear and Poiseuille flows as external wind

Let us solve Eq. (8) under the presence of shear and Poiseuille flows modeling the effect of wind on the spherical droplets. These profiles are given by

$$\mathbf{u} = \left( U_{\max} \frac{z}{h} + U_0 z (z - h) \right) \mathbf{j}. \quad (9)$$

Notice that the shear flow profile correspond to the case $U_0 = 0$. Given Eq. (9), we solve Eq. (8) whose solution including dimensions and subject to $\mathbf{r}(0) = (0, 0, h)$ and $\mathbf{v}(0) = (v_{x0}, v_{y0}, v_{z0})$ can be shown to be

$$v_x(t) = v_{x0} e^{-Kt}, \quad v_z(t) = \gamma_z e^{-Kt} - \frac{g}{K}, \quad (10)$$

$$v_y(t) = \gamma_0 e^{-Kt} - U_0 \left( \frac{\gamma_y}{K} \right)^2 e^{-2Kt} + U_0 \frac{\gamma_y g}{K} t e^{-Kt} - A t e^{-Kt} + B - Ct + U_0 \frac{g^2}{K^2} t^2, \quad (11)$$

where $K = R_T/m$ and $\beta = K a^2 U_0/3$. Integrating the
velocities, we get
\[ x(t) = -\frac{v_{z0}}{K} e^{-Kt} + s_x, \quad z(t) = -\frac{\gamma z}{K} e^{-Kt} - \frac{g}{K} t + s_z, \]
\[ y(t) = \frac{1}{K} \left( A - \gamma y - \frac{2U_0 \gamma z g}{K^3} \right) e^{-Kt} \]
\[ + \frac{1}{K} \left( A - \frac{2U_0 \gamma z g}{K^2} + \frac{U_0 \gamma z g}{K^2} t^2 e^{-Kt} + \frac{U_0 \gamma z g}{2K^2} e^{-2Kt} + s_y + Bt - \frac{C}{2} t^2 + \frac{U_0 g^2}{3K^2} t^3 \right), \]
where constants \( A, B, C, \gamma, \gamma_z, s_x, s_y, s_z \) are defined in Appendix IX.

**B. Unsteady shear flow as external wind**

A more realistic situation, is to consider the fact that air flows for a while, and then stops, and then flows again. The simplest model is to assume a time-dependent shear flow scenario. This unsteady vector flow field \( \mathbf{u}(\mathbf{r}, t) \) should satisfy the time-dependent Navier-Stokes equations, which after assuming \( \mathbf{u}(\mathbf{r}, t) = (0, u_y(z, t), 0) \), incompressibility, and a null pressure, reduce to
\[ \frac{\partial u_y}{\partial t} = v_a \frac{\partial^2 u_y}{\partial z^2}. \]

This parabolic equation must satisfy \( u_y(z, 0) = (U_{\text{max}}/h) z, u_y(0, t) = 0 \), and \( u_y(h, t) = U_{\text{max}}/2(1 + \cos \omega t) \). Its solution can be verified to be
\[ u_y(z, t) = \sum_{m=1}^{\infty} D_m g(t) \sin \frac{m\pi z}{h} + \frac{U_{\text{max}}}{2h} (1 + \cos \omega t) z, \]
where
\[ g(t) = e^{-d_m t} \frac{d_m}{\omega + \omega t}, \]
\[ D_m = \frac{\omega f_m}{a_m}, \]
\[ f_m = (-1)^{m+1} \omega U_{\text{max}}/m\pi, \]
\[ d_m = v_a \left( \frac{m\pi}{h} \right)^2. \]

Once Eq. [15] is available, it is plugged into Eq. [8] and its solution subject to \( \mathbf{r}(0) = (0, 0, h) \) and \( \mathbf{v}(0) = (v_{z0}, v_{y0}, v_{z0}) \) although very lengthy, can be analytically found. The dominant terms of the solution along the \( y \)-component for long times read.
FIG. 4. (a)-(c) Effect of higher order terms on the dispersion along the $y$-direction of spherical droplets as a function of radius $a$ and an external flow. Here $\Delta y = y_{MB} - y_m$, where $y_{MB}$ is the maximum travelled distance by a droplet along the $y$-direction containing first order terms ($a \leq 20 \mu m$) or the full terms in the M-R equation ($a > 20 \mu m$), whereas $y_m$ represents the solution directly obtained from Eqs. (12) and (13). (d) Effect of higher order terms on the flying time of spherical droplets as a function of radius $a$. Here $\Delta t_F = t_{F_B} - t_F$, where $t_{F_B}$ is the flying time by a droplet containing first order terms ($a \leq 20 \mu m$) or the full terms in the M-R equation ($a > 20 \mu m$); whereas $t_F$ represents the the solution directly obtained from Eqs. (12). (e)-(f) Linear convergence of $\Delta y = y_{MB} - y_m$ as the time-step in the simulations decreases. (e) Convergence for a droplet of radius 30$\mu m$ after discretizing the full M-R equation 5. (f) Convergence for a droplet of radius 19$\mu m$ after discretizing the overdamped M-R equation 22.

\[
v_y(t) = -\frac{U_{max}g}{2hK}t - \frac{U_{max}g[K\cos(\omega t) + \omega \sin(\omega t)]}{2h(K^2 + \omega^2)}t.
\]

(20)

\[
y(t) = \frac{U_{max}}{2h}\left\{ \frac{g}{K}\frac{t^2}{2} - \frac{g}{2K}t^2 \right. \\
- \left. \frac{g}{(K^2 + \omega^2)}\left[ \frac{K}{\omega}t\sin(\omega t) - t\cos(\omega t) \right] \right\}.
\]

(21)

Once Eq. (12), Eq. (13), and Eq. (21) are available, we can now exemplify a typical droplets’ dispersion (cloud) after a person talks and generates micrometric droplets (radius between 5$\mu m$ to 100$\mu m$), which are subject to either a shear flow (Fig. 3(a)), a Poiseuille flow (Fig. 3(b)), or an unsteady shear flow (Fig. 3(c)). We simulate the situation of a normal and a strong talk by imposing an exit speed of droplets (from a cone shape of velocities representing a person’s mouth) of $v_0 = 1m/s$ and $v_0 = 5m/s$, respectively. The cloud is made of 1000 droplets of random size between 5$\mu m$ to 100$\mu m$. Additionally, we impose a moderate ($U_{max} = 5m/s$) and a calm ($U_{max} = 1m/s$) wind scenario. The cone shape as the initial velocity distribution, and as observed from experiments 10, is implemented by imposing $\psi(0) = (v_0 \cos \varphi_0 \sin \theta_0, v_0 \sin \varphi_0 \sin \theta_0, v_0 \cos \theta_0)$, whose angular polar and azimuthal initial extremum are set to $\theta_{0\text{max}} = 110^\circ$, $\theta_{0\text{min}} = 81^\circ$, $\varphi_0 = 110^\circ$, and $\varphi_0 = 81^\circ$. The case of a Poiseuille flow profile considers for a calm flow $\{U_{max}, U_0\} = \{1m/s, -0.4m^{-1}s^{-1}\}$ whereas for a moderate flow $\{U_{max}, U_0\} = \{5m/s, -2m^{-1}s^{-1}\}$. The simulations for the unsteady flow profile consider $\omega = 0.5s^{-1}$. Finally, ($\varphi, \theta$) will randomly vary between their extremum. The results can be visualized in Fig. 3 which shows the distribution of droplets at three different times namely, $t = 0.08s, t = 0.4s,$ and $t = 1.33s$. A color code indicating the droplets’ sizes is also introduced. The safe distancing is indicated as a vertical dotted-black line. Clearly, droplets move beyond the safe distancing. As it can be seen, droplets under a moderate wind and after only 1.33s, are already 6m away from the person’s mouth. Droplets under a calm wind and less than 50$\mu m$ in radius will overpass the safe distancing in the next second. These results indicate a potential danger for people in a public space since the safe distancing has been easily surpassed. We finally take Eq. (12), Eq. (13), and Eq. (21) to find the maximum travelled distance ($y_M$) along the $y$-direction as a function of droplets’ size, as well as a function of different external flows namely, shear, Poiseuille, and unsteady shear flow. The results are shown in Fig. 3(a)-(c). Figures 3(a)-(c) show that droplets under a Poiseuille profile reached the the longest distance compared to the other analyzed profiles. This figure also indicates that for $U_{max} = 5m/s$, droplets
of radius 5 µm, and under a Poiseuille flow, can travel up
until 2000 m, while around 1500 m under a shear flow. As
expected, the condition of having a wind blowing and
ceasing (unsteady flow) reduces the droplets maximum
dispersion to around 750 m for \( U_{\text{max}} = 5 \text{ m/s} \). On
the other hand, our results to leading order indicate that
the maximum travelled distance for small droplets (5 µm ra-
dius) under a shear/Poiseuille external flow with a maxi-
mum speed of \( U_{\text{max}} = 1 \text{ m/s} \) may easily reach more than
250 meters. The long distance achieved by these small
droplets is because they remain in the air for about 600
seconds, see Figs. 4(d)-(e). In these figures, the flying
time obtained from Eq. (12), is shown as a solid-black
line; while the red circles represent the approximated
equation \( t_F \approx 9 \mu g h / 2 \rho g a^2 \). These figures also
indicate that the largest (100 µm radius) droplets can only stay in
the air for about 1.5 s.

It is worth mentioning that all the
studied droplets \((a \in [5 \mu m, 100 \mu m])\) reached a constant
vertical terminal velocity \( U \approx mg / R_T \).

### IV. DYNAMICS WITH BOUSSINESQ-BASSET MEMORY

In the literature, the Boussinesq-Basset (B-B) mem-
ory force has been less frequently considered, probably
because of its order-of-magnitude and because of the re-
quired computational effort. However, there exist some
theoretical\[5,17-21\], numerical\[22-23\] and experimental\[23-27\]
works dealing with this force. In this section we analyze
the effect of the memory force term on the dispersion an
time of spherical droplets.

Consider first small droplets ranging between \( a \in
[5 \mu m, 20 \mu m] \). According to Table I, their motion can
be modeled by the M-R equation under the overdamped
approximation. However, by keeping terms of order \( \sqrt {Re} \)
to see the effect of the B-B memory force, Eq. (5) reads

\[
\nabla_T = -(1 - \lambda) \mathbf{k} - \frac{\sqrt {Re}}{\pi} \int_0^T \frac{d \nabla_T}{d \tau} \frac{d \tau}{(t - \tau)}.
\]

To solve this integro-differential equation, a first order
Euler method is chosen. Under this method, a component
of the B-B force term can be shown to be:

\[
\int_0^T \frac{d \tau}{\sqrt{(t - \tau)}} = \sum_{i=1}^{k-1} \frac{v_{i+1} - v_i}{\sqrt{\Delta \tau}} \alpha(k, i),
\]

where we have assumed that \( \Delta \tau = \Delta \tau \) and defined
\( \alpha(k, i) = 2 \sqrt{k - i} - 2 \sqrt{k - 1 - i} \). After certain
steps, one can prove that the overdamped M-R Eq. (22),
on the z-direction and in dimensional variables, acquires
the following discrete form for \( k = 3, \ldots N \):

\[
v_{z,k} = -\sqrt{\Delta \tau} \frac{c_0}{c_1} + \frac{c_2}{c_1} v_{z,k-1} - \frac{c_3}{c_1} \sum_{i=1}^{k-2} (v_{z,i+1} - v_{z,i}) \alpha(k, i),
\]

where constants \( C_0, C_1, C_2, C_3 \) are defined in Appendix X.
A similar expression as Eq. (24) is obtained for the other
spatial components. In the case of larger droplets and
from Table I one notices that the order-of-magnitude of all
terms in Eq. (5) is the same. Therefore, one has to
solve for the full M-R Eq. (5). Using the Euler method
together with Eq. (25), one can show that the discretized
\( z \)-component of Eq. (5) in dimensional variables reads

\[
v_{z,k} = -\frac{\Delta t}{b_1} Rg + \frac{b_2}{b_1} v_{z,k-1} - \frac{b_3}{b_1} \sum_{i=1}^{k-2} (v_{z,i+1} - v_{z,i}) \alpha(k, i),
\]

where constants \( R, b_0, b_1, b_2, b_3 \) are defined in Appendix X.
The discretization of the other components in Eq. (5) is
similar to Eq. (25). After posing Eq. (24) and Eq. (25),
we are ready to find the droplets’ dynamics under higher
order terms.

Because of the external flows we have chosen and from
the simulations in Sec. II, we observe that the dynami-
cs mostly occurs along the \( z \)-\( y \) plane, and that initial
conditions are not relevant for long times (at least for \( v_T(0) = 0 \));
thus from now on, a 2D problem with initial
conditions \( v_z(0) = 0 \) and \( v_y(0) = U_{\text{max}} \) will be consid-
ered. Equations (5) and (22) are then numerically solved
using the discretized scheme in Eq. (24) and Eq. (25),
under the presence of a shear flow (the other flows ba-
sically share the same features), and for two typical exit
initial speeds while talking respectively, \( 1 \text{ m/s} \) and \( 5 \text{ m/s} \).
The time-step used for solving Eq. (5) and Eq. (22)
was \( 4 \times 10^{-4} \text{s} \) and \( 5 \times 10^{-3} \text{s} \), respectively. The results
of the simulations are shown in Figs. 4(a)-(c). In these
figures, \( \Delta y = y_{MB} - y_t \), where \( y_{MB} \) is the maximum
travelled distance along the \( y \)-direction by droplets of size

![FIG. 5. Effect of the Boussinesq-Basset memory force term on the sedimentation velocity \((v_z(0) = 0)\), for two spherical
droplets reaching their terminal velocity \( U \). The black-solid lines indicate an exponential decay towards \( U \) when the B-B
term is absent. The red-dashed lines indicate a \( t^{-1/2} \) decay
when the B-B term is present.](image-url)
\(a \leq 20\mu m\) and obtained from Eq. [22]. For droplets of size \(a > 20\mu m\), \(y_{MB}\) represents the maximum travelled distance along the \(y\)-direction obtained after solving the full M-R equation [1]; whereas \(y_{MR}\) represents the solution directly obtained from Eqs. [12] and [13]. One can observe that for a low wind speed, the effect of higher order terms barely enhance the droplets’ dispersion. However, for a wind speed of \(5m/s\) and for the smallest considered droplet, higher order terms can increase its dispersion around \(2m\). Figures [4]a)-(c) also indicate that as the size of the droplets increases, higher order terms effects become smaller until they finally disappear.

The effect of first order terms and the full terms in the M-R equation, on the flying time of spherical droplets as a function of the radius \(a\) is also analyzed. Using Eq. [12], the numerical solutions from Eq. [24] and Eq. [25], and defining \(\Delta t_F = t_{FB} - t_F\); where for \(a \leq 20\mu m\), \(t_{FB}\) represents the flying time calculated using first order terms (Eq. [22]). For \(a > 20\mu m\), \(t_{FB}\) represents the flying time calculated using the full terms in the M-R equation. On the other hand, \(t_F\) is the flying time directly obtained from Eqs. [12]. This analysis is shown in Fig. [3]d). It can be seen that \(\Delta t_F\) increases as the droplets’ sizes decrease. In fact, for \(a = 5\mu m\) there is a 0.8s flying time difference between the dynamics of Eq. [5] and Eq. [8]. This extra time also contributes to the observed enhancement of dispersion of small droplets. A numerical analysis on the convergence of \(\Delta y = y_{MB} - y_M\), as the time-step \(\Delta t\) in the simulations decreases, is also performed. Fig. [4]f) shows this convergence for a droplet of radius \(30\mu m\) after discretizing the full M-R equation [6]. As expected, a linear convergence can be appreciated. The convergence for a droplet of radius \(19\mu m\) and after using the overdamped M-R equation [22] is shown in Fig. [3]b). Once again a linear convergence is achieved. Therefore, the latter analysis validates our employed first order numerical algorithm.

Finally, the implications of the Boussinesq-Basset memory force term on the sedimentation velocity component \(v_z(t)\) with \(v_z(0) = 0\) is also studied. The numerical results are shown in Fig. [5] which illustrates the dynamics of \(v_z(t) + U\) versus time in a log-log plot, and for two different spherical droplets reaching their terminal velocity \(U\). The black-solid lines belong to an exponential decay of \(v_z(t)\) towards \(U\); whereas the red-dashed lines indicate a \(t^{-1/2}\) decay. It can be observed that for short times, \(v_z(t)\) exponentially decays towards \(U\); however, \(v_z(t)\) decays according to the scaling \(t^{-1/2}\) for long times. This is a rather surprising result, since the order-of-magnitude of the B-B term is really small. This \(t^{-1/2}\) decay of the sedimenting velocity has been also recently reported[23]. Further consequences of the B-B term on the motion of particles at low Reynolds numbers may be search for in future investigations.

![Fig. 6](image_url)

**V. DISCUSSIONS AND CONCLUSIONS**

According to Dbouk and Drikakis[11] and others, evaporation and relative humidity play a role on droplets’ dispersion. These factors may reduce or increase the size of droplets and their cloud shape as it travels. Therefore, our results could be improved to account for a rocket-like dynamics (drops varying mass). Following Dbouk and Drikakis[11] who considered a conjugated flow-heat-mass transfer problem and CFD dynamics, it can be concluded that evaporation of droplets takes place at high relative humidities, and high temperatures. Thus for the case of countries with an annual average of relative humidity, \(RH = 50\%\), a wind speed of \(4km/h = 1.1m/s\), a temperature less than \(30C\), and five seconds later since a cloud of droplets originated, there will be a null evaporation[11] Other works also supporting a long time survival of infected droplets is Stadnytskyi et al.[28].

Based on this information, Fig. [6]a) shows four droplets’ distributions (cloud) five seconds later since the cloud originated under a shear (S) and an unsteady shear flow (UNS) at calm (1m/s) and moderate winds (5m/s). The rest of the parameters are the same as in Fig. 2. Under these parameters, together with \(RH = 50\%\) and temperature les than \(30C\), evaporation does not play a role[11]. (b) Comparison of droplets’ position under an uniform flow \(\{h, U_{max}, v_0\} = \{h = 1.7m, 1.1m/s, 1m/s\}\) either using single particle dynamics (this work), or using a more elaborated CFD analysis[11]. (c) Droplets’ distribution at \(t = \{1s, 2s, 3s, 4s, 5s\}\), and linear paths followed by some droplets of different sizes under an uniform flow. See Fig. 2 for color code.
however, the smallest droplets under a calm wind (1 m/s) for both \(UNS\) and \(S\), have surpassed the social distancing (vertical black-dashed lines). The same droplets under a moderate wind (5 m/s) for both \(UNS\) and \(S\), reached 15 m and 25 m, respectively. Thus from this figure, one can explicitly visualize a more real scale to which people may be in risk of contagion. Fig. 6(b) compares the position of droplets’ distribution (cloud) under an uniform flow \(\{h, U_{max}, v_0\} = \{h = 1.7 m, 1.1 m/s, 1 m/s\}\), either using single particle dynamics (this work) or employing a more elaborated CFD analysis\(^{[1]}\). Both methods result in a similar cloud’s position. Finally, using the latter parameters, the droplets’ distribution at \(t = \{1 s, 2 s, 3 s, 4 s, 5 s\}\) and some paths followed by droplets of different sizes are shown in Fig 6(c). These paths have a linear behavior since the cloud dynamics is practically overdamped, implying \(y(t) \approx U_{max} t\) and \(z(t) \approx h - U t\), thus particles will follow the function \(z \approx h - \frac{U}{U_{max}} y\).

In summary, using single particle dynamics, which has the advantage of requiring a minimum computational cost, this paper provided an estimate of the maximum dispersion of micrometric droplets generated after talking. Briefly, under conditions of a null evaporation and only five seconds later since droplets were originated, it was found that an unsteady shear calm wind (1 m/s) can disperse droplets beyond the social distancing, and until more than 15 m when droplets are subject to a moderate unsteady shear flow (5 m/s). As expected, an unsteady shear profile is less efficient to disperse droplets than a constant Poiseuille or shear profile. These constants profiles modeling a calm wind (1 m/s) were found to provide a maximum dispersion beyond 250 m for the smallest droplets (5 \(\mu m\)). The effect of the Boussinesq–Basset force term was also analyzed. Although its order-of magnitude is small, it was found to be enough to change the behavior of the sedimentation velocity (from exponentially decaying towards its terminal velocity, to proportionally decaying as \(t^{-1/2}\)) of a micrometric particle and slightly increase droplets’ dispersion and their flying time.

Future research would be to consider external flows in the presence of buildings and to find the complex streamlines generated and their effects on droplets’ distribution. It may be inferred for example that the presence of a corner on a common street, could generate stagnation points, that may risk areas of infection, since those points could storage for a while infected droplets. Furthermore, walls may induce three-dimensional flows that may drag particles away from the floor, thus increasing its flying time and hence their capability of traveling longer. We hope this study helps people to be more aware about the effect of daily wind currents on the propagation of potentially infected droplets. Based on our findings of droplets easily dispersing beyond the social distancing when subject to wind currents, we recommend the use of masks\(^{[5]}\) to contain the virus, as well as flex seal googles, since droplets dragged by wind may reach eyes. We also recommend to wash all your wearing clothes and shoes, and to shower after being out from home, since infected droplets may be attached to clothes or hair\(^{[13]}\). Direct exposure to wind currents, mainly in crowded cities, should also be avoided.

VI. AUTHOR CONTRIBUTIONS

All authors contributed equally to this research.

VII. ACKNOWLEDGEMENTS

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VIII. DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

IX. APPENDIX 1: CONSTANTS ADDED

The constants \(A, B, C, \gamma_y, \gamma_z, s_x, s_y, s_z\) appearing in Eqs. (10)–(13) are defined as

\[
A = \gamma_z \left( \frac{U_{max}}{h} + U_0 (2s_z - h) \right),
\]

\[
B = U_0 s_z^2 + \frac{\beta}{K}
\] + \left( U_0 \left[ \frac{2g}{K^2} - h \right] + \frac{U_{max}}{h} \right) \left( \frac{g}{K^2} + s_z \right),
\]

\[
C = \frac{g}{K} U_0 \left[ 2 \left( \frac{g}{K^2} + s_z \right) - \frac{U_{max}}{h} \right],
\]

\[
\gamma_y = v_{y0} + U_0 \left[ \left( \frac{\gamma_z}{K} \right)^2 - s_z^2 \right]
\]

\[
- \left( U_0 \left( \frac{2g}{K^2} - h \right) + \frac{U_{max}}{h} \right) \left( \frac{g}{K^2} + s_z \right) - \frac{\beta}{K},
\]

\[
\gamma_z = v_{z0} + \frac{g}{K}, s_x = \frac{v_{y0}}{K}, s_z = \frac{v_{z0}}{K}, \quad s_x = h + \frac{\gamma_z}{K},
\]

\[
s_y = -\frac{1}{K} \left[ \frac{A}{K} - \gamma_y + \frac{U_0 \gamma_z}{K^2} \left( \frac{\gamma_z}{2} - \frac{2g}{K} \right) \right].
\]
X. APPENDIX 2: NUMERICAL PART FOR THE B-B EQUATION

The constants $c_0, c_1, c_2, c_3$ appearing in Eq. \( (24) \) are defined as

$$c_0 = \frac{g (m - m_a)}{R_T}, \tag{32}$$

$$c_1 = \frac{2a}{\sqrt{\pi u_a}} + \sqrt{\Delta t}, \tag{33}$$

$$c_2 = \frac{2a}{\sqrt{\pi u_a}}, c_3 = \frac{c_2}{2}, \tag{34}$$

whereas constants $R, b_1, b_2, b_3$ appearing in Eq. \( (25) \) are

$$R = \frac{(1 - \lambda)}{D} \text{ with } D = 1 + \lambda/2, \tag{35}$$

$$b_1 = \left(1 + \sqrt{\Delta t} \frac{2Ka}{D\sqrt{\pi u_a}}\right), \tag{36}$$

$$b_2 = \sqrt{\Delta t} \frac{2Ka}{D\sqrt{\pi u_a}} - \Delta t \frac{K}{D} + 1, \tag{37}$$

$$b_3 = \sqrt{\Delta t} \frac{Ka}{D\sqrt{\pi u_a}}. \tag{38}$$

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