Chaotic Time Series Prediction Using LSTM with CEEMDAN

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Abstract. Chaotic systems are complex dynamical systems that play a very important role in the study of the atmosphere, aerospace engineering, finance, etc. To improve the accuracy of chaotic time series prediction, this study proposes a hybrid model CEEMDAN-LSTM which combines Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) and long short-term memory (LSTM). In the model, the original time series is decomposed into several intrinsic mode functions (IMFs) and a residual component. To reduce the difficulty of predicting chaotic time series and provide a high level of predictive accuracy, the LSTM prediction model is built for all each characteristic series from CEEMDAN deposition. Finally, the final prediction results are obtained by combining all the prediction sequences. To test the effectiveness of this model we proposed, we examined the CEEMDAN-LSTM model using the Lorenz-63 system. Further compared to Autoregressive Integrated Moving Average (ARIMA), Support Vector Regression (SVR), multilayer perceptron (MLP), and the single LSTM model, the results of the experiment show that the proposed model performs better in the prediction of chaotic time series. Besides, the hybrid model proposed in this paper has better results than the LSTM model alone. Therefore, hybrid models based on deep learning methods and signal decomposition methods have great potential in the field of chaotic time series prediction.

1. Introduction
The prediction of chaotic systems is one of the most important areas of research in recent years [1,2]. The fundamental characteristic of chaotic systems is their extreme sensitivity to initial values, which determines the long-term unpredictability of changes in the system, making it impossible to predict chaotic time series over long periods [3]. However, the certainty of the chaotic system, i.e., the small degree of dispersion of the motion trajectory in the short term, determines its short-term predictability [4]. Currently, the prediction of chaotic systems is accomplished in two main ways. One is that over the years, many scientists have built models of mechanisms such as differential equations by analyzing the internal laws of natural systems and simplifying the main features of natural systems. This approach requires a large reserve of expert knowledge. Another approach to prediction is the machine learning approach based on data-driven, i.e. using historical data sets to build predictive models. In recent years, deep learning, as a new research direction in the field of machine learning, has gained significant advantages in different application areas [5-7]. Among the many deep learning models, the Recurrent Neural Networks (RNN) introduce the concept of time step length and show greater adaptability in the analysis of time-series data. Since recurrent neural networks (RNNs) tend to ignore
long-term dependencies within time series and have problems such as gradient explosion, the Long short-term memory (LSTM) [8], a variant of RNNs, has emerged with a special memory structure and gate structure that enhances long-term dependent memory. Numerous studies show that Long short-term memory (LSTM) is an effective method for achieving a chaotic time series prediction [9-11]. Therefore, this paper uses LSTM as a prediction algorithm to construct a chaotic time series prediction model.

In recent years, the “decomposition before reconstruction” framework has achieved good results in many forecasting areas, such as PM2.5 prediction, financial time series prediction, and long-term streamflow forecasting. Among the many data decomposition methods, the ensemble empirical mode decomposition (EEMD) inherits the advantages of EMD (empirical mode decomposition) and solves the problem of mode mixing of EMD, while complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) [12] further improves based on EEMD, not only overcoming the problem of low decomposition efficiency but also the reconstruction error is nearly zero, more suitable for the decomposition of nonlinear time series. Therefore, in this study, we chose CEEMDAN to decompose the time series.

Based on CEEMDAN and LSTM, this paper proposes a hybrid model, which first pre-processes the chaotic time series through CEEMDAN and then predicts its components through LSTM. The final forecast is obtained by integrating the component forecasts. We apply this model to predict the Lorenz-63 system and compare the predicted results with other predictive models. The experimental results show that the model proposed in this paper is significantly superior in chaotic time series prediction compared to other models.

2. Proposed Method

Previous studies have found that CEEMDAN has advantages in time series decomposition, while LSTM does well in the long time series prediction. Therefore, in this paper, we integrated these two methods and proposed a hybrid approach, so-called CEEMDAN-LSTM, for forecasting chaotic time-series prediction. The proposed CEEMDAN-LSTM includes three stages: decomposition, individual forecasting, and ensemble. In the first stage, we use the CEEMDAN method to decompose the original time series of the chaotic system into \( k + 1 \) components, including \( k \) IMF and a residual component. Of these components, some exhibit high-frequency characteristics while others exhibit low-frequency characteristics. In the second stage, we build a forecasting model for each component using long short-term memory (LSTM). Then we use the model built to make predictions for each component. So we ended up with predictions for different frequency components. In the third stage, we aggregate the forecast results of all components as the final result. Although there are many ways to aggregate the forecast results for all components, in this study we add up the forecast results for all components using equal weights. The flowchart of the CEEMDAN-LSTM is shown in Figure 1.

![Figure 1 The flowchart of the proposed CEEMDAN-LSTM.](image-url)
3. Experiments and Results

3.1 Datasets and Pre-process
To measure and compare the forecasting performance of different models, the chaotic time series of the Lorenz-63 system is employed as the sample data set for the experiment. Lorenz-63 system described a model where fluid flows in a container whose top and bottom are cooled, then heated to create a temperature similar to the atmosphere. Specifically, Lorenz-63 system, which is a set of coupled ordinary differential equations with three components, is given by:

\[
\begin{align*}
\frac{dx}{dt} &= \sigma(y - x) \\
\frac{dy}{dt} &= x(\rho - z) - y \\
\frac{dz}{dt} &= xy - \beta z
\end{align*}
\]  

(1)

where \(x\), \(y\), and \(z\) are the state variables and \(\sigma = 10\), \(\beta = 8/3\), and \(\rho = 28\). The study of the Lorenz-63 system has important implications for the discovery and understanding of the characteristic properties of sensitivity to the initial condition for a deterministic system [13,14]. Therefore, the Lorenz-63 system has been extensively studied as an important prototype for studying the special properties of chaotic systems. We set the initial state of this chaotic system as \(x(0) = -0.2028\), \(y(0) = 3.5418\), \(z(0) = 25.0873\), the time step is 0.01, and the integration time length is 50, and we finally got 5000 sample data.

3.2 Parameters Details
Our implementation was based on the Tensorflow framework [15]. To prevent the problem of gradient explosion during training, we made the model training more robust by reducing the learning rate and increasing the number of batches. The process of model training is to minimize the loss function, so after defining the loss function, setting up a suitable optimizer to solve the parameter optimization problem is very important for experimental results. There are a great number of optimization algorithms to choose from in current deep learning libraries, such as Stochastic Gradient Descent (SGD), Adaptive Moment Estimation (Adam), and root mean square prop (RMSProp). The ideal optimizer can not only get the best model as fast as possible with the training samples but also prevent overfitting. To choose the best optimizer, we have carried out comparative experiments, which compared the mean squared error (MSE) loss of different optimizers when training the LSTM model. From comparative experiments, we found that the Adam method is the best. Adam has been widely used as an effective stochastic optimization method. Therefore, Adam optimizer is used in the process of training the model, which can converge to good results faster. The Rectified Linear Unit (ReLU) does not have any backpropagation error compared to the sigmoid function. And for larger neural networks, it is faster to build models based on ReLU. Therefore, when selecting the activation function in the experiment, we chose ReLU as the activation function. In this study, the input of LSTM is 100 × 1 data. What's more, we applied dropout to the LSTM and set the dropout rate to 0.1 to prevent overfitting.

3.3 Analysis of Experimental Results
First, the CEEMDAN method is used to decompose the original chaotic time series. The amplitude and ensemble number of the added noise are 0.4 and 500, respectively. Figure 2 shows the IMFs and the residual(trend item) of the chaotic time series decomposed by CEEMDAN. The result contains 1 original sequence, 12 IMF components, and 1 trend item.
Figure 2 The decomposition on the x-axis of the Lorenz series by CEEMDAN.

It can be seen that the original sequence of variable x in the Lorenz-63 system exhibits highly nonlinear and nonstationary, the fluctuating frequency of the IMF component is gradually reduced from IMF1 to IMF12. The left of the first line of Figure 2 displays the original chaotic time series of variable x in the Lorenz-63 system. First, the different LSTM prediction model was built for each of the 12 IMF components and one trend item respectively, and the forecast results for each component were obtained. Next, the forecast results for each component were summed with equal weights to produce the final forecast results for the original variable.

A prediction result for the lorenz-63 system made by the CEEMDAN-LSTM is shown in Figure 3. The black dash-dot curves are historical data. The red dots are predictions from the CEEMDAN-LSTM. The black curves are the true state value of the Lorenz-63 system. As can be seen from Figure 3, based on the historical data of 2 units of Lyapunov time, we can predict the trajectory of about 4 units of Lyapunov time [16-18] using the CEEMD-LSTM prediction model. In other words, with short-term historical information as input, the CEEMD-LSTM prediction model can predict the trajectory of chaotic dynamic systems over longer periods.
To study the performance of the proposed method, a rigorous quantitative analysis of the prediction and true values was performed, and the specific statistical results are shown in Table 1.

| Model            | RMSE | MAE  | MAPE  |
|------------------|------|------|-------|
| ARIMA            | 6.014| 5.631| 46.2% |
| SVR              | 4.352| 3.379| 40.3% |
| MLP              | 3.541| 2.617| 32.6% |
| LSTM             | 2.042| 1.527| 21.4% |
| CEEMDAN-LSTM     | 1.327| 1.124| 11.9% |

According to the specific statistical results, we can find that the model proposed in this paper has a smaller error in the prediction results and has obvious advantages over other models.

4. Conclusions
To improve the performance of predicting chaotic time series, a hybrid prediction model combining CEEMDAN and LSTM, called CEEMDAN-LSTM, is proposed in this paper. The method decomposes a complex chaotic time series into several components using the CEEMDAN method and then predicts each component separately using an LSTM-based prediction model. In the end, the sum of the predicted results of all the components is taken as the final forecast result. The results of the forecasting experiments on the Lorenz-63 system show that the proposed hybrid model outperforms the other forecasting models in the experiment. From the experimental results, we conclude that the “Decomposition and ensemble” framework can significantly improve the performance of chaotic time series prediction. Reasonable decomposition results can significantly improve the forecast performance, and the choice of CEEMDAN parameters has a significant impact on the forecast results. Therefore, in future studies, we will focus on how to select reasonable parameters efficiently during the time series decomposition process.

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