Introducing Nine-Point Circle to Junior High School Students

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Abstract. The concept of circles is an ancient concept that has appeared since Ancient Egypt from which this concept gives many significant contributions in mathematics’ development until now. Nevertheless, the concept of circles hides many uncover mysterious features that are of applications in mathematics. One of the mysterious features is the Nine-Point Circle. This Nine-point circle is also known as Euler's circle, six-point circle, Feuerbach's circle, the twelve-point circle, and many others. Because of these different names, there have been misunderstand among mathematicians about the Nine-Point Circle’s history. Besides, the discussion of Nine-Point Circle can be used to be an initial material to explain elementary geometry topic in junior high school’s level curriculum of 2013. Therefore, this concept needs to be delivered to the students as a geometry introduction. A possible form of the integration historical aspect of Nine-point circle is suggested in this paper as well as its importance in the curriculum of 2013.

1. Introduction

Based on the government regulation issued by the of ministry educational and cultural of Republic Indonesia number 24 in 2016 about curriculum, the curriculum to be used for junior high school is Curriculum 2013[1]. Curriculum 2013 is a curriculum that covers three important domains namely cognitive domain, psychomotor domain, and affective domain. In the recent Curriculum 2013, the mathematics textbook has implemented content with perspective of history in mathematics. This historical content is implemented in order to make sure that the mathematics concept is taught meaningfully. Nevertheless, the improvement of mathematics curriculum in Curriculum 2013 has omitted several competencies and mathematics materials. This development should be administered properly such that important competencies would not disappear.

In general, the integrating of history of mathematics (HOM) in classroom mathematics has become an issue in mathematics education research [2]–[5]. One of the implementation of history of mathematics in the classroom practice is the integration of HOM in the mathematics text book. In the mathematics textbook, the perspectives of history may be presented into three ways. They are (1) the perspectives of history are attached in the beginning part of chapter before the main part as motivational resources, (2) the perspectives of history are attached in the main part of chapter as conceptual development activities resource, and (3) the perspectives of history are attached in the end part of chapters alternative fun game resources[6]. In addition, the perspective of history can also be used for introducing some mathematics material in school. One of historical content that can be introduced is Nine-Point Circle. Nine-Point Circle is a circle that passing through nine significant concyclic points defined from any triangle. The related material of Nine-Point Circle to junior high school material is the topic of circumcircle of triangle. Unfortunately, this material has been removed from the textbook and curriculum basic
competence. However, in trigonometry material of senior high school mathematics, the concept of triangle circumcircle is used for proving sine rule. Therefore, the circumcircle of triangle needs to be taught to student before teaching about the sine rule in senior high school.

As the preparation for the teaching the circumcircle of triangle, student needs to know the construction of Nine-Point Circle. Nine-Point Circle can be constructed by the special line of triangle, altitude line, which has been taught in triangle material of 7th grade. With the fact that the Nine-Point Circle construction can be introduced before the teacher explanation about circumcircle of triangle in 8th grade in circle topic, easier content could be accepted by the students and restore the assumption of Nine-Point Circle historical discovery to student. This paper would discuss the opportunity to improve the material of excircle of triangle using HOM in the recent curriculum 2013.

2. Method

Literature review is used in this study to develop possible implementation of Nine-Point Circle. The materials are collected by reviewing and combining some related sources of Nine-Point Circle and school mathematics through books, journals, proceedings, and internet. these collected materials are processed into some steps as shown in diagram bellow (Figure 1):

![Figure 1. Steps to processed the collected materials](image)

In the first step, the basic competence of senior and junior high school was reviewed to elicit information about suitable material for the topic of Nine-Point Circle. Review is conducted from not only the basic competence stated on the textbook, but also the delivery package of the textbooks. Subsequently, both of basic competence and books are compared to elicit the correlation between mathematics textbooks with the basic competence. In this step, all materials, questions, and quiz are also reviewed. Then, the last step is finding suitable place for inserting Nine-Point Circle as introduction based on review results. In this paper is also added High Order Thinking Skills concept related to curriculum 2013 as the government suggests to exist in every material.

3. Results

3.1 Construction of Nine-Point Circle

As the introduction of circumcircle material, students can construct Nine-Point Circle which is used to explore problems related to altitude line and perpendicular bisector of triangle in 7th grade. The steps of Nine-Point Circle’s construction as follows:

- a. Draw a triangle ABC. (see Figure 2)
- b. Construct the midpoints of the three sides. Label them as L, M, N. (see Figure 3)
c. Construct the altitudes of the triangle ABC. Label the intersections of them with the triangle as D, E, F. Label the point of intersection of the three altitudes as H. (see Figure 4)

d. Construct the midpoints of the segments AH, HB, CH. Label them as X, Y, Z. (see Figure 5)

e. Find the center of the Nine-Point Circle. Construct triangle LMN and find intersection point of three perpendicular bisectors of triangle LMN as the center of Nine-Point Circle. Label the center as U. (see Figure 6)

f. Draw circle passing through point L, M, N, D, E, F, X, Y and Z. (see Figure 7)

3.2 Properties

3.2.1 Medial triangle and orthic triangle

The Nine-Point Circle of a reference triangle is the circumcircle of both the reference triangle's medial triangle (with vertices at the midpoints of the sides of the reference triangle) (see Figure 8) and its orthic triangle (with vertices at the feet of the reference triangle's altitudes) (see Figure 9).

This properties help students to conclude the definition of circumcircle that “circumcircle is circle that is outside the triangle and passing through all of vertices of the triangle”[9].
3.2.2 The center point and the radius of Nine-Point Circle

The center point of Nine-Point Circle is also the center point of circumcircle. The radius of a triangle's circumcircle is twice the radius of that triangle's Nine-Point Circle. (see Figure 10)

Based on this property, student can discover the center point and the radius of circumcircle. Then, they can directly construct the circumcircle.

3.3 History

The earliest author to whom the discovery of the Nine-Point Circle has been attributed is Euler, but no one has ever given a reference to any passage in Euler's writings where the characteristic property of this circle is either stated or implied [10]. Historical development of the Nine-Point Circle appeared first in 1804 until 1860 which covered a wide range of territory such English, French, German, and Swiss. Start from 1804 Bevan's theorem appears in Leybourn's Mathematical Repository. First enunciation of Feuerbach's Theorem, including the first published proof, appears in Karl Wilhelm Feuerbach's EigenschafteneinigermerkwürdigenPunkte des geradlinigenDreiecks, along with many other interesting proofs relating to the Nine-Point Circle in 1822 discovered as the former Nine-Point Circle. However, Feuerbach only found the six point circle. These points are the midpoints of the three sides of the triangle and the feet of the triangle perpendicular bisectors (MA, MB, MC, HA, HB, and HC). The circle is officially designated the "Nine-Point Circle" (le cercle des neufpoints) by Olry Terquem in 1842. Terquem published the second analytical proof of the theorem that the Nine-Point Circle touches the incircle and the circumcircle of triangle. He was the first person that recognized the added significance of the three midpoints between the triangle's vertices and the orthocenter. And he also called the circle as the Nine-Point Circle.
4. Discussion

Permendikbud Republic Indonesia number 24 in 2016 states that curriculum 2013 (K13) is the official curriculum used in elementary school, junior high school, and senior high school [1]. This government regulation declares the structure and basic design of curriculum 2013. There are core competence (KI) and basic competence (KD) that should be taught to the students. In curriculum 2013, 2 core competences and 60 basic competences for mathematics’ subject in junior high school is stated to be taught. Furthermore, students need to improve their High-Order Thinking to satisfy knowledge aspect. Therefore, problems that require student to think creatively and analytically are needed.

Basic competence (KD) 3.11 of curriculum 2013 for 7th grade students of junior high school is associate the formula of circumferences and areas of various types of rectangles (square, rectangle, rhombus, parallelogram, trapezoid, and kit) and triangle. Understanding special lines of triangle is part of this competence. In mathematics’ textbook for junior high school student 7th grade 2nd semester 2016 revise edition, this material is taught in learning activity 8.6 in which the definition of altitude line, median line, perpendicular bisector, angle bisector, and how to construct them are included [7]. The use of historical perspective in mathematics’ learning, giving non-routine problems can improve motivation and intellectual wealth of students [8]. Therefore, based on the historical aspect of Nine-Point Circle, the material can be developed as non-routine problem for students after learning about special lines of triangle. Students can construct Nine-Point Circle by applying how to construct altitude line and perpendicular bisector that they have learned. In detail, before students construct it, history of Nine-Point Circle can be inserted to gain the student interest.

The related material of Nine-Point Circle to junior high school material is circumcircle of triangle’s material. Unfortunately, this material has been removed from the textbook and curriculum basic competence. Removing incircle and circumcircle in curriculum 2013 is not appropriate because they will be used as important prerequisite topic for the student in senior high school. In 11th grade of senior high school, basic competence (KD) 3.9 state that the students are able to explain sine and cosine rule. In proofing these rules, the students need to use the idea of circumcircle (see the Figure 11 for the example of problem and Figure 12 for the solution of the problem). Therefore, circumcircle material needs to be taught in Junior High School as the prerequisite before sine and cosine is taught. This circumcircle material is suggested to be inserted to curriculum inside circle chapter before basic competence (KD) 3.8 explain and construct interior tangent line and exterior tangent line of two circle. Then, Nine-Point Circle can be inserted as introduction of this new basic competence (KD).

![Figure 11. Problem in mathematics textbook 11th grade (obtained from [7])](image-url)
5. Conclusion

The perspective of history also can be used to introduce some mathematics material in school, for instance Nine-Point Circle. Nine-Point Circle is a circle that passing through nine significant concyclic points defined from any triangle. The related material of Nine-Point Circle to junior high school material is circumcircle of triangle’s material. Unfortunately, this material has been removed from the textbook and curriculum basic competence. Nevertheless, in trigonometry material of senior high school mathematics are found that the excircle of triangle concept is used to prove sine rule. Therefore, this excircle material is suggested to be inserted to curriculum inside circle chapter before basic competence (KD) 3.8. Then, Nine-Point Circle can be inserted as introduction of this new basic competence (KD).

References
[1] Kemendikbud, Peraturan Pemerintah Kementerian Pendidikan dan Kebudayaan (Permendikbud) Republik Indonesia nomor 24 tahun 2016. Indonesia, 2016.
[2] L. Radford, Of Love, Frustration, and Mathematics: A Cultural-Historical Approach to Emotions in Mathematics Teaching and Learning, From beliefs to dynamic affect systems in mathematics education, 2015, pp. 25–49.
[3] M. Doorman and J. van Maanen, A historical perspective on teaching and learning calculus, Aust. Sr. Math. Teach., vol. 22, no. 2, pp. 4–14, 2008.
[4] A. S. Ozdemir, S. Goktepe, and I. Kepceoglu, Using Mathematics History to Strengthen Geometric Proof Skills, Procedia - Soc. Behav. Sci., vol. 46, pp. 1177–1181, 2012.
[5] B. Savizi, Applicable problems in the history of mathematics: practical examples for the classroom, Teach. Math. its Appl., vol. 26, no. 1, pp. 45–50, Mar. 2007.
[6] C. Tzanakis, A. Arcavi, C. C. de Sa, M. Isoda, C. Lit, M. Niss, J. P. de Carvalho, M. Rodriguez, and M.-K. Siu, Integrating History of Mathematics in the Classroom: An Analytic Survey, Hist. Math. Educ. ICMI Study, pp. 201–240, 2000.
[7] Kemendikbud, Matematika untuk SMP/MTs kelas VII: Buku Guru, seri edisi lengkap. Jakarta: Kemendikbud, 2013.
[8] U. T. Jankvist, A categorization of the ‘whys’ and ‘hows’ of using history in mathematics education, Educ. Stud. Math., vol. 71, no. 3, pp. 235–261, 2009.
[9] G. A. Venema, Exploring Advanced Euclidean Geometry with GeoGebra USA. USA: Mathematical Association of America, Inc, 2013.
[10] J. S. MacKay, History of the Nine Point Circle. Proceedings of the Edinburgh, 1892. [Online]. Available:http://jwilson.coe.uga.edu/emt668/emt668.folders.f97/anderson/geometry/geometr y1project/historyofninepointcircle/history.html.