The relationships within the mathematical content of teachers’ lesson sequences

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Abstract. This study explored how mathematics content is carried through by means of the problems presented during lessons. Following the definitions and the coding criteria from the TIMSS 1999 Video Study, a total of 163 mathematics problems were identified in the video-recorded lesson sequences of four Bruneian mathematics teachers teaching at the Year 8 level. These problems were classified according to the four basic kinds of relationships: mathematically related, thematically related, repetition and unrelated. Drawing on the mathematical content of the teachers’ lesson sequences, the findings revealed variations among the mathematical problems coded as repetition and thematically related, between the four Brunei classes. The aggregated results obtained from the four classes highlighted several points of discussion, such as the relatively higher proportion of repetition problems (52%) from one teacher in particular; the percentage similarities of thematically related problems for all four classes (ranging from 26% to 33%); and the incredibly varied results for mathematically related problems across the four Brunei classes.

1. Introduction
A teacher can attest that what happens when they teach is not just about conveying new or even recalling previously taught information to students. A lot of planning will occur prior to stepping foot in the classroom, for instance, how best to teach a specific topic, which relevant resources to use as reference, what examples to show, the amount of exercises to give and so on [1-6]. The same can be said about teaching mathematics. Much of the content in a single mathematics lesson involves solving mathematical questions or problems assigned to students by the teacher.

From a researcher’s perspective, how we view the content of a mathematics lesson may help us understand the factors influencing the clarity and the coherence of the lesson. And one such influence is on how the problems are related [5-11]. A thematic approach to teaching can also provide coherence to the classroom occupants [12], and there may be obstacles in implementing mathematics courses based on a thematic approach [13]. It was suggested that if a thematic approach is to be practical, then the thematic application must serve the purpose by assimilating it fully across the syllabus [14].

The general aims of the 1998-2000 Third International Mathematics and Science Video Study (henceforward mentioned as the TIMSS 1999 Video Study) was to examine and provide teaching practice descriptions concerning Year 8 mathematics classrooms. The seven countries that participated in the TIMSS 1999 Video Study were Japan, Hong Kong SAR, the United States, Switzerland,
Australia, the Netherlands and the Czech Republic. In the TIMSS 1999 Video Study, random samples of Year 8 video-recorded mathematics lessons were collected from these seven countries. The six countries Japan, Hong Kong SAR, Switzerland, Australia, the Netherlands and the Czech Republic were considered as ‘high-achieving’ countries because of their relatively high achievement performance in comparison to the United States on the TIMSS 1995 Year 8 mathematics assessment [7, 15].

One of the features investigated in the TIMSS 1999 Video Study was on the mathematical content of lessons. Furthermore within this certain feature, the mathematics problem analysis specialist group coded [8] the mathematical relationship among all the problems that were presented during the lessons. In the TIMSS 1999 video study, the term ‘problem’ was used to indicate any mathematical questions assigned by the teacher to the students. According to Jacobs and her colleagues, “The main requirement of a problem was that a mathematical operation must be necessary in order to arrive at the intended answer, and the problem needed to require some degree of thought by an eight-grade student” [8]. Subsequently, each problem, with the exception of the first problem, was categorised using strictly one of the four basic types of relationships: mathematically related, thematically related, repetition and unrelated. The descriptions for the four kinds of relationships are provided in Figure 1.

**Figure 1.** Classifications of the four basic kinds of relationships (taken from [7]).

Furthermore, Hiebert et al. [5] reported that problems that are mathematically related links the content of the mathematics lesson through diverse mathematical connections, while problems classified as repetition involves students to think less since they were able to solve the preceding problem. In contrast, any coded unrelated problems imply lessons being split into mathematically unrelated fragments.

### 2. Method

This study explores in what way mathematics content is carried through by means of the mathematics problems presented during lessons. Although the design of the present study followed the style of the TIMSS 1999 Video Study [7, 8], for example in relation to the codings and its subsequent analyses, several aspects of the research design from the Learner’s Perspective Study [16] was also incorporated in the data collection approach, such as video-recording each competent teacher [17] for a relatively longer period.

In relation to the data collection approach, several continuous series of mathematics lessons, ranging from four to six consecutive lessons or lesson sequences, by four teachers teaching at the Year 8 level in two of the secondary schools in Brunei Darussalam were video-recorded [11, 18]. In the first school, teachers Andy and Henri each had 5 lessons that were video-recorded consecutively, and from the second school, Natalia with 4 and Yasmin had 6 lessons. Presented in Table 1 are the
characteristics of the classes and the lessons belonging to the four teachers. Among them, only Henri was considered more experienced, at the time of study, in terms of mathematics teaching experience and qualifications. For Andy in particular, he was teaching the Year 8 express class, which meant that his students completed three years of syllabus within two years. Furthermore, at the time of recording, all his lessons were on reviewing or revising the entire topic that had been covered.

Table 1. Characteristics pertaining to the four mathematics teachers.

| Class/Lesson characteristics | Andy         | Henri         | Natalia       | Yasmin       |
|------------------------------|--------------|---------------|---------------|--------------|
| Teaching experience (at time of study) | Two years | Six years | Two years | Two years |
| Qualification background | Bachelor* | Bachelor & Postgrad Cert | Bachelor* | Bachelor* |
| Classes responsible & Students’ ability (Express Class) | High-achieving | Average | High-achieving | High-achieving |
| Nature of lesson content | 5 AR | 2 HR & HN + 3 N | 2 HR & HN + 2 AR | 3 HR & HN + 2 N + 1 AR |
| Delivery pace | High | Relaxed | High | High |

NOTE: * denotes the field of the degree is specifically in Education. The nature of the lesson content with AR stands for ‘All Review’ lessons, HR & HN stands for ‘Half Review and Half New’ lessons, and N stands for ‘New’ lesson.

In total, twenty video-recorded lessons were collected for further analysis. Relevant supplementary documents were also obtained, such as photocopied lesson plans, notes and exercise worksheets given to their students and the mathematics syllabus. These collected materials were necessary in providing as reference guides in analysing the data. The topic areas covered by the teachers categorised in Table 2 were based on the five major and sub-category topics as listed by Hiebert et al. [7] in Figure 2.

- **Number**: Whole numbers, fractions, decimals, ratio, proportion, percent, and integers;
- **Geometry**: Measurement (perimeter and area), two-dimensional geometry (polygons, angles, lines, transformations, and constructions), and three-dimensional geometry;
- **Statistics**: Probability, statistics, and graphical representation of data;
- **Algebra**: Operations with linear expressions, linear equations, inequalities and graphs of linear functions, and quadratic and higher degree equations; and
- **Trigonometry**: Trigonometric identities, equations with trigonometric expressions.

Figure 2. The five major and sub-category topics (taken from Hiebert et al. [7, p 68]).

Table 2. The mathematics topic areas covered by the teachers.

| Topic Area       | Andy | Henri | Natalia | Yasmin |
|------------------|------|-------|---------|--------|
| Number           | ✓    | ✓     | ✓       | ✓      |
| Geometry         | ✓    | ✗     | ✓       | ✓      |
| Algebra          | ✓    | ✗     | ✓       | ✓      |
| Statistics       | ✓    | ✓     | ✓       | ✓      |
| Trigonometry     | ✗    | ✗     | ✗       | ✗      |
This paper focuses mainly in addressing the relationships of the mathematics problems presented by the four teachers in their lessons. A problem is defined as a statement that requires one to apply a mathematical operation to solve. Subsequently, using the definitions and the coding criteria and schemes provided in the technical report by the mathematics problem analysis specialist coding group [8] and the main report [7] of the TIMSS 1999 Video Study, initially there were 183 mathematical problems coded and identified from the present study’s video-recorded lesson sequences data and the collected relevant supplementary documents. However, in order to proceed in coding for relationship, Hiebert et al. [7] indicated, “The first problem in each lesson was not coded for a relationship because the coding scheme defined relationship in terms of problems that preceded a given problem, and no problem preceded the first problem”. Since the first problem in each lesson was not taken into account, the total number of mathematical problems to be analysed from the coded data was then reduced to 163. Each of the 163 problems was subsequently classified according to the four basic kinds of relationships. The classifications were done following the original definitions shown earlier in Figure 1. A reliability check was also conducted to confirm the agreement in the coding categories. The examples on how problems were coded as Repetition and Thematically Related are provided in Table 3.

**Table 3.** Examples of the two-dimensional geometry problems coded as Repetition and Thematically Related (from Andy’s fourth lesson).

| Problem | Description |
|---------|-------------|
| 1(a) In each of the following figures, O is the centre of the circle. Find the unknown length in each case. (All lengths are in cm) |
| (i) < Image of a circle with a known length of 4 and unknown lengths of 3 and 2 connected to point O> |
| 1a (i) Finding the unknown length ‘a’ is the first problem in Andy’s fourth lesson; hence it was not coded for a relationship. |
| (ii) < Image of a circle with a known length of 2 and unknown length of 2.5 connected to point O> |
| 1a (ii) Finding the unknown length ‘b’ is the second problem and it was coded as Repetition because it was essentially the same as the first problem. |
| 1(b) Find the unknown angle marked in each of the following figures where O is the centre of the circle. |
| (i) < Image of an angle marked x with a known angle of 110°> |
| 1b (i) Finding the unknown angle ‘x’ is the third problem in Andy’s fourth lesson and it was coded as Thematically Related because this third problem was related to the first and second problems above by means of it being of a similar topic (two-dimensional geometry problem) and the operation to solve this problem was different to preceding problems. |
| (ii) < Image of an angle marked y with a known angle of 25°> |
| 1b (ii) Finding the unknown angle ‘y’ is the fourth problem and it was coded as Repetition because this fourth problem was essentially the same as third problem. |
Displayed in Table 3 above are examples of four problems (under the topic two-dimensional geometry), taken from Andy’s fourth video-recorded lesson, coded as Repetition and Thematically Related. These examples were obtained from the ‘Revision Paper 8’ exercise worksheet depicted in the textbook by Tay et al. [19]. The examples shown in Table 3 are typical of the problems coded as Repetition or Thematically Related in the coded data, especially within the topics statistics and two-dimensional geometry.

3. Result and discussion

3.1 How mathematics is related within the lesson sequences

From the 163 mathematics problems within the twenty video-recorded lessons there were a total of 32 problems coded as ‘Mathematically Related’, while ‘Thematically Related’ and ‘Repetition’ each had 54, and ‘Unrelated’ was 23. Table 4 shows the breakdown of the mean (or average) percentage of problems for each lesson in relation to the four kinds of relationship for the four teachers studied in Brunei. From Table 4, apart from the thematically related results, significant differences can be seen between the results of individual teachers. As a reminder, the difference between Mathematically Related and Thematically Related was that a problem was coded as Mathematically Related because it was related to a preceding problem in a mathematically significant way (for example, using the solution of a previous problem to solve the next problem), whereas for a Thematically Related problem to be coded, it required a completely different procedure from the previous problem.

| Relationship                  | Classroom (N=163) |
|-------------------------------|-------------------|
|                               | Andy (n = 69)     | Henri (n = 19)   | Natalia (n = 31) | Yasmin (n = 44) |
| Mathematically Related        | 16                | 28               | 5                | 35              |
| Thematically Related          | 32                | 33               | 32               | 26              |
| Repetition                    | 52                | 39               | 35               | 31              |
| Unrelated                     | 0                 | 0                | 28               | 8               |

NOTE: ‘N’ stands for the overall number of mathematical problems (except the first problem in each video-recorded lesson) in all four classrooms and, ‘n’ is the overall number of mathematical problems (except the first problem in each video-recorded lesson) in each classroom. Due to rounding, the percentages may not amount to 100%. For each teacher, the average or mean percentage was calculated as the sum of the percentage within each lesson, divided by the number of lessons.

From Table 4, there are four identified points that will be discussed. Firstly, the relative proportion of Repetition problems for Andy was higher (52%) compared to the other three teachers because most of the mathematical problems that he distributed to his students in all five of his review lessons were related to a single topic (either ‘two dimensional geometry’, ‘statistics’ or ‘solutions and graphs of linear equalities and inequalities’). The operations to solve these problems were essentially the same (or mostly the same) as the previous problem (within the particular topic area). It seems reasonable that a revision lesson would have a higher concentration of repetition problems, so Andy’s high repetition proportion should not be unexpected. The difference between Andy’s lessons and the other three teachers’ lessons was that their lessons were a mixture of reviewing and introducing or practicing new content (refer also to the ‘Nature of lesson content’ in Table 1). However, consistent with earlier findings where the lessons were short involving fast-paced utterances or exchanges [11, 20], the students in the classrooms of these three teachers were given less opportunity to practice procedures for solving the mathematical problems [20, 21].

Secondly, the percentages of Thematically Related problems were very similar for all four classrooms. Further investigation revealed that 34 out of the 54 problems coded as Thematically Related belonged to the mathematics topics of two-dimensional geometry (n = 15, but for Andy only)
and statistics ($n = 18$, for all four teachers). The problems within these two topics were required to be solved using different types of procedures, for example, finding the mean, mode and median of a set of numbers (for statistics), or finding the image of an object under reflection, rotation or vector translation (for two-dimensional geometry). Additionally, referencing the coding criteria given in Table 3 above as examples, each of the 34 Thematically Related problems was coded based on the criteria of having an apparent thematic connection where “the problem was related to a preceding problem only by virtue of it being a problem of a similar topic” [7] or, “the problem required operations that were much different than the first. However the mathematics topic was similar” [8]. It should be noted that Repetition and Thematically Related are mutually exclusive codes, where for each problem, with the exception of the first problem, was categorised using strictly one of the four basic types of relationships. Based on this analysis, each teacher appeared to have a certain number of Thematically Related problems in lessons covering these two topics. Nonetheless, since the lessons were short and fast-paced, along with the students getting less mathematical work time in a week, this would indicate that the teachers had to expose their students to many different types of procedures with which to solve the mathematical problems.

The third point is that the results for Mathematically Related problems were incredibly varied across the four Brunei classrooms. Accordingly, 27 out of the 32 Mathematically Related problems were coded as ‘Extension’, a Mathematically Related code category that indicated “the problem required many of the same operations as the preceding problem plus some important additional operations” [8]. Andy and Yasmin typically gave two or three problems of this Mathematically Related-Extension code category type within their single topic lessons, whereas for Henri, a majority of the problems were confined within the topic ‘integers’. In contrast, there was only one Mathematically Related problem coded for Natalia.

For the final point, the result attained from Natalia’s video-recorded lesson was unusual because her lessons contained a much larger proportion of Unrelated tasks (28%) in comparison to the other three teachers. The reason was, unlike Andy’s review lessons where each review lesson had a specific topic that was covered, Natalia’s third and fourth review lessons consisted of several review problems from a variety of Year 8 topics. Thus, the revision problems were mathematically unrelated or entirely different operations had to be used in the process of solving the previous problems.

### 3.2 Mathematics problems related to a single topic

Similar to Hiebert et al. [7], another kind of investigation into the mathematical relatedness within lessons was carried out, but this time it was in relation to the mathematical topics and the shifts among topics. The number mathematics lessons for each of the four teachers containing problems associated to a one particular major topic area are presented in Table 5.

| Major Topic Area | Andy | Henri | Natalia | Yasmin |
|------------------|------|-------|---------|--------|
| Number           | 0    | 1     | 0       | 0      |
| Geometry         | 2    | 0     | 0       | 0      |
| Algebra          | 1    | 0     | 0       | 0      |
| Statistics       | 1    | 4     | 2       | 5      |
| Trigonometry     | 0    | 0     | 0       | 0      |
| **Total (N=16)** | **4**| **5** | **2**   | **5**  |
| **Total of video-recorded lessons (N=20)** | **5**| **5** | **4**   | **6**  |

The results reveal that a large percentage of the lessons (16 out of 20 lessons or 80%) included problems associated to a one specific topic. Thus indicating that at least 20% of the overall video-
recorded lessons contained problems associated to two topics or more. Although there were fewer shifts among the topics made by these teachers, what is of concern here is whether the topic shifts within a lesson may interrupt (or break) the flow of the mathematics content being taught. Hiebert et al. [7] stated “…parts of lessons can become unrelated when teachers switch from one topic to another…Of course, a teacher might also connect different topics mathematically or thematically”. Comparing the results obtained here with the number of unrelated problems for each lesson for all four teachers combined, the result is negligible. Thus, taking these results into consideration, the topic shifts within a lesson did not necessarily lead to content fragmentation mainly because a majority of the mathematics problems per lesson were related.

4. Conclusion
Drawing on the mathematical content of the teachers’ lesson sequences, the findings revealed variations among the mathematical problems coded as repetition and thematically related, between the four Brunei classes. The aggregated results obtained from the four classes highlighted several points of discussion, such as the relatively higher proportion of repetition problems (52%) from Andy’s lessons; the percentage similarities of thematically related problems for all four classes (ranging from 26% to 33%); and the incredibly varied results for mathematically related problems across the four Brunei classes. The additional question prompted from the above findings is, what are the advantages for students, in their mathematics lessons, if they are given more thematically related problems in comparison to mathematically related problems or repetition problems, and how does this affect or even contribute to the coherence and the clarity of the lessons in Brunei? Possibly, an advantage for having more thematically related problems in the lessons will be, if the mathematics content of the lesson is contained in a single topic, then students will be able to work on the different types of related-single-topic-problems given within that lesson, such as the difference between the examples given in Table 4. Nevertheless, to make full use of the mathematics content so as to contribute to the clarity and coherence of a lesson, perhaps, teachers should incorporate tasks that are mathematically related as well. By doing so, students will be able to see how those problems are related to one another in a more elaborate mathematically significant way [22, 23].

There were observations of a teacher’s practice that would have been misleading if only one of the lessons were video-recorded and analysed [11, 16-18, 24, 25]. For example, the results relating to the four kinds of relationships presented in Table 4 for each teacher would have been vastly different if they had not been averaged out over the five lessons. In the case of Andy, if only the results of his first video-recorded lesson were analysed, it would have shown 33%, 0%, 67% and 0% for the types Mathematically Related, Thematically Related, Repetition and Unrelated respectively, as opposed to 16%, 32%, 52% and 0% where each kind of relationship had been averaged out over his five lessons. However, as his lesson sequence progressed, variations were observed in the proportions of the mathematics problems coded as Mathematically Related, Thematically Related and Repetition. A much more representative picture of Andy’s practice was obtained by documenting a sequences of lessons instead of only one lesson. As shown in this present study, video-recording a teacher’s lesson sequence provided a better indication of the diversity needed to characterise a teacher’s practice [11, 18, 26-28].

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