Revisiting the symmetric reactions for synthesis of super heavy nuclei of $Z \geq 120$

R. K. Choudhury and Y. K. Gupta

Nuclear Physics Division, Bhabha Atomic Research Centre, Trombay, Mumbai, 400085, India

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Abstract – Extensive efforts have been made experimentally to reach nuclei in the super heavy mass region of $Z = 110$ and above with suitable choices of projectile and target nuclei. The cross sections for production of these nuclei are seen to be in the range of a few picobarn or less, and pose great experimental challenges. Theoretically, there have been extensive calculations for highly asymmetric (hot-fusion) and moderately asymmetric (cold-fusion) collisions and only a few theoretical studies are available for near symmetric collisions to estimate the cross sections for production of super-heavy nuclei. In the present article, we revisit the symmetric heavy ion reactions with suitable combinations of projectile and target nuclei in the rare-earth region, that will lead to compound systems with very low excitation energy and with better neutron-to-proton ratio for higher stability.

Introduction. – An island of super heavy nuclei, with half lives ranging from a few seconds to a few thousands of years has been predicted by calculations based on macroscopic-microscopic theories [1–5]. The large stability arises due to strong shell effects in the range of proton numbers ($Z = 114 - 126$) and neutron numbers ($N = 170 - 188$), which in turn gives rise to large fission barriers ($5 - 8$ MeV) in this mass region. There have been extensive efforts experimentally to synthesize super-heavy elements (SHE) through heavy ion reactions with suitable choice of projectile and target nuclei. However, the compound nuclei are formed in the excitation energy of few tens of MeV, and due to washing out of shell effects with increasing excitation energy, the production cross sections are usually quite low (in the range of picobarn or less) for compound nuclei with $Z = 110$ and above. Nevertheless, nuclei with $Z$ up to 118 have been synthesized in laboratory by various experiments so far [6–10]. The two main routes followed are: ‘hot fusion’ with actinide target nuclei and highly asymmetric reaction channels [6–13], and ‘cold fusion’ with Pb, Bi target nuclei with moderately asymmetric reaction channels [14–16]. In all these experiments, the compound nucleus (CN) is formed with relatively less neutron numbers as compared to that needed for the extra stability due to the shell effects.

Theoretically, there have been many attempts to understand the reaction mechanism leading to the formation of the super heavy nuclei [17–23]. Based upon the various theoretical formalisms, different reaction routes for the synthesis of super heavy nuclei have been proposed [24–29]. The main considerations in selecting a reaction channel for producing super-heavy nuclei are the following:

1. Large fusion cross section.
2. Low excitation energy of CN for optimum survival probability.
3. Proper neutron-to-proton ratio ($n/p$) of CN for better stability.
4. High beam intensity and target concentration for good yield.

One of the main reasons for poor success of the experiments is that the reaction $|Q|$-value is much lower than (for ‘hot-fusion’), or similar to (for ‘cold-fusion’) the Coulomb barrier ($V_{\text{Coul}}$) of the fusing target and projectile nuclei. Hence, at beam energies just above Coulomb barrier, the CN is formed with high excitation energy which is already larger than the neutron emission threshold. For example, for $^{48}\text{Ca} + ^{249}\text{Cf}$ ($\{Z, A\}_{\text{CN}} = \{118, 297\}$), $Q = -174.48$ MeV, $V_{\text{Coul}} = 205.4$ MeV and for $^{76}\text{Ge} + ^{208}\text{Pb}$ ($\{Z, A\}_{\text{CN}}$}
Table 1: Relevant data for the new reaction routes using the rare-earth nuclei.

| Reaction | $Z_PZ_T$ | g.s. deformations | $Q$- Value | $V_{\text{Coul}}$ | $S_n$ |
|----------|----------|------------------|------------|-----------------|------|
| $^{154}_{\text{Sm}}+^{150}_{\text{Nd}}$ | 3720 | (0.27, 0.24) | -377.5 | 373.9 | 7.1 |
| (122, 304) | (122, 304) | | | | |
| $^{154}_{\text{Sm}}+^{154}_{\text{Sm}}$ | 3844 | (0.27, 0.27) | -394.9 | 385.5 | 7.1 |
| (124, 308) | (124, 308) | | | | |
| $^{160}_{\text{Gd}}+^{154}_{\text{Sm}}$ | 3968 | (0.28, 0.27) | -412.2 | 396.2 | 7.3 |
| (126, 314) | (126, 314) | | | | |

Fig. 1: (Color online) (a) Sticking cross section at two different $E_{\text{c.m.}}$ values as a function of $Z_PZ_T$ for the reactions discussed in the present work. Solid circles are for $E_{\text{c.m.}}$ values for which $E_X=10$ MeV and squares are for $E_{\text{c.m.}}=V_{\text{Coul}}$, corresponding center-of-mass energies as a function of $Z_PZ_T$ in (b). The lines in (a) and (b) are shown to guide the eye.

$= (114, 284)$, $Q= -260.25$ MeV, $V_{\text{Coul}} = 272.1$ MeV. Similar is the case for other target-projectile combinations of hot- and cold-fusion reactions being used for the SHE synthesis.

As mentioned above, the cross-sections for SHE production have been found to be in the range of only a few picobarn or less in the experiments carried out so far. Recent reviews \[20, 30–32\] have emphasized on radioactive-ion-beam routes for producing $Z_{\text{CN}} \geq 120$. In order to have better survival probability, radioactive neutron rich beams ($^{96}_{\text{Sr}}, ^{132}_{\text{Sn}}$) are being suggested to reach a more suitable neutron/ proton combination. However, these reactions will have severe limitation on beam intensity.

Symmetric heavy-ion collisions using rare-earth nuclei. – There have been some attempts using nearly symmetric collisions such as $^{136}_{\text{Xe}}+^{136}_{\text{Xe}}$ to synthesize Hs nuclei for which upper limit of the production cross section was obtained to be 4 picobarn \[33\]. The symmetric collisions using deformed projectile and target nuclei have also been suggested earlier \[20, 28\] to synthesize super heavy nuclei. For $^{149}_{\text{La}}+^{149}_{\text{La}}$ collision to produce $Z=114$ nuclei, the upper limit of cross section was estimated from theoretical consideration to be around 10 picobarn \[29\]. However, there is no experimental data available for this system. In the following, we revisit the near symmetric collisions involving rare-earth nuclei that might prove useful for synthesis of cold super-heavy nuclei.

Table I shows some relevant data such as the $Z_PZ_T$ value, ground state (g.s.) deformations of projectile and target nuclei (from Ref. \[34\]), the fusion $Q$-value, $V_{\text{Coul}}$ and the neutron separation energy ($S_n$) for certain reaction routes using rare-earth nuclei fusion channels. The $Q$ and $S_n$ values are calculated using the predicted masses by Möller and Nix \[34\]. The $V_{\text{Coul}}$ values are taken from the NRV code \[35\] which are consistent with the parameterizations of mean value of the barrier distribution given in Ref. \[24\]. In addition to the reactions shown in the Table I many more fusion reaction channels are feasible using other different rare-earth target/projectile combinations. The advantages that these reactions offer are:

1. $V_{\text{Coul}} < |Q|$ value.
2. Large g.s. deformations of both target and projectile nuclei that might enhance near barrier fusion cross section by channel coupling and lowering of fusion barrier, $B_{fus}$.
3. Good n/p ratio of CN.
4. Stable beams for large beam intensity.
5. Large elemental abundances of rare-earth elements.
6. Large center-of-mass velocity for better collection of CN residues in forward direction.
7. Low neutron background at optimum low bombarding energy.

For example, in case of $^{160}$Gd + $^{154}$Sm reaction, the CN is (126, 314) where $V_{\text{Coul}}$ is 16 MeV lower than the energy required ($|Q|$-value) for initiating the reaction. With optimum above barrier bombarding energy, the CN can be produced with relatively low excitation energy.

**Theoretical estimates.** – We will now describe some method to calculate the fusion/survival probability of the above rare-earth reaction channels. One expects that due to large $Z_P Z_T$ product, fusion will be largely hindered. However, for deformed nuclei there is no clear cut understanding of the fusion hindrance (except the extra push effects suggested by W. Swiatecki [36,37]). There are calculations reported in literature, where only target deformation is considered [22]. We discuss below the basic method to have approximate estimates for the formation cross section of the super-heavy nuclei using rare-earth nuclear collisions.

In case of heavy colliding systems typically used for super-heavy mass-region, overcoming the Coulomb barrier is not enough to form the super-heavy compound nucleus. There are two avenues for estimating the compound nuclear formation cross section for heavy colliding nuclei similar to the ones discussed in the present article. These are: (i) extra-extra push model [36,37] and (ii) Fusion by Diffusion model (FBD) [18]. According to the extra-extra-push model, an extra energy (‘extra-extra-push’) with respect to the Coulomb barrier is needed to land inside the unconditional saddle point which guards the colliding system against re-separation before forming the compound nucleus. The ‘extra-extra-push’ energy increases rapidly with effective fissility, given by [37]:

$$\chi_{\text{eff}} = \frac{(Z^2/A)_{\text{eff}}}{(Z^2/A)_{\text{crit}}}$$  \hspace{1cm} (1)

For the present reactions where deformed projectile and target nuclei are considered, due to broad Coulomb barrier distribution, a large amount of ‘extra-extra-push’ energy could be available (~100 MeV) at very low probability for certain orientations of the colliding deformed nuclei. However, the entrance channel barrier distributions for these kind of heavy deformed nuclei with inclusion of dynamical effects are not easily calculable.

On the other hand, the FBD model has been successfully employed in reproducing the measured excitation function of the super-heavy element synthesis [18]. A set of twelve fusion reactions has been analyzed with the original version of the FBD model by Swiatecki et al. [18]. With the improved version of the FBD model, the experimental excitation functions of a complete set of 27 cold fusion reactions have been reproduced by Cap et al. [21]. In the FBD model, the evaporation residue cross section $\sigma_{ER}$ for production of a given final nucleus in its ground state is factorized as the product of the partial sticking cross-section $\sigma_{\text{stick}}(\ell)$, the diffusion probability $P_{\text{Diffus}}(\ell)$, and the survival probability $P_{\text{surv}}(\ell)$ [21]:

$$\sigma_{ER} = \sum_{\ell=0}^{\infty} \sigma_{\text{stick}}(\ell) P_{\text{Diffus}}(\ell) P_{\text{surv}}(\ell)$$  \hspace{1cm} (2)

$$= \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) P_{\text{Diffus}}(\ell) P_{\text{surv}}(\ell)$$  \hspace{1cm} (3)

By replacing the summation in above equation by an in-
of di-nuclear shape, the parameter $s$ of nuclei, contours. The diffusion probability $P$ parabolic barrier [18]. If fusion is calculated using the diffusion process over a tem injected at a point outside the saddle point achieves $L$ by [18, 21]:

$$Z_{\text{c.m.}} - B_0,$$  

where the diffusion probability decreases very rapidly (de-

$$H$$ (MeV) increases with $Z$ c.m. values for which initial excitation energy of the CN, $E$ = 10 MeV. Corresponding center-of-mass energies as a function of barrier height opposing fusion along the asym-

$$P_{\text{inj}}$$ is barrier height opposing fusion along the asym-

$$\ell_{\text{max}}$$ is determined by the “diffused barrier formula” based on assumption of Gaussian distribution of the bar-

The macroscopic deformation energies are calculated as a function of the parameter $s$ using the improved version of algebraic equations [21]. In order to estimate the barrier height, $H$, $s_{\text{inj}}$ is a crucial parameter. In the FBD model this parameter $s_{\text{inj}}$ is a free parameter which is adjusted to reproduce the measured fusion cross section. In the work by Cap et al. [21], $s_{\text{inj}}$ has been deduced for 27 cold fusion reactions including GSI, LBNL and RIKEN data. In that work, the $s_{\text{inj}}$ values are plotted as a function of the excess of kinetic energy above the Coulomb barrier, $E_{\text{c.m.}} - B_0$, where $B_0$ is the mean value of the Coulomb barrier ($V_{\text{Coul}}$). The overall trend of $s_{\text{inj}}$ is of decreasing nature with increasing $E_{\text{c.m.}} - B_0$. It is seen from Ref. [21] that except the GSI data, all other data are scattered. For the purpose of present reactions, $s_{\text{inj}}$ values for GSI data (from Ref. [21]) are considered and a linear least-square fit is obtained as shown in Fig. 5 given by:

$$s_{\text{inj}} = 1.5985 - 0.23587(E_{\text{c.m.}} - B_0) \text{ fm/MeV}. \quad (6)$$

Since the present projectile-target nuclei are deformed ones, the fusion barrier distribution is expected to be quite broad [38]. Even at $E_X < 8$ MeV, a large fraction of the barrier distribution will have $(E_{\text{c.m.}} - B_0) > 30$ MeV, which will lead to $s_{\text{inj}} \sim -5$ fm as reflected from Fig. 3. For the present reactions, the barrier height, $H$ is calculated at $s_{\text{inj}} = -5, -4, -3$ fm as shown in Fig. 4 using the algebraic equations of macroscopic energies from Ref. [21]. It is seen from Fig. 4 that the value of $H$ increases with $Z_{\text{c.m.}}$ and it is lower for smaller value of $s_{\text{inj}}$. At $s_{\text{inj}} = -4$ fm, $H$ value is around $5.5 \pm 1.5$ MeV for all three reactions considered in the present work (see Table I). In the estimation of the diffusion probability using Eq. 5 the parameters $H$ and $T$ are crucial. At the excitation energy $E_X < 8$ MeV, the temperature $T$ is expected to be $< 1.0$ MeV but definitely $> 0.5$ MeV. Fig. 2 indicates that at $H = 5 \pm 1.5$ MeV the diffusion probability will be in between of $10^{-6}$ and $10^{-3}$ for $0.5$ MeV $\leq T \leq 1.0$ MeV. Using Eq. (2) and Fig. 1 it appears that for the present reactions, lower limit of $\sigma_{\text{stick}} \times P_{\text{diffus}}$ is $\sim 10^{-7}$ barn. Present reactions using the rare-earth nuclei offers a gain factor of the order of $\sim 10^4$ for $\sigma_{\text{stick}} \times P_{\text{diffus}}$ over the reactions of cold fusion, as can be seen from Fig. 2 of Ref. [18].

As far as survival probability ($P_{\text{surv}}$) is concerned, in cold fusion reactions when only one neutron is emitted from the compound nucleus, the $P_{\text{surv}}$ is the product of probability to emit a neutron rather than fission in the first stage of de-excitation process times the probability $P_<$ that the excitation energy (after the emission of neu-

$$P_{\text{surv}} = \frac{\Gamma_n}{\Gamma_n + \Gamma_f} P_< \quad (7)$$

where $\Gamma_n$ and $\Gamma_f$ are the partial decay widths for first chance neutron emission and fission, respectively. $P_{\text{surv}}$
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Fig. 5: (Color online) Survival probability ($P_{\text{surv}}$) as derived from Fig. 2 of Ref. [18] as a function of (a) atomic number and (b) corresponding fission barrier of the compound nucleus (see text). The horizontal bars in each panel are drawn to indicate the range of $P_{\text{surv}}$ from $0.2 \times 10^{-4}$ to $6 \times 10^{-4}$.

is expected to be influenced by the properties of the CN such as its ground state mass, its fission barrier, excitation energy, neutron separation energy, shell effects and the level density [18]. As mentioned earlier, a set of cold fusion reactions with $^{208}\text{Pb}$ and $^{209}\text{Bi}$ targets has been analyzed with FBD model by Swiatecki et al. [18]. From their work (Fig. 2 of Ref. [18]), we have derived the value of $P_{\text{surv}}$ for the elements with atomic numbers $Z=104$ to 119 which are seen to lie in a narrow range of $0.2 \times 10^{-4}$ to $6 \times 10^{-4}$. In Fig. 5(a), we show the values of $P_{\text{surv}}$ as a function of $Z$ of the compound nucleus. The fission barriers for these compound nuclei vary from 5.5 to 9 MeV [39]. In Fig. 5(b), we have plotted the value of $P_{\text{surv}}$ against the fission barrier taken from Ref. [39]. It is seen from Figs. 5(a) and (b) that the survival probability has a very weak dependence on the properties of compound nucleus in case of cold fusion reactions (for $Z=104\text{--}119$).

The values of $P_{\text{surv}}$ can differ in other mass regions, in particular if regions with shell closures are entered like is the case in the present work. For the present reactions it will be larger than cold as well as hot fusion reactions due to following reasons: (i) initial excitation energies can be tuned to be very small ($\sim 8$ MeV). Therefore, shell effects are expected to be more prominent and (ii) good $n/p$ ratio required for the stability of the super-heavy elements.

The fission barriers for $Z \geq 120$ have been calculated by different authors of Refs. [39]--[41] and are seen to be varying widely between 2 to 8 MeV depending on the model parameters used in their work. However, since the value of $P_{\text{surv}}$ for the cold fusion reactions (for $Z=104\text{--}119$) corresponding to the excitation energy range of 10 to 15 MeV (below $2n$ threshold) is not sensitive to the value of fission barrier, we consider a lower limit of $10^{-4}$ for the survival probability for all the systems considered in the present work. Using this value of $P_{\text{surv}}$ and $P_{\text{Diffus}}$ to be $10^{-6}$, the lower limit of final cross sections for the synthesis of super-heavy nuclei for the present systems having $Z \geq 120$ is arrived to be in the range of $1.7 \times 10^{-11}$ barn to $3.0 \times 10^{-11}$ (see Fig. 1). Even if we allow some uncertainties in the parameter values, the results seem to be quite encouraging. Present work suggests it to be definitely worth for experimental investigations using rare-earth nuclear collisions. It is also necessary to carry out full microscopic calculations to understand the fusion mechanism for these heavy systems.

**Summary.** – In the present work, we have made a case for the use of rare-earth projectile and target nuclei to produce super-heavy nuclei in the range of $Z \sim 120$ and above using cold fusion reactions. The advantages offered by these near symmetric collisions have been outlined. The cross sections for production of the super-heavy nuclei in these collisions have been estimated within the framework of the Fusion by Diffusion model with empirically derived parameter values and are seen to be quite encouraging. It is, however, necessary to carry out experiments to explore these possibilities of using rare-earth nuclei in cold fusion reactions for production of super-heavy elements.

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