Relativistic kinetic theory of magnetoplasmas

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Abstract. Recently, an increasing interest in astrophysical as well as laboratory plasmas has been manifested in reference to the existence of relativistic flows, related in turn to the production of intense electric fields in magnetized systems [1]. Such phenomena require their description in the framework of a consistent relativistic kinetic theory, rather than on relativistic MHD equations, subject to specific closure conditions. The purpose of this work is to apply the relativistic single-particle guiding-center theory developed by Beklemishev and Tessarotto [2], including the nonlinear treatment of small-wavelength EM perturbations which may naturally arise in such systems [3]. As a result, a closed set of relativistic gyrokinetic equations, consisting of the collisionless relativistic kinetic equation, expressed in hybrid gyrokinetic variables, and the averaged Maxwell’s equations, is derived for an arbitrary four-dimensional coordinate system.

INTRODUCTION

A basic prerequisite for the formulation of a consistent relativistic kinetic for strongly magnetized plasmas in astrophysical problems, is the formulation of single-particle dynamics in the context of a relativistic, fully covariant, formulation of gyrokinetic theory [2, 3, 4]. As is well known, this regards the so-called “gyrokinetic problem”, i.e., the description of the dynamics of a charged particle in the presence of suitably “intense” electromagnetic (EM) fields realized by means of appropriate perturbative expansions for its equations of motion. The expansions, usually performed with respect to the ratio \( \varepsilon = r_L/L << 1 \), where \( L \) and \( r_L \) are respectively a characteristic scale length of the EM fields and the velocity-dependent particle Larmor radius \( r_L = \frac{w}{\Omega_s} \), with \( \Omega_s = \frac{qB}{mc} \) the Larmor frequency and \( w \) the orthogonal component of a suitable particle velocity. The goal of gyrokinetic theory is to construct with prescribed accuracy in \( \varepsilon \) the so called “gyrokinetic” or “guiding center variables”, by means of an appropriate “gyrokinetic” transformation, such that the equations of motion result independent of the gyrophase \( \phi \), \( \phi \) being the angle of fast gyration, which characterizes the motion of a charged particle subject to the presence of a strong magnetic field. In non-relativistic theory the gyrokinetic transformation can be constructed by means of a perturbative expansion of the form:

\[
\mathbf{z} \rightarrow \mathbf{z}' = \mathbf{z}'_0 + \varepsilon \mathbf{z}'_1 + \varepsilon^2 \mathbf{z}'_2 + \ldots,
\]

which in terms of the Newtonian state \( \mathbf{x} = (\mathbf{r}, \mathbf{v}) \) reads:

\[
\mathbf{r} = \mathbf{r}' + \varepsilon \mathbf{r}' + \varepsilon^2 \mathbf{r}'_2(\mathbf{z}', t, \varepsilon),
\]

\[
\mathbf{v} = u' \mathbf{b}' + \mathbf{w}' + \mathbf{v}' + \varepsilon \mathbf{v}'_1(\mathbf{z}', t, \varepsilon),
\]

where \( \varepsilon \mathbf{r}' \) is the Larmor radius,

\[
\varepsilon \mathbf{r}' = -\varepsilon \frac{\mathbf{w} \times \mathbf{b}'}{\Omega'_s},
\]

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while \( \mathbf{V}' \) is the electric drift velocity
\[
\mathbf{V}' = \frac{c \mathbf{E}' \times \mathbf{B}'}{B'{}^2},
\]
and the phyrophase gyrophase \( \phi' \) is defined:
\[
\phi' = \arctg \left\{ \frac{(\mathbf{V}' - \mathbf{V}{}')_y \cdot \hat{e}_2}{(\mathbf{V}' - \mathbf{V}{}')_x \cdot \hat{e}_1} \right\}.
\]

In the past several methods have been devised to construct hybrid gyrokinetic variables. These include perturbative theories based, respectively, on non-canonical methods (see for example [5]), canonical perturbation theory [6, 7], canonical and non-canonical Lie-transform approaches [8, 9], as well as Lagrangian non-canonical formulations which make use of the so-called hybrid Hamilton variational principle [10, 4, 2].

FIGURE 1. Guiding center and Larmor radius in non-relativistic theory. Here \( \hat{e}_1', \hat{e}_2', \hat{e}_3' = b' \) denotes a right-handed basis of unit vectors.

**RELATIVISTIC GYROKINETIC THEORY: MOTIVATIONS**

For a certain class of problems in plasma-physics and astrophysics, existing limitations of the standard gyrokinetic theory [16, 17, 18, 19] make its use difficult or impossible. In particular, this involves the description of experiments in which the electric field may become comparable in strength to the magnetic field (so that the drift velocity becomes relativistic), and the study of relativistic plasma flows in gravitational fields, which are observed or assumed to exist in accretion disks and related plasma jets around neutron stars, black holes, and active galactic nuclei. The finite Larmor radius effects and the influence of short wavelength electromagnetic perturbations are also expected to play a fundamental role in particle dynamics. In many respects, previous relativistic theory results inadequate for such a task. In fact, some of meearlier treatments consider the particle velocity as relativistic while its drift velocity is not [11, 12, 13, 14, 15]. This deficiency has been pointed out by Pozzo and Tessarotto [4], who developed a special-relativistic theory which includes the effect of relativistic drifts. However, the self-consistent inclusion of the gravitational fields, a prerequisite to make the theory suitable for astrophysical applications, as well the treatment of nonlinear EM perturbations of relativistic plasmas, requires a relativistic covariant formulation. This has been investigated by Beklemishev and Tessarotto [2, 3]. In this theory the appropriate relativistic gyrokinetic theory has been carried out through second order in the expansion parameter, including wave fields, based on a Lagrangian approach making use of the hybrid Hamilton variational principle. In such a case the variational functional for a charged point particle with the rest-mass \( m_a \) and charge \( q_a \) in prescribed fields can be written:
\[
S = \int Q \mu d\mathbf{u}^{\mu} = \int (q A_{\mu}(x^\nu) + u_{\mu}) d\mathbf{u}^{\mu},
\]
where \( q = q_a/m_a c^2 \), and variations of \( u_{\mu} \) occur on the seven-dimensional hypersurface \( u_{\mu} u^{\mu} = 1 \), being \( u_{\mu} \) the four-velocity \( u^{\mu} = \frac{dx^{\mu}}{ds} \) and the relevant tensor notations are standard. Thus, \( g_{\mu\nu} \) denotes the metric tensor components, characterizing the coordinate system (and the underlying space-time structure) which provides the connection between
the co- and countervariant components of four-vectors (henceforth referred to as 4-vectors) \( A_\mu = g_{\mu \nu} A^\nu \), while the invariant interval \( ds^2 = g_{\mu \nu} dx^\mu dx^\nu \), (8)

where the Greek indices are assumed to go through \( \mu, \nu = 0, \ldots, 3 \).

**THE RELATIVISTIC GYROKINETIC TRANSFORMATION**

The object of gyrokinetics is to introduce a new set of phase-space variables (called the “gyrokinetic variables”) such that the variable describing the rotation angle along the Larmor orbit (i.e. the gyrophase \( \phi \)) becomes ignorable. This happens, by definition, when the Lagrangian (or, more generally, the functional) is independent of \( \phi \). Once an ignorable variable is found, the number of corresponding Euler equations is reduced by one, and the new variables allow simplified numerical calculations, as the motion is effectively integrated over the fast Larmor rotation. The one-to-one transformation from the original set of phase-space variables \( (x^\mu, u^\nu) \) to the gyrokinetic variables is called the “gyrokinetic transformation”. In what follows, we use the Lagrangian perturbative approach to find those variables, which is equivalent (in broad terms), to the Lie-transform method, though more direct.

First, we assume that the curvature radius of the space-time and the gradient lengths of the background electromagnetic fields are much larger than the Larmor radius characterizing the particle path. However, we allow for existence of wave-fields with sharp gradients \([k_{p_\lambda} \sim O(1)], \) including \( k_{p_\lambda} \sim O(1) \), and rapidly varying in time \([\omega p_{\lambda} / c \sim O(1)]\), while such fields are assumed suitably smaller in strength than the background field. We stress that, unlike in conventional formulations of gyrokinetic theory, this type of ordering is required in a covariant theory due to the reference-frame dependence of the ordering assumptions involving space and time scale lengths of the perturbations.) For this purpose we introduce the ordering scheme following the notation of [16]:

\[
Q_\mu dx^\mu = \{u_\mu + q(\frac{1}{\epsilon} A_\mu + \lambda a_\mu)\} dx^\mu, \tag{9}
\]

where \( \epsilon \) and \( \lambda \) are formal small parameters (they should be set to 1 in the final results) allowing distinction between the large-scale background field \( A_\mu \), and the wave-fields given by \( a_\mu \). We search for the gyrokinetic transformation \( (y') \equiv (x'^\alpha, \phi, \hat{\mu}, u|_l) \leftrightarrow (x_\alpha, u^0) \) in the form of an expansion in powers of \( \epsilon \):

\[
x^\nu = x'^\nu + \sum_{s=1}^{3} \epsilon^s r^s_{\nu} (y'), \tag{10}
\]

where \( \phi \) is the ignorable phase variable (gyrophase), \( \hat{\mu} \) and \( u|_l \) represent two other independent characteristics of velocity (to be defined later), \( x'^\nu \) is the 4-vector “guiding center” position, \( r_{\nu} \) are arbitrary 4-vector functions of the new variables \( (y') \) to be determined. We require that \( r_{\nu} \) are purely oscillatory in \( \phi \), i.e., the \( \phi \)-averages of \( r_{\nu} \) are zero, as a part of the \( x'^\nu \) - definition. Note that the above descriptions of the new variables \( (x'^\alpha, \phi, \hat{\mu}, u|_l) \) will acquire precise mathematical meaning only as a result of the search for the gyrokinetic transformation.

This search consists in applying the expansion (10) to the fundamental 1-form (9) and imposing the requirement that it is independent of \( \phi \) in each order. A convenient framework is provided by projecting all 4-vectors and 4-tensors along the directions of a suitable fundamental tetrad \( (\tau, l, l', l'') \): i.e., an orthogonal basis of unit 4-vectors so that the last three are space-like, and

\[
\sqrt{-g} e^{\alpha}_{\lambda \mu \nu} \tau^\alpha l^\lambda l'^\mu l'^\nu = 1, \tag{11}
\]

where \( \sqrt{-g} e^{\alpha}_{\lambda \mu \nu} \) is the purely antisymmetric tensor. As a consequence the four-velocity can be represented in the form:

\[
u_\mu = w (l'_\mu \cos \phi + l''_\mu \sin \phi) + \bar{u}_\mu, \tag{12}
\]

which can be also regarded as the definition for the gyrophase \( \phi \); it is defined as an angle in the velocity-subspace, where we introduce the cylindrical coordinate system. This definition is covariant with respect to transformations of the space-time coordinate system, which may change the vector components, but not the vectors themselves. Furthermore, we assume that \( w \) and \( \bar{u}_\mu \) are independent of \( \phi \). Validity of this assumption is justified by existence of the solution (at least for a non-degenerate Faraday tensor).

The \( \phi \)-independent part of the 4-velocity \( \bar{u} \) is not completely arbitrary, but satisfies certain restrictions following from the requirement \( u_\mu \bar{u}^\mu = 1 \) for all \( \phi \):

\[
\bar{u}_\mu = u_\mu l'_\mu + u_\mu l''_\mu, \tag{13}
\]
Any two of three scalar functions $w, u_\theta,$ or $u_\phi$ can be considered independent characteristics of velocity, while the third can be expressed via (14). It is now straightforward to eliminate from $\delta G$ terms oscillating in $\phi$ by properly defining displacements $\mathbf{r}_i$. This task can, in principle, be carried out systematically at any order in the relevant expansion parameters (in particular in $\epsilon$). Thus, to leading order in $\epsilon$ to eliminate the gyrophase-dependent terms in the fundamental differential 1-form one must impose constraint:

$$\tilde{a}_\mu + \lambda a_\mu - qr_\mu F_{\mu\nu} = 0,$$

where $\tilde{y}$ denotes the oscillating part of $y$, namely $\tilde{y} = y - \bar{y}, \bar{y} = \langle y \rangle_\phi$ is the gyrophase-averaged part of $y$ and $F_{\mu\nu}$ is the EM field tensor. If the above requirement (15) is satisfied, the gyrophase $\phi$ is ignorable and the hybrid variational principle in our approximation can be expressed as $\delta S' = 0$. As a result, the $\phi$-independent functional $S''$ becomes

$$S'' = \int \left\{ \left( \frac{\partial}{\partial \epsilon} \lambda A_{\mu} + \lambda q_{\mu} - q_{\mu} F_{\rho\nu} \right) du^\mu + \hat{\mu} d\phi \right\},$$

where $\hat{\mu}$ is the relativistic wave-field-modified magnetic moment, accurate to order $\epsilon^1$ and $u_\theta = \sqrt{1 + u^2 + u_\phi^2}$.

The equations of motion, expressed in terms of the relationships between differentials tangent to the particle orbit, can be obtained as Euler equations of the transformed variational principle [2, 3]. Using the $\phi$-independent functional (16) in the variational principle $\delta S'' = 0$ defines the particle trajectory in terms of the new gyrokineic variables $(\chi^\mu, \hat{\mu}, u_\theta, \phi)$. This set is non-canonical, but further transformations of variables (not involving $\phi$) also lead to $\phi$-independent functionals and can be used for this purpose.

### The Relativistic Gyrokinetic Vlasov Kinetic Equation

The single-particle distribution function can be written in general relativity either in the eight-dimensional phase space $\Phi(x^\mu, u_\nu), \mu, \nu = 0, \ldots, 3$, or in the seven-dimensional phase space $f(y^\mu, u_\nu)$, where only 3 components of the 4-velocity $u_\nu$ are independent, so that

$$\Phi(x^\mu, u_\nu) = f(x^\mu, u_\nu) [\sqrt{u_\xi u^\xi} - 1] \theta(u^0).$$

The $\delta-$function here reflects the fact that $u_\xi u^\xi = 1$ is the first integral of motion in the case of the eight-dimensional representation.

The kinetic equation in both cases retains the same form and yields the collisionless Vlasov kinetic equation, namely

$$u^\mu \frac{\partial f}{\partial \chi^\mu} + \left( \frac{du_\nu}{ds} \frac{\partial f}{\partial u_\nu} \right) = 0,$$

although in the 7-dimensional case $\nu = 1, 2, 3$ only, while $u^0$ is the dependent variable. Here $(du_\nu/ds)$ is a function of independent variables $(\chi^\mu, u_\nu)$ found as the right-hand side of the single-particle dynamics equations. The kinetic equation can be multiplied by $ds$ in this way it can also be represented in the parametrization-independent form as follows:

$$\frac{\partial f}{\partial \chi^\mu} du^\mu + \frac{\partial f}{\partial u_\nu} du_\nu = 0,$$

where the differentials are tangent to the particle orbit.

Due to general properties of variable transformations it is obvious that any non-degenerate transformation of the phase-space variables $(\chi^\mu, u_\nu) \to (y^i)$ will lead to the same form of the kinetic equation

$$\frac{\partial f}{\partial y^j} dy^j = 0,$$

where the differentials are tangent to the particle orbit. In particular, this property is useful for transformation to the gyrokinetic variables.
Let \((y) \equiv (x^\alpha, \phi, \tilde{\mu}, u_\|)\), then the kinetic equation becomes

\[
\frac{\partial f}{\partial x^\mu} dx^\mu + \frac{\partial f}{\partial u_\|} du_\| + \frac{\partial f}{\partial \tilde{\mu}} d\tilde{\mu} + \frac{\partial f}{\partial \phi} d\phi = 0. \tag{21}
\]

By definition of the gyrokinetic variables the dynamic equations should be independent of \(\phi\), i.e., expressions for \(dx^\mu/\partial \phi\), \(du_\|/\partial \phi\), \((d\tilde{\mu}/\partial \phi)\) are independent of \(\phi\), while \(\partial f/\partial \phi\) is periodic in \(\phi\). It follows that \(\partial f/\partial \phi = 0\), and, if \(\tilde{\mu}\) is the integral of motion, \(d\tilde{\mu} = 0\), we get the kinetic equation expressed in the gyrokinetic variables as

\[
\frac{\partial f}{\partial x^\mu} dx^\mu + \frac{\partial f}{\partial u_\|} du_\| = 0, \tag{22}
\]

which we shall call relativistic gyrokinetic Vlasov kinetic equation. Here the coefficients \(dx^\mu\) and \(du_\|\) must be determined from the equations of motion in the gyrokinetic variables.

**THE MAXWELL’S EQUATIONS**

Finally we point out another important feature of the present formulation of the gyrokinetic theory. Namely, the Jacobian of the transformation is simple enough to allow explicit integration in the gyrophase, needed for evaluation of the charge and current densities. The general form of the Maxwell’s equations in presence of an arbitrary gravitational field is well known[20]. The first pair of equations in presence of an arbitrary gravitational field is well known[20]. The first pair of equations can be written as

\[
e^{\gamma \lambda \mu \nu} \frac{\partial F_{\mu \nu}}{\partial x^\lambda} = 0, \tag{23}
\]

while the second as

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left( \sqrt{-g} F_{\mu \nu} \right) = -\frac{4\pi}{c} j^\mu, \tag{24}
\]

where

\[
 j^\mu = e \sum_\alpha q_\alpha \int u^\mu f_\alpha (x, u) \delta \left( \sqrt{u^\nu u_\nu - 1} \right) \frac{d^4 u}{\sqrt{-g}}, \tag{25}
\]

is the current density, expressed via the distribution function of particle species \(\alpha\), and the signed particle charge \(q_\alpha\). The \(\delta\)-function under the integral allows to make partial integration, for example over \(u_0\), and arrive at a more widely used form

\[
\delta \left( \sqrt{u^\nu u_\nu - 1} \right) d^4 u \rightarrow \frac{du_1 du_2 du_3}{u_0}. \tag{26}
\]

However, in the gyrokinetic transformation the four-velocity is expressed via Eqs. (12)-(14) as

\[
u_\mu = \nu (l^\mu_{\|} \cos \phi + l^\mu_\| \sin \phi) + u_\| l^\mu_{\|} + u_o \tau_\mu, \tag{27}
\]

so that \(d^4 u = w d\omega d\phi du_\| du_o\) [the sign is positive due to Eq.(11)], while the partial integration over \(du_o\) leads to

\[
\delta \left( \sqrt{u^\nu u_\nu - 1} \right) d^4 u \rightarrow \frac{w d\omega d\phi du_\|}{u_o}. \tag{28}
\]

where \(u_o = \sqrt{1 + w^2 + u_\|^2}\). As a result, the expression for components of the current density can be rewritten as

\[
 j^\mu = e \sum_\alpha q_\alpha \int \left( w (l^\mu_{\|} \cos \phi + l^\mu_\| \sin \phi) + u_\| l^\mu_{\|} + u_o \tau_\mu \right) f_\alpha (x, u) \frac{w d\omega d\phi du_\|}{\sqrt{-g} u_o}. \tag{29}
\]

Further, the distribution function \(f_\alpha\) is expressed as the function of the gyrokinetic variables

\[
f_\alpha = f_\alpha \left( x^\mu, \tilde{\mu}, u_\| \right), \tag{30}
\]
and it is necessary to transform it back to particle coordinates before integrating, as in Eq.(29) the particle position \( \mathbf{x} \), rather than its gyrocenter position \( \mathbf{x}' \), is kept constant while integrating over the particle velocity. This makes it convenient to rewrite Equation (24) as
\[
\frac{\partial}{\partial x^\nu} \left( \sqrt{-g} F^\mu_{\nu} \right) = -\frac{4\pi}{c} j^\mu \sqrt{-g} = Q^\mu (\mathbf{x}),
\] (31)
where the right-hand side is also evaluated at \( \mathbf{x} \). Then
\[
Q^\mu (\mathbf{x}) = -4\pi \sum_\alpha q_\alpha \int \left[ w \left( l^\mu \cos \phi + l'^\mu \sin \phi \right) + u_\parallel l^\mu + u_\varphi \tau^\mu \right] f_\alpha \left( \mathbf{x} - \sum_{i=1}^k \epsilon^i \mathbf{r}_i \right) \frac{w d\omega d\phi du_\parallel}{u_\varphi}. \tag{32}
\]

**CONCLUSION**

A closed set of relativistic gyrokinetic equations, consisting of the collisionless gyrokinetic equation and the averaged Maxwell’s equations, is derived for an arbitrary four-dimensional coordinate system.

In several respects the theory here developed represents a significant improvement with respect to kinetic equations derived by other authors. The present covariant kinetic theory adopts a set of hybrid gyrokinetic variables, two of which include the Lorentz-invariant magnetic moment and gyrophase angle. The theory, allows \( E/B \sim O(1) \) and therefore permits relativistic drifts (\( V_d \sim c \)) and moreover takes into account nonlinear effects of the EM wave-fields. Moreover, since the gyrokinetic transformation is obtained to the second order in terms of the ratio of the Larmor radius to the inhomogeneity scale, the theory can be applied also to the investigation of finite-Larmor radius effects. Another interesting aspect is that in the present theory the wave field is no longer limited in frequency and the wavelength, i.e., \( \omega/\Omega_c \sim O(1) \), \( k_{\parallel}/\rho_L \sim O(1) \), so that the class of admissible waves is broader than the usual “drift-Alfven perturbations” and can include the magneto-sonic waves, for example.

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