ONSET OF SHEAR WAVES IN A BACTERIAL BATH: A NOVEL EFFECT

SUPURNA SINHA
Raman Research Institute, Bangalore 560080, India.

Received (received date)
Revised (revised date)
Accepted (accepted date)

Recent experiments on particle diffusion in bacterial baths indicate the formation of correlated structures in the form of bacterial swirls. Here we predict that such a structural ordering would give rise to the new effect of propagating shear waves in a bacterial bath at length scales of the order of a swirl, which corresponds to time scales of the order of the lifetime of a swirl. Our prediction can be tested against future experiments in bacterial baths.

Keywords: diffusion, viscoelasticity, shear waves, Brownian motion.

1. Introduction

A simple liquid cannot sustain shear stress under a static condition. However, at molecular length and time scales (i.e. time scales of the order of the relaxation time of the liquid), it is known that a liquid behaves much like a solid with a viscoelastic character \[1\]. One can understand such a phenomenon qualitatively as follows. On molecular length scales a dense liquid develops solid-like structures due to slowing down of diffusional structural relaxation and consequently it can support a shear wave. Molecular dynamics simulations of hard sphere liquids and experiments on liquid Argon support this picture \[2,3\]. On the theoretical end, such a behavior can be understood in terms of viscoelastic models \[1,3,4\].

In recent years there have been experiments probing ‘Brownian Motion’ of polystyrene spheres in bacterial baths \[5\]. These experiments point to a crossover from superdiffusive to diffusive behavior of the mean-square displacement of the polystyrene beads suspended in a bacterial bath. This effect is similar to ballistic to diffusive transition in colloids \[5,6,7\], where the borderline for such a transition is given by the viscous damping time \[t_0 = \frac{m_b}{\gamma}\] (\(m_b\) being the mass of the Brownian particle and \(\gamma\) the friction coefficient), which for a polystyrene bead of diameter 10\(\mu m\) is about \(10^{-5}\) sec. However, in the case of a bacterial bath one notices that such a transition takes place at a time of about \(\tau_c \approx 1\) sec. This characteristic time
scale $\tau_c$ increases with the number density $n$ of bacterial cells in the bath. The length scale $l_c = \sqrt{\langle \Delta r^2(\tau_c) \rangle}$, which is the root mean square displacement at $t = \tau_c$ also increases with $n$. These observations point to the formation of correlated structures in the form of bacterial swirls which have been observed in these systems. In fact, one finds that $l_c$ is indeed of the order of the spatial scales of these swirls $(10 - 20 \mu m)$. To sum up, one of the key observations coming out of these experimental studies is the emergence of a time scale and a corresponding length scale in the Brownian motion of a polystyrene bead suspended in a bacterial bath. This time scale $\tau_c$ increases with the number density $n$ of the bacterial bath, indicating the appearance of solid-like ordering over length scales around $l_c = \sqrt{\langle \Delta r^2(\tau_c) \rangle}$. In a dense liquid such ordering takes place on molecular length scales. In the present experiments the bacterial bath can be viewed as a scaled up liquid with a typical relaxation time scale $\approx 1$ sec in contrast to a colloidal liquid characterized by a time scale $\approx 10^{-5}$ sec.

2. Viscoelasticity in Bacterial Bath

We now describe in some more quantitative detail the crossover from super-diffusive to diffusive behavior in a bacterial bath. Consider the dynamics of a bead of mass $m_b$ suspended in a bacterial bath in terms of a phenomenological Langevin-type equation:

$$m_b \frac{d\vec{v}}{dt} = -\gamma \vec{v} + \vec{f}(t)$$

(1)

where $\gamma$ is the friction coefficient of the fluid. The collisional force $f(t)$ experienced by the bead due to “kicks” from the swirls of bacteria has a zero mean and is exponentially correlated in time with a time scale $\tau_c$ corresponding to the lifetime of a swirl. Thus

$$<\vec{f}(t) >= 0; <\vec{f}(0) \cdot \vec{f}(t) >= <f^2(0) > exp(-t/\tau_c).$$

(2)

where $<f^2(0) > = D\gamma^2/\tau_c$ with $D = k_B T/\gamma$ the Einstein diffusion coefficient. Such an analysis leads to the following behavior for the mean square displacement of the bead. At short times ($t << \tau_c$) the motion of the bead is ballistic with $<\Delta r^2(t) > \sim t^2$ whereas at long times ($t >> \tau_c$) it is diffusive $<\Delta r^2(t) > \sim t$. More recent experiments on correlated bacterial motion also indicate the emergence of a long time scale $\tau_c$ increasing with the number density $n$ of bacteria. One interesting observation that comes out of the study is an enhancement of the viscosity $\eta$ of the bacterial bath. This enhanced viscosity is proportional to the large time scale $\tau_c$ emerging in the problem. In this paper we show that this crossover time scale $\tau_c$ is connected to a viscous to an elastic transition in a bacterial bath. 1

Let us consider the following model which interpolates between a purely viscous response and a purely elastic one for a liquid of viscosity $\eta$ and high frequency rigidity modulus $G$.

$$\left( \frac{1}{\eta} + \frac{1}{G} \frac{\partial}{\partial t} \right) \sigma_{zz} = \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}.$$  

(3)

1We emphasize that the main aspect of a bacterial bath that we are interested in is the presence of correlated structures in the form of swirls. Such structures give rise to a large spatial scale in the problem. This in turn leads to a large crossover time scale $\tau_c$ which is of the order of the lifetime of a swirl.
Here $\sigma_{xz}$ is the $xz$ component of the shearing stress tensor corresponding to the strain rate $\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}$. On taking the Fourier-Laplace transform of Eq. (3) one notices that the viscoelastic approximation corresponds to replacing the zero frequency shear viscosity $\eta$ by a frequency dependent shear viscosity:

$$\tilde{\eta}(\omega) = \frac{G}{(-i\omega + \tau_c)}.$$  

(4)

Defining $\tau_c = \frac{G}{\eta}$ to be the relaxation time, we can rewrite Eq. (4) as

$$\tilde{\eta}(\omega) = \frac{G}{(-i\omega + \frac{1}{\tau_c})}.$$  

(5)

Therefore on long time scales for which $\omega \tau_c \ll 1$ one expects the flow to be viscosity dominated. As one starts probing shorter time scales, one expects a transition from a viscous response to an elastic one to take place for $\omega \tau_c \geq 1$. Since for a bacterial bath $\tau_c \approx 1 \text{sec}$ we arrive at the following important quantitative point. Unlike a colloidal bath where the onset of propagating shear waves takes place on time scales $\approx 10^{-5} \text{sec}$, in a bacterial bath we predict that this would happen on time scales $\approx 1 \text{sec}$. Thus, if a bacterial bath is perturbed at a frequency of $\approx 1 \text{Hz}$ one would notice a transition from a viscous relaxing behavior to an elastic propagating one. Piezoelectric bimorphs can be used to produce and detect controlled shear motions at a frequency $\approx 1 \text{Hz}$ [10] in a bacterial bath. In contrast, in order to see the propagation of shear waves in a colloidal liquid one needs to perturb it at a high frequency $\approx 10^5 \text{Hz}$. A bacterial bath thus provides us with a “scaled up” non-equilibrium statistical mechanical system. This is the key qualitatively and quantitatively interesting observation made in this paper. We will analyze the emergence of shear waves in some more detail below.

An analysis based on memory function formalism can be used to obtain a statistical mechanical description of the transverse current fluctuations in a bacterial bath. In this formalism the spectrum $\tilde{C}_T(k, \omega)$ of transverse current fluctuations (i.e. the Fourier-Laplace transform of the transverse current autocorrelation function $C_T(k, t) = \langle j_{-k}^T(t) j_k^T(0) \rangle$) takes the form:

$$\tilde{C}_T(k, \omega) = \frac{C_T(k, t = 0)}{[-i\omega + \frac{k^2}{\rho} \tilde{M}_T(k, \omega)]}.$$  

(6)

where $\tilde{M}_T(k, \omega)$ is the pertinent memory function which converges to the hydrodynamic viscosity $\eta$ in the long wavelength and long time limit. Modelling the memory function in the time domain as $M_T(k, t) = \tilde{M}_T(k, \omega) e^{-t/\tau_c(k)}$ we arrive at the following expression for the real part of the spectrum $\tilde{C}_T(k, \omega)$ of transverse current fluctuations:

$$C_T(k, \omega) = \frac{\omega_0^2 \omega_1^2 \tau_c(k)}{\pi [\omega^2 + \tau_c^2(k)(\omega_1^2 - \omega^2)^2]}.$$  

(7)

where $\tau_c(k)$ is the wave vector dependent lifetime of correlated structures in the bacterial bath; $\omega_0^2 = C_T(k, t = 0)$ and $\omega_1^2 = \frac{k^2 G}{\rho}$, with $\rho$ the mass density of a
bacterial swirl. We can now establish the criterion for the existence of propagating transverse modes. From Eq. (7) it is clear that the condition for $C_T(k, ω)$ to have a peak at a nonzero frequency, i.e. the threshold condition for shear waves to propagate at a finite speed is $\omega^2 = \frac{1}{\tau_c^2} \frac{1}{c(k)}$. The corresponding value for the critical wavevector is $k_c = \frac{2\pi}{l_c} = \frac{1}{\tau_c^2} \sqrt{\frac{2}{\rho}}$ which corresponds to a length scale of the order of the spatial scale $l_c$ of a bacterial swirl. Thus, as one starts sweeping over wavevectors, the form of the spectrum $C_T(k, ω)$ changes from a profile peaked around $ωτ_c = 0$ for $kl_c << 1$ to one peaked around a finite nonzero frequency $ωτ_c ≈ 1$ for $kl_c ≈ 1$ (See Fig. 1).

Fig 1. Schematic representation of spectrum of transverse current autocorrelation function $C_T(k, ω)$ as a function of $ωτ_c$. The figure shows transition from a profile centered around $ωτ_c = 0$ (dashed curve) for $kl_c << 1$ to a profile with a side peak at $ωτ_c = 1$ for $kl_c = 1$ illustrating the phenomenon of onset of shear waves.

Using $τ_c ≈ 2 sec$, $l_c ≈ 10 µm$ and an appropriate value of $ρ$ based on experimental observations $^5$ one can thus calculate the high frequency rigidity modulus $G$ for a bacterial bath.

3. Discussion

There have been some interesting studies of Brownian motion of spherical probes in wormlike micelles $^{11}$ where they study the role of polymer solutions in determining diffusion behavior of probes. The focus of the work presented here is quite different from that. We focus on a super-diffusive to diffusive behavior of the type that is observed, for instance, in a simple liquid where a ballistic to diffusive transition takes place at a typical relaxation time scale $^{1, 12}$. This leads to an interesting viscoelastic effect. To summarize, we predict an onset of shear waves in a bacterial bath at $\approx 1 Hz$ as a result of structural ordering on larger length scales compared to that in a dense viscous colloidal liquid. This is a consequence of a large spatial scale arising from the swirl size of the bacteria forming the bath. This enables us to view a bacterial bath as a ‘scaled up’ dense viscous liquid. Furthermore, our work points to an independent test of structural ordering in a bacterial bath. We expect this work to generate interest in testing the predictions through computer simulations and experiments in bacterial baths.

Acknowledgements

It is a pleasure to thank E. G. D. Cohen and M. C. Marchetti for insightful comments on propagation effects in dense liquids. I also thank V. Shenoy, Abhishek Dhar and G. V. Shivashankar for discussions.
References

[1] J. P. Hansen and I. R. McDonald, *Theory of Simple Liquids*, Academic Press, London, New York, San Francisco (1976).

[2] W. E. Alley, B. J. Alder and S. Yip, *Phys. Rev. A* 27, 3174 (1983).

[3] T. R. Kirkpatrick *Phys. Rev. Lett.* 53, 1735 (1984).

[4] Supurna Sinha and M. C. Marchetti, *Phys. Rev. A* 46, 4942 (1992).

[5] X. L. Wu and A. Libchaber, *Phys. Rev. Lett* 84, 3017 (2000).

[6] P. N. Pusey, H. N. W. Lekkerkerker, E. G. D. Cohen and I. M. de Schepper *Physica A* 164, 12 (1990).

[7] J. Schofield, A. H. Marcus and S. A. Rice *J. Phys. Chem* 100, 18950 (1996).

[8] C. Cheung, Y. H. Hwang, X. L. Wu and H. J. Choi, *Phys. Rev. Lett* 76, 2531 (1996).

[9] G. V. Soni, B. M. Jaffar Ali, Y. Hatwalne and G. V. Shivashankar, *Biophysical Journal* 84, 2634 (2003).

[10] X. Zhang, Y. Zhu and S. Granick, *Science* 295, 663 (2002).

[11] T. G. Mason, K. Ganesan, J. H. Zanten, D. Wirtz and S. C. Kuo, *Physical Review Letters* 79, 3282 (1997); J. H. van Zanten and K. P. Rufener, *Physical Review E* 62, 5389 (2000).

[12] J. K. G. Dhont, *An Introduction to Dynamics of Colloids*, Elsevier, Amsterdam (1996).