N = 8 Superconformal Algebra and the Superstring

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ABSTRACT

The superstring in $D=3, 4$ and $6$ is invariant under an $N = D-2$ superconformal algebra based on $S^{D-3}$. There is a direct relationship between this (world-sheet) symmetry and the super-Poincaré (target space) symmetry. We establish this relationship using the light-cone gauge, show how the statement generalizes to $D=10$ and examine the properties of the $N = 8$ superconformal algebra and the possible implications of its existence.

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In order to understand the physics of a certain model, we need to find its symmetries. The crucial step in the Veneziano model [1] was the discovery of the Virasoro algebra [2], the infinite-dimensional conformal symmetry. It led to the no-ghost theorem, and in the modern era it has been used e.g. to construct string field theory. It was also the extension of this symmetry to the superconformal one by Ramond [3] that led to the Ramond-Neveu-Schwarz model [3,4] we use today. However, in this formalism global supersymmetry is somewhat hidden and appears only after the GSO-projection [5]. To remedy this fact, Green and Schwarz proposed their formulation [6], where space-time supersymmetry is manifest. The superconformal symmetry, however, is unconventional [7], and leads to difficulties when one tries to quantize the theory covariantly [8]. It is not directly related to the conventional extended superconformal structures [9].

In both these formulations, the target space symmetries are divorced from the world-sheet ones. This is really not satisfactory. If there is a “fundamental string theory”, we expect the two sets of symmetries to have a common origin. There should be two equivalent principles for formulating the theory. On the one hand, starting with enough physical requirements on the space-time theory, the world-sheet symmetries should follow, while on the other hand the correct assumptions about the two-dimensional physics on the world-sheet should give the right target space behaviour.

In this letter we will show that in the light-cone gauge there is indeed such a correlation. The light-cone formulation can be obtained by using either principle. The resulting theory is described by the transverse coordinates and is explicitly unitary. The difficulties arise in the (super-)Poincaré algebra which is non-linearly implemented [10]. In the quantum case anomalies occur, unless the critical dimension is chosen.

The Green-Schwarz string [6] in the light-cone gauge is described by an action (in a heterotic form)

\[ S = \frac{1}{\pi} \int dz d\bar{z} (\partial \phi^I \overline{\partial} \phi^I + S^a \overline{S}^a) \]  

(1)
The index $I=0, \ldots, D-3$ is vectorial, while $a=0, \ldots, D-3$ is spinorial. The action (1) is classically super-Poincaré invariant for $D = 3, 4, 6$ and 10 [10]. Quantum mechanically matter has to be added for $D=3, 4$ and 6 in order to avoid anomalies. Already at this point we would like to remark that the potential occurrence of an anomaly in the super-Poincaré algebra points towards a close connection between some of its generators and the world-sheet symmetry that carries the anomaly in a covariant formulation. The most important issue of this paper is to specify this connection.

Before going into details on the super-Poincaré and superconformal algebras, we would like to introduce some division algebra formalism, by now well known to be related to the space-times with $D = 3, 4, 6$ and 10 and to the supersymmetric structures appearing in these dimensionalities [11-15]. We denote by $K_\nu$ the division algebra of dimension $\nu$: $K_1 = \mathbb{R}$, the reals, $K_2 = \mathbb{C}$, the complex numbers, $K_4 = \mathbb{H}$, the quaternions, and $K_8 = \mathbb{O}$, the octonions. In the following, $\nu$ and $D-2$ are exchangeable. Conjugation of an element $x \in K_\nu$ is denoted $x^*$, not to confuse with the complex structure of the world-sheet. Division algebra multiplication encodes the Clifford algebra of transverse space-time. So for example, is the equation $c^* = vs$ equivalent to $c_a = v^I \gamma_{Ia} s_a$, where $v \in s_\nu$, $s \in s_s$ and $c \in s_c$ of $SO(8)$, and analogously for lower dimensionalities. We also use the notation $[x] = \frac{1}{2} (x + x^*)$ and $\{x\} = \frac{1}{2} (x - x^*)$. Structure constants and associator coefficients are defined by $[e_i, e_j] = 2 \sigma_{ijk} e_k$, $[e_i, e_j, e_k] = 2 \rho_{ijkl} e_l$, where $\{e_i, i = 1, \ldots, 7\}$ are the imaginary units.

The action (1) can trivially be rewritten in this notation as

$$S = \frac{1}{2} \int d\tau d\bar{\tau} \left[ \partial \phi^* \partial \phi + S^* \partial S \right]$$

In $D=3, 4$ and 6, the light-cone superstring action is invariant under an $N=D-2$ extended superconformal algebra [15]. We only consider the part of the
generators containing holomorphic fields, with
\[ \varphi^I(z)\varphi^J(\zeta) \sim \delta^{IJ} \ln(z - \zeta) \]
\[ S^a(z) S^b(\zeta) \sim \frac{\delta^{ab}}{z - \zeta} \] (3)

The generators of the algebras are [15]
\[ \mathcal{J} = \frac{1}{2} S^* S \]
\[ \mathcal{G} = \partial \varphi S \]
\[ \mathcal{L} = \frac{1}{2} [\partial \varphi^* \partial \varphi - S^* \partial S] \] (4)

with the operator product expansions
\[ \mathcal{J}_\alpha(z) \mathcal{J}_{\alpha'}(\zeta) \sim \frac{c/3}{(z - \zeta)^2} [\alpha \alpha'] + \frac{2}{z - \zeta} \mathcal{J}_{\alpha \alpha'} \]
\[ \mathcal{J}_\alpha(z) \mathcal{G}_{\Omega}(\zeta) \sim -\frac{1}{z - \zeta} \mathcal{G}_{\Omega \alpha} \]
\[ \mathcal{G}_\Omega(z) \mathcal{G}_{\Omega'}(\zeta) \sim \frac{2c/3}{(z - \zeta)^3} [\Omega \Omega'] + \frac{2}{(z - \zeta)^2} \mathcal{J}_{\Omega \Omega'}(\zeta) + \frac{1}{z - \zeta} (\partial \mathcal{J}_{\Omega \Omega'} + 2 [\Omega \Omega'] \mathcal{L}) \]
\[ \mathcal{L}(z) \mathcal{J}(\zeta) \sim \frac{1}{(z - \zeta)^2} \mathcal{J}(\zeta) + \frac{1}{z - \zeta} \partial \mathcal{J} \]
\[ \mathcal{L}(z) \mathcal{G}(\zeta) \sim \frac{3/2}{(z - \zeta)^2} \mathcal{G}(\zeta) + \frac{1}{z - \zeta} \partial \mathcal{G} \]
\[ \mathcal{L}(z) \mathcal{L}(\zeta) \sim \frac{c/2}{(z - \zeta)^4} + \frac{2}{(z - \zeta)^2} \mathcal{L}(\zeta) + \frac{1}{z - \zeta} \partial \mathcal{L} \] (5)

where fields in \( \nu \)- or \((\nu - 1)\)-dimensional representations are given indices by contractions \( X_a = [a^* X] \), and where the anomaly \( c \) with these minimal field contents take the value \( 3\nu/2 \). The Kac-Moody part of this algebra is \( S^{\nu - 1} \). A similar statement applies to \( D = 10 \), as we will soon describe. By examining the gauge-fixing procedure of the Green-Schwarz superstring [6] to the light-cone gauge, and in particular the non-linear realization of the super-Poincaré algebra, we will give an interpretation of the world-sheet superconformal algebra in terms of space-time supersymmetry.
Consider the constraints derived from the Green-Schwarz action for the left-moving variables of a heterotic string (in \(SL(2; K)\)-notation):

\[
\begin{align*}
D_\alpha &\equiv \pi_{\dot{\alpha}} - \frac{1}{\sqrt{2}} \Pi_{\dot{\alpha}\alpha} \theta^\alpha + \frac{1}{4} (\theta \partial \theta^\dagger - \partial \theta \theta^\dagger)_{\dot{\alpha}\alpha} \theta^\alpha \approx 0 \\
L &\equiv \frac{1}{4} \Pi_{\dot{\alpha}\alpha} \Pi^{\dot{\alpha}\alpha} \approx 0
\end{align*}
\]

(6)

where \(\Pi = \partial \varphi + \frac{1}{\sqrt{2}} (\theta \partial \theta^\dagger - \partial \theta \theta^\dagger)\) and \(\pi\) is the conjugate momentum of \(\theta\). The special property of the spinorial constraint is that it contains an equal number of first and second class constraints [7,8]. When a light-cone gauge is chosen, the separation comes about naturally, splitting the \(SO(1, D-1)\) spinor into two spinors of the transverse group. The light-cone gauge amounts to choosing

\[
\varphi^+(z) = \alpha^+ \ln z \\
\theta^1 = 0
\]

(7)

The remaining part of the spinorial constraint reads \(\pi^2 + \alpha^+ \theta^2 \approx 0\) and is obviously second class. When we eliminate it and define \(S = \sqrt{2 \alpha^+ \theta^2}\), the spinor correlator in eq. (3) is recovered. Then, one can solve for \(\varphi^-\) and \(\pi^1\) through eq. (6) to obtain

\[
\begin{align*}
\partial \varphi^- &= \frac{z}{2 \alpha^+} [\partial \varphi^* \partial \varphi - S^* \partial S] \\
\pi^1 &= \frac{1}{2} \sqrt{\frac{z}{\alpha^+}} \partial \varphi S
\end{align*}
\]

(8)

When we now go back to the superconformal generators of eq. (4), we notice that the variables eliminated by the gauge choices are exactly the superconformal generators \(L\) and \(G\). From the light-cone variables, we can construct the now non-linearly realized super-Poincaré generators. The crucial part, \(i.e.\) the part where anomalies may appear, contains \(P^-, J^{+-}, J^-\) and \(Q^-,\) the generators that take us
out of the quantization surface. The complete set of generators is [10]

\[
P^+ = \alpha^+
\]

\[
P^- = \frac{1}{2\alpha^+} \int \frac{dz}{2\pi i} z [\partial \phi^* \partial \phi - S^* \partial S] = \frac{\mathcal{L}_0}{\alpha^+}
\]

\[
P = p
\]

\[
J^{+-} = x^+ \frac{1}{2\alpha^+} \int \frac{dz}{2\pi i} z [\partial \phi^* \partial \phi - S^* \partial S] + \alpha^+ \frac{\partial}{\partial \alpha^+} = \frac{x^+}{\alpha^+} (\mathcal{L}_0 - \frac{1}{2}) + \alpha^+ \frac{\partial}{\partial \alpha^+}
\]

\[
J^+ = x^+ p - \alpha^+ x
\]

\[
J^- = -p \frac{\partial}{\partial \alpha^+} - \frac{1}{2\alpha^+} \int \frac{dz}{2\pi i} z \left( \tilde{\phi} [\partial \phi^* \partial \phi - S^* \partial S - \frac{1}{2}] - \frac{1}{2} (\partial \phi S) S^* \right) =
\]

\[
= -p \frac{\partial}{\partial \alpha^+} - \frac{1}{\alpha^+} \int \frac{dz}{2\pi i} z \left( \tilde{\phi} (\mathcal{L} - \frac{1}{2z^2}) - \frac{1}{4} \mathcal{G} S^* \right)
\]

\[
J^{1J} = 2 x^+ [p, J] + \int \frac{dz}{2\pi i} \left( \tilde{\phi}^I \partial \tilde{\phi}^J + \frac{1}{4} [S^* e^*_I (e_J S)] \right)
\]

\[
Q^+ = 2^{1/4} \sqrt{\alpha^+} \int \frac{dz}{2\pi i} z^{-1/2} S
\]

\[
Q^- = \frac{1}{2^{1/4} \sqrt{\alpha^+}} \int \frac{dz}{2\pi i} z^{1/2} \partial \phi S = \frac{1}{2^{1/4} \sqrt{\alpha^+}} \mathcal{G}_0
\]

(9)

(transverse indices are again suppressed, so that \(J^-\) contains the components \(J^{-I}\) etc.). For convenience, we have separated out the logarithmic mode of \(\phi^\mu\) according to \(\tilde{\phi}^\mu(z) = \phi^\mu(z) - \ln z \int \frac{d\zeta}{2\pi i} \partial \phi^\mu(\zeta)\), and defined \(x^\mu = \int \frac{dz}{2\pi i} \tilde{\phi}^\mu(z)\). The important point is that knowledge of the super-Poincaré generators provides us with information about the superconformal generators, and vice versa. Explicit calculation of the anomaly in \([J^{-I}, J^{-J}]\) of course gives the result \(\nu = 8\).

Compactification to \(D=6\) yields the following changes:
\[ P^- = \frac{\mathcal{L}_0 + \mathcal{L}^\text{int}_0}{\alpha^+} \]

\[ J^{+-} = \frac{x^+}{\alpha^+} (\mathcal{L}_0 + \mathcal{L}^\text{int}_0 - \frac{1}{2}) + \alpha^+ \frac{\partial}{\partial \alpha^+} \]

\[ Q^- = \frac{1}{2^{1/4} \sqrt{\alpha^+}} (\mathcal{G}_0 + \mathcal{G}^\text{int}_0) \]

\[ J^- = -p \frac{\partial}{\partial \alpha^+} - \frac{1}{\alpha^+} \oint \frac{dz}{2\pi i} z \left( \tilde{\varphi}(\mathcal{L} + \mathcal{L}^\text{int} - \frac{1}{2} \frac{1}{z^2}) - \frac{1}{4}(\mathcal{G} + 2\mathcal{G}^\text{int})S^* + \frac{1}{2}\mathcal{J}^\text{int} \partial \varphi \right) \]

\[ J^{IJ} = 2x^{[I} p^{J]} + \oint \frac{dz}{2\pi i} \left( \tilde{\varphi}^I \partial \tilde{\varphi}^J + \frac{1}{4}[S^* e_I^*(e_J S)] + \frac{1}{2}[e_I^* e_J \mathcal{J}^\text{int}] \right) \]

where \( \mathcal{J}^\text{int}, \mathcal{G}^\text{int} \) and \( \mathcal{L}^\text{int} \) is an \( N = 4 \) superconformal algebra for the internal degrees of freedom, and we are working with quaternions instead of octonions. An anomaly-free theory arises if \( c^\text{int} = 6 \) and if the nullmode condition

\[ \oint \frac{dz}{2\pi i} z \left( J^I_{\text{int}} J^K_{\text{int}} - \frac{1}{3} \delta_{IK} J^L_{\text{int}} J^L_{\text{int}} \right) = 0 \]

is satisfied. We note that while \( \mathcal{J} = \frac{1}{2} S^* S \) contains the antiselfdual combination of spinors, we find in \( J^{IJ} \) the selfdual combination. Hence the internal algebra has the structure corresponding to an antiselfdual multiplet, while the transverse spacetime algebra corresponds to a selfdual multiplet. These two \( N = 4 \) algebras are independent, and when one goes about constructing sigma-models, one will have to consider two independent hyperkähler structures, one in the internal sector and one in the noncompact sector.

If we compactify down to \( D = 4 \), we obtain the same operators as in (10), except for \( J^- \), which now reads

\[ J^- = -p \frac{\partial}{\partial \alpha^+} - \frac{1}{\alpha^+} \oint \frac{dz}{2\pi i} z \left( \tilde{\varphi}(\mathcal{L} + \mathcal{L}^\text{int} - \frac{1}{2} \frac{1}{z^2}) - \frac{1}{4}(\mathcal{G} + 2\mathcal{G}^\text{int})S^* - \frac{1}{2}\mathcal{J}^\text{int} \partial \varphi + \frac{1}{2} A^* \right) \]

Now the internal algebra is more complicated. It contains the \( N = 2, c = 9 \) superconformal algebra \( \mathcal{J}^\text{int}, \mathcal{G}^\text{int} \) and \( \mathcal{L}^\text{int} \) as well as a complex chiral multiplet \((A, \mathcal{R})\)
of conformal weights \((2, \frac{3}{2})\) with the following operator products:

\[
\begin{align*}
\mathcal{G}^\text{int}_a(z)\mathcal{G}^\text{int}(\zeta) &\sim \frac{6e_a}{(z-\zeta)^3} + \frac{2e_a}{z-\zeta}L^\text{int} - \frac{e_a}{(z-\zeta)^2}(\mathcal{J}^\text{int}(z) + \mathcal{J}^\text{int}(\zeta)) \\
\mathcal{J}^\text{int}(z)\mathcal{G}^\text{int}(\zeta) &\sim \frac{i}{z-\zeta}\mathcal{G}^\text{int} \\
\mathcal{J}^\text{int}(z)\mathcal{J}^\text{int}(\zeta) &\sim \frac{1}{(z-\zeta)^2} \\
\mathcal{G}^\text{int}_a(z)\mathcal{A}(\zeta) &\sim \frac{3e_a^*}{(z-\zeta)^2}\mathcal{R}(\zeta) + \frac{e_a^*}{z-\zeta}\partial\mathcal{R} \\
\mathcal{G}^\text{int}_a(z)\mathcal{R}(\zeta) &\sim \frac{e_a}{z-\zeta}\mathcal{A} \\
\mathcal{J}^\text{int}(z)\mathcal{A}(\zeta) &\sim \frac{i}{z-\zeta}\mathcal{A} \\
\mathcal{J}^\text{int}(z)\mathcal{R}(\zeta) &\sim \frac{2i}{z-\zeta}\mathcal{R}
\end{align*}
\]

Expressions like \((\mathcal{G}^\text{int})^2\) in the above expressions are normal ordered with respect to the modes of the currents. This prescription differs from the normal ordering with respect to the modes of, say, free component fields. The operator algebra in (13) replaces the nullmode condition (11). We do not know whether it is peculiar to some compactifications to \(D = 4\) or a more general property of the internal sector of \(D = 4\) superstrings. We have here an algebraic structure on the internal
space without explicit appearance of coordinates. We have not checked, but it may well be possible to do so, whether this algebra, or some algebra of this type has to appear for $J^-$ to be non-anomalous. If that is the case, one will have an instrument for treating the internal manifold in an abstract algebraic manner, that might be useful for extracting the physical consequences of specific choices for internal manifolds, and possibly for demonstrating “equivalence” between different manifolds with respect to their properties concerning string propagation.

What we still have not shown is that the interpretation of the super-Poincaré generators in terms of superconformal generators is valid also for the case $D=10$. That will be the subject of the rest of this letter.

Let us now turn to the generalization to $N=8$. We will give an intuitive step by step construction leading to the final form of the $N=8$ superconformal generators and their algebra. Since on the light cone the spacetime supersymmetry algebra for $D=10$ has the same structure as for $D=6$ without a compact sector, one feels compelled to simply replace quaternions with octonions. The operator product of the imaginary currents $J = \frac{1}{2} S^a S$ is then given by

$$J^i(z) J^j(\zeta) = -\frac{4}{(z-\zeta)^2} \delta^{ij} + \frac{2}{z-\zeta} \left( \sigma_{ijk} J^k + \rho_{ij\alpha\beta} S^\alpha S^\beta \right)$$  \(14\)

Hence this current algebra does not close, and we may attribute this fact to the nonassociativity of the octonions, or equivalently to the fact that $S^7$ is not a group manifold. Using octonions, the 7-sphere is economically described by $S^7 = \{ X \in O \mid |X| = 1 \}$, with tangent vectors $X e_i$ and normal $X$ [16]. This defines a connection without curvature and torsion $T_{ijk}(X) = [(X e_i)^*(X e_j)e_k]$. Note that

$$T_{ijk}(X)|_{X=1} = \sigma_{ijk} \quad \nabla_i T_{jkl}(X)|_{X=1} = 2 \rho_{ijkl}$$  \(15\)

Hence we may move from the north pole $X=1$ and form $J$ by multiplication in a
basis corresponding to another point on \( S^7 \), \( J = \frac{1}{2} (XS)^*(XS) \), to obtain

\[
J^i(z) J^j(\zeta) = -\frac{4}{(z-\zeta)^2} \delta^{ij} + \frac{2}{z-\zeta} \left( T^{ijk}(X) J^k + \nabla^i J^j \right)
\]  

(16)

The rest of the algebra has a similar structure: for \( G = (X\varphi)^*(XS) = \sigma_{ab}^{I}(X)S_{b}^{I}\varphi^{J} \), we obtain an algebra that closes modulo infinitesimal shifts on \( S^7 \), i.e. besides the “expected” terms there are terms containing \( \nabla^i \).

By considering finite transformations generated by \( J \), we see that we transform \( Xe_a \rightarrow Y(Xe_a) \) for another unit octonion \( Y \), i.e. we obtain a rotated basis of tangent vectors at \( YX \in S^7 \). Clearly we can also generate the basis \((X^*Y^*)(Y(Xe_i))\) at the northpole \( Z = 1 \). This basis is rotated with respect to the \( Xe_i \). We conclude that the shifts operate not on the 7-sphere, but on an \( SO(7) \)-bundle over \( S^7 \), i.e. on \( SO(8) \). The operator product of, say, \( J_X = \frac{1}{2} (XS)^*(XS) \) and \( J_{YX} = \frac{1}{2} (Y(XS))^*(Y(XS)) \), does not fit into the simple scheme displayed above. But then, we would expect it to contain terms with an infinite number of \( S^7 \)-derivatives. We will call the structure we found a current algebra that is “local on \( S^7 \).

Up to this point we have treated \( X \) as a number. For the algebra to close, we need a mechanism that takes care of the infinitesimal shifts on \( S^7 \). This is accomplished by letting the \( S^7 \) coordinate \( X \) be an operator, and adding an \( S^7 \) translation generator to \( J \). More precisely, we introduce a pair of octonionic bosons \((\lambda, \omega)\) with conformal weights \((\frac{1}{2}, \frac{1}{2})\) and their superpartners \((\theta, \pi)\) of weights \((0, 1)\), set \( X = \lambda/|\lambda| \) and define

\[
\mathcal{J} = \{ \omega^*\lambda \} + \frac{1}{2} (XS)^*(XS) \\
\mathcal{G} = \pi^*\lambda - \partial\theta^*\omega + (X\partial\varphi)^*(XS) + \frac{1}{2} (XS)^*(\Lambda S) - \frac{1}{2} (\Lambda S)^*(XS) \\
\mathcal{L} = \frac{1}{2} [\partial\lambda^*\omega - \lambda^*\partial\omega] - [\pi^*\partial\theta] + \frac{1}{2} [\partial\varphi^*\partial\varphi] - \frac{1}{2} [S^*\partial S]
\]  

(17)

where \( \Lambda = |\lambda|^{-1}(\partial\theta - X[X^*\partial\theta]) \) is the tangential part of \( \partial\theta \). The algebra of these operators is soft [17], i.e. it closes with field dependent structure “constants” and
anomaly terms. The classical algebra is

\[ J_\alpha(z) J_{\alpha'}(\zeta) \sim \begin{cases} \frac{2}{z - \zeta} J_{(\alpha X^*)(X\alpha')} & \text{if } \alpha = \alpha' \\ \frac{1}{z - \zeta} \left( G_{(\Omega X^*)(X\alpha)} + J_{(\lambda^{-1}((\partial\theta\Omega)\alpha - \partial\theta((\Omega X^*)(X\alpha))))} \right) & \text{if } \alpha \neq \alpha' \end{cases} \]

\[ G_{\Omega}(\zeta) G_{\Omega'}(\zeta) \sim \begin{cases} \frac{2}{(z - \zeta)^2} J_{(\Omega^* X^*)(X\Omega)} + \frac{2}{z - \zeta} \left( \frac{1}{2} \partial(\lambda^{-1}((\partial\theta\Omega)\alpha - \partial\theta((\Omega X^*)(X\alpha)))) + [\Omega^* \Omega'] L \right) & \text{if } \Omega = \Omega' \\ \frac{1}{z - \zeta} \left( G_{(\Omega X^*)(X\alpha)} + J_{(\lambda^{-1}((\partial\theta\Omega)\alpha - \partial\theta((\Omega X^*)(X\alpha))))} \right) & \text{if } \Omega \neq \Omega' \end{cases} \]  

(18)

(the quantum algebra involves some subtleties that will be addressed in a forthcoming paper [19]). We note that only \( \lambda \) and \( \theta \) enter into the field dependence, so that the structure functions (anti)commute. They have a natural interpretation in terms of the torsion tensor and its superpartner on \( S^7 \). The reduction to the \( N < 8 \) algebras of eq.(5) is obvious: just remove all associator terms. If one replaces the term \( \frac{1}{2} (X S)^*(X S) \) in \( J \) by \( \{ (X \omega)^*(X \lambda) \} \), where \( (\lambda', \omega') \) is a conjugate pair of bosons of weights \( (1/2, 1/2) \), and makes the corresponding replacements in \( G \) and \( L \), one finds the soft algebra Berkovits describes in the context of the twistor formulation of the superstring [20]. Thus we have found a natural generalization of the \( N=4 \) free field constructions.

We want to emphasize that we are working with explicit generators, and therefore automatically have the Jacobi identities fulfilled. If we on the right hand side of eq. (18) set \( X = 1, \partial \theta = 0 \), we get a non-associative algebra like the ones in [17,18,21]. The present formulation is stronger. A non-trivial feature is that, unlike what could be expected from a naive consideration of the properties of the octonions, the Kac-Moody part \( \hat{S}^7 \) actually commutes with the \( SO(8) \) of space rotations, and that our seven-sphere is therefore not the quotient of this group with an \( SO(7) \) subgroup, but an additional symmetry. This and other issues concerning the \( N = 8 \) algebra are to be developed in detail in a forthcoming publication [19].

The \( N = 8 \) generators of eq. (17) as they stand are not the ones that enter in the super-Poincaré generators (9). First the “parameter fields” \( (\lambda, \omega) \) and \( (\theta, \pi) \) must be removed — they are not physical fields. With our present understanding of the role of these fields we cannot make any certain statements about their
physical interpretation in a covariant theory. We do not for example know what
the constraints are that eliminate the parameter fields. A possible interpretation
is that they are a remnant of a set of super-twistor variables from a combined
space-time/twistorial formulation. For the moment we will take a very pragmatic
point of view and note that in order to reduce the field content to that of the light-
cone superstring, we need some quantum mechanically consistent set of constraints
(note that the superconformal generators cannot be set to zero with a quantum-
mechanically nilpotent BRST charge). We may state \( \omega \approx 0 \) and \( \pi \approx 0 \), allowing
for the gauge choices \( X = 1, \theta = 0 \), which of course takes us back to the situation in
eqs. (14) and (16). The closure of the algebra is obstructed by the gauge choices.
However, the role of the generators of the superconformal algebra in the super-
Poincaré algebra is identical to that in the lower dimensionalities.

Finally, one may speculate in the ultimate role of the superconformal algebra
in some kind of “covariant” formulation. We have a strong belief that the \( N = D-2 \)
superconformal algebras have a fundamental significance, yet their generators enter
very asymmetrically \( e.g. \) in the super-Poincaré generators, where \( \mathcal{J} \) is not seen at
all. It is tempting to think that the relation between space-time and worldsheet
symmetries established in this paper gives a glimpse of the structure of a bigger
symmetry.

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