A special model of a massless vector-field is presented, which has an extra modified-gravity-type interaction term in the action. The cosmology of the model is studied with standard noninteracting relativistic matter added. It is found that this cosmology can have an early phase where the vector-field starts to compensate a (Planck-scale) cosmological constant and a late Friedmann–Robertson–Walker (FRW) phase where the relativistic-matter energy density dominates the dynamic vacuum energy density.

Journal: Int. J. Mod. Phys. D 21 (2012) 1250025

Preprint: [arXiv:1108.1995]

Keywords: general relativity; early universe; cosmological constant.
1. Introduction

The cosmological constant problem\cite{12} is perhaps the most important outstanding question of modern physics. In a nutshell, the problem is to explain why the energy scale corresponding to the measured value of the cosmological constant $\Lambda$ is negligible compared to the energy scales of elementary particle physics, $|\Lambda|^{1/4} \ll E_{\text{QCD}} \ll E_{\text{ew}} \ll E_{\text{Planck}} \sim 10^{18}$ GeV.

The cosmological constant $\Lambda$ can be canceled dynamically and without fine-tuning in a particular massless-vector-field model\cite{31}. But this model runs into two obstacles. First, the local Newtonian dynamics is ruined\cite{5}. Second, the final phase of the model universe expands so rapidly (scale factor $a \propto t$) that any standard matter contribution ($\rho_M \propto 1/a^p$ with $p = 4$ for relativistic matter or $p = 3$ for non-relativistic matter) becomes irrelevant compared to the decreasing remnant vacuum energy density ($\rho_V \propto 1/t^2$).

Following previous work on the $q$–theory approach\cite{6,7,8,9,10,11,12} to the main cosmological constant problem, an extended model with two massless vector-fields has been found\cite{13,14}, which evades the first obstacle regarding the local Newtonian dynamics. Here, we present another model with a single vector-field and a modified-gravity-type interaction term, which circumvents the second obstacle by having a final phase with slower expansion ($a \propto t^{1/2}$).

2. Model and Ansatz

Consider the following model of a massless vector-field $A_\alpha(x)$ with an effective action ($\hbar = c = 1$)

$$ S_{\text{eff}} = -\int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \epsilon(Q_3) + \Lambda + f(R)A_\alpha A^\alpha + L_M \right], \quad (1a) $$

where $R$ is the Ricci scalar, $\Lambda$ the effective cosmological constant, and $f(R)$ a function to be specified later. For the moment, the contribution of other matter fields is set to zero, $L_M = 0$. The $q$–theory-type scalar entering the above action is defined as follows:

$$ Q_3(x) \equiv g^{\alpha\beta}(x) A_\alpha;\beta(x), \quad (1b) $$

with the spacetime coordinate $(x) = (x^1, x^2, x^3, t)$ as argument and the semicolon standing for covariant derivation. The particular contraction (1b) was mentioned in
Vector-field model with compensated $\Lambda$ and FRW phase

Ref. [3] but not studied in detail.

In the following, we consider a spatially-flat Friedmann–Robertson–Walker (FRW) universe with scale factor $a(t)$ for cosmic time $t$ and take the isotropic Ansatz [3] for the vector-field:

$$\left( g_{\alpha \beta}(t) \right) = \left( \text{diag}[1, -a^2(t), -a^2(t), -a^2(t)] \right), \quad (2a)$$

$$A_{\alpha}(t) = A_0(t) \delta^0_{\alpha}. \quad (2b)$$

The Ansätze (2) give

$$R = -6 \left( \dot{H} + 2 H^2 \right), \quad (3a)$$

$$Q_3 = \dot{A}_0 + 3 H A_0, \quad (3b)$$

where the overdot stands for differentiation with respect to the cosmic time $t$ and the Hubble parameter $H(t)$ is defined by $H(t) \equiv \dot{a}(t)/a(t)$.

Note that the variable $(Q_3)^2 = (\dot{A}_0 + 3 H A_0)^2$ differs from the variable $(Q_1)^2 \equiv A_{\alpha \beta} A^{\alpha \beta} = A_0^2 + 3 H^2 A_0^2$ discussed in Refs. [8] and [13]. For the statics of $q$–theory, the precise realization of the $q$-type vacuum variable is irrelevant [here, either $Q_3$ or $Q_1$]. But for the dynamics, the realization matters [here, giving a FRW universe with $t H(t)$ approaching either the value $1/2$ or the value $1$]. See App. A of Ref. [12] for a brief introduction to the basic ideas of $q$–theory. This introduction to $q$–theory may give the reader some background information but is not needed to understand the present article, which is entirely self-contained.

3. Reduced Field Equations

3.1. Vector-field equation

The variational principle applied to the vector-field of action (1a) gives the following equation:

$$\nabla_{\alpha} \left( \frac{d \epsilon}{d Q_3} \right) = 2 f(R) A_{\alpha}. \quad (4)$$

The Ansätze (2) reduce this field equation to a single ordinary differential equation (ODE),

$$\zeta \ddot{A}_0 + \left( 3 H \zeta + \dot{\zeta} \right) \dot{A}_0 + \left[ 3 \dot{H} \zeta + 3 H \dot{\zeta} - f(R) \right] A_0 = 0, \quad (5)$$

with $\zeta \equiv (2 Q_3)^{-1} d \epsilon/dQ_3$. 
3.2. Generalized FRW equations

The energy-momentum tensor of the vector-field \( A_\alpha \) follows from the variation of the action with respect to the metric field \( g_{\alpha\beta}(x) \). The result is

\[
T_{\alpha\beta} = \left[ \epsilon - Q_3 \frac{d\epsilon}{dQ_3} - A^A \nabla_\lambda \left( \frac{d\epsilon}{dQ_3} \right) \right] g_{\alpha\beta} + 2 A_{(\alpha} \nabla_{\beta)} \left( \frac{d\epsilon}{dQ_3} \right) \\
-2 \left( A^2 f' R_{\alpha\beta} - \frac{1}{2} A^2 f g_{\alpha\beta} + f A_\alpha A_\beta - \nabla_\alpha \nabla_\beta (f' A^2) + g_{\alpha\beta} \nabla^2 (f' A^2) \right),
\]

where the round brackets around spacetime indices denote symmetrization and the prime on \( f \) stands for differentiation with respect to \( R \). With the vector-field equation (4), the energy-momentum tensor (6) becomes

\[
T_{\alpha\beta} = \left[ \epsilon - Q_3 \frac{d\epsilon}{dQ_3} \right] g_{\alpha\beta} \\
-2 \left( A^2 f' R_{\alpha\beta} + \frac{1}{2} A^2 f g_{\alpha\beta} - f A_\alpha A_\beta - \nabla_\alpha \nabla_\beta (f' A^2) + g_{\alpha\beta} \nabla^2 (f' A^2) \right).
\]

From the gravitational field equations, the Ansatz (2), and the energy-momentum tensor (7), the generalized FRW equations are

\[
3 H^2 = (8\pi G) \left[ \Lambda + \rho(A) + \rho_M \right],
\]

\[
2 \dot{H} + 3 H^2 = (8\pi G) \left[ \Lambda - P(A) - w_M \rho_M \right],
\]

having added a further matter contribution from a nonzero term \( \mathcal{L}_M \) in the action (1a), with \( w_M \equiv P_M/\rho_M \) the constant equation-of-state (EOS) parameter of the corresponding homogeneous perfect fluid. The vector-field contributions on the right-hand sides of (8) are given by

\[
\rho(A) = +\epsilon - Q_3 \frac{d\epsilon}{dQ_3} + 2 \left[ 3 (\dot{H} + H^2) f' A_0^2 + \frac{1}{2} f A_0^2 - 3 H \frac{d}{dt} \left( f' A_0^2 \right) \right],
\]

\[
P(A) = -\epsilon + Q_3 \frac{d\epsilon}{dQ_3} \\
-2 \left[ (\dot{H} + 3 H^2) f' A_0^2 - \frac{1}{2} f A_0^2 - \frac{d^2}{dt^2} \left( f' A_0^2 \right) - 2 H \frac{d}{dt} \left( f' A_0^2 \right) \right].
\]
4. Special Vector-Field Model

4.1. Linear $f(R)$ and quadratic $\epsilon(Q_3)$

The special model considered in this article has a linear function $f(R)$ in the interaction term of the action (1a),

$$\mathcal{F}(R) = \kappa R,$$

(10a)

and a quadratic vector-field function $\epsilon(Q_3)$,

$$\tau(Q_3) = \zeta_0 Q_3^2,$$

(10b)

for nonzero dimensionless coefficients $\kappa$ and $\zeta_0$. In order to be able to cancel a nonzero cosmological constant, these two coefficients must obey the condition

$$\text{sgn}(6\kappa + 5\zeta_0) = \text{sgn}(\Lambda),$$

(10c)

in terms of the sign function $\text{sgn}(x) \equiv x/|x|$ for $x \neq 0$ and $\text{sgn}(0) \equiv 0$. The particular form of condition (10c) will be derived at the end of this subsection.

The Ansatz (10a) reduces the energy density and the isotropic pressure from (9) to the following expressions:

$$\rho(A) = +\epsilon - Q_3 \frac{d\epsilon}{dQ_3} - 6\kappa \left( H^2 A_0^2 + H \frac{d}{dt} A_0^2 \right),$$

(11a)

$$P(A) = -\epsilon + Q_3 \frac{d\epsilon}{dQ_3} - 2\kappa \left( (4\dot{H} + 9H^2) A_0^2 - \frac{d^2}{dt^2} A_0^2 - 2H \frac{d}{dt} A_0^2 \right).$$

(11b)

Similarly, the Ansatz (10b) reduces the ODE (5) to the following equation:

$$\ddot{A}_0 + 3H \dot{A}_0 + \left[ 3\dot{H} - \frac{1}{\zeta_0} f(R) \right] A_0 = 0.$$

(12)

Inserting the function (10a) in the last ODE and using (3a) gives

$$\ddot{A}_0 + 3H \dot{A}_0 + 3\dot{H} A_0 + 6\frac{\kappa}{\zeta_0} \left( \dot{H} + 2H^2 \right) A_0 = 0.$$

(13)

Equation (13) is the core result of this article, as will become clear in Sec. 4.2.

For $t \to \infty$, the asymptotic solution of (13) is given by

$$A_{\text{asymp}}(t) = \overline{\mathbf{t}}, \quad H_{\text{asymp}}(t) = (1/2) t^{-1},$$

(14)

*Condition (10c) is purely technical and appears because of the simple $\epsilon$–function used in (10b). For an appropriate, more complicated, $\epsilon$–function (cf. Refs. 6, 7, 8, 13), the cancelation of the cosmological constant $\Lambda$ can be expected to hold for any sign of $\Lambda*.}
V. Emelyanov, F.R. Klinkhamer

with a constant $\overline{\theta}$ to be determined by the solution of the combined field equations (see Sec. 4.3). Substituting this asymptotic solution into the energy density $\rho(A)$ and the pressure $P(A)$ from (11), we obtain

$$\rho_\Lambda(A_{\text{asympt}}) = +\Lambda + \left(\tau - Q_3 \frac{d\tau}{dQ_3}\right)_{Q_3 = 5\overline{\theta}/2} - \frac{15}{2} \kappa \overline{\theta}^2, \quad (15a)$$

$$P_\Lambda(A_{\text{asympt}}) = -\Lambda - \left(\tau - Q_3 \frac{d\tau}{dQ_3}\right)_{Q_3 = 5\overline{\theta}/2} + \frac{15}{2} \kappa \overline{\theta}^2, \quad (15b)$$

having added the contributions from the cosmological constant $\Lambda$ in the action (1a).

Observe that $\rho_\Lambda(A_{\text{asympt}}) + P_\Lambda(A_{\text{asympt}}) = 0$. In fact, (15) has the particular structure of $q$–theory.6 It is even possible to define a new effective $\epsilon$–function by making a $Q_3$–independent shift, $\tau_{\text{eff}}(Q_3) = \tau(Q_3) - (15/2) \kappa \overline{\theta}^2$.

Equation (15) also explains condition (10c): for the simple quadratic function (10b), it is possible to cancel $\Lambda$ with an appropriate value of the linear $\theta$–coefficient from (14) only if condition (10c) is satisfied. As mentioned before, the actual value $\overline{\theta}$ follows from the solution of the combined field equations. Remark, finally, that the expression on the right-hand side of (15a) may be called the ‘dressed’ cosmological constant if $\Lambda$ from the action (1a) is considered to be the ‘bare’ cosmological constant.

4.2. Heuristic argument

Returning to the vector-field ODE (13), we can give the following heuristic argument for the appearance of the FRW-like asymptotic solution (14), even with a nonzero cosmological constant $\Lambda$ present in the action (1a). The physically relevant case has both coefficients $\kappa$ and $\zeta_0$ nonvanishing, but, for completeness, also the other cases will be briefly mentioned.

For $\kappa \neq 0, \zeta_0 \neq 0$, and a linear time-dependence of the vector component $A_0(t) \propto t$, the $H^2A_0$ term in (13) excludes having $H(t) = \text{const}$. Purely for dimensional reasons, assume $H(t) = \tilde{\gamma} t^{-1}$ with a numerical constant $\tilde{\gamma} > 0$ [excluding the case of a static or contracting Universe with $\tilde{\gamma} \leq 0$]. The first three terms of the left-hand side of (13) then cancel by themselves. This leaves the $\kappa$ term in (13), which vanishes for $\tilde{\gamma} = 1/2$ corresponding to a radiation-dominated FRW universe. Indeed, the functions (14) allow for a compensation of the cosmological constant $\Lambda$, as shown by (15) and the final ODEs to be presented in Sec. 4.3.
For $\kappa = 0$ and $\zeta_0 \neq 0$, the ODE (13) also has the de-Sitter solution with constant $A_0$ and $H$. As said before, precisely this solution is forbidden by having the $\kappa H^2 A_0$ term in (13) if $\kappa \neq 0$.

For $\kappa \neq 0$ and $\zeta_0 = 0$, (13) implies $A_0 = 0$ (i.e., $\vec{\theta} = 0$) for generic $H(t)$ and the cosmological constant can no longer be canceled, as exemplified by (15).

For $\kappa = \zeta_0 = 0$ in the Ansätze (10), the vector-fields have disappeared from the action (1a) altogether and the cosmological constant problem remains unsolved.

Returning to the case of $\kappa \neq 0$ and $\zeta_0 \neq 0$, the above heuristic argument for obtaining a radiation-dominated FRW universe, thus, relies on having an action employing the $Q_3$ field [which gives the $3 H A_0 + 3 \dot{H} A_0$ combination in (13)] and the Ricci scalar (which vanishes precisely for $H = \frac{1}{2} t^{-1}$).

An entirely different question is whether or not solution (14) appears dynamically as an attractor. Here, this question will be addressed numerically, leaving a proper mathematical treatment for the future (see Note Added).

### 4.3. Additional relativistic matter and dimensionless ODEs

From now on, we also consider an additional standard-matter component with energy density $\rho_M$ and pressure $P_M$. These and other dimensional variables can be replaced by the following dimensionless variables:

\[
\{\Lambda, \epsilon, \rho_M, P_M\} \rightarrow \{\lambda, e, r_M, p_M\}, \quad (16a)
\]

\[
\{t, H, Q_3, A_0\} \rightarrow \{\tau, h, q_3, v\}, \quad (16b)
\]

if appropriate powers of the (Planck-type) energy scale $(8\pi G)^{-1/2}$ are used, without further numerical factors. Henceforth, an overdot stands for differentiation with respect to $\tau$, for example, $h(\tau) \equiv \dot{a}(\tau)/a(\tau)$.

The previous results (13) and (14), together with the extra standard-matter component, give the following dimensionless ODEs:

\[
2 \dot{h} + 3 h^2 = \lambda - \zeta_0 \left( \dot{v} + 3 h \dot{v} \right)^2 - w_M r_M \\
+ 2 \kappa \left[ v^2 \left( 4 \dot{h} + 9 h^2 \right) - 2 \dot{v}^2 - 2 v \ddot{v} - 4 h v \dot{v} \right], \quad (17a)
\]

\[
\ddot{v} + 3 h \dot{v} + \left[ 3 \dot{h} + 6 (\kappa/\zeta_0)(\ddot{h} + 2 h^2) \right] v = 0, \quad (17b)
\]

\[
\dot{r}_M + 3 (1 + w_M) h r_M = 0, \quad (17c)
\]
where the last equation describes the evolution of the additional homogeneous perfect fluid of the standard-matter component, with constant EOS parameter \( w_M \equiv p_M/r_M = 1/3 \) for ultrarelativistic particles. The corresponding generalized Friedmann equation from (8a) is given by

\[
3 h^2 = \lambda - \zeta_0 (\dot{\varphi} + 3 h \varphi)^2 + r_M - 6 \kappa (h^2 \varphi^2 + 2 h \varphi \dot{\varphi}).
\]  

(18)

Observe that the right-hand side of (18), evaluated for the asymptotic solution (14), has canceling \( O(t^0) \) terms for one particular value of \( \vartheta \), provided condition (10c) holds. Specifically, the two possible \( \vartheta \) values are given by

\[
\vartheta = \pm \sqrt{\frac{4 \lambda}{5 \kappa + 5 \zeta_0}},
\]

(19)
in terms of the dimensionless cosmological constant \( \lambda \equiv (8\pi G)^2 \Lambda \). The actual sign of \( \vartheta \) will be determined by the initial boundary conditions.

In fact, the ODEs (17) are to be solved with boundary conditions on \( v(\tau) \), \( \dot{v}(\tau) \), \( h(\tau) \), and \( r_M(\tau) \) at \( \tau = \tau_{\text{start}} \), where these particular function values must satisfy the constraint equation (18). The corresponding physical quantities must be small enough for classical gravity to be relevant, e.g., \( r_M(\tau_{\text{start}}) \ll 1 \).

### 4.4. Numerical results

Numerical calculations have been performed for the model defined by (1) and (10), with the dimensionless cosmological constant \( \lambda = 0.02 \) and model parameters \( \kappa = -\zeta_0/2 = -1/2 \). The results are presented in two figures, the first without and the second with dynamical effects from the vector-field.

With special boundary conditions to ensure the exact vanishing of the vector-field, Fig. 1 shows that the cosmological constant \( \lambda \) rapidly dominates the matter component and that exponential expansion sets in. Asymptotically, this model universe approaches de Sitter space having \( \dot{h} = r_M = 0 \). The behavior of \( h(\tau) \) and \( r_M(\tau) \) shown in Fig. 1 is completely standard and can also be obtained analytically.

With boundary conditions of the vector-field in an appropriate domain, Fig. 2 shows that the vector-field is able to compensate a Planck-scale cosmological constant \( \lambda = O(1) \) and that Minkowski spacetime is approached asymptotically, \( h(\tau) \to 0 \) for \( \tau \to \infty \). The numerical solution \( v(\tau) \) asymptotically takes the form (14), with the precise linear coefficient needed for the complete compensation of the
Vector-field model with compensated $\Lambda$ and FRW phase

Fig. 1. Numerical solution of ODEs (17) for parameters $\lambda = 0.02$, $\zeta_0 = 1$, $\kappa = -1/2$, and $w_M = 1/3$. The boundary conditions are $v(10) = \dot{v}(10) = 0$, $r_M(10) = 0.02$, and $h(10) = 0.115470$. With these boundary conditions, the ODEs have the exact solution $v(\tau) = 0$ for $\tau \geq 10$.

Fig. 2. Same as Fig. 1 but now with nonzero starting values for the vector-field, $v(10) = 1 \pm 0.1$ and $\dot{v}(10) = 0$, where the dashed curves correspond to the starting value $v(10) = 0.9$. The corresponding starting values for $h$ follow from (18) and are, respectively, $h(10) = 0.0624391$ and $h(10) = 0.0713376$. The scaling in the $v$-panel uses the constant $\tilde{\theta}$ from (19), which, for $\lambda = 0.02$, takes the value $\tilde{\theta} = \sqrt{2\lambda/5} = 1/(5\sqrt{5})$. The numerical solutions of $v(\tau)$ shown in the left panel approach asymptotically the function $\theta_{\text{num}}\tau$, with $\theta_{\text{num}} = \tilde{\theta}$ within the numerical accuracy of the calculation (the rescaled functions plotted in the left panel run towards the value 1 as $\tau \to \infty$). The corresponding numerical solutions of $h(\tau)$ shown in the middle panel approach asymptotically the function $\gamma_{\text{num}}\tau^{-1}$, with $\gamma_{\text{num}} = 1/2$ within the numerical accuracy of the calculation (the rescaled functions plotted in the middle panel run towards the value 1 as $\tau \to \infty$).

cosmological constant $\lambda$. It needs to be emphasized that this linear coefficient of $v(\tau)$ is not put in by hand but arises dynamically; cf. Ref. 8. See also the remark below (18) and the details given in the caption of Fig. 2.

The final phase shown in Fig. 2 corresponds to an FRW universe ($h \sim \frac{1}{3}\tau^{-1}$) dominated by relativistic matter. The solid curve in the right panel of Fig. 2 corresponds to an asymptotic universe with $r_M \sim 1.09\tau^{-2}$ and $r_V \equiv \left(3h^2 - r_M\right) \sim -0.34\tau^{-2}$ (hence, ratio $r_M/|r_V| \sim 3$) and the dashed curve to an asymptotic universe with $r_M \sim 0.687\tau^{-2}$ and $r_V \sim 0.063\tau^{-2}$ (hence, ratio $r_M/r_V \sim 11$). For these numerical solutions, the action density term $RA_0^2 \propto -6(\dot{h} + 2h^2)v^2$ has also been found to drop to zero faster than $\tau^{-1}$. 

5. Discussion

The vector-field model defined by (1) and (10) provides an attractor solution which compensates an arbitrary positive cosmological constant $\Lambda$ and gives a final universe with Hubble parameter $H(t) = \frac{1}{2} t^{-1}$. This final state resembles a standard radiation-dominated FRW universe.

If the model is extended by the addition of a standard noninteracting-relativistic-matter component, the final state can be a genuine radiation-dominated FRW universe with a subleading (time-dependent) vacuum-energy-density component. Figure 2 shows two possible model universes, which start out with equal matter and vacuum energy densities, $\rho_M = \Lambda \sim (E_{\text{Planck}})^4$, but end up with the matter component dominating, $\rho_M > |\rho_V|$, both components $\rho_M$ and $|\rho_V|$ decreasing as $t^{-2}$ (specific numbers are given in the last paragraph of Sec. 4.4).

In this way, there is a more or less realistic physical description of the earliest cosmological phase as a radiation-dominated FRW universe with a dynamically canceled cosmological constant. Possibly, this description needs to be augmented with the effects from inflation.\textsuperscript{14}

As discussed in Refs. 9, 10, 11, quantum-dissipative processes can be expected to lead to a further (exponential) reduction of $|\rho_V(t)|$ in the very early universe ($kT \gg E_{\text{ew}} \sim \text{TeV}$) and related processes at the electroweak scale can perhaps generate a finite remnant value of order $\rho_V(\infty) \sim (E_{\text{ew}}^2/E_{\text{Planck}})^4 \sim (\text{meV})^4$. Alternative explanations of the remnant vacuum energy density in the $q$-theory framework have been reviewed in Ref. 12.

To conclude, both obstacles mentioned in the Introduction have been dealt with separately, the first in our previous article of Ref. 13 and the second in the present article. It remains to find a joint solution, provided such a solution exists.

\textsuperscript{b}Most likely, the model can be extended to allow for a similar compensation of a cosmological constant $\Lambda$ of arbitrary sign for the case of positive $\Lambda$, also a final cosmological phase with $H(t) = (2/3) t^{-1}$ can be obtained by replacing $\epsilon(Q_3)$ in (12) with $\epsilon(Q_1, Q_2) = \zeta_1 (Q_1)^2 + \zeta_2 (Q_2)^2$, for the contractions $(Q_1)^2 \equiv A_{\alpha, \beta} A^{\alpha, \beta}$ and $(Q_2)^2 \equiv A_{\alpha, \beta} A^{\beta, \alpha}$, and by taking the coefficient $\kappa = - (\zeta_1 + \zeta_2)/2$ in (10). This type of model may be relevant to inflationary effects in vector-field theories with a dynamically canceled cosmological constant\textsuperscript{13}. At this point, it should be mentioned that certain scalar-tensor theories (with a fundamental scalar field $\phi$) have also been argued to give both a compensation of the cosmological constant and a final FRW-like phase.\textsuperscript{15}
Acknowledgments

It is a pleasure to thank M. Kopp and G.E. Volovik for helpful discussions and the referee for useful remarks.

Note Added

For the case of $\lambda > 0$ and $\kappa = -\zeta_0/2 = -1/2$, it is possible to prove the existence of an asymptotically stable (attractor) solution. The proof relies on an appropriate change of variables, knowledge of the solution $r_M(a)$, and the Poincaré–Lyapunov theorem [Theorem 7.1 in Ref. 16]. Most likely, the attractor solution can be established rigorously also for general $\kappa \neq 0$ and $\zeta_0 \neq 0$ with $6\kappa + 5\zeta_0 > 0$.

The present article is the second of a trilogy of articles, the first one being Ref. 13. The third article of the trilogy (Ref. 17) finds the joint solution mentioned in the last sentence of Sec. 5. This third article also gives a detailed mathematical discussion of the attractor behavior in the type of vector-field models considered.

References

1. S. Weinberg, “The cosmological constant problem,” Rev. Mod. Phys. 61 (1989) 1.
2. V. Sahni and A.A. Starobinsky, “The case for a positive cosmological $\Lambda$–term,” Int. J. Mod. Phys. D 9 (2000) 373, arXiv:astro-ph/9904398.
3. A.D. Dolgov, “Field model with a dynamic cancellation of the cosmological constant,” JETP Lett. 41 (1985) 345.
4. A.D. Dolgov, “Higher spin fields and the problem of cosmological constant,” Phys. Rev. D 55 (1997) 5881, arXiv:astro-ph/9608175.
5. V.A. Rubakov and P.G. Tinyakov, “Ruling out a higher spin field solution to the cosmological constant problem,” Phys. Rev. D 61 (2000) 087503, arXiv:hep-ph/9906239.
6. F.R. Klinkhamer and G.E. Volovik, “Self-tuning vacuum variable and cosmological constant,” Phys. Rev. D 77 (2008) 085015, arXiv:0711.3170.
7. F.R. Klinkhamer and G.E. Volovik, “Dynamic vacuum variable and equilibrium approach in cosmology,” Phys. Rev. D 78 (2008) 063528, arXiv:0806.2805.
8. F.R. Klinkhamer and G.E. Volovik, “Towards a solution of the cosmological constant problem,” JETP Lett. 91 (2010) 259, arXiv:0907.4887.
9. F.R. Klinkhamer and G.E. Volovik, “Vacuum energy density kicked by the electroweak crossover,” Phys. Rev. D 80 (2009) 083001, arXiv:0905.1919.
10. F.R. Klinkhamer, “Effective cosmological constant from TeV-scale physics,” Phys. Rev. D 82 (2010) 083006, arXiv:1001.1939.
12. V. Emelyanov, F.R. Klinkhamer

11. F.R. Klinkhamer, “Effective cosmological constant from TeV-scale physics: Simple field-theoretic model,” *Phys. Rev. D* **84** (2011) 023011, arXiv:1101.1281

12. F.R. Klinkhamer and G.E. Volovik, “Dynamics of the quantum vacuum: Cosmology as relaxation to the equilibrium state,” *J. Phys. Conf. Ser.* **314** (2011) 012004, arXiv:1102.3152

13. V. Emelyanov and F.R. Klinkhamer, “Reconsidering a higher-spin-field solution to the main cosmological constant problem,” *Phys. Rev. D* **85** (2012) 063522, arXiv:1107.0961

14. F.R. Klinkhamer, “Inflation and the cosmological constant,” *Phys. Rev. D* **85** (2012) 023509, arXiv:1107.4063

15. C. Charmousis, E.J. Copeland, A. Padilla, and P.M. Saffin, “General second order scalar-tensor theory, self tuning, and the Fab Four,” arXiv:1106.2000

16. F. Verhulst, *Nonlinear Differential Equations and Dynamical Systems* (Springer, Berlin, 1996).

17. V. Emelyanov and F.R. Klinkhamer, “Possible solution to the main cosmological constant problem,” to appear in *Phys. Rev. D*, arXiv:1109.4915