Strange Quark Matter Objects Excited in Stellar Systems

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February 21, 2020

Abstract

It is shown that strange quark matter (SQM) objects, stars and planets, can very efficiently convert the mechanical energy into hadronic energy when they oscillate. This is because the mass density at the edge of SQM objects, \( \rho_0 = 4.7 \times 10^{14} \frac{g}{cm^3} \), is the critical density below which strange quark matter is unstable with respect to decay into photons, leptons. We consider here radial oscillations of SQM objects that could be induced in stellar or planetary systems where tidal interactions are ubiquitous. Already oscillations of 0.1% radius amplitude result in 1 keV per unit baryon number excitation near the surface of SQM stars. The excitation energy is converted into electromagnetic energy in a short time of 1 msec, during a few oscillations. Higher amplitude oscillations result in faster energy release that could lead to fragmentation or dissolution of SQM stars. This would have significant consequences for hypothetical SQM star binaries and planetary systems of SQM planets with regard to gravitational wave emission.
Recent papers \[1,2\] renew the interest in strange quark matter SQM objects of planetary masses \[3\] as candidates for high density planets orbiting some pulsars. Also, SQM planets around neutron stars are proposed to be new sources of gravitational radiation that could be detected by new generation detectors \[4\]. The idea that SQM forms the ground state of baryon matter was put forward by Witten \[5\]. Neutron stars as SQM stars were studied soon \[6\]. It was pointed out that there could exist SQM objects of any mass down to planetary values \[7\] and even smaller objects, called strange nuggets \[8\], could be present in the space.

In \[1,2\] the pulsar PSR B1257+12 is proposed as one of the hosts of candidates for SQM planets. It is the first stellar object discovered to harbour three planets \[9\] of 4.3 M\(_\odot\), 3.9 M\(_\odot\) and 0.025 M\(_\odot\). This system was conjectured in \[3\] to be composed of SQM.

Here our aim is to study the dynamical behaviour of SQM stars and planets interacting with other objects in stellar systems, in particular, stability of SQM objects with respect to radial oscillations. Such oscillations occur when these objects are subject to tidal interactions in stellar or planetary systems, especially during the formation of binary systems and close encounters with other stars. Here we assume that the oscillations of an SQM star are excited during the close encounter with another neutron star or a black hole \[10\].

Radial oscillations of compact stars (i.e. neutron stars and quark stars) have been studied in \[11,12\] (see also \[13,14\]). For a number of equations of state of dense matter the fundamental mode frequencies are listed as functions of stellar mass \[11,12\]. For the SQM star of 1.4 M\(_\odot\) the fundamental mode frequency is calculated for the SQM equation of state due to Glendenning \[15\] to be \(\omega=16900\) sec\(^{-1}\) and the period of oscillations is \(T=0.37\) msec \[12\]. Periods of a few tenths of millisecond are typical for other equations of state. The energy of such oscillations can be estimated to be \(\left(\xi/R\right)^2\times10^{53}\) erg, where \(R\) is the stellar radius and \(\xi\) is the amplitude of surface displacement \[11\].

The SQM stars and planets are very compact objects of radius of 10.3 km for a star of 1.4 M\(_\odot\) to 145 m for the planet of 1 M\(_\odot\). SQM stars have much more uniform distribution of mass as compared to normal neutron stars. The central densities for both branches of stars are similar, of order \(10^{15}\) \(\frac{g}{cm^3}\). However, neutron stars have low density crust and the surface (defined by the condition the pressure vanishes there) is of density of a few \(\frac{g}{cm^3}\) \[16\]. In contrast, the surface of SQM stars is of density \(\rho_0=4.665\times10^{14}\) \(\frac{g}{cm^3}\). Of course, the same is for SQM planets (we consider here bare SQM stars for simplicity with no crust, which could be present \[8\]). The density \(\rho_0\) is the saturation density of strange quark matter \[8\] and it is the crucial parameter for SQM objects.

1 PROPERTIES OF SQM STARS AND PLANETS

The properties of SQM are calculated here in a simple model approach within MIT-bag scenario \[3\] and are parametrized mainly by the value of the bag constant \(B\). The energy density, \(\rho c^2=\varepsilon+B\), includes relativistic energy density of quarks \(\varepsilon\) defined by the requirement that chemical potentials of electrically neutral matter satisfy the beta-stability condition. In what follows we assume \(B=60\) MeV fm\(^{-3}\), and strange quark mass \(m_s=150\) MeV (with up/down quarks and electrons being massless). The resulting energy per baryon is shown in Figure 1. In our model the saturation density (at the minimum) is \(\rho_0=4.665\times10^{14}\) \(\frac{g}{cm^3}\).

The saturation density \(\rho_0\) is the critical density for SQM because baryon matter is stable only as long as its density \(\rho>\rho_0\). At lower densities, \(\rho<\rho_0\), baryon matter decays into leptons, photons and hadrons conserving the baryon number. The baryon density at the saturation is \(n_0=0.2961\) fm\(^{-3}\) and the energy per baryon has its minimum \(E_{min}=883.6\) MeV. For lower baryon densities, \(n<n_0\), the energy per baryon increases.

In Figure 2 we show distribution of the baryon density in the SQM generally-relativistic spherical star of mass 1.4 M\(_\odot\). The areal radius of the star is \(R=10.31\) km. Also the scaled distribution corresponding to the maximum expansion of the star undergoing 10\% radial oscillations is shown. The oscillation amplitude of 10\% (which is quite large) is chosen for a sake of illustration. In the following we focus on smaller amplitudes. In the scaled distribution which is meant to model a uniform expansion of the oscillating star, the baryon density drops below the saturation value \(n_0\) in the outer shell of the star with radii \(R>r>r_c\) (here, \(r_c=7.535\) km). In the top panel in Figure 2 one can see the energy per baryon as a function of areal radius. The minimum energy per baryon occurs at \(r=r_c\) and then the energy increases toward the surface where the value higher by 9.450 MeV is found.
One can imagine how vulnerable the surface of SQM objects is to even small fluctuations. The energy per unit baryon number at the surface for 0.1% amplitude radial oscillation is calculated to be 1.221 keV above the minimum energy $E_{\text{min}}$. This corresponds to radius expansion to $R+\xi$, where $\xi=10.31$ m for $R=10.31$ km, or $x=\xi/R=0.001$. The baryon density decreases to roughly $n_0/(1+x)^3 = 0.2953 \text{ fm}^{-3}$ at the surface. The shell of $n<n_0$ comprises the outer 26 m of the star’s areal radius.

Generally, SQM becomes excited as baryon density is lowered below saturation density $n_0$ and the excess energy could finally be radiated away. We wish to stress here that the excitation effect occurs only in radial oscillations of SQM stars. It is absent for normal neutron stars. For a real star one can estimate the change of the surface density by a factor $xR$, where $x=0.001$, the surface displacement $\xi=xR=14.51$ cm, and $\Delta n=8.866 \times 10^{-4} \text{ fm}^{-3}$. The surface density decreases to $n_0-\Delta n$ and the baryon density inside the whole planet, $r<R$, decreases below the saturation density by almost the same amount. This is because the density gradient in the planet is very small and the central density differs little from the surface density $n_0$. Thus the whole planet becomes excited at the maximum expansion. The (mean) excitation energy is $E_{\text{exc}}=1.205$ keV per baryon. The baryon number of the planet $N_B=(4\pi/3)n_0 R^3=3.790 \times 10^{21}$. One can thus find that there is $7.315 \times 10^{42}$ erg energy that could eventually be radiated away.

For a real star one can estimate the change of the surface density by dividing the star into $N$ radial zones of thickness $\Delta r=R/N$. The baryon number of each zone is conserved. The last zone has baryon number $N=4\pi n_0 R^2 \cdot \Gamma_G(0)\Delta r$ (here, $\Gamma_G$ is a relativistic correction factor). At the maximum expansion the radius changes to $R+\xi$ and $N=4\pi n(\xi)(R+\xi)^2(1+\xi/R)\Gamma_G(\xi/R)\Delta r$. We thus find the baryon density at the surface to be $n(\xi)=n(0) \left( \frac{n(0)}{\Gamma_G(0)} - \frac{\Gamma_G(x)}{\Gamma_G(0)} \right)$. For $R=10.31$ km, with the 10% radius increase, $x=0.1$ ($\xi=1.031$ km), the baryon surface density is $0.2292 \text{ fm}^{-3}$.

The amount of energy that can be released during one oscillation period is calculated within the MIT-bag model as follows. The value of the baryon density at the maximum expansion, $n(\xi)=0.2292 \text{ fm}^{-3}$, corresponds to the energy per baryon $E=893.07$ MeV. At the saturation density the energy per baryon of the ground state of SQM is $883.62$ MeV. The SQM in the last shell is thus unbound with respect to the minimum energy by $9.450$ MeV per baryon. To calculate all quark matter energy available for conversion into electromagnetic energy, one should add all zones with baryon density less than $\sim 0.39 \text{ fm}^{-3}$ (reduced by a small and position dependent relativistic correction) in the equilibrium star as all these will have lower densities by a factor $\sim 0.75$, that is below saturation density $n_0$ at systems, where astrophysical objects interact with one another, fully static surface is difficult to imagine. Here we consider what can happen when the surface of SQM star oscillates radially (in so called monopole or breathing mode).

The radial oscillations engulf the whole star. All elements of mass oscillate in phase with the same frequency around equilibrium positions. At the maximum expansion the radius of the star increases to $R+\xi$ and at the maximum compression it decreases to $R-\xi$, where $R$ is the equilibrium radius of the star. The volume of the star oscillates and so does the density. At the maximum expansion the density is lowest.

One can calculate the change in mean baryon density for the SQM object of constant density $n_0$ as $\Delta n=n_0(1-1/(1+x)^3)$. The SQM planets of Earth-like masses are essentially such objects. In Table I some properties of the Earth mass SQM planet are listed. The radius of this planet is $R=145.1$ m. For 0.1% radial oscillations, $x=0.001$, the surface displacement $\xi=xR=14.51$ cm, and $\Delta n=8.866 \times 10^{-4} \text{ fm}^{-3}$. The surface density decreases to $n_0-\Delta n$ and the baryon density inside the whole planet, $r<R$, decreases below the saturation density by almost the same amount. This is because the density gradient in the planet is very small and the central density differs little from the surface density $n_0$. Thus the whole planet becomes excited at the maximum expansion. The (mean) excitation energy is $E_{\text{exc}}=1.205$ keV per baryon. The baryon number of the planet $N_B=(4\pi/3)n_0 R^3=3.790 \times 10^{21}$. One can thus find that there is $7.315 \times 10^{42}$ erg energy that could eventually be radiated away.

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the maximum expansion. These zones comprise \(\sim 57\%\) of the star’s baryon number and store the excitation energy of \(6.16\times10^{54}\text{ erg}\). The excitation energy contained in the outermost shell of the star at the maximum expansion is \(7.95\times10^{49}\text{ erg}\) for division of the star in 1000 zones.

## 2 THE FATE OF EXCITATION ENERGY

Our main interest here is the fate of this excitation energy. The process of excitation of quark matter is an irreversible one. The excitation energy will be dissipated eventually into heat and radiation. Here we first focus on an intermediate step which is conversion of hadronic excitation energy into electromagnetic energy. To calculate the generation rate of electromagnetic energy we need a time scale of the radial movement of the SQM star. We assume that the frequency of radial oscillations calculated in [12] without considering any excitation of the SQM matter can be used to obtain the first time scale of radial movement of the stellar matter just at the moment of beginning of the first cycle of oscillations. We calculate the mean radial velocity of the surface, \(v_r\), in the expansion phase. In a quarter of period the surface moves outward a distance \(\xi\), thus \(v_r=\xi/(T/4)=xR/(T/4)=111 \text{ km sec}^{-1}\) for 0.1% \((x=0.001)\) oscillations. In terms of the radial coordinate of the stationary star in the same time the critical surface dividing excited and non-excited matter moves inward reaching the radius of \(r_c=10.29\text{ km}\) with velocity \(v_c=(R-r_c)/(T/4)=220 \text{ km sec}^{-1}\).

The time scale for electromagnetic interactions is \(10^{-16}\) sec which is much shorter than the quarter of period \(T/4=9.25\times10^{-5}\) sec. Thus the excitation energy can be assumed to be converted into electromagnetic energy instantaneously. The rate of electromagnetic energy generation is \(\varepsilon=\dot{E}_T/(T/4)\) were \(\dot{E}_T=6.63\times10^{45}\text{ erg sec}^{-1}\) is the whole energy available for conversion, see Table 1. We find \(\varepsilon=7.17\times10^{45}\text{ erg sec}^{-1}\). The mean excitation energy per baryon is \(\bar{E}_{\text{exc}}=\dot{E}_T/N_B(r>r_c)\). Here \(N_B(r_c)=1.288\times10^{55}\) is the number of baryons in the shell of excited SQM, \(r>r_c\). We find \(\bar{E}_{\text{exc}}=0.3213\text{ keV}\). Similarly, the numbers for \(x=0.1\), as shown in Table 1 are: the total excitation energy \(E_T=6.16\times10^{51}\text{ erg}\), the critical surface radius \(r_c=7.535\text{ km}\), the number of excited baryons \(N_B(r>r_c)=1.158\times10^{52}\), the mean excitation energy per baryon is \(\bar{E}_{\text{exc}}=3.321\text{ MeV}\), the velocity \(v_c=3.004\times10^4\text{ km sec}^{-1}\), and finally the energy deposition rate is \(\varepsilon=6.66\times10^{55}\text{ erg sec}^{-1}\).

The above analysis concerned the process of energy conversion inside the SQM star. Astrophysically, we would like to know how much of the electromagnetic energy will be radiated away and how fast this will proceed. This problem requires further investigation. Here we estimate conservatively the luminosity assuming that one half of the released energy in the outermost shell of thickness \(\lambda\) is radiated away and the other half is absorbed by inner layers, where \(\lambda\) is the photon mean free path.

The luminosity for \(x=0.001\) amplitude oscillations is \(L=3.29\times10^{48}\text{ erg sec}^{-1}\). For \(\lambda=3.95\text{ fm}\), \(L=1.30\times10^{34}\text{ erg sec}^{-1}\). The effective temperature defined through \(L=4\pi R^2\sigma T^4\) is \(T=\sqrt{2}\times10^8\text{ K}\). The photon mean free path \(\lambda=3.95\text{ fm}\) corresponds to non-degenerate quark matter and reduced Thomson cross section \(\sigma_T=(2/3)^4(m_e/m_u)^2=665\text{ mb}\), where the up quark mass is \(m_u=2\text{ MeV}\) and the electron mass \(m_e=\text{ electron mass}\).\(=0.511\text{ MeV}\). Thus \(\sigma_T=8.58\text{ mb}\) and \(\lambda=1/(\sigma_T n_0)\). Contributions from strange and down quarks are neglected here. When the quark matter is degenerate \((T=0)\) the mean free path can be longer, however it depends on the photon energy. The total energy released in the star, \(E_T=6.6\times10^{45}\text{ erg}\) would sustain radiation with the luminosity \(L=1.3\times10^{34}\text{ erg sec}^{-1}\).

For \(\tau\sim\dot{E}_T/L=1.6\times10^4\text{ years}\) however, neutrino cooling will switch on in \(10^{-6}\text{ sec}\) and the star will cool fast. One can estimate order of magnitude of neutrino cooling time by using standard value of neutrino emissivity of SQM \(\varepsilon_\nu=10^{44}\frac{\text{ erg sec}^{-1} cm^{-2}}{\text{keV}}\). The neutrino cooling time \(\tau_\nu=\dot{E}_T/E_\nu\) is \(\tau_\nu=1.4\times10^4\text{ sec}\) for the volume of the star, \(\tau_\nu=1.5\times10^5\text{ sec}\) for the volume of the initial energy deposition shell. More accurate calculation requires taking into account the evolution of temperature of the SQM star, which is beyond the scope of this research.

For \(x=0.1\) oscillations the surface luminosity is \(L=1.5\times10^{38}\text{ erg sec}^{-1}\) and time of radiation of \(E_T=6.16\times10^{51}\text{ erg}\) is \(\tau=1.3\times10^6\text{ years}\). However, neutrino cooling time is \(\tau_\nu=42\text{ years}\). As any excitation is damped in real stars, after some time the oscillations would cease to exist. In [12] damping times for radial oscillations of compact stars are calculated. For the considered SQM star of 1.4 \(M_\odot\), the damping time is 5.47 sec [12]. However if the excitation mechanism considered here is accounted for, the whole energy of 10% amplitude oscillations, of order of \(10^{51}\text{ erg}\) is less than the value of excitation energy generated during the time of a
single oscillation. Thus oscillations would be damped during the first cycle. Physically, this means that no radial oscillations of the SQM star of such an amplitude would exist in nature. For lower amplitudes the energy generation time is longer.

There are many-fold astrophysical consequences of the instability of SQM stars. The strongest tidal excitations are expected in very close neutron star binaries before their merger [35]. Such objects are prime candidates for gravitational wave (GW) sources. If one of the stars is the SQM star then the radial oscillations excited by the companion star could destroy the SQM star before the merger and the GW emitted during last orbits would differ from the templates. The neutrino cooling may not be fast enough to prevent the fragmentation of the excited SQM matter into smaller SQM objects. Fragmentation would proceed down to fragments that can cool effectively. Distribution of fragments in SQM objects decay is calculated in [19]. If both stars are SQM stars then they mutually excite one another and thus both can be destroyed before the merger.

For 1 M$_\odot$ SQM planet of radius $R=145$ m radial oscillation of 10% amplitude, $\xi=1.45$ m, give baryon density 0.7513 $n_0$ and excitation energy of 11.84 MeV. Total excitation energy is $E_T=7.187 \times 10^{16}$ erg. This makes 1.34% of the rest energy. Also radial oscillations of SQM planets excited in binary systems could lead to disastrous consequences if they are of the same relative amplitude that for massive SQM stars.

In conclusion, SQM stars and planets are very sensitive to radial oscillations. Even 0.1% oscillations of the radius result in about 1 keV per unit baryon number excitation energy in the surface layer of every SQM object, equally for stars of pulsar masses and planetary-mass SQM objects. In the evolution of binary systems with the SQM objects the energy loss due to excitations of SQM stars and planets must be accounted for and this can change significantly the predictions obtained with unexcited SQM objects. This is particularly relevant to binary gravitational wave sources as the exact template of the system gravitational amplitude can not be calculated without including excitations of SQM objects.

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