Two approaches to testing general relativity in the
strong-field regime

Dimitrios Psaltis
Departments of Astronomy and Physics, University of Arizona, 933 N. Cherry Ave., Tucson,
AZ 85721
E-mail: dpsaltis@email.arizona.edu

Abstract. Observations of compact objects in the electromagnetic spectrum and the detection
of gravitational waves from them can lead to quantitative tests of the theory of general relativity
in the strong-field regime following two very different approaches. In the first approach, the
general relativistic field equations are modified at a fundamental level and the magnitudes
of the potential deviations are constrained by comparison with observations. In the second
approach, the exterior spacetimes of compact objects are parametrized in a phenomenological
way, the various parameters are measured observationally, and the results are finally compared
against the general relativistic predictions. In this article, I discuss the current status of both
approaches, focusing on the lessons learned from a large number of recent investigations.

1. Introduction
Testing the theory of general relativity is the key science objective of many future astrophysical
missions as well as of NASA’s Beyond Einstein program, as a whole. However, there has been
very little consensus, as of now, on the particular direction along which quantitative tests of the
theory will be performed.

The parametric post-Newtonian framework, which has been crucial in testing general
relativity at solar-system scales, is no longer valid in the strong-field regime. In fact, in the
absence of an accepted alternative theory of gravity, there are very few a priori constraints on
the potential deviations from general relativistic predictions. However, there are a number of
arguments that may guide us in the effort to reach this goal.

As in most other branches of physics, there are two distinct avenues we may follow in testing
general relativity in the strong-field regime. In a top-down approach, we may modify the theory
at a fundamental level and calculate the consequences of the modifications that can be tested
observationally. In a bottom-up approach, we will use a phenomenological description of the
observations in order to obtain clues for how to modify the theory at the fundamental level.
In this article, I draw a roadmap for testing the theory of general relativity in the strong-field
regime along these two avenues, based on general theoretical expectation as well as on a large
number of recent investigations.
2. The top-down approach: From Theory to Observations

2.1. The Equivalence Principle and the Einstein Field Equations

The theory of general relativity has two distinct ingredients. The equivalence principle that describes how matter moves in the presence of a gravitational field and the Einstein field equations that describes how the gravitational field is generated in the presence of matter.

The validity of the equivalence principle gives rise to the geometric aspect of the theory (see Will 2006). According to this principle, it is impossible to tell the difference between a reference frame at rest and one free-falling in a gravitational field, by performing local experiments. Moreover, the equivalence principle encompasses the Lorentz symmetry, as well as our beliefs that there is no preferred frame and position anywhere in the universe.

The Einstein equivalence principle has been tested to a very high degree during the last century, mostly in the weak-field regime. The upper bound on possible violations are as low as one part in $10^{12}$ (Will 2006). It is very unlikely that astrophysical observations of black holes and neutron stars can lead to tests of comparable accuracy. For this reason, most approaches to date have concentrated only on plausible modifications of the field equations although frameworks (such as the Standard Model Extension; see Kostelecky 2003 and references therein) exist that can lead, in principle, to strong-field tests of this principle (see Stairs 2003 for a discussion of testing the strong equivalence principle with double neutron stars; see also Eling & Jacobson 2006 for black-hole solutions in an Einstein-aether theory).

2.2. Modifying the Einstein Field Equations

Contrary to the case of the equivalence principle, there are no arguments one can make that lead uniquely to the Einstein field equations. Indeed, Einstein reached the field equation, more or less, by reverse engineering. For this reason, his approach cannot be easily modified to include additional terms. On the other hand, the “derivation” of the field equations from a Lagrangian action provided by Hilbert allow us to modify the theory, while preserving many of its symmetries and conservation laws.

The Einstein-Hilbert action that leads to the field equations of general relativity is directly proportional to the Ricci scalar, $R$,

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda),$$

where $g$ is the spacetime metric, $c$ is the speed of light, $G$ is the gravitational constant, and $\Lambda$ is the cosmological constant.

We can construct a self-consistent theory of gravity from this starting point using any other action that obeys the following four simple requirements (see Misner et al. 1973; p. 410). It has to: (i) reproduce the Minkowski spacetime in the absence of matter and cosmological constants, (ii) be constructed from only the Riemann curvature tensor and the metric, (iii) follow the symmetries and conservation laws of the stress-energy tensor of matter, and (iv) reproduce Poisson’s equation in the Newtonian limit. Of all the possibilities that meet these requirements, the field equations that are derived from the Einstein-Hilbert action are the only ones that are also linear in the Riemann tensor.

The function form of the Einstein-Hilbert action suggests that we can modify it in one of four ways: In principle, we can (i) change the number of spacetime dimensions, (ii) change the functional dependence of the Lagrangian density on the Ricci scalar $R$, (iii) allow for the Lagrangian density to depend on other scalars generated from the Riemann curvature tensor, or (iv) introduce additional scalar, vector, or tensor fields.

(i) The number of spacetime dimensions.—The possibility that we live in a Universe with more than four spacetime dimensions has been the subject of intense research during the last decade,
primarily as a solution to the hierarchy problem in physics (Arkani-Hamed et al. 1998a; Randall & Sundrum 1999; Maartens 2005). In these braneworld gravity models, all non-gravitational fields and particles live on a four dimensional spacetime, whereas the gravitational field is allowed to propagate in the extra dimensions. In order for the theory to be consistent with Newton’s law of gravity down to the sub-mm level, the effective size of the extra dimensions has to be smaller than $\sim 50 \mu m$ (Kapner et al. 2007). As a result, such a modification to gravity is not expected to have any observable classical consequences even for neutron stars, which are the smallest known astrophysical objects with strong gravitational fields (Arkani-Hamed et al. 1998b).

A plausible consequence of one variant of braneworld gravity (the RS2 scenario of Randall & Sundrum) that can be constrained with astrophysical observations is the rapid evaporation of black holes caused by the emission of gravitons in the extra dimensions (Emparan et al. 2003 and references therein). The rate of rapid evaporation was inferred using the AdS/CFT correspondence and is a matter of debate (Fitzpatrick et al. 2006). If proven accurate, however,
it can lead to astrophysical bounds on the asymptotic scale of the large extra dimensions that are comparable to those obtained in table-top experiments (Johannsen et al. 2009 and references therein).

(ii) The functional dependence of the Langrangian density on the Ricci scalar.— A Langrangian action that depends on a non-linear function of the Ricci scalar $R$, i.e.,

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} f(R),$$  

leads to field equations that obey all the requirements outlined above. Indeed, the action (2) results in a field equation that allows for the Minkowski solution in the absence of matter, is constructed only from the Riemann tensor, obeys the usual symmetries and conservation laws, and can be made to produce negligible corrections at the small curvatures probed by weak-field gravitational experiments. Such non-linear actions for the gravitational field, albeit with negligibly small coefficients, occur naturally in quantum gravity theories and in string theory. This is true because of radiative corrections to the usual Einstein-Hilbert action that induce an infinite series of counterterms that cannot be reabsorbed into the original Lagrangian by adjusting its bare parameters (see discussion in Burgess 2004).

Despite their simplicity and appeal, a number of important limitations to $f(R)$ gravity theories have been recently explored in the cosmological context (see Sotiriou & Faraoni 2009 for an extensive review). Modifications to the Lagrangian action that appear negligible at solar-system scales may lead to order unity deviations of the PPN parameters that are inconsistent with current experiments (Chiba 2003). Moreover, astrophysical objects in non-linear $f(R)$ gravity, including the Universe as a whole, can be violently unstable, depending on the sign of $d^2 f / dR^2$ (Dolgov & Kawasaki 2003). Finally, special attention has to be paid to ensure that a well posed initial value problem can be defined under all physical conditions within the non-linear $f(R)$ theory (Lanahan-Tremblay & Faraoni 2007).

Resolving the above problems requires a carefully constructed $f(R)$ theory with fine-tuned coefficients of the non-linear terms (e.g., Nojiri & Odintsov 2003) or one that leads to the so-called chameleon mechanism (see Hu & Sawicki 2007 and references therein). Alternatively, a simple $f(R)$ function may be considered as an expansion of an unknown, more general theory and its consequences for astrophysical phenomena may be calculated within the context of perturbative constraints (Simon 1990; see also Cooney et al. 2009 and references therein).

In adding terms to the Lagrangian action that are of non-linear order in the curvature, we have the choice of performing the derivation of the parametric Einstein equation either in the metric formalism or in the Palatini formalism. The first corresponds to extremizing the action under variations in the metric only, whereas the second corresponds to extremizing the action under variations in the metric and the connection, as well. For the simple Einstein-Hilbert action, both approaches are equivalent and give rise to the same field equations. However, when the action has non-linear terms in the Ricci scalar, the two approaches diverge. Recent studies of $f(R)$ gravity in the Palatini formalism, however, have revealed a number of serious problems with this approach that are related to the suppression of the new dynamical degrees of freedom and render it a rather unlikely alternative to the usual derivation of the field equations from a Lagrangian action (see Sotiriou & Faraoni 2009 and references therein for a detailed discussion).

(iii) Additional scalar terms constructed from the Riemann curvature tensor.— It is remarkable that the Einstein field equations can be derived from a first-order Lagrangian action that does not involve the Riemann tensor (see Landau & Lifshitz 1992). However, higher-order field equations that obey the Lorenz symmetry have been derived only from Lagrangian actions that
Involves different scalar quantities constructed from the Riemann tensor $R_{\alpha\beta\gamma\delta}$, e.g.,

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left( R + \alpha R^2 + \beta R_{\sigma\tau} R^{\sigma\tau} + \gamma R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} \right),$$

where $R_{\sigma\tau}$ is the Ricci tensor and $\alpha$, $\beta$, and $\gamma$ the three parameters of the theory.

Similar to the case of $f(R)$ gravity, such terms arise naturally as high-order corrections in quantum gravity and string theory and their relative importance increases with the curvature of the metric. The resulting field equations suffer, however, from the Ostrogradski instability, which can be avoided only by fine tuning of the free parameters of the theory (see Woodard 2006 and references therein).

(iv) The presence of additional gravitational fields.—The single, rank-2 tensor field $g_{\mu\nu}$ (i.e., the metric) of the Einstein-Hilbert action may also not be adequate to describe completely the gravitational force (although, if additional fields are introduced, then the strong equivalence principle is violated, with important implications for the frame- and time- dependence of gravitational experiments).

The simplest case of a theory with additional gravitational fields involves a single additional scalar and has been studied for more than 40 years in the form of Brans-Dicke gravity (e.g., Will 2006). Because of its particular properties, the fractional deviation of the Brans-Dicke predictions from general relativity are comparable in both the weak and strong-field regimes. In a more general case, however, depending on the coupling between the metric, the scalar field, and matter, the relative contribution of such additional fields may become significant only at the high curvatures found in the vicinity of compact objects.

The general form of the Lagrangian of a scalar-tensor theory is given, in the Einstein frame, by the Bregmann-Wagoner action (see Will 2006 for details)

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g_{\ast}} \left[ R_{\ast} \pm g^{\ast\mu\nu} \partial_\mu \phi \partial_\nu \phi + 2\lambda(\phi) \right] + S_m[\phi_m, A^2(\phi)g_{\mu\nu}],$$

where $A(\phi)$ and $\lambda(\phi)$ are two arbitrary functions, and $S_m$ is the action for the matter field $\phi_m$. In the strong-field regime, the potential term $\lambda(\phi)$ in the action (4) is typically negligible. The single coupling function $A(\phi)$ encodes the properties of the theory and may be parametrized as $A(\phi) = \exp[\phi_0 + \alpha_0 \phi + (1/2)\beta_0 \phi^2]$, with $\phi_0 \rightarrow 0$ being the cosmological value of the field (Damour & Esposito-Farese 1993).

2.3. Studying the Properties of a Parametric Lagrangian Theory

The parametric addition of terms in the Einstein-Hilbert action offers a systematic and self-consistent way of characterizing potential deviations from general relativistic predictions that can be constructed to become arbitrarily large in the strong-field regime. It may also lead, however, to a number of pitfalls that need to be studied and understood before embarking onto a detailed comparison to observational data.

First, a parametric extension to the Einstein-Hilbert action is guided only by the mathematical properties of the form that gave rise to the field equations of general relativity and not by a new requirement of the physics. As such, the extensions discussed above will not cover the entire range of possibilities, let alone a theory that is fundamentally different from general relativity in the strong-field regime. In some sense, using the parametric extension of the Einstein-Hilbert action will allow us to look for deviations from general relativity albeit not necessarily the ones that are described by the particular parameters measured to differ from their general relativistic values. If future experiments do measure such a deviation, then a more fundamental study will be required to account for it. There is a close similarity between
this situation and the current measurements of the cosmological constant in the very weak field regime. Indeed, the non-zero measured value of the cosmological constant may just be an indication for the presence of a dynamical scalar field (such as quintessence) as opposed to a constant in the Einstein field equations. However, until a non-zero cosmological constant was measured, it made little sense to introduce the additional complication of a dynamical equation for the scalar field in the Friedman equation.

Second, taken at face value, the predictions of a parametric Lagrangian theory may describe unstable physical systems, as discussed above. Albeit challenging to deal with, such theories need not be \textit{a priori} excluded from quantitative comparison with astrophysical data. There have been, indeed, many cases in physics where unstable solutions have been tested against data (e.g., the cosmological solution with $\Omega \neq 1$ is highly unstable but is still being tested against WMAP data) and even some situations in which the unstable solution was preferred over the stable one (e.g., the Rutherford model of the atom). When the experimental data prefer a steady-state solution that is unstable, it is almost always an indication of new physics, which would have been missed had such a solution been excluded.

The same argument can also be made for predictions that appear to be unphysical in the context of the parametric theory but are not at a more fundamental level. This is similar to some predictions of the PPN formalism related, e.g., to non-zero values of the parameters $\alpha_3$, and $\zeta_1$ through $\zeta_4$ that describe violations of conservation of total momentum. Measuring any of these “unphysical” parameters to be anything other than zero would be interpreted simply as a smoking gun for the presence of an additional field that has not been incorporated in the PPN formalism but carries some momentum away from the system under study.

Finally, it is also worth emphasizing that there is significant degeneracy between theories that arise from different Lagrangian actions and, therefore, not every astrophysical observation can distinguish between them. For example, it is well known that the field equations of gravity theories with higher-order terms in the Ricci scalar are completely equivalent to those of a scalar-tensor theory (see, e.g., Magnano & Sokolowski 1994) although this is no longer true when additional terms are introduced to the action that involve the Riemann and Ricci tensors, i.e., $R^\mu_\nu R_\mu^\nu$ and $R^{\alpha_3\mu_\nu}R_\alpha^{\beta_3\mu_\nu}$. Even in the latter case, however, the particular predictions of the theory for specific astrophysical systems is not always different from their general relativistic counterparts.

Consider, for example, the theory given by the Lagrangian action (3). Because of the Gauss-Bonnet identity,

$$\frac{\delta}{\delta g_{\mu\nu}} \int d^4x \left( R^2 - 4R_{\sigma\tau}R^{\sigma\tau} + R^{\alpha_3\beta_3\gamma_3\delta}R_{\alpha_3\beta_3\gamma_3\delta} \right) = 0 ,$$

variations of the term proportional to $\gamma$ in equation (3), with respect to the metric, can be expressed as variations of the terms proportional to $\alpha$ and $\beta$. Therefore, for all classical tests, the predictions of the theory described by the Lagrangian action (3) are identical to those of the Lagrangian

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[ R + (\alpha - \gamma)R^2 + (\beta + 4\gamma)R_{\sigma\tau}R^{\sigma\tau} \right] .$$

As a result, classical astrophysical tests can only constrain a particular combination of the parameters, i.e., $\alpha - \gamma$ and $\beta + 4\gamma$. It is only through phenomena related to quantum gravity, such as the evaporation timescale of black holes, that the parameter $\gamma$ may be constrained.

When the spacetime is isotropic and homogeneous, as in the case of cosmological tests, an additional identity is satisfied, i.e.,

$$\frac{\delta}{\delta g_{\mu\nu}} \int d^4x \left( R^2 - 3R_{\sigma\tau}R^{\sigma\tau} \right) = 0 .$$
This implies that, for cosmological tests, the predictions of the theory described by the Lagrangian action (3) are identical to those of the Lagrangian

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[ R + \left( \alpha + \frac{1}{3}\beta + \frac{1}{3}\gamma \right) R^2 \right].$$

(8)

As a result, cosmological tests of gravity can only constrain a particular combination of the parameters, i.e., $\alpha + 1/3\beta + 1/3\gamma$.

2.4. Reducing the Size of the Parameter Space of Alternative Theories

The general relativistic field equations in the presence of matter are highly non-linear and the same is true for all alternatives explored so far. Because of this non-linearity, modifications to the theory that are apparently significant, e.g., only in the strong-field regime may have observable consequences at very different astrophysical or cosmological scales and vice versa. Recent studies of $f(R) = R \pm \mu^4/R$ gravity theories have actually provided one of the most straightforward demonstrations of this effect. Although the term $\mu^4/R$ in the action appears to become relevant only when the curvature of the field is $R \lesssim \mu^2$, the PPN parameter $\gamma$ for such a theory is equal to $1/2$, is independent of the value of the small parameter $\mu^2$, and is demonstratively different that the general relativistic prediction of $\gamma = 1$ (Chiba 2003).

Because of this non-linearity in the field equations, the vast parameter space of alternatives to general relativity can be reduced by examining their behavior in the weak-field regime or even at cosmological scales. In particular, every modification to general relativity has to account for

(i) the observed constraints on the PPN parameters.— In particular, the PPN parameters $\beta$ and $\gamma$ have been constrained to lie within $\simeq 10^{-5}$ of their general relativistic values using tests involving bodies in the solar-system as well as man-made spacecraft (Will 2006 and references therein). Calculations of the PPN parameters in scalar-tensor theories specifically designed to perform strong-field tests have been performed by Damour & Esposito-Farese (1993), whereas a large number of similar studies have been carried out for $f(R)$ gravity theories (Barrause & Sotiriou 2009).

(ii) the orbital period decay of double neutron stars.— This indirect evidence for the emission of gravitational waves offers a unique setting in which the predictions of a gravity theory for the time dependence of a dynamical spacetime can be compared against highly accurate observations (Stairs 2003). Damour & Esposito-Farese (1993) have exploited these observations to place strong constraints on strong-field modifications to general relativity that involve dynamical scalar fields.

(iii) the evolution of the Universe predicted by the modified Friedman equation. Reversing the argument given earlier for cosmologically motivated theories, it is important that a modification to gravity constructed for use in strong-field tests does not contradict the constraints imposed by the observations of the high-z supernovae (which probe the nearby universe, up to a redshift of $\sim 1$), of the cosmic microwave background (which is sensitive to the expansion of the universe up to a redshift of $\sim 1000$), and of big-bang nucleosynthesis (which is highly sensitive to the rate of the expansion of the universe at a much earlier time).

2.5. Black Holes and Neutron Stars in Alternative Gravity Theories

The final step in setting up tests of strong-field gravity in the top-bottom approach discussed here involves calculating the properties of black holes and neutron stars in the parametric Lagrangian theories.
In the case of neutron stars, the presence of matter in the domain of solution guarantees that the addition of terms to the Einstein field equations will lead to modifications in the structure of the stars and their exterior spacetimes. Indeed, studies of neutron stars in alternative gravity theories have invariably resulted in larger and more massive neutron stars compared to their general relativistic counterparts. Zaglauer (1992) and Damour & Esposito-Farese (1993) studied the structure of neutron stars in gravity theories that involve a parametric addition of a scalar field to the Einstein-Hilbert action. More recently, a number of studies investigated compact objects in $f(R)$ gravity and in particular the role of the chameleon mechanism in allowing neutron stars to exist in a large class of theories (see Upadhye & Hu 2009 and references therein).

Contrary to the case of neutron stars, most parametric extensions to the Einstein-Hilbert action lead to field equations that admit as a solution the Kerr spacetime of a black hole (Psaltis et al. 2008). This is formally true only for steady-state black holes, as the time-dependent spacetime of a perturbed black hole that can be probed with observations of gravitational waves is different in different theories (Barausse & Sotiriou 2008). The reason behind the similarity of black hole solutions among widely different theories is the absence of matter everywhere in the domain of solution. Indeed, the entire spacetime of a black hole as well as that of vacuum are characterized by a constant Ricci scalar curvature $R$. As a result, calculating the spacetime of a black hole requires simply solving the equation $R = \text{constant}$, independent of the underlying field equations of the theory.

Recently, Yunes & Pretorius (2009) as well as Konno et al. (2009) showed that, in Chern-Simons gravity, rotating black-hole solutions are different from the Kerr spacetimes even though the theory admits the normal vacuum solution. They key difference between Chern-Simons gravity and general relativity is the addition of a term in the Einstein-Hilbert action that allows for solutions that violate parity. The spacetime for the vacuum and for a Schwarzschild black hole have zero parity and are, therefore, the same in both theories. On the other hand, the spacetime of a rotating black hole has non-zero parity and, in Chern-Simons gravity, is parametrically different from the Kerr solution.

Having obtained the structure of neutron stars and exterior spacetimes of black holes, the theory can now be put to test against observations that probe strong gravitational fields. In the electromagnetic spectrum, such tests can be provided by observations of the images of black-hole accretion flows, of the continuum and line spectra of neutron stars and black holes, as well as of their rapid variability properties (see Psaltis 2008 for a review). In the near future, gravitational wave observations of coalescing compact objects, of extreme mass-ratio inspirals, as well as of the ringing of perturbed black holes and of the global oscillatory modes of neutron stars will open a new window to the study of the strong-field properties of gravity (see Flanagan & Hughes 2005 for a review).

3. The bottom-up approach: From Phenomenology to Observations
The top-down approach to testing the strong-field regime of gravity discussed above is appealing for a number of reasons. First, the terms added to the Einstein-Hilbert action are often motivated by quantum gravity and string theory, offering therefore the possibility of testing predictions of a fundamental modification to gravity. Second, starting from a Lagrangian action guarantees that the theory will encompass important symmetries and conservation laws. Finally, the parameters of the same theory can be constrained by a wide range of observations of neutron stars and black holes, in the electromagnetic spectrum and with gravitational waves, or even with solar-system experiments and in cosmological settings. As a result, very tight constraints can be obtained and degeneracies between parameters can be broken.

The key drawback of the top-down approach, however, is the fact that observational data are interpreted within the narrow confines of a particular theoretical framework. The case of black-hole spacetimes discussed earlier is the most striking example of the limitations of this
approach: a very wide range of gravity theories lead to only one black-hole solution potentially modified by a parity violating term, thereby allowing only for a very limited range of possibilities to be tested against observations.

As an alternative to this approach, we might follow a different, phenomenologically driven path. We will start again by assuming the validity of the equivalence principle and hence of the fact that test particles and photons follow geodesics of the spacetime. In this approach, however, we will not aim to obtain the spacetime as a solution to a field equation. Instead, we will prescribe the external spacetime of a black hole or a neutron star in a way that is parametrically different from their general relativistic counterparts. Using these spacetimes we can then predict observational signatures of deviations from general relativistic predictions to be tested observationally. This approach has been followed only recently and for a limited number of cases, which I will discuss below.

3.1. Parametric Spacetimes of Black Holes and Neutron Stars
The exterior spacetime of an astrophysical black hole in general relativity is completely described by only two parameters: its mass and its spin (any charge on the black hole will be very quickly neutralized by attracting opposite charges from the surrounding medium). This is the so-called
no-hair theorem that provides the framework on which to build a phenomenological model of black-hole spacetimes (this approach was pioneered by Ryan 1995 and was further developed more recently by Collins & Hughes 2004 and Glampedakis & Babak 2006).

Because of the no-hair theorem, if we expand the exterior spacetime of a black hole in multipole moments, the coefficients of only two of them are independent quantities: the monopole is related to the black-hole mass $M$ and the dipole to its spin $a$. All higher-order moments depend in a particular way on these two. For example, the dimensionless quadrupole moment of a black hole in general relativity is equal to $q = -a^2$. We can, therefore, write the most general spacetime with arbitrary coefficients of its multipole moments and use it to predict observables such as the spectra of accretion flows around black holes or the waveforms of gravitational waves during extreme mass-ratio inspirals. Measuring the coefficients of at least three multipole moments of a black hole spacetime by comparing these predictions to observations we can, therefore, test this fundamental property of general relativistic black-hole spacetimes and search for potential deviations.

This approach is very powerful for performing tests of strong-field gravity with black holes and perhaps the one that is the least model dependent. This is especially true for tests that will use observations in the electromagnetic spectrum, because the calculation of the propagation of photons from the strong gravitational fields to the distant observers does not require the knowledge of the underlying field equations that gave rise to the parametric spacetimes. In order, however, to predict the waveforms of gravitational waves emitted, for example, from an extreme mass-ratio inspiral into such a spacetime one needs to use a field equation to describe the time-dependence of the parametric spacetime. As a result, gravitational wave observations can be used within this parametric framework only as null hypothesis tests of general relativistic predictions (see discussion in Hughes 2006).

Contrary to the case of black holes, the exterior spacetimes of neutron stars depend, in general, on the mass distribution in their interiors and are not uniquely described by only two coefficients of their multiple moments. On the other hand, a number of phenomena have been observed from neutron stars that take place in their atmospheres, which is a very thin shell on their surfaces. In order to predict observable quantities related to these phenomena we need to parametrize only the values of the metric elements on their surfaces and not throughout their exterior spacetimes. We can, therefore, use observations to read directly different combinations of the metric elements on the surfaces of the stars and compare them against general relativistic predictions.

As an example, the atmospheric spectrum from a slowly spinning bursting neutron star is determined by various metric elements that depend on three parameters: the coordinate radius of the neutron-star surface, the redshift from it, and the effective gravitational acceleration at the same place. In general relativity all three parameters depend only on the mass and radius of the neutron star. Using, therefore, at least three atmospheric phenomena to measure these three parameters independently can lead to a quantitative test of this general relativistic prediction (Psaltis 2008).

Conclusions
Testing general relativity in the strong-field regime may follow one of two main approaches that are familiar from other areas of physics or astrophysics. We can again draw a parallel between the directions described in this article with current investigations of gravity at cosmological scales. In one set of studies that aim to characterize the accelerated expansion of the universe, the Friedman equation is modified at a fundamental level by postulating, e.g., a scalar field such as quintessence, and the parameters of the theory are constrained against observations. In a different set of studies, the field that provides the acceleration of the universe is described phenomenologically by an equation of state, with parameters that can be measured.
observationally. The results of these two sets of studies offer a complimentary view of the observational data. At the same time, the lessons learned and the interplay between the two approaches allows for both to be further developed. The quality of high-energy and of gravitational-wave data that are expected from future missions and observatories require that both approaches be exploited as well in the case of strong-field tests of general relativity.

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