Interaction-Free Measurement in mesoscopic systems and the reduction postulate

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We show that a noninvasive, “negative-result measurement” can be realized in quantum dot systems. The measurement process is studied by applying the Schrödinger equation to the whole system (including the detector). We demonstrate that the possibility of observing a particular state out of coherent superposition leads to collapse of the corresponding nondiagonal density-matrix elements of the measured system. No additional reduction postulate is needed. Experimental consequences of the collapse time and the relativistic requirement are discussed for mesoscopic and optical systems.

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According to the principles of quantum mechanics, the density-matrix of a system in a linear superposition of several states collapses to the statistical mixture after measurement, \( \sum_m |m\rangle \rho_{m,m} |m\rangle \rightarrow \sum_m |m\rangle \rho_{m,m} |m\rangle \). This is the von Neumann projection postulate in terms of the density-matrix formalism [1]. Since both the measuring device (the detector) and the measured system are described by the Schrödinger equation, the question arises of how such a non-unitary process takes place. It was suggested that the dissipative interaction of a measured system with the detector might be responsible for the density-matrix collapse [2]. Yet, such an explanation would not be valid if the detector does not distort the measured system, for instance, for “negative result” measurements, when the information on the measured system is obtained by non-observing it at a given place [3]. In this case the density-matrix collapse should be generated by pure “informative” process. It leads to another question of how fast such a collapse takes place and whether it can be accommodated with the relativistic requirement.

A weak point of many studies of the measurement problem is the lack of a detailed quantum mechanical treatment of the entire system, that is, of the detector and the measured system together. The reason is that the detector is usually a macroscopic system, the quantum mechanical analysis of which is very complicated. Mesoscopic systems might be more useful for study of the measurement problem. We thus consider measurement of an electron in coupled quantum dots [4] by a detector showing single electron charging of a quantum dot [5]. We demonstrate how the “negative result” measurement can be designed in this case. Since the entire system is rather simple for detailed quantum mechanical treatment, the collapse of the density-matrix due to such an interaction-free measurement and its influence on the resonant current can be followed in great detail. Although the investigated system looks rather specific, it bears all essential physics of the measurement process. The method can be also used for analysis of different systems, in particular for optical “negative result” measurements [6].

We start with description of a measurement of single electron charging of a quantum dot, which eventually determines the resonant current, flowing through this dot. The system is shown schematically in Fig. 1 [4]. The detector (the upper dot) is in close proximity to the lower dot (the measured system). Both dots are coupled to two separate reservoirs at zero temperature. The resonant levels \( E_0 \) and \( E_1 \) are below the corresponding Fermi levels, \( \tilde{E}_F^L, \tilde{E}_F^R \) in the left reservoirs. In the absence of electrostatic interaction between electrons, the dc resonant currents in the detector and the measured system are respectively [6]

\[
I_D^{(0)} = e \frac{\gamma_L \gamma_R}{\gamma_L + \gamma_R}, \quad I_S^{(0)} = e \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R},
\]

where \( \gamma_{L,R} \) and \( \Gamma_{L,R} \) are the tunneling partial widths of the levels \( E_0 \) and \( E_1 \) respectively, due to coupling with left and right reservoirs. The situation is different in the presence of electron-electron interaction between the dots, \( H_{int} = U n_0 n_1 \), where \( n_{0,1} \) are the occupancies of the upper and the lower dots and \( U \) is the Coulomb repulsion energy. If \( E_0 + U > \tilde{E}_F^L, \) an electron from the left reservoir cannot enter the upper dot when the lower dot is occupied [Fig. 1 (b)]. On the other hand, if an electron occupies the upper dot [Fig. 1 (a’,b‘)], the displacement of the level \( E_1 \) of the lower dot is much less important, since it remains below the Fermi level, \( E_1 + U < \tilde{E}_F^L \). The upper dot can thus be considered as a detector, registering electrons entering the lower dot [3]. For instance, by measuring the detector current, \( I_D \), one can determine the current in the lower dot, \( I_S \).

![FIG. 1. The measurement of resonant current in a single-dot structure by another, nearby dot. All possible electron states of the detector (the upper well) and the measured system (the lower well) are shown. Also indicated are the tunneling rates (\( \gamma \) and \( \Gamma \)) of the detector and the measured system respectively.](image_url)

Actually, the described detector affects the measured system. It happens when the detector is occupied [Fig. 1 (a’,b‘)]. In this case an electron enters the lower dot at the energy \( E_1 + U \). As a result, the corresponding tunneling rates are modified (\( \Gamma_{L,R} \rightarrow \Gamma_{L,R}' \)), so that the measured current is distorted. One finds, however, that the states with empty detector, \( |a\rangle \) and \( |b\rangle \) [Fig. 1 (a,b)], do not distort the measured system. Nevertheless, the measurement process does take place: the detector current is interrupted whenever an electron occupies the measured system, but it flows freely when the measured system is empty. Such a measurement is in fact the negative result measurement [3]. Therefore, in order to
transform the above detector to a noninvasive one, we need to diminish the role of the states \(|a'\rangle\) and \(|b'\rangle\) in the measurement process. It can be done by varying penetrability of the detector barriers, \(\gamma_R \gg \gamma_L\), so that the dwelling time of an electron inside the detector decreases. (Indeed, the average charge inside double-barrier structure \(\langle \frac{e}{\Delta} \rangle \approx \frac{e\gamma_L}{\gamma_R+\gamma_L} \approx 0\) for \(\gamma_L/\gamma_R \rightarrow 0\). In this case an electron entering the detector leaves it immediately, remaining the measured system undistorted.

Now we confirm it explicitly by direct evaluation of the measured current that the above measurement is a non-invasive one. The currents through the detector and the measured system are determined by the density-matrix for the entire system \(\rho(t)\), which obeys the Schrödinger equation \(i\dot{\rho}(t) = [\mathcal{H}, \rho]\) for \(\mathcal{H} = \mathcal{H}_D + \mathcal{H}_S + \mathcal{H}_{int}\), where \(\mathcal{H}_D,\mathcal{H}_S\) are the tunneling Hamiltonians of the detector and the measured system, respectively, and \(\mathcal{H}_{int} = U_nn_1\). The current in the detector (or in the measured system) is the time derivative of the total average charge \(\dot{Q}(t)\) accumulated in the corresponding right reservoir (collector): \(I(t) = \dot{Q}(t)\), where \(\dot{Q}(t) = e\mathcal{T}\rho^R(t)\) and \(\rho^R(t)\) is the density-matrix of the collector. It was shown \(\text{[3]}\) that \(I(t)\) is directly related to the density-matrix of the multidot system \(\rho(t)\), obtained from the total density-matrix \(\rho(t)\) by tracing out the reservoir states. One finds that the current in the detector or in the measured system is given by

\[
I(t) = e \sum_c \sigma_{cc}(t) \Gamma_R^{(c)},
\]

where \(\sigma_{cc} \equiv \langle c|\sigma|c\rangle\) and the sum is over states \(|c\rangle\) in which the well adjacent to the corresponding collector is occupied. \(\Gamma_R^{(c)}\) is the partial width of the state \(|c\rangle\) due to tunneling to the collector (\(\gamma_R\) or \(\Gamma_R\)). In turn, \(\sigma(t)\) obeys the following system of the rate equations

\[
\begin{align*}
\dot{\sigma}_{aa} &= -\langle \gamma_L + \Gamma_L\rangle \sigma_{aa} + \gamma_R \sigma_{a'a'} + \Gamma_R \sigma_{bb} \quad (3a) \\
\dot{\sigma}_{bb} &= -\Gamma_R \sigma_{bb} + \langle \gamma_L + \Gamma_L\rangle \sigma_{aa} + \langle \gamma_R + \gamma'_{R}\rangle \sigma_{b'b'} \quad (3b) \\
\dot{\sigma}_{a'a'} &= -\langle \gamma_R + \Gamma_L\rangle \sigma_{a'a'} + \gamma_L \sigma_{aa} + \Gamma_R \sigma_{b'b'} \quad (3c) \\
\dot{\sigma}_{b'b'} &= -\langle \gamma_L + \gamma'_{R}\rangle \sigma_{b'b'} + \Gamma_L \sigma_{a'a'}, \quad (3d)
\end{align*}
\]

where the states \(|a\rangle, |b\rangle, |a'\rangle, |b'\rangle\) are the available states of the entire system, Fig. 1.

The currents in the detector and in the lower dot are \(I_D(t)/e = \gamma_R \sigma_{a'a'}(t) + \gamma_L \sigma_{b'b'}(t)\) and \(I_S(t)/e = \gamma_R \sigma_{bb}(t) + \Gamma_R \sigma_{b'b'}(t)\), respectively, Eq. (2). The stationary (dc) current corresponds to \(I = I(t \rightarrow \infty)\). Solving Eqs. (3) in the limit \(\gamma_R, \gamma_L \gg \gamma_L, \gamma'_R\) we find

\[
\frac{I_D}{I_S} = \frac{\gamma_L}{\Gamma_L}, \quad I_S = e \frac{\Gamma_L \Gamma_R}{\gamma_L + \Gamma_R} = \frac{I_S^{(0)}}{e}. \quad (4)
\]

The first equation shows that one can measure \(I_S\) by measuring the detector current \(I_D\), no matter how low the current \(I_S\) is (providing that the ratio \(\gamma_L/\Gamma_L\) is large enough). On the other hand, the current \(I_S\) is not distorted by the detector, as follows from the second equation.

Consider now resonant transport in the coupled dot structure shown in Fig. 2, where the upper dot is the detector. For simplicity, we assume strong Coulomb repulsion inside the coupled-dot, so only one electron can occupy it \([\text{Fig. 1}]\). \(U_{1,2}\) is the Coulomb repulsion energy between the detector and the measured system when an electron occupies the first or the second dot of the measured system. Similar to the previous case, Fig. 1, each of the states, \(|a\rangle, |b\rangle, |c\rangle\) has its counterpart \(|a'\rangle, |b'\rangle, |c'\rangle\) (not shown in Fig. 2) corresponding to the occupied detector. We consider \(E_0+U_1 > E_F^R\), but \(E_0+U_2 < E_F^R\). It means that the detector is blocked only when an electron occupies the first dot. We assume that \(U_2\) is very small so that the corresponding tunneling rates for entering the dots are not modified.

\[
\sigma_{aa} = -\Gamma_L \sigma_{aa} + \Gamma_R \sigma_{cc} \quad (5a) \\
\sigma_{bb} = \Gamma_L \sigma_{aa} + i\Omega (\sigma_{bc} - \sigma_{cb}) \quad (5b) \\
\sigma_{cc} = -\Gamma_R \sigma_{cc} - i\Omega (\sigma_{bc} - \sigma_{cb}) \quad (5c) \\
\sigma_{bc} = i\epsilon \sigma_{bc} + i\Omega (\sigma_{bc} - \sigma_{cc}) - \frac{1}{2} \Gamma_R \sigma_{bc}, \quad (5d)
\]

where \(\epsilon = E_2 - E_1\) and \(\sigma_{cb} = \sigma_{bc}^*\). The diagonal density-matrix elements \(\sigma_{ii}\) are the probabilities of finding the system in one of the states, \(|a\rangle, |b\rangle\) and \(|c\rangle\). In the distinction with resonant tunneling through a single dot, the diagonal density-matrix elements are coupled with
the non-diagonal elements $\sigma_{bc}$, $\sigma_{cb}$ (“coherences”) that provide the hopping between two isolated levels, $E_1$ and $E_2$. The total resonant dc flowing through this system is $I_S^{(0)} = e\Gamma_R \sigma_{cc}(t \to \infty)$, Eq. (2). One obtains [10]

$$I_S^{(0)} = \frac{e\Gamma_R \Omega^2}{\epsilon^2 + \Gamma_R^2/4 + \Omega^2(2 + \Gamma_R/\Gamma_L)}$$

(6)

Note that the dissipation of the “coherences”, $\sigma_{bc}$, is generated by the last term in Eq. (5a), proportional to the half of decay rates of the states $|b\rangle$ and $|c\rangle$ due to their coupling with the reservoirs. (In our case the state $|b\rangle$ cannot decay, but only the state $|c\rangle$, so the dissipation is proportional to $\Gamma_R$). Since the resonant current proceeds via hopping between two dots, generated by $\sigma_{bc}$, it should decrease with dissipation: $I_S^{(0)} \to 0$ for $\Gamma_R \to \infty$, Eq. (3).

Now we “switch on” the detector. The available states in the entire system are $|a\rangle$, $|b\rangle$, $|c\rangle$, Fig. 2, and $|a'\rangle$, $|b'\rangle$, $|c'\rangle$, corresponding to empty and occupied detector, respectively. The rate equations describing the transport in the entire system are [3]

$$\dot{\sigma}_{aa} = -(\Gamma_L + \gamma_L)\sigma_{aa} + \gamma_R \sigma_{a'a'} + \Gamma_R \sigma_{cc}$$

(7a)

$$\dot{\sigma}_{a'a'} = -(\Gamma_L + \gamma_L)\sigma_{a'a'} + \gamma_L \sigma_{aa} + \Gamma_R \sigma_{c'c'}$$

(7b)

$$\dot{\sigma}_{bb} = \Gamma_L \sigma_{aa} + i\Omega(\sigma_{bc} - \sigma_{cb}) + (\gamma_L' + \gamma_R')\sigma_{bb'}$$

(7c)

$$\dot{\sigma}_{bb'} = \Gamma_L \sigma_{a'a'} + i\Omega(\sigma_{c'c} - \sigma_{cc'}) - (\gamma_L' + \gamma_R')\sigma_{bb'}$$

(7d)

$$\dot{\sigma}_{cc} = -(\Gamma_R + \gamma_L)\sigma_{cc} - i\Omega(\sigma_{bc} - \sigma_{cb}) + \gamma_R \sigma_{c'c'}$$

(7e)

$$\dot{\sigma}_{c'c'} = -(\Gamma_R + \gamma_L')\sigma_{c'c'} - i\Omega(\sigma_{bc} - \sigma_{cb}) + \gamma_L \sigma_{cc'}$$

(7f)

$$\dot{\sigma}_{bc} = i\epsilon \sigma_{bc} + i\Omega(\sigma_{bb} - \sigma_{cc})$$

(7g)

$$\dot{\sigma}_{b'c'} = \frac{1}{2}(\Gamma_R + \gamma_L + \gamma_L')\sigma_{b'c'} - \frac{1}{2}(\gamma_R + \gamma_R')\sigma_{cc'}$$

(7h)

Let us again take the limit of the negative result measurement, $\gamma_R, \gamma_R' \gg \gamma_L, \gamma_L'$, when the detector is expected not to influence the measured system. If so, the density-matrix of the entire system, traced over the detector states would coincide with the density-matrix for the double-dot system without detector, Eqs. (3). We thus introduce $\bar{\sigma}_{aa} = \sigma_{aa} + \sigma_{a'a'}$, $\bar{\sigma}_{bb} = \sigma_{bb} + \sigma_{bb'}$, $\bar{\sigma}_{cc} = \sigma_{cc} + \sigma_{c'c'}$, and $\bar{\sigma}_{bc} = \sigma_{bc} + \sigma_{bc'}$, which is the density matrix traced over the detector states. One finds from Eqs. (3) that $\bar{\sigma}_{ij}$ obeys the following equations

$$\dot{\bar{\sigma}}_{aa} = -\Gamma_L \bar{\sigma}_{aa} + \Gamma_R \bar{\sigma}_{cc}$$

(8a)

$$\dot{\bar{\sigma}}_{bb} = \Gamma_L \bar{\sigma}_{aa} + i\Omega(\bar{\sigma}_{bc} - \bar{\sigma}_{cb})$$

(8b)

$$\dot{\bar{\sigma}}_{cc} = -\Gamma_R \bar{\sigma}_{cc} - i\Omega(\bar{\sigma}_{bc} - \bar{\sigma}_{cb})$$

(8c)

$$\dot{\bar{\sigma}}_{bc} = i\epsilon \bar{\sigma}_{bc} + i\Omega(\bar{\sigma}_{bb} - \bar{\sigma}_{cc}) \frac{1}{2}(\Gamma_R + \gamma_L)\bar{\sigma}_{bc}$$

(8d)

Let us compare Eqs. (8) with Eqs. (3). Although the equations for the diagonal matrix elements are the same, the equations for the off-diagonal matrix element are not. The difference is in the dissipative term, which includes now the detector tunneling rate $\gamma_L$. It is easy to trace its origin. In accordance with the Bloch equations the dissipation of the nondiagonal density-matrix elements $\tilde{\sigma}_{bc}$ is the half of all possible decay rates of each of the states $|b\rangle$ and $|c\rangle$. In the presence of the detector, the state $|c\rangle$, Fig. 2, has an additional decay channel, corresponding to the possibility for an electron to enter the detector. Although the time which an electron spends in the detector tends to zero, so that the related detector state does not distort the measured system, the possibility of such a process may influence the measured current very drastically. Indeed, if $\gamma_L \gg \Gamma_L, \Gamma_R, \Omega$, one finds from Eq. (5d) that $\tilde{\sigma}_{bc} \to 0$ and therefore $I_S = e\Gamma_R \tilde{\sigma}_{bc} \to 0$.

The described strong damping of the nondiagonal density-matrix elements takes place only when the detector can distinguish a particular dot occupied by an electron. Such an “observation” effect disappears if $\tilde{E}_F^L < E_0 + U_2$. In this case an electron cannot enter the detector no matter which of the dots of the measured system is occupied. Then the additional decay channel for the state $|c\rangle$ is blocked and Eq. (5c) coincides with Eq. (5a), i.e. the measured current remains undistorted, $I_S = I_S^{(0)}$, Eq. (3). Such a peculiar dependence of the measure current $I_S$ on $\tilde{E}_F^L$ is shown in Fig. 3.

Our analysis demonstrates that the disappearance of the non-diagonal matrix elements during the measurement process is attributed to the dissipation term. The latter is always generated by the Schrödinger equation by tracing out the continuum spectrum states [8]. It is therefore no need to introduce an additional projection postulate to describe the measurement process. Most interesting aspect, which reveals our analysis is that the reduction postulate is not necessary even for interaction-free (negative-result) measurement.

Although the negative-result measurement which influences the measured system without actual interaction is similar to the famous EPR paradox, some features are different. First, the above “observation” effect appears as a stationary state phenomenon. Second, we do not need any special initial correlations between electrons in the detector and the measured system. It leads to a pos-

FIG. 3. Maximal current in the double dot structure ($E_1 = E_2$) as a function of the Fermi energy of the left reservoir adjacent to the detector.
sibility of influencing the measured current by switching the detector on (or off). Such a process which has a relaxation time $\sim 1/\gamma$, can also be studied using the same rate equations (4). Most interesting problem would arise when the distance between the detector and the measured system is larger then $c/\gamma$, so that the density-matrix reduction, generated by the negative result measurement might contradict the relativistic requirement.

Finally we like to mention that a similar description of the negative result measurement can be used for the optical experiment of Kwiat et al. [6]. In this experiment the non-observance of the "object" modifies the interference of the photon. In terms of our description it corresponds to the collapse of the photon density-matrix due to the possibility of the photon absorption by the "object". Notice that the "object" would play a role of our "detector" for a constant flux of photons. The time which takes such a collapse due to the non-observance of the object is therefore $\sim 1/\bar{\gamma}$, where $\bar{\gamma}$ is the photon absorption width. One thus can investigate a possible contradiction between the density-matrix reduction rate and the relativistic requirement also in the optical negative-result measurements.

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