Multi-quark baryons and color screening at finite temperature

Kazuo Ghoroku†, Masafumi Ishihara‡, §Akihiro Nakamura³
¶Fumihiko Toyoda⁴

†Fukuoka Institute of Technology, Wajiro, Higashi-ku
Fukuoka 811-0295, Japan
‡Department of Physics, Kyushu University, Hakozaki, Higashi-ku
Fukuoka 812-8581, Japan
§Department of Physics, Kagoshima University, Korimoto 1-21-35,
Kagoshima 890-0065, Japan
¶School of Humanity-Oriented Science and Engineering, Kinki University,
Iizuka 820-8555, Japan

Abstract

We study baryons in $SU(N)$ gauge theories at finite temperature according to the gauge/string correspondence based on IIB string theory. The baryon is constructed out of D5 brane and $N$ fundamental strings to form a color singlet $N$-quark bound state. At finite temperature and in the deconfining phase, we could find $k(< N)$-quark “baryons”. Thermal properties of such $k$-quark baryons and also of the $N$-quark baryon are examined. We study the temperature dependence of color screening distance and Debye length of the baryon of $k$-quark and $N$-quark. We also estimate the melting temperature, where the baryons decay into quarks and gluons completely.
1 Introduction

In the context of string/gauge theory correspondence the baryons related to the D5-branes has been proposed [1, 2, 3, 4, 11, 12, 7, 8, 9] and studied using the Born-Infeld approach [5, 6, 10]. In these approaches, the fundamental strings (F-string) are dissolved in the D5 brane in order to make a baryon vertex. Here the D5-brane is embedded as a probe in a 10D background in which the conformal invariance is broken by the non-trivial dilaton. As a result of non-conformal invariance, the embedded configurations of the D5-brane have in general cusps, which are the singular points to be cancelled out by some external objects. This is performed by introducing F-strings whose surface term cancels out with this cusp singularity. This is represented by the no-force condition between the F-strings and the D5-brane. From a viewpoint of smoothed picture of the baryon configuration, these added F-strings could be regarded as the one which flow from the D5-brane through the cusps.

According to the above idea, an analysis for the baryon configurations has been performed in [13] based on a simple holographic model. At finite temperature and in the quark deconfinement phase, we could find a possible bound states of small number \( k(< N) \) of quarks with D5 vertex. Here we study the thermal properties of these states and also the usual \( N \)-quarks baryons. The quarks in the baryon are connected to the vertex through F-string. This F-string is common to the one of mesons which are the bound states of a quark and an anti-quark. So we expect, for the part related to the quarks in the baryon, that the thermal properties are similar to the one of the mesons. But the situation is different from the meson case in the following two points. (i) Since the baryon is constructed from \( N \) quarks, various configurations are possible in the quark deconfinement phase. They are discriminated by the number of quarks retained in the state. (ii) The second point is the non-trivial configuration of the vertex given by D5 brane, which is determined by the dynamics of the gauge theory.

These properties depend on the temperature (T). At low temperature, we find four types \( ((A) \sim (D)) \) of D5-brane configurations. Two of them \( ((C) \text{ and } (D)) \) appear for the first time at finite temperature. When the temperature exceeds a value \( T_{c1} \), which is given in [13], the solution is reduced to the one type \( ((A)) \). But before arriving at this temperature, any baryon state decays into quarks and gluons since its energy \( (E) \) exceeds over the sum of the effective mass of free quarks. We call this temperature as melting point, \( T_{\text{melt}} \), which is slightly smaller than \( T_{c1} \) as estimated below.

The baryon energy \( E \) varies with its size. We study here the relation between the size and the energy \( E \) of the baryon. As for the size of the baryon, we could measure it as the F-string length observed in the real three dimensional space. The configurations of the F-strings are intimately related to the vertex configurations or the D5 brane structure, which varies with the temperature. Through the study of the F-string configurations, we find that the quarks in the baryon can not stay far from the vertex exceeding a distance \( (L_{\text{max}}) \) which increases with decreasing \( T \). This is the reflection of the thermal
screening of the color force and it is seen by measuring \(L_{q-v}\), the distance of the quarks from the vertex, as a function of \(E\). At low temperature, we find \(E - L_{q-v}\) relation as three connected curves. They are a little complicated compared to the case of mesons. The first curve starts from \(L_{q-v} = 0\) and ends at \(L_{\text{max}}\), then it comes back toward the small \(L_{q-v}\) side along the second curve with larger \(E\) and it stops at \(L_{q-v} = L_1(> 0)\). Then it turns to the larger \(L_{q-v}\) again along the third curve with more large \(E\), and it stops at \(L_{q-v} = L_2(< L_{\text{max}})\). (See Fig. 5.) This behavior is common to all the baryon configurations of different \(k'\)s, and the slopes of the curves of different \(k\)s are proportional to \(k\).

However we find a real screening point at \(L^*(\ll L_{\text{max}})\), where the baryon decays to the quarks and gluons, before arriving at \(L_{\text{max}}\). So we could not see the above complicated properties as stable baryon states, and for \(L^* < L_{q-v}\), they decay into the quarks and gluons. When \(T\) increases and approaches to \(T_{\text{melt}}\), then \(L^*\) vanishes to zero and all the baryons melt down.

In the section 2 we review our model briefly, and the equation of motion for D5 branes is solved to show four types of solutions and to discuss the stability of the baryon. In the section 3, the balance condition of the forces is given, and in the next section 4, \(E - L_{q-v}\) relations and the estimation of the screening mass are given. In the section 5, we discuss the stability of the baryon state from the viewpoint of baryon vertex energy and the gauge condensate parameter. The summary and discussions are given in the final section.

## 2 D5 baryon vertex at finite temperature

Here, we consider the baryon configurations in the non-confining, finite temperature Yang-Mills theory. Such a model is given by the AdS blackhole solution, which represents the high temperature gauge theory. In our theory with dilaton, the corresponding background solution is given as [15]

\[
\begin{align*}
&d\Sigma_{10}^2 = e^{\Phi/2} \left( \frac{r^2}{R^2} \left[ -f^2 dt^2 + (dx^i)^2 \right] + \frac{R^2}{r^2 f^2} dr^2 + R^2 d\Omega_5^2 \right), \\
&f = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \quad e^\Phi = 1 + \frac{q}{r_T} \log \left(\frac{1}{f^2}\right), \quad \chi = -e^{-\Phi} + \chi_0 ,
\end{align*}
\]

The temperature \((T)\) is denoted by \(T = r_T/(\pi R^2)\). The world volume action of D5 brane is rewritten by eliminating the \(U(1)\) flux in terms of its equation of motion as above, then we get its energy as [13]

\[
U_{D5} = \frac{N}{3\pi^2 \alpha'} \int d\theta \ e^{\Phi/2} f \sqrt{r^2 + r'^2 / f^2 + (r/R)^4 x'^2} \sqrt{V_\nu(\theta)} .
\]

where

\[
V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8 \theta
\]
Here, $D$ denotes the electric flux at $\theta$-cross section. And the integration constant $\nu$ is expressed as $0 \leq \nu = n/N \leq 1$, where $n$ corresponds to the number of F-strings emerging from one of the pole of the $S^5$ at $\theta = 0$. Then $N - n$ comes out from the other pole at $\theta = \pi$.

### 2.1 Embedded solutions of the D5 brane

The vertex configuration is obtained from the above Legendre transformed action (3). Here for simplicity, we concentrate on the point vertex configuration and $\nu = 0$, which means the quarks come from the cusp at $\theta = \pi$. There is no other cusp. Then, we set as $x' = 0$, and we obtain

$$U_{D5} = \frac{N}{3\pi^2 \alpha'} \int d\theta \, e^{\Phi/2} \sqrt{r^2 + r'^2} \sqrt{\frac{V_0(\theta)}{f^2}}.$$  \hspace{1cm} \text{(6)}

From this action, the equation of motion of $r(\theta)$ is obtained as follows

$$\partial_\theta \left( \frac{r'}{\sqrt{r^2 f^2 + (r')^2}} \right) \sqrt{V_0(\theta)} - \frac{P(r)}{\sqrt{r^2 f^2 + (r')^2}} \sqrt{V_0(\theta)} = 0.$$  \hspace{1cm} \text{(7)}

$$P(r) = \frac{1}{2} e^\Phi \partial_r \left( e^\Phi \, r^2 f^2 \right).$$  \hspace{1cm} \text{(8)}

Fig. 1: $P(r)$ for $q = 2$ and $r_T = (\alpha) 0.1$, $(\beta) 0.45$, $(\gamma) 0.8$, $(\delta) 0.689$. The curve $(\delta)$ is given at the critical point $T_{c_1}$. The curve of $T = 0$ shows $3P|_{r_t=0} = 3r(1 - q/r^4)/(1 + q/r^4)$.

The solutions $r(\theta)$ of equation (7) are characterized by the prefactor $P(r)$ in the second term. Its behavior depends on the temperature. For the comparison, $P(r)$ at $T=0$ is also shown in the Fig. 1 with the one of the various finite temperatures.
At any temperature, $P(r)$ is positive at large $r$ and changes its sign to the negative value for the low temperature cases according to the curves ($\alpha$) and ($\beta$) in the Fig. 1. When we take the value of $r(0)(\equiv r_0)$ at the point of $P > 0 \ (P < 0)$, then we obtain the typical configuration (A) ((B)) of the baryon as shown in the Fig. 2 where F-strings are added according to the condition given below. Here we notice only the configuration of the vertex part. For the case of zero temperature, the two types solutions (A) and (B) are enough correspondingly to the region of $P > 0$ and $P < 0$ respectively. However, at finite and low temperature, the behavior of $P$ is not so simple since the zero point appears at smaller $r$. When $r_0$ is chosen near this zero point, we find two typical solutions (C) and (D) for $P < 0$ and $P > 0$ respectively as shown in the Fig. 3. They oscillate once with respect to $\theta$, and these solutions can not be seen at zero temperature.

Fig. 2: $k$-quarks baryons at finite temperature. The triangles represent the end point $r(\pi)$ of D5 vertex and the direction of the cusp at $\theta = \pi$. The left (right) corresponds to the solutions (A) and (C) ((B) and (D)).

At higher temperature, $T > T_{c_1}$, the zero points of $P(r)$ disappear as shown by the curve ($\gamma$) in the Fig. 1. In this case, only the type (A) solutions are obtained, and several solutions with different $r_0$ are shown in the Fig. 4. We can see that the end point value $r(\pi) \equiv r_c$ of the solutions $r(\theta)$ takes its minimum for some initial value $r(0)$, then it rises when $r(\pi)$ decreases furthermore and approaches to the horizon $r_T$.

Next, we turn to the F-strings. In the present finite $T$ model, the effective quark mass ($\tilde{m}_q$) is finite and it diverges at $T = 0$. In this sense, the present theory is in the quark deconfinement phase for finite $T$. So free quarks or independent F-strings could exist, and the end points of these F-strings touch on the horizon $r_T$ and $r_{\text{max}}$. 

Fig. 4: Several solutions of $r(\theta)$ with different $r_0$ at $T > T_{c_1}$.
Fig. 3: The four types of solution of \( r(\theta) \) for \( q = 2 \) at \( r_T = 0.45 \). The curves represent for (A) \( r_0 = 1.5 \), (B) \( r_0 = 1.1 \), (C) \( r_0 = 0.6 \) and (D) \( r_0 = 0.5 \) respectively.

Fig. 4: The type (A) solutions of \( r(\theta) \) for \( q = 2 \), \( r_T = 0.8 \) and (Aa) \( r_0 = 0.80001 \) (Ab) \( r_0 = 1.0 \) (Ac) \( r_0 = 1.2 \) (Ad) \( r_0 = 1.5 \).

Then, some of the F-strings, which are originally attached to the baryon, could escape from the baryon to form such free F-strings. And as shown in the Fig. 2, we could find \( k(< N) \)-quarks baryon configuration with extra \( N - k \) F-strings which connect the vertex and the horizon. These \( N - k \) F-strings are not observed as quarks in the gauge theory, but they contribute to the energy of the baryon. As a result, some number of \( N - k \) quark disappear from the original baryon, and we would find it as a \( k \)-quarks baryon. Then, this implies that we may find color non-singlet baryons as depicted in the Fig. 2.

3 No force condition and \( k \)-quarks baryon

Full baryon configuration is given by the vertex and F-strings. The configurations are determined through the no force condition at the cusp point of the vertex and the profiles of the vertex and the F-strings. They are characterized by the energy of the total system, the baryon energy, and the distance between the quark and the vertex.
The tension of D5-brane at \( r_c \) is given as
\[
\frac{\partial U_{D5}}{\partial r_c} = N T_F e^{\Phi/2} \frac{r'_c}{\sqrt{r_c^2 + r_c^2 f(r_c)^2}} \tag{9}
\]

As for the F-string The energy is written as
\[
U_F = k U_F^{(1)} + (N - k) U_F^{(2)} \tag{10}
\]

Here, \( U_F^{(1)} \) is the energy of string which extends to the \( x \)-direction and its action is written as
\[
S_F^{(1)} = -\frac{1}{2\pi \alpha'} \int dt dx e^{\Phi/2} \sqrt{r_x^2 + f^2(r)(r/R)^4} \tag{11}
\]
where \( r_x = \frac{\partial}{\partial x} \) and the world sheet coordinates are set as \((t, x)\). Then, its energy and \( r \)-directed tension at the point \( r = r_c \) are obtained as follows
\[
U_F^{(1)} = T_F \int dx e^{\Phi/2} \sqrt{r_x^2 + f^2(r)(r/R)^4} \tag{12}
\]
\[
\frac{\partial U_F^{(1)}}{\partial r_c} = T_F \frac{r_x}{\sqrt{r_x^2 + (r_c/R)^4 f(r_c)^2}} \tag{13}
\]
\( U_F^{(2)} \) is the energy of string extending from horizon \( r_T \) to \( r_c \), and it is written as
\[
U_F^{(2)} = T_F \int_{r_T}^{r_c} dr e^{\Phi/2} \tag{14}
\]

Then the no force condition is obtained as
\[
N \frac{r'_c}{\sqrt{r_c^2 + r_c^2 f(r_c)^2}} + (N - k) = k \frac{r_x}{\sqrt{r_x^2 + (r_c/R)^4 f(r_c)^2}} \tag{15}
\]

### 3.1 Lower bound on \( k \)

From this equation, we can estimate the lower bound for the \( k \), which is written as
\[
k \geq \frac{N}{2} (1 + Q_5), \quad Q_5 \equiv \frac{r'_c}{\sqrt{r_c^2 + r_c^2 f(r_c)^2}}. \tag{16}
\]

The value of \( Q_5 \) depends on the parameters, but we can see it does not arrive at -1. Then \( k > 0 \), however, \( k \) could decrease and approaches to zero by choosing the parameters. For example, we find \( Q_5 = -0.99916 \) for \((q, r_0, r_T) = (2.0, 0.6, 0.01)\).

In our model, the F-string configuration must satisfies Eq.(17) with a constant \( h \). This implies \( h = 0 \) for the string which touches on the horizon \( r_T \) since \( f(r_T) = 0 \). So we obtain a vertical straight line as the configuration. However, it would be possible to obtain a slightly curved string configuration in a more realistic model. Then the configuration of \( k = 0 \) would be possible in principle at finite temperature. But it would not realized energetically as a stable state.
3.2 Energy of $k$-quarks baryon

The energy of baryons are constructed by the F-strings energy and that of the vertex. In obtaining the F-string configuration from Eq. (12), we introduce the constant $h$ as follows,

$$e^{\Phi/2} \frac{r^4 f^2(r)}{R^4 \sqrt{r_x^2 + (r/R)^4 f^2(r)}} = h$$  \hspace{1cm} (17)

For $r_x > 0(< 0)$, $k$-quarks baryons configuration becomes the type (A)((B)). First, consider the solution of (A). In this case, the energy of the F-strings is given as

$$U_F = T_F \left[ k \int_{r_c}^{r_{\text{max}}} dr \frac{e^{\Phi/2}}{\sqrt{1 - \frac{h^2 R^4}{f^2(r)}}} + (N - k) \int_{r_T}^{r_c} dr e^{\Phi/2} \right]$$  \hspace{1cm} (18)

The second term denotes the F-strings stretching between horizon and $r_c$. And the distance $L_{q-v}$ is given as

$$L_{q-v} = R^2 \int_{r_c}^{r_{\text{max}}} dr \frac{1}{r^2 f^2 \sqrt{e^{\Phi/2} f^2 - 1}}$$  \hspace{1cm} (19)

Next we turn to the configuration of (B). In this case, the string configuration includes a bottom part of U-shaped configuration and the bottom point is seen at $r = r_{\text{min}}$, where we find

$$\frac{dr}{dx} \bigg|_{r=r_{\text{min}}} = 0$$  \hspace{1cm} (20)

Since (17) can be used at any $r$, we obtain

$$h = e^{\Phi(r_{\text{min}})/2} f(r_{\text{min}}) \left( \frac{r_{\text{min}}}{R} \right)^2 = e^{\Phi(r_c)/2} \frac{r_c^4 f^2(r_c)}{R^4 \sqrt{r_c^2 + (r_c/R)^4 f^2(r_c)}}$$  \hspace{1cm} (21)

From this, $r_{\text{min}}$ is determined by $r_c$, then by $r_0$. On the other hand, $r_x$ is determined by using $r_c$ from (15). As a result, we obtain the string energy and the distance between the quark and the vertex as follows,

$$U_F = T_F \left[ k \left( \int_{r_{\text{min}}}^{r_{\text{max}}} dr \frac{e^{\Phi/2}}{\sqrt{1 - \frac{h^2 R^4}{f^2(r)}}} + \int_{r_c}^{r_{\text{min}}} dr \frac{e^{\Phi/2}}{\sqrt{1 - \frac{h^2 R^4}{f^2(r)}}} \right) + (N - k) \int_{r_T}^{r_c} dr e^{\Phi/2} \right]$$  \hspace{1cm} (22)

$$L_{q-v} = R^2 \left[ \int_{r_{\text{min}}}^{r_{\text{max}}} dr \frac{1}{r^2 f \sqrt{e^{\Phi/2} f^2 - 1}} + \int_{r_{\text{min}}}^{r_c} dr \frac{1}{r^2 f \sqrt{e^{\Phi/2} f^2 - 1}} \right]$$  \hspace{1cm} (23)

Here we consider the energy of $k$-quark (for any $k$) baryon and $N - k$ free quarks to compare the energy with that of the $N$ free quarks which form the baryon as a singlet
state. So we add \((N - k)\) free quarks, which are strings stretching between horizon and \(r_{\text{max}}\), to the \(k\)-quark baryon state. We estimate the energy of the baryon in this way as a function of the distance \(L_{q-v}\). Then the total \(k\)-quark baryon energy is obtained as

\[
E_k = U_F + U_{D5} + (N - k)\tilde{m}_q ,
\]

where notice that the first two terms \(U_F\) and \(U_{D5}\) are dependent on \(k\), and \(\tilde{m}_q\) is the energy of a free quark,

\[
\tilde{m}_q = T_F \int_{r_T}^{r_{\text{max}}} dr e^{\phi/2} .
\]

4 Color screening and baryon melting

Here we consider the stability of the various baryon configurations in the deconfining thermal medium. Although the baryon is in the deconfining phase some of the baryon states could be stable until the temperature arrives at the critical point where all the baryons melt down to free quarks. This melting temperature is estimated below by comparing the baryon energy and the energy of free quarks.

In the followings, we estimate numerically the energy \(E\) and the distance between quark and the vertex, \(L_{q-v}\) for \(k\)-quark baryon \((k \leq N)\) in order to cover various cases. The color screening due to the temperature is observed as the existence of a maximum value of \(L_{q-v}\) for each baryon states. However, the real maximum value of \(L_{q-v}\) is determined energetically by comparing the energy of the baryon state and the one of the summation of the free quarks.

4.1 For \(k = N\) case

Typical \(E - L_{q-v}\) curves are shown in the Fig. 5 for two temperatures. The beginning point at \(L = 0\) corresponds to the solution of \(r(\theta)\) with \((r(\pi) = r_c = r_{\text{max}}\) and \((r(0) = r_0) > r_T\). Then \(E\) increases almost linearly with \(L_{q-v}\). We notice two turn over points in these curves, and the second one has not been seen in the case of the mesons. We could see that the second turn over is related to the existence of two zero points of \(P(r)\) and new two type of solutions ((C) and (D)) as seen in the section 2. However, this intriguing behavior of \(E - L_{q-v}\) curves is seen in the unstable region of the baryon since the baryon energy in this region is higher than the sum of the free quarks energy, which are shown by the horizontal lines. They cross at \(L = L^*\), which depends on the temperature, then the baryon, which has the energy below \(N\tilde{m}_q\), is allowed as a stable state. The point \(L^*\) is therefore interpreted as the maximum size of the baryon. Within this distance, the color force works on the quark, and the color gauge force is screened outside this distance. As a result, all the baryons decay into quarks and gluons for \(L^* < L_{q-v}\).
Fig. 5: Energy of $N$-quarks baryon versus $L_{q-v}$ (which is denoted simply by $L$ in the figure) at $r_T = 0.05$ (left) and $r_T = 0.1$ (right) for $q=2$. In these figures two turnovers are seen as mentioned in the text. However, physically allowed region is only the part below the horizontal lines of $N\tilde{m}_q$.

Of course, the value of $L^*$ depends on the temperature. Their values for various temperatures are plotted in the Fig. 6. The numerical results show that $L^*$ is proportional to $1/r_T$ or $1/T$, and it approaches to zero at some limiting temperature. Above this temperature, any baryon state is not allowed.

The value of $L^*$ should be related to the Debye screening. In the gauge theory, the Debye screening has been considered as the correction to the gluon propagator. Then the potential between two colored particles separating $r$ is given by the form $e^{-m_D r}/r$. However, in the present model, the force at short distance is given by the strong coupling CFT and a long range force which leads to linear potential. And we find a linear-like potential near $L^*$ even if it is still not so large. So the potential screened by the thermal effect is just this linear like potential, which is considered as

Fig. 6: The left hand side shows the Debye screening mass $m_D$ estimated for $k/N = 1$ and 0.8. And the line is fitted as $m_D = 6.81r_T$ ($m_D = 10.5$ $r_T$) for $k/N = 1$ ($k/N = 0.8$). In the right hand figure, melting point $L^*$ are shown by the curves for $k/N = 1$ and 0.8 baryon versus $1/r_T$ at $q = 2$. The dots represent the superposition of the data for $m_D$ given in the left hand figure. They are given as $L^* = 6.8/m_D$ for both $k/N = 1$ and 0.8.
the tail of the linear potential seen at large $L_{q-v}$ in the case of $T = 0$. When we denote this tail as $V_0(L)$ at zero temperature, it is modified at finite temperature as,

$$V \equiv E - N\tilde{m}_q = V_0(L) \ e^{-m_D(T)L},$$

(26)

where $m_D(T)$ denotes the Debye mass. Using this formula, the numerical curves of $E - L$ are fitted by this form, and we obtain $m_D(T)$. However we used here $V_{T_{\text{min}}}(L)$ instead of $V_0(L)$, where $V_{T_{\text{min}}}(L)$ denotes the potential of the lowest temperature. So we corrected the data of $m_D(T)$ obtained by shifting the temperature $T \to T'$ such that we could obtain $m_D(T') \to 0$ for $T' \to 0$. The results are shown in the left hand of the Fig.6.

Then we could find the relation $L^* = c/m_D(T)$ with a definite constant $c = 6.8$. And we obtain the result that $m_R(T)$ increases linearly with the temperature $T$.

In [16], using the AdS/CFT correspondence, Bak, Karch and Yaffe examined the behavior of correlators of Polyakov loops and other operators in $N = 4$ supersymmetic Yang-Mills theory at non-zero temperature. And the implications for Debye screening in that strongly coupled non-Abelian plasma, and comparisons with available results for thermal QCD, were discussed. The connection with our result is that the proportionality $m_D \propto T$ is common with [16]. In [17], however, $m_D/T$ is predicted to decrease slowly with $T/T_c$ on the basis of nonperturbative computation in the deconfined phase of QCD, using the method called background perturbation theory. This prediction is in good agreement with the data of lattice QCD [18]. In order to reproduce this decrease, string-string interactions are taken into of, in string side. This is a future problem.

4.2 For $k < N$ case

In this subsection, we discuss the properties discussed above for the baryon with $k < N$. Several $E - L_{q-v}$ curves for different $k$-baryons are shown in the Fig. 7. The melting points $L^*$ are obtained from these results, and they are shown for $k/N = 0.8$ in the

![Fig. 7: Energy of $k$-quarks baryon versus $L_{q-v}$ for $r_T = 0.05$. The curves represents $k/N = 1$ (a), 0.8 (b) and 0.67 (c). And the flat line represents $N\tilde{m}_q = 48.8066$.](image-url)
Fig. 6. Compared to the case of $k/N = 1$, it starts from a larger $1/r_T$ (smaller temperature) with a smaller grade in the $L^*-1/r_T$ plane. For smaller $k$, the starting point moves to larger $1/r_T$ with smaller slope. Then we could find small $k$-baryon at low temperature as stated in the subsection 3.1.

However, the energy of $k$-baryon is larger than the $k'$-baryon for $k < k'$ at the same $L$. Then the $k$-baryon ($k < N$) is always observed as an exited state of the $N$-quark baryon. One more point to be noticed is that our model is in the deconfinement phase as far as the temperature is finite. However, the confinement phase would be realized at a finite temperature, in other words, the critical temperature of quark confinement and deconfinement must be finite. In this sense, the lower bound of $k$ would be existing at some finite value. But this bound is not given here.

5 Baryon stability

We discuss here more about the stability of the baryon from two viewpoints in order to get physical insights. Since the vertex is an important part of the baryon, it would be important to see its role for the baryon stability. Another important ingredient in our model is the gauge condensate which is the main source of the confinement force. So we like to see its role in the deconfinement phase considered here.

5.1 Vertex energy and stability

First, we give a comment from the viewpoint of the vertex energy. In [15], a comment on the stability of the baryon at finite temperature has been given through the vertex energy without any inner structure. We give a similar kind of comment on this point in terms of the energy of the vertex with structure given above. Before doing it, we firstly notice that the energy of the vertex without structure. Assuming the baryon is at $r$, the energy is given from equation (6) by setting $r' = 0$ as,

$$U_{D5} = \tilde{N} e^{\Phi/2} r f(r)$$

(27)

where $\tilde{N} = \frac{N}{3\pi^2 a^3} \int d\theta \sqrt{V_0(\theta)}$ denotes a $r$-independent constant. Then we find

$$P(r) \propto \frac{\partial U_{D5}}{\partial r}.$$  

(28)

From this, we can say that the zero point of $P(r)$ is needed to find a minimum of $U_{D5}$, and we actually find such a point at low energy. However, there is a temperature $T_{c1}$ above which the zero point disappears, then the vertex becomes unstable as mentioned in [15]. However, this statement should be modified when the structure of the vertex is taken into account of.
By substituting the solution of (7) into (3), we obtain the vertex energy $U_{D5}$ as a function of $r_0$. In this case, we find the minimum even if the temperature is taken to be larger than $T_{c1}$, as seen in the Fig. 8. Then we need another criterion to see the stability of the baryon.

Fig. 8: The vertex energy $U_{D5}$ versus $r_0$. The minimum of $U_{D5}$ exists near $r_0 = q^{1/4}$. For $r_T \geq 0.8$, the minimum increases rapidly.

Fig. 9: The T dependence of the minimum energy of the $N$-quark baryon ($E_{D5}$) are shown by the top, middle and the bottom lines for $q = 2.0$, 1.0 and 0.2 respectively. The three curves show $\tilde{N}\tilde{m}_q$ for $q = 2.0$, 1.0 and 0.2.

The new criterion can be stated as follows. The stability of the baryon is assured when its energy is smaller than the sum of free effective-quark mass, $\tilde{N}\tilde{m}_q$. Because the baryon decays to free quarks when its energy exceeds $\tilde{N}\tilde{m}_q$.

The energy of the baryon is given by the sum of $E_{D5}$ and F-strings. Then the minimum value of the baryon energy is given by $E_{D5}^{\min}$ when F-strings part vanishes due to $L = 0$. Then, the stability of the baryon is found by comparing $E_{D5}^{\min}$ with $\tilde{N}\tilde{m}_q$. The value of $E_{D5}^{\min}$ is given by the D5 energy with a configuration of $r_T < r(o) < r < r(\pi) = r_{\max}$. In this configuration, the F-strings are reduced to a point at $r_c$. The higher energy states are obtained by adding F-strings with finite length to the D5 brane with $r(\pi) < r_{\max}$ and $r(0) \geq r_T$. 

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In the Fig. 9, the minimum energy of the $N$-quark baryon ($E_{D5}$) and $N\tilde{m}_q$ are plotted as functions of $r_T$. We find the melting point ($T_{melt}$) near $T_{c1}$, above which $E_{D5}$ exceeds $N\tilde{m}_q$. Then the baryon decays to quarks and gluons. It is an interesting point that this temperature is very near to $T_{c1}$. This temperature $T_{melt}$ depends sensitively on the gauge condensate $q$, so $q$ is also related to the stability of the baryon state. On this point, we discuss in the following subsection.

5.2 Gauge condensate and Baryon at high temperature

As shown in the Fig. 9, $T_{melt}$ increases with $q$. Then an unstable baryon at a temperature can be stabilized by increasing the gauge condensate $q$. This point is understood in terms of the role of the gauge condensate in the gauge theory.

The role of $q$ has been made clear as a source of quark confinement force. At zero temperature with finite $q$, we find the linear potential between quark and anti-quark in the mesons [14] and also between the vertex and the quark in the baryons [13]. The tension of this linear potential is proportional to $q^{1/2}$ in both cases. On the other hand, the role of the temperature is to screen the color force and to prevent making the bound states of quarks. The highly exited state has a large size, then it would be destabilized by the screening effect at high temperature. However, this temperature would depend on the magnitude of the force or the value of $q$.

Actually, we find that the strength of the linear potential preserved even in the short range is proportional to $q^{1/2}$, so the force to make the baryon at finite $T$ is also proportional to $q^{1/2}$. Then we need higher temperature to destroy the baryon formed at larger $q$. Since the effects of $T$ and $q$ are opposite, then the melting point $T_{melt}$ increases with $q$.

This implies that the baryon configuration constructed by the vertex and F-strings can not be obtained for $q = 0$. In other words, the gauge condensate is essential to find $k(\leq N)$-quark baryon at finite temperature. On the other hand, mesons are expected to be observed even in the deconfinement high temperature phase of $q = 0$ [15].

This is understood also from the vertex energy given by (27) which is represented for $q = 0$ as follows,

$$U_{D5} = N\sqrt{r^2 - \frac{r_T^4}{r^2}}. \quad (29)$$

This implies that the vertex is not stable and it vanishes into the horizon. But this result is given for the structure less vertex [15]. When the structure is considered, the end point of the vertex increases infinitely toward the boundary when we solve the configuration of the D5 brane, $r(\theta)$. The reason is that the mass scale to retain $r_c$ at a finite value is given by $q$. For $q = 0$, the configurations given above do not exist. In other words, the confinement force given at zero temperature should be retained at finite temperature in order to get $k(\leq N)$-quark baryon. This point should be verified by the experiments at the finite temperature.
6 Summary and Discussion

The baryon is studied here from a holographic approach. It has a complicated structure since it is constructed by the $N$-quarks and their vertex, which would not be a point but an extended object. From the holographic viewpoint, the baryon vertex is identified with the D5 brane and its configuration is obtained by solving the equation of motion given by D5 brane action which includes $N$ fundamental string flux. At zero temperature, this vertex has two typical structures. When we observe it in our real space-time, its shape is a point like or an one dimensionally extended object. From 10D supergravity side, both configurations have further complicated structure due to the extra six dimensional coordinates.

For the simplicity, here, we study the baryon with the vertex, which appears as a point in the 4D space-time, at finite temperature. In our model, the quarks are not confined in the sense that the effective free quark mass is finite. In this sense, the theory is in the deconfinement phase. Then, the fundamental strings could touch on the horizon, then the baryon configuration with $k(<N)$-quark is possible in this phase. The configuration of the $k$-quark baryon is formed by the D5 brane vertex and $k$ fundamental strings which connect the cusp of the vertex and the boundary. The boundary is here set at the finite radial coordinate of the extra dimension as a cutoff point of the string energy or the position of the flavor quark brane. And the remaining $N-k$ fundamental strings are connecting the vertex and the horizon. Then the $k$-quark baryons with $k \leq N$ in a thermal medium are studied here.

The structure of the vertex becomes a little rich at finite temperature since two extra new solutions of vertex configuration are found. They are not seen at zero temperature. However, we find that these new configurations can not be observed since they are unstable against to the decay into the free quarks. This instability is studied through the calculation of the baryon energy for its fixed size $L$. The size is defined by the distance between the vertex and a quark. We assumed that the quarks in the baryon are distributed maximally symmetric, then we impose the no force condition given at the connecting point of the fundamental strings and the vertex. The energy of $k$-quark baryon is obtained in this way and it is compared with the one of the free $k$-quarks.

From these analyses, the baryon energy is smaller than the one of the free quarks at small $L$. And we find a point $L = L^*$, where the baryon energy exceeds the one of the free quarks. Then the baryon with large size $(L^* <)L$ decays into the free quarks, we call this point as melting point. The size depends on the vertex configuration through the no force condition, and the configurations newly found at finite temperature give the larger size than $L^*$.

The value of $L^*$ can be also related to the Debye length, which has been introduced originally for the Coulomb potential in the electro-magnetism, and it depends on the temperature. This is estimated here by multiplying the Gauss damping factor to the linear-like potential, which appear as the tail of the zero temperature linear potential.
And we find the expected behavior of the Debye mass gap, which is proportional to the temperature. This temperature dependence is seen for all the state of $k$-quark baryon, but small $k$-quark baryons are restricted to low temperature.

Finally, we find that the baryon stability at finite temperature is supported by the gauge condensate $q$. The thermal medium destabilizes the baryon by the screening effect of the color force, but $q$ support the color force to make a linear potential to retain the baryon at not so high temperature, and we could observe the baryon even at finite temperature.

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