Reflective Scattering and Unitarity

S.M. Troshin and N.E. Tyurin

Institute for High Energy Physics, Protvino, Moscow Region, 142281, Russia

Abstract. Interpretation of unitarity saturation as reflective scattering is discussed. Analogies with optics and Berry phase alongside with the experimental consequences of the proposed interpretation at the LHC energies are considered.

Keywords: Elastic scattering, unitarity saturation, reflective scattering

PACS: 11.80.-m, 13.85.Dz

UNITARITY: ABSORPTIVE VS REFLECTIVE SCATTERING

The essential and persisting problems of QCD are related to confinement and spontaneous chiral symmetry breaking phenomena. It is the field of collective, coherent interactions of quarks and gluons resulting in formation of the asymptotic states — the colorless, experimentally observable particles. Closely related — the total cross-section growth with energy — constitutes one of the important unanswered question in the theory. Elastic scattering gives significant contribution to the total cross-section, which by means of optical theorem, is related to the forward elastic scattering amplitude. Thus, the process of elastic scattering, where all hadron constituents interact coherently, can serve as a tool in the confinement studies. Relative strength of elastic and inelastic processes is regulated by unitarity. It is important to note here that unitarity is formulated for the asymptotic colorless hadron on-mass shell states and is not directly connected to the fundamental fields of QCD — quarks and gluons. The same is valid for the analyticity which is relevant for the scattering amplitudes of the observable particles only. It is not clear what these fundamental principles imply for the confined objects.

Unitarity or conservation of probability, which can be written in terms of the scattering matrix as following

\[ SS^+ = 1, \]

implies an existence at high energies of the two scattering modes - shadow one (absorptive scattering) and antishadow (reflective scattering). Indeed, writing unitarity relation for the partial wave amplitudes \( f_l(s) \):

\[ \text{Im} f_l(s) = |f_l(s)|^2 + \eta_l(s), \]

where elastic \( S \)-matrix is related to the amplitude as

\[ S_l(s) = 1 + 2i f_l(s) \]

and \( \eta_l(s) \) stands for the contribution of the intermediate inelastic channels to the elastic scattering with the orbital angular momentum \( l \), we can easily observe that the relation...
turns out to be a quadratic equation in the case of the pure imaginary scattering amplitude and the elastic amplitude appears to be not a singe-valued function of $\eta_l$. But, only one of the two solutions of the equation corresponding to the relation (2) is considered almost everywhere:

$$f_l(s) = \frac{i}{2} (1 - \sqrt{1 - 4\eta_l(s)}), \quad \text{i.e.} \quad |f_l| \leq 1/2,$$

while another one:

$$f_l(s) = \frac{i}{2} (1 + \sqrt{1 - 4\eta_l(s)}), \quad \text{i.e.} \quad 1/2 \leq |f_l| \leq 1$$

is neglected. However, there are no reasons for its neglecting at small and moderate values of orbital angular momentum $l$ [1]. The two above mentioned solutions can be easily reconciled in the uniform way in the $U$-matrix unitarization approach [2], which represents elastic $2 \rightarrow 2$ scattering matrix $S(s,b)$ in the impact parameter picture in the following form

$$S(s,b) = \frac{1 + iU(s,b)}{1 - iU(s,b)}.$$

$U(s,b)$ is the generalized reaction matrix, which is considered to be an input dynamical quantity. The transform (6) is one-to-one and easily invertible. Inelastic overlap function $\eta(s,b)$ can also be expressed through the function $U(s,b)$ by the relation

$$\eta(s,b) = \frac{\text{Im}U(s,b)}{|1 - iU(s,b)|^2},$$

and the only condition to obey unitarity is $\text{Im}U(s,b) \geq 0$.

In what follows we consider for simplicity the case of pure imaginary $U$-matrix and make the replacement $U \rightarrow iU$. The value of energy corresponding to the full absorption at central collisions $S(s,b)|_{b=0} = 0$ will be denoted as $s_0$ and it is determined by the equation $U(s,b)|_{b=0} = 1$. In the energy region $s \leq s_0$ the scattering in the whole range of impact parameter variation has a shadow nature and correspond to solution (4), the $S$ matrix varies in the range $0 \leq S(s,b) < 1$. But when the energy is higher than the threshold value $s_0$, the scattering picture at small values of impact parameter $b \leq R(s)$ corresponds to the solution Eq. (5), where $R(s)$ is determined by solution of equation $S(s,b = R) = 0$. The $S$-matrix variation region is then $-1 < S(s,b) \leq 0$ at $s \geq s_0$ and $b \leq R(s)$.

**ANALOGIES WITH OPTICS AND BERRY PHASE**

There is a close analogy here with the light reflection off a dense medium, when the phase of the reflected light is changed by 180°. Therefore, using the optical concepts [3], the above behavior of $S(s,b)$ should be interpreted as an appearance of a reflecting ability of scatterer due to increase of its density beyond some critical value, corresponding to refraction index noticeably greater than unity. In another words, the scatterer has now
not only absorption ability (due to presence of inelastic channels), but it starts to be reflective at very high energies and its central part \((b = 0)\) approaches to the completely reflecting limit \((S = -1)\) at \(s \to \infty\). In order to combine shadowing at large values of \(b\) with antishadowing in central collisions the real part of the phase shift should have the dependence

\[
\delta_R(s, b) = \frac{\pi}{2} \theta(R(s) - b).
\]

Such a behavior of \(\delta_R(s, b)\) just takes place in the \(U\)-matrix form of unitarization. Indeed, the phase shift \(\delta(s, b)\) can be expressed in terms of the function \(U(s, b)\) as following

\[
\delta(s, b) = \frac{1}{2i} \ln \frac{1 - U(s, b)}{1 + U(s, b)}.
\]

(8)

It is clear that in the region \(s > s_0\) the function \(\delta(s, b)\) has a real part \(\pi/2\) in the region \(0 < b < R(s)\), while \(\delta_I(s, b)\) goes to infinity at \(b = R(s)\).

It also leads to another interesting similarity, namely it allows one to consider \(\delta_R\) as an analog of the geometric Berry phase in quantum mechanics which appears as a result of a cyclic time evolution of the Hamiltonian parameters \([4]\). This interesting phenomenon can be observed in many physical systems and is, in fact, a feature of a system that depends only on the path it evolves along of. In the case of pure imaginary elastic scattering amplitude the contribution of the inelastic channels \(\eta\) can be considered as a parameter which determines due to unitarity (but not in a unique way) the elastic \(S\)-matrix. We can vary variable \(s\) (and/or \(b\)) in a way that the parameter \(\eta\) (which has a peripheral \(b\)-dependence) evolves cyclically from \(\eta_i < 1/4\) to \(\eta_{\text{max}} = 1/4\) and again to the value \(\eta_f\), where \(\eta_f = \eta_i\) (loop variation). As a result the non-zero phase appears \(\delta_R = \pi/2\) at \(b < R(s)\) and this phase is independent of the details of the energy evolution (Fig. 1). Since the Berry phase has a geometrical origin it is tempting to relate the step energy dependence of \(\delta_R(s)\) with existence of an extra dimension at TeV energy scale.

Thus, we can summarize that the physical scattering picture beyond the black disk limit evolves with energy by simultaneous increase of the reflective ability (i.e. \(|S(s, b)|\) increases with energy and \(\delta_R = \pi/2\)), and decrease of the absorptive ability \(1 - |S(s, b)|^2\) at the impact parameters \(b < R(s)\).

Having in mind a quark-gluon structure of hadrons it would be interesting to find a particular microscopic mechanisms related to the collective quark-gluon dynamics in head-on collisions which can be envisaged as an origin of the reflection phenomenon. One can speculate at this point and relate the appearance of the reflective scattering to the Color-Glass Condensate in QCD (cf. e.g. \([5]\) and references therein) merely ascribing the reflective ability to Glazma.

The particular model for \(U\)-matrix \([6]\) allows to give a rather good description of the observables in elastic hadron scattering and provide relevant predictions for them at the LHC energies. It is interesting that power-like dependence of the differential cross-sections at large angles appeared to be related to the total cross-section growth. Transition to the reflective scattering regime is also responsible for the existence of the knee in the energy spectrum of cosmic rays. There is an interesting possibility that the reflective scattering mode could be discovered at the LHC by measuring \(\sigma_{el}/\sigma_{tot}\) ratio
which would be greater than the black disk value $1/2$ [7]. However, the asymptotical regime is expected in the model at $\sqrt{s} > 100$ $TeV$ only. Increasing weight of the reflective scattering at the LHC energies would lead to the less prominent dip-bump structure in the $d\sigma/dt$ in $pp$ scattering at large values of $-t$ and hadronic glory effect would be observed. The concept of reflective scattering itself is rather general, and results from the unitarity saturation for $S$-matrix which is related to the necessity to reproduce a total cross section growth at $s \to \infty$.

ACKNOWLEDGMENTS

Authors are grateful to N. Buttimore, L. Jenkovszky, U. Maor, A. Martin, V. Petrov and O. Selyugin for interesting discussions. One of the authors (S.T.) is also grateful to the Organizers of Diffraction 2008, in particular, to A. Bravar, R. Fiore, A. Papa and J. Soffer for support and warm hospitality at La Londe-les-Maures.

REFERENCES

1. S. M. Troshin, and N. E. Tyurin, *Int. J. Mod. Phys. A* 22, 4437–4449 (2007).
2. A. A. Logunov, V. I. Savrin, N. E. Tyurin, and O. A. Khrustalev, *Teor. Mat. Fiz.* 6, 157–165 (1971).
3. K. Gottfried, *CERN–72–20*, 27p. (1972).
4. M. V. Berry, *Proc. R. Soc. London, Ser. A* 392, 45–57 (1984).
5. L. McLerran, *Acta Phys. Polon. B* 37, 3237–3252 (2006).
6. S. M. Troshin, and N. E. Tyurin, *Phys. Rev. D* 49, 4427–4433 (1994).
7. S. M. Troshin, and N. E. Tyurin, *Phys. Lett. B* 316, 175–177 (1993).