Real Tunneling Solutions
and the Hartle-Hawking Wave Function

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Abstract

A real tunneling solution is an instanton for the Hartle-Hawking path integral with vanishing extrinsic curvature (vanishing “momentum”) at the boundary. Since the final momentum is fixed, its conjugate cannot be specified freely; consequently, such an instanton will contribute to the wave function at only one or a few isolated spatial geometries. I show that these geometries are the extrema of the Hartle-Hawking wave function in the semiclassical approximation, and provide some evidence that with a suitable choice of time parameter, these extrema are the maxima of the wave function at a fixed time.

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1. Introduction

In the Hartle-Hawking approach to quantum cosmology [1,2], the wave function of the universe is obtained from a Euclidean path integral over metrics and matter fields on a manifold $M$ with a single boundary component $\partial M$. Such a path integral depends on the induced metric $h_{ab}$ and the matter configuration $\varphi|_{\partial M}$ on $\partial M$, thus determining a functional

$$
\Psi[h, \varphi|_{\partial M}] = \sum_M \int [dg][d\varphi] \exp \{-I_E[g, \varphi]\},
$$

(1.1)

where the summation represents a sum over topologies. The functional $\Psi$ is the Hartle-Hawking wave function, which we are instructed to interpret as an amplitude for finding a universe characterized by $h$ and $\varphi|_{\partial M}$. The Hartle-Hawking approach neatly finesse the question of initial conditions by omitting any initial boundary, and it postpones the question of the nature of time in quantum gravity: information about time is hidden in the boundary geometry $h$, but the path integral can be formulated without making a choice of time explicit.

In references [3] and [4], it was shown that the sum over topologies in (1.1) can have a drastic effect on the behavior of the wave function. For most of this paper, this complication will be ignored; we shall focus on the contribution of a single fixed topology $M$ to the wave function of the universe. In particular, we shall look for the maxima of $\Psi$, the most probable spatial geometries and matter configurations of the universe.

In the saddle point approximation, the wave function (1.1) takes the standard form

$$
\Psi_M[h, \varphi|_{\partial M}] \sim \sum_{\{\bar{g}, \bar{\varphi}\}} D_M[\bar{g}, \bar{\varphi}] \exp \{-I_E[\bar{g}, \bar{\varphi}]\},
$$

(1.2)

where the sum is now over classical solutions $\{\bar{g}, \bar{\varphi}\}$ with the prescribed boundary values, and the prefactor $D_M$ is a combination of determinants coming from gauge-fixing and from small fluctuations around the extrema. In this approximation, it may not always be possible to specify $h$ and $\varphi|_{\partial M}$ arbitrarily — for a given topology $M$, some boundary values may not lead to classical solutions — but we certainly have a great deal of flexibility. For instance, given one classical solution, we can cut off a small neighborhood of the boundary to obtain a new solution with the same topology but different boundary data. A given manifold $M$ will therefore contribute to the wave function over a wide range of values of $h$ and $\varphi|_{\partial M}$.

A particularly interesting class of saddle points consists of the so-called real tunneling geometries [5], solutions of the field equations for which the metric has Riemannian signature in $M$ and vanishing extrinsic curvature at the boundary $\partial M$. Classically, these are the solutions for which the metric can be smoothly joined to a metric with Lorentzian signature to the future of $\partial M$. Real tunneling geometries clearly give the right classical picture of the Hartle-Hawking “no boundary” boundary condition, describing universes characterized by a Lorentzian metric “now” but no initial boundary. Moreover, by analogy with more tractable problems such as pair production in a constant electric field [6], we expect these solutions to play an important role in the overall behavior of the path integral.

* “Smoothly” here means “with finite action.”
Note, however, that a real tunneling solution will typically contribute to $\Psi[h, \varphi|\partial M]$ at only a few isolated values of $h_{ab}$. In the Hamiltonian formulation of general relativity, the extrinsic curvature is canonically conjugate to $h_{ab}$, and we should not expect to be able to freely specify $h_{ab}$ while simultaneously maintaining $K_{ab} = 0$. This conclusion can be checked explicitly in three dimensions with $\varphi = 0$ and $\Lambda < 0$: a real tunneling solution is then a hyperbolic three-manifold with a totally geodesic boundary, and for any given topology $M$, the boundary metric $h_{ab}$ either does not exist (if there is no classical extremum) or is unique.

We shall see below that the boundary values of real tunneling solutions are in fact the extrema of the wave function $\Psi$. In general, these extrema are saddle points rather than maxima. But we shall find some evidence that with a suitable choice of time parameter, the real tunneling solutions determine the genuine maxima of the Hartle-Hawking wave function at a fixed time.

2. Scalar Fields

Before tackling the more difficult problem of gravity, it is instructive to look at the behavior of the “Hartle-Hawking wave function” for a free scalar field. The Euclidean action for a field $\varphi$ in $n$ spacetime dimensions is

$$I_E[\varphi] = \frac{1}{2} \int_M d^n x \sqrt{g} \left( g^{ab} \partial_a \varphi \partial_b \varphi + m^2 \varphi^2 \right), \quad (2.1)$$

where for the moment the metric $g$ is fixed. The first variation of the action is

$$\delta I_E[\varphi] = - \int_M d^n x \sqrt{g} (\delta \varphi) (\Delta_g - m^2) \varphi + \int_{\partial M} d^{n-1} x \sqrt{h} (\delta \varphi) \nabla_n \varphi, \quad (2.2)$$

where $\nabla_n$ is the normal derivative at $\partial M$ and $\Delta_g = \nabla_a \nabla^a$ is the Laplacian. Solutions of the classical equations of motion are thus genuine extrema as long as $\varphi$ is fixed at the boundary.

For a free scalar field, the path integral (1.1) is trivial; the saddle point approximation (1.2) is exact, with $D_M = \det(\Delta_g - m^2)^{-1/2}$. In this context, equation (2.2) now has a new interpretation: it tells us that if we vary the boundary data $\varphi|\partial M$, the classical action $I_E[\bar{\varphi}]$ is extremal when $\nabla_n \bar{\varphi}$ vanishes. This is the scalar analog of the condition for a real tunneling solution — a classical solution $\bar{\varphi}$ has a finite-action extension across $\partial M$ to a Lorentzian spacetime precisely when $\nabla_n \bar{\varphi} = 0$. We thus see from (1.2) that the scalar Hartle-Hawking wave function has its extrema at precisely the boundary values of the “scalar real tunneling solutions.”

We can now ask whether these extrema are maxima. If we restrict our attention to the space of classical solutions, the second variation of the Euclidean action, evaluated at the point $\nabla_n \bar{\varphi} = 0$, is

$$\delta^2 I_E[\varphi] = \int_{\partial M} d^{n-1} x \sqrt{h} (\delta \varphi) \nabla_n (\delta \varphi) = \int_M d^n x \sqrt{g} \left( g^{ab} \partial_a (\delta \varphi) \partial_b (\delta \varphi) + (\delta \varphi) \Delta_g (\delta \varphi) \right). \quad (2.3)$$

But for variations in the space of classical solutions,

$$(\Delta_g - m^2)(\delta \varphi) = 0, \quad (2.4)$$
so
\[
\delta^2 I_E[\varphi] = \int_M d^n x \sqrt{g} \left( g^{ab} \partial_a (\delta \varphi) \partial_b (\delta \varphi) + m^2 (\delta \varphi)^2 \right) > 0. \tag{2.5}
\]
Scalar real tunneling solutions are therefore minima of the Euclidean action, and hence maxima of the wave function.

The scalar Hartle-Hawking wave function is thus peaked at the boundary value \( \varphi|_{\partial M} \) for which the extension \( \tilde{\varphi} \) to \( M \) has a vanishing normal derivative. The map from \( \varphi|_{\partial M} \) to \( \nabla_n \tilde{\varphi} \) is known as the Poisson map \([7,8]\), and we can summarize our results by stating that the maxima of the Hartle-Hawking wave function occur at the zeros of the Poisson map. For the more complicated case of gravity, the analog to \( \varphi|_{\partial M} \) is the boundary metric \( h_{ab} \), while the normal derivative \( \nabla_n \tilde{\varphi} \) corresponds at least roughly to the extrinsic curvature \( K_{ab} \), and the Poisson map takes the metric boundary data to the corresponding extrinsic curvature. Let us now ask whether our results for the scalar field can be extended to this situation.

### 3. Gravity

For simplicity, we shall concentrate on the case of pure gravity, with a nonvanishing cosmological constant acting as a stand-in for matter fields. The Euclidean action \( I_E \) is
\[
I_E[g] = -\frac{1}{16\pi G} \int_M d^n x \sqrt{g} (R[g] - 2\Lambda) - \frac{1}{8\pi G} \int_{\partial M} d^{n-1} x \sqrt{h} K, \tag{3.1}
\]
where \( R[g] \) is the scalar curvature, \( \Lambda \) is the cosmological constant, and \( K \) is the trace of the intrinsic curvature of \( \partial M \). Let \( n^a \) be a unit vector field in a neighborhood of \( \partial M \) whose restriction to \( \partial M \) is the outward unit normal; for simplicity, we can choose \( n^a \) to satisfy
\[
n^b \nabla_b n^a = 0 \tag{3.2}
\]
near \( \partial M \). By a slight abuse of notation, let
\[
h_{ab} = g_{ab} - n_a n_b. \tag{3.3}
\]
(The restriction of \( h_{ab} \) to \( \partial M \) is the induced spatial metric on the boundary.) An easy extension of Wald’s calculation in \([9]\) then gives
\[
-16\pi G \delta I_E[g] = \int_M d^n x \sqrt{g} (G_{ab} + \Lambda g_{ab}) \delta g^{ab} + \int_{\partial M} d^{n-1} x \sqrt{h} \left( (K_{ab} - h_{ab} K) \delta g^{ab} + h^{ab} \nabla_a (n^c \delta g_{bc} + 2g_{bc} \delta n^c) \right). \tag{3.4}
\]
Now consider the vector \( v_b = n^c \delta g_{bc} + 2g_{bc} \delta n^c \) in a neighborhood of \( \partial M \). Using the fact that \( n^a \) is a unit vector, one may easily check that \( n^a v_a = 0 \), so \( v_a \) is tangential on \( \partial M \). Thus
\[
h^{ab} \nabla_a v_b = D_a v^a, \tag{3.5}
\]
where \( D_a \) denotes the covariant derivative on \( \partial M \) \([9]\). The last term in \((3.4)\) is therefore the integral of a total derivative on \( \partial M \), and hence vanishes, giving
\[
-16\pi G \delta I_E[g] = \int_M d^n x \sqrt{g} (G_{ab} + \Lambda g_{ab}) \delta g^{ab} + \int_{\partial M} d^{n-1} x \sqrt{h} (K_{ab} - h_{ab} K) \delta g^{ab}, \tag{3.6}
\]
3
the gravitational version of equation (2.2).

As in the scalar case, this relation tells us that solutions of the classical equations of motion are genuine extrema as long as the metric is fixed on \( \partial M \). But as in the scalar case, it also tells us more: if we vary the boundary data, the extrema of the classical action are precisely the real tunneling solutions, for which \( K_{ab} = 0 \) on \( \partial M \). For the scalar field, the saddle point approximation was exact, and these extrema were also the extrema of the Hartle-Hawking wave function. For gravity, this is no longer true — the prefactor \( D_M \) in (1.2) can depend nontrivially on the boundary data, and there will be higher loop corrections — but as a first approximation, the real tunneling solutions will again be the extrema of \( \Psi[h] \), with corrections suppressed by powers of Planck’s constant.

To determine whether these extrema are maxima, we must again compute the second variation of the Euclidean action at the point \( K_{ab} = 0 \):

\[
16\pi G \delta^2 I_E[g] = \int_{\partial M} d^{n-1}x \sqrt{h} \gamma^{ab} \delta(K_{ab} - h_{ab}K),
\]

where the variation of the metric has been denoted by \( \delta g_{ab} = \gamma_{ab} \) (note that \( \delta g^{ab} = -\gamma^{ab} \)).

A set of rather routine calculations is described in the Appendix; the end result is that with the gauge choice \( (n^a \gamma_{ab} h^b) |_{\partial M} = 0 \),

\[
32\pi G \delta^2 I_E[g] = \int_{\partial M} d^{n-1}x \sqrt{h} \left[ \gamma^{ab} \nabla_n \gamma_{ab} - \frac{1}{2} \gamma \nabla_n \gamma + n^a \beta_a (\gamma - 2\gamma_{ab} h^a h^b) \right],
\]

where \( \gamma = g^{ab} \gamma_{ab} \) and \( \beta^a = \nabla_b (\gamma^{ab} - \frac{1}{2} g^{ab} \gamma) \). This relation is of the same general form as (2.3), but in contrast to the scalar case, it is not manifestly positive. In particular, let us assume that we can consistently choose harmonic gauge, \( \beta_a = 0 \) (see the Appendix for a brief discussion of this choice). Then using Lichnerowicz’s results for the variation of the Ricci tensor [10],

\[
\delta R_{ab} = -\frac{1}{2} \Delta g \gamma_{ab} + \frac{1}{2} (\gamma^c R_{bc} + \gamma^c R_{ac}) - \gamma^{cd} R_{acbd} = \frac{2\Lambda}{n-2} \gamma_{ab},
\]

we obtain

\[
32\pi G \delta^2 I_E[g] = \int_M d^n x \sqrt{g} \left[ \nabla_c \gamma_{ab} \nabla^c \gamma^{ab} - \frac{1}{2} \nabla_c \gamma \nabla^c \gamma \right.

\left. - 2\gamma^{ab} \gamma^{cd} C_{acbd} + \frac{4\Lambda}{(n-1)(n-2)} \left( \gamma_{ab} \gamma^{ab} + \frac{n-3}{2} \gamma^2 \right) \right],
\]

which has no clear positivity properties.

Indeed, it is easy to see that the boundary value of a real tunneling solution is generically a saddle point of the Hartle-Hawking wave function, and not a maximum. For an empty space solution of the gravitational field equations with \( \Lambda \neq 0 \), the Euclidean action \( I_E \) is proportional to the volume of \( M \). But we can always decrease this volume by cutting off a small neighborhood of the boundary \( \partial M \), or increase it by extending \( M \) slightly past \( \partial M \).

Intuitively, though, such a movement of the boundary is not the kind of deformation we should be concerned about. It is well known that the spatial geometry described by the
metric \( h_{ab} \) implicitly contains information about time \([1]\), and a movement of the entire boundary \( \partial M \) corresponds physically to a time translation. It is not so surprising that the wave function has no maxima if such translations are allowed; the more interesting question is whether \( \Psi[h] \) has any maxima \textit{at a fixed time}.

To answer this question, we must extract information about time from the spatial metric \( h_{ab} \) and the extrinsic curvature \( K_{ab} \) at the boundary \( \partial M \). This is not easy to do, since there is no unique way to parametrize time, but the structure of the real tunneling geometries suggests one natural approach: we can use York’s “extrinsic time” \( K \) as a time parameter \([12]\).

If we restrict ourselves to “fixed time” variations of the metric, those for which \( K \) remains zero, we can eliminate at least some of the negative terms from \((3.10)\). From equations \((A.9)\) and \((A.12)\) of the Appendix, we see that in harmonic gauge, \( \delta K = 0 \) implies that
\[
\nabla_n \gamma = 0. \tag{3.11}
\]
On the other hand, equation \((3.9)\) tells us that
\[
\Delta_g \gamma + \frac{4\Lambda}{(n-2)} \gamma = 0. \tag{3.12}
\]
For a metric \( g_{ab} \) with Riemannian signature, the Laplacian \( \Delta_g \) with boundary conditions \((3.11)\) has no positive eigenvalues. Hence for \( \Lambda < 0 \), \( \gamma \) must vanish, eliminating the term \( \nabla_c \gamma \nabla^c \gamma \) from \((3.10)\). For \( \Lambda > 0 \), equation \((3.12)\) may sometimes have nontrivial solutions, but the spectrum of the Laplacian is discrete, so there will be at most finitely many of them; for most variations of the metric, \( \gamma \) will again vanish.

The term in \((3.10)\) involving the Weyl tensor is harder to control. To temporarily avoid this problem, it is interesting to look at the simple model of quantum gravity in three spacetime dimensions, where \( C_{abcd} \) is identically zero. Here, at least, it may be possible to prove that \( \delta^2 I_E \) is strictly positive when \( \delta K = 0 \).

If \( \Lambda < 0 \), our task is somewhat harder, since the last term in \((3.10)\) is no longer positive. In this case, the question can be reformulated in terms of hyperbolic geometry: the Euclidean
action is proportional to the volume of $M$, and the problem is to show that among hyperbolic metrics on $M$ with $K = 0$ on $\partial M$, the metric for which $\partial M$ is totally geodesic has the smallest volume. One possible approach is the following. Hyperbolic structures on $M$ — metrics with constant curvature $-1$ — are parametrized by a finite-dimensional moduli space $\mathcal{M}$. It is plausible that the volume of $M$ becomes large outside a compact region of $\mathcal{M}$, where the metric on $\partial M$ starts to degenerate. If this is true, and if the volume is a reasonably well-behaved function on $\mathcal{M}$, then it must have a minimum. But among hyperbolic metrics on a fixed manifold $M$, the real tunneling solution is unique, and is the only extremum of the volume; it must therefore be the minimum. It should be stressed that this argument is at best a strategy for a proof; work on this question is in progress.

In four dimensions the situation is more complex, since the Weyl tensor in (3.10) need not vanish. The inclusion of matter is likely to provide an added complication. For the moment, it seems difficult to make any definitive statements about the maxima of the Hartle-Hawking wave function for realistic cosmologies. But the partial results from three dimensions suggest a conjecture: that the maxima of the Hartle-Hawking wave function at “time” $K = 0$ are precisely the boundary values of the real tunneling solutions.

4. Conclusion

The Hartle-Hawking program offers an intriguing approach to quantum cosmology, but it has not been easy to extract predictions from the formalism. Up to now, most work has involved either simple minisuperspace models (for example, [15]) or classical or semiclassical approximations [3,4,16,17]. It is therefore of some importance to understand how good these approximations are.

If the boundary values of real tunneling solutions are the maxima of the Hartle-Hawking wave function, then classical and semiclassical approximations may be reasonably reliable. In particular, the classical signature-changing solutions of the Einstein equations are precisely the real tunneling geometries, and the interesting predictions based on these solutions [17] presuppose that they dominate the quantum wave function. Similarly, investigations of the sum over topologies [3,4,16] have been based on the properties of real tunneling solutions, and may be invalid if the peaks of the wave function lie elsewhere. On the other hand, if the real tunneling contributions are not the maxima of the Hartle-Hawking wave function, this would have interesting implications as well: the wave function would then have no maxima, and would presumably be dominated by behavior near some boundary of the space of spatial geometries.

This paper has not answered the question of whether the Hartle-Hawking wave function has maxima. We have seen, however, that if any such maxima exist, they must come from the contributions of real tunneling geometries. In the simple model of quantum gravity in three spacetime dimensions, we have found some reasonably strong evidence that these extrema are indeed maxima, but in realistic four-dimensional gravity the question remains open.
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Appendix

In this appendix, we fill in some of the details in the derivation of equation (3.8) from (3.7). Note first that the tangent directions to \( \partial M \) are defined without reference to the metric on \( M \), so the direction of normal \( n_a \) is also independent of the metric. We can therefore write

\[
\delta n_a = \alpha n_a = \frac{1}{2}(\gamma_{bc} n^b n^c) n_a \quad \text{on} \quad \partial M.
\]  

(A.1)

To proceed further, it is useful to partially fix the gauge. An infinitesimal diffeomorphism corresponds to a transformation

\[
\gamma_{ab} \rightarrow \gamma_{ab} + \nabla_a \xi_b + \nabla_b \xi_a, \quad n^a \xi_a = 0 \quad \text{on} \quad \partial M,
\]

(A.2)

and metrics that differ by such transformations are physically equivalent. We can use this freedom to set

\[
n^a \gamma_{ab} h^b_c = 0 \quad \text{on} \quad \partial M,
\]

(A.3)

implying that

\[
\gamma_{ab} n^b = 2 \alpha n_a \quad \text{on} \quad \partial M.
\]

(A.4)

The remaining diffeomorphisms are now restricted to those that preserve (A.3), i.e.,

\[
n^a \xi_a = 0, \quad \nabla_n (n^a h^c_b) = 0 \quad \text{on} \quad \partial M.
\]

(A.5)

(We have used the fact that \( K_{ab} = 0 \), which together with (3.2) implies that \( \nabla_a n_b = 0 \) on \( \partial M \).)

We could fix the remaining freedom by using the fact that

\[
\beta_a = \nabla^b (\gamma_{ab} - \frac{1}{2} g_{ab} \gamma) \rightarrow \beta_a + (\Delta g \xi_a + R^b_a \xi_b).
\]

(A.6)

Given the boundary conditions (A.3), \( D^b_a = \Delta g \delta^b_a + R^a_b \) is a self-adjoint elliptic operator. With appropriate assumptions of smoothness, this means that the equation \( D^b_a \xi_b = -\beta_a \) can be solved, thus transforming \( \beta_a \) to zero, unless \( \beta_a \) is itself a zero-mode of \( D^b_a \). Such zero-modes are rare: in the case of empty space with a negative cosmological constant, for instance, the eigenvalues of \( D \) are strictly negative. To postpone dealing with the exceptional cases when \( D \) has zero-modes, however, we shall not yet impose any condition on \( \beta_a \).

We can now evaluate the variation of the extrinsic curvature \( K_{ab} \):

\[
\delta K_{ab} = h_a^c \nabla_c \delta n_b - \frac{1}{2} h_a^c n^d (\nabla_c \gamma_{bd} + \nabla_b \gamma_{cd} - \nabla_d \gamma_{bc})
\]

\[
= -h_a^c \nabla_b (\alpha n_c) + \frac{1}{2} h_a^c \nabla_n \gamma_{bc} = \frac{1}{2} h_a^c \nabla_n \gamma_{bc},
\]

(A.7)
where the fact that $\nabla_a n_b = 0$ on $\partial M$ has been used repeatedly. Thus

\[
\gamma^{ab} \delta K_{ab} = \frac{1}{2} (\gamma^{ab} - n^a n^b \gamma^{c}) \nabla_n \gamma_{ab} = \frac{1}{2} \gamma^{ab} \nabla_n \gamma_{ab} - 2\alpha \nabla_n \alpha,
\]  
(A.8)

and

\[
\delta K = g^{ab} \delta K_{ab} = \frac{1}{2} \nabla_n \gamma - \nabla_n \alpha.
\]  
(A.9)

Note also that

\[
h_{ab} \gamma^{ab} = \gamma - n^a n_b \gamma^{ab} = \gamma - 2\alpha.
\]  
(A.10)

Inserting these results into equation (3.7), we find that

\[
32\pi G \delta^2 I_E = \int_{\partial M} d^{n-1}x \sqrt{h} \left[ \gamma^{ab} \nabla_n \gamma_{ab} - \gamma \nabla_n \gamma + 2\gamma \nabla_n \alpha + 2\alpha \nabla_n \gamma - 8\alpha \nabla_n \alpha \right].
\]  
(A.11)

This equation can be further simplified by noting that

\[
n^a \beta_u = n_a \nabla_b \gamma^{ab} - \frac{1}{2} \nabla_n \gamma = n_a n_b \nabla_n \gamma^{ab} + n_a h_{bc} \nabla_c \gamma^{ab} - \frac{1}{2} \nabla_n \gamma
\]

\[
= 2\nabla_n \alpha + h_{bc} \nabla_c (2\alpha n^b) - \frac{1}{2} \nabla_n \gamma = 2\nabla_n \alpha - \frac{1}{2} \nabla_n \gamma,
\]  
(A.12)

leading directly to equation (3.8).

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