We propose measurement of transverse single spin asymmetry (SSA) in charmonium production as a probe of gluon Sivers function. We estimate SSA in low virtuality electroproduction of $J/\psi$ using color evaporation model of charmonium production and existing models of the gluon Sivers function and find sizable asymmetry at JLab, HERMES, COMPASS and eRHIC energies.
1. INTRODUCTION

Transverse Single Spin Asymmetry (SSA) arises in the scattering of a transversely polarized proton off an unpolarized hadron or nucleon if the scattering cross section depends on the direction of polarization. The Single Spin Asymmetry for inclusive process \( A^+ + B \rightarrow C + X \) is defined as

\[
A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \tag{1.1}
\]

where \( d\sigma^{\uparrow(\downarrow)} \) denotes the cross section for scattering of a transversely polarized hadron A off an unpolarized hadron B, with A upwards (downwards) transversely polarized w.r.t. the production plane. SSA’s significantly different from zero have been observed over last 35 years starting with pion production in scattering of polarized protons off unpolarised proton target \([1]\). Large SSA’s have been measured in pion production at Fermilab \([2]\) as well as at BNL-RHIC in \( pp^\uparrow \) collisions \([3]\). SSA’s have also been observed by the HERMES \([4]\) and COMPASS \([5]\) collaborations, in polarized semi-inclusive deep inelastic scattering (SIDIS). The magnitude of the observed asymmetries has been found to be larger than what is predicted by perturbative quantum chromodynamics (pQCD) \([6]\).

Theoretically there are two major approaches to explain the SSA’s. One is the twist three approach and other is the transverse momentum dependent (TMD) approach which we have used in the present work. The TMD approach is based on a pQCD factorization scheme in which spin and intrinsic transverse momentum effects are included in parton distribution functions (pdf’s) and fragmentation functions (ff’s). One of the difficulties in getting information about the spin and transverse momentum dependent pdf’s and ff’s is that very often two or more of these functions contribute to the same physical observable making it difficult to estimate each single one separately.

The study of spin asymmetries requires extension of TMD factorization scheme to polarized case. Sivers in early 90’s proposed that there exists a correlation between the azimuthal distribution of an unpolarized parton and spin of its parent hadron \([7]\). Number density of partons inside proton with transverse polarization \( S \), three momentum \( p \) and intrinsic transverse momentum \( k_\perp \) of partons is expressed in terms of Sivers function \( \Delta^N f_{a/p^\uparrow}(x,k_\perp) \)

\[
\hat{f}_{a/p^\uparrow}(x,k_\perp,S) = \hat{f}_{a/p}(x,k_\perp) + \frac{1}{2} \Delta^N f_{a/p^\uparrow}(x,k_\perp)S \cdot (\hat{p} \times \hat{k}_\perp) \tag{1.2}
\]

\( S \cdot (\hat{p} \times \hat{k}_\perp) \) gives the correlation between the spin of the proton and intrinsic transverse momentum of the unpolarised quarks and gluons. There have been studies on the quark Sivers function in SIDIS and the gluon Sivers function in the process \( p^\uparrow p \rightarrow DX \) \([8, 9]\). In this work, we propose charmonium electrodroduction as another probe of the gluon Sivers function.

2. FORMALISM FOR ASYMMETRY IN \( J/\psi \) PRODUCTION

We have estimated SSA in photoproduction of charmonium in the process \( e + p^\uparrow \rightarrow e + J/\psi + X \). At leading order (LO), there is contribution only from a subprocess \( \gamma g \rightarrow c\bar{c} \). In addition, since we are using color evaporation model (CEM) for charmonium production, only one pdf is involved. Thus the process under consideration can be used as a clean probe of the gluon Sivers function.
Also since the charmonium production mechanism can have implications for this SSA, its study can help throw some light on the production mechanism of charmonium as well.

Charmonium production process can be understood in terms of two distinct steps: production of a $c\bar{c}$ pair (a short distance process) and a subsequent binding of this pair in to charmonium (a long distance process). Various methods to describe this non-perturbative evolution of the $c\bar{c}$ pair into charmonium lead to different models of charmonium production. As a first step in our investigations of SSA in charmonium production, we have used the Color Evaporation model (CEM) of charmonium production. According to CEM, the cross section for charmonium production is proportional to the rate of production of $c\bar{c}$ pair integrated over the mass range $2m_c$ to $2m_D$ [10, 11, 12]

$$\sigma = \frac{1}{9} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}} \quad (2.1)$$

where $m_c$ is the charm quark mass and $2m_D$ is the $D\bar{D}$ threshold.

The cross section for the low virtuality electroproduction within CEM is

$$\sigma_{ep\rightarrow e/J/\psi +X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 \int dy dx f_{\gamma/e}(y) f_{g/p}(x) \frac{d\hat{\sigma}_{\gamma\rightarrow c\bar{c}}}{dM_{c\bar{c}}^2} \quad (2.2)$$

where $f_{\gamma/e}(y)$ is the distribution function of the photon in the electron which, in the Weizsacker-Williams approximation [13], is given by

$$f_{\gamma/e}(y,E) = \frac{\alpha}{\pi} \left( \frac{1 + (1-y)^2}{y} \right) \left( \frac{\ln E}{m} - \frac{1}{2} \right) + \frac{y}{2} \left[ \ln \left( \frac{2}{y} \right) - 1 \right] + \frac{(2-y)^2}{2y} \ln \left( \frac{2-2y}{2-y} \right) \right} \). \quad (2.3)$$

To calculate SSA in scattering of electrons off a polarized proton target, we assume generalization of this CEM expression for low virtuality electroproduction of $J/\psi$ by taking into account the transverse momentum dependence of the Weizsacker-Williams (WW) function and the gluon distribution function:

$$\sigma_{e+p^1\rightarrow e+J/\psi +X} = \int_{4m_c^2}^{4m_D^2} dM_{c\bar{c}}^2 \int dx_{g} d^2k_{1\perp} d^2k_{2\perp} \left[ 2^2 d^2k_{1\perp} d^2k_{2\perp} \right] f_{g/p}(x_g, k_{1\perp}) \times f_{\gamma/e}(x_{\gamma}, k_{1\perp}) \frac{d\hat{\sigma}_{\gamma\rightarrow c\bar{c}}}{dM_{c\bar{c}}^2} \quad (2.4)$$

We assume $k_{1\perp}$ dependence of pdf’s and WW function to be factorized in Gaussian form [8]

$$f(x, k_{1\perp}) = f(x) \frac{1}{\pi(k_{1\perp}^2)} e^{-k_{1\perp}^2/\langle k_{1\perp}^2 \rangle} \quad (2.5)$$

with $\langle k_{1\perp}^2 \rangle = 0.25 \; GeV^2$.

The expression for the numerator of the asymmetry is

$$\frac{d^4\sigma^+}{dy d^2q_T} - \frac{d^4\sigma^-}{dy d^2q_T} = \frac{1}{2} \int_{4m_c^2}^{4m_D^2} dM^2 \int [dx_{g} d^2k_{1\perp} d^2k_{2\perp}] \Delta_N f_{g/p}(x_g, k_{1\perp}) \times f_{\gamma/e}(x_{\gamma}, k_{1\perp}) \delta^4(p_{g} + p_{\gamma} - q) \frac{d^4\hat{\sigma}_{0\rightarrow c\bar{c}}(M^2)}{dM_{c\bar{c}}^2} \quad (2.6)$$
where \( q = p_c + p_{\bar{c}}, \Delta f_{g/p}^N(x_g, k_{\perp}) \) is the gluon Sivers function and \( M^2 \) is invariant mass of the \( c\bar{c} \) pair.

The partonic cross section is \([14]\)

\[
\hat{\sigma}_0^{N^c \rightarrow c\bar{c}}(M^2) = \frac{1}{2} e^2 4\pi \alpha s \frac{\alpha_s}{M^2} \left[ (1 + \gamma - \frac{1}{2} \gamma^2) \ln \frac{1 + \sqrt{1 - \gamma}}{1 - \sqrt{1 - \gamma}} - (1 + \gamma) \sqrt{1 - \gamma} \right] \tag{2.7}
\]

where \( \gamma = 4m_c^2/M^2 \).

Sivers asymmetry integrated over the azimuthal angle of \( J/\psi \) with a weight factor \( \sin(\phi_{q_T} - \phi_S) \) is defined as

\[
A_N = \frac{\int d\phi_{q_T} \left[ \int_{4m_c^2}^{M^2} \int [d^2k_{\perp}] \Delta^N f_{g/p}(x_g, k_{\perp}) f_{\gamma/e}(x_{\gamma}, q_{T} - k_{\perp}) \hat{\sigma}_0 \right] \sin(\phi_{q_T} - \phi_S)}{2 \int d\phi_{q_T} \left[ \int_{4m_c^2}^{M^2} \int [d^2k_{\perp}] \Delta^N f_{g/p}(x_g, k_{\perp}) f_{\gamma/e}(x_{\gamma}, q_{T} - k_{\perp}) \hat{\sigma}_0 \right]} \tag{2.8}
\]

where \( \phi_{q_T} \) and \( \phi_S \) are azimuthal angles of \( J/\psi \) and proton spin respectively and \( x_{\gamma, y} = \frac{M}{\sqrt{s}} e^{\pm y} \).

### 3. MODELS FOR SIVERS FUNCTION

We have used the following parameterization for the gluon Sivers function \([8]\)

\[
\Delta^N f_{g/p}(x, k_{\perp}) = 2 \mathcal{N}_g(x) h(k_{\perp}) f_{g/p}(x) e^{-k_{\perp}^2/(\langle k_{\perp}^2 \rangle)} \cos \phi_{k_{\perp}}. \tag{3.1}
\]

There is no information available about the gluon Sivers function from experimental data. The valance and sea quark Sivers distribution functions used are the ones extracted from the HERMES and COMPASS experimental data in SIDIS processes \([15]\).

The \( x \) dependent normalization for \( u \) and \( d \) quarks is given by,

\[
\mathcal{N}_f(x) = N_f x^{\alpha_f}(1 - x)^{\beta_f} \frac{(a_f + b_f)(a_f + b_f)}{a_f^{a_f}b_f^{b_f}} \tag{3.2}
\]

where \( a_f, b_f, N_f \) and \( M_1 \) are best fit parameters obtained by fitting SIDIS, HERMES and COMPASS data \([8]\).

For \( \mathcal{N}_g(x) \), we have used two choices \([16]\)

(a) \( \mathcal{N}_g(x) = (\mathcal{N}_u(x) + \mathcal{N}_d(x))/2 \).

(b) \( \mathcal{N}_g(x) = \mathcal{N}_d(x) \).

For \( h(k_{\perp}) \), we have used following two choices proposed by Anselmino etal \([8, 9]\):

- **Model I**
  \[
  h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_1} e^{-k_{\perp}^2/M_1^2}, \tag{3.3}
  \]

- **Model II**
  \[
  h(k_{\perp}) = \frac{2k_{\perp}M_0}{k_{\perp}^2 + M_0^2}, \tag{3.4}
  \]

where \( M_0 = \sqrt{\langle k_{\perp}^2 \rangle} \) and \( M_1 \) are best fit parameters.
its

Figure 1: (Color online) The single spin asymmetry $A_N^{\sin(\phi_{qT}-\phi_s)}$ for the $e + p^+ \to e + J/\psi + X$ as a function of $y$ (left panel) and $q_T$ (right panel). The plots are for model I with parameterization (a) compared for JLab ($\sqrt{s} = 4.7$ GeV) [solid red line], HERMES ($\sqrt{s} = 7.2$ GeV) [dashed green line], COMPASS ($\sqrt{s} = 17.33$ GeV) [dotted blue line], eRHIC-1 ($\sqrt{s} = 31.6$ GeV) [long dashed pink line] and eRHIC-2 ($\sqrt{s} = 158.1$ GeV) [dot-dashed black line].

4. NUMERICAL ESTIMATES

We have used the following best fit parameters from the recent HERMES and COMPASS data \[17\]

$$
N_u = 0.40, \quad a_u = 0.35, \quad b_u = 2.6, \\
N_d = -0.97, \quad a_d = 0.44, \quad b_d = 0.90, \\
M_2^2 = 0.19 \text{ GeV}^2.
$$

(4.1)

In figure 1 we have shown the comparison of $y$ and $q_T$ distribution of estimated SSA at JLab, HERMES, COMPASS and eRHIC for model I and parameterization (a) of the gluon Sivers function. The estimates are obtained using GRV98LO for gluon distribution function and Weizsaker-Williams function for photon distribution. The results for model II and parametrization (b) are given in reference \[18\]. The hard scale involved in the calculation for all experiments is between $4m_c^2$ and $4m_D^2$ as we are using color evaporation model. Hence the scale evolution of TMD’s is not expected to affect much our estimates for the experiments at higher energies.

According to our estimates sizable asymmetry is expected at various experiments covering different kinematical regions. Hence it is worthwhile to look at SSA’s in charmonium production both from the point of view of comparing different models of charmonium production as well as comparing the different models of gluon Sivers function.

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