Conformal Symmetry for Rotating D-branes

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Abstract

We apply the Kerr/CFT correspondence to the rotating black $p$-brane solutions. These solutions give the simplest examples from string theory point of view. Their near horizon geometries have structures of AdS, even though black $p$-brane solutions do not have AdS-like structures in the non-rotating case. The microscopic entropy which can be calculated via the Cardy formula exactly agrees with Bekenstein-Hawking entropy.

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1 Introduction

Recently, the Bekenstein-Hawking entropy of Kerr black hole is calculated from the asymptotic symmetry [1]. In this calculation, the Cardy formula is used to relate the central charge of algebra to the entropy. This fact implies that there exists a correspondence between the Kerr black hole and some conformal field theory (CFT). The correspondence obtained is realized through a parallel way of the work by Brown and Henneaux [2] for $\text{AdS}_3$. The authors in [1] used the near horizon extremal Kerr geometry which was found by Bardeen and Horowitz [6] instead.

After the work [1], the analysis was applied to various rotating black holes [7–20]. However, it is difficult to obtain more information about the corresponding CFT than their symmetries. There are also some works on the geometries which can be related to the string theory [21–27], but the explicit structures of their CFT sides are still unclear.

In this paper, we apply the analysis of the Kerr/CFT correspondence to the rotating black $p$-brane solutions. These solutions are expected to describe the geometry in the presence of rotating D-branes. The corresponding field theory might be obtained from the low energy effective theory of D-branes, and will give one of the simplest examples from the viewpoint of string theory.

This paper is organized as follows: In section 2, we show the general form of the rotating black $p$-brane solutions, and their structure in the near horizon limit. A briefly review on the asymptotic symmetry and the formulation of the central charge is given in section 3. We then derive the asymptotic symmetry of the rotating black $p$-brane solutions and calculate their central charges in section 4. In section 5, we show the correspondence between the Bekenstein-Hawking entropy and the microscopic entropy which is obtained from the Cardy formula. Section 6 is devoted to conclusions and discussions.

2 Rotating black $p$-brane solutions

We consider the rotating black $p$-brane solutions in type II supergravity. The D$p$-brane has a charge associated with RR $(p + 1)$-form field $A_{p+1}(x)$. Relevant parts of the action are

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} (R + 4(\partial\Phi)^2) - \frac{1}{2} |F_{p+2}|^2 \right],$$

(2.1)

There are also some works in which the entropy is calculated in a different formulation [3–5].
where $\Phi(x)$ is the dilaton and $F_{p+2}(x) = dA_{p+1}(x)$ is the field strength of the RR $(p+1)$-form field. The rotating black $p$-brane solution is given in [28–30] (see also [31]). The metric has the form of

$$ds^2 = H^{-1/2} \left[ -H dt^2 + dx^2 + \frac{2m}{r^{D-3} f_D} \left( \cosh \delta dt + \sum_i a_i \mu_i^2 d\phi_i \right)^2 \right] + H^{1/2} \left[ \frac{dr^2}{f_D (\Pi_D - \frac{2m}{r^{D-3}})} + \left( \sum_i (r^2 + a_i^2) (d\mu_i^2 + \mu_i^2 d\phi_i^2) \right) \right],$$

(2.2)

where $D = 10 - p$. For even $D$, we have to take $a_i = (10 - p)/2 = 0$ and $d\phi_i = (10 - p)/2 = 0$, since there are $\frac{10-p}{2}$ $\mu$'s but number of $\phi_i$ is $\frac{8-p}{2}$. Angular coordinates $\mu_i$ are not independent and satisfy a constraint $\sum_i \mu_i^2 = 1$. The functions $H(r, \mu)$, $f_D(r, \mu)$ and $\Pi_D(r)$ are defined as

$$H = 1 + f_D \frac{2m \sinh^2 \delta}{r^{D-3}}, \quad f_D^{-1} = \Pi_D \left[ \sum_i \frac{\mu_i^2}{g_i} \right], \quad \Pi_D = \prod_i g_i, \quad g_i = 1 + \frac{a_i^2}{r^2}.$$

The RR $(p+1)$-form field $A_{p+1}(x)$ and the dilaton $\Phi(x)$ are given by

$$A_{p+1} = \frac{1}{\sinh \delta} (H^{-1} - 1) \left( \cosh \delta dt + \sum_i a_i \mu_i^2 d\phi_i \right) \wedge dx^1 \wedge \cdots \wedge dx^p,$$

$$e^\Phi = g_s H^{\frac{2-p}{4}}.$$

We now consider the extremal case. The extremal limit can be realized through the degeneracy of inner and outer horizons. This degeneracy requires the following condition at the horizon radius $r_H$,

$$\Pi_D \big|_{r=r_H} = \frac{2m}{r_H^{D-3}} = 0,$$

$$\Pi_D' \big|_{r=r_H} = (D-3) \frac{2m}{r_H^{D-2}} = 0,$$

where the prime (“′”) stands for the derivative with respect to $r$.

Next we shall consider the near horizon geometry. The near horizon metric has an AdS like structure in the components of $g_{tt}(x)$ and $g_{rr}(x)$ with $g_{\phi_i(x)}$ [6]. In order to see this, we need to consider the following coordinate transformations and the limit,

$$r \rightarrow r_H + \epsilon \hat{r}, \quad t \rightarrow \epsilon^{-1} \hat{t}, \quad \phi_i \rightarrow \varphi_i - \frac{a_i}{r_H^2 + a_i^2} \frac{1}{\cosh \delta} \epsilon^{-1} \hat{t} \quad \text{with} \quad \epsilon \rightarrow 0.$$
The coordinate transformation of $\phi_i$ can be easily fixed by the condition of $g_{\phi_i}(x) \sim \mathcal{O}(\epsilon^0)$. Then this transformation makes all inverse powers of $\epsilon$ vanish. Finally, we arrive at the near horizon geometry of the rotating black $p$-brane solution:

$$
\begin{align*}
\text{d}s^2 &= f_0(\mu)\hat{r}^2\text{d}\hat{t}^2 + \gamma_{ij}(\mu)\left(\text{d}\varphi_i + k_i\hat{r}\text{d}\hat{t}\right)\left(\text{d}\varphi_j + k_j\hat{r}\text{d}\hat{t}\right)
&+ c_r^2 f_0(\mu)\frac{\text{d}\hat{r}^2}{\hat{r}^2} + \sum_i f_{\mu i}(\mu)\text{d}\mu_i^2 + f_x(\mu)\text{d}\vec{x}^2, \\
\end{align*}
$$

(2.5)

where

$$
\begin{align*}
&f_0(\mu) = -\frac{k_0\hat{H}^{1/2}}{2\hat{F}_D \cosh^2 \delta}, \\
&f_{\mu i}(\mu) = \hat{H}^{1/2} (r_H^2 + a_i^2), \\
&f_x(\mu) = \hat{H}^{-1/2}, \\
&\gamma_{ij}(\mu) = \hat{H}^{-1/2} \hat{F}_D a_i \mu_i^2 a_j \mu_j^2 + \delta_{ij} \hat{H}^{1/2} (r_H^2 + a_i^2) \mu_i^2,
\end{align*}
$$

with

$$
\begin{align*}
&\hat{H}(\mu) = H\bigg|_{r=r_H} = 1 + \hat{F}_D \sinh^2 \delta, \\
&\hat{F}_D(\mu) = f_D \Pi_D \bigg|_{r=r_H} = \frac{2m}{r_D - 3f_D} \bigg|_{r=r_H} = \left(\sum_i \frac{r_H^2 a_i^2}{r_H^2 + a_i^2}\right)^{-1}.
\end{align*}
$$

The RR field should be also transformed as

$$
A_{p+1} = \epsilon^{-1} \tanh \delta \text{d}\hat{r} \land \text{d}x^1 \land \cdots \land \text{d}x^p + \hat{A}_{p+1},
$$

where

$$
\hat{A}_{p+1} = \frac{\hat{F}_D}{\hat{H}} \sinh \delta \left[\sum_i a_i \mu_i^2 \left(\text{d}\varphi_i + k_i\hat{r}\text{d}\hat{t}\right)\right] \land \text{d}x^1 \land \cdots \land \text{d}x^p.
$$

(2.6)

The first term of $A_{p+1}$ is constant and can be absorbed by the gauge transformation. Then only the second term $\hat{A}_{p+1}$ should be take into account in the near horizon limit. Hereafter, we will consider this near horizon metric (2.5) and the RR field (2.6) and omit the hat (“^”).
3 Asymptotic symmetry

In this section, we briefly review on the covariant formalism of the asymptotic symmetry [32, 33].

Let us consider a system with local fields $\phi^i(x)$ on $n$-dimensional spacetime. The variation of Lagrangian density can be divided into the “left hand side” of the equation of motion (EOM) and surface term:

$$\delta I(\ast L) = (\text{EOM})_i \delta \phi^i + d\Theta_I,$$  \hspace{1cm} (3.1)

where $\delta I$ is the variation with respect to the fields, $\phi^i(x) \to \phi^i(x) + \Delta \varphi^i \delta \phi^i(x)$, and $\Theta_I(x)$ is an $(n-1)$-form. The equation of motion is just given by $(\text{EOM})_i = 0$. If one restricts the configuration space to that of on-shell, the first term in (3.1) $(\text{EOM})_i$ vanishes. The conserved current $J_\xi$ is given by the variation of the Lagrangian with respect to a symmetry:

$$d \tilde{J}_\xi = -(\text{EOM})_i \delta \xi \phi^i,$$  \hspace{1cm} (3.2)

where, $\tilde{J}_\xi = \ast J_\xi$. If fields $\phi^i(x)$ satisfy the equations of motion, the current satisfies a condition $d \tilde{J}_\xi = 0$. Except for singularities, the current is expressed in terms of exact forms:

$$\tilde{J}_\xi = dk_\xi,$$  \hspace{1cm} (3.3)

where $k_\xi$ is an arbitrary $(n-2)$-form. The charge is given by integrating this $(n-2)$-form,

$$Q_\xi = \int_\Sigma \tilde{J}_\xi = \int_{\partial \Sigma} k_\xi[\phi].$$  \hspace{1cm} (3.4)

Since $k_\xi$ is arbitrary, this charge cannot be determined without fixing reference.

For asymptotic symmetry, we allow small deviations of the fields up to some asymptotic conditions:\footnote{The $O(\chi^i)$ are symbolic forms to specify the asymptotic conditions. Later we will write this as $O(r^\alpha)$ with some power $\alpha$, where $r$ is a radial coordinate.}

$$\phi^i \to \bar{\phi}^i + O(\chi^i),$$  \hspace{1cm} (3.5)

where $\bar{\phi}^i(x)$ are background fields. The current conservation is now satisfied up to some asymptotic condition,

$$d \tilde{J}_\xi = O(\chi),$$  \hspace{1cm} (3.6)

and the current is fixed up to the boundary term and the asymptotic condition,

$$\tilde{J}_\xi = dk_\xi + O(\chi^\mu).$$  \hspace{1cm} (3.7)
In order to fix the \((n-2)\)-form \(k_\xi\), it might be useful to introduce the homotopy operator which is defined such that

\[
\delta \omega_n = \delta \phi^i E_i + d I^n \omega_n, \tag{3.8}
\]

\[
\delta \omega_p = I^{p+1} d \omega_p + d I^p \omega_p, \tag{3.9}
\]

where \(\omega_p\) is an arbitrary \(p\)-form. The homotopy operator \(I^p\) maps \(p\)-form to \((p-1)\)-form,

\[
I^p \omega_n = \delta \phi^i \frac{\partial (i \partial_\mu \omega_n)}{\partial \phi_{,\mu}} + \delta \phi^i \frac{\partial (i \partial_\nu \omega_n)}{\partial \phi_{,\nu}} - \delta \phi^i \frac{\partial (i \partial_\nu \omega_n)}{\partial \phi_{,\mu}} + \cdots, \tag{3.10}
\]

\[
I^{n-1} \omega_{n-1} = \frac{1}{2} \delta \phi^i \frac{\partial (i \partial_\mu \omega_{n-1})}{\partial \phi_{,\mu}} + \frac{2}{3} \delta \phi^i \frac{\partial (i \partial_\nu \omega_{n-1})}{\partial \phi_{,\nu}} - \frac{1}{3} \delta \phi^i \frac{\partial (i \partial_\nu \omega_{n-1})}{\partial \phi_{,\mu}} + \cdots, \tag{3.11}
\]

where \(\phi_{,\mu \cdots \nu}(x) \equiv \partial_\mu \cdots \partial_\nu \phi(x)\). One can define an asymptotic charge which could be observed through the difference from the background \(\bar{\phi}(x)\). This definition corresponds to \(J_\xi[\bar{\phi}] = 0\), and therefore we choose \(k_\xi[\bar{\phi}] = 0\) by using the ambiguity of \(k_\xi\). Then the current of asymptotic symmetry can be expanded as

\[
J_\mu \xi[\bar{\phi}] = J_\mu \xi[\bar{\phi}] + \delta J_\mu \xi[\delta \phi, \bar{\phi}] + \cdots = \delta J_\mu \xi[\delta \phi, \bar{\phi}] + O(\chi^\mu), \tag{3.12}
\]

where \(\delta \phi^i(x) = \phi^i(x) - \bar{\phi}^i(x)\). From the equations (3.7), (3.9) and (3.12), we obtain the following expression:

\[
k_\xi[\phi - \bar{\phi}, \bar{\phi}] = I^{n-1} \bar{J}_\xi + O(\chi^{\mu \nu}). \tag{3.13}
\]

The asymptotic charge is then given by using this \(k_\xi\) (3.13),

\[
Q_\xi = \int_{\partial \Sigma} k_\xi[\delta \phi, \bar{\phi}]. \tag{3.14}
\]

The commutation relation of the asymptotic charges can be calculated by taking the variation of the field \(\phi^i(x)\):

\[
\delta_\zeta Q_\xi = \int k_\xi[\delta_\zeta \phi, \bar{\phi}]. \tag{3.15}
\]

Since the charges are linear order in \(\delta \phi^i(x) = \phi^i(x) - \bar{\phi}^i(x)\), this can be expressed as

\[
\delta_\zeta Q_\xi = \int k_\xi[\delta_\zeta \phi, \bar{\phi}] + \int k_\xi[\delta_\zeta \phi, \bar{\phi}] + \int k_\xi[\delta_\zeta \phi, \bar{\phi}]. \tag{3.16}
\]

The second term which is linear in \(\delta \phi^i(x)\) gives the commutator of \(\xi(x)\) and \(\zeta(x)\). An additional constant term appears in the first term, which gives the central charge of this algebra.
Before closing this section, it would be worth mentioning a symplectic structure of the configuration space [34–36]. We start by introducing the symplectic form

$$\Omega = \frac{1}{2} \Omega_{IJ} d\varphi^I \wedge d\varphi^J,$$

(3.17)

where $d\varphi^I$ are the basis of one-form on the configuration space and $\Omega_{IJ}$ is given by

$$\Omega_{IJ} = \int \omega_{IJ},$$

(3.18)

with

$$\omega_{IJ} = \delta_I \Theta_J - \delta_J \Theta_I.$$

(3.19)

If one add the boundary term $K$ to the Lagrangian, the symplectic form gets an additional term, $[\delta_I, \delta_J]K$. Therefore this definition of the symplectic form depends on choice of the boundary term. By definition, a function $H_\xi$ defines a vector field $V_I^\xi(\phi^i)$ as

$$\delta_J H_\xi = \Omega_{IJ} V^I_\xi,$$

(3.20)

therefore $H_\xi$ is the charge which generates the flow in the configuration space,

$$\delta_\xi \phi^i = V^I_\xi \delta_I \phi^i.$$

(3.21)

In order to relate this symplectic structure to the asymptotic charge, one could introduce an additional term and use the following definition of the symplectic form instead:

$$\omega_{IJ} = \delta_I \Theta_J - \delta_J \Theta_I - dE_{IJ},$$

(3.22)

with

$$E_{IJ} = -\frac{1}{2} (J^n_I \Theta_J - J^n_J \Theta_I).$$

(3.23)

Then, it was shown in [32,33] that the symplectic form is related to the $(n-2)$-form $k_\xi$ which is defined in (3.13) as

$$dk_\xi[\delta_J \phi, \bar{\phi}] = I^n_J d\bar{J}_\xi - \delta_J \bar{J}_\xi = \omega_{IJ} V^I_\xi,$$

(3.24)

if the variation $\delta_J$ is tangent to the space of solutions. The charge $H_\xi$ which is defined in (3.20) is identical to the one in (3.14).
We now study the asymptotic symmetry of the rotating black $p$-brane solution. Here we generalize the analysis in [1], and consider a geometry of the following form which might be extended from the one in (2.5),

$$\begin{align*}
ds^2 &= f_0(y) r^2 dt^2 + \gamma_{ij}(y) (d\varphi_i + k_i r dt) (d\varphi_j + k_j r dt) \\
&\quad + c_r^2 f_0(y) \frac{dr^2}{r^2} + \sigma_{ab}(y) dx^a dx^b + \tau_{\alpha\beta}(y) dy^\alpha dy^\beta,
\end{align*}$$

(4.1)

where $i, j = 1, \ldots, d$, $a, b = 1, \ldots, \tilde{d}$ and $\alpha, \beta = 1, \ldots, \tilde{d}$. We impose the following asymptotic boundary condition for the deviation of the metric $h_{\mu\nu}(x)$:

$$h_{\mu\nu} = \begin{pmatrix}
t & i & k \neq i & r & b & \beta \\
t & \mathcal{O}(r^2) & \mathcal{O}(r^0) & \mathcal{O}(r^-2) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) \\
i & \mathcal{O}(r^0) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) \\
r & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) \\
a & \mathcal{O}(r^-3) & \mathcal{O}(r^-2) & \mathcal{O}(r^-2) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) \\
\alpha & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1) & \mathcal{O}(r^-1)
\end{pmatrix}$$

(4.2)

where the index “$i$” stands for a specific direction of the Killing $\varphi_i$, and “$j$” and “$k$” are other directions of $\varphi$. Indices of $a, b, \alpha, \beta$ indicate $x^a, x^b, y^\alpha, y^\beta$ direction, respectively.

We can also introduce a $(p + 1)$-form gauge field of the form

$$A_{p+1} = \sum_i A_i(y) (d\varphi_i + k_i r dt) \wedge dx^1 \wedge \cdots \wedge dx^p.$$  

(4.3)

In [15], an asymptotic condition is imposed on the gauge field and a part of the transformation is absorbed by using the $U(1)$ gauge transformation. However we do not impose any condition on the gauge field here. Since constraints on the geometry are enough to fix the asymptotic Killing vectors, we do not need to impose any more constraints on the gauge field. Since asymptotic symmetry is defined up to some asymptotic condition, the theory could be symmetric even though it is not invariant. In fact, the gauge field does not contribute to the central charge even if we do not impose asymptotic condition. As we will see later, these contributions can be absorbed by redefinition of the asymptotic charge. Therefore we do not need to absorb the transformation of gauge fields by using $U(1)$ gauge transformation.

We introduce a dilaton $\Phi(y)$. Due to the isometry of the geometry, the dilaton does not depends on $t, \varphi$, and $x$. We also assume that the dilaton does not have any
singularity or zero at the horizon of the original geometry. Then the dilaton depends on only \( y^\alpha \) coordinates. Since it is rather natural to consider the D-brane effective theory in string frame, we consider the asymptotic charge and the Virasoro algebra in the string frame.

The diffeomorphism which satisfies this asymptotic condition (4.2) is given for each rotating directions

\[
\xi^{(i)} = \epsilon(\phi_i) \partial_{\phi_i} - r' \epsilon(\phi_i) \partial_r. \tag{4.4}
\]

Using the mode expansion for \( \epsilon(\phi_i) \), we define \( \xi^{(i)}_n \) as

\[
\xi^{(i)}_n = \sum_n \epsilon_n \xi^{(i)}_n \quad \text{with} \quad \xi^{(i)}_n = e^{-in\phi_i} (\partial_{\phi_i} + inr \partial_r). \tag{4.5}
\]

These vectors satisfy the Virasoro algebras,

\[
i[\xi^{(i)}_n, \xi^{(j)}_m] = \delta_{ij} (n - m) \xi^{(i)}_{n+m}. \tag{4.6}
\]

Let us proceed to calculate the central charges. In our case, the dilaton is invariant under this diffeomorphism. Therefore contributions to the central charges come from the conserved \((n-2)\)-form associated to the gravity and the RR \((p+1)\)-form. The (on-shell vanishing) current is thus given by

\[
J^\mu_\xi = J^\mu_{\xi(g)} + J^\mu_{\xi(A)}, \tag{4.7}
\]

where \( J^\mu_{\xi(g)} \) and \( J^\mu_{\xi(A)} \) are the currents associated to the equations of motion of gravity and RR \((p+1)\)-form, respectively. Their explicit forms are

\[
J^\mu_{\xi(g)} = e^{-2\Phi} \left[ R^\mu\nu - \frac{1}{2} g^\mu\nu R + 2 \nabla^\mu \nabla^\nu \Phi - 2 g^\mu\nu \nabla_\rho \nabla^\rho \Phi \right. \\
+ 2 g^\mu\nu \nabla_\rho \Phi \nabla^\rho \Phi \left. \right] \xi_\nu - T^\mu\nu \xi_\nu, \tag{4.8}
\]

\[
J^\mu_{\xi(A)} = (p + 2)(\nabla_\nu F^{\nu\mu\lambda_1...\lambda_p}) \xi^\rho A_{\rho\lambda_1...\lambda_p}, \tag{4.9}
\]

where \( \nabla_\mu \) is the covariant derivative. An energy-momentum tensor for the RR field is expressed by \( T^\mu\nu(x) \). We introduce a small perturbation of the metric and the RR \((p+1)\)-form, so that the conserved \((n-2)\)-forms \( k_\xi \) are given by (3.13). We define \( \tilde{k}_\xi \) as the dual of \( k_\xi \), which can be derived from the current \( J_\xi \) by using (3.11) and (3.13).

The relevant formula is given by

\[
\tilde{k}^\mu\nu_\xi [\phi] = \frac{1}{2} \delta_\phi I^\mu_{(\nu)} \partial I^\nu_{(\mu)} + \frac{2}{3} \delta_\phi I^\mu_{(\lambda)} \partial I^\nu_{(\mu,\lambda)} - \frac{1}{3} \delta_\phi I^\mu_{(\nu,\lambda)} \partial I^\nu_{(\mu,\lambda)} + (\mu \leftrightarrow \nu). \tag{4.10}
\]

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The gravity part of $\tilde{k}_\xi$ with the metric perturbation $\delta g_{\mu\nu}(x) = h_{\mu\nu}(x)$ is given by

$$\tilde{k}_{\xi(g)}^{\mu\nu}[h] = -\frac{p+2}{2\kappa^2} \left[ 2F^{\nu\rho\lambda_1 \cdots \lambda_p} h^\rho_{\mu\xi} A_{\sigma \lambda_1 \cdots \lambda_p} + pF^{\nu\rho\lambda_1 \cdots \lambda_p-1} h^\rho_{\mu\xi} A_{\sigma \lambda_1 \cdots \lambda_p-1} \right] - (\mu \leftrightarrow \nu), \quad (4.11)$$

where $D_\mu$ is the covariant derivative for the background $\bar{g}_{\mu\nu}(x)$, while the RR field part with metric perturbation is expressed by

$$\tilde{k}_{\xi(A)}^{\mu\nu}[h] = -\frac{p+2}{2\kappa^2} \left[ 2F^{\nu\rho\lambda_1 \cdots \lambda_p} h^\rho_{\mu\xi} A_{\sigma \lambda_1 \cdots \lambda_p} + pF^{\nu\rho\lambda_1 \cdots \lambda_p-1} h^\rho_{\mu\xi} A_{\sigma \lambda_1 \cdots \lambda_p-1} \right] - (\mu \leftrightarrow \nu). \quad (4.12)$$

The gravity part with the gauge perturbation $\delta A_{\mu_1 \cdots \mu_{p+1}}(x) = a_{\mu_1 \cdots \mu_{p+1}}(x)$ is

$$\tilde{k}_{\xi(g)}^{\mu\nu}[a] = \frac{p+2}{2\kappa^2} \left[ (p+1)\xi_\sigma F^{\nu\rho_1 \cdots \rho_p} a^{\mu \rho_1 \cdots \rho_p} + (p+1)F^{\nu\rho_1 \cdots \rho_p} A^{\sigma \rho_1 \cdots \rho_p} \right] - (\mu \leftrightarrow \nu), \quad (4.13)$$

and the RR $(p+1)$ form part is given by

$$\tilde{k}_{\xi(A)}^{\mu\nu}[a] = \frac{p+2}{2\kappa^2} \left[ (D^\nu a^{\mu \lambda_1 \cdots \lambda_p}) \xi^\sigma A_{\sigma \lambda_1 \cdots \lambda_p} - \frac{1}{2} a^{\mu \lambda_1 \cdots \lambda_p} D^\nu (\xi^\sigma A_{\sigma \lambda_1 \cdots \lambda_p}) \right] \quad (4.14)$$

The central extensions could be calculated by the first term in l.h.s. of (3.10),

$$\frac{c^{(i)}}{12} \equiv \int_{\partial \Sigma} \tilde{k}_{\xi^{(i)}}^{\mu\nu} [\bar{\delta}_{\xi^{(i)}} \bar{\Phi}, \bar{\Phi}], \quad (4.15)$$

where fields $\phi^i(x)$ are the metric and the RR $(p+1)$-form. We set the variations as their Lie derivatives of the background fields $h_{\mu\nu}(x) = L_\xi \bar{g}_{\mu\nu}(x)$ and $a_{\mu_1 \cdots \mu_{p+1}}(x) = L_\xi A_{\mu_1 \cdots \mu_{p+1}}(x)$. Explicit forms of the Lie derivatives for the metric are written down as

$$L_{\xi^{(i)}} \bar{g}_{IJ} = ine^{-in\varphi_i}(\delta_{II} + \delta_{JJ} - \delta_{II} - \delta_{JJ}) \bar{g}_{IJ}, \quad (4.16)$$

$$L_{\xi^{(i)}} \bar{g}_{ir} = L_{\xi^{(i)}} \bar{g}_{ri} = n^2 re^{-in\varphi_i} \bar{g}_{rr}, \quad (4.17)$$

$$L_{\xi^{(i)}} \bar{g}_{\mu\nu} = 0, \quad \text{for other components} \quad (4.18)$$
where indices $I$ and $J$ stand for $(t, \varphi_i, \varphi_j)$, and for the RR $(p + 1)$-form as

$$\mathcal{L}_{\xi_n} A = i e^{-i \varphi} \sum_j A_j(y) (k_j r dt - \delta_{ij} d\varphi_i) \wedge dx^1 \wedge \cdots \wedge dx^p. \quad (4.19)$$

Since two vectors $\xi_i^{(i)}$ and $\xi_m^{(i)}$ are manifestly antisymmetric, the central charge has only the odd power of $n$. In this case, we can easily see that it contains only terms of order $n$ and $n^3$. The linear part can be absorbed by taking an appropriate definition of $Q_0$. Hence we consider only the terms of order $n^3$. Using the fact that $i \xi_i A = O(n^0) \times e^{-i \varphi}$ and $\mathcal{L}_{\xi_n} A = O(n) \times e^{-i \varphi}$, we see that there are no contributions to $O(n^3)$ terms. As a result, the remaining relevant part is $k_{\xi(g)}[\mathcal{L}_{\xi} g]$ and can be rewritten as

$$k_{\xi}^{\mu \nu} [\mathcal{L}_{\xi} g] = \frac{1}{2 \kappa^2} e^{-2 \Phi} \left[ - D_\rho \xi^\rho D^\nu \xi^\mu + D_\rho \xi^\rho D^\nu \xi^\mu + 2 D_\rho \xi^\rho D^\mu \xi^\mu \\
+ \frac{1}{2} (D^\rho \xi^\nu + D^\nu \xi^\rho) (D_\rho \xi^\mu + D^\mu \xi^\rho) - 2 R^{\mu \nu \rho \sigma} \xi^\rho \xi^\sigma \\
- \frac{1}{2} \xi^\mu (D^\nu \xi^\rho + D^\rho \xi^\nu) \partial_\rho \Phi - \frac{3}{2} (D^\mu \xi^\rho + D^\rho \xi^\mu) \xi^\rho \partial^\nu \Phi \right] - (\mu \leftrightarrow \nu), \quad (4.20)$$

up to exact forms. By using the explicit expressions of the Lie derivatives with respect to the vector field $\xi_i$, it turns out that the terms to contribute to the central charges are only

$$c_i^{12} = \frac{1}{2 \kappa^2} \int d^4 \varphi \, d^4 x \, d^4 y \, \sqrt{-g} \, e^{-2 \Phi} \left[ D_\rho \xi_n^{(i)} D^\rho \xi_m^{(i)} - (m \leftrightarrow n) \right]. \quad (4.21)$$

Using (4.1), we finally obtain the following expressions,

$$c_i^{12} = \frac{c_i k_i}{\kappa^2} \int d^4 \varphi \, d^4 x \, d^4 y \, e^{-2 \Phi} \sqrt{(\det \gamma)(\det \sigma)(\det \tau)}. \quad (4.22)$$

### 5 Correspondence of the entropy

In order to see the correspondence, we have to define the quantum vacuum in the rotating black $p$-brane background. In the case of rotating black hole background, we have to measure the energy from the viewpoint of the zero angular momentum observer (ZAMO) due to the ergoregion of the geometry. Let us consider, for example, a field with an energy $\omega$ and angular momenta $m_i$. This field has an energy

$$\omega_{ZAMO} = \omega - \Omega_i m_i, \quad (5.1)$$
where $\Omega_i$ is the angular velocity at the horizon. Then the Boltman factor for this field is given by

$$e^{-\beta_H \omega_{\text{ZAMO}}} = e^{-\beta_H \omega + \beta_m \Omega_i},$$

(5.2)

where $\beta_H = 1/T_H$ is the inverse of the Hawking temperature. The Frolov-Thorne temperature $T_i = 1/\beta_i$ [37] is defined by

$$\beta_i = \beta_H \Omega_i.$$  

(5.3)

This field could have a nontrivial Boltzmann factor even if the Hawking temperature is zero.

In order to calculate the Frolov-Thorne temperature, we first consider the near-extremal case, and then take the extremal limit. The near extremal metric has the form of

$$ds^2 = -r^2 \left(1 - \frac{r^2}{r^2_+}\right) f_0 dt^2 + \gamma_{ij} \left( d\varphi_i + k_i r dt \right) \left( d\varphi_j + k_j r dt \right) + \frac{f_r dr^2}{r^2 \left(1 - \frac{r^2}{r^2_+}\right)} + \cdots,$$

(5.4)

where $r_+$ is the position of the horizon. Since this is the near horizon geometry of the rotating black $p$-brane, this horizon corresponds to the outer horizon of original metric and the inner horizon is placed at $r = 0$. The temperature and the velocity are given by

$$T_H = \frac{r_+}{2\pi} \sqrt{\frac{f_0}{f_r}}, \quad \Omega_i = r_+ k_i,$$

(5.5)

so that the Frolov-Thorne temperature can be estimated as

$$T_i = \frac{T_H}{\Omega_i} = \frac{1}{\pi c_s k_i},$$

(5.6)

By using the Cardy formula, the microscopic entropy is given by

$$S = \frac{\pi^2}{3} c_i T_i = \frac{4\pi}{\kappa^2} \int d^d \varphi \, d^d x \, d^d y \, e^{-2\Phi} \sqrt{(\det \gamma)(\det \sigma)(\det \tau)},$$

(5.7)

for each rotating directions $i$. The additional factor $e^{-2\Phi}$ is absorbed by the redefinition of the metric in the Einstein frame. This entropy exactly agrees with the Bekenstein-Hawking entropy.
6 Conclusions and discussions

In this paper, we applied the Kerr/CFT correspondence to the rotating black $p$-brane solutions. Near horizon geometries of these solutions are given by $\text{AdS}_2$ with $U(1)$ fibers. By imposing appropriate boundary conditions, we obtained the asymptotic symmetry which forms the Virasoro algebra. Assuming that there exists a corresponding CFT, we applied the Cardy formula and obtained the entropy from the central charge of the algebra. This microscopic entropy exactly agrees with the Bekenstein-Hawking entropy.

These solutions are expected to be describe the geometry in presence of $D_p$-brane. Hence, if there exist corresponding CFTs, they might be related to the low energy effective theory of D-branes. For the $D0$-brane, the effective theory is expected to be a one-dimensional field theory. This implies that the corresponding field theories of rotating black holes are not two-dimensional theories but one-dimensional field theories (or spacetime is compactified to one dimension). In other words, the effective theory of $D0$-brane might be a one-dimensional conformal field theory for a special configuration which corresponds to the rotating case. Then, the conformal symmetry should be that on the direction of worldline. Actually, time direction of the black-$p$-brane geometry is included into the angular coordinate of their near horizon geometry. The RR field basically lies on the time direction, but on the rotational direction at the origin of the near horizon geometry on which it couples to the black $p$-brane. Hence all of the Virasoro algebras associated with the rotational directions pick up the same degree of freedom in the field theory side. In fact, the Bekenstein-Hawking entropy is reproduced by each of these Virasoro symmetries, not sum of these contributions. Therefore, the degree of freedom associated to these Virasoro symmetries should be identical to each other.

It is interesting that the effective theories of rotating D-branes might be a CFT even though general non-rotating D-brane effective theories are not CFT. There exists a difference between the near horizon geometry of the rotating black $p$-brane and that of the non-rotating case. The non-rotating black $p$-brane solution does not have an $\text{AdS}_2$-like structure while we obtain the $\text{AdS}_2$ geometry in the rotating case. This implies that there should be some links between the rotating and non-rotating cases also in the effective theory of the D-branes. It is also interesting to consider how this conformal symmetry appears in the effective theories. This is left for future studies.

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