Towards Multiple-Star Population Synthesis

P. P. Eggleton

1Lawrence Livermore National Laboratory, 7000 East Ave, Livermore, CA94551, USA

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ABSTRACT

The multiplicities of stars, and some other properties, were collected recently by Eggleton & Tokovinin, for the set of 4559 stars with Hipparcos magnitude brighter than 6.0 (4558 excluding the Sun). In this paper I give a numerical recipe for constructing, by a Monte Carlo technique, a theoretical ensemble of multiple stars that resembles the observed sample. Only multiplicities up to 8 are allowed; the observed set contains only multiplicities up to 7. In addition, recipes are suggested for dealing with the selection effects and observational uncertainties that attend the determination of multiplicity. These recipes imply, for example, that to achieve the observed average multiplicity of 1.53, it would be necessary to suppose that the real population has an average multiplicity slightly over 2.0.

This numerical model may be useful for (a) comparison with the results of star and star cluster formation theory, (b) population synthesis that does not ignore multiplicity above 2, and (c) initial conditions for dynamical cluster simulations.

Key words: stars: statistics; stars: general

1 INTRODUCTION

Recently Eggleton & Tokovinin (2008; ET08) counted the stellar systems with multiplicity from 1 to 7 in the set of stars brighter than mag. 6 on the Hipparcos scale. There are 4558 such systems in total (excluding the Sun), with 2716, 1438, 285, 86, 20, 11 and 2 having multiplicity 1 to 7. We refer to this as the Hipparcos-Bright Multiple System Catalogue (HBMSC). It is, of course, provisional: a few new components are found every year. The present paper proposes a simple model distribution that will generate the observed population, including the multiplicities. Selection effects are modeled, so that allowance can be made for the fact that some potential visual binaries or sub-binaries may not be resolvable (depending partly on distance), and some potential spectroscopic binaries may be undetectable because of unfavorable luminosity or mass ratios, or broad lines as in many OB stars. I estimate that to obtain the observed multiplicity fraction of 1.53 (ET08), i.e. an average of 1.53 components per system, I need an actual multiplicity fraction of about 2.0 or more. I attempt to fit the distribution of periods (and sub-periods), as well as of multiplicity and period.

Statistics on multiplicity are useful for at least three reasons.

(a) They are a statement about the final product of star formation, and thus something of a check on theoretical models of this difficult process.

(b) There are some stellar evolutionary scenarios that are importantly modified by, or even wholly dependent on, the existence of triple companions; thus we need to know something of the probability of triple (or higher multiple) companions.

(c) They represent a starting point for N-body dynamical simulations of clusters, and more generally for any population-synthesis project.

Note however that the HBMSC can provide statistics only for systems with mass above about \(1 \, M_{\odot}\), since very few systems of lower mass are included among the bright stars. Although many low-mass multiples have been recognised, there does not exist a comparable wealth of data on, say, the 5000 nearest stars.

In relation to point (a), it can be questioned how representative are the systems in the general Galactic field of what is produced directly in Star Formation Regions (SFRs). This is not the place to attempt a detailed discussion of star formation, but there seems no doubt that the great majority of stars form in localised, relatively dense, SFRs, and are then released or scattered into the Galactic field. Since only about 1% of stars are currently in clusters, it follows that the field stars are the great bulk of what is produced in SFRs, and so it is not unreasonable to see them as representative of those products. Some of the systems have probably been modified, by N-body encounters, during both the formation process and the later escape process. One might attempt to distinguish between the formation of stars and the formation of systems, but although a semantic distinction might be made I doubt if any clear physical distinction can be made. The widest systems that can be found in the field are usually much too wide to have had a chance to form in the relatively compact SFRs where they were born, but presumably formed during the process of evaporation that put them into the field. Those field systems that are not very wide can
reasonably be supposed to be much the same as when they formed in the SFR. Numerical attempts to model star formation in SFRs will presumably have to address the N-body interactions and the dissipation into the field as well as the gas-condensation stage, and so it is not unreasonable that they should be constrained to some extent by the statistics of multiplicity in field stars.

These remarks also relate to point (c). The distribution of multiples in a cluster will certainly differ from that of the field, by missing the widest field multiples. But if one starts an N-body cluster with the multiplicity distribution suggested below, the widest systems will be very quickly broken up. It is not unreasonable to suppose that the closer systems, which do not get broken up quickly, might be representative of what should be expected in a young cluster.

In Section 2 I describe the Monte Carlo procedure I use, applied to a model in which stellar evolution, but only so far of single stars, is taken from a table of evolved stellar models. In Section 3 the procedure is generalised from single to multiple stars; a procedure called Create constructs a theoretical population of multiple systems. Although the issue of interactive binary and triple systems is briefly discussed, it is left for future work. In Section 4 I describe a ‘theoretical observatory’, a procedure called Observe, which looks at the multiple systems generated by Create and decides which systems and subsystems should be recognisable as visual or spectroscopic or eclipsing binaries, and which not. Thus a ‘raw’ model from Create is turned into a model that can in practice be compared with what is actually seen, in the HBMSC. In Section 5 a comparison is made, and I discuss some applications of the model to the problems mentioned in the second paragraph.

I would like to emphasise that I am not claiming to find a definitive model of multiplicity, but only a reasonable model. For instance, a definitive model would need many more mass ratios than are currently well-known, and in particular rather extreme mass ratios, such as are very difficult to determine reliably at present. In addition, there is an obvious degeneracy between the Create stage and the Observe stage, since one can imagine that many more binaries (and higher multiples) are created that have parameters that are hard to observe, and so are excluded at the Observe stage.

2 METHOD

We would like to model the statistics with some set of (cumulative) distribution functions. Suppose for the sake of argument that all stars are single stars. Each star is determined by two parameters, mass (m) and age (t). I ignore metallicity for the time being – and in fact throughout this paper – although it would not be difficult in principle to include it. For determining which stars are visible above some apparent magnitude, a third parameter, distance (d), has also to be assigned. The cumulative distributions of these parameters will be some functions of the three variables. There is no a priori reason to suppose separability, i.e. that the distribution is the product of three elementary distributions, each in one variable only. We select three random numbers X, Y, and Z, uniform in the interval (0, 1), and then generate m from some global (cumulative) IMF, m = m(X), t from an age distribution t = t(Y, m), and finally d from a distance distribution d = d(Z, m, t). The order is immaterial, although of course the actual functional forms will depend on the order selected. The distribution over distance will have to fall off at large distances fast enough to avoid Olbers’ paradox; the formulation given below – equation (10) or its inverse equations (12) and (13) – is roughly spherical within ~ 100 pc, roughly disk-like to ~ 1000 pc and becomes negligible beyond ~ 3 kpc.

The number of stars brighter than a star of luminosity L0 at distance d0, i.e. that have

\[ \frac{L}{d^2} \geq \frac{L_0}{d_0^2}. \]

out of a total population of Ns stars is an integral

\[ N = N_s \int_{\text{unit cube}} H(L(X,Y) - L_0 \frac{d^2(Z)}{d_0^2}) \, dX \, dY \, dZ, \]

where H is the Heaviside step function. The integral can be rewritten as

\[ N = N_s \int_{\text{unit square}} Z_v(X,Y) \, dX \, dY, \]

where \(Z_v\) is the fraction of stars at distance less than \(d_v\), and \(d_v\) is the distance out to which a particular star as a function of \(X, Y\) is visible. This distance \(d_v\) is given by

\[ d_v^2 = d_0^2 \frac{L(X,Y)}{L_0}. \]

Given that stars are discrete rather than continuous, we replace the integral by a Monte Carlo approximation. Suppose we select \(n^2\) points randomly in the \(X, Y\) unit square. Then we approximate the integral (3) by

\[ N = \frac{N_s}{n^2} \sum_{i=1}^{n^2} Z_v(X_i, Y_i). \]

It is clear that only stars that come from a rather small fraction of the \(X, Y\) plane will populate the night sky visibly to the naked eye: we know from experience that only stars more massive than about 0.8 \(M_\odot\), i.e. \(\lesssim 10%\) of stars from a reasonable IMF, can be bright enough, and the most massive and brightest stars must be very young, e.g. age less than \(\sim 10^7\) yrs at masses \(\gtrsim 10\, M_\odot\). We therefore use ‘importance sampling’, i.e. a transformation that maps the unit square in \(X, Y\) from the unit square in \(X', Y'\) so that the region near \(X = 1, Y = 0\), where most visible stars come from, is sampled more densely. Following Eggleton et al. (1989; EFT89) I use

\[ X = 1 - (1 - X')^2, \quad Y = 1 - (1 - Y')(1 - X'). \]

In equation (5) we have to insert an extra factor, the Jacobian of the transformation:

\[ N = \frac{N_s}{n^2} \sum_{i=1}^{n^2} Z_v(X_i, Y_i) \frac{\partial(L(X,Y))}{\partial((X',Y'))}. \]

Equation (7) gives a fractional number of stars at each sampling point. We quantise this in the following way. At the \(i\)th element the value \(N_s Z_v J/n^2\) in equation (7), where \(J\) is the Jacobian, is some fractional number, say \(k + f\), where \(k\) is an integer and \(0 \leq f < 1\). Then select a further random
number, uniform in \([0, 1]\). If this is less than \(f\) then \(k + 1\) ‘cloned’ stars are allowed, but if greater, then \(k\) clones. A value for \(n\) of 200 was found to give a reasonable compromise between speed and detail.

The formula

\[ m = 0.3 \left( \frac{X}{1 - X} \right)^{0.85} , \tag{8} \]

gives a distribution which, in terms of \(\log m\), is peaked at \(X = 0.5\), and falls off with a moderately Salpeter-like slope of 2.23 for \(m \gtrsim 0.8 M_\odot\). The median mass, at \(X = 0.5\), is \(m = 0.3 M_\odot\). For age we use the simplest distribution

\[ t = Y , \tag{9} \]

t being the age in units of \(10^{10}\) yrs. Although for a total Galactic population such a uniform rate of star production is unlikely to be realistic, most of the bright stars are less than a tenth of the Galaxy’s age, and a uniform birth rate is not so unreasonable in this span.

Formulae (8) and (9) can be seen as ‘plucked out of thin air’. However a distribution like (8) has the desirable quality of resembling the Salpeter distribution at large masses, but turning over in a reasonable way at low masses, below \(0.3 M_\odot\). The value of the turnover mass and of the slope below that mass are not determinable from the HBMSC, because the latter includes hardly any stars below \(\sim 1 M_\odot\). The slope at the high-mass end is in principle determinable, say by doing some least-squares fitting; but my initial value of 0.75 seemed to produce too many high-mass relative to intermediate-mass stars, and a second guess at 0.85 seemed to be about right, as quantified by a \(\chi^2\) test given near the end of this Section. The assumption in (9) of a reasonably uniform star formation rate over the last Gyr is similarly bound to be incorrect in detail (i.e. on a time scale of Myrs, and a lengthscale of tens of parsecs), and yet is a fairly reasonable model on longer timescales and lengthscales.

The cumulative distribution \(Z(d)\) is approximated by

\[ \frac{1}{Z} = \frac{h_0 h_1^2}{d^3} + \frac{h_1^2}{d^2} + 1 , \tag{10} \]

so that

\[ Z \propto d^3 \quad \text{if} \quad d \ll h_0 \ll h_1 \]

\[ \propto d^2 \quad \text{if} \quad h_0 \ll d \ll h_1 \]

\[ \sim 1 \quad \text{if} \quad d \gtrsim h_1 . \tag{11} \]

We need the inverse of this, \(d(Z)\). If

\[ x = \frac{3 h_0}{2 h_1} \sqrt{\frac{3(1 - Z)}{Z}} , \tag{12} \]

then

\[ d = \frac{3 h_0}{x} \cos \left( \frac{1}{3} \cos^{-1} x \right) \quad \text{if} \quad x \leq 1 \]

\[ = \frac{3 h_0}{x} \cosh \left( \frac{1}{3} \cosh^{-1} x \right) \quad \text{if} \quad x \geq 1 . \tag{13} \]

A refinement that we make to this \(d(Z)\) model is that we take \(h_0\), which is essentially the scale height of the disc, to be dependent on age:

\[ h_0(t) = h t^{0.3} , \quad h = 200 , \tag{14} \]

with \(h, h_0\) in parsecs and \(t\) in units of \(10^{10}\) yrs, as before. The constant \(h_1\) is taken as 1 kpc. Another refinement is that reddening, at 1 mag. per kpc, is included.

Once again formulae and coefficients are apparently plucked out of thin air, but are based nevertheless on a general picture of the Solar neighbourhood that I believe is widely accepted: that the distribution is roughly spherical for old stars but more disc-like for young stars. I have simply adopted what I believe is about the simplest formulation that describes this. I make no attempt to improve the fit by varying some coefficients.

\[ \text{(a) Theoretical HRD} \]

Fig 1 – Hertzsprung-Russel diagram for 171 stellar models with initial masses in the range \(0.1 \rightarrow 63 M_\odot\). Evolution was terminated when it became rapid, approaching a final compact remnant; except that an upper age limit of 10 Gyr was also imposed.

I used my stellar-evolution code (Eggleton 1971, 1972, Eggleton et al. 1973, Eggleton 2006; Pols et al. 1995) to evolve 171 single-star models with (log) masses of

\[ \log m = -1.0 \ (0.02) \ 0.0 \ (0.01) \ 0.6 \ (0.02) \ 1.8 \ . \tag{15} \]

All had a roughly solar composition: \(X = 0.70, Y = 0.28, Z = 0.02\). They were evolved from the Zero-Age Main Sequence (ZAMS) either until \(10^{10}\) yrs old or until fairly near the end of their lives. For the lowest masses this meant very little evolution in practice. For intermediate masses I stopped the evolution either at the very rapid phase of evolution towards the white-dwarf region, or shortly after
the onset of carbon burning in the core. A rate of mass loss by stellar wind was adopted (Eggleton 2006), so that intermediate-mass stars less massive originally than 4.2 $M_\odot$ would become white dwarfs less massive than 1.05 $M_\odot$. All estimates of mass-loss rates for red supergiants are very tentative; the one used here gave relatively little mass loss in the (initial) mass range 6–20 $M_\odot$, but proportionately much increased mass loss at still higher masses.

For every tenth timestep of each evolutionary sequence, the following six numbers were stored: $t$, $m(t)$, $\log L$, $\log R$, $\log R_{\text{max}}$, and $L_\nu$. $R_{\text{max}}$ is the maximal radius in the evolution prior to age $t$, useful for knowing whether binary interaction with a fairly close companion will have occurred, and $L_\nu$ is an integer that describes crudely the evolutionary state, from 1 for an MS star to 10 for a neutron star or black hole. Fig. 1 shows a theoretical Hertzsprung-Russell diagram (HRD) for this collection of stars. To avoid some interpolation which would often be of a highly non-linear nature, when the Monte Carlo procedure of equation (8) selects an initial mass $I$ replace it by the nearest value in the sequence (15).

Table 1 presents some results. The distributions (8) for mass, (9) for age and (10) for distance were used, and only $N_e$, the total population, was varied to get about the right number (to much better than 1%) of visible stars. The distribution over spectral type (Table 1, row labeled ‘single’), to some extent equivalent to the distribution over mass at least for types F and earlier, is not very satisfactory compared to the observed distribution. The ratio computed/observed is nearly 3 for O stars, and ranges between 1.4 and 0.7 for the remaining spectral types.

Although we might try to ameliorate this by playing with the IMF, it is not difficult to see that much of the disagreement is really a consequence of the fact that the distributions of mass, age and distance are not independent of each other, as is implicit in equations (8)–(10); although equation (10) for the distance dependence does include an age dependent term via equation (14). The regions that form massive stars are very clumpy, with several hundred parsecs between them, and the Sun appears to be located within a ‘bubble’ (Cox & Reynolds 1987) of 100–250 pc in which no formation of massive stars has taken place recently, i.e. within about 75 Myr, the lifetime of a 6 $M_\odot$ star. O stars have formed much more recently than that within say the Orion SFR (Star Forming Region) at about 450 pc, and are easily above $H_p = 6$, but there is no O star nearer than about 250 pc. Thus one way to improve the agreement at O and early B is to exclude stars above 6 $M_\odot$ that are nearer than 250 pc, and that was used in the line of Table 5 labeled ‘OB down’. This is equivalent to saying that the mass, age and distance distributions are not in fact disjoint; but it is numerically easiest simply to omit systems that have mass above a threshold and distance below another threshold.

We might try some more alterations, particularly of the IMF, to improve agreement further, but in practice much of the remaining disagreement disappears when we include multiple systems, in the next Section. The row labeled ‘multiple’ in Table 1 is essentially the same model as the ‘OB down’ model, but with multiplicity included as will be discussed in the next Section. The agreement is either slightly or in some cases considerably better. The overall improvement is at least partly because massive systems, as we will see shortly, appear to be more highly multiple than less massive systems, and so some O and eB systems are broken down to later types. A χ² test comparing the four distributions over spectral type listed in Table 1 gives, of course, very poor agreement between the observed distribution and either of the first two theoretical distributions, ‘single’ and ‘OB down’. But we get a χ² of 11.2 when comparing the ‘multiple’ with the ‘observed’ distribution, which is acceptable at the 10% level. No doubt this could be improved by tweaking some coefficients, but this does not seem necessary.

For each of the ‘single’, ‘OB down’ and ‘multiple’ lines in Table 1, ten Monte Carlo models were run differing only in the random numbers generated. What is listed is the average of these ten, and the rms variation. A few preliminary runs were done in order to estimate the value of the normalisation constant $N_e$ of equation (7) that would give a final count (averaged over the ten runs) of approximately the number 4558 observed.

### 3 A MODEL OF MULTIPURITY

When deciding on the multiplicity of a computed stellar system, I continue to use random numbers, thinking in terms of a process of successive bifurcation; although for practical reasons I allow no more than 3 successive bifurcations so that only multiplicity up to octuple is considered. Note however that this is not a claim that real multiples are formed by successive fragmentation during a pre-main-sequence contraction phase. It is simply an acknowledgment of the well-established fact that the great majority of multiple systems are highly hierarchical (Evans 1968), and such systems by definition consist of binaries nested inside wider binaries. A rather small proportion of systems is non-hierarchical, in the sense that 3 or more components are found at roughly equal distances from each other. Seventeen such systems were identified in the HBMSC of ET08, but these are among the least certain. Their inclusion or exclusion will make little difference to the following.

We have already selected a mass $m$, which we now interpret as the total mass, and the age $t$. When considering whether to bifurcate or not, we choose two more random numbers uniformly probable in (0,1), $U$ and $V$ say, to determine (i) the mass ratio $Q$ (with a finite probability of zero, i.e. no bifurcation) and (ii) orbital period $P$. Of course, both $P$ and $Q$ are irrelevant if the choice of $U$ is found to imply no bifurcation after all. $P$ and $Q$ might, indeed probably will, depend on $m$ as well as $U, V$, and also on the hierarchical depth $l$, i.e. on whether we are creating a binary, a sub-binary or a sub-sub-binary. We allow only $l = 0, 1$ or 2. At each potential bifurcation we choose a fresh $U$ and $V$, so that we may need as many as 7 $U, V$ pairs, although with some probability that there may be no bifurcation at all, and only one $U$ is used.

First we determine whether the current hierarchical level $l$ bifurcates or not. I specify (Table 2) a tabular function $U_0(m, l)$, $l$ being the hierarchical depth. Then $U \geq U_0$ implies no bifurcation, and $U < U_0$ implies bifurcation with a mass ratio that we can suppose depends in some manner on $(U_0 - U)/U_0$, a ratio which is always between zero and unity, and with uniform probability. Actually the entries in Table 2 are treated as functions of a variable $j$ related to
P. I take the period to be imply bifurcation, a new random number V integer values of to infinity, and I interpolate linearly in the Table for non-

\[ P \]

The variable \( P \) in days. This expression gives a distribution of \( \log \) \( P \) by

\[ j = 5 + 2 \log \left( \frac{1 + 100m}{100 + m} \right) \]. \quad (16)

The variable \( j \) ranges from 1 to 9 as \( m \) ranges from zero to infinity, and I interpolate linearly in the Table for non-

\[ \text{Table 1. Distribution over spectral type.} \]

|          | O  | eB | IB | A  | F  | G  | K  | M  | tot. |
|----------|----|----|----|----|----|----|----|----|------|
| observed | 38 | 401| 653| 928| 578| 587| 1042| 331| 4558 |
| single   | 97 | 620| 897| 886| 468| 492| 881 | 225 | 4565 |
| rms      | 16 | 44 | 46 | 65 | 25 | 28 | 79  | 87  | 138  |
| OB down  | 83 | 440| 929| 915| 491| 509| 923 | 260 | 4550 |
| rms      | 18 | 42 | 60 | 41 | 42 | 24 | 95  | 102 | 123  |
| multiple | 47 | 402| 699| 917| 581| 529| 1041| 335 | 4550 |
| rms      | 9  | 45 | 55 | 39 | 34 | 35 | 101 | 56  | 148  |

\( eB \) stands for early B (B0 – B3.5), and IB for later B.

‘Single’ refers to the single-star model of Section 2.

‘OB down’ is a model, described at the end of Section 2, where stars over 6 \( M_\odot \) and nearer than 250 pc are discounted.

‘Multiple’ is the same as ‘OB down’, but with multiplicities up to 8 allowed as in Section 3.

\[ \text{Table 2. Bifurcation Probability } U_0 \text{ as a Function of Mass and Hierarchical Level.} \]

| \( m \) = | 0 | .01 | .09 | .32 | 1  | 3.2 | 11 | 32 | \( \infty \) |
|----------|----|----|----|----|----|----|----|----|--------|
| \( j \) = | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9      |
| \( l \) = 0 | 0.40| 0.40| 0.40| 0.40| 0.50| 0.75| 0.88| 0.94| 0.96   |
|           | 1  | 0.18| 0.18| 0.18| 0.18| 0.18| 0.20| 0.60| 0.80   |
|           | 2  | 0.00| 0.00| 0.00| 0.00| 0.00| 0.20| 0.33| 0.82   |
|           | 3  | 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00| 0.00   |

The values for \( m < 1 \) are notional, because very few systems of this low a total mass appear in the HBMSC. However they are reasonably consistent with what is known about the nearest 66 systems.

The median period ratio is \( 10^{-3} \). These determinations of outer or inner periods may well be expected to depend also on all previous choices, in particular on \( m \) and \( l \). They might depend on \( t \) as well, if we are talking about a cluster in which orbits can soften or harden as time advances; but for the time being I do not allow such complexity.

The mass ratio \( Q \) can be determined from the previous \( U \) and corresponding bifurcation probability \( U_0 \) by, for instance,

\[ Q = \left( \frac{U_0 - U}{U_0} \right)^\alpha, \quad \alpha = f(m, l, P_t), \] \quad (19)

whether \( l \) is 0, 1 or 2. Values of \( Q \) less than 0.01 are replaced by 0.01. Because, as noted above, \( (U_0 - U)/U_0 \) is uniformly distributed in \((0, 1)\), distribution (19) is just a power law. Obviously with enough information it will be better to have a more complicated distribution, but several attempted determinations of the mass-ratio distribution have been presented as power laws. For example, Kouwenhoven (2006) obtains, for near-infrared companions to IB/A stars in the Sco OB2 association, a differential distribution \( \propto Q^{-0.33} \). This corresponds to \( \alpha \sim 1.5 \) in equation (19). Tokovinin (2000), following Lucy & Ricco (1979), finds a marked preference for ‘twins’, i.e near-equal-mass pairs, at fairly short periods (\( \lesssim 25 \) d). This would require a lower power like \( \alpha \sim 0.15 \) if one wanted 50% of systems to have \( Q > 0.9 \). For the present I assume, following EFT89, that \( \alpha = 0.8 \) for all periods above \( 25 \) d, and for 75% of those with shorter periods, but for the remaining 25% of the latter I take

\[ Q = 0.9 + 0.09 \left( \frac{U_0 - U}{U_0} \right) \]. \quad (20)

One would like to do the same analysis for the 4500 nearest stars, but although information on these stars, particularly on L and T dwarfs, has increased by leaps and bounds over the last decade, there is nothing like as complete a study of their multiplicity as already exists for the
of the 18 remaining constants were varied, from guessed initial values, to try and improve the agreement. The 12 not varied include, for instance, almost all of the coefficients and exponents in equations (8) – (14) and (17) – (20). In most cases it was possible to estimate from an eyeball analysis of the HBMSC what values would be plausible, and these values did indeed appear to work without further tweaking. It would be only a slight exaggeration to say that the coefficient and two exponents of equation (17), for example, were ‘plucked out of thin air’. But they were based on an eyeball analysis of the HBMSC section of Table 6, along with the suspicion that the rather marked shortage of systems there with periods of 1 – 100 yr at early spectral types might be an effect of observational selection, as is indeed found in the next Section.

Once an n-tuple system has been created, the various components can be evolved, all according to the same prescription as described in Section 2. I do not (yet) put in a binary interaction, such as Roche-lobe overflow (RLOF). It is feasible to put in back-of-the-envelope prescriptions (Hurley et al. 2002), but I prefer, in a future version, to utilise the capacity of my stellar evolution code to follow RLOF in some circumstances (Nelson & Eggleton 2001, de Mink, Pols & Hilditch 2007). But a substantial proportion of potentially interacting systems are quite difficult to follow numerically, because the RLOF can become very rapid (Paczyński & Sienkiewicz 1972). In addition winds, including those linked magnetically to a component and capable of draining angular momentum from the component, and via tidal friction from the binary, can affect the orbit. Such winds are included in my binary evolution code, which can work in a mode where both components, and the orbital period, and in addition the eccentricity, which is also modified by tidal friction, can be solved for in a single implicit time step. This process even extends to models in which both stars overfill their Roche lobes, and influence each other’s structure via internal luminosity transfer as is frequently hypothesised for contact binaries (Lucy 1968, 1976, Yakut & Eggleton 2005); but this mechanism still contains some very uncertain physics, and it is also unusually expensive to compute since evolution progresses in cycles on a thermal timescale, while lasting for something like a nuclear timescale. Thus the number of timesteps is larger than usual by a factor of several hundred.

The worst cases of rapid mass transfer, usually hypothesised to lead to common-envelope evolution (Paczyński 1976), and to either very close white dwarf + MS star binaries or else to a merger, are not dealt with yet, even by the most sophisticated version of my evolution code. In practice, these are likely to be the majority of RLOF cases, and so for the present I simply annotate those systems in which one component is, or has been at some time, larger than its Roche lobe.

Quite a few massive systems will have produced supernovae by now, even though one or two OB stars (and even AF stars) remain in the system. This is partly why we need multiplicity significantly higher in O stars than in much later stars. Although we expect that such neutron stars should have ejected themselves from their natal systems by means of asymmetric mass ejection (Shklovskii 1970, Lyne & Lorimer 1994, Hansen & Phinney 1997), I choose in the present work to keep them in the system so that, for ex-
ample, I can estimate from the RTBMSC how many neutron stars should have been produced by the systems of the HBMSC; 220 ± 24. Many more systems will have produced white dwarfs, and these are rather more likely to have remained in their original systems, although some, perhaps the majority, may have merged with a companion star. The number of white dwarf companions, seen or not, and merged or not, is 371 ± 60 in the RTBMSC. Only 20 are known in the HBMSC, which suggests that many WD companions have not so far been recognised as such.

4 THE THEORETICAL OBSERVATORY

I use the code described in the previous Section and called Create to create a ‘raw’ catalogue of theoretical bright multiple systems, the RTBMSC. Then I use a code called Observe which reads in the output catalogue from Create and decides which components would actually be identifiable, either as visual systems, spectroscopic systems or eclipsing systems. This works on very simple principles, outlined below, and converts the kind of statistics seen in the middle sections of Tables 4, 5 and 6 into those of the bottom sections. Let us call this the ‘Theoretically Observed BMSC’ or TOBMSC.

For visual binaries to be resolvable we require that one or other of

\[ \Delta H_p < 2.5(\rho - 0.1'') , \]

\[ \Delta H_p < 4.5 \log(\rho/0.05'') , \]

be satisfied, where \( \rho \) is the separation in arcsecs and \( \Delta H_p \) is the modulus of the difference in Hipparcos magnitudes. A further condition is that the fainter (sub)component should be brighter than 14th magnitude. Criterion (21) is roughly appropriate to normal visual measurements, and the much stronger criterion (22) is roughly appropriate for high-resolution measurements with speckle or adaptive optics (Sterzik & Tokovinin 2002). For the present I assume simply that 50% of stars have been examined by one process and 50% by the other, chosen at random.

For a spectroscopic binary (SB) to be identifiable I assume that the radial velocity amplitude of the brighter component, \( K_1 \), is above a threshold value \( V_{cr} \) that is given in Table 3, and further that the period is less than \( \sim 50 \, \text{yr} \):

\[ K_1 > V_{cr} , \quad P < 18000 \, \text{d} . \]  \hspace{1cm} (23)

The threshold \( V_{cr} \) depends on spectral type. Theoretical SBs can be single or double lined, depending on the luminosity ratio. The value of \( K_1 \) includes an inclination which was assigned randomly to each sub-binary in the Create stage. This implies the assumption that inclinations at different hierarchical depths are uncorrelated. It is hard to test this: Muterspaugh et al. (2006) showed that only six triples have unambiguous determinations of the inclination of one orbit to the other (as distinct from the inclination of each orbit to the line of sight). It is unlikely that 6 data points would give a clear determination of the statistics. In fact the inclinations were not very consistent with a complete lack of correlation, nor with complete correlation. Sterzik & Tokovinin (2002) considered 135 visual triples, and estimated the mean angle between orbits as 67° ± 9°. This value would be 90° if the two angular momenta were uncorrelated; there is no evidence for such a correlation. However, for most of these systems there was a 180° ambiguity in the position angle of one or other line of nodes, which creates a further ambiguity in the relative angle of the angular momenta; this ambiguity was missing in the much smaller sample of Muterspaugh et al. (2006).

The inclination assigned in Create also says, along with the period and radii, whether a system or subsystem will eclipse, and further gives an estimate of the depth of eclipse. Provided the eclipse depth is sufficient, such eclipsers are added to the tally of recognised theoretical binaries. However all theoretical eclipsers turn out to be theoretical SBs, whether single or double-lined. I do not at present count ellipsoidal variables, although they are included in the
Table 5. Multiplicity Frequency by Spectral Type.

| sp     | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | tot. | av. |
|--------|----|----|----|----|----|----|----|----|-----|-----|
| HBMSC  |    |    |    |    |    |    |    |    |     |     |
| O      | 13 | 10 | 7  | 4  | 2  | 2  | 0  | 0  | 38  | 2.42|
| eB     | 213| 125| 46 | 8  | 6  | 1  | 2  | 0  | 401 | 1.70|
| IB     | 353| 215| 61 | 21 | 1  | 2  | 0  | 0  | 653 | 1.63|
| A      | 501| 336| 63 | 17 | 6  | 5  | 0  | 0  | 928 | 1.61|
| F      | 304| 219| 36 | 15 | 4  | 0  | 0  | 0  | 578 | 1.61|
| G      | 323| 217| 33 | 12 | 1  | 1  | 0  | 0  | 587 | 1.56|
| K      | 739| 263| 32 | 8  | 0  | 0  | 0  | 0  | 1042| 1.34|
| M      | 270| 53 | 7  | 1  | 0  | 0  | 0  | 0  | 331 | 1.21|
| tot.   | 2716| 1438| 285| 86 | 20 | 11 | 2  | 0  | 4558| 1.53|
| RTBMSC |    |    |    |    |    |    |    |    |     |     |
| O      | 7  | 15 | 6  | 7  | 6  | 2  | 2  | 2  | 47  | 3.31|
| eB     | 76 | 162| 56 | 39 | 38 | 15 | 10 | 6  | 402 | 2.80|
| IB     | 247| 339| 71 | 25 | 9  | 4  | 3  | 2  | 699 | 1.92|
| A      | 320| 453| 102| 27 | 9  | 3  | 3  | 1  | 917 | 1.88|
| F      | 211| 264| 67 | 26 | 6  | 4  | 2  | 1  | 581 | 1.93|
| G      | 172| 269| 58 | 19 | 7  | 3  | 1  | 1  | 529 | 1.94|
| K      | 351| 490| 122| 45 | 17 | 7  | 6  | 4  | 1041| 1.99|
| M      | 77 | 188| 36 | 15 | 9  | 6  | 4  | 1  | 335 | 2.19|
| tot.   | 1459| 2179| 517| 202| 101| 44 | 30 | 18 | 4550| 2.04|
| TOBMSC |    |    |    |    |    |    |    |    |     |     |
| O      | 14 | 15 | 8  | 6  | 2  | 1  | 0  | 0  | 47  | 2.41|
| eB     | 192| 126| 44 | 24 | 12 | 3  | 1  | 0  | 402 | 1.88|
| IB     | 458| 205| 28 | 6  | 2  | 0  | 0  | 0  | 699 | 1.41|
| A      | 561| 300| 47 | 9  | 1  | 0  | 0  | 0  | 917 | 1.46|
| F      | 333| 193| 41 | 11 | 3  | 1  | 0  | 0  | 581 | 1.56|
| G      | 278| 205| 36 | 8  | 1  | 0  | 0  | 0  | 529 | 1.58|
| K      | 616| 325| 74 | 16 | 6  | 2  | 1  | 0  | 1041| 1.54|
| M      | 198| 102| 23 | 6  | 3  | 2  | 1  | 0  | 335 | 1.56|
| tot.   | 2649| 1472| 302| 85 | 29 | 9  | 3  | 0  | 4550| 1.55|

Early B stars (B0 – B3.5) are called ‘eB’; later B stars are called ‘lB’. Wolf-Rayet stars are included under O; S and C stars under M. The second last column is the total number; the last column is the average multiplicity.

HBMSC: the observed Hipparcos-Bright Multiple Star Catalogue (ET08). RTBMSC: a Raw Theoretical Bright Multiple Star Catalogue (Section 3). TOBMSC: a Theoretically Observed Bright Multiple Star Catalogue (Section 4). Each row of RTBMSC and TOBMSC is an average of 10 simulations, rounded to the nearest integer. Consequently the row or column for ‘total’ is not necessarily the exact total.

5 DISCUSSION

Tables 4, 5 and 6 present some details of the multiplicity model that appears to represent reasonably well the observed data of the HBMSC. Table 4 shows the distribution over multiplicity. The TOBMSC has somewhat too many triples and quintuples, somewhat too few binaries and sextuples, and roughly the right average. The last, of course, is not by accident; the entries in Table 2 were varied from ones initially guessed until the average came out about right. I should emphasise again that only those values in Table 2 that relate to \( m_{\text{total}} \) \( \geq \) unity are tested by the HBMSC.

The top third of Tables 5 and 6 gives the HBMSC data from ET08. The middle third gives the equivalent data that emerges from the Create stage described in Section 3. The bottom third gives the result of processing the middle third through the selection effects of the Observe stage of Section 4. I ran ten cases differing only in the random number selection. Tables 4 – 6 present the average of these ten, and in some cases the rms scatter. The 10 cases gave an average total of 4550 ± 148 systems. Because the middle and lower thirds are both averages of 10 cases, the rows and columns labelled ‘total’ in Tables 5 and 6 are not necessarily the exact totals but only the approximate totals of the integers in the bodies of these Tables. Note that the RTBMSC in Table 4 produced 18 ± 7 octuple systems, but none of these fully survived the scrutiny of the Theoretical Observatory.

When in Table 5 we compare the distribution over spectral type (second-last column) of the TOBMSC with the HBMSC we get a \( \chi^2 \) of 11.2, as already quoted in Section 2. When we compare the distribution over multiplicity (last row) of the TOBMSC with the HBMSC we get a \( \chi^2 \) of 7.3. The first value is on the margin of significant or non-significant discrepancy at the 10% level, while the second favours no significant discrepancy. However a \( \chi^2 \) test is not really going to validate (or contradict) the model, since we could have the same \( \chi^2 \) using either (a) a model of multiplicity that gives more multiples, combined with a model of selection effects that makes them harder to recognise, or
Table 6. Period Distribution in Systems and Subsystems.

| log \( P(\text{yr}) \) | -3.0 | -2.0 | -1.0 | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | tot. |
|------------------------|-----|-----|-----|----|----|----|----|----|----|----|----|----|-----|
| HBMSC                  |     |     |     |    |    |    |    |    |    |    |    |    |     |
| O                      | 0   | 5   | 11  | 5  | 0  | 4  | 3  | 6  | 12 | 1  | 0  | 0  | 47  |
| eB                     | 0   | 25  | 41  | 21 | 14 | 26 | 29 | 33 | 21 | 3  | 0  | 1   | 235 |
| IB                     | 0   | 25  | 53  | 24 | 20 | 43 | 62 | 63 | 48 | 16 | 8  | 0   | 362 |
| A                      | 0   | 27  | 62  | 25 | 24 | 24 | 46 | 61 | 44 | 26 | 14 | 3   | 305 |
| F                      | 1   | 7   | 9   | 20 | 46 | 78 | 78 | 66 | 47 | 24 | 9  | 0   | 264 |
| G                      | 0   | 4   | 11  | 56 | 35 | 35 | 42 | 35 | 26 | 4  | 0   | 0   | 252 |
| K                      | 0   | 27  | 62  | 25 | 24 | 24 | 46 | 61 | 44 | 26 | 14 | 3   | 305 |
| M                      | 1   | 0   | 0   | 4  | 9  | 10 | 4  | 9  | 9  | 3  | 3  | 0   | 52  |
| tot.                   | 2   | 107 | 213 | 134| 224| 275| 318| 297| 242| 128| 36 | 2   | 1979 |
| RTBMSC                 |     |     |     |    |    |    |    |    |    |    |    |    |     |
| O                      | 0   | 16  | 21  | 12 | 15 | 12 | 11 | 5  | 2  | 1  | 1  | 0   | 107 |
| eB                     | 1   | 84  | 96  | 87 | 89 | 101| 99 | 77 | 48 | 25 | 10 | 6   | 724 |
| IB                     | 5   | 29  | 42  | 53 | 79 | 106| 101| 57 | 27 | 9  | 4   | 0   | 641 |
| A                      | 8   | 34  | 52  | 62 | 97 | 142| 128| 77 | 31 | 12 | 9   | 0   | 810 |
| F                      | 8   | 29  | 38  | 51 | 64 | 86 | 104| 74 | 50 | 21 | 8   | 7   | 539 |
| G                      | 5   | 19  | 33  | 48 | 58 | 92 | 87 | 78 | 47 | 19 | 7   | 5   | 497 |
| K                      | 4   | 50  | 77  | 86 | 122| 166| 174| 164| 94 | 56 | 25 | 13  | 1032|
| M                      | 1   | 18  | 35  | 36 | 56 | 48 | 72 | 60 | 25 | 6  | 7   | 0   | 399 |
| tot.                   | 32  | 278 | 394 | 434| 556| 764| 811| 705| 439| 206| 77 | 52  | 2315|
| TOBMSC                 |     |     |     |    |    |    |    |    |    |    |    |    |     |
| O                      | 0   | 10  | 21  | 12 | 15 | 12 | 11 | 5  | 2  | 1  | 1  | 0   | 49  |
| eB                     | 0   | 42  | 56  | 28 | 8  | 7  | 35 | 47 | 36 | 16 | 7  | 5   | 286 |
| IB                     | 0   | 21  | 17  | 7  | 18 | 51 | 66 | 38 | 20 | 7  | 3   | 0   | 263 |
| A                      | 0   | 29  | 36  | 27 | 33 | 79 | 85 | 56 | 22 | 10 | 7   | 0   | 399 |
| F                      | 1   | 18  | 37  | 36 | 30 | 52 | 52 | 33 | 12 | 5  | 5   | 0   | 307 |
| G                      | 1   | 13  | 34  | 42 | 59 | 33 | 44 | 29 | 12 | 5  | 3   | 0   | 297 |
| K                      | 0   | 37  | 57  | 95 | 93 | 35 | 69 | 45 | 30 | 11 | 5   | 0   | 536 |
| M                      | 0   | 12  | 25  | 19 | 17 | 28 | 6  | 24 | 35 | 6  | 4   | 2   | 178 |
| tot.                   | 2   | 164 | 249 | 232| 233| 270| 297| 394| 275| 120| 48 | 31  | 2315|

HBMSC: the observed Hipparcos-Bright Multiple Star Catalogue (ET08).
RTBMSC: a Raw Theoretical Bright Multiple Star Catalogue (Section 3).
TOBMSC: a Theoretically Observed Bright Multiple Star Catalogue (Section 4).
Each row of RTBMSC and TOBMSC is an average of 10 simulations, rounded to the nearest integer. Consequently the row or column for ‘total’ is not necessarily the exact total.

(b) the converse of that. I am not claiming that both halves of the model are correct, but only that they represent a reasonable starting point. Most stellar population synthesis calculations discount entirely multiplicity greater than two, and I believe the present model is somewhat better than that.

The distribution over spectral type (second-last column of Table 5) is the same as in the last two rows (‘multiple’) in Table 1, apart from the fact that Table 1 contains the rms spread as well as the average. The only difference between the model in Table 5 and the ‘OB down’ model of Table 1 is that multiples are now allowed. Probably the remaining discrepancies can be improved by varying the IMF a little. However there are at least two other ways of altering them: (i) ‘convective overshooting’, an effect modeled in my stellar-evolution code on the lines of Schröder et al. (1997) and Pols et al. (1997), can vary the length of time spent on the MS, differentially as a function of mass; and (ii) small changes, of order 50K, in the temperature boundaries of spectral types in Table 3 can send quite a lot of systems from say type A to type F. I prefer for the time being the rather considerable simplicity of the model used here, without trying to further improve the agreement.

The selection effects implemented in Observe are very crude, but they seem to imply that in order to get approximately the observed average multiplicity of 2.42 for O stars, we need to start with an average well in excess of 3.0. This average has to drop fairly rapidly to about 2 at later spectral types in order that the observed multiplicity should drop to ∼ 1.6, and even lower for M stars. The average multiplicity in the entire ensemble, before the Observe stage, was 2.04 ± 0.03. After Observe, it dropped to 1.55 ± 0.02. In the HBMSC, and of course also the TOBMSC, triples and higher multiples constitute about 9% of systems, but in the RTBMSC they constitute 20%, which I suggest is a more realistic assessment of their frequency.

In Table 6, the ‘observed’ periods of the HBMSC that are fairly long, in particular those over ∼ 200 yr, are estimates that are based on Kepler’s Law, a measured angular separation and parallax, and a mass based on spectral type. They are obviously very uncertain, but perhaps by not much worse than one bin (a factor of 10) either way. We see that the effect of selection, for OB stars, is to turn a largely unimodal period distribution as seen in the RTBMSC section of Table 6 into a strongly bimodal distribution as seen in the TOBMSC and HBMSC sections. The ‘missing’ systems at periods of ∼ 1 – 100 yr are presumably too wide (and broad-lined) for radial velocity curves, and too close for visual resolution.

In fact a slight bimodality can be seen even in the
RTBMSC period distribution for type O, which comes from the fact the shorter periods are more probably from inner pairs via equation (18) than from outer pairs via equation (17). But the bimodality is much more marked when the selection effects are allowed for, and runs all the way from type O to type A, with a minimum at about 10 – 100 yrs as observed.

The most obvious discrepancy between the HBMSC and TOBMSC sections of Table 6 is in the bottom left-hand corner, for types G/K/M and periods ≲ 1 yr. The fact that there are many in the TOBMSC and and few in the HBMSC is due to binary interaction. I have emphasised that this is not (yet) included in the Create code, and so the model is comfortable producing say a 0.01 yr binary containing a well-evolved red giant, that ought to have filled its Roche lobe some time ago. Such a system might now resemble a well-evolved red giant, that ought to have filled its Roche mass companion that is (probably) a white dwarf, in a 40.1 d circular orbit. This orbit was not known previously, and is not included in ET08. 204 ± 27 such systems can be expected. Some will probably have merged into single stars, and others to some other part of the Table. 17 binaries or sub-binaries are known in the HBMSC to be semidetached, i.e. currently undergoing RLOF. A further handful, such as α Per, γ Cas, θ Tuc and ζ Cet, and now α Leo, are believed to be post-RLOF objects. But we can be confident that there are many more in this category that would be hard to recognise by any means.

Leaving aside the bottom left-hand corner, we can see that there is reasonable agreement over the rest of the Table. I do not believe it would be helpful to try and quantify this by some χ² statistic, or to overegg the pudding by trying to make the agreement even better with adjustments to, for instance, the period distributions (17) and (18).

There appears to be another discrepancy: the total number of objects in the HBMSC is 1979, and in the TOBMSC 2315. But this is because there are several systems or subsystems (in fact 384) in the HBMSC that do not have a measured, or even estimated, period. These are for example the ‘astrometric accelerators’ of Makarov & Kaplan (2005), radial velocity variables where probably the motion is orbital but no orbit has been determined (Nordström & Andersen 1985) and about 40 Ba stars for which a white-dwarf companion is expected and yet no orbit has yet been measured.

One of the more important uncertainties in the model is the mass-ratio distribution (19). It is unlikely that a single value of the exponent α will apply independently of m, P and hierarchical depth l. It would be desirable to check this much more rigorously, by using observed mass ratios. However in observational analysis the mass ratio is one of the more difficult quantities to measure. A spectroscopic binary has to be double-lined, with the secondary preferably not very faint so that its amplitude can be well determined. This means that mostly only systems with Q fairly close to unity are measurable, and so the regime of Q ≲ 0.5 is not well tested. The TOBMSC model of Tables 5 and 6 produced 198 ± 21 double-lined and 854 ± 56 single-lined spectroscopic systems or subsystems. In the HBMSC 638 systems or subsystems are SB1 or SB2. The discrepancy is fairly large, but is quite probably because all (theoretical) systems or subsystems with P ≲ 50 yrs and large enough radial velo-

ity amplitude (Table 3) are recognised as SB, whereas it is likely that many real systems in the period range of say 10 – 50 yrs have not been recognised. I am not claiming that a global choice of α = 0.8 is correct, but only that it works reasonably well to produce an ensemble like that observed.

The TOBMSC contained 85 ± 12 eclipsing binaries, against 130 in the HBMSC. The RTBMSC (Table 6) contained as many as 32 very short-period (P < 0.001 yr) binaries, most of which were eclipsing; but these were all double-M-dwarf sub-binaries of systems whose primary had the type listed, and all but two on average were undetectable in the Theoretical Observatory. The HBMSC contains two systems at such short periods, both being subsystems in fact. The well-known M-dwarf eclipsing sub-binary in the sextuple α Gem is almost in the same category, but its slightly longer period puts it in the 0.001 – 0.01 yr bin.

Barium-rich (Ba) stars are thought (McClure 1983, Boffin & Jorissen 1988) to be binaries where the primary is now a white dwarf, and where the secondary, a G/K giant, has been contaminated from s-processed material in the envelope of the white dwarf’s precursor when it was near the tip of the Asymptotic Giant Branch (AGB). There are 53 Ba stars in the HBMSC: 11 of them have known SB orbits, with periods ranging from 285 to 6500 days; and 3 of them have known white dwarf companions, notwithstanding the difficulty of recognising a white dwarf companion whose luminosity will usually be one part in 10³ or 10⁴ of the red giant. In the TOBMSC, all systems contain a white dwarf, a G/K giant, and with period in the range 100 – 10000 d were identified as potential Ba stars. There were 37 ± 15, as compared with the 53 in the HBMSC. But many Ba stars in the HBMSC are only mildly enriched in Ba, and if this can be achieved in wider binaries with periods up to 20000 d then there is no discrepancy (56 ± 25).

I emphasise again that the HBMSC only includes systems more massive than about 1 M⊙ in total. Coincidentally, but conveniently, this is also the set of stars that are capable of evolving significantly in the course of the Galaxy’s lifetime. Thus this model may be quite adequate for exploring the evolution of an ensemble of multiples (including by definition singles) on the scale of a galaxy, or a galactic cluster. In the context of a cluster, we can be reasonably sure that the wider systems will be broken up quite quickly, provided that the cluster remains rich for a reasonably long time. However, current thinking is that all stars form in rich or fairly rich SFRs. This must therefore include the rather highly-multiple, yet also quite wide, O star multiples of the HBMSC, which may indeed be the result of (a) gravitational diffusion inwards of the most massive stars of a cluster, and (b) dissipation of the cluster because of the ejection of significant mass by, for instance, supernova explosions. I believe that one of the more rigorous tests of star-formation models will be that they produce the right spectrum of multiplicity, as well as the right spectrum of masses and orbital periods.

A striking result of rather recent analyses (Tokovinin et al. 2006, Pribulla & Rucinski 2006), confirmed by ET08, is that short-period binaries (P < 3d) are very commonly in triples, and arguably are exclusively in triples (or higher multiples); and this then implies that tripleness is necessary for the production of a short-period binary. The necessary mechanism may be the combination of Kozai cycles and tidal friction: KCTF (Mazeh & Shaham 1979, Kiseleva
et al. 1998, Eggleton & Kisseleva-Eggleton 2006, Fabrycky & Tremaine 2007). It may be that the process of star formation does not directly produce binaries with periods of less than say 0.1 – 1 yr, but it produces enough triples, with suitably oblique orbits, that these short periods can be generated by the KCTF mechanism, in the course of 10^3 – 10^9 yr depending on the initial parameters. Once an orbital period has been reduced to 2 – 3 d by KCTF, those systems which contain late-type dwarfs (F/G/K/M) may evolve to still shorter periods by the quite different process of magnetic braking and (again) tidal friction, or MBTF. This can reduce the period to the point where a contact binary is formed, on a timescale that may be roughly 10^7 – 10^10 yr.

The reader will note that there is something of a contradiction between
(a) offering a formulation that is intended to give roughly the right distribution of periods at age zero,
(b) generating arbitrarily short periods via equations (17) and (18), although with the distribution truncated when the period is so short that the stars would be touching on the ZAMS, and
(c) hypothesising that the shortest periods (<3 d) are in fact due to the KCTF and MBTF mechanisms, and not to formation at such short periods.

I would hope, in a future analysis, to investigate whether by considering the interactive evolution of triples hypothesis (c) can be supported, but until then it seems desirable to produce about the right frequency of such short-period systems ab initio.

Part of the above apparent contradiction is because ‘age zero’ is rather hard to define. Returning to the brief discussion of star formation in the introduction, the bright stars in the Solar neighbourhood presumably formed in SFRs, and then escaped as the SFRs evaporated; but they may have been subject to dynamical interaction within their parent SFRs for at least several million years, so that their multiplicities and periods may have been modified in that interval. ‘Age zero’ can be surprisingly well defined for individual stars when their nuclear evolution alone is being discussed. But ‘age zero’ is much harder to define when binaries and higher multiples are being considered.

The purpose of this paper has been to produce a model for the multiplicity statistics of an observed complete magnitude-limited sample of substantial size. Such a model must include selection effects, but even the best such model will not be definitive. In particular, one can add to almost every observed system a T dwarf that would hardly be recognisable by current technology, if located at a suitable distance from the primary component. We can only attempt to find something like a lower limit to multiplicity; but I believe that the model proposed here is something like a lower limit.

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