Nonequilibrium Aspects of Quantum Field Theory

Travis R. Miller $^a$, Michael C. Ogilvie $^a$

$^a$Dept. of Physics, Washington University, St. Louis, MO 63130 USA

We have developed a method for extracting equilibrium observables from non-equilibrium simulations by rapidly changing the temperature and recording the subsequent evolution of the Polyakov loop. Both nucleation and spinodal decomposition are observed to occur. In the latter case the Polyakov loop correlation function shows exponential growth for wavenumbers less than or equal to the critical wavenumber $k_c$. We have constructed the bare as well as the effective potential for the Polyakov loop, from which $k_c$ and $m_D/k_c$ can be extracted as a function of temperature. The shift from spinodal decomposition to nucleation as the dominant equilibration mechanism occurs at the spinodal temperature that separates these two regimes.

1. Introduction

Spinodal decomposition is characterized by the appearance of explosive growth in the low momentum modes of the system, while the high momentum modes relax to their equilibrium distributions. The critical wavenumber $k_c$ which separates low and high momentum behaviors is determined by the thermodynamic potential which governs the long wavelength behavior of the system. We have previously shown that the confined phase of pure $SU(2)$ gauge theory decays via spinodal decomposition after a rapid change from below the deconfinement temperature to temperatures above $T_d$. The first order character of the deconfinement transition in pure $SU(3)$ gauge theory implies that the confined phase is metastable for a range of temperatures above the deconfinement temperature $T_d$. At higher temperatures $T > T_s > T_d$ the confined phase is unstable and decays via spinodal decomposition. The spinodal temperature $T_s$ separates the metastable and spinodal temperature ranges.

2. Dynamics of the Polyakov Loop

As a consequence of asymptotic freedom, the perturbative form of the effective potential for the Polyakov loop at high temperatures is valid. Writing the Polyakov loop in the form $P_F = 1 + 2cos(2\pi/3 - \psi)$, the perturbative form of the effective action is at 1 loop $\mathcal{S}_{eff}$

$$S_{eff} = \frac{4T^2}{g^2} \int d^3x \left( \frac{1}{2} (\nabla \psi)^2 - \frac{1}{2} \frac{g^2T^2}{3} \psi^2 - \frac{g^2T^2}{6\pi} \psi^3 + \frac{3g^2T^2}{8\pi^2} \psi^4 \right),$$

where the constant black body term has been discarded.

We assume that the long distance behavior of the Polyakov loop 2-point function can be calculated using the 1-loop perturbative effective action $S_{eff}$ and the Langevin equation,

$$\frac{\partial P}{\partial t} = -\frac{\delta S_{eff}}{\delta P^*} + \eta$$

where $\eta$, the Gaussian noise term, is such that the equipartition theorem holds. A linearized solution to the Langevin equation gives for the Polyakov loop 2-point function (also referred to as the structure function)

$$\left\langle \bar{\psi}(k) \psi(-k) \right\rangle = \left| \bar{\psi}(k,0) \right|^2 e^{-2\alpha(k^2+m^2)t} + \frac{T}{k^2+m^2} \left(1-e^{-2\alpha(k^2+m^2)t}\right),$$

where $\alpha = 4\Gamma T^2/g^2$.

When the equation is linearized about $\psi = 0$ the mass squared is negative $m^2 = -k_c^2$. 

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\(-g^2T^2/3\) causing exponential growth for values of the wavenumber less than the critical value \(k_c\).

At late times, the equation is linearized about the non-trivial minimum at \(\psi = 2\pi/3\). Here the mass is the Debye screening mass \(m_D = gT\), and the system relaxes to the non-trivial minimum.

3. Method

3.1. The Bare Potential

After integrating out all other fields at the scale \(a\) we are left with an action just in terms of the Polyakov loop. The minimal Landau-Ginsberg action which is gauge invariant, Z(3) invariant, and only involves terms up to the fourth power in the Polyakov loop is

\[
S = \int d^3x[|\nabla P_F|^2 + a_2(P_F^2P_F^*) + \frac{a_4}{2}(P_F^3 + (P_F^*)^3)] + b_3(P^3 \langle P \rangle + (P^*)^3),
\]

where \(P^* = tr_F(P)\) is the trace of the Polyakov loop in the fundamental representation. The Langevin equation without noise is then used as an ansatz for the evolution of the low momentum modes. In Fourier space this yields an equation of the form

\[
\frac{d}{dt} F[P_F] = c_1(k^2)F[P_F] + c_2(k^2)F[(P_F^*P_F)P_F^*P_F] + c_3(k^2)F[(P_F^*)^2].
\]

The behavior of the lattice simulation is fit to the above equation for each \(k\) mode yielding the parameters. A fit of \(c_1(k^2)\) versus \(k^2\) gives the critical wavenumber \(a_2 = -k_c^2\).

3.2. The Effective Potential

The potential obtained above has a minimum which is not the equilibrium value of the vacuum expectation value of the Polyakov loop (see figure 2). This is the bare potential defined at the scale \(a\), while the effective potential can be defined as the generator at zero momentum of 1PI vertices. To find the effective potential, the behavior of the 1-point function is fit to the ansatz

\[
V(P_{cl}) = a_2|P_{cl}|^2 + \frac{a_4}{2}|P_{cl}|^4 + b_3 \left(P_{cl}^3 + (P_{cl}^*)^3\right)
\]

where \(P_{cl} = \langle P \rangle\).

Without multiple momentum modes to fit, the absolute normalization of the coefficients of the effective potential can not be determined. However the ratio of the curvature at the nontrivial minimum to the curvature at zero is equal to the ratio of the square of the Debye mass to the square of the critical wavenumber

\[
\frac{V''(P_{min})}{V''(0)} = \frac{m_D^2}{k_c^2} \sim 3,
\]

which is comparable to the perturbative result for this ratio. The minimum of the effective potential is located at the final equilibrium value.

4. Results

We start from equilibrated lattices of size \(64^3 \times 4\) at \(\beta = 5.5\) and change the value of \(\beta\) to a range of values above the deconfinement value. We use the Cabbibo-Marinari heat bath algorithm both for equilibration and for the evolution. The large lattice sizes were required to resolve the long wavelength modes, and to reduce the effects of changing the physical volume.

Quenches to just above the deconfinement temperature show evidence of metastability. However more extreme changes in the temperature yield behavior consistent with spinodal decomposition. Typical behavior of the structure function \(S(k,t)\) after a quench to \(\beta = 6.0\) for the three lowest non-trivial wavenumbers is displayed in figure 1.
for a single run. The initial exponential rise in the structure function is evident. At intermediate times the growth levels off, with the appearance of “shelves” in some of the modes which is presumably a consequence of mode-mode coupling. At late times the modes relax to their final equilibrium values.

![Figure 2](image1)

**Figure 2.** $V(P)$ vs. $P$

In figure 2 the bare and the effective potential are plotted. As the overall normalization of the effective potential cannot be fixed by the present methods, the normalization was chosen so that both curves could be plotted side by side. These potentials were obtained by averaging over several runs and the error bars were estimated using a jackknife analysis. Note the significant shift in the location of the minimum from the bare to the effective potential. This indicates that fluctuations are large even at temperatures of the order of $2T_d$.

In figure 3 the value of the critical wavenumber determined from the bare potential is plotted versus temperature for the four temperatures considered. The temperature dependence is fit to the ansatz $k_c^2 = aT^2 + bT_d^2$, which is a form suggested by a model for the pure gauge thermodynamics\[5\]. The fit is used to determine the spinodal temperature, which is defined by $k_c(T_s) = 0$. The spinodal temperature found is $T_s = 1.29(7)T_d$ and corresponds to $\beta_s \approx 5.81$.

![Figure 3](image2)

**Figure 3.** $k_c(T)$ vs. $T/T_d$

5. Conclusion

Pure SU(3) gauge theory exhibits both metastability and spinodal decomposition following a quench to temperatures above the deconfinement temperature. In the spinodal region the evolution can be modeled using Langevin dynamics, and the bare and effective potentials for the Polyakov loop field can be constructed. These potentials determine $k_c$ and $m_D/k_c$. We find a nontrivial temperature dependence for the critical wavenumber. As the temperature is lowered the critical wave number decreases. The extrapolation to zero of $k_c(T)$ gives a spinodal temperature slightly above the deconfinement temperature.

REFERENCES

1. T. Miller and M. Ogilvie, Phys. Lett. B488 (2000) 313; Nucl. Phys. B(Proc. Suppl.)94 (2001) 419.
2. D. Gross, R. Pisarski and L. Yaffe, Rev. Mod. Phys. 53 (1981) 43.
3. N. Weiss, Phys. Rev. D24, 475 (1981); Phys. Rev. D25 (1982) 2667.
4. T. Bhattacharya, A. Gocksch, C. Korthals Altes and R. Pisarski, Phys. Rev. Lett. 66 (1991) 998; Nucl. Phys. B383 (1992) 497.
5. P. Meissenger, T. Miller and M. Ogilvie, hep-ph/0108009