Accuracy of Geopotential Models Used in Smartphone Positioning in the Territory of Poland

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Accuracy of Geopotential Models Used in Smartphone Positioning in the Territory of Poland

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Abstract. Effective realisation of both land and sea transport has always required the determination of safe and fast routes. In the era of technological progress and modern technologies, the classic methods of navigation and paper maps have been practically replaced by satellite positioning and marine electronic maps. The degree of occurring changes is evident in the widespread use of mobile phones and tablets in car navigation and recreation. Among the determined geolocation data in these fields, only the ellipsoidal longitude and latitude of the receiver are most commonly used. Knowledge of these coordinates is sufficient to present the position on the maps available online or offline. The third of the coordinates, ellipsoidal height, finds particular application in some areas of recreation, such as mountain climbing or cycling to present the differences in elevation along the track. However, the ellipsoidal heights related to the reference ellipsoid, excluding some regions of the Earth, are not identical to the normal heights used widely in many European countries. The conversion of ellipsoidal heights into normal heights takes place through the use of geopotential models that are currently commonly implemented in the GNSS modules of smartphones. Geopotential models, which are discrete sets of quasigeoid spacing values from the reference ellipsoid, allow interpolation of the local undulation value added later on to the determined ellipsoidal height of the receiver. The publication will present the results of the conducted comparative analysis of geopotential models of selected telephones with the EGM2008 model developed by The National Geospatial-Intelligence Agency (NGA). The analysis will concern the results of measurements carried out in selected regions of Poland. The cohesion of both geopotential models will be shown, which directly translates into the correctness of the heights computed by the phone receiver. The conducted research proves the occurrence of significant distortions in the calculated values of undulations. The examined issue, in the perspective of the rapid development of personal navigation and the increasing accuracy of the receivers of mobile phones, is an important factor affecting the credibility of the positioning results.

1. Introduction

Widespread miniaturization and constantly upgrading electronic modules enables the creation of portable devices and smartphones with very high computational parameters. Adding the possibility of implementing a series of sensors in the devices [1] results in extremely efficient, useful and universal devices that can be adapted to user’s individual needs. Hence the rapid development of the market of mobile applications [2] for everyday navigation [3], sports and recreation [4], tourism [5] and many other areas of human activity. One can hypothesize that nowadays it would be extremely difficult to part with positioning dedicated services. The conducted research proves the relatively high accuracy of the position determination in relation to the price of mobile devices [6]. In addition, the continuous
development of the global navigation satellite systems (GNSS) and the modernisation of the satellites constellation [7] leads to increasing positioning accuracy [8], [9]. Hence, the GNSS receivers can be used, among others, in construction [10] or in surveying [11]. It is worth noting that the majority of research concerns determining the smartphones horizontal position accuracy with the minor mentioning of the 3D position [12], [13]. There is definitely the demand response for horizontal positioning, which can then be applied to the map available both on-line and off-line.

The GNSS receivers are smartphone sensors that transmit data to the applications via NMEA messages. The abbreviation comes from the National Marine Electronics Association - an American organization that creates communication standards between various types of marine electronics [14]. NMEA 0183 is a standard commonly used in navigational and marine devices such as sonars, GNSS receivers, gyro-compasses and others. A feature of the standard is the presence of numerous types of messages containing specific parameters registered or calculated by a GPS or GNSS receiver. The GPGGA message contains the global positioning system fix data. The GPS/GNSS receiver, after determining the position, generates a string of ASCII characters with a strictly defined structure containing the type of the sentence, data logging time, latitude, longitude, GPS/GNSS fix quality, number of visible satellites, Horizontal Dilution of Precision (HDOP) value, altitude, height of geoid above ellipsoid of reference (undulation), DGPS additional parameters, and finally a checksum for transmission errors detection. The NMEA 0183 standard has been implemented in GNSS receivers of popular smartphones. Users can choose from a range of mobile applications to display NMEA sentences generated by the receiver. In addition to GPGGA, numerous other types of sentences containing additional detailed information exist for example describing visible satellites (GPGSV), vessel track made good and ground speed (GPVTG), DOP values (GPGSA) or vessel course parameters (GPRMC).

The publication focuses on the undulation parameter defining the spatial relation of two vertical surfaces of reference - ellipsoid of reference and geoid [15]. The Earth Gravitational Model 2008 (EGM2008) was used as a reference to the undulation values generated by the GNSS receivers of smartphones. The EGM2008 model was created by National Geospatial-Intelligence Agency (NGA) based on several previous models: Preliminary Gravitational Model 2007A (PGM2007A), GRACE and Dynamic Ocean Topography (DOT) [16]. Hence, it is an important source of information about the Earth's geopotential and the geoid.

2. Materials and methods

The high resolution of the EGM2008 model achieved by the reiteration of a number of separate data sets is a reliable reference object to the geoid models implemented in the smartphones' GNSS receivers. The global range of the EGM2008 model was limited in the first stage to the coordinates of Poland's borders with a minor external buffer (figure 1). This resulted in a regular 5' grid of geoid-ellipsoid separation values.

To illustrate the models accuracy, the NMEA 0183 (GPGGA) message was recorded in several regions of Poland. Measurements were made along several routes (profiles) crossing the country's territory from north to south (Gdynia - Krosno, Ilawa - Zamosc), from south to north (Poznan - Mielno), from west to east (Poznan - Warszawa) and from east to west (Gdansk - Szczecin). In addition, several single measurements were made to cover the largest possible research area: in the north-east (Pisz) and in the south (Krakow, Zakopane, Katowice, Raciborz, Laziska).
The sentences were recorded using the GPS Info & NMEA Logging application [17] using several smartphone models. The recording of the NMEA 0183 message on the profiles took place at selected locations on the route. The smartphones used came from different manufacturers and often differed in terms of technical parameters (table 1). From the point of view of positioning, the number of GNSS systems used is of particular importance, which determines the number of satellites visible in the sky and DOP coefficients values that translate into the obtained positioning accuracy.

Table 1. Selected technical parameters of used smartphones.

| Device                  | Market launch | GPS | A-GPS | GLONASS | BeiDou |
|-------------------------|---------------|-----|-------|---------|--------|
| Xiaomi Redmi 4X          | 2017          | YES | YES   | YES     | YES    |
| HTC One M9 Prime Camera Edition | 2016          | YES | YES   | YES     | YES    |
| Huawei Nova             | 2016          | YES | YES   | YES     | NO     |
| Samsung Galaxy Xcover 4 | 2017          | YES | YES   | YES     | YES    |
| Samsung Galaxy J5       | 2015          | YES | YES   | YES     | YES    |
| Xiaomi Redmi 4           | 2017          | YES | YES   | YES     | YES    |
| LG G3                   | 2014          | YES | YES   | YES     | NO     |

The publication focuses on the vertical coordinate of ellipsoidal height h which is determined iteratively from the Earth-Centered Earth-Fixed (ECEF) Cartesian coordinates [18]. It should be noted that introducing the orthometric heights system with orthometric height H requires the determination of the geoid-ellipsoid separations (undulations) N. To achieve this goal, the designated horizontal
coordinates of longitude and latitude are used to interpolate the local undulation value from the geoid model implemented in the GNSS receiver.

In the GPGGA messages, two out of three mentioned parameters appear: orthometric height and undulation. As the orthometric height value depends on a number of technical conditions (including GNSS receiver clock accuracy and the number of GNSS systems used) and terrain conditions (the presence of obstacles, satellite signal multipath), an assessment of the measuring device vertical accuracy is a rather complex topic. In the case of undulations, the situation is simpler, as it is determined from a discrete geoid model only and, within an area limited by the model resolution, it takes similar or even equal values. Hence, it is possible to unambiguously determine the undulation determination error.

The reference values for the GPGGA messages undulations were obtained by interpolating the EGM2008 model according to the horizontal coordinates measured by the smartphone GNSS receiver. For a clearer presentation of the research results, the interpolation along the profiles was used. As a result, the approximate trend of the NMEA-derived undulations errors values is shown. Comparison of the undulation values generated by the smartphones GNSS receivers with the reference EGM2008 model undulation values allowed to determine the local error of the parameter.

3. Results and discussions

Tables and figures present the results of the conducted analysis. Measured undulation values (NMEA) are represented by points in the graphs (figures 2-6). The reference values (the EGM2008 model) are marked in orange. Both of the mentioned values are presented as a function of the profile mileage. The first profile crossed Poland towards the south and south-east (figure 1) from the Baltic Sea (Gdynia) to the highlands located on the side of the Bieszczady mountains (Krosno). Therefore, it included a significant range of terrain heights in Poland. It is worth noting the change in the undulation value (EGM2008 model) from 29.1 m to 37.1 m along the measurement profile, which covers a significant range of the geoid-ellipsoid separations in this region (figure 1).

![Figure 2. NMEA and EGM2008 undulations along Gdynia – Krosno profile.](image)

Tables 2-7 contain detailed information on the calculated undulation error values. The tables columns contain the names of the nearest major cities, the mileage of the measurement profile, the undulation value registered by the smartphone (NMEA) and interpolated from the EGM2008 model, the undulation error value and the model of the smartphone used.

As a result of the profile comparison, an average error value of 4.8 m was calculated with a standard deviation of 1.6 m. The median of the analysed set of values was 4.6 m. On the basis of table 2 it can be noticed that the undulation errors of the smartphone geoid model take lower values in the highland regions than in the lowlands.
Table 2. Gdynia – Krosno profile undulations differences.

| No. | Nearest City     | Mileage [km] | EGM2008 undulation [m] | NMEA undulation [m] | Difference [m] | Smartphone |
|-----|------------------|--------------|------------------------|---------------------|----------------|------------|
| 1   | Gdynia           | 0.0          | 29.4                   | 34                  | 4.6            |            |
| 2   | Pruszcz Gd.      | 39.8         | 29.6                   | 35                  | 5.4            |            |
| 3   | Starogard Gd.    | 70.4         | 29.5                   | 35                  | 5.5            |            |
| 4   | Kwidyn           | 94.6         | 29.2                   | 36                  | 6.8            |            |
| 5   | Grudziadz       | 125.7        | 29.1                   | 36                  | 6.9            |            |
| 6   | Chelmza          | 155.9        | 29.4                   | 37                  | 7.6            |            |
| 7   | Torun            | 181.1        | 29.9                   | 37                  | 7.1            |            |
| 8   | Wloclawek        | 250.6        | 31.6                   | 38                  | 6.4            |            |
| 9   | Kutno            | 293.5        | 32.3                   | 38                  | 5.7            |            |
| 10  | Lodz             | 338.2        | 33.4                   | 38                  | 4.6            |            |
| 11  | Lodz             | 361.4        | 34.1                   | 39                  | 4.9            |            |
| 12  | Piotrkow Tryb.   | 399.2        | 35.1                   | 39                  | 3.9            |            |
| 13  | Zarnow           | 439.2        | 35.9                   | 39                  | 3.1            |            |
| 14  | Kielce           | 479.5        | 36.9                   | 39                  | 2.4            |            |
| 15  | Piotrkowice      | 527.1        | 36.9                   | 40                  | 3.1            |            |
| 16  | Busko Zdroj      | 551.5        | 37.1                   | 40                  | 2.9            |            |
| 17  | Dabrowa Tarn.    | 602.5        | 36.9                   | 40                  | 3.1            |            |
| 18  | Tarnow           | 638.7        | 36.6                   | 40                  | 3.4            |            |
| 19  | Krosno           | 695.3        | 36.0                   | 39                  | 3.0            |            |

The second profile was directed to the south-east (figure 1) from Warmia (Ilawa) to Zamosc. Similarly to the previous profile of Gdynia - Krosno, there is a gradual increase in the terrain height. The undulation values changed to much smaller extent than previously in the range from 29.7 m to 31.5 m (figure 3).

Figure 3 shows the presence of similar undulations errors, which may indicate a systematic error in geoid models implemented in smartphones. An increase in undulation values between Ilawa and Pilawa cities were noted. The decreasing values were recorded on the Lublin - Zamosc part of the profile. A similar trend was observed in the reference model EGM2008 (table 3).

Detailed analysis of undulation errors on this profile proves the presence of small differences in value within the statistical sample. A mean error value of 7.1 m was obtained with a standard deviation of 0.7 m. The median of the set was 7.2 m.
Table 3. Ilawa - Zamosc profile undulations differences.

| No. | Nearest City | Mileage [km] | EGM2008 undulation [m] | NMEA undulation [m] | Difference [m] | Smartphone |
|-----|--------------|--------------|------------------------|---------------------|----------------|------------|
| 1   | Ilawa        | 2.8          | 29.7                   | 35.9                | 6.2            |            |
| 2   | Dzialdowo    | 52.2         | 30.4                   | 36.6                | 6.2            |            |
| 3   | Regimin      | 103.7        | 30.9                   | 37.5                | 6.6            |            |
| 4   | Sonsk        | 127.2        | 31.1                   | 37.9                | 6.8            |            |
| 5   | Sulejowek    | 227.9        | 30.9                   | 38.8                | 7.9            | HTC One M9 Prime Camera Edition |
| 6   | Pilawa       | 257.5        | 31.3                   | 39                  | 7.7            |            |
| 7   | Lublin       | 388.5        | 31.5                   | 38.7                | 7.2            |            |
| 8   | Piaski       | 416.6        | 30.9                   | 38.5                | 7.6            |            |
| 9   | Krasnystaw  | 440.6        | 30.4                   | 38.2                | 7.8            |            |
| 10  | Zamosc       | 471.9        | 30.9                   | 38.2                | 7.3            |            |

The third measurement profile was located along the Polish border of the Baltic Sea (figure 1) from Eastern Pomerania (Gdansk) to Western Pomerania (Szczecin). Hence, the registration took place in lowland areas with low altitudes. The growing trend of undulation along the northern sea border of Poland towards the west is worth noting (figure 4).

Figure 4. NMEA and EGM2008 undulations along Gdansk - Szczecin profile.

On the Gdansk - Szczecin profile, a large number of measurements were made in relation to other measuring profiles. Figure 4 shows abrupt changes in undulations recorded at a short distance from two measurement locations (e.g. Lisowo and Ploty). This is due to the rounding of the undulation values to one decimal place by the GNSS receiver of the Huawei Nova smartphone (table 4). Thus, an error of up to 0.5 m should be expected only due to rounding of the geoid-ellipsoid separation values in NMEA messages to integers.

The largest statistical sample in the conducted study was characterized by an average value of 4.5 m, standard deviation equal to 0.3 m and an error median of 4.4 m. It should be noted that despite the undulation values rounding, which significantly reduce the reliability of the determined heights, there is a similar trend between the recorded point values and the reference EGM2008 model. In both cases a gradual increase in value was noted. This may confirm the hypothesis regarding the presence of systematic errors in smartphone geoid models.

Another measurement profile crossed Great Poland and Pomerania from Poznan to Mielno (figure 1). In a large part, it ran along the 33.5 m undulation isoline (figure 5). As in the case of previous profiles, there were similar changes both in the NMEA and EGM2008 model undulation values.
Table 4. Gdansk - Szczecin profile undulations differences

| No. | Nearest City | Mileage [km] | EGM2008 undulation [m] | NMEA undulation [m] | Difference [m] | Smartphone |
|-----|--------------|--------------|------------------------|---------------------|----------------|------------|
| 1   | Gdansk       | 0.0          | 29.6                   | 34                  | 4.4            |            |
| 2   | Gdynia       | 9.0          | 29.6                   | 34                  | 4.4            |            |
| 3   | Gdynia       | 20.8         | 29.5                   | 34                  | 4.5            |            |
| 4   | Rumia        | 26.5         | 29.6                   | 34                  | 4.4            |            |
| 5   | Wejherowo    | 38.0         | 29.8                   | 35                  | 5.2            |            |
| 6   | Godetowo     | 60.6         | 30.5                   | 35                  | 4.5            |            |
| 7   | Lebork       | 72.7         | 30.7                   | 35                  | 4.3            |            |
| 8   | Karznica     | 100.4        | 31.4                   | 36                  | 4.6            |            |
| 9   | Redzikowo    | 114.7        | 31.8                   | 36                  | 4.2            |            |
| 10  | Sycewice     | 136.1        | 32.2                   | 37                  | 4.8            |            |
| 11  | Slawno       | 151.5        | 32.5                   | 37                  | 4.5            | Huawei Nova|
| 12  | Pekanino     | 169.9        | 32.9                   | 37                  | 4.1            |            |
| 13  | Koszalin     | 190.4        | 33.4                   | 38                  | 4.6            |            |
| 14  | Malonowo     | 223.7        | 34.2                   | 38                  | 3.8            |            |
| 15  | Plonino      | 245.8        | 34.8                   | 39                  | 4.2            |            |
| 16  | Modlimowo    | 260.2        | 35.0                   | 39                  | 4.0            |            |
| 17  | Lisowo       | 274.3        | 35.0                   | 39                  | 4.0            |            |
| 18  | Ploty        | 276.6        | 35.0                   | 40                  | 5.0            |            |
| 19  | Glewice      | 305.0        | 35.2                   | 40                  | 4.8            |            |
| 20  | Rzesnia      | 333.2        | 35.7                   | 40                  | 4.3            |            |
| 21  | Szczecin     | 352.8        | 36.1                   | 41                  | 4.9            |            |

Figure 5. NMEA and EGM2008 undulations along Poznan - Mielno profile.

It is worth noting that the measuring device used (Samsung Galaxy Xcover 4 smartphone) rounded the undulations to the nearest decimetres, not to meters as the previous model Huawei Nova (table 5).

Table 5. Poznan - Mielno profile undulations differences.

| No. | Nearest City | Mileage [km] | EGM2008 undulation [m] | NMEA undulation [m] | Difference [m] | Smartphone |
|-----|--------------|--------------|------------------------|---------------------|----------------|------------|
| 1   | Poznan       | 0.0          | 35.4                   | 40.8                | 5.4            |            |
| 2   | Poznan       | 49.5         | 34.9                   | 40.6                | 5.7            |            |
| 3   | Oborniki     | 64.9         | 34.3                   | 40.4                | 6.1            |            |
| 4   | Chodzież     | 108.1        | 33.6                   | 39.6                | 6.0            |            |
| 5   | Pila         | 140.0        | 33.6                   | 39.4                | 5.8            |            |
| 6   | Szczecinek   | 209.4        | 33.0                   | 38.2                | 5.2            |            |
| 7   | Koszalin     | 269.3        | 33.4                   | 37.7                | 4.3            |            |
| 8   | Mielno       | 288.5        | 33.6                   | 37.8                | 4.2            | Samsung Galaxy Xcover 4 |
The calculated error values were characterized by a mean value of 5.3 m and a standard deviation of 0.7 m. The median error was close to an average of 5.5 m.

The same phone model was used on the last measurement profile Poznan - Warsaw (figure 1). It ran through the central part of Poland from the west (Great Poland) to the east (Mazovia). The undulation values in the majority of cases had a decreasing trend visible both in the registered NMEA undulation values and in the reference EGM2008 model (figure 6).

Similarly to the previous profile measured by the Samsung Galaxy Xcover 4 smartphone (Poznan - Mielno), a small standard deviation value of 0.8 m was recorded. The average error value on the profile was 6.7 m with a median of 6.8 m.

To increase the reliability of the study the NMEA messages were recorded in several Polish cities located in different regions of Poland (figure 1). Particularly noteworthy are the values recorded in the southern highlands and mountainous regions of Poland. Another three smartphone models were used to register the NMEA messages, which similarly to the previously presented Xiaomi Redmi 4X (table 2) and Huawei Nova (table 4), rounded the undulations to integers. This indicates significantly lower vertical accuracy compared to the models HTC One M9 Prime Camera Edition (table 3) and Samsung Galaxy Xcover 4 (tables 5-6) which used the decimetres rounding (table 7).

**Table 6.** Poznan – Warszawa profile undulations differences.

| No. | Nearest City | Mileage [km] | EGM2008 undulation [m] | NMEA undulation [m] | Difference [m] |
|-----|--------------|--------------|------------------------|---------------------|----------------|
| 1   | Poznan       | 9.0          | 35.5                   | 40.8                | 5.3            |
| 2   | Wrzesnia     | 52.6         | 34.4                   | 40.2                | 5.8            |
| 3   | Swinice Warckie | 159.3      | 33.1                   | 39.9                | 6.8            |
| 4   | Wartkowice   | 168.4        | 33.1                   | 40                  | 6.9            |
| 5   | Strykow      | 207.3        | 33.3                   | 40.1                | 6.8            |
| 6   | Dmosin       | 222.4        | 33.1                   | 40                  | 6.9            |
| 7   | Brwinow      | 292.7        | 31.9                   | 39.3                | 7.4            |
| 8   | Konotopa     | 303.4        | 31.7                   | 39.2                | 7.5            |

**Table 7.** Undulations differences in selected Polish cities.

| No. | City         | EGM2008 undulation [m] | NMEA undulation [m] | Difference [m] | Smartphone   |
|-----|--------------|------------------------|---------------------|----------------|--------------|
| 1   | Pisz         | 29.2                   | 33                   | 3.8            | Samsung Galaxy J5 |
| 2   | Kraków       | 39.9                   | 42                   | 2.1            | Xiaomi Redmi 4 |
| 3   | Zakopane     | 42.4                   | 42                   | -0.4           | Xiaomi Redmi 4 |
| 4   | Racibórz     | 42.0                   | 43                   | 1.0            | LG G3        |
| 5   | Katowice     | 41.1                   | 42                   | 0.9            | LG G3        |
| 6   | Łaziska      | 42.3                   | 43                   | 0.7            | LG G3        |
In Krakow, there was an undulation determination error of 2.1 m. Even lower values were recorded in Silesia in Katowice (0.9 m), Raciborz (0.9 m) and border Laziska (0.7 m). The three locations are located in the area where undulations take one of the highest values in Poland territory (figure 1). In Zakopane (Tatra mountains), the only negative error value in the analysis of -0.4 m was obtained.

4. Conclusions

The publication presents the results of the analysis of smartphones geopotential models accuracy in Poland. Seven geoid models implemented in the smartphones GNSS receivers of several popular manufacturers were tested. The geoid-ellipsoid separations values were recorded in the NMEA (GPGGA) messages. These sentences contain a number of data on the position determined by GNSS receivers of different types of navigational equipment. The EGM2008 model was chosen according to the high degree of completion (2160) as a reliable reference.

For the needs of the study, measurements were made along five routes (profiles) crossing the territory of Poland in various directions. To increase the credibility of the work, additional measurements were added in several Polish cities. The EGM2008 model provided highly accurate reference undulation values along the measured profiles and locations. It enabled the calculation of the differences between the two geopotential models and determination of undulation error values recorded by the smartphones GNSS receivers.

The results of the conducted analyses prove the existence of significant differences in undulation values between the models implemented in smartphones and the reference EGM2008 model. The smartphones used for registering on the measurement profiles obtained the mean values of the undulation determination error of 4.8 m (Xiaomi Redmi 4X), 7.1 m (HTC One M9 Prime Camera Edition), 4.5 m (Huawei Nova), 5.3 m and 6.7 m (Samsung Galaxy Xcover 4) with the values of standard deviations equal to 1.6 m, 0.7 m, 0.3 m, 0.7 m and 0.8 m respectively. Hence the conclusion of several-meter errors of the geoid-ellipsoid separations, which directly translates into the height determination error by smartphones GNSS receivers, can be formed.

Depending on the altitude, individual regions of Poland have lower values of undulation determination error in highland and mountainous areas than in the lowlands and coastal areas. This is indicated by the point measurements carried out independently of the profiles in selected cities in Poland. In Silesia (the area of the largest undulation value in Poland) there was a decrease in the error value to individual meters (e.g. 0.9 m in Katowice). In the Tatra the only negative error value of the undulation in the analysis was found (-0.4 m in Zakopane).

On the basis of the conducted research it can be hypothesized that in the majority of Poland territory there are positive errors values in determining the undulation by GNSS receivers of popular smartphones. Thus, the altitudes are initially (excluding the ellipsoidal height error) increased by several meters, and in some places by nearly 8 meters. Error values generally decrease with the altitude increase. However, this is not the rule, as in the case of the highland area of the Zamosc region some of the highest undulation error values were recorded. The highest compliance of geopotential models was found in the Polish high mountains - the Tatras.

Some observation regarding the recording of NMEA messages generated by smartphones GNSS receivers needs to be emphasized. The undulations values are recoded in NMEA sentences with different values rounding. In five of the seven tested smartphone models, it is a meter precision (integer numbers). Two smartphone models recorded undulations with a precision of decimetres. However, it does not significantly improve the accuracy of geopotential models implemented in GNSS receivers.
In conclusion, it should be noted that in mobile applications installed on smartphones and using the vertical coordinates, the discrepancies with the world's leading global geopotential model were observed. At present, the majority of mobile applications are based on a horizontal position displayed on the electronic map, but the ever-growing spectrum of applications can soon go from two to three dimensions. Given the ubiquitous presence of positioning technology implemented in widely available smartphones, it is worth keeping in mind the identified undulations errors, for example in order to ensure safety of transportation.

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