Quantum entanglement makes it possible to solve some quantum communication tasks better. This situation is encountered for instance in the teleportation of quantum states. In this task, one party called Alice, is requested to transmit an *unknown* pure state $|\psi\rangle_A$ of a qubit $A$ to the second party called Bob without sending the qubit itself. There are two ways how she can accomplish the task. First, she can perform an optimal measurement on the state and send the results to Bob. After receiving the classical information from Alice Bob is able to estimate the input state on another qubit $B$. Such a protocol, sometimes called classical teleportation, however allows Alice to transmit the input state only approximately. Using the mean fidelity as a characterization of the quality of the transfer the fidelity $F_B$ of Bob’s estimated state can be at most $F_B = 2/3$ [1]. In addition, the measurement on Alice’s side disturbs the original qubit $A$ in such a way that it does not carry any further information on the input state and therefore the mean fidelity of Alice’s output state $F_A$ cannot exceed $F_A = 2/3$ [2]. In what follows we denote the fidelities $F_A$ and $F_B$ as operation and teleportation fidelity, respectively. It is possible to consider a more general classical teleportation in which the information on the input state $|\psi\rangle_A$ is distributed asymmetrically between the qubits $A$ and $B$. This regime is achieved if Alice performs less demolishing measurement after which her qubit carries more information on the input state and therefore $F_A > 2/3$ while she gains less information and therefore $F_B < 2/3$. Quantum mechanics imposes a fundamental constraint on the fidelities $F_A$ and $F_B$ in the *partial* classical teleportation (PCT) in the form of the following inequality [2]:

$$\sqrt{F_A - 1/3} \leq \sqrt{F_B - 1/3} + \sqrt{2/3 - F_B}.$$  

(1)

The equality in inequality [1] corresponds to an *optimal* PCT in the sense, that for a given operation fidelity $F_A$ a larger teleportation fidelity $F_B$ cannot be obtained. The optimal fidelities lie on the fragment of the ellipse depicted by the dashed curve in Fig. 1 [2].

Better and even perfect transfer of an unknown state can be achieved using another procedure now known as quantum teleportation [3]. In this protocol, Alice and Bob share a maximally entangled state of two qubits $a$ and $B$. Alice then performs a specific joint measurement, called Bell measurement, on the input qubit $A$ whose state is to be transferred and on a qubit $a$. As a result, Bob’s qubit $B$ collapses into the state that can be brought into the original state by an appropriate unitary transformation. Thus, at the output of the quantum teleportation the qubit $B$ carries a perfect replica of the original state and therefore the teleportation fidelity is $F_B = 1$ while the qubit $A$ has to carry no information on the input state and therefore the operation fidelity is $F_A = 1/2$.

![FIG. 1: Bounds for the fidelities $F_A$ and $F_B$ in the partial classical teleportation (dashed curve) and the partial quantum teleportation (solid curve).](image)

Obviously, Bell measurement is the crucial ingredient of quantum teleportation. This is the two-qubit projection measurement discriminating perfectly among four orthogonal maximally entangled Bell states. Motivated by the classical partial teleportation we can think about a *partial* non-demolition Bell measurement that discriminates only partially among the Bell states and simultaneously preserves all Bell states. Such a measurement
would realize just a partial quantum teleportation (PQT) of an unknown input state onto qubit $B$ while preserving some information on the state in the input qubit $A$. In addition, in the PQT one can expect a larger teleportation fidelity $F_B$ for a given operation fidelity $F_A$ than in the PCT. Also for PQT quantum mechanics sets a non-trivial bound between the fidelities $F_A$ and $F_B$ in the form of the cloning inequality [2]

$$(1 - F_A)(1 - F_B) \geq \frac{1}{2} - (1 - F_A) - (1 - F_B),$$

where the equality corresponds to an optimal PQT in the sense that a larger teleportation fidelity $F_B$ cannot be obtained for a given operation fidelity $F_A$. The bound for the fidelities is again given by the fragment of the ellipse (2) depicted by the solid curve in Fig. 1. The Fig. 1 reveals that the PQT really provides a larger teleportation fidelity $F_B$ for a given operation fidelity $F_A$ than the PCT.

In this article we construct such a partial non-demolition Bell measurement (PNBM) for qubits that allows optimal PQT. The measurement rests on the scheme for complete non-demolition Bell measurement proposed in [1]. The novel point is that two qubits used as ancillas in the measurement are prepared in a specific partially entangled state depending on a single parameter whose change allows to continuously control the degree of discrimination of the Bell states. Further we show, that the PNBM is also an optimal two-qubit operation providing maximum estimation fidelity $F_A$, i.e. the fidelities saturate the two-qubit analogy of the inequality (1) [2]

$$\sqrt{F_A - 1} / \sqrt{5} \leq \sqrt{F_B - 1} / \sqrt{5} + \sqrt{3}(2 / 5 - F_B).$$

Note, that the PQT differs from the telecloning or cloning “at a distance” [3] as it realizes rather the “non-local” cloning as the qubits carrying the information on the input state emerge at different possibly distant locations. Recently, a conditional scheme realizing optimal PQT was proposed in [4] based on the partial discrimination of a singlet state the degree of which being controlled by the splitting ratio of the beam splitter used for the discrimination. Here, we concentrate on the deterministic scheme. The performance of our scheme relies on the measurement discriminating partially among all four Bell states. In our scheme, the degree of discrimination is controlled by the preparation of the pair of qubits used as ancillas in the measurement.

The paper is structured as follows. In Section I a PNBM for qubits is designed and an optimal PQT protocol is constructed. Section II contains interpretation of the PNBM. Section III deals with continuous-variable (CV) partial teleportation of coherent states. Section IV contains conclusion.

FIG. 2: Schematic of the optimal partial quantum teleportation. The lines connecting the qubits represent CNOT gates with control qubit indicated by full circle and target qubit by empty circle; $H$ stands for the Hadamard gate. The unitary transformations converting the qubits $B$ and $a$ into the approximate replicas of the state $|\psi\rangle$ and $|\psi_{\perp}\rangle$ are denoted as $U_{ij}^{(B)}$ and $U_{ij}^{(a)}$, respectively. See text for details.

I. PARTIAL NON-DEMOLITION BELL MEASUREMENT FOR QUBITS

Let us consider the following task. Alice has at her disposal a qubit $A$ in an unknown pure normalized state $|\psi\rangle_A = a|0\rangle_A + b|1\rangle_A$ ($|0\rangle$ and $|1\rangle$ denote the eigenvectors of the Pauli diagonal matrix $\sigma_z = \text{diag}(1, -1)$ corresponding to the eigenvalues $+1$ and $-1$, respectively) and she would like to optimally partially teleport the state to Bob. According to the definition in introduction optimal PQT means that after the teleportation Alice and Bob each hold an imperfect copy $\rho_A$ and $\rho_B$, respectively, of the original state whose fidelities $F_A$ and $F_B$ saturate the inequality [2].

Alice can perform the task via the teleportation scheme depicted in Fig. 2. For this purpose she needs four CNOT gates, four Hadamard gates, a pair of properly prepared auxiliary qubits and a shared entangled state. Initially, Alice and Bob share a pair of qubits $a$ and $B$ prepared in a singlet state $|\Psi_{-}\rangle_{aB} = (|01\rangle_{aB} - |10\rangle_{aB}) / \sqrt{2}$. The key ingredient of the protocol is the PNBM that is performed by Alice. This is in fact the perfect non-demolition Bell measurement [5] on the qubits $A$ and $a$ in which the ancillary qubits 1 and 2 are prepared in the state of the form

$$|\Sigma\rangle_{12} = \alpha|00\rangle_{12} + \beta|+\rangle_{12},$$

where $\alpha, \beta \geq 0$, $\alpha^2 + \alpha \beta + \beta^2 = 1$ and $|\pm\rangle = (|0\rangle \pm |1\rangle) / \sqrt{2}$. For $\alpha \beta \neq 0$ the state (4) is partially entangled. This follows from the fact that the purity $P_1 = \text{Tr}(\rho_1^2) = 1 - \alpha^2 \beta^2 / 2$ of the reduced density ma-
trix $\rho_1 = \text{Tr}_2(\Sigma_{12}\Sigma)$ of the qubit 1 is less than one for $\alpha\beta \neq 0$. Further, since $\alpha\beta \leq 1/3$, where the equality holds for $\alpha = \beta = 1/\sqrt{3}$, $P_1 \geq 17/18$ for any $\alpha$ and $\beta$ and therefore the state (11) is never entangled maximally the largest amount of entanglement being achieved for $\alpha = \beta = 1/\sqrt{3}$.

The logical network in the PNBM (see dashed box in Fig. 2) transforms the input state $|\psi\rangle_A|\Psi_{-\alpha\beta}\rangle_{AB}|\Sigma_{12}\rangle$ to

$$\frac{1}{2} \sum_{i,j=0}^1 |ij\rangle 12U_{ij}|\Psi_{-\alpha\beta}\rangle_{AB}|\Psi_{-\alpha\beta}\rangle_{AB}. \tag{5}$$

Here $U_{ij} = U_{ij}^{(a)} \otimes U_{ij}^{(B)}$ is the product of local unitary transformations on qubits $a$ and $B$, where $U_{00}^{(i)} = U_{11}^{(i)} = \sigma_x^{(i)}$, $U_{01}^{(i)} = U_{10}^{(i)} = \sigma_y^{(i)}$, $U_{11}^{(i)} = -I^{(a)}$ and $U_{13}^{(i)} = I^{(B)}$, where $\sigma_k^{(i)}$, $k = 1, 2, 3$ and $I^{(i)}$ are standard Pauli matrices and the identity matrix, respectively, of the qubit $l$. Measuring the ancillary qubits 1 and 2 in the basis $|0\rangle$, one finds with probability 1/4 one of four results 00, 01, 10 and 11 where the first (second) digit is the result of the measurement on the qubit 1(2). Communicating the result to Bob via classical channel, performing the corresponding unitary transformations $(U_{ij}^{(i)})^{-1} = U_{ij}^{(l)}$, $l = a, B$ on qubits $a$ and $B$, and taking into account the formula $U_{ij}^{(l)}|\Psi\rangle_{AB} = -|\Psi\rangle_{AB}$ we find that Alice and Bob in each run of the teleportation establish between themselves a normalized state of three qubits $A, a$ and $B$ of the form

$$|\Phi\rangle = \alpha|\Psi_{-\alpha\beta}\rangle_{AB} - |\Psi_{-\alpha\beta}\rangle_{AB}. \tag{6}$$

The interpretation of the resulting state is straightforward. If $\alpha = 0$, i.e. the ancillary qubits are prepared in the separable state $|++\rangle_{12}$ and therefore the PNBM does not discriminate among the Bell states, the input state is completely preserved in the original qubit. If, on the other hand, $\beta = 0$, i.e. the ancillary qubits are prepared in the separable state $|00\rangle_{12}$ therefore the PNBM perfectly discriminates among the Bell states, the input state $|\psi\rangle_A$ is completely teleported to Bob. For $\alpha\beta \neq 0$, i.e. for partially entangled ancillary qubits when PNBM discriminates partially among the Bell states, the state (6) represents a coherent superposition of two previous possibilities the probability amplitudes of which being controlled by the choice of the parameter $\alpha$ in the state (4). In other words, this intermediate case corresponds to the partial teleportation of the qubit $A$. In what follows we show that the quantum teleportation scheme is optimal in the sense of the inequality (4).

Tracing the state (6) over two qubits one finds the remaining qubit $i$ in the mixed state

$$\rho_i = F_i|\psi\rangle_i \langle \psi| + (1 - F_i)|\psi_\perp\rangle_i \langle \psi_\perp|, \quad i = A, B, a, \tag{7}$$

where $|\psi_\perp\rangle = b^*|0\rangle - a^*|1\rangle$ is the state orthogonal to the input state $|\psi\rangle$ and

$$F_A = 1 - \frac{\alpha^2}{2}, \quad F_B = 1 - \frac{\beta^2}{2}, \quad F_a = \frac{\alpha^2 + \beta^2}{2}. \tag{8}$$

Since the inequality (4) becomes equality for the latter fidelities our scheme really realizes an optimal PQT. Viewed from the point of view of quantum cloning, our scheme realizes a measurement induced and entanglement assisted optimal $1 \to 2$ universal asymmetric quantum cloning machine (4). In particular, if the superposition in the state (11) is balanced with $\alpha = \beta = 1/\sqrt{3}$, then the information is distributed symmetrically between the qubits $A$ and $B$ with the fidelities $F_A = F_B = 5/6$. In the language of quantum cloning this regime corresponds to an optimal symmetric universal quantum cloning scheme. Moreover, the third qubit $a$ leaves the teleportation in the state $\rho_a$ for which the fidelity $F_{a_{\perp}} = \langle a|\psi_\perp\rangle \langle \psi_\perp|a\rangle$ attains its maximum possible value $F_{a_{\perp}} = 2/3$. Therefore, our scheme also realizes deterministically the optimal universal NOT gate (4).

It remains to prepare the state of ancillae $|\Sigma_{12}\rangle$. This can be done with the help of a single CNOT gate and four local unitary transformations as is depicted in Fig. 3. The transformations $U$ and $V$ are represented in the basis $|0\rangle$, $|1\rangle$ by the rotations $U$ and $V$ with elements

$$U_{11} = U_{22} = (\alpha + \beta + \sqrt{\alpha^2 + \beta^2})/2, \quad U_{12} = -U_{21} = - (\alpha + \beta - \sqrt{\alpha^2 + \beta^2})/2,$$

$$V_{11} = V_{22} = (\alpha + \sqrt{\alpha^2 + \beta^2})/K, \quad V_{12} = -V_{21} = -\beta/K,$$

where $K = \sqrt{2(\alpha^2 + \beta^2 + \alpha \sqrt{\alpha^2 + \beta^2})}$. $H$ is the Hadamard gate and $W$ is represented in the basis $|\pm\rangle$ by the orthogonal matrix $W$ with elements

$$W_{11} = \sqrt{2} U_{11}/K, \quad W_{12} = \beta(\sqrt{\alpha^2 + \beta^2} - \beta)/\sqrt{2 K U_{21}},$$

$$W_{21} = \alpha(\sqrt{\alpha^2 + \beta^2} + \alpha)/\sqrt{2 K U_{11}}, \quad W_{22} = -\alpha/\sqrt{2 K U_{21}}.$$

II. INTERpretation of the PARTIAL Bell MEasurEment

As it was already mentioned the standard Bell measurement is a projection measurement on a system of two qubits $A$ and $a$ discriminating perfectly among four orthogonal Bell states. Consequently, the measurement

![FIG. 3: Logical network for preparation of the state $|\Sigma_{12}\rangle$. The line connecting the qubits represents CNOT gate with control qubit indicated by full circle and target qubit by empty circle. $U$, $V$, are rotations, $W$ is the orthogonal transformation, and $H$ is the Hadamard gate.](image-url)
provides maximum information on an unknown pure two-qubit state and therefore the mean fidelity of the estimated state is \( \mathcal{F}_B = 2/5 \) while disturbing the input state in such a way that it does not carry any more information on the state and therefore the mean fidelity of the output state is also \( \mathcal{F}_A = 2/5 \). On the other hand, the partial Bell measurement considered in the preceding section discriminates among the Bell states only partially thus providing less information on the input state of qubits \( A \) and \( a \) while preserving larger information in the outgoing qubits. As the degree of the Bell state discrimination can be controlled by the parameter \( \alpha \) in the ancillary state \( (4) \), one can increase continuously the information gain at the expense of a larger disturbance of the measured system.

To express this behaviour quantitatively let us assume the ancillary qubits 1 and 2 to be again prepared in the state \( (1) \). For such a state the measurement scheme in the dashed box in Fig. 2 realizes a two-qubit quantum operation on qubits \( A \) and \( a \) described by the set of four operators \( A_k, k = 1, 2, 3, 4 \) where we have identified the indices 1, 2, 3 and 4 with the measurement outcomes 00, 01, 10 and 11. In the Bell basis \( \{\Phi_{1,2}\} = (|00\rangle \pm |11\rangle)\sqrt{2}, \{\Phi_{3,4}\} = (|01\rangle \pm |10\rangle)\sqrt{2} \) the operators are represented by the diagonal matrices

\[
A_1 = \text{diag}(\alpha + \beta/2, \beta/2, \beta/2, \beta/2), \\
A_2 = \text{diag}(\beta/2, \alpha + \beta/2, \beta/2, \beta/2), \\
A_3 = \text{diag}(\beta/2, \beta/2, \alpha + \beta/2, \beta/2), \\
A_4 = \text{diag}(\beta/2, \beta/2, \beta/2, \alpha + \beta/2).
\]

Consequently, the PNBM preserves the Bell states and therefore it really realizes a non-demolition measurement of these states. Further, the operators \( (2) \) satisfy the completeness relation \( \sum_{i=1}^{4} A_i^\dagger A_i = 1 \) due to the normalization condition \( \alpha^2 + \beta^2 + \beta^2 = 1 \) and hence the PNBM performs partial discrimination of the Bell states deterministically. Restricting ourselves to pure two-qubit input states, the information gain in the PNBM can be characterized by the mean estimation fidelity \( \mathcal{F}_B \) while the disturbance of the state caused by the measurement can be characterized by the mean operation fidelity \( \mathcal{F}_A \). The fidelities can be calculated from the formulas \( (2) \)

\[
\mathcal{F}_A = \frac{1}{20} \left( 4 + \sum_{i=1}^{4} |\text{Tr} A_i|^2 \right), \quad \mathcal{F}_B = \frac{1}{20} \left( 4 + \sum_{i=1}^{4} \lambda_i \right),
\]

where \( \lambda_i \) is the maximum eigenvalue of the matrix \( A_i^\dagger A_i \). On inserting the Eqs. \((3)\) into the formulas \((10)\) we finally arrive at the fidelities of the PNBM

\[
\mathcal{F}_A = \frac{1}{6} \left( 1 + (\alpha + 2\beta)^2 \right), \quad \mathcal{F}_B = \frac{1}{6} \left( 1 + (\alpha + \beta/2)^2 \right).
\]

Now an interesting question arises what is the trade-off between the obtained fidelities \( (11) \). Substituting the fidelities \( (11) \) into the inequality \( (3) \) one finds that they saturate the inequality. Hence, the PNBM is an optimal two-qubit quantum operation in the sense that for a given operation fidelity \( \mathcal{F}_A \) a larger estimation fidelity \( \mathcal{F}_B \) cannot be obtained. Therefore, the measurement allows an optimal PCT of two-qubit states. Moreover, as the same optimal distribution of information can be achieved in quantum teleportation with nonmaximally entangled states \( (2) \), our scheme represents a local counterpart to the quantum teleportation based on nonmaximally entangled states.

### III. PARTIAL TELEPORTATION FOR CONTINUOUS VARIABLES

Let us now study the extension of the optimal PQT to continuous-variables (CVs). In this case the five relevant qubits are replaced with the five modes of electromagnetic field described by the quadrature operators \( x_i, p_i \), \( i = A, B, a, 1, 2 \) with the commutators \( [x_j, p_k] = i\delta_{jk} \). The state of the mode \( A \) that is to be teleported is an unknown coherent state \( |\alpha\rangle_A \). For CVs the optimal PQT means that after the teleportation Alice and Bob hold imperfect copies \( \rho_A \) and \( \rho_B \), respectively, of the original coherent state having the following fidelities \( F_i = \langle \alpha | \rho_i | \alpha \rangle \), \( i = A, B \):

\[
F_A = \frac{2}{2 + e^{2\gamma}}, \quad F_B = \frac{2}{2 + e^{-2\gamma}}, \quad (12)
\]

where \( \gamma = \text{asymmetry parameter} \) \( (11) \).

The CV extension of the qubit protocol presented in Sec. II is as follows. Initially, Alice and Bob share a two-mode squeezed vacuum state of modes \( a \) and \( B \) produced in the non-degenerate optical parametric amplifier (NOPA)

\[
|\text{NOPA}\rangle_{aB} = \sqrt{1 - \lambda^2} \sum_{n=0}^{\infty} \lambda^n |mn\rangle_{aB}, \quad (13)
\]

where \( \lambda = \tanh r \), \( r \) is the squeezing parameter. Alice then performs the CV partial non-demolition Bell measurement on the mode \( A \) whose state is to be teleported and on the mode \( a \) of the NOPA state. Recently, the measurement was used for partial reversion of the CV cloning \( (12) \). The measurement exploits two ancillary vacuum modes 1, 2 and a sequence of four CV CNOT gates followed by the homodyne detections of the \( x \) quadrature on mode 1 and \( p \) quadrature on mode 2 \( (12) \). The CNOT gates are realized by the quantum non-demolition (QND) interaction

\[
x_j' = x_j, \quad x_k' = x_k + \kappa x_j, \quad p_j' = p_j - \kappa p_k, \quad p_k' = p_k, \quad (14)
\]

where \( j(k) \) is the control (target) mode and \( \kappa \) is the coupling constant. By the measurement Alice detects the quadratures \( x_a = x_1 - \kappa(x_A - x_a) \) and \( p_a = p_2 + \kappa(p_A + p_a) \) for which she obtains two classical results \( \bar{x}_a \) and \( \bar{p}_a \). The quadratures of mode \( A \) transform as

\[
x_{A,\text{out}} = x_A - \kappa x_2, \quad p_{A,\text{out}} = p_A + \kappa p_1, \quad (15)
\]
while the quadratures of the remaining two modes $a$ and $B$ can be written as

$$\begin{align*}
x'_a &= x_A - \frac{1}{\kappa} \bar{x}_1 - \kappa x_2 + \frac{1}{\kappa} \bar{x}_u, \\
p'_a &= -p_A - \kappa p_1 - \frac{1}{\kappa} \bar{p}_2 + \frac{1}{\kappa} \bar{p}_v, \\
x_B &= x_A - (x_a - x_B) - \frac{1}{\kappa} \bar{x}_1 + \frac{1}{\kappa} \bar{x}_u, \\
p_B &= p_A + (p_a + p_B) + \frac{1}{\kappa} \bar{p}_2 - \frac{1}{\kappa} \bar{p}_v.
\end{align*}$$

(16)

As a result of Alice’s measurement the operators $x_u$ and $p_v$ collapse in modes $a$ and $B$ into the classical variables $\bar{x}_u$ and $\bar{p}_v$. After the measurement Alice and Bob perform on the modes the displacements

$$\begin{align*}
x_{a,\text{out}} &= x'_a - \frac{1}{\kappa} \bar{x}_u, & p_{a,\text{out}} &= p'_a - \frac{1}{\kappa} \bar{p}_v, \\
x_{B,\text{out}} &= x_B - \frac{1}{\kappa} \bar{x}_u, & p_{B,\text{out}} &= p_B + \frac{1}{\kappa} \bar{p}_v.
\end{align*}$$

(17)

thus obtaining the following output quadratures:

$$\begin{align*}
x_{a,\text{out}} &= x_A - \frac{1}{\kappa} \bar{x}_1 - \kappa x_2, \\
p_{a,\text{out}} &= -p_A - \kappa p_1 - \frac{1}{\kappa} \bar{p}_2, \\
x_{B,\text{out}} &= x_A - (x_a - x_B) - \frac{1}{\kappa} \bar{x}_1, \\
p_{B,\text{out}} &= p_A + (p_a + p_B) + \frac{1}{\kappa} \bar{p}_2.
\end{align*}$$

(18)

The fidelities $F_A$ and $F_B$ between the input coherent state and the output states $\rho_A$ and $\rho_B$ can be calculated from the formula $F_i = 1/(1 + \langle n_{ch,i} \rangle)$, where $\langle n_{ch,i} \rangle$ is the mean number of chaotic photons in $i$th mode. Hence, one finally arrives using the Eqs. (15) and (18) at the following formulas:

$$\begin{align*}
F_A &= \frac{2}{1 + e^{2\gamma}}, & F_B &= \frac{2}{2(1 + e^{-2\gamma}) + e^{-2\gamma}},
\end{align*}$$

(19)

where $\gamma = \ln \kappa$. Comparing the obtained fidelities with those of given in Eq. (12), we find that while the operation fidelity $F_A$ attains optimal value the teleportation fidelity $F_B$ is reduced in comparison with the optimal value due to the finiteness of the shared entanglement. Therefore, in contrast with the qubit protocol our CV teleportation scheme realizes only approximately optimal PQT of coherent states. The teleportation fidelity $F_B$ can approach the optimum (12) arbitrarily closely if sufficiently large CV entanglement (sufficiently large squeezing $r$) is available.

IV. CONCLUSION

In conclusion, we have explicitly constructed a joint two-qubit non-demolition measurement allowing to partially deterministically discriminate among all Bell states. Moreover, this partial non-demolition Bell measurement allows to continuously control the degree of the discrimination by the preparation of the state of ancilla. We have then proposed the teleportation scheme based on the partial Bell measurement making it possible to continuously control the flow of information between the output qubits and it was proved that such a distribution of information is optimal in the sense that the teleportation and operation fidelity saturate the cloning inequality. Further, the measurement was shown to be optimal two-qubit quantum operation from the point of view of the trade-off between the gain of information and the state disturbance. Analogic partial non-demolition Bell measurement can be constructed for CVs. However, in contrast with qubits the CV partial quantum teleportation based on the measurement does not distribute the information optimally while the optimum is approached with increasing shared entanglement.

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