A Dynamical Theory for Massive Supergravity$^1$

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ABSTRACT

We present a new massive theory of superspin $Y = 3/2$ which has non-minimal supergravity as it's massless limit. The new result will illuminate the underlying structure of auxiliary fields required for the description of arbitrary massive half-integer superspin systems.

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1 Introduction

After four decades of exploring the topic of supersymmetry (SUSY), the problem of writing a manifestly susy-invariant action that describes a free, off-shell massive arbitrary superspin irreducible representation of the Super-Poincaré group still possesses puzzles. Although the non-supersymmetric case of massive higher spin theory has been developed [1], [2] and is well understood, the off-shell supersymmetric case has yet to be understood with a comparable level of clarity. There has been progress for on-shell supersymmetry [3], but these results don’t capture the rich off-shell structure of supersymmetric theories. There is a need for a manifestly susy invariant theory of massive integer and half-integer superspins which includes all the auxiliary superfields a theory of this nature is expected to possess.

Progress in this direction was made with the works presented in [4], [5], [6]. These results provided a proof of concept that constructions like these are possible, but in these cases, the results don’t shed light to the heart of the problem which is to determine the set of auxiliary superfields required to describe an arbitrary superspin system with a proper massless limit. Specifically in [4] the focus was on massive extension of theories such as old-minimal supergravity, new-minimal supergravity, whose massless limits don’t generalize to the arbitrary spin case. Therefore they do not provide clues about the underlying structure of auxiliary superfields for the arbitrary superspin case.

This is not the case with the work presented in [6] where a massive extension of non-minimal supergravity is derived. The massless limit of that theory is non-minimal supergravity which is a member of an arbitrary super-helicity tower and that makes it a good starting point. However the derivation used a lagrange multiplier technique in order to impose constraints that were not be derived in a dynamical way.

We will show in the following that there is an alternative formulation of the theory where all the superfields are dynamical and the desired constraints follow from the equations of motion of these superfields. For this to work we require the presence of two fermionic auxiliary superfields. In the massless limit one of these decouples and the other one will play the role of the compensator in non-minimal supergravity.

Our presentation is organized as follows: In section 2, we quickly review the representation theory of the Super-Poincaré group for a massive half-integer superspin system. In section 3, we present the constraints imposed in the theory in order to have a proper massless limit. In the following section 4, we start with a warm up exercise by quickly reproducing the massive theory for superspin $Y = 1/2$. In the
last section 5 we present the new massive theory for \( Y = \frac{3}{2} \).

## 2 Half-Integer Superspin Representation Theory

The irreducible representations of the Super-Poincaré group are labeled by it’s two Casimir operators. The first one is the mass and the other one is a supersymmetric extension of the Poincaré Spin operator. For the massive case the Super-spin casimir operator takes the form

\[
C_2 = \frac{W^2}{m^2} + \left( \frac{3}{4} + \lambda \right) P_{(o)} ,
\]

where \( W^2 \) is the ordinary spin operator, \( P_{(o)} \) is a projection operator and the parameter \( \lambda \) satisfies the equation

\[
\lambda^2 + \lambda = \frac{W^2}{m^2} .
\]

In order to diagonalize \( C_2 \) we want to diagonalize both \( W^2, P_{(o)} \). The superfield \( \Phi_{\alpha(n)\dot{\alpha}(m)} \) that does this and describes the highest possible representation (highest superspin)

\[
C_2 \Phi_{\alpha(n)\dot{\alpha}(m)} = Y(Y+1)\Phi_{\alpha(n)\dot{\alpha}(m)}, \quad Y = \frac{n+m+1}{2},
\]

has to satisfy the following:

\[
\begin{align*}
\Delta^2 \Phi_{\alpha(n)\dot{\alpha}(m)} &= 0 , \\
\bar{\Delta}^2 \Phi_{\alpha(n)\dot{\alpha}(m)} &= 0 , \\
\Delta^\gamma \Phi_{\gamma\alpha(n-1)\dot{\alpha}(m)} &= 0 , \\
\partial^\gamma \Phi_{\gamma\alpha(n-1)\dot{\gamma}(m-1)} &= 0 , \\
\Box \Phi_{\alpha(n)\dot{\alpha}(m)} &= m^2 \Phi_{\alpha(n)\dot{\alpha}(m)} ,
\end{align*}
\]

where all dotted and undotted indices are fully symmetrized and the spin content of this supermultiplet is \( j = Y + 1/2, \ Y, \ Y, \ Y - 1/2 \).

A superfield that describes a superspin \( Y \) system has index structure such that \( n + m = 2Y - 1 \) where \( n, m \) are integers. This Diophantine equation has a finite number of different solutions for \( (n, m) \) pairs but the corresponding superfields are all equivalent because we can use the \( \partial_{\beta\dot{\beta}} \) operator to convert one kind of index to another. So we can pick one of them to represent the entire class.

One last comment has to be made about the reality of the representation. The reality condition imposed on the superfield differs with the character of the superfield. For bosonic ones (even total number of indices), we can pick them to have \( n = m \).
and the reality condition is $\Phi_{\alpha(n)}\dot{\alpha}(n) = \bar{\Phi}_{\alpha(n)}\dot{\alpha}(n)$. For fermionic superfields (odd total number of indices) we can pick $n = m + 1$ and the reality condition is the Dirac equation $i\partial_{\alpha_n}^{\alpha_n}\dot{\Phi}_{\alpha(n-1)}\dot{\alpha}(n) + m\Phi_{\alpha(n)}\dot{\alpha}(n-1)$.

3 **The Massless Limit**

Representation theory tells us the type of superfield and constraints we need in order to describe a specific irreducible representation. We would like to have a dynamical way to derive these constraints, through a lagrangian. That means we need a set of auxiliary superfields to help us generate these constraints. This is the core of the problem, to find the set of auxiliary fields and their interactions that accomplish these goals. That sounds like an intuitive trial-and-error process, but there is a hidden clue and...the massless limit of the theory.

As was illustrated in [7], [8], [9], [10], and [11] there is one infinite tower for theories of integer superhelicity and two different infinite towers for theories of half integer super-helicities.

\[
\begin{array}{cccc}
\vdots & \vdots & \vdots \\
\hline \\
\hline
s = 2 & \text{Ogievetsky-Sokatchev} & \text{non-minimal supergravity} & \text{Old/New/Minimal supergravity} \\
\hline
s = 1 & \{\Psi_{\alpha(s)}\dot{\alpha}(s)\mid V_{\alpha(s-1)}\dot{\alpha}(s-1)\} & \{H_{\alpha(s)}\dot{\alpha}(s)\mid \chi_{\alpha(s-1)}\dot{\alpha}(s-1)\} & \{H_{\alpha(s-1)}\dot{\alpha}(s-1)\mid \chi_{\alpha(s)}\dot{\alpha}(s-1)\} \\
\hline
s = 0 & \{\Phi\} & \text{Integer Superhelicity} & \text{Half-Integer Superhelicity} \\
\hline
\end{array}
\]

Figure One: Towers of Massive Higher Spin Supermultiplets

These theories were constructed under the requirement that the massless limit of a massive superspin $Y$ theory will give the massless theory of superhelicity $Y$ plus things that decouple. Therefore now that we want to build the massive theory we know what
its massless limit must be. The conclusion is that the construction of the massive theories must start with the massless action and the addition of (self)interaction terms proportional to $m$ and $m^2$, so the massless theory decouples in the massless limit. Hence immediately and for free we obtain the first auxiliary field required. The massless theories are formulated in terms of a main superfield and a compensator. For the massive extension the compensator will become the first auxiliary superfield needed.

4 Warming up with $Y = \frac{1}{2}$

So if we want to construct the theory of superspin $1/2$ we start with the theory of superhelicity $1/2$, add terms proportional to $m$ and $m^2$ and check if we can generate the desired constraints. If not then we add extra auxiliary fields until we do. The starting action is:

$$S = \int d^8z \left\{ a_1 H D^\gamma D^2 \gamma H + a_2 m \ H \left(D^2 H + D^2 H\right) + a_3 m^2 \ H^2 \right\} \ . \quad (5)$$

To describe $Y = \frac{1}{2}$, $H$ must satisfy $D^2 H = 0$ and $\Box H = m^2 H$. The equation of motion is

$$E^{(H)} = \frac{\delta S}{\delta H} = 2 a_1 D^\gamma \bar{D}^2 \gamma H + 2 a_2 m \left(D^2 H + \bar{D}^2 H\right) + 2 a_3 m^2 H \ , \quad (6)$$

which gives

$$D^2 E^{(H)} = 2 a_2 m D^2 \bar{D}^2 H + 2 a_3 m^2 D^2 H \ , \quad (7)$$

so by choosing $a_2 = 0$, $a_3 \neq 0$ we find $D^2 H = 0 \leadsto \bar{D}^2 H = 0$ (reality) and if this is substituted back into $E^{(H)}$ we get $\Box H = \frac{a_3}{a_1} m^2 H$ which fixes $a_3 = a_1$ for compatibility with the Klein-Gordon equation.

There is also another way to obtain these results and that is à la Stückelberg. The observation is that at least on-shell the massive superspin $\frac{1}{2}$ can be seen as the result of the combination of the massless superhelicity $\frac{1}{2}$ plus the massless superhelicity 0. So we start with the actions for superhelicity $\frac{1}{2}$ and 0 and we introduce (self)interaction terms proportional to $m$ and $m^2$

$$S = \int d^8z \left\{ a_1 H D^\gamma D^2 \gamma H + a_2 m \ H \left(D^2 H + D^2 H\right) + a_3 m^2 \ H^2 \right\} + \gamma m \ H \left(\Phi + \bar{\Phi}\right) + b_1 \ \Phi \bar{\Phi} \ + \int d^6z \ b_2 m \ \Phi \bar{\Phi} \ . \quad (8)$$

The equations of motion are:

$$E^{(H)} = 2 a_1 D^\gamma D^2 \gamma H + 2 a_2 m \left(D^2 H + \bar{D}^2 H\right) + \gamma m \left(\Phi + \bar{\Phi}\right) + 2 a_3 m^2 H \ , \quad (9)$$
\[ \mathcal{E}^{(\Phi)} = -b_1 \bar{\mathcal{D}}^2 \Phi - \gamma m \bar{\mathcal{D}}^2 H + 2b_2 m \Phi \quad . \]  

(10)

If we manage to show that on-shell \( \Phi = 0 \) then \( \mathcal{E}^{(\Phi)} = 0 \Rightarrow \bar{\mathcal{D}}^2 H = 0 \Rightarrow \Box H = m^2 H \ (a_3 = a_1) \). With that goal in mind we attempt to eliminate \( H \) from the equation of \( \Phi \) and choose coefficients in such a way to find \( \Phi = 0 \). We begin by defining \( I = \bar{\mathcal{D}}^2 \mathcal{E}^{(H)} + m \mathcal{E}^{(\Phi)} \) and then notice

\[ I = \bar{\mathcal{D}}^2 \mathcal{E}^{(H)} + m \mathcal{E}^{(\Phi)} = (\gamma - b_1) m \bar{\mathcal{D}}^2 \Phi + (2a_3 - \gamma) m^2 \bar{\mathcal{D}}^2 H + 2a_2 \bar{\mathcal{D}}^2 \bar{\mathcal{D}}^2 H + 2b_2 m^2 \Phi \quad . \]  

(11)

If we choose \( \gamma = b_1 = 2a_3 = 2a_1 \), \( a_2 = 0 \) we obtain \( I = 2b_2 m^2 \Phi \). Now we can follow two possible routes

1. \( b_2 \neq 0 \): \( b_2 \) can be anything besides zero and in that case on-shell \( I = 0 \Rightarrow \Phi = 0 \) we find all the desired constraints for \( H \) and the action is

\[ S = \int d^8z \left\{ c \ H \bar{\mathcal{D}}^i \bar{\mathcal{D}}^j H + cm^2 H^2 + 2cm \ H (\Phi + \bar{\Phi}) + 2c \ \Phi \bar{\Phi} \right\} + \int d^6z \ b_2 m \ \Phi \bar{\Phi} \quad . \]  

(12)

2. \( b_2 = 0 \): If we set \( b_2 \) to zero, then \( I \) identically vanish. That means the \( \bar{\mathcal{D}}^2 \mathcal{E}^{(H)} + m \mathcal{E}^{(\Phi)} = 0 \) can be treated as a Bianchi identity and the corresponding action is invariant under a symmetry. The symmetry of the action that generates the above Bianchi identity is

\[ \delta_G H \sim \bar{\mathcal{D}}^2 L + D^2 \bar{L} \quad , \]

\[ \delta_G \Phi \sim m \bar{\mathcal{D}}^2 L \quad . \]  

(13)

(14)

Due to this symmetry, the chiral superfield \( \Phi \) can be gauged away completely and therefore it’s equation of motion (or the Bianchi identity) will give the desired constraint of \( \bar{\mathcal{D}}^2 H = 0 \). The action for this case is

\[ S = \int d^8z \left\{ c \ H \bar{\mathcal{D}}^i \bar{\mathcal{D}}^j H + cm^2 H^2 + 2cm \ H (\Phi + \bar{\Phi}) + 2c \ \Phi \bar{\Phi} \right\} \quad , \]  

(15)

and the gauge fixed action is identical with the action obtained from the first derivation. We would like to know if similar ‘Stückelberg’ constructions can occur for the higher superspin theories, like it is the case for the higher spin theories.
5 New Massive $Y = 3/2$ Theory

Now we will follow a similar strategy to build a theory of superspin $\frac{3}{2}$. The starting point is the theory of superhelicity $\frac{3}{2}$, in specific we choose the theory of non-minimal supergravity ($s = 1$ in [9]). Non-minimal supergravity is formulated in terms of $H_{a\dot{a}}$ and $\chi_{\alpha}$. We will add mass corrections to that action and check if 1) we can make $\chi_{\alpha}$ vanish on-shell (auxiliary status) and 2) we can generate the constraints on $H_{a\dot{a}}$ demanded by representation theory $D^{\alpha}H_{a\dot{a}} = 0$, $\Box H_{a\dot{a}} = m^2 H_{a\dot{a}}$. The starting action is given by

$$S = \int d^8z \left\{ H^{\alpha\dot{\alpha}}D^\gamma D_\gamma H_{a\dot{a}} + a_1mH^{\alpha\dot{a}}(\bar{D}_\alpha \chi_{\alpha} - D_a \bar{\chi}_{\dot{a}}) 
- 2 H^{\alpha\dot{\alpha}}D_\alpha D^2 \chi_{\alpha} + c.c. + a_2mH^{\alpha\dot{a}}(D^2 H_{a\dot{a}} + \bar{D}^2 H_{a\dot{a}}) 
- 2 \chi^{\alpha}D^2 \chi_{\alpha} + c.c. + a_3m\chi^{\alpha} \bar{\chi}_{\dot{a}} + c.c. 
+ 2 \chi^{\alpha}D_a \bar{D}^\dot{\alpha} \bar{\chi}_{\dot{a}} + a_4m^2 H^{\alpha\dot{a}} H_{a\dot{a}} \right\},$$

and the equations of motion are:

$$\mathcal{E}^{(H)}_{a\dot{a}} = 2D\gamma D^2 D_\gamma H_{a\dot{a}} + 2(D_a \bar{D}^\dot{\alpha} \bar{\chi}_{\dot{a}} - D_a D^2 \chi_{\alpha}) + a_1m(\bar{D}_\alpha \chi_{\alpha} - D_a \bar{\chi}_{\dot{a}})$$

$$+ 2a_2m(D^2 H_{a\dot{a}} + \bar{D}^2 H_{a\dot{a}}) + 2a_4m^2 H_{a\dot{a}};$$

$$\mathcal{E}^{(\chi)}_{\alpha} = -4D^2 \chi_{\alpha} + 2D_a \bar{D}^\dot{\alpha} \bar{\chi}_{\dot{a}} - 2D^2 \bar{D}^\dot{\alpha} H_{a\dot{a}} + a_1m\bar{D}^\dot{\alpha} H_{a\dot{a}} + 2a_3m\chi_{\alpha}. \quad (18)$$

Now we may use these equations and attempt to remove any $H_{a\dot{a}}$-dependence to derive one equation that depends solely on $\chi_{\alpha}$. That will tell us if we can pick coefficients in a way that $\chi_{\alpha}$ vanishes on-shell. Consider the following linear combination of equations of motion where each such equation of motion is obtained by the variation of the action with regard to the respective superfields indicated by the subscripts in the first equation below:

$$I_{\alpha} = AD^2 \bar{D}^\dot{\alpha} \mathcal{E}^{(H)}_{a\dot{a}} + BD^2 \bar{D}^2 \mathcal{E}^{(\chi)}_{\alpha} + m^2 \mathcal{E}^{(\chi)}_{\alpha}$$

$$= -2(A + B) \Box D^2 \bar{D}^\dot{\alpha} H_{a\dot{a}} + 2(A + B) D^2 \bar{D}^\dot{\alpha} D_a \bar{D}^\dot{\alpha} \bar{\chi}_{\dot{a}} - Aa_1mD^2 \bar{D}^\dot{\alpha} D_a \bar{\chi}_{\dot{a}}$$

$$+ 2(Aa_4 - 1)m^2 D^2 \bar{D}^\dot{\alpha} H_{a\dot{a}} - 4(A + B) \Box D^2 \chi_{\alpha} - 4m^2 D^2 \chi_{\alpha}$$

$$+ (a_1)m^3 \bar{D}^\dot{\alpha} H_{a\dot{a}} + 2(Aa_1 + Ba_3)mD^2 D^2 \chi_{\alpha} + 2m^2 D_a \bar{D}^\dot{\alpha} \bar{\chi}_{\dot{a}}$$

$$+ 2a_3m^3 \chi_{\alpha}. \quad (19)$$
The following choice of coefficients will remove any $H_{\alpha\dot{\alpha}}$ dependence from the equation above:

$$A + B = 0 , \quad Aa_4 - 1 = 0 , \quad a_1 = 0 ,$$

and imposing these leads to the form of $I_\alpha$ to be given by

$$I_\alpha = -4m^2 D^2 \chi_\alpha + 2B a_3 m D^2 \bar{D}^2 \chi_\alpha + 2m^2 D_\alpha \bar{D}^\dot{\alpha} \bar{\chi}_\dot{\alpha} + 2a_3 m^2 \chi_\alpha .$$

From this we see there is no choice of coefficients that will make $\chi_\alpha$ vanish on-shell. Therefore we must introduce an auxiliary superfield. Its purpose will be to impose a constraint on $\chi_\alpha$ when it vanishes. That constraint will be used to simplify the above expression for $I_\alpha$ and set $\chi_\alpha$ to zero. But a more careful examination of $I_\alpha$ will convince us that there is no unique constraint on $\chi_\alpha$ that will make all terms (except the last one) vanish. The inescapable conclusion is that we have to treat $\chi_\alpha = 0$ as the desired constraint. This suggests that we must introduce a spinorial superfield $u_\alpha$ that couples with $\chi_\alpha$ through only a mass term $\sim m u^\alpha \chi_\alpha$. Hence when $u_\alpha = 0$ then immediately we see $\chi_\alpha = 0$.

We must update the action with the introduction of a few new terms: the interaction term $m u^\alpha \chi_\alpha$ and the kinetic energy terms for $u_\alpha$ (the most general quadratic action). The new action is

$$S = \int d^8 z \left\{ H^{\alpha\dot{\alpha}} D^\gamma \bar{D}^2 D_\gamma H_{\alpha\dot{\alpha}} + \gamma m u^\alpha \chi_\alpha + c.c. \
- 2 H^{\alpha\dot{\alpha}} \bar{D}_\dot{\alpha} D^2 \chi_\alpha + c.c. + a_2 m H^{\alpha\dot{\alpha}} \bar{D}^2 H_{\alpha\dot{\alpha}} + c.c. + b_1 u^\alpha \bar{D}^2 u_\alpha + c.c. \
- 2 \chi^\alpha D^2 \chi_\alpha + c.c. + a_3 m \chi^\alpha \chi_\alpha + c.c. + b_2 u^\alpha \bar{D}^2 u_\alpha + c.c. \
+ 2 \chi^\alpha D_\alpha \bar{D}^\dot{\alpha} \bar{\chi}_\dot{\alpha} + a_4 m^2 H^{\alpha\dot{\alpha}} H_{\alpha\dot{\alpha}} + b_3 u^\alpha \bar{D}^\dot{\alpha} \bar{D}_\dot{\alpha} \bar{u}_\dot{\alpha} + b_4 u^\alpha \bar{D}_\alpha \bar{D}^\dot{\alpha} \bar{u}_\dot{\alpha} + b_5 m u^\alpha u_\alpha \right\} ,$$

and the updated equations of motion are

$$\mathcal{E}^{(H)}_{\alpha\dot{\alpha}} = 2D^\gamma \bar{D}^2 D_\gamma H_{\alpha\dot{\alpha}} + 2(D_\alpha \bar{D}^2 \bar{\chi}_\dot{\alpha} - \bar{D}_\dot{\alpha} D^2 \chi_\alpha) + 2a_2 m(D^2 H_{\alpha\dot{\alpha}} + \bar{D}^2 H_{\alpha\dot{\alpha}}) + 2a_4 m^2 H_{\alpha\dot{\alpha}} ,$$

$$\mathcal{E}^{(\chi)}_\alpha = -4D^2 \chi_\alpha + 2D_\alpha \bar{D}^\dot{\alpha} \bar{\chi}_\dot{\alpha} - 2D^2 \bar{D}^\dot{\alpha} H_{\alpha\dot{\alpha}} + 2a_3 m \chi_\alpha + \gamma m u_\alpha ,$$

$$\mathcal{E}^{(u)}_\alpha = 2b_1 D^2 u_\alpha + 2b_2 \bar{D}^2 u_\alpha + b_3 \bar{D}^\dot{\alpha} D_\alpha \bar{u}_\dot{\alpha} + b_4 D_\alpha \bar{D}^\dot{\alpha} \bar{u}_\dot{\alpha} + 2b_5 m u_\alpha + \gamma m \chi_\alpha .$$
Now we repeat the process of eliminating $H_{\alpha \dot{\alpha}}$, but since $u_\alpha$ doesn’t couple to $H_{\alpha \dot{\alpha}}$ nothing will be changed regarding the $H_{\alpha \dot{\alpha}}$-dependent terms. The same choice of coefficients as in (24) must be made to remove $H_{\alpha \dot{\alpha}}$. So the updated expression for $I_\alpha$ is

$$I_\alpha = 2Ba_3mD^2\bar{D}^2\chi_\alpha - 4m^2D^2\chi_\alpha$$

$$+ B\gamma mD^2u_\alpha + 2mD\bar{\alpha}\bar{\alpha}D^2u_\bar{\alpha}$$

$$+ \gamma m^3u_\alpha + 2a_3m^3\chi_\alpha \ .$$

(26)

Now we want to use the equation of motion of $u_\alpha$ to remove any dependences on $\chi_\alpha$ in order to derive an equation of $u_\alpha$. For that we calculate the updated version of $I_\alpha$ which we denote by $J_\alpha$ whose explicit form is given by

$$J_\alpha = I_\alpha + mK\bar{D}^2\bar{\epsilon}_\alpha(u) + m\Lambda D\bar{\alpha}\bar{\alpha}\bar{\epsilon}_\alpha(u)$$

$$= [2Ba_3]D^2\bar{D}^2\chi_\alpha$$

$$+ [B\gamma + 2Kb_2 + \Lambda b_3]mD^2\bar{\alpha}u_\bar{\alpha}$$

$$- [4 - K\gamma]m^2D^2\chi_\alpha$$

$$+ [Kb_3 + 2\Lambda b_2]mD^2\bar{\alpha}\bar{\alpha}D\bar{\alpha}u_\bar{\alpha}$$

$$+ [2 + \Lambda \gamma]m^2D\bar{\alpha}\bar{\alpha}$$

$$+ [Kb_5]m^2D^2u_\alpha$$

$$+ [\Lambda b_5]m^2D\bar{\alpha}\bar{\alpha}u_\bar{\alpha} \ .$$

(27)

If we choose

$$a_3 = 0 \ , \ - 4 + K\gamma = 0 \ , \ 2 + \Lambda \gamma = 0 \ ,$$

(28)

we derive an equation of motion for $u_\alpha$ in the form

$$J_\alpha = [B\gamma + 2Kb_2 + \Lambda b_3]mD^2\bar{\alpha}u_\bar{\alpha}$$

$$+ [Kb_3 + 2\Lambda b_2]mD^2\bar{\alpha}\bar{\alpha}D\bar{\alpha}u_\bar{\alpha} + [\Lambda (2b_4 - b_3)]D\alpha D^2\beta u_\beta$$

$$+ \gamma m^3u_\alpha$$

(29)

Now we are in position to choose coefficients so as to make $u_\alpha$ vanish on-shell by selecting

$$B\gamma + 2Kb_2 + \Lambda b_3 = 0 \ , \ Kb_3 + 2\Lambda b_2 = 0 \ , \ 2b_4 - b_3 = 0 \ , \ b_5 = 0 \ , \ \gamma \neq 0$$

(30)

Since $u_\alpha = 0$ on-shell, now we can reverse the arguments. Its equation of motion will give $\chi_\alpha = 0$ and that will put constraints on $H_{\alpha \dot{\alpha}}$: $D^2\bar{\alpha}H_{\alpha \dot{\alpha}} = 0$

$$\bar{\epsilon}_\alpha^{(H)} = 2D\gamma \bar{D}^2\alpha H_{\alpha \dot{\alpha}} + 2a_2m(D^2H_{\alpha \dot{\alpha}} + \bar{D}^2H_{\alpha \dot{\alpha}}) + 2a_4m^2H_{\alpha \dot{\alpha}} \ ,$$

$$\bar{\epsilon}_\alpha^{(\chi)} = -2D^2\bar{\alpha}H_{\alpha \dot{\alpha}} \ .$$

(31)
Finally because of $D^{2}D^{\dot{\alpha}}H_{\alpha\dot{\alpha}} = 0$ we see that
\begin{equation}
D^{\alpha}E^{(H)}_{\alpha\dot{\alpha}} = 2a_{2}mD^{\alpha}D^{2}H_{\alpha\dot{\alpha}} + 2a_{4}m^{2}D^{\alpha}H_{\alpha\dot{\alpha}} .
\end{equation}
For $a_{2} = 0$, $a_{4} \neq 0$ this gives $D^{\alpha}H_{\alpha\dot{\alpha}} = 0$. Thus the equation of motion for $H_{\alpha\dot{\alpha}}$ becomes the Klein-Gordon equation with $a_{4} = 1$
\begin{equation}
\Box H_{\alpha\dot{\alpha}} = m^{2}H_{\alpha\dot{\alpha}}
\end{equation}
To complete the analysis we look for the consistency and non-trivial solution of the systems of equations (20), (28), (30), $a_{2} = 0$, and $a_{4} = 1$. A solution exists and it is
\begin{align*}
a_{1} &= 0 , \quad b_{1} = \text{free, can be set to zero} , \quad \gamma = 1 , \quad \Lambda = -2 , \\
a_{2} &= 0 , \quad b_{2} = \frac{1}{6} \\
a_{3} &= 0 , \quad b_{3} = \frac{1}{6} , \quad \quad A = 1 , \\
a_{4} &= 1 , \quad b_{4} = \frac{1}{12} , \quad \quad B = -1 , \\
b_{5} &= 0 .
\end{align*}
The final action takes the form
\begin{equation}
S = \int d^{8}z \left\{ H^{\alpha\dot{\alpha}}D^{\gamma}D^{2}D_{\gamma}H_{\alpha\dot{\alpha}} + mu^{\alpha}\chi_{\alpha} + c.c. \\
- 2 H^{\alpha\dot{\alpha}}D_{\dot{\alpha}}D^{2}\chi_{\alpha} + c.c. + \frac{1}{6}u^{\alpha}D^{2}u_{\alpha} + c.c. \\
- 2 \chi^{\alpha}D^{2}\chi_{\alpha} + c.c. + \frac{1}{6}u^{\alpha}D^{\dot{\alpha}}u_{\dot{\alpha}} \\
+ 2 \chi^{\alpha}D_{\alpha}D^{\dot{\alpha}}\chi_{\dot{\alpha}} + \frac{1}{12}u^{\alpha}D_{\alpha}D^{\dot{\alpha}}u_{\dot{\alpha}} \\
+ m^{2}H^{\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} \right\} .
\end{equation}
This is the superspace action that describes a superspin $Y = \frac{3}{2}$ system with the minimum number of auxiliary superfields and has a massless limit that gives non-minimal supergravity. This action is a representative of a family of actions that are all equivalent and connected through superfields redefinitions of the form
\begin{align}
\chi_{\alpha} &\rightarrow \chi_{\alpha} + z_{1}u_{\alpha} + w_{1}D^{\alpha}H_{\alpha\dot{\alpha}} \\
u_{\alpha} &\rightarrow u_{\alpha} + z_{2}\chi_{\alpha} + w_{2}D^{\dot{\alpha}}H_{\alpha\dot{\alpha}}, \text{ where } z_{i}, w_{i} \text{ are complex}
\end{align}

6 Summary

We started with the $\frac{3}{2}$ superhelicity theory of non-minimal supergravity, formulated in terms of a real vector superfield $H_{\alpha\dot{\alpha}}$ and a fermionic compensator $\chi_{\alpha}$. We then
added mass terms to it in an attempt to discover a theory for massive superspin $\frac{3}{2}$ system, only to find that it is not possible and we need the help of an extra fermionic auxiliary superfield $u_\alpha$ which must couple only to $\chi_\alpha$ through a mass term. Finally using the equations of motion we manage to show that on-shell $u_\alpha = 0 \leadsto \chi_\alpha = 0 \leadsto \Delta^\alpha H_{\alpha\dot{\alpha}} = 0 \leadsto \square H_{\alpha\dot{\alpha}} = m^2 H_{\alpha\dot{\alpha}}$

We have managed to derive yet another formulation of massive supergravity and most importantly probe the set of auxiliary superfields required for the construction of higher superspin theories.

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