Container slot allocation and dynamic pricing considering overbooking and market segmentation

Mingzhu Yu1* and Jiajing Liao2

1 College of Management, Shenzhen University, Shenzhen, Guangdong, 518060, China
2 College of Civil and Transportation Engineering, Shenzhen University, Shenzhen, Guangdong, 518060, China
*Corresponding author’s e-mail: mzyu@szu.edu.cn

Abstract. In this paper, we study the problem of slot allocation and dynamic pricing for container liner with uncertain demand. We consider the carrier’s overbooking strategy and market segmentation. A two-stage mathematical model is proposed for container liner slot allocation and dynamic pricing. After linearizing the model, CPLEX can be used to solve it. The results of numerical experiments show that the model considering both overbooking and market segmentation can significantly improve the revenue of container liner companies.

1. Introduction
Maritime transportation is the most important part of the overall transport system in modern society and plays an important role in international trade. More than four-fifths of the world merchandise trade is carried out by ocean transportation, especially ocean container transportation. Marine transport mainly includes liner transport and charter transport. This paper focuses on liner transport. According to the latest data, in September 2020, 6,135 container ships were in operation globally, with a total capacity of 24,062,901 TEU, which is equivalent to about 290 million DWT.

The container liner will sail according to a certain schedule and port rotation. The slots on container liners are limited. Container slot demand is very unstable. Many factors will impact the container slot demand. For example, during the COVID-19 outbreak, the container slot demand declined and there was surplus capacity. When COVID-19 is under control, the demand may increase. Due to the volatility of demand, the price of container slot also changes dynamically. Therefore, how to allocate the container slot capacity and to set the container slot price to the consignees is important for the ocean carriers.

Overbooking strategy means that the merchant sells more products than the actual available capacity to hedge against customer cancellations before service is offered or non-presence when the service is offered (no-shows) [1]. The overbooking strategy originated in the airline passenger transport industry. At present, it is widely used in air passenger transport, hotel rooms and car rental industry (e.g., Yoon et al. [2]; Xu and Gang [3]; Li and Pang [4]; Pimentel [5]). In fact, the overbooking strategy is also useful in the allocation and dynamic pricing of container liner slot. However, there are few studies considering the overbooking strategy in the allocation and dynamic pricing of container liner slot. Wang et al. [6]

1 UNCTAD. (2019) United nations conference on trade and development, Review of maritime transport 2019, Geneva. pp. 19-20.
2 Alphaliner. (2020) Alphaliner TOP 100. https://alphaliner.axsmarine.com/PublicTop100/
established the profit maximization MINLP model to solve the seasonal revenue management problem of liner shipping companies. Liu and Yang[7] proposed a two-stage optimal model based on revenue management to solve the problem of slot allocation and dynamic pricing for multimodal transport of container sea rail. None of these studies used an overbooking strategy. Wang et al.[8] adopted overbooking (OB) and delivery-postponed (DP) strategies to solve the container slot allocation problem and proposed a surrogate subgradient algorithm. Their study considered the overbooking strategy but did not consider market segmentation and empty container scheduling, which could not be ignored in the actual transportation. Market segmentation means that the container slots could be separately sold through contract market (pre-booking) and spot market. This study considers overbooking strategy and market segmentation and establishes a two-stage mathematical model for container liner slot allocation and dynamic pricing. At the same time, the results of numerical experiments show that compared with the basic model and the model with only the overbooking strategy, the two-stage mathematical model with both the overbooking strategy and the market segmentation can significantly improve the earnings of container liner companies.

2. Problem description and assumptions

Suppose that a container liner shipping company operates a container liner with the capacity of $Q$ and the container liner sails in a one-way cycle between ports. A number of ports is denoted by set $P$, $p = 1, 2, \ldots, |P|$. A service route consists of several sea voyage segments. $l$ is the index for a sea voyage segment between two adjacent ports. Segments $l$ refers to the sea voyage segment from the $p_l$ to $p_{l+1}$, $l = 1, 2, \ldots, |P|$. Especially, $l = |P|$ refers to the sea voyage segment from the $p_{|P|}$ to $p_1$.

Customers who need to transport containers will apply to container liner shipping company for reserved slots. These requirements will be expressed as a certain amount of cargo needs to be transported by container liner from the port of origin to the port of destination. A certain number of slot required is denoted by $D_{ij}$, where $i$ is the port of origin, $j$ is the port of destination.

The objective of this paper is to allocate the limited capacity of shipping slots reasonably and sell them to customers at an appropriate price, so as to maximize the benefits of container liner shipping company while meeting the market demand and empty container transportation.

Container slots are perishable assets, which mean if they cannot be sold timely, their value disappears as they couldn't be stored. Some orders may not arrive on time. This will lead to some empty slots during the voyage of container liner [8]. In order to simplify the model, this paper assumes that cancellations are not considered. Overbooking is an effective strategy to avoid no-show risk. This paper defines the actual arrival rate $\eta$ as the ratio of the actual arrival number of orders to the total number of orders. Through the overbooking strategy, container liner shipping company can sell more slots than their capacity (but no more than $U$). But if the number of orders arrived on time exceed the capacity, some orders will be rejected with unit penalty cost $c^a$.

The customers of container liner shipping companies can be divided into contract customers and spot customers[9]. The contract customers are usually big customers with high demand for container transportation and stronger bargaining power. And spot customers are generally small customers. This paper considers market segmentation to study these two types of customers separately. We found that the overbooking occurred mostly in the contract market, while the spot market freight rate can be adjusted more flexibly. Therefore, for the contract market, this paper will consider empty container transportation and the strategy of overbooking to establish a container slot allocation model. For the spot market, this paper will consider both slot allocation and dynamic pricing.

3. Mathematical Models

In this section, we construct three mathematical models to solve the problem of slot allocation and dynamic pricing.
3.1. basic model

Without overbooking and market segmentation strategies, the basic model of liner company revenue maximization is established by considering only the slot allocation and empty container scheduling. The notations used in the basic model are defined as follows. The number of shipping slots allocated to customers from \( p_l \) to \( p_j \) is \( x_{ij} \) (TEU). The number of shipping slots allocated to empty slot from \( p_l \) to \( p_j \) is \( y_{ij} \) (TEU). Freight rate for cargoes from \( p_l \) to \( p_j \) is \( e_{ij} \) (dollar/TEU). Total slot capacity of container liner is \( Q \) (TEU). Demand for empty slot in port \( p_j \) is \( D_j^E \) (TEU). Demand for cargoes from \( p_l \) to \( p_j \) is \( D_j^H \) (TEU). Transportation cost for cargoes from \( p_l \) to \( p_j \) is \( c_{ij}^H \) (dollar/TEU). Transportation cost for empty slot from \( p_l \) to \( p_j \) is \( c_{ij}^E \) (dollar/TEU). When the voyage from the origin port to the destination port passes through segment \( l \), \( n_{ijl} = 1 \), otherwise \( n_{ijl} = 0 \).

\[
\begin{align*}
\text{max } & Z^B = \sum_{i,j \in P} (e_{ij}x_{ij} - c_{ij}^H x_{ij} - c_{ij}^E y_{ij}) \\
\text{s.t.} & \quad \sum_{i,j \in P} n_{ijl}x_{ij} + \sum_{i,j \in P} n_{ijl}y_{ij} \leq Q \quad \forall l \in L \\
& \quad x_{ij} \leq D_j^H \quad \forall i, j \in P \\
& \quad \sum_{l \in P} y_{ij} \geq D_j^E \quad \forall j \in P \\
& \quad x_{ij}, y_{ij} \in N \quad \forall i, j \in P
\end{align*}
\]

Objective function (1) maximizes the revenue of shipping company, which equals to the incomes minus transportation costs. Constraints (2) represent the capacity constraint. Constraints (3) represent the demand constraint of cargoes. Constraints (4) represent the demand constraint of empty containers. Constraints (5) indicate that the decision variables are natural numbers.

3.2. Overbooking (OB) model

The overbooking model is as follows.

\[
\begin{align*}
\text{max } & Z^{OB} = \sum_{i,j \in P} (e_{ij}x_{ij} - c_{ij}^H x_{ij} - c_{ij}^E y_{ij}) - c^o \sum_{l \in L} \left[ \sum_{i,j \in P} n_{ijl}(x_{ij} + y_{ij}) - Q \right]^+ \\
\text{s.t.} & \quad \sum_{i,j \in P} n_{ijl}x_{ij} + \sum_{i,j \in P} n_{ijl}y_{ij} \leq Q + U \quad \forall l \in L \\
& \quad x_{ij} \leq D_j^H \quad \forall i, j \in P \\
& \quad \sum_{l \in P} y_{ij} \geq D_j^E \quad \forall j \in P \\
& \quad x_{ij}, y_{ij} \in N \quad \forall i, j \in P
\end{align*}
\]

Objective function (6) maximizes the revenue of shipping company, which is equal to the incomes minus transportation costs and overbooking penalties. Constraints (7) represent the capacity constraint of slot allocation. Constraints (8) represent the demand constraint of cargoes. Constraints (9) represent the demand constraint of empty containers. Constraints (10) indicate that the decision variables are natural number.

3.3. Overbooking and market segmentation(OBMS) model

The new notations used in the overbooking and market segmentation(OBMS) model are defined as follows. The number of shipping slots allocated to customers from \( p_l \) to \( p_j \) at the first stage is \( x_{ij}^a \) (TEU). The number of shipping slots allocated to customers from \( p_l \) to \( p_j \) with pre-booking time \( w \) at
the second stage is $x_{ijw}^b$ (TEU). Freight rates for cargoes from $p_i$ to $p_j$ with pre-booking time $w$ at the second stage is $e_{ijw}^b$ (dollar/TEU). The set of the pre-booking period is $W$ (weeks), $w \in W$. The coefficient of the spot market demand function represents the potential demand in the spot market and can be estimated from historical data is $\alpha$. The coefficient of the spot market demand function represents the price sensitivity and can be estimated from historical evidence $\beta$. Freight rates for cargoes from $p_i$ to $p_j$ at the first stage is $e_{ij}^a$ (dollar/TEU). Demand for cargoes from $p_i$ to $p_j$ at the first stage is $D_{ij}^a$ (TEU). Demand for cargoes from $p_i$ to $p_j$ with pre-booking time $w$ at the second stage is $D_{ijw}^b$ (TEU).

**The first stage objective function (contract market)**

$$
\text{max } Z^c = \sum_{i,j \in P} \left( e_{ij}^c x_{ij}^a - c_{ij}^h x_{ij}^a - c_{ij}^E y_{ij} - c^o \sum_{l \in L} \sum_{i,j \in P} n_{ijl}(\eta x_{ijl}^a + y_{ijl}) - Q \right)^+ 
$$

s.t.

1. $\sum_{i,j \in P} n_{ijl}x_{ijl}^a + \sum_{i,j \in P} n_{ijl}y_{ijl} \leq Q + U \quad \forall l \in L$ (12)
2. $x_{ijl}^a \leq D_{ij}^a \quad \forall i, j \in P$ (13)
3. $\sum_{i \in P} y_{ijl} \geq D_{ijl}^E \quad \forall j \in P$ (14)
4. $x_{ijl}^a, y_{ijl} \in N \quad \forall i, j \in P$ (15)

Objective function (11) maximizes the revenue of shipping company at contract market, which is equal to the incomes minus transportation costs and overbooking penalties. Constraints (12) represent the capacity constraint of slot allocation. Constraints (13) represent the demand constraint of cargoes. Constraints (14) represent the demand constraint of empty containers. Constraints (15) indicate that the decision variables are natural number.

**The second stage objective function (spot market)**

$$
\text{max } Z^b = \sum_{w \in W} \sum_{i,j \in P} (e_{ijw}^b x_{ijw}^b - c_{ijw}^h y_{ijw}^b)
$$

s.t.

1. $D_{ijw}^b = \alpha - \beta e_{ijw}^b \quad \forall i, j \in P, \forall w \in W$ (17)
2. $\sum_{i,j \in P} n_{ijw}x_{ijw}^a + \sum_{i \in P} \sum_{j \in P} n_{ijw}y_{ijw}^a + \sum_{w \in W} \sum_{i,j \in P} n_{ijw}x_{ijw}^b \leq Q \quad \forall l \in L$ (18)
3. $x_{ijw}^b \leq D_{ijw}^b \quad \forall i, j \in P, \forall w \in W$ (19)
4. $x_{ijw}^b \in N \quad \forall i, j \in P, \forall w \in W$ (20)
5. $e_{ijw}^b \geq 0, e_{ijw}^b \in R \quad \forall i, j \in P, \forall w \in W$ (21)

Objective function (16) maximizes the revenue of shipping company at spot market, which is equal to the incomes minus transportation costs. Constraints (17) represent the linear relationship between spot market demand and freight rates. Constraints (18) represent the capacity constraint of slot allocation ($x_{ijw}^a$ and $y_{ijw}^a$ are the optimal solutions got from the first stage). Constraints (19) represent the demand constraint of cargoes. Constraints (20) and Constraints (21) indicate that the range of decision variables.

### 4. Model Solution and numerical analysis

Since both the OB model and the OBMS model have nonlinear parts (Equation 6 and 11), we first need to linearize these parts.
4.1. Linearization

The nonlinear parts \( \sum_{i,j \in P} n_{ij}(\eta x_{ij} + y_{ij}) - Q\) (Equation 6) and \( \sum_{i,j \in P} n_{ij}(\eta x_{ij}^d + y_{ij}) - Q\) (Equation 11) can be linearized by following steps. Above all, we introduce the process variable \( \Delta x_{ij} \) (Equation 22, 23) and \( \Delta x_{ij}^d \) (Equation 24, 25).

\[
\Delta x_{ij} \geq \sum_{i,j \in P} n_{ij}(\eta x_{ij} + y_{ij}) - Q^+ \quad \forall i \in L \\
\Delta x_{ij} \geq 0 \\
\Delta x_{ij}^d \geq \sum_{i,j \in P} n_{ij}(\eta x_{ij}^d + y_{ij}) - Q^+ \quad \forall i \in L \\
\Delta x_{ij}^d \geq 0
\]

Then the nonlinear equation (6), (12) can become the linear equation (26), (27):

\[
\begin{align*}
\max Z^{OB} &= \sum_{i,j \in P} (e_{ij} x_{ij} - c_{ij}^H \eta x_{ij} - c_{ij}^E y_{ij}) - c^o \sum_{i \in L} \Delta x_{ij} \\
\max Z^d &= \sum_{i,j \in P} (e_{ij}^d x_{ij}^d - c_{ij}^H \eta x_{ij}^d - c_{ij}^E y_{ij}) - c^o \sum_{i \in L} \Delta x_{ij}^d
\end{align*}
\]

The linearized models can be solved by commercial optimization software, such as CPLEX.

4.2. Numerical example

In this subsection, the validity of the model will be verified by numerical example. In this example, a container liner shipping company operates a container liner with a capacity of \( Q = 5000 \) TEUs. The liner calls at three ports in turn, and the operating route is \( p_1 \rightarrow p_2 \rightarrow p_3 \rightarrow p_4 \). It receives the container slot freight demand from six different port pairs. At the same time, this paper considers three different contract market demands: high, low and medium. When the contract market demand is high, container slot demand is 1.5 times that of the medium demand. When the contract market demand is low, container slot demand is 0.8 times that of the medium demand. Assuming that the container slot medium market demand has been predicted based on historical information, that \( \eta = 0.9 + \text{rand}(-0.03, 0.03) \), \( w = \{1, 2, 3\} \), \( c^o = 2500 \), \( \alpha = 600 \), \( U = 80 \), \( \beta = 0.1 \), \( D_{ijw}^H = 0.9D_{ij}^H < 0.2, 0.25, 0.3 \) >.

The Data for numerical experiments are shown in Table 1 (Some data are adopted from [8]).

| Table 1. Data for numerical experiments. |
|-----------------------------------------|
| from \( p_1 \) to \( p_j \) | \( p_1 \rightarrow p_2 \) | \( p_1 \rightarrow p_3 \) | \( p_2 \rightarrow p_3 \) | \( p_2 \rightarrow p_1 \) | \( p_1 \rightarrow p_3 \) | \( p_3 \rightarrow p_1 \) | \( p_3 \rightarrow p_2 \) |
|-----------------------------------------|
| Basic model | \( e_{ij} \) | 1370 | 2960 | 2750 | 2050 | 1210 | 1980 |
| OB model | \( \mu_{ij} \) | 1980 | 1200 | 980 | 2080 | 2000 | 1290 |
| OBMS model | \( \sigma_{ij} \) | 130 | 120 | 95 | 115 | 110 | 135 |
| All model | \( e_{ij}^H \) | 1300 | 2900 | 2700 | 2000 | 1200 | 1900 |
| | \( c_{ij}^E \) | 1900 | 1200 | 900 | 2000 | 1900 | 1200 |
| | \( \sigma_{ij}^E \) | 120 | 110 | 90 | 110 | 105 | 125 |
| Contract Market Demand Situation | Case1 | medium | medium | medium | medium | medium | medium |
| Case2 | high | high | high | high | high | high |
| Case3 | low | low | low | low | low | low |

\( D_{ij}^{H} = 0 \), \( D_{ij}^{E} = 280 + \text{rand}(-20, 20) \), \( D_{ij}^{F} = 0 \)
4.3. Result analysis

Using CPLEX to solve the three models separately, the optimal solution can be obtained as shown in the following table.

| Case   | The revenue of shipping company (Unit: dollar) | Improvement rate | Improvement rate |
|--------|-----------------------------------------------|------------------|------------------|
|        | basic            | OB         | OBMS       |                  |
| Case1  | 18267640         | 20481359   | 22168194   | 21.35%           |
| Case2  | 19236360         | 22496923   | 23521657   | 22.28%           |
| Case3  | 17880105         | 19981108   | 21286898   | 19.05%           |

It can be seen from Table 2 that, in general, compared with the basic model, both the OB model and the OBMS model have improved the revenue of shipping companies. Among them, the OB model increased the revenue of shipping company by more than 11.75%, the OBMS model increased the revenue of shipping company by more than 19.05%. The OBMS model has the higher increase rate than the OB model. In the OB model, the demand of contract market changes has a great impact on the results. In the case of strong market demand, the revenue improvement rate is the highest, with an increase of 16.95%. In the case of weak market demand, the revenue improvement rate is the lowest, with an increase of 11.75%.

For the OBMS model, in the case of strong market demand, the revenue improvement rate is the highest, with an increase of 22.28%. In the case of weak market demand, the revenue improvement rate is the lowest, with an increase of 19.05%. To sum up, the results of numerical experiments show that compared with the basic model and the OB model, the OBMS model can significantly improve the revenue of container liner companies.

4.4. Sensitivity analysis

Figure 1 shows the impact of the change in the actual arrival rate, \( \eta \), on the revenue of shipping company in OBMS model. We can find that the revenue of the shipping company decreases with the increase of the actual arrival rate \( \eta \). In addition, when the market demand is strong, the change of the actual arrival rate has a smaller impact on the earnings of shipping company, and the lower the market demand is, the greater the impact of the change of the actual arrival rate on the earnings of shipping company.
Next, we conducted a sensitivity analysis on parameters $\alpha$ (The potential demand in the spot market). It can be seen from Figure 2 that the revenue of shipping company increases with the increase of parameter $\alpha$. At the same time, the stronger the market demand is, the smaller the effect of parameter $\alpha$ on earnings of shipping company will be, while the weaker the market demand is, the larger the effect of parameter $\alpha$ on earnings of shipping company will be.

5. Conclusions
Based on the strategy of overbooking and market segmentation, this study establishes a two-stage mathematical model to rationally allocate limited slot capacity and set reasonable price while meeting the market demand and empty container scheduling demand. The linearized models can be solved by CPLEX. The results of numerical experiments show that compared with the basic model and the model using only the overbooking strategy, the two-stage mathematical model using both the overbooking strategy and the market segmentation strategy can significantly improve the earnings of container liner companies. In addition, this paper also has some limitations. In order to simplify the model, this study assumes that the route is determined. Our future work may consider slot allocation and dynamic pricing when the route is uncertain.

References
[1] Klein, R., Koch, S., Steinhardt, C., Strauss, A.K. (2020) A review of revenue management: recent generalizations and advances in industry applications. European Journal of Operational Research, 284: 397-412.
[2] Yoon, M.G., Lee, H.Y., Song, Y.S. (2012) Linear approximation approach for a stochastic seat allocation problem with cancellation refund policy in airlines. Journal of Air Transport Management, 23: 41-46.
[3] Xu, C., Gang, H. (2013) Co-opetition alliance models of parallel flights for determining optimal overbooking policies. Mathematical Computer Modelling, 57: 1101-1111.
[4] Li, D., Pang, Z. (2017) Dynamic booking control for car rental revenue management: a decomposition approach. European Journal of Operational Research, 256: 850-967.
[5] Pimentel, V., Aizezikali, A., Baker, T. (2019) Hotel revenue management: benefits of simultaneous overbooking and allocation problem formulation in price optimization. Computers Industrial Engineering, 137: 106073-.
[6] Wang, Y., Meng, Q., Du, Y. (2015) Liner container seasonal shipping revenue management. Transportation Research Part B: Methodological, 82: 141-161.
[7] Liu, D., Yang, H. (2015) Joint slot allocation and dynamic pricing of container sea–rail multimodal transportation. Journal of Traffic Transportation Engineering, 2: 198-208.
[8] Wang, T., Xing, Z., Hu, H., Qu, X. (2019) Overbooking and delivery-delay-allowed strategies for container slot allocation. Transportation Research Part E: Logistics and Transportation Review, 122: 433-447.
[9] Meng, Q., Zhao, H., Wang, Y. (2019) Revenue management for container liner shipping services: critical review and future research directions. Transportation Research Part E: Logistics and Transportation Review, 128: 280-292.