Quantum oscillations of spin current through a III-V semiconductor loop

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We have investigated the transport of spin polarization through a classically chaotic semiconductor loop with a strong Rashba spin-orbit interaction. We found that if the escape time of a particle is long enough, the configuration averaged spin conductance oscillates strongly with the geometric spin phase. We predict a sizable rotation of spin polarization along its flowing path across the loop from the injector to the collector. We have also discovered a quantized universal spin relaxation in a 2D reservoir connected to such a semiconductor loop.

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In the emerging field of spin electronics, the recent achievement of spin injection into paramagnetic semiconductors [1] makes it an urgent task to control the spin current in semiconductor nanostructures. The spin current in a 2D channel of narrow gap III-V semiconductor can be manipulated by taking advantage of the strong spin-orbit splitting of the conduction electron energy, because the mechanism of such splitting produces a spin precession which depends on the electron quasimomentum. One example is the spin valve transistor [2], in which spin polarisation precesses in a 2D semiconductor channel between a ferromagnet spin injector and a ferromagnetic spin collector. The measured resistance is determined by the angle of spin rotation along the propagating path. This angle can be varied by adjusting the spin-orbit interaction (SOI) strength in the semiconductor with an external gate [3].

In this Letter we will investigate an interesting phenomenon which can be observed in a 2D semiconductor loop as shown in Fig. 1. It is well known [4] that due to the SOI, when an electron travels along a closed path, its wave function accumulates an additional phase $\psi$. If the SOI is a linear function of the electron quasimomentum, this phase depends only on the shape and the length of the path. In multiconnected conductors the effect of this phase on electron transport is similar to the Aharonov-Bohm (AB) effect. For example, in a disordered 2D ring, $\psi$ adds itself to the AB phase in the Aronov-Al'tshuler-Spivak oscillation of the DC electric conductance [5], as well as to the AB oscillation of the electric conductance mesoscopic fluctuations [6].

In this Letter, instead of the oscillation of the electric current, we will study the effect of the spin phase on the quantum oscillation of the spin current. However, we will assume the motion of electrons as ballistic along their classical trajectories, rather than a diffusive transport inside the loop [4, 5]. One such trajectory is schematically illustrated by the zigzag-lined path in Fig. 1, although it can be curved by a smooth random potential produced by the modulation doped impurities.

We assume that the motion of a particle inside the loop is classically chaotic, and so the quasiclassic approach of Ref. [7] can be applied. Nevertheless, to calculate the spin current through the loop we need to generalize this method by taking into account the spin degree of freedom and the SOI. With this approach, we will calculate the average spin current. This implies that its mesoscopic fluctuations will be averaged out. The corresponding experimental performance is, for example, to average the results measured with several gate voltage sweeps. We will also ignore the weak localization correction, which is small because our system has a large number of transport channels through the loop. We will show that the so calculated spin conductance oscillates as a function of the SOI strength, and consequently can be controlled by varying the gate voltage. We would like to emphasize that these quantum oscillations appear in the classical spin conductance, which is a drastically different phenomenon from the AB effect. The AB effect is absent in the average spin current when the weak localization effects are ignored.

Following the Landauer approach, to study the spin dependent conductance we define

$$g_{\alpha\beta\gamma\delta} = \frac{e^2}{h} \sum_{n,m} t_{n\alpha m\beta} t^*_{n\gamma m\delta}, \tag{1}$$

where $t_{n\alpha m\beta}$ is the transmission amplitude of an electron.
at Fermi energy $E_F$ propagating from the channel $n$ and the spin state $\alpha$ in the injector to the channel $m$ and the spin state $\beta$ in the collector. $\rho_{nm}^{\alpha\beta}$ itself is the $\alpha\beta$-element of the matrix $t_{mn}$ which operates on spin states. The usual spin independent electric conductance is simply $g=(e^2/h)\sum_{n,m,\alpha,\beta}|t_{nm}^{\alpha\beta}|^2$. If a spin oriented along the $x$-axis is injected from the injector, and its orientation becomes along the $y$-axis when the spin is collected at the collector, let $g_{xy}$ represent this spin current passing through the loop. The matrix elements can then be written as

$$g_{ij} = \frac{e^2}{h} \sum_{n,m} Tr \{ \sigma_i t_{nm} \sigma_j t_{nm}^\dagger \} ,$$

(2)

where $\sigma_i$ are Pauli matrices with $i=x, y, z$. The spin orientation can be detected by measuring the polarization of the emitted photons. In such an experiment, the polarization matrix of the emitted photons can be derived if we know $g_{ij}$.

In narrow gap III-V semiconductor quantum wells, the SOI is dominated by the Rashba interaction $[\delta]$ with the interaction Hamiltonian

$$H_{so} = \alpha \sigma \times p ,$$

(3)

where $p$ is the momentum operator, and the vector $\sigma$ has components $\sigma_x$, $\sigma_y$, and $\sigma_z$. Then the quasiclassical expression of $g_{a\beta\gamma\delta}$ can be easily obtained as

$$g_{a\beta\gamma\delta} = \frac{e^2}{\hbar} \sum_{a,b} t_0(a) t_0^*(b) S_a^{\alpha\beta} S_b^{\gamma\delta} ,$$

(4)

where $t_0(s)$ is the spin independent transmission amplitude for a classical trajectory labelled by $s$. One such trajectory is schematically plotted in Fig. 1 as the zigzag line. The explicit expression of $t_0(s)$ as well as the boundary conditions are given in Ref. [\delta]. The spin evolution operator $S_a$ along the $a$-trajectory is defined as

$$S_a = T \left[ \exp \left\{ -i \frac{\hbar}{\alpha m^*} \int_a z \times \sigma(r) \, dr \right\} \right] ,$$

(5)

where the integration is along the $a$-trajectory, $m^*$ is the effective mass, and $z$ is a unit vector parallel to the $z$-axis. The symbol $T$ means that the Pauli matrices must be ordered along the path.

The $g_{a\beta\gamma\delta}$ given in (4) must be averaged over the mesoscopic fluctuations, using the procedure described in Ref. [\delta]. Such averaging may be considered as a temperature effect, or, in accordance with the ergodic hypothesis, as an average over an ensemble of loops of slightly different shapes. Because of the rapidly oscillating phase factors in the quasiclassical amplitudes $t_0$, after the averaging procedure, in (4) only the terms with $a=b$ remain. Accordingly, we obtain from (4) the so averaged spin conductance $\langle g_{ij} \rangle$ as

$$\langle g_{ij} \rangle = \frac{e}{\hbar} \sum_a |t_0(a)|^2 D_{ij}^a ,$$

(6)

where

$$D_{ij}^a = Tr \{ \sigma_i S_a \sigma_j S_a^\dagger \} .$$

Similarly, the so averaged electrical conductance is simply $(2e^2/h)\sum |t_0(a)|^2$, and is spin independent.

The evolution matrix can be parametrized using its property that it is a SU(2) representation of 3D rotations. In fact, we can express $S_a$ as a time ordered product of infinitesimal rotations corresponding to small shifts $dr$ along the trajectory $a$. Each infinitesimal rotation is along the axis $dr \times z$ through an angle $2|dr \times z|/L_{so}$. These infinitesimal rotations are represented by operators $\exp[-i/L_{so}(d r \times z) \sigma]$, and they sum up to make $S_a$ for a finite rotation through the angle $2\psi_a$ around a unit vector $N_a$. Hence, the evolution matrix in (5) can be represented as

$$S_a = e^{i\psi_a N_a \sigma} .$$

(8)

We should notice that $\psi_a$ and $N_a$ are uniquely determined by the geometric shape and the length of the trajectory $a$.

From now on we will consider a particular sample geometry that the area occupied by the 2D electron gas in the loop is much less than $L_{so}^2$, and the linear dimension of the loop can be larger than $L_{so}$, where $L_{so}=\hbar/\alpha m^*$. In other word, both the upper path and the lower path of the loop are narrow. In this case one can show that each trajectory $a$ in (3), as indicated by the zigzag line in Fig. 1, can be replaced by a smooth trajectory, which is shown in Fig. 1 as the arrowed curve. We will label this smooth curve as $l$-path. Let $\theta_a$ be the area enclosed by the classical trajectory $a$ making one turn around the loop. Then, the area enclosed by the $l$-path is the average of $\theta_a$ over classical trajectories. Deviations of real paths from the $l$-path can be treated perturbatively, which will be reported elsewhere.

After the trajectory $a$ is replaced by the $l$-path, the evolution matrix (5) becomes a simple function of the number $w$ of windings the trajectory $a$ makes around the loop until a particle escapes into the collector. $w$ is positive if the winding is counterclockwise. The corresponding evolution operator for the smooth $l$-path, denoted as $S(w)$, can be expressed as

$$S(w) = e^{i\psi_0 N_0 \sigma} e^{i\psi w N \sigma} .$$

(9)

Here $\psi_0$ and $N_0$ are the 3D rotation parameters for the $l$-path in the lower half of the loop, and $\psi$ and $N$ are the 3D rotation parameters for the $l$-path around the complete loop. Using Eq. (5) we obtain the general dependence of the trace in (7) on the winding number

$$D_{ij}(w) = M_{ij}^{(1)} e^{i2\psi w} + M_{ij}^{(-1)} e^{-i2\psi w} + M_{ij}^{(0)} ,$$

(10)

where the matrix elements $M_{ij}^{(1)} = M_{ij}^{(-1)*}$ depend only on the geometric shape of the $l$-path.
Based on the above expressions, we can follow the approach used in Ref. [3] to calculate the Aharonov-Bohm effect on mesoscopic electric conductance fluctuations in doubly connected classically chaotic loop. Let $T$ be the time interval that a particle spends inside the loop, and $T_0$ be such a duration for the shortest trajectory. Then, according to Ref. [3], we average the winding number with the Gaussian distribution function

$$ P(w|T) = \sqrt{\frac{T_0}{2\pi \alpha T}} \exp \left\{ -\frac{w^2}{2\beta^2 T/T_0} \right\}, \quad (11) $$

where $\beta$ is the system dependent dimensionless constant. For classically chaotic systems, it has been shown [7] that $T$ obeys the distribution function $P(T)=\pi^{-1} \exp[-(T-T_0)/\tau]$, where $\tau$ is the mean escape time of the particles. We should point out that this kind of statistical approach is valid only for large $\tau$, such that the particle can travel around the loop many times before it escapes from the loop. Hence, we assume $\kappa \ll \sqrt{2T_0/\tau \beta} \ll 1$. After averaging [11] over $w$ and $T$ we arrive at

$$ \langle D_{ij}(0) \rangle_{w,T} = M^{(0)}_{ij} + 2 \text{Re}[M^{(1)}_{ij}] A(\psi) \quad \text{(12)} $$

$$ A(\psi) = \kappa^2 / (\kappa^2 + 4 \sin^2 \psi). $$

The oscillation pattern of the spin current, caused by the $A(\psi)$ term, gets sharper as $\kappa$ becomes smaller, and eventually transforms into a periodic array of narrow peaks at positions $\psi = \pi n$. Eq. (12) provides a general relation between the spin conductance and the spin phase $\psi$. Here we will consider a specific example that the loop is nearly circular with radius $r$. In such a case, the loop becomes smaller, and even-though the SOI can transform into a periodic array of narrow peaks.

Considering the SOI in the loop gives rise to a spin dependent reflectance. A particle which enters the loop from the injector-connected lead with a given spin orientation can be reflected back into the same lead with an opposite spin direction. This provides an additional relaxation process of the spin polarization this lead, and the oscillatory dependence of such a relaxation on the spin phase $\psi$ is expected. A very suitable system for studying this relaxation mechanism is a reservoir connected to a loop via a point contact which has $N$ transmitting channels. The reservoir needs not to be very big. At a certain time, a nonequilibrium spin polarization $\Sigma=(N_\uparrow-N_\downarrow)/2$ is created in the reservoir, where $N_\sigma$ is the number of particles with spin projection $\sigma$ onto the quantization axis. Let $R_{\uparrow\downarrow}$ be the reflectance associated to the electron spin flip reflection. Then, the time rate of change of $\Sigma$ is given by

$$ \frac{d\Sigma}{dt} = -\frac{1}{2\hbar} R_{\uparrow\downarrow} (\mu_\uparrow - \mu_\downarrow) \frac{\Sigma}{r_s}, \quad (15) $$

where $\mu_\sigma$ is the chemical potential of the $\sigma$-spin state, and $r_s$ is the spin relaxation time within the reservoir. Let the reservoir be a 2D degenerate electron gas of volume $V$, with the density of states at the Fermi level $N(E_F)=m^*/2\pi\hbar^2$. Then, $(\mu_\uparrow-\mu_\downarrow)=2\Sigma/N(E_F)V$, and we find from (15) an additional spin relaxation rate in the reservoir

$$ \Gamma = (\hbar/m^*V) R_{\uparrow\downarrow}, \quad \text{(16)} $$

due to its connection to the loop. Following the same quasiclassical approach which leads to the results (14), we obtain

$$ R_{\uparrow\downarrow} = N \left\langle \left| \langle \exp[iw\psi N\sigma] \rangle_{\uparrow\downarrow} \right|^2 \right\rangle_w, \quad \text{(17)} $$

where $w<\cdots>w$ is an average over the winding number $w$. If the direction of spin polarization is parallel to $N$, then, the evolution matrix in (17) is diagonal, and hence $\Gamma=0$. On the other hand, if the spin polarization is perpendicular to $N$, we get $\langle \exp[iw\psi N\sigma] \rangle_{\uparrow\downarrow}^2 = \sin^2 \psi$. Consequently, averaging over $w$ gives the final form for the relaxation rate

$$ \Gamma = (\hbar/m^*V) \left[ 1 - A(\psi) \right], \quad \text{(18)} $$
For small $\kappa$ the function $[1-A(\psi)]$ oscillates between zero for $\psi=n\pi$ and a value very close to 1 for $\psi=(n+\frac{1}{2})\pi$. Hence, around $\psi=(n+\frac{1}{2})\pi$ the spin relaxation rate $\Gamma$ is determined mainly by the ratio $N/V$. Taking a typical value $V=4 \mu m^2$, and the electron effective mass $m^*/m=0.03$ as in InAs, we obtain $\Gamma \simeq N \cdot 10^{-9}$ sec$^{-1}$. The ”quantum” of relaxation time, $10^{-9}$ sec, is comparable to the intrinsic relaxation time $\tau_s$. We would like to emphasize that at this fixed spin phase $\psi=(n+\frac{1}{2})\pi$, the relaxation rate is a universal value in the sense that it does not depend on a shape and other parameters of the loop. On the other hand, we should also point out that (18) is valid only in the quasiclassic limit when $N \gg 1$. Therefore, one can not extrapolate this result to $N \simeq 1$ where this quantization effect may be observed experimentally.

In order to be able to observe the spin relaxation due to the SOI in the loop, it is necessary to have a long intrinsic spin relaxation time in the reservoir. Therefore, the SOI in the reservoir must be sufficiently small while the SOI in the loop must be sufficiently large. Such a condition can be created by proper modulation doping, resulting in a highly asymmetric confinement potential near the loop but a nearly symmetric confinement potential in the vicinity of reservoir. A strong Rashba SOI will then appear only in the loop.

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[1] R. Fiederling, M. Keim, G. Reuscher, W. Ossau, G. Schmidt, A. Waag, L.W. Molenkamp, Nature 402, 787 (1999); Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura, H. Ohno, D. D. Awschalom, Nature 402, 790 (1999); A. T. Hanbicki, B. T. Jonker, G. Itskos, G. Kioseoglou, A. Petrou, preprint [cond-mat/0110059].
[2] S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).
[3] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).
[4] H. Mathur and A. D. Stone, Phys. Rev. Lett. 68, 2964 (1992).
[5] B. L. Altshuler and A. G. Aronov, in Electron-Electron Interactions in Disordered Systems, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
[6] A. G. Mal’shukov, V. V. Shlyapin, K. A. Chao, Phys. Rev. B 60, R2161 (1999); Hans-Andreas Engel and Daniel Loss, Phys. Rev. B 62, 10238 (2000).
[7] R. A. Jalabert, H. U. Baranger and A. D. Stone, Phys. Rev. Lett. 65, 2442 (1990); H. U. Jalabert, R. A. Baranger, and A. D. Stone, Chaos 3, 665 (1993); R. Blümel, U. Smilansky, Phys. Rev. Lett. 60, 477 (1988).
[8] Yu. L. Bychkov and E. I. Rashba, J. Phys. C 17, 6093 (1984).
[9] S. Kawabata, Phys. Rev. B 58, 6704 (1998); S. Kawabata and K. Nakamura, Phys. Rev. B 57, 6282 (1998).
[10] L. D. Landau and E. M. Lifshitz, Quantum Mechanics, 3rd edition (Pergamon Press, 1977).