Nucleation of domain walls by \( Z_2 \) symmetry breaking transition in \( p_x + ip_y \) superconductors.

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We show that time reversal symmetry breaking \( p_x + ip_y \) wave superconductors undergo several phase transitions subjected to external magnetic field or supercurrent. In such system the discrete \( Z_2 \) symmetry can recover before the complete destruction of the order parameter. The topological defects associated with \( Z_2 \) symmetry - domain walls can be created in a controllable way by magnetic field or current sweep according to the Kibble-Zurek scenario. Such domain wall generation can take place in exotic superconductors like \( Sr_2RuO_4 \) and some heavy fermion compounds.

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Topological defect formation in the systems which undergo non-equilibrium phase transitions has become a subject to interdisciplinary research between high energy and condensed matter physics.\(^1\)\(^2\). Commonly accepted cosmological model suggests that cosmic strings can form according to the Kibble-Zurek (KZ) scenario through the nonequilibrium phase transition in expanding Universe.\(^3\) The KZ mechanism was confirmed in experiments with quantized vortices in superfluid \(^4\)Ho\(^6\) and \(^3\)He\(^7\)\(^8\) which can be produced by rapid quench or pressure sweep driving the system through the second order \( U(1) \) symmetry breaking phase transition.\(^4\)

The physics of domain walls (DWs) is less studied and remains a large enigma both in cosmology and condensed matter systems.\(^4\)\(^5\)\(^6\) Indeed the observational constraints require to accept the fact that DWs have disappeared at the early history of the Universe. A plausible explanation involves assumptions of the initial baryon asymmetry or time inversion symmetry violation which finally totally removes the domains of one kind.\(^7\)\(^8\) However these speculations remains yet unconfirmed which make theorists to rule out the models with discrete symmetry breaking since the mechanism of DWs disappearance remains a mystery.\(^9\)

One of the few known condensed matter systems which allows studying quench induced formation of cosmiclike DWs is superfluid \(^3\)He\(^10\). Experimentally DW generation was detected during the cooling into A-phase\(^11\). However with rapid temperature sweep one can hardly fine tune the parameters in order to produce exclusively DWs without producing vortices and composite defects.\(^12\) Moreover in real system quench is always spatially inhomogeneous which provides important modifications to the physics of defect formation.\(^13\)\(^14\)\(^15\) In this Letter we propose a unique selective mechanism of DWs formation during spatially homogeneous phase transition in exotic superconductors with chiral \( p_x + ip_y \) wave superconductor with Cooper pairs having an effective internal orbital momentum projection on the crystal anisotropy axis \( L_z = \pm 1 \). Such superconducting state has a broken time reversal symmetry (TRS) so the superconducting phase transition is determined by the spontaneous \( U(1) \times Z_2 \) symmetry violation. Recently such state was suggested to appear also in multiband superconductors.\(^21\)

The two different TRS breaking vacuum states can be separated by DWs which are known to support spontaneous supercurrent generating magnetic field.\(^22\) However high resolution scanning SQUID microscopy experiments detected no stray fields which should be generated by DWs above the surface of superconducting \( Sr_2RuO_4 \). Moreover polar Kerr effect measurements also did not reveal chiral domains. Thus up to now no direct observation of DWs in \( Sr_2RuO_4 \) was obtained although phase-sensitive Josephson spectroscopy experiments revealed some evidences of dynamical domain structure. This enigma of DWs stimulated further theoretical research. It has been suggested that in some cases the DW generates only very weak stray field.\(^25\) The stray fields suppression can result also from the multiband superconductivity which on the other hand can stimulate the proposed unconventional mixed state with vortex coalescence in \( Sr_2RuO_4 \).\(^26\)

In addition to the above mentioned hypotheses the possibility still remains that DWs disappear at some stage of the superconducting transition in \( Sr_2RuO_4 \). Therefore the proposed method to create in controllable way an arbitrary initial concentration of DWs in \( Sr_2RuO_4 \) can prompt experimental identification of this defects which has been recently one of the most intriguing problems in the field of low temperature physics. Moreover it can shed a new light on the fate of cosmic DWs during the early history of the Universe.

To describe DWs separating different \( L_z = \pm 1 \) vacuum states we use Ginzburg-Landau (GL) model of superconducting state in \( Sr_2RuO_4 \). This material belongs to the tetragonal crystallographic symmetry group \( D_{4h} \) and has strong crystal anisotropy which keeps both spin and orbital momentum of Cooper pairs parallel to the c axis.\(^18\) The coordinate system is chosen so that the crystal
The GL model \( \Gamma_5 \) yields two degenerate ground states \( (\eta_+, \eta_-) = (0, 1) \) and \( (1, 0) \). Here we implement numerical minimization of the GL energy \( \Gamma_5 \) choosing the \( x \) axis perpendicular to the DW plane. In Fig. 1 we plot the calculated order parameters and equilibrium density of supercurrent which flows along the DW.

Let us now consider \( p_x + ip_y \) superconducting film in \( xy \) plane so that the crystal anisotropy axis is \( z \parallel c \). The film is supposed to be thin \( d \ll \xi, \lambda \) where \( \xi \) and \( \lambda \) are coherence and London penetration lengths. This condition ensures that we can use the standard approximation when the magnetic field and order parameter are homogeneous along the \( z \) axis inside the film.

First we assume that the film is subjected to the magnetic field parallel to the film plane \( \mathbf{H} = H\mathbf{y} \) as shown in Fig. 2a. In a thin film of conventional superconductor the \( U(1) \) symmetry braking phase transition is known to be of the second order and the critical field \( H_c = \sqrt{\xi H_{cm}}/d \). However in \( U(1) \times Z_2 \) superconductor one can expect qualitatively new features. Indeed

FIG. 1: Domain wall structure in \( p_x + ip_y \) superconductor described by GL model \( \Gamma_5 \). The DW plane is \( yz \). We choose anisotropy parameters \( \nu_1 = \nu_2 = 0.1 \). By solid and dashed lines the distributions \( \eta_+(x) \) and \( \eta_-(x) \) are shown. By dashed-dotted line the longitudinal superfluid current density \( j_y(x) \) is shown normalized to \( j_0 = (c/4\pi)H_{cm}/\sqrt{2}\xi \). The overall order parameter magnitude \( \sqrt{\eta_+^2 + \eta_-^2} \) is shown by dotted line.

FIG. 2: Phase transitions in a thin film of \( p_x + ip_y \) superconductor. (a,b) Second order \( Z_2 \) and \( U(1) \) transitions under the action of external magnetic field and (c,d) First order transitions in external current. By red solid and dashed lines the order parameter amplitudes \( \eta_+ \) and \( -\eta_- \) are shown in the \( U(1) \times Z_2 \) phase. The energetically equivalent state is obtained by interchanging values of \( \eta_+ \) and \( -\eta_- \). The dotted blue line corresponds to the non-degenerate \( U(1) \) phase with order parameter components \( \eta_+ = -\eta_- \). Magnetic field and current is normalized to \( H_0 = H_{cm}d/(2\sqrt{3}\lambda) \) and \( j_0 = (c/4\pi)H_{cm}/\sqrt{2}\lambda \) correspondingly.
energy (1) by the amplitudes $\psi_{\pm}$ at fixed $k$ we obtain two stable branches of the order parameter.

(i) On the first branch the magnitude of order parameter components is different $|\psi_+| \neq |\psi_-|$ and they have opposite signs

$$|\psi_{\pm}|^2 = \left[1 - k^2 \pm \sqrt{(1 - k^2)^2 - k^4 \left(\frac{1 + \nu_2}{1 + \nu_1}\right)^2}\right]$$

Due to the invariance of GL theory (1) with respect to the replacement $\psi_+ \to \psi_-$, and vice versa, the found solution is twice degenerate and corresponds to the superconducting $U(1) \times Z_2$ phase. This solution is stable if the velocity of Cooper pairs smaller than the critical value $k < k_{Z_2} = \sqrt{(1 + \nu_1)/(2 + \nu_1 + \nu_2)}$. Note that $k_{Z_2} < k_c = \sqrt{2/(1 - \nu_2)}$ where $k_c$ is the depairing superfluid velocity which destroys superconducting state completely.

(ii) On the second branch the magnitudes of order parameter components are the same $\psi_+ = -\psi_-$ where

$$|\psi_{\pm}|^2 = \left[1 - k^2(1 - \nu_2)/(3 + \nu_1)\right]$$

Unlike the previous case, this solution is nondegenerate. Therefore it corresponds to usual $U(1)$ superconducting state. This phase is stable in the interval $k_{Z_2} < |k| < k_c$.

That is we obtain an additional phase transition at $k = k_{Z_2}$, when the ground state double degeneracy is removed and the corresponding discrete $Z_2$ symmetry is restored. The order parameter components change continuously while we shift the $k$ value through the $Z_2$ critical point therefore this is a second order phase transition.

The solution of an auxiliary problem considered above can be applied to find the critical fields of a thin $p_x + ip_y$ superconducting film. Indeed we choose Landau gauge $A_z = Bxyz$ [see Fig.2(a)] and use a standard thin film approximation assuming $\eta_{\pm}$ to be constants with respect to $z$ coordinate. Taking the $z$ average of the free energy yields an effective superfluid velocity $k = \sqrt{A_z^2} = dH/\sqrt{6}$. Then one immediately finds the critical fields values:

$$H_{Z_2} = (2\sqrt{3}\lambda/d)k_{Z_2}H_{cm}$$

$$H_c = (2\sqrt{3}\lambda/d)k_cH_{cm}$$

The critical field $H_{Z_2}$ restores discrete $Z_2$ symmetry and the field $H_c$ is a standard critical field of thin superconducting film which suppresses superconductivity completely. The evolution of order parameter components as functions of applied magnetic field is shown in Fig.2(b). In this case both $Z_2$ and $U(1)$ phase transitions are of the second order and characterized by vanishing order parameters and divergent coherence lengths.

Naturally the order parameter of $Z_2$ phase transition can be chosen in the form $\eta_1 = (\eta_+ + \eta_-)/2$. Indeed $\eta_1$ vanishes near $H_{Z_2}$ in the first phase and is identical zero in the second phase. To reveal the physical origin of $Z_2$ coherence length let us consider the structure of DW in the vicinity of the critical point. Here we can derive the equation for the order parameter $\eta_1$ taking the other component $\eta_2 = (\eta_+ - \eta_-)/2$ to be constant $\eta_2 = \eta_2(H = H_{Z_2})$. In this way we assume the order parameter amplitude to be slowly varying real valued function $\eta_1 = \eta_1(x,y)$ and obtain single component GL equation:

$$-D\nabla^2\eta_1 + a\eta_1/2 + b\eta_1^3 = 0$$

with coefficients $D = (3 + \nu_2)\kappa^{-2}$, $a = (1 + \nu_2/3)(H^2 - H_{Z_2}^2)d^2$ and $b = 2(3 + \nu_1)$. We can find a DW structure as the topological soliton in Eq. (6) $\eta_1 = \sqrt{a/b}\tan\left(\sqrt{a/b}\nabla\right)$. Since $a \sim (H_{Z_2} - H)$ we see that the DW dissolves near the critical field $H_{Z_2}$ and the size of DW proportional to $\xi_{Z_2} \sim (H_{Z_2} - H)^{-1/2}$.

The obtained $Z_2$ symmetry breaking phase transition provides a unique possibility to create arbitrary concentration of DW in $p_x + ip_y$ superconductor. We employ a generalization of Kibble-Zurek defect formation mechanism to explore the DW appearance during nonequilibrium $Z_2$ symmetry breaking phase transition. Let us assume that the external field decreases with the constant rate $\tau_H$ so that $H(t) = (1 - t/\tau_H)H_{Z_2}$. Just below the $Z_2$ critical point $H < H_{Z_2}$ the growth of $Z_2$ order parameter fluctuations can be described by linearized TDGL equation

$$\tau_{\eta_1} = \left[H_{Z_2}^2 - H^2(t)\right]\eta_1 + 6(d\nu)^2\nabla^2\eta_1$$

Eq. (7) describes two competing effects: exponential growth and diffusive spreading due to the last term in the r.h.s. Comparing these times we can obtain the distance between defects just after the phase transition as the minimum length scale which can grow. The characteristic growth time is $t_{2} \sim \frac{\gamma}{d^2H_{Z_2}}$, also known as Zurek time. This time should be much less than diffusive time $(kd)^2\tau_2$, where $l$ is characteristic length scale. So we obtain the condition on the distance between defects immediately after the system has been driven through $Z_2$ phase transition $l \sim (\tau_H/\tau)^{1/4}$. Thus varying the rate $\tau_H$ it is possible to create arbitrary concentration of DWs.

Applying an external transport current $J_x$ along the film plane [see Fig.4(c)] it is possible to obtain the first order $Z_2$ symmetry breaking phase transition. To study this case we introduce a new thermodynamic potential performing Legendre transformation to the free energy $f = f - kJ_x$ where $k$ is dimensionless superfluid velocity. In this case stable state can be found only numerically. An example of resulting stable branches is shown in Fig.4(d) where we plot order parameter components $\eta_+$ and $-\eta_-$ for $Z_2$ symmetry breaking phase. By blue dotted line the non-degenerate state with $\eta_+ = -\eta_-$ in $Z_2$ symmetric phase is shown.

From Fig.4(d) one can see that $Z_2$ transition is of the first order so that $U(1) \times Z_2$ and $U(1)$ phases can coexist. At the same time it is well known that $U(1)$ phase
transition in thin superconducting film with external current is also of the first order\textsuperscript{22}. Thus to have an additional \( Z_2 \) symmetry braking phase transition the critical current of \( U(1) \times Z_2 \) state should be smaller than that of \( U(1) \). Otherwise the system will fall into normal phase directly from \( U(1) \times Z_2 \) state produced by magnetic field suggesting to employ transport measurements in the mixed phase transition. The first order \( Z_2 \) phase transition discussed above occurs through the growth of the nuclei with sizes larger than the critical one\textsuperscript{20}. It can be easily estimated as \( f_s/\Delta f_b \), where \( f_s \) is the surface free energy density and \( \Delta f_b \) is difference of bulk free energy densities in two phases. Thus the critical size is determined by the external current through the bulk energy dependence \( \Delta f_b = \Delta f_b(j_s) \). It is natural to expect that the distance between DW after the first order transition should be determined by the critical size which can vary from 0 to \( \infty \) by setting the current \( j_s \).

Finally let us discuss a way to measure the residual DW concentration which survives after the transient processes after the non-equilibrium \( Z_2 \) phase transition. The DW can be stabilized by geometrical confinement in mesoscopic samples\textsuperscript{22}, pinning on vortices and defects\textsuperscript{30–32}. Besides several known experimental approaches\textsuperscript{22}–\textsuperscript{26} we suggest to employ transport measurements in the mixed state produced by magnetic field \( H \parallel c \) where \( c \) is anisotropy axis. The proposed method is based on the observation that such field creates Abrikosov vortices which are known to remove \( Z_2 \) degeneracy of superconducting vacuum in \( p_x + ip_y \) superconductor. That is vortices have different core structures in the chiral domains with \( \langle \text{HL} \rangle > (\langle \text{L} \rangle)^2 \text{\textsuperscript{43–45}} \) where \( \text{L} \) denotes the direction of the internal orbital momentum of Cooper pairs which in our case is \( \text{L} \parallel c \). We denote these vortex structures \( N_+ \) and \( N_- \) vortices correspondingly.

In isotropic case \( \nu_1 = \nu_2 = 0 \) the order parameter in axially symmetric vortices has form \( \eta_\pm = |\eta_\pm|e^{im_\pm} \) where \((r, \theta)\) are polar coordinates with the origin at the vortex center. Axial symmetry is preserved the choice of the vorticities \( m_+ = 1, m_- = 3 \) for \( N_+ \) and \( m_+ = 1, m_- = -1 \) for \( N_- \) vortices.

Here we note that \( N_+ \) and \( N_- \) vortices have different viscosities due to the difference in their core structures. Hence the flux flow conductivity has a chirality sensitive contribution \( \sigma = \sigma_0 + \sigma_1(\text{HL}) \). The flux flow conductivity can be calculated within the framework of time dependent GL theory\textsuperscript{22}. In this way we obtain

\[
\sigma/\dot{\sigma} = \int_0^\infty \sum_\alpha \left[ \rho |\eta_\alpha|^2 + |\eta_\alpha|^2 (m_\alpha^2 + \rho \mu_\alpha) \right] d\rho \quad (8)
\]

Here we normalize conductivity by \( \dot{\sigma} = \sigma_n\Phi_0 H_{cm}/\sqrt{2} H \), where \( \ell \) is the electric field penetration length \( \ell^2 = (\sigma_n \Phi_0^2)/8\pi^2 e^2 r \), \( u = (\xi/\ell)^2 \), \( \rho = r/\ell \) and \( \sigma_n \) is a normal metal conductivity. The function \( \mu_0 = \mu_0(r) \) determines electrostatic potential around moving vortex \( \varphi = \mu_0(r)(e_r[v, z_0]) \) where \( v \) is vortex velocity and \( e_r = r/r \). It satisfies the Poisson equation

\[
\left( \nabla^2 + \rho^2 - |\eta_+|^2 - |\eta_-|^2 \right) \mu_0 = \rho^{-1}m_\alpha |\eta_\alpha|^2 \quad (9)
\]

For example taking the parameters \( \kappa = 2.3 \) and \( u = 6 \) we obtain the flux-flow conductivities \( \sigma_+ = 13.5\sigma_n H_{cm}/H \) and \( \sigma_- = 14.6\sigma_n H_{cm}/H \) for \( N_+ \) and \( N_- \) vortices correspondingly so that the chirality sensitive part is \( \sigma_1 = (\sigma_+ - \sigma_-)/2 = 0.018\sigma_0 \). Averaged over the sample flux flow conductivity is given by \( \dot{\sigma} = \sigma_0 S_+ + \sigma_- S_- \) where \( S_\pm \) are the measures of the parts occupied by domains of positive and negative chiralities. Thus measuring flux flow conductivity \( \sigma \) it is possible to study the evolution of domain structure in \( St_2RuO_4 \) generated through the nonequilibrium \( Z_2 \) phase transition.

To conclude we have found discrete symmetry breaking phase transition in \( p_x + ip_y \) superconductors. The transition can be of the first order if driven by external current and of the second order under the action of external field. That is applying in-plane magnetic field to the thin superconducting film one can drive it continuously from \( U(1) \times Z_2 \) to the simple \( U(1) \) state. Such \( Z_2 \) symmetry restoration is marked by dissolution of DWs. Decreasing the field through \( Z_2 \) critical point at a constant rate one can create a particular concentration of DWs according to the Kibble-Zurek scenario. This possibility can facilitate experimental identification of DWs. Results on the present paper have been derived for a thin superconducting film. Our approach can be generalized to describe surface layers with thickness of the order of London penetration length in superconducting single crystals.

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