Thermal rectification of electrons in hybrid normal metal-superconductor nanojunctions

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We theoretically investigate heat transport in hybrid normal metal-superconductor (NS) nanojunctions focusing on the effect of thermal rectification. We show that the heat diode effect in the junction strongly depends on the transmissivity and the nature of the NS contact. Thermal rectification efficiency can reach up to \( \sim 123\% \) for a fully-transmissive ballistic junction and up to \( 84\% \) in diffusive NS contacts. Both values exceed the rectification efficiency of a NIS tunnel junction (I stands for an insulator) by a factor close to \( \sim 5 \) and \( \sim 3 \), respectively. Furthermore, we show that for NS point-contacts with low transmissivity, inversion of the heat diode effect can take place. Our results could prove useful for tailoring heat management at the nanoscale, and for mastering thermal flux propagation in low-temperature caloritronic nanocircuitry.

Control of the heat flow at the nanoscale has been attracting the attention of several research groups in the last decade.1,2 An accurate understanding of heat transport is essential, for instance, for a fine control of ultrasensitive cryogenic radiation detectors,3,4 nanocoolers,5 and caloritronic circuits.5–11 In several cases such devices contain superconductors as building block elements which introduce phase coherence to the heat transport. Examples include Josephson heat interferometers12 and thermal quantum diffractors13 in which the heat current is controlled by a magnetic flux, or electronic refrigeration in normal metal-superconductor (NS)1 and ferromagnet-superconductor (FS)14,15 structures whose efficiency depends on Andreev reflection16 at the interface with the superconductor.

In a voltage-biased NS junction the charge current consists of two contributions: the quasiparticle and the Andreev current16. For voltages \( V \) below the superconducting energy gap the latter may dominate, and the amplitude of the current depends on the transmissivity and the nature of the contact. Due to the electron-hole symmetry and for a spatially-symmetric barrier at the SN interface, the amplitude of the electric current does not depend on the sign of \( V \). The same holds as well for the heat current flowing through the junction in a voltage-biased configuration1. By contrast, the electronic contribution to the heat current in the presence of a temperature bias across the NS junction depends on the sign of the temperature drop17. This property stems from the strong temperature dependence of the superconducting density of states at high temperatures. In this regard, a NS junction therefore behaves as a thermal diode18,19 with this meaning that heat conduction along one direction is preferred with respect to that occurring upon temperature bias reversal. Strong effort has been devoted so far to envision and to realize thermal rectifiers dealing, for instance, either with phonons20–24, electrons25–29 or with photons.30

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FIG. 1. Scheme of a hybrid normal metal-superconductor (NS) heat diode under forward, (a), and reverse, (b), thermal bias configuration. The NS junction is temperature-biased with \( T_N \neq T_S \), and \( \dot{Q}_+ \) and \( \dot{Q}_- \) denote the heat current flowing through the structure in the forward \( (T_N > T_S) \) and reverse \( (T_N < T_S) \) thermal-bias setup, respectively. The circular hatched regions indicates the NS interface which, as discussed in the text, can describe a ballistic or tunnel junction as well as a diffusive or dirty contact.

In this Letter we address the heat diode effect in NS nanojunctions and explore how thermal rectification depends on the interface properties. We show that a perfectly-transparent point contact can provide a large rectification coefficient up to \( \sim 123\% \) which exceeds by a factor close to 5 the one predicted to occur in NIS tunnel junctions17. In more realistic diffusive junctions the maximum heat rectification efficiency can be as large as \( \sim 84\% \). Furthermore, in a NS point-contact thermal rectification can change sign as a function of temperature for a low enough interface transmissivity. Our predictions for the heat diode effect in hybrid NS junctions could prove useful for developing future caloritronic nanodevices.

The system under investigation is schematized in Fig. 1 and
consists of a temperature-biased NS junction. The electronic temperature in N and S is set to $T_N$ and $T_S$, respectively. We assume a spatially uniform temperature in the electrodes so to avoid the generation of any thermal gradient within each of them. In the forward thermal bias configuration [see Fig. 1(a)] a thermal gradient is intentionally created developing at the NS interface by setting $T_N = T_{hot} > T_S = T_{cold}$ which gives rise to a total heat flux $\dot{Q}_+$ through the system. By contrast, in the reverse thermal bias configuration the thermal gradient is inverted so that $T_N = T_{cold} < T_S = T_{hot}$ which yields a total heat current $\dot{Q}_-$ flowing from S to N [see Fig. 1(b)]. We note that by definition $\dot{Q}_+$ and $\dot{Q}_-$ have opposite sign. The hatched circles in the figure indicate the NS contact region which, as discussed below, can be ballistic or tunnel as well as diffusive or dirty. The thermal rectification coefficient ($R$) can be defined as

$$R(\%) = \frac{\dot{Q}_- - \dot{Q}_+}{\dot{Q}_+} \times 100. \quad (1)$$

According to Eq. (1), $R = 0$ implies the absence of a heat rectification whereas $R > 0$ implies a thermal current flowing preferentially from the S toward the N side of the junction.

For a quantitive description of the charge and heat transport through the NS junction it is convenient to introduce the Keldysh Green’s functions

$$\check{G}_{S(N)} = \begin{pmatrix} \hat{G}^R & \hat{G}^A \\ 0 & \hat{G}^A \end{pmatrix}, \quad (2)$$

where the retarded ($R$), advanced ($A$) and Keldysh components in the S and N electrodes are given by

$$\hat{G}^{R(A)}_N = \pm \tau_3 \quad (3)$$
$$\hat{G}^K_3 = 2 \tau_3 \tanh \left( \frac{E}{2k_B T_N} \right) \quad (4)$$
$$\hat{G}^{R(A)}_S = g^{R(A)} \tau_3 + f^{R(A)}_N t_2 \quad (5)$$
$$\hat{G}^K_3 = (\hat{G}^R - \hat{G}^A) \tanh \left( \frac{E}{2k_B T_S} \right). \quad (6)$$

In the above expressions, $\tau_i$ are the Pauli matrices in the Nambu space, $g^{R(A)} = (E/\Delta(T_S)) f^{R(A)} = E/\xi^{R(A)}$, $\xi^{R(A)} = \sqrt{(E \pm i \eta)^2 - \Delta^2(T_S)}$, $\Delta(T_S)$ is the BCS temperature-dependent superconducting order parameter, $T_{N(S)}$ is the temperature of the N (S) electrode, and $k_B$ is the Boltzmann constant. The parameter $\eta$ accounts for the inelastic scattering rate within the relaxation time approximation.

The electronic transport through the NS junction can be described using the matrix current $(\hat{I})$ introduced by Nazarov,

$$\hat{I} = -\frac{2e^2}{\pi \hbar} \sum_n \tau_n \left[ \check{G}_S, \check{G}_N \right] \left[ 4 - \tau_n (2 - \left\{ \check{G}_N, \check{G}_S \right\}) \right]^{-1} \quad (7)$$

Here, $\tau_n$ is the transmission of the nth junction channel, and the sum goes over the junction conducting channels. In our analysis we shall focus on the electronic contribution to the heat current only, $\dot{Q}$, which is defined as

$$\dot{Q} = \frac{1}{8e^2} \int_{-\infty}^{\infty} E \tau_R dE, \quad (8)$$

and we do not take into account neither the heat exchanged between electrons and phonons nor a pure phononic heat current.

From Eqs. (3-9) we get for $T_N \neq T_S$ the following expression for the heat current flowing through the contact

$$\dot{Q} = \frac{1}{2\pi \hbar} \sum_n \int_{-\infty}^{\infty} dE \left[ \frac{\tau_n}{2 - \tau_n (1 + g^A)} \right. \times \left[ (g^R - g^A) - \frac{2 \tau_n (f^R - f^A)}{4 - 2 \tau_n (1 - g^A)} \right] \times \left( \tanh \left( \frac{E}{2k_B T_S} \right) - \tanh \left( \frac{E}{2k_B T_N} \right) \right). \quad (10)$$

In our notation $\dot{Q} > 0$ represents the heat current flowing out of the N lead when $T_N > T_S$. Equation (10) is a general expression that describes the heat flow for an arbitrary contact. For example, a point-contact is defined by a unique conducting channel with transmission $\tau$. A ballistic junction is described by setting all channel transmissions $\tau_n = 1$, whereas in the
case of a tunnel contact all $\tau_n \ll 1$. In the latter case, from Eq. (9) we recover the well-known expression for the heat current ($\dot{Q}_{\text{tunnel}}$) flowing through a temperature-biased superconducting tunnel junction$^1$, i.e.,

$$\dot{Q}_{\text{tunnel}} = \frac{G_N}{e^2} \int_{\Delta(T_S)} \frac{E^2}{\sqrt{E^2 - \Delta(T_S)^2}} dE \times \left[ \tanh \frac{E}{2k_BT_S} - \tanh \frac{E}{2k_BT_N} \right],$$

where $G_N = (e^2 / \pi \hbar) \sum_n \tau_n$ is the contact normal-state conductance.

In the case of an extended NS interface with a continuous distribution of channels one can replace in Eq. (10) the sum $\sum_n$ with the integral $\int d\tau \mathcal{P}(\tau)$, where $\mathcal{P}(\tau)$ is the interface transmission distribution function. Realistic interfaces between metals are typically dirty, and can be described by a scattering region of a certain characteristic length. If this length is larger than the Fermi wave length, the interface is called to be diffusive, and is characterized by the following distribution function$^3$

$$\mathcal{P}(\tau) = \frac{\hbar G_N}{e^2} \frac{1}{\tau \sqrt{1 - \tau}}.$$  

By contrast, if the characteristic scattering region is much smaller than the Fermi wave length (i.e., a sharp interface), the distribution function reads$^5$

$$\mathcal{P}(\tau) = \frac{\hbar G_N}{e^2} \frac{1}{\tau^{3/2} \sqrt{1 - \tau}}.$$  

Thus, with the help of Eqs. (10,12,13) we are able to describe heat transport through a large variety of junctions and obtain the thermal rectification coefficient $R$. In the normal state, i.e., for temperatures larger than the superconducting critical one, $\Delta = 0$ and Eq. (10) reduces to

$$\dot{Q} = \frac{k_N^2 G_N \pi^2}{6e^2} (T_N^2 - T_S^2).$$

This expression shows that no thermal rectification occurs in a full normal-metal junction.

We are now able to explore the thermal diode properties of the NS contact by calculating the rectification coefficient [see Eq.(1)]. To this end it is illustrative to start our discussion considering first the heat rectification characteristics of a point-contact ballistic NS junction characterized by a unique channel of transmission $\tau$. Figure 2(a) shows the rectification efficiency $R$ vs $T_{hot}$ for $T_{cold} = 0.1T_c$ and for several values of $\tau$. Above, $T_c = (1.764k_B)^{-1}A_0$ is the superconducting critical temperature while $A_0$ is the zero-temperature energy gap. In general, for any transmission, $R$ is a non-monotonic function of the temperature peaked at a specific $T_{hot}$ which depends on $\tau$, and rapidly decreasing at higher temperature. In particular, for a perfect transmissive interface ($\tau = 1$) a maximum thermal rectification coefficient as high as $\sim 123\%$ is obtained at $T_{hot} = T_c$. This large $R$ value stems from ideal Andreev reflection$^6$ at the NS interface. For $\tau \gtrsim 0.1$ the heat rectification turns out to be always positive in the whole range of temperatures. By reducing $\tau$ yields a suppression of the maximum of $R$ which is attained for smaller values of $T_{hot}$. Notably, negative $R$ values are obtained at large $T_{hot}$ temperatures, i.e., for $T_{hot} > T_c$. This sign inversion of the thermal rectification coefficient implies that the heat current flows preferentially from N to S. For low interface transmissivity (i.e., $\tau = 10^{-4}$), which describes a NIS tunnel junction, $R$ reaches values as large as $\sim 26\%$ at $T_{hot} \approx 0.85T_c$. We stress that the latter value is around $\sim 20\%$ of the maximum reached in the the $\tau = 1$ limit. It is worthwhile to mention that thermal rectification is a fully non linear effect, and that is absent in the linear response regime. The dependence of the maximum thermal rectification efficiency ($R_{\text{max}}$) as a function of the transmission coefficient for a point-contact is shown in Fig. 2(b). In particular, the plot shows that for $\tau = 0.5$ thermal rectification is reduced by almost a factor of two with respect to the ideal junction, whereas the lowest saturation limit is already reached for $\tau \lesssim 10^{-3}$.

The effect of the smaller temperature $T_{cold}$ onto $R$ for a perfectly-transmitting NS point-contact is displayed in Fig. 2(c) as a function of $T_{hot}$. In particular, the increase of $T_{cold}$
leads to a suppression of $R$. We emphasize that the sign of thermal rectification turns out to be positive in the whole range of temperatures, while $R$ obtains its maximum values always for $T_{\text{hot}} = T_c$. The evolution of the maximum rectification efficiency $R_{\text{max}}$ with $T_{\text{cold}}$ is shown in Fig. 2(d). It can be noted how $R$ it is reduced by increasing the temperature. In particular, $R$ reaches $\sim 57\%$ of the maximum at $T_{\text{cold}} = 0.5T_c$.

In order to assess the full applicability of heat rectifiers based on NS junctions we consider now less ideal hybrid contacts, i.e., NS junctions with diffusive or dirty interfaces. These are characterized by distributions of transmissivities described by Eqs. (12) or (13), respectively. Figure 3(a) shows the comparison of the thermal rectification coefficient $R$ versus $T_{\text{hot}}$ calculated at $T_{\text{cold}} = 0.1T_c$ for four representative different types of NS interfaces: ballistic ($\tau_n = 1$), diffusive, dirty and tunnel ($\tau_n \ll 1$). In particular, for diffusive and dirty interfaces $R$ turns out to be always positive, with a shape strongly resembling that of the ballistic case. The maximum values for $R$ are $\sim 84\%$ and $\sim 63\%$ for a diffusive and dirty interface, respectively, and occur at $T_{\text{hot}} = T_c$. Such a reduction of the $R$ coefficient stems from a substantial suppression of the Andreev reflection transmission occurring in diffusive or dirty contacts in comparison to the fully-transmitting ballistic case. In spite of such a reduction, both diffusive and dirty junctions are still able to provide a sizeable thermal rectification efficiency which obtains values up to factor of 3 larger than the maximum achievable with a NIS tunnel junction.

In Fig. 3(b) we show the behavior of $R$ for a diffusive NS junction calculated against $T_{\text{hot}}$ for several values of $T_{\text{cold}}$ (for dirty interfaces similar results, not shown here, are obtained). By increasing $T_{\text{cold}}$ yields a reduction of the maximum rectification efficiency, being the sign always positive. The dependence of $R_{\text{max}}$ on $T_{\text{cold}}$ is displayed in Fig. 3(c), and shows that at $T_{\text{cold}} = 0.5T_c$ the coefficient $R$ can obtain values as large as the $\sim 54\%$ of the maximum achievable. The behavior described above for a diffusive NS contact therefore confirms the picture that this kind of junctions can provide a substantially large $R$ in a wide range of temperatures.

From a practical point of view and in light of a realistic implementation, superconducting aluminum (Al) or vanadium (V) combined, for instance, with copper (Cu) as a normal metal would allow the fabrication of diffusive NS nanojunctions. On the other side, InAs-based two-dimensional electron gases combined with niobium (Nb) would enable the realization of Schottky barrier-free highly-transmissive semiconductor-superconductor ballistic junctions. These predictions for thermal rectifications could be tested experimentally in a prototype hybrid microstructure designed along the lines of that presented in Ref. 17, in particular by symmetrically tunnel-coupling two additional identical normal metal electrodes to the NS junction. Electron heating and thermometry can be performed through NIS tunnel or SNS Josephson junctions coupled to the N leads, therefore allowing to realize selectively the forward and reverse thermal-bias configuration in the structure. Concerning potential applications, NS thermal rectifiers could be exploited in the field of electronic cooling, or for thermal isolation and heat management at the nanoscale. Moreover, other caloritronic devices such as heat interferometers, sensitive radiation detectors or magnetic sensors might likely benefit from the combination with NS thermal diodes to improve their performance.

In summary, we have theoretically analyzed thermal rectification in normal metal-superconductor nanojunctions comparing different types of NS contacts. We have shown, in particular, that by increasing the interface transmissivity leads to a substantial enhancement of the heat diode effect whereas the sign of rectification can be changed in a suitable range of temperatures for low junction transparency. For perfectly-transmissive ballistic contacts, thermal rectification can be as high as 123% thus exceeding by a factor close to $\sim 5$ that achievable in NIS tunnel junctions. For diffusive contacts, the rectification efficiency can obtain values as high as $\sim 84\%$. Because of the above results and of the ease intrinsic in their fabrication, NS junctions appear therefore as a promising building block for the implementation of effective heat diodes to be exploited in low-temperature caloritronic nanocircuitry.

F.G. acknowledges the Italian Ministry of Defense through the PNRM project “TERASUPER”, and the Marie Curie Initial Training Action (ITN) Q-NET 264034 for partial financial support. The work of F.S.B was supported by the Spanish Ministry of Economy and Competitiveness under Project FIS2011-28851-C02-02. F.S.B thanks Prof. Martin Holthaus and his group for their kind hospitality at the Physics Institute of the Oldenburg University.

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