THE STUDY ON THE SEMI-EMPIRICAL SOIL MODELS FOR IMPROVING THE GNSS-R SM RETRIEVAL

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ABSTRACT:

Soil moisture (SM) is one of the important physical properties of soil and plays an important role in agricultural production. Global Navigation Satellite System Reflectometry (GNSS-R) SM retrieval is based on the concept of receiving GPS signals reflected by the ground using a passive receiver. Although many research has been reported showing the capability of the GNSS-R technique on SM application. Due to the diversity of soil types and the complexity of soil composition, the study of soil permittivity is still attracting many interesting. This paper presented the investigations on characteristic and performances for different semi-empirical soil models, to reveal and verify the relationship between dielectric constant and SM under different realistic soil compositions. This study is helpful to improve the accuracy for the GNSS-R SM retrieval and can be used to other SM related studies.

1. INTRODUCTION

SM directly affects crop growth, farmland microclimate, and soil mechanical properties (Craig, 2005; Lunt et al., 2005; Schmugge et al., 1980; Terzaghi, 1943). Moreover, the factors that affect soil permittivity are mainly soil moisture content (SMC) and soil texture (the mechanical composition of the soil). This paper mainly analyses and compares the performance of four soil models with different soil textures, namely the Topp model (Topp et al., 1980), Hallikainen model (Hallikainen et al., 1985), Dobson model (Dobson et al., 1985), and Wang model (Lundien, 1971; Wang et al., 1980) with the same temperature and frequency. The aim of the paper is to clarify the relationship between SM and soil dielectric constant, and able to reveal and verify the availability and scope of the soil semi-empirical model for SMC retrieval.

Soil texture refers to the percentage of soil weight of each particle size in the soil, also known as the mechanical composition of the soil. Soil texture is one of the most basic physical properties of soil. It has a great impact on various soil properties, such as soil permeability, preservation, tillage, and nutrient content. It is also an evaluation of soil fertility and crop suitability. The measurement methods mainly include the densitometer method, pipette method, and laser particle size analyzer method, among which the most common one is the pipette method. Soils with different soil textures generally have different agricultural production features, so understanding the texture type of each soil type is helpful for agricultural production. Soil is generally divided into three-grain grades, namely clay, silt, and sand. Each soil particle classification standard is different, as well as the soil texture classification. Meanwhile, the soil texture classification standards adopted by each country in the world are inconsistent. There are four commonly used soil texture classification standards, namely the international system, the American system, the Kachinsky system, and the Chinese tentative classification system. Among them, there are two versions of soil texture classification standards in China, namely, the 1978 and 1985 classification standards.

2. METHODOLOGY AND DATA SET

2.1 Topp Model

In 1980, Topp et al., used the method of Time Domain Reflection (TDR) to measure the soil dielectric constant, respectively, to study the influence of soil volumetric water content, frequency, and soil texture on the soil dielectric characteristic. Using the method of data fitting, it is believed that factors such as soil texture, soil bulk density, and frequency have almost no influence on the soil dielectric constant, and the SMC can be determined directly according to the soil dielectric constant. By inputting the measured soil dielectric constant into the Topp model, the volumetric water content of soil can be calculated by the modeled formula. The expression of the Topp empirical model (Topp et al., 1980) is as follows:

\[
\theta = -5.3 \times 10^{-2} + 2.92 \times 10^{-3} \varepsilon - 5.5 \times 10^{-4} \varepsilon^2 + 4.3 \times 10^{-5} \varepsilon^3
\]

where \( \varepsilon \) is the dielectric constant of soil, \( \theta \) is the volumetric water content of the soil.

2.2 Hallikainen Model

The Hallikainen empirical model was proposed by Hallikainen in 1985. He used the waveguide method and the free space method to measure the dielectric constants of soils with different soil textures under different water contents. A quadratic polynomial fitting model was proposed based on the measured data under different soil types, which is the Hallikainen empirical model. The quadratic polynomial fitting model is obtained at a few fixed frequency points, and these frequency points are not commonly used in microwave remote sensors, so a large amount of data is needed to calculate the calibration coefficients corresponding to different frequency bands. The expression of the Hallikainen empirical model is:

\[
\varepsilon = (a_0 + a_2 \varepsilon + a_4 \varepsilon^2) + (b_0 + b_2 \varepsilon + b_4 \varepsilon^2)m + (c_0 + c_2 \varepsilon + c_4 \varepsilon^2)m^2
\]
In this expression, \( \varepsilon \) represents the soil dielectric constant, \( S \) represents the volume percentage of sand in the soil, \( C \) represents the volume percentage of clay in the soil, \( \varepsilon_0 \) and \( \varepsilon_a \) are determined by the mechanical composition of the soil (soil texture). Moreover, \( m_v \) represents the volumetric moisture content of the soil, \( a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2 \) represent calibration coefficients. The calibration coefficients are different at different frequencies and the dielectric constant changes with the water content of five soils at 1.4 GHz in the Hallikainen model (Hallikainen et al., 1985).

### 2.3 Dobson Model

Dobson Model is a semi-empirical model of soil dielectric constant and water content established by Dobson et al. in 1985. Based on the Refract-Index Model, the model was proposed using free-space propagation technology and a waveguide permittivity measurement system to measure the data of five different soil types under 1.4GHz – 18GHz. Dobson Model has been widely used in the retrieval of soil dielectric constant to soil moisture because it is applicable to a wide range of electromagnetic frequencies. The parameters in the model do not depend too much on the specific soil type, and the simulation results show high accuracy. Dobson Model expresses the relationship between the soil dielectric constant and the volume percentage of sand and clay, the volume moisture content of the soil, the frequency of the incident wave, the soil temperature, and the soil volume mass under 1.4GHz to 18GHz. The permittivity of soil calculated by Dobson Model can be expressed as follows (Dobson et al., 1985):

\[
e = \frac{1}{\varepsilon_0} + \frac{\rho_b}{\rho_a} \varepsilon_a \left( \varepsilon_a \varepsilon_r - 1 \right) + m_v \varepsilon_r \varepsilon_m - m_v \varepsilon_m^{-1} \varepsilon_0
\]

\[
\varepsilon_r = (1.01 + 0.44 \rho_a)^2 - 0.002
\]

\[
\varepsilon_m = \varepsilon_0 + \frac{\varepsilon_0 \varepsilon_r - \varepsilon_m}{1 + 2 (2\pi \tau)^2}
\]

\[
\varepsilon_{vis} = 8.8045 - 0.4117T + 6.295 \times 10^{-5}T^2 + 1.075 \times 10^{-5}T^3
\]

\[
2\pi \tau = 1.1109 \times 10^{-10} - 3.824 \times 10^{-13}T + 6.938 \times 10^{-15}T^2 - 5.096 \times 10^{-18}T^3
\]

\[
\beta = 1.2748 - 0.51959 - 0.152C
\]

For the sake of simple calculation, only the real part of soil complex permittivity was calculated in this experiment to stand for soil permittivity. \( \varepsilon_r \) is the real part of the soil complex dielectric constant, \( \rho_s \) is the density of soil solids, \( \rho_b \) is the density of soil total solid particles, \( \beta \) is the density of soil volume. In order to facilitate calculation, generally taking \( \rho_b = 1.4g/cm^3, \rho_s = 2.65g/cm^3, \varepsilon_a = 4.7 \) stands for the constant factor, and the optimal value of \( a \) for all types of soils is 0.65. \( m_v \) stands for the volumetric water content of the soil, and \( \beta \) is the real part of the complex coefficient introduced when bound water and free water are combined. \( \varepsilon_{vis} \) stands for the real part of the free water permittivity. \( \varepsilon_{vis} \) stands for the dielectric constant of high frequency, generally taking \( \varepsilon_{vis} = 4.9 \) is the static dielectric constant of pure water, and \( T \) is the temperature, in the unit of °C. \( \varepsilon_m \) is the relaxation time of pure water in seconds and is related to temperature. \( f \) is the electric field frequency, in Hz. \( S \) and \( C \) represent the volume percentage of sand and clay in the soil; \( S \) and \( C \) are determined by the mechanical composition of the soil (soil texture). For simple calculation, \( T = 20°C, f = 1.4ghz, \rho_s = 1.4g/cm^3, 2\pi\tau \approx 5.8285 \times 10^{-11}, \varepsilon_a = 80.1488, \varepsilon_{vis} = 79.8940 \) (Dobson et al., 1985).

### 2.4 Wang and Schmugge Model

The Wang and Schmugge model (hereinafter referred to as the Wang model) is a semi-empirical model established by Wang and Schmugge in 1980 through the research and analysis of numerous measured data. They found that the soil dielectric constant increased slowly with the increase of soil water content when the soil water content was relatively small, and the soil dielectric constant increased rapidly with the increase of water content when the water content rises to a threshold. So, there are two different situations in the Wang model. 1) When the volumetric water content of the soil is less than or equal to the threshold, the water in the soil is mainly bound water, and water molecules are bound by soil particles. 2) When the volumetric water content of the soil is greater than the threshold, the soil particles cannot absorb more water molecules, and the water molecules are freed from the shackles of the soil particles and behave as free water. This moisture content threshold is called filtered moisture. The expression of the Wang model is as follows (Wang et al., 1980):

\[
WP = 0.06774 - 0.000064S + 0.00478C
\]

\[
\gamma = -0.57WP + 0.481
\]

\[
W_F = 0.49WP + 0.165
\]

\[
\rho_s = \left( 25 - 0.21S + 0.22C \right)^{1/3}
\]

\[
P = 1 - \rho_s
\]

\[
\text{When } Wc < Wt
\]

\[
\varepsilon = \varepsilon_r + \left( \varepsilon_w - \varepsilon_r \right)WP
\]

\[
\varepsilon = \varepsilon_r + \left( \varepsilon_w - \varepsilon_r \right)\varepsilon_w + \left( 1 - P \right)\varepsilon_c
\]

\[
\text{When } Wc > Wt
\]

\[
\varepsilon = \varepsilon_r + \left( \varepsilon_w - \varepsilon_r \right)\varepsilon_w + \left( \varepsilon_w - \varepsilon_c \right)\varepsilon_r + \left( 1 - P \right)\varepsilon_c
\]

Among these, \( WP \) represents the wilting point, \( S \) represents the percentage of sandy soil content to soil dry weight, \( C \) represents the percentage of clay content to soil dry weight, \( \gamma \) represents the adjustment parameter when calculating the dielectric constant of combined water, \( WP \) represents the filtration humidity (water content threshold), \( P \) represents the porosity of the dry soil, \( ps \) represents the dry soil density, \( \rho_r \) represents the associated solid rock density, \( \varepsilon_s \) represents the volumetric water content of the soil, \( \varepsilon_a, \varepsilon_w, \varepsilon_r, \varepsilon_i \) represent the dielectric constants of air, free water, soil matrix rock minerals, and ice crystals, respectively, \( \varepsilon_s \) represents the dielectric constant of bound water. The values of \( ps \) at 5 GHz and 1.4 GHz ps range from 1.1 to 1.7 g/cm3 and the values of \( pr \) range from 2.6 to 2.75 g/cm3. For simplicity, we take \( pr = 2.65/cm^3 \), so we can obtain \( P = 0.47 \), where \( ea = 1, ew = 81, er = 5, e_w = 3.2 \) (Wang et al., 1980).

### 2.5 Data Set

It is noted that the content of clay and sand in the soil texture data is extremely important. In order to conduct the experiment, the soil texture data was collected and reprocessed, making the repeated soil composition reduced to one kind, and then the average content of clay and sand was filled in the following table according to the various classification standards that were introduced before. The employed experimental data are presented in Table 1.

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The results are shown in the following figures. The experimental results of the Topp model are listed in Table 2 since the texture of the soil does not have many effects on the water content. For the other models, the results for changing different soil textures are presented, and the results are shown in Fig. 2-17.

| Soil types       | Content of clay (%) | Content of sandy soil (%) |
|------------------|---------------------|---------------------------|
| Silty loam       | 13                  | 25                        |
| Silty clay loam  | 33                  | 10                        |
| Clay             | 70                  | 23                        |
| Clay loam        | 33                  | 33                        |
| Loamy clay       | 35                  | 33                        |
| Sandy clay loam  | 20                  | 70                        |
| Sandy loam       | 7                   | 70                        |
| Silty sandy loam | 7                   | 28                        |
| Sandy clay       | 7                   | 93                        |
| Clay             | 20                  | 33                        |
| Loamy clay       | 55                  | 45                        |
| Heavy clay loam  | 45                  | 55                        |
| Loamy clay       | 37                  | 55                        |
| Silty clay loam  | 30                  | 25                        |
| Loam             | 45                  | 15                        |

Table 1. The employed dataset corresponds to different soil classification standards.

2.6 The Adopted Approach for Comparing the Semi-empirical Soil Models

The Topp model only considers the water content in the soil. Other factors such as soil mechanical composition and soil bulk density are not considered. So there is only one case in the Topp model. Since the frequency used in this experiment is 1.4 GHz, and the temperature is set to 20°C, the calibration coefficient in the Hallikainen model, that is, \( a_0=2.378, a_1=0.326, a_2=-0.046, b_0=10.750, b_1=59.894, b_2=15.704, c_{01}=73.555, c_{11}=-58.372, c_{21}=-14.154 \). The experimental results are obtained by bringing in data from the different soil textures to compare the obtained SM results. For simple calculation, in the Dobson model experiment, the bulk density of soil \( \rho=1.4 g/cm^3 \), the density of soil solid particles \( \rho_s=2.65 g/cm^3 \). Then \( 2\pi\varepsilon_0=5.8285*10^{-11} \), \( \varepsilon_{oTopp}=80.1488, \varepsilon_r=79.8940 \). Again, the soil dielectric constants under different soil moisture contents were obtained by using the data of different soil textures. For simple calculation of this Wang model experiment, we set \( \varepsilon_{oTopp}=1, \varepsilon_r=79.894, \varepsilon=5, \varepsilon=3.2, \rho=2.65 g/cm^3 \). The input data are the volumetric water content of the soil and the content of clay and sand in the soil texture to obtain the dielectric constant of the soil.

As mentioned previously, the Topp model only requires soil permittivity as input and the soil water content is the output. For the other three models, the inputs for soil models include the soil dielectric constant, the clay and sand content in the soil, and the output is SMC. The experimental results are shown in the following figures. The experimental results of the Topp model are listed in Table 2 since the texture of the soil does not have many effects on the water content. For the other models, the results for changing different soil textures are presented, and the results are shown in Fig. 2-17.

| SMC      | 0.080 | 0.188 | 0.276 | 0.345 | 0.400 | 0.444 | 0.480 | 0.510 |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| Dielectric constant | 5     | 10    | 15    | 20    | 25    | 30    | 35    | 40    |

Table 2. The experiment results for the Topp model.

3.1 The performance of four models for changing the texture of the soils

Here we presented some preliminary results. The data on silty loam mainly come from the Henan and Hubei provinces of China. From Fig.2, it can be seen that the dielectric constant values of the Hallikainen model and Topp model are always very high when the water content of silty loam is 0.1-0.25. The dielectric constant value of the Hallikainen model is consistently higher than that of the Dobson model. All models show a positive correlation between dielectric constant and SMC.
The data on silty clay loam is mainly from Chongqing, China. It can be seen from Fig. 3 that when the clay content increases and the sand content decreases, the dielectric constant values of the Hallikainen model, Dobson model, and Wang model decrease. However, the value of the dielectric constant of the Topp model does not change.

Figure 3. The four soil models on American silty clay loam soil \( (C=0.33, S=0.1) \).

The data on clay products mainly come from the three northeastern provinces, Hunan Province and Hainan Province China. It can be seen from Fig. 4 that the clay content in the soil texture is relatively high. The gaps between the models are relatively small, and the dielectric constant value of the Hallikainen model is slightly larger than that of the Dobson model.

Figure 4. The four soil models on American clay soil \( (C=0.7, S=0.23) \).

The data of clay loam mainly comes from Tibet, China. It can be seen from Fig. 5 that when the soil texture is one-third of clay and sand, respectively, the values of the models alternately lead when the value is between 0.15 and 0.45.

Figure 5. The four soil models on American clay loam \( (C=0.33, S=0.33) \).

The data of loamy clay mainly comes from Yunnan, Guizhou, Jiangxi, and Hebei, China. Because the texture difference between clay loam of the international system and clay loam of the American system is not obvious, the experimental results are similar to those of loam of the American system.

Figure 6. The four soil models on international loamy clay \( (C=0.35, S=0.33) \).

The data on sandy clay loam mainly comes from Beijing, Guangxi, and Fujian, China. It can be seen from Fig. 7 that the dielectric constant value of the Wang model is less than that of the Dobson model, which is less than that of the Hallikainen model. With the increase in water content, the gap between these models becomes smaller, and the Topp model is in the middle of the three models.

Figure 7. The four soil models on international sandy clay loam \( (C=0.2, S=0.7) \).

The data on sandy loam mainly comes from Shandong, Inner Mongolia, and Ningxia, China. It can be seen from figure 8 that the trend of the Wang model, Dobson model, and Hallikainen model is similar to that of international sandy loam. Among them, the dielectric constant value of the Dobson model and Hallikainen model decrease with the decrease of clay under the same sand content.

Figure 8. The four soil models on international sandy loam \( (C=0.07, S=0.7) \).
The data on silty sandy loam mainly comes from Guangdong, China. Compared with silty loam of the international system, it shows that when the clay content is the same, the dielectric constant value of the Dobson model and Hallikainen model decrease with the decrease of the content of sand, while the dielectric constant value of the Wang model is not affected much by the content of sand.

The data on sandy soil mainly comes from Qinghai, China. The dielectric constant value of the Wang model and Topp model are much smaller than that of the Dobson model and Hallikainen model, and the dielectric constant value of the Hallikainen model is still higher than that of the Dobson model, but the gap is increasing with the increase of the SMC.

The data on loam mainly comes from Xinjiang, China. It can be seen from Fig. 11 that the gap between the dielectric constant value of the four models is relatively small. The content of water in the Hallikainen model and Wang model is very small between 0.3 and 0.45, and the dielectric constant value of the Hallikainen model is still higher than that of the Dobson model.

The data on clay loam is mainly from Jiangsu Province, China. It can be seen from Fig. 12 that the dielectric constant value of the Wang model is the highest in the range of 0.1-0.45 when the content of water is larger than 0.25. Compared to the clay loam of the American system (C= 0.33, S=0.33), the Wang and Hallikainen models changed very little, and the Dobson model decreased when the SMC is between 0.1 and 0.35.

The data on silty clay mainly comes from Tianjin, China. It can be seen from Fig. 13 that the dielectric constant of the Dobson model is still less than that of the Hallikainen model. When the SMC is between 0.2 and 0.45, the trend of the four models is similar, especially at 0.45, the gap is very small.

The data on heavy loam mainly comes from Zhejiang Province, China. Compared to the content of clay and sandy soil in silty clay, it can be seen from Fig. 14 that the values of the three models increase, among which the dielectric constant values of the Dobson model and Hallikainen model increase significantly.
The data on loamy clay is mainly from Anhui province, China. Compared with the heavy loam of the Kachinsky system, the dielectric constant value of the Dobson model and Hallikainen model decline with the decrease of the content of clay, while the dielectric constant value of the Wang model increases slightly.

3.2 The comparison results of four models for changing the inputs variables

In order to further verify the experimental results, we use the method of changing the inputs variables, making the SMC in the soil fixed to 20%, the content of sand in the soil fixed to 50%, and the content of clay in the soil controlled from 0% to 50%. Since the Topp model was not affected by soil texture, only the Dobson model, Wang model, and Hallikainen model are considered, and the experimental results are obtained and shown in Fig 18.

As we can see from Fig. 18, the Dobson model and Hallikainen model are greatly affected by clay, and the dielectric constant value of the Dobson model and Hallikainen model increases with the increase of the content of clay. Wang model is slightly affected by clay and the dielectric constant value of the Wang model decrease with the increase of the content of clay.

Similarly, we use the method of control variate to make the SMC in the soil fixed to 20%, the content of clay in the soil fixed to 50%, and the content of sand in the soil controlled from 0% to 50%. The experimental results on the influence of sand on the Dobson model, Wang model, and Hallikainen model are obtained and shown in Fig. 19.

As we can see from Fig. 19, the dielectric constant value of Dobson model, Wang model, and Hallikainen model all increase with the increase of the content of sand. Wang model is very slightly affected by sand, while the Dobson model and Hallikainen model are also greatly affected by sand.

3.3 The comparison results of four models for using the maximum control method
In order to understand the influence of clay, silt, and sand on the soil model better, we set the content of clay, silt, and sand to 100% respectively, and that of the other two soils as 0. The results of these three models are shown in Fig. 20.

4. CONCLUSIONS

In the Topp model, Wang model, Hallikainen model, and Dobson model, the soil moisture content is positively correlated with the soil dielectric constant. Because the Topp model only considers the effect of soil moisture on soil dielectric constant, it is the most stable model and is not affected by soil texture. The application is relatively simple. Compared with other models, it lacks a theoretical basis with poor universality. The Hallikainen model and Dobson model are greatly affected by soil texture, among which the Hallikainen model is most affected by soil texture. The Wang model was slightly influenced by soil texture. When the content of sand soil is the same, the dielectric constant values of the Dobson model and Hallikainen model increase with the increase of clay content, while the dielectric constant values of the Wang model decrease with the increase of clay content. When the clay content in the soil is equal, the dielectric constant values of the Wang model, Dobson model, and Hallikainen model increase with the increase of sand content. Moreover, when there is only clay and sandy soil in the soil, the higher the content of sand is, the higher the dielectric constant values of the Dobson model, Wang model, and Hallikainen model are.

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