Aharonov-Bohm Effect in the
Abelian-Projected $SU(3)$-QCD with $\Theta$-term

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Abstract

By making use of the path-integral duality transformation, string representation of the Abelian-projected $SU(3)$-QCD with the $\Theta$-term is derived. Besides the short-range (self-)interactions of quarks (which due to the $\Theta$-term acquire a nonvanishing magnetic charge, i.e. become dyons) and electric Abrikosov-Nielsen-Olesen strings, the resulting effective action contains also a long-range topological interaction of dyons with strings. This interaction, which has the form of the 4D Gauss linking number of the trajectory of a dyon with the world-sheet of a closed string, is shown to become nontrivial at $\Theta \neq 3\pi \times \text{integer}$. At these values of $\Theta$, closed electric Abrikosov-Nielsen-Olesen strings in the model under study can be viewed as solenoids scattering dyons, which is the 4D analogue of the Aharonov-Bohm effect.

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1 Introduction

Recently, by making use of the method of Abelian projections \cite{1}, a remarkable progress in the description of confinement in QCD has been achieved (for a recent review see \cite{2}). In this way, most of the results have been obtained for the simplest case of the SU(2)-group. That is because in this case the respective Cartan subgroup is just $U(1)$. Owing to this fact, under the assumptions on the irrelevance of the off-diagonal degrees of freedom in the IR limit (the so-called Abelian dominance hypothesis) \cite{3} and condensation of monopoles arising during the Abelian projection, the resulting effective theory takes the form of the dual Abelian Higgs model. In the latter model, confinement of external electrically charged particles is provided by the formation of electric Abrikosov-Nielsel-Olesen (ANO) strings \cite{4} between these particles. This is just the essence of the 't Hooft-Mandelstam scenario of confinement \cite{5}.

Besides this simplest SU(2)-case, an effective $[U(1)]^2$ gauge invariant Abelian-projected theory of the realistic SU(3)-QCD, based on the Abelian dominance hypothesis, has also been proposed \cite{6}. Quite recently, confining properties of this model were investigated by a derivation of its string representation \cite{7} (for a review see \cite{8}). Moreover, in Ref. \cite{7} the collective effects in the grand canonical ensemble of strings in this model were investigated. In particular, a crucial difference of this ensemble from the one of the SU(2) Abelian-projected theory \cite{10} (following from the appearance of two types of (self-)interacting strings in the SU(3)-case) has been found.

The present Letter is devoted to the investigation of Abelian-projected SU(3)-QCD extended by the introduction of the $\Theta$-term. The relevance of this term to the instanton physics is a well known issue, and we will not discuss it further, referring the reader to classical monographs (see e.g. \cite{11}). Within our investigations, it will be demonstrated that the introduction of the $\Theta$-term leads to the appearance of a long-range topological interaction of quarks (which due to this term become dyons by acquiring a nonvanishing magnetic charge \cite{12}) with the world-sheets of closed electric ANO strings. This interaction has the form of the Gauss linking number of these objects and can be interpreted as the 4D analogue \cite{13} of the Aharonov-Bohm effect \cite{14}, where strings play the rôle of solenoids, which scatter dyons. In particular, it will be demonstrated that at certain discrete critical values of $\Theta$ this effect disappears. This can be interpreted in such a way that at these values of $\Theta$, the relation between the electric flux in the ANO string (viewed as a solenoid) and the magnetic charge of the dyon is so, that the scattering of dyons over strings is absent. Note that the long-range interaction of dyons with magnetic ANO strings in the Abelian Higgs model (which corresponds to the theory dual to the Abelian-projected SU(2)-QCD with the $\Theta$-term) has been studied in Ref. \cite{15}. Similar interaction of an external electrically charged particle with magnetic ANO strings in the same model but without $\Theta$-term has been found in Ref. \cite{16} by the evaluation of the Wilson loop describing this particle.

It will be also shown that, as it can be expected from the very beginning, besides this long-range interaction of strings and dyons, the resulting effective action contains the short-range (self-)interactions of these objects. The short-rangeness of all these interactions is due to the (self-)exchanges of strings and dyons by massive dual gauge bosons. Among these short-range interactions, the selfinteraction of the open world-sheets of strings, which end up at dyons, is the most important one, since it is this interaction, which provides the linear growth of the confining dyon-antidyon potential.

The organization of the Letter is as follows. In the next Section, we shall discuss an effective theory of the Abelian-projected SU(3)-QCD with the $\Theta$-term. In Section 3, a derivation of the announced string representation for the partition function of this model will be presented, and the
obtained results will be briefly discussed. Finally, in three Appendices, some technical details of calculations, which are necessary for a derivation of the formulae from the main text, are outlined.

2 The Model

In the present Section, we shall discuss the model, whose string representation will be derived below. To start with, let us consider the partition function of an effective $[U(1)]^2$ gauge invariant Abelian-projected theory of the pure $SU(3)$-gluodynamics (i.e. a theory without quarks and the $\Theta$-term) [3]. It reads [1]

$$
Z = \int |\Phi_a| D\Phi_a |D\theta_a D\Phi_a|^2 \delta \left( \sum_{a=1}^3 \theta_a \right) \times \exp \left\{ - \int d^4x \left[ \frac{1}{4} F_{\mu\nu}^2 + \sum_{a=1}^3 \left[ (\partial_\mu - ig_m e_a B_\mu) \Phi_a \right]^2 + \lambda \left( |\Phi_a|^2 - \eta^2 \right)^2 \right] \right\}.
$$

Here, $\Phi_a = |\Phi_a| e^{i\theta_a}$ are the dual Higgs fields, which describe the condensates of monopoles emerging after the Abelian projection, and $g_m$ is the magnetic coupling constant related to the electric one via the topological quantization condition $g_m g = 4\pi k$. In what follows, we shall for simplicity restrict ourselves to the monopoles possessing the minimal charge only, i.e. set $k = 1$. Next, $B_\mu$ is the “magnetic” potential dual to the “electric” potential $a_\mu \equiv (A_3^\mu, A_8^\mu)$, where $A_3^{3,8}$ are just the diagonal components of the gluonic field. On the R.H.S. of Eq. (1), there have also been introduced the so-called root vectors $e_a$’s, which have the form

$$
e_1 = (1, 0), \quad e_2 = \left( -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right), \quad e_3 = \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right).
$$

These vectors play the rôle of the structural constants in the first of the following commutation relations

$$
[H, E_{\pm a}] = \pm e_a E_{\pm a}, \quad [E_{\pm a}, E_{\pm b}] = \mp \frac{1}{\sqrt{2}} \epsilon_{abc} E_{\pm c}, \quad [E_a, E_{-b}] = \delta_{ab} e_a H.
$$

In these relations, $H \equiv (H_1, H_2) = (T^3, T^8)$ are the diagonal $SU(3)$-generators, which generate the Cartan subalgebra, where from now on $T^i = \frac{\lambda^i}{2}$, $i = 1, \ldots, 8$, are just the $SU(3)$-generators with $\lambda^i$’s denoting the Gell-Mann matrices. We have also introduced the so-called step operators $E_{\pm a}$’s (else called raising operators for positive $a$’s and lowering operators otherwise) by redefining the rest (non-diagonal) $SU(3)$-generators as follows

$$
E_{\pm 1} = \frac{1}{\sqrt{2}} \left( T^1 \pm iT^2 \right), \quad E_{\pm 2} = \frac{1}{\sqrt{2}} \left( T^4 \mp iT^5 \right), \quad E_{\pm 3} = \frac{1}{\sqrt{2}} \left( T^6 \pm iT^7 \right).
$$

(Clearly, these operators are non-Hermitean in the sense that $(E_a)^\dagger = E_{-a}$.) It is also worth noting that owing to the fact that the original $SU(3)$ group is special, the phases $\theta_a$’s of the dual Higgs fields are related to each other by the constraint $\sum_{a=1}^3 \theta_a = 0$. This constraint was imposed by introducing the corresponding $\delta$-function into the R.H.S. of Eq. (1).

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1 Throughout the present Letter, all the investigations will be performed in the Euclidean space-time.
In what follows, we shall be interested in the study of the model (1) in the London limit, i.e. the limit of infinitely large coupling constant $\lambda$ of the dual Higgs fields (Clearly, this limit corresponds to the case of infinitely large masses of the dual Higgs bosons.). It is just this limit, where the model under study allows for an exact reformulation in terms of the integral over closed electric ANO strings $\Sigma^a$. In the London limit, the radial parts of the Higgs fields can be integrated out, and the partition function (1) takes the form

$$Z = \int DB_\mu D\theta^\text{sing}_a D\theta^\text{reg}_a Dk\delta \left( \sum_{a=1}^{3} \theta^\text{sing}_a \right) \times$$

$$\times \exp \left\{ \int d^4 x \left[ -\frac{1}{4} F_{\mu\nu}^2 - \eta^2 \sum_{a=1}^{3} \left( \partial_\mu \theta_a - g_m e_a B_\mu \right)^2 + ik \sum_{a=1}^{3} \theta^\text{reg}_a \right] \right\}.$$  

(2)

Since in the model (1) there exist string-like singularities of the ANO type, in Eq. (2) we have decomposed the total phases of the dual Higgs fields into multivalued and singlevalued (else oftenly called singular and regular, respectively) parts, $\theta_a = \theta^\text{sing}_a + \theta^\text{reg}_a$, and imposed the constraint of vanishing of the sum of regular parts by introducing the integration over the Lagrange multiplier $k(x)$. Analogously to the dual Abelian Higgs model, in the model (2), $\theta^\text{sing}_a$'s describe a certain electric string configuration and are related to the closed world-sheets $\Sigma_a$'s of strings of three types via the equation

$$\varepsilon_{\mu\nu\lambda\rho} \partial_\lambda \partial_\rho \theta^\text{sing}_a(x) = 2\pi \sum_{a=1}^{3} \theta^\text{sing}_a(x) \equiv 2\pi \int_{\Sigma_a} d\sigma_{\mu\nu} \left( x^{(a)}(\xi) \right) \delta \left( x - x^{(a)}(\xi) \right).$$  

(3)

This equation is nothing else, but the covariant formulation of the 4D analogue of the Stokes theorem for the gradient of the field $\theta_a$, written in the local form. In Eq. (3), $x^{(a)}(\xi) \equiv x^{(a)}(\xi)$ is a vector, which parametrizes the world-sheet $\Sigma_a$ with $\xi = (\xi^1, \xi^2)$ standing for the 2D coordinate. On the other hand, the regular parts of the phases, $\theta^\text{reg}_a$'s, describe a singlevalued fluctuation around the above mentioned given string configuration. Note that owing to the one-to-one correspondence between $\theta^\text{sing}_a$'s and $\Sigma_a$'s, established by Eq. (3), the integration over $\theta^\text{sing}_a$'s is implied in the sense of a certain prescription of the summation over string world-sheets. One of the possible concrete forms of such a prescription, corresponding to the summation over the grand canonical ensemble of virtual pairs of strings with opposite winding numbers, has been considered in Refs. 4-7. It is also worth noting that due to Eq. (3) the integration measure over the full phases $\theta_a$'s can be shown to become factorized into the product of measures over the fields $\theta^\text{sing}_a$'s and $\theta^\text{reg}_a$'s.

As it was announced in the Introduction, the model whose string representation will be of our interest is not the London limit of pure Abelian-projected SU(3)-gluodynamics (3), but rather the full SU(3)-QCD extended by the $\Theta$-term. For a certain quark colour $c = R, B, G$ (red, blue, green, respectively), the partition function of such a theory reads [3]

$$Z_c = \int DB_\mu D\theta^\text{sing}_a D\theta^\text{reg}_a Dk\delta \left( \sum_{a=1}^{3} \theta^\text{sing}_a \right) \exp \left\{ \int d^4 x \left[ -\frac{1}{4} \left( F_{\mu\nu} + F^{(c)}_{\mu\nu} \right)^2 \right. \right.$$  

$$\left. - \eta^2 \sum_{a=1}^{3} \left( \partial_\mu \theta_a - g_m e_a B_\mu \right)^2 + ik \sum_{a=1}^{3} \theta^\text{reg}_a + \frac{i \Theta g_m}{16\pi^2} \left( F_{\mu\nu} + F^{(c)}_{\mu\nu} \right) \left( F_{\mu\nu} + F^{(c)}_{\mu\nu} \right) \right\}.$$  

(4)

Note that the size of the core of the string (vortex in 3D) is equal to the inverse mass of the dual Higgs fields, which means that the London limit corresponds to infinitely thin strings.
Appendix A. The result of this procedure reads
3
presented for the simplest case of the (dual) Abelian Higgs model), whose details are outlined in
so-called path-integral duality transformation (see e.g. Ref. [17], where this transformation is
employed the

with the world-sheet Σ of a certain open electric ANO string, bounded by the contour C.

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or

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and introduced the following

Here, Σ

is the vorticity tensor current associated with the world-sheet Σ of a certain open electric ANO string, bounded by the contour C.

Aharonov-Bohm Effect and Confinement

In order to proceed with the string representation of the model [3], it is useful to employ the
so-called path-integral duality transformation (see e.g. Ref. [17], where this transformation is
presented for the simplest case of the (dual) Abelian Higgs model), whose details are outlined in
Appendix A. The result of this procedure reads

Here, \( H_{\mu\nu}^a =\partial_\mu h_{a\lambda}^\nu + \partial_\nu h_{a\lambda}^\mu + \partial_\lambda h_{a\mu}^\nu + \partial_\lambda h_{a\nu}^\mu \) is the field strength tensor of the Kalb-Ramond field of the
a-th type, \( F_{\mu\nu} \) is defined by Eq. (3), and \( \hat{L}(\Sigma, C) \equiv \frac{1}{8\pi^2} \varepsilon_{\mu\nu\lambda\rho} \int dx dy \Sigma_{\mu\nu}(x) \Sigma_{\lambda\rho}(y) \frac{\partial_x}{\partial_x(x-y)^2} \) is the (formal expression for the) 4D Gauss linking number of the surface Σ with its boundary C,
which will be shown to become cancelled from the final expression for the partition function. In
Eq. (3), we have also denoted \( \Sigma_{\mu\nu} \equiv \sum_{a} \Sigma_{\mu\nu}^a - s_3^a \Sigma_{\mu\nu}^a - \frac{i g_a^2}{4\pi^2} s_3^a \hat{F}_{\mu\nu}^a \) and introduced the following numbers \( s_3^a \)'s:

As the next step, one needs to carry out the integration over the Kalb-Ramond fields \( h_{a\mu}^\nu \)'s. Referring the reader for details of this procedure to Appendix B, we shall present here only the outcome of the respective calculation, which has the form

From now on, we omit everywhere the normalization constant, implying for every colour c the normalization condition \( Z_c[C = 0] = 1 \).
Here, \( \mathcal{D}^{(4)}_m(x) \equiv \frac{m}{4\pi^2} K_1(m|x|) \) is the propagator of a dual vector boson of the mass \( m = g_m \eta \sqrt{3} \) with \( K_1 \) standing for the modified Bessel function.

Further, it is necessary to carry out the integral

\[
2 \left( \frac{\Theta g_m \eta}{4\pi} \right)^2 (s_a^{(c)})^2 \int d^4x d^4y \bar{F}_{\mu\nu}(x) \mathcal{D}^{(4)}_m(x - y) \bar{F}_{\mu\nu}(y),
\]

emerging among other terms in the expression \(-2\pi^2 \eta^2 \int d^4x d^4y \bar{\Sigma}^{a\mu}_\nu(x) \mathcal{D}^{(4)}_m(x - y) \bar{\Sigma}^{a}_\mu\nu(y)\), which stands in the argument of the exponent on the R.H.S. of Eq. (7). This calculation is outlined in Appendix C, and the result reads

\[
\frac{1}{6} \left( \frac{\Theta g_m}{2\pi^2} \right)^2 \int d^4x d^4y j_\mu(x) \left[ \frac{1}{(x - y)^2} - 4\pi^2 \mathcal{D}^{(4)}_m(x - y) \right] j_\mu(y).
\]

On the other hand, after substituting Eq. (3) into the term \(-\frac{(\Theta g_m)^2}{12\pi^2} \int d^4x F_{\mu\nu}^2\), which stands in the first exponent on the R.H.S. of Eq. (3), we get for this term the expression \(-\frac{(\Theta g_m)^2}{24\pi^2} \int d^4x d^4y j_\mu(x) \frac{1}{(x - y)^2} j_\mu(y)\), which precisely cancels the corresponding Coulomb interaction of currents in Eq. (3). Clearly, this cancellation could be anticipated from the very beginning just from physical principles, since in the resulting theory there does not remain any massless particles, by which the quarks might exchange.

Substituting now Eqs. (7) and (3) into Eq. (3), we arrive at the following intermediate result for the partition function

\[
\mathcal{Z}_c = \exp \left[ -\frac{4i\Theta}{3} \hat{L}(\Sigma, C) - \left( \frac{8\pi^2}{3g_m^2} + \frac{(\Theta g_m)^2}{6\pi^2} \right) \int d^4x d^4y j_\mu(x) \mathcal{D}^{(4)}_m(x - y) j_\mu(y) \right] \times
\]

\[
\times \int D(x)^{(a)}(\xi) \delta \left( \sum_{a=1}^3 \Sigma^{a\mu}_\nu \right) \exp \left[ -2(\pi\eta)^2 \int d^4x d^4y \hat{\Sigma}^{a\mu}_\nu(x) \mathcal{D}^{(4)}_m(x - y) \hat{\Sigma}^{a}_\mu\nu(y) + i\Theta g_m^2 \eta^2 s_a^{(c)} \int d^4x d^4y \hat{\Sigma}^{a\mu}_\nu(x) \mathcal{D}^{(4)}_m(x - y) \bar{F}_{\mu\nu}(y) \right],
\]

where it has been denoted \( \hat{\Sigma}^{a\mu}_\nu \equiv \Sigma^{a\mu}_\nu - s_a^{(c)} \Sigma^{a\mu}. \) Clearly, the last term in the argument of the second exponent on the R.H.S. of Eq. (11) requires further simplifications. By virtue of Eq. (3) and a partial integration, this term can be rewritten as

\[
i\Theta g_m^2 \eta^2 \frac{m}{(4\pi^2)^2} s_a^{(c)} \varepsilon_{\mu\nu\rho} \int d^4x d^4y d^4z \hat{\Sigma}^{a\mu}_\nu(x) \frac{j_\lambda(z)}{(y - z)^2} \frac{\partial}{\partial x_\rho} K_1(m|x - y|) \]

The integral over \( y \) has the same form as the integral \( I(\lambda) \) from Appendix C with \( \lambda = x - z \) and reads \( \frac{4\pi^2}{m^2} \left[ \frac{1}{(x - z)^2} - 4\pi^2 \mathcal{D}^{(4)}_m(x - z) \right] \). Owing to this result and the definition of \( \hat{\Sigma}^{a\mu}_\nu \), the term under study takes the form

\[
- \frac{i\Theta}{3} \left[ 2s_a^{(c)} \hat{L}(\Sigma, C) - 4\hat{L}(\Sigma, C) - s_a^{(c)} \varepsilon_{\mu\nu\rho} \int d^4x d^4y \mathcal{D}^{(4)}_m(x - y) j_\mu(x) \frac{\partial}{\partial y_\nu} \Sigma^{a\lambda\rho}_\nu(y) \right].
\]

One can now see that, as it was expected, the singular term \(-\frac{i\Theta}{3} \hat{L}(\Sigma, C)\) from the first exponent on the R.H.S. of Eq. (11) becomes cancelled by the corresponding term from Eq. (3). Finally, we arrive at the following expression for the partition function
\[ Z_c = \exp \left[-\left(\frac{8\pi^2}{3g_m^2} + \frac{(\Theta g_m)^2}{6\pi^2}\right)\int d^4xd^4y j_\mu(x) D_m^{(4)}(x - y) j_\mu(y) \right] \int D x^{(a)}(\xi) \delta \left(3 \sum_{a=1} \Sigma_{\mu\nu}^a \right) \times \\
\times \exp\left[-2(\pi\eta)^2 \int d^4xd^4y \hat{\Sigma}_{\mu\nu}^a(x) D_m^{(4)}(x - y) \hat{\Sigma}_{\mu\nu}^a(y) + \right. \\
+i\Theta \sum_{a=1} s_{a(c)}^\epsilon_{\mu\nu\lambda\rho} \int d^4xd^4y D_m^{(4)}(x - y) j_\mu(x) \frac{\partial}{\partial y^\nu} \hat{\Sigma}_{\lambda\rho}^a(y) - \frac{2i\Theta}{3} s_a(c) \hat{L}(\Sigma^a, C) \right], \tag{12} \]

which is the main result of the present Letter. Notice that for every colour \( c \), it is straightforward to integrate one of the world-sheets \( \Sigma^a \)'s out of Eq. (12) by resolving the constraint imposed by the corresponding \( \delta \)-function. This procedure leads to an obvious expression for the partition function in terms of two independent string world-sheets.

The first exponent on the R.H.S. of Eq. (12) represents a short-ranged interaction of quarks, which due to the \( \Theta \)-term have now acquired also a nonvanishing magnetic charge, i.e. became dyons. Clearly, the first term in the second exponent on the R.H.S. of Eq. (12) is the short-ranged (self-)interaction of four different world-sheets: three closed ones \( \Sigma^a \)'s and an open one \( \Sigma \). This term is responsible for the linearly rising potential, which confines dyons. Upon the expansion in powers of the derivatives \( w.r.t. \, \xi^a \)'s, this term yields the coupling constants of the local string effective action (see the first paper from Ref. [7] for details). The second term is just the short-range interaction of dyons and strings. The most nontrivial term in the obtained expression is the last, third one, which describes a long-range interaction of dyons with closed world-sheets. Such an interaction represents the 4D-analogue of the Aharonov-Bohm effect [13], which means that at \( \Theta \neq 3\pi \times \text{integer} \) (cf. the explicit form of the numbers \( s_{a(c)}^\epsilon \)'s), the dyons become scattered by the closed electric ANO strings. Contrary to that, these critical values of \( \Theta \) correspond to such a relation between the magnetic charge of a dyon and the electric flux inside the string when the scattering is absent.

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5 Appendices

A. Path-Integral Duality Transformation

In the present Appendix, we shall outline some details of a derivation of Eq. (13) from the main text. To proceed with, we shall firstly rewrite a \( \theta_{a \text{reg}} \)-dependent part of the statistical weight, entering the partition function (14), as follows

\[ \int D \theta_{a \text{reg}} D k \exp \left\{ \int d^4 x \left[ -\eta^2 \sum_{a=1}^3 (\partial_\mu \theta_a - g_m e_a B_\mu)^2 + ik \sum_{a=1}^3 \theta_{a \text{reg}}^3 \right] \right\} = \]
and carry out the integration over $\theta^\text{reg.}_a$. In this way, one needs to solve the equation $\partial_\mu C^a_\mu = k$, which should hold for an arbitrary index $a$. The solution to this equation reads

$$C^a_\mu(x) = \partial_\mu \hat{h}^a_\mu(x) - \frac{1}{4\pi^2} \frac{\partial}{\partial x_\mu} \int d^4 y \frac{k(y)}{(x-y)^2},$$

where $h^a_\mu \nu$ stands for the Kalb-Ramond field of the $a$-th type.

Secondly, replacing the integrals over $\theta^\text{sing.}_a$'s with the integrals over $x^{(a)}(\xi)$'s by virtue of Eq. (3) and discarding for simplicity the Jacobians $[16]$ emerging during such changes of the integration variables $[16]$, we arrive at the following representation for the $\theta_a$-dependent part of the partition function $[14]$.

$$\int D\theta^\text{sing.}_a D\theta^\text{reg.}_a Dk \delta \left( \sum_{a=1}^3 \theta^\text{sing.}_a \right) \exp \left\{ \int d^4 x \left[ -\eta^2 \sum_{a=1}^3 (\partial_\mu \theta_a - g_m e_a B_\mu)^2 + ik \sum_{a=1}^3 \theta^\text{reg.}_a \right] \right\} =$$

$$= \int Dk \exp \left\{ \frac{1}{4\pi^2} \int d^4 x d^4 y \left[ -\frac{3}{4\pi^2} \frac{k(x) k(y)}{(x-y)^2} + i g_m \left( \frac{\partial}{\partial x_\mu} \frac{k(y)}{(x-y)^2} \right) \sum_{a=1}^3 e_a B_\mu(x) \right] \right\} \times \int D x^{(a)}(\xi) \delta \left( \sum_{a=1}^3 \Sigma^a_\mu_\nu \right) D h^a_\mu \nu \exp \left\{ \int d^4 x \left[ -\frac{1}{24\pi^2} (H^a_{\mu\nu\lambda})^2 + i \pi h^a_\mu \nu \Sigma^a_\mu_\nu - ig_m e_a B_\mu \partial_\nu \hat{h}^a_\mu \right] \right\}. \ (A.1)$$

Here, $H^a_{\mu\nu\lambda} = \partial_\mu h^a_\lambda + \partial_\lambda h^a_\mu + \partial_\nu h^a_\lambda \mu$ stands for the field strength tensor of the Kalb-Ramond field $h^a_\mu \nu$. Clearly, due to the explicit form of the root vectors, the sum $\sum_{a=1}^3 e_a B_\mu$ vanishes, and the integration over the Lagrange multiplier $k$ thus yields an inessential (field-independent) determinant factor. Notice also that due to Eq. (3), the constraint $\sum_{a=1}^3 \theta^\text{sing.}_a = 0$ has gone over into a constraint for the world-sheets of closed electric ANO strings of three types $\sum_{a=1}^3 \Sigma^a_\mu_\nu = 0$. This means that actually only the world-sheets of two types are independent of each other, whereas the third one is unambiguously fixed by the demand that the above constraint holds.

Let us now turn ourselves to the pure gauge field sector of the partition function $[14]$. In this way, firstly the term $-\frac{1}{2} \int d^4 x F^\mu_\nu F^{(c)}_{\mu\nu}$ can be rewritten as $Q^{(c)} \int d^4 x B_\mu \partial_\nu \tilde{\Sigma}_\mu$. Secondly, the $\Theta$-term is equal to $\frac{i g_n}{8\pi^2} \varepsilon^{\mu\nu_\lambda\rho} \int d^4 x B_\mu \partial_\nu \tilde{F}^{(c)}_{\lambda\rho}$. (It is worth noting that this expression can further be written as $-\frac{i g_n}{\pi} q^{(c)} \int d^4 x B_\mu j_\mu$ with $q^{(c)} \equiv Q^{(c)}_d$ being just a $g$-independent vector. The last equation means that due to the $\Theta$-term the quarks acquire a nonvanishing magnetic charge, i.e. become dyons. The $\Theta$-term then describes an interaction of a dyon with the dual gauge field $[14]$. After that, the gauge field sector takes the form

$\text{4}^4$Alternatively, these Jacobians can be referred to the integration measures $D x^{(a)}$'s.
\[
\int DB_\mu \exp \left\{ \int d^4x \left[ -\frac{1}{4} \left( F_{\mu\nu} + F^{(c)}_{\mu\nu} \right)^2 + \frac{i\Theta g_m^2}{16\pi^2} \left( F_{\mu\nu} + F^{(c)}_{\mu\nu} \right) \left( \tilde{F}_{\mu\nu} + \tilde{F}^{(c)}_{\mu\nu} \right) - ig_m e_a B_\mu \partial_\nu a^a_{\mu\nu} \right] \right\} = \\
= \int DB_\mu \exp \left\{ -\int d^4x \left[ \frac{1}{4} F^2_{\mu\nu} + \frac{1}{4} \left( F^{(c)}_{\mu\nu} \right)^2 + \varepsilon_{\mu\nu\lambda\rho} B_\mu \partial_\nu \left( \frac{ig_m e_a h^a_{\lambda\rho}}{2} - \frac{Q^{(c)}_{\mu\nu}}{2\Sigma_{\lambda\rho}} - \frac{i\Theta g_m^2}{8\pi^2} \tilde{F}_{\lambda\rho} \right) \right] \right\}.
\]

It is further convenient to pass from the integration over the \( B_\mu \)-fields to the integration over the fields \( B^a_\mu \equiv e_a B_\mu \). Then, performing the rescaling \( B^a_\mu (\text{new}) = \sqrt{\frac{2}{3}} B^a_\mu (\text{old}) \) (in order to restore the standard factor 1/4 in front of \( (F^a_{\mu\nu})^2 \)) and taking into account that \( Q^{(c)} = \frac{4}{3} e_a s^a_{\mu\nu} \) (see the notations after Eq. (6)), we get for Eq. (A.2) the following expression

\[
\int DB^a_\mu \exp \left\{ -\int d^4x \left[ \frac{1}{4} (F^a_{\mu\nu})^2 + \frac{1}{4} \left( F^{(c)}_{\mu\nu} \right)^2 + \frac{1}{2} \sqrt{\frac{3}{2}} \varepsilon_{\mu\nu\lambda\rho} B^a_\mu \partial_\nu \left( \frac{ig_m h^a_{\lambda\rho}}{3} - \frac{2}{3} \frac{s^a_{\lambda\rho}}{\Sigma \tilde{F}_{\lambda\rho}} \right) \right] \right\}.
\]

After that, the integration over the \( B^a_\mu \)-fields can be carried out by making use of the first-order formalism, \textit{i.e.} by representing the factor \( e^{-\frac{1}{4} \int d^4x (F^a_{\mu\nu})^2} \) as an integral over an auxiliary antisymmetric tensor field as follows

\[
\exp \left[ -\frac{1}{4} \int d^4x (F^a_{\mu\nu})^2 \right] = \int DG^a_{\mu\nu} \exp \left\{ \int d^4x \left[ (G^a_{\mu\nu})^2 + i\varepsilon_{\mu\nu\lambda\rho} B^a_\mu \partial_\nu G^a_{\lambda\rho} \right] \right\}.
\] (A.3)

The \( B^a_\mu \)-integration then yields the following expression for the field \( G^a_{\mu\nu} \)

\[
G^a_{\mu\nu} = \frac{1}{2} \sqrt{\frac{3}{2}} \left[ g_m h^a_{\mu\nu} + \frac{ig_m s^a_{\mu\nu}}{3} - \frac{\Theta g_m}{3\pi} s^a_{\lambda\rho} \tilde{F}_{\lambda\rho} \right] + \partial_\nu \lambda^a_\mu - \partial_\mu \lambda^a_\nu
\]

with \( \lambda^a_\mu \)'s standing for the new fields, resulting from the resolution of the corresponding constraint.

Bringing now together Eqs. (A.1) and (A.3), we arrive at the following expression for the full partition function

\[
Z_c = \int D \xi^{(a)}(\xi) \delta \left( \sum_{a=1}^{3} \Sigma^a_{\mu\nu} \right) \int d^4x \int D h^a_{\mu\nu} D \lambda^a_\mu \exp \left\{ \int d^4x \left[ -\frac{1}{24\sqrt{2}} \left( H^a_{\mu\nu\lambda} \right)^2 - \frac{1}{2} \sqrt{\frac{3}{2}} \left( g_m h^a_{\mu\nu} + \frac{ig_m s^a_{\mu\nu}}{3} - \frac{\Theta g_m}{3\pi} s^a_{\lambda\rho} \tilde{F}_{\lambda\rho} \right) + \partial_\nu \lambda^a_\mu - \partial_\mu \lambda^a_\nu \right]^2 + i\pi h^a_{\mu\nu\lambda} \Sigma^a_{\mu\nu} - \frac{1}{4} \left( F^{(c)}_{\mu\nu} \right)^2 \right\}.
\] (A.4)

Clearly, due to the closeness of the world-sheets \( \Sigma^a \)'s, the obtained action standing in the exponent on the R.H.S. of Eq. (A.4) is hypergauge-invariant, \textit{i.e.} it is invariant under the transformations \( h^a_{\mu\nu} \rightarrow h^a_{\mu\nu} + \partial_\mu \gamma^a_\nu - \partial_\nu \gamma^a_\mu, \lambda^a_\mu \rightarrow \lambda^a_\mu - \frac{\gamma^a_\mu}{2 \Sigma^a_{\mu\nu}}, \). Owing to that, the fields \( \gamma^a_\mu \)'s can be completely eliminated by setting \( \gamma^a_\mu = \frac{1}{3} \sqrt{2} \lambda^a_\mu \), which leads to the following expression
\[ Z_c = \int \mathcal{D}x^{(a)}(\xi) \delta \left( \sum_{a=1}^{3} \Sigma_{\mu \nu}^a \right) \mathcal{D}h_{\mu \nu}^a \exp \left\{ -\int d^4x \left[ \frac{1}{24\eta^2} \left( H_{\mu \nu \lambda}^a \right)^2 + \frac{3}{8} \left( g_m h_{\mu \nu}^a + \frac{ig}{3} s_a^{(c)} \Sigma_{\mu \nu} - \Theta g_m s_a^{(c)} \tilde{F}_{\mu \nu} \right)^2 - i\pi h_{\mu \nu}^a \Sigma_{\mu \nu}^a + \frac{1}{4} \left( F_{\mu \nu}^{(c)} \right)^2 \right] \right\}. \tag{A.5} \]

Next, a straightforward algebra yields

\[ \frac{3}{8} \left( g_m h_{\mu \nu}^a + \frac{ig}{3} s_a^{(c)} \Sigma_{\mu \nu} - \Theta g_m s_a^{(c)} \tilde{F}_{\mu \nu} \right)^2 = \frac{(\Theta g_m)^2}{12\pi^2} F_{\mu \nu}^2 + \frac{3g_m^2}{8} \left( h_{\mu \nu}^a \right)^2 - \frac{g^2}{12} \Sigma_{\mu \nu}^a + i\pi s_a^{(c)} h_{\mu \nu}^a \Sigma_{\mu \nu}^a - \frac{\Theta g_m^2}{4\pi} s_a^{(c)} h_{\mu \nu}^a \tilde{F}_{\mu \nu} - \frac{2i\Theta}{3} \Sigma_{\mu \nu} \tilde{F}_{\mu \nu}, \]

where we have taken into account that for every \( c, \left( s_a^{(c)} \right)^2 = 2 \). Clearly, the singular term \(-\frac{1}{4} \left( F_{\mu \nu}^{(c)} \right)^2\) becomes cancelled by the term \( \frac{g^2}{12} \Sigma_{\mu \nu}^a \). Finally, substituting for \( \tilde{F}_{\mu \nu} \) Eq. (B), we can rewrite the term \( \frac{2i\Theta}{3} \int d^4x \Sigma_{\mu \nu} \tilde{F}_{\mu \nu} \) as \(-\frac{2i\Theta}{3} \hat{L}(\Sigma, C)\), where \( \hat{L}(\Sigma, C) \equiv \frac{1}{8\pi^2} \epsilon_{\mu \nu \lambda \rho} \int d^4x d^4y \Sigma_{\mu \nu}(x) j_\rho(y) \frac{\partial}{\partial x_\lambda} \frac{1}{|x-y|} \) is the formal expression for the 4D Gauss linking number of the surface \( \Sigma \) with its boundary \( C \). As it will be demonstrated below, this singular term eventually cancels out.

Accounting for the above considerations in Eq. (A.5), we arrive at Eq. (B) of the main text.

**B. Integration over the Kalb-Ramond Fields**

In this Appendix, we shall present some details of a derivation of Eq. (B) from the main text. Namely, we shall carry out the following functional integral over the Kalb-Ramond fields

\[ \int \mathcal{D}h_{\mu \nu}^a \exp \left\{ -\int d^4x \left[ \frac{1}{24\eta^2} \left( H_{\mu \nu \lambda}^a \right)^2 + \frac{3g_m^2}{8} \left( h_{\mu \nu}^a \right)^2 - i\pi h_{\mu \nu}^a \Sigma_{\mu \nu}^a \right] \right\}. \tag{B.1} \]

Clearly, such a Gaussian integral can be calculated upon the substitution of its saddle-point value back into the action. The saddle-point equation in the momentum representation reads

\[ \frac{1}{4\eta^2} \left( p^2 h_{\nu \lambda}^{a(\text{extr.})} - p_\lambda p_\nu h_{\mu \nu}^{a(\text{extr.})} + p_\mu p_\nu h_{\lambda \mu}^{a(\text{extr.})} \right) + \frac{3g_m^2}{4} s_a^{(c)} h_{\nu \lambda}^{a(\text{extr.})} = i\pi \Sigma_{\nu \lambda}^a(p). \]

This equation can most easily be solved by rewriting it in the following way

\[ \left( p^2 \hat{P}_{\lambda \nu, \alpha \beta} + m^2 \hat{1}_{\lambda \nu, \alpha \beta} \right) h_{\alpha \beta}^{a(\text{extr.})}(p) = 4\pi i\eta^2 \Sigma_{\lambda \nu}^a(p), \tag{B.2} \]

where \( m = g_m \eta \sqrt{3} \) is just the mass of the dual gauge bosons, equal to the mass of the Kalb-Ramond fields. In Eq. (B.2), we have also introduced the following projection operators

\[ \hat{P}_{\mu \nu, \lambda \rho} \equiv \frac{1}{2} \left( P_{\mu \lambda} P_{\nu \rho} - P_{\mu \rho} P_{\nu \lambda} \right) \quad \text{and} \quad \hat{1}_{\mu \nu, \lambda \rho} \equiv \frac{1}{2} \left( \delta_{\mu \lambda} \delta_{\nu \rho} - \delta_{\mu \rho} \delta_{\nu \lambda} \right) \]

with \( P_{\mu \nu} \equiv \delta_{\mu \nu} - \frac{\mu \rho}{\rho \nu} P_{\mu \rho} P_{\nu \mu} \). These projection operators obey the following relations
\[ \hat{1}_{\mu\nu,\lambda\rho} = -\hat{1}_{\nu\mu,\lambda\rho} = -\hat{1}_{\mu\rho,\lambda\nu} = \hat{1}_{\lambda\rho,\mu\nu}, \quad \hat{1}_{\mu\nu,\lambda\rho} \hat{1}_{\lambda\rho,\alpha\beta} = \hat{1}_{\mu\nu,\alpha\beta} \]  
\hspace{1cm} (B.3)

(the same relations hold for \( \hat{P}_{\mu\nu,\lambda\rho} \), and)

\[ \hat{P}_{\mu\nu,\lambda\rho} \left( \hat{1} - \hat{P} \right)_{\lambda\rho,\alpha\beta} = 0. \]  
\hspace{1cm} (B.4)

By virtue of the properties (B.3) and (B.4), the solution to the saddle-point equation (B.2) reads

\[ h_{\alpha\nu}^{(\text{extr.})}(p) = \frac{4\pi i\eta}{p^2 + m^2} \left[ \hat{1} + \frac{p^2}{m^2} \left( \hat{1} - \hat{P} \right) \right]_{\lambda\rho,\alpha\beta} \Sigma_{\alpha\beta}^{a}(p), \]

which, once being substituted back into the functional integral (B.1), yields for it the following expression

\[ \exp \left\{ -2\pi^2 \eta^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m^2} \left[ \hat{1} + \frac{p^2}{m^2} \left( \hat{1} - \hat{P} \right) \right]_{\mu\nu,\alpha\beta} \Sigma_{\mu\nu}^{a}(-p) \Sigma_{\alpha\beta}^{a}(p) \right\}. \]  
\hspace{1cm} (B.5)

Rewriting now the term from Eq. (B.5), which is proportional to the projection operator \( \hat{1} \), in the coordinate representation we obtain the expression

\[ 2\pi^2 \eta^2 \int d^4 x d^4 y \Sigma_{\mu\nu}^{a}(x) D_{m}^{(4)}(x - y) \Sigma_{\alpha\beta}^{a}(y) \]

from the action standing in the exponent on the R.H.S. of Eq. (5).

Let us now consider the term proportional to the projection operator \( \left( \hat{1} - \hat{P} \right) \) on the R.H.S. of Eq. (B.5). In this way, by making use of the following equation

\[ p^2 \left( \hat{1} - \hat{P} \right)_{\mu\nu,\alpha\beta} = \frac{1}{2} (\delta_{\nu\beta} p_{\mu} p_{\alpha} + \delta_{\mu\alpha} p_{\nu} p_{\beta} - \delta_{\nu\alpha} p_{\mu} p_{\beta} - \delta_{\mu\beta} p_{\nu} p_{\alpha}), \]

we see that the term under study,

\[ -2\pi^2 \eta^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m^2} \left[ \hat{1} + \frac{p^2}{m^2} \left( \hat{1} - \hat{P} \right) \right]_{\mu\nu,\alpha\beta} \int d^4 x d^4 y e^{ip(y - x)} \Sigma_{\mu\nu}^{a}(x) \Sigma_{\alpha\beta}^{a}(y), \]

after carrying out the integration over \( p \), reads

\[ \left( \frac{2\pi \eta}{m} \right)^2 \int d^4 x d^4 y \Sigma_{\mu\nu}^{a}(x) \Sigma_{\alpha\beta}^{a}(y) \frac{\partial^2}{\partial x_{\mu} \partial y_{\beta}} D_{m}^{(4)}(x - y). \]

Performing twice the partial integration and making use of the equation \( \partial_{\mu} \Sigma_{\mu\nu}^{a} = s_{\alpha}^{(c)} j_{\nu} \), we can further rewrite this expression as

\[ -\frac{8\pi^2}{3g_{m}^2} \int d^4 x d^4 y j_{\mu}(x) D_{m}^{(4)}(x - y) j_{\mu}(y), \]

which is just the last term in the exponent on the R.H.S. of Eq. (5).

**C. Calculation of the Integral (8)**

Below in this Appendix, we shall present some details of a calculation of the integral (8). Owing to Eq. (3) and the fact that for every \( c \), \( \left( s_{\alpha}^{(c)} \right)^2 = 2 \), this integral can be rewritten as follows

\[ \frac{m}{8\pi^4} \left( \frac{\Theta g_{m}^2 \eta}{4\pi^2} \right)^2 \int d^4 x d^4 y \frac{K_{1}(m|x - y|)}{|x - y|} \frac{\partial}{\partial x_{\mu}} \int d^4 z \frac{j_{\nu}(z)}{(x - z)^2} \frac{\partial}{\partial y_{\mu}} \int d^4 u \frac{j_{\nu}(u)}{(y - u)^2}. \]  
\hspace{1cm} (C.1)

Next, by making use of the following sequence of partial integrations

\[ \frac{m}{8\pi^4} \left( \frac{\Theta g_{m}^2 \eta}{4\pi^2} \right)^2 \int d^4 x d^4 y K_{1}(m|x - y|) \frac{\partial}{\partial x_{\mu}} \int d^4 z \frac{j_{\nu}(z)}{(x - z)^2} \frac{\partial}{\partial y_{\mu}} \int d^4 u \frac{j_{\nu}(u)}{(y - u)^2}. \]
\[
\frac{\partial}{\partial y} \frac{1}{(y-u)^2} \rightarrow -\frac{\partial}{\partial y} \frac{K_1(m|x-y|)}{|x-y|} = \frac{\partial}{\partial x} \frac{K_1(m|x-y|)}{|x-y|} \rightarrow -\frac{\partial}{\partial x} \frac{1}{(x-z)^2},
\]

one can carry out the integration over \( z \). After that, the integral (C.1) takes the form

\[
\frac{m}{2\pi^2} \left( \frac{\Theta g^2}{4\pi^2} \right)^2 \int d^4 x d^4 u \mu(x)(x-u) I(x-u) j_{\mu}(u), \tag{C.2}
\]

where \( I(x-u) \equiv \int d^4 y \frac{K_1(m|x-y|)}{|x-y|^2} \) is just the integral, which is left to be calculated.

In order to proceed with this calculation, let us pass to the integration over the variable \( z \equiv y-u \) and introduce the vector \( \lambda \equiv x-u \), after which we obtain

\[
I(\lambda) = \int d^4 z \frac{K_1(m|z-\lambda|)}{|z-\lambda|} \frac{1}{|z-\lambda|} = \frac{\pi^2}{m} \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip(z-\lambda)}}{p^2 + m^2} \quad \text{and} \quad \frac{1}{z^2} = 4\pi^2 \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iqz}}{q^2}.
\]

Then we have

\[
I(\lambda) = \frac{16\pi^4}{m} \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ipx}}{(p^2 + m^2)p^2}
= \frac{16\pi^4}{m} \int \frac{d^4 p}{(2\pi)^4} \int_0^\infty d\alpha \int_0^\infty d\beta e^{-ip\alpha + \beta (p^2 + m^2)} = \frac{\pi^2}{m} \int_0^\infty d\alpha \int_0^\infty d\beta e^{-\beta (p^2 + m^2)} \frac{\lambda^2}{(\alpha + \beta)^2}. \tag{C.3}
\]

It is further convenient to introduce new integration variables \( a \in [0, +\infty) \) and \( t \in [0, 1] \) according to the formulae \( \alpha = at \) and \( \beta = a(1-t) \). Straightforward integration over \( t \) then reduces the integral (C.3) to the following expression

\[
I(\lambda) = \frac{\pi^2}{m^3} \int_0^\infty \frac{da}{a^2} e^{-\frac{\lambda^2}{a^2}} \left( 1 - e^{-am^2} \right).
\]

Such an integral can be carried out by virtue of the formula

\[
\int_0^{\infty} x^{\nu-1} e^{-\beta x^{\gamma}} dx = 2 \left( \frac{\beta}{\gamma} \right)^{\frac{\nu}{2}} K_{\nu} \left( 2\sqrt{\beta \gamma} \right), \quad \Re \beta > 0, \quad \Re \gamma > 0,
\]

and the result has the form

\[
I(\lambda) = \frac{4\pi^2}{m^3 |\lambda|} \left[ \frac{1}{|\lambda|} - m K_1(m|\lambda|) \right].
\]

Finally, substituting this expression into Eq. (C.2), we arrive at Eq. (12) of the main text.
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