Non-universal power laws in transport properties of one-dimensional quantum dots

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Abstract. We investigate discrepancies between recent experimental results on transport through one-dimensional quantum dots and universal power laws predicted by an idealized Luttinger Liquid description. The temperature dependence of Coulomb blockade peaks in one-dimensional quantum dots obeys non-universal power-laws from which different values of the interaction strength can be deduced. We find that, depending on the temperature range, measurements probe local or global properties of the interaction. In particular, we investigate the role of contacting semiconductor quantum wires and nanotubes connected to leads through tunnel junctions and compare to recent experiments. We conclude that a conventional Luttinger Liquid description of the quantum wire does explain the observed behaviour if specific properties of either experimental setup are carefully taken into account.

1. Introduction

Rapid advances have been made in the fabrication of one-dimensional electronic nanostructures in recent years. In one dimension the usual Fermi liquid picture of quasiparticles is not applicable anymore. Instead, the state is described by collective charge density fluctuations in terms of the Tomonaga–Luttinger liquid model (TLL). The electronic interactions are then manifest in the typical power-law exponent $g$. In particular one-dimensional quantum dots are ideal systems to examine this TLL parameter. One can estimate the $g$ from the charging energy, and also extract the interaction parameter from the temperature dependence of the conductance peaks in the quantum Coulomb blockade (CB) regime. However, in recent experiments probing the transport through quantum dots in semiconductor quantum wires as well as in nanotubes contradictory findings for the interaction parameter where reported [1, 2, 3].

When the electron density in the GaAs/AlGaAs-quantum wires fabricated by using the cleaved-edge-overgrowth technique (CEO) is decreased by applying a voltage to an external gate, eventually even the lowest electronic subband can be depopulated [1]. Here, the mean electron density is so low that only very few maxima of the random potential of the impurities are higher than the Fermi level. A one-dimensional quantum island can be formed between two potential maxima in such a wire. At temperatures lower than the charging energy, the linear conductance shows discrete peaks that correspond to transferring exactly one electron through the quantum island and hence a one-dimensional single electron transistor (SET) is created. In this regime, it has been detected that the temperature dependence of the intrinsic width of several conductance peaks (the area below the peaks) is modified by the correlations between the electrons and shows a power-law behaviour in temperature.

Carbon nanotubes are cylindrical graphene sheets with a diameter of only a few nanometres and a length of several microns. In the radial direction electrons are confined by the monolayer thickness of the graphene sheet. The unique electronic properties of
these nanostructures are due to the quantum confinement of electrons normal to the tube axis. The electrons can only propagate along the nanotube axis and form a truly one-dimensional system. Tubes can be metallic or semiconducting depending on the actual molecular configuration. SET’s were made by placing a metallic nanotube between two metal electrodes [2]. Then, the nanotube itself acts as the island of the SET and the contact resistances form the tunnel junctions of the transistor. Recently, an SET operated at room temperature was fabricated by putting a metallic nanotube between Au contacts and manipulating the tube with an atomic force microscope [3]. In this way, two buckles in a distance of 25 nm were created. Buckles in a nanotube behave much like electronic tunnel junctions [5, 6], and hence a quantum dot is formed between the two. However, the interaction parameter which was extracted from the temperature dependence of the CB peaks did not correspond to the theoretically expected one.

We suggest two mechanisms, each appropriate for a quantum dot immersed in a CEO quantum wire, and nanotube, respectively. To explain the discrepancy between expected and measured power-law exponents we focus on the nature of the contacts which connect the quantum wire to the leads.

2. Luttinger liquid with impurities

Using the bosonisation method the excitations of one-dimensional interacting electron system are described by density waves in terms of the conjugate fields [7], [∂(x), Π(x’)] = iδ(x – x’). In the simplest case, one band of spinless electrons, the Hamiltonian is

\[ H_0 = \frac{v_F}{2} \int dx \left( \Pi^2(x) + \frac{1}{g_0^2} [\partial_x \Pi(x)]^2 \right), \]

where \( v_F \) is the Fermi velocity. We set \( \hbar = k_B = 1 \) throughout this paper. The fields are related to the electron density \( \rho \) via \( \partial_x \Pi(x) = \sqrt{\pi} [\rho(x) - \rho_0] \) where \( \rho_0 = k_F/\pi \) is the mean density. The parameter \( g_0 = [1 + V_0/\pi v_F]^{-1/2} \) is the interaction constant that arises from the Fourier transform of any one-dimensional interaction potential in the limit of vanishing screening length. \( 0 < g_0 < 1 \) corresponds to repulsive interactions, \( g_0 = 1 \) no interaction, and \( g_0 > 1 \) attractive. A single localized impurity at \( x_b \) contributes a periodic potential term

\[ H_b = U_b \cos \left[ 2k_Fx_b + 2\sqrt{\pi} \partial \Pi(x_b) \right]. \]

The Hamiltonian \( H_0 + H_b \) can be interpreted as a potential model with variable \( \Pi(x_b) \), coupled to a harmonic field described by \( H_0 \). At low energies, traversal of the potential barriers is by tunneling, corresponding to the transport of electrons through a tunnel junction [8]. This electronic transport is characterized by the evolution of \( \Pi(x_b) \), influenced by the bulk modes away from the barrier position. Deriving an effective action [2] for \( \Pi(x_b) \) the forward tunneling rate through the junction can be written in the limit of high barriers [9] as

\[ \gamma(V) = \left( \frac{\Delta}{2} \right)^2 \int_{-\infty}^{\infty} dt \exp \left[ iVt - \int_{-\infty}^{\infty} d\omega \frac{J(\omega) - 1 - e^{-i\omega t}}{-\omega^2/2} \frac{1 - e^{\omega/\Theta}}{1 - e^{-\omega/\Theta}} e^{-\omega/\Theta} \right]. \]

Here, the tunneling amplitude \( \Delta \) is related to \( U_b \) via the WKB-method, \( V \) the associate energy of the tunneling event (e.g. voltage across the junction), and \( \Theta \) denotes a plasmon bandwidth cutoff. The spectral density \( J(\omega) \) contains the information due to the plasmon excitations and the electronic interactions in the system. It depends in general on the retarded Green’s function \( G(x,x’;t,t’) = -i\Theta(t-t’)[\Pi(x,t)\Pi(x’,t’)]_{H_0} \). In the case of a single barrier we find

\[ J_b(\omega, x_b) = -\text{Im} \frac{1}{G(\omega; x_b, x_b)}. \]
Here, the spectral density simply becomes $J(\omega) = 2\omega/g_0$. Using the detailed balance relation for the rate $\gamma(-V) = \exp(-V/k_B T)\gamma(V)$ we obtain the current $I(V)$ through the junction from the difference between forward and backward tunneling rates. Then the linear conductance $\lim_{V\to 0} I(V)/V$ of the tunnel junction reads

$$G_b(T) = \frac{1}{R_\Delta} \frac{\Gamma^2(1/g_0)}{\Gamma(2/g_0)} \left( \frac{2\pi T}{\omega_c} \right)^{2/g_0-2}.$$

(5)

Here, $R_\Delta = 2\omega_c^2/\pi e^2 \Delta^2$ and $\Gamma$ is the gamma function. The conductance shows a typical power-law in temperature dependence explicitly on the interaction through $g_0$. If electrons tunnel from a metallic lead ($g_0 = 1$) into the end of a TLL one needs to replace $2/g_0 \to 1 + 1/g_0$ in equation (5), and the power-law for the conductance of such a contact becomes $G_c(T) \propto T^{1/g_0-1}$.[8]

A quantum dot is created by two such impurities located at $x_d^\pm = x_d \pm a/2$, whose barrier contributions to the Hamiltonian can be combined to form the term

$$H_d = U_d \cos(\pi N_+) \cos[\pi(n_0 + N_-)],$$

(6)

where $N_\pm = (\theta(x_d^+) \pm \theta(x_d^-))/\sqrt{\pi}$. Changes of the quantities $N_+$ and $N_-$ are associated with the transfer of particles between left and right leads, and the fluctuations of the particle number in the dot, respectively. One can write the spectral density extracted from the effective theory for $N_\pm$ in terms of $J_b$ in [4][10]

$$J_d(\omega) = \frac{1}{2} J_b(\omega, x_d) \left[ 1 + \epsilon \sum_{n=1}^{\infty} \delta(\omega - n\epsilon) \right].$$

(7)

The energy $\epsilon = \pi v_F/a g_0 = 2g_0 E_c$ is the discrete level spacing of the plasmon states in the quantum dot and $E_c$ the charging energy. In the limit of linear transport the chemical potentials in the left and right leads and the dot are aligned. Then Coulomb blockade is relaxed and the conductance versus the gate voltage shows a peak. For sequential tunneling we can use the master equation method [[11]] for calculating the conductance for $T \ll \epsilon, E_c$ and obtain

$$G_d(\mu, T) = \frac{e^2}{4T} \frac{e^{-\mu/2T}}{\cosh \mu/2T} \gamma(\mu).$$

(8)

Here, $\mu$ is the distance from the resonance energy and $\gamma$ is defined in (3) with the appropriate $J_d(\omega)$ and $\Delta$ for $U_d$. The conductance of the CB peak reads

$$G_d(T, \mu) = \frac{1}{4R_\Delta} \frac{\Gamma(1/2g_0 + i\mu/2\pi T)^2}{\cosh \mu/2T} \left( \frac{\epsilon}{\omega_c} \right)^{1/g_0} \left( \frac{2\pi T}{\omega_c} \right)^{1/g_0-2} e^{-|\mu|/\omega_c} \Gamma(1/g_0).$$

(9)

From (9) the maximum of the CB peak ($\mu = 0$) scales as $G_d^{\text{max}} \propto T^{1/g_0-2}$ in temperature.

3. CEO wires and SET – inhomogeneous interaction

In the CEO wires the entire one-dimensional system is situated along the edge of the sample. Electrons travel from the "leads" region below the two-dimensional electron gas (2DEG) into the wire region where the electronic density is kept extremely low and the Coulomb interaction is very strong (see left sketch in figure [1]). We characterize a larger wire region containing the quantum dot by a spatially varying, short-ranged interaction,

$$V(x,y) = [V_0 + \varphi(x)] \delta(x-y).$$

(10)
\( \phi(x) \) is assumed to be smooth, with a maximum near \( x = 0 \) and a characteristic length \( L^* \), and \( \phi(x) \to 0 \) when \( |x| \to \infty \). The inhomogeneity also introduces a characteristic frequency \( \omega^* = \nu_F g_0 L^* \). With [10] the interaction parameter \( g \) varies as a function of \( x \),

\[
g(x) = g_0 \left[ 1 + \frac{\phi(x)}{V_0 + \nu_F} \right]^{-1/2}.
\]  

To calculate the spectral density for the dot we need the Green’s function which obeys the equation of motion

\[
\left[ \frac{\omega^2}{\nu_F} + \frac{\partial}{\partial x} \frac{\nu_F}{g^2(x)} \frac{\partial}{\partial x} \right] G(\omega; x, x') = -\delta(x-x')
\]

with outgoing-wave boundary conditions. One can immediately solve [12] for the asymptotic cases of short or long plasmon wavelength, \( \omega \gg \omega^* \) and \( \omega \ll \omega^* \). In the first case, a WKB-like solution yields

\[
G(\omega; x, x') = \frac{i g_0}{2 \omega} \exp \left( \frac{i \omega}{\nu_F} \int_{x'}^x dy g(y) \right),
\]

and in the low-frequency case we obtain

\[
G(\omega; x, x') = \frac{i g_0}{2 \omega} \exp \left( \frac{i \omega}{\nu_F g_d} \int_{x'}^x dy g^2(y) \right).
\]

This defines the asymptotic behaviour of the corresponding spectral density for the single barrier, which we later generalize for the case of a quantum dot with extension \( a \ll L^* \),

\[
J_b(\omega) = \begin{cases} 
\frac{\omega}{g_0} & \text{for } \omega \ll \omega^* \\
\frac{\omega}{g_d} & \text{for } \omega \gg \omega^* 
\end{cases}.
\]

Here \( g_d \) is the local value of the interaction parameter \( g(x) \) at the position of the barrier or dot. The charging energy of a small dot can easily be deduced[2] from [14],

\[
E_c = \frac{\pi \nu_F}{2ag_d^2}.
\]

Thus, \( E_c \) is a local quantity which depends on the length of the dot and the interaction strength in the dot region through \( g_d \). Likewise, the level spacing \( \varepsilon = 2g_d E_c \) is a local probe of the interaction that depends only on \( g_d \).

In order to calculate the conductance [8], the entire spectral density, i.e., the Green’s function at arbitrary frequency, is needed. Treating \( \phi(x) \) perturbatively (up to first order) in [12] we get

\[
G(\omega; x_d, x_d) = \frac{i g_0}{2 \omega} \left( 1 + c \int_0^\infty dy [\phi(y + x_d) + \phi(x_d - y)] e^{i\eta y} \right),
\]

where \( \eta = 2g_0 \omega/\nu_F \) and \( c = i 2g_0^3 \omega/2\pi \nu_F^2 \). Equation [1] remains valid for a small dot in an inhomogeneous wire [12], but \( J_b(\omega) \) is now deduced from [17]. The rates entering expression [8] for the conductance must be computed numerically in the inhomogeneous case. Figure [1] shows the temperature dependence of the CB peak maximum for \( \phi(x) = [1 + (2x/L^*)^2]^{-1} \), \( x_d = 0, g_0 = 0.6 \) and \( g_d = 0.3 \). We observe a crossover from a power-law with exponent \( 1/g_d - 2 \) at high temperatures to one with exponent \( 1/g_0 - 2 \) at low temperature. The measurement of the linear conductance at low temperatures thus reflects the interaction far away from the dot (global probe). Measurements at high temperature, on the other hand, act as a local probe of the interaction close to the dot. It is to be noted that the transition region around \( T = \omega^* \) appears to cover at least one order of magnitude. Fitting to a power-law in a narrow interval within the transition region may yield any value \( g_{\text{eff}} \) between \( g_d \) and \( g_0 \), related to the function \( \gamma_d^{\text{max}}(T) \) through \( 1/g_{\text{eff}} = \partial \ln \gamma_d^{\text{max}}/\partial \ln T + 2 \).
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Figure 1. Left: sketch of the CEO arrangement, electrons travel from the 2DEG into the quantum wire situated along the edge. Inset: CB peaks at temperatures $T/\omega^* = 2.8, 2.3, 1.9, 1.3, 0.9, 0.4$ (from top to bottom). $\mu$ denotes the distance from the resonance energy. Both are given in units of the characteristic energy induced by the inhomogeneity in the interaction strength $\omega^* = v_F/g_0L$. Main: temperature dependence of a conductance peak for a one-dimensional quantum dot in a TLL with inhomogeneous interaction strength. The conductance is given in units $G_0 = e^2(\Delta/4\omega_c)^2(\varepsilon/\omega_c)^{1/g_d}$ with the tunneling matrix element $\Delta$ and the high-energy cutoff $\omega_c$ (here: $\omega^*/\omega_c = 10^{-3}$). The conductance maximum obeys a non-universal power-law $G_{\text{max}} \propto T^{1/g_{\text{eff}}-2}$ with a crossover from $g_{\text{eff}} = g_0 = 0.6$ (interaction strength in the leads) at low temperatures to $g_{\text{eff}} = g_d = 0.3$ (local strength at the dot position) at high temperatures. The asymptotic power-laws are plotted with dashed lines.

4. Carbon nanotube SET

In metallic nanotubes two one-dimensional bands intersect the Fermi energy, hence two charge and spin channels are available for transport. Only the symmetric combination of the two charge modes is affected by the interaction \cite{13}, characterized by the interaction parameter $g_{\text{nt}} \approx 0.27$ \cite{14}. Taking into account the three non-interacting channels (due to the spin modes and the antisymmetric combination of the charge modes), one can account for this particular band structure by substituting \cite{13} $1/g_0 \rightarrow (3 + 1/g_{\text{nt}})/4$ in the above result \cite{9} for the dot-conductance $G_d$.

Unlike CEO wires, nanotubes are very difficult to connect to leads \cite{14}; the connection between the nanotube and a metallic contact is usually regarded as a tunnel junction \cite{8,13,15}. In the experiments done so far, two kinds of contacts have been used. On the one hand, a contact can be created by depositing metallic leads on top of the tube (figure 2 right sketch). In this case, the one-dimensional conductor terminates at the contact, and tunneling occurs from the external lead into the end of the nanotube. On the other hand, nanotubes can be placed on top of predefined metallic leads. Then electrons can tunnel from the metal into the one-dimensional bulk of a tube (figure 2 left). In both cases the temperature dependence of the contact resistance is given by a TLL power-law of the form $T^{\alpha}$ \cite{8,13,15}. Tunneling into the bulk of the nanotube is described by the exponent $\alpha_{\text{bulk}} = (1/g_{\text{nt}} + g_{\text{nt}} - 2)/8$ while tunneling into the end corresponds to an exponent $\alpha_{\text{end}} = (1/g_{\text{nt}} - 1)/4$. 
We write the contribution of the contacts through tunneling into the nanotube as the conductance

\[ G_{\text{nt}}(T) = \frac{1}{R_c} \left( \frac{T}{T_0} \right)^{\alpha} \tag{18} \]

where is \( R_c \) the resistance of the junction at an arbitrary reference temperature \( T_0 \) (e.g., room temperature), and \( \alpha \) is either the end or bulk exponent.

The temperature dependence of the CB peak conductance is governed by a different power-law,

\[ G_{\text{d}}^{\max}(T) = \frac{1}{R_d} \left( \frac{T}{T_0} \right)^{\alpha_d} \tag{19} \]

where \( \alpha_d = (1/g_{\text{nt}} - 5)/4 \). Note that \( \alpha_d < 0 \) for \( g_{\text{nt}} > 1/5 \), while \( \alpha_{\text{bulk}} \) and \( \alpha_{\text{end}} \) are always positive for repulsive interaction.

Assuming the electronic coherence length to be short compared to the extension of the nanotube, the total resistance is given by the sum of the resistances of the contacts and the dot,

\[ \frac{1}{G(T, \mu)} = \frac{2}{G_{\text{nt}}(T)} + \frac{1}{G_{\text{d}}(T, \mu)} \tag{20} \]
At room temperature the experimentally observed resistance $R_d$ of the dot is one order of magnitude larger than the combined contact resistance $2R_d$ [3]. The high-temperature behaviour of the observed conductance peaks is given by the exponent $\alpha_d$, and the transport properties of the wire are dominated by the SET physics of the dot for arbitrary gate voltage. At low temperatures, however, the resistance of the contacts dominates over the relatively small resistance of the dot near a conductance peak. This means that a two-point conductance measurement observes a ‘clipped’ peak, whose peak height is no longer governed by (19) but by (18). Between the CB peaks the resistance of the dot of the dot is always higher than that of the contacts, with measured currents limited mostly by the Coulomb blockade. Thus the temperature behaviour of the CB conductance maxima can be given by the power-law of the contact conductance (18) in spite of the fact that CB conductance peaks are observed as a function of the gate voltage.

This implies a crossover between the two power-laws in the temperature dependence of the CB peak, with low-temperature exponents, $\alpha_{\text{end}}$ or $\alpha_{\text{bulk}}$, and crossover temperatures

$$\frac{T^*}{T_0} = \left(\frac{2R_c}{R_d}\right)^{1/(\alpha - \alpha_d)}$$

where $\alpha$ is $\alpha_{\text{end}}$ or $\alpha_{\text{bulk}}$, depending on the type of contact used in the experiment. For given parameters $R_c$ and $R_d$ at the reference temperature $T_0$, the crossover temperatures are related to each other by

$$\left(\frac{T^*}{T_0}\right)_{\text{bulk}} = \left(\frac{T^*}{T_0}\right)_{\text{end}}^{1/(\alpha_{\text{bulk}} - \alpha_d)}.$$  

For the experimental value $g_{\text{nt}} = 0.27$ and $R_c \ll R_d$ this means $T^*_{\text{bulk}} \ll T^*_{\text{end}}$. The behaviour of the conductance maximum as a function of temperature is shown in figure 2.

With the above model, the temperature behaviour of the Coulomb peak described in [3] can be understood using the interaction parameter $g_{\text{nt}} = 0.27$ reported earlier. Assuming metal-to-end tunneling at the contacts, we find a quantitative agreement with the observed increase of the peak conductance with rising temperature. The authors of [3] discuss an alternative explanation of their data, postulating a correlated tunneling process depending only on intrinsic properties of the quantum dot. However, in view of the existing knowledge about the behaviour of metal-to-nanotube contacts, we consider the straightforward explanation given here rather persuasive.

5. Conclusion

We demonstrate that contacts to quantum wires strongly influence transport through a one-dimensional quantum dot system. Recent experiments on both cleaved-edge overgrowth and carbon nanotube quantum wires can thus be reconciled with the Tomonaga-Luttinger Model of the one-dimensional electron liquid. Non-Fermi-liquid transport properties such as power-laws governing the height of Coulomb blockade peaks were previously interpreted only through characteristics of the quantum dot itself. However, the power-laws in the temperature dependence of the CB maxima can be modified or superseded by the effect of inhomogeneous contacts (CEO’s) or tunnel junctions (nanotubes). Depending on the energy regimes where the measurements are carried out, the linear conductance is either a local probe establishing the interaction parameter near the dot, or a global probe of a larger system including extended one-dimensional excitations and external contacts. The temperature distinguishing between these two regimes is either determined by the length scale on which an inhomogeneous interaction changes (CEO’s), or by the ratio of the contact and buckle tunneling resistances (nanotubes).
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