Weighted variational regularization image dehazing algorithm based on non-local prior

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Abstract. In hazy weather conditions, the quality of images taken outdoors will be degraded, so researching image dehazing has great practical significance. Therefore, an image dehazing algorithm based on non-local prior weighted variational regularization is proposed. Since the performance of the image dehazing algorithm depends largely on the accuracy of the estimated transmission, a classical non-local prior algorithm is first adopted to make a rough initial estimate of the transmission. Then a weighted variational regularization model is proposed to further optimize the initial transmission and solve the model by the alternating direction multiplier method. Finally, a haze-free image can be recovered directly from the atmospheric scattering model by using the optimized transmission. The experimental results verify that the proposed algorithm has higher image dehazing performance than several commonly used algorithms.

1. Introduction

Haze is a common weather phenomenon, which will reduce the visibility of the atmosphere and lead to a degradation in the quality of the images taken outdoors. When the haze in the atmosphere is severe, it may even affect the normal operation of the outdoor intelligent transportation system. Therefore, the research on image dehazing algorithm has important practical significance and broad application prospects. The atmospheric scattering model in [1] is generally used to represent the degradation process of hazy images

\[ I(x) = J(x)tr(x) + [(1 - t(x))A \]

where \( I(x) \) is the pixel value, \( I(x) \) and \( J(x) \) are the hazy image and the real scene radiance, respectively, \( t(x) \) is the transmission map, and \( A \) is the atmospheric illumination. The airlight \( A \) is the atmospheric illuminance representing a single color, \( t(x) \) is the transmission map depending on scene depth. Image dehazing needs to restore \( J(x) \) from \( I(x) \) with two unknown variables which is an ill-posed problem.

Narasimhan et al. [2] proposed a scattering model which a haze-free image can be recovered from two or more hazy images. For the problem of dehazing a single image, the [3,4] proposed several methods based on image enhancement, such as histogram equalization algorithm and Retinex theory. [5,6] adopted the method of deep learning to make the image dehazing effect significantly improved. Meng et al. [7] found that a geometric model for image dehazing can be derived from the boundary constraints of the transmission map, and the transmission can be optimized in the form of a filter using contextual regular terms. Berman et al. [8] assumed that haze-free images converge into
hundreds of clusters in the RGB space, and proposed a non-local image dehazing method, which combines the global information of the image. The method effectively avoids the halo effect. Shu Qiaoling et al. [9] proposed a variational regularized transmission refinement model for image dehazing. The dehazing effect of the model is better than others, but it needs to be improved when the image contains thick haze.

Therefore, combining the nonlocal information of the image and the variational regularized theory, this paper designs a weighted variational regularization image dehazing algorithm based on non-local prior. The non-local prior knowledge is used to make a rough estimate of the transmission map and the weighted variational regularization model is adopted to further optimize the initial estimated transmission map. The final haze-free image is calculated from the optimized transmission map and atmospheric scattering model.

2. Initial Transmission Estimation Based on Non-Partial Prior

Berman et al. [5] used K-means to cluster RGB values for a single image to several hundreds clusters in Berkeley segmentation database which is a diverse data set containing clear outdoor images. And replace the pixel values in the image with the center values of their respective clusters. It turns out that a single image can be represented by up to 500 different RGB values (This number is much smaller than the number of pixels of the original image). For a given cluster, the pixels belonging to a cluster are non-local because they are distributed at different locations throughout the image. Under the influence of haze, since the pixels in the same color cluster in the original haze-free image are located in various image regions and the distances of these pixels from the camera are diverse, there are some differences between the observed color value and the original color value in the haze-free image. Therefore, these pixels are no longer clustered into one cluster but form a straight haze-line in the RGB space.

In order to estimate the transmission more accurately, it is necessary to first calculate the haze-line composed of image pixels, \( \mathbf{A} \) in this paper is calculated by the method provided in [10] and the distance between the color of the airlight \( \mathbf{A} \) and the pixel on the haze-line is defined as

\[
I_a(x) = |\mathbf{A} - \mathbf{J}(x)| \cdot t(x)
\]  

(2)

The Eq. (2) expressed in a spherical coordinate system, i.e

\[
I_a(x) = [r(x), \alpha(x), \mu(x)]
\]  

(3)

The \( r(x) = \| I - \mathbf{A} \| \) is the radius from the pixel to the center \( \mathbf{A} \) of the spherical coordinate system, \( \alpha(x) \) and \( \mu(x) \) represent longitude and latitude, respectively.

It can be seen from Eq.(3) that, for a given \( \mathbf{A} \) and \( \mathbf{J} \), the different distances of the pixels in the image from the camera only cause the transmission \( t \) values of the respective pixels to be different. If the \( x \) and \( y \) values of pixels \( \alpha \) and \( \mu \) are similar, they have similar color value \( \mathbf{J} \) in the haze-free image [8]. Therefore, if the \( \alpha(x) \) and \( \mu(x) \) values of two pixels are similar, the two pixels are on the same haze-line.

In order to determine whether each pixel is on the same haze-line, the K-D tree is builded to cluster the \( \phi \) and \( \theta \) of the pixel in the image. After clustering to obtain the haze-line to which each pixel belongs. On the same haze-line, the distance from each pixel to the center of the sphere can be calculated from \( \mathbf{J} \) and \( \mathbf{A} \)

\[
r(x) = t(x) \| \mathbf{A} - \mathbf{J}(x) \|
\]  

(4)

In Eq.(4), \( 0 \leq t(x) \leq 1 \). In the case of \( t = 1 \), the maximum distance radius between the pixel and the center of the sphere can be calculated as \( r_{\text{max}}^{\text{def}} = \| \mathbf{J} - \mathbf{A} \| \) and the maximum radius can be estimated when the haze-line \( \mathbf{H} \) contains haze-free pixels, i.e.

\[
\hat{r}_{\text{max}}(x) = \max_{x \in \mathbf{H}}(r(x)).
\]

A rough initial transmission can be obtained by combining Eq.5 and \( \hat{r}_{\text{max}}(x) \).

\[
\hat{t}(x) = r(x) / \hat{r}_{\text{max}}(x)
\]  

(5)
3. Transmission optimization model based on weighted variational regularization

3.1. Weighted variational regularization model

From Eq. (1), the performance of estimated transmission can directly affect the accuracy of image dehazing. For the purpose of obtaining a more precise transmission estimation map to avoid color distortion in the process of image dehazing, a transmission optimization model based on weighted variational regularization is proposed. At the aim of facilitating the calculation, rewrite the original image model to

\[ \tilde{I}(x) = J(x) \cdot r(x) \]  

For every \( x \in \Omega \), there is \( \tilde{I}(x) = A - I(x) \) and \( \tilde{J}(x) = A - J(x) \). Besides, the initial value of \( \tilde{J} \) is set to \( J_0 = A - I / \max(t_\varepsilon) \). The small constant \( t_\varepsilon \) prevents the denominator from being zero. Then the transmission optimization model based on weighted variational regularization can be defined as

\[
\begin{align*}
\min_{J} \{ g \cdot \| \nabla \tilde{J} \|_0 + \frac{\lambda_1}{2} \| \tilde{J} - \tilde{t} \|_0^2 \\
+ \frac{\lambda_2}{2} \| \nabla \tilde{t} \|_0 + \frac{\lambda_3}{2} \| \tilde{t} - \tilde{r} \|_0^2 + \lambda_4 \| \nabla \tilde{t} - \nabla \tilde{r} \|_0 \}
\end{align*}
\]

(7)

In Eq. (7), \( I = I_\varepsilon, \tilde{I} = \tilde{I} \), and \( \tilde{J} = \tilde{J}_\varepsilon \), \( c \in \{r, g, b\} \), \( (\lambda_1, \lambda_2, \lambda_3, \lambda_4) \) are the regularization parameters. The first and third items are the regularization items used to constrain the estimation process, and the second and fourth items are data fidelity items to protect the edges of the transmission map. \( g \) is a weight function, defining its expression as

\[ g = \frac{1}{1 + K \| \nabla G \ast \tilde{J} \|} \]

(8)

Where \( K \geq 0 \), \( \nabla G \ast \tilde{J} \) means Gaussian convolution processing for \( \tilde{J} \). The weight function \( g \) can acquire different regularization parameters according to different pixels in the image location, and can better distinguish the edge region and the flat region of image, thereby better protecting the edges of the map while suppressing unwanted artifacts in the uniform region.

3.2. Numerical algorithm

The stable solution can not be calculated by the traditional numerical algorithm, since there are non-smooth optimization problems [9] in Eq. (7). Therefore, an alternating direction method (ADMM) which can effectively processed the problems is applied to solve this model. Let \( X = \nabla \tilde{J}, Y = \nabla \tilde{t} \) and \( Z = \nabla \tilde{t} - \nabla \tilde{r} \), then convert the original model (7) into

\[
\begin{align*}
\min_{x, t, z, \lambda} \{ g \cdot \| X \|_1 + \frac{\lambda_1}{2} \| \tilde{J} - \tilde{t} \|_1^2 \\
+ \lambda_2 \| Y \|_1 + \frac{\lambda_3}{2} \| \tilde{t} - \tilde{r} \|_1^2 + \lambda_4 \| Z \|_1 \}
\end{align*}
\]

(9)

The augmented Lagrangian expression of Eq. (9) can be expressed as

\[
\begin{align*}
L_\alpha = g \cdot \| X \|_1 + \frac{\lambda_1}{2} \| \tilde{J} - \tilde{t} \|_1^2 + \lambda_2 \| Y \|_1 + \frac{\lambda_3}{2} \| \tilde{t} - \tilde{r} \|_1^2 + \lambda_4 \| Z \|_1 + \frac{\beta_1}{2} \| X - \nabla \tilde{J} - \eta \|_2^2 \\
+ \frac{\beta_2}{2} \| Y - \nabla \tilde{t} - \zeta \|_2^2 + \frac{\beta_3}{2} \| Z - (\nabla \tilde{t} - \nabla \tilde{r}) - \xi \|_2^2
\end{align*}
\]

(10)

In Eq.(10), \( \eta, \zeta \) and \( \xi \) represent Lagrangian multipliers, and \( (\beta_1, \beta_2, \beta_3) \) are predefined positive parameters. In order to solve the model effectively, \( L_\alpha \) decomposed into multiple sub-problems about \( X, Y, Z, \tilde{J} \) and \( t \), and these sub-problems are solved iteratively until the optimal value of the result is obtained.

In the case given by \( \tilde{J} \) and \( t \), the \((X, Y, Z)\) subproblem can essentially be regarded as the L1 regularized least squares problem. When \((X, Y, Z)\) is fixed, solving the optimal value of \( L_\alpha \) related to
\( J \) and \( t \) is equivalent to solving the least squares optimization problem. The specific method has been summarized in \textbf{Algorithm 1}.  

\textbf{Algorithm 1:} ADMM for optimization Eq. (10)  

\textbf{Input:} \( J_0, I_0, t_0 = \tilde{t} \)  

\textbf{Parameter:} \( \lambda_1, \lambda_2, \lambda_3, \beta_1, \beta_2, \beta_3, K \)  

\textbf{Iteration:}  

\begin{align*}
X & = \min_{\mathbf{x}} \{ g \cdot \| \mathbf{x} \| + \frac{\beta_1}{2} \| \mathbf{x} - \nabla J \| + \frac{\beta_2}{2} \| \mathbf{x} - \nabla I \| - \frac{\eta}{\beta_1} \} \\
Y & = \min_{\lambda_1} \{ \lambda_2 \| \lambda_2 \| + \frac{\beta_3}{2} \| \lambda_1 - \nabla t \| - \frac{\eta}{\beta_3} \} \\
Z & = \min_{\lambda_2} \{ \lambda_3 \| Z \| + \frac{\beta_4}{2} \| Z - (\nabla t - \nabla I) \| - \frac{\xi}{\beta_4} \} \\
\tilde{J} & = \min_{\tilde{t}} \{ \frac{\lambda_1}{2} \| \tilde{J} - \tilde{t} \| \| + \frac{\beta_2}{2} \| \tilde{J} - \nabla I \| - \frac{\eta}{\beta_2} \} \\
\tilde{t} & = \min_{\tilde{t}} \{ \frac{\lambda_2}{2} \| \tilde{t} - \tilde{I} \| \| + \frac{\lambda_3}{2} \| \tilde{t} - \tilde{I} \| \| + \frac{\beta_2 + \beta_3}{2} \| \tilde{t} - \nabla I \| \} \\
\phi & = \frac{\beta_2 Y + \beta_3 \hat{Z}}{\beta_2 + \beta_3}, \quad \hat{Y} = Y - \frac{\xi}{\beta_3}, \quad \hat{Z} = Z + \nabla I - \frac{\xi}{\beta_3} \\
\end{align*}  

\textbf{Return:} \( \tilde{J}, \tilde{t}, \phi \)  

For the \((X,Y,Z)\) subproblem can be directly solved by the soft threshold algorithm:  

\begin{equation}
\begin{aligned}
X & = \text{shrinkage}(\nabla \tilde{J} + \eta / \beta_1, g / \beta_1) \\
Y & = \text{shrinkage}(\nabla t + \zeta / \beta_2, \lambda_2 / \beta_2) \\
Z & = \text{shrinkage}(\nabla t - \nabla I + \zeta / \beta_3, \lambda_3 / \beta_3) \\
\end{aligned}
\end{equation}  

During each iteration, the Lagrangian multipliers \((\eta, \zeta, \xi)\) can be simply updated by the following formula:  

\begin{equation}
\begin{aligned}
\eta & = \eta - \nabla \lambda_1 (X - \nabla \tilde{J}) \\
\zeta & = \zeta - \nabla \lambda_2 (Y - \nabla t) \\
\xi & = \xi - \nabla \lambda_3 (Z - (\nabla t - \nabla I)) \\
\end{aligned}
\end{equation}  

The forward fast Fourier transform (FFT) can be directly used to solve the \((\tilde{J}, \tilde{t})\) problem  

\begin{equation}
\begin{aligned}
\tilde{J} &= \frac{\lambda_1 F(I / \tilde{t}) + \beta_1 F(V)F(X - \nabla \tilde{I})}{\lambda_1 F(I) + \beta_1 F(V)F(V)} \\
\tilde{t} &= \tilde{F}^{-1} \frac{\lambda_2 F(I / \tilde{J}) + \lambda_3 F(I) + (\beta_2 + \beta_3) F(V)F(\tilde{V})}{(\lambda_2 + \lambda_3) F(I) + (\beta_2 + \beta_3) F(V)F(V)} \\
\end{aligned}
\end{equation}  

In Eq.(13), \( I \) is the identity matrix, \( F^{-1}(\cdot) \) and \( \overline{F(\cdot)} \) represent the inverse transform and complex conjugate operation of the FFT, respectively.  

(a) Hazy image \hspace{1cm} (b) Initial transmission \hspace{1cm} (c) Optimized transmission  

Figure 1. Transmission map estimation of hazy images
On the basis of the above theory, the initial estimated transmission map and an optimized transmission map can be obtained, as shown in Figure 1. It can be found that after the two estimates of the transmission by the algorithm in this paper, the final transmission map is more accurate and contains more detailed components, so that the image dehazing effect can be better improved.

Through the above analysis, the final haze-free image will be calculated according to atmospheric scattering model Eq. (1):

$$ J_{\text{final}}(x) = \mathbf{A} + \frac{I(x) - A}{\max(t_s, t_p(x))} $$

(14)

4. Analysis of experiment results

For the purpose of comparing the difference in dehazing effect between various algorithms, the four hazy images in Fig. 2a were tested. The dehazing effects of our algorithm and the algorithms of NL [8], RCCR [7] and the VRTR [9] are compared and analyzed from the visual effects and objective quantitative indicators. The experimental parameters in this paper are set to: $$ \lambda_0 = 0.2, \lambda_2 = 1, \lambda_1 = 0.5, \beta_1 = 0.25, \beta_1 = 0.65, \beta_2 = 0.5. $$ All experiments were programmed on MATLAB R2016b with hardware parameters of Intel Core i5-7300HQ, CPU 2.5GHz, and 8GB of memory.

![Figure 2. Image dehazing results for each algorithm](image)

By comparison, it can be found that the image in Figure 2b the sky area has obvious color distortion phenomenon, the overall contrast of the image is reduced; in Figure 2c, the dehazing effect on the near scene in the image is better, but some images have obvious halo effect; the overall color of Fig. 2d is consistent, most haze of some images has been removed, but when the image contains thick haze and the dehazing effects are not very well. The results in Figure 2e show that the dehazing effect of our algorithm is more ideal, and the image with thick haze can effectively remove most of the haze.

The peak signal-to-noise ratio (PSNR) and image structure similarity (SSIM) at the Table 1 are used to objectively illustrate the dehazing performance of each algorithm. It can be found that, overall,
the PSNR value and SSIM value of the algorithm in this paper are improved compared with the NL algorithm, RCCR algorithm and VRTR algorithm.

| Original image | NL        | BCCR     | VRTR     | Ours     |
|----------------|-----------|----------|----------|----------|
| Cityscape      | 9.58/0.48 | 11.36/0.52 | 15.39/0.63 | 17.72/0.76 |
| Straw          | 13.50/0.66 | 14.08/0.73 | 16.66/0.68 | 16.89/0.74 |
| Forest         | 13.32/0.72 | 16.25/0.83 | 16.81/0.91 | 19.05/0.93 |
| Town           | 12.49/0.51 | 10.50/0.48 | 12.77/0.49 | 12.99/0.52 |

5. Conclusion
In this paper, a weighted variation regularization image dehazing based on non-local prior is proposed. The non-local prior and variational regularization are used to estimate and optimize the transmission of the image, and the haze-free image was restored from the atmospheric scattering model by using the optimized transmission. The experimental results show that the algorithm has better dehazing effect when the image contains thick haze and effectively avoids color distortion. Further research is needed for noise reduction in the image dehazing process.

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