Leptogenesis in a seesaw model
with Fritzsch type lepton mass matrices

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Abstract

We investigate how the baryon asymmetry of our universe via leptogenesis can be achieved within the framework of the seesaw model with Fritzsch type lepton mass matrices proposed by Fukugita \textit{et al.} We study the cases with CP-violating phases in charged lepton Yukawa matrix, however, with and without Dirac neutrino Yukawa phases. We consider both flavor independent and flavor dependent leptogenesis, and demonstrate how they lead to different amounts of lepton asymmetries in detail. In particular, it is shown that flavor dependent leptogenesis in this model can be worked out only when the CP phases in Dirac neutrino Yukawa matrix become zero at the GUT scale. In addition to the CP phases, for successful leptogenesis in the model it is required that the degeneracy of the heavy Majorana neutrino mass spectrum should be broken and we also show that the breakdown of the degeneracy can be radiatively induced.

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I. INTRODUCTION

Recent precise neutrino experiments appear to show robust evidence for the neutrino oscillation. The present neutrino experimental data \[1, 2, 3\] exhibit that the atmospheric neutrino deficit points toward a maximal mixing between the tau and muon neutrinos. However, the solar neutrino deficit favors a not-so-maximal mixing between the electron and muon neutrinos. In addition, although we do not have yet any firm evidence for the neutrino oscillation arisen from the 1st and 3rd generation flavor mixing, there is a bound on the mixing element \(U_{e3}\) from CHOOZ reactor experiment, \(|U_{e3}| < 0.2\) \[4\]. Although neutrinos have gradually revealed their properties in various experiments since the historic Super-Kamiokande confirmation of neutrino oscillations \[1\], properties related to the leptonic CP violation are completely unknown yet. To understand the neutrino mixings observed in various oscillation experiments is one of the most interesting issues in particle physics.

The phenomenon of lepton flavor mixing can be described by a \(3 \times 3\) unitary matrix \(U\), the Maki-Nakagawa-Sakata (MNS) matrix \[5\], which contains three mixing angles (\(\theta_{12}, \theta_{23}, \theta_{13}\)) and three CP-violating phases (\(\delta, \rho, \sigma\)). Four of these six parameters (i.e., \(\theta_{12}, \theta_{23}, \theta_{13}\) and \(\delta\)), together with two neutrino mass-squared differences (\(\Delta m_{21}^2 \equiv m_2^2 - m_1^2\) and \(\Delta m_{32}^2 \equiv m_3^2 - m_2^2\)), can be extracted from the measurements of neutrino oscillations. At present, a global analysis of current experimental data yields \[6\]

\[
0.26 \leq \sin^2 \theta_{12} \leq 0.40, \quad 0.34 \leq \sin^2 \theta_{23} \leq 0.67, \quad \sin^2 \theta_{13} \leq 0.050
\]
\[
2.0 \leq \Delta m_{\text{Atm}}^2[10^{-3}\text{eV}^2] \leq 2.8, \quad 7.1 \leq \Delta m_{\text{Sol}}^2[10^{-5}\text{eV}^2] \leq 8.3,
\]

at the 3\(\sigma\) confidence level, but the Dirac CP-violating phase \(\delta\) is entirely unrestricted at present. More accurate neutrino oscillation experiments are going to determine the size of \(\theta_{13}\), the sign of \(\Delta m_{32}^2\) and the magnitude of \(\delta\). The proposed precision experiments for the tritium beta decay \[7\] and the neutrinoless double-beta decay \[8\] will help to probe the absolute mass scale of three light neutrinos and to constrain the Majorana CP-violating phases \(\rho\) and \(\sigma\).

To understand the neutrino mass spectrum and the neutrino mixing pattern indicated by Eq. (1), Fukugita, Tanimoto and Yanagida (FTY) have proposed \[9\] an interesting ansätze to account for current neutrino oscillation data by combining the Fritzsch texture \[10\] in the seesaw mechanism \[11\] with three degenerate right-handed Majorana neutrinos. In the
FTY ansätze, charged-lepton and Dirac neutrino Yukawa coupling matrices are also of the Fritsch texture, but the heavy Majorana neutrino mass $M_R = MI$ with $I$ being the $3 \times 3$ unit matrix ($i.e.$, $M_i = M$ for $i = 1, 2$ and $3$) has been assumed. Then the effective (left-handed) neutrino mass matrix $m_{\text{eff}}$ in the FTY ansatz is no more of the Fritsch form. Ref. [9] has shown that the FTY ansätze is compatible very well with current experimental data on solar and atmospheric neutrino oscillations. And also there have been many phenomenological analysis [12] of FTY model compatible with current neutrino data.

It is also worthwhile to examine if baryon asymmetry of our universe (BAU) [13] can be viable in the context of FTY model. In this work, we study how BAU via leptogenesis can be achieved within the framework of FTY model with possible CP-violating phases in Dirac neutrino Yukawa matrix and charged lepton Yukawa matrix. We consider both flavor independent and dependent leptogenesis, and show how they lead to different amounts of lepton asymmetries in detail. As will be shown later, in particular, flavor dependent leptogenesis in the FTY model can be worked only when the CP phases in Dirac neutrino Yukawa matrix becomes zero at GUT scale. In addition to the CP phases, for successful leptogenesis in the FTY model, it is required that the degeneracy of the heavy Majorana neutrino mass spectrum should be broken and we show that it can be radiatively induced.

II. FTY MODEL REALIZED AT GUT SCALE AND CP VIOLATION

Let us begin by considering the Standard Model (SM) of the seesaw mechanism, which is given by

$$\mathcal{L} \supset e^{cT} Y_L L \cdot \varphi + N^{cT}_R Y_{\nu} \nu L - \frac{1}{2} N^{cT}_R M_R N^c_R + h.c. \tag{2}$$

where the family indices have been omitted and $L_{\alpha} (\alpha = e, \mu, \tau)$ stand for the left-handed lepton doublets, $(e^{c}_R)_{\alpha}$ are the charged lepton singlets, $N_{R\alpha}$ the right-handed neutrino singlets and $\varphi$ is the Higgs doublet fields. In the above lagrangian, $Y_L$ and $Y_{\nu}$ are the $3 \times 3$ charged lepton and neutrino Dirac Yukawa matrices, respectively. After spontaneous symmetry breaking, the seesaw mechanism leads to a following effective light neutrino mass term,

$$m_{\text{eff}} = -Y^{cT}_\nu M^{-1}_R Y_{\nu} \nu^2, \tag{3}$$

where $\nu$ is a vacuum expectation value of the Higgs field $\varphi$ with $\nu \simeq 174$ GeV.
Let us assume that the charged-lepton mass matrix $m_l = v Y_l$ and the Dirac neutrino mass matrix $m_D = v Y_\nu$ are both symmetric and of the Fritzsch texture, at the high energy scale, where

$$
Y_l = \begin{pmatrix}
0 & A_t e^{i \phi_A} & 0 \\
A_t e^{i \phi_A} & 0 & B_t e^{i \phi_B} \\
0 & B_t e^{i \phi_B} & C_l
\end{pmatrix}
\quad Y_\nu = \begin{pmatrix}
0 & A_\nu e^{i \phi_A} & 0 \\
A_\nu e^{i \phi_A} & 0 & B_\nu e^{i \phi_B} \\
0 & B_\nu e^{i \phi_B} & C_\nu
\end{pmatrix}.
$$

Here $A_{l(\nu)}, B_{l(\nu)}, C_{l(\nu)}, \phi_A, \phi_B, \phi_A$ and $\phi_B$ are taken to be all real and positive without loss of generality and then only the off-diagonal elements of $Y_{l(\nu)}$ are complex. Following the FTY ansatz, we take the right-handed Majorana neutrino mass matrix to be,

$$
M_R = M I.
$$

In the basis where the charged lepton Yukawa coupling matrix and the mass matrix of the right-handed neutrino singlets are diagonal,

$$
e_R \rightarrow V_R e_R, \quad \nu_L \rightarrow V_L \nu_L,
$$

and the Yukawa matrices of $Y_l$ and $Y_\nu$ transform as

$$
Y_l \rightarrow V_R^T Y_l V_L, \quad Y_\nu \rightarrow Y_\nu V_L
$$

where $V_{R(L)}$ are the unitary matrices to diagonalize $Y_l$. Since the charged-lepton Yukawa matrix $Y_l$ is symmetric in the present framework, only one unitary matrix, $V_L = V_R \equiv V$, is sufficient to diagonalize $Y_l$. Then, the transformed Yukawa matrices $Y_l'$ and $Y_\nu'$ are given by

$$
Y_l' = V^T Y_l V = \begin{pmatrix}
Y_e & 0 & 0 \\
0 & Y_\mu & 0 \\
0 & 0 & Y_\tau
\end{pmatrix}, \quad Y_\nu' = \begin{pmatrix}
0 & A_\nu e^{i \phi_A} & 0 \\
A_\nu e^{i \phi_A} & 0 & B_\nu e^{i \phi_B} \\
0 & B_\nu e^{i \phi_B} & C_\nu
\end{pmatrix} V.
$$

In addition, $Y_l$ can be decomposed as $Y_l = P^T \hat{Y}_l P$ with $P = \text{diag}(e^{i (\phi_A - \phi_B)}, e^{i \phi_B}, 1)$ and

$$
\hat{Y}_l = \begin{pmatrix}
0 & A_l & 0 \\
A_l & 0 & B_l \\
0 & B_l & C_l
\end{pmatrix}.
$$
Then, the mass matrix $Y_l$ can finally be diagonalized by the unitary matrix $V = PO$ where the elements of the orthogonal matrix $O$ can be presented in terms of two parameters $x \equiv y_e/y_\mu$ and $y \equiv y_\mu/y_\tau$ as follows,

$$
O_{11} = + \left[ \frac{1 - y}{(1 + x)(1 - xy)(1 - y + xy)} \right]^{1/2}, \quad O_{12} = -i \left[ \frac{x(1 + xy)}{(1 + x)(1 + y)(1 - y + xy)} \right]^{1/2}, \\
O_{13} = + \left[ \frac{xy^3(1 - x)}{(1 - xy)(1 + y)(1 - y + xy)} \right]^{1/2}, \quad O_{21} = + \left[ \frac{x(1 - y)}{(1 + x)(1 - xy)} \right]^{1/2}, \\
O_{22} = +i \left[ \frac{1 + xy}{(1 + x)(1 + y)} \right]^{1/2}, \quad O_{23} = + \left[ \frac{y(1 - x)}{(1 - xy)(1 + y)} \right]^{1/2}, \\
O_{31} = - \left[ \frac{xy(1 - x)(1 + xy)}{(1 + x)(1 - xy)(1 - y + xy)} \right]^{1/2}, \quad O_{32} = -i \left[ \frac{y(1 - x)(1 - y)}{(1 + x)(1 + y)(1 - y + xy)} \right]^{1/2}, \\
O_{33} = + \left[ \frac{(1 - y)(1 + xy)}{(1 - xy)(1 + y)(1 - y + xy)} \right]^{1/2}.
$$

(10)

The Dirac neutrino Yukawa matrix can also be written in the basis we consider as,

$$
Y'_\nu = B_{\nu} \begin{pmatrix}
0 & \omega e^{i\phi_A} & 0 \\
\omega e^{i\phi_A} & 0 & e^{i\phi_B} \\
0 & e^{i\phi_B} & \kappa
\end{pmatrix} \begin{pmatrix}
e^{i(\phi_A - \phi_B)} & 0 & 0 \\
0 & e^{i\phi_B} & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
O_{11} & O_{12} & O_{13} \\
O_{21} & O_{22} & O_{23} \\
O_{31} & O_{32} & O_{33}
\end{pmatrix}
$$

(11)

where the parameters $\omega$ and $\kappa$ are defined by

$$
\omega \equiv \frac{A_{\nu}}{B_{\nu}}, \quad \kappa \equiv \frac{C_{\nu}}{B_{\nu}}.
$$

(12)

Then, we are led to the effective light neutrino mass matrix as follows,

$$
m_{\text{eff}} = -v^2 M^{-1} Y^T_{\nu} Y'_{\nu} = \frac{-B_{\nu}^2 v^2}{M} O \begin{pmatrix}
e^{2i(\phi_A + \phi_B - \phi_B)} \omega^2 & 0 & e^{i(\phi_A + \phi_B + \phi_A - \phi_B)} \omega \\
0 & e^{2i\phi_B} (e^{2i\phi_B} + e^{2i\phi_A} \omega^2) & e^{i(\phi_B + \phi_B)} \kappa \\
e^{i(\phi_A + \phi_B + \phi_A - \phi_B)} \omega & e^{i(\phi_B + \phi_B)} \kappa & e^{2i\phi_B} + \kappa^2
\end{pmatrix}.
$$

(13)

Concerned with CP violation, we notice from Eq. (13) that the CP phases $\phi_{A,B}$ coming from $Y_{\nu}$ as well as the CP phases $\varphi_{A,B}$ from $Y_l$ obviously take part in low energy CP violation because low energy CP violation is associated with the form $Y'^T_{\nu} Y'_{\nu}$. On the other hand, flavor independent leptogenesis is associated with the form given by

$$
Y'_{\nu} Y'^T_{\nu} = Y_{\nu} Y^T_{\nu} = B_{\nu}^2 \begin{pmatrix}
\omega^2 & 0 & \omega e^{i(\phi_A - \phi_B)} \\
0 & 1 + \omega^2 & \kappa e^{i\phi_B} \\
\omega e^{-i(\phi_A - \phi_B)} & \kappa e^{-i\phi_B} & 1 + \kappa^2
\end{pmatrix}.
$$

(14)
From this, we find that only the phases $\phi_A$, $\phi_B$ in $Y_\nu$ take part in leptogenesis. However, the situation is changed when we consider the scenario of \textit{flavored leptogenesis} \cite{14}, where flavor effects become important. As will be shown later, the magnitude of CP asymmetry in the scenario of flavored leptogenesis crucially depends on the following quantity

\[
\text{Im}\{[Y_\nu, Y_\nu^\dagger]_{jk}(Y_\nu)_{j\alpha}(Y_\nu)^\dagger_{\alpha k}\}
= \text{Im}[Y_\nu, Y_\nu^\dagger]_{jk}\text{Re}[(Y_\nu)_{j\alpha}(Y_\nu)^\dagger_{\alpha k}] + \text{Re}[Y_\nu, Y_\nu^\dagger]_{jk}\text{Im}[(Y_\nu)_{j\alpha}(Y_\nu)^\dagger_{\alpha k}].
\]

This quantity implies that both CP phases in $Y_\nu$ and $Y_l$ take part in flavored leptogenesis. Contrary to the case of flavor independent leptogenesis, flavored leptogenesis can be realized without the CP phases appeared in $Y'_\nu Y'_\nu^\dagger$ as long as the phases $\varphi_{A,B}$ are non-zero. In addition, we expect that in the FTY model, there may exist a connection between flavored leptogenesis with low energy CP violation, contrary to the observation from the generic seesaw model with three generations \cite{15}.

\section{III. CONFRONTING WITH LOW-ENERGY NEUTRINO DATA}

Before discussing how to achieve leptogenesis in FTY model, we first examine if it is consistent with low energy neutrino data. To do so, we need renormalization group (RG) evolution \cite{16, 17, 18} of neutrino Dirac-Yukawa matrix and heavy Majorana neutrino masses with the FTY forms from the GUT scale to the seesaw scale by numerically solving all the relevant RG equations presented in Ref. \cite{17}. For our purpose, we consider two cases, one is the case with non-vanishing CP phases in both $Y_\nu$ and $Y_l$, $\phi_{A,B} \neq 0$ and $\varphi_{A,B} \neq 0$, and the other is the case that only the phases $\varphi_{A,B}$ are non-zero, \textit{i.e.} $\phi_{A,B} = 0$ and $\varphi_{A,B} \neq 0$. Then, we solve the RGE’s by varying input values of the parameter set \{\[B_\nu, \kappa, \omega, \varphi_A, \varphi_B, \phi_A, \phi_B, M\]\}, and \{\[B_\nu, \kappa, \omega, \varphi_A, \varphi_B, \varphi_A, \varphi_B, M\]\} given at the GUT scale, respectively, and determine the parameter set which is in consistent with low energy neutrino data. In our numerical calculation, we use five experimental results for neutrino mixing parameters and mass squared differences at 3$\sigma$ \cite{6} by Eq. (11) as inputs.

In Fig. (11) the two figures of upper panel exhibit how the parameter $\omega$ (left-panel) and the mixing angle $\theta_{23}$ (right-panel) are related with the phase $\varphi_B$ for the case of $\phi_{A,B} = 0$ at the GUT scale. In this case we find that the parameters $\kappa$ and $\omega$ strongly depend on the phase $\varphi_B$, not $\varphi_A$. The two figures of lower panel present the predictions of $\theta_{23}$ (left-panel) and
\( \theta_{12} \) (right-panel) in terms of \( \omega \). The horizontal lines correspond to the bounds of present experimental values for \( \theta_{23} \) and \( \theta_{12} \) given at 3\( \sigma \) range, Eq. (1), respectively. From the results, it is interesting to see that most predictions of \( \theta_{23} \) lie below 45\( ^{\circ} \). In fact, the experimental result for \( \theta_{12} \) gives at 3\( \sigma \) constraint the values of parameter 0.4 \( \lesssim \omega \lesssim 1.05 \). We find that the constraint of \( \omega \) prevents the prediction of \( \theta_{23} \) from lying above 45\( ^{\circ} \).

Fig. 2 shows how the mixing angle \( \theta_{13} \) is predicted in terms of the parameters \( \kappa, \omega \) (upper-panel) and \( \varphi_{B} \) (lower-panel) whose sizes are constrained, as in Fig. 1, by the experimental results of \( \theta_{23} \) and \( \theta_{12} \). In each figures, we draw the current reactor experimental upper bound on \( \theta_{13} \). We see from Fig. 2 that very small values of \( \theta_{13} \) are not predicted in FTY model. In

![Graphs showing mixing angles and phases](image)

**FIG. 1:** (Upper-panel:) Left-figure represents that the parameter \( \omega \) over the charged lepton phase \( \varphi_{B} \). Right-figure represents the relation between the mixing angle \( \theta_{23} \) and the charged-lepton phase \( \varphi_{B} \). Here the horizontal dotted lines represent the experimental lower and upper bounds of the mixing angle \( \theta_{23} \). (Lower-panel:) Left-figure shows the mixing angle \( \theta_{23} \) as a function of the parameter \( \omega \). Here the horizontal dotted lines represent the experimental upper and lower bounds of the mixing angle \( \theta_{23} \). Right-figure shows the mixing angle \( \theta_{12} \) as a function of the parameter \( \omega \).
lower right panel, we present the predicted regions for $\theta_{13}$ and $\theta_{23}$ in FTY model.

Fig. 3 shows the parameter spaces allowed by the $3\sigma$ experimental constraints given in Eq. (1) for $10^6 \lesssim M[GeV] \lesssim 10^{12}$ when the CP phases $\phi_A$ and $\phi_B$ are turned on at the GUT scale. The upper left panel plots the correlation between $\kappa$ and $\omega$, and the upper right panel presents the predictions of $\theta_{23}$ in terms of $\phi_B$. The lower left (right) panel shows the prediction of $\theta_{23}(\theta_{12})$ in terms of $\omega$. Contrary to the previous case with vanishing CP phases $\phi_{A,B}$, the values above $45^\circ$ for $\theta_{23}$ are possibly predicted.

**FIG. 2**: In the case of $\phi_{A,B} = 0$, $\varphi_{A,B} \neq 0$ at the GUT scale, the parameter regions allowed by the $3\sigma$ experimental constraints for $10^6 \lesssim M[GeV] \lesssim 10^{12}$. (Upper-panel:) Left-figure represents that the parameter $\kappa$ over the mixing angle $\theta_{23}$ and right-figure $\omega$ over $\theta_{23}$, where the vertical dotted line indicates the upper bound of $\theta_{13}$. (Lower-panel:) Left-figure shows the charged-lepton phase $\varphi_B$ over the mixing angle $\theta_{13}$, and the vertical line corresponds to the upper bound on $\theta_{13}$. Right-figure shows the predicted parameter space for $\theta_{13}$ and $\theta_{23}$ in FTY model and the horizontal dotted lines indicate the experimental upper bound on $\theta_{13}$ and the vertical dotted line represents the experimental lower and upper bound on $\theta_{23}$.
Similar to Fig. 2, we present in Fig. 4 how the mixing angle $\theta_{13}$ is predicted in terms of the parameters $\kappa$, $\omega$ (upper-panel) and $\varphi_B$ (down-panel), whose sizes are constrained, as in Fig. 3, by the experimental results of $\theta_{23}$ and $\theta_{12}$. In each figures, we draw the current reactor experimental upper bound on $\theta_{13}$. We see from Fig. 4 that very small values of $\theta_{13}$ are allowed in FTY model, which is contrary to the previous case with $\phi_{A,B} = 0$. In lower right panel, we present the predicted regions for $\theta_{13}$ and $\theta_{23}$ in FTY model.

![Fig. 3: (Upper-panel:) Left-figure represents the allowed parameter space, $\kappa$ vs. $\omega$. Right-figure represents the mixing angle $\theta_{23}$ as a function of $\varphi_B$. (Lower-panel:) Left-figure shows how the mixing angle $\theta_{23}$ predicted in terms of $\omega$. Right-figure shows how $\theta_{12}$ predicted in terms of $\omega$. Here the horizontal dotted lines represent the experimental upper and lower bound of the mixing angle $\theta_{23}$ and $\theta_{12}$, respectively.](image-url)
IV. RADIATIVELY INDUCED RESONANT LEPTOGENESIS

It is well known that if heavy Majorana neutrinos are exact degenerate as in FTY model, the generated lepton asymmetry is zero [19]. A non-zero leptonic asymmetry can be generated if and only if the $CP$-odd invariant $\Delta_{CP} = \text{Im} \text{Tr}[Y_\nu Y_\nu^\dagger M_R M_R^\dagger M_R^* Y_\nu^T M^R]$ does not vanish [20]. The exact mass degeneracy of three right-handed neutrinos implies that the $CP$-odd invariant

$$\Delta_{CP} = 2 \sum_{i<j} \left\{ M_i M_j (M_j^2 - M_i^2) \text{Im}[H_{ij}] \text{Re}[H_{ij}] \right\}, \quad H \equiv Y_\nu Y_\nu^\dagger,$$

(16)

which is relevant for leptogenesis [21], is actually vanishing. In order to accommodate leptogenesis, it requires not only $M_i \neq M_j$ but also $\text{Im}[H_{ij}] \text{Re}[H_{ij}] \neq 0$. Even if we have exactly degenerate heavy Majorana neutrinos at a certain high energy scale, it is likely that some splitting in the mass spectrum could be induced at a different scale through RG running effect. If this is the case, we will get the splittings of heavy Majorana neutrino masses i.e. a slightly broken $SO(3)$ symmetry in the right-hand sector with $|M_1| \simeq |M_2| \simeq |M_3|$. And

\[0.2\]

\[0.2\]

\[0.2\]

\[0.2\]

FIG. 4: The same as Fig. 2 except for $\phi_{A,B} \neq 0$ at the GUT scale.
the Dirac neutrino Yukawa matrix $Y_{\nu}$ is also modified by the same RG effect, which is very important to get non-zero $\text{Im}[H_{ij}]\text{Re}[H_{ij}] \neq 0$, as will be shown later.

Let us consider the evolution of the right-handed heavy Majorana neutrinos masses and the matrix $\Omega$ which diagonalizes the heavy Majorana mass matrix $M$ in lagrangian $\text{Eq. (2)}$, whose RGEs can be written by $\text{Ref. \[18\]}$:

\[
\frac{d}{dt}M = (Y'_{\nu}Y'^{\dagger}_{\nu})M + M(Y'_{\nu}Y'^{\dagger}_{\nu})^T, \tag{17}
\]
\[
\frac{d}{dt}\Omega = \Omega A, \tag{18}
\]

where $t = \frac{1}{16\pi^2}\ln(\mu/\Lambda)$ with renormalizable scale $\mu$ and degenerate seesaw scale $\Lambda$ and $Y'_{\nu}$ is the re-basing form in $\text{Eq. (8)}$. With the use of unitary transformation $N_j \rightarrow \Omega_{ji}N_i$, one can obtain

$$\Omega^T M \Omega = \text{diag}(M_1, M_2, M_3). \tag{19}$$

Since $(d/dt)\Omega = \Omega A$, $A$ satisfies $A + A^\dagger = 0$, and then from Eq. (19) we can obtain:

$$\frac{dM_{ij}\delta_{ij}}{dt} = A_{ij}^T M_j + M_i A_{ij} + \{\Omega^T([(Y'_{\nu}Y'^{\dagger}_{\nu})M + M(Y'_{\nu}Y'^{\dagger}_{\nu})^T]\Omega)_{ij}. \tag{20}$$

Thus, the RG evolutions for the right-handed heavy Majorana neutrino masses are governed by the diagonal part in the above equation:

$$\frac{dM_i}{dt} = 2M_i(Y_{\nu}Y^\dagger_{\nu})_{ii}, \quad \text{with } Y_{\nu} = \Omega^T Y'_{\nu} \tag{21}$$

and the anti-hermitian property of the imaginary part of the matrix $A$ leads to

$$\text{Im}[A_{ii}] = 0. \tag{22}$$

In addition, the off-diagonal part in Eq. (20) leads to

$$A_{jk} = \frac{M_k + M_j}{M_k - M_j} \text{Re}[(Y_{\nu}Y^\dagger_{\nu})_{jk}] + i\frac{M_j - M_k}{M_j + M_k} \text{Im}[(Y_{\nu}Y^\dagger_{\nu})_{jk}] = -A^*_{kj}, \quad (j \neq k). \tag{23}$$

The RG equation for the Dirac neutrino Yukawa matrix is given by

$$\frac{dY_{\nu}}{dt} = Y_{\nu}[(T - \frac{3}{4}g_Y^2 - \frac{9}{4}g_2^2) - \frac{3}{2}(Y_l^\dagger Y_l - Y_{\nu}^\dagger Y_{\nu})] + A^T Y_{\nu}, \tag{24}$$

\footnote{Actually, Ref. $[18]$ follows bottom-up approach, that is, from electroweak scale to seesaw scale. On the contrary, we apply top-down approach.}
where $T = Tr(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_\nu^\dagger Y_\nu + Y_l^\dagger Y_l)$, $Y_u$ ($Y_d$) and $Y_l$ are the Yukawa matrices for up-type (down-type) quarks and charged leptons and $g_{2,Y}$ are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants. The RG evolution for the quantity $H$ relevant for leptogenesis can be written as

$$\frac{d}{dt}H = 2Y_\nu\{Q + P_\nu\}Y_\nu^\dagger + A^T H + HA^*, \quad (25)$$

where

$$Q = T - \frac{3}{4}g_2^2 - \frac{9}{4}g_1^2, \quad P_\nu = -\frac{3}{2}(Y_{l_1}^\dagger Y_l - Y_{\nu_1}^\dagger Y_\nu).$$

From Eq. (22), we see that there exists a singularity in $A_{jk}$. The singularity in $A_{jk}$ can be eliminated with the help of an appropriate rotation between degenerate heavy Majorana neutrino states. Such a rotation does not change any physics and it is equivalent to absorb the rotation matrix $R$ into the neutrino Dirac Yukawa matrix $Y_\nu$,

$$Y_\nu \rightarrow \tilde{Y}_\nu = RY_\nu, \quad (26)$$

where the matrix $R$ is an $3 \times 3$ orthogonal matrix which can be parameterized in terms of angles $\theta_i$ as $R(\theta_i, \theta_j, \theta_k) = R(\theta_i) \cdot R(\theta_j) \cdot R(\theta_k)$

$$R(\theta_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{pmatrix}, \quad R(\theta_2) = \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix}, \quad R(\theta_3) = \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (27)$$

where $s_i \equiv \sin \theta_i$, $c_i \equiv \cos \theta_i$. Then, the singularity in real part of $A_{jk}$ can be indeed removed when the rotation angles $\theta_i$ are taken to be satisfied with the condition,

$$\text{Re}[(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{jk}] = 0 \quad \text{for any pair } j, k \text{ corresponding to } M_j = M_k. \quad (28)$$

At the degeneracy scale of $M_R$ there is a freedom to rotate the right-handed neutrino fields $N_{1,2,3}$ with a real orthogonal matrix that does not change $M_R$, but rotates $Y_\nu$ to the appropriate basis, which allows the use of an $SO(3)$ transformation to remove the off-diagonal elements of $\text{Re}[H]$, and thus we can obtain a matrix $\tilde{H}$ satisfying the condition Eq. (28) as follows,

$$\tilde{H} \equiv (\tilde{Y}_\nu \tilde{Y}_\nu^\dagger) = RHR^T = B_\nu^2 \begin{pmatrix} \tilde{h}_{11} & i\text{Im}[\tilde{h}_{12}] & i\text{Im}[\tilde{h}_{13}] \\ -i\text{Im}[\tilde{h}_{12}] & \tilde{h}_{22} & i\text{Im}[\tilde{h}_{23}] \\ -i\text{Im}[\tilde{h}_{13}] & -i\text{Im}[\tilde{h}_{23}] & \tilde{h}_{33} \end{pmatrix}, \quad (29)$$
where \( \tilde{h}_{jj} \) and \( \text{Im}[\tilde{h}_{jk}] \) \((j \neq k = 1, 2, 3)\) are given by
\[
\tilde{h}_{11} = \omega^2 + \sin^2 \theta_3 + q_1 \tan \theta_2,
\]
\[
\tilde{h}_{22} = (\omega^2 + \cos^2 \theta_3) \cos^2 \theta_1 + \{1 + \kappa^2 - q_1 \tan \theta_2\} \sin^2 \theta_1 + q_2 \frac{\sin 2\theta_1}{\cos \theta_2},
\]
\[
\tilde{h}_{33} = (\omega^2 + \cos^2 \theta_3) \sin^2 \theta_1 + \{1 + \kappa^2 - q_1 \tan \theta_2\} \cos^2 \theta_1 - q_2 \frac{\sin 2\theta_1}{\cos \theta_2},
\]
\[
\text{Im}[\tilde{h}_{12}] = \cos \theta_3 \{\omega \sin(\phi_A - \phi_B) \sin \theta_1 - \kappa \sin \phi_B \cos \theta_1 \sin \theta_2\}
+ \sin \theta_3 \{\kappa \sin \phi_B \sin \theta_1 + \omega \sin(\phi_A - \phi_B) \cos \theta_1 \sin \theta_2\},
\]
\[
\text{Im}[\tilde{h}_{13}] = \cos \theta_3 \{\omega \sin(\phi_A - \phi_B) \cos \theta_1 + \kappa \sin \phi_B \sin \theta_1 \sin \theta_2\}
+ \sin \theta_3 \{\kappa \sin \phi_B \cos \theta_1 - \omega \sin(\phi_A - \phi_B) \sin \theta_1 \sin \theta_2\},
\]
\[
\text{Im}[\tilde{h}_{23}] = \cos \theta_3 \{\kappa \sin \phi_B \cos \theta_3 - \omega \sin(\phi_A - \phi_B) \sin \theta_3\}. \tag{30}
\]

Here, the parameters \( q_1 \) and \( q_2 \) are given by
\[
q_1 = \kappa \cos \phi_B \sin \theta_3 + \omega \cos(\phi_A - \phi_B) \cos \theta_3,
\]
\[
q_2 = \kappa \cos \phi_B \cos \theta_3 - \omega \cos(\phi_A - \phi_B) \sin \theta_3. \tag{31}
\]

The angle \( \theta_i \) in the real \( 3 \times 3 \) orthogonal matrix \( R \) and CP-violating parameters \( \phi_A, \phi_B \) in the matrix \( Y_\nu \) make \( d\tilde{Y}/dt \) non-singular, \( i.e. \) when the degeneracy is exact, \( Y_\nu \) changes rapidly from its unperturbed form at \( t = 0 \) to a stable form that makes \( d\tilde{Y}_\nu/dt \) non-singular in Eq. \( \text{(24)} \). In the case of exact degenerate heavy Majorana neutrinos, \( i.e. \), \( M_R = M_I \), the rotation matrix \( R \) must be used to remove singularities at the degeneracy scale, therefore \( \theta_i \) \((i = 1, 2, 3)\) is no longer free parameters, \( i.e. \), it is constrained by the conditions Eq. \( \text{(28)} \) from which we can obtain the following relations,
\[
\tan 2\theta_1 = \frac{2q_2}{\cos \theta_2(\omega^2 + \cos^2 \theta_3 + q_1 \tan \theta_2 - 1 - \kappa^2)},
\]
\[
\tan 2\theta_2 = \frac{2q_1}{\omega^2 + \sin^2 \theta_3 - 1 - \kappa^2}, \quad \text{(or} \quad \tan \theta_2 = -\frac{\sin 2\theta_3}{2q_2} \text{)}, \tag{32}
\]
which show the initial stable conditions of angles at the GUT scale. Note that \( \theta_1, \theta_2 \) and \( \theta_3 \) have scale dependence when RG running from GUT to seesaw scale, Eq. \( \text{(18)} \).

\textbf{A. Flavor Independent Leptogenesis}

In a basis where the right-handed Majorana neutrino mass matrix is diagonal, ignoring flavor effects in the Boltzmann evolution of charged leptons, the CP asymmetry generated
through the interference between tree and one-loop diagrams of heavy singlet Majorana neutrino decay is given by \[22, 23\]:

\[
\varepsilon_i = \frac{\sum_\alpha [\Gamma(N_i \to l_\alpha \varphi) - \Gamma(N_i \to \bar{l}_\alpha \varphi^\dagger)]}{\sum_\alpha [\Gamma(N_i \to l_\alpha \varphi) + \Gamma(N_i \to \bar{l}_\alpha \varphi^\dagger)]} = \frac{1}{8\pi (Y_\nu Y_\nu^\dagger)_{ii}} \sum_{j \neq i} \text{Im}\{ (Y_\nu Y_\nu^\dagger)_{ij}^2 \} g\left( \frac{M_j^2}{M_i^2} \right), \quad (33)
\]

where the function \(g(x)\) is given by

\[
g(x) = \sqrt{x} \left[ \frac{1}{1-x} + 1 - (1+x) \ln \frac{1+x}{x} \right] \quad (34)
\]

with \(x = M_j^2/M_i^2\). In the case that the mass splitting of the heavy Majorana neutrinos is very small, the CP asymmetries \(\varepsilon_i\) can be simplified by \[23, 24\] as

\[
\varepsilon_i \simeq \frac{\text{Im}\{ (Y_\nu Y_\nu^\dagger)_{ij}^2 \}}{16\pi (Y_\nu Y_\nu^\dagger)_{ii} \delta_{N}^{ij}} \left( 1 + \frac{\Gamma_j^2}{4M_j^2\delta_{N}^{jj}} \right)^{-1}, \quad \text{with} \quad \Gamma_j = \frac{(Y_\nu Y_\nu^\dagger)_{jj} M_j}{8\pi} \quad (i \neq j = 1, 2, 3), \quad (35)
\]

where \(j\) denotes a generation number and \(\Gamma_j\) is the decay width of the \(j\)th-generation right-handed neutrino. We notice from Eq. \(35\) that \(\varepsilon_i\) is resonantly enhanced when \(\Gamma_j \simeq (M_i^2 - M_j^2)/M_i\). Here, the parameter \(\delta_{N}^{jk} (= 1 - |M_k|/|M_j| \ll 1)\) reflecting the mass splitting of the degenerate heavy Majorana neutrinos is governed by the following RGE derived from Eq. \(20\),

\[
\frac{d\delta_{N}^{jk}}{dt} = 2(1 - \delta_{N}^{ik})[\tilde{H}_{jj} - \tilde{H}_{kk}], \quad (36)
\]

which represents that radiative corrections induce mass-splittings proportional to the neutrino couplings. In the limit \(\delta_{N}^{jk} \ll 1\), the leading-log approximation for \(\delta_{N}^{jk}\) can be easily found to be

\[
\delta_{N}^{jk} \simeq 2[\tilde{H}_{jj} - \tilde{H}_{kk}] \cdot t. \quad (37)
\]

In order for Eq. \(33\) to give successful leptogenesis, not only the degeneracy of right-handed neutrinos should be broken but also the non-vanishing \(\text{Im}\{ (Y_\nu Y_\nu^\dagger)_{ik}^2 \}\) is required at seesaw scale \(M\).

To see how leptogenesis can successfully be achieved, let us first consider the case that \(\phi_A = \phi_B = 0\) in \(Y_\nu\) Eq. \(4\) at the GUT scale, while keeping CP phases arisen from the charged-lepton Yukawa matrix \(Y_l\) which move to \(Y_\nu\) through re-basing, \(i.e., \tilde{Y}_\nu = RY'_\nu = RY_\nu V\). In this case, the off-diagonal elements of \(\tilde{H} \equiv (\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)\) becomes zero, so that CP
asymmetry could not be generated. However, the RG effects mainly due to $Y_\tau$ lead to non-vanishing off-diagonal elements in $\tilde{H}_{jk}$, whose forms are approximately given by

$$\text{Re}[\tilde{H}_{jk}(t)] \simeq -\frac{3}{2}y_\tau^2 \text{Re}[\tilde{Y}_\nu^\dagger j3(\tilde{Y}_\nu^\dagger)_{3k}] \cdot t,$$

$$\text{Im}[\tilde{H}_{jk}(t)] \simeq -3y_\tau^2 \text{Im}[\tilde{Y}_\nu^\dagger j3(\tilde{Y}_\nu^\dagger)_{3k}] \cdot t. \quad (38)$$

From these results, we see that CP-violating effects are induced by RG corrections due to the charged-lepton Yukawa couplings, which can play a crucial role in leptogenesis [25]. With the help of Eqs. (33,38), the CP-asymmetry for each heavy Majorana neutrino is given as

$$\varepsilon_i \simeq \frac{9y_\tau^4}{512\pi^3 \tilde{H}_{ii}} \cdot \ln \left( \frac{M_i}{\Lambda} \right) \sum_j \text{Re}[\tilde{Y}_\nu^j j3(\tilde{Y}_\nu^j)_{3i}] \text{Im}[\tilde{H}_{ji}] \tilde{H}_{jj} - \tilde{H}_{ii}. \quad (39)$$

Now, let us consider the case that $\phi_A \neq 0$ and $\phi_B \neq 0$ of $Y_\nu$ in Eq. (4) at the GUT scale. In this case, from Eq. (25), it is easy to find that $\text{Re}[\tilde{H}_{jk}(0)] = 0$ and $\text{Im}[\tilde{H}_{jk}(0)] \neq 0$, and thus RG effects on the off-diagonal elements $\tilde{H}_{jk}$ may be prominent in the real part as given by

$$\text{Re}[\tilde{H}_{jk}] \simeq -\frac{3}{2}y_\tau^2 \text{Re}[\tilde{Y}_\nu^j j3(\tilde{Y}_\nu^j)_{3k}] \cdot t. \quad (40)$$

With the help of Eqs. (33,40), the CP-asymmetry can be written as

$$\varepsilon_i \simeq \frac{3y_\tau^2}{32\pi \tilde{H}_{ii}} \sum_j \text{Re}[\tilde{Y}_\nu^j j3(\tilde{Y}_\nu^j)_{3i}] \text{Im}[\tilde{H}_{ji}] \tilde{H}_{ji} - \tilde{H}_{ii}. \quad (41)$$

In addition to $\varepsilon_i$, it is well-known that he baryon asymmetry depends on the parameters

$$K_i \equiv \frac{\tilde{m}_i}{m_*}, \quad \tilde{m}_i \equiv \frac{\tilde{H}_{ii}}{M_i} y^2, \quad (42)$$

where $m_* \simeq 10^{-3}eV$ is the so-called equilibrium neutrino mass and the effective neutrino mass $\tilde{m}_i$ is a measure of the strength of the coupling of $N_i$ to the thermal bath. After reprocessing by sphaleron transitions, the baryon asymmetry is related to the $(B-L)$ asymmetry by $Y_B = (12/37)(Y_{B-L})$ [26]. In flavor independent leptogenesis we are always in the strong wash-out regime with $K_i \gg 1$ and the right-handed neutrinos $N_i$’s are nearly in thermal equilibrium. Then, the generated $B-L$ asymmetry in the strong wash-out regime is given [27] as

$$Y_{B-L} \simeq \sum_i 0.3 \frac{\varepsilon_i}{g_*} \left( \frac{0.55 \times 10^{-3}eV}{\tilde{m}_i} \right)^{1.16}, \quad (43)$$

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where $g_*$ is the effective number of degrees of freedom. Therefore, the resulting baryon-to-photon ratio becomes

$$
\eta_B = 7.0394 \cdot Y_B,
$$

where

$$
Y_B \simeq 7.0394 \cdot \frac{0.55 \times 10^{-3} eV}{m_i}^{1.16}.
$$

(44)

Here the value 7.0394 comes out from the present ratio of entropy density to photon density [28].

**B. Flavor Dependent Leptogenesis**

Considering flavor effects, the CP asymmetry generated through the interference between tree and one-loop diagrams of heavy singlet Majorana neutrino $N_i$ decay is given for each lepton flavor $\alpha (= e, \mu, \tau)$ by [14, 29]:

$$
\varepsilon_\alpha^i = \frac{\Gamma(N_i \rightarrow l_\alpha \varphi) - \Gamma(N_i \rightarrow \bar{l}_\alpha \varphi^\dagger)}{\sum_\alpha [\Gamma(N_i \rightarrow l_\alpha \varphi) + \Gamma(N_i \rightarrow \bar{l}_\alpha \varphi^\dagger)]} = \frac{1}{8\pi (Y_\nu Y_\nu^\dagger)_{ii}} \sum_j \text{Im} \left\{ (Y_\nu Y_\nu^\dagger)_{ij} (Y_\nu)_{ia} (Y_\nu^*)^{ja} \right\} g \left( \frac{M_j^2}{M_i^2} \right),
$$

(45)

where $j$ runs over 1, 2 and 3 but $i \neq j$ and the function $g(M_j^2/M_i^2)$ is given by Eq. (34). We note that the total CP asymmetries $\varepsilon_i$ in Eq. (33) are obtained by summing over the lepton flavors $\alpha$. From Eq. (44), we see that leptogenesis reflecting flavor effects depends not only on $Y_\nu Y_\nu^\dagger$ but also on the individual $Y_\nu$, which makes it different from the conventional leptogenesis. The CP asymmetry $\varepsilon_i^\alpha$ is resonantly enhanced when the decay width of the $j$th-generation right-handed neutrino $\Gamma_j \simeq (M_j^2 - M_i^2)/M_i$. Once the initial values of $\varepsilon_\alpha^i$ are fixed, the final result of $\eta_B$ or $Y_B$ will be governed by a set of flavor-dependent Boltzmann equations including the (inverse) decay and scattering processes as well as the nonperturbative sphaleron interaction [14, 30, 31].

In the case of $\phi_A = \phi_B = 0$ at the GUT scale, the CP-asymmetry of a single flavor $\alpha$ including RG effects from high-energy scale to seesaw scale is approximately written as

$$
\varepsilon_i^\alpha \simeq \frac{3g^2}{32\pi H_{ii}} \sum_j \text{Im} \left[ (Y_\nu Y_\nu^*)_{ij} (Y_\nu^*)_{ja} \right] \frac{\text{Re} \left[ (Y_\nu)_{ia} (Y_\nu^*)^{ja} \right]}{H_{jj} - H_{ii}} + \frac{3g^2}{64\pi H_{ii}} \sum_j \text{Re} \left[ (Y_\nu Y_\nu^*)_{ij} (Y_\nu^*)_{ja} \right] \frac{\text{Im} \left[ (Y_\nu)_{ia} (Y_\nu^*)^{ja} \right]}{H_{jj} - H_{ii}}.
$$

(46)
Here, we note that \((Y_\nu)_{ia}(Y_{\nu}^\dagger)_{ai}\) contains the CP-phases \(\varphi_A\) and \(\varphi_B\) which may enhance the CP asymmetry.

On the other hand, in the case of \((\phi_A \neq 0, \phi_B \neq 0)\) at the GUT scale, the imaginary part in Eq. (45) including RG effects becomes

\[
\text{Im}\left\{\tilde{H}_{jk}(Y_{\nu})_{ja}(Y_{\nu}^\dagger)_{ka}\right\} \\
\simeq \text{Im}[\tilde{H}_{jk}]\frac{\text{Re}[(\tilde{Y}_{\nu})_{ja}(\tilde{Y}_{\nu}^\dagger)_{ka}]{\text{Im}[\tilde{H}_{ij}]} \\
\times \text{Re}[(\tilde{Y}_{\nu})_{ja}(\tilde{Y}_{\nu}^\dagger)_{ka}]}{\tilde{H}_{ii}} - \frac{3y_\tau^2}{2} \text{Re}[(\tilde{Y}_{\nu})_{ja}(\tilde{Y}_{\nu}^\dagger)_{ka}]}{t} \quad (j \neq k),
\]

where the first term in the second line dominates over the second one. We see from Eq. (47) that CP asymmetry can be generated without the CP phases \(\varphi_{A,B}\) in this case. Neglecting the second term in Eq. (47), the CP-asymmetry of a single flavor \(\alpha\) is approximately written as

\[
\varepsilon_{i}^{\alpha} \approx \frac{\pi}{2\tilde{H}_{ii} \cdot \text{ln}(M_i/\Lambda)} \sum_{j} \frac{\text{Re}[(\tilde{Y}_{\nu})_{ja}(\tilde{Y}_{\nu}^\dagger)_{ja}]{\text{Im}[\tilde{H}_{ij}]} }{\tilde{H}_{ii} - \tilde{H}_{jj}}.
\]

In order to estimate the washout effects, one may introduce the parameter \(K_i^{\alpha}\) which is the washout factor due to the inverse decay of the Majorana neutrino \(N_i\) into the lepton flavor \(\alpha(= e, \mu, \tau)\) [27]

\[
K_i^{\alpha} = \frac{\Gamma(N_i \to l_{\alpha}\varphi) + \Gamma(N_i \to \bar{l}_{\alpha}\varphi)}{\sum_{\alpha}[\Gamma(N_i \to l_{\alpha}\varphi) + \Gamma(N_i \to \bar{l}_{\alpha}\varphi)]} K_i = \frac{(Y_{\nu})_{ia}(Y_{\nu}^\dagger)_{ai}^{\alpha}}{(Y_{\nu}^\dagger)_{ii}} K_i,
\]

where

\[
K_i = \sum_{\alpha=e,\mu,\tau} K_i^{\alpha} = \frac{\Gamma_i}{H(T = M_i)}, \quad K^{\alpha} = \sum_{i=1}^{3} K_i^{\alpha},
\]

with \(\Gamma_i = \sum_{\alpha} \Gamma^{\alpha}\) denoting the total decay width of \(N_i\) at tree level where \(\Gamma^{\alpha}_i\) is the partial decay rate of the process \(N_i \to l_{\alpha} + \varphi\). The washing out of a given flavor \(l_{\alpha}\) is operated by the \(\Delta L = 1\) scattering involving all three right-handed neutrinos, which is parameterized by

\[
\tilde{m}_{i}^{\alpha} = (Y_{\nu}^\dagger)_{\alpha i} (Y_{\nu})_{ia} \frac{v^2}{M_i}, \quad \tilde{m}_{i}^{\alpha} = \frac{\Gamma(N_i \to \varphi l_{\alpha})}{H(M_i)},
\]

where \(\tilde{m}_{i}^{\alpha}\) parameterizes the decay rate of \(N_i\) to the leptons of flavor \(l_{\alpha}\) and the trace \(\sum_{\alpha} \tilde{m}_{i}^{\alpha}\) coincides with the \(\tilde{m}_{i}\) parameter defined in the previous section. The each lepton asymmetries are washed out differently by the corresponding washout parameter which is given by Eq. (49), and appear with different weights in the final formula for the baryon.
asymmetry \[27\], as will be shown later (see Eqs. \((54, 55)\)). Indeed the lepton asymmetry for each flavor \(l_\alpha\) generated through \(N_i\) decay is given by

\[
Y_i^\alpha \simeq 0.3 \frac{\varepsilon_i^\alpha}{g_*} \left( \frac{0.55 \times 10^{-3} \text{eV}}{\tilde{m}_i^\alpha} \right)^{1.16}
\]  

in the strong wash-out regime (\(K_i^\alpha \gg 1\)), and

\[
Y_i^\alpha \simeq 1.5 \frac{\varepsilon_i^\alpha}{g_*} \left( \frac{\tilde{m}_i^\alpha}{3.3 \times 10^{-3} \text{eV}} \right) \left( \frac{\tilde{m}_i^2}{3.3 \times 10^{-3} \text{eV}} \right)
\]

in the weak wash-out regime (\(K_i^\alpha \ll 1\)).

For temperatures \(10^9 \text{ GeV} \lesssim T \sim M_i \lesssim 10^{12} \text{ GeV}\), the interactions mediated by the \(\tau\) Yukawa coupling are in equilibrium, whereas those by the other Yukawa couplings are out of equilibrium. Then, the lepton asymmetries for the electron and muon flavors can be treated as a linear combination: \(Y_i^2 \equiv Y_i^e + Y_i^\mu\). Finally, the baryon asymmetry is given by \[27\]

\[
Y_B \simeq \frac{12}{37} \sum_{N_i} \left[ Y_i^2 \left( \varepsilon_i^2, \frac{417}{589} \tilde{m}_i^2 \right) + Y_i^\tau \left( \varepsilon_i^\tau, \frac{390}{589} \tilde{m}_i^\tau \right) \right],
\]

where \(\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu\), and the corresponding wash-out parameter is \(K_i^2 = K_i^e + K_i^\mu\).

Below temperatures \(T \sim M_i \lesssim 10^9 \text{ GeV}\), muon and tau charged lepton Yukawa interactions are much faster than the Hubble expansion parameter rendering the \(\mu\) and \(\tau\) Yukawa couplings in equilibrium. Then, in this case the final baryon asymmetry is given \[27\] as

\[
Y_B \simeq \frac{12}{37} \sum_{N_i} \left[ Y_i^e \left( \varepsilon_i^e, \frac{151}{179} \tilde{m}_i^e \right) + Y_i^\mu \left( \varepsilon_i^\mu, \frac{344}{537} \tilde{m}_i^\mu \right) + Y_i^\tau \left( \varepsilon_i^\tau, \frac{344}{537} \tilde{m}_i^\tau \right) \right].
\]

Notice that the CP-asymmetries of a single flavor given in Eqs. \((54, 55)\) are weighted separately due to the different values of \(\tilde{m}_i^\alpha\).

In the strong washout regime, which corresponds to our case, given the initial thermal abundance of \(N_i\) and the condition \(K_i^\alpha \gg 1\), the baryon asymmetry including lepton flavor effects is given \[31\] as

\[
\eta_B \simeq -0.96 \times 10^{-2} \sum_i \sum_\alpha \varepsilon_i^\alpha \frac{K_i^\alpha}{K_i K_i^\alpha}.
\]

The ratio of \(\eta_B\), generated through flavor independent leptogenesis, to \(\eta_B^{\text{flavor}}\), generated through flavor dependent leptogenesis, in \(10^9 \text{ GeV} \lesssim M \lesssim 10^{12} \text{ GeV}\) region yields

\[
\frac{\eta_B}{\eta_B^{\text{flavor}}} \simeq \frac{\varepsilon_3}{\varepsilon_3^3} \frac{K_3}{K_3} \approx \frac{y_T^2}{8\pi^2} \ln \left( \frac{M}{\Lambda} \right) \frac{K_3}{K_3^3},
\]

18
where the orders of magnitude of $K^\tau$ and $K_3^\tau$ are $\sim O(100)$ and $\sim O(1)$, respectively. Thus, without taking lepton flavor effects into account, in this region the prediction of $\eta_B$ is suppressed by $4 \sim 5$ orders of magnitude compared with $\eta_B^{\text{flavor}}$.

Below the temperature $M \sim 10^9$ GeV, all cases of parameter spaces can contribute to leptogenesis with different washout-factors. As indicated in Eqs. (39,48), the CP-asymmetries $\varepsilon_i$ and $\varepsilon^\alpha_i$ are weakly dependent on the heavy Majorana neutrino scale $M$. Without taking account of wash-out factors, since there is no CP-violation phases at the degeneracy scale, in this case we can obtain approximately $\varepsilon_i \propto y_i^4 t$ and $\varepsilon^\alpha_i \propto y^2_t$, see Eqs. (39,46), and the CP-asymmetry $\varepsilon^\alpha_i$ gets enhanced by $\varepsilon^\alpha_i/\varepsilon_i \sim 1/y^2_t$ due to flavor effects.

V. NUMERICAL ANALYSIS

Confronting neutrino masses and mixing in the context of our scheme with low energy neutrino experimental data given in Eq. (1), we determine the allowed regions of the model parameters for which we estimate the lepton asymmetry. For the case of $\phi_A = \phi_B = 0$ at the GUT scale, in left figure of Fig. 5, we plot the predictions of baryon asymmetry $\eta_B$ for $10^6 \lesssim M[\text{GeV}] \lesssim 10^{12}$. The horizontal dotted lines correspond to the bounds on $\eta_B$ measured from current astrophysical observations, $(2 \times 10^{-10} < \eta_B < 10 \times 10^{-10})$. The asters
FIG. 6: The predictions of the BAU $\eta_B$ for $10^6 \lesssim M\,[\text{GeV}] \lesssim 10^{12}$. The horizontal dotted lines correspond to the current observation from WMAP \cite{13}. The crosses correspond to flavored leptogenesis and the triangles correspond to flavor independent leptogenesis.

correspond to flavored leptogenesis, whereas the crosses correspond to flavor independent leptogenesis. We see from left figure of Fig. 5 that successful leptogenesis in the FTY model is possible only when lepton flavor effects are included, and the required values of $\eta_B$ can be achievable for the temperature ranges of $M \gtrsim 10^9 \text{GeV}$. As explained before, for $10^9 \lesssim M\,[\text{GeV}] \lesssim 10^{12}$, only the interactions mediated by the $\tau$ Yukawa coupling are in equilibrium and thus only the $\tau$-flavor is treated separately in the Boltzmann equations while the $e$ and $\mu$ flavors are indistinguishable. Left-figure of Fig. 5 shows that FTY structure reaches maximal $\eta_B$ near $10^7 \text{GeV}$ (seesaw scale) running down from GUT scale, corresponding to $M_1 \lesssim M_2 \lesssim M_3$, which is related with the stable angle $\theta_i$ in Eq. (18) (see also \cite{18}).

For $10^9 \lesssim M\,[\text{GeV}] \lesssim 10^{12}$, right figure of Fig. 5 represents how the predictions of $\eta_B$ in flavored leptogenesis depend on the initial value of the phase $\varphi_B$ imposed at GUT scale. In the same region of $M$, we find that $\eta_B^\tau$ dominates over $\eta_B^e = \eta_B^\mu + \eta_B^\tau$, and thus the successful leptogenesis in the FTY model is approximately equal to tau-resonant leptogenesis \cite{31}.

In the case of $\phi_A \neq 0$ and $\phi_B \neq 0$ at the GUT scale, Fig. 6 presents the predictions of $\eta_B$ generated through flavor independent leptogenesis (the triangles) and those of $\eta_B^{\text{flavor}}$ through flavor dependent leptogenesis (the crosses) for $10^6 \lesssim M\,[\text{GeV}] \lesssim 10^{12}$. Note that we vary the values of $\phi_{A,B}$ as well as $\varphi_{A,B}$ from $0$ to $2\pi$ without fixing certain values. The horizontal dotted lines correspond to the current bounds on $\eta_B$. We see from Fig. 6 that
flavor independent leptogenesis leads to the right amount of baryon asymmetry required from the current observational result, whereas the predictions for $\eta_B^{\text{flavor}}$ are too large for flavor dependent leptogenesis to be a desirable candidate for baryogenesis. The reason why $\eta_B^{\text{flavor}}$ get enhanced compared with $\eta_B$ generated through flavor independent leptogenesis is that the first contribution in Eq. (47) dominates over the second one, so that $\varepsilon_i^x / \varepsilon_i \sim 1/y^2 t$ which is much less than one.

VI. SUMMARY

As a summary, we have considered FTY model [9] realized at the GUT scale. By considering RG evolution from GUT scale to low energy scale, we have confronted light neutrino masses and mixing with low energy experimental data, and found the allowed parameter space. We have investigated how BAU can be achieved via leptogenesis in FTY model. In particular, we considered two scenarios, one is to include lepton flavor effects and the other is to ignore them. In FTY model we consider, there are two types of CP phases, $\phi_{A,B}$ appeared in $Y_\nu$ and $\varphi_{A,B}$ in $Y_l$. Besides those CP phases, we need to splitting of the heavy Majorana neutrino spectrum in order to generate lepton asymmetry in FTY model. We have shown that the desirable splitting of the heavy Majorana neutrino spectrum could be radiatively induced at the seesaw scale by using the RG evolution from GUT to seesaw scale. In the case of $\phi_A = 0, \phi_B = 0$ at GUT scale, we have found that the predictions of $\eta_B$ through flavor independent leptogenesis are not enough to achieve successful baryogenesis, whereas it can be achieved by flavor dependent leptogenesis for $10^9 \lesssim M[\text{GeV}] \lesssim 10^{12}$.

In the case of the phases $\phi_A \neq 0, \phi_B \neq 0$ at the GUT scale, contrary to the previous case, the successful leptogenesis can be achieved by ignoring the lepton flavor effects because flavor effects greatly enhance the lepton asymmetry so that they are not desirable to achieve baryon asymmetry of our universe.

We note that in both cases of our work, leptogenesis can be viable for

$$10^9 \lesssim M[\text{GeV}] \lesssim 10^{12}. \tag{58}$$

In particular, in the FTY model, flavor dependent leptogenesis can be worked when $Y_\nu$ does not contain CP-phases, but $Y_l$ contains CP-phases.
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