A Comparative Study of Equation of State and Conductivity for Warm Dense Matter using Pulsed-power Wire Discharges in Water

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Abstract. A method of measuring equation of state and electrical conductivity is proposed relevant to conditions in warm dense matter (WDM). The WDM was produced by exploding wire discharges in water. The wire/plasma parameters were measured spectroscopically and also by a shadow graph method. The density and the temperature were estimated to be typically $0.01\rho_{\text{solid}}$ and $15\text{ kK}$, respectively, at $2\mu\text{sec}$ from beginning of the discharges. We have compared the behaviors of the plasma boundary and an accompanied shock wave in water to a numerical estimation based on a model of equation of state (EOS). We indicate that the comparative study of the experimental and the numerical results, is a possible approach for a 'semi-empirical' scaling of EOS.

1. Introduction
A plasma has a variety of forms in the universe. However, a dense-low-temperature plasma, such as the structure of giant planets (e.g., Jupiter) [1], do not obey the existing models for equation of state (EOS) and transport coefficients. On the other hand, to understand a hydrodynamics of inertial confinement fusion [2, 3], better formulations for EOS and transport coefficients are in demand. The warm dense matter (WDM) is defined for a region of the density and temperature, $10^{-3}\rho_s \sim \rho_s$ ($\rho_s$: solid density) and $10^{3} \sim 10^5 \text{ K}$, respectively.

To make WDM, we might use ultra-short pulse laser [4, 5], pulsed-power discharges [6, 7], and heavy ion beams [8, 9, 10]. These methods are different, namely, the typical time scale and the achievable plasma parameters are not the same[11]. In this study, we measured electrical conductivities of WDM, and proposed a method that semi-empirically evaluates the EOS from the comparison of experimental results to numerical simulations [6, 12, 13]. For that purpose, the state of matter should be well-defined by temperature and density together with the input energy history for a long time. The accuracy of wire/plasma temperature, which is method dependent (i.e., it is sometimes estimated from input energy with conventional EOS), might be affected by density-temperature dependency for discharge produced plasma. We characterized the well-defined condition by a method of measuring the temperature. With this method, we can investigate the conductivity and EOS of WDM almost accurately over a wide range of parameter on density-temperature diagram.
2. Experimental Setup

A schematic diagram of the experimental apparatus is shown in Fig. 1. To drive a wire explosion, we arranged low inductance capacitors cylindrically \((8 \times 0.4 \ \mu F)\). The capacitor bank was charged up to 10 kV to ensure vaporization of the wire. The behaviors of wire/plasma radius and the shock wave in water were measured by a shadow-graph method using a fast streak camera with a laser \((\lambda_L = 532 \ \text{nm})\). From the time evolution of radius, we can evaluate a wire/plasma density and conductivity with the voltage and the current measurements.

We observed the spectrum of plasma emission using a fast framing camera. The resolution of spectrum was estimated to be \(\Delta \lambda \sim 0.1 \ \text{nm}\). Figure 2 shows typical spectra of copper plasma over a range of 500 nm to 550 nm at 2 \(\mu\text{sec}\), 3 \(\mu\text{sec}\), and 4 \(\mu\text{sec}\), from beginning of the discharge. As is well known, thin copper plasma has emission or absorption lines at 510 nm, 515 nm, 521 nm. However, as shown, the broad Plankian spectra were observed during 4 \(\mu\text{sec}\). We assumed that the plasma temperature is in local thermodynamic equilibrium (LTE) state during the observation time.

Under aforementioned assumption, we assembled a spectrometer which is made of a polychrometer with photo-diodes array for evolution of plasma temperature. The spectrometer, which was calibrated by a xenon lamp (L7810: Hamamatsu Photonics), detects the emission at 458 nm, 558 nm, and 658 nm. An example of intensity versus time data at these wavelengths is shown in Fig. 2 (d). The plasma emission in LTE situation obeys the Planck’s formula, that is, a black-body radiation as

\[
I(\lambda, T) \ d\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/k_B T \lambda} - 1} \ d\lambda,
\]

where \(\lambda\) is the wavelength, \(T\) is the plasma temperature, \(c\) is the speed of light, \(h\) is the Planck’s constant, and \(k_B\) is the Boltzmann constant. The plasma temperature is estimated using the intensity ratio from these signals with eq. (1).

3. Results and Discussions

Figure 3 shows typical evolutions of voltage, current, input energy, conductivity, hydrodynamic behavior and temperature of aluminum wire with 100 \(\mu\text{m}\) in diameter and 18 mm in length. At 0.2 \(\mu\text{sec}\) from beginning of the discharge, the wire expands with the shock wave in water. At this time, the voltage and current waveforms show peaks, and the input energy corresponds
Figure 3. Typical waveforms of (a) voltage, (b) current, (c) input energy history, (d) conductivity, (e) evolution of hydrodynamic behaviors, and (f) temperature of aluminum wire discharges with 100 µm in diameter and 18 mm in length.

to the vaporization energy (5.1 J). After the vaporization, the conductivity suddenly decreased to less than $10^3$ S/m. From 1 µsec, we can see that the current waveform again increases, and then enhances the input energy, and temperature. The plasma density $\rho(t)$ is derived from the plasma radius $r(t)$ as $\rho(t) = \rho_s(r(t)/r_0)$, where $r_0$ denotes the initial radius of wire. At 2 µsec, the plasma density and temperature were estimated to be $\sim 0.01 \rho_s$, and $\sim 15$ kK, respectively, under assumption of homogeneous plasma state. Simultaneously, the conductivity was estimated to be $\sim 10^4$ S/m.

The consistency of wire/plasma behavior is essential to evaluate EOS and transport coefficients. The numerical hydrodynamic evolution of wire/plasma is affected by the models of material’s and water’s EOS. Concerning the water’s EOS effect, we compared the hydrodynamic behaviors. For the material, QEOS [14] has been used, and the water’s EOS has been taken from IAPWS95 [15] or Cole’s semi-empirical formula [16]. To estimate the hydrodynamic evolution of discharge, we have used a one-dimensional magneto-hydrodynamic (MHD) code in cylindrical geometry with IAPWS95. This calculation used the experimentally obtained input energy history. Other hand, the hydrodynamic calculation using a piston model was given by the experimental behavior of plasma boundary with Cole’s EOS.

Figure 4 shows the shock wave and the plasma boundary evaluated by the numerical results and the experimental observation. As shown in the figure, the experimentally obtained velocity of shock wave (3) is slower than the numerical ones. In MHD calculation, the condition of shock behind was estimated to be about $T \sim 400$ K, $\rho \sim 1.3$ g/cc and $P \sim 2$ GPa. The water’s EOS at the dense and high pressure regime is extrapolated to the region using IAPWS95. Therefore, the driven shock wave velocity at the beginning of expansion may be over estimated by the extrapolated water’s EOS. In hydrodynamic calculation using a piston model, the velocity of shock wave is faster than in MHD case. The difference of shock velocity indicates that the hydrodynamics is affected by sound velocity, that is, the EOS of water is needed to improve accuracy in the high pressure region. All of the comparative results of the hydrodynamic behavior indicate a requirement of more accurate EOS models both for water and dense plasma at high density regime.

Note that through the comparison and fittings of the shock surface, the plasma boundary,
Figure 4. Comparison of shock wave; (1), (2), (3), and wire/plasma boundary; (4), (5), trajectories. Circles and Squares denote the experimental results, and the line (1) expresses the numerical result of a piston model using Cole’s EOS, (2) and (5) reveal the MHD calculation results using QEOS for wire/plasma and IAPWS95 for water.

and the experimentally observed temperature, we can expect to extract ’semi-empirical’ scaling of EOS and transport coefficients, consistently.

4. Summary
We investigated behavior of the exploding aluminum wire discharge in water as a method for study on the WDM physics. The time evolutions of voltage, current, input energy, conductivity, hydrodynamic behavior, and temperature were measured at the same time. The density and the temperature were estimated to be $0.01\rho_{\text{solid}}$ and 15 kK, respectively, at 2 $\mu$sec from the beginning of discharge. At that time, the conductivity was estimated to be $\sim 10^4$ S/m. We have compared the behaviors of the plasma boundary and the shock wave propagation in water to a numerical estimation based on models of EOS. Based on the experimental and numerical results, we indicate that the EOS accuracy of water is needed to be improved in high pressure region. Those hydrodynamical behaviors basically reflect the evolution of wire plasma at WDM. Then, using these hydrodynamical evolutions, together with the wire/plasma temperature history and conductivity measurements, we can derive a ’semi-empirical’ EOS scaling.

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