Development of measurement quantity determination procedure for estimation of uncertainty of influence of conditions during calibration of thermal engineering measurement instruments

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Abstract. Calculation of uncertainty and identification of influencing factors is one of the current problems of measuring instruments (MI) operation, because testing programs of MI including only typical influence and don’t take into account specific measuring instruments. Calibration of measuring instruments during their operation and calculation of uncertainty of measurement results will allow to establish in the long term the actual values of measurement results traced to the national primary standard. One of the tasks in assessing the uncertainty of measurements is to assess the impact of calibration conditions, which is evaluated based on parallel measurements at each calibration point. The aim of the study is to develop a method for determining the number of parallel measurements when calibrating thermal measuring instruments on the basis of statistical information based on the results of verification of measuring instruments. In that research, was determined and experimentally tested method for determining the number of parallel measurements during calibration of thermal measuring instruments. The method was tested on the calculator of a quantity of heat VKT-7. Was identify strengths and weaknesses parts of the technique, as well as its field of application, have been revealed.

1. Introduction

Many oil and gas industry organizations use non-commercial thermal energy accounting nodes. That measurements are outside the sphere of state regulation, and therefore, measurement tools that are installed on accounting nodes can be calibrated. In addition, the results of calibration can be recognized during verification in accordance with Government Decision No. 311. Through this, non-commercial accounting nodes can be transformed into commercial.

ISO/IEC 17025 [1] contains a requirement for metrological traceability of measuring instruments and reliability of results. In calibration, by setting the actual values and information on the stability and drift of the measurement instrument readings, it is possible to reduce the cost of the final measurement by calibrating the measurement instrument with all the influencing values, it is possible to obtain the measurement instrument with the highest accuracy values compared to its claimed characteristics. Also, generation of reliable measurement information with constant monitoring of stability and reliability of measurement results makes it possible to optimally form composition (nomenclature) of measurement devices used at non-commercial nodes of thermal energy calculation.

At the same time reliability of measurement results is increased, which depends on accuracy and uncertainty of measurements. As a result, it is possible to change the inter-calibration interval in accordance with RMG 74 [2], which will also save funds spent on metrological support of non-commercial accounting nodes.
One of the important factors in solving the presented tasks is the task of developing a methodology for determining the number of measurements for estimating uncertainty of the influence of the conditions during calibration of thermal engineering I & C.

2. Problem definition
One of the tasks in estimation of measurement uncertainty during calibration of thermotechnical measuring tools is estimation of uncertainty of influence of calibration conditions.

When calibrating the measuring instruments, the calibration conditions can be extended as compared to the calibration based on the operating conditions of the measuring instruments. Accordingly, the impact of calibration conditions should be taken into account when estimating measurement uncertainty during calibration. Laboratories accredited to the right of verification and calibration have information on the results of verification of measuring instruments.

Taking into account that when estimating the uncertainty of measurements of the effects of calibration conditions at each calibration point of the measuring instrument, it is necessary to carry out a series of parallel measurements, one of the current problems in estimating the uncertainty of measurements is the task of determining the number of measurements at each calibration point on the basis of information from the results of verification of the measuring instruments.

3. Theory
In order to develop a methodology for determining the number of measurements in order to estimate the influencing factors, it is necessary to draw up a measurement model for the calibration of the measuring instruments.

According to the recommendations of COOMET/GM/32:2017 [3], a measurement model for temperature calibration with the help of a VKT-7 heat quantity calculator has been developed, which includes the average value of the measured value, the systematic error of the measuring instrument and the standard and the random component and is represented by the formula:

\[ t = \bar{i}_{mes} + f(\Delta_{vkt-7}) + f(\Delta_{sk-6}) + f(\Delta_s) \]

\( \bar{i}_{mes} \) : average measured value;

\( f(\Delta_{vkt-7}) \) : limit blundered VKT-7 equal to ±0.01 °C;

\( f(\Delta_{sk-6}) \) : the limit of permissible SKS-6 error of ± 0.015 Ohm;

\( f(\Delta_s) \) : accidental component of an error;

\( f \) : translated function.

Temperature is measured by setting the resistance at the reference. Assume that the temperature change from the resistance will vary by linear law:

\[ t(R) = a_0 + a_1 R \]

Where \( a_0 \) and \( a_1 \) are the translation coefficients of the linear equation.

Then the dependence of temperature error on resistance error will be as per GOST 8.736 [4] by formula:

\[ \Delta t(\Delta R) = \left| \frac{\partial t(R)}{\partial R} \right| \]

When we find the R derivative, we get the formula:

\[ \Delta t(\Delta R) = \sqrt{(a_1 \cdot \Delta R)^2} = a_1 \cdot \Delta R. \]

Taking into account this formula, we obtain a refined model for measuring the temperature channel calibration of the VKT-7 heat exchanger:

\[ t = \bar{i}_{mes} + \Delta_{vkt-7} + a_1 \cdot \Delta_{sk-6} + \Delta_s \]

According to the measurement uncertainty expression manual [6], the following are estimated:
- Standard uncertainties for each component by formulas:

\[
U_b[\Delta_{\text{VKT-7}}] = \frac{\Delta_{\text{VKT-7}}}{\sqrt{3}}
\]

\[
U_b[\Delta R_{\text{SKS-6}}] = \frac{\Delta R_{\text{SKS-6}}}{\sqrt{3}}
\]

\[
U_A[\Delta_z] = \sqrt{\frac{\sum (t_i - t_{\text{meas}})^2}{n(n-1)}}
\]

- Total indeterminacy:

\[
U, [t] = \sqrt{(U_b[\Delta_{\text{VKT-7}}])^2 + (a_1 \cdot U_b[\Delta R_{\text{SKS-6}}])^2 + (U_A[\Delta_z])^2}
\]

- Expanded indeterminacy:

\[
U = k \cdot U, [t]
\]

k, the coverage factor.

The standard Type A uncertainty for the random component is broken down according to influencing factors, such as calibration conditions: temperature, humidity and pressure. The accuracy of the uncertainty estimate from the random component depends on the number of measurements at each calibration point. The more measurements, the more accurate the estimate. At the same time, the smaller the measurements, the lower the cost and time of calibration.

The procedure for determining the number of measurements to estimate the uncertainty of the influence of the conditions during calibration of thermal measuring instruments is shown in Figure 1.
The method of determining the number of measurements for estimation of uncertainty of influence of conditions during calibration of thermotechnical measuring instruments is carried out in several stages.

Stage 1. The calibration model and total and extended uncertainty are determined.

Stage 2. Random error is found based on verification protocols. If the random error is greater than zero, the hypothesis of systematic error equations is tested. If the value is less than zero, step 3 is performed.

Stage 3. Gross errors on Grabbs 'grittery are eliminated.

Stage 4. Data is broken down into intervals.

Stage 5. The hypothesis of uniformity of systematic errors according to the Kohren criterion is tested. If the hypothesis is rejected, the group with the maximum value of systematic error is excluded, if accepted, step 6 is performed.

Stage 6. The hypothesis of equality of systematic errors on Fisher 's criterion is tested. If the hypothesis is rejected, step 4 is performed, if received, step 7.

Stage 7. The average group sample variance is calculated.
Stage 8. Number of parallel measurements is selected depending on required accuracy, random error estimation and level of confidence probability of random error estimation.

4. Results of experiments
On the basis of the developed methodology taking into account the protocols VKT-7 verification for 2018 and 2019, an evaluation of the developed methodology was carried out.
At the first stage the value $\Delta_i$ and $\Delta_{cp}$ is calculated:

$$\Delta_i = t_{VKT-7} - t_{SKS-6},$$

$$\bar{\Delta} = \frac{\sum_{i=1}^{N} \Delta_i}{N},$$

$t_{VKT-7}$, temperature values output by the VKT-7,
$t_{SKS-6}$, temperature values supplied by reference SKS-6,
$N$, number of protocols.

Then total variance is calculated, variance of systematic component and random component of error is estimated by formulas:

$$S_0^2 = \frac{\sum_{i=1}^{N} (t_{VKT-7} - t_{SKS-6})^2}{N-1},$$

$$S_{VKT-7}^2 = \frac{\Delta^2}{3},$$

$$S_e^2 = S_0^2 - S_{VKT-7}^2$$

According to formulas (1) to (3):

$$S_0^2 = \frac{(66.64 - 66.65)^2 + (66.63 - 66.65)^2 + \ldots + (66.65 - 66.65)^2}{425 - 1} = 0.000111,$$

$$S_{VKT-7}^2 = \frac{-0.028089^2}{3} = 0.000263,$$

$$S_e^2 = 0.000111 - 0.001884 = -0.001773.$$

As a result of the calculation, the random component has a negative value, which means that the sample shows data with different systematic errors of the measuring means, they cannot be averaged, so it is necessary to break the data into several samples.

Assuming that the measurement results belong to the normal law of distribution, gross errors on the Grabb criterion are excluded [5]. The Grabb criterion is $G_1$ and $G_2$ calculated by the following formulas:

$$G_1 = \frac{|\Delta_{max} - \bar{\Delta}|}{S},$$

$$G_2 = \frac{|\Delta - \Delta_{min}|}{S},$$

$$S = \sqrt{\frac{\sum_{i=1}^{N} (\Delta_i - \bar{\Delta})^2}{N-1}}.$$

The $G_1$ and $G_2$ are compared to the theoretical value of $G_t$ at a value level of 5%. If $G_1 > G_t$ and $G_2 > G_t$, $\Delta_{max}$ and $\Delta_{min}$ are excluded as unlikely values respectively. Next, $S$ is recalculated again and the procedure is repeated.
According to the presented methodology, four iterations were made, according to which six results containing gross errors were excluded.

The protocol data, taking into account the Grabbs test, are broken down into intervals according to the Sterges formula:

\[ k = 1 + 3.322 \cdot \log N. \]

The work is divided into eight intervals, for each of which the number of hits in interval \( h \) is determined and values are calculated according to formulas (1) - (3). Calculation results are presented at Table 1.

In order to make sure that the intervals obtained no longer contain gross misses and errors, the hypothesis of uniformity of systematic errors according to the Kohren criterion is tested:

\[ K_p = \frac{\max(S^2_{VKT - \gamma_i})}{\sum S^2_{VKT - \gamma_i}}. \]

If \( K_p < K_t \), the hypothesis of uniformity of the dispersions of systematic errors is accepted. In operation, \( K_p = 0.42 \) and \( K_t = 0.68 \) at \( m = 8 \) (number of intervals), \( q = 1 \) (single measurements). Since \( K_p < K_t \), the hypothesis is accepted.

### Table 1. Results of the dispersive analysis

| Sample | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    |
|--------|------|------|------|------|------|------|------|------|
| \( \bar{H}_h \) | 9    | 8    | 134  | 59   | 72   | 108  | 16   | 13   |
| \( S^2_{VKT - \gamma_i} \) | 0.000523 | 0.000114 | 0.000062 | 0.000407 | 0.000913 | 0.002036 | 0.003840 | 0.005808 |
| \( S^2 \) | 0.000152 | 0.000033 | 0.000013 | 0.000133 | 0.000300 | 0.000664 | 0.001200 | 0.001780 |
| \( F_p \) | 0.408579 | 0.411765 | 0.257746 | 0.487395 | 0.489655 | 0.480466 | 0.454545 | 0.441898 |
| \( F_m \) | 3.25 | 3.9 | 1.4 | 1.4 | 1.4 | 1.4 | 2.3 | 2.6 |
| Hypothesis of equality of systematic errors | accepted | accepted | accepted | accepted | accepted | accepted | accepted | accepted |

In order to consider the obtained values of systematic errors in each interval to be the same, the hypothesis of equality of systematic errors according to Fisher’s criterion is tested (see Table 1):

\[ F_p = \frac{S^2_{VKT - \gamma_i}}{S^2_{\varepsilon}}. \]

If \( F_p < F_t \), the systematic error equality hypothesis is accepted. In the presented calculations for each interval this hypothesis is accepted.

The variance of random errors will converge to the weighted average of the variance. The average group sample variance of random errors is calculated as an estimate of the most probable value of random error:

\[ \hat{S}^2_{\varepsilon} = \frac{\sum_{i=1}^{n} (h_i - 1) \cdot S^2_{\varepsilon_i}}{\sum_{i=1}^{n} (h_i - 1)}. \]

The obtained value equally 0.000740 is further used to determine the number of parallel measurements at each calibration point.

Taking into account that the group selective variance of random errors is a random value and in turn is given by a confidence interval depending on the natural variance of the \( \hat{S}^2 \) and the given boundaries of the measurements that are commensurate with the interval.
The confidence level is determined from the equation:

\[ P \left( \frac{\Delta S}{\chi^2_{\alpha,n}} \leq S^2 \leq \frac{\Delta S}{\chi^2_{1-\alpha,n}} \right) = \alpha \]

\( \Delta S \), dispersion deviation commensurate with interval of performed measurements;
\( P \), is the confidence level;
\( \alpha \), quantile of distribution of \( \chi^2 \);
\( n_{\alpha} \), number of samples according to verification protocols.

Based on the results of the calculation, the confidence level of the variance estimate was 0.62.

The number of parallel tests is calculated using the formula:

\[ n_p = \frac{t^2 S^2}{\delta^2} \]  \hspace{1cm} (4)

\( T \), parameter of Laplace function;
\( \Delta \), the required accuracy.

With a confidence of 0.62, the number of parallel tests at each calibration point according to formula (4):

\[ n_p = \frac{0.88^2 \cdot 0.000740}{0.01^2} = 5.73 \approx 6 \]

Based on the obtained and verification results of the heat exchanger VKT-7 6 measurements should be carried out at each calibration point to assess the impact of calibration conditions.

5. Discussion of results
The result of the carried out studies is the method of determining the number of measurements to estimate uncertainty of influence of the conditions during calibration of thermotechnical measuring instruments.

The advantage of the developed methodology is the possibility of using statistical information based on the results of verification of measuring instruments.

The disadvantage of the developed technique is a low level of confidence and, as a result, an overestimated number of measurements at each calibration point.

In view of the disadvantage of the developed technique, the application of this technique is reduced to the stage of preliminary experimental studies of determination of the number of measurements at each calibration point.

6. Conclusions
The study developed a method of determining the number of measurements to estimate the uncertainty of the influence of the conditions during calibration of thermotechnical measuring instruments.

Based on the proposed procedure, the number of parallel measurements at each calibration point of the VKT-7 heat exchanger is calculated.

Advantages, disadvantages and scope of application of the developed method are defined.

Financing source. Thanks
The authors thank the laboratory of verification of measuring instruments of thermal technical values of OmGTU in the person of engineer Fadeeva Natalia Lviv for providing statistical data.

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