CP violation and the CKM angle $\gamma$ from angular distributions of untagged $B_s$ decays governed by $\bar{b} \to \bar{c}u\bar{s}$

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Abstract

We demonstrate that time-dependent studies of angular distributions for $B_s$ decays caused by $\bar{b} \to \bar{c}u\bar{s}$ quark-level transitions extract cleanly and model-independently the CKM angle $\gamma$. This CKM angle could be cleanly determined from untagged $B_s$ decays alone, if the lifetime difference between the $B_s$ mass eigenstates $B_s^L$ and $B_s^H$ is sizable. The time-dependences for the relevant tagged and untagged observables are given both in a general notation and in terms of linear polarization states and should exhibit large CP-violating effects. These observables may furthermore provide insights into the hadronization dynamics of the corresponding exclusive $B_s$ decays thereby allowing tests of the factorization hypothesis.

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1 Introduction

Some time ago it was pointed out in [1] that a clean measurement of the angle $\gamma$ of the usual “non-squashed” unitarity triangle [2] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [3] is possible by studying the time dependence of the color-allowed decays $B_s \rightarrow D_s^\pm K^\mp$. A similar analysis of the color-suppressed modes $B_s \rightarrow D^0 \phi$ provides in principle also clean information about $\gamma$ [4]. Because current detectors have difficulties in observing the soft photon in $D_s^* \rightarrow D_s \gamma$ decays, Aleksan, Le Yaouanc, Oliver, Pene and Raynal employed several plausible assumptions to show that the CKM angle $\gamma$ can still be extracted from partially reconstructed $B_s$ modes where that soft photon could be missed [5].

Unfortunately in all these strategies tagging, i.e. the distinction between initially present $B_s$ and $\bar{B_s}$ mesons, is essential. Moreover one has to resolve the rapid $B_s - \bar{B_s}$ oscillations, which may arise from the expected large mass difference $\Delta m \equiv m_H - m_L > 0$ between the mass eigenstates $B_s^H$ (“heavy”) and $B_s^L$ (“light”) [3]. This is a formidable experimental task. In a recent paper [6] these methods have been re-considered in light of the expected perceptible lifetime difference [8] between $B_s^H$ and $B_s^L$. There it has been shown that the rapid $\Delta m t$–oscillations cancel in untagged data samples. Whereas the extraction of $\gamma$ from untagged $B_s \rightarrow D_s^\pm K^\mp$ requires some mild additional theoretical input, it does not require any theory beyond the validity of the CKM model from untagged $B_s \rightarrow D^0 \phi$ decays [7].

In a recent publication [9] we have investigated quasi two body modes $B_s \rightarrow X_1 X_2$ into admixtures of different CP eigenstates where both $X_1$ and $X_2$ carry spin and continue to decay through CP-conserving interactions. The time-dependent angular distributions for the untagged decays $B_s \rightarrow D_s^{+} D_s^{-}$ and $B_s \rightarrow J/\psi \phi$ determine the Wolfenstein parameter $\eta$ [10]. If one uses $|V_{ub}|/|V_{cb}|$ as an additional input, the CKM angle $\gamma$ can be fixed. That input allows, however, also the determination of $\eta$ (or $\gamma$) from the mixing-induced CP asymmetry of $B_d \rightarrow J/\psi K_S$ measuring $\sin 2\beta$ ($\beta$ is another angle of the unitarity triangle [2]). Comparing these two results for $\eta$ (or $\gamma$) obtained from $B_s$ and $B_d$ modes, respectively, an interesting test whether the $B_s - \bar{B_s}$ and $B_d - \bar{B_d}$ mixing phases are described by the Standard Model or receive additional contributions from “New Physics” can be performed. Another application of the formalism developed in [9] is the point that a determination of $\gamma$ is possible by using the $SU(2)$ isospin symmetry of strong interactions to relate untagged data samples of $B_s \rightarrow K^{*+} K^{*-}$ and $B_s \rightarrow K^{*0} \overline{K^{*0}}$.

Having all these results in mind it is quite natural to ask what can be learned from time-dependent untagged measurements of the angular distributions for $B_s \rightarrow D_s^{\pm} K^{*\mp}$,
$D_{s1}(2536)^{\pm}K^{*\mp}$, $D_{s}^{*\pm}K^{*\mp}$ and $(\overline{B}_{s}\rightarrow D^{*0} \phi$, $(\overline{D}_{1}(2420)^{0}\phi$, $D^{*0} \phi$ or – more generally – from $B_{s}$ modes governed by $\bar{b} \rightarrow \bar{c}u\bar{s}$ quark-level transitions. Since the photon(s) in the strong or electromagnetic decays of $D_{s}$ and $D^{*0}$ are more difficult to detect than charged particles for generic detectors, we listed also higher resonances because of their significant all-charged final states, such as $D_{s1}(2536)^{+}\rightarrow D^{*+}K^{0}$, $D^{*+}\rightarrow \pi^{+}D^{0}$ or $D_{1}(2420)^{0}\rightarrow D^{*+}\pi^{-}$, $D^{*+}\rightarrow \pi^{+}D^{0}$. The $K^{*\mp}$ in the above $B_{s}$-decays can be substituted by either a strange resonance or a collection of strange resonances with common spin and parity quantum numbers.

While our note focuses on quasi two body modes where each body has a well-defined spin and parity, a complementary report discusses the effects when several resonances contribute to the final state [11]. In the former case the final states cannot be classified by their CP eigenvalues as in [9]. However, they can instead be classified by their parities. To this end linear polarization states [12] are particularly useful. As we will demonstrate in the present paper, the untagged angular distributions for such $B_{s}$ decays may inform us in a clean way about $\gamma$, if the lifetime difference between $B_{s}^{H}$ and $B_{s}^{L}$ is in fact sizable. In particular we do not need any theoretical input to extract this quantity from the untagged data samples which exhibit in addition interesting CP-violating effects. Furthermore essentially the whole hadronization dynamics can be extracted from these angular correlations. Since, for example, the $(\overline{B}_{s}\rightarrow D_{s}^{*\pm}K^{*\mp}$, $D_{s1}(2536)^{\pm}K^{*\mp}$, $D_{s}^{*\pm}K^{*\mp}$ modes are color-allowed whereas the $B_{s}\rightarrow D^{*0}\phi$, $D_{1}(2420)^{0}\phi$, $D^{*0}\phi$ channels are color-suppressed, the factorization hypothesis [13, 14], which has some justification within the $1/N_{C}$-expansion [13], should work quite well in the former case and should be very questionable in the latter case [16]. Therefore we expect significant non-factorizable contributions to the angular distributions for the $B_{s}\rightarrow D^{*0}\phi$, $D_{1}(2420)^{0}\phi$, $D^{*0}\phi$ decays. The explicit angular distributions for some of these decays will be given in a separate publication [17]. There also appropriate weighting functions are given allowing an efficient extraction of the corresponding observables from experimental data with the help of a moment analysis (see [18, 19]).

Our paper is organized as follows: The time-dependences of the observables of the angular distributions are calculated in Section 2 in terms of a general notation that allows an easy comparison with the results presented in [9]. In Section 3 these time-dependences are given in terms of linear polarization states which provide a useful tool to calculate the explicit angular distributions for final state configurations having definite parities. There we demonstrate explicitly that the observables of the untagged angular distributions for the $\bar{b} \rightarrow \bar{c}u\bar{s}$ (and $\bar{b} \rightarrow \bar{u}c\bar{s}$) decays suffice to extract the CKM angle $\gamma$. The issue of CP violation in untagged data samples is discussed in Section 4 and the main results of our paper are summarized briefly in Section 5.
2 Calculation of the general time-evolutions

In the case of the decays considered in this paper, the transition amplitudes for the quasi two body modes $\overline{B}_s \to X_1 X_2$ and $B_s \to X_1 X_2$ can be expressed as hadronic matrix elements of low energy effective Hamiltonians having the following structures:

$$H_{\text{eff}}(\overline{B}_s \to X_1 X_2) = \frac{G_F}{\sqrt{2}} \left[ C_1(\mu)\overline{O}_1 + C_2(\mu)\overline{O}_2 \right]$$

$$H_{\text{eff}}(B_s \to X_1 X_2) = \frac{G_F}{\sqrt{2}} v^* \left[ C_1(\mu)\overline{O}_1^1 + C_2(\mu)\overline{O}_2^1 \right],$$

where $\overline{v}$ and $v$ denote appropriate CKM factors, $\overline{O}_k$ and $O_k$ ($k \in \{1, 2\}$) are four-quark operators ("current-current" operators in our case) and $C_1(\mu)$ and $C_2(\mu)$ are the Wilson coefficient functions of these operators. They can be calculated perturbatively and contain the whole short distance dynamics. As usual $\mu = \mathcal{O}(m_b)$ is a renormalization scale. To be definite, for $X_1 X_2 \in \{D_s^0 K^0, D_{s1}(2536)^+ K^0, D_{s}^{*+} K^0, D^{*0} \phi, D_1(2420)^0 \phi, D_{s}^{**} \phi\}$ we have

$$\overline{O}_1 = (\bar{s}_a u_\beta)_{\text{V-A}} (\bar{c}_b d_\alpha)_{\text{V-A}}$$
$$\overline{O}_2 = (\bar{s}_a u_\alpha)_{\text{V-A}} (\bar{c}_b d_\beta)_{\text{V-A}}$$
$$O_1 = (\bar{s}_a c_\beta)_{\text{V-A}} (\bar{u}_b d_\alpha)_{\text{V-A}}$$
$$O_2 = (\bar{s}_a c_\alpha)_{\text{V-A}} (\bar{u}_b d_\beta)_{\text{V-A}},$$

where the greek indices denote $SU(3)_C$ color indices, and the CKM factors are given by

$$\overline{v} = V_{ts}^* V_{cb}$$
$$v = V_{cs}^* V_{ub}. \tag{5}$$

Nowadays the Wilson coefficients $C_1(\mu)$ and $C_2(\mu)$ are available beyond the leading logarithmic approximation $[20, 21]$. A nice review of such next-to-leading order calculations has been given recently in $[22]$, and we refer the reader to that publication for the details of such calculations.

Applying a similar notation as in $[4]$, we obtain the following transition amplitudes for decays of $B_s$ and $\overline{B}_s$ mesons into a configuration $f$ of the quasi two body state $X_1 X_2$, where $f$ is a label that defines the relative polarizations of the two hadrons $X_1$ and $X_2$:

$$\overline{M}_f \equiv \langle (X_1 X_2)_f | H_{\text{eff}}(\overline{B}_s \to X_1 X_2) | B_s \rangle = \frac{G_F}{\sqrt{2}} \overline{v} \overline{M}_f \tag{6}$$
$$A_f \equiv \langle (X_1 X_2)_f | H_{\text{eff}}(B_s \to X_1 X_2) | B_s \rangle = \eta_f e^{i\phi_{CP}(B_s)} \frac{G_F}{\sqrt{2}} v^* M_f \tag{7}$$

with

$$\overline{M}_f \equiv \langle (X_1 X_2)_f | C_1(\mu)\overline{O}_1 + C_2(\mu)\overline{O}_2 | B_s \rangle \tag{8}$$
$$M_f \equiv \langle (X_1 X_2)_f | C_1(\mu)O_1 + C_2(\mu)O_2 | B_s \rangle. \tag{9}$$
In order to evaluate (10) we have performed the CP transformations

\[
\langle (X_1 X_2)_f | C_1(\mu) O_1^f + C_2(\mu) O_2^f | B_s \rangle 
= \langle (X_1 X_2)_f | (\mathcal{CP})^\dagger (\mathcal{CP}) \left[ C_1(\mu) O_1^f + C_2(\mu) O_2^f \right] (\mathcal{CP})^\dagger (\mathcal{CP}) | B_s \rangle 
= \eta_f^\mu \, e^{i\phi_{\mathcal{CP}}(B_s)} \langle (X_1 X_2)_f^C | C_1(\mu) O_1 + C_2(\mu) O_2 | B_s \rangle
\]

by taking into account the relations

\[
(\mathcal{CP})^\dagger (\mathcal{CP}) = O_k
\]

and

\[
(\mathcal{CP}) | B_s \rangle = e^{i\phi_{\mathcal{CP}}(B_s)} | B_s \rangle
\]

\[
(\mathcal{CP}) | (X_1 X_2)_f \rangle = \eta_f^\mu | (X_1 X_2)_f \rangle^C.
\]

Here \(\phi_{\mathcal{CP}}(B_s)\) parametrizes the applied CP phase convention and \(\eta_f^\mu \in \{-1,+1\}\) denotes the parity eigenvalues of the configurations \(f\) of \(X_1 X_2\). In terms of linear polarization amplitudes \([22]\) (see also \([23]\)) we have \(\eta_f^0 = \eta_f^\parallel = +1\) and \(\eta_f^\perp = -1\) for \(X_1 X_2 \in \{D_s^+ K^-, D_{s1}(2536)^0 K^-, D_{s1}(2420)^0 \}\). In contrast, for \(X_1 X_2 \in \{D_{s1}(2536)^+ K^+, D_{s1}(2420)^+ \}\) we have \(\eta_f^0 = \eta_f^\parallel = -1\) and \(\eta_f^\perp = +1\).

Let us now consider the \(\overline{B}_s\) and \(B_s\) decays into the charge-conjugate quasi two body states \((X_1 X_2)^C\). In the case relevant for the present paper corresponding to \(X_1 X_2 \in \{D_s^+ K^-, D_{s1}(2536)^0 K^-, D_{s1}(2420)^0 \}\) we have \((X_1 X_2)^C \in \{D_s^- K^+, D_{s1}(2536)^0 K^+, D_{s1}(2420)^0 \}\), respectively. If the charge-conjugate states are present in a configuration \(f\) with parity eigenvalue \(\eta_f^\mu\), a similar calculation as sketched above yields

\[
\overline{\mathcal{T}}_f^\mu \equiv \langle (X_1 X_2)_f^C | H_{\text{eff}} (\overline{B}_s \rightarrow (X_1 X_2)^C) | B_s \rangle = \frac{G_F}{\sqrt{2}} v M_f
\]

\[
A_f^\mu \equiv \langle (X_1 X_2)_f^C | H_{\text{eff}} (B_s \rightarrow (X_1 X_2)^C) | B_s \rangle = \eta_f^\mu \, e^{i\phi_{\mathcal{CP}}(B_s)} \frac{G_F}{\sqrt{2}} v^* M_f.
\]

Using these results and the well-known formalism describing \(B_s - \overline{B}_s\) mixing \([1, 24]\), we obtain the following expressions for initially, i.e. at \(t = 0\), present \(B_s\) and \(\overline{B}_s\) mesons:

\[
A_f^*(t) A_f(t) = \frac{G_F^2}{2} |v|^2 \eta_f^\mu \eta_f^\ell M_f^j M_f
\]

\[
\times \left[ |g_+(t)|^2 + \eta_f^\ell \lambda_f^j g_+(t) g_-^*(t) + \eta_f^\mu \lambda_f^j g_+^*(t) g_-(t) + \eta_f^\mu \eta_f^\ell \lambda_f^j \lambda_f^j |g_-(t)|^2 \right]
\]

\[
\overline{\mathcal{T}}_f^*(t) \overline{\mathcal{T}}_f(t) = \frac{G_F^2}{2} |v|^2 \eta_f^\mu \eta_f^\ell M_f^j M_f
\]

\[
\times \left[ |g_-(t)|^2 + \eta_f^\ell \lambda_f^j g_-^*(t) g_+(t) + \eta_f^\mu \lambda_f^j g_+(t) g_-^*(t) + \eta_f^\mu \eta_f^\ell \lambda_f^j \lambda_f^j |g_+(t)|^2 \right]
\]
\[ A_f^c(t) A_f(t) = \frac{G_F^2}{2} |v|^2 M_f^2 \]
\[ \times \left[ |g_-(t)|^2 + \eta_f^\dagger \lambda_f^c g_+^\ast(t) g_-(t) + \eta_f \lambda_f^c g_+(t) g_-^\ast(t) + \eta_f^\dagger \eta_f \lambda_f^c \lambda_f^c |g_+(t)|^2 \right] \]
\[ \overline{A}_f^c(t) \overline{A}_f(t) = \frac{G_F^2}{2} |v|^2 M_f^2 \]
\[ \times \left[ |g_+(t)|^2 + \eta_f^\dagger \lambda_f^c g_+^\ast(t) g_+(t) + \eta_f \lambda_f^c g_+^\ast(t) g_-(t) + \eta_f^\dagger \eta_f \lambda_f^c \lambda_f^c |g_-(t)|^2 \right], \]

where
\[ |g_\pm(t)|^2 = \frac{1}{4} \left[ e^{-\Gamma_L t} + e^{-\Gamma_H t} \pm 2e^{-\Gamma t} \cos(\Delta t) \right] \]
\[ g_+(t)g_-^\ast(t) = \frac{1}{4} \left[ e^{-\Gamma_L t} - e^{-\Gamma_H t} - 2ie^{-\Gamma t} \sin(\Delta t) \right] \]

with $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$. The observable $\lambda_f$ is defined through
\[ \lambda_f \equiv -\eta_f^\dagger e^{-i\Theta^{(s)}_{M_{h2}}} \overline{A}_f \]
\[ \equiv -\eta_f^\dagger e^{-i\Theta^{(s)}_{M_{h2}}} \overline{A}_f \]
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\[ \equiv -\eta_f^\dagger e^{-i\Theta^{(s)}_{M_{h2}}} \overline{A}_f \]

with
\[ \Theta^{(s)}_{M_{h2}} = \pi + 2 \arg(V_{ts}^\ast V_{tb}) - \phi_{CP}(B_s) \]

(23)

denoting the phase of the off-diagonal element of the $B_s - \overline{B}_s$ mass matrix. Combining (22) with (23) and (6) and (7), we observe explicitly that the convention dependent phases $\phi_{CP}(B_s)$ cancel (as they have to!) and arrive at
\[ \lambda_f = \exp \left( -2i \arg(V_{ts}^\ast V_{tb}) \right) \frac{\overline{v}}{v^\ast} \frac{M_f}{\overline{M}_f}. \]

(24)

Correspondingly we have introduced
\[ \lambda_f^c \equiv -\frac{1}{\eta_f^\dagger e^{-i\Theta^{(s)}_{M_{h2}}} \overline{A}_f} \left[ \exp \left( -2i \arg(V_{ts}^\ast V_{tb}) \right) \frac{\overline{v}}{v^\ast} \frac{M_f}{\overline{M}_f}. \right] \]

(25)

Note that $\lambda_f$ and $\lambda_f^c$ can be obtained easily from (24) and (23) by replacing $f$ with $\tilde{f}$.

Real or imaginary parts of bilinear combinations of decay amplitudes like those given in (14)-(19) govern the angular distributions for the decay products of $X_1$ and $X_2$. In this paper we are focussing on untagged angular distributions, where one does not distinguish between initially present $B_s$ and $\overline{B}_s$ mesons. The corresponding observables for $B_s \to X_1 X_2$ and $\overline{B}_s \to (X_1 X_2)^c$ are related to real or imaginary parts of

\[ \left[ A_f^c(t) A_f(t) \right] \equiv \overline{A}_f^c(t) \overline{A}_f(t) + A_f^c(t) A_f(t) = \frac{G_F^2}{4} |v|^2 \eta_f^\dagger \eta_f M_f^2 \]
\[ \times \left[ \left( 1 + \eta_f^\dagger \eta_f \lambda_f^c \lambda_f \right) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + \left( \eta_f^\dagger \lambda_f^c + \eta_f \lambda_f \right) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \]

(26)
and
\[ [A^c_f(t) A^c_f(t)] \equiv \overline{A^c_f}(t) A^c_f(t) + A^c_f(t) A^c_f(t) = \frac{G_F^2}{4} |v|^2 M_f^* M_f \] (27)
\times \left[ \left( 1 + \eta_f \eta_f^* \lambda_f \lambda_f^* \right) (e^{-\Gamma_L t} + e^{-\Gamma_H t}) + \left( \eta_f \lambda_f^* + \eta_f^* \lambda_f \right) (e^{-\Gamma_L t} - e^{-\Gamma_H t}) \right],

respectively. In order to evaluate these equations we have combined (16)-(19) with the explicit time-dependences of (20) and (21). At present such untagged studies are obviously much more efficient from an experimental point of view than tagged analyses. In the distant future it will be feasible to collect also tagged data samples of Bs decays and to resolve the rapid oscillatory $\Delta m_t$-terms. The corresponding tagged observables are given in (16)-(19).

Let us after these general considerations become more specific in the following section. There we give the time-evolutions in terms of linear polarization states and demonstrate that the untagged observables evolving as real or imaginary parts of (26) and (27) suffice to extract the CKM angle $\gamma$.

3 The extraction of the CKM angle $\gamma$

Since it is convenient to give the angular distributions in terms of the linear polarization states $f \in \{0, \|, \perp\}$ (see [12, 23]), let us summarize the corresponding time-dependences in this section. The linear polarization states are characterized by the parity eigenvalues $\eta_f^\pm$. If we introduce the quantity
\[ R_f \equiv |R_f| e^{i\rho_f} \equiv \frac{|\tau|}{v} \frac{M_f}{M_f}, \] (28)
where $\rho_f$ is a CP-conserving strong phase originating from strong final state interaction processes, we have in our specific case $X_1 X_2 \in \{D_s^+K^-S, D_{s1}(2536)^+K^-S, D_{s2}^+K^-S, D^{*0}_b, D_1(2420)^0, D^{*0}_b \}$

\[ \lambda_f = e^{-i\gamma} R_f \] (29)
\[ \lambda_f^c = e^{+i\gamma} R_f, \] (30)

where $\gamma$ is the notoriously difficult to measure CKM angle of the unitarity triangle $[2]$. Using (5) and the Wolfenstein expansion $[10]$ of the CKM matrix by neglecting terms of $O(\lambda^2)$, where $\lambda = \sin \theta_C = 0.22$ is related to the Cabibbo angle, we obtain
\[ R_f = \frac{1}{R_b} \frac{M_f}{M_f} \] (31)
with

\[ R_b \equiv \frac{1}{X} \frac{|V_{ub}|}{|V_{cb}|} \]  \hspace{1cm} (32)

The CKM factor \( R_b \) is constrained by present experimental data to lie within the range \( R_b = 0.36 \pm 0.08 \) \cite{23, 24, 25}.

If we express the hadronic matrix elements \( M_f \) defined by \cite{9} in the form

\[ M_f = |M_f|e^{i\vartheta_f}, \]  \hspace{1cm} (33)

where \( \vartheta_f \) denotes a CP-conserving strong phase shift, the time-dependent untagged observables corresponding to the linear polarization states \cite{12} are in the case of \( B_s \to X_1 X_2 \) given by

\[ [A_0(t)]^2 = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0|^2 \times \left[ (1 + |R_0|^2) \left(e^{-i\lambda t} + e^{-i\mu t}\right) + 2|R_0| \cos(\rho_0 - \gamma) \left(e^{-i\mu t} - e^{-i\mu t}\right) \right] \hspace{1cm} (34) \]

\[ [A_\| (t)]^2 = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_\||^2 \times \left[ (1 + |R_\||^2) \left(e^{-i\lambda t} + e^{-i\mu t}\right) + 2|R_\|| \cos(\rho_\| - \gamma) \left(e^{-i\mu t} - e^{-i\mu t}\right) \right] \hspace{1cm} (35) \]

\[ [A_\bot (t)]^2 = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_\bot|^2 \times \left[ (1 + |R_\bot|^2) \left(e^{-i\lambda t} + e^{-i\mu t}\right) - 2|R_\bot| \cos(\rho_\bot - \gamma) \left(e^{-i\mu t} - e^{-i\mu t}\right) \right] \hspace{1cm} (36) \]

\[ A_\| A_\bot (t) = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_\| |M_\bot| e^{i(\vartheta_\| - \vartheta_\bot)} \left[ (1 + |R_\| |R_\bot| e^{i(\rho_\| - \rho_\bot)}) \right] \times \left( e^{-i\lambda t} + e^{-i\mu t} \right) + \left( |R_\| e^{i(\gamma - \rho_\|)} + |R_\bot| e^{-i(\gamma - \rho_\bot)} \right) \left( e^{-i\mu t} - e^{-i\mu t} \right) \]  \hspace{1cm} (37)

\[ A_\| A_\bot (t) = -\frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_\| |M_\bot| e^{i(\vartheta_\| - \vartheta_\bot)} \left[ (1 - |R_\| |R_\bot| e^{i(\rho_\| - \rho_\bot)}) \right] \times \left( e^{-i\lambda t} + e^{-i\mu t} \right) + \left( |R_\| e^{i(\gamma - \rho_\|)} - |R_\bot| e^{-i(\gamma - \rho_\bot)} \right) \left( e^{-i\mu t} - e^{-i\mu t} \right) \]  \hspace{1cm} (38)

\[ A_0^* (t) A_\bot (t) = -\frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0| |M_\bot| e^{i(\vartheta_\bot - \vartheta_0)} \left[ (1 - |R_\bot| |R_\bot| e^{i(\rho_\bot - \rho_\bot)}) \right] \times \left( e^{-i\lambda t} + e^{-i\mu t} \right) + \left( |R_\bot| e^{i(\gamma - \rho_\bot)} - |R_\bot| e^{-i(\gamma - \rho_\bot)} \right) \left( e^{-i\mu t} - e^{-i\mu t} \right) \]  \hspace{1cm} (39)
For the untagged decays into the charge conjugate two body states we obtain on the other hand the following expressions:

\[ [A_0^c(t)]^2 = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0|^2 \]
\[ \times \left[ (1 + |R_0|^2) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + 2|R_0| \cos(\rho_0 + \gamma) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \quad (40) \]

\[ [A_0^c(t)]^2 = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0|^2 \]
\[ \times \left[ (1 + |R_0|^2) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + 2|R_0| \cos(\rho_0 + \gamma) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \quad (41) \]

\[ [A_0^c(t)]^2 = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0|^2 \]
\[ \times \left[ (1 + |R_0|^2) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + 2|R_0| \cos(\rho_0 + \gamma) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \quad (42) \]

\[ [A_0^c(t)A_0^c(t)] = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0||M_0| e^{i(\theta_1 - \theta_0)} \left[ (1 + |R_0||R_0|e^{i(\rho_0 - \rho_0)}) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + (|R_0|e^{-i(\gamma + \rho_0)} + |R_0|e^{i(\gamma + \rho_0)}) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \quad (43) \]

\[ [A_0^c(t)A_0^c(t)] = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0||M_0| e^{i(\theta_1 - \theta_0)} \left[ (1 + |R_0||R_0|e^{i(\rho_0 - \rho_0)}) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + (|R_0|e^{-i(\gamma + \rho_0)} - |R_0|e^{i(\gamma + \rho_0)}) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \quad (44) \]

\[ [A_0^c(t)A_0^c(t)] = \frac{G_F^2}{4} |V_{ub}V_{cs}|^2 |M_0||M_0| e^{i(\theta_1 - \theta_0)} \left[ (1 + |R_0||R_0|e^{i(\rho_0 - \rho_0)}) \left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) + (|R_0|e^{-i(\gamma + \rho_0)} - |R_0|e^{i(\gamma + \rho_0)}) \left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) \right] \quad (45) \]

Combining these equations appropriately – each of them represents a certain measurement – a determination of \( \gamma \) and of the strong phase shifts is possible without using any additional input. This can be seen as follows:

Let us consider the untagged observables corresponding to (34), (35) and to the real part of (37). From these rates the ratios of the coefficients of \( e^{-\Gamma_L t} - e^{-\Gamma_H t} \) and of \( e^{-\Gamma_L t} + e^{-\Gamma_H t} \) can be determined. The overall normalizations of these rates cancel in the ratios which are given by

\[ u_f \equiv \frac{2|R_f| \cos(\rho_f - \gamma)}{1 + |R_f|^2} \quad (f \in \{0, |\}) \quad (46) \]
and

\[ u_{0,||} \equiv \frac{|R_0| \cos(\vartheta_\parallel - \vartheta_0 - \rho_0 + \gamma) + |R_\parallel| \cos(\vartheta_\parallel - \vartheta_0 + \rho_0 - \gamma)}{\cos(\vartheta_\parallel - \vartheta_0) + |R_0||R_\parallel| \cos(\vartheta_\parallel - \vartheta_0 + \rho_0 - \rho_0)}, \quad (47) \]

respectively, and depend thus only on \(|R_0|, \rho_0, |R_\parallel|, \rho_\parallel, \vartheta_\parallel - \vartheta_0\) and on the CKM angle \(\gamma\). Using in addition the observables of the untagged \(B_s\) decays into the charge conjugate final states that are related to (40), (41) and to the real part of (43), we can determine similar ratios of the coefficients of \(e^{-\Gamma_L t} - e^{-\Gamma_H t}\) and \(e^{-\Gamma_L t} + e^{-\Gamma_H t}\). These charge conjugate ratios, which are given by

\[ u^C_f \equiv \frac{2|R_f| \cos(\rho_f + \gamma)}{1 + |R_f|^2} \quad (f \in \{0, ||\}) \quad (48) \]

and

\[ u^C_{0,||} \equiv \frac{|R_0| \cos(\vartheta_\parallel - \vartheta_0 - \rho_0 - \gamma) + |R_\parallel| \cos(\vartheta_\parallel - \vartheta_0 + \rho_\parallel + \gamma)}{\cos(\vartheta_\parallel - \vartheta_0) + |R_0||R_\parallel| \cos(\vartheta_\parallel - \vartheta_0 + \rho_\parallel - \rho_0)}, \quad (49) \]

respectively, depend on the same six “unknowns” as (40) and (43) determined from (54), (55) and (57). We have therefore six observables at our disposal to determine the six “unknowns” \(|R_0|, \rho_0, |R_\parallel|, \rho_\parallel, \vartheta_\parallel - \vartheta_0, \gamma\). In particular we are in a position to extract the CKM angle \(\gamma\). Using furthermore the observables we have not considered so far, certain discrete ambiguities are resolved and also \(|R_\perp|, \rho_\perp, \vartheta_\perp - \vartheta_0\) can be determined. Note that the overall normalizations of the rates corresponding to (54)-(56) inform us about \(|V_{ub}V_{cs}^*}|\cdot|M_f|\), where \(f \in \{0, ||, \perp\}\).

Obviously the major goal of this approach is the extraction of the CKM angle \(\gamma\). However, also the the quantities \(|R_f|\) and the strong phase shifts \(\rho_f, \vartheta_f\) are of interest, since they allow insights into the hadronization dynamics of the corresponding four-quark operators.

## 4 CP violation

There are many CP-violating observables that can be constructed from tagged time-dependent measurements. Some of them survive even when only untagged data samples are used. The most striking untagged CP-violating observable is

\[
\text{Im}\left\{ [A_f^+(t) A_\perp(t)] \right\} + \text{Im}\left\{ [A_f^+(t) A_\parallel(t)] \right\} = -\frac{G_f^2}{2} |V_{ub}V_{cs}|^2 |M_f| |M_\perp| \\
\times \{|R_f| \cos(\rho_f + \vartheta_f - \vartheta_\perp) + |R_\perp| \cos(\rho_\perp + \vartheta_\perp - \vartheta_f)\} \left(e^{-\Gamma_L t} - e^{-\Gamma_H t}\right) \sin \gamma, \quad (50)
\]

where \(f \in \{0, ||\}\). Note that the plus sign on the l.h.s. of that equation is due to the fact that the parity eigenvalues of the final state configurations \(f\) and \(\perp\) arising in the “mixed” combinations are different. The CP observable (50) is proportional to \(\sin \gamma\) and occurs...
even when all strong phase shifts vanish. This CP-violating effect can be potentially very large as can be seen by employing the factorization assumption which implies vanishing strong phase shifts.

In contrast, to observe CP violation in the untagged interference term involving final state configurations with equal parity eigenvalues requires non-vanishing strong phase shifts as can be seen from the corresponding CP-violating observable

\[
\text{Re}\left\{[A_0^e(t) A_||^e(t)] - [A_0^e(t) A_||^e(t)]\right\} = \frac{G_F^2}{2} |V_{ub}V_{cs}|^2 |M_0||M_||
\times \left\{|R_0| \sin(\rho_0 + \vartheta_0 - \vartheta_||) + |R_||| \sin(\rho_|| + \vartheta_|| - \vartheta_0)\right\} \left(e^{-\Gamma_{Lt}} - e^{-\Gamma_{Ht}}\right) \sin\gamma.
\]

The last category of CP-violating effects in untagged data samples is related to

\[
\left|[A_f(t)]^2 \right| - \left|[A_f^e(t)]^2 \right| = \eta_f^G G_F^2 |V_{ub}V_{cs}|^2 |M_f|^2 |R_f| \sin \rho_f \left(e^{-\Gamma_{Lt}} - e^{-\Gamma_{Ht}}\right) \sin\gamma
\]

with \( f \in \{0, ||, \perp\} \) and requires also non-vanishing strong phase shifts. This last category is the only one that has been considered so far in the literature [7].

5 Summary

We have calculated the time-dependences of the observables of angular distributions for \(B_s\) decays caused by \(\bar{b} \rightarrow \bar{c}u\bar{s}\) quark-level transitions both in a general notation and in terms of linear polarization states. Examples for exclusive modes belonging to this decay category are the color-allowed and color-suppressed channels \((B_s \rightarrow D_s^{*\pm} K^{*\mp}, D_{s1}(2536)^{\pm} K^{*\mp}, D_s^{*\pm} K^{*\mp}\) and \((B_s \rightarrow D_s^{*0} \phi, D_1(2420)^0 \phi, D_s^{*0} \phi\)

\)

respectively. Since charged particles are easier to detect for generic detectors than the photon(s) in the strong or electromagnetic decays of \(D_s^*\) and \(D_s^{*0}\), we have also listed higher resonances exhibiting significant all-charged final states. The information that is provided by the corresponding angular correlations allows – without any theoretical input – the extraction both of the notoriously difficult to measure CKM angle \(\gamma\) and of the whole hadronization dynamics of these decays thereby allowing e.g. tests of the factorization hypothesis.

If the lifetime difference between the \(B_s\) mass eigenstates \(B_L^s\) and \(B_H^s\) is sizable, as is indicated by certain present theoretical analyses, even untagged \(B_s\) data samples suffice to accomplish this ambitious task. Interestingly, some of the many CP-violating observables that can be constructed from tagged measurements survive also in that untagged case and are potentially very large. One class of these untagged CP-violating observables is proportional to \(\sin\gamma\) and arises even when all strong phase shifts vanish.
From an experimental point of view, untagged analyses of $B_s$-meson decays are obviously much more efficient than tagged studies. The feasibility of our untagged strategies for extracting $\gamma$ in a clean way depends, however, crucially on a sizable lifetime difference of the $B_s$ system. Even if this lifetime splitting should turn out to be too small for untagged analyses, once a non-vanishing lifetime difference has been established experimentally, the formalism presented in our paper must be used in the case of tagged measurements in order to extract $\gamma$ correctly. Clearly time will tell and an exciting future may lie ahead of us.

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