Excited Heavy Baryons and Their Symmetries III: Phenomenology

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Phenomenological applications of an effective theory of low-lying excited states of charm and bottom isoscalar baryons are discussed at leading and next-to-leading order in the combined heavy quark and large $N_c$ expansion. The combined expansion is formulated in terms of the counting parameter $\lambda \sim 1/m_Q, 1/N_c$; the combined expansion is in powers of $\lambda^{1/2}$. We work up to next-to-leading order. We obtain model-independent predictions for the excitation energies, the semileptonic form factors and electromagnetic decay rates. At leading order in the combined expansion these observables are given in terms of one phenomenological constant which can be determined from the excitation energy of the first excited state of $\Lambda_c$ baryon. At next-to-leading order an additional phenomenological constant is required. The spin-averaged mass of the doublet of the first orbitally excited state of $\Lambda_b$ is predicted to be approximately 5920 MeV. It is shown that in the combined limit at leading and next-to-leading order there is only one independent form factor describing $\Lambda_b \to \Lambda_c \ell \bar{\nu}$; similarly, $\Lambda_b \to \Lambda_c^* \ell \bar{\nu}$ and $\Lambda_b \to \Lambda_{c1} \ell \bar{\nu}$ decays are described by a single independent form factor. These form factors are calculated at leading and next-to-leading order in the combined expansion. The value of the $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ form factor at zero recoil is predicted to be 0.998 at leading order which is very close to HQET value of unity. The electromagnetic decay rates of the first excited states of $\Lambda_c$ and $\Lambda_b$ are determined at leading and next-to leading order. The ratio of radiative decay rates $\Gamma(\Lambda_c^* \to \Lambda_c \gamma)/\Gamma(\Lambda_{c1} \to \Lambda_b \gamma)$ is predicted to be approximately 0.2, greatly different from the heavy quark effective theory value of unity.

I. INTRODUCTION AND REVIEW

This is a third in a series of articles in which heavy baryons (baryons containing a single heavy quark) are treated in the combined heavy quark and large $N_c$ expansion. In this limit, the heavy baryon subspace of the QCD Hilbert space, spanned by low-lying states with baryon number one and heavy quark number one, exhibits an additional symmetry, viz. a contracted $O(8)$ symmetry. This symmetry connects orbitally excited states of heavy baryons to the ground state. This symmetry is distinct from the well-known $SU(2N_f)$ light quark spin-flavor symmetry which connects heavy baryon states with light degrees of freedom in different spin and isospin states, as well as from the well-known $SU(2N_c)$ heavy quark spin-flavor symmetry. An effective theory based on the contracted $O(8)$ symmetry was developed in Ref. [1]. In the real world neither the heavy quark mass, $m_Q$, nor the number of colors, $N_c$, is infinite. However, if they are large enough, the heavy baryons should possess an approximate symmetry and the effective theory should reliably describe heavy baryon phenomenology. It is the goal of this paper to make predictions about spectroscopy and electroweak decays of heavy baryons based on the effective theory. The focus here will be on the phenomenology of $\Lambda_c$ and $\Lambda_b$ baryons and their excited states. The light degrees of freedom in these baryons are in the spin and isospin ground state which simplifies the formalism. The generalization to the higher spin and isospin states presents no conceptual difficulties and can be done by combining the contracted $O(8)$ symmetry with the light quark spin-flavor symmetry.

The emergence of the new symmetry can be seen in a class of models of a heavy baryon (which will be referred to as the bound state picture) in which the heavy baryon is described as a bound state of a heavy meson and an ordinary baryon. For example, the $\Lambda_c$ baryon is described as a bound state of a $D$ or $D^*$ meson plus the nucleon. In the combined limit, the heavy meson and the nucleon have large masses (the nucleon mass $m_N$ goes to infinity in the large $N_c$ limit). The reduced mass of the meson-nucleon system is large, which in turn leads to a highly localized wave function. As a result, the potential for the low-lying excited states of the heavy baryon can be approximated by an harmonic potential. While the reduced mass of the system is of order $N_c$, the potential is of order unity. Hence, excitation energies are of order $N_c^{-1/2}$ and vanish as $N_c$ goes to infinity. The contracted $O(8)$ group is generated by the harmonic oscillator creation and annihilation operators and their bilinears. In the limit where excited states become degenerate with the ground state, the contracted $O(8)$ group becomes the symmetry group of the system.

The connection between QCD and the bound state picture is obscure. However, as was shown in Ref. [1], the contracted $O(8)$ symmetry also emerges from QCD in the combined limit in a model-independent manner. This emergent symmetry along with self-consistent counting rules of Ref. [2] can be used to write down an effective
Hamiltonian which describes the low-energy excited states of heavy baryons [2]. The effective expansion is formulated in terms of a small parameter $\lambda$ which scales as:

$$\lambda \sim 1/N_c, \quad \Lambda/m_Q,$$

(1.1)

where $\Lambda$ is a typical hadronic scale. The actual expansion parameter turns out to be $\lambda^{1/2}$. The ordering of heavy quark and large $N_c$ limits is formally irrelevant; the expansion is valid for an arbitrary value of $N_c, \Lambda/m_Q$.

In the combined limit the heavy baryons contain two types of excitations—collective and non-collective. The collective excitations can be interpreted as the coherent motion of the brown muck (the technical term for the light degrees of freedom) relative to the heavy quark. In the bound state picture, these excitations are described as the motion of the heavy meson relative to an ordinary baryon. The non-collective excitations are internal excitations of the brown muck itself. In the bound state picture this corresponds to using an excited baryon in place of the nucleon (e.g. $N^*$). As shown in Ref. [2], the typical excitation energy of the collective degrees of freedom near the combined limit is of order $\lambda^{1/2}$, while the typical non-collective excitations are of order $\lambda^0$. The Hilbert space near the combined limit is spanned by states of the form:

$$|C, I\rangle = |C\rangle \otimes |I\rangle$$

(1.2)

where $C$ and $I$ stand for collective and intrinsic excitations, respectively. The low-energy excited heavy baryons can be identified with states of the form of eq. (1.2) in which only collective degrees of freedom are excited, i.e. $|C, 0_I\rangle$ ($0_I$ denotes the ground state of the brown muck).

The effective Hamiltonian which describes the low-energy degrees of freedom contains operators which only excite the collective degrees of freedom. It turns out that these operators can be defined in a model-independent way in terms of the QCD operators near the combined limit [3].

One group of such operators consists of the total momentum and position operators of the heavy baryon: $\vec{P}$ and $\vec{X}$. These operators are conserved quantities and are well defined; they are the translation operators in position and momentum space. Another useful pair of operators—$(\vec{P}_Q, \vec{X}_Q)$—is defined as,

$$\vec{P}_Q = \int d^3x \, Q^\dagger(x)(-i\vec{D})Q(x),$$

$$\vec{X}_Q = \int d^3x \, Q^\dagger(x)\vec{x}Q(x),$$

(1.3)

where $Q(x)$ is the heavy quark field, and $\vec{D}$ is the three-dimensional covariant derivative. The definitions in eq. (1.3) are valid in the combined limit for the subspace of states with a heavy quark number one. The operators $\vec{P}_Q$ and $\vec{X}_Q$ are (up to corrections of order $\lambda$) the translation operators for the heavy quark in position and momentum space. The corrections arise because the heavy quark can not be translated independently from the rest of the heavy baryon. These corrections are of relative order $\lambda$ and will be neglected in this paper, since we are working only up to relative order $\lambda^{1/2}$. These two pairs of operators can be used to define additional collective variables. The operator $\vec{P}_t = \vec{P} - \vec{P}_Q$ and its canonical conjugate operator $\vec{X}_t$ can be interpreted as the momentum and position of the brown muck. This interpretation of $\vec{X}_t$ makes sense when the coherent motion of the brown muck is considered. The total position operator $\vec{X}$ can be expressed in terms of the operators $\vec{X}_Q$ and $\vec{X}_t$ [3]:

$$\vec{X} = \left(\frac{m_H}{m_N + m_H} \vec{X}_Q + \frac{m_N}{m_N + m_H} \vec{X}_t\right)(1 + \mathcal{O}(\lambda)),$$

(1.4)

where $m_H$ and $m_N$ are the masses of the heavy meson and the nucleon. In the combined limit these masses diverge as $\lambda^{-1}$.

There exists an additional pair of useful conjugate operators which can be defined in a model-independent way [3]:

$$\vec{p} = \left(\frac{m_H}{m_H + m_N} \vec{p} - \vec{P}_Q\right)(1 + \mathcal{O}(\lambda)),$$

$$\vec{x} = \left(1 + \frac{m_H}{m_N}\right) \left(\vec{X} - \vec{X}_Q\right)(1 + \mathcal{O}(\lambda)),$$

(1.5)

The operators $\vec{x}$ and $\vec{p}$ can be interpreted as the position and momentum operators of the brown muck relative to the heavy quark. These operators commute with operators $\vec{X}$ and $\vec{P}$ (up to corrections of order $\lambda$). The operators defined in eq. (1.5) can be re-expressed in terms of $\vec{X}_Q$, $\vec{P}_Q$, $\vec{X}_t$ and $\vec{P}_t$.

2
\[ \ddot{x} = \dddot{X}_t - \dddot{X}_Q, \]
\[ \ddot{p} = \left( \frac{m_N}{m_N + m_H} \dddot{P}_t - \frac{m_H}{m_N + m_H} \dddot{P}_Q \right) (1 + O(\lambda)). \]  

(1.6)

While the definitions in eqs. (1.3), (1.5) and (1.6) closely resemble similar relations in the bound state picture, they are well-defined (up to higher order corrections) QCD operators that excite the collective degrees of freedom. As shown in Ref. [1], the power counting rules which lead to the effective Hamiltonian consistent with the contracted \( O(8) \) symmetry are:

\[
(x, X, X_Q, X_t) \sim \lambda^{1/4},
\]
\[
(p, P, P_Q, P_t) \sim \lambda^{-1/4}.
\]  

(1.7)

This means that the typical matrix elements of these operators between the low-lying states scale as \( \lambda^{1/4} \) and \( \lambda^{-1/4} \) (for position and momentum operators, respectively). Based on these counting rules and the structure of the Hilbert space in the combined limit, the most general effective Hamiltonian up to terms of order \( \lambda \) is given by [3]:

\[
H_{\text{eff}} = (m_H + m_N) + c_0 + \left( \frac{c^2}{2(m_N + m_H)} + \frac{c^2}{m_Q} + \frac{1}{2} \kappa x^2 \right) + \frac{1}{4} \alpha x^4 + O(\lambda^{3/2}),
\]  

(1.8)

where \( H_{\lambda^0} \) refers to the piece of the Hamiltonian whose contribution is of order \( \lambda^0 \); the reduced mass is given by,

\[
\mu_Q = \frac{m_N m_H}{m_N + m_H},
\]  

(1.9)

which is of order \( \lambda^{-1} \), since \( m_H \) and \( m_N \) scale as \( \lambda^{-1} \). The coefficients \( c_0, \kappa \) and \( \alpha \) are of order \( \lambda^0 \). These parameters are flavor independent at order \( \lambda^0 \). However, they contain \( 1/m_Q \) corrections due to the order \( \lambda \) ambiguity both in threshold mass, \( (m_N + m_H) \), and in the interaction of the brown muck and the heavy quark. The \( 1/m_Q \sim \lambda \) corrections to the constants \( \kappa \) and \( \alpha \) do not contribute at order \( \lambda \) as seen from eq. (1.8). Hence, they can be neglected when we work up to terms of order \( \lambda \) in the effective Hamiltonian. On the other hand, the \( 1/m_Q \) correction to \( c_0 \) contribute at order \( \lambda \), i.e. at the same order as the \( \alpha x^4/4! \). Since \( c_0 \) is an overall constant, the dynamics is determined by the last three terms in the effective Hamiltonian containing only two phenomenological constants—\( \kappa \) and \( \alpha \).

The first term in \( H_{\lambda^{1/2}} \) describes the center of mass motion of the entire system and is irrelevant for the internal dynamics of the collective degrees of freedom. However, the counting rules require this kinetic term to be of order \( \lambda^{1/2} \). As discussed later, this restriction on \( \dddot{P} \) means that the predictions of our effective theory for electroweak observables are valid only for velocity transfers of order \( \lambda^{3/4} \). The terms of order \( \lambda^{1/2} \) will be referred to as leading order (LO). The last term is of order \( \lambda \); it will be referred to as next-to-leading order (NLO). In this paper, we work up to NLO, i.e. we include corrections of relative order \( \lambda^{1/2} \). The corrections of relative order \( \lambda \), which contribute only at next-to-next-to-leading order (NNLO) may be neglected. As can be seen from eq. (1.8), the effective expansion is an expansion in powers of \( \lambda^{1/2} \) and not \( \lambda \). This is ultimately connected to the intrinsic scales that determine the dynamics: the heavy quark and the brown muck masses are of order \( \lambda^{-1} \) while the coupling is of order unity, so that the harmonic-like excitations are of order \( \lambda^{1/2} \).

In addition to the effective Hamiltonian, a number of other important operators (which determine the electroweak decays of heavy baryons) can be expressed in terms of the collective variables of eqs. (1.3), (1.4), (1.5) and (1.6). In subsequent sections, we will apply the effective theory to study the phenomenology of the low-lying heavy baryon states. The leading order results of our effective theory reproduce those obtained in the bound state model of heavy baryons [7, 25]. However, the effective theory treatment has the advantage of being model independent. Moreover, it can be consistently extended to higher orders. In this work, we extend the predictions to next-to-leading order. These corrections contribute at relative order \( \lambda^{1/2} \) and not \( \lambda \) as was previously believed [13].

It is important to emphasize that while \( \lambda \) is formally a small number, in the real world \( N_c = 3 \), so that the expansion parameter \( \lambda^{1/2} \sim 1/\sqrt{3} \) is not particularly small. Therefore, predictions of the effective theory even at NLO may be relatively crude. Moreover, the effective theory can only be useful for describing the ground and the first excited state of heavy baryons. The second orbitally excited state is either unbound or very near the threshold and clearly beyond the harmonic limit implicit in the effective theory. One also has to keep in mind that in the combined expansion \( 1/m_Q \) and \( 1/N_c \) corrections are formally of the same order. However, on phenomenological grounds, the heavy quark limit seems to be more closely reproduced in nature than the large \( N_c \) limit. Of course, the ultimate test of the effectiveness of the combined expansion is how well it reproduces experiment. At present, there is not enough data to critically test the theory, especially in the bottom sector.
In the next section, we discuss the spectroscopy of the low-lying states of \( \Lambda_c \) and \( \Lambda_b \) baryons based on the effective Hamiltonian in eq. (1.8). In Sec. III, we consider the semileptonic decays of these baryons. In Sec. IV, we further apply the effective theory to determine the radiative decay rates of the heavy baryons. In Sec. V, we present a summary of our results.

II. SPECTROSCOPY OF HEAVY BARYONS

The effective Hamiltonian in eq. (1.4) exhibits a number of symmetries which determine the quantum numbers and wave functions of \( \Lambda_c \) and \( \Lambda_b \) baryons and their low-energy excited states. The contracted \( O(8) \) symmetry is generated by the set of 28 generators \( \{ a_i, a_i^\dagger, T_{ij}, S_{ij}, S_{ij}^3 \} \), where \( T_{ij} = a_i^\dagger a_j^\dagger \) and \( S_{ij} = a_i a_j \) and \( i, j = 1, 2, 3 \). The operators \( a_j \) and \( a_j^\dagger \)—creation and annihilation operators of the three-dimensional harmonic oscillator—are defined as:

\[
a_j = \sqrt{\mu Q \omega Q} \frac{1}{2} \hbar \omega Q \right) x_j + i \sqrt{\frac{1}{2\mu Q \omega Q}} \hbar p_j , \quad a_j^\dagger = \sqrt{\mu Q \omega Q} \frac{1}{2} \hbar \omega Q \right) x_j - i \sqrt{\frac{1}{2\mu Q \omega Q}} \hbar p_j ,
\]

where \( \mu, \omega, \bar{x} \) are defined in eqs. (1.3) and (1.5). \( \omega_Q = (\kappa/\mu Q)^{1/2} \). This symmetry is broken by terms of order \( \lambda^{1/2} \) in the effective Hamiltonian of eq. (1.8). The \( U(3) \) symmetry—a subgroup of \( O(8) \) and the symmetry group of the three dimensional harmonic oscillator—is broken only at next-to-leading order. Therefore, a useful basis to compute physical observables is an eigenbasis of the effective Hamiltonian at order \( \lambda^{1/2} \), namely the harmonic oscillator basis. This basis can be parameterized by the eigenvalues of the Casimir operators of the chain of the subgroups of \( U(3) \supset O(3) \supset U(1) \), i.e., \( N = T_{11} + T_{22} + T_{33}, L^2 = L_1^2 + L_2^2 + L_3^2 \) and \( L_3 \) (where \( L_3 \equiv -it_{ij}T_{jk} \)).

In the heavy quark effective theory (HQET), \( \Lambda_c \) and \( \Lambda_b \) baryons and their excited states exhibit the \( SU(4) \) heavy quark spin-flavor symmetry. This symmetry is broken by corrections of order \( 1/m_Q \). As a result, at leading order in the pure heavy quark expansion, the excitation energies of \( \Lambda_c \) and \( \Lambda_b \) baryons are equal. In addition, the states which differ by a heavy quark spin flip are degenerate. The physical origin of this symmetry is quite simple. At leading order in HQET, the heavy quark acts as a source of static color field which is independent of the heavy quark spin and flavor. The heavy quark pair creation and spin-dependent chromomagnetic interactions—effects that break spin-flavor symmetry—appear only at the next-to-leading order in HQET (i.e. at \( O(1/m_Q) \)). However, in the combined heavy quark and large \( N_c \) expansion, the flavor symmetry is already broken at leading nontrivial order via the flavor dependence of the reduced mass \( \mu_Q \). The heavy quark is no longer a static color source but is coupled to the brown muck at leading order in the dynamics. However, this leading order dynamics is still independent of the heavy quark spin. The spin-dependent chromomagnetic effects contribute at NNLO. Thus, the heavy quark spin, \( s_Q \), is a good quantum number up to NNLO.

The eigenstates of the leading order effective Hamiltonian, eq. (1.4), can be labeled by \( |\Lambda_Q; N, l, m; \sigma \rangle \), where \( N, l, m \) are the quantum numbers of the three-dimensional harmonic oscillator and \( \sigma \) is the third component of the heavy quark spin \( (\sigma = \pm \frac{1}{2}) \). The wave functions of these eigenstates, which will be referred to as the collective wave functions, are:

\[
\Psi_{Nlm,\sigma}(\bar{x}) = Ar^l e^{-r^2/2\mu_Q} F\left( -\frac{N-l}{2}, l + \frac{3}{2}, r^2/2\mu_Q \right) \chi(\sigma),
\]

where \( A \) is a normalization constant, \( F(n, k; z) \) is a hypergeometric function and \( \chi(\sigma) \) is a normalized spinor wave function. The spatial part of the wave function in eq. (2.2) is the wave function of the three-dimensional harmonic oscillator. There are \( (N+1)(N+2)/2 \) degenerate eigenstates for a given \( N \).

The heavy baryon states have definite total angular momentum \( J \) and can be labeled as, \( |\Lambda_Q; N, J, J_z \rangle \). These states can be written as linear superpositions of the states \( |\Lambda_Q; N, l, m; \sigma \rangle \). The brown muck in isoscalar charm and bottom baryons—\( \Lambda_c \) and \( \Lambda_b \) baryons and their excited states—has zero spin and isospin. For these states, the total angular momentum \( J = l + s_Q = |l| + 1/2 \). The collective wave functions for these states are the linear superposition of the wave functions in eq. (2.2):

\[
\Phi_{N,l+1/2,J_z} = \sqrt{\frac{J + J_z}{2J_z}} \Psi_{N,J_z-1/2,\sigma=1/2} + \sqrt{\frac{J - J_z}{2J_z}} \Psi_{N,J_z+1/2,\sigma=-1/2},
\]

\[
\Phi_{N,l-1/2,J_z} = \sqrt{\frac{J - J_z + 1}{2J_z + 2}} \Psi_{N,J_z-1/2,\sigma=1/2} + \sqrt{\frac{J + J_z + 1}{2J_z + 2}} \Psi_{N,J_z+1/2,\sigma=-1/2},
\]

where the square factors are the appropriate Clebsch-Gordan coefficients.
In the charm sector, the states $|\Lambda_c;0,\frac{1}{2}, J_z\rangle$, $|\Lambda_c;1,\frac{1}{2}, J_z\rangle$ and $|\Lambda_c;1,\frac{3}{2}, J_z\rangle$ correspond to the experimentally observed states $\Lambda_c (J^P = \frac{1}{2}^+)\), $\Lambda_c (2593) (J^P = \frac{1}{2}^-)$ and $\Lambda_c (2625) (J^P = \frac{3}{2}^-)$, respectively [26,28]. The states $|\Lambda_c;0,\frac{1}{2}, J_z\rangle$ and $|\Lambda_c;1,\frac{1}{2}, J_z\rangle$ are degenerate in the combined heavy quark and large $N_c$ limit due to the heavy quark spin symmetry valid up to NNLO. The same parameterization of states exist in the bottom sector; however only the ground state, $\Lambda_b (J^P = \frac{1}{2}^+)$, has been observed to date. The lack of the observation of the excited states limits our ability to test the theory at present. For further convenience a shorthand notation for heavy baryon states will be used. The ground state will be denoted as,

$$|\Lambda_Q\rangle \equiv |\Lambda_Q; 0, \frac{1}{2}, J_z\rangle,$$

and the doublet of the first excited state will be denoted as,

$$|\Lambda_{Q1}\rangle \equiv |\Lambda_Q; 1, \frac{1}{2}, J_z\rangle,$$

$$|\Lambda_{Q1}^*\rangle \equiv |\Lambda_Q; 1, \frac{3}{2}, J_z\rangle.$$

(2.4)

(2.5)

The expansion of the effective Hamiltonian, eq. (1.8), including terms of order $\lambda$ contains three phenomenological parameters—$c_0$, $\kappa$, $\alpha$. The first two terms in eq. (1.8) determine the dissociation threshold of the heavy baryon into a heavy meson and an ordinary baryon. There are two heavy mesons for each heavy quark flavor—pseudoscalar and pseudovector mesons. In the charm sector they correspond to $D$ and $D^*$ mesons and in the bottom sector to $B$ and $B^*$. The pseudoscalar and pseudovector mesons differ due to the heavy quark spin flip. In the combined limit these mesons are degenerate up to corrections of relative order $\lambda$. This leads to an ambiguity in the energy of the dissociation threshold. One natural way to define this threshold is to use the spin-averaged mass of the meson doublet:

$$m_B \equiv \frac{1}{4} m_D + \frac{3}{4} m_{D^*} \approx 1980 \text{ MeV},$$

$$m_B \equiv \frac{1}{4} m_B + \frac{3}{4} m_{B^*} \approx 5310 \text{ MeV},$$

(2.6)

where the masses are averaged over one spin state of a pseudoscalar meson and three states of the pseudovector meson. Similarly, the spin-averaged mass of the doublet of the first orbitally excited states $\Lambda_{Q1}$ and $\Lambda_{Q1}^*$ is given by,

$$m_{\Lambda_Q^*} \equiv m_{\Lambda_Q^*} + \frac{2}{3} m_{\Lambda_{Q1}},$$

(2.7)

where the mass is averaged over the two spin states with $J^P = \frac{1}{2}^-$ and four possible states with $J^P = \frac{3}{2}^-$. The spin-averaged mass of the $\Lambda_{c1}$ and $\Lambda_{c1}^*$ is:

$$m_{\Lambda_{c1}} = \frac{1}{3} m_{\Lambda_{c1}} + \frac{2}{3} m_{\Lambda_{c1}^*} \approx 2610 \text{ MeV}.$$

(2.8)

The ground state mass and the spin-averaged mass of the first excited state can be determined from the effective Hamiltonian in eq. (1.8) including terms of order $\lambda$:

$$m_{\Lambda_Q} = m_N + m_H + c_0 + \frac{3}{2} \sqrt{\frac{\kappa}{\mu_Q}} + \frac{15}{96} \alpha \kappa \mu_Q + \mathcal{O}(\lambda^{3/2}),$$

$$m_{\Lambda_Q^*} = m_N + m_H + c_0 + \frac{5}{2} \sqrt{\frac{\kappa}{\mu_Q}} + \frac{35}{96} \alpha \kappa \mu_Q + \mathcal{O}(\lambda^{3/2}),$$

(2.9)

where the mass $m_H$ is the spin-averaged mass of the meson doublet, eq. (2.6). The $\mathcal{O}(\lambda^{1/2})$ term in eq. (2.9) corresponds to the energy of the ground and the first excited states of the three dimensional harmonic oscillator. The correction of order $\lambda$ is obtained by treating the term $\alpha \lambda x^4/4!$ perturbatively.

The expressions for the heavy baryon masses in eq. (2.9) lead to a number of predictions at each order in the combined expansion. At order $\lambda^0$, i.e. neglecting all nontrivial terms, we have the following relation:

$$(m_{\Lambda_b} - m_{\Lambda_c}) - (m_B - m_D) = \mathcal{O}(\lambda^{1/2}).$$

(2.10)

This is a well-known result in HQET which is a consequence of the heavy quark flavor symmetry. In the combined limit, the flavor symmetry is already broken at order $\lambda^{1/2}$. It is important to emphasize that the prediction in eq. (2.10) is a
One of them is a relation between the excitation energies of $\Lambda_c$ and $40\text{MeV}$ can be as high as $140\text{MeV}$ in the charm and bottom sectors at this order are:

\begin{equation}
O\to\text{NLO})\text{ the first excited state of }\Lambda_c\text{ and }\Lambda_b\text{ baryons. Equations (2.9) imply,}
\end{equation}

\begin{equation}
m_{\bar{\Lambda}_b} - m_{\Lambda_b} = (m_{\bar{\Lambda}_c} - m_{\Lambda_c})\sqrt{\frac{\mu_c}{\mu_b}}(1 + O(\lambda)) .
\end{equation}

In HQET the excitation energies of $\Lambda_c$ and $\Lambda_b$ baryons are equal up to $1/m_Q$ corrections. Equation (2.11) predicts (up to NLO) the first excited state of $\Lambda_b$ (or more precisely, the spin-averaged mass) to lie approximately $300\text{MeV}$ above the ground state. Hence, the model independent prediction at LO for the spin-averaged mass of the first orbitally excited state of $\Lambda_b$ is:

\begin{equation}
m_{\bar{\Lambda}_b} \approx 5920\text{MeV} (1 + O(\lambda)) .
\end{equation}

Other relations can be obtained at this order by combining eq.(2.9) for two heavy quark flavors and re-expressing the results in terms of physical observables:

\begin{equation}
(m_{\Lambda_b} - m_{\Lambda_c}) - (m_B - m_B) - \frac{3}{2}(m_{\bar{\Lambda}_c} - m_{\Lambda_c})\sqrt{\frac{\mu_c}{\mu_b}} - 1 = O(\lambda),
\end{equation}

\begin{equation}
(m_{\bar{\Lambda}_b} - m_{\bar{\Lambda}_c}) - (m_B - m_B) - \frac{5}{2}(m_{\bar{\Lambda}_c} - m_{\Lambda_c})\sqrt{\frac{\mu_c}{\mu_b}} - 1 = O(\lambda),
\end{equation}

which are the NLO corrections to eq. (2.10). The first of the relations in eq. (2.13) shows that the leading order prediction of HQET (eq. (2.10)) in the combined limit is already broken at order $\lambda^{1/2}$. It is interesting to note that this relation is satisfied with a bigger error than that in eq. (2.10). Indeed, $(m_{\Lambda_c} - m_{\Lambda_b}) - (m_B - m_B) - (3/2)(m_{\bar{\Lambda}_c} - m_{\Lambda_c})(\mu_c/\mu_b)^{1/2} - 1 = 60\text{MeV}$. However, this is still consistent with zero within the theoretical uncertainty which can be as high as $140\text{MeV}$.

At order $\lambda$ there is one additional parameter in the effective Hamiltonian—the constant $\lambda$. The excitation energies in the charm and bottom sectors at this order are:

\begin{equation}
m_{\bar{\Lambda}_c} - m_{\Lambda_c} = \sqrt{\frac{\kappa}{\mu_c}} + \frac{5}{\sqrt{4!\kappa\mu_c}} + O(\lambda^{3/2}),
\end{equation}

\begin{equation}
m_{\bar{\Lambda}_b} - m_{\Lambda_b} = \sqrt{\frac{\kappa}{\mu_b}} + \frac{5}{\sqrt{4!\kappa\mu_b}} + O(\lambda^{3/2}).
\end{equation}

The excitation energies of $\Lambda_c$ and $\Lambda_b$ completely determine two phenomenological parameters $\kappa$ and $\alpha$. These constants can then be used to predict the semileptonic form factors and radiative decay rates which will be discussed in the following sections.

III. SEMILEPTONIC DECAYS

A. Decays of the Ground State Heavy Baryons

The semileptonic decays of heavy baryons were extensively analyzed within the framework of HQET [11–14]. The heavy quark spin-flavor symmetry imposes very strict constraints on the number of independent semileptonic form factors, on their normalization and their functional dependence. For example, there is only one form factor for the $\Lambda_b \to \Lambda_c \ell\nu$ decay at leading order in the $1/m_Q$ expansion. This form factor is a smooth function of the recoil parameter $w = v \cdot v'$, where $v$ and $v'$ are the 4-velocities of the initial and final baryons. Indeed, as $m_Q \to \infty$, the effect of the heavy quark current on the heavy baryon is a boost of the heavy quark from an initial velocity $v$ to a final velocity $v'$. Due to the heavy quark spin-flavor symmetry, the transition matrix element is independent of the heavy quark spin and flavor. In the limit as $m_Q \to \infty$, the matrix element is determined by the overlap of the final and initial states of the brown muck, which depends only on the heavy quark velocity. Thus, the form factors can depend only on the recoil parameter, $w$. Moreover, at zero recoil ($w = 1$) the brown muck does not feel any change—it
is insensitive to the heavy quark spin and flavor—so that the transition matrix element (with properly normalized initial and final states) is unity. This gives a unique nonperturbative normalization to the form factors \[11-29\]. By Luke’s theorem, this normalization is unchanged at next-to-leading order in the heavy quark expansion \[30\]. From what follows, it will become clear that the situation is quite different in the combined limit.

Let us first consider the semileptonic decay of the ground state of the \( \Lambda_b \) to the ground state of \( \Lambda_c \) in which the initial and final baryons have 4-velocities \( v \) and \( v' \). The hadronic part of the invariant amplitude of such a transition is determined by the matrix element of the left-handed current \( J = \bar{c} \gamma^{\mu}(1 - \gamma_5) b \). This matrix element is conventionally parameterized by six form factors:

\[
\langle \Lambda_c(\vec{v}) | \bar{c} \gamma^{\mu}(1 - \gamma_5) b | \Lambda_b(\vec{v}) \rangle = \bar{u}_c(\vec{v}') (\Gamma_V - \Gamma_A) u_b(\vec{v}) ,
\]

where

\[
\Gamma_V = f_1 \gamma^{\mu} + i f_2 \sigma^{\mu\nu} q_{\nu} + f_3 q^{\mu} ,
\]

\[
\Gamma_A = (g_1 \gamma^{\mu} + i g_2 \sigma^{\mu\nu} q_{\nu} + g_3 q^{\mu}) \gamma_5 ,
\]

(3.2)

where \( q = m_{\Lambda_c} v' - m_{\Lambda_b} v \) is the momentum transfer and \( u_b(\vec{v}') , u_c(\vec{v}) \) are the Dirac spinors corresponding to \( \Lambda_c \) and \( \Lambda_b \) baryons. The spinors are normalized as,

\[
u^\dagger_Q(\vec{v}, s') u_Q(\vec{v}, s) = \frac{E_p}{M_Q} \delta^{ss'} .
\]

(3.3)

In HQET, the form factors (as functions of the velocity transfer) have a smooth expansion in powers of \( 1/m_Q \). As will be seen from the analytic expressions, this is no longer true for the \( \lambda \)-expansion in the combined limit. In fact, the form factors as functions of the velocity transfer have an essential singularity in the combined limit. Intuitively, this can be seen by considering the semileptonic decay in the bound state picture. The amplitude is an overlap of the harmonic oscillator wave functions. The final wave function contains a factor \( e^{-im_N \vec{x} \cdot \vec{v}} \) due to the boost operator. As a result, the amplitude in the combined limit is proportional to

\[
\exp \left( -\frac{m_N^2 |\delta \vec{v}|^2}{2(\sqrt{\kappa_{15}} + \sqrt{\kappa_{15}})} \right) \sim \exp \left( -\lambda^{-3/2} |\delta \vec{v}|^2 \right) ,
\]

(3.4)

where \( |\delta \vec{v}| \equiv |\vec{v}' - \vec{v}| \) is the velocity transfer. Thus, the amplitude is exponentially small for velocity transfers of order unity and can not be reliably determined using an expansion in powers of \( \lambda^{1/2} \). On the other hand, for velocity transfers of order \( \lambda^{3/4} \) the semileptonic matrix is of order unity \[13\].

There is another reason to consider only velocity transfers of order \( \lambda^{3/4} \). The effective Hamiltonian in eq. (1.8) is based on the self-consistent counting rules of eq. (1.7) according to which all the momentum operators (including \( \vec{P} \)) are of order \( \lambda^{-1/4} \). As was discussed in Sec. 1, the kinetic term corresponding to the center-of-mass motion is of order \( \lambda^{1/2} \). Thus, the counting rules require a typical heavy baryon velocity to be of order \( \lambda^{3/4} \) for the expansion to be valid. In what follows, we will only consider \( |\delta \vec{v}| \) of order \( \lambda^{3/4} \).

In HQET, the heavy quark spin symmetry is used to reduce the six form factors in eq. (3.2) to a single independent Isgur-Wise function with corrections of order \( 1/m_Q \) \[13-18\]. In the combined limit, the heavy quark symmetry generates corrections of order \( \lambda \), while, as was discussed above, the hadronic matrix element as a function of the velocity transfer is exponentially suppressed. Nevertheless, as will be shown shortly, for the velocities of order \( \lambda^{3/4} \), (i.e in the regime of the applicability of the combined expansion) one form factor determines the hadronic amplitude in eq. (3.2); moreover, this form factor is calculable in a model-independent way near the combined limit. As will become clear, this is a consequence of the self-consistent scaling of the Lorentz components of the amplitude. In this velocity regime, it is more convenient to use a different parameterization of the matrix element of left-handed current, eq. (3.2). The form factors can be re-expressed using two Dirac-like equations,

\[
\bar{u}_c(\vec{v}') (i \sigma^{\mu\nu} p_{\nu} + q^{\mu} + \gamma^{\mu}(m_{\Lambda_c} - m_{\Lambda_b})) u_b(\vec{v}) = 0 ,
\]

\[
\bar{u}_c(\vec{v}') (i \sigma^{\mu\nu} q_{\nu} + p^{\mu} + \gamma^{\mu}(m_{\Lambda_c} - m_{\Lambda_b})) \gamma_5 u_b(\vec{v}) = 0 ,
\]

(3.5)

where \( p = m_{\Lambda_c} v' + m_{\Lambda_b} v \). This 4-vector \( p \) should not be confused with the collective variable \( \vec{p} \) introduced in Sec. 1.

Using these equations, the vector and axial currents in eq. (3.2) can be written as,

\[
\Gamma_V = F_1 \gamma^{\mu} + i F_2 \sigma^{\mu\nu} q_{\nu} + i F_3 \sigma^{\mu\nu} p_{\nu} ,
\]

\[
\Gamma_A = (G_1 \gamma^{\mu} + G_2 p^{\mu} + G_3 q^{\mu}) \gamma_5 ,
\]

(3.6)
where the form factors \( f_i \) and \( g_i \) are expressed in terms of \( F_i \) and \( G_i \) by,

\[
\begin{align*}
    f_1 &= F_1 + F_3 (m_{\Lambda_c} - m_{\Lambda_b}), & f_2 &= F_2, & f_3 &= -F_3, \\
    g_1 &= G_1 + G_2 (m_{\Lambda_c} - m_{\Lambda_b}), & g_2 &= -G_2, & g_3 &= G_3.
\end{align*}
\]  

(3.7)

Furthermore, it is useful to consider separately the time and space components of the vector and axial currents separately:

\[
\begin{align*}
    \langle \Lambda_c(v') | \bar{c} \gamma^0 b | \Lambda_b(v) \rangle &= \bar{u}_c(v') \left( F_1 \gamma^0 + 2F_2 \alpha^i q^j + F_3 \alpha^j p^i \right) u_b(v), \\
    \langle \Lambda_c(v') | \bar{c} \gamma^5 b | \Lambda_b(v) \rangle &= \bar{u}_c(v') \left( F_1 \gamma^5 + 2F_2 \left( \alpha^0 q^0 + \epsilon^{ijk} \Sigma^j q^k \right) + F_3 \left( \alpha^0 p^0 + \epsilon^{ijk} \Sigma^j p^k \right) \right) u_b(v), \\
    \langle \Lambda_c(v') | \bar{c} \gamma^0 \gamma_5 b | \Lambda_b(v) \rangle &= \bar{u}_c(v') \left( G_1 \gamma^0 + G_2 p^0 + G_3 q^0 \right) \gamma_5 u_b(v), \\
    \langle \Lambda_c(v') | \bar{c} \gamma^5 \gamma_5 b | \Lambda_b(v) \rangle &= \bar{u}_c(v') \left( G_1 \gamma^5 + G_2 p^5 + G_3 q^5 \right) \gamma_5 u_b(v).
\end{align*}
\]  

(3.8) (3.9) (3.10) (3.11)

where \( \alpha^i = \gamma^0 \gamma^i \) and \( \Sigma^k = i \epsilon^{ijk} \gamma^j = \alpha^k \gamma_5 \).

Out of the six form factors in eqs. (3.8), (3.9), (3.10) and (3.11), only two, namely \( F_1 \) and \( G_1 \), contribute at leading order in the combined expansion if the velocity transfer is of order \( \lambda^{3/4} \). Indeed, as shown in the Appendix, the matrix elements satisfy the following scaling rules in the combined limit:

\[
\begin{align*}
    \langle \Lambda_c(v') | \bar{c} \gamma^0 b | \Lambda_b(v) \rangle &\sim \lambda^0, \\
    \langle \Lambda_c(v') | \bar{c} \gamma^5 b | \Lambda_b(v) \rangle &\sim \lambda^{3/4}, \\
    \langle \Lambda_c(v') | \bar{c} \gamma^0 \gamma_5 b | \Lambda_b(v) \rangle &\sim \lambda^{3/4}, \\
    \langle \Lambda_c(v') | \bar{c} \gamma^5 \gamma_5 b | \Lambda_b(v) \rangle &\sim \lambda^3.
\end{align*}
\]  

(3.12)

These rules lead to a particular scaling of the form factors which can be determined by requiring a consistent scaling of the left- and right-hand side of eqs. (3.8), (3.9), (3.10) and (3.11). The invariant kinematic factors containing Dirac spinors and Dirac matrices scale as:

\[
\begin{align*}
    \bar{u}_c(v') \gamma^0 u_b(v) &\sim \lambda^0, \\
    \bar{u}_c(v') \alpha^i q^j u_b(v) &\sim \lambda^{1/2}, \\
    \bar{u}_c(v') \alpha^i p^j u_b(v) &\sim \lambda^{1/2}, \\
    \bar{u}_c(v') \gamma^i u_b(v) &\sim \lambda^{3/4}, \\
    \bar{u}_c(v') \left( \alpha^i q^0 + \epsilon^{ijk} \Sigma^j q^k \right) u_b(v) &\sim \lambda^{-1/4}, \\
    \bar{u}_c(v') \left( \alpha^0 q^0 + \epsilon^{ijk} \Sigma^j p^k \right) u_b(v) &\sim \lambda^{-1/4}, \\
    \bar{u}_c(v') \gamma^5 \gamma_5 u_b(v) &\sim \lambda^0.
\end{align*}
\]  

(3.13)

Combining the scaling rules of eq. (3.13) with the scaling of the matrix elements in eqs. (3.8) and (3.10), we obtain:

\[
\begin{align*}
    F_1 &\sim \lambda^0, & F_2 &\sim \lambda, & F_3 &\sim \lambda, \\
    G_1 &\sim \lambda^0, & G_2 &\sim \lambda, & G_3 &\sim \lambda.
\end{align*}
\]  

(3.14)

It is easy to see that these scaling rules are consistent with scaling of the amplitudes in eqs. (3.8) and (3.11).

Using the \( \lambda \)-scaling of the form factors in eq. (3.14), the only dominant matrix elements at NLO are given by,

\[
\begin{align*}
    \langle \Lambda_c(v') | \bar{c} \gamma^0 b | \Lambda_b(v) \rangle &= F_1 \bar{u}_c(v') \gamma^0 u_b(v) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right), \\
    \langle \Lambda_c(v') | \bar{c} \gamma^5 \gamma_5 b | \Lambda_b(v) \rangle &= G_1 \bar{u}_c(v') \gamma^5 \gamma_5 u_b(v) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right).
\end{align*}
\]  

(3.15)

As shown in the Appendix, in the combined limit the form factors \( F_1 \) and \( G_1 \) are equal up to corrections of order \( \lambda \). Thus, only a single function describes the entire electroweak matrix element. We denote this function as \( \Theta \) defined by,

\[
\Theta \equiv F_1 = G_1 \left( 1 + \mathcal{O}(\lambda) \right).
\]  

(3.16)
This function should be distinguished from the Isgur-Wise universal function $\eta(w)$ [14,15]. The latter is a smooth function of the velocity transfer parameter $w$ in the pure heavy quark expansion. The function $\Theta$ is only defined for $|\delta\vec{v}| \sim \lambda^{3/4}$; for this range of the velocity transfer the function $\Theta$ can be determined as an expansion in powers of $\lambda^{1/2}$. As discussed below, $\Theta$ is not a smooth function of $|\delta\vec{v}|$ in the combined limit. However, another kinematic variable, $z$, can be introduced so that $\Theta$ is a smooth function of $z$ in the combined limit.

Let us focus on the following matrix element:

$$\langle \Lambda_c(\vec{c})|c\bar{b}\Lambda_b(\vec{c})\rangle = \Theta \; u_{\Lambda_c}(\vec{c}) u_{\Lambda_b}(\vec{c}) \left(1 + \mathcal{O}(\lambda^{3/2})\right).$$

(3.17)

This matrix element can be determined using the collective wave functions of the initial and final heavy baryons and the effective operator corresponding to the operator $c\bar{b}$ in the combined limit. The collective wave functions are determined in the rest frame of the heavy baryon by the effective Hamiltonian in eq. (1.3). At leading order, they are given in eq. (2.3). The next-to-leading order corrections come from the term $\alpha x^4/4!$ treated perturbatively. A collective wave function of the heavy baryon moving with velocity $\vec{v}$ can be determined using the boost operator. In the combined limit the effective boost operator acting on the heavy baryon state is [4]:

$$B_{\vec{v}} = e^{-i(m_N + m_H)\vec{X} \cdot \vec{v}} \left(1 + \mathcal{O}(\lambda)\right).$$

(3.18)

As shown in the Appendix, the effective operator corresponding to $c\bar{b}$ at leading and next-to-leading order in the combined expansion is $h_c^{(v)}(\bar{y}) h_b^{(v)}(y)$, where the heavy quark field $h_Q^{(v)}$ is defined in eq. (A1). Moreover, in the combined limit this effective operator can be expressed in terms of the collective operator $\tilde{X}_b$ defined in eq. (1.3), [2]:

$$h_c^{(v)}(\bar{y}) h_b^{(v)}(y) = \delta^3(\tilde{X}_b - \bar{y}) (1 + \mathcal{O}(\lambda)).$$

(3.19)

Using the effective operator (eq. (3.19)), the boost operator (eq. (3.18)) and the collective wave functions of the ground state heavy baryons (eq. (2.3)), the form factor $\Theta$ at leading order in the combined expansion is:

$$\Theta = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \exp\left(-\frac{m_N^2 |\delta\vec{v}|^2}{2(\sqrt{\mu_b} + \sqrt{\mu_c})}\right) (1 + \mathcal{O}(\lambda)).$$

(3.20)

where $|\delta\vec{v}| \sim \lambda^{3/4}$. This result agrees with one obtained using the bound state picture [14,15].

As a function of $|\delta\vec{v}|$, $\Theta$ has an essential singularity in the combined limit; it vanishes faster than any power of $\lambda$. However, as a function of the kinematic variable $z$ defined as,

$$z = \frac{m_N |\delta\vec{v}|}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} = \frac{m_N \sqrt{2}(w - 1)^{1/2}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{1/2}} \left(1 + \mathcal{O}(\lambda^{3/2})\right),$$

(3.21)

$\Theta$ is well behaved in the combined limit. For velocity transfers of order $\lambda^{3/4}$, variable $z$ can be expressed in terms of a Lorentz scalar, $w$, since $w = (1 + |\delta\vec{v}|^2/2) (1 + \mathcal{O}(\lambda^{3/2}))$. As a function of $z$, the form factor $\Theta$ at LO has the form:

$$\Theta(z) = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \exp\left(-\frac{z^2}{2\sqrt{k}}\right) (1 + \mathcal{O}(\lambda)).$$

(3.22)

The form factor $\Theta$ as a function of $z$ at LO is plotted in Fig. 1 for the physical values of the nucleon mass, $m_N$, and reduced masses, $\mu_c$ and $\mu_b$.

The experimentally measurable quantities are the value and derivatives of the form factors at zero recoil, or equivalently at $z = 0$. The value of the form factor at zero recoil is given at LO by,

$$\Theta(z = 0) = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \approx 0.998.$$

(3.23)

Thus, the form factor at zero recoil is completely determined at LO in the combined expansion in terms of the known masses, $\mu_c$ and $\mu_b$. In HQET, due to the heavy quark spin-flavor symmetry, the form factor at zero recoil is unity up to corrections of order $1/m_Q^2$ [21]. The flavor symmetry is broken in the combined limit already at LO due to the dependence of kinetic energy of the collective motion on the flavor of the heavy quark via the reduced mass $\mu_Q$. However, the deviation from unity is small for the $\Lambda_b \to \Lambda_c \ell\bar{v}$ decay.

The second derivative of $\Theta(z)$ at zero recoil is given at LO in the combined limit by,
Expressing $\kappa$ in terms of the excitation energy of $\Lambda_c$, we get:

$$
\rho = \frac{-2\sqrt{2} \mu_b^{3/8} \mu_c^{-1/8}}{(m_{\Lambda_c} - m_{\Lambda_{c1}})(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \approx -1.197 \times 10^{-4} \text{MeV}^{-3/2}.
$$

The NLO correction to the leading order expressions in eqs. (3.20), (3.23) and (3.25) are obtained by treating the next-to-leading order term in eq. (1.8) perturbatively. The form factor $\Theta(z)$, its value and second derivative at zero recoil are given at NLO by,

$$
\Theta(z) = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \exp \left( \frac{-2z^4}{\sqrt{\mu_b} + \sqrt{\mu_c}} \right)
$$

$$
\left( 1 + \frac{\alpha}{4! \sqrt{\mu_b} \sqrt{\mu_c} (\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left( \frac{45\kappa(\sqrt{\mu_b} - \sqrt{\mu_c})^2}{16\kappa^{3/2} \sqrt{\mu_b} \sqrt{\mu_c}} + 5 \kappa^{1/2} z^2 - \frac{1}{4} z^4 \right) \right) \left( 1 + O(\lambda^{3/2}) \right),
$$

$$
\Theta(z = 0) = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left( 1 + \frac{\alpha}{4! \sqrt{\mu_b} \sqrt{\mu_c} (\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left( 45(\sqrt{\mu_b} - \sqrt{\mu_c})^2 \right) \right) \left( 1 + O(\lambda^{3/2}) \right),
$$

$$
\rho = \frac{-2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left( 1 + \frac{\alpha}{4! \sqrt{\mu_b} \sqrt{\mu_c} (\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left( 45(\sqrt{\mu_b} - \sqrt{\mu_c})^2 - 160(\mu_b \sqrt{\mu_c}) \right) \right) \left( 1 + O(\lambda^{3/2}) \right).
$$

As in the case with the spectroscopic observables in Sec. III A, the semileptonic observables in eqs. (3.27) and (3.28) are completely determined up to NNLO corrections in terms of two phenomenological constants—$\kappa$ and $\alpha$.

B. Decays of the Ground State to the Excited States of Heavy Baryons

There are two electroweak decay channels of the ground state of $\Lambda_b$ to the low-lying excited states of $\Lambda_c$: $\Lambda_b \to \Lambda_{c1} \ell \bar{\nu}$ and $\Lambda_b \to \Lambda_{c2} \ell \bar{\nu}$. As will be shown, at LO and NLO in the combined limit there is a single independent form factor that determines the hadronic matrix elements for each of the decay channels. The approach here is similar to the one used in Sec. III A. The $\lambda$-scaling of the form factors is determined by the self-consistent counting rules of the hadronic matrix elements when the velocity transfer is of order $\lambda^{3/4}$.

The form factors for the $\Lambda_b \to \Lambda_{c1} \ell \bar{\nu}$ decay are given by,
\begin{equation}
\langle \lambda c_1(\vec{v})|\bar{c}\gamma^\mu (1 - \gamma_5)b|\Lambda_b(\vec{v})\rangle = \bar{u}_c(\vec{v}) (\Gamma_V - \Gamma_A) u_b(\vec{v}) ,
\end{equation}

where

\begin{align}
\Gamma_V &= K_1 \gamma^\mu + iK_2 \sigma^{\mu\nu} q_\nu + iK_3 \sigma^{\mu\nu} p_\nu , \\
\Gamma_A &= (L_1 \gamma^\mu + L_2 \eta^\mu + L_3 q^\mu) \gamma_5 ,
\end{align}

so that the parameterization is the same as in eq. (3.4).

The \(\lambda\)-scaling of the time and space components of the vector and axial current matrix elements in eq. (3.29) is the same as for the corresponding amplitudes in \(\Lambda_b \to \Lambda_c\ell\bar{\nu}\) decay eq. (3.12). Combining these scaling rules with the scaling of the factors containing Dirac spinors and Dirac matrices, eq. (3.13), we get:

\begin{align}
K_1 &\sim \lambda^0 , & K_2 &\sim \lambda , & K_3 &\sim \lambda , \\
L_1 &\sim \lambda^0 , & L_2 &\sim \lambda , & L_3 &\sim \lambda .
\end{align}

The vector and axial form factors \(K_1\) and \(L_1\) are equal up to corrections of order \(\lambda\) as are the form factors \(F_1\) and \(G_1\). Thus, the dominant decay matrix element is determined by the single form factor:

\begin{equation}
\langle \lambda c_1(\vec{v})|c^\dagger b|\Lambda_b(\vec{v})\rangle = K_1 u_1^c(\vec{v}) u_b(\vec{v}) \left(1 + \mathcal{O}(\lambda^{3/2})\right) .
\end{equation}

This matrix element can be calculated at LO and NLO in the combined expansion.

The hadronic matrix element of the \(\Lambda_b \to \Lambda_c\ell\bar{\nu}\) decay is parameterized by eight form factors:

\begin{equation}
\langle \lambda c_1(\vec{v})|\bar{c}\gamma^\mu (1 - \gamma_5)b|\Lambda_b(\vec{v})\rangle = \bar{u}_{c\nu}(\vec{v}) (\Gamma_V - \Gamma_A) u_b(\vec{v}) ,
\end{equation}

where

\begin{align}
\Gamma_V &= (N_1 \sigma^\mu^\nu + N_2 \gamma^\mu q^\nu + iN_3 \sigma^\mu^\nu p_\nu q^\nu + iN_4 \sigma^{\mu\nu\rho} p^\rho q^\nu) \gamma_5 , \\
\Gamma_A &= M_1 \eta^\mu + M_2 \gamma^\mu q^\nu + M_3 q^\mu q^\nu + M_4 q^\mu p^\nu ,
\end{align}

and \(u_\nu^c(\vec{v}, s)\) is the Rarita-Schwinger spinors corresponding to the heavy baryons with total spin \(3/2\). They are normalized so that \(\bar{u}_{Q\nu} u^Q_{\nu} = -1\). The time component of \(u_\nu\) is suppressed by \(\lambda^{3/4}\) relative to the special components for the velocities of order \(\lambda^{3/4}\).

The \(\lambda\)-scaling for the velocity transfers of order \(\lambda^{3/4}\) of the vector and axial matrix elements in eq. (3.33) are the same as in eq. (3.12). The kinematic factors scale as:

\begin{align}
\bar{u}_{c\nu}(\vec{v})\sigma^{\nu\iota} \gamma_5 u_b(\vec{v}) &\sim \lambda^{3/4} , \\
\bar{u}_{c\nu}(\vec{v})\gamma^\iota q^\nu \gamma_5 u_b(\vec{v}) &\sim \lambda^{-1/4} , \\
\bar{u}_{c\nu}(\vec{v})\sigma^{\iota\rho} p_\rho q^\nu \gamma_5 u_b(\vec{v}) &\sim \lambda^{-5/4} , \\
\bar{u}_{c\nu}(\vec{v})\sigma^{\iota\rho} p_\rho q^\nu \gamma_5 u_b(\vec{v}) &\sim \lambda^{-5/4}
\end{align}

for the spatial components, and,

\begin{align}
\bar{u}_{c\nu}(\vec{v})\sigma^{\nu\rho} \gamma_5 u_b(\vec{v}) &\sim \lambda^0 , \\
\bar{u}_{c\nu}(\vec{v})\gamma^\rho \gamma_5 u_b(\vec{v}) &\sim \lambda^{1/2} , \\
\bar{u}_{c\nu}(\vec{v})\sigma^{\rho\iota} q_\iota q^\nu \gamma_5 u_b(\vec{v}) &\sim \lambda^{-1/2} , \\
\bar{u}_{c\nu}(\vec{v})\sigma^{\rho\iota} q_\iota q^\nu \gamma_5 u_b(\vec{v}) &\sim \lambda^{-1/2}
\end{align}

for the time component. Since the spatial components of the vector current matrix elements are of order \(\lambda^{3/4}\) (eq. (3.12)), the scaling rules in eq. (3.33) lead to the following scaling rules of the vector form factors:

\begin{equation}
N_1 \sim \lambda^0 , & N_2 \sim \lambda , & N_3 \sim \lambda^2 , & N_4 \sim \lambda^2 .
\end{equation}

These scaling rules are consistent with the scaling of the time component of the vector current amplitude (eqs. (3.12) and (3.36)). Combining eqs. (3.37) and (3.36) we get,
Hence, the terms on the right-hand side in eq. (3.33) scale as,

\[ N_1 \bar{u}_{c\nu}(\bar{v}) \sigma^{\mu \nu} \gamma_5 u_b(\bar{v}) \sim \lambda^0, \]
\[ N_2 \bar{u}_{c\nu}(\bar{v}) \gamma^0 q^\nu \gamma_5 u_b(\bar{v}) \sim \lambda^{3/2}, \]
\[ N_3 \bar{u}_{c\nu}(\bar{v}) \sigma^{\mu \nu} q^\nu \gamma_5 u_b(\bar{v}) \sim \lambda^{3/2}, \]
\[ N_4 \bar{u}_{c\nu}(\bar{v}) \sigma^{\mu \nu} \rho^\nu q^\nu \gamma_5 u_b(\bar{v}) \sim \lambda^{3/2}. \]

Thus, only \( N_1 \) contributes to the vector current matrix element at leading and next-to-leading order in the combined expansion. The dominant matrix element is:

\[
\langle \Lambda^{*}_{c1}(\bar{v})  c^\dagger b|\Lambda_b(\bar{v}) \rangle = i N_1 \bar{u}_{c\nu}(\bar{v}) \sigma^{\mu \nu} u_b(\bar{v}) + \mathcal{O}(\lambda^{3/2}) = i \sqrt{2} N_1 \bar{u}_{c\nu}(\bar{v}) u_b(\bar{v}) \left(1 + \mathcal{O}(\lambda^{3/2})\right),
\]

where the expressions for the Rarita-Schwinger spinors in terms of Dirac spinors were used.

The scaling of the axial form factors is determined by the scaling of the time component of the axial current matrix element and by the scaling of the corresponding kinematic factors:

\[
\bar{u}_{c\nu}(\bar{v}) g^{0 \nu} u_b(\bar{v}) \sim \lambda^{3/4}, \quad \bar{u}_{c\nu}(\bar{v}) \gamma^0 q^\nu u_b(\bar{v}) \sim \lambda^{-1/4}, \quad \bar{u}_{c\nu}(\bar{v}) q^0 q^\nu u_b(\bar{v}) \sim \lambda^{-5/4}, \quad \bar{u}_{c\nu}(\bar{v}) q^0 p^\nu u_b(\bar{v}) \sim \lambda^{-5/4}. \]

These scaling rules lead to the following scaling of the axial form factors:

\[ M_1 \sim \lambda^0, \quad M_2 \sim \lambda, \quad M_3 \sim \lambda^2, \quad M_4 \sim \lambda^2. \]

Hence, the terms on the right-hand side in eq. (3.33) scale as,

\[
M_1 \bar{u}_{c\nu}(\bar{v}) g^{0 \nu} u_b(\bar{v}) \sim \lambda^0, \quad M_2 \bar{u}_{c\nu}(\bar{v}) \gamma^0 q^\nu u_b(\bar{v}) \sim \lambda^{3/2}, \quad M_3 \bar{u}_{c\nu}(\bar{v}) q^0 q^\nu u_b(\bar{v}) \sim \lambda^{3/2}, \quad M_4 \bar{u}_{c\nu}(\bar{v}) q^0 p^\nu u_b(\bar{v}) \sim \lambda^{3/2},
\]

so that the dominant axial matrix element for \( \Lambda_b \to \Lambda^{*}_{c1} \ell \bar{\nu} \) decay is determined by a single form factor at LO and NLO in the combined limit:

\[
\langle \Lambda^{*}_{c1}(\bar{v})  c^\dagger b|\Lambda_b(\bar{v}) \rangle = M_1 \bar{u}_{c\nu}(\bar{v}) u_b(\bar{v}) + \mathcal{O}(\lambda^{3/2}) = \sqrt{2} M_1 \bar{u}_{c\nu}(\bar{v}) u_b(\bar{v}) \delta^{ij} + \mathcal{O}(\lambda^{3/2}).
\]

As shown in the Appendix the form factors \( N_1 \) and \( M_1 \) are equal up to corrections of order \( \lambda \). Hence, the hadronic matrix element of \( \Lambda_b \to \Lambda^{*}_{c1} \ell \bar{\nu} \) decay is determined by a single independent form factor given by,

\[
\langle \Lambda^{*}_{c1}(\bar{v})  c^\dagger b|\Lambda_b(\bar{v}) \rangle = \sqrt{2} \frac{2}{3} N_1 u_{c\nu}(\bar{v}) u_b(\bar{v}) \left(1 + \mathcal{O}(\lambda^{3/2})\right).
\]

The form factors \( N_1 \) and \( K_1 \) are not independent since states \( |\Lambda_{c1} \rangle \) and \( |\Lambda^{*}_{c1} \rangle \) are degenerate in the combined limit (they are related by the heavy quark spin flip) and the effective operator in eq. (3.19) is independent of the heavy quark spin. The boost operator and the effective operator in eq. (3.19) connect states with definite orbital angular momentum \( l \) and heavy quark spin \( s_Q \). Using appropriate Clebsch-Gordan coefficients, the form factor \( K_1 \) is given in terms of the form factor \( N_1 \) by,

\[
K_1 = \sqrt{\frac{1}{3}} N_1 (1 + \mathcal{O}(\lambda)).
\]

As in the case of the \( \Lambda_b \to \Lambda_c \ell \bar{\nu} \), the electroweak decays of the ground state of \( \Lambda_b \) into the doublet of the first orbitally excited state is determined by a single form factor at LO and NLO in the combined limit:

\[
\Xi \equiv N_1 = M_1 (1 + \mathcal{O}(\lambda)) = \sqrt{3} K_1 (1 + \mathcal{O}(\lambda)) = \sqrt{3} L_1 (1 + \mathcal{O}(\lambda)).
\]
We denote this function as $\Xi$ to distinguish it from the universal heavy quark form factor in HQET, Ref. [9–13], since it applies in the combined limit. The function $\Xi$ can be calculated from eq. (3.44) using the effective boost operator in eq. (3.18), the heavy quark effective operator in eq. (3.19) and the collective wave functions of the ground and the first excited states of heavy baryons in eq. (2.3). Hence, at leading order the function $\Xi$ is given by,

$$
\Xi = \frac{4 m_N |\delta\bar{\ell}|^{3/8} \mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b} + \sqrt{\mu_c})^2} \exp \left( -\frac{m_N^2 |\delta\bar{\ell}|^2}{2(\sqrt{\mu_b} + \sqrt{\mu_c})} \right) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right),
$$

(3.47)

where the velocity transfer $|\delta\bar{\ell}|$ is of order $\lambda^{3/4}$. The form factor can be expressed as a function of the kinematic variable $z$ defined in eq. (3.21):

$$
\Xi(z) = \frac{4 z \mu_b^{3/8} \mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b} + \sqrt{\mu_c})^2} \exp \left( -\frac{z^2}{2\sqrt{\kappa}} \right) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right). \tag{3.48}
$$

As a function of $z$, $\Xi(z)$ has a smooth expansion in powers of $\lambda^{1/2}$ near zero recoil. The function $\Xi(z)$ is plotted in Fig. 3.

The slope of $\Xi$ at zero recoil at LO is given by,

$$
\sigma = \frac{\partial \Xi}{\partial z} (z = 0) = \frac{4 \mu_b^{3/8} \mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b} + \sqrt{\mu_c})^2} (1 + \mathcal{O}(|\lambda|))
$$

$$
= \frac{4 \mu_b^{3/8} \mu_c^{5/8}}{\sqrt{m_{\Lambda_c}^2 - m_{\Lambda_c}(\sqrt{\mu_b} + \sqrt{\mu_c})^2}} (1 + \mathcal{O}(|\lambda|)) \approx 0.011 \text{ MeV}^{-3/4}, \tag{3.49}
$$

where in the third equality the constant $\kappa$ at LO is expressed in terms of the excitation energy of the first excited state of $\Lambda_c$.

At NLO the function $\Xi(z)$ and its slope at zero recoil are given by,

$$
\Xi(z) = \frac{4 z \mu_b^{3/8} \mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b} + \sqrt{\mu_c})^2} \exp \left( -\frac{z^2}{2\sqrt{\kappa}} \right) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right)
$$

$$
\left( 1 + \frac{\alpha}{4 \mu_b^{3/8}} \left( \frac{105 \mu_b - 230 \sqrt{\mu_b \mu_c} + 45 \mu_c}{16 \sqrt{\mu_b \mu_c}} \right) - \frac{1}{4} \kappa^{1/2} z^2 \right) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right), \tag{3.50}
$$

$$
\sigma = \frac{4 \mu_b^{3/8} \mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b} + \sqrt{\mu_c})^2} \left( 1 + \frac{\alpha}{4 \mu_b^{3/8}} \left( \frac{105 \mu_b - 230 \sqrt{\mu_b \mu_c} + 45 \mu_c}{16 \sqrt{\mu_b \mu_c}} \right) \right) \left( 1 + \mathcal{O}(\lambda^{3/2}) \right). \tag{3.51}
$$

The slope is completely determined in terms of the constants $\kappa$ and $\alpha$. 

FIG. 2. The dominant semileptonic form factor for the $\Lambda_b \to \Lambda_{c1}^+(\Lambda_{c1})\ell\bar{\nu}$ decays at LO defined in eq. (3.40) plotted as a function of the natural variable $z$ defined in eq. (3.21) with physical values of masses $m_N$, $\mu_c$ and $\mu_b$. 

IV. RADIATIVE DECAYS OF EXCITED HEAVY BARYONS

The effective theory near the combined heavy quark and large $N_c$ limit can be used to predict electromagnetic decay rates of excited heavy baryons. The strong decays of excited $\Lambda_c$ baryons are dominated by three-body isospin conserving decays with two final pions. However, because of the small energy splitting between these states and the ground state, the phase space for these decays is restricted. Due to this phase space restriction, the radiative transitions may even be the dominant decay mode in the bottom sector. Thus, the strong decays are greatly suppressed and electromagnetic decays will have a substantial branching ratio. The radiative decays of excited $\Lambda_c$ and $\Lambda_b$ baryons within the framework of the bound state picture of heavy baryons were considered in [31]. Here we will use the effective theory to perform a model independent analysis up to NLO. The effective theory gives the form of the matrix elements of a dipole operator between the low-lying bound states of a heavy baryon which determine the leading order contribution to the decay amplitude. At leading order, the radiative decay rates are completely determined in terms of the constant $\kappa$. At next-to-leading order, an additional constant——$\alpha$——is needed. As will be shown, no additional phenomenological parameters associated with the electromagnetic current arise.

An electromagnetic decay amplitude is determined by the interaction Hamiltonian, $\mathcal{H}_{\text{int}} = e \int d^4x j^\mu(\vec{x})A_\mu(\vec{x})$, where $j^\mu(\vec{x})$ is a current operator that couples to a photon field $A^\mu(\vec{x})$ with coupling constant $e$. According to Fermi’s golden rule, the decay rate is proportional to the square of the absolute value of the matrix element of $\mathcal{H}_{\text{int}}$ between initial and final states. If the wavelength of a radiated photon is much larger than the typical size of the system, $\omega a \ll 1$ ($\omega$——the energy of a photon, $a$——typical size), then the matrix element can be expanded in terms of multipole operators [32]:

$$
(\langle f | H_{\text{int}} | i \rangle = (-1)^{m+1}J \sqrt{\frac{(2J+1)(J+1)}{\pi J}} \frac{\omega^{J+\frac{3}{2}}}{(2J+1)!!} e((M^{(e)}_{J,-m})_{fi} + (M^{(m)}_{J,-m})_{fi}),
$$

(4.1)

where the initial state consists of a single heavy baryon in an excited state; the final state consists of a heavy baryon in the ground state and a photon with a definite angular momentum, $J$, and its $z$-component, $m$. The matrix elements $(M^{(e)}_{J,m})_{fi}$ and $(M^{(m)}_{J,m})_{fi}$ correspond to electric, $E_J$, and magnetic, $M_J$, transitions, respectively; they are given by,

$$
(M^{(e)}_{J,m})_{fi} = \sqrt{\frac{4\pi}{(2J+1)}} \int d^3x \rho_{fi}(\vec{x})|\vec{x}|^J Y_{JM}(\frac{\vec{x}}{|\vec{x}|}),
$$

(4.2)

$$
(M^{(m)}_{J,m})_{fi} = \frac{1}{(J+1)} \sqrt{\frac{4\pi}{(2J+1)}} \int d^3x (\vec{x} \times \vec{j}_{fi}(\vec{x})) \cdot \nabla(|\vec{x}|^J Y_{JM}(\frac{\vec{x}}{|\vec{x}|})),
$$

(4.3)

where $\rho_{fi}(\vec{x})$, $\vec{j}_{fi}(\vec{x}) = j_{fi}^{\mu}(\vec{x})$.

Fermi’s golden rule gives the total decay rates in which photons with given values of $J$ and $m$ are radiated:

$$
\Gamma^{(e)} = \frac{2(2J+1)(J+1)}{J((2J+1)!!)^2} \omega^{2J+1}e^2 |(M^{(e)}_{J,-m})_{fi}|^2
$$

(4.4)

for $E_J$ transitions, and,

$$
\Gamma^{(m)} = \frac{2(2J+1)(J+1)}{J((2J+1)!!)^2} \omega^{2J+1}e^2 |(M^{(m)}_{J,-m})_{fi}|^2
$$

(4.5)

for $M_J$ transitions.

In order to arrive at eqs. (4.2) and (4.3), only the first term in the expansion of the photon radial wave functions is kept. These wave functions are spherical Bessel functions, $g_J(|\vec{x}|)$, which equal to $\pi/2k|\vec{x}|^{1/2}J_{J+1/2}(k|\vec{x}|)$, where $J_{J+1/2}(k|\vec{x}|)$ is the Bessel function of the first kind. This is valid when $k|\vec{x}| \sim k a \ll 1$. The next-to-leading order term in the expansion of the radial wave function is suppressed by $(k|\vec{x}|)^2$. For the decay of the first excited state of a heavy baryon to its ground state, this condition is satisfied in the combined limit. From the counting rules of our effective theory the momentum $k = \omega \sim \lambda^{1/4}$, and the typical size is determined by the expectation value of the operator $\vec{x}$, which is of order $\lambda^{1/4}$. Thus, $(k|\vec{x}|)^2 \sim \lambda^{3/2} \ll 1$ near the combined limit validating the multipole expansion. In this approximation, decay rates corresponding to higher order $E_J$ and $M_J$ operators are suppressed by $(k|\vec{x}|)^2 \sim \lambda^{3/2}$ (as is evident from eqs. (4.2) and (4.3)). The $M_J$ decay rate is suppressed relative to the $E_J$ decay with the same $J$ by $|\vec{v}|^2 \sim \lambda^{3/2}$. In addition, the transitions between states with definite total angular momentum
can only occur either through $E_j$ or $M_j$ decays due to parity conservation in the electromagnetic interaction; $E_j$ photons have parity $(-1)^j$, while $M_j$ photons have parity $(-1)^{j+1}$. Hence, at LO and NLO in the combined limit the dominant mode for the electromagnetic decay of the first excited state of a heavy baryon (with negative parity) into the ground state (with positive parity) is the $E1$ decay. It is determined by the matrix element of the dipole operator of the heavy baryon, $\vec{d}$. Summing eq. (4.4) over all possible values of $m$ for $J = 1$, i.e. $m = 0, \pm 1$, we get the decay rate,
\[
\Gamma = \frac{4\omega^3}{3}|\vec{d}_{fi}|^2,
\]
with the dipole matrix element defined as,
\[
\vec{d}_{fi} = e \int d^3x \rho_{fi}(\vec{x})\vec{x}.
\]

When recoil is taken into consideration, eq. (4.6) is multiplied by $(1 - \omega/2M)$, where $M$ is the total mass of the initial baryon. However, because this correction is of relative order $\lambda^{3/2}$, it can be neglected.

The combined expansion can be used to determine the dipole matrix element in eq. (4.7). The charge density in eq. (4.7) is the time component of the current $j^0$:
\[
\vec{d}_{fi} = \langle f | e_Q \int d^3x Q^\dagger(x) \vec{Q}(x) | i \rangle + \langle f | \sum_{i = u, d} e_i \int d^3x q_i^\dagger(x) \vec{x} q_i(x) | i \rangle \tag{4.8}
\]
where $e_Q$ and $e_i$ are the heavy quark and light quark charges in units of $e$; the sum is over the light quark flavors $u$ and $d$. The first term in eq. (4.8) is the matrix element of heavy quark position operator in the combined limit, eq. (4.3), times the heavy quark charge. The second matrix element—the brown muck contribution to the total dipole moment—can be written as,
\[
\vec{d}_\ell = \langle f | \sum_{i = u, d} (I_{3i} + B_i) \int d^3x q_i^\dagger(x) \vec{x} q_i(x) | i \rangle \tag{4.9}
\]
where $I_{3i}$ is the third component of the isospin and $B_i$ is the baryon number of a quark which is the same for every flavor. The first term in eq. (4.9) vanishes since it is an isospin one operator between isospin zero states. The baryon number of a single quark can be taken to be $1/N_c$, so that the total baryon number of the heavy baryon is still unity in the combined limit. Thus, the light quark contribution to the dipole moment is:
\[
\vec{d}_\ell = \frac{1}{2N_c} \langle f | \sum_{i = u, d} \int d^3x q_i^\dagger(x) \vec{x} q_i(x) | i \rangle \tag{4.10}
\]

In analogy to the heavy quark part, one would expect that the brown muck contribution to the dipole moment is proportional to the operator $\vec{X}_\ell$. However, the operator in eq. (4.10) contains only quark fields while $\vec{X}_\ell$ acts on gluon fields as well. Nevertheless, as was shown in Ref. [3], in the subspace of the low-lying states the dipole moment of the brown muck is proportional to $\vec{X}_\ell$ with proportionality constant completely determined up to corrections of order $\lambda$:
\[
\frac{1}{N_c - 1} \langle f | \sum_{i = u, d} \int d^3x q_i^\dagger(x) \vec{x} q_i | i \rangle = \langle f | \vec{X}_\ell | i \rangle (1 + \mathcal{O}(\lambda)) \tag{4.11}
\]
Combining eqs. (4.8), (4.9), (4.10) and (4.11), we get:
\[
\vec{d}_{fi} = e \langle f | e_Q \vec{X}_Q + e_\ell \vec{X}_\ell | i \rangle (1 + \mathcal{O}(\lambda)) \tag{4.12}
\]
where the effective brown muck charge is given by,
\[
e_\ell = (N_c - 1)/2N_c. \tag{4.13}
\]
To get the correct total charges for $\Lambda_c$ and $\Lambda_b$ baryons, the heavy quark charge is given by,
\begin{equation}
e_Q = (1 \pm N_c)/2N_c \tag{4.14}
\end{equation}
for $\Lambda_c$ and $\Lambda_b$ baryons, respectively.

The charge assignment for the heavy quark and the brown muck is somewhat ambiguous. One possibility is to use the same charge assignment as for $N_c = 3$, namely $e_Q = e_1 = +2/3$, $e_\ell = -1/3$, and for the brown muck $e_\ell = +1/3$. Another way is to choose an assignment valid in the combined limit:
\begin{equation}
e_Q = \pm 1/2 (1 + O(\lambda)) , \quad e_\ell = 1/2 (1 + O(\lambda)) . \tag{4.15}
\end{equation}
This charge assignment is used in the bound state picture of the heavy baryon, where the heavy meson and the nucleon are in the superposition of two isospin states $[31]$. The ambiguity between two charge assignments is of order $\lambda$ and hence can not be resolved at the order to which we are working. The charge assignment in eq. (4.15) is also obtained by considering anomaly cancelation of the $SU(N_c) \times SU(2) \times U(1)$ generalization of the standard model $[33,34]$.

Using eqs. (4.14) and (4.13), the dipole matrix element in eq. (4.12) can be expressed in terms of the relative position operator $\vec{x}$:
\begin{equation}
d_{fi} = e \mu_Q (\frac{e_\ell}{m_N} - \frac{e_Q}{m_Q}) \langle f | \vec{x} | i \rangle . \tag{4.16}
\end{equation}
Note, this operator is identical to the one used in the bound state picture $[31]$. However, in the present context it emerges from QCD in the combined limit without additional assumptions.

The dipole matrix element between the first excited state and the ground state in eq. (4.16) can be evaluated up to NLO using the collective wave functions discussed in Sec. [4]. The decay rate is given by eq. (4.17). To find a total decay rate one needs to sum over all final states and average over the initial states. This gives the same result for both $\Lambda_Q \to \Lambda_Q \gamma$ and $\Lambda^*_Q \to \Lambda_Q \gamma$ decays. At order $\lambda^{1/2}$ the total decay rate is completely determined in terms of the constant $\kappa$:
\begin{equation}
\Gamma(\Lambda_Q \to \Lambda_Q \gamma) = \frac{2}{3} \kappa^2 \left( \frac{e_\ell}{m_N} - \frac{e_Q}{m_N} \right)^2 (1 + O(\lambda)) . \tag{4.17}
\end{equation}
As was shown in Sec. [4], at LO $\kappa$ is equal to the square of the excitation energy of $\Lambda_{c1}$ times the reduced mass, $\kappa = \omega^2 \mu_Q = (m_{\Lambda_c} - m_{\Lambda_{c1}})^2 \mu_c$. Using the charge assignment in eq. (4.15), the total decay rates are given by:
\begin{equation}
\Gamma(\Lambda_{c1} \to \Lambda_c \gamma) = \frac{1}{6} \kappa e_\ell e_\ell \left( m_{\Lambda_{c1}} - m_{\Lambda_c} \right)^2 m_{\Lambda_c} m_N m_{\Lambda_c} m_N \approx 0.025 \text{ MeV} (1 + O(\lambda)) , \tag{4.18}
\end{equation}
\begin{equation}
\Gamma(\Lambda_{b1} \to \Lambda_b \gamma) = \frac{1}{6} \kappa m_{\Lambda_{b1}} m_{\Lambda_b} \left( m_{\Lambda_{b1}} - m_{\Lambda_b} \right)^2 m_{\Lambda_b} m_N m_{\Lambda_b} m_N \approx 0.130 \text{ MeV} (1 + O(\lambda)) . \tag{4.19}
\end{equation}

It is interesting to consider the ratio of the decays in eqs. (4.18) and (4.19):
\begin{equation}
\frac{\Gamma(\Lambda_{c1} \to \Lambda_c \gamma)}{\Gamma(\Lambda_{b1} \to \Lambda_b \gamma)} = \left( \frac{m_{\Lambda_b} - m_{\Lambda_c}}{m_{\Lambda_b} + m_{\Lambda_c}} \right)^2 \frac{m_{\Lambda_{b1}}^2}{m_{\Lambda_{c1}}^2} \approx 0.2 (1 + O(\lambda)) . \tag{4.20}
\end{equation}
This is striking since in the pure heavy quark limit this ratio is unity. In this limit, the heavy quark is located at the center of mass of the heavy baryon. As a result, the dipole moment is determined by the motion of the brown muck alone relative to the center-of-mass. The pure heavy quark expansion is valid if the recoil of the heavy quark due to its finite mass is suppressed (it is an order $1/m_Q$ effect). Thus, one might expect the ratio in eq. (4.20) to be close to 1. However, if the mass of the brown muck is not much smaller than the mass of the heavy quark, as is the case for $\Lambda_c$ baryons and their excited states, the recoil of the heavy quark is not greatly suppressed. As a result, the motion of the heavy quark relative to the center of mass of a heavy baryon gives a significant contribution to the dipole moment of the system. The contributions to the dipole moment due to the heavy quark and the brown muck motions around the center of mass partially cancel when their respective charges have the same sign. If the charges have opposite signs, the two contributions add together. The partial cancelation happens for $\Lambda_c$ baryons, while for $\Lambda_b$ baryons the two contributions add $[31]$. This increases the suppression of the ratio of the decay rates in eq. (4.20).
It is a general feature of effective theories that a nonperturbative expansion (such as the $1/m_Q$ expansion in HQET) can work well for a number of observables, although it may have a very slow convergence for others. The failure of the pure heavy quark expansion of radiative decay rates of heavy baryons can be traced to a particular type of correction, i.e., $m_N/m_H$, which are suppressed in the pure heavy quark limit but are of order $\lambda^0$ in the combined limit. Phenomenologically, the ratio $m_N/m_H$ is not very small for heavy baryons, particularly for charm baryons. One would like to sum these types of corrections to all orders to improve the convergence. This is accomplished by using the combined expansion.

At next-to-leading order in the combined expansion, an additional constant—$\alpha$—is needed to determine the total decay rate. Once this constant is fixed from the spectroscopic or the semileptonic observables, the total decay rate can be determined by treating perturbatively the term of order $\lambda$ in the effective Hamiltonian (eq. (1.8)). The total decay rate at NLO is given by,

$$\Gamma(\Lambda_{c1}\to\Lambda_c\gamma) = \frac{1}{6} e^2 \kappa \left(\frac{m_\gamma - m_N}{m_B m_N} \right)^2 \left(1 - \frac{\alpha}{4! \sqrt{k^3 \mu_e}} \right) (1 + O(\lambda)),$$  \hspace{1cm} (4.21)

where the correction of order $\lambda$ arises from the $O(\lambda)$ ambiguity in the charge assignment. A similar expression is obtained for the $\Lambda_{b1}$ decay:

$$\Gamma(\Lambda_{b1}\to\Lambda_b\gamma) = \frac{1}{6} e^2 \kappa \left(\frac{m_B + m_N}{m_B m_N} \right)^2 \left(1 - \frac{\alpha}{4! \sqrt{k^3 \mu_b}} \right) (1 + O(\lambda)).$$  \hspace{1cm} (4.22)

Conversely, by measuring the radiative decay rates of the first excited states of $\Lambda_c$ and $\Lambda_b$ baryons, the constants $\kappa$ and $\alpha$ can be fixed and used to predict the spectroscopic and semileptonic observables.

V. CONCLUSION

We have studied the phenomenology of isoscalar heavy baryons within the framework of an effective theory based on the contracted $O(8)$ in the combined heavy quark and large $N_c$ limit. This symmetry emerges in the subspace of the QCD Hilbert space with baryon number one and heavy quark number one, i.e., the heavy baryon subspace. The low-energy excited states of heavy baryons described by the effective theory are the collective excitations of the brown muck relative to the heavy quark.

The effective Hamiltonian, eq. (1.8), describes the low-lying excited states of isoscalar heavy baryons with excitation energies of order $\lambda^{1/2}$. Reliable predictions can be expected only for the ground state and a doublet of the first orbitally excited state. At leading nontrivial order, the spin-averaged sum of the first excited state of $\Lambda_c$ baryons is predicted to be approximately $5920 \, \text{MeV}$. The next-to-leading corrections to the excitation energies, eq. (2.14), are found by treating perturbatively the $\alpha x^4/4!$ term in the effective Hamiltonian eq. (1.8).

In addition to the spectroscopic observables, dominant semileptonic form factors of the electroweak decays of $\Lambda_c$ are determined at LO and NLO. We have shown, that at leading and next-to-leading order in the combined expansion there are only two independent form factors: one for $\Lambda_c \to \Lambda_c \ell \bar{\nu}$ transition, and one for $\Lambda_b \to \Lambda_{c1} \ell \bar{\nu}$ and $\Lambda_b \to \Lambda_{b1} \ell \bar{\nu}$ decays. These form factors are calculated for the velocity transfers of order $\lambda^{3/4}$. The form factors are exponentially suppressed for velocity transfers of order unity.

Experimentally useful quantities include values and derivatives of the form factors at zero recoil. We have determined these observables at leading and next-to-leading order. At LO, the form factor for $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ at zero recoil is 0.998 which is very close to the HQET normalization of unity. In the combined limit, the heavy quark is no longer the static source of the color magnetic field (in the baryon rest frame). As a result, we expect a deviation from the HQET result for the form factor normalization. However, as seen here this effect is very small.

We have calculated the total radiative decay rates of the excited heavy baryons. As we have shown, at LO and NLO in the combined expansion, these decays are dominated by the dipole radiation. There is a significant deviation of the ratio of these decay rates (0.2 at LO in the combined limit) from the HQET value of unity (eq. (4.21)).

At leading order ($O(\lambda^{1/2})$) our predictions are the same as those obtained in the bound state picture of heavy baryons [17, 20, 31]. However, our predictions are model independent and based on an effective theory with self-consistent counting rules. In addition, we have extended the treatment to next-to-leading order corrections which appear at relative order $\lambda^{1/2}$ and not at order $\lambda$ as was previously thought. In subsequent publications, we will extend the treatment of the effective theory to the excited states of non-zero isospin heavy baryons, such as $\Sigma^*_c$ and $\Sigma^*_b$. This can be done by combining the contracted $O(8)$ symmetry with the $SU(2N_f)$ symmetry which emerges in QCD in the large $N_c$ limit.
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APPENDIX:

In this appendix we show that only one effective heavy quark operator contributes to the electroweak matrix element (eqs. (3.6), (2.29) and (3.33)) at LO and NLO in the combined expansion. This operator is the first term in the combined expansion of the left-handed heavy quark current. We also obtain the $\lambda$-scaling of the Lorentz components of the effective operator which determine the self-consistent $\lambda$-scaling of the electroweak form factors discussed in Sec. [HI].

The combined expansion of the heavy quark current $J = \bar{Q}_j(y)\Gamma Q_i(y)$ was developed in Ref. [2]. The expansion is achieved by separating the total heavy quark field $Q(y)$ into two parts:

\begin{equation}
\begin{align*}
\hat{h}_Q^{(v)}(y) &= e^{-i(m_Q + m_N)\nu \gamma_\mu} P_+ Q(y), \\
\hat{H}_Q^{(v)}(y) &= e^{-i(m_Q + m_N)\nu \gamma_\mu} P_- Q(y),
\end{align*}
\end{equation}

where the projection operators $P_+$ and $P_-$ are defined as $P_+ = (1 + \gamma^\nu)/2$ and $P_- = (1 - \gamma^\nu)/2$. The fields $\hat{h}_Q^{(v)}$ and $\hat{H}_Q^{(v)}$ satisfy the following conditions:

\begin{equation}
\gamma h^{(v)} = h^{(v)}, \quad \gamma H^{(v)} = -H^{(v)}.
\end{equation}

In HQET, the contribution of the effective field $H_Q^{(v)}$ is suppressed by $1/m_Q$ relative to the field $h_Q^{(v)}$. The process of the heavy quark pair creation is suppressed at leading order in HQET. We certainly expect the same in the combined limit. While, the heavy quark is bound to the massive brown muck, the heavy quark is still nonrelativistic at LO and NLO. Hence, the field $H_Q^{(v)}$ should be suppressed in the combined limit as well. However, it is not immediately clear due to the phase redefinition in eq. (A3). The heavy quark part of the total Lagrangian density can be expressed in terms of $h_Q^{(v)}$ and $H_Q^{(v)}$ using the conditions in eq. (A2):

\begin{equation}
\begin{align*}
\mathcal{L}_Q &= \bar{Q}(i\not\!D - m_Q)Q = \bar{h}_Q^{(v)}(i\gamma^\mu D_\mu h_Q^{(v)} - \bar{H}_Q^{(v)}(i\gamma^\mu D_\mu + 2m_Q)H_Q^{(v)} + \bar{h}_Q^{(v)}(\not\!D_\perp)H_Q^{(v)} \\
&\quad + \bar{H}_Q^{(v)}(\not\!D_\perp)h_Q^{(v)} + m_N\left(\bar{h}_Q^{(v)}h_Q^{(v)} - \bar{H}_Q^{(v)}H_Q^{(v)}\right)
\end{align*}
\end{equation}

where $D_\perp = D - (\gamma^\nu D_\nu)v$ is the “transverse part” of the covariant derivative $D$. The heavy quark Lagrangian in eq. (A3) written in terms of the fields $h_Q^{(v)}$ and $H_Q^{(v)}$ differs from its analog in HQET due to the additional term proportional to $m_N$. This $m_N$ term would indicate that the fields $h_Q^{(v)}$ and $H_Q^{(v)}$ are both heavy with masses $m_N$ and $m_Q + m_N$, respectively, which apparently prevents integrating out the “heavy” field $H_Q^{(v)}$.

To get the correct scaling of the fields $h_Q^{(v)}$ and $H_Q^{(v)}$ one needs to consider the total QCD Lagrangian density which includes contributions from the light degrees of freedom:

\begin{equation}
\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_q + \mathcal{L}_M = \bar{Q}(i\not\!D - m_Q)Q + \sum_j \bar{q}_j(i\not\!D - m_j)q_j + \mathcal{L}_YM,
\end{equation}

where the sum is over all light quarks, and $\mathcal{L}_YM$ is the Yang-Mills Lagrangian density. The total Lagrangian density in eq. (A4) can be re-expressed so as to build the brown muck contribution into the heavy degrees of freedom. In the Hamiltonian, eq. (A3), it is done by adding and subtracting a quantity which is an overall constant $m_N$ and regrouping terms according to their $\lambda$ counting scaling. In the Lagrangian formalism this can be accomplished by using a Lorentz covariant operator $m_N \not\!Q \not\!\gamma Q$:

\begin{equation}
\mathcal{L} = (\mathcal{L}_Q - m_N \bar{Q} \not\!\gamma Q) + (\mathcal{L}_q + m_N \bar{Q} \not\!\gamma Q + \mathcal{L}_YM) = \mathcal{L}_H + \mathcal{L}_\ell,
\end{equation}

where

\begin{equation}
\begin{align*}
\mathcal{L}_H &\equiv \mathcal{L}_Q - m_N \bar{Q} \not\!\gamma Q, \\
\mathcal{L}_\ell &\equiv \mathcal{L}_q + m_N \bar{Q} \not\!\gamma Q + \mathcal{L}_YM.
\end{align*}
\end{equation}
In the rest frame of a heavy baryon operator, \( \tilde{Q} \neq Q \) is equal to the heavy quark density operator \( Q^\dagger Q \) and its spatial integral equals to 1 in the Hilbert space of heavy baryons. The heavy quark dynamics in the combined limit is determined by \( \mathcal{L}_H \). The additional mass term (with minus sign) corresponds to the brown muck mass, the \( m_N \) term in the effective Hamiltonian, eq. (A3).

In terms of the fields \( h_Q^{(v)} \) and \( H_Q^{(v)} \), the operator \( m_N \tilde{Q} \neq Q \) is given by,

\[
m_N \tilde{Q} \neq Q = m_N \left( \tilde{h}_Q^{(v)} - \tilde{H}_Q^{(v)} H_Q^{(v)} \right).
\]

This operator cancels the last term in eq. (A3), so that the Lagrangian density \( \mathcal{L}_H \) has the form:

\[
\mathcal{L}_H = \tilde{h}_Q^{(v)} (i v^\mu D^\nu) h_Q^{(v)} - \tilde{H}_Q^{(v)} (i v^\mu D^\nu + 2m_Q) H_Q^{(v)} + \tilde{h}_Q^{(v)} (i \partial \perp) H_Q^{(v)} + \tilde{H}_Q^{(v)} (i \partial \perp) h_Q^{(v)}. \tag{A8}
\]

Hence, the “large” component \( h_Q^{(v)} \) describes a massless field while the “small” component \( H_Q^{(v)} \) has the mass \( 2m_Q \).

Using equations of motion the “heavy” field \( H_Q^{(v)} \) can be expressed in terms of the “light” field \( h_Q^{(v)} \) as follows:

\[
H_Q^{(v)} = \frac{1}{2m_Q + i v^\mu D_\mu} i \partial \perp h_Q^{(v)}. \tag{A9}
\]

The relation between fields \( h_Q^{(v)} \) and \( H_Q^{(v)} \) in eq. (A3) is identical in form to that in HQET between the “large” and “small” components of the heavy quark field \( Q \).

We can now expand the field \( Q = e^{i(m_Q+m_N)v^\mu y_\mu} (h_Q^{(v)} + H_Q^{(v)}) \), the Lagrangian density in eq. (A3), and the current \( J = Q_i \Gamma Q_i \), in powers of \( \lambda \). This can be done by eliminating fields \( H_Q^{(v)} \) (using eq. (A9)) and expanding the numerator in powers of \( (i v^\mu D_\mu)/2m_Q \). The combined expansion of the heavy quark field including terms up to order \( \lambda \) is,

\[
Q(y) = e^{-i(m_Q+m_N) v^\mu y_\mu} \left[ 1 + \left( \frac{1-\gamma^5}{2} \right) \frac{i \partial}{2m_Q} \right] h_Q^{(v)}(y) + \mathcal{O}(\lambda^2). \tag{A10}
\]

Using this expansion, the Lagrangian density \( \mathcal{L}_H \) including terms up to order \( \lambda \) has the form:

\[
\mathcal{L}_H = \tilde{h}_Q^{(v)} (i v^\mu D_\mu) h_Q^{(v)} + \frac{1}{2m_Q} \tilde{h}_Q^{(v)} \left[ -(i v^\mu D_\mu)^2 + (i D)^2 - \frac{1}{2} g_s \sigma_{\mu\nu} G^{\mu\nu} \right] h_Q^{(v)} + \mathcal{O}(\lambda^2), \tag{A11}
\]

where \( g_s \) is the strong coupling constant and \( G^{\mu\nu} = [D^\mu, D^\nu] \) is the gluon field strength tensor. In a similar way, we can expand the current \( J = \tilde{Q}_i \Gamma Q_i \); keeping terms up to \( \lambda \), we get,

\[
J(y) = \tilde{h}_Q^{(v)} \Gamma \tilde{Q}_i^{(v)} + \frac{1}{2m_Q} \tilde{h}_Q^{(v)} \Gamma (i \partial) \tilde{Q}_i^{(v)} + \frac{1}{2m_Q} \tilde{h}_Q^{(v)} (-i \partial \perp) \Gamma \tilde{Q}_i^{(v)} + \mathcal{O}(\lambda^2), \tag{A12}
\]

where the covariant derivative \( \tilde{\partial} \) acts on the field on the left. The expressions in eqs. (A10), (A11) and (A12) are identical in form to the analogous quantities in HQET. However, there is one important difference; these expressions represent the combined expansions in powers of \( \lambda \) and not the \( 1/m_Q \) expansion of HQET. In other words, by defining fields \( h_Q^{(v)} \) and \( H_Q^{(v)} \) appropriately, we were able to re-sum implicitly all the \( m_N/m_Q \) corrections.

A typical magnitude of the momentum, \( k \), carried by the effective field \( h_Q^{(v)} \) is of order \( \tilde{\lambda} \equiv m_{\Lambda_Q} - (m_Q + m_N) \sim \lambda^0 \). As a result, operators containing \( n \) powers of the covariant derivative \( D \) are of order \( (k/m_Q)^n \sim \lambda^n \) in the combined limit. For example, the second and third terms in eq. (A12) are of relative order \( \lambda \), so that they contribute only at NNLO in the combined expansion. Therefore, only one operator—\( \tilde{h}_Q^{(v)} \Gamma h_Q^{(v)} \)—contributes at LO and NLO.

The effective vector and axial currents at leading and next-to-leading order are given by:

\[
\tilde{v}_j^{(v)} b (y = 0) = \tilde{h}_Q^{(v)} \gamma^\mu h_b^{(v)} (y = 0) + \mathcal{O}(\lambda),
\]

\[
\tilde{v}_j^{(v)} \gamma_5 b (y = 0) = \tilde{h}_Q^{(v)} \gamma^\mu \gamma_5 h_b^{(v)} (y = 0) + \mathcal{O}(\lambda). \tag{A13}
\]

The heavy quark effective currents, eq. (A13), have the same form at \( y = 0 \) regardless of whether the heavy quark effective fields \( \tilde{h}_Q^{(v)} \) and \( h_b^{(v)} \) are defined at the same 4-velocity, \( v' = v \), or at two different values, \( v' \neq v \).

Now it is easy to show the self-consistent scaling rules of the electroweak matrix elements, eq. (A12). The effective operators corresponding to the time component of the vector current and spatial components of the axial current scale as \( \lambda^0 \).
\[ h_{\Sigma}^{(v)} h_{b}^{(v)} \sim \alpha^i, \]
\[ \hat{h}_{\Sigma}^{(v)} \sim \lambda^0, \]
\[ \hat{h}_{b}^{(v)} \sim \lambda^0, \]

where the scaling of the axial operators follow because \( \Sigma^i = \gamma^0 \gamma^i \gamma_5 \) act only on the upper components of the effective heavy quark fields which are of order \( \lambda \). On the other hand, spatial components of the vector current and time component of the axial current are of order \( \lambda^{3/4} \):

\[ \hat{h}_{c}^{(v)} \gamma^i \hat{h}_{b}^{(v)} = \hat{h}_{c}^{(v)} \alpha^i \hat{h}_{b}^{(v)} \sim \lambda^{3/4}, \]
\[ \hat{h}_{c}^{(v)} \gamma_5 \hat{h}_{b}^{(v)} = \hat{h}_{c}^{(v)} \gamma_5 \hat{h}_{b}^{(v)} \sim \lambda^{3/4} \]

(A15)

since the matrices \( \alpha^i \) and \( \gamma_5 \) mix the upper and lower components of the effective heavy quark fields which brings an additional power of \( \lambda^{3/4} \).

As shown in Sec. II, the counting rules in eqs. (A14) and (A13) lead to the self-consistent scaling rules for the semileptonic form factors, eqs. (A14), (3.31), (3.37) and (3.41). As a result, the dominant form factors at LO and NLO for the \( \Lambda_b \rightarrow \Lambda_c \ell \bar{\nu} \) decay are \( F_1 \) and \( G_1 \) given by,

\[ \langle \Lambda_c(\vec{v})|\bar{c}\gamma_0 b|\Lambda_b(\vec{v})\rangle = \langle \Lambda_c(\vec{v})|\hat{h}_{\Sigma}^{(v)} h_{b}^{(v)} |\Lambda_b(\vec{v})\rangle = F_1 u_\lambda^c(\vec{v}) u_\lambda^b(\vec{v}) (1 + O(\lambda)), \]
\[ \langle \Lambda_c(\vec{v})|\bar{c}\gamma_i \gamma_5 b|\Lambda_b(\vec{v})\rangle = \langle \Lambda_c(\vec{v})|\hat{h}_{\Sigma}^{(v)} \Sigma^i h_{b}^{(v)} |\Lambda_b(\vec{v})\rangle = G_1 u_\lambda^c(\vec{v}) \Sigma^i u_\lambda^b(\vec{v}) (1 + O(\lambda)), \]

(A16)

where the effective heavy quark operators, eq. (A13), were used. The effective operator \( \hat{h}_{\Sigma}^{(v)} \Sigma^i h_{b}^{(v)} \) acts on the heavy quark spin degrees of freedom due to the \( \Sigma^i \) matrix. As discussed in Sec. II, the heavy quark spin decouples from the dynamics of the collective degrees of freedom—the motion of the brown muck relative to the heavy quark—at LO and NLO in the combined limit. Hence, up to NNLO the effect of the heavy quark effective operator \( \hat{h}_{\Sigma}^{(v)} \Sigma^i h_{b}^{(v)} \) in the Hilbert space of the low-lying heavy baryon states is identical to that of the effective operator \( \hat{h}_{\Sigma}^{(v)} h_{b}^{(v)} \). As a result, the dominant vector and axial form factors—\( F_1 \) and \( G_1 \)—are equal up to NNLO in the combined limit.

Similarly, the pair of the form factors \( K_1 \) and \( L_1 \) (dominant in the \( \Lambda_b \rightarrow \Lambda_c^* \ell \bar{\nu} \) decay), and \( N_1 \) and \( M_1 \) (dominant in the \( \Lambda_b \rightarrow \Lambda_c^* \ell \bar{\nu} \) decay) are equal up NNLO in the combined limit.
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