Algorithm of Medical Image Fusion based on Laplasse Pyramid and PCA

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Abstract. The paper firstly expounds the principle and method of image fusion of Gauss Pyramid, Laplasse Pyramid and principal component transform. After that, it explains in detail that the image is decomposed by Laplasse and Pyramid, and then the image fusion is carried out by principal component analysis and average gradient method for high frequency part and low frequency part. Finally, Laplasse's inverse transform is used to get the final fused image. The algorithm is tested with medical images, and the effect of the algorithm is not very ideal, but it is more effective than the simple PCA and Laplasse Pyramid image fusion method.

1. Introduction
Image Pyramid is a multi-scale representation of images. It is an effective but simple in concept structure for multi-resolution interpretation of images. The theory of the image Pyramid method is to decompose each image of the fusion into a multi-scale Pyramid image sequence. A low-resolution image is in the upper level, a high-resolution image is in the lower level, and the upper image is 1/4 of the size of the previous level. The number of layers is 0, 1, 2... N. Combining all the images of Pyramid on the corresponding level with certain rules, the synthetic Pyramid can be obtained, and then the synthetic Pyramid is reconstructed according to the inverse process generated by Pyramid, and the fusion of Pyramid is obtained.

2. Image Decomposition Based on Gauss Pyramid (Gaussian Pyramid, GP)
Gauss Pyramid is a technology used in image processing, computer vision and signal processing. Gauss Pyramid is through Gauss smoothing and subsampling to obtain some subsampling images. That is, the K layer Gauss Pyramid can get the K+1 layer Gauss image by smooth, subsampling. Gauss Pyramid contains a series of low-pass filters, whose up to the bottom is gradually increasing from the upper to the next layer by factor 2. Gauss Pyramid can span a very large frequency range.
The source image is $G_0$, $G_0$ is used as the zeroth layer (bottom layer) of Gauss Pyramid. The original input picture is sampled by Gauss low pass filter and interlaced interlaced, and the first level of Gauss's Pyramid is obtained. In the first level image low pass filtering and lower sampling, the next level of the pyramid is obtained, and the above-mentioned process is repeated to form the high level pyramid. The current layer image of Gauss Pyramid is generated by the low pass filtering of the previous image, Gauss's low pass filter, and then the 2 sampling of interlaced and separated rows. The size of the current layer image is in turn the 1/4 of the previous image size.

3. Image Reconstruction Based on Laplasse Pyramid (Laplasse Pyramid, LP)
The composition of Laplasse Pyramid is evolved on the basis of Gauss Pyramid, which describes some of the high frequency details lost by convolution and lower sampling operations during the operation of Gauss Pyramid. Laplasse Pyramid is composed of a series of images that are first subtracted and then enlarged by the source image.

We can interpret Laplasse Pyramid as an inverse form of Gauss and Gauss:

$$G_l^*(i,j) = 4G_l = \sum_{m=-n}^{n} \sum_{n=-m}^{m} \omega(m, n) G_l \left( \frac{i+m, j+n}{2} \right) \quad 0 \leq l \leq N, 0 \leq l \leq R_i, 0 \leq j \leq C_i$$

When \( \left( \frac{i+m, j+n}{2} \right) \) are integers

$$G_l \left( \frac{i+m, j+n}{2} \right) = G_l \left( \frac{i+m, j+n}{2} \right)$$

Otherwise

$$G_l \left( \frac{i+m, j+n}{2} \right)$$

The above $G_l$ interpolation method is used to get the amplified image $G_l^*$, which is inserted 0 in the even and even number, and then the lower Gauss kernel is used to filter, so that the size of the $G_l^*$ is the same as that of the $G_{l-1}$. The image $G_l^*$ is magnified and $G_l$ is reduced. Although the image $G_l^*$ of the first layer of Pyramid image $G_l$ is amplified and the size of the Pyramid second layer image $G_{l-1}$ is the same, the two are actually different. We can see from formula (2) that $G_{l-1}$ contains more details than $G_l^*$.

Laplasse's image in Pyramid can be obtained by subtracting approximate two adjacent images in Gauss Pyramid:

$$\begin{align*}
LP_l &= G_l - G_{l+1}^* \\
LP_N &= G_N
\end{align*}$$

In formula (3), $N$ is the top layer of Laplasse's Pyramid; $LP_l$ is the L image of Laplasse and Pyramid. The images of $LP_0$, $LP_1$, ..., $LP_N$ is made up of Pyramid by Laplasse Pyramid. The $LP_0$ image of each layer is the difference between Gauss's Pyramid $G_0$ image and its high level image $G_1$ after interpolated and enlarged $G_1^*$.

Through the inverse process of formula (3), we get the formula (4). For the above mentioned inverse operation of Pyramid Laplasse, we can add the noise and the up sampling to restore the corresponding Gauss Pyramid to get the final source image.

$$\begin{align*}
G_N &= LP_N \\
G_l &= LP_l + G_l^* \quad 0 \leq l < N
\end{align*}$$

After the fusion of Laplasse Pyramid, we have inferred the source images from the top to bottom.
4. Image Fusion Based on PCA (Principal component analysis, PCA)

PCA is an optimal orthogonal transformation based on target characteristics. In statistics, principal component analysis is a multivariate statistical method. PCA transform aims to transform multiple indicators into a few comprehensive indicators by using the idea of dimensionality reduction. PCA technology can often get the most important elements and structures from too "rich" data information, remove the noise and redundancy of data, reduce the original complexity of data, and reveal the simple structure hidden behind the complex data.

The goal of PCA method is to look for $r(r<n)$ new variables to reflect the main features of things, to compress the size of the original data matrix, to reduce the dimension of the eigenvectors and to select the least dimension to summarize the most important features. Each new variable is a linear combination of the original variables, reflecting the comprehensive effect of the original variables, and has certain practical implications. These $r$ new variables are called "principal components", which can reflect the effect of the original $n$ variables to a large extent, and these new variables are interrelated and orthogonal. Through principal component analysis, the data space is compressed, and the characteristics of multivariate data are intuitively expressed in low dimensional space.

(1) According to the original image data matrix $X$, we find out its covariance matrix $C$:

$$C = \frac{1}{n} \left[ (x - \bar{x})(x - \bar{x})^T \right] \in \mathbb{R}^{n \times n} \tag{5}$$

(2) To obtain the eigenvalues and eigenvectors of the covariance matrix, and to form the transformation matrix. The characteristic equation is as follows:

$$(A\mathbf{I} - C)\mathbf{U} = \mathbf{0} \tag{6}$$

In the form: I is a unit matrix, and $\mathbf{U}$ is an eigenvector.

(3) To Calculate transform matrix $T: T = \mathbf{U}^T$. It is a matrix composed of various eigenvectors, and the $\mathbf{U}$ matrix is an orthogonal matrix, that is, the $\mathbf{U}$ matrix satisfies:

$$\mathbf{U}^T \mathbf{U} = \mathbf{U}\mathbf{U}^T = \mathbf{I} \quad \text{(Unit Matrix)}. \tag{7}$$

(4) To replace the transformation matrix $T: Y = TX$, the specific expression of the PCA transformation will be obtained.

$$Y = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1r} \\ u_{21} & u_{22} & \cdots & u_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nr} \end{bmatrix} \mathbf{X} \tag{8}$$

5. Image Fusion of Laplasse Transform Pyramid based on PCA

Assuming that $L_{A_1}$ and $L_{B_1}$ are the $l$ layer images after the decomposition of Laplasse Pyramid of the source image $A$ and $B$, the fusion result is $L_{F_1}$. When $l = N$, $L_{AN}$ and $L_{BN}$ are the top-level images obtained from the source image $A$ and $B$ after the decomposition of Laplasse Pyramid. Since PCA can combine the most important information of the two components, the principal component analysis is used to fuse the top level images. The algorithm is as follows:

There are $N$ source images, each image regards as a one-dimensional vector, records as $x_k$, $k=1,2,\ldots,N$.

(1) From the source image constructing the data matrix $X=(x_1,x_2,\ldots,x_N)^T$;
(2) To calculate the covariance matrix $C$ of the data matrix $X$;
(3) To calculate the eigenvalue of $\lambda$ and the corresponding eigenvector $\zeta_i$ of the covariance matrix C. To get the eigenvalue $\lambda_i$ and the corresponding eigenvector $\zeta_i$ from the characteristic equation $|\lambda I - C| = 0$.

(4) To determine the weighting coefficient $\omega$:

$$
\omega_i = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i}
$$

(9)

(5) To calculate the final fusion image $F$:

$$
F = \sum_{i=1}^{n} \omega_i \zeta_i
$$

(10)

So, the Laplace image at the top level is $LF_N(i,j)=F(i,j)$, and the result after fusion is $LF_1$.

For the fusion of other layer images, when $0 \leq l < N$, for the $L$ layer that is decomposed by Laplace Pyramid, first calculates the regional average gradient of $M*N$ ($M, N$ are odd number and $M\geq3$, $N\geq3$) with its pixels as the center.

$$
G = \frac{1}{(M-1)(N-1)} \sum_{i=2}^{M} \sum_{j=2}^{N-1} \sqrt{I_x^2 + I_y^2}/2
$$

(11)

Among them, $I_x$ and $I_y$ are the first difference of pixel $f(x, y)$ in the direction of $x$ and $y$ respectively, as follows:

$$
\begin{align*}
I_x &= f(x, y) - f(x - 1, y) \\
I_y &= f(x, y) - f(x, y - 1)
\end{align*}
$$

(12)

Therefore, for every pixel $LA_L(i,j)$ and $LB_L(i,j)$ in the $L$ level image, we can get the corresponding regional average gradient $GA(i,j)$ and $GB(i,j)$. Because the average gradient reflects the small details and texture changes in the image, it also reflects the sharpness of the image. Generally speaking, the larger the average gradient, the richer the image level is, and the clearer the image is. Therefore, the fusion results of each layer are as follows:

$$
LF_L(i,j) = \begin{cases} 
LA_L(i,j), & GA(i,j) \geq GB(i,j) \\
LB_L(i,j), & GA(i,j) < GB(i,j)
\end{cases}
$$

(13)

After get the fusion images of Pyramid at all levels $LF_1, LF_2, \ldots, LF_N$, we can obtain the final fusion image reconstructing by the previous formula (4).

6. Experimental Results

The medical MRI image is regarded as a sample image, and the effectiveness of the method is evaluated on MatLab7.0. The experimental results are shown in figures 1, 2, 3, 4 and 5.
7. Conclusion
This paper introduces the basic principle, thought and algorithm steps of Laplasse Pyramid image fusion based on PCA, and carries out a simulation experiment on MatLab7.0 with medical image as the data. It can be seen from the experimental results that the image effect of the fusion method is not very ideal and obviously blurred, but it is better than the simple PCA image fusion and the Laplasse Pyramid image fusion algorithm. At the same time, the research of the algorithm lays a solid foundation for the research of image fusion algorithm in later wavelet transform.

Acknowledgement
This research was financially funded by the ‘Kaiwu’ Innovation Team Support Project of Lanzhou Institute of Technology, the Colleges and Universities Scientific Research Project of Gansu Province “Research on Key Technologies of medical image fusion based on transform domain”(2016A-097), the National Natural Science Foundation of China "compressed domain massive video concentrated key technology research"(61462057).

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