In order to construct accurate point sources simulations at the frequencies relevant to 21 cm experiments, the angular correlation of radio sources must be taken into account. Using the 74 MHz VLSS survey, we measured the angular 2-point correlation function, $w(\theta)$. We obtain the first measurement of clustering at the low frequencies relevant to 21 cm tomography. We find that a single power law with shape $w(\theta) = A\theta^{-\gamma}$ fits well the data. For a galactic cut of $|b| > 10^\circ$, with a data cut of $\delta > -10^\circ$, and a flux limit of $S = 770$ mJy, we obtain a slope of $\gamma = (-1.2 \pm 0.35)$. This value of $\gamma$ is consistent with that measured from other radio catalogues at the millimeter wavelengths. The amplitude of clustering has a length of $0.2^\circ - 0.6^\circ$, and it is independent of the flux-density threshold.

I. INTRODUCTION

Progress in detector, space and computer technology has triggered an avalanche of high-quality cosmological data, removing cosmology from the realm of philosophy and transforming it into a quantitative empirical science. In the past few years, many authors have argued that the 21cm tomography, i.e., the three-dimensional mapping of highly redshifted 21cm emission, will be the ultimate cosmological probe — see, e.g., [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Although this signal has yet to be detected, there is a theoretical consensus that the 21 cm signal must be out there and would be extremely useful if measured. [12, 13, 14, 15, 16, 17]

Although ambitious experimental efforts in 21cm tomography are now in progress across the globe (see Table I), it is widely known/understood that the cosmological results of these experiments will only be as good as our ability to deal with (or to remove) foreground contamination [18, 19, 20, 21, 22, 23, 24]. The goal of this work is to support these worldwide experimental efforts by tackling the foreground issue.

Understanding the physical origin of Galactic metre wavelength emission is interesting for two reasons: to determine the fundamental properties of the Galactic components, and to refine the modeling of foreground emission for cosmological 21 cm experiments. At metre wavelengths, the bulk of foreground contamination is due to synchrotron emission. When coming from extragalactic objects, this radiation is usually referred to as point source contamination and affects mainly small angular scales. When coming from the Milky Way, this diffuse Galactic emission fluctuates mainly on large angular scales [21].

Normal galaxies, radio galaxies and active galactic nuclei form the majority of extragalactic continuum sources [25]. A number of surveys of radio sources have been performed at frequencies relevant to the 21 cm tomography — see Table III and analysis of these catalogs have helped to bring some understanding about their statistical properties: the distribution of radio sources is found to obey Poisson statistics with very weak observed angular clustering — see Table III.

Some aspects of both experimental design optimization and actual data analysis require full-blown simulations of the sky signal and knowledge about how it propagates through the instrument and the data analysis pipeline — this has motivated the ambitious simulation efforts carried out by, e.g., WMAP and Planck. End-to-end simulations are at least as important for 21 cm experiments because of the many complicated issues related to instrumental performance, ionospheric turbulence corrections, etc. [19, 26, 27, 28, 29, 30]. In order to construct accurate simulations at the metre wavelengths, the angular correlation of radio sources must be taken into account [31]. It is important to point out that the relative importance of the clustering contribution increases and may eventually become dominant if sources are identified and subtracted down to faint flux limits [28] — which are exactly the limits involved in the point source removal of 21 cm experiments.
TABLE I: 21 cm Tomography Experiments.

| Experiment | FWHM | ν | Receiver | Sensitivity | Effective Area | Site-yr |
|------------|------|---|----------|-------------|----------------|---------|
| GMRT       | 3.8°-0.4° | 50–1420 | 30 dishes | 15 mK/√day | 5.10^4 | India - 2007 |
| PAST/21CMA | 3' | 50–200 | 10,000 antennas | 7.10^4 | 10,000 antennas | Ulastai, CH - 2007 |
| LOFAR      | 25°-3.5° | 10–240 | 25,000 dipole antennas | 1.10^4 | 25,000 dipole antennas | Drenthe, NL - 2007 |
| MWA        | 15' | 80–300 | 8,192 dipole antennas | 1.10^4 | 8,192 dipole antennas | Murchison, AU - 2007 |
| PAPER      | 110–200 | 16 antennas | 1.10^4 | 16 antennas | USA/AU - 2008 |
| SKA        | 0.1'' | 100–25GHz | 1.10^4 | 100–25GHz | AU(?) - 2015(?) |

GMRT = Giant Metrewave Radio Telescope, see [http://www.gmrt.ncra.tifr.res.in/](http://www.gmrt.ncra.tifr.res.in/).
PaST/21CMA = PrimevAl Structure Telescope, see [http://web.phys.cmu.edu/~past/](http://web.phys.cmu.edu/~past/).
LOFAR = LOw F requency ARray, see [http://www.lofar.org](http://www.lofar.org).
MWA = Murchison Widefield Array, see [http://www.haystack.mit.edu/ast/arrays/mwa/index.html](http://www.haystack.mit.edu/ast/arrays/mwa/index.html).
PAPER = Precision Array to Probe Epoch of Reionization, see [http://astro.berkeley.edu/~dbacker/eor/](http://astro.berkeley.edu/~dbacker/eor/).
SKA = Square Kilometer Array, see [http://www.skatelescope.org](http://www.skatelescope.org).

TABLE II: Publicly available point source catalogues at the frequencies relevant to 21-cm tomography.

| Ref | ν [MHz] | Region | FWHM [arcmin] | S_comp [Jy] | S_min [Jy] | N_obj | Observatory | Status |
|-----|---------|--------|---------------|-------------|------------|-------|-------------|--------|
| 31  | 38      | 00°<α<24° +60°<δ<+90° | 4.5 | 5859 | CLFST, ENG | A |
| 32  | 60      | 00°<α<24° +55°<δ<+55° | 450 | 100 | Pushchino, RUS | B |
| 33  | 74      | 00°<α<24° -30°<δ<+90° | 1.33 | 68311 | VLA, USA | A |
| 34  | 80      | 00°<α<24° -49°<δ<+37° | 3.7 | 999 | Pushchino, RUS | A |
| 35  | 31      | 00°<α<24° +70°<δ<+90° | 10 | 558 | Cambridge, ENG | B |
| 36  | 102     | 00°<α<24° +27°<δ<+70° | 60 | 920 | LPA, RUS | A |
| 37  | 150     | 18°<α<24° -70°<δ<+10° | 4.6 | 2784 | MRT, India | A |
| 38  | 158     | 18°<α<24° -30°<δ<+90° | 4.2 | 34418 | CLFST, ENG | A |
| 39  | 150     | 00°<α<24° -22°<δ<+71° | 10.0 | 43689 | CLFST, ENG | A |
| 40  | 160     | 00°<α<24° -49°<δ<+37° | 1.85 | 471 | Cambridge, ENG | A |
| 41  | 178     | 00°<α<24° -90°<δ<+5° | 6.0 | 2041 | CLFST, ENG | A |
| 42  | 178     | 00°<α<24° -90°<δ<+5° | 0.1 | 51000 | Cambridge, ENG | A |
| 43  | 232     | 00°<α<24° +30°<δ<+90° | 3.8 | 4844 | 4C Array, ENG | A |
| 44  | 325     | 00°<α<24° +30°<δ<+90° | 0.9 | 84481 | WSRT, NLD | A |
| 45  | 352     | 00°<α<24° -90°<δ<+26° | 0.9 | 229420 | WSRT, NLD | A |
| 46  | 365     | 00°<α<24° +36°<δ<+72° | 0.1 | 66841 | UTRAO, USA | A |

S_comp = Limit of completeness.
S_min = Smallest flux value.
N_obj = Number of sources in the catalogue.
A = Publicly available in digital form.
B = Available as printed table (which we will OCR).
In this paper, we present measurements of the angular 2-point correlation function, \( w(\theta) \), from the 74 MHz VLSS survey. We obtain the first measurement of clustering at the low frequencies relevant to 21 cm tomography. In Section III we described the statistical tools used in this analysis, as well as the 74 MHz VLSS survey. In Section IV we present our results, and in Section V we present our conclusions.

II. DATA ANALYSIS TOOLS

A. The Angular 2-point Correlation Function

In recent years, the analysis of the correlation-function has become the standard way of quantifying the clustering of different populations of astronomical sources. Specifically, the angular two-point correlation function \( w(\theta) \) gives the excess probability \( \delta P \), in comparison to a random Poisson distribution, of finding two sources in a solid angle \( \delta \Omega_1 \) and \( \delta \Omega_2 \) separated by the angle \( \theta \). \( \delta P \) is defined as

\[
\delta P = N^2 \delta \Omega_1 \delta \Omega_2 [1 + w(\theta)],
\]

where \( N \) is the mean number density of objects in the catalogue under consideration.

Many derivations for estimators of \( w(\theta) \) can be found in the literature (see, e.g., \[54, 60, 61\]). One way to estimate this function is to compare the distribution of the objects in the real catalogue to the distribution of points in a random Poisson distributed catalogue with the same boundaries, or

\[
w(\theta) = \frac{DD(\theta) \ast RR(\theta)}{(DR(\theta))^2} - 1
\quad (2)
\]

where \( DD(\theta) \), \( RR(\theta) \) and \( DR(\theta) \) are the numbers of data-data, random-random and data-random pairs separated by the distance \( \theta + \delta \theta \). It is important to remember that the estimation of \( RR(\theta) \) and \( DR(\theta) \) requires a catalogue of objects scattered uniformly over an area with the same angular boundaries of the data catalogue.

B. Mock Catalogues

We used the “Sphere Point Picking Algorithm” \[62\] to generate random cartesian vectors equally distributed on the surface of a unit sphere (to avoid having vectors “bunched” around the poles, as it would happen if one chooses to plot the vectors in spherical coordinates instead). Accordingly, we calculate these vectors by doing

\[
x = \sqrt{1 - u^2} \cos \theta
\quad (3)
\]
\[
y = \sqrt{1 - u^2} \sin \theta
\quad (4)
\]
\[
z = u
\quad (5)
\]

where \( u = \cos \phi \), with \( \theta \in [0, 2\pi] \) and \( u \in [-1, 1] \). In order to obtain points such that any small area on the sphere is expected to contain the same number of points, we choose \( u \) and \( v \) to be random variates in the interval

\[
\delta \Omega_1 \delta \Omega_2
\]

\[
S_{\text{lim}} \text{ = Smallest flux value.}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Ref} & \nu & A \times 10^{-3} & \gamma & w(\theta) & S_{\text{lim}} \\
\hline
[47] & 0.178 & 1.0 \pm 0.4 & 1.22 \pm 0.33 & > 0.2 & 3000 \\
[48] & 0.325 & 1.2 \pm 0.1 & 1.5 \pm 0.1 & > 0.07 & 3 \\
[50] & 0.408 & 1.0 \pm 0.1 & 0.9 \pm 0.2 & > 0.07 & 3 \\
[51] & 1.400 & 0.8 \pm 0.2 & 1.05 \pm 0.10 & > 0.3 & 10 \\
[52] & 1.400 & 1.0 \pm 0.2 & 0.7 \pm 0.1 & > 0.1 & 10 \\
[53] & 1.400 & 1.0 \pm 0.2 & 0.7 \pm 0.1 & > 0.1 & 10 \\
[54] & 1.400 & 1.0 \pm 0.2 & 0.7 \pm 0.1 & > 0.1 & 10 \\
[55] & 4.850 & 0.8 \pm 0.2 & 1.05 \pm 0.10 & > 0.3 & 10 \\
[56] & 4.850 & 0.8 \pm 0.2 & 1.05 \pm 0.10 & > 0.3 & 10 \\
[57] & 4.850 & 10.0 \pm 5.0 & 0.8 \pm 0.2 & 1.0 \pm 1.0 & 50 \\
\hline
\end{array}
\]

FIG. 2: A comparison between point source sensitivity and resolution of the 74 MHz VLSS survey (in red) and other low frequency surveys (see Table II).
FIG. 3: Measured $w(\theta)$ for different galactic cuts. All angular correlations are calculated at the flux limit of $S = 770$ mJy. The red lines are single power law fits to the data, where $w(\theta) = A \theta^{-\gamma}$; and the yellow shaded regions are $w(\theta)$ calculated using solely mocks.

Using the equations above we generate a position in the random catalogue. If this position is inside the boundaries of the data catalogue, then a temperature of the data catalogue is associated with that random vector. This procedure is repeated until the random catalogue has the same number of “objects” as the data catalogue. This method, also known as “bootstrapping”, involves resampling the data with replacement and, at random, to construct a new data set which has population distribution identical to that of the original dataset. Figure 1 shows a realization of one of our mock catalogues.

C. VLSS: The VLA Low-Frequency Survey

The VLA Low-frequency Sky Survey (VLSS, formerly known as 4MASS) is a 74 MHz (or 4 meter wavelength) continuum survey carried out by the National Radio Astronomy Observatory (NRAO) and the Naval Research Laboratory (NRL). The aim of the survey is to map an area of $3\pi$ sr covering the entire sky north of $-30^\circ$ declination at resolution $80''$ (FWHM), with an average noise level of 0.1 Jy/beam. The principal data product is a set of 358 continuum images of $(14^\circ \times 14^\circ)$, and a catalogue with 68,311 discrete sources [58]. The VLSS catalogue was created by fitting elliptical Gaussians to all the sources that are detected at the 5 sigma level or higher [64], and it is complete at the 770 mJy level [65]. The 74 MHz catalogue is shown in Figure 1 top, and a comparison of this survey with other low-frequency surveys can be seen in Table II and Figure 2.

III. RESULTS

In Figure 3, we present our measurement of $w(\theta)$ for the flux limit of $S = 770$ mJy (black squares), which is the completeness limit of the VLSS catalogue. Distances between data and/or random sources are measured in bins of 0.09\degree, which is safely above the VLSS resolution limit of 0.02\degree. We also investigated if $w(\theta)$ changes with bin size, and we found no indication that any change in bin size affects our results.

As shown in Figure 4 there are sources in the VLSS catalogue that may be galactic in origin. In this figure, we plot the source fluxes at galactic longitude $\ell = 120^\circ$ as a function of galactic latitude $b$. The green and yellow shades enclose the regions $|b| \leq 20^\circ$ and $|b| \leq 10^\circ$, respectively. To reduce contamination from galactic sources, we discarded regions inside chosen Galactic cuts; we also discarded regions below $\delta < -10^\circ$, due to the patch sky coverage of VLSS – see Figure 1 top.

From top-to-bottom, Figure 3 shows the measured $w(\theta)$ for different galactic cuts. We detected no correlation for cuts smaller than 10\degree and, above this limit, there are no large variations in $w(\theta)$. Since we want to maximize the number of sources used in our statistics, from here on, all final calculations are for a 10\degree galactic cut (i.e., for 39,118 sources). A galactic cut of 10\degree (or bigger) also excludes the “blank” regions in the VLSS survey – see Figure 1. They are regions around, e.g., Cas A and Cyg A.

We construct 100 mock catalogues using the procedure described in [113] with flux values above the sensi-
TABLE IV: $w(\theta)$\(^1\) results.

| $|b|$ | $A$ | $\gamma$ | $w(\theta)$ | $S_{\text{lim}}$ | $\chi^2$ |
|-----|-----|-----|-----|-----|-----|
| 10° | 0.103±0.026 | -1.21±0.35 | 0.2–0.6 | 770 | 0.62 |
| 15° | 0.062±0.011 | -1.81±0.47 | 0.2–0.6 | 770 | 0.73 |
| 20° | 0.041±0.007 | -2.22±0.78 | 0.2–0.6 | 770 | 0.57 |
| 25° | 0.066±0.011 | -1.81±0.28 | 0.2–0.5 | 770 | 0.58 |
| 10° | 0.113±0.029 | -1.09±0.20 | 0.2–0.6 | 850 | 0.86 |
| 10° | 0.104±0.028 | -1.26±0.38 | 0.2–0.6 | 900 | 0.63 |

$S_{\text{lim}}$ = Smallest flux value.

\(^1w(\theta)$ is fitted by a power-law of the form $A\theta^{-\gamma}$.

![FIG. 5: Measured amplitudes of A and $\gamma$ for various flux-density limits at 770 mJy, 850 mJy and 900 mJy. Note that the amplitude of clustering does not depend on flux density.](image)

The authors wish to thank Joseph Lazio and Mike Matejek for helpful comments. Support for this work was provided by NSF through grants AST-0607597 and AST-0908950. JC acknowledges the Center for Excellence in Education for holding the Research Science Institute (RSI) at MIT to support this work.

**ACKNOWLEDGMENTS:**

The literature that instrumental effects in radio surveys manifest themselves on particular characteristic scales, and are usually rendered transparent by the $w(\theta)$ analysis (see, e.g., \([66]\)). If the anomaly described above is caused by such effects, this is something that should be carefully studied, but it is outside the scope of this paper.

**IV. DISCUSSION**

In order to construct accurate simulations at the metre wavelengths, the angular correlation of radio sources must be taken into account. The relative importance of the clustering contribution increases and may eventually become dominant if sources are identified and subtracted down to faint flux limits – which are exactly the limits involved in the point source removal of 21 cm experiments.

Using the 74 MHz VLSS survey, we measured the angular 2-point correlation function, $w(\theta)$. We obtain the first measurement of clustering at the low frequencies relevant to 21 cm tomography. We find that a single power law with shape $w(\theta) = A\theta^{-\gamma}$ fits the data well. For a galactic cut of $|b| > 10^\circ$, with a data cut of $\delta > -10^\circ$, and a flux limit of $S = 770$ mJy, we obtain a slope of $\gamma = (−1.2±0.35)$ with $\chi^2=0.62$. This value of $\gamma$ is consistent with that measured from other radio catalogues – see Table [III]. The amplitude of clustering has a length of $0.2^\circ–0.6^\circ$, and it is independent of the flux-density threshold.

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