The Phase Transition of the Two Higgs extension of the Standard Model

Vasilios Zarikas

Department of Physics, University of Newcastle Upon Tyne, NE1 7RU
U.K.

Abstract

We present the analysis of the phase transition for the two Higgs electroweak model. We have found that for a wide range of parameters the universe first tunnels to a new intermediate phase. This feature not only is very important by itself, but also provides the essential requirements for producing baryon asymmetry with only small explicit $CP$-violating terms in the two Higgs tree Lagrangian.

Pacs numbers: 12.60.Fr, 11.15.Ex, 98.80.Cq
I. INTRODUCTION

The mechanism of spontaneous symmetry breaking plays a big role in the success of
the standard model of the electroweak interactions. It also leads to the possibility of phase
transitions and important cosmological phenomena. This symmetry breaking mechanism is
based ultimately on the scalar Higgs potential.

The current experimental situation with respect to the standard electroweak model’s
Higgs sector gives the possibility for modifications. A model with two scalar boson doublets
can be a sensible extension of the standard model, and may be necessary if one requires low
energy supersymmetry or sufficient baryon asymmetry in the electroweak transition [3], [5].

In the two-Higgs model the desired baryogenesis at the electroweak scale has been
achieved by the introduction of a term that breaks the $CP$ invariance explicitly at tree
level. The amount of $CP$ violation required is uncomfortably large, particularly if the two
Higgs model is the relic of a symmetry breaking at scales above the electroweak transition,
when it is more natural that any explicit $CP$ violating terms be small [1], [2].

Here it is reported that, using the $CP$ invariant finite temperature effective potential,
domains of the universe can first tunnel towards a new minimum before evolving to the usual
minimum. When small $CP$ violating terms are added to the potential the $CP$ violation is
amplified in the intermediate phase. This happens for a significant range of parameters that
cover also the minimal supersymmetric model. The result is that we expect reasonably large
baryon asymmetry even from a two Higgs scalar potential with very small $CP$ violation at
low temperatures.

II. THE TREE POTENTIAL

The two Higgs scalar potential can be written [9] using doublets of hypercharge +1 as
follows

\[ V_{tr} = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \]

\[ \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \frac{1}{2} \lambda_5[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2] \]  

where \( \lambda_i \) real numbers and

\[ \Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix} \]  

The above form is the most general one satisfying the following discrete symmetry,

\[ \Phi_2 \rightarrow -\Phi_2, \quad \Phi_1 \rightarrow \Phi_1, \quad d_R^i \rightarrow -d_R^i, \quad u_R^i \rightarrow u_R^i \]  

where \( u_R^i \) and \( d_R^i \) represent the charge \( \frac{2}{3} \) and \( -\frac{1}{3} \) right-handed quarks in the weak eigenstates. This symmetry forces all the quarks of a given charge to interact with only one doublet and thus avoids flavour-changing neutral currents.

If we stick to this potential there is no CP violation in the symmetric phase. It is possible however to introduce “soft” discrete symmetry breaking terms of the form

\[ \Delta V = \text{Re}\{2\mu_3^2\Phi_1^\dagger \Phi_2 + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2)\} \]  

The parameters \( \mu_3, \lambda_6 \) and \( \lambda_7 \) can be complex numbers, providing explicit CP violation at tree level through the developed phase between the two VEV’s. Although their presence will be crucial for achieving the desired baryon asymmetry, it is possible to ignore them for the study of the phase transition, assuming that these terms are small.

We can perform an SU(2) rotation that puts the VEV’s of the field for \( i = 1, 2, 3 \) equal to zero. Solving the system \( \partial V/\partial \phi_i = 0 \) implies several different stationary points. One of them is the usual asymmetric minimum that respects the \( U(1) \) of electromagnetism.

\[ \Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_I \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_I \end{pmatrix} \]  

Another already known stationary point is the following

\[ \Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u_{II} \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{II} \\ 0 \end{pmatrix} e^{i\xi} \]  

where \( u_I, v_I, u_{II}, v_{II} \) real numbers. In this extremum the charge invariance is broken (the upper component of the doublet is charged).
The unknown free parameters in this model can be reduced to the five $\lambda_i$ and the ratio $\beta = u_f/v_f$. We have the following constraints on the parameters (Note that there are some differences with [9].)

- Stationary point $I$ is the minimum if and only if
  \begin{align}
  \lambda_1 &> 0, \quad \lambda_2 > 0 \\
  \lambda_4 + \lambda_5 &< 0, \quad \lambda_5 < 0 \\
  2\sqrt{\lambda_1 \lambda_2} &> |\lambda_3 + \lambda_4 + \lambda_5| \tag{7}
  \end{align}

- The potential is bounded below if and only if
  \begin{align}
  \lambda_1 &> 0, \quad \lambda_2 > 0, \quad 2\sqrt{\lambda_1 \lambda_2} > -\lambda_3 \\
  2\sqrt{\lambda_1 \lambda_2} &> -(\lambda_3 + \lambda_4 - \lambda_5) \\
  2\sqrt{\lambda_1 \lambda_2} &> -(\lambda_3 + \lambda_4 + \lambda_5) \tag{8}
  \end{align}

- Assuming that the potential is bounded below, the stationary point $II$ exists if and only if
  \begin{align}
  \frac{-2\lambda_2(\lambda_4 + \lambda_5)}{4\lambda_1 \lambda_2 - \lambda_3(\lambda_3 + \lambda_4 + \lambda_5)} < \beta^2 < \frac{4\lambda_1 \lambda_2 - \lambda_3(\lambda_3 + \lambda_4 + \lambda_5)}{-2\lambda_1(\lambda_4 + \lambda_5)} \\
  4\lambda_1 \lambda_2 > \lambda_3(\lambda_3 + \lambda_4 + \lambda_5) \tag{9}
  \end{align}

Investigating the whole parameter space would be too time consuming. We consider representative values of the coupling constants with absolute value equal to $g^2$ of $SU(2)$ or $10^{-n}g^2$ and we take all the possible combinations: some of them be equal to $\pm g^2$ and some equal to $\pm 10^{-n}g^2$ ($n$ a fixed positive integer).

\footnote{between equation (7) and equation (5.25) in reference [9].}
It turns out that for all the cases with $\beta = 0.1$ or $\beta = 10$ and $\forall n$ the scalar bosons are too light. This suggests taking a range of values near $\beta = 1$. Also, we have found that even for $\beta \simeq 1$ and $n > 1$ in only a few cases do the scalar bosons have masses above the experimental lower bounds. The set of parameters given in Table I is allowed by constraints (4), (8), (9) and experimental limits on the Higgs masses.

III. THE PHASE TRANSITION

In order to study the cosmological phase transition of the model we use the finite temperature effective potential. We include one loop radiative corrections to the tree potential using the temperature corrected fermion and gauge boson masses (We assume that the scalar masses do not alter the potential significantly). This gives the following potential,

$$V_\beta = V_{tr} + \frac{1}{8} \left[ \sum_i (M^2_i) + 2 m_t^2 \right] T^2 - \frac{1}{4\pi} \sum_i (M^2_i)^{3/2} \frac{T}{i}$$

(10)

$m_t$ is the top quark mass and $(M^2_i)$ are the eigenvalues of the gauge boson mass matrix,

$$(M^2_A)^{ab} = g^2 \sum_{k=1}^2 \Phi^i_k T^a T^b \Phi_k$$

(11)

The Lie algebra matrices

$$T^a = \sigma^a \quad \text{for} \quad a = 1, 2, 3,$$

$$T^a = t I \quad \text{for} \quad a = 4.$$  

(12)

with $t = g'/g$, $g'$ is the $U(1)$ coupling constant.

Note that the top quark is coupled to the $\Phi_1$ doublet only. This is an essential requirement in order to avoid flavor changing neutral currents otherwise one must tune the Yukawa couplings.

It is necessary to explore the full range of the potential in order to have a complete picture of the transition. We checked the shape of the potential in every two dimensional plane passing through the origin, $\phi_i = c \phi_j$ and $\phi_k = 0$ in order to identify local and global minima. These were verified by evaluating the first derivatives of the potential.
At very high temperatures the symmetry is restored and as the temperature drops, we eventually start to get the first asymmetric minimum. There is a barrier between this minimum and the symmetric one, so it seems likely to have a first order transition.

When $\beta = 1$ the first asymmetric true minimum appears in the plane where $\phi_3 = \phi_7$ and $\phi_i = 0$, and everything looks conventional. For $\beta < 1$ the picture alters dramatically. Taking, for example, the sixth set of parameters of Table I and setting $\beta = 0.8$, one finds that for $T_1 = 259.1$ GeV the symmetric minimum ceases to be an absolute minimum in favour of a new asymmetric minimum. To be precise, there is a manifold of absolute minima satisfying $\phi_i = 0$ for $i = 1, \ldots, 4$ and $\phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2 = w(T)^2$.

As the temperature drops to $T_2 = 104$ GeV there is a new absolute minimum $\phi_3 = u(T)$ and $\phi_7 = v(T)$ and $\phi_i = 0$ separated from the symmetric one by a barrier. The part of the universe that had already tunnelled in the previous minimum rolls towards this new one. The position of this minimum as the universe cools shifts to the zero temperature one.

In fact one can find an $SU(2)$ gauge that simplifies the picture considerably. The Higgs fields tunnel first towards the new absolute minimum III,

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w(T) \end{pmatrix}$$  \hspace{1cm} (13)

After the temperature drops further the fields roll down toward the next absolute minimum:

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u(T) \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(T) \end{pmatrix}$$  \hspace{1cm} (14)

finally reaching the absolute minimum $I$, equation (4).

The matrix,

$$(M^2_S)_{ij} = \frac{\partial^2 V_\beta}{\partial \phi_i \partial \phi_j}$$  \hspace{1cm} (15)

of second derivatives of the potential can be found analytically. Figure I shows the evolution of the eigenvalues of $M^2_S$ evaluated numerically at point III. One of them starts positive and, as temperature drops, becomes negative. The expectation values plotted in Figure 2 show that the initial phase transition towards the minimum is a first order one, while the following is a second order transition.
One can not gauge transform minimum III to a minimum of the form of the stationary point II, equation (3). Point II is not a minimum at high temperatures, even though the parameters we have taken fulfill expression (3) and II is an extremum at zero temperature. Also, for completeness, it is worth to mentioning that for $\beta > 1$ the universe first tunnels towards the following absolute minimum $\phi_i = 0$ for $i \neq 3$ and $\phi_3 \neq 0$ and eventually rests at the usual zero temperature minimum.

It is well known that when a discrete symmetry is broken during a cosmological phase transition it will produce stable domain walls. This would be a problem because the discrete symmetry $\Phi \rightarrow -\Phi$ is broken at minimum I but unbroken at minimum III. This problem is overcome with small additional terms, $\Delta V$ which break the discrete symmetry and provide at the same time the explicit $CP$ violation.

The value of $\Phi_1$ at minimum III is shifted to non–zero values by $\Delta V$,

$$\Phi_1 = \begin{pmatrix} 0 \\ z(T) \end{pmatrix} e^{i\theta}. \quad (16)$$

The angle $\theta$ is a source of $CP$ violation in the fermion interactions. It depends strongly on the phases of the extra terms,

$$\theta \approx \frac{(M_S^2)_{33}}{(M_S^2)_{44}} \arg(\mu_3^2 + \frac{w^2}{4}\lambda_7). \quad (17)$$

It does not depend strongly on the magnitudes of the parameters in $\Delta V$. At minimum I however,

$$\theta \approx \frac{1}{4}(\mu_3^2 + \lambda_5 uv)^{-1} \text{Im}\left(\mu_3^2 + \lambda_6 u^2 + \lambda_7 v^2\right). \quad (18)$$

The $CP$ violation in the lowest temperature phase is determined not just by the arguments of the extra terms, but also by the ratios $|\lambda_7/\lambda_5|$ etc., which can be small.

**IV. CONCLUSIONS**

Kuzmin, Rubakov and Shaposhnikov [4] have argued that anomalous baryon number violation in the electroweak interactions can explain the origin of the cosmological baryon
asymmetry. The problem is that in the standard model the Kobayashi-Maskawa phases give too little $CP$ violation to explain the observed baryon asymmetry \cite{5}. In a two Higgs extension of the standard model, an extra source of $CP$ violation can come directly from a $CP$ non-invariant scalar potential \cite{3}, \cite{7}, \cite{8}, \cite{4}. The amount of $CP$ violation necessary for the baryon asymmetry to entropy ratio inferred from nucleosynthesis is difficult to reconcile with tests of the electroweak model. The present work relaxes the $CP$ requirement making more plausible the explanation of the baryogenesis coming from a realistic model.

What we have found is that if $\beta < 1$ regions of the universe first tunnels towards a minimum in the Higgs potential which is very sensitive to small $CP$ violating terms in the potential. The bubbles of the new phase have significant $CP$ violation depending only on the phase of the extra terms in the potential.

For the non-local mechanisms of baryogenesis which seem to work efficiently, the fermions as they reflect from the bubble wall experience a space dependent phase. The result is that the right-handed fermion particles have different reflection coefficients from the left-handed anti-particles. From ref \cite{8}, this leads to the baryon asymmetry which for small velocities $v_w$ of the bubble walls is given by

$$\frac{n_B}{s} \approx \frac{15}{2g_s \pi^4} v_w f^2 \left( \frac{m}{T_c} \right) \theta$$

(19)

for Yukawa coupling $f$ and $g_s$ spin states.

The issue of how thin the walls are depends upon the details of the transition. A handy feature of the two Higgs model is that it can give a stronger first order phase transition and thus thinner walls. In this case the theory of tunneling rates using the finite temperature effective potential up to one loop order \cite{10} suffices and there is no need for out of equilibrium techniques.
ACKNOWLEDGMENTS

I am grateful to Ian Moss for several enlightening discussions and for suggesting I study this model. I wish to acknowledge the support of the State’s Scholarship Foundation of Greece.
### TABLE I. Sample of allowed parameters from the various theoretical and experimental constraints

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\beta$ |
|--------------|--------------|--------------|--------------|--------------|--------|
| $g^2$        | $g^2$        | $g^2$        | $-g^2$       | $-g^2$       | $\simeq 1$ |
| $g^2$        | $g^2$        | $-0.1g^2$    | $-g^2$       | $-0.1g^2$    | $\simeq 1$ |
| $g^2$        | $g^2$        | $0.1g^2$     | $-g^2$       | $-0.1g^2$    | $\simeq 1$ |
| $g^2$        | $g^2$        | $g^2$        | $-g^2$       | $-0.1g^2$    | $\simeq 1$ |
| $g^2$        | $g^2$        | $-0.1g^2$    | $0.1g^2$     | $-g^2$       | $\simeq 1$ |
| $g^2$        | $g^2$        | $0.1g^2$     | $0.1g^2$     | $-g^2$       | $\simeq 1$ |
| $g^2$        | $g^2$        | $g^2$        | $0.1g^2$     | $-g^2$       | $\simeq 1$ |
FIGURES

FIG. 1. The evolution of the non zero eigenvalues of the second derivative of the potential, evaluated at point $III$, as a function of the temperature.

FIG. 2. The vacuum expectation values described in the text are plotted as a function of the temperature.
REFERENCES

[1] D.Comelli, M.Pietroni, A.Riotto, Nucl. Phys. 412B, 441 (1994)

[2] M.Dugan, B.Grinstein, L.Hall, Nucl. Phys. 25B, 413 (1985)

[3] N.Turok N, J.Zadrozny J, Phys. Rev. Lett. 65 ,2331 (1990) ; Nucl. Phys. 358B, 471 (1991)

[4] J.Cline, Kainulainen, Vischer, preprint hep-ph 9506283

[5] MB.Gavela ,P.Hernandez , J.Orloff , O.Pene , C.Quimbay , Nucl.Phys. 430B, 382 (1994)

[6] VA.Kuzmin , VA.Rubakov , ME.Shaposhnikov , Phys. Lett. 155B ,36 (1985)

[7] A.Cohen , D.Caplan , A.Nelson , Phys.Lett. 263B , 86 (1991) ; Nucl.Phys 373B , 453 (1992)

[8] M.Joyce , T.Prokopec , N.Turok ,preprint PUPT-94-1495, hep-ph/9410281 ; preprint PUPT-94-1496, hep-ph/9410282

[9] M.Sher, Physics Reports 179 ,No 5-6 ,273 (1989)

[10] I.Moss, V.Zarikas , preprint NCL95-TP4

[11] I.Moss, V.Zarikas , in progress
The diagram shows the functions $v(T)$, $u(T)$, and $w(T)$ plotted against $T$ in GeV.

- $v(T)$ starts high and decreases as $T$ increases.
- $u(T)$ has a sharp peak and then decreases rapidly as $T$ increases.
- $w(T)$ decreases steadily as $T$ increases.
