Renormalization of \((2+1)D\) scalar Weyl spinors interactions on lattices using the Clifford groups

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We consider symplectic quaternions instead of unitary spinors sitting on a lattice, and calculate the fixed point Wilson action on a finite 2D plane expanded by \(u_1a_1 + u_2a_2\) and on two 2D planes separated by \(a_1 \wedge e_2\). Only the nearest neighbor interactions are considered. Following Migdal and Kadanoff, we perform the renormalization of the Wilson action by making the lattice spacing \((\frac{1}{2})^h a\ (h = 0, \cdots, 11)\), in order to simulate bosonic and solitonic phonon propagation in materials. Renormalization group method of Benfatto and Gallavotti for \((2+1)D\) scalar \(\varphi^4\) system for sound propagation in Fermi sea is applied and feasibility of numerical simulation is discussed.

I. INTRODUCTION

In [1], an outline of performing a simulation of phonetic solitons propagating on \((2+1)D\) plane was proposed. We start from \(4 \times 4\) lattices surrounded by Clifford pairs, and by adopting the renormalization group method, calculate the correlations of ultra-sonic phonetic solitons. A main difference from standard methods is on each lattice site quaternions following symplectic groups are sitting.

We considered the fixed point Wilson actions in one loop adopted by deGrand et al. [2]. They considered in the \((3+1)D\) lattice, 28\,Fixed point (FP) actions of length less than or equal to 8 lattice lengths. We classify the loops on two parallel 2D planes connected by two links in the direction \(e_1 \wedge e_2\) and in the direction \(e_2 \wedge e_1\). The Loop1, 2, 5, 6, 11, 12, 18 and 28 belong to the LoopC and the Loop\(3, 4, 7, 8, 9, 10, 13, 14, 15, 16, 17\) and 26, 27 belong to the LoopD. Loop19, \cdots, 25 are irrelevant in \((2+1)D\).

In abstract graph theory of Luescher [3] a loop in a graph is a non-empty subset of lines with the property that there exists a sequence \(v_1, \cdots, v_N\) of pairwise different vertices and a labelling \(\ell_1, \cdots, \ell_N\) of the lines in a loop, such that \(v_k, v_{k+1}\) are the end points of \(\ell_k\) \((k = 1, \cdots, N - 1)\) and \(v_N, v_1\) are end points of \(\ell_N\). On every loop there are two orientations, and a crossing of two loops produces a new vertex.

In this sense, the Loop28 is not a proper loop. In [2], the loop did not play important roles, and we also observed that the eigenvalues of the action are large. We did not consider the Loop26 and 27 which are shown in Fig.1 since eigenvalues were large, but we consider these loops in \((2+1)D\) space, since they are proper.

![FIG. 1. Loop26 (left) and Loop27 (right).](image)

Depending on the direction of the link between two planes, we consider paths

\[\text{Loop26a: } u \to u + ae_1 \to u + ae_1 + ae_2 \to u + ae_1 + ae_2 + ae_1 \wedge e_2 \to u + ae_1 + ae_1 \wedge e_2 \to u + ae_1 \to u + ae_1 + ae_2 \to u + ae_2 \to u,\]

and

\[\text{Loop26b: } u \to u + ae_1 \to u + ae_1 + ae_2 \to u + ae_1 + ae_2 + ae_2 \wedge e_1 \to u + ae_1 + ae_2 \wedge e_1 \to u + ae_1 \to u + ae_1 + ae_2 \to u + ae_2 \to u.\]
Similarly the paths

\text{Loop27a: } u \rightarrow u + ae_1 \rightarrow u + ae_1 + ae_2 \rightarrow u + ae_1 + ae_2 + ae_1 \wedge e_2 \rightarrow u + ae_1 + ae_1 \wedge e_2 \rightarrow u + ae_1 \wedge e_2 \rightarrow u + ae_1 \wedge e_1 \rightarrow u + ae_2 \rightarrow u.

\text{Loop27b: } u \rightarrow u + ae_1 \rightarrow u + ae_1 + ae_2 \rightarrow u + ae_1 + ae_2 + ae_1 \wedge e_2 \rightarrow u + ae_1 + ae_1 \wedge e_2 \rightarrow u + ae_1 \wedge e_2 \rightarrow u + ae_2 \rightarrow u.

In the two loops \( \alpha \) and \( \beta \), the blue circle and the red circle are to be interchanged.

The eigenvalues of loops have dependence on the direction of paths. Difference of eigenvalues between Loop26\( \alpha \) and Loop26\( \beta \) is larger than that of Loop27\( \alpha \) and Loop27\( \beta \). Their eigenvalues are about the same as those of Loop28.

The dependence of eigenvalues on the direction of \( e_1 \wedge e_2 \) shows a presence of time reversal symmetric but rotational symmetry breaking phase.

We evaluate Wilson’s optimum plaquet actions by making a linear combination of eigenvalues of selected FP actions. The structure of this presentation is as follows. In Sec. II, we compare eigenvalues \( \chi(L^{(h)}) \) of Loop\( C \) and Loop\( D \) for a lattice spacing \( a \) used in [16] and \( a/2 \). The similar analysis for the trace of link variables are given in Sec. III. In Sec. IV, a perspective of the renormalization group analysis using supercomputer are given.

II. LATTICE SPACING DEPENDENCE OF EIGENVALUES OF PLAQUETTES

We consider the case in which the spacing between the lattice \( \Delta x = \frac{a}{4} \) which is called Loop\( C \) and they are \( \Delta x = \frac{a}{8} \) which are called Loop\( D \).

A. Paths on one 2D plane expanded by \( e_1 \) and \( e_2 \)

The Loop\( 1 \) consists of 4 sides of a square, whose eigenvalue is the smallest among the FP actions. We characterized the path by \( u \rightarrow u + \frac{1}{4} e_1 \rightarrow u + \frac{1}{4} + \frac{1}{4} \rightarrow u + \frac{1}{4} \rightarrow u \) where \( u \) are mesh points \( (u_1e_1 + u_2e_2) \), \( 0 \leq u_1, u_2 \leq 3 \) \( (u_i \in \mathbb{Z}) \). We call the path Loop1c.

We compare eigenvalues of the path of \( u \rightarrow u + \frac{1}{4} e_1 \rightarrow u + \frac{1}{4} + \frac{1}{4} \rightarrow u + \frac{1}{4} \rightarrow u - \frac{1}{4} e_2 \rightarrow u \), which we call Loop1d.

In the left figure of Fig.2, eigenvalues of \( u_1 = 0 \) (blue), \( 1 \) (orange), \( 2 \) (green), \( 3 \) (red) as a function of \( u_2 \) are plotted, and in the right figure of Fig.2 eigenvalues of \( u_1 = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3 \) as a function of \( u_2 \) are plotted. When the lattice spacing is smaller, eigenvalues are smaller.

![Fig. 2. Absolute values of eigenvalues for fixed \( u_1 \) in Loop1c(left) and in Loop1d (right).](image)

The eigenvalues of Loop28 are shown in Fig.4. The loop is not proper in the sense of Luescher[3]. Fluctuations are large for small \( u \) or in infrared regions.
The eigenvalues of Loop2 are shown in Fig. 4. Eigenvalues of the smaller lattice spacing Loop2d is smaller than those of Loop2c.

The eigenvalues of Loop5 are shown in Fig. 5. The Loop5 has a bending point of the path at the center of the loop. It produces a large eigenvalues for small $u_1$. 
The eigenvalues of Loop6 are shown in Fig. 6. They have $u$ dependence in the infrared region.

![Fig. 6](image1)

The eigenvalues of Loop11 are shown in Fig. 7. The Loop11 has a bending due to presence of overlapping links in the center.

![Fig. 7](image2)

The eigenvalues of Loop12 are shown in Fig. 8. The Loop12 is similar to the Loop5, but there is a self-crossing of two long links at the center. The $u$ dependence of eigenvalues of Loop5 and Loop12 are similar, but absolute values of eigenvalues of Loop12 are smaller, due to presence of long links. In the $(3 + 1)$D lattice simulation of [2], the Loop5 is more effective than Loop12.

![Fig. 8](image3)

In [1], the starting point of the Loop18 was chosen to be same as the Loop1, and only the scale was changed. In this paper we replace the vertices as those of the paper [2],

$$L18[u_1, u_2] = t1[-\frac{1}{4}, u_1 + \frac{1}{4}, u_2] \times t2[-\frac{1}{2}, u_1 + \frac{1}{4}, u_2 + \frac{1}{2}] \times t1[-\frac{1}{2}, u_1 + \frac{1}{4}, u_2 + \frac{1}{2}] \times t2[\frac{1}{2}, u_1 + \frac{1}{4}, u_2] \times t1[\frac{1}{4}, u_1, u_2].$$

The eigenvalues of the new Loop18 and its lattice scalings halved are shown in Fig. 9. The Loop18 contains long straight links parallel to $e_2$. 

![Fig. 9](image4)
When there are links between two 2D planes, a problem of choice of scale of length between the two 2D planes appears. We leave it as a future study, and calculate eigenvalues using the same lattice spacing between the 2D planes and on the 2D plane.

The paths on two planes we considered in [16] were restricted to α type. Since eigenvalues in the large $u$ region are almost independent of $\alpha$ and $\beta$, we restrict loops to be α type and consider β type near the final stage of Monte Carlo simulation.

The path of Loop3c consists of $u \rightarrow u + \frac{1}{4} e_1 \rightarrow u + \frac{1}{4} e_1 + \frac{1}{4} e_2 \rightarrow u + \frac{1}{4} e_1 + \frac{1}{4} e_2 + \frac{1}{4} e_1 \land e_2 \rightarrow u + \frac{1}{4} e_1 + \frac{1}{4} e_1 \land e_2 \rightarrow u + \frac{1}{4} e_1 \land e_2 \rightarrow u$.

The path of Loop3d consists of $u \rightarrow u + \frac{1}{8} e_1 \rightarrow u + \frac{1}{8} e_1 + \frac{1}{8} e_2 \rightarrow u + \frac{1}{8} e_1 + \frac{1}{8} e_2 + \frac{1}{8} e_1 \land e_2 \rightarrow u + \frac{1}{8} e_1 + \frac{1}{8} e_1 \land e_2 \rightarrow u + \frac{1}{8} e_1 \land e_2 \rightarrow u$.

The eigenvalues of Loop3 are shown in Fig. 10. We observe the eigenvalues of action of Loop3 is roughly about twice of those of Loop1.

The eigenvalues of Loop4 are shown in Fig. 11.
The eigenvalues of Loop7 are shown in Fig. 12.

FIG. 12. Absolute values of eigenvalues for a fixed $u_1$ in Loop7c (left) and in Loop7d (right).

The eigenvalues of Loop8 are shown in Fig. 13.

FIG. 13. Absolute values of eigenvalues for a fixed $u_1$ in Loop8c (left) and in Loop8d (right).

The eigenvalues of Loop9 are shown in Fig. 14.

FIG. 14. Absolute values of eigenvalues for a fixed $u_1$ in Loop9c (left) and in Loop9d (right).
The eigenvalues of Loop10 are shown in Fig. 15.

The eigenvalues of Loop13 are shown in Fig. 16.

The eigenvalues of Loop14 are shown in Fig. 17.

FIG. 15. Absolute values of eigenvalues for a fixed $u_3$ in Loop10c (left) and in Loop10d (right).

FIG. 16. Absolute values of eigenvalues for a fixed $u_3$ in Loop13c (left) and in Loop13d (right).

FIG. 17. Absolute values of eigenvalues for a fixed $u_3$ in Loop14c (left) and in Loop14d (right).
The eigenvalues of Loop15 are shown in Fig. 18. The eigenvalues of Loop16 are shown in Fig. 19.

FIG. 18. Absolute values of eigenvalues for a fixed $u_1$ in Loop15c (left) and in Loop15d (right).

FIG. 19. Absolute values of eigenvalues for a fixed $u_1$ in Loop16c (left) and in Loop16d (right).

The eigenvalues of Loop17 are shown in Fig. 20.

FIG. 20. Absolute values of eigenvalues for a fixed $u_1$ in Loop17c (left) and in Loop17d (right).
The eigenvalues of the Loop26$\alpha$ and Loop26$\beta$ of $a = 1/8$ are shown in Fig.21

![Fig. 21. Absolute values of eigenvalues for a fixed $u_1$ in Loop26$\alpha$ (left) and in Loop26$\beta$ (right).](image1)

The eigenvalues of the Loop27$\alpha$ and Loop27$\beta$ are shown in Fig.22

![Fig. 22. Absolute values of eigenvalues for a fixed $u_1$ in Loop27d (left) and in Loop27f (right).](image2)

III. LATTICE SPACING DEPENDENCE OF TRACES OF LINK VARIABLES

In Clifford algebra, transformation of a coordinate $X$ is represented by

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} x & xx^- \\ 1 & x^- \end{pmatrix} \begin{pmatrix} d^- & c^- \\ b^- & a^- \end{pmatrix} = \lambda \begin{pmatrix} x' & x'x'^- \\ 1 & x'^- \end{pmatrix}$$

and the term $xx^-$ yields link products.

In lattice simulations of scalar fields, we consider Feynman path integrals is $Z = \int[d\phi]e^{-S}$ with $x = a(u_1e_1 + u_2e_2) + au_3e_1 \wedge e_2$, $-N/2 < u_1, u_2, u_3 \leq N/2$, $\mu = 1, 2, 3$. $N \sim 2^{11} = 2048$. The scale of $e_1 \wedge e_2$ is chosen to be the same as $e_1, e_2$, for Wilson loops, but it can be complex for Polyakov loops.

The expectation values of Wilson or Polyakov action $S$ consists of eigenvalues of left lower components of the Loop matrices $Lk[u_1, u_2]$ where $k$ specifies the FP actions of $[2]$, and the trace of $DS(Lk[u_1, u_2])$ which consists of the sum of $dS_1$ and $dS_2$ along the loops.

The $4 \times 4$ matrix of the Loop1 contribution

$$DS(L1[u_1, u_2]) = dS_1[u_1, u_2] + dS_2[u_1 + a, u_2] - dS_1[u_1 + a, u_2 + a] - dS_2[u_1, u_2 + a]$$

has non-zero components in the right upper component. We measure the trace of the $2 \times 2$ matrices.

In the case of Loop2,

$$DS(L2[u_1, u_2]) = dS_1[u_1, u_2] + 2dS_2[u_1 + a, u_2] - dS_1[u_1 + a, u_2 + 2a] - 2dS_2[u_1, u_2 + 2a]$$

has non-zero components only in the right upper corner.
A. Paths on one 2D plane expanded by $e_1$ and $e_2$

The traces of the $2 \times 2$ matrix, which is twice the real part of the diagonal component in the case of Loop1 as a function of $u_2$ for fixed $u_1$ are shown in Fig. 23.

FIG. 23. Trace of $dS(L1[u_1, u_2])$ for $\Delta u_i = 1$(left) and $\Delta u_i = \frac{1}{2}$(right). ($i = 1, 2$)

The traces of the matrix in the case of Loop2 are shown in Fig. 24.

FIG. 24. Trace of $dS(L2[u_1, u_2])$ for $\Delta u_i = 1$(left) and $\Delta u_i = \frac{1}{2}$(right). ($i = 1, 2$)

The traces of the matrix in the case of Loop5 are shown in Fig. 25.

FIG. 25. Trace of $dS(L5[u_1, u_2])$ for $\Delta u_i = 1$(left) and $\Delta u_i = \frac{1}{2}$(right). ($i = 1, 2$)
The traces of the matrix in the case of Loop5 are shown in Fig. 26.

FIG. 26. Trace of $dS(L6[u_1, u_2])$ for $\Delta u_i = 1$(left) and $\Delta u_i = \frac{1}{2}$(right). $(i = 1, 2)$

The traces of the matrix in the case of Loop11 are shown in Fig. 27.

FIG. 27. Trace of $dS(L11[u_1, u_2])$ for $\Delta u_i = 1$(left) and $\Delta u_i = \frac{1}{2}$(right). $(i = 1, 2)$

The traces of the matrix in the case of Loop12 are shown in Fig. 28.

FIG. 28. Trace of $dS(L12[u_1, u_2])$ for $\Delta u_i = 1$(left) and $\Delta u_i = \frac{1}{2}$(right). $(i = 1, 2)$
The traces of the matrix in the case of Loop18 are shown in Fig. 29.

\[ \begin{align*}
\text{FIG. 29. Trace of } dS(L18[u_1, u_2]) \text{ for } \Delta u_i = 1 \text{(left) and } \Delta u_i = \frac{1}{2} \text{(right). (} i = 1, 2) \end{align*} \]

B. Paths on two planes connected by \( e_1 \wedge e_2 \)

The traces of the matrix in the case of Loop3 are shown in Fig. 30.

\[ \begin{align*}
\text{FIG. 30. Trace of } dS(L3[u_1, u_2]) \text{ for } \Delta u_i = 1 \text{(left) and } \Delta u_i = \frac{1}{2} \text{(right). (} i = 1, 2) \end{align*} \]

The traces of the matrix in the case of Loop4 are shown in Fig. 31.

\[ \begin{align*}
\text{FIG. 31. Trace of } dS(L4[u_1, u_2]) \text{ for } \Delta u_i = 1 \text{(left) and } \Delta u_i = \frac{1}{2} \text{(right). (} i = 1, 2) \end{align*} \]
The traces of the matrix in the case of Loop7 are shown in Fig. 32.

![Fig. 32](image)

**FIG. 32.** Trace of $dS(L7[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)

The traces of the matrix in the case of Loop8 are shown in Fig. 33.

![Fig. 33](image)

**FIG. 33.** Trace of $dS(L8[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)

The traces of the matrix in the case of Loop9 are shown in Fig. 34.

![Fig. 34](image)

**FIG. 34.** Trace of $dS(L9[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)
The traces of the matrix in the case of \textit{Loop}10 are shown in Fig.35.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig35}
\caption{Trace of $dS(L10[u_1,u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)}
\end{figure}

The traces of the matrix in the case of \textit{Loop}13 are shown in Fig.36.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig36}
\caption{Trace of $dS(L13[u_1,u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)}
\end{figure}

The traces of the matrix in the case of \textit{Loop}14 are shown in Fig.37.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig37}
\caption{Trace of $dS(L14[u_1,u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)}
\end{figure}
The traces of the matrix in the case of Loop15 are shown in Fig. 38.

FIG. 38. Trace of $dS(L15[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)

The traces of the matrix in the case of Loop16 are shown in Fig. 39.

FIG. 39. Trace of $dS(L16[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)

The traces of the matrix in the case of Loop17 are shown in Fig. 40.

FIG. 40. Trace of $dS(L17[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)
The traces of the matrix in the case of Loop26 are shown in Fig. 41.

![Fig. 41. Trace of $dS(L.26[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)](image1)

The traces of the matrix in the case of Loop27 are shown in Fig. 42.

![Fig. 42. Trace of $dS(L.27[u_1, u_2])$ for $\Delta u_i = 1$ (left) and $\Delta u_i = \frac{1}{2}$ (right). ($i = 1, 2$)](image2)

**IV. DISCUSSION AND PERSPECTIVE**

We expected that correlation of phonons and its time-reversed phonons propagating on a 2D plane can be simulated by a model of Bosonic quasiparticle propagating in the Fermionic sea of Weyl spinors.

In an exploratory analysis using Clifford algebra, we observed that as the lattice spacing $a$ is halved, eigenvalues of the FP Wilson action and the trace of the matrix representing links of Loops surrounding each FP Wilson action are reduced. Monte-Carlo simulations using a small lattice constant $a$ will allow fixing the optimal Wilson action or the Polyakov action as a linear combination of the FP actions with correction terms derived by the renormalization group.

There are works on the study of Quantum spin systems associated with time translations and space translations of lattice spaces[14, 15]. Solitons are integrable systems induced by nonlinear interactions. Detailed comparison between experiments and simulation will clarify the Kubo-Martin-Schwinger boundary condition.

We have a plan of reducing $a$ and perform the renormalization group analysis using supercomputers which allow parallel computations. Whether one can connects the chiral anomaly and gravitation anomaly by simulating the ultrasonic waves remains as a future study.

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