Cosmological signatures of ultra-light dark matter with an axion-like potential

Francisco X. Linares Cedeño,1 Alma X. González-Morales,1,2 and L. Arturo Ureña-López1,†

1Departamento de Física, DCI, Campus León, Universidad de Guanajuato, 37150, León, Guanajuato, México
2Consejo Nacional de Ciencia y Tecnología, Av. Insurgentes Sur 1582. Colonia Crédito Constructor, Del. Benito Juárez, C.P. 03940, México D.F. México
(Dated: March 31, 2017)

Abstract

Axion dark matter models have been thoroughly studied in the recent literature, in particular under the prescription of a free scalar field, but a full treatment of the axion field is still required mainly because nonlinearities in a more realistic potential may play an important role in the cosmological dynamics. In this paper, we show how to solve the cosmological equations of an axion field for both the background and the linear perturbations with the aid of an amended version of the Boltzmann code CLASS, and contrast our results with those of cold dark matter and the free axion case. We conclude that there is a slight delay in the onset of the axion field oscillations when nonlinearities in the axion potential are taken into account, and the characteristic cut-off in the mass power spectrum shifts towards larger scales (smaller wavenumbers). We quantified the differences between the axion and free cases and discuss how the two models could be distinguished by the properties of their mass power spectrum.

PACS numbers: 98.80.-k, 95.35.+d

Introduction.– Modern cosmological observations have brought about a large amount of data [1,2], making possible to constrain, with high accuracy, theoretical models describing the Universe at large scales. The so-called Lambda Cold Dark Matter (ΛCDM) paradigm is very successful at reproducing observations such as the cosmic microwave background anisotropies, and the galaxy distribution at several redshifts [1], among others. It requires the existence of a cosmological constant accounting for most of the present energy density (≈ 69%); a dark matter (DM) component (≈ 26%), effectively described by collisionless particles that interacts mostly gravitationally with other matter components [3,5]; and only a small fraction of baryonic and radiation-like matter (≈ 5%). However, there are longstanding discussions about how well the ΛCDM model describes the Universe at small scales. The solution to these problems may come from the specific properties of the DM, or from an interplay between the DM properties and kinematic effects with baryons, but still the incompleteness of galactic observations may impair our ability to infer the DM distribution properties from them. Given the current status, the further development of theoretical models still is very much desirable if one is to elucidate the properties of this matter component of the Universe.

According to recent studies, axion DM has become a compelling candidate to replace CDM [8,10], and even some experiments have been already set up to have a direct detection of this elusive particle [11,13]. Nonetheless, there are still many open questions such as what is the right axion mass limits one can place by using, for instance, galactic kinematics [19,23], and Lyman-α observations [24,25]. At the cosmological level, axion models have been studied considering it as a free scalar field, i.e., as a scalar field endowed with a quadratic potential $V(\phi) = m^2 \phi^2/2$ [26–31]. However, a more realistic form of the axion potential is the trigonometric one,

$$V(\phi) = M^4 \left[1 + \cos (\phi/f)\right],$$

where $M$ is the energy scale of the potential and $f$ is the decay constant of the axion. The axion mass scale $m_\phi$ is a derived quantity from the physical parameters in the potential: $m_\phi = M^2/f$.

The axion potential [1] originally arose in QCD with the aim to solve the strong CP problem [32–35]: the parameter $\theta_{QCD}$ in the topological term of the QCD Lagrangian is promoted to a scalar field (the axion field), and the potential of such field arises from nonperturbative effects which generate a periodic behavior after the breaking of the Peccei-Quinn symmetry $U(1)_{PQ}$ due to instantons. [36,38]. Because of the periodicity of the axion field, the potential is usually parametrized as a trigonometric function with the generic form $V(\theta_{QCD}) \propto \cos \theta_{QCD}$. More recently, it has been argued that axions emerge in string theories from the compactification of extra dimensions, and their interactions with stringy instantons induce an exponentially suppressed axion potential given by $V(a) \propto e^{-S_{\text{inst}} \cos(a)}$, where $a$ is the axion field and $S_{\text{inst}}$ is the instanton action [39,41].

Given the motivations above, our aim in this paper is to study the axion field as source of DM with its corresponding trigonometric potential [1]. For that purpose, we present for the first time an analysis of the cosmological evolution of an axion field taking into account the whole properties of the potential [1]. This is accomplished by: 1) transforming the standard cosmological equations for both, the background and the linear perturbations into a dynamical system, and 2) using an amended version of the Boltzmann code CLASS [42] (with high precision settings, see also Ref. [20] for more technical details) to obtain accurate numerical solutions. We
use this to analyze the differences in the linear process of structure formation of the axion field with respect to the free (quadratic potential) and the CDM cases. As we shall see, there is a delay in the oscillations of the axion around the minimum of the potential $V_{\phi}$, and the MPS is further suppressed when compared to that of the free case. For the sake of concreteness we present all the results for a fiducial model with axion mass $m_\phi = 10^{-22} \, \text{eV}$, but we have verified that the qualitative features hold for other masses in the range $10^{-26} < m_\phi/\text{eV} < 10^{-20}$.

**Background Dynamics.**—The equations of motion for a scalar field $\phi$ endowed with the potential $V_{\phi}$, in a homogeneous and isotropic space-time with null spatial curvature, are given by

$$H^2 = \frac{\kappa^2}{3} \left( \sum_j \rho_j + \rho_\phi \right), \quad \dot{\rho}_j = -3H(\rho_j + p_j), \quad (2a)$$

$$\dot{H} = -\frac{\kappa^2}{2} \left[ \sum_j (\rho_j + p_j) + (\rho_\phi + p_\phi) \right], \quad (2b)$$

$$\ddot{\phi} = -3H\dot{\phi} + m_\phi^2 \sin(\phi/f), \quad (2c)$$

where $\kappa^2 = 8\pi G$, $\rho_j$ and $p_j$ are the energy and pressure density of ordinary matter, a dot denotes derivative with respect to cosmic time $t$ and $H = \dot{a}/a$ is the Hubble parameter. The scalar field energy density and pressure are given by the canonical expressions $\rho_\phi = (1/2)\dot{\phi}^2 + V(\phi)$ and $p_\phi = (1/2)\dot{\phi}^2 - V(\phi)$.

We define a new set of polar coordinates as in \[26, 43, 44, \]

$$\frac{\kappa\dot{\phi}}{\sqrt{6}H} \equiv \Omega_{\phi}^{1/2} \sin(\theta/2), \quad \frac{\kappa V^{1/2}}{\sqrt{3}H} \equiv \Omega_{\phi}^{1/2} \cos(\theta/2), \quad (3a)$$

$$y_1 = -2\sqrt{2} \frac{\partial_{\phi} V^{1/2}}{H}, \quad (3b)$$

with which the Klein-Gordon equation \[2c\] takes the form of the following dynamical system,

$$\dot{\theta} = -3\sin \theta + y_1, \quad (4a)$$

$$\dot{y}_1 = \frac{3}{2} \left( 1 + \omega_{\text{tot}} \right) y_1 + \frac{\lambda}{2} \Omega_{\phi} \sin \theta, \quad (4b)$$

$$\Omega_{\phi} = 3(\omega_{\text{tot}} - w_\phi)\Omega_{\phi}, \quad (4c)$$

where $\lambda = 3/\kappa^2 f^2$ and $\Omega_{\phi} = \kappa^2 \rho_\phi/3H^2$ is the standard scalar field density parameter. Here, a prime denotes derivative with respect to the number of e-foldings $N = \ln(a/a_0)$, with $a$ the scale factor of the Universe and $a_0$ its initial value. Other quantities in Eqs. \[4\] are the total equation of state $w_{\text{tot}} = p_{\text{tot}}/\rho_{\text{tot}}$, and the equation of state associated to the scalar field $w_\phi = p_\phi/\rho_\phi = -\cos \theta$.

For $\lambda = 0$ in Eq. \[4c\] the dynamical system for the free case is recovered, see Ref. \[26\].

One critical step in the numerical solution of Eqs. \[2\] and \[4\] is to find the correct initial conditions of the dynamical variables. In general, we expect the initial values to depend on the physical parameters of the model, that is $A_i = A_i(\lambda, m_\phi, \Omega_{\phi0})$, where $A_i$ represents the initial value of any of the dynamical variables. For the axion case, it can be shown that we must satisfy the following constraints,

$$\Omega_{\phi0} = a_{\text{osc}}^{-3} \frac{\Omega_{\phi0}}{\Omega_{\gamma0}}, \quad y_{1i} = 5\theta_1, \quad \frac{m_\phi^2}{H_i^2} = \frac{y_{1i}^2}{4} + \lambda \Omega_{\phi0} \quad (5)$$

where $a_{\text{osc}}$ is the value of the scale factor at the onset of the oscillations of the field $\phi$ around the minimum of the potential $V_{\phi}$. The last equality in Eqs. \[5\] is the usual Pythagorean identity, which arises because of the trigonometric functions involved in the axion potential $V_{\phi}$ and its field derivatives.

The solution of Eqs. \[5\] provides appropriate seed values that the CLASS code adjusts through a shooting procedure to obtain the correct value of the axion density parameter $\Omega_{\phi0}$ at the present time. As expected, Eqs. \[5\] also provide the right seed values for the free case if $\lambda = 0$. The importance of the relation between the initial conditions and the values of the physical parameters to obtain the right DM abundance at the present time cannot be overestimated, as otherwise the comparison of our results with the free case, or with the standard CDM one, would be meaningless.

In Fig. \[4\] we show the evolution of the axion energy density $\rho_\phi$ in comparison with that of CDM (all other cosmological quantities are the same as in the fiducial $\Lambda$CDM model \[1\]). The numerical examples correspond to an axion field with mass $m_\phi = 10^{-22} \, \text{eV}$ and $\lambda = 0, 10, 10^2, 10^3, 10^4, 10^5$. We can clearly see that $\rho_\phi$ evolves just like CDM after the onset of the field oscillations. The latter are delayed by the presence of the decay parameter $\lambda$, and also the transition to the CDM behavior occurs more abruptly for larger values of $\lambda$. This is just a consequence of the increase in the steepness of the potential $V_{\phi}$ for $\lambda \gg 1$, which in turn makes it more difficult to find a reliable numerical solution of Eqs. \[4\].

For the axion mass $m_\phi = 10^{-22} \, \text{eV}$ shown in Fig. \[4\] the largest value considered was $\lambda = 10^5$. Although larger values would be desirable, we are already close to the expected upper bound on $\lambda$. As estimated in Ref. \[45\], the axion field can provide the whole of the DM budget as long as $m_\phi/\sqrt{\lambda} > 6 \times 10^{-27} \, \text{eV}$. In particular, a conservative estimate is that $\lambda \lesssim 10^8$ if $m_\phi = 10^{-22} \, \text{eV}$, although other considerations can provide stronger constraints \[45, 47\].

**Linear Density Perturbations and Mass Power Spectrum.**—Let us now consider the case of linear perturbations $\varphi$ of the axion field in the form $\phi(x,t) = \phi(t) + \varphi(x,t)$. As for the metric, we choose the synchronous gauge with the line element $ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$, where $h_{ij}$ is the tensor of metric perturbations. The linearized Klein-Gordon equation
for a given Fourier mode $\varphi(k,t)$ reads [48–51]:

$$\ddot{\varphi} = -3H\dot{\varphi} - \left[ \frac{k^2}{a^2} - m_\phi^2 \cos(\phi/f) \right] \varphi - \frac{1}{2} \dot{\varphi} \dot{h},$$  

(6)

where a dot means derivative with respect the cosmic time, $\dot{h} = h/J$ and $k$ is a comoving wavenumber.

As shown in Ref. [26], we can transform Eq. (6) into a dynamical system by means of the following (generalized) change of variables,

$$\sqrt{\frac{2}{3}} H_0 \dot{\chi} = -\Omega_\phi^{1/2} e^\alpha \cos(\theta/2), \quad \kappa \dot{\chi}_i \equiv -\Omega_\phi^{1/2} \frac{e^\alpha}{\sqrt{6}} \sin(\theta/2),$$  

(7)

with $\alpha$ and $\theta$ the new variables needed for the evolution of the scalar field perturbations. But if we further define $\delta_0 = -e^\alpha \sin(\theta/2 - \vartheta/2)$ and $\delta_1 = -e^\alpha \cos(\theta/2 - \vartheta/2)$, then Eq. (6) takes on a more manageable form,

$$\delta_0' = \left[ -3 \sin \theta - \frac{k^2}{k_J^2} (1 - \cos \theta) \right] \delta_1 + \frac{k^2}{k_J^2} \sin \theta \delta_0 - \frac{\dot{h}}{2} (1 - \cos \theta),$$  

(8a)

$$\delta_1' = \left[ -3 \cos \theta - \frac{k^2}{k_J^2} \sin \theta + \frac{\Lambda \Omega_\phi}{2y_1} \sin \theta \right] \delta_1 + \left( \frac{k^2}{k_J^2} - \frac{\Lambda \Omega_\phi}{2y_1} \right) (1 + \cos \theta) \delta_0 - \frac{\dot{h}}{2} \sin \theta,$$  

(8b)

where $k_J^2 \equiv \alpha^2 H_0^2 y_1$ is the (squared) Jeans wavenumber and a prime again denotes derivative with respect to the number of e-folds $N$. Notice that the new dynamical variable $\delta_0$ is the axion density contrast, as a straightforward calculation using Eqs. (9) and (7) shows that $\delta_0 = \left( \dot{\phi} + \partial_\phi V \varphi \right)/\rho_{\phi}$. This implies that Eq. (8a) is the closest expression one can find to a fluid equation for the evolution of the axion density contrast. The physical interpretation of $\delta_1$ is by no means as direct as that of $\delta_0$, and then Eq. (8b) tells us of the difficulties to match the equations of motion of scalar field linear perturbations to those of a standard fluid [52]. For the initial conditions, we use the attractor solutions at early times [26] given by $\delta_{0i} = -\dot{h}_i \theta_i^2/84$ and $\delta_{1i} = -\dot{h}_i \theta_i/7$, where $\dot{h}_i$ and $\theta_i$ are, respectively, the initial values of the trace of metric perturbations $h$ and the background angular variable $\theta$.

The solution of Eqs. (8) are useful to build up cosmological observables such as the MPS, which we show for the axion field and CDM in Fig. 2. It is well known that there is a characteristic cut-off in the MPS of a free field, and this feature is also present for the axion case, see Figure 2 although with differences that we are to explain now.

To quantify the differences we determined the wavenumbers $k_{50\%}$ and $k_{50\%}$ at which the amplitude of the MPS changes by 5% and 50%, respectively, with respect to that of CDM. The value of $k_{50\%}$ can be considered an estimate of the cut-off wavenumber, whereas that of $k_{50\%}$ will help us to estimate further deviations
of the MPS at larger $k$. The results for the free case are $k_{50\%} = 3.9\, \text{Mpc}^{-1}$ and $k_{50\%} = 6.4\, \text{Mpc}^{-1}$, whereas the corresponding ones for $\lambda = 10^5$ are $k_{50\%} = 3.2\, \text{Mpc}^{-1}$ and $k_{50\%} = 5.7\, \text{Mpc}^{-1}$. As we can see, there is a consistent shift on both $k_{50\%}$ and $k_{50\%}$ towards smaller values as the value of $\lambda$ increases.

The shift on $k_{50\%}$ seems to indicate that there must be an equivalent free case with a smaller axion mass $m_{\phi}^*$ that produces an equivalent MPS to the case with $\lambda = 10^5$. Indeed, for the free case with $m_{\phi}^* = 6.78 \times 10^{-23} \text{eV}$ it is possible to obtain the same cut-off at $k_{50\%} = 3.2\, \text{Mpc}^{-1}$; however, the profile of the its MPS differs from that of $m = 10^{-22} \text{eV}$ with $\lambda = 10^5$ at larger values of $k$. This ultimately means that the axion MPS is non-degenerate with respect to that of the free case. This shows that the axion case ($m_{\phi}, \lambda \neq 0$) cannot be matched onto another one with ($m_{\phi}^*, \lambda = 0$).

One parameter of special relevance to understand the evolution of linear perturbations and the special features of the MPS in Fig. 2 is the so-defined Jeans wavenumber $k_J$. In general terms, a perturbation with a given wavenumber $k$ can grow as long as $k < k_J$, in which case it will be equal to that of the CDM after the onset of the axion oscillations. In contrast, linear perturbations will appear suppressed with respect to those of CDM if $k > k_J$.

The Jeans wavenumber is a function only of background quantities, and then its evolution is shown in Fig. 3 for different values of $\lambda$. In general, $k_J$ remains constant during radiation domination, but it grows steadily afterwards. The interpretation of the free case ($\lambda = 0$) is the simplest of all: we see from Fig. 2 that all wavenumbers $k < 8.2\, \text{Mpc}^{-1}$ are always smaller than the Jeans wavenumber, and then their corresponding linear perturbations must be the same as those of CDM. In contrast, linear perturbations with a wavenumber $k > 8.2\, \text{Mpc}^{-1}$ must appear suppressed when compared to those of CDM (for more details see Ref. 26).

We can also see in Fig. 3 the main effect induced by the presence of $\lambda$ in the evolution of linear perturbations: the value of the Jeans wavenumber $k_J$ during radiation domination is smaller than in the free case. This means that extra suppression of the MPS appears for smaller values of $k$ (down to 0.53 Mpc$^{-1}$) when compared to the free case. The effect is not quite noticeable, as we have already seen in Fig. 2 mainly because the suppression of power in the aforementioned scales happens well before the time of radiation-matter equality.

**Discussion and conclusions.**—In this work we have considered a DM model with an ultra-light scalar field endowed with a trigonometric potential, which can be identified as an ultra-light axion field. The evolution of the axion energy density and its linear perturbation was obtained numerically from an amended version of the Boltzmann code CLASS, with which we were able to find the effect of the axion physical parameters on the cosmological solutions.

We computed the MPS for the axion potential and showed that its features do not change significantly, within the range of physical parameters that we were able to explore, with respect to those of the free case (quadratic potential); nonetheless we could quantify the small differences between the two cases. Our results are in agreement with the semi-analytical studies of the axion field in Refs. 45, 54. It would be interesting to know whether future surveys as the Dark Energy Spectroscopic Instrument (DESI) 55 and the Large Synoptic Survey Telescope (LSST) 56, in case a cut-off in the MPS is detected, will also be able to spot the small differences between the free case ($\lambda = 0$) and the full axion one.

Just recently a set of new constraints on the axion mass based on the analysis of Lyman-α forest had been presented. The strongest constraint comes from high resolution spectra, implying $m_{\phi} > 37.5 \times 10^{-22} \text{eV}$ 24 and $m_{\phi} > 29 \times 10^{-22} \text{eV}$ 25, at the 2-σ confidence level. Our results would then imply that the same constraints will apply to the axion mass, even under the consideration of a more realistic axion potential.

We would like to thank Alberto Diez-Tejedor for useful discussions and comments. FXLC acknowledges CONACYT for financial support. AXGM acknowledges support from Cátedras CONACYT and UCMEXUS-CONACYT collaborative project funding. This work was partially...
[53] L. Anderson et al. (BOSS), Mon. Not. Roy. Astron. Soc. 441, 24 (2014), arXiv:1312.4877 [astro-ph.CO]

[54] U.-H. Zhang and T. Chiueh, (2017), arXiv:1702.07065 [astro-ph.CO]

[55] M. Levi et al. (DESI), (2013), arXiv:1308.0847 [astro-ph.CO]

[56] Z. Ivezic, J. A. Tyson, R. Allsman, J. Andrew, and R. Angel (LSST), (2008), arXiv:0805.2366 [astro-ph]