Sub-milliwatt-level microresonator solitons with extended access range using an auxiliary laser: supplementary material

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Mathematical description of the doubly pumped resonance system. Here, we define a parameter \( \Gamma \) for the resonance shift per unit intracavity optical power (unit: Hz/W), which depends on the amount of intracavity power as well as the geometry of the resonator. With the presence of the auxiliary laser, the 1550 nm resonance shift \( (K_5) \) originates from two parts: one is the shift induced by the 1550 nm laser itself and the other part is the thermal shift induced by the 1330 nm auxiliary laser:

\[
K_5 = \Gamma_5 P_3 + \Gamma_{55} P_5,
\]

where \( P_3 \) and \( P_5 \) are the intracavity optical power of the pump and auxiliary lasers, respectively; \( \Gamma_5 \) and \( \Gamma_{55} \) represent the 1550 nm self resonance shift and cross resonance shift from the thermal effect of the auxiliary laser, respectively. The two coefficients are both negative. Similarly, the 1330 nm resonance shift \( (K_3) \) can be written

\[
K_3 = \Gamma_3 P_3 + \Gamma_{35} P_5,
\]

where \( \Gamma_3 \) and \( \Gamma_{35} \) represent the 1330 nm self resonance shift and cross resonance shift from the thermal effect of the auxiliary laser, respectively. From Eqs. S1 and S2, we obtain

\[
\frac{\partial P_5}{\partial K_5} = \frac{\Gamma_{35}}{\Gamma_3 \Gamma_{55} - \Gamma_3 \Gamma_{35}}, \tag{S3}
\]

The measurement of the resonance shift. To characterize the cross thermal effect of the auxiliary and pump lasers, the coefficient for the 1550 nm resonance frequency shift per unit 1330 nm resonance frequency shift is measured here. In this measurement, the 1330 nm auxiliary laser operates at high power (tens of mW) and is scanned by a 1 Hz triangle wave signal while the 1550 nm pump laser operates at only ~1 µW, permitting its thermal effect on the resonator to be ignored. The 1550 nm pump laser is scanned by a 50 Hz triangle signal, as a result, there are about 10 thermally shifted 1550 nm resonance peaks (blue trace) within the 1330 nm triangle (black trace) as shown in Fig. S1. From this equation, it can be inferred that, due to the presence of the 1330 nm auxiliary laser, the cross thermal effect (the product of \( \Gamma_{35} \) and \( \Gamma_{53} \)) makes the slope of the 1550 nm intracavity power versus the resonance shift much larger than \( 1/\Gamma_5 \). As a result, the 1550 nm thermal triangle is narrowed by the auxiliary laser.

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These thermally-shifted resonance peaks occurring on the rising edge of the 50 Hz triangle signal (red trace in Fig. S1) are extracted with their corresponding times \( (t_1, \ldots, t_n) \). The 1550 nm resonance shift \( K_{5n} \) caused by the thermal effect of the 1330 nm auxiliary laser can be extracted from

\[
K_{5n} = K_{51} - (t_n - t_1 - (n - 1)T)\nu_{53}, \tag{S4}
\]

where \( T \) is the scan period (20 ms) of the pump laser, \( \nu_{53} \) is the pump laser scanning speed (unit: MHz/s), and \( K_{51} \) is the 1550 nm
resonance shift at $t$, where the first resonance peak appears in the 1330 nm thermal triangle. The latter can be determined from fitting the results. The corresponding values ($V_1 \ldots V_n$) of the 1330 nm thermal triangle (black line in Fig. S1) at the times of the resonance-peaks ($t_1 \ldots t_n$) are also recorded and used to extract the 1330 nm resonance shift $K_{30}$

$$K_{30} = -\frac{\Delta V(V_n - V_1)}{A} \quad \text{(5)}$$

where $\Delta V$ and $A$ are the width (in frequency) and amplitude (measured as photo detector voltage) of the 1330 nm thermal triangle, respectively. $V_0$ is the maximum value of the triangle waveform when the auxiliary laser is not coupled into the resonator. Figure 4(d) in the main paper shows the measurement results of the resonance shift for the 1550 nm resonance as a function of the 1330 nm resonance shift, from which the resonance shift coefficient can be determined by linear fitting.

![Fig. S1. Experimental measurement of the resonance shift coefficient for the 1550 nm resonance frequency shift per unit 1330 nm resonance frequency shift. The black line is the 1330 nm thermal triangle when the auxiliary laser is scanned by a 1 Hz triangle wave signal. The blue line is the transmission signal when the pump laser is scanned at a low power by a 50 Hz triangle signal (red line).](image)

**Measurement and numerical calculation of the width of the 1554-nm broadened resonance and characterization of the 1334-nm auxiliary laser detuning.** As shown in Fig. 4 (a), the 1554 nm broadened resonance is recorded by an oscilloscope and its width is measured from the start point of the triangle to the end edge. Knowing the pump laser scanning rate, the width can be expressed in frequency. In the simulation, the thermal broadening is calculated directly based on the pump resonance shift from the position of the cold cavity mode. To characterize the 1334 nm laser detuning from its resonance, the 1334 nm thermal triangle is also recorded with the amplitude $\Delta V$ and $V_0$ as shown in Fig. S2. During the measurement of the 1554-nm triangle, the 1334-nm transmission voltage ($V_1$) is also simultaneously recorded. Knowing the linewidth of the Lorentzian shaped 1334 nm resonance, the recorded voltage $V_1$ can be mapped to the laser detuning ($\Delta f$). The same scheme can be used for the numerical calculation of the 1334-nm laser detuning.

![Fig. S2. Scheme to determine the auxiliary laser detuning from its resonance based on the transmission signal of the thermally broadened resonance.](image)

**Procedure for reducing the thermal broadening and extending the soliton existence range using an auxiliary laser.**

1. Switch the pump laser on with any desired optical power, and set the frequency around the selected comb-generating mode.
2. Set the optical power of the auxiliary laser with an initial value $P_0$, which depends on the Q factor of the auxiliary resonance and the pump power. Higher pump power needs higher auxiliary laser power to compensate the temperature changes in the resonator.
3. Modulate the frequency of the auxiliary laser with a low frequency signal (7 Hz in our experiment), choose a high-Q mode as the auxiliary mode and optimize the polarization of the laser.
4. Stop scanning the auxiliary laser frequency, and set the frequency of the auxiliary laser on the blue side of the selected auxiliary mode.
5. Scan the frequency of the pump laser with a low frequency signal (7 Hz in our experiment), monitor the transmission signal, so that a thermal broadening triangle of the 1550 nm resonance appears on the oscilloscope. Optimize the polarization of the pump.
6. Tune the frequency of the auxiliary laser further into the auxiliary resonance while monitoring the thermally broadened pump resonance. When tuning into the auxiliary resonance, the width of the thermally broadened pump mode is getting smaller, meanwhile the soliton steps are getting more and more visible on the oscilloscope.
7. If the soliton steps are not long enough to access the soliton before the auxiliary laser jumps out its resonance, increase the optical power $P_0$ of the auxiliary laser.
8. Finally, when tuning the auxiliary laser further into resonance, the width of the broadened pump mode resonance will reach its smallest value and will not get narrower anymore (as shown in Fig. 4(c) of the main manuscript when the auxiliary laser detuning is smaller than 1 MHz).
9. Stop tuning the auxiliary laser frequency and also stop scanning the pump laser frequency. Slowly tune the pump laser frequency from blue side into the soliton step, until the multi-/single- soliton states are generated. A single soliton can be accessed by further tuning the laser frequency if a multi-soliton state emerges first.
Detuning-power map of the auxiliary laser for 20 mW pump power. Figure S3 shows a colormap of the resonance broadening as a function of the auxiliary laser power and detuning (with a pump laser power of 20 mW). It can be seen that a smaller auxiliary laser can be compensated by a smaller detuning of the auxiliary laser in order to achieve a similar thermal compensation effect on the 1550 nm pump mode.

**Fig. S3.** Colormap of the resonance broadening of the 1550-nm-resonance as a function of the optical power and detuning of the 1330 nm laser. The pump laser power is fixed at 20 mW.