Yukawa Alignment in the Higgs Basis

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Abstract

We implement a comprehensive and detailed study of the alignment of Yukawa couplings in the so-called Higgs basis taking the framework of general two Higgs doublet models (2HDMs). The alignment of the Yukawa couplings implies the alignment of them in the flavor space and the alignment of the lightest Higgs-boson couplings to a pair of the SM fermions in the decoupling limit. We clarify the model input parameters and derive the Yukawa couplings considering the two sources of CP violation: one from the Higgs potential and the other from the three complex alignment parameters \( \zeta_f = u, d, e \). We consider the theoretical constraints from the perturbative unitarity and for the Higgs potential to be bounded from below. Also considered are the constraints from electroweak precision observables. Identifying the lightest neutral Higgs boson as the 125 GeV one discovered at the LHC, we show that the decoupling of its couplings to a pair of the SM fermion \( f \) is delayed by the amount of \( |\zeta_f| (1 - g_{h_1 VV}^2 V V)^{1/2} \) compared to the \( g_{h_1 VV} \) coupling of the lightest Higgs boson to a pair of massive vector bosons. The delay factor could be sizable even when the coupling \( g_{h_1 VV} \) is very close to its SM value of 1 if \( |\zeta_f| \) can be arbitrarily large. Via a numerical study, it is demonstrated that the precision LHC Higgs data constrain \( |\zeta_f| \) to be small when the lightest Higgs-boson Yukawa coupling to each fermion pair \( f \bar{f} \) takes the value near the corresponding SM one. On the other hand, if the Yukawa couplings are equal in strength but opposite in sign to the SM ones, the corresponding alignment parameters could be large in general.
I. INTRODUCTION

Since the discovery of the 125 GeV Higgs boson in 2012 at the LHC [1, 2], it has been scrutinized very closely and extensively. At the early stage, several model-independent studies [3–25] show that there were some rooms for it to be unlike the one predicted in the Standard Model (SM) but, after combining all the LHC Higgs data at 7 and 8 TeV [26] and especially those at 13 TeV [27–45], it turns out that it is best described by the SM Higgs boson. Specifically, the third-generation Yukawa couplings have been established. And the most recent model-independent study [46] shows that the $1\sigma$ error of the top-quark Yukawa coupling is about 6% while those of the bottom-quark and tau-lepton ones are about 10%. In addition, the possibility of negative top-quark Yukawa coupling has been completely ruled out and the bottom-quark Yukawa coupling shows a preference of the positive sign $1$ at about $1.5\sigma$ level. For the tau-Yukawa coupling, the current data still do not show any preference for its sign yet. On the other hand, the coupling to a pair of massive vector bosons is constrained to be consistent with the SM value within about 5% at $1\sigma$ level.

Even though we have not seen any direct hint or evidence of new physics beyond the SM (BSM), we are eagerly anticipating it with various compelling motivations such as the tiny but non-vanishing neutrino masses, matter dominance of our Universe and its evolution driven by dark energy and dark matters, etc [47]. In many BSM models, the Higgs sector is extended and it results in existence of several neutral and charged Higgs bosons. Their distinctive features depending on new theoretical frameworks could be directly probed through their productions and decays at future high-energy and high-precision experiments [48–62].

By the alignment of the Yukawa couplings in general 2HDMs [63–72], first of all, we imply that the Yukawa matrices describing the couplings of the two Higgs doublets to the SM fermions should be aligned in the flavor space to avoid the tree-level Higgs-mediated flavor-changing neutral current (FCNC). In 2HDMs, there are three neutral Higgs bosons and one of them should be identified as the observed one at the LHC which weighs 125.5 GeV [73]. In this case, by the alignment of the Yukawa couplings, we also mean that the couplings of this SM-like Higgs boson to the SM fermions should be the same as those of the SM Higgs boson itself or its couplings are strongly constrained to be very SM-like by the current LHC data as outlined above. One of the popular ways to achieve this alignment is to identify the lightest neutral Higgs boson as the 125.5 GeV one and assume

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1 Precisely speaking, here the sign of the bottom-quark Yukawa coupling is relative to the top-quark Yukawa coupling configured through the $b$- and $t$-quark loop contributions to the $Hgg$ vertex.
that all the other Higgs bosons are heavier or much heavier than the lightest one \[74, 75\]. But this decoupling scenario is not phenomenologically interesting and another scenario is suggested in which all the couplings of the SM-like Higgs candidate are (almost) aligned with those of the SM Higgs while the other Higgs bosons are not so heavy \[76, 79\].

The alignment of Yukawa couplings are previously discussed and studied \[77, 80\]. For some recent works, see, for example, Refs. \[81–83\]. In this work, taking the framework of general 2HDMs, we implement comprehensive and detailed study of the alignment of Yukawa couplings in the so-called Higgs basis \[70, 71, 84–88\] in which only the doublet containing the SM-like Higgs boson develops the non-vanishing vacuum expectation value (vev) \(v\). We identify the lightest neutral Higgs boson as the 125.5 GeV one and consider the alignment of its Yukawa couplings as the masses of the heavier Higgs bosons increase or as the heavy Higgs bosons decouple. For the alignment of the Yukawa matrices, we assume that the Yukawa matrices are aligned in the flavor space \[89–91\] by introducing the three alignment parameters \(\zeta_f\) with \(f = u, d, e\) for the couplings to the up-type quarks, the down-type quarks, and the charged leptons, respectively. We quantify that the decoupling of the Yukawa couplings of the lightest Higgs boson is delayed by the amount of \(|\zeta_f|\left(1 - g_{H_1VV}^2\right)^{1/2}\) compared to its coupling to a pair of massive vector bosons, \(g_{H_1VV}\). We demonstrate that \(|\zeta_f|\)’s are constrained to be small by the precision LHC Higgs data unless the so-called wrong-sign alignment of the Yukawa couplings \[75, 92–95\] occurs. \(^2\)

This paper is organized as follows. Section II is devoted to a brief review of the 2HDM Higgs potential, the mixing among neutral Higgs bosons and their couplings to the SM particles in the Higgs basis. In Section III, we elaborate on the constraints from the perturbative unitarity, the Higgs potential bounded from below, and the electroweak precision observables. And we carry out numerical analysis of the three constraints and the alignment of Yukawa couplings in Section IV. A brief summary and conclusions are made in Section V.

II. TWO HIGGS DOUBLET MODEL IN THE HIGGS BASIS

In this section, we study the two Higgs doublet model taking the so-called Higgs basis \[70, 71, 84–88\]. We consider the general potential containing 3 dimensionful quadratic and 7 dimensionless quartic parameters, of which four parameters are complex. We closely examine the relations among the potential parameters, Higgs-boson masses, and the neutral Higgs-boson mixing so as to figure

\(^2\) In the wrong-sign alignment limit, the Yukawa couplings are equal in strength but opposite in sign to the SM ones.
out the set of input parameters to be used in the next Section. We further work out the Yukawa couplings in the Higgs basis together with the interactions of the neutral and charged Higgs bosons with massive gauge bosons.

### A. Higgs Potential

The general 2HDM scalar potential containing two complex SU(2)_L doublets of Φ₁ and Φ₂ with the same hypercharge \( Y = 1/2 \) may be given by

\[
V_\Phi = \mu_1^2(\Phi_1^+ \Phi_1^-) + \mu_2^2(\Phi_2^+ \Phi_2^-) + m_{12}^2(\Phi_1^+ \Phi_2^-) + m_{12}^2(\Phi_2^+ \Phi_1^-) \\
+ \lambda_1(\Phi_1^+ \Phi_1^-)^2 + \lambda_2(\Phi_2^+ \Phi_2^-)^2 + \lambda_3(\Phi_1^+ \Phi_1^-)(\Phi_2^+ \Phi_2^-) + \lambda_4(\Phi_1^+ \Phi_2^-)(\Phi_2^+ \Phi_1^-) \\
+ \lambda_5(\Phi_1^+ \Phi_2^-)^2 + \lambda_6(\Phi_2^+ \Phi_1^-)^2 + \lambda_7(\Phi_2^+ \Phi_2^-)(\Phi_1^+ \Phi_1^-) + \lambda_8(\Phi_1^+ \Phi_1^-)(\Phi_2^+ \Phi_1^-) \\
+ \lambda_9(\Phi_2^+ \Phi_2^-)(\Phi_1^+ \Phi_2^-) + \lambda_{10}(\Phi_2^+ \Phi_2^-)(\Phi_2^+ \Phi_1^-),
\]

(1)
in terms of 2 real and 1 complex dimensionful quadratic couplings and 4 real and 3 complex dimensionless quartic couplings. Note that the \( \mathbb{Z}_2 \) symmetry under \( \Phi_1 \to \pm \Phi_1 \) and \( \Phi_2 \to \mp \Phi_2 \) is hardly broken by the non-vanishing quartic couplings \( \lambda_6 \) and \( \lambda_7 \) and, in this case, we have three rephasing-invariant CPV phases in the potential. With the general parameterization of two scalar doublets \( \Phi_{1,2} \) as

\[
\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + ia_1) \end{pmatrix}; \quad \Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + ia_2) \end{pmatrix},
\]

(2)
and denoting \( v_1 = v \cos \beta = v_c \beta \) and \( v_2 = v \sin \beta = v_s \beta \) with \( v = \sqrt{v_1^2 + v_2^2} \), one may remove \( \mu_1^2, \mu_2^2 \), and \( \Im(m_{12} e^{i\xi}) \) from the 2HDM potential using three tadpole conditions:

\[
\mu_1^2 = -v^2 \left[ \lambda_1 c_\beta^2 + \frac{1}{2} \lambda_3 s_\beta^2 + c_\beta s_\beta \Re(\lambda_6 e^{i\xi}) \right] + s_\beta^2 M_H^2 \pm, \\
\mu_2^2 = -v^2 \left[ \lambda_7 s_\beta^2 + \frac{1}{2} \lambda_3 c_\beta^2 + c_\beta s_\beta \Re(\lambda_7 e^{i\xi}) \right] + c_\beta^2 M_H^2 \pm, \\
\Im(m_{12}^2 e^{i\xi}) = -\frac{v^2}{2} \left[ 2 c_\beta s_\beta \Im(\lambda_6 e^{2i\xi}) + c_\beta^2 \Im(\lambda_7 e^{i\xi}) + s_\beta^2 \Im(\lambda_7 e^{i\xi}) \right],
\]

(3)

\[3\text{ In contrast with the Higgs basis which has been taken for this work, we call it the } \Phi \text{ basis.} \]
with the square of the charged Higgs-boson mass

\[ M_{H^\pm}^2 = -\frac{\text{Re}(m_{H_2}^2e^{i\xi})}{c_\beta s_\beta} - \frac{v^2}{2c_\beta s_\beta} \left[ \lambda_4 c_\beta s_\beta + 2 c_\beta s_\beta \text{Re}(\lambda_5 e^{2i\xi}) + c_\beta^2 \text{Re}(\lambda_6 e^{i\xi}) + s_\beta^2 \text{Re}(\lambda_7 e^{i\xi}) \right]. \] (4)

On the other hand, in the Higgs basis where only one doublet contains the non-vanishing vev \( v \), the general 2HDM scalar potential again contains three (two real and one complex) massive parameters and four real and three complex dimensionless quartic couplings and it might take the same form as in the \( \Phi \) basis:

\[
V_H = Y_1(\mathcal{H}_1^\dagger \mathcal{H}_1) + Y_2(\mathcal{H}_2^\dagger \mathcal{H}_2) + Y_3(\mathcal{H}_1^\dagger \mathcal{H}_2) + Y_3^*(\mathcal{H}_2^\dagger \mathcal{H}_1) \\
+ Z_1(\mathcal{H}_1^\dagger \mathcal{H}_1)^2 + Z_2(\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + Z_3(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_1^\dagger \mathcal{H}_2) + Z_4(\mathcal{H}_1^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1) \\
+ Z_5(\mathcal{H}_1^\dagger \mathcal{H}_1)^2 + Z_6(\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + Z_7(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_1^\dagger \mathcal{H}_2) + Z_8(\mathcal{H}_1^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1) \\
+ Z_9(\mathcal{H}_2^\dagger \mathcal{H}_2)(\mathcal{H}_1^\dagger \mathcal{H}_1) + Z_{10}(\mathcal{H}_2^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1) ,
\] (5)

where the new complex SU(2)\(_L\) doublets of \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) are given by the linear combinations of \( \Phi_1 \) and \( \Phi_2 \) as follows

\[
\mathcal{H}_1 = c_\beta \Phi_1 + e^{-i\xi} s_\beta \Phi_2 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} (v + \varphi_1 + iG^0) \end{pmatrix}; \\
\mathcal{H}_2 = -s_\beta \Phi_1 + e^{-i\xi} c_\beta \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (\varphi_2 + ia) \end{pmatrix},
\] (6)

with the relations

\[
\varphi_1 \equiv c_\beta \varphi_1 + s_\beta \varphi_2 , \quad \varphi_2 \equiv -s_\beta \varphi_1 + c_\beta \varphi_2 ; \quad a = -s_\beta a_1 + c_\beta a_2,
\] (7)

in terms of \( \varphi_{1,2} \) and \( a_{1,2} \) in Eq. (2). Incidentally, we have that \( G^0 = c_\beta a_1 + s_\beta a_2 \), \( G^+ = c_\beta \phi_1^+ + s_\beta \phi_2^+ \), and \( H^+ = -s_\beta \phi_1^+ + c_\beta \phi_2^+ \). Note that only the neutral component of the \( \mathcal{H}_1 \) doublet develops the non-vanishing vacuum expectation value \( v \) and it contains only one physical degree of freedom let alone the Goldstone modes. In the so-called decoupling limit, \( \mathcal{H}_1 \) take over the role of the SM SU(2)\(_L\) doublet and the remaining three Higgs states are accommodated only by the \( \mathcal{H}_2 \) doublet. \(^4\)

\(^4\) In the notations taken for a numerical study later, \( \varphi_1 = h \), \( \varphi_2 = H \), and \( a = A \) in the decoupling limit.
The potential parameters $Y_{1,2,3}$ and $Z_{1-7}$ in the Higgs basis could be related to those in the $\Phi$ basis through:

$$Y_1 = \mu_1^2 c_\beta^2 + \mu_2^2 s_\beta^2 + \Re(m_{12}^2 e^{i\xi}) s_\beta,$$

$$Y_2 = \mu_1^2 s_\beta^2 + \mu_2^2 c_\beta^2 - \Re(m_{12}^2 e^{i\xi}) s_\beta,$$

$$Y_3 = -(\mu_1^2 - \mu_2^2) c_\beta s_\beta + \Re(m_{12}^2 e^{i\xi}) c_\beta + i \Im(m_{12}^2 e^{i\xi}), \quad (8)$$

for two real and one complex dimensionful parameters and $^5$

$$Z_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2\lambda_{345} c_\beta^2 s_\beta^2 + [\Re(\lambda_6 e^{i\xi}) c_\beta^2 + \Re(\lambda_7 e^{i\xi}) s_\beta^2] s_\beta,$$

$$Z_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + 2\lambda_{345} c_\beta^2 s_\beta^2 - [\Re(\lambda_6 e^{i\xi}) s_\beta^2 + \Re(\lambda_7 e^{i\xi}) c_\beta^2] s_\beta,$$

$$Z_3 = \lambda_3 + 2(\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 - [\Re(\lambda_6 e^{i\xi}) - \Re(\lambda_7 e^{i\xi})] c_\beta s_\beta,$$

$$Z_4 = \lambda_4 + 2(\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 - [\Re(\lambda_6 e^{i\xi}) - \Re(\lambda_7 e^{i\xi})] c_\beta s_\beta,$$

$$Z_5 = (\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 + \Re(\lambda_5 e^{2i\xi}) - [\Re(\lambda_6 e^{i\xi}) - \Re(\lambda_7 e^{i\xi})] c_\beta s_\beta$$

$$+ i [\Im(\lambda_5 e^{2i\xi}) c_\beta s_\beta - \Im(\lambda_6 e^{i\xi}) c_\beta s_\beta + \Im(\lambda_7 e^{i\xi}) c_\beta^2],$$

$$Z_6 = (\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2) s_\beta + 2\lambda_{345} c_\beta s_\beta - [\Re(\lambda_6 e^{i\xi}) c_\beta^2 - 3s_\beta^2] c_\beta^2 + \Re(\lambda_7 e^{i\xi})(3c_\beta^2 - s_\beta^2) s_\beta^2$$

$$+ i [\Im(\lambda_5 e^{2i\xi}) s_\beta^2 + \Im(\lambda_6 e^{i\xi}) c_\beta^2 + \Im(\lambda_7 e^{i\xi}) s_\beta^2],$$

$$Z_7 = (\lambda_1 s_\beta^2 + \lambda_2 c_\beta^2) s_\beta - 2\lambda_{345} c_\beta s_\beta - [\Re(\lambda_6 e^{i\xi})(3c_\beta^2 - s_\beta^2) s_\beta^2 + \Re(\lambda_7 e^{i\xi})(c_\beta^2 - 3s_\beta^2) s_\beta^2$$

$$+ i [-\Im(\lambda_5 e^{2i\xi}) s_\beta^2 + \Im(\lambda_6 e^{i\xi}) s_\beta^2 + \Im(\lambda_7 e^{i\xi}) c_\beta^2], \quad (9)$$

for four real and three complex dimensionless parameters with $\lambda_{345} \equiv (\lambda_3 + \lambda_4)/2 + \Re(\lambda_5 e^{2i\xi}).$

We note that $Z_1 \leftrightarrow Z_2$, $Z_3 \leftrightarrow Z_4$, $Z_6 \leftrightarrow Z_7$ and $Z_5$ is invariant under the exchanges $c_\beta \leftrightarrow s_\beta$, $\lambda_3 \leftrightarrow \lambda_4$, $(\lambda_5 e^{2i\xi}) \leftrightarrow (\lambda_5 e^{2i\xi})^*$, $(\lambda_6 e^{i\xi}) \leftrightarrow -(\lambda_6 e^{i\xi})^*$. The tadpole conditions in the Higgs basis, which are much simpler than those in the $\Phi$ basis as shown in Eq. (3), are

$$Y_1 + Z_1 v^2 = 0; \quad Y_3 + \frac{1}{2} Z_6 v^2 = 0, \quad (10)$$

where the first condition comes from $\langle \frac{\partial V}{\partial \varphi_1} \rangle = 0$ and the second one from $\langle \frac{\partial V}{\partial v^2} \rangle = 0$ and $\langle \frac{\partial V}{\partial \varphi} \rangle = 0.$

$^5$ We find that our results are consistent with those presented in, for example, Ref. [82].
Note that the second condition relates the two complex parameters of $Y_3$ and $Z_6$.

B. Masses, Mixing, and Potential Parameters in the Higgs Basis

In the Higgs basis, the 2HDM Higgs potential includes the mass terms which can be cast into the form consisting of two parts

$$V_{H,\text{mass}} = M_{H^\pm}^2 H^+ H^- + \frac{1}{2} (\varphi_1 \varphi_2 a) M_{\varphi}^2 \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ a \end{pmatrix},$$

(11)

in terms of the charged Higgs bosons $H^\pm$, two neutral scalars $\varphi_{1,2}$, and one neutral pseudoscalar $a$. The charged Higgs boson mass is given by

$$M_{H^\pm}^2 = Y_2 + \frac{1}{2} Z_3 v^2.$$  

(12)

while the $3 \times 3$ mass-squared matrix of the neutral Higgs bosons $M_0^2$ takes the form

$$M_0^2 = M_A^2 \text{diag}(0,1,1) + M_Z^2,$$

(13)

where $M_A^2 = M_{H^\pm}^2 + \frac{1}{2} Z_4 - \text{Re}(Z_5) v^2$ and the $3 \times 3$ real and symmetric mass-squared matrix $M_Z^2$ is given by

$$\frac{M_Z^2}{v^2} = \begin{pmatrix} 2Z_1 & \text{Re}(Z_6) & -3\text{m}(Z_6) \\ \text{Re}(Z_6) & 2\text{Re}(Z_5) & -3\text{m}(Z_5) \\ -3\text{m}(Z_6) & -3\text{m}(Z_5) & 0 \end{pmatrix}.$$  

(14)

Note that the quartic couplings $Z_2$ and $Z_7$ have nothing to do with the masses of Higgs bosons and the mixing of the neutral ones. They can be probed only through the cubic and quartic Higgs self-couplings, see Eq. (5) while noting that only the $H_1$ doublet contains the vev $v \simeq 246$ GeV. We further note that $\varphi_1$ decouples from the mixing with the other two neutral states of $\varphi_2$ and $a$ in the $Z_6 = 0$ limit, and its mass squared is simply given by $2 Z_1 v^2$ which gives $Z_1 \simeq 0.13 (M_{H^\pm}/125.5 \text{GeV})^2$. And, in this decoupling limit, the CP-violating mixing between the two states of $\varphi_2$ and $a$ is dictated only by $\text{Im}(Z_5)$. 

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Once the $3 \times 3$ real and symmetric mass-squared matrix $\mathcal{M}_0^2$ is given, the orthogonal $3 \times 3$ mixing matrix $O$ is defined through

$$(\varphi_1, \varphi_2, a)^T_{\alpha} = O_{\alpha i} (H_1, H_2, H_3)_i^T,$$  

(15)
such that $O^T \mathcal{M}_0^2 O = \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2)$ with the increasing ordering of $M_{H_1} \leq M_{H_2} \leq M_{H_3}$, if necessary. Note that the mass-squared matrix $\mathcal{M}_0^2$ involves only the four (two real and two complex) quartic couplings $\{Z_1, Z_4, Z_5, Z_6\}$ once $v$ and $M_{H^\pm}$ are given. And then, using the matrix relation $O^T \mathcal{M}_0^2 O = \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2)$, one may find the following expressions for the quartic couplings of $\{Z_1, Z_4, Z_5, Z_6\}$ in terms of the three masses of neutral Higgs bosons and the components of the $3 \times 3$ orthogonal mixing matrix $O$: \(^6\)

\[ \begin{align*}
Z_1 &= \frac{1}{2v^2} \left( M_{H_1}^2 O_{\varphi_{11}}^2 + M_{H_2}^2 O_{\varphi_{12}}^2 + M_{H_3}^2 O_{\varphi_{13}}^2 \right), \\
Z_4 &= \frac{1}{v^2} \left[ M_{H_1}^2 (O_{\varphi_{21}}^2 + O_{\varphi_{a1}}^2) + M_{H_2}^2 (O_{\varphi_{22}}^2 + O_{\varphi_{a2}}^2) + M_{H_3}^2 (O_{\varphi_{23}}^2 + O_{\varphi_{a3}}^2) - 2M_{H^\pm}^2 \right], \\
Z_5 &= \frac{1}{2v^2} \left[ M_{H_1}^2 (O_{\varphi_{21}}^2 - O_{\varphi_{a1}}^2) + M_{H_2}^2 (O_{\varphi_{22}}^2 - O_{\varphi_{a2}}^2) + M_{H_3}^2 (O_{\varphi_{23}}^2 - O_{\varphi_{a3}}^2) \right] \\
& \quad - \frac{i}{v^2} \left( M_{H_1}^2 O_{\varphi_{21}} O_{\varphi_{a1}} + M_{H_2}^2 O_{\varphi_{22}} O_{\varphi_{a2}} + M_{H_3}^2 O_{\varphi_{23}} O_{\varphi_{a3}} \right), \\
Z_6 &= \frac{1}{v^2} \left( M_{H_1}^2 O_{\varphi_{11}} O_{\varphi_{21}} + M_{H_2}^2 O_{\varphi_{12}} O_{\varphi_{22}} + M_{H_3}^2 O_{\varphi_{13}} O_{\varphi_{23}} \right) \\
& \quad - \frac{i}{v^2} \left( M_{H_1}^2 O_{\varphi_{11}} O_{\varphi_{a1}} + M_{H_2}^2 O_{\varphi_{12}} O_{\varphi_{a2}} + M_{H_3}^2 O_{\varphi_{13}} O_{\varphi_{a3}} \right),
\end{align*} \]

(16)
for given $v$ and $M_{H^\pm}$.

Now we are ready to consider the input parameters for 2HDM in the Higgs basis. First of all, the input parameters for the Higgs potential Eq. (5) are

$$\{Y_1, Y_2, Y_3; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}. \quad (17)$$

Using the tadpole conditions in Eq. (10), the dimensionful parameters $Y_1$ and $Y_3$ can be removed from the set in favor of $v$ and observing that the quartic couplings $Z_2$ and $Z_7$ do not contribute

\(^6\) Note that we reserve the notations of $H_{i=1,2,3}$ for the mass eigenstates of three neutral Higgs bosons taking account of CP-violating mixing in the neutral Higgs-boson sector when $\text{Im}(Z_{5,6}) \neq 0$. In general, the neutral Higgs bosons do not carry definite CP parities and they become mixtures of CP-even and CP-odd states.

\(^7\) The $3 \times 3$ orthogonal mixing matrix $O$ contains three independent degrees of freedom represented by the three rotation angles.
to the mass terms, one may consider the following set of input parameters

\[ \{v, Y_2; M_{H^\pm}, Z_1, Z_5, Z_6; Z_2, Z_7\} , \] (18)

where we trade the quartic coupling \( Z_3 \) with the charged Higgs mass \( M_{H^\pm} \) using the relation

\[ Z_3 = \frac{2 (M_{H^\pm}^2 - Y_2^2)}{v^2} \]

with \( Y_2 \) given. Further using \( M_{H_1} \) and \( O \) instead of \( \{Z_1, Z_4, Z_5, Z_6\} \), we end up with the following set of input parameters:

\[ \mathcal{I} = \{v, Y_2; M_{H^\pm}, M_{H_1}, M_{H_2}, M_{H_3}, \{O_3\times3\}; Z_2, Z_7\} , \] (19)

which contains 12 real degrees of freedom. If desirable, one may remove the unphysical massive parameter \( Y_2 \) in favor of the dimensionless quartic coupling \( Z_3 \) by having an alternative set

\[ \mathcal{I}' = \{v; M_{H^\pm}, M_{H_1}, M_{H_2}, M_{H_3}, \{O_3\times3\}; Z_3; Z_2, Z_7\} , \] (20)

consisting of 12 real parameters as well.

For example, in the CP-conserving (CPC) case with \( 3mZ_5 = 3mZ_6 = 0 \), one may denote the masses of the three neutral Higgs bosons by \( M_h, M_H, \) and \( M_A \) or \( O^T M_0^2 O = \text{diag}(M_h^2, M_H^2, M_A^2) \). Note that \( M_h \) is for the SM Higgs boson in the decoupling limit. The mixing \( O \) can be parameterized as

\[ O_{\text{CPC}} = \begin{pmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} , \] (21)

introducing the mixing angle \( \gamma \) between the two CP-even states \( \varphi_1 \) and \( \varphi_2 \). In this CP-conserving case, the relations Eq. (16) simplify into

\[
\begin{align*}
Z_1 &= \frac{1}{2v^2} \left( c_\gamma^2 M_h^2 + s_\gamma^2 M_H^2 \right) , \\
Z_4 &= \frac{1}{v^2} \left( s_\gamma^2 M_h^2 + c_\gamma^2 M_H^2 + M_A^2 - 2M_{H^\pm}^2 \right) , \\
Z_5 &= \frac{1}{2v^2} \left( s_\gamma^2 M_h^2 + c_\gamma^2 M_H^2 - M_A^2 \right) , \\
Z_6 &= \frac{1}{v^2} \left( -M_h^2 + M_H^2 \right) c_\gamma s_\gamma .
\end{align*}
\] (22)

We observe that, in the decoupling limit of \( \sin \gamma = 0 \), \( Z_1 = M_h^2 / 2v^2 \) and \( Z_6 = 0 \), and \( Z_4 \) and \( Z_5 \) are determined by the mass differences of \( M_H^2 + M_A^2 - 2M_{H^\pm}^2 \) and \( M_H^2 - M_A^2 \), respectively. Finally,
for the study of the CPC case, one may choose one of the following two equivalent sets:

\[ \mathcal{I}_{\text{CPC}} = \{ v, Y_2; M_{H^\pm}, M_h, M_H, M_A, \gamma; Z_2, Z_7 \} \, , \]

\[ \mathcal{I}^{\prime}_{\text{CPC}} = \{ v; M_{H^\pm}, M_h, M_H, M_A, \gamma; Z_3; Z_2, Z_7 \} \, , \]

(23)
each of which contains 9 real degrees of freedom, and the convention of \(|\gamma| \leq \pi/2\) without loss of
generality resulting in \(c_\gamma \geq 0\) and \(\text{sign}(s_\gamma) = \text{sign}(Z_6)\) if \(M_H > M_h\) GeV.

In the presence of non-vanishing \(\Im Z_5\) and/or \(\Im Z_6\), the mixing between the two CP-even
states \(\varphi_{1,2}\) and the CP-odd one \(a\) arises leading to CP violation in the neutral Higgs sector. In
this CP-violating (CPV) case, one may parameterize the mixing matrix \(O\) by introducing three
mixing angles of \(\gamma, \eta,\) and \(\omega\) as follow:

\[
O = O_\gamma O_\eta O_\omega \equiv \begin{pmatrix} O_\gamma T \end{pmatrix} O_\eta \equiv \begin{pmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\eta & 0 & s_\eta \\ 0 & 1 & 0 \\ -s_\eta & 0 & c_\eta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\omega & s_\omega \\ 0 & -s_\omega & c_\omega \end{pmatrix}
\]

\[
= \begin{pmatrix} c_\gamma c_\eta s_\omega - c_\gamma s_\eta s_\omega & s_\gamma s_\eta s_\omega + c_\gamma s_\eta c_\omega & -s_\gamma c_\eta s_\omega - c_\gamma s_\eta c_\omega \\ -s_\gamma c_\eta c_\omega + s_\gamma s_\eta s_\omega & c_\gamma s_\omega - s_\gamma s_\eta c_\omega & c_\gamma c_\eta \omega \\ -s_\eta & -c_\eta s_\omega & c_\eta c_\omega \end{pmatrix} .
\]

(24)

And then, using this form of the mixing matrix \(O\) and the relations in Eq. (16) or, equivalently,
\(O_\gamma T M_0^2 O_\gamma = O_\eta O_\omega \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2) O_\omega T O_\eta\), we obtain

\[
\begin{pmatrix} O_\gamma T \frac{M_0^2}{v^2} O_\gamma \end{pmatrix} \begin{pmatrix} (Z_1 - \frac{Z_4}{4} - \Re(Z_5)) - \frac{M_{H^+}^2}{2v^2} \end{pmatrix} \begin{pmatrix} s_2 \gamma + \Re(Z_6) c_2 \gamma = \frac{M_{H_3}^2 - M_{H_2}^2}{v^2} s_\eta c_\omega s_\omega \end{pmatrix} ,
\]

\[
\begin{pmatrix} O_\gamma T \frac{M_0^2}{v^2} O_\gamma \end{pmatrix} \begin{pmatrix} \Im(Z_5) s_\gamma - \Im(Z_6) c_\gamma = \frac{M_{H_3}^2 c_\omega^2 + M_{H_2}^2 s_\omega^2 - M_{H_1}^2}{v^2} c_\eta s_\eta \end{pmatrix} ,
\]

\[
\begin{pmatrix} O_\gamma T \frac{M_0^2}{v^2} O_\gamma \end{pmatrix} \begin{pmatrix} \Im(Z_5) c_\gamma - \Im(Z_6) s_\gamma = \frac{M_{H_3}^2 - M_{H_2}^2}{v^2} c_\eta c_\omega s_\omega \end{pmatrix} .
\]

(25)

In the CPC case with \(s_2 = s_2 = 0\) or \(\Im(Z_5) = \Im(Z_6) = 0\), we observe that the first relation
in Eq. (25) becomes \(^8\)

\[
\begin{pmatrix} \left( -\frac{M_H^2 + M_h^2}{2v^2} \right) c_2 \gamma \end{pmatrix} s_2 \gamma + \Re(Z_6) c_2 \gamma = 0 ,
\]

(26)

\({}^8\) We use the three relations for \(Z_{1,4,5}\) given in Eq. (22) to calculate the quantity in the square bracket.
which is the same as the fourth relation in Eq. (22), as it should be. On the other hand, in the
limit of \( c_\gamma = 1 \) and \( s_\gamma = 0 \), the mixing matrix takes the simple form

\[
O_{\eta \bar{\eta} \omega} \equiv O_{\eta} O_{\omega} = \begin{pmatrix}
  c_\eta & -s_\eta s_\omega & s_\eta c_\omega \\
  0 & c_\omega & s_\omega \\
  -s_\eta & -c_\eta s_\omega & c_\eta c_\omega
\end{pmatrix}.
\]  

(27)

When \( c_\eta \simeq 1 - \eta^2/2 \) and \( s_\eta \simeq \eta \), the lightest \( H_1 \) is SM like and the heavier ones \( H_{2,3} \) are mostly arbitrary mixtures of \( \varphi_2 \) and \( a \). And, in this case, we have

\[
|\text{Re}(Z_6)| \propto |\eta| \ll 1, \quad |s_{2\eta}| \propto |\Im m(Z_6)| \ll 1, \quad s_{2\omega} \propto -\Im m(Z_5) .
\]  

(28)

On the other hand, when \( c_\eta \simeq |\eta| \) and \( |s_\eta| \simeq 1 - \eta^2/2 \), the lightest \( H_1 \) is mostly CP odd \( (H_1 \sim a) \) and \( H_2 \) \( (H_3) \) is SM like when \( |s_\omega| \simeq 1 \) \( (|c_\omega| \simeq 1) \). And, in this case, we have\(^9\)

\[
|s_{2\omega}| \propto |\text{Re}(Z_6)| \ll 1, \quad |s_{2\eta}| \propto |\Im m(Z_6)| \ll 1, \quad \Im m(Z_5) \simeq 0 .
\]  

(29)

Incidentally, using \( O^T \gamma M_0^2 O_\gamma = O_{\eta} O_{\omega} \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2) O^T \omega O^T _{\eta} \), we also have

\[
\begin{align*}
(O^T_{\gamma} \frac{M_0^2}{v^2} O_\gamma)_{(1,1)} & : \left[ \left( Z_1 + \left( \frac{Z_4}{4} + \frac{\text{Re}(Z_5)}{2} \right) \right) + M_{H_1}^2 \right] - \text{Re}(Z_6) s_{2\gamma} + \left( Z_1 - \left( \frac{Z_4}{4} - \frac{\text{Re}(Z_5)}{2} \right) - \frac{M_{H_1}^2}{2v^2} \right) c_{2\gamma} \\
& = \frac{(M_{H_1}^2 c^2_\eta + M_{H_2}^2 s^2_\eta) s^2_\omega + M_{H_1}^2 c^2_\eta}{v^2},
\end{align*}
\]

\[
\begin{align*}
(O^T_{\gamma} \frac{M_0^2}{v^2} O_\gamma)_{(2,2)} & : \left[ \left( Z_1 + \left( \frac{Z_4}{4} + \frac{\text{Re}(Z_5)}{2} \right) \right) + M_{H_2}^2 \right] + \text{Re}(Z_6) s_{2\gamma} - \left( Z_1 - \left( \frac{Z_4}{4} - \frac{\text{Re}(Z_5)}{2} \right) - \frac{M_{H_2}^2}{2v^2} \right) c_{2\gamma} \\
& = \frac{M_{H_2}^2 c^2_\omega + M_{H_1}^2 s^2_\omega}{v^2},
\end{align*}
\]

\[
\begin{align*}
(O^T_{\gamma} \frac{M_0^2}{v^2} O_\gamma)_{(3,3)} & : \frac{Z_4}{2} - \text{Re}(Z_5) + \frac{M_{H_3}^2}{2v^2} = \frac{(M_{H_3}^2 c^2_\omega + M_{H_2}^2 s^2_\omega) c^2_\eta + M_{H_1}^2 s^2_\eta}{v^2} .
\end{align*}
\]

(30)

Note that one of the CP phases of \( \Im m(Z_5) \), \( \Im m(Z_6) \), and \( \Im m(Z_7) \) can be rotated away by rephasing the Higgs fields \( H_{1,2} \). By introducing a rotation \( H_2 \rightarrow e^{i\xi} H_2 \),\(^{10}\) the complex potential

\(^9\) When \( M_{H_2} \sim M_{H_3} \), a mixture of the two very degenerate scalar states plays the role of the SM Higgs boson with \( |s_{2\omega}| \sim 1 \) but still with \( |\text{Re}(Z_6)| \ll 1 \).

\(^{10}\) Or, equivalently, \( H_1^* H_2 \rightarrow e^{i\xi} H_1^* H_2 \).
parameters change

\[ Y_3 \rightarrow Y_3 e^{i\zeta}; \quad Z_5 \rightarrow Z_5 e^{2i\zeta}, \quad Z_6 \rightarrow Z_6 e^{i\zeta}, \quad Z_7 \rightarrow Z_7 e^{i\zeta}, \quad (31) \]

and then one can make, for example, \( \Im m(Z_5 e^{2i\zeta}) = 0 \) by choosing \( \phi_5 + 2\zeta = 0 \) with \( \phi_5 \equiv \arg(Z_5) \).

By exploiting this rephasing transformation, we observe that one may always choose a basis in which \( c_{\omega} s_{\omega} = 0 \) by taking \( \zeta \) satisfying \( \sin(\phi_5 + 2\zeta) c_{\gamma} + \sin(\phi_6 + \zeta) s_{\gamma} = 0 \), see the third relation in Eq. (25). In this case, one ends up with the following set of input parameters:

\[ I_{\text{CPV}} = \{ v, Y_2; M_{H_1}, M_{H_2}, M_{H_3}, \gamma, \eta; Z_2, \Re(e^{i\zeta}Z_7), \Im m(Z_7) \} . \quad (32) \]

which contains 11 real degrees of freedom. Under the rotation \( H_2 \rightarrow e^{i\zeta} H_2 \), the originally CP-even neutral state \( \varphi_2 \) and the CP-odd one \( a \) mixes:

\[ \begin{pmatrix} \varphi_2 \\ a \end{pmatrix} \rightarrow \begin{pmatrix} c_{\zeta} & -s_{\zeta} \\ s_{\zeta} & c_{\zeta} \end{pmatrix} \begin{pmatrix} \varphi_2 \\ a \end{pmatrix} . \quad (33) \]

However, the phase of \( H_2 \) cannot be freely rotated if the Yukawa interactions are to be considered together with the Higgs potential since the scalar and pseudo-scalar components exclusively couple to \( \bar{f} f \) and \( \bar{f} (i\gamma_5)f \), respectively \[88\]. In passing, in the rotated basis where \( s_{2\omega} = 0 \), we note that \( O_{a2} (O_{a3}) = 0 \) when \( s_{\omega}(c_{\omega}) = 0 \), see Eq. (24). This might imply that one may freely choose a basis in which either \( H_2 \) or \( H_3 \) is purely CP even if one can free rotate \( H_2 \).

C. Yukawa Couplings in Higgs basis

In the 2HDM, the Yukawa couplings might be given by

\[ -\mathcal{L}_Y = \sum_{k=1,2} \overline{Q_L^0} y_{uk}^y \tilde{H}_k u_R^0 + \overline{Q_L^0} y_{dk}^d \tilde{H}_k d_R^0 + \overline{L_L^0} y_{e_k}^e \tilde{H}_k e_R^0 + \text{h.c.} \quad (34) \]

in terms of the six \( 3 \times 3 \) Yukawa matrices \( y_{1,2}^{u,d,e} \) with the electroweak eigenstates \( Q_L^0 = (u_L^0, d_L^0)^T \), \( L_L^0 = (\nu_L^0, e_L^0)^T \), and two Higgs doublets in the Higgs basis

\[ H_1 = \left( G^+, \frac{1}{\sqrt{2}}(v + \varphi_1 + iG^0) \right)^T, \quad H_2 = \left( H^+, \frac{1}{\sqrt{2}}(\varphi_2 + ia) \right)^T , \quad (35) \]
and their SU(2)-conjugated doublets

\[ \tilde{\mathcal{H}}_1 = i\tau_2 H_1^* = \left( \frac{1}{\sqrt{2}}(v + \varphi_1 - iG^0), -G^- \right)^T, \quad \tilde{\mathcal{H}}_2 = i\tau_2 H_2^* = \left( \frac{1}{\sqrt{2}}(\varphi_2 - ia), -H^- \right)^T. \] (36)

The Yukawa interactions include the following mass terms

\[ -\mathcal{L}_{Y,\text{mass}} = \frac{v}{\sqrt{2}} \left( \overline{u_L^0} y_1^u u_R^0 + \overline{d_L^0} y_1^d d_R^0 + \overline{e_L^0} y_1^e e_R^0 + \text{h.c.} \right). \] (37)

Therefore, introducing two unitary matrices relating the left/right-handed electroweak eigenstates \( f_{L,R}^0 \) to the left/right-handed mass eigenstates \( f_{L,R} \) with \( f = u, d, e \) as follows

\[ u_L^0 = \mathcal{U}_{uL} u_L, \quad d_L^0 = \mathcal{U}_{dL} d_L, \quad e_L^0 = \mathcal{U}_{eL} e_L; \]
\[ u_R^0 = \mathcal{U}_{uR} u_R, \quad d_R^0 = \mathcal{U}_{dR} d_R, \quad e_R^0 = \mathcal{U}_{eR} e_R, \] (38)

we have, for the mass terms,

\[ -\mathcal{L}_{Y,\text{mass}} = \overline{u_L} M_u u_R + \overline{d_L} M_d d_R + \overline{e_L} M_e e_R + \text{h.c.}, \] (39)

where the three diagonal matrices are

\[ M_u = \frac{v}{\sqrt{2}} \mathcal{U}_{uL}^\dagger y_1^u \mathcal{U}_{uR} = \text{diag}(m_u, m_c, m_t), \]
\[ M_d = \frac{v}{\sqrt{2}} \mathcal{U}_{dL}^\dagger y_1^d \mathcal{U}_{dR} = \text{diag}(m_d, m_s, m_b), \]
\[ M_e = \frac{v}{\sqrt{2}} \mathcal{U}_{eL}^\dagger y_1^e \mathcal{U}_{eR} = \text{diag}(m_e, m_\mu, m_\tau), \] (40)

in terms of the six quark and three charged-lepton masses. We note that \( \mathcal{U}_{uL}^\dagger \mathcal{U}_{dL} = V_{\text{CKM}} \equiv V \) is nothing but the CKM matrix, in term of which the SU(2)_L quark doublets in the electroweak basis can be related to those in the mass basis in the following two ways:

\[ Q_L^0 = \mathcal{U}_{uL} \begin{pmatrix} u_L \\ V d_L \end{pmatrix} \quad \text{or} \quad Q_L^0 = \mathcal{U}_{dL} \begin{pmatrix} V^\dagger u_L \\ d_L \end{pmatrix}. \] (41)
The first relation is used for the Yukawa interactions with the right-handed up-type quarks and the second one for those with the right-handed down-type quarks. Incidentally, we also have

$$L_L^0 = \mathcal{U}_{eL} \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right)$$

by re-defining $$\nu_L \equiv \mathcal{U}_{eL}^\dagger \nu_L^0$$ with no physical effects in the case with vanishing neutrino masses.

Collecting all the parameterizations, unitary rotations, and re-parameterizations, the couplings of the neutral Higgs bosons to two fermions are given by

$$-\mathcal{L}_{Hff} = \frac{1}{v} [\bar{u} M_u u] \varphi_1 + [\bar{u} \left( h_f^H + h_f^A \gamma_5 \right) u] \varphi_2 + [\bar{u} \left( -i h_f^d - i h_f^H \gamma_5 \right) u] a$$

$$+ \frac{1}{v} [\bar{d} M_d d] \varphi_1 + [\bar{d} \left( h_f^d + h_f^A \gamma_5 \right) d] \varphi_2 + [\bar{d} \left( i h_f^d + i h_f^H \gamma_5 \right) d] a$$

$$+ \frac{1}{v} [\bar{e} M_e e] \varphi_1 + [\bar{e} \left( h_f^e + h_f^A \gamma_5 \right) e] \varphi_2 + [\bar{e} \left( i h_f^e + i h_f^H \gamma_5 \right) e] a$$

where three Hermitian and three anti-Hermitian Yukawa coupling matrices are

$$h_f^H \equiv \frac{h_f + h_f^\dagger}{2}, \quad h_f^A \equiv \frac{h_f - h_f^\dagger}{2},$$

with $$h_f$$ given in terms of the $$3 \times 3$$ Yukawa matrix $$y_f^2$$ and two unitary matrices as

$$h_f \equiv \frac{1}{\sqrt{2}} \mathcal{U}_{fL} y_f^2 \mathcal{U}_{fR}.$$ 

We observe that the couplings of the $$\varphi_1$$ field are diagonal in the flavor space and their sizes are directly proportional to the masses of the fermions to which it couples. In contrast, those of the $$\varphi_2$$ and $$a$$ fields are not diagonal in the flavor space leading to the tree-level Higgs-mediated FCNC and their magnitudes are arbitrary in principle.

To avoid the tree-level FCNC, the matrices $$h_{f=u,d,e}$$ are desired to be diagonal which is guaranteed by requiring

$$y_f^2 = \zeta_f y_f^1,$$

along with introducing the three complex alignment parameters $$\zeta_{f=u,d,e}$$. In this case, the two aligned Yukawa matrices $$y_1^f$$ and $$y_2^f$$ can be diagonalized simultaneously and the Yukawa matrices
describing the couplings of $\varphi_2$ and $a$ fields to the fermion mass eigenstates are given by

$$h_f = \zeta_f \frac{M_f}{v}, \quad (47)$$

which leads to the Hermitian and anti-Hermitian Yukawa matrices

$$h^H_f = \Re(\zeta_f) \frac{M_f}{v}, \quad h^A_f = i \Im(\zeta_f) \frac{M_f}{v}. \quad (48)$$

When $\Im(\zeta_f) = 0$, the conventional 2HDMs based on the Glashow-Weinberg condition [96] can be obtained by choosing $\zeta_f$ as shown in Table I. Otherwise, the couplings of the mass eigenstates of the neutral Higgs bosons $H_{i=1,2,3}$ to two fermions are given by

$$-L_{H_i \bar{f}f} = \sum_{i=1}^{3} \sum_{f=u,c,t,d,s,b,e,\mu,\tau} \frac{m_f}{v} \bar{f} \left( g^S_{H_i \bar{f}f} + i g^P_{H_i \bar{f}f} \gamma_5 \right) f \ H_i \quad (49)$$

with the scalar and pseudoscalar couplings given by

$$g^S_{H_i \bar{f}f} = O_{\varphi_1i} + \Re(\zeta_f) O_{\varphi_2i} \pm \Im(\zeta_f) O_{ai},$$

$$g^P_{H_i \bar{f}f} = \Im(\zeta_f) O_{\varphi_2i} \mp \Re(\zeta_f) O_{ai}, \quad (50)$$

where the upper and lower signs are for the up-type fermions $f = u, c, t$ and the down-type fermions $f = d, s, b, e, \mu, \tau$, respectively. The simultaneous existence of the scalar $g^S_{H_i \bar{f}f}$ and pseudoscalar $g^P_{H_i \bar{f}f}$ couplings signals the CP violation in the neutral Higgs sector. We figure out that there are two different sources of the neutral Higgs-sector CP violation: (i) one is the CP-violating mixing between the CP-even and CP-odd states arising in the presence of non-vanishing $\Im(Z_{5,6})$ in the Higgs potential and (ii) the other one is the complex alignment parameters of $\zeta_f$’s. Note that the second source is absent in the conventional four types of 2HDMs since $\zeta_f$’s are assigned to be real in those models.

The couplings of charged Higgs bosons to two fermions are given by

$$-L_{H^+ \bar{f}_1 f_2} = -\sqrt{2} \left[ \bar{\nu}_R(h^u_d V) d_L \right] H^+ + \sqrt{2} \left[ \bar{\nu}_L(V h_d) d_R \right] H^+ + \sqrt{2} \left[ \bar{\nu}_L h_e e_R \right] H^+ + \text{h.c.}. \quad (51)$$

in terms of the CKM matrix $V$ and the $3 \times 3$ Yukawa matrices $h_{u,d,e}$.

Incidentally, we note that the rotation $H_2 \rightarrow e^{i\zeta} H_2$ induces rephasing of the $y^f_2$ matrices such
TABLE I. Classification of the conventional 2HDMs satisfying the Glashow-Weinberg condition \[96\] which guarantees the absence of tree-level Higgs-mediated flavor-changing neutral current (FCNC). For the four types of 2HDM, we follow the conventions found in, for example, Ref. \[97\].

|      | 2HDM I | 2HDM II | 2HDM III | 2HDM IV |
|------|--------|---------|----------|---------|
| $\zeta_u$ | $1/t_\beta$ | $1/t_\beta$ | $1/t_\beta$ | $1/t_\beta$ |
| $\zeta_d$ | $1/t_\beta$ | $-t_\beta$ | $1/t_\beta$ | $-t_\beta$ |
| $\zeta_e$ | $1/t_\beta$ | $-t_\beta$ | $-t_\beta$ | $1/t_\beta$ |

$\zeta_d = \zeta_e = \zeta_u$  $\zeta_d = \zeta_e = -1/\zeta_u$  $\zeta_d = -1/\zeta_e = \zeta_u$  $\zeta_d = -1/\zeta_e = -1/\zeta_u$

as $y^f_2 \rightarrow e^{\pm i\zeta} y^f_2$ with the upper and lower signs being for the down-type massive fermions $f = d, e$ and the up-type massive fermions $f = u$, respectively, see Eq. (34). Therefore, the rotation leads to an overall rephasing of the $h_f$ matrices such as $h_u \rightarrow e^{-i\zeta} h_u$ and $h_{d,e} \rightarrow e^{+i\zeta} h_{d,e}$ and can eventually alter the Yukawa couplings $g_{H_i ff}^{S,P}$, see Eqs. (47) and (50).

The rotation $H_2 \rightarrow e^{i\zeta} H_2$ might induce the following transformation of the mixing matrix $O$ of the neutral Higgs bosons:

$$O \rightarrow O^T \zeta O \quad \text{with} \quad O_\zeta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\zeta & s_\zeta \\ 0 & -s_\zeta & c_\zeta \end{pmatrix},$$

(52)

under which the components of the mixing matrix $O$ change into

$$O_{\varphi_{1i}} \rightarrow O_{\varphi_{1i}} , \quad O_{\varphi_{2i}} \rightarrow c_\zeta O_{\varphi_{2i}} - s_\zeta O_{a_i} , \quad O_{ai} \rightarrow s_\zeta O_{\varphi_{2i}} + c_\zeta O_{ai} .$$

(53)

From Eq. (50), we observe that the above changes due to the rotation $H_2 \rightarrow e^{i\zeta} H_2$ are translated into the following corresponding redefinitions of the three alignment parameters $(f = u, d, e)$:

$$\Re(\zeta_f) \rightarrow c_\zeta \Re(\zeta_f) \pm s_\zeta \Im(\zeta_f) , \quad \Im(\zeta_f) \rightarrow c_\zeta \Im(\zeta_f) \mp s_\zeta \Re(\zeta_f) ,$$

(54)

which are nothing but the rephasing $\zeta_f \rightarrow e^{\pm i\zeta} \zeta_f$ with the upper and lower signs being for the up-type massive fermions $f = u$ and the down-type massive fermions $f = d, e$, respectively, see

11 This might be derived from the fact that $(H_1, H_2, H_3)^T = O^T (\varphi_1, \varphi_2, a)^T$ should remain the same under the rotation: see Eq. (33) which could be equivalently written as $(\varphi_1, \varphi_2, a)^T \rightarrow O^{T_f} (\varphi_1, \varphi_2, a)^T$. Or, equivalently, one may obtain this by observing that $M_0 \rightarrow O^T \zeta M_0 O_\zeta$ under the rotation while $\text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2) = O^T M_0^2 O$ should remain the same.
D. Interactions with Massive Vector Bosons

The cubic interactions of the neutral and charged Higgs bosons with the massive gauge bosons $Z$ and $W^\pm$ are described by the three interaction Lagrangians:

$$\mathcal{L}_{HVV} = g M^W \left( W^\mu_\mu W^{-\mu} + \frac{1}{2 c^2 W} Z^\mu Z^\mu \right) \sum_i g_{H_{iV}^V} H_i,$$

$$\mathcal{L}_{HHZ} = \frac{g}{2 c_W} \sum_{i>j} g_{H_iH_jZ} Z^\mu (H_i \overset{\leftrightarrow}{\partial^\mu} H_j),$$

$$\mathcal{L}_{HH^\pm W^\mp} = -\frac{g}{2} \sum_i g_{H_iH^+W^-} W^{-\mu} (H_i \overset{\leftrightarrow}{\partial^\mu} H^+) + \text{h.c.},$$

respectively, where $X \overset{\leftrightarrow}{\partial^\mu} Y = X\partial^\mu Y - (\partial^\mu X)Y$, $i, j = 1, 2, 3$ and the normalized couplings $g_{H_{iV}^V}$, $g_{H_iH_jZ}$ and $g_{H_iH^+W^-}$ are given in terms of the neutral Higgs-boson $3 \times 3$ mixing matrix $O$ by (note that $\det(O) = \pm 1$ for any orthogonal matrix $O$):

$$g_{H_{iV}^V} = O \varphi_i,$$

$$g_{H_iH_jZ} = \text{sign}[\det(O)] \epsilon_{ijk} g_{H_{kV}^V} = \text{sign}[\det(O)] \epsilon_{ijk} O \varphi_k,$$

$$g_{H_iH^+W^-} = -O \varphi_2 + iO a_i,$$

leading to the following sum rules:

$$\sum_{i=1}^3 g_{H_{iV}^V}^2 = 1 \quad \text{and} \quad g_{H_{iV}^V}^2 + |g_{H_iH^+W^-}|^2 = 1 \quad \text{for each } i = 1, 2, 3.$$

On the other hand, the quartic interactions of the neutral and charged Higgs bosons with the massive gauge bosons $Z$ and $W^\pm$ and massless photons are given by

$$\mathcal{L}_{HHVV} = \frac{1}{v^2} \left( M^2_W W^\mu_\mu W^{-\mu} + \frac{M^2_Z}{2} Z^\mu Z^\mu \right) \sum_{i,j=1}^3 g_{H_{iV}^V} H_i H_j,$$
with \( g_{H,H,V} = \delta_{ij} \) and

\[
\mathcal{L}_{H^+H^-VV} = \left( \frac{g^2}{2} W_\mu^+W_\mu^- + \frac{g_Z^2 c_w^2}{4} Z^\mu Z_\mu + e^2 A_\mu A_\mu + e g_Z c_{2W} A^\mu A_\mu \right) H^+H^-,
\]

\[
\mathcal{L}_{H^+HZW^+} = \frac{g_Z g s^2 W}{2} \left( Z_\mu W^- \mu - \sum_{i=1}^{3} g_{zw-H+h_i} H^+ H_i + \text{h.c.} \right),
\]

\[
\mathcal{L}_{H^+HAVW^+} = -\frac{e g}{2} \left( A_\mu W^- \mu - \sum_{i=1}^{3} g_{aw-H+h_i} H^+ H_i + \text{h.c.} \right),
\]

with \( g_{zw-H+h_i} = g_{aw-H+h_i} = -O_{\varphi 2i} - iO_{ai}, c_{2W} = \cos 2\theta_W, \) and \( g_Z = g/c_W = e/(s_W c_W) \).

### III. Constraints

In this Section, we consider the perturbative unitarity (UNIT) conditions and those for the Higgs potential to be bounded from below (BFB) to obtain the primary theoretical constraints on the potential parameters or, equivalently, the constraints on the Higgs-boson masses including correlations among them and the mixing among the three neutral Higgs bosons. We further consider the constraints on the Higgs masses and their couplings with vector bosons taking into account the electroweak oblique corrections to the so-called \( S \) and \( T \) parameters.

We emphasize that all the three types of constraints from the perturbative unitarity, the Higgs potential bounded from below, and the electroweak precision observables (EWPOs) are independent of the basis chosen and working in the Higgs basis does not invoke any restrictions.

#### A. Perturbative Unitarity

For the unitarity conditions, we closely follow Ref. \[98]\[12\] considering the three scattering matrices of \( \mathcal{M}_{1,2,3}^S \) which are expressed in terms of the quartic couplings \( Z_{1-7} \), see also Ref. \[99\]. The two \( 4 \times 4 \) real and symmetric scattering matrices \( \mathcal{M}_1^S \) and \( \mathcal{M}_2^S \) are given by

\[
\mathcal{M}_1^S = \begin{pmatrix}
\eta_{00} - I & \eta^T \\
\eta & E + I \times \mathbf{1}_{3 \times 3}
\end{pmatrix}; \quad \mathcal{M}_2^S = \begin{pmatrix}
3\eta_{00} - I & 3\eta^T \\
3\eta & 3E + I \times \mathbf{1}_{3 \times 3}
\end{pmatrix},
\]

\[12\] We keep our conventions for the potential parameters.
where \( \eta_{00} = Z_1 + Z_2 + Z_3 \) and \( I = Z_3 - Z_4 \). The row vector \( \eta^T \) is given by

\[
\eta^T = (\Re(Z_6 + Z_7), -\Im(Z_6 + Z_7), Z_1 - Z_2).
\]  

(61)

and the \( 3 \times 3 \) real and symmetric matrix \( E \) by

\[
E = \begin{pmatrix}
Z_4 + 2\Re(Z_5) & -2\Im(Z_5) & \Re(Z_6 - Z_7) \\
-2\Im(Z_5) & Z_4 - 2\Re(Z_5) & -\Im(Z_6 - Z_7) \\
\Re(Z_6 - Z_7) & -\Im(Z_6 - Z_7) & Z_1 + Z_2 - Z_3
\end{pmatrix}.
\]  

(62)

The third \( 3 \times 3 \) scattering matrix \( M_3^S \) is Hermitian which takes the form of

\[
M_3^S = \begin{pmatrix}
2Z_1 & 2Z_5 & \sqrt{2}Z_6 \\
2Z_5^* & 2Z_2 & \sqrt{2}Z_7^* \\
\sqrt{2}Z_6^* & \sqrt{2}Z_7 & Z_3 + Z_4
\end{pmatrix}.
\]  

(63)

And then, the unitarity conditions are imposed by requiring that the 11 eigenvalues of the three scattering matrices \( M_{1,2,3}^S \) and the quantity \( I \) should have their moduli smaller than \( 4\pi \).

When \( Z_6 = Z_7 = 0 \), the 12 unitarity conditions simplify into

\[
|Z_3 \pm Z_4| < 4\pi,
\]

\[
|Z_3 \pm 2|Z_5|| < 4\pi,
\]

\[
|Z_3 + 2Z_4 \pm 6|Z_5|| < 4\pi,
\]

\[
|Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + 4|Z_5|^2}| < 4\pi,
\]

\[
|Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + Z_4^2}| < 4\pi,
\]

\[
|3Z_1 + 3Z_2 \pm \sqrt{9(Z_1 - Z_2)^2 + (2Z_3 + Z_4)^2}| < 4\pi.
\]  

(64)

While taking \( Z_1 = Z_2 = Z_3 = Z_4 = Z_5 = 0 \), one may have

\[
\sqrt{|Z_6|^2 + |Z_7|^2} < 2\sqrt{2}\pi,
\]

\[
\sqrt{|Z_6|^2 + |Z_7|^2 + |Z_6^2 + Z_7^2|} < \frac{4\pi}{3}.
\]  

(65)

Then, by combining them, one may arrive at the following UNIT conditions for individual param-
\[ |Z_{1,2,5}| < 2\pi/3, \quad |Z_{6,7}| < 2\sqrt{2}\pi/3, \]
\[ |Z_3 - Z_4| < 4\pi \cup |2Z_3 + Z_4| < 4\pi \cup |Z_3 + 2Z_4| < 4\pi. \tag{66} \]

B. Higgs Potential Bounded-from-below

We consider the following 5 necessary conditions for the Higgs potential to be bounded-from-below in a marginal sense \[72, 98\]:

\[ Z_1 \geq 0, \quad Z_2 \geq 0; \]
\[ 2\sqrt{Z_1Z_2} + Z_3 \geq 0, \quad 2\sqrt{Z_1Z_2} + Z_3 + Z_4 - 2|Z_5| \geq 0; \]
\[ Z_1 + Z_2 + Z_3 + Z_4 + 2|Z_5| - 2|Z_6 + Z_7| \geq 0. \tag{67} \]

Note that though the quartic couplings \(Z_2\) and \(Z_7\) have no direct relations to the masses and mixing of Higgs bosons but they are interrelated with the other five quartic couplings of \(Z_{1,3-6}\) through the UNIT and BFB conditions as shown.

C. Electroweak Precision Observables

The electroweak oblique corrections to the so-called \(S, T\) and \(U\) parameters \[100, 101\] provide significant constraints on the quartic couplings of the 2HDM. Fixing \(U = 0\) which is suppressed by an additional factor \(M_Z^2/M_{NP}^2\) \[13\] compared to \(S\) and \(T\), the \(S\) and \(T\) parameters are constrained as follows

\[ \frac{(S - \hat{S}_0)^2}{\sigma_S^2} + \frac{(T - \hat{T}_0)^2}{\sigma_T^2} - 2\rho_{ST} \frac{(S - \hat{S}_0)(T - \hat{T}_0)}{\sigma_S\sigma_T} \leq R^2 (1 - \rho_{ST}^2), \tag{68} \]

with \(R^2 = 2.3, 4.61, 5.99, 9.21, 11.83 \) at 68.3\%, 90\%, 95\%, 99\%, and 99.7\% confidence levels (CLs), respectively. For our numerical analysis, we adopt the 95\% CL limits. The central values

\[^{13}\] The marginal stability bound means that it comes with the equality sign.

\[^{14}\] Here, \(M_{NP}\) denotes the heavy mass scale involved with New Physics (NP).
and their standard deviations are given by \[15\]

\[
\left( \widetilde{S}_0, \sigma_S \right) = (0.00, 0.07), \quad \left( \widetilde{T}_0, \sigma_T \right) = (0.05, 0.06), \quad \tag{69}
\]

with a strong correlation \( \rho_{ST} = 0.92 \) between \( S \) and \( T \) parameters. The electroweak oblique parameters, which are defined to arise from new physics (NP) only, are in excellent agreement with the SM values of zero for the reference values of \( M_{H_{SM}} = 125.25 \text{ GeV} \) and \( M_t = 172.5 \text{ GeV} \) \[102\].

In 2HDM, the \( S \) and \( T \) parameters might be estimated using the following expressions \[103, 104\]

\[
S_\Phi = -\frac{1}{4\pi} \left[ (1 + \delta_{tZ})^2 F'_\Delta(M_{H^\pm}, M_{H^\pm}) - \sum_{(i,j)=(1,2)} (g_{H, H, j} + \delta_{Z, H}^T)^2 F'_{\Delta}(M_{H_i}, M_{H_j}) \right], \quad \tag{70}
\]

\[
T_\Phi = -\frac{\sqrt{2} G_F}{16 \pi^2 \alpha_{EM}} \left[ -\sum_{i=1}^{3} |g_{H, W, W^+} + \delta_{H}^T|^2 F_{\Delta}(M_i, M_{H^\pm}) + \sum_{(i,j)=(1,2)} (g_{H, H, j} + \delta_{Z, H}^T)^2 F_{\Delta}(M_{H_i}, M_{H_j}) \right].
\]

We observe that \( g_{H, W, W^+}^2 = |\epsilon_{ijk}| g_{H_{k, W, W^+}}^2 = |\epsilon_{ijk}| O^2_{\varphi_{i,2}} \text{ and } \frac{g_{H, W, W^+}}{m_{W}^2} = 1 - g_{H_{k, W, W^+}}^2 = 1 - O^2_{\varphi_{i,2}} \) and, therefore, all the relevant couplings are determined by the three physical couplings of \( g_{H_{k, W, W^+}} \). In this work, we ignore the vertex corrections \( \delta_{tZ}^T, \delta_{Z, H}^T, \text{ and } \delta_{Z}^T \) since the size of the most of the quartic couplings are smaller than 3 and the quantum corrections proportional to \( \sim Z^2/16 \pi^2 \) are negligible. On the other hand, the one-loop functions are given by \[16\]

\[
F_{\Delta}(m_0, m_1) = F_{\Delta}(m_1, m_0) = \frac{m_0^2 + m_1^2}{2} - \frac{m_0^2 m_1^2}{m_0^2 - m_1^2} \ln \frac{m_0^2}{m_1^2},
\]

\[
F'_{\Delta}(m_0, m_1) = F'_{\Delta}(m_1, m_0) = -\frac{1}{3} \left[ \frac{4}{3} - \frac{m_0^2 \ln m_0^2 - m_1^2 \ln m_1^2}{m_0^2 - m_1^2} - \frac{m_0^2 + m_1^2}{(m_0^2 - m_1^2)^2} F_{\Delta}(m_0, m_1) \right]. \quad \tag{71}
\]

We note that \( F_{\Delta}(m, m) = 0 \) and \( F'_{\Delta}(m, m) = \frac{1}{3} \ln m^2 \). \[17\] When \( g_{H_{k, W, W^+}}^2 = 1 \), neglecting the \( Z^2 \)-dependent vertex correction factors \( \delta_{tZ}^T, \delta_{W}^T \text{ and } \delta_{Z, H}^T \), \( S_\Phi \) and \( T_\Phi \) are symmetric under the exchange \( M_{H_2} \leftrightarrow M_{H_3} \) and they are identically vanishing when \( M_{H_2} = M_{H_3} = M_{H^\pm} \). \[18\]

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\[15\] See the 2020 edition of the review “10. Electroweak Model and Constraints on New Physics” by J. Erler and A. Freitas in Ref. \[102\].

\[16\] See, for example, Ref. \[105\].

\[17\] Here and after, \( \ln m^2 \) could be understood as, for example, \( \ln [m^2 / (1 \text{ GeV})^2] \) if necessary.

\[18\] The \( S_\Phi \) and \( T_\Phi \) parameters are independent of \( M_{H_i} \) when \( g_{H_{k, W, W^+}}^2 = 1 \).
IV. NUMERICAL ANALYSIS

From the relation $g_{H_1VV} = O_{\varphi_1V}$ given in Eq. (56) and the expressions for the $H_i$ couplings to the two SM fermions given in Eq. (50), one might define the delay factor $\Delta_{H_1\bar{f}f}$ by the amount of which the decoupling of the Yukawa couplings of the lightest Higgs boson is delayed compared to its coupling to a pair of massive vector bosons:

$$\Delta_{H_1\bar{f}f} \equiv \sqrt{\left(g_{H_1\bar{f}f}^S - g_{H_1VV}^S\right)^2 + \left(g_{H_1\bar{f}f}^P\right)^2} = |\zeta_f| \left(1 - g_{H_1VV}^2\right)^{1/2}, \quad (72)$$

where we use the relation $\sum_{\alpha=\varphi_1,\varphi_2,a} O_{\alpha i}^2 = 1$ for $i = 1$. Therefore, anticipating that the impacts due to the CP-violating phases of $Z_{5,6,7}$ and $\zeta_{u,d,e}$ on the alignment of Yukawa couplings are redundant, we consider the CP-conserving (CPC) case for our numerical study for simplicity.

A. UNIT and BFB constraints

First of all, we consider the UNIT and BFB constraints. Observing that the two conditions depend only on the quartic couplings $Z_{1-7}$, we take the following set of input parameters:

$$\mathcal{I}_{\text{CPC}}^Z = \{v, Y_2; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}. \quad (73)$$

In the left panel of Fig. 1, we show the scatter plots of $Z_2$ versus $Z_1$ (upper left), $Z_4$ versus $Z_3$ (upper right), $Z_5$ versus $Z_1$ (lower left), and $Z_7$ versus $Z_6$ (lower right). The plots are produced by randomly generating the quartic couplings in the $\mathcal{I}_{\text{CPC}}^Z$ set. In each plot, the black points are obtained by imposing only the simplified UNIT conditions of Eqs. (64) and (65). The full consideration of the UNIT conditions based on the scattering matrices $\mathcal{M}^{S}_{1,2,3}$ produces the red points. The results obtained by simultaneously imposing the full UNIT and BFB conditions (UNIT⊕BFB) are denoted by the blue points. After imposing the UNIT and UNIT⊕BFB conditions, we note that the normalized distributions of the quartic couplings are no longer flat as shown in the right panel of Fig. 1. As in the left panel, the distributions of the quartic couplings obtained by requiring only the UNIT (red) conditions and the combined UNIT⊕BFB conditions are in red and blue, respectively. We note that the smaller $|Z_6 + Z_7|$ and the positive $Z_3$ values are preferred by further imposing the BFB conditions in addition to the UNIT ones, see Eq. (67).

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19 To have $\mathcal{I}_{\text{CPC}}^Z$ from Eq. (18), we trade $M_{H\pm}$ with $Z_3$. 

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22
FIG. 1. The UNIT and BFB constraints using $I_{\text{CPC}}^\prime$, see Eq. (73): (Left) Scatter plots (red) of $|Z_5|$ versus $|Z_1|$ (upper left), $|Z_7|$ versus $|Z_6|$ (upper right), and $Z_3$ versus $Z_4$ with the unitarity conditions imposed. For the blue points, the necessary BFB conditions additionally imposed. Also shown are the points in black which are obtained by requiring only the simplified conditions in Eqs. (64) and (65). (Right) The normalized distributions of the quartic couplings obtained by requiring only the UNIT (red) and the combined UNIT$\oplus$BFB conditions.

B. Electroweak constraints

Coming to the electroweak (ELW) constraints, since the oblique corrections are expressed in terms of the masses and couplings of Higgs bosons, it is more natural and convenient to take the following set of input parameters:

$$I_{\text{CPC}}^\prime = \{v; M_{H^\pm}, M_h = M_{H_1}, M_H, M_A, \gamma; Z_3; Z_2, Z_7\} ,$$  \hfill (74)

referring to Eq. (23). In the $I_{\text{CPC}}^\prime$ set, all the massive parameters are physical Higgs masses except $v = (\sqrt{2}G_F)^{-1/2} \simeq 246.22$ GeV. We assume that the neutral state $h$ is the lightest Higgs boson and plays the role of the SM Higgs boson in the decoupling limit of $s_\gamma = 0$ by taking $M_{H_1} = 125.5$ GeV \cite{73}. And, for the masses of heavy Higgs bosons, we randomly generate their masses squared between $M_{H_1}^2$ and $(1.5 \text{ TeV})^2$. For the mixing angle $\gamma$, we take the convention of $|\gamma| \leq \pi/2$ without loss of generality resulting in $c_\gamma \geq 0$ and sign$(s_\gamma) = \text{sign}(Z_6)$. For the implementation of the UNIT and BFB constraints using the set $I_{\text{CPC}}^\prime$, we recall that the quartic couplings $Z_{1,4,5,6}$ are given by Eq. (22) in terms of the Higgs masses and the mixing angle $\gamma$ in the CPC case.

Using the set $I_{\text{CPC}}^\prime$ for the input parameters in the CPC case, Eq. (70) for the $S$ and $T$
parameters takes the following simpler form:

\[
S^\text{CPC}_\Phi = -\frac{1}{4\pi} \left[ F'_\Delta(M_{H^\pm}, M_{H^\pm}) - c^2_\gamma F'_\Delta(M_A, M_H) - s^2_\gamma F'_\Delta(M_A, M_h) \right],
\]

\[
T^\text{CPC}_\Phi = \frac{\sqrt{2}G_F}{16\pi^2\alpha_{\text{EM}}} \left[ F_\Delta(M_A, M_{H^\pm}) + c^2_\gamma F_\Delta(M_H, M_{H^\pm}) + s^2_\gamma F_\Delta(M_h, M_{H^\pm}) \right.
\]

\[
- c^2_\gamma F_\Delta(M_A, M_H) - s^2_\gamma F_\Delta(M_A, M_h) \right],
\]

(75)

ignoring the vertex corrections. We observe that \(T^\text{CPC}_\Phi\) is identically vanishing when \(M_{H^\pm} = M_A\) and, when \(M_{H^\pm} \sim M_A \sim M_H \gg M_h\), we obtain\(^{20}\)

\[
S^\text{CPC}_\Phi \simeq -\frac{1}{4\pi} \left[ \frac{\ln M^2_{H^\pm}}{3} - c^2_\gamma \left( \frac{\ln M^2_A}{3} + \frac{M_H - M_A}{3M_A} \right) - s^2_\gamma \left( \frac{\ln M^2_A}{3} - \frac{5}{18} \right) \right],
\]

\[
T^\text{CPC}_\Phi \simeq \frac{\sqrt{2}G_F}{16\pi^2\alpha_{\text{EM}}} \left[ \frac{2(M_A - M_{H^\pm})^2}{3} + c^2_\gamma \frac{2(M_H - M_{H^\pm})^2}{3} + s^2_\gamma \frac{M^2_{H^\pm}}{2} \right.
\]

\[
- c^2_\gamma \frac{2(M_A - M_H)^2}{3} - s^2_\gamma \frac{M^2_A}{2} \right],
\]

(76)

keeping the leading terms. To obtain Eq. (76) for the approximated expressions of the \(S\) and \(T\) parameters, we use

\[
F_\Delta(m_0, m_1) = \frac{2(m_0 - m_1)^2}{3} - \frac{(m_0 - m_1)^4}{30 m_1^2} + \mathcal{O} \left[ \frac{(m_0 - m_1)^5}{m_1^3} \right],
\]

\[
F'_\Delta(m_0, m_1) = \frac{\ln m_1^2}{3} + \frac{(m_0 - m_1)}{3m_1} - \frac{(m_0 - m_1)^2}{30 m_1^2} + \mathcal{O} \left[ \frac{(m_0 - m_1)^3}{m_1^3} \right],
\]

(77)

for \(m_0 \sim m_1\) and

\[
F_\Delta(m_0, m_1) = \frac{m_1^2}{2} + \left( \frac{1}{2} + \ln \frac{m_0^2}{m_1^2} \right) m_0^2 + \mathcal{O} \left[ \left( \frac{m_0^4}{m_1^4} \right) \ln \frac{m_0^2}{m_1^2} \right],
\]

\[
F'_\Delta(m_0, m_1) = \frac{\ln m_1^2}{3} - \frac{5}{18} + \frac{2m_0^2}{3m_1^2} + \mathcal{O} \left[ \left( \frac{m_0^4}{m_1^4} \right) \ln \frac{m_0^2}{m_1^2} \right],
\]

(78)

for \(m_1 \gg m_0\).

In the left panel of Fig. 2 we show the \(S\) and \(T\) parameters imposing the UNIT, BFB, and ELW constraints abbreviated by UNIT⊕BFB⊕ELW\(_{95\%}\). Note that the 95\% CL ELW limits are adopted and the heavy Higgs masses are scanned up to 1.5 TeV. We find that \(S\) takes values in the range between \(-0.02\) and \(0.05\) whose absolute values are smaller than \(\sigma_S = 0.07\), see Eq. (69). Actually,

\(^{20}\) For \(S\), note that \[\ln \frac{M^2_A}{3} + (M_H - M_A)/3M_A\] \[\ln \frac{M^2_H}{3} + (M_A - M_H)/3M_H\] \(\simeq (M_H - M_A)^3/9M^3_A\).
we find that $|S| < \sigma_S$ even with only the UNIT and BFB constraints imposed. Note that $S$ is mostly negative (positive) when $M_{H^\pm} > (<) M_A$. Specifically, we find that $S \simeq -1/4\pi (5/18) \simeq -0.02$ when $M_{H^\pm} - M_A = 0$ and $\gamma = \pi/2$. The $T$ parameter takes its value between $-0.02$ and $0.13$ which are given by the delimited range determined by $-0.02 < S < 0.05$, the strong correlation $\rho_{ST} = 0.92$ and $R_{95\%}^2 = 5.99$, see Eqs. $[68]$ and $[69]$ and the lower-right plot in the left panel of Fig. [2]. Incidentally, we observe that $T = 0$ when $M_{H^\pm} = M_A$ though it quickly deviates from $0$ when $M_{H^\pm} \neq M_A$. In the right panel of Fig. [2] we show the correlations among the mass differences and the mixing angle $\gamma$ using the set $\mathcal{I}'_{CPC}$. We find that

$$|M_H - M_A|/\text{GeV} \lesssim 200 \,(100) , \quad |M_{H^\pm} - M_H|/\text{GeV} \lesssim 200 \,(110) ,$$

$$|M_{H^\pm} - M_A|/\text{GeV} \lesssim 200 \,(110) , \quad |\gamma| \lesssim 0.8 \,(0.14) ,$$

when $M_{H^\pm} \gtrsim 500$ GeV (1 TeV).

We show the correlations among the heavy Higgs-boson masses and the mixing angle $\gamma$ in the
FIG. 3. The \( \text{UNIT} \oplus \text{BFB} \oplus \text{ELW}_{95\%} \) constraints (magenta) using \( I_{\text{CPC}} \), see Eq. (74). For comparisons, we also show the results after applying only the \( \text{UNIT} \oplus \text{BFB} \) constraints (blue). (Left) Scatter plots of \( M_A \) versus \( M_H \) (upper left), \( M_{H^\pm} \) versus \( M_H \) (upper right), \( M_{H^\pm} \) versus \( M_A \) (lower left), and \( M_{H^\pm} \) versus \( \gamma \) (lower right). (Right) The normalized distributions of the quartic couplings and the mixing angle \( \gamma \).

left panel of Fig. 3. Requiring the ELW constraint in addition to the \( \text{UNIT} \oplus \text{BFB} \) ones, we find that \( Z_1 \) and \( \gamma \) take values near to 0 more likely and \( Z_4 \) and \( Z_5 \) positive ones, see the right panel of Fig. 3. We find that the \( \text{UNIT} \) and BFB conditions combined with the ELW constraint restrict the quartic couplings as follows:

\[
0.1 \lesssim Z_1 \lesssim 2.0, \quad 0 \lesssim Z_2 \lesssim 2.1, \quad -2.4 \lesssim Z_3 \lesssim 8.0, \quad -6.3 \lesssim Z_4 \lesssim 6.0, \\
-1.9 \lesssim Z_5 \lesssim 1.6, \quad -2.7 \lesssim Z_6 \lesssim 2.7, \quad -2.7 \lesssim Z_7 \lesssim 2.7. \quad (80)
\]

C. Alignment of Yukawa couplings

At last, we have come to the point to address the main issue of this work or the alignment of Yukawa couplings. When we talk about the alignment of the Yukawa couplings in general 2HDMs, we imply: (i) the alignment of them in the flavor space and (ii) the alignment of the lightest Higgs-boson couplings to a pair of the SM fermions in the decoupling limit of \( M_{H,A,H^\pm} \to \infty \). By (i), we precisely mean the assumption that the two Yukawa matrices of \( y_1^f \) and \( y_2^f \) are aligned in the
FIG. 4. (Left) Scatter plots of $\zeta_u$ versus $\gamma$ (upper left), $\zeta_d$ versus $\gamma$ (upper right), and $\zeta_e$ versus $\gamma$ (lower left) obtained by scanning $-\pi/2 \leq \gamma \leq \pi/2$ and the three real parameters of the set $\mathcal{I}^\prime_{CPC}$ in the ranges of $-2 < \zeta_u < 2$ and $-10 < \zeta_{d,e} < 10$. On each $\zeta_f$-$\gamma$ plane, the regions satisfying $|g^{S}_{H_{1}ff} - 1| < 0.1$ and $|g^{S}_{H_{1}ff} + 1| < 0.1$ are denoted in red and blue, respectively. (Right) Scatter plot of $\zeta_d$ versus $\zeta_e$ with $1/100 < \zeta_u < 2$. The four lines represent the four conventional 2HDMs as denoted.

flavor space or $y_2^f = \zeta_f y_1^f$, see Eq. (46), which, in the CPC case, leads to

$$g^{S}_{H_{1}ff} = O_{\varphi_{1}1} + \zeta_f O_{\varphi_{2}1} = c_{\gamma} - \zeta_f s_{\gamma},$$

(81)

with $f = u$ and $d$ for the up- and down-type quarks, respectively, and $f = e$ for the three charged leptons. Then, by $(ii)$, one might mean

$$g^{S}_{H_{1}ff} \to 1 \quad \text{as} \quad M_{H,A,H^{\pm}} \to \infty.$$

(82)

In Eq. (81), we note that the quantity $c_{\gamma}$ is nothing but the coupling $g_{H_{1}VV} = O_{\varphi_{1}1} = c_{\gamma}$ which is driven to take the SM value of 1 by the combined UNIT, BFB, and ELW constraints as $M_{H,A,H^{\pm}}$ increases. Therefore, the alignment of the lightest Higgs-boson couplings to the SM fermions in the decoupling limit is delayed by the amount of $\zeta_f s_{\gamma}$, which can be not ignored even when $|s_{\gamma}| \ll 1$ if $|\zeta_f|$ is significantly larger than 1.

For a quantitative study, in addition to ${\mathcal{I}}_{CPC}^\prime$ given by Eq. (74), we have added the following
The choice of \( \zeta \) within 10% range of the SM value of 1 or \( \zeta \approx 1 \) except the type-I 2HDM, \( g_{H_1,gg} \) between 1 and 2. Otherwise, at least one of them is limitless in principle. Therefore, except the type-I 2HDM, \( g_{H_1,gg} \) and/or \( g_{H_1,ee} \) could be largely deviated from 1 in the decoupling limit even when \( \zeta_u \) is limited.

To concentrate on the alignment of the lightest Higgs-boson couplings to a pair of the SM fermions in the decoupling limit of \( M_{H,A,H^\pm} \to \infty \) under the assumption of \( y_2^f \propto y_1^f \) as in Eq. (46), we consider a simplified scenario in which the mixing angle \( |\sin \gamma| \) is inversely proportional to \( 1/M_{H^\pm}^2 \) reflecting the behavior of \( |\sin \gamma| = |g_{HVV}| \) being suppressed by the quartic powers of the heavy Higgs-boson masses at leading order [106]. In the upper-left frame of Fig. 5, we show the scatter plot for \( |\gamma| \) versus \( M_{H^\pm} \) together with the three curves showing the cases of \( \sin \gamma = (125 \text{ GeV}/M_{H^\pm})^2 \) (black), \( \sin \gamma = (200 \text{ GeV}/M_{H^\pm})^2 \) (red), and \( \sin \gamma = (350 \text{ GeV}/M_{H^\pm})^2 \) (blue) from bottom to top. The input parameters are the same as in Fig. 3 and the combined UNIT\( \oplus \)BFB\( \oplus \)ELW\(_{95\%}\) constraints are imposed. For our study, we take the case of \( \sin \gamma = (200 \text{ GeV}/M_{H^\pm})^2 \). The coupling of the lightest Higgs boson \( H_1 \) to a pair of massive vector bosons are constrained by the precision LHC Higgs data [46]. We note that, for example, \( \cos \gamma = g_{HVV} \gtrsim 0.95 \) or \( |\sin \gamma| = |g_{HVV}| \lesssim 0.3 \) can be satisfied when \( M_{H^\pm} \gtrsim 400 \text{ GeV} \) for this choice. We further assume that the masses of the heavy Higgs bosons of \( H, A, \) and \( H^\pm \) are degenerate. This assumption reflects the fact that the combined UNIT \( \oplus \) BFB \( \oplus \) ELW\(_{95\%}\) constraints prefers quite degenerate heavy-Higgs bosons when they weigh more than about 400 GeV as shown

\[ \mathcal{I}_{\text{CPC}} = \{ \zeta_u, \zeta_d, \zeta_e \} . \] (83)

In the left panel of Fig. 4, we show the correlations between each of the three alignment parameters \( \zeta_{f=u,d,e} \) and the mixing angle \( \gamma \) when the absolute value of the corresponding coupling \( g_{H_1,gg} \) is within 10% range of the SM value of 1 or \( \zeta_{f=u,d,e} \approx 1 \) \( \zeta_{f=u,d,e} \approx 1 \). Scanning \( |\gamma| \leq \pi/2, \) \( g_{H_1,gg} \approx 1 \) near \( \gamma = 0 \). When \( \gamma = \pm \pi/2, \) it takes the value of 1 when \( \zeta_{f=u,d,e} \approx 1 \). We also note \( g_{H_1,gg} = -1 \) if \( \gamma = \pm \pi/2 \) and \( \zeta_{f=u,d,e} \approx 1 \). In the right panel of Fig. 4 by the four lines, we show the correlations between \( \zeta_d \) and \( \zeta_e \) in the four conventional 2HDMs\( ^{21} \) based on appropriately defined discrete \( Z_2 \) symmetries taking 1/100 \( \zeta_u = 1/t_\beta < 2 \), see Table I. We observe that both \( \zeta_d \) and \( \zeta_e \) are bounded only in the type-I 2HDM between 1/100 and 2. Otherwise, at least one of them is limitless in principle. Therefore, except the type-I 2HDM, \( g_{H_1,gg} \) and/or \( g_{H_1,ee} \) could be largely deviated from 1 in the decoupling limit even when \( \zeta_u \) is limited.

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\(^{21}\) The parameters \( \zeta_d \) and \( \zeta_e \) are completely uncorrelated in the general 2HDM based on the relation Eq. (46) as shown by the scattered black dots in the right panel of Fig. 4.

\(^{22}\) The choice of \( \sin \gamma = (m_6/M_{H^\pm})^2 \) is equivalent to fix \( Z_6 = (M_H^2 - m_h^2) \cos \gamma \sin \gamma/v^2 \sim (m_6/v)^2 \) when \( M_H \sim M_{H^\pm} \gg M_h \) and \( \sin(\gamma) \ll 1 \), see Eq. (22).
FIG. 5. (Upper Left) The same as in the lower-right plot in the left panel of Fig. 3 but for $|\gamma|$ versus $M_{H\pm}$ with the UNIT$\oplus$BFB$\oplus$ELW$95\%$ constraints imposed. The three curves show the cases of $\sin \gamma = (125 \text{ GeV}/M_{H\pm})^2$ (black), $\sin \gamma = (200 \text{ GeV}/M_{H\pm})^2$ (red), and $\sin \gamma = (350 \text{ GeV}/M_{H\pm})^2$ (blue) from bottom to top. (Upper Right) Scatter plot of $g_{S_{H_1\bar{u}u}}$ versus $M_{H\pm}$ taking SCN200, see Eq. (84). The blue (red) lines are for $\cos \gamma + (-)\zeta_u |\sin \gamma|$ for $\zeta_u = 2, 1, 0.2$ from the outermost lines to the magenta one which is for $g_{H_1V\bar{V}V} = \cos \gamma$. (Lower Left) Scatter plot of $g_{S_{H_1\bar{d}d}}$ versus $M_{H\pm}$ taking SCN200. The blue (red) lines are for $\cos \gamma + (-)\zeta_d |\sin \gamma|$ for $\zeta_d = 100, 50, 20, 10, 0.5$ from the outermost lines to the magenta one which is for $g_{H_1V\bar{V}V} = \cos \gamma$. (Lower Right) The same as in the lower-left plot but for $g_{S_{H_1\bar{e}e}}$ versus $M_{H\pm}$ with the lines for $\cos \gamma \pm |\zeta_e \sin \gamma|$ for $\zeta_e = 100, 50, 20, 10, 0.5$.  

in the left panel of Fig. 3. We dub this scenario SCN200 in which we precisely fix and vary the
input parameters in the two sets of $\mathcal{I}_{\text{CPC}}$ and $\mathcal{I}_{\text{CPC}}^\zeta$ as follows:

$$\text{SCN200} : \{M_h = M_{H_1} = 125.5 \text{ GeV}, M_H = M_A = M_{H^\pm} = [200..1500] \text{ GeV};$$
$$\sin \gamma = \pm (200 \text{ GeV}/M_{H^\pm})^2; Z_2 = [0..2], Z_3 = [-3..8], Z_7 = [-3..3]\}$$
$$\oplus \{\zeta_u = [1/100..2], \zeta_d = [-100..100], \zeta_e = [-100..100]\},$$

(equation 84)

together with the combined UNIT $\oplus$ BFB $\oplus$ ELW\text{95\%} constraints imposed.

In the upper-right frame of Fig. 5, we show the scatter plot of $g_{H_1\bar{u}u}^S$ versus $M_{H^\pm}$ taking SCN200. We observe that the coupling $g_{H_1\bar{u}u}^S$ is within about 30% and 10% ranges of the SM value of 1 when $M_{H^\pm} > 500$ GeV and $M_{H^\pm} > 1$ TeV, respectively. As previously discussed, the alignment of the coupling $g_{H_1ff}^S$ is delayed by the amount of $\zeta_f \sin \gamma$ compared to the coupling $g_{H_1VV}^S$ and $g_{H_1\bar{u}u}^S$ is most deviated from its SM value of 1 when $|\zeta_u| = |\zeta_u|^{\text{MAX}} = 2$. To make this point clear, we add the blue and red lines showing $g_{H_1\bar{u}u}^S$ taking $\zeta_u = 0.2, 1,$ and $2$ and the magenta one showing $g_{H_1VV}^S$. We indeed see that $g_{H_1\bar{u}u}^S$ is most close to $g_{H_1VV}^S$ when $\zeta_u = 0.2$ and the two lines taking $\zeta_u = 2$ provide the envelope which includes all the scattered points. In the lower frames of Fig. 5, the scatter plots of $g_{H_1\bar{d}d}^S$ versus $M_{H^\pm}$ (left) and $g_{H_1\bar{e}e}^S$ versus $M_{H^\pm}$ (right) are shown. They are basically the same since $\zeta_d$ and $\zeta_e$ are varied in the same range of $[-100, 100]$. And the same arguments are applied as in the case of $g_{H_1\bar{u}u}^S$; they are most close to $g_{H_1VV}^S$ when $\zeta_{d,e} = 0.5$ among the blue and red lines and the lines taking $|\zeta_{d,e}| = |\zeta_{d,e}|^{\text{MAX}} = 100$ provide the envelopes which include all the scattered points. We see that $g_{H_1\bar{d}d}^S$ and $g_{H_1\bar{e}e}^S$ can be largely deviated from their SM values of 1 when $|\zeta_{d,e}|$ is large. For example, when $M_{H^\pm} = 1.5$ TeV, $|g_{H_1\bar{d}d}^S|$ and $|g_{H_1\bar{e}e}^S|$ can deviate from 1 by the amount of $|\zeta_{d,e}^{\text{MAX}} \sin \gamma| = 100 \left(\frac{200}{1500}\right)^2 \simeq 1.8$.

Of course, the alignment parameters $\zeta_{d,e}$ are constrained by the precision LHC Higgs data. From the observation that the absolute values of the couplings of the SM-like $H_1$ to a pair of bottom quarks and tau leptons are required to be consistent with 1 within about 10% at 1$\sigma$ level \text{46}, one might have $|g_{H_1\bar{d}d}^S \pm 1| \lesssim 0.1$ and $|g_{H_1\bar{e}e}^S \pm 1| \lesssim 0.1$. \text{23} For the positive sign, the condition $|g_{H_1\bar{d}d}^S - 1| < 0.1$ constrains $|\zeta_d| \lesssim 6$, see the red points in the left panel of Fig. 6. On the other hand $g_{H_1\bar{d}d}^S \sim -1$ allows larger values of $\zeta_d$ given by $\zeta_d = (1 + \cos \gamma) / \sin \gamma \simeq \pm 2 M_\pm^2 / (200 \text{ GeV})^2$, see the blue points in the left panel of Fig. 6. The same arguments are applied for $g_{H_1\bar{e}e}^S$, see the right panel of Fig. 6.

\text{23} The negative value of $g_{H_1\bar{d}d}^S \sim -1$ is less preferred than the positive one $g_{H_1\bar{d}d}^S \sim +1$ at the level of about 1.5$\sigma$ considering the $b$-quark loop contributions to the $H_1$ coupling to two gluons \text{46}. While, for $g_{H_1\bar{e}e}^S$, the current data precision is yet insufficient to tell its sign. In this work, we consider both signs for $g_{H_1\bar{d}d}^S$ and $g_{H_1\bar{e}e}^S$. 

30
Lastly, we comment on the wrong-sign alignment limit in the four types of conventional 2HDMs in which the $H_1$ couplings to the down-type quarks and/or those to the charged leptons are equal in strength but opposite in sign to the corresponding SM ones. The two couplings $g_{H_1\bar{d}d}^S$ and $g_{H_1\bar{e}e}^S$ are completely independent from each other in general 2HDM. But, in the conventional four types of 2HDMs, they are related. We observe that the couplings are given by either $\cos\gamma - \sin\gamma/t_\beta$ or $\cos\gamma + t_\beta\sin\gamma$ in any type of 2HDMs, see Table I. In this case, $\cos\gamma - \sin\gamma/t_\beta = \pm 1$ for the $t_\beta$ value which makes $\cos\gamma + t_\beta\sin\gamma = \mp 1$. This implies that, independently of 2HDM type and regardless of the heavy Higgs-mass scale, all four types of 2HDMs could be viable against the LHC Higgs precision data in the wrong-sign alignment limit.

V. CONCLUSIONS

We have studied the alignment of Yukawa couplings in the framework of general 2HDMs identifying the lightest neutral Higgs boson as the 125 GeV one discovered at the LHC. We take the so-called Higgs basis [70, 71, 84-88] for the Higgs potential in which only one of the two doublets contains the non-vanishing vacuum expectation value $v$. For the Yukawa couplings, rather than invoking the Glashow-Weinberg condition [96] based on appropriately defined discrete $Z_2$ symme-
tries, we merely require the absence of tree-level FCNCs \[89–91\] by assuming that the Yukawa matrices describing the couplings of the two Higgs doublets to the SM fermions are aligned in the flavor space.

For a numerical study, we further assume that the seven quartic couplings $Z_{i=1−7}$ appearing in the Higgs potential and the three alignment parameters $\zeta_{f=u,d,e}$ for Yukawa couplings are all real by anticipating that the impacts due to CP-violating phases of $Z_{5,6,7}$ and $\zeta_f$’s on the alignment of Yukawa couplings are redundant. In this case, in addition to the vev $v$ and masses of the SM fermions, the model can be fully described by specifying the following set of 11 free parameters:

$$\{Y_2; M_h = M_{H_1}, M_H, M_A, M_{H^\pm}; \gamma; Z_2, Z_7; \zeta_u, \zeta_d, \zeta_e\},$$

where $M_{h,H}$ and $M_A$ denote the masses of CP-even and CP-odd neutral Higgs bosons with $M_h < M_H$ respectively, the mixing between the two CP-even neutral states is described by the angle $\gamma$. The quartic couplings $Z_{1,4,5,6}$ are determined in terms of $M_{h,H,A}$, $M_{H^\pm}$, $\gamma$, and $v$. And, instead of the massive parameter $Y_2$, one may adopt $Z_3$ by noting the relation $Y_2 = M_{H^\pm}^2 - Z_3 v^2/2$. The quartic couplings $Z_2$ and $Z_7$ with no direct relations to the masses and mixing of Higgs bosons can be measured only through the cubic and quartic Higgs self-couplings. But, we observe that they are interrelated with the other five quartic couplings of $Z_{1,3,6}$ through the perturbative unitarity (UNIT) conditions and those for being bounded from below (BFB). We note that the UNIT and BFB conditions are basis-independent, i.e., the same in any basis \[98\]. Also considered are the constraints from the electroweak (ELW) oblique corrections to the $S$ and $T$ parameters which are expressed in terms of the physical observable quantities of $M_{h,H,A}$, $M_{H^\pm}$, and $g_{H,VV}$ which are again invariant under a change of basis \[81\].

We summarize our major findings as follows:

1. By scanning the heavy Higgs masses up to 1.5 TeV, we find that the UNIT and BFB conditions combined with the ELW constraint restrict the quartic couplings as follows:

$$0.1 \lesssim Z_1 \lesssim 2.0, \quad 0 \lesssim Z_2 \lesssim 2.1, \quad -2.4 \lesssim Z_3 \lesssim 8.0, \quad -6.3 \lesssim Z_4 \lesssim 6.0,$$
$$-1.9 \lesssim Z_5 \lesssim 1.6, \quad -2.7 \lesssim Z_6 \lesssim 2.7, \quad -2.7 \lesssim Z_7 \lesssim 2.7.$$  \[85\]

24 For a numerical analysis, we fix $M_h = 125.5$ GeV and use the quartic coupling $Z_3$ instead of $Y_2$ by exploiting the relation $Y_2 = M_{H^\pm}^2 - Z_3 v^2/2$.
And, when $M_{H^\pm} \gtrsim 500$ GeV (1 TeV), we also find that

$$
|M_H - M_A|/\text{GeV} \lesssim 200\ (100), \quad |M_{H^\pm} - M_H|/\text{GeV} \lesssim 200\ (110),
$$

$$
|M_{H^\pm} - M_A|/\text{GeV} \lesssim 200\ (110), \quad |\gamma| \lesssim 0.8\ (0.14).
$$

2. As the masses of heavy Higgs bosons increase, compared to the $g_{H_1 V V}$ coupling of the lightest Higgs boson to a pair of massive vector bosons, the decoupling of the Yukawa couplings to the lightest Higgs boson is delayed by the amount of $|\zeta_f| (1 - g_{H_1 V V}^2)^{1/2}$. Therefore, though $g_{H_1 V V}$ approaches its SM value of 1 very quickly as the masses of heavy Higgs bosons increase, the coupling of $H_1$ to a pair of fermions can significantly deviate from its SM value if $|\zeta_f|$ is large. Note that $|\zeta_f|$ is constrained to be small by the LHC precision Higgs data when the corresponding Yukawa coupling is with the similar strength and the same sign as the SM one. But it could be larger when the Yukawa coupling takes the wrong sign.

3. In the type-I 2HDM, the 3 alignment parameters are the same $\zeta_u = \zeta_d = \zeta_e = 1/t_\beta$ and cannot be significantly large than 1 since $t_\beta \ll 1$ leads to a non-perturbative top-quark Yukawa coupling and a Landau pole close to the TeV scale. Therefore, in the type-I model among the 4 conventional 2HDMs, all the Yukawa couplings of the lightest Higgs boson most quickly approach the corresponding SM values as the masses of the heavy neutral Higgs bosons increase and their decouplings are least delayed.

4. The wrong-sign alignment, in which the $H_1$ couplings to a pair of $f$-type fermions are equal in strength but opposite in sign to the corresponding SM ones, occurs when $\zeta_f = (1 + \cos \gamma) / \sin \gamma$ independently of the heavy Higgs-boson masses. We observe that, in the wrong-sign alignment limit, any type of conventional 2HDMs is viable against the LHC Higgs precision data since $\cos \gamma + (1/\zeta_f) \sin \gamma = +1$ when $\cos \gamma - \zeta_f \sin \gamma = -1$ with $\zeta_f = -t_\beta$ or $1/t_\beta$.

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[1] G. Aad et al. [ATLAS], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B 716 (2012), 1-29 doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan et al. [CMS], “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” Phys. Lett. B 716 (2012), 30-61 doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[3] D. Carmi, A. Falkowski, E. Kuflik and T. Volansky, “Interpreting LHC Higgs Results from Natural New Physics Perspective,” JHEP 07 (2012), 136 doi:10.1007/JHEP07(2012)136 [arXiv:1202.3144 [hep-ph]].

[4] A. Azatov, R. Contino and J. Galloway, “Model-Independent Bounds on a Light Higgs,” JHEP 04 (2012), 127 [erratum: JHEP 04 (2013), 140] doi:10.1007/JHEP04(2012)127 [arXiv:1202.3415 [hep-ph]].

[5] J. R. Espinosa, C. Grojean, M. Muhlleitner and M. Trott, “Fingerprinting Higgs Suspects at the LHC,” JHEP 05 (2012), 097 doi:10.1007/JHEP05(2012)097 [arXiv:1202.3697 [hep-ph]].

[6] M. Klute, R. Lafaye, T. Plehn, M. Rauch and D. Zerwas, “Measuring Higgs Couplings from LHC Data,” Phys. Rev. Lett. 109 (2012), 101801 doi:10.1103/PhysRevLett.109.101801 [arXiv:1205.2699 [hep-ph]].

[7] D. Carmi, A. Falkowski, E. Kuflik and T. Volansky, “Interpreting the 125 GeV Higgs,” Nuovo Cim. C 035 (2012) no.6, 315-322 doi:10.1393/ncc/i2012-11386-2 [arXiv:1206.4201 [hep-ph]].

[8] I. Low, J. Lykken and G. Shaughnessy, “Have We Observed the Higgs (Imposter)?,” Phys. Rev. D 86 (2012), 093012 doi:10.1103/PhysRevD.86.093012 [arXiv:1207.1093 [hep-ph]].

[9] P. P. Giardino, K. Kannike, M. Raidal and A. Strumia, “Is the resonance at 125 GeV the Higgs boson?,” Phys. Lett. B 718 (2012), 469-474 doi:10.1016/j.physletb.2012.10.042 [arXiv:1207.1347 [hep-ph]].

[10] J. Ellis and T. You, “Global Analysis of the Higgs Candidate with Mass “ 125 GeV,” JHEP 09 (2012), 123 doi:10.1007/JHEP09(2012)123 [arXiv:1207.1693 [hep-ph]].
[11] J. R. Espinosa, C. Grojean, M. Muhlleitner and M. Trott, “First Glimpses at Higgs’ face,” JHEP 12 (2012), 045 doi:10.1007/JHEP12(2012)045 [arXiv:1207.1717 [hep-ph]].
[12] D. Carmi, A. Falkowski, E. Kuflik, T. Volansky and J. Zupan, “Higgs After the Discovery: A Status Report,” JHEP 10 (2012), 196 doi:10.1007/JHEP10(2012)196 [arXiv:1207.1718 [hep-ph]].
[13] S. Banerjee, S. Mukhopadhyay and B. Mukhopadhyaya, “New Higgs interactions and recent data from the LHC and the Tevatron,” JHEP 10 (2012), 062 doi:10.1007/JHEP10(2012)062 [arXiv:1207.3588 [hep-ph]].
[14] F. Bonnet, T. Ota, M. Rauch and W. Winter, “Interpretation of precision tests in the Higgs sector in terms of physics beyond the Standard Model,” Phys. Rev. D 86 (2012), 093014 doi:10.1103/PhysRevD.86.093014 [arXiv:1207.4599 [hep-ph]].
[15] T. Plehn and M. Rauch, “Higgs Couplings after the Discovery,” EPL 100 (2012) no.1, 11002 doi:10.1209/0295-5075/100/11002 [arXiv:1207.6108 [hep-ph]].
[16] A. Djouadi, “Precision Higgs coupling measurements at the LHC through ratios of production cross sections,” Eur. Phys. J. C 73 (2013), 2498 doi:10.1140/epjc/s10052-013-2498-3 [arXiv:1208.3436 [hep-ph]].
[17] B. A. Dobrescu and J. D. Lykken, “Coupling spans of the Higgs-like boson,” JHEP 02 (2013), 073 doi:10.1007/JHEP02(2013)073 [arXiv:1210.3342 [hep-ph]].
[18] G. Cacciapaglia, A. Deandrea, G. Drieu La Rochelle and J. B. Flament, “Higgs couplings beyond the Standard Model,” JHEP 03 (2013), 029 doi:10.1007/JHEP03(2013)029 [arXiv:1210.8120 [hep-ph]].
[19] G. Belanger, B. Dumont, U. Ellwanger, J. F. Gunion and S. Kraml, “Higgs Couplings at the End of 2012,” JHEP 02 (2013), 053 doi:10.1007/JHEP02(2013)053 [arXiv:1212.5244 [hep-ph]].
[20] G. Moreau, “Constraining extra-fermion(s) from the Higgs boson data,” Phys. Rev. D 87 (2013) no.1, 015027 doi:10.1103/PhysRevD.87.015027 [arXiv:1210.3977 [hep-ph]].
[21] T. Corbett, O. J. P. Eboli, J. Gonzalez-Fraile and M. C. Gonzalez-Garcia, “Constraining anomalous Higgs interactions,” Phys. Rev. D 86 (2012), 075013 doi:10.1103/PhysRevD.86.075013 [arXiv:1207.1344 [hep-ph]].
[22] T. Corbett, O. J. P. Eboli, J. Gonzalez-Fraile and M. C. Gonzalez-Garcia, “Robust Determination of the Higgs Couplings: Power to the Data,” Phys. Rev. D 87 (2013), 015022 doi:10.1103/PhysRevD.87.015022 [arXiv:1211.4580 [hep-ph]].
[23] E. Massó and V. Sanz, “Limits on anomalous couplings of the Higgs boson to electroweak gauge bosons from LEP and the LHC,” Phys. Rev. D 87 (2013) no.3, 033001 doi:10.1103/PhysRevD.87.033001 [arXiv:1211.1320 [hep-ph]].

[24] K. Cheung, J. S. Lee and P. Y. Tseng, “Higgs Precision (Higgcision) Era begins,” JHEP 05 (2013), 134 doi:10.1007/JHEP05(2013)134 [arXiv:1302.3794 [hep-ph]].

[25] K. Cheung, J. S. Lee and P. Y. Tseng, “Higgs precision analysis updates 2014,” Phys. Rev. D 90 (2014), 095009 doi:10.1103/PhysRevD.90.095009 [arXiv:1407.8236 [hep-ph]].

[26] G. Aad et al. [ATLAS and CMS Collaborations], “Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at \(\sqrt{s} = 7\) and \(8\) TeV,” JHEP 1608, 045 (2016), [arXiv:1606.02266 [hep-ex]].

[27] The ATLAS collaboration [ATLAS Collaboration], “Measurements of Higgs boson properties in the diphoton decay channel using 80 fb\(^{-1}\) of pp collision data at \(\sqrt{s} = 13\) TeV with the ATLAS detector,” ATLAS-CONF-2018-028.

[28] A. M. Sirunyan et al. [CMS], “Measurements of Higgs boson properties in the diphoton decay channel in proton-proton collisions at \(\sqrt{s} = 13\) TeV,” JHEP 11 (2018), 185 doi:10.1007/JHEP11(2018)185 [arXiv:1804.02716 [hep-ex]].

[29] The ATLAS collaboration [ATLAS Collaboration], “Measurements of the Higgs boson production, fiducial and differential cross sections in the 4\(\ell\) decay channel at \(\sqrt{s} = 13\) TeV with the ATLAS detector,” ATLAS-CONF-2018-018.

[30] CMS Collaboration [CMS Collaboration], “Measurements of properties of the Higgs boson in the four-lepton final state at \(\sqrt{s} = 13\) TeV,” CMS-PAS-HIG-18-001.

[31] The ATLAS collaboration [ATLAS Collaboration], “Measurement of gluon fusion and vector boson fusion Higgs boson production cross-sections in the \(H \rightarrow WW^* \rightarrow e\mu\nu\nu\) decay channel in pp collisions at \(\sqrt{s} = 13\) TeV with the ATLAS detector,” ATLAS-CONF-2018-004.

[32] CMS Collaboration [CMS Collaboration], “Measurements of properties of the Higgs boson decaying to a W boson pair in pp collisions at \(\sqrt{s} = 13\) TeV,” CMS-PAS-HIG-16-042.

[33] M. Aaboud et al. [ATLAS], “Observation of \(H \rightarrow b\bar{b}\) decays and \(VH\) production with the ATLAS detector,” Phys. Lett. B 786 (2018), 59-86 doi:10.1016/j.physletb.2018.09.013 [arXiv:1808.08238 [hep-ex]].

[34] CMS Collaboration [CMS Collaboration], “Combined measurements of the Higgs boson’s couplings at \(\sqrt{s} = 13\) TeV,” CMS-PAS-HIG-17-031.
[35] A. M. Sirunyan et al. [CMS], “Observation of Higgs boson decay to bottom quarks,” Phys. Rev. Lett. 121 (2018) no.12, 121801 doi:10.1103/PhysRevLett.121.121801 [arXiv:1808.08242 [hep-ex]].

[36] The ATLAS collaboration [ATLAS Collaboration], “Cross-section measurements of the Higgs boson decaying to a pair of tau leptons in proton-proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” ATLAS-CONF-2018-021.

[37] CMS Collaboration [CMS Collaboration], “Search for the standard model Higgs boson decaying to a pair of $\tau$ leptons and produced in association with a W or a Z boson in proton-proton collisions at $\sqrt{s} = 13$ TeV,” CMS-PAS-HIG-18-007.

[38] M. Aaboud et al. [ATLAS], “Observation of Higgs boson production in association with a top quark pair at the LHC with the ATLAS detector,” Phys. Lett. B 784 (2018), 173-191 doi:10.1016/j.physletb.2018.07.035 [arXiv:1806.00425 [hep-ex]].

[39] M. Aaboud et al. [ATLAS Collaboration], “Evidence for the associated production of the Higgs boson and a top quark pair with the ATLAS detector,” Phys. Rev. D 97, no. 7, 072003 (2018), [arXiv:1712.08891 [hep-ex]].

[40] M. Aaboud et al. [ATLAS], “Search for the standard model Higgs boson produced in association with top quarks and decaying into a $b\bar{b}$ pair in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” Phys. Rev. D 97 (2018) no.7, 072016 doi:10.1103/PhysRevD.97.072016 [arXiv:1712.08895 [hep-ex]].

[41] A. M. Sirunyan et al. [CMS], “Evidence for associated production of a Higgs boson with a top quark pair in final states with electrons, muons, and hadronically decaying $\tau$ leptons at $\sqrt{s} = 13$ TeV,” JHEP 08 (2018), 066 doi:10.1007/JHEP08(2018)066 [arXiv:1803.05485 [hep-ex]].

[42] A. M. Sirunyan et al. [CMS], “Search for $t\bar{t}H$ production in the all-jet final state in proton-proton collisions at $\sqrt{s} = 13$ TeV,” JHEP 06 (2018), 101 doi:10.1007/JHEP06(2018)101 [arXiv:1803.06986 [hep-ex]].

[43] A. M. Sirunyan et al. [CMS], “Search for $t\bar{t}H$ production in the $H \to b\bar{b}$ decay channel with leptonic $t\bar{t}$ decays in proton-proton collisions at $\sqrt{s} = 13$ TeV,” JHEP 03 (2019), 026 doi:10.1007/JHEP03(2019)026 [arXiv:1804.03682 [hep-ex]].

[44] A. M. Sirunyan et al. [CMS], “Combined measurements of Higgs boson couplings in proton–proton collisions at $\sqrt{s} = 13$ TeV,” Eur. Phys. J. C 79 (2019) no.5, 421 doi:10.1140/epjc/s10052-019-6909-y [arXiv:1809.10733 [hep-ex]].
[45] G. Aad et al. [ATLAS], “Combined measurements of Higgs boson production and decay using up to 80 fb$^{-1}$ of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment,” Phys. Rev. D 101 (2020) no.1, 012002 doi:10.1103/PhysRevD.101.012002 [arXiv:1909.02845 [hep-ex]].

[46] K. Cheung, J. S. Lee and P. Y. Tseng, “New Emerging Results in Higgs Precision Analysis Updates 2018 after Establishment of Third-Generation Yukawa Couplings,” JHEP 09 (2019), 098 doi:10.1007/JHEP09(2019)098 [arXiv:1810.02521 [hep-ph]].

[47] For a recent review, see, for example,
M. Khlopov, “What comes after the Standard Model?,” Prog. Part. Nucl. Phys. 116 (2021), 103824 doi:10.1016/j.ppnp.2020.103824

[48] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, “The Higgs Hunter’s Guide,” Front. Phys. 80 (2000), 1-404 SCIPP-89/13.

[49] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, “Errata for the Higgs hunter’s guide,” [arXiv:hep-ph/9302272 [hep-ph]].

[50] M. Carena and H. E. Haber, “Higgs Boson Theory and Phenomenology,” Prog. Part. Nucl. Phys. 50 (2003), 63-152 doi:10.1016/S0146-6410(02)00177-1 [arXiv:hep-ph/0208209 [hep-ph]].

[51] A. Djouadi, “The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model,” Phys. Rept. 457 (2008), 1-216 doi:10.1016/j.physrep.2007.10.004 [arXiv:hep-ph/0503172 [hep-ph]].

[52] A. Djouadi, “The Anatomy of electro-weak symmetry breaking. II. The Higgs bosons in the minimal supersymmetric model,” Phys. Rept. 459 (2008), 1-241 doi:10.1016/j.physrep.2007.10.005 [arXiv:hep-ph/0503173 [hep-ph]].

[53] E. Accomando, A. G. Akeroyd, E. Akhmetzyanova, J. Albert, A. Alves, N. Amapane, M. Aoki, G. Azuelos, S. Baffioni and A. Ballestrero, et al. “Workshop on CP Studies and Non-Standard Higgs Physics,” doi:10.5170/CERN-2006-009 [arXiv:hep-ph/0608079 [hep-ph]].

[54] S. Dittmaier et al. [LHC Higgs Cross Section Working Group], “Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables,” doi:10.5170/CERN-2011-002 [arXiv:1101.0593 [hep-ph]].

[55] S. Dittmaier, C. Mariotti, G. Passarino, R. Tanaka, S. Alekhin, J. Alwall, E. A. Bagnaschi, A. Banfi, J. Blumlein and S. Bolognesi, et al. “Handbook of LHC Higgs Cross Sections: 2. Differential Distributions,” doi:10.5170/CERN-2012-002 [arXiv:1201.3084 [hep-ph]].
[56] S. Heinemeyer et al. [LHC Higgs Cross Section Working Group], “Handbook of LHC Higgs Cross Sections: 3. Higgs Properties,” doi:10.5170/CERN-2013-004 [arXiv:1307.1347 [hep-ph]].

[57] D. de Florian et al. [LHC Higgs Cross Section Working Group], “Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector,” doi:10.2172/1345634, 10.23731/CYRM-2017-002 [arXiv:1610.07922 [hep-ph]].

[58] S. Dawson, A. Gritsan, H. Logan, J. Qian, C. Tully, R. Van Kooten, A. Ajaib, A. Anastassov, I. Anderson and D. Asner, et al. “Working Group Report: Higgs Boson,” [arXiv:1310.8361 [hep-ex]].

[59] M. Spira, “QCD effects in Higgs physics,” Fortsch. Phys. 46 (1998), 203-284 doi:10.1002/(SICI)1521-3978(199804)46:3<203::AID-PROP203>3.0.CO;2-4 [arXiv:hep-ph/9705337 [hep-ph]].

[60] M. Spira, “Higgs Boson Production and Decay at Hadron Colliders,” Prog. Part. Nucl. Phys. 95 (2017), 98-159 doi:10.1016/j.ppnp.2017.04.001 [arXiv:1612.07651 [hep-ph]].

[61] S. Dawson, C. Englert and T. Plehn, “Higgs Physics: It ain’t over till it’s over,” Phys. Rept. 816 (2019), 1-85 doi:10.1016/j.physrep.2019.05.001 [arXiv:1808.01324 [hep-ph]].

[62] S. Y. Choi, J. S. Lee and J. Park, “Decays of Higgs Bosons in the Standard Model and Beyond,” Prog. Part. Nucl. Phys. 120 (2021), 103880 doi:10.1016/j.ppnp.2021.103880 [arXiv:2101.12435 [hep-ph]].

[63] T. D. Lee, “A Theory of Spontaneous T Violation,” Phys. Rev. D 8 (1973), 1226-1239 doi:10.1103/PhysRevD.8.1226.

[64] T. D. Lee, “CP Nonconservation and Spontaneous Symmetry Breaking,” Phys. Rept. 9 (1974), 143-177 doi:10.1016/0370-1573(74)90020-9.

[65] R. D. Peccei and H. R. Quinn, “CP Conservation in the Presence of Instantons,” Phys. Rev. Lett. 38 (1977), 1440-1443 doi:10.1103/PhysRevLett.38.1440.

[66] P. Fayet, “A Gauge Theory of Weak and Electromagnetic Interactions with Spontaneous Parity Breaking,” Nucl. Phys. B 78 (1974), 14-28 doi:10.1016/0550-3213(74)90113-8.

[67] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, “Low-Energy Parameters and Particle Masses in a Supersymmetric Grand Unified Model,” Prog. Theor. Phys. 67 (1982), 1889 doi:10.1143/PTP.67.1889.

[68] R. A. Flores and M. Sher, “Higgs Masses in the Standard, Multi-Higgs and Supersymmetric Models,” Annals Phys. 148 (1983), 95 doi:10.1016/0003-4916(83)90331-7.
[69] J. F. Gunion and H. E. Haber, “Higgs Bosons in Supersymmetric Models. 1.,” Nucl. Phys. B 272 (1986), 1 [erratum: Nucl. Phys. B 402 (1993), 567-569] doi:10.1016/0550-3213(86)90340-8.

[70] F. J. Botella and J. P. Silva, “Jarlskog - like invariants for theories with scalars and fermions,” Phys. Rev. D 51 (1995), 3870-3875 doi:10.1103/PhysRevD.51.3870 [arXiv:hep-ph/9411288 [hep-ph]].

[71] G. C. Branco, L. Lavoura and J. P. Silva, “CP Violation,” Int. Ser. Monogr. Phys. 103 (1999), 1-536, Chapters 22 and 23.

[72] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, “Theory and phenomenology of two-Higgs-doublet models,” Phys. Rept. 516 (2012), 1-102 doi:10.1016/j.physrep.2012.02.002 [arXiv:1106.0034 [hep-ph]].

[73] A. M. Sirunyan et al. [CMS], “A measurement of the Higgs boson mass in the diphoton decay channel,” Phys. Lett. B 805 (2020), 135425 doi:10.1016/j.physletb.2020.135425 [arXiv:2002.06398 [hep-ex]].

[74] H. E. Haber and Y. Nir, “Multiscalar Models With a High-energy Scale,” Nucl. Phys. B 335 (1990), 363-394 doi:10.1016/0550-3213(90)90499-4.

[75] J. F. Gunion and H. E. Haber, “The CP conserving two Higgs doublet model: The Approach to the decoupling limit,” Phys. Rev. D 67 (2003), 075019 doi:10.1103/PhysRevD.67.075019 [arXiv:hep-ph/0207010 [hep-ph]].

[76] N. Craig, J. Galloway and S. Thomas, “Searching for Signs of the Second Higgs Doublet,” [arXiv:1305.2424 [hep-ph]].

[77] M. Carena, I. Low, N. R. Shah and C. E. M. Wagner, “Impersonating the Standard Model Higgs Boson: Alignment without Decoupling,” JHEP 04 (2014), 015 doi:10.1007/JHEP04(2014)015 [arXiv:1310.2248 [hep-ph]].

[78] P. S. Bhupal Dev and A. Pilaftsis, “Maximally Symmetric Two Higgs Doublet Model with Natural Standard Model Alignment,” JHEP 12 (2014), 024 [erratum: JHEP 11 (2015), 147] doi:10.1007/JHEP12(2014)024 [arXiv:1408.3405 [hep-ph]].

[79] J. Bernon, J. F. Gunion, H. E. Haber, Y. Jiang and S. Kraml, “Scrutinizing the alignment limit in two-Higgs-doublet models: \( m_h = 125 \) GeV,” Phys. Rev. D 92 (2015) no.7, 075004 doi:10.1103/PhysRevD.92.075004 [arXiv:1507.00933 [hep-ph]].

[80] M. Carena, H. E. Haber, I. Low, N. R. Shah and C. E. M. Wagner, “Complementarity between Nonstandard Higgs Boson Searches and Precision Higgs Boson Measurements in the MSSM,” Phys. Rev. D 91 (2015) no.3, 035003 doi:10.1103/PhysRevD.91.035003 [arXiv:1410.4969 [hep-ph]].
[81] B. Grzadkowski, H. E. Haber, O. M. Ogreid and P. Osland, “Heavy Higgs boson decays in the alignment limit of the 2HDM,” JHEP 12 (2018), 056 doi:10.1007/JHEP12(2018)056 arXiv:1808.01472 [hep-ph]].

[82] S. Kanemura, M. Kubota and K. Yagyu, “Aligned CP-violating Higgs sector canceling the electric dipole moment,” JHEP 08 (2020), 026 doi:10.1007/JHEP08(2020)026 arXiv:2004.03943 [hep-ph]].

[83] I. Low, N. R. Shah and X. P. Wang, “Higgs Alignment and Novel CP-Violating Observables in 2HDM,” arXiv:2012.00773 [hep-ph]].

[84] J. F. Donoghue and L. F. Li, “Properties of Charged Higgs Bosons,” Phys. Rev. D 19 (1979), 945 doi:10.1103/PhysRevD.19.945

[85] H. Georgi and D. V. Nanopoulos, “Suppression of Flavor Changing Effects From Neutral Spinless Meson Exchange in Gauge Theories,” Phys. Lett. B 82 (1979), 95-96 doi:10.1016/0370-2693(79)90433-7

[86] S. Davidson and H. E. Haber, “Basis-independent methods for the two-Higgs-doublet model,” Phys. Rev. D 72 (2005), 035004 [erratum: Phys. Rev. D 72 (2005), 099902] doi:10.1103/PhysRevD.72.099902 arXiv:hep-ph/0504050 [hep-ph]].

[87] H. E. Haber and D. O’Neil, “Basis-independent methods for the two-Higgs-doublet model. II. The Significance of tanβ,” Phys. Rev. D 74 (2006), 015018 [erratum: Phys. Rev. D 74 (2006) no.5, 059905] doi:10.1103/PhysRevD.74.015018 arXiv:hep-ph/0602242 [hep-ph]].

[88] R. Boto, T. V. Fernandes, H. E. Haber, J. C. Romão and J. P. Silva, “Basis-independent treatment of the complex 2HDM,” Phys. Rev. D 101 (2020) no.5, 055023 doi:10.1103/PhysRevD.101.055023 arXiv:2001.01430 [hep-ph]].

[89] A. V. Manohar and M. B. Wise, “Flavor changing neutral currents, an extended scalar sector, and the Higgs production rate at the CERN LHC,” Phys. Rev. D 74 (2006), 035009 doi:10.1103/PhysRevD.74.035009 arXiv:hep-ph/0606172 [hep-ph]].

[90] A. Pich and P. Tuzon, “Yukawa Alignment in the Two-Higgs-Doublet Model,” Phys. Rev. D 80 (2009), 091702 doi:10.1103/PhysRevD.80.091702 arXiv:0908.1554 [hep-ph]].

[91] A. Peñuelas and A. Pich, “Flavour alignment in multi-Higgs-doublet models,” JHEP 12 (2017), 084 doi:10.1007/JHEP12(2017)084 arXiv:1710.02040 [hep-ph]].

[92] P. M. Ferreira, J. F. Gunion, H. E. Haber and R. Santos, “Probing wrong-sign Yukawa couplings at the LHC and a future linear collider,” Phys. Rev. D 89 (2014) no.11, 115003 doi:10.1103/PhysRevD.89.115003 arXiv:1403.4736 [hep-ph]].
[93] P. M. Ferreira, R. Guedes, M. O. P. Sampaio and R. Santos, “Wrong sign and symmetric limits and non-decoupling in 2HDMs,” JHEP 12 (2014), 067 doi:10.1007/JHEP12(2014)067 [arXiv:1409.6723 [hep-ph]].

[94] A. Biswas and A. Lahiri, “Alignment, reverse alignment, and wrong sign Yukawa couplings in two Higgs doublet models,” Phys. Rev. D 93 (2016) no.11, 115017 doi:10.1103/PhysRevD.93.115017 [arXiv:1511.07159 [hep-ph]].

[95] N. M. Coyle, B. Li and C. E. M. Wagner, “Wrong sign bottom Yukawa coupling in low energy supersymmetry,” Phys. Rev. D 97 (2018) no.11, 115028 doi:10.1103/PhysRevD.97.115028 [arXiv:1802.09122 [hep-ph]].

[96] S. L. Glashow and S. Weinberg, “Natural Conservation Laws for Neutral Currents,” Phys. Rev. D 15 (1977) 1958.

[97] K. Cheung, J. S. Lee and P. Y. Tseng, “Higgcision in the Two-Higgs Doublet Models,” JHEP 01 (2014), 085 doi:10.1007/JHEP01(2014)085 [arXiv:1310.3937 [hep-ph]].

[98] D. Jurčiukonis and L. Lavoura, “The three- and four-Higgs couplings in the general two-Higgs-doublet model,” JHEP 12 (2018), 004 doi:10.1007/JHEP12(2018)004 [arXiv:1807.04244 [hep-ph]].

[99] S. Kanemura and K. Yagyu, “Unitarity bound in the most general two Higgs doublet model,” Phys. Lett. B 751 (2015), 289-296 doi:10.1016/j.physletb.2015.10.047 [arXiv:1509.06060 [hep-ph]].

[100] M. E. Peskin and T. Takeuchi, “A New constraint on a strongly interacting Higgs sector,” Phys. Rev. Lett. 65 (1990), 964-967 doi:10.1103/PhysRevLett.65.964

[101] M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys. Rev. D 46 (1992), 381-409 doi:10.1103/PhysRevD.46.381

[102] P. A. Zyla et al. [Particle Data Group], “Review of Particle Physics,” PTEP 2020 (2020) no.8, 083C01 doi:10.1093/ptep/ptaa104

[103] D. Toussaint, “Renormalization Effects From Superheavy Higgs Particles,” Phys. Rev. D 18 (1978), 1626 doi:10.1103/PhysRevD.18.1626

[104] J. S. Lee and A. Pilaftsis, “Radiative Corrections to Scalar Masses and Mixing in a Scale Invariant Two Higgs Doublet Model,” Phys. Rev. D 86 (2012), 035004 doi:10.1103/PhysRevD.86.035004 [arXiv:1201.4891 [hep-ph]].

[105] S. Kanemura, Y. Okada, H. Taniguchi and K. Tsumura, “Indirect bounds on heavy scalar masses of the two-Higgs-doublet model in light of recent Higgs boson searches,” Phys. Lett. B 704 (2011), 303-307 doi:10.1016/j.physletb.2011.09.035 [arXiv:1108.3297 [hep-ph]].
[106] S. Y. Choi, J. S. Lee and J. Park, “Alignment of Yukawa couplings in two Higgs doublet models,” arXiv:2011.04978 [hep-ph].