State-of-charge estimation technique for lithium-ion batteries by means of second-order extended Kalman filter and equivalent circuit model: Great temperature robustness state-of-charge estimation

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Abstract
The present work focuses on the state-of-charge (SOC) estimation of a lithium-ion battery in terms of a second-order extended Kalman filter (EKF). First, an equivalent circuit model is introduced to describe the performance of lithium-ion batteries. The model parameters are then identified through hybrid pulse power characterization experiments conducted over a wide range of temperatures (−10 to 55°C). A two-dimensional mathematical relationship is established with respect to the SOC and temperature based on a dual-fifth polynomial expression. The main effects and sensitivities of the SOC and temperature on the parameters are analysed according to the principle of variance analysis and partial derivatives. An estimation algorithm is developed, which combines the two-dimensional parameter model and second-order EKF. Finally, the proposed approach is validated compared to other estimation schemes through discharge experiments under extreme temperatures and dynamic loading profiles, which yields experimental results that estimate the SOC with an absolute error of less than 4.5% under harsh conditions. This not only demonstrates that it can characterize dependency of the model parameters on the operating conditions and address the uncertainty of model parameters, but also verifies the advantage of present method at low temperatures especially at sub-zero temperatures.

1 INTRODUCTION

Electric vehicles mainly represent cars without CO₂ emissions due to the use of power batteries instead of gasoline. Lithium-ion batteries are extensively employed as power batteries for electric vehicles, on account of their long lifespan, high energy density, and high efficiency [1,2]. To present precise information in relation to the battery states, it is essential to develop an effective battery management system (BMS) for lithium-ion batteries. This particular system can increase service efficiency and extend the service life of batteries, thus reducing the operating costs and increasing the reliability of battery packs. The BMS involves three main measurable parameters: voltage, temperature, and current. The state of charge (SOC), cell consistency, and state of health as additional vital information, can be obtained from these parameters by estimation techniques [3,4], or the co-estimation of SOC and state of health [5]. The estimation of SOC plays a key role in the appropriate operation and control of batteries’ performance, safety, and reliability. However, the accurate estimation of SOC remains challenging because of the non-linear time-varying characteristics and the electrochemical reactions in lithium-ion batteries.

Previous studies on SOC estimation algorithms have provided a complete assessment of model construction and battery SOC estimation approaches, as reviewed in detail in [6] and [7]. In addition to the conventional ampere-hour counting...
technique [8,9] and open-circuit voltage (OCV) technique [10–13], filter algorithms have been proposed, such as Kalman filter (KF) [14], extended Kalman filter (EKF) [15,16], unscented Kalman filter (UKF) [17,18], sigma point KF [19,20], particle filter [21,22], and H filter [23], as well as machine learning approaches, that is, neural network [24,25], fuzzy logic [26,27], genetic algorithm [28] and data-driven method based on Gaussian process regression [29]. The conventional methods are in general simple, direct and feasible. However, the ampere-hour counting method involves an open-loop estimation, which increases the estimation error because of the accumulated error originated from current measurement noise, while the battery is left unused for a long time (more than 1 h) for restoring the terminal voltage to the OCV. Exploring machine learning methods for SOC estimation are promising, however, unsuitable for error estimation with poor interpretability. Thus, the most commonly utilized SOC estimation technique is the filter algorithm, which typically estimates the OCV or the SOC from the current and voltage through the equivalent circuit model (ECM) where the circuit parameters can be identified either offline or online.

One of the challenges in utilizing an ECM is that the model parameters depend on the operating conditions of batteries, such as SOC and temperature. For instance, the internal ohmic resistance of a LiFePO4 cell is almost doubled when the temperature decreases from 25 to 0°C [30], which may significantly degrade battery performance. This is why battery heating technology has been developed to make sure that the battery works at suitable temperature under low ambient temperature, even sub-zero [31].

On the one hand, the dependency of the model parameters on the operating conditions can be characterized by a look-up table (LUT). Feng et al. [32] employed a 201 × 41-point LUT of the OCV versus temperature and SOC and an adaptive joint EKF to estimate the SOC at various operating temperatures. Xing et al. [33] also adopted an OCV–SOC–temperature LUT and UKF for SOC estimation at various temperatures. This approach although takes into account the effect of temperature on the internal resistance and OCV of batteries, it cannot completely reflect the influence of temperature on the polarization effects. However, only few studies have established a two-dimensional mathematical relationship of all the model parameters with respect to temperature and SOC. Moreover, most studies only considered the OCV versus temperature and SOC, despite the fact that the LUT undertake much more storage space than the coefficients of a mathematical relationship.

On the other hand, several algorithms with online parameter identification are proposed to accommodate condition changes. He et al. [16] proposes an online estimation method of the OCV, internal resistance and other model parameters based on EKF. A multiple adaptive forgetting factors recursive least-squares method is used to capture the real-time parameters accurately by Duong et al. [34]. Li et al. [35] establish a second-order ECM for cell used in electric vehicles, and a method of recursive least squares is used to update the model parameters according to fuzzy adaptive variable weighting forgetting factor. Zhang et al. [36] present a decoupled weighted recursive least-squares method for online estimation of both the ECM parameters and battery SOC. In addition, the aforementioned online parameter identification methods can be biased under noise corruptive conditions. Wei et al. [37] propose a novel parameterization method combining instrumental variable estimation and a bilinear principle is proposed to compensate for the noise-induced biases of model identification. These methods can effectively address the variability and uncertainty of model parameters, but it usually takes a certain period to obtain stable results when identifying model parameters, which depends on the complexity of the identification method used.

Therefore, it is worth considering an approach via the combination of offline and online technique, which identifies all the model parameters offline, supplemented by parameter adjustment and correction from online identification periodically or irregularly as needed for the effect of ageing. As the first step, a practicable estimation method needs to be established, which can characterize dependency of the model parameters on the operating conditions and address the uncertainty of model parameters, and is verified under dynamic loading profiles as well as extreme temperatures to guarantee its robustness.

Thus, the present work has mainly the following objectives:

1. To establish a two-order RC EMC with temperature compensation to create the robust battery SOC estimation against ambient temperature alterations. A dual-fifth polynomial expression is proposed to establish a mathematical relationship of all the model parameters with respect to temperature and SOC.
2. Based on the two-dimensional relationships, the main effects and the sensitivity of the SOC and temperature in relation to investigated parameters are analysed according to the principle of variance analysis and partial derivatives.
3. The second-order EKF (SOEKF)-based SOC estimator is designed at various temperatures in terms of the ECM with two-dimensional relationship parameters. Ultimately, the robustness and accuracy of the presented estimator are confirmed by conducting discharge experiments under extreme temperatures and dynamic loading profiles.

2 | ESTIMATION METHOD

2.1 | Battery modelling

Among the different models of lithium-ion batteries, the RC EMC is extensively utilized due to its practicability [38]. The model is composed of a voltage source (OCV), an ohmic resistance ($R_0$), and several RC (resistance–capacitance) networks. Figure 1 shows the structure of the model, where $U$ represents the battery terminal voltage, $U_{OCV}$ indicates the OCV, $R_0$ is the ohmic internal resistance, $R_1$ and $R_2$ are the polarization resistances, and $C_1$ and $C_2$ denote the polarization capacitances.

The voltage across the ohmic resistance and RC networks is represented by $U_0$, $U_1$, and $U_2$, and $I$ denotes the current. Based on the circuit structure, a state-space model can be
Estimation algorithm, we first set the state vector as
accuracy, the SOEKF is applied. To implement the filter algo-
employed instead of the original KF . To ensure the estimation
also exhibit a non-linear relationship. This is why the EKF is
relationship, implying that the observation and state variables
Generally, the OCV and SOC exhibit an obvious non-linear
expressions of the model parameters with respect to the SOC
will be interpreted in the next section, where the mathematical
were considered in the battery modelling. These parameters
were interpreted in the next section, where the mathematical
expressions of the model parameters with respect to the SOC
and temperature are established.

2.2 Estimation algorithm

Generally, the OCV and SOC exhibit an obvious non-linear
relationship, implying that the observation and state variables
also exhibit a non-linear relationship. This is why the EKF is
employed instead of the original KF . To ensure the estimation
accuracy, the SOEKF is applied. To implement the filter algo-
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Generally, the OCV and SOC exhibit an obvious non-linear
expressions of the model parameters with respect to the SOC
covariance trace.

Notably, Equation (3) is strictly linear, which signifies that the
time update equation is the same as that in the standard KF: Therefore, a measurement update equation needs to be de-
veloped.
The superscript ‘+’ represents the estimated value corrected
by the observed value at the current moment (posteriori estimation), whereas ‘−’ indicates a value that is yet to be corrected
(priori estimation). We assume that the measurement update
equation for the state estimation is given as

\[ x_k^+ = x_k^- + K_k[\beta_k - b(x_k^-)] - \beta_k, \]

where \( K_k \) represents the determined Kalman gain, and \( \beta_k \)
denotes the determined correction term. We select \( \beta_k \) such that \( x_k^+ \) is unbiased, and then \( K_k \) is selected to minimise the estimation
covariance trace.

The estimation errors are calculated as

\[ e_k^- = x_k^- - x_k^-, \]

\[ e_k^+ = x_k^+ - x_k^+. \]

Now, a second-order Taylor-series expression \( b(x_k, \beta_k) \) around
the nominal point \( x_k^- \) is considered in order to obtain

\[ b(x_k) = b(x_k^-) \]

\[ + \frac{\partial b}{\partial x_k} \Big|_{x_k^-} (x_k - x_k^-) \]

\[ + \frac{1}{2} (x_k - x_k^-)^T \frac{\partial^2 b}{\partial x_k^2} \Big|_{x_k^-} (x_k - x_k^-). \]

By combining the above three equations, we can obtain the post-
eriori estimation error as

\[ e_k^+ = e_k^- - K_k H_k e_k^- - \frac{1}{2} K_k (e_k^-)^T D_k e_k^- - K_k \beta_k + \beta_k, \]

where \( w_k \) represents the process noise vector, \( \nu_k \) denotes the
measurement noise. It is worth noting that we use bold letters
for vectors or matrices in equations. The matrices \( F \) and \( G \) are
given by

\[ F_k = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 - \frac{\Delta t}{R_{1,k} C_{1,k}} & 0 & 0 \\
0 & 0 & 1 - \frac{\Delta t}{R_{2,k} C_{2,k}} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \]

\[ G_k = \begin{bmatrix}
0 \\
\Delta t \frac{1}{C_{1,k}} \\
\Delta t \frac{1}{C_{2,k}} \\
\Delta t \frac{1}{3600 Q}
\end{bmatrix}. \]
Experimental details

where

\[
H_k = \frac{\partial b}{\partial x_{k=x_k^{-}}} = \begin{bmatrix} -1 & -1 & -1 \\ \frac{\partial U_{\text{ocv}}}{\partial \text{SOC}_{k=\text{SOC}}} \end{bmatrix}
\]

\[
D_k = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \]

Assuming that \( E(e_{k}^{-}) = 0 \), we must set [39]

\[
\beta_k = \frac{1}{2} K_k (e_k^{-})^T D_k e_k^{-} \approx \frac{1}{2} K_k \text{Tr}(D_k P_k^{-}),
\]

where \( \text{Tr} \) represents the trace of the matrix. Then, we replace the third term on the right of Equation (10) with its estimated value, which is the covariance of KF.

The posterior covariance matrix is

\[
P_k^+ = E[(e_k^+)^T (e_k^+)]
\]

\[
= (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k (R_k + \Lambda_k) K_k^T,
\]

where \( R_k \) is the variance of \( r_k \), and \( \Lambda_k \) can be expressed as [40]:

\[
\Lambda_k = \frac{1}{4} E[\text{Tr}(D_k (e_k^-)^T - P_k^-)]^2 \text{Tr}(D_k (e_k^-)^T - P_k^-)]
\]

\[
= \frac{1}{2} \text{Tr}(D_k P_k^- D_k P_k^-). \]

Now, a function \( f_k \) is defined with the aim to minimise a weighted summation of the estimation errors:

\[
f_k = E[(e_k^+)^T S_k e_k^+],
\]

where \( S_k \) represents any positively weighted matrix. \( K_k \) that minimises the function \( f_k \) can be found by the equation:

\[
K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k + \Lambda_k)^{-1}.
\]

This gives the posterior covariance matrix from Equation (13):

\[
P_k^+ = P_k^- - P_k^- H_k^T (H_k P_k^- H_k^T + R_k + \Lambda_k)^{-1} H_k P_k^-.
\]

This equation, along with Equations (7), (12), and (16), determine the measurement update equations for the SOEKF. All the steps for the SOEKF are as follows:

1. Set initial values as

\[
x_k^+ = E(x_0),
\]

\[
P_k^+ = E[(x_0 - x_0^+)(x_0 - x_0^+)^T]. \]

2. Time update:

\[
P_k^- = F_{k-1} P_{k-1} F_{k-1}^T,
\]

\[
x_k^- = F_{k-1} x_{k-1}^+ + G_{k-1} u_{k-1}.
\]

3. Measurement update:

\[
\Lambda_k = \frac{1}{2} \text{Tr}(D_k P_k^- D_k P_k^-),
\]

\[
K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k + \Lambda_k)^{-1},
\]

\[
b_k = \frac{1}{2} K_k \text{Tr}(D_k P_k^-),
\]

\[
x_k^+ = x_k^- + K_k [y_k - b(x_k^-)] - \beta_k,
\]

\[
P_k^+ = P_k^- - P_k^- H_k^T (H_k P_k^- H_k^T + R_k + \Lambda_k)^{-1} H_k P_k^-.
\]

The SOC estimation is conducted through the combination of the ECM and SOEKF algorithm, which is shown in the flowchart in Figure 2.

3 | EXPERIMENTAL

3.1 | Experimental details

A test bench and 50 Ah lithium-ion batteries with a lithium nickel cobalt manganese oxide (NCM) cathode were configured to obtain the experimental data of voltage, temperature, and current. The information of batteries is shown in Table 1. The test bench setup consists of the following: (1) A programmable temperature chamber allowing the operating temperature control in the range of -40 to 130°C; (2) A battery test system (Arbin BT-2000 tester, Arbin, College Town, TX, US) with a measurement inaccuracy within 0.1% for the current and voltage transducers; and (3) A host computer with Arbin MITS Pro Software for battery charging or discharging control. The Arbin BT-2000 is attached to the battery cell located into the thermal chamber for preserving the temperature. The experimental protocol is repeatedly employed at various temperatures, namely -10, -5, 0, 5, 10, 25, 35, 45, and 55°C. The battery is maintained at each temperature for a certain period prior to any test in order to obtain a stable temperature. The temperature interval is small at low temperatures since the performance of lithium-ion batteries significantly varies at low temperatures.

The temperature and SOC influence the polarization resistance, ohmic resistance, and polarization capacitance. Thus, the OCV and capacity must be examined, and the battery parameters can be determined by conducting a hybrid pulse power characterization (HPPC) test at temperatures ranging from -10 to 55°C. The test procedure is indicated in Figure 3, which involves both characterization as well verification test, and the latter will be interpreted in Section 5.
### Table 1 Battery specifications

| Parameter (unit) | Values         |
|------------------|----------------|
| Nominal voltage (V) | 3.65           |
| Nominal capacity (Ah) | 50            |
| Weight (g)      | 735            |
| Dimension (mm)  | 310 (length) × 103 (width) × 11 (thickness) |
| Cathode material | NCM            |
| Anode material   | Si/C           |

### 3.2 Available capacity test

The available capacity of the battery is determined at temperatures ranging from \(-10\) to \(55^\circ\text{C}\). At each temperature, the fully charged battery is discharged at a particular temperature in the constant current mode until reaching the cut-off voltage, with a discharge cut-off voltage of \(2.8\) V and charge cut-off voltage of \(4.2\) V [41].

Figure 4 shows the obtained experimental results. It is evident that the temperature has a significant effect on the available capacity of battery, which increases with temperature. By contrast, the available capacity significantly decreases at low temperatures. The available capacity reduces by 9.3% at \(-10^\circ\text{C}\) as compared to that at \(25^\circ\text{C}\). 

### 3.3 OCV test

An open-circuit test is performed to investigate the relationship between the OCV and the SOC at different temperatures. During this test, the battery is initially completely charged utilizing the constant current and constant voltage (CC–CV) technique. Subsequently, the battery is sufficiently rested in order for the measured voltage to reach the equilibrium, and then the OCV is recorded, thus obtaining the OCV values. Consequently,
3.4 | HPPC test

The data from the HPPC test are utilized to determine the model parameters of the battery. The HPPC test procedure applied in this work is reported in the FreedomCAR Battery Test Manual. However, at each temperature, the SOC interval is adjusted from 10% to 5%. Moreover, the discharge capacity at low temperatures is lower than that at room temperature. Therefore, the standard SOC cannot be considered as the discharge capacity at room temperature but at the current temperature; otherwise, the battery will reach the cut-off voltage before the experimental procedure is completed.

4 | PARAMETER IDENTIFICATION AND ANALYSIS

4.1 | Parameter identification

Figure 5a demonstrates that the battery OCV is associated with the SOC, affected by the temperature as well. The effect of temperature on the OCV is inferior compared to that on the SOC;
Figure 5b–f shows the battery parameters at different temperatures used to determine the rest parameters of the RC circuit model.

On the other hand, the absence of relationship expressions is not conducive for an in-depth parameter analysis. Specifically, Equation (21) is universal to a certain extent and can be used to determine the relationship between the parameters of the following dual-fifth polynomial:

$$f(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{03}y^3 + p_{40}x^4 + p_{31}x^3y + p_{22}x^2y^2 + p_{13}xy^3 + p_{41}x^4y + p_{32}xy^3 + p_{50}y^5 + p_{42}x^3y^2 + p_{14}x^4y + p_{51}y^6 \quad (x = T, y = SOC).$$  (21)

The expression contains not only power terms of $x$ and $y$ but also their product terms, thereby reflecting the interaction between them. The undetermined coefficients of each parameter can be obtained through the MATLAB software, which is based on the recursive least-squares method. Table 2 summarizes the results obtained.

All the $R$-square values are greater than 0.9 (which is acceptable for surface fitting), thus indicating that the expression in Equation (21) can sufficiently describe the relationship of each parameter with the SOC and temperature, nonetheless that the variation phenomena of each parameter are distinctly different. Specifically, Equation (21) is universal to a certain extent and can be used to determine the relationship between the parameters of other similar models with SOC and temperature. Furthermore, the only 126 coefficients included in Table 2 can fully describe the changes of the six parameters in ECM as a function of temperature and SOC. When necessary, due to the effect of ageing, these coefficients can be adjusted and corrected by combining with online identification in the later stage. In practical application, to avoid storage space occupation, the original coefficients can be replaced by the newly corrected ones.

### Table 2

| Coefficient | OCV | $R_0$ | $R_1$ | $R_2$ | $C_1$ | $C_2$ |
|-------------|-----|-------|-------|-------|-------|-------|
| $p_{00}$    | 2.974 | 5.277 | 0.79  | -5.699 | 7.1   | 0.02324 |
| $p_{10}$    | -0.00258 | -0.6299 | -0.01615 | 1.109 | -0.3562 | -0.00091 |
| $p_{11}$    | 4.292 | -5.448 | -4.759 | 386.3 | -29.35 | -0.05069 |
| $p_{20}$    | 0.00027 | 0.02939 | 0.000471 | 0.01828 | 0.000596 | 0.000225 |
| $p_{12}$    | 0.02158 | 0.2335 | 0.08171 | 0.6992 | 0.6953 | -0.00302 |
| $p_{22}$    | -13.95 | 15.17 | 19.85 | -1204 | 100.1 | 1.813 |
| $p_{30}$    | -8.51 $\times 10^{-6}$ | -0.00033 | -6.25 $\times 10^{-6}$ | -0.00082 | 0.000547 | -3.46 $\times 10^{-6}$ |
| $p_{32}$    | -0.00061 | -0.01176 | -0.00089 | -0.4806 | -0.00132 | -0.00107 |
| $p_{33}$    | -0.0434 | -0.1316 | -0.1837 | 3.822 | -1.282 | 0.04341 |
| $p_{40}$    | 24.89 | -24.8 | -37.62 | 1709 | -162.2 | -6.873 |
| $p_{42}$    | 5.17 $\times 10^{-8}$ | -5.30 $\times 10^{-6}$ | -7.44 $\times 10^{-8}$ | 1.52 $\times 10^{-5}$ | -1.52 $\times 10^{-5}$ | 6.23 $\times 10^{-9}$ |
| $p_{44}$    | 1.47 $\times 10^{-5}$ | 0.000225 | -5.77 $\times 10^{-6}$ | 0.007209 | -0.00035 | 3.46 $\times 10^{-5}$ |
| $p_{50}$    | 0.000439 | 0.00443 | 0.001176 | 0.6164 | 0.009164 | 0.001209 |
| $p_{52}$    | 0.03872 | 0.03403 | 0.2099 | -14.03 | 1.245 | -0.119 |
| $p_{54}$    | -20.2 | 20.02 | 32.85 | -1098 | 122.3 | 9.179 |
| $p_{60}$    | 3.28 $\times 10^{-10}$ | 9.11 $\times 10^{-8}$ | 1.84 $\times 10^{-9}$ | -1.29 $\times 10^{-7}$ | 1.18 $\times 10^{-7}$ | 3.65 $\times 10^{-10}$ |
| $p_{62}$    | -1.03 $\times 10^{-7}$ | -1.27 $\times 10^{-6}$ | 7.32 $\times 10^{-8}$ | -1.12 $\times 10^{-5}$ | 4.83 $\times 10^{-6}$ | -3.93 $\times 10^{-7}$ |
| $p_{64}$    | -3.83 $\times 10^{-6}$ | -6.32 $\times 10^{-5}$ | -1.50 $\times 10^{-6}$ | -0.00547 | -3.94 $\times 10^{-5}$ | -3.00 $\times 10^{-7}$ |
| $p_{66}$    | -0.00014 | 0.000573 | -0.00032 | -0.2066 | -0.00192 | -0.00082 |
| $p_{68}$    | -0.01238 | -0.0284 | -0.09623 | 9.174 | -0.5437 | 0.07999 |
| $p_{70}$    | 6.063 | -5.928 | -10.66 | 243.8 | -33.77 | -4.044 |
| $p_{72}$    | 0.999 | 0.926 | 0.953 | 0.918 | 0.932 | 0.930 |

However, it is difficult to perform a quantitative evaluation from the figure’s results.

The recursive least squares with a forgetting factor [42] is used to determine the rest parameters of the RC circuit model. Figure 5b–f shows the battery parameters at different temperatures.  

### 4.2 Mathematical expressions

The numerical matrix of each parameter varying with SOC and temperature is then obtained. The SOC can be also estimated through the LUT based on the numerical matrices. However, this leads to the following disadvantages that no mathematical relationship could resolve. On the one hand, the LUT must be large enough to ensure accuracy, which could require a large amount of storage space in the BMS. In comparison, mathematical relationships need to store only few expression coefficients. On the other hand, the absence of relationship expressions is not conducive for an in-depth parameter analysis.

The mathematical expressions of all model parameters with respect to the temperature and SOC are established on the basis of the following dual-fifth polynomial:

$$f(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{03}y^3 + p_{40}x^4 + p_{31}x^3y + p_{22}x^2y^2 + p_{13}xy^3 + p_{41}x^4y + p_{32}xy^3 + p_{50}y^5 + p_{42}x^3y^2 + p_{14}x^4y + p_{51}y^6 \quad (x = T, y = SOC).$$  (21)
4.3 Main effect analysis

Figure 5 shows that the SOC and temperature affect each parameter, but to a different extent which cannot be quantitatively evaluated. Therefore, an approach based on variance analysis is established to analyse the influence degree of each factor.

In variance analysis [43], the $F$ value is applied to assess the effect of each factor on the response variable, which is expressed as:

$$ F = \frac{\text{MSF}}{\text{MSE}}, $$

where MSF represents the mean square of the factor, and the MSE indicates the mean square of the error. For each parameter, the $F$ values of the SOC and temperature can be calculated through Equation (22), and the influence degree of the two factors on each parameter can be analysed. However, the absolute $F$ values calculated for each parameter are significantly different due to the variation in parameter dimensions. This renders it impossible to compare the influence degrees of SOC and temperature on the different parameters. Thus, the $F$ value is normalized as follows:

$$ F_{i,SOC} = \frac{F_{i,SOC}}{F_{i,SOC} + F_{i,T}}, $$

$$ F_{i,T} = \frac{F_{i,T}}{F_{i,SOC} + F_{i,T}}, $$

where $i$ denotes the six parameters, namely, $U_{OCV}$, $R_0$, $R_1$, $R_2$, $C_1$, and $C_2$.

Figure 6 shows that the temperature has a more dominant effect on the model parameters, except on the OCV. In theory, OCV versus SOC curves are temperature based, as estimated from the entropy contribution; however, the experimental results show that within the operating conditions, they are very similar. In a similar manner, a temperature change of $10^\circ\text{C}$ could exert a variation in the OCV of about $3\text{ mV}$ [44].

4.4 Sensitivity analysis

Figure 6 depicts the effect of temperature and SOC on each parameter in terms of the overall average, without showing the specific influence in different ranges. However, a two-dimensional mathematical relationship of each parameter with respect to the temperature and SOC, has been obtained in Section 4.2; therefore, the partial derivatives can be used in order to analyse the sensitivity of the parameters to the temperature or SOC at any point, since the partial derivative is the slope of the tangent line at a point on a fixed surface.

According to Equation (21), the partial derivative can be represented as

$$ \frac{\partial f}{\partial x} = (p_{01} + p_{11}x + p_{12}x^2 + p_{13}x^3 + p_{14}x^4) \left. \right|_{x=SOC} $$

$$ + 2(p_{20} + p_{21}x + p_{22}x^2 + p_{23}x^3) $$

$$ + 3(p_{30} + p_{31}x + p_{32}x^2) $$

$$ + 4(p_{40} + p_{41}x) $$

$$ + 5p_{50}x^4 $$

$$ \frac{\partial f}{\partial y} = (p_{01} + p_{11}x + p_{21}x^2 + p_{31}x^3 + p_{41}x^4) $$

$$ + 2(p_{02} + p_{12}x + p_{22}x^2 + p_{32}x^3) $$

$$ + 3(p_{03} + p_{13}x + p_{23}x^2) $$

$$ + 4(p_{04} + p_{14}x) $$

$$ + 5p_{05}x^4 $$

(24)
5 | ESTIMATION VERIFICATION

As shown in Figure 3, several experiments were conducted in order to validate the proposed estimation method. The experimental setup was described in Section 2. The SOC estimation results are compared with the ampere-hour counting values at different constant temperatures, variable temperatures, and dynamic loading profiles [45].

5.1 | Verification at constant temperatures

While verifying the proposed method, we first compared the estimated results with the ampere-hour counting values at different constant temperatures. Meanwhile, the estimation results of present method were also compared with that of other estimation schemes, the details of which are indicated in the footnote of Table 3. The initial values of all algorithms applied for comparison are uniformly set to \([0.075 \ 0.022 \ 0.082 \ 1]^T\). It should be noted that no initial SOC error is considered, since the robustness analysis in terms of initial SOC error for KF has been confirmed [46], which is not the focus of the present study.

Figure 9 presents the comparison of the results and estimated errors at \(-10, 0, 25, \text{and } 45^\circ C\). By calculating the maximum absolute estimation errors (MAAEs) and root-mean-square errors (RMSEs), the accuracy of this technique is assessed. Table 3 presents the results. Furthermore, in order to evaluate the practical perspective of the present method, the computing time of estimation methods are normalized to that of \(CP + EKF\) method. The reason for adopting normalized time is that the computing hardware used in this study must be different from that in BMS, which makes the absolute time not of
### TABLE 3  RMSE and MAEE at different temperatures

| Estimation schemes | −10°C | 0°C  | 25°C | 45°C | Variable |
|--------------------|-------|------|------|------|----------|
| RMSE<sup>a</sup>   |       |      |      |      |          |
| CP + EKF           | 37.2% | 9.97%| 3.44%| 2.87%| 13.2%    |
| VP + EKFD<sup>b</sup> | 7.92% | 5.08%| 3.12%| 2.71%| 5.48%    |
| CP + SOEKF<sup>c</sup> | 31.1%| 9.46%| 3.31%| 2.69%| 12.8%    |
| VP + SOEKF<sup>d</sup> | 4.48%| 3.98%| 2.93%| 2.66%| 3.00%    |
| MAEE<sup>e</sup>   |       |      |      |      |          |
| CP + EKF           | 14.2% | 3.69%| 1.42%| 1.06%| 5.07%    |
| VP + EKF           | 3.35% | 1.92%| 1.36%| 1.02%| 2.41%    |
| CP + SOEKF         | 14.0% | 3.69%| 1.38%| 1.03%| 5.08%    |
| VP + SOEKF<sup>f</sup> | 2.15%| 1.91%| 1.27%| 1.00%| 0.80%    |
| Normalized computing time | CP + EKF | —— | —— | —— | —— |
| VP + EKF           | 1.25  | 1.00 | 1.02 | 1.02 | 1.01     |
| CP + SOEKF         | 1.32  | 1.18 | 1.02 | 1.03 | 1.05     |
| VP + SOEKF         | 1.45  | 1.23 | 1.03 | 1.11 | 1.20     |

<sup>a</sup>Root-mean-square error.
<sup>b</sup>Maximum absolute estimation error.
<sup>c</sup>Combination of ECM with constant temperature (25°C) parameters and EKF.
<sup>d</sup>Combination of ECM with variable temperature parameters and EKF.
<sup>e</sup>Combination of ECM with constant temperature (25°C) parameters and SOEKF.
<sup>f</sup>Combination of ECM with variable temperature parameters and SOEKF (present method).

**FIGURE 9** The SOC estimation results at (a) −10°C, (b) 0°C, (c) 25°C, and (d) 45°C.
FIGURE 10  (a) The temperature profile and (b) SOC estimation results of variable temperature experiment

5.2  |  Verification at variable temperatures

A discharge experiment is conducted under temperature variation to further assess the performance of the presented estimation technique. The temperature of the battery is maintained at approximately 0°C for the first 60 min of discharge, and is then increased to 20 and 40°C. Figure 10a shows the temperature profile during the experiment. Despite the poor regularity of the temperature variation, the curve validates the adaptability of the proposed estimation method to temperature changes.

Figure 10b shows the obtained results. The estimated errors are slightly higher compared to those of the constant temperature test. This is mainly due to the fact that the temperature profiles are dynamic. Nevertheless, the RMSE and MAEE are 3.0% and 0.8%, respectively. These results further demonstrate the adaptability of the proposed estimation method to temperature changes.

5.3  |  Verification under dynamic loading profiles

Figure 11a shows the current profile in operation mode cycle experiment. The load current is heavily dynamic and varies continuously with changing magnitude. Figure 11b shows the SOC estimation results. The typical operating SOC range is employed for the validation. The estimation error of the battery SOC is consistently within 4.5%, and the average error is less than 1.5%. Therefore, the proposed method can be used to perform accurate SOC estimation by frequently varying operating conditions.

6  |  CONCLUSION

A model-based approach was developed to assess the effect of temperature variations on the SOC estimation of a Li-ion battery with an NCM cathode. A dual-fifth polynomial expression was proposed, and a two-dimensional mathematical relationship of the OCV and the other EMC parameters with the SOC and temperature was established on the basis of experimental investigation. The main effects and sensitivity of the SOC and temperature on the model parameters were thoroughly analysed. The results revealed that the temperature has a more dominant effect on the model parameters, except on OCV. The SOC was estimated through a combination of the ECM and SOEKF algorithm. The effectiveness of the proposed approach was validated experimentally compared to other estimation schemes, which demonstrates its superiority in accuracy, especially in low temperature adaptability. The experimental data show that the proposed estimation method can efficiently limit the error below 4.5% under extreme temperatures and dynamic loading.
profiles. The obtained advancements in relation to temperature effects are critical towards retaining modelling simplicity and enhancing the reliability and accuracy of battery SOC estimation under dynamic working conditions. As a future work, the estimation method established in this study will be further improved and validated under various states of battery ageing, being also extended to other battery systems and combined with online parameter adjustment and correction periodically or irregularly.

ACKNOWLEDGEMENT
This study is supported by Beijing Natural Science Foundation (2214066).

FUNDING
Beijing Natural Science Foundation (2214066).

Abbreviations and nomenclature

- **BMS**: Battery management system
- **C₁/C₂**: Polarisation capacitances
- **D**: Coefficient matrix defined by Eq.(11)
- **e**: Estimation error vector
- **E**: Expectation of the random variable
- **EKF**: Extended Kalman filter
- **F**: Value defined by Eq.(22)
- **F̄**: Normalised value of F
- **F**: Coefficient matrix defined by Eq.(5)
- **G**: Coefficient matrix defined by Eq.(6)
- **b**: Observation function defined by Eq.(4)
- **H**: Partial derivative of b with respect to the state vector
- **HPPC**: Hybrid pulse power characterisation
- **I**: Unit matrix
- **Iₑ**: Current
- **k**: Weighted summation of the estimation errors defined by Eq.(15)
- **K**: Kalman gain matrix
- **KF**: Kalman filter
- **LUT**: Look-up table
- **MAEE**: maximum absolute estimation error
- **MSE**: Mean square of the error
- **MSF**: Mean square of the factor
- **OCV**: Open-circuit voltage
- **P**: Covariance matrix
- **Q**: Available capacity
- **R**: Variance of measurement noise
- **Δ**: Defined by Eq.(14)
- **R₀**: Ohmic resistance
- **R₁/R₂**: Polarisation resistances
- **RMSE**: Root-mean-square error
- **S**: Positively weighted matrix
- **SOC**: State of charge
- **SOEKF**: Second-order EKF
- **t**: Time
- **T**: Temperature
- **Tr**: Trace of the matrix
- **u**: Control variable
- **UKF**: Unscented Kalman filter
- **U**: Battery terminal voltage
- **U₀**: The voltage across R₀
- **U₁**: The voltage across R₁
- **U₂**: The voltage across R₂
- **U_ocv**: The voltage value of OCV
- **v**: Measurement noise
- **w**: Process noise vector
- **x**: State vector
- **z**: Observation variable
- **Δt**: Sampling time
- **Δ**: Defined by Eq.(12)

Subscripts

- **₀**: Initial values
+ Estimated value corrected by the observed (posteriori estimation)
- Value that has yet to be corrected (priori estimation)
k Serial number of sampling interval

CONFLICT OF INTEREST
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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How to cite this article: Fang, Y., et al.: State-of-charge estimation technique for lithium-ion batteries by means of second-order extended Kalman filter and equivalent circuit model: Great temperature robustness state-of-charge estimation. IET Power Electron. 14, 1515–1528 (2021).
https://doi.org/10.1049/pel2.12129