Subbarrier fusion of carbon isotopes: from resonance structure to fusion oscillations

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Abstract. At energies below the Coulomb barrier, the fusion excitation function for the \textsuperscript{12}C+\textsuperscript{12}C system shows prominent fine structures, whereas that for the \textsuperscript{12}C+\textsuperscript{13}C system behaves more smoothly as a function of energy. We demonstrate that these different behaviors can be simultaneously reproduced using an optical potential in which the strength of the imaginary part is proportional to the level density of each compound nucleus. We also discuss the oscillatory behavior of fusion excitation function for these systems observed at energies above the Coulomb barrier from a viewpoint of quantum mechanical systems with identical particles.

1. Introduction
The \textsuperscript{12}C+\textsuperscript{12}C fusion reaction plays an important role in several astrophysical phenomena, such as the carbon burning in stellar evolution, type Ia supernovae, and the X-ray superburst of an accreting neutron star. Even though there is a long history of research on this reaction, the reaction still attracts lots of attention and new experimental and theoretical works have continuously been carried out \cite{1,2,3,4,5}. A characteristic feature of the \textsuperscript{12}C+\textsuperscript{12}C fusion reaction is that the cross sections have many fine structures, at energies both below and above the Coulomb barrier, while the cross sections for the neighboring systems, \textsuperscript{12}C+\textsuperscript{13}C and \textsuperscript{13}C+\textsuperscript{13}C, are much less structured. In this contribution, we simultaneously analyze fusion reactions for the \textsuperscript{12}C+\textsuperscript{12}C and \textsuperscript{12}C+\textsuperscript{13}C systems and discuss possible origins for the different behavior of fusion excitation functions for these systems. Notice that most of the previous studies, except for Ref. \cite{4}, have concentrated only on the \textsuperscript{12}C+\textsuperscript{12}C system. In contrast, we shall analyze both the \textsuperscript{12}C+\textsuperscript{12}C and \textsuperscript{12}C+\textsuperscript{13}C systems and clarify the dynamics of subbarrier fusion of two carbon isotopes.

2. Subbarrier molecular resonances
We first discuss the resonance behavior of fusion cross sections for the \textsuperscript{12}C+\textsuperscript{13}C system at energies below the Coulomb barrier. Figure 1(a) shows the experimental data for the modified astrophysical S-factor, defined as $S^*(E) = \sigma(E)E \exp(87.21/\sqrt{E} + 0.46E)$, where $\sigma(E)$ is the fusion cross section and the energy $E$ is in units of MeV, for the \textsuperscript{12}C+\textsuperscript{12}C and the \textsuperscript{12}C+\textsuperscript{13}C systems \cite{2}. As has been well known \cite{6}, the fusion cross sections for the \textsuperscript{12}C+\textsuperscript{13}C system exhibit a few resonance peaks. Those relatively narrow resonances are known as molecular resonances, for which the excitation of \textsuperscript{12}C to the first $2^+$ state plays an important role \cite{7,8,9}.
As has been pointed out in Ref. [2], the fusion cross sections for the $^{12}$C+$^{12}$C system show inhibitions as compared to those for the $^{12}$C+$^{13}$C system except at a few resonance energies, at which the fusion cross sections for the two systems somehow match with each other. Recently, Jiang et al. have argued that the fusion inhibition in the $^{12}$C+$^{12}$C system is attributed to i) the smaller fusion $Q$-value in the entrance channel compared to that for the $^{12}$C+$^{13}$C and $^{12}$C+$^{13}$C systems, ii) a smaller level density of the compound nucleus $^{24}$Mg than $^{25}$Mg and $^{26}$Mg at a given excitation energy, and iii) the fact that only states with positive parity and even spin of the compound nucleus are populated in fusion because the entrance channel consists of identical spin-zero bosons [3]. Jiang et al. have succeeded to explain the average behavior of fusion cross sections for the $^{12}$C+$^{12}$C system based on this idea.

Our first aim in this contribution is to implement the idea of Jiang et al. into coupled-channels calculations and discuss the difference between the $^{12}$C+$^{12}$C and $^{12}$C+$^{13}$C systems. To this end, we carry out coupled-channels calculations with an optical potential, in which the strength of the imaginary part is proportional to the level density of the compound nucleus. That is, the imaginary part of the optical potential is assumed to be

$$W(r) = -w_0\rho_J(E^*)f(r),$$  \hspace{1cm} (1)

where $w_0$ is an overall strength, $\rho_J(E^*)$ is the level density of the compound nucleus at the excitation energy of $E^*$ with the angular momentum $J$, and $f(r)$ determines the radial dependence of the imaginary potential, which we assume to be a Woods-Saxon form. The level-density dependent imaginary potential has been employed in Refs. [9, 12], which has further been investigated in Refs. [13, 14]. Eq. (1) may be justified in terms of the Fermi’s golden rule for a transition from the entrance channel to compound nucleus states [9, 12, 13, 14]. In this approach, the energy, the angular momentum, and the system dependences of the imaginary potential are taken into account through the level density of the compound nucleus.

In order to perform the coupled-channels calculations for the $^{12}$C+$^{12}$C and $^{12}$C+$^{13}$C systems, we closely follow the calculations presented in Ref. [8]. That is, we use the same geometry for the optical potential as in Ref. [8], and we include the excitations of both the projectile and the target nuclei up to the first member of the ground state rotational band (including also the mutual excitation channels). We take the deformation parameters from Ref. [4], while we
take the empirical level density parameters from Ref. [3]. The overall strength $w_0$ in Eq. (1) is determined so as to reproduce the results of Ref. [8] if all the parameters are set identical to those in Ref. [8]. We use the same value of $w_0$ both for the $^{12}$C+$^{12}$C and $^{12}$C+$^{13}$C systems. Most of the channels are closed, and the coupled-channels equations are solved with the variational method [8, 15, 16] in order to avoid a numerical instability.

Figure 1(b) shows the results so obtained. The solid and the dashed lines show the modified astrophysical $S$-factors for the $^{12}$C+$^{12}$C and the $^{12}$C+$^{13}$C systems, respectively. One can see that the system dependence of fusion cross sections is qualitatively well reproduced. That is, the calculated $S$-factors for the $^{12}$C+$^{12}$C system show resonance peaks, while those for the $^{12}$C+$^{13}$C system behave rather smoothly. This indicates that a promising origin for the resonance behavior in the $^{12}$C+$^{12}$C system is attributed to the properties of the compound nucleus $^{24}$Mg, as has been suggested by Jiang et al. [3].

The calculations still underestimate the fusion cross sections below 5 MeV for both the systems. This would be due to those channels which are not included in the present calculations. Possible candidates are the first $3^-$ state and the second $0^+$ state (that is, the Hoyle state) [4, 5]. The $\alpha$-transfer channel may also play an important role [17, 18, 19]. It would be an interesting future work to repeat the present calculation by including those channels.

### 3. Fusion oscillations above the Coulomb barrier

We next discuss fusion cross sections at energies above the Coulomb barrier. The experimental data for the $^{12}$C+$^{12}$C system are shown in Fig. 2 (a). One can see that the fusion cross sections significantly oscillate as a function of energy. The origin for the fusion oscillations may be different from the origin for the resonance behavior at subbarrier energies shown in Fig. 1, however.

At energies above the barrier, the level density is large enough so that resonances in the compound nucleus appreciably overlap with each other. This results in the picture of strong absorption, with which fusion cross sections are given by,

$$\sigma(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E) = \sum_l \sigma_l(E),$$

where $k$ is the wave number related to the incident energy in the center of mass frame, $P_l$ is the penetrability of the Coulomb barrier with the partial wave $l$, and $\sigma_l(E)$ is a partial fusion cross
section. For each partial wave, the penetrability $P_l(E)$ is close to unity and does not change significantly at energies well above the barrier. The partial cross section $\sigma_l(E)$ then decreases as $1/k^2$ as a function of energy. Since the partial waves are discrete, one would then obtain a structure in fusion cross sections, which are associated with the energy dependence of partial fusion cross sections. This is the origin for the fusion oscillations advocated in Refs. [21, 22] (see also Ref. [23]). That is, the fusion oscillations are due to the addition of successive individual partial waves as the energy increases.

For the $^{12}\text{C}+^{12}\text{C}$ system, since the wave function of the whole system has to be symmetric with respect to the interchange of two identical spin-zero bosons, only even partial waves contribute to fusion cross sections. That is,

$$\sigma(E) = \frac{\pi}{k^2} \sum_l (1 + (-1)^l)(2l + 1)P_l(E) = \sum_l (1 + (-1)^l)\sigma_l(E).$$  \hspace{1cm} (3)

This leads to an enhancement of oscillations, since the energy spacing between successive contributing partial waves then increases.

The solid line in Fig. 2(a) shows a potential model fit to the experimental fusion cross sections. Since the effect of channel couplings are small for this system at energies above the Coulomb barrier, we perform single-channel calculations for simplicity. In order to account for the observed decrease of fusion cross sections at energies higher than 25 MeV, we have reduced the penetrability for $l = 14$ by a factor of 2 and set the penetrabilities for all the higher partial waves to be zero. Such an assumption may be justified from a consideration based on the excitation energy of the compound nucleus relative to the yrast energy [24, 25]. This calculation fairly well reproduces the experimental data, including the fusion oscillations. The structure of fusion cross sections is evident if the corresponding partial fusion cross sections are plotted individually, as is done in Fig. 2(b).

Within the parabolic approximation to the Coulomb barrier, one can derive a compact formula for the oscillatory part of fusion cross sections [21, 24, 25, 26],

$$\sigma_{osc}(E) = 2\pi R_E^2 \frac{\hbar \Omega_E}{E} \exp \left( -\frac{\pi \mu R_E^2 \hbar \Omega_E}{(2l_g + 1)\hbar^2} \right) \sin(\pi l_g),$$ \hspace{1cm} (4)

which is added to the smooth part of fusion cross sections given by the well known Wong formula [27]. In this equation, $\mu$ is the reduced mass of the system, and $R_E$ and $\hbar \Omega_E$ are the barrier position and the curvature of the Coulomb barrier, respectively, evaluated at the grazing angular momentum, $l_g$. For a spatially anti-symmetric configuration, the negative sign has to be multiplied to this formula [24, 25]. This formula indicates that the amplitude of fusion oscillations exponentially decrease for heavy systems (with large values of $\mu$), and one has a better chance to see the fusion oscillations in light systems, such as $^{12}\text{C}+^{12}\text{C}$ and $^{16}\text{O}+^{16}\text{O}$.

The experimental fusion cross sections for the $^{13}\text{C}+^{13}\text{C}$ are shown in Fig. 3(a). One can see that the amplitude of fusion oscillation is much smaller than that for the $^{12}\text{C}+^{12}\text{C}$ system shown in Fig. 2(a). The $^{13}\text{C}+^{13}\text{C}$ system is a system of two identical spin-1/2 fermions, and both the $S=0$ (which gives even $l$ with weight 1/4) the $S=1$ (which gives odd $l$ with weight 3/4) configurations contribute. Since the oscillatory cross section for odd partial waves has the opposite sign to that for even partial waves, the oscillation is reduced by a factor 2 and has the opposite phase from the symmetric system. The solid line in Fig. 3(a) is obtained in this way.

For the $^{12}\text{C}+^{13}\text{C}$ system, since this is not a system with identical particles, the oscillations from even $l$ and odd $l$ would cancel out without any further effect. However, the presence of the elastic neutron transfer channel will introduce a parity dependence into the problem. This is most easily seen by considering the total elastic scattering, where one must add an exchange term $f_{trans}(\pi - \theta)$ for elastic transfer to the amplitude $f_{el}(\theta)$ for direct elastic scattering. This yields a
Figure 3. (The left panel) Same as Fig. 2(a), but for the $^{13}\text{C}+^{13}\text{C}$ system. The experimental data are taken from Ref. [28]. (The right panel) The experimental fusion cross sections for the $^{12}\text{C}+^{13}\text{C}$ system taken from Ref. [20]. The three curves are obtained with a parity dependent potential with the depth parameter of $V = (1 + (-1)^l\epsilon)V_0$, where $V_0$ is a negative value, with different values of $\epsilon$ as indicated in the figure.

total scattering amplitude, $f_{\text{total}}(\theta) = f_{\text{el}}(\theta) + f_{\text{trans}}(\pi - \theta)$. Using $P_l(\cos(\pi - \theta)) = (-1)^l P_l(\cos \theta)$, one obtains $S^{\text{eff}}_l = S^{\text{el}}_l + (-1)^l S^{\text{trans}}_l$, that is, different effective $S$-matrix elements for the odd and even partial waves. The same mechanism with $\alpha$ transfer has been discussed in Ref. [22] for the fusion oscillations observed in the $^{12}\text{C}+^{16}\text{O}$ system. As is well known, the parity dependence due to elastic transfer can be well mocked up with a parity dependent potential [22, 29, 30, 31]. The three different lines in Fig. 3(b) are obtained with a parity dependent potential, whose depth parameter is defined as $V = (1 + (-1)^l\epsilon)V_0$ (with a negative value of $V_0$). Treating $\epsilon$ as a free parameter, we obtain a good fit to the experimental data with $\epsilon = -0.15$, that is, a shallower potential (and thus a higher Coulomb barrier) for even partial waves. We mention that the sign of $\epsilon$ found for the $^{12}\text{C}+^{13}\text{C}$ system is consistent with a simple rule proposed by Baye based on the resonating group method (RGM) with a two-center harmonic oscillator shell model [32, 33].

Notice that, around $E \sim 13$ MeV, the calculation with $\epsilon = +0.15$ appears more consistent with the experimental data compared with the result with $\epsilon = -0.15$. It would be interesting to remeasure fusion cross sections in this energy region with higher precision to clarify whether there indeed exists a possible shift in phase of the oscillations as a function of energy.

4. Summary

The fusion reaction of carbon isotopes is important from the astrophysical point of view, and at the same time it makes also an interesting quantum mechanical problem. We have discussed this reaction from the subbarrier to the above barrier regions by comparing the $^{12}\text{C}+^{12}\text{C}$ system to the neighboring $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$ systems.

At subbarrier energies, the fusion cross sections for the $^{12}\text{C}+^{12}\text{C}$ system show many resonance peaks whereas those for the other systems behave smoothly. We have demonstrated that this fact can be naturally explained if one considers properties of the compound nuclei, as had been conjectured by Jiang et al.. To this end, we have carried out coupled-channels calculations by including excitations to the first excited state in the ground state rotational band. In these calculations, we have employed an optical potential whose strength is directly proportional to the level density of each compound nucleus.

At energies above the Coulomb barrier, the fusion cross sections for the $^{12}\text{C}+^{12}\text{C}$ system oscillate as a function of energy. We have demonstrated that this oscillation can be interpreted...
as due to the addition of successive individual partial waves. The oscillation is stronger in the $^{12}\text{C}+^{12}\text{C}$ system than in the $^{13}\text{C}+^{13}\text{C}$ system, because only even partial waves contribute to fusion cross sections for the former system while both even and odd partial waves contribute (with a statistical weight of 1:3) for the latter system. We have also shown that the fusion oscillation observed in the $^{12}\text{C}+^{13}\text{C}$ system is due to elastic transfer of neutron, whose effect is well mocked up in terms of a parity dependent potential.

It still remains as a challenging problem to quantitatively explain observed fusion cross sections for the C+C systems at deep subbarrier energies. In order to achieve this goal, one would have to take into account the effects of several collective excitations as well as the alpha transfer channel.

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