Research of Toppling Criterion and Analysis of Instability Evolution of Toppling Deformation Slope

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Abstract. Toppling failure is one of the main types of rock slope instability. Based on the Goodman-Bray toppling stability analysis model, the starting mechanism and criteria for toppling deformation are deduced. Combined with the discrete element numerical simulation method, the evolution failure mode of the toppling slope is studied, which can better reflect the basic regularity of the toppling failure, and provide support for slope engineering. In order to predict the influence of the foot displacement of the leading edge on the slope surface displacement, several monitoring points are set at the boundary of the slope and the bend of the toppling body. It is of great engineering significance and practical value to make safety early warning in time.

1. Introduction
Toppling deformation, as a special deformation and failure phenomenon of slopes, has long attracted the attention of the engineering community. It mainly occurs in layered rock or similar layered rock that is massive but densely developed along the slope towards fractures, especially in layered soft rock or alternating layers of soft and hard rock[1]. Generally, small-scale toppling occurs in a small area with a shallow front of the slope, with limited development depth and a short evolutionary process. Most of them are mainly local slumps and collapses[2]. Large-scale toppling deformation failure has a long breeding evolution process, and rock mass deformation will not cause rapid slope failure in a short period of time, but once damage occurs, it must be deep and large-scale[3]. The problem of toppling deformation has become an extremely important engineering geological problem.

Currently, the main methods for studying toppling stability are limit equilibrium method and numerical analysis methods (including finite element method, discrete element method, discontinuous deformation method, and manifold method. )[4,5]. Goodman and Bray first proposed the analysis method based on the principle of limit equilibrium[6]. The landslide was cut into multiple rectangular bars of equal width by using inversely inclined structural planes. The blocks in different states divide the toppling body into three parts: the stable area at the top of the slope, the dump area in the middle, and the sliding area at the foot of the slope. However, it is found in practical application that the Goodman-Bray analysis model of toppling stability cannot reflect the geological characteristics and failure modes of the toppling rock mass well. Based on the analysis of the structure and deformation characteristics of the toppling rock mass and the collection of relevant data, the instability mode of the
toppling deformation is studied, and the crack initiation conditions for the toppling deformation are derived by the formula.

2. Theoretical basis and formula derivation

According to the mechanical state of a toppling block of the toppling deformation slope, the crack initiation conditions are derived. The schematic diagram of the mechanical relationship is shown in Figure 1. The block rotates along point O and bends. Among them, $G_0$ is the weight of block ADF, $G_1$ is the weight of block DFOE, and $G_2$ is the weight block OECD; $\sigma_c$ and $\tau_c$ are the equivalent normal stress and the equivalent tangential stress of the upper interface respectively; $\sigma_{bc}$ and $\sigma_{oc}$ are the tensile stresses at the block ends CB and CD respectively. $\gamma$ is the weight of the rock, $\sigma_1$ and $\sigma_3$ are the major and minor stresses at point B respectively. The principal stress at A is assumed to be approximately 0. Then, the normal stress $\sigma_{oe}$ and the tangential stress $\tau_{oe}$ on the rock mass are assumed to be linearly distributed.

![Figure 1. Sketch of mechanical relationship](image)

Table 1. Formulas of the force and the arm of force of the toppling body

| Function categories | Formulas of force | Formulas of the arm of force |
|---------------------|-------------------|-------------------------------|
| $G_0$               | $(l-l_0-l_t)\cdot t\cdot \frac{1}{2}\gamma$ | $\frac{1}{3}(2l+l-l_0-2t\cdot \tan \beta)\cdot \cos \beta$ |
| $G_1$               | $l_1\cdot t\cdot \gamma$ | $\frac{l-t\cdot \tan \beta}{2}\cdot \cos \beta$ |
| $G_2$               | $l_0\cdot t\cdot \gamma$ | $\frac{t}{2\sin \beta} - \frac{t\cdot \cot \beta - l_0}{2}\cdot \cos \beta$ |
| $\sigma_c$          | $\frac{2(l-l_0)}{3}\left[\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cdot \cos 2\cdot (90-\beta)\right]$ | $\frac{l-l_0}{3}$ |
| $\tau_c$            | $\frac{2(l-l_0)}{3l}\frac{\sigma_1-\sigma_3}{2}\cdot \sin 2\cdot (90-\beta)$ | $t$ |
| $\sigma_{bc}$       | $\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cdot \cos 2\beta$ | $\frac{t}{3}$ |
| $\sigma_{oc}$       | $\frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2}\cdot \cos 2\beta$ | $\frac{l_0}{3}$ |

(1) Overall rotation limit equilibrium condition of toppling deformed rock mass:
Taking the O point as the axis of rotation, the self-weight of the block ADOE and the total bending moment \( M \) of the external load can be expressed as:

\[
M = G_{x} \cdot x_{o} + G_{y} \cdot y_{o} + \sigma_{c} \cdot \frac{1}{3}(l-l_{o})^2 - \tau_{c} \cdot (l-l_{o}) \cdot t
\]  

(1)

Block ADOE is the sliding segment, block OEB is the anti-sliding segment, and O point is the motorized articulation rotation point of the toppling body. Considering its own weight and external load, and taking the distance from O point according to its mechanical relationship diagram, the limit equilibrium State can be expressed as:

\[
\sum M_{o} = 0
\]  

(2)

Introducing the bending moment \( M \) in equation (1), \( M_{o} \) in equation (2) can be expressed as:

\[
\sum M_{o} = M - G_{x} \cdot x_{o} - \frac{1}{3} \sigma_{c} \cdot t^2 - \frac{1}{3} \sigma_{c} \cdot l_{o}^2 = 0
\]  

(3)

Equation (3) is the approximate limit equilibrium equation considering the overall rotation of the toppling deformation body. According to the known geometrical conditions such as \( l_{1}, t, \) and \( \beta \) in Equation (3), combined with stress conditions such as \( \sigma_{1} \) and \( \sigma_{3} \), the distance between the hinge support point O and the embedding point \( l_{0} \) can be obtained, which is the critical length of the restraint section where the toppling deformed rock mass reaches the limit equilibrium state.

(2) Toppling deformation cracking starting conditions:

The OE section is the critical fracture surface assumed by toppling and bending. Under the action of gravity and the upper load, the OE section is also the maximum bending moment section of the overhanging rock mass. Point O is the assumed turning point, and point E, where the tensile force is the largest, is the crack initiation point. When the tensile stress \( \sigma_{e} \) of the rock mass at point E in the OE section ≥ the ultimate tensile stress of the rock mass \([\sigma_{t}]\) is satisfied, tensile cracking of the rock mass occurs. Take the ADOE block as the research object, and under the limit state meet:

\[
[\sigma_{e}] = [\sigma_{t}]
\]  

(4)

Among them, \( M \) is the maximum bending moment of the OE section, \( y \) is \( t/2 \) that the half of the section height, and \( I \) is the moment of inertia of the section, which is calculated in unit width. In formula (4), \( M \) is the self-weight of the ADOE block and the total bending moment of the external load with the point O as the rotation axis. It can be obtained according to (1), so

\[
K = \frac{[\sigma_{e}]}{\sigma_{t}} = \frac{[\sigma_{e}]}{M \cdot y} = \frac{[\sigma_{e}]}{M \cdot y} \cdot \frac{t^2}{6M}
\]  

(5)

According to the relationship between \( M \) and the tensile strength \([\sigma_{t}]\) of the rock mass, it can be determined whether the rock mass has reached the critical condition of bending initiation.

3. Study of project simulation

Based on the toppling deformation management project of a water conservancy and hydropower project, this paper studies the failure process of toppling deformation slope relied on the geological survey and rock mass physical and mechanical test results. The rock mass adopts the Mohr-Coulomb elastoplastic model, and the interface adopts a Mohr-Coulomb slip model.

In this case, based on the in-situ surveying and mapping of engineering geology, the basic occurrence distribution of the toppling body fracture zone is determined. The failure slip generally occurs between preset joint units. When the tensile force is reached at the preset joint to reach its tensile strength, the contact surfaces are separated. The contact force is zero. The simulation evolution diagram is shown in Figure 2. The first stage is the deformation of the toe of slope position. The second stage is the deformation of the fracture zone of the toppling layer, and the toppling deformation starts. The third stage and the fourth stage are the constant increasing deformation of the toppling body and the formation of traction damage of the toppling body.
For such toppling deformation slopes, the cracking and destruction are not allowed to the above degree. In the first stage of the failure mode above, the stability may have been lost and the instability limit state has been reached. At this time, there is no safety reserve value for the project. Therefore, the research status of the impact on the deformation safety of the slope should be before the first stage.

Qualitative analysis of the failure mode of the toppling slope can clearly find: 1) It has a large deformation on the surface of the toppling body, and it is less likely to have a deep sliding phenomenon. 2) The starting point of its deformation basically coincides with the cracking point of the site monitoring. 3) It may be related to boundary constraints or mechanical parameters. The slope basically slides down along the boundary of the toppling body, and the sliding occurs at the waist of the slope. 4) It is possible that the excavation of the slope foot caused the deformation of the upper toppling body, forming a progressive traction continuous toppling deformation.

4. Analysis of numerical model
Taking an ideal numerical model as an example, set a number of monitoring points on the slope of a toppling slope (including the top, middle, and toe) and the boundary of the bending section of the toppling body (upper and lower bends). Monitoring the dynamic real-time deformation during the calculation process, with a view to achieve the purpose of analyzing the overall displacement and stability of the slope.
With the increase of the number of iterative calculation steps, under the action of gravity and the compression of the upper rock mass, the toppling and bending of the slope body will penetrate at the preset joint surface. Under the action of this complex and comprehensive discontinuous mechanical mechanism, when a certain amount of cumulative deformation is reached, the ultimate toppling failure mode of such slopes is generated. Typical monitoring points 1, 2, and 3 are set at intervals on the slope of the toppling slope to dynamically capture the development of displacement change during the calculation process. The initial displacement value ($S_0$) of a certain range of the slope foot block is dynamically given. It is set to 2cm, 5cm, 10cm, and 15cm respectively. The displacement value ($S_i$) of the toppling slope monitoring point in the corresponding calculation process can be monitored and extracted to qualitatively and quantitatively analyze the displacement changes of the control point of the toppling slope surface. It can predict the influence of the foot displacement of the leading edge on the slope surface displacement. By fitting the data function of the monitoring point displacement and the slope foot displacement, it is possible to grasp the displacement change of the slope control monitoring points inward and outward, so as to make a timely safety warning.

Through exponential function fitting, a more suitable prediction equation for displacement ($S_i$) of each calculation point (monitoring point) on the slope surface can be obtained. With the increase of the displacement of the slope foot block, the prediction curve has an exponential growth trend.
At the same time, considering the insufficiency of statistical sample data, the accuracy error of the complex correlation coefficient \( R^2 \) of the fitting curve, the complexity of engineering geological conditions and the uncertainty of model construction and calculation parameters, the above fitting formula has certain applicable conditions \((S_0 \leq 20\text{cm})\). If the displacement of the toe of slope block increases to a certain value, the slope foot will have a large free surface, and the mechanical development mechanism and tendency of the slope's toppling deformation will be greatly different. Toppling instability may occur in local slopes. The local or overall instability failure caused by the slope displacement change regularity is worth further research. Its displacement monitoring control and early warning analysis research before instability has great engineering significance and practical value.

5. Conclusion
Limit equilibrium theory and discrete element numerical simulation methods are used to analyze the slope deformation-failure mechanism and stability. Combined with the results of numerical simulation, the instability failure mode of the toppling rock mass is discussed.

Through engineering geomechanics derivation and simulation analysis, the following conclusions can be obtained: 1) Analytical solutions based on the instability criterion of the block were obtained through detailed mechanical and mathematical analysis. 2) The failure of an actual toppling slope engineering was initially judged through the above analytical solutions. Combined discrete element numerical simulation to analyze the evolution process of slope instability. 3) Combined with the numerical test model, by setting monitoring points, the influence of the foot displacement of the leading edge on the slope surface displacement was studied.

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