Neutrino transitions $\nu \rightarrow \nu \gamma$, $\nu \rightarrow \nu e^+ e^-$ in a strong magnetic field as a possible origin of cosmological $\gamma$-burst

A.A. Gvozdev$^1$, A.V. Kuznetsov$^1$, N.V. Mikheev$^1$, L.A. Vassilevskaya$^{1,2}$
1. Yaroslavl P.G. Demidov State University, Yaroslavl, Russia
2. Moscow M.V. Lomonosov State University, Moscow, Russia

Abstract
The high energy neutrino transitions with the photon and electron-positron pair creation in a strong magnetic field in the framework of the Standard Model are investigated. The process probabilities and the mean values of the neutrino energy and momentum loss are presented. The asymmetry of outgoing neutrinos, as a possible source of sufficient recoil “kick” velocity of a remnant and the emission of $e^+ e^-$-pairs and $\gamma$-quanta in a “polar cap” region of a remnant, as a possible origin of cosmological $\gamma$-burst are discussed.

Talk presented at the International Workshop on Non-Accelerator New Physics (NANP-97), Dubna, Russia, July 7-11, 1997
1 Introduction

The absorption, emission, and scattering of neutrinos are of great importance in astrophysics. These combined processes occur in a star medium. Hot and dense star matter or plasma is usually considered as the medium. Of particular conceptual interest are those neutrino reactions in dense matter (or plasma) which are forbidden or suppressed substantially in a vacuum. Thus under favorable conditions the conversion of one neutrino flavor into another is greatly enhanced by the presence of the background. Some time ago it was observed[1] that the electromagnetic properties of neutrinos are also drastically modified within a dense matter when compared to the properties in vacuum. At the present time the neutrino propagation in magnetized dense matter is the subject of intensive investigations[2]. The effect of the background matter upon the neutrino transitions can lead to such interesting phenomena as the Cherenkov radiation by massless neutrinos[3] and the possible explanation of pulsar “kick” velocities by neutrino oscillations biased by a magnetic field[4].

The Cherenkov radiation $\nu(p) \rightarrow \nu(p') + \gamma(q)^*$ by massless neutrinos as well as the plasmon decay into $\nu\bar{\nu}$-pair is forbidden in vacuum. In the presence of a matter however neutrinos acquire an effective $\nu\nu\gamma$-coupling and the processes are kinematically allowed because the photon acquire essentially an effective mass square (positive or negative).

We stress that a strong magnetic field itself can also play the role of an active medium and in the presence of the magnetic field neutrinos also acquire an effective $\nu\nu\gamma$-coupling just as in a dense matter. It was observed that a field-induced amplitude of the massive neutrino radiative decay $\nu_i \rightarrow \nu_j + \gamma$ was not suppressed by the smallness of neutrino masses and did not vanish even in the case of the massless neutrino as opposed to the vacuum amplitude[5]. The photon dispersion in the strong magnetic field differs significantly from the vacuum dispersion with increasing of the photon energy[6], so the on-shell photon 4-momentum can appear as the space-like with sufficiently large value of $q^2$, $(|q^2| \gg m^2_\gamma)$. In this case the phase space for the neutrino transition $\nu_i \rightarrow \nu_j + \gamma$ with $m_i < m_j$ is opened also. It means that the decay probability of ultrarelativistic neutrino becomes insensitive to the neutrino mass spectrum due to the photon dispersion relation in the strong magnetic field. This phenomenon results in a strong suppression $(\sim m^2_\nu/E^2_\nu)$ of the neutrino transition with flavour violation, so a diagonal process $\nu_l \rightarrow \nu_l + \gamma$ $(l = e, \mu, \tau)$ is realized only. Thus, this diagonal radiative neutrino transition does not contain uncertainties associated with a possible mixing in the lepton sector of

---

[1] Here $i$ and $j$ enumerate the neutrino mass eigenstates but not the neutrinos with definite flavors.
the Standard Model, and can lead to observable physical effects in the strong magnetic fields.

Another decay channel also exists, $\nu_i \rightarrow \nu_j e^- e^+$, which is forbidden in vacuum when $m_i < m_j + 2m_e$. However, the kinematics of a charged particle in a magnetic field is that which allows to have a sufficiently large space-like total momentum for the electron-positron pair, and this process is possible even for very light neutrinos. It means that a flavor of the ultrarelativistic neutrino is also conserved in this transition in a magnetic field, to the terms of the order of $m_\nu^2/E_\nu^2$ regardless of the lepton mixing angles. Consequently, a question of neutrino mixing is not pertinent in this case and the process $\nu \rightarrow \nu e^- e^+$ can be considered in the frame of the Standard Model without lepton mixing.

Here we investigate the processes $\nu \rightarrow \nu \gamma$ and $\nu \rightarrow \nu e^- e^+$ for the high energy neutrino, $E_\nu \gg m_e$, in a strong constant magnetic field. We consider a magnetic field as the strong one if it is much greater than the known Schwinger value $B_e = m_e^2/e \approx 4.41 \cdot 10^{13} G$. These processes could be of importance in astrophysical applications, e.g. in an analysis of cataclysms like a supernova explosion or a coalescence of neutron stars, where the strong magnetic fields can exist, and where neutrino processes play the central physical role. At the present time there is some evidence that neutron stars with unusually strong magnetic field strength $B \approx 10^{15} \div 10^{16} G$ and even up to $3 \cdot 10^{17} G$ both for toroidal and for poloidal fields can exist in nature. Such high-field neutron stars (so-called “magnetars”) known as a model of SGRR – pulsars.

Here we calculate the probabilities of the processes and the mean values of the neutrino energy and momentum loss through the production of electron-positron pairs and photons in such a “magnetar”.

If the momentum transferred is relatively small, $|q^2| \ll m_\nu^2$, the weak interaction of neutrinos with electrons could be described in the local limit by the effective Lagrangian of the form

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \left[ \bar{e}_\gamma \gamma^\alpha (g_V - g_A \gamma_5) e \right] \left[ \bar{\nu}_\gamma \gamma^\alpha (1 - \gamma_5) \nu \right], \quad (1)$$

g_V = \pm \frac{1}{2} + 2 \sin^2 \theta_W, \quad g_A = \pm \frac{1}{2}.

Here the upper signs correspond to the electron neutrino ($\nu = \nu_e$) when both neutral and charged current interaction takes part in a process. The lower signs correspond to $\mu$ and $\tau$ neutrinos ($\nu = \nu_\mu, \nu_\tau$), when the neutral current interaction is only presented in the Lagrangian.

\[b\text{As the analysis shows, it corresponds in this case to the neutrino energy } E \ll m_W^3/eB.\]
The strong magnetic field is the only exotic we use.

2 The probability of the process $\nu \rightarrow \nu\gamma$

The field-induced $\nu\nu\gamma$-vertex can be calculated using an effective four-fermion weak interaction (1) of the left neutrino with the electron only, because the electron is the most sensitive fermion to the external field. By this means, the diagrams describing this process are reduced to an effective diagram with the electron in the loop.

The calculation technique for the loop diagram of this type was described in detail in the paper. We note that this effective $\nu\nu\gamma$ amplitude is enhanced substantially in the vicinity of the so-called photon cyclotronic frequencies. The same phenomenon in the field-induced vacuum polarization is known as the cyclotronic resonance.

As was first shown by Adler, two eigenmodes of the photon propagation with polarization vectors

$$\varepsilon^{(||)}_\mu = \frac{(q\varphi)_\mu}{\sqrt{q^2_\perp}}, \quad \varepsilon^{(\perp)}_\mu = \frac{(q\hat{\varphi})_\mu}{\sqrt{q^2_\parallel}}$$

are realized in the magnetic field, the so-called parallel ($||$) and perpendicular ($\perp$) polarizations (Adler’s notations). Here $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\hat{\varphi}_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\mu\nu} \varphi_{\mu\nu}$ are the dimensionless tensor and dual tensor of the external magnetic field with the strength $\vec{B} = (0, 0, B)$; $q^2_\parallel = (q\hat{\varphi}\hat{\varphi}q) = q_\alpha\hat{\varphi}_{\alpha\beta}\hat{\varphi}_{\beta\mu}q_\mu = q^2_0 - q^2_\parallel$, $q^2_\perp = (q\varphi\varphi q) = q^2_1 + q^2_2$.

It is of interest for some astrophysical applications the case of relatively high neutrino energy $E_\nu \simeq 10 \div 20 MeV \gg m_e$ and strong magnetic field $eB > E^2_\nu$. As the analysis of the photon dispersion in a strong magnetic field shows, a region of the cyclotronic resonance $q^2_\parallel \sim 4m_e^2$ in the phase space of the final photon, corresponding to the ground Landau level of virtual electrons dominates in the process $\nu \rightarrow \nu\gamma$. It is particularly remarkable that the $\perp$ photon mode only acquires a large space-like 4-momentum in the vicinity of the resonance. The corresponding amplitude of the process contains the enhancement due to the square-root singularity when $q^2_\parallel \rightarrow 4m^2_e$. It means that taking account of the many-loop radiative corrections is needed. A detailed description of this procedure will be published elsewhere. A general expression of the probability of the process $\nu \rightarrow \nu\gamma^{(\perp)}$ has a rather complicated form. Here we present the result of our calculation in the limit $eB \gg E^2_\nu \sin^2 \theta$:

$$W^{(\gamma)} \simeq \frac{\alpha G^2_F}{8\pi^2} (g^2_V + g^2_A) e^2 B^2 E \sin^2 \theta. \quad (3)$$
Here $E$ is the initial neutrino energy, $\theta$ is the angle between the vectors of the magnetic field strength $\vec{B}$ and the momentum of the initial neutrino $\vec{p}$. The dispersion relation for the $\parallel$ photon mode is close to the vacuum one, $q^2 = 0$, and it gives a negligibly small contribution to the probability in the limit considered.

The probability of the process $\nu \rightarrow \nu \gamma(\perp)$ is also non-zero above the threshold point of the $e^-e^+$-pair creation, $q^2 > 4m_e^2$, due to an imaginary part of the amplitude. However, the $\perp$-photon mode is unstable and another channel $\nu \rightarrow \nu e^-e^+$ dominates in this region.

3 The probability of the process $\nu \rightarrow \nu e^-e^+$

An amplitude of the process $\nu \rightarrow \nu e^-e^+$ could be immediately obtained from the Lagrangian (1) where the known solutions of the Dirac equation in a magnetic field should be used. We present here the results of our calculations of the probability in the strong field limit $eB \gg E^2 \sin^2 \theta$ which is of more importance in some astrophysical applications.

In the case when the field strength $B$ appears to be the largest physical parameter, the electron and the positron could be born only in the states corresponding to the lowest Landau level. Integrating over the phase space, one obtains the following expression for the probability in the limit $eB \gg E^2 \sin^2 \theta$

$$W^{(ee)} = \frac{G_F^2 (g_V^2 + g_A^2)}{16\pi^3} eBE^3 \sin^4 \theta. \quad (4)$$

4 The neutrino energy and momentum losses

It should be noted that a practical significance of these processes for astrophysics could be in the mean values of the neutrino energy and momentum losses rather than in the process probabilities. These mean values could be found from the four-vector

$$Q^a = E \int dW q^a = (I, \vec{F})E. \quad (5)$$

Its zero component is connected with the mean neutrino energy loss in a unit time, $I = dE/dt$. The space components of the four-vector (5) are connected similarly with the neutrino momentum loss in unit time, $\vec{F} = d\vec{p}/dt$. Here we present the expressions for $Q^a$ in the strong field limit for both processes:

$$I = EW C_1 \left( 1 + \frac{2g_V g_A}{g_V + g_A} \cos \theta \right), \quad (6)$$
\[ F_z = EW C_1 \left( \cos \theta + \frac{2gv g_A}{g_V^2 + g_A^2} \right), \quad F_\perp = EW C_2 \sin \theta, \]  

where the \( z \) axis is directed along the field, the vector \( \vec{F}_\perp \), transverse to the field, lies in the plane of the vectors \( \vec{B} \) and \( \vec{p} \),

\[ C_1^{(\gamma)} = \frac{1}{4}, \quad C_2^{(\gamma)} = \frac{1}{2}, \quad C_1^{(ee)} = \frac{1}{3}, \quad C_2^{(ee)} = 1. \]

We emphasize that all expressions for the processes \( \nu \to \nu e^- e^+, \nu \to \nu \gamma \) are applicable for the processes with antineutrino due to the \( CP \)-invariance of the weak interaction.

It should be mentioned also that our results are valid in the presence of plasma with the electron density \( n \sim 10^{33} \div 10^{34} cm^{-3} \) \( (n = n_{e^-} - n_{e^+}) \). This is due to a peculiarity of the statistics of the relativistic electron gas in a magnetic field.\( ^{15} \) As the analysis shows, the suppressing statistical factors in integrating over the phase space of the final particles do not arise at the conditions \( B > 10^{15} G (T/3 \text{ MeV})^2 \) and \( B > 5 \cdot 10^{15} G (n/10^{33} \text{ cm}^{-3})^{2/3} \).

### 5 Possible astrophysical consequences

To illustrate a possible application of our results we estimate below the neutrino energy and momentum losses in some astrophysical cataclysm of type of a supernova explosion or a merger of neutron stars. We assume that for some reasons a compact remnant has a very strong poloidal magnetic field, \( B \sim 10^{15} \div 10^{17} G \). Objects of such a type, so-called “magnetars”, were investigated in the paper.\( ^{12} \) According to standard astrophysical models\( ^{16} \) the neutrinos of all species with the typical mean energy \( \bar{E}_\nu \sim 20 \text{MeV} \) are radiated from a neutrinosphere in above mentioned astrophysical cataclysm. The electron density in the vicinity of neutrinosphere will be considered to be not too high, so a creation of the \( e^- e^+ \) pairs is not suppressed by statistical factors. In this case the neutrino propagating through the magnetic field will loose the energy and the momentum in accordance with our formulas. A dominant contribution to the total energy lost by neutrinos in the field due to the process of the \( e^- e^+ \) pair creation could be estimated from Eq. (6):

\[ \frac{\Delta E^{(ee)}}{E_{\text{tot}}} \sim 10^{-2} \left( \frac{B}{10^{17} G} \right) \left( \frac{\bar{E}}{20 \text{ MeV}} \right)^3 \left( \frac{\Delta \ell}{10 \text{ km}} \right), \]  

here \( \Delta \ell \) is a characteristic size of the region where the field strength varies insignificantly, \( E_{\text{tot}} \) is the total energy carried off by neutrinos in a supernova explosion, \( \bar{E} \) is the neutrino energy averaged over the neutrino spectrum. Here we take the energy scales which are believed to be typical for supernova explosions.\( ^{14} \)
An asymmetry of outgoing neutrinos is another interesting manifestation

\[ A = \frac{\left| \sum_i p_i \right|}{\sum_i |p_i|}. \]  

(9)

In the same limit of the strong field we obtain

\[ A^{(ee)} \sim 10^{-2} \left( \frac{B}{10^{17} G} \right) \left( \frac{E}{20 \text{MeV}} \right)^3 \left( \frac{\Delta \ell}{10 \text{km}} \right). \]  

(10)

One can see from Eqs. (8), (10) that the effect could manifest itself at a level of about percent. In principle, it could be essential in a detailed theoretical description of the process of supernova explosion. For the process \( \nu \rightarrow \nu \gamma \) one obtains

\[ A^{(\gamma)} \sim 2\pi \alpha \frac{eB}{E^2} A^{(ee)}. \]  

(11)

It is seen that two processes considered could be comparable for some values of physical parameters.

We note that \( e^+e^- \)-pairs and \( \gamma \)-quanta produced are captured by a strong magnetic field and propagate along the field. At first glance it seems that these particles are confined. However, the magnetosphere of a “magnetar” has a “polar cap” region which is defined as a narrow cone along the magnetic field axis with open lines of the magnetic field strength. So the particles which are created in the mentioned above neutrino reactions within a narrow cone can escape outside. We estimate below the neutrino energy loss in a “polar cap” regions of a millisecond “magnetar” taking into account that the both processes are comparable for values of the physical parameters used:

\[ \mathcal{E} \sim 10^{48} \left( \frac{\mathcal{E}_{\text{tot}}}{3 \times 10^{53} \text{erg}} \right) \left( \frac{B}{10^{17} G} \right)^2 \left( \frac{E}{20 \text{MeV}} \right) \left( \frac{R}{10 \text{km}} \right)^3 \left( \frac{10^{-3} \text{sec}}{P} \right)^2 \text{erg}, \]  

(12)

where \( \mathcal{E}_{\text{tot}} \sim 10^{53} \text{erg} \) is the typical total neutrino radiation energy; \( P \) is the “magnetar” rotation period; \( B \) is the magnetic field strength in the vicinity of the neutrinosphere of radius \( R \).

We pointed out that the energy loss (12) in terms of \( 4\pi \)-geometry is close to deposition of energy observed as \( \gamma \)-ray bursts (GRB’s). In a standard “fireball” model energy of order of \( 10^{51} \text{erg} \) is deposited in a small volume and results in an ultrarelativistic ejecta. A collision of the ejecta with intergalactic medium can be a source of the GRB (14). It is interesting that a rapid rotation of the remnant combined with the strong magnetic field becomes popular for understanding GRB’s afterglow.
6 Conclusions

• If the physical parameters would have the above-mentioned values, the effect of neutrino energy and momentum losses could manifest itself at a percent level. It could be essential in a detailed analysis of the process of a supernova explosion or merger of neutron stars.

• An origin of the asymmetry of the neutrino momentum loss with respect to the magnetic field direction is a manifestation of the parity violation in weak interaction (proportional to $g_V g_A$).

• This asymmetry results in the recoil “kick” velocity of a rest of the cataclysm. For the parameters used, it would provide a “kick” velocity of order 1000 km/s for a pulsar with mass of order of the solar mass.

• The $\gamma$ and $e^+e^-$ emission from the “polar cap” region of newborn “magnetar” could be observed as an anisotropic $\gamma$-burst with the duration of order of the neutrino emission time and of the energy $\sim 10^{50}$ erg in terms of $4\pi$-geometry.

Acknowledgements

The authors are grateful to Arnon Dar for useful critical remarks. This work is supported in part by the International Soros Science Education Program under the Grants N d97-872 (A.K.) and N d97-900 (A.G.).

References

1. L. Wolfenstein, Phys. Rev. D17 (1978) 2369; S.P. Mikheyev and A.Y. Smirnov, Nuovo Cimento 9C (1986) 17.
2. V.N. Oraevsky, V.B. Semikoz and Ya.A. Smorodinsky, JETP Lett. 43 (1986), 709;
J.C. D’Olivo, J.F. Nieves and P.B. Pal, Phys. Rev. D40 (1989) 3679.
3. V.B. Semikoz and J.W.F. Valle, Nucl. Phys. B425 (1994) 651; erratum Nucl. Phys. B485 (1997) 542;
H. Nunokawa, V.B. Semikoz, A.Yu. Smirnov and J.W.F. Valle, Preprint [hep-ph/9701420].
4. J.C. D’Olivo, J.F. Nieves and P.B. Pal, Phys. Lett. B365 (1996) 178.
5. A. Kusenko and G. Segre, Phys. Rev. Lett 77 (1996) 4872.
6. G.G. Raffelt, Phys. Rept. 198 (1990) 1.
7. A.A. Gvozdev, N.V. Mikheev and L.A. Vassilevskaya, Phys. Lett. B289 (1992) 103; B321 (1994) 108; Yad. Fiz. 57 (1994) 124 [Phys. At. Nucl. 57 (1994) 117].
8. A.A. Gvozdev, N.V. Mikheev and L.A. Vassilevskaya, Phys. Rev. D54 (1996) 5674.
9. S.L. Adler, Ann. Phys. N.Y. 67 (1971) 599;
   I.A. Batalin and A.E. Shabad, Zh. Eksp. Teor. Fiz. 60 (1971) 894 [JETP 33 (1971) 483];
   W.-Y. Tsai, Phys. Rev. D10 (1974) 2699.
10. G.S. Bisnovatyi-Kogan and S.G. Moiseenko, Astron. Zh. 69 (1992) 563
    [Sov. Astron. 36 (1992) 285];
    G.S. Bisnovatyi-Kogan, Astron. Astrophys. Trans. 3 (1993) 287.
11. C. Thompson and R.C. Duncan, Astrophys. J. 408 (1993) 194.
12. R.C. Duncan and C. Thompson, Astrophys. J. 392 (1992) L9.
13. A.E. Shabad, Ann. Phys. N.Y. 90 (1975) 166;
    A.E. Shabad and V.V. Usov, Astrophys. and Space Sci. 102 (1984) 327.
14. L.A. Vassilevskaya, A.A. Gvozdev and N.V. Mikheev, submitted to Yad. Fiz. [Sov. J. Nucl. Phys.]
15. L.D. Landau and E.M. Lifshitz, *Statistical Physics*, Pergamon Press, Oxford, 1980.
16. V.S. Imshennik and D.K. Nadyozhin, Usp. Fiz. Nauk 156 (1988) 561
    [Sov. Sci. Rev., Sect. E 8 (1989) 1];
    D.K. Nadyozhin, in: *Particles and Cosmology*, Proc. Baksan Int. School, eds. V.A. Matveev et al., World Sci., 1992, p.153;
    H.T. Janka and M. Ruffert, Astr. and Ap. 307 (1996) L33.
17. A. Dar and N.J. Shaviv, MNRAS. 277 (1995) 287;
    M.J. Rees and P. Meszaros, MNRAS. 258 (1992) 41.
18. B. Paszynski. Preprint astro-ph/9706232.