Effect of entropy changes on time to failure caused by tribological causes

V. V. Ryazanov

Institute for Nuclear Research, pr. Nauki, 47 Kiev, Ukraine, e-mail: vryazan19@gmail.com

The time until the failure of some node of the system or until the end of some stage of the operation of the tribological system is associated with the change in entropy in the system that occurs during this time. Methods of the first-passage time by a random process of some given level are used. A statistical distribution containing the first-passage time is introduced. The thermodynamic parameter associated with the random variable of the first-passage time is expressed in terms of the change in entropy. This parameter is also included in the expression for the time to failure. This approach allows arbitrary reasons for failures to be included in the consideration. The possibilities of the proposed approach are discussed.

Keywords: time to failure, entropy change, first-passage time.

1. Introduction

In works [1-17] the relations of entropy with problems of tribology are considered. In [13-17], it is shown how methods of statistical physics and probability theory are applied to tribological problems. However, the first-passage time (FPT) methods [18-21], which are effective in application to problems of the time to reach failure or the end of a certain stage of the process, have not been applied to tribological studies. For example, definitions of time to failure in tribology are directly related to FPT. The aim of this work is to apply the FPT methods to tribological problems. The methods proposed in the paper are based on the work [20]. In real physical systems, changes in entropy are constantly occurring. In this paper, we consider the effect of entropy changes on the time to failure caused by tribological reasons.

In work [20], the first-passage time (FPT) is associated with the entropy changes in the system that occur during this time. The impact of entropy changes on the mean FPT can be significant [21]. Such estimates can be made for the time until the failure of some node of the system or until the end of some stage of the operation process. Data on entropy changes are taken from tribological studies. The key issue is the choice of distribution for the FPT. One of the possible solutions can be the maximum entropy method with restrictions corresponding to the specifics of the process. This approach makes it possible to isolate the contributions from various processes (friction, wear, structural-phase transformations, physico-chemical processes, etc.), evaluate them, and choose the optimal ways of influencing the system.

We assume that the first-passage time is equal to the time to failure. What is first-passage time (FPT)? This is the time for the random process to reach some limit.

In the second section, the general scheme for taking into account entropy effects in the FPT method is considered. The third section considers the possibility of determining the thermodynamic parameter associated with FPT, which contains the entropy changes during the FPT. It is noted that a similar value is present in tribology. In the fourth section, a specific stochastic process model is selected, for which explicit expressions are written that characterize the achievement of a given level. In the fifth section, brief conclusions are given.

2. Statistical description of the expected service life
In [20–24], a generalization of the Gibbs distribution to the nonequilibrium case is introduced, containing FPT as a thermodynamic parameter. The introduction of such a distribution is based on the theory of large deviations and the maximum entropy method. In triboology, in addition to a large number of particles in a system, there are a large number of influences. For example, friction is accompanied by multiple influences. Both the initial size of tribological elements and their wear rates depend on a large number of independent factors. In this case, a statistical distribution is used, which contains the time it takes for a random process to reach a certain level (first-passage time [22, 23] or lifetime in terms of [24]).

In [20, 21, 22–24] the statistical distribution of the random variable of the system the first passage time (lifetime) \( T_\gamma \) was obtained. In the theory of random processes, a more general concept is used - the first-passage time it takes for a random process to reach a given level. The microscopic density of this distribution in the phase space of variables \( z \) (coordinates and momenta of all particles of the system) has the form

\[
\rho(z;u, T_\gamma) = \frac{\exp[-\beta u - \gamma T_\gamma]}{Z(\beta, \gamma)}, \quad \beta = 1/T_1, \tag{1}
\]

where \( u \) is the energy of the system (we assume this value to be a random process), \( T_1 \) is the temperature (in energy units),

\[
Z(\beta, \gamma) = \int \! \! d\mu \exp[-\beta u - \gamma T_\gamma] = \frac{1}{Z(\beta, \gamma)} \int \! \! d\mu \exp[-\beta u - \gamma T_\gamma], \quad \beta = 1/T_1, \tag{2}
\]

is the partition function. The transition from the left side of expression (2) to the right one corresponds to the transition from the microscopic description to the macroscopic one. The Lagrange parameters \( \beta \) (related to the reciprocal temperature) and \( \gamma \) are determined from the equations for the averages:

\[
\langle u \rangle = -\partial \ln Z / \partial \beta, \quad \langle T_\gamma \rangle = -\partial \ln Z / \partial \gamma, \quad \beta = 1/T_1. \tag{3}
\]

It is possible to introduce cells of an “extended” (compared to the Gibbsian) phase space with constant values \((u, T_\gamma)\) (instead of phase cells with constant values of \(u\)). The structure factor \( \omega(u) \) is replaced by \( \omega(u, T_\gamma) \) - the volume of the hypersurface in the phase space containing fixed values of \(u\) and \(T_\gamma\). If \( \mu(u, T_\gamma) \) is the number of states in the phase space with parameters less than \(u\) and \(T_\gamma\), then \( \omega(u, T_\gamma) = d^2 \mu(u, T_\gamma) / d\mu dT_\gamma \). In this case, \( \int \omega(u, T_\gamma) dT_\gamma = \omega(u) \).

The function \( \omega(u, T_\gamma) \) reflects the internal properties of the system. At the mesoscopic level, this function corresponds to a random process, and \( \omega(u, T_\gamma) \) is the joint probability density of unperturbed (when \( \gamma = 0 \) and \( \beta = \beta_0 \) is an equilibrium state (or some stationary non-equilibrium) and without external influences) values \(u, T_\gamma\), understood as a stationary distribution of this process. Distribution (1)-(2) generalizes the Gibbs distribution, in which \( \gamma = 0 \), and is valid for stationary non-equilibrium systems, including the ensemble of neutrons in the reactor (in the stationary state).

The joint distribution density of random variables \(u\) and \(T_\gamma\) is equal to

\[
p(u, T_\gamma) = \int \! \! d\omega (u-u(z)) \delta (T_\gamma - T_\gamma(z)) \rho(z;u(z), T_\gamma(z)) dz =
\]

\[
= \rho(u, T_\gamma) \omega(u, T_\gamma) \exp[-\beta u - \gamma T_\gamma \omega(u, T_\gamma)/Z(\beta, \gamma)]. \tag{4}
\]

Integrating (4) over \(T_\gamma\), we obtain a distribution for the energy \(u\) of the form

\[
p(u) = \int p(u, T_\gamma) dT_\gamma = \frac{e^{u\beta}}{Z(\beta, \gamma)} \int_0^{\infty} \omega(u, T_\gamma) e^{\gamma T_\gamma} dT_\gamma, \quad \beta = 1/T_1. \tag{5}
\]

In accordance with the assumptions about the form of the function \(\omega(u, T_\gamma)\), we rewrite it in the form
\[ \omega(u, T_r) = \omega(u) \omega_r(u, T_r) = \omega(u) \sum_{i=0}^{n} P_i f_i(T_r, u). \]  

(6)

Expression (6) assumes that the system has \( n+1 \) classes of ergodic states, \( P_i \) is the probability that the system is in the \( i \)-th class of ergodic states, \( f_i(T_r, u) \) is the probability density that in a system that is in ergodic states of the \( i \)-th class, the time of the first passage is \( T_r \). The expression under the integral on the right side of (5) determines the non-equilibrium part of the distribution.

If we choose the function \( f_i \) in (6) in the form of a gamma distribution

\[ f_i(x) = \frac{1}{\Gamma(\alpha_i)} b_i^{\alpha_i} x^{\alpha_i - 1} e^{-b_i x}, \quad x > 0, \quad f_i(x) = 0; \quad x < 0; \quad \int_0^\infty e^{-\gamma_i x} f_i(x) dx = (1 + \gamma_i b_i)^{-\alpha_i}, \quad (7) \]

where \( \Gamma(\alpha) \) is the gamma function, and put \( b_i \alpha_i = T_{y,0} \) (\( T_{y,0} \) are the unperturbed average lifetimes of the system in \( i \)-th states [20-24]), \( \alpha_i = \gamma_i / \lambda_i \), where \( \lambda_i \) is the intensity of entry into the \( i \)-th subsystem in a state of dynamic equilibrium, \( \gamma_i = \gamma \) in the \( i \)-th subsystem, then from (4)-(7) we obtain

\[ (1 + \gamma_i b_i)^{-\alpha_i} = (1 + \lambda_i T_{y,0})^{-\gamma_i / \lambda_i}; \quad p(u) = \exp[-\theta^{-1} u] Z(\omega(u) \sum_{i=0}^{n} P_i / (1 + \lambda_i T_{y,0})^{\alpha_i}. \quad \text{(8)} \]

If \( f_i \) is an exponential distribution, then \( \alpha_i = 1, \gamma_i = \lambda_i, b_i = T_{y,0}. \)

For the exponential distribution when \( \alpha_i = 1 \) in (7)-(8) and one class of ergodic states when \( n = 0, \)

\[ (1 + \gamma T_{y,0})^{-1} = <T_r>/T_{y,0}, \quad \text{where} \quad <T_r> \quad \text{is the average lifetime of the system obtained from (2)-(3), (6)-(7) [20-24].} \]

Let us define partition function (2). In this case, we use expressions (2)-(3) and the approximation that the distribution of the first passage time does not depend on the random value of energy, the variables are separated, and [20]

\[ \bar{u} = -\frac{\partial \ln Z}{\partial \beta} = u_{\mu} + u_{\gamma}, \quad u_{\mu} = -\frac{\partial \ln Z_{\beta}}{\partial \beta}, \quad u_{\gamma} = -\frac{\partial \ln Z_{\gamma}}{\partial \beta} = \int_0^\infty e^{\gamma T_{y}} \frac{\partial f(T_{y})}{\partial \theta^{-1}} dT_{y} \frac{1}{Z_{\gamma}}, \]

\[ Z(\beta, \gamma) = Z_{\beta} Z_{\gamma}, \quad Z_{\beta} = \int e^{-\mu u} \omega(u) du \quad Z_{\gamma} = \int_0^\infty e^{-\gamma T_{y}} f(T_{y}) dT_{y} \quad \beta = 1/T_{1}, \quad \text{(10)} \]

\[ Z_{\gamma} = \int_0^\infty e^{-\gamma T_{y}} \sum_{j=0}^{n} P_j f_j(T_{y}, \Phi) dT_{y}. \quad \text{(11)} \]

In (10), we assume that the parameters included in the distribution of the first passage time depend on the average values of the energy and flux, and not on their random values. In accordance with expressions (10)-(11), the partition function factor describing the nonequilibrium behavior of the system is expressed through the Laplace transform of the probability density that in a system that is in ergodic states of the \( i \)-th class, the time of the first passage is \( T_r \).

Besides distribution (7), many other distributions for FPT can be used. It depends on the specific task.

The parameter \( \gamma \) of distribution (1) will be related to entropy using the generally accepted definition of entropy in statistical physics as the logarithm of the distribution density (1) averaged over this distribution, \( s = \langle \ln \rho(z, u, T_r) \rangle \), where brackets denote averaging. Variables are separated as in (10). Wherein \( s = s_{\gamma} + s_{eq} \) (\( s \) is the entropy density)

\[ s_{\gamma} + s_{eq} = s = s_{eq} - \Delta = \gamma \bar{u} + \beta \bar{u} + \ln Z = \beta u_{\mu} + \ln Z_{\beta} - \Delta, \quad -\Delta = s_{\gamma}. \quad \text{(12)} \]
Stationary part of entropy is \( s_{eq} = s_\beta = \theta^{-1} u_\beta + \ln Z_\beta \). The Shannon entropy is equal to (13) \( \ln P(\Gamma) \) is the distribution of the random value \( \Gamma \),
\[
s_{\gamma} = - \int P(\Gamma) \ln P(\Gamma) d\Gamma.
\]
We equate expression (13) to the nonequilibrium part of the entropy from (12). Taking into account (10)-(13) and (1)-(3), we obtain
\[
-\Delta = s_{\gamma} = \gamma T_{\gamma} + \ln Z_{\gamma} + s_\beta = \gamma T_{\gamma} + \ln Z_{\gamma} + \beta \bar{u}_{\gamma},
\]
where the term \( s_\beta \) is determined by the explicit form of the distribution \( P(\Gamma) \). The right side of expression (14) was also obtained in [20] and follows from (10)-(13) and the definition of entropy as the average logarithm of the distribution (13) [20] of the relation
\[
-\Delta = s_{\gamma} = \gamma T_{\gamma} + \ln Z_{\gamma} + \beta \bar{u}_{\gamma}.
\]

3. Determination of the conjugate FPT thermodynamic parameter \( \gamma \).

The thermodynamic parameter \( \gamma \) of distribution (1), expressed in terms of entropy changes, can be determined in various ways. In this section, we will indicate three such methods. Let's start with a direct analogy between the parameter \( \gamma \) and the wear parameter in tribology. In [16, 17], the controlled parameter of the state of the tribosystem \( X_t \), which changes with wear, the linear size of one of the parts or their combination, is assumed to be a random variable. In the linear wear model, the state parameter \( X_t \) changes as
\[
X_t = X_0 + \Gamma_{yt},
\]
where \( \Gamma_y \) is a random variable with mean value \( \gamma_y \). The limiting (normative) value of the state parameter \( X_t \) is equal to \( X_{lim} \). In [16], the parameter \( \gamma_y \) is expressed in terms of the ratio of the nominal areas of contact and friction of the elements, the critical value of the measure of damage to materials, the speed of the relative movement of the surfaces, and the coefficients of conversion (absorption) of external energy by the surface layer of triboelements. In [17], this parameter is expressed through the volumetric wear of each interface element on the friction path and the friction area of the elements. In [17], the degradation or linear wear rate parameter is defined as
\[
\gamma_y = \nu \frac{p_{max} f_{mech} V_{sl}}{\Delta u_c^*}.
\]
Here \( \nu \) is the coefficient of absorption of external energy by the surface layer of the sealing material, determined as a function of the physical and microgeometric characteristics of the friction pair elements [17]; \( p_{max} \) is the maximum nominal pressure at the contact of the plunger and the seal during operation, determined using a known technique for interference mates; \( f_{mech} \) is the mechanical component of the friction coefficient; \( V_{sl} \) - speed of relative sliding of elements; \( \Delta u_c^* \) is critical density of internal potential energy (critical energy capacity) of the material of the contact volumes of the seal. The critical energy intensity is defined as [17]:

\[
\Delta u_c^* = \Delta H_S - \Delta H_T. \text{ Here } \Delta H_S \text{ is the enthalpy of melting of the sealing material in the liquid state at the melting temperature } T_S; \Delta H_T = \int_0^T \rho c dT \text{ are enthalpy of the material at temperature } T
\]

of the steady friction mode, which, in turn, is determined by known methods; \( \rho \), \( c \) – density and heat capacity of the sealing material at temperature \( T \). The numerical characteristics of the random variables \( \Gamma_y \) included in the conditions (15) are determined by equation (16) as a function of the numerical characteristics of the mechanical component \( f_{mech} \) of the friction coefficient. They, in turn, are determined as a function of the numerical characteristics of the
complex index of plunger surface roughness over the range of reference data for run-in surfaces according to the “three sigma” rule [25].

The second approach to determining this quantity is related to the expression for the distribution entropy for FPT, as in [26]. This equation is written below as (20).

In the third approach, as in [20], an explicit expression for the change in entropy is used, which is assumed to be known, and the equation (14).

In [20], the equation for determining the parameter was written in terms of the change in entropy. For example, for the case of a phase transition, the change in entropy is

$$\Delta S = \frac{\Delta H}{T_{\text{phase}}}.$$  \hspace{1cm} (17)

Phase transitions are associated with surface melting and recrystallization of metals, occurring with a change in entropy. Here $\Delta H$ is the change in enthalpy, the latent heat absorbed or shed during the phase transition, and $T_{\text{phase}}$ is the temperature associated with the phase transition. In [27], the entropy change is written in a more general form

$$\Delta S = \Lambda_{D,PH} \left[ \frac{U_c}{T_m} \Delta V + \frac{G - 2\gamma_o}{T_{cr}} \Lambda_D + \frac{\Delta H}{T_{\text{phase}}} \right],$$  \hspace{1cm} (18)

where $\Lambda_D$ is the function (coefficient) of the interaction between the entropy components during surface plastic deformation and cracking, $\Lambda_{D,PH}$ is the function (coefficient) of the interaction between the entropy components during mechanical failure and changes in material properties during the phase transition. Plastic deformation associated with abrasive wear, frictional ploughing, and/or cutting, with a change in entropy. Here $U_c$ is the work (plastic deformation or cutting) spent per unit of affected volume $\Delta V$, and $T_m$ is the temperature of the acting material medium. Fracture is associated with fatigue wear and surface damage, with a change in entropy. Here $a$ is the crack length, $G = -\partial U_S / \partial a$ is the energy release rate depending on the strain energy $U_S$, $\gamma_o$ is the surface energy, and $T_{cr}$ is the temperature of the cracked material at the crack tip.

The equation for determining the parameter $\gamma$ is associated with equation (14) or with the expression

$$-\Delta = s_{\gamma} = \gamma \bar{T}_\gamma + \ln Z_{\gamma} + \beta \bar{u}_\gamma,$$  \hspace{1cm} (19)

This equation after substituting the expression for entropy (13) into it and in the case of using the explicit form of distribution (22)-(23) has a complex form

$$s_{\gamma} = A_0 - \frac{1}{Z_{\gamma}} \sqrt{\frac{J_{\sigma_s} \langle t \rangle}{4\pi}} e^{\frac{J_{\sigma_s} \langle t \rangle}{4}} \left[ \frac{3}{2} A_1 + \frac{J_{\sigma_s}}{4} (A_2 + (t^2) A_3) \right],$$  \hspace{1cm} (20)

$$A_0 = \ln \sqrt{\frac{J_{\sigma_s} \langle t \rangle}{4\pi}} + \frac{J_{\sigma_s}}{2} \langle t \rangle,$$

$$A_1 = \pi [\ln((J, \sigma_s / 4) + \gamma I_{1/2}(y) + \ln((J, \sigma_s / 4) \langle t \rangle^2) I_{-1/2}(y) -$$

$$-((J, \sigma_s / 4) + \gamma)^{1/2} \sum_{k=0}^{\infty} \frac{((J, \sigma_s / 4) + \gamma)((J, \sigma_s / 4) \langle t \rangle^2)^k}{k! \Gamma(k + 3/2)},$$

$$\times \Psi(k + 3/2) - ((J, \sigma_s / 4) \langle t \rangle^2)^{-1/2} \sum_{k=0}^{\infty} \frac{((J, \sigma_s / 4) + \gamma)((J, \sigma_s / 4) \langle t \rangle^2)^k}{k! \Gamma(k + 1/2)} \Psi(k + 1/2)],$$

$$A_2 = \sqrt{\frac{\pi}{((J, \sigma_s / 4) + \gamma)}} \exp[-2 \langle t \rangle \sqrt{(J, \sigma_s / 4)((J, \sigma_s / 4) + \gamma)}].$$
Determining the parameter $\gamma$ from expression (1) from this equation (20) is a difficult task and is possible under certain approximations.

If expression (18) is substituted into (19) and a stochastic model of the form (22)--(25) is used, then we obtain an equation for $\gamma$ with solution (26).

4. Stochastic process model

It is required to establish an adequate correspondence between a physical phenomenon and a random process used for its mathematical modeling. There are many random processes that can be adapted to a specific task. The properties of the process can be significantly affected by various circumstances, for example, changes in the boundary conditions [28, 29].

The same phenomenon under different conditions can exhibit different properties and be described by different random processes, distributions and their Laplace transforms. Examples of this kind are given in [30]. For our purposes, the Laplace transforms of the FPT distributions are more important than the distributions themselves. This facilitates the analytical solution of the problem.

The phenomenon under study is modeled by some random process. The functional of this random process is the FPT statistics. The exact analytical form of the solution for the FPT is determined in rare cases. It is possible to use general results for the FPT, for example, the first few terms of the expansions obtained in [31-32] for the Laplace transform of the FPT. There are other options for selecting and configuring FPT statistics models. The use of various types of approximations also significantly depends on the stage of system evolution [33, 34].

The proposed approach is based on probabilistic mathematical relations and has great generality. The area of applicability of the results is large and, possibly, corresponds to the area of applicability of the Gibbs statistics. This generality allows one to consider arbitrary distributions rather than being restricted to any one particular form of FPT.

Let us illustrate the application of the above approach by examples. In [35] and [36] FPTs are considered for achieving a certain level of entropy production and flux values. The above possibilities of connecting the FPT distribution with the thermodynamic characteristics of the system are applicable to this situation, as well as to others cases. Consider reaching a certain value of the parameter limit value $X$ (15) $X_{\text{lim}}$ level. As an independent thermodynamic parameter, we choose a random time mark $T_{\text{pt}}$ of the first achievement of a given $X$ value $X_{\text{lim}}$. Adding this parameter, along with the energy, to the exponential distribution, as in (8), we write the relation for the entropy of the form (12), (14) taking into account (10), (11)

$$s_{\text{pt}} = \beta u + \gamma T_{\text{pt}} + \ln Z = \beta u_T + \ln Z_T - \Delta, \quad u_T = u_T + u, \quad -\Delta = \beta u_T + \gamma T_{\text{pt}} + \ln Z_T, \quad \Delta = s_T - s_{\text{pt}}.$$  

As in [36], for the FPT distribution density of the X level $X_{\text{lim}}$, we use the inverse Gaussian distribution (Wald's distribution) with a density of the form

$$F_{\text{lim}}(t) = \frac{X_{\text{lim}} \sigma_T (t)}{4\pi t^3} \exp \left[ -\frac{X_{\text{lim}} \sigma_T (t - \langle t \rangle)^2}{4t} \right],$$  

where $\sigma_T$ is entropy production rate (with Boltzmann's constant $k_B = 1$), $\langle t \rangle$ is average (with $\gamma = 0$) value of FPT. The quantity $X$ from (22) in this case is equal to the value $X$ (15). The Laplace transform of distribution (22) is
\[ Z_{\gamma} = \int_{0}^{\infty} e^{-\gamma T} F_{0}\left(T_{0}\right) dT = \exp\left\{ \frac{aT_{0}}{2} \left(1 - \sqrt{1 + 4\gamma/a}\right)\right\}, \quad a = X_{\text{lim}} \sigma_{\gamma}, \quad (23) \]

where \( \langle t \rangle = T_{0} = T_{0} \) is the value of \( T_{0} \) in the absence of disturbances, at \( \gamma = 0 \). The expression obtained from relation (23) for the average time to reach the level \( X_{\text{lim}} \) is

\[ T_{\gamma} = -\gamma \frac{\partial \ln Z_{\gamma}}{\partial \gamma} = \frac{T_{0}}{\sqrt{1 + 4\gamma/a}}, \quad T_{0} = \langle t \rangle. \quad (24) \]

In expression (21) it is necessary to use the value \( u_{\gamma} = -\gamma \frac{\partial \ln Z_{\gamma}}{\partial \beta} \) \( (3), (10), (11). \)

Considering \( \partial a / \partial \beta, \partial T_{0} / \partial \beta \), we obtain

\[ -\beta u_{\gamma} = \frac{1}{2} \left(1 - \sqrt{1 + 4\gamma/a}\right) \beta(T_{0} \frac{\partial a}{\partial \beta} + a \frac{\partial T_{0}}{\partial \beta}) + \beta \frac{\partial a}{\partial \beta} a(\sqrt{1 + 4\gamma/a}). \quad (25) \]

Substituting (24), (25) into (21), we obtain, taking into account (11), a quadratic equation to determine the dependence \( \gamma(\Delta) \) form \( a_{\gamma}(\gamma)^{2} + b_{\gamma}(\gamma) + c_{\gamma} = 0 \) with the solution

\[ \gamma = \frac{b_{\gamma}}{2a_{\gamma}} \left[\sqrt{1 - 4a_{\gamma}c_{\gamma}/b_{\gamma}^{2}} - 1\right]. \quad (26) \]

The “+” sign is selected to match the condition \( \gamma|_{\Delta=0} = 0 \). In (26)

\[ a_{\gamma} = a_{\gamma}^{2}, \quad a_{\gamma} = \beta \frac{\partial a}{\partial \beta} T_{0} + 2\beta \frac{\partial T_{0}}{\partial \beta} - T_{0}, \quad b_{\gamma} = -k_{3} \beta \frac{\partial T_{0}}{\partial \beta} + 4c_{\gamma}, \quad c_{\gamma} = -k_{3}\Delta - \Delta^{2}, \quad k_{3} = -\beta \frac{\partial(aT_{0})}{\partial \beta} + aT_{0} \]

Substitution of (26) into (24) gives the relation

\[ T_{\gamma} = \frac{T_{0}}{\sqrt{1 + 4\Delta(\Delta + k_{3}) + ak_{3}\beta T_{0} / \partial \beta} \left[1 - \frac{4\Delta(\Delta + k_{3})k_{3} a \beta T_{0} / \partial \beta} {4\Delta(\Delta + k_{3}) + ak_{3}\beta T_{0} / \partial \beta} \right]}]. \]

Knowledge of the value \( \gamma \) (26) also makes it possible to use the probabilistic interpretation of the Laplace transform

\[ Z_{\gamma} = \int_{0}^{\infty} e^{-\gamma T} P(T_{\gamma}) dT_{\gamma} = P(T_{\gamma} \leq \tau), \quad P(\tau > t) = e^{-\gamma t}. \]

The same distribution (22) \([38]\) is written in \([39]\) and is applied in \([40]\) to various problems, including for FPT level \( L \) in drift-diffusion process. The Langevin equation for drift-diffusion process is

\[ \frac{dX_{i}(t)}{dt} = v + \zeta(t), \]

where \( X_{i}(t) \) is the coordinate, \( v = F/\zeta \) is the drift velocity, \( F \) is an external force, \( \zeta \) is a friction coefficient, and \( \zeta(t) \) is a Gaussian white noise with zero mean \( \langle \zeta(t) \rangle = 0 \) and with autocorrelation \( \langle \zeta(t) \zeta(t') \rangle = D \delta(t-t') \), \( D = k_{b}T/\zeta \) is the diffusion coefficient. We consider a system containing \( N \) different constituents labeled \( k = 1, 2, ..., N \) with mass density \( \rho_{k} \) and velocity \( v_{k} \);

\[ \rho = \sum_{k=1}^{N} \rho_{k} \]

is total density and the center-of-mass velocity is \( \rho v = \sum_{k=1}^{N} \rho_{k} v_{k} \), \( J_{k} \) is diffusion flux. In [35] the FPT distribution for \( X_{i} \) to pass at time \( T_{\gamma} = t \) for the FPT the threshold \( L>0 \), starting from the initial condition \( X_{i}(0) = 0 \), is given by Wald's distribution \([38, 35]\)

\[ F_{\gamma}(t) = \frac{|L|}{4\pi D t^{3}} \exp\left\{-\frac{(L-vt)^{2}}{4Dt}\right\}. \quad (27) \]

Distributions (22) and (27) coincide for \( \langle t \rangle = L/v, \ a = v^{2}/D \) in (28) (in (22)-(23) \( a = X_{\text{lim}} \sigma_{\gamma} \)). For the deviation of entropy from the equilibrium value during diffusion in the time

\( T_{\gamma} \) can write the expression from EIT \([37]\)
\[ \Delta = s_{eq} - s = \dot{s} T_\gamma / \rho, \quad \dot{s} = \sum_{k=1}^{N} \frac{\mu_k}{T} \nabla J_k, \quad J_k = \rho_k (v_k - v), \quad (28) \]

\( \mu_k \) is the chemical potential. Expression for \( T_\gamma \), we write from (23) - (24), (27). From the ratio

\[ \beta u_x + \gamma \dot{T}_\gamma + \ln Z_\gamma = -\Delta = -\dot{s} T_\gamma / \rho \quad (29) \]

we obtain a quadratic equation for \( \gamma \) with the solution

\[ \gamma = \frac{1}{M_1} \left[ c_1 \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta) \right] (1 - \sqrt{1 + \frac{\dot{s}}{\rho} c_1 \beta^2 (\dot{\gamma} T_0 / \dot{\gamma} \beta)^2 M_1 (M_1 + \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta)) + \frac{\dot{s}}{\rho} \dot{T}_0}, \]

\[ T_0 = \frac{T_0}{\gamma^2 + \gamma a} \left[ c_1 \frac{a T_0}{2} - \beta \frac{\partial(a T_0)}{\partial \beta}, \quad M_1 = T_0 (1 - \frac{\beta}{a} \frac{\partial a}{\partial \beta}) - 2 \beta \frac{\partial (a T_0)}{\partial \beta}. \quad (30) \]

At small \( \dot{s} \), when

\[ \left[ 1 + \frac{\dot{s}}{\rho} \frac{2 \dot{\gamma} T_0 M_1 (M_1 + \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta))}{\dot{\gamma} \beta} \right] \approx 1 + \frac{\dot{s}}{\rho} \frac{2 \dot{\gamma} T_0 M_1 (M_1 + \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta))}{\dot{\gamma} \beta}, \quad \gamma \approx -\frac{\dot{s}}{\rho} \frac{\dot{T}_0}{\gamma \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta)}. \]

If we assume that the energy of the diffusing particle is \( u = m v^2 / 2 = u_\gamma \), then \( u_\gamma = 0 \).

Derivatives of \( v \) and \( a \) are equal

\[ \frac{\partial T_0}{\partial \beta} = -\frac{T_0}{v} \frac{\partial v}{\partial \beta}, \quad \frac{\partial a}{\partial \beta} = a \left( \frac{2 \partial v}{v} - \frac{1}{D} \frac{\partial D}{\partial \beta} \right), \quad \frac{\partial D}{\partial \beta} \approx -E_a D, \quad E_a \]

the activation energy. From the condition \( u_\gamma = 0 \) we find

\[ \frac{1}{v} \frac{\partial v}{\partial \beta} = -E_a \frac{\sqrt{1 + 4 \gamma/a - 1 - 2 \gamma/a}}{\sqrt{1 + 4 \gamma/a - 1}}. \quad (31) \]

By \( \gamma \rightarrow 0 \), \( \partial v / \partial \beta \rightarrow 0 \);

\[ \frac{1}{T_0} \frac{\partial T_0}{\partial \beta} = -E_a \frac{\sqrt{1 + 4 \gamma/a - 1 - 2 \gamma/a}}{\sqrt{1 + 4 \gamma/a - 1}} - \frac{1}{a} \frac{\partial a}{\partial \beta} = -E_a \frac{\sqrt{1 + 4 \gamma/a - 1 - 4 \gamma/a}}{\sqrt{1 + 4 \gamma/a - 1}}, \quad M_1 = T_0 (1 - \beta E_a). \quad (32) \]

For small \( \gamma \), \( \frac{1}{T_0} \frac{\partial T_0}{\partial \beta} \approx -E_a \frac{2 \gamma}{a}, \quad \gamma^2 \approx -\frac{\dot{s}}{\rho} \frac{a}{2 \beta E_a} \). Substitution of (30) into (24) gives the relation

\[ \dot{T}_\gamma = \sqrt{1 + \frac{4}{a M_1} \left[ c_1 \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta) \right] \left(1 - \sqrt{1 + \frac{\dot{s}}{\rho} c_1 \beta^2 (\dot{\gamma} T_0 / \dot{\gamma} \beta)^2 M_1 (M_1 + \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta)) + \frac{\dot{s}}{\rho} \dot{T}_0} \right]} \quad (33). \]

Substitution of expressions (31) - (32) into relation (30) leads to an equation for \( y = \sqrt{1 + 4 \gamma/a - 1}, \quad \gamma = ay(y + 2) / 4 \), of the form

\[ y^3 \beta E_a + y^2 (5 \beta E_a / 2 - 1) - y(1 - \beta E_a) - 4(\frac{\dot{s}}{a \rho}) = 0. \quad (34) \]

When \( \gamma \rightarrow 0, \quad y \rightarrow 0 \). The diffusion coefficient can be written as

\[ D = \frac{L^2}{\dot{T}_\gamma} = D_0 \frac{T_0}{\dot{T}_\gamma} = D_0 \sqrt{1 + \frac{4}{a M_1} \left[ c_1 \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta) \right] \left(1 - \sqrt{1 + \frac{\dot{s}}{\rho} c_1 \beta^2 (\dot{\gamma} T_0 / \dot{\gamma} \beta)^2 M_1 (M_1 + \beta (\dot{\gamma} T_0 / \dot{\gamma} \beta)) + \frac{\dot{s}}{\rho} \dot{T}_0} \right]}, \quad D_0 = \frac{L^2}{T_0} \quad (35) \]

For small \( \gamma \), when \( y \sim \gamma \), neglecting in (34) powers \( y \) higher than two, we obtain a quadratic equation for \( y \) with the solution

\[ y = \frac{1 - \beta E_a}{2 (5 \beta E_a / 2 - 1)} \left[1 - \sqrt{1 + \frac{16 \dot{s}}{a \rho} \left(5 \beta E_a / 2 - 1 \right)^2} \right]. \quad (36) \]

The “–“ sign is selected to get \( y > 0, \quad \gamma > 0 \). Then expressions (33) and (35) take on the form
\[
\gamma = R \left[ \frac{2R}{aT_0^2} + \frac{1}{T_0} \sqrt{1 + \frac{4R^2}{(aT_0)^2}} \right], \quad R = \sqrt{K(K+aT_0)}, \quad \bar{T}_\gamma = T_0 / \sqrt{1 + \frac{4R^2}{aT_0^2} + \frac{2R}{aT_0} + \frac{1}{1 + \frac{4R^2}{(aT_0)^2}}}.
\]

Relation (38) can be rewritten as
\[
\bar{T}_\gamma = \frac{T_0}{\sqrt{1 + 2p(p + \sqrt{1 + p^2})}}, \quad D = D_0 \sqrt{1 + 2p(p + \sqrt{1 + p^2})}, \quad p = 2R/aT_0.
\]

Once again, we note that for each specific task, it is necessary to look for that random process that most fully corresponds to this particular task.

5. Conclusion

Distribution (22) has entropy production rate \( \sigma_s \) as a parameter. There are several ways to set this option. Thus, the expression for \( \Delta S \) (18) can be divided by \( \bar{T}_\gamma \) (24), obtaining a third-order equation for \( \sigma_s \). A simpler approach involves using the time until the value \( X_{\text{lim}} \) is reached, [16, 17, 25].

\[
\bar{T} = (x_{\text{lim}} - x_0) / \bar{T}.
\]

Then we put in (22)
\[
\sigma_s = \Delta S / \bar{T} = \Delta S / (x_{\text{lim}} - x_0).
\]

If we substitute (41) into (24) and use expression (16) for the parameter \( \gamma \), then
\[
\bar{T}_\gamma = -\frac{\partial \ln Z_\gamma}{\partial \gamma} = \frac{1}{\sqrt{1 + 4p^2}} / a, \quad T_0 = \langle \tau \rangle, \quad a = \sigma_s x_{\text{lim}}.
\]

Here expression (24) is used. You can also use expressions (33), (37), (39), etc. The value of the lifetime \( T_0 = \langle \tau \rangle \) is assumed to be known, its change is determined under the influence of the processes occurring in the system. For distribution (22)-(23), the mean time to failure can only decrease. There are distributions for which an increase \( T_0 = \langle \tau \rangle \) is possible in some intervals of entropy, for example, the Weibull distribution, Feller processes, etc. This raises the following problem. We use the quantity (40), which is the desired one. However, the value (40) was obtained in [16, 17, 25] in other ways. We consider this value as some zero approximation.

In [41] the entropy change rate which reflects the relation between the entropy and cumulative plastic strain can be determined:
\[
\dot{s} = -\frac{N_s k_n \alpha}{m_s \ln(p_{cr} / p) f(\sigma_m / \sigma_{eq})} \left[ \dot{f} (\sigma_m / \sigma_{eq}) \ln(p_{cr} / p) - f(\sigma_m / \sigma_{eq}) \dot{p} / p \right],
\]

where \( \alpha \) is the damage exponent; \( \sigma_{eq} \) is the equivalent von Mises stress, \( \sigma_m \) is the hydrostatic stress or mean stress (\( \sigma_m = \sigma_{kk} / 3 \)). For uniaxial loading, the factor \( f(\sigma_m/\sigma_{eq}) = 1 \), cumulative
plastic strain rate is $\dot{p}$, $p$ is the cumulative plastic strain. The corresponding cumulative plastic strain when $D = D_0$ and $D = D_{cr}$ are $p_{th}$ and $p_{cr}$, respectively; $D$ is the damage variable; $D_{cr}$ is the value of damage variable when final failure occurs, $N_0$ is the Avogadro constant, $m_s$ is the specific mass, $k_0$ is the Boltzmann constant. One can equate $\dot{s} (42)$ to $\sigma_s$. The expression for $\sigma_s$ was also obtained in [43] and is also written using the results of [42].

The problems of determining the parameters $\gamma$ and $\sigma_s$ are related. If the values $\sigma_s$ from distributions (22) and (23) (or $\dot{s}$ from (42)) are known, then multiplying this value by $\bar{T}_s$ and substituting into equation (21), we obtain an equation for determining the parameter $\gamma$, for example, expression (29) with the solution (30). If the value $\Delta S$ is known, then substitution of this value into relation (21) gives an equation for the parameter $\gamma$ with solution (26). Instead of distribution (22)-(23), many other distributions can be used, in some situations more adequately corresponding to one or another problem being solved. For example, models of random walks, death and birth, etc. It is possible to combine different methods for determining the parameters $\gamma$ and $\sigma_s$, depending on which method is more convenient, more accurately or better corresponds to the task. Thus, the proposed approach provides considerable freedom in solving specific problems.

As in [25], using the proposed methods, it is possible to consider the reliability of plunger hydraulic cylinders of the system for balancing rolls of rolling mills according to the criterion of wear resistance of sealing elements.

The relations obtained are applied to these and many other problems of tribology. Entropy changes include all possible effects on the system. This allows the description to include the behavior and use of lubricants, corrosion-fatigue degradation mechanism, the processes of friction, wear and fatigue, the processes of sliding friction with wear under sliding and mechano-sliding fatigue, etc.

Similar methods that do not use a distribution containing FPT were used in [44] and in [45]. It should be noted the general similarity between the methods of applying FPT and tribological approaches. So, in this work, a comparison of the parameter of the rate of degradation or linear wear of triboelements with the thermodynamic parameter associated with FPT is noted and used.

It was shown in [16] that the rate of degradation of a loaded element is determined by the rate of damage to the structure of its material, i.e., the rate of accumulation of the latent energy density of defects. This value is expressed through the activation energy of the process of destruction of interatomic bonds and other parameters. These relations can be used in expressions (12)-(14) along with non-equilibrium energy of the form (10).

In [16], the design evaluation of the reconstruction of the rotary drive of the kiln according to the criterion of the strength of the foundation bolts was considered; - to increase the durability of the balancing system of rolls according to the criteria of wear resistance of the sealing elements of the actuating hydraulic motors; - to extend the service life of the system for cleaning hot-rolled strips from scale according to the criteria of wear resistance of friction pairs of hydraulic distributors. Problems of this kind can be solved using the methods of this work.

One of the most common reasons for the downtime of metallurgical units is the failure of spur gears. In some cases, the failure is not due to chipping or breakage of the teeth, but to the limit of wear. If the methods of design assessment of the efficiency of the spur gears according to the criterion of contact and bending strength are widely known, then predicting their durability according to the criterion of wear resistance is difficult. The results of this work are applicable to the solution of this problem.

An example of the application of the above approach can be the use of the above considerations to the probabilistic method implemented in [25] for predicting the reliability of plunger hydraulic cylinders of the system for balancing rolls of rolling mills according to the criterion of wear resistance of sealing elements.
The equations obtained on the basis of a general methodological approach to predicting the reliability of plunger hydraulic cylinders according to the criterion of wear resistance of seals make it possible, even at the design stage, to study the process of formation of their wear failures, to track the change in the level of their reliability indicators and to evaluate the resource characteristics in the expected operating conditions. Given the change in entropy, the time to failure can be determined using the above approach.

The use of an FPT-based approach and the relationship of FPT to entropy change provides many new and promising possibilities for the study of tribological systems.

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