Dynamic modeling and simulation of 4X4 wheeled vehicle

Yijie Chen, Yafeng Zhang, Fu Du, Le Wang and Yiqiang Wan
China North Vehicle Research Institute, No.4, Huaishuling, Fengtai District, Beijing, China

1 E-mail: chenyijie1206@163.com

Abstract. In view of the requirements of military vehicles for high-mobility off-road performance, the dynamic modeling of 4X4 wheeled armored vehicles was carried out. The dynamic model expression expressed in matrix form was derived, and the decoupling calculation was performed by Fourier transform. After that, the power spectral density function was used to characterize the random road input of the vehicle, and the coherence of each wheel excitation in the four-wheeled vehicle was analyzed to derive the coherence coefficient. Finally, the calculation method of vibration response of vehicles under typical roads was formed by programming analysis and reasonable algorithm, and it provided theoretical support for suspension vibration isolation performance design.

1. Introduction
A large number of test data showed that the suspension characteristics determine the ride comfort and steering stability of the vehicle, which is the key to the high-mobility off-road performance [1]. Therefore, it is necessary to carry out matching optimization of suspension characteristics and analyze the vibration response of the vehicle. Based on this, it is especially necessary to construct the vehicle dynamic model and compile the random road spectrum as the design input. It is proposed to evaluate the matching effect of the elastic and damping characteristics by monitoring the vibration acceleration response of the vehicle body to achieve the purpose of improving the driving performance of the vehicle.

Figure 1. Physical model of the vehicle.
2. Dynamic model of the vehicle

2.1. Physical model
Taking the 4X4 wheeled armored vehicle as an example, the physical model is constructed as shown in Figure 1, which has 7 freedom degrees including 4 degrees of freedom for vertical direction of the unsprung mass, and 3 degrees of freedom for vertical, pitch and roll directions of the car body. Point O is mass center of body, and the vehicle travels along the X direction. The suspension system consists of elastic and damping elements, and the stiffness of the tire is the main consideration of the unsprung mass.

2.2. Model deduction
According to the vibration theory, the differential equation of vehicle dynamics is: [2]

$$M \ddot{X} + C \dot{X} + KX = F = K_yQ$$

In which, $$X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T$$, $$Q = [q_1, q_2, q_3, q_4]^T$$.

Taking into account the symmetry of the vehicle and the actual use, make the following assumptions about suspension stiffness and damping:

$$k_{s1} = k_{s2} = k_{s3}; k_{s4} = k_{s}; c_{d1} = c_{d2} = c_{d3}; c_{d4} = c_{d5}; k_{t1} = k_{t2} = k_{t3}; k_{t4} = k_{tr}$$

Spread differential equations in a matrix, among the mass matrix is:

$$M = \begin{bmatrix}
    m_p & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & I_r & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & I_r & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & m_{s1} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & m_{s2} & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & m_{s3} & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & m_{s4}
\end{bmatrix} = \text{diag}(m_p, I_r, m_{s1}, m_{s2}, m_{s3}, m_{s4})$$

The damping matrix is:

$$C = \begin{bmatrix}
    2(c_{s1} + c_{s2}) & 0 & -2(l_{c1}c_{s2} - l_{c1}c_{s1}) & -c_{s1} & -c_{s2} & -c_{s3} & -c_{s4} \\
    0 & 2(l_{c1}c_{s2} + l_{c1}c_{s1}) & 0 & -l_{c1}c_{s2} & l_{c1}c_{s1} & 0 & 0 \\
    -c_{s1} & -l_{c1}c_{s2} & l_{c1}c_{s1} & c_{s1} & 0 & 0 & 0 \\
    -c_{s2} & l_{c1}c_{s2} & l_{c1}c_{s1} & 0 & c_{s2} & 0 & 0 \\
    -c_{s3} & 0 & 0 & 0 & c_{s3} & 0 & 0 \\
    -c_{s4} & 0 & 0 & 0 & 0 & c_{s4} & 0
\end{bmatrix}$$

The stiffness matrix is:

$$K = \begin{bmatrix}
    2(k_{s1} + k_{s2}) & 0 & -2(l_{k1}k_{s2} - l_{k1}k_{s1}) & -k_{s1} & -k_{s2} & -k_{s3} & -k_{s4} \\
    0 & 2(l_{k1}k_{s2} + l_{k1}k_{s1}) & 0 & -l_{k1}k_{s2} & l_{k1}k_{s1} & 0 & 0 \\
    -k_{s1} & -l_{k1}k_{s2} & l_{k1}k_{s1} & 0 & 0 & 0 & 0 \\
    -k_{s2} & l_{k1}k_{s2} & l_{k1}k_{s1} & 0 & 0 & 0 & 0 \\
    -k_{s3} & -l_{k1}k_{s2} & -l_{k1}k_{s1} & 0 & 0 & 0 & 0 \\
    -k_{s4} & l_{k1}k_{s2} & -l_{k1}k_{s1} & 0 & 0 & 0 & 0
\end{bmatrix}$$

The force matrix is:
Fourier transform to the Formula (1): [3]

\[-w^2 MX(w) + jwCX(w) + KX(w) = F(w) = K_q Q(w)\]  \hspace{1cm} (6)

2.3. Random excitation of road

By using power spectral density function, the characterization of random pavement is expressed as follows: [4-5]

\[G_q(n) = G_q(n_0)((n/n_0)^W) \hspace{1cm} n_l < n < n_u \]  \hspace{1cm} (7)

In which, \(n\) is time frequency, \(n_0\) and \(n_l\) are upper and lower limit of spatial frequency of road spectrum, \(n_0 = 0.1 \text m^{-1}\) is reference space frequency, \(W\) is frequency index.

The road spatial power spectrum density could be changed to time power spectrum density as follows:

\[G_q(w) = G_q(f) = \frac{1}{u} G_q(n) = (2\pi)^2 n_0^2 G_q(n_0) \frac{u}{w^2} \]  \hspace{1cm} (8)

And power spectrum matrix of 4 wheels is:

\[
\begin{bmatrix}
1 & coh(n) & e^{-j2\pi n_t d} & coh(n)e^{-j2\pi n_t d} \\
coh(n) & 1 & coh(n)e^{-j2\pi n_t d} & e^{-j2\pi n_t d} \\
e^{j2\pi n_t d} & coh(n)e^{j2\pi n_t d} & 1 & coh(n) \\
e^{j2\pi n_t d} & coh(n) & e^{j2\pi n_t d} & 1
\end{bmatrix} = G_q(n) \]  \hspace{1cm} (9)

In which, \(coh_{xy}(n)\) is the coherence coefficient for the wheels, it can be expressed as: [6]

\[coh_{xy}^2(n) = \begin{cases} 
(a-0.1b)^r & 0 \leq |n| \leq 0.1 \\
(a-0.1b)^r & 0.1 < |n| \leq 2 \\
0 & 2 < |n| \end{cases} \]  \hspace{1cm} (10)

In which, \(a = 1, b = 10(1-0.1^{1/2}), r = 4^B\), \(B\) is the tread.

2.4. The evaluation index system of suspension performance

The vehicle ride comfort and stability is determined by suspension, the performance of which can be evaluated through the following three basic parameters:

(1) The vibration acceleration response

The calculation of the frequency response function of the second derivative of the output to the displacement input is:

\[H_{\dddot{x}_q}(jw) = -w^2 H_{\ddot{x}_q}(jw) \]  \hspace{1cm} (11)

The time power spectrum of the vertical vibration acceleration of the vehicle body is:

\[S_{\ddot{x}_q}(w) = w^4 G_{x1} = w^4 [H_{11} L_{14} G_{44} H_{14}^*] \]  \hspace{1cm} (12)
It is expressed through root mean square of vertical body acceleration:

\[
\sigma_{\text{rms}} = \frac{1}{2\pi} \int_0^{\pi} \left[ 2 \sigma_{\text{rms},w}(w) + 2w^2 G_{11}(w) \right] \frac{dw}{2} \quad (13)
\]

(2) Suspension stroke

Suspension stroke parameter is defined as the wheels and the body displacement difference RMS value, which can be used to describe the degree of relative displacement of the static state. Suspension stroke parameter has great influence on handling stability of vehicle.

The four suspension stroke is defined as:

\[
\begin{align*}
 f_{d1} &= x_1 + L_{x1} x_2 - L_4 x_3 - x_4 \\
 f_{d2} &= x_1 - L_{x1} x_2 - L_4 x_3 - x_5 \\
 f_{d3} &= x_1 + L_{x2} x_2 + L_6 x_3 - x_6 \\
 f_{d4} &= x_1 - L_{x2} x_2 + L_6 x_3 - x_7 
\end{align*}
\]

And the four suspension stroke RMS is calculated as follows:

\[
\begin{align*}
 \sigma_{f_{d1}} &= \left( \sigma_{x1} + b_1 \sigma_{x2} + L_1 \sigma_{x3} + \sigma_{x4} \right)^2 \\
 \sigma_{f_{d2}} &= \left( \sigma_{x1} + b_2 \sigma_{x2} + L_1 \sigma_{x3} + \sigma_{x5} \right)^2 \\
 \sigma_{f_{d3}} &= \left( \sigma_{x1} + b_3 \sigma_{x2} + L_2 \sigma_{x3} + \sigma_{x6} \right)^2 \\
 \sigma_{f_{d4}} &= \left( \sigma_{x1} + b_4 \sigma_{x2} + L_2 \sigma_{x3} + \sigma_{x7} \right)^2
\end{align*}
\]  

\[
\frac{1}{2\pi} \int_0^{\pi} 2(2G_{11} + b_1^2 G_{22} + 2L_1^2 G_{33} + G_{44}) dw \right]^\frac{1}{2} \quad (15)
\]

(3) Relative dynamic load of wheels

The relative dynamic load parameters are defined as the variation root mean square relative to the static load of the tires.

The four wheels dynamic load are:

\[
\begin{align*}
 F_{d1}/G_1 &= k_1 (x_4 - q_1) \\
 F_{d2}/G_2 &= k_2 (x_5 - q_2) \\
 F_{d3}/G_3 &= k_3 (x_6 - q_3) \\
 F_{d4}/G_4 &= k_4 (x_7 - q_4)
\end{align*}
\]

In which, \( G_i, i = 1,2,3,4 \) is the static load of four wheels on the ground obtained by mechanical analysis:

\[
\begin{align*}
 G_1 &= L_m g (2L_1)^{-1} + m_{u1} g \\\n G_2 &= L_m g (2L_1)^{-1} + m_{u2} g \\
 G_3 &= L_m g (2L_1)^{-1} + m_{u3} g \\
 G_4 &= L_m g (2L_1)^{-1} + m_{u4} g
\end{align*}
\]

The frequency response function of relative dynamic load on the input excitation is:

\[
H_{F_{d1}/G_1} = \begin{bmatrix}
     c_1 (H_{x1} - 1) & c_1 H_{x2} & c_1 H_{x3} & c_1 H_{x4} \\
     c_2 H_{x1} & c_2 (H_{x2} - 1) & c_2 H_{x3} & c_2 H_{x4} \\
     c_3 H_{x1} & c_3 H_{x2} & c_3 (H_{x3} - 1) & c_3 H_{x4} \\
     c_4 H_{x1} & c_4 H_{x2} & c_4 H_{x3} & c_4 (H_{x4} - 1)
\end{bmatrix}
\]

Power spectral density of dynamic load response and input excitation have the following relationship:
\[
\begin{bmatrix}
G_{F_y}(w) \\
H_{F_y}^*
\end{bmatrix} = \begin{bmatrix}
H_{F_y} \\
G_{y}(w)
\end{bmatrix}^T \begin{bmatrix}
H_{F_y} \\
G_{y}(w)
\end{bmatrix} \tag{19}
\]

After that, the root mean square value of dynamic tire load is:
\[
\sigma_{F_y} = \left[ \frac{1}{2\pi} \int_0^{\infty} 2G_{F_y}(i,i)dw \right]^{1/2} \tag{20}
\]

3. Vehicle dynamic simulation

The vehicle dynamic model is calculated through Matlab program, as shown in Figure 2, the unevenness of the military undulating soil road with time and the spatial frequency distribution characterized by power spectral density are given, which can be used as an input load for vehicle dynamic models. Figure 3 shows a two-dimensional map of the vibration acceleration of the driver's position with the suspension damping ratio, and a three-dimensional map with the front and rear suspension damping coefficients, it provides basis for the optimization characteristics and the evaluation of vehicle ride comfort. Similarly, Figure 4 shows the variation of the relative stroke of the suspension with the damping coefficient, which can be used to evaluate the steering stability of the vehicle.

Figure 2. Frequency domain characteristics of undulating pavement.
Figure 3. Vertical vibration acceleration response of different damping.
4. Conclusions

(1) Based on the 4X4 military wheeled vehicle, the vehicle dynamic model was constructed, and the analytical calculation method was developed to clarify the index system for evaluating vehicle comfort.

(2) A random road spectrum of off-road pavement was prepared, and the grade of the pavement was evaluated by power spectral density, which provides quantifiable input excitation for the dynamic model.

(3) Based on the vibration acceleration of the vehicle body, the optimal matching strategy of suspension elasticity and damping characteristics under different road surface and vehicle speed is established, which provides theoretical support for the development of vehicle chassis system.
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