Vanishing of Cosmological Constant
in Dual Supergravity

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Abstract
The vacuum configuration of dual supergravity in ten dimensions with one-loop fivebrane corrections is analyzed. It is shown that the compactification of this theory with rather general conditions to six dimensional space leads to zero value of cosmological constant.

1 Introduction
The cosmological constant $C$ can be defined as a vacuum expectation value (VEV) of an effective lagrangian:

$$C = < \mathcal{L}_{\text{eff}} >$$

(1)

It vanishes ($C = 0$) in any theory with global supersymmetry, - that is a direct consequence of a supersymmetry algebra. The situation is not so unambiguous in a supergravity, where in general $C \neq 0$. But $C = 0$ for the heterotic superstring in ten dimensions (D=10). So, one may expect that the Type I (N=1, D=10) supergravity considered as a field-theory limit of a heterotic superstring also leads to $C = 0$. The same must be true for the dual N=1, D=10 supergravity considered as a field-theory limit of a fivebrane [1], [2], [3] because this theory can be obtained from a Type I supergravity by dual transformation of the axionic field.

It is not evident, that the condition $C = 0$ will persist in a compactification for lower dimensions because the part of the supersymmetry can be lost and vacuum properties are rather specific in the process.

We demonstrate in the present paper, that rather general and realistic compactification procedure from the space $M_{10}$ to the $M_6$ leads to vanishing of the cosmological constant. (Here $M_D$ is a $D$-dimensional space-time with Minkowsky signature). Really the case is considered:

$$M_{10} \rightarrow M_6 \otimes E_4$$

(2)

where the compact Euclidian D=4 space $E_4$. We require that vacuum configuration leaves only half of the D=10 supersymmetry unbroken and leads to chiral theory in D=6. This assumption greatly simplifies the equations for vacuum configuration and uniquely fixes the topological structure of $E_4$ manifold. It follows, that $E_4$ is related to the $K_3$-space (see [4] for references on $K_3$-space).

We concentrate on the dual supergravity because the supersymmetric lagrangian for this theory is constructed including terms of the next order in the string-tension parameter $\alpha'$ (see [5], [6]).

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2 Lagrangian

The lagrangian of dual supergravity in $M_{10}$ is equal to:

$$\mathcal{L} = \mathcal{L}^{(gauge)} + \mathcal{L}^{(grav)}$$

where $\mathcal{L}^{(gauge)}$ and $\mathcal{L}^{(grav)}$ are lagrangians for gauge-matter and for supergravity multiplet. The term $\mathcal{L}^{(gauge)}$ takes the form [7]:

$$E^{-1} \mathcal{L}^{(gauge)} = \frac{1}{g^2} \text{tr} \left[ \frac{1}{4} F_{AB} F^{AB} - \frac{1}{8 \cdot 6!} \varepsilon^{A_1 \ldots A_6} C_{A_1 \ldots A_6} F_{A_7 A_8} F_{A_9 A_{10}} \right]$$

Here $C_{A_1 \ldots A_6}$ is an axionic potential, $F_{AB}$ is a gauge field which is in the algebra of internal symmetry group $G$, $F_{AB} = E_A^M E_B^N F_{MN}$, where $E_A^M$ is the veilbein in $M_{10}$.

One must consider $G = SO(32)$ or $G = E_8 \times E_8$ and $4 g^2 = -1/\alpha'$ as it follows from the superstring consideration [8]. The symbol $\text{tr}$ in (4) means the trace in a vectorial representation of $SO(32)$. It can be changed to $(1/30) \text{Tr}$, where $\text{Tr}$ means the trace in the adjoint representation of $E_8 \times E_8$ or $SO(32)$.

We consider only bosonic terms in the lagrangian (3) because we are interested in the vacuum configuration. Notations correspond in general to [4] with some differences which are or self-evident or explained in the text. In particular, the following index notations are used: $A, B, C, \ldots, a, b, c, \ldots$ and $\alpha, \beta, \gamma, \ldots$ are flat indices respectively in $M_{10}, M_6$ and $E_4$; $M, N, P, \ldots, m, n, p, \ldots$ and $\mu, \nu, \lambda, \ldots$ are corresponding world indices; $Z^M = (x^\mu, y^m)$ is the coordinate in $M_{10}, x, y$ are coordinates in $M_6$ and $E_4$ respectively.

We present the gravity part of the lagrangian as an expansion in $\alpha'$:

$$\mathcal{L}^{(grav)} = \mathcal{L}^{(grav)}_0 + \alpha' \mathcal{L}^{(grav)}_1$$

where $\mathcal{L}^{(grav)}_0$ is equal to [6] (see [4] for further references on the subject):

$$E^{-1} \mathcal{L}^{(grav)}_0 = \phi \left( \mathcal{R} - \frac{1}{12} \tilde{M}_{ABC} \right)$$

Here $\mathcal{R}$ is the curvature scalar, $\phi$ is the dilatonic field, $\tilde{M}_{ABC}$ is defined by:

$$\tilde{M}_{ABC} = \frac{1}{7!} \varepsilon_{ABC}^{A_1 \ldots A_7} M_{A_1 \ldots A_7}$$

where $M_{N_1 \ldots N_7} = 7 \partial_{[N_1} C_{N_2 \ldots N_7]}$ is the axionic field-strength.

The result for $\mathcal{L}^{(grav)}_1$ was obtained in [5], [6] in the form:

$$\mathcal{L}^{(grav)}_1 = 2 \mathcal{R}^2_{AB} - \mathcal{R}^2_{ABCD} + \frac{1}{2 \cdot 6!} \varepsilon^{ABCDF_1 \ldots F_6} \mathcal{R}^{IJ}_{AB} \mathcal{R}_{IJ} C_{F_1 \ldots F_6} -$$

$$- \frac{1}{2} \mathcal{R}^{AB} (\tilde{M}^2)_{AB} - \frac{1}{6} \tilde{M}^{ABC} D^2_I \tilde{M}_{ABC} +$$

$$+ \frac{1}{2} \tilde{M}^{ABC,D} (\tilde{M}^2)_{ABCD} - \frac{1}{24} (\tilde{M}^2)_{ABCD} (\tilde{M}^2)_{ACBD}$$
Here $\mathcal{R}_{ABCD}$ is the curvature tensor, $\mathcal{R}_{AB}$ is the Ricci tensor, $;B$ means the covariant derivative $D_B$. The following notations are introduced here and below:

$$M^2 = (\tilde{M}_{ABC})^2, \quad (M^2)_{AB} = \tilde{M}_{A}^{CP} \tilde{M}_{BCD}$$

$$\tilde{M}_{ABCD} = \tilde{M}_{AB} \tilde{M}_{CDF}, \quad (\tilde{M}^3)_{ABC} = \tilde{M}_{A}^{ij} \tilde{M}_{BJ}^{K} \tilde{M}_{CKI}$$

The $\tilde{M}_{ABC}$ field is connected by the dual transformation with the 3-form axionic field of standard Type I supergravity (see below).

### 3 Vacuum Configuration

The most general anzatz for VEV of the veilbein in $M_{10}$, which corresponds to the compactification according to eq.(2), takes the form:

$$E_M^A = \left( e^{2\xi(y)} \partial^\mu \, \delta^\alpha_\mu \, 0 \right) e^{-2\xi(y)} e_m^a(y)$$

where $e_m^a \equiv \exp(-2\xi) \, e_m^a$ is the veilbein in $E^*_4$. (the factor $\exp(-2\xi)$ is extracted for future convenience). $\xi$ is an arbitrary function. The analogous ansatz was considered [4] in the study of compactification scheme $M_{10} \rightarrow M_4 \otimes E_6$ with only partial account of $\alpha'$ corrections.

Only the gauge-field component $F_{ab} = \exp(2\xi) \, F_{ab}$ survive in the vacuum configuration ($F_{ab} = e_a^m \, e_b^a \, F_{mn}$).

The following VEV's of curvature tensor components survive ($\mathcal{R} = d\omega + \omega \wedge \omega$ where $\omega$ is the connection related with $E_M^A$):

$$R_{ab}^{\alpha \beta} = 2 \exp(-2\xi) e_{m}^a \, e_{m}^b$$

$$R_{ab}^{\alpha} = \exp(-4\xi) e_{m}^a \, e_{m}^b$$

$$R_{ab}^{cd} = e^{-4\xi} (R_{ab}^{cd} - 8 \delta_{[a}^{[c} \delta_{b]}^{d]} - 16 \delta_{[a}^{[c} j_{b]}^{d]} + 8 \delta_{[a}^{c} \delta_{b]}^{d} j_{f}^{f})$$

where $R_{mnab}$ is the curvature defined in terms of $e_m^a$, $\xi_b = \nabla_b \xi = e_b^m \partial_m \xi$, etc.

Let us start now to study equations defining vacuum configuration: $<\delta_Q \Phi> = 0$, where $\Phi$ is some field but $\delta_Q$ is a supersymmetry transformation. When $\Phi$ is a boson, such an equation satisfied identically. It has nontrivial content for $\Phi = \psi_A$, $\chi$, $\lambda$, i.e. for gravitino, dilatino and gaugino fields respectively.

We get [10] (see also [12], [13] where another parametrization is used):

$$<\delta_Q \psi_A> = \epsilon \, A + \frac{1}{144} \left( 3 \tilde{M}_{BCD} \tilde{\Gamma}^{BCD} \tilde{\Gamma}_A + \Gamma_A \tilde{M}_{BCD} \tilde{\Gamma}^{BCD} \right) \epsilon = 0$$

$$<\delta_Q \chi> = \frac{1}{2} \partial_A \phi \, \Gamma^A \epsilon - \left( \frac{\phi}{36} \tilde{M}_{ABC} \tilde{\Gamma}^{ABC} - \alpha' \, A_{ABC} \tilde{\Gamma}^{ABC} \right) \epsilon = 0$$

$$<\delta_Q \lambda> = \frac{1}{4} F_{AB} \Gamma^{AB} \epsilon = 0$$
Here $\epsilon$ is a 32-component Dirac spinor, - the parameter of supersymmetry transformation. It is subjected to the Majorana-Weyl condition: $\epsilon_c = \bar{\epsilon}, \epsilon = \Gamma \epsilon$, where $\Gamma$ is the chirality matrix (see below), $\epsilon_c$ is the charge-conjugated spinor. We suppose that $\epsilon$ is independent on the coordinates in $M_6$: $\epsilon = \epsilon(y)$, that corresponds to the symmetry of vacuum configuration. All the fields in the r.h.s. of (11)-(13) depend only on $y$.

The 3-form field $A_{ABC}$ in eq.(12) is equal to [5], [6]:

$$A_{ABC} = -\frac{1}{18} \Box \tilde{M}_{ABC} + \frac{7}{36} (\tilde{M}^2_{D[ABC]}))^{[D} + \frac{1}{36} \tilde{M}_{DE[ABC]} \tilde{M}_{DE}^{D] - }$$
$$- \frac{5}{8 \cdot 243} \tilde{M}^2 M_{ABC} + \frac{5}{8 \cdot 27} \tilde{M}^2_{D[ABC]} M_{D] - } - \frac{5}{27} \tilde{M}^3_{ABC} -$$
$$- \frac{1}{4 \cdot 972} \varepsilon_{ABC}^{DEFGHIJ} M_{DEF} (\tilde{M}_{HIJ,G} + \tilde{M}^2_{GHIJ})$$  

(14)

The following representation of $\Gamma$-matrices $M_{10}$ is convenient:

$$\Gamma^\alpha = \gamma^\alpha \otimes \mathcal{T}, \quad \alpha = 0, 1, \ldots, 5$$
$$\Gamma^{a+5} = I \otimes \mathcal{T}^a, \quad a = 1, 2, 3, 4$$  

(15)

where $\gamma^\alpha$ are $8 \times 8$ Dirac matrices in $M_6$, but $\mathcal{T}^a$ are $4 \times 4$ Dirac matrices in $E_4$. The chirality matrices are:

$$\gamma = \gamma^0 \gamma^1 \ldots \gamma^5, \quad \mathcal{T} = \mathcal{T}^1 \mathcal{T}^2 \mathcal{T}^3 \mathcal{T}^4$$  

(16)

Then

$$\Gamma = \Gamma^0 \Gamma^1 \ldots \Gamma^9 = \gamma \otimes \mathcal{T}, \quad (\Gamma)^2 = (\gamma)^2 = (\mathcal{T})^2 = 1$$  

(17)

We use also the standard notation: $\Gamma_{A_1\ldots A_k} = \Gamma_{[A_1 A_2 \ldots A_k]}$.

We begin our study from eq.(11). For $A = \alpha$ it reduces to:

$$\gamma_\alpha \gamma \otimes \left( \epsilon \xi_a \mathcal{T}^a + \frac{1}{36} \tilde{M}_{abc} \mathcal{T}^{abc} \right) \epsilon = 0$$  

(18)

Let us suppose, that $\epsilon$ has definite chirality in $E_4$:

$$\mathcal{T} \epsilon = \nu \epsilon, \quad \text{where } \nu = 1 \text{ or } -1$$  

(19)

It means that we keep only ”one-half” of the supersymmetry in $M_{10}$. Using the relation:

$$\mathcal{T}^{abc} = -\varepsilon^{abcd} \mathcal{T}_d \mathcal{T}$$

where $\varepsilon^{abcd}$ is a completely antisymmetric tensor ($\varepsilon^{1234} = 1$), we get the solution of (18) in the form:

$$\tilde{M}_{abc} = 6 \nu \epsilon^2 \varepsilon_{abcd} \xi^d$$  

(20)

It means that the only nonzero component of the axion potential with all flat $M_6$ indices appears to be a constant:

$$C_{\beta_1 \ldots \beta_6} = -\nu \varepsilon_{\beta_1 \ldots \beta_6}$$  

(21)

where $\varepsilon_{\beta_1 \ldots \beta_6}$ is a completely antisymmetric tensor ($\varepsilon^{01\ldots6} = 1$).
Having all this, one can reduce eq. (11) with $A = a$ to the following:

$$\nabla_a \epsilon' = 0,$$

where $\epsilon' = e^{-\xi/2} \epsilon$  \hspace{1cm} (22)

Equation $[\nabla_a, \nabla_b] \epsilon' = 0$ leads to:

$$R_{abcd} T^{cd} \epsilon = 0$$  \hspace{1cm} (23)

Eq. (23) is fulfilled for arbitrary chiral $\epsilon$ if the curvature defined by the veilbein $e_m^a$ is (anti)selfdual for the second pair of indices:

$$R_{abcd} = \nu^2 \varepsilon_{ef} R_{abef}$$  \hspace{1cm} (24)

This equation immediately follows from (23) because $T^{ab} = -(1/2) e^{abcd} T_{cd} T$. It follows from (24) with the help of symmetry properties of the curvature tensor that curvature is (anti)selfdual for the first pair of indices too. The compact 4-dimensional space with the (anti) selfdual curvature is the $K_3$ space. It means that the space $E_4$ which is defined by the veilbein $e_m^a$ is the $K_3$-space. This result follows from the ansatz (9) with the constraint (10).

Now we turn to the eq.(13). It is analogous to eq.(23), so it leads to the selfduality of the gauge-field $F_{ab}$:

$$F_{ab} = \nu^2 \epsilon_{ab} \mathcal{F}_{cd}$$  \hspace{1cm} (25)

We are left with eq.(12) which is the most complicated one. Taking into account eqs. (10), (20) one is able to present the $A_{abc}$-tensor from eq.(14) in a rather simple form:

$$A_{abc} = -\frac{\nu}{3} \varepsilon_{abcd} \left[ e^{6\xi} (\xi_f^f + \xi_f^f \xi_f) \right]^{;d}.$$  \hspace{1cm} (26)

where $;d$ means the covariant derivative $\nabla_d$ defined by the veilbein $e_m^a$.

Expression (26) allows us to drop one derivative in eq.(12) reducing the order of equation from third to second. Then it follows immediately from eq.(13):

$$e^{2\xi} \phi + 4 \alpha' e^{6\xi} (\xi_f^f + \xi_f^f \xi_f) = C_0$$  \hspace{1cm} (27)

where $C_0$ is an arbitrary constant. This equation connects the dilaton VEV with function $\xi(y)$ introduced by the anzatz (9).

## 4 Equations of Motion

We examine here whether equations of motion impose some additional constraints on the parameters of vacuum configuration. Equations for $M_{10}$ supergravity obtained in the lowest and next order in $\alpha'$ are used (see [3], [8]), see also [14] where another parametrization was considered):

One can easily see that gauge-field equations of motions are fulfilled because of selfduality condition and Bianchi Identity $\nabla_{[a} F_{bc]} = 0$.

The dilaton equation of motion can be presented in the form:
\[ \square \phi + \frac{1}{12} \phi \tilde{M}^2_{ABC} + \frac{1}{12} g^2 tr (F_{AB})^2 - \alpha' \left[ -\frac{2}{3} (R_{AB})^2 + \frac{1}{3} (R_{ABCD})^2 - \frac{1}{6} R^{AB} \tilde{M}^2_{AB} - \frac{1}{18} M^{ABC} \Box \tilde{M}_{ABC} + \frac{1}{3} M^{ABC;D} \tilde{M}^2_{ABCD} - \right. 
\left. - \frac{1}{24} \tilde{M}^2_{ABCD} \tilde{M}^2_{ACBD} - \frac{1}{12} \Box \tilde{M}^2 + \frac{1}{6} (\tilde{M}^2_{AB})^{AB} \right] = 0 \]  
(28)

where \( \Box = D^2 \).

Calculating all the terms in eq. (28) with the help of relations obtained before and using eq. (27), one obtains the following result:

\[
\left( e^{-6\xi} C_0 - 12 \alpha' \xi^a \right)^b + \frac{1}{4 g^2} tr (F_{ab})^2 - \alpha' (R_{abcd})^2 = 0 
\]  
(29)

where \( F_{ab} = e^a_m e^b_n F_{mn} \), \( R_{abcd} = e^a_m e^b_n R_{mncd} \).

One can expect (cf. [15]) that VEV’s of supersymmetry transformations (11)-(13) provide the complete information on the vacuum configuration, i.e. they are equivalent to equations of motion. In such a case, one must be able to derive eq. (29) starting immediately from (11)-(13). But up to now we were not able to do this.

One gets immediately from (29) the topological constraint [16] (in form notations):

\[
\int_{K_3} (tr F \wedge F - tr R \wedge R) = 0 
\]  
(30)

Here \( tr \) is calculated over indices of vectorial representation of corresponding group, i.e \( tr R \wedge R = R_{ab} \wedge R^{ba} \). The integral is fulfilled over the compact space with the vierbein \( e^a_m \) (i.e the \( K_3 \)-space).

Now we turn to the equation of motion for the \( C_{A_1 \ldots A_6} \)-field. One can write it in the form [3]:

\[
H_{[ABC;D]} + 3 \alpha' \left( tr F_{[AB} F_{CD]} - R_{[AB}^{\ EF} R_{CD]}^{\ FE} \right) = 0 
\]  
(31)

where

\[
H_{ABC} = \phi \tilde{M}_{ABC} - 2 \alpha' \left( -\Box \tilde{M}_{ABC} + 3 (\tilde{M}^2_{D[ABC]} \right)^{;D} + 
\frac{3}{2} \tilde{M}_{DF[A;B} \tilde{M}^{DF}_{C]} + 3 R_{D[ABC]}^{\ AB} \tilde{M}_{BC]}^{;D} - \frac{1}{2} (\tilde{M}^2_{[ABC]} \right) 
\]  
(32)

Only the \( H_{abc} \)-component survive in the vacuum configuration. The calculation of this component is similar to that performed for \( A_{abc} \). The result is:

\[
H_{abc} = 6 \nu \epsilon_{abcd} (C_0 \xi^d + 2 \alpha' e^{6\xi} \xi^{df}) 
\]  
(33)

Then, one can check with the help of selfduality conditions that eq. (31) is equivalent to eq. (29), i.e. no new constrains are produced.

So we obtain, that eq. (29) is the VEV of usual \( M_{10} \) supergravity Bianchi Identity:

\[
dH' = 2\alpha' (-tr F \wedge F + tr R \wedge R) 
\]  
(34)
where $H'_{abc}$ is interpreted as a VEV of axionic field-strength of usual Type I supergravity. It takes the form

$$H'_{abc} = \nu \epsilon_{abcd} H^d$$  (35)

where $H(y)$ is a scalar field. It follows from (29):

$$(H - e^{-6\xi} C_0' + 12 \alpha' \xi_a \xi^a)_{;b} = 0$$  (36)

One can easily find the relation between $H'_{abc}$ and $H_{abc}$-field in (33).

A long study of a rather complicated graviton equation of motion ($\phi \mathcal{R}_{AB} + \ldots = 0$) does not produce additional constraints for vacuum configuration.

5 Zero Cosmological Constant

In this section we demonstrate that the action VEV vanishes. The internal space assumed to be closed, so surface integrals do not give any contribution.

Using eqs. (10) and (20) one gets immediately:

$$< L^{(grav)}_0 >= 0$$  (37)

where $L^{(grav)}_0$ is defined in eq. (6).

The gauge-matter part of the lagrangian (3) can be transformed to the form:

$$< E^{-1} L^{(gauge)} >= \frac{1}{4g^2} tr (F^2_{ab} - \frac{\nu}{2} \epsilon^{abcd} F_{ab} F_{cd})$$  (38)

where eq. (21) was used. Then, the selfduality condition (25) leads immediately to:

$$< L^{(gauge)} >= 0$$  (39)

The most difficult term in (3) is the $L^{(grav)}_1$. Using (8) and relations obtained before we are able to transform this term to the form of complete derivative:

$$< E^{-1} L^{(grav)}_1 >= \nabla_a \left[ e^{6\xi} (12 \xi^a \xi^b - 3 \xi_a \xi^b - 12 \xi^a \xi^b \xi_b) \right]$$  (40)

So, one can put:

$$< \int d^{10} Z L^{(grav)}_1 >= 0$$  (41)

Then

$$C = < \int d^{10} Z (L^{(gauge)} + L^{(grav)}_0 + \alpha' L^{(grav)}_1) >= 0.$$  (42)

We conclude, that no cosmological constant is generated.
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