Operating Room Scheduling Optimization Based on a Fuzzy Uncertainty Approach and Metaheuristic Algorithms

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**A B S T R A C T**

Today, planning and scheduling problems are the most significant issues in the world and make a great impact on improving organizational productivity and serving systems such as medical and healthcare providers. Since operating room planning is a major problem in healthcare organizations, the optimization of medical staff and equipment plays an essential role. Thus, this study presents a multi-objective mathematical model with a new categorization (preoperative, intraoperative, and postoperative) to minimize operating room scheduling and the risk of using equipment. Time constraints in healthcare systems and medical equipment limited capacity are the most significant considered limitation in the present study. In this regard, since the duration of patient preparation and implementation of treatment processes occur in three states of optimistic, pessimistic, and normal, the introduced parameters are examined relying on a fuzzy uncertainty analysis of the problem. Hence, the model is measured in a real numerical solution sample in a medical center to evaluate and confirm the proposed mathematical model. Then, two meta-heuristic algorithms (NRGA and NSGA-II) are implemented on the mathematical model to analyze the proposed model. Finally, the research results indicate that the NSGA-II is more efficient in the operating room scheduling problem.

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1. INTRODUCTION

Nowadays, changes in people’s lifestyles, socio-cultural structures, diseases patterns, people’s health requirements along with increasing growth of population, have proved a variety of problems in the provision of health services and facilities. The rapid rise in healthcare costs is considerable and the main problem of medical and healthcare systems in various countries, even the most developed countries in the world is how to control the costs. Accordingly, healthcare organizations are trying to reduce healthcare costs while improving the quality of their services. Considering the healthcare system condition in the current era, it has been undergoing major changes and developments such as rising numbers of patients, aging population, and increasing number of the elderly people [1]. Therefore, the investigation of efficient strategic and operational decisions by healthcare executives is of great significance for costing and risk perspectives in order to minimize the expected costs [2]. Taking into account, the technical limitations of the human resources available in medical centers [3], a large proportion of patients cannot immediately be given treatment [4], and consequently, the longer patients wait for treatment, the less satisfaction will be obtained, and thus, the more lengthy treatment process will move ahead [5]. This is while other medical centers, especially public hospitals face funding crisis, shortage of medical staff, and appropriate equipment [6]. These factors have caused medical centers to improve efficiency performance and reduce healthcare costs [7]. Considering the obtained information, more than 70% of medical referrals are allocated to surgical operations, additionally, more than 15% of wasted time in medical...
centers is related to operating rooms [8]. It verifies the importance of operating room and its proper utilization [9, 10]. The operating room, especially surgical room, has a particular sensitivity [11] and is considered as the most vital unit in medical centers [12]. Therefore, the smallest defect or non-compliance with a specific planning and pre-determined standards will cause problems [13]. According to the above-mentioned discussion, operating room can be likened to medical center engine [14]. Operating rooms affect other wards of medical centers, as well as having repercussions on medical staff. So, the increasingly growing demand for surgeries and resource constraints have caused medical centers to efficiently manage their resources, especially expensive resources such as operating rooms and special medical staff. Scheduling surgical operations, including sequencing surgeries, allocating resources, and determining start times, is a complicated task for the medical center manager [15]. The problem of operating room planning and scheduling seeks to propose a solution to reduce costs, increase the efficiency of operating room utilization, and attain other related goals [16]. Moreover, one of the main applications of operations research in health systems management is the improvement of resources allocation mechanism [17]. Therefore, staff scheduling, including nurses and doctors, is regarded as one of the most important and sensitive challenges of medical centers. The importance of medical staff’s quality of work has caused scheduling-related problems, preserving the interests of patients and the medical center along with creating staff satisfaction and paying attention to their demands in scheduling that are some of the main points [18]. Some of healthcare operational challenges such as staff scheduling and surgical operation scheduling in operating rooms are highly complicated [19]. The proper scheduling of surgical operations in operating room can reduce idle time, personnel overtime costs, and fixed costs of operating room equipment [20]. Beside these achievements, the proper scheduling will bring patients higher level of healthcare facilities by reducing patient wait time and providing rapid access to emergency medical services for emergency patients [21]. Uncertainties level existing in operating room scheduling, plays an important role in increasing system efficiency and reducing costs. Therefore, it has become difficult to address operating room planning and scheduling [22]. Considering the above-mentioned issues, addressing the uncertainties in operating room scheduling and planning is the most important factor affecting the optimization of the system.

The present study investigates a multi-objective mathematical model by minimizing operating room scheduling based on the MAKESPAN model by considering the problems with earliness/tardiness issues, which occur in the real problem. Evaluating earliness/tardiness in the operation process is critical because the activities before and after the operation are highly significant in operating room scheduling to the extent that the slightest inconsistency between the processes leads to patient death. As a result, this study aims to minimize earliness, which causes inconsistency in recovery room (postoperative units), and tardiness which causes overlap in treatment processes after the operation is done based on the executed plan. In addition, this study, seeks to minimize the risk of performing treatment processes on medical equipment in a two-objective-function model. For instance, if a patient requires a special surgery, the equipment which increases the risk of treatment will be minimized and replaced by a less risky equipment. Accordingly, since the treatment processes can be predicted in three states of optimistic, pessimistic, and normal, in this study triangular fuzzy uncertainty is used to evaluate and estimate the parameters of the mathematical model to integrate the modeling condition with operating room problem in reality. Since the proposed mathematical model is NP-HARD type, two metaheuristic algorithms of NRGA and NSGA-II are evaluated through epsilon-constraint method to confirm and solve the mathematical model in the studies case, and evaluate the validity of the proposed model.

### 2. LITERATURE REVIEW

Operating room planning is of great importance in medical centers for two reasons. First, the precise scheduling in operating room ensures timely and accurate follow-up treatment, which quickens patients’ medical process period. Second, medical administrators always emphasize the optimal use of operating room equipment due to the high cost of such equipment. Thus, operating room planning and scheduling is a strategic concept and key factor to manage medical centers.

In this section different strategies of operating room scheduling and planning such as theoretical foundations and research background, levels of operating room planning and scheduling (strategic level, tactical level, and operational planning level) are discussed.

Today, there is a rapid growth in life expectancy in developed countries, leading to an increase in the population of the elderly [23]. Therefore, it should be noted that the problem of operating room planning and scheduling seeks to propose a solution to reduce costs, improve operating room efficiency, and attain other goals to fulfill health centers’ expectations [24]. Operating room planning and scheduling is classified into long-term, mid-term, and short-term solutions at the strategic, tactical, and operational levels [10]. Strategic planning includes long-term decisions, which are usually made on an annual basis [25]. The mid-term decisions are made at tactical level. The two mentioned levels are provided with master surgical scheduling (MSS). At the operational planning level, decisions are made on daily
and weekly basis. In daily scheduling, the sequence of operations in each operation room is determined. There are various criteria for this level of planning including the preferences of surgeons, resource availability, maximum admission capacity, operating room utilization and efficiency, and reducing overtime hours [26]. Operating room planning has begun in 1935. Research studies on this subject have had an upward trend since 1960's that many researchers have joined the healthcare scheduling community [27]. These research studies also continued their incremental trend in the 1970's [28]. Since 2000 onward, the operating room planning has gradually developed and achieved remarkable accomplishments including multi-level operating room planning, presenting different strategies for operating room scheduling, generating more effective solutions based on different methods, which have been added to the operating room planning over recent years. Testi et al. [29] proposed a hierarchical three-phase approach for operating room weekly scheduling. The models aim to maximize the specialists’ preferences and profit motives. In another research, overtime issues, ward beds and ICU were also considered. The model was in the definitive state that was solved by a heuristic algorithm [30]. Banditori et al. [31] proposed a model, by which the patients who are on the waiting list of the hospital can be classified in identical surgery groups based on the resources. The objective function in this model is composed of three parts. This model proposed a simulation-based-optimization method with real data. Lappas and Gounaris [32] proposed a multi-stage robust optimization method in which uncertainties during scheduling decision making are considered. Jittamai and Kangwansura [33] presented a patient admission scheduling model that improves the operating room efficiency by reducing resource losses. The model considers the existing uncertainties in demand, such as the stay duration, emergency admissions, and non-referral of patients. Al-Refaie et al. [34] developed three optimization models to optimize operating room scheduling during unexpected events and adjust emergency operations to a determined schedule. The first model schedules the emergency patients to the newly established rooms, while the second model aims to allocate emergency patients to unused and empty rooms. The third model specifies emergency patients and emergency units based on the rooms with the freest condition. Abedini et al. [35] presented a model for minimizing block time in preoperative and postoperative care units. In addition, this study deals with an integer model with definitive data for this problem and evaluates the model efficiency under different conditions and variable input of patients by means of simulation. In another study, Aftahi et al. [36] investigated the scheduling problem of prioritized outpatients in a medical center. The objective function of the problem is to minimize the total length of time the patients spend in the medical center. Additionally, a heuristic algorithm was proposed for medium-sized problems. The computational results indicate an increase in the patient satisfaction level, as well as the improvement of the efficiency and productivity of the medical center. Yazdi et al. [37] proposed a mathematical model for the problem of scheduling elective and emergency surgeries. This model considers surgeries as multi-activities projects. They implemented the Break-in-Moments (BIMs) technique in this structure and observed that this method is capable of reducing the waiting time in emergency rooms to be included into the schedule without specifically assigning any operating room to emergencies. Therefore, this method establishes a balance between the efficient use of operating rooms and taking responsibility for emergency surgeries. Khaniyev et al. [37] provided a new mathematical model for calculating the total amount of patient waiting time, operating room idle time, and overtime. The presented model improves the mean of performance gap up to 1.22% and the worst average gap up to 2.77%. Abdeljaouad et al. [38] introduced daily operation planning in hospitals based on a multi-objective mathematical model. The studied problem includes a set of operations, which should be planned in similar operating rooms. Such operations have no specific duration, so that they require adjustment activities before and after operation that are assigned to the surgeons who perform the operation. This study aims to introduce the best sequence of operations in each room to optimize the start time and waiting time for surgeons. Mousavi and Ebrahimnejad [39] presented the problem of operating room scheduling at tactical and operational levels of decision-making by considering the processes before and after the operation. For this purpose, a multi-objective mathematical programming model was proposed for scheduling the specialized operations considering the uncertainty of the operation duration, length of stay, and emergency needs. Finally, a two-step meta-heuristic algorithm was created to solve the large-scale cases. Zhou and Yue [40] introduced a common selection and scheduling problem between random service time and non-display in multi-stage service systems. Thus, the total expected costs for patient waiting time and service providers’ idle time were minimized in a few stages, and a two-stage random optimization program and a standard Benders decomposition algorithm were developed. Finally, it was found that the efficiency of the algorithm improved by 6%. Sun et al. [41] modeled the problem of proper appointment scheduling in an emergency hospital system and analyzed the model using simulation algorithm. It was found that block scheduling improved by 11.6%. Due to the multi-objective mathematical model, medical centers could find solutions to accurately reflect their appropriate transactions.

In the final section of literature review, besides taking account of the researches in the field of health
management and medical centers scheduling, the researches of recent years are summarized and reviewed. Health system management in the world involves different subjects such as supply chain management [42, 43], data management [44, 45, 46], risk management [47], routing and scheduling management [48, 49] and home healthcare management [50, 51, 52].

Therefore, as mentioned above, the present study is designated to the scheduling of medical centers, especially operating rooms. Thus, in this section before evaluating the research gap, the relevant studies in recent years are mentioned. Naderi et al. [52] in their study included a new and exact solution method with the aim of integrating personnel, allocating, routing, planning and scheduling a home care center under uncertain conditions, modelling and problem solving with the aim of creating a weekly program with the presence of a special mathematical structure in the model, as well as developing Banders decomposition algorithm. The results of solving the model and the problem indicate the optimization and efficient results. In another study, they solved the generalized operating room planning and scheduling (CORPS) problem in a hospital in Toronto with the aim of creating a weekly schedule and the primary goal of reducing fixed and variable costs by modeling and Banders algorithm. Regarding the outbreak of COVID-19, the intended problem was highly optimal [53], and in the next study, they presented the integrated planning and scheduling of the operating room with the development of the Banders solution method for maximizing the total planned surgery time that the efficiency of it was properly validated [54]. Tsai et al. [55] developed a stochastic optimization model for the operating room planning by considering a two-step integer model in uncertain conditions. In addition, they proposed two stochastic algorithms for this problem. In addition, the experimental results indicated that the proposed algorithms are more desirable compared to the present methods. Lin et al. [56] developed an operating room scheduling model for preparing a weekly surgery plan with an open planning strategy aimed at reducing cost. In addition, they developed two heuristic algorithms and finally developed the ant colony algorithm. Furthermore, the results of computational solution indicated that the ant colony method is highly efficient in large-scale problems. Bovim et al. [57] proposed the development of an MSS scheduling model and a simulation-optimization method for evaluating emergency patients with different clinical scenarios. Zhu et al. [58] presented a dynamic operating room planning problem for reducing total costs and developing a mathematical model. In addition, two heuristic algorithms were developed. Ultimately, some computational tests were performed to test the efficiency, stability, and convergence rate of the proposed algorithm. The results of comparison with other main algorithms indicated that their proposed algorithm gives a better performance than the compared algorithms. Park et al. [59] evaluated the efficient operating room scheduling for cost optimization. The scheduling model improved the local treatment processes by 120%, which was a great improvement in defined processes. The proposed model was solved with the branch and bound algorithm. Roshanaei et al. [60] addressed the operating room scheduling problem using a MIP mathematical model that select patients for the treatment process. They developed heuristic approach to solve the proposed NP-HARD problem. Roshanaei et al. [61] formulated a deterministic four-stage model based on the Benders decomposition algorithm in order to solve the operating room scheduling problem. They showed that in the developed four-stage model, Benders algorithm reduced the computational time by 11%.

As discussed in the literature, the research gap observed in these studies is the limitation of capacity in medical staff and equipment, as well as the availability of medical staff in unlimited conditions. Thus, in the present study, the limitation of resources (medical staff and equipment) is considered and a new mathematical model is developed. As mentioned in the introduction, a new fuzzy uncertainty model is evaluated in this study. In addition, an exact solution method (epsilon-constraint) along with two meta-heuristic algorithms of NRGA and NSGA-II are used in order to validate the mathematical model and finally the evaluation of the mathematical model is presented.

3. MATHEMATICAL MODELING

3.1. Statement of the Problem In this section, the problem and the new mathematical model are proposed. The proposed model is meant to schedule the patient arrival and departure in different wards of the medical center such as operating room, pre-operative and post-operative sections. Therefore, the sequence of providing healthcare services in different stages and the regular queuing system are considered. Besides, in this study, the proposed formulation is established based upon the research by Naderi et al. [62]. In the codified model, the conditions for receiving services from medical personnel are considered in a way that at each stage the patient is visited while receiving required services all at once. In this way, the operating rooms and wards of a medical center are scheduled on a daily basis. Effective scheduling of executive activities is an important subject in the research studies that has been neglected due to the uncertainty of the effective parameters in the mathematical model. So, using fuzzy approach for the mathematical model, the challenge ahead of the scheduling tasks is addressed. In this approach, the patient preparation and the time duration to present services are represented by triangular fuzzy membership functions. Using this new methodology in this study, the
fuzzy uncertainty parameters are definite. Finally, since the introduced mathematical model is of NP-HARD type, two metaheuristic algorithms of NRGA and NSGA-II are evaluated through epsilon-constraint method to confirm, solve, and evaluate the validity of the proposed model.

3. 2. Notation

| Indices (Sets & index) |
|----------------------|
| I                    | Set of patients |
| E(i)                 | Set of common patients |
| W(i)                 | Set of special patients |
| J                    | Set of stages |
| K                    | Set of equipment |
| i & i’                | Patient index |
| j & j’                | Stage index |
| k & k’                | Equipment index |
| p                    | Medical center personnel index |

| Parameters |
|------------|
| ST_{ijk}  | Time to prepare patient i at stage j on equipment k |
| RSK_{ijk} | Risk to service to patient i at stage j on equipment k |
| SST_{ijk} | The patient i enter time at stage j on equipment k |
| PT_{ijk}  | Time to service to patient i at stage j on equipment k |
| PPT_{pijk} | Time to providing health service by medical personnel p for patient i at stage j on equipment k |
| TW_{i}    | The waiting time of patient i from operation time to recovery |

| Variables |
|-----------|
| C_{ijk}   | End time of service process to patient i at stage j on equipment k |
| Y_{ijk}   | If medical personnel p provides service on equipment k for patient i at stage j, 1, otherwise it is zero |
| Q_{pijk}  | If medical center p provides service on equipment k for patient i at stage j it is 1, otherwise it is zero |
| X_{iijk}  | If patient i after patient i’ uses equipment k at stage j it is 1, otherwise it is zero |

3. 3. Mathematical Model

\[
\begin{align*}
\text{MIN C1} & = \left( \Sigma (\text{MAX} \Sigma_{j} \Sigma_{k} C_{ijk} - SST_{ij}) \right) \quad (1) \\
\text{MIN C2} & = \Sigma_{i} \Sigma_{j} \Sigma_{k} Y_{ijk} \ast \text{RSK}_{ijk} \quad (2) \\
\Sigma_{i \in I} \{ \Sigma_{j} \Sigma_{k} X_{iijk} \} & \leq 1 \ \forall \ j, k \quad (3) \\
\Sigma_{k} Y_{ijk} & = 1 \ \forall \ i, j \quad (4) \\
\Sigma_{k} Q_{pijk} & = 1 \ \forall \ i, j, p \quad (5) \\
C_{ijk} & \leq M \ast Y_{ijk} \ \forall \ i, j, k \in k_{ij} \quad (6) \\
C_{ijk} & \geq \Sigma_{i} Y_{ijk} \ast \text{SST}_{ij} + \text{PT}_{ijk} + \Sigma_{p} \text{PPT}_{pijk} - M(1 - Y_{ijk}) \ i \in W(i) \quad (7) \\
C_{ijk} & \geq \Sigma_{i} Y_{ijk} \ast \text{SST}_{ij} + \text{PT}_{ijk} + \Sigma_{p} \text{PPT}_{pijk} - M(1 - Y_{ijk}) \ i \in E(i), j \neq s_{3} \quad (8) \\
C_{ijk} & \geq \Sigma_{i} Y_{ijk} \ast \text{SST}_{ij} + \text{PT}_{ijk} + \Sigma_{p} \text{PPT}_{pijk} - M(1 - Y_{ijk}) \ i \in E(i), j = s_{3} \quad (9) \\
\Sigma_{k} C_{ijk} + \Sigma_{k} C_{ijk} & \leq TW_{i} \ \forall \ i \in W(i), j = s_{2}, i’ = s_{3} \quad (10) \\
\Sigma_{k} C_{ijk} + \Sigma_{k} C_{ijk} & \leq TW_{i} \ \forall \ i \in E(i), j = s_{2}, i’ = s_{3} \quad (11) \\
X_{iijk} + X_{iijk} & \leq 1 \ \forall \ i < i’, j, k \quad (12) \\
2X_{iijk} & \leq Y_{ijk} + X_{ijk} \ \forall \ i < i’, j, k \quad (13) \\
Y_{ijk} + Y_{ijk} & \leq X_{iijk} + X_{iijk} + 1 \ \forall \ i < i’, j, k \quad (14) \\
C_{ijk} & \geq C_{i’jk} + \text{SST}_{ij} + \text{PT}_{ijk} - M \ast X_{i’ijk} - 2M + \text{MY}_{ijk} + \text{MY}_{ijk} \ V_{k, j, i’, i’}, i’ < l’ \quad (15) \\
C_{ijk} & \geq C_{i’jk} + \text{SST}_{ij} + \text{PT}_{ijk} - M \ast X_{i’ijk} - 2M + \text{MY}_{ijk} + \text{MY}_{ijk} \ V_{k, j, i’, i’}, i’ < l’ \quad (16) \\
\text{MAX} \Sigma_{k} C_{ijk} = t_{1} \ t_{1} \geq \text{MAX} \Sigma_{i} \Sigma_{k} C_{ijk} - \text{SST}_{ij} \quad (17)
\end{align*}
\]

According to Equation (1), the first objective function of the problem is to minimize patient waiting time in all stages of medical center wards. On the basis of Equation (2), the second objective function is to minimize the risk of using medical equipment. Constraint (3) states that if both patient i and patient i’ are in a similar condition, one of the patients enters the stage of receiving treatment process, while the other patient stays in the queue. Constraint (4) states that according to the patient information table defined upon the patient arrival, each patient should pass all the prescribed stages. To put it differently, the patients cannot skip the procedures. Constraint (5) states that in accordance with a pre-considered procedure, as soon as the patient is registered, the patient certainly receives treatment process. Constraint (6) states that there is a relationship between the time of receiving treatment and its binary variable. Constraint (7) states that if the patient receives treatment at each stage, then the corresponding binary variable related to the medical center personnel will get a value. Constraint (8) states that the end of receiving treatment for each patient includes the time of receiving treatment in the previous stages, the time of patient preparation at that stage, the time of receiving treatment using medical equipment, and the time of receiving treatment from personnel in addition to the final condition that the patient
needs services and personnel at that stage. Constraint (9) states that the end time of receiving a service for each common patient includes the time of receiving services in the previous stages (except recovery stage), the time of patient preparation at that stage, the time of receiving services from equipment, and the time of receiving services from personnel, as well as the final condition that the patient needs services and personnel at that stage. Constraint (10) indicates that the time of receiving treatment at different stages includes the patient’s preparation time, the time of providing service by medical center personnel, the duration of service, the time of entering into the system in case the given stage and ward are defined in the patient treatment description (this constraint checks if the patient needs to receive treatment at that stage, then the end time of patient process will be the sum of preparation time and receiving treatment). Constraint (11) states that the waiting time to receive recovery services should be less than a fixed value for special patients who have undergone an operation. Constraint (12) states that for common patients who have undergone an operation, the waiting time to receive recovery services should be less than the fixed value. Constraint (13) states that if two patients in a ward ask to receive equipment, only one of the patients receives treatment and the other patient is put in the queue. Constraint (14) states that if a patient is in the queue before another patient, they wait until that specific equipment will be unused. In this way, there is a queue for each medical equipment. Constraint (15) completes constraints (7) and (8). Constraint (16) states that if patient \( i \) receives treatment sooner than patient \( i \) and patient \( i \) is in the queue before patient \( i \), the time of receiving treatment for patient \( i \) is longer than patient \( i \), and constraint (17) is in contradiction to constraint (16). In constraint (18), considering the non-linearity of the model, it is required to transform the variable to make the model linear.

3. FUZZY APPROACH

The defuzzification of regular weighted points was introduced by Opricovic and Tzeng [63] for the first time. In their work, they investigated several methods of defuzzification, and finally, CFCS technique was proposed as the proper method of defuzzification in MCDM techniques. The implementation of this approach which involves a five-step algorithm is as follows:

First step: Normalization of values

The maximum value of the upper bound is subtracted from the lowest value of the lower bound to calculate the lowest to the highest interval.

\[
\Phi_{\text{max}}^{\text{min}} = \max u_{ij} - \min l_{ij}
\]

where, \( \Phi_{\text{max}}^{\text{min}} \) is the upper and lower bound of the fuzzy value, \( \max u_{ij} \) is the greatest value of the upper bound in the triangular fuzzy number, and \( \min l_{ij} \) is the smallest bound value of triangular fuzzy number.

Second step: Each bound is separately subtracted from the range

\[
l_{ij} = \frac{(l_{ij} - \min l_{ij})}{\Phi_{\text{max}}^{\text{min}}} (20)
\]

\[
m_{ij} = \frac{(m_{ij} - \min l_{ij})}{\Phi_{\text{max}}^{\text{min}}} (21)
\]

\[
u_{ij} = \frac{(u_{ij} - \min l_{ij})}{\Phi_{\text{max}}^{\text{min}}} (22)
\]

In the above mentioned equation, \( l_{ij} \) is the correction of the lower bound of the fuzzy number, \( m_{ij} \) is the correction of the middle bound of the fuzzy number, and \( u_{ij} \) is the correction of the upper bound of triangular fuzzy number.

Third step: Calculation of normal values of the upper and lower bound

\[
l_{ij} = \frac{m_{ij}}{1 + m_{ij} - l_{ij}} (23)
\]

\[
u_{ij} = \frac{u_{ij}}{1 + u_{ij} - m_{ij}} (24)
\]

At this step, based on the modified bounds in the second step, the normal lower bound and the normal upper bound of the fuzzy numbers are obtained.

Fourth step: Calculation of the total normalized definite values

\[
x_{ij} = \frac{l_{ij}(1 - l_{ij}) + u_{ij} - u_{ij}}{1 - l_{ij} + u_{ij}} (25)
\]

At this step, based on the previous step, the final normalized values of each fuzzy number are obtained and in the next step the definite number for triangular fuzzy numbers is obtained.

Fifth step: Calculation of the definite values

\[
Z_{ij} = \min l_{ij} + (x_{ij} + \Phi_{\text{max}}^{\text{min}}) (26)
\]

3.1. Data analysis

The performance of the proposed mathematical model is verified corresponding real and random data. The present study addresses a specific scheduling problem in a medical center considering 5 patients and 3 different sections including operating room, surgery preparation section and recovery room (note that all 3 sections are not necessarily used by all 5 patients) wherein 9 types of medical equipment, and 9 members of the medical personnel including nurses, doctors and recovery room personnel are deployed. The main parameters are thus as shown in Table I.
Therefore, fuzzy values of the patient preparation time (ST), and time of health care provision (PT), are presented in Tables 2 and 4.

| Parameter | Data Range |
|-----------|------------|
| SST_{ijk} | Uniform – [1,10] |
| PPT_{ijk} | Uniform – [1, 5] |
| TW_j | Uniform – [1, 3] |
| RSK_{ijk} | Uniform – [0, 1] |

TABLE 1. Numerical example data

Considering the fuzzy values and implementation of CFCS method, the definite values obtained from the patients preparation time and time of health care provision are summarized in Tables 3 and 5.

As it can be observed in Figure 1, the operation for all patients is due for completion in 12 hours. Accordingly, the health service procedure of all patients is completed within the certain time. In addition, the deployment of medical staff allocation is presented in Figure 2.

TABLE 2. Fuzzy data of the patients’ preparation time

| Patient. Section | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 | E9 |
|------------------|----|----|----|----|----|----|----|----|----|
| i1.s1            | 2  | 4  | 5  | 2  | 4  | 6  | 1  | 3  | 5  |
| i1.s2            | 3  | 3  | 5  | 2  | 3  | 7  | 2  | 4  | 2  |
| i2.s2            | 2  | 3  | 5  | 3  | 3  | 4  | 2  | 4  | 1  |
| i3.s1            | 4  | 5  | 5  | 1  | 1  | 3  | 2  | 3  | 5  |
| i3.s2            | 2  | 3  | 3  | 2  | 3  | 4  | 3  | 6  | 2  |
| i3.s3            | 1  | 3  | 4  | 1  | 2  | 3  | 1  | 2  | 3  |
| i4.s1            | 1  | 3  | 4  | 2  | 3  | 4  | 2  | 3  | 5  |
| i4.s2            | 2  | 3  | 6  | 3  | 4  | 5  | 5  | 7  | 3  |
| i4.s3            | 3  | 3  | 5  | 2  | 2  | 3  | 5  | 2  | 3  |
| i5.s1            | 2  | 4  | 6  | 3  | 4  | 5  | 3  | 4  | 6  |
| i5.s2            | 3  | 4  | 5  | 2  | 3  | 4  | 2  | 3  | 5  |

TABLE 3. Definite data of the patients’ preparation time

| Patient. Section | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 | E9 |
|------------------|----|----|----|----|----|----|----|----|----|
| i1.s1            | 4  | 4  | 1  | 3  | 1  | 4  | 4  | 3  | 1  |
| i1.s2            | 3  | 4  | 4  | 2  | 4  | 4  | 3  | 2  | 4  |
| i2.s2            | 3  | 3  | 4  | 3  | 4  | 3  | 3  | 3  | 4  |
| i3.s1            | 5  | 1  | 3  | 3  | 1  | 5  | 3  | 3  | 3  |
| i3.s2            | 3  | 3  | 4  | 3  | 4  | 3  | 3  | 3  | 4  |
| i3.s3            | 3  | 2  | 2  | 4  | 2  | 2  | 3  | 4  | 2  |
| i4.s1            | 3  | 3  | 3  | 4  | 3  | 3  | 3  | 4  | 3  |
| i4.s2            | 3  | 4  | 5  | 5  | 5  | 4  | 3  | 5  | 5  |
| i4.s3            | 3  | 2  | 3  | 3  | 2  | 3  | 3  | 4  | 3  |
| i5.s1            | 4  | 4  | 4  | 3  | 4  | 4  | 3  | 4  | 3  |
| i5.s2            | 4  | 3  | 3  | 2  | 3  | 3  | 4  | 2  | 3  |
TABLE 4. Fuzzy data of the time of health care provision

| Patient. Section | l | m | u | l | m | u | l | m | u | l | m | u | l | m | u | l | m | u | l | m | u | l | m | u | l | m | u |
|------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11.s1            | 2 | 2 | 3 | 2 | 3 | 3 | 3 | 5 | 3 | 4 | 6 | 2 | 2 | 3 | 3 | 4 | 6 | 3 | 4 | 5 | 2 | 3 | 5 | 2 | 2 | 3 |
| 11.s2            | 2 | 3 | 4 | 2 | 3 | 6 | 3 | 6 | 7 | 4 | 5 | 7 | 2 | 4 | 6 | 3 | 3 | 7 | 3 | 4 | 5 | 2 | 2 | 3 | 2 | 4 | 6 |
| 12.s2            | 5 | 6 | 7 | 2 | 6 | 8 | 4 | 5 | 7 | 4 | 5 | 9 | 2 | 4 | 6 | 3 | 3 | 6 | 2 | 4 | 5 | 2 | 3 | 5 | 2 | 4 | 4 |
| 13.s1            | 3 | 4 | 5 | 3 | 5 | 6 | 4 | 5 | 6 | 3 | 4 | 8 | 2 | 3 | 5 | 2 | 2 | 3 | 4 | 5 | 8 | 2 | 3 | 5 | 2 | 3 | 5 |
| 13.s2            | 3 | 4 | 5 | 3 | 7 | 3 | 6 | 3 | 3 | 5 | 3 | 4 | 6 | 2 | 3 | 7 | 2 | 3 | 3 | 2 | 3 | 3 | 3 | 6 |
| 13.s3            | 2 | 5 | 7 | 3 | 4 | 7 | 2 | 2 | 3 | 3 | 4 | 6 | 2 | 2 | 3 | 2 | 2 | 3 | 2 | 3 | 4 | 3 | 4 | 6 | 1 | 2 | 3 |
| 14.s1            | 3 | 5 | 6 | 3 | 4 | 8 | 3 | 5 | 3 | 6 | 2 | 3 | 5 | 2 | 3 | 5 | 2 | 3 | 4 | 3 | 4 | 6 | 2 | 3 | 5 |
| 14.s2            | 1 | 1 | 3 | 2 | 3 | 4 | 3 | 4 | 3 | 3 | 5 | 5 | 7 | 3 | 4 | 5 | 2 | 3 | 6 | 3 | 5 | 5 | 5 | 5 | 7 |
| 14.s3            | 3 | 4 | 6 | 2 | 3 | 4 | 2 | 2 | 4 | 2 | 3 | 3 | 2 | 3 | 5 | 2 | 2 | 3 | 3 | 5 | 2 | 3 | 6 | 2 | 3 | 5 |
| 15.s1            | 4 | 5 | 6 | 3 | 4 | 5 | 2 | 3 | 4 | 2 | 3 | 3 | 4 | 6 | 3 | 4 | 5 | 2 | 4 | 6 | 2 | 3 | 4 | 3 | 4 | 6 |
| 15.s2            | 4 | 6 | 9 | 2 | 3 | 4 | 3 | 4 | 6 | 2 | 3 | 3 | 2 | 3 | 3 | 2 | 3 | 4 | 3 | 4 | 5 | 2 | 2 | 3 | 2 | 3 |

TABLE 5. Definite data of the time of health care provision

| Patient. Section | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 | E9 |
|------------------|----|----|----|----|----|----|----|----|----|
| 11.s1            | 2  | 2  | 3  | 4  | 2  | 4  | 4  | 3  | 2  |
| 11.s2            | 3  | 3  | 6  | 5  | 4  | 4  | 4  | 2  | 4  |
| 12.s2            | 6  | 6  | 5  | 6  | 4  | 4  | 4  | 3  | 4  |
| 13.s1            | 4  | 5  | 5  | 5  | 3  | 2  | 5  | 3  | 3  |
| 13.s2            | 4  | 4  | 4  | 3  | 4  | 4  | 3  | 3  | 4  |
| 13.s3            | 5  | 4  | 2  | 4  | 2  | 3  | 4  | 2  | 2  |
| 14.s1            | 5  | 5  | 3  | 4  | 3  | 3  | 4  | 3  | 3  |
| 14.s2            | 1  | 3  | 3  | 3  | 5  | 4  | 3  | 5  | 5  |
| 14.s3            | 4  | 3  | 2  | 3  | 2  | 3  | 3  | 3  | 3  |
| 15.s1            | 5  | 4  | 3  | 4  | 4  | 4  | 3  | 4  | 3  |
| 15.s2            | 6  | 3  | 4  | 3  | 3  | 3  | 4  | 2  | 3  |

Figure 1. Patient scheduling results

Figure 2. The deployment of medical center personnel allocation
Considering Figure 2, the status of medical staff allocation to the patient’s treatment process is obvious.

4. METAHEURISTIC APPROACHES

In this study, a multi-objective mathematical model is presented which is utilized in a NP-HARD operating room scheduling problem based on a fuzzy uncertainty approach. In addition, real problems are generally big in their size and consequently create additional complexity. Regarding the NP-HARD nature of the problem, it would be impossible to solve the problem using exact methods as they are time-consuming and unreliable in order to find an optimal solution. Therefore, two different metaheuristic approaches and the epsilon-constraint method were employed. Moreover, NRGA and NSGA-II are exerted to find the best Pareto solutions for the proposed multi-objective model.

4.1. The Multi-objective Approach Based on NSGA-II and NRGA

Taking into account of NSGA-II and NRGA proposed by Deb et al. [64], Al Jadaan et al. [65] and all above-mentioned studies related to genetic algorithm, it is demonstrated that both algorithms are identical in most of their procedures. They initiate by producing the population. Then, this population is evaluated. In the next steps, two operators are applied to make the selected population even better. The difference between these two algorithms depends on their selection mechanisms as NRGA uses roulette wheel selection while NSGA-II uses binary tournament selection. Figure 3 depicts the pseudo-code of these two algorithms.

4.1.1. Parameter Tuning and Computational Results

In this section, Taguchi method is employed to increase the efficiency of the proposed meta-heuristic algorithm. This method involves various test experiments along with parameter tuning for each algorithm in order to get the best results regarding the given problem.

4.1.1.1. Parameter Settings

In order to prove the applicability of the proposed model, the problem must be solved under various conditions and settings. Hence, here, 12 test problems with various sizes and parameter settings are considered. Different dimensions, parameters and their values to initiate the problem are represented in Table 6.

4.1.2. The Obtained Results of Parameter Tuning

As mentioned before, Taguchi method is employed to set the algorithms parameters in order to get the optimum results. It also decreases the number of total experiments by eliminating unnecessary ones. In this regard, it uses a cluster of factors, which are based on orthogonal arrays. These factors are categorized into two essential groups namely control and noise factors.

| Table 6. Test problems settings |
|---------------------------------|
| Test problem | i | j | k |
| 1 | 8 | 2 | 2 |
| 2 | 10 | 4 | 4 |
| 3 | 20 | 6 | 7 |
| 4 | 25 | 8 | 8 |
| 5 | 30 | 10 | 12 |
| 6 | 41 | 12 | 15 |
| 7 | 45 | 14 | 18 |
| 8 | 50 | 16 | 20 |
| 9 | 55 | 18 | 22 |
| 10 | 60 | 20 | 30 |
| 11 | 65 | 22 | 35 |
| 12 | 70 | 30 | 40 |
| 13 | 75 | 32 | 45 |
| 14 | 80 | 34 | 50 |
| 15 | 85 | 36 | 55 |

The initial step to implement the Taguchi experimental design is to identify the levels for each factor of the algorithm. The next step is to use Minitab software to analyze the experiment with its Taguchi experiment

| Table 7. Algorithm factors and levels |
|--------------------------------------|
| Algorithms | Factor | Level 1 | Level 2 | Level 3 |
| NSGA-II    | A: Pc  | 0.65    | 0.75    | 0.85    |
|           | B: Pm  | 0.04    | 0.06    | 0.08    |
|           | C: N-pop | 90     | 100     | 130     |
|           | D: Max-iteration | 2x   | 3x    | 4x |
| NRGA       | A: Pm  | 0.65    | 0.75    | 0.85    |
|           | B: Pm  | 0.04    | 0.12    | 0.18    |
|           | C: N-pop | 50     | 100     | 150     |
|           | D: Max-iteration | 2x   | 3x    | 4x |
toolbar. In this respect, the L9 design is used for NSGA-II, NRGA, as aforementioned, to identify the best levels for each algorithm. For this purpose, the evaluation of signal-to-noise-ratio is required. Equation (27) represents the selected signal-to-noise-ratio and its evaluation method.

$$\text{Signal/Noise} = -10 \log \left( \frac{\sum(Y^2)}{n} \right)$$  

(27)

Here, Y and n are the response time and number of orthogonal arrays respectively. To verify the best levels for each algorithm, the signal-to-noise plot for each algorithm is depicted in Figures 4 and 5.

Considering Figures 4 and 5, the best levels for each algorithm factors are described in Table 8.

4.1.3. Metrics for Comparing Algorithms In this section, two different metric indices including Mean Ideal Distance (MID) and CPU time are used to verify the validity of the proposed investigation. The reason for choosing these metric indices is that they measure the different issues of each proposed algorithms. MID considers the convergence rate of each algorithm and CPU time calculates the time to get the optimize solution and answer. In addition, these two metrics are frequently used in the similar studies. So, MID calculates the distance between Pareto solutions and the ideal solution.

The formulation of MID can be described as Equation (28) for two-objective-function model.

$$\text{MID} = \sum_{i=1}^{n} \left( \frac{n_{\text{min}} - n_{\text{best}}}{n_{\text{max}} - n_{\text{total}}} \right)^2 + \left( \frac{c_{\text{best}} - c_{\text{total}}}{c_{\text{max}} - c_{\text{total}}} \right)^2$$  

(28)

In this equation, n, c1 and c2 are the number of non-dominated answers and the value of ith non-dominated answers, respectively. The ideal, smallest and biggest values between all non-dominated answers are represented by $c_{\text{best}}$, $c_{\text{min}}$, $c_{\text{max}}$, and $c_{\text{total}}$, respectively. Smaller values of MID indicate that the algorithm has a better performance. CPU time calculates problem processing time. This value determines how fast an algorithm can get its optimum values that is highly useful in more complex problems in which the running time is very important.

Here, to verify the applicability of the proposed algorithms, the aforementioned metrics are applied to the considered test problems. These 12 test problems are divided into small-sized (1-4), medium-sized (5-8), and large-sized (9-15).

In addition, the Pareto solutions of the considered metaheuristics are depicted in Figure 6.

After designing the test and regulating the parameters, the appropriate parameters in both NSGAII and NRGA have been determined. Now, it is time to implement and compare the algorithms to each other in case of the generated problems. First, an overview of the

| Test Problem | Test Problem 2 | Test Problem 3 |
|--------------|----------------|----------------|
| Test Problem 1 | 16372, 0.05 | 33346, 0.05 | 125219, 90.93 |
| Test Problem 2 | 25105, 0.05 | 24549, 0.06 | 146309, 94.16 |
| Test Problem 3 | 25895, 0.05 | 24681, 0.06 | 136259, 77.72 |
| Test Problem 4 | 14741, 0.05 | 33960, 0.05 | 154396, 90.86 |
| Test Problem 5 | 18880, 0.04 | 26724, 0.05 | 136777, 105.56 |
| Test Problem 6 | 24888, 0.04 | 22543, 0.08 | 115376, 86.45 |
| Test Problem 7 | 16372, 0.05 | 33346, 0.05 | 125219, 90.93 |
| Test Problem 8 | 25105, 0.05 | 24549, 0.06 | 146309, 94.16 |
| Test Problem 9 | 25895, 0.05 | 24681, 0.06 | 136259, 77.72 |

TABLE 9. The exact method's results

![Figure 6. The Pareto solution](image-url)
problem chromosome and allocation algorithms is displayed in Figures 7, 8 and 9.

As indicated in Figure 7 the problem chromosome has two parts that all numbers of which are filled with random numbers between 0 and 1. Then, they are sorted out and the obtained sequences are used as allocation sequences. It should be noted that this study uses a priority-based coding method.

4.2. Algorithm Analysis

Evaluating the T-Test on the means of the first objective function:

Table 10 indicates the results of the T-Test on the means of the first objective function. In addition, Figures 10 and 11 display a comparative diagram showing the means of the first objective function in each sample test, as well as showing the boxplot for rejecting or accepting the null hypothesis in the T-Test.

**Figure 7.** An overview of the problem chromosome

**Figure 8.** Allocation algorithm for the first part of the chromosome

**Figure 9.** Allocation algorithm for the second part of the chromosome

**TABLE 10.** T-Test results on the means of the first objective function

| Index          | Algorithm | NSGA-II | NRGA |
|----------------|-----------|---------|------|
| Sample Test    | 15        | 15      |
| Mean           | 5626072   | 4497208 |
| S.D            | 3852039   | 3821250 |
| Confidence Interval 95% | (4041, 53686) | (4041, 53686) |
| T-Test         | 2.49      |         |
| P-Test         | 0.026     |         |

**Figure 10.** Comparison of the means of the first objective function for sample tests using meta-heuristic algorithms

Based on Table 10 and the p-value, there is a significant difference between the means of the first objective...
function obtained from NSGA-II and NRGA. According to the minimization state of the first objective function, it can be concluded that NRGA achieves better results than NSGA-II in this index.

Based on Figure 10, NRGA obtains better results than NSGA-II in regard to the sample test 12-15. As a result, NRGA will be more efficient in obtaining the results of the first objective function in larger scales.

Based on the boxplot in Figure 11, since the null hypothesis is not in the obtained range, there is a significant difference between the means of the first objective function obtained from NSGA-II and NRGA.

Evaluating the T-Test on the means of the second objective function:

Table 11 indicates the results of the T-Test on the means of the second objective function. In addition, Figures 12 and 13 indicate the comparative diagram showing the means of the second objective function in each sample test, as well as the boxplot for rejecting or accepting the null hypothesis in the T-Test.

Regarding the statistics of P-Test at 0.584 obtained from Table 11, it can be concluded that there is no significant difference between the means of the second objective function. Thus, multi-criteria decision making methods such as TOPSIS should be used for comparing the most efficient algorithms.

**TABLE 11.** T-Test results on the means of the second objective function

| Index               | Algorithm | Algorithm |
|---------------------|-----------|-----------|
| Sample Test         | NSGA-II   | NRGA      |
| Mean                | 134440    | 135094    |
| S.D                 | 45240     | 45418     |
| Confidence Interval | (-3156, 1848) | (-3156, 1848) |
| T-Test              | 0.56      |           |
| P-Test              | 0.584     |           |

Figure 12 displays a comparative diagram for the means of the second objective function in different sample tests. It reveals that the results obtained from the sample tests are not different from each other. Thus, it is not easy to comment on the efficiency of algorithms in the second objective function.

Figure 13 completes the results of Table 11 and since the null hypothesis is in a 95% confidence interval, it can be concluded that there is no significant difference between the means of the second objective function obtained by NSGA-II and NRGA.

Evaluating the T-Test on the means of the number of efficient answers:

In this section, statistical comparisons are performed on the comparison indices of meta-heuristic algorithms. Table 12 indicates the results of the T-Test on the means of the number of efficient answers establishing a 95% confidence level.

Since the p-value is more than 0.05 as the base value, it can be concluded that the null hypothesis is accepted. Thus, there is no significant difference between the means of the number of efficient answers obtained from the meta-heuristic algorithms.

Figure 14 compares the means of the number of efficient answers for NSGA-II and NRGA. Based on Figure 14, the number of efficient answers in each sample test is different and no correct answer of the algorithm
TABLE 12. T-Test results on the means of the number of efficient answers

| Index          | Algorithm       | NSGA-II | NRGA |
|----------------|-----------------|---------|------|
| Sample Test    | 15              | 15      |
| Mean           | 20.27           | 1727    |
| S.D            | 5.04            | 6.91    |
| Confidence Interval 95% | (-1.48, 7.48)         |
| T-Test         | 1.43            |
| P-Test         | 0.173           |

Figure 14. Comparing the means of the number of efficient answers in sample tests using meta-heuristic algorithms

Efficiency can be inferred from the obtained results of this index.

Figure 15 demonstrates a boxplot for confirming or rejecting the null hypothesis for the means of the number of efficient answers. It can be concluded that the null hypothesis is accepted due to its location at the confidence interval while hypothesis 1 is rejected.

Evaluating the T-Test on the means of the most extended deviation:

Table 13 represents the statistical comparison of the T-Test on the means of the most extended deviation index.

| Index          | Algorithm       | NSGA-II | NRGA |
|----------------|-----------------|---------|------|
| Sample Test    | 15              | 15      |
| Mean           | 2478417         | 2260667 |
| S.D            | 1496766         | 1661211 |
| Confidence Interval 95% | (-88459, 523960)         |
| T-Test         | 1.53            |
| P-Test         | 0.149           |

Table 13 shows no significant difference in the means of the most extended deviation index obtained from NSGA-II and NRGA. In this test, the p-value is higher than the considered confidence level.

As indicated in Figure 16, NRGA operates more efficiently than NSGA-II in most sample tests. This reveals that the answers obtained from the first and second objective functions using NRGA are more extended than NSGA-II.

Figure 17 shows that the value of the null hypothesis is in a 95% confidence interval for the most extended deviation index.

Figure 15. Boxplot for confirming or rejecting the null hypothesis for the means of the number of efficient answers

Figure 16. Comparing the means of the most extended deviation index in sample tests using meta-heuristic algorithms

Figure 17. Boxplot for confirming or rejecting the null hypothesis for the means of the most extended deviation index
Evaluating T-Test on the means of Gap index:

Table 14 shows the statistical comparison of the T-Test on the means of gap index. In addition, Figure 18 shows the comparison of the means of gap index in all sample tests employing NSGA-II and NRGA.

The results presented in Table 14 and the obtained p-value (0.205) indicate no significant difference between the means of gap index using NSGA-II and NRGA.

Figure 18 displays the efficiency of NSGA-II compared to NRGA in terms of obtaining the results of gap index. This means that the dispersion of the results of the first and second objective functions in NSGA-II is more systematic than the NRGA.

Furthermore, Figure 19 completes the results of Table 14 and shows the rejection of hypothesis I and non-significant difference in comparing the means of gap index.

Evaluating the T-Test on the means of computational time:

Table 15 indicates the results of the T-Test on the means of computational time. In addition, Figures 20 and 21 show the comparative diagram of the means of computational time in each sample test, as well as a boxplot for rejecting or accepting the null hypothesis in the T-Test.

Based on the p-value obtained from the comparisons of T-Test on the means of computational time, it can be acknowledged that there is no significant difference between the means of computational time obtained from NSGA-II and NRGA.

| Index          | Algorithm   | NSGA-II | NRGA |
|----------------|-------------|---------|------|
| Sample Test    |             | 15      | 15   |
| Mean           |             | 0.623   | 0.557|
| S.D            |             | 0.152   | 0.148|
| Confidence Interval 95% |       | (-0.0405, 0.1725) | (-0.0405, 0.1725) |
| T-Test         |             | 1.33    |      |
| P-Test         |             | 0.205   |      |

Table 15. T-Test results on the means of computational time

| Index          | Algorithm   | NSGA-II | NRGA |
|----------------|-------------|---------|------|
| Sample Test    |             | 15      | 15   |
| Mean           |             | 844     | 1041 |
| S.D            |             | 730     | 1220 |
| Confidence Interval 95% |       | (-483, 88) | (-483, 88) |
| T-Test         |             | 1.48    |      |
| P-Test         |             | 0.160   |      |

Figure 18. Comparing the means of gap index in sample tests using meta-heuristic algorithms

Figure 19. Boxplot for confirming or rejecting the null hypothesis for the means of gap index

Figure 20. Comparing the means of computational time in sample tests using meta-heuristic algorithms

Figure 21. Boxplot for confirming or rejecting the null hypothesis for the means of computational time
Figure 20 more accurately shows that the computational time exponentially increases with the increase of sample size, which is a reason for the NP-hardness of the problem. Nevertheless, it is revealed that NRGA is more efficient than NSGA-II in terms of computational time in medium problems, while the computational time obtained by this algorithm significantly increases with the increase of the problem size.

Finally, Figure 21 shows that the null hypothesis is in a 95% confidence interval that is a reason for the first hypothesis. Thus, it can be concluded that there is no significant difference between the means of computational time obtained from NSGA-II and NRGA.

Therefore, Table 16 shows an overview of the significant differences between the means of the comparison indices. Based on the previous results and the following table, it can be stated that there is a significant difference only between the means of the first objective function obtained from solving the sample tests using NSGA-II and NRGA.

Selecting the most efficient algorithm using TOPSIS method:

In the previous section, the comparisons were conducted to determine the significant difference between the means of computational index obtained from NSGA-II and NRGA and the results indicate a significant difference only between the means of the first objective function. In this section, the TOPSIS multi-criteria decision making method was used to select the most efficient algorithm. Thus, Table 17 indicates the total means obtained from 15 sample problems.

### TABLE 16. Overview of the significant difference between the mean of comparison indices

| Index                                           | Significant Difference |
|-------------------------------------------------|------------------------|
| The mean of the first objective function        | Yes                    |
| The mean of the second objective function       | No                     |
| Number of efficient answers                     | No                     |
| The most extended deviation index               | No                     |
| Gap index                                       | No                     |
| Computational time index                        | No                     |

### TABLE 17. Means of the obtained indices using meta-heuristic algorithms

| Index     | NSGA-II  | NRGA    | Weight |
|-----------|----------|---------|--------|
| Obj1      | 4526072  | 4497208 | 0.4    |
| Obj2      | 45240    | 45418   | 0.4    |
| Efficiency Answer | 20.27 | 17.27   | 0.05   |
| Deviated Extended | 2478417 | 2260667 | 0.05   |
| Gap       | 0.623    | 0.557   | 0.05   |
| Time      | 844      | 1041    | 0.05   |

After descaling the results of Table 17, the data were entered into MCDM engine software and the output indicates the efficiency of the NSGA-II with a weight of 0.6945 compared to NRGA with a weight of 0.3055. Consequently, the use of NSGA-II is recommended according to all indices and results.

5. CONCLUSION AND SUGGESTION FOR FURTHER RESEARCH

Numerous studies have been conducted on reducing costs and increasing efficiency of the operating rooms in medical centers. This paper presents a fuzzy model for operating room planning and scheduling as well as concerning the limitations of access to medical staff and patient prioritization. The results reveal the validity of the model by solving a number of problems with regard to different number of patients at small and medium scales with random data. Additionally, in order to solve the problem, defuzzification of values is performed based on the CFCS method. Furthermore, based on the fuzzy mathematical model, two meta-heuristic algorithms of NSGA-II and NRGA are developed. So, after adjusting the parameters based on the Taguchi approach, 15 samples of operating room scheduling problems are designed and the evaluation of two meta-heuristic algorithms are analyzed. The analysis indicates that both meta-heuristic algorithms have a good efficiency compared to the gaps of the obtained solutions. Thus, the hypothetical test is presented for validating the algorithms. In the conducted analysis, no significant difference is observed in other models except between the means of the first objective function in the NSGA-II and NRGA. Ultimately, the algorithms are evaluated using the TOPSIS method indicating that the NSGA-II with a weight of 0.6945 is more effective. Based on the evaluation, it is suggested to implement the mathematical model in medical staff scheduling in order to increase personnel satisfaction. In addition, for further research in this area, the use of recent meta-heuristic algorithms such as red deer and social engineering optimization algorithms is worth being undertaken in a future research project in order to develop and solve problems. Moreover, in this study, Benders’ decomposition and Lagrangian relaxation algorithms are deployed.

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چکیده
امروزه، مسائل برنامه‌ریزی و زمان‌بندی مهم‌ترین مسائل در جهان هستند و تاثیر چشمگیری بر بهبود بهره‌وری سازمان‌ها و سرویس‌دهی به سیستم‌های ارائه دهنده خدمات درمانی و پزشکی دارند. از آنجا که برنامه‌ریزی اتاق‌های عمل جراحی یک مساله عمده در سازمان‌های مراقبت از سلامت می‌باشد، بهینه‌سازی کارکنان مراکز پزشکی و تجهیزات پزشکی نقشی اساسی ایفا می‌کند. از این رو، این مطالعه یک مدل ریاضی چندمنظوره با طبقه‌بندی (پیش از عمل، حین عمل، و پس از عمل) به منظور حداکثر سایر مسائل مربوط به اتاق‌های عمل و ریسک استفاده از تجهیزات ارائه می‌باشد. محدودیت‌های زمانی در سیستم‌های سلامت و ظرفیت محدود تجهیزات پزشکی مهم‌ترین محدودیت‌های اساسی این مساله هستند. همچنین، از آنجا که مدت انتظار بیمار و پیامدهای فراوان‌یابی درمان در سه حالت خوشبینه، بدبینه و نرمال رخ می‌دهد، مدل‌الگوریتمی معرفی شده با کیفیت بالا، با توجه موارد و رفتار زمان‌بندی و تخصیص تجهیزات و پزشکی به منظور ارزیابی و تقابل مدل‌های پیشنهادی مورد بررسی قرار می‌گیرد. با توجه به اینکه مدل‌های پیشنهادی و روش‌های حل کردن مسئله مزین، حداکثری از میزان مزایا و کاهش نیازهای اطلاعاتی در مدل‌های پیشنهادی، از الگوریتم‌های مبتنی بر نظریه و ریاضی، برای حل مسئله مورد بررسی قرار گرفته است. از این رو، این مقاله این الگوریتم‌ها را برای حل مسئله مورد بررسی قرار می‌دهد و نتایج نشان می‌دهد که الگوریتم NSGA-II در مسئله زمان‌بندی اتاق عمل کارکنان است.