Numerical simulation of the propagation of a shock wave from a neutral and electrically charged dusty medium into a homogeneous gas

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Abstract. In this work, the expansion of a shock wave from a dusty medium into a pure gas is simulated. The carrier medium is described as a viscous compressible and heat-conducting gas. A mathematical model of the dusty medium dynamics allowed for force interaction. Also, the mathematical model took into account inter-component heat transfer. The aim of the work was to study the effect of the electric charge of the dispersed component on the parameters of a shock wave during the passage of a shock wave from an inhomogeneous medium to a homogeneous gas. Differences between processes in neutral and electrically charged dusty media were revealed.

1. Introduction
One of the important sections of liquid and gas mechanics is the dynamics of the inhomogeneous mediums [1, 2]. The most complicated is the dynamics simulation of the medium which consists of different aggregate states components [3-11]. For example, such mediums as the bubble, vapour-droplet mediums or solid particle suspensions – gas mixtures. Interest to such mediums can be attracted by technical tasks bound to powder mediums transport, problems of the industrial explosions screening and also a supersonic dusting of the powder coverings in an electric field. In the latter case for such processes simulation, it is necessary to use a mathematical model which would consider interfacial force and thermal interactions as well as the presence of the aerodynamic and electromagnetic nature forces [4-7].

2. Mathematical model.
The set of equations of the two-phase two-temperature two-speed monodisperse mixture moving in a two-dimensional case is [1, 2, 6, 7]:

\[
\begin{align*}
\frac{\partial \rho_1}{\partial t} + \nabla \rho_1 V_1 &= 0, \\
\frac{\partial \rho_2}{\partial t} + \nabla \rho_2 V_2 &= 0,
\end{align*}
\]

(1)
\[
\frac{\partial \rho V_i}{\partial t} + \nabla \cdot (\rho_i V_i^k) - \nabla \cdot \tau_i + \nabla p = -F + \alpha_2 \nabla p ,
\]

\[
\frac{\partial \rho \alpha V_2}{\partial t} + \nabla \cdot (\rho_2 V_2^k) = F - \alpha_2 \nabla p ,
\]

\[
\frac{\partial (e_i)}{\partial t} + \nabla \cdot ((e_i + p - \tau_i)V_i) + \Delta T_i = -Q - |F_i| (V_i^k - V_2^k) + \alpha_2 \nabla \cdot (pV_i^k) ,
\]

\[
\frac{\partial (e_2)}{\partial t} + \nabla \cdot e_2 V_2 = Q.
\]

Here and below \( V_i = [u_i, v_i] \) is a velocity vector of the carrying and disperse components of the multiphase medium, \( i = 1, 2; \tau_i = \tau_{xi, k} \) – viscous stress tensor of the carrying component [12, 13]:

\[
\tau_{xi, 1} = \mu(2 \frac{\partial u_i}{\partial x_1} - \frac{2}{3} D), \quad \tau_{x2, 2} = \mu(2 \frac{\partial v_i}{\partial x_2} - \frac{2}{3} D), \quad \tau_{x1, 2} = \mu(\frac{\partial u_i}{\partial x_1} + \frac{\partial v_i}{\partial x_2}) , \quad D = \frac{\partial u_i}{\partial x_1} + \frac{\partial v_i}{\partial x_2} \quad (2)
\]

Interphase force interaction was described by the equations:

\[
F_{xl} = \frac{3}{4} \frac{a_2}{(2r)} C_d \rho_1 \left( \frac{u_i - u_2}{2} + \frac{v_i - v_2}{2} (u_i - u_2) + \alpha_2 \rho_1 \left( \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x_1} + v_i \frac{\partial u_i}{\partial x_2} - \frac{\partial u_2}{\partial x_1} \right) \right) + 0.5 \alpha_2 \rho_2 \left( \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x_1} + v_i \frac{\partial u_i}{\partial x_2} - \frac{\partial u_2}{\partial x_1} \right) - q_0 \frac{\partial \varphi}{\partial x_1} ,
\]

\[
F_{x2} = \frac{3}{4} \frac{a_2}{(2r)} C_d \rho_1 \left( \frac{v_i - v_2}{2} (v_i - v_2) + \alpha_2 \rho_1 \left( \frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x_1} + v_i \frac{\partial v_i}{\partial x_2} - \frac{\partial v_2}{\partial x_1} \right) \right) + 0.5 \alpha_2 \rho_2 \left( \frac{\partial v_i}{\partial t} + u_i \frac{\partial v_i}{\partial x_1} + v_i \frac{\partial v_i}{\partial x_2} - \frac{\partial v_2}{\partial x_1} \right) - \alpha_2 \rho_2 (q_0 \frac{\partial \varphi}{\partial x_2} + g).
\]

The heat flow between the components of the mixture is described by the expression: \( Q = 6 \alpha \text{Nu}_{12} \lambda (T_1 - T_2)/(2r)^2 \).

\( T_1 = (\gamma - 1) (e_1 / \rho_1 - 0.5(u_1^2 + v_1^2)) / R, \) \( R \) – gas constant of the carrier phase, \( \gamma \) – heat capacity ratio, \( r \) – radius of the particle, \( \lambda \) – the heat capacity of the gas, \( \mu \) – the viscosity of the gas.

Here \( p \) – is the gas pressure; \( T_1, T_2 \) is the temperature of the carrier and dispersed components; \( \rho_1, \rho_2 = \alpha_2 \rho_2 \) – gas density and the "average" density of the dispersed component; \( e_1, e_2 \) – total gas energy and internal energy of the dispersed component. Internal energy of the solid phase fractions suspended in gas is defined as \( e_2 = \rho_2 C_p T_2, \) \( C_p \) – the specific heat of the dispersed component; \( a_2 \) – volume content of the dispersed component; \( \rho_2 \) – physical density of the substance of the dispersed component.

The Nusselt number is determined using the well-known approximation depending on the relative Mach, Reynolds numbers and the Prandtl number [2]:

\[
\text{M}_{12} = \frac{|V_1 - V_2|}{c}, \quad \text{Re}_{12} = \rho_1 |V_1 - V_2| 2r / \mu, \quad \Pr = C_p \mu / \lambda , \quad (4)
\]

\[
\text{Nu}_{12} = 2 \exp(-M_{12}) + 0.459 \text{Re}_{12}^{0.55} \Pr^{0.33}, \quad C_d = \frac{24}{\text{Re}_{12}} + \frac{4}{\text{Re}_{12}^{1.5}} + 0.4 .
\]

The set of equations was solved by a second-order explicit MacCormack method [13] with the subsequent application of the decision nonlinear correction scheme [14].
Electric field potential in the computational area was found from the solution of the Poisson equation with second-type boundary conditions [15, 16]:

\[
\Delta \varphi = \rho_2 q_2.
\] (5)

In the right side of the Poisson equation the gas mixture charge density normalized to an absolute capacitity of the carrying medium was placed. At the realization of a numerical algorithm for the carrying and disperse phases velocity components the Dirichlet boundary conditions were set. For other two-phase mixture variable parameters homogeneous Neumann boundary conditions were used. The procedure of the Poisson equation solution assumed the definition of the Dirichlet boundary conditions. In the article [9] the physical experiment [3] on the distribution of a shockwave from pure gas in a gas mixture was numerically modelled. The physical experiment made in work [3] revealed the quantitative and qualitative differences of a shockwave moving velocity dependence from pure gas and from a gas mixture. The analysis of numerical simulation shows satisfactory compliance of numerical calculations results with the results received in a physical experiment. In this work gas parameters and a disperse phase concentration comparison at the shockwave distribution from electrically neutral and the charged gas mixture to pure gas is carried out.

3. Calculation results.

In figure 1 the shock channel [3, 9], in which the high-pressure chamber is filled with compressed gas comprising a disperse phase – an compressed gas mixture and the low-pressure chamber is filled with pure gas, is schematically represented. Initial volume content of a disperse phase was supposed equal to \( \alpha_2 = 0.0001 \). The true physical density of a disperse phase was equal to nichrome density – \( \rho_{20} = 8400 \text{ kg/m}^3 \).

![Figure 1: Schematic image of a shock channel, one of the compartments of which is filled with the charged gas mixture.](image-url)

The intensity of an initial pressure difference was assumed equal to two: \( \rho_2 / \rho_1 = 2 \). In figure 2 (a – d) spatial distributions of the gas pressure in various time instants during moving of direct shock wave from the electrically neutral and the charged gas mixture in pure gas are presented. In the scattering of the electrically charged gas mixture increase of the gas pressure on the compression section is observed that there is no in the course of the scattering of the electrically neutral gas mixture, where the largest pressure value is reached immediately at the pressure-shock front. Along with it in presence of electrically charged particles of a disperse phase, gas pressure in an exhaustion wave has the essentially smaller value, than when the scattering of the gas mixture with electrically neutral disperse component. It is also possible to note that with time the difference of the pressure values of gas in the compression and exhaustion waves during the scattering of the electrically neutral and charged gas mixtures increases, what displayed in table 1: \( \Delta p_1 \) – maximum pressure difference in calculations of neutral and electrically charged dust media scattering in the area between compression wave and exhaustion wave; \( \Delta p_2 \) – maximum pressure difference in calculations of neutral and electrically charged dust media scattering in the exhaustion wave area.
Table 1. The maximum values of gas pressure differences on identical areas of compression and exhaustion waves in case of neutral and electrically charged disperse phase scattering in various time instants.

| Time instants in milliseconds | $\Delta p_1$        | $\Delta p_2$        |
|-----------------------------|---------------------|---------------------|
| $t = 5.83$ ms               | $\Delta p_1 = 680$ Pa | $\Delta p_2 = 2967$ Pa |
| $t = 11.6$ ms              | $\Delta p_1 = 1824$ Pa | $\Delta p_2 = 2509$ Pa |
| $t = 17.4$ ms              | $\Delta p_1 = 3019$ Pa | $\Delta p_2 = 1248$ Pa |
| $t = 23.3$ ms              | $\Delta p_1 = 3780$ Pa | $\Delta p_2 = 1248$ Pa |

In figure 3 comparison of the density spatial distributions of the disperse phase received in the numerical calculations with considering and without considering Coulomb force is presented. As appears from the figure the charged gas mixture extends in the low pressure chamber quicker that can be explained with the fact that particles of the charged disperse phase have a charge of the identical sign and repel from each other.

Thus due to repulsion of particles from each other the disperse phase accelerates in the direction of the compression wave moving, at the same time at the expense of an electric field concentration of a disperse phase in the area of the exhaustion wave becomes less, and in the area of the compression wave the high concentration of a disperse phase is observed. In this regard there is a pressure increase of gas on sites to the increased and reduced concentration of a disperse phase [2]. The revealed lows can be used in dusting processes optimization of the charged disperse metal inclusions in supersonic streams.

Figure 2. Spatial distributions of gas pressure in case of decay of discontinuity from electrically neutral gas mixture – curve 1 and from the charged gas mixture – curve 2. In
time instants $t = 5.83$ ms – (a), $t = 11.6$ ms – (b), $t = 17.4$ ms – (c), $t = 23.3$ ms – (d).

Figure 3. Spatial distribution of average density of a disperse phase in time instant $t = 23.3$ ms.

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