Cost Functions Over Feasible Power Transfer Regions of Virtual Power Plants

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Abstract—A virtual power plant (VPP) facilitates the integration of distributed energy resources (DERs) for transmission-level operations. A challenge in operating a VPP is to characterize the cost function over its feasible power transfer region under DERs’ uncertainties. To address this challenge, a characterization method is proposed for the intraday operation of a VPP with nonanticipativity and robustness considerations to DERs’ uncertainties. The characterization stems from designing operational requirements subject to nonanticipativity and robustness, by which a feasible power transfer region across all the time periods is constructed by boundary points at each time period and time-coupling constraints.

Furthermore, a cost function over the feasible power transfer region is formulated as the combination of convex piecewise surfaces and a constant compensation cost. Finally, a perspective to analyze the impacts of DERs’ uncertainties on feasible power transfer regions is studied. Numerical experiments in the IEEE 33-bus and 136-bus systems show that 1) our feasible power transfer region fulfills nonanticipativity and robustness to face DERs’ uncertainties, 2) our cost function covers the true operational cost of a VPP, and 3) impacts of DERs’ uncertainties on feasible power transfer regions can be measured by the product of their fluctuation intervals and system parameters.

Index Terms—Cost function, distributed energy resources (DERs), feasible power transfer region, virtual power plant (VPP).

I. INTRODUCTION

THE penetration of distributed energy resources (DERs) has kept growing in distribution networks [1], [2]. DERs have been observed to bring benefits (e.g., the improvement of system flexibility [3], and reduction of carbon emission [4]). To facilitate the utilization of DERs, the concept of a virtual power plant (VPP) has drawn much attention, leading to practical projects (e.g., FENNIX [5], and EDISON [6]). A VPP is essentially a unit portfolio which aggregates an entire distribution network with DERs, and it can participate in the transmission-level operation by adjusting its power exchanges at the point of common coupling (PCC) [7]. However, the operators of VPPs and transmission networks are usually different. This makes their coordination difficult in a centralized manner. As an alternative, the coordination based on the feasible power transfer region of a VPP was shown to be a promising direction [8].

The feasible power transfer region of a VPP is the feasible region in the domain of PCC power exchanges (i.e., \( P_{\text{PCC}}^0 \) and \( Q_{\text{PCC}}^0 \) in Fig. 1). Given any such feasible power exchanges, the VPP operational requirements (e.g., power balance and voltage limits) are not violated. The feasible power transfer region can be used as constraints in the dispatch problem of a transmission network [9]. Also, a VPP can bid its PCC power exchanges in a transmission electricity market, which motivates the formulation of a cost function over its feasible power transfer region. Such a cost function can be embedded in the objective function of the transmission dispatch problem to reflect the operational cost of a VPP.

Representative methods can be discussed based on what are characterized. First, the methods which only characterize feasible power transfer regions are discussed, including the Fourier–Motzkin elimination methods [10]–[12], the vertex search methods [13], [14], and the Stackelberg-game methods [15]–[17]. Fourier–Motzkin elimination was first used in [10], [11] to project a generation-demand space onto the demand space. Furthermore, Fourier–Motzkin elimination was used in [12] to project the power flow and operational constraints onto the space of \((P_{\text{PCC}}^0, Q_{\text{PCC}}^0)\). In that process, the variables except \((P_{\text{PCC}}^0, Q_{\text{PCC}}^0)\) are iteratively bounded in terms of other variables and thus eliminated. A vertex search method was performed in [13] by solving linear programs (LPs) to find boundary hyperplanes of the feasible power transfer region. Uncertainties of DERs were further incorporated in the vertex search method [14] by robust optimization. The Stackelberg-game methods [15], [16] formulated min–max problems to construct the feasible power transfer region. Stemming from an outer approximation to the feasible power transfer region, Refs. [15] and [16] solved the min–max problems to find the worse point, which violates the operational requirements, and then a hyperplane was constructed to remove the worse point from the outer approximation. DERs’ uncertainties were further incorporated in the Stackelberg-game method [17] by chance constraints.

However, cost functions over power transfer regions were neglected in the methods above. This prevents that a VPP bids its PCC power exchanges in a transmission electricity market. Also, most methods mentioned above neglected DERs’ uncertainties, except [14] and [17]. Ref. [14] converted uncertainties into deterministic intervals by a two-stage robust optimization, and thus the vertex search method [13] can be applied to get a feasible power transfer region under uncertainties. Ref. [17]
employed chance constraints to consider uncertainties modeled by probabilistic distribution functions. Based on a quantile approach, these chance constraints were converted into deterministic constraints. However, the methods in [14] and [17] preset the power exchanges of a VPP in a day-ahead manner, whereas the VPP operational decisions such as generation levels can only be determined intraday after observing the realizations of uncertain DER outputs at different time periods successively. In other words, the methods in [14] and [17] ignored the critical requirement of nonanticipativity for stochastic cases. As reported in [18], current decisions without nonanticipativity may not satisfy operational requirements at future periods for all uncertainty scenarios, i.e., robustness may not be guaranteed. This motivates us to study VPP operations based on nonanticipativity and robustness with which cost functions over feasible power transfer regions of VPPs are further characterized.

Different from [12]–[17], which only characterize feasible power transfer regions, multiparametric programming was introduced in [19] and [20] to calculate a feasible power transfer region and its cost function in a regional transmission network. The combination of active and inactive constraints in an economic dispatch problem varies with the border power flows that serve as programming parameters. For each such parameter, the Karush–Kuhn–Tucker conditions can be divided into linear equalities and inequalities. A facet of the cost function over the feasible power transfer region was solved from the set of linear equalities, while a subset of the feasible power transfer region was specified by the inequalities. A modified method in [21] only enumerated the boundaries of the feasible power transfer region to reduce the computational burden compared to [19] and [20], without characterizing the cost function over the whole feasible power transfer region. However, uncertainties were neglected in [19]–[21]. Also, Refs. [19]–[21] rely on the linearity of a dc power flow model to calculate cost functions over feasible power transfer regions. However, a nonlinear convex second-order-cone-based power flow model is usually used for VPPs in distribution networks, as done in VPP studies [22], [23]. Consequently, the multiparametric programming methods [19]–[21], which are the unique type to calculate cost functions over feasible power transfer regions, do not work for VPPs. This motivates us to develop a novel method to characterize cost functions over feasible power transfer regions of VPPs.

To overcome the limitations mentioned above, this article studies the cost function over the feasible power transfer region of a VPP with nonanticipativity and robustness considerations, as sketched in Fig. 1 and compared in Table I with the previous methods. Given any pair \((P_{0}^{\text{PCC}}, Q_{0}^{\text{PCC}})\) within the feasible power transfer region in the day-ahead horizon, the intraday decisions of a VPP always turn out to satisfy operational requirements as the realizations of DERs’ outputs are successively observed. The major contributions of the proposed characterization method can be summarized as follows:

1) The proposed characterization method can provide a feasible power transfer region, which is found by exploring boundary points at each time period and establishing time-coupling constraints. Particularly, such a feasible power transfer region is calculated through an optimization problem, which is designed for the intraday decisions of a VPP to fulfill nonanticipativity and robustness. Compared with the current methods that neglect nonanticipativity and robustness, the proposed characterization method guarantees the nonanticipative and robust intraday operation of a VPP.

2) The proposed characterization method can provide the cost function over the feasible power transfer region of a VPP operated in a nonlinear convex model. The cost function is explicitly formulated as convex piecewise surfaces superposed on a constant compensation cost to guarantee an upper estimation of the true cost. Compared with the current methods which rely on linear models, our method provides a general idea to calculate cost functions.
over feasible power transfer regions based on nonlinear convex models.

In addition to the proposed characterization method, this article provides a perspective to analyze the impacts of DERs' uncertainties on feasible power transfer regions. Particularly, it was found that such impacts can be measured by the production of system parameters and DER fluctuation intervals.

The rest of this article is organized as follows. An optimization problem is designed in Section II to fulfill intraday decisions of VPPs under nonanticipativity and robustness. Based on the optimization problem above, the proposed characterization method is illustrated in Section III, together with the impact analysis of DERs' uncertainties. Numerical studies are given in Section IV in the IEEE 33-bus and 136-bus test systems. Section V concludes this article.

II. INTRADAY VPP OPERATIONS UNDER NONANTICIPATIVITY AND ROBUSTNESS

The intraday operational requirements of a VPP are illustrated in Section II-A, followed by an optimization problem in Section II-B to fulfill nonanticipativity and robustness in the VPP’s intraday operation.

A. Intraday Operational Requirements of a VPP

In our discussed VPP that encompasses a radial distribution network, we use the Dist-Flow equations [24] and the second-order-cone-based power flow model [25] to describe the intraday operational requirements of a VPP, as done in [22], [23]. For illustration purposes, only fuel DERs, energy storage DERs, and renewable DERs are incorporated in our discussed VPP. Uncertainties are introduced by renewable DERs. The intraday operational requirements can be presented based on whether they are coupled over time periods as following.

1) Requirements without time coupling at time period $t$

a) Power balance at each node, i.e.,

\[
P_{ij,t} - R_{ij}i_{ij,t} + \sum_{g \in G} e_{jg} P_{g,t}^G + \sum_{n \in N} e_{jn} \left( P_{n,t}^{N,d} - P_{n,t}^{N,c} \right) + e_{j} P_{ij,t}^{PCC} = \sum_{k \in L_i} P_{jk,t} + \sum_{r \in R} e_{jr} P_{r,t}^{R}, \forall i, \forall j, \forall t
\]

(1)

\[
Q_{ij,t} - X_{ij} i_{ij,t} + \sum_{g \in G} e_{jg} Q_{g,t}^G + e_{j} Q_{ij,t}^{PCC} = \sum_{k \in L_i} Q_{jk,t} + \sum_{r \in R} e_{jr} \tan \beta_r P_{r,t}^{R}, \forall i, \forall j, \forall t
\]

(2)

where $P_{ij,t}$ and $Q_{ij,t}$ are active and reactive branch power flows from node $i$ to node $j$ at time period $t$, respectively; $i_{ij,t}$ is the square of current magnitude through the branch $i \rightarrow j$; $P_{g,t}^G$ and $Q_{g,t}^G$ are active and reactive generation levels of unit $g$ at time period $t$, respectively; $P_{ij,t}^{PCC}$ and $Q_{ij,t}^{PCC}$ are active and reactive power exchanges at PCC at time period $t$, respectively; $P_{i,t}^R$ is net active demand at DER node $i$ at time period $t$; $P_{n,t}^{N,d}$ and $P_{n,t}^{N,c}$ are discharging and charging power of energy storage $n$ at time period $t$, respectively; $G$ is the set of unit nodes; $N$ is the set of energy storage nodes; $R$ is the set of renewable nodes; $L_j$ is the set of nodes that are connected to node $j$; $R_{ij}$ and $X_{ij}$ are resistance and reactance of the branch $i \rightarrow j$; $\beta_r$ is the constant power factor angle at node $r$; $e_{jg}, e_{jn}, e_{jr}$ are incident indicators.

b) The voltage change along the branch $i \rightarrow j$ is given as follows:

\[
v_{j,t} - v_{i,t} = -2 \left( R_{ij} P_{ij,t} + X_{ij} Q_{ij,t} \right) + \left( (R_{ij})^2 + (X_{ij})^2 \right) i_{ij,t}, \forall i, \forall j, \forall t
\]

(3)

where $v_{i,t}$ is the square of the voltage magnitude at node $i$ at time period $t$.

c) The active branch flow should not exceed its upper bound $P_{ij,t}$ and lower bound $P_{ij,t}$, i.e.,

\[
P_{ij,t} \leq P_{ij,t} \leq P_{ij,t}, \forall i, \forall j, \forall t.
\]

(4)

d) Relationship among branch flows, voltages, and currents can be described as follows:

\[
v_{i,t}, i_{ij,t} \geq \left( P_{ij,t} \right)^2 + \left( Q_{ij,t} \right)^2, \forall i, \forall j, \forall t.
\]

(5)

e) Active generation levels should not exceed the upper bound $P_{g,t}^{G_{\text{max}}}$ and lower bound $P_{g,t}^{G_{\text{min}}}$, i.e.,

\[
P_{g,t}^{G_{\text{max}}} \leq P_{g,t}^G \leq P_{g,t}^{G_{\text{min}}}, \forall g, \forall t.
\]

(6)

where $P_{g,t}^{G_{\text{max}}}$ and $P_{g,t}^{G_{\text{min}}}$ are the parameters to indicate the allowed maximum and minimum outputs of unit $g$ (how to determine these parameters will be discussed in Appendix A in [26]).

Similarly, the reactive generation levels are limited by its upper bound $Q_{g,t}^{G_{\text{max}}}$ and lower bound $Q_{g,t}^{G_{\text{min}}}$, i.e.,

\[
Q_{g,t}^{G_{\text{max}}} \leq Q_{g,t}^G \leq Q_{g,t}^{G_{\text{min}}}, \forall g, \forall t.
\]

(7)

f) The squared voltage magnitude at every node $i$ and time period $t$ should not exceed its upper bound $\bar{v}_i$ and lower bound $\underline{v}_i$, i.e.,

\[
\underline{v}_i \leq v_{i,t} \leq \bar{v}_i, \forall i, \forall t.
\]

(8)
i.e.,

\[-\Delta P_{CC} \leq P_{t,0}^{PCC} - P_{t,0}^{QCC} \leq \Delta P_{CC}, \forall t \]  \hspace{1cm} (9)

\[-\Delta Q_{CC} \leq Q_{t,0}^{PCC} - Q_{t,0}^{QCC} \leq \Delta Q_{CC}, \forall t \]  \hspace{1cm} (10)

where $P_{t,0}^{PCC}$ and $Q_{t,0}^{QCC}$ are the power exchanges pre-set in the day-head horizon; $\Delta P_{CC}$ and $\Delta Q_{CC}$ are given thresholds.

h) The state of charge (SoC) should not exceed its upper bound $S_{g,t}^{\text{max}}$ and lower bound $S_{g,t}^{\text{min}}$, i.e.,

\[S_{g,t}^{\text{min}} \leq S_{g,t} \leq S_{g,t}^{\text{max}}, \forall n, \forall t \]  \hspace{1cm} (11)

where $S_{g,t}$ is the SoC of energy storage $n$ at time period $t$; $S_{g,t}^{\text{max}}$ and $S_{g,t}^{\text{min}}$ are the parameters to indicate the allowed maximum and minimum SoC (how to determine these parameters will be discussed in Appendix A in [26]).

i) Charge and discharge power $P_{n,t}^{N,c}$ and $P_{n,t}^{N,d}$ of energy storage $n$ should not exceed their upper bounds $\bar{P}_{n,t}^{N,c}$ and $\bar{P}_{n,t}^{N,d}$, respectively, i.e.,

\[0 \leq P_{n,t}^{N,c} \leq \bar{P}_{n,t}^{N,c}, \forall n, \forall t \]  \hspace{1cm} (12)

\[0 \leq P_{n,t}^{N,d} \leq \bar{P}_{n,t}^{N,d}, \forall n, \forall t \]  \hspace{1cm} (13)

j) The SoC at the last time period $S_{n,|T|}$ should equal the initial SoC $S_{n,\text{initial}}$, i.e.,

\[S_{n,\text{initial}} = S_{n,|T|}, \forall n \]  \hspace{1cm} (14)

k) The charge and discharge of energy storage $n$ cannot simultaneously occur at time period $t$, i.e., the following complementarity constraint should be fulfilled

\[P_{n,t}^{N,c} \times P_{n,t}^{N,d} = 0, \forall n, \forall t \]  \hspace{1cm} (15)

2) Requirements with time coupling between time period $t$ and time period $t-1$

a) Changes of active generation levels between consecutive periods should not exceed ramp rate limits $R_{g}^{\text{down}} < 0$ and $R_{g}^{\text{up}} > 0$, i.e.,

\[R_{g}^{\text{down}} \leq P_{g,t}^{G} - P_{g,t-1}^{G} \leq R_{g}^{\text{up}}, \forall g, \forall t \]  \hspace{1cm} (16)

b) The SoC of energy storage $n$ is affected by its charge and discharge behaviors, as described as follows:

\[S_{n,t} = S_{n,t-1} + \tau_{n,c} P_{n,t}^{N,c} - \tau_{n,d} P_{n,t}^{N,d}, \forall n, \forall t \]  \hspace{1cm} (17)

where $\tau_{n,c}$ and $\tau_{n,d}$ are charge and discharge efficiencies of energy storage $n$ respectively.

For ease of discussions, the intraday decisions at time period $t$ are stacked as $x_{t} = [P_{t,0}^{PCC} Q_{t,0}^{QCC} P_{t,0}^{G} Q_{t,0}^{G} P_{i,j,t} Q_{i,j,t} v_{i,j,t} i_{i,j,t} S_{n,t} P_{n,t}^{N,c} P_{n,t}^{N,d}]^{T} \in X_{t}^{(n)} \times X_{t}^{(c)}$

where $X_{t}^{(n)}$ is described by (1)-(15) without time coupling, and $X_{t}^{(c)}$ is described by (16) and (17) with time coupling. The uncertain quantities $P_{R,t}$ are confined in a box-shaped set in the following:

\[P_{R} = \left\{ P_{R,t} \mid P_{R,t}^{\text{up}} \leq P_{R,t} \leq P_{R,t}^{\text{down}}, \forall r, \forall t \right\} \]  \hspace{1cm} (18)

where the fluctuation interval [$P_{R,t}^{\text{up}}, P_{R,t}^{\text{down}}$] can be obtained by interval forecasting [28].

Also, define $w_{t} = [P_{f,t}^{PCC} Q_{f,t}^{QCC}]^{T}$ and $w = [w_{1} \ w_{2} \ . \ . \ . \ w_{|T|}]^{T}$. Given a $w$ in the day-ahead horizon, the decision vector $x$ should be successively determined when the realizations of uncertainties in (18) are observed during the intraday horizon. This is the nonanticipativity requirement. In addition, the decision vector $x_{t}$ at time period $t$ should be feasible at time period $t$ and at future time periods under all possible DERs’ realizations. This is the robustness requirement. In the next section, a second order cone program (SOCP) will be designed to show existence of a nonanticipative and robust $w$.

### B. SOCP to Fulfill Nonanticipativity and Robustness

Our designed SOCP contains the constraints under the expected scenario and the constraints under the extreme scenario $s$. Particularly, $P_{R,t}$ equals its predicted value under the expected scenario, and $P_{R,t}^{k}$ under the extreme scenario $s$ is located at the st h vertex of (18). The goal of the designed problem is to minimize the operational cost under the expected scenario. Details are described as follows:

1) The objective function is to minimize the cost under the expected scenario, i.e.,

\[
\min_{t \in T} \left( \sum_{g \in G} f_{g}^{G} (P_{g,t,0}^{G}) + \sum_{n \in N} c_{n}^{N,c} P_{n,t,0}^{N,c} + \sum_{n \in N} c_{n}^{N,d} P_{n,t,0}^{N,d} \right) 
\]

over $P_{g,t,0}^{G}, Q_{g,t,0}^{G}, P_{i,j,t}^{G}, Q_{i,j,t}^{G}, v_{i,j,t}^{G}, i_{i,j,t}^{G}, S_{n,t,0}^{N,c}, P_{n,t,0}^{N,d}, F_{n,t,0}^{G}, F_{n,t,0}^{QCC}, F_{n,t,0}^{G}, Q_{g,t}^{G}, P_{i,j,t}^{G}, Q_{i,j,t}^{G}, v_{i,j,t}^{G}, i_{i,j,t}^{G}, S_{n,t,s,0}^{N,c}, P_{n,t,s,0}^{N,d}, \forall i, \forall j, \forall s, \forall n$; where $f_{g}^{G}$ is the convex piecewise linear cost function of unit $g$ [29]; $c_{n}^{N,c}$ and $c_{n}^{N,d}$ are the nonnegative cost parameters of energy storage $n$ to charge and discharge, respectively [30]; the subscript “$0$” indicates the variables under the expected scenario; the subscript “$s$” indicates the variables under the extreme scenario $s$.

2) The operating region under the expected scenario at time period $t$ is the Cartesian product of the feasible set $\tilde{X}_{t}^{(nc)}$ without time coupling and the feasible set $\tilde{X}_{t-1}^{(c)}$ with time coupling. $\tilde{X}_{t,0}^{(c)}$ is defined by the constraints (1)-(14) while 1) replacing $P_{R,t}$ with its forecast $P_{R,t}^{k}$; and 2) replacing the variable $x_{t}$ with $w_{t} \times \tilde{X}_{t,0}^{(c)}$, where $\tilde{X}_{t,0}^{(c)} = [P_{t,0}^{G} Q_{t,0}^{G} P_{i,j,t}^{G} Q_{i,j,t}^{G} v_{i,j,t}^{G} i_{i,j,t}^{G} S_{n,t,0}^{N,c} P_{n,t,0}^{N,d}]^{T}$. In addition, the following constraints are incorporated in $\tilde{X}_{t,0}^{(c)}$ to guarantee that charge and discharge will not simultaneously occur

\[P_{n,t,0}^{N,c} = 0, \forall n \in \{ N_{t,d} \cup N_{t,c} \} \]  \hspace{1cm} (20)

\[P_{n,t,0}^{N,d} = 0, \forall n \in \{ N_{t,c} \cup N_{t,z} \} \]  \hspace{1cm} (21)

where $N_{t,c}$ is the set of energy storage that is assigned to charge at time period $t$; $N_{t,d}$ is the set of energy storage that is assigned to discharge at time period $t$; $N_{t,z}$ is the set of energy storage that is assigned not to charge or discharge at time period $t$. The sets $N_{t,c}, N_{t,d}$, and $N_{t,z}$ are determined in Appendix A in [26]. In particular, they are mutually exclusive and satisfy $N_{t,d} \cup N_{t,c} \cup N_{t,z} = N$.
\[ \bar{x}_{t,t-1,0}^{(c)} \] is obtained from (17) by 1) replacing the variable \( x_t \) with \( \bar{w}_t \times \bar{x}_{t,0} \) and 2) fixing charging and discharging states based on (20) and (21). Note that 1) \( \bar{x}_{t,0}^{(nc)} \) and \( \bar{x}_{t,t-1,0}^{(c)} \) are in the domain of \( \bar{x}_{t,0} \) when \( \bar{w}_t \) is given as a parameter, and 2) \( \bar{x}_{t,0}^{(nc)} \) and \( \bar{x}_{t,t-1,0}^{(c)} \) make \( \bar{x}_{t,0} \).

3) The operating region under the extreme scenario \( s \) at time period \( t \) is the Cartesian product of \( \bar{x}_{t,0}^{(nc)} \) and \( \bar{x}_{t,t-1,1,s}^{(c)} \) by 1) revising \( \bar{x}_{t,0} \) with \( \bar{x}_{t,0} = [P_t^{G}, Q_t^{G}, P_{t,t-1}, Q_{t,t-1}, P_{t,t,s}, Q_{t,t,s}, P_{t,t,s}^{PCC}, Q_{t,t,s}^{PCC}]^T \), and 2) further replacing \( P_{t,t,s}^{PCC} \) with \( P_{t,t,s}^{R} \), i.e., the realization of \( P_{t,t,s}^{R} \) at the \( s \) h vertex of (18). Also, we design two additional constraints (22)–(23) in \( \bar{x}_{t,0}^{(nc)} \) to fulfill the operational requirement (5) for each pair \((n, s)\) of scenarios. Two these constraints can guarantee robustness to fulfill the operational requirement (5) when facing any realization of DER outputs (how to guarantee such robustness will be proven in Appendix B in [26]).

\[
v_{t,s}^{ij,t,n} = (P_{t,s}^{PCC})^2 + (Q_{t,s}^{PCC})^2, \forall i, j, \forall t, \forall n, \forall s \quad (22)
\]

\[
v_{t,s}^{ij,t,n} = (Q_{t,s}^{PCC})^2 + (P_{t,s}^{PCC})^2, \forall i, j, \forall t, \forall n, \forall s. \quad (23)
\]

Note that the parameters \( P_{G, g}^{G}, P_{G, g}^{Max}, S_{g, t}^{Min}, S_{g, t}^{Max}, n_{g, t} \) and \( S_{g, t}^{Max} \) in the SOCP above satisfy the following rules (how to satisfy such rules is elaborated in Appendix A in [26]).

\[
P_g^G \leq P_{g}^{G, g} \leq P_{g}^{Max, g,t}, \forall t, \forall g \quad (24)
\]

\[
P_{down} \leq P_{g}^{Min, g,t}, \forall t, \forall g \quad (25)
\]

\[
P_{g, t}^{Max, g,t} - P_{g, t}^{Min, g,t} \leq P_{g, t}^{PCC}, \forall t, \forall g \quad (26)
\]

\[
S_{g, t}^{N} \leq S_{g, t}^{Min, n}, \forall t, \forall n \quad (27)
\]

\[
S_{g, t}^{Max, n} - S_{g, t}^{Min, n} \leq S_{g, t}^{N}, \forall t, \forall n \quad (28)
\]

\[
S_{g, t}^{Max, n} - S_{g, t}^{Min, n} \leq S_{g, t}^{N}, \forall t, \forall n \quad (29)
\]

where \( P_{g}^{Max, g,t} \) and \( P_{g}^{Max, g,t} \) are maximum and minimum physical capacities of unit \( g \), respectively; \( S_{g}^{N} \) and \( S_{g}^{N} \) are maximum and minimum physical capacities of energy storage \( n \), respectively.

For ease of discussions, the SOCP above is given in a compact form as follows:

\[
z = \min_{\bar{x}_{t,0}^{(nc)}, \bar{x}_{t,1}^{(c)}} \sum_{t} f_t(\bar{x}_{t,0}^{(c)}) \quad (30)
\]

subject to

\[
\bar{w}_t \times \bar{x}_{t,0} \in \bar{x}_{t,0}^{(nc)} \times \bar{x}_{t,1}^{(c)}, \bar{x}_{t,s} \in \bar{x}_{t,s}^{(nc)} \times \bar{x}_{t,s}^{(c)}, \forall t, \forall s \quad (31)
\]

where \( \sum_{t} f_t(\bar{x}_{t,0}) \) is the cost in (19).

For our SOCP designed in (30) and (31), a proposition is provided below to show a nonanticipative and robust intraday decision:

**Proposition 1:** Given \( w = [w_1 \ w_2 \ . \ w_{|T|}]^T \) with which the SOCP problem (30)–(31) is feasible, there is a nonanticipative and robust intraday decision vector \( x_t \)

\[
x_t = \sum_{s \in S} \lambda_{t,s} x_{t,s} \quad (32)
\]

where \( \lambda_{t,s} \) is calculated as follows from the realization \( P_{R, t}^{R, o} \) of \( P_{R, t} \) observed at time period \( t \)

\[
\sum_{s \in S} \lambda_{t,s} P_{R, t, s}^{R, o} = 1, \lambda_{t,s} \geq 0, \forall r, \forall t, \forall s. \quad (33)
\]

The proof of Proposition 1 is given in Appendix B in [26]. Note that the SOCP (30)–(31) is formulated for each given \( w \). Any such a \( w \) that makes (1)–(17) feasible under nonanticipativity and robustness is called a feasible power transfer of the VPP in this article, and the minimum objective value (19) is treated as a cost function in the domain of \( w \). It is critical for a VPP to characterize its feasible power transfer region and cost function in the domain of \( w \), from which the transmission operator can solve for its day-ahead dispatch [9]. This characterization will be elaborated in the following section.

III. CHARACTERIZATION OF COST FUNCTIONS OVER FEASIBLE POWER TRANSFER REGIONS OF VPPs

The characterization of the feasible power transfer region will be explained in Section III-A, followed by the formulation of the cost function in Section III-B. Also, the impacts of DERs’ uncertainties on feasible power transfer regions are studied in Section III-C.

A. Characterization of the Feasible Power Transfer Region

The formal definition of a feasible power transfer region is

\[
\Omega \triangleq \left\{ w \in [\bar{w}_t] \times \bar{x}_{t,0} \in \bar{x}_{t,0}^{(nc)} \times \bar{x}_{t,1,0}^{(c)} \right\}. \quad (34)
\]

The feasible power transfer region \( \Omega \) can be regarded as a projection of the constraint (31) onto the space of PCC power exchanges \( [P_{0}^{PCC}, Q_{0}^{PCC}] \). However, the dimension of \( [P_{0}^{PCC}, Q_{0}^{PCC}] \) is high, and the calculation of a high-dimension polytope usually faces a heavy computational burden [31]. Consequently, a successive determination strategy is proposed in this section for a fast calculation. The key idea of the proposed strategy to calculate the temporally coupled feasible power transfer region lies in 1) calculating the feasible power transfer region \( \Omega_{t-1} \) at time period \( t-1 \) by finding the vertices of the constraints at time period \( t-1 \), 2) incorporating the time coupling constraints given \( \Omega_{t-1} \) to connect the constraints at time period \( t \). Finally, the intersection of feasible power transfer regions across all the time periods is employed as an estimation of \( \Omega \). The produce above is elaborated as follows.

1) Calculation of \( \Omega_{t-1} \)

The feasible power transfer region \( \Omega_{t-1} \) at time period \( t-1 \) is defined as follow:

\[
\Omega_{t-1} \triangleq \left\{ w \in [\bar{w}_t] \times \bar{x}_{t,0} \in \bar{x}_{t,0}^{(nc)} \times \bar{x}_{t,1,t-2,0}^{(c)} \right\}. \quad (35)
\]
where $\mu_{t-2}$ is the vector of ancillary variables in real space; $X_{t-1,t-2,0}$ and $X_{t-1,t-2,s}$ are convex feasible sets that reflect the time coupling of $X_{t-1,t-2,0}$ and $X_{t-1,t-2,s}$, respectively. The explicit construction of $X_{1,t-1,1,0}$ and $X_{1,t-1,1,s}$ will be found in (39).

Considering the convexity of $\tilde{X}_{t-1,1,0} \times \tilde{X}_{t-1,1,2,0}$ and $X_{t-1,1,s} \times X_{t-1,1,2,0}$, $\Omega_{t-1}$ can be computed using the following vertex search method.

Step 1: Initialization. SOCPs are solved subject to $\tilde{X}_{t-1,1,0} \times \tilde{X}_{t-1,1,2,0}$ and $X_{t-1,1,s} \times X_{t-1,1,2,0}$ for all the scenarios $s$. To obtain the minimum and maximum of each element in $w_{t-1}$. Collect those minimum and maximum points in a set $\mathbf{V}_0$.

Step 2: Polytope construction. A polytope $\mathbf{R}$ can be constructed with $\mathbf{V}_0$. The half-space representation of the polytope $\mathbf{R}$ is $A_{t-1} w_{t-1} \leq B_{t-1}$.

Step 3: Exploration. For each facet of the polytope $\mathbf{R}$, the following SOCP is solved:

$$\max \{w_{t-1}, x_{t-1,0}, x_{t-1,s}, ..., \mu_{t-2}\}$$

s.t. $w_{t-1} \times (X_{t-1,0}, \mu_{t-2}) \in \tilde{X}_{t-1,1,0} \times \tilde{X}_{t-1,1,2,0}$

$(x_{t-1,s}, \mu_{t-2}) \in X_{t-1,s} \times X_{t-1,1,2,0}$, $\forall s$

where $A_{t-1}$ is the $j$th $h$-row of $A_{t-1}$.

Intuitively, the SOCP (36)–(38) move the facet $A_{t-1,j} w_{t-1} = B_{t-1,j}$ as far as possible away from the center of $\mathbf{R}$. After solving (36)–(38) for all rows $j$, collect all the optimal $w_{t-1}$ in a set $\mathbf{V}_{\text{new}}$, such that $\{\mathbf{V}_{\text{new}} \cup \mathbf{V}_0\}$ replaces $\mathbf{V}_0$ as the vertices of a new polytope $\mathbf{R}_{\text{new}}$.

Step 4: Termination check. Check the difference between the volumes of $\mathbf{R}$ and $\mathbf{R}_{\text{new}}$. Once the difference is smaller than a given threshold, terminate the algorithm and return $\mathbf{R}_{\text{new}}$ as $\Omega_{t-1}$; otherwise, let $\mathbf{R}_{\text{new}} \rightarrow \mathbf{R}$ and go back to Step 3.

2) Explicit construction of $X_{1,t-1,1,0}$ and $X_{1,t-1,1,s}$.

Observed from (34), $\Omega$ is coupled across all the time periods by $X_{t-1,0}$ and $X_{t-1,1,s}$, time coupling arises from the SoC changes. The SoC changes between two consecutive time periods can be compacted as $\mathbf{U}_t^\text{SoC} \mathbf{y}_t = \mathbf{U}_0^\text{SoC} \mathbf{y}_0 + \mathbf{U}_t^P \mathbf{y}_t$, where $\mathbf{y}_t = [\mathbf{x}_{t,0}^T T \mathbf{x}_{t,1}^T T ... \mathbf{x}_{t,|S|}^T T]^T$, $\mathbf{U}_t^\text{SoC}$ are constant matrices to extract variables of the SoC at time period $t$ from $\mathbf{y}_t$ and $\mathbf{U}_t^P$ are constant matrices to extract charge and discharge variables at time period $t$ from $\mathbf{y}_t$.

Motivated by the convexity of $\tilde{X}_{t-1,0} \times \tilde{X}_{t-1,1,0}$ and $\tilde{X}_{t-1,1,0} \times \tilde{X}_{t-1,1,0}$ for all the scenarios $s$, $\tilde{X}_{t,1,1,0}$ and $\tilde{X}_{t,1,1,0}$ can be explicitly constructed as follows:

$$\begin{align*}
\mathbf{U}_t^\text{SoC} \mathbf{y}_t &= \mathbf{U}_0^\text{SoC} \mathbf{y}_0 + \mathbf{U}_t^P \mathbf{y}_t \\
\sum_{j \in J_{t-1}} \mu_{t-1,j} y_{t-1,j} + 1 &\geq 0
\end{align*}$$

(39)

where $y_{t-1,j}$ is the optimal solution of $y_{t-1}$ that corresponds to the $j$th vertex of $\Omega_{t-1}$ obtained in Steps 1–4.

Once the strategy above is implemented, $\Omega$ in (34) can be estimated by $\Omega$ in the following:

$$\begin{align*}
\tilde{\Omega} \triangleq \left\{ \langle w, \mu \rangle : \right. &\\
& w_t = \sum_{j \in J_t} \mu_{t,j} w_{t,j}, \forall t \\
& U_1^\text{SoC} \sum_{j \in J_t} \mu_{t,j} y_{t,j} = U_0^\text{SoC} \sum_{j \in J_1} \mu_{1,j} y_{1,j}, \forall t \\
& \left. \sum_{j \in J_t} \mu_{t,j} = 1, \mu_{t,j} \geq 0, \forall t \right\}
\end{align*}$$

(40)

where $w$ is a vector that is composed of $\mu = [\mu_1, ..., \mu_{|J_t|}]^T$ across all the time periods.

Particularly, $\tilde{\Omega}$ is an inner estimation of $\Omega$. This can be explained by regarding $\tilde{\Omega}$ as a projection after two stages. In the first stage, the constraint (31) is projected onto the space of $\{w_1, ..., w_T\}$, $U_0^\text{SoC} y_1, ..., U_T^\text{SoC} y_T$. Due to the convexity of (31), this projection leads to a convex polytope. Some vertices of this projection are found based on Steps 1–4 and (39). In the second stage, these vertices are further projected onto the space of $w$ to get an inner estimation $\tilde{\Omega}$.

B. Calculation of the Cost Function

A VPP can be required to bid its PCC power exchanges at each time period [32]. However, time coupling in (30) and (31) leads to difficulties for individually formulating the cost function at each time period. To resolve this issue, this article proposes a method to calculate the cost function composed of two parts. The first part includes a piecewise convex surface at each time period, and the second part is a compensation cost with which the summation of all the surfaces in the first part can be above the true total cost function. Our method is elaborated as follows:

1) Calculation of the convex surface at time period $t$ considering the following optimization problem to minimize the cost at time period $t$ when the operational requirement (31) is fulfilled:

$$z_t(w_t) = \min_{\{g, x_{t,0}, x_{t,s}\}} g$$

s.t. $f_t$ is $\{\mathbf{x}_{t,0}, \mathbf{x}_{t,s}\}$ and (31)

(41)

where $x_{t,0}$ is the upper bound of $x_{t,0}$; $g$ is a cost variable.

Note that $w_t$ is involved in (31) as coverage parameters. This is why the minimum (41) can be expressed as a function of $w_t$.

Also, $z_t(w_t)$ is actually convex in the domain of $w_t$, as proven in Appendix C in [26]. Motivated by the convexity of $z_t(w_t)$, the following auxiliary region $\Psi_t$ is constructed to calculate the cost function $z_t(w_t)$ over the feasible power transfer region $\Omega_t$:

$$\Psi_t \triangleq \left\{ \{g, w, \mu_t\} : \mathbf{x}_{t,0}, \mathbf{x}_{t,s}, \forall t, \forall s \right\}$$

(42)

where $\mathbf{x}_{t,0}$ is the optimal solution of $\mathbf{x}_{t,0}$; $g$ is a cost variable.

Let $\Phi_t$ denote the projection of $\Psi_t$ to the domain of $(g, w_t)$. The values of $q_t$ at some boundary points of $\Phi_t$ are $z_t(w_t)$. Consequently, the remaining work is to identify the right facets from $\Phi_t$ whose intersections are those boundary points. For the $j$th facet $q = C_j \mathbf{w}_t + E_j$ in $\Phi_t$, a point $w_{t,j}$ located on this facet can be easily found. Given $w_{t,j}$, the following SOCP is solved:

$$\min_{\{g, x_{t,0}, x_{t,s}\}} g, \text{s.t. } w_t = w_{t,j}$$

(44)
Denote the minimum objective value of (44) as $g^*_t$. If $C_{ij}w_{ij} + E_{ij} = g^*_t$, the $j$th facet attains $z_t(w_t)$, otherwise (if $C_{ij}w_{ij} + E_{ij} > g^*_t$) the infimum is not attained. Fig. 2 illustrates these two cases.

2) Calculation of a compensation cost $\sigma$

If the same $w$ is given in (30) and (31) and (41) and (42), the summation of $z_t(w_t)$ in (41) and (42) across all the time periods will not exceed the true total cost $z$ in (30) and (31). Therefore, a compensation cost $\sigma$ is necessary to guarantee the coverage of the true total cost when $z_t(w_t)$ is employed to bid PCC power exchanges at each time period $t$. To achieve this goal, an LP is designed as follows:

$$\max \left\{ w, \mu, x_{t,0}, x_{t,r} \right\}, \forall t \in T$$

$$\sum_{t \in T} f^\text{new}_t \left( x_{t,0} \right) - \sum_{t \in T} z_t \left( w_t \right) \tag{45}$$

s.t. \( (w, \mu) \in \bar{\Omega} \) and (31) \( \tag{46} \)

where $f^\text{new}_t$ is the same function $f_t$ in (30) except that the cost function $f_g$ in $f_t$ is revised as a linear function by directly connecting the beginning point and the endpoint of $f_t$.

In Appendix D in [26], it is proven that the optimal objective value of the LP (45), (46) can be taken as $\sigma$ that satisfies $\sum_{t \in T} z_t(w_t) + \sigma \geq z, \forall (w, \mu) \in \bar{\Omega}$, i.e., the true total cost can be fully covered by our method.

In practical implementations, the proposed method above is used by operators of VPPs to get their cost functions over feasible power transfer regions; then, these results can be submitted to the transmission network to solve its dispatch problem to coordinate the power exchanges with VPPs. Particularly, the feasible power transfer regions can be treated as constraints in the transmission dispatch problem, and the cost functions can be embedded in the objective function to reflect the operational costs of VPPs.

Three major steps are contained in the proposed method, and the flowchart of the proposed method is provided in Fig. 3.

Step 1: VPP operators prepare data (e.g., topology, line parameters) to formulate the optimization problem (30), (31) for VPP operations based on the concepts of nonanticipativity and robustness, as described in Section II.

Step 2: Based on the optimization problem formulated in Step 1, VPP operators employ our method in Section III-A to get their feasible power transfer regions $\Omega$ which are shown in (40).

Step 3: VPP operators calculate cost functions [including the facets selected from Fig. 2, and the compensation cost from (45) to (46)] over feasible power transfer regions, as demonstrated in Section III-B.

C. Impacts of DERs on Feasible Power Transfer Regions

This article provides a perspective to study such impacts of DERs’ uncertainties on feasible power transfer regions. Intuitively, DERs’ uncertainties may shrink the feasible power transfer region, since the VPP sacrifices certain abilities of PCC power adjustments to face DERs’ uncertainties. DERs’ uncertainties appear in power balance constraints (1)–(2), and the impact of a DER lies in its fluctuation interval $\left[ p^R_{r,t}, p^R_{r,t} \right]$ and its coefficients in (1)–(2), i.e., $(1 + \tan \beta_r)$. Therefore, the impacts of DER at time period $t$ can be ranked by $(1 + \tan \beta_r) \times \left( p^R_{r,t} - p^R_{r,t} \right)$.

Based on the rationale above, the operator can identify the DERs with the most significant impacts. This finding is believed to be applied in some scenarios. For example, the calculation above may suffer a heavy computational burden brought by numerous DERs in a VPP. This issue arises from repeatedly solving SOCPs for all the $2^M$ extreme scenarios of the box uncertainty set across $M$ DERs. To alleviate the computational burden, one of our future works is to study the fast calculation method by aggregating DERs based on their different impacts on feasible power transfer regions.

IV. CASE STUDIES

The proposed methods are verified in the IEEE 33-bus and 136-bus systems. All the parameters of networks and DERs can

Fig. 2. Two cases regarding a facet of $\Phi_2$ checked through (44). (a) The infimum is not attained. (b) The infimum is attained.

Fig. 3. Flowchart of the proposed method.
be found in [33]. Particularly, DERs’ uncertainties in the IEEE 33-bus system are introduced at node 2 and node 3, while DER’s uncertainties in the IEEE 136-bus system occur at node 8, node 41, node 44, node 45, and node 106.

First, the feasible power transfer region obtained by our method is verified to guarantee nonanticipativity and robustness, as will be shown in Section IV-A. Second, the cost function obtained by our method is verified to guarantee the coverage of the true costs, as will be shown in Section IV-B. Finally, the impacts of DERs’ uncertainties on feasible power transfer region are examined, as will be shown in Section IV-C.

A. Validation of Nonanticipativity and Robustness in the Feasible Power Transfer Region

In this section, it will be first shown that any pair \((P^{\text{PCC}}_0, Q^{\text{PCC}}_0)\) selected from our feasible power transfer region can guarantee the nonanticipative and robust intraday operation of a VPP. The IEEE 33-bus system over 24 time periods in [33] is used. Conventional units are installed at node 5 and node 33. Energy storage is installed at node 20. To verify the advantage of the proposed method, the following three methods will be compared:

M1: The vertex search method [13] directly projects the constraint (31) onto the space of PCC power exchanges \((P^{\text{PCC}}_0, Q^{\text{PCC}}_0)\).

M2: The proposed method in Section III-A.

M3: The method in [14], which considers DERs’ uncertainties by robust optimization based on the vertex search method [13].

Since the M1 method directly projects the constraint (31), which considers nonanticipativity and robustness, the M1 method can be regarded as a benchmark. Also, the M3 method is selected as a representative of the current works which considers DERs’ uncertainties. The computational time of the three methods is listed in Table II.

| Method | M1 (Second) | M2 (Second) | M3 (Second) |
|--------|-------------|-------------|-------------|
| Computational time | > 3600 | 510.33 | > 3600 |

In our simulation results, only the M2 method (i.e., the proposed method) can fast calculate a feasible power transfer region (part of its visualization is shown in Fig. 4), while the M1 and M3 methods fail to get results within 1 h. This is because M1 and M3 methods rely on the vertex search method [13] to directly project a constraint set onto the high-dimension space of PCC power exchanges \((P^{\text{PCC}}_0, Q^{\text{PCC}}_0)\) over 24 h. The vertex search method [13] needs to convert the found vertices into a half-space representation to find new vertices. However, such a conversion into a half-space representation may be difficult for a high-dimension projection, as also observed in [31]. In contrast, the proposed method in Section III-A successively calculates the feasible power transfer region in the domain of PCC power exchanges \((P^{\text{PCC}}_t, Q^{\text{PCC}}_t)\) at each time period \(t\) and then establishes their time-coupling constraints. In other words, our method only handles a two-dimension \((P^{\text{PCC}}_t, Q^{\text{PCC}}_t)\) for each time period \(t\). This explains the advantage of the proposed method on the computational speed. It should be pointed out that the M3 method relies on robust optimization, which only considers the worst case without the consideration of nonanticipativity and robustness. Consequently, even if a feasible power transfer region can be obtained by the M3 method, nonanticipativity, and robustness may not be fulfilled. A similar conclusion can be also found in [18], which shows that robust optimization cannot guarantee nonanticipativity and robustness in a dc-power-flow-based unit commitment problem. In the current practical implementations, the feasible power transfer region obtained in Fig. 4 is calculated by a VPP operator after the operator collects all the parameters from DERs. Then, such a feasible power transfer region can be submitted as constraints in the dispatch problem of a transmission network.

Although we cannot obtain the benchmark by the M1 method, we propose the following procedure to test whether our feasible power transfer region can fulfill nonanticipativity and robustness: 1) 10 000 points in our feasible power transfer region are randomly sampled; 2) for each sample, ten realizations of DERs’ uncertainties at each time period are randomly made from the uncertainty set (18); 3) given each uncertainty realization as time goes, a feasible decision vector \(x_t\) at time period \(t\) is successively obtained by solving the optimization problem, where the objective function is set as zero and the constraints are (1)–(17). Our result is that each sample in the feasible power transfer region with each realization of DERs’ uncertainties always leads to a feasible decision of the VPP. Simulation results with different realizations indicate that our feasible power transfer region can guarantee robustness. Also, the decisions are successively made upon realizations of DER outputs, so that nonanticipativity is fulfilled by our simulation results. In other words, even if decisions are successively made as time goes, changes of active generation levels between consecutive time periods always satisfy ramp rate limits, and the charge and discharge rates of energy storage do not exceed their limits. This phenomenon can be observed in our simulation results, as shown in Figs. 5 and 6.

B. Validation of the Cost Function

Our cost function over the feasible power transfer region is examined in the previous IEEE 33-bus test system. As illustrated...
in Section III-B, our cost function is composed of two parts. One is the convex piecewise surface \( z_t(P_{\text{PCC}}^t, Q_{\text{PCC}}^t) \) at each time period \( t \), and the other one is the compensation cost \( \sigma \). The compensation cost \( \sigma \) is obtained as $6855 by solving the LP (45), (46), and a convex piecewise surface at time period 1 is plotted in Fig. 7 as an example.

In a transmission electricity market, a VPP can bid its PCC power exchanges and its bid can be embedded into the objective function of a transmission dispatch problem. For the results (i.e., the piecewise convex surface \( z_t \) in Fig. 7 and the compensation cost \( \sigma \)) obtained above, their summation \( \sum_{t \in T} z_t(P_{\text{PCC}}^t, Q_{\text{PCC}}^t) + \sigma \) can be set as the bid of a VPP. Furthermore, we test whether such a bid can cover the true cost a VPP. For samples in our feasible power transfer region, our bids are compared with the true total costs that are obtained by solving the SOCP (30)–(31). The numerical results are shown in Fig. 8. Our bids are always above the true total costs. Consequently, our method can guarantee to cover the true total cost with the proposed cost function for the bid at each time period.

Note that Refs. [19]–[21] are the unique type of the current works, which can calculate cost functions over feasible regions. However, they strongly rely on the linearity of a dc power flow model, while our model (30)–(31) employs a nonlinear model to delineate the operational requirements of a VPPs. Consequently, Refs. [19], [20], [21] cannot be applied to calculate cost functions over feasible power transfer regions for such VPPs,
while our proposed method provides a general idea to calculate such cost functions over feasible power transfer regions under nonlinear convex models.

### C. Impacts of DERs’ Uncertainties on Feasible Power Transfer Regions

In the IEEE 136-bus system with five DERs over one time period [33], we evaluate the following cases for their impacts on feasible power transfer regions.

**Case 1:** Consider expected values of DER outputs at all the nodes.

**Case 2:** Consider only the uncertainties at node 106 and expected values at all the other nodes.

**Case 3:** Consider only the uncertainties at node 44 and expected values at all the other nodes.

**Case 4:** Consider only the uncertainties at node 8 and expected values at all the other nodes.

The volumes of feasible power transfer regions under four cases are calculated, since a larger volume indicates that the feasible power transfer region can provide an opportunity for PCC power exchanges in a broader range. Our numerical experiments are shown in Table III. Two resulting phenomena can be found.

1) The volume of case 1 is the largest. Note that no uncertainties are involved in case 1, while uncertainties are introduced in cases 2–4. If case 1 is regarded as the situation, where outputs of DERs are accurately predicted, the feasible power transfer region can provide the broadest range for PCC power exchanges. In other words, it is revealed from a new perspective that a good prediction accuracy on DER outputs is beneficial for PCC power exchanges.

2) Based on the criterion in Section III-C, we calculate \((1 + \tan \beta_r) \times (P_{r,t} - P_{R,t})\) as an index to reflect the impact of DER \(r\) at time period \(t\). In our simulation settings, the uncertainties at node 106 make the most significant impact and those at node 8 are most insignificant. For Case 2, where uncertainties at node 106 are considered, its volume of feasible power transfer region is smaller than that under Case 4 where uncertainties at node 8. This matches the impacts ranked by our index.

### V. Conclusion

This article proposed a characterization method for cost functions over feasible power transfer regions of VPPs, when VPPs are operated based on nonanticipativity and robustness under uncertainties of DERs. Also, the impacts of DERs’ uncertainties on feasible power transfer regions were studied. Based on the numerical results in the IEEE 33-bus and 136-bus systems, we found that 1) each sample generated from our feasible power transfer region can lead to feasible decisions made successively when facing any realizations of DER outputs, as required by nonanticipativity and robustness, 2) each bid of VPPs based on our cost function can guarantee the coverage of the true cost, i.e., our cost function is always above the true cost function, and 3) impacts of DERs’ uncertainties on feasible power transfer regions can be measured by their fluctuation intervals and system parameters, and the DERs which have larger impacts shrink more feasible power transfer regions.

In our future work, our method can be improved from the following two aspects: 1) aggregating numerous DERs based on their different impacts on feasible power transfer regions to accelerate our method, and 2) deriving the conditions which exactly relax the complementarity constraint of energy storage to release more flexibility in our feasible power transfer region.

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