Bose–Einstein condensation versus localization of bosonic quasiparticles in disordered weakly-coupled dimer antiferromagnets

Tommaso Roscilde\(^1,2\) and Stephan Haas\(^1\)

\(^1\) Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484, USA
\(^2\) Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-strasse 1, 85748 Garching, Germany

Received 12 January 2005, in final form 23 January 2006
Published 2 May 2006
Online at stacks.iop.org/JPhysB/39/S153

Abstract

We investigate the field-induced insulator-to-superfluid transition of bosonic quasiparticles in \(S = 1/2\) weakly-coupled dimer antiferromagnets. In the presence of realistic disorder due to site dilution of the magnetic lattice, we show that the system displays an extended Bose-glass phase characterized by the localization of the hard-core quasiparticles.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The quantum phase transition from the insulating to the superfluid state in strongly correlated bosonic systems is realized in a number of model systems, ranging from Josephson-junction arrays [1] to cold bosons in optical lattices [2]. Recently, it has been realized that quantum antiferromagnets also offer a clear example of such transition [3], and this has stimulated an intense experimental and theoretical activity on the subject [4–6]. In the context of quantum magnets the bosonic degrees of freedom correspond to the local deviations from a given ground state, either ordered or disordered, and their condensation corresponds to the occurrence of spontaneous antiferromagnetic long-range order breaking a planar rotational symmetry.

A particularly intriguing question in the context of bosonic quantum phase transitions is their fate in presence of lattice disorder. General considerations [7] and specific examples coming from microscopic Hamiltonians [8] show that a novel Bose-glass phase is induced by disorder between the superfluid and the insulating phase, and it corresponds to a localized phase for the bosonic degrees of freedom. Nonetheless, the experimental observation of such a phase has been so far elusive. It appears therefore tempting to imagine its realization in quantum magnets [9, 10], where disorder can be introduced in a highly controlled manner through site dilution of the magnetic lattice [11] and a rich variety of experimental probes are available to detect the specific features of this phase.
In this paper we show the emergence of a Bose-glass phase in a site-diluted $S = 1/2$ Heisenberg bilayer in a magnetic field. After a general description of the superfluid-insulator transition(s) in quantum magnets, we show how to recast the site-diluted bilayer in terms of a disordered Bose–Hubbard model for hard-core bosons. Then we present quantum Monte Carlo data on the original spin Hamiltonian, clearly supporting the scenario of an extended Bose-glass phase in a realistic quantum magnetic model.

2. Superfluid-insulator transitions in quantum magnets

Although quantum magnets are nearly perfect Mott insulators in terms of electronic properties, they admit a well-known bosonic representation in terms of local elementary spin deviations with respect to a given spin configuration. In the case of classically ordered magnets, such spin deviations are the so-called magnons, namely bosonic quasiparticles carrying a spin 1. A perfectly ordered magnet, such as a Heisenberg ferromagnet in its ground state, represents therefore a trivial insulating state (the vacuum) for such quasiparticles. More complex ordered ground states, in which none of the spin components reaches its saturation value, can be mapped onto bosonic states with a macroscopic number of bosons displaying long-range phase coherence, namely on superfluid bosonic states. The simplest example of such a state—and the one relevant for the remainder of the present paper—is that of a quantum Heisenberg antiferromagnet in a uniform magnetic field, displaying canted antiferromagnetic order. If the field is applied along the $z$ axis, using, e.g., the Villain spin-boson transformation \[ 12 \] the deviation of the $z$ spin component from its saturation value at site $i$, $S_{z,i}$, represents the local number of bosons $n_i$, and the transverse spin components $S_\pm_i \sim \sqrt{S} \exp(\pm j \phi_i)$ (in the limit of large spin $S$) bring the information on the local phase $\phi_i$. Long-range antiferromagnetic order transverse to the field corresponds to long-range phase coherence of the bosons \[ 13 \]. The transition from a fully ferromagnetic state to a canted antiferromagnetic state is realized for instance in a spin-$S$ quantum Heisenberg antiferromagnet on a hypercube of dimensions $D \geq 2$ when an applied uniform field $h = g\mu_B H/J$ (in units of the Heisenberg exchange coupling $J$) is driven below the critical value $h_c = 4DS$. This transition can be conveniently described as a band-insulator-to-superfluid transition in the bosonic language.

A more complex bosonic insulating state is realized by quantum magnets having a quantum-disordered ground state. This is the case of systems of $S = 1/2$ antiferromagnetic dimers with intradimer coupling $J$, coupled together through weaker antiferromagnetic couplings $J' < J$, with the general Hamiltonian

\[
H = J \sum_{(ij)} S_i \cdot S_j + J' \sum_{(lm)} S_l \cdot S_m - hJ \sum_i S_i^z,
\]

where the $(ij)$ nearest-neighbour bonds are intra-dimer bonds and the $(lm)$ bonds are inter-dimer ones. In the limit of $J' = 0$ each dimer is obviously in a singlet state $|s\rangle = |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle)/\sqrt{2}$ with total spin $S_{\text{tot}} = 0$. This state naturally represents the vacuum for three $S_{\text{tot}} = 1$ bosonic triplets $|t_\pm\rangle = |\uparrow \uparrow \downarrow \rangle$, $|\downarrow \downarrow \uparrow \rangle$, and $|t_0\rangle = (|\uparrow \downarrow \rangle + |\downarrow \uparrow \rangle)/\sqrt{2}$, separated by a gap of $J$ from the singlet ground state. A weak coupling $J'$ between the dimers slightly perturbs the simple picture of dimer singlets, and the state of each dimer is properly described as an incoherent mixture of the singlet state with the triplet states, retaining the full $SU(2)$ symmetry of the Hamiltonian: the pure-state probabilities, corresponding to the diagonal elements of the dimer reduced density matrix $\rho^{(\text{dimer})}$, are $p(|s\rangle) \gg p(|t_0\rangle) = p(|t_+\rangle) = p(|t_-\rangle)$, and all off-diagonal terms are vanishing. This picture of a so-called dimer-singlet ground state is valid below a critical value of the ratio $g = J'/J$, strongly dependent on the dimensionality.
of the resulting lattice and on the specific geometry of the weak couplings. In what follows we assume the system to be well inside the dimer-singlet regime in zero field.

Applying a magnetic field to each isolated dimer \((J’ = 0)\) along the quantization axis, the two triplets \(|r^+\rangle\) and \(|r^-\rangle\) remain well separated from the singlet ground state, while the \(|r^0\rangle\) triplet is brought to degeneracy with the singlet for a field \(h = 1\), at which the system jumps to a fully magnetized state. In presence of a finite interdimer coupling \(J > 0\) the application of a weak field \(h \lesssim 1\) reduces the symmetry of the Hamiltonian from \(SU(2)\) to \(U(1)\), and correspondingly in the dimer-singlet phase the reduced density matrix \(\rho^{(dim)}\) for each dimer has \(p(|t_0\rangle) \neq p(|t_+\rangle) = p(|t_-\rangle)\), but the last equality guarantees that the system does not develop a finite magnetization along the field, so that the ground state has a higher symmetry than the Hamiltonian. Moreover the mixture remains incoherent to maintain the \(U(1)\) symmetry.

Nonetheless, the interdimer \(J’\) interaction is clearly seen to couple effectively the singlet state with the \(t_{a−}\) triplets through terms of the type \(J’ S_i^{+} S_m^{-}\), where \(l\) is one of the two sites of the dimer and \(m\) is a site of a neighbouring dimer (actually the \(J’\) term couples coherently the \(t_{a−}\) triplets and the singlet on both neighbouring dimers). When the singlet and the \(t_{a}\) triplet are brought close to degeneracy by the field, this coupling becomes resonant and it changes drastically the property of the ground state. A critical field \(h_{c1}^{(0)} \sim 1 - aJ’/J\) (where \(a\) is a model-dependent constant) suffices to match the resonance conditions. The resonant coupling between the two states causes the breaking of all the symmetries previously retained by the reduced density matrix. A transfer of population from the singlet to the \(t_{a}\) triplet breaks the symmetry between \(t_{a}\) and \(t_{−}\) thereby leading to the appearance of a finite magnetization along the field. Moreover, the reduced density matrix acquires finite off-diagonal terms \(\langle s|\rho^{(dim)}|t_{a}\rangle = \langle t_{a}|\rho^{(dim)}|s\rangle \neq 0\), which implies that, defining creation/annihilation operators for the \(t_{a}\) bosons \(h_{+} |s\rangle = |t_{a}\rangle, h_{−} |t_{a}\rangle = |s\rangle\), the expectation values \(\langle h_{+}\rangle\) and \(\langle h_{−}\rangle\) become non-zero. This clearly identifies the breaking of the \(U(1)\) symmetry in the plane (i.e. the appearance of antiferromagnetic order transverse to the field) with the appearance of a condensate of \(t_{a}\) bosons.

From the magnetic point of view, the field-induced ordered ground state is a cantanted ferromagnetic state of the type described at the beginning of the section, with finite uniform magnetization along the field and finite transverse staggered magnetization. By further increasing the field, the system is eventually brought to a fully polarized ferromagnetic state when reaching a critical field \(h_{c2}^{(0)} \sim 1 + aJ’/J\) (with \(a\) again model dependent) which destroys the coherent mixing between the singlet and the \(t_{a}\) triplet. Around this field the dimer-reduced density matrix has \(p(|t_{a}\rangle) \gg p(|s\rangle)\), so the ordered state is conveniently described as a superfluid state of bosonic singlet holes in the triplet ‘sea’. The fully polarized state corresponds simply to the vacuum of such holes. Therefore we can conclude that a weakly coupled dimer system in a field realizes two successive insulator/superfluid transitions.

3. Bilayer Heisenberg antiferromagnet in a field

In what follows we consider the specific case of a \(S = 1/2\) Heisenberg bilayer in a uniform magnetic field (figure 1(a)):

\[
\mathcal{H} = J' \sum_{(ij)} \sum_{a=1,2} S_{i,a} \cdot S_{j,a} + J \sum_i S_{i,1} \cdot S_{i,2} - J h \sum_{i,a} \epsilon_{i,a} S_{i,a}^z.
\]

where the index \(i\) runs over the sites of a square lattice, \((ij)\) are pairs of nearest neighbours on the square lattice and \(\alpha\) is the layer index. This model has been intensively investigated.
recently synthetized compound BaCuSi$_2$O$_6$[16].

The superfluid density $\Upsilon$ with respect to the field.

On each dimer location $i$, $|0\rangle = |s\rangle$ and $|1\rangle = |t_i\rangle$. For $h < h_{c1}^{(0)} = 1 - z J'/2J$ the ground state is the vacuum, while for $h > h_{c2}^{(0)} = 1 + z J'/2J$ the ground state has one hard-core triplet per dimer ($z = 4$ is the coordination number of the square lattice). Both insulating states have a finite particle gap. For any intermediate field value $h_{c1}^{(0)} < h < h_{c2}^{(0)}$ the system is gapless with bosons having a fractional filling (which corresponds to a finite uniform magnetization away from its saturation value, $0 < m^z_{c1} < S^z_{c1} < 1/2$), and forming a condensate with long-range phase coherence (which corresponds to staggered antiferromagnetic order transverse to the field $m^z_{c1} = (-1)^i S^z_{c1}$).

Figure 2 shows the succession of ground-state bosonic phases in a bilayer Heisenberg antiferromagnet with $g = J/J' = 4$. Shown are the results of a quantum Monte Carlo calculation based on the stochastic series expansion method with directed-loop update [18]. The superfluid density $\Upsilon$ is estimated through winding number fluctuations [19], while the uniform susceptibility $\chi_u$ is obtained by numerical derivation of the uniform magnetization with respect to the field.
4. Bilayer Heisenberg antiferromagnet with site dilution: Bose-glass phase

A well-controlled way of introducing lattice disorder in quantum spin systems is by doping the magnetic ions with non-magnetic ones with the same valence, so that the main effect is the removal of some of the spins in the magnetic Hamiltonian. In the case of BaCuSi$_2$O$_6$, e.g., this can be achieved by doping the Cu$^{2+}$ ions with non-magnetic ions as Zn$^{2+}$ or Mg$^{2+}$. The magnetic Hamiltonian of the Heisenberg bilayer in presence of site dilution (figure 1(b)) reads:

\[ H = J \sum_i \epsilon_i S_i \cdot S_i + J' \sum_{\langle ij \rangle} \sum_{a=1,2} \epsilon_{i,a} \epsilon_{j,a} S_i,1 \cdot S_j,2 - Jh \sum_{i,a} \epsilon_{i,a} S_i,1 \cdot \cdot \cdot \]  

Here the variables $\epsilon_{i,a}$ are random numbers taking value 0 with probability $p$ (corresponding to the concentration of non-magnetic dopants) and 1 with probability $1-p$.

In presence of site dilution, some of the dimers are completely missing, and some are reduced to single spins. We can nonetheless repeat the same approximate bosonic mapping as discussed in the previous section by truncating the Hilbert space of the intact dimers while retaining the full space of the dangling spins. The $J'$ coupling between two intact dimers has obviously the same expression in terms of bosonic operators as before. The coupling between a dimer and a dangling spin on the $\alpha$th layer is instead of the form

\[ H_{\text{dimer}} - \text{spin} = \left( J' / 2 \right) (\sigma^x_i S^x_j,\alpha + \sigma^y_i S^y_j,\alpha) + \left( J' / 2 \right) (\sigma^z_i + 1/2) S^z_j,\alpha. \]  

Finally, mapping the pseudospins onto $b, b^\dagger$ hard-core bosons and the dangling $S = 1/2$ spins onto $c, c^\dagger$ hard-core bosons, we end up with the following (rather complex) disordered bosonic model

\[ H_{\text{boson}} = H_{bb} + H_{cc} + H_{bc} \]

where

\[ H_{bb} = - J' / 2 \sum_{\langle ij \rangle} \omega_{i,j} (b_i b_j + \text{h.c.}) + J' \sum_{\langle ij \rangle} \sum_{a=1,2} \epsilon_{i,a} \epsilon_{j,a} S_i,1 \cdot S_j,2 - Jh \sum_{i,a} \epsilon_{i,a} S_i,1 \cdot \cdot \cdot \]  

\[ H_{cc} = - J' \sum_{\langle ij \rangle,\alpha} \gamma_{i,j,\alpha} (c_i c_j + \text{h.c.}) + J' \sum_{\langle ij \rangle,\alpha} \gamma_{i,j,\alpha} M_i M_j \]

\[ - \left[ Jh + J' / 2 \sum_{d} (\gamma_{i,+d} - \lambda_{i,d}) \right] \sum_i \gamma_i M_i. \]
\[ \mathcal{H}_{bc} = -\frac{J'}{2\sqrt{2}} \sum_{\langle ij \rangle} \left[ \lambda_i \gamma_j (b_i c_i^\dagger + \text{h.c.}) + \gamma_i \lambda_j (c_i b_i^\dagger + \text{h.c.}) \right] + J' \sum_{\langle ij \rangle} (\lambda_i \gamma_j n_i M_j + \gamma_i \lambda_j M_i n_j). \]

Here \( M_i = c_i^\dagger c_i, \gamma_i, \lambda_i = \epsilon_{i,a}(1 - \epsilon_{i,a}), \gamma_i = \sum_\alpha \gamma_{i,a}, \) and \( \lambda_i = \epsilon_{i,1} \epsilon_{i,2} \) and \( \sum_d \) runs over the four lattice vectors.

For site dilution well below the bilayer percolation threshold \((p < p^* = 0.5244(2) [20])\), the above model contains a network of \( b \)-sites \((\lambda_i = 1, \gamma_i = 0)\), corresponding to intact dimers in the original spin Hamiltonian, interrupted by \( c \)-sites \((\lambda_i = 0, \gamma_i = 1)\), corresponding to dangling spins in the original model, and by empty sites \((\lambda_i = 0, \gamma_i = 0)\) corresponding to missing dimers. The most relevant feature of this complex model is that the chemical potential for the \( b \)-bosons, \( \mu_b = J(h - 1) \) is quite different from that of the \( c \)-bosons, \( \mu_c = J h + \frac{J'}{2} \sum_d (\gamma_{i+d} - \lambda_{i+d}) \). In fact, if \( J \gg J' \), the chemical potential for the \( b \)-bosons is always positive, and when \( h J \sim J' \) the \( c \)-sites are almost all filled, which corresponds to field polarization of the dangling spins. On the contrary the chemical potential for the \( b \)-bosons can be negative for \( h < 1 \), which means that, for the same field \( h \), a system with only \( b \)-sites can be empty of bosons, while a system with only \( c \)-sites would be completely filled. When \( b \)- and \( c \)-sites are combined together, \( c \)-bosons can hop to neighbouring \( b \)-sites, so that there is always a finite, albeit small, population of \( b \)-bosons around \( c \)-sites, but, for negative enough \( \mu_b \), bosons are prevented from travelling deep inside continuous regions of \( b \)-sites. Therefore for \( h \leq h_{c1}^0 \) the bosonic model has all \( c \)-sites essentially filled and \( b \)-sites only partially filled when neighbouring a \( c \)-site, but continuous regions of \( b \)-sites are essentially empty of particles. If \( J \gg J' \), in this situation the system has a gap to the addition of further particles, and it is in a quantum disordered state very similar to the insulating states in the clean limit.

When \( h \geq h_{c1}^0 \) the first \( b \)-bosons appear in a homogeneous system made exclusively of \( b \)-sites, and they form a coherent condensate. In presence of disorder, instead, \( b \)-bosons are induced only away from \( c \)-sites in those rare regions which are locally well approximating a clean system, namely where the local coordination number is very close to \( z = 4 \) of the clean system. In regions with a lower local coordination number the local critical field is effectively higher, given its dependence on \( z \). This simply means that the first bosons to appear in the bulk of the \( b \) regions are localized, although the \( b \)-sites might form a percolating cluster for low enough dilution of the lattice. Therefore, we are in presence of a phenomenon of quantum localization of the \( b \)-bosons, introducing a new phase in the disordered system, namely a Bose-glass phase. In such a phase there is no gap to the addition of a particle because, for any value of \( h \geq h_{c1}^0 \), we will find a region in the system where the local critical field equals \( h \), so we can inject a boson there. This is therefore an unconventional quantum-disordered insulating phase with short-range correlations and a gapless spectrum. To overcome localization and enter the condensate phase, it is necessary to reach a higher critical field \( h_{c1} > h_{c1}^0 \) at which a percolating network of \( b \)-sites becomes accessible to bosons, so that long-range phase coherence can be established throughout the system.

A similar phenomenological description can be repeated when approaching the upper critical field, \( h \sim h_{c2}^0 \). Here the description is simplified by the fact that, for such a high field, the \( c \)-sites can be thought of as perfectly filled, and bosonic holes are all on \( b \)-sites (a bosonic hole on a \( c \)-site is energetically quite unfavourable). Therefore the \( c \)-sites can be simply quenched in the \((M_i = 1)\) state, and we get the simplified bosonic model

\[ \mathcal{H}_{\text{boson}} \approx -\frac{J'}{2} \sum_{\langle ij \rangle} \lambda_i \lambda_j (b_i b_i^\dagger + \text{h.c.}) + \frac{J'}{2} \sum_{\langle ij \rangle} \lambda_i \lambda_j n_i n_j - \sum_i \lambda_i \left[ J(h - 1) - J' \sum_d \gamma_{i+d} \right] n_i. \]

(9)
Bose–Einstein condensation versus localization of bosonic quasiparticles S159

Figure 3. Left panel. Zero-temperature field scan in the site-diluted bilayer Heisenberg antiferromagnet with $J/J' = 4$ and $p = 0.2$. Note that the apparently finite magnetization $m_z$ in the quantum disordered phase is the result of a strong finite size effect for the smallest size considered ($L = 24$). Right panel. Scaling of the transverse structure factor $S^{xx}(\pi, \pi) = (1/N) \sum_{ij,\alpha\beta} (-1)^{i+j} \langle S^{\alpha}(i,\beta) S^{\beta}(j,\beta) \rangle$ in the different phases of the system. When $S^{xx}(\pi, \pi)$ scales slower than linearly in the volume, superfluid order is absent in the system. The dashed lines are linear fits to the finite-size data.

which is the hard-core Bose–Hubbard model on a site-diluted square lattice with random on-site chemical potential (fluctuations in the chemical potential appear only close to a site vacancy). As before, the regions of $b$-sites with a lower local coordination number will have a local upper critical field which is less than $h_{c2}(0)$, which means that they tend to expel bosonic holes. For $h \lesssim h_{c2}(0)$ such holes remain localized on regions with local coordination close to that of the clean system, implying that, for a critical field $h_{c2} < h_{c2}(0)$, the system loses the hole condensate and it enters a Bose-glass phase of bosonic holes.

5. Diluted Heisenberg bilayer: quantum Monte Carlo data

The approximate bosonic picture we put forward in the previous section for the physics of the site-diluted Heisenberg bilayer is confirmed by extensive quantum Monte Carlo simulations done on the original spin model. Making use of stochastic-series-expansion quantum Monte Carlo, we were able to efficiently reach the physical $T = 0$ behaviour by a successive increase of the inverse temperature [21]. We investigated system sizes up to $48 \times 48 \times 2$, averaging typically over 200 disorder realizations.

Figure 3 shows the succession of phases in a bilayer Heisenberg model with $J/J' = 4$ and site dilution $p = 0.2$ when the applied field $h$ is changed from zero to the saturation value $h = 2$. This picture has to be contrasted with the analogous field scan in the clean limit, as shown in figure 2.

At low field, a particular feature of the spin model, not discussed previously in the bosonic mapping, shows up. Doping the dimer-singlet phase, we get long-range antiferromagnetic ordering of the free moments appearing around each of the vacancies, a well-known phenomenon of order-by-disorder [20, 22, 23]. This corresponds approximately to bosons being created around $c$-sites but largely fluctuating in number given that the chemical potential $\mu_c$ is very small, and coherently propagating between $c$-sites through effective long-range hoppings which decay exponentially with the inter-site distance, so that tenuous long-range phase coherence is established.
Increasing the field leads to a strong increase in the $\mu_c$ chemical potential, which quenches particle number fluctuations and it destroys the disorder-induced superfluid order. This quenching is strongly inhomogeneous, reflecting the inhomogeneity of the random couplings between the $c$-sites: some of the $c$-sites, which are far from any other $c$-site and thus weakly coupled to the remainder, are easily filled with bosons up to saturation, while more coordinated $c$-sites resist density saturation in favour of the local kinetic energy. Nonetheless it can happen that partially filled regions of $c$-sites are separated from each other by fully filled $c$-sites, and therefore coherent propagation throughout the system is no longer possible. The system enters then a gapless quantum-disordered phase, whose gapless nature, reflected in the finite compressibility of the bosons (susceptibility of the magnetic system), is due to the fact that the $c$-sites are not all completely filled. For a larger ratio $J/J'$ than the one considered here, by further increasing the field one can access the regime where $c$-sites are essentially all filled and the system does not admit any further particle, namely the compressibility vanishes and the system acquires a gap as in the clean case [24]. In this regime, either gapful or gapless, bosons are mostly pinned around $c$-sites, and they cannot propagate in $b$-site regions due to the local finite particle gap in such regions. Given that the local particle gap in the $b$ regions is lower bounded by $h_{c1}(0)$, this regime persists as long as $h < h_{c1}(0)$.

When $h \geq h_{c1}(0) \approx 0.47(1)$, nonetheless, large clean $b$-regions accept the appearance of bosons, which remain quantum localized, giving rise to a Bose-glass phase. This is clearly seen in the real-space picture of the dimer magnetization $m_i = \langle S_{i,1}^z + S_{i,2}^z \rangle$ in figure 4(a).

Figure 4. (a) Real-space images of the dimer magnetization $m_i = \langle S_{i,1}^z + S_{i,2}^z \rangle$ on intact dimers in a $40 \times 40 \times 2$ bilayer with $J/J' = 4$, dilution $p = 0.1$ and at inverse temperature $\beta J = 256$, for $h = 0.56$ (left) and $h = 0.6$ (right). The radius of the dots is proportional to the dimer magnetization. The magnetization of unpaired spins ($c$-sites) is omitted for clarity. The most visible localized states are highlighted in the left panel, while the backbone of the percolating magnetized network is highlighted in the right one. The central panel shows the superfluid density as a function of the field for the specific sample considered. (b) Real-space images of the distance of the dimer magnetization from its saturation value, $1 - m_i$, for $h = 1.87$ (left) and $h = 1.93$ (right). Other symbols, parameters and explanations as in (a).
Bose–Einstein condensation versus localization of bosonic quasiparticles

(left panel), representing the local density of bosons on \( b \)-sites within the approximate bosonic mapping. We observe that, in this regime, the order parameter \( m_x \) and the superfluid density are zero, and the number of bosons, corresponding to the uniform magnetization, increases extremely slowly with increasing field, corresponding to the gradual appearance of particles in the rare clean regions of the system.

Increasing the field beyond \( h_{c1} \approx 0.69(2) \) a superfluid state is finally established in the system through delocalization of the collective state of the bosons (figure 4(a) (right panel))\(^3\)

Switching then to a description in terms of bosonic holes when the filling exceeds \( 1/4 \), we observe that the delocalized superfluid state of holes, whose local concentration is proportional to \( 1 - m_t \) (figure 4(b) (left panel)), is destroyed at a critical field \( h_{c2} \approx 1.78(2) \) lower than the saturation one through a mechanism of quantum localization (figure 4(b) (right panel)), which gives rise to the hole Bose-glass phase at high fields. Even more clearly than in the lower-field case, this phase shows a finite compressibility and hence a gapless spectrum, but absence of long-range phase coherence due to quantum localization.

6. Conclusions

In this work we have investigated the emergence of a rich physical scenario of strongly correlated bosons in weakly-coupled dimer antiferromagnets in a magnetic field. In presence of site dilution of the magnetic lattice, extended Bose-glass phases appear around the field-induced ordered phase, suggesting the possibility of an experimental realization of such a phase in quantum magnets. The particular case of a bilayer antiferromagnet we considered in this paper is realized by \( \text{BaCuSi}_2\text{O}_6 \) \([16]\). Nonetheless the picture of quantum localization of bosonic quasiparticles applies straightforwardly to other spin gap systems with different geometries, such as \( \text{Sr}_2\text{Cu(BO}_3)_2 \) \([26]\) or \( \text{Tl(K)CuCl}_3 \) \([4]\), where site dilution can be analogously realized by doping the \( \text{Cu} \) sites. The availability of high magnetic fields allows us to scan the whole succession of phases discussed in the paper, and different probes, such as elastic neutron scattering and magnetometry measurements, can directly access the magnetic observables characterizing the various phases. An open question is how to directly probe the superfluid nature of the bosonic quasiparticles condensed in the ordered-ground state, given that a supercurrent of such particles corresponds to a pure spin current with no energy or charge transport associated with it.

Acknowledgments

We thank M Vojta for insightful discussions, and all the organizers of the Cortona BEC workshop 2005 for the exciting program they put together. This work was supported by DOE under grant DE-FG02-05ER46240. Computational facilities have been generously provided by the HPC Center at USC.

References

[1] Fazio R and van der Zant H 2001 Phys. Rep. 355 235
[2] Greiner M, Mandel O, Esslinger T, Hänsch T W and Bloch I 2002 Nature 415 39
[3] Nikuni T, Oshikawa M, Oosawa A and Tanaka H 2000 Phys. Rev. Lett. 84 5868

\(^3\) Note that this figure refers to a different dilution value \( p = 0.1 \), for which we estimate a disorder-averaged value \( h_{c1} \approx 0.553(5) \). This value of \( h_{c1} \) differs only slightly from the sample-specific value \( h_{c1} \approx 0.56 \) that one can derive from figure 4(a). Similarly we estimate \( h_{c2} \approx 1.95(2) \) for the upper critical field at \( p = 0.1 \), differing from the sample-specific value \( h_{c2} \approx 1.91 \) suggested by figure 4(b).
[4] Rüegg Ch, Cavadini N, Furrer A, Güdel H-U, Krämer K, Mutka H, Wildes A, Habicht K and Vorderwisch P 2003 Nature 423 62
[5] Rice T M 2002 Science 298 760
[6] Matsumoto M, Normand B, Rice T M and Sigrist M 2002 Phys. Rev. Lett. 89 077203
Matsumoto M, Normand B, Rice T M and Sigrist M 2004 Phys. Rev. B 69 054423
[7] Fisher M P A, Weichman P B, Grinstein G and Fisher D S 1989 Phys. Rev. B 40 546
[8] Scalettar R T, Batrouni O G and Ziman J T 1991 Phys. Rev. Lett. 66 3144
[9] Oosawa A and Tanaka H 2002 Phys. Rev. B 65 184437
Shindo Y and Tanaka H 2004 J. Phys. Soc. Japan 73 2642
[10] Nohadani O, Wessel S and Haas S 2005 Phys. Rev. Lett. 95 227201
[11] Vajk O P, Mang P K, Greven M, Gehring P M and Lynn J W 2002 Science 295 1691
Villain J 1974 J. Phys. (Paris) 35 27
[12] Leggett A J 1995 Bose–Einstein Condensation ed A Griffin, D W Snoke and S Stringari (NY: Cambridge) p 452
[13] Sandvik A W and Scalapino D J 1994 Phys. Rev. Lett. 72 2777
[14] Sommer T, Vojta M and Becker K W 2001 Eur. Phys. J. B 23 329
[15] Jaime M et al 2004 Phys. Rev. Lett. 93 087203
Sebastian S E, Sharma P A, Jaime M, Harrison N, Correa V, Balicas L, Kawashima N, Batista C D and Fisher I R 2005 Phys. Rev. B 72 100404
[16] Tachiki M and Yamada T 1970 J. Phys. Soc. Japan 28 1413
[17] Syljuåsen O F and Sandvik A W 2002 Phys. Rev. E 66 046701
[18] Pollock E L and Ceperley D M 1987 Phys. Rev. B 36 8343
[19] Roscilde T and Haas S 2005 Phys. Rev. Lett. 95 207206
[20] Sandvik A W 2002 Phys. Rev. B 66 024418
[21] Shender E F and Kivelson S A 1991 Phys. Rev. Lett. 66 2384
[22] Sigrist M and Furusaki A 1996 J. Phys. Soc. Japan 65 2385
[23] Roscilde T and Haas S in preparation
[24] Mikeska H-J, Ghosh A and Koleszhuk A K 2004 Phys. Rev. Lett. 93 217204
[25] Sebastian S E et al 2005 Phys. Rev. B 71 212405