Testing \( CPT \) Invariance in \( B_d^0-\bar{B}_d^0 \) and \( B_s^0-\bar{B}_s^0 \) Oscillations

Ping Ren and Zhi-zhong Xing

Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918, Beijing 100049, China

Abstract

Recent CDF and D0 measurements of \( B_s^0-\bar{B}_s^0 \) mixing make it possible to search for \( CP \) violation and test \( CPT \) symmetry in a variety of \( B_s \) decays. Considering both coherent \( B_d^0-\bar{B}_d^0 \) decays at the \( \Upsilon(4S) \) resonance and coherent \( B_s^0-\bar{B}_s^0 \) decays at the \( \Upsilon(5S) \) resonance, we formulate their time-dependent and time-integrated rates by postulating small \( CPT \) violation in \( B_d^0-\bar{B}_d^0 \) and \( B_s^0-\bar{B}_s^0 \) oscillations. We show that the opposite-sign dilepton events from either \( C \)-odd or \( C \)-even \( B^0_q-\bar{B}^0_q \) states (for \( q = d \) or \( s \)) can be used to determine or constrain the \( CPT \)-violating parameter at a super-\( B \) factory. The possibility of distinguishing between the effect of \( CPT \) violation and that of \( \Delta B = -\Delta Q \) transitions is also discussed.

\(^1\text{E-mail: xingzz@ihep.ac.cn}\)
1 Introduction

A correlated $P^0 \bar{P}^0$ system, where $P$ may be either $K$, $D$, $B_d$ or $B_s$, has been of great interest for the study of $CP$, $T$ and $CPT$ symmetries in particle physics. In the $K^0 - \bar{K}^0$ mixing system, for instance, both indirect $CP$ violation [1] and direct $CP$ violation [2] have unambiguously been observed; the evidence for $T$ violation [3] has been achieved; and $CPT$ invariance has been tested to an impressive degree of accuracy [4]. The $\Delta Q = \Delta S$ rule has also been examined in the semileptonic $K^0$ and $\bar{K}^0$ transitions. Beyond the $K^0 - \bar{K}^0$ system, both indirect and direct signals of $CP$ violation have been observed in a number of neutral $B_d$ decays [4]; and possible $CPT$ violation in $B^0_d - \bar{B}^0_d$ mixing has been searched for at the KEK and SLAC $B$-meson factories [5]. Although the phenomena of $CP$ violation have not been seen in the $B^0_s - \bar{B}^0_s$ and $D^0 - \bar{D}^0$ mixing systems, they may show up and even surprise us in the near future at the LHC-$B$ [6], $\tau$-charm [7] and super-$B$ [8] factories.

The CDF [9] and D0 [10] Collaborations have recently reported their measurements of $B^0_s - \bar{B}^0_s$ mixing (both the mass and width differences between the light and heavy $B_s$ mass eigenstates) at the Fermilab Tevatron Collider:

$$\Delta M_s = 17.77 \pm 0.10 \text{(stat)} \pm 0.07 \text{(syst)} \text{ ps}^{-1} \text{ (CDF [9])},$$

$$\Delta \Gamma_s = 0.13 \pm 0.09 \text{ ps}^{-1} \text{ (D0 [10])}. \quad (1)$$

This remarkable progress in experimental $B$ physics makes it possible to search for $CP$ violation and test $CPT$ symmetry in the $B^0_s - \bar{B}^0_s$ system. As the magnitude of $B^0_s - \bar{B}^0_s$ mixing is much larger than that of $B^0_d - \bar{B}^0_d$ mixing [4], its impact on the time-dependent and time-integrated rates of $B^0_s$ and $\bar{B}^0_s$ decays deserves attention. In particular, the $CP$- and $CPT$-violating signals might get enhanced or suppressed due to large $B^0_s - \bar{B}^0_s$ mixing. At a super-$B$ factory with the luminosity $L \sim a \times 10^{36} \text{cm}^{-2}\text{s}^{-1}$ [8], both coherent $B^0_d - \bar{B}^0_d$ decays at the $\Upsilon(4S)$ resonance and coherent $B^0_s - \bar{B}^0_s$ decays at the $\Upsilon(5S)$ resonance will be studied down to the last detail.

The main purpose of this work is to formulate the decay rates of a correlated $B^0_q - \bar{B}^0_q$ state (either $q = d$ or $q = s$) in the assumption that there are both small $CPT$ violation in $B^0_q - \bar{B}^0_q$ oscillation and small $\Delta B = \Delta Q$ violation in semileptonic $B^0_q$ and $\bar{B}^0_q$ decays. Note that possible effects of $CPT$ violation in neutral $B_q$ and $D$ decays have been analyzed in Refs. [11, 12, 13] and Refs. [14, 15], respectively; and possible effects of $\Delta B = -\Delta Q$ and $\Delta C = -\Delta Q$ transitions have been discussed in Ref. [16] and Ref. [17], respectively. The present paper is different from the previous ones not only because we are dealing with a new heavy meson-antimeson mixing system (i.e., the $B^0_s - \bar{B}^0_s$ system) but also because our discussions are essentially new in two aspects. (1) We shall take into account both $C$-odd and $C$-even $B^0_q - \bar{B}^0_q$ pairs with $C$ being the charge-conjugation parity of this coherent
system, and calculate both time-dependent and time-integrated rates of \((B_q^0\bar{B}_q^0)_C\) decays by assuming slight \(CPT\) violation and small \(\Delta B = -\Delta Q\) effects. Our analytical results are more general and more useful than those obtained in Refs. [11]–[16]. (2) We shall concentrate on the opposite-sign dilepton asymmetries of \((B_q^0\bar{B}_q^0)_C\) decays to investigate possible effects of \(CPT\) violation and \(\Delta B = -\Delta Q\) transitions. It is worth remarking that an opposite-sign dilepton event may be either \(l_1^+l_2^-\) (for \(l_1 \neq l_2\)) or \(l^+l^-\). Although the opposite-sign dilepton events of \((B_q^0\bar{B}_q^0)_C\) decays have been considered in Refs. [12, 13], our results for the \(B_s^0\bar{B}_s^0\) mixing system have their own features and implications due to the small \(CPT\)-violating phase of \(B_s^0\bar{B}_s^0\) mixing and the large values of \(\Delta M_s\) and \(\Delta \Gamma_s\).

So far some interest has been paid to the possibilities of exploring \(CPT\) violation and probing new physics in weak \(B_s\) decays at the \(\Upsilon(5S)\) resonance [18]–[21], although the experimental feasibility remains an open question. We expect that the future super-\(B\) factory can run at this interesting energy threshold [22]. Then it will be possible to test \(CPT\) invariance in both \(B_d^0\bar{B}_d^0\) and \(B_s^0\bar{B}_s^0\) oscillations by studying the coherent \(B_d^0\bar{B}_d^0\) and \(B_s^0\bar{B}_s^0\) decays.

2 \(CPT\) violation in coherent \((B_q^0\bar{B}_q^0)_C\) decays

The mixing or oscillation between \(B_q^0\) and \(\bar{B}_q^0\) mesons can naturally arise from their common coupling to a subset of real and virtual intermediate states. Hence the mass eigenstates \(|B_L\rangle\) and \(|B_H\rangle\), where “L” (“H”) denotes “light” (“heavy”), are different from the flavor (weak interaction) eigenstates \(|B_q^0\rangle\) and \(|\bar{B}_q^0\rangle\). Taking account of both \(CPT\)- and \(CPT\)-violating effects in \(B_q^0\bar{B}_q^0\) mixing, one may parametrize the correlation between \(|B_L\rangle\) and \(|B_q^0\rangle\) and \(|B_H\rangle\) in Eq. (2) have been omitted. \(CPT\) invariance requires \(\cos \theta = 0\) or equivalently \(\theta = \pi/2\), while \(CPT\) conservation requires both \(\theta = \pi/2\) and \(\phi = 0\) [23]. \(^2\) The proper-time

\(^2\)As \(CPT\) violation may simultaneously imply the violation of Lorentz covariance in a quantum field theory [24], the dependence of \(\theta\) on the sidereal time should in general be taken into account [25]. For simplicity, here we take \(\theta\) as a constant by assuming that the boost parameters of \(B_q^0\) and \(\bar{B}_q^0\) mesons are small and the corresponding Lorentz violation is rotationally invariant in the laboratory frame. In this approximation, our results are valid as averages over the sidereal time, such that the effect of Lorentz violation due to the direction of motion can be neglected. A complete analysis, which requires incorporating Lorentz-violating parameters directly into the phenomenology to account for possible \(CPT\) violation [26], is beyond the scope of this paper and will be done elsewhere.
evolution of an initially pure $|B_{q}^{0}\rangle$ or $|\bar{B}_{q}^{0}\rangle$ state is given by [11]

\[
|B_{q}^{0}(t)\rangle = e^{-(iM_{q}+\frac{1}{2}\Delta\Gamma_{q})t} \left[ g_{+}(t)|B_{q}^{0}\rangle + \bar{g}_{+}(t)|\bar{B}_{q}^{0}\rangle \right],
\]

\[
|\bar{B}_{q}^{0}(t)\rangle = e^{-(iM_{q}+\frac{1}{2}\Delta\Gamma_{q})t} \left[ g_{-}(t)|\bar{B}_{q}^{0}\rangle + \bar{g}_{-}(t)|B_{q}^{0}\rangle \right],
\]

where

\[
g_{\pm}(t) = \cosh \left( \frac{ix_{q} - y_{q}\Gamma}{2} \right) \pm \cos \theta \sinh \left( \frac{ix_{q} - y_{q}\Gamma}{2} \right),
\]

\[
\bar{g}_{\pm}(t) = \sin \theta e^{\mp i\phi} \sinh \left( \frac{ix_{q} - y_{q}\Gamma}{2} \right).
\]

The definitions in Eqs. (3) and (4) are $M \equiv (M_{L} + M_{H})/2$, $\Gamma \equiv (\Gamma_{L} + \Gamma_{H})/2$, $x_{q} \equiv \Delta M_{q}/\Gamma$ with $\Delta M_{q} \equiv M_{H} - M_{L}$, and $y_{q} \equiv \Delta\Gamma_{q}/(2\Gamma)$ with $\Delta\Gamma_{q} \equiv \Gamma_{L} - \Gamma_{H}$, where $M_{L,H}$ ($\Gamma_{L,H}$) denotes the mass (width) of $B_{L,H}$. Taking account of $1/\Gamma \approx 1.52$ ps [10], we approximately obtain $x_{s} \approx 27$ and $y_{s} \approx 0.1$ from the central values of $\Delta M_{s}$ and $\Delta\Gamma_{s}$ given in Eq. (1). In contrast, $x_{d} \approx 0.78$ [4] has been known but $y_{d}$ has not been measured for the $B_{d}^{0}$-$\bar{B}_{d}^{0}$ mixing system. We stress that the experimental values of $x_{d}$, $x_{s}$ and $y_{s}$ are in good agreement with the standard-model predictions [27] $^{3}$, although the uncertainty associated with $y_{s}$ remains quite large.

In order to calculate the proper-time distribution of coherent $(B_{q}^{0}\bar{B}_{q}^{0})_{C}$ decays, we neglect the tiny final-state electromagnetic interactions and assume CPT invariance in the direct transition amplitudes of semileptonic or nonleptonic $B_{q}^{0}$ and $\bar{B}_{q}^{0}$ decays. Such an assumption can be examined, without the mixing-induced complexity, by detecting the charge asymmetry of semileptonic $B^{\pm}$ decays [12]. We shall take into account possible $\Delta B = -\Delta Q$ transitions in our calculations. The latter can be described by using a small parameter $\sigma_{t}$ for a given semileptonic decay mode,

\[
\langle l^{+}X_{l}^{-}|\mathcal{H}|B_{q}^{0}\rangle \equiv A_{l}, \quad \langle l^{+}X_{l}^{-}|\mathcal{H}|\bar{B}_{q}^{0}\rangle = \sigma_{t}A_{l};
\]

\[
\langle l^{-}X_{l}^{+}|\mathcal{H}|\bar{B}_{q}^{0}\rangle \equiv A_{l}^{*}, \quad \langle l^{-}X_{l}^{+}|\mathcal{H}|B_{q}^{0}\rangle = \sigma_{t}^{*}A_{l}^{*},
\]

where $\sigma_{t}$ measures the $\Delta B = -\Delta Q$ effect and $|\sigma_{t}| \ll 1$ is expected to hold. $|\sigma_{t}| \neq 0$ implies that it is in practice impossible to have a pure tagging of the $B_{q}^{0}$ or $\bar{B}_{q}^{0}$ state through its semileptonic decay (to $l^{+}X_{l}^{-}$ or $l^{-}X_{l}^{+}$). In general, the amplitudes of $B_{q}^{0}$ and $\bar{B}_{q}^{0}$ decays into the final-state $f_{i}$ (either semileptonic or nonleptonic) are denoted as $A_{f_{i}} \equiv \langle f_{i}|\mathcal{H}|B_{q}^{0}\rangle$ and $\bar{A}_{f_{i}} \equiv \langle f_{i}|\mathcal{H}|\bar{B}_{q}^{0}\rangle$. When the coherent $(B_{q}^{0}\bar{B}_{q}^{0})_{C} \rightarrow f_{1}f_{2}$ decays are concerned, the following combinations

\[
\xi_{C} = e^{-i\phi} + Ce^{i\phi} \frac{\bar{A}_{f_{1}}A_{f_{2}}}{A_{f_{1}}^{*}A_{f_{2}}}, \quad \zeta_{C} = \frac{\bar{A}_{f_{2}}}{A_{f_{2}}} + \frac{\bar{A}_{f_{1}}}{A_{f_{1}}},
\]

$^{3}$Note that $y_{d} \approx 0.002$ and $y_{s} \approx 0.06\cdots0.08$ are the updated predictions [27]. The theoretical expectation $x_{s}/x_{d} \sim y_{s}/y_{d} \sim 35$ has partly been confirmed by current experimental data.
where $C = \pm 1$, will be frequently used.

Now let us consider a correlated $B^0_q \bar{B}^0_q$ state at rest. Its time-dependent wave function can be written as

$$
\frac{1}{\sqrt{2}} \left[ |B^0_q(\mathbf{K}, t)|\bar{B}^0_q(-\mathbf{K}, t) + C|B^0_q(-\mathbf{K}, t)|\bar{B}^0_q(\mathbf{K}, t) \right],
$$

(7)

where $\mathbf{K}$ is the three-momentum vector of $B^0_q$ and $\bar{B}^0_q$, and $C = \pm 1$ is the charge-conjugation parity of this coherent system. If one of the two $B_q$ mesons (with momentum $\mathbf{K}$) decays to a final state $f_1$ at proper time $t_1$ and the other (with $-\mathbf{K}$) to $f_2$ at $t_2$, the amplitude of their joint decays is given by

$$
A(f_1, t_1; f_2, t_2)_C = \frac{1}{\sqrt{2}} e^{-(i\mathcal{M} + \xi)(t_1 + t_2)} \left\{ A_{f_1} A_{f_2} \left[ g_+(t_1)\bar{g}_-(t_2) + C\bar{g}_-(t_1)g_+(t_2) \right]
+ A_{f_1} \bar{A}_{f_2} \left[ g_+(t_1)g_-(t_2) + C\bar{g}_-(t_1)\bar{g}_+(t_2) \right]
+ \bar{A}_{f_1} A_{f_2} \left[ \bar{g}_+(t_1)\bar{g}_-(t_2) + Cg_-(t_1)g_+(t_2) \right]
+ \bar{A}_{f_1} \bar{A}_{f_2} \left[ \bar{g}_+(t_1)g_-(t_2) + C\bar{g}_-(t_1)\bar{g}_+(t_2) \right] \right\}.
$$

(8)

The calculation of the decay rate $R(f_1, t_1; f_2, t_2)_C \propto |A(f_1, t_1; f_2, t_2)_C|^2$ is straightforward but lengthy. Our result is

$$
R(f_1, t_1; f_2, t_2)_C \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t} \left[ \left| \xi_C \right|^2 + \left| \zeta_C \right|^2 \right] \cosh \left( y_q \Gamma t_C \right)
- \left( \left| \xi_C \right|^2 - \left| \zeta_C \right|^2 \right) \cos \left( x_q \Gamma t_C \right)
- 2\text{Re} \left( \xi_C^{*} \zeta_C \right) \sinh \left( y_q \Gamma t_C \right)
+ 2\text{Im} \left( \xi_C^{*} \zeta_C \right) \sin \left( x_q \Gamma t_C \right) + \mathcal{V}(f_1, t_1; f_2, t_2)_C,
$$

(9)

where $t_C = t_2 + Ct_1$ is defined, $\xi_C$ and $\zeta_C$ have been given in Eq. (6), and $\mathcal{V}(f_1, t_1; f_2, t_2)_C$ denotes the CPT-violating term:

$$
\mathcal{V}(f_1, t_1; f_2, t_2)_C = -2\text{Re} \left[ \left( \xi_+^{*} \zeta^- + \xi^- \zeta_+^{*} \right) \cos \theta \right] \cosh \left( y_q \Gamma t_+ \right)
-2\text{Re} \left[ \left( \xi_+^{*} \zeta^- - \xi^- \zeta_+^{*} \right) \cos \theta \right] \cos \left( x_q \Gamma t_- \right)
+ 2\text{Re} \left( \left| \xi_+ \right|^2 + \zeta^- \zeta_+^{*} \right) \cos \theta \sin \left( y_q \Gamma t_- \right)
- 2\text{Im} \left( \left| \xi_+ \right|^2 - \zeta^- \zeta_+^{*} \right) \cos \theta \sin \left( x_q \Gamma t_- \right)
+ \left| \xi_- \left( \xi_- + \zeta_- \right) \right| \cos \theta \left| e^{+y_q \Gamma t_2} \cos \left( x_q \Gamma t_2 - \Theta_+ \right) \right|
- \left| \xi_- \left( \xi_- - \zeta_- \right) \right| \cos \theta \left| e^{-y_q \Gamma t_1} \cos \left( x_q \Gamma t_1 + \Theta_- \right) \right|
- \left| \xi_- \left( \xi_- - \zeta_- \right) \right| \cos \theta \left| e^{+y_q \Gamma t_2} \cos \left( x_q \Gamma t_1 - \Theta_- \right) \right|
+ \left| \xi_- \left( \xi_- + \zeta_- \right) \right| \cos \theta \left| e^{-y_q \Gamma t_1} \cos \left( x_q \Gamma t_1 + \Theta_+ \right) \right|
$$

(10)

with

$$
\tan \Theta_\pm = \frac{\text{Im} \left[ \xi_- \left( \xi_- \pm \zeta_- \right) \cos \theta \right]}{\text{Re} \left[ \xi_- \left( \xi_- \pm \zeta_- \right) \cos \theta \right]},
$$

(11)
or

\[ V(f_1, t_1; f_2, t_2) = +2\text{Re} \left( \xi_- \zeta^* \cos \theta \right) \cosh (y_q \Gamma t_+) \]
\[ +2\text{Re} \left( \xi_- \zeta^* \cos \theta \right) \cos (x_q \Gamma t_+) \]
\[ -2\text{Re} \left( \xi_- \zeta^* \cos \theta \right) \sinh (y_q \Gamma t_+) \]
\[ +2\text{Im} \left( \xi_- \zeta^* \cos \theta \right) \sin (x_q \Gamma t_+) \]
\[ + \left( \xi_- \zeta^- (\xi_+ \zeta^-) \right) \cos \theta e^{y_q \Gamma t_1} \cos (x_q \Gamma t_2 + \Theta_{-+}) \]
\[ - \left( \xi_+ \zeta^- \right) \cos \theta e^{-y_q \Gamma t_1} \cos (x_q \Gamma t_2 - \Theta_{++}) \]
\[ + \left( \xi_- \zeta^- \right) \cos \theta e^{y_q \Gamma t_2} \cos (x_q \Gamma t_1 + \Theta_{--}) \]
\[ - \left( \xi_+ \zeta^- \right) \cos \theta e^{-y_q \Gamma t_2} \cos (x_q \Gamma t_1 - \Theta_{++}) \] (12)

with

\[ \tan \Theta_{\pm} = \frac{\text{Im} \left[ \left( \xi^* \pm \zeta^* \right) \left( \xi \pm \zeta \right) \cos \theta \right]}{\text{Re} \left[ \left( \xi^* \pm \zeta^* \right) \left( \xi \pm \zeta \right) \cos \theta \right]} . \] (13)

We observe that the CPT-violating term has a more complicated time-dependent behavior than the CPT-conserving term. For completeness, the time-integrated form of \( R(f_1, t_1; f_2, t_2)_C \) is given by

\[ R(f_1, f_2)_C \propto |A_{f_1}|^2 |A_{f_2}|^2 \left[ \frac{1 + Cy_q^2}{1 - y_q^2} \left( |\xi_C|^2 + |\zeta_C|^2 \right) - \frac{1 - Cx_q^2}{1 + x_q^2} \left( |\xi_C|^2 - |\zeta_C|^2 \right) \right] \]
\[ - \frac{2(1 + C)y_qx_q \text{Re}(\xi_C \zeta_C) + 2(1 + C)x_q^2 \text{Im}(\xi_C \zeta_C) + 2V(f_1, f_2)_C}{(1 + x_q^2)(1 - y_q^2)}, \] (14)

where the CPT-violating term reads

\[ V(f_1, f_2)_- = - \frac{1}{1 - y_q^2} \text{Re} \left[ (\xi_- \zeta^* + \xi^* \zeta_+) \cos \theta \right] \]
\[ - \frac{1}{1 + x_q^2} \text{Re} \left[ (\xi_- \zeta^* - \xi^* \zeta_+) \cos \theta \right] \]
\[ + \frac{2}{(1 + x_q^2)(1 - y_q^2)} \left[ \text{Re} \left( \xi_- \zeta^* \cos \theta \right) + x_q y_q \text{Im} \left( \xi_- \zeta^* \cos \theta \right) \right], \] (15)

or

\[ V(f_1, f_2)_+ = + \left[ \frac{1 + y_q^2}{(1 - y_q^2)^2} + \frac{1 - x_q^2}{(1 + x_q^2)^2} \right] \text{Re} \left( \xi_- \zeta^* \cos \theta \right) \]
\[ - \frac{2y_q}{(1 - y_q^2)^2} \frac{x_q^2 + y_q^2}{1 + x_q^2} \text{Re} \left( \xi_- \zeta^* \cos \theta \right) \]
\[ - \frac{2x_q}{(1 + x_q^2)^2} \frac{x_q^2 + y_q^2}{1 - y_q^2} \text{Im} \left( \xi_- \zeta^* \cos \theta \right) \]
When CPT is a good symmetry (cosθ = 0), Eqs. (9) and (14) are in agreement with the formulas obtained in Refs. [15, 28].

We stress that Eqs. (9)–(16) are new results and may serve as the master equations for the study of CPT violation in coherent \(B_q^0 \bar{B}_q^0\) decays. They are also valid for other correlated meson-antimeson systems with CPT violation, in particular useful to describe coherent \((D^0 \bar{D}^0)_C\) decays at the \(\psi(3770)\) and \(\psi(4140)\) resonances.

### 3 Example: opposite-sign dilepton events

To be specific, we consider a particularly simple and interesting possibility of testing CPT invariance in \(B_q^0 \bar{B}_q^0\) mixing: the opposite-sign dilepton events from coherent \((B_q^0 \bar{B}_q^0)_C\) decays. One may take \(f_1 = l_1^+ X_1^-\) and \(f_2 = l_2^+ X_2^+\) with either \(l_1 = l_2\) (e.g., \(l_1 = l_2 = \mu\)) or \(l_1 \neq l_2\) (e.g., \(l_1 = e\) and \(l_2 = \mu\)). The amplitude of each semileptonic decay mode can be parameterized in analogy with Eq. (5). Since the \(\Delta B = -\Delta Q\) transitions must be strongly suppressed, it is reasonable to take \(|\sigma_i| \ll 1\) (for \(i = 1\) or 2) in our calculations.

We first look at the \(C = -1\) case. By simplifying Eqs. (9), (10) and (11), we obtain the time-dependent rates of \((B_q^0 \bar{B}_q^0)_- \rightarrow (l_1^+ X_1^+, t_1) (l_2^+ X_2^+, t_2)\) decays as follows:

\[
R \left(l_1^+ X_1^-, t_1; l_2^+ X_2^+, t_2\right)_- \propto |A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma_{l_2} t_2} \left[\cosh(y_q \Gamma_{l_2}) + \cos(x_q \Gamma_{l_2})\right] + 2 \text{Re} \Omega \sinh(y_q \Gamma_{l_2}) + 2 \text{Im} \Omega \sin(x_q \Gamma_{l_2})
\]

\[
R \left(l_1^- X_1^+, t_1; l_2^- X_2^-, t_2\right)_- \propto |A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma_{l_2} t_2} \left[\cosh(y_q \Gamma_{l_2}) + \cos(x_q \Gamma_{l_2})\right] - 2 \text{Re} \Omega \sin(x_q \Gamma_{l_2}) - 2 \text{Im} \Omega \sin(x_q \Gamma_{l_2})
\]

where \(t_\pm = t_2 \pm t_1\), and

\[
\Omega = \cos \theta + \sigma_t e^{+i\phi} - \sigma_t^* e^{-i\phi}, \quad \overline{\Omega} = \cos \theta - \sigma_t^* e^{-i\phi} + \sigma_t e^{+i\phi}
\]

have been derived by keeping the leading terms of CPT violation and \(\Delta B = -\Delta Q\) effects.

Note that \(e^{i\phi} = (V_{tb}^* V_{ts})/(V_{tb} V_{ts}) \approx 1 + 2i\lambda^2 \eta\) holds for \(B_s^0 - \bar{B}_s^0\) mixing described by the box diagrams in the standard model, where \(\lambda \approx 0.22\) and \(\eta \approx 0.34\) are the Wolfenstein parameters [4]. If \(e^{\pm i\phi} \approx 1\) is taken in the leading-order approximation, Eq. (18) can be simplified to \(\Omega \approx \cos \theta + (\sigma_t - \sigma_t^*)\) and \(\overline{\Omega} \approx \cos \theta - (\sigma_t^* - \sigma_t)\). If \(l_1 = l_2\) is further taken, then we have \(\Omega \approx \overline{\Omega} \approx \cos \theta + 2i \text{Im}(\sigma_t)\). It is remarkable that the same simplification cannot be made for the \(B_d^0 - \bar{B}_d^0\) mixing system, where \(e^{i\phi} = (V_{tb}^* V_{td})/(V_{tb} V_{td}) \approx e^{-2i\beta}\) with \(\beta \approx 22^\circ\) being one of the inner angles of the Cabibbo-Kobayashi-Maskawa unitarity triangle in the standard phase convention [4]. Of course, \(e^{i\phi}\) might deviate from the standard-model
expectation if $B^0_q-\bar{B}^0_q$ mixing (for $q = d$ or $s$) involves a kind of new physics. The latter has also been included into the parameters $\Omega$ and $\overline{\Omega}$. Thus these two parameters serve for an effective description of possible new physics ($CPT$ violation, $\Delta B = -\Delta Q$ transitions and new $\Delta B = 2$ effects) in $B^0_q-\bar{B}^0_q$ mixing.

Eqs. (17) and (18) clearly show that the $\Delta B = -\Delta Q$ parameters have the same time-dependent behavior as the $CPT$-violating parameter $\cos \theta$ in the opposite-sign dilepton events of the correlated $B^0_q \bar{B}^0_q$ state with $C = -1$. Hence it is in general impossible to distinguish between the effect of $CPT$ violation and that of $\Delta B = -\Delta Q$ transitions in this kind of events, unless one of them is remarkably smaller than the other. If the decays of the correlated $B^0_q \bar{B}^0_q$ state with $C = +1$ are taken into account, however, it is in principle possible to cleanly extract the $CPT$-violating parameter [13]. To illustrate this point in a transparent way, we simplify Eqs. (9), (12) and (13) to obtain the time-dependent rates of $(B^0_q \bar{B}^0_q)_+ \to (l^+_1 X^+_1)_{t_1} (l^+_2 X^+_2)_{t_2}$ decays. The result is

$$R \left( l^+_1 X^-_1, t_1; l^+_2 X^+_2, t_2 \right)_+ \propto |A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t_+} \left[ \cos(y_q \Gamma t_+) + \cos(x_q \Gamma t_-) - 2Re\Omega' \sin(y_q \Gamma t_+) - 2i\Omega \sin(x_q \Gamma t_-) + 2\Delta(t_1, t_2) \right],$$

$$R \left( l^-_1 X^+_1, t_1; l^+_2 X^+_2, t_2 \right)_+ \propto |A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t_+} \left[ \cos(y_q \Gamma t_+) + \cos(x_q \Gamma t_-) - 2Re\overline{\Omega}' \sin(y_q \Gamma t_+) - 2i\overline{\Omega} \sin(x_q \Gamma t_-) - 2\Delta(t_1, t_2) \right].$$ (19)

where $\Omega' = \sigma_{l_1} e^{+i\phi} + \sigma_{l_2} e^{-i\phi}$ and $\overline{\Omega}' = \sigma_{l_1} e^{-i\phi} + \sigma_{l_2} e^{+i\phi}$ do not contain the $CPT$-violating effect, but $\Delta(t_1, t_2)$ is purely a $CPT$-violating term:

$$\Delta(t_1, t_2) = + \left[ \cos(x_q \Gamma t_1) \sinh(y_q \Gamma t_2) - \sin(y_q \Gamma t_1) \cos(x_q \Gamma t_2) \right] \text{Re}(\cos \theta)$$

$$- \left[ \sin(x_q \Gamma t_1) \cosh(y_q \Gamma t_2) - \sin(y_q \Gamma t_1) \sin(x_q \Gamma t_2) \right] \text{Im}(\cos \theta).$$ (20)

One can easily see that $\Delta(t_2, t_1) = -\Delta(t_1, t_2)$ holds, but the $CPT$-conserving terms in Eq. (19) do not change with the exchange of $t_1$ and $t_2$. This interesting feature implies that $\Delta(t_1, t_2)$ can in principle be extracted from the rate differences

$$R \left( l^+_1 X^-_1, t_1; l^+_2 X^+_2, t_2 \right)_+ - R \left( l^+_1 X^-_1, t_2; l^+_2 X^+_2, t_1 \right)_+ \propto 4|A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t_+} \Delta(t_1, t_2),$$

$$R \left( l^-_1 X^+_1, t_2; l^+_2 X^+_2, t_1 \right)_+ - R \left( l^-_1 X^+_1, t_1; l^+_2 X^+_2, t_2 \right)_+ \propto 4|A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t_+} \Delta(t_1, t_2).$$ (21)

As $\text{Im}\phi \approx 0$ is a good approximation for $B^0_q-\bar{B}^0_q$ mixing in the standard model, we have $\text{Re}\overline{\Omega}' \approx \text{Re}\Omega'$ and $\text{Im}\overline{\Omega}' \approx -\text{Im}\Omega'$. In this case, $\text{Im}\Omega'$ can in principle be extracted from the rate differences

$$R \left( l^+_1 X^-_1, t_1; l^+_2 X^-_2, t_2 \right)_+ - R \left( l^+_1 X^-_1, t_2; l^+_2 X^-_2, t_1 \right)_+ \propto 4|A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t_+} \text{Im}\Omega',$$

$$R \left( l^-_1 X^+_1, t_2; l^+_2 X^-_2, t_1 \right)_+ - R \left( l^-_1 X^+_1, t_1; l^+_2 X^-_2, t_2 \right)_+ \propto 4|A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t_+} \text{Im}\Omega'.$$ (22)
For $B_s^0$-$\bar{B}_s^0$ mixing with $e^{\pm i\phi} \approx 1$, $\text{Im} \Omega' \approx \text{Im} \sigma_1 - \text{Im} \sigma_2$ holds; and for $B_d^0$-$\bar{B}_d^0$ mixing with $e^{\pm i\phi} \approx e^{\pm 2i\beta}$, we obtain $\text{Im} \Omega' \approx (\text{Re} \sigma_2 - \text{Re} \sigma_1) \sin 2\beta - (\text{Im} \sigma_2 - \text{Im} \sigma_1) \cos 2\beta$. Thus the dilepton events of coherent $(B_q^0 \bar{B}_q^0)_C$ decays are very useful to probe possible $CPT$ violation and $\Delta B = -\Delta Q$ effects.

If the forthcoming super-$B$ factory is also an asymmetric $e^+e^-$ collider as the present KEK and SLAC $B$-meson factories, it will be easier to measure the proper-time difference $t_+ = (t_1 - t_2)$ of a dilepton event. A measurement of the $t_+$ distribution might be difficult in either linacs or storage rings, unless the bunch lengths are much shorter than the decay lengths [29]. Hence we may calculate the $t_+$ distribution of the dilepton events by integrating $R(l_1^\mp X_1^\mp, t_1; l_2^\mp X_2^\mp, t_2)_C$ over $t_+$. For simplicity, here we assume $\Delta B = \Delta Q$ to be a perfect rule and use $t$ to denote $t_+$. We take $t > 0$ by convention. Our results are

$$
R(l_1^\mp X_1^\mp, l_2^\mp X_2^\pm, t)_- \propto |A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t} \left[ \cosh(y_q \Gamma t) + \cos(x_q \Gamma t) ight] 
\pm 2\text{Re}(\cos \theta) \sinh(y_q \Gamma t) \pm 2\text{Im}(\cos \theta) \sin(x_q \Gamma t) ,
$$

and

$$
R(l_1^\mp X_1^\mp, l_2^\mp X_2^\pm, t)_+ \propto |A_{l_1}|^2 |A_{l_2}|^2 e^{-\Gamma t} \left[ \frac{\cosh(y_q \Gamma t + \varphi_y)}{\sqrt{1 - y_q^2}} + \frac{\cos(x_q \Gamma t + \varphi_x)}{\sqrt{1 + x_q^2}} ight] 
\pm \frac{2|\cos \theta|}{x_q^2 + (2 - y_q)^2} \left[ \cos(\Theta + \omega_-) e^{y_q \Gamma t} - \cos(\Theta + \omega_+ + x_q \Gamma t) \right] 
+ \frac{2|\cos \theta|}{x_q^2 + (2 + y_q)^2} \left[ \cos(\Theta - \omega_- e^{-y_q \Gamma t} - \cos(\Theta - \omega_+ - x_q \Gamma t) \right] ,
$$

where the parameters $\varphi_x$, $\varphi_y$, $\omega_\pm$ and $\Theta$ are defined through $\tan \varphi_x = x_q$, $\tan \varphi_y = y_q$, $\tan \omega_\pm = x_q/(2 \pm y_q)$ and $\tan \Theta = \text{Im}(\cos \theta)/\text{Re}(\cos \theta)$, respectively. Taking $x_q \sim 27$ and $y_q \sim 0.07$ for example, we have $\varphi_x \sim 1.53$, $\varphi_y \sim 0.07$, $\omega_+ \sim 1.49$ and $\omega_- \sim 1.50$; while taking $x_d \sim 0.78$ and $y_d \sim 0.002$ for example, we have $\varphi_x \sim 0.66$, $\varphi_y \sim 0.002$ and $\omega_+ \sim \omega_- \sim 0.37$. Eqs. (23) and (24) show that both $\text{Re}(\cos \theta)$ and $\text{Im}(\cos \theta)$ can in principle be determined or constrained by measuring the decay rates $R(l_1^\mp X_1^\mp, t_1; l_2^\mp X_2^\pm, t_2)_C$, provided the $\Delta B = -\Delta Q$ transitions and other new-physics effects are negligibly small. These formulas are also applicable for the $D^0$-$\bar{D}^0$ mixing system.

**4 Summary**

Keeping with the great experimental interest in testing discrete symmetries and conservation laws at the present and future $B$-meson factories, we have reformulated the time-dependent and time-integrated rates of coherent $(B_q^0 \bar{B}_q^0)_C$ decays ($q = d$ or $s$) by assuming small $CPT$ violation in $B_q^0$-$\bar{B}_q^0$ oscillation. Our results are new and generic,
and thus they can serve as the master formulas for the analysis of possible $CPT$-violating effects in both the $B^0 \bar{B}^0$ mixing system at the $\Upsilon(4S)$ resonance and the $B^0_s \bar{B}^0_s$ mixing system at the $\Upsilon(5S)$ resonance. Taking the opposite-sign dilepton events for example, we have shown that it is possible to separately determine or constrain the parameters of $CPT$ violation and $\Delta B = - \Delta Q$ transitions by measuring their time distributions in the $C = +1$ case. In the $C = -1$ case, however, the $CPT$-violating and $\Delta B = - \Delta Q$ effects have the same time-dependent behavior and are in general indistinguishable.

We expect that a stringent test of $CPT$ symmetry and the $\Delta B = \Delta Q$ rule will finally be realized at a super-$B$ factory with the luminosity $\mathcal{L} \sim a \times 10^{36} \text{cm}^{-2}\text{s}^{-1}$, where other kinds of new physics may also be explored. The prospect of such ambitious experiments is by no means dim, indeed.

Finally let us mention the evidence for $D^0 \bar{D}^0$ mixing achieved in the BaBar [30] and Belle [31] experiments. It turns out that the mixing parameter $y_D$ is at the one percent level and $|x_D| < |y_D|$ is expected to hold [32]. Therefore it seems possible to test $CP$, $T$ and $CPT$ symmetries in the charm system in the (far) future. As we have emphasized before, the master formulas obtained in this paper can all be used to describe coherent $D^0 \bar{D}^0$ decays with small $CPT$ violation and (or) small $\Delta C = - \Delta Q$ effects.

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References

[1] J.H. Christenson et al., Phys. Rev. Lett. 13, 138 (1964).

[2] NA31 Collaboration, H. Burkhardt et al., Phys. Lett. B 206, 169 (1988); NA48 Collaboration, V. Fanti et al., Phys. Lett. B 465, 335 (1999); KTeV Collaboration, A. Alavi-Harati et al., Phys. Rev. Lett. 83, 22 (1999).

[3] CPLEAR Collaboration, A. Angelopoulos et al., Phys. Lett. B 444, 43 (1998).

[4] Particle Data Group, W.M. Yao et al., J. Phys. G 33, 1 (2006).

[5] See, e.g., R. Covarelli (BaBar Collaboration), hep-ex/0702040; C. Leonidopoulos (Belle Collaboration), hep-ex/0107001.

[6] T. Nakada, plenary talk given at the 4th International Workshop on the CKM Unitarity Triangle, December 2006, Nagoya, Japan.

[7] Y.F. Wang et al., in the Proceedings of the International Workshop on Tau-Charm Physics, Beijing, 2006, edited by Y.F. Wang and H.B. Li, Int. J. Mod. Phys. A 21, 5371 (2006); D. Asner, plenary talk given at the 4th International Workshop on the CKM Unitarity Triangle, December 2006, Nagoya, Japan.

[8] M. Yamauchi, plenary talk given at the 4th International Workshop on the CKM Unitarity Triangle, December 2006, Nagoya, Japan.

[9] CDF Collaboration, A. Abulencia et al., Phys. Rev. Lett. 97, 242003 (2006).

[10] D0 Collaboration, V.M. Abazov et al., Phys. Rev. Lett. 98, 121801 (2007); Phys. Rev. D 76, 057101 (2007).

[11] M. Kobayashi and A.I. Sanda, Phys. Rev. Lett. 69, 3139 (1992); Z.Z. Xing, Phys. Rev. D 50, 2957 (1994); D. Colladay and V.A. Kostelecký, Phys. Lett. B 344, 259 (1995); V.A. Kostelecký and R. Van Kooten, Phys. Rev. D 54, 5585 (1996); P. Colangelo and G. Corcella, Eur. Phys. J. C 1, 515 (1998); A. Mohapatra, M. Satpathy, K. Abe, and Y. Sakai, Phys. Rev. D 58, 036003 (1998); D. Du and Z.T. Wei, Eur. Phys. J. C 14, 479 (2000).

[12] Z.Z. Xing, Phys. Lett. B 450, 202 (1999).

[13] G.V. Dass, W. Grimus, and L. Lavoura, JHEP 0102, 044 (2001); G.V. Dass and W. Grimus, Eur. Phys. J. C 26, 201 (2002).

[14] D. Colladay and V.A. Kostelecký, Phys. Rev. D 52, 6224 (1995).
[15] Z.Z. Xing, Phys. Rev. D 55, 196 (1997).

[16] G.V. Dass and K.V.L. Sarma, Phys. Rev. Lett. 72, 191 (1994); erratum ibid. 72, 1573 (1994); Z.Z. Xing, in Ref. [12]; G.V. Dass and W. Grimus, in Ref. [13].

[17] Some comments and discussions have been made by D. Colladay and V.A. Kostelecký in Ref. [14].

[18] Z.Z. Xing, Eur. Phys. J. C 4, 283 (1998); Phys. Lett. B 443, 365 (1998).

[19] L. Randall and S.F. Su, Nucl. Phys. B 540, 37 (1999).

[20] A.F. Falk and A.A. Petrov, Phys. Rev. Lett. 85, 252 (2000)

[21] Z.Z. Xing, Phys. Lett. B 488, 162 (2000).

[22] Belle Collaboration, A. Drutskoy et al., Phys. Rev. Lett. 98, 052001 (2007).

[23] T.D. Lee and C.S. Wu, Annu. Rev. Nucl. Sci. 16, 471 (1966).

[24] O.W. Greenberg, Phys. Rev. Lett. 89, 231602 (2002).

[25] V.A. Kostelecký, Phys. Rev. Lett. 80, 1818 (1998).

[26] V.A. Kostelecký, Phys. Rev. D 64, 076001 (2001).

[27] A. Lenz and U. Nierste, JHEP 0706, 072 (2007); C. Tarantino, hep-ph/0702235; and references therein.

[28] Z.Z. Xing, Phys. Rev. D 53, 204 (1996).

[29] See, e.g., G.J. Feldman et al., in the Proceedings of High Energy Physics in the 1990s, Snowmass, Colorado, 1988, edited by S. Jensen (World Scientific, Singapore, 1988), p. 561; K. Berkelman et al., Report No. CLNS 91-1050, 1991.

[30] BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 98, 211802 (2007).

[31] M. Staric (Belle Collaboration), talk given at the XLII Recontres de Moriond, March 2007, La Thuille, Italy.

[32] Y. Nir, JHEP 0705, 102 (2007); and references therein.