The emergence of symmetries in complex nuclei

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Abstract. After a brief review of early developments, two recent examples of applications of symmetries in nuclear physics are presented: (i) pseudo-spin symmetry which can be used to extend Wigner’s supermultiplet model to heavy nuclei; (ii) symmetries of the interacting boson model with isoscalar and isovector bosons which can be used to analyze the problem of deuteron transfer in $N = Z$ nuclei.

1. Introduction
Symmetry considerations have played an important role in nuclear physics, starting from the birth of the discipline, and have continued to do so throughout its development. Particularly noteworthy applications of symmetry ideas are Heisenberg’s isospin SU(2) symmetry [1], Wigner’s supermultiplet model of spin–isospin SU(4) symmetry [2] and Racah’s pairing model [3] based on SU(2) quasi-spin symmetry [4]. All these symmetries can be understood as a result of an invariance property of the hamiltonian of the spherical shell model. A crucial ingredient for the emergence of more complex symmetry classifications (of the spherical shell model) is that of configuration mixing proposed in the 1950s by Arima and Horie [5]. Building on the idea of configuration mixing, It was shown subsequently by Elliott [6] that rotational features of atomic nuclei can be understood on the basis of an underlying SU(3) symmetry generated by the nuclear quadrupole interaction. Another important landmark in the application of symmetries to nuclei was the interacting boson model proposed by Arima and Iachello [7] as an alternative to the shell model to understand and interpret a wide variety of collective structures observed in nuclei.

Many of the basic features of the structure of nuclei can be understood from a few essential characteristics of the nuclear mean field and its residual interactions. One important aspect is the attractive, short-range nature of the residual interaction. In the extreme short-range limit of a delta interaction $\delta(r_1 - r_2)$ without spin dependence, the many-body nuclear wave function conserves total orbital angular momentum $L$ and total spin $S$, besides total angular momentum $J$ associated with rotational invariance. This classification (LS or Russell–Saunders coupling) is broken by the spin–orbit term in the nuclear mean field. The conflicting tendency between the short-range character of the residual interaction, which favours LS coupling, and the spin–orbit term in the average potential, which leads to a $jj$-coupled classification, is a crucial element in the structural determination of the nucleus. This conflict was recognized and studied in the early days of the nuclear shell model [8]. The generally accepted conclusion is that, while the LS classification is appropriate for very light nuclei, with increasing mass it is gradually replaced by $jj$ coupling which is relevant for the vast majority of nuclei [9].
The second important feature that determines the structure of the nucleus is the number of neutrons and protons in the valence shell. The residual interaction between identical nucleons has a pairing character which favours the formation of pairs of nucleons in time-reversed orbits. This is no longer true when the valence shell contains both neutrons and protons, in which case the interaction acquires an important quadrupole component. Hence, nuclei display a wide variety of spectra, from pairing-type towards rotational-like.

The strong spin–orbit coupling in nuclei has brought about the demise of many a nuclear model and the supermultiplet scheme is but one example of it. In the first part of this contribution a modification is proposed of Wigner’s SU(4) model based on pseudo-spin symmetry which extends the validity of this scheme to heavier nuclei. The spin–orbit coupling is also of central importance to the topic of $T = 0$ and $T = 1$ deuteron transfer between self-conjugate $N = Z$ nuclei which is discussed in the second part of this contribution. This is done in the context of a simplified boson model which considers bosons without orbital angular momentum but with full isospin–spin structure. These transfer predictions can be correlated with nuclear binding energies in specific regions of the mass table.

2. SU(4) and pseudo-SU(4) symmetry

Wigner’s isospin–spin or SU(4) symmetry [2] involves an extension from the isospin SU(2) algebra to the larger SU(4) algebra that includes spin. A nuclear hamiltonian with SU(4) symmetry satisfies

$$[H, \sum_i \tau_i] = [H, \sum_i \sigma_i] = [H, \sum_i \tau_i \sigma_i] = 0,$$

where $\tau_i$ and $\sigma_i$ are Pauli matrices for the $i^{th}$ particle in the nucleus acting in isospin and spin space, respectively. The breaking of SU(4) is caused by several effects but mainly by the spin–orbit term in the nuclear mean field [10]. The breaking is illustrated by the data on summed Gamow–Teller strength from $N = Z - 2$ into $N = Z$ nuclei. In the SU(4) model this strength is predicted to be zero for $A = 4n$ and large for $A = 4n + 2$. This pattern is indeed observed in the very light nuclei but it gradually disappears with increasing mass [11]. A similar conclusion regarding the disappearance of SU(4) symmetry in heavy nuclei is reached from nuclear binding energies [12].

Nuclei beyond $^{56}$Ni, although strongly admixed in SU(4), are more amenable to a classification in terms of pseudo-SU(4). This symmetry arises by treating the $p_{3/2}$, $p_{3/2}$ and $f_{5/2}$ orbits, which are the dominant ones above $N, Z = 28$, as a pseudo-sd shell. This scheme is based on the idea of pseudo-spin symmetry suggested by Arima et al. [13] and Hecht and Adler [14], and which only recently was given an explanation in the context of relativistic mean field theory [15]. A nuclear hamiltonian with pseudo-SU(4) symmetry satisfies

$$[H, \sum_i \tilde{\tau}_i] = [H, \sum_i \tilde{\sigma}_i] = [H, \sum_i \tilde{\tau}_i \tilde{\sigma}_i] = 0,$$

where $\tilde{\tau}_i$ are Pauli matrices for the $i^{th}$ particle in pseudo-spin space. Since the pseudo-spin–orbit splitting (between $p_{3/2}$ and $f_{5/2}$) is substantially smaller than the normal spin–orbit splitting (between $p_{1/2}$ and $p_{3/2}$, or between $f_{5/2}$ and $f_{7/2}$) the violation of the second commutator in (2) is correspondingly smaller.

The hamiltonian in (2) has eigenstates labelled by $|n(\tilde{\lambda}\tilde{\mu}\tilde{\nu})\tilde{L}\tilde{S}JT\rangle$, where $n$ is the number of nucleons in the valence shell which is taken to be the entire pseudo-oscillator shell. The total pseudo-orbital angular momentum $\tilde{L}$ and the total pseudo-spin $\tilde{S}$, which result from the separate coupling of all individual pseudo-orbital angular momenta $\tilde{L}_i$ and pseudo-spins $\tilde{s}_i$ [13], are conserved. The labels $(\tilde{\lambda}\tilde{\mu}\tilde{\nu})$ are associated with pseudo-SU(4) in direct analogy with Wigner’s supermultiplet scheme.
The validity of the hypothesis (2) has been tested with several shell-model interactions for nuclei at the beginning of the 28–50 shell. The conclusion of this analysis [17] is that, at low energies, the pseudo-SU(4) classification holds to within a 10–20\% level of approximation.

The predictions of pseudo-SU(4) symmetry concerning Gamow–Teller $\beta$ decay should now be contrasted with those of SU(4). A typical example of the latter, the $\beta$ decay of $^{18}$Ne, is shown in Fig. 1: the Gamow–Teller decay proceeds to two $1^+$ states in $^{16}$O with log $ft$ values of 3.1 and 4.5, respectively. In SU(4) these correspond to transitions within the supermultiplet (010); the first occurs without change of orbital structure ($^1S_0 \rightarrow S_1^+$) and is fast, while the second ($^1S_0 \rightarrow D_1$) is forbidden. The SU(4) classification thus provides a qualitative understanding of Gamow–Teller $\beta$ decay in $^{18}$Ne.

The analysis of the analogous problem in pseudo-SU(4) is more complicated because the Gamow–Teller operator is not a generator of pseudo-SU(4). Details are given in [17]. It is seen that the transition without change in orbital structure ($^1S_0 \rightarrow S_1^+$) is about one order of magnitude weaker than the corresponding one in SU(4) (4.1 versus 3.1). Furthermore, the second transition ($^1S_0 \rightarrow D_1$) no longer is forbidden. Its matrix element depends upon the amount and character of orbital mixing; the numbers shown in the figure (3.3 $\sim$ 3.8) represent a typical range between prolate and oblate deformation. It is also seen that the observed $ft$ values in the decay of $^{58}$Zn [18] strongly differ from those for $^{18}$Ne, and are more in line with the pseudo-SU(4) prediction.

3. Deuteron transfer in $N = Z$ nuclei

While effects of pairing are well understood if this interaction acts among identical nucleons, they are less so in the presence of neutrons and protons. Neutron–proton pairing can be either isoscalar $T = 0$ (spin-triplet) or isovector $T = 1$ (spin-singlet) and this leads to a generalized structure of the condensate, typical of superfluidity in a two-component system. The most obvious experimental signature for the existence and character of this generalized condensate is an enhancement in the probability for the transfer of a pair (either $T = 0$ or $T = 1$) which can be obtained by contrasting the reaction ($d,^4$He) where only $T = 0$ deuteron transfer is possible with reactions like ($p,^3$He) which allow both $T = 0$ as well as $T = 1$ deuteron(-like) transfer. The relevant experiments will require the new generation of radioactive beam facilities.

The interacting boson model (IBM) [19] provides a description of nuclei in terms of correlated pairs of nucleons which are treated as bosons. As such, it offers a natural framework to discuss...
the issue of two-nucleon transfer. Two-neutron and two-proton transfer have been analyzed in the early days of the model (see, e.g., Refs. [20, 21]) using the neutron–proton (np) version of the model, IBM-2 [22], which includes neutron–neutron (nn) and proton–proton (pp) bosons. A description of deuteron transfer requires a more complicated version of IBM which involves bosons corresponding to np pairs. Such extensions have been considered in the past and of particular relevance is the so-called IBM-4 [23] since it contains np pairs with isospin $T = 0$ and $T = 1$. The full IBM-4 is a rich spectroscopic model [24] with bosons with orbital angular momentum $L = 0$ (s boson) or $L = 2$ (d boson), with intrinsic spin $S = 0$ or $S = 1$, and with isospin $T = 0$ (if $S = 1$) or $T = 1$ (if $S = 0$). This particular choice of bosons is justified on the basis of the nuclear shell model [25].

To avoid the complexity of the full IBM-4, it is instructive to confine the analysis to $L = 0$ bosons. This simplification preserves the complete isospin–spin structure of the model—crucial for the study of deuteron-transfer properties—and can be put to use in the analysis of the competition between isoscalar and isovector pairing in self-conjugate nuclei [26]. The dynamical algebra of the $L = 0$ IBM-4 is U(6), obtained from two vector bosons. One is vector in isospin while scalar in spin and the other boson is vector in spin while scalar in isospin. Based on a comparison with the full IBM-4 and its interpretation in terms of the shell model, one can justify the use of a simplified IBM-4 in all $N = Z$ nuclei and also in even–even $N \neq Z$ but not in odd–odd $N \neq Z$ nuclei. In the latter case the favoured U(6) representation of the full IBM-4 is non-symmetric [23] and is not contained in the simplified $L = 0$ IBM-4.

Two different symmetry classifications occur in the $L = 0$ IBM-4:

$$U(6) \supset \left\{ \begin{array}{c} SU(4) \\ U_T(3) \otimes U_S(3) \end{array} \right\} \supset SO_T(3) \otimes SO_S(3).$$

(3)

The total number of bosons $[N_b]$ labels U(6) while $SO_T(3)$ and $SO_S(3)$ are associated with the total isospin $T$ and the total spin $S$ of the bosons. Mathematical details on the two limits in (3) can be found in Ref. [27] where also the correspondence is studied between the U(6) model and its fermionic analogue, the SO(8) model of $T = 0$ and $T = 1$ pairing with neutrons and protons [28]. A simple Hamiltonian that describes the transition from one limit of (3) to the other is of the form

$$H = a C_2[SU(4)] + b C_1[U_S(3)],$$

(4)

where $C_n[G]$ denotes a linear or quadratic ($n = 1, 2$) Casimir operator of the algebra $G$. The first term in (4) is associated with SU(4) and implies equal single-boson energies and boson–boson interactions in the two isospin–spin channels $(T, S) = (0, 1)$ and $(1, 0)$. The physical origin of SU(4) symmetry is the attractive short-range interaction between nucleons which favours spatially symmetric states. The operator $C_1[U_S(3)]$ arises from a mapping of the spin–orbit term $v_{so} \sum_i l_i \cdot s_i$ of the shell model into the boson space. The transition from SU(4) to $U_T(3) \otimes U_S(3)$ is governed by the single parameter $b/a$ and intermediate results can be obtained after diagonalization [26].

Deuteron transfer is described in this model by the operators $b_{T0}^b$ ($b_{T0}^a$) for $T = 0$ ($T = 1$) transfer, which create bosons with $T = 0$ and $S = 1$ ($T = 1$ and $S = 0$), both with orbital angular momentum $L = 0$. It can be shown that the matrix elements of $b_{TS}^\dagger$ enter into the expression of the amplitude for two-nucleon transfer [29]. In the derivation of this dependence care must be taken of different reaction as well as structure aspects such as the application of the Talmi–Moshinsky transformation or the inclusion of Pauli factors. The conclusion is that the transfer intensity between the states $\langle N_b | \phi_A T_A S_A \rangle$ and $\langle N_b + 1 | \phi_B T_B S_B \rangle$ (where $\phi_A$ and $\phi_B$ are additional labels of initial and final states in the reaction $A + a \rightarrow B + b$) is proportional to the square of the matrix element of $b_{TS}^\dagger$

$$C_T^2 \equiv \langle N_b + 1 | \phi_B T_B S_B | b_{TS}^\dagger | N_b | \phi_A T_A S_A \rangle^2,$$

(5)
Figure 2. The $T=0$ and $T=1$ deuteron-transfer intensities $C^2_T$ between $N=Z$ nuclei with boson numbers $N_b$ and $N_b+1$ as a function of $b/a$ between the lowest $T=0$ and $T=1$ eigenstates of the hamiltonian (4). The even–even to odd–odd and odd–odd to even–even intensities are displayed for specific values of $N_b$. In odd–odd nuclei the $T=0$ and $T=1$ states are close in energy and the figure shows the isospin-allowed intensity to or from both, i.e., to/from $T=0$ for $C^2_0$ and to/from $T=1$ for $C^2_1$. The dashed line indicates the value of $b/a$ obtained from nuclear masses in the first half of the 28–50 shell.

where, for convenience, the matrix element is reduced in isospin and spin.

The properties of the hamiltonian (4) have been studied previously [26]. The ground state of even–even nuclei ($N_b$ even) has $T=0$ while the ground state of an odd–odd nucleus has $T=0$ for $b/a < 0$ and $T=1$ for $b/a > 0; b/a = 0$ yields degeneracy between the lowest $T=0$ and $T=1$ states and corresponds to the SU(4) limit. Thus by varying the ratio $b/a$ one can study the qualitative features of deuteron transfer with changing $T=0$ versus $T=1$ pairing correlations. The result for $N=Z$ nuclei is shown in Fig. 2.

Deuteron transfer at the $N=Z$ line has unique properties since, starting from an even–even $N=Z$ nucleus with $T=0$ ground state, one finds two states excited in the low-energy region of the odd–odd nucleus corresponding to $T=0$ and $T=1$ transfer, respectively. Not surprisingly, the two states are equally excited in the SU(4) limit ($b/a = 0$) whilst otherwise the sign of $b/a$ determines which of the two transfer intensities is strongest. It is evident from Fig. 2 that the transfer intensities change rapidly around the SU(4) limit but saturate quickly at large values of $|b/a|$. Since the $L=0$ IBM-4 can be used to calculate binding energies of $N=Z$ nuclei [30], $b/a$ can be obtained from nuclear masses and an estimate of $b/a \approx 5$ is found for the first half of the 28–50 shell. Even if there is considerable uncertainty in the value of this ratio, the fact that the deuteron-transfer intensities quickly saturate for large $b/a$ leads to a clear prediction of this analysis: the favoured deuteron-transfer mode in this mass region has $T=1$ rather than $T=0$ character. Some appreciable strength of the latter can only be expected in the transfer from an even–even to an (excited) $T=0$ state of an odd–odd nucleus.

In considering future experiments to determine the relative contributions of the different pairing modes on the $N=Z$ line, it is important to recall that the $N=Z$ nuclei are located increasingly far from stability as mass increases so that the study of deuteron transfer mandates the use of radioactive beams and inverse kinematics, either at classical transfer energies or through knockout reactions at the higher energies available at fragmentation facilities. The possibility to extract two-particle spectroscopic factors from the latter method has recently been demonstrated for the first time for two-proton transfer [31, 32]. The extraction of spectroscopic factors describing deuteron transfer poses particular additional problems in either approach and will require an enhancement in both the beam intensity and experimental sensitivity currently available in this type of study to achieve meaningful results. Nevertheless, the new generation of exotic beam accelerators currently proposed or under construction promises just such a degree of enhancement and the study presented here provide a first prediction of what may be observed in this new class of experiments.
4. Concluding remark
The common thread of the material presented in this contribution is the spin–orbit coupling in
the nuclear mean field and its consequences for SU(4) supermultiplet symmetry. Two symmetry-
based approaches were advocated to deal with its effect. The first is entirely phrased in shell-
model language and involves an extension of Wigner’s idea based on pseudo-spin symmetry.
A shell-model analysis confirms its approximate validity but only in very limited regions of
the nuclear chart. To carry the analysis further a mapping to a boson hamiltonian can be
proposed in which case the spin–orbit coupling again appears as a SU(4) symmetry-breaking
term. The resulting schematic boson hamiltonian can be used to make simple predictions
concerning deuteron transfer between $N = Z$ nuclei.

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