The greedy randomized adaptive search procedure method in formulating set covering model on cutting stock problem

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Abstract. This study aims to apply the Greedy Randomized Adaptive Search Procedure method in formulating the Set Covering model. Selected cutting patterns in the Cutting Stock Problem are generated by the Greedy Randomized Adaptive Search Procedure method and then formulating them to the Set Covering model. This study used a single stock with four types of items. The Greedy Randomized Adaptive Search Procedure method can show the maximum number of cutting patterns with minimum trim loss. Based on data analysis, it can be concluded that the Greedy Randomized Adaptive Search Procedure method will give different patterns depends on the number of the upper limits of demand. The Set Covering model, which was solved by LINGO 13.0, showed the optimal cutting patterns with minimum trim loss, but still has the lack of the product in one of the items.

1. Introduction
Cutting Stock Problem (CSP) is a problem to determine cutting patterns from a stock size to the demand of item size, in order to minimize the stock. The cutting process generally results in the remainder of the cut, which is called trim loss. Trim loss can’t be used anymore to fulfill another demand. The proper and optimal cutting pattern must be generated to overcome this problem. Many types of research were conducted to create the optimal cutting pattern, such as pattern generation algorithm \cite{1}\cite{2}\cite{3} and Modified Branch and Bound Algorithm \cite{4}\cite{5}\cite{6}. The search for cutting patterns required a lot of time and a high level of accuracy, so it took an application using the Modified Branch and Bound Algorithm in two-dimensional CSP \cite{2}\cite{3}\cite{6}\cite{7}.

Also, to minimize the trim loss, CSP aims to maximize the number of cutting on items to be produced. The method usually used to solve this CSP is the heuristic and meta-heuristic methods. The most common is the heuristic method \cite{8}, but this method often shows improper solutions so that the meta-heuristic method is used. The solution of the meta-heuristic method is not the optimal local solution as in the heuristic method \cite{9}\cite{10}\cite{11}. One of the meta-heuristic methods used in CSP resolution is the Greedy Randomized Adaptive Search Procedure (GRASP) method. The GRASP method is one of the meta-heuristic using two stages in the process of completion, namely the construction stage and the stage of finding local solutions.

The GRASP method in 2D-CSP is used to produce a maximum number of items \cite{12} and can solve CSP by providing optimum solutions in a timely manner \cite{13}. Resulting cutting patterns were formed into a model to obtain the optimal trim loss. The Set Covering model can solve problems involving
multiple rows and columns. The Set Covering model was one of the combinatorial optimization problems and can solve various difficult problems [14][15]. The heuristic method for completing the Set Covering model was also developed [16]. There was no appropriate algorithm in the literature for the Set Covering model rather than using a suitable programming tool.

Therefore, this research intends to implement the GRASP method in CSP to generate the cutting pattern and formulate it to the Set Covering model, which was completed by the LINGO 13.0 program.

2. Methods
The first step in this study is describing the data needed in forming cutting patterns, which include stock size (length and width), item size, and the number of demand for each item. The data used in this study was secondary, whereas the stock size is 300 cm × 150 cm which consists of 4 types of items. The size of each type of thing and the number of demand can be seen in table 1.

Table 1. Size of items and number of demands.

| No | Size of items (cm²) | Number of Demands (pieces) |
|----|---------------------|----------------------------|
| 1  | 50 × 40             | 8                          |
| 2  | 40 × 30             | 11                         |
| 3  | 30 × 20             | 14                         |
| 4  | 30 × 15             | 20                         |

The second step is processing data using the GRASP method to determine cutting patterns with minimum trim loss. The next step is forming a picture with a proper pattern so that cut loss can be obtained. Step four is formulating the Set Covering model: define the variables, determine the function that produces the minimum amount of stock to fulfill the demand for each item, and determine constraints by ensuring that all requests are fulfilled. The next step is solving the Set Covering model using the LINGO 13.0 application, and the last is analyzing the final results.

3. Result and discussion

3.1. Implementation of the greedy randomized adaptive search procedure method
The GRASP method is one of the iterative meta-heuristic methods, where each iteration consists of two stages, namely the construction phase and the stage of finding local solutions. The construction phase builds feasible solutions using the randomized GRASP method. Whereas the local solution searches phase performs iterative repetitions by replacing the current solution with a better one, and iteration ends if no better solution is found.

The variables to implement the GRASP method in this study are defined as follows:

$L$ is the standard stock length,
$W$ is the standard stock width,
$P_i$ is the lower limit for the number of deductions,
$Q_i$ is the upper limit of the number of deductions $i$-th item,
$e_i$ is the $i$-item efficiency value,
$L$ is the stock to be deducted based on a collection of items $i$,
$L^*$ is the size of stock that can be cut by item $I$,
$P$ is a collection of items $i$ to cut, and
$C$ is the list of item $i$ that will be cut

Stock sizes $(L, W)$ are cut according to item of sizes $(l_i,w_i)$, $i = 1, 2, \ldots, n$. Cutting on each item is oriented, i.e., cutting between length and width cannot be reversed. The deduction amount of each item must meet the limits of $P_i$ and $Q_i$ with $0 \leq P_i \leq Q_i$, $P_i$ and $Q_i$ respectively being the lower and upper limits for each item $i$.

Based on the value of $P_i$ and $Q_i$, there are 3 types of problems that can be distinguished as follows:
1. Unconstrained
\[ \forall i, \ P_i = 0 \text{ and } Q_i = \frac{LW}{l_i w_i} \] (1)

2. Constrained
\[ \forall i, \ P_i = 0 \text{ and } \exists i, \ Q_i = \frac{LW}{l_i w_i} \] (2)

3. Doubly constrained
\[ \exists i, \ P_i > 0 \text{ and } \exists j, \ Q_j < \frac{LW}{l_j w_j}, j \in \mathbb{Z}^+, j = 1, 2, \ldots, m \] (3)

Based on the efficiency value of each item \( i (e_i) \), there are two types of problems that can be distinguished as follows: 1) Unweighted \( \forall i, e_i = 1 \). The size of each item \( i \) is the same as the size in stock and 2) Weighted \( \forall i, e_i \neq 1 \). Some item \( i \) sizes are not the same size as stock. The efficiency value of each item can be found by equation (4)
\[ e_i = \frac{v_i}{l_i w_i} \] (4)

In general, the steps of GRASP method are as follows:

1. Step 0 (Initialization)
   \( \mathcal{L} \) as stock to be deducted based on a collection of items \( i, \mathcal{P} = \{p_1, p_2, \ldots, p_n\} \) list item \( i \) still to be cut. \( \mathcal{C} = \emptyset \), list of item \( i \) cut.

2. Step 1 (Selecting a rectangle on \( \mathcal{L} \))
   Take \( \mathcal{L}^* \), which is the smallest rectangle on \( \mathcal{L} \) which can load items on \( \mathcal{P} \). If \( \mathcal{L}^* \) does not exist, then stop. Otherwise, proceed to Step 2.

3. Sep 2 (Selecting item \( i \) to cut)
   Selecting item \( p_i \) with \( n_i \leq Q_i \), constraint, the selected \( p_i \) items are then formed into the block \( b^* \) to be cut in \( \mathcal{L}^* \). Select a position on \( \mathcal{L}^* \) to cut \( b^* \). Usually the block \( b^* \) to be cut doesn’t completely fill \( \mathcal{L}^* \), so the cut is cut at a close angle. Update \( \mathcal{P}, \mathcal{C}, \) and \( Q_i \) values. Update \( \mathcal{C} \) with type \( i \) and the number \( n_i \) of items cut. Set \( Q_i = Q_i - n_i \) if \( Q_i = 0 \) then remove item \( i \) from \( \mathcal{P} \). Move the block \( b^* \) towards the nearest corner of the stock.

4. Step 3 (Updating \( \mathcal{L} \))
   Combine unused stock to cut new items from \( \mathcal{P} \). So update the new \( \mathcal{L} \), then proceed to Step 1.

Cutting \( \mathcal{P} \) on items that are cut can be divided into three criteria, as follows: 1) Cut \( P_i l_i w_i \), prioritizing pieces that must be cut; 2) Cut \( e_i, \forall i, P_i = 0 \), and 3) Cut \( l_i w_i, \forall i, e_i = 1 \).

Based on the value of \( P_i \) and \( Q_i \), the type of problems in this study are constrained, \( \forall i, P_i = 0 \), and \( \exists i, Q_i < \frac{LW}{l_i w_i} \). The type of problem in this study has a value of \( P_i = 0 \), so the deduction made in this study is based on the deduction of \( e_i \). The \( e_i \) values for each item are sorted from the largest, so cutting items that take precedence are items that have the largest \( e_i \) values. Based on equation (4), the efficiency values for each item \( i (e_i) \) can be calculated as follows:
Furthermore, the data used for the GRASP method can be seen in table 2.

Table 2. The data used for the GRASP method.

| Item | $l_i$ | $w_i$ | $P_i$ | $Q_i$ | $v_i$ | $e_i$ |
|------|------|------|------|------|------|------|
| 1    | 50   | 40   | 0    | 8    | 21   | 0.0105 |
| 2    | 40   | 30   | 0    | 11   | 37   | 0.0308 |
| 3    | 30   | 20   | 0    | 14   | 75   | 0.125  |
| 4    | 30   | 15   | 0    | 20   | 100  | 0.2222 |

Table 2 indicates that the first item has a length $(l_1) = 50$ cm, a width $(w_1) = 40$ cm, the lower limit of the number of deductions $(P_1) = 0$, the upper limit of the number of deductions $(Q_1) = 8$, the number of demand $(v_1) = 21$, and the value of efficiency $(e_1) = 0.0105$ and so on for other items. By using the data in table 2 and implementing the GRASP method, the steps are as follows:

1. The first cutting

   Initialize $\mathcal{L} = \{0,0,300,150\}$, $\mathcal{P} = \{4,3,2,1\}$, and $\mathcal{C} = \emptyset$. Selecting $\mathcal{L}^*$ on $\mathcal{L}$ that is possible to cut and fit with the item's size on $\mathcal{P}$, $\mathcal{L}^* = \{0,0,300,150\}$. The item that is cut based on the value item $e_i$ is $p_4$ with $n_4 \leq 20$, then forms a block $b^*$ to cut at $\mathcal{L}^*$. The number of the fourth item is $n_4 = 20$, so $Q_4 = Q_4 - n_4 = 20 - 20 = 0$. Next, update the value of $\mathcal{P}, \mathcal{C}$, and $Q_4$, $\mathcal{P} = \{3,2,1\}$, $\mathcal{C} = 4$, and $Q_4 = 0$. Update $\mathcal{L}$ for the next cutting and go to step 1, $\mathcal{L} = \{60,0,300,150\}$.

2. The second cutting

   Choose $\mathcal{L}^*$ on $\mathcal{L}$ that is possible to be cut according to the item's size on $\mathcal{P}$, so $\mathcal{L}^* = \{60,0,300,150\}$. The item that is cut based on the value item $e_i$ is $p_3$ with $n_3 \leq 14$, then forms a block $b^*$ to cut at $\mathcal{L}^*$. The number of the third item is $n_3 = 4$, so $Q_3 = 14 - 14 = 0$. Next, update the value of $\mathcal{P}, \mathcal{C}$, and $Q_3$, $\mathcal{P} = \{2,1\}$, $\mathcal{C} = 3$, and $Q_3 = 0$. Update $\mathcal{L}$ for the next cutting and go to step 1, $\mathcal{L} = \{60,140,120,150\}$. The steps in GRASP method continue until $\mathcal{L}^* = \emptyset$, then stop cutting.

3.2. The set covering model

   The Set Covering model can be seen in equation (5).

   Minimize:

   \[ z = \sum_{j=1}^{n} x_j \]  

Subject to:
\[
\sum_{j=1}^{n} a_{ij} x_j \geq 1; \quad i = 1, 2, \ldots, m \\
x_j \in \{0,1\}; \quad j = 1, 2, \ldots, n
\]

Where

- \( a_{ij} \) is the number of \( j^{th} \) cutting pattern,
- \( x_j \) is the number of \( i^{th} \) items which cut in \( j^{th} \) cutting pattern, the variable \( x_j \) equals to 1 if the \( j^{th} \) pattern is selected, and 0 otherwise.

From the GRASP method, there are a totally 85 cutting patterns (31 cutting patterns according to length and 54 cutting patterns according to width). Then all patterns are formed to the Set Covering model, which can be seen in equation (6).

Minimize:

\[
z = \sum_{j=1}^{54} x_j
\]

Subject to:

\[
\sum_{j=1}^{54} a_{ij} x_j \geq 1; \quad i = 1, 2, \ldots, 31 \\
x_j \in \{0,1\}, \quad j = 1, 2, \ldots, 54
\]

The model is solved by LINGO 13.0, and the optimal cutting pattern can be seen in figure 1.

![Figure 1. Optimal cutting pattern.](image)

The optimum solution, as shown in Figure 1 shows that there are 6 pieces for the first item (50 cm × 40 cm), 11 pieces for the second item (40 cm × 30 cm), 14 pieces for the third item (30 cm × 20 cm), and 20 pieces for the fourth item (30 cm × 15 cm). The description of the optimal solution can be seen in table 3.
Table 3. The description of the optimal solution.

| No. | Item Size (cm) | Upper limit of demand-i | Number of Items from Patterns | Number of Excess Product |
|-----|----------------|--------------------------|-------------------------------|--------------------------|
| 1   | 50 x 40        | 8                        | 6                             | 2                        |
| 2   | 40 x 30        | 11                       | 11                            | 0                        |
| 3   | 30 x 20        | 14                       | 14                            | 0                        |
| 4   | 30 x 15        | 20                       | 20                            | 0                        |

From table 3 it can be seen that item 1 is not fulfilled, causing a lack of products. There are still two pieces of excess production of item 1. For the second, third, and fourth items, all of the demands are fulfilled. The GRASP method can meet the demand and give different patterns depends on the number of the upper limits of demand. All of the possible patterns that formed to Set Covering model show the minimum trim loss.

4. Conclusion

From the result and discussion, it can be concluded that the GRASP method can be used to formulate the Set Covering model in the Cutting Stock Problem. The set Covering model, which was solved by LINGO 13.0, showed the optimal cutting patterns with the minimum trim loss but still has the lack of products in one of the items. For further research, the more extensions of the Set Covering Problem with heuristics methods are critically important to be studied and tested in some computational tests.

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