Modelling of the forming devices of high-current pulsed accelerators

G P Averyanov, V V Dmitrieva, N P Kornev
National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe Highway 31, Moscow, 115409, Russia

E-mails: GPAveryanov@mephi.ru, VVDmitrieva@mephi.ru, NPKornev@mephi.ru

Abstract. The report provides a comparison of the mathematical model of the forming devices of high-current pulsed accelerators based on the solution of unsteady problems for electric circuits with the distributed parameters and the model implementing direct calculation methods obtained on the basis of operator transformations.

1. The first section in your paper
The devices forming the high-voltage pulses of short duration are one of the most important components of high-current pulsed accelerators, with a wide range of applications. Such accelerators consist of three main parts – the primary storage, the sharpening agent (a device forming nanosecond pulses) and the accelerating diode.

As a rule, in such installations, as the primary storage, an impulse voltage generator based on the cascade voltage multiplication circuits (an Arkadiev-Marx generator) is used. A feature of those generators is a parallel connection of capacitors during charging and a serial connection of capacitors during the load operation. Switching is accomplished using spark gaps. At the output of such generators (in an accelerating diode), it is possible to obtain pulsed currents of accelerated electrons of tens of amperes at a pulse duration of several microseconds. Obtaining shorter pulses, and hence higher pulsed power, is difficult due to the influence of parasitic capacitances and inductances in the design of the pulse voltage generator. To form shorter pulses to increase the pulse power with a minimum energy loss, at the output of the pulse voltage generator, a forming device on the basis of a long line is set up, which ensures operation with a spectrum width up to gigahertz (for a nanosecond pulse duration). The design of these devices (a structure selection and optimization of scheme parameters) is associated with the use of traditional models for the analysis of electrical circuits, including methods of the theory of directed graphs. A difficulty of using traditional methods of analysis of electrical circuits is aggravated by the usage of non-typical circuitry elements (long lines, arresters etc.).

Long lines as forming devices may have different configurations (the double forming line, the alternating wave impedance line, etc.) and the elements with distributed parameters and are described by the evolution equation in partial derivatives. Component equations of these elements require an individual analysis, the complexity of which exceeds the traditional analysis of electrical circuits, being its integral part.
2. Modeling features of forming devices

Mathematical model of formation can be obtained by combining the component and topological equations; and it is necessary to consider the essential feature of high-power pulsed devices, which are the elements of the scheme, which, along with the standard elements with lumped parameters, include also the elements with distributed parameters.

\[
\begin{align*}
    u &= u_v + Z(i - i_v) \\
    i &= i_g + Y(u - u_g)
\end{align*}
\]

(1)

where \(u_g\) and \(i_g\) are independent initial conditions; \(a, u_v\) and \(i_v\) are external impacts on the circuit.

As it is known, in general, the topology of the electrical circuit is presented in the form of graphs, the use of which allows to consider the processes in electric circuits on two levels: topological and component. The nature of connections between the circuit elements is determined by the topological properties of the graph elements (branches, nodes, and paths). Topological features determine the order of connections of electric circuit elements and are not associated with their specific parameters. The topology of the circuit is described by topological matrices (an incidence matrix, a matrix of main circuits, a matrix of main cross-sections).

The component equations are mathematical models of the circuit branches and express the voltage and current in each branch through the parameters of the elements of this branch. The most difficult part of the mathematical analysis of electrical circuits is the formation of equations of electrical balance.

Of the greatest interest is the choice of the algorithm allowing to automate this process. During the formation of equations of electrical balance for an extended description of electric circuits is used, and each circuit element is a separate branch of the tree. In highly branched circuits, it is first necessary to find the optimal variant of the construction of the tree of a connected graph, which is the most convenient for analysis of the circuit. For the selected tree, at the subsequent stages of automation of computations, two methods are used – the method of node voltages (2) and the method of loop currents (3).

\[
Y_{ij}u_{i0} = j_{i0}
\]

(2)

where is the matrix of nodal conductances \(Y_{ij}\), the voltage of the i-th node relative to the 0-th node is \(u_{i0}\), and the matrix-column of the nodal currents is \(j_{i0}\).

\[
Z_{ij}j_{i0} = e_{ii}
\]

(3)

here, the matrix of contour resistance is \(Z_{ij}\), the matrix-column of loop currents – \(j_{ii}\), and the matrix-column of contour electromotive force – \(e_{ii}\).

Equations compiled on the basis of the first and second Kirchhoff laws are topological equations for electrical circuits with lumped parameters, but analysis of systems based on lines with distributed parameters requires a different approach, arising from the method of representation of the analyzed system in the form of a directed graph, the branches of which are line segments.

Regions of bifurcation of the branches of the graph are not "nodes" in the standard view of graph theory, so to avoid confusion it is advisable to consider a new mathematical object, marking it as a "connection". Moreover, the change of currents and voltages in the branches of such a graph cannot be described by Kirchhoff laws as for circuits with lumped parameters, because, in a general case, these branches may be transmission lines for which it is required to account for resistance losses and the conductivity of the leakage.

The most effective method of construction of optimal mathematical models of devices of formation of nanosecond high-voltage pulses in high-current accelerators based on the respective topology of electrical circuits is the use of the directed graphs theory, however, the inclusion in the scheme of more complex elements including traditional devices with well-described models and methods, such as diode, transistor, etc., complicates the study of forming device models and requires a preliminary study of the individual nonlinear elements and presenting them in the form of external software modules.
The inclusion in the forming device of specific switches and segments of non-uniform transmission lines, which are quadrupoles from the point of view of circuit design, transforms and greatly complicates the structure of the graph; thus there is a need for specialized methods of solution of the unsteady problems describing the transient processes in systems with distributed parameters.

3. Modeling features of subsystems with distributed parameters
As practice has shown, various variants of the method of characteristics (in particular, the Dvorak circuit) for solving telegraph equations are the most effective for solving non-stationary problems related to the description of processes in lines with distributed parameters. Ultimately, this allows us to consider transmission line segments as quadrupoles, and to incorporate the developed models into specialized CAD systems, while allowing to significantly expand the range of tasks compared to analogue circuit modeling systems (MicroCAP, SPICE, P-CAD, etc.).

The passage of a pulse in a segment of a long line with losses can be described by a system of telegraph equations (4):

\[
\begin{align*}
-\frac{\partial}{\partial x} u &= (R + \rho \tau \frac{\partial}{\partial t}) i \\
-\rho \frac{\partial}{\partial x} i &= (\rho G + \tau \frac{\partial}{\partial t}) u
\end{align*}
\]

where \( u = u(x,t) \) is a function of voltage and \( i = i(x,t) \) is a function of current with the following distributed electrical parameters: \( L \) – inductance, \( C \) – capacitance, \( R \) – resistance, \( G \) – conductance, \( \tau \) is the delay (or electrical length), \( \rho \) is the wave resistance.

The solution of the system (4) can be obtained in the form of a recurrent finite-difference schemes for functions of current and voltage (5) depending on time:

\[
\begin{align*}
 &u_{k+1}(t + T_k) + \rho_k i_{k+1}(t + T_k) = (1 - \rho_k g_k) u_k(t) + (\rho_k - \tau_k) i_k(t) \\
 &u_k(t + T_k) - \rho_k i_k(t + T_k) = (1 - \rho_k g_k) u_{k+1}(t) + (\rho_k - \tau_k) i_{k+1}(t)
\end{align*}
\]

here, the magnitude of current – \( i_{k+1} \) and voltage – \( u_{k+1} \) in the section of the line with the number \((k+1)\) are determined via the amount of current – \( i_k \) and voltage \( u_k \), as well as through the electric length \( T_k \), resistance \( \tau_k \), and conductivity \( g_k \), computed for the previous line segment (with number \( k \)).

The method of characteristics assumes consideration of a long line as a sequence of sections with smoothly varying wave resistance and the same electrical length.

The analyzed circuit along with the distributed parameters line segments with can contain discrete elements (such as \( R, G, L, C \)), as well as keys and programmable sources. Any of the discrete elements can be non-linear and defined as a function of the time and state of the circuit (currents and voltages). In this connection, in the process of computation, there arises the problem of joining distributed parameters lines and blocks of a circuit with lumped parameters.

The Dvorak scheme for solving a system of telegraph equations actually considers distributed parameters line as a sequence of quadrupoles with electrical characteristics calculated at each time step. Thus, the solution of (8) is reduced to solving a system of \( 2N + 2 \) linear algebraic equations by splitting a line with distributed parameters into a sequence of \( N \) sections with smoothly varying characteristic impedance. Since the application of the method of nodal voltages or loop currents to calculate the characteristics of discrete elements of the circuit also reduces to the construction of a system of linear algebraic equations, it is possible to construct and solve a generalized system of linear algebraic equations for each time step for the whole scheme. The method of characteristics, which has a high accuracy, is the most suitable for use in specialized CAD systems. To model long lines with small losses in conductors and dielectrics, it is expedient to use direct calculations based on the Laplace transform.

Expressions for the voltage functions \( \tilde{V}_x(p) \) and \( \tilde{V}_y(p) \), represented in an operator form in the system of equations (6) are standard one-dimensional solutions of the Telegraph equation for the line segment:
where $p$ is the Laplace operator, $Z_0 = const$ is the characteristic impedance of the line, $V_i$ is the function of the voltage generator, $Z_s$ is the internal resistance of the voltage generator, $I'_0 = const$ is a reflection coefficient at the input of the line, $k$ is the number of the reflected wave, $I_k = I_i(p)$ is a coefficient of the load line reflection, and presenting it so it is possible to obtain a universal formula of Laplace transformation:

$$\bar{I}_k(p) = \lambda / (p + \beta) + \delta$$

where constants $\lambda$, $\beta$ and $\delta$ depend on the parameters of the load and the type of connection (serial or parallel). By substituting (7) into (6), it is simple enough to convert back and get the formula for direct calculation.

The verification of the results of the modeling of transient processes, based on the method of characteristics (in accordance with the modified Dvorak scheme) and the operator method (based on the Laplace transformation), given in Figure 1, shows that the maximum variance in the values of the voltage calculation results does not exceed 5%.

![Figure 1. Results of the modelling of transient processes](image)

4. Conclusion

Modeling of transients in shaper circuits can be useful in the development of specialized software for evaluating the factors that cause distortion of pulses in existing installations (including parasitic
capacitances and inductances), as well as in the development of virtual powerful pulse technology laboratories of the 5th generation for conducting educational and scientific research in the framework of training, retraining and advanced training of specialists in the field of charged particle beams and accelerator technology.

Presented in this paper, methods of modeling transient processes in distributed-parameter lines, implemented on the basis of two methods for solving telegraph equations, can be used in two successive stages of designing devices for the formation of subnanosecond pulses of high-current high-voltage accelerators.

- At the stage of pre-project studies of transient processes in the lines for preliminary estimation of their main parameters, as well as for estimating the influence of parasitic factors on pulse formation.
- Construction of schemes for the implementation of pulse shapers, as well as optimization and refinement of design parameters of the element base on the basis of preliminary pre-project studies results.

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