Joint influence of electric field and vibrations on the instability of fluid dielectric layer with free boundary

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Abstract. The joint influence of normal electric field and normal vibrations on the instability of the infinitely deep fluid dielectric layer with free boundary in the approximation of small viscosity was studied. It was shown that normal vibrations increase the value of electrical field strength exciting. The electric field effect on the first resonance zone was also studied. It was shown that for sufficiently small values of Weber number there were such values of electrical field strength that had three or two resonance zones existed instead of one first resonance zone.

1. Introduction
The influence of electric field on the instability of free surface of electrically conducting fluid was first considered by Tonks [1] and Frenkel [2]. The influence of electric field on the instability of interface between two fluid dielectrics (and magnetics) was investigated in the paper of Schliomis [3].

The influence of normal vibrations on the instability of free surface in the absence of electric field was first investigated in the experimental work of Faraday [4]. Rayleigh considered this problem theoretically under the assumption of zero viscosity. In the paper [6] viscosity was first considered phenomenologically. Consistent viscosity accounting was taken into account in the work [7].

In the paper [8] was investigated the joint influence of electric field and normal vibrations on the instability of two-layer system for the case when upper fluid is dielectric and lower fluid is ideal conductor.

In the present work, the joint influence of normal electric field and normal vibrations on the instability of dielectric fluid layer with free boundary is investigated.

2. Mathematical formulation
The infinitely deep layer of dielectric fluid with upper free boundary is investigated. Outside can be fluid with zero density and zero viscosity. We are only interested in the electric properties of this fluid. Normal electric field and normal vibrations affect the layer. The problem is described by the following system of equations and boundary conditions:
\[ \frac{\partial \mathbf{v}}{\partial t} + A(\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p' + \frac{2}{\mathcal{F} \text{We}} \left( \nabla \cdot (\mathbf{E}_i \otimes \mathbf{E}_i) - \frac{1}{2} \nabla \mathbf{E}_i^2 \right) \]

\[ + \frac{N}{\sqrt{\text{We}}} \nabla^2 \mathbf{v} - \frac{1}{\text{We}} \frac{\partial}{\partial t} - 4 \cos 2t \right) \mathbf{e}_z, \]

\[ \mathbf{v} \cdot \mathbf{v} = 0, \]

\[ \nabla^2 \phi_1 = 0, \quad \nabla^2 \phi_2 = 0, \]

\[ z \to -\infty : \quad \frac{\partial \phi_1}{\partial z} = -1, \]

\[ z \to \infty : \quad \mathbf{v} = 0, \quad \frac{\partial \phi_2}{\partial z} = -\mathcal{F}. \]

\[ z = \zeta : \quad \mathbf{n} \cdot \nabla \phi_1 = \mathcal{E} \mathbf{n} \cdot \nabla \phi_2, \quad \mathbf{e}_z \cdot \nabla \phi_1 = \mathbf{e} \cdot \nabla \phi_2, \quad \frac{\partial \zeta}{\partial t} + A(\mathbf{v} \cdot \nabla) \zeta = \mathcal{A} \mathbf{v}_z, \]

\[ p' + \frac{F}{\mathcal{F} \text{We}} [\mathcal{E}(\mathbf{n} \cdot \nabla \phi_1)^2 - (\mathbf{n} \cdot \nabla \phi_2)^2] + \frac{N}{\sqrt{\text{We}}} \sigma = \frac{1}{\text{We}} \nabla \cdot \mathbf{n}, \quad \frac{\partial \mathbf{v}_z}{\partial z} = 0, \]

where (for plane disturbances)

\[ \mathbf{n} = \frac{\mathbf{e} \cdot \left( -\frac{\partial \zeta}{\partial x} \right) \mathbf{e}_z}{\sqrt{1 + \left( -\frac{\partial \zeta}{\partial x} \right)^2}}, \quad \mathbf{e}_z = \frac{\mathbf{e} \cdot \left( \frac{\partial \zeta}{\partial x} \right) \mathbf{e}_z}{\sqrt{1 + \left( \frac{\partial \zeta}{\partial x} \right)^2}}, \quad p' = \frac{2}{\mathcal{F} \text{We}} \left( \frac{\partial \mathbf{e}}{\partial t} \right)_z, \]

\[ \sigma_\parallel = \nabla \mathbf{v}_z + \nabla \times \mathbf{v}_z, \quad \mathbf{E}_i = -\nabla \phi_1, \quad \mathbf{E}_z = -\nabla \phi_2, \quad \mathbf{v}_z = (\mathbf{v} \cdot \mathbf{e}) \mathbf{e}_z, \quad \mathbf{v}_z = (\mathbf{v} \cdot \mathbf{e}) \mathbf{e}_z. \]

It is seen that equations of motion are written for the lower medium only, and equations for electric field are written for both media.

Here the following dimensionless parameters appear:

\[ A = \frac{a}{l_i}, \quad \text{We} = \frac{\alpha \rho}{g}, \quad F = \frac{\mathbf{E}_i \mathbf{E}_i^2}{\mathcal{E}_i}, \quad N = \frac{V}{\sqrt{g \rho^2}}, \quad \mathcal{F} = \frac{\mathcal{F}_i}{\mathcal{F}_i}, \]

\[ l_i = \sqrt{\bar{a}/(g \rho)}. \]

These are dimensionless amplitude of vibrations, Weber number, and parameter, characterizing intensity of electrical field, dimensionless viscosity, and ratio of dielectric constants.

### 3. Instability boundary

For investigation of instability of base state (\( \zeta = 0, \quad \mathbf{v} = 0 \)) was used the multi-scale method. We suppose that it is the smallness of viscosity. For consideration of viscosity, the quick vertical coordinate is used (it allows describing viscous boundary layer).

#### 3.1. Electric mode

First, we considered the influence of normal vibrations on the electric field strength for instability started. We called this instability mode the “electric mode”. We used the following expansions in formal small parameter \( \delta \):\n
\[ A = \delta \mathcal{A}^{(0)}, \quad N = \delta \mathcal{N}^{(2)}, \]

\[ F = \mathcal{F}^{(0)} + \delta \mathcal{F}^{(2)} + \ldots, \]

\[ \zeta = \mathcal{Z}^{(0)} + \delta \mathcal{Z}^{(2)} + \ldots, \]

\[ \mathbf{v}_z = \mathcal{V}_z^{(0)} + \delta \mathcal{V}_z^{(2)} + \ldots, \]

\[ \mathbf{q} = \mathcal{Q}^{(0)} + \delta \mathcal{Q}^{(2)} + \ldots. \]
\( \tilde{\phi}_2^{(1)} = \delta \phi_2^{(1)} + \delta^2 \phi_2^{(2)} + \ldots, \)  
(13)

\( \xi = \frac{z}{\delta}, \quad \xi_0 = z, \quad \frac{\partial}{\partial z} = \frac{1}{\delta} \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \xi_0} \) 
(14)

(tilde means that we consider deviations from the basic state).

After the use of the multiscale method, we obtain the following stability boundary:

\[ F - F^{(0)} = \delta^2 f^{(1)} = \frac{1+\varepsilon}{1-\varepsilon} \text{We} A^2, \]
(15)

\[ F^{(0)} = \frac{1+\varepsilon}{2(1-\varepsilon)} \left( \frac{k+\frac{1}{k}}{k} \right) \]
(16)

From these formulae we see, that vibrations increase critical value of electric field strength. These formulas are presented in Figure 1(a) (curves 0, 1) and in Figure 2(a) (curve 1).

3.2. Resonant mode

After the study of electric mode we investigated the influence of electric field on first resonant mode for which frequency of disturbances in the main order of smallness is equal one half of vibration frequency (higher modes are less dangerous and in this reason were not investigated). We used the following expansions in formal small parameter \( \delta \) :

\[ A = \delta A^{(2)}, \quad N = \delta N^{(2)}, \]
(17)

\[ F = F^{(0)} + \delta F^{(2)} + \ldots, \]
(18)

\[ \zeta = \delta \zeta^{(3)} + \delta^2 \zeta^{(5)} + \ldots, \]
(19)

\[ \nu = \delta \nu^{(3)} + \delta^2 \nu^{(5)} + \ldots, \]
(20)

\[ \tilde{\phi}_1 = \delta^3 \phi_1^{(3)} + \delta^2 \phi_1^{(5)} + \ldots, \]
(21)

\[ \tilde{\phi}_2 = \delta^2 \phi_1^{(3)} + \delta \phi_2^{(3)} + \ldots, \]
(22)

\[ \frac{\partial}{\partial \xi} = \frac{z}{\delta}, \quad \xi_0 = z, \quad \frac{\partial}{\partial z} = \frac{1}{\delta} \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \xi_0}. \]
(23)

Using the multiscale method, we obtained the following instability boundary:

\[ A = \delta^2 A^{(2)} = \pm \sqrt{\frac{(1-F)^4}{(1+F)^4} \left( \frac{4(F-F^{(0)})^4 k^2 + k^2 N}{\text{We}} \right)} \]
(24)

\[ F^{(0)} = \frac{1+\varepsilon}{2(1-\varepsilon)} \left( \frac{1+\frac{\text{We}}{k^2}}{k} \right) \]
(25)

( \( F - F^{(0)} = \delta^2 f^{(2)}, \quad N = \delta^2 N^{(2)} \)). From the equations it can be seen that the critical value of \( A \) is

\[ A_s = |\delta^2 A^{(2)}|_{k_x} = \pm k_x \sqrt{\frac{N}{\text{We}}}. \]
(26)

where \( k_x \) is defined from equation \( F - F^{(0)}(k_x) = 0 \). Relations (26) shows that viscosity suppressed the resonant mode.

The analysis of formulas (24), (25) shows at first, that viscosity stabilizes the basic state, and at second, that for sufficiently small values of Weber number we have such values of parameter \( F \) (it is responsible for intensity of electric field), for which we have three or two resonant zones instead one. These zones are not connected with higher resonant modes (in the main order they have frequency of disturbances, which is equal to one-half of the vibration frequency).
Figures 1 and 2 illustrate this analysis. The areas of instability are marked with symbols “+”. We considered the case when fluid is decane at the temperature 200°C and normal pressure. Outside of decane we considered air (we neglected their density and viscosity). In this case we have the following values of material parameters: $N = 0.002972$, $\tau = 0.5023$. Figure 1 corresponds to the vibration circular frequency $10$ s$^{-1}$ ($We = 1.052$). For this value of $We$ we observe only one minimum for first resonance mode for dependence $A(k)$. Fig. 2 corresponds to the vibration frequency $3$ s$^{-1}$ ($We \approx 0.09469$). For this Weber number we can observe three cases. The first case corresponds to three minimums for first resonance mode (Figs. 2(b, c)). The second case corresponds to two minimums (Fig. 2(d)). It can also be seen from Fig. 1(a). In these cases, the horizontal line $F = F_c = \text{const}$ intersects resonant zones thrice or twice. For sufficiently small and large values of $F$ we have one minimum. It can be seen from Fig. 2(a). In this case, horizontal line $F = F_c = \text{const}$ intersects resonant zone only once.

Figures 1(c), 2(c) demonstrate that critical vibration amplitude for resonant modes is not equal to zero (in Figures 1(b), 2(b, d) it is not visible due to used scale). It is correct for all cases when viscosity is not zero.

**Figure 1.** (a) Dependence of parameter $F$ on wave number $k$. Curves 0, 1 describe electric modes ($A = 0, 0.4$). The curves a, b describe the boundaries of the instability of resonant modes ($A = 0.2, 0.4$). Dotted line describes resonant mode without viscosity and without vibrations. (b) Dependence of vibration amplitude on wave number $k$. The curves 1, 2 describe electric modes ($F = 6.5, 7$). The curves a, b, c describe resonant modes ($F = 6, 6.5, 7$). (c) It is Fig. 1(b) without electric modes and for other scale.
4. Conclusion
We have found that normal vibrations acting on the infinitely deep layer of dielectric fluid layer with free boundary increase the critical value of electric field strength. For electric field effect on the first resonant mode (mode with frequency of disturbances in the main order equaling to one-half of the vibration frequency) we have found that for Weber number less a critical one is such values of electric field strength that have three or two resonant zones instead one resonant mode. These zones are not connected with higher resonant modes, since their disturbance frequencies are equal to one-half of the vibration frequency mode. The viscosity for resonant mode plays stabilizing role (as for the case of absence of electric field).
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