We theoretically consider the formation of bright solitons in a mixture of Bose and Fermi degenerate gases. While we assume the forces between atoms in a pure Bose component to be effectively repulsive, their character can be changed from repulsive to attractive in the presence of fermions. Experimentally, this situation is for example accessible in Bose-Fermi mixtures of bosonic $^{87}\text{Rb}$ atoms and fermionic $^{50}\text{K}$ atoms.

We consider the bare interaction between bosonic atoms to be repulsive ($g_B > 0$) whereas the particle interaction between bosonic and fermionic atoms is assumed to be strongly attractive ($g_{BF} < 0$). Bright solitons in Bose-Fermi gas mixtures are then produced as a result of a competition between two interparticle interactions: boson-boson repulsion versus boson-fermion attraction. Experimentally, this situation is for example accessible in Bose-Fermi mixtures of bosonic $^{87}\text{Rb}$ atoms and fermionic $^{50}\text{K}$ atoms.

We determine the critical strength of attraction between bosons and fermions necessary for the formation of bright solitons and show that these parameter regimes might be achievable in present experiments. We study the formation of bright solitons by different excitation mechanisms: either increasing the attractive boson-fermion interaction by Feshbach resonance techniques or by radial squeezing of the mixture which corresponds to increasing the effective one-dimensional scattering length. We contrast the response of the system following adiabatic and fast increase of the boson-fermion interaction strength.

We consider a Bose-Fermi mixture confined in a trap at zero temperature and describe this system in terms of the many-body wave function $\Psi(x_1, ..., x_{N_B}; y_1, ..., y_{N_F})$, where $N_B$ and $N_F$ are the numbers of bosons and fermions, respectively. However, instead of direct (and of course approximate) solving of the many-body Schrödinger equation we start with the equivalent approach based on the Lagrangian density. Since the fermionic sample is spin-polarized only boson-boson and boson-fermion interactions are included. At zero temperature we assume that the wave function of Bose-Fermi mixture is a product of the Hartree ansatz for bosons and the Slater determinant (antisymmetric wave function) for fermions.
\[\Psi(x_1, ..., x_{N_B}; y_1, ..., y_{N_F}) = \prod_{i=1}^{N_B} \varphi^{(B)}(x_i) \times \frac{1}{\sqrt{N_F}} \left[ \begin{array}{c} \varphi^{(F)}_1(y_1) \cdots \varphi^{(F)}_{N_F}(y_{N_F}) \\ \vdots \\ \varphi^{(F)}_{N_F}(y_1) \cdots \varphi^{(F)}_{N_F}(y_{N_F}) \end{array} \right]. \quad (1)\]

Now, all the single-particle wave functions \((\varphi^{(B)}, \varphi^{(F)}_1, ..., \varphi^{(F)}_{N_F})\) have to be determined. Therefore the many-body wave function \(\Psi\) is inserted into the Lagrangian density and integrated over the spatial coordinates to get the Lagrangian. In fact, this integration is performed only over \(N_B - 1\) bosonic and \(N_F - 1\) fermionic coordinates and leads in this way to the mean-field single-particle Lagrangian. The Euler-Lagrange equations corresponding to this Lagrangian are the basic equations of the presented approach

\[i\hbar \frac{\partial \varphi^{(B)}(y)}{\partial t} = -\frac{\hbar^2}{2m_B} \nabla^2 \varphi^{(B)} + V_{\text{trap}} \varphi^{(B)} + g_B N_B |\varphi^{(B)}|^2 \varphi^{(B)} + g_{BF} \sum_{i=1}^{N_F} \varphi^{(F)}_i \varphi^{(F)}_i \quad (2)\]

\[i\hbar \frac{\partial \varphi^{(F)}_i}{\partial t} = -\frac{\hbar^2}{2m_F} \nabla^2 \varphi^{(F)}_i + V_{\text{trap}} \varphi^{(F)}_i + g_{BF} N_B |\varphi^{(B)}|^2 \varphi^{(F)}_i \quad i = 1, 2, ..., N_F \quad (2)\]

All of the above equations have simple interpretation. Removing the last terms in these equations (i.e., neglecting the mean-field interaction energy between Bose and Fermi components in comparison with other energies) one recovers the Gross-Pitaevskii equation for degenerate Bose gas and the set of Schrödinger equations describing the noninteracting Fermi system. It is easy to notice that when the Bose and Fermi gases attract each other strongly enough the mean-field energy connected with this attraction can overcome the repulsive mean-field energy for bosons. This can happen provided both phases have a significant overlap. It means that the presence of a degenerate Fermi gas changes the character of the interaction between bosonic atoms from repulsive to attractive. Therefore the formation of bright solitons becomes possible.

Based on the above considerations one can easily write the condition for the value of critical strength of attraction between bosons and fermions. It is given by \(g_B n_B = |g_{BF}^c|^2 n_F\), where \(n_B = N_B |\varphi^{(B)}|^2\) and \(n_F = \sum_{i=1}^{N_F} |\varphi^{(F)}_i|^2\) are the densities of both fractions, normalized to the number of particles, taken at the center of the trap. A rough estimation of \(|g_{BF}^c|^2\) can be found assuming the densities of both components are calculated within the Thomas-Fermi approximation and components do not interact. Then we have

\[|g_{BF}^c|^2 = C \frac{N_B^{2/5} N_F^{1/2}}{g_B}, \quad (3)\]

where \(C = C_1 (a_B^2/a_B)^{3/5} (a_F^2/a_F)^3 \lambda_B^{2/5} / \lambda_F^{1/2}\), \(C_1 = 3^{9/10} 5^{2/5} \pi / 16 \approx 1.0\), \(a_B\) is the radial harmonic oscillator length, \(\lambda = \omega_z / \omega_\perp\) defines the aspect ratio of axially symmetric trap, and \(a_B\) is the s-wave scattering length for the pure Bose gas related to the interaction strength through \(g_B = 4\pi \hbar^2 a_B / m_B\). The condition \(c\) has several implications. Squeezing radially both Bose and Fermi components decreases the value of critical \(g_{BF}\). The same happens when the number of fermions is getting bigger in comparison with the number of bosons. For a particular trap numbers of atoms are limited by the occurrence of a collapse \(c\). In the case of experiment of Ref. \(c\), it was found that the system was stable if the number of atoms in both species \((^{87}\text{Rb} \text{ and } ^{40}\text{K})\) in \([2, 2] > \text{and } [9/2, 9/2]\) hyperfine states, respectively) were smaller than \(2 \times 10^4\). Taking parameters of that experiment \((\omega_B^2 = 2\pi \times 215\text{ Hz} \text{ and } \omega_F^2 = 2\pi \times 16.3\text{ Hz})\) and assuming the following numbers of atoms \(N_B = 10^3\) and \(N_F = 10^4\), one gets the critical coupling \(|g_{BF}^c| = 6.7 g_B\) which equals the natural value of \(g_{BF}\) for \(^{87}\text{Rb} - ^{40}\text{K}\) mixture in the double-polarized state mentioned above. It is understood that the relation \(c\) is only the necessary condition for creation of bright solitons. Another important factor is the geometry of the system.

![FIG. 1: Density profiles (normalized to one) of one-dimensional Bose-Fermi mixture consisting of \(N_B = 1000\) bosons and \(N_F = 100\) fermions for various strengths of attraction between components as described in the label. The effective repulsion for bosons is governed by \(g_B = 0.0163\). Solid lines correspond to the Bose fraction whereas the dashed ones indicate fermions. By increasing the attraction between components some fermions are pulled inside the Bose cloud. All quantities are given in oscillatory units calculated based on fermionic component.](image)
of solitons. In Fig. 1 we plot the density profiles of Fermi (dashed lines) and Bose (solid lines) components of one-dimensional mixture in its ground state. The gases are confined in traps with frequencies $2\pi \times 30$ Hz (for fermions) and $2\pi \times 20$ Hz (for bosons). To get these curves we solve numerically the set of Eqs. (2) by evolving adiabatically the coupling constant $g_{BF}$ from zero to the given value. Even though the number of bosons is 10 times larger than the number of fermions, due to the Pauli exclusion principle the size of the fermionic cloud is much bigger. We see that after turning on the attractive forces between components some number of fermions is drawn inside the bosonic cloud. When the attraction is increasing further the fermionic cloud is clearly divided into two distinguishable parts. One of them is a broad background gas whereas the second is the narrow density peak hidden within the bosonic peak. Both peaks get higher and narrower when the attraction is getting stronger. This is a sign of effective attraction.

![Fig. 2](image-url)  
**FIG. 2:** Illustration of the solitonic character of the ground state of Bose-Fermi mixture. Both frames show the densities 34 ms after switching off the trapping potential. Only when the attraction between the species is stronger than the critical one (the lower frame) the system forms double-peak structure which does not spread in time.

It turns out that for strong enough attraction between Bose and Fermi components the central peaks (bosonic and fermionic ones) in Fig. 1 form, in fact, the soliton. After switching off the trapping potential the fermionic background is lost but the bosonic peak persists without changing its shape and confines fermionic density whose profile is also preserved. It is illustrated in Fig. 2 where fermionic and bosonic densities are plotted some time after the opening the trap. Upper and lower frames differ by the value of $g_{BF}$. In the case of the upper frame the strength of attraction is weaker than the critical one and no solitonic behavior is observed - the bosonic and fermionic clouds spread out. For the lower frame the attraction is strong enough and the formation of a bright soliton is observed. Such a structure can be forced to move by imposing a momentum on it (realized experimentally by applying the Bragg scattering technique) and its shape again does not change. The critical value of the coupling constant is in our one-dimensional model approximately equal to $g_{BF}^c \approx -1.0$ osc. units and can be compared to the value obtained based on the one-dimensional counterpart of condition (3). Taking from Fig. 1 the ratio $n_B(0)/n_F(0) = 57.5$ one gets $g_{BF}^c = -0.94$ osc. units what remains in agreement with numerical estimation.

![Fig. 3](image-url)  
**FIG. 3:** Density profiles of one-dimensional Bose-Fermi mixture after switching the attraction between Bose and Fermi components from $g_{BF} = -0.5$ to $g_{BF} = -2.0$ oscillatory units. The interaction strength is changed linearly during 8.5 ms (the upper frame) and 17 ms (the lower frame). The snapshots are taken at 20 ms. More adiabatic change of the strength of attraction results in lower number of solitons.

In Figs. 4 and 5 we show the densities of Bose and Fermi components after the strength of attraction between bosons and fermions has been increased by using the Feshbach resonance technique. The other way of changing the interaction strength could be the firm radial squeezing of both samples. The basic observation is that after some time after switching the mutual interaction the bosonic cloud breaks into several peaks provided the final coupling (attraction) between both components is strong enough. Stronger attraction results in bigger number of peaks. Each bosonic peak (marked by the solid line) contains the fermionic density (marked by the dashed line). Such double-peak structures oscillate almost without changing their shapes. Switching off the trap shows that the observed structures are indeed solitons. In the case of Fig. 5 the strength of the coupling between Bose and Fermi gases is changed instantaneously.
The contrast of the peaks depends on the initial width of the bosonic cloud and it gets higher for bigger number of bosons or smaller bosonic trap frequency. The latter can be realized only in the optical trap. On the other hand, in Fig. 3 we show the response of the system when the mutual attraction is increased within the finite time which is of the order of trap period. Slower switching of the interaction leads to a smaller number of solitons. Based on numerical results, we propose two schemes for generating bright solitons in degenerate Bose-Fermi mixtures. First of all, the system has to be pushed in the range of strong enough attraction between fermions and bosons which is of the order of trap period. Slower switching of the interaction leads to a smaller number of solitons. Another way could be performing the evaporation already under favorable conditions, i.e., at the presence of the appropriate magnetic field or strong enough one-dimensional geometry. However, in this case only one, placed at the center of the trap double-peak soliton is formed.

In conclusion, we have shown that bright solitons can be generated in a Bose-Fermi mixture as a result of a competition between two interparticle interactions: boson-boson collisions which are effectively repulsive and boson-fermion collisions which are attractive. Assuming the strength of attraction is large enough both kinds of atoms start to mediate in the other species interaction introducing the system into a new regime where locally Bose and Fermi gases become gases of effectively attractive atoms. Therefore it becomes possible to generate bright solitons in the system under such conditions. Depending on how fast the change of the attraction strength is performed the system responses forming the train of solitons (fast change) or the single soliton (adiabatic change). Each soliton is, in fact, the double-peak structure with the fermionic cloud hidden within the bosonic one.

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