Is the Optical Conductivity of Heavy Fermion Strange Metals Planckian?

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ABSTRACT

Strange metal behavior appears across a variety of condensed matter settings and beyond, and achieving a universal understanding is an exciting prospect. The beyond-Landau quantum criticality of Kondo destruction has had considerable success in describing the behavior of strange metal heavy fermion compounds, and there is some evidence that the associated partial localization-delocalization nature can be generalized to diverse materials classes. Other potential overarching principles at play are also being explored. An intriguing proposal is that Planckian scattering, with a rate of $\frac{k_B T}{\hbar}$, captures the linear temperature dependence of the (dc) electrical resistivity, which is a hallmark of strange metal behavior. Here we extend a previously introduced analysis scheme based of the Drude description of the dc resistivity to optical conductivity data. When they are well described by a simple (ac) Drude model, the scattering rate can be directly extracted. This avoids the need to determine the ratio of charge carrier concentration to effective mass, which has complicated previous analyses based on the dc resistivity. However, we point out that strange metals may exhibit strong deviations from Drude behavior, as exemplified by the “extreme” strange metal YbRh\textsubscript{2}Si\textsubscript{2}. This calls for alternative approaches, and we point to the power of scaling relationships in terms of temperature and energy (or frequency).

Keywords: strange metals, Planckian scattering, optical conductivity, Drude model, heavy fermion compounds, quantum criticality, non-Fermi liquid, YbRh\textsubscript{2}Si\textsubscript{2}
1 INTRODUCTION

The standard theory of metals is Fermi liquid theory. It describes materials across the correlation spectrum, from the simplest metals such as sodium or aluminum to heavy fermion compounds (Stewart, 1984) with mass renormalizations of more than three orders of magnitude (Kadowaki and Woods, 1986; Jacko et al., 2009). However, under certain circumstances, behavior at odds with this theory, dubbed non-Fermi liquid behavior, is observed. From the perspective of Fermi liquid theory, this regime arises when the quasiparticle weight factor $Z$, which decreases with increasing correlation strength, vanishes. This is where Fermi liquid theory breaks down and alternative descriptions are needed. In spite of tremendous efforts and much progress, a full understanding of non-Fermi liquids is an outstanding challenge.

One of the best understood settings in which non-Fermi liquid behavior arises are continuous quantum phase transitions (Sachdev, 1999). Heavy fermion compounds have proven particularly suitable to study them because the competing energy scales governing these materials make them readily tunable by external control parameters (v. Lohneysen et al., 2007; Kirchner et al., 2020; Paschen and Si, 2021a,b). Through a large body of investigations on many different compounds it became clear that two broad categories can be distinguished. In the first one, the quantum critical behavior, typically observed at the border of antiferromagnetic order in these materials, is dominated by fluctuations of the vanishing order parameter. This leads to strong scattering at hot spots on the Fermi surface that are associated with the ordering wave vector, for which the resistivity in the clean case goes as $T^2$ given that the current is primarily carried by Landau quasiparticles that reside in the “cold regions” of the Fermi surface (Hlubina and Rice, 1995). In the presence of disorder, the electrical resistivity is predicted to vary as $T^\epsilon$, with $1 \leq \epsilon \leq 1.5$ depending on the amount of disorder (Rosch, 1999). Predictions for such order parameter fluctuation or spin density wave (SDW)-type quantum critical points (QCPs) have also been made for other physical properties (Hertz, 1976; Millis, 1993; Moriya and Takimoto, 1995; v. Lohneysen et al., 2007; Zhu et al., 2003), and have been neatly verified in some cases (Kuechler et al., 2003).

In the second category of materials, however, observations at odds with these predictions have been made (Paschen and Si, 2021a,b), which have provided support for the beyond-Landau quantum criticality of Kondo destruction (Si et al., 2001; Coleman et al., 2001; Senthil et al., 2004). These observations include energy over temperature scaling of inelastic neutron scattering (Schroeder et al., 2000) and optical conductivity data (Prochaska et al., 2020), and an abrupt change of the Fermi surface volume across the QCP as evidenced by de Haas–van Alphen (Shishido et al., 2005) and Hall effect measurements (Paschen et al., 2004; Friedemann et al., 2010; Custers et al., 2012; Martelli et al., 2019). The sudden reconstruction from a small to a large Fermi surface characterizes a (partial, 4f selective) localization-delocalization transition, and is emerging as a potential universal organizing principle with supporting evidence in a variety of other correlated materials classes (Paschen and Si, 2021a), including the cuprates (Badoux et al., 2016; Fang et al., 2022). The interacting nature of the fixed point describing a Kondo-destruction QCP (Si et al., 2001; Coleman et al., 2001; Senthil et al., 2004) means that $k_B T$ is the only energy scale, suggesting that the scattering rate as appearing in the electrical resistivity would be linearly proportional to $k_B T$. Quite remarkably, all heavy fermion compounds of this second category indeed exhibit a linear-in-temperature dc electrical resistivity (Taupin and Paschen, 2022). Thus, quantum criticality beyond order parameter fluctuations appears to be tied to the “strange metal” linear-in-temperature resistivity in heavy fermion compounds.

The most striking case in this novel category is YbRh$_2$Si$_2$, where the electrical resistivity is linear in temperature over 3.5 orders of magnitude in temperature, from above 10 K down to the onset of unconventional superconductivity near 3 mK at the quantum critical field (Nguyen et al., 2021). Recently,
an attempt has been made (Taupin and Paschen, 2022) to answer the question whether this extreme linear-in-temperature resistivity might be characterized by “Planckian dissipation” (Zaanen, 2004)—scattering at a rate equal to the inverse of the “Planckian” time scale (the shortest possible time scale in analogy with the “Planck” time in quantum gravity). While the physics and implications of the Planckian form of scattering remain unclear, its applicability to strange metals is an intriguing question to ascertain empirically. As proposed in (Bruin et al., 2013) and more recently taken up by others (Legros et al., 2019; Cao et al., 2020; Ghiotto et al., 2021; Grissonnanche et al., 2021; Mousatov and Hartnoll, 2021), this has been done by assuming a Drude description of the (dc) resistivity. For YbRh$_2$Si$_2$, the scattering rate was found to be much smaller, unless the charge carriers were assumed to have astonishingly light masses. The same analysis was also carried out for other strange metal heavy fermion compounds, with similar results (Taupin and Paschen, 2022). As such light carriers are absent in the heavy Fermi liquid ground state of these materials, even at tuning parameter values far away from the quantum critical value, it was concluded that dissipation was not Planckian (Taupin and Paschen, 2022). Nevertheless, a technique that could independently determine the effective mass (more precisely, the ratio of charge carrier concentration to effective mass) and the scattering rate is highly desirable. Here we show that the optical conductivity can in principle accomplish this goal, albeit only if the data can be described by the simple Drude form.

In this perspective paper, we start by introducing the Drude model of the optical conductivity in the notation used here (section 2), and the Planckian scattering analysis based on this model (section 3). Next, we use this description to first analyse the optical conductivity of simple, noninteraction materials at high temperatures, where the scattering from phonons leads to linear-in-temperature dc resistivity (sections 4). Then we try to apply this Drude-based Planckian scattering scheme to the optical conductivity of YbRh$_2$Si$_2$ (section 5). As anticipated, the strong deviation from Drude behavior in the compound’s strange metal regime limits this analysis to temperatures and frequencies outside this regime, where Drude behavior is recovered. Section 6 is devoted to scaling analyses, which are the appropriate tool to characterize strange metal optical conductivities. Before closing with a discussion and outlook (section 8), we comment on the relation of the temperature and frequency dependences in the Fermi liquid regime of the optical conductivity (section 7).

## 2 DRUDE FORMULATION OF THE OPTICAL CONDUCTIVITY

In the Drude model, conduction electrons in solids are described as particles of a classical gas executing diffusive motion with an average relaxation time $\tau$. The equation of motion in the presence of a dc electric field leads to the Drude form of the dc electrical conductivity

$$\sigma = \frac{ne^2\tau}{m},$$

(1)

where $n$ is the charge carrier concentration, $m$ their effective mass, and $-e$ the electronic charge. In an ac field, a complex, frequency-dependent optical conductivity

$$\sigma(\omega) = \frac{ne^2\tau}{m} \frac{1}{1 - i\omega\tau} = \frac{ne^2\tau}{m} \frac{1 + i\omega\tau}{1 + \omega^2\tau^2}$$

(2)

results, with the real part

$$\text{Re}[\sigma(\omega)] = \sigma_1 = \frac{ne^2\tau}{m} \frac{1}{1 + \omega^2\tau^2}$$

(3)
and the imaginary part
\[ \text{Im}[\sigma(\omega)] = \sigma_2 = \frac{ne^2\tau}{m} \frac{\omega\tau}{1 + \omega^2\tau^2} . \quad (4) \]

We also introduce the plasma frequency
\[ \omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}} \quad (5) \]
with the permittivity of free space \( \epsilon_0 \).

Frequently, it is useful to discriminate between residual scattering due to defects and intrinsic scattering. In the case of dc resistivity, this is what underlies the usually adopted forms
\[ \rho = \rho_0 + AT^2 = \frac{1}{\sigma_{\text{res}}} + \frac{1}{\sigma_{\text{in}}} = \rho_{\text{res}} + \rho_{\text{in}} \quad (6) \]
and
\[ \rho = \rho'_0 + AT = \frac{1}{\sigma_{\text{res}}} + \frac{1}{\sigma_{\text{in}}} = \rho_{\text{res}} + \rho_{\text{in}} \quad (7) \]
for Fermi liquids and strange metals, respectively. Here \( \rho_0 \) and \( \rho'_0 \) are the extrapolations to \( T = 0 \) of a quadratic-in-temperature and a linear-in-temperature dependence with slope \( A \) and \( A' \), respectively; \( \sigma_{\text{res}} \) and \( \sigma_{\text{in}} \) (or \( \rho_{\text{res}} \) and \( \rho_{\text{in}} \)) are the zero-temperature residual (usually elastic) and the temperature-dependent intrinsic (typically inelastic) contributions, respectively. The underlying assumption is that the Matthiessen rule, which states that for independent scattering channels the scattering rates and thus the corresponding resistivities (or inverse conductivities) add, holds. Making the same assumption for the optical conductivity leads to
\[ \frac{1}{\sigma(\omega)} = \frac{1}{\sigma_{\text{res}}(\omega)} + \frac{1}{\sigma_{\text{in}}(\omega)} . \quad (8) \]
Within the Drude approach, both \( \sigma_{\text{res}} \) and \( \sigma_{\text{in}} \) would have the Drude form of equation\(^2\) with \( \tau = \tau_{\text{res}} \) and \( \tau = \tau_{\text{in}} \), respectively.

In special cases, discussed further below, the situation may arise that the residual scattering rate is much smaller than the intrinsic one, such that
\[ \frac{1}{\tau} = \frac{1}{\tau_{\text{res}}} + \frac{1}{\tau_{\text{in}}} \approx \frac{1}{\tau_{\text{in}}} . \quad (9) \]
One can then rewrite equation\(^2\) as
\[ \sigma(\omega) = \frac{ne^2}{m} \frac{\tau}{(1 - i\omega\tau)} = \frac{ne^2}{m} \frac{1}{(1/\tau - i\omega)} \approx \frac{ne^2}{m} \frac{1}{(1/\tau_{\text{in}} - i\omega)} = \sigma_{\text{in}}(\omega) . \quad (10) \]

### 3 PLANCKIAN SCATTERING ANALYSIS OF THE OPTICAL CONDUCTIVITY

Previous attempts to characterize strange metal behavior in terms of Planckian scattering using dc conductivity data faced the problem that the scattering time \( \tau \) appears in a product with the ratio of charge carrier concentration to mass, \( n/m \); see equation\(^1\). Thus, to estimate \( \tau \) or, more precisely, \( \tau_{\text{in}} \) and compare it with the Planckian time
\[ \tau_P = \frac{\hbar}{k_B T} , \quad (11) \]
as typically done by quantifying the coefficient

$$\alpha = \frac{\tau_p}{\tau_m} = \frac{\hbar}{k_B T} \frac{1}{\tau_m} \equiv \alpha_T,$$  \hspace{1cm} (12)

required the knowledge of $n/m$, as seen explicitly by inserting equations [1] and [7] to obtain

$$\alpha = \frac{n}{m} \frac{e^2 \hbar}{k_B A'} \equiv \alpha_{n/m},$$  \hspace{1cm} (13)

Because extracting $n$ and $m$ from different physical quantities gave different results, this led to conflicting conclusions, as discussed in (Taupin and Paschen, 2022).

Here we propose an analysis of the optical conductivity that avoids this problem. As can be seen from equation [2] in addition to appearing in a product with $n/m$ (in the prefactor), $\tau$ enters as the only parameter in the second factor of $\sigma(\omega)$. Thus, fitting the Drude form to the (intrinsic) optical conductivity data allows us to extract $\tau_m$ and $n/m$ independently. We can therefore determine $\alpha$ directly via equation [12] or, in combination with dc data in the linear-in-temperature regime (which gives $A'$), using equation [13].

4 DRUDE-LIKE OPTICAL CONDUCTIVITY OF LEAD, ALUMINUM, AND SILICON

As discussed in section[1] strange metal behavior refers to the unusual properties exhibited by a number of strongly correlated electron systems at low temperatures, most notably a linear-in-temperature electrical resistivity where a normal metal would exhibit Fermi liquid behavior. However, this temperature dependence can also arise from entirely different physics, that of electrons scattered by classical (macroscopically populated) phonons at sufficiently high temperatures. This case was included in a previous Planckian dissipation analysis (Bruin et al., 2013), and we revisit it here with two metals, lead and aluminum, from the perspective of the optical conductivity. Combining the $A'$ coefficients (0.071 $\mu\Omega$cm/K and 0.01 $\mu\Omega$cm/K, respectively) of their linear-in-temperature dc resistivities at high temperatures with $n/m$ values estimated from quantum oscillation data, $\alpha$ values of 2.7 and 1.1 were obtained (Bruin et al., 2013).

Here we use tabulated optical conductivity data at 300 K (Ordal et al., 1983; Shiles et al., 1980; Brändli and Sievers, 1972; Golovashkin and Motulevich, 1968) to obtain a unique parameter set of $n/m$ and $\tau$ (figure 1); the process minimizes the joint standard deviation, which is calculated by adding up the fitting residuals associated with both the real and imaginary parts. Because lead and aluminum are good metals, we can assume equations [9] and [10] to be fulfilled and thus used the total conductivity without residual resistivity corrections.

For lead, we obtain $\tau = 4.08 \cdot 10^{-15}$ s/rad and $n/m = 4.23 \cdot 10^{58}$ m$^{-3}$kg$^{-1}$, which gives $\alpha_T = 6.3$ and $\alpha_{n/m} = 5.9$ (with the above value for $A'$). Using the same method for aluminum, we obtain $\tau = 1.10 \cdot 10^{-14}$ s/rad and $n/m = 1.13 \cdot 10^{59}$ m$^{-3}$kg$^{-1}$, which gives $\alpha_T = 2.3$ and $\alpha_{n/m} = 2.2$. The good agreement between $\alpha_T$ and $\alpha_{n/m}$ indicates internal consistency. For both metals, this is about two times larger than the values obtained in (Bruin et al., 2013), pointing to some inaccuracy in the estimation of $n/m$ from quantum oscillation experiments even for simple metals.

For comparison, we also include published optical conductivity data of moderately doped silicon (van Exter and Grischkowsky, 1990) (figure 2). At room temperature, where the scattering in such samples is known to be dominated by electron-phonon scattering, the scattering times $\tau$ obtained from the Drude fits in (van Exter and Grischkowsky, 1990) are $1.33 \cdot 10^{-13}$ s/rad and $2.65 \cdot 10^{-13}$ s/rad for p- and n-type
samples of the carrier concentrations \(1.1 \cdot 10^{15} \text{ cm}^{-3}\) and \(0.42 \cdot 10^{15} \text{ cm}^{-3}\) and the effective masses \(0.37m_0\) and \(0.26m_0\), respectively ([van Exter and Grischkowsky, 1990]), where \(m_0\) is the free electron mass. This results in \(\alpha_\tau = 19.2\) and 9.6, respectively, at least an order of magnitude above the Planckian limit.

Our results for both simple metals and simple semiconductors thus indicate that the scattering of electrons by classical phonons is not subject to the Planckian dissipation bound of \(\alpha \approx 1\).

### 5 DRUDE CONTRIBUTION TO THE OPTICAL CONDUCTIVITY OF YBRH\(_2\)Si\(_2\)

Next we address the optical conductivity of the heavy fermion compound YbRh\(_2\)Si\(_2\). Strong correlations are known to push the Drude response of metals to low frequencies, frequently outside the range of standard spectrometers, making experiments on these materials challenging. When optical reflectivity measurements are used, the Kramers-Kronig transformation is needed to extract the optical conductivity. This may induce considerable uncertainty at low frequencies. These problems were overcome in a recent study on YbRh\(_2\)Si\(_2\) thin films grown by molecular beam epitaxy (MBE) and measured by terahertz (THz) time-domain transmission spectroscopy ([Prochaska et al., 2020]). The data were shown to exhibit dynamical scaling in the strange metal regime, which we will come back to in section 6. Here, we first examine whether a simple Drude description is possible in any frequency and temperature range such that the above Planckian dissipation analysis can be performed.

In figure 3A, we replot the real part of the frequency dependent intrinsic optical conductivity from [Prochaska et al., 2020] at different fixed temperatures between 20 and 150 K. The black lines are Drude fits to data at frequencies above 1.5 THz. At lower frequencies, pronounced deviations from Drude behavior are seen, which become more striking with decreasing temperature. Below 20 K, the data deviate so strongly from the Drude form that such fits lose any significance (and are therefore discarded). The extracted parameters \(n/m\) and \(1/\tau\) are shown as functions of temperature in figure 3B and C, respectively. The parameter \(n/m\) appears to saturate to \(2.3 \cdot 10^{56} \text{ m}^{-3} \text{ kg}^{-1}\) at the lowest temperatures, which corresponds to a plasma frequency of 0.54 eV. This is much larger than the expectation of about 45 meV for the background heavy Fermi liquid state, as estimated from \(\sqrt{Dk_BT_K}\) ([Millis and Lee, 1987], where \(D \sim 1 \text{ eV}\) is the

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Drude fits to the optical conductivity of (A) lead (Pb) and (B) aluminum (Al) at room temperature. The real and imaginary parts of the conductivity are represented as red and blue markers, respectively. The Drude fits are shown in black solid lines. See text for the results. The raw data were obtained from ([Ordal et al., 1983; Shiles et al., 1980; Brändli and Sievers, 1972; Golovashkin and Motulevich, 1968]).
Figure 2. Frequency dependence of the (A) real and (B) imaginary parts of the optical conductivity of silicon, weakly doped by holes ($1.1 \cdot 10^{15} \text{ cm}^{-3}$, open circles) or electrons ($4.2 \cdot 10^{14} \text{ cm}^{-3}$, solid circles). The experiments were performed at $T = 293$ K in the time domain; the full lines correspond to Drude fits (see text). Figure adapted from (Dressel and Grünert 2002), with original data and Drude fits from (van Exter and Grischkowsky 1990).

(bare) conduction electron bandwidth and $T_K = 25$ K is the single ion Kondo temperature of YbRh$_2$Si$_2$ (Troverelli et al. 2000). Using equation 13 with $A' = 1.42 \mu \Omega \text{cm}$, the slope of the linear-in-temperature dc resistivity of the MBE film used for the THz spectroscopy experiments, yields $\alpha_{n/m} = 0.64$. On the other hand, using equation 12 and the result for the scattering rate at 20 K, $1/\tau = 1.03 \cdot 10^{13} \text{ rad/s}$, yields $\alpha_\tau = 3.93$, which is more than a factor of 6 larger than $\alpha_{n/m}$. Compared to the cases of Pb and Al, the discrepancy between $\alpha_{n/m}$ and $\alpha_\tau$ is significant. Furthermore, the temperature dependence of the scattering rate is clearly at odds with that of a strange metal. We thus conclude that a Drude description fails to capture even the high-temperature and high-frequency onset of strange metal behavior in YbRh$_2$Si$_2$.

We summarize the results of the Planckian analyses of the Drude fits in figure 4, which is an expanded version of Fig. 4 of (Taupin and Paschen 2022). The results obtained there are included as dark grey squares and broad light grey lines. They were obtained by combining the effective mass (determined from the $A$ coefficient of the low-temperature dc electrical resistivity using the Kadowaki-Woods ratio) and the charge carrier concentration (from Hall effect measurements) with the $A'$ coefficient via equation 13. The squares represent the data point closest to the QCP, where the effective mass is largest. The broad lines represent the range of $A$ values measured away from the QCP. As discussed in (Taupin and Paschen 2022), it is clear from these results that the strange metal resistivity of quantum critical heavy fermion compounds cannot be understood as heavy quasiparticles undergoing Planckian dissipation. Also included are $\alpha_{n/m}$ data of selected materials obtained in (Bruin et al. 2013) using published quantum oscillation data (grey circles). The effective masses of these results are much smaller than those determined through the resistivity, suggesting that quantum oscillation experiments fail to detect the heaviest masses. The $\alpha$ values from (Bruin et al. 2013) thus do not represent the heavy (or heaviest) quasiparticles of the respective compounds, but only lighter ones.

The results of the present analyses are shown as colored symbols. The ambiguity of which effective mass and which charge carrier concentration to use is removed in this analysis because both $\tau$ and $n/m$ are obtained from the Drude fits. For the simple metals (Pb and Al) and semiconductors (p- and n-type Si) at room temperature, where the resistivities are governed by linear-in-temperature electron-phonon
Figure 3. (A) Drude fits to the real part of the intrinsic optical conductivity of YbRh$_2$Si$_2$ at various temperatures. The subtraction of the residual resistivity is described in (Prochaska et al., 2020). The fits (black solid lines) are anchored to the high-frequency range of the experimental data (red circles). (B) and (C) Temperature dependence of the ratio \(n/m\) and the scattering rate extracted from the Drude fits. The black dashed line is a guide to the eyes for extrapolating the \(n/m\) value to \(T = 0\).

scattering rates, \(\alpha_{n/m}\) and \(\alpha_\tau\) agree well (when both are available), confirming that the analysis is robust. At least some of the \(\alpha\) values are sizably larger than 1, showing that the Planckian bound is not obeyed in the phonon scattering case. For YbRh$_2$Si$_2$, there is a large discrepancy between \(\alpha_{n/m}\) and \(\alpha_\tau\), indicating that the analysis is not meaningful. Indeed, the Drude fits describe the optical conductivity only at high temperatures and frequencies, but not in the strange metal regime. The large \(\alpha\) values thus correspond to only weakly renormalized quasiparticles with Drude behavior. For UPt$_3$, for instance, the large effective mass renormalization found in transport and thermodynamic measurements was early on confirmed by optical conductivity measurements to be above 160 (Degiorgi et al., 1997), whereas the value found in (Bruin et al., 2013) is less than 50.

An interesting observation is that the value \(n/m = 2.2 \cdot 10^{56} \text{m}^{-3} \text{kg}^{-1}\) determined in (Prochaska et al., 2020) for the residual resistivity, which was obtained by using \(n = 2.6 \cdot 10^{28} \text{m}^{-3}\) from Hall effect measurements in (Paschen et al., 2004) and a deviation (\(\chi^2\)) minimization procedure performed within the dynamical scaling analysis of the data [see Fig. S5 of the Supplementary Material of (Prochaska et al., 2020)] that resulted in \(m/m_0 \approx 130\), is very similar to the low-temperature limit we obtained here (2.3 \(\cdot 10^{56} \text{m}^{-3} \text{kg}^{-1}\), see above) by fitting \(\text{Re}[\sigma_{\text{in}}(\omega)]\) with the Drude form. This suggests that the quasiparticles that undergo residual (likely elastic) scattering and those described by the low-temperature extrapolation of the Drude fits to \(\text{Re}[\sigma_{\text{in}}(\omega)]\) are the same particles. Their \(n/m\) ratios suggest only moderate mass renormalization (130 if the single-band interpretation of the Hall coefficient is trusted). The much larger mass renormalizations as extracted from the electronic specific heat coefficient \(\gamma\) or the linear-in-\(T^2\) resistivity coefficient \(A\) via the Kadowaki-Woods ratio, as done in (Taupin and Paschen, 2022), thus appear to be entirely dynamically generated, and to be part of the non-Drude regime in the optical conductivity even though extracted from Fermi liquid relations of the specific heat and dc resistivity.
Figure 4. Results of various Planckian analyses, in a double-logarithmic plot of $\alpha$ vs $(m/m_0)/n$. The dark grey squares and the broad light grey lines were obtained in (Taupin and Paschen, 2022) from low-temperature transport measurements. The dashed grey lines are extrapolations to the dashed black $\alpha = 1$ line of Planckian dissipation. The circles are $\alpha_{n/m}$ values obtained in (Bruin et al., 2013) from quantum oscillation data. The red and blue symbols are $\alpha_{\tau}$ and $\alpha_{n/m}$, respectively, determined in the present work from Drude fits to published optical conductivity data (see text). The fits for Pb, Al, and Si are of good quality in the temperature range where the scattering rates are linear-in-temperature due to electron-phonon scattering. For YbRh$_2$Si$_2$, by contrast, Drude fits fail in the strange metal regime (below 10 K in the dc resistivity). The $\alpha$ values (stars) were therefore extracted from Drude fits to data above 20 K and 1.5 THz, where the quality is reasonable. The discrepancy between $\alpha_{\tau}$ and $\alpha_{n/m}$ is large in this case, which casts doubt on their significance.

6 OPTICAL CONDUCTIVITY IN THE STRANGE METAL REGIME OF YBRH$_2$SI$_2$

As seen from figure 3 and also demonstrated in (Prochaska et al., 2020), Re[$\sigma_{\text{in}}(\omega)$] of YbRh$_2$Si$_2$ shows pronounced deviations from Drude behavior at low temperatures and frequencies. These deviations appear to be related to the strange metal behavior of YbRh$_2$Si$_2$, evidenced early on by the low-temperature upturn of the electronic specific heat coefficient setting on somewhat below 30 K and the linear-in-temperature dc (intrinsic) resistivity $\rho_{\text{in}}$ below 10 K (Trovarelli et al., 2000). Indeed, at the lowest temperature of 1.4 K reached in the THz spectroscopy experiment (Prochaska et al., 2020), which corresponds to a frequency $k_B T/h = 0.03$ THz well below the lowest accessed frequency of 0.23 THz, Re[$\sigma_{\text{in}}(\omega)$] is linear in frequency just as $\rho_{\text{in}}$ is linear in temperature (figure 5).

One can ask whether the coefficients of the two linear dependences are related, perhaps in a similar way to what is expected for Fermi liquids (see section 7), namely

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{res}}} + a'(k_B T) + b'(h\omega). \quad (14)$$
From the data in figure 5 we determine the ratio of $a'/b'$ to be about 0.3. In analogy with the ratio $a/b = 4\pi^2$ predicted for the Fermi liquid case (section 7) one might have expected $a'/b' = \sqrt{a/b} = 2\pi$, which is sizably larger.

When the energy equivalents of temperature ($k_B T$) and frequency ($\hbar \omega$) are similar in magnitude, neither the temperature nor the frequency dependence alone can characterize the conductivity of a material. Here, testing scaling relationships in terms of frequency and temperature becomes of crucial importance. In figure 6 we show two such attempts. For the cuprate Bi$_2$Sr$_2$Ca$_{0.92}$Y$_{0.08}$Cu$_2$O$_{8+\delta}$ (van der Marel et al., 2003), the scaling behavior $\text{Re}(\sigma_{in}) \cdot \sqrt{\omega/\omega_0} = f(\hbar \omega/k_B T)$ is seen for $3k_B T < \hbar \omega < 30k_B T$ and a different one, $\text{Re}(\sigma_{in}) \cdot T = f(\hbar \omega/k_B T)$, is observed in the relatively narrow range $0.7k_B T < \hbar \omega < 1.7k_B T$ (figure 6A). In YbRh$_2$Si$_2$, the latter scaling relationship holds in the entire accessed part of the material’s strange metal regime (figure 6B). The linear temperature exponent is consistent with the linear-in-temperature strange metal behavior of the dc resistivity and indicates its dynamical nature. It will be extremely important to see whether the same law holds also for $\hbar \omega < k_B T$. This will require microwave experiments.

## 7 RELATION OF TEMPERATURE AND FREQUENCY IN THE DRUDE MODEL

As a side aspect, we comment here on the relation between temperature and frequency in Fermi liquids. That a temperature dependent scattering rate should go along with a corresponding frequency dependence was realized early on, and formulated for the Fermi liquid case as (Gurzhi, 1959)

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{res}}} + a(k_B T)^2 + b(h \omega)^2 \quad \text{with} \quad \frac{a}{b} = 4\pi^2. \quad (15)$$

Surprizingly, according to (Scheffler et al., 2013), this relation has remained untested. The challenge is to resolve the frequency dependence of equation 15 in the Fermi liquid regime, i.e., at sufficiently low temperatures and frequencies. As shown in (Scheffler et al., 2013), this has not even been possible in heavy fermion compounds, where the prefactors are strongly enhanced (Kadowaki and Woods, 1986; Jacko et al., 2009). In a tour de force effort, the optical conductivity of MBE grown thin films of the heavy fermion compound UPd$_2$Al$_3$ was measured at low temperatures in the microwave regime (Scheffler et al., 2005).

![Figure 5](image-url)

**Figure 5.** DC and THz resistivity of YbRh$_2$Si$_2$. (A) The dc resistivity is linear in temperature. The slopes of the curve for an MBE film and a bulk single crystal are similar. (B) The inverse of the real part of the low-temperature intrinsic optical conductivity, $1/\text{Re}(\sigma_{in})$, of the MBE film is approximately linear in frequency. Figures from (Prochaska et al., 2020).
Figure 6. Energy over temperature scaling of the real part of the optical conductivity of (A) the cuprate $\text{Bi}_2\text{Sr}_2\text{Ca}_{0.92}\text{Y}_{0.08}\text{Cu}_2\text{O}_{8+d}$ and (B) an MBE film of $\text{YbRh}_2\text{Si}_2$. For the latter, the residual conductivity was subtracted as explained in (Prochaska et al., 2020). Note the different temperature and frequency ranges, above 100 K (above the compound’s superconducting transition temperature $T_c = 96$ K) and 1.5 THz (far infrared) in the former, and above 1.4 K and 0.23 THz (THz range) in the latter. Panel A is adapted from (van der Marel et al., 2003), panel B is taken from (Prochaska et al., 2020).

The data at 2.75 K (figure 7B), however, closely follow a simple Drude law, with a frequency independent scattering rate. As a consistency check, the magnitude of the third term in equation 15 was estimated from the $A$ coefficient of a bulk polycrystalline sample [$A = 0.23\mu\Omega\text{cm/K}^2$ (Dalichaouch et al., 1992)] which, assuming the validity of the Kadowaki–Woods ratio (Kadowaki and Woods, 1986), is consistent with the Sommerfeld coefficient $\gamma_0 = 150$ mJ/molK$^2$ (Geibel et al., 1991). The relative increase of the inverse optical conductivity (figure 7B) due to the third term was found to be well below the resolution limit of the experiment [less than 0.1% at 20 GHz (Scheffler, 2021)]. Using the somewhat larger value $A = 0.51\mu\Omega\text{cm/K}^2$ of a UPd$_2$Al$_3$ MBE film (Huth et al., 1993) would not have changed the conclusion.

Another heavy fermion compound on which optical conductivity measurements have been performed to rather low frequencies (10 GHz), and down to 1.2 K, is CeAl$_3$ (Awasthi et al., 1993). Fermi liquid behavior in the dc resistivity, with a very large $A$ coefficient of $35\mu\Omega\text{cm/K}^2$, was reported below 300 mK (Andres et al., 1975). The scattering rate (and the effective mass), extracted using an extended Drude model (see section 8), indeed show pronounced frequency dependence at low temperatures (Awasthi et al., 1993). Our analysis of these data reveals that, even at the lowest frequencies and temperatures, no clear $\omega^2$ dependence is seen. Assuming that the two lowest-frequency data points at 1.2 K represent the slope of an $\omega^2$ dependence, we estimate $a/b \approx 0.24$, much smaller than $4\pi^2$. Data at even lower frequencies and temperatures will be needed to confirm this result.

8 SUMMARY, DISCUSSION, AND OUTLOOK

In this perspective paper, we have examined published optical conductivity data of various materials to address the question whether strange metal behavior is captured by Planckian dissipation. The idea of using the optical conductivity instead of the dc resistivity, as was done in previous work (Bruin et al., 2013; Legros et al., 2019; Cao et al., 2020; Ghiotto et al., 2021; Grissonnanche et al., 2021; Mousatov and Hartnoll, 2021; Taupin and Paschen, 2022), was to remove the uncertainty created by estimating the charge carrier concentration and effective mass (or, more precisely, their ratio $n/m$). Using the Drude form of the
optical conductivity can, in principle, overcome this problem because here the scattering time $\tau$ enters not only in a product with $n/m$ (as in the dc conductivity $\sigma = n e^2 \tau/m$) but also in a second term. This allows one to fit $n/m$ and $\tau$ independently. The ratio $\alpha$ of the experimental (inelastic) scattering rate $1/\tau$ and the Planckian scattering rate $1/\tau_P = k_B T/h$ can then be determined.

We tested the method with high-temperature data of simple metals and semiconductors in the range where the resistivity is governed by a linear-in-temperature scattering rate due to electron-phonon scattering, and found it to be reliable. Notably, $\alpha$ can be obtained in two different ways, directly from $\tau_m$ (which we then call $\alpha_{\tau}$) or by combining $n/m$ and the linear-in-temperature dc resistivity coefficient $A'$ (which we call $\alpha_{n/m}$), and both gave very similar results for the simple metals.

We then attempted to apply the method to the extreme strange metal YbRh$_2$Si$_2$. However, as already pointed out in Prochaska et al. (2020), the Drude form of the optical conductivity fails to describe the data at low temperatures and frequencies, which we would have deemed the most appropriate range to characterize the compound’s strange metal state. To achieve reasonable fits, the fitting range has to be constrained to sufficiently high temperatures and frequencies. In this case, a much more modest mass renormalization results than what is obtained by using the dc resistivity at low temperatures, even at tuning parameter values far away from the quantum critical point (Taupin and Paschen, 2022). In consequence, also much larger $\alpha$ values are found. The large discrepancy between $\alpha_{\tau}$ and $\alpha_{n/m}$ resulting from this procedure points to the need for alternative approaches—not based on dc or ac Drude models—to classify strange metal behavior.

Some researchers have used the “extended” Drude model (Dressel and Grüner, 2002) to describe optical conductivity data of strongly correlated electron systems. It assumes a complex frequency-dependent relaxation rate $1/\tau(\omega) = 1/\tau_1(\omega) + i/\tau_2(\omega)$ and, via

$$\frac{1}{\tau_1(\omega)} = \frac{1}{\tau^*(\omega)} \cdot \frac{m^*(\omega)}{m} \quad \text{and} \quad \frac{1}{\tau_2(\omega)} = \omega \cdot \left[1 - \frac{m^*(\omega)}{m}\right],$$ (16)
a frequency-dependent effective mass related to it. When introduced into equation 2, this can be brought into the generalized Drude form

\[
\sigma(\omega) = \frac{ne^2\tau^*(\omega)}{m^*(\omega)} \frac{1}{1 - i\omega\tau^*(\omega)}.
\]

(17)

Plotted and analysed are typically the quantities

\[
\frac{1}{\tau_1(\omega)} = \frac{ne^2}{m} \frac{\sigma_1(\omega)}{|\sigma(\omega)|^2}
\]

(18)

and

\[
\frac{m^*(\omega)}{m} = \frac{ne^2}{m} \frac{\sigma_2(\omega)/\omega}{|\sigma(\omega)|^2},
\]

(19)

but this requires the knowledge of the (unrenormalized) plasma frequency

\[
\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}},
\]

(20)

where \(\epsilon_0\) is the vacuum permittivity. At low frequencies and temperatures, residual scattering may play an important role and should be subtracted, as described in section 2. The charge carrier concentration \(n\) is assumed to be frequency independent in this approach, which may not be true in heavy fermion strange metals (Paschen et al., 2004; Shishido et al., 2005; Friedemann et al., 2010; Custers et al., 2012; Jiao et al., 2015; Martelli et al., 2019) and related materials (Anisimov et al., 2002; Oike et al., 2015; Badoux et al., 2016; Cao et al., 2018; Jia et al., 2020). More generally, in a strange metal, any a priori assumption on \(n\), \(m\), and \(\tau\) limits the generality and may bias the conclusions. Forcing (extended) Drude forms to the data bears the risk to overlook the essential physics. Strange metals may be governed by exotic excitations or even the absence of any well-defined quasiparticles (Si et al., 2001; Coleman et al., 2001; Senthil et al., 2004; Phillips, 2011; Chang et al., 2018; Patel and Sachdev, 2018; Komijani and Coleman, 2019; Banerjee et al., 2021; Cai et al., 2020; Cha et al., 2020; Guo et al., 2020; Balm et al., 2020; Lee, 2021; Else and Senthil, 2021; Wang et al., 2022; Caprara et al., 2022), so they defy description by the above models and require alternative approaches.

We consider scaling analyses of the (intrinsic) optical conductivity to be the most promising way forward. Here, it will be essential to access, with high-resolution data, both the regime \(\hbar\omega/(k_B T) > 1\) and the regime \(\hbar\omega/(k_B T) < 1\). In strange metals with low energy scales, such as the heavy fermion compounds, the latter will require challenging low-temperature experiments in the microwave regime. When high-quality thin films are available, this can in principle be achieved with the broadband Corbino technique (Scheffler and Dressel, 2005), as demonstrated for the Fermi liquid heavy fermion compound UPd₂Al₃ (Scheffler et al., 2005) (figure 7). We note that even in state-of-the-art THz experiments on MBE grown thin-films of YbRh₂Si₂ (Prochaska et al., 2020), this regime was not accessed. This calls for future studies to advance the field.

CONFLICT OF INTEREST STATEMENT

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.
AUTHOR CONTRIBUTIONS

XL performed the Drude analyses, SP conceived the work and wrote the paper, with input from XL, JK, and QS. All authors contributed to the discussion.

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