Probing Gluonic Spin-Orbit Correlations in Photon Pair Production

Jian-Wei Qiu\textsuperscript{1,2}, Marc Schlegel\textsuperscript{3} and Werner Vogelsang\textsuperscript{3}

\textsuperscript{1}Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA
\textsuperscript{2}C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA and
\textsuperscript{3}Institute for Theoretical Physics, Tübingen University, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

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We consider photon pair production in hadronic collisions at large mass and small transverse momentum of the pair, assuming that factorization in terms of transverse-momentum dependent parton distributions applies. The unpolarized cross section is found to have azimuthal angular dependencies that are generated by a gluonic version of the Boer-Mulders function. In addition, the single-transversely polarized cross section is sensitive to the gluon Sivers function. We present simple numerical estimates for the Boer-Mulders and Sivers effects in diphoton production at RHIC and find that the process would offer unique opportunities for exploring transverse-momentum dependent gluon distributions.

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Introduction. — Hard hadronic processes with small transverse momentum $q_T$ of an observed final-state system have attracted a lot of interest because of their sensitivity to intrinsic parton transverse momenta. Such processes may hence offer detailed insights into the partonic substructure of hadrons, in terms of transverse-momentum dependent parton distributions (TMDs). Of particular interest are correlations of the parton transverse momentum with the nucleon or quark spin, which are expressed by the Sivers [1] and Boer-Mulders (BM) [2] functions. From these, one ultimately hopes to learn about spin-orbit correlations and orbital angular momenta of partons confined in a nucleon. So far, the main focus of the field has been on quark TMDs. This is due to the fact that quark TMDs are primarily probed in semi-inclusive deep-inelastic scattering (SIDIS) and the Drell-Yan (DY) dilepton production process, which have been accessible experimentally [3]. On the theoretical side, the relative simplicity of these two reactions has allowed to derive factorization theorems involving TMDs [4, 5].

Gluon TMDs [6] and processes sensitive to them have received closer attention only quite recently, at least for cases where nucleon or gluon polarization matter. Several processes for accessing the Sivers [7, 8] gluon distribution, or the gluonic version of the BM function (more appropriately described as the TMD distribution of linearly polarized gluons in an unpolarized nucleon) [9, 10], have been proposed for high-energy hadronic collisions, in particular, at the Relativistic Heavy Ion Collider (RHIC), or for $ep$ scattering at a future Electron Ion Collider (EIC). There is a generic dilemma concerning the processes considered so far: in cases where experiments can be carried out at today’s hadron colliders, factorization is known to be broken for TMDs [11], or to hold at best for weighted asymmetries that only give information on integrated TMDs with certain transverse momentum weights. On the other hand, while transverse-momentum dependent factorization is expected to hold for reactions such as $ep \to ccX$ or $ep \to jetX$ [9], realization of an EIC is still a decade or so away.

In this Letter we argue that the process $pp \to \gamma\gamma X$ can be used to study spin-dependent gluonic TMDs in a theoretically clean process already at RHIC. In proposing this process, we are motivated by the following observations. First of all, since the final state is a color singlet, the diphoton process is expected to share many features with the DY process, as far as factorization is concerned. Indeed, like the DY process, its lowest-order contribution comes from $qq$ annihilation, $qq \to \gamma\gamma$, which can be shown to give rise to the same Wilson lines as the DY subprocess $qq \to \gamma^*$, and hence involves the same quark and antiquark TMDs. Second, it has been known for a long time that in the spin-averaged case [12] at colliders photon pair production is in fact dominated by the process $gg \to \gamma\gamma$, that is, gluon-gluon fusion to a photon pair via a quark box. Even though this process is formally down by two powers of the strong coupling constant $\alpha_s$ with respect to $q\bar{q} \to \gamma\gamma$, the suppression is compensated by the structure of the associated hard-scattering function, and by the size of the gluon distribution function. Hence, an experimental study of gluon TMDs should in principle be possible in this process. Finally, in order to study TMDs, precise measurement of the (small) transverse momentum of a final state is crucial. It seems to us that this should be easier to achieve for a photon pair than, for example, for the jet pair in the reaction $ep \to jetX$.

Being a background to a possible Higgs boson decay into two photons, QCD diphoton production has received a lot of attention in theoretical studies, in particular, for the diphoton pair transverse-momentum distributions based on perturbative all-order resummation of Sudakov logarithms [13, 14]. In fact, these studies pointed out that the resummation formalism naturally suggests the presence of gluon TMDs, among them a perturbative spin-flip distribution akin to the gluonic BM function. In our present Letter, we examine the diphoton process...
entirely from the point of view of TMD factorization. Focusing on the gluonic Sivers and BM functions, we restrict ourselves to the application of an effective tree-level TMD formalism in the spirit of Refs. [15, 16]. At present, we are not able to present a proof that TMD factorization indeed holds for this process. Given the color-singlet nature of the final state and its similarity to DY kinematics, it appears plausible that such a factorization could be established if \( Q \sim p_T \gg q_T \), where \( Q \) (\( q_T \)) is the photon pair mass (transverse momentum) and \( p_T \) the transverse momentum of one photon. We hope that our study will motivate work in this direction.

Measurements of diphoton production have been carried out at the Tevatron [17]. Detection of diphoton signals should be well feasible in polarized \( pp \) collisions at RHIC [18] – statistics for the reaction will depend of nals should be well feasible in polarized antiquark annihilation, a detailed discussion of this issue to a future publication.

Kinematics. — We analyze the diphoton process \( h(P_a) + h(P_b) \rightarrow \gamma(q_a) + \gamma(q_b) + X \) in the center-of-mass (c.m.) frame of the incoming hadrons with momenta \( P_a^\mu = \sqrt{s}/2 \left[ 0, 1, \vec{0}_T \right] \) and \( P_b^\mu = \sqrt{s}/2 \left[ 1, 0, \vec{0}_T \right] \), where we used the light-cone notation \( a^\pm = [a^-, a^+, \vec{a}_T] \), with \( a^\pm = (a^0 \pm a^3)/\sqrt{2}, \vec{a}_T = (a^1, a^2), \) and \( S = (P_a + P_b)^2 \). The expressions for the photon momenta are simplified for \( q_T \ll Q [16] \)

\[
q^\mu_{a(b)} = \frac{\sqrt{2}}{S} \left[ x_a \frac{1+\cos \theta}{\sqrt{2}}, x_a \frac{1-\cos \theta}{\sqrt{2}}, \pm \sqrt{x_a x_b} \sin \theta \vec{e}_\phi \right],
\]

where the upper (lower) sign in the above expression refers to photon \( a (b) \), \( x_{a/b} = q^2/(2P_{a/b} \cdot q) \) with the photon pair momentum \( q = q_a + q_b \), and the spatial orientations of the photons are fixed by their angles \( \theta, \phi \) in the Collins-Soper (CS) frame [4, 16]. The Lorentz transformation between the c.m. frame and CS-frame has been worked out in Ref. [16] for the (kinematically identical) DY process \( hh \rightarrow \ell^+ \ell^- X \). In Eq. (1), \( \vec{e}_\phi = (\cos \phi, \sin \phi) \). Additional azimuthal dependence may be introduced by transverse spin vectors of the hadrons, \( \vec{S}_{a/bT} = (\cos \phi_a, \sin \phi_a) \). The partonic Mandelstam variables expressed in the c.m.-frame read \( s = 2k_a \cdot k_b = Q^2, t = -2k_a \cdot q_a = -Q^2 \sin^2 \frac{\theta}{2} \) and \( u = -2k_b \cdot q_a = -Q^2 \cos^2 \frac{\theta}{2} \), where \( k_{a/b} \) are the incoming parton momenta.

Photon pair production in \( q\bar{q} \) annihilation. — At lowest order, photon pairs are produced through quark-antiquark annihilation, \( q\bar{q} \rightarrow \gamma\gamma \). By following the same steps of the DY calculation in [16], we find for \( q_T \ll Q \)

\[
\frac{d\sigma^{q\bar{q} \rightarrow \gamma\gamma}}{d^2q d\Omega} \bigg|_{q_T \ll Q} = \frac{2}{\sin^2 \theta} \frac{d\sigma^{q\bar{q} \rightarrow t\bar{t}^{\pm}}}{{d^2q d\Omega}} \bigg|_{q_T \ll Q} (e_q^2 \rightarrow e_q^2),
\]

where the expression for \( d\sigma^{q\bar{q} \rightarrow t\bar{t}^{\pm}} \) can be found in Ref. [16]. In Eq. (2), the overall factor \( 2/\sin^2 \theta \) is caused by the fact that the process \( q\bar{q} \rightarrow \gamma\gamma \) proceeds via \( t \) and \( u \) channels while the DY process is via the \( s \)-channel. As pointed out above, the diphoton production and DY processes share the same quark and antiquark TMDs because both have only initial state interactions.

Gluon TMDs and photon pair production. — Gluon TMDs are defined through the correlator [6]

\[
\Gamma_{\mu\nu;\lambda_0}(x, k_T) = \frac{1}{x F^{+}_T} \int \frac{d^2z d^2k_T}{(2\pi)^2} e^{ik_T z} <(P, S) F^a_{\mu\nu}(0) W_{ab}(0; z) F_b^{\lambda_0}(z) |P, S)>_{z^+ = 0},
\]

which is a diagonal hadronic matrix element of two field strength tensors \( F^{\mu\nu} \) between nucleon states with large momentum component \( P^+ \) and spin vector \( S \). The Wilson line \( W_{ab} \) with color indices \( a, b \) in the adjoint representation makes the correlator gauge invariant [5, 11]. However, the explicit form of the Wilson line depends on the color structure of partonic scattering that the gluon TMDs are convoluted with. For the \( gg \rightarrow \gamma\gamma \) subprocess of photon pair production, we have all gluon TMDs with past-pointing Wilson lines.

The leading terms in a \( 1/P^+ \) expansion of \( \Gamma_{\mu\nu;\lambda_0} \) are given by \( \Gamma^{i;i+j} \) with transverse indices \( i, j \in \{1, 2\} \). For an unpolarized \( (U) \) or a transversely polarized \( (T) \) hadron of mass \( M \) one has the following decompositions of the correlator \( \Gamma^{i;i+j} \) into gluon TMDs [6, 19]:

\[
\Gamma^{i;i+j}_U(x, k_T) = \frac{\lambda_{ij}^U}{2} f_1^q + \frac{\lambda_{ij}^{T \perp}}{2} \frac{1}{2M^2} \kappa_{ij}^q S^T + \frac{\lambda_{ij}^{T \parallel}}{2} \frac{1}{2M^2} \kappa_{ij}^q S^T + \frac{\lambda_{ij}^{T \perp}}{2M^2} \kappa_{ij}^q S^T + \frac{\lambda_{ij}^{T \parallel}}{2M^2} \kappa_{ij}^q S^T + h_{1,1}^g \},
\]

\[
\Gamma^{i;i+j}_T(x, k_T) = \frac{\lambda_{ij}^{T \perp}}{2} \frac{\epsilon_T^{ij} \epsilon_T^{ij}}{2M^2} f_1^q + \frac{\lambda_{ij}^{T \parallel}}{2} \frac{\epsilon_T^{ij} \epsilon_T^{ij}}{2M^2} f_1^q + \frac{\lambda_{ij}^{T \perp}}{2M^2} \kappa_{ij}^q S^T + \frac{\lambda_{ij}^{T \parallel}}{2M^2} \kappa_{ij}^q S^T + h_{1,1}^g \},
\]

where \( \epsilon_T^{ij} \equiv e^{-ij} + d_i d_j \) and \( d_i d_j \). Each of the TMDs in (4) is a function of \( x \) and \( k_T^2 \). The unpolarized correlator \( \Gamma_U \) contains two gluon TMDs, the unpolarized gluon distribution \( f_1^g \) and the gluonic BM function \( h_{1,1}^g \). \( \Gamma_T \) is parameterized by the gluon Sivers function \( f_1^g \), and several other gluon TMDs [6]. Among these, \( h_{1,1}^g \) is the gluon "pretzelosity" TMD.
The leading order diagrams for $gg \to \gamma\gamma$ are shown in Fig. 1. Helicity amplitudes for them have been presented in Refs. [20, 21]. While these could be combined in a way suitable for projecting onto transverse external gluon indices, we choose to compute the diagrams directly, defining “semicontracted” amplitudes $\mathcal{M}_{\pm \pm}^{(1)} \equiv \mathcal{M}_{\rho \rho}^{(1)} \left( \varepsilon_+^a(q_a) \right)^*\left( \varepsilon_-^b(q_b) \right)^*$, with transverse gluon indices $i, j$, and contracted with photon polarization vectors $\varepsilon_{\pm}(q_{a/b})$. To perform the loop integrals, we use standard Mellin-Barnes space methods. The semicontracted amplitudes $\mathcal{M}_{\pm \pm}^{(1)}$ are finite. We obtain

$$
\mathcal{M}_{\pm \pm}^{(1)} = -K \left( \varepsilon_+^a \varepsilon_-^b - \varepsilon_+^a \varepsilon_-^b + f_s \varepsilon_+^a \varepsilon_-^b \right),
$$

$$
\mathcal{M}_{\pm \mp}^{(1)} = K \left( f_t \varepsilon_+^a \varepsilon_-^b + f_u \varepsilon_+^a \varepsilon_-^b + \varepsilon_-^a \varepsilon_+^b \right),
$$

with $\varepsilon_{\pm}^a = (1, \mp i) e^{\pm i\theta}$, $K \equiv 2g_\alpha \alpha_s \alpha_{em} \sum_q e_q^2$, where $e_q$ denotes the fraction of the charge of the quark in the box in units of the elementary charge $e$ and $a, b$ are the gluon adjoint color indices. Furthermore, $f_t \equiv L(t, u)$, $f_s \equiv L(s, u)$, $f_u \equiv L(s, t)$, with $L$ defined as

$$
L(x, y) = 1 - \frac{1}{4\pi x} \left[ \ln \left( |x| - i\pi \theta \left( -\frac{y}{x} \right) \right) \right] + \frac{x^2 + y^2}{(x^2 + y^2)^2} \left( \pi^2 + \ln \left( |x| - i\pi \theta \left( -\frac{y}{x} \right) \right) - 2 \right).
$$

Note that we have $f_s = -M_{\pm \mp}^{(1)}$, $f_t = -M_{\pm \pm}^{(1)}$, $f_u = -M_{\mp \mp}^{(1)}$ in terms of the helicity amplitudes of [21].

Our semicontracted amplitudes and the correlator $\Gamma^{+i;+j}$ of Eq. (3) can now be used to compute the cross section for $gg \to \gamma\gamma$ in the TMD formalism:

$$
\frac{\sigma^{gg}}{d^4q} = H \int d^2 k_T d^2 k_T' \delta^{(2)}(k_T + k_T' - q_T) \times
$$

$$
\Gamma^{+i;+j}(x_a, k_T) \Gamma^{-i;-j}(x_b, k_T') \sum_{\lambda_1, \lambda_2} \mathcal{M}_{\lambda_1 \lambda_2}^{(1)} \mathcal{M}_{\lambda_1 \lambda_2}^{(1)}^*,
$$

where $H = (128\pi^2) x_a x_b S^2 - 1$, and where we sum over the photon helicities.

Using the decomposition in Eq. (4) we derive from Eq. (7) the following result for the unpolarized and single transverse spin polarized cross sections in terms of the CS-frame angles:

$$
\frac{\sigma^{gg}}{d^4q} = \sigma_0^{gg} \left[ F_1(\theta) C + \sqrt{f_1^2} \right] + F_2(\theta) C \left[ w_5 h_{1g} h_{1g} \right] + \cos(2\phi) \left[ F_3(\theta) \left( C + \sqrt{f_1^2} \right) + C \left[ w_2 f_1 h_{1g} \right] \right] + \cos(4\phi) \left[ F_4(\theta) C \left[ w_4 h_{1g} h_{1g} \right] \right],
$$

$$
\frac{\sigma^{gg}}{d^4q} = \sigma_0^{gg} \left[ S_T \right] \sin \phi_a \left[ F_1(\theta) C \left[ w_3 f_{11g} f_{11g} \right] + F_2(\theta) \left[ C \left[ w_6 h_{1g} h_{1g} \right] + C \left[ w_7 h_{1g} h_{1g} \right] \right] + \ldots \right],
$$

where $\sigma_0^{gg} \equiv 2K^2 \mathcal{H}$, and where the ellipses denote additional terms that vanish upon $\phi$-integration. We have defined $F_1(\theta) = f_1^2 + \frac{1}{\rho} f_1^2 + \frac{1}{\rho} f_1^2 + 5$, $F_2(\theta) = 2(f_s - 1)$, $F_3(\theta) = f_s + \sqrt{f_1^2} - \frac{1}{\rho} f_1^2 + \frac{1}{\rho} f_1^2 + 2$, and

$$
\mathcal{C} \left[ w_1 f_2 \right] \equiv \int d^2 k_T d^2 k_T' \delta^{(2)}(k_T + k_T' - q_T) \times w_1(k_T, k_T') f_2(x_a, k_T^2) f_2(x_b, k_T'^2).
$$

Note that the angular structure of the unpolarized cross section shown in (8) is identical to that found in the context of collinear factorization for perturbative soft-gluon radiation from the LO process $gg \to \gamma\gamma$ [13, 14]. This may hint at a possible matching of the TMD and collinear formalisms in the intermediate $q_T$ region $\Lambda_{QCD} \ll q_T \ll \infty$. We also note that weighted cross sections of the form $\langle \mathcal{F} \rangle \equiv \int d^2 q_T d\phi \mathcal{F}(q_T, \phi) (\sigma/d^4q)$ may help in disentangling the various terms in (8) and (9). For instance, $(q_T^2 \cos(2\phi)) \propto h_{1g}^{(1)}(x_a) h_{1g}^{(1)}(x_b)$, and $(q_T^2 \cos(2\phi)) \propto h_{1g}^{(1)}(x_a) f_1^{(0)}(x_b)$, with $q_T$ the transverse momentum of $f_1^{(0)}$ and $h_{1g}^{(1)}$.

IV. Numerical estimates.—In order to estimate the size of the various contributions to (8), (9), we use a Gaussian model for the TMDs as frequently chosen for the analysis of SIDIS or DY data [3, 22]. For the unpolarized quark and gluon TMDs $f_1^{(0)}$ we make the ansatz

$$
f_1^{(0)}(x, k_T^2) = \frac{f_0^{(0)}(x)}{m^2} e^{-\frac{k_T^2}{m^2}}, \quad f_1^{(0)}(x, k_T^2) = \frac{G(x)}{m^2} e^{-\frac{k_T^2}{m^2}},
$$

with widths $\beta$ and $\gamma$ for which we assume $\beta = \gamma = 0.5$ GeV$^2$ at RHIC, and with the $k_T$-integrated parton distributions [23]. Very little is known about the other quark and gluon TMDs at RHIC energies. Model-independent positivity bounds for them were derived in Refs. [6, 24]. To estimate the maximally possible effects in the diphoton process we assume saturation of these positivity bounds for both quarks and gluons. For the gluon Sivers function this gives approximately

$$
|f_1^{(0)}(x) | \approx 0.5 f_1^{(0)}(x).
$$

Similarly the positivity bounds lead to the following approximations for the other TMDs: $|h_{1g}^{(1)}| \approx (2M^2)/k_T^2 f_1^{(0)}$, $|h_{1g}^{(1)}| \approx M/k_T f_1^{(0)}$, $|h_{1g}^{(1)}| \approx M/k_T f_1^{(0)}$ (with $h_{1g}^{(1)} = h_{1g}^{(1)} + k_T^2/(2M^2) h_{1g}^{(1)}$, and $|h_{1g}^{(1)}| \approx (2M^2)/k_T^2 f_1^{(0)}$).

In Fig. 2 we present our numerical estimates from our Gaussian ansatz. In generating those curves, we required each photon to have a transverse momentum of at least.
1 GeV, and we integrated over $4 \leq Q^2 \leq 30$ GeV$^2$, $0 \leq q_T \leq 1$ GeV, and the CS angles with appropriate azimuthal weightings. For the unpolarized cross section (upper panel), the $gg \to \gamma \gamma$ channel dominates at midrapidity while the $q\bar{q} \to \gamma \gamma$ channel is more important at forward/backward rapidity ($|y| > 2$) of the photon pair. The contribution by the gluon BM effect to the $\phi$-independent cross section turns out to be rather small. On the other hand, the $\cos 2\phi$ and $\cos 4\phi$ contributions induced by gluons could be at the percent level for TMDs saturated by the positivity bounds. Realistically, however, one may expect smaller effects depending on the actual size of the TMDs. In order to estimate the maximum size of quark Sivers contribution to the spin-dependent cross section, we kept only the positivity bound saturated up-quark Sivers function since the up- and down-quark Sivers functions have an opposite sign. From the lower panel in Fig. 2, it is important to note that the gluon Sivers effect exceed the quark Sivers effect by a factor seven or so at midrapidity, and dominates the contribution for a wide range of rapidity. That is, the single transverse spin asymmetry of the diphoton production at RHIC could offer excellent opportunities for exploring the gluon Sivers function. Other effects caused by the gluon TMDs $h_1^{+g}$ and $h_1^{-g}$ are negligible.

V. Conclusion. — We have investigated photon pair production in hadronic collisions in the framework of TMD factorization. We have shown that this process may be suited for studying gluon TMDs at RHIC. The $\cos(4\phi)$ modulation can be used to extract the gluon Boer-Mulders function. Even a small effect can be significant since this modulation is absent in the $q\bar{q}$ channel. The $\cos(2\phi)$ modulation ultimately gives information on the sign of $h_1^{+g}$. Such measurements may also be performed at the LHC where the production rate from gluon fusion is much larger. Another unique feature of the diphoton process is its sensitivity to the gluon Sivers function in polarized proton collisions. Measurements at RHIC could hence provide important clues about the correlation between gluon motion and hadron spin.

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