Formation of annular cracks in glass during contact interaction

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Abstract. Experimental results are presented on the indentation of steel balls of various diameters into glass samples having a rectangular parallelepiped shape. The ultimate load in the formation of an annular crack in the vicinity of the contact region and the radius of this crack were determined experimentally. The annular crack appeared outside the contact area almost in all tests. Due to the fact that the diameter of the contact area was much smaller than the dimensions of the samples, the glass samples were considered as an elastic half-space. Huber's solution for Hertz's problem of pressing a ball into an elastic half-space was used to determine the field of contact stresses in the fracture zone. Local criterion for maximum stresses and nonlocal failure criteria (average stress criterion, the Nuismer criterion, and the gradient criterion) were used to model fracture in a contact interaction. The ultimate tensile stress and the critical stress intensity factor of the glass were experimentally determined on beams without a notch and with a notch for calculating a parameter having a dimension of length and entering nonlocal failure criteria. It is shown that the estimates of the radius of the annular crack that are closest to the experimental data give a gradient criterion among the applied criteria. Estimates of the ultimate tensile stress according to this criterion exceed the values obtained when the beams are bent, which can be explained by the scale factor.

1. Introduction

According to the solution of the Hertz problem on the ball pressing into the elastic half-space [1-3], the maximum tensile stress is reached on the surface of the half-space on the boundary of the circular contact area and acts on the section perpendicular to the radius from the point of initial contact, that is, the radial stress. Consequently, according to the local criterion of maximum stresses, failure must begin at the boundary of the contact area. However A.S. Argon et al. [4] experimentally established that when the balls are pressed into glass samples, the radius of the annular crack on the surface of the samples is greater than the radius of the contact region. This circumstance is a motivation for the experimental investigation of brittle fracture in the vicinity of the contact region and the modeling of this failure using nonlocal failure criteria, taking into account the inhomogeneity of the contact stress field. It is necessary to know the stress field not only on the surface, but also inside the half-space for the application of these criteria. The stress distribution for the Hertz problem of pressing a ball into an elastic half-space was obtained by M.T. Huber [2].

The values of the limiting load on the ball in the formation of an annular crack and the radius of this crack will be determined in an experimental study. Estimates of the ultimate load in the formation of an annular crack and the radius of this crack will be obtained by modeling the failure in the zone of contact stresses. A comparison of the obtained estimates with the experimental data will be carried out...
and a conclusion will be made about the consistency of the fracture criteria with the experimental results. The main attention will be paid to the correspondence of the theoretical and experimental values of the radius of annular cracks. It is known that annular cracks appear outside the contact area from experiments [4-8]. Therefore, the theoretical estimation of the radius of cracks can also be checked by comparison with the radius of the contact area.

2. Experiments and methods

The real-time video method was used to determine the force at which a crack is formed. The tests were carried out on a Zwick / Roell Z100 machine at the Lavrentyev Institute of Hydrodynamics SB RAS. USB-microscope was installed under steel protection. The objective of the microscope is directed to the center of the through hole Ø10 mm in the cover of protection. The glass sample is placed on top of this 10 mm thick protection. The steel ball is pressed into this glass sample. The microscope transmits the image to the computer in real time, and the installed software allows to videotape and take photographs. The operator notices the formation of a crack on the computer monitor, stops the experiment and fixes the ultimate cracking force. Glass samples were in the form of rectangular parallelepipeds, having a height of 10 mm and sides of 20 mm. Results of experiments are discussed in detail in [9, 10].

The radius of ring-shaped cracks $r_c$ was determined using a measuring microscope. The radius of the contact region $a$ was calculated from the formula [3]:

$$a = \frac{3RP}{4E}.$$  

(1)

Here $R$ is the radius of the ball; $P$ is the force at which a crack is formed; $E^*$ is the effective modulus of elasticity upon contact of a steel ball with a glass sample, which is calculated from the known values of the elasticity modulus and Poisson's ratio of the steel $E_{st}$, $\nu_{st}$ and glasses $E_{gl}$, $\nu_{gl}$:

$$E^* = \left(1 - \nu_{gl}^2\right)\left(1 - \nu_{st}^2\right)^{-1}.$$  

(2)

For steel $E_{st} = 2.11 \times 10^{11}$ Pa, $\nu_{st} = 0.28$. For glass $E_{gl} = 7 \times 10^{10}$ Pa, $\nu_{gl} = 0.2$.

3. Results and discussion

The local criterion of maximum tensile stress applied to simulate the fracture. When the ball is pressed into an elastic half-space, the tensile radial forces take place in the contact vicinity. The maximum value of the radial stress on the surface of the half-space is concentrated at the points of the boundary circular contact region. Therefore, according to the local criterion of maximum stresses, failure must begin at the boundary contact area. The radial stress depends on the radius $r$ on the surface of the half-space outside the contact region in accordance with the formula [2, 5, 6]

$$\sigma_r = \frac{p_m}{\pi a^2} \left(1 - 2\nu\right) \frac{a^2}{r^2}.$$  

(3)

Here $p_m = \frac{P}{\pi a^2}$ is average pressure in the contact area; $\nu$ is Poisson's ratio.

The averaged experimental data obtained by indentation of balls with different diameters and the results of calculation from these data are summarized in Table 1. Figure 1 shows the dependence of the maximum tensile stresses at the surface of a half-space on the radius. Figure 1 depicts points whose values of coordinates are equal to the value of the limiting tensile stress of the glass obtained during the bending of the beams and the radii of annular cracks, respectively, for each of the ball’s diameters. The numbers of the curves and points in Figure 1 correspond to the numbering of the data in Table 1. Table 1 and Figure 1 show that, firstly, the maximum tensile stresses with ball pressing are an order of magnitude higher than the limiting tensile stress for glass obtained when bending the beams, and secondly, the radii of annular cracks are larger than the radii of the contact area.
Table 1. Averaged experimental data and calculation results.

| No | Diameter of the ball, Mm | Breaking Load, P, N | Radius of the annular crack, \( r_c \), mm | Radius of the contact area, \( a \), mm | Average pressure, \( p_m \), GPa | Maximum radial stress, \( \sigma_{max} \), MPa |
|----|--------------------------|---------------------|------------------------------------------|--------------------------------|-----------------------------|-------------------------------------|
| 1  | 5.5                      | 692                 | 0.376                                    | 0.281                          | 2.79                        | 837                               |
| 2  | 10                       | 717                 | 0.456                                    | 0.355                          | 1.81                        | 543                               |
| 3  | 17                       | 1115                | 0.670                                    | 0.498                          | 1.43                        | 429                               |

Figure 1. Dependencies of the maximum tensile stresses on the radius for different diameters of balls.

Taking into account the poor description of the experimental data by the criterion of maximum stresses, nonlocal failure criteria were used to estimate the ultimate tensile stress and the crack radius when the ball was pressed into glass samples. It is necessary to know the distribution of stresses not only on the surface, but also within the half-space for the application of these criteria. Formulas that can be used to calculate stresses at any point in a half-space for Hertz’s problem of ball indentation were obtained by Huber [2]. The radial stress in Huber’s solution is calculated by formula

\[
\sigma_r = \frac{3}{2} p_m \left[ \frac{1-2\nu}{3} \frac{a^2 z^3}{r^2} \left(1 - \frac{z^3}{u^3}\right) + \frac{a^2 z^3}{(u^2 + a^2 z^2)^{3/2}} + \frac{z}{(u^2 + a^2 z^2)^{3/2}} \left( \frac{1-\nu}{u^2 + a^2} + \frac{1+\nu}{u^2} \sqrt{u} \arctg \left( \frac{a}{\sqrt{u}} \right) - 2 \right) \right].
\] (4)

Here \( z \) is the coordinate, measured from the surface of the half-space along its normal;

\[
u = \frac{1}{2} \left( r^2 + z^2 - a^2 + \sqrt{(r^2 + z^2 - a^2)^2 + 4 a^2 z^2} \right).
\]

3.1. Estimation by the criterion of average stresses. According to this criterion, the ultimate strength of the material \( \sigma_b \) is compared not with the maximum positive value of the first principal stress \( \sigma_1 \), but with the average normal stress

\[
\sigma_{av} = \frac{1}{L_n} \int_0^L \sigma_n \, dz
\] (5)

which is calculated on area of size \( L_n \), including an infinitesimal area \( \sigma_n \) at the considered point of the body, where both areas have a common normal \( n \). The stress along the other side of the area is assumed to be constant. At the moment of the beginning of destruction \( \sigma_{av} = \sigma_b \). The size of the averaging area is given by formula [11, 12]

\[
L_n = \frac{2 K^2_s}{\pi \sigma_b^2}.
\] (6)
Here $K_c$ is the critical stress intensity factor. For sodium-calcium-silicate glass of grade M1, after annealing, the following was obtained: $K_c = 1.271 \text{ MPa m}^{1/2}$, $\sigma_s = 41.85 \text{ MPa}$. Then $L_* = 0.587 \text{ mm}$.

According to this criterion, the dependence of the average tensile stress on the radius is obtained upon indentation of balls of three diameters. In this problem, these are the dependences of the radial stress averaged over the segment $L_*$ on the radius, which are shown in Figure 2. In calculating the average radial stress, integration is performed over $z$ from 0 to $L_*$. The obtained dependences have maxima, the coordinates of which along the $r$ axis give estimates of the radius of annular cracks, and the values of the maxima give estimates of the ultimate tensile stress of the glass. The estimates obtained are shown in Table 2. Figure 2 shows the points which values of coordinates are equal to the value of the ultimate tensile stress of the glass obtained during the bending of the beams and the radii of annular cracks, respectively, for each of the diameters of the balls. The numbers of curves and points on the graph correspond to the numbering of the data in Table 2.

![Figure 2. Dependencies of the average radial stresses on the radius for different diameters of balls.](image)

**Table 2. Results predicted by the criterion of average stresses**

| No | Diameter of the ball | Estimate of the radius of an annular crack along the coordinate of the maximum $\sigma_{av}$, mm | Maximum average radial stress $\sigma_{av}^{\text{max}}$, MPa |
|----|----------------------|-------------------------------------------------|--------------------------------------------------|
| 1  | 5.5                  | 2.441                                           | 3.5                                              |
| 2  | 10                   | 2.430                                           | 3.6                                              |
| 3  | 17                   | 2.482                                           | 5.5                                              |

Table 2 and Figure 2 show that, firstly, the estimates of the ultimate tensile stress by the criterion of average stresses are an order of magnitude smaller than the ultimate tensile stress of the glass obtained when the beams are bent, and secondly, estimates of the radius of annular cracks are many times larger than the experimental values. Consequently, the criterion of average stresses poorly describes the experimental data on the destruction of glass when indenting steel balls.

3.2. **Estimation by the Nuismer failure criterion for stresses at a distant point.** In this criterion [11], the ultimate strength of material is not compared with the maximum tensile stress at the considered point of the body, but with the stress at the point considered at a distance $C_*$:

\[
\sigma_N = \sigma_r (r, z = C_*),
\]

\[
C_* = L_*/4.
\]

According to this criterion, the dependence of the distant stress on the radius is obtained by indenting balls of three diameters. In this problem, this is the dependence of the radial stress distant from the surface of a half-space on the distance $C_*$ on the radius, which are presented in Figure 3. The
obtained dependencies have maxima, the coordinates of which along the $r$ axis give estimates of the radius of annular cracks, and the values of the maxima give estimates of the ultimate tensile stress of the glass. The obtained estimates are shown in Table 3. Figure 3 shows the points which coordinates are equal to the value of the ultimate tensile stress of the glass obtained when the beams are bent and the radii of annular cracks for each of the ball diameters, respectively. The numbers of curves and points on the graph correspond to the numbering of the data in Table 3.

Table 3 and Fig. 3 show that, firstly, the estimates of the ultimate tensile stress by Nuismer's failure criterion are lower than the ultimate tensile stresses of the glass obtained when the beams are bent, and secondly, estimates of the radius of annular cracks are larger than the experimental values. Consequently, the Nuismer criterion does not satisfactorily describe the experimental data on the destruction of glass when pressing steel balls.

Table 3. Results predicted by the Nuismer's failure criterion

| No | Diameter of the ball | Estimate of the radius of an annular crack along the coordinate of the maximum $\sigma_N$, mm | Maximum average radial stress $\sigma_{max}^{\sigma_N}$, MPa |
|----|----------------------|-------------------------------------------------|---------------------------------------------------|
| 1  | 5.5                  | 1.321                                           | 12.6                                              |
| 2  | 10                   | 1.357                                           | 12.6                                              |
| 3  | 17                   | 1.442                                           | 18.2                                              |

3.3. *Estimation by the gradient failure criterion.* In the gradient criterion [12-14], the strength of the material $\sigma_0$ is compared not with the maximum stress, but with an effective stress $\sigma_e$ for determining the start of failure. The effective stress is proportional to the first principal stress $\sigma_1$ at the considered point of the body and depends on the relative gradient $g_n$ of the positive normal stress $\sigma_n$ acting perpendicular to the plane including the area of the first principal stress at the considered point of the body where the plane and the area have a common normal $n$:

$$g_n = \frac{|\text{grad} \sigma_n|}{\sigma_n},$$

(9)

The relative gradient is found using the solution of the corresponding elasticity problem. The expression for the effective stress is written in the form:

$$\sigma_e = \frac{\sigma_1}{1 - \beta + \sqrt{\beta^2 + L \cdot g_n}},$$

(10)

where $L$ is a parameter having the dimension of length, which is expressed in terms of the known characteristics of the material $\sigma_0$ and $K_{ic}$ by formula (6); $\beta$ is a non-negative dimensionless parameter ($\beta \geq 0$), which can be considered as an approximation parameter. It is believed that the destruction in
the vicinity of the consideration point begins when the effective stress is reaches the material ultimate strength and initially spreads over the action area of the first principal stress, and the failure condition is written in the form:

$$
\sigma_y = \sigma_y.
$$

(11)

For \( z = 0 \), on the surface of the half-space in the vicinity of the contact region, the first principal stress is the radial stress \( \sigma_r \), which is used in the gradient criterion. We use formulas (3) and (4) to calculate the radial stress and its derivatives with respect to the coordinates \( r \) and \( z \). Next, determine the relative gradient of the radial stress and the effective stress. We find on the surface of the half-space from (3):

$$
\frac{\partial \sigma_r}{\partial z} = \frac{p_m}{r^3} (1-2v)a^2.
$$

Find the derivative \( \frac{\partial \sigma_r}{\partial z} \), using the expression (4):

$$
\frac{\partial \sigma_r}{\partial z} = \frac{3}{2} p_m \left[ 1 - \frac{2v}{3} a^2 \frac{z^2}{r^2} + \frac{3}{2} \frac{\partial u}{\partial z} \right] + \frac{3}{2} a^2 z^2 \left( \frac{u^{1/2}}{a u^{1/2}} + \frac{z}{2a u^{1/2}} \frac{\partial u}{\partial z} \right) \left( a^2 + u \right) + \left( 1 + \frac{v}{2} \right) \left( \frac{\partial u}{\partial z} \right) \left( 1 + \frac{r^2 + z^2 - a^2}{u^{1/2}} \right) \right] \left( a^2 + u \right) \right]
$$

When \( z = 0 \) and \( r \neq a \) then \( \frac{\partial u}{\partial z} = 0 \) and \( \frac{\partial \sigma_r}{\partial z} = \frac{3}{2} p_m \left[ \frac{(1-v)\sqrt{a}}{a^2 + u} + \frac{1+v}{a} \frac{\partial u}{\partial z} \right] - \frac{2v}{3} a^2 \frac{z^2}{r^2} \left( a^2 + u \right) \right]

$$
\sigma_y \left( \frac{1}{2} \right) \left( r^2 - a^2 + \sqrt{r^2 - a^2} \right)^2.
$$

The relative gradient is calculated from formula:

$$
g_y = \left| \frac{\text{grad} \sigma_y}{\sigma_y} \right| = \sqrt{\left( \frac{\partial \sigma_y}{\partial r} \right)^2 + \left( \frac{\partial \sigma_y}{\partial z} \right)^2}.
$$

(12)

Substituting formula (12) into (10), we find the effective stress at \( \beta = 0 \). In accordance with the gradient criterion, the dependencies of the effective stress on the radius are plotted for indentation of balls of three diameters in Figure 4. These dependencies have maxima, the coordinates of which along the \( r \) axis give estimates of the radius of annular cracks, and the values of the maxima give estimates of the ultimate tensile stress of the glass. The obtained estimates are shown in Table 4. Figure 4 shows the points whose coordinates are equal to the value of the ultimate tensile stress of the glass obtained during the bending of the beams and the radii of annular cracks, respectively, for each of the diameters of the balls. The numbers of curves and points on the graph correspond to the numbering of the data in Table 4.

![Figure 4. Dependencies of the effective stresses on the radius for different diameters of balls.](image-url)
Table 4. Results predicted by the gradient failure criterion

| No | Diameter of the ball | Estimate of the radius of an annular crack along the coordinate of the maximum $\sigma_e$, mm | Maximum average radial stress $\sigma_{\text{max}}, \text{MPa}$ |
|----|---------------------|-----------------------------------------------|-------------------------------------------------|
| 1  | 5.5                 | 0.322                                         | 102.6                                           |
| 2  | 10                  | 0.407                                         | 73.4                                            |
| 3  | 17                  | 0.568                                         | 66.6                                            |

Table 4 and Figure 4 show that, firstly, estimates of the ultimate tensile stress with respect to the gradient criterion are higher than the ultimate tensile stress of glass obtained from the bending of beams, which can be explained by the scale factor, and secondly, estimates of the radius of annular cracks are close to experimental data, but less than them. Thus, the gradient criterion is better describing experimental data on the destruction of glass when indenting steel balls than the other considered criteria.

4. Conclusion
The series of experiments were carried out to indentation of steel balls of different diameters into glass samples in the form of a rectangular parallelepiped. The ultimate forces in the formation of annular cracks in the vicinity of the contact region and the radii of these cracks were determined experimentally. Almost in all tests, annular cracks appeared outside the contact area. Since the diameter of the contact region was much smaller than the dimensions of the samples, the glass samples were considered as an elastic half-space. Huber’s solution of Hertz’s problem of indentation ball into an elastic half-space was used to determine the field of contact stresses in the fracture zone. Local maximum stress criterion and non-local failure criteria: the criterion of average stresses, the Nuismer criterion and the gradient criterion were used to simulate contact fracture of the samples. It is shown that the gradient criterion is most close to the experimental data from the all considered failure criteria. Estimates of the ultimate tensile stress according to this criterion exceed the values obtained in the bending of beams, which can be explained by the scale factor.

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