Screening Masses in $SU(2)$ Pure Gauge Theory

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Abstract

We perform a systematic scaling study of screening masses in pure gauge $SU(2)$ theory at temperatures above the phase transition temperature. The major finite volume effect is seen to be spatial deconfinement. We extract the screening masses in the infinite volume and zero lattice spacing limit. We find that these physical results can be deduced from runs on rather coarse lattices. Dimensional reduction is clearly seen in the spectrum.

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In a recent paper [1] we examined the spectrum of screening masses at finite temperature in four dimensional $SU(2)$ and $SU(3)$ pure gauge theories. Our primary result was that dimensional reduction could be seen in the (gauge invariant) spectrum of the spatial transfer matrix of the theory. In addition, we had shown that the specific details of the spectrum precluded any attempt to understand it perturbatively. In this paper we present a complete set of non-perturbative constraints on the effective dimensionally reduced theory [2, 3, 4] at a temperature ($T$) above the phase transition temperature ($T_c$) for the $SU(2)$ case in the zero-lattice spacing and infinite volume limit.

The study of screening masses is interesting for two reasons. First, they are crucial to phenomenology because they determine whether the fireball obtained in a relativistic heavy-ion collision is large enough for thermodynamics. Second, the problem of understanding screening masses impinges on several long-standing problems concerning the infrared behaviour of the $T > T_c$ physics of non-Abelian gauge theories.

It is known that electric polarisations of gluons get a mass in perturbation theory, whereas magnetic polarisations do not. Long ago, Linde pointed out [5] that $T > 0$ perturbation theory breaks down at a finite order due to this insufficient screening of the infrared in non-Abelian theories. The most straightforward way to cure this infrared divergence would be if the magnetic polarisations also get a mass non-perturbatively. There have been recent attempts to measure such a mass in gauge-fixed lattice computations [6].

It was found long back that the solution could be more complicated and intimately related to the dynamics of dimensionally reduced theories. Jackiw and Templeton analysed perturbative expansions in massless and super-renormalisable three dimensional theories [7] and found that subtle non-perturbative effects screen the infrared singularities in such theories. In a companion paper, Applequist and Pisarski discussed the possibility that such effects might, among other things, also give rise to magnetic masses [8]. In fact the recent suggestion of Arnold and Yaffe that non-perturbative terms and logarithms of the gauge coupling may be important in an expansion of the Debye screening mass in powers of the coupling [9] may be seen as an example of such non-perturbative effects. The generation of the other screening masses are also non-perturbative. We discuss these issues further after presenting our main results.

In this paper we report our measurements of the screening masses in the
infinite volume and zero lattice spacing limit of $SU(2)$ pure gauge theory at temperatures of $2-4T_c$. We found a strong finite volume movement of one of the screening masses due to spatial deconfinement. However, the lack of finite volume effects in the remaining channels allowed us to extract infinite volume results from small lattices. The effect of a finite lattice spacing turned out to be small. We were able to pin down all the available screening masses with an accuracy of about 5%.

It is necessary to set out our notation for the quantum numbers of the screening masses. The transfer matrix in the spatial direction, $z$, has the dihedral symmetry, $D_h^4$ of a slice of the lattice which contains the orthogonal $x$, $y$ and $t$ directions. The irreducible representations (irreps) are labelled by charge conjugation parity, $C$, the 3-dimensional ($x, y, t$) parity, $P$, and the irrep labels of $D_4$ (four one-dimensional irreps $A_{1,2}$, $B_{1,2}$ and one two-dimensional irrep $E$). In $SU(2)$ gauge theory, only the $C = 1$ irreps are realised; hence we lighten the notation by dropping this quantum number.

Dimensional reduction implies the following pair-wise degeneracies of screening masses—

$$m(A_1^P) = m(A_2^{-P}), \quad m(B_1^P) = m(B_2^{-P}), \quad m(E^P) = m(E^{-P}). \quad (1)$$

After this reduction, the symmetry group becomes $C^4_v$ on the lattice and $O(2)$ in the continuum. The latter group has two real one-dimensional irreps— $0_+$ and $0_-$. The first comes from the $J_z = 0$ components of even spin irreps of $O(3)$, and the second from the $J_z = 0$ components of the odd spins. There are also an infinite number of real two dimensional irreps, $M$, corresponding to the $J_z = \pm M$ pair coming from any spin of $O(3)$. Dimensional reduction associates the irreps of $D_h^4$ with those of $O(2)$ according to

$$m(0_+) = m(A_1^1), \quad m(0_-) = m(A_1^{-1}), \quad m(1) = m(E), \quad m(2) = m(B_1^+) = m(B_1^-). \quad (2)$$

The final double equality is valid only when all lattice artifacts disappear. Although $O(2)$ has an infinite tower of states, only these four masses are measurable in a lattice simulation of the $SU(2)$ theory. Some of the equalities in eqs. (1,2) may be broken by dynamical lattice artifacts.

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3There has been a first attempt to disentangle these lattice effects and measure the higher irreps [10].
We studied these artifacts using “torelon” correlators \([11]\). These are correlation functions of Polyakov loop operators in the spatial \((P_x \text{ and } P_y)\) and temporal \((P_t)\) directions. \(P_t\) and \(P_x + P_y\) transform as the scalar \((A_1^+)\) of \(D_4^h\), whereas \(P_x - P_y\) transforms as \(B_1^+\). At zero temperature, a major part of finite volume effects in masses can be understood (for moderate \(mL\)) in terms of torelons.

The status of the \(A_1^+\) torelon, \(P_t\), at \(T > 0\) is very different from that at \(T = 0\). Here, \(P_t\) is the order parameter for the phase transition, and its correlations have genuine physical meaning— giving the static quark-antiquark potential, and hence defining the Debye screening mass, \(M_D\). This is identical to \(m(A_1^+)\) obtained from the Wilson loop operators \([12]\). In this respect, the finite temperature theory is nothing but a finite size effect.

We believe that the major part of finite volume effects in screening masses can be understood in terms of finite temperature physics. In simulations of \(N_t \times L^2 \times N_z\) lattices at a given coupling \(\beta\), when the transverse direction, \(L\), is small enough, the spatial gauge fields are deconfined. The spatial torelons \(P_{x,y}\) are order parameters for this effect. In general, large lattices, \(L/N_t \gg T/T_c\) have to be used to obtain the thermodynamic limit. Below this limiting value of \(L\), we should find strong finite volume effects, but only in the \(A_1^+\) and \(B_1^+\) sectors. When such effects can be directly measured, the \(B_1^+\) loop mass is expected to be twice the torelon mass. Whether or not similar effects are seen in the \(A_1^+\) sector depends on whether the spatial torelon mass is less than \(M_D/2\). If it is, then finite volume effects should be strong, otherwise not. We look upon torelons as convenient probes of finite volume effects, not their cause.

We have studied screening masses for \(SU(2)\) gauge theory on \(N_t \times L^2 \times N_z\) lattices with \(N_z = 4N_t\) at a temperature of \(T = 2T_c\). We studied two series of lattices, one for \(N_t = 4\) and another for \(N_t = 6\). For the first, we took \(L = 8, 10, 12\) and \(16\). For the second, we chose \(L = 16, 20\) and \(24\). For \(N_t = 4\), a temperature of \(2T_c\) is obtained by working with \(\beta = 2.51\). On \(N_t = 6\), the choice \(\beta = 2.64\) gives \(T = 2T_c\). The choice of lattice sizes allowed us to investigate finite volume as well as finite lattice spacing effects at constant physics.

We have also carried out measurements at \(T = 3T_c\) and \(4T_c\). Since our measurements at \(2T_c\) showed that lattice spacing effects are quite small for \(N_t = 4\), we restricted ourselves to this size at higher temperatures. At \(3T_c\), we worked with a \(4 \times 24^3\) lattice. At \(4T_c\), we supplemented our earlier
measurements on small \((4 \times 8^2 \times 16\) and \(4 \times 12^2 \times 16\) \() lattices \([1]\) with measurements on \(4 \times 24^3\) and \(4 \times 32^3\) lattices. For \(N_t = 4\), temperatures of \(3T_c\) and \(4T_c\) are attained by working at \(\beta = 2.64\) and \(2.74\), respectively.

We used a hybrid over-relaxation algorithm for the Monte-Carlo simulation, with 5 steps of OR followed by 1 step of a heat-bath algorithm. The autocorrelations of plaquettes and Polyakov loops were found to be less than two such composite sweeps; hence measurements were taken every fifth such sweep. We took \(10^4\) measurements in each simulation except on the \(6 \times 24^3\) lattice where we took twice as much, and the \(4 \times 32^3\) lattice where we took 4000.

Noise reduction involved fuzzing. The full set of loop operators measured on some of the smaller lattices can be found in \([1]\). Since analyses of subsets of these operators gave identical results, we saved CPU time on the larger lattices by measuring a smaller number of operators. The full matrix of cross correlations was constructed, between all operators at all levels of fuzzing, in each irrep. A variational procedure was used along with jack-knife estimators for the local masses. Torelons were also subjected to a similar analysis.

Our measurements at \(2T_c\) for \(N_t = 4\) are reported in Table 1. We can measure torelons for fairly large values of \(L/N_t\). Twice the \(A_1^+\) spatial torelon screening mass is greater than that obtained from \(P_t\). Hence \(m(A_1^+)\) obtained from loops is equal to the latter and therefore shows no finite volume effect. The \(A_1^+\) and \(B_1^+\) spatial torelons have equal screening masses. The \(B_1^+\) loop screening mass closely equals twice the \(B_1^+\) torelon mass, and hence shows a systematic dependence on \(L\). Finite volume effects are absent in all the other channels, as expected. For the \(L = 16\) lattice for \(N_t = 4\), the torelon is not measurable, and finite volume effects are under control. At this largest volume dimensional reduction and continuum physics is visible since the equalities in eqs. \((1,2)\) are satisfied.

We have investigated finite lattice spacing effects by making the same measurements at the same physical temperature on lattices with \(N_t = 6\). The measurement of \(m(E^-)\) turns out to be rather noisy. Since we had observed on the coarser lattice that \(m(E^+) = m(E^-)\), we saved on CPU time by dropping the measurement of the \(E^-\) screening mass on the \(N_t = 6\) lattices.

Our results on the finer lattice are collected in Table 2. Again, dimensional reduction and continuum physics is visible because the equalities in eqs. \((1,2)\) are satisfied on the largest lattice.

From the data collected in Tables 1 and 2 it is clear that the physical ratio
$$L = 8 \quad L = 10 \quad L = 12 \quad L = 16$$

| Operators | $L = 8$ | $L = 10$ | $L = 12$ | $L = 16$ |
|-----------|---------|---------|---------|---------|
| $A_1^+$   | $0.71 \pm 0.05$ | $0.73 \pm 0.06$ | $0.69 \pm 0.04$ | $0.73 \pm 0.05$ |
| $A_1^-$   | $1.14 \pm 0.02$ | $1.01 \pm 0.03$ | $1.02 \pm 0.02$ | $0.99 \pm 0.02$ |
| $B_1^+$   | $1.18 \pm 0.03$ | $1.45 \pm 0.05$ | $1.62 \pm 0.06$ | $1.73 \pm 0.08$ |
| $B_1^-$   | $1.9 \pm 0.2$   | $1.9 \pm 0.1$   | $1.8 \pm 0.1$   | $1.8 \pm 0.1$   |
| $B_2^+$   | $1.8 \pm 0.1$   | $1.8 \pm 0.1$   | $1.75 \pm 0.05$ | $1.76 \pm 0.09$ |
| $B_2^-$   | $1.9 \pm 0.2$   | $1.9 \pm 0.2$   | $1.8 \pm 0.2$   | $1.9 \pm 0.2$   |
| $E^+$     | $1.8 \pm 0.07$  | $1.8 \pm 0.1$   | $1.68 \pm 0.07$ | $1.7 \pm 0.2$   |
| $E^-$     | $1.8 \pm 0.07$  | $1.8 \pm 0.1$   | $1.68 \pm 0.07$ | $1.7 \pm 0.2$   |

Table 1: Values of $m$ on $4 \times L^2 \times 16$ lattices in units of inverse lattice spacing, $1/a = 4T$ at $T = 2T_c$. Local masses could be followed to distance $z \approx 4/m$. Blanks in the table mean that the operators were not measured, and an entry of "-" denotes that the measurements were too noisy to yield a screening mass.
| Operators | $L = 16$ | $L = 20$ | $L = 24$ |
|-----------|---------|---------|---------|
| Loops     |         |         |         |
|           | $A_1^+$ | 0.46 ± 0.02 | 0.51 ± 0.01 | 0.51 ± 0.02 |
|           | $A_1$   | 0.69 ± 0.01 | 0.68 ± 0.02 | 0.68 ± 0.02 |
|           | $B_1^+$ | 0.98 ± 0.03 | 1.07 ± 0.06 | 1.08 ± 0.09 |
|           | $B_1^-$ | 1.23 ± 0.06 | 1.17 ± 0.07 | 1.19 ± 0.08 |
|           | $B_2^+$ | 1.23 ± 0.08 | 1.23 ± 0.09 | 1.22 ± 0.06 |
|           | $B_2^-$ | 1.28 ± 0.09 | 1.27 ± 0.05 | 1.25 ± 0.11 |
|           | $E^+$   | 1.25 ± 0.03 | 1.3 ± 0.1 | 1.24 ± 0.08 |
| Torelon   | $P_t$   | 0.46 ± 0.02 | 0.52 ± 0.02 | 0.51 ± 0.02 |
|           | $P_x + P_y$ | 0.32 ± 0.17 | - | - |
|           | $P_x - P_y$ | 0.37 ± 0.16 | - | - |

Table 2: Values of $m$ on $6 \times L^2 \times 24$ lattices in units of the inverse lattice spacing $1/a = 6T$ at $T = 2T_c$. Local masses could be followed to distance $z \approx 4/m$. An entry of “-” in the table denotes that the measurements were too noisy to yield a screening mass.

$m/T$ is the same with both lattice spacings, for loop masses. Hence finite lattice spacing effects are under control. This result is consistent with zero temperature lattice measurements which show that at these lattice spacings, ratios of physical quantities are independent of the spacing.

In Figure 1 we illustrate the nature of the finite volume effects. The lack of movement in $m(A_1^+)$, $m(A_1^-)$ and $m(B_1^+)$ is obvious. We have used the fact of dimensional reduction to prune the amount of data that has to be displayed in the graph. Note that the data show that $m(2)$ can be estimated by measuring any of the $B$ irrep screening masses, apart from the $B_1^+$, at small volumes. Note also that $m(B_1^+)/T$ scales with either $L/N_t$ or $L/N_z$. However, for $N_t = 6$, it becomes difficult to measure the torelon correlator at fairly small value of $L/N_t$. This supports our earlier statement that the torelon is a measure, not the cause, of finite volume effects.

In the SU(3) pure gauge theory, which has a first order phase transition, the simple equalities $N_z/N_t$, $L/N_t > T/T_c$ are sufficient to remove finite volume effects. The observed slow finite volume movement of $m(B_1^+)$ is special to the SU(2) gauge theory, which has a second order finite temperature phase transition. As a result, the above constraints on the lattice sizes
Figure 1: Values of $m/T$ using different operators as a function of $L/N_t$ (or $L/N_z$), the scaled transverse spatial size of the lattice. The open symbols are for $N_t = 6$ lattices, filled symbols for $N_t = 4$. In order to improve visibility, the data at $L/N_t = 4$ for $N_t = 6$ has been displaced slightly to the right.

are compounded by two separate systematics of second order phase transitions. The first is that there are precursor effects which cause masses to decrease at temperatures less than $T_c$; the second that part of this decrease is power-law singular in $N_z$ at fixed $\beta$. Consequently, in the SU(2) theory we can at best state the more stringent conditions $L/N_t = N_z/N_t \gg T/T_c$.

In our measurements with $N_t \times N^3$ lattices at higher temperatures, we found that the lattice artifact in $m(B^+_1)$ persists for the largest values of $N_t/N_t$ that we had. At both the higher temperatures no other finite volume effects were seen within the precision of the measurements. As a result, we were able to estimate all the four screening masses listed in eq. (2). The ratios $m/T$ are seen to be approximately constant in this temperature range. This is illustrated in Fig. 2.

The four masses that we have extracted from simulations of the 4-d theory represent the maximum information available non-perturbatively to constrain the effective 3-d theory. We found it instructive to display the same data
Figure 2: Values of $m/T$ for $2T_c \leq T \leq 4T_c$, showing the near-constancy of the ratios in this temperature range. In order to improve visibility, the points for $m(\mathbf{1})/T$ have been shifted slightly to the left and those for $m(\mathbf{2})/T$ slightly to the right.

in Figure 3 as a plot of the ratio $m(0_+)/m(\mathbf{2})$ against $m(0_-)/m(\mathbf{2})$. The finite volume movement in these numbers is fairly large if the denominators are estimated through $m(B_1^+)$. However, as shown, the movement is much reduced if $m(B_1^-)$ is used as an estimator of $m(\mathbf{2})$. Since the continuum and thermodynamic limit is pretty well pinned down, the figure also serves well to compare the 4-d theory with different 3-d theories.

The point for the 3-d $SU(2)$ pure gauge theory in the infinite volume and for zero lattice spacing is shown in the figure. It is clear that this is not the appropriate effective theory. This result is expected, since a perturbative mode counting shows that the effective three dimensional theory must contain a gauge field and a scalar field that transforms adjointly under gauge transformations, and the scalar field does not decouple completely from the theory even at high temperatures.

The vertical bands in Figure 3 come from measurements in a 3-d $SU(2)$ gauge theory with a fundamental scalar in both the symmetric and Higgs
Figure 3: Finite size movement of ratios of screening masses with two different identifications of the screening mass $m(2)$. The physical point shown should be understood to have error bars compatible with those for the other points. The point for the 3-d pure gauge theory [13] is denoted by the cross. The vertical bands show the measured $1-\sigma$ ranges for the mass ratio $m(0_+)/m(2)$ in an $SU(2)$ scalar-gauge theory [14] in the symmetric and Higgs phases. It is not surprising that the ratio $m(0_+)/m(2)$ in either phase of this theory does not agree with our measurements at $2T_c$, in view of the arguments already presented.

In [3] a super-renormalisable 3-d theory of $SU(2)$ gauge fields and an adjoint scalar, with three couplings, was suggested as the effective theory. Matching two of these couplings in a perturbation expansion, the screening masses were computed through a simulation of the 3-d theory. It turned out that at couplings corresponding to a temperature of $2T_c$, $m(1)/m(0_+) = 1.6 \pm 0.2$ as opposed to the value $2.4 \pm 0.2$ that we measure. Whether better agreement can be obtained by fine-tuning the third coupling remains as a future exercise. If three couplings can be tuned to reproduce four masses, then this would vindicate the perturbative approach to matching espoused in [3].
However, until such a demonstration is made, there are questions whether this procedure is viable at $T \approx 2T_c$. A direct measurement suggests that the gauge coupling is larger than unity, $g^2/2\pi \approx 0.53$, even for $T = 2T_c$ [10]. A related statement has been made based on a recent study of the Debye mass—that higher orders in the perturbative series become numerically smaller only at $T \approx 10^7 T_c$ [17]. A similar statement comes from attempts to find the region of validity of the perturbative expansion of the free energy in a non-Abelian plasma [18], which give $T > 10^5 T_c$. It has recently been suggested [13] that effects associated with screening and damping should be resummed to all orders in $g$, if perturbation theory is to behave reasonably at $T \approx 2T_c$.

We have earlier concluded that the screening masses we observe cannot be obtained perturbatively [1]. The fact that our measurements show $m/T > 2\pi$ in some channels also indicates that the perturbative matching procedure may not be useful, since dimensional reduction works only if modes with energy $2\pi T$ or more decouple [2, 3].

There are alternatives to perturbation theory. One interesting method would be to use gauge invariant composites directly to construct the effective theory. Phenomenology of this kind was used long ago to examine lattice data on the energy density for $T > T_c$ SU(3) gauge theory [20]. A more sophisticated attempt of this kind was tried in [21], but needed the machinery of large-N theories to control the expansion.

The question of the compositeness of screening masses is closely related. Note that the 3-d adjoint Higgs, $A_t$, is in the 0−irrep of $O(2)$, and the 3-d gauge field, $A$, in the 1. The gauge invariant 0+ can be seen in correlations of the composite operators $O_1 = \text{Tr}(A_t^2)$ and $O_2 = \text{Tr}(A \cdot A)$, as well as higher dimensional operators. The gauge invariant 0−screening mass can be seen, for example, in correlations of $O_3 = \text{Tr}(A_t^3)$. The composite operators corresponding to the remaining gauge invariant screening masses can also be easily written down. Nadkarni had shown by explicit computation that $O_1$ and $O_2$ mix at order $g^4$, where $g$ is the 4-d gauge coupling [22]. Hence, the characterisation of $m(0_\pm)$ as being due to electric phenomena is a perturbative statement, and is more or less correct according to how large $g$ is at the temperature that concerns us. Similar problems occur in the other channels as well.

In summary, we identified the only source of large finite volume effects in the determination of screening masses at $T > T_c$ in SU(2) pure gauge theory. These are due to spatial deconfinement and can be conveniently studied using...
torelons. Finite lattice spacing effects turn out to be easy to control. We found that rather small and coarse lattices can be used to obtain a good measurement of the physical screening masses, provided one ignores the $B_1^+$ channel. Our best estimates are shown in Figure 2. Dimensional reduction, as expressed non-perturbatively in eqs. (1,2), is seen in the temperature range 2–4$T_c$.

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