Kondo effect driven by chirality imbalance

Daiki Suenaga,1,* Kei Suzuki,2,† Yasufumi Araki,2,† and Shigeiho Yasui3,§

1Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
2Advanced Science Research Center, Japan Atomic Energy Agency (JAEA), Tokai 319-1195, Japan
3Research and Education Center for Natural Sciences, Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan

(Dated: January 1, 2020)

We propose a novel mechanism of the Kondo effect driven by a chirality imbalance (or chiral chemical potential) of relativistic light fermions. This effect is realized by the mixing between a right- or left-handed fermion and a heavy impurity in the chirality imbalanced matter even at zero density. This is different from the usual Kondo effect induced by finite density. We derive the Kondo effect from both a perturbative calculation and a mean-field approach. We also discuss the temperature dependence of the Kondo effect. The Kondo effect at nonzero chiral chemical potential can be tested by future lattice simulations.

I. INTRODUCTION

The Kondo effect [1–5] is known as a phenomenon which occurs in metal including heavy impurities. It leads to drastic modifications of the transport properties of conducting (or itinerant) electrons at low temperature. While, in the conventional case, itinerant electrons are treated as nonrelativistic fermions, recent studies show that the Kondo effect can be realized also in systems with relativistic fermions.

One example of the Kondo effect realized in the relativistic system is the isospin Kondo effect. This effect can be induced near the Fermi surface of nucleons with a heavy hadron such as Σ or ¯ Σ existing as an impurity, where the non-Abelian SU(2) interaction between the light nucleon and the heavy hadron is supplied by an isospin exchange [6–9]. In the context of Quantum Chromodynamics (QCD) in which the non-Abelian SU(3) interaction is governed by the color interaction mediated by gluons, the so-called QCD Kondo effect which may be realized in quark matter composed of up and down (and also often strange) quarks with a heavy (charm or bottom) quark, has been studied in the literatures [6, 10–22]. Moreover, in the context of electron systems with Dirac/Weyl dispersion in solid states, called Dirac/Weyl semimetals, it has been seen that the vanishing density of states around the Dirac/Weyl points leads to an anomalous Kondo screening behavior, distinct from normal metals [14, 23–35].

In relativistic massless fermions, one of the interesting characteristics is their chirality i.e. the left-handed and right-handed degrees of freedoms. In this paper, we propose a novel type of Kondo effect: the Kondo effect driven by a chirality imbalance (or chiral chemical potential µ5). This is different from the “usual” Kondo effect induced on the Fermi surface but slightly different in the sense that it occurs even at zero chemical potential, µ = 0. We particularly study the Kondo effect at finite µ5 perturbatively and non-perturbatively: The former is accomplished by the renormalization group (RG) analysis at one loop, and the latter is by the mean-field analysis.

To investigate systems with µ5 will give a motivation for Monte Carlo (lattice) simulations of strongly correlated quantum systems such as the Kondo effect and quark-gluon dynamics, which is one of the promising tools to nonperturbatively study them. While Monte Carlo simulations with a finite chemical potential µ suffer from the sign problem, at finite chiral chemical potential µ5, the sign problem is absent [36] (also see Refs. [37–41]). Therefore, when the Kondo effects are induced by finite µ5, we expect that Monte Carlo simulations with µ5 would be promising for measuring the Kondo effect.

Our analyses can also be extended to Dirac/Weyl semimetals with energy splitting among Dirac/Weyl cones, such as in Weyl semimetals with broken inversion symmetry [42–44]. Such an effect may also be reproduced by Zeeman splitting of spin-degenerate Dirac cones in topological Dirac semimetals [45], such as Cd₃As₂ [46, 47].

This paper is organized as follows. In Sec. II, we consider the Kondo effect at finite µ5 from an effective Lagrangian and a perturbation calculation. In Sec. III, to study the Kondo effect in the nonperturbative region, we formalize a mean field approach, and show the phase diagram of the Kondo effect on the plane of temperature and µ5. Section IV is devoted to our conclusion and outlook.

* suenaga@mail.ccnu.edu.cn
† k.suzuki.2010@th.phys.titech.ac.jp
‡ araki.yasufumi@jaea.go.jp
§ yasuis@keio.jp
II. PERTURBATIVE APPROACH

In this section, we show the emergence of the Kondo effect at finite $\mu_5$ within a perturbative scheme, which can be signaled by existence of a Landau pole in the renormalization group (RG) flow for the effective coupling between a light fermion and a heavy fermion [48].

We start our discussion by the following Lagrangian to describe a scattering between a light fermion and a heavy fermion:

$$\mathcal{L} = \bar{\psi}(i\partial + \mu_5 \gamma_0 \gamma_5)\psi + \bar{\Psi}(i\partial - M_Q)\Psi + G(\bar{\psi}^a \gamma^\mu \psi)(\bar{\Psi}^a \gamma_\mu \Psi),$$  \hspace{1cm} (1)

in which $\psi$ and $\Psi$ denote the light fermion and heavy fermion fields, respectively. $\mu_5$ is the chiral chemical potential and $M_Q$ is the heavy fermion mass whose value is significantly larger than the typical scale of the theory. $t^a$ with an index $a = 1, \ldots, N^2 - 1$ is the generator of the $SU(N)$ group characterizing a non-Abelian interaction. In terms of the interaction manner between the light fermion and the heavy fermion, we have employed a vector-type contact interaction. $G > 0$ is the coupling constant. We notice that, in this section, we introduce the heavy fermion field ($\Psi$) as a Dirac spinor which includes an anti-particle as well as a particle component. However, later, we will take a limit of $M_Q \to \infty$ to describe the emergence of the Kondo effect in more transparent way.

The scattering amplitude between the light fermion and the heavy fermion up to one loop is of the form

$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(1)},$$  \hspace{1cm} (2)

where $\mathcal{M}^{(0)}$ and $\mathcal{M}^{(1)}$ are the amplitude at tree level and at one-loop level, respectively. Explicitly, $\mathcal{M}^{(0)}$ and $\mathcal{M}^{(1)}$ are obtained as

$$\mathcal{M}^{(0)} = G\bar{u}(p_f)t^a\gamma^\mu u(p_i)\bar{U}(q_f)t^a\gamma_\mu U(q_i),$$  \hspace{1cm} (3)

and

$$\mathcal{M}^{(1)} = \mathcal{M}^{(1a)} + \mathcal{M}^{(1b)},$$  \hspace{1cm} (4)

with

$$\mathcal{M}^{(1a)} = -iG^2T\sum_n \int \frac{d^3k}{(2\pi)^3}\bar{u}(p_f)t^a\gamma^\mu S_l(k)t^b\gamma_\nu u(p_i)\times\bar{U}(q_f)t^a\gamma_\nu S_h(q_i - k + p_f)t^b\gamma_\mu U(q_i),$$  \hspace{1cm} (5)

and

$$\mathcal{M}^{(1b)} = -iG^2T\sum_n \int \frac{d^3k}{(2\pi)^3}\bar{u}(p_f)t^a\gamma^\mu S_l(k)t^b\gamma_\nu u(p_i)\times\bar{U}(q_f)t^a\gamma_\nu S_h(q_i + k - p_f)t^b\gamma_\mu U(q_i),$$

respectively, which are diagrammatically indicated in Fig. 1. $u(p)$ and $U(q)$ are the Dirac wavefunctions for the light and heavy fermions, respectively, with $p = p_i$ ($p_f$) and $q = q_f$ ($q_i$) the initial (final) momenta. In Eq. (4), we have employed the imaginary-time formalism to take into account the finite temperature effect, so that the propagators $S_l(k)$ and $S_h(k)$ take the form

$$S_l(k) = \sum_{\epsilon_5 = \pm} P_\epsilon \Delta_l(i(\omega_n - i\epsilon_5\mu_5)), \hspace{1cm} (7)$$

and

$$S_h(k) = \tilde{\Delta}_h(i\omega_n), \hspace{1cm} (8)$$

with

$$\Delta_l(i(\omega_n - i\epsilon_5\mu_5)) = -\frac{i(-\omega_n + i\epsilon_5\mu_5)\gamma_0 + \vec{k} \cdot \vec{\gamma}}{\omega_n - i\epsilon_5\mu_5} - \frac{\omega_n^2 + |\vec{k}|^2 - M_Q^2}{\omega_n^2 + |\vec{k}|^2 + M_Q^2}, \hspace{1cm} (9)$$

and

$$\tilde{\Delta}_h(i\omega_n) = \frac{-i\omega_n \gamma_0 + \vec{k} \cdot \vec{\gamma} - M_Q}{\omega_n^2 + |\vec{k}|^2 - M_Q^2}.$$  \hspace{1cm} (10)

where $\vec{\gamma} = (\gamma^1, \gamma^2, \gamma^3)$ is the spatial components of the Dirac gamma matrices. In these expressions, $P_\pm = (1 \pm \gamma_5)/2$ is the right-handed or left-handed projection operator, and the Matsubara frequency is $\omega_n = (2n + 1)\pi T$ ($n = 0, \pm1, \pm2, \cdots$). The detailed calculation of the one loops in Eqs. (5) and (6) within the imaginary-time formalism is provided in Appendix A.

Before showing the results of Eq. (2), we notice some important points about the fermion wavefunctions $u(p)$ ($\bar{u}(p)$) or $U(q)$ ($\bar{U}(q)$). First, in terms of the light fermion wavefunction, it is useful to separate the light fermion transition part in Eq. (2) into the right-handed and left-handed ones by defining $u_R = P_+ u$ and $u_L = P_- u$, since the Lagrangian (1) preserves the axial current. Next, in terms of the heavy fermion wavefunction, as is well known, the free Dirac spinor can be decomposed into

$$U(q) = \Lambda_+ U(q) + \Lambda_- U(q) = U_+(q) + U_-(q),$$  \hspace{1cm} (11)

with $U_\pm(q) \equiv \Lambda_\pm U(q)$, by defining the projection operator with respect to the positive-energy (+) and negative-energy (−) solutions of the Dirac equation:

$$\Lambda_\pm = \frac{M_Q \pm (q_0 \gamma_0 - \vec{q} \cdot \vec{\gamma})}{2M_Q},$$  \hspace{1cm} (12)

with $q_0 = \sqrt{|\vec{q}|^2 + M_Q^2}$. When we measure the energy of the fermion from $M_Q$ as in the non-relativistic system,

---

1 A perturbative calculation of the QCD Kondo effect at finite $\mu_5$ was also done in an early work by Ozaki and Itakura (unpublished).

2 In the context of QCD, the interaction term in Eq. (1) can be motivated by a one-gluon exchange interaction between the light quark and the heavy quark with a large Debye mass, as demonstrated in Ref. [10].
i.e., by shifting the energy of the positive-energy and negative-energy components commonly as $q_0 \rightarrow q_0 - M_Q$, we need to cost at least $2M_Q$ for the excitation of the negative-energy component, which can be ignored in the limit of $M_Q \rightarrow \infty$. Therefore, when we consider such a situation, we can drop $U_-(q)$ in Eq. (11), and replace $U(q)$ by $U_+(q)$.

By taking the above arguments into account, the tree-level amplitude in Eq. (3) can be reduced to

$$\mathcal{M}^{(0)} = G \bar{u}_R(p_f)t^a\gamma_0u_R(p_i)\bar{U}_+(q_f)t^aU_+(q_i) + G \bar{u}_L(p_f)t^a\gamma_0u_L(p_i)\bar{U}_+(q_f)t^aU_+(q_i),$$

in which we have used a fact of $\bar{U}_+(q_f)t^a\gamma U_+(q_i) = 0$ with $M_Q \rightarrow \infty$. The one-loop amplitude $\mathcal{M}^{(1)}$ in Eq. (4) is calculated in detail in Appendix A. According to Eq. (A24), the resulting $\mathcal{M}^{(1)}$ is of the form

$$\mathcal{M}^{(1)} \approx \frac{G^2 N\rho_0}{2} \int_{-\mu_5}^{\infty} dE \frac{1 - \tilde{f}_\beta(E)}{E} \times \bar{u}_R(p_f)t^a\gamma_0u_R(p_i)\bar{U}_+(q_f)t^aU_+(q_i) + \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\mu_5 - |k|} \times \bar{u}_L(p_f)t^b\gamma_0u_L(p_i)\bar{U}_+(q_f)t^bU_+(q_i),$$

in the limit of $M_Q \rightarrow \infty$ with the initial- and final-state light fermions inhabiting the “Fermi surface”, i.e. the initial- and final-state light fermions satisfy the kinematics of $(p^0, |\vec{p}|) = (0, \mu_5)$ for the right-handed fermion while $(p^0, |\vec{p}|) = (2\mu_5, \mu_5)$ for the left-handed fermion (as $p^\mu$ stands for $p^0$ and $p^i$ collectively), due to the Dirac equation. $\tilde{f}_\beta(E)$ is the Fermi distribution function, $\tilde{f}_\beta(E) = 1/(e^{\beta E} + 1)$ with inverse temperature $\beta = 1/T$, and $\rho_0 = \mu_5^2/(2\pi^2)$ is the density of state on the Fermi surface.

From the above considerations, it turns out that Eqs. (13) and (14) lead to the RG equation [48] as

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{N\rho_0 G^2(\Lambda)}{4} (1 - \tilde{f}_\beta(\Lambda)),$$

for the coupling $G(\Lambda)$ of only the right-handed fermion, where the effective coupling $G(\Lambda)$ depends on the energy scale $\Lambda$ measured from the Fermi surface. Alternatively, the RG equation (15) can be converted into the dimensionless one as

$$\tilde{\Lambda} \frac{d\tilde{G}(\tilde{\Lambda})}{d\tilde{\Lambda}} = -\frac{NG(\tilde{\Lambda})^2}{8\pi^2} (1 - \tilde{f}_\beta(\tilde{\Lambda})),\tag{16}$$

by defining $\tilde{\Lambda} = \Lambda/\mu_5$, $G(\Lambda) = G(\Lambda)\mu_5^2$, and $\tilde{\beta} = \beta\mu_5$ ($\tilde{T} = T/\mu_5$). We comment that Eq. (16) is reduced to the simple form,

$$\tilde{\Lambda} \frac{d\tilde{G}(\tilde{\Lambda})}{d\tilde{\Lambda}} = -\frac{NG(\tilde{\Lambda})^2}{8\pi^2},\tag{17}$$

at $\tilde{T} = 0$.

The resulting RG flow of the dimensionless coupling $\tilde{G}$ with $N = 3$ is shown in Fig. 2. In this plot, the results with $T = 0$, $T = 0.02$, and $T = 0.2$ are shown. As an example, the initial values are taken to be $G_0 = 3$ at $\Lambda_0 = 0.2$. The results clearly show the logarithmic divergences at lower-energy scales and the emergence of the Landau poles at the energy scale $\Lambda = \Lambda_K$ lower than $\Lambda_0$ (or temperature), implying the appearance of the Kondo effect. We call $\Lambda_K$ the Kondo scale. This behavior is easily understood by the fact that the right-hand side of Eq. (16) is always negative. It is important to note that the Kondo scale is generated dynamically through the quantum processes.
accompanying the non-Abelian interaction\textsuperscript{3}. The existence of the Kondo scale is more clearly confirmed in the case of zero temperature ($T = 0$). In fact, from Eq. (17), we obtain the analytic form of the solution as

$$G(\bar{\Lambda}) = \frac{G_0}{1 + \frac{N G_0}{8 \pi^2} \log \frac{\Lambda}{\Lambda_0}}, \quad (18)$$

leading to

$$\bar{\Lambda}_K = \bar{\Lambda}_0 e^{-\frac{8 \pi^2}{N G_0}} \ll \bar{\Lambda}_0. \quad (19)$$

The last inequality indicates that the Kondo scale is the low-energy scale, so that it is exponentially smaller than the high-energy scale $\bar{\Lambda}_0$. At finite temperature, we notice that, as the temperature becomes higher, the value of $\Lambda_K$ becomes smaller. Thus, this behavior implies the suppression of the Kondo effect by finite temperature effects.

### III. MEAN-FIELD APPROACH

At the low-energy scale below the Kondo scale, we need to describe the Kondo effect in a nonperturbative way. For this purpose, we adopt a mean-field approach describing a mixing between a light relativistic fermion and a heavy fermion based on the treatment in Refs. [12, 16].

#### A. Mean-field Lagrangian

For the light relativistic fermions, we use the one-flavor light-fermion field $\psi$ with a chemical potential $\mu$ and a chiral chemical potential $\mu_5$.\textsuperscript{4} For the heavy fermions, we use a redefined field based on the so-called heavy-quark formalism to multi-flavor fermions, $\Psi \equiv (\psi_1, \psi_2, \cdots, \psi_{N_f})$ [12, 16].

\textsuperscript{3} If there is no non-Abelian interaction (or the generator $t^a$) in the Lagrangian (1), all the logarithmic divergences from $M(1)$ in Eqs. (14) are canceled, and hence the Kondo scale disappears.

\textsuperscript{4} The one flavor is a simplified setup, but we can easily extend our formalism to multi-flavor fermions, $\psi \equiv (\psi_1, \psi_2, \cdots, \psi_{N_f})$ [12, 16].

\textsuperscript{5} Note that the four-point interaction in Eq. (20) can be obtained by the Fiertz transformation from Eq. (1). See e.g., Refs. [12, 16]. Using the projection operators for the chirality of the light fermions, $\psi_R = \frac{1 + \gamma_5}{2} \psi$ and $\psi_L = \frac{1 - \gamma_5}{2} \psi$, we can easily check the chiral symmetry for the four-point interaction terms:

$$|\bar{\psi} \gamma_5 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_2 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_3 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_4 \psi|^2 = 2 \left[ |\bar{\psi} \gamma_5 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_2 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_3 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_4 \psi|^2 \right].$$

Dirac spinor of the heavy-fermion field survives by the projection operator $\frac{1}{2}(1 + \gamma_5)$. The original mass $M_Q$ is subtracted by the factor $e^{i M_Q v \cdot x}$.

As a result, the effective Lagrangian is given by

$$L = \bar{\psi} \left(i \hat{\cal D} + \mu \gamma_0 + \mu_5 \gamma_5 \right) \psi + \bar{\psi} \gamma_5 \left[i v^\mu \partial_\mu \Psi \right] + \bar{\Psi} \gamma_5 \left[i v^\mu \partial_\mu \bar{\psi} \right] + \bar{\psi} \gamma_5 \bar{\psi} \gamma_5 \psi + \bar{\psi} \gamma_5 \bar{\psi} \gamma_5 \psi$$

leading to

$$\bar{\Lambda}_K = \bar{\Lambda}_0 e^{-\frac{8 \pi^2}{N G_0}} \ll \bar{\Lambda}_0. \quad (19)$$

The last inequality indicates that the Kondo scale is the low-energy scale, so that it is exponentially smaller than the high-energy scale $\bar{\Lambda}_0$. At finite temperature, we notice that, as the temperature becomes higher, the value of $\Lambda_K$ becomes smaller. Thus, this behavior implies the suppression of the Kondo effect by finite temperature effects.

As a mean-field approximation, we assume the following form of the condensate, which is the so-called Kondo condensate [12, 16]:

$$G(\bar{\psi}_R \Psi_v) = \Delta_R, \quad \bar{G}(\bar{\psi}_L \psi_v) = \Delta_L, \quad (20)$$

$$G(\bar{\psi}_R \gamma_5 \psi_v) = \Delta_R \bar{p}, \quad \bar{G}(\bar{\psi}_L \gamma_5 \psi_v) = \Delta_L \bar{p}, \quad (22)$$

where $\bar{p} \equiv \bar{p}/p$ ($p \equiv |\vec{p}|$) is the unit vector for the three-dimensional momentum $\vec{p}$. The angle brackets $\langle O \rangle$ denote the vacuum expectation value for an operator $O$. Note that $\Delta_R(L)$ is a complex number, which indicates the mixing between the light fermion and the heavy particle. Thus, $|\Delta_R(L)|$ gives the absolute value of the Kondo condensate. From Eq. (20), as a result, the mean-field Lagrangian is written as

$$\mathcal{L}_{\text{MF}} = \bar{\phi} G(p_0, \bar{p})^{-1} \phi - \frac{2|\Delta_R|^2}{G} - \frac{2|\Delta_L|^2}{G} + \lambda n_Q \Psi_v^2 \quad (23)$$

where $\phi \equiv (\psi^t, (\Psi_{\text{pos}}^t)^t)$ contains the six components with the Dirac four-spinor of the light-fermion field $\psi$ and the positive-energy projected components (two-spinor) of the heavy-particle field $\Psi_v^t = (\Psi_{\text{pos}}^t, 0)$. The factor 2 in front of $|\Delta_R(L)|^2$ comes from the ansatz (21) and (22). The inverse propagator of $\phi$ is given by

\textsuperscript{6} Notice that $\bar{\Psi}_v = \Psi_v^t$ in the rest frame.

\textsuperscript{7} The Kondo effect for a single heavy particle within the same mean-field ansatz is formalized in Ref. [13].

\textsuperscript{8} The momentum dependence in Eq. (22) is called the hedgehog solution. We assumed the scalar and hedgehog condensate have the same value of $\Delta_R(L)$. 

---

\textsuperscript{3} If there is no non-Abelian interaction (or the generator $t^a$) in the Lagrangian (1), all the logarithmic divergences from $M(1)$ in Eqs. (14) are canceled, and hence the Kondo scale disappears.

\textsuperscript{4} The one flavor is a simplified setup, but we can easily extend our formalism to multi-flavor fermions, $\psi \equiv (\psi_1, \psi_2, \cdots, \psi_{N_f})$ [12, 16].

\textsuperscript{5} Note that the four-point interaction in Eq. (20) can be obtained by the Fiertz transformation from Eq. (1). See e.g., Refs. [12, 16]. Using the projection operators for the chirality of the light fermions, $\psi_R = \frac{1 + \gamma_5}{2} \psi$ and $\psi_L = \frac{1 - \gamma_5}{2} \psi$, we can easily check the chiral symmetry for the four-point interaction terms:

$$|\bar{\psi} \gamma_5 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_2 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_3 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_4 \psi|^2 = 2 \left[ |\bar{\psi} \gamma_5 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_2 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_3 \psi|^2 + |\bar{\psi} \gamma_5 \gamma_4 \psi|^2 \right].$$
\[ G(p_0, \vec{p})^{-1} \equiv \begin{pmatrix} p_0 + \mu & -\vec{p} \cdot \vec{\sigma} + \mu_5 \\ \vec{p} \cdot \vec{\sigma} - \mu_5 & -(p_0 + \mu) \end{pmatrix} \begin{pmatrix} \frac{\Delta_R}{2} (1 + \hat{p} \cdot \vec{\sigma}) + \frac{\Delta_L}{2} (1 - \hat{p} \cdot \vec{\sigma}) & \frac{\Delta_R}{2} (1 + \hat{p} \cdot \vec{\sigma}) - \frac{\Delta_L}{2} (1 - \hat{p} \cdot \vec{\sigma}) \\ \frac{\Delta_L}{2} (1 + \hat{p} \cdot \vec{\sigma}) + \frac{\Delta_R}{2} (1 - \hat{p} \cdot \vec{\sigma}) & p_0 - \lambda \end{pmatrix} \],

(24)

in the standard representation of the Dirac matrices.

**B. Dispersion relations**

By solving \( \det[G(p_0, \vec{p})^{-1}] = 0 \), we obtain the six energy-momentum dispersion relations

\[ E_{R\pm}(p) = \frac{1}{2} \left( p + \lambda - \mu_R \pm \sqrt{(p - \lambda - \mu_R)^2 + 8|\Delta_R|^2} \right), \]

(25)

\[ E_{L\pm}(p) = \frac{1}{2} \left( p + \lambda - \mu_L \pm \sqrt{(p - \lambda - \mu_L)^2 + 8|\Delta_L|^2} \right), \]

(26)

\[ \tilde{E}_R(p) = -p - \mu_R, \]

(27)

\[ \tilde{E}_L(p) = -p - \mu_L. \]

(28)

with \( \mu_{R,L} \equiv \mu \pm \mu_5 \). The four modes, \( E_{R\pm} \) and \( E_{L\pm} \), are the mixing modes (quasiparticles) between the light fermion and the heavy particle, which are induced by the nonzero value of the Kondo condensate \( \Delta_R, \Delta_L \). On the other hand, \( \tilde{E}_R \) and \( \tilde{E}_L \) are the decoupling antiparticle modes. The obtained dispersion relations and the wave functions lead to the quasiparticles, but they preserve the topological properties for the original massless Dirac fermions, where the Berry’s curvature induces the monopoles in momentum space [16].

A schematic figure of these dispersion relations is shown in Fig. 3. Among them, the quasiparticles with \( E_{R-} \) and \( E_{L-} \) are essential for the Kondo effect because the Kondo condensate is induced by the occupation of quasiparticles under \( E(p) = 0 \).

**C. Thermodynamic potential**

From the modes in Eqs. (25)-(28), the thermodynamic potential at finite temperature \( T \) is obtained as

\[ \Omega(T, \mu, \mu_5, \lambda; \Delta_R(L)) = N \int_0^{\Lambda^2_{\text{cut}}} f(T, \mu, \mu_5, \lambda; p) \frac{p^2 dp}{2\pi^2} \]

\[ + \frac{2|\Delta_R|^2}{G} + \frac{2|\Delta_L|^2}{G} - \lambda n_Q \]

(29)

where \( \Lambda_{\text{cut}} \) is an ultraviolet cutoff parameter of the momentum integral, and the integrand is

\[ f(T, \mu, \mu_5, \lambda; p) = -\frac{1}{2} \sum_{i=R,L} \left[ E_{i+}(p) + E_{i-}(p) + \tilde{E}_i(p) \right] \]

\[ -\frac{1}{\beta} \ln \left[ \prod_{i=R,L} \left( 1 + e^{-\beta E_{i+}(p)} \right) \left( 1 + e^{-\beta E_{i-}(p)} \right) \right] \]

(30)

From the minimization condition of Eq. (29) or the gap equation \( \partial \Omega / \partial \Delta_R = \partial \Omega / \partial \Delta_L = 0 \), we can determine \( \Delta_{R(L)} \) in a self-consistent way. In this model setting, the free parameters are \( \tilde{G} \) and \( \Lambda_{\text{cut}} \), and they can be tuned for a specific system, as it will be explained later.

**D. Numerical results**

The Kondo condensate \( \Delta_R \) as a function of \( \mu_5 > 0 \) is plotted in Fig. 4. Here we use, for example, \( \tilde{G} = 2 / \Lambda^2_{\text{cut}} \) and \( 4 / \Lambda^2_{\text{cut}} \) at \( N = 3 \). We find that \( \Delta_R \) is enhanced as \( \mu_5 \) increases. This behavior indicates that the (relativistic) Kondo effect is induced by finite \( \mu_5 \). This is consistent with the result from the perturbative analysis in Sec. II. We emphasize that the usual (nonrelativistic
and relativistic) Kondo effects occur at finite $\mu$, but the Kondo effect at finite $\mu_5$ appears even when $\mu = 0$. This is a unique property of relativistic fermions composing matter including impurities. Such Kondo effects can be realized in relativistic-fermion matter, i.e. Weyl/Dirac metal/semimetals and quark matter.

Furthermore, within our model, the phase transition along the $\mu_5$ axis is a crossover. This is the same result as the transition along the $\mu$ axis is also a crossover (see Appendix B).

Within our parameters, we numerically find that, for $\mu_5 > 0$, the Kondo effect is dominated by the right-handed condensate $\Delta_R$, and the value of the left-handed condensate $\Delta_L$ is almost zero. On the other hand, in the case of $\mu_5 < 0$, $\Delta_L$ dominates the Kondo effect.

For a typical parameter in the QCD Kondo effect, we apply the coupling constant, $G = G_c$, where $G_c \equiv 2/\Lambda_{\text{cut}}^2$ and $\Lambda_{\text{cut}} = 0.65$ GeV, and the number of the colors is $N = 3$. These parameters are the same as those used in the Nambu–Jona-Lasinio model with a four-point interaction between a light quark and a light antiquark [53]. When we use $G_c$, we find $\Delta_R = 7.9$ MeV at $\mu_5 = 0.5$ GeV. If we use a stronger coupling constant, the Kondo effect is increasingly enhanced, as shown by the blue curve in Fig. 4. Note that, if we extrapolate the results to $0.75 \lesssim \mu_5/\Lambda_{\text{cut}}$, then we find a sudden decrease of $\Delta_R$, but this behavior is an artifact from the cutoff $\Lambda_{\text{cut}}$ in our model.

We comment the possible setup on lattice QCD simulations. At finite $\mu$, the Monte-Carlo simulations suffer from the sign problem, so that it is difficult to measure the QCD Kondo effect (by finite $\mu$) by using lattice simulations. On the other hand, at finite $\mu_5$, we can escape from the sign problem [36–41], and the QCD Kondo effect (by finite $\mu_5$) will be observed.

Finally, we give a discussion on the temperature dependence of $\Delta_R$ at finite $\mu_5$. In Fig. 5, we show $\Delta_R$ on the $T$-$\mu_5$ plane. We observe that, when a finite $T$ is switched on, the value of $\Delta_R$ decreases: the Kondo effect is suppressed by finite-temperature effects, which is again consistent with the perturbative analysis in Sec. II. While the transition along the $\mu_5$ axis at $T = 0$ is a crossover, the transition at finite $T > 0$ is a second order.

IV. CONCLUSION AND OUTLOOK

In this paper, we proposed the Kondo effect driven by a chirality imbalance (or chiral chemical potential $\mu_5$) from the point of view of the two theoretical approaches. Using the perturbative approach, we found the infrared divergence of scattering amplitude as a signal of the Kondo effect. Using the mean-field approach, we found that the Kondo condensate is enhanced by finite $\mu_5$. These are universal properties in relativistic-fermion matter with heavy impurities and a chirality imbalance, which can be attributed to the enhancement of the density of states at the Fermi surface. Our findings generalize the analysis of the Kondo effect in Dirac/Weyl electron systems with an energy splitting among Dirac cones [26], involving various types of $SU(N)$ exchange interactions, such as spin, isospin, and color. The interplay effect between the exchange interaction and particular spin-orbit coupling in crystalline electron systems, such as topological Dirac semimetal Cd$_3$As$_2$, is left for further analysis.

As a topics not covered in the present study, we comment that the response to magnetic and electric fields would be interesting. For example, when $\mu_5$ is coupled to a magnetic field, an electric current can be induced, which is the so-called chiral magnetic effect [36, 54]. The correlation between the chiral transport phenomena and the Kondo effects will be worth to be studied. See for example the discussion of the transport coefficients in the
Kondo effect in relativistic fermion gas [18].

In the context of QCD, lattice simulations at finite \( \mu_5 \) evade from the sign problem [36–41], so that we can numerically measure the QCD Kondo effects in a fully non-perturbative way. The ground state of QCD in the low-temperature and/or low-chemical potential region is the chiral-symmetry breaking phase characterized by the chiral condensate, and the ground state in the high-chemical potential region is expected to be the color superconducting phase characterized by diquark condensate. These condensates could exclude the Kondo condensate [14, 17] or might induce a “coexistence” phase with two order parameters [17]. The topological properties of the QCD Kondo effect is also an interesting issue [16]. However, the conclusion from the effective models depends on the coupling constants of the interactions, and in the future it should be checked based on QCD.

In particular, the properties of chiral condensates at finite \( \mu_5 \) have been studied from chiral effective models [36, 55–75], Schwinger-Dyson equations [76, 77], and lattice QCD simulations [39–41]. One of the characteristic properties is the catalysis effect of the chiral symmetry breaking by finite \( \mu_5 \). Therefore, in matter with a chirality imbalance and impurities, the two catalysis effects of the chiral symmetry breaking and Kondo effect could be correlated.

In addition, in two- (or multi-) component fermion systems, the situation including an imbalance between the chemical potentials of different fermions would be also important. In QCD, the isospin chemical potential \( \mu_I \), an imbalance between up- and down- quark chemical potentials, is realized in neutron-rich nuclei and neutron stars, and lattice QCD simulations are also applicable [78–81]. For a similar external parameter to \( \mu_5 \), the effects from the chiral isospin chemical potential \( \mu_I \) could be also interesting [71, 72, 82–86].

ACKNOWLEDGMENTS

Y. A. is supported by the Leading Initiative for Excellent Young Researchers (LEADER). This work is supported by NSFC Grant 20201191997 (D. S.), and by JSPS Grant-in-Aid for Scientific Research (KAKENHI Grants No. JP17K14316 (Y. A.), No. JP17K14277 (K. S.) and No. JP17K05435 (S. Y.)), and by the Ministry of Education, Culture, Sports, Science (MEXT)-Supported Program for the Strategic Research Foundation at Private Universities “Topological Science” (Grant No. S1511006) (S. Y.).

Appendix A: Matsubara summation in Eqs. (5) and (6).

In this appendix, we show a detailed calculation of Matsubara summation in the one-loop amplitudes in Eqs. (5) and (6).

Within the imaginary-time formalism, Eqs. (5) and (6) are rewritten to

\[
\mathcal{M}^{(1a)} = -iG^2 \sum_{\epsilon_5 = \pm} T \sum_n \int \frac{d^3k}{(2\pi)^3} \bar{u}(p_f) t^a P_{\epsilon_5} \tilde{\Delta}_l(i(\omega_n - i\epsilon_5\mu_5)) t^b u(p_i) \bar{U}(q_f) t^a \Delta_h(i\omega_{q_i} - i\omega_n + i\omega_{p_f}) t^b U(q_i), \tag{A1}
\]

and

\[
\mathcal{M}^{(1b)} = -iG^2 \sum_{\epsilon_5 = \pm} T \sum_n \int \frac{d^3k}{(2\pi)^3} \bar{u}(p_f) t^a P_{\epsilon_5} \tilde{\Delta}_l(i(\omega_n - i\epsilon_5\mu_5)) t^b u(p_i) \bar{U}(q_f) t^b \Delta_h(i\omega_{q_i} + i\omega_n - i\omega_{p_f}) t^b U(q_i). \tag{A2}
\]

respectively, where the Matsubara Green’s functions for the light and heavy fermions are given by

\[
\tilde{\Delta}_l(i(\omega_n - i\epsilon_5\mu_5)) = -i(\omega_n + i\epsilon_5\mu_5) \gamma_0 + \vec{k} \cdot \vec{\gamma}, \\
(\omega_n - i\epsilon_5\mu_5)^2 + |\vec{k}|^2 \tag{A3}
\]

and

\[
\Delta_h(i\omega_n) = -\frac{i\omega_n \gamma_0 + \vec{k} \cdot \vec{\gamma} - M_Q}{\omega_n^2 + |\vec{k}|^2 - M_Q^2}, \tag{A4}
\]

with the Matsubara frequency \( \omega_n = (2n + 1)\pi T \) (\( n = 0, \pm 1, \pm 2, \cdots \)). Therefore, apart from the spinor and

\[
SU(N) \text{ non-Abelian algebras, we need to calculate }
\]

\[
\mathcal{I}_1 \equiv T \sum_n \int \frac{d^3k}{(2\pi)^3} \tilde{\Delta}_l(i(\omega_n - i\epsilon_5\mu_5)) \otimes \Delta_h(i\omega_{q_i} - i\omega_n + i\omega_{p_f}), \tag{A5}
\]

and

\[
\mathcal{I}_2 \equiv T \sum_n \int \frac{d^3k}{(2\pi)^3} \tilde{\Delta}_l(i(\omega_n - i\epsilon_5\mu_5)) \otimes \Delta_h(i\omega_{q_i} + i\omega_n - i\omega_{p_f}), \tag{A6}
\]

for the evaluation of \( \mathcal{M}^{(1a)} \) and \( \mathcal{M}^{(1b)} \).

First, let us demonstrate a detailed calculation of \( \mathcal{I}_1 \). The three-momentum integral in Eq. (A5) is performed
by the conventional procedure as in the vacuum, such that we show only the zeroth components of the momentum or coordinate space explicitly below. The inverse Fourier transformations of the Matsubara Green’s functions \( \bar{\Delta}_l(i\omega_n - i\epsilon_5\mu_5) \) and \( \bar{\Delta}_h(i\omega_n) \) given in Eqs. (A3) and (A4) can be defined by

\[
\bar{\Delta}_l(i\omega_n - i\epsilon_5\mu_5) = \int_0^\beta d\tau \bar{\Delta}_l(\tau)e^{(\omega_n - i\epsilon_5\mu_5)\tau},
\]

\[
\bar{\Delta}_h(i\omega_n) = \int_0^\beta d\tau \bar{\Delta}_h(\tau)e^{i\omega_n\tau}.
\]

(A7)

Then, by making use of the Poisson summation formula

\[
\sum_n \delta(x + \beta n) = T \sum_{n=-\infty}^{\infty} e^{\frac{2\pi inx}{\beta}},
\]

we get the following equation:

\[
\mathcal{J}_1 \equiv T \sum_n \bar{\Delta}_l(i\omega_n - i\epsilon_5\mu_5) \otimes \bar{\Delta}_h(i\omega_q - i\omega_n + i\omega_p)
\]

\[
= \int_0^\beta d\tau \bar{\Delta}_l(\tau) \otimes \bar{\Delta}_h(\tau) e^{i\epsilon_5\mu_5\tau + (i\omega_q + i\omega_p)\tau}.
\]

(A9)

The Matsubara Green’s function \( \bar{\Delta}_{l(h)}(\tau) \) is defined by an analytic continuation of the greater Green’s function

\[
S_{l(h)}^\tau(t) = \langle \psi(t)\bar{\psi}(0) \rangle_{l(h)},
\]

\[
S_{l(h)}^\tau_h(t) = \langle \bar{\psi}(t)\psi(0) \rangle_{l(h)},
\]

(A10)

as

\[
\bar{\Delta}_{l(h)}(\tau) = S_{l(h)}^\tau(-i\tau).
\]

Here, we remind that the Fourier transformation of the greater Green’s function \( S_{l(h)}(t) \) can be expressed as [87]

\[
S_{l(h)}(t) = \int \frac{dk_0}{2\pi} (1 - \hat{f}_\beta(k_0))\tilde{\rho}(k_0)e^{-ikt},
\]

(A12)

in which \( \hat{f}_\beta(k_0) \) is the Fermi distribution function, \( \hat{f}_\beta(k_0) = 1/(e^{\beta k_0} + 1) \) (\( \beta = 1/T \)), and \( \tilde{\rho}(k_0) \) is the spectral function

\[
\tilde{\rho}_l(k_0) = 2\pi \epsilon(k_0)\tilde{\delta}(k^2),
\]

\[
\tilde{\rho}_h(k_0) = 2\pi \epsilon(k_0)(\bar{k} + M_Q)\delta(k^2 - M_Q^2).
\]

(A13)

Then, we find that Eq. (A9) can be rewritten to

\[
\mathcal{J}_1 \approx \frac{1}{2|\bar{k}|} \left[ \frac{1 - \hat{f}_\beta(\bar{k}_0 - \epsilon_5\mu_5)}{p_0^\gamma + \epsilon_5\mu_5 - |\bar{k}| + i\epsilon} \otimes \bar{\Lambda}_+ - \frac{\hat{f}_\beta(\bar{k} + \epsilon_5\mu_5)}{p_0^\gamma + \epsilon_5\mu_5 + |\bar{k}| + i\epsilon} \otimes \bar{\Lambda}_+ \right],
\]

(A15)

with \( \bar{\Lambda}_+ \equiv \lim_{M_Q \to \infty}(\bar{q} + M_Q)/(2M_Q) = (1 + \gamma_0)/2 \). Hence, by combining the three-momentum integral, finally we can evaluate \( \mathcal{I}_1 \) in Eq. (A5) as

\[
\mathcal{I}_1 \approx -\frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \hat{f}_\beta(\bar{k} - \epsilon_5\mu_5)}{p_0^\gamma + \epsilon_5\mu_5 - |\bar{k}| + i\epsilon} + \frac{\hat{f}_\beta(\bar{k} + \epsilon_5\mu_5)}{p_0^\gamma + \epsilon_5\mu_5 + |\bar{k}| + i\epsilon} \right] \gamma^0 \otimes \bar{\Lambda}_+.
\]

(A16)

Note that we are interested in only the real part of the amplitude, so that the imaginary parts have been omitted.
Therefore, $\mathcal{M}^{(1a)}$ in Eq. (A1) becomes
\[
\mathcal{M}^{(1a)} \approx \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} - \mu_5) + \tilde{f}_\beta(\mathbf{k} + \mu_5)}{p^0_\mu + \mu + |\mathbf{k}|} \right] \bar{u}(p_f) t^a \gamma^\mu P_+ \gamma_0 t^b \gamma^\nu u(p_i) \mathcal{U}_+(q_f) t^a \gamma_\mu \mathcal{A}_+ t^b \gamma_\nu U_+(q_i)
\]
\[
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} + \mu_5) + \tilde{f}_\beta(\mathbf{k} - \mu_5)}{p^0_\mu - \mu + |\mathbf{k}|} \right] \bar{u}(p_f) t^a \gamma^\mu P_- \gamma_0 t^b \gamma^\nu u(p_i) \mathcal{U}_+(q_f) t^a \gamma_\mu \mathcal{A}_+ t^b \gamma_\nu U_+(q_i) \quad (A17)
\]
by replacing $U(q_i) \to U_+(q_i)$ ($\bar{U}(q) \to \bar{U}_+(q_f)$) together with the $M_Q \to \infty$ limit.

In a similar manner, we can evaluate $\mathcal{I}_2$ in Eq. (A6) as
\[
\mathcal{I}_2 \approx -\frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\tilde{f}_\beta(\mathbf{k} - \epsilon_5\mu_5) - 1 - \tilde{f}_\beta(\mathbf{k} + \epsilon_5\mu_5)}{p^0_\mu + \epsilon_5\mu_5 + |\mathbf{k}|} \right] \bar{u}(p_f) t^a \gamma^\mu P_+ \gamma_0 \bar{\gamma}^\nu u(p_i) \mathcal{U}_+(q_f) t^b \gamma_\nu \mathcal{A}_+ t^a \gamma_\mu U_+(q_i),
\]
which yields that $\mathcal{M}^{(1b)}$ in Eq. (A2) becomes
\[
\mathcal{M}^{(1b)} \approx \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} - \mu_5) + \tilde{f}_\beta(\mathbf{k} + \mu_5)}{p^0_\mu - \mu + |\mathbf{k}|} \right] \bar{u}(p_f) t^a \gamma^\mu P_- \gamma_0 \bar{\gamma}^\nu u(p_i) \mathcal{U}_+(q_f) t^b \gamma_\nu \mathcal{A}_+ t^a \gamma_\mu U_+(q_i)
\]
\[
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} + \mu_5) + \tilde{f}_\beta(\mathbf{k} - \mu_5)}{p^0_\mu + \mu + |\mathbf{k}|} \right] \bar{u}(p_f) t^a \gamma^\mu P_+ \gamma_0 \bar{\gamma}^\nu u(p_i) \mathcal{U}_+(q_f) t^b \gamma_\nu \mathcal{A}_+ t^a \gamma_\mu U_+(q_i) \quad (A19)
\]
in the same limit. The total one-loop amplitude is given by the sum of Eqs. (A17) and (A19): $\mathcal{M}^{(1)} = \mathcal{M}^{(1a)} + \mathcal{M}^{(1b)}$.

In the present study, we are interested only in the vicinity of the “Fermi surface” defined for the right-handed fermion with $\mu_5 > 0$. Namely, we assume that the initial- and final-state light fermions satisfy the kinematics of $(p^0, |p|) = (0, \mu_5)$ for the right-handed fermion while $(p^0, |p|) = (2\mu_5, \mu_5)$ for the left-handed fermion ($p^\mu$ stands for $p^\mu$ and $p^\mu_\mu$ collectively), due to the Dirac equation. Thus, upon this assumption, $\mathcal{M}^{(1)}$ reads
\[
\mathcal{M}^{(1)} \approx \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} - \mu_5) + \tilde{f}_\beta(\mathbf{k} + \mu_5)}{\mu_5 - |\mathbf{k}|} \right] \bar{u}_R(p_f) t^a \gamma^\mu u_R(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i)
\]
\[
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} + \mu_5) + \tilde{f}_\beta(\mathbf{k} - \mu_5)}{\mu_5 + |\mathbf{k}|} \right] \bar{u}_L(p_f) t^a \gamma^\mu u_L(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i)
\]
\[
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} - \mu_5) + \tilde{f}_\beta(\mathbf{k} + \mu_5)}{\mu_5 - |\mathbf{k}|} \right] \bar{u}_R(p_f) t^a \gamma^\mu u_R(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i)
\]
\[
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} + \mu_5) + \tilde{f}_\beta(\mathbf{k} - \mu_5)}{\mu_5 + |\mathbf{k}|} \right] \bar{u}_L(p_f) t^a \gamma^\mu u_L(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i) \quad (A20)
\]
When we choose $\mu_5 > 0$ and assume its value is large compared to the temperature $T$, but small enough so that $M_Q \to \infty$ limit is justified, the terms including $1/(|\mathbf{k}| + \mu_5)$ or $\tilde{f}_\beta(|\mathbf{k}| + \mu_5)$ in Eqs. (A17) and (A19) can be neglected. Hence, we find that $\mathcal{M}^{(1)}$ is reduced to
\[
\mathcal{M}^{(1)} \approx \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - \tilde{f}_\beta(\mathbf{k} - \mu_5)}{\mu_5 - |\mathbf{k}|} \right] \bar{u}_R(p_f) t^a \gamma^\mu u_R(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i)
\]
\[
- \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{\mu_5 - |\mathbf{k}|} \right] \bar{u}_L(p_f) t^a \gamma^\mu u_L(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i)
\]
\[
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{\mu_5 - |\mathbf{k}|} \right] \bar{u}_L(p_f) t^a \gamma^\mu u_L(p_i) \mathcal{U}_+(q_f) t^b \gamma_\mu U_+(q_i) \quad (A21)
\]
This expression clearly shows that the transition amplitude of the left-handed fermion is not affected by the Fermi surface, as naively anticipated. Then, by defining $E = |\mathbf{k}| - \mu_5$ for the first line in Eq. (A21) while $E = \mu_5 - |\mathbf{k}|$ for
the second line, we find

$$
\mathcal{M}^{(1)} \approx -\frac{G^2}{2} \rho_0 \int_{-\mu_5}^{\infty} dE \frac{1 - \bar{f}_\beta(E)}{E} \bar{u}_R(p_f) t^a t^b \gamma_0 u_R(p_i) \bar{U}_+(q_f) t^a t^b U_+(q_i) \\
+ \frac{G^2}{2} \rho_0 \int_{-\mu_5}^{\infty} dE \frac{\bar{f}_\beta(-E)}{E} \bar{u}_R(p_f) t^a t^b \gamma_0 u_R(p_i) \bar{U}_+(q_f) t^b t^a U_+(q_i) \\
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\mu_5 - |\vec{k}|} \bar{u}_L(p_f) t^a t^b \gamma_0 u_L(p_i) \bar{U}_+(q_f) t^b t^a U_+(q_i),
$$

(A22)

where we have replaced the density of states by that on the Fermi surface, \(\rho_0 = \mu_5^2/(2\pi^2)\), since this study is based on the assumption that \(\mu_5\) is sufficiently large. By using a relation \(\bar{f}_\beta(-E) = 1 - \bar{f}_\beta(E)\) and the identities

\[
(t^a t^b)_{kl}(t^a t^b)_{ij} = \frac{N^2 - 1}{4N^2} \delta_{kl} \delta_{ij} - \frac{1}{N} (t^a)_{kl} (t^a)_{ij},
\]

\[
(t^a t^b)_{kl}(t^b t^a)_{ij} = \frac{N^2 - 1}{4N^2} \delta_{kl} \delta_{ij} - \frac{2 - N^2}{2N} (t^a)_{kl} (t^a)_{ij},
\]

(A23)

finally we arrive at

$$
\mathcal{M}^{(1)} \approx \frac{G^2}{2} \frac{N \rho_0}{\mu_5} \int_{-\mu_5}^{\infty} dE \frac{1 - \bar{f}_\beta(E)}{E} \times \bar{u}_R(p_f) t^a \gamma_0 u_R(p_i) \bar{U}_+(q_f) t^a U_+(q_i) \\
+ \frac{G^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\mu_5 - |\vec{k}|} \times \bar{u}_L(p_f) t^a t^b \gamma_0 u_L(p_i) \bar{U}_+(q_f) t^b t^a U_+(q_i),
$$

(A24)

which yields Eq. (14).

**Appendix B: Mean-field approach for Kondo effect at finite \(\mu\)**

In this appendix, in order to compare the Kondo effects at finite \(\mu\) and \(\mu_5\), we show the phase diagram at finite \(\mu\) using the same formalism as those in the main text. In the upper panel of Fig. 6, we show the \(\mu-\mu_5\) phase diagram of \(\Delta_R\). In the region with large \(\mu\) and/or \(\mu_5\), we find the appearance of the Kondo phase with nonzero \(\Delta_R\). The transitions along either the \(\mu\) axis or the \(\mu_5\) axis are still a crossover. Note that, in the region with large \(\mu + \mu_5\), \(\Delta_R\) is suddenly suppressed and becomes zero, but this behavior is an artifact from the ultraviolet cutoff, as mentioned in the main text. Therefore, we cannot conclude the true physics in this region, which is beyond the scope of this model. As shown in the lower panel of Fig. 6, in the region with large \(\mu\) but small \(\mu_5\), we find that \(\Delta_L\) is also enhanced. This behavior indicates that the “usual” Kondo effect induced by only finite chemical potential is realized, where both the right-handed and left-handed condensates contribute to the Kondo effect (namely, \(\Delta_R \approx \Delta_L\)).

[1] J. Kondo, “Resistance Minimum in Dilute Magnetic Alloys,” Prog. Theor. Phys. **32**, 37–49 (1964).

[2] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, 1993).
[3] K. Yosida, *Theory of Magnetism* (Springer-Verlag Berlin Heidelberg, 1996).
[4] K. Yamada, *Electron Correlation in Metals* (Cambridge University Press, 2004).
[5] Piers Coleman, *Introduction to Many-Body Physics* (Cambridge University Press, 2015).
[6] S. Yasui and K. Sudoh, “Heavy-quark dynamics for charm and bottom flavor on the Fermi surface at zero temperature,” Phys. Rev. C88, 015201 (2013), arXiv:1301.6830 [hep-ph].
[7] Shigehiro Yasui, “Kondo effect in charm and bottom nuclei,” Phys. Rev. C93, 065204 (2016), arXiv:1602.00227 [hep-ph].
[8] Shigehiro Yasui and Kazutaka Sudoh, “Kondo effect of $D_c$ and $D_s$ mesons in nuclear matter,” Phys. Rev. C95, 035204 (2017), arXiv:1607.07948 [hep-ph].
[9] Shigehiro Yasui and Tomokazu Miyamoto, “Spin-isospin Kondo effects for $\Sigma_c$ and $\Sigma_b$ baryons and $D$ and $D^*$ mesons,” Phys. Rev. C100, 045201 (2019), arXiv:1905.02478 [hep-ph].
[10] Koichi Hattori, Kazuomii Itakura, Sho Ozaki, and Shigehiro Yasui, “QCD Kondo effect: quark matter with heavy-flavor impurities,” Phys. Rev. D92, 065003 (2015), arXiv:1504.07619 [hep-ph].
[11] Sho Ozaki, Kazuomii Itakura, and Yoshihii Kuramoto, “Magnetically induced QCD Kondo effect,” Phys. Rev. D94, 074013 (2016), arXiv:1509.06966 [hep-ph].
[12] Shigehiro Yasui, Kei Suzuki, and Kazuomii Itakura, “Kondo phase diagram of quark matter,” Nucl. Phys. A983, 90–102 (2019), arXiv:1604.07208 [hep-ph].
[13] Shigehiro Yasui, “Kondo cloud of single heavy quark in cold and dense matter,” Phys. Lett. B773, 428–434 (2017), arXiv:1608.06450 [hep-ph].
[14] Takuya Kanazawa and Shun Uchino, “Overscreened Kondo effect, (color) superconductivity and Shiba states in Dirac metals and quark matter,” Phys. Rev. D94, 114005 (2016), arXiv:1609.00033 [cond-mat.str-el].
[15] Taro Kimura and Sho Ozaki, “Fermi/non-Fermi mixing in SU(N) Kondo effect,” J. Phys. Soc. Jap. 86, 084703 (2017), arXiv:1611.07284 [cond-mat.str-el].
[16] Shigehiro Yasui, Kei Suzuki, and Kazuomii Itakura, “Topology and stability of the Kondo phase in quark matter,” Phys. Rev. D96, 014016 (2017), arXiv:1703.04124 [hep-ph].
[17] Kei Suzuki, Shigehiro Yasui, and Kazuomii Itakura, “Interplay between chiral symmetry breaking and the QCD Kondo effect,” Phys. Rev. D96, 114007 (2017), arXiv:1708.06930 [hep-ph].
[18] Shigehiro Yasui and Sho Ozaki, “Transport coefficients from the QCD Kondo effect,” Phys. Rev. D96, 114027 (2017), arXiv:1710.03434 [hep-ph].
[19] Taro Kimura and Sho Ozaki, “Conformal field theory analysis of the QCD Kondo effect,” Phys. Rev. D99, 014040 (2019), arXiv:1806.06486 [hep-ph].
[20] Juan C. Macías and F. S. Navarra, “Kondo QCD effect in stellar matter,” arXiv:1901.01623 [nucl-th].
[21] Koichi Hattori, Xu-Guang Huang, and Robert D. Pisarski, “Emergent QCD Kondo effect in two-flavor color superconducting phase,” Phys. Rev. D99, 094044 (2019), arXiv:1903.10953 [hep-ph].
[22] Daiki Suenaga, Kei Suzuki, and Shigehiro Yasui, “QCD Kondo excitons,” arXiv:1909.07573 [nucl-th].
[23] A. Principi, G. Vignale, and E. Rossi, “Kondo effect and non-Fermi-liquid behavior in Dirac and Weyl semimetals,” Phys. Rev. B92, 041107 (2015), arXiv:1410.8532 [cond-mat.mes-hall].
[24] T. Yanagisawa, “Dirac fermions and Kondo effect,” J. Phys.: Conference Series 603, 012014 (2015), arXiv:1502.07898 [cond-mat.str-el].
[25] T. Yanagisawa, “Kondo Effect in Dirac Systems,” J. Phys. Soc. Jpn. 84, 074705 (2015), arXiv:1505.05295 [cond-mat.str-el].
[26] Andrew K. Mitchell and Lars Fritz, “Kondo effect in three-dimensional Dirac and Weyl systems,” Phys. Rev. B92, 121109 (2015), arXiv:1506.05491 [cond-mat.str-el].
[27] Jin-Hua Sun, Dong-Hui Xu, Fu-Chun Zhang, and Yi Zhou, “A magnetic Impurity in a Weyl semimetal,” Phys. Rev. B92, 195124 (2015), arXiv:1509.05180 [cond-mat.str-el].
[28] Xiao-Yong Feng, Hanting Zhong, Jianhui Dai, and Qimiao Si, “Dirac-Kondo semimetals and topological Kondo insulators in the dilute carrier limit,” arXiv:1605.02380 [cond-mat.str-el].
[29] Hsin-Hua Lai, Sarah E. Grefe, Silke Paschen, and Qimiao Si, “Weyl-Kondo semimetal in heavy-fermion systems,” Proc. Natl. Acad. Sci. 115, 93–97 (2018), arXiv:1612.03899 [cond-mat.str-el].
[30] Seulgi Ok, Markus Legner, Titus Neupert, and Ashley M. Cook, “Magnetic Weyl and Dirac Kondo semimetal phases in heterostructures,” arXiv:1703.03804 [cond-mat.str-el].
[31] Da Ma, Hua Chen, Haiwen Liu, and X. C. Xie, “Kondo effect with weyl semimetal fermi arcs,” Phys. Rev. B 97, 045148 (2018), arXiv:1709.08008 [cond-mat.str-el].
[32] Lin Li, Jin-Hua Sun, Zhen-Hua Wang, Dong-Hui Xu, Hong-Gang Luo, and Wei-Qiang Chen, “Magnetic states and kondo screening in weyl semimetals with chiral anomaly,” Phys. Rev. B 98, 075110 (2018).
[33] Hai-Feng Li, Ying-Hua Deng, Sha-Sha Ke, Yong Guo, and Hui-Wu Zhang, “Quantum impurity in topological multi-veyl semimetals,” Phys. Rev. B 99, 115109 (2019).
[34] Ki-Seok Kim and Jae-Ho Han, “Interplay between chiral magnetic and kondo effects in weyl metal phase,” Curr. Appl. Phys. 19, 236 – 240 (2019).
[35] Sarah E. Grefe, Hsin-Hua Lai, Silke Paschen, and Qimiao Si, “Weyl-Kondo semimetals in non-symmetric systems,” arXiv:1911.01400 [cond-mat.str-el].
[36] Kenji Fukushima, Dmitri E. Kharzeev, and Harmen J. Warringa, “The Chiral Magnetic Effect,” Phys. Rev. D78, 074033 (2008), arXiv:0808.3382 [hep-ph].
[37] Arata Yamamoto, “Chiral magnetic effect in lattice QCD with a chiral chemical potential,” Phys. Rev. Lett. 107, 031601 (2011), arXiv:1105.0385 [hep-lat].
[38] Arata Yamamoto, “Lattice study of the chiral magnetic effect in a chirally imbalanced matter,” Phys. Rev. D84, 114504 (2011), arXiv:1111.4681 [hep-lat].
[39] V. V. Braguta, V. A. Goy, E. M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, M. Muller-Preussker, and B. Petersson, “Two-Color QCD with Non-Zero Chiral Chemical Potential,” Phys. Rev. D99, 094506 (2019), arXiv:1903.08670 [hep-lat].
[40] V. V. Braguta, E. M. Ilgenfritz, A. Yu. Kotov, B. Petersson, and S. A. Skinderev, “Study of QCD Phase Diagram with Non-Zero Chiral Chemical Potential,” JHEP 06, 094 (2015), arXiv:1503.06670 [hep-lat].
[41] N. Yu. Astrakhankhtev, V. V. Braguta, A. Yu. Kotov, D. D. Kuznedev, and A. A. Nikolaev, “Lattice study of QCD at finite chiral density: topology and confinement.”
(2019), arXiv:1902.09325 [hep-lat].

[42] A. A. Zyuzin, Si Wu, and A. A. Burkov, “Weyl semimetal with broken time reversal and inversion symmetries,” Phys. Rev. B 85, 165110 (2012), arXiv:1201.3624 [cond-mat.mes-hall].

[43] A. A. Zyuzin and A. A. Burkov, “Topological response in weyl semimetals and the chiral anomaly,” Phys. Rev. B 86, 115133 (2012), arXiv:1206.1868 [cond-mat.mes-hall].

[44] M. M. Vazifeh and M. Franz, “Electromagnetic Response of Weyl Semimetals,” Phys. Rev. Lett. 111, 027201 (2013), arXiv:1303.5784 [cond-mat.mes-hall].

[45] Anton A. Burkov and Yong Baek Kim, “$Z_2$ and chiral anomalies in topological dirac semimetals,” Phys. Rev. Lett. 117, 136602 (2016), arXiv:1606.08446 [cond-mat.mes-hall].

[46] Zhijun Wang, Hongming Weng, Quansheng Wu, Xi Dai, and Zhong Fang, “Three-dimensional dirac semimetal and quantum transport in Cd$_3$As$_2$,” Phys. Rev. B 88, 125427 (2013), arXiv:1305.6780 [cond-mat.mtrl-sci].

[47] A. A. Zyuzin, Si Wu, and A. A. Burkov, “Topological response in weyl semimetals and the chiral anomaly,” Phys. Rev. B 86, 115133 (2012), arXiv:1201.3624 [cond-mat.mes-hall].

[48] P.W. Anderson, “A poor man’s derivation of scaling laws for the Kondo problem,” J. Phys. C 3, 2436 (1970).

[49] Ettia Eichten and Brian Russell Hill, “An Effective Field Theory for the Calculation of Matrix Elements Involving Heavy Quarks,” Phys. Lett. B234, 511–516 (1990).

[50] Howard Georgi, “An Effective Field Theory for Heavy Quarks at Low-energies,” Phys. Lett. B240, 447–450 (1990).

[51] Matthias Neubert, “Heavy quark symmetry,” Phys. Rept. 245, 259–396 (1994), arXiv:hep-ph/9306320 [hep-ph].

[52] Aneesh V. Manohar and Mark B. Wise, *Heavy Quark Physics*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (Cambridge University Press, 2000).

[53] S. P. Klevansky, “The Nambu-Jona-Lasinio model of quantum chromodynamics,” Rev. Mod. Phys. 64, 649–708 (1992).

[54] Dmitri E. Kharzeev, Larry D. McLerran, and Harman J. Warringa, “The Effects of topological charge change in heavy ion collisions: “Event by event P and CP violation”,” Nucl. Phys. A803, 227–253 (2008), arXiv:0711.0950 [hep-ph].

[55] Kenji Fukushima, Marco Ruggieri, and Raoul Gatto, “Chiral magnetic effect in the PNJL model,” Phys. Rev. D81, 114031 (2010), arXiv:1003.0047 [hep-ph].

[56] M. N. Chernodub and A. S. Nedelin, “Phase diagram of chirally imbalanced QCD matter,” Phys. Rev. D83, 105008 (2011), arXiv:1102.0188 [hep-ph].

[57] Marco Ruggieri, “The Critical End Point of Quantum Chromodynamics Detected by Chirally Imbalanced Quark Matter,” Phys. Rev. D84, 014011 (2011), arXiv:1103.6186 [hep-ph].

[58] Raoul Gatto and Marco Ruggieri, “Hot Quark Matter with an Axial Chemical Potential,” Phys. Rev. D85, 054013 (2012), arXiv:1110.4904 [hep-ph].

[59] Alexander A. Andrianov, Domenec Espriu, and Xumeu Planells, “An effective QCD Lagrangian in the presence of an axial chemical potential,” Eur. Phys. J. C73, 2294 (2013), arXiv:1210.7712 [hep-ph].

[60] Alexander A. Andrianov, Domenec Espriu, and Xumeu Planells, “Chemical potentials and parity breaking: the Nambu-Jona-Lasinio model,” Eur. Phys. J. C74, 2776 (2014), arXiv:1310.4416 [hep-ph].

[61] P. V. Buividovich, “Spontaneous chiral symmetry breaking and the Chiral Magnetic Effect for interacting Dirac fermions with chiral imbalance,” Phys. Rev. D90, 125025 (2014), arXiv:1408.4573 [hep-th].

[62] Lang Yu, Hao Liu, and Mei Huang, “Effect of the chiral chemical potential on the chiral phase transition in the NJL model with different regularization schemes,” Phys. Rev. D94, 041026 (2016), arXiv:1511.03073 [hep-ph].

[63] V. V. Braguta and A. Yu. Kotov, “Catalysis of Dynamical Chiral Symmetry Breaking by Chiral Chemical Potential,” Phys. Rev. D93, 105025 (2016), arXiv:1601.04957 [hep-th].

[64] Marco Frasca, “Nonlocal Nambu–Jona-Lasinio model and chiral chemical potential,” Eur. Phys. J. C78, 790 (2018), arXiv:1602.04654 [hep-ph].

[65] M. Ruggieri and G. X. Peng, “Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential,” J. Phys. G43, 125101 (2016), arXiv:1602.05250 [hep-ph].

[66] R. L. S. Farias, Dyana C. Duarte, Gastao Krein, and Rudnei O. Ramos, “Thermodynamics of quark matter with a chiral imbalance,” Phys. Rev. D94, 074011 (2016), arXiv:1604.04518 [hep-ph].

[67] Zhu-Fang Cui, Ian C. Cloet, Ya Lu, Craig D. Roberts, Sebastian M. Schmidt, Shu-Sheng Xu, and Hong-Shi Zong, “Critical endpoint in the presence of a chiral chemical potential,” Phys. Rev. D94, 071503 (2016), arXiv:1604.08454 [nucl-th].

[68] Ya Lu, Zhu-Fang Cui, Zan Pan, Chao-Hsi Chang, and Hong-Shi Zong, “QCD phase diagram with a chiral chemical potential,” Phys. Rev. D93, 074037 (2016).

[69] M. Ruggieri, Z. Y. Lu, and G. X. Peng, “Influence of chiral chemical potential, parallel electric, and magnetic fields on the critical temperature of QCD,” Phys. Rev. D94, 116003 (2016), arXiv:1608.08310 [hep-ph].

[70] Zan Pan, Zhu-Fang Cui, Chao-Hsi Chang, and Hong-Shi Zong, “Finite-volume effects on phase transition in the Polyakov-loop extended Nambu–Jona-Lasinio model with a chiral chemical potential,” Int. J. Mod. Phys. A32, 1750067 (2017), arXiv:1611.07370 [hep-ph].

[71] T. G. Khunjua, K. G. Klimenko, and R. N. Zhokhov, “Dualities in dense quark matter with isospin, chiral, and chiral isospin imbalance in the framework of the large-$N_c$ limit of the NJL$_4$ model,” Phys. Rev. D98, 054030 (2018), arXiv:1804.01014 [hep-ph].

[72] T. G. Khunjua, K. G. Klimenko, and R. N. Zhokhov, “Dualities and inhomogeneous phases in dense quark matter with chiral and isospin imbalances in the framework of effective model,” JHEP 06, 006 (2019), arXiv:1901.02855 [hep-ph].

[73] V. V. Braguta, M. I. Katsnelson, A. Yu. Kotov, and A. M. Trunin, “Catalysis of Dynamical Chiral Symmetry Breaking by Chiral Chemical Potential in Dirac semimetals,” Phys. Rev. B100, 085117 (2019), arXiv:1904.07003 [cond-mat.str-el].

[74] Arpan Das, Deepak Kumar, and Hiranmaya Mishra, “Chiral susceptibility in the Nambu–Jona-Lasinio model: A Wigner function approach,” Phys. Rev. D100, 094030 (2019), arXiv:1907.12332 [hep-ph].
[75] Li-Kang Yang, Xiaofeng Luo, and Hong-Shi Zong, “QCD phase diagram in chiral imbalance with self-consistent mean field approximation,” Phys. Rev. D100, 094012 (2019), arXiv:1910.13185 [nucl-th].

[76] Bin Wang, Yong-Long Wang, Zhu-Fang Cui, and Hong-Shi Zong, “Effect of the chiral chemical potential on the position of the critical endpoint,” Phys. Rev. D91, 034017 (2015).

[77] Shu-Sheng Xu, Zhu-Fang Cui, Bin Wang, Yuan-Mei Shi, You-Chang Yang, and Hong-Shi Zong, “Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations,” Phys. Rev. D91, 056003 (2015), arXiv:1505.00316 [hep-ph].

[78] J. B. Kogut and D. K. Sinclair, “Quenched lattice QCD at finite isospin density and related theories,” Phys. Rev. D66, 014508 (2002), arXiv:hep-lat/0201017 [hep-lat].

[79] J. B. Kogut and D. K. Sinclair, “Lattice QCD at finite isospin density at zero and finite temperature,” Phys. Rev. D66, 034505 (2002), arXiv:hep-lat/0202028 [hep-lat].

[80] J. B. Kogut and D. K. Sinclair, “The Finite temperature transition for 2-flavor lattice QCD at finite isospin density,” Phys. Rev. D70, 094501 (2004), arXiv:hep-lat/0407027 [hep-lat].

[81] B. B. Brandt, G. Endrődi, and S. Schmalzbauer, “QCD phase diagram for nonzero isospin-asymmetry,” Phys. Rev. D97, 054514 (2018), arXiv:1712.08190 [hep-lat].

[82] D. Ebert, T. G. Khunjua, and K. G. Klimentko, “Duality between chiral symmetry breaking and charged pion condensation at large \( N_c \): Consideration of an NJL_2 model with baryon, isospin, and chiral isospin chemical potentials,” Phys. Rev. D94, 116016 (2016), arXiv:1608.07688 [hep-ph].

[83] T. G. Khunjua, K. G. Klimentko, R. N. Zhokhov, and V. C. Zhukovsky, “Inhomogeneous charged pion condensation in chiral asymmetric dense quark matter in the framework of NJL_2 model,” Phys. Rev. D95, 105010 (2017), arXiv:1704.01477 [hep-ph].

[84] T. G. Khunjua, K. G. Klimentko, and R. N. Zhokhov, “Dense baryon matter with isospin and chiral imbalance in the framework of NJL_4 model at large \( N_c \): duality between chiral symmetry breaking and charged pion condensation,” Phys. Rev. D97, 054036 (2018), arXiv:1710.09706 [hep-ph].

[85] T. G. Khunjua, K. G. Klimentko, and R. N. Zhokhov, “Chiral imbalanced hot and dense quark matter: NJL analysis at the physical point and comparison with lattice QCD,” Eur. Phys. J. C79, 151 (2019), arXiv:1812.00772 [hep-ph].

[86] T. G. Khunjua, K. G. Klimentko, and R. N. Zhokhov, “Charged pion condensation and duality in dense and hot chirally and isospin asymmetric quark matter in the framework of the NJL_2 model,” Phys. Rev. D100, 034009 (2019), arXiv:1907.04151 [hep-ph].

[87] Michel Le Bellac, *Thermal Field Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2011).