Interferometers from single-atom mirrors

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Abstract. Interferometers are ubiquitous devices in optics, consisting of arrangements of totally reflective and semitransparent mirrors, fiber optics, detectors, etc. The smallest possible mirror consists of a single atom, and with recent advances in nanotechnology it is possible to fabricate them and couple them to transmission lines. This can be realized for example with superconducting qubits and superconducting coplanar waveguide resonators or with dipole emitters coupled to surface-plasmon nanowires. Based on a recent proposal [G. S. Paraoanu, Phys. Rev. A 82, 023802 (2010)], here we give a brief overview of the two-atom Mach-Zender interferometer. Here we show that both the phase and the amplitude of the output field can be used to extract information about the phase difference between the two arms of the interferometer. Also, we point out that this device can be used as well in the reflection mode.

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When the size of the two semitransparent mirrors in a Mach-Zender interferometer is reduced to that of two atoms, the classical description of the device in terms of rays has to be supplemented with a careful quantum-mechanical analysis of the effects of absorption and re-emission of the electromagnetic field. If the fields are guided through and confined into transmission lines and the coupling of the atoms results in high Purcell factors, the field emitted by the atoms can interfere with the incident field, giving rise to non-negligible interference effects [1]. This gives rise to the concept of "fluorescence interferometry" [2]. Such effects can also be used for building devices such as single-photon detectors [3].

A general description of a quantum network with any number of modes and atoms as nodes can be given [2]. In this paper, we will consider the case of only two atoms and two modes with a single input field, realizing a two-atom Mach-Zender interferometer. Such a device can be fabricated either with artificial atoms such as superconducting qubits placed along coplanar waveguides [4] or with "natural" atoms - for example dipolar atoms or molecules coupled to nanowires [5]. A schematic of the device is shown in Fig. 1, representing the simplification of the network in Fig. 1 of [2] for the case of two modes, two atoms (qubits) and a single input field \( a_1^{(\text{in})} \).

We denote by \( \theta \) the phase accumulated by the field as it travels between the two qubits in the case when the two paths corresponding to modes 1 and 2 are equal. It is possible to induce a phase difference between the two paths for example by placing a material in one of the arms; in the following we will consider that the arm "2" of the interferometer (the mode with subscript 2) experiences a phase shift \( \phi \).

Following the theoretical treatment of [2] for a general quantum network, the output field amplitudes \( a_1^{(\text{out})} \), \( a_2^{(\text{out})} \), \( b_1^{(\text{out})} \), and \( b_2^{(\text{out})} \) can be determined in the stationary state as a function...
Figure 1. Schematic of a fluorescence interferometer with two qubits $A$ and $B$ and two modes 1 and 2. The amplitudes of the right-propagating modes are denoted by $a$ and the amplitudes of the left-propagating modes are denoted by $b$. The phase difference between the two arms is denoted by $\varphi$.

Figure 2. The argument of the complex transmission coefficient in the mode 1 as a function of (a) the phases $\varphi$, $\theta$ for $\gamma = 1$, and (b) the real and imaginary parts of $\gamma$ (for $\theta = \pi/2$ and $\varphi = 0$).

of the input (or probe) field amplitude $a_1^{(\text{in})}$ by the method of transfer matrices. We will also denote by $\gamma$ the combined effects of qubit dissipation and of the detuning between the qubit frequency and that of the radiation field in a bandwidth centered around the qubit’s resonant frequency [2]. Much like in the case of vector network analysis, information about the phase $\varphi$ can be extracted either from the transmitted or reflected signal.

*Transmission mode*

The amplitudes of the transmitted field are found to be [2],

$$
a_1^{(\text{out})} = e^{i\theta} \frac{-e^{i\varphi} + e^{2i(\varphi+\theta)} + e^{i(\varphi+2\theta)} - (1 + \gamma)^2}{4e^{i(\varphi+2\theta)} \cos^2 \varphi/2 - (2 + \gamma)^2} a_1^{(\text{in})},
$$

(1)

$$
a_2^{(\text{out})} = e^{i\theta} \frac{(1 + e^{i\varphi}) \left[-1 - \gamma + e^{i(\varphi+2\theta)}\right]}{4e^{i(\varphi+2\theta)} \cos^2 \varphi/2 - (2 + \gamma)^2} a_1^{(\text{in})}.
$$

(2)

A plot of the argument of the transmission coefficient of mode 1 $a_1^{(\text{out})}/a_1^{(\text{in})}$ is shown in Fig. 2. The transmission coefficient for the second mode has a similar behavior.
Figure 3. A plot of the absolute value of the reflection in the mode 1 as a function of (a) the phases \( \varphi \), \( \theta \) for \( \gamma = 1 \), and (b) the real and imaginary parts of \( \gamma \) (for \( \theta = \pi/2 \) and \( \varphi = 0 \)).

Reflection mode

In the reflection mode, the information about the phases \( \varphi \) and \( \theta \) is extracted from the reflected fields \( b_1^{(\text{out})} \) and \( b_2^{(\text{out})} \), which in the stationary state can be obtained as [2]

\[
\begin{align*}
  b_1^{(\text{out})} &= \frac{2 - 2e^{i(\varphi + 2\theta)} + \gamma(1 + e^{2i\theta})}{4e^{i(\varphi + 2\theta)} \cos^2 \varphi/2 - (2 + \gamma)^2} a_1^{(\text{in})}, \\
  b_2^{(\text{out})} &= -\frac{-2 + e^{2i\theta} + e^{2i(\varphi + \theta)} - \gamma \left[ 1 + e^{i(\varphi + 2\theta)} \right]}{4e^{i(\varphi + 2\theta)} \cos^2 \varphi/2 - (2 + \gamma)^2} a_1^{(\text{in})}.
\end{align*}
\]

A plot of the absolute value of the reflection coefficient of mode 1 \( \left| b_1^{(\text{out})}/a_1^{(\text{in})} \right| \) is shown in Fig. 3. The reflection coefficient for the second mode has a similar behavior.

In conclusion, we analyze an interference device consisting of two atoms coupled to transmission lines and show that both the transmitted and reflected fields contain information about the phase difference between the two arms.

Acknowledgments

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