Performance Analysis of Time Frequency Resolution Techniques for Non-Stationary Signals

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Abstract

Background/Objectives: All Traditional data analysis methods are based on linear and stationary assumptions. One of the important attributes of the signal processing techniques are applicable for non stationary signal environments is an improvement in time-frequency resolution or localization, relative to classical techniques like Fourier Transform (FT) which overcome these drawbacks of FT. The main objective of this paper is to improve the target detection performance in active sonar and radar systems. Methods/Statistical Analysis: In this paper a study of time-frequency resolution analysis of non stationary signals using different transform techniques like Short Time Fourier Transform (STFT), Continuous Wavelet Transform (CWT) and Fractional Fourier Transform (FrFT) is presented. Findings: STFT provides the time information by computing multiple FFT’s for consecutive intervals of time and then putting them together. The spectrogram can able to resolve the temporal frequency evolution content from the signal. Spectrogram has a trade-off in time and frequency resolution in accordance with the uncertainty principle of Heisenberg. For the Short Time Fourier Transform, time-frequency resolution is fixed. It can be varied in the Continuous Wavelet Transform as a function of an analyzing frequency. The Continuous Wavelet Transform having the analysis function can be chosen with more freedom. Fractional Fourier Transform is a time-frequency distribution which provides us with an external degree of freedom. It can allow signal to be transformed into a fractional domain with a fractional order parameter α. Application/Improvement: Time Frequency resolution or localization transform techniques are used to improve the performance of target detection in active sonar and radar systems.

Keywords: Continuous Wavelet Transform, Fractional Fourier Transform, Linear Frequency Modulated Signal, Short Time Fourier Transform

1. Introduction

A new method has been introduced for analyzing the non linear and non stationary data. For example, Wavelet Analysis, Wigner Ville Distribution (WVD), Hilbert Huang Transform (HHT) and Fractional Fourier Transform techniques were introduced for analysis of linear but non-stationary data.¹² One of the key attributes of the signal processing techniques is applicable for non stationary signal environments is an improvement in the time frequency resolution or localization. These are relative to classical techniques like Fourier transforms which overcome these drawbacks of FT.

There are some alternative methods to classical STFT signal analysis technique which was proposed and first introduced by Dennis Gabor in 1946³–⁵. The techniques have received considerable attention in recent times are Time Frequency Localization (TFL) techniques such as the Wigner Ville Distribution which is a non-stationary and quadratic time frequency signal analysis tool, introduced in 1948 by Ville and its alternatives⁶–⁸. The example of an empirical based data analysis method is Hilbert Huang Transform. Basis of expansion for this transform is adaptive. So it can produce physically meaningful representations of data from non stationary and nonlinear processes. The purpose of Hilbert Huang Transform is to
demonstrate an alternative method to present spectral analysis tools for providing the time-frequency-energy description of time data. The advantage of this transform is to describe non stationary data locally. Based on Fourier or wavelet transforms, the Hilbert transform was used, in order to compute instantaneous frequencies and amplitudes and describe the signal more locally. Continuous Wavelet Transform (CWT) is one of the Time Scale Analysis (TSA) techniques. In this technique scale is inversely proportional to the frequency. CWT has capable of improving the signal time frequency characterization. However, following are a few performance issues found in practice when the actual signals are used for sonar and radar applications:

- In case of multi component signal analysis, WVD generates significant cross over terms.
- It also obscures signal components due to the additive noise.
- Negative TFL energy magnitude is very difficult to interpret, if it is related to signal component.
- All TFL techniques are developed to handle a single component data from finite signal duration.

So, it’s desirable to handle non-stationary signals persistently. The STFT and WVD have time localization or resolution which is fundamentally defined by the sliding window duration, τ, which is chosen for the Short Time Fourier Transform. The STFT has a frequency resolution or localization which is fundamentally defined by the window, v, which is the reciprocal of τ. Thus, the Duration-Bandwidth (DB) product, v.τ≥1, is a measure of the complete TFL or resolution of STFT. The WVD is a quadratic TFL that achieves DB product v.τ≥1/2 compared to STFT. However, this improvement comes by price which interprets that the cross over products introduced by the WVD that enables us to distinguish actual signal component from the presence of multiple signal frequency components and noise. Jean Morlet in 1982 introduced the idea of wavelet transform, an adjustable window analysis, which is similar to the Fourier spectral analysis method. There are different kinds of Wavelet Transforms (WTs) introduced: Wavelet transform designed for being easily invertible, signal analysis and decompose a signal into a set of scales by projecting the signal onto an element of a set of basis orthogonal functions. The CWT method computed time domain signals separately to overcome the resolution problem in STFT. In quantum mechanics field, the FrFT was first invented by Namias in 1980. Fractional Fourier Transform is a time frequency distribution method. It is a generalization of the ordinary Fourier transform with a fractional order parameter ‘alpha’ and it is identical to the ordinary Fourier transform when the fractional order is equal to π/2. It can provide us with an external degree of freedom, which gives more benefits over conventional Fourier transform.

2. Non Stationary Signals

Non stationary signal is a signal whose frequency changes with respect to time. Linear frequency modulation is a technique used to increase the waveform bandwidth while maintaining pulse durations such that τ>>1/β. Time-bandwidth product is βτ>>1. A Linear Frequency Modulated (LFM) signal is defined as

\[ I_{lm}(t) = e^{j2\pi\left(\left(\frac{f_s}{2} - \frac{B_w}{2}\right)t + \frac{B_w}{2}t^2\right)} \quad \text{for } 0 \leq t \leq 1 \]

Where B_w is the bandwidth, f_s is the central frequency, f_s is the starting frequency, and T is the duration. The instantaneous frequency, f_i, of the LFM signal reveals the time frequency relationship. The instantaneous frequency of a narrowband signal can be calculated by the following expression, \( f_i(t) = \frac{1}{2\pi} \phi'(t) \) where \( \phi'(t) \) is the instantaneous phase of the signal.

3. Short Time Fourier Transform

The Short Time Fourier Transform is used to analyze windowed portion of the signal in time domain. This concept is referred as windowing. Time domain data of the signal is divided in different number of shorter sequences, to overlap. Fourier transform is used to show the frequency spectrum and calculate the magnitude of the signal for each sequence. Frequency spectrum is then ordered on a corresponding time-scale domain and a time, frequency and magnitude dimensional picture is formed. A spectrogram is a spectral representation of a signal. It shows how the spectral density of a signal varies with respect to time.

\[ \text{STFT}_P(t, \omega) = \int_{-\infty}^{\infty} P(\tau) \delta'(\tau - t) \exp(-j\omega\tau) d\tau \quad (2) \]

The STFT spectrogram is defined as the normalized squared magnitude of the STFT. Normalization makes the STFT spectrogram to obey property of Parseval’s energy conservation. The energy in the STFT spectrogram is equal to the energy in the original time domain signal.
Spectrogram is used very frequently for analyzing non stationary and time varying signals.

\[
\text{Spectrogram}[\text{STFT}_y[t, \omega]]
\]

The time-frequency resolution is achieved by choosing the proper window length about the same size as the stationary or time-invariance of the individual signal components.

\section{Continuous Wavelet Transform}

All time-frequency resolution or localization analysis techniques are based on the principle of computing correlation between the signal and analysis. Since the wavelet transform is a new time scale localization technique and the basic differences between the WT and the STFT are,

- Window size can be varied in Wavelet Transform as a function of the analyzing frequency.
- The Wavelet Transform having the analysis function it can be chosen with more freedom.

Time-Frequency analysis of a non stationary signal using Fourier and Short Time Fourier Transforms does not give any satisfactory results. Wavelet analysis method is ability to perform local analysis and reveal signal aspects. This technique does not provide the frequency contents of breakdown points and discontinuities. As compared with the STFT, wavelet analysis methods make it possible to perform a Multi-Resolution Analysis (MRA). The time-frequency resolution problem is caused by the uncertainty principle of Heisenberg and exists regardless of the used analysis technique. STFT method uses a fixed time frequency resolution. By using an approach called Multi Resolution Analysis (MRA) it is possible to analyze a signal at different frequencies with different resolutions. WT can be applied based on the "\(t\)" application. The wavelet analysis method is used to calculate the correlation between the signal and the respective wavelet function \(\Phi(t)\). The actual signal properties can be computed for different time intervals using the different time scale wavelet functions and the resulting WT is called a two dimensional TSL. The analyzing wavelet function \(\Phi(t)\) is also referred as the mother wavelet.

An analyzing shape and window are the important parameters in wavelet function. The CWT is given in the following equation,

\[
\text{CWT}(\tau, q) = \frac{1}{|q|} \int_{-\infty}^{\infty} p(t) \Phi^*\left(\frac{t-\tau}{q}\right) dt
\]

The function CWT \((\tau, q)\) is a transformed signal. It is a function of the translation parameter \(\tau\), scaling parameter \(q\), mother wavelet \(\Phi\), and complex conjugate parameter \(*\). The CWT also has an inverse transformation is also called ICWT:

\[
p(t) = \text{ICWT}(\text{CWT}(\tau, q)) = \frac{1}{|C_q|} \int_{-\infty}^{\infty} \text{CWT}(\tau, q) \frac{1}{s^2} \Phi^*\left(\frac{t-\tau}{q}\right) dq
\]

A wavelet window has its central frequency at each scale. The scale is inversely proportional to that frequency. If the large scale is corresponding to the low frequency then it gives the global information of the signal and a small scale is corresponding to the higher frequencies then it gives the detailed information.

\section{Fractional Fourier Transform}

Fractional Fourier Transform is a time frequency distribution technique. It provides us with an external degree of freedom, which gives more benefits over conventional methods like Fourier transform. It can allow chirp signals to be transformed into a fractional domain with a fractional order parameter. The ability of this transform is to process chirp signals better than the conventional Fourier Transform. The transform absorbs the chirp parameters in its kernel by a fractional parameter ‘alpha’. If magnitude response of the FrFT reaches its maximum, then the axis of rotation is matched to the signal chirp rate. It is defined as transform optimization.

The Fourier transform is expressed as,

\[
Y_\alpha(f) = \int_{-\infty}^{\infty} B_\alpha(f, t) y(t) dt,
\]

\[
Y(t) = \int_{-\infty}^{\infty} Y_\alpha(f) B_\alpha(f, t) df
\]

Where \(B_\alpha(f, t)\) the Fourier transform kernel, \(t\) and \(f\) are time and frequency parameters respectively.

\[
R_\alpha(\text{ft}) = \exp\left(\frac{-i\pi ft}{2}\right)
\]

The modified Fourier kernel equation for Fractional Fourier Transform technique is\textsuperscript{14,17}

\[
B_\alpha: B_\alpha = R_\alpha \exp\left[i\pi(a^2 \cot(\alpha x) - 2ab \csc(\alpha x) + b^2 \cot(\alpha x))\right]
\]

Where,

\[
R_\alpha = \exp\left[-\frac{i\pi \text{sgn} (\sin(\alpha x))}{4} + i\frac{\alpha x}{2} \text{sin}(\alpha x)\right]^{1/2}
\]

Where \(a, b\) defines the fractional domain axes. Rather than defining the fraction of the transform as an angle \(\alpha\), in the interval \([-\pi, \pi]\) radians, and another new variable defined as the transform order, \(\alpha_{\text{int}}\), in the interval \([-2, 2]\).
The fractional Fourier transform have many applications like signal processing, signal restoration and noise removal.

6. Results

6.1 Simulation Results

Time domain plot of input signal which is Sinusoidal signal plus LFM signal is shown in Figure 1. Graph was plotted between time and amplitude of the signal. The FFT of the input signal is having 2 kHz frequency for sinusoidal signal and LFM signal having frequency range from 5 kHz to 7 kHz is shown in Figure 2. Graph was plotted between frequency and magnitude of the input signal. Spectrogram will gives the spectral properties in time frequency plane having the signal frequencies and its time over different time intervals. Spectrogram of STFT is shown in Figure 3. Graph was plotted between time and frequency of the input signal. Wigner Ville Distribution technique is also giving signal properties in time and frequency plane even better than STFT in terms of energy and it is generating an additional interference cross terms apart from signal is shown in Figure 4.

Hilbert Huang Transform (HHT) technique will provide the time-frequency-energy description of time series data rather than a Fourier and wavelet transforms. Hilbert transform was used to compute instantaneous frequencies and amplitudes and describe the signal more locally. Time Frequency plot of HHT technique is shown in Figure 5. It’s also giving signal properties time-frequency plane even better than STFT in terms of energy. Continuous Wavelet Transform (CWT) gives the signal properties in time scale plane instead of giving in time-frequency plane is show in Figure 6. At different scales signal is analyzed like STFT and WVD.
Fractional Fourier Transform technique is also giving signal properties in time frequency plane. The frequency plot for LFM signal at $a = 1.036$ is shown in Figure 7. Graph was plotted between frequency and magnitude of the input signal. It will give better performance for time frequency resolution or localization for non stationary signals compared with WVD and HHT. FrFT can operate with a fractional parameter $\alpha$. From the simulation result figures we conclude that STFT has fixed time-frequency localization or resolution over different fixed time intervals. Fixed resolution is achieved by fixed window length chosen by STFT. CWT gives multi-resolution at different time scales. Using both the techniques we extracted the signal without any additional frequency terms but using WVD along with the signal frequency components additional frequency components also extracted. Thus it is concluded that WVD works better when the signal is single frequency component. Thus it is proposed that the STFT, CWT and FrFT methods are better for extracting or analysing the signal frequency components when it is more than one.

6.2 Experimental Results
The performance of CWT and FrFT over STFT is to be established in a practical scenario. For this purpose an experiment was conducted in an acoustic tank of dimension $10 \times 10 \times 10 \text{ m}^3$. The acoustic source and the receiver are placed at a depth of 4m and they are separated by a distance of 3m. A standard hydrophone is used as the transmitter and an array antenna with $6 \times 6$ elements are used to receive the signals. The received signal is signal conditioned by a preamplifier broadband beam former. The beam former output is digitized by ADC at a sampling frequency of 18 kHz to obtain the baseband component. The data was collected and analyzed in PC using MATLAB software. Pre-amplified and normalized range time domain of collected beam data 1 in acoustic tank for an input LFM signal without echo is shown in Figure 8. It represents the time domain plot of acoustic received signal tank data 1 without echo. Graph was plotted between pulse repetition time and ADC level of the signal.

Figure 8. Acoustic tank data1 plot.

Figure 9 represents the spectrogram plot of acoustic received signal tank data 1. Graph was plotted between pulse repetition time and frequency of the signal using STFT technique. Figure 10 represents the time-scale-energy plot of acoustic received signal tank data 1 using CWT technique. After applying the FrFT method on acoustic received tank data 1 and then the spectrogram plot is shown in Figure 11. Graph was plotted between pulse repetition time and frequency of the signal using FrFT technique. Figure 12 represents the pre-amplified and range normalized time domain of beam collected data 2 in acoustic tank for an input LFM signal with echo. Graph was plotted between pulse repetition time and ADC level of the signal.
Figure 9. Acoustic tank data1 plot of STFT.

Figure 10. Acoustic tank data1 plot of CWT.

Figure 11. Acoustic tank data1 plot of FrFT.

Figure 12. Acoustic tank data2 plot.

Figure 13. Acoustic tank data2 plot of STFT.

Figure 14. Acoustic tank data2 plot of CWT.

Figure 15. Graph was plotted between pulse repetition time and frequency of the signal using FrFT technique.

Figure 16. Graph was plotted between pulse repetition time and frequency of the signal using CWT technique. After applying the FrFT method on acoustic received tank data 2 and then the spectrogram plot is shown.
7. Conclusion

In this article the performances of different time frequency localization techniques using non stationary sonar signal for improving the underwater signal detection has been explored. The different time-frequency localization techniques that were used FFT, STFT, WVD, HHT, CWT and FrFT. Simulation results show that STFT, CWT and FrFT gives better spectral resolution or localization compared to the WVD and HHT. The experimental results are analyzed by performing STFT, CWT and FrFT on the underwater acoustic data. These three techniques give good results for non stationary signals. It is noticed that the work done in this paper is treated as bench mark for the underwater signal processing through the different time-frequency localization techniques of non-stationary sonar signals.

8. References

1. Boashash B. Time-frequency signal analysis. 1st ed. In: S. Haykin editor. NJ: Prentice Hall, Englewood Cliffs. 1991.
2. Boashash B. Time frequency signal analysis and processing: A comprehensive reference. Amsterdam the Netherlands: Elsevier; 2003.
3. Gabor D. Theory of communication. Journal of the Institution of Electrical Engineers, Part III: Radio and Communication. 1946; 93(26):429–57.
4. Cohen L. Time frequency distributions - a review. Proceedings of the IEEE. 1989; 77(7):941–81.
5. Cohen L. Time–frequency analysis. 1st ed. New York: Springer-Science Business Media. 1995.
6. Wigner EP. On the quantum correction for thermodynamic equilibrium. Phys Rev. 1932; 40(1):749–59.
7. Hlawatsch F, Boudreaux Bartels GF. Linear and quadratic time-frequency signal representations. IEEE Signal Processing Magazine. 1992; 9(2):21–67.
8. Sandsten MM. Time-frequency analysis of non-stationary processes an introduction. Lund University: Centre for Mathematical Sciences; 2013.
9. Huang NE, Shen SPS. Hilbert huang transform and its applications. 5th ed. Singapore: Interdisciplinary Mathematical Sciences World Scientific; 2005.
10. Sudheera K, Nanditha NM. Application of hilbert transform for flaw characterization in ultrasonic signals. Indian Journal of Science and Technology. 2015; 8(13):1–6.
11. Qian S, Morris JM. Wigner distribution decomposition and cross-terms deleted representation. Signal Processing. 1992; 27(2):125–44.
12. Schneider's MGE. Wavelets in control engineering [Master's thesis]. Netherland: Eindhoven University of Technology; 2001.
13. Merry RJE. Wavelet theory and applications a literature studies DCT 2005.3. Eindhoven University, Mechanical Engineering Control Systems Technology Group; 2005.
14. Almeida LB. The fractional Fourier transform and time - frequency representations. IEEE Transactions on Signal Processing. 1994; 42(11):3084–93.
15. Cowell David MJ, Freear S. Separation of overlapping Linear Frequency Modulated (LFM) signals using the fractional Fourier transform. IEEE Transactions on Ultrasonic's, Ferroelectrics and Frequency Control. 2010; 57(10):2324–33.
16. Ashok Narayanan V, Prabhu KMM. The fractional Fourier transform: Theory, implementation and error analysis, microprocessors and Microsystems. 2003; 27(10):511–21.
17. Venkata Suman J, Beatrice Seventline J. Separation of HFM and NLFM signals for radar using fractional Fourier transform. IEEE International Conference on Communication and Network Technologies; Sivakasi: Mepco Schlenk Engineering College; 2014. p. 193–7.
18. Suma bindu Y, Naidu Cheepurupalli CH. Time frequency localization techniques for non-stationary signals. NCAC'T14; Visakhapatnam, India: GITAM Institute of Technology; 2014. p. 12–3.