Damping Term Related to Ground Velocity Affects Structural Response Calculated by Displacement Excitation

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Abstract. The structural seismic responses obtained by a displacement excitation approach and an acceleration excitation approach respectively are sometimes significantly different. Regarding the causes of this difference, researchers have not provided a plausible explanation. For the aim of determining the effects of damping terms related to ground velocity on the structural seismic response calculated by the displacement excitation approach, the mode superposition method and Duhamel’s integral are applied to solving the dynamic equilibrium equations. The analysis indicates that the difference of structural seismic response caused by the method of displacement excitation, obtained by integral acceleration is small. When calculating the structural seismic response, a time step for displacement excitation and acceleration excitation can be assumed as the same value.

1. Introduction
For most structures, the structural seismic response can be calculated by acceleration excitation. When ground motions of all points at the base of the structure are different or the structure is subjected to displacement excitation, the structural seismic response must be calculated by displacement excitation. However, for the same structure, difference between the response obtained by displacement excitation and that of acceleration excitation respectively is significant. The causes of this difference have been followed by researchers for many years, but no widely accepted explanation has been adopted so far.

Using acceleration excitation to calculate structural seismic response is based on relative coordinates to establish the dynamic equilibrium equation. In this equation, the damping force is proportional to the relative velocity while restoring force is proportional to the relative displacement. Using displacement excitation to calculate the structural seismic response is based on global coordinates to establish the dynamic equilibrium equation. In this equation, the damping force is proportional to the absolute velocity while the restoring force is proportional to the absolute displacement. Therefore, difference exists between the dynamic equilibrium equations established on the basis on different coordinates (Lin & Zhang, 2004). Tsai (1998) indicates that if a specific damping term related to ground velocity is selected, the dynamic equilibrium equations based on different coordinates would be consistent. Here the structural response obtained by different kinds of excitation would be alike. However, when solving the dynamic equilibrium equation based on global coordinates, the structural response needs to be calculated by using ground velocity excitation and ground displacement excitation. And this solution process can be difficult (Clough & Penzien, 1995). If the damping term related to ground velocity is neglected, the dynamic equilibrium equation can be solved just by using ground displacement excitation (Li & Gui, 2012; Li et al. 2012; EL, 2004; Lou & Li, 2008). Li (2012) points out that if this method is used, the amplitude of difference under different kinds of excitation will be influenced by the excitation frequency and structural damping ratio. EL (2004) points out that the difference of the structural...
response under different kinds of excitation is significant at the base of the structure. This is because under the action of displacement excitation the damping force of the structure comes from the absolute velocity, and the rigid body motion at the base of the structure increases the damping force. Meanwhile, researchers also propose that by using higher mode static correction (Shi, 2011) and narrowing the time step (EL, 2004), the difference could be narrowed.

Although some studies have been carried out on the regular patterns of the difference, researchers have seldom made theoretical analysis on the relationship between the inconsistency of dynamic equilibrium equations and the difference of structural seismic response from a theoretical perspective. Therefore, in this paper, displacement is obtained from the acceleration integral as the excitation based on the linear hypothesis. The structural seismic response is calculated by using modal superposition method and Duhamel’s integral. The effects of displacement excitation obtained by acceleration excitation integral and reducing displacement excitation time step on the structural seismic response are analyzed and a formulation based on displacement excitation approach to calculate structural seismic response is presented.

2. Multiple Degree of Freedom (MDOF)

Based on global coordinates, the dynamic equilibrium equation established by using the lumped mass matrix, can be expressed as follows:

\[
\begin{bmatrix}
M & 0 \\
0 & M_{gg}
\end{bmatrix} \begin{bmatrix}
\ddot{u}_i \\
\ddot{u}_g
\end{bmatrix} + \begin{bmatrix}
C & C_g \\
C_g^T & C_{gg}
\end{bmatrix} \begin{bmatrix}
\dot{u}_i \\
\dot{u}_g
\end{bmatrix} + \begin{bmatrix}
K & K_g \\
K_g^T & K_{gg}
\end{bmatrix} \begin{bmatrix}
u_i \\
u_g
\end{bmatrix} = \begin{bmatrix}0 \\
P_i
\end{bmatrix}
\]

(1)

Where m-dimensional column vectors \( \{u_i, \dot{u}_i, \ddot{u}_i\} \) are displacement, velocity and acceleration of the ground motions of the support structure, respectively. And n-dimensional column vectors \( \{u_g, \dot{u}_g, \ddot{u}_g\} \) are the motions of the superstructure. The m-dimensional column vector \( \{P_i\} \) is the force applying to the support structure. M, C and K are the mass, damping and stiffness matrix of the superstructure, respectively. While M_{gg}, C_{gg} and K_{gg} are the mass, damping and stiffness matrix of the support structure, respectively. C_g and K_g are the damping and stiffness matrix, which are related to ground velocity and ground displacement. If the ground motions are known, the upper part of Equation (1) is the only part to be solved, and Equation (1) can be rewritten as follows:

\[
M\ddot{u} + C\dot{u} + Ku = -C_g\ddot{u}_g - K_gu_g
\]

(2)

When solving Equation (2), if the damping matrix related to the ground velocity is neglected, instead, take \( C_g = [0]_{m \times m} \), the dynamic equilibrium equation can be rewritten as:

\[
M\ddot{u} + C\dot{u} + Ku = KRu_g
\]

(3)

The dynamic equilibrium equation is solved by using modal superposition method, assuming structural response being represented by the equation below:

\[
u = \sum_{i=1}^{\text{dim}} \Phi_i x_i
\]

(4)

\( \Phi_i \) is i-th vibration mode, and \( x_i \) is the corresponding amplitude. Decoupling the dynamic equilibrium equation, Equation (3) can be expressed as follows:

\[
\ddot{x}_i + 2\xi_i w_i \dot{x}_i + w_i^2 x_i = -w_i^2 P_i u_g
\]

(5)

Applying Duhamel’s integral to solve Equation (5), the corresponding amplitude can be obtained as:

\[
x_i = \frac{w_i P_i}{\sqrt{1-\xi_i^2}} \int_0^t u_i(\tau)e^{-\xi_i w_i(t-\tau)} \sin w_{di}(t - \tau) d\tau
\]

(6)

Structural response can be obtained by displacement excitation, expressed as follows:
\[ x_i = -Ru_i(t) + \sum_{i=1}^{n} \Phi_i \frac{w_1}{1 - \xi_1^2} \int_0^t u_s(\tau) e^{-\xi_1(t-\tau)} \sin w_1(t-\tau) d\tau \]  

(7)

There is no equipment recording earthquake using displacement. Most earthquakes are recorded by using acceleration. For analyzing structural response, the common method to obtain displacement excitation is integrating acceleration twice.

Assuming acceleration is linear hypothesis during a time step, shown as follows:

\[ u_s(\tau) = u_{i-1} + \frac{1}{\Delta t} (u_i - u_{i-1}) \tau \quad 0 \leq \tau \leq \Delta t \]  

(8)

Integrating Equation (8), Equation (9) can be obtained, which is used to get displacement excitation.

\[ u_s(\tau) = u_{i-1} + \tau u_{i-1} + \frac{\tau^2}{2} u_i - u_{i-1} + \frac{\tau^2}{6} \frac{u_i - u_{i-1}}{\Delta t} \quad 0 \leq \tau \leq \Delta t \]  

(9)

3. Numerical Examples

When comparing the response of MDOF under different kinds of excitation, a ten-storey building is used as an example of calculation. In this example, each mass point of the structural can only implement horizontal displacement and the structural damping ratios are selected as 0.02, 0.05 and 0.07 respectively. Figure 1 shows the structural height, quality, and stiffness.

![Figure 1 Structural Parameters for MDOF Example](image)

**Table 1 Sections of building**

| level | column | beam |
|-------|--------|------|
| 1-4   | H 450×450×16×16 | H 600×200×12×16 |
| 5-10  | H 400×400×14×14 | H 600×200×12×16 |
When making a structural response contrast, three seismic records are selected according to the different characteristics of sites, shown in Table 2.

| number | Earthquake Name | Station Name                | $A_{pg}$/gal | $V_{pg}$/(cm/s $^1$) | $D_{pg}$/cm |
|--------|-----------------|-----------------------------|--------------|----------------------|------------|
| 1      | Kern Country    | 1096 Taft Lincoln School    | 3.810        | 64.253               | 111.822    |
| 2      | Cape Mendocin   | Cape Mendocin               | 10.194       | 43.960               | 34.850     |
| 3      | EL-Centro       | El Centro Array #9          | 3.417        | 38.130               | 140.050    |

Figure 2 indicates the maximum displacement response of each storey under Cape Mendocin excitation. For both DE (displacement excitation) and AE (acceleration excitation), time step is 0.02s, for DES (displacement excitation with small step) time step is 0.002s. As can be seen from Figure 2, the calculated result does not change when time step is reduced. Under other four excitations, reduced time step does not change the calculated results as well. Therefore, when calculating structural seismic response with displacement excitation, we should not use a time step smaller than that of acceleration excitation, instead, the same time step can be used. This is different with the suggestion provided in literature (6).

4. Conclusion

When solving the dynamic equilibrium equation which is established based on the absolute coordinates, neglecting the damping term related to the ground velocity has an advantage of solving the dynamic equilibrium equation. However, this will cause inconsistency of the dynamic equilibrium equations corresponding to the acceleration excitation and displacement excitation. The study shows that the inconsistency will lead to slightly different structural seismic response under different kinds of excitation in MDOF system, all modes used in calculation. Meanwhile, the study points out that the method to get displacement excitation and reducing the displacement excitation time step have negligible effect on structural seismic response under the displacement excitation.

Reference
[1] Lin jia hao, Zhang Yahui(2004). Pseudo-excitation method of random vibration[M], Beijing: Science press, in Chinese.
[2] HSIANG-CHUAN TSAI. Modal superposition method for dynamic analysis of structures excited by prescribed support displacements [J]. Computers and Structures, 1998, 66 (5):675-683.
[3] Ray Clough, Joseph Penzien, Dynamics of Structures[M].1995, Berkley: Computers and Structures, Inc.
[4] Li Yonghua, Gui Guoqing(2012). Relative motion error analysis method and absolute displacement direct solving method algorithm [J], The Civil Construction and Environmental Engineering, 31 (12): 112 -119, in Chinese.
[5] Li Yonghua, Zhuo Pingshan, Hu Yuanhui(2012). Absolute displacement algorithm error of frequency domain analysis directly under multipoint excitation[J]. Vibration and Impact.
[6] Wilson EL, Three Dimensional Static and Dynamic Analysis of Structures: A Physical Approach with Emphasis on Earthquake Engineering [M]. 2004, Berkley: Computers and Structures, Inc.

[7] Lou Menglin, Li Qiang (2008). Earthquake input pattern discussion about structure system [J], The Earthquake Engineering, 24 (2): 21-25, in Chinese.

[8] SHI Yong-jiu, JIANG Yang, WANG Yuan-qing. Application and improvement of direct solving method in seismic response analysis of structures under multi-support excitations [J]. Engineering Mechanics, 2011, 28 (1): 75-81.

[9] G. V. Berg and G. W. Housner. Integrated velocity and displacement of strong earthquake ground motion [J]. Bulletin of the Seismological Society of America. 1961, 51(2): 175-189.