Optimizing single-photon-source heralding efficiency and detection efficiency metrology at 1550 nm using periodically poled lithium niobate

S Castelletto¹, I P Degiovanni¹, V Schettini¹ and A Migdall²

¹ I.N.R.I.M., Turin 10135, Italy
² NIST, Optical Technology Division, Gaithersburg, MD 20899-8441, USA

Received 26 October 2005
Published 23 March 2006
Online at stacks.iop.org/Met/43/S56

Abstract
We explore the feasibility of using high conversion-efficiency periodically-poled crystals to produce photon pairs for photon-counting detector calibrations at 1550 nm. The goal is the development of an appropriate parametric down-conversion (PDC) source at telecom wavelengths meeting the requirements of high-efficiency pair production and collection in single spectral and spatial modes (single-mode fibres). We propose a protocol to optimize the photon collection, noise levels and the uncertainty evaluation. This study ties together the results of our efforts to model the single-mode heralding efficiency of a two-photon PDC source and to estimate the heralding uncertainty of such a source.

1. Introduction
Parametric down-conversion (PDC) consumes pump photons and produces light with a two-photon field description [1]. This two-photon light, which allows one photon to indicate or herald the existence of its twin, has a key role in applications such as quantum radiometry [2], where it makes an independent primary standard method possible for direct photon-counting detector calibrations. In addition, this two-photon light is key for improving single-photon source schemes that have significant advantages over the attenuated laser sources often employed in quantum information applications [3, 4]. To realize a single-photon source [5] appropriate for these applications, we consider two key issues: (a) efficient photon pair production and (b) collection of the heralded photon in a single-mode fibre (SMF), which is of interest for telecom applications. Periodically poled crystals (PPLs) have been used for efficient pair production [6] but have not yet been used for metrology. While metrology applications would certainly benefit from the high pump-conversion efficiency of PPLs, characteristics such as the level of background fluorescence may limit the uncertainty that can be achieved.

As for the collection efficiency of PDC light into a SMF, the goal has been to understand how the detection of one photon of a PDC pair in a well-defined spatial and spectral mode defines its partner’s spatial and spectral mode that can then be efficiently collected. Recent efforts have included experimental and theoretical work in both continuous wave (CW) [7–9] and pulse-pumped [10–12] bulk crystal PDC and have highlighted the critical problem of coupling the PDC photons into SMFs due to the difficulty of spatial and spectral matching of the photon pairs. The results show that optimization of the heralding efficiency is very sensitive to spectral and spatial mode selection.

Here we describe a PDC source and a measurement technique and compare them with other configurations. We present theoretical and experimental characterizations of the photon collection problem and suggest possible solutions.

2. Experimental setup
We pumped (figure 1) a 5 mm long periodically poled MgO-doped lithium niobate (PPLN) crystal with a CW laser at 532 nm. We used noncritical phase-matching (90° phase-matching angle) with a 7.36 µm poling period to produce 810 nm and 1550 nm photon pairs at external angles of 1° and 2°. We achieved fine tuning by adjusting the crystal temperature near 131 °C. Lens Lp, in the pump beam produced
3. Measurement technique

The heralded D₂ detector efficiency as measured by the PDC source is given by

\[ \eta_{det} = \frac{\chi_D}{\chi_T \cdot \tau_{opt} \cdot \tau_{SMF-lens}}, \]

where \( \chi_T \) is the heralded arm optical transmittance (including PPLN, Fc and DM) and \( \tau_{SMF-lens} \) is the optical transmittance of L₂ and the SMF of the heralded arm. \( \chi_D \) is the raw D₂ channel detection efficiency, directly measured according to

\[ \chi_D = \frac{P_{coinc} - P_{uncorr}}{(1 - P_{uncorr})(1 - P_{backgnd})}, \]

where \( P_{coinc} \) is the probability of coincidence counts per gate, \( P_{uncorr} \) is the probability of uncorrelated or accidental coincidence counts per gate (determined by changing the heralding delay so the detection gate misses heralded photons) and \( P_{backgnd} \) is the probability of gating counts produced by uncorrelated photons and dark counts in the heralding arm.

3.1. Raw detection efficiency

To understand equation (2) we note that when D₁ fires, D₂ is gated for a duration \( T \). We define the probability of D₂ not firing during \( T \) as \( p_T^0 \). We assume that the 'events' that make D₂ fire have a poissonian distribution (i.e. uniform in time). D₁ can fire for either background or PDC photons, and the associated probabilities are \( P_{heralding}^0 \) and \( P_{heralding}^1 \). The probability that D₂ fires for true correlated photons is

\[ P_{coinc} = P_{heralding}^0 P_{heralding}^1 P_{PDC}, \]

where \( P_{heralding}^0 \) is the probability for the heralded detector to fire for the correlated photon arriving at \( T/2 \), as opposed to firing during the first half of the gating time. We assume that correlated photons arrive exactly at \( T/2 \).

The probability that the heralded detector fires for accidental events is

\[ P_{acc} = (1 - P_{heralding}^0)(1 - P_{heralding}^1)P_{PDC} + p_T^0 P_{heralding}^0 (1 - P_{heralding}^1)(1 - P_{PDC}). \]

The first term is the probability of an accidental coincidence due to D₁ firing for a background event and D₂ firing for any event. The second and third terms are for D₁ firing for a PDC event and D₂ firing due to a background event either before or after the arrival of the PDC photon at \( T/2 \). For D₂ to fire after \( T/2 \), it cannot have fired due to a background photon before \( T/2 \) or a PDC photon at \( T/2 \). These factors make up the third term. The total coincidence probability is \( P_{coinc} = P_{heralding}^0 P_{heralding}^1 + P_{acc} \). Considering that for poissonian statistics \( p_T^0 = \frac{P_{PDC}}{T/2} \times \frac{P_{PDC}}{T/2} \) and inverting the above equations, we obtain equation (2), where \( P_{uncorr} = 1 - p_T^0 \).

3.2. Estimate and uncertainty

We develop an estimate of the raw detection efficiency and the associated uncertainty following [14]. The terms \( P_{coinc}, P_{uncorr} \) and \( P_{heralding}^0 \) in equation (2) are all independent statistical variables; therefore the estimate of the raw detection efficiency is

\[ \langle \chi_D \rangle = \left( \frac{1}{1 - P_{heralding}^0} \right) \times \left[ 1 - \left( \frac{1}{1 - P_{uncorr}} \right)(1 - P_{coinc}) \right], \]

(4)

Applying the maximum likelihood (ML) model estimator to the random variables \( P_{coinc} \) and \( P_{uncorr} \), the probability of \( M_{coinc} \) coincidence counts given \( M_{heralding} \) heralding counts is

\[ P(M_{coinc} | M_{heralding}, p) = \frac{M_{heralding}^1}{M_{coinc}^1} \times p^{M_{coinc}} (1 - p)^{M_{heralding} - M_{coinc}}, \]

(5)

where \( p \) is the probability of a correlated photon being detected. The uncertainty in \( \chi_D \) is propagated through the ML model and

\[ \chi_D \sim \text{Beta} \left( \frac{M_{coinc} + 1}{2}, \frac{M_{heralding} - M_{coinc} + 1}{2} \right). \]
where \( p \) is the parameter to estimate. The ML function is then

\[
L(M_{\text{conc}}, M_{\text{heralding}}|p) = P(M_{\text{conc}}|M_{\text{heralding}}|p) 
\times P(M_{\text{heralding}}),
\]

where \( P(M_{\text{heralding}}) \) is the distribution probability of heralding counts, which is not known. The minimum function of the unknown probability \( L(M_{\text{conc}}|M_{\text{heralding}}, p) \) gives the estimate of \( P_{\text{conc}} \), without the need for \( P(M_{\text{heralding}}) \). It can be shown that \( (P_{\text{conc}}) = (M_{\text{conc}}|M_{\text{heralding}}) \). The estimate of \( (P_{\text{uncorr}}) \) is less trivial. We must estimate \( 1/(1 - P_{\text{uncorr}}) \), as is clear from equation (4), so we introduce the parameter \( p' \) as \( p = (p' - 1)/p' \). The ML function in this case is

\[
L(M_{\text{uncorr}}, M_{\text{delayed}}(p') = P(M_{\text{uncorr}}|M_{\text{heralding}}|p) 
\times P(M_{\text{heralding}}),
\]

which we minimize to obtain

\[
(1/(1 - P_{\text{uncorr}})) = (M_{\text{heralding}}(M_{\text{delayed}} - M_{\text{uncorr}})).
\]

We distinguished \( M_{\text{heralding}} \) from \( M_{\text{delayed}} \) because we measure heralding counts in two configurations, one in the presence of true coincidence photons and one without coincidences when an electronic delay line is added to the heralding arm. \( M_{\text{uncorr}} \) are the coincidences corresponding to uncorrelated events. For \( P_{\text{heralding}} \) the ML model is not applicable because we cannot measure it at the same time as the background counts \( M_{\text{background}} \) and \( M_{\text{heralding}} \). Therefore we apply \( (P_{\text{background}}(M_{\text{background}})/M_{\text{heralding}}) \). This implies that \( (P_{\text{background}}(M_{\text{background}}) \) must be close to 0, to have no impact on the estimate of the raw detection efficiency. We finally estimate

\[
\langle \Delta \rangle = \frac{1}{1 - (M_{\text{background}}/M_{\text{heralding}})} 
\times \left[ 1 - \left( \frac{M_{\text{delayed}}}{M_{\text{heralding}} - M_{\text{uncorr}}} \right) \right] \times \left( \frac{1}{1 - (M_{\text{heralding}}/M_{\text{heralding}})} \right),
\]

with the uncertainty given by Gaussian uncertainty propagation for each random variable considered independently.

### 3.3. Heraldng efficiency

The heralding efficiency \( \chi_p \) is the efficiency of preparing a photon in the heralded channel in a definite spatial and spatial mode, by specific mode selection of the heralding arm. It quantifies how well the collection system geometrically catches photons correlated to those seen by \( D_1 \). To calibrate a SMF-coupled detector, the heralding efficiency must be optimized and estimated. It has been estimated in [8] for a bulk crystal. For PPLN [13] it is

\[
\chi_p = \frac{4 w_{0,1}^2 w_{0,2}^2 (w_{0,1}^2 + w_p^2)}{(w_{0,2}^2 w_p^2 + w_{0,1}^2 (w_{0,2}^2 + w_p^2))^2} \times \frac{\Delta_2}{(\Delta_1^2 + \Delta_2^2)^2} \times f(c_1, c_2),
\]

where \( w_{0,1,2} \) are the mode waist of the fibres at the crystal and \( \Delta_1, \Delta_2 \) are the FWHM of the spectral distribution selected geometrically by the SMFs. \( f \) is a correction function accounting for crystal length \( (L) \) and PDC crystal frequency dispersion given by [12]

\[
f(p, q) = \left\{ \int_0^1 dx e^{-p x^2 + (q x^2) + 4 p} (\text{Erf}(q x / 2 \sqrt{p}) - \text{Erf}((p - q) x / 2 \sqrt{p})) \right\} \sqrt{p}^{-1}.
\]

The parameters for this case are

\[
\begin{align*}
c_1 &= c_2 = \frac{L^2 (w_{0,2}^2 \alpha_2 + w_{0,1}^2 \tan \theta_1 + w_{0,1}^2 (\alpha_1 + \tan \theta_1)^2)}{w_{0,2}^2 w_p^2 + w_{0,1}^2 (w_p^2 + w_{0,1}^2)}, \\
c_2 &= \frac{L^2 D_{2n}^2 \Delta_2^2}{\Delta_1^2 + \Delta_2^2}, \\
s_1 &= \frac{L^2 D_{2n}^2 \Delta_2^2}{\alpha_2^2} + \frac{L^2 (w_{0,2}^2 \alpha_2^2 + w_{0,1}^2 (\alpha_1 + \tan \theta_1)^2)}{2 w_{0,1}^2 w_p^2 (w_p^2 + w_{0,1}^2)} + \frac{L^2 (2 w_{0,1}^2 w_p^2 (\alpha_2^2 + w_{0,1}^2 (\alpha_1 + \tan \theta_1)^2))}{2 w_{0,1}^2 w_{0,1}^2 (w_p^2 + w_{0,1}^2)}, \\
s_2 &= \frac{2 L^2 D_{2n}^2 \Delta_2^2}{\alpha_2^2} + \frac{L^2 (2 w_{0,1}^2 w_p^2 (\alpha_2^2 + \alpha_1 + \tan \theta_1 + \tan \theta_2)^2)}{w_{0,1}^2 w_{0,1}^2 (w_p^2 + w_{0,1}^2)}.
\end{align*}
\]

Here, \( \alpha = 2 \sqrt{\ln(2)} \) converts between the FWHM and the Gaussian profile 1/e² radius. The other terms are

\[
\begin{align*}
D_{1n} &= \mathcal{D}_1 (\cos \theta_1 - \sin \theta_1 \tan \theta_1) - \mathcal{D}_1 (\cos \theta_1 + \sin \theta_1 \tan \theta_1), \\
D_{1p} &= -\mathcal{D}_1 (\cos \theta_1 + \sin \theta_1 \tan \theta_1) + \mathcal{D}_p, \\
\mathcal{D}_s &= (\delta_{s_1} (\alpha_2, \phi_p) \omega_{s_1} / c) / (\omega_{s_1} \Omega_{s_1}), \\
\mathcal{D}_p &= (\delta_{p_0} (\alpha_2, \phi_0) \omega_{p_0} / c) / (\omega_{p_0} \Omega_{p_0}).
\end{align*}
\]

\( \alpha_s = -\cos \theta_1 \tan \theta_1 + \sin \theta_1 \tan \theta_1 \), \( \theta_{1,s} \) are the associated idler and signal emission angles around \( \phi_0 = \pi / 2 \) (the phase-matching angle in a non-classical phase-matching configuration) and \( n_{s,p} (\alpha_2, \phi_0 \) are the indices of refraction (all are \( \text{e}^{-r} \) at the three frequencies. \( \Omega_{s,p} \) are the central angular frequencies.

For our experiment the fibre mode field diameters are MFD\(_1\) = 3.9 \( \mu \)m for the heralding arm and MFD\(_2\) = 5.6 \( \mu \)m for the heralded arm, giving mode waists \( w_{0,1,2} = M_{1,2} \) MFD\(_1,2\) at the crystal according to the magnification (\( M_{1,2} \)) used. Because of this, we identify a SMF as a spectral filter with a gaussian spectral distribution given by \( \mathcal{I}_s (\omega_s - \Omega_s) \propto e^{-(\omega_s^2 / 2 \Delta_s^2)} \text{mod}(\omega_s, \Omega_s)^2 \). The component of spectral width due to geometric selection, \( \Delta_{1,2} \), is given by the FWHM angular collection \( \Delta \theta_{1,2} = \mathcal{D}_s (\pi w_{0,1,2}) \) and the spectral/angular spread of the PDC, \( \theta_{1,s} (\mathcal{D}_p) \) around the central wavelength \( \lambda_{1,s} \). Here, because the non-degenerate PPLN configuration and the bandwidth estimation are critical, we measured it with a monochromator in the heralding arm. We spectrally scanned the heralding single counts at three different values of \( w_{0,1} \) (table 1). We also theoretically estimated these bandwidths by evaluating the FWHM of the phase-matching function according to [15], extended to the case of PPLN. Here we consider the PDC phase-matching function including the pump transverse distribution

\[
\Phi(\omega_\alpha, \theta_1, \theta_s) = \exp\left( -\frac{w_{0,1}^2 (\Delta k_1^2 + \Delta k_2^2)}{4} \right) \times \left( \frac{\sin \Delta k \cdot L}{\Delta k_2 \cdot L} \right)^2.
\]

\[\text{S58}\]

Metrologia, 43 (2006) S56–S60
$\Delta k_{x,y,z} = (k_\text{p} - k_i - k_s)_{x,y,z}$, in terms of the pump, signal and idler $k$-vectors:

$$
\Delta k_c = \frac{n(\omega_s) k_0}{c} \cos \theta_i - \frac{n(\omega_o) k_0}{c} \cos \theta_o = \frac{2\pi}{\Lambda}
$$

$$
\Delta k_s = \frac{n(\omega_s) k_0}{c} \sin \theta_i + \frac{n(\omega_o) k_0}{c} \sin \theta_o,
$$

where $\Lambda$ is the poling period. To estimate the bandwidth selected by the heralding arm fibre, we first find $\theta_{\text{max}}$ that maximizes equation (11) and then we consider for each wavelength the function $f(\theta_i, \omega_i) = 1$ for $\Phi(\omega_s, \theta_{\text{max}}, \omega_o) > 0.5$ and $f(\theta_i, \omega_i) = 0$ for $\Phi(\omega_s, \theta_{\text{max}}, \omega_o) < 0.5$. Finally we invert the functions $f(\theta_i + \Delta \theta_i/2, \omega_i)$ and $f(\theta_i - \Delta \theta_i/2, \omega_i)$ to determine $\Delta \lambda$.

We estimated the heralded arm bandwidth in the same way. This estimation, which was found to be in agreement with the experimental values, allowed us to extrapolate the bandwidths of the heralded arm for other configurations. The discrepancy between the estimated and measured values is likely due to using Sellmeier’s equations [16] for undoped PPLN. Moreover, we do not have precise knowledge of the crystal poling region length (it maybe slightly shorter than the full 5 mm crystal length), a crucial engineering parameter. In figure 2 we report the estimated $\chi^D$ versus $w_{o,1,2}$ for our experiment’s fixed pump waist, and crystal length in two spectral-selection configurations on the heralding arm: (a) the SMF acts as a spectral filter with an average FWHM of about 2 nm and (b) a FWHM 0.1 nm spectral filter (a monochromator was used for this narrow bandwidth). The two experimental conditions for the measurements in table 2 are roughly indicated in figure 2(a) by squares, while the conditions for the narrow band measurement are indicated in figure 2(b). While the heralded arm is spectrally selected only by the fibre, the large improvement achieved by narrowing the heralding bandwidth indicates that spectral mode matching is more critical than spatial mode matching (the only spatial requirement is that $w_{o,2} \geq w_{p,1} > w_{p,2}$). Using this we should be able to improve $\chi^D$ by further narrowing the heralding arm bandwidth.

4. Experimental results

Following the approach in section 3.2, we measured the raw detection efficiency in our setup at the best signal-to-noise ratio of the heralding arm. Table 2 shows $\chi^D$ measured with $w_{o,1} = 82 \mu m$ fixed ($d_1 = 270 \text{ mm}$) for two different positions of the heralded arm, corresponding to $w_{o,2} = 158(197) \mu m$ ($d_2 = 300(380) \text{ mm}$). The estimates and uncertainties calculated with this model and the measured intrinsic statistical fluctuations show good agreement. For each setup we tested the repeatability by applying our alignment procedure [2] over several days. Figure 3 shows the measurement repeatability in agreement with the estimated uncertainty at the $2\sigma_{\text{stat}}$ level. However, we point out that the repeatability test was limited to collection lens positions close to the source. We obtained theoretical values of $\chi^D = 20\%$ and $13\%$, respectively, for the waist in table 2 using only the SMF spectral selectivity. The decrease in $\chi^D$ for bigger $w_{o,2}$ is in agreement with the measured decrease in $\chi^D$. Our theoretical model for $\chi^D$ is very sensitive to the measurements of the beam waists and the bandwidth estimates. Estimating the optical transmittances at $\tau_{\text{opt}} = 65\%$ and $\tau_{\text{SMF-lens}} = 83\%$ by an independent calibration and using the measured $\chi^D$ in table 2, we were able to consistently obtain $\chi^D = 48\%, 37\%$. On occasion we measured $\chi^D$ much closer to one by inserting a monochromator in the heralding arm; however, the stability of our setup did not allow repeatable results at this level.

At this stage, we note that the model can only be considered qualitative due to the spectral approximation adopted. In our approach we limited the calculation of the mismatch term to first order in the transverse $k$-vector and the frequency, losing some of the correlation between the frequency and the spatial variables in the biphoton field [12]. To compensate, we introduced a selection term due to the SMF spatial selection, but this works best for the degenerate
5. Conclusions

We have shown the feasibility of using PPLN for single-photon detector calibration at 1550 nm, as well as for a single-photon single-mode source. We highlighted the need to maintain low dark counts in the heralding arm to reduce statistical fluctuations. We have shown that proper spatial mode matching and spectral mode selection in the heralding arm is of paramount importance in achieving the highest heralding efficiency.

Acknowledgment

This work was supported in part by DTO, ARO and DARPA/QUIST.

References

[1] Klyshko D N 1988 Photons and Nonlinear Optics (London: Gordon and Breach)
[2] Migdall A L, Castelletto S, Degiovaninni I P and Rastello M L 2002 Appl. Opt. 41 2914–22
[3] Tittel W, Brendel J, Zbinden H and Gisin N 2000 Phys. Rev. Lett. 84 4737–40
[4] Knill E, Laflamme R and Milburn G J 2001 Nature 409 46–52
[5] Fasel S, Alibart O, Tanzilli S, Baldi P, Beveratos A, Gisin N and Zbinden H 2004 New J. Phys. 6 163
[6] Albota M A and Dauler E 2004 J. Mod. Opt. 51 1417–32
[7] Kurtz N, Oberparlieter M and Weinfurter H 2001 Phys. Rev. A 64 023802
[8] Castelletto S, Degiovaninni I P, Migdall A, Schettini V and Ware M 2004 Proc. SPIE 5551 60–72
[9] Ljunggren D and Tengner M 2005 Preprint quant-ph/0507046
[10] Bovino F A, Varisco P, Colla A M, Castagnoli G, Di Giuseppe G and Sergienko A V 2003 Opt. Commun. 227 343–8
[11] Pittmann T B, Jacobs B C and Franson J D 2005 Opt. Commun. 246 545–50
[12] Castelletto S, Degiovaninni I P, Schettini V and Migdall A 2005 Opt. Express 13 6709–22
[13] Castelletto S, Degiovaninni I P, Schettini V and Migdall A 2005 Proc. SPIE 5893 288–99
[14] Castelletto S, Degiovaninni I P and Rastello M I 2002 J. Opt. Soc. Am. B 19 1247–58
[15] Boeuf N, Branning D, Chaperot I, Dauler E, Guerin S, Jaeger G, Muller A, Migdall A 2000 Opt. Eng. 39 1016–24
[16] Jundt D H 1997 Opt. Lett. 22 1553–5