Heterotic Action in SUGRA-SYM Background

Jaemo Park\textsuperscript{1}, Cheol Ryou\textsuperscript{1} and Woojoo Sim\textsuperscript{3}

\textsuperscript{1}Department of Physics, POSTECH, Pohang 790-784, Korea
\textsuperscript{2}Postech Center for Theoretical Physics (PCTP), POSTECH, Pohang 790-784, Korea
\textsuperscript{3}Asia Pacific Center for Theoretical Physics, Pohang 790-784, Korea

Abstract

We consider the generalization of the heterotic action considered by Cherkis and Schwarz where the chiral bosons are introduced in a manifestly covariant way using an auxiliary field. In particular, we construct the kappa-symmetric heterotic action in ten-dimensional supergravity background coupled to super Yang-Mills theory and prove its kappa-symmetry. The usual Bianchi identity of Type I supergravity with super Yang-Mills $dH_3 = -\text{Tr} F \wedge F$ is crucially used. For technical reason, the Yang-Mills field is restricted to be abelian.
1 Introduction

In [1], supersymmetric heterotic action with kappa-symmetry in flat space is constructed motivated by the bosonic truncation of the heterotic action obtained by the M5 brane action wrapping K3 [2]. In the construction of [1, 2], the realization of the degrees of freedom associated with the current algebra is novel, which is generalization of PST approach of the realization of chiral two-form in M5-brane [3]. Thus it would be interesting to work out various aspects of this new heterotic action with kappa-symmetry. As one such attempt we generalize [1] to the kappa-symmetric heterotic action in arbitrary background of Type I coupled to super Yang-Mills. Due to the technical subtleties, we consider only abelian gauge fields. The form of the heterotic action can be guessed from the M5 brane action wrapping on K3. Once written, it is straightforward to work out the kappa-symmetry of such action. One point is worthy of mention. Along the proof of the kappa-symmetry the usual relation of the Type I supergravity coupled to super Yang-Mills

\[ dH_3 = - \text{Tr} F \wedge F \]  

(1.1)

is crucially used. Similar feature appears in the kappa-symmetric action of the membrane with the Horava-Witten boundary worked out by Cederwall [4].

In the current formalism of the heterotic string theory, it is easy to couple the abelian gauge field to the worldsheet of the heterotic string but the non-abelian generalization is not straightforward. Related fact is that if we wrap M5 brane on K3, we assume that involved K3 is smooth so that we can carry out the usual Kaluza-Klein compactification so that we obtain the heterotic string in 2-dimensions. Thus the non-abelian generalization is related to figuring out the additional degrees of freedom we should keep when we try to wrap M5 brane on singular K3. This is also related to working out the full chiral current algebra of the heterotic string when we assume the gauge group \( E_8 \times E_8 \) or \( SO(32) \) in the current formalism. Thus if we assume that the heterotic string realizes \( E_8 \times E_8 \) or \( SO(32) \) current algebra, abelian gauge field configuration we are considering is a subclass of the general field configuration. In the proof of the kappa symmetry, the background equation of motion we obtain has straightforward generalization to non-abelian gauge field background so we can guess the general form of the background equation of motion once we figure out the correct coupling of the general gauge field configurations to the worldsheet of the heterotic string. This is an interesting topic per se, but it’s beyond the scope of the current letter.
In the below we assign coordinate indices as follows:

- 10-d and 11-d target (tangent): $Z^A = (X^a, \theta^a)$.
- 10-d and 11-d target (curved): $Z^M = (X^m, \theta^\mu)$.
- 2-d and 6-d worldvolume: $\sigma^i, \sigma^j, \cdots$.
- K3: $X^r = \sigma^r, X^s = \sigma^s, \cdots$ (static gauge).

## 2 Heterotic string in SUGRA-SYM background from M5-brane

In [1], a 10-d supersymmetric heterotic action in flat background has been constructed from the M5-brane action doubly reduced on K3 surface. Here we extend the 10-d action to have the SUGRA-SYM background. As in the flat case, the action with generic background can be guessed from the M5-brane action with 11-d SUGRA background. Then, we can justify the action by working out the gauge symmetry and the kappa symmetry as given in section 3.

The M5-brane action in 11-d supergravity background is given as [5, 6]

$$\mathcal{L}_1 = -\sqrt{-\det(G_{ij} + i\tilde{H}_{ij}\sqrt{-Gu^2})},$$

$$\mathcal{L}_2 = -\frac{1}{4u^2}\tilde{H}^{ij}\mathcal{H}_{ijk}G^{kl}u_l,$$

$$S_{WZ} = \int \left(c_6 + \frac{1}{2}\mathcal{H}_3 \wedge c_3\right). \quad (2.1)$$

Here the worldvolume fields are $Z^M, A_2$ and auxiliary scalar $a (u_i = \partial_ia)$ which makes the action covariant. In the action, $\mathcal{H}_3$ is the supersymmetrized field strength of $A_2$:

$$\mathcal{H}_3 = dA_2 - c_3, \quad \tilde{H}^{\mu\nu} = \frac{1}{6}e_{ijklmn}\mathcal{H}_{klm}u_n, \quad (2.2)$$

and $\Pi_i^A, c_3, c_6$ are the pullbacks of the supervielbein and the superforms of the background:

$$\Pi_i^A = \partial_i Z^M E_M^A,$$

$$c_{ijk} = \partial_i Z^M \partial_j Z^N \partial_k Z^P C_{PNM},$$

$$G_{ij} = \Pi_i^A \Pi_j^B \eta_{AB}. \quad (2.3)$$

From the above action, we can infer the 10-d heterotic action in SUGRA-SYM background. This has been done for a simpler case in [1, 2] by wrapping the M5-brane
on K3 surface to get a 7-d action and then by lifting it to 10-d. Here we adopt the same scheme but do not perform the reduction in a precise way. Instead, we guess the reduction and the 10-d action from the past experience of the calculations in [1].

First, the worldvolume fields in eq. (2.1) can be reduced to the worldsheet fields as

\[(dA_2)_{ijk} \rightarrow \partial_i Y^I b_{Irs},\]
\[c_{ijk} \rightarrow -A^I_i b_{Irs},\]
\[c_{i_1...i_6} \rightarrow -B^{ij} \omega_{rstu},\]  
(2.4)

where \(b_I\) and \(\omega\) are the harmonic 2-forms and the volume form on K3, respectively. For instance, the reduction of \(c_3\) can be inferred as

\[c_{irs} = \partial_i Z^M \partial_r Z^N \partial_s Z^P C_{P,N,M} \rightarrow -\partial_i Z^M A^I_M b_{Irs} = -A^I_i b_{Irs}.\]  
(2.5)

Note that among 22 harmonic 2-forms on K3, 19 are anti-self-dual and 3 are self-dual, which yields the correct degrees of freedom of chiral bosons \(Y^I\) (upon gauge fixing) for the heterotic string action compactified on \(T^3\).

In eq. (2.4), as the action is lifted up to 10-d, \(A_i\) and \(B_{ij}\) are understood as the pullbacks of 10-d superforms:

\[A^I_i = \partial_i Z^M A^I_M,\]
\[B_{ij} = \partial_i Z^M \partial_j Z^N B_{NM}.\]  
(2.6)

Then, \(\mathcal{H}_3 = dA_2 - c_3\) reduces to

\[\mathcal{H}_{ijk} \rightarrow (\partial_i Y^I + A^I_i) b_{Irs} = D_i Y^I b_{Irs},\]
\[\tilde{\mathcal{H}}_{ij} \rightarrow \tilde{Y}^I b_{Irs},\]  
(2.7)

where \(\tilde{Y}^I = \epsilon^{ij} D_i Y^I u_j\) and \(D_i\) is a gauge covariant derivative given that

\[\delta A^I_i = \partial_i \Lambda^I, \quad \delta Y^I = -\Lambda^I.\]  
(2.8)

Note that in this way we obtain the coupling of the abelian gauge field to the heterotic string.
In eq. (2.4), the harmonic 2-forms $b_I$ contribute to the reduced action in terms of

$$L_{IJ} = \int_{K3} b_I \wedge b_J, \quad M_{IJ} = \int_{K3} b_I \wedge *b_J. \quad (2.9)$$

Here, $L$ is the intersection matrix of the 2-cycles on $K3$:

$$L = -\Gamma_8 \oplus -\Gamma_8 \oplus \sigma \oplus \sigma \oplus \sigma, \quad (2.10)$$

where $\Gamma_8$ is the same as the Cartan matrix of $E_8$ and $\sigma = (\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix})$. Then, as lifted to 10-d, $L$ can be interpreted as the Cartan matrix of $E_8 \times E_8$ or $SO(32)$, which is one of the two gauge groups of 10-d heterotic theory. (Note, in 10-d, $L = -M$.) Note that at the tree level, the possible gauge groups are not determined. Only if we consider one-loop consistency condition, the possible gauge groups can be fixed.

In this sense, $Y^I$ in eq. (2.4) are understood as the scalars which represent the 16 bosonic degree of freedom compactified on $T^{16}$ in 10-d heterotic theory, yielding the Narain lattice which is the same as the root lattice of $E_8 \times E_8$ due to the one-loop modular invariance. In addition, the gauge field $A^I$, which couples to $L_{IJ}$, is an element of the Cartan subalgebra of $E_8 \times E_8$, so as to be abelian ($F = dA$), being consistent with the form of the action given below. We can also take $L = -\Gamma_{16}$, root lattice of $SO(32)$ gauge group for $SO(32)$ heterotic string theory [2].

Finally, taking above field reductions into account, we propose the heterotic action whose background is 10-d SUGRA-SYM:

$$L_1 = -\sqrt{-G} \sqrt{1 - \frac{\tilde{Y}^I L_{IJ} \tilde{Y}^J}{G u^2} + \left( \frac{\tilde{Y}^I L_{IJ} \tilde{Y}^J}{2G u^2} \right)^2},$$

$$L_2 = -\frac{\tilde{Y}^I L_{IJ} D_i Y^J u^i}{2 u^2},$$

$$L_{WZ} = -\frac{1}{4} \epsilon^{ij} Y^I L_{IJ} F_{ij}^I - \frac{1}{2} \epsilon^{ij} B_{ij}. \quad (2.11)$$

This action is invariant under gauge transformations

$$\delta A^I_i = \partial_i \Lambda^I, \quad \delta Y^I = -\Lambda^I, \quad \delta B_{ij} = \frac{1}{2} \Lambda^I L_{IJ} F_{ij}^J. \quad (2.12)$$
3 Kappa symmetry of 10-d heterotic action

Now let us show the kappa invariance of the action. First, we propose the following kappa variations of the worldsheet fields in the heterotic action.

\[ \delta \kappa Z^M = \Delta^\alpha E^M_{\alpha} \quad (\delta \kappa Z^M E^\alpha_M = \Delta^\alpha, \quad \delta \kappa Z^M E^\alpha_M = 0), \]

\[ \delta \kappa Y^I = K^I, \]

\[ \delta \kappa A^I_i = -\partial_i K^I + F^I_i, \]

\[ \delta \kappa B_{ij} = -\Delta^\alpha \Pi^A_i \Pi^B_j (dB)_{\alpha AB} + \text{total derivative}, \]  

where \( \Delta^\alpha = \kappa^\beta (1 - \Gamma)^\alpha_\beta \) and

\[ K^I = -\delta \kappa Z^M A^I_M = -\Delta^\alpha E^M_{\alpha} A^I_M, \]

\[ F^I_i = \delta \kappa Z^M \partial_i Z^N F^I_{MN} = \Delta^\alpha \Pi^A_i F^I_{A}. \]  

(3.1)

Then, the variations of related fields in the action are evaluated as

\[ \delta \kappa \Pi^a_i = \Delta^\alpha \Pi^B_i (T_{aB} - \Omega_{aB}), \]

\[ \delta \kappa G_{ij} = \Delta^\alpha T_{aA} \Pi^A_i \Pi^B_j \eta_{ab}, \]

\[ \delta \kappa (D_i Y^I) = F^I_i, \]

\[ \delta \kappa \tilde{Y}^I = \varepsilon^{ij} F^I_i u_j, \]  

(3.3)

where \( T \) and \( \Omega \) are the torsion and the connection one-form of the background superspace, respectively.

Note here that the kappa variation of \( Y^I \) in eq. (3.1) can be inferred from the variation of \( A_2 \) in M5-brane side:

\[ \delta \kappa A_2 = -\delta \kappa Z^M dZ^N dZ^P C_{PMN} \]

\[ \rightarrow \delta \kappa Y^I b_{Irs} = -\delta \kappa Z^M A^I_M b_{Irs}, \]  

(3.4)

yielding

\[ \delta \kappa Y^I = -\delta \kappa Z^M A^I_M = -\Delta^\alpha E^M_{\alpha} A^I_M \equiv K^I. \]  

(3.5)

For the details of the kappa variations of the fields, see appendix \[A\].
3.1 Kappa Invariance: scheme

The method we use in the proof of the kappa symmetry is similar to the one used in [1] for the flat case without $A_i$. We organize terms in $\delta_\kappa \mathcal{L}$ in the order of $\tilde{\mathcal{Y}}^I = \epsilon^{ij} D_i Y^I u_j$ and check the invariance order by order. This means that all terms in the variation should be recast in terms of the powers of $\tilde{\mathcal{Y}}$. For the case of $\delta_\kappa \mathcal{L}_1$, this is evident since all terms in $\mathcal{L}_1$ are already written in terms of $\tilde{\mathcal{Y}}$.

Then we can expect the terms in $\delta_\kappa (\mathcal{L}_2 + \mathcal{L}_{WZ})$ to be recast in terms of $\tilde{\mathcal{Y}}$ as well, considering the Nambu-Goto type action we are dealing with here. Indeed, as advertised in the introduction, this can be realized in the aid of the $H_3$ in eq. (1.1) and (3.10). For the details, see section 3.3.

In the proof, $U$ and $T$ are defined as

$$4\Delta U = \delta_\kappa (\mathcal{L}_1)^2, \quad 2\Delta T = \delta_\kappa \mathcal{L}_2 + \delta_\kappa \mathcal{L}_{WZ},$$

(3.6)

where $\Delta = \bar{\kappa}(1 - \Gamma)$ and we have suppressed the spinor indices. Then, we can take a quantity $\rho$ satisfying

$$U = \rho T, \quad \rho^2 = (\mathcal{L}_1)^2,$$

(3.7)

which ensures the kappa invariance taking $\Gamma = \rho/\mathcal{L}_1$:

$$\delta_\kappa \mathcal{L} = 2\Delta \left( \frac{U}{\mathcal{L}_1} + T \right) = 2\bar{\kappa}(1 - \Gamma)(1 + \Gamma)T = 0.$$

(3.8)

The details of the proof will be given in section 3.3.

In the proof of kappa symmetry, we need to consider the on-shell constraints on the SUGRA-SYM backgrounds. Using the constraints, we will see that the proof goes in the same way as the flat case with the addition of gauge field $A_i$. Therefore, let us first look through the SUGRA-SYM constraints.

3.2 SUGRA-SYM constraints

The on-shell constraints on the 10-d heterotic background read [7, 8, 9]

$$T_{\alpha\beta}^a = 2(\Gamma_a)^{\alpha\beta}, \quad T_{ab}^a = 0,$$

$$H_{\alpha\beta a} = 2(\Gamma_a)^{\alpha\beta}, \quad H_{aba} = H_{a\beta}\gamma = 0,$$

$$F_{\alpha a} = 2(\Gamma_a)^{\alpha\beta} \chi^\beta, \quad F_{a\beta} = 0.$$

(3.9)
Here, $H_3$ is written as

$$H_3 = dB_2 - \frac{1}{2} A^I \wedge F^J L_{IJ} = dB_2 - \operatorname{Tr}(A \wedge F)$$  \hspace{1cm} (3.10)$$

Note that $H_3$ is a gauge invariant field strength, which can be seen from eq. (2.12). In addition, reminding that $A$ is an element of the Cartan subalgebra of $E_8 \times E_8$ and that $L_{IJ}$ is the Cartan matrix of the algebra, we see that eq. (3.10) is satisfied since

$$\operatorname{Tr}(A \wedge F) = A^I \wedge F^J \operatorname{Tr}(h_I h_J) = \frac{1}{2} A^I \wedge F^J L_{IJ}.$$  \hspace{1cm} (3.11)$$

for a suitable choice of the basis $h_I$ of the Cartan subalgebra.

Using the constraints in eq. (3.9), the kappa variations of related fields reduce as

$$\delta_\kappa G_{ij} = \Delta^a T_{a;i} \Pi^A (i) \Pi^B (j) = 2 \Delta \gamma (i \Pi_j),$$

$$\delta_\kappa (D_i Y^I) = F^I_i = \Delta^a \Pi^A_a F^I_{\alpha A} = -2 \Delta \gamma (i \chi^I),$$  \hspace{1cm} (3.12)$$

and, up to a total derivative,

$$\delta_\kappa B_{ij} = -\Delta^a \Pi^A_a \Pi^B_i (dB)_{\alpha AB}$$

$$= -\Delta^a \Pi^A_a \Pi^B_i (H_{\alpha AB} + \operatorname{Tr}(A \wedge F)_{\alpha AB})$$

$$= 2 \Delta \gamma [i \Pi_j] - \frac{1}{2} \Delta^a \Pi^A_a \Pi^B_i L_{IJ} (A^I \wedge F^J)_{\alpha AB}.$$  \hspace{1cm} (3.13)$$

with $\gamma_i = \Pi^a_i \Gamma_a$ in the final expression of each formula. (We have not suppressed the spinor indices for the 3-superforms to be explicit.) Here the beauty lies in the fact that the second term of $\delta_\kappa B_{ij}$ exactly cancels some terms that appears in $\delta_\kappa (L_2 + L_W Z)$, which is needed to make every term of $T$ written in terms of the powers of $\tilde{Y}$. (See the detailed derivations given below.)

Note in eq. (3.12) that $\delta_\kappa G_{ij}$ has the same form as the flat case [1] where $\Pi_j$ reduces to $\partial_j \theta$. Likewise, the first term of $\delta_\kappa B_{ij}$ has the same form as the flat case. Consequently, as mentioned above, this implies that the proof works in the same way as the flat case. However, in [1] the induced gauge field $A_i$ was not considered. Therefore, while we describe the whole proof in the followings, we concentrate on the part where the gauge field has an effect on.

### 3.3 Details of the proof

Now let us complete the proof by showing $U = \rho T$ with $\rho^2 = (\mathcal{L}_1)^2$. 

First, $\rho$ can be determined as

$$\rho = \gamma - \frac{\tilde{Y}L\tilde{Y}}{2Gu^2}\gamma. \quad (3.14)$$

Then, we will show $U = \rho T$ order by order in $\tilde{Y}$:

$$U_0 = \rho_0 T_0,$$
$$U_1 = \rho_0 T_1,$$
$$U_2 = \rho_0 T_2 + \rho_2 T_0,$$
$$U_3 = \rho_2 T_1,$$
$$U_4 = \rho_2 T_2, \quad (3.15)$$

where the subscripts denote the order of $\tilde{Y}$ in the terms and we have used $\rho_1 = 0$.

Here $U$ and $T$ are determined from eq. (3.6) using the kappa variations of the worldsheet fields in eq. (3.12) and (3.13) where SUGRA-SYM constraints have been considered. First, from $\delta_k(L_1)^2$ we get $U$ as

$$U_0 = -G\gamma^i \Pi_i,$$
$$U_1 = \frac{\tilde{Y}L}{u^2} \epsilon^{ij} u_i \gamma^j \chi,$$
$$U_2 = \frac{\tilde{Y}L\tilde{Y}}{(u^2)^2} u^i u^j \gamma^j \Pi_j,$$
$$U_3 = -\frac{\tilde{Y}L\tilde{Y}}{2G(u^2)^2} \tilde{Y}L\epsilon^{ij} u_i \gamma^j \chi,$$
$$U_4 = -\frac{\tilde{Y}L\tilde{Y}}{4G(u^2)^3} (2\gamma^i u^j - \gamma^j u^i) u_i \Pi_j. \quad (3.16)$$

Next, $T$ is determined by $\delta_k L_2$ and $\delta_k L_{WZ}$:

$$\delta_k L_2 = -\frac{1}{2u^2} \tilde{Y}LF_i u^i - \frac{\epsilon^{ij}}{2u^2} F_i u_j LD_k Y u^k + \frac{\tilde{Y}L\tilde{Y}}{G(u^2)^2} \epsilon^{ik} u_k u^j \Delta^i \Pi_j,$$
$$\delta_k L_{WZ} = \frac{\epsilon^{ij}}{2} (KL\partial_i A_j + F_i L \partial_j Y) - 2\epsilon^{ij} \Delta^i \Pi_j - \frac{\epsilon^{ij}}{4} \Delta^a \Pi^A_i \Pi^B_j (A \wedge F)_{aAB}, \quad (3.17)$$

where we have used the identity

$$\epsilon^{ij} \epsilon_{kl} = -\delta^i_k \delta^j_l + \delta^i_l \delta^j_k. \quad (3.18)$$
Here, adding the first two terms of $\delta_\kappa \mathcal{L}_2$ to the first two terms of $\delta_\kappa \mathcal{L}_{WZ}$ and using eq. (3.18) yield
\begin{equation}
- \frac{1}{u^2} \tilde{Y} LF_i u^i + \frac{\epsilon^{ij}}{2} (KL \partial_i A_j - F_i L A_j). \tag{3.19}
\end{equation}

Note that the last two terms in eq. (3.19) are not written in terms of $\tilde{Y}$ and we can expect it to be canceled out with some other term in the variation. In fact, those two terms are recast as
\begin{equation}
-\frac{\epsilon^{ij}}{4} \Delta^a \Pi_i^A \Pi_j^B L_{I,J} (A^I \wedge F^J)_{\alpha AB}, \tag{3.20}
\end{equation}
which exactly cancels out the last term of $\delta_\kappa \mathcal{L}_{WZ}$, making all terms in $\delta_\kappa (\mathcal{L}_2 + \mathcal{L}_{WZ})$ written in terms of $\tilde{Y}$. With eq. (3.20), we are led to
\begin{align*}
T_0 &= -\epsilon^{ij} \gamma_i \Pi_j, \\
T_1 &= \frac{\tilde{Y} L}{u^2} u^i \gamma_i \chi, \\
T_2 &= \frac{\tilde{Y} L}{2G(u^2)^2} \epsilon^{ik} u_k u^j \gamma_i \Pi_j). \tag{3.21}
\end{align*}

Now, we are left with the verification of $U = \rho T$. However, we have seen in section 3.2 that the kappa variations of the worldsheet fields have the same forms, the SUGRA-SYM constraints being considered, as the ones in the flat background. Then, since in [1] the kappa symmetry for the flat case without gauge field $A$ has been proved, it is sufficient here to consider only the terms related with the variation of $A_i$ in showing $U = \rho T$.

Considering the kappa variations of the fields in eq. (3.12) and (3.13), we notice that only $\delta_\kappa D_i Y$ is related to $\delta_\kappa A_i$ and that the terms including $\delta_\kappa (D_i Y)$ in $U = \rho T$ are the 1st and the 3rd order terms. As a result, we only need to show $U = \rho T$ in the odd orders.\footnote{For the proof of $U = \rho T$ in even orders, refer to the appendix of [1].}

However, note that $U_3$ can be recast as, for $[\rho_0, \rho_2] = 0$,
\begin{equation}
U_3 = \rho_2 \rho_0^{-1} U_1, \tag{3.22}
\end{equation}
and that if $U_1 = \rho_0 T_1$ is satisfied, $U_3 = \rho_2 T_1$ is automatic. Therefore, we only need to
show $U_1 = \rho_0 T_1$ to complete the proof. Then, using $\rho_0 = \bar{\gamma}$,

$$\rho_0 T_1 = \frac{\bar{Y} L}{u^2} \bar{\gamma} \gamma^i u_i \chi = \frac{\bar{Y} L}{u^2} \epsilon^{ij} u_i \gamma_j \chi = U_1,$$

(3.23)

where we have used the identity $\bar{\gamma} \gamma^i = \epsilon^{ij} \gamma_j$.

Thus, we have completed the proof of the kappa invariance.

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Appendix

A Kappa Variations

The kappa variations of the pullback $A^I_i$ is evaluated as

$$\delta_\kappa A^I_i = \delta_\kappa (\partial_i Z^M A^I_M)$$

$$= \partial_i \delta_\kappa Z^M A^I_M + \partial_i Z^M \delta_\kappa A^I_M$$

$$= \partial_i (\delta_\kappa Z^M A^I_M) - \delta_\kappa Z^M \partial_i A^I_M + \partial_i Z^M \delta_\kappa A^I_M$$

$$= \partial_i (\delta_\kappa Z^M A^I_M) - \delta_\kappa Z^M \partial_i Z^N \partial_N A^I_M + \partial_i Z^M \delta_\kappa Z^N \partial_N A^I_M$$

$$= -\partial_i K^I + F^I_i,$$

(A.1)

where, for $F^I = dA^I$,

$$F^I_i = \delta_\kappa Z^M \partial_i Z^N F^I_{MN} = \Delta^\alpha \Pi_i^A F_{\alpha A}.$$ 

(A.2)

Note, from $\delta_\kappa Y^I$ and $\delta_\kappa A^I_i$ above,

$$\delta_\kappa (D_i Y^I) = \partial_i \delta_\kappa Y^I + \delta_\kappa A^I_i = F^I_i,$$

$$\delta_\kappa \bar{Y}^I = \epsilon^{ij} F^I_i u_j,$$

(A.3)
which is crucial for the kappa symmetry in the sense that $\Delta$ can be factorized as an overall factor.

The kappa variation of $B_{ij}$ is evaluated similarly to $\delta_\kappa A_i$, up to a total derivative:

$$
\delta_\kappa B_{ij} = -\delta_\kappa Z^M \partial_i Z^N \partial_j Z^P (dB)_{MNP} = -\Delta^\alpha \Pi_i^A \Pi_j^B (dB)_{\alpha AB}.
$$

(A.4)

Finally, we need to evaluate $\delta_\kappa \Pi_i^A$:

$$
\delta_\kappa \Pi_i^A = \delta_\kappa (\partial_i Z^M E_M^A)
$$

$$
= \partial_i \delta_\kappa Z^M E_M^A + \partial_i Z^M \delta_\kappa E_M^A
$$

$$
= \partial_i (\delta_\kappa Z^M E_M^A) - \delta_\kappa Z^M \partial_i E_M^A + \partial_i Z^M \delta_\kappa E_M^A
$$

$$
= \partial_i \Delta^A + \Delta^\alpha \Pi_i^B (T_{\alpha B}^A - \Omega_{\alpha B}^A),
$$

(A.5)

where we have used $\Delta^a \equiv 0$ (as a notational convention), which is consistent with

$$
\delta_\kappa Z^M E_M^a = \Delta^a, \quad \delta_\kappa Z^M E_M^a = 0.
$$

(A.6)

As a result,

$$
\delta_\kappa \Pi_i^a = \Delta^\alpha \Pi_i^B (T_{\alpha B}^a - \Omega_{\alpha B}^a),
$$

(A.7)

which yields

$$
\delta_\kappa G_{ij} = \delta_\kappa \Pi_i^a \Pi_j^b \eta_{ab}
$$

$$
= \Delta^\alpha \Pi_i^B \Pi_j^b (T_{\alpha B}^a - \Omega_{\alpha B}^a) \eta_{ab}
$$

$$
= \Delta^\alpha T_{alpha B} \Pi_i^B \Pi_j^b \eta_{ab}.
$$

(A.8)

Note here that the connection $\Omega$ has not contributed since $\Omega_{\alpha Bb} + \Omega_{\alpha bb} = 0$.

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