Excess current in superconducting Sr$_2$RuO$_4$

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Introduction.—The discovery of superconductivity below $T_c \sim 1.5$ K in Sr$_2$RuO$_4$ has quickly triggered a large amount of interest because of the unconventional properties and the initially proposed analogy to $^3$He. The enhanced specific heat, magnetic susceptibility, and electronic mass indicate the presence of significant correlations [1,2]. For a more detailed overview see Refs. 1 and 2. The exact symmetry of the superconducting order parameter (OP) [3,4,5,6] and notably the pairing mechanism [7,8,9,10,11,12] are still controversial. The shape of the spectra previously obtained from point-contact measurements in superconducting Sr$_2$RuO$_4$ where satisfactorily reproduced by an analysis of a $p$-wave pairing state with OP $d(k) = \tilde{z}(k_x \pm ik_y)$ [13].

It has been shown that the excess current in s-wave superconductors is proportional to the superconducting gap [14] and consequently contains further information on the superconducting state. The present paper discusses excess-current measurements in Sr$_2$RuO$_4$ in greater detail. In particular, we present experimental results for the excess current in applied magnetic fields and find a systematic linear behavior as a function of field over a surprisingly wide range. This finding is compared with the experimental excess-current data from Ref. 10 (squares in Fig. 1), which exhibit a strikingly linear temperature dependence as well. We show that these two findings imply a well defined functional relationship between the excess current and the OP, and discuss the resulting implications in the framework of the $p$-wave picture [15,16,17] extended to include effects of low-energy fluctuations [18,19]. Our measurements also suggest the presence of a normal-conducting surface layer in Sr$_2$RuO$_4$. We model such a layer by an enhanced scattering rate near the surface and obtain qualitative agreement with the experimental point-contact spectra within the $p$-wave picture.

FIG. 1: Temperature dependence of the normalized excess current across a point contact in Sr$_2$RuO$_4$. Experimental results (squares) are taken from Ref. 10. Open symbols show the results of our calculation for the excess current from a $p$-wave analysis, without (diamonds) and with (circles) the effects of an inelastic scattering channel $\Gamma_{\text{in}}(T)$. The dashed thick curve illustrates the scaling relation $I_{\text{exc},2}(T) \propto \Delta_{\text{bulk},2}(T)^2$ for the inelastic scattering model.

Experiment.—Our measurements were performed on two single crystals grown both by a floating zone technique in different groups. $T_c$ was obtained via bulk resistivity measurements. One crystal, labeled #5 (Ref. 20), shows a midpoint transition temperature $T_c^{50\%} = 1.02$ K with a transition width $\Delta T^{90\% - 10\%} = 0.035$ K and the other, #C8SB5 (Ref. 21), has $T_c^{50\%} = 1.54$ K and $\Delta T^{90\% - 10\%} = 0.15$ K. Heterocontacts between superconducting Sr$_2$RuO$_4$ and a sharpened Pt needle as a counter-electrode were realized inside the mixing chamber of a $^3$He/$^4$He dilution refrigerator. The differential resistance $dV/dI$ vs. $V$ was recorded by a standard lock-in technique. The differential conductance, $dI/dV$, is obtained by numerical inversion of the measured $dV/dI$ data. Measurements were performed in different configurations with respect to the predominant current injection.
relative to the crystallographic axis of Sr$_2$RuO$_4$, the applied magnetic field, and the surface treatment. Here we focus on results obtained for contacts with $j$$\parallel$ab and $H$$\parallel$c within an accuracy of about 5-10$^\circ$.

**Linear field dependence.**—We focus on high transmission contacts, which exhibit a double-minimum structure in the differential resistance, i.e., a double-maximum in the differential conductance $dI/dV$ vs $V$ (see inset of Fig. 2). In this metallic regime, in contrast to the tunneling regime on any theoretical approach to the superconducting OP near the field dependent critical temperature $T_c(H)$ as a function of the reduced temperature $t(H) = 1 - T/T_c(H)$ at a given magnetic field:

$$\Delta(T) |_{t<1} = A_H t^{\nu}.$$  \(1\)

The mean-field exponent ($p$-wave approach) is $\nu = 1/2$ while in the recently introduced third-order transition picture one has $\nu = 1$. The proportionality factor $A_H$ depends on the magnetic field $H$. In order to find the resulting field dependence of the OP modulus consider the phenomenological interpolation formula

$$\frac{H_{c2}(T)}{H_{c2}(T = 0)} = 1 - \left(\frac{T}{T_c(H = 0)}\right)^{2}$$  \(2\)

determining the upper critical magnetic field, $H_{c2}$, as a function of temperature. Eq. (2) with $\mu_0H_{c2}(T = 0) = 1.5$ T and $T_c(H = 0) = 1.5$ K reproduces the experimental data satisfactorily. The inverse of Eq. (2) determines $T_c(H)$. Defining the reduced field $h(T) = 1 - \frac{H}{H_{c2}(T)}$ at a given temperature and expanding Eq. (2) for $t \ll 1$ one finds the relation

$$t(H) \frac{T_c^2(H)}{T_c^2(0)} \approx \frac{1}{2} \frac{H_{c2}(T)}{H_{c2}(0)} h(T)$$  \(3\)

between the reduced temperature and the reduced field. Consequently the reduced field dependence of the gap at a given temperature is

$$\Delta_T(H) |_{h<1} = A_H \frac{[H_{c2}(T)/2]^{\nu}}{[H_{c2}(0) - H]^{\nu}} h^{\nu}.$$  \(4\)

For $A_H = constant$ the pre-factor of the right hand side of Eq. (4) implies anomalies for low temperatures near the critical field, notably $\Delta_T \rightarrow 0$ at $H_{c2}(T) \rightarrow 0$. Since $\Delta_T(H)$ and $I_{exc}$ are closely related (Ref. 20 and below), the observed linearity in Fig. 2 requires that the divergence is compensated by the pre-factor through $A_H \sim [H_{c2}(0) - H]^{\nu}$ and hence

$$I_{exc} = constant \times \Delta^{1/\nu}.$$  \(5\)



FIG. 2: Field dependence of the normalized excess current across several point contacts in Sr$_2$RuO$_4$. The magnetic field $H$ is aligned almost parallel to the c-axis, and the current across the point contact is applied in the ab-plane. Each symbol represents one point contact on one of the two studied samples. The full line is a guide to the eye. For explanation of $I_{exc}(H = 0)$ and $H^\text{fit}(I_{exc} = 0)$ see text. The inset shows for one point contact typical $dI/dV$ curves from which the excess current was determined as a function of magnetic field.
determined from the calculated conductance for Andreev type spectra, in the framework of the p-wave analysis is also shown in Fig. 1 as $I_{\text{exc,1}}$ (diamonds). The calculations are performed for a mean free path of 15 coherence lengths ($\xi_0 = v_f/2\pi T_c$) and for a diffusely scattering surface modelled as in Ref. 19. It is clear that this model is insufficient to describe the experimental data. Nevertheless, it is interesting to note that unlike in the s-wave case in unconventional superconductors the excess current is not necessarily proportional to the OP; we find near $T_c$ a temperature variation of the excess current linear in $t$, in contrast to the $t^{1/2}$ variation of the OP. This is because impurities and disorder strongly affect the surface properties of unconventional superconductors.23

Pair-breaking by low-frequency bosonic fluctuations.—As seen above, the p-wave scenario alone does not account for the observed temperature dependence of the experimentally obtained $I_{\text{exc}}(T)$. This is true also for the overall magnitude of the bulk gap, $\Delta(0) = 1.1\text{meV} = 6 \times 1.76k_BT_c$, extracted from tunneling spectra.17 To reconcile the measured $\Delta(0)$ and $I_{\text{exc}}(T)$ with a p-wave OP we consider an additional pair-breaking channel. It was shown by Millis et. al.23 that a low-frequency bosonic mode at a characteristic frequency $\omega_p$ described by an Einstein spectrum $A_p(\omega) = \frac{\Gamma}{\pi} J_p \omega_p \delta(\omega - \omega_p)$ leads to a temperature dependent pair-breaking parameter

$$\Gamma_{\text{in}}(T) = \frac{(1 - g)}{4} J_p \omega_p \coth\left(\frac{\omega_p}{2T}\right),$$

where $g$ is the coupling-constant appearing in the gap equation. The assumptions are that $\omega_p < T_c \ll \omega_E$, $\omega_E$ being the frequency of the pairing mode, and that $A_p(\omega)$ is unaffected by the transition into the superconducting state. We performed calculations using the quasiclassical Green’s functions technique and included the pair-breaking parameter as a self-energy within a self-consistent Born approximation, i.e. $\Sigma_{\text{in}}(R, \epsilon, T) = \Gamma_{\text{in}}(T) \langle \hat{g}(p_f, R, \epsilon) \rangle_{p_f}$, where $\epsilon$ is the energy of the quasiparticles, $R$ the position with respect to the interface, and $p_f$ the Fermi momentum; the $p_f$-average is over the Fermi surface. The Green’s function $\hat{g}(p_f, R, \epsilon)$ is a functional of the self-energy $\Sigma_{\text{in}}(R, \epsilon, T)$ in the usual way. The order parameter profile $\Delta(R, T)$ near the interface was then obtained by iterating the weak-coupling gap equation and $\Sigma_{\text{in}}(R, \epsilon, T)$ until convergence.

For the excess current this model gives an excellent agreement with experimental data, as shown by $I_{\text{exc,2}}$ (circles) in Fig. 1 for $\omega_p = 0.5 T_c$ and $\frac{1}{4} J_p = 2\pi \times 0.25$. The almost linear temperature dependence over the whole temperature range is reproduced within our model, and furthermore, as shown as the dashed thick line in Fig. 1 the above introduced scaling relation between the calculated $I_{\text{exc,2}}(T)$ and the theoretically obtained order parameter $\Delta_{\text{bulk,2}}(T)$ is fulfilled to remarkable accuracy with the scaling exponent $\nu = 1/3$.

Another effect of $\Gamma_{\text{in}}(T)$ is that the enhanced scattering at higher temperatures reduces the observed $T_c$ substantially from its ideal value while the gap at $T \to 0$ is much less affected, giving $\Delta(0)/k_BT_c$-ratios much larger than the BCS-ratio 1.76. Our calculations give the correct absolute magnitude, $\Delta(0) = 5.6\Delta_{\text{BCS}}(0)$. Notably, also the functional form of $\Delta(T)/\Delta(0)$ is modified compared to the pure p-wave case (see $\Delta_{\text{bulk,2}}$ thick line in Fig. 1). The conductances calculated with the present model have the same qualitative features, both for the Andreev and the tunnel limit, as those displayed in Ref. 19, and can still account for the measured data.

Normal-state surface layer.—In order to obtain more detailed insight about the nature of the pairing state in Sr$_2$RuO$_4$ it would be instructive to quantify empirically the field dependence of $A_H$ in Eqs. 11 and 14. Unfortunately, obtaining data from the necessary temperature scans at different fields for a given point contact is difficult because of the sensitivity of the large background resistivity 19 in the $dV/dI$ data to very small changes in the configuration. A possible reason for the presence of a large background can be found in a normal-state surface layer due to surface reconstructions 23 that leads to an additive resistivity in the point contact as $\sigma_{\text{measured}} = R_N + \sigma_{\text{in-S}}^{-1}$. Here $\sigma_{\text{in-S}}$ is the conductivity of the normal-superconductor interface, $R_N$ is the normal layer resistivity, and typically $\sigma_{\text{in-S}}^{-1}/R_N \sim 10\%$. Note that the thickness of the normal-state surface layer appears to be independent of the sample quality since the observed values of $0.5 \Omega \leq R_N \leq 25 \Omega$ vary from point contact to point contact but are in the same range for both samples.26

Such a normal-state surface layer has a natural explanation in a p-wave triplet scenario because the p-wave OP is very sensitive to scattering. We assume a region near the interface in which scattering is enhanced. In Fig. 8 we show the self-consistent OP, $\Delta(R, T = 0.05T_c)$, for a mean free path of 0.3 coherence lengths in the shaded region, and of 15 coherence lengths elsewhere. The bulk OP is of the form $k_x + ik_y$, and near the surface a secondary OP component, $k_x - ik_y$, is induced. As can be seen from Fig. 8 both components are suppressed in the surface layer where scattering is enhanced, leading effectively to a normal-conducting layer near the interface.

The presence of a normal-state layer affects strongly point-contact and tunneling spectra. However, as we show in the insets of Fig. 4 the form of the spectra in the presence of a normal-state layer is in agreement with experiment (c.f. inset in Fig. 2II and Ref. 15). The excess current is reduced by such a surface layer (see lower inset in Fig. 4). Also, the tunneling spectra show a pronounced zero-energy anomaly in contrast to the clean surface. The temperature dependence of the excess current near $T_c$ is only weakly affected by a normal-conducting surface layer, leaving the results discussed above unaltered.

Conclusions.—We presented point-contact measure-
FIG. 3: Creation of a normal-conducting surface layer in a \( p \)-wave superconductor due to an increased scattering rate near the surface. For comparison, as dashed lines are shown the wave superconductor due to an increased scattering rate near the surface. Both OP components are suppressed in a layer with increased scattering, leading effectively to a normal-conducting surface layer. The calculations are for \( T = 0.05 T_c \). The insets show the corresponding point-contact spectra (bottom) and tunneling spectra (top).

ments on the unconventional superconductor \( \text{Sr}_2\text{RuO}_4 \) as a function of temperature and applied magnetic field. The excess current exhibits linear behavior both in temperature and magnetic field over a large range. Using these findings we derive a scaling relation between the excess current and the order parameter [Eq. 8].

We discuss this result within the theory of \( p \)-wave spin-triplet superconductivity. We find that the excess current in conventional superconductors is not necessarily proportional to the order parameter. In order to account for the wide range over which the linear behavior of the excess current holds experimentally we extend the pure \( p \)-wave theory to take into account scattering between quasiparticles and low-energy bosonic fluctuations, probably originating from spin fluctuations [21]. The extended theory yields a very good agreement with the measured excess current and yields a scaling exponent \( \nu = 1/3 \). Furthermore it can account for the large \( \Delta(0)/k_B T_c \)-ratios obtained from point-contact measurements [19]. Finally, we show that surface effects should be considered for a satisfactory reproduction of the point-contact spectra.

In closing, we mention that a recent Ginzburg-Landau analysis, assuming a third order phase transition induced by gapless excitations in the superconducting phase, yields the correct temperature dependence to account for the data, at least close to \( T_c \) [22, 20]. As shown in Ref. 8 the \( p \)-wave channel of superconductivity may be only marginally dominant assuming that pairing in \( \text{Sr}_2\text{RuO}_4 \) is mediated by incommensurate spin fluctuations. In this case the presence of fluctuations in the OP is not unlikely.

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