A local-optimizing surface reconstruction method based on point cloud data

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Abstract. Surface reconstruction based on discrete point cloud data is a frequently encountered problem in the field of CAD/CAGD and computer graphics. In this paper, a local-optimizing surface reconstruction method based on discrete point cloud data is proposed. Firstly, the moving least square method is used to fit the discrete point cloud data to generate a gridded point clouds. Then the global interpolation method is used to generate the initial NURBS surface based on the gridded point clouds. Additionally, the knots are inserted into the regions with excessive errors. Ultimately, a multi-objective optimization function is established to optimize the regions with excessive errors and their neighbourhoods simultaneously. Compared with the existing methods, the proposed method can improve the fitting accuracy of the reconstructed surface locally and has higher efficiency.

1. Introduction
The measurement of machining errors of complex curved surface parts is of great significance to ensure the machining accuracy[1]. In the actual measurement process, a certain number of discrete points can be obtained through reasonable measurement strategy, and then it is fitted with a continuous surface, which is used to characterize the surface of the complex curved surface parts[2]. Free-form surface reconstruction technology based on NURBS (Non-Uniform Rational B-Splines) surface has become a focus in the field of surface modeling[3].

The surface reconstruction method based on B-Spline includes the following four steps: parameterization, choice of the number of control points, placement of knots, and calculation of control points[4]. A good parameterization should reflect the position of the data point on the ideal curve or surface[5]. At present, there is no reliable and universal parameterization method before obtaining the reconstructed surface[6]. Hoschek[7] adopted the idea of parameter correction to solve this problem and achieved good results. The existing B-spline surface fitting methods can be divided into three kinds: point distance minimization (PDM), tangent distance minimization (TDM) and square distance minimization (SDM)[8]. For a long time, the PDM algorithm has been widely used[9,10]. The disadvantage of PDM algorithm is that the convergence speed is slow[11]. Wang Wenping[12] compared PDM, TDM and SDM algorithms, and pointed out that TDM algorithm converges fast but diverges easily, while SDM algorithm has high convergence speed and stability. The B-spline surface can be divided into a corresponding number of meshes according to the knot
vector, and each mesh corresponding to the same number of control points[13]. However, in the optimization process of SDM algorithm, the knot vector is relatively uniform and remains unchanged in the process of optimization, which means that the complex regions usually have excessive fitting errors. What’s more, the SDM algorithm takes a large computational overhead.

In this paper, a local-optimizing surface reconstruction method based on discrete point cloud data is proposed to address this problem. A multi-objective optimization function is established to optimize the regions with excessive errors and their neighborhoods simultaneously.

2. Surface reconstruction algorithm

2.1. Optimization problem statement
Fitting a series of discrete point cloud data can be expressed as the following optimization problem: let the point cloud data be \( \{ X_k \} \), \( k = 1,2,\cdots,N \), where \( N \) is the number of discrete points, calculate the appropriate knot vectors and optimize the control points \( P_{i,j} \) to minimize the function shown in equation (1):

\[
F = \sum_{k=1}^{N} d^2(S(u,v),X_k) + \lambda_s f_s
\]

Where \( d(S(u,v),X_k) \) represents the fitting error between the reconstructed surface and data point \( X_k \), \( f_s \) is the smoothing term, and \( \lambda_s \) is the weight of the smoothing term. We choose the integral as shown in equation (2) as the smoothing term:

\[
f_s = \int_{\Omega} (S_{uu}^2 + 2SS_{uv}^2 + S_{vv}^2) dudv
\]

Where \( S_{uu} \), \( S_{vv} \) and \( S_{uv} \) are the second derivative and mixed derivative of the surface parameters \( u \) and \( v \), respectively.

2.2. Determine the required fitting accuracy
In this paper, the required fitting accuracy is set in advance as the termination condition of the surface reconstruction algorithm. In the process of actual surface reconstruction, due to the noise of point cloud data, it will lead to unwanted variation when the reconstructed surface is too close to the discrete point cloud data. This paper considers how to determine the required fitting accuracy in the presence of random deviation of the measured point. Firstly, an ideal NURBS curve is designed, on which a group of discrete points are taken to simulate measured points by the CMM. Then a set of random deviations are added to the discrete points to simulate the measurement error. The random deviations are given by

\[
e = 0.0005 \times \text{rand}(0,1) \times a
\]

Where \( a \) is a three-dimensional unit vector with random direction. After that, different required fitting accuracy are set and the curve fitting algorithm is carried out to obtain the reconstruct curves. In order to evaluate the quality of curve reconstruction, the curvature error between the reconstructed curve and the ideal curve is calculated eventually. Figure 1 shows the maximum curvature errors between the reconstructed curve and the ideal curve with different fitting error.

As can be seen from figure 1, when the fitting error is smaller than the maximum random deviation of the data point, the maximum curvature error between the reconstructed curve and the ideal curve increases sharply, indicating that the unwanted variation of the reconstructed curve occurs due to the existence of random deviation. On the other hand, when the fitting error increases, the maximum curvature error between the reconstructed curve and the ideal curve increases slowly. In order to
ensure that the reconstructed curve do not produce unwanted variation and obtain higher fitting accuracy, the required fitting accuracy can be set to equal to the maximum random deviation.

2.3. Local-optimizing algorithm

Our method creates an initial surface by global interpolation and optimize it with the SDM method. Then the knots are inserted into the regions with excessive errors. The local-optimizing algorithm is established to further optimize the reconstructed surface.

The NURBS surface has the character of local support. Give a NURBS surface \( S(u,v) \), the patch corresponding to the parameter region \( [u_{i}, u_{i+p+1}] \times [v_{j}, v_{j+q+1}] \) will change when moving the control point \( P_{i,j} \). Assume that the error between data point \( X_k \) and the reconstructed surface exceeds the required precision, the local patch \( S(u_{k}, v_{k}) \) is called the target patch \( \Omega_1 \), on which located the foot point of \( X_k \). Then all the control points that affect the local patch \( S(u_{k}, v_{k}) \) can be calculated as \( \{P_{m,n} | m = i-p, \cdots, i; n = j-q, \cdots, j \} \). When adjusting \( \{P_{m,n} \} \), the patch corresponding to the parameter region \( [u_{i-p}, u_{i+p+1}] \times [v_{j-q}, v_{j+q+1}] \) will change, which is expressed as \( \Omega_2 \). Let \( \Omega_2 = \Omega_2 - \Omega_1 \) be the affected patch.

Our algorithm takes reducing the reconstruction error of target patch \( \Omega_1 \) as the first goal. At the same time, it introduces a constraint by optimizing the affected patch \( \Omega_2 \):

\[
\min F_1(\tilde{P}) = \sum E(X_i) + \lambda_1 f_{s1}, X_i \in \{X_{s1}\} \tag{4}
\]

\[
\min F_2(\tilde{P}) = \sum E(X_j) + \lambda_2 f_{s2}, X_j \in \{X_{s2}\} \tag{5}
\]

Where \( \{X_{s1}\} \) denotes the set of data points whose foot points locate on \( \Omega_1 \), \( \{X_{s2}\} \) denotes the set of data points whose foot points locate on \( \Omega_2 \). \( E \) denotes the squared distance function. Then the multi-objective optimization problem is obtained, and its standard form is:

\[
V = \min [F_1, F_2]^T \tag{6}
\]

The linear weighting method is used to transform it into a single objective optimization problem:
\[
\min v(F(\hat{P})) = \omega_1 \left( \sum E(X_i) + \lambda_{s1} f_{s1} \right) + \omega_2 \left( \sum E(X_j) + \lambda_{s2} f_{s2} \right), \quad X_i \in \{X_{i1}\}, \quad X_j \in \{X_{i2}\}
\] (7)

The value of weight \(\omega_F\) satisfies:

\[
\sum \omega_F = 1, \quad F = 1, 2
\]
\[
\omega_F \geq 0
\] (8)

Figure 2. Flow chart of the algorithm.

The weight \(\omega_F\) can be calculated by \(\alpha\)-method. Then equation (7) can be solved by the quasi-Newton method. Optimize all the patches with excessive errors and calculate the reconstruction error of the local-optimized surface. The local-optimizing method will be carried out again if the reconstruction error of the local-optimized surface exceed the required accuracy. The overall process of the algorithm is shown in figure 2:

3. Experiment on the accuracy of Surface fitting

In order to evaluate the effectiveness of the proposed surface reconstruction algorithm in this paper, a NURBS surface is designed as an ideal surface. As can be seen in figure 3, the ideal surface has uneven distribution of geometric features.
In order to simulate the data measured by CMM, a group of scanning points are obtained to simulate the data obtained by coordinate measuring machines and random deviations are added to it. The random deviations are given by

$$e = 0.001 \times \text{rand}(0,1) \times a$$  (9)

The required fitting accuracy is set to 1 micrometre according to the method in Section 2.2. Then the surface reconstruction algorithm shown in figure 2 is carried out to obtain the reconstructed surface. The surface obtained by interpolation is called the initial surface while the global-optimized surface and the local-optimized surface are obtained by the SDM method and the proposed method, separately. The fitting errors of them are compared to evaluate the validity of the proposed algorithm.
Figure 4 shows the overall results of surface reconstruction. As can be seen from the figure, the SDM algorithm generate surface with excessive errors. Therefore, the global-optimized surface needs to be further optimized. And the local-optimizing algorithm can generate surface which meets the required accuracy.

Table 1. Reconstruction results.

|                      | Maximum fitting error(μm) | Average fitting error(μm) | Total number of regions | Number of overproof regions |
|----------------------|----------------------------|---------------------------|-------------------------|-----------------------------|
| Initial surface      | 54.595                     | 1.563                     | 4761                    | 3213                        |
| Global-optimized     | 5.774                      | 0.304                     | 16896                   | 278                         |
| surface              |                            |                           |                         |                             |
| Local-optimized      | 0.997                      | 0.267                     | 16896                   | 0                           |
| surface              |                            |                           |                         |                             |

Table 1 shows the details of the surface reconstruction results. It is worth noting that the number of regions with excessive errors is much smaller than the total number after the global optimization, thus means that most parts of the global-optimized surface meet the required accuracy. The local-optimizing algorithm is more efficient because it only calculates a small number of regions with excess errors instead of optimizing the whole surface again.

4. Conclusion
In this paper, a local-optimizing surface reconstruction method based on discrete point cloud data is proposed. Our method creates an initial surface by global interpolation and then optimize it by adjusting the position of control points with the SDM method. Additionally, the knots are inserted into the regions with excessive errors to increase the number of control points of such area. Ultimately, a multi-objective optimization function is established to optimize the regions with excessive errors and their neighborhoods simultaneously. A group of scanning points are obtained to simulate the data obtained by coordinate measuring machines. Then the local-optimizing surface reconstruction method is carried out. The results indicate that the proposed method can obviously improve the fitting accuracy of the regions with excessive errors and it saves computation in a sense.

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