Long-distance effects in Rare and radiative K decays

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• Outline

Introduction

I - \( K \rightarrow \pi \nu \bar{\nu} \)

II - \( K_L \rightarrow \pi^0 \ell^+ \ell^- \)

III - \( K_L \rightarrow \ell^+ \ell^- \)

Conclusion
New round of experiments aiming at very rare K decays

Prime targets because of - the *cleanness of their SM predictions*,
- their *sensitivity to New Physics*.

But, *long-distance effects are nevertheless present*.

*How to deal with these effects?*

*As usual in ChPT*, by relating them to other, well measured observables.

These inputs come essentially from *radiative K decays*.

Needed to learn about the *QCD – EW interplay at low-energy*. 
A. Electroweak anatomy of rare & radiative $K$ decays

| CPV: top dominates | CPC: top & charm (+ small correction from up) |
|--------------------|---------------------------------------------|
| $u, c, t$ | $V \rightarrow Z$ |
| $V$ | $W^\pm$ |
| $\bar{s}$ | $d$ |

$$K_2 \rightarrow \pi^0 \nu \bar{\nu}$$
$$K_1 \rightarrow \ell^+ \ell^-, K_2 \rightarrow \pi^0 \ell^+ \ell^-$$

| CPV: only top & charm (Im $V_{ud} V_{us}^\dagger = 0$) |
|--------------------------|
| CPC: up dominates |
| $u, c, t$ | $V \rightarrow \gamma$ |
| $V$ | $W^\pm$ |
| $\bar{s}$ | $d$ |

$$K_1 \rightarrow \pi^0 \ell^+ \ell^-$$
$$K \rightarrow \pi \pi \gamma$$
$$K_2 \rightarrow \pi^0 \ell^+ \ell^-$$

| CPV: only top & charm (suppressed $\sim 1/m_{c,t}$) |
|--------------------------|
| CPC: up dominates |
| $u, c, t$ | $V \rightarrow \gamma \gamma$ |
| $V$ | $W^\pm$ |
| $\bar{s}$ | $d$ |

$$K_{1,2} \rightarrow \gamma \gamma, K_{1,2} \rightarrow \pi^0 \gamma \gamma$$
$$K_{1,2} \rightarrow \pi^0 \ell^+ \ell^-$$
$$K_{1,2} \rightarrow \ell^+ \ell^-$$

Mass states are combinations of CP states: $K_L \sim K_2 + \varepsilon K_1, K_S \sim K_1 + \varepsilon K_2$

→ neutral modes have two contributions: direct and ($\varepsilon$-suppressed) indirect.
A. Electroweak anatomy of rare & radiative K decays

| Decay                                      | CPV: top dominates         | CPC: top & charm (+ small correction from up) |
|--------------------------------------------|----------------------------|-----------------------------------------------|
| $K_2 \rightarrow \pi^0 \nu \bar{\nu}$     | $K_1 \rightarrow \ell^+ \ell^-$, $K_2 \rightarrow \pi^0 \ell^+ \ell^-$ |
| $K_1 \rightarrow \pi^0 \nu \bar{\nu}$     | $K_2 \rightarrow \ell^+ \ell^-$, $K_1 \rightarrow \pi^0 \ell^+ \ell^-$ |
| $K \rightarrow \pi \nu \nu$               | $K \rightarrow \pi \pi \gamma$ |
| $K_2 \rightarrow \pi^0 \ell^+ \ell^-$     | $K_2 \rightarrow \pi^0 \ell^+ \ell^-$ |

Decays of the $K^+$ proceed through both the “CPC” and “CPV” contributions. Except for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, there is always a dominant up-quark contribution.
A. Electroweak anatomy of rare & radiative K decays

| CPV: top dominates | CPC: top & charm (+ small correction from up) |
|-------------------|---------------------------------------------|
| $K_1 \rightarrow \ell^+\ell^-$, $K_2 \rightarrow \pi^0\ell^+\ell^-$ | $K_1 \rightarrow \pi^0\ell^+\ell^-$, $K_2 \rightarrow \ell^+\ell^-$ |

| CPV: only top & charm (Im $V_{ud}V_{us}^\dagger = 0$) |
|-------------------|
| $K \rightarrow \pi\pi\gamma$ |
| $K_2 \rightarrow \pi^0\ell^+\ell^-$ |

When there are direct LD contributions, they usually dominate. **New Physics** can be significant when SD is significant (exception: *asymmetries!*).
B. Probing EW structures with rare K decays

| EW Penguin | SM and/or example of SUSY diagram | Contributes to |
|------------|-----------------------------------|---------------|
| $Z$        | $u^i_L$ $Z$ $V$ $d_L$ $W^\pm$ $\bar{s}_L$ $d_L$ | $K \to \pi\nu\bar{\nu}$ $K_L \to \pi^0\ell^+\ell^-$ $K_L \to \ell^+\ell^-$ |
| $\gamma$   | $u^i_L$ $\gamma$ $V$ $d_L$ $W^\pm$ $\bar{s}_L$ $d_L$ | $K_L \to \pi^0\ell^+\ell^-$ $K \to \pi\pi\gamma$ |
| $H^0$      | $h^0, H^0, A^0$ $H^0_{\mu}$ $\tilde{u}_R$ $\tilde{u}_L$ $h_L^0\tilde{u}_R$ $d_L$ $A^U$ $d_R^i$ $\chi^\pm$ $\bar{s}_{L,R}$ $d_{R,L}$ | $K_L \to \pi^0\mu^+\mu^-$ $K_L \to \mu^+\mu^-$ (helicity-suppressed) |

New Physics to be identified by looking at patterns of deviations!
\[ K \rightarrow \pi \nu \bar{\nu} \]
### A. Where are the long-distance effects?

| CPV: | CPC: | \( K \rightarrow \pi\nu\bar{\nu} \) |
|------|------|----------------------------------|
| top dominates | top & charm (+ small correction from up) | \( K_2 \rightarrow \pi^0\nu\bar{\nu} \) \( K_1 \rightarrow \ell^+\ell^-, K_2 \rightarrow \pi^0\ell^+\ell^- \) |
| top dominates | only top & charm (\( \text{Im} V_{ud} V_{us}^\dagger = 0 \)) | \( K_1 \rightarrow \pi^0\ell^+\ell^- \) |
| top dominates | only top & charm (suppressed \( \sim 1/m_{c,t} \)) | \( K_{1,2} \rightarrow \gamma\gamma, K_{1,2} \rightarrow \pi^0\gamma\gamma \) \( K_{1,2} \rightarrow \pi^0\ell^+\ell^- \) \( K_{1,2} \rightarrow \ell^+\ell^- \) |

These modes probe exclusively the Z penguin (and W box). Dominated by short-distance physics, but…
A. Where are the long-distance effects?

1. LD effects for the top/charm “pure” SD contribution = matrix elements

\[ Q_{\text{eff}} = (\bar{s}d)_V \otimes (\bar{\nu}\nu)_{V-A} \rightarrow \langle \pi | (\bar{s}d)_V | K \rangle \]

2. The up-quark pure LD contribution (\textit{CP-conserving})
B. Matrix elements of the dimension-six operator

The “mesonic dressings” of $Q_{\text{eff}}$ is very similar to those for the Fermi operator:

The vector and scalar form-factors are needed (values at zero and slopes).

Isospin-breaking effects, $\varepsilon^{(2)} \sim m_d - m_u \sim 1\%$, must be included!

For that, two very clean ratios can be used:

$$r(q^2) = \frac{f_+^{K^+\pi^0}(q^2)f_+^{K^0\pi^0}(q^2)}{f_+^{K^+\pi^+}(q^2)f_+^{K^0\pi^+}(q^2)} = 1 + \mathcal{O}(\left(\varepsilon^{(2)}\right)^2) = 1.0000(2)$$

$$r_K = \frac{f_+^{K^+\pi^+}(0)}{f_+^{K^0\pi^+}(0)} = 1.00027(8) + \varepsilon^{(2)}0.12(7) = 1.0015(7)$$

($\text{NLO + partial NNLO}$)
For the slopes: \[
\frac{\lambda_+^{FCNC}}{\lambda_+^{CC}} = \frac{M^2(K^{*+})}{M^2(K^{*0})} = 0.990 \ (\pm 0.005)
\]

The Flavianet fit to \( K_\ell 3 \) form-factors & slopes (2008) leads to

\[
\kappa_v \sim \int d\Phi_3 \left| \langle \pi \nu \bar{\nu} | Q_{eff} | K \rangle \right|^2
\]

| \( \kappa_v^+ \) | \( \tau_+ \) | \( f(0) \) | slopes | \( r_K \) | \( r \) | Future? |
|---|---|---|---|---|---|---|
| \( \kappa_v^+ \) | 0.5168(25) | 19\% | 43\% | 21\% | 17\% | - | \( \pm 0.0023 \) |
| \( \kappa_v^0 \) | 2.190(18) | - | 77\% | 12\% | 9\% | 2\% | \( \pm 0.013 \) |

\[
\frac{\kappa_v^+}{\kappa_v^0} = 0.2359(17) \quad (\text{Future?} \pm 0.0008)
\]

Still room for improvement on the experimental side.
C. Long-distance up-quark contribution

Naïve inclusion of the $Z$ through the covariant derivative in ChPT produces

\[ K^+ \rightarrow \pi^+ \]

\[ K^+ \rightarrow K \pi \]

\[ K^+ \rightarrow \pi^+ \]

\[ K^+ \rightarrow K, \pi \]

How to \textit{disentangle the genuine up-quark contribution}?

Remove from the $Z$ coupling any $Q_{\text{eff}}$ structure.

Ask that the $Z$ coupling does not induce a local $K_L \rightarrow Z$ coupling.

Many unknown counterterms, part of them occurring in $K^+ \rightarrow \pi^+ \gamma^* \rightarrow \pi^+ \ell^+ \ell^-.$

Overall, these contributions are small, about 10% of the charm contribution.

(expected from the behavior of the $Z$ penguin $\sim m_q^2$).
$K_L \rightarrow \pi^0 \ell^+ \ell^-$
A. Where are the long-distance effects?

| CPV: top dominates | CPC: top & charm (+ small correction from up) |
|---------------------|-----------------------------------------------|
| \( Z \)             | \( W^\pm \)                                   |
| \( u, c, t \)       | \( V \)                                       |
| \( \bar{s} \)       | \( d \)                                       |

\( K \to \pi \ell \ell \)

| CPV: top dominates | CPC: up dominates |
|---------------------|-------------------|
| \( Z \)             | \( W^\pm \)       |
| \( u, c, t \)       | \( V \)           |
| \( \bar{s} \)       | \( d \)           |

| CPV: only top & charm (suppressed \( \sim 1/m_{c,t} \)) | CPC: up dominates |
|-------------------------------------------------------|-------------------|
| \( Z \)                                               | \( W^\pm \)       |
| \( u, c, t \)                                         | \( V \)           |
| \( \bar{s} \)                                         | \( d \)           |

Direct CPV (Short-distance)

Indirect CPV (Long-distance)

CPC (Long-distance)
B. Direct CPV: Matrix elements of the dimension-six operators

LD effects for the top/charm “pure” SD contribution = matrix elements

\[ Q_{\text{eff}}^V = (\bar{s}d)_V \otimes (\bar{\ell}\ell)_V, \quad Q_{\text{eff}}^A = (\bar{s}d)_V \otimes (\bar{\ell}\ell)_A \]

As for \( K \to \pi\nu\bar{\nu} \), those are extracted from \( K_{\ell3} \) decays:

| \( \kappa_{e,A}^V \) | \( \kappa_{\mu}^V \) | \( \kappa_{\mu}^A \) |
|---|---|---|
| \( \tau_+ \) | \( f(0) \) | slopes | \( r_K \) | \( r \) | Future? |
| 0.7691(64) | - | 77% | 12% | 9% | 2% | ±0.0046 |
| 0.1805(16) | - | 73% | 16% | 8% | 2% | ±0.0011 |
| 0.4132(51) | - | 54% | 38% | 6% | 2% | ±0.0031 |

\[ \kappa_{\ell}^{V,A} \sim \int d\Phi_3 \left| \langle \pi^0 \ell\ell | Q_{\text{eff}}^{V,A} | K_L \rangle \right|^2 \]

Already very precise compared the other contributions.
C. Indirect CPV: Long-distance photon penguin

Indirect CP-violation is $K_L \rightarrow \varepsilon K_1 \rightarrow \pi^0 \ell^+ \ell^-$, related to $K_S \rightarrow K_1 \rightarrow \pi^0 \ell^+ \ell^-$:

Loops are rather small, a single counterterm $a_s$ dominates.

It is fixed from $K_S \rightarrow \pi^0 \ell^+ \ell^-$ (up to its sign) measured by NA48:

$$Br(K_S \rightarrow \pi^0 e^+ e^-)_{m_{ee} > 165 \text{MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9}\left\{\begin{array}{l}
Br(K_S \rightarrow \pi^0 \mu^+ \mu^-) = (2.9^{+1.4}_{-1.2} \pm 0.2) \times 10^{-9}
\end{array}\right\} \rightarrow |a_s| = 1.2 \pm 0.2$$
C. Indirect CPV: Long-distance photon penguin

This CT is the main source of error for

\[ K \rightarrow \pi^0 \ell^+ \ell^- \]

Besides \( K_S \rightarrow \pi^0 \ell^+ \ell^- \), the paths to constrain or measure \( a_S \) are:

- The decay \( K^+ \rightarrow \pi^+ \ell^+ \ell^- \) is similar, dominated by \( a_+ \), theory can approximately relate the two \( (a_S \approx 2N_{14} + N_{15}, \ a_+ \approx N_{14} - N_{15}) \).

  \text{e.g. Buchalla,D’Ambrosio,Isidori ’03, Greynat,Friot,de Rafael ’04; see also Bruno,Prades ’03}

- \( K_L \rightarrow \pi^0 \pi^0 \ell^+ \ell^- \) depends on the same \( a_S \) and is sensitive to its sign. However, its branching is \( \leq 10^{-9} \) for \( \ell = e \) (KTeV limit: < 6.6 \times 10^{-9} ).

  \text{Funck,Kambor ’93}

- FB asymmetries for \( K_L \rightarrow \pi^0 \mu^+ \mu^- \) could fix the sign.

  \text{Mescia,Trine,C.S. ’06}
D. CPC: Long-distance double photon penguin

LO ($p^4$) is finite, produces $\ell^+ \ell^-$ in a scalar state only (helicity-suppressed),

Higher order estimated using the $K_L \rightarrow \pi^0 \gamma \gamma$ rate and spectrum:

- Production of $(\mu^+ \mu^-)_{0^{++}}$ under control within 30%.

- No signal of $(\gamma \gamma)_{2^{++}}$ implies $(e^+ e^-)_{2^{++}}$ is negligible.

($K_S \rightarrow \gamma \gamma$ is also useful to constrain the $p^6$ CT structure)
E. Indirect accesses to the photon penguin

1. Direct CP-asymmetry $A_{CP} = \frac{\Gamma(K^+ \rightarrow \pi^+ \ell^+ \ell^-) - \Gamma(K^- \rightarrow \pi^- \ell^+ \ell^-)}{\Gamma(K^+ \rightarrow \pi^+ \ell^+ \ell^-) + \Gamma(K^- \rightarrow \pi^- \ell^+ \ell^-)}$

Sensitive to the interference between the up $\gamma$ penguin and charm, top contributions. Expected to be in the $10^{-5}$ range in the SM.

\textit{e.g. D'Ambrosio et al. '98}

2. Direct CP-asymmetry $A_{CP} = \frac{\Gamma(K^+ \rightarrow \pi^+ \pi^0 \gamma) - \Gamma(K^- \rightarrow \pi^- \pi^0 \gamma)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0 \gamma) + \Gamma(K^- \rightarrow \pi^- \pi^0 \gamma)}$

Sensitive to EM operator, again expected to be small in the SM ($10^{-5}$).

\textit{e.g. D'Ambrosio,Isidori. '95}

3. Phase-space asymmetries for $K_L \rightarrow \pi^+ \pi^- \gamma^*$

Large, but dominated by indirect CPV effects ($K_L \rightarrow \varepsilon K_1 \rightarrow \pi^+ \pi^-$)

\textit{e.g. D'Ambrosio,Isidori. '95}

4. BUT: $K_L \rightarrow \pi^0 \ell^+ \ell^-$ is richer since it probes also the Higgs penguins.

\textit{Mescia,Trine,C.S. '06}
$K_L \rightarrow \ell^+ \ell^-$
A. Where are the long-distance effects?

| Diagram |
| --- |
| **CPC:** top dominates |
| $K_2 \rightarrow \pi^0 \nu \bar{\nu}$ |
| $K_1 \rightarrow \ell^+ \ell^-$, $K_2 \rightarrow \pi^0 \ell^+ \ell^-$ |
| **CPV:** only top & charm |
| (+ small correction from up) |
| $K_1 \rightarrow \pi^0 \nu \bar{\nu}$ |
| $K_2 \rightarrow \ell^+ \ell^-$, $K_1 \rightarrow \pi^0 \ell^+ \ell^-$ |

| Diagram |
| --- |
| **CPC:** top & charm |
| (Im $V_{ud} V_{us}^\dagger = 0$) |
| $K \rightarrow \pi \pi \gamma$ |
| $K_2 \rightarrow \pi^0 \ell^+ \ell^-$ |

| Diagram |
| --- |
| **CPC:** up dominates |
| $K_{1,2} \rightarrow \gamma \gamma$, $K_{1,2} \rightarrow \pi^0 \gamma \gamma$ |
| $K_{1,2} \rightarrow \pi^0 \ell^+ \ell^-$ |
| **CPV:** only top & charm |
| (suppressed $\sim 1/m_{c,t}$) |
| $K_{1,2} \rightarrow \ell^+ \ell^-$ |

- **Direct CPC** (Short-distance)
- **Indirect CPV** (Negligible)
- **CPC** (Long-distance)
B. Detailed structure of the $K_L \to \ell^+ \ell^-$ process

Matrix element from $K_{\ell 2}$:

$Br(K_L \to \mu^+ \mu^-) \approx ((-0.95 \pm ???)^2 + 6.7) \cdot 10^{-9}$

- Nearly saturated by $Abs(\gamma\gamma)$ since $B^{\exp} = 6.87(11) \cdot 10^{-9}$ (smaller exp. error ?)
- Short-distance is CPC, and interfere with the $\gamma\gamma$ contribution (sign?)
- The dispersive part $Disp(\gamma\gamma)$ diverges (how to estimate it reliably?)
C. The two-photon decay $K_L \to \gamma\gamma$

The $SU(3)$ pole amplitude vanishes:

\[
\begin{array}{c}
\text{SU(3) pole amplitude}
\end{array}
\]

The decay is driven by $Q_1^\mu = (\bar{s}d) \otimes (\bar{u}u)$, but there is no linear combinations such that $\alpha \pi^0 + \beta \eta_8 = \bar{u}u$!

Same mechanism at play in $K_L \to \pi^+\pi^-\gamma$ & $\Delta M_K$:

To consistently account for NLO corrections (unknown CTs), go first to $U(3)$.

Leading $N_c$ $SU(3)$-$O(p^6)$ CTs all collapse to a single parameter $G_8^s$.

Using the experimental value $B(K_L \to \gamma\gamma)^{exp} \Rightarrow G_8^s / G_8 \approx \pm \frac{1}{3}$.
D. The SD-LD interference sign in $K_L \rightarrow \ell^+ \ell^-$

Requires the sign of the $K_L \rightarrow \gamma\gamma$ amplitude $\Leftrightarrow$ Sign of $G_8^s$.

1- Theoretical clues:

\[
H_{\text{eff}} (\mu > 1 \text{GeV}) = z_1 Q_1^u + z_2 Q_2^u + z_6 Q_6^u + ...
\]

\[
H_{\text{eff}} (\mu_{\text{hadr.}}) = -(G_8^s + \frac{2}{3} G_{27}) \tilde{Q}_1 + (G_8^s - G_{27}) \tilde{Q}_2 - (G_8 + G_8^s - \frac{1}{3} G_{27}) \tilde{Q}_6 + ...
\]

If the non-perturbative evolution of $Q_1^u$ & $Q_2^u$ is \~smooth (no sign change):

\[
(z_1 + z_2)^2 (z_2 - z_1) = 1.0 \pm 0.3 \Rightarrow G_8^s / G_8 = -0.38(12)
\]

One can then resolve the current-current vs. penguin fraction in $K \rightarrow \pi\pi$:

\[
\tilde{Q}_{1,2} : 35\% \Leftrightarrow \tilde{Q}_6 : 65\%
\]

Penguins account for \~2/3 of the $\Delta I = \frac{1}{2}$ rule (at the hadronic scale, not at $m_c$!).
2- Experimentally, $G_8^s$ could be fixed from $K_S \rightarrow \pi^0 \gamma \gamma$:

$$B(K_S \rightarrow \pi^0 \gamma \gamma)_{z>0.2}^{\text{exp}} = (4.9 \pm 1.8) \cdot 10^{-8}$$

or from pole contributions to $K^+ \rightarrow \pi^+ \gamma \gamma$

(even more constraining at the low-energy end of the $\gamma$ spectrum)
E. The dispersive two-photon contribution to $K_L \rightarrow \ell^+ \ell^-$

The $\gamma\gamma$ loop diverges (requires *unknown CTs*) for a constant vertex:

$$f(q_1^2, q_2^2) = \sum_i \left( 1 + \alpha_i \left( \frac{q_1^2}{q_1^2 - m_i^2} + \frac{q_2^2}{q_2^2 - m_i^2} \right) + \beta_i \frac{q_1^2 q_2^2}{(q_1^2 - m_i^2)(q_2^2 - m_i^2)} \right)$$

With two resonances: $\rho +$ one around $J/\psi$.

*Low-energy contraints* from the $K_L \rightarrow \gamma e^+ e^-, \gamma \mu^+ \mu^-, e^+ e^- \mu^+ \mu^-$ linear slope.

(We would need also the quadratic slope, and other modes like $\mu^+ \mu^- \mu^+ \mu^-$ !)

*High-energy constraints* from the perturbative up & charm-quark $\gamma\gamma$ penguin.
F. $K_L \rightarrow \mu^+\mu^-$ summary --- Preliminary ---

- $G_8^s / G_8 < 0 \Rightarrow$ constructive interference between SD and LD.

- Updating the analysis, we find $\text{Disp}(\gamma\gamma) = -0 \pm 1.5$,

  Compared to $\text{Disp}(\gamma\gamma) = \pm 0.7 \pm 1.15$

Compared to Isidori & Unterdorfer ‘03

\begin{align*}
K_L & \rightarrow \pi^0 \nu \bar{\nu} : \\
\bar{\eta} & < 17 \\
K_L & \rightarrow \pi^0 e^+ e^- : \\
\bar{\eta} & < 3.3 \\
K_L & \rightarrow \pi^0 \mu^+ \mu^- : \\
\bar{\eta} & < 5.4
\end{align*}
Conclusion
