\(K_2(1770)\) and \(K_4(2500)\) dynamically generated in the \(K\)-multi-\(\rho\) interactions

C. W. Xiao\(^1,2\)

\(^1\)Institute for Advanced Simulation and Institut für Kernphysik (Theorie), Forschungszentrum Jülich, D-52425 Jülich, Germany
\(^2\)Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Instituto de Investigación de Paterna, Apartado 22085, 46071 Valencia, Spain

(Dated: February 4, 2015)

Abstract

In the present work we use three-body interaction formalism of the Faddeev equations under the fixed center approximation to investigate the many-body interaction in the \(K\)-multi-\(\rho\) systems. Based on the local hidden gauge formalism and the coupled channel approach, we reproduced dynamically the resonances \(f_2(1270)\) and \(K_1(1270)\) in the two-body interactions of \(\rho\rho\) and \(\rho K\) respectively, which are the clusters of the fixed center approximation. Then, we let a third particle, \(K\) or \(\rho\), collide with them, and thus, calculate the three-body scattering amplitudes with the fixed center approximation of the Faddeev equations. Similarly, we extrapolate the formalism to study the systems of four-body, five-body and six-body interactions, containing one \(K\) meson and multi-\(\rho\) mesons. In our research, we dynamically generate the \(K_2(1770)\) state in the three-body interaction with the mass of 1710 MeV and a width about 100 MeV, which are consistent with the experiments. The \(K_4(2500)\) state is also reproduced in our results of the five-body interaction, with a mass 2505 MeV and a width about 30 MeV or more, obtaining consistent results with the experimental findings. Furthermore, we predict some new states in the other many body interactions, \(K_3(2050)\), \(K_5(2600)\) (isospin \(I = 1/2\)), and \(K_4(2620)\) (isospin \(I = 3/2\)), with uncertainties.

PACS numbers:
I. INTRODUCTION

In nowadays particle physics, one of the topics is to understand the nature and structure of the particles found in the experiments, and to search for new hadronic states, which is the issue both for the theory and experiment. In modern high energy physics it has been accepted generally that quarks are the basic building blocks of matter. Within the Gell-Mann-Zweig quark model \[1,2\] for the normal hadron states, mesons are made of a quark-antiquark pair, \(q\bar{q}\), and baryons made of three quark components, \(qqq\). On the other hand, there are some states found in the experiments, such as the mesons, \(f_0(500)\), \(f_0(980)\), \(a_0(980)\), \(\kappa(800)\), the baryons, \(\Lambda(1405)\), \(N(1440)\), \(N(1535)\), with structure and properties difficult to explain by the normal quark model. These states might therefore be called “exotic” states (recent experimental discussions are given in Refs. [3,4]). To understand the structure and properties of these exotic states in the strong interaction, we need to exploit other theories or approaches, for example chiral perturbative theory [5–11], effective field theory [12–14], Lattice QCD [15–17], QCD sum rule [18–21], Dyson-Schwinger equations [22–24], chiral quark model [25–27], chiral unitary approach (ChUA) [28–33], and so on. But, some discovered particles, such as the \(\phi(2170)\) (also called \(X(2175), Y(2175)\)), \(Y(4260)\), \(N^*(1710)\), look like having a more complicated structure and could come from multi-body hadron interaction, which is a subject in hadron physics drawing much attention for a long time [34–38]. With this motivation, the work of Ref. [33] develops the ChUA for the three-body interaction, which combines the three-body Faddeev equations with on shell approximation of the ChUA and has reported several \(S\)-wave \(J^P = \frac{1}{2}^+\) resonances qualifying as two mesons-one baryon molecular states. In Ref. [40] this combination of Faddeev equations and chiral dynamics in the \(DKK\) system obtains consistent results with QCD sum rules. Furthermore, when in some cases there are resonances (or bound states) appearing in the two-body subsystem of the three-body interaction, Ref. [41] takes the fixed center approximation (FCA) [35,42–46] to the Faddeev equations, where several multi-\(\rho(770)\) states are dynamically produced, and the resonances \(f_2(1270)(2^{++})\), \(\rho_3(1690)(3^{--})\), \(f_4(2050)(4^{++})\), \(\rho_5(2350)(5^{--})\), and \(f_6(2510)(6^{++})\) are theoretically found as basically molecules of an increasing number of \(\rho(770)\) particles with parallel spins. Analogously, the resonances \(K_2^*(1430)\), \(K_3^*(1780)\), \(K_4^*(2045)\), \(K_6^*(2380)\) and a new \(K_6^*\) are produced in the \(K^*\)-multi-\(\rho\) systems and could be explained as molecules with the components of an increasing number of \(\rho(770)\) and one \(K^*(892)\) meson in Ref. [47]. Also in Ref. [48], charmed resonances \(D_3^+, D_1^+, D_2^+\) and \(D_6^+\) are predicted in the \(D^*\)-multi-\(\rho\) interaction. Following the same direction, in the present work we investigate the \(K\)-multi-\(\rho\) interaction.

The FCA to the Faddeev equations is a useful and effective tool in the multi-body hadronic interaction. The study of the \(KNN\) system in Refs. [49,51] has proved accuracy when dealing with bound states, and obtains consistent results with the full Faddeev equation evaluation without taking FCA [51], or a variational calculation with a nonrelativistic three-body potential model [52], which is also confirmed by the recent results with new Faddeev calculations in Refs. [53,54]. Even though there is a different results claimed in Ref. [55] on the \(KNN\) system, the work of Ref. [56] clarified the different kinematical between them using two different approaches in their investigations, the Watson approach and the truncated Fadddeev approach. A further study of the \(KNN\) system is done in the recent work of [57], which has investigated the \(\bar{K}d\) scattering length with the first-order recoil correction using the non-relativistic effective field theory approach. A narrow quasibound state of 3500 MeV in the \(DNN\) system is predicted in Ref. [58] by both FCA to Faddeev equations calculation and the variational method approach with the effective one-channel Hamiltonian. Therefore, in the present work, we use the FCA to the Faddeev equations to investigate the \(K\)-multi-\(\rho\) interaction. There are some possible \(K\) excited states with strangeness \(S = \pm 1\) and aligned with increasing spin number in the Particle Data Group (PDG) [59], such as \(K_1(1270)(1^+)\) or \(K_1(1400)(1^+)\) or \(K_1(1650)(1^+)\), \(K_2(1580)(2^-)\) or \(K_2(1770)(2^-)\) or \(K_2(1820)(2^-)\)
or $K_2(2250)(2^-)$, $K_3(2320)(3^+)$, $K_4(2500)(4^-)$. In experiment, some of these states still need more confirmation. This is the motivation of the present work, that some them would be reproduced in our theoretic model. To apply the FCA to the Faddeev equations, there should be resonances or bound states in the two-body interaction of the subsystem. For the $K$-multi-$\rho$ systems, the basic two-body subsystems are the $\rho \rho$ and $K \rho$ interactions. Starting from the local hidden gauge Lagrangians \[60–63\], the two-body $\rho \rho$ interaction is studied in Ref. \[64\] with the coupled channel approach in ChUA and found that a $\rho \rho$ quasibound state or molecule could be associated to the $f_2(1270)$ found in the PDG \[59\]. With the same on-shell Bethe-Salpeter equation of ChUA, and a chiral invariant Lagrangian, the two-body $K \rho$ interaction is studied in Ref. \[65\] and dynamically produced the $K_1(1270)$ resonance. Furthermore, the $K_1(1270)$ state is reinvestigated with detail using ChUA to analyse the experimental data in the later work of Ref. \[66\]. Thus, the resonances $f_2(1270)$ in the $\rho \rho$ interaction and $K_1(1270)$ in the $K \rho$ interaction are the needed clusters in the subsystems of the $K$-multi-$\rho$ systems.

In the present work, we will first talk about the formalism of the FCA to the Faddeev equations. Then, in the next section, the resonances $f_2(1270)$ and $K_1(1270)$ are reproduced dynamically with the on-shell Bethe-Salpeter equation of ChUA in the $\rho \rho$ interaction and the $K \rho$ interaction. Our investigation results are shown in the next-next section. Finally, we finish with some conclusions.

II. FORMALISM

For the three-body interaction as Faddeev suggested in Ref. \[35\], the scattering amplitude of $T$-matrix can be written as a sum of three partitions,

$$ T = T^1 + T^2 + T^3, $$

where partition amplitude $T^i$ ($i = 1, 2, 3$) includes all the possible interactions contributing to the three-body $T$-matrix with the particle $i$ being a spectator in the last interaction. But, if there are resonances (or bound states) as clusters appearing in the two-body subsystem interaction, for example the cluster coming from $T^3$, thus, we can assume that a cluster is formed by the two particles (named as particle 1, 2) and is not much modified by the interaction of a third particle (particle 3) with this cluster. Therefore, assuming the cluster as the fixed center of the three-body system, we can take the FCA \[35, 42–46\] to the Faddeev equations. Then, the $T^3$ partition amplitude contributes to the cluster for the FCA, and the FCA multiple scattering of the third particle with the components of the cluster is taken into account. Thus, we can write the Faddeev equations of Eq. \[3\] easily (which is also developed by the ChUA),

$$ T_1 = t_1 + t_1 G_0 T_2, \quad T_2 = t_2 + t_2 G_0 T_1, \quad T = T_1 + T_2, $$

where $T$ is the total three-body scattering amplitude. The amplitudes $t_1$ and $t_2$ represent the unitary scattering amplitudes with coupled channels for the interactions of particle 3 with particle 1 and 2, respectively. And $G_0$ is the propagator of particle 3 between the components of the two-body subsystem, as depicted in Fig. 4. From this figure, we can see that the Faddeev equations under the FCA are first a pair of particles (1 and 2) forming a cluster, and then particle 3 interacts with the components of the cluster, undergoing all possible multiple scattering with those components. Thus, the two partition amplitudes $T_1$ and $T_2$ sum all diagrams of the series of Fig. 4 which begin with the interaction of particle 3 with particle 1 of the cluster ($T_1$), or with the particle 2 ($T_2$). Finally, the scattering amplitude $T$ is the total three-body interaction amplitude that we look for.

3
The propagator $G_0$ in Eq. (4) is given by,

$$G_0(s) = \frac{1}{2M_R} \int \frac{d^3q}{(2\pi)^3} F_R(q) \frac{1}{q^{0^2} - \vec{q}^2 - m_3^2 + i\epsilon},$$  \hspace{1cm} (5)$$

where $F_R(q)$ is the form factor of the cluster of particles 1 and 2. More detail about the definition of $G_0$ function will be discussed later. We must use the form factor of the cluster consistently with the theory used to generate the cluster as a dynamically generated resonance. This requires to extend to the wave functions the formalism of the chiral unitary approach developed for scattering amplitudes, which has been done in Refs. [67–69] for $S$-wave bound states, $S$-wave resonant states and states with arbitrary angular momentum, respectively. Here we only need the expressions for $S$-wave bound states, and then the expression for the form factors is given by [68],

$$F_R(q) = \frac{1}{N} \int_{|\vec{p}|<\Lambda'} |\vec{p}| < \Lambda' \frac{d^3\vec{p}}{1} \frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})}  \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})},$$  \hspace{1cm} (6)$$

$$N = \int_{|\vec{p}|<\Lambda'} d^3\vec{p} \left( \frac{1}{2E_1(\vec{p})} \frac{1}{2E_2(\vec{p})} \frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \right)^2,$$  \hspace{1cm} (7)$$

where $E_1$ and $E_2$ are the energies of the particles 1, 2 and $M_R$ the mass of the cluster. The parameter $\Lambda'$ is a cut off that regularizes the integrals of Eqs. (6) and (7). This cut off is the same that one needs in the regularization of the loop function of the two particle propagators in the study of the interaction of the two particles of the cluster [68]. As done in Refs. [47, 70], we take the value of $\Lambda'$ the same as the cutoff $q_{\text{max}}$ used to generate the resonance in the two-body interaction. Thus we do not introduce any free parameters in the present procedure.

Now we continue to discuss how to define the $G_0$ function. We can start from the normalization of the field theory. Taking the normalization of Mandl and Shaw [71] which has different weight factors for the particle fields, we must take into account how these factors appear in the single scattering and double scattering and in the total amplitude [47, 70]. We show below the details for the case of a meson cluster (also particles 1 and 2) and a meson as scattering particle (the third particle). In this case, following the field normalization of Ref. [71] we find for the $S$ matrix of
single scattering, Fig. 1 (a),

\[ S^{(1)}_1 = -i t_1 (2\pi)^4 \delta(k + k_R - k' - k'_R) \times \frac{1}{V^2} \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3'}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega_1'}} \]

\[ S^{(1)}_2 = -i t_2 (2\pi)^4 \delta(k + k_R - k' - k'_R) \times \frac{1}{V^2} \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3'}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega_2'}} \]

(8)

where, \( k, k' \) \((k_R, k'_R)\) refer to the momentum of initial, final scattering particle \((R \text{ for the cluster})\), \( \omega_i, \omega'_i \) are the energies of the initial, final particles, \( V \) is the volume of the box where the states are normalized to unity and the subscripts 1, 2 refer to scattering with particle 1 or 2 of the cluster.

Next, the double scattering diagram, Fig. 1 (b), is given by,

\[ S^{(2)} = -i (2\pi)^4 \delta(k + k_R - k' - k'_R) \times \frac{1}{V^2} \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3'}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega_1'}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega_2'}} \]

\[ \times \int \frac{d^3 q}{(2\pi)^3} F_R(q) q^{02} - \vec{q}^2 - m_3^2 + i \epsilon \]

\[ t_1 t_2, \]

(10)

where \( F_R(q) \) is the cluster form factor that we have discussed above, seen in Eq. (6), and then we can define the propagator of the third particle, \( G_0 \), seen in Eq. (5). Besides, \( q^0 \), the energy carried by particle 3 in the rest frame of the three particle system, is given by,

\[ q^0(s) = \frac{s + m_3^2 - M_R^2}{2\sqrt{s}}. \]

(11)

Similarly, the full \( S \) matrix for scattering of particle 3 with the cluster will be given by,

\[ S = -i T (2\pi)^4 \delta(k + k_R - k' - k'_R) \times \frac{1}{V^2} \frac{1}{\sqrt{2\omega_3}} \frac{1}{\sqrt{2\omega_3'}} \frac{1}{\sqrt{2\omega_1}} \frac{1}{\sqrt{2\omega_1'}} \frac{1}{\sqrt{2\omega_2}} \frac{1}{\sqrt{2\omega_2'}} \]

(12)

Now, we can see that for the unitary amplitudes corresponding to single-scattering contribution, one must take into account the isospin structure of the cluster and write the \( t_1 \) and \( t_2 \) amplitudes in terms of the isospin amplitudes of the \((3,1)\) and \((3,2)\) systems. In view of the different normalization of these terms by comparing Eqs. (8), (9), (10) and (12), we can introduce suitable factors in the elementary amplitudes,

\[ \tilde{t}_1 = \frac{2 M_R}{2m_1} t_1, \quad \tilde{t}_2 = \frac{2 M_R}{2m_2} t_2, \]

(13)

with \( m_1, m_2, M_R \) the masses of the particles 1,2 and the cluster, respectively, where we have taken the approximations, suitable for bound states, \( \frac{1}{\sqrt{2\omega_i}} = \frac{1}{\sqrt{2m_i}} \), and sum all the diagrams by means of,

\[ T = T_1 + T_2 = \frac{\tilde{t}_1 + \tilde{t}_2 + 2 \tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}. \]

(14)

When \( \tilde{t}_1 = \tilde{t}_2 \) in some cases, it can be simplified as,

\[ T = \frac{2 \tilde{t}_1}{1 - t_1 G_0}. \]

(15)
Note that, when the cluster, the particles 1 and 2, one of them is a baryon, the factors in Eqs. (5) and (13), $2M_R$ and $2m_i$ ($i = 1, 2$) should be replaced by 1 for taking the baryonic field factor approximation $\sqrt{\frac{2M_R}{2E_B}} \approx 1$.

From the $S$ matrix of single scattering, Eqs. (8), (9), and the full $S$ matrix, Eq. (12), we should note that the arguments of the amplitudes $T_i(s)$ and $t_i(s_i)$ are different, where $s$ is the total invariant mass of the three-body system, and $s_i$ are the invariant masses in the two-body subsystems. The value of $s_i$ is given by (16),

$$s_i = m_3^2 + m_i^2 + \frac{(M_R^2 + m_i^2 - m_j^2)(s - m_3^2 - M_R^2)}{2M_R^2}, (i, j = 1, 2, i \neq j)$$

where $m_l$ ($l = 1, 2, 3$) are the masses of the corresponding particles in the three-body system and $M_R$ the mass of two body resonance or bound state (cluster).

The FCA to Faddeev equations are useful and effective for the study of the three-body interaction, which are particularly suited to study system with the subsystem bound or even loose bound, as discussion in Refs. [49, 50]. But, there are some limitation on the case when the cluster of the two particles in the subsystem is excited in the intermediate states (more discussions seen in Ref. [72] for the study of $\phi K\bar{K}$ system).

III. TWO-BODY INTERACTION

Using the Faddeev equations under the FCA, we first need to look for the bound states in the two-body interaction as the cluster in the fixed center, and then let the third particle collide with the cluster and interact with the components of the forming cluster. Thus, we need to start investigating the subsystems of the $K$-multi-$\rho$ system. We know that the two-body $\rho\rho$ and $\rho K$ interactions in the subsystems are studied in Refs. [64] and [66]. We briefly summarize the works of Refs. [64, 66] here to reproduce the resonances $f_2(1270)$ and $K_1(1270)$, the clusters in the present work, and also obtain the scattering amplitudes for the subsystem of two-body interaction.

A. $\rho\rho$ interaction

In Ref. [64], the $\rho\rho$ interaction is studied with the local hidden gauge formalism [60, 63] and the unitary coupled channels method of ChUA, and dynamically produced the $f_2(1270)$ ($I(J^{PC}) = 0(2^{++})$) state. Starting from the local hidden gauge Lagrangians, the potential of spin $S = 2$ for $\rho\rho$ interaction is obtained, which is also needed in the present work,

$$V_{\rho\rho}^{(l=0,S=2)}(s_i) = -4g^2 - 8g^2 \left(\frac{3s_i}{4m_\rho^2} - 1\right),$$

$$V_{\rho\rho}^{(l=2,S=2)}(s_i) = 2g^2 + 4g^2 \left(\frac{3s_i}{4m_\rho^2} - 1\right),$$

where $g = M_V/2f_\pi$, with $M_V$ the vector meson mass and $f_\pi$ the pion decay constant. Then, the scattering amplitude of $\rho\rho$ interaction is calculated by the coupled channels unitary approach, the on-shell Bethe-Salpeter equation,

$$t = [1 - VG]^{-1}V,$$
where the kernel $V$ is a matrix of the interaction potentials in each channel, which is calculated from the hidden gauge Lagrangian. $G$ is a diagonal matrix of the loop function of every channel.

Following the work of Ref. [64], we also should take into account the contribution of the box diagram with two pseudoscalar mesons in the intermediate state. We only consider the imaginary part of the box diagram contribution as the correction of the potential $V$, and neglect its real part which is very small. Note that we do not take these intermediate channels in the box diagram (more detail seen in Ref. [64]) as accounting for the coupled channels. On the other hand, as done in Ref. [64], we also consider the $\rho$ mass distribution by replacing the $G$ function in the corresponding channel by its convoluted form. Then, we can obtain consistent results with Ref. [64], as shown in Fig. 2 where the structure of the resonance $f_2(1270)$ is found in the peak of the modulus squared of the amplitude. We have successfully reproduced the $f_2(1270)$ state as the cluster for our procedure. Besides, the nonresonant amplitude $t^{(I=0,S=2)}_{\rho\rho}$ is not shown here, which is needed when we evaluate the total three-body amplitude for considering the isospin structure of the two-body interaction in the subsystem as discussed before.

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Modulus squared of the scattering amplitudes: $|t^{I=0}_{\rho\rho}|^2$, $f_2(1270)$.}
\end{figure}

B. $\rho K$ interaction

The $\rho K$ interaction is investigated by the ChUA in Refs. [65, 66]. Following the work of Ref. [66], we can dynamically generate the $K_1(1270)$ resonance in the interaction of $\rho K$ and its couple channels, $\phi K$, $\omega K$, $K^*\eta$ and $K^*\pi$. Also starting from the local hidden gauge Lagrangians, the parts of Lagrangian for the vector-pseudoscalar interaction can be derived, then, the vector-pseudoscalar potential projected over $s$-wave can be obtained as

\[
V_{ij}(s_i) = -\frac{\tilde{e} \cdot \tilde{e}'}{8f_{\pi}^2} C_{ij} \left[ 3s_i - (M_i^2 + m_i^2 + M_j^2 + m_j^2) \right.
- \frac{1}{s_i} (M_i^2 - m_i^2)(M_j^2 - m_j^2) \left. \right],
\]

where $M_{i(j)}$ and $m_{i(j)}$ represent the masses of $i (j)$ channel of the incoming (outgoing) particles, and the coefficients of $C_{ij}$ can be found in Ref. [66]. Having the interaction potential, we can input them into the kernel $V$ of the on-shell Bethe-Salpeter equation to evaluate the interaction.
amplitude,

\[ t = [1 + V \hat{G}]^{-1}(-V)\vec{e} \cdot \vec{e}', \]  

(21)

where \( \hat{G} \) is \( (1 + \frac{q^2}{4M^2})G \) being a diagonal matrix \( (G \) as the normal loop function in Eq. 19) and \( \vec{e} (\vec{e}') \) represents a polarization vector of the incoming (outgoing) vector-meson. As done in Ref. 66, we also take into account the large width of the vector mesons, and consider the convolution of the vector mesons as intermediate state in the loop function \( G \). In Fig. 3 we show our calculation results for the modulus squared of \( t_{\rho K}^{I=1/2} \), which are consistent with Ref. 66. From the peak position, we can see that the \( K_1(1270) \) is successfully reproduced in our work as the cluster in the FCA. For the consideration of isospin structure of the subsystem, we also need the amplitude \( t_{\rho K}^{I=3/2} \). In \( I = 3/2 \) sector, there only two coupled channels, \( \rho K \) and \( K^*\pi \), which is no resonance appeared and not showed in the figure.

IV. RESULTS

In the present work, we study the \( K \)-multi-\( \rho \) interactions. In the former section, we have reproduced the resonances \( f_2(1270) \) and \( K_1(1270) \) in the \( \rho \rho \) and \( \rho K \) two-body interactions, which are the clusters of the FCA to Faddeev equations for the three-body interaction. Therefore, based on the possible clusters in the two-body interaction as above, the possible cases for the \( K \)-multi-\( \rho \) interactions are listed in Table 1 and explained as follows. Thus, for the three-body interaction, we have two options: (i) particle 3 = \( K \), cluster or resonance \( R = f_2 \) (particle 1 = \( \rho \), 2 = \( \rho \)) and (ii) 3 = \( \rho \), \( R = K_1 \) (1 = \( \rho \), 2 = \( K \)). For four-body interaction, we can extrapolate the FCA ideas and also have two cases: (i) 3 = \( f_2 \), \( R = K_1 \) (1 = \( \rho \), 2 = \( K \)) and (ii) 3 = \( K_1 \), \( R = f_2 \) (1 = \( \rho \), 2 = \( \rho \)). For five-body interaction, (i) 3 = \( K \), \( R = f_4 \) (1 = \( f_2 \), 2 = \( f_2 \)) and (ii) 3 = \( \rho \), \( R = K_3 \) (1 = \( f_2 \), 2 = \( K_1 \)). For six-body interaction, (i) 3 = \( K_1 \), \( R = f_4 \) (1 = \( f_2 \), 2 = \( f_2 \)) and (ii) 3 = \( f_2 \), \( R = K_3 \) (1 = \( f_2 \), 2 = \( K_1 \)). We show our investigation results for all these cases as below.
TABLE I: The cases considered in the $K$-multi-$\rho$ interactions.

| particles: | 3 | R (1,2) | amplitudes |
|------------|---|---------|------------|
| Two-body   | $\rho$ | $K$       | $t_{\rho K}$ |
|            | $\rho$ | $\rho$    | $t_{\rho \rho}$ |
| Three-body | $K$    | $f_2(\rho \rho)$ | $T_{K-f_2}$ |
|            | $\rho$ | $K_1(\rho K)$ | $T_{\rho-K_1}$ |
| Four-body  | $K_1$ | $f_2(\rho \rho)$ | $T_{K_1-f_2}$ |
|            | $f_2$ | $K_1(\rho K)$ | $T_{f_2-K_1}$ |
| Five-body  | $K$    | $f_4(f_2f_2)$ | $T_{K-f_4}$ |
|            | $\rho$ | $K_3(f_2K_1)$ | $T_{\rho-K_3}$ |
| Six-body   | $K_1$ | $f_4(f_2f_2)$ | $T_{K_1-f_4}$ |
|            | $f_2$ | $K_3(f_2K_1)$ | $T_{f_2-K_3}$ |

A. Three-body interaction

First, we start from the three-body interaction. There are two options of structure: $K - f_2(\rho \rho)$ and $\rho - K_1(\rho K)$, which means (i) $3 = K$, $R = f_2$ ($1 = \rho$, $2 = \rho$) and (ii) $3 = \rho$, $R = K_1$ ($1 = \rho$, $2 = K$). Thus, to evaluate these scattering amplitudes, we need as input the $t_1$ and $t_2$ amplitudes of the (3,1) and (3,2) subsystems, $t_1 = t_2 = t_{\rho K}$ for $K - f_2(\rho \rho)$ and $t_1 = t_{\rho \rho}$, $t_2 = t_{\rho K}$ for $\rho - K_1(\rho K)$. Thus, we should calculate the two-body $\rho \rho$ and $\rho K$ amplitudes, which are discussed in the former section, following the work of Refs. [64, 66]. But, note that, in their work, the dimensional regularization scheme is used for the loop functions. To evaluate the form factor of the cluster, we need a cutoff $\Lambda'$, which is the same as the one $q_{\text{max}}$ used in the loop function for the two-body interaction. As discussed in Ref. [75], we can compare the value of the $G$ function at threshold using the dimensional regularization formula [76] with the one of the cutoff which can be taken from Ref. [28] or the analytic expression in Ref. [77]. Then, equivalently to the parameters in the dimensional expression, we obtain $q_{\text{max}} = 875$ MeV for the $f_2(1270)$ cluster and $q_{\text{max}} = 1035$ MeV for the $K_1(1270)$ cluster. In fact, we do not introduce any free parameter.

We have mentioned in the formalism that we should take into account the isospin structure of the two-body amplitudes $t_1$ and $t_2$ for the third particle interacting with the cluster. For the first case of $K - f_2(\rho \rho)$, the cluster of $f_2$ resonance has isospin $I = 0$. Therefore the two $\rho$ mesons are in an $I = 0$ state, and we have the isospin components

$$|\rho \rho >^{(0,0)} = \frac{1}{\sqrt{3}} \left(|(1,-1) > + |(-1,1) > - |(0,0) > \right),$$

where $|(1,-1) >$ denote $|(I_1, I_2) >$ which shows the $I_z$ components of particles 1 and 2, and
\(|\rho \rho >^{(0,0)}\) means \(|\rho \rho >^{(I_f L_f)}\). Then, the third particle is a \(K\) meson taken \(|I_f^2 > = |1/2 >\), obtained

\[
T_{K-f_2}^{(1/2,1)} = < K \rho \rho | \hat{t} | K \rho \rho >^{(1/2,1)}
\]
\[
= < K |(1/2,1) \otimes < \rho \rho |^{(0,0)} (\hat{t}_{31} + \hat{t}_{32}) (|K >^{(1/2,1)} \otimes |\rho \rho >^{(0,0)})
\]
\[
= \left[ < 1/2 | \otimes \frac{1}{\sqrt{3}} \left( (1/2,1) + < (1,-1)| + < (-1,1) | - < (0,0) > \right) \right] (\hat{t}_{31} + \hat{t}_{32}) \left[ 1/2 > \right.
\]
\[
\left. \otimes \frac{1}{\sqrt{3}} \left( |(1,-1) > + |(-1,1) > - |(0,0) > \right) \right]
\]
\[
= \left[ \left( < (3/2,3/2), -1 | + \sqrt{7}/3 < (3/2,1/2), 1 | + \sqrt{2}/3 < (1/2,1/2), 1 | - \sqrt{2}/3 < (3/2,1/2), 0 | \right.ight.
\]
\[
- \left. \sqrt{7}/3 < (3/2,1/2), 0 | \right) \right] \hat{t}_{31} \left[ \left( < (3/2,3/2), -1 | + \sqrt{7}/3 < (3/2,1/2), 1 | + \sqrt{2}/3 < (1/2,1/2), 1 | + \sqrt{2}/3 < (3/2,1/2), 0 | \right.ight.
\]
\[
- \left. \sqrt{7}/3 < (3/2,1/2), 0 | \right) \right] \hat{t}_{32} \left[ \left( < (3/2,3/2), -1 | + \sqrt{7}/3 < (3/2,1/2), 1 | + \sqrt{2}/3 < (1/2,1/2), 1 | + \sqrt{2}/3 < (3/2,1/2), 0 | \right.ight.
\]
\[
+ \sqrt{7}/3 < (3/2,1/2), 0 | \right) \right] \hat{t}_{32} \left[ \left( < (3/2,3/2), -1 | + \sqrt{7}/3 < (3/2,1/2), 1 | + \sqrt{2}/3 < (1/2,1/2), 1 | + \sqrt{2}/3 < (3/2,1/2), 0 | \right.ight.
\]
\[
- \left. \sqrt{7}/3 < (3/2,1/2), 0 | \right) \right], \tag{23}
\]
where the notation of the states followed in the terms is \(|\frac{3}{2}, \frac{3}{2} >, -1 > = |I_{31}, I_{31}^1 >, I_{32}^1 > > t_{31}, \) and \(|I_{32}^1, I_{32}^2, I_{32} > > t_{32}. \) Finally, we obtain the isospin structure relationship

\[
t_1 = t_{\rho K} = \frac{1}{3} (2 t_{I^3=3/2}^f t_{I^3=1/2}^f), \quad t_2 = t_1. \tag{24}
\]

But for the second case of \(\rho - K_1 (\rho K)\), the isospin structure relationship is different. Now, the isospins of \(\rho\) and \(K_1\) are \(I_\rho = 1\) and \(I_{K_1} = \frac{1}{2}\), thus, the total isospin of the three-body system are two cases \(I_{total} \equiv I_{\rho K} = \frac{1}{2}\) or \(I_{total} \equiv I_{\rho K} = \frac{3}{2}\). Therefore we have

\[
|\rho K >^{(1/2,1)} = |\rho \rho K >^{(1/2,1)} = \sqrt{2}/3 |(1, -1/2) > - \sqrt{1}/3 |(0, 1/2) >, \tag{25}
\]
\[
|\rho K >^{(3/2,1)} = |\rho \rho K >^{(3/2,1)} = \sqrt{1}/3 |(1, -1/2) > + \sqrt{2}/3 |(0, 1/2) >,
\]
where we have taken the third isospin component \(I_z = \frac{1}{2}\) for convenience. Therefore the \(|\rho K >\) states inside the \(K_1\) for the \(I_z = -\frac{1}{2}\) and \(I_z = +\frac{1}{2}\) are given by

\[
|\rho K >^{(1/2,-1/2)} = \sqrt{1}/3 |(0, -1/2) > - \sqrt{2}/3 |(-1, 1/2) >, \tag{26}
\]
\[
|\rho K >^{(3/2,-1/2)} = \sqrt{2}/3 |(1, -1/2) > - \sqrt{1}/3 |(0, 1/2) >.
\]

For the two possibilities, using Eqs. (25) and (26) and performing a similar derivation of Eq. (23), we obtain

\[
T_{\rho - K_1}^{(I=1/2)} : \quad t_1 = t_{\rho \rho} = \frac{2}{3} t_{31}^{(I=0)}; \quad t_2 = t_{\rho K} = \frac{1}{9} (8 t_{32}^{I=3/2} + t_{32}^{I=1/2});
\]
\[
T_{\rho - K_1}^{(I=3/2)} : \quad t_1 = t_{\rho \rho} = \frac{5}{6} t_{31}^{(I=2)}; \quad t_2 = t_{\rho K} = \frac{1}{9} (5 t_{32}^{I=3/2} + 4 t_{32}^{I=1/2}). \tag{27}
\]
In Fig. 4 we show our results of the modulus squared of the amplitude for $|T_{K-f_2}^{I=1/2}|^2$. There is a clear and sharp peak around 1770 MeV, which is close to the threshold of $K - f_2$ and similar to a cusp in the case of the $\eta'KK$ system [28]. Therefore, this peak in $|T_{K-f_2}^{I=1/2}|^2$ would be affected by the threshold effect, and the width of this resonance structure has large uncertainties. There would be the cusp corresponded to a real resonance in some cases, like $a_0(980)$ [28] (more discussions about the states appearing near the threshold can be found in the recent works of [79, 80]). In Fig. 5 we show the results of $|T_{p-K_1}^{I=1/2}|^2$ (left) and $|T_{p-K_1}^{I=3/2}|^2$ (right). From the $|T_{p-K_1}^{I=1/2}|^2$ results, we find that there is a clear peak around the energy 1710 MeV with a width about 100 MeV, which is about 340 MeV below the $\rho - K_1$ threshold and the $\rho\rho K$ threshold. Because of the large width of the $\rho$ meson, for a system with two $\rho$ mesons, the large bindings in the present case will be acceptable. In PDG [54], the $K_2(1770)$ of $J^P = 2^-$ strangeness state is the mass of $1773 \pm 8$ MeV and the width $186 \pm 14$ MeV. But, from the analysis of the $K\omega$ spectrum in the reaction $K^-p \rightarrow K^-\omega p$, the work of Ref. [51] obtains its mass as $1710 \pm 15$ MeV and width $110 \pm 50$ MeV, which is consistent with our results. Our results are also consistent with the other experimental results [82–84]. Thus, considering the uncertainties in our study (which will be discussed later), the peak appearing in the $|T_{p-K_1}^{I=1/2}|^2$ corresponds to the $K_2(1770)$, which would be the $\rho - K_1$ molecular state in our model. The strength of the peak of $|T_{K-f_2}^{I=1/2}|^2$ is about 25 times smaller than for $|T_{p-K_1}^{I=1/2}|^2$, thus, we could not expect a state structure in Fig. 4 even though the two-body interaction of $pK$ is not strong as $\rho\rho$ by comparing the results of Fig. 2 and Fig. 3. From the results of $|T_{p-K_1}^{I=3/2}|^2$ in Fig. 5 (right), we can see that there is a clear resonant structure about 2075 MeV with the strength 20 times smaller than that of $|T_{p-K_1}^{I=1/2}|^2$ in the left figure, which is a little above the $\rho - K(1270)$ threshold. We are looking for the lowest lying states bound in the $K$-multi-$\rho$ system, and hence, we could not expect a new state in the $|T_{p-K_1}^{I=3/2}|^2$ results.

B. Four-body interaction

For the four-body interaction as shown in Table II we also have two possibilities: (i) particle 3 = $f_2$, cluster $R = K_1$ (1 = $\rho$, 2 = $K$), or (ii) particle 3 = $K_1$, resonance $R = f_2$ (1 = $\rho$, 2 = $\rho$). Because the isospins of the two cluster are $I_{f_2} = 0$ and $I_{K_1} = \frac{1}{2}$, the total isospin of the four-body system is only $I_{total} = \frac{1}{2}$. Using the FCA formalism as discussed in section III for the four-body interaction, for the first option, $f_2$ interacting with the $K_1$, we need to evaluate the amplitudes $t_1 = t_{f_2\rho} = T_{\rho-f_2}$, which has been done in Ref. [41], and $t_2 = t_{f_2K} = T_{K-f_2}$, which has been
calculated in the former subsection [IVA]. Similarly, for the second case, \( K_1 \) collides with the \( f_2 \), and the amplitudes \( t_1 = t_2 = t_{K_1 K_2} = T_{p-K_1} \) have been evaluated in the former subsection [IVA]. Note that the three-body amplitude \( T_{p-K_1} \) should be also written in terms of the isospin structure as discussed in the section [II] and its amplitudes of isospin components evaluated in the last subsection [IVA]. Since the isospins of both the \( K_1 \) and \( K \) are \( I = \frac{1}{2} \), the isospin structure is similar to the case when the \( K \) collides with the \( f_2 \). Thus, analogously to Eq. (24) we have

\[
T_{pK_1} = \frac{1}{3}(2T_{I=3/2}^{I=3/2} + T_{I=1/2}^{I=1/2}) , \quad t_2 = t_1.
\]  

(28)

We show our results in Fig. 6, of which the left is \( |T_{I=1/2}^{f_2-K_1}|^2 \) and the right \( |T_{I=1/2}^{I=1/2}|^2 \), where the clusters are resonances \( K_1 \) and \( f_2 \) respectively. In the left of Fig. 6, there is a clear peak at an energy of about 2075 MeV, the width of which is about 200 MeV. We also find another sharp peak at the position about 2550 MeV, which comes from the contribution of the threshold effect in the \( T_{I=1/2}^{f_2-K_1} \), seen in Fig. 4. In the right of Fig. 6, we also find that there is a resonant peak around the energy 2025 MeV with a width of 200 MeV in the \( |T_{I=1/2}^{f_2-K_1}|^2 \). The strength of the peak of \( |T_{I=1/2}^{f_2-K_1}|^2 \) is at the same magnitude as the one of \( |T_{I=1/2}^{I=1/2}|^2 \) and just about 25% bigger, and the energy of the peak is a little more bound too. In PDG, there is only one \( J^P = 3^+ \ K_3 \) state found in the experiments, \( K_3(2320) \), with a mass 2324 ± 24 MeV and width 150 ± 30 MeV. This \( K_3(2320) \) state was just found in the reactions \( K^+p \to (\Lambda p)p \) and \( K^-p \to (\Lambda\bar{p})p \) [35, 80]. But, its mass is too far away from our results. Therefore, we find a new \( K_3 \) state, with a mass about 2025–2075 MeV and a width about 200 MeV.

C. Five-body interaction

As we expected before, we also find one \( K_3 \) state in the four-body interaction in the former subsection [IVB]. Thus, for the five-body interaction, there also are two options for the cluster, one of which is the \( f_4 \) state found in the PDG and studied in Ref. [41], and the other one the resonance \( K_3 \) obtained in the four-body interaction above. Then following the idea of FCA and letting the third particle (\( K \) or \( \rho \)) collide with them, the two possibilities are (i) particle 3 = \( K \), cluster \( R = f_4 \) \( (1 = f_2, 2 = f_2) \), or (ii) 3 = \( \rho \), \( R = K_3 \) \( (1 = f_2, 2 = K_1) \). Since the isospin \( I_{f_4} = 0 \) and \( I_{K_3} = \frac{1}{2} \), for the first case, the total isospin of the \( K - f_4 \) system is only \( I_{total} = \frac{1}{2} \), but for the second option \( \rho - K_3 \), the total isospin of this structure is \( I_{total} = \frac{1}{2} \) or \( I_{total} = \frac{3}{2} \). Therefore, the situation of \( K \) interacting with \( f_4 \) is similar to the three-body interaction discussed before, \( K \)
colliding with $f_2$, and $\rho - K_3$ analogous to the one of $\rho - K_1$. Thus, in the first case, the $K$ collides with the $f_4$, and the amplitudes $t_1 = t_2 = t_{Kf_2} = T^{I=1/2}_{K-f_2}$ have been evaluated in subsection IV A for the three-body interaction. For the second case, the $\rho$ interacts with the $K_3$, which is similar to $\rho - K_1$ in subsection IV A thus, doing a similar derivation as in Eq. (23), we obtain

$$
\begin{align*}
T^{I=1/2}_{\rho-K_3} & : & t_1 = t_{\rho f_2} = T^{I=1}_{31}, & t_2 = t_{\rho K_1} = T^{I=1/2}_{32}; \\
T^{I=3/2}_{\rho-K_3} & : & t_1 = t_{\rho f_2} = T^{I=1}_{31}, & t_2 = t_{\rho K_1} = T^{I=3/2}_{32},
\end{align*}
$$

(29)

where the $T^{I=1}_{31}$ is the amplitude of $T^{I=1}_{\rho-f_2}$, which is the same as calculated in the subsection IV B reproducing the results of Ref. [41], and the amplitudes $T^{I=1/2}_{\rho-K_1}$ and $T^{I=3/2}_{\rho-K_1}$ have also been evaluated in subsection IV A.

We show our results for the two cases of the five-body interaction in Fig. 7. The results of $|T^{I=1/2}_{K-f_4}|^2$ is shown on the left of Fig. 7 where we can see a resonant peak around the energy 2505 MeV with a width of about 30 MeV. The right of Fig. 7 is the results of $|T^{I=1/2}_{\rho-K_3}|^2$ and $|T^{I=3/2}_{\rho-K_3}|^2$. We observe that there are clear peaks for both of them. A resonant structure in $|T^{I=1/2}_{\rho-K_3}|^2$ is found at the energy 2375 MeV with the width about 400 MeV, which is about 100 MeV more bound than the one of $|T^{I=1/2}_{K-f_4}|^2$ and much larger width too. But the strength of $|T^{I=1/2}_{\rho-K_3}|^2$ is just one half smaller than the one $|T^{I=1/2}_{K-f_4}|^2$. Thus, the more bound energy and larger width in $|T^{I=1/2}_{\rho-K_3}|^2$ come from the stronger $\rho$ interaction in the components of $K_3$ state, where we can see from the Fig. 2 and Fig. 4. In experiments, one $J^P = 4^-$ state was also found in Ref. [85], $K_4(2500)$, with the mass $2490 \pm 20$ MeV and width around 250 MeV, which is not well confirmed in PDG because of the lack of more experimental information. This $K_4(2500)$ state is dynamically generated as a molecular state of $K - f_4$ in our results of $|T^{I=1/2}_{K-f_4}|^2$, with a mass 2505 MeV and a width about 30 MeV. The predicted width of our results is eight times smaller than the one reported. We should admit that there are also uncertainties in our results, seen from the results of $|T^{I=1/2}_{\rho-K_3}|^2$ on the right of Fig. 7 and discussed later. For $|T^{I=3/2}_{\rho-K_3}|^2$ there is also resonant structure at the position about 2620 MeV, the width of which is about 150 MeV, which is not found for any $I = 3/2$ $K_4$ state in the PDG. Thus, within uncertainties, we predict a new $K_4$ resonance of isospin $I = 3/2$, with a mass about 2620 MeV and a width about 150 MeV.
IV B. the amplitudes $t$ of the particles,

Since $T$ IV B, the amplitude reproducing the results of Ref. [41], and for the second case, the $K$ 3 \( = \frac{1}{2} \), we find that the total isospin of the six-body system is only $I_{\text{total}} = \frac{1}{2}$. Thus, for the first case, the $K_1$ colliding with the $f_4$, we need calculate the amplitudes $t_1 = t_2 = t_{K_1 f_2} = T_{K_1 f_2}^{(I=1/2)}$, which have been evaluated in subsection IV B. For the second case, the $f_2$ colliding with the $K_3$, we evaluate the amplitudes $t_1 = t_{f_2 f_2} = T_{f_2 f_2}^{(I=1/2)}$ by reproducing the results of Ref. [41], and $t_2 = t_{f_2 K_1} = T_{f_2 K_1}$ taking from the results of subsection IV B.

We show our results for the six-body interaction in Fig. 8. The left of Fig. 8 is $|T_{K_1 f_4}^{I=1/2}|^2$. Since $K_1$ collides with the $f_4$, the amplitudes $t_1 = t_2 = t_{K_1 f_2} = T_{K_1 f_2}^{(I=1/2)}$, as shown in subsection IV B the amplitude $T_{K_1 f_2}^{(I=1/2)} \neq T_{f_2 K_1}^{(I=1/2)}$, for a test we also take $t_1 = t_2 = t_{K_1 f_2} = T_{f_2 K_1}^{(I=1/2)}$ to evaluate Faddeev equation with the FCA. By taking $t_1 = t_2 = t_{K_1 f_2} = T_{K_1 f_2}^{(I=1/2)}$, we find a peak around the energy 2550 MeV with a large width of about 500 MeV. For the results of taking $t_1 = t_2 = t_{K_1 f_2} = T_{f_2 K_1}^{(I=1/2)}$, there also is a resonant peak in the position about 2675 MeV, the width of which is nearly 500 MeV too. We can see that this test just gives us some uncertainties for our results. The right of Fig. 8 is $|T_{f_2 K_3}^{I=1/2}|^2$, where we can observe that there is not a clear peak at the position 2675 MeV. This peak looks like the resonant structure of $f_2 - K_3$ is less stable comparing with the one of $K_1 - f_4$. Since there is no $K_5$ particle found in PDG, there could be a new $K_5$ resonance which is more uncertainty from our results, with a mass of about 2550 $\sim$ 2675 MeV and a large width about 500 MeV.

D. Six-body interaction

From Table I if we predict a $K_3$ state, seen in the subsection IV B analogously to the five-body interaction, there also are two options of the cluster for the six-body interaction, the particle $f_4$ found in PDG and studied in Ref. [41] and the resonance $K_3$ predicted above. Under the FCA, we let a third particle of the composite resonance ($K_1$ or $f_2$) to collide with them, having (i) particle $3 = K_1$, cluster $R = f_4$ (1 = $f_2$, 2 = $f_2$), or (ii) 3 = $f_2$, $R = K_3$ (1 = $f_2$, 2 = $K_1$). Since the isospin of the particles, $I_{f_2} = I_{f_4} = 0$ and $I_{K_1} = I_{K_3} = \frac{1}{2}$, we find that the total isospin of the six-body system is only $I_{\text{total}} = \frac{1}{2}$. We show our results for the six-body interaction in Fig. 8. The left of Fig. 8 is $|T_{K_1 f_4}^{I=1/2}|^2$. Since $K_1$ collides with the $f_4$, the amplitudes $t_1 = t_2 = t_{K_1 f_2} = T_{K_1 f_2}^{(I=1/2)}$, as shown in subsection IV B the amplitude $T_{K_1 f_2}^{(I=1/2)} \neq T_{f_2 K_1}^{(I=1/2)}$, for a test we also take $t_1 = t_2 = t_{K_1 f_2} = T_{f_2 K_1}^{(I=1/2)}$ to evaluate Faddeev equation with the FCA. By taking $t_1 = t_2 = t_{K_1 f_2} = T_{K_1 f_2}^{(I=1/2)}$, we find a peak around the energy 2550 MeV with a large width of about 500 MeV. For the results of taking $t_1 = t_2 = t_{K_1 f_2} = T_{f_2 K_1}^{(I=1/2)}$, there also is a resonant peak in the position about 2675 MeV, the width of which is nearly 500 MeV too. We can see that this test just gives us some uncertainties for our results. The right of Fig. 8 is $|T_{f_2 K_3}^{I=1/2}|^2$, where we can observe that there is not a clear peak at the position 2675 MeV. This peak looks like the resonant structure of $f_2 - K_3$ is less stable comparing with the one of $K_1 - f_4$. Since there is no $K_5$ particle found in PDG, there could be a new $K_5$ resonance which is more uncertainty from our results, with a mass of about 2550 $\sim$ 2675 MeV and a large width about 500 MeV.

E. Discussions

Now we would like to make a discussion about the uncertainties of our results. As discussed in section III we have considered the width of the vector mesons in the evaluation of the two-body interaction amplitudes, $t_{\rho \rho}$ and $t_{\rho K}$, by taking into account the convolution of the loop function $G$. On the other hand, for the $K$-multi-$\rho$ system in the present work, the third particle in many
cases, seen in Table. \[ \text{Table} \], is a vector meson or a resonance, and has a certain width, which should be taken into account. Thus, in principle, as done in Ref. [48], we can roughly consider the width of the third particle in the propagator function \( G_0 \), Eq. [5], by replacing the term in the denominator \( i\epsilon \) with \( im_3\Gamma_3 \). Or, we can do the convolution of \( G_0 \) function for considering more exactly the contribution of the width of the third particle as done in Ref. [37]. But, as shown in the results of Refs. [48, 87], the final results for the position of the generated states are not altered after considering the contribution of the width of the third particle, to which we assign some uncertainties on the width for the generated results. Furthermore, there is also certain width for the clusters, for example, the resonances of \( f_2(1270) \) and \( K_1(1270) \) in the three-body interaction, which have a big width. As done in Refs. [48, 87], we also would take into account roughly the contribution of the width of the cluster by replacing the mass of the cluster \( M_R \) in Eqs. [6] and [7] with \( M_R - i\Gamma_R/2 \). In fact, the contribution for the width of the cluster is just a small effect on the masses and widths of the generated results (seen more discussions in Refs. [48, 87]). For checking this contribution, we also can consider the convolution for the \( t_i \) since Eq. [10] is dependent on the mass of the cluster, seen in Fig. [9] taking \( K - \rho \rho \) scattering for example, where we can see that the effect of the width of the cluster is small for the position of the peak and just makes the width of the peak a bit larger. Therefore, in our present work, we ignore all these effects in our investigation, just keeping in mind that there also some uncertainties from considering the contribution of the width of the third particle and the cluster. Besides, according to the discussions in Ref. [88], there may be some uncertain effect in the denominator term of \( \tilde{t}_1 \tilde{t}_2 G_0^2 (\tilde{t}_1 G_0) \) in Eq. [14] (Eq. [15]).
V. CONCLUSIONS

In the present work, we investigate the many body interactions between $K$-multi-$\rho$ systems, using the formalism of the fixed center approximation to the Faddeev equations. We start from the two-body interaction of $\rho\rho$ and $\rho K$ with the combination of dynamics of the local hidden gauge Lagrangian and the coupled channel effect, to reproduce the resonances of $f_2(1270)$ and $K_1(1270)$ as the clusters for our formalism. In the three-body $K\rho\rho$ system, we dynamically generate the $K_2(1770)$ state in our formalism, obtained a resonant peak in the modulus squared of the scattering amplitudes around the position in 1710 MeV with a width about 100 MeV and explained its structure as a $\rho - K_1(1270)$ molecular state. Continuing with the three-body interaction formalism, we extrapolate the fixed center approximation assumption for the four-body interaction. We observe a new $K_3$ state, with a mass about 2025 − 2075 MeV and a width about 200 MeV, which would be a $K_1 - f_2$ molecular resonance and not found in the PDG yet. For the five-body interaction, we successfully generate the $K_4(2500)$ state in our results of $|T_{K-f_4}^{I=1/2}|^2$, with a mass 2505 MeV and a width about 30 MeV, even though our theoretically predicted width is smaller than the one of unconfirmed experimental results of about 250 MeV. Thus, we also explain the particle $K_4(2500)$ as a molecular state of $K - f_4$. Besides, we find a new $K_4$ resonance of isospin $I = 3/2$ in the $\rho-K_3$ interaction, with a mass about 2620 MeV and a width about 150 MeV. Finally, analogously we predict a new $K_5$ state in the six-body interactions of the $K_1 - f_2$ and $f_2 - K_3$, with a mass of about 2550 ∼ 2675 MeV and a large width about 500 MeV, with more uncertainties. We hope that in future experiments, our predicted states of $K_3(2050)$, $K_5(2600)$ (isospin $I = 1/2$), and $K_4(2620)$ (isospin $I = 3/2$) are found.

Acknowledgements

We thank M. J. Vicente Vacas for useful discussions, also appreciate E. Oset, C. Hanhart and Ulf-G. Meiβner for careful reading the paper and useful comments. This work is supported, in part, by NSFC and DFG through funds provided to the Sino-German CRC 110 Symmetries and the Emergence of Structure in QCD (NSFC Grant No. 11261130311), NSFC (Grant Nos. 11035006 and 11165005). This work is also partly supported by the Spanish Ministerio de Economia y Competitividad and European FEDER funds under Contract No. FIS2011-28853-C02-01 and the Generalitat Valenciana in the program Prometeo, 2009/090. We acknowledge the support of the European Community-Research Infrastructure Integrating Activity Study of Strongly Interacting Matter (Hadron Physics 3, Grant No. 283286) under the Seventh Framework Programme of the European Union.

[1] R. P. Feynman, M. Gell-Mann and G. Zweig, Phys. Rev. Lett. 13, 678 (1964).
[2] M. Gell-Mann, Phys. Lett. 8, 214 (1964).
[3] S. L. Olsen, Hyperfine Interact. 229, no. 1-3, 7 (2014).
[4] S. Choi, Int. J. Mod. Phys. Conf. Ser. 31, 1460293 (2014).
[5] J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985).
[6] U. -G. Meißner, Rept. Prog. Phys. 56, 903 (1993).
[7] V. Bernard, N. Kaiser and U.-G. Meißner, Int. J. Mod. Phys. E 4, 193 (1995).
[8] A. Pich, Rept. Prog. Phys. 58, 563 (1995).
[9] G. Ecker, Prog. Part. Nucl. Phys. 35, 1 (1995).
[10] S. Scherer, Adv. Nucl. Phys. 27, 277 (2003).
[11] V. Bernard, Prog. Part. Nucl. Phys. 60, 82 (2008).
[64] R. Molina, D. Nicmorus and E. Oset, Phys. Rev. D 78, 114018 (2008).
[65] L. Roca, E. Oset and J. Singh, Phys. Rev. D 72, 014002 (2005).
[66] L. S. Geng, E. Oset, L. Roca and J. A. Oller, Phys. Rev. D 75, 014017 (2007).
[67] D. Garmemann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D 81, 014029 (2010).
[68] J. Yamagata-Sekihara, J. Nieves, E. Oset, Phys. Rev. D 83, 014003 (2011).
[69] F. Aceti and E. Oset, Phys. Rev. D 86, 014012 (2012).
[70] J.-J. Xie, A. Martinez Torres and E. Oset, Phys. Rev. C 83, 065207 (2011).
[71] F. Mandl and G. Shaw, Quantum Field Theory (Wiley-Interscience, New York, 1984).
[72] A. Martinez Torres, E. J. Garzon, E. Oset and L. R. Dai, Phys. Rev. D 83, 116002 (2011).
[73] J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010).
[74] J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. C 84, 015202 (2011).
[75] C. W. Xiao, M. Bayar and E. Oset, Phys. Rev. D 84, 034037 (2011).
[76] E. Oset, A. Ramos and C. Bennhold, Phys. Lett. B 527, 99 (2002) [Erratum-ibid. B 530, 260 (2002)].
[77] F. K. Guo, R. G. Ping, P. N. Shen, H. C. Chiang and B. S. Zou, Nucl. Phys. A 773, 78 (2006).
[78] W. Liang, C. W. Xiao and E. Oset, Phys. Rev. D 88, no. 11, 114024 (2013).
[79] F. K. Guo, C. Hanhart, Q. Wang and Q. Zhao, arXiv:1411.5584 [hep-ph].
[80] T. Hyodo, Phys. Rev. C 90, no. 5, 055208 (2014).
[81] S. U. Chung, R. L. Eisner, S. D. Protopopescu, N. P. Samios and R. C. Strand, Phys. Lett. B 51, 413 (1974).
[82] M. Aguilar-Benitez, V. E. Barnes, D. Bassano, S. U. Chung, R. L. Eisner, E. Flaminio, J. B. Kinson and R. B. Palmer et al., Phys. Rev. Lett. 25, 54 (1970).
[83] H. R. Blieden, G. Finocchiaro, J. Kirz, C. Nef, R. Thun, D. Bowen, D. Earles and W. Faissler et al., Phys. Lett. B 39, 668 (1972).
[84] G. D. Tikhomirov, I. A. Erofeev, O. N. Erofeeva and V. N. Luzin, Phys. Atom. Nucl. 66, 828 (2003) [Yad. Fiz. 66, 860 (2003)].
[85] W. E. Cleland, A. Delfosse, P. A. Dorsaz, J. L. Gloor, M. N. Kienzle-Focacci, G. Mancarella, A. D. Martin and M. Martin et al., Nucl. Phys. B 184, 1 (1981).
[86] T. Armstrong et al. [Bari-Birmingham-CERN-Milan-Paris-Pavia Collaboration], Nucl. Phys. B 227, 365 (1983).
[87] M. Bayar, W. H. Liang, T. Uchino and C. W. Xiao, Eur. Phys. J. A 50, 67 (2014).
[88] V. Lensky, V. Baru, J. Haidenbauer, C. Hanhart, A. E. Kudryavtsev and U.-G. Meißner, Eur. Phys. J. A 26, 107 (2005).