Light Sterile Neutrino in the Minimal Extended Seesaw

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Motivated by the recent observations on sterile neutrinos, we present a minimal extension of the canonical type-I seesaw by adding one extra singlet fermion. After the decoupling of right-handed neutrinos, an eV-scale mass eigenstate is obtained without the need of artificially inserting tiny mass scales or Yukawa couplings for sterile neutrinos. In particular, the active-sterile mixing is predicted to be of the order of 0.1. Moreover, we show a concrete flavor $A_4$ model, in which the required structures of the minimal extended seesaw are realized. We also comment on the feasibility of accommodating a keV sterile neutrino as an attractive candidate for warm dark matter.

INTRODUCTION

During the past decade, various neutrino oscillation experiments have shown very solid evidence of non-vanishing neutrino masses and lepton flavor mixing. Apart from neutrino oscillations within three active flavors, recent re-evaluations of the anti-neutrino spectra suggest that there exists a flux deficit in nuclear reactors, which could be explained if anti-electron neutrinos oscillate to sterile neutrinos [1]. Such a picture would require one or more sterile states with masses at the eV scale, together with sizable admixtures [i.e., $\mathcal{O}(0.1)$] with active neutrinos. Moreover, the light-element abundances from precision cosmology and Big Bang nucleosynthesis favor extra radiation in the Universe, which could be interpreted with the help of additional sterile neutrinos [2].

From the theoretical side, it is unclear why the energy scales related to electroweak symmetry breaking and sterile neutrinos are different by many order of magnitude. In the canonical type-I seesaw [3], right-handed neutrinos could in principle play the role of sterile neutrinos if their masses lie in the eV ranges. This could be technically natural since right-handed neutrinos are Standard Model (SM) gauge singlets [4]. However, in such a case, the Yukawa couplings relating lepton doublets and right-handed neutrinos should be of the order $10^{-12}$ (namely, the Dirac mass should be at the sub-eV scale) for correct mixings between active and sterile neutrinos. It is therefore more appealing to consider a natural and consistent framework yielding both low-scale sterile neutrino masses and sizable active-sterile mixings. In this respect, models simultaneously suppressing the Majorana and Dirac mass terms have been proposed in the literature, e.g., the split seesaw models in extra dimensions [5], the Froggatt-Nielsen mechanism [6–9], and flavor symmetries [10].

Recall that the seesaw mechanism is among one of the most popular theoretical attempts that gives a natural way to understand the smallness of neutrino masses. This motivates us to look for the possibility of generating eV-scale sterile neutrino masses by using a similar approach. Such an idea has been briefly mentioned in Ref. [7], in which the type-I seesaw is extended by adding only one singlet fermion [i.e., the minimal extended seesaw (MES)] acting as a sterile neutrino, without the need of imposing tiny Yukawa couplings or mass scales. A similar idea was also employed in Ref. [12] to accommodate a sterile neutrino of mass $\sim 10^{-3}$ eV in order to explain the solar neutrino problem. In this note, we exploit in detail the properties of the MES. Especially, we will show that the sterile neutrino mass is stabilized at the eV scale, while a sizable active-sterile mixing accounting for the rector neutrino anomaly is predicted. Furthermore, we will discuss how to realize the MES structure in flavor symmetries, i.e., a flavor model based on the tetrahedral group $A_4$. We also comment that the model could be generalized in order to include a keV sterile neutrino playing the role of warm dark matter.

THE MINIMAL EXTENDED SEESEAW

Here we describe the basic structure of the MES, in which three right-handed neutrinos and one additional gauge singlet chiral field $S$ are introduced besides the SM particle content. We will show that there could be a natural eV-scale sterile neutrino in this picture, without the need of inserting a small mass term or tiny Yukawa couplings. Explicitly, the Lagrangian of neutrino mass terms is given by

$$\mathcal{L}_m = \bar{\nu}_L M_D \nu_R + \bar{S} M_S \nu_R + \frac{1}{2} \bar{\nu}_R M_R \nu_R + \text{h.c.}, \quad (1)$$

where $M_D$ and $M_R$ denote the Dirac and Majorana mass matrices, respectively. Note that, $M_S$ is a $1 \times 3$ matrix, since we only introduce one extra singlet. The full $7 \times 7$ neutrino mass matrix in the basis $(\nu_L, \nu_R^c, S^c)$ reads

$$M_\nu^{7 \times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^c \\ 0 & M_S & 0 \end{pmatrix}.$$  \quad (2)

Similar to the typical type-I seesaw model, $M_D$ is assumed to be around the electroweak scale, i.e., $10^2$ GeV,
while the right-handed neutrino Majorana masses are chosen to be not far away from the typical grand unification scale, $M_R \sim 10^{14}$ GeV. Furthermore, there is no bare Majorana mass term assumed for $S$, while $S$ is only involved in the $M_S$ term, which may originate from certain Yukawa interactions with right-handed neutrinos and a SM singlet scalar. There is essentially no constraint on the scale of $M_S$. In the remaining parts, we will consider the interesting situation $M_R \gg M_S$.

In analogy to the type-I seesaw, the right-handed neutrinos are much heavier than the electroweak scale, and thus they should be decoupled at low scales. Effectively, one can block-diagonalize the full mass matrix $M^4_{\nu} \times 7$ by using the seesaw formula, and arrive at a $4 \times 4$ neutrino mass matrix in the basis $(\nu_L, S^c)$, i.e.,

$$M^4_{\nu} = -\begin{pmatrix} M_{D\nu} M^{-1}_{R} M^{T}_{D} & M_{D\nu} M^{-1}_{R} M^{T}_{S} \\ M_{S\nu} (M^{-1}_{R})^{T} & M_{S\nu} M^{-1}_{R} M^{T}_{S} \end{pmatrix}.$$  \hspace{1cm} (3)

One observes from Eq. (3) that there exist in total four light eigenstates corresponding to three active neutrinos and one sterile neutrino, and their masses are all suppressed by a factor $M^{-1}_{R}$ in the spirit of the seesaw mechanism. Moreover, $M^4_{\nu}$ is at most of rank 3, since

$$\det (M^4_{\nu}) = \det (M_{D\nu} M^{-1}_{R} M^{T}_{D}) \det [-M_{S\nu} M^{-1}_{R} M^{T}_{S}$$

$$+ M_{S\nu} M^{-1}_{R} M^{T}_{D} (M_{D\nu} M^{-1}_{R} M^{T}_{D})^{-1} M_{D\nu} M^{-1}_{R} M^{T}_{S}$$

$$= \det (M_{D\nu} M^{-1}_{R} M^{T}_{D}) \det [M_{S\nu} (M^{-1}_{R} - M^{-1}_{R}) M^{T}_{S}$$

$$= 0,$$ \hspace{1cm} (4)

where we have assumed both $M_R$ and $M_D$ are invertible. Therefore, at least one of the light neutrinos is massless.

We proceed to diagonalize $M^4_{\nu}$, $M^4_{\nu}$ could be in general three choices of the scale of $M_S$: 1) $M_D \sim M_S$; 2) $M_D > M_S$; 3) $M_D < M_S$. For case 1, $M^4_{\nu}$ is nearly democratic, indicating a maximal mixing between active and sterile neutrinos, and therefore is not compatible with neutrino oscillation data. In the second case, active neutrinos are heavier than the sterile one. Such a scenario results in more tension with cosmological constraints on the summation of light neutrino masses. We will comment on this case later on. In what follows, we shall concentrate on the third case, and study the properties of the sterile neutrino in detail.

Since in case 3 $M_D$ is larger than $M_D$ by definition, one can apply the seesaw formula once again to Eq. (3), and obtain at leading order the active neutrino mass matrix

$$m_{\nu} \simeq M_{D\nu} M^{-1}_{R} M^{T}_{D} (M_{S\nu} M^{-1}_{R} M^{T}_{S})^{-1} M_{S\nu} (M^{-1}_{R})^{T} M^{T}_{D},$$ \hspace{1cm} (5)

as well as the sterile neutrino mass

$$m_s \simeq -M_{S\nu} M^{-1}_{R} M^{T}_{S}.$$ \hspace{1cm} (6)

Note that the right-hand-side of Eq. (5) does not vanish since $M_S$ is a vector rather than a square matrix; if $M_S$ were a square matrix this would lead to an exact cancelation between the two terms of Eq. (5). Here, $m_{\nu}$ can be diagonalized by means of a $3 \times 3$ unitary matrix as

$$m_{\nu} = U \text{diag}(m_1, m_2, m_3) U^{T},$$ \hspace{1cm} (7)

where $m_i$ (for $i = 1, 2, 3$) denote the masses of three active neutrinos. The full neutrino mixing matrix then takes a $4 \times 4$ form \cite{13},

$$V \simeq \begin{pmatrix} (1 - \frac{1}{2}R R^T) U & \frac{R}{1 - \frac{1}{2}R R^T} \end{pmatrix},$$ \hspace{1cm} (8)

where $R$ is a $3 \times 1$ matrix governing the strength of active-sterile mixing,

$$R = M_{D} M^{-1}_{R} M^{T}_{S} (M_{S} M^{-1}_{R} M^{T}_{S})^{-1}.$$ \hspace{1cm} (9)

Essentially, the $R$ matrix (i.e., $V_{44}$ for $\alpha = e, \mu, \tau$) is suppressed by the ratio $O(M_D)/O(M_S)$.

As a naive numerical example, for $M_D \sim 10^2$ GeV, $M_S \sim 5 \times 10^2$ GeV and $M_R \sim 2 \times 10^{14}$ GeV, one may estimate that $m_\nu \sim 0.05 \text{eV}$, $m_s \sim 1.3 \text{eV}$ together with $R \sim 0.2$. This is in very good agreement with the fitted sterile neutrino parameters [14, 15]. We further stress that the MES structure is stable against radiative corrections since the loop contributions to the light neutrino masses are suppressed by both the heavy right-handed neutrino masses and loop factors.

The MES picture described above is a minimal extension of the type-I seesaw in the sense that one could allow for at most one extra singlet in order to account for neutrino oscillation phenomena (see a recent analysis in Ref. [16]). In other words, three heavy right-handed neutrinos can lead to at most three massive light neutrinos (the “seesaw-fair-play-rule” [17]), out of which two are active and needed to account for the solar and atmospheric neutrino mixing.

Unfortunately, it is not possible to accommodate two eV-scale sterile neutrinos in the MES, unless the number of right-handed neutrinos is increased. Furthermore, if this scenario is embedded into certain grand unified theory framework, e.g. SO(10), it cannot be anomaly free due to the lacking of other two generations of S. Apart from these shortcomings, the MES possesses the following features:

- apart from the electroweak and seesaw scales, one does not artificially insert small mass scales for sterile neutrino masses. As in the canonical type-I seesaw, one can take $M_S > M_D \sim O(10^2 \text{GeV})$, while $M_R$ can be chosen close to the $B - L$ scale, not far from the grand unification scale;

- it is more predictive owing to the absence of one active neutrino mass, while it does explain all the experimental data. Neutrino-less double beta decay is also allowed because not all of the neutrinos are light;
• there exist heavy right-handed neutrinos that could be responsible for thermal leptogenesis. Note that, in the setup we considered, right-handed neutrinos would preferably decay to the sterile neutrino since their coupling to \(S\) is larger than active neutrinos. However, this drawback could easily be circumvented since \(S\) enters in the one-loop self-energy diagram of the decay of right-handed neutrinos, which could compensate for this.

We finally comment on the second case, i.e., \(M_D \gg M_S\). Now that \(M_\delta^{1 \times 4}\) possesses a hierarchical texture along the inverted direction, one may still apply the seesaw formula to Eq. (3), and obtain that, at leading order, the active neutrino mass matrix is the same as that given in the type-I seesaw, i.e., \(m_\nu \simeq -M_D M_R^{-1} M_D^T\), whereas the sterile neutrino mass is vanishing. In viewing of the experimental results on the active-sterile mass-squared difference, one would expect all the three active neutrinos to be located at the eV scale, which is however challenged by standard cosmology since that leads to a large total mass of neutrinos. Note also that, despite the cosmological constraints, one can in principle add more singlets since they do not affect active neutrino masses. Especially, in case of three additional singlets \((S_1, S_2, S_3)\), the particle contents are analogous to those in the inverse seesaw or double seesaw \cite{ref18}, although the mass matrix structures are clearly different.

**REALIZATION IN A FLAVOR \(A_4\) MODEL**

In this section, we focus on a simple flavor \(A_4\) model giving rise to the exact mass structures depicted in Eq. (1). In addition to the SM Higgs boson, we introduce three sets of flavons \(\varphi, \xi\) and \(\chi\). An extra discrete abelian symmetry \(Z_4\) has been introduced in order to avoid interferences between the neutrino and charged-lepton sectors. The particle assignments are shown in Table I.

**TABLE I: Particle assignments in the flavor \(A_4\) model.**

| Field | \(\ell\) | \(e_R\) | \(\mu_R\) | \(\tau_R\) | \(H\) | \(\varphi\) | \(\varphi'\) | \(\varphi''\) | \(\xi\) | \(\xi'\) | \(\chi\) | \(\nu_{R1}\) | \(\nu_{R2}\) | \(\nu_{R3}\) | \(S\) |
|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| \(SU(2)\) | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| \(A_4\) | 3 | 1 | \(\dfrac{1}{2}\) | 1 | 1 | 3 | 3 | 3 | 1 | \(\dfrac{1}{2}\) | 1 | 1 | \(\dfrac{1}{2}\) | 1 | 1 |
| \(Z_4\) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

At leading order, the \(A_4 \otimes Z_4\) invariant Lagrangian for the lepton sector is given by

\[
\mathcal{L} = \frac{y_1}{\Lambda} (\bar{\ell} H \varphi) \ell_R e_R + \frac{y_2}{\Lambda} (\bar{\ell} H \varphi') \ell_R \mu_R + \frac{y_3}{\Lambda} (\bar{\ell} H \varphi'') \ell_R \tau_R + \frac{y_4}{\Lambda} (\bar{\ell} H \varphi'') \ell_R \nu_{R1} + \frac{y_5}{\Lambda} (\bar{\ell} H \varphi'') \ell_R \nu_{R2} + \frac{y_6}{\Lambda} (\bar{\ell} H \varphi'') \ell_R \nu_{R3} + \frac{1}{2} \lambda_1 \xi \nu_{R1} \nu_{R1} + \frac{1}{2} \lambda_2 \xi \nu_{R2} \nu_{R2} + \frac{1}{2} \lambda_3 \xi \nu_{R3} \nu_{R3} + \frac{1}{2} \rho \chi S \nu_{R1} + \text{h.c.},
\]

where \(\Lambda\) denotes the cut-off scale and \(\tilde{H} \equiv i \tau_2 H\). If we choose the real basis for \(A_4\), along with the flavon alignments

\[
\langle \varphi \rangle = (v, 0, 0), \quad \langle \varphi' \rangle = (v, v, v), \quad \langle \varphi'' \rangle = (0, -v, v),
\]

\[
\langle \xi \rangle = \langle \xi' \rangle = v, \quad \langle \chi \rangle = u,
\]

then the charged-lepton mass matrix is diagonal \(^4\), i.e.,

\[
m_\ell = \frac{\langle H \rangle v}{\Lambda} \text{diag} (y_e, y_\mu, y_\tau),
\]

while the Dirac mass term is given by

\[
M_D = \frac{\langle H \rangle v}{\Lambda} \begin{pmatrix} y_1 & y_2 & 0 \\ 0 & y_2 & y_3 \\ 0 & y_2 & -y_3 \end{pmatrix}.
\]

Due to the additional \(Z_4\) symmetry, the right-handed neutrino mass matrix is diagonal as well, viz.

\[
M_R = \text{diag} (\lambda_1 v, \lambda_2 v, \lambda_3 v).
\]

Furthermore, the singlet fermion \(S\) does not acquire a Majorana mass term, at least at leading order. The coupling matrix between \(S\) and right-handed neutrinos reads

\[
M_S = (\rho u 0 0).
\]

As a rough numerical example, we assume the following mass scales: \(v \simeq 10^{13}\) GeV, \(\Lambda \simeq 10^{14}\) GeV, and \(u \simeq 10^2\) GeV. One can then estimate that, by assuming order

\(^2\) Here the non-renormalizable interactions for the neutrino sector are not included since they are suppressed by \(1/\Lambda\). In principle, dimension-five operators like \(\dfrac{1}{\Lambda} \varphi \varphi' SS\) may spoil the desired MES structure, since it leads to an unacceptable large mass term for \(S\) after the symmetry breaking. Such a drawback could be easily avoided by introducing an additional global \(U(1)_F\) symmetry, under which only \(\chi\) and \(S\) are charged but with opposite sign. The \(\varphi' SS\) term is then forbidden by \(U(1)_F\), whereas the \(\chi S\mu R\) interaction in the Lagrangian remains.

\(^3\) We do not expand our discussions on how the vacuum alignment of flavons is achieved, whereas we refer readers to Ref. [19], in which the same flavon vacuum alignment is acquired by assuming a radiative symmetry breaking mechanism.

\(^4\) Note that, the hierarchies between charged-lepton masses can be obtained by using the Froggatt–Nielsen mechanism, viz., assigning different \(U(1)_{FN}\) charges to the right-handed fields.
1 Yukawa couplings, the condition $M_R \gg M_S > M_D$ can be satisfied. Compared to Eq. (5), we obtain

$$m_\nu = -(H)^2 v \left( \frac{v_y^2}{\lambda_3} \frac{v_y^2}{\lambda_3} \frac{v_y^2}{\lambda_2} \frac{v_y^2}{\lambda_2} \right).$$

(16)

It is straightforward to see that $m_\nu$ features a $\mu - \tau$ symmetry, which generally predicts a maximal mixing in the 2–3 sector and a vanishing $\theta_{13}$. Indeed, $m_\nu$ can be analytically diagonalized by using the tri-bimaximal mixing [20] matrix $V_{TB}$ as

$$m_\nu = -V_{TB} \text{ diag} \left(0, \frac{3\alpha(H)^2 v}{\lambda_3^3}, \frac{2\alpha(H)^2 v}{\lambda_3^3} \right) \left( V_{TB}^T \right)^T,$$

(17)

with $V_{TB}$ being

$$V_{TB} = \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{\sqrt{3}-1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \\
-\frac{\sqrt{6}}{2} & \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{2} \\
\end{array} \right).$$

(18)

Therefore, the normal mass ordering ($m_1 \ll m_2 \ll m_3$) together with the tri-bimaximal mixing pattern are obtained. Taking for example $y_3 = 0.91$, $y_2 = 0.31$, and $\lambda_2 = \lambda_3 = 1$, one obtains $\Delta m^2_{32} \simeq 7.6 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{21} \simeq 2.5 \times 10^{-3}$ eV$^2$, being consistent with current global-fit data of neutrino mass-squared differences [21].

The sterile neutrino mass is obtained from Eq. (7) as

$$m_s \approx \frac{\rho^2 \mu^2}{\lambda_1 v}.$$  

(19)

Fitting to the sterile neutrino mass from a recent best-fit given in [14], one can get $m_s \approx 1.2$ eV (corresponding to $\Delta m^2_{31} \approx 1.5$ eV$^2$) for $\rho = 1.1$ and $\lambda_1 = 1$. By choosing $y_1 = 1$ and inserting the above parameters to Eq. (9), we arrive at the active-sterile mixing, i.e.,

$$R \simeq \left( \frac{y_1(H) v}{\rho \lambda_1} \right)^T \simeq \left( 0.16 \ 0 \ 0 \right)^T,$$

(20)

corresponding to $|V_{e4}|^2 \simeq 0.025$ together with $|V_{\mu 4}| = |V_{\tau 4}| = 0$, in good agreement with the best-fit value of active-sterile mixing [22] in the four neutrino mixing scenario. Therefore, in this simple model, both the tri-bimaximal mixing pattern in the active neutrino sector and a sizable active-sterile neutrino mixing are predicted without the need of fine-tuning the Yukawa couplings.

Alternatively, the flavor model described above can be slightly changed in order to admit the inverted mass ordering of active neutrinos (i.e., $m_3 \gtrsim m_1 \gtrsim m_2$). For this purpose, one could instead take the VEV alignment $\langle \varphi' \rangle = (v, -v, v, -v)$, which retains the tri-bimaximal mixing in the active neutrino mixing, and leads to a vanishing mass $m_3 = 0$. Note that, in case of the inverted mass ordering, the next-to-leading seesaw corrections [23] should be included in the diagonalization of $M^{4 \times 4}_{\nu}$, because of the degeneracy between $m_1$ and $m_2$.

As mentioned in the previous section, in this model, both active and sterile neutrinos may mediate the neutrino-less double beta decay processes, and their contributions to the effective mass are not cancelled. Concretely, we have $\langle m_{ee} \rangle \simeq |m_2 V_{e2}^2 + m_s V_{e4}^2| \simeq |m_{4e}|^2$ in the normal mass ordering case, and $\langle m_{ee} \rangle \simeq |m_1 V_{e1}^2 + m_s V_{e2}^2 + m_s V_{e3}^2| \simeq |m_{2e}|^2$ in the inverted mass ordering case. In addition, effects from right-handed neutrinos are negligibly small since they are highly suppressed by $M_R$ (see e.g. Ref. [24] for detailed discussions). This is a very distinctive feature, in particular compared to models with only eV-scale right-handed neutrinos, in which neutrino-less double beta decays are forbidden.

One may also wonder if the model could be modified to allow for a keV sterile neutrino warm dark matter candidate. Indeed, the sterile neutrino mass can be chosen at the keV ranges by setting, e.g. $u \sim 4$ TeV. Using the same Yukawa coupling parameters in the previous discussions, one then arrives at $m_s \simeq 1.9$ keV. Unfortunately, the active-sterile mixing $\theta_s = R_{11} \simeq 4 \times 10^{-3}$ turns out to exceed the current X-ray constraint [25],

$$\theta_s^2 \lesssim 1.8 \times 10^{-5} \left( \frac{1 \text{ keV}}{m_s} \right)^5.$$  

(21)

In order to keep $\theta_s$ small enough, we need a mild tuning of the Yukawa coupling, i.e., $y_1 < 0.2$. For example, taking $y_1 = 0.15$, we get $\theta_s^2 \simeq 1.2 \times 10^{-5} \left( \frac{1 \text{ keV}}{m_s} \right)^5$, satisfying the bound in Eq. (21).

Finally, we note that the reactor experiments Double Chooz [26], Daya Bay [27] and RENO [28] has found the smallest mixing angle $\theta_{13}$, i.e., $\sin^2 \theta_{13} \simeq 0.025$ from a recent global-fit [29]. Therefore, the exact tri-bimaximal mixing pattern should be modified in order to accommodate non-vanishing $\theta_{13}$. This can be achieved by including the next-to-leading order corrections to the flavor model. For example, in Ref. [30], the higher dimensional corrections to the vacuum alignments would result in a sizable $\theta_{13}$ compatible with the current experimental observation. Moreover, perturbations to the charged-lepton sector may also lead to a large $\theta_{13}$ (see discussions in Ref. [9]). Since the main purpose of this work is to present a novel mechanism generating light sterile neutrinos, we will not further expand our discussion on $\theta_{13}$. 

Note that various mechanisms for the production of sterile neutrino warm dark matter have been proposed in the literature, which, however, will not be discussed in this note.
CONCLUSION

In this note, we have studied a minimal extension of the type-I seesaw, which contains an extra singlet fermion coupled purely to the right-handed neutrinos. In such a framework, both active and sterile neutrino masses are suppressed via the seesaw mechanism, and thus, an eV-scale sterile neutrino together with sizable active-sterile mixing is accommodated without the need of artificially inserting small mass scales or Yukawa couplings. Furthermore, we have presented a flavor $A_4$ model, in which both the MES structures and the tri-bimaximal mixing pattern are realized. In particular, for a sterile neutrino with mass being around the eV scale, the active-sterile mixing (i.e., $|V_{e4}|$) is found to be of the order of 0.1, in good agreement with current experimental observations. The model may also be modified to take keV sterile neutrino warm dark matter into account. We hope this note serves as a useful guide for future model building works on low-scale sterile neutrinos.

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