SUSY: NEW PERSPECTIVES AND VARIANTS

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Although supersymmetry (SUSY) is thirty five years old, it is still one of the most attractive theories for physics beyond the standard model. Assuming that SUSY will be discovered at the LHC, the key question is: What SUSY model do we expect to be the correct one? After reviewing briefly the advantages and problems of SUSY, several interesting models that have been proposed in the literature will be discussed. In particular, models such as the MSSM, BRpV, NMSSM, and possible extensions. We will also introduce the $\mu$SSM whose interest resides in the fact that it generates a solution to the (famous) $\mu$ problem of SUSY models that is connected to the (nowadays very popular) neutrino physics.

1 Introduction

We know from the past that symmetries are crucial in physics. The laws of modern physics are invariant under certain symmetries: Invariance under Lorentz transformations is the origin of special relativity, and invariance under local gauge transformations is the key point of the Standard Model (SM), $SU(3)_c \times SU(2)_L \times U(1)_Y$. Supersymmetry (SUSY) was proposed in the early 1970’s as an invariance of the theory under the interchange of fermions and bosons. However, knowns fermions like quarks, electron and neutrino, and bosons like gluons, $W^\pm$ and photon are not married up in this fashion. Instead, every known particle should have a (super) partner. Thus the spectrum of elementary particles is doubled. There are squarks and sleptons as superpartners of quarks and leptons, and gluinos, Winos, Zino and photino as superpartners of gluons, $W^\pm$, $Z$ and photon. Obviously, we have not detected SUSY particles with the same masses as their SM partners (e.g. there is no selectron with mass $\sim$ 0.5 MeV), and actually accelerator physics imposes important lower bounds on their masses. This also implies that SUSY cannot be an exact symmetry of Nature and should be broken.

The SM particles together with their massive SUSY partners constitute the so-called Minimal Supersymmetric Standard Model (MSSM). Since the Higgs has a fermionic superpartner, the Higgsino, two Higgs doublets with opposite hypercharges, $H_1$ and $H_2$, must be present in order to avoid anomalies. In addition, because of the structure of SUSY, these two Higgs doublet superfields are necessary to give masses to all quarks and leptons. Clearly the MSSM has potentially a very rich phenomenology. Let us also remark that the joining of the three gauge coupling constants at a single unification scale agrees with the LEP experimental results. Whether this is a hint or just a coincidence, will be clarified by the LHC.

In any case, despite the absence of experimental verification, relevant theoretical arguments can be given in favour of SUSY. First of all, SUSY solves the so-called gauge hierarchy problem. If we believe that the SM should be embedded within a Grand Unified Theory (GUT) with a
typical scale $M_{\text{GUT}} \sim 10^{15}$ GeV or within a more fundamental theory including gravity with a characteristic scale $M_{\text{Planck}} \sim 10^{19}$ GeV, then we are faced with the hierarchy problem. There is no symmetry protecting the masses of the scalar particles against quadratic divergences in perturbation theory. Therefore they will be proportional to the huge cut-off scale $\sim M_{\text{GUT}}$ or $M_P$. The Higgs particle is included in the SM because of its good properties: it can have a vacuum expectation value (VEV) without breaking Lorentz invariance, inducing the spontaneous breaking of the electroweak (EW) symmetry at the same time that generates the gauge boson masses, and the fermion masses through Yukawa couplings. But, of course, all these properties are due to the fact that the Higgs is a scalar particle. As mentioned above, this leads to a huge mass for it and, as a consequence of the minimization, for the $W$ and $Z$ gauge bosons. This problem of naturalness, to stabilize $M_W \ll M_{\text{GUT}}, M_P$ against quantum corrections, is solved in SUSY since now the masses of the scalars and the masses of their partners, the fermions, are related. As a consequence, only a logarithmic divergence in the Higgs scalar mass is left. In diagrammatic language, the dangerous diagrams of SM particles are cancelled with new ones which are present due to the existence of the additional partners and couplings.

Nevertheless, let us recall that SUSY was not invented to solve this hierarchy problem. As mentioned above, it was created as a new kind of symmetry which relates bosons and fermions. Notice that the latter also involves that the Higgs is no longer a mysterious particle as it stands in the SM: the only fundamental scalar particle which exists. Now, the Supersymmetric Standard Model (SSM) is naturally plenty of fundamental scalars (squarks, sleptons and Higgses) related through SUSY with their fermionic partners (quarks, leptons and Higgsinos).

Let us also mention that SUSY allows us to understand better how the EW symmetry, $SU(2)_L \times U(1)_Y$, is broken. Whereas in the case of the SM the quadratic and quartic terms of the Higgs potential, $V = m^2 H^* H + \lambda (H^* H)^2$, with $m^2 < 0$, have to be postulated "ad hoc", in the context of the SSM they appear in a natural way. The quartic terms arise from the usual $D$-term contributions and $\lambda$ is given by the EW coupling constants, and the quadratic terms arise once SUSY is broken and masses are generated for all scalar particles. Fortunately, these mass terms are ‘soft’ in the sense that they do not induce quadratic divergences, and therefore do not spoil the SUSY solution to the gauge hierarchy problem. In addition, the large top mass produces radiatively a negative mass square for the Higgs $H_2$ inducing the breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$.

Besides, we should not forget that the local version of SUSY leads to a partial unification of the SM with gravity, the so-called Supergravity (SUGRA). In this theory the graviton has a SUSY partner, the gravitino, and actually the breaking of SUGRA in order to generate a mass for the gravitino is an interesting mechanism to generate also the (soft) masses of the superpartners of the SM\textsuperscript{5}. Last but not least, SUSY seems to be a crucial ingredient of string theory.

Another (more model dependent) advantage of SUSY is related to the issue of the existence of dark matter in the Universe. If the superpotential of the theory conserves a discrete symmetry called $R$-parity (+1 for particles and -1 for superpartners), SUSY particles are produced or destroyed only in pairs and, as a consequence, the lightest supersymmetric particle (LSP) is absolutely stable. This implies that the LSP is a possible candidate for dark matter. Concerning this point, it is remarkable that in most of the parameter space of the MSSM the LSP is the lightest neutralino, a physical superposition of the Bino, and neutral Wino and Higgsinos. The neutralino is obviously an electrically neutral (also with no strong interactions) particle, and this is welcome since otherwise the LSP would bind to nuclei and would be excluded as a candidate for dark matter from unsuccessful searches for exotic heavy isotopes\textsuperscript{6}. Therefore, in the MSSM, typically the lightest neutralino is a very good dark matter candidate\textsuperscript{7}. Actually, the neutralino is a Weakly Interacting Massive Particle (WIMP), and therefore it is able to produce in some regions of the SUSY parameter space a value of the relic density of the correct order
of magnitude, $\Omega_{DM}h^2 \sim 0.1$. Let us finally remark that the fact that the LSP is stable, and typically neutral, implies that a major signature in accelerator experiments for $R$-parity conserving models is represented by events with missing energy.

On the other hand, there is no a fundamental reason to impose $R$-parity conservation in SUSY models. Actually, lepton and baryon number violating terms in the superpotential like $\epsilon_{ab} \left( \lambda^{ijk} \hat{L}_i^a \hat{L}_j^b \hat{c}_k^c + \lambda^{ijk} \hat{L}_i^a \hat{E}_j^b \hat{e}_k^c + \mu^j \hat{L}_i^j \hat{H}_2^c \right)$, and $\lambda^a_{ijk} \hat{d}_i^a \hat{c}_j^b \hat{\nu}_k^c$, respectively, with $i, j = 1, 2, 3$ generation indices and $a, b = 1, 2$ $SU(2)$ indices, which break explicitly $R$-parity, are in principle allowed by gauge invariance. As is well known, to avoid too fast proton decay mediated by the exchange of squarks of masses of order the EW scale, the presence together of terms of the type $\hat{\nu}_i^c \hat{\nu}_j^b \hat{\nu}_k^a$ and $\hat{d}_i^c \hat{d}_j^c \hat{\nu}_k^c$ must be forbidden, unless we impose very stringent bounds such as e.g. $\chi'^{a}_{112} \chi'^{b}_{112} \lesssim 2 \times 10^{-27}$. However the latter values for the couplings are not very natural, and for constructing viable SUSY models one usually forbids at least one of the operators $LQd^c$ or $Ld^cL^c$. The other type of operators above are not so stringently supressed, and therefore still a lot of freedom remains.\(^8\)

There is a large number of works in the literature\(^9\) exploring the possibility of $R$-parity breaking in SUSY models, and its consequences for the detection of SUSY at the LHC\(^10\). A popular model is the so-called Bilinear $R$-parity Violation (BRpV) model\(^11\), where bilinear terms of the above type, $\hat{L} \hat{H}_2$, are added to the MSSM. In this way it is in principle possible to generate neutrino masses without including in the model right-handed neutrinos, unlike the MSSM. Analyses of mass matrices\(^12\) in the BRpV, as well as studies of signals at accelerators\(^13\) has been extensively carried out in the literature. Other interesting models are those producing the spontaneous breaking of $R$-parity through the VEVs of singlet fields\(^14\).

Of course, the phenomenology of models where $R$-parity is broken is going to be very different from that of models where $R$-parity is conserved. Needless to mention, the LSP is no longer stable, and therefore not all SUSY chains must yield missing energy events at colliders. Obviously, in this context the neutralino is no longer a candidate for dark matter. Nevertheless, other candidates can be found in the literature, such as the gravitino\(^15\), the well-known axion, and many other (exotic) particles\(^7\).

Up to now we have mentioned several advantages of SUSY, but it is fair to say that problems are also present in this theory. For example, it is not clear yet the mechanism of SUSY breaking generating the soft masses for scalar particles of order the EW scale. As mentioned above, dangerous baryon and lepton number violation operators may be present, and they must be supressed by some mechanism. Dangerous charge and colour breaking minima may also be present in the parameter space of SUSY model\(^16\). There is the possibility of having too large contributions to Flavour Changing Neutral Currents (FCNC), as well as to the Neutron Electric Dipole Moment (EDM). There might be a fine-tuning problem in SUSY models. Finally, there is also the so-called $\mu$ problem\(^17\), arising from the presence of a mass term for the Higgs fields in the superpotential $\mu \hat{H}_1 \hat{H}_2$.

Concerning the latter, the Next-to-Minimal Supersymmetric Standard Model\(^18\) (NMSSM), provides an elegant solution via the introduction of a singlet superfield $\hat{S}$. In the simplest form of the superpotential, which is scale invariant and contains the $\hat{S} \hat{H}_1 \hat{H}_2$ coupling, an effective $\mu$ term is generated when the scalar component of $\hat{S}$ acquires a VEV of order the SUSY breaking scale. The NMSSM has been extensively analysed in the literature\(^18\)\(^19\)\(^20\), as well as its possible extensions generating neutrino masses, and also $R$-parity breaking. In Sects. 2 and 3 we will review the MSSM, and the NMSSM, respectively, and their extensions.

Let us finally mention that a model, the so-called $\mu$SSM, has been proposed\(^21\)\(^22\) to solve the $\mu$ problem of the MSSM without having to introduce an extra singlet superfield as in the NMSSM. Interestingly, the solution is connected to the neutrino physics. In particular, terms of the type $\nu^c \hat{H}_1 \hat{H}_2$ in the superpotential generate the $\mu$ term spontaneously through right-handed sneutrino VEVs. In addition, terms of the type $(\nu^c)^3$ forbid a global $U(1)$ symmetry in
the superpotential, avoiding therefore the existence of a Goldstone boson. Besides, the latter contribute to generate effective Majorana masses for neutrinos at the EW scale. On the other hand, these two type of terms break lepton number and $R$-parity explicitly implying that the phenomenology of this model is very different from the one of the MSSM/NMSSM. For example, the usual neutralinos are now mixed with the neutrinos. Since we have a generalized see-saw mechanism at the EW scale, for a Dirac mass of the heaviest neutrino of order the mass of the electron, 0.1 MeV, an eigenvalue reproducing the correct scale of the heaviest neutrino mass, 0.01 eV, is obtained. Playing with the hierarchies in the Dirac masses one can obtain the other neutrino masses. In Sect. 4 we will review the $\mu\nu$SSM.

We have mentioned in this Introduction several interesting SUSY models, but let us remark that others have been proposed during the years, and therefore our list is by no means complete. In any case, pretty soon, at the end of this year 2007, the LHC will start operations, thus the crucial question is by now: What SUSY model do we expect to be discovered? The reader can find in the rest of the paper several of the above models discussed in some detail, and pick up her/his preferred one.

2 MSSM and extensions

The MSSM is the most popular SUSY extension of the SM. It was the first SUSY model studied in detail in the literature, and it is the simplest extension: no extra fields are included apart from the SUSY partners of the SM fields. Let us discuss briefly the structure of the MSSM.

Working for the moment with massless neutrinos, in addition to the Yukawa couplings for quarks and charged leptons, the MSSM superpotential contains the so-called $\mu$-term involving the Higgs doublet superfields, $\hat{H}_1$ and $\hat{H}_2$,

$$W = \epsilon_{ab} \left( Y_{ij}^u \hat{H}_2^a \hat{Q}_i^b \hat{u}_j^c + Y_{ij}^d \hat{H}_1^a \hat{Q}_i^b \hat{d}_j^c + Y_{ij}^e \hat{H}_1^a \hat{L}_i^b \hat{e}_j^c \right) - \epsilon_{ab} \mu \hat{H}_1^a \hat{H}_2^b,$$

where we take $\hat{H}_1^T = (\hat{H}_1^0, \hat{H}_1^1)$, $\hat{H}_2^T = (\hat{H}_2^0, \hat{H}_2^1)$, $\hat{Q}_i^T = (\hat{u}_i, \hat{d}_i)$, $\hat{L}_i^T = (\hat{\nu}_i, \hat{\ell}_i)$, $i, j = 1, 2, 3$ are generation indices, $a, b$ are $SU(2)$ indices, and $\epsilon_{12} = 1$. It is worth noticing that this superpotential conserves $R$-parity. The presence of the $\mu$-term in (1) is essential to avoid the appearance of an unacceptable Goldstone boson associated to a global $U(1)$ symmetry $H_{1,2} \rightarrow e^{i\alpha} H_{1,2}$, when this is broken by the VEVs of the Higgses giving masses to all quarks and leptons. In addition, the minimum of the Higgs potential without including this term occurs for a vanishing VEV for $H_1$, and therefore $d$-type quarks and $e$-type leptons remain massless. Unfortunately, the $\mu$-term introduces a naturalness problem, the so-called $\mu$ problem\cite{232125}. Note that, to this respect, the $\mu$-term is purely supersymmetric, and therefore the natural scale of $\mu$ would be $M_{GUT}$ or $M_{Planck}$. Thus, any complete explanation of the EW scale must justify the origin of $\mu$, i.e. why its value is of order $M_W$ and not $M_{GUT}$ or $M_P$.

This problem has been considered by several authors and different possible solutions have been proposed, producing an effective $\mu$-term\cite{232125}. On the other hand, there are also very interesting solutions in the literature that necessarily introduce new structure beyond the MSSM at low energies. Several of these solutions, and the associated SUSY models, will be discussed in the next sections.

Concerning the spectrum of the MSSM, as is well known, after the EW breaking the model is left with three neutral Higgses and two charged ones. Although at tree level the mass of the lightest Higgs, $m_h$, is bounded by $M_Z$, loop corrections allow a larger bound, $m_h \lesssim 135$ GeV, still fulfilling the experimental constraint $m_h \gtrsim 114$ GeV. In the MSSM there are also four neutralinos, the physical superpositions of the Bino, and neutral Wino and Higgsinos. As discussed in the Introduction, the lightest one is a good candidate for dark matter.
2.1 Neutrino masses

Neutrino experiments have confirmed during the last years that neutrinos are massive. As a consequence, all theoretical models must be modified in order to reproduce this result. In particular, it is natural in the context of the MSSM to supplement the ordinary neutrino superfields, $\hat{\nu}_i$, contained in the $SU(2)_L$-doublet, $\hat{L}_i$, with gauge-singlet neutrino superfields, $\hat{\nu}_c^i$. Thus, in addition to the usual Yukawa couplings for quarks and charged leptons, and the $\mu$-term, the superpotential (1) may contain new terms such as Yukawa couplings for neutrinos, and possible Majorana mass terms:

$$\delta W = \epsilon_{ab}^i Y_{ij}^\nu \hat{H}_2^b \hat{L}_a^i \hat{\nu}_c^j + m_{ij}^\nu \hat{\nu}_c^i \hat{\nu}_c^j. \quad (2)$$

Clearly, one has that the couplings $Y_{ij}^\nu$ determine the Dirac masses for the neutrinos, $m_D = Y_{ij}^\nu v_2$, whereas $m_M$ are the Majorana masses. If $m_D << m_M$, both type of contributions induce very light neutrino masses of order

$$m_\nu \sim \frac{m_D^2}{m_M}. \quad (3)$$

This is the SUSY extension of the well-known see-saw mechanism studied in the context of the SM. Notice also that there will be a heavy neutrino with mass of order $m_M$. Result (3) was considered very interesting in the context of the early attempts to connect the standard model with GUTs where $M_{GUT} \sim 10^{15}$ GeV. Indeed, since $v_2 \sim 10^2$ GeV, with a neutrino Yukawa coupling of order one, and $m_M \sim 10^{15}$ GeV, one is able to obtain neutrino masses as small as $10^{-2}$ eV. Of course, this can be consider an improvement with respect to the use of a purely Dirac mass for the neutrino, which would imply the necessity of explaining a Yukawa coupling of order $10^{-13}$, i.e. thirteen orders of magnitude smaller than the one that we need with a GUT-scale see-saw. Recall that one chooses to forbid couplings with such small values, when discussing R-parity violating couplings producing proton decay, because they seem not very natural.

Nevertheless, let us remark that a see-saw at the EW scale is also a very interesting possibility. Since we know that the Yukawa coupling of the electron has to be of order $10^{-6}$, why the one of the neutrino should be six orders of magnitude larger? With a EW-scale see-saw, i.e. $m_M \sim 1$ TeV, a neutrino Yukawa coupling of order of the one of the electron generating $m_D \sim 10^{-4}$ GeV, is sufficient to produce a neutrino mass of order $10^{-2}$ eV (see eq. (3)). This possibility is also interesting because one does not need to introduce in the game any ad-hoc high-energy scale. It is worth mentioning here that in some string constructions, where SUSY standard-like models can be obtained without the necessity of a GUT, and Yukawa couplings can be explicitly computed, those for neutrinos cannot be as small as $10^{-13}$, and therefore the presence of a see-saw at the EW scale is helpful.

On the other hand, it is worth remarking that neutrino masses can also be obtained without using the singlet superfields $\hat{\nu}_c^i$. Adding to the superpotential (1) the bilinear terms

$$\delta W = \epsilon_{ab}^i \mu^j \hat{H}_2^b \hat{L}_a^i \hat{\nu}_c^j, \quad (4)$$

neutrino masses are induced through the mixing with the neutralinos (actually only one mass at tree level and the other two at one loop). The above terms break R-parity explicitly, as discussed in the Introduction, and together with the superpotential (1) they constitute the BRpV. Although this is an interesting mechanism for generating neutrino masses, notice that the $\mu$ problem is augmented with the three new bilinear terms.

3 NMSSM and extensions

The NMSSM provides an elegant solution to the $\mu$ problem of the MSSM via the introduction of a singlet superfield $\hat{S}$ under the SM gauge group. The simplest form of the superpotential,
which is scale invariant, is given by:

\[
W = \epsilon_{ab} \left( Y_{ij}^u \hat{H}_2^b \hat{Q}_i^a \hat{u}_j^c + Y_{ij}^d \hat{H}_1^b \hat{d}_j^a \hat{d}_i^c + Y_{ij}^e \hat{H}_1^b \hat{e}_j^a \hat{e}_i^c \right) - \epsilon_{ab} \lambda \hat{S} \hat{H}_1^a \hat{H}_2^b + \frac{1}{3} k \hat{S} \hat{S} \hat{S}.
\] (5)

In this model, the usual \(\mu\) term is absent from the superpotential, and only dimensionless trilinear couplings are present in \(W\). For this to happen it is usually invoked a \(Z_3\) symmetry. On the other hand, let us recall that this is actually what happens in the low-energy limit of string constructions, where all fields are massless, and, as a consequence, only trilinear couplings are present in the superpotential. Since string theory seems to be relevant for the unification of all interactions, including gravity, this argument \(^{24}\) in favour of the absence of a bare \(\mu\) term in this kind of superpotentials is robust.

When the scalar component of the superfield \(\hat{S}\), denoted by \(S\), acquires a VEV of order the SUSY breaking scale, an effective interaction \(\mu \hat{H}_1 \hat{H}_2\) is generated through the fourth term in (5), with \(\mu = \lambda \langle S \rangle\). This effective coupling is naturally of order the EW scale if the SUSY breaking scale is not too large compared with \(M_W\). In fact, the NMSSM is the simplest SUSY extension of the SM in which the EW scale exclusively originates from the SUSY breaking scale. The last term in (5) is allowed by all symmetries, and avoids, as the \(\mu\)-term in the MSSM, the presence of a Goldstone boson, in this case associated to the global \(U(1)\) symmetry \(H_1 H_2 \rightarrow e^{i\alpha} H_1 H_2, S \rightarrow e^{-i\alpha} S\).

In addition to the MSSM fields, the NMSSM contains an extra CP-even and CP-odd neutral Higgs bosons, as well as one additional neutralino. These new fields mix with the corresponding MSSM ones, giving rise to a richer and more complex phenomenology. For example, a light neutralino may be present. The mass of the lightest neutral Higgs state can be raised arbitrarily by increasing the value of the new Higgs self-coupling parameter \(\lambda\). Imposing that the coupling remains perturbative up to the Planck scale, then the upper bound is larger than in the MSSM, \(m_h \lesssim 150\) GeV. Moreover, a very light Higgs boson is not experimentally excluded. In particular, such a particle has a significant singlet composition, thus scaping detection and being in agreement with accelerator data. The latter may modify the results concerning the possible detection of neutralino dark matter with respect to those of the MSSM \(^{20,30}\).

Let us remark that the superpotential (5) has a \(Z_3\) symmetry. Therefore, one expects to have a cosmological domain wall problem \(^{31,32}\) in this model. Nevertheless, there is a solution to this problem \(^{33}\) in the superpotential can break explicitly the dangerous \(Z_3\) symmetry, lifting the degeneracy of the three original vacua, and this can be done without introducing hierarchy problems. In addition, these operators can be chosen small enough as not to alter the low-energy phenomenology. An alternative solution \(^{35,36,37}\) uses an extra \(U(1)\). Gauge invariance of \(W\) under the new \(U(1)\) forbids not only the \(\mu\)-term, but also the term \(\hat{S} \hat{S} \hat{S}\), and thus the model is free from the domain wall problem. Notice that the Goldstone boson is eaten by the extra \(Z\). The extra \(U(1)\) can also be very useful to forbid R-parity violating terms producing proton decay \(^{38,39}\). Unless some quark and lepton Yukawa couplings are forbidden at tree level \(^{37}\), cancellation of anomalies with the new \(U(1)\) requires the introduction of new fermions charged under the SM group \(^{40}\). See ref. \(^{40}\) for a brief review of other variants of this type of models.

### 3.1 Neutrino masses

Other interesting extensions of the NMSSM, which in addition can help us to explain the origin of neutrino masses, have been considered in the literature. Inspired by the BRpV, the simplest possibility consists of extending the superpotential (5) with the bilinear terms in (4) \(^{11}\).

\(^{a}\)We thank J.R. Espinosa for useful comments about this point.
On the other hand, if right-handed neutrino superfields are added, an interesting extension of the superpotential (5) can be obtained\cite{22}

$$\delta W = \epsilon_{ab}Y_{\nu}^{ij}\hat{H}_2^b\hat{L}_i^a\hat{\nu}_j^c + k^{ij}\hat{S}\hat{\nu}_i^c\hat{\nu}_j^c. \quad (6)$$

Here Majorana masses for neutrinos are generated dynamically through the VEV of the singlet $S$. This is an example of a see-saw at the EW scale. Actually, in this model the scalar components of the neutrino superfields may acquire VEVs, and therefore R-parity may be broken spontaneously through the first term in (6). A similar superpotential was proposed in ref.\cite{20} including three families of Higgses, which may be naturally obtained in the context of string constructions.

Another possibility discussed in ref.\cite{23} extends the superpotential (5) with only terms of the type $\hat{\nu}\hat{\nu}$ gauge-singlet neutrino superfields, $\hat{\nu}$. As discussed in the context of the MSSM and NMSSM, experiments may induce us to introduce terms of the type $\mu\nu$\,4, this would allow us to solve the $\mu$ problem at tree level, the authors of ref.\cite{41} proposed to add also bilinear terms in (7):

$$\delta W = \epsilon_{ab}\hat{\mu}\hat{H}_2^b\hat{L}_i^a S + \epsilon_{ab} \hat{\mu}\hat{H}_2^b\hat{\nu}_i^c. \quad (7)$$

In this model R-parity is explicitly broken, and neutrino masses arise similarly to the case of BRpV (notice that three effective bilinear terms are generated through the VEV of $S$). Therefore, only one mass arises at tree level, and the other two at one loop. To generate all masses at tree level, the authors of ref.\cite{31} proposed to add also bilinear terms in (7):

$$\delta W = \epsilon_{ab}\hat{\mu}\hat{H}_2^b\hat{L}_i^a S + \epsilon_{ab} \hat{\mu}\hat{H}_2^b\hat{L}_i^a. \quad (8)$$

4 $\mu\nu$SSM

As discussed in the context of the MSSM and NMSSM, experiments may induce us to introduce gauge-singlet neutrino superfields, $\hat{\nu}_i^c$. Now, given the fact that sneutrinos are allowed to get VEVs, we may wonder why not to use terms of the type $\hat{\nu}\hat{H}_1\hat{H}_2$ to produce an effective $\mu$ term. This would allow us to solve the $\mu$ problem of the MSSM, without having to introduce an extra singlet superfield as in case of the NMSSM. Thus the aim of what follows is to analyse the “$\mu$ from $\nu$” Supersymmetric Standard Model ($\mu\nu$SSM) arising from this proposal\cite{21,22}, natural particle content without $\mu$ problem.

In addition to the MSSM Yukawa couplings for quarks and charged leptons, the $\mu\nu$SSM superpotential contains Yukawa couplings for neutrinos, and two additional type of terms involving the Higgs doublet superfields, $\hat{H}_1$ and $\hat{H}_2$, and the three neutrino superfields, $\hat{\nu}_i^c$,

$$W = \epsilon_{ab} \left( Y_{\nu}^{ij}\hat{H}_2^b\hat{Q}_i^a\hat{\nu}_j^c + Y_{\nu}^{ij}\hat{H}_2^b\hat{\nu}_i^c\hat{\nu}_j^c + Y_{\nu}^{ij}\hat{H}_2^b\hat{\nu}_i^c\hat{\nu}_j^c + Y_{\nu}^{ij}\hat{H}_2^b\hat{\nu}_i^c\hat{\nu}_j^c \right)$$

$$- \epsilon_{ab} \lambda^i\hat{\nu}_i^c\hat{H}_2^a\hat{H}_2^b + \frac{1}{\sqrt{3}} \epsilon_{ijk}\hat{\nu}_i^c\hat{\nu}_j^c\hat{\nu}_k^c. \quad (9)$$

In this model, the usual MSSM bilinear $\mu$-term is absent from the superpotential, and only dimensionless trilinear couplings are present in $W$. As argued in the previous section, this is a natural situation in string constructions. When the scalar components of the superfields $\hat{\nu}_i^c$, denoted by $\hat{\nu}_i^c$, acquire VEVs of order the EW scale, an effective interaction $\mu\hat{H}_1\hat{H}_2$ is generated through the fifth term in (9), with $\mu \equiv \lambda^i(\hat{\nu}_i^c)$. The last type of terms in (9) is allowed by all symmetries, and avoids the presence of a Goldstone boson associated to a global $U(1)$ symmetry, similarly to the case of the NMSSM. In addition, it contributes to generate effective Majorana masses for neutrinos at the EW scale. These two type of terms replace the two NMSSM terms $\hat{S}\hat{H}_1\hat{H}_2$ and $\hat{S}\hat{S}\hat{S}$.

It is worth noticing that these terms break explicitly lepton number, and therefore, after spontaneous symmetry breaking, a massless Goldstone boson (Majoron) does not appear. On the other hand, R-parity is also explicitly broken and this means that the phenomenology of the $\mu\nu$SSM is going to be very different from the one of the MSSM. It is also interesting to realise that the Yukawa couplings producing Dirac masses for neutrinos, the fourth term in (9), generate through the VEVs of $\hat{\nu}_i^c$, three effective bilinear terms $\hat{H}_2\hat{L}_i^a$. As mentioned above these
characterize the BRpV. Let us finally mention that the terms $\nu^c H_1 H_2$ and $\nu^c \nu^c \nu^c$ have also been analysed as sources of the observed baryon asymmetry in the Universe, and of neutrino masses and bilarge mixing, respectively.

Notice that the superpotential has a $Z_3$ symmetry, just like the NMSSM. Therefore, one expects to have also a cosmological domain wall problem in this model. Nevertheless, the usual solutions to this problem discussed for the NMSSM above, will also work in this case.

Working in the framework of SUGRA, we will discuss now in more detail the phenomenology of the $\mu\nu$SSM. Let us write first the soft terms appearing in the Lagrangian, $\mathcal{L}_{soft}$, after SUSY breaking, which in our conventions is given by

$$-\mathcal{L}_{soft} = (m_Q^2)^{ij} \tilde{Q}_i^a \tilde{Q}_j^a + (m_{W}^2)^{ij} \tilde{\nu}_i^c \tilde{\nu}_j^c + (m_{\tilde{\nu}_e}^2)^{ij} \tilde{\nu}_i^{c*} \tilde{\nu}_j^{c*} + (m_{H_d}^2)^{ij} \tilde{L}_i^a \tilde{L}_j^a + (m_{\tilde{\nu}}^2)^{ij} \tilde{\nu}_i^c \tilde{\nu}_j^c$$

$$+ m_{H_u}^2 H_1^i H_1^j + m_{H_2}^2 H_2^i H_2^j + (m_{\nu}^2)^{ij} \tilde{\nu}_i^{c*} \tilde{\nu}_j^c + \epsilon_{ab} \left[ (A_d Y_u)^{ij} H_1^b \tilde{Q}_i^a \tilde{\nu}_j^c + (A_d Y_d)^{ij} H_1^b \tilde{Q}_i^a \tilde{\nu}_j^c + (A_e Y_e)^{ij} H_1^b \tilde{L}_i^a \tilde{\nu}_j^c \right]$$

$$+ \epsilon_{ab} \left[ (A_d Y_u)^{ij} H_1^b \tilde{Q}_i^a \tilde{\nu}_j^c + (A_d Y_d)^{ij} H_1^b \tilde{Q}_i^a \tilde{\nu}_j^c + (A_e Y_e)^{ij} H_1^b \tilde{L}_i^a \tilde{\nu}_j^c \right]$$

$$- \frac{1}{2} \left( M_3 \tilde{\lambda}_3 \tilde{\lambda}_3 + M_2 \tilde{\lambda}_2 \tilde{\lambda}_2 + M_1 \tilde{\lambda}_1 \tilde{\lambda}_1 + H.c. \right).$$

(10)

In addition to terms from $\mathcal{L}_{soft}$, the tree-level scalar potential receives the usual $D$ and $F$ term contributions. Once the EW symmetry is spontaneously broken, the neutral scalars develop in general the following VEVs:

$$\langle H_1^0 \rangle = v_1, \quad \langle H_2^0 \rangle = v_2, \quad \langle \tilde{\nu}_1 \rangle = \nu_1, \quad \langle \tilde{\nu}_2^c \rangle = \nu_2^c.$$

(11)

In what follows it will be enough for our purposes to neglect mixing between generations in (9) and (10), and to assume that only one generation of sneutrinos gets VEVs, $\nu, \nu^c$. The extension of the analysis to all generations is straightforward, and the conclusions are similar. We then obtain for the tree-level neutral scalar potential:

$$\langle V_{\text{neutral}} \rangle = \frac{g_1^2 + g_2^2}{8} \left( |\nu|^2 + |v_1|^2 - |v_2|^2 \right)^2$$

$$+ |\lambda|^2 \left( |\nu^c|^2 |v_1|^2 + |\nu^c|^2 |v_2|^2 + |v_1|^2 |v_2|^2 \right) + |\kappa|^2 |\nu^c|^4$$

$$+ |Y_\nu|^2 \left( |\nu^c|^2 |v_1|^2 + |\nu^c|^2 |v_2|^2 + |\nu^c|^2 |v_2|^2 \right)$$

$$+ m_{H_1}^2 |v_1|^2 + m_{H_2}^2 |v_2|^2 + m_{\tilde{\nu}}^2 |\nu^c|^2 + m_{\nu}^2 |\nu^c|^2$$

$$+ \left( -\lambda \kappa v_1 v_2 \nu^c \right) - \lambda \nu^c v_1 v_2 \nu^c - \lambda \nu^c v_1 v_2 \nu^c + k \nu^c v_1 v_2 \nu^c$$

$$+ \left( -\frac{1}{3} \lambda \kappa A_{ \nu^c} v_1 v_2 + Y_\nu A_{ \nu^c} v_1 v_2 + 1 + \lambda A_{ \nu^c} v_1 v_2 + H.c. \right).$$

(12)

In the following, we assume for simplicity that all parameters in the potential are real. One can derive the four minimization conditions with respect to the VEVs $v_1, v_2, \nu^c, \nu$. With the result

$$\frac{1}{4} (g_1^2 + g_2^2) (\nu^2 + v_1^2 - v_2^2) v_1 + \lambda^2 v_1 \left( \nu^2 + v_2^2 \right) + m_{H_1}^2 v_1 - \lambda \nu^c v_2 (\kappa \nu^c + A_{\lambda})$$

$$- \lambda Y_\nu \nu \left( \nu^2 + v_2^2 \right) = 0,$n

$$- \frac{1}{4} (g_1^2 + g_2^2) (\nu^2 + v_1^2 - v_2^2) v_2 + \lambda^2 v_2 \left( \nu^2 + v_1^2 \right) + m_{H_2}^2 v_2 - \lambda \nu^c v_1 (\kappa \nu^c + A_{\lambda})$$

$$+ Y_\nu v_2 \left( \nu^2 + v_2^2 \right) + Y_\nu \nu \left( -2 \lambda v_1 v_2 + \kappa \nu^c + A_{\nu^c} \right) = 0,$n

$$\lambda \left( v_1^2 + v_2^2 \right) \nu^c + 2 \kappa \nu^c v_1 v_2 + m_{\nu^c}^2 \nu^c - 2 \lambda \nu^c v_1 v_2 + \kappa A_{\nu^c} \nu^c$$

$$- \lambda A_{\nu^c} v_1 v_2 + H.c. = 0.$$
the contributions \(\nu \propto \nu^c\) proportional to \(\nu\), we can also get an estimate of the value, \(m\), Dirac mass for the neutrinos, \(m_D \equiv Y_{\nu} v_2\), \(\nu\) has to be very small. Using this rough argument we can also get an estimate of the value, \(\nu \lesssim m_D\). This also implies that, neglecting terms proportional to \(Y_{\nu}\), we can approximate the other three equations as the ones defining the minimization conditions for the NMSSM, with the substitution \(\nu^c \leftrightarrow s\). Thus one can carry out the analysis of the model similarly to the NMSSM case, where many solutions in the parameter space \(\lambda, \kappa, \mu(\equiv \lambda s)\), \(\tan \beta, A_{\lambda}, A_{\kappa}\), can be found (see e.g. ref \ref{20} and references therein).

Once we know that solutions are available in this model, we have to discuss in some detail the important issue of mass matrices. Concerning this point, the breaking of \(R\)-parity makes the \(\mu\nu\)SSM very different from MSSM and NMSSM. In particular, neutral gauginos and Higgsinos are now mixed with the neutrinos. Not only the fermionic component of \(\tilde{\nu}\) mixes with the neutral Higgsinos (similarly to the fermionic component of \(\tilde{S}\) in the NMSSM), but also the fermionic component of \(\tilde{\nu}\) enters in the game, giving rise to a sixth state. Of course, now we have to be sure that one eigenvalue of this matrix is very small, reproducing the experimental results about neutrino masses.

The neutral fermion mass matrix is

\[
\mathcal{M}_n = \begin{pmatrix} M & m \\ m^T & 0 \end{pmatrix},
\]

(14)

where

\[
M = \begin{pmatrix}
M_1 & 0 & -M_2 \sin \theta_W \cos \beta & M_2 \sin \theta_W \sin \beta & 0 \\
0 & M_2 \cos \theta_W \cos \beta & -M_2 \cos \theta_W \sin \beta & -\lambda \nu^c & 0 \\
-M_2 \sin \theta_W \cos \beta & M_2 \cos \theta_W \cos \beta & 0 & -\lambda \nu^c & -\lambda v_2 \\
M_2 \sin \theta_W \sin \beta & -M_2 \cos \theta_W \sin \beta & -\lambda v_2 & -\lambda v_1 + Y_{\nu} v_2 & 2 \kappa \nu^c \\
0 & 0 & -\lambda v_2 & -\lambda v_1 + Y_{\nu} v_2 & 2 \kappa \nu^c
\end{pmatrix},
\]

(15)

is very similar to the neutralino mass matrix of the NMSSM (substituting \(\nu^c \leftrightarrow s\) and neglecting the contributions \(Y_{\nu} \nu\)), and

\[
m^T = \begin{pmatrix}
-\frac{g_1 \nu}{\sqrt{2}} & \frac{g_2 \nu}{\sqrt{2}} & 0 & Y_{\nu} \nu^c & Y_{\nu} v_2
\end{pmatrix}.
\]

(16)

Matrix (14) is a matrix of the see-saw type that will give rise to a very light eigenvalue if the entries of the matrix \(M\) are much larger than the entries of the matrix \(m\). This is generically the case since the entries of \(M\) are of order the EW scale, but for the entries of \(m, \nu\) is small and \(Y_{\nu} v_2\) is the Dirac mass for the neutrinos \(m_D\) as discussed above \((Y_{\nu} \nu^c\) has the same order of magnitude of \(m_D\)). We have checked numerically that correct neutrino masses can easily be obtained. For example, using typical EW-scale values in (15), and a Dirac mass of order \(10^{-4}\) GeV in (16), one obtains that the lightest eigenvalue of (14) is of order \(10^{-2}\) eV. Including the three generations in the analysis we can obtain different neutrino mass hierarchies playing with the hierarchies in the Dirac masses. It is worth noticing here that because of the matrix (15), the presence of the last type of terms in (9) is not essential to generate Majorana masses for the neutrinos.

On the other hand, the charginos mix with the charged leptons. One can check that there will always be a light eigenvalue corresponding to the electron mass \(Y_e v_1\). The extension of the analysis to three generations is again straightforward.
Of course, other mass matrices are also modified. This is the case for example of the Higgs boson mass matrices. The presence of the VEVs \( \nu, \nu^c \), leads to mixing of the neutral Higgses with the sneutrinos. Likewise the charged Higgses will be mixed with the charged sleptons. On the other hand, when compared to the MSSM case, the structure of squark mass terms is essentially unaffected, provided that one uses \( \mu = \lambda \nu^c \), and neglects the contribution of the fourth term in (9).

5 Conclusions

We are all very lucky to live in a historic moment for particle physics and science in general, the moment when the LHC is switched on. This huge machine, under construction for more than eight years, will finally start operations at the end of this year. The LHC will be able to answer not only a crucial question such as the origin of the mass, but also to clarify whether or not a new symmetry in Nature with spectacular experimental implications exists. Of course, if the Higgs is finally found, this will be a great success for everybody, but let us be more ambitious and hope that also one of the SUSY models described here will be confirmed.

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