A method for detecting weak signals using chaotic systems based on multiple correlation and phase-locked loop

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Abstract. In order to improve the detection ability of chaotic system to detect weak periodic signal frequency, phase and amplitude, a chaotic system based on multiple correlation and phase-locked loop is established. Firstly, the signal to be measured is input into the detection frequency of chaotic system. Also, the reference signal of multiple correlation technology is constructed, and the signal to be tested is processed by multiple cross-correlation. Then, the signal processed by multiple correlation is input into the phase-locked loop to obtain the phase of the signal to be measured. Finally, the signal to be measured is input into the chaotic system again, and the amplitude of the signal to be measured is detected according to the change of the chaotic oscillator in critical periodic state and large-scale periodic state. The experimental results show that the detection system can detect the frequency, phase and amplitude of weak periodic signal, which is easy to operate and has high detection ability.

1. Introduction

Weak signal detection technology is a technology that applies physics, computer, information theory, etc., to study the causes and characteristics of noise and signal to be measured, and then extract useful information under strong background noise. With the development of engineering application, the technology plays a paramount role in the field of measurement technology. At present, weak signal detection technology is widely used in radar, communication, biomedicine, vibration measurement, system identification, fault diagnosis, etc. It is an important technical foundation for the development of high-tech and related cutting-edge fields. The research of weak signal detection technology with high detection ability is of great practical significance to promote scientific and technological progress and production development [1-2].

Weak signal detection technology based on chaotic system is an important branch of chaos theory. Compared with other traditional linear and deterministic weak signal detection techniques, chaotic systems have better sensitivity to periodic signals and immunity to strong noises. Based on the frequency and amplitude of the built-in policy force in chaotic system, the phase state change rule of the chaotic oscillator can be judged, and the frequency and amplitude of weak periodic signal can be extracted under strong noise [2]. However, the trisection symmetric phase detection method is usually used for the phase detection of the periodic signal to be measured. Although the phase information can be effectively extracted, the workload is heavy, and the efficiency and accuracy need to be improved.
With the development of weak signal detection technology, phase-locked loop technology has been developed rapidly, and it has unique advantages in measuring the phase of the signal to be measured [4]. Therefore, the hybrid detection system based on phase-locked loop and chaotic detection technology can detect the frequency, amplitude and phase of weak periodic signal at the same time, and the operation is simple and easy [3]. However, the detection ability of phase-locked loop is far lower than that of chaotic system, which limits the detection ability of the whole hybrid detection system. At present, cross-correlation technology is a traditional signal detection technology. According to the different characteristics of correlation between noise and signal to be measured, the purpose of noise suppression is achieved. Therefore, in order to detect the frequency, phase and amplitude of weak periodic signal in strong noise background, a chaotic system based on multiple correlation and phase-locked loop is proposed to detect weak periodic signal.

2. Chaotic system based on multiple correlation and phase-locked loop

2.1. Multiple correlation technology

In the field of signal processing, cross-correlation method is a common time-domain processing method. When the frequency of the signal to be measured is known, the reference signal with the same frequency can be used for correlation processing with the signal to be tested in the noise background. According to the different characteristics of correlation between the noise and the signal to be measured, the purpose of noise suppression can be realized [4]. The signal to be measured with the known frequency is:

\[ f_1(t) = s_1(t) + n(t) \]  

(1)

\[ f_2(t) = s_2(t) \]

is set to local signal. Among them, \( n(t) \) is noise signal, and \( s_1(t) \) and \( s_2(t) \) are periodic signals. The correlation between \( f_1(t) \) and \( f_2(t) \) can be obtained as follows:

\[
R_{12}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t)f_2(t-\tau)dt
\]

\[
= \lim_{T \to \infty} \left[ \frac{1}{2T} \int_{-T}^{T} s_1(t)s_2(t-\tau)dt + \frac{1}{2T} \int_{-T}^{T} n(t)s_2(t-\tau)dt \right]
\]

(2)

In equation (2), \( R_{S_1S_2}(\tau) \) is the cross-correlation function of \( s_1(t) \) and \( s_2(t) \), and \( R_{nS_2}(\tau) \) is the cross-correlation function of \( n(t) \) and \( s_2(t) \). Ideally, if the mean value of noise \( n(t) \) or \( s_2(t) \) is zero, then \( R_{nS_2}(\tau) \) is zero. However, the whiteness of noise is not always ideal, which leads to the fact that \( R_{nS_2}(\tau) \) is not equal to zero. If the local signal \( s_2(t) \) is related to the signal \( s_1(t) \) to be measured, then \( R_{12}(\tau) = R_{S_1S_2}(\tau) \). Theoretically, when the integration time \( T \) is always long enough, \( R_{nS_2}(\tau) \) is zero. In fact, \( T \) is finite and \( R_{nS_2}(\tau) \) is not zero. Therefore, the output signal \( f_1'(t) \) still has residual noise after a correlation processing, which can be expressed as

\[ f_1'(t) = s_1(t) + n'(t) \]

Among them, \( n'(t) \) is the residual noise after a correlation processing. In order to minimize the residual noise, this work uses multiple correlation technology to process the measured signal (i.e., \( f_1'(t) \) is used to replace \( f_1(t) \) in equation (2) for multiple correlation processing). With the increase of correlation processing times, the residual noise is further reduced.

2.2. Phase-locked loop detection method

Phase-locked loop is composed of voltage-controlled oscillator (VCO), phase detector and loop filter. It is a phase feedback tracking system that can realize periodic signal. When phase-locked loop is
locked, phase parameters of periodic signal to be detected can be extracted [4]. The specific process of detecting signal phase is as follows. After the phase error signal is output by the phase detector, it is input to the loop filter. After being output by the loop filter, it is used as the control signal of VCO, and then fed back to the phase detector from the VCO. In the phase detector, the phase is compared with the input signal. When the frequency of VCO output signal is equal to the frequency of input signal, the phase-locked loop is in locked state. The phase of VCO output signal will track the phase change of input signal to realize signal phase detection.

Based on the analysis of linear system theory, $\phi(t)$ is taken as the input signal of the system, and the phase signal $\hat{\phi}(t)$ of VCO is taken as the output of the system. Then, the transfer function of phase-locked loop system can be expressed as:

$$H(s) = \frac{\hat{\phi}}{\phi} = \frac{G(s)K/s}{1+G(s)K/s}$$ (3)

Where $K$ is the loop gain, and the unit is $(rad/s)/V$. $G(s)$ is the transfer function of the loop filter. When $G(s) = 1$, it can be expressed as a first-order low-pass filter. At the same time, $H(s) = \frac{K/s}{1+k/s}$ is first-order.

In this work, the first-order phase-locked loop is used to detect the phase of the signal to be measured. Then, the specific method of first-order phase-locked loop system to detect the phase of the periodic signal to be measured is as follows. The center frequency $f_c$ of VCO is adjusted to be equal to the frequency of the signal to be measured, so that the phase-locked loop is in the locked state. Then, the phase of the signal to be measured is equal to the phase of the VCO control signal output by the phase detector. When the phase of the periodic signal to be measured is set to zero, it can be seen from figure 1 that the phases of the signal to be measured and the VCO control signal output by the phase detector are both zero, and the period is T. When the phase of the signal to be measured is $\pi/6$, the first unstable waveform of the signal is discarded in the VCO control signal output by the phase detector. In the second cycle (i.e., at time T), the signal waveform can be observed and its phase is equivalent to that of the periodic signal to be measured, both of which are $\pi/6$. However, it can be seen from figure 3 that the VCO control signal output by the phase detector fails to determine the phase when there is noise in the signal to be measured. At this time, the noise in the VCO control signal can be filtered out by using the band-pass filter, thus improving the detection ability of phase-locked loop to detect the phase of the signal to be measured. As shown in figure 4, the phase detection of the signal to be measured is realized.

**Figure 1.** The phase of the signal is 0

**Figure 2.** The phase of the signal is $\pi/6
2.3. Signal parameter detection method based on chaotic oscillator

In the field of signal processing, the Duffing oscillator is a common chaotic system model [5-6]. Duffing equation is expressed as follows:

\[
\begin{align*}
    x' &= \omega y \\
    y' &= \omega(-ky + x^3 - x^5 + \gamma \cos(\omega t))
\end{align*}
\]

(4)

Where \(\gamma \cos(\omega t)\) is the built-in force, and \(k\) is the damping ratio. \(x^3 - x^5\) is the nonlinear restoring force. The dynamic behavior of the chaotic system can be observed by adjusting the amplitude of the built-in force. When \(\gamma\) is small, the phase trajectories of chaotic oscillators are attractors in the sense of Poincare mapping, and the phase points oscillate periodically around the focus. When the value of \(\gamma\) is gradually increased, the system experiences homoclinic orbit, periodic doubling bifurcation, chaotic state, critical periodic state and large-scale periodic state. Then, the periodic signal can be detected based on the different state changes of the system phase diagram. When the frequency of the signal to be measured is equal to the frequency of the built-in driving force, the change of built-in driving force amplitude \(\gamma\) will lead to the change of Duffing oscillator phase state according to the detection requirements of the periodic signal to be measured. There is always a threshold \(\gamma_d\), and the Duffing oscillator system enters the critical periodic state when \(\gamma = \gamma_d\). When \(\gamma\) continues to increase, the system enters the large-scale periodic state [7].

For the frequency detection of the signal to be measured, the gain \(k\) is set for \(f(t)\) when the signal \(f(t)\) in equation (1) is taken as the built-in driving force of Duffing oscillator system. By adjusting the value of \(k\), the oscillator can enter the large-scale periodic state. At this time, the period of the oscillator is equal to the period of the periodic signal (i.e., the frequency of the signal to be measured is obtained). In order to detect the amplitude of the signal to be measured, a built-in driving force with the same frequency as the signal to be measured is constructed. When the sum of periodic signal amplitude and built-in driving force amplitude is greater than the threshold value \(\gamma_d\), the chaotic oscillator will enter a large-scale periodic state. Finally, the amplitude of the built-in driving force is adjusted again to make the chaotic oscillator return to the critical period state. At this time, another threshold \(\gamma_z\) is obtained. Consequently, the difference between \(\gamma_d\) and \(\gamma_z\) is the amplitude of the periodic signal to be measured.

2.4. A method for detecting weak signals in chaotic systems based on multiple correlation and phase-locked loop

Firstly, the signal to be measured is used as the built-in driving force of chaotic system to make the chaotic oscillator enter the critical period state and obtain the frequency of the signal to be measured. Secondly, the reference signal of the multiple correlation technology is constructed based on the frequency of the signal to be measured in step 1, and the signal to be measured is processed by
multiple correlation to suppress part of the noise in the signal to be measured. Then, the signal to be measured after multiple correlation processing is input into the phase-locked loop, and the center frequency of VCO is adjusted to make it equal to the measured signal frequency. Consequently, the phase-locked loop is locked and the phase of the signal to be measured is obtained. Finally, the frequency and phase of the signal to be measured are measured respectively to construct the built-in driving force of chaotic system according to step 1 and step 3. Based on the phase change of chaotic oscillator, the amplitude of the signal to be measured is obtained. The model block diagram is shown in figure 5.

![Model block diagram](image)

Figure 5. Model block diagram

3. System simulation and analysis

In Simulink environment, a chaotic system based on multiple correlation and phase-locked loop is established according to the model block diagram. Among them, the damping ratio \( k \) in Duffing equation is 1. The simulation time is 500s, and the step size is 0.001s. In this work, the proposed chaotic system based on multiple correlation and phase-locked loop is validated by using the Nan voltmeter level sinusoidal signal \( m(t) = m \cos(\omega t) \) mixed with Gaussian white noise \( n(t) \). Where \( m = 10^{-10} \) and \( \omega = 1 \text{rad/s} \), and the noise power of \( n(t) \) is set to \( 1.5 \times 10^{-15} \text{W} \). Combined with a large number of observation experiments, the threshold \( \gamma_d \) of chaotic system can be obtained by Melnikov method, which is 0.7195966111.

Frequency detection of the signal to be measured: the gain of the signal to be measured is set to \( g \), and the value of \( g \) is adjusted. When \( g = 7195966111 \), the chaotic system is stable in the large-scale periodic state. The calculated time of phase point running for one week is \( T = 6.2823676472 \text{s} \), and thus the measured frequency is \( 1.0001301 \text{rad/s} \). The relative error is 0.013%.

Phase detection of the signal to be measured: the multiple correlation local signals \( s(t) = \sin(1.0001301t) \) is constructed based on the frequency of the measured signal. The signal to be measured and the signal to be measured after five times of correlation processing are shown in figure 6 and figure 7, respectively. By comparing figure 6 and figure 7, it can be seen that the noise in the signal to be measured is greatly suppressed after multiple correlation processing. The signal to be measured is input into the phase-locked loop after multiple correlation processing, and the phase-locked loop is locked by the frequency of the measured signal. The phase of the signal to be measured is determined to be 0 according to the VCO control signal after locking.

Detection of signal amplitude to be measured: based on the measured frequency and phase, the built-in driving force which is equal to the frequency and phase of the signal to be measured is constructed, and the amplitude \( \gamma \) of the built-in driving force is adjusted. When \( \gamma = \gamma_d = 0.7195966111 \), the system enters a critical state. When the signal \( m(t) \) to be measured is added, the chaotic system jumps to a large-scale periodic state, as shown in figure 8. When \( \gamma = \gamma_d = 0.7195966112 \), the system jumps to a critical state, as shown in figure 9. Then, the amplitude of signal \( m(t) \) to be measured is \( \gamma_x - \gamma_d = 0.0000000001 \).
Through the detection of the signal to be measured, the measured amplitude is $m = 1.0 \times 10^{-10} \text{v}$. If the noise power is increased to $1.8 \times 10^{-15} \text{W}$, the chaotic system cannot identify the system state. Therefore, the signal-to-noise ratio of the system can be detected as:

$$SNR = 10 \log \frac{\text{Signal power to be measured}}{\text{Noise power}} = 10 \log \frac{1 \times (10^{-10})^2}{1.8 \times 10^{-15}} = -55.563 \text{dB}$$

4. Conclusion

Based on multiple correlation and phase-locked loop, the frequency of the signal to be measured can be obtained by using the chaotic system. According to the frequency of the signal to be measured, the local signal with multiple correlation is established, and the noise in the signal to be measured is suppressed. The signal processed by multiple correlation is input to the phase-locked loop to obtain the phase of the signal to be measured. Finally, the built-in driving force of the chaotic system is constructed based on the frequency and phase of the measured signal, and the amplitude detection of the signal to be measured is realized. The system can simultaneously detect the frequency, phase and amplitude of the Nan voltmeter level periodic signal, and has high detection ability and is easy to operate. However, although the chaotic system has strong immunity to noise, its system characteristics cannot effectively distinguish multiple periodic signals when the signal to be measured is mixed with multiple weak periodic signals. This will lead to the failure of chaotic system detection. Therefore, the next step is to realize the detection of multiple periodic signals in strong noise background.

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