RECONSTRUCTION OF BASIC PARAMETERS FROM CHARGINO PRODUCTION

S.Y. CHOI\textsuperscript{1} and J. KALINOWSKI\textsuperscript{2}
\textsuperscript{1} Korea Institute for Advanced Study, Seoul 130-012, Korea
\textsuperscript{2} Instytut Fizyki Teoretycznej UW, Hoża 69, 00681 Warsaw, Poland

The event characteristics of chargino pair production at $e^+e^-$ collisions are explored to determine gaugino-higgsino mixing angles. We demonstrate that by measuring total cross sections and left-right asymmetries with polarized beams or angular correlations among chargino decay products the fundamental SUSY parameters $M_2$, $\mu$ and $\tan\beta$ can be uniquely determined in a model independent way.

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1 Introduction

The low-energy supersymmetric model (SUSY) involves a large number of arbitrary parameters reflecting our ignorance of the supersymmetry breaking mechanisms. Once the supersymmetric particles are discovered, the priority will be to measure the low-energy SUSY parameters independently of theoretical assumptions and confront them with relations following from e.g. grand unification theories. A clear strategy is needed to deal with so many a priori arbitrary parameters.

In many scenarios charginos, mixtures of spin 1/2 partners of the $W$ and charged Higgs bosons, $\tilde{W}^\pm$ and $\tilde{H}^\pm$, are among the lightest supersymmetric particles. The chargino sector depends on only three soft SUSY breaking parameters: the SU(2) gaugino mass $M_2$, the higgsino mass parameter $\mu$, and the ratio $\tan \beta (= v_2/v_1)$ of the vacuum expectation values of the two neutral Higgs fields which enter the chargino mass matrix ($s_\beta = \sin \beta, c_\beta = \cos \beta$)

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W c_\beta \\ \sqrt{2} m_W s_\beta & \mu \end{pmatrix}$$

written in the ($\tilde{W}, \tilde{H}$) basis. Therefore the chargino production processes

$$e^+ e^- \rightarrow \tilde{\chi}_i^- \tilde{\chi}_j^+ \quad [i, j = 1, 2]$$

may serve as a good starting point towards a systematic and model–independent determination of the fundamental SUSY parameters.

2 Exploiting chargino production processes

Since the mass matrix is asymmetric, two different mixing matrices acting on the left– and right–chiral ($\tilde{W}, \tilde{H}$) states are needed to diagonalize it. The two eigenvalues are given by

$$m_{\chi_{1,2}^\pm}^2 = \frac{1}{2} [M_2^2 + \mu^2 + 2m_W^2 \mp \Delta]$$

and the left– and right–chiral components of the light mass eigenstate $\tilde{\chi}_1^-$ are related to the wino and higgsino components in the following way

$$\tilde{\chi}_1^- = \tilde{W}_L^- \cos \phi_L + \tilde{H}_1^- \sin \phi_L, \quad \tilde{\chi}_1^- = \tilde{W}_R^- \cos \phi_R + \tilde{H}_2^- \sin \phi_R,$$

$$\cos 2\phi_L = - (M_2^2 - \mu^2 - 2m_W^2 \cos 2\beta)/\Delta, \quad \sin 2\phi_L = - 2\sqrt{2} m_W (M_2 \cos \beta + \mu \sin \beta)/\Delta$$

$$\cos 2\phi_R = - (M_2^2 - \mu^2 + 2m_W^2 \cos 2\beta)/\Delta, \quad \sin 2\phi_R = - 2\sqrt{2} m_W (M_2 \sin \beta + \mu \cos \beta)/\Delta$$

$$\Delta = [(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2 \mu - m_W^2 \sin 2\beta)^2]^{1/2}$$

2.1 Production of light charginos

With only light charginos $\tilde{\chi}_1^\pm$ accessible kinematically, the parameters $M_2, \mu$ and $\tan \beta$ can be determined up to at most a two-fold discrete ambiguity. To this end measurements of the chargino mass, the total production cross section and angular correlations among the chargino decay products are necessary. The angular correlations depend on the polarizations of the produced charginos and thus on the gaugino–higgsino mixing angles $\phi_L$ and $\phi_R$. Beam polarization is helpful but not necessarily required.

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6In CP–noninvariant theories, $M_2$ and $\mu$ can be complex. However, by reparametrization of the fields, $M_2$ can be assumed real and positive without loss of generality so that only $\mu = |\mu| \exp i \Phi_\mu$ may have a non–trivial invariant phase $\Phi_\mu$. Here we will consider a CP–invariant scenario.
Since charginos decay through a $W$ boson or scalar partners of leptons or quarks, the decay matrix depends on additional parameters, like scalar masses and couplings to neutralinos. The presence of two invisible neutralinos in the process $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 (f_1 \bar{f}_2) (f_3 \bar{f}_4)$ makes it impossible to measure directly the chargino production angle $\Theta$. Integrating over this angle and also over the invariant masses of the fermionic systems $(f_1 \bar{f}_2)$ and $(f_3 \bar{f}_4)$, the fully correlated angular distribution $\Sigma(\theta^*, \phi^*, \theta^R, \phi^R)$ can be expressed in terms of sixteen independent angular combinations of helicity production amplitudes. The $(\theta^*, \phi^*)$ are the polar and azimuthal angles of the $(f_1 \bar{f}_2)$ system in the $\tilde{\chi}_1^-$ rest frame with respect to the chargino’s flight direction in the lab frame; quantities with a bar refer to the $\tilde{\chi}_1^+$ decay. Out of the sixteen terms, corresponding to the unpolarized, $2 \times 3$ polarization components and $3 \times 3$ spin–spin correlations, only 7 are independent if small effects from the $Z$-boson width and loop corrections are neglected.

The crucial observation of is that four terms: $\Sigma_{un}, \mathcal{P}, \mathcal{Q}$ and $\mathcal{Y}$, can be measured directly by means of simple kinematical projections since $\cos \theta^*, \cos \theta^R$ and $\sin \theta^* \sin \theta^R \cos (\phi^* + \phi^R)$ are simple functions of energies and momenta of the decay systems $(f_i \bar{f}_j)$ in the lab frame. Moreover, three physical observables, $\Sigma_{un}, \mathcal{P}^2 / \mathcal{Q}$ and $\mathcal{P}^2 / \mathcal{Y}$ by construction are independent of the details of the chargino decay dynamics and of the structure of (potentially more complex) neutralino and sfermion sectors. The measurements of $\sigma_{tot}$ and either of the ratios $\mathcal{P}^2 / \mathcal{Q}$ or $\mathcal{P}^2 / \mathcal{Y}$ therefore allow us to determine $\cos 2\phi_L$ and $\cos 2\phi_R$. If polarized beams are available, the left-right asymmetry $A_{LR}$ can provide an alternative way to extract these quantities (or serve as a consistency check). From the “measured” values of $\{\cos 2\phi_L, \cos 2\phi_R\}$ and the chargino mass, the Lagrangian parameters $M_2, \mu$ and $\tan \beta$ can be obtained up to a two-fold ambiguity.

### 2.2 Above the heavy chargino threshold

It has been recently demonstrated that if the collider energy is sufficient to produce the light and heavy chargino states in pairs, the underlying fundamental SUSY parameters, $M_2, \mu$ and $\tan \beta$, can be extracted unambiguously from chargino masses, production cross sections and left-right asymmetries with polarized electron beams. The new ingredient is the heavier chargino mass which, like for the lighter chargino, can be determined very precisely from the sharp rise of the cross sections $\sigma(e^+ e^- \rightarrow \tilde{\chi}_i^- \tilde{\chi}_j^+)$). The value of $\tan \beta$ is uniquely determined in terms of the mass difference of two chargino states, $\Delta = m_{\tilde{\chi}_2^\pm}^2 - m_{\tilde{\chi}_1^\pm}^2$, and two mixing angles as follows

$$\tan \beta = [(4m_W^2 + \Delta (\cos 2\phi_R - \cos 2\phi_L))/(4m_W^2 - \Delta (\cos 2\phi_R - \cos 2\phi_L))]^{1/2}$$

and $M_2, |\mu|$ and $\text{sign}(\mu)$ are given by

$$M_2 = \frac{1}{2} [2(m_{\tilde{\chi}_2^\pm}^2 + m_{\tilde{\chi}_1^\pm}^2 - 2m_W^2) - \Delta (\cos 2\phi_R + \cos 2\phi_L)]^{1/2}$$

$$|\mu| = \frac{1}{2} [2(m_{\tilde{\chi}_2^\pm}^2 + m_{\tilde{\chi}_1^\pm}^2 - 2m_W^2) + \Delta (\cos 2\phi_R + \cos 2\phi_L)]^{1/2}$$

$$\text{sign}(\mu) = (\Delta^2 - (M_2^2 - \mu^2)^2 - 4m_W^2(M_2^2 + \mu^2) - 4m_W^4 \cos^2 2\beta)/8m_W^2 M_2 |\mu| \sin 2\beta$$

### 2.3 Expected errors

An important question is what precision one might expect in the above procedure due to the propagation of experimental errors. It turns out that $M_2$ and $|\mu|$ can be determined very well from eqs. (6). However for large $\tan \beta$ case, the eq. (6) is not very useful to obtain the value of $\tan \beta$ due to error propagation. Instead, we can determine first $\cos 2\beta = \Delta (\cos 2\phi_L - \cos 2\phi_R)/4m_W^2$ and check for the sign of $\mu$ from the numerator of eq. (6). For this purpose the error of $\cos 2\beta$ does not matter. Then the eq. (7) can be used to extract $\sin 2\beta$

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*In the CP–noninvariant theories, the eq. (6) determines the $\cos \Phi$, where $\Phi$ is the mixing angle between the light and heavy charginos.*
and finally tan β obtained. As an example we consider two reference points in the parameter space, denoted by RR1 and RR2 in \( \mathbb{R}^4 \), corresponding to \((M_2, \mu, \tan \beta) = (152, 316, 3)\) and \((150, 263, 30)\), respectively (mass parameters are in GeV). We consider a high luminosity option of 500 fb\(^{-1}\) at the \( e^+e^- \) c.m. energy \( \sqrt{s} = 800 \) GeV and only statistical errors are taken into account. From measured total cross sections and LR asymmetries the mixing angles can be determined with the expected errors as follows: \( \delta \cos 2\phi_L = 0.02 \) and \( \delta \cos 2\phi_R = 0.005 \). Assuming the error of 100 MeV for the chargino masses, we arrive at the values shown in the Table:

|         | RR1            | RR2            |
|---------|----------------|----------------|
| \( M_2 \) [GeV] | 152 \(\pm 1.75\) | 150 \(\pm 1.2\) |
| \( \mu \) [GeV]   | 316 \(\pm 0.85\) | 263 \(\pm 0.68\) |
| \( \cos 2\beta \) | \(-0.8 \pm 0.08\) | \(-0.998 \pm 0.056\) |
| \( \sin 2\beta \) | 0.6 \(\pm 0.058\) | 0.066 \(\pm 0.033\) |

It is important to stress that for a discrimination between small and large tan β scenarios a high luminosity option is required. From the numbers for \( \sin 2\beta \) one finds the “measured” values of tan β in the range \( \{2.46, 4.01\} \) and \( \{20.2, 59.6\} \) for RR1 and RR2, respectively.

### 3 Summary

We explored event characteristics to isolate the chargino sector instead of relying on global fits to chargino/neutralino system\(^5\). The angular correlations among chargino decay products provide two independent observables. Thus, even from the light chargino pair production in \( e^+e^- \) annihilation, tan β, \( M_2 \) and \( \mu \) are determined up to at most a two-fold discrete ambiguity. If the collider energy is sufficient to produce the two chargino states in pairs, the above ambiguity is removed. The production cross sections and LR asymmetries allow us to determine chargino masses and mixing angles very precisely and extract \( \tan \beta, M_2 \) and \( \mu \) unambiguously.

Note that from the energy distribution of chargino decay products the mass of the lightest neutralino can be measured. As a result, the parameter \( M_1 \) and the neutralino mass matrix can be also reconstructed\(^\text{\#}\). An alternative way to determine \( M_1, M_2, \tan \beta \) and \( \mu \), based only on some of the masses of charginos and neutralinos, can be found in\(^\text{\#}\)\(^\text{\#}\). In short, \( e^+e^- \rightarrow \tilde{\chi}_i^- \tilde{\chi}_j^+ \) with polarized beams allows us to extract the fundamental parameters in the chargino sector.

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\(^\#\)Some of the material presented here goes through unaltered if the phase \( \Phi_\mu \) is allowed\(^\text{\#}\), although extra information is needed to determine the phase.