Large Deviation Delay Analysis of Queue-Aware Multi-user MIMO Systems with Two-timescale Mobile-Driven Feedback

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Abstract—Multi-user multi-input-multi-output (MU-MIMO) systems transmit data to multiple users simultaneously using the spatial degrees of freedom with user feedback channel state information (CSI). Most of the existing literatures on the reduced feedback user scheduling focus on the throughput performance and the user queuing delay is usually ignored. As the delay is very important for real-time applications, a low feedback queue-aware user scheduling algorithm is desired for the MU-MIMO system. This paper proposed a two-stage queue-aware user scheduling algorithm, which consists of a queue-aware mobile-driven feedback filtering stage and a user scheduling stage, where the feedback filtering policy is obtained from an optimization. We evaluate the queuing performance of the proposed scheduling algorithm by using the sample path large deviation analysis. We show that the large deviation decay rate for the proposed algorithm is much larger than that of the CSI-only user scheduling algorithm. The numerical results also demonstrate that the proposed algorithm performs much better than the CSI-only algorithm requiring only a small amount of feedback.

Index Terms—MU-MIMO, Limited Feedback, Queue-aware, Large Deviation, Random Beamforming

I. INTRODUCTION

MIMO is an important core technology for next generation wireless systems. In particular, in multi-user MIMO (MU-MIMO) systems, a base station (BS) (with \( M \) transmit antennas) communicates with multiple mobile users simultaneously using the spatial degrees of freedom at the expense of knowledge of channel states at the transmitter (CSIT). It is shown in [1], [2] that using simple zero-forcing precoder and near orthogonal user selection, a sum rate of \( M \log \log K \) can be achieved with full CSIT knowledge over \( K \) users. Yet, full CSIT knowledge is difficult to achieve in practice and there are a lot of works focusing on reducing the feedback overhead in MIMO systems [3]–[8]. For instance, in [3], [4], the authors have focused on the codebook design and performance analysis under limited-rate feedback schemes. In [5]–[7], on the other hand, a threshold based feedback control is adopted where users attempt to feedback only when its channel quality exceeds a threshold. It was further shown that a sum rate capacity \( O(M \log \log K) \) can be achieved when only \( O(M \log \log K) \) users feeding back to the BS [5].

While there are a lot of works that consider reduced feedback design for MU-MIMO, all these existing works focused on the throughput performance. They have assumed infinite backlog at the base station and therefore, ignored the bursty arrival of the data source as well as the associated delay performance, which is very important for real-time applications. For instance, the CSI information indicates good opportunity to transmit whereas the Queue State Information (QSI) indicates the urgency of the data flow. A delay-aware MU-MIMO should incorporate both the CSI and QSI in the user scheduling. However, it is far from trivial to integrate these information in determining the user priority. There are some works considering QSI in the user scheduling of MU-MIMO systems. In [9], the author considered a queue-aware power control and dynamic clustering in downlink MIMO systems. In [10], the authors considered MU-MIMO user scheduling to maximize queue-weighted sum rate. Due to the exponentially large solution space, heuristic greedy-based algorithm is proposed. However, these works required the BS to have global CSI knowledge of all the users, which is hard to achieve in practice. Furthermore, the delay performance in [10] is obtained by simulation only and not much design insights can be obtained in these works. In general, there are still a number of first order technical challenges associated with designing delay-aware MU-MIMO systems.

- **Challenges in User Scheduling Design**: For real-time applications, it is important to exploit CSI and QSI in the user scheduling. Yet, it is highly non-trivial to design a priority metric that strike a balance between transmission opportunity and urgency. One one hand, the Markov decision process (MDP) based methods [11], [12] result in high complexity (exponential w.r.t. \( K \)). On the other, brute-force application of Lyapunov optimization techniques [13] in MU-MIMO is also not feasible because of the associated exponential complexity of user selection for MU-MIMO.

- **Challenges in Delay Analysis**: Due to the QSI-aware control algorithm, the service rate of the data queues are state-dependent and the queue dynamics from these \( K \) data flows are coupled together. This makes the queuing delay analysis extremely difficult. There is no closed form results on the steady state distributions of the queue length in such complex queueing systems. In [14], the authors characterized the stability region of the MU-MIMO systems under limited CSI feedback. Yet, stability
is only a weak form of delay performance.

In this paper, we consider a MU-MIMO downlink system with a $M$-antenna BS and $K$ multi-antenna mobile users. The BS applies the random beamforming for MU-MIMO to exploit the multi-user diversity. To overcome the complexity challenge of user scheduling, we propose a two-timescale delay-aware user scheduling policy for the MU-MIMO system. The proposed policy consists of two stages, namely the queue-aware user-driven feedback filtering stage and the dynamic queue-weighted user scheduling stage. At the first stage (slower timescale), the BS broadcasts a QSI-dependent user feedback candidate list and only the mobiles in the list are allowed to feedback the CSI to the BS. At the second stage (faster timescale), the BS selects the best user according to the queue-weighted metric among the users selected in the first stage. Based on the two-timescale user scheduling policy, we then analyze the delay performance of the MU-MIMO system. It is in general difficult to analyze the delay for state-dependent coupled queues. To overcome this challenge, we consider the large deviation tail for the maximum queue length among all the users, which reflects the worst case delay performance in the system. Using large deviation theory for state-dependent coupled queues. To overcome this challenge, we consider the large deviation tail for the maximum queue length among all the users, which reflects the worst case delay performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system. Using large deviation theory for random process [15], we derive the asymptotic exponential performance in the system.

The above channel assumptions have captured many practical channel models, such as the i.i.d. model and the AR($n$) model [16].

At the BS, random beamforming is used to support near-orthogonal data streams transmissions to the selected users without knowing the full CSI. The BS chooses $M$ random orthonormal vectors $\{\phi_1, \ldots, \phi_M\}$, where $\phi_m \in \mathbb{C}^{M \times 1}$ are generated according to an isotropic distribution. Let $s(t) = (s_1(t), \ldots, s_M(t))$ be the vector of the transmit symbols. The transmit signal is given by

$$x(t) = \sum_{m=1}^{M} \phi_m s_m(t).$$

Therefore, the receive signal at the $k$-th user is

$$y_k(t) = \sum_{m=1}^{M} \sqrt{P} H_{k} \phi_m s_m(t) + n_k.$$

We assume the receivers know the beamforming vectors $\{\phi_m\}$. The effective SINR of the $i$-th beam on the $n$-th antenna of the $k$-th user can be calculated as follows,

$$\text{SINR}_{k,n}^i = \frac{|H_k^{(n)} \phi_i|^2}{\sum_{j, j \neq i} |H_k^{(n)} \phi_j|^2 + 1/P},$$

where $H_k^{(n)}$ denotes the $n$-th row of the channel matrix $H_k$ of user $k$. By selecting the users with the highest SINR on each beam, the transmitter can support near-orthogonal transmissions and exploit multi-user diversity without the global CSI $\{H_k\}$ [17].

### B. Bursty Data Source and Queue Model

Data arrives in packets randomly for different users. Let $A_k(t)$ denote the number of packets that arrive at the BS for user $k$ during time slot $t$, and $A(t) = (A_1(t), \ldots, A_K(t))$. We assume that the arrivals $A_k(t)$ are i.i.d over different time slot $t$. We have the following assumptions regarding the bursty arrival processes $A_k(t)$.

**Assumption 2 (Bursty Source Model):** The packet arrival $A_k(t)$ are identically and independently distributed (i.i.d.) with

1Note that the proposed two-timescale framework can also work for other beamforming schemes, such as zero-forcing. One may derive the corresponding control policy using similar techniques presented in this paper.
respect to (w.r.t.) $t$ and independent w.r.t. $k$ according to a general distribution with mean $\mathbb{E}[A_k(t)] = \lambda_k$ and finite moment generating function (MGF) $\psi_{A_k}(\theta) = \mathbb{E}[e^{\theta A_k}]$. The packet length is assumed to be constant $L$ bits.

The BS maintains queueing backlogs $Q_{k}(t)$ for each user $k$. Let $D_{k}(Q(t),\mathbf{H}(t))$ represents the amount of departures in packets for user $k$ at time slot $t$, where $Q(t) = (Q_1(t),\ldots,Q_K(t))$ and $\mathbf{H}(t) = (H_1(t),\ldots,H_K(t)).$ $D_{k}(t)$ depends on the specific user scheduling policy. The queueing dynamics for user $k$ is given by

$$Q_k(t+1) = [Q_k(t) - D_k(Q(t),\mathbf{H}(t))]_+ + A_k(t)$$

(3)

where the operator $[w]_+$ represents $\max\{0,w\}$. Here we do not consider packet drops or retransmissions. Using Little’s Law\cite{13}, the average delay of the $k$-th user is given by $T_k = \frac{Q_k}{D_k}$, where $Q_k$ is the average backlog for the $k$-th queue and $D_k$ is the average departure at each time slot. As a result, there is no loss of generality to study the queue length $Q_k$ for the purpose of understanding the delay. Obviously, the queue length (or the delay) of the MU-MIMO system depends on how we use the channel resources. Hence the goal of the user scheduling controller is to adjust the channel access opportunity for all the users so that their queue lengths (or delay) are minimized while maintaining a high system throughput.

C. Two-timescale User Scheduling with Reduced Feedback for MU-MIMO

A reasonable delay-aware user scheduling algorithm should jointly adapt to both the CSI (to capture good transmission opportunity) and the QSI (to capture the urgency). In particular, we are interested in the control policy that can maximize queue stability region. However, conventional throughput optimal (in stability sense) user scheduling policies such as max-weighted-queue (MWQ) algorithms\cite{13} require global CSI and QSI knowledge. However, the CSI is available at the mobile user side while the QSI is available at the BS. Furthermore, the MWQ policy requires solving a queue weighted sum rate combinatorial optimization problem, which has exponential searching space. Hence, a brute-force solution of the MWQ problem requires huge signaling overhead as well as huge complexity. To overcome these challenges, we propose a two-timescale user scheduling solution as follows.

- **Stage I: Queue-aware user-driven feedback filtering.** The BS determines and broadcasts the user feedback probability $\{p_1(Q),\ldots,p_K(Q)\}$ based on the user queueing backlogs $Q(t)$ for every $T$ time slots. Mobile user $k$ randomly feedback to the BS in the stage II with probability $p_k$. We denote $\chi_k \in \{0,1\}$ as the stochastic feedback filtering policy with $P(\chi_k = 1) = p_k$, and a user $k$ feeds back when $\chi_k(t) = 1$. The motivation of the mobile feedback filtering is to save the feedback cost by reducing the lower priority users from feedback back.

- **Stage II: Dynamic Queue-Weighted User Scheduling.** If the feedback indicator $\chi_k = 1$, then user $k$ measures the effective SINR vector $\{\text{SINR}_{k,n}^1,\ldots,\text{SINR}_{k,n}^L\}$ on each receive antenna $n$ according to\cite{2} and finds the strongest beam $i^*(k,n) = \arg\max_{1\leq j\leq M} \text{SINR}_{k,n}^j$. The mobile then feeds back the selected beam index $i^*(k,n)$ and the associated SINR$_{k,n}^j$ to the BS on each $n$. The set of feedback users at time slot $t$ is denoted by $F(t)$. The BS schedules user $k^*(i)$ to transmit at the $i$-th beam to maximize the queue-weighted throughput, i.e., $k^*(i) = \arg\max_{k \in F(t)} Q_k \log (1 + \gamma_k)$, where $\gamma_k = \max_{n \in N(k,i)} \text{SINR}_{k,n}^j$ denotes the highest SINR of user $k$ on the $i$-th beam over $n \in N(k,i)$. Here $N(k,i) = \{ n : 1 \leq n \leq N, i^*(k,n) = i \}$ denotes the set of receive antennas of user $k$ that have fed back the SINR for the $i$-th beam.\cite{2} As a result, the stage II user scheduling exploits the multi-user diversity among the set of users attempting to feedback $F(t)$.

The following lemma shows that, in a MU-MIMO system, it is sufficient for each user feedback back only the beam with the highest SINR as Stage II policy suggests.

**Lemma 1 (SINR property of a MU-MIMO channel\cite{2}):** If $\max_{k \in F,1 \leq n \leq N} \text{SINR}_{k,n}^j \geq 1$, then it is impossible for a user to have maximum SINRs for more than two beams on one antenna, i.e., for $(k^*,n^*) = \arg\max_{k \in F,1 \leq n \leq N} \text{SINR}_{k,n}^j$, we have $\text{SINR}_{k^*,n^*} = \max_{1 \leq j \leq M} \text{SINR}_{k^*,n^*}^j$, \forall i.

One may easily see that the probability for violating the condition in Lemma 1 exponentially decreases w.r.t. the number of feedback users, and hence is negligible.

Fig.\cite{1} depicts an illustration of the two stages user scheduling policy. The policy tries to balance the transmission opportunity and urgency with a low complexity and low feedback cost strategy. For the user with a long queue, it will be given priority to feedback during the stage I feedback filtering phase. Users who have passed the stage I filtering will compete for channel access based on the stage II queue weighted scheduling in which users with better queue weighted metric will be served. Moreover, the two stages processing can be implemented on different timescales. The SINR feedback and user scheduling in stage II is done at every time slot $t$, while the user feedback probability $\{p_k(Q)\}$ determined in stage I can be updated once every $T$ time slots. The update period $T$ trades the performance of the two-timescale policy with the control signaling overhead. With a larger $T$, there is a smaller signaling overhead associated with broadcasting $\{p_k(Q)\}$ in stage I but then the feedback priority may be driven by outdated QSI.

D. Queue-Aware Feedback Filtering (Stage I) Optimization

The feedback filtering control in stage I plays a critical role in the overall delay performance of the MU-MIMO system. In the following, we adopt a Lyapunov optimization technique to derive the stage I feedback filtering policy to achieve the maximum queue stability region in the MU-MIMO system.

\footnote{We define $\gamma_k^j = 0$ if $N(k,i) = \emptyset$.}

\footnote{Although we have assumed the fading channels are i.i.d. among users, the two-timescale algorithm framework can also be applied to non-i.i.d. users, using a similar feedback policy in stage I. However, the analysis in this case is much more complicated, and we shall leave it to the future work.}
1) Queue Stability: We first define the queue stability and the stability region formally below.

Definition 1 (Queue Stability): The queueing system is called stable if \( \lim \sup_{t \to \infty} \frac{1}{T} E[\max_k Q_k(t)] < \infty \).

Definition 2 (Stability region and Throughput Optimal): The stability region \( C \) is the closure of the set of all the arrival rate vectors \( \{\lambda_k\} \) that can be stabilized in a MU-MIMO system for some feedback probability vector \( \{p_k\} \) in the two-timescale scheduling framework. A throughput optimal feedback control is a feedback probability vector \( \{p_k\} \) that stabilizes all the arrival rate vectors \( \{\lambda_k\} \) within the stability region \( C \).

2) The Data Rate and the Amount of Feedback: Let \( J_k(Q, H, \chi) \in \{0,1\} \) be the scheduling indicator of the \( k \)-th user on the \( i \)-th beam according to the Stage II policy. Therefore, the instantaneous data rate for user \( k \) is given by

\[
R_k(Q, H, \chi) = \sum_{i=1}^{M} J_k(Q, H, \chi) \chi_k \log(1 + \gamma_k^i). \tag{4}
\]

We define the conditional feedback cost \( S(Q) \) and the average feedback cost \( \overline{S} \) as follows,

\[
S(Q) = E \left[ \sum_k \chi_k | Q \right] = \sum_k p_k(Q), \quad \text{and} \quad \overline{S} = E[S(Q)]. \tag{5}
\]

In addition, the minimum average feedback cost to achieve the maximum queue stability region \( C \) in the MU-MIMO system is denoted as \( \overline{S}^* \).

3) The Feedback Filtering Optimization: The feedback filtering control policy is derived from the Lyapunov technique and to achieve the throughput optimality.

Define \( L(Q) = \sum_k Q_k^2 \) as the Lyapunov function. Then the one-step conditional Lyapunov drift \( \Delta L(Q(t)) \) is given by,

\[
\Delta L(Q(t)) = E[L(Q(t+1) - L(Q(t))|Q(t)]. \tag{6}
\]

The following lemma establishes the relationship between the Lyapunov drift \( \overline{S} \) and the queue stability.

Lemma 2 (Lyapunov drift and the queue stability): Given positive constants \( V \) and \( \epsilon \), the \( K \) queues of the MU-MIMO system \( \{Q_1(t), \ldots, Q_K(t)\} \) are stable if the following condition is satisfied,

\[
\Delta L(Q(t)) + V E[\mathcal{S}(Q(t)|Q(t)) \leq C_0 K - \epsilon \sum_k Q_k(t) + V \overline{S} \tag{7}
\]

for some constant \( C_0 < \infty \) and all \( Q(t) \). The average queue length satisfies

\[
\lim_{t \to \infty} \frac{1}{T} \sum_{\tau=0}^{T-1} E[Q_k(\tau)] \leq \frac{C_0 K + V \overline{S}}{\epsilon} \tag{8}
\]

and the average feedback cost satisfies

\[
\overline{S} \triangleq \lim_{T \to \infty} \mathbb{E} \left[ \frac{1}{T} \sum_{\tau=0}^{T-1} S(Q(\tau)) \leq \overline{S}^* + C_0 K/V. \tag{9}
\]

Proof: The proof can be extended from [19, Lemma 1] by replacing the power cost function with the feedback cost function \( S(Q) \) defined in \( (5) \).

Lemma 2 motivates us to minimize the Lyapunov drift in \( (7) \) to achieve the maximum queue stability region. With this insight, we have the feedback filtering control problem as follows.

Feedback Filtering Control Problem (FFCP): Observing the current queue length \( Q(t) \), users feedback their CSI according to the probability vector \( p^*(Q(t)) = \{p^*_1(Q(t)), \ldots, p^*_K(Q(t))\} \), where \( p^*(Q(t)) \) is obtained from the solution of the following optimization problem,

\[
\max_{\{0 \leq p_k \leq 1\}} E \left[ \sum_{k=1}^{K} Q_k(t) R_k(Q, H, \chi) - V S(Q(t)) \right]. \tag{10}
\]

The parameter \( V \) in \( (10) \) trades off the average queue length (delay) and the feedback cost. A large parameter \( V \) reduces the average feedback cost in \( (9) \) but results in a larger average queue length \( (8) \). Note that due to the feedback filtering variable \( \chi \in \{0,1\}^K \), we have an exponential complexity (w.r.t. \( K \)) to evaluate the expectation in \( (10) \). This makes the problem difficult to solve. In the next section, we try to derive the solution of the FFCP problem by exploiting the specific problem structure.

III. THE QUEUE-AWARE USER FEEDBACK FILTERING ALGORITHM

In this section, we focus on deriving the FFCP solution to \( (10) \). Towards this end, we first decompose FFCP into two-level subproblems and study their properties. We then proceed to find the optimal solution to the inner problem and derive a low complexity algorithm to find an approximate solution to the outer problem.

A. Property of the FFCP problem

Using primal decomposition techniques, \( (10) \) can be transformed into the following two subproblems

- Inner subproblem:

\[
W(S) = \max_{\{p_k\}} E \left[ \sum_{k=1}^{K} Q_k(t) R_k(Q, H, \chi) \right] \tag{11}
\]

subject to

\[
0 \leq p_k \leq 1, \quad \forall k = 1, \ldots, K \tag{12}
\]

\[
\sum_{k=1}^{K} p_k = S \tag{13}
\]
where $S$ is an auxiliary variable with the meaning of the average feedback cost (the number of feedback users).

- Outer subproblem:

$$\max_S W(S) - VS. \quad (14)$$

The objective function (11) of the inner problem can be written as

$$\mathbb{E}\left\{ \sum_{k=1}^{K} Q_k(t) R_k(Q, H, \chi) | \chi \right\} = \sum_{j=1}^{2^K} w_j(Q) \Pr(\chi = \chi^{(j)})$$

where $w_j(Q) = \mathbb{E}[\sum_{k=1}^{K} Q_k(t) R_k(Q, H, \chi) | \chi^{(j)}]$ is a deterministic parameter independent of $\{p_k\}$, and $\Pr(\chi = \chi^{(j)}) = \prod_{k} p_k^{(j)} (1 - p_k)_{\chi^{(j)}}$ is the probability of a particular feedback indicator vector $\chi^{(j)}$, $j = 1, \ldots, 2^K$.

The above expression is a posynomial w.r.t. $\{p_k\}$. Moreover, the constraints (12)-(13) are monomials. Therefore, the inner problem (14) can be solved using the Benders decomposition algorithm.

**B. Solution to the inner problem**

Let $\Pi = \{\pi(1), \ldots, \pi(K)\}$ be a permutation of $Q$ such that $Q_{\pi(1)} \geq Q_{\pi(2)} \geq \ldots \geq Q_{\pi(K)}$. We find the optimal solution of the inner problem under the average feedback amount $\mathbb{E}[\sum \chi_k] = \sum_k p_k = S$ as follows.

**Theorem 1 (The optimal solution to the inner problem):**

The feedback probability $\{p_k\}$ to solve (11) is given by

$$p_{\pi(k)} = 1, \quad 1 \leq k \leq |S| \quad (15)$$

$$p_{\pi(k_0)} = S - |S|, \quad k_0 = |S| + 1 \quad (16)$$

$$p_{\pi(k)} = 0, \quad \text{otherwise}. \quad (17)$$

**Proof:** Please refer to Appendix A for the proof.

Although an intuition may argue that it might be better to allow more than $S$ users to feed back (each with lower $p_k$) in order to boost up the opportunistic utility in stage II, the above result shows that the best strategy is actually allowing only the users with the $S$ largest queues to feed back, while keeping the others inactive.

**C. Solution to the outer subproblem**

To derive the optimal feedback cost $S^*$, we first study the mean data rate $\mathbb{E}[R_k(Q, H, \chi)]$ (denoted as $R_k$) in the utility function (11). Define $\eta_k(S) \triangleq \mathbb{E}[R_k(Q, H, \chi) | \chi_k = S, \sum \chi_k = S]$ as the average data rate for user $k$, conditioned on the feedback amount being $|F| = S$. We characterize $\eta_k(S)$ in the following lemma.

**Lemma 3 (Data rate under heavy traffic approximation):**

Given the set of feedback users $\mathcal{F}$, where $|\mathcal{F}| = S$. If $Q_{\pi(2)} : Q_{\pi(1)} \approx 1$, then we have for $k \in \mathcal{F}$,

$$\eta_k(S) \approx M \int_0^\infty \log(1 + x) N f(x) F(x)^{NS-1} dx \triangleq \hat{\eta}(S) \quad (18)$$

where

$$F(x) = 1 - \frac{e^{-x/P}}{(1 + x)^{M-1}}. \quad (19)$$

is the cumulative distribution function (CDF) of $\text{SINR}_k^{i,n}$ in (2) and $f(x)$ is the corresponding probability distribution function (PDF).

**Proof:** Please refer to Appendix B for the proof.

The approximation is accurate when the ratio $Q_{\pi(2)} : Q_{\pi(1)}$ is close to 1, which means all the feedback users have comparable queue lengths. This can usually happen in heavy traffic scenarios where most of the users have large queues. As such, we have

$$W(S) = \mathbb{E}\left[ \sum_{k=1}^{S} Q_{\pi(k)} R_{\pi(k)} | \chi_{\pi(k)} = 0 \right] (1 - p_{\pi(k_0)})$$

$$+ \mathbb{E}\left[ \sum_{k=1}^{S+1} Q_{\pi(k)} R_{\pi(k)} | \chi_{\pi(k_0)} = 1 \right] p_{\pi(k_0)}$$

$$\approx \sum_{k=1}^{S} Q_{\pi(k)} \hat{\eta}(S) \left[ 1 - (S - |S|) \right]$$

$$+ \sum_{k=1}^{S+1} Q_{\pi(k)} \hat{\eta}(S+1) (S - |S|) \triangleq \hat{W}(S).$$

and we obtain an approximation to the outer problem (14) as

$$\max_{S \leq K} \hat{U}(S) \triangleq \hat{W}(S) - VS. \quad (21)$$

**Problem (21) is concave and has a nice property as shown in the following.**

**Theorem 2 (Solution property of (21)):** The objective function $\hat{U}(S)$ in (21) is concave. Moreover, the optimal solution $S^*$ is an integer.

**Proof:** Please refer to Appendix C for the proof.

**Theorem 2** suggests that a bisection algorithm can be applied to find the unique solution $S^*$ in (21) in at most $\log_2(K)$ steps, where the optimality condition can be expressed as

$$\hat{U}(S^*) \geq \hat{U}(S^* + 1) \quad \text{and} \quad \hat{U}(S^*) \geq \hat{U}(S^* - 1) \quad (22)$$

for a unique $S^* \in \{1, \ldots, K\}$.

Using Theorem 1 for solving the inner problem and the optimality condition (22) for solving the outer problem (14) under heavy traffic approximation, Algorithm 1 summarizes the Feedback Filtering Control Algorithm (FFCA), which finds the feedback probability vector $\{p_k^*\}$ in Stage I.

The proposed two-timescale user scheduling algorithm can be summarized as follows. First of all, determine the optimal user feedback amount $S^*$ by solving (14) using the FFCA. Secondly, choose $S^*$ users who have the longest queues among all the $K$ users to feed back to the BS according to the policy decision $\{p_k^*\}$ in (15). Thirdly, the selected users feed back their effective SINRs based on $\{p_k^*(Q)\}$ and the BS schedules the users to maximize the queuel-weighted throughput as described in the stage II policy.
Algorithm 1 Feedback Filtering Control Algorithm (FFCA)

1) Initialization: $S := \left\lceil \frac{K}{2} \right\rceil$. $S_{\min} = 1$, $S_{\max} = K$.
2) Evaluate the condition in (22). If $\mathcal{U}(S^*) \geq \mathcal{U}(S^* - 1)$, then $S_{\min} := S$. Otherwise, $S_{\max} := S$.
3) Repeat Step 2 by setting $S := \frac{(S_{\min} + S_{\max})}{2}$, until $S_{\max} - S_{\min} \leq 1$.
4) Find the optimal user feedback probability vector $p$ according to (13) in Theorem 1 by setting $S = S^*$. The algorithm thus finishes.

Algorithm 1 Feedback Filtering Control Algorithm (FFCA)

Although the FFCA is derived using heavy traffic approximation, it is in fact throughput optimal as summarized below.

Theorem 3 (Throughput optimality of the FFCA): Suppose \{\text{B}_k(t)\} are i.i.d. over $k$ and $t$. The feedback control $p^*(Q)$ given by FFCA achieves the maximum stability region $C$ in the MU-MIMO system.

Proof: Please refer to Appendix A for the proof.

IV. LARGE DEVIATION DELAY ANALYSIS FOR THE WORST CASE USER

In this section, we will study the queueing delay performance of the proposed solution and illustrate the gain of having queue-aware policy. We are interested in the steady state distribution of the worst case queueing performance, i.e.,

$$\lim_{t \to \infty} \Pr \left( \max_{1 \leq k \leq K} Q_k(t) > B \right)$$

where $B$ is the buffer size. We denote $Q_{\max}(t) = \max_k Q_k(t)$ as the maximum queue length process and $Q_{\max}(\infty)$ as the steady state of the $Q_{\max}(t)$. To overcome the technical challenges associated with delay analysis of MU-MIMO system, we consider the large deviation approach \[\text{[21]}\]. Specifically, we focus on the asymptotic overflow probability for the maximum queue $Q_{\max}(\infty)$ over a large buffer size $B$, which is captured by the large deviation decay rate of the tail probability of $Q_{\max}(\infty)$. In the next section, we shall introduce the decay rate function for $Q_{\max}(\infty)$.

A. Large Deviation Decay Rate for $Q_{\max}(\infty)$ Using Sample Path Analysis

The large deviation decay rate function $I^*$ for the tail probability of $Q_{\max}(\infty)$ is defined as

$$I^* \triangleq \lim_{B \to \infty} \frac{1}{B} \log \Pr \left( Q_{\max}(\infty) > B \right).$$

(23)

Note that, with the notion of the large deviation rate function, the queue overflow probability can be written as

$$\Pr \left( Q_{\max}(\infty) > B \right) = e^{-I^* B + o(B)}$$

(24)

where the component $I^*$ controls how fast the queue overflow probability drops when the buffer size $B$ grows. A larger decay rate $I^*$ corresponds to a better performance of the scheduling algorithm in the sense of reducing the worst case delay $Q_{\max}$ in the system.

To find the large deviation decay rate $I^*$, we first study the packet departure process $D_{\max}(t)$ associated with the maximum queue $Q_{\max}(t)$. Denote $D_{\max}(t) = R_{\max}(t, Q(t))/L$, where $R_{\max}(t, Q(t))$ is the transmission data rate in bits. Define the $r$-range logarithm moment generating function (LMF) as

$$\Lambda_r(\theta) = \frac{1}{r} \log \mathbb{E} \left[ \exp \left( \theta \sum_{t=1}^{\infty} D_{\max}(t) \right) \right].$$

We consider a “near i.i.d.” property for the departure process $D_{\max}(t)$, which is captured in the following:

Assumption 3 (Existence of the LMF): The limit of the $r$-range LMF exists as an extended real number $\mathbb{R} \cup \{+\infty\}$ for each $\theta \in \mathbb{R}$, i.e., $\lim_{r \to \infty} \Lambda_r(\theta) = \Lambda_D(\theta)$.

Note that, a simple example to satisfy the above assumption is $D_{\max}(t)$ being i.i.d., where $\Lambda_D(\theta) = \log \mathbb{E} \left[ \exp \left( \theta D_{\max} \right) \right]$.

For easy discussion, consider i.i.d. arrivals $A_k(t)$ with mean $\mathbb{E}[A_k] = \lambda$ and LMF $\log \psi_A(\theta) = \Lambda_A(\theta)$. Denote $g(x, \theta) = \Lambda_A(\theta) + \Lambda_D(\theta, -\theta)$, where $x$ represents some system state according to the scheduling policy. We carry out a sample path analysis as follows.

Consider a scaled sample path $q_{\max}(B) = \frac{1}{B} \max_k Q_k(B)$, which starts from $q_{\max}(0) = 0$ and reaches $q_{\max}(T_s) = 1$, for some $T_s$. With the scaling, we have $\Pr(Q_{\max}(\infty) > B) = \Pr(q_{\max}(\infty) > 1)$. Let $w(t)$ be a continuous sample path following $q_{\max}(t)$, as $w(t) \approx q_{\max}(t)$. We focus on the rate function $I_0$ defined as $I_0 = \inf_{w(\cdot)} \left\{ \int_0^{T_s} l(w(\tau), w'(\tau)) d\tau : w(0) = 0, w(T_s) = 1, T_s > 0 \right\}$

where

$$l(x = w(\tau), y = w'(\tau)) \triangleq \sup_{\theta} \{\theta y - g(x, \theta)\}$$

(25)

is the local rate function \[\text{[21]}\]. As an intuitive illustration, $I_0$ corresponds to finding a “least cost” path $w^*(t)$ that goes overlow at $w(T_s) = 1$. In other words, the $q_{\max}(t)$ “most likely” follows the path $w^*(t)$ to overflow, if it would.

We then connect the $I_0$ defined above with the large deviation principle of $Q_{\max}(\infty)$ in the following results.

Theorem 4 (The large deviation principle for $Q_{\max}(\infty)$): Suppose $g(x, \theta)$ is Lipschitz continuous on $x \in [0, 1]$. Then

$$\lim_{B \to \infty} \frac{1}{B} \log \mathbb{E} \left[ \Pr(q_{\max}(\infty) > 1) \right] = -I_0.$$  

In addition, assume that $l(x, y)$ in (25) is differentiable in $y$ at all $x$, which is non-degenerate in $[0, 1]$. For each $x$, the equation $g(x, \theta^*(x)) = 0$ has at most two solutions. Then with the appropriate choice of $\theta^*(x)$, we have

$$I_0 = \int_0^1 \theta^*(x) dx.$$  

(26)

Proof: Please refer to Appendix B for the proof.

As an application example for the above result, we calculate the rate function for a CSI-only baseline scheduling algorithm: Each user $k$ feeds back the SINR for the $i^*(k, n)$-th beam on each antenna $n$, where $i^*(k, n) = \max_{1 \leq i \leq M} \text{SINR}_{k,n}^i$.

A comprehensive technique to verify the assumption is given in \[\text{[23]}\]. For easy discussion, we omit the details here.
On the other hand, the BS schedules the user with the highest SINR on each beam $i$, for $i = 1, \ldots, M$. Consider i.i.d Poisson arrivals $A(t)$ with parameter $\lambda = \lambda_{tot}/K$, and i.i.d. CSI $\{H_k\}$. We have the following results.

**Corollary 1 (Decay rate for the CSI-only algorithm):** Assume $\mu_b \triangleq \frac{M \log (P \log N K)}{L_{tot}} > \lambda$. The large deviation decay rate for $Q_{\max}(\infty)$ under the CSI-only baseline algorithm can be expressed as

$$I^*_\text{baseline} \approx \log \frac{M \log (P \log N K)}{\lambda_{tot} L_{tot}},$$

which is asymptotically accurate at large $M$ and $K$.

**Proof:** Please refer to Appendix [H] for the proof.

The above result shows that the CSI-only baseline algorithm has a decay rate $I^*_\text{baseline} = O(\log \log \log K)$. We will show later that, by taking into account the QSI in the user scheduling, the proposed two-timescale algorithm achieves a much larger decay rate of the overflow probability.

**B. Asymptotic Data Rate of the Proposed Algorithm**

To derive the large deviation decay rate $I^*$ for $Q_{\max}(t)$ under the proposed algorithm, we need to understand the corresponding packet departure rate $D_{\max,p}(t)$. Denote $D_{\max,b}(t; S)$ as the packet departure rate under the CSI-only algorithm for a group of $S$ users. We have the following property.

**Lemma 4 (Property of $D_{\max,p}(t)$):** Given $|F| = S$ users feedback, we have

$$D_{\max,b}(t; S) \leq D_{\max,p}(t; S) \leq \frac{1}{L} \sum_{n=1}^{N} \log(1 + \text{SINR}^{*}_{m(t),n}),$$

where $\text{SINR}^{*}_{m(t),n}$ is the SINR on the $n$-th receive antenna of the $k = m(t)$ user who has the longest queue and feeds back the $i^*(n)$-th beam.

The left hand side of (28) is due to the fact that the maximum queue user has a higher probability to get scheduled under the Stage II queue-weighted scheduling policy. The equality holds when all the feedback users have similar queue length, i.e., $Q_{\pi(1)} = Q_{\pi(S)}$. The equality on the right hand side of (28) holds when the maximum queue user has dominating queue length, i.e., $Q_{\pi(1)} \gg Q_{\pi(2)}$, and hence must be scheduled.

In addition, we derive the following result for evaluating the feedback amount $S^*$.

**Lemma 5 (Upper bound of $S^*$):** The upper bound of $S^*(t)$ which solves (21) is given by

$$S^*(Q(t); K) \leq \min \left\{ e^{W\left(c_1\right)} / N, K \right\} \triangleq \hat{S}^*(Q_{\max})$$

where $c_1 = \frac{M N Q_{\max}}{L_{tot}}$, and $W(x)$ is the Lambert W function defined as $W(x)e^{W(x)} = x$. The equality holds when $Q_{\pi(k)} = Q_{\max}$ for all $k$.

**Proof:** Please refer to Appendix [H] for the proof.

**Remark 1 (Interpretation of $S^*$):** The results provides an important insight that, when $Q_{\max}$ is large, it is better to have more user feedback to boost up the system throughput. On the other hand, when $Q_{\max}$ is small, we can have less user feedback and give higher priorities to the urgent users.

With the results of Lemma 4 and 5 we can obtain the packet departure rate for $Q_{\max}(t)$. We thus study the large deviation decay rate for the proposed algorithm in the next subsection.

**C. Rate Function for the Proposed Algorithm under $T = 1$**

To gain more insight from the general results in Theorem 4, we consider a special case where the CSI $\{H_k\}$ are i.i.d., and the arrivals $A_k$ follow the Poisson distribution with parameter $\lambda_k = \lambda_{tot}/K$.

We first consider the case $T = 1$, where the BS broadcasts the updated feedback policy $\hat{q}_k(Q)$ at every time slot. We obtain the following results for the large deviation decay rate of $Q_{\max}(\infty)$ under the proposed two-timescale user scheduling algorithm.

**Theorem 5 (Decay rate for the proposed algorithm):** Let $\mu_p(x) = \frac{M \log (P \log N S^*(x))}{LS^*(x)}$. Assume that $\lambda < \inf_{x \in [0,1]} \mu_p(x)$. Then the large deviation decay rate of $Q_{\max}(\infty)$ under the two-timescale user scheduling algorithm can be expressed as

$$I^*_\text{prop} \geq (1 - \epsilon) \log K + \log \frac{M}{\lambda_{tot} L_{tot}} + \epsilon \log r_0 + C \triangleq I^*_\text{LB} (30)$$

where $\epsilon > 0$ is a small constant, $r_0 = \int_1^1 \log (1 + x) dF(x) \text{ and } C = \int_1^1 \left\{ \log \left[ N \log \left( \frac{PW(MN_x)}{V} \right) \right] - W \left( \frac{MN_x}{V} \right) \right\} dx$.

**Proof:** Please refer to Appendix [H] for the proof.

Based on the results in Corollary 1 and Theorem 5 we conclude the following for the CSI-only user scheduling algorithm and the proposed two-timescale algorithm.

- **Gain of the queue-aware policy:** Large deviation decay rates $I^*_\text{prop} \gg I^*_\text{baseline}$, when the number of users $K$ grows large. This demonstrates that it is important to utilize the queue information in the user scheduling algorithm to minimize the worst case delay.

- **Impact of the multi-user diversity:** In addition, both of the schemes benefit from the increase of the number of users $K$, as seen from the terms $\log (P \log N K)$ in (27) and $\log (K)$ in (30). The decay rate increases when the number of users increases, and the rate $I^*_\text{prop}$ increases faster than the baseline.

- **Impact of the multi-antenna transmission:** Furthermore, both of the schemes benefit from the MU-MIMO channel. It is demonstrated that, when increasing the number of data streams $M$ and the receive antennas $N$, the large deviation decay rates $I^*_\text{prop}$ and $I^*_\text{baseline}$ both increase as $O(\log M \log \log N)$.

In summary, by carefully exploiting the queue information in the stage I feedback filtering, the proposed MU-MIMO algorithm has significant delay performance gain compared with conventional CSI-only schemes.

**D. Rate Function for $T > 1$**

Now we consider the $T$-step feedback policy, where the BS updates the $\hat{q}_k(Q)$ for every $T > 1$ time slot. Denote the
corresponding maximum queue process as $Q_{\text{max}}(t)$ with $t > 0$. We are interested in the case where the process $Q_{\text{max}}(t)$ is stable and assume the large deviation principle exists.

Define the rate function as

$$I_{\text{prop}}^{(T)} = \lim_{B \to \infty} -\frac{1}{B} \log \Pr \left( Q_{\text{max}}(\infty) > B \right).$$

For easy discussion, we consider i.i.d. arrivals $A_k(t)$ and i.i.d. CSI $\{H_k(t)\}$. Consider a random process $\nu(t) = A_1(t) - A_2(t) - d(t)$, where $A_1$ and $A_2$ are two i.i.d. arrival sequences, and $d(t)$ has probability distribution function given by $F((P-1)2^x - 1))$ and $F(x)$ is defined in [19]. We have the following result for the decay rate of the $T$-step feedback policy.

**Theorem 6** (Decay rate for the $T$-step feedback policy): Assume the conditions in Theorem 5, we have

$$I_{\text{prop}}^{(T)} \geq I_{\text{prop}}^{LB} - \int_0^1 \rho(x) dx$$

where $\rho(x) \triangleq -\frac{1}{\lambda_0} \log (e^{\lambda_0} - \lambda) - (e^{\lambda_0} - \lambda - 1)P_0^{(T)}$ and $P_0^{(T)} \triangleq \Pr\{ \sum_{\tau=1}^{T-1} \nu(\tau) > 0 \}$.

**Proof:** Please refer to Appendix 1 for the proof.

**Remark 2** (Impact of $T$ and the arrival distribution): Note that $P_0^{(T)}$ represents a lower bound probability for the maximum queue user remaining in the outdated feedback group $F(t_0)$ during $t \in [t_0, t_0 + T)$; the larger the $T$, the smaller the $P_0^{(T)}$. The lower bound becomes tight when $P_0^{(T)}$ is close to 1. The above result shows that the decay rate function $I_{\text{prop}}^{(T)}$ decreases when the QSI update period $T$ increases. Moreover, the distribution of arrival plays an important role in $T > 1$. With a heavier tail for the arrival, $P_0^{(T)}$ decreases, resulting in a higher performance penalty for $T > 1$. Finally, the performance in terms of the overflow probability for the two-timescale algorithm is sensitive to the timely queue-aware feedback under heavy loading when $\mu_p - \lambda$ is small.

V. NUMERICAL RESULTS

In this section, we simulate the queuing delay performance of the proposed two-timescale user scheduling algorithm. We consider a MU-MIMO system with $K$ users, and packets arrive to the queue of each user according to a Poisson distribution with rate $\lambda = \lambda_{\text{tot}}/K$, where the total arrival rate is $\lambda_{\text{tot}} = 7500$ packets/second. Each packet has $L = 8000$ bits. The system bandwidth is 10 MHz and the SNR is 10 dB. The number of transmit and receive antennas are $M = 4$ and $N = 2$, respectively. The scheduling time slot is $\tau = 1$ ms and the simulation is run over $T_{\text{tot}} = 100$ seconds. We compare the performance of proposed algorithm against the following reference baselines.

- **Baseline 1**: CSI-only user scheduling (CSIO) [6]. At each time slot, all the users feedback the CSI to the BS, and the BS schedules a set of users who respectively have the highest SINR on each beam (see Section IV.C).
- **Baseline 2**: CSI-only user scheduling with limited feedback (CSIO-LF) [6]. The scheme is similar to baseline 1 except that the user feeds back to the BS only when its SINR exceeds a threshold $t_{\text{SINR}} = 1$ dB.

**Baseline 3**: Proportional fair user scheduling (PFS) [11]. At each time slot, all the users feedback the CSI to the BS, and the BS transmits data to the users using proportional fair scheduling with window size $t_w = 100$ ms.

**Baseline 4**: Max weighted queue user scheduling (MWQ) [13]. At each time slot, all the users feedback their CSI to the BS, and the BS selects a set of users so that the instantaneous queue-weighted sum rate $\sum Q_k R_k$ is maximized.

Note that the associated user scheduling problem in baseline 4 has much higher complexity for user scheduling and feedback from all the users are required. Hence, baseline 4 serves for performance benchmarking purpose only.

A. Queueing Performance and Feedback Comparisons

Fig. 2 shows the overflow probability for the worst case queue $\Pr(Q_{\text{max}}(\infty) > B)$ versus the buffer size $B$. The number of users is $K = 40$. The feedback policy $\chi$ updates on every $T = 1, 5, 10$ time slots. The proposed scheme significantly outperforms over baselines 1 - 3. It also performs closely to baseline 4.

**Baseline 4** has much higher complexity for user scheduling and feedback from all the users are required. Hence, baseline 4 serves for performance benchmarking purpose only.

B. Large Deviation Decay Rate for a Large Number of Users

Fig. 4 shows the large deviation decay rate over the number of users. The decay rate for the proposed scheme grows much faster than those of baselines 1 - 3 with the number of users $K$. Moreover, the theoretical rate functions are plotted. These are consistent with the results in Corollary 1 and Theorem 5.
The large deviation decay rate for the proposed algorithm, which means that the proposed scheme performs better in reducing the worst case delay. The numerical results demonstrated a significant performance gain over the CSI-only algorithm and a huge feedback reduction over the MWQ algorithm.

**Appendix A**

**Proof of Theorem 1**

Note that the amount of feedback $s = \sum_k \chi_k$ follows the Poisson Binomial distribution, which is insensitive of individual $p_k$ given a fixed $\sum_k p_k = S$. For an easy elaboration, consider a Poisson distribution (which is close to the Poisson Binomial distribution) with parameter $\sum_k p_k = S$ to approximate the distribution of $s$. The approximation error is upper bounded by $2 \sum_k p_k^2$.

We first find the optimal solution under the heavy traffic approximation, and then we generalize the result into the normal case. In the heavy traffic case where $Q_\pi(1) \approx Q_\pi(K)$, the objective in (11) can be written as $f(p) = \sum_k Q_k \mathbb{E}\{\chi_k \eta(s) | \chi_k\} \approx \sum_k p_k \mathbb{E}\eta(s)$, where $\mathbb{E}\chi_k \eta(s) | \chi_k = p_k \mathbb{E}\eta(s) + o(\sum_k p_k) \approx p_k \mathbb{E}\eta(s)$, and $\eta(s)$ does not depend on $Q$ since all $Q_k$ are almost the same. Thus $\mathbb{E}\eta(s)$ can be computed by an approximated Poisson distribution which does not depend on $\chi_k$.

As such, the inner subproblem becomes a linear program with constraints $\sum p_k \leq S$ and $0 \leq p_k \leq 1, \forall k$. The solution is given by $p_\pi(k) = 1, 1 \leq k \leq |S|, p_\pi(k_0) = S - |S|, k_0 = |S| + 1$, and $p_\pi(k) = 0$, otherwise, where the permutation $\Pi = \{\pi(k)\}$ is such that $Q_{\pi(1)} \geq \cdots \geq Q_{\pi(K)}$.

Now we show that the above solution is also a local optimum under general queueing profiles. Consider an arbitrary feasible probability vector $\bar{p} = p^* + p^*$ lies in a small neighborhood of $p^*$. Since $\sum_k \bar{p}_k = S$, we must decrease a probability of $\bar{p}_k$ for some user $k = \pi(j), j \leq S$, in order to increase a probability $\bar{p}_k$ for a user $k' = \pi(j'), j' > S$. The differential utility $\mathcal{W}(\bar{p}; S) - \mathcal{W}(p; S)$ then becomes

$$\Delta \mathcal{W}(S) = -p_0 Q_k \mathbb{E}[R_k | Q_k R_k \in \max \{Q_i R_i, i \in F\}]$$

$$\times \text{Pr}(Q_k R_k \in \max \{Q_i R_i, i \in F\})$$

$$+ p_0 Q_k' \mathbb{E}[R_k' | Q_k' R_k' \in \max \{Q_i R_i, i \in F\}]$$

$$\times \text{Pr}(Q_k R_k' \in \max \{Q_i R_i, i \in F\})$$

where $\max \{A\}$ means a subset of $A$ with $M$ elements which are the largest. Since $Q_k \geq Q_k'$, and $R_k$ and $R_k'$ are identical, we must have $\text{Pr}(Q_k R_k \in \max \{Q_i R_i, i \in F\}) \geq \text{Pr}(Q_k R_k' \in \max \{Q_i R_i, i \in F\})$. Therefore, the differential utility cannot be positive. As $p^*$ can be arbitrary, the vector $p^*$ must achieve the local maximum utility.

Moreover, as the inner problem is a GP, $p^*$ is also a global optimum.

**Appendix B**

**Proof of Lemma 3**

Consider $Q_\pi(1) \approx Q_\pi(S)$. The queue weighted user scheduling algorithm degenerates to a max-SINR based algorithm. Then the order statistics can be applied to study the expected data rate, and each user has around $1/S$ probability to be scheduled independently on each beam.
From the effective SINR expression in (2), as \( \phi_i \) are unitary vectors, \( |H_k^{(n)} \phi_i|^2 \) are i.i.d. over \( i \) with chi-square distribution with degrees of freedom 2. Consequently, the term \( \sum_{j=1}^{|F|} |H_k^{(n)} \phi_i|^2 \) is chi-square distributed with degrees of freedom \( 2M-2 \). Thus, the PDF \( f(x) \) and CDF \( F(x) \) of \( \text{SINR}_k,n^x \) are given by \( f(x) = \frac{e^{-x}}{(1+x)^2} \left( \frac{1}{1+x} \right) (1+x) + M-1 \) and \( F(x) = 1 - \frac{1}{(1+x)^2} \), respectively [2]. Thus, for a particular user \( k \in F \), as \( \text{SINR}_k,n \) are i.i.d. over different users \( k \) and antennas \( n \), the probability that user \( k \) has the largest SINR on the \( i \)-th beam and the \( n \)-th antenna is given by \( 1/NS \). The corresponding CDF of the maximum SINR is

\[
P \left( \max_{k\in F, 1 \leq n \leq N} \text{SINR}_k,n \leq x \right) = (F(x))^{NS}
\]

and hence, the data rate can be given by

\[
\hat{R} = \int_0^\infty \log(1+x)d(F(x))^{NS} = \int_0^\infty \log(1+x)NSf(x)F(x)^{NS-1}dx.
\]

As each user equips with \( N \) antennas, the average data rate for user \( k \in F \), given \( |F| = S \) is \( \eta_k(S) = \sum_{n=1}^N \Pr \left( \text{SINR}_k,n = \max_{k\in F, 1 \leq n \leq N} \text{SINR}_k,n \right) \hat{R} = \frac{NM}{NS} \hat{R} = \hat{\eta}(S) \).

APPENDIX C
PROOF OF THEOREM [2]

We first note that the function \( \mathcal{V}(S) \) is piece-wise linear and so does \( \mathcal{U}(S) \). Then the function \( \mathcal{U}(S) \) is concave if we can find a a smooth and concave upper envelope function that passes through every corner point of \( \mathcal{U}(S) \).

Let \( \mathcal{I} \) denote the space of twice-differentiable positively non-decreasing concave functions, i.e., \( \mathcal{I} \equiv \left\{ \phi \in C^2(0, +\infty): \phi > 0, \phi' > 0, \phi'' \leq 0 \right\} \). Let \( \eta_k(s) = \hat{\eta}(s) \), where \( \eta_k(s) \) is allowed to take real values. Given \( g \in \mathcal{I} \), define \( G(s) = g(s)\eta_k(s) - Vs \). We have the following result.

Lemma 6: \( G(s) \) is concave for any \( g \in \mathcal{I} \).

Proof: To show \( G(s) \) is concave is equivalent to showing \( G''(s) = g''(s)\eta_k(s) + 2g'(s)\eta'_k(s) + g(s)\eta''_k(s) \leq 0 \).

From the property of \( g \in \mathcal{I} \), we have \( g(s) \leq \hat{g}(s) \). Thus

\[
G''(s) \leq g''(s)\eta_k(s) + \frac{g(s)}{s} \left[ 2\eta'_k(s) + s\eta''_k(s) \right].
\]

The first term is negative by the definition of \( g \in \mathcal{I} \). In the second term, \( g''(s) \) is positive. Now, let \( \Gamma(s) = 2\eta'_k(s) + s\eta''_k(s) \). Note that, from (33), \( \eta_k(s) \) is twice differentiable on \( s \in (0, +\infty) \), and we have the following two equations

\[
\eta'_k(s) = M \int_0^\infty \log(1+x)N^2f(x)f(x)F(x)^{NS-1}dx,
\]

\[
\eta''_k(s) = M \int_0^\infty \log(1+x)N^3f(x)f(x)^2F(x)^{NS-1}dx.
\]

One can easily verify that, \( \Gamma(s; N = 1) \leq 0 \) for all \( s > 0 \). This can be seen by first numerically verifying \( \Gamma(s; N = 1) \leq 0 \) for small \( s \) (e.g., \( s < 1000 \)), and then verifying \( \Gamma(s)' > 0 \) for large \( s \) through analyzing the dominating components \( F(x)^{S-1} \) in the integrand as \( F(x) \) sufficiently close to 1. Moreover, for \( s \to \infty \), \( \Gamma(s; N = 1) \to 0 \).

For \( N > 1 \), let \( t = Ns \). From the above two equations, we have \( \Gamma(s; N) = N^2\Gamma(t; N = 1) \leq 0 \). With \( \Gamma(s) \leq 0 \), we have \( G''(s) \leq 0 \) in (33).

Hence \( G(s) \) is concave.

Now notice that the sequence \( \sum_{k=1}^N Q_k^{(n)} \) is non-decreasing for \( S = 1, \ldots, K \), and the increment is non-increasing. Then there must exist a function \( g_E(S) \) in \( \mathcal{I} \), such that \( g_E(S) \) passes through every point of the sequence \( \sum_{k=1}^S Q_k^{(n)} \), i.e., \( g_E(S) = \sum_{k=1}^S Q_k^{(n)} \) for \( S = 1, \ldots, K \). According to Lemma 6, the function \( G_E(S) \) is \( g_E(S)\eta_k(s) - Vs \) is concave. Moreover, \( G_E(S) \) is an upper envelope function that passes throughout every corner point of \( \mathcal{U}(S) \). This proves that \( \mathcal{U}(S) \) is concave.

To show the optimal solution appears at one the integer point, we take derivative of \( \mathcal{U}(S) \) and obtain

\[
\frac{d}{dS} \mathcal{U}(S) = -\sum_{k=1}^{|F|} \eta_k(S) + \sum_{k=1}^{|S|+1} Q_k^{(n)} \hat{\eta}(S) - V.
\]

It is observed that, given any integer \( S_0 \), the gradient \( \frac{d}{dS} \mathcal{U}(S) \) remains constant for any \( S \in (S_0, S_0 + 1) \). If \( \frac{d}{dS} \mathcal{U}(S) = 0 \), we can consider \( S_0 \) or \( S_0 + 1 \) to be the local maximum. If \( \frac{d}{dS} \mathcal{U}(S) \neq 0 \), using the optimality condition (25), \( S \in (S_0, S_0 + 1) \) cannot be the maximum. It concludes that, the maximum should be an integer.

APPENDIX D
PROOF OF THEOREM [5]

Consider the queue dynamic in (3). By squaring the equation on both sides and using the property \( \lfloor x \rfloor = \lfloor y \rfloor \), we obtain \( \forall k \),

\[
Q_k^2(t + 1) \leq Q_k^2(t) + \mu_k^2(t) - 2Q_k(t)(D_k(t) - A_k(t)) + A_k^2(t) \quad (33)
\]

Following the definition of conditional Lyapunov drift \( \Delta L(Q(t)) \) in (6), taking conditional expectations and summing over all \( k \) inequalities in (33) yields

\[
\Delta L(Q(t)) \leq \mathbb{E} \left[ \sum_k \mu_k^2(t) + A_k^2(t) | Q(t) \right] - 2 \sum_k Q_k(t) \mathbb{E} [D_k(t) - A_k(t) | Q(t) ] .
\]

Denote positive constants \( \bar{\mu}_{\max}^2 \) and \( \bar{A}_{\max}^2 \) such that \( \mathbb{E} [D_k^2(t) | Q(t) ] \leq \bar{\mu}_{\max}^2 \) and \( \mathbb{E} [A_k^2(t) | Q(t) ] \leq \bar{A}_{\max}^2 \). Let \( C_0 = \bar{\mu}_{\max}^2 + \bar{A}_{\max}^2 \). Adding \( V \mathbb{E} \{ S(Q(t)) | Q(t) \} \) on both sides, the drift (34) is bounded by

\[
\Delta L(Q(t)) + V \mathbb{E} \{ S(Q(t)) | Q(t) \} \leq C_0 K + 2 \sum_k Q_k(t) \lambda_k \quad (35)
\]

Suppose now that the arrival \( \lambda = (\lambda_1, \ldots, \lambda_K) \) is strictly interior to the stability region \( \mathcal{C} \) such that \( \lambda + \epsilon \mathbf{1} \in \mathcal{C} \), for \( \epsilon > 0 \). Since channel states are i.i.d. over time slots, using
the result in [19 Corollary 1], it follows that there exists a stationary randomized feedback control policy that schedules user to feedback independent of queue $Q(t)$ and yields $\mathbb{E}[D_k(t) | Q(t)] = \mathbb{E}[R_k(t)] \geq \delta \epsilon + \epsilon$ and $\mathbb{E}[S(Q(t)) | Q(t)] = S(\epsilon)$. Because the stationary policy is simply a particular feedback policy and note that the FFCA maximizes the term $\sum_k \mathbb{E}[Q_k(t) R_k(t)]$ under and approximated feedback cost $S \leq K$, the right hand side of (35) under FFCA is thus upper bounded by $C_\eta K - 2\epsilon \sum_k Q_k(t) + V K$.

Using the results in Lemma 2 it follows that $\sum_k Q_k(t) \leq C_\eta K + V S < \infty$, which proves that the FFCA policy stabilizes all the queues.

**APPENDIX E
PROOF OF THEOREM 3**

Consider the scaled sample path $q^B_{\max}(t) = \frac{1}{B} Q_{\max}(\lfloor Bt \rfloor)$, where the jumps can be given by $\tilde{q}^B_{\max}(t) - q^B_{\max}(0)$

$$= \frac{1}{B} \sum_{s \in [Bt_0]} A_{s}(s) - \frac{1}{B} \sum_{s \in [Bt_0]} D_{m_{s}}(s)$$

for $0 \leq t_0 < t \leq T$, where $m(s) = \arg \max Q_{k}(s)$ denotes the index of the maximum queue at time $s$. Note that, for $|t-t_0|$ small, the jump $q^B_{\max}(t) - q^B_{\max}(t_0)$ is a sum of sequence of random variables $\nu(s) = A_{m_{s}}(s) - D_{m_{s}}(s)$, whose $\tau$-step LMF is given by

$$\lambda^\gamma_{\nu} = \sum_{s} \sum_{t=1}^{t} \log \mathbb{E} \left[ \exp \left( \theta \sum_{s=1}^{t} (A_{m_{s}}(s) - D_{m_{s}}(s)) \right) \right]$$

$$= \log \mathbb{E}[\exp(\theta A)] + \frac{1}{\tau} \log \mathbb{E} \left[ \exp \left( - \theta \sum_{s=1}^{t} D_{m_{s}}(s) \right) \right]$$

Under Assumption 3 taking $\tau \to \infty$, we obtain $\lambda^\gamma_{\nu} \to g(x, \theta)$, which defines the local rate function in (25).

Thus one can use the Gartner-Ellis theory [26 Theorem 2.3.6] to show the large deviation principle associated with the local rate function (25) for the non-i.i.d. sequence $u(t)$ on each $(w(t), w'(t))$ pair following the path $w(t)$. Then we consider the escape time $\tau_B = \inf \{ t > 0 : q^B_{\max}(t) \leq 1 \}$.

Using the Freidlin-Wentzell theory [13 Theorem 6.17], we thus obtain the large deviation principle $\lim_{B \to \infty} \frac{1}{B} \log \mathbb{E}[\tau_B] = I_0$ for the random process $q^B_{\max}(t)$.

Note that the mean escape time $\tau_B$ implies the steady state probability for $q^B_{\max}(\infty)$ staying in the set $\{q^B_{\max}(\infty) > 1\}$, i.e., $\lim_{B \to \infty} \frac{1}{B} \log \mathbb{E}[\tau_B] = \lim_{B \to \infty} \frac{1}{B} \log \mathbb{P}(q^B_{\max}(\infty) > 1)$. Therefore, the first part of the theorem is established.

The second part of the theorem completely follows [21 Lemma C.9] and thus we omit the details here.

APPENDIX F
PROOF OF COROLLARY 1

For the $i$-th beam, the CSI-only algorithm selects the user with the highest SINR for transmission. Denote $R_i^{(1)}$ as the corresponding transmission data rate. We have $\mathbb{E} R_i^{(1)} = K \hat{\eta}(K)$, where $\hat{\eta}(s)$ is given in (13).

Note that we have $D_k = \sum_i R_i^{(1)} / L$, where $\nu = 0, \ldots, \min \{M, N \}$ is the number of beams assigned to user $k$ and $D_k = \frac{M \hat{\eta}(K)}{L \mu_k}$ since SINR's are i.i.d. over $k$ and $n = 1, \ldots, N$, the probability for a user being assigned $\nu$ beams approximately follows a binomial distribution $B(M, \rho)$, with $\rho = \frac{1}{K}$. It is well-known that $B(M, \rho) \sim \text{Poiss}(\rho)$ with $\rho = \frac{M}{KL}$, as $M, K \to \infty$. Therefore, $D_k$ approximately follows the distribution of

$$\hat{D}_k(K) = \frac{\xi}{L} K \eta(K)$$

(36)

where $\xi \sim \text{Poiss}(\rho)$. The LMF of $\hat{D}_k$ can be easily obtained as $\lambda_D^K(\theta) = \mu_b (e^{\theta} - 1)$. Note that $Q_{\max}(t)$ and $Q_k(t)$ are identical under the CSI-only algorithm. Therefore, we have an explicit expression of the LMF as

$$g(x, \theta) = \Lambda_A(\theta) + \lambda_D(x, -\theta) = \lambda(e^{\theta} - 1) + \mu_b (e^{\theta} - 1).$$

Using Theorem 4 and solving $g(x, \theta) = 0$, we obtain $e^{\theta} = 1$ and $e^{\theta} = \frac{\mu_b}{\lambda}$. One can verify that $e^{\theta}$ yields trivial solution $I^* = 0$. Then we have

$$I^* \approx \frac{\mu_b}{\lambda} = \log \frac{M \hat{\eta}(K)}{\lambda_{tot} L}.$$ (37)

Moreover, using the extreme value theorem, we obtain $\mathbb{E} R_i^{(1)} / \log (P \log N) \to 1$, as $K \to \infty$ [2], which implies $\hat{K}^\eta(K) \to \log (P \log N K)$. Therefore, we further have $I^* \approx \frac{\mu_b}{\lambda_{tot}} \log \frac{M \hat{\eta}(K)}{\lambda_{tot} L}$. The conditions of Theorem 4 are satisfied when $\mu_b > \lambda$, or approximately, $\mu_b \approx \frac{M \hat{\eta}(K)}{\lambda_{tot} L}$. Therefore, we have

APPENDIX G
PROOF OF LEMMA 5

Consider an upper bound ordered queue length profile as follows, $Q_{\pi(1)} = Q_{\max}$ and $Q_{\pi(j)} = Q_{\max}(1 - \frac{j-1}{K})$, where $\delta \geq 0$ is chosen such that $Q_{\pi(j)} \leq Q_{\pi(j)}$ for all $j = 1, \ldots, K$. We first note that using the extreme value theorem, we have $\hat{K}^\eta(K) / \log (P \log N K) \to 1$, as $K \to \infty$ [2], which implies that $\hat{K}^\eta(K) \to M \log (P \log N K)$. Focusing on large $K$, we may typically obtain a large $S^*$ which can validate the asymptotic approximation of $\hat{\eta}(S)$. Thus we solve the outer subproblem (21) by substituting $Q_{\pi(k)}$ with $Q_{\pi(k)}$ and $\eta_{\pi(k)}(S) = \frac{M \hat{\eta}(K)}{S} \log (P \log N S)$ as follows,

$$\max_{\delta} g(\tilde{S}) = \frac{Q_{\max}}{2K} (2K + \delta - 3\delta \tilde{S}) \log (P \log N \tilde{S}) - V \tilde{S}.$$ (38)

It can be shown that $g(\tilde{S})$ is concave. Taking derivative of $g(\tilde{S})$, and setting $g'(\tilde{S}^*) = 0$, we have $\tilde{S}^* = \log (P \log NS^*) + \frac{1}{\log NS^*} - \frac{1}{S^* \log NS^*}$.
Therefore, we have \( N \hat{S}^* \log N \hat{S}^* \leq \left( \frac{V}{Mq_{\text{max}}} \right)^{-1} N = \frac{Mq_{\text{max}}}{N} \hat{c}_1 \), for \( S^* \geq 3 \) and all \( \delta \geq 0 \). Thus we have \( \hat{S}^* \leq \frac{1}{\epsilon} e^{W(c_1)} \). Note that, under \( \delta \rightarrow 0 \), we have \( \bar{Q}_{\pi(k)} \downarrow Q_{\text{opt}}(k) \) and \( \hat{\delta} \), which means the upper bound is achieved when \( Q_{\pi(k)} \approx Q_{\text{max}} \).

Note that, in the outer subproblem (21), increasing \( Q_{\pi(k)} \) to \( \bar{Q}_{\pi(k)} \) for every \( k \) yields a larger solution point \( S^*(Q_{\text{max}}) \geq S^*(Q) \) [due the term \( \sum_{k=1}^{S} Q_{\pi(k)} \)]. Hence, we have \( S^*(Q) \leq S^*(Q_{\text{max}}) \leq \frac{1}{\epsilon} e^{W(c_1)} \).

**APPENDIX H**

**PROOF OF THEOREM 6**

In Lemma 4 the departure rate \( D_{\text{max},b}(t; S) \) can be approximately given in (30), which is a decreasing function of \( S \) and has a Poisson distribution with mean \( D_{\text{max},b}(t; S) = \frac{M[S]}{T} \).

With Lemma 4[5] we have \( D_{\text{max},b}(t; S^*) \geq D_{\text{max},b}(t; S^*) \geq D_{\text{max},b}(t; S^*(Q_{\text{max}})) \), since \( S^* \leq S^* \). Moreover, using the extreme value theorem, we have \( D_{\text{max},b}(t; S^*(Q_{\text{max}})) \rightarrow 1 \), as \( K \rightarrow \infty \) [2], which implies \( D_{\text{max},b}(t; S^*(Q)) \)

\[
\frac{M}{LS^*(Q_{\text{max}})} \log(P \log N \hat{S}^*(Q_{\text{max}})) \doteq \hat{\mu}_p(Q_{\text{max}}).
\]

Consider the performance lower bound driven by the packet arrival process \( A(t) \) and departure process \( D_{\text{max},b}(t, S^*(Q_{\text{max}})) \), which are both Poisson processes. The corresponding LMF is given by

\[
\hat{g}(x, \theta) = \lambda(e^\theta - 1) + \hat{\mu}_p(x)(e^{-\theta} - 1) \tag{38}
\]

where \( x = Q_{\text{max}} \). Using Theorem 4 and solving \( \hat{g}(x, \theta) = 0 \), we obtain \( e^\theta = 1 \) and \( e^\theta = \hat{\mu}_p(x) \). One can verify that \( e^\theta = 2 \) only yields a trivial solution \( \hat{I}^* = 0 \). We thus calculate the lower bound rate function by \( \hat{I}^* = \int_0^1 \log \hat{\lambda}_p(x) dx \).

Here, additional tricks should be used to complete the integral. Note that when \( Q_{\text{max}} \) is small, \( S^*(Q_{\text{max}}) \) is small, which violates the large \( S \) asymptotic assumption to obtain the approximated departure rate \( D_{\text{max},b}(t, S^*(Q_{\text{max}})) \). To fix this, we use the following augmented approximation, \( \hat{\mu}_p(Q_{\text{max}}) = \max \{ \hat{\mu}_p(Q_{\text{max}}), \frac{M_r}{LK} \} \), where \( r_0 = \int_0^{\infty} \log(1 + x) dF\).

Note that \( r_0 \) is the average per-beam data rate, and hence \( \frac{M_r}{LK} \) is a lower bound average package departure rate for the maximum queue process \( Q_{\text{max}}(t) \).

Note that \( \hat{\mu}_p(x) \) is monotonically increasing. Define \( \epsilon_k \) as the solution to \( \hat{\mu}_p(x) = \frac{M_r}{LK} \), and \( \epsilon = \inf \{ \epsilon_k : K \geq K_0 \} \) for some \( K_0 < \infty \). Using Theorem 3 we have

\[
\hat{I}^* = \int_0^1 \log \frac{\hat{\lambda}_p(x)}{\bar{\lambda}_k} dx = \int_0^1 \log \left( \frac{1}{\bar{\lambda}_k} \right) \max \left\{ \frac{M \log(P \log N \hat{S}^*(x))}{LS^*(x)}, \frac{M r_0}{LK} \right\} dx
\]

\[
= \log \left( \frac{M}{\bar{\lambda}_k L} \right) + \int_0^\epsilon \log r_0 dx + \int_\epsilon^1 \log \left( \frac{P \log N \hat{S}^*(x)}{S^*(x)} \right) dx
\]

\[
= \log \left( \frac{M}{\bar{\lambda}_k L} \right) + \int_0^\epsilon \log r_0 dx + \int_\epsilon^1 \log \left( \frac{P \log N \hat{S}^*(x)}{S^*(x)} \right) dx
\]

\[
= \log \frac{M}{\bar{\lambda}_k L} + \epsilon \log r_0 + (1 - \epsilon) \log K = C \doteq I_{\text{LIM}}
\]

where \( C = \int_0^1 \{ \log \left[ N \log (P \log \left( \frac{M}{\bar{\lambda}_k L} \right)) \right] - W \left( \frac{M}{\bar{\lambda}_k L} \right) \} dx \).

The first inequality is because \( \hat{\mu}_p(Q_{\text{max}}) \) is a lower bound estimation for the departure.

Since \( D_{\text{max},b}(t; S^*) \geq D_{\text{max},b}(t; S^*) \), we have \( I_{\text{LIM}} \geq I^* \).

Thus we have proven the result.

**APPENDIX I**

**PROOF OF THEOREM 6**

We first study the effect of the outdated QSI. Let \( m(t) = \arg \max_k Q_k(t) \) be the user who has the longest queue at time \( t \). Let \( F(t) \) denote the feedback group under the proposed FFCA with \( T = 1 \). We concern with whether the feedback group \( F(t) \) still contains the longest queue user \( m(t) \) at time \( t \), i.e., the event \( m(t) \in F(t) \) happens at time \( t \).

Consider the “best effort” event: the user \( m(t) \) is scheduled at every time slot but is still in the feedback group \( F(t) \) at time \( t \),

\[
E_{BE}(t) \doteq \left\{ Q_{\max}(t_0) - \sum_{\tau=0}^{t} d_{m(t_0)}(\tau) \right\} \doteq \left\{ Q_{\max}(t_0) - \sum_{\tau=0}^{t} A_{m(t_0)}(\tau) \right\}
\]

where \( d_{m(t_0)}(H_{m(t_0)}(\tau)) \) is the packet departure rate under a fictitious “best effort” policy that schedules user \( m(t_0) \) at every time slot regardless of \( Q(\tau) \). Specifically, according to [19], the distribution of \( d \) is given by

\[
Pr(d \leq x) = \Pr(\log(1 + P \text{SINR}) \leq x)
\]

\[
= \Pr(\text{SINR} \leq P^{-1}(2^x - 1))
\]

\[
= F(P^{-1}(2^x - 1)).
\]

In addition, \( \pi^-(t_0) = \pi(S^*(Q(t_0)) + 1) \) is the user who just cannot be selected in the feedback set \( F(t_0) \) at \( t_0 \). (Just recall that \( \pi(\cdot) \) is the ordered permutation of \( Q \).) In \( E_{BE} \), one schedules the outdated longest queue user \( m(t_0) \) at every time slot, but still, no user from outside \( F(t_0) \) has the longest queue at time \( t \). Note that we must have \( Q_{m(t_0)}(t_0) \geq Q_{BE}(t_0) \) almost surely, where \( Q_{m(t_0)}(t) \) is the queue length for user \( m(t_0) \) under the queue-weighted scheduling in Stage II, and \( Q_{BE}(t_0) \) is under the “best effort” scheduling. Therefore, we must have \( \Pr(m(t) \in F(t)) \geq \Pr(E_{BE}(t)) \), for \( t \leq t_0 + T - 1 \). The upper bound is tight in the heavy queue region for small \( T \).

Moreover, since \( Q_{m(t_0)}(t_0) > Q_{BE}(t_0) \), under the i.i.d. assumption for the arrivals \( A_k(t) \) and the CSI \( H_k(t) \) respectively, we have

\[
Pr(E_{BE}(t)) \geq \Pr\left\{ \sum_{\tau=0}^{t} v(\tau) > 0 \right\} \doteq P_{0} t_0 \geq P_{0}^T
\]

where \( v(\tau) = A_1(\tau) - A_2(\tau) - d(\tau) \). The last inequality holds, since \( E[v(\tau)] < 0 \) and \( \sum_{\tau=1}^{\infty} v(\tau) \) is more negative as \( t - t_0 \) increases.
We then study the departure rate for the process $Q_{\text{max}}(t)$. Denote $D_{\text{max}}(T)(h, t, Q(t); S^*(Q(t), F(t)))$ as the packet departure for $Q_{\text{max}}(t)$ under the $T$-step feedback policy in $t_0 \leq t \leq t_0 + T - 1$, where the feedback probability is updated at time $t_0$. Similarly, denote $D_{\text{max}}(T)(h, t, Q(t); S^*(Q(t), F(t)))$ as the packet departure under the per time slot feedback policy update ($T = 1$). We have,
\[
D_{\text{max}}(T)(h, t, Q(t); S^*(Q(t), F(t))) \\
\approx D_{\text{max}}(h, t, Q(t); S^*(Q(t), F(t))) \cdot 1\{m(t) \in F(t_0)\} \\
\geq D_{\text{max}}(h, t, Q(t); S^*(Q(t), F(t))) \cdot 1\{\mathcal{E}_{\text{BE}}(t)\}
\]

where the lower bound is tight in heavy queue region and $T$ is small. The first approximate equality holds, since when the user with the maximum queue is outside the feedback group under outdated QSI, $Q_{\text{max}}(t)$ cannot be served at all.

According to Theorem 1, we then need to find the solution of the LMF $\bar{g}(x, \theta_T) = 0$ under the $T$-step policy. The LMF of the random variable $D_{\text{max}}(T) \cdot 1\{\mathcal{E}_{\text{BE}}\}$ is given by
\[
\Lambda_{\text{D}}^T(\theta) = \log \mathbb{E}[\exp(\theta D_{\text{max}}(T)\cdot 1\{\mathcal{E}_{\text{BE}}\})] = \log \mathbb{E}\{\exp(\theta D_{\text{max}}(T)\cdot 1\{\mathcal{E}_{\text{BE}}\})\mid 1\{\mathcal{E}_{\text{BE}}(t)\}\}
\]
\[
= \log (1 - P_0^T + P_0^T \Lambda_{\text{D}}(\theta))
\]

and the local LMF for the queuing process $Q_{\text{max}}(t)$ is
\[
\bar{g}(x, \theta_T) = \Lambda_{\text{A}}(\theta) + \log (1 - P_0^T + P_0^T \mathcal{M}_{\text{D}}(x, -\theta))
\]

where $\mathcal{M}_{\text{D}}(x, -\theta)$ is the MGF of $D_{\text{max}}(T)$. 

To find the root $\beta_T(\theta)$ of the above function, we consider a linearization,
\[
\bar{g}_L(x, \theta_T) = \bar{g}(x, \theta_T) + \nabla_\theta \bar{g}(x, \theta_T) \Delta \theta,
\]
and where $\theta_0(x)$ is the solution to $\bar{g}_L(x, \theta_T) = 0$ in $\mathbf{R}^T$ under the $T = 1$ policy. Let $\beta_0 \triangleq \epsilon_0^\theta$ and $\Delta \beta \approx \epsilon_\theta \beta_0 - \beta_0$. Setting $\bar{g}_L(x, \theta_T) = 0$, we obtain,
\[
\Delta \beta \approx \frac{\mu(x) - \lambda}{\beta_0(x)} \log \left(1 - P_0^T + P_0^T e^{(\lambda - \mu(x))} \right)
\]
\[
\Delta \beta \approx \frac{\mu(x) - \lambda}{\beta_0(x)} \log \left(1 - P_0^T + P_0^T e^{(\lambda - \mu(x))} \right)
\]
\[
= \frac{\mu(x) - \lambda}{\beta_0(x)} \log \left(e^{\mu(x)} - e^{\lambda} - 1\right) P_0^T
\]
\[
\Delta \beta \approx \rho(x).
\]

The approximation, which is obtained by linearization, becomes accurate when $P_0^T$ is close to 1. Therefore, using Theorem 1, the rate function under the $T$-step feedback policy is bounded by
\[
I_{\text{prop}}^{(T)} \geq \int_0^1 \theta_T^*(x) dx = \int_0^1 \log \beta_0 \left(1 + \frac{\Delta \beta(x)}{\beta_0(x)}\right) dx
\]
\[
\geq I_{\text{LB}} - \int_0^1 \rho(x) dx.
\]

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