Generalized rational blow-down, torus knots, and Euclidean algorithm

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Abstract

We construct a Kirby diagram of the rational homology ball used in “generalized rational blow-down” developed by Jongil Park. The diagram consists of a dotted circle and a torus knot. The link is simpler, but the parameters are a little complicated. Euclidean Algorithm is used three times in the construction and the proof.

1 Main theorem

For a coprime pair \((m, n)\) of positive integers, we take a simple closed curve \(k(m, n)\) in the standardly embedded once-punctured torus \(F\) in \(S^3\) as in Figure 1. We study the Kirby diagram \(k(m, n) \cup u\): the component \(k(m, n)\) is a torus knot \(T(m, n)\) with \((mn)\)-framing, and \(u\) is a dotted unknoted circle (It is a 1-handle, see [A], [AK] and [GS, p.168]) in the complement of \(F\). This diagram defines a rational homology ball that has cyclic fundamental group of order \((m + n)\). It has a symmetry: \(k(n, m) \cup u = k(m, n) \cup u\).

In the next section, for a given coprime pair \((p, q)\) of positive integers with \(1 \leq q < p\), we will construct an involutive symmetric function \(A\) by Algorithm, to decide (another) coprime pair \((m, n) = A(p - q, q)\) satisfying \(m + n = p\). It holds that \(A(p - 1, 1) = (p - 1, 1)\),

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Figure 1: \(F, k(m, n)\) in \(F\) ex. \(k(2, 3) \cup u\) (\(= L_{5,2}\)
Figure 2: Plumbed manifold $C_{p,q}$

$A(p-2,2) = ((p-1)/2, (p+1)/2)$ for odd $p$, see Lemma 2.3 and 2.5. Now, we let $L_{p,q}$ denote Kirby diagram $k(A(p-2, q)) \cup u$. Our main theorem is:

**Theorem 1.1** For any coprime pair $(p, q)$ of positive integers with $1 \leq q < p$, the boundary of the rational homology ball described by $L_{p,q}$ defined above, is a lens space $L(p^2, pq-1)$.

Thus, we can regard $L_{p,q}$ as a description of the rational homology ball $B_{p,q}$ in general rational blow-down defined by J. Park in [P] applying [FS2] via [CH]. It is the operation “cut out $C_{p,q}$ and paste $B_{p,q}$” on a 4-manifold, where $C_{p,q}$ is the negative definite plumbed 4-manifold corresponding to the weighted graph in Figure 2. The weights ($-c_i$)’s ($-c_i \leq -2$ for each $i$) are defined by the continued fraction expansion:\n
$$p^2/(pq-1) = [c_0, c_1, \cdots, c_N],$$

Thus $\partial C_{p,q} = L(p^2, pq-1) = \partial B_{p,q}$. Note that $C_{p,q}$ has a symmetry: $C_{p,p-q} = C_{p,q}$, corresponding to the reverse of the continued fraction $[c_N, \cdots, c_1, c_0]$, and also to the homeomorphism $L(p^2, p(p-q)-1) \cong L(p^2, pq-1)$.

Our strategy of the proof is: First, in the next section, we will present Algorithm A, based on Euclidean algorithm of the pair $(p-q, q)$. In the process, we construct a word $w(p-q, q)$ of $L$ and $R$, its “reverse” $W(p-q, q)$, decide integers $n_L, n_R$, and a finite sequence

$$(a_1, a_2, \cdots, a_n, c, a_{-n}, \cdots, a_{-2}, a_{-1})$$

satisfying $a_i \leq -2$ (for each $i$) and $c \leq -4$. Next in Section 3, we show that this sequence agrees to $(-c_0, -c_1, \cdots, -c_N)$, or its reverse $(-c_N, \cdots, -c_1, -c_0)$. Finally, in Section 4, we prove Theorem 1.1 by a sequence of Kirby calculus, guided by the word $W(p-q, q)$ constructed in Algorithm A. The process is related to the resolution (HIKK [L]) of the singularity of the complex curve of type $z^m - w^n = 0$, or unknotting twisting sequence on torus knots, that is, Euclidean algorithm.

Note that $(mn)$-framed $T(m, n)$ is a kind of the most “exceptional” Dehn surgery, see [MI]. Similar algorithm has been already discussed by the author in [Y3] (whose older version is in [Y1]) in the study of exceptional Dehn surgery. The operation “reverse” of the word at Step(2) in Algorithm A, is in contrast to the old results, and cause difficulty in the construction and the proof. Some parts (ex. Figure 7) are modification from the manuscript of [Y3], but we rewrite them to make the present paper self-contained.
To the author’s knowledge, descriptions of $B_{p,q}$ of some concrete $(p,q)$ can be seen, in \cite{SS} and \cite{R}. Our method is different from theirs: Non-trivial component of the diagram of $B_{28,9}$ in \cite{SS} is 251-framed $T(28,9)$ ($251 = 28 \cdot 9 - 1$), see also Remark \ref{r}. Difference between theirs and ours looks like a kind of “dual”, or “complemental” in the sense that our function $A$ is involutive.

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2 Algorithm

Here we present the algorithm to define $(m, n) = A(p - q, q)$ via words $w(a, b)$, its reverse $W(a, b)$ (Here $a = p - q, b = q$), define the integers $n_R, n_L$, and the sequence

$$(a_1, a_2, \cdots, a_{n_L}, c, a_{n_R}, \cdots, a_2, a_1), \quad c = a_{n_L + 1} + a_1 - (n_R + 1) - 2.$$ 

This algorithm is closely related to the resolution \cite{HKK} of the singularity of the complex curve of type $z^a - w^b = 0$, that is, Euclidean algorithm. We also show some formulas on $A$.

It may be curious, but we start with an example, which would help the readers.

Example 2.1 $(a, b) = (7, 2)$ (corresponding to $(p, q) = (9, 2)$)

$$(a_1, b_1) : \quad \begin{array}{llllll}
7, 2 & \leftarrow & L & (5, 2) & \leftarrow & L & (3, 2) & \leftarrow & L & (1, 2) & \rightarrow & R & (1, 1).
\end{array}

w(7, 2) = LLLR.

$(m_i, n_i) : \quad (1, 1) \leftarrow L & (2, 1) & \leftarrow L & (3, 1) & \leftarrow L & (4, 1) & \rightarrow R & (4, 5).$

Thus $A(7, 2) = (4, 5)$.

$(s_i, t_i) : \quad (1, 0) \leftarrow L & (1, 0) & \leftarrow L & (1, 0) & \rightarrow R & (1, 1).

n_L = 3, n_R = 1, W(7, 2) = RLLL.

$$

\begin{array}{cccccc|cccccc}
 i & a_1^{(i)} & a_2^{(i)} & a_3^{(i)} & a_4^{(i)} & a_5^{(i)} & i & \overline{a}_1^{(i)} & \overline{a}_2^{(i)} & \overline{a}_3^{(i)} & \overline{a}_4^{(i)} & \overline{a}_5^{(i)} \\
\hline
 0 & -1 & -1 & -1 & & & 0 & -4 & & & & \\
 1 & -1 & -2 & -1 & -2 & & 1 & -5 & -2 & & & \\
 2 & -1 & -3 & -1 & -2 & -2 & 2 & -5 & -3 & & & \\
 3 & -1 & -4 & -1 & -2 & -2 & -2 & -5 & -4 & & & \\
 4 & -1 & -5 & -1 & -2 & -2 & -2 & -2 & -5 & -5 & & \\
\end{array}

\begin{array}{cccc}
 & a_1^{(i)} & a_2^{(i)} & a_3^{(i)} & a_4^{(i)} & a_5^{(i)} \\
\hline
\end{array}
\begin{array}{cccc}
 i & \overline{a}_1^{(i)} & \overline{a}_2^{(i)} & \overline{a}_3^{(i)} & \overline{a}_4^{(i)} & \overline{a}_5^{(i)} \\
\hline
 0 & -4 & & & & \\
 1 & -5 & -2 & & & \\
 2 & -5 & -3 & & & \\
 3 & -5 & -4 & & & \\
 4 & -5 & -5 & & & \\
\end{array}

We get the sequence $(-2, -2, -2, -5, -5) = C_{9,2}$, and $[5, 5, 2, 2, 2] = 81/17$. See Figure 4.

Algorithm A

(1) Euclidean algorithm: From the pair $(a, b) = (p - q, q)$, we construct a word $w(a, b) = w_1 w_2 \cdots w_n$ of two letters $L(left)$ and $R(right)$, and a sequence of the pair $\{(m_i, n_i)\}$, inductively, by the rule below:

Start with $(a_0, b_0) := (a, b), (m_0, n_0) := (1, 1)$

If $a_i > b_i$, then $w_{i+1} := L$ and

$$(a_{i+1}, b_{i+1}) := (a_i - b_i, b_i), \quad (m_{i+1}, n_{i+1}) := (m_i + n_i, n_i).$$

(LR Rule) If $a_i < b_i$, then $w_{i+1} := R$ and

$$(a_{i+1}, b_{i+1}) := (a_i, b_i - a_i), \quad (m_{i+1}, n_{i+1}) := (m_i, n_i + m_i).$$
By coprime-ness of \((a, b)\), after some \(N\) steps, the pair \((a_N, b_N)\) becomes to \((1, 1)\), which is the end of this step.

**Definition 2.2** We define \(n_R\) (and \(n_L\), respectively) as the number of \(R\) (and \(L\)) in the word \(w(a, b)\). Thus \(n_R + n_L = N\). We define

\[
A(a, b) := (m_N, n_N).
\]

(2) Let \(W(a, b) = W_1W_2 \cdots W_N\) be the reverse of \(w(a, b)\), i.e., \(W_i = w_{N+1-i}\) for each \(i\). It is easy to see

**Lemma 2.3** Let \((a, b)\) a coprime pair of positive integers.

1. If \(A(a, b) = (m, n)\), then \(W(a, b) = w(m, n)\), i.e., \(W(a, b) = w(A(a, b))\).
2. \(A\) is involutive; If \(A(a, b) = (m, n)\), then \(A(m, n) = (a, b)\).
3. \(A\) is symmetric; If \(A(a, b) = (m, n)\), then \(A(b, a) = (n, m)\).
4. \(A(a, 1) = (a, 1)\). If \(a\) is odd, \(A(a, 2) = \left(\frac{a+1}{2}, \frac{a+3}{2}\right)\).

We go back to Algorithm A.
Next, starting with

\[ \{a_s^{(0)}\} = (a_{-1}^{(0)}, a_0^{(0)}, a_1^{(0)}) := (-1, -1, -1), \]

based on the blow-up diagram in Figure 3 (see also Figure 4), we define the sequences \( \{a_s^{(i)}\} \) and \( c^{(i)} \) \((i = 1, 2, \cdots, N)\) inductively: For each \( i \), \( a_0^{(i)} = -1 \) and \( c^{(i)} = a_{M(i)}^{(i)} + a_{m(i)}^{(i)} - 2 \), where \( M(i) \) (and \( m(i) \), respectively) is the maximum (or the minimum) in \( \{ j \in \mathbb{Z} | a_j^{(i)} \text{ is defined} \} \).

Now, using \( W_i \)'s (contrast to \([Y3]\)),

If \( W_i = R \), then we define \( \{a_s^{(i)}\} \) as

\[
\begin{cases}
  a_j^{(i)} := a_j^{(i-1)} & \text{if } 1 < j \leq M(i-1) \\
  a_1^{(i)} := a_1^{(i-1)} - 1, \\
  a_{-1}^{(i)} := -2, \\
  a_j^{(i)} := a_{j+1}^{(i-1)} & \text{if } m(i-1) - 1 \leq j < -1
\end{cases}
\]

If \( W_i = L \), then we define \( \{a_s^{(i)}\} \) as

\[
\begin{cases}
  a_j^{(i)} := a_j^{(i-1)} & \text{if } m(i-1) \leq j < -1 \\
  a_1^{(i)} := a_{-1}^{(i-1)} - 1, \\
  a_{-1}^{(i)} := -2, \\
  a_j^{(i)} := a_{j-1}^{(i-1)} & \text{if } 1 < j \leq M(i-1) + 1
\end{cases}
\]

(4) For each integer \( j \) with \(-n_R \leq j \leq n_L \), we define \( a_j \) as \( a_j^{(N)} \) in the sequence \( \{a_s^{(N)}\} \) obtained after the \( N \)-th step, where \( N \) is the length of the word \( W(a, b) \). We also define \( c := c^{(N)} = a_{nL+1} - a_{-(nR+1)} - 2 \). This is the end of Algorithm A \( \Box \)

Related to Seifert fibration of \( S^3 \) whose regular fiber is the torus knot \( T(m, n) \), it is well-known:

**Lemma 2.4** If \( m < n \) (or if \( m > n \), respectively), then \( a_{-(nR+1)} = -1 \) (or \( a_{nL+1} = -1 \), and

\[
|a_{-(nR+1)}|, |a_{-nR}|, \cdots, |a_2|, |a_{-1}| = \begin{cases} 
  m/n & \text{if } m < n \\
  n/m & \text{if } m > n
\end{cases},
\]

\[
|a_{nL+1}|, |a_{nL}|, \cdots, |a_2|, |a_1| = \begin{cases} 
  n/m & \text{if } m < n \\
  m/n & \text{if } m > n
\end{cases}.
\]

Here, we add two formulas on \( A \).

**Lemma 2.5** Let \((a, b)\) a coprime pair of positive integers. Suppose \( A(a, b) = (m, n) \), then

(1) \( m + n = a + b \)

(2) Let \( s, t \) be the unique positive integers that satisfies \( mt - ns = -1 \) and \( 0 < s, t < a + b \), then \( s + t = b \).
Proof. We go back to (LR Rule) in the construction of the function $A$. (1) If $w_i = L$, then we took $(a_{i+1}, b_{i+1}) = (a_i - b_i, b_i)$ and $(m_{i+1}, n_{i+1}) = (m_i + n_i, n_i)$. The equality $a_i n_i + b_i m_i = a + b$ is kept, in the process. It is kept also in the case $w_i = R$.

(2) Similarly to $(m_i, n_i)$, we define $(s_i, t_i)$ inductively as: Starting $(s_0, t_0) = (1, 0)$, if $w_i = L$ (or $R$, respectively), then we take $(s_{i+1}, t_{i+1}) = (s_i + t_i, t_i)$ (or $(s_{i+1}, t_{i+1}) = (s_i, t_i + s_i)$). Then the equality $m_i t_i - n_i s_i = -1$ is kept. We have $(s, t) = (s_N, t_N)$. The equality $a_i t_i + b_i s_i = b$ is also kept. □

(3) In addition, we present another algorithm to construct a sequence $\{\overline{\alpha}_s\}$. The author has been informed by J. Park that this is a resolution graph of a quotient singularity of class $T$. The resulting sequence $\{\overline{\alpha}_s\}$ will be agree to the sequence we have constructed in Algorithm A.

Starting with
\[
\{\overline{\alpha}_s^{(0)}\} = (\overline{\alpha}_0^{(0)}) := (-4),
\]
we define the sequences $\{\overline{\alpha}_s^{(i)}\}$ ($i = 1, 2, \ldots, n$) inductively. We set $M(i) := \max\{j \in \mathbb{Z} | \overline{\alpha}_j^{(i)} \text{ is defined}\}$, and $m(i) := \min\{j \in \mathbb{Z} | \overline{\alpha}_j^{(i)} \text{ is defined}\}$.

If $W_i = R$, then we define $\{a_s^{(i)}\}$ as
\[
\begin{align*}
\overline{\alpha}_j^{(i+1)} &:= -2 & \text{if } j = M(i - 1) \\
\overline{\alpha}_j^{(i)} &:= \overline{\alpha}_j^{(i-1)} - 1 & \text{if } j = m(i - 1) \\
\overline{\alpha}_j^{(i)} &:= \overline{\alpha}_j^{(i-1)} & \text{if } m(i - 1) < j \leq M(i - 1)
\end{align*}
\]

If $W_i = L$, then we define $\{a_s^{(i)}\}$ as
\[
\begin{align*}
\overline{\alpha}_j^{(i)} &:= \overline{\alpha}_j^{(i-1)} - 1 & \text{if } j = M(i - 1) \\
\overline{\alpha}_j^{(i)} &:= -2 & \text{if } j = m(i - 1) \\
\overline{\alpha}_j^{(i)} &:= \overline{\alpha}_j^{(i-1)} & \text{if } m(i - 1) \leq j < M(i - 1)
\end{align*}
\]

Finally, we define $\overline{\alpha}_j$ as $\overline{\alpha}_j^{(N)}$ in the sequence $\{\overline{\alpha}_s^{(N)}\}$ obtained after the $N$-th step. Note that $\overline{\alpha}_0^{(i)} = c^{(i)}$ and $\overline{\alpha}_0 = c(= a_{nL+1} + a_{-(nR+1)} - 2)$.

Lemma 2.6 Two resulting sequence agrees to each other, i.e.,
\[(a_1, a_2, \ldots, a_{nL}, c, a_{-nR}, \ldots, a_{-2}, a_{-1}) = (\overline{\alpha}_{-nL}, \ldots, \overline{\alpha}_0, \ldots, \overline{\alpha}_{nR}).\]

Proof. (See Figure 4 again.) As a cyclic diagram, $(a_1, a_2, \ldots, a_{nL}, c, a_{-nR}, \ldots, a_{-2}, a_{-1})$ connected by $a_0 = (-1)$ in natural order, agrees to that of $(\overline{\alpha}_{-nL}, \ldots, \overline{\alpha}_{nR})$ connected by $\overline{\alpha}_{-(nL+1)} := -1 =: \overline{\alpha}_{nR+1}$. We have the lemma. □

3 Sequence and lens space

From a given coprime pair $(p, q)$, the sequence $(a_1, a_2, \ldots, a_{nL}, c, a_{-nR}, \ldots, a_{-2}, a_{-1})$ has been constructed in the previous section. Now, we show
Figure 5: Lens space \((C_{p,q})\)

**Lemma 3.1** The plumbed 4-manifold of the weighted tree of the above sequence is diffeomorphic to \(C_{p,q}\). In other words, the continued fraction expansion of \(p^2/(pq-1)\) agrees to the sequence of the absolute values, up to reverse, i.e., it holds

\[
|a_1|, \ldots, |a_{n_L}|, |c|, |a_{-n_R}|, \ldots, |a_{-1}| = \frac{p^2}{pq-1},
\]

or

\[
|a_{-1}|, \ldots, |a_{-n_R}|, |c|, |a_{n_L}|, \ldots, |a_1| = \frac{p^2}{pq-1}.
\]

**Proof.** We also use \((m, n) = A(p - q, q)\), and \((s, t)\) satisfying \(mt - ns = -1\) defined in Section 2. By Lemma 2.4 up to reverse, we can contract the weighted graph as in Figure 5. In general, if two fractions at the vertices are \(-\frac{a_1}{\beta_1}\) and \(-\frac{a_2}{\beta_2}\), the corresponding lens space is \(L(P, Q)\) with \(P = \alpha_1 \alpha_2 - \beta_1 \beta_2, \ Q \equiv \alpha_1 \gamma_2 - \beta_1 \delta_2 \pmod{P}\), where \(\gamma_2, \delta_2\) are integers satisfying \(\alpha_2 \delta_2 - \beta_2 \gamma_2 = -1\). In our case \((\alpha_2, \beta_2) = (m, n)\), by Lemma 2.5(2), we set \((\gamma_2, \delta_2) = (s, t)\). Thus,

\[
P = m \cdot m + n \cdot (2m + n) = (m + n)^2, \quad Q = m \cdot s + (2m + n) \cdot t = m(s + t) + nt + nt = m(s + t) + (ns - 1) + nt = (m + n)(s + t) - 1.
\]

By Lemma 2.5(1) \(m + n = a + b = p\), and (2) \(s + t = b = q\), we have \(P = p^2, Q = pq - 1\). By the uniqueness of the continued fraction expansion (with \(a_i < -2, c < -2, \) and \(c_i < -2\)), we have the lemma. \(\Box\)

### 4 Proof of the Main Theorem

Let \((a, b) = (p - q, q)\) and \(A(a, b) = A(p - q, q) = (m, n)\) as before. Here we prove that the boundary of the rational ball described by Kirby diagram \(L_{p,q} = k(m, n) \cup u\) is homeomorphic to \(L(p^2, pq - 1)\).

We have defined \(F\) as a standardly embedded once-punctured torus in \(S^3\), see Figure 1 again. It consists of a disk \(D\) and two bands \(b_L\) and \(b_R\). We took a simple closed curve \(k(m, n)\) in \(F\) as in Figure 1. The framing of \(k(m, n)\) defined by the surface \(F\) is \((mn)\). From now on, we call such a framing \(F\)-framing ("surface framing").

Our first Kirby move is in Figure 6 where, and from now on, we draw neither \(D\) nor the components \(k(m, n)\)'s: (1) Exchange the dotted circle \(u\) to a 0-framed same component, say
Figure 6: First Kirby move

\( u' \). This operation corresponds to a surgery (cut out \( S^1 \times D^3 \) and paste \( D^2 \times S^2 \)) in the interior of the rational ball, see [K, p.7] or [GS, p.168]. Thus the boundary is unchanged. (2) Blow-up. The central crossing is changed.

Before starting the next step, we define a notation: \((m'_i, n'_i) := (m_{N+1-i}, n_{N+1-i})\), where \(\{(m_j, n_j)\}\) is the sequence of the pair constructed in Step(1) in Algorithm A. Thus, \((m_0, n_0) = (m, n)\) decreases to \((m_N, n_N) = (1, 1)\) guided by \(w(m, n) = W(a, b) = W_1W_2 \cdots W_N\) (Lemma 2.3(1)), i.e., it holds that

If \(m'_i < n'_i\), then \(W_{i+1} = R\) and \((m_{i+1}, n_{i+1}) = (m_i, n_i - m_i),\) and

If \(m'_i > n'_i\), then \(W_{i+1} = L\) and \((m_{i+1}, n_{i+1}) = (m_i - n_i, n_i)\).

Next, guided by \(W_1W_2 \cdots W_N\), we move \(F\) and the curve \(k(m, n) = k(m_0, n_0)\) simultaneously in the total space \(S^3\), inductively \((i = 0, 1, 2, \cdots, N)\): If \(W_{i+1} = R\) (i.e., \(m_i < n_i\)), we move the left band \(b_L\) over the central \((-1)\)-component and slide over \(b_R\) as in Figure 7. In the black box, in the first step, we take a tangle \(T\) (two sub-arcs of the same component \((-4)\)-framed \(u'\)), and in the second or later steps, we take the tangle that appeared in the gray box at the end of the previous step, inductively. In the case \(W_{i+1} = L\), exchange the right and the left, but it is similar by symmetry. Note that after a set of operation in Figure 7 which includes one blow-up, \(F\) comes back to the starting position and \(k(m, n)\) is changed to \(k(m_i, n_i)\) in \(R\) case or to \(k(m_i - n_i, n_i)\) in \(L\) case, that is, \(k(m_{i+1}, n_{i+1})\) in either case and new \((-1)\)-component appears for the next step. Note that the relation “\(F\)-framing of \(k(m_i, n_i)\) is \(k(m, n)\)” is kept during the process.

After \(N\) steps \((N\) is the length of the word \(w(m, n)\), see also Lemma 2.3(1)), the diagram that we hoped appears at the black box, because this sequence of blow-ups exactly same with the construction \((3)'\) of \(\{\pi_s\}\) in Section 2. By Lemma 2.6 and 3.1, it is the diagram of \(C_{p,q}\), up to reverse. The final \((-1)\)-curve \(\gamma\) and a \((+1)\)-framed curve \(\gamma' := k(m_N, n_N) = k(1, 1)\) in \(F\). Sliding \(\gamma'\) over \(\gamma\), we can cancel them. The proof of Theorem 1.1 is completed. □

Our proof shows:

Remark 4.1 By the Kirby calculus in our proof, the \(0\)-framed meridian of \(k(m, n)\) in \(L_{p,q}\) comes to \((-1)\)-framed \(\gamma\) in the diagram of \(C_{p,q}\) in Figure 8. This is (as a link component, at least) different from the example in [SS], or the one obtained by the method in [R] and [CH].
Figure 7: Operation (R case)

Figure 8: $C_{p,q}$ with $\gamma$
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