From digital image correlation to damage law identification

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Abstract. In this paper, it is proposed to identify a damage law based upon kinematic fields. The equilibrium gap method is used to determine the parameters of an anisotropic damage model. The approach is applied to a practical biaxial test performed on a C/C composite.

1. Introduction
Composite materials, as many other materials, have a complex anisotropic behavior induced by matrix cracking and fiber breakage. Many models have been proposed to describe these mechanisms [1]. Unfortunately, the identification of the numerous parameters introduced in these models often requires many tests performed at various scales. Furthermore, the validity of the identification result is usually difficult to check against experimental data.

The recent advances of full field measurements and simulation tools are opening the way for convenient and robust identification methods. With these methods, one may envision using them not only to identify parameters but also to check the real contribution of a supposed refinement. A continuous pathway from images to mechanical properties may then be possible [2]. Based on a new Digital Image Correlation scheme (Q4-DIC [3]) and on the Equilibrium Gap Method (EGM [4]), the so-called Digital Image Mechanical Identification (DIMI) approach allows one to retrieve an isotropic damage law directly from pictures acquired during a single multiaxial test. It is proposed to extend DIMI to anisotropic damage cases.

In the first part, the two main inverse problems to go from digital images to an anisotropic damage law are presented. It is first shown how displacement fields are retrieved when a Finite Element (FE) kinematics is introduced in the optical flow conservation. The EGM is then presented and solved when considering both a FE kinematics and a specific decomposition of the anisotropic damage law. In the second part, the very first results of such approach are shown. Both synthetic and real experiments are considered. Non linear simulations of a biaxial shear test performed on a [±45°] laminate are first used to check the ability of the method to identify a given damage law. It is then applied to a similar experimental test performed on a C/C composite.

2. Digital Image Mechanical Identification
When performing an experiment, pictures are taken at different levels of loading. By matching pixels in different pictures, it is possible to measure displacement fields. The latter are
subsequently used to identify the parameters of a damage law.

2.1. From images to displacement fields

The correlation of two gray level images \( f \) and \( g \) (\( f \) is the reference picture and \( g \) the deformed one) is recast as the local passive advection of the texture of two images by a displacement field \( \mathbf{u} \)

\[
g(x) = f(x + \mathbf{u}(x))
\]

(1)

The problem consists in identifying the best displacement field by minimizing the correlation residual

\[
\varphi(x) = |f(x + \mathbf{u}(x)) - g(x)|
\]

(2)

The minimization of \( \varphi \) is intrinsically a non-linear and ill-posed problem. For these reasons, a discrete and weak format is preferred by adopting a general discretization scheme

\[
\mathbf{u}(x) = \sum_{n \in \mathbb{N}} u_n \psi_n(x) = [\psi(x)]\{\mathbf{u}\}
\]

(3)

where \( \psi_n \) are the vector shape functions, and \( u_n \) their associated degrees of freedom. In a matrix-vector format, \([\psi]\) is a row vector containing the values of the shape functions \( \psi_n \) and \( \{\mathbf{u}\} \) the column vector of the degrees of freedom. After integration over the domain \( \Omega \), the global residual is defined as

\[
\Phi = \int_\Omega |f(x + [\psi(x)]\{\mathbf{u}\}) - g(x)|^2 \, dx
\]

(4)

At this level of generality, many choices can be made to measure displacement fields. In the following, classical bilinear shape functions associated with quadrilateral 4-node elements (or Q4) [3] are chosen. It is referred to as Q4 Digital Image Correlation (or Q4-DIC).

2.2. From displacement fields to an anisotropic damage law

2.2.1. Constitutive law and state variables

Let us consider a homogeneous and orthotropic material. Let us first assume that only shear properties vary because of gradual damage \( d \). Some [0,90] composites, as a first order approximation and at a certain scale, behave in a such a way [5]. In the following, the indices (1, 2) refer to the ply coordinate system (i.e., material frame), here coinciding with the fiber directions. With these notations, \( E_1 \) and \( E_2 \) denote the initial young moduli (here in the fiber directions), \( G_{12} \) the initial shear modulus and \( \nu_{12} \) a Poisson’s ratio. The angle between this frame and that of the camera coordinate system is \( \theta \). Under plane stress assumption, the constitutive law is written as

\[
\begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{pmatrix} = \begin{bmatrix}
K_1 & K_{12} & 0 \\
K_{12} & K_2 & 0 \\
0 & 0 & K_3
\end{bmatrix} \begin{pmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
2\epsilon_{12}
\end{pmatrix}
\]

(5)

where \( K_1 = \frac{E_1}{E_1 - 2\nu_{12}E_2} \), \( K_2 = \frac{E_2}{E_1 - 2\nu_{12}E_2} \), \( K_3 = G_{12} (1 - d) \) and \( K_{12} = \nu_{12}K_2 \).

A continuum thermodynamics framework is used [6]. The Helmholtz free energy density \( \Psi \) is used to define the driving force \( Y \) associated to the shear damage, and an equivalent strain \( \epsilon_{eq} \)

\[
Y = -\frac{\partial \Psi}{\partial d} = 2G_{12}\epsilon_{12}^2 \quad \text{and} \quad \epsilon_{eq}^2 = \frac{Y}{G_{12}} = \frac{1}{2} [(\epsilon_y - \epsilon_x) \sin(2\theta) + \epsilon_{xy} \cos(2\theta)]^2
\]

(6)

In the present case, the equivalent strain only depends on the shear strain in the material frame, and it is assumed that no coupling exists with the normal strains in the fiber directions. The goal is now to identify the damage law relating the shear damage variable \( d \) and the maximum over the elapsed time of its associated equivalent strain \( \epsilon_{eq} \).
2.2.2. Identification of a damage law

The Equilibrium Gap Method (EGM) is followed herein. It consists in minimizing the force residuals associated with a mismatch of local elastic properties from elements to elements. In absence of body forces, it is written in a weak form by using a Finite Element discretization

\[ [K(\{d\})]\{u_{\text{meas}}\} = \{f_{\text{res}}\} \]  

where \{f_{\text{res}}\} is the residual vector associated with measured displacement fields \{u_{\text{meas}}\}. Unlike classical FE problems, the aim is to determine the damage fields \{d\} for known (i.e., measured) displacement fields \{u_{\text{meas}}\} (e.g., by digital image correlation) and the nodal force vector \{f\} here assumed to vanish since only interior nodes are considered. To be consistent with the measured displacements, Q4 elements are used again. The damage variable \(d\) is assumed to be element-wise uniform. In the case of anisotropic damage, the elementary stiffness matrix \(K^{\text{el}}\) is no longer linear in \(d\) as was the case for an isotropic damage description [2]. This matrix is expressed as an affine function of the damage variable \(d\)

\[ K^{\text{el}}_{ij} = M^0_{ij} + M^1_{ij}(1 - d) \]  

where \([M^0]\) and \([M^1]\) are matrices dependent upon the initial elastic parameters (of the undamaged element). These elastic constants may also be identified using full field measurements. This point will not be developed herein. To simplify the expressions, an element-wise decomposition of the stiffness matrix is introduced $M^0_{ij}$ $M^1_{ij}(1 - d)$

The solution to such problem would then lead to one map of shear modulus contrasts for each loading step. The difficulty is to identify a damage growth law, namely, it is necessary to link the maps obtained at different loading steps. One prescribes that all the maps result from the same damage law. By enforcing that damage grows according to the same expression everywhere in the region of interest, the damage law minimizes the equilibrium gap. The regularization of the problem consists in using, at the very beginning of the procedure, a specific decomposition \(H\) of the damage law (a kind of Laplace transform)

\[ d = H(\varepsilon_{\text{eq}}, C_k) = \sum_k C_k \varphi_k(\varepsilon_{\text{eq}}) \]  

with \(\varphi_k(\varepsilon_{\text{eq}}) = 1 - \exp\left(\frac{\varepsilon_{\text{eq}}}{\epsilon_k}\right)\) and \(\varepsilon_{\text{eq}} = \max_{0<\tau<t}(\varepsilon_{\text{eq}}(\tau))\)

The objective function \(E_g\) then depends quadratically on the coefficients \(C_k\) defining the damage law for the given set of characteristic strains \(\epsilon_k\)

\[ E_g(C_k) = \sum_i \left( \sum_e (L^0_{ie} + L^1_{ie}(1 - H(\varepsilon_{\text{eq}}, C_k))) \right)^2 \]  

The system obtained above is not well conditioned because the \([M^n]\) matrices correspond within the chosen discretization scheme to second order differential operators acting on the displacement.
field in the continuum limit. Because the displacement field is obtained experimentally, it is inevitably prone to noise and hence the above scheme may amplify it in particular at short wavelengths.

In order to mend this potential deficiency, it is proposed to introduce the operator $S$ such that $[S]{\{L\}} = \{u_{\text{meas}}\}$, where $S$ solves the homogeneous elastic problem for the undamaged medium, with Dirichlet boundary conditions as determined experimentally, and body forces $\{L\}$. The “reconditioned” equilibrium gap objective function $\tilde{E}_g$ is proposed as given by the following expression

$$\tilde{E}_g(C_k) = \sum_i \left( \sum_j S_{ij} \sum_e \left( L^0_{je} + L^1_{je}(1 - \sum_k C_k \varphi_k(\tilde{\epsilon}^e_{eq})) \right) \right)^2 = \sum_i \left( u_i - \sum_k C_k \sum_j S_{ij} \sum_e L^1_{ie} \varphi_k(\tilde{\epsilon}^e_{eq}) \right)^2,$$

which can be read as the quadratic norm of a nodal vector homogeneous to a displacement field. In practice, the inverse operator $S$ is not computed, but rather the “vectors” $[S]{\{L\varphi_k\}}$, which is simply obtained as the solution of a homogeneous elastic problem for the undamaged solid.

3. Validation and first application

3.1. Validation on a synthetic experiment

As a first step, the identification procedure is tested using kinematic data obtained from a non linear FE computation performed with a known damage law. The FE model is implemented in an in-house code [7]. The chosen “virtual test” consists in subjecting a flat cruciform specimen to a biaxial load. It corresponds to the experiment that will be treated next. A plane stress state is assumed. The whole cruciform specimen is meshed (see Figure 1a). The central part of the specimen is uniformly meshed using $20 \times 20$ Q4 elements. The displacements computed on this part of the mesh are used as input data for the identification procedure. The arms and the material surrounding the region of interest are meshed using T3 elements. The angle between the local material coordinate system and the camera coordinate system is set to 45°.

The chosen damage law is decomposed on the basis described in the previous paragraph. The chosen parameters are $\epsilon_k = 10^{-3} \times [1248]$ and $C_k = [0.50.30.150.05]$. The resulting curve $d$ versus $\epsilon_k$ is shown in Figure 1b. A non-local approach is used to limit numerical localization effects induced by strain softening. In practice, a mean force $Y$ is computed over each element. The coupon is subjected to tension with respect to $y$ and to compression with respect to $x$. The loading consists in a uniform displacement prescribed at the end of the arms. Normal displacements are increased step by step, while tangential displacements are forced to 0.

The damage map obtained at the last stage of loading for a given set of elastic parameters ($E_1 = 35$ GPa, $E_2 = 30$ GPa, $G_{12} = 7$ GPa and $\nu_{12} = 0.4$) is shown in Figure 1a. To quantify the quality of the identification, it is proposed to compare the reconstructed displacement field (using the identified damage field) to the reference data, i.e., the “measured” displacement field. A residue $R$ is defined as the ratio of the root mean square (denoted by $\chi(\cdot)$) of the difference between the measured and reconstructed displacement field, normalized by the root mean square of the measured displacement field

$$R = \frac{\chi(u_{\text{reconstructed}} - u_{\text{meas}})}{\chi(u_{\text{meas}})}$$

The smaller the residue $R$, the better the result. In the following, this quantity is reported for the last load level. Different trial functions are tested. First, the trial function of the imposed law is
used, i.e., $\epsilon_k = 10^{-3} \times [1 2 4 8]$. The identified parameters are $C_k = [0.5502 0.1325 0.3159 0.0015]$. Although the numerical values of the $C_k$ coefficients are different from the imposed ones, it is seen in Figure 1b that the identified and prescribed damage laws with the chosen equivalent strain are undistinguishable over the range of strains covered by the experiment. This excellent agreement is confirmed by the residue which is equal to $R \approx 3 \times 10^{-3}$. Different parameters of the trial functions have then been tested to check the sensitivity of the method to the damage decomposition (Figure 1b). For example, identifications achieved with $\epsilon_k = 5 \times 10^{-4} \times [1 2 4 8]$ and $\epsilon_k = 15 \times 10^{-4} \times [1 2 4 8]$ give respectively $C_k = [0.0280 0.4899 0.1846 0.2976]$ and $C_k = [0.8821 0.0 0.0 0.1185]$. In both cases, the agreement is good, even if for larger values of $\epsilon_k$ the results are less accurate (respectively $R = 3.0 \times 10^{-3}$ and $R = 3.3 \times 10^{-3}$). Parameters $\epsilon_k$ that do not follow a geometric sequence have also been tested. As an example, the result for $\epsilon_k = 8 \times 10^{-4} \times [1 2 3 4]$, amounts to $C_k = [0.3732 0.2076 0.4192]$, and is also presented in Figure 1b. Again the result is satisfactory ($R = 3.0 \times 10^{-3}$). For all the presented identification cases, a relative difference of less than 1% is measured between computed and identified displacement fields. Moreover, the identified shear damage maps are almost identical to the FE computed ones.

Further work on the performance of the method is currently being carried out, and will be reported elsewhere. In particular the noise robustness, or the use of other algebraic forms for the damage laws, will be analyzed in details.

3.2. Experimental test

The first experiment used to test the method with real data consists in a biaxial test performed on a so-called 2.5D C/C composite. The test has been carried out on the multiaxial machine ASTREE (see Figure 2). This woven material has a non-linear behavior when subjected to shear or tension with respect to the fiber direction. A flat (10 mm thick) cruciform specimen, considered as a $[\pm 45^\circ]$ laminate, is subjected to a shear test. Tabs glued on the arms (100 mm large) allow for a transmission of the loads to the specimen. Due to the specimen aspect ratio, a plane stress state is assumed. For each ply direction, the same damage mesomodel is proposed. This test was designed, in particular thanks to FE computations, to induce a high value of shear damage in the center part of the specimen [8].

Digital images of the surface (resolution: $1016 \times 1008$ pixels, 8-bit depth) are shot at various

Figure 1. -a-Mesh used in the virtual test and damage map at the last load level. -b-Comparison between the prescribed shear damage law and various identifications.
steps of the loading. The pictures are analyzed by using a Q4-DIC algorithm. The element size is set to 32 pixels (≈ 3 mm). The damage field inside each layer within the ROI was computed by using a damage post-processor [8] (Figure 3a,b). One notes a very good agreement between these post-processed damage maps and that determined by following the present procedure (Figure 3c).

Figure 4 shows a comparison between the measured and reconstructed displacement fields for the last loading step. The corresponding residue is here estimated to be $R = 0.06$, naturally higher than for the artificial case, which was deprived of noise, but still quite low. It is to be underlined that unlike the classical tensile test usually used, the present approach allows one to identify the damage law corresponding to post-peak data.

In practice, the equivalent strain is directly based on the total measured strains and therefore includes a non negligible inelastic part. Future developments will be performed to account for both damage and inelasticity in the constitutive law.
Figure 4. Comparison between measured and reconstructed displacements (expressed in pixels), and corresponding differences for the last loading level.

4. Summary and perspectives
A way of identifying anisotropic damage laws using images shot during a mechanical test has been presented. The proposed approach is based on recent developments of two (inverse) methods, namely, finite element digital image correlation and identification based on the equilibrium gap method. The first one allows one to retrieve full-field (FE formatted) displacement fields from images along the loading history. The second one consists in solving a FE problem for which the data are measured displacements and the unknowns the parameters of the chosen trial damage law. The performance of the method is first evaluated using displacement fields resulting from FE non-linear computations. A biaxial test on a cruciform specimen made of an orthotropic material is simulated. The results of the procedure, in terms of identified law and of displacement fields, are excellent, and only weakly sensitive to the basis of chosen trial functions. The presented results are very encouraging.

The procedure is then applied to analyze a real biaxial test performed on a woven composite. In that case, it is possible to identify in a reliable way a damage pattern quite similar to the one obtained by post-processing the measurements with a classically identified model. The reconstructed displacement field is then very close to the measured one.

This work corresponds to the very first step towards the identification of more general constitutive laws when considering anisotropic materials. Future developments will include coupled multiaxial damage and inelasticity, which are important for a full account of the behavior of, say, composite materials.

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5. References

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