Two-criteria technical and economic optimization of forest transportation problem

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Abstract. The systematic analysis of optimizing the producer-consumer structure transportation problem is carried out in paper. It is in demand for the timber industry complex. The well-known minimum transportation cost objective is solved as a static problem using the linear programming method. Due to complicated market development conditions, it is necessary to set up and solve the problem of minimizing transportation time (quick performance) as a dynamic problem. Solving it in general case, the transportation cost may be different from the minimum cost determined by the static problem solution. The dynamic problem solution is based on the carrier's vehicle performance information used for determining the functional time spent per transported product unit. The rational decision making under market conditions is only possible by adopting a compromise obtained on the basis of the multi-criteria optimization theory. The task of determining the compromise area of two-criteria optimization (cost-time minimum ratio) is decided by experts. It is solved using the linear programming method for static and dynamic problems and their linear approximation. To develop a compromise transportation plan becomes the subject of solving the inverse transportation problem. The systemic “producer – transport – consumer” approach is necessary for sustainable timber industry complex development, when the producer-transport system makes up a single production process. Its parameters are determined on the basis of systems theory.

1. Introduction

The product transportation plays an important role in the forest industry system. Its costs make up a significant part of general expenses structure. So, in practice the rational transportation organization is in demand. At the same time, the market dynamics require quick satisfaction of consumer needs by the manufacturer. Thus, it is relevant to find compromise solutions in conflicting objective functions.

There are two characteristic types of classical transport problems, namely, static and dynamic. The static problem is formulated on the basis of the cost criterion, as minimization of transportation costs, and its solution determines the economic efficiency.

The dynamic task is set on the basis of the time criterion as transportation time minimization. In general case, its costs are different from the transportation ones in the static task.

The solution of the first task is widely used in the timber industry. The second task is not represented analytically, although its formulation and solution is relevant. It is in demand in dynamically changing conditions of the forest industry market development.
The existence of two conflicting optimization criteria in the transportation problem - minimum costs and maximum performance – requires considering general multi-criteria optimization problems [1-3].

The formulation of a multi-criteria optimization problem is as follows: in a given domain of defining variables \( x_1, x_2, ..., x_n \), the optimization of the system of two or more conflicting objective functions is performed

\[
Z_j = f_j(x_1, x_2, ..., x_n), \quad Z_j = f_j(x_1, x_2, ..., x_n),
\]

under restrictions

\[
g_i(x_1, x_2, ..., x_n) = 0, \quad i = 1, 2, ..., I,
\]

The purpose of the study is to formulate analytically the dynamic transport problem in the forest complex using linear programming method, to analyze the optimization of competing static and dynamic problems, and to find their trade-off relatedness using the system approach.

2. Materials and methods

The formulation of a static transportation problem is well-known [4-7]. It is a task of planning minimum transportation costs of identical products from production points to consumption points, whereas the static cost data of transporting a unit of production in a linear formulation is used.

The wood material measured in cubic meters is the subject of labor in the logging industry. So, the classical timber transport problem takes the following form: the corresponding volumes of wood \( a_1, a_2, ..., a_m \) are produced by the enterprises \( A_1, A_2, ..., A_m \). Those volumes are to be delivered to consumers \( B_1, B_2, ..., B_n \) with the corresponding demand in quantities \( b_1, b_2, ..., b_n \), in case the total volume of production is equal to the total demand

\[
\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j
\]

whence the model is considered balanced. The transportation cost of a production unit from \( A_i \) to \( B_j \) is \( c_{ij} \); the process of reducing unbalanced problems to balanced ones is presented in [7].

The balanced static timber transport problem is formulated as follows: to minimize the cost of transportation

\[
Z_c = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

where, \( x_{ij} \) - the number of production units transported from \( A_i \) to \( B_j \) with restrictions

\[
\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, ..., n;
\]

\[
\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, ..., m.
\]

A variety of methods for solving this problem is presented in [5, 7].

3. Results

In order to set the timber transport problem according to the dynamic criterion (delivery time minimization), it is necessary to enter into consideration the vehicle transportation operation performance \( \Pi_{ij} \) (the number of production units transported per time unit) from \( A_i \) to \( B_j \) and its dual representation of the functional time
characterizing the amount of time per production units transported from $A_i$ to $B_j$.

Performance and functional transportation time form a multiplication group multiplied by the set of real numbers $[8, 9]$.

The dynamic timber transport problem is formulated as follows: to minimize the transportation time

$$ t = \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} x_{ij} $$

or

$$ t = \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} \rightarrow \min, $$

under restrictions

$$ \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n, $$

$$ \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m, $$

where $t_{ij} = q_{ij} x_{ij}$ - product transportation time from $A_i$ to $B_j$, $x_{ij}$ - number of product units transported from $A_i$ to $B_j$.

Comparing the formulation of static (4) and dynamic (9) problems, it is clear that the transition from the first to the second is connected with the replacement of the transported unit cost with the functional transportation time $c_{ij} \rightarrow q_{ij}$.

The connection of static and dynamic problems is performed representing the product unit transportation cost from $A_i$ to $B_j$, in the form

$$ c_{ij} = c_{ij} / \Pi_{ij} = c_{ij} q_{ij}, $$

where, $c_{ij}$ - cost of unit transportation time.

Based on (11), the static problem (4) can be presented as dynamic (8)

$$ z_c = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} q_{ij} x_{ij}, $$

with delivery time

$$ t_c = \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} x_{ij}, $$

and the dynamic problem in the form of static

$$ z_f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} q_{ij} x_{ij}, $$
at a constant cost per transportation hour $c_{ij} = c_t = \text{const}$,

$$z_c = c_t \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} x_{ij} = c_t t_c ,$$  

(15)

the delivery time to customers is equal to

$$t_c = z_c / c_t ,$$

in this case, there is an ideal two-criteria optimization solution of conflict objective functions, where the static (cost, economic) and dynamic (time, technical) criteria for transport minimization are coincided.

In the general case, formulas (4) and (8) allow determining the average cost value per unit transportation time in the “producer – transport – consumer” system

$$c_{ek} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} q_{ij} x_{ij} / \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} x_{ij} ,$$  

(16)

and the cost transportation time

$$t_{ek} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} q_{ij} x_{ij} / \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} ,$$  

(17)

The static (4) and dynamic (8) solutions of the timber transport problem allow determining the compromise decision area in a linear approximation

$$z_k = z_c + t_k (z_c - t_c) / (t_c - t),$$

(18)

in which the transportation strategy choice is made by a professional person. The reverse transportation problem is solved on its basis, where the total cost and transportation time make up a compromise transportation plan.

In order to obtain sustainable development of timber enterprises in the dynamic market conditions, it is necessary to move from a differential approach to solving a problem (combining separate structures of producer, transport and consumer) to an integral (systemic) approach. The timber production processes and its transportation to a consumer should be considered as a single cycle with consistently performed corresponding operations determined by the formula [10]

$$\Pi_{12} = \frac{1}{(q_1 + q_2)} ,$$

in this case $q_1 = \frac{1}{\Pi_1}$, $q_2 = \frac{1}{\Pi_2}$

(19)

where $\Pi_1$, $\Pi_2$ - production processes and forest transport performance, respectively.

If the timber products are stored, their performance can be represented by the formula

$$\Pi_c = \frac{V_c}{t_c} ,$$  

(20)

where, $V_c$ - volume, $t_c$ - time, then formula (19) turns into

$$\Pi_{12c} = \frac{1}{(q_1 + q_2 + q_c)} ,$$  

(21)

where, the functional storage time is equal to $q_c = \frac{1}{\Pi_c}$,
It should be noted that the functional cycle time is increased when the product is stored before transportation. Respectively, its productivity is reduced and the total timber delivery time to a consumer is increased.

In the “producer - timber transport – consumer” system, the functional time per unit volume in a continuous process \( q_c = 0 \) in the “producer - timber transport” subsystem is equal to \( q_{c2} = \frac{1}{2} (q_1 + q_2) \). It means that the productivity is increased in a continuous process in comparison with a cyclical one reducing the product delivery time.

Using the systemic approach of “producer - timber transport – consumer” integrity, the minimization of production time and timber volume transportation is considered to be the dynamic task optimization in the following formulation, namely, to minimize the total timber production time and its transportation to a consumer:

\[
T = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ij} x_{ij} + \sum_{j=1}^{n} q_s x_{ij} \right] \to \min, \; i = 1,2,\ldots,m, \tag{22}
\]

under restrictions

\[
\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1,2,\ldots,n,
\]

\[
\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,\ldots,m. \tag{23}
\]

The first sum component (22) determines the transportation time, whereas the second - the time of production.

The static task of minimizing the timber production cost and its transportation in the single “producer - timber transport – consumer” system takes the form

\[
C = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{j=1}^{n} c_s x_{ij} \right] \to \min, \; i = 1,2,\ldots,m, \tag{24}
\]

under restrictions

\[
\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1,2,\ldots,n,
\]

\[
\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,\ldots,m. \tag{25}
\]

In this case the first term in (22) defines the timber transport cost and the second - the production cost.

The relationship of static and dynamic problems in the system approach is written in the following form

\[
C = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{j=1}^{n} c_s x_{ij} \right] = \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} q_{ij} x_{ij} + \sum_{j=1}^{n} c_s q_{ij} x_{ij} \right] \to \min, \; i = 1,2,\ldots,m, \tag{26}
\]

By virtue of optimizing conflicting objective functions (22) and (24), it is possible to determine the compromise decision area by analogy with linear approximation (18). A professional person takes decisions on its basis and solves the inverse problems of appropriate dynamic system status strategies.
4. Conclusions
The minimum cost transportation plan is drawn up optimizing the static transport problem in the “producer-transport-consumer” triad. The dynamic minimum time transportation plan comes into conflict to it due to the fact that their costs do not coincide in general.

Dynamic conditions of market development make it necessary for timber transport enterprises find compromise solutions based on the result analysis optimizing two conflicting objective functions (minimum cost and minimum transportation time). The transportation plan for the accepted compromise is considered to be a solution of the inverse transportation problem.

In order to achieve sustainable development of timber enterprises in market conditions, it is necessary to combine a systematic approach to the “producer-transport-consumer” triad as a deeply integrated structure with the basics of multicriteria optimization theory.

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