Violation of lepton number in 3 units

R M Fonseca

E-mail: fonseca@ipnp.mff.cuni.cz
Institute of Particle and Nuclear Physics Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 18000 Prague 8, Czech Republic

Abstract. The number of leptons may or may not be a conserved quantity. The Standard Model predicts that it is (in perturbative processes), but there is the well known possibility that new physics violates lepton number in one or two units. The first case ($\Delta L = 1$) is associated to proton decay into mesons plus a lepton or an anti-lepton, while the second one ($\Delta L = 2$) is usually associated to Majorana neutrino masses and neutrinoless double beta decay. It is also conceivable that leptons can only be created or destroyed in groups of three ($\Delta L = 3$). Colliders and proton decay experiments can explore this possibility.

1. Lepton number violation: looking beyond the usual scenarios
There is currently no experimental hint of physical processes where the total number of leptons (minus anti-leptons) changes, such as

$$ p \rightarrow \pi^0 e^+ $$

and

$$ nn \rightarrow ppe^- e^- . $$

(1)

Indeed, there are stringent bounds on the proton decay lifetime [2] as well as on the neutrinoless double beta decay lifetime of several isotopes [3, 4]. Note that these two processes are very different: the first destroys a baryon and creates an anti-lepton ($\Delta B = \Delta L = -1$) so $B - L$ is preserved, while the second creates two leptons ($\Delta L = 2$) without changing the number of baryons ($\Delta B = 0$). Therefore it is possible that one of these processes occurs and the other does not: if $B - L$ is a conserved quantity then the proton may decay as indicated in (1) but neutrinoless double beta decay is forbidden, while $B$ conservation implies the converse.

These $\Delta L = \pm 1, \pm 2$ signals, as well as Majorana neutrino masses and the production of pairs of same-sign leptons at colliders, have been extensively studied over the years. But what if leptons can only be created or destroyed in groups of three ($\Delta L = 3$)? In this case all the processes mentioned so far would never be seen, but the proton could still decay into three leptons,

$$ p \rightarrow \ell^+ \ell^+ \ell^+ + \text{mesons}, $$

(2)

and in a proton-proton collider it would be possible to produce three same-sign leptons:

$$ pp \rightarrow \ell^+ \ell^+ \ell^+ + \text{jets}. $$

(3)

One could object to the possibility of simultaneously observing lepton number violation in three units at colliders and at proton decay experiments with the argument that the current bounds on the proton’s lifetime rule out mediators of these processes with masses significantly
below $10^{16}$ GeV, hence these particles would not be produced at colliders. However, this large mass bound applies only to proton decay channels of the type shown in (1) which are induced by an effective 4-fermion interaction. The coefficient $c$ of such a dimension 6 operator is proportional to the inverse of the square of some new physics scale $\Lambda$,

$$c \sim \frac{1}{\Lambda^2},$$

and in turn the proton decay width $\Gamma$ must be proportional to the square of this quantity. We have to insert powers of the only other energy scale in this process, the proton mass $m_p$, in order to have an expression for $\Gamma$ with dimensions of energy, so

$$\tau (\text{proton})^{-1} = \Gamma \sim \frac{m_p^5}{\Lambda^4} \sim 10^{32} \left(\frac{m_p}{\Lambda}\right)^4 \text{years}^{-1}.$$ (5)

Super-Kamiokande established that $\tau (\nu_e \rightarrow e^+\pi^0) > 1.6 \times 10^{34}$ years at 90% confidence level [2], so from this back-of-the-envelop calculation, it is clear why the mediator of this process must have a large mass $\Lambda \gtrsim 10^{16}$ GeV. However, the effective interaction inducing the decay of a proton into three charged leptons (or anti-leptons) has dimension 13 at the very least, so instead of expression (4) we have $c \sim \Lambda^{-9}$, and the correct estimate for the proton’s decay width as a function of $\Lambda$ is

$$\tau (\text{proton})^{-1} \sim 10^{32} \left(\frac{m_p}{\Lambda}\right)^{18} \text{years}^{-1}.$$ (6)

This formula shows that even for values of $\Lambda$ as low as a few TeVs, the proton might still be sufficiently stable. It is worth noting as well that at least 5 tracks will be produced in a Cherenkov detector whenever a proton decays into 3 same-sign charged leptons, which means that this signal should stand out cleanly over a very low background. Nonetheless, no publicly available bounds on this type of events seems to exist, in which case one must rely on very old inclusive searches: $\tau (\nu_e \rightarrow e^+ + \text{anything}) > 0.6 \times 10^{30}$ years and $\tau (\nu_\mu \rightarrow \mu^+ + \text{anything}) > 1.2 \times 10^{31}$ years [5, 6].

2. The effective operator point of view

In order to make model-independent statements about lepton number violation in 3 units, it is convenient to look at non-renormalizable interactions of the Standard Model (SM) fields with this property. It is well known that all the renormalizable ones preserve baryon and lepton number. Interestingly, non-perturbative effects associated to an energy $E_{sph}$ of around 9 TeV violate both $L$ and $B$ in 3 units [7, 8]. It would certainly be fascinating to produce for the first time in a laboratory an excess of matter over anti-matter. But it is not certain that these processes would be observable even if a parton center-of-mass energy $E_{sph}$ is reached in proton-proton collisions in the not-so-distant future (see for example [9–11]).

Having said this, let us now go through non-renormalizable interactions of Standard Model fields which violate $L$ and $B$. By convention SM leptons have $U(1)_L$ charges of $\pm 1$, so obvious any given operator can only violate $L$ by an integer quantity. And the same is true for baryon number, even though quarks carry a $\pm \frac{1}{3}$ charge under $U(1)_B$ — one way to see this is by realizing that color invariance requires an excess/deficit of quarks over anti-quarks in each operator which must be a multiple of 3. In addition to this, Lorentz invariance requires $\Delta B - \Delta L$ to be an even number.\(^2\) For example, this implies that if the proton is unstable, it must decay into an odd

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\(^1\) This statement is not entirely true since there is also the QCD scale $\Lambda_{QCD} \sim 0.2$ GeV. However, given that $\Lambda_{QCD}$ is of the same order of magnitude as the proton mass, I will ignore it.

\(^2\) The author of [12] goes further, showing that $\Delta B - \Delta L + d$ is always even ($d$ is the operator dimension).
number leptons. The lowest dimensional operators which can induce this process are associated to \( \Delta B = \Delta L = \pm 1 \):\(^3\)

\[
qqql, \ q(u^c)^* (d^c)^* l, \ qq (u^c)^* (e^c)^* \quad \text{or} \quad (u^c)^* (u^c)^* (d^c)^* (e^c)^* .
\] (7)

As for \( \Delta L = \pm 3 \), the simplest operators are expected to be associated with \( \Delta B = 1 \),\(^4\) in which case their dimension \( d \) cannot be smaller than

\[
\frac{3}{2} (n_{\text{leptons}} + n_{\text{quarks}}) \geq \frac{3}{2} (|\Delta L| + 3|\Delta B|) = 9 .
\] (8)

In fact there are two operators at the lower limit of this bound [13]:

\[
(u^c)^* (u^c)^* (e^c)^* l l \quad \text{and} \quad (u^c)^* (u^c)^* q l l .
\] (9)

However, in both cases the right-handed up quarks \( u^c \) cannot all be from the first generation (otherwise the operators become identically zero), hence they do not induce proton decay. On the other hand, the unique \( d = 10 \) operator

\[
(d^c)^* (d^c)^* (d^c)^* l^+ l^+ l^+ h^*
\] (10)
can induce the decay \( p \rightarrow e^- \nu \nu \pi^+ \pi^+ \) (\( \Delta L = -3 \)). Note however that electric charge conservation implies that there will necessarily be two neutrinos plus a charged lepton in the final state. It is also worth point out that, from a theoretical point of view, it is possible to forbid this operator with some discrete or continuous symmetry: for example, if the laws of physics are \( U(1)_{3B-L} \) symmetric, proton decay would be possible, but only through operators of dimension 11, 13 or higher.

At \( d = 11 \) one finds many \( \Delta L = 3 \) operators with different structure, for example

\[
\partial \partial (u^c)^* (u^c)^* q l l \quad \text{and} \quad \partial q (u^c)^* (u^c)^* l l (e^c)^* h ,
\] (11)

where the derivatives must be covariantly applied to the fields (or alternatively a pair of derivatives can stand for a field strength tensor). The problem with these operators is that experimentally they also cannot be used to establish lepton number violation, since they involve neutrinos. They induce processes such as \( p \rightarrow e^+ \nu \overline{\nu} \) and \( p \rightarrow \pi^- e^+ e^+ \overline{\nu} \).

If we insist on observing three charged leptons with the same sign, then in order to conserve electric charge at least 5 quarks are needed in the operator as well. That is an 8-fermion, dimension 12 operator, but in reality it is easy to check that under the full Standard Model group, there must also be either a derivative or a Higgs field in the interaction, raising the operator dimension to 13. Two (out of many) possibilities are

\[
\partial (u^c)^* (u^c)^* (u^c)^* d^c (e^c)^* (e^c)^* (e^c)^* \quad \text{and} \quad \partial (u^c)^* (u^c)^* d^c q q (e^c)^* l ,
\] (12)

both inducing the decay \( p \rightarrow \ell^+ \ell^+ \ell^+ \pi^- \pi^- \).

\(^3\) Spinor and gauge indices will be suppressed throughout the text. The fermion fields \( q, u^c, d^c, l \) and \( e^c \) are all left-handed, which means that their conjugation (indicated with a ‘\(^*\)’) produces a right-handed field.

\(^4\) Without loss of generality, I will henceforth assume that \( \Delta B \) is positive.
3. A specific model

The paper [1] presents a specific model where lepton number is violated in multiples of 3 units only. It contains some fields that are not in the Standard Model (see table 1), including some scalars which induce the normal $\Delta L = \pm 1$ proton decay modes under normal circumstances. However, a $Z_3 (L)$ symmetry

$$\psi \rightarrow \omega^L(\psi) \psi$$

with $\omega = \exp(2\pi i/3)$ will forbid the combinations of interactions leading to such decays. For example, the scalar $s_d$ has gauge quantum numbers both of a leptoquark and of a diquark since it couples to $q l$ and $q^* q^*$. Therefore, it would generate the effective operator $qqql$ were it not for the $Z_3 (L)$ symmetry which forbids the diquark coupling.

| Field(s) | Spin | $SU(3)_C \times SU(2)_L \times U(1)_Y$ | $L$ |
|---|---|---|---|
| $N \times 3$ | Left-fermion | $(1, 1, 0)$ | $+1$ |
| $N^c \times 6$ | Left-fermion | $(1, 1, 0)$ | $-1$ |
| $s_u$ | Scalar | $(\mathbf{3}, 1, -\frac{2}{3})$ | $-1$ |
| $s_d, s'_d$ | Scalar | $(\mathbf{3}, 1, \frac{1}{3})$ | $-1$ |

Table 1. New fields in the model presented in [1].

The interactions of the new fields allowed by all symmetries are

$$\mathcal{L} = Y_u l N^c h + Y_1 (u^c)^* (N^c)^* s_u + Y_2 N^c d^c s_d^* + Y_3 (e^c)^* (u^c)^* s'_d + Y_4 q l s_d + \mu s_u s_d s'_d + m_N N N^c.$$ (14)

There are several important remarks to be made about this Lagrangian:

(i) In the absence of the $\mu$ term, lepton and baryon number are conserved. Therefore the scalar trilinear interaction $s_u s_d s'_d$ must appear in any diagram with a net number of external baryons/leptons different from zero.

(ii) By construction the model is $Z_3 (L)$ symmetric, but it turns out that the Lagrangian is invariant under the bigger $U(1)_{3B-L}$ symmetry group. This is significant because the $Z_3 (L)$ symmetry would allow in principle $\Delta (B, L) = (1, -3)$ processes, while the $U(1)_{3B-L}$ symmetry does not. Hence, in accordance with what was discussed previously, proton decay is induced only by operators of dimension 11, 13, or higher.

(iii) Majorana masses are forbidden so, as expected, neutrinos are Dirac particles. In this model the three small masses of the neutrinos observed in oscillation experiments can be obtained through a rather delicate choice of the matrices $Y_e$ and $m_N$.

(iv) The fields $s_d$ and $s'_d$ have the same quantum numbers, therefore they can have the same interactions. However, meson decay and atomic parity violation experiments place stringent bounds on the product of some couplings [14, 15]. To avoid them, one might consider that $s_d$ interacts only with left-handed Standard Model fermions, while $s'_d$ couples exclusively with right-handed ones. The Lagrangian in equation (14) assumes so.

The most important diagrams for proton decay are shown in figure 1. An interesting way to see that the proton can only decay through these complicated diagrams is to start with the crucial trilinear interaction $s_u s_d s'_d$ and connect each of these scalars to Standard Model fermions through the available Yukawa interactions. Using some optimistic values for the various parameters of the model (1 TeV scalar masses, and order 1 Yukawa couplings) and leaving free the heavy neutrino mass $m_N$ one obtains estimates for the partial lifetimes $\tau (p \rightarrow e^+ e^+ e^+ \pi^- \pi^-)$ and
Figure 1. On the left: diagrams of the two main contributions to proton decay in the model described in the main text. On the right: rough proton lifetime estimates as a function of the mass \( m_N \), for 1 TeV scalar masses and order 1 Yukawa couplings. This figure was adapted from [1].

\[ \tau (p \rightarrow \pi^0 e^+ \pi^-) \] which might be within reach of future proton decay experiments. It depends on \( m_N \), as shown in figure 1.

Lepton number violation can be observed at the LHC through the pair production of one of the scalar leptoquarks (\( s_u, s_d \) or \( s'_d \)), followed by their decay into two different final states. In order to see several of these events, each of the two decay modes should have a large branching fraction. The leptoquarks \( s'_d \) do not fulfill this requirement because they yield either a 2-body or a 6-body final state, hence the branching fraction of the second decay mode is very small. On the other hand, \( s_u \) has two distinct 4-body final states: \( s_u \rightarrow s'_d + s'_d \rightarrow 2 \ell^- + 2\text{jets} \) and \( s_u \rightarrow N^c + u^c \rightarrow \ell^+ + 3\text{jets} \). The branching fractions are similar if \( s_d, s'_d \) and \( N^c \) are produced off-shell. Overall, one would see three same-sign leptons in a proton-proton collision: \( pp \rightarrow s_u s_u^* \rightarrow 3 \ell^\pm + \text{jets} \).

Summary

Lepton number violation is almost always associated with processes where one or two leptons are created or destroyed. Well known examples are proton decay into a (anti)lepton plus a meson, and neutrinoless double beta decay. However, it is conceivable that lepton number can only be changed in multiples of 3, in which case the signals to look for are different. In fact, this is what happens in the Standard Model since non-perturbative effects can change the net lepton and baryon numbers by 3 units only.

Following [1], I presented here the possibility that there might be perturbative new physics at the TeV scale associated to \( \Delta L = 3 \) and \( \Delta B = 1 \). Even though the new fields responsible for lepton and baryon number violation are much lighter than usually assumed, a simple estimate indicates that the proton decay lifetime can easily exceed the current experimental bounds. The reason is simple: \( U (1)_{3B−L} \) invariance implies that proton decay is induced by high dimensional operators only, which means that the proton lifetime is enhanced by many powers of the ratio of the new physics scale over the proton mass. Hyper-Kamiokande and DUNE might be able to observe the decay \( p \rightarrow e^+e^+e^−\pi^-\pi^- \), and at the LHC it might be possible to produce 3 leptons with the same charge plus jets and no missing energy.

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