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Frequency analysis of acoustic signal using the Fast Fourier Transformation in MATLAB

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Abstract

The paper deals with frequency analysis of acoustic signals using the Fast Fourier Transformation (FFT). Most manufacturers that are producing domestic appliances such as washing machines, dishwashers or refrigerators have a problem with the final product because these machines can make noise and vibrations during the running. Manufacturers try to decrease this unpleasant and noisy event. However, at first we need to know natural frequencies of vibrated system and then we can take measures to reduce the vibrations. Recording sound to a digital file and transforming the data by the Fast Fourier Transformation is one of the ways how to accomplish that.

Keywords: FFT; MATLAB; acoustic signal; frequency analysis

Nomenclature

| $T$ | period of the signal function (s) |
| $t$ | time variable (s) |
| $x(t)$ | time-dependent function |
| $X(\omega)$, $Y(\omega)$, $Z(\omega)$ | frequency-dependent functions |
| $W$ | variable |
| $a$, $b$ | coefficients |
| $N$ | number of samples |
| $\omega$ | angular frequency (rad.s$^{-1}$) |

Greek symbols

$\omega$ coefficient of the frequency

1. Introduction

Fourier series can decompose any periodic signal or function into the sum of simple goniometric functions, namely function sinus and cosines. This decomposition of a complex function to set of simple function is the main advantage of the Fourier method [1]. The main reason for using the FFT in mechanical engineering science is to transform some time-domain
digital signal into the frequency-domain signal. This approach is very useful for determining modal parameters of vibrating systems. If the vibrating system generates noise during its vibration, it is possible to record noise to the digital wave file and use the data for further processing.

This paper describes some of the basics of FFT and discusses an example how eigenfrequencies of noisy vibrating system can be recorded to a digital sound file. Fourier analysis was performed using software MATLAB. The results are compared with the other ones, received using the Finite Element Method (FEM) program.

2. Discrete signals

In electronic signal and information processing and transmission, digital technology is increasingly being used because, in various applications, digital signal transmission has many advantages over analog signal transmission.

Unlike analog technology which uses continuous signals, digital technology encodes the information into discrete signal states (Fig. 1a; 1b). A binary signal representing only two states contains very little information compared to an analog signal. If a quantity to be represented digitally requires a wider range of values, it must be described by several bits (Fig. 1c) [4].

![Fig. 1. Analog and discrete signal in time domain](image)

3. Fourier Series and Fourier transformation

Assume that we have a periodic function. For period \(-T \leq t \leq T\) we can write this function by the infinite series

\[
x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos \frac{2\pi k t}{T} + b_k \sin \frac{2\pi k t}{T} \right)
\]

where \(x(t)\) is the function (signal) in time domain, and \(a_k\) and \(b_k\) are coefficients of the series to be determined. The integer \(k\) corresponds to the frequency of the wave.

With using Euler’s formula and Fourier integral we get

\[
X(\omega) = \int_{-T}^{T} x(t) e^{-i\omega t} \, dt
\]

where \(\omega\) is angular frequency and \(X(\omega)\) is function of amplitude spectrum of \(x(t)\). Equation (2) is called Fourier transformation of \(x(t)\).

3.1. The Fast Fourier Transformation

The Fast Fourier Transformation (FFT) is an effective algorithm of Discrete Fourier Transformation (DFT) which decreases calculating time from \(N^2\) to \(N \log_2 N\) where \(N\) is number of samples of the discrete signal. That fact means the enormous time saving in comparison with DFT. Discretization of the time signal needed for Discrete Fourier Transform is shown in Fig.2.

FFT algorithm is based on the fact that every discrete Fourier transformation with \(N\) samples can be divided into the two Fourier transforms, each with \(N/2\) samples (first with even samples and second with odd samples), Fig.3.
Fourier transformation is then sum of two new Fourier transforms:

\[ X_k = \sum_{r=0}^{N-1} x_r e^{-\frac{2\pi ir}{N}} = \]
\[ = \sum_{r=0}^{\frac{N}{2}-1} x_{2r} e^{\frac{2\pi i(2r)}{N}} + \sum_{r=\frac{N}{2}}^{N-1} x_{(2r+1)} e^{\frac{2\pi i(2r+1)}{N}} = \]
\[ = \sum_{r=0}^{\frac{N}{2}-1} x_{2r} e^{\frac{2\pi ir}{N/2}} + e^{\frac{\pi i}{N/2}} \sum_{r=0}^{\frac{N}{2}-1} x_{(2r+1)} e^{\frac{2\pi ir}{N/2}} \]  

(3)

where \( r \) is sample number. We have two new Fourier transforms in equation (3) so we can define real variables

\[ Y_k = \sum_{r=0}^{\frac{N}{2}-1} x_{2r} e^{\frac{2\pi ir}{N/2}} \]  

(4)

\[ Z_k = \sum_{r=0}^{\frac{N}{2}-1} x_{(2r+1)} e^{\frac{2\pi ir}{N/2}} \]  

(5)

And the complex variable

\[ W = e^{\frac{\pi i}{N}} \]  

(6)

With regard to equations (4), (5) and (6) we can write equation (3) as
Equation (7) is included in most of digital signal processing software which use FFT.

4. Eigenfrequencies determination in MATLAB

We will determine eigenfrequencies of the beam with rectangular cross-section area shown in Fig. 4.

![Beam used in experiment](image)

Fig. 4. Beam used in experiment

Boundary conditions of the beam are with both free ends. We have to excite eigenfrequencies by appropriate device in order to make audible sound (tinkle). We can use impact hammer for excitation. MATLAB allows us to record sound directly to a digital wave file (.wav) by command \texttt{wavrecord}:

\begin{verbatim}
>>wave=wavrecord(n,Fs)
\end{verbatim}

records \( n \) samples of an audio signal, sampled at rate of \( F_s \) (Hz).

Recorded sound is shown in Fig. 5. We have to transform this signal from time-domain to frequency domain in order to see frequency spectrum.

![Recorded sound in MATLAB](image)

Fig. 5. Recorded sound in MATLAB.

In order to transform the time signal, FFT instruction command

\begin{verbatim}
>>p=fft(wave)
\end{verbatim}

was used.

The command for plotting frequency spectrum is

\begin{verbatim}
>>semilogy(f,p)
\end{verbatim}

where \( f \) is frequency in Hz. In Fig. 6 is shown frequency spectrum from 0 to 1600 Hz.

![Frequency spectrum of the recorded sound](image)

Fig. 6. Frequency spectrum of the recorded sound
We can see there are four eigenfrequencies in this picture: 1: 218.90 Hz; 2: 601.2 Hz; 3: 650.70 Hz; 4: 1176.00 Hz.

5. Comparison of the FFT results with FEM values

Usually it is necessary to compare results with another method. In this case we will perform frequency analysis by Finite Element Method (FEM). FEM computing has been realized by SolidWorks 2009. The beam model and its meshed geometry are shown in Fig.7 and Fig.8. There were set conditions equal to real situation and the analysis was accomplished in SolidWorks Simulation module.

Fig. 7. Solid model                         Fig. 8. Meshed model

In Fig.9, 10, 11, 12 are shown first four eigenshapes associated with eigenfrequencies. These eigenfrequencies are listed in Table 1. As we can see the values are similar enough.

Fig. 9. Eigenshape no. 1                     Fig. 10. Eigenshape no. 2

Fig. 11. Eigenshape no. 3                   Fig. 12. Eigenshape no. 4
Table 1. First four Eigenvalues of the beam

| Mode No. | FFT frequency (Hz) | FEA frequency (Hz) | difference (%) |
|----------|--------------------|--------------------|----------------|
| 1        | 218.90             | 223.36             | 2.04           |
| 2        | 601.20             | 612.78             | 1.93           |
| 3        | 650.70             | 653.69             | 0.46           |
| 4        | 1176.00            | 1195.50            | 1.66           |

6. Conclusions

The paper deals with possibilities of using FFT at the analysis of mechanical vibration. The time-dependent series of different physical quantities from experiments provide us with a basis for investigation of mechanical system properties. FFT analysis is good choice for transforming some digital signal from time-domain to frequency domain. We can transform also different data from acoustic sound such as acceleration, velocity or displacement obtained from sensors.

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References

[1] Bracewell, R., N., 1991. The Fourier Transform & Its Applications, McGraw-Hill.
[2] Hanna, J., Ray, Rowland, John, H., 1990. Fourier Series, Transforms and Boundary Value Problems, Braun-Brumfield, USA, ISBN 0-471-61983-3
[3] Ivančo, V., Kubín, K., Kostolný, K., 1994. Metóda konečných prvkov I, Košice: Elfa, ISBN 80-96731-4-0
[4] Samson, AG, Digital signals, Frankfurt, Germany <http://www.samson.de/pdf_en/l150en.pdf>
[5] Trebuha, F., Šimčák, F., 2007. Príručka experimentálnej mechaniky, TypoPress Košice, ISBN 970-80-8073-816-7