Pumping and Cooling of Nanomechanical Vibrations Generated by Cooper-Pair Exchange

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Abstract
We consider a nanoelectromechanical system composed of a carbon nanotube suspended between two normal leads and coupled to a superconducting scanning tunneling microscope (STM) tip via vacuum tunnel barrier. Treating the nanotube as a single-level quantum dot, it is shown that an applied voltage between the superconducting STM tip and normal leads gives rise to a pumping or a cooling of the mechanical subsystem depending on the direction of the electronic flow. It is also demonstrated that the transition between these two regimes is controlled by the strength of the tunnel coupling between the nanotube and superconducting STM tip and the relative position of the electronic level. Such phenomena are realized due to a specific electromechanical coupling that is fully governed by the quantum dynamics of the Cooper pairs. The amplitude of the self-sustained oscillations in the pumping regime is analyzed numerically, and the effective temperature of the mechanical subsystem in the cooling regime is obtained.

1 Introduction

Nanoelectromechanical systems (NEMS) provide a promising platform for investigations of the quantum mechanical interplay between mechanical and electronic subsystems [1, 2]. The generation of self-driven mechanical oscillations by a dc electronic flow [3, 4] is a bright exhibition of such interplay. The study of self-driven phenomena is itself an interesting problem from a fundamental
point of view since such phenomena are promising for mass and force sensing, while the underlying physical processes have potential applications for mechanical cooling [5–9]. Self-driven mechanical oscillations were first observed in a carbon nanotube (CNT)-based transistor [10], and their transport signatures were later verified in further studies [11, 12]. Recently, an experimental observation of self-driven oscillations in a nanomechanical-suspended CNT-based resonator in the Coulomb blockade regime were reported [13, 14].

Recently, much attention has been paid to effects based on the excitation of mechanical modes due to coherent quantum dynamics of the electronic subsystem. Such dynamics are naturally induced by the macroscopic phase coherence of the superconducting (SC) elements incorporated in the mesoscopic system. Several interesting physical effects have been predicted in various superconducting NEMS, see, e.g., [15–26].

The crucial point for NEMS performance is the nature of the coupling between the nanomechanical and electronic subsystems. Usually, the coupling is associated with the localization of electronic charge or spin on the movable parts [27, 28], while at the same time it can also be associated with the so-called covalent coupling [29] (or covalent bonding) based on the sharing of electrons between atoms and molecules that are well known in chemistry. Superconducting elements incorporated into NEMS, the ground state of which may be considered as a macroscopic “molecule”, open the possibility to consider this type of coupling as an origin for electromechanical performance. Namely, a SC lead located near a moving quantum dot (QD) can affect its electronic state due to the tunneling exchange of Cooper pairs (SC proximity effect). Moreover, if the tunneling amplitude depends on the distance between the QD and the SC lead, such exchange also provides a coupling between the electronic and mechanical degrees of freedom. It is associated only with the coherent quantum superposition between empty and double-occupied states of the QD and does not admit the classical interpretation. This is in contrast with previous works, see, e.g., [22–24], where electromechanical coupling has been established through the charge or spin localized in the QD.

In this paper, we address a question of how coherent electron-pair exchange between the movable quantum dot and superconducting electrode affects mechanical oscillations in a particular NEMS. By calculating the Wigner function of the stationary state of the mechanical subsystem, we show that the interplay between coherent two-electron (Cooper pair) and an incoherent one-electron tunneling to/from the moving part of the NEMS can lead to pumping or cooling effects depending on the parameters.

The paper is organized as follows. Section 2 is devoted to the formulation of the model describing a hybrid superconducting–normal metal nanoelectromechanical device. The reduced density matrix technique and a Wigner function description of the mechanical subsystem are discussed in Sect. 3. Details about the mechanical subsystem in the two regimes of cooling and pumping are provided in Sect. 4, and a summary and discussions are given in Sect. 5.
2 Model

A sketch of the NEMS under investigation is presented in Fig. 1. We consider a nanoelectromechanical device consisting of a single-wall CNT suspended between two equally voltage-biased normal electrodes and coupled to a grounded superconducting scanning tunneling microscope (STM) tip via vacuum tunnel barrier. We assume that the CNT is short enough to operate in the regime where the electronic mean-level spacing $\Delta E \sim \frac{\hbar v_F}{L}$, where $L$ is the CNT length, is greater than the temperature $k_B T$ and bias voltage $eV$. The CNT bending, which crucially affects the tunnel coupling with the STM tip, is described by the fundamental flexural mode. These assumptions allow us to treat the CNT as a movable single-level QD whose position relative to the SC tip is free to vary.

The Hamiltonian of the model reads as follows:

$$H = H_N + H_S + H_{QD} + H_{tun}.$$  \hspace{1cm} (1)

The first two terms in Eq. (1) describe all the electrodes in the system: the two normal metal leads that are equally biased by voltage $V$, and the SC STM tip characterized by an order parameter $\Delta$:

$$H_N = \sum_{k\sigma} (\epsilon_k - eV) c_{k\sigma}^{\dagger} c_{k\sigma},$$  \hspace{1cm} (2)
\[ H_S = \sum_{k\sigma} \left\{ \xi_k a_{k\sigma}^+ a_{k\sigma} - \Delta (a_{k\uparrow}^+ a_{-k\downarrow}^+ + H.c.) \right\}. \] (3)

Here, \( c_{k\sigma}^\dagger \) and \( a_{k\sigma} \) are annihilation (creation) operators of the electrons with spin (\( \sigma = \uparrow, \downarrow \)) in the normal (\( j = L \) and \( j = R \) stand for the left and right electrodes, respectively) and SC leads with energies \( \epsilon_k \) and \( \xi_k \), correspondingly.

The Hamiltonian of the single-level vibrating CNT-QD reads as follows:

\[ H_{QD} = \sum_{\sigma} \varepsilon_{\sigma} d_{\sigma}^\dagger d_{\sigma} + \frac{\hbar \omega_0}{2} (\chi^2 + \hat{p}^2) - \alpha \hat{x} \sum_{\sigma} d_{\sigma}^\dagger d_{\sigma}. \] (4)

The first term in Eq. (4) describes the quantum dynamics of the electronic degree of freedom: \( \varepsilon_{\sigma} \) is the QD electron energy level, and \( d_{\sigma}^\dagger \) and \( d_{\sigma} \) are annihilation and creation operators of the electrons in the QD. The second term in Eq. (4) characterizes the linear dynamics of the fundamental flexural mode, which we treat as a mechanical oscillator with frequency \( \omega_0 \). Dimensionless operators \( \hat{x} = \hat{X}/x_0, \hat{p} = x_0 \hat{P}/\hbar \) are canonically conjugated displacement and momentum, where \( x_0 = \sqrt{\hbar/2m \omega_0} \) is the amplitude of the zero-point oscillations with \( m \) the effective mass of the CNT. The third term in the Eq. (4) is an electromechanical coupling induced via electrostatic interaction of the charged CNT with a gate electrode. In Eq. (4) we neglect an electron–electron interaction in the QD \( Ud_{\sigma}^\dagger d_{\sigma}^\dagger d_{\sigma}^\dagger d_{\sigma}^\dagger \) in the case when \( U < eV \), because it only results in renormalization of the QD energy level \( \varepsilon_0 \rightarrow \varepsilon_0 + U/2 \) in equations below, and as the results give only quantitative correction to the considered phenomena.

The last term in Eq. (1),

\[ \begin{align*}
H_{\text{tun}} = & \sum_{k\sigma} e^{e\zeta/2} \left( r_{k\sigma}^\dagger c_{k\sigma}^\dagger a_{k\sigma}^\dagger (r_{k\sigma}^\dagger)^* d_{\sigma}^\dagger a_{k\sigma} \right) \\
& + \sum_{kj\sigma} \left( r_{k\sigma}^\dagger c_{j\sigma}^\dagger a_{k\sigma}^\dagger (r_{k\sigma}^\dagger)^* d_{\sigma}^\dagger c_{j\sigma} \right),
\end{align*} \] (5)

describes the tunneling processes between the CNT and i) the STM tip with a deflection-dependent hopping amplitude \( r_{k\sigma}^\dagger \exp(\zeta \hat{x}) \) (\( \zeta \sim x_0/\xi \) and \( l \approx 10^{-10} \text{m} \) is the tunneling length of the barrier, \( x_0 \approx 2 \text{pm} \) for the typical CNT-resonator of the length \( L \sim 1 \text{nm} \), see the first line in Eq. (5), and ii) the normal leads with a tunnel amplitude \( r_{k\sigma}^\dagger \), see the second line in Eq. (5). Note that the deflection-dependent tunneling amplitude causes the electromechanical coupling in the above model. 3

The electromechanical coupling induced by the long-range electrostatic interaction, \( a \sim eE x_0 \sim x_0 (eV/L) \) is determined by the voltage drop \( V \) between bulk electrodes and the characteristic size of the system. Meanwhile, the covalent coupling is determined by the short-range proximity effect \( \Delta_{pr} \sim x_0 \Gamma_S/l \), where \( \Gamma_S \sim \nu_{\sigma} |r_{k\sigma}^\dagger|^2 \) is tunnel coupling with SC electrode (it determines the QD energy-level width if SC tip would be in a normal state), \( \nu_{\sigma} \) is a normal density of states of electrons in the SC STM tip. Taking the minimal required value for the voltage drop \( eV \sim \Delta E \sim 1 \text{K} \), we estimate \( a/\Delta_{pr} \sim (eV/\Gamma_S)(l/L) \sim 10^{-1} \div 10^{-2} \). As the result, one can see that for the certain set of parameters the SC proximity effect-induced NEM coupling may

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exceed the coupling induced by the electrostatic force. In this paper we consider effects associated only with the covalent bonding assuming $\alpha = 0$. Nevertheless, the influence of the electromechanical coupling induced by electrostatic force ($\alpha \neq 0$) is investigated in Appendix A using a semiclassical treatment.

3 Reduced Density Matrix and Wigner Function Description

To analyze the stationary state of the mechanical subsystem in the SC–normal metal hybrid junction described by the Hamiltonian in Eq. (1), we use the reduced density matrix technique [30–32]. First, we obtain the quantum master equation for the total density matrix $\hat{\rho}$ by considering the tunneling Hamiltonian in Eq. (5) as a perturbation. Then we look for the solution of this equation in the form $\hat{\rho} = \hat{\rho}_S \otimes \hat{\rho}_N \otimes \hat{\rho}$, where $\hat{\rho}_S$ and $\hat{\rho}_N$ are density matrices describing the thermodynamic state of the SC and normal electrodes, while density matrix $\hat{\rho}$ describes the mechanical and electronic states of the movable single-level QD. After substituting this ansatz into the obtained master equation and tracing out the electronic degrees of freedom in both normal and SC leads, we get the equation for the reduced density matrix $\hat{\rho}$, which reads as follows (in $\hbar = k_B = 1$ units):

$$\dot{\hat{\rho}} = -i[H_{QD}, \hat{\rho}] + i\Delta_d [e^{\xi_x}(d^\dagger_\uparrow d^\dagger_\downarrow + h.c.), \hat{\rho}] - \mathcal{L}_e[\hat{\rho}].$$

Here, $\Delta_d = 2\pi v^0 |r^\dagger_0 k|^2$ is the strength of the intra-QD electron pairing generated by the proximity effect with the SC STM tip, and $\mathcal{L}_e[\hat{\rho}]$ is a Lindbladian superoperator describing the incoherent electronic exchange between the CNT and normal biased electrodes, which in the high-voltage regime $eV \gg 2|\epsilon_0|, \omega_0, T$ considered in this paper takes the form:

$$\mathcal{L}_e[\hat{\rho}] = \Gamma \sum_{\sigma} \left( \frac{\sigma}{2} \left( (d^\dagger_\sigma d^\dagger_\sigma, \hat{\rho}) - 2d^\dagger_\sigma \hat{\rho} d^\dagger_\sigma \right), V > 0, \right.$$  

$$\left. \frac{\sigma}{2} \left( (d^\dagger_\sigma d^\dagger_\sigma, \hat{\rho}) - 2d^\dagger_\sigma \hat{\rho} d^\dagger_\sigma \right), V < 0, \right)$$

where $\Gamma = 2\pi v^0 |r^\dagger_0 k|^2$ is the QD energy level width and $v^0_\sigma$ is the electron density of states in the normal electrodes. The direction of the electronic flow (sign of the bias voltage) is characterized by the parameter $\kappa = \text{sign}(V)$ in what follows.

The quantum master equation [Eq. (6)] is justified in the so-called deep sub-gap regime [22, 23], which assumes that all energies are smaller than the SC gap ($eV, T, \epsilon_0 \ll \Delta$) and disregards all scattering processes above the SC gap. Interestingly, the nanoelectromechanical covalent coupling, see the second term in Eq. (6), only exists as long as there is a tunnel connection with the SC lead, $\Delta_d \neq 0$. Assuming typical gap value for the Nb superconductor $T_c \approx \Delta \sim 10K$ [33], one has the limitation for applied bias voltage $\lesssim 1\text{meV}$, and as the result on temperature and the QD energy level, $T, \epsilon_0 \ll 1\text{meV}$. The latter limitation arises because the NEM effects considered in the manuscript are attenuated when quasi-particle tunneling processes are involved.

The reduced density matrix $\hat{\rho}$ acts in a Hilbert space, being a tensor product of a finite Fock space of the two-fold degenerate single-electron level quantum state and

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$L^2(x)$ space describing the mechanical degree of freedom. This Hilbert space is treated as a direct sum of four $L^2(x)$ sub-spaces: $|0, x\rangle = |0\rangle \otimes |x\rangle$, $|\sigma, x\rangle = d_\sigma^*|0\rangle \otimes |x\rangle$ ($\sigma = \uparrow, \downarrow$), and $|2, x\rangle = d_1^d d_1^d|0\rangle \otimes |x\rangle$.

Below we investigate the stationary solutions of the master equation, Eq. (6). In order to do this it is convenient to use the Wigner function representation $w_{i,i'}(x,p) = \int dy \rho^{ij}(x - \frac{y}{2}, x + \frac{y}{2}) e^{iyp}$ for the matrix elements of the density operator $\rho^{ij}(x, x') = \langle i, x| \rho |i', x'\rangle$ ($i, i' = 0, \uparrow, \downarrow$). In general, Eq. (6) generates a system of linear partial differential equations for the 16 real functions of $x$ and $p$. However, the number of relevant functions may be reduced by taking into account that $\rho^{\sigma,0}(x, x') = \rho^{\sigma,2}(x, x') = \rho^{1,1}(x, x') = 0$. We utilize this condition because of the superselection rule that forbids a quantum superposition of states with different parity. The time-reversal symmetry additionally results in $\rho^{1,1}(x, x') = \rho^{1,1}(x, x')$.

As a consequence, the system of the remaining equations can be presented as a linear combination of five functions: $W_{\Sigma}(x,p) = \sum_i w_{i,i}(x,p)$, $W_0(x,p) = w_{0,0}(x,p)$, $W_1(x,p) = 2\Re[w_{2,0}(x,p)]$, $W_2(x,p) = -2\Im[w_{2,0}(x,p)]$, and $W_3(x,p) = w_{2,2}(x,p) - w_{0,0}(x,p)$, and reads as follows:

$$\hat{L}W_{\Sigma} = \zeta \Delta_d(x) \frac{\partial W_1}{\partial p},$$  \hspace{1cm} (8)

$$(\hat{L} - 2\Gamma)W_0 = \zeta \Delta_d(x) \frac{\partial W_1}{\partial p} - \Gamma(\kappa W_3 + W_{\Sigma}),$$  \hspace{1cm} (9)

$$(\hat{L} + \hat{M})|W\rangle = \begin{pmatrix} \Delta_d(x) \zeta \frac{\partial W_0}{\partial p} \\ 0 \\ -\kappa \Gamma W_{\Sigma} \end{pmatrix}.$$  \hspace{1cm} (10)

Here, $\hat{L}$ is a differential operator that reads as

$$\hat{L} = \omega_0 \left( x \frac{\partial}{\partial p} + p \frac{\partial}{\partial x} \right),$$  \hspace{1cm} (11)

and we define the vector $|W\rangle = (W_1, W_2, W_3)^T$ and use a shorthand notation $\Delta_d(x) = \Delta_d \exp(\zeta x)$. In Eq. (10) we introduce a matrix $\hat{M} = -\Gamma \hat{1} - i2\epsilon_0 \hat{\lambda}_2 + i2\Delta_d(x)\hat{\lambda}_7$, where $\hat{1}$ is the unit matrix and $\hat{\lambda}_2, \hat{\lambda}_7$ are the Gell-Mann matrices.

To analyze the system of Eqs. (8)–(10), we use the perturbation theory over small mechanical frequency $\omega_0$ and present the vector $|W\rangle$ approximately as $|W\rangle \approx |W^{(0)}\rangle + (\omega_0/\Gamma)|W^{(1)}\rangle$. Thus, in the first order of perturbation theory over $\omega_0/\Gamma$, one can combine Eqs. (8)–(10) into a form of the Fokker–Planck equation for $W_{\Sigma}(x,p)$:

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\[
\left[ \hat{L} - d(x) \frac{\partial}{\partial p} - \zeta \Delta_d(x) \frac{\partial}{\partial p} \left( f(x) \frac{\partial}{\partial p} + \gamma(x) + R(x) \hat{L} \right) \right] W_\Sigma(x, p) = 0,
\]

(12)

where \( d(x) = -\kappa \zeta \Gamma \Delta_d(x) \langle e_1 | \hat{M}^{-1} | e_3 \rangle \) is the equilibrium displacement of the CNT induced by the SC proximity effect, and

\[
f(x) = \zeta \langle e_1 | \hat{M}^{-1} | e_1 \rangle \Delta_d(x) C_0,
\]

(13)

\[
\gamma(x) = \kappa \Gamma \langle e_1 | \hat{M}^{-1} \hat{L} \hat{M}^{-1} | e_3 \rangle,
\]

(14)

\[
R(x) = \kappa \Gamma \langle e_1 | \hat{M}^{-2} | e_3 \rangle.
\]

(15)

Here, we denote the vectors \( | e_1 \rangle = (1, 0, 0)^T, | e_3 \rangle = (0, 0, 1)^T \), and consider the fact that \( \hat{M}|W^{(0)}\rangle = -\kappa \Gamma |e_3\rangle W_\Sigma \) and \( W_0^{(0)} = C_0 W_\Sigma \) [here \( C_0 = (4\epsilon_0^2 + 2\Delta_d^2(x) + \Gamma^2)/(4\epsilon_0^2 + 4\Delta_d^2(x) + \Gamma^2) \)].

Further, we develop the perturbation theory over small parameter \( \zeta \sim 10^{-2} \sim 10^{-3} \) and expand \( \Delta_d(x) \) in Eq. (12) over the CNT’s displacement. We eliminate the shift \( \propto d(x) \) by redefining the CNT’s displacement coordinate: \( x \to x + x_{eq} \), where \( x_{eq} \) can be found by solving the equation \( \omega_0 x_{eq} = \zeta d(x + x_{eq}) \). Since the equilibrium displacement \( x_{eq} \approx -\zeta (\kappa \epsilon_0 / \omega_0) [\Delta_d^2 / (\epsilon_0^2 + \Delta_d^2 + \Gamma^2 / 4)] \) appears as a consequence of the SC proximity effect, it disappears if the coupling with the SC lead vanishes, and it additionally depends on the sign of both the CNT-QD energy level \( \epsilon_0 \) and bias voltage polarity \( \kappa \). Since \( x_{eq} \sim \zeta \), it gives only a small renormalization of \( \Delta_d(x) \), which we ignore in what follows.

As the result, one can find the solution of Eq. (12) in the form:

\[
W_\Sigma \approx W_\Sigma^{(0)}(x^2 + p^2) + O(\zeta^2).
\]

(16)

Solving Eq. (12) consistently in all orders of perturbation theory and using for convenience an action-angle representation \( x = A \cos \varphi, p = A \sin \varphi \), one can simplify Eq. (12) as follows,

\[
\left\{ \hat{\gamma}(A) A + \hat{D}(A) \frac{d}{dA} \right\} W_\Sigma^{(0)}(A) = 0.
\]

(17)

Here, the new notations introduced in Eq. (17), \( \hat{\gamma}(A) = \int_{-\pi}^\pi d\varphi \sin^2 \varphi \Delta_d(A \cos \varphi) \gamma(A \cos \varphi) \), \( \hat{D}(A) = \zeta^2 \int_{-\pi}^\pi d\varphi \sin^2 \varphi \Delta_d(A \cos \varphi) D(A \cos \varphi) \) mean the "drift" and "diffusive" coefficients, respectively, and \( D(x) \) reads as \( D(x) = f(x) + \omega_0 x_{eq} R(x) \). For simplicity, we also redefine the dimensionless amplitude taking \( A \zeta \to A \). The solution of Eq. (17) is easy to find and reads as follows,
where $Z$ is a normalization constant that should be defined from the condition $2\pi \int_0^\infty dA W_\Sigma^{(0)}(A) = 1$. Equation (18) shows the probability of the CNT to vibrate with an amplitude $A$ in the hybrid SC nanoelectromechanical device presented in Fig. 1. In the following, we will analyze the stationary solution of Eq. (18) in different regimes.

4 Cooling and Pumping Regimes

As one can see from Eq. (17), the Wigner function $W_\Sigma^{(0)}(A)$ always has an extremum at $A = 0$, and at such amplitudes when the effective damping coefficient $\tilde{\gamma}(A)$ induced by the nonlinear coupling with the electronic degree of freedom vanishes, $\tilde{\gamma}(A) = 0$. First, we would like to check the stability of the solution Eq. (18) for $A = 0$. This can be done by checking the sign of the effective inverse (dimensionless) temperature, $\beta_{\text{eff}} = (T_{\text{eff}}/\omega_0)^{-1} = Z(d^2/dA^2) \log W_\Sigma^{(0)}(A)|_{A=0}$. Note that the sign of $\beta_{\text{eff}}$ is fully determined by the effective damping $\tilde{\gamma}(0)$ since the sign of the coefficient $\tilde{D}$ is fixed for any parameter. The probability density $W_\Sigma(A)$ has a maximum at $A = 0$, i.e., the ground state with $A = 0$ is stable, if $\beta_{\text{eff}} > 0$ (or when $\tilde{\gamma}(0) > 0$), and it has a minimum, i.e. the ground state with $A = 0$ is unstable, if $\beta_{\text{eff}} < 0$ (or when $\tilde{\gamma}(0) < 0$). More precisely, the sign of $\beta_{\text{eff}}$ should be defined by the comparison of the effective damping $\tilde{\gamma}(0)$ with the real one $\propto Q^{-1}$, see footnote 1, where $Q$ is the quality factor of the CNT. For the realistic values of the quality factor $Q \sim 10^5 \div 10^6$, see [34, 35], one can find that $Q^{-1} \ll \zeta^2$. As a consequence, we ignore real damping for simplicity.

We refer to the regime with $\beta_{\text{eff}} < 0$ as a pumping regime and that with $\beta_{\text{eff}} > 0$ as a cooling regime. Using Eqs. (13)–(15) and (18), it is easy to find the effective inverse temperature as follows:

$$\beta_{\text{eff}} = 8\kappa \frac{\omega_0\epsilon_0}{(4\epsilon_0^2 + 2\Delta^2_d + \Gamma^2)(\Gamma + 4\epsilon_0^2\Gamma^2 + 4\Delta^2_d\epsilon^2 + 8\Delta^2_d\Gamma^2) - 16\Delta^2_d\epsilon^2(\epsilon^2 + 3\Gamma^2)} \left[2\Delta^2_d(3\Gamma^2 + \xi^2) - \Gamma^2(4\epsilon_0^2 + \Gamma^2)\right].$$

(19)

1 Note, coefficients in Eq. (18) should be replaced as follows $\tilde{\gamma}(A) \rightarrow \tilde{\gamma}(A) + (\zeta^2Q)^{-1}$ and $\tilde{D}(A) \rightarrow \tilde{D}(A) + (2\zeta^2Q)^{-1} \coth(\omega_0/2T)$ due to interaction with the thermodynamic mechanical environment. Here, $Q$ is a quality factor of the CNT and $T$ is the temperature of the environment.

2 Note, an effective inverse temperature $\beta_{\text{eff}}$ has the physical meaning only in the cooling regime since it corresponds to the width of the Gibbs-like distribution function. In the pumping regime, such coefficient indicates that the probability to find system at the state $A \neq 0$ is larger than probability to find system at the state $A = 0$. The coefficient $\beta_{\text{eff}}$ in the pumping regime should not be confused with the inverse temperature associated with the inverse energy-level population introduced, e.g., in [38].
where $\xi = 2\sqrt{\varepsilon_0^2 + \Delta_d^2}$ is the Andreev energy level difference [36, 37]. In Fig. 2, one can see a stability diagram (effective inverse temperature) as a function of different system parameters: the relative position of the electronic level in the QD and the strength of the tunnel coupling between the CNT and SC lead. The red and blue color schemes correspond to the pumping and cooling regimes, respectively. Interestingly, the stability diagram may be obtained “quasiclassically” by considering the CNT’s displacement and momentum as classical variables; see details in the Appendix.

The bias voltage affects the stationary regime of the CNT’s oscillations only by resolving the direction of the electronic flow since $\bar{\gamma}(A) \propto \kappa$. Below, we explore the case of $V > 0$ ($\kappa = +1$) only. The inverse effective temperature changes its sign when $\varepsilon_0 = 0$ and when the condition $2(4\varepsilon_0^2 + 4\Delta_d^2 + 3\Gamma^2) = (\Gamma/\Delta_d)^2(4\varepsilon_0^2 + \Gamma^2)$ is fulfilled, see the numerator of Eq. (19). More specifically, one has a transition between cooling and pumping at $\Gamma/\Delta_d \approx 2.37$ if $\varepsilon_0/\Delta_d \to 0$, and at $\Gamma/\Delta_d \approx \sqrt{2}$ if $\varepsilon_0/\Delta_d \to \infty$. The latter coincides with the instability condition in similar hybrid SC–normal metal NEMS, see Eq. (13) in [25].

Next, we analyze the value of the minimum effective temperature in the cooling regime (blue regions in Fig. 2). This regime corresponds to a bell-shaped Wigner function with a maximum at $A = 0$ and width determined by $\beta_{\text{eff}}$, as shown in the left panel of Fig. 3a. We note that the effective damping coefficient $\bar{\gamma}(A)$ is always a positively defined function, as one can see in Fig. 3b with the dashed and dash-dotted curves. In the adiabatic regime $\omega_0 \ll \Gamma$, we obtain the minimal value of the effective temperature $T_{\text{eff}}$ using Eq. (19): $T_{\text{eff}} \approx \Gamma/2$ if $\Gamma \gg \Delta_d$ achieved at $\varepsilon_0 = -\Gamma/2$, and $T_{\text{eff}} \approx \Gamma/\sqrt{2}$ if $\Gamma \ll \Delta_d$ achieved at $\varepsilon_0 = \sqrt{2}\Delta_d^2/\Gamma$. In the non-adiabatic regime, $T_{\text{eff}}$ may be obtained directly from Eqs. (8)–(10) using the perturbation

![Fig. 2](image-url) Effective inverse temperature $\beta_{\text{eff}}(0)/(\omega_0/\Gamma)$, see Eq. (19), as a function of the CNT-QD’s energy level position $\varepsilon_0$ and normalized tunnel coupling with the superconducting STM tip $\Delta_d/\Gamma$. The blue color scheme indicates the regime of cooling (i.e., a ground state solution with the amplitude of mechanical oscillations $A = 0$ is stable), while the red color scheme indicates the regime of self-sustained oscillations (the solution with $A = 0$ is unstable). White regions separate the regimes. The diagram is plotted at fixed bias voltage polarity $\kappa = +1$. 
theory for small amplitudes. In the most interesting case of resonance, when the frequency of the mechanical oscillations coincides with the Andreev energy level difference, \( \omega_0 = \sqrt{\epsilon_0^2 + \Delta_d} \), the minimum of the effective temperature reads

\[
T_{\text{eff}} = \Gamma / \sqrt{2} \quad \text{at} \quad \epsilon_0 = \Gamma / 2 \sqrt{2}
\]

in the limit when \( \Delta_d \ll \Gamma \).

The stationary state of Eq. (18) with \( A = 0 \) in the pumping regime becomes unstable when \( \beta_{\text{eff}} < 0 \), and the CNT vibrations develop into pronounced self-sustained oscillations of finite amplitude \( A_M \). Interestingly, the appearance of the finite amplitude \( A_M \) even in the absence of real damping \( Q^{-1} = 0 \) is associated with the emergence of a new extremum of \( W^{(0)}_\Sigma \) that corresponds to the condition \( \tilde{\gamma}(A_M) = 0 \).

Fig. 3  a Examples of the Wigner function obtained using Eq. (18) as a function of dimensionless displacement and momentum for the cooling regime (left panel) and pumping regime (right panel). The maximum value of the Wigner function corresponds to the stable solution with \( A = 0 \) in the cooling regime and to some finite \( A_M \) in the pumping regime. b Values of the effective damping term \( \tilde{\gamma}(A) \) as a function of the dimensionless amplitude \( \zeta A \) for different sets of parameters: \( \epsilon_0 / \Delta_d = 4, \Gamma / \Delta_d = 10 \) (red solid line), \( \epsilon_0 / \Delta_d = 4, \Gamma / \Delta_d = \sqrt{2} \) (blue dashed line), \( \epsilon_0 / \Delta_d = -4, \Gamma / \Delta_d = 0.5 \) (black dotted line), and \( \epsilon_0 / \Delta_d = 4, \Gamma / \Delta_d = 0.5 \) (black dash-dotted line). Only the case of \( \kappa = +1 \) and \( \zeta = 10^{-2} \) was considered.
We refer to such a phenomenon as a self-saturation effect. In this regime, the Wigner function is ring-shaped and has a maximum at $A_M$, as shown in the right panel of Fig. 3a.

In Fig. 3b, one can see the effective damping coefficient $\tilde{\gamma}(A)$ as a function of the CNT bending amplitude for different sets of system parameters. Here, one can recognize two different scenarios for reaching the self-saturation effect depending on the sign of the product of the QD energy level and direction of electronic flow, $\kappa \epsilon_0$. The first scenario corresponds to the case when $\Delta_d \ll \Gamma$ and $\kappa \epsilon_0 > 0$, see the red solid line in Fig. 3b. The CNT-QD vibrational ground state with $A = 0$ is unstable (since $\tilde{\gamma}(0) < 0$) and the amplitude of the CNT bending grows until the condition $\tilde{\gamma}(A) > 0$ is fulfilled. The mechanical subsystem enters the cooling regime accompanied by CNT amplitude saturation until the condition $\tilde{\gamma}(A) < 0$ is achieved, after which the mechanical subsystem enters the pumping regime again. As a consequence of such repetitions, a stationary regime with $\tilde{\gamma}(A_M) = 0$ is established. Thus, the value of the self-saturated amplitude $A_M$ is determined by the parameters of the system. In Fig. 4, one can see values of the self-saturation amplitude as a function of the ratio between the tunnel coupling with the SC lead and the energy level width $\Gamma/\Delta_d$. The gray region in Fig. 4 corresponds to the cooling regime.

In the second scenario occurring when $\Delta_d \gg \Gamma$ and $\kappa \epsilon_0 < 0$, see the black dotted line in Fig. 3b, the mechanical subsystem with $A = 0$ is unstable, while the development of CNT bending is not intrinsically limited. Thus, the saturation of the CNT amplitude is fully determined by the coupling with the thermodynamic environment, $Q \neq 0$.

Note, considered in this paper pumping regime relates to the so-called lasing effect. We address the readers to study Ref. [23], where the lasing phenomenon for similar normal metal-SC hybrid system at the limit $\Gamma \lesssim \omega_0$ was investigated. To see this analogy, it is rather useful to treat the Wigner distribution function $W_{\Sigma}^{(0)}(A)$ as the averaged occupation number of the vibrational modes, $P(N)$ with $N = A^2/2$.

Finally, we would like to qualitatively discuss the origin of the pumping and cooling in the system under consideration. The mechanism of these two processes is based on the asymmetry in transition between different electronic states in the QD. The single-particle injection from the normal electrodes leads to the consecutive transitions from empty to double occupied state, $|0\rangle \rightarrow |\sigma\rangle \rightarrow |2\rangle$ (if $\kappa = 1$). Inverse processes, $|\sigma\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |\sigma\rangle$, are prohibited because of large bias voltage. Meanwhile, SC proximity effect allows for coherent electron pair tunneling between the QD and SC lead. It results in coherent Rabi oscillations between empty
and double-occupied states $|0\rangle \leftrightarrow |2\rangle$, which may assist by emitting/absorbing the energy of the mechanical subsystem. It occurs since two electronic states $|0\rangle, |2\rangle$ are not the eigenstates of the QD-SC subsystem (while the Andreev states are). Nevertheless, at positive QD energy level offset, $\epsilon_0 > 0$, the emission rate dominates over the absorption rate during $|2\rangle \rightarrow |0\rangle$ transition. It corresponds to the pumping of mechanical subsystem. At negative dot’s level, $\epsilon_0 < 0$, situation is inverse: Intensity of the energy absorption larger than intensity of the energy emission. Nanoelectromechanical system is in the cooling regime. Competition between coherent dynamics of the electron pairs and incoherent single-electron tunneling results in more subtle condition for the pumping-to-cooling crossover summarized in Fig. 2.

In the end, we shortly discuss the self-saturation phenomenon. Its origin is connected with the “quantum” nature of the electromechanical coupling induced by the SC proximity effect (Cooper-pair tunneling). The “quantumness” means that the interaction between charge and mechanical degrees of freedom is defined by the superposition of the two states of the QD: empty and double occupied. Formally, it can be seen by writing the Newton’s equation for the average value of the CNT displacement, see the derivation in Appendix A. The quantity in the r.h.s of the Newton’s equation, $\propto \Delta d[\rho^{0,2}(t) + \rho^{2,0}(t)]$, can be treated as the averaged nano-mechanical force. As a consequence, the electromechanical coupling and, what is more important, back-action have quite sophisticated dependencies on the CNT’s displacement, $\epsilon_0$, and $\Delta_d$.

5 Conclusions

In conclusion, we investigated the stationary regimes of the mechanical subsystem in a hybrid nanoelectromechanical device comprising a carbon nanotube (CNT) suspended between two normal metal leads, equally biased by a voltage, and weakly coupled to a nearby superconducting (SC) STM tip. The proximity effect with the SC lead determined by the CNT displacement-dependent tunneling processes provides a coupling between the electronic and mechanical degrees of freedom. As a consequence, the exchange of two electrons (Cooper pair) between the SC lead and the single-level quantum dot (QD) formed by the CNT results in the appearance of a specific force that acts on the CNT bending, which is similar to the force in the covalent bonds known in chemistry. Our analysis showed that the existence of two distinct cooling and pumping regimes of the mechanical subsystem is fully determined by the direction of the electronic flow, the relative position of the single electron energy level, and the ratio between the energy level width and the strength of the intra-QD electron pairing. The effective temperature of the mechanical subsystem was predicted in the regime of mechanical cooling. This work demonstrated that peculiar quantum dynamics of the Cooper pair state forming in the QD in the pumping regime results in self-saturated bending oscillations of the CNT resonator.
6 Appendix A: “Semiclassical” Treatment

In this Appendix, we derive the Newton’s equation for the CNT’s displacement by using the reduced density matrix technique and accounting for the effects of electro-mechanical coupling induced by both a gate voltage (external electric field) and superconducting proximity effect. For this we use the Liouville-von Neumann equation Eq. (6) together with the Hamiltonian of the QD:

\[ H_{QD} = \sum_{\sigma} (\varepsilon_{0} - \alpha \hat{x}) d_{\sigma}^{\dagger} d_{\sigma} + \frac{\hbar \omega_{0}}{2} (\hat{p}^{2} + \hat{x}^{2}), \]  

(A.1)

where \( \alpha \) is the electro-mechanical interaction strength determined by the electrostatic interaction of the charged CNT-QD with the gate electrode.

Since we are interested in a classical consideration, we treat the operators of the CNT displacement \( \hat{x} \) and momentum \( \hat{p} \) as numbers after tracing them with the reduced density matrix:

\[ x_{c} = \text{Tr}\{\hat{x}\rho\}, \quad p_{c} = \text{Tr}\{\hat{p}\rho\}. \]

The system of the relevant equations for the reduced density matrices \( R_{1} = \langle 0 | \hat{\rho} | 2 \rangle + \langle 2 | \hat{\rho} | 0 \rangle, \quad R_{2} = i\langle 2 | \hat{\rho} | 0 \rangle - i\langle 0 | \hat{\rho} | 2 \rangle, \) and \( R_{3} = \langle 2 | \hat{\rho} | 2 \rangle - \langle 0 | \hat{\rho} | 0 \rangle \) read as follows:

\[ \dot{R}_{1} = -\Gamma R_{1} - 2(\varepsilon_{0} - \alpha x_{c}) R_{2}, \]  

(A.2)

\[ \dot{R}_{2} = 2(\varepsilon_{0} - \alpha x_{c}) R_{1} - \Gamma R_{2} + 2\Delta_{d} e^{\xi x_{c}} R_{3}, \]  

(A.3)

\[ \dot{R}_{3} = -2\Delta_{d} e^{\xi x_{c}} R_{2} - \Gamma R_{3} + \kappa \Gamma. \]  

(A.4)

Using the perturbation theory over small CNT displacement, the reduced density matrices can be expanded as \( R_{i}(t) = R_{i}^{(0)} + R_{i}^{(1)}(t) (i = 1, 2, 3) \). We find the stationary solution of the system in Eqs. (A.2)–(A.4) as

\[ R_{1}^{(0)} = -\kappa \frac{4\varepsilon_{0} \Delta_{d}}{D}, \quad R_{2}^{(0)} = \kappa \frac{2\Gamma \Delta_{d}}{D}, \quad R_{3}^{(0)} = \frac{\kappa}{D}, \]  

(A.5)

where \( D = \xi^{2} + \Gamma^{2} \) and \( \xi = 2\sqrt{\varepsilon_{0}^{2} + \Delta_{d}^{2}} \) is the Andreev energy level distance. Introducing the vector \( |R^{(1)}\rangle = (R_{1}^{(1)}, R_{2}^{(1)}, R_{3}^{(1)})^{T} \), we obtain in the first order of perturbation theory:

\[ |\dot{R}^{(1)}\rangle = -\hat{m}|R^{(1)}\rangle + 2\xi \Delta_{d} x_{c}(t)|e_{\xi}\rangle + 2\alpha x_{c}|e_{a}\rangle, \]  

(A.6)

where \( |e_{\xi}\rangle = (0, R_{3}^{(0)}, -R_{2}^{(0)})^{T}, |e_{a}\rangle = (R_{2}^{(0)}, -R_{1}^{(0)}, 0)^{T} \) and

\[ \hat{m} = \begin{pmatrix} \Gamma & 2\varepsilon_{0} & 0 \\ -2\varepsilon_{0} & \Gamma & -2\Delta_{d} \\ 0 & 2\Delta_{d} & \Gamma \end{pmatrix}. \]  

(A.7)

The solution of Eq. (A.6) can be written in the adiabatic regime, \( \omega_{0}/\Gamma \ll 1 \), as follows:
\[
|\mathbf{R}^{(1)}(t)| = \int_0^\infty d\tau [x_c(t) - \tau \dot{x}_c(t)] \hat{U} e^{-\hat{E} \tau} \hat{U}^{-1} \{4 \zeta \Delta_d \varepsilon \mathbf{c} + 2\alpha e_\alpha \} 
\]  \quad (A.8)

where
\[
\hat{U} = \begin{pmatrix} -\Delta_d & 2\varepsilon_0 & 2\varepsilon_0 \\ 0 & i\xi & -i\xi \\ \varepsilon_0 & 2\Delta_d & 2\Delta_d \end{pmatrix} \quad \text{and} \quad \hat{U}^{-1} = \frac{1}{\xi^2} \begin{pmatrix} -4\Delta_d & 0 & 4\varepsilon_0 \\ \varepsilon_0 & -i\frac{\xi}{2} & \Delta_d \\ \varepsilon_0 & i\frac{\xi}{2} & \Delta_d \end{pmatrix} 
\]  \quad (A.9)

and \(\hat{E}\) is a matrix with the eigenvalues of the matrix \(\hat{m} = \Gamma\), \(\lambda_{2,3} = \Gamma \pm i\xi\) as diagonal elements. Here, a small retardation effect of the CNT displacement \(x_c(t - \tau) \approx x_c(t) - \tau \dot{x}_c(t)\) is taken into account (adiabatic regime). As a result, after taking the integral in Eq. (A.8) we obtain
\[
\begin{align*}
R^{(1)}_1(t) & \approx \dot{x}_c(t)2\kappa \zeta \varepsilon_0 \Delta_d \frac{\{4\Gamma^2(\Gamma^2 + 4\varepsilon_0^2) - 8\Delta_d^2(\xi^2 + 3\Gamma^2)\}}{\Gamma D^3} \\
& \quad - \dot{x}_c(t)4\kappa \alpha \Delta_d \frac{\{4\Delta_d^2(2\Gamma^2 + \xi^2) + \Gamma^2(\Gamma^2 - 12\varepsilon_0^2)\}}{\Gamma D^3},
\end{align*} 
\]  \quad (A.10)

and
\[
\begin{align*}
R^{(1)}_3(t) & \approx \dot{x}_c(t)4\kappa \zeta \Delta_d^2 \frac{\{4\varepsilon_0^2(4\Gamma^2 + \xi^2) + \Gamma^2(3\Gamma^2 - 4\Delta_d^2)\}}{\Gamma D^3} \\
& \quad + \dot{x}_c(t)4\kappa \alpha \varepsilon_0 \Delta_d^2 \frac{\{4\xi^2 + 20\Gamma^2\}}{\Gamma D^3},
\end{align*} 
\]  \quad (A.11)

From here, let’s operate with the dimensionless variables by performing the following replacement: \(\omega_0 t \rightarrow t\) and \(\{\varepsilon_0, \Gamma, \Delta_d, \alpha\}/\omega_0 \rightarrow \{\epsilon_0, \Gamma, \Delta_d, \alpha\}\).

The equations of motion for the CNT’s displacement and momentum read as follows:
\[
\ddot{x}_c = p_c, \quad (A.12)
\]
\[
\dot{p}_c = -x_c + \zeta \Delta_d e^{\xi x} R_1(t) + \alpha + \alpha R_3(t). \quad (A.13)
\]

As the result, one obtains the Newton’s equation (accounting for small displacement of the CNT):
\[
\ddot{x}_c + x_c = \alpha + \zeta \Delta_d R_1(t) + \alpha R_3(t). \quad (A.14)
\]

Combining the Newton’s equation (A.14) with the results obtained above in Eqs. (A.10, A.11) we find
\[
\ddot{x}_c + \frac{\Delta_d^2}{\Gamma D^3} (\alpha^2 f_1 + \alpha \zeta f_2 + \zeta^2 f_3) \dot{x}_c + x_c = \alpha - \zeta \kappa \frac{\Delta_d^2 4\epsilon_0}{D} + \alpha \kappa \frac{4\epsilon_0^2 + \Gamma^2}{D}, \quad (A.15)
\]

where
Interestingly, Eq. (A.16) coincides with the mechanical friction obtained for closely related SC-normal metal hybrid NEM system in [25], see Eqs. (12, 13) therein.

In relevant for the current paper regime when \( a = 0 \), the effective damping induced by interaction with the electronic degree of freedom coincides with the coefficient \( \tilde{\gamma}(0) \) from the Eq. (18). The imaginary part of the eigenfrequencies of Eq. (A.15) is negative at \( \Delta_d \ll \Gamma \) and \( \kappa \varepsilon_0 > 0 \), which corresponds to the existence of instability in the mechanical subsystem. This regime corresponds to the pumping regime discussed in the main text. On the other hand, the positive imaginary part of the eigenfrequencies occurring at \( \Delta_d \gg \Gamma \) and \( \kappa \varepsilon_0 > 0 \) results in the damping regime, which is connected with the cooling regime in the main text.

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