Numerical Simulation on Characteristics of Heat Transfer with Different Elliptic Tube Cross Sections

Quanfu Gao and Sen Ma
School of Mechanical Engineering, Lanzhou Jiaotong University, Lanzhou, 730070, China.
Email: gaoqf@mail.lzjtu.cn

Abstract. With the developing of manufacturing technique and the lack of material in recent years, the elliptic tube-fin bank heat exchangers have been paid more and more attentions. The numerical simulation has become an important mean and effective way for studying the flow and heat transfer. The heat transfer capacity of the heat exchanger is affected by multiple factors, among which the cross sections of the elliptic tube is a major factor. In this article, we take numerical analysis methods and SIMPLE algorithm in body-fitted coordinate system to analyze the effect of the different elliptic cross sections on heat transfer characteristics, the results will provide basis for designing and manufacturing compact heat exchangers.

1. Introduction
In many fields of modern science, heat exchangers are the indispensably important device. The design and production of high efficiency heat exchangers become more and more important. The circular and flat tube bank fin heat exchangers are widely employed early in industrial and civil applications due to the restricting of the manufacturing technique. In practice, it is found that elliptic tube bank fin heat exchanger has better aerodynamic characteristics, lower pressure drop, more thermal length at the same cross area and more compact. More and more studies of the heat exchanger were carried out. The conventional method for obtaining the fin pattern with good heat transfer performance is to experiment [1-3]. Bordalo and Saboya [1] experimentally investigated the pressure drop coefficients of the different arrangements for elliptic and circular sections. The average heat transfer coefficient didn’t vary too much when the circular tubes was replaced with elliptical ones [2]. The influence on the heat exchanger with different pitches of fin and diameters of tube was discussed [3]. Experimental research methods are very believable, but such technique can be used only to a special configuration. In recent years, with developing of numerical method, numerical heat transfer has been developed rapidly, solving the problems of flow and heat transfer in heat exchanger by numerical analysis becomes more and more effective [4-7], such as standard model [4], 3D numerical simulations [5], and realizable k-ε turbulence model [6]. The advantages of numerical methods is it has good applicability, but the efficiency and accuracy of the calculated results is very critical for researches in practical application. Our group have studied a lot of research about the numerical model [7]. In this paper, the numerical analysis methods [7] are taken to analyze the effects of the different cross sections of elliptic tube on heat transfer.

2. Physical Models
As shown in figure 1, the heat-transfer characteristic of the air-side is investigated. For the multi-row tube cases, only the staggered arrangement is considered. The schematic diagram of the physical models for elliptic tube of one-row and two-row are presented figure 2(a) and (b), respectively, and the
actual computational domain are marked by shadow regions. The geometric parameters are $S$, $L$, $F_p$, $2a$ and $2b$. where $S$ and $L$ represent the lateral and longitudinal tube pitch, $F_p$ is fin spacing, $2a$ and $2b$ mean major axis and minor axis of the elliptic tube. The simulation domain in physical space is transformed into a rectangular parallelepiped in the computational space by using the multi-surface algebraic method [8].

![Geometric characteristics of plate fin and tube heat exchangers](image)

**Figure 1.** Geometric characteristics of plate fin and tube heat exchangers.

![Elliptic tube bank fin parameters and simulation domain](image)

**Figure 2.** Elliptic tube bank fin parameters and simulation domain

### 3. Mathematical formulation and boundary conditions

The numerical model is assumed two-dimensional, steady, incompressible, and laminar, the governing equations as follows.

\[
\frac{\partial}{\partial x_i} (\rho u_i) = 0
\]  

(1)

\[
\frac{\partial}{\partial x_i} (\rho u_i u_k) = \frac{\partial}{\partial x_i} (\mu \frac{\partial u_k}{\partial x_i}) - \frac{\partial p}{\partial x_k} \quad (k = 1, 2, 3)
\]  

(2)

\[
\frac{\partial}{\partial x_i} (\rho c_p u_i T) = \frac{\partial}{\partial x_i} (\lambda \frac{\partial T}{\partial x_i})
\]  

(3)

Equation (1), (2) and (3) are continuity equation, momentum equation and energy equation respectively, where $x_i$ represent coordinates axes, consisting of $x$, $y$, $z$; $u_i$ are components of velocity vector, consisting of $u$, $v$, $w$; $c_p$ is specific heat capacity, $T$ is temperature, $p$ is static pressure and $\rho$ is density. The local and average Nusselt numbers, bulk temperature, and friction factor are as follows.

\[
Nu_{\text{local}} = \frac{D}{\lambda_f} \frac{q_w}{(T_{\text{wall}} - T_{\text{bulk}})}; \quad Nu_{\text{m}} = \frac{h_m D}{\lambda}, \quad f = \frac{\Delta p}{L_x \rho u_{\text{m}}^2}
\]  

(4)

Where $q_w$ is the local wall heat flux on fin surfaces and is calculated by Fourier’s law. Subscript signs “W” and “bulk” denote the wall surface and cross-averaged value, “m” denotes average value. The $h_m$ is determined by:
4. Simulation Results and Discussion

The detailed derivation and the validity of the algorithm had been described in [7], here it is would not be repeated again. Six configurations, three for one-row and three for two-row, are employed to investigate the effects of the elliptic configurations on heat transfer characteristics. The Reynolds number varies from 250 to 1200 with the following geometric parameters: $S/2b=2.50$, $e=b/a=0.5$, 0.65 and 1.0, $S/L=1.1541$, $F/L=0.0919$. The actual values of the geometric parameters are $S=21.35$ mm, $L=18.5$ mm, $2a=17.06$ mm, $13.12$ mm and $8.53$ mm, $2b=8.53$ mm, and the net fin spacing $F_{n}=1.70$ mm. Here the $e=b/a$ is the aspect ratio of the elliptical tubes.

The simulation results of the different cross section are shown as figures 3(a)-(f). As shown in figures 3(a)-(d), for the same $Re$, the $Nu$ and $f$ increase with the decrease of $e$. For the Nusselt number, the difference among the three is relatively obvious compared with friction factor. For the friction factor, it is much bigger for $e=0.5$ than that $e=0.65$ and $e=1.0$, but friction factor differential is relatively small between $e=0.65$ and $e=1.0$, especially for the large $Re$, it has little impact on the $f$.

In summary, when the elliptic tubes take the place of circular tubes, the transfer coefficients is affected little and the friction factor is affected significantly. But for the elliptic tubes, the changing of ration $e$ has little effect on friction factor. When the ratio $e$ decreases, the major axis $a$ becomes bigger due to the minor axis $b$ is invariable, thus the perimeter of elliptic tube increase, which result in heat exchange areas increase, and the Nusselt number increase. But for friction factor, it depends on the combination of pressure drag and frictional drag.

The overall performance of the heat exchanger include high transfer coefficients and less pressure loss, the $Nu^{1/3}$ is selected as the evaluation indexes. Figure 3(e) and (f) show the relations of the aspect ratio $e$ with the overall heat exchanger characteristics. The $Nu^{1/3}$ of the two elliptic tube structures with $e=0.5$ and 0.65 are larger than that of $e=1.0$, which shows the overall performance of the heat exchanger with elliptic tubes is better than that with circular tubes.
Figure 3. Effects of the aspect ratio on average Nusselt number and friction factor. (a) $Nu$, (c) $f$, (e) $Nu/f^{1/3}$ for one-row; (b) $Nu$, (d) $f$, (f) $Nu/f^{1/3}$ for two-row.
Figure 4. Effects of aspect ratio on span average $Nu$ on fin surface

Figure 5. Distribution of local $Nu$ on fin surface at $F_p=1.7$mm and $Re=700$

In the above discuss, overall performance of the heat exchanger are analyzed from the global, the effects of the tube cross section on local heat transfer performance would be investigated in the next. Figure 4 illustrates the effects of the aspect ratio $\epsilon$ on the span averaged $Nu$ of fin surface with $Re=700$. The trends of the $Nu$ for any structure tube with the same aspect ratio $\epsilon$ is the same, in the region of the inlet, the $Nu$ is much higher than the other region, in the region ahead of the second tube occurs the second peak of the $Nu$. The valleys occurs in the region behind the first and the second tube. At the entrance, the flow area reduces suddenly, the fluid is perturbed by the channel, the flow velocity and direction change sharp, the convection heat transfer coefficients increase and the first peak of the $Nu$ appears. The heat transfer coefficient decreases gradually because of the convective boundary layer beginning to grow up with the flow distance increase along the flow direction and the first valley occurs behind the first tube. Then the convection heat transfer coefficients increase again in the front of the second tube, which is similar to the first tube. In general, the local $Nu$ of the fin-side increase
with the aspect ratio decreasing. The reason is that when the aspect ratio reduces, it means the major axis increasing, the flow around characteristics become better, the recirculation zone decreases, so the global heat transfer of fin surface is improved.

As discussed in the above, on the whole, the span average $Nu$ of the elliptic tube is higher than the circular tube. In contrary, it can be seen that in the front of the tube, the average $Nu$ of the circular tube is higher than the elliptic one as shown in Figure 4. The reason is when the ration $e$ increase, it means the shape of the tube change larger, the direction of the fluid velocity changes more than the elliptic one, the perturbation becomes bigger and the heat transfer of the region is higher as shown in Figure 5.

5. Conclusion
In this article, we take numerical analysis methods and SIMPLE algorithm in body-fitted coordinate system to analyze flow and heat transfer characteristics of elliptic tube bank fin heat exchanger. The effects of the Reynolds number and cross sections on the heat transfer and friction characteristic are investigate through overall and local stability analysis. In summary, when the elliptic tubes take the place of circular tubes, the transfer coefficients is affected little and the friction factor is affected significantly. But for the elliptic tubes, the changing of ration $e$ has little effect on friction factor, the elliptic tube with $e=0.5$ and $e=0.65$ have overall heat transfer performance as compared to that of circular tube. In contrary, in some local zones as the front of the tube, the average $Nu$ of the circular tube is higher than the elliptic one.

6. Acknowledgment
The financial supports from the Chinese National Natural Science Foundation under Grants No. 51866008 and Gansu Higher Education Institutions Ability Improvement Plan under Grants No. 2018A-024 are gratefully acknowledged.

7. References
[1] S.N. Bordalo, F.E.M. Saboya, Pressure drop coefficients for elliptic and circular sections in one, two and three-row arrangements of plate fin and tube heat exchangers, Journal of the Brazilian Society of Mechanical Sciences 21(4) (1999) 600-610.
[2] S.M. Saboya, F.E.M. Saboya, Experiments on elliptic sections in one-and two-row arrangements of plate fin and tube heat exchangers, Experimental Thermal Fluid Science 24 (2001) 67-75.
[3] X. Wu, Z.M. Lin, S. Liu, M. Su, L.C. Wang and L. B. Wang, Experimental study on the effects of fin pitches and tube diameters on the heat transfer and fluid flow characteristics of a fin punched with curved delta-winglet vortex generators, Applied Thermal Engineering 119 (2017) 560-572.
[4] L.P. Zhao, Z.G. Yang, Effect of tube pitches on performance of rectangular finned elliptical tube bundles, Journal of Tongji University 44(1) (2016) 150-154.
[5] L. Zhao, X. Gu, L. Gao, Z. Yang, Numerical study on airside thermal-hydraulic performance of rectangular finned elliptical tube heat exchanger with large row number in turbulent flow regime, International Journal of Heat and Mass Transfer 114 (2017) 1314-1330.
[6] A. Gholami, H.A. Mohammed, M.A. Wahid, M. Khiadani, Parametric design exploration of fin-and-oval tube compact heat exchangers performance with a new type of corrugated fin patterns, International Journal of Thermal Sciences 144 (2019) 173-190.
[7] Z.M. Lin, L.B Wang, Q.F. Gao, A new method to specify the outlet boundary condition of fluid flow in the channel formed tube bank fins, Computational Thermal Sciences 3(6) (2011) 445-459.
[8] P.R. Eiseman, A Multi-Surface Method of Coordinate Generation, Journal of Computational Physics 33(1) (1979) 118–150.