In this work, we present the results of the magnetic, critical, and magnetocaloric properties of the rhombohedral-structured $\text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3$ cobaltite. Based on the modified Arrott plot, Kouvel–Fisher, and critical isotherm analyses, we obtained the values of critical exponents ($\delta$, $\gamma$, and $\beta$) as well as Curie temperature ($T_C$) for the investigated compound. These components were consistent with their corresponding values and they were validated by the Widom scaling law and scaling theory. The obtained critical exponents were close to the theoretical prediction of the mean-field model values, revealing the characteristic of long-range ferromagnetic interactions. The magnetic entropy, heat capacity, and local exponent $n(T, \mu, H)$ of the $\text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3$ compound collapsed to a single universal curve, confirming its universal behaviour. The estimated spontaneous magnetization value extracted through the analysis of the magnetic entropy change was consistent with that deduced through the classical extrapolation of the Arrott curves. Thus, the magnetic entropy change is a valid and useful approach to estimate the spontaneous magnetization of $\text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3$.

1. Introduction

As a member of perovskite oxides, cobaltites are important agile and multifunctional materials that are very promising for several applications, including high-temperature oxygen separation membranes, cathodes in solid oxide fuel cells (SOFCs), magnetic storage, and magnetic refrigeration.\(^1\)\(^-\)\(^3\) The Co-based sister compounds of manganites have been less intensively studied.\(^4\)\(^-\)\(^6\) It is well-known that the similar Hund’s rule exchange energy and crystal-field energy both lead to an additional spin-state degree of freedom in cobaltites, which results in close competition between the multiple ground states. This, in turn, leads to phenomena such as magneto-electronic phase separation, colossal magnetoresistance, and magnetocaloric effect. The spin state (ST) of cobalt ions is very sensitive to the application of external stimuli such as magnetic and electrical fields, temperature, pressure, or compositional doping.\(^7\)\(^-\)\(^8\) This sensitivity is due to the very small energy difference between the $t_{2g}$ and $e_g$ levels. Consequently, the state of Co can be presented in a low-spin (LS), intermediate-spin (IS), or high-spin (HS) state.\(^4\)\(^-\)\(^5\)\(^,\)\(^8\) This ability of Co to exist in several STs given a principal property distinguishes the cobalt oxides from other transition metals such as manganese and makes the physical phenomena observed in the cobalt oxides very complex, due to which they have not been entirely comprehended so far. The complexity of STs provides promising opportunities for basic science as well as electronic applications, inducing a multifunctional characteristic to cobalt oxides. The latter property derives from the fact that the crystal-field splitting of the 3d energy level of Co ions in cobalt oxides is of the same order of magnitude as the Hund’s rule intraatomic exchange energy and 3d orbital bandwidth.\(^6\)

Physical effects such as magnetoresistance and magnetocaloric effects observed in manganites\(^6\)\(^,\)\(^11\) and cobaltites\(^3\)\(^-\)\(^14\) have been the subject of several investigations in the last few years. Double exchange (DE), phase separation (PS), Jahn–Teller distortion (JT), and Griffiths phase (GP) have been found to explain the aforementioned effects. Moreover, these compounds are also interesting for applications since they present low costs and longer usage times. This family of materials can be easily elaborated, and grain growth can be achieved to the desired size; moreover, they possess tunable Curie temperature and high chemical stability. La-based cobaltite is one of the perovskite oxides and it shows a wide variety of physical properties with relatively high $T_C$ values. Like
manganese, iron, and copper, cobalt exhibits various possible oxidation states (Co²⁺, Co³⁺, and Co⁴⁺) and several types of coordinations (tetrahedral, pyramidal, and octahedral). Consequently, cobalt oxides offer a wide gamut of opportunities for the creation of many frameworks involving a mixed valence state of cobalt. Similar to manganites, the substitution of La⁴⁺ by Sr²⁺ in La₁₋₀.₅Sr₀.₅CoO₃ converts an adapted number of Co⁵⁺ to Co⁴⁺, introducing a predominantly ferromagnetic (FM) order due to the DE interactions between Co³⁺ and Co⁴⁺ ions. In addition, the substitution at the A-site of cobaltite oxides induces changes in the chemical internal pressure that locally affects the Co–O–Co networks and easily modifies the ST of cobalt ions due to the fact that crystal-field splitting is very sensitive to changes in the Co–O–Co angle and Co–O distance. Therefore, Co²⁺ adopts three possible STs, namely, LS, IS, and HS, whereas Co³⁺ usually exhibits only the LS.

It is believed that doped cobaltalites are not a homogeneous ferromagnet. This inhomogeneity might affect the cooperative behavior of the Co sublattice, and therefore, the nature of FM-paramagnetic (PM) phase transition and the class of universality of this magnet. Therefore, the analysis of critical exponents (β, γ, and δ) of a magnetic system can yield valuable information about the magnetic phase transitions and can be classified into different universal classes, such as mean field, 3D Ising, and 3D Heisenberg, depending on the exponent values. Recently, the theory of critical phenomena justified the existence of a universal MCE behavior in materials exhibiting second-order magnetic phase transitions. Recent studies have revealed the impact of Bi⁺ substitution on several properties in La-based manganites. It is believed that the BiMnO₃ system is a special and promising compound, which exhibits multiferroic properties where phases like ferroelectric, FM, and ferroelastic coexist in this oxide. Earlier reports have suggested that Bi doping in manganite systems exhibit a high charge sensitivity of this magnet.

2. Experimental details

The ceramic with a nominal composition of La₀.₅₅Bi₀.₀₅Sr₀.₄CoO₃ was synthesized by the solid-state reaction method. The precursors of La₂O₃ (Aldrich 99.9%; USA), Bi₂O₃ (Aldrich 99.9%; USA), SrCO₃ (Aldrich 99.9%; USA), and Co₃O₄ (Aldrich 99.9%; USA) were mixed in an agate mortar with the desired proportion according to the La₀.₅₅Bi₀.₀₅Sr₀.₄CoO₃ system. Then, the obtained powder was heated at 800 °C for 24 h. After cooling to the ambient temperature, the bulk was ground and pressed into pellets and then sintered at 900, 1000, 1100, and 1200 °C for 24 h with intermediate regrounding and repelling to ensure homogenization. Finally, the obtained disk-shaped samples were slowly cooled to room temperature in air. As the sample was exposed to air, it consequently became stoichiometric with respect to oxygen. The X-ray diffraction (XRD) pattern was recorded at room temperature on a PANalytical XPert Pro MPD diffractometer using θ/2θ Bragg–Brentano geometry with diffracted beam monochromatized CuKα radiation (λ = 1.5406 Å). The diffraction patterns were collected at steps of 0.017° over the angle range of 20–80°. Rietveld refinement was performed to determine the structural parameters by using the FullProf software. A vibrating-sample magnetometer developed at NEEL Institute was used to investigate the thermomagnetic properties of the sample. The temperature and field dependencies of the magnetization, $M(T, \mu_0H)$, were recorded in a temperature range around the $T_C$. To accurately extract the critical exponents of the sample, the corrected magnetic isotherms were measured in the range of 0–5 T and within a temperature interval of 2 K in the vicinity of $T_C$.

3. Scaling analysis

For continuous phase transition, near $T_C$, the critical behavior for a second-order magnetic phase transition can be investigated via a series of critical exponents (β, γ, and δ). According to the scaling hypothesis, these exponents are expressed as follows:\textsuperscript{24,25}

$$M_{SP} = \lim_{H \to 0} (M) = M_0(-\varepsilon)^\beta, \quad \varepsilon < 0; \; T < T_C$$

$$\chi_0^{-1} = \lim_{H \to 0} (\mu_0H/M) = (h_0/M_0)\varepsilon^\gamma, \quad \varepsilon > 0; \; T > T_C$$

$$M = D(\mu_0H)^{1/\delta}, \quad \varepsilon = 0; \; T = T_C$$

where $\varepsilon = (T - T_C)/T_C$ is the reduced temperature, $M_0$, $h_0$, and $D$ are the critical amplitudes; $\beta$ (associated with $M_{SP}$), $\gamma$ (associated with magnetic susceptibility $\chi_0^{-1}$), and $\delta$ (associated with the field-dependent magnetization at $T_C$) are the critical parameters.

The critical exponents ($\beta$, $\gamma$, and $\delta$) should obey the scaling equations:\textsuperscript{24,25} A formulation was used in this work, which was based on the scaling equations of state. In the asymptotic critical region and according to the scaling equations, the magnetic equation can be expressed as follows:

$$M(\mu_0H, \varepsilon) = \varepsilon^{\beta}f_\pm(\mu_0H)\varepsilon^{\beta+\gamma}$$

where $f_\pm$ denote regular analytic functions with $f_\pm$ denoting the FM state for $T$ below $T_C$ and $f_\mp$ denoting the PM state for $T$ above $T_C$. Eqn (4) shows that for true scaling relations and for the right
choice of $\beta$, $\gamma$, and $\delta$, $M(\mu_0H, T)e^{-\beta} \text{vs. } \mu_0He^{-(\beta+\gamma)}$ yields two universal curves of temperature $T$ above and below $T_C$.

4. Results and discussions

The XRD $\theta$–$2\theta$ patterns measured at room temperature show that La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ can be refined in the perovskite structure with a rhombohedral structure with the $R3c$ space group. This demonstrates that the substitution of La by 5% Bi does not induce the relevant effect on the crystal structure as compared to those in the undoped La$_{0.5}$Sr$_{0.5}$CoO$_3$ compound prepared by the solid-state reaction method or sol-gel technique. The structural information of the prepared sample is obtained after fitting the XRD data using the Rietveld refinement technique by means of the FullProf software. During the initial stages of refinement, only the following parameters were changed: scale factor, background coefficients, unit-cell parameters, full-width parameters, sample displacement, and peak asymmetry. First, only the scale factor was refined; then, the remaining parameters were gradually included in the successive least-squares cycles. The background was fit using a third-order polynomial, and the observed peak shapes were approximated by using a pseudo-Voigt profile function, limited to ten full-widths on each side of the peak maximum. Fig. 1 shows the XRD diffraction pattern for the investigated sample, where the open circle dots represent the measured XRD reflections and the solid lines denote the Rietveld-refined results. Evidently, a marginal difference between the measured spectra and refined ones can be observed. The quality of refinement is evaluated through the goodness of the fit indicator $\chi^2$, which is equal to 1.98. From Fig. 1, it is clear that a small secondary phase can be observed, which can be attributed to the presence of the CoO impurity. This impurity is identified with the X’Pert HighScore Plus software. It is interesting to note that the most intense peak in the XRD spectra for La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ exhibits a double peak, representing the rhombohedral phase.

The intense peak is shown in the inset of Fig. 1. The lattice parameters are found to be $a = b = 5.415(7)$ Å, $c = 13.326(9)$ Å, and unit cell volume $V = 338.51$ Å$^3$ for La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$. For the undoped compound prepared by using the conventional solid-state reaction method, the lattice parameters can be determined as $a = b = 5.436(3)$ Å, $c = 13.226(3)$ Å, and unit cell volume $V = 338.49(1)$ Å$^3$. It is clear that the lattice parameter “$c$” and unit cell volume marginally increase with the addition of 5% Bi content, while parameter “$a$” slightly decreases with the addition of 5% Bi content. This almost-perfect match can be explained considering the similar ionic radii of La$^{3+}$ ions (1.16 Å, 8-coordinate) and Bi$^{3+}$ ions (1.17 Å, 8-coordinate).

In order to understand the general magnetic behavior and to estimate $T_C$, low-field magnetization vs. temperature ($M(T)$) is obtained in the field-cooled (FC) mode for the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ sample. Fig. 2 shows the $M(T)$ curve under an applied magnetic field of 0.05 T. This curve exhibits a sharp FM–PM phase transition, where $T_C$, defined from the inflexion point of the $dM/dT$ vs. $T$ curve (inset, Fig. 2), is found to be 210 K. Here, $T_C$ is suppressed by 20 K as compared to undoped La$_{0.5}$Sr$_{0.5}$CoO$_3$. It is noteworthy that even a higher value of $T_C$ = 237 K was found by T. A. Ho et al. for a sample prepared under different conditions. This $T_C$ suppression correlates with the competition between the DE interaction and superexchange interactions altered by the incorporation of Bi ion in the Co–O–Co networks. The observed $M(T)$ curve reveals a strong variation in the magnetization around $T_C$, which indicates that there is possibly a large magnetic entropy change around the magnetic transition.

When showing the inverse of the magnetic susceptibility ($1/\chi$) curve in the PM state (insets, Fig. 2), a linear behavior with temperature is observed above $T_C$, which can be fitted with the Curie–Weiss law:

$$\chi = C(1/T - \theta)$$

![Fig. 1](Image) Room-temperature XRD pattern for the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ sample.

![Fig. 2](Image) Magnetization measurements as a function of temperature for the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ sample under 0.05 T. Insets: $dM/dT$ as a function of temperature and temperature dependence of the inverse of magnetic susceptibility. The red line presents a linear fit at higher temperatures.
where $\theta_p$ is the PM $T_C$ and C is the Curie constant. The obtained $\theta_p$ is determined to be 230 K, which is higher than $T_C$, where $\Delta T = \theta_p - T_C = 20$ K. This $\theta_p$ value is smaller than the one observed in the undoped compound (245 K). The decrease in $\theta_p$ with 5% Bi doping can be explained by considering the DE mechanism in cobaltites. It is known that Bi$^{3+}$, with the 6s$^2$ lone pair, is a highly polarizable ion and it induces local distortions in the Co–O–Co bond angles, resulting in a decrease in the DE interaction strength. For our sample, the positive values of $\theta_p$ and $\Delta T$ reveal the existence of a FM exchange interaction between the nearest neighbors in the PM region and confirm the presence of a magnetic inhomogeneity. Similar results are observed in several manganites and cobaltite perovskites, such as La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$. 

Fig. 3 shows the magnetization vs. applied magnetic field curve up to 8 T, $M(\mu_0 H)$, recorded at 10 K. This curve shows the typical FM nature of La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$. The magnetization sharply increases with the applied magnetic field for $\mu_0 H = 1$ T, but does not saturate even up to a field of 8 T. Such results confirm the existence of FM clusters, which, in cobaltites, are related to the presence of a spin disorder in the system, with different S's of Co$^{3+}$ and Co$^{4+}$ ions. Further, the $M(\mu_0 H, T = 10 \text{ K})$ curve can be used for the evaluation of the characteristic magnetization values by means of the procedures described elsewhere. Namely, the saturation magnetization ($M_{sat}$) calculated from the $M$ vs. $1/H$ plot at $1/H \rightarrow 0$ (inset, Fig. 3). The obtained $M_{sat}$ values reach 1.75 $\mu_B$/Co in the same order as the other perovskite systems. At low temperatures, the spontaneous magnetization $M_{sp}(\text{exp})$ determined by the extrapolation of $M(\mu_0 H)$ from the high field to zero field is determined to be 1.37 $\mu_B$/Co (inset, Fig. 3).

Fig. 4 shows the isothermal curves $M(\mu_0 H)$ over a wide range of temperatures and external fields up to 5 T. These curves are used to determine the changes in the magnetic entropy ($-\Delta S_m$) and critical exponents ($\beta$, $\gamma$, and $\delta$). From the data, it is clear that the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ sample shows FM behavior below $T_C$ and PM behavior considerably above $T_C$. The $M(\mu_0 H)$ curves of the sample in the temperature range of 100–300 K are shown in the inset of Fig. 4. At lower temperatures, the $M(\mu_0 H)$ curve reveals that the magnetizations rise sharply in weak applied magnetic fields and then progressively increase with the $\mu_0 H$ value. However, a diminution of magnetization with increasing temperature is clearly observed. Further, it is clear that at $T_C$, the investigated material transits from the FM state to the PM state. This transition is due to the magnetic disorder established as the temperature increases. In this case, the deflection of magnetic momentum occurs, and hence, the total magnetic moment of the entire system decreases and compound magnetization gets diminished. Therefore, once the temperature reaches $T_C$, the thermal motion of the molecules of the material affects the ordered spin at the zero field and the PM behavior is observed instead of the FM behavior.

In order to determine the type of magnetic phase transition in the vicinity of $T_C$, Fig. 5a shows the Arrott curves ($M^2-\mu_0 H/M$) for the prepared La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ compound. In these curves, it is assumed that the critical exponents follow the mean-field theory (MFT), where $\beta = 0.5$ and $\gamma = 1$. Fig. 5a shows that in the low-field region, the nonlinear and curvature characters in $M^2-\mu_0 H/M$ parts at $T > T_C$ and $T < T_C$ are driven toward two opposite directions. The latter phenomenon is essentially due to the misaligned magnetic domains, which reveal the FM–PM separation and indicate that the values of $\beta = 0.5$ and $\gamma = 1$ are inaccurate.

The characteristics of the magnetic phase transition in the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ cobaltite can be determined by assessing the feature of the Arrott plots around $T_C$. In our case, no inflection or negative slope is observed as a signature of the metamagnetic transition above $T_C$, indicating the nature of the second-order phase transition (SOP). This is in agreement with those observed in earlier studies. In general, for SOP, its thermodynamic function can be expressed in the form of a power law with the aforementioned critical exponents,
namely, $\beta$, $\gamma$, and $\delta$. This transition obeys the following asymptotic relations. By using eqn (1) and (2), as well as the so-called Arrott–Noakes equation of state

$$\left[ \frac{\mu_0 H}{M} \right]^{1/\gamma} = a(T - T_c)/T + bM^{1/\beta}$$

(where $a$ and $b$ are constant parameters), we can obtain the values of $\beta$ and $\gamma$; this approach is known as the modified Arrott plots (MAPs). Therefore, to determine the correct values of $\beta$ and $\gamma$, MAPs should be used. In this context, it is possible to initially use the tri-critical mean-field model ($\beta = 0.25$ and $\gamma = 1$), 3D Ising model ($\beta = 0.325$ and $\gamma = 1.24$), 3D Heisenberg model ($\beta = 0.365$ and $\gamma = 1.386$), and mean-field model ($\beta = 0.5$ and $\gamma = 1$) to construct tentative Arrott plots and then select the best one to be the initial Arrott plot for fitting the data. The so-called normalized slope (NS) defined as $NS = S(T)/S(T_c)$ at the critical point can be used for effecting further comparisons. Since the modified Arrott plots are a series of parallel lines, the NS of the most satisfactory model should be close to 1 (unity) regardless of temperature. As shown in Fig. 5b, the mean-field model provides an NS value closest to 1 in the temperature range under investigation. The latter model is the best one to determine the critical exponents as well as to describe the material. The temperature dependencies of $\chi_0^{-1}(T)$ and $M_{SP}(T)$ are shown in Fig. 6a; eqn (1) and (2) are used for fitting these data. These fits yielded the critical parameters as $\beta = 0.486 \pm 0.017$ with $T_c = 211.572 \pm 0.1 \text{ K}$ and $\gamma = 1.109 \pm 0.065$ with $T_c = 212.024 \pm 0.107 \text{ K}$. These results are very close to the exponents of the mean-field model ($\beta = 0.5$ and $\gamma = 1$). It is evident that the obtained value of $T_c$ agrees well with that obtained from the $M(T)$ curve.

Alternatively, in order to more accurately determine the $\beta$, $\gamma$, and $T_c$ parameters, we can use the Kouvel–Fisher (KF) method expressed as

$$\frac{M_{SP}}{dT} = \left\{ \frac{T - T_c}{\beta} \right\} \frac{\chi_0^{-1}}{dT} = \frac{T - T_c}{\gamma}$$

According to the above equations, the $1/\beta$ and $1/\gamma$ slopes are obtained by linear fitting and the value of $T_c$ is obtained from the intercepts on the temperature axis. The results of the best fits are shown in Fig. 6b, where $\beta = 0.482 \pm 0.052$, $T_c = 212.71 \pm 0.13 \text{ K}$ and $\gamma = 0.965 \pm 0.441$, $T_c = 212.116 \pm 0.023 \text{ K}$ for the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ compound. It is worth noting that these new values are also in agreement with those obtained by the MAPs, which indicates that the estimated values are self-consistent and unambiguous.

The third critical exponent, namely, $\delta$, is determined from the Widom scaling relation, i.e., $\delta = 1 + \gamma/\beta$. Here, $\delta$ is calculated to be 3.281 $\pm$ 0.084 as obtained by the MAPs and 3.002 $\pm$ 0.493 as obtained by KF. Fig. 6c shows the magnetic isotherm $M(\mu_0 H)$ for the same plot on the $T$ vs. $M(\mu_0 H)$ curve would be a straight line with a slope of $1/\delta$. The values of $\delta$ obtained through the MAP and KF methods are close to that observed from the fitting of the isotherm at $T = T_c$ to eqn (3) ($\delta = 3.125 \pm 0.012$). Moreover, we note that these values of the critical exponents are in good agreement with those obtained from the mean-field model ($\beta = 0.5$, $\gamma = 1$, and $\delta = 3$). When compared with La$_{0.5}$Sr$_{0.5}$CoO$_3$, it is evident that the critical exponents of the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ sample are different from those observed for the undoped sample. It has been reported that the critical exponents for La$_{0.5}$Sr$_{0.5}$CoO$_3$ prepared by the sol–gel process are in good agreement with those predicted by the 3D Heisenberg model. However, many inconsistent results have been reported on La$_{0.5}$Sr$_{0.5}$CoO$_3$ made by the solid–solid reaction. For example, T. A. Ho et al. found a complex scenario in the La$_{0.5}$Sr$_{0.5}$CoO$_3$ sample.
influence of the replacement of La$^{3+}$ ion (with zero magnetic moment) by Bi$^{3+}$ ion (with nonzero magnetic moment). Therefore, the insertion of 5% Bi may contribute toward the magnetic interactions along with Co$^{3+}$ and Co$^{4+}$ ions and causes a difference in the critical behavior in the aforementioned samples.

In addition, the scaling equation stipulates that the $M(\mu_0H, T)$ data in the critical region obeys the scaling relation expressed as:

$$\Delta S_{m}(T, \mu_0 H_{\text{max}}) = S(T, \mu_0 H_{\text{max}}) - S(T, 0)$$

where $S$ is the magnetic entropy change, which is obtained from the $M(\mu_0H)$ curves at various temperatures according to the classical thermodynamic theory based on Maxwell’s relations by using the following expression:

$$\Delta S_{m}(T, \mu_0 H_{\text{max}}) = \int_{0}^{\mu_0 H_{\text{max}}} \frac{\partial S}{\partial H} d\mu_0 H$$

The isothermal magnetic entropy change ($\Delta S_m$), which results from the spin ordering under the influence of a magnetic field, can be obtained from the $M(\mu_0H)$ curves at various temperatures according to the classical thermodynamic theory based on Maxwell’s relations by using the following expression:

$$\Delta S_{m}(T, \mu_0 H_{\text{max}}) = S(T, \mu_0 H_{\text{max}}) - S(T, 0)$$

where $f_+$ and $f_-$ are the analytical functions for $T > T_C$ and $T < T_C$, respectively. Eqn (8) shows that $M(\mu_0H, T)$ yields two distinct curves: one for $T > T_C$ and the other for $T < T_C$.

By using the correct (KF)$\beta$ and (KF)$\gamma$ values for La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$, the $M(\mu_0H)\beta$ vs. $H/|\mu_0H_{\text{max}}|$ curves are plotted, as shown in Fig. 7. Evidently, they collapse onto two sides: one above $T_C$ and the other below $T_C$. The obedience of the scaling equation over the entire range of normalized variables proves that the critical exponents and $T_C$ are reasonably accurate and unambiguous with the scaling theory.

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In this work, the magnetization measurements are made under discrete magnetic fields and temperature intervals. Therefore, the magnetic entropy change can be approximately given by

\[ \Delta S_m(T, \mu_0 H) = \sum \left( \frac{M_t - M_{t+1}}{T_{t+1} - T_t} \right) \Delta \mu_0 H_t \]  \hfill (9)

In this equation, \( M_t \) and \( M_{t+1} \) are the experimental values of magnetization measured at temperatures \( T_t \) and \( T_{t+1} \), respectively, under the applied magnetic field \( \mu_0 H_t \).

Fig. 8a shows the behavior of \( \Delta S_m \) for the \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \) sample as a function of temperature. These curves exhibit peaks around \( T_C \). Immediately below and above \( T_C \), the \( -\Delta S_m \) value monotonically increases with an increasing magnetic field, which corresponds to a magnetic FM–PM transition. The dependence of the magnetic entropy changes on the value of \( \left( \frac{\delta M}{\delta T} \right)_H \) has been clearly indicated in eqn (8). Therefore, a large magnetic entropy change usually occurs near \( T_C \), where the magnetization changes swiftly with a variation in temperature. Therefore, the negative sign of the magnetic entropy change confirms the FM character of our sample.\(^{46}\) The large values of \( -\Delta S_m \) for the \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \) system are due to a second-order magnetic transition.\(^{46}\) The magnitude of \( \Delta S_m \) increases with an increasing strength of \( \mu_0 H \). The maximum value of \( \Delta S \) decreases from 2.66 J kg\(^{-1}\) K\(^{-1}\) for the \( \text{La}_{0.6}\text{Sr}_{0.4}\text{CoO}_3 \) sample prepared by the solid–solid reaction\(^{49} \) (2.10 J kg\(^{-1}\) K\(^{-1}\) for \( \text{La}_{0.6}\text{Sr}_{0.4}\text{CoO}_3 \) made by the sol–gel method\(^{48} \)) to 1.45 J kg\(^{-1}\) K\(^{-1}\) under an applied magnetic field of 5 T. This indicates that Bi substitution leads to a marginal decrease in the MCE properties with a reduced transition temperature. On the other hand, the obtained value of Bi-doped cobaltite is comparable to those obtained in other cobaltites,\(^{3,14,50,51} \) indicating that our sample could be used as a refrigerant material in magnetic cooling devices.

In order to determine the magnetic refrigeration efficiency, only the magnitude of the magnetic entropy is insufficient. The relative cooling power (RCP) is another decisive parameter that can be used to select materials for practical applications. RCP can be calculated by the following expression:

\[ \text{RCP} = |\Delta S_m^{\text{max}}| \times \delta T_{\text{FWHM}} \]  \hfill (10)

where \( \delta T_{\text{FWHM}} \) is the full width at half maximum of \( \Delta S_m(T) \). For \( \mu_0 H = 5 \) T, the RCP is \( \sim 115.5 \) J kg\(^{-1}\), which is of the same order of magnitude as those found in other cobaltites, such as \( \text{La}_{0.6}\text{Sr}_{0.4}\text{CoO}_3 \), \( \text{La}_{0.6}\text{Sr}_{0.5}\text{CoO}_3 \), and \( \text{La}_{2/3}\text{Sr}_{1/3}\text{CoO}_3 \).\(^{49,50,52} \) Evidently, substituting La\(^{3+} \) by Bi\(^{3+} \), \( \Delta S_m(T) \) shows a considerably broad

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**Fig. 8** (a) Temperature and field dependence of the magnetic entropy change for \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \). (b) Universal behavior of the scaled entropy change curves under the influence of several magnetic fields. Inset: the dependencies of the two reference temperatures (\( T_{r1} \) and \( T_{r2} \)) at various magnetic fields. (c) Scaled magnetic entropy change vs. scaled temperature using the critical exponents.
variation with temperature around $T_C$ as compared with the parent sample. Such an effect is beneficial for magnetic refrigeration.

Moreover, we can use the obtained $\Delta S_m$ curves to accurately distinguish between the order of the PM–FM phase transition according to a phenomenological universal curve of the field dependence of magnetic entropy change. Such a method has been successfully applied to FM perovskites, such as cobaltites and manganites. The universal curve could be plotted by means of the normalized entropy change ($\Delta S_m / \Delta S_{m}^{\text{peak}}$) and rescaling temperature ($\theta$) as follows:

$$\theta = \begin{cases} 
-\frac{(T - T_C)}{(T_{11} - T_C)}; & T \leq T_C \\
\frac{(T - T_C)}{(T_{12} - T_C)}; & T \leq T_C 
\end{cases}$$

where $T_{11}$ and $T_{12}$ denote the temperatures of the two reference points that were selected as those corresponding to $\Delta S_{m}^{\text{peak}}/2$. If all the universal curves of $\Delta S_m(\theta)$ at various magnetic fields collapse onto a single universal curve, the nature of the second-order transition would be confirmed. As shown in Fig. 8b, it is evident that all the $(\Delta S_m/\Delta S_{m}^{\text{peak}})$ values fall onto one universal curve, which is consistent with the analysis based on the Arrott plots. The field dependencies of $T_{11}$ and $T_{12}$ for the investigated sample are shown in the inset of Fig. 8b.

Based on the relationship between the critical exponents and the scaled equation of state defined as $\frac{\mu_0 H}{M_T} = f\left(\frac{\varepsilon}{M_T^{1/2}}\right)$, the $\Delta S_m$ value can be described in the form of the following scaling relationship:

$$\Delta S(\mu_0 H, T) = (\mu_0 H)^{1-\alpha/\Delta} \left[ \frac{\varepsilon}{(\mu_0 H)^{1/2}} \right]$$

where $\alpha$ and $\Delta$ are the usual critical exponents, which can be obtained by using $\Delta = \beta + \gamma$ and $\alpha + 2\beta + \gamma = 2$. The latter relation has been successfully applied to several manganite systems. This scaling relation is used to confirm the validity of the estimated critical exponents for La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$. Fig. 8c shows that all the $\Delta S_m(T)$ values fall on a universal curve for several applied magnetic fields. The excellent overlap of the experimental data points confirms that the estimated critical exponents and $T_C$ for the investigated compound are in obedience with the scaling theory.

Based on the MCE data, the dependence of the magnetic entropy change on the external magnetic field is analyzed. $\Delta S_m$ can be expressed as a power law of the following form:

$$\Delta S_{m,\text{max}}(T, \mu_0 H) = a(T)(\mu_0 H)^{n(T, H)}$$

where “$a$” depends on the temperature and exponent “$n$” depends, in general, on both the temperature and field. Fig. 9a shows the temperature dependencies of the exponents $a(T)$ and $n(T)$, which were determined by using eqn (13) from the $\Delta S_m$ values at several magnetic fields. From Fig. 9a, it is evident that the value of $a(T)$ increases with the temperature, reaching the value of $0.3$ at 210 K. This value is very low as compared to those observed in manganite systems. The value of $n$ reaches the minimum of $n = 0.989$ at $T_C = 210$ K.

Basicallly, the MFT predicts that for materials with SOPT, the $n(T)$ curve exhibits three regimes: well below $T_C$ ($n = 1$), well above $T_C$ ($n = 2$), and at $T_C$ ($n = 2/3$, which is related to the critical exponents of the transition). The exponent “$n$” depends on both temperature and field and can be locally estimated using the following formula:

$$n(T, \mu_0 H) = \frac{d \ln(|\Delta S|)}{d \ln(\mu_0 H)}$$

From the curves of $n(T, \mu_0 H)$ shown in Fig. 9b for the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ compound, it is evident that all the curves exhibit the minimum value of $n$ at $T_C$, which is different from the MFT value of $n = 2/3$. In addition, we observed that the $n$ values are unstable under varying temperature $T$ and field $\mu_0 H$. The minima of the curves with changes in $T_C$ with the applied magnetic field range within 0.861–1.021. This behavior is related to the magnetic disorder and FM clusters in the vicinity of $T_C$. A similar behavior has been previously reported in other perovskites. In order to verify the collapse or
breakdown of \( n(T, \mu_0H) \) curves under the influence of different applied fields, we first arbitrarily selected the reference temperatures \( T_i \) as those that have \( n(T_i) = 1.5 \) (ref. 55) and constructed \( n(\theta, \mu_0H) \) as a function of the rescaled temperature \( \theta \), which is, in turn, obtained as follows:

\[
\theta = \frac{(T - T_C)(T_i - T_C)}{T_i - T_C}
\]

From the inset of Fig. 9b, it is evident that all the \( n(\theta, \mu_0H) \) values collapse onto a single universal curve, revealing a universal behavior in \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \).

Using the MCE data, we calculated the specific heat capacity change as a function of the rescaled temperature for all the magnetic fields. The scaling of \( D \) exponents. The scaling method is a result of the scaling behavior has also been observed in other FM systems. The worthwhile overlap of the data points obviously suggests that the obtained exponents (\( \beta \) and \( \gamma \)) and \( T_C \) for the \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \) sample are in agreement with the scaling hypothesis at various magnetic fields.

Moreover, the obtained magnetic entropy change is used to deduce the spontaneous magnetization in the \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \) sample. According to the MFT and relationship between the magnetic entropy (\( S \)) and magnetization (\( M \)), \( S(\sigma) \) can be expressed as follows:75,76

\[
S(\sigma) = -Nk_B \left[ \ln(2J + 1) - \ln \left( \frac{\sinh \left( \frac{2J + 1}{2} \right)}{\sinh \left( \frac{1}{2} \right)} \right) \right] + B_1^{-1}(\sigma)\sigma
\]

where \( M \) is the magnetization, \( N \) is the number of spins, \( J \) is the electron spin number, \( \sigma = M/g\mu_BJN \) is the reduced magnetization, \( k_B \) is the Boltzmann constant, and \( B_1 \) is the Brillouin function. For small \( M \) values, \( \Delta S \) is proportional to \( M^2 \):75

\[
S(\sigma) = \frac{3}{2} \frac{J}{J+1} Nk_B\sigma^2 + O(\sigma^4)
\]

Furthermore, it should be noted that the compound exhibits spontaneous magnetization below \( T_C \) (FM state), and consequently, the \( \sigma = 0 \) state is certainly not obtained. By considering only the first term of eqn (18), \( \Delta S_m \) may be expressed as

\[
S(\sigma) = \frac{3}{2} \frac{J}{J+1} Nk_B\left( \sigma^2 + \sigma_{\text{Spon}}^2 \right)
\]

The latter equation indicates that \( \Delta S_m \) vs. \( M^2 \) plots show a linear variation with a constant slope in the FM region. At

Fig. 10 \( \Delta C_p \) for the \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \) system. Inset: normalized heat capacity change as a function of the rescaled temperature for all the magnetic fields.

Fig. 11 Spontaneous magnetization of \( \text{La}_{0.55}\text{Bi}_{0.05}\text{Sr}_{0.4}\text{CoO}_3 \) determined from the extrapolation of \( -\Delta S_m \) vs. \( M^2 \) curves and from the mean-field results. Inset: \( -\Delta S_m \) vs. \( M^2 \) curves. The dashed lines are linear fits to the data.
different temperatures, all the curves exhibit a horizontal drift from the origin corresponding to a value of $M_{\text{spin}}(T)$. For the PM region, the $\Delta S_m$ vs. $M^2$ plots start at a null $M$ value. Fig. 11 shows the $M_{\text{sp}}(T)$ data obtained from the $\Delta S_m$ vs. $M^2$ curves by the intersection of the linearly extrapolated curve with the $M^2$ axis (inset, Fig. 11). The linear behavior of $-\Delta S_m$ vs. $M^2$ confirmed the validity of the linear expansion of eqn (19). In the same figure, we show a comparison between the estimated $M_{\text{sp}}(T)$ values obtained from the isothermal ($-\Delta S_m$) vs. $M^2$ plots and those obtained from the Arrott plots ($\mu_bH$ vs. $M^2$). The worthwhile agreement between the aforementioned methods confirms the validity of this process in order to determine the $M_{\text{sp}}$ value using a mean-field analysis of $\Delta S_m$ in the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ system. The magnetic behavior of our sample is effectively described by the classical MFT.

Finally, the investigation of the scaling hypotheses of the thermomagnetic properties of the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ sample offers the opportunity of using the universal curve in the investigations of novel FM materials on various applied functionalities. Such methods present a simple screening procedure of the performance of FM compounds, simple way to extrapolate results to magnetic fields or temperatures not available in the laboratories, remade the impact of non-saturating conditions, reduce experimental noise, or eliminate the effects of minority magnetic phases.

5. Conclusion

In summary, we conducted an in-depth investigation on magnetic, critical behavior, and MCE effect on La$_{0.52}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ cobaltite synthesized via the solid-state reaction. The XRD analysis shows that the sample exhibits a rhombohedral structure with the $R3c$ space group. The magnetic properties reveal that our sample undergoes a FM–PM second-order magnetic transition, which can be confirmed from the Arrott curves and universal curves of magnetic entropy changes. The critical behavior of the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ cobaltite is studied by means of various techniques and validated by using the scaling theory and Widom scaling relation. The obtained critical exponents are close to the theoretical prediction of the mean-field model values, which implies that long-range interactions dominate the critical behavior in La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$. Moreover, the experimental magnetic entropy changes, specific heat capacity changes, and local exponents $n$ obtained for several magnetic fields collapse onto a universal curve, confirming the universal behavior of the MCE properties in this oxide. The estimated spontaneous magnetization value extracted through the analysis of the magnetic entropy change ($-\Delta S_m$ vs. $M^2$) is consistent with that extracted through the classical extrapolation of the Arrott plots ($\mu_bH/M$ vs. $M^2$). Consequently, the magnetic entropy change is a valid approach to determine the spontaneous magnetization of the La$_{0.55}$Bi$_{0.05}$Sr$_{0.4}$CoO$_3$ compound.

Conflicts of interest

There are no conflicts to declare.

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