Application of multi response optimization with grey relational analysis and fuzzy logic method

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Abstract. Multi-response optimization is an optimization process by considering multiple responses simultaneously. The purpose of this research is to get the optimum point on multi-response optimization process using grey relational analysis and fuzzy logic method. The optimum point is determined from the Fuzzy-GRG (Grey Relational Grade) variable which is the conversion of the Signal to Noise Ratio of the responses involved. The case study used in this research are case optimization of electrical process parameters in electrical discharge machining. It was found that the combination of treatments resulting to optimum MRR and SR was a 70 V gap voltage factor, peak current 9 A and duty factor 0.8.

1. Introduction
Optimization of quality is an effort to get factors that produce optimum quality. There are two optimization approaches, namely single response optimization and multi-response optimization. The single response optimization approach is used if the quality of the product generated only considers a single product characteristic. When product quality not only considers a product characteristic, the optimization approach used is multi-response optimization. On the single response optimization approach, the optimum point obtained is not necessarily the optimum point simultaneously, [1]. The quality of a product often not only considers a single quality characteristic, but some characteristics are considered simultaneously. The optimization process by considering more than one response (product characteristics) is called multi responses optimization. It can be illustrated in Fig. 1.

![Figure 1. Single and multi responses optimization](image)

In the scheme of Figure 1, The method analysis that be used in single response approach are the Signal to Noise Ratio (SNR) and response surface method. The analysis on the SNR method is relatively
easy, so this method is the most commonly used for single response optimization. In the multi responses optimization approach there are several analytical methods used. Multiple SNR is a development of the SNR method. Overlaid surface plot and desirability function method is the development of response surface method. Other methods that can be used in multi-response optimization are principal component analysis, grey relational analysis, and fuzzy logic. Fuzzy Logic and grey relational analysis is a decision-making technique used to optimize multiresponse problem. This research will be carried out the development of multi-response optimization analysis using grey relational analysis and fuzzy logic method [3]. The purpose of this research is to get the optimum point in the case of taguchi design multi responses optimization using grey relational analysis, and fuzzy logic method.

2. Methods and material
The case studies used in this research were cases of optimization on Electrical Discharge Machining (EDM), 'Taguchi-grey relational based multi response optimization of electrical process parameters in electrical discharge machining [4] In this study grey relational analysis will be combined with fuzzy logic method into fuzzy-grey relational analysis.

2.1. The Case Study
Electrical Discharge Machining (EDM) is a thermal erosion process, in which the material removal from the work piece is obtained by high thermal energy due to the ionization of dielectric medium between tool and electrode. Figure 2 show the set up of electrical parameters are mainly contributed factor to produce thermal energy, its needed to find optimal combination of electrical parameters on machinability.

![Figure 2. Case of EDM optimization](image)

| Factors       | Level 1 | Level 2 | Level 3 |
|---------------|---------|---------|---------|
| A : Gap Voltage (V) | 40      | 60      | 70      |
| B : Peak current (A) | 9       | 12      | 15      |
| C : Duty factor  | 0.4     | 0.6     | 0.8     |

Respon :
- Material Removal Rate (mm³/min)
- Surface Roughness (mm)

Since the discharge energy is mainly determined by gap voltage, peak current and duty factor, these factors have been taken as the input parameters. Duty factor is defined as ratio of pulse duration to total pulse cycle. The process variables used in the present study 40 V, 60 V and 70 V have been considered as gap voltage variables. 0.4, 0.6 and 0.8 have been chosen as duty factor with the discharge current value of 9 A, 12 A, and 15 A.

Engine performance is measured by response of Material Removal Rate (MRR) and Surface Roughness (SR). Both of these responses will be considered simultaneously to obtain the treatment composition which yields the optimum MRR and SR. The experimental design used in this case is the taguchi L9 design. Experimental data coding are given in Table 1.

| Table 1. Data experiment for case of EDM optimization |
|-----------|----------|---------|----------|
| Ru  | Factors | Responses |
| n   | A | B | C | MRR | SR |
| 1   | 1 | 1 | 1 | 4.74 | 3.26 |
| 2   | 1 | 2 | 2 | 8.60 | 6.98 |
| 3   | 1 | 3 | 3 | 9.51 | 11.78 |
| 4   | 2 | 1 | 2 | 7.46 | 4.58 |
### 2.2. Fuzzy-Grey Relational Analysis

The method of analysis used in this research is fuzzy-grey relational analysis method. The fuzzy-grey relational analysis method is a combination of fuzzy logic and grey relational analysis methods. Stages of analysis begins with the orthogonal array design that is the input of fuzzy logic. The analysis step is given in Fig. 3.

![Figure 3. Stages of fuzzy-grey relational analysis method](image)

The fuzzy logic GRG (Grey Relational Grade) analysis stage consists of three stages: fuzzifier, fuzzy inference engine, and defuzzifier. The output of fuzzy logic GRG analysis is Fuzzy-GRG. The optimum solution is determined from the influence of the factor with the highest Fuzzy-GRG value. The complete analysis phase can be explained as follows, [3]:

(a) **Step 1: Calculation of S/N Ratios**

There are three types of S/N ratio: larger-the-better, smaller-the-better and nominal the best. SNR larger-the-better is used when the goal of response optimization is maximize response. The material removal rate is a larger-the-better performance characteristic, since the maximization of the quality characteristic of interest is sought and can be expressed as Equation 1, [5].

\[
\eta_{ij} = -10 \log \left( \frac{1}{n} \sum_{j=1}^{n} y_{ij}^2 \right)
\]

Where \( n = \) number of replicas, \( y_{ij} = \) observed value, and \( i = 1,2, \ldots n, j = 1,2, \ldots k \)

SNR smaller-the-better is used when the goal of response optimization is minimize response. The surface roughness is the smaller-the-better performance characteristic and the loss function for the same can be expressed as Equation 2.

\[
\eta_{ij} = -10 \log \left( \frac{1}{n} \sum_{j=1}^{n} \frac{1}{y_{ij}^2} \right)
\]

SNR nominal-the-best is used when the goal of response optimization is target response. In the case study used in this research there was no response with nominal-the-best purposes.

(b) **Step 2: Normalization of S/N Ratios**

The beginning step in the GRA is normalization of the S/N ratio, which is performed to prepare raw data for the analysis where the original sequence is transferred to a comparable sequence. Linear normalization is usually demanded since the range and unit in one data sequence may differ from the others. A linear normalization of the S/N ratio in the range between zero and unity is also called as the

| Ru | Factors | Responses |
|----|---------|-----------|
| n  | A       | B C       | MRR  | SR  |
| 5  | 2       | 2 3       | 12.8 | 9.62|
| 6  | 2       | 3 1       | 8.38 | 6.99|
| 7  | 3       | 1 3       | 15.51| 6.53|
| 8  | 3       | 2 1       | 10.90| 4.82|
| 9  | 3       | 3 2       | 13.36| 8.98|

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grey relational generation. [3]. Formula to normalize SNR with greater the better criterion given in
Equation 3.

$$ \eta^*_ij = \frac{\eta_{ij} - \min \eta_{ij}}{\max \eta_{ij} - \min \eta_{ij}} $$ (3)

With $\eta^*_ij$ is the normalized value of the SNR of the i-th response and the j-th treatment. $\min \eta_{ij}$ is
the minimum value of $\eta_{ij}$ and $\max \eta_{ij}$ is the maximum value of $\eta_{ij}$. SNR normalization with smaller
the better criteria is given in the Equation 4.

$$ \eta^*ij = \frac{\max \eta_{ij} - \eta_{ij}}{\max \eta_{ij} - \min \eta_{ij}} $$ (4)

Normalized SNR values range from 0 to 1. Value 1 indicates that the response is close to the target.

Step 3: Determination of Grey Relational Coefficient (GRC)

After the SNR normalization process, the next step is to determine the GRC value. GRC for all the
sequences expresses the relationship between the ideal (best) and actual normalized S/N ratio. If the
two sequences agree at all points, then their GRC is 1, [3]. The formulation for GRC is given in the
Equation 5.

$$ \gamma(x_{0j}, x_{ij}) = \frac{\Delta \min + \zeta \Delta \max}{\Delta_{ij} + \zeta \Delta \max} $$ (5)

with $i = 1,2,\ldots,m$ $j = 1,2,\ldots,n$
$$ \Delta \min = \min\{\Delta_{ij}, i = 1,2,\ldots,m; j = 1,2,\ldots,n\} $$
$$ \Delta \max = \max\{\Delta_{ij}, i = 1,2,\ldots,m; j = 1,2,\ldots,n\} $$

$\Delta_{ij} = x_{0j} - x_{ij}$ is an absolute difference between values $x_0(j)$ and $x_i(j)$. $\zeta$ is the identification
coefficient and is used to offset the influence $\Delta \max$ when it has a value too large. In this case the value
is taken $\zeta = 0.5$.

Step 4: Determination of Fuzzy-Grey Relational Grade (Fuzzy-GRG)

Fuzzy logic GRG analysis is divided into fuzzifier, fuzzy engine inference, defuzzifier. At the fuzzifier
stage, the conversion of linguistic variables into fuzzy variables. in the fuzzy engine inference, fuzzy
rules are established based on the "IF and THEN" rules. For two input variables and one output variable,
the fuzzy rule can be formed as follows. [6]:

Rule 1 : if $x_1$ is $A_1$ and $x_2$ is $B_1$ then $y$ is $C_1$ else,
Rule 2 : if $x_1$ is $A_2$ and $x_2$ is $B_1$ then $y$ is $C_2$ else,
Rule 3 : if $x_1$ is $A_3$ and $x_2$ is $B_3$ then $y$ is $C_3$ else,

Rule n : if $x_1$ is $A_n$ and $x_2$ is $B_n$ then $y$ is $C_n$ else.

The variables $Ai$, $Bi$, and $Ci$ are fuzzy subsets that connect the MRR normalization input variables,
SR normalization and Fuzzy-GRG output variables. In this research input variables are formed in three
subset, namely low (L), medium (M), and Hight (H). While the output variables are formed in five
subsets, namely very small (VS), small (S), medium (M), large (L) and very large (VL). With fuzzy sets of input and output variables can be formed fuzzy rule given in Table 2.

| Table 2. Fuzzy Rule |
|---------------------|
| **SR Normalized**   | **MRR Normalized** |
| Low                 | Very Small         |
| Medium              | Small              |
| High                | Medium             |
|                     | High               |
|                     | Very Large         |

If normalization of MRR low and normalization of SR low then fuzzy-GRG very small, if normalization of MRR medium and normalization of SR low then fuzzy-GRG small, and so on. This rule can be simulated with mamdani operators as follows, [7]:

$$\mu_{C_0}(y) = (\mu_{A_1}(x_1) \land \mu_{B_1}(x_2)) \lor (\mu_{A_2}(x_1) \land \mu_{B_2}(x_2)) \lor ... \lor (\mu_{A_n}(x_1) \land \mu_{B_n}(x_2))$$  \hspace{1cm} (6)

Next is the defuzzifier stage, which transforms the fuzzy output variable into a non fuzzy variable. The transformation uses Equation 7.

$$y_0 = \frac{\sum y \mu_{C_0}(y)}{\sum \mu_{C_0}(y)}$$  \hspace{1cm} (7)

The fuzzy-GRG variable is used to determine the optimum point.

**Step 5: Determination of The Optimum Point**

The optimum point is determined from the influence of the largest fuzzy-GRG main factor. Determination of the influence of the main factors using the main influence plot, [8].

3. **Results and discussion**

Fuzzy-grey relational analysis is done with the initial phase of data preparation. The response value is converted to the SNR value which is then normalized. Through the grey relational analysis process, the normalized SNR are converted to GRC value. The obtained GRC values will be used as inputs to the fuzzy process and produce the MPCI output used to determine the optimum solution.

3.1. **Results Determination of SNR, SNR Normalized and GRC Values**

In the case study of EDM optimization considers MRR response with larger the better criteria and SR response with smaller the better criteria. The value of SNR of response, its normalization, and GRC results are given in Table 3.

| TABLE 3. SNR, Normalized of SNR, GRC for MRR and SR Response |
|---------------------|---------------------|---------------------|
| Run | MRR | SR | MRR | SR | MRR | SR | MRR | SR |
| 1   | 4.74 | 3.26 | 13.516 | -10.264 | 0.000 | 1.000 | 0.333 | 1.000 |
| 2   | 8.60 | 6.98 | 18.690 | -16.877 | 0.502 | 0.408 | 0.501 | 0.458 |
| 3   | 9.51 | 11.78 | 19.564 | -21.423 | 0.587 | 0.000 | 0.548 | 0.333 |
| 4   | 7.46 | 4.58 | 17.455 | -13.217 | 0.383 | 0.736 | 0.448 | 0.654 |
| 5   | 12.80 | 9.62 | 22.144 | -19.663 | 0.838 | 0.159 | 0.755 | 0.373 |
| 6   | 8.38 | 6.99 | 18.469 | -16.889 | 0.481 | 0.407 | 0.491 | 0.457 |
| 7   | 15.51 | 6.53 | 23.812 | -16.298 | 1.000 | 0.460 | 1.000 | 0.481 |
SNR value of MRR and SR is obtained from Equation 1 and Equation 2. Normalization of SNR is done by Equation 3 and 4. Normalization value is in the interval value 0 to 1. The normalized SNR close to 1 indicates that the response value is close to the target. In MRR with larger the better criteria, a large response value will result the SNR normalized close to 1. As for the SR response with smaller the better criteria, the normalized SNR that are worth close to 1 are for small value responses. In contrast to the SNR normalized close to 0 are for MRR of small value and SR of great value. Determination of GRC value is done by first determining the value of $\Delta_{ij}$ which is the absolute difference between the value of $x_0(\tilde{j})$ with $x_i(\tilde{j})$. Value $x_0(\tilde{j})$ is the largest normalized SNR value in each response. The GRC value is determined by Equation 5.

3.2. Results Analysis of Fuzzy-Grey Relational Analysis

In this research, fuzzy-grey relational analysis is done with matlab software. The initial stage is defining the input and output variables given in Fig. 4.

![FIGURE 4. Input and output fuzzy logic](image)

The input variable used is GRC value for MRR and SR from Table 3, while its output is fuzzy-GRG. The fuzzy variable definition is given in Fig. 5.

![FIGURE 5. Fuzzy variable for input (a) and output (b)](image)
Fuzzy variables for GRC of MRR and GRC of SR are divided into three subsets; Low (L), Medium (M) and Hight (H), figure 5(a). As for the fuzzy variables MPCI output is divided into five subsets; Very Small (VS), Small (S), Medium (M), Large (L) and Very Large (VL), figure 5(b). Fuzzy rule at the fuzzy engine inference stage and the defuzzifier stage is given in Fig. 6.

At the fuzzy engine inference stage there are nine fuzzy rules that are formed (Equation 6). The rule transforms the input fuzzy variable into fuzzy output. Next is the defuzzifier stage, transforming fuzzy output into fuzzy-GRG output (Equation 7). In Figure 6 if the GRC of MRR is 0.333 and GRC of SR is 1 then fuzzy-GRG will be 0.690. fuzzy-GRG results are given in Table 4.

| Factors | Fuzzy-GRG |
|---------|-----------|
| A B C   |    |
| 1 1 1   | 0.690    |
| 1 2 2   | 0.500    |
| 1 3 3   | 0.440    |
| 2 1 2   | 0.549    |
| 2 2 3   | 0.597    |
| 2 3 1   | 0.500    |
| 3 1 3   | 0.750    |
| 3 2 1   | 0.527    |
| 3 3 2   | 0.649    |

Fuzzy-GRG value close to 1 indicates that the response value is close to the target value, otherwise fuzzy-GRG value close to 0 indicates that the response value is far from its target value. Fuzzy-GRG is a conversion variable of some of the responses considered in the optimization process. Furthermore fuzzy-GRG is used to determine the optimum point.

3.3. Result of Determination of Optimum Solution

The optimum solution is determined from the influence of the factors of each experimental factor. The determination of the optimum solution visually done with the main effect plot given in Fig. 7.
FIGURE 7. Main effect plot for fuzzy-GRG

It is found that for factor A the greatest influence of fuzzy-GRG is given by level 3. In factor B, the greatest influence of fuzzy-GRG is given by level 1. Whereas in the C factor, the largest effect of fuzzy-GRG is given by level 3. Thus the optimum solution is A3B1C3. This means that the optimum MRR and SR can be generated by the treatment of A3B1C3.

4. Conclusion

The conclusion of this research is that fuzzy-grey relational analysis method can be used as an alternative method on multi-response optimization. In the case of EDM optimization used in this study it was found that the combination of treatments resulting in optimum MRR and SR was a 70 V gap voltage factor, peak current 9 A and duty factor 0.8.

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