Mathematical modelling of microwave photonic time-stretch system

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Abstract. The paper highlights the mathematical modeling results of optical pulse stretching process in fiber due to group velocity dispersion effect.

1. Introduction
Investigations described in the present article are aimed to mathematical modeling of broadband optical pulses propagation in single mode fiber and explore the temporal stretching of pulses due to GVD (group velocity dispersion) effect. GVD induced stretching of broadband optical pulses allows realization of bandwidth compression method that can be used in different information processing systems such as wideband PhADC (photonic analogue-to-digital converters) [1-4]. Advantage of this method is the possibility to construct a system capable to capture the wideband (up to 1THz) radiofrequency (RF) signal using limited bandwidth ADCs (analogue-to-digital converters) with record resolution [3].

Figure 1 illustrates the principle of PhADC based on optical pulses time stretching. Super-continuum (SC) source generates wideband optical pulse; firstly a pulse propagates through the first dispersion module D1 with fiber length L1. GVD causes the temporal shift of pulse spectral components such that each lower wavelength component goes after higher wavelength components. Electro-optic light modulator modulates the chirped super-continuum pulse by input analogue RF signal. Second dispersion module D2 with fiber length L2 provides next temporal shift of modulated pulse optical components resulting the widening of pulse RF modulated envelope. This operation provides the compression of encoded RF signal bandwidth, thus photo diode and low-bandwidth electronic ADC can be used to capture and digitize the stretched sample.

Figure 1. SC – super-continuum light source, D1 – first dispersion module, D2 – second dispersion module, M – electro-optic modulator, PD – photo-diode, ADC – electronic “slow” analogue to digital converter.
Input RF sample stretch factor can be determined as
\[ N = \frac{\tau_2}{\tau_1} = \frac{L_1 + L_1}{L_1} \quad (1) \]
where \( \tau_1, \tau_2 \) are the lengths of an optical pulse after first and second dispersion modules correspondingly. This determines the length of radio signal to be processed according to
\[ \tau_1 > \tau_{RF} \quad \text{or} \quad \tau_1 > \frac{1}{\omega_{minRF}} \quad (2) \]
where \( \tau_{RF}, \omega_{minRF} \) are the temporal length and the lowest frequency of RF signal. Another one limitation of this system is that optical bandwidth must be wider than input RF bandwidth:
\[ \Delta \omega_{opt} \gg \omega_{maxRF} \quad (3) \]
In the case of continuous time signal processing a multichannel version of the described optical scheme can be used [2-4].

2. Mathematical models

2.1. Super-continuum light source and fiber
The theory of electro-magnetic wave propagation in dispersive medium was used to provide the analytical model of proposed system. Using the generalized nonlinear Schrödinger equation, slowly varying amplitude approximation (SVAA), and single-mode fiber model in coordinate system moving with optical field group velocity the variation of optical amplitude envelope [5] \( A \) can be described by:
\[ \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} = i\gamma \left( |A|^2 A + \frac{2}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_R \right) \quad (4) \]
For successful performance the parameters of a system must be chosen such that the nonlinear effects will be negligible comparing to dispersion effects. Also it is useful to introduce the normalized amplitude in order to exclude the signal decay from equation (4):
\[ A(z, T) = \sqrt{P_0} e^{-\frac{az}{T}} U(z, T) \quad (5) \]
Where \( P_0 \) – peak impulse power. The propagation equation can be written as:
\[ \frac{\partial U}{\partial z} = -i \beta_2 \frac{\partial^2 U}{\partial T^2} + \frac{1}{6} \beta_3 \frac{\partial^3 U}{\partial T^3} \quad (6) \]
Where \( \beta_2, \beta_3 \) – 2nd and 3rd order propagation constants (DVG-2, 3).
Using Fourier method one can obtain:
\[ U(T, z) = \left( \frac{1}{2\pi} \right) \int_{-\infty}^{\infty} \tilde{U}(\omega, 0) K(\omega, z) e^{-i\omega T} d\omega \quad (7) \]
\[ K'(\omega, z) = \exp \left( \frac{i \beta_2 \omega^2 z}{2} + \frac{i \beta_3 \omega^3 z}{6} \right) \quad (8) \]
\( K'(\omega, z) \) – fiber impulse response function depending on \( z \), \( \tilde{U}(\omega, 0) \) – Fourier transformation of a signal in fiber input, \( U(T, z) \) – signal function at the fiber point \( z \).

2.2. Modulator
We used a Mach–Zehnder modulator in analytical model. Suppose it biased in the central point of operating performance curve linear region, in this case the envelope function of an optical pulse passed through the modulator can be written as:
\[ U_{out}(t) = U_{in}(t) \cos \left( \frac{\pi}{4} + \frac{m}{2} Y(t) \right) \quad (9) \]
where \( m \) – is a modulation depth, \( Y(t) = \frac{U_{max}}{U_{\pi}} S(t) - \text{RF signal function}, \frac{U}{U_{\pi}} = \frac{m}{2}, \quad U_{\pi} \) – half-wave voltage, \( U_{max} \) – is a maximum value of \( Y(t) \).
2.3. Photo diode
The current of a photo diode illuminated by optical field $E_{in}(T)$ can be described as

$$I(t) = KE_{in}^*(T)E_{in}(T),$$

where

$$K = \frac{c\varepsilon_0 n}{2} R_{pd} A_{ef},$$

$A_{ef}$ – fiber modal efficiency, $R_{pd}$ – photo diode sensitivity, $\varepsilon_0$ – dielectric constant, $n$ – light permittivity, $c$ – light speed in vacuum. Factually a photo diode performs as quadratic detector. The equations introduced in this section describe all the modules of optical system presented in figure 1.

3. Modeling examples
We used a Gaussian shaped super-continuum pulse with temporal length of 833fs in the model of optical signal stretch system (figure 2).

![Figure 2. SC pulse intensity envelope](image)

The first dispersion module D1 has the fiber length $L_1 = 2$km and 2nd and 3rd order dispersion constants $\beta_2 = 20 \text{ps}^2/\text{km}, \beta_3 = 0.2 \text{ps}^3/\text{km}$. Figure 3 represents the envelope of source pulse after propagation through D1.
After D1 the stretched SC pulse is modulated by sinusoidal RF signal with the frequency of 30GHz and modulation depth $m = 20\%$ (see figure 4a).

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Being modulated the optical pulse is sent to second dispersion module D2 with fiber length $L_2 = 8\text{km}$, dispersion parameters are chosen the same as for D1 module. The modulated and stretched in D2 optical pulse envelope is demonstrated in figure 4b. Figures 5a and 5b represent the normalized amplitude function and spectrum of a signal in the output of PD after subtraction of Gaussian pulse envelope function.
The view of a spectral distribution of RF signal at PD output demonstrates N times stretch of the signal corresponding to (1), also it shows the appearance of spur 2nd harmonic.

4. Conclusion
The reviewed analytical model was used in development of a special software capable the simulation of time-stretch PhADC system. Modeling results demonstrate the stretch of RF signal using the system under research and appearance of spur harmonic caused by electro-optic modulator nonlinearity.

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