Influence of quantum correction on the Jeans instability of strongly coupled magnetized plasma

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Abstract. It is shown that the Jeans instability condition of strongly coupled finitely conducting gaseous plasma is reduced in the presence of magnetic field and quantum correction. The linearized equations are constructed by using the Quantum magneto-hydrodynamic (QMHD) model. With the help of the normal mode analysis method, a general dispersion relation is obtained. This dispersion relation is discussed by a different mode of propagation. In this paper, it has been found that, quantum correction stabilizes the growth rate of the organization but the relaxation time and electrical resistivity destabilize the growth rate in the transverse mode of propagation. Further, the modified Jeans criterion of the instability of the magnetized plasma system has been also found.

1. Introduction

The plasma physics has ubiquitous research fields of knowledge such as space and astrophysical systems, semiconductor devices, nuclear fusion collision, biomedical sources, etc. In the interstellar astrophysical system, when high temperature is given to the substance it becomes sufficiently hot and forms a plasma state. In this field, many scientists have studied the instability conditions of plasma physics under various assumptions. There has been a wide area of research to understand the phenomena of astrophysical problems (like the formation of stars, supernova white dwarfs, etc.) under the influence of gravitational instability. In this sequence, first of all, Jeans \cite{1} has studied the gravitational instability of infinite homogeneous plasma with the help of classical equations of hydrodynamic and Newtonian gravitational theory. Chandrasekhar \cite{2} has investigated a comprehensive survey on the Jeans instability of an infinite uniform plasma system under the various parameters. In many investigations \cite{3-12}, self-gravitational instability of homogeneous plasma has been studied considering the effect of various parameters.

It is a fact that the study of quantum plasma provides a wide range of investigation of astrophysical and laboratory verifications such as low-temperature plasma, micro, and nano-electronic devices, star formation, etc. In this connection, Haas \cite{13} has predicted the modified quantum magneto-hydrodynamic model. Using this QMHD model, the self-gravitational instability of magnetized quantum plasma including various parameters has been discussed by many authors \cite{14-18}. It is a well-known fact that the effect of a strong correlation on the low-frequency collective model of gaseous plasma plays an important role to understand the astrophysical problems. Apart from these above studies, the Jeans instability of plasma system considering the effects of quantum correction and finite electrical resistivity has been investigated.
2. Basic linearized equations

To study the effect of quantum correction and finite electrical conductivity on homogeneous viscoelastic quantum plasma with uniform magnetic field \( H (0, 0, H) \), this is along to the \( z \)-direction. To analyze the Jeans criterion for finite electrical conductivity homogeneous viscoelastic plasma, we are using the QMHD model. We can write generalized perturbation equations as follows

\[
\begin{align*}
\left( 1 + \tau \frac{\partial}{\partial t} \right) \left[ \rho_0 \frac{\partial \vec{U}}{\partial t} + \rho_0 \vec{V} \psi + \vec{V} \rho - \frac{1}{4\pi} \left( \vec{\nabla} \times \vec{h} \right) \times \vec{h} - \frac{\hbar^2}{4m_e m_i} \vec{V}(\vec{\nabla}^2 \rho_i) \right] \\
= \eta \nabla^2 \vec{U} + \left( \nu + \frac{\eta}{3} \right) \vec{V} \left( \vec{V} \cdot \vec{U} \right)
\end{align*}
\]

\( (1) \)

\[
\frac{\partial \rho}{\partial t} + (\vec{V} \cdot \vec{U}) \rho_0 = 0
\]

\( (2) \)

\[
\nabla^2 \psi = -4\pi G \rho_0
\]

\( (3) \)

\[
\frac{\partial \vec{h}}{\partial t} = \vec{V} \times (\vec{V} \times \vec{h}) + \chi \nabla^2 \vec{h}
\]

\( (4) \)

The above equation (1)-(4) represents the force equation, continuity equation, Poisson’s equation, and magnetic field equations (in terms of electrical resistivity) respectively. Where terms \( \eta, \nu, \tau, G, U, P, \vec{U}, \vec{h}(U_x, U_y, U_z), h(h_x, h_y, h_z), \chi, m_e \) and \( m_i \) are the shear viscosity, bulk viscosity, relaxation time, gravitational constant, gravitational potential, pressure, velocity, magnetic field, electrical resistivity, electron mass and ion mass respectively and \( \rho_0 \) and \( \rho_i \) denote the unperturbed and perturbed quantities of density. Where \( \hbar = \frac{h}{2\pi} \), \( h \) is Planck’s constant. The term \( \left( 1 + \tau \frac{\partial}{\partial t} \right) \) of equation (1) is the Frenkel’s term and used as a viscoelastic operator. To study the stability of the viscoelastic system, assume that the perturbation equation’s varied as-

\[
\exp[i(k_x x + k_z z + \sigma t)]
\]

\( (5) \)

Where \( \sigma \) indicates the frequency of harmonic disturbance, \( k_x \) and \( k_z \) are wave propagation in perpendicular and parallel direction to the magnetic field respectively. Such that, \( k_x^2 + k_z^2 = k^2 \)

On solving equations (1)-(4) the following matrix relation is,

\[
X_i Y_j = 0 \quad ij = 1, 2, 3
\]

Where \( X_i \) is a 3×3 matrix with following components

\[
X_{ij} = \begin{cases} (1 + \tau \sigma) \left\{ (\sigma + \chi k^2) \left( \sigma^2 + c^2 k^2_x - \frac{\omega_j^2 k_x^2}{k^2} + \frac{\chi^2 k_x^2}{4m_e m_i} k_x^2 \right) + V^2 k^2 \sigma \right\} + \frac{\sigma}{\rho_0}(\sigma + \chi k^2) \left\{ \eta k^2 + \left( \nu + \frac{\eta}{3} \right) k_x k_z \right\}, & X_{12} = 0, \\
(1 + \tau \sigma) \left\{ (\sigma + \chi k^2) \left[ 1 + \frac{\eta}{3} \right] k_x k_z \right\}, & X_{22} = (\sigma + \chi k^2) \left[ 1 + \frac{\eta}{3} \right] k_x k_z \end{cases}
\]

\( X_{12} = 0, \quad X_{22} = [\left( 1 + \tau \sigma \right) \left( \sigma + \chi k^2 \sigma + \nu^2 k^2 \right) + \frac{\eta}{3} \nu k^2] (\sigma + \chi k^2), \quad X_{32} = 0, \)

\[
X_{12} = \left[ (1 + \tau \sigma) \left( \sigma^2 + c^2 k^2_x - \frac{\omega_j^2 k_x^2}{k^2} + \frac{\chi^2 k_x^2}{4m_e m_i} k_x^2 \right) + \frac{\sigma}{\rho_0} \left( \nu + \frac{\eta}{3} \right) k_x k_z \right], \quad X_{32} = 0,
\]

\[
X_{12} = \left[ (1 + \tau \sigma) \left( \sigma^2 + c^2 k^2_z - \frac{\omega_j^2 k_z^2}{k^2} + \frac{\chi^2 k_z^2}{4m_e m_i} k_z^2 \right) + \frac{\sigma}{\rho_0} \left( \eta k^2 + \left( \nu + \frac{\eta}{3} \right) k_z^2 \right) \right].
\]

Where \( V = \frac{H}{(4\pi G \rho_0)^{1/2}} \) is Alfven velocity and \( \omega_j = \sqrt{4\pi G \rho_0} \) is Jean’s frequency. On solving equations, then the following dispersion relation is
\[
(1 + \tau \sigma)\left\{(\sigma + \chi k^2)\sigma^2 + V^2 k_z^2\right\}
+ \frac{\eta k^2}{\rho_0} (\sigma + \chi k^2) + \frac{\eta k^2}{\rho_0} (\sigma + \chi k^2) \left\{(1 + \tau \sigma)\left\{\sigma^2 + c^2 k_z^2 - \frac{\omega^2_k k_x k_z}{k^2} + \frac{\hbar^2 k^2}{4m_e m_i} k_z^2\right\} + V^2 k^2 \sigma^2\right\}
+ \sigma \frac{\rho_0}{(\sigma + \chi k^2)} \left\{(1 + \tau \sigma)\left\{\sigma^2 + c^2 k_z^2 - \frac{\omega^2_k k_x k_z}{k^2} + \frac{\hbar^2 k^2}{4m_e m_i} k_z^2\right\} + \frac{\sigma}{\rho_0} (\nu + \frac{\eta}{3}) k_x k_z\right\}
\] 

Equation (6) gives general dispersion relation that shows the combined effect of quantum correction, magnetic field, and electrical conductivity on the Jeans instability of the homogeneous viscoelastic plasma. If ignoring the quantum effect in our dispersion relation, shear and bulk viscosity in equation (6) is relevant to the Chandrasekhar [2] in many aspects. Now the dispersion relation (6) has been discussed in different directions of the magnetic field.

3. Discussion

Now the study on the above general dispersion relation in two different modes of propagation is as follows.

3.1. Parallel mode of propagation

In this mode, the axis of wave propagation is along the direction of the magnetic field. If taking \(k_x = 0\) and \(k_x = k\), then the following general dispersion relation is,

\[
\left[(1 + \tau \sigma)\left\{(\sigma + \chi k^2)\sigma^2 + V^2 k^2\right\} + \frac{\eta k^2}{\rho_0} (\sigma + \chi k^2) \right\]^2 \left[(1 + \tau \sigma)\left\{\sigma^2 + c^2 k^2 - \omega^2_j + \frac{\hbar^2 k^4}{4m_e m_i}\right\}\right. \\
+ \frac{\sigma}{\rho_0} (\nu + \frac{4\eta}{3}) k^2 \right] = 0
\] 

(7)

In this manner, the dispersion relation (7) shows combined influence of the presence of quantum correction and electrical resistivity on the magnetized viscoelastic plasma. The above equation (7) has two components. The first component shows the Alfven non-gravitational mode of propagation for the viscoelastic plasma which represents the effect of electrical resistivity and stable condition of the system. Now if the second component is equal to zero. Then

\[
\tau \sigma^3 + \sigma^2 + \sigma \left\{\frac{\tau c^2 k^2 - \tau \omega^2_j}{4m_e m_i} + \frac{k^2}{\rho_0} (\nu + \frac{4\eta}{3})\right\} + \left(\frac{\hbar^2 k^4}{4m_e m_i}\right) = 0
\] 

(8)

If taking the constant term of above equation is negative then the Jean’s criteria of the system as following

\[
k_{f1} < k_f \left(\frac{4\pi G \rho_0}{c^2 + \frac{\hbar^2 k^2}{4m_e m_i}} \right)^{1/2}
\] 

(9)
The following condition shows that the Jeans instability of the system is totally influenced by the quantum term. Where $k_j$ the Jeans wave number and this Jean’s instability criterion is relevant to the obtained by Chandrasekhar [2] without the quantum correction term.

3.2. Transverse mode of propagation
In this mode of propagation, the axis of wave propagation is along the perpendicular to the direction of a magnetic field. When putting $k_x = k$ and $k_x = 0$, then the following general dispersion relation is

$$
\left[(1 + \tau \sigma)\sigma^2 + \frac{\eta k^2}{\rho_0} \right] \left[(1 + \tau \sigma)(\sigma + \chi k^2)\sigma + \frac{\eta k^2(\sigma + \chi k^2)}{\rho_0} \right] \left((1 + \tau \sigma) \left(\left(\sigma + \chi k^2\right) \left(\sigma^2 + c^2 k^2 - \omega_f^2 + \frac{\hbar^2 k^4}{4m_e m_i} + V^2 k^2 \sigma\right) \right) + \frac{\sigma k^2}{\rho_0} (\sigma + \chi k^2) \left(\nu + \frac{4\pi}{3}\right) \right] = 0
$$

(10)

In this mode of propagation, the dispersion relation (10) shows the combined effect of quantum correction and electrical resistivity on magnetized viscoelastic plasma. The equation (10) has three factors. The first factor represents a shear viscous mode of propagation for the viscoelastic plasma and the second factor of relation gives a viscous mode of propagation with effect of electrical conductivity. When third factor is taken equal to zero, then

$$
\tau \sigma^2 + (1 + \chi k^2 \tau) \sigma^3 + \left(\chi k^2 + c^2 k^2 \tau - \omega_f^2 \tau + \frac{\hbar^2 k^4}{4m_e m_i} \tau + V^2 k^2 \tau\right) \sigma^2 + \left(c^2 k^2 - \omega_f^2 + \frac{\hbar^2 k^4}{4m_e m_i} + V^2 k^2 + \tau \chi k^4 c^2 - \omega_f^2 \chi k^2 \tau\right) \sigma + \left(c^2 k^4 \chi - \omega_f^2 k^2 \chi + \frac{\hbar^2 k^6}{4m_e m_i} \chi\right) = 0
$$

(11)

The above equation shows the combined effect of quantum correction, electrical resistivity and viscosity. If taking constant term of equation (11) is negative, then the Jeans criterion of the system as follow.

$$
k_{j2} \leq k_j \left(\frac{4\pi G \rho_0}{c^2 + \frac{\hbar^2 k^2}{4m_e m_i}}\right)^{1/2}
$$

(12)

This is modified Jeans criteria, which is affected by the quantum correction. For infinitely conducting system Jeans criteria are obtained as,

$$
k_{j3} \leq k_j \left(\frac{4\pi G \rho_0}{c^2 + V^2 + \frac{\hbar^2 k^2}{4m_e m_i}}\right)^{1/2}
$$

(13)

From equation (12) & (13) it is observe that in the absence of electrical resistivity Jeans criteria is modified by the magnetic field and quantum correction.

To study the effect of quantum term, relaxation time and electrical resistivity on the growth rate of the Jeans instability, on converting the dispersion relation (11) in dimensionless form dividing by $\sqrt{4\pi G \rho}$ thus equation (11) becomes

$$
\tau^* \sigma^{*4} + (1 + \chi^* k^2 \tau^*) \sigma^{*3} + \left(\chi^* k^2 + \tau^*(k^2 - 1 + Q^* k^2 + k^2 V^2)\right) \sigma^{*2} + \left(k^2 - 1 + Q^* k^2 + k^2 V^2 + \chi^* \tau^* k^2 (k^2 - 1)\right) \sigma^* + \chi^* k^2 (k^2 - 1 + Q^* k^2)
$$

(14)
Here the following non-dimensional parameters are written as

\[ \sigma^* = \frac{\sigma}{\sqrt{4\pi G \rho}}, \quad k^* = \frac{kc}{\sqrt{4\pi G \rho}}, \quad Q^* = \frac{h^2 k^*}{4m_e m_i}, \quad V^* = \frac{V \sqrt{4\pi G \rho}}{c}, \quad \chi^* = \frac{\chi \sqrt{4\pi G \rho}}{c^2}, \quad \tau^* = \tau \sqrt{4\pi G \rho} \tag{15} \]

![Figure 1](image_url)

**Figure 1.** The growth rate of system versus wave number has shown with values of electrical resistivity \((\chi^* = 0.0, 1.0, 2.0, 3.0)\), keeping the values of other parameters to be fixed (i.e. 0.5).
Figure 2. The growth rate of system versus wave number has shown with values of quantum correction ($Q^* = 0.0, 1.0, 2.0, 3.0$), keeping the values of other parameters to be fixed ($\epsilon = 0.5$).

Figure 3. The growth rate of system versus wave number has shown with values of relaxation time ($\tau^* = 0.0, 1.0, 2.0, 3.0$), keeping the values of other parameters to be fixed ($\epsilon = 0.5$).
In figures 1, 2 and 3, we've shown non-dimensional growth rate verses of non-dimensional wave number for variation values of electrical resistivity, quantum correction and relaxation time, and fixed value of other parameters. In figure 1, we observed that the increasing value of electrical resistivity increasing the growth rate of instability, thus we can say the electrical resistivity destabilized the system. It is clear form figure 2 that the growth rate decrease with increasing the value of quantum correction, thus the influence of quantum correction is stabilizing. Figure 3 shown the effect of relaxation time on the growth rate gravitational instability increases as the value of relaxation time increases. Hence, it is destabilized the system.

4. Conclusion

In this paper, it is analyzed that Jeans instability condition of the strongly coupled finitely conducting gaseous plasma is more affected by the quantum term. It is found that Jeans criterion of instability is modified by the parameter of quantum correction in both the manner of propagation as a longitudinal and transverse manner. But in the transverse propagation, Jean’s wave number decreased by the magnetic field in the absence of electrical resistivity but this effect of the magnetic field is removed by finite electrical resistivity. It is illuminated from the graphical presentations that quantum correction gives stabilized effect but the relaxation time and electrical resistivity is destabilized the growth rate of the organization. This result demonstrates that the relaxation time, electrical resistivity, and quantum corrections affect the dense molecular clouds configuration and star formation.

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