Formation of guided spin-wave bullets in ferrimagnetic film stripes

A.A. Serga, M.P. Kostylev, and B. Hillebrands

Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany
School of Physics, The University of Western Australia, Crawley WA 6009, Australia

The formation of quasi-2D nonlinear spin-wave eigenmodes in longitudinally magnetized stripes of a ferrimagnetic film, so-called guided spin-wave bullets, was experimentally observed by using time- and space-resolved Brillouin light scattering spectroscopy and confirmed by numerical simulation. They represent stable spin-wave packets propagating along a waveguide structure, for which both transversal instability and interaction with the side edges of the waveguide are important. The experiments and the numerical simulation of the evolution of the spin-wave excitations show that the shape of the formed packets and their behavior are strongly influenced by the confinement conditions. The discovery of these modes demonstrates the existence of quasi-stable nonlinear solutions in the transition regime between one-dimensional and two-dimensional wave packet propagation.

Stable two-dimensional localized nonlinear spin-wave excitations, so-called spin-wave bullets, have been previously observed in thin ferrimagnetic films of yttrium-iron-garnet (YIG) magnetized along the propagation direction. These films were practically unbounded in both in-plane directions compared to the transversal size of the spin-wave packets and the wavelength of the carrier spin wave. In contrary, in a one-dimensional waveguide structure, where the width is comparable to or smaller than the spin-wave wavelength, only quasi-one-dimensional nonlinear spin-wave objects were observed, which are spin-wave envelope solitons. Both for solitons and bullets linear pulse spreading in the direction of propagation (so called longitudinal direction) due to wave dispersion is compensated by longitudinal nonlinear compression. As for the transverse in-plane direction, solitons are meant to have a stable transverse distribution of their dynamic magnetization coinciding with the profile of the lowest linear spin-wave eigenmode of the waveguide. On the contrary, bullets show transverse nonlinear instability of attractive type which overcompensates transverse diffraction broadening of the wave packet. The nonlinear compression would lead to a wave packet collapse if the medium is lossless. Weak magnetic losses in a real magnetic film ensure a fine balance of nonlinear narrowing of the packet and of its diffraction spreading for some distance of propagation. This results in a quasi-2D spatially localized bell-shaped waveform which is stable during the lifetime of the bullet.

Here we report on the experimental observation of a stable spin-wave packet propagating along a waveguide structure, for which both transversal instability and interaction with the side edges of the film waveguide are crucial. The structure can be considered as a transitional case between the 2D case of a continuous film and the quasi-1D case of a narrow stripe. We show that in this case the nonlinear wave dynamics is distinctively different from both the soliton and the bullet cases, and can be considered as efficient coherent nonlinear mixing of spin-wave eigenmodes of the waveguide. Our theory shows that this mixing is due to a specific interaction – the pseudo-linear generation of higher order modes by the fundamental one.

The experiment was carried out using a longitudinally magnetized long YIG film stripe of 2.5 mm width and 7 µm thickness. The magnetizing field was 1831 Oe. The spin waves were excited by a rf magnetic field created with a microstrip antenna of 25 µm width placed across the stripe and driven by input microwave current pulses 20 ns in duration at a carrier frequency of 7.125 GHz. The spatio-temporal behavior of the traveling spin-wave packets was investigated by means of space- and time-resolved Brillouin light scattering spectroscopy.

The results of our measurements are demonstrated in Fig. 1. The panels show the spatial distribution of the intensity of spin-wave packets at different times of their propagation from the left to the right along the stripe waveguide. The left vertical set of diagrams corresponds to the linear case. The input microwave power for this set is 20 mW. The right set is for nonlinear propagation. These data were collected applying an input driving power of 376 mW.

Differences between these two cases are clearly seen. The linear spin-wave packet is characterized by a transverse profile very similar to one half of the period of the sine function, while the cross section of the nonlinear packet has a pronounced bell-like shape. Furthermore, the intensity of the linear packet decays monotonically with time. That is obviously due to magnetic damping in the film. The intensity of the nonlinear packet initially increases because of its strong transverse compression (see the second diagram from the top in Fig. 1), then gradually decays as the damping overcomes the nonlinear compression for larger propagation times.

Formation of a stable nonlinear spin-wave object is also evidenced in Fig. 2 which shows the measured peak in-
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lutions which will be discussed below.

Both figures provide a clear proof of development of transverse instability and bullet formation. Interestingly, the 2D bell-like shape survives even at the end of the propagation path (z > 4 mm) when the packet intensity has decreased more than ten times and the nonlinearity contribution to the spin-wave dynamics has been considerably diminished.

Importantly, in Fig. 2 one observes superimposed oscillatory variations in the intensity and in the packet width, both in the quasi-linear regime of the packet propagation (z > 4 mm) and in the highly nonlinear regime z < 4 mm. In the entire range the oscillations in intensity and in width are in anti-phase to each other. This effect has not been observed in case of conventional bullet formation [1].

On the contrary, a similar picture is usually observed for linear guided spin waves in narrow film waveguides [7, 8], where it is explained as a beat of phase-correlated linear waveguide width modes propagating at the same carrier frequency. This effect evidences the importance of the

fluence of confinement on the nonlinear evolution of the packet transverse profile and suggests that interaction of the linear eigenwaves of the waveguide – the so-called width modes – underlies nonlinear wave dynamics. Therefore we term this nonlinear object a “guided spin-wave bullet”.

The theoretical description of the observed phenomena is based on concepts developed in Refs. [9] and [10]. The ingredients of the model are (i) effective dipole pinning of magnetization, which results in a tri-linear interaction as the initial interaction for development of the transverse instability of the wave packet profile, (ii) the width-mode group velocity matching, and (iii) the nonlinear extension of spectrum of width modes. In Ref. [8], we theoretically studied linear propagating eigenwaves of a magnetic stripe. The eigenwaves represent guided modes with discrete transverse wavenumbers. For a stripe of rectangular cross-section the modes are characterized by a standing-wave type dynamic magnetization distribution across the stripe cross-section and by a monochromatic propagating wave with the longitudinal wavenumber \( k_z \) along the stripe. Importantly, for stripes with a large ratio \( p \) of width to thickness the thickness distribution of the dynamic magnetization is practically homogeneous, whereas in the direction of the stripe width the standing spin waves possess considerably decreased amplitudes at the edges due to dynamic demagnetization effects [11]. Previous calculations (see Fig. 3 in [9]) show that the assumption of a totally pinned magnetization at the stripe edges results in good approximation for the transverse profiles of the propagating eigenmodes. Adopting the assumption of totally pinned edge spins, the transverse profile of the dynamic magnetization is described by an integer number of half-periods \( n \) of the sine function with the lowest transverse wavenumber \( k_n \equiv k_n \) being \( k_{n=1} = 1 \cdot \pi/w \), where \( w \) is the stripe width. For

![FIG. 1: Observation of linear propagation and of formation of the guided spin-wave bullet in an YIG film stripe waveguide using time- and space-resolved Brillouin light scattering spectroscopy. For parameters see main text.](image)

![FIG. 2: Measured peak intensity and width for the nonlinear wave packet shown in the right panels of Fig. 1 as a function of propagation path. The dashed vertical lines show positions of local minima of intensity and of the local maxima of the packet width.](image)
the aspect ratio of our waveguide, \( p = 357 \), this works with good accuracy.

As one sees from Fig. 1 this conclusion is in a good agreement with the packet profiles measured in the linear regime. The nonlinear dynamics is now described by a theory which is analogous to the one developed in Ref. [10]. We assume that the dynamic pinning of magnetization is conserved in the weakly nonlinear regime. Then the transverse evolution of the nonlinear packet can be considered as interaction of linear eigenmodes which are pinned at the stripe edges. Indeed, in Fig. 1 one clearly sees that in the nonlinear regime the dynamic magnetization at the stripe edges practically vanishes. Then the description for the propagating modes results in an evolutional equation for the spin-wave precession angle \( \phi \) which reads:

\[
i\partial \phi / \partial t + (\omega_{n,k} + i\eta - \omega) F_{n,k}[\phi(y, z)] + T F_{n,k}[|\phi(y, z)|^2 \phi(y, z)] = f_{n,k,t}.
\]

In this expression \( k \equiv k_z, \omega_{n,k} \) is the eigenfrequency of the \( n \)-th guided mode for the longitudinal wavenumber \( k \), \( \eta \) is the relaxation frequency for the film, and \( \omega \) is the carrier frequency of the microwave signal \( f(y, z, t) \exp(i\omega t) \) applied at the vicinity of \( z = 0 \) which excites the input spin-wave packet, and \( T \) is the nonlinear coefficient analogous to the nonlinear coefficient of the spin-wave version of the Nonlinear Schrödinger Equation [12]. The operation \( F_{n,k} \) denotes a 2D Fourier transform. It is the discrete sine transform with the basis functions \( \sin(k_n y) \) in the direction of stripe width \( y \), and is the Fourier integral over continuous wavenumbers \( k \) with the basis functions \( \exp(ik z) \) along the stripe.

The expression Eq. (1) can be easily transformed into a system of dynamic equations for amplitudes of guided modes \( \phi_n(z, t) = F_n[\phi(y, z, t)] \) coupled by the four-wave nonlinear interaction. The analysis of the system shows that the formation of the two-dimensional waveform can be considered as an extension of the spectrum of the width modes. The partial waveforms carried by the individual width modes have the same carrier frequencies equal to that of the external excitation signal \( \omega \) and the carrier wave numbers which satisfy the dispersion relations for the modes \( \omega_{n,k} = \omega \). In the linear regime all the modes are independent. In the nonlinear (high amplitude) regime the width modes become mutually coupled which ensures intermodal coherent energy transfer.

The efficiency of mode coupling in the pulse regime depends on two major factors: the mode group velocity matching and the type of nonlinear interaction. The geometry of a relatively wide stripe is very favorable for having maximum contributions from both.

Let us first discuss the type of nonlinear mode interaction. The spin-wave packet immediately after having been launched into the stripe is carried by the lowest (fundamental) width mode \( (n = 1) \phi_1(z, t) \). Therefore it is necessary to consider the nonlinear interaction of higher-order width modes with this particular mode.

The interactions of the fundamental width mode with all even modes is not important for symmetry reasons. The nonlinear interaction of modes of the same type of symmetry is described by the parametric term as well as by an additional pseudo-linear (tri-linear) term. The parametric interaction is of convective instability type and is the same parametric instability which triggers formation of conventional bullets in continuous films. This conventional process is described by a pair of complex-conjugated equations with a parametric term proportional to the square of amplitude of the pumping wave (see e.g. Eqs. (4) and (5) in [13]). In this interaction the packet carried by the fundamental mode plays the role of the pumping wave. Its energy is transferred to the partial waveforms carried by the higher-order width modes. There is a threshold associated with this parametric process due to natural damping in the medium. In the waveguide structure the threshold is of the same order of magnitude as the modulation instability threshold in continuous films (see e.g. Eq. (10) in Ref. [1]). Furthermore, an initial perturbation in the form of non-vanishing amplitude of a higher-order mode is needed to start the parametric amplification. This perturbation usually is provided by thermal excitation. Therefore an amplified higher-order mode, which is group velocity matched with the pumping wave, needs a large distance of propagation in order to reach a noticeable level. The energy of the pumping wave decreases down the propagation path due to losses in the medium. If the parametric amplification gain is small because of small supercriticality the higher-order mode cannot reach an amplitude comparable with that of the fundamental mode before the amplitude of the pumping wave packet carried by the fundamental mode falls below the threshold of parametric instability and the gain ceases.

What distinguishes the confined waveguide geometry is that the nonlinear mixing starts as a pseudo-linear (tri-linear) interaction of the fundamental with the next-order symmetric mode which is the third mode \( \phi_3(z, t) \): \( \partial \phi_3 / \partial t + v_3 \partial \phi_3 / \partial z + i\omega_3^{nl} \phi_3 = S_{13}(z, t) \). Here \( v_3 \) is the group velocity of the third mode, \( \omega_3^{nl} \) is its nonlinear frequency shift, and \( S_{13} \) is the tri-linear inhomogeneous term. This term has the form of a linear source of excitation with amplitude proportional to \( |\phi_1(z, t)|^3 \) moving with the group velocity of the fundamental mode. The presence of this pseudo-linear interaction at the early stage of the bullet formation is entirely due to the effective dipolar pinning of the magnetization at the stripe edges. If the edge spins were unpinned, the interaction of all the width modes would be purely parametric. The pseudo-linear excitation introduced by this term into the dynamic equations is a threshold-free process, so it works even below the threshold of parametric instability. Furthermore, in contrast to the parametric process this pro-
cess does not require non-zero initial amplitude of the amplified waveform to start the amplitude growth. This mechanism ensures rapid growth of the symmetric \( n = 3 \) mode driven by the \( n = 1 \) mode up to the level where the parametric mechanism starts to work efficiently. After that the fundamental mode jointly with the \( n = 3 \) mode is capable to rapidly generate a large set of modes with yet higher odd numbers \( n \) through both pseudo-linear and parametric mechanisms.

Let us now discuss the importance of the mode group velocity matching. Our theory shows that the efficiency of both nonlinear interaction mechanisms (parametric and tri-linear) strongly depends on the group velocity difference of interacting modes and the initial length of the nonlinear packet. In wider stripes the group velocities of the width modes are closer to each other. As the nonlinearity is of attractive type, nonlinear corrections to the group velocities partially compensate for the group velocity mismatch. As a result the nonlinearly generated higher-order partial waveforms remain for some time within the pump packet. Again, due to the attractive character of the nonlinearity the modes have initial phases such as their transverse profiles are summed up constructively in the middle of the stripe width and a bullet-like total wave packet is formed as confirmed by direct numerical solution of Eq.(1) shown in Fig. 3.

In the narrower stripes (1 mm in width as in Fig. 3 panel 2) the mode group velocity difference is larger and cannot be compensated by the nonlinear corrections to the group velocities for the same initial intensity of the wave packet. Thus the group velocity matching is not ensured. As a result the nonlinearly generated higher-order modes leave the area of the interaction before they reach significant amplitudes. For the same length of the initial packet \( \phi_1(t = 0, z) \) the extension of the spectrum of width modes does not occur. The waveform \( \phi_1(t, z) \) remains unaffected by the interactions with the higher-order modes. It undergoes only a nonlinear longitudinal compression and forms a quasi-1D wave packet – the spin-wave envelope soliton – which has a stable sine-like profile in the \( y \)-direction.

The excellent agreement of the simulation results with the experimental data shown in Fig. 3 provides evidence for the validity of the developed theory.

In conclusion, the formation of quasi-2D nonlinear localized wave packets – guided spin-wave bullets – was studied in spin-wave waveguides. Our experimental and theoretical investigations show that formation of these stable nonlinear objects is strongly affected by the transverse confinement of the medium. A specific magneto-static effect – the effective dipole pinning of the magnetization at the edges of the stripe, the width-mode group velocity matching of different discrete waveguide modes, and the extension of the width mode spectrum due to nonlinear mode-mode energy transfer are essential for the nonlinear evolution of the initial spin-wave excitation. Both the experimentally detected properties of the evolution and the theoretically revealed mechanism of formation show that the observed nonlinear wave packets can be treated as “guided spin-wave bullets” which are specific for laterally confined magnetic films.

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* on leave from St.Petersburg Electrotechnical University, 197376, St.Petersburg, Russia

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