A note on the relations between elastic and inelastic interactions and increasing ratio

\[ \frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \] at the LHC

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Abstract

We comment briefly on relations between the elastic and inelastic cross-sections valid for the shadow and reflective modes of the elastic scattering. Those are based on the unitarity arguments. It is shown that the redistribution of the probabilities of the elastic and inelastic interactions under the reflective scattering mode can lead to increasing ratio \( \sigma_{el}(s)/\sigma_{tot}(s) \) at the LHC energy range. A short notice is given on the slope parameter and the leading contributions to its energy dependence in the both modes.

Keywords: Impact parameter; Elastic scattering; Unitarity.
It is known that the upper bounds for the inelastic cross-section [1, 2] differ from the well-known Froissart–Martin bound for the total cross-sections. This fact can be considered as a hint for a different asymptotic energy dependence of the inelastic cross-section compared to the total and elastic ones. Such difference indeed takes place in case of the reflective scattering mode [3, 4] at high values of the collision energy. The reflective scattering mode implies unitarity saturation at asymptotics. However, this mode is relevant for the energy region of very high energies. There are corroborative indications of its existence [5] despite the data at the LHC energy of \( \sqrt{s} = 13 \) TeV [6] analyses are still not convergent and do not provide therefore a commonly accepted confirmation of its observation [7].

First, we briefly mention what the shadow and reflective scattering modes are. Matrix element of the elastic scattering is related to the corresponding partial amplitude \( f_l(s) \) by the relation

\[
S_l(s) = 1 + 2i f_l(s),
\]

and the partial amplitude \( f_l(s) \) is constrained by the unitarity equation:

\[
\text{Im} f_l(s) = |f_l(s)|^2 + h_{l,\text{inel}}(s)
\]

Eq. (2) means that there are profound interrelations between elastic and inelastic interactions, see for discussion e.g. [8]. At high energies, it is a common practice to use an impact parameter representation making replacement \( l = b \sqrt{s}/2 \) and use assumption on a smallness of the real part of the elastic scattering amplitude. In fact, the problem of the real part cannot be considered as finally solved.

Unitarity written in the impact parameter representation (the real part of the scattering amplitude is neglected) provides relations for the differential distributions of the elastic and inelastic collisions over \( b \):

\[
h_{el}(s, b) = f^2(s, b),
\]

\[
h_{inel}(s, b) = f(s, b)(1 - f(s, b)).
\]

Thus, the amplitude \( f(s, b) \) variation is limited by the values from the interval \( 0 \leq f \leq 1 \) and \( f = 1/2 \) means the complete absorption of the initial state, i.e. \( S = 0 \), and \( h_{el} \leq 1 \), but \( h_{inel} \leq 1/4 \).

Shadow scattering mode corresponds to the interval \( 0 < f \leq 1/2 \), while the reflective scattering mode appears in the region of the energies \( s \) and impact parameters \( b \) such that the amplitude \( f \) has the values in the range of \( 1/2 < f \leq 1 \).

\[1\text{Note, that impact parameter is a conserved quantity at high energies [9].}\]
As it follows from the analysis \cite{5}, one can safely assume that the elastic scattering has a shadow nature in the energy region $\sqrt{s} \leq 5$ TeV, i.e. in this energy region: $f \leq 1/2$ at all values of the impact parameter $b$. In this energy region the following inequality takes place since $f \leq 1/2$:

$$h_{el}(s, b) \leq h_{inel}(s, b) \leq 1/4,$$

(6)

where $h_{el, inel}$ are the respective overlap functions. Therefore, in the above energy region, the elastic and inelastic cross-section should obey the relation

$$\sigma_{inel}(s) \geq \sigma_{el}(s),$$

(7)

since

$$\sigma_{el, inel}(s) = 8\pi \int_{0}^{\infty} b db h_{el, inel}(s, b).$$

(8)

Eq. (7) is an important inequality for discussion of the shadow versus reflective modes, but not quite new, e.g. the Pumplin bound \cite{10} for inelastic diffraction presupposes such a relation between elastic and inelastic cross-sections since it has been obtained in a shadow approach\textsuperscript{2}.

Thus, the shadowing is the reason for the inelastic interactions dominance. But the opposite claim is not valid, from dominance of the inelastic interactions one cannot conclude that the shadowing takes place in the whole region of the impact parameter variation.

When the energy becomes greater than the threshold value $s_{r}$\textsuperscript{3}, the scattering picture at small values of the impact parameter ($b \leq r(s)$, where $S(s, b = r(s)) = 0$ and $S(s, b)$ becomes negative at $b \leq r(s)$\textsuperscript{4}, starts to acquire a reflective contribution and the Eq. (7) transforms into the two inequalities:

$$\int_{r(s)}^{\infty} b db h_{el}(s, b) < \int_{r(s)}^{\infty} b db h_{inel}(s, b)$$

(9)

and

$$\int_{0}^{r(s)} b db h_{el}(s, b) > \frac{r^2(s)}{8} > \int_{0}^{r(s)} b db h_{inel}(s, b).$$

(10)

It happens due to appearance of the new relation between $h_{el}$ and $h_{inel}$ at $b < r(s)$, i.e.

$$h_{el}(s, b) > 1/4 > h_{inel}(s, b),$$

(11)

(see Fig. 1).

\textsuperscript{2}Generalized upper bound when both – shadow and reflective – modes are presented has been obtained in \cite{11}.

\textsuperscript{3}The threshold value $s_{r}$ is determined by $S(s_{r}, b = 0) = 0$.

\textsuperscript{4}Note, that $r(s) \sim \ln s$ at $s \to \infty$\textsuperscript{4}.
To confront above inequalities with the data, the explicit forms of the impact parameter dependencies of the functions $h_{el, inel}$ are needed. Currently, such information cannot be unambiguously extracted from the available data. But Eq. (10) can be used for a qualitative explanation of the ratio $\sigma_{el}(s) / \sigma_{tot}(s)$ increase with energy at the LHC [13]. Indeed, we observe the redistribution of the total probability increase with the energy growth in favor of elastic interactions under transition to a reflective mode (see Fig. 1). In this regard, it is helpful to recall that

$$\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{inel}(s),$$

where $\sigma_{el}(s) \sim r^2(s)$, but $\sigma_{inel}(s) \sim r(s)$ and $\sigma_{el}(s) / \sigma_{tot}(s) \to 1$ at $s \to \infty$ [4].

The above results have some implications for the slope parameter $B(s)$ of the forward elastic peak,

$$B(s) = \frac{d}{dt} \ln \frac{d\sigma}{dt}\bigg|_{t=0}. \quad (12)$$

This quantity is determined by the average value of the impact parameter squared, i.e.

$$\langle b^2 \rangle_{tot} = \int_0^\infty b^2 f(s, b) db / \int_0^\infty f(s, b) db. \quad (13)$$

In its turn, according to unitarity the energy dependence of $\langle b^2 \rangle_{tot}$ is determined by the ones of cross–sections $\sigma_{el, inel}$ and average values $\langle b^2 \rangle_{el, inel}$. 
In the shadow scattering mode the major contribution to $B(s)$ comes from inelastic processes:

$$\sigma_{\text{inel}}(s) \langle b^2 \rangle_{\text{inel}} \geq \sigma_{\text{el}}(s) \langle b^2 \rangle_{\text{el}}, \quad (14)$$

where

$$\langle b^2 \rangle_{\text{el,inel}} = \int_0^{\infty} b^2 h_{\text{el,inel}}(s, b) db \int_0^{\infty} h_{\text{el,inel}}(s, b) db. \quad (15)$$

But the elastic interactions give a subdominant contribution to the slope parameter in this mode. The relation Eq. (14) is a direct result of Eq. (6).

Contrary, in the reflective scattering mode, due to unitarity saturation, the main contribution to $B(s)$ comes from the elastic scattering decoupled from the inelastic interactions [12] and the following limiting dependence at $s \to \infty$ should take place due to the elastic scattering asymptotic dominance:

$$B(s) \to \langle b^2 \rangle_{\text{el}}/2. \quad (16)$$

It should be noted again that elastic scattering dominance at $s \to \infty$ is a natural result of a self–damping of the inelastic channels’ contributions in this limit [14], i.e. it is a consequence of the aforementioned redistribution between the probabilities of elastic and inelastic interactions due to unitarity. At the same time, the average values $\langle b^2 \rangle_{\text{el,inel}}(s)$ have similar dependence in the limit of $s \to \infty$ [12]:

$$\langle b^2 \rangle_{\text{el,inel}}(s) \sim r^2(s).$$

Redistribution of probability is a result of the reflective scattering mode appearance at high energies. The mode is called reflective since the phases of incoming and outgoing waves differ by $\pi$. In optics, its appearance is associated with density increase of a reflecting medium beyond some threshold value. Such medium has a higher refractive index than the one incoming wave arrived from. In QCD, one can assume that the color conducting medium is formed in the transient state of hadron interactions and it is responsible for reflective scattering mode [15].

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