Quantum Zeno effect on Quantum Discord

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We examine the quantum Zeno effect on the dynamics of quantum discord in two initially entangled qubits which are subjected to frequent measurements via decoherent coupling with independent reservoirs. The links between characteristic parameters such as system bias, measurement time duration, strength of initial entanglement between the two qubit systems and the dynamics of quantum discord are examined for two initial state configurations. At weak or unsharp measurements, the quantum discord, which is an intrinsically distinct entity from concurrence, serves as a reliable indicator of the crossover point in Zeno to anti-Zeno transitions. However at highly precise quantum measurements, the monitoring device interferes significantly with the evolution dynamics of the monitored system, and the quantum discord yields indeterminate values in a reference frame where the observer is not an active constituent of the subsystems.

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I. INTRODUCTION

Recently, studies of separable and therefore non entangled states containing other kinds of non classical correlations has attracted increased attention. One such correlation measure, the quantum discord [1,8], based on the difference between quantum and classical information theories, incorporates more generalized correlations not seen in other non-classical correlations such as entanglement. In particular quantum states with zero entanglement properties are seen to possess quantum discord and classical-quantum states which are necessarily separable have zero quantum discord. Two positive discord states can be mixed to obtain a zero-discord classical state, and two zero-discord classical states in orthogonal directions can be merged to form a non-zero discord state [4]. Moreover the quantum discord is not restricted by the monogamy rule [5] which is obeyed by the concurrence measure during entanglement sharing. Such intriguing features of quantum discord has opened up avenues for variety of attributes and applications in non-markovian open quantum systems [6,10], spin array systems [11] detection of quantum phase transitions [12], quantum information processing [13] and quantum communication [14].

The quantum Zeno effect (QZE) describes the retarded time evolution of a quantum state subjected to frequent measurements [15,17]. In the limiting case of continuous measurement, the time evolution of the state comes to a standstill. The opposite effect which leads to enhancement in time evolution is known as anti-Zeno effect (AZE) and has been observed to be much more ubiquitous than Zeno effect [18]. In unstable systems, the occurrence of both QZE and AZE effects depends on critical parameters like measurement frequencies and environmental noise [19]. Quantum systems exhibiting both effects include the nanomechanical oscillator [20], two-state system coupling to a spin chain environment in transverse magnetic fields [21], the non equilibrium steady state spin-fermion model a variant of the Kondo model [22], damped quantum harmonic oscillator [19], disordered spin systems [23] and trapped atomic systems [24]. The nanomechanical oscillator system, in particular is of increased interest as it provides an ideal medium for testing quantum effects on a macroscopic scale.

An inherent feature in determining quantum discord involves the one-sided projective measurements on a selected subsystem of the composite quantum state. As is well known, this introduces various counter-intuitive features linked with the measurement process itself with associated controversies linked with the collapse of the wave function of the measured system. For instance the term “subjective reality” was introduced by Wiseman [25] to describe the dependence of quantum trajectories on the observer’s measurement frame, hence there are several ways that quantum systems which are monitored can be interpreted. A well-known approach to the widely used collapse postulate involves its replacement by the decoherence process subjected by a detector on the system under study [26]. An alternative scheme involves the idea of quantum Zeno subspaces [17] which provides a convenient platform for interpreting the Zeno effect. In this regard, we note that the active presence of the measuring device is not a requirement for quantum Zeno effects to be seen. This is due to the fact that the Zeno effect is linked to the evolution of the non-Hermitian Schrödinger equation associated with any irreversible mechanism, with the act of measurement being a well known one.

For low precision or unsharp measurements, the device D introduces minimal disturbance on the measured system, S, with state \(|S_u\rangle + v|S_v\rangle\). The state of the measuring device can be \(|D_u\rangle\) or \(|D_v\rangle\) after the measurement, and is different from its state before measurement, \(|D_i\rangle\). The composite system \(S \otimes D\) proceeds in an approximately unitary fashion as \(U|S_u\rangle|D_i\rangle = |S_u\rangle|D_u\rangle\), \(U|S_v\rangle|D_i\rangle = |S_v\rangle|D_v\rangle\). In the case of ideal measurements, the resulting state of the system after measurement generally belongs to the set of the orthonormal basis...
of the quantum system. Thus for weak or unsharp measurements, the non-Hermitian term can be ignored and simplified approaches such as that based on the Kofman and Kurizky’s formalism [18] can be employed to analyze the effect of measurements.

For highly precise measurements, any analysis of the quantum evolution becomes complicated due to the influence of the non-Hermitian term, which can be interfere strongly with the dynamics of the measured system. Accordingly, we provide an analysis of the evolution of a measured system involving a non-Hermitian term which appears due to highly precise measurements or a strong monitoring device here. This is performed by applying the results of the non-Hermitian Hamiltonian of a two-level system originally solved in the context of the link between a decay term and Berry’s phases by Garrison and Wright [27] to our measurement model. We note that a analogous decay term is explicitly linked with the precision of quantum measurements, a higher measurement precision results in a larger magnitude of this decay term. Some ideas introduced in this work may thus be extended to study the links between Berry phases and the quantum measurement problem.

For ideal or weak measurements, the Von Neumann projection operator \( P \) [15, 17] is convenient to formulate measurement procedures in Hilbert space \( \mathcal{H} \) of a quantum system, \( S \). The initial density matrix \( \rho_0 \) of system \( S \) is constrained within \( \mathcal{H}_P \) as \( \rho_0 = P \rho_0 P \), \( \text{Tr}[\rho_0 P] = 1 \). In the absence of any measurement, the state evolves as \( \rho(t) = U(t)\rho_0 U^\dagger(t) \) where \( U(t) = \exp(-iH^*t) \), and \( H^* \) is a time-independent Hamiltonian. The probability that the system remains within \( \mathcal{H}_P \) is given by \( P(t) = \text{Tr}(U(t)\rho_0 U^\dagger(t)P) \). In the event of measurement at time \( \tau \), density matrix \( \rho(\tau) \) transforms as \( \rho(\tau) = \frac{1}{P(\tau)} PU(\tau)\rho_0 U^\dagger(\tau)P \). The survival probability in \( \mathcal{H}_P \) is given by \( P(\tau) = \text{Tr}(V(\tau)\rho_0 V^\dagger(\tau)) \) where \( V(\tau) \equiv PU(\tau)P \). For measurements taken at time intervals \( \tau = t/N \), the survival probability is given by

\[
P^{(N)}(t) = \text{Tr} \left( V_N(t)\rho_0 V_N^\dagger(t) \right),
\]

\[
V_N(t) = \left[ V \left( \frac{t}{N} \right) \right]^N
\]

At very large \( N \), no transitions allowed outside \( \mathcal{H}_P \) occur and \( P^{(N)}(t) \to 1 \), the culmination of the mathematical formulation of the Zeno effect. Eqs. (1) embodies the intriguing effect of a measurement process, where a system monitored to determine whether it remains in a particular state persists to remain in that state. This idea has been examined via the adiabatic theorem [17] in which different outcomes are eliminated and the system evolves as a group of exclusive quantum Zeno subspaces within the total Hilbert space. The measurement procedure therefore has a decomposing effect on the total Hilbert space which is partitioned into orthogonal quantum Zeno subspaces [17]. The initial state remains in a particular invariant subspace, and its survival probability remains unchanged over a period of time.

The effect of measurement on the dynamics of quantum discord can be examined in one of several ways. An obvious one involves examining the role of Zeno effect associated with measurements introduced in one subsystem in order to obtain the conditional entropy, and enabling determination of the classical correlation measure based on optimal measurements. This procedure forms the basis of determining the quantum discord, as shown in earlier mathematical formulations [12]. In order to evaluate the quantum discord, a set of positive-operator-valued measurements (POVM) need to be performed in a neighboring partition. Does the measurement process itself induce a distinct category of quantum discord? How exactly can such optimal measurements be performed without incurring the quantum Zeno effect? What are the key attributes of an optimal measurement and the possible role played by the Zeno effect in POVM? In this regard, the consideration of distinct measurement techniques in separate sub-systems will introduce greater depth to the analysis of the quantum discord present in the global system. This includes the effects due to the asymmetry of measurement procedures. However such detailed investigations is not an easy task, as the difficulty in determining the quantum discord even for simpler systems is well known. So far analytical form has been derived only under restricted conditions [29, 31].

For the sake of obtaining analytical expressions, it is generally assumed that the measurement time duration or frequency of measurements is the same for subsystems not in contact with any reservoir system. We continue to assume this model for simplicity in analytical treatment, however we opt to examine the effect of measurement from a different perspective. This involves examining the influence of the Zeno-like effect associated with acts of continuous measurements by the environment that is in contact with the qubit subsystem [32]. The well-known model of the solid-state qubit interacting with a reservoir system presents a convenient platform for examining the complicated link between quantum Zeno effect, quantum discord and the dynamics of Zeno subspaces. The reservoir may be viewed as providing the “back-action” needed for the dynamical collapse of the wave-function collapse.

In order to keep the problem tractable, we consider in the first instance, the well-known model of a pair of initially entangled spin-boson system with independent harmonic reservoirs found useful in quantifying salient aspects of dissipative dynamics of many quantum systems [33–34]. Factors such as spectral density, bias and temperature are considered to play important roles in the overall dynamics of the qubit-reservoir system. We follow Prezhdo’s approach involving the quantum control of chemical reactivity by a solvent acting as the environment [35], the anti-Zeno mechanism therefore occurs by loss of electronic coherence in some chemical systems. The interplay of various quantum interactions (non-local and local) between the environment and the qubit system results in the reservoir acting as continuous detector.
Our paper is organized as follows. In Section II we provide a brief review of the concept of quantum discord and highlight the role of measurements in its formulation. In Section III we describe salient features of Zeno dynamics of the spin-boson system using Kofman and Kurizky’s formalism which yields the effective decay of a quantum system under ideal measurements. In Section IV we investigate the influence of quantum Zeno effect on the dynamics of the quantum discord for X-type qubit states with two initial state configurations. We present our main results and make comparisons between the quantum discord and the concurrence measure. In Section V we analyze the non-Hermitian dynamics resulting from higher precision measurements on a two-level quantum system, and highlight the appearance of exceptional points. A brief discussion and conclusions are then presented in Section VI.

II. MEASUREMENTS AND QUANTUM DISCORD

Following the formulation of quantum discord in Refs.1-6, we express the quantum mutual information of a composite state $\rho$ of two subsystems $A$ and $B$ as $I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho)$ for a density operator in $\mathcal{H}_A \otimes \mathcal{H}_B$. $\rho_A$ and $\rho_B$ are reduced density matrices and $S(\rho_i) = -\text{tr}(\rho_i \log \rho_i)$ is the entropy of the density operator $\rho_i$. The mutual information can also be written in terms of quantum conditional entropy $S(\rho|\rho_A) = S(\rho) - S(\rho|\rho_A)$.

The quantum Zeno effect appears as a result of the measurement process intrinsic in the definition of the conditional entropy. A series of one-dimensional orthogonal projectors $\{\Pi_k\}$ induce in $\mathcal{H}_A$ leads to different outcomes of the measurement. We are presented with the post measurement conditional state $\rho_{B|k} = \frac{1}{p_k} (\Pi_k \otimes I_B) \rho (\Pi_k \otimes I_B)$ where the probability $p_k = \text{tr}[\rho_{B|k}(\Pi_k \otimes I_B)]$ and $\{\Pi_k\}$ denote the one-dimensional projector indexed by the outcome $k$. A conditional entropy of the subsystem $B$ can be attached to $\rho_{B|k}$ based on the cumulative effect of the mutually exclusive measurements on $A$ as $S(\rho|\{\Pi_k\}) = \sum_k p_k S(\rho_{B|k})$.

The measurement induced mutual information is therefore $I(\rho|\{\Pi_k\}) = S(\rho) - S(\rho|\{\Pi_k\})$ while the classical correlation measure based on optimal measurements is obtained as $C_A(\rho) = \sup_{\{\Pi_k\}} I(\rho|\{\Pi_k\})$. The difference in $I(\rho)$ and $C_A(\rho)$ yields the non symmetric term known as quantum discord $D_A(\rho) = I(\rho) - C_A(\rho)$. A discord $D_B(\rho)$ corresponding to measurements made on $B$ can likewise be obtained and need not be the same as $D_A(\rho)$. As to be expected, the quantum discord is not symmetric with respect to $A$ and $B$, particularly if attributes such as the measurement duration employed in either subsystems differ.

III. ZENO DYNAMICS OF THE SPIN-BOSON SYSTEM

In order to examine the dynamics of the spin-boson system, we utilize the density matrix associated with the Liouville equation $\frac{\partial \rho}{\partial t} = -i[H_T, \rho(t)]$, where the total Hamiltonian $\hat{H}_T = \hat{H}_{qb} + \hat{H}_{os} + \hat{H}_{qb-os}$. The Pauli matrices are expressed in terms of the two possible states $|\downarrow\rangle, |\uparrow\rangle$, and $\sigma_z = |\downarrow\rangle\langle\downarrow| - |\uparrow\rangle\langle\uparrow|$. $\Delta \Omega$ is the biasing energy while $\Delta$ is the tunneling amplitude.

We consider that the two uncoupled qubits are coupled to independent reservoirs of harmonic oscillators, $\hat{H}_{os} = \sum_i \hbar \omega_q \hat{b}_q^\dagger \hat{b}_q$, where $\hat{b}_q^\dagger$ and $\hat{b}_q$ are the respective creation and annihilation operators of the quantum oscillator with wave vector $q$. The qubit-oscillator interaction Hamiltonian is linear in terms of oscillator creation and annihilation operators $\hat{H}_{qb-os} = \sum_n \lambda_n (\hat{b}_q^\dagger + \hat{b}_q) \sigma_z$. The term $\lambda_n$ denotes the coupling between the qubit and the environment and is characterized by the spectral density function, $J(\omega) = \sum_q \lambda^2 \delta (\omega - \omega_q)$, which we assume to be of the ohmic form $J(\omega) = 2\pi \eta \omega e^{-\frac{\omega}{\Delta}}$. $\eta$ is the dimensionless reservoir coupling function, and $\omega_c$ is the reservoir cutoff frequency. We consider the measuring device to be an active constituent of the total Hamiltonian $\hat{H}_T = \hat{H}_{qb} + \hat{H}_{os} + \hat{H}_{qb-os}$. The reservoir assumes the role of the measuring device, by inducing a projection operation that disrupts the normal evolution of Hamiltonian $\hat{H}_T$. The reservoir here serves the same role as the solvent in Prezhdo’s work on the quantum control of chemical reactivity [35].

Each qubit decays to oscillator states in the reservoir when measurements are made, making a transition from its excited state $|\uparrow\rangle_q$ to ground state $|\downarrow\rangle_q$. We consider an initial state of the qubit with its corresponding reservoir in the vacuum state, existing in equilibrium at temperature $T = 0K$ $|\phi_i\rangle = |\downarrow\rangle_q \otimes \prod_k \{0\}_k$, where $|\downarrow\rangle_q$ implies that all $N'$ wavevector modes of the reservoir are unoccupied in the initial state. $|\phi_i\rangle$ then undergoes the following mode of decay $|\phi_i\rangle \longrightarrow u(t) \ |\downarrow\rangle_q |0\rangle_c + v(t) \ |\uparrow\rangle_q |1\rangle_c$, (2)

In order to keep the problem tractable we consider that $|\downarrow\rangle_c$ denotes a collective state of the reservoir, $|\downarrow\rangle_c = \frac{1}{\sqrt{N}} \sum_n \lambda_n \{n\}$ where $\{n\}$ denotes an occupation scheme in which there are $n_i$ oscillators with wavevector $k = i$ in the reservoir and we define the state $\{n\}$ as $\{|n\rangle\rangle = |n_0, n_1, n_2, \ldots, n_{N'}\rangle$. For ideal measurements, the functions $u(t)$ and $v(t)$ in Eq. (2) satisfy the relation $u(t)^2 + v(t)^2 = 1$, and can be considered to be approximately satisfied for unsharp or weak measurements which introduce minimal disturbance to the system being monitored. The square of the function $u(t)$ yields the survival probability associated with $N$ measurements performed at regular intervals $\tau$,.
\( P(t) = u(t)^2 = \exp(-N\Delta^2\tau^2/4) \) where \( t = N\tau \). In the extreme limit \( \tau \to 0 \), \( u(t) \to 1 \) and the decay into phonon states is totally inhibited. For small \( \tau, N \) values and a weak qubit-reservoir coupling, we assume that the state of the collective reservoir at time \( t = \tau \) is equivalent to that at \( t = N\tau \). The second order processes giving rise to exchanges between oscillators and hence changes in the ensemble configuration of oscillators in can be considered minimal and neglected at small \( t \). At very short times, the effective relaxation rate for the two-level qubit is given by \( \gamma(\tau) = (\Delta/2)^2\tau \) so that \( u(\tau)^2 = \exp(-\gamma(\tau)\tau) \). The decay of a quantum state interacting with a reservoir is almost zero at the beginning of the decay process, a typical behaviour in quantum Zeno effect. At intermediate measurement time intervals, the decay of quantum state may be accelerated as is the case in anti-Zeno effects.

It is to be noted that Eq. (2) does not provide a dynamic description of the measurement process such as the evolution of the system during or after the effects. The state may be accelerated as is the case in anti-Zeno effect. A typical behaviour in quantum Zeno effect. At intermediate measurement time intervals, the decay of quantum state may be accelerated as is the case in anti-Zeno effects.

\[ \gamma(\tau) = 2 \left( \frac{\Delta}{2} \right)^2 \int_0^\infty d\omega K(\omega)F_\tau(\omega - \Delta\Omega), \quad (3) \]

The function \( F_\tau(\omega - \Delta\Omega) = \frac{\pi}{2\pi} \text{sin}^2 \left[ \frac{(\omega - \Delta\Omega)\pi}{2} \right] \) and is associated with measurements at intervals of \( \tau \). The reservoir coupling function \( K(\omega) \) is evaluated using \( K(\omega) = \int_0^\infty e^{-i\omega t} \cos[\Delta\Omega + G_1(t)] e^{-G_2(t)} dt \) where \( G_1(t) = \int_0^\infty d\omega J(\omega) \sin\omega t \) and \( G_2(t) = \int_0^\infty d\omega J(\omega) \text{coth}(\beta\omega/2)[1 - \cos\omega t] \), where \( \beta = \frac{1}{k_B T} \) and \( T \) is the lattice temperature. Explicit expressions for \( G_1(t) \) and \( G_2(t) \) in Refs. [33, 34] for an ohmic \( J(\omega) \) show the strong dependence of \( K(\omega) \) on the reservoir coupling function \( \eta \) and the exponential cutoff frequency \( \omega_c \). The occurrence of QZE or AZE is determined by changes in the overlap between functions \( F_\tau(\omega) \) and \( K(\omega) \) as \( \tau \) is varied. QZE (AZE) occurs when the overlap of functions decreases (increases) with decrease in \( \tau \). The crossover from QZE to AZE is most pronounced when \( \tau \) is increased in systems with weak spin-boson coupling [37], and also when bias \( \Delta\Omega \) is increased as well.

### A. Approximate relations of the Zeno-anti Zeno crossover point

To obtain approximate analytical relations, we employ the an effective decay rate applicable at short times [22], \( \gamma(\tau) = \frac{2\pi}{\Delta} \int_0^\tau dt_1 \int_0^{t_1} K(t') dt'' \) where \( K(t) \) is the Fourier transform of the coupling function \( K(\omega) \) defined below Eq. (3). Using explicit expressions for \( G_1(t) \) and \( G_2(t) \) given in Refs. [33, 34] for an ohmic \( J(\omega) \), we obtain \( \gamma(\tau) \) as follows

\[ \gamma(\tau) = \frac{2\pi}{\Delta} \int_0^\tau dt \cos[\Delta\Omega + 2\eta \tan^{-1} \omega_c t] \times \left( \frac{\pi t^2}{1 + (\omega_c t)^2} \right)^{2\eta} \cos\left[ \frac{\pi t^2}{\beta} \right] \]

where \( \beta = \frac{1}{k_B T} \), \( T \) is the lattice temperature and \( \text{csch}(x) \) is the hyperbolic cosecant function. Using \( \Delta\Omega = 0 \), \( T = 0K \), \( \Delta^2 = 2 \) and \( \omega_c = 1 \), we obtain simple expressions for \( \gamma(\tau) \) and \( \frac{\partial\gamma(\tau)}{\partial\tau} \)

\[ \gamma(\tau) = \left( 1 + \frac{\tau}{\gamma} \right)^{-\eta} \left( \sin[2\eta \tan^{-1} \tau] - \tau \cos[2\eta \tan^{-1} \tau] \right) \]

\[ \frac{\partial\gamma(\tau)}{\partial\tau} = \left( 1 + \frac{\tau}{\gamma} \right)^{-\eta} \cos[2\eta \tan^{-1} \tau] \]

At very short time intervals \( \omega_c \tau < 1 \), \( \gamma(\tau) \approx \tau \) whereas at very large times \( \omega_c \tau \to \infty \) and for \( \eta \neq 1/2 \), \( \gamma(\tau) \approx \frac{\cos\pi\gamma}{2\eta - 1} \tau^{1-2\eta} \). At \( \eta = 1/2 \), \( \frac{\cos\pi\gamma}{2\eta - 1} \to \frac{\pi}{2} \) and we get a rate which is independent of the measuring device, \( \gamma(\tau) \approx \frac{\pi\Delta^2}{4\omega_c} \).

At the point of Zeno-anti-Zeno transition, \( \frac{\partial\gamma(\tau)}{\partial\tau} = 0 \), and using Eq. (3) we obtain an explicit expression for the measurement interval \( \tau_c \) at which Zeno to anti-Zeno transition occurs \((\eta \neq 1/2)\)

\[ \tau_c = \frac{\pi}{4\eta} \]

At non-zero values of \( \Delta\Omega \) where the spin-boson system exist under biased conditions, the measurement interval \( \tau_c \) at which Zeno to anti-Zeno transition occurs is modified to

\[ \tau_c = \tan \left[ \frac{1}{2\eta} \left( \frac{\pi}{2} - \mu\Delta\Omega \right) \right] \]

where the factor \( 2 < \mu < 3 \) and depends on the bias \( \Delta\Omega \). Eq. (8) is consistent with the fact that an increase in biasing energy \( \Delta\Omega \) increases the probability of Zeno-anti-Zeno transition.

It is important to note that the definition of the Zeno-anti-Zeno transition is based on the properties of the decay rate, \( \gamma(\tau) \) and not a fixed natural rate. \( \tau_c \) can be
viewed as the duration of one of many other pulses, and therefore specific to the local dynamics of the quantum system being monitored. We point out the difference in settings between the current work and an earlier work in which the reservoir constitutes a part of the dynamical system that is monitored by the measuring device. In Ref. [37], Zeno to anti-Zeno features were revealed even with the first (of many) measurement by a distant observer due to the continuous measurement effect by the reservoir of oscillators.

IV. DYNAMICS OF QUANTUM DISCORD FOR X-TYPE QUBIT STATES

In order to examine the joint evolution of a pair of two-level qubit systems in uncorrelated reservoirs, we consider the following Bell-like initial state

$$|\Phi_0\rangle = \left[ a|0\rangle_{q_1}|0\rangle_{q_2} + b|1\rangle_{q_1}|1\rangle_{q_2} \right]|0\rangle_{r_1}|0\rangle_{r_2}, \quad (9)$$

where $i=1,2$ denote the two qubit-reservoir systems associated function $u_i(t)$ in Eq.(2), $a,b$ are real coefficients and satisfy, $a^2 + b^2 = 1$. Using Eq.(2) and tracing out the reservoir states we obtain a time-dependent qubit-qubit reduced density matrix in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ which evolves with time duration $\tau$ as

$$\rho_{q_1,q_2}(t) = \begin{pmatrix} f_1 & 0 & 0 & f_5 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & 0 \\ f_5 & 0 & 0 & f_1 \end{pmatrix}, \quad (10)$$

where $f_1 = a^2 + b^2v_1(\tau)^2v_2(\tau)^2$, $f_5 = abu_1(\tau)u_2(\tau)$, $f_2 = b^2v_1(\tau)^2u_2(\tau)^2$, $f_3 = b^2u_1(\tau)^2v_2(\tau)^2$, $f_4 = b^{-2}u_1(\tau)^2u_2(\tau)^2$. We assume that the usual unit trace and positivity conditions of the density operator $\rho_{q_1,q_2}$ are satisfied, however these may not constitute strict requirement for the determination of the quantum discord. The reservoir-reservoir reduced density matrix $\rho_{r_1,r_2}$ is similarly obtained by by tracing out qubit states. Each non-zero matrix term of $\rho_{r_1,r_2}$ is easily obtained from the corresponding term $\rho_{q_1,q_2}(t)$ by swapping $u_i \leftrightarrow v_i$. Both matrices possess the well-known X-state structure which preserve its form during evolution. Complete Wooters concurrence [2] for the density matrix in Eq.(10) is $C_{q_1,q_2}(\tau) = 2\sqrt{\frac{a - b(1 - e^{-\gamma_1\tau})}{\gamma_1 + \gamma_2\tau}} \times \left\{ \begin{array}{c} a - b(1 - e^{-\gamma_1\tau}) \\ (1 - e^{-\gamma_2\tau}) \end{array} \right\}$ and $C_{r_1,r_2}(\tau) = 2b(1 - e^{-\gamma_1\tau}) \times \left\{ \begin{array}{c} a - b(1 - e^{-\gamma_1\tau}) \\ (1 - e^{-\gamma_2\tau}) \end{array} \right\}$. The density matrix in Eq.(10) yields $S(\rho_{q_2}) = -b^2u_2^2\log_2[b^2u_2^2] - (a^2 + b^2v_1^2)\log_2[a^2 + b^2v_1^2]$ with explicit dependence on the measurement time duration $\tau$, and system bias, $\Delta\alpha$ and tunneling amplitude $\Delta$ via the functions $u_i$. The condition $S(\rho_{q_1}) = S(\rho_{q_2})$ is therefore satisfied only if these parameters are the same for both subsystems. The quantum discord present in the two-qubit $D_{q_1,q_2}(\tau)$ and two-reservoir $D_{r_1,r_2}(\tau)$ partitions for the subclass of density matrix for which $\gamma_1 = \gamma_2$ (i.e. $f_2 = f_3$) are evaluated following Fancini et al. [8]. We obtain $D_{q_1,q_2}(\tau) = H(b^2u_2^2) - H(\frac{1}{2}(1 + (1 - 4b^2u_2^2v_2^2)^{1/2}))$, and from which $D_{r_1,r_2}(\tau)$ is obtained by swapping $u \leftrightarrow v$. The function $H(x) = -x\log_2 x - (1 - x)\log_2(1 - x)$, and the difference in quantum discords, $D_{q_1,q_2} - D_{r_1,r_2} = H(b^2u_2^2) - H(b^2v_2^2)$. For $\gamma_1 \neq \gamma_2 = \gamma$ or unequal $u_1,u_2$ values, the quantum discord of the density matrix in Eq.(10) can evaluated following the main results in Ref. [30, 31] where the quantum conditional entropy is generalized as $S(\rho_i(\Pi_k)) = p_0 S(\rho_0) + p_1 S(\rho_1)$, based on the earlier work of Luo [28]. The terms $p_0 = \left[ (f_1 + f_3)k + (f_2 + f_4)l \right]$, $p_1 = \left[ (f_1 + f_3)l + (f_2 + f_4)k \right]$ and $S(\rho_0), S(\rho_1)$ are dependent on generalized angles $\theta, \theta'$. The generally cumbersome procedure of determining $S(\rho_i(\Pi_k))$ and the classical correlation is greatly simplified if cross terms $\rho_{23} = 0$ (following the notation in Ref.[30]) as is the case in the density matrix in Eq.(10). The problem reduces to minimization with just one parameter $k$ or $l = 1 - k$, instead of the set of three parameters.

\[\text{FIG. 1: a) Two-qubit concurrence } C_{q_1,q_2}(\tau) \text{ (solid line) as function of measurement time duration } \tau \text{ with initial amplitude parameter } a = \sqrt{1/5} \text{ with same subsystem bias } \Delta\Omega_1 = \Delta\Omega_2 = 0.65 \text{ (upper solid line), dissimilar subsystem bias } \Delta\Omega_1 = 0.65, \Delta\Omega_2 = 0.15 \text{ (lower solid line), tunneling amplitude } \Delta\Omega = 0.6, \eta = 0.05, \omega = 1 \text{. The two-reservoir concurrence } C_{r_1,r_2}(\tau) \text{ is computed for } \Delta\Omega_1 = \Delta\Omega_2 = 0.65 \text{ (lower dashed line)} \text{ and } \Delta\Omega_1 = 0.65, \Delta\Omega_2 = 0.15 \text{ (upper dashed line).} \text{ b) Quantum discord present in the two-qubit partition, } D_{q_1,q_2}(\tau) \text{ (solid lines) and two-reservoir discord } D_{r_1,r_2}(\tau) \text{ (dashed lines) as function of measurement time duration } \tau \text{. All other parameters and positioning of lines with respect to bias configurations are the same as in (a).} \]
which clearly displays the transition point at the similar bias configuration in Figure 1b. Due to coupling with a system of lower bias, a transition point is not present at the dissimilar bias configuration.

The crossover or transition point which occurs at the minimum (maximum) in the two-qubit partition (two-reservoir partition) can be numerically verified using Eqs. (4). We noted that the crossover point at a Zeno/anti-Zeno transition coincides with the equivalent point for the quantum discord, thus a decrease to increase and then a subsequent decrease in quantum discord can be interpreted as a sign of the Zeno/anti-Zeno transition.

![Image 54x407 to 172x590](image)

FIG. 2: (a),(b) Two-qubit concurrence $C_{q1-q2}(\tau)$ and two-reservoir concurrence $C_{r1-r2}(\tau)$ as function of measurement time duration $\tau$ and initial amplitude parameter $a$ with same subsystem bias $\Delta \Omega_1=\Delta \Omega_2=0.65$. All other parameters are the same as in Figures 1a,b.

(c),(d) Quantum discord present in the two-qubit partition, $D_{q1-q2}(\tau)$ and two-reservoir discord $D_{r1-r2}(\tau)$ as function of measurement time duration $\tau$. All other parameters are the same as in Figures 1a,b.

Figures 2b,c,d which incorporates a change in the initial state parameter $a$, show that the two-reservoir discord best captures the Zeno-anti Zeno transition point. While it is known that the quantum discord remains non-zero under various conditions [1,2], these results show that the quantum discord is reliable in being able to display Zeno-anti-Zeno dynamics occurring in separate qubit-reservoir subsystems, and which are also weakly coupled (small values of $a$).

A. Quantum discord in an initial state with single excitation

The analysis of quantum discord can be extended to the initial state of the Bell-like state with just a single excited state residing in either of the qubit

$$|\Phi \rangle_0 = [c|0\rangle_{ex1}|1\rangle_{ex2} + d|1\rangle_{ex1}|0\rangle_{ex2}][0]_r1|0\rangle_r2 \quad (11)$$

where $i=1,2$ denote the two qubit-reservoir systems with associated functions $u_i(t)$. As in the case in Eq. (10), we trace out the reservoir states to obtain a time-dependent qubit-qubit reduced density matrix

$$\rho_{q1-q2}(t) = \begin{pmatrix} g_1 & 0 & 0 & 0 \\ 0 & g_2 & g_4 & 0 \\ 0 & 0 & g_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (12)$$

where for $t \geq 0$, the matrix elements evolve as $g_1(t) = c^2 u_2(t)^2 + d^2 v_1(t)^2$, $g_2(t) = c^2 u_2(t)^2$, $g_3(t) = d^2 u_1(t)^2$ and $g_4(t) = c d u_1(t) u_2(t)$. Following Fancini et al. [8], we obtain $D_{q1-q2}(\tau) = H(a^2 u^2) - H(u^2) + H(\frac{1}{2}(1 - 4 b^2 u^2 v^2)^{1/2})$, from which $D_{r1-r2}(\tau)$ is obtained by swapping $u \leftrightarrow v$ and the difference in quantum discords, $D_{q1-q2} - D_{r1-r2} = H(a^2 u^2) + H(v^2) - H(u^2) - H(a^2 v^2)$.

Figures 3a,b show the dynamics of the two-qubit quantum discord, $D_{q1-q2}(\tau)$ and two-reservoir quantum discord $D_{r1-r2}(\tau)$ at different subsystem bias configurations, $\Delta \Omega_1=\Delta \Omega_2=0.75, 0.25$. The quantum discord displays anti-crossing behavior at the higher system bias value for the two different states given in Eqs. (9) and (11). The two-reservoir quantum discord is however enhanced in Eq. (11), due to greater participation from the two-reservoir partition. The slight differences in the quantum discord due to the two different initial states in Eqs. (9), (11) are mainly due to variations in classical correlations, $C_i(\rho)$ where $i$ denotes the subsystem under consideration.

FIG. 3: a) Quantum discord in the two-qubit partition, $D_{q1-q2}(\tau)$ (solid lines) as function of measurement time duration $\tau$ with same subsystem bias $\Delta \Omega_1=\Delta \Omega_2=0.75$, upper solid line and 0.25, solid line), tunneling amplitude $\Delta=0.6$, $\eta=0.05$, $\omega_c=1$. Quantum discord present in the two-reservoir partition, $D_{r1-r2}(\tau)$ is denoted by dashed lines, $\Delta \Omega_1=\Delta \Omega_2=0.75$ corresponds to the lower dashed line, $\Delta \Omega_1=\Delta \Omega_2=0.15$ corresponds to the upper dashed line. All measures are evaluated using the initial state in Eq. (11) with initial amplitude parameter $a = \sqrt{1/2}$.

b) Quantum discord present in the two-qubit partition, $D_{q1-q2}(\tau)$ (solid lines) and two-reservoir discord $D_{r1-r2}(\tau)$ (dashed lines) as function of measurement time duration $\tau$. All other parameters and positioning of lines with respect to bias configurations are the same as in (a). All measures are evaluated using the initial Bell-like state in Eq. (11) with initial amplitude parameter $c = \sqrt{1/2}$. 
V. QUANTUM DISCORD AND EXCEPTIONAL POINTS AT HIGH PRECISION MEASUREMENTS

While the quantum Zeno effect is viewed as the effect of repeated measurements on a quantum system, it can be studied in the wider context of the dynamical time evolution of quantum systems. The Zeno effect appears even if the information regarding the state of the observed system manifests in the form of an external degree of freedom such as the spontaneous emission process. It would be interesting to examine whether the features of the Zeno effect, and the quantum discord are retained if the monitoring device imparts a significant disturbance on the system under study and itself dominates the time evolution of the quantum system.

For a two-level system with energies $E_1$ ($E_2$) at state $|0\rangle$ ($|1\rangle$) subjected to a continuous measurement process, its original Hamiltonian $\hat{H}_0$ transforms via the non-Hermitian Hamiltonian $\hat{H}_{eff} = \hat{H}_0 - i\frac{\hbar}{\tau} (\hat{H}_0 - E)^2$, $E$ is the selected measurement output after a time $\tau$ and $E_r$ is the error made during the measurement of the energy. $E_r$ can also be considered as a measure of the precision of the monitoring device. A large error made during the measurement can be viewed as a weak or unsharp measurement and $\hat{H}_{eff} \rightarrow \hat{H}_0$, whereas one made with very small error can be considered a highly precise measurement. The system therefore evolves as $i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H}_{eff} |\psi(t)\rangle$ during measurement due to the constraining effect of the selected readout $E_r$.

The state of the system being measured can be expanded within the unperturbed basis states $|n\rangle$ of the unmeasured system with Hamiltonian $\hat{H}_0$ as $|\psi(t)\rangle = \sum C_n(t) |n\rangle$. The coefficients $C_n(t)$ can be determined using the Schrödinger equation and the non-Hermitian $\hat{H}_{eff}$. In the presence of an external potential of the form $V_{21} = V_{11} = 0$ and $V_{12} = V_{21} = V_0 e^{i\omega t}$ with $V_0$ real, the system evolves as $|\psi(t)\rangle = e^{-i(E_1-2\lambda_1)t} C_1(0) |0\rangle + e^{-i(E_2-2\lambda_2)t} C_2(0) |1\rangle$ where $\lambda_1 = (E_1-E_2)^2/2\kappa\epsilon_2$ and $\lambda_2 = (E_2-E_1)^2/2\kappa\epsilon_2$.

The coefficients $C_1(t), C_2(t)$ can be recast as

$$
\begin{bmatrix}
C_1(t) \\
C_2(t)
\end{bmatrix}
= \begin{bmatrix}
\cos \kappa t - i\alpha_1 \\
-\alpha_2 \\
\cos \kappa t + i\alpha_1
\end{bmatrix}
\begin{bmatrix}
C_1(0) \\
C_2(0)
\end{bmatrix}
,$$

where $\alpha_1 = \cos \theta \sin \kappa t$, $\alpha_2 = \sin \theta \sin \kappa t$, $\cos \theta = \frac{\pi}{\kappa}$, $\kappa = \sqrt{p^2 + q^2}$, $q = \frac{1}{\sqrt{2}} (\omega - \Delta E + 2i\Omega)$, $\Delta E = (E_2 - E_1)$, $p = V_0$ and $\Omega = \lambda_2 - \lambda_1$. The terms $\lambda_2$ and $\lambda_1$ as defined in the earlier paragraph are dependent on the measurement precision, $E_r$ as well as the energy $E$ to be measured.

For a system in which the initial state at $t = 0$ is $|1\rangle$ and the final state at time $t$ is either $|1\rangle$ or $|0\rangle$, the probability $P_{11}$ of the system to be in the state $|1\rangle$ can be obtained following Ref. [40] as $P_{11} = |\cos \theta \sin \kappa t - i \cos \theta \sin \kappa t| e^{-\lambda_1 t}$ where $\lambda_1 = \frac{(E_2-E_1)^2}{2\kappa\epsilon_2}$. Likewise the probability $P_{10}$ that the system is present in the state $|0\rangle$ is given by $P_{10} = |\sin \theta \sin \kappa t| e^{-\lambda_1 t}$.

The total probabilities, $P_{11} + P_{10} \leq 1$, the loss of normalization is dependent on the measurement precision, $E_r$ as expected, and further evaluation of the quantum discord will be significantly affected in the case of highly precise measurements.

At the resonance frequencies, $\omega = \Delta E$, the Rabi frequency $\kappa_0 = 2(V_0^2 - \lambda_1^2)^{1/2}$, and $\cos \theta = -i\lambda_1/\kappa_0$. There are two regimes, depending on the relation between $V_0$ and $\lambda_1$. The range where $V_0 > \lambda_1$ applies to the coherent tunneling regime where

$$
P_{11} = e^{-\lambda_1 t} \left[ \cos \kappa_0 t - \frac{\lambda_1}{\kappa_0} \sin \kappa_0 t \right]^2,
$$

$$
P_{10} = e^{-\lambda_1 t} \frac{V_0^2}{\kappa_0^2} \sin^2 \kappa_0 t,
$$

For $V_0 < \lambda_1$, the system undergoes incoherent tunneling

$$
P_{11} = e^{-\lambda_1 t} \left[ \cos \kappa_0 t - \frac{\lambda_1}{\kappa_0} \sin \kappa_0 t \right]^2,
$$

$$
P_{10} = e^{-\lambda_1 t} \frac{V_0^2}{\kappa_0^2} \sin^2 \kappa_0 t.
$$

At the exceptional point, $\kappa_0 = 0$, and both regimes merge and we obtain $P_{11} = \left(1 - \frac{\lambda_1^2}{V_0^2}\right)^2 e^{-\lambda_1 t}$ and $P_{10} = \left(\frac{V_0^2}{\kappa_0^2}\right)^2 e^{-\lambda_1 t}$. Exceptional points are singularities [41] which appear at the branch point of eigenfunctions due to changes in parameters which govern the non-Hermitian Hamiltonian operator. These points are known to be located in the vicinity of a level repulsion [41] and unlike degenerate points, only one eigenfunction exists at the exceptional point due to the merging of two eigenvalues. In the case of the quantum measurements considered in this work, the exceptional point appears at a critical measurement precision $E_r = \frac{\sqrt{c_0}}{\sqrt{2V_0}}$. Considering a unit system in which $h = V_0 = \Delta E = 1$, $\tau = \pi/V_0$ and a unitless time $t = t'/\tau$, we obtain $E_r = \frac{\sqrt{c_0}}{\sqrt{2\pi}}$. Using $r$ to denote the unitless measurement precision parameter, we note that at $r > \frac{1}{\sqrt{2\pi}} (r < \frac{1}{\sqrt{2\pi}})$, the quantum system undergoes coherent (incoherent) tunneling.

A. Entangled qubits subjected to high precision measurements

Similar to the model adopted in the Section IV, we consider two uncoupled qubits which are entangled initially, but which differ from the earlier treatment in being monitored by independent observers. These observers assume the role of the reservoirs of harmonic oscillators. We consider functions $u(t)$ and $v(t)$ which previously were associated with the decay of the two-level qubit in Eq. [2]. The influence of the measurement precision $E_r$ on the quantum discord is investigated by setting $u(t)^2 = P_{11}$, $v(t)^2 = P_{10}$, and evaluating $D_{\tilde{u},\tilde{v}}(t)$ and $D_{\tilde{u},\tilde{v}'}(t)$ as described in Section IV. Unlike in Sections IIIA III here we examine the dynamics of the quantum discord in the
context of a Zeno effect manifesting itself even before a measurement outcome is reached and therefore time, $t$ satisfies $0 \leq t \leq \tau$ where $\tau$ is the measurement duration. As the relation $u(t)^2 + v(t)^2 = 1$ is not satisfied for high precise measurements, the widely accepted definition of the quantum discord discussed in Section I may be considered as a limiting case of a more generalized definition that may apply in the case of quantum systems which undergo non-Hermitian evolution dynamics. With the inclusion of a non-Hermitian term, the unit trace and strict positivity conditions of the density operator of the quantum system will not be satisfied as well. With the view of realizing qualitative results of the quantum discord, we therefore relax conditions needed for more rigorous and accurate quantitative approach to evaluating the quantum discord for non-Hermitian systems. We illustrate the dynamics of the quantum discord in the two regimes specified by Eqs. (14) and (15) in Figures 4a,b and 5a,b. These figures show the explicit dependence of the quantum discord on the measurement precision, with appearance of indeterminate values of the quantum discord at very high precision measurements (low values of $r$). The figures also indicate that a highly precise observer can diminish the non-classical correlation shared between two subsystems, with the tendency to do so increasing with the measurement precision. It has to be noted that the quantum discord is evaluated in a reference frame where the observer is not under active consideration as one of the subsystems. The results will therefore be modified if the monitoring system is included and the quantum system then expands to a group of three subsystems.

![FIG. 4: a) Quantum discord $D$ present in the two-qubit partition, as function of normalized time $t$ and measurement precision $r$ in the coherent tunneling regime. All parameters are the same as in (a). b) Quantum discord $D$ present in the two-reservoir partition, as function of normalized time $t$ and measurement precision $r$. All other parameters are the same as in (a).](image)

**VI. DISCUSSION AND CONCLUSIONS**

We have examined the influence of quantum measurements on quantum discord with consideration of two types of measurements, weak or low precision measurements and highly precise measurements. In the case of ideal weak measurements, the results show that the quantum discord present in a two qubit or reservoir system responds to characteristic parameters such as the system bias, duration and frequency of the measurement induced by the decoherence processes as well as the strength of initial entanglement between the two qubit systems. Unlike the reservoir-reservoir concurrence $C_{\rho_{12}}(\tau)$, its quantum discord counterpart is more resilient to changes in the measurement duration, $\tau$. For weak measurements, the quantum discord therefore presents as a suitable measure to identify and quantify Zeno-anti Zeno crossover dynamics in the spin-boson system. The quantum discord may be used as a reliable measure of quantum processes influenced by the quantum Zeno effect such as quantum switching and preparation of decoherence-free states and cluster states. Another potential application is the possibility of using quantum discord as an efficiency measure of the purification of qubit states which occurs via extraction of a pure state through a series of Zeno-like measurements. The model used in this work is generic to most quantum systems which undergo Zeno-anti Zeno crossover dynamics, and can therefore be extended to other quantum systems displaying such crossover effects, as mentioned earlier in the text.

For the class of highly precise measurements which introduce maximal interference in the dynamics of quantum systems, the appearance of singularities introduce complications in the quantum evolution of a measured system. The quantum discord becomes indeterminate for highly precise quantum measurements. Importantly, the Zeno effect fails at very precise measurements as the system does not reside at one level, but possibly transfers available information to the unspecified level of the observer. In future works, the direct influence of the Zeno effect due to measurements made in one subsystem in order to obtain the conditional entropy in a second subsystem will be considered. Such an approach will allow determination of the influence of the measurement precision on the classical correlation measure in a neighboring
partition. This alternative perspective of the influence of a monitoring device will also allow convenient analysis of the Berry phase due to quantum measurements.

Finally, we have presented results of the influence of the quantum Zeno effect on the concurrence and quantum discord for various biased configurations of the qubit-reservoir system. We have demonstrated the resilience of the quantum discord measure, in particular it is more robust than the concurrence in the reservoir-reservoir partition subsystem. The quantum discord which is an intrinsically distinct entity from entanglement, therefore serves as a better indicator, of the crossover point in Zeno to anti-Zeno transition evident in some spin-boson systems under suitable conditions and for weak measurements. As to whether this applies to other quantum systems which display both Zeno and anti-Zeno effects needs further investigation. For highly precise measurements, the monitoring device can significantly interfere with the evaluation of the quantum discord and produce indeterminate values of the quantum discord. With progress in experimental techniques and studies of quantum measurement in optics and nanostructure systems [44, 45], investigations involving the quantum discord of entangled systems are expected to play a greater role in future experimental works.

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