Energy extraction and particle acceleration around a rotating dyonic black hole in $\mathcal{N} = 2$, $U(1)^2$ gauged supergravity

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Abstract

In the present paper, we explore various gravitational aspects such as energy extraction, particle collision around $U(1)^2$ dyonic rotating black hole spacetime. The impact parameter of the rotation parameter and the gauge coupling constant on the behavior of horizon and ergoregion of the black hole is studied. The energy extraction mechanism via Penrose process and superradiant modes is also presented numerically for the black hole. Interestingly, our result indicates that for a strongly coupled extremal black hole under certain constraint, the maximum efficiency for this case becomes almost double while compared with the case of extremal Kerr black hole spacetime in general relativity (GR). Under the same constraint, the maximum amount can be extracted from an extreme black hole which increases with respect to the coupling constant and comes to have an upper bound which is $\sim 60.75\%$ of its total energy. It is worth noticing that the amount of energy extracted is extremely high as compared to the extreme Kerr black hole spacetime in GR. The limit of energy extraction in terms of the local speeds of the fragments is also examined with the help of the Wald inequality. We identify an upper limit on the gauge coupling constant up to which the phenomenon of Superradiance is likely to occur. We have also computed the energy for the collision of two particles having equal rest mass accelerated by the black hole spacetime in the center-of-mass (CM) frame. Our study also aims to sensitise the CM energy to the rotation parameter and the gauge coupling constant for both extremal and nonextremal black hole spacetimes. For the extremal spacetime, an infinitely large amount of CM energy can be achieved closer to the horizon which allows the $\mathcal{N} = 2$, gauged super gravity dyonic black hole to serve as a more powerful Planck-energy-scale collider as compared to Kerr and any other generalized Kerr black hole explored so far in GR and in other alternative theories of gravity. The

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CM energy for the nonextremal spacetime is however shown to be finite.

**Keywords**: Supergravity, Dyonic black hole, Penrose process, Superradiance.

### 1. Introduction

The existence of black holes (BHs) is considered one of the most astonishing consequences of general relativity (GR) proposed by Einstein in 1915. One can use BHs to study gravity in strong regime and they can also be used as probes of quantum effects of any fundamental theory, such as supergravity theories. It is thus important to have exact BH solutions derived in the context of supergravity which could be fully analysed by extracting all possible differences from the view of their geometrical structure with the Kerr black hole (KBH) spacetime in GR. In particular, if a BH is asymptotically AdS and extremal, it plays a crucial role in the AdS/CFT correspondence, principally if the model is embedded in a supergravity model. It is therefore quite interesting to further investigate such BH solutions emerging in a gauged supergravity model due to their geometrical structure. An exact rotating BH solution with dyonic charges \( N = 2, U(1)^2 \) gauged supergravity is investigated in [1] along with the detailed study of geodesic motion [2].

It is worth mentioning that the most powerful conceivable source of energy for galactic nuclei, X-ray binaries, quasars has always been considered of crucial interest in high energy astrophysics. Among several energy extraction mechanisms to explain high energy cosmic events [3],[4], the Penrose process (particle splitting inside the ergosphere) [5]-[8] and the BlandfordZnajek mechanism (manipulating magnetic field inside the Ergosphere) [9] are quite famous. The Penrose process has been extensively studied for various spacetimes time and again [10]-[18]. Further a similar mechanism known as superradiance is customarily thought as the wave analogue of the Penrose process [19],[20]. In this case, a scalar field is boosted while scattering over a BH by extracting energy and angular momentum after reflection off a spinning BH for some range of frequencies [21]. The BH superradiance have been studied from the viewpoint of thermodynamics [22],[23] and BH evaporation as well [24]. The scenario becomes more fascinating if the bosonic waves were repeatedly back onto the BH by a runaway process and thus inducing a BH bomb [25]-[27].

In view of the above motivation for different energy extraction schemes from a BH, we intend to investigate a dyonic BH emerging in a supergravity model as a particle accelerator. It is now well acquainted that the energy of two colliding particles calculated in center of mass frame, \( E_{CM} \) can be arbitrarily high in a rotating extremal spacetime of a BH. In this context, the BSW (Baados, Silk and West) mechanism has a fertile impact on the viewpoint of ultra high energy collisions as it may generate new interesting physics at Planck-scale (see [28]-[43] for recent developments).

The present paper is aimed to investigate the energy extraction via Penrose process, superradiance and particle acceleration in the background of a dyonic rotating BH spacetime in \( N = 2, U(1)^2 \) gauged supergravity model. The paper is organised as follows. We first discuss the horizon structure and ergosphere in detail for the above BH spacetime in Section 2 followed by the geodesic motion in Section 3. Various energy extraction mechanisms are then presented in the Sections 4 and 5. The particle acceleration is discussed afterward in view of Baados, Silk and West (BSW) mechanism in Section 6. Finally, the results obtained are summarised in Section 7.
2. The structure of spacetime

The spacetime corresponding to the general solution for $\mathcal{N} = 2$, $U(1)^2$ gauged supergravity BH with dyonic charges $[1, 2]$, reads as,

$$ ds^2 = -\frac{R_g}{B - aA} \left( dt - \frac{A}{\Xi} d\phi \right)^2 + \frac{B - aA}{R_g} dr^2 + \Theta_g a^2 \sin^2 \theta \left( dt - \frac{B}{a^2 \Xi} d\phi \right)^2 + \frac{B - aA}{\Theta_g} d\theta^2, $$  

(1)

where,

$$ R_g = r^2 - 2mr + a^2 + e^2 - N_g^2 + g^2 [r^4 + (a^2 + 6N_g^2 - 2v^2)r^2 + 3N_g^2 (a^2 - N_g^2)], $$

$$ \Theta_g = 1 - a^2 g^2 \cos^2 \theta - 4a^2 N_g \cos \theta, $$

$$ A = a \sin^2 \theta + 4N_g \sin^2 \theta, $$

$$ B = r^2 + (N_g + a)^2 - v^2, $$

$$ \Xi = 1 - 4N_g ag^2 - a^2 g^2. $$

The notations used are as in [1], [2] and the BH spacetime mentioned above has in general six parameters which are parameterized by mass ($m$), rotation ($a$), electric charge ($e$), magnetic charge ($v$), the gauge coupling constant ($g$) and NUT charge ($N_g$). It is worth mentioning that the BH solution (1) has a number of limiting cases having their individual implications as discussed extensively in [1].

2.1. The Structure of Horizons and Ergosphere

The horizon of the spacetime given by equation (1) can be found by looking at the zeros of $R_g$ (i.e. $R_g = 0$) then leads to,

$$ r^2 - 2mr + a^2 + e^2 - N_g^2 + g^2 [r^4 + (a^2 + 6N_g^2 - 2v^2)r^2 + 3N_g^2 (a^2 - N_g^2)] = 0, $$  

(2)

which can be further re-written as below in the structuraly similar form as for Kerr-Newman-AdS BH,

$$ (1 + g^2 r^2)(\alpha + r^2) - 2mr + z = 0, $$  

(3)

where

$$ \alpha = a^2 + 6N_g^2 - 2v^2, $$

(4)

$$ z = e^2 - 7N_g^2 + 2v^2 + 3N_g^2 g^2 (a^2 - N_g^2). $$

(5)

These types of quartic equations can always be reduced to a depressed quartic equation which can be solved by Ferrari’s method and following the same, one can define a critical mass,

$$ m_c = \frac{1}{3\sqrt{6g}} \left( 2 + 2\alpha g^2 + \sqrt{x} \right) \left( \sqrt{x} - 1 - \alpha g^2 \right)^{1/2}, $$  

(6)

where $x = 12g^2(\alpha + z) + (1 + g^2 \alpha)^2$. A study of the positive zeros of the function $R_g$ shows that the line element (1) describes a naked singularity for $m < m_c$ and a BH with an outer event
horizon and an inner Cauchy horizon for $m > m_c$. Finally, for $m = m_c$, $R_g$ has a double root and (1) represents an extremal BH spacetime. One can notice an important difference with the case of Kerr-Newman-AdS BH for which $\alpha = a^2 \geq 0$ and $z = e^2 \geq 0$. In Kerr-Newman-AdS BH case, $m_c > 0$ and therefore we have a minimum mass of the BH, while the situation for the metric (1) is different because of undefined signature of $A$ and $z$. One can notice that the condition (6) reduces in the ungauged case ($g = 0$) to $m_c = \sqrt{a^2 + e^2 - N_g^2}$ which is similar to the Kerr-Newman-Taub-NUT BH condition. One of the important features of the rotating BHs is the existence of the region between the outer horizon and the stationary limit surface (which satisfies $g_{tt} = 0$) and by definition, inside this particular region, the asymptotic time translation killing vector becomes spacelike. For the spacetime given by equation (1), the static limit surface requires to satisfy,

$$R_g - \Theta_g a^2 \sin^2 \theta = 0. \quad (7)$$

In order to have the ergosphere outside the event horizon, one must impose $\Theta_g > 0$, which in turn implies $1 - a^2 g^2 - 4 a^2 N_g^2 > 0$. It is found numerically that the ergosphere depends in a non-trivial way on the parameters of the spacetime. In general, the ergosphere is found to become smaller when the value of $g$ or $a$ increases. One can also notice that the structure of the singularity is very complex as noticed in [2]. It is not only a ring singularity but for different parameters, this spacetime can have a more complicated three-dimensional structure, which is defined by $r = \sqrt{v^2 - (N_g + a \cos \theta)^2}$. The complexity is actually due to the parameters $(N_g, v)$. Even if we have secured the existence of a horizon and an ergosphere exterior to the horizon, the singularity could pop-up outside the horizon. One therefore always needs to impose an additional condition that the singularity is inside the horizon, and therefore that $r_E > \sqrt{v^2 - (N_g + a \cos \theta)^2}$ where $r_E$ is the position of the horizon.

### 3. The Geodesic Motion

In the Schwarzschild BH spacetime background, one can study the geodesic motion through the Lagrangian $g_{\mu\nu} u^\mu u^\nu$, from which one defines two conserved quantities along geodesics, the energy at infinity per mass unit and the angular momentum per mass unit, along with the normalization relation $g_{\mu\nu} u^\mu u^\nu = (-1, 0)$. In the Hamilton-Jacobi formalism, one can look for a function of the coordinates and of the curve parameter $\lambda$, $S = S(x^\mu, \lambda)$, which is solution of the Hamilton-Jacobi equation,

$$H \left( x^\mu, \frac{\partial S}{\partial x^\mu}, \lambda \right) = -\frac{\partial S}{\partial \lambda}, \quad (8)$$

where $H(x^\mu, p_\nu) = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu$ which leads to,

$$\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -\frac{\partial S}{\partial \lambda}, \quad (9)$$

where $p_\mu$ is the momentum.

In order to derive Hamiltonian-Jacobi equation, we need to compute the inverse metric $g^{\mu\nu}$ and using standard algebra techniques for metric inversion, the inverse metric components can be written as,

$$g^{tt} = -\frac{\Xi^2}{\Theta_g R_g \sin^2 \theta} \left[ \frac{\Theta_g a^2 \sin^2 \theta B^2}{(B - aA) a^2 \Xi^2} - \frac{R_g A^2}{(B - aA) \Xi^2} \right], \quad (10)$$
\[ g^{\phi \phi} = \frac{\Xi^2}{\Theta_g R_g \sin^2 \theta} \left[ \frac{R_g}{B - aA} - \frac{\Theta_g a^2 \sin^2 \theta}{B - aA} \right], \]  
\[ g^{\phi \phi} = \frac{\Xi^2}{\Theta_g R_g \sin^2 \theta} \left[ \frac{R_g}{B - aA} - \frac{\Theta_g a^2 \sin^2 \theta}{B - aA} \right], \]  
\[ g^{rr} = \frac{R_g}{B - aA}, \]  
\[ g^{\phi \phi} = \frac{\Theta_g}{B - aA}. \]  
Let us define constants \( m_0, E \) and \( L \) which corresponds to rest mass, conserved and axial part of the angular momentum of the particle respectively. These constants are related via \( m_0^2 = -p_\mu p^\mu, \ E = -p_t \) and \( L = p_\phi \) (with \( c = 1 \) and \( G = 1 \)). The Jacobi action is therefore given by,

\[ S = \frac{1}{2} m_0^2 \lambda - Et + L\phi + S_r(r) + S_\theta(\theta). \]  
Here, we are looking for a separable solution with \( S_r \) a function of \( r \) and \( S_\theta \) a function of \( \theta \) only. Substituting eq. (15) into the Hamilton-Jacobi eq. (9), we obtain,

\[ R_g S'_r(r)^2 + m_0^2 B - \frac{(aL\Xi - EB)^2}{R_g} = -\Theta_g S'_\theta(\theta)^2 + am_0^2 A - \frac{(L\Xi - EA)^2}{\sin^2(\theta)\Theta_g}. \]  
One can notice that the left-hand side of eq. (16) does not depend on \( \theta \), and it is equal to the right-hand side which does not depend on \( r \); therefore, this quantity must be a constant \( C \) (popularly known as the Carter constant) which is not related to any isometry of the spacetime contrary to other constants of the problem such as \( E \) and \( L \) such that,

\[ R_g S'_r(r)^2 + m_0^2 B - \frac{(aL\Xi - EB)^2}{R_g} + K = -C, \]  
\[ \Theta_g S'_\theta(\theta)^2 - am_0^2 A + \frac{(L\Xi - EA)^2}{\sin^2(\theta)\Theta_g} - K = C, \]  
here \( K = -am_0^2 (a + 2N_\theta) + (E(a + 2N_\theta) - L\Xi)^2. \) The conjugate momenta \( p_\mu = \partial S/\partial x^\mu \) may then be written as below,

\[ p_r^2 = (g_{rr} \dot{r})^2 = S'_r(r)^2 = \frac{-C - K - m_0^2 B + \frac{(aL\Xi - EB)^2}{R_g}}{R_g}, \]  
\[ p_\phi^2 = (g_{\phi \phi} \dot{\phi})^2 = S'_\phi(\theta)^2 = \frac{C + K + am_0^2 A - \frac{(L\Xi - EA)^2}{\sin^2(\theta)\Theta_g}}{\Theta_g}. \]  
The following conjugate momenta are conserved : \( p_t = -E \) and \( p_\phi = L \) and the equation of motion for \( t \) amd \( \phi \) reads as below,

\[ \frac{dt}{d\tau} = \frac{B(EB - aL\Xi)}{R_g} + \frac{A(L\Xi - EA)}{\Theta_g \sin^2 \theta}, \]  
\[ \frac{d\phi}{d\tau} = \frac{a\Xi(EB - aL\Xi)}{R_g} + \frac{\Xi(L\Xi - EA)}{\Theta_g \sin^2 \theta}. \]
Figure 1: Various orbits from the ergosphere to the event horizon for different values of $g$ (with $g = 0.5$ in black, $g = 1$ in dashed blue and $g = 2$ in red dotted), while other parameters are fixed to have a unit value except $e = 0.1$. For the different values of $g$, the ergosphere and the event horizon have been rescaled to coincide.

In the equatorial plane (i.e. $\theta = \pi/2$), $p_\theta^2 = C$, with the introduction of the constant $K$ and with $C = 0$, we have $\dot{\theta} = 0$ which in turn implies a planar orbit. The equation (19) then reads as given below,

$$p_r^2 = \left( \frac{B - aA}{R_g} \right)^2 = \frac{-K - m_0^2B + \frac{(aL - EB)^2}{R_g}}{R_g},$$

(23)

which can be re-written in the following form

$$(B - aA)\dot{r} = \pm \sqrt{R},$$

(24)

where

$$R \equiv P^2 - R_g(K + m_0^2B),$$

(25)

and

$$P^2 \equiv (EB - aL)^2.$$  

(26)

With the so-called Mino time as $d\lambda = (B - aA)d\tau$, we have,

$$\frac{dr}{d\tau} = \pm \sqrt{R},$$

(27)

from which one can obtain $dr/d\phi$ and integrate it. One can notice in the Fig.1 that orbits are falling faster into the BH when $g$ is smaller which is consistent because $g$ plays the same role than the inverse of the AdS scale and thus produces a repulsive effect.

4. Energy Extraction: Penrose Process

The existence of an ergosphere was first pointed out by Penrose in 1969 [5] which provides a way to extract energy from a rotating BH by sending a test particle to the ergosphere where it decays into two identical particles at a turning point, $\dot{r} = 0$, in its geodesic trajectory: one with
positive energy which escapes the BH and the other with negative energy absorbed by the BH. It is thus important to find the limits on the energy which a particle at a particular location can have. Here, we will consider the possibilities of energy extraction from a BH being used by us and immediately afterward, we will discuss the original Penrose process along with the study of the bounds from Wald inequality.

From eq. (27), with the condition \( \dot{r} = 0 \), we have \( R = 0 \) which leads to,

\[
E = \frac{aB - (a + 2N_g)R_g \pm \sqrt{R_g(r^2 + N_g^2 - v^2)\sqrt{1 + m_0^2\Delta}}}{\Delta},
\]

where

\[
\Delta = \frac{B^2 - R_g(a + 2N_g)^2}{L}\Xi,
\]

or alternatively,

\[
L = \frac{E}{\Xi(a^2 - R_g)}[aB - (a + 2N_g)R_g \pm \sqrt{R_g(r^2 + N_g^2 - v^2)\frac{1}{2}\frac{a^2 - R_g^2}{(E(0))^2(r^2 + N_g^2 - v^2)}}].
\]

The ’\( \mp \)’ signs in eq. (30) corresponds to co-rotating and counter rotating orbits respectively. In order to have positive energy in the Kerr limit, we must retain only the positive sign. We can also see easily that a necessary condition for negative energy is \( L < 0 \) which means only counter rotating particles can possess negative energy.

In an energy extraction process, the incident massive particle with \((E(0), L(0))\) breaks up into two massless particles with energy and angular momentum: \((E(1), L(1))\) for a particle falling into the BH and \((E(2), L(2))\) for the particle leaving the ergosphere. The angular momentum of the incident particle as well as two disintegrated photons can be written as follows:

\[
L(0) = E(0) \frac{aB - (a + 2N_g)R_g + (r^2 + N_g^2 - v^2)\sqrt{R_g(1 + \frac{a^2 - R_g^2}{(E(0))^2(r^2 + N_g^2 - v^2)})^{1/2}}}{\Xi(a^2 - R_g)},
\]

\[
L(1) = E(1) \frac{aB - (a + 2N_g)R_g - (r^2 + N_g^2 - v^2)\sqrt{R_g}}{\Xi(a^2 - R_g)},
\]

\[
L(2) = E(2) \frac{aB - (a + 2N_g)R_g + (r^2 + N_g^2 - v^2)\sqrt{R_g}}{\Xi(a^2 - R_g)}.
\]

We consider total energy and angular momentum as conserved at the point of break which then reads as,

\[
E(0) = E(1) + E(2), \quad (34)
\]

\[
L(0) = L(1) + L(2). \quad (35)
\]

Solving the equations (34) and (35) along with the equations (31) - (33), one may obtain,

\[
E(1) = \frac{E(0)}{2} \left(1 - \sqrt{1 + \frac{a^2 - R_g^2}{(E(0))^2(r^2 + N_g^2 - v^2)}}\right),
\]

\[
E(2) = \frac{E(0)}{2} \left(1 + \sqrt{1 + \frac{a^2 - R_g^2}{(E(0))^2(r^2 + N_g^2 - v^2)}}\right).
\]
Figure 2: Efficiency of the Penrose process as a function of $N_g$ for different values of $g$ (i.e. $g = 0$ in blue, $g = 0.5$ in dashed red and $g = 1$ in dotted green). Here $v = 0$ and the mass is fixed to be the critical mass i.e. $m = m_c$ and $E^{(0)} = 1$.

The amount of energy extracted is given by $-E^{(1)}$ and the efficiency therefore reads,

$$\eta = -\frac{E^{(1)}}{E^{(0)}} = \frac{1}{2} \left( \sqrt{1 + \frac{a^2 - R_g^2}{(E^{(0)})^2(r^2 + N_g^2 - v^2)} - 1} \right).$$  \hspace{1cm} (38)

Further, when the incident particle splits at the horizon $r_E$, one can obtain the maximum efficiency as below,

$$\eta_{\text{max}} = \frac{1}{2} \left[ \sqrt{1 + \frac{a^2}{(E^{(0)})^2(r_E^2 + N_g^2 - v^2)} - 1} \right].$$  \hspace{1cm} (39)

It is interesting to note that this result do not depend explicitly on the parameters $(g, e, m)$. Of course these parameters appear in the equation for the horizon, $r_E$. One can also easily recover the standard results for KBH case with $N_g = v = e = g = 0$ for which $\eta_{\text{max}} = 0.207$.

One can see in Fig. 2 that efficiency is similar to KBH for $g = 0$ and for $N_g < 0.65$ and smaller for $N_g > 0.65$. It may therefore conclude that Kerr-Newman-NUT spacetime is less efficient than its Kerr counterpart in GR as already discussed in [41]. One can also notice that for larger $g$, it has always a better efficiency.

For $v = 0.08$, (see (1)), few values of the efficiency, where we considered $(v, N_g)$ as 2 parameters while the other parameters remain fixed and in Table 2. we have looked to efficiency as a function of $a$, in the very particular case where $N_g = v$. In this case, the formula (39) reduces to the Kerr formula. But because the horizon can have different values compared to Kerr spacetime, the efficiency will be different as marked in Table 1 and Table 2.

| $v$ | $N_g$ | $r_E$ | $\eta$ | $N_g$ | $v$ | $r_+$ | $\eta$ |
|-----|-------|-------|--------|-------|-----|-------|--------|
| 0.08| 0     | 0.883 | 0.049  | 0.3   | 0.4 | 0.842 | 0.591  |
|     | 0.1   | 0.869 | 0.050  | 0.7   | 1.028| 0.595 |
|     | 0.2   | 0.829 | 0.052  | 0.9   | 1.2  | 0.052 |
|     | 0.3   | 0.769 | 0.561  | 1     | 1.305| 0.048 |
|     | 0.4   | 0.709 | 0.576  | 1.1   | 1.42 | 0.042 |

Table 1: The efficiency of the energy extraction via Penrose process in the nonextremal BH case for different values of $N_g$ and $v$ with $a = 0.4$, $e = 0.3$ and $g = 1$.

One may note that the energy extraction by Penrose process can be enhanced as the parameter $g$ increases and decreases with $N_g$. 

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Table 2: The efficiency of the energy extraction via Penrose process in the nonextremal BH case for different values of $g$ and $a$ with $e = 0.3$ and $N_g = v = 0.2$.

However, the question on how much energy could maximally be extracted not from a particle orbiting towards the rotating BH but from the BH itself is still to be answered which is discussed in the next section.

### 4.1. The Maximum Energy

Milking energy from a rotating BH changes its mass $M$ (along with angular momentum $L$), which is identified as irreducible mass ($M_{irr}$) of the BH. The mass of a BH (asymptotic mass) depends on the location of the observer. According to the new horizon mass theorem [45], the horizon mass is always twice of the irreducible mass observed at infinity. The irreducible mass may therefore be defined as,

$$M_{irr} = \sqrt{\frac{1}{2}Mr_+}.$$  \hspace{1cm} (40)

The maximum amount of energy that can therefore be extracted from an extremal $U(1)^2$ supergravity BH is given by,

$$M - M_{irr} = M - \sqrt{\frac{Mr_+}{2}}.$$  \hspace{1cm} (41)

For extreme strongly coupled supergravity BH, it is possible to extract approximately 60.7% of the total energy while it is only 29.3 % for the extreme KBH as mentioned in Table 3. Although the amount of energy extraction increases with the increase in $g$, but $g$ has an upper limit (as discussed earlier).

Table 3: The net extracted energy ($M - M_{irr}$) from the extremal BH for different values of $g$, where $M_{irr}$ is the irreducible mass of the BH.

### 4.2. The Wald Inequality

The Wald inequality [46] further establishes lower bounds on the local speeds of the fragments in order the Penrose process to take place. It explains the origin and limitations of the Penrose process depending on the geometry of a particular spacetime as well as the velocity components of the fragments. Here the detailed derivation of Wald inequality is not presented as it is almost a parallel treatment to that of the KBH case as already developed in [6] and we follow the Ref.
6. Let us imagine, a particle with four-velocity $U^\mu$ and conserved energy $\tilde{E}$ that splits up and emits a fragment with energy $\tilde{E}'$ and four-velocity $u^\mu$. Now, the Wald inequility imposing the limits on $\tilde{E}'$, gives three velocities of the fragment $\vec{v}$ as measured in the rest frame of the incident body becomes,

$$\gamma \tilde{E} - \gamma v(\tilde{E}^2 + g_{tt})^{\frac{1}{2}} \leq \tilde{E}' \leq \gamma \tilde{E} + \gamma v(\tilde{E}^2 + g_{tt})^{\frac{1}{2}},$$  

(42)

where $\gamma$ is the Lorentz factor, i.e. $\gamma = \frac{1}{\sqrt{1-v^2}}$. For $\tilde{E}'$ to be negative, we must have for our spacetime at $\theta = \frac{\pi}{2}$ and on the horizon,

$$|v| > \frac{1}{\sqrt{1 + \tilde{E}r^2_{+}(N_0^2+a)^2 - v^2}}.$$  

(43)

Before any extraction of energy, the fragments must possess relativistic energies as evident from Table 4. We can consider the extreme cases as well. For the extreme Kerr spacetime $|v| > 0.707$, as derived in [6], considering the customary value $\tilde{E} = 1, 2$, it is observed that $|v|$ decreases as the gauge-coupling constant $g$ becomes stronger.

| $g$ | $a_E$ | $r_E$ | $|v|$ |
|-----|-------|-------|-----|
| 0.1 | 0.9638131893 | 0.9711 | 0.8417 |
| 0.5 | 0.8235924578 | 0.6096 | 0.8284 |
| 1.0 | 0.6635002999 | 0.4835 | 0.8251 |

Table 4: Lower bounds on the local speed ($|v|$) of fragments for different values of $g$ and $a$ in the extremal BH case.

5. Superradiance

The Penrose mechanism adheres only about a possibility that allows for a particle to come out of a rotating BH with more energy than its ‘parent particle’. Practically, Penrose processes are not likely to be important in astrophysics because the required conditions can not easily realized. A more general situation could occur where one has to put some medium or some matter field in some background spacetime that provides the arena for superradiance because superradiance requires dissipation. It is thus important to remember that superradiance may occur in vacuum provided the given spacetime is curved. As compared to Penrose process, superradiance has an analogous effect for the waves. A part of the wave is absorbed while it reaches to BH and a part of the wave is reflected. In some cases, the absorbed wave carries negative energy while the reflected wave is amplified. We explore here the superradiant scattering of radiation by an $U(1)^2$ dyonic rotating spacetime. Let us start, with current continuity equation,

$$\Box \Phi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \Phi),$$  

(44)

which is associated with the energy flux vector field $\Phi$ (i.e. a test scalar field). Considering a simple wave mode of frequency $\omega$,

$$\Phi = e^{-i \omega t} e^{i m \phi} \partial(\theta) R(r).$$  

(45)
Here we closely follow the derivations for superradiance as presented in [41] and the expression obtained for the energy flux lost per unit time (power) is given below,

\[ dP = \omega(\omega - m\Omega_H) \left( \frac{B}{B - aA} \right) r_E \int \int (B - aA) r_E \Theta(\theta)^2 \sin^2 \theta d\theta d\phi. \] (46)

\[ P \sim \omega(\omega - m\Omega_H)[r_E^2 + (N_g + a)^2 - v^2] = \text{constant.} \] (47)

where \( \Omega_H = \frac{\omega_0}{B} \) is the angular velocity of the outer horizon.

If \( \omega > m\Omega_H \), \( P \) is positive, then the superradiance is not possible. However, on the other hand, the superradiance occurs if \( \omega \) lies in the range \( 0 < \omega < m\Omega_H \). Within this inequality range, it is evident from equation (47) that a wave mode is amplified indeed by the BH. The angular momentum quantum number (\( m \)) must be non-zero as it has to take away angular momentum from the BH. The value of \( \Omega_H \) is important and for extremal \( U(1)^2 \) supergravity spacetime with constraint \( N_g = v \) the Table 5. clearly indicates that \( \Omega_H \) essentially changes sign as \( g \) becomes larger. So in order to have superradiance \( g \) must not exceed 1 in the Table 5. below, that makes the frequency range smaller. The prefactor \( r_E^2(N_g + a)^2 - v^2 \) increases and decreases with respect to \( N_g \) and \( v \) respectively. The prefactor is related in modifying the magnitude of the amplification.

| \( g \) | \( \Xi \) | \( r_E \) | \( a \) | \( \Omega_H = \frac{\omega_0}{B} \) |
|---|---|---|---|---|
| 0.1 | 0.9136 | 0.9711 | 0.9638131893 | 0.3900 |
| 0.5 | 0.6657 | 0.6906 | 0.8235924578 | 0.3692 |
| 1 | 0.0289 | 0.4835 | 0.6635002999 | 0.0204 |
| 2 | -1.31 | 0.3080 | 0.4600967101 | -1.2285 |

Table 5: The angular velocity of the extremal BH at the horizon for different values of \( g \) and \( a \).

6. CM Energy and Particle Collision

In this section, the CM energy of the two particles colliding in the equatorial plane of \( U(1)^2 \) dyonic rotating BH is investigated. We further assume that the particles with some rest mass \( m_0 \) but having different energies coming from the infinity has \( E_1 = E_2 = m_0 \) where they were initially at rest. Finally, the particles are approaching towards the event horizon of the BH (mentioned above) with different angular momenta \( L_1 \) and \( L_2 \). The general form of CM energy of two colliding particles \( i(i = 1, 2) \) [28] is given by,

\[ \frac{E_{CM}^2}{2m_0^2} = 1 - g_{\mu\nu}u^{a}_{(1)}u^{b}_{(2)}, \] (48)

where, \( u^{a}_{(1)} \) and \( u^{b}_{(2)} \) are the four velocities of two particles respectively. For the spacetime considered here the above formula reads as,

\[ \frac{E_{CM}^2}{2m_0^2} = \frac{1}{R_g(B - aA)}[R_g(B - aA) + (B^2 - R_gA^2) - (R_g - a^2)\Xi^2L_1L_2
- (aB - R_gA)\Xi(L_1 + L_2) - \sqrt{(B - \Xi aL_1)^2 - R_g(A - \Xi L_1)^2 - R_gB}
\sqrt{(B - \Xi aL_2)^2 - R_g(A - \Xi L_2)^2 - R_gB}], \] (49)
which is similar to the KBH in GR [28] for \( g = 0, N_g = 0, e = 0 \) and \( v = 0 \).

Since CM is an invariant scalar, it serves as an observable, therefore independent of coordinate choice. This ensures the validity of the formula given in equation (49) in special as well as GR.

### 6.1. Near-Horizon Collision in Extremal Spacetime

We now analyze the CM energy formulated in equation (49) of two colliding particles as \( r \to r_E \) for the extremal gauged supergravity BH. With the limit \( r \to r_E \), the equation (49) becomes indeterminate. By applying L’Hôpital’s rule, one can easily calculate the limiting value of \( E_{CM} \) (as \( r \to r_E \)) with critical angular momentum. The numerical data for the critical values of angular momenta is listed in Table 6.

| \( g \) | \( r_E \) | \( a \) | \( L_c = \frac{R(r_E)}{\alpha \Xi} \) |
|-----|-------|------|-----------------|
| 0.1 | 0.9711 | 0.9638131893 | 2.5640 |
| 1   | 0.4835 | 0.6635002999 | 48.990 |
| 2.44406907 | 0.2660 | 0.4 | -0.5252 |

Table 6: Numerical estimation of critical angular momentum (\( L_c \)) for an extremal BH for different values of \( g \) and \( a \).

From Fig. 3 it can be easily observed that when the angular momentum of any one of the colliding particles gets the critical value (\( L_1 = L_c \) or, \( L_2 = L_c \)), the CM energy diverges then and there, but it remains finite if \( L_1 \neq L_c \) or, \( L_2 \neq L_c \). The same argument can also be verified with the help of Fig.4 where the variation of \( E_{CM} \) with \( a \) has been taken into consideration. Most importantly to have an unlimited \( E_{CM} \) one can bring out the effect of gauge coupling constant \( g \) with the help of the data as in Table 6. \( E_{CM} \) blows off at \( r = r_E \) when \( g = g_E \) as suggested in Fig.5.

The unbounded nature of \( E_{CM} \) reveals the highly energetic particle collisions near the event horizon of the extremal BH. This in turn implies the fact that extremal \( U(1)^2 \) dyonic rotating BH can act as a particle accelerator at very high energy scales.

![Figure 3: The variation of the CM energy \( E_{CM} \) with \( r \) for extremal BH where the vertical black dashed line corresponds to the degenerate horizon.](image-url)
6.2. Near-Horizon Collision in Nonextremal Spacetime

A nonextremal BH satisfies the condition $r_+ \neq r_-$ where $r_+$ and $r_-$ denote the outer and the inner horizon respectively. Both the denominator and the numerator in equation (49) will become zero as $r \to r_+$. At this point, one has to use L’Hôpital’s rule to eliminate this uncertainty. The list of maximum/minimum angular momenta for this case are presented in Table 7. Using this data, one can study the behaviour of $E_{CM}$ as given by equation (49). For $r \to r_+$, the nonextremal gauged supergravity BH, Fig. 6 illustrates the variation of $E_{CM}$ with $r$, where other parameters are held fixed. It is evident from this that $E_{CM}$ is not divergent for the nonextremal BH case.

Now, the BSW (Baados, Silk and West) effect which is dependent on $g$ is estimated numerically from the data given in Table 8. Initially, it seems that $E_{CM}$ decreases with an increase in the value of $g$ and comes to have a lower bound as illustrated in Fig. 7. But $E_{CM}$ approaches an upper bound if one moves slightly beyond $g = 1.2$ as highlighted in the embedded diagram in Fig. 7.
Table 7: Numerical evaluation of maximum and minimum values of angular momentum for the nonextremal BH spacetime for two different values of $g$.

| $g = 0.1$ | $g = 0.5$ |
|---------|---------|
| $a$ | $r_+$ | $r_-$ | $L_{\text{max}}$ | $L_{\text{min}}$ | $a$ | $r_+$ | $r_-$ | $L_{\text{max}}$ | $L_{\text{min}}$ |
| 0.1 | 1.906 | 0.027 | 36.86 | -0.5077 | 0.1 | 0.94 | 0.027 | 10.25 | -0.557 |
| 0.3 | 1.859 | 0.08 | 12.59 | -0.7434 | 0.3 | 0.893 | 0.077 | 5.01 | -1.07 |
| 0.5 | 1.532 | 0.167 | 7.27 | -0.962 | 0.5 | 0.779 | 0.182 | 6.03 | -2.76 |

Table 8: The maximum and minimum values of the angular momentum for the nonextremal BH for different values of $g$ with $a = a_E = 0.4600967101$.

| $g$ | $r_+$ | $r_-$ | $L_{\text{max}}$ | $L_{\text{min}}$ |
|-----|-------|-------|----------------|----------------|
| 0   | 1.890 | 0.107 | 8.623 | 0.8849 |
| 0.1 | 1.811 | 0.107 | 8.028 | 0.8894 |
| 0.3 | 1.491 | 0.107 | 5.953 | 0.9257 |
| 0.5 | 1.234 | 0.110 | 4.73  | 1.007  |

Figure 6: The variation of $E_{CM}$ with $g$ for the nonextremal BH case. The vertical black dashed line corresponds to the event horizon.

7. Summary and Conclusions

The spacetime investigated in this paper are the spacetime of a rotating BH in $N = 2$, $U(1)^2$ gauged supergravity model with dyonic charges. To conclude, we provide below, in a systematic way, a summary of the results obtained.

(i) The structure of the horizon for this nontrivial spacetime has been reviewed briefly in order to find the critical value of the mass which can be translated to critical (or extreme) values of parameters $a = a_E$ and $g = g_E$ signifying an extremal BH with degenerate horizon.

(ii) The equations of motion of the energy extraction processes are derived and have been examined accordingly. The most interesting results are observed when $g$ becomes stronger in extremal BH spacetime constrained with $N_g = v$. The Penrose process is found to be more efficient ($\sim 39\%$) than that is for the KBH in this scenario. Moreover, by the Penrose process, it is found that one can extract about 60% of the initial mass from an
extremal BH.

(iii) In superradiant scattering for $\omega_m \Omega_H < 0$, the flux of the energy momentum going through the outer horizon turns out to be negative but the flux is positive in infinity. It therefore indeed extracts energy from the BH. Next, the influence of the gauge coupling constant $g$ on $\Omega_H$ and the angular velocity of the BH at outer horizon have been inquired thoroughly. It is strikingly noticed that no superradiance occurs for strong enough coupling.

(iv) In view of original BSW mechanism fitted for the spacetime (1), we estimated $E_{CM}$ for a pair of colliding particles moving near the horizon for both extremal and nonextremal BH cases. In case of an extremal BH, $E_{CM}$ blows up under some restrictions on the angular momentum. This unleashed nature of $E_{CM}$ can be envisaged to open up a new window to explain physics at the Planck energy scale. On the other hand, for nonextremal BH case $E_{CM}$ found to remain finite with a finite upper bound.

(v) We have seen Kerr black hole can act as the accelerators of neutral particles. But here in this article we are dealing with dyonic black holes which incorporate the magnetic field as well and can therefore act as the accelerators of charged particle to unboundedly high energy.

In addition, the supergravity theory, based on the particle symmetry, includes a collection of fields that together have a long-range gravity force with a superpartner. So it is natural to believe gravitational particle acceleration takes significantly extremes to be robust for the black holes in supergravity scenario than that is for the Kerr and other generalized Kerr black holes.

From such a consideration, rotating dyonic black hole in $N = 2$ gauged supergravity can be regarded to be more influential super collider ever which further deepen the understanding of new physics with astrophysical applications e.g. ultra-high energetic dark.
matter collision at the galactic center, some indirectly observable signatures on the spectra of cosmic rays, neutrinos and gravitational waves in a more obvious way.

In continuing this work, a more detailed and general analysis for a BH in $N = 8$, $SO(8)$ gauged supergravity theory with having a maximal number of supercharges is postponed to future work.

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