Could the near–threshold $XYZ$ states be simply kinematic effects?

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We demonstrate that the spectacular structures discovered recently in various experiments and named as $X$, $Y$ and $Z$ states cannot be purely kinematic effects. Their existence necessarily calls for nearby poles in the $S$–matrix and they therefore qualify as states.

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In recent years various narrow (widths from well below 100 MeV down to values even below 1 MeV) peaks were discovered both in the charmonium as well as in the bottomonium mass range that do not fit into the so far very successful quark model. For instance, the most prominent ones include $X(3872)$ [1], $Z_c(3900)$ [2–4], $Z_c(4020)$ [5,6], $Z_b(10610)$ and $Z_b(10650)$ [7], which are located close to $DD^*$, $D\bar{D}^*$, $D^*\bar{D}^*$, $BB^*$ and $B^*\bar{B}^*$ thresholds in relative $S$–waves, respectively. Apart from other interpretations, such as hadro–quarkonia [10,11], hybrids [12,14], and tetraquarks [15,16] (for recent reviews we refer to Refs. [17,18]), due to their proximity to the thresholds these five states were proposed to be of a molecular nature [19–37]. As an alternative explanation various groups conclude from the mentioned proximity of the states to the thresholds that the structures are simply kinematic effects [38–45] that necessarily occur near every $S$–wave threshold. Especially, it has been claimed that the structures are not related to a pole in the $S$–matrix and therefore should not be interpreted as states.

In this letter we show that the latter statement is based on calculations performed within an inconsistent formalism. In particular, we demonstrate that, while there is always a cusp at the opening of an $S$–wave threshold, in order to produce peaks as pronounced and narrow as observed in experiment non–perturbative interactions amongst the heavy mesons are necessary, and as a consequence, there is to be a near–by pole. Or, formulated the other way around: if one assumes the two–particle interactions to be perturbative, as it is implicitly done in Refs. [38–45], the cusp should not appear as a prominent narrow peak. This statement is probably best illustrated by the famous $K^{\pm}\to\pi^+\pi^0\pi^0$ data [19]: the cusp that appears in the $\pi^0\pi^0$ invariant mass distribution at the $\pi^+\pi^-$ threshold is a very moderate kink, since the $\pi\pi$ interactions are sufficiently weak to allow for a perturbative treatment (for a comprehensive theoretical framework and related references we refer to Ref. [17]).

To be concrete, in this paper we demonstrate our argument on the example of an analysis of the existing data on the $Z_c(3900)$, but it should be clear that the reasoning as such is general and applies to all structures observed very near $S$–wave thresholds such as those above–mentioned $XYZ$ states. To illustrate our point, we here do not aim for field theoretical rigor but use a very simple separable interaction for all vertices accompanied by loops regularized with a Gaussian regulator. This regulator will at the same time control the drop–off of the amplitudes as will be discussed below. Accordingly, we write for the Lagrangian that produces the tree–level vertices (here and in what follows we generically write $DD^*$ for the proper

\[ \text{FIG. 1: The tree–level, one–loop and two–loop Feynman diagrams for } Y(4260) \to \pi DD^*. \]
linear combination of $D\bar{D}^*$ and $\bar{D}D^*$

$$L_1 = g_Y \pi (D\bar{D}^*_\mu)^\dagger Y^\mu + \frac{C}{2} (D\bar{D}^*)^\dagger (D\bar{D}^*)$$

$$+ g_{\psi Y} \psi \mu \pi \pi Y^\mu + g_{\psi Y} \psi \mu \pi D\bar{D}^*_\mu + ... , \quad (1)$$

where $Y, D, D^*, \pi$ and $\psi$ denote the fields for the $Y(4260), D, D^*, \pi$ and $J/\psi$, respectively. The dots indicate terms not needed for this study like the one where the $Y$-field is created. All fields but the pion field are non-relativistic and accordingly the couplings $g_Y$ and $g_{\psi Y}$ have dimension GeV$^{-3/2}$, $g_{\psi Y}$ has dimension GeV$^{-1}$, while $C$ has dimension GeV$^{-2}$. The loops are regularized with the cutoff function $f_\Lambda(\vec{p}^2)$, which for convenience we choose as

$$f_\Lambda(\vec{p}^2) = \exp \left( -2\vec{p}^2/\Lambda^2 \right) , \quad (2)$$

where here and below $\vec{p}$ denotes the three-momentum of the $D$-meson in the center-of-mass frame of the $D\bar{D}^*$ system. Therefore the loop function reads

$$G_\Lambda(E) = \int \frac{d^3q}{(2\pi)^3} \frac{f_\Lambda(q^2)}{E - m_1 - m_2 - q^2/(2\mu)} , \quad (3)$$

where $m_{1,2}$ denote the masses of the charmed mesons, $\mu$ is the reduced mass and $E$ is the total energy. With the regulator specified in Eq. (2), the analytic expression for the loop function for $E \geq m_1 + m_2$ is given by

$$G_\Lambda(E) = \frac{\mu \Lambda}{(2\pi)^3} + \frac{\mu k}{2\pi} e^{-2k^2/\Lambda^2} \left[ \text{erfi} \left( \frac{\sqrt{2k}}{\Lambda} \right) - i \right] \quad (4)$$

where $k = \sqrt{2\mu(E - m_1 - m_2)}$, and

$$\text{erfi}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{t^2} dt \quad (5)$$

is the imaginary error function.

With the ingredients of the model fixed it is straightforward to derive the explicit expressions for the transition matrix elements. Within this model the $Y(4260) \rightarrow \pi D\bar{D}^*$ amplitude reads to one-loop order (cf. the diagrams of Fig. 1(a)+(b))

$$g_Y \left[ 1 - G_\Lambda(E)C \right] . \quad (6)$$

The analogous result for the $Y(4260) \rightarrow \pi \pi J/\psi$ amplitude is (cf. the diagrams of Fig. 2(a)+(b))

$$g_{\psi Y} - g_Y G_\Lambda(E)g_{\psi} . \quad (7)$$

We now proceed as follows: We first confirm the claims of Refs. [39, 43, 45], namely, that the data available for both $Y(4260) \rightarrow \pi D\bar{D}^*$ as well as $Y(4260) \rightarrow \pi \pi J/\psi$ can at least qualitatively be described by a sum of the tree-level and one-loop diagrams shown in Fig. 1(a)+(b) and Fig. 2(a)+(b), respectively. Note that diagram (b) in either Fig. 1 or 2 explicitly contains the above mentioned cusp. It was this observation that lead the authors of Refs. [39, 43, 45] to interpret the near-threshold structure for values below this invariant mass, see Fig. 3, as purely kinematical effect. To fix the parameters we first fix $g_Y, \Lambda$ and $C$ by a fit to the $D\bar{D}^*$ spectrum. The fit result is shown by the solid line in Fig. 3, corresponding to the number of events, and a factor of $10^{-3}$ (1000). Since this fitting procedure leads us to the same structure for values below this invariant mass, see Fig. 3.

Next we keep $g_Y$ and $\Lambda$ fixed and fit $g_{\psi}$ and $g_{\psi Y}$ to the $J/\psi \pi$ spectrum. The best fit gives $g_{\psi Y} = 46.4$ GeV$^{-3/2}$ and $g_{\psi} = 0.44$ GeV$^{-3/2}$ which are also not normalized to the physical values due to fitting to the event numbers. The result of this fit is shown as the solid line in Fig. 3. In this work we only aim at a qualitative description of the data. It should be mentioned that we can get a perfect fit of the $J/\psi \pi$ spectrum, if we also fit $\Lambda$, but then we have to compromise on the fit quality for the $D\bar{D}^*$ channel.

1. Note that the cut-off function $f_\Lambda(\vec{p}^2)$ is needed in phenomenological studies not only to regularize the real parts of the loops.
As mentioned above, the intrinsic assumption of the approaches outlined in Refs. [39, 43, 45] is that the interactions are perturbative, and consequently, the amplitude is properly represented by the one-loop result. With the parameters fixed we can now calculate the amplitudes to two-loop order from

\[ g_Y \left[ 1 - G_\Lambda(E)C + (G_\Lambda(E)C)^2 \right], \]  

(9)

for the \( \pi D \bar{D}^* \) channel (cf. Fig. 1 (a)+(b)+(c)) and

\[ g_{\psi Y} - g_Y G_\Lambda(E)g_\psi + g_\psi G_\Lambda(E)CG_\Lambda(E)g_\psi \]  

(10)

for the \( \pi \pi J/\psi \) channel (cf. Fig. 2 (a)+(b)+(c)). The results are shown as the dashed lines in Figs. 3 and 4 respectively. As one can see, in both cases the two-loop results are shown as the dashed lines in Figs. 3 and 4, respectively. The dot–dashed line shows the one-loop result with the strength of the rescattering requested to be small to justify a perturbative treatment as described in the text.

In fact, when we sum all loops in the \( \bar{D}D^* \) channel using the parameters of Eqs. [9, 39, 43], the series produces a bound state pole right below threshold. This means the following for the results of Refs. [39, 43, 45]: if one wants to fit the available data for the near-threshold \( Z_{\psi}(3900) \) states within a perturbative approach, the presence of a pronounced near-threshold structure calls for such a large coupling constant that the use of a perturbative approach is not justified. This demonstrates explicitly that the approach used in Refs. [39, 43, 45] is intrinsically inconsistent.

This argument also works in the other direction: we may constrain the coupling \( C \) for \( D\bar{D}^* \) elastic scattering to a value where it might still be justified to treat \( D\bar{D}^* \) scattering perturbative, e.g. one may require in the full kinematic regime \( |G_\Lambda(E)C| \ll 1 \). Since \( G_\Lambda(E) \) is maximal for \( E = 2M \), we may demand \( |CG_\Lambda(m_1 + m_2)| = a \) with \( a \ll 1 \). For \( \Lambda \) as given in Eq. (8) and \( a = 1/2 \) we can again calculate the amplitude to one–loop order. The resulting \( D\bar{D}^* \) spectrum is shown by the dot–dashed line in Fig. 3. Clearly, such a small coupling is not able to produce the pronounced structure in the data.

In the calculation described above we used a Gaussian form factor to regularize the loop. We checked that a different regulator leads to qualitatively similar results. Especially the conclusions stay unchanged. In fact, any other form factor which is commonly used drops off more slowly for higher momenta. As a result an even larger value of \( |CG_\Lambda(m_1 + m_2)| \) will be connected to a narrow near–threshold structure. From this point of view, the use of a Gaussian form factor as employed above already leads to the most conservative estimate of the higher loop effects.

To distinguish an \( S \)–matrix pole from a simple cusp effect it is necessary to fix the strength of the production vertex and of the meson–meson rescattering separately. This is possible only for the elastic channel, as can be clearly seen from comparing Eqs. (6) and (7): the term \( CG_\Lambda(E) \) which controls the elastic interaction strength can be fixed from the peak since it interferes with 1, while the inelastic coupling strength \( g_\psi \) in Eq. (7) always appears in a product with \( g_Y \).

Although in this work all calculations are tuned to the

![Figure 3: Results for the \( D\bar{D}^* \) invariant mass distribution in \( Y(4260) \rightarrow \pi D\bar{D}^* \). The data are from Ref. [2] and the results from the tree level, full one-loop and full two-loop calculations are shown by the dotted, solid and dashed curves, respectively. The dot–dashed line shows the one-loop result with the strength of the rescattering requested to be small to justify a perturbative treatment as described in the text.](image1)

![Figure 4: Results for the \( \pi J/\psi \) invariant mass distribution in \( Y(4260) \rightarrow \pi \pi J/\psi \). The data are from Ref. [2] and the results from the tree level, full one-loop and full two-loop calculations are shown by the dotted, solid and dashed curves, respectively, with the cutoff as well as \( g_Y \) from the fit to the \( D\bar{D}^* \) spectrum.](image2)
production of $Z_c(3900)$ seen in $Y(4260) \rightarrow \pi Z_c(3900)$ it should be understood that the arguments are indeed very general: any consistent treatment of the spectacular very near-threshold structures, namely some of those $X\ Y\ Z$ states, necessarily needs the inclusion of a near-by pole, which was done, e.g., in Refs. [14, 19, 37, 48]. For each individual state a detailed high-quality fit to the data is necessary to decide if this pole is located on the first sheet (bound state) or on the second sheet (virtual state or resonance). It also requires additional research to decide on the origin of that pole, which might, e.g., come from short-ranged four-quark interactions or from meson–meson interactions. All we can conclude from the results of this paper is that there has to be a near-threshold pole.

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