Phased-MIMO Radar : Angular resolution

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Abstract. Phased-MIMO is very suitable for multi-target detection such as radars for bird migration, radars for autonomous cars, fish radars and others. The problem is the ability of radar to distinguish the first target, the second target and the next target. This study reports the ability of Phased-MIMO radars to be able to distinguish two or more objects that are close known as angular resolution of radar.

Keywords : Angular, Radar, Resolution

1. Introduction
Phased-MIMO radar is a combination of two types of radars, namely Phased-array radar which has an excess coherent gain and MIMO radar that has advantages diversity. Waveform diversity can improve the number of targets and coherent gain can improve range of target that can be detected by radar [1] [2] [3] [4].
The structure of Phased-MIMO radar is arrange antennas to several sub-arrays. All sub-array consists of the same number of antennas, some of antenna which are used together with other sub-arrays, thus forming overlapping sub-arrays. The antennas in one sub-array transmit the same signals so that they produce a coherent gain. Different sub-arrays emit mutually orthogonal signals to form diversity [5] [6].
The Phased-MIMO radar is suitable for multi-target [1] detection where the number of targets and their distances can vary e.g radar for bird migration, radar for autonomous car, fish radar for fisherman and so on. When less number of target, and longer target's distance, Phased-MIMO arrange the number of sub-arrays is less therefore that the number of antenna elements in each sub-array is more in number. As a result, Phased-MIMO radar generates greater coherent gain. Conversely, if more number of targets, and the target distance is closer then Phased-MIMO will form more number of sub-arrays, thus the resulting better diversity.
This paper reports angular resolution of Phased-MIMO radar. Angular resolution is the angle difference in degrees, between two adjacent targets that are capable detected by radar. Phased-MIMO radar can increase angular resolution by improving the number of sub-arrays or by increasing the number of antenna elements.
Phased-MIMO radar is described in Chapter 2. Chapter 3 discuss the Least square estimation of Phased-MIMO radar to evaluate Angular resolution. The results of the numerical examples are reported and discussed in chapter 4. Chapter 5 contains conclusions from this study.

2. Phased-Mimo Radar
2.1. Antenna Configuration
The structure of Phased-MIMO radar is arrange antennas to several sub-arrays. All sub-array consists of the same number of antennas, some of which are shared with the other sub-arrays, thus forming
overlapping sub-arrays. Assuming a radar system with co-located antennas has \( M_t \) antennas on the transmit array and \( M_r \) antennas on the receive array. Spaces between transmit and receive antenna elements, respectively \( dt \) and \( dr \). Transmit arrays and receive arrays have been partitioned into \( T \) and \( R \) subarray overlapping one another. The number of antenna elements in each subarray is \( M - K + 1 \). The first sub-array, the second sub-array and subsequent sub-arrays generate orthogonal waveforms known as diversity. While each element in one sub-array generates the same waveform signal, so a coherent gain is achieved.

As an illustration in Figure 1, Phased-MIMO radar uses 4 antennas element \( (M = 4) \) with the number of sub-arrays equal to two \( (K = 2) \). Thus the number of elements in one sub-array is three \( (M - K + 1) \) [7]. The waveform generator of phased-MIMO equal to number of sub-arrays \( K \). Each sub-array will produce orthogonal waveform as a function of MIMO radar. In one sub-array: elements 1, 2 and 3 will transmit same waveform to produce coherent processing which is a function of the phased-array radar. The recipients used are non-coherent who use matched-filters on each recipient element, thus will produce the four same output.

![Figure 1](image)

Figure 1 Antenna configuration of phased-MIMO radars with \( M = 4, K = 2 \) with non-coherent receiver

2.2. Phased-MIMO radar Signal [1]

The Phased-MIMO radar transmit baseband signal on \( K \) sub-arrays can be expressed as:

\[
\mathbf{s}_k(t) = \frac{\sqrt{M}}{K} \mathbf{\Phi}_k(t) \mathbf{\bar{w}}_k^* \quad k = 1, \ldots, K \quad (1)
\]

where \( M \) is the number of Tx antennas, \( K \) is the number of element antenna on every each sub-arrays, \((.)^*\) is the complex conjugate operator, \( \mathbf{w}_k \) is the \( M \)-element unit-norm complex weight vector for the \( k \)-th of \( M - K + 1 \) sub-arrays in the transmit array. The beamforming weight equals to the active elements in the \( k \)-th sub-array, thus the number of non-zero elements in \( \mathbf{w}_m \) is equal to \( M - K + 1 \) and the others as many as \( K + 1 \) are zero [10]. In (1), \( M/K \) is the coefficient of power normalization therefore that the energy sent in one pulse is \( M \).

Phased-MIMO radar transmitted signal matrix \( \mathbf{S}(t) \) of \( K \times M \) dimension has the orthogonal each other, so that:

\[
\int_{T_p} \mathbf{S}(t)\mathbf{S}^H(t) \, dt = \mathbf{I}_{MM} \quad (2)
\]

with \( T_p \) being the pulse repetition interval (PRI), \((.)^H\) matrix transpose hermitian operation and \( \mathbf{I}_{MM} \) the \( M \times M \) identity matrix.

Assumed that the total energy transmitted by \( M \) elements in one PRI is equal to \( M \), then the energy of signal \( \mathbf{s}_k(t) \) on the \( k \)-th sub-array in one PRI is
Signal energy of the k-th sub-array from the equation (3) is $M/K$, thus the signal energy of one element in one sub-array is $M/(K \times (M-K+1))$.

The reflected signal with target coefficient reflection $\sigma(\theta)$ at the angle $\theta$, at time $t$ in the far field can be expressed by:

$$r(t, \theta) = \frac{\sqrt{M}}{\sqrt{K}} \sigma(\theta) \sum_{k=1}^{K} \bar{\mathbf{w}}_k^H \mathbf{\tilde{a}}_k(\theta) \mathbf{\varphi}_k(t) = \frac{\sqrt{M}}{\sqrt{K}} \sigma(\theta) \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{a}_k(\theta)e^{-2\pi j \tau_k(\theta)} \mathbf{\varphi}_k(t)$$  (4)

where $\sigma(\theta)$ is the target reflection coefficient at angle $\theta$, $\mathbf{a}_k(\theta)$ is $M \times 1$ steering vector of in the $k$-th sub-array of transmitter antennas, $\tau_k(\theta) = kd_{M} \sin(\frac{\theta}{c})$ is the time needed from transmitted signal until arrived at the first antenna element of the $k$-th receiver sub-array, $d_{M}$ is the distance between antenna elements and $c$ is the speed of light $3 \times 10^8$ m/s.

If the vector of $K \times 1$ coherent processing on the transmit antenna is

$$\mathbf{c}(\theta) = [\mathbf{w}_1^H \mathbf{a}_1(\theta), ..., \mathbf{w}_K^H \mathbf{a}_K(\theta)]^T$$  (5)

where

$$\mathbf{a}(\theta) = [e^{j2\pi d \sin(\theta)/\lambda}, e^{j2\pi d \sin(\theta)/\lambda}, ..., e^{j2\pi (M-K+1)d \sin(\theta)/\lambda}]^T$$  (6)

and $K \times 1$ vector of waveform diversity $[\mathbf{d}]$

$$\mathbf{d}(\theta) = [e^{-j2\pi \tau_1(\theta)}, ..., e^{-j2\pi \tau_K(\theta)}]^T$$  (7)

Then, the reflected signal can be simplified to

$$r(t, \theta) = \frac{\sqrt{M}}{\sqrt{K}} \sigma(\theta) (\mathbf{c}(\theta) \odot \mathbf{d}(\theta))^T \mathbf{\varphi}_k(t)$$  (8)

where $\odot$ is the Hadamard product operator.

If the $p$-th target is in the direction of $\theta_p$ with $p = 1, 2, ..., P$, then the vector of received complex signal is

$$\mathbf{y}_{PMIMO}(n) = r(t, \theta_p) \mathbf{b}(\theta_p) + \mathbf{z}(t)$$  (9)

$$\mathbf{y}_{PMIMO}(n) = \frac{\sqrt{M}}{\sqrt{K}} \sum_{p=1}^{P} \sigma(\theta_p) (\mathbf{c}(\theta_p) \odot \mathbf{d}(\theta_p))^T \mathbf{\varphi}(n) \mathbf{b}(\theta_p) + \mathbf{z}(t)$$

$$= \frac{\sqrt{M}}{\sqrt{K}} \sum_{p=1}^{P} \sigma(\theta_p) \mathbf{b}(\theta_p) (\mathbf{c}(\theta_p) \odot \mathbf{d}(\theta_p))^T \mathbf{\varphi}(n) + \mathbf{z}(t)$$  (10)

where $\sigma_p(\theta_p)$ is reflection coefficient of the $p$-th target and $\mathbf{z}$ is the vector of noise including interference on radar receiver. Assumed that noise and interference are uncorrelated with the signal $\varphi_k(t)$.

2.3. Least-squares Parameter estimation

Least-Square (LS) estimation was chosen because it is an easy detection method to estimate target parameters. LS estimation of the phased-MIMO radar can be obtained with derive reflected signal from equation (10):

$$\mathbf{y}_{PMIMO}(n) = \frac{\sqrt{M}}{\sqrt{K}} \sum_{p=1}^{P} \sigma(\theta_p) \mathbf{b}(\theta_p) (\mathbf{c}(\theta_p) \odot \mathbf{d}(\theta_p))^T \mathbf{\varphi}(n)$$  (11)

By multiplying right and left sides of (11) with the corresponding hermitians, we get:
Estimation of LS spectrum from (12) can be simplified into

\[
\hat{\sigma}(\theta)_{\text{PMIMO}} = \frac{\sqrt{\frac{K}{M}} \sum_{p=1}^{P} \sigma(\theta_p) \mathbf{b}^H(\theta_p) \mathbf{b}(\theta_p) (c(\theta_p) \odot d(\theta_p))^T \Phi(n) }{ (c(\theta_p) \odot d(\theta_p))^T \Phi(n) }^H \tag{13}
\]

where (\cdot)^* indicates the complex conjugate.

By following the normalization of MIMO [5], then:

\[
\hat{\sigma}(\theta)_{\text{PMIMO}} = \frac{\sqrt{\frac{K}{M}} \sum_{n=1}^{N} \mathbf{b}^H(\theta) \mathbf{y}_{\text{PMIMO}}(n) \Phi^H(n) (c(\theta) \odot d(\theta))^T }{ (c(\theta) \odot d(\theta))^T \Phi(n) \Phi^H(n) (c(\theta) \odot d(\theta))^T } \tag{14}
\]

### 2.3.1 Angular Resolution

Radar angular resolution is the minimum distance between two equally large targets at the same range which radar is able to distinguish and separate to each other [8].

In figure 2, the first and second targets located at the same or closest range in km; and closest range in degrees. On a radar system, the angle resolution shows distance in degree between the first target and the second target that can be detected as two objects. If the distance in degrees between the two targets is too close, then the radar cannot distinguish the first target and the second target; in other words the two targets only detected as one target. However, if the distance of the two targets in degrees; can still be detected by the radar, the first target and second target can be recognized as different objects. This is the meaning Angular resolution of radar.

![Figure 2 Illustration of Angular Resolution](image)

### 2.3.2 Hadamard Matrices [9]

A hadamard matrix with order n is an H matrix which contains ±1 which fulfills

\[
HH^T = nI_n
\]

The hadamard matrix was introduced by Sylvester in 1867, introducing mutually orthogonal matrices
The example of hadamard code with order $n = 1, 2$ and $4$ are:

$$\begin{bmatrix}
\mathbf{H} & \mathbf{H} \\
\mathbf{H} & -\mathbf{H}
\end{bmatrix}$$

The hadamard matrix is used to generate the low pass equivalent signal on the Phased-MIMO radar antenna $[10]$. Row of hadamard matrix indicate the signal emitted by the antenna sub-array. The first row of the hadamard matrix is the signal emitted by the first sub-array, the second row for the second sub-array and the $n$th row for the $n$th sub array.

### 3. Numerical Example

The To prove the angular resolution of Phased-MIMO radar, equation (14) is used. The two close targets are separated by distance in degrees; began from $1^\circ, 2^\circ, 3^\circ$, ... until Phased-MIMO radars can distinguish between these two targets. The detection value of Complex amplitude expressed as a target is more than $0.5$ or $-3$dB in dB. By increasing the number of sub-arrays it is expected to increasing angular resolution. The number of sub-arrays is determined based on the value that makes it possible to generate hadamard signals. Furthermore, increasing the number of antenna elements is also carried out to prove the increase in angular resolution by using equation (14). The increment of elements starts from $M = 10$, $M = 20$ and $M = 100$. It is expected from this variable that the number of elements and the number of sub-arrays is proportional to the angular resolution.

First case on figure 3, consider a $10 \times 10$ Phased-MIMO radar system with ULA structure, i.e., $M_t = M_r = 10$ and sub-arrays increases from $K=2$, $K=4$ and $K=8$; then the number of elements ($K=2$) on sub-array ($M-K+1$) equal 9, $K=4$ equal 7 and $K=8$ equal 3. The Hadamard code generated as a transmitted signal based on the number of subarray $[10][9].$

When the sub-array $K=2$, the angular resolution of Phased-MIMO radar is 8 $^\circ$ where the first target located at -20 $^\circ$ and the second target located at -28 $^\circ$. If the distance in degrees; less than 8 $^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

When the sub-array $K=4$, the angular resolution of Phased-MIMO radar is 6 $^\circ$ where the first target located at 0 $^\circ$ and the second target located at 6 $^\circ$. If the distance in degrees; less than 6 $^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

When the sub-array $K=8$, the angular resolution of Phased-MIMO radar is 6 $^\circ$ equal to $K=4$; where the first target located at 0 $^\circ$ and the second target located at 6 $^\circ$. If the distance in degrees; less than 6 $^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

From this experiment, can be concluded that by increasing the number of sub-arrays of Phased-MIMO radars be able increase the angular resolution. However when $K = 4$ and $K=8$, angular resolution is not increase.
From figure 3, it can be observed that increasing the number of sub-arrays on the Phased-MIMO radar will decrease the sidelobe of detection results. This means that the greater the number of sub-arrays, the smaller the sidelobe of detection results. The best RCS value which approaches one; obtained when $K = 4$ or when the number of sub-arrays $K$ approaches half the number of antenna elements $M$.

Second case on figure 4, consider a $20 \times 20$ Phased-MIMO radar system with ULA structure, i.e., $M_t = M_r = 20$ and sub-arrays increases from $K=2$, $K=8$ and $K=12$; then the number of elements $(K=2)$ per sub-array $(M-K+1)$ equal 19, $K=8$ equal 13 and $K=12$ equal 9.

When the sub-array $K=2$, the angular resolution of Phased-MIMO radar is $5^\circ$ where the first target located at $-20^\circ$ and the second target located at $-25^\circ$. If the distance in degrees; less than $8^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

When the sub-array $K=8$, the angular resolution of Phased-MIMO radar is $5^\circ$ equal to $K=4$; where the first target located at $0^\circ$ and the second target located at $5^\circ$. If the distance in degrees; less than $5^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

When the sub-array $K=12$, the angular resolution of Phased-MIMO radar is $4^\circ$ where the first target located at $0^\circ$ and the second target located at $6^\circ$. If the distance in degrees; less than $4^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

From this experiment, can be concluded that by increasing the number of sub-arrays of Phased-MIMO radars be able increase the angular resolution and confirm the hypothesis in the first case. However when $K = 2$ and K=8, angular resolution is not increase.

From figure 4, it can be observed that increasing the number of sub-arrays on the Phased-MIMO radar will decrease the sidelobe of detection results. This means that the greater the number of sub-arrays, the smaller the sidelobe of detection results. The best RCS value which approaches one; obtained when $K = 4$ or when the number of sub-arrays $K$ approaches half the number of antenna elements $M$. This result strengthens the hypothesis in the first case.

Third case; Phased-MIMO radar uses the same number of sub-arrays, which are 4 sub-arrays. Changes were made to the number of antenna elements are 10, 20 and 100 antenna elements. It aims to determine the impact of changes in the number of elements $M$ on the angular resolution of Phased-MIMO radar.

When the antenna element $M=10$, the angular resolution of Phased-MIMO radar is $8^\circ$ where the first target located at $-20^\circ$ and the second target located at $-28^\circ$. If the distance in degrees; less than $8^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

When the antenna element $M=20$, the angular resolution of Phased-MIMO radar is $5^\circ$ where the first target located at $0^\circ$ and the second target located at $5^\circ$. If the distance in degrees; less than $8^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.
When the antenna element $M = 100$, the angular resolution of Phased-MIMO radar is $2^\circ$ where the first target located at $30^\circ$ and the second target located at $32^\circ$. If the distance in degrees; less than $8^\circ$ then the Phased-MIMO radar cannot distinguish the two objects.

From this experiment, can be concluded that by increasing the number of antenna elements Phased-MIMO radars be able increase the angular resolution. Also from figure 5, it can be observed that increasing the number of antenna elements Phased-MIMO radar will decrease the sidelobe of detection results.

Next case, Phased-MIMO radar for multi-targets with antenna elements $M = 20$ and the number of sub-arrays varies from $K = 2$, $K = 8$ and $K = 16$. Nine targets are located at $-40^\circ$, $-35^\circ$, $-30^\circ$, $0^\circ$, $5^\circ$, $10^\circ$, $30^\circ$, $34^\circ$, and $38^\circ$. When the antenna element $K = 2$, the radar is able to detect 3 targets located at $-40^\circ$, $-35^\circ$ and $-30^\circ$. Three targets separated by a distance of $5^\circ$ can be detected precisely. For the next three targets located at $0^\circ$, $5^\circ$, $10^\circ$; Phased-MIMO radars are also able to detect all three targets correctly. But in the last three targets, Phased-MIMO radars with $K = 2$ are not appropriate for detection. Radar only detects two targets whose lobes are close to $8^\circ$ or greater than their angular resolution. This becomes a recommendation if the detection lobe width is greater than the angular resolution, it is necessary to suspect a target detection error therefore increasement of the angular resolution is needed to improve detection.

When the antenna element $K = 8$, Angular resolution of Phased-MIMO radar increases. Radar radar can detect 3 targets located at $-40^\circ$, $-35^\circ$, and $-30^\circ$. Three targets separated by a distance of $5^\circ$ can be detected properly. For the next three targets located at $0^\circ$, $5^\circ$, $10^\circ$; Phased-MIMO radars are also
able to detect all three targets correctly. However, in the last three targets, Phased-MIMO radar with \(K = 8\) made wrong detection. The radar only detected two targets because one of the lower lobes did not reach 0.5. This is recommended if the lowest value of the mainlobe is greater than 0.5, it is necessary to suspect a target detection error therefore increase the angular resolution is needed to improve detection.

Unlike when the number of antenna elements \(K = 16\), Phased-MIMO radars are able to detect all targets at an angle of -38 °, -30 °, 22 °, 0 °, 5 °, 10 °, 30 °, 32 °, and 34 ° with correct. This is because the Phased-MIMO radar with the number of elements \(K = 16\) has a high angular resolution.

Last case, Phased-MIMO radar with antenna elements \(M=10, M=20,\) and \(M=100\) with the number of sub-arrays \(K=4\). The nine targets are located at -38°, -30°, 22°, 0°, 5°, 10°, 30°, 32°, and 34°. When the antenna element \(M = 10\), the radar is able to detect 3 targets located at -38 °, -30 ° and 22 °. Three targets separated by a distance of 8 ° can be detected precisely. But on the next three targets which are located at 0°, 5°, 10°; error detection occurred. Phased-MIMO radars only detect two targets which should have three targets. Likewise the last three targets, Phased-MIMO radars with \(M=10\) are not appropriate for detection. Radar only detects one target with a very large complex amplitude value, because all three targets are outside of angular resolution. As if the complex amplitude of one detected target is the sum of the three targets.

When the antenna element \(M = 20\), Angular resolution of Phased-MIMO radar increases. Radar is able to detect three targets located at -38 °, -30 ° and 22 °. Three targets separated by a distance of 8 ° can be detected precisely. The next three targets are located at 0 °, 5 °, 10 ° also can be detected correctly. But the last 3 targets, Phased-MIMO radars with \(M=20\) are not appropriate for detection. Phased-MIMO radars can only detect two targets, because the distance between targets is outside their angular resolution.

Unlike when the number of antenna elements \(M = 100\). Phased MIMO radars are able to detect all targets at an angle of -38 °, -30 °, 22 °, 0 °, 5 °, 10 °, 30 °, 32 °, and 34 ° with correct. This is because the Phased-MIMO radar with the number of elements \(M = 100\) has a high angular resolution.
4. Conclusions
The From this research, it can be concluded :

1. Estimates for Angular Resolution of Phased-MIMO radar has been proposed.
2. The Angular resolution of Phased-MIMO radar 10x10 when K=2, K = 4 and K = 8 are 8°, 8° and 6°
3. The Angular resolution of Phased-MIMO radar 20x20 when K=2, K=8 dan K=16 are 5°, 5° dan 4°
4. The Angular resolution of Phased-MIMO radar 10x10, 20x20 and 100x100 when K=4 are 8°, 5° dan 2°
5. There are two ways to increase the angular resolution of Phased-MIMO radars by increasing the number of sub-arrays and increasing the number of antenna elements.
6. Phased-MIMO radars will achieve the best angular resolution and complex amplitude values when the number of K sub-arrays approaches half the number of M antenna elements.
7. Increasing the number of sub-arrays improves the phased-MIMO radar detection sidelobe.
8. Increasing the number of antenna elements Phased-MIMO radar improves the detection sidelobe and improves the detection mainlobe width.
9. If the complex amplitude value is greater than one, it needs suspectif there is an error detection by the Phased-MIMO radar thus increasement of angular resolution is needed. This error was allegedly the sum of the complex amplitude values of two or more targets.
10. If the detection width of the mainlobe is greater than the angular resolution of the Phased-MIMO radar, it is necessary to suspect a detection error so that an increase in angular resolution is needed. This error is allegedly the existence of two or more close targets, but not detected correctly.

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