LEARNING γ FROM $B \rightarrow K\pi$ DECAYS

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Current information on $\gamma = \text{Arg}(V_{ub}^*)$ from other CKM constraints is still in need of improvement, with $39^\circ < \gamma < 80^\circ$ at 95% c.l. Direct probes of $\gamma$ can tighten these bounds, possibly indicating new physics effects in case that an inconsistency with this range is observed. In order to study $\gamma$ directly in charmless two-body $B$ decays, which involve a $b$ to $u$ transition, one must generally separate strong and weak phases from one another. We describe several cases of $B \rightarrow K\pi$ decays in which progress in this work has been accomplished, and what improvements lie ahead. Some additional details are noted in earlier reviews and in Refs. and Refs. [5] and [6].

A great deal of information can be obtained from $B \rightarrow K\pi$ decay rates averaged over CP, supplemented with measurements of direct CP asymmetries. One probes in this manner tree-penguin interference in various processes. The data which are used in these analyses are summarized in Table 1. The $B^+ \rightarrow B^0$ lifetime ratio is taken to be $\tau^+ / \tau_0 = 1.078 \pm 0.013$, based on $\tau_B = 1.653 \pm 0.014$ ps and $\tau_0 = 1.534 \pm 0.013$ ps [8]. Table 1 also contains contributions to the four $B \rightarrow K\pi$ decay processes of penguin ($P'$), electroweak penguin ($P_{\text{EW}}'$), tree ($T'$) and color-suppressed tree ($C'$) amplitudes. These contributions are hierarchical and can be classified using flavor symmetries. Smaller contributions, from color-suppressed electroweak penguin amplitudes, annihilation and exchange amplitudes, are not shown in Table 1. All four $B \rightarrow K\pi$ decays are dominated by penguin amplitudes, which are related to each other by isospin. Tree amplitudes $T' + C'$ and electroweak penguin amplitudes $P_{\text{EW}}'$ are subdominant and can be related to each other by flavor SU(3) [13]. SU(3) breaking in tree amplitudes is introduced assuming factorization.

| Decay mode | Amplitude | $B$ (units of $10^{-6}$) | $A_{CP}$ |
|------------|-----------|------------------------|----------|
| $B^+ \rightarrow K^0 \pi^+$ | $P'$ | 21.78 ± 1.40 | 0.016 ± 0.057 |
| $B^+ \rightarrow K^+ \pi^0$ | $-(P' + P_{\text{EW}}' + T' + C')/\sqrt{2}$ | 12.53 ± 1.04 | 0.00 ± 0.12 |
| $B^0 \rightarrow K^+ \pi^-$ | $-(P' + T')$ | 18.16 ± 0.79 | −0.095 ± 0.029 |
| $B^0 \rightarrow K^0 \pi^0$ | $(P' - P_{\text{EW}}' - C')/\sqrt{2}$ | 11.68 ± 1.42 | 0.03 ± 0.37 |

Several comparisons between pairs of processes can be made:

- $B^0 \rightarrow K^+ \pi^-(P' + T')$ vs. $B^+ \rightarrow K^0 \pi^+(P')$ [5, 13, 15, 16];
- $B^+ \rightarrow K^+ \pi^0(P' + P_{\text{EW}}' + T' + C')$ vs. $B^+ \rightarrow K^0 \pi^+(P')$ [5, 13, 17, 18];
- $B^0 \rightarrow K^0 \pi^0$ vs. other modes [5, 19, 20, 21, 22, 23].

We give the example of $B^0 \rightarrow K^+ \pi^-$ in detail. The tree amplitude for this process is $T' \sim V_{ub} V_{us}^*$, with weak phase $\gamma$, while the penguin amplitude is $P' \sim V_{ts} V_{tb}^*$ with weak phase $\pi$. We denote the penguin-tree

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relative strong phase by $\delta$ and define $r \equiv |T'/P'|$. Then we may write

\begin{align}
A(B^0 \to K^+\pi^-) &= |P'| [1 - re^{i(\gamma + \delta)}], \\
A(\overline{B}^0 \to K^-\pi^+) &= |P'| [1 - re^{i(-\gamma + \delta)}], \\
A(B^+ \to K^0\pi^+) &= A(B^- \to \overline{K}^0\pi^-) = -|P'|.
\end{align}

In the last two amplitudes we neglect small annihilation contributions with weak phase $\gamma$, assuming that rescattering effects are not largely enhanced. A test for this assumption is the absence of a CP asymmetry in $B^+ \to K^0\pi^+$, and a U-spin relation between this process and $B^+ \to \overline{K}^0 K^+$ [24], in which a corresponding amplitude with weak phase $\gamma$ is expected to be much larger. One also neglects small color-suppressed electroweak contributions, for which experimental tests were proposed in [25].

One now forms the ratio

$$R \equiv \frac{\Gamma(B^0 \to K^+\pi^-) + \Gamma(\overline{B}^0 \to K^-\pi^+)}{2\Gamma(B^+ \to K^0\pi^+)} = 1 - 2r \cos \gamma \cos \delta + r^2.$$  

(4)

Fleischer and Mannel [14] pointed out that $R \geq \sin^2 \gamma$ for any $r, \delta$ so if $1 > R$ one can get a useful bound. Moreover, if one uses

$$R A_{CP}(K^+\pi^-) = -2r \sin \gamma \sin \delta$$  

(5)

as well and eliminates $\delta$ one can get a more powerful constraint, illustrated in Fig. 1.

We have used $R = 0.898 \pm 0.071$ and $A_{CP} = -0.095 \pm 0.029$ based on recent averages [24] of CLEO, BaBar, and Belle data, and $r = |T'/P'| = 0.142 \pm 0.012$. In order to estimate the tree amplitude and the ratio of amplitudes $r$, we have used factorization in $B^0 \to \pi^-\ell^+\nu_\ell$ at low $q^2$ [26] and $\Gamma(B^0 \to \pi^-\ell^+\nu_\ell) \approx (0.12)(0.23) = 0.28$. One could also use processes in which $T$ dominates, such as $B^0 \to \pi^+\pi^-$ or $B^+ \to \pi^+\pi^0$, but these are contaminated by contributions from $P$ and $C$, respectively. The $1\sigma$ allowed region lies between the curves $A_{CP} = 0$ and $|A_{CP}| = 0.124$. The most conservative upper bound on $\gamma$ arises for the smallest value of $|A_{CP}|$ and the largest value of $r$, while the most conservative lower bound would correspond to the largest $|A_{CP}|$ and the smallest $r$. Currently no such lower bound is obtained at a $1\sigma$ level. At this level one has $R < 1$, leading to an upper bound $\gamma < 80^\circ$.

We note that for the current average value of $R$ the $1\sigma$ upper bound, $\gamma < 80^\circ$, happens to coincide with that of Ref. [1]. This bound does not depend much on the value of $r$, for which we assumed factorization of $T$ in order to introduce SU(3) breaking. The upper bound on $\gamma$ varies only slightly, $\gamma < 78^\circ - 80^\circ$, for a wide range of values $r = 0.1 - 0.3$. On the other hand, a potential lower bound on $\gamma$ depends more sensitively on the value of $r$, and would result if small values of this parameter could be excluded. For instance, Fig. 1 shows that a value $r = 0.166$ implies $\gamma > 49^\circ$ at $1\sigma$. Thus, it is crucial to improve our knowledge of $r$.

The process $B^+ \to K^+\pi^0$ also provides constraints on $\gamma$. The deviation of the ratio

$$R_c \equiv \frac{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^- \to K^-\pi^0)}{\Gamma(B^+ \to K^0\pi^+)} = 1.15 \pm 0.12$$  

(6)

from 1, when combined with $A_{CP}(K^+\pi^0) = 0.00 \pm 0.12$, $r_c = |(T' + C')/P'| = 0.195 \pm 0.016$ and an estimate of the electroweak penguin amplitude $\delta_{EW} \equiv |P'_{EW}|/(T' + C') = 0.65 \pm 0.15$, leads to a $1\sigma$ lower bound $\gamma > 40^\circ$. Details of the method may be found in Refs. [27, 28, 21, 14, 15]; the present bound represents an update of previous valued quoted values. The most conservative lower bound on $\gamma$ arises for smallest $A_{CP}$, largest $r_c$, and largest $|P'_{EW}|$, and is shown in Fig. 2. These values of $r_c$ and $|P'_{EW}|$ would also imply an upper

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Figure 1. Behavior of $R$ for $r = 0.166$ and $A_{CP} = 0$ (dashed curves) or $|A_{CP}| = 0.124$ (solid curve) as a function of the weak phase $\gamma$. Horizontal dashed lines denote $\pm 1\sigma$ experimental limits on $R$, while dot-dashed lines denote 95% c.l. ($\pm 1.96\sigma$) limits. The short-dashed curve denotes the Fleischer-Mannel bound $\sin^2 \gamma \leq R$. The upper branches of the curves correspond to the case $\cos \gamma \cos \delta < 0$, while the lower branches correspond to $\cos \gamma \cos \delta > 0$. 

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Figure 2. Behavior of $R_c$ for $r_c = 0.21$ (1σ upper limit) and $A_{CP}(K^+π^0) = 0$ (dashed curves) or $|A_{CP}(K^+π^0)| = 0.125$ (solid curve) as a function of the weak phase $γ$. Horizontal dashed lines denote ±1σ experimental limits on $R_c$, while dotdashed lines denote 95% c.l. (±1.96σ) limits. We have taken $δ_{EW} = 0.80$ (its 1σ upper limit), which leads to the most conservative bound on $γ$. Upper branches of curves correspond to $\cos δ_c(\cos γ - δ_{EW}) < 0$, while lower branches correspond to $\cos δ_c(\cos γ - δ_{EW}) > 0$. Here $δ_c$ is a strong phase.
bound, $\gamma < 77^\circ$, which demonstrates the importance of improving our knowledge of these two hadronic parameters.

Another ratio

$$R_n = \frac{\Gamma(B^0 \to K^+\pi^-) + \Gamma(B^0 \to K^-\pi^+)}{2 \left[ \Gamma(B^0 \to K^0\pi^0) + \Gamma(B^0 \to K^0\pi^0) \right]} = 0.78 \pm 0.10$$

(7)

involves the decay $B^0 \to K^0\pi^0$. This ratio should be equal to $R_c$ since to leading order in $T'/P'$, $C'/P'$, and $P_{EW}'/P'$ one has

$$\left| \frac{P' + T'}{P' - P_{EW}' - C'} \right|^2 \approx \left| \frac{P' + P_{EW}' + T' + C'}{P'} \right|^2,$$

(8)

but the two ratios differ by $2.4\sigma$. Possibilities for explaining this apparent discrepancy (see, e.g., Refs. [5, 27]) include (1) new physics, e.g., in the EWP amplitude, and (2) an underestimate of the $\pi^0$ detection efficiency in all experiments, leading to an overestimate of any branching ratio involving a $\pi^0$. The latter possibility can be taken into account by considering the ratio $(R_n R_c)^{1/2} = 0.96 \pm 0.08$, in which the $\pi^0$ efficiency cancels. As shown in Fig. 4 this ratio leads only to the conservative bound $\gamma \leq 88^\circ$. A future discrepancy between $R_c$ and $R_n$ at a statistically significant level implying new physics effects would clearly raise questions about the validity of constraints on $\gamma$ obtained from these quantities.

Recently a time-dependent asymmetry measurement in $B^0(t) \to K_S\pi^0$ was reported [28]

$$S_{\pi K} = 0.48^{+0.38}_{-0.47} \pm 0.11, \quad C_{\pi K} = 0.40^{+0.27}_{-0.28} \pm 0.10$$

(9)

where $S_{\pi K}$ and $-C_{\pi K}$ are coefficients of $\sin \Delta m t$ and $\cos \Delta m t$ terms in the asymmetry. In the limit of a pure penguin amplitude, $A(B^0 \to K^0\pi^0) = (P' - P_{EW}')/\sqrt{2}$, one expects $S_{\pi K} = \sin 2\beta, C_{\pi K} = 0$. The color-suppressed amplitude, $C'$, contributing to this process involves a weak phase $\gamma$. Its effect was studied recently [8] by relating these two amplitudes within flavor SU(3) symmetry to corresponding amplitudes in $B^0 \to \pi^0\pi^0$. Correlated deviations from $S_{\pi K} = \sin 2\beta, C_{\pi K} = 0$, at a level of $0.1-0.2$ in the two asymmetries, were calculated and shown to be sensitive to values of $\gamma$ in the currently allowed range. Observing such deviations and probing the value of $\gamma$ requires reducing errors in the two asymmetries by about an order of magnitude.

To summarize, promising bounds on $\gamma$ stemming from various $B \to K\pi$ decays have been mentioned. So far all are statistics-limited. At $1\sigma$ we have found

- $R$ ($K^+\pi^-$ vs. $K^0\pi^+$) gives $\gamma \leq 80^\circ$;
- $R_c$ ($K^+\pi^0$ vs. $K^0\pi^+$) gives $\gamma \geq 40^\circ$;
- $R_n$ ($K^+\pi^-$ vs. $K^0\pi^0$) should equal $R_c$; $(R_c R_n)^{1/2}$ gives $\gamma \leq 88^\circ$.

The future of most such $\gamma$ determinations remains for now in experimentalists’ hands, as one can see from the Figures. We have noted (see, e.g., [15]) that measurements of rate ratios in $B \to K\pi$ can ultimately pinpoint $\gamma$ to within about $10^\circ$. The required accuracies in $R$, $R_c$, and $R_n$ to achieve this goal can be estimated from the Figures. For example, knowing $(R_c R_n)^{1/2}$ to within 0.05 would pin down $\gamma$ to within $10^\circ$ if this ratio lies in the most sensitive range of Fig. 3. A significant discrepancy between the values of $R_c$ and $R_n$ would be evidence for new physics.

It is difficult to extrapolate the usefulness of $R$, $R_c$, and $R_n$ measurements to very high luminosities without knowing ultimate limitations associated with systematic errors. The averages in Table 1 are based on
Figure 3. Behavior of $(R_c R_n)^{1/2}$ for $r_c = 0.18$ (1σ lower limit) and $A_{CP}(K^+\pi^0) = 0$ (dashed curves) or $|A_{CP}(K^+\pi^0)| = 0.125$ (solid curve) as a function of the weak phase $\gamma$. Horizontal dashed lines denote $\pm 1\sigma$ experimental limits on $(R_c R_n)^{1/2}$, while dotdashed lines denote 95% c.l. ($\pm 1.96\sigma$) limits. Upper branches of curves correspond to $\cos \delta_c (\cos \gamma - \delta_{EW}) < 0$, while lower branches correspond to $\cos \delta_c (\cos \gamma - \delta_{EW}) > 0$. Here we have taken $\delta_{EW} = 0.50$ (its 1σ lower limit), which leads to the most conservative bound on $\gamma$. 

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individual measurements in which the statistical errors exceed the systematic ones by at most a factor of about 2 (in the case of $B^0 \rightarrow K^0\pi^0$) \cite{7}. For $B^+ \rightarrow K^+\pi^0$ the statistical and systematic errors are nearly equal. Thus, the clearest path to improvements in these measurements is associated with the next factor of roughly 4 increase in the total data sample. Thereafter, reductions in systematic errors must accompany increased statistics in order for these methods to yield improved accuracies in $\gamma$.

In our study we used the most pessimistic values of the parameters $r$, $r_c$ and $\delta_{EW}$ leading to the weakest bounds on $\gamma$. The theoretical uncertainties in these parameters can be further reduced, and the assumption of negligible rescattering can be tested. This progress will rely on improving branching ratio measurements for $B \rightarrow K\pi$, $B \rightarrow \pi\pi$ and $B^0 \rightarrow \pi^-\ell^+\nu_\ell$, on an observation of penguin-dominated $B \rightarrow K\bar{K}$ decays, and on various tests of factorization which imply relations between CP-violating rate differences \cite{29, 30}.

A complementary approach to the flavor-SU(3) method is the QCD factorization formalism of Refs. \cite{21, 22, 23}. It predicts small strong phases (as found in our analysis) and deals directly with flavor-SU(3) breaking; however, it involves some unknown form factors and meson wave functions and appears to underestimate the magnitude of $B \rightarrow VP$ penguin amplitudes. Combining the two approaches seems to be the right way to proceed.

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References

[1] A. Hocker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C 21, 225 (2001) [arXiv:hep-ph/0104062]. For periodic updates see http://ckmfitter.in2p3.fr/.

[2] M. Gronau, Presented at Flavor Physics and CP Violation (FPCP 2003), Paris, France, 3–6 June 2003 [arXiv:hep-ph/0306308].

[3] J. L. Rosner, AIP Conf. Proc. 689, 150 (2003) [arXiv:hep-ph/0306284].

[4] J. L. Rosner, Presented at the 9th International Conference on B-Physics at Hadron Machines (BEAUTY 2003), CMU, Pittsburgh, October 2003 [arXiv:hep-ph/0311170].

[5] M. Gronau and J. L. Rosner, Phys. Lett. B 572, 43 (2003) [arXiv:hep-ph/0307095].

[6] M. Gronau, Y. Grossman and J. L. Rosner, to be published in Phys. Lett. B [arXiv:hep-ph/0310020].

[7] C.-W. Chiang, M. Gronau, J. L. Rosner, and D. A. Suprun, 2003, manuscript in preparation; see also Heavy Flavor Averaging Group, Lepton-Photon 2003 branching ratios and CP asymmetries, at http://www.slac.stanford.edu/xorg/hfag/rare/.

[8] LEP B Oscillation Working Group, results for the summer 2003 conferences, http://lepbosonic.web.cern.ch/LEPBOSC/.

[9] D. Zeppenfeld, Z. Phys. C 8, 77 (1981).

[10] M. J. Savage and M. B. Wise, Phys. Rev. D 39, 3346 (1989) [Erratum-ibid. D 40, 3127 (1989)].

[11] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 50, 4529 (1994) [arXiv:hep-ph/9404283].

[12] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 52, 6374 (1995) [arXiv:hep-ph/9504327].

[13] M. Neubert and J. L. Rosner, Phys. Lett. B 441, 403 (1998) [arXiv:hep-ph/9808493].

[14] R. Fleischer and T. Mannel, Phys. Rev. D 57, 2752 (1998) [arXiv:hep-ph/9704423].

[15] M. Gronau and J. L. Rosner, Phys. Rev. D 57, 6843 (1998) [arXiv:hep-ph/9711246].

[16] M. Gronau and J. L. Rosner, Phys. Rev. D 65, 013004 (2002) [Erratum-ibid. D 65, 079901 (2002)] [arXiv:hep-ph/0109238].

[17] M. Neubert and J. L. Rosner, Phys. Rev. Lett. 81, 5076 (1998) [arXiv:hep-ph/9809311].

[18] M. Neubert, JHEP 9902, 014 (1999) [arXiv:hep-ph/9812396].

[19] A. J. Buras and R. Fleischer, Eur. Phys. J. C 11, 93 (1999) [arXiv:hep-ph/9810260].

[20] A. J. Buras and R. Fleischer, Eur. Phys. J. C 16, 97 (2000) [arXiv:hep-ph/0003323].

[21] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001) [arXiv:hep-ph/0104110].

[22] M. Beneke and M. Neubert, Nucl. Phys. B 651, 225 (2003) [arXiv:hep-ph/0210085].

[23] M. Beneke and M. Neubert, arXiv:hep-ph/0308039.

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[24] A. F. Falk, A. L. Kagan, Y. Nir and A. A. Petrov, Phys. Rev. D 57, 4290 (1998) [arXiv:hep-ph/9712225].
[25] R. Fleischer, Eur. Phys. J. C 6, 451 (1999) [arXiv:hep-ph/9802433].
[26] Z. Luo and J. L. Rosner, Phys. Rev. D 68, 074010 (2003) [arXiv:hep-ph/0305262].
[27] Y. Grossman, Presented at LP03, XXI International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, 2003 [hep-ph/0310229].
[28] BaBar Collaboration, A. Farbin et al., PLOT–0053, contribution to LP03, Ref. 27; https://oraweb.slac.stanford.edu:8080/pls/slacquery/babar_documents.startup.
[29] N. G. Deshpande and X. G. He, Phys. Rev. Lett. 75, 1703 (1995) [arXiv:hep-ph/9412393].
[30] M. Gronau, Phys. Lett. B 492, 297 (2000) [arXiv:hep-ph/0008292].