Nuclear transparency of the charged hadron produced in the inclusive electronuclear reaction

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Abstract

The nuclear transparency of the charged hadron produced in the inclusive \((e, e')\) reaction on the nucleus has been calculated using Glauber model for the nuclear reaction. The color transparency (CT) of the produced hadron and the short-range correlation (SRC) of the nucleons in the nucleus have been incorporated in the Glauber model to investigate their effects on the nuclear transparency of the hadron. The calculated nuclear transparencies for the proton and pion are compared with the data.

The hadron-nucleus cross section is less than that in the plane wave impulse approximation (PWIA) because the initial and/or final state interactions of the hadron with the nucleus are neglected in PWIA. The difference in the cross sections can be characterized by the nuclear transparency \(T_A\), defined [1] as

\[
T_A = \frac{\sigma_{hA}}{\sigma_{hA(PWIA)}},
\]

where \(\sigma_{hA}\) represents the hadron-nucleus cross section.

The transverse size \(d_\perp\) of the hadron produced in the nucleus due to the space-like high momentum transfer \(Q^2\) is reduced as \(d_\perp \sim 1/Q\) [1, 2]. The reduced (in size) hadron is referred as point like configuration (PLC) [1]. According to Quantum Chromodynamics, a color neutral PLC has reduced interaction with the nucleon in the nucleus because the sum of its gluon emission amplitudes cancel [1, 3]. The PLC expands to the size of physical hadron, as it moves up to a length \((\sim 1\ \text{fm})\) called hadron formation length \(l_h\) [1, 4]:

\[
l_h = \frac{2k_h}{\Delta M^2},
\]

where \(k_h\) is the momentum of hadron in the laboratory frame. \(\Delta M^2\) is related to the mass difference between the hadronic states originating due to the (anti)quarks fluctuation in PLC. The interaction of PLC with the nucleon in the nucleus increases, as its size increases during its passage \(l_h\) through the nucleus. The decrease in the hadron-nucleon

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cross section in the nucleus, as explained by Glauber model [5], leads to the increase in the hadron-nucleus cross section $\sigma_{hA}$. Therefore, the transparency $T_A$ in Eq. (1) of the hadron raises. The enhancement in $T_A$ due to the above phenomenon is referred as color transparency (CT) of the hadron. The physics of CT for hadrons have been discussed elaborately in Refs. [3, 6].

The experiments to search the CT of proton ($p$CT) in the $A(p, pp)$ reactions done at Brookhaven National Laboratory (BNL) [7] could not confirm the $p$CT. In fact, the experimental results are not understood [8]. The $p$CT is not also seen in the $A(e, e'p)$ experiment done at Standford Linear Accelerator Center (SLAC) [9] and Jefferson Laboratory (JLab) [10] for $0.64 \leq Q^2 \leq 8.1 \text{ GeV}^2$. This experiment done for $8 \leq Q^2(\text{GeV}^2) \leq 14.2$ [11] at the upgraded JLab facility agrees with the previous observation [9, 10]. Therefore, it appears the PLC required for CT is unlikely to form for three quarks ($qqq$) system, such as proton.

Since the meson is a bound state of two quarks (i.e., quark-antiquark) the PLC formation of it can be more probable than that of the baryon, a three quarks ($qqq$) system. The color transparency is unambiguously reported from Fermi National Accelerator Laboratory (FNAL) [12] in the experiment of the nuclear diffractive dissociation of pion (of $500 \text{ GeV/c}$) to dijets. The color transparency is also illustrated in the $\pi^-$ meson photoproduction [13] and $\rho^0$ meson electroproduction (from nuclei) experiments [14]. Several authors studied the $\rho$-meson color transparency in the energy region available at JLab [1, 15].

The nuclear transparency of the $\pi^+$ meson produced in the $A(e, e')$ process was measured at JLab for the photon virtuality $Q^2 = 1.1 - 4.7 \text{ GeV}^2$ [16]. The data have been understood by the pionic color transparency ($\pi$CT) [17]. Larson et al., [18] described the momentum dependence of $\pi$CT in the above reaction. Cosyn et al., [19] studied the effects of $\pi$CT and nucleon short-range correlation in the pion photoproduction from nuclei. Larionov et al., [4] estimated the $\pi$CT in the $(\pi^-, l^+l^-)$ reaction on nuclei for $p_\pi = 5 - 20 \text{ GeV/c}$, which can be measured at the forth-coming facilities in Japan Proton Accelerator Research Complex (J-PARC) [20]. This reaction provides the informations complementary to those obtained from the $A(\gamma^*, \pi)$ reaction. Miller and Strikman [21] illustrated large CT in the pionic knockout of the proton off nuclei at the energy $200 \text{ GeV}$ available at CERN COMPASS experiment.

The enhancement in $T_A$ due to $\sigma_{hA}$ in Eq. (1) can also occur because of the short-range correlation (SRC) of nucleon in the nucleus. The SRC arises because of the repulsive (short-range) interaction between the nucleons bound in the nucleus. This interaction keeps the bound nucleons apart ($\sim 1 \text{ fm}$), which is called nuclear granularity [8]. Therefore, the SRC prevents the shadowing of the hadron-nucleon interaction due to the surrounding nucleons present in the nucleus. This occurrence, as elucidated by Glauber model [5], leads to the enhancement in $\sigma_{hA}$. The SRC is widely used to investigate various aspects in the nuclear physics [22].
The hadron \( h \) is produced in the inclusive \( A(e, e')X \) reaction because of the interaction of the virtual photon \( \gamma^* \) (emitted at the \( ee' \) vertex) with the nucleus \( A \). In this reaction, the nucleus in the final state denoted by \( X \) is unspecified. The scattering amplitude for the \( \gamma^*A \rightarrow hX \) transition, according to Glauber model [1], can be written as

\[
F_{X0}(q - k_h) = \frac{iq}{2\pi} \int d\mathbf{b} e^{i(q - k_h) \cdot \mathbf{b}} \Gamma_{X0}^{\gamma^*h}(\mathbf{b}),
\]

where \( q \) and \( k_h \) are the momenta of \( \gamma^* \) and \( h \) respectively. \( \Gamma_{X0}^{\gamma^*h}(\mathbf{b}) \) describes the matrix element for the transition of the nucleus from its initial to final states, i.e.,

\[
\Gamma_{X0}^{\gamma^*h}(\mathbf{b}) = <X|\Gamma_{hX}^{\gamma^*A}(\mathbf{b}, \mathbf{r}_1, ..., \mathbf{r}_A)|0>.
\]

where \( |0> \) denotes the ground state of the target nucleus and \( |X> \) represents the unspecified nuclear state in the exit channel. The nuclear profile operator \( \Gamma_{A}^{\gamma^*h}(\mathbf{b}, \mathbf{r}_1, ..., \mathbf{r}_A) \) [1, 23] is given by

\[
\Gamma_{A}^{\gamma^*h}(\mathbf{b}, \mathbf{r}_1, ..., \mathbf{r}_A) = \sum_i \Gamma_{A}^{\gamma^*h}(\mathbf{b} - \mathbf{b}_i) e^{i(q - k_h) \cdot z_i} \Pi_{j \neq i}^{A-1}[1 - \Gamma^{hN}(\mathbf{b} - \mathbf{b}_j) \theta(z_j - z_i)].
\]

The summation \( i \) is taken over the number of nucleons in the nucleus participated for the hadron production, e.g., the protons in the nucleus take part to produced charged hadron in the reaction.

\( \Gamma_{A}^{\gamma^*h}(\mathbf{b}) \) is the two-body profile function for the hadron produced from the nucleon, i.e., \( \gamma^*N \rightarrow hN \) process. It is related to the reaction amplitude \( f_{\gamma^*h}(\tilde{q}_\perp) \) [1] as

\[
\Gamma_{A}^{\gamma^*h}(\mathbf{b}, \mathbf{r}_1, ..., \mathbf{r}_A) = \frac{1}{i2\pi q} \int d\tilde{q}_\perp e^{-i\tilde{q}_\perp \cdot \hat{b}} f_{\gamma^*h}(\tilde{q}_\perp).
\]

The two-body profile function \( \Gamma^{hN}(\tilde{b}) \) is connected to \( hN \) (hadron-nucleon) elastic scattering amplitude \( f_{hN}(\tilde{q}_\perp) \) [1, 5] as

\[
f_{hN}(\tilde{q}_\perp') = \frac{ik_h}{2\pi} \int d\tilde{b}' e^{i\tilde{q}_\perp \cdot \tilde{b}'} \Gamma^{hN}(\tilde{b}').
\]

The nuclear states, assuming the independent particle model [24], can be written in terms of the single particle state \( \Phi \) as \( |0> = \Pi_{i=1}^{A} |\Phi_0(\mathbf{r}_i)> \) and \( |X> = |\Phi_X(\mathbf{r}_m)> \Pi_{n \neq m}^{A-1} |\Phi_0(\mathbf{r}_n)> \). Using those, \( \Gamma_{X0}^{\gamma^*h}(\mathbf{b}) \) in Eq. (4) can be written as

\[
\Gamma_{X0}^{\gamma^*h}(\mathbf{b}) = \sum_i \int d\mathbf{r}_i \Phi_X^*(\mathbf{r}_i) \Gamma_{X0}^{\gamma^*h}(\mathbf{b} - \mathbf{b}_i) e^{i(q - k_h) \cdot z_i} \Phi_0(\mathbf{r}_i) D(\mathbf{b}, z_i),
\]

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where $D(b, z_i)$ is given by

$$D(b, z_i) = \frac{\Pi_{j \neq i}^{-1}}{A} \int d\mathbf{r}_j \Phi_0^*(\mathbf{r}_j) \left[ 1 - \Gamma^h_{\mathbf{r}_j}(b - b_j) \theta(z_j - z_i) \right] \Phi_0(\mathbf{r}_j)$$

$$= \left[ 1 - \frac{1}{A} \int db_J \Gamma^h_{\mathbf{r}_j}(b - b_j) \int dz_J \theta(z_J - z_i) \rho(\mathbf{r}_j) \right]^{A-1}. \tag{9}$$

In this equation, $\rho(\mathbf{r}_j)$ is the matter density distribution of the nucleus, i.e., $\rho(\mathbf{r}_j) = A |\Phi_0(\mathbf{r}_j)|^2$. $\rho(b_j, z_j)$ can be replaced by $\rho(b_j, z_j)$, since $\Gamma^h_{\mathbf{r}_j}(b - b_j)$ varies much rapidly than $\rho(b_j, z_j)$ [1]. Using Eq. (7) and $\mathcal{L}_{n \to \infty} (1 + \frac{r}{n})^n = e^r$, the above equation can be simplified to

$$D(b, z_i) \simeq e^{-\frac{i}{2} t^h b |1 - i \alpha_{hN}| T(b, z_i)}, \tag{10}$$

where $\alpha_{hN}$ denotes the ratio of the real to imaginary part of the hadron-nucleon scattering amplitude $f_{hN}(0)$, and $\sigma^h_{tN} = \frac{\gamma}{k_h} Im [f_{hN}(0)]$ is the hadron-nucleon total cross section. $T(b, z_i)$ is the partial thickness function of the nucleus, i.e.,

$$T(b, z_i) = \int_{z_i}^{\infty} dz_j \rho(b, z_j). \tag{11}$$

Using Eq. (8), $F_{X0}[(q - k_h)_\perp]$ in Eq. (3) can be expressed as

$$F_{X0} = \frac{iq}{2\pi} \int d\mathbf{r} e^{i(q - k_h)_\perp \cdot \mathbf{r}} \sum_i \int d\mathbf{r}_i \Phi^*_X(\mathbf{r}_i) \Gamma^{\gamma h}(b - b_i) e^{i(q - k_h)_\perp \cdot \mathbf{r}_i} \Phi_0(\mathbf{r}_i) D(b, z_i),$$

$$= \sum_i \int d\mathbf{r}_i \Phi^*_X(\mathbf{r}_i) f_{hN}^{(i)}[(q - k_h)_\perp] e^{i(q - k_h)_\perp \cdot \mathbf{r}_i} \Phi_0(\mathbf{r}_i) D(b, z_i). \tag{12}$$

$f_{hN}^{(i)}$, defined in Eq. (7), can be considered identically equal for all nucleons.

The nucleus in the final state $|X>$ differs from its initial state $|0>$ (i.e., ground state) for the charged hadron production, i.e., $\Phi_X \neq \Phi_0$ and $F_{00} = 0$. To calculate the cross section, $|F_{X0}|^2$ is to multiply by the phase-space of the reaction and that is to divide by the incident flux. Since the final state $|X>$ of the nucleus is not detected in the inclusive reaction, the summation over all states $X$ has to carry out. In the multi-GeV region, the phase space of the reaction can be considered independent of the state $X$, and therefore, the nuclear transparency $T_A$ can be written [1] as

$$T_A = \frac{\sum_{X \neq 0} |F_{X0}|^2}{\sum_{X \neq 0} |F_{X0}|_{PWA}^2}. \tag{13}$$

The hadron-nucleon cross section $\sigma^h_{tN}$ in the free-space is used in Eq. (10) to evaluate $T_A$ in Glauber model. To look for the color transparency (CT), $\sigma^h_{tN}$ (according to quantum diffusion model [2, 18]) has to replace by $\sigma^h_{tCT}$:

$$\sigma^h_{t, CT}(Q^2, l_z) = \sigma^h_{tN} \left\{ \frac{l_z}{l_h} + \frac{n^2}{Q^2} > \left( 1 - \frac{l_z}{l_h} \right) \right\} \theta(l_h - l_z) + \theta(l_z - l_h), \tag{14}$$
where $Q^2$ is the space-like four-momentum transfer, i.e., photon virtuality. $n_q$ denotes the number of valence quark-(anti)quark present in the hadron, e.g., $n_q = 2\,(3)$ for pion (proton) [2]. $k_t$ illustrates the transverse momentum of the (anti)quark: $< k_t^2 >^{1/2} = 0.35$ GeV/c. $l_x$ is the path length traversed by the hadron after its production. The hadron formation length $l_h(\propto \frac{1}{\Delta M^2})$ is already defined in Eq. (2).

The short-range correlation (SRC) can be incorporated by replacing the nuclear density distribution $\varrho$ in Eq. (11) by

$$\varrho(b, z_j) \rightarrow \varrho(b, z_j)C(|z_j - z_i|),$$

where $C(u)$ represents the correlation function [8]. Using the nuclear matter estimate, it can be written as

$$C(u) = \left[1 - \frac{h(u)^2}{4}\right]^{1/2} [1 + f(u)],$$

with $h(u) = \frac{3n(k_Fu)}{k_Fu}$ and $f(u) = -e^{-\alpha u^2}(1 - \beta u^2)$. The Fermi momentum $k_F$ is chosen equal to 1.36 fm$^{-1}$. $C(u)$ with the parameters $\alpha = 1.1$ fm$^{-2}$ and $\beta = 0.68$ fm$^{-2}$ agrees well that derived from the many-body calculations [8].

The nuclear transparency $T_A$ of the charged hadron, i.e., proton and $\pi^+$ meson, produced in the inclusive electronuclear reaction has been calculated using Glauber model (GM), where the measured nuclear density distribution $\varrho(r)$ [25], and hadron-nucleon cross section $\sigma_h^{hN}$ [26] are used. As shown later, the calculated results due to GM (presented by the dashed curves) underestimate the measured $T_A$ for both proton and pion. Therefore, GM has been modified by taking account of CT and SRC. Since the CT is energy dependent, the calculated $T_A(\pi^+)$ increases with $Q^2$ due to the inclusion of CT in GM. Unlike CT, the SRC is independent of energy. Therefore, the calculated $T_A(\pi^+)$ due to SRC added in GM does not illustrate the $Q^2$ dependence. The dot-dot-dashed and dot-dashed curves arise because of the inclusion of CT in GM for $\Delta M^2$, defined in Eq. (2), taken equal to 0.7 and 1.4 GeV$^2$ respectively. The calculated $T_A$ due to SRC incorporated in GM are presented by the solid curves.

The calculated proton transparency $T_A(p)$ vs photon virtuality $Q^2$ in the $A(e, e'p)X$ reaction is compared with the data in Fig. 1. The data reported from SLAC [9] and JLab [10, 11] are represented by the white squares and black circles respectively. Fig. 1(a) shows CT does not exist for the proton moving through $^{12}$C for a wide range of the photon virtuality, i.e., $0.64 \leq Q^2 \leq 14.2$ GeV$^2$. This is corroborated by the results for other nuclei shown in Figs. 1(b) and (c), where the data are available for lesser range of $Q^2$, i.e., $0.64 \leq Q^2 \leq 8.1$ GeV$^2$ for $^{56}$Fe and $0.64 \leq Q^2 \leq 6.77$ GeV$^2$ for $^{197}$Au. Therefore, the CT of proton is distinctly ruled out. Fig. 1 shows the calculated $T_A(p)$ due to the inclusion of SRC in GM reproduce the data reasonably well for all nuclei.

The measured pionic transparency $T_A(\pi^+)$ for $1.1 \leq Q^2 \leq 4.69$ GeV$^2$ in the $A(e, e'\pi^+)X$ reaction have been reported from JLab [16] for $^{12}$C, $^{27}$Al, $^{63}$Cu and $^{197}$Au nuclei. The data
for all nuclei (except $^{12}$C) show the enhancement of $T_A(\pi^+)$ with $Q^2$. Proposals are there to measure $T_A(\pi^+)$ at JLab for higher $Q^2$, i.e., $5 \leq Q^2 \leq 9.5$ GeV$^2$ [3, 27]. Therefore, $T_A(\pi^+)$ for $1.1 \leq Q^2 \leq 9.5$ GeV$^2$ have been calculated and those are presented in Fig. 2 along with the available data [16]. The calculated results due to GM+CT (i.e., $\pi$CT) are accord with both the $Q^2$ dependence and magnitude of the data. This is in concurrence with the earlier calculations [4, 17]. The calculated $T_A(\pi^+)$ due to GM+SRC do not describe the $Q^2$ dependence of the data but the calculated results agree with the large number of data points within the errors. Therefore, the data of $T_A(\pi^+)$ in the region of $Q^2 = 5 - 9.5$ GeV$^2$ are necessary to prove the existence of $\pi$CT.

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Figure 1: (color online). The calculated nuclear transparency of the proton $T_A(p)$ vs. photon virtuality $Q^2$. The dashed curve denotes $T_A(p)$ calculated using Glauber model (GM). The dot-dot-dashed and dot-dashed curves illustrate the proton color transparency ($pCT$) for two different values of $\Delta M^2$, see text. The solid curves arise due to the inclusion of the short-range correlation (SRC) in GM. The data are taken from Refs. [9]-[11].
Figure 2: (color online). Same as those presented in Fig. 1 but for the pion. The data are taken from Refs. [16].