Parametrically excited surface waves in magnetic fluids:
observation of domain structures

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Abstract

Observations of parametrically excited surface waves in a magnetic fluid are presented. Under the influence of a magnetic field these waves have a non-monotonic dispersion relation, which leads to a richer behavior than in ordinary liquids. We report observation of three novel effects, namely: i) domain structures, ii) oscillating defects and iii) relaxational phase oscillations.
“Nam caelo terras et terris abscidit undas et liquidum spisso secrevit ab aere caelum” [1]. Ovid describes here a vision of the origin of the earth, arising from an unstructured ground state. The idea appears amazingly modern: A physicist would use the term “spontaneous symmetry–breaking” to describe such a process, and his vision of the origin of the universe is apparently similar. Spontaneous formation of patterns from unstructured ground states are investigated systematically within the field of nonequilibrium pattern formation [2]. The most popular example are surface waves generated by wind blowing over water. Such waves have to be excited in a spatially homogenous manner — e. g. via parametric excitation generated by vertical vibration — in order to allow for spontaneous symmetry–breaking processes [3].

A separation of the surface wave into different phases has recently been predicted for parametrically excited surface waves [4]. This intriguing result is due to a non–monotonic dispersion relation for surface waves, which means that up to three different wave numbers can be excited with one single driving frequency. The ensuing competition between these different wave numbers is the reason for a phase separation in the form of domain structures. A magnetic fluid is an experimental system with a non–monotonic dispersion relation for surface waves [5]. This Letter reports the first experimental observation of the theoretically predicted domain structures using that material. Furthermore, two novel dynamic states caused by the nonlinear interaction of the surface waves are described, which are presumably due to the unusual dispersion relation of that complex fluid.

The experimental setup is shown in Fig. 1. A V–shaped circular channel was machined into a Teflon dish. The channel has a diameter of 60 mm, a depth of 5 mm and an upper width of 4 mm, which is smaller than the typical wavelength in the experiment [5]. The upper part of the channel has a slope of 15° to the horizontal in order to suppress surface waves in radial direction. The border of the dish has a slope of 45°, and is used as a screen, where the shadow of the surface waves of the fluid is projected. A light bulb placed in the center of the channel casts a shadow of the surface on the screen.

The channel is filled with the magnetic fluid EMG 909, Lot. No. F081996A (Ferrofluidics).
The properties of the fluid are: density $\rho = 1020 \text{kgm}^{-3}$, surface tension $\sigma = 26.5 \cdot 10^{-3} \text{Nm}^{-1}$, initial magnetic permeability $\mu = 1.8$, magnetic saturation $M_s = 1.6 \cdot 10^4 \text{Am}^{-1}$ and dynamic viscosity $\eta = 6 \cdot 10^{-3} \text{Nsm}^{-2}$. The channel is placed in the center of a pair of Helmholtz–coils (Oswald–Magnetfeldtechnik), with an inner diameter of 40 cm. Each coil consists of 474 windings of flat copper wire with a width of 4.5 mm and thickness 2.5 mm. A current of about 10 A is supplied by a linear amplifier (fug NLN 5200 M-260). The magnetic field is monitored by means of a hall probe (Group 3 DTM-141 Digital Teslameter) located near the surface of the channel.

Control and analysis of the experiment is done with a 90 MHz Pentium–PC, equipped with a $512 \times 512$ 8–bit frame grabber (Data Translation DT2853), and a synthesizer–board (WSB-10). The sine–signal of the synthesizer–board is amplified by a linear amplifier (Brüel & Kjaer Power Amplifier Type 2712) and controls the oscillation of the vibration–exciter (Brüel & Kjaer PM Vibration Exciter Type 4808), which drives the channel in vertical direction. For sufficiently large driving amplitudes the flat surface becomes unstable and standing waves ensue.

The shadow of the standing waves is detected by a CCD–camera (Philips LDH 0600/00) placed 100 cm above the center of the channel. The camera operates in the interlaced mode at 50 Hz using an exposure time of 40 ms and supplies the frame grabber with images. A typical image is shown in Fig. 2. One can see the bulb in the center of the dish. A screen on the top of the bulb avoids direct light emission into the camera. The real channel is only seen as a black ring in the image, but the shadow of the surface waves can be seen clearly.

To observe the time evolution of the surface we use a phase–locked technique between the driving and the sampling, where the synthesizer–board is triggered by the $64 \mu s$ signal of the horizontal synchronization of the camera. This phase–locking enables us to analyze the dynamics of the fluid surface in a Poincare section, where the sampling phase is chosen to observe maximum amplitude of the standing waves. The data reduction of the two–dimensional camera image to one line is described in Ref. [5].

In Ref. [4] it is suggested, that one could obtain domains of coexisting wave numbers in a
fluid with non–monotonic dispersion relation for surface waves by quenching. In our system three parameters could be quenched: the driving frequency, the amplitude, and the magnetic field. The last one is most convenient experimentally. Therefore, in our measurements we fix the driving frequency \( f_D \) and start with a magnetic field of \( H = 0.85 H_C \), where \( H_C \) is the critical field for the onset of the Rosensweig–Instability [6], where the flat surface is unstable in a static magnetic field. Then, the mechanical driving is increased to a value 10% above the onset of standing waves, which oscillate at a frequency \( f = f_D / 2 \). For fixed driving frequency and amplitude the magnetic field \( H \) serves as the control parameter. We perform jumps of \( H \) to quench the system into a bistable regime. The jumps are random since there is no prediction for the parameter values where the domains should arise.

A camera snapshot of the channel with domains of excited surface waves with different wavelengths is shown in Fig. 2. In the upper right quadrant the wavelength is larger than it is in the rest of the channel. From images like this we extract the amplitude of the standing wave as a function of azimuthal position and time [5]. One example, measured at \( f_D = 20.8 \) Hz, is presented in Fig. 3. The left diagram shows the time evolution of the surface along the azimuthal position at a constant phase. Black parts in this diagram correspond to wave crests, where white parts correspond to wave troughs or a flat surface. The right diagram presents the Fourier–spectrum obtained by a one–dimensional Fast Fourier Transformation [7]. The scale at the top gives the wave number \( k_{SI} \) in SI–units, while the bottom scale indicates the modes \( k \) in the Fourier–space. For a homogeneous system \( k \) is the number of waves in the channel. At \( t = 0 \) the magnetic field is \( H = 0.86 H_C \), and steady stable waves are present. The standing wave is homogeneous, 31 wave crests are counted. A jump to \( H = 0.99 H_C \) leads to a flat surface almost instantaneously, and subsequently to the creation of two domains with different wave numbers. At \( t = 200 \) s the two wave numbers are \( k_1 = 34 \) and \( k_2 = 46 \). The two domains can be seen in the space–time–diagram left. The Fourier–spectrum right indicates the two corresponding modes. A jump back to \( H = 0.86 H_C \) leads to the destruction of the two domains. Only one homogeneous wave number \( k = 30 \) results. The dynamics is similar to that of the first transition only in
the short wave domain: the amplitude vanishes almost instantaneously. The long wave
domain on the other hand reorganizes comparatively slowly. This is the first experimental
demonstration of the existence of domain structures, which stem from the non-monotonic
dispersion relation for surface waves in a ferrofluid. This phenomenon was predicted in
Ref. [4].

Domains of coexisting wave numbers are not only observed for magnetic fields below
$H_C$, where this theory should be applicable, but also for higher values of the magnetic field.
A corresponding observation is presented in Fig. [1]. Here, the driving frequency again is
$20.8$ Hz and the magnetic field $H = 1.13 H_C$. A jump of the field to an even higher value
of $H = 1.16 H_C$ again induces domains, and the dynamics of the transition is similar to
that observed at smaller field. The wave numbers in the steady state are now very different,
$k_1 = 51$ and $k_2 = 87$. The ratio of the two wave numbers is larger than for Fig. [3] and the
amplitude modulation is even stronger, thus the waves in the $k_2 = 87$ – domain are hardly
visible in the plot.

The domains presented so far are in principal explained by theoretical considerations. In
addition we would like to present two novel oscillatory modes, which go beyond that theory.
The first example is shown in Fig. [5] starting at a homogeneous state at wave number $k = 33$
and $H = 0.92 H_C$ the field is increased to $H = 0.94 H_C$. The resulting state is now time–
dependent, a defect occurs, oscillating in its position with a period of about 45 s. In addition
other states have been observed, where the oscillation amplitude is modulated in time.

Another time dependent state, named relaxational phase oscillation, is presented in
Fig. [6]. Here we start from homogeneous state with wave number $k = 27$ at the mag-
netic field $H = 1.01 H_C$. Increasing the field to $H = 1.08 H_C$ leads to a time–dependent
state with an interesting dynamics. The first step consists of a fast decay of the amplitude.
A short wave subsequently grows, which is unstable leading to the destruction of a few wave
crests. The resulting homogeneous state with a smaller wave number is again unstable. It
leads to a sudden crash of the standing wave amplitude and the cycle repeats. The oscillat-
ing behavior of the phase with its fast and slow time scales is very reminiscent of a relaxation
oscillation.

The oscillations presented here are two examples of many different oscillatory modes. Systematic measurements have been made to investigate the range of existence of domains and the phase diagram is presented in Fig. 7. Apparently domains are created near $H = H_C$ for a driving frequency of 20 Hz. This corresponds to 10 Hz of the parametrically driven surface waves, which oscillate at half the driving frequency. 10 Hz is approximately the frequency, where the non-monotonic dispersion relation leads to competing wave numbers. This phase diagram is intended to provoke more quantitative theoretical studies, which might also lead to an understanding of the novel wave phenomena presented here, namely the oscillating defects and the relaxational phase oscillations.

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FIG. 1. Experimental setup.
FIG. 2. Snapshot of the channel. The shadows of the standing waves are visible. In this image there are two domains of different wave numbers, a longer wave in the upper right quadrant of the channel and a smaller wave in the rest of the channel. The magnetic field is $H = 0.96 \, H_C$, wave number at the onset of the static instability $k_C = 56$. 
FIG. 3. Observation of domains for $H < H_C$ at $f_D = 20.8$ Hz. Left diagram: the surface is observed at constant phase. Black parts correspond to wave crests, whereas white parts correspond to wave troughs or a flat surface. Right diagram: the corresponding spectrum is shown, obtained by a FFT. Between both diagrams the value of the magnetic field is indicated. One can clearly see that domains of different wavelengths exist at $H = 0.99 H_C$, $H_C = 19000$ Am$^{-1}$.
FIG. 4. Observation of domains for $H > H_C$ at $f_D = 20.8$ Hz. See figure capture of Fig. 3.
FIG. 5. Observation of oscillating defects at $f_D = 20.8$ Hz. See figure capture of Fig. [3].
FIG. 6. Observation of relaxational phase oscillations. No external parameter is changed for $t > 40s$. The driving frequency is $f_D = 17.8$ Hz. See figure capture of Fig. [3].
FIG. 7. Phase-diagram of coexisting domains with different wave numbers as function of the driving frequency $f_D$ and the magnetic field $H$. 