A mechanism for the $T$-odd pion fragmentation function

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We consider a simple rescattering mechanism to calculate a leading twist $T$-odd pion fragmentation function, a favored candidate for filtering the transversity properties of the nucleon. We evaluate the single spin azimuthal asymmetry for a transversely polarized target in semi-inclusive deep inelastic scattering (for HERMES kinematics). Additionally, we calculate the double $T$-odd $\cos 2\phi$ asymmetry in this framework.

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I. INTRODUCTION

The transversity distribution, $h_1$ (also known as $\delta q$), which measures the probability to find a transversely polarized quark in the transversely polarized nucleon, is as important for the description of the internal nucleon structure and its spin properties as the more familiar longitudinal distribution function, $g_1$. However, it still remains unmeasured, unlike the spin-average and helicity distribution functions, which are known experimentally and extensively modeled theoretically. The difficulty is that $h_1$ is a chiral-odd function, and consequently suppressed in inclusive deep inelastic scattering (DIS) processes; it has to be accompanied by a second chiral-odd quantity. In semi-inclusive deep inelastic scattering (SIDIS) of transversely polarized nucleons several methods have been proposed to access transversity distributions. The more promising one relies on the so called Collins fragmentation function $F_2$, which correlates the transverse spin of the fragmenting quark to the transverse momentum of the produced hadron. Beside being chiral-odd, this fragmentation function is also time-reversal odd ($T$-odd, see e.g. [3, 4]), which makes its calculation challenging. Earlier, in addition to the Collins parameterization [5], a theoretical attempt was made to estimate the Collins function for pions [6]. More recently, a non-vanishing $T$-odd fragmentation function was obtained through a consistent one-loop calculation, where massive constituent quarks and pions are the only effective degrees of freedom [7]. In addition to parameterizations from data indicating a non-zero Collins function $F_2$, the non-zero single spin asymmetries in recent measurements [8, 9, 10] signal the existence of a non-trivial $T$-odd effects. These indications of the $T$-odd fragmentation functions taken together call for deeper investigations, both theoretical and experimental.

In this paper we explore an alternative one-gluon exchange mechanism, for the fragmentation of a transversely polarized quark into a spinless hadron similar to the approach we applied [11, 12] to the distribution of the transversely polarized quarks in the both unpolarized and transversely polarized nucleon (in this context see also [13, 14, 15]). Within this consistent framework we now make predictions of the leading twist $T$-odd pion fragmentation function, and the resulting single transverse spin azimuthal asymmetry, $\sin(\phi + \phi_s)$, as well as the spin-independent, $\cos 2\phi$, asymmetry in SIDIS.

II. THE MECHANISM

The non-perturbative information about the quark content of the target and the fragmentation of quarks into hadrons in SIDIS is encoded in the general form of the factorized cross sections in terms of the quark distributions $\Phi(p)$ and fragmentation functions $\Delta(k)$, entering the hadronic tensor

$$M_{\mu\nu}(P, P_h, q) = \int d^4k d^4p d^4(k + q - p) \times \text{Tr} (\gamma^\mu \Phi(k) \gamma^\nu \Delta(p)) + \left( \begin{array}{c} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{array} \right) ,$$

(1)

to leading order in $1/Q^2$ [16]. Here $k$ and $p$ are the quark scattering and fragmenting momenta and $P$ and $P_h$ are the target and fragmented hadron momenta respectively. Further, $\Phi$ is given by

$$\Phi(p, P) = \frac{1}{2} \int dp^- \Phi(p, P)|_{p^+ = \pm P^+}$$

$$= \sum_X \int \frac{d^2\xi^+ d^2\xi^-}{(2\pi)^3} e^{i\nu \xi^+} \langle P|\psi(\xi^+, \xi^-)G^{\dagger}_{(\xi^+, -\xi^-)}|X \rangle$$

$$\langle X|G_{(0, \infty)}\psi(0)|P \rangle |_{\xi^+ = 0}.$$  

(2)
The path ordered gauge link operator in Eq. (4) comes from the first non-trivial term in expanding asymmetries in semi-inclusive pion electroproduction. This generates the rules for such processes \cite{17, 20}. Momentum dependent quark distributions, implicit in those \cite{11, 12, 13, 14, 15, 18, 19}, have been reported in the literature \cite{11, 12, 18, 19}. Metries and single spin asymmetries (SSAs) that have \cite{5} \( \phi \) \( \text{III. PION FRAGMENTATION FUNCTION} \)

With the tree level contribution vanishing for \( T \)-odd functions, the leading order contributions to the (see \cite{12} for the quark distribution) \( T \)-odd fragmentation functions come from the first non-trivial term in expanding the path ordered gauge link operator in Eq. (4)

\[
\Delta(k, P_h) = \frac{1}{4 z} \int \frac{d k^+ \Delta(k, P_h)}{k^-} = \sum_X \int \frac{d \xi^+ d^2 \xi}{2 z (2 \pi)^3} e^{i k^+ \xi} \langle X; P_h | 0 \rangle \langle 0 | \mathcal{G}_{[\xi^+, -\infty]} \psi(\xi) | X; P_h \rangle \langle X; P_h | \overline{\psi}(0) \mathcal{G}_{[0, -\infty]}^\dagger | 0 \rangle |_{\xi^- = 0}, \tag{3}
\]

where the path ordered exponential along the light like direction \( \xi^- \) is

\[
\mathcal{G}_{[\xi, -\infty]} = \mathcal{P} \exp \left( -i g \int_{\xi}^{\infty} d \xi^- A^+(\xi) \right), \tag{4}
\]

and \( \{ |X \rangle \} \) represents a complete set of states.

The path ordered light-cone link operator is necessary to maintain gauge invariance and appears to resolve the momentum dependence of the initial state quark depicted in Fig. 1, is given by the following equation:

\[
\langle 0 | \psi(0) | P; X \rangle = \left( \frac{i}{k - m} \right) \Upsilon(k_i^2) u(k - P_e, s), \tag{6}
\]

where

\[
\Upsilon(k_i^2) = i \gamma_5 f_{qq\pi} \mathcal{N} e^{-b' k^2}. \tag{7}
\]

We have introduced a Gaussian distribution in the transverse momentum dependence of the quark-spectator-pion vertex \cite{12, 21} in order to address the log divergence that arises from the moments of fragmentation functions. Here, \( f_{qq\pi} \) (defined henceforth as \( f \)) is the quark-pion coupling and \( k \) is the momentum of the off-shell quark, \( k_1 \) and \( b' = 1/ < k_1^2 > \), are the intrinsic transverse momentum and its inverse mean square, respectively, \( \mathcal{N} \) is a dimensionful normalization, and finally, \( u(p, s) \) is the on-shell quark spinor. The Collins function, is projected from Eq. (6)

\[
\Delta^{e^{-i + \gamma_5}}(z, k_\perp) = \frac{1}{4 z} \int d k^+ \text{Tr} \left( \gamma^- - \gamma^\perp \gamma_5 \Delta \right) |_{k^- = 0}. \tag{8}
\]

The leading order (in \( 1/Q \)) one loop contribution to Eq. (5), which arises in the limit that the - light-cone component of the quark's momentum goes to infinity (see e.g. \cite{13}), corresponding to the rescattering of the initial state quark depicted in Fig. 1 is given by the expression

\[
\Delta(k, P_e) = \int \frac{d^4 q}{(2 \pi)^4} \frac{\not{k} - \not{q} + m}{(k - q)^2 - m^2} \frac{N' f_{qq\pi} (\not{k} + \not{P_e} + \mu)}{(k - q - P_e)^2 - \mu^2} \frac{1}{q^- + i \epsilon q^2 - \lambda^2_q + i \epsilon} \delta((k - P_e)^2 - \mu^2) \frac{N' f_{qq\pi} (\not{k} + m)}{k^2 - m^2}. \tag{9}
\]
where we have taken the sum over states to be saturated by a single spectator quark with effective mass $\mu$. The off-shell fragmenting quark’s momentum is given by

$$k^2 = \frac{z k_f^2}{1-z} + \frac{\mu^2}{1-z} + \frac{M^2_{\perp}}{z},$$

(10)

which results in the correct counting rules for scalar meson production in limit $z \to 1$. The details of the loop integration are similar to those performed in [11, 12]. We evaluate the projection $\Delta^{(\pi^+ + \pi^-)}$, which results in the leading twist, $T$-odd contribution

$$H^{\perp}_{1}(z, k^2) = \frac{N_q q^2}{(2\pi)^2} \frac{g^2}{4 z} \frac{1}{\Lambda'^{(k^2_{\perp})}} \frac{M_{\perp}}{k^2_{\perp}} e^{-b' \left(k^2_{\perp} - \Lambda'^{(0)}\right)} \left[\Gamma(0, b \Lambda'(0)) - \Gamma(0, b' \Lambda'(k^2_{\perp}))\right],$$

(11)

where, $\Lambda'(k^2_{\perp}) = k^2_{\perp} + \frac{1-z}{z^2} M^2_{\perp} + \frac{\mu^2}{z} - \frac{1}{z} m^2$. The average $< k^2_{\perp} >$ or $b'$ is a regulating scale which we fix to the expression for the integrated unpolarized fragmentation function

$$D_1(z) = \frac{N_q q^2}{4 (2\pi)^2} \frac{g^2}{z} \left( \frac{m^2 - \Lambda'(0)}{\Lambda'(0)} \right) \left\{ - 2b' \left( m^2 - \Lambda'(0) \right) - 1 \right\} e^{2b \Lambda'(0)} \Gamma(0, b' \Lambda'(0)),$$

(12)

which, multiplied by $z$ at $< k^2_{\perp} > = (0.5)^2 \text{GeV}^2$ and $\mu = \mu$, is in good agreement with the distribution of Ref. [27].

It is important to emphasize that the kinematics of the decaying quark Eq. [10] enforces the proper dimensional counting rules [24, 26].

The chiral odd transversity distribution, $h_1$, for a scalar spectator diquark in the quark-diquark model of the nucleon is

$$h_1(x) = \frac{g^2 N_c^2}{4(2\pi)^2} \left(1 - x\right) \times (m + x M)^2 \left\{ \frac{1}{\Lambda(0)} - 2 e^{2b \Lambda(0)} \Gamma(0, 2b \Lambda(0)) \right\}.$$

(13)

and the previously calculated [12] unpolarized distribution is

$$f(x) = \frac{g^2 N_c^2}{4(2\pi)^2} \left(1 - x\right) \left\{ \frac{(m + x M)^2 - \Lambda(0)}{\Lambda(0)} \right. \left. - 2b \left( (m + x M)^2 - \Lambda(0) \right) - 1 \right\} e^{2b \Lambda(0)} \Gamma(0, 2b \Lambda(0)),$$

(14)

where $g$ contains the gluon-scalar diquark coupling, and $\Lambda(0) = (1 - x) m^2 + x \lambda^2 - x(1 - x) M^2$, while $M$, $m$, and $\lambda$

are the nucleon, quark, and diquark masses respectively. Choosing $< p^2_{\perp} > = (0.4)^2 \text{GeV}^2 = 1/b$ yields good agreement with the valence distribution of Ref. [28].

We consider this to be a reasonable phenomenological framework, which avoids the log divergence [12, 24] involved in integrating over $k_{\perp}$ and $p_{\perp}$, while introducing an average transverse momentum determined from spin averaged scattering [24, 28]. Additionally, this form factor approach is compatible with the parameterization of the fragmentation functions employed in references [31, 32] to set the Gaussian width for the fragmentation function. In Fig. 2 the weighted analyzing power, $H^{(1)}_{1}(z)/D_1(z)$, is displayed. The resulting behavior is similar to a previous model ansatz proposed by Collins and calculated in Ref. [31].

IV. AZIMUTHAL ASYMMETRIES

We discuss the explicit result and numerical evaluation of the single transverse-spin $\sin(\phi + \phi_s)$ and double $T$-odd $\cos 2\phi$ asymmetries for $\pi^+$ production in SIDIS.

The $\cos 2\phi$ asymmetry of SIDIS is projected out of the cross section and depends on a leading double $T$-odd product,

$$\left| \frac{P_{h_{1}}^2}{MM_{\pi}} \cos 2\phi \right|_{UU} = \frac{1}{\int d^2 P_{h_{1}} \cos 2\phi d\sigma} \int d^2 P_{h_{1}} \cos 2\phi d\sigma$$

$$= \frac{8(1 - y) \sum_{q} e_{q}^{2} f_{1}(x) z^{2} H^{(1)}_{1}(z)}{(1 + (1 - y)^2) \sum_{q} e_{q}^{2} f_{1}(x) D_{1}(z)},$$

(15)

where the subscript $UU$ indicates unpolarized beam and target (Note: The non-vanishing $\cos 2\phi$ asymmetry originating from the $T$-even distribution and fragmentation function appears only at order $1/Q^2 [32, 33, 34]$).
on variables \(z\). The SIDIS differential cross section depends respectively.

For a transversely polarized target nucleon, the chiral-odd (both azimuthal integration, yielding, the convolution of two azimuthal angles, of the target spin projection \(S\) the virtual photon momentum direction. \(\phi_s\) and \(\phi\) are the azimuthal angles, of the target spin projection \(S_T\) and \(P_{h\perp}\) respectively.

\[
h_1^{(1)}(z) = \text{the weighted moment of the distribution function} \quad (3, 12). \quad \text{The SIDIS differential cross section depends on variables } x, y, z \text{ and azimuthal angles } \phi \text{ and } \phi_s \text{ (see Fig. 3). For a transversely polarized target nucleon, the } \sin(\phi + \phi_s) \text{ asymmetry can be projected out with an azimuthal integration, yielding, the convolution of two chiral-odd (both } T\text{-odd and } T\text{-even) structures,}
\]

\[
\frac{\langle P_{h\perp} \sin(\phi + \phi_s) \rangle_{UT}}{M_\pi} = \frac{\int d\phi_s \int d^2 P_{h\perp} P_{h\perp} \sin(\phi + \phi_s) (d\sigma^+ - d\sigma^-)}{\int d\phi_s \int d^2 P_{h\perp} (d\sigma^+ + d\sigma^-)} = |S_T| \frac{2(1 - y) \sum_q e_q^2 h_1(x) z H_1^{(1)}(z)}{(1 + (1 - y)^2) \sum_q e_q^2 f_1(x) D_1(z)}.
\]

\[
(16)
\]

We define the variable range to coincide with the HERMES kinematics, 1 GeV\(^2\) \(\leq Q^2 \leq 15\) GeV\(^2\), 4.5 GeV \(\leq E_x \leq 13.5\) GeV, 0.2 \(\leq z \leq 0.7\), 0.2 \(\leq y \leq 0.8\). In Fig. 3 the \(\frac{\langle P_{h\perp} \sin(\phi + \phi_s) \rangle_{UT}}{M_\pi}\) asymmetry for \(\pi^+\) production on a proton target is presented as a function of \(x\) and \(z\), respectively (using \(\Lambda_{QCD} = 0.2\) GeV) Fig. 4 indicates approximately a 10 - 15\% \(P_{h\perp}/M_\pi\) weighted \(\sin(\phi + \phi_s)\) asymmetry. Similarly, in Fig. 5 the \(P_{h\perp}^2/(M_\pi M_\pi)\) weighted \(\cos2\phi\) asymmetry of Eqs. 16 for \(\pi^+\) production on an unpolarized proton target is presented as a function of \(x\) and \(z\), respectively. Fig. 6 indicates a few percent asymmetry.

\[\text{FIG. 3: The kinematics of semi-inclusive DIS: } k_1 (k_2) \text{ is the 4-momentum of the incoming (outgoing) charged lepton, where } q = k_1 - k_2, \text{ the 4-momentum of the virtual photon. } P (P_{h\perp}) \text{ is the momentum of the target (observed) hadron. The scaling variables are } x = Q^2/2P \cdot q, \text{ } y = P \cdot q/P \cdot k_1, \text{ and } z = P \cdot P_{h\perp}/P \cdot q. \text{ The momentum } k_{1T} (P_{h\perp}) \text{ is the incoming lepton (observed hadron) momentum component perpendicular to the virtual photon momentum direction. } \phi_s \text{ and } \phi \text{ are the azimuthal angles, of the target spin projection } S_T \text{ and } P_{h\perp} \text{ respectively.}
\]

\[\text{FIG. 4: Upper Panel: The } \langle \frac{P_{h\perp}}{M_\pi} \sin(\phi + \phi_s) \rangle_{UT} \text{ asymmetry for } \pi^+ \text{ production as a function of } x. \text{ Lower Panel: The } \langle \frac{P_{h\perp}}{M_\pi} \sin(\phi + \phi_s) \rangle_{UT} \text{ asymmetry for } \pi^+ \text{ production as a function of } z.\]

\[\text{V. CONCLUSION}\]

A mechanism to generate the \(T\)-odd Collins fragmentation function that is derived from the gauge link has been considered. This approach complements and is consistent with the approach that was employed to generate the Sivers \(f_{1T}^{qg}\) and the chiral odd unpolarized \(h_1^{(1)}\) distribution functions that fuel the Sivers and \(\cos2\phi\) asymmetries. In order to consistently calculate these asymmetries it is advantageous to generate the Collins fragmentation function from this framework. The derivation of the Collins function is consistent with the observation that intrinsic transverse quark momenta and angular momentum conservation are intimately tied with studies of transversity \(h_1^{(1)}\). This was demonstrated previously from analyzes of the tensor charge in the context of the axial-vector dominance approach to exclusive meson photo-
thermore, this approach is interesting in that it does not suffer from the possible cancellation of the Collins effect cited in [37]; namely, that phases from final state interactions of the pions with the spectator remnant will sum to zero. This mechanism does not rely on multiple interactions with the outgoing pion. On the contrary, the effect is generated in the non-trivial phase associated with the gauge link operator.

In the case of unpolarized beam and target, we have predicted that at HERMES energies there is a non-trivial $\cos 2\phi$ asymmetry associated with the asymmetric distributions of transversely polarized quarks inside unpolarized hadrons. We have evaluated the analyzing power and predicted the $P_{h\bot}/M_\pi$ weighted $\sin(\phi + \phi_S)$ asymmetry. We note also that the analyzing power, $H_{\bot}^{(1)}(z)/D_1(z)$ displays behavior characteristic of previous results where other methods were used to characterize the Collins mechanism. This ratio is consistent with the Collins ansatz [2, 31]. Generalizing from these model calculations, it is clear that initial and final state interactions can account for leading twist $T$-odd contributions to SSAs. In addition, while it has been shown that other mechanisms, ranging from initial state interactions to the non-trivial phases of light-cone wave functions [13, 22, 39], can give rise to SSAs, these various mechanisms can be understood in the context of gauge fixing as it impacts the gauge link operator in the transverse momentum quark distribution functions.

Note added in proof: After our manuscript was submitted for publication a paper appeared on a similar subject [40].

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