Leptogenesis in realistic SO(10) models

Xiangdong Ji\textsuperscript{a,b}, Yingchuan Li\textsuperscript{a,*}, R.N. Mohapatra\textsuperscript{a}, S. Nasri\textsuperscript{a}, Yue Zhang\textsuperscript{b}

\textsuperscript{a} Department of Physics, University of Maryland, College Park, MD 20742, USA
\textsuperscript{b} Institute for Theoretical Physics, Peking University, Beijing, China

Received 16 May 2007; received in revised form 4 June 2007; accepted 10 June 2007
Available online 13 June 2007
Editor: T. Yanagida

Abstract

We study the origin of baryonic matter via leptogenesis in realistic SO(10) models, in particular, in a new lopsided mass matrix model introduced recently by three of the authors. It is shown that the model generates sufficient baryon asymmetry in addition to fitting to fermion masses and mixing. We compare this result with other realistic SO(10) models.

© 2007 Elsevier B.V. Open access under CC BY license.

1. Introduction

One of the most fundamental questions in modern cosmology is where the baryon number asymmetry in today’s Universe comes from. It is an attractive idea to assume that it arises dynamically in a symmetric big-bang model through C and CP violating processes out of thermal equilibrium. Of the several proposals that use this scenario, leptogenesis \cite{1} has emerged as one of the most interesting scenarios for baryon generation. Here the baryon number is produced through the so-called sphaleron processes from a residual lepton number density left over from an early stage of the universe through lepton number violating decays of right-handed neutrinos. Such connection is extremely interesting since the same heavy right-handed neutrinos are also important ingredients for understanding the small left-handed neutrino masses through the so-called see-saw mechanism \cite{2}.

Recently much study has been made in the literature about the feasibility of such a scenario \cite{3}. Although one can discuss leptogenesis and neutrino mass texture in models without introducing quark degrees of freedom, the most interesting theoretical frameworks involve quark–lepton unification, possibly under supersymmetric formalism. In addition to connecting the origin of matter to the big picture of particle physics, the grand unified theory (GUT) models build in more constraints and have better predictive power \cite{4}. For various reasons, SUSY SO(10) has been a favored framework to unify physics beyond the Standard Model (for a recent review of SO(10) models, see \cite{5}). Depending upon which set of Higgs multiplets is chosen to break the GUT group and electroweak symmetry, two classes of SO(10) models are most studied in the literature: One uses 10\textsubscript{H}, 126\textsubscript{H}, 126\textsubscript{H} and 210\textsubscript{H} \cite{6–9}, and the other uses 10\textsubscript{H}, 16\textsubscript{H}, 16\textsubscript{H} and 45\textsubscript{H} \cite{10–13}. While most of these models are quite successful in fitting and predicting the known experimental masses and mixing angles of leptons and quarks, they predict very different values for the poorly-known neutrino mixing angle $\theta_{13}$. Quite interestingly, they also paint different pictures for leptogenesis.

In this Letter, we are mostly interested in the leptogenesis in a lopsided SO(10) mass matrix model proposed recently by three of us (X.J, Y.L. and R.N.M.) \cite{13}. The model is a modification of the lopsided model originally proposed by Albright, Babu and Barr \cite{11}. The lopsidedness built within the Yukawa couplings between the second and third families generates, among other interesting physical consequences, the large atmospheric-neutrino mixing angle $\theta_{23}$ while keeping $V_{cb}$ in the Cabibbo–Kobayashi–Maskawa (CKM) matrix small. The most characteristic property of the model with lopsided structure is that there is a large mixing

\* Corresponding author.
E-mail address: yli@physics.umd.edu (Y. Li).

0370-2693 © 2007 Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2007.06.024
of right-handed down-type quarks associated with the large atmospheric-neutrino mixing. This large mixing of right-handed down-type quarks could induce large $b \to s$ transitions in the supersymmetric theory, which makes the model testable in the B decays [14].

In the modified version, the right-handed neutrino mass matrix has a simple diagonal form. The large solar mixing angle is mainly generated from the neutrino Dirac mass matrices. The prediction of this model for $\sin \theta_{13}$ extending to the first family, $m$ approximately the same as those in the original lopsided model, and thus the successful prediction for the mass ratios $\sigma$ parameters in the CKM matrix are used to determine 10 parameters: $\epsilon$, $\delta$, $\delta'$ generated for left-handed up-type quarks and neutrinos are proportional to the ratios $\rho/\omega$. Produced without requiring the two heavy right-handed neutrino masses, $M_1$ and $M_2$, to be quasi-degenerate.

The presentation of the Letter is as follows. In Section 2, we review the new lopsided SO(10) model. In Section 3, we consider in detail lepton genesis in this model. In Section 4, we make a comparison of the leptonogenesis with other SO(10) models, emphasizing similarities and differences. We present our conclusions in Section 5.

2. A new lopsided SO(10) model

A new SUSY SO(10) GUT model with lopsided mass matrices was introduced in [13]. In this section, we briefly describe its physics. The model was obtained by modifying the right-handed neutrino, up-quark, and left-handed neutrino mass matrices of the original lopsided model of Albright and Barr [11,15,16], which we will describe briefly in Section 4. The modified lopsided model assumes that the right-handed neutrino Majorana mass matrix $M_R$ has a simple diagonal structure, and introduces additional off-diagonal couplings in the upper-type-quark and neutrino Dirac mass matrices to generate 1–2 (solar angle) rotation. All the fermion masses and mixing angles can be fitted well in the new model. The mixing angle $\theta_{13}$, however, is close to the upper limit from the CHOOZ experiment and therefore definitely within the range of next generation reactor experiments.

We use the convention that Yukawa couplings in the Lagrangian appear as

$$\mathcal{L} = - \bar{Q}_i H Y_{ij} d_{Rj} + \cdots.$$  

Then the fermion mass matrices are $M_{ij} = v Y_{ij}$ with $v = 174$ GeV. Through couplings with a set of Higgs multiplets $10_H$, $16_H$, $\overline{10}_H$ and $45_H$, the up-type-quark, down-type-quark, charged-lepton and neutrino Dirac and Majorana mass matrices in the model of Ref. [13] have the following forms,

$$M_u = \begin{pmatrix} \eta & \kappa - \rho/3 & \kappa + \rho/3 \\ 0 & \omega_1 & \omega_2 \\ \kappa + \rho/3 & \omega_1 & \omega_2 \end{pmatrix} A_U, \quad M_{vD} = \begin{pmatrix} \eta & 0 & \kappa + \rho \\ 0 & \omega_1 & \omega_2 \\ \kappa + \rho & \omega_1 & \omega_2 \end{pmatrix} A_U, \quad M_d = \begin{pmatrix} \eta & \delta & \delta e^{-i\phi} \\ \delta & 0 & \delta e^{-i\phi} \\ \delta e^{-i\phi} & \delta e^{-i\phi} & \omega \end{pmatrix} A_D, \quad M_l = \begin{pmatrix} \eta \delta & \delta e^{-i\phi} \\ \delta e^{-i\phi} & \omega \end{pmatrix} A_D. \quad M_{vR} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} A_R. \quad (2)$$

From the above, we get the Majorana mass matrix of left-handed neutrinos from the see-saw formula, $m_{vL} = -M_{vD} M_{vR}^{-1} M_{vD}^T$,

$$m_{vL} = -\begin{pmatrix} \frac{\eta^2}{\omega} + (\kappa + \rho) & \frac{\kappa + \rho}{\omega} & \frac{\eta(\kappa - \rho)}{\omega} \\ \frac{\kappa + \rho}{\omega} & \omega & \frac{\omega(\kappa - \rho)}{\omega} \\ \frac{\eta(\kappa - \rho)}{\omega} & \frac{\omega(\kappa - \rho)}{\omega} & \omega \end{pmatrix} A_D^{-1} / A_D. \quad (3)$$

The parameter $\sigma$ is one order-of-magnitude smaller than $\sigma$ and generates the hierarchy between the second and third families. In extending to the first family, $\delta$ and $\delta'$ were introduced into the $M_d$ and $M_l$. Usually a large rotation in the 1–2 sector is in conflict with the hierarchical property of the quark masses. However, in the above texture, the 1–2 rotation angle from $M_d$ to obtain the Cabibbo angle $\theta_c$, and thus the constraint from the up-type quark spectrum is avoided. The first two families in the $M_d$ and $M_{vR}$ are not coupled to each other directly but through couplings with the third family. The rotations in 1–2 sector generated for left-handed up-type quarks and neutrinos are proportional to the ratios $\gamma \equiv (\kappa - \rho/3)/\omega$ and $\gamma' \equiv (\kappa + \rho)/\omega$, respectively.

The procedure to fit various parameters to experimental data is as follows. First, the up-type quark and lepton spectra and the parameters in the CKM matrix are used to determine 10 parameters: $\sigma$, $\epsilon$, $\delta$, $\delta'$, $\phi$, $\omega$, $\gamma$, $\eta$, $A_U$ and $A_D$. The best fit yields $\sigma$ and $\epsilon$ approximately the same as those in the original lopsided model, and thus the successful prediction for the mass ratios $M^0_\nu / m^0_{\nu_1}$ and $m^0_{\nu_2} / m^0_{\nu_3}$ are kept. The down-type quark mass spectrum comes out as predictions. The present model constrains the neutrino mass spectrum as hierarchical. The mass difference $\Delta m^2_{\nu_{12}}$ is used to fix the right-handed neutrino mass scale $A_R$. 

X. Ji et al. / Physics Letters B 651 (2007) 195–207
We summarize our input and detailed fits as follows. For CKM matrix elements, we take $|V_{us}| = 0.224$, $|V_{ub}| = 0.0037$, $|V_{cb}| = 0.042$, and $\delta_{CP} = 60^\circ$ as inputs at the electroweak scale. With a running factor of 0.8853 for $|V_{ub}|$, and $|V_{cb}|$ taken into account, we have $|V_{ub}|^0 = 0.0033$ and $|V_{cb}|^0 = 0.037$ at the GUT scale. For charged lepton masses and up quark masses, we take the values at the GUT scale corresponding to $\tan \beta = 10$ from Ref. [17]. For neutrino oscillation data, we take the solar-neutrino angle to be $\theta_{\odot} = 32.5^\circ$ and mass square differences as $\Delta m^2_{12} = 7.9 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{23} = 2.4 \times 10^{-3}$ eV$^2$. The result for the 12 fitted parameters is

$$\begin{align*}
\sigma &= 1.83, \quad \epsilon = 0.1446, \quad \delta = 0.01, \\
\delta' &= 0.014, \quad \phi = 27.9^\circ, \quad \eta = 1.02 \times 10^{-5}, \\
\omega &= -0.0466, \quad \rho = 0.0092, \quad \kappa = 0.0191, \\
M_U &= 82.2 \text{ GeV}, \quad M_D = 583.5 \text{ MeV}, \quad \Lambda_R = 1.85 \times 10^{13} \text{ GeV}.
\end{align*}$$

There is a combined constraint on $a$ and $b$, and thus the right-handed Majorana mass spectrum is not well determined. As examples, if $a = b$, $a = -2.039 \times 10^{-3}$; and if $a = 1$, $b = -1.951 \times 10^{-3}$.

In our previous paper, we have taken $a = b$ and real. The results for the down-type quark masses and right-handed Majorana neutrino masses are as follows,

$$\begin{align*}
m_0^d &= 1.08 \text{ MeV}, \quad m_0^e = 25.97 \text{ MeV}, \quad m_0^\nu = 1.242 \text{ GeV}, \\
M_1 &= 3.77 \times 10^{10} \text{ GeV}, \quad M_2 = 3.77 \times 10^{10} \text{ GeV}, \quad M_3 = 1.85 \times 10^{13} \text{ GeV}.
\end{align*}$$

The predictions for the mixing angles in the PMNS matrix are,

$$\sin^2 \theta_{\text{atm}} = 0.49, \quad \sin^2 \theta_{13} = 0.074.$$  

(6)

If one releases the best-fit value of $\Delta m^2_{12}$ and $\Delta m^2_{23}$ and impose only the $3\sigma$ constraint as $7.1 \times 10^{-5}$ eV$^2 \leq \Delta m^2_{12} \leq 8.9 \times 10^{-5}$ eV$^2$ and $1.4 \times 10^{-3}$ eV$^2 \leq \Delta m^2_{23} \leq 3.3 \times 10^{-3}$ eV$^2$, one would obtain, $0.44 \leq \sin^2 \theta_{\text{atm}} \leq 0.52$ which is well within the $1\sigma$ limit, and $0.055 \leq \sin^2 2\theta_{13} \leq 0.110$ which, as a whole region, lies in the scope of the next generation of reactor experiments.

3. Leptogenesis

The baryon asymmetry of the universe is customarily defined as the ratio of the baryon density and the photon density after recombination, and has been measured to very good precision from the WMAP experiment [18]:

$$\eta_B = \frac{n_B}{n_\gamma} = (6.1 \pm 0.2) \times 10^{-10}.$$  

(7)

Interestingly, the big-bang nucleosynthesis is completely consistent with this determination.

To produce this asymmetry through leptogenesis, several considerations have to be addressed. First, what is the number of right-handed neutrinos decaying out of thermal equilibrium? The answer to this question is in principle depends on the thermal history of the right-handed neutrinos. In our model, it turns out that this dependence is rather weak because of the strong washout. Second, what is the lepton density generated from the right-handed neutrino decay? This, of course, is related to the CP asymmetry of the decay which depends on complex phases in the Yukawa interactions. Third, some of the generated lepton density gets washed out by inverse-decay processes and scattering. This effect can be rather important, particularly in the so-called strong washout region. Finally, one must calculate the percentage of lepton number density converted into the baryon number density through the electroweak sphaleron process. The answers to some of the questions are less model-dependent and are standard in the literature [19]. Here we focus on the parts depending on particular models for the right-handed neutrinos.

The density of leptons from right-handed neutrino decays is

$$n_L = \frac{3 \xi(3) g_N T^3}{4 \pi^2} \sum_{i=1}^{3} k_i \epsilon_i,$$

(8)

where the first factor is the thermal density of a relativistic fermion with $g_N = 2$ and the sum is over the number of right-handed neutrinos. The $\epsilon_i$ is the decay CP asymmetry of the $i$th right-handed neutrino; $k_i$ is the corresponding efficiency factor, taking into account the fraction of out-of-equilibrium decays and the washout effect. Both factors depend on the effective mass defined as

$$\bar{m}_i = \frac{(M_{\nu i} M_{\nu i}^\dagger)_{ii}}{M_i},$$

(9)
where $M'$ denotes the Dirac neutrino mass in the basis in which the right-handed neutrino matrix is diagonal, $M'_\nu = M_i U^*$. If $U$ diagonalizes $M_{\nu R}$, then

$$M_{\nu R} = U M U^T = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} U^T,$$

and the right-handed neutrino mass eigenstates are $\chi_i = \sum_f U_{fi} \chi_f$, where $\chi_f$ is the family basis. Reversing this relation, one has $\chi_f = \sum_i U_{fi}^* \chi_i$. Thus if the right-handed field $\chi_f$ is replaced by $\chi_i$ in the Yukawa coupling, the Yukawa matrix is multiplied on the right by $U^*$. Hence the above mass relation follows.

The lepton number is converted into the baryon number through the $B-L$ conserving electroweak sphaleron effect \[20\],

$$n_B = - \frac{8 N_G + 4 N_H}{22 N_G + 13 N_H} n_L,$$  \tag{11}

where $N_G = 3$ is the number of fermion families and $N_H$ is the number of Higgs doublets. In the Standard Model $N = 3$ and $N_H = 1$.

The photon density can be calculated from the entropy density $s = \frac{2}{3} g_* \pi^2 T^3$, where $g_*$ is the effective number of degrees of freedom, through the relation

$$s = \frac{4 \pi^2}{30} \left( \frac{2}{11} \pi^2 - 3 \right) n_\gamma,$$  \tag{12}

where the second factor takes into account the neutrino contribution. Ignoring the lightest right-handed neutrino contribution, $g_*$ is 106.75 in the Standard Model.

The final ratio of baryon to photon number density through leptogenesis is

$$\eta_B = \frac{n_B}{n_\gamma} = - \frac{516}{53009} \sum_i \kappa_i \epsilon_i = -0.0096 \sum_i \kappa_i \epsilon_i.$$  \tag{13}

Now we turn to the decay asymmetry and efficiency factors.

### 3.1. The CP asymmetry from right-handed neutrino decay

The right-handed neutrinos are assumed to be CP eigenstates in the absence of the Yukawa type of weak interactions. In the presence of the interactions, they can decay into both left-handed leptons (neutrino and charged leptons) plus Higgs bosons and right-handed antileptons plus Higgs bosons. In the leading order, the decay rate is

$$\Gamma_i = \frac{1}{8 \pi} (Y^\dagger Y)_{ii} M_i,$$  \tag{14}

where again, $Y' = Y U^*$ is the Yukawa matrix in the basis where the right-handed neutrinos are in mass eigenstates.

At next-to-leading order, the decay rates into leptons and antileptons are different due to the complex phases in the Yukawa couplings. The decay CP asymmetry is defined as

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow l_j H) - \Gamma(N_i \rightarrow \bar{l}_j H^\dagger)}{\Gamma(N_i \rightarrow l_j H) + \Gamma(N_i \rightarrow \bar{l}_j H^\dagger)}.$$  \tag{15}

In one-loop approximation, one finds,

$$\epsilon_i = \frac{1}{8 \pi} \sum_{j \neq i} F \left( \frac{M_j^2}{M_i^2} \right) \frac{\text{Im}[(Y^\dagger Y')_{ij}]}{(Y^\dagger Y')_{ii}},$$  \tag{16}

where the decay function is given by \[21\]

$$F(x) = \sqrt{x} \left[ \frac{1}{1-x} + 1 - (1+x) \ln \frac{1+x}{x} \right].$$  \tag{17}

In the limit of large $x$, this become $-3/2 \sqrt{x}$. The first term in $F$ is singular when two right-handed neutrinos become degenerate in mass, in which case, one must resum the self-energy corrections which lead to the so-called resonant leptogenesis.

To get a non-zero CP asymmetry, one needs to have complex phases in the mass matrices. In the model presented in the last section, we have assumed the right-handed neutrino mass matrix is real. Now we relax this assumption, and choose the simplest
Thus the masses of three right-handed neutrinos are

\[ M_{\nu_R} = \begin{pmatrix} a e^{i \alpha} & 0 & 0 \\ 0 & b e^{i \beta} & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R. \] (18)

We refit the parameters \( a, b, \alpha \) and \( \beta \) to reproduce the light-neutrino masses and mixing. Requiring to satisfy the 3\( \sigma \) range of neutrino oscillation data: \( 0.7 \leq \sin^2 2 \theta_{12} \leq 0.92; \sin^2 2 \theta_{23} \geq 0.87; \sin^2 \theta_{13} \leq 0.051; 7.1 \times 10^{-5} \text{ eV}^2 \leq \Delta m_{21}^2 \leq 8.9 \times 10^{-5} \text{ eV}^2; \) \( 1.4 \times 10^{-3} \text{ eV}^2 \leq \Delta m_{32}^2 \leq 3.3 \times 10^{-3} \text{ eV}^2 \), we find that the parameter \( a, b, \alpha \) and \( \beta \) need to be within the following regions:

\[
\begin{align*}
0.0005 & \leq a \leq 0.0013, & 0.0019 & \leq b \leq 0.0023, \\
-2.2 & \leq \alpha \leq -1.4, & -3.22 & \leq \beta \leq -3.20. \\
\end{align*}
\] (19)

To give an example of numerical values for CP asymmetries from the model, we choose a particular set of parameters

\[
\begin{align*}
a &= 0.0013, & b &= 0.00198, \\
\alpha &= -1.808, & \beta &= -3.210. \\
\end{align*}
\] (20)

Thus the masses of three right-handed neutrinos are

\[ M_1 = 2.27 \times 10^{10} \text{ GeV}, \quad M_2 = 3.61 \times 10^{10} \text{ GeV}, \quad M_3 = 1.85 \times 10^{13} \text{ GeV}. \] (21)

We see that \( M_1 \) and \( M_2 \) are fairly close to each other, with \( \delta = (M_2 - M_1)/M_1 = 0.59 \). The Yukawa matrix in the basis in which the right-handed neutrino mass matrix is diagonal and real looks like

\[
Y_{ij}' = \begin{pmatrix} 4.8 \times 10^{-6} \exp(0.9i) & 0 & 0.013 \\
0 & -2.022 \exp(1.6i) & 0.472 \\
0.0046 \exp(0.9i) & 0 & -0.022 \end{pmatrix},
\] (22)

where the signs of the phases are chosen to reproduce the right sign for the CP asymmetry.

Plugging the Yukawa matrix and mass ratios, we find the following CP asymmetries,

\[
\epsilon_1 = -0.92 \times 10^{-5}, \quad \epsilon_2 = -0.24 \times 10^{-5}. \] (23)

Here we have also shown the CP asymmetry from the second right-handed neutrino because its mass is close to the first one and is potentially important for leptogenesis. The result for \( \epsilon_1 \) exceeds slightly the bound derived by Davidson and Ibarra [22] because the masses are not so hierarchical.

### 3.2. Effective out-of-equilibrium decays

In our model, \( M_1 \) is close to \( M_2 \), and \( M_3 \) is much heavier. Thus, it is a good approximation to neglect the CP asymmetries and lepton number generated from the heaviest right-handed neutrinos (those with mass \( M_3 \)). However, since \( \delta = (M_2 - M_1)/M_1 \) is less than 1, one has to consider the full decay and washout effects from the two light right-handed neutrinos.

The efficiency factor can be calculated by solving the Boltzmann equation for the right-handed neutrinos and lepton densities. The result depends on the effective mass \( \tilde{m}_i \). In the present case, we find,

\[
\tilde{m}_1 = 29.1 \text{ meV}, \quad \tilde{m}_2 = 406 \text{ meV}. \] (24)

The effective masses determine the so-called decay parameters \( K_i = \tilde{m}_i/m^* \) where \( m^* = 16\pi^5/\sqrt{g^*}v^2/(3\sqrt{3}M_{pl}) = 1.08 \times 10^{-3} \text{ eV} \). In our case

\[
K_1 = 27.0, \quad K_2 = 376.2. \] (25)

Since \( K_i \gg 1 \), we are in the so-called strong washout region. In this region, the effective factor has little dependence on the thermal history of the right-handed neutrinos. One can assume for instance that they are not present in the beginning but are produced purely by the inverse scattering process.

Since the \( M_1 \) and \( M_2 \) are close to each other, one expects that the existence of \( N_2 \) will strongly modify the washout of \( N_1 \). This situation has been discussed recently in Ref. [23], where analytical formulas have been derived from numerical solutions of the Boltzmann equations,

\[
\kappa_1 = \frac{2}{z_B(K_1 + K_2^{(1-\delta)/3}) \cdot (K_1 + K_2^{(1-\delta)})}, \] (26)
\[ \kappa_2 = \frac{[1 + 2 \ln |\frac{1+\delta}{1-\delta}|]^2}{z_B (K_2 + K_1 (1-\delta)^3) \cdot (K_2 + K_1 (1-\delta))} e^{-\frac{2\pi}{3} K_1 (\frac{1}{z_B})^2}, \]  
\tag{27} 

where \( z_B = M_1/T_B \) is the inverse temperature at which the washout effects are minimized and \( \kappa_2 \) is valid when \( \delta < 1 \) \cite{3}. Plugging in the parameters, we find,

\[ \kappa_1 = 6.8 \times 10^{-3}, \quad \kappa_2 = 1.3 \times 10^{-4}. \]  
\tag{28} 

Thus, because \( K_2 \gg K_1 \), one has \( \kappa_1 \gg \kappa_2 \). Therefore, the number of out-of-equilibrium decays from \( N_2 \) is more than an order of magnitude smaller.

Putting everything together, the baryon asymmetry in our model is

\[ \eta_B = -0.96 \times 10^{-2} \sum_i \kappa_i \epsilon_i = 6.0 \times 10^{-10}, \]  
\tag{29} 

which is close to the observation data.

We have also numerically solved the Boltzmann equations to check the accuracy of the above formula,

\[ \frac{dn_{N_i}}{dz} = -(D_i + S)(n_{N_i} - n_{N_i}^{eq}), \]  
\tag{30} 

\[ \frac{dn_{N_i}}{dz} = -\epsilon_{N_1} D_1 (n_{N_1} - n_{N_1}^{eq}) - \epsilon_{N_2} D_2 (n_{N_2} - n_{N_2}^{eq}) - W N_L, \]  
\tag{31} 

where \( z = M_1/T, n_{N_i} \) is in unit of \( n_\gamma \) in a co-moving volume. \( D_i \) is the decay width measured in Hubble expansion rate \( H \).

\[ D_1 = K_1 z K_1 (z), \quad D_2 = K_2 z K_1 ((1+\delta)z) K_2 ((1+\delta)z), \]  
\tag{32} 

where \( K_{1,2} \) are modified Bessel functions. \( S \) is the scattering rate of the right-handed neutrinos off the Higgs bosons and gauge bosons. The washout rate \( W \) depends on the inverse decay, right-handed neutrino scattering, and processes involving the right-handed neutrino in the intermediate state (the \( \Delta L = 2 \) process). Since we are in the strong washout regime, only inverse decay itself can bring the right-handed neutrinos to thermal equilibrium, so we may neglect scattering and consider only decays and inverse decays,

\[ W = \frac{1}{4} z^3 [K_1 \cdot K_1 (z) + K_2 \cdot (1+\delta)^2 K_1 ((1+\delta)z)]. \]  
\tag{33} 

The final baryon asymmetry can be calculated via

\[ \kappa_i = -\int_{\frac{1}{z_i}}^{\infty} \frac{dN_i}{dz} e^{-\int_{z_i}^{z'} dz'' W(z'')} dz'. \]  
\tag{34} 

From Eq. (29), we solve the time evolution of the \( N_i \) distribution.

It was pointed out \cite{23} that the main contribution to \( \kappa_i \) comes from a Gaussian-like peak of the integrand around \( z_B \). Integrands of both \( \kappa_1 \) and \( \kappa_2 \) are depicted in Fig. 1. We find there is such a peak at around \( z_B = 8 > z^{eq} \) (\( z^{eq} \) represents the time that distribution of the i\( ^{th} \) RH neutrino equals the equilibrium distribution and then follows closely to it in strong washout regime). The asymmetries generated before thermalization of \( N_i \) are negligible, since they experiences longer washout. The dashed line, which shows the

![Fig. 1. (Color online.) The efficiency rate for out-of-equilibrium decays of the lightest (solid line) and the next-to-lightest (dashed line) right-handed neutrinos as a function of inverse temperature \( z = M_1/T \).](image-url)
integrand for $\kappa_2$, is much smaller than the solid line for $\kappa_1$. So in our model, we can basically neglect the effect of $N_2$. The numerical results are

$$\kappa_1 = 5.9 \times 10^{-3}, \quad \kappa_2 = 1.4 \times 10^{-4}. \quad (35)$$

Compared with Eq. (27), we find that the numerical solution and the analytic approximation are reasonably close.

### 3.3. Adding supersymmetry

In the presence of supersymmetry, for example in minimal supersymmetric Standard Model (MSSM), the above leptogenesis calculation must be modified in several ways.

First, the lepton number density now must include the contribution from the decay of right-handed sneutrinos,

$$n_L = \frac{3\xi(3)g_\nu T^3}{4\pi} \sum_{i=1}^{3} \left( \kappa_i \epsilon_i + \tilde{\kappa}_i \tilde{\epsilon}_i \right), \quad (36)$$

where, $\tilde{\kappa}_i$ and $\tilde{\epsilon}_i$ are the efficient factor and decay asymmetries of the sneutrinos. Because of supersymmetry, the second term in the sum is the same as the first term.

Second, when the lepton number is converted into the baryon number through the sphaleron process, one has now $n_B = -8/23 n_L$.

Third, the entropy density $s = \frac{2}{3} g^* \pi^2 T^3$ now has an effective $g^*$ factor 228.75 in MSSM.

Combining the above, one has

$$\eta_B = -\frac{344}{7165} \sum_i \left( \epsilon_i \kappa_i + \tilde{\epsilon}_i \tilde{\kappa}_i \right) = -4.46 \times 10^{-3} \sum_i \left( \epsilon_i \kappa_i + \tilde{\epsilon}_i \tilde{\kappa}_i \right), \quad (37)$$

which has a coefficient roughly a factor of 2 smaller.

Now consider the decay asymmetry. Eq. (16) for the right-handed neutrino still applies, except now we have to take into account the sneutrino intermediate state contribution, the decay function becomes,

$$F(x) = -\sqrt{x} \left[ \frac{2}{x-1} + \ln \frac{1+x}{x} \right]. \quad (38)$$

In the limit of large-$x$, the above becomes $3/\sqrt{x}$. Thus the asymmetry is a factor of 2 bigger. In our model, this is roughly the case:

$$\epsilon_1 = -1.78 \times 10^{-5}, \quad \epsilon_2 = -4.9 \times 10^{-6}. \quad (39)$$

The decay asymmetry for the sneutrinos $\tilde{\epsilon}_i$ are the same as $\epsilon_i$ because of supersymmetry.

To get the efficiency factors, we turn to the Boltzmann equations. Not only do we now have an equation for the right-handed neutrinos, but also for the sneutrinos. Ignoring the effects from scattering and taking into account the decay and inverse decay, we have

$$\frac{dn_{N_i}}{dz} = -D_i (n_{N_i} - n_{N_i}^{eq}), \quad \frac{dn_{\tilde{N}_i}}{dz} = -\tilde{D}_i (n_{\tilde{N}_i} - n_{\tilde{N}_i}^{eq}),$$
$$\frac{dn_{\tilde{L}}}{dz} = -\epsilon_{N_i} D_i (n_{N_i} - n_{N_i}^{eq}) - \epsilon_{\tilde{N}_i} \tilde{D}_i (n_{\tilde{N}_i} - n_{\tilde{N}_i}^{eq}) - W n_{L}, \quad (40)$$

where $n_{\tilde{N}_i}$ stands for the density of sneutrinos. The above equations are similar to the non-supersymmetric case, except for the additional contribution from the sneutrinos. Because of supersymmetry, the latter contribution to the lepton density is the same as that of the neutrino. However, the decay width of the particles is enhanced by a factor of 2, which leads to a factor of 2 larger $K$-factors,

$$K_1 = 53.9, \quad K_2 = 752.5. \quad (41)$$

We numerically solve the Boltzmann equations and find that

$$\kappa_1 = 3.1 \times 10^{-3}, \quad \kappa_2 = 4.8 \times 10^{-5}. \quad (42)$$

So $\kappa_i$ are about half of what we find in the non-supersymmetric case.

Tallying all the changes above, we find the final baryon asymmetry

$$\eta_B = 4.9 \times 10^{-10}, \quad (43)$$

which is slightly smaller than that of the non-supersymmetric case. But the difference is well within theoretical uncertainties.
In this model, the lightest right-handed neutrino can be produced thermally if the temperature of the universe after the inflaton decays is higher than $10^{10}$ GeV. However, if one takes the cosmological gravitino problem seriously, there is an upper bound on the reheating temperature $T_R \leq 10^6 - 10^8$ GeV when the gravitino mass is in the range $100$ GeV $\leq m_{3/2} \leq 1$ TeV [24]. In this case the right-handed neutrinos will be produced non-thermally from the decay of the inflaton. For example, one may consider the superpotential [25]

$$W_{\phi N} = \frac{1}{2} m_{\phi} \phi^2 + g \phi N N,$$

(44)

where $\phi$ is the inflaton field with mass $m_{\phi} > 2M_1$ and $g$ is a dimensionless coupling. The reheating temperature is given by

$$T_R \simeq |g| \sqrt{m_{\phi} M_{pl}}.$$

(45)

Assuming that the branching ratio of the inflaton decay is of order 1, the produced baryon asymmetry is given by

$$\eta_B \simeq 10^{-10} \frac{\epsilon}{2 \times 10^{-5}} \frac{T_R}{10^6 \text{ GeV}} \frac{5 \times 10^{10} \text{ GeV}}{m_{\phi}}.$$

(46)

Taking the reheating temperature $T_R \sim 10^7$ GeV and $m_{\phi} \sim 2M_1$ one can still obtain the desired baryon asymmetry of the universe.

Having shown that our model can generate enough CP violation at high energy for leptogenesis, it is interesting to calculate the size of the CP violation at low energy. The low-energy CP violation is encoded into one Dirac CP phase $\delta_{CP}$, which is multiplied by $\sin^2 \theta_{13}$ in the standard convention, and two Majorana phases $\phi_1$ and $\phi_2$, which appear in the form $\text{diag}(e^{i\phi_1}, e^{i\phi_1}, 1)$ in the PMNS matrix. It has been shown by Branco, Morozumi, Nobre and Rebelo in [26] that there is no model-independent relation between the CP violation at high and low energies. However, we do have predictions of the low-energy CP phases from our model, and it turns out that they are all small. The scatter plot of these CP phases versus the $\sin^2 \theta_{13}$ are shown in Fig. 2. Those points are from the parameter space given in Eq. (19). As shown in these scatter plots, $\delta_{CP}$ is constrained to be around 3°, and the $\phi_1$ and $\phi_2$ are constrained to be within 3 degree and 5 degree deviation from $-180°$ and 90°, respectively, indicating small CP violations at low energy. The prediction of the Jarlskog factor $J_{CP} \equiv \sin \theta_{12} \cos \theta_{12} \sin \theta_{23} \cos \theta_{23} \sin \theta_{13} \cos^2 \theta_{13} \sin \delta_{CP}$ which can be measured in the long baseline neutrino experiment lies in the range $0.0013 \leq J_{CP} \leq 0.0018$. The prediction of $\sin^2 2\theta_{13}$ is shown to be within the range $0.06 \leq \sin^2 2\theta_{13} \leq 0.085$.

4. Leptogenesis in other SO(10) models

There are a number of other realistic SO(10) models in the literature which fit well the quark and charged-lepton properties, and are consistent with the recent experimental data on the neutrino mass differences and mixings. However, they provide very different pictures on leptogenesis. The key parameter which controls the main features of the leptogenesis is the effective mass $\tilde{m}_1$: For a small $\tilde{m}_1$, the efficiency factor is large, and one only needs a moderate value of the decay asymmetry $\epsilon$ to accomplish leptogenesis. For a large values of $\tilde{m}_1$, the out-of-equilibrium decays are rare, and a successful leptogenesis requires a large decay asymmetry, which is possible when the masses of right-handed neutrinos become degenerate (resonant leptogenesis).

In this section, we compare leptogenesis scenarios in different SO(10) models and comment on their strong and weak points.

4.1. AB model

The AB model [15] utilizes Higgs fields $10_H$, $16_H$, $\overline{16}_H$ and $45_H$. The Dirac mass matrices of fermions are as follows:

$$M_u = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} A_U, \quad M_d = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon/3 \\ \delta' e^{i\phi} & \sigma + \epsilon/3 & 1 \end{pmatrix} A_D,$$

where $A_U$ and $A_D$ are appropriate matrices.

Fig. 2. The predictions of $\delta_{CP}$, $\phi_1$, and $\phi_2$ plotted against $\sin^2 \theta_{13}$. The points are chosen according to the requirement of producing enough leptogenesis and satisfying the 3σ range of neutrino oscillation data as described in the context.
These parameters lead to the following right-handed neutrino masses,

\[ M_{\nu R} = \begin{pmatrix} c^2 \eta^2 & -b \epsilon \eta & a \eta \\ -b \epsilon \eta & c^2 & -\epsilon \\ a \eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \]

where \( \eta \) and \( \epsilon \) are the same parameters as those in the Dirac mass matrices, and \( a, b \) and \( c \) are additional parameters of order 1.

A set of parameters which reproduce the quark and charged-lepton spectra and mixings are,

\[ \epsilon = 0.147, \quad \eta = 6 \times 10^{-6}, \]
\[ \delta = 0.00946, \quad \delta' = 0.00827, \]
\[ \sigma = 1.83, \quad \phi = 2\pi/3, \]
\[ m_U = 113 \text{ GeV}, \quad m_D = 1 \text{ GeV}. \]

Given the above, additional parameters, \( a, b, c \) and \( \Lambda_R \), can easily be found to fit the neutrino mass differences and mixings. However, the model usually generates a very large \( m_l \), which in turn produces a very large decay width for the lightest right-handed neutrino. As a consequence, the efficiency factor \( \kappa \) is too small. To enhance the lepton number production, the masses of the two lightest right-handed neutrinos are forced to a near degeneracy, yielding a large resonant decay asymmetry.

In a recent publication, a very extensive search in the parameter space was conducted to find a viable leptogenesis in the model [27]. One of the solutions is described by the following parameters,

\[ \eta = 1.1 \times 10^{-5}, \quad \delta_N = -1.0 \times 10^{-5}, \quad \delta_N' = -1.5 \times 10^{-5}, \]
\[ \Lambda_R = 2.85 \times 10^{14} \text{ GeV}, \]
\[ a = c = 0.5828i, \quad b = 1.7670i. \]

These parameters lead to the following right-handed neutrino masses,

\[ M_1 \sim M_2 = 5.40 \times 10^8 \text{ GeV}, \quad M_3 = 2.91 \times 10^{14} \text{ GeV}. \]

The \( \eta_B \) we calculate from these parameters, however, is \( 2.6 \times 10^{-10} \), roughly a factor of 2 smaller than that quoted in Ref. [27]. The difference comes from the CP asymmetry of the decay. When the masses of the two right-handed neutrinos are close, one cannot use the one-loop result in Eq. (16) directly. One has to resum the self-energy correction [28] to arrive at

\[ \epsilon_1 \approx \frac{\text{Im}(Y^T Y')_{21}^2}{8\pi (Y^T Y')_{11}^2} \frac{r_N}{r_N^2 + [(Y^T Y')_{11}^2/8\pi]^2}, \]
\[ \epsilon_2 \approx \frac{\text{Im}(Y^T Y')_{22}^2}{8\pi (Y^T Y')_{21}^2} \frac{r_N}{r_N^2 + [(Y^T Y')_{22}^2/8\pi]^2}, \]

where \( Y' \) is the \( \tilde{\nu}_L H \nu_R \) Yukawa couplings in the mass eigenstate basis of right-handed neutrinos, and \( r_N = (M_1^2 - M_2^2)/(M_1 M_2) = -2\delta \) is the degeneracy parameter.

It is worth pointing out that although the CP asymmetry tends to be enhanced due to the resonance in the case of two lightest right-handed neutrinos being quasi-degenerate, the washout effect is also enlarged in this case. Fortunately, in the present model, \( \tilde{m}_2 \sim \tilde{m}_1 \), so the effect is not particularly large. The modified numerical results are listed in Table 1.

4.2. The BPW model

In a model proposed by Babu, Pati, and Wilczek [10], the fermion Dirac and Majorana mass matrices have the following form,

\[ M_R = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon + \sigma \\ 0 & -\epsilon + \sigma & 1 \end{pmatrix} \Lambda_U, \quad M_d = \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' + \eta' & 0 & \epsilon + \eta \\ 0 & -\epsilon + \eta & 1 \end{pmatrix} \Lambda_D, \]
Introducing an intrinsic mass term for the light neutrinos might be physically motivated. However, it dilutes the relation between the produced, which stems from operators experimentally-preferred large-mixing angle solution. To remedy this, a small intrinsic mass for left-handed neutrinos is introduced, where

\[ \tilde{\kappa} \]

and the atmospheric neutrino oscillation parameters. With the above, a number of successful predictions follow, including the masses for bottom and down quarks, CKM matrix elements, and the atmospheric neutrino oscillation parameters.

Ref. [30], we take \( \tilde{m}_1 = 3 \text{ meV} \), and the efficiency factor is then about \( 6 \times 10^{-2} \). The baryon asymmetry is about \( 12 \sin^2 2\theta_{13} \times 10^{-10} \), where \( \phi \) is a CP-violation phase. With a reasonable choice of \( \phi \), the experimental \( \eta_B \) is produced.

The result of leptogenesis in this model is summarized under “BPW” in Table 1. Instead of showing a range of results, as in Ref. [30], we take \( \tilde{m}_1 = 3 \text{ meV} \), and the efficiency factor is then about \( 6 \times 10^{-2} \). The baryon asymmetry is about \( 12 \sin^2 2\phi \times 10^{-10} \), where \( \phi \) is a CP-violation phase. With a reasonable choice of \( \phi \), the experimental \( \eta_B \) is produced.

| Parameter | Value |
|-----------|--------|
| \( \sin^2 2\theta_{13} \) | \( 0.12 \) |
| \( \tilde{m}_1 \) (eV) | \( 12 \times 10^{-10} \) |
| \( \kappa \) | \( 6 \times 10^{-2} \) |
| \( \eta_B \) | \( 10^{-4} \) |
| \( \eta' \) | \( 4.4 \times 10^{-3} \) |
| \( \epsilon' \) | \( 2 \times 10^{-4} \) |
| \( \epsilon \) | \( -0.095 \) |
| \( \theta_{12} \) | \( 0.04 \) |

With the above, a number of successful predictions follow, including the masses for bottom and down quarks, CKM matrix elements, and the atmospheric neutrino oscillation parameters.

However, the solar-neutrino mixing angle from this model comes out too small: \( \sin \theta_{12} = 0.04 \), in contradiction with the experimentally-preferred large-mixing angle solution. To remedy this, a small intrinsic mass for left-handed neutrinos is introduced, which stems from operators \( \kappa_{12}16_116_216_H16_H10_H10_H/10^5 \). With this modification, the solar mixing angle changes to \( \sin^2 \theta_{13} \approx 0.6 \) [29].

The result of leptogenesis in this model is summarized under “BPW” in Table 1. Instead of showing a range of results, as in Ref. [30], we take \( \tilde{m}_1 = 3 \text{ meV} \), and the efficiency factor is then about \( 6 \times 10^{-2} \). The baryon asymmetry is about \( 12 \sin^2 2\phi \times 10^{-10} \), where \( \phi \) is a CP-violation phase. With a reasonable choice of \( \phi \), the experimental \( \eta_B \) is produced.

Clearly the model does not provide a tight constraint on the relation between the low-energy neutrino properties and leptogenesis. Introducing an intrinsic mass term for the light neutrinos might be physically motivated. However, it dilutes the relation between \( M_R \) and low-energy neutrino observables. In an extreme case, the constraint on the right-handed neutrino properties will be lost if the low-energy neutrino mass matrix is entirely “intrinsic”, i.e., of non-see-saw origin. A mild reflection of this “decoupling” is that \( \tilde{m}_1 \) in this model is particularly small, which is hard to achieve in a complete first-type see-saw model which fits the low-energy data. A small \( \tilde{m}_1 \) yields a large efficiency factor which certainly aids the leptogenesis here.

Table 1

| \( M_1 \) (GeV) | BPW | GMN | JLM | DMM | AB |
|----------------|-----|-----|-----|-----|----|
| \( -\epsilon \) | \( 10^{10} \) | \( 10^{13} \) | \( 3.77 \times 10^{10} \) | \( 10^{13} \) | \( 5.4 \times 10^8 \) |
| \( \tilde{m}_1 \) (eV) | \( 2.0 \times 10^{-6} \) | \( 1.94 \times 10^{-6} \) | \( 1.0 \times 10^{-5} \) | \( 10^{-4} \) | \( 9.4 \times 10^{-4} \) |
| \( \kappa \) | \( 0.003 \) | \( 0.006 \) | \( 0.026 \) | \( 0.1-0.4 \) | \( 5.4 \) |
| \( \eta_B \) | \( 12 \times 10^{-10} \) | \( 4.97 \times 10^{-10} \) | \( 6.2 \times 10^{-10} \) | \( 10^{-9} \) | \( 2.6 \times 10^{-10} \) |
| \( \sin^2 \theta_{13} \) | \( \leq 0.1 \) | \( 0.12 \) | \( 0.014-0.048 \) | \( 0.01 \) | \( 0.01 \) |

One set of parameters which produces good phenomenology without CP violation is

\[
M_{\nu_R} = \begin{pmatrix}
0 & -3\epsilon' & 0 \\
3\epsilon' & 0 & -3\epsilon + \sigma \\
0 & 3\epsilon + \sigma & 1
\end{pmatrix}
A_U, \quad M_I = \begin{pmatrix}
0 & -3\epsilon' + \eta' & 0 \\
3\epsilon' + \eta' & 0 & -3\epsilon + \eta \\
0 & 3\epsilon + \eta & 1
\end{pmatrix}
A_D.
\]

(52)

With the above, a number of successful predictions follow, including the masses for bottom and down quarks, CKM matrix elements, and the atmospheric neutrino oscillation parameters.

4.3. The minimal 126-Higgs model

This model [7,31] (referred to as GMN model in the table) uses the 126\(_H\) to break \( B-L \) symmetry and the 10\(_H\), 210\(_H\) to break gauge symmetries [7], and allows all couplings to be complex. 10\(_H\) and \( \bar{126}_H \) are used to give fermion masses through superpotential

\[ W_Y = h_{ij} \psi_i \psi_j H_{10} + f_{ij} \psi_i \psi_j \Delta \bar{126}. \]

(53)

So the following mass matrices are obtained:

\[
M_u = \tilde{h} + \tilde{f}, \quad M_d = \tilde{h} r_1 + \tilde{f} r_2, \\
M_l = \tilde{h} r_1 - 3 \tilde{f} r_2, \quad M_{\nu_R} = \tilde{h} - 3 \tilde{f},
\]

(54)

where \( \tilde{h} \) and \( \tilde{f} \) are matrices

\[
\tilde{h} = h^* \cos \alpha_u \sin \beta, \quad \tilde{f} = f^* e^{i\phi_u} \sin \alpha_u \sin \beta.
\]
and the parameters are
\[ r_1 = \frac{\cos \alpha_d}{\cos \alpha_u} \cot \beta, \quad r_2 = e^{i(\gamma_d - \gamma_u)} \frac{\sin \alpha_d}{\sin \alpha_u} \cot \beta. \] (55)

Hence, we can get a sumrule for quark and charged-lepton masses
\[ \frac{k M_t}{m_t} = \frac{M_d}{m_b} + \frac{M_u}{m_t}. \] (56)

The see-saw formula in this model is given by \( M_{\nu L} = f v_{B-L} - M_{\nu R} (M_{\nu R})^{-1} (M_{\nu D})^T \) and the calculations are done assuming that the first term dominates over the second. The left-handed neutrino Majorana mass matrix satisfies a sumrule
\[ M_{\nu L} = \alpha (M_t - M_d)^* \beta. \] (57)

The right-handed Majorana neutrino mass matrix is given by choosing the \( B-L \) (seesaw) scale \( v_{B-L} = 2 \times 10^{14} \text{ GeV} \) and \( \gamma_u = 0 \), \( \sin \alpha_u \sim \sin \beta \sim O(1) \),
\[ M_{\nu R} = f v_{B-L} = \tilde{f} - \frac{e^{-iy_u}}{\sin \alpha_u \sin \beta} v_{B-L} \simeq (2 \times 10^{14} \text{ GeV}) \tilde{f}. \] (58)

Therefore, the left- and right-handed neutrino Majorana masses have the same texture, and there are some nontrivial relations between the low-energy phenomena and leptogenesis, when the first term is assumed to dominate in \( M_{\nu L} \).

We use the following parameters and matrices,
\[ k = -0.846, \quad r = -1.846, \] (59)
\[ r_1 = 0.0116, \quad r_2 = 0.00337, \] (60)
\[ \tilde{h} = \frac{r_2 M_u - M_d}{r_2 - r_1} = \begin{pmatrix} 0.1766 & -0.0207 + 0.0038i & -0.058 + 0.097i \\ -0.0207 - 0.0038i & 3.49 & -1.332 \\ -0.058 + 0.097i & -1.332 & 94.7 \end{pmatrix}, \] (61)
\[ \tilde{f} = M_u - \tilde{h} = \begin{pmatrix} 0.165 & 0.0714 - 0.0133i & 0.2 - 0.335i \\ 0.0714 + 0.0133i & -3.161 & 4.596 \\ 0.2 + 0.335i & 4.596 & -12.28 \end{pmatrix}. \] (62)

The predicted baryon asymmetry through thermal leptogenesis is listed in Table 1 as “GMN”.

This model is also characterized by a small \( \tilde{m}_1 \), which is not constrained by the low-energy neutrino mass spectra because the first type of see-saw mass contribution is assumed to be small. Note that the neutrino Dirac mass texture in this model is completely untested at low energy and can only be effective in the right-handed neutrino decay process.

### 4.4. The DMM model

The DMM model [9] is an extension of the GMN model by enlarging the Higgs sector to include 120(A) which gives an additional contribution to the fermion mass matrices through the coupling
\[ W_{120} = \frac{1}{2} h_{ij}^\prime \psi_i \psi_j D. \] (63)

where \( h_{ij}^\prime \) is an antisymmetric matrix due to the SO(10) symmetry. Furthermore, the \( 10 \) and \( 126 \) couplings are chosen real and the \( 120 \) imaginary by using a \( Z_2 \) symmetry. In this case there are six pairs of Higgs doublets: \( \phi_d = (H_d^{10}, A_d^1, A_d^2, \Delta_d, \Delta_d, \Phi_d) \), \( \phi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \Delta_u, \Phi_u) \), where superscripts 1, 2 of \( A_{u,d} \) stand for SU(4) singlet and adjoint pieces under the \( G_{422} = SU(4) \times SU(2) \times SU(2) \) decomposition.

The MSSM Higgs doublets are given by
\[ H_d = U_{1a}^\dagger (\phi_d)_a, \] (64)
\[ H_u = V_{1a}^\dagger (\phi_u)_a, \]

where \( a = 1, \ldots, 6 \), \( U \) and \( V \) are unitary matrices which diagonalize the Higgs mass matrix. The Yukawa coupling matrices for fermions are given by
\[ Y_u = \tilde{h} + r_2 \tilde{f} + r_3 \tilde{h}^\prime, \] (65)
\[ Y_d = r_1 (\tilde{h} + \tilde{f} + \tilde{h}^\prime), \] (66)
Y_e = r_1(\tilde{h} - 3\tilde{f} + c_e\tilde{h}'),
Y_\nu = \tilde{h} - 3r_2\tilde{f} + c_v\tilde{h}',
\tag{67}
\end{equation}
\begin{equation}
where the subscripts u, d, e, v denote for up-type quark, down-type quark, charged-lepton, and Dirac neutrino Yukawa couplings, respectively, and
\begin{equation}
\begin{aligned}
&\tilde{h} = V_{11}h, \\
&\tilde{f} = U_{14}/(\sqrt{3}r_1)f, \\
&\tilde{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1h',
\end{aligned}
\tag{69}
\end{equation}
\begin{equation}
\begin{aligned}
r_1 &= \frac{V_{11}}{V_{11}}, \\
r_2 &= r_1\frac{V_{15}}{U_{14}}, \\
r_3 &= r_1\frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}},
\end{aligned}
\tag{70}
\end{equation}
\begin{equation}
\begin{aligned}
c_e &= \frac{U_{12} - \sqrt{3}U_{13}}{U_{12} + U_{13}/\sqrt{3}}, \\
c_v &= r_1\frac{V_{12} + \sqrt{3}V_{13}}{U_{12} + U_{13}/\sqrt{3}}.
\end{aligned}
\tag{71}
\end{equation}
\begin{equation}
The light neutrino mass is obtained as
\begin{equation}
m_{\nu}^{light} = M_L - M_D^\nu M^{-1}_R (M_D^\nu)^T,
\tag{72}
\end{equation}
\end{equation}
\begin{equation}
where M_D^\nu = Y_\nu(H_u), \quad M_L = 2\sqrt{2} f \langle \Delta_L \rangle, \quad \text{and} \quad M_R = 2\sqrt{2} f \langle \Delta_R \rangle.
\end{equation}
\begin{equation}
\text{It has been shown} \ [9] \ \text{that the Dirac neutrino coupling matrix}
\begin{equation}
\hat{Y}_\nu = \begin{pmatrix}
0.002 & 0.003 \exp(-1.54i) & 0.0026 \exp(-0.344i) \\
-0.0167 & 0.021 \exp(-1.53i) & 0.025 \exp(3.37i) \\
-0.229 & 0.417 \exp(4.70i) & 0.422 \exp(-3.019i)
\end{pmatrix},
\tag{73}
\end{equation}
\end{equation}
gives a good fit to $\Delta m^2_{atm}$, $\Delta m^2_{sol}$ and mixing angles with the lightest right-handed neutrino mass $M_1 \simeq 10^{13}$ GeV. In this model there is a correlation between $U_{e3}$ and $V_{ub}$ as well as $U_{e3}$ and $\Delta m^2_{sol}/\Delta m^2_{atm}$. The former imposes upper bound on $U_{e3}$, while the latter gives a lower bound. The predicted values of $U_{e3}$, lepton asymmetry and the washout factor are given in Table 1.

5. Conclusion

In summary, we have discussed leptogenesis in a SUSY SO(10) GUT model for the fermion masses and mixings, which is developed from the original lopsided model of Albright and Barr [11]. We have done a detailed analysis of the washout factor and find that the model predicts a value for the baryon to photon ratio ($\eta_B$) of the universe in good agreement with observations from WMAP as well as the requirement of a successful nucleosynthesis. We then compare with the same predictions for other successful SO(10) models in the literature.

X. Ji and Y. Li are partially supported by the US Department of Energy via grant DE-FG02-93ER-40762 and by National Natural Science Foundation of China (NSFC). R. Mohapatra and S. Nasri are supported by National Science Foundation (NSF) Grant No. PHY-0354401.

References

[1] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
[2] P. Minkowski, Phys. Lett. B 67 (1977) 421;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, et al. (Eds.), Supergravity, North-Holland, Amsterdam, 1980, p. 315;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, KEK, Tsukuba, Japan, 1979, p. 95;
S.L. Glashow, The future of elementary particle physics, in: M. Lévy, et al. (Eds.), Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons, Plenum Press, New York, 1980, p. 687;
R.N. Mohapatra, G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.
[3] W. Buchmuller, P. Di Bari, M. Plumacher, Ann. Phys. 315 (2005) 305, hep-ph/0401240.
[4] For some recent reviews on neutrino mass models, see R.N. Mohapatra, A.Y. Smirnov, hep-ph/0603118;
G. Altarelli, F. Feruglio, New J. Phys. 6 (2004) 106, hep-ph/0405048;
Z.Z. Xing, Int. J. Mod. Phys. A 19 (2004) 1, hep-ph/0307359;
A.Y. Smirnov, Int. J. Mod. Phys. A 19 (2004) 1180, hep-ph/0311259.
[5] M.C. Chen, K.T. Mahanthappa, Int. J. Mod. Phys. A 18 (2003) 5819, hep-ph/0305088;
B. Bajc, A. Melfo, G. Senjanovic, F. Vissani, Phys. Lett. B 634 (2006) 272;
B. Bajc, A. Melfo, G. Senjanovic, F. Vissani, AIP Conf. Proc. 805 (2006) 152, hep-ph/0511352.
[6] K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. 70 (1993) 2845, hep-ph/9209215;
M.C. Chen, K.T. Mahanthappa, Phys. Rev. D 62 (2000) 113007, hep-ph/0005292;
M.C. Chen, K.T. Mahanthappa, Phys. Rev. D 68 (2003) 017301, hep-ph/0212375;
T. Fukuyama, N. Okada, JHEP 0211 (2002) 011, hep-ph/0205066;
G.G. Ross, L. Velasco-Sevilla, Nucl. Phys. B 653 (2003) 3, hep-ph/0208218;
M. Bando, S. Kaneko, M. Obara, M. Tanimoto, hep-ph/0405071.
[7] B. Bajc, G. Senjanovic, F. Vissani, Phys. Rev. Lett. 90 (2003) 051802;
H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Lett. B 570 (2003) 215;
H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Rev. D 68 (2003) 115008.
[8] H.S. Goh, R.N. Mohapatra, S. Nasri, S.P. Ng, Phys. Lett. B 587 (2004) 105, hep-ph/0311330;
H.S. Goh, R.N. Mohapatra, S. Nasri, Phys. Rev. D 70 (2004) 075022, hep-ph/0408139;
W.M. Yang, Z.G. Wang, Nucl. Phys. B 707 (2005) 87, hep-ph/0406221;
S. Nasri, J. Schechter, S. Moussa, Phys. Rev. D 70 (2004) 053005.
[9] B. Dutta, Y. Mimura, R.N. Mohapatra, Phys. Rev. D 72 (2005) 075009.
[10] K.S. Babu, J.C. Pati, F. Wilczek, Nucl. Phys. B 566 (2000) 33, hep-ph/9812538;
K.S. Babu, J.C. Pati, P. Rastogi, Phys. Rev. D 71 (2005) 015005, hep-ph/0410200;
K.S. Babu, J.C. Pati, P. Rastogi, Phys. Lett. B 621 (2005) 160, hep-ph/0502152.
[11] C.H. Albright, K.S. Babu, S.M. Barr, Phys. Rev. Lett. 81 (1998) 1167, hep-ph/9802314;
C.H. Albright, S.M. Barr, Phys. Rev. D 58 (1998) 013002, hep-ph/9712488;
C.H. Albright, S.M. Barr, Phys. Rev. D 62 (2000) 093008, hep-ph/0003251.
[12] T. Blazek, S. Raby, K. Tobe, Phys. Rev. D 62 (2000) 055001, hep-ph/9912482;
Z. Berezhiani, A. Rossi, Nucl. Phys. B 594 (2001) 113, hep-ph/0003084;
R. Kitano, Y. Mimura, Phys. Rev. D 63 (2001) 016008, hep-ph/0008269;
T. Asaka, Phys. Lett. B 562 (2003) 291, hep-ph/0304124;
R. Dermisek, S. Raby, Phys. Lett. B 622 (2005) 327, hep-ph/0507045.
[13] X. Ji, Y. Li, R.N. Mohapatra, Phys. Lett. B 633 (2006) 755.
[14] X. Ji, Y. Li, Y. Zhang, Phys. Rev. D 75 (2007) 055016, hep-ph/0612114.
[15] C.H. Albright, S.M. Barr, Phys. Rev. D 64 (2001) 073010, hep-ph/0104294.
[16] C.H. Albright, S. Geer, Phys. Rev. D 65 (2002) 073004, hep-ph/0108070;
C.H. Albright, S. Geer, Phys. Lett. B 532 (2002) 311, hep-ph/0112171;
C.H. Albright, Phys. Rev. D 72 (2005) 013001, hep-ph/0502161.
[17] C.R. Das, M.K. Parida, Eur. Phys. J. C 20 (2001) 121, hep-ph/0010004.
[18] D.N. Spergel, et al., astro-ph/0603449.
[19] G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, Nucl. Phys. B 685 (2004) 89.
[20] J.A. Harvey, M.S. Turner, Phys. Rev. D 42 (1990) 3344.
[21] L. Covi, E. Roulet, F. Vissani, Phys. Lett. B 384 (1996) 169;
M. Plumacher, Nucl. Phys. B 530 (1998) 207.
[22] S. Davidson, A. Ibarra, Nucl. Phys. B 648 (2003) 345.
[23] S. Blanchet, P. Di Bari, JCAP 0606 (2006) 023, hep-ph/0603107.
[24] M. Kawasaki, K. Kohri, T. Moroi, Phys. Rev. D 71 (2005) 083502.
[25] T. Fukuyama, T. Kikuchi, T. Osaka, JCAP 0506 (2005) 005.
[26] G.C. Branco, T. Morozumi, B.M. Nobre, M.N. Rebelo, Nucl. Phys. B 617 (2001) 475.
[27] C.H. Albright, Phys. Rev. D 72 (2005) 013001.
[28] A. Pilaftsis, Phys. Rev. D 56 (1997) 5431, hep-ph/9707235.
[29] J.C. Pati, hep-ph/0204240.
[30] J.C. Pati, Phys. Rev. D 68 (2003) 072002.
[31] K.S. Babu, C. Macesanu, Phys. Rev. D 72 (2005) 115003;
S. Bertolini, M. Malinsky, Phys. Rev. D 72 (2005) 055021.