3D Dynamic Crack under Cyclic Loading using XFEM: Numerical Treatment

Tengfei Lyu1*, Stefan Löhnert2, and Peter Wriggers1
1 Institute of Continuum Mechanics, Leibniz University Hannover, Appelstraße 11, 30167 Hannover
2 Institute of Mechanics and Shell Structures, TU Dresden, August-Bebel-Str. 30, 01219 Dresden

The eXtended Finite Element Method (XFEM) is a special numerical method to handle arbitrary discontinuities in the displacement field independent of the finite element mesh. This is advantageous during crack initiation, growth and propagation processes. In the range of continuum damage mechanics, gradient-enhanced damage models can be used to model damage and fracture without spurious mesh dependencies. Gradient-enhanced damage models have been investigated extensively in the context of quasi-brittle and elasto-plastic materials. To avoid fracture and failure of materials, modelling the component under cyclic loading is significant for fatigue lifetime prediction. The focus of this contribution is set on algorithmic issues. The numerical treatment of 3d cracks under cyclic loading is investigated. The domain is discretized with ten-node tetrahedral elements. Discrete cracks are captured using XFEM and updated by level set methods. In order to take advantage of the explicit time discretization scheme, a modified differential equation for the gradient-enhanced-damage is presented and the central difference explicit time stepping method is employed to obtain 2nd order accuracy of the solved equations.

1 Introduction

Fatigue lifetime prediction is one of the most significant and critical issues among engineering design and industrial manufacturing [1]. Many factors could lead to structural failure such as dynamic loadings, thermal effects and imperfect materials. Fatigue damage increases in a cumulative manner, along with the number of applied cycles, resulting in crack formation and final failure of the component. The traditional lifetime predictions are generally based on stress-cycles curves or stain-cycles curves, which are obtained from numerous laboratory experiments. Metal specimens are subjected to thousands of load cycles until a macroscopic crack or failure occurs. Both, low-cycle fatigue testing and high-cycle fatigue testing are expensive and time-consuming. Therefore, a computationally efficient and accurate numerical approach for fatigue lifetime prediction is of significant interest. In this study the numerical treatment for accelerating the computational time of dynamic XFEM-based fatigue simulations is investigated, preparing for the further numerical predictions of the fatigue lifetime.

2 Numerical treatment

2.1 XFEM and standard gradiend-enhanced damage

The primary unknowns in this work are displacement field variable \( \mathbf{u} \) and gradient-enhanced equivalent strain \( \bar{\varepsilon} \), which are governed by the linear momentum equation and the inhomogeneous Helmholtz equation [2] respectively,

\[
\rho \ddot{\mathbf{u}} = \nabla \cdot \mathbf{\sigma} + \mathbf{b}, \quad \bar{\varepsilon} = c \nabla^2 \bar{\varepsilon} = \bar{\varepsilon}_{eq}
\]

with boundary conditions \( \mathbf{u}(x, t) = \bar{\mathbf{u}}(x, t) \) on \( \Gamma_u \), \( \mathbf{\sigma} \cdot \mathbf{n} = \mathbf{t} \) on \( \Gamma_t \), \( \mathbf{\sigma} \cdot \mathbf{n} = \mathbf{0} \) on \( \Gamma_\alpha \), and \( \nabla \bar{\varepsilon} \cdot \mathbf{n} = 0 \) on \( \Gamma_\phi \). In order to capture discrete cracks in the domain during the cyclic loading, the primary unknowns are discretized using XFEM. The additional degrees of freedom are associated with the Heaviside \( H(\phi) \) and a front enrichment function \( B_j(\phi, \psi) \) as suggested in [3, 4]:

\[
\bar{\varepsilon}(x, t) = \sum_{i \in N} N_i \bar{\varepsilon}_i(t) + \sum_{i \in N_H} N_i H(\phi) \mathbf{a}_i(t) + \sum_{i \in N} \sum_{j=1}^{4} N_i B_j(\phi, \psi) R \mathbf{b}_{ij}(t)
\]

2.2 Modified gradient-enhanced damage

The traditional method to solve the linear momentum equation and standard gradient-enhanced damage is to use a Newton-Raphson iteration combined with a staggered scheme. With respect to the dynamic cyclic loading, small time step sizes are necessary to maintain the stability and accuracy of the solution. Therefore it is reasonable to adopt an explicit time stepping method to accelerate the computational time. However the standard gradient-enhanced damage equation is a Helmholtz type equation without any time derivative. Here we introduce an artificial time derivative part of the gradient-enhanced equivalent strain \( \bar{\varepsilon} \) and use a scalar value \( \alpha \) with unit [time] to describe its contribution to the standard model,

\[
\alpha \left( \dot{\bar{\varepsilon}} - c \nabla^2 \bar{\varepsilon} \right) + \left( \bar{\varepsilon} - c \nabla^2 \bar{\varepsilon} \right) = \alpha \bar{\varepsilon}_{eq} + \bar{\varepsilon}_{eq}
\]

* Corresponding author: e-mail lyu@ikm.uni-hannover.de, phone +49 511 762 4297, fax +49 511 762 5496
with additional boundary conditions $\nabla \dot{\varepsilon} \cdot n = 0$ on $\Gamma_0$. The derivation of the weak form of equation (3) is analog to the procedure in [2]. Hence the matrix form of the discretized modified gradient-enhanced damage equation is

$$C_d \ddot{\varepsilon} + K_d \dot{\varepsilon} = F_d$$

with

$$C_d = \int_{\Omega} \alpha \left( N_e^T N_e + B_c^T c B_c \right) \, d\Omega, \quad K_d = \int_{\Omega} \left( N_e^T N_e + B_c^T c B_c \right) \, d\Omega, \quad F_d = \int_{\Omega} \left( \alpha N_e^T \dot{\varepsilon} + N_e^T \varepsilon_{eq} \right) \, d\Omega$$

### 2.3 Explicit time discretization

Equation (4) together with $M_u \dddot{u} + K_u \ddot{u} = F_u$ can be solved simultaneously through an explicit time stepping method. A lumped mass matrix $M_u = [M_{ii}]$ and a lumped damage capacity matrix $C_d = [C_{ii}]$ is beneficial for the solution process. Since we use quadratic shape functions for the tetrahedral elements, row sum lumping would introduce negative terms in the lumped mass matrix. Therefore here we adopt the diagonal scaling lumping procedure. The corresponding XFEM part of the mass and the damage capacity matrix is computed according to the strategy in [5]. A central difference explicit time stepping method is employed for both equation systems to obtain 2nd order accuracy for the solution.

$$\begin{bmatrix} \ddot{u} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} \frac{1}{M_{ii}} (F_u - K_u u) \\ \frac{1}{C_{ii}} (F_d - K_d \dot{\varepsilon}) \end{bmatrix}$$

### 3 Numerical example

To evaluate the proposed modified damage model a simple numerical test of model I fracture is performed. As shown in Fig.1 a 3D block with pre-existent crack is under displacement controlled load. The domain is discretized with tetrahedral elements with quadratic shape functions. Linear elasticity including modified gradient-enhanced damage with $\alpha = 10^{-5}$ is considered. Comparing the results for the staggered implicit time integration and the explicit time discretization scheme, we can conclude that the modified damage model can achieve similar results as the standard damage model.

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