From de Sitter to de Sitter

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ABSTRACT

We obtain $D = 6$, $N = (1, 1)$ de Sitter supergravity from a hyperbolic reduction of the massive type IIA* theory. We construct a smooth cosmological solution in which the co-moving time runs from an infinite past, which is $dS_4 \times S^2$, to an infinite future, which is a $dS_6$-type spacetime with the boundary $R^3 \times S^2$. This provides an effective four-dimensional cosmological model with two compact extra dimensions forming an $S^2$. Interestingly enough, although the solution is time-dependent, it arises from a first-order system via a superpotential construction. We lift the solutions back to $D = 10$, and in particular obtain two smooth embeddings of $dS_4$ in massive type IIA*, with the internal space being either $H^4 \times S^2$ or an $H^4$ bundle over $S^2$. We also obtain the analogous $D = 5$ and $D = 4$ solutions. We show that there exist cosmological solutions that describe an expanding universe with the expansion rate significantly larger in the past than in the future.

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1 Introduction

Target spacetime duality [1] is the most established duality in string theory. It states that a string on a circle with radius $R$ is equivalent to the same, or another, string theory on a circle with radius $2\pi \alpha' / R$. For the heterotic string theory, the duality was extended to a time-like circle [2]. However, as was first observed in [3], the T-duality that relates the type IIA/IIB theories breaks down in a time-like direction, due to the Ramond-Ramond fields. Specifically, when reduced on a time-like circle, the kinetic terms for the R-R fields have opposite signs in the type IIA and type IIB effective field theories. In order to extend time-like T-duality to the type II theories, type IIA$^*$ and type IIB$^*$ theories were introduced in [4, 5], where the kinetic terms of the R-R fields have the opposite sign to those in the usual type II theories. In particular, the proposed time-like T-duality relates type IIA to type IIB$^*$ and type IIB to type IIA$^*$.

Lifting type IIA$^*$ to eleven dimensions leads to an M$^*$-theory which has signature $(2,9)$. Just as anti-de Sitter spacetimes arise naturally in type IIB and M-theories, so de Sitter spacetimes arise in the * theories. The advantage of such embeddings of dS is that the * theories can still be viewed as “supersymmetric,” at a price that the anti-commutators of the super-charges are no longer positive definite. In this paper, we continue referring this as supersymmetry. De Sitter supergravities as hyperbolic reductions of type IIB$^*$ and M$^*$-theories have recently been obtained in [6]. In this paper, we consider the massive type IIA$^*$ theory. We show that there is a smooth solution in which the metric is a warped product of dS$_6$ and hyperbolic 4-space $H^4$. This enables us to perform a Kaluza-Klein hyperbolic reduction to an $\mathcal{N} = (1,1)$ gauged supergravity that admits dS$_6$ as a vacuum solution. The gauge group is $SU(2) \times U(1)$, which is the same as that of AdS$_6$ gauged supergravity. However, the signs of the kinetic terms of the gauge fields in dS supergravity are opposite to those in AdS supergravity. The dilaton and the 3-form field strength have the standard sign for their kinetic terms.

Our purpose in studying such a theory is to construct cosmological solutions. Recent experimental evidence suggests that our universe might be de Sitter [7, 8]. We obtain a smooth six-dimensional cosmological solution in which the co-moving
time runs from an infinite past, which is $dS_4 \times S^2$, to an infinite future, which is a $dS_6$-type of geometry with the boundary $R^3 \times S^2$. The Hubble constant in the infinite past is of the same order of magnitude as that in the infinite future, but the value decreases. The increased value of the Hubble constant in the past is due to the contribution from the 2-form flux. This solution provides an effective four-dimensional cosmological model, with two extra compact dimensions forming an $S^2$.

The cosmological flow is analogous to BPS domain walls in gauged supergravities, for which there is a renormalization group (RG) flow [9] in the context of the AdS/CFT correspondence [10, 11, 12]. In particular, RG flows in massive type IIA supergravity were constructed in [13, 14]. These are the AdS analogs of the solutions discussed in the present paper. Cosmological RG flows have recently been discussed in [15]. However, at least in the truncation or approximation considered, the cosmology flows from de Sitter to a singularity. In fact, cosmological solutions in string and M-theory were constructed sometime ago [16, 17], which are now known as S-branes [18]. These solutions are typically singular. It has been argued that one way to resolve the singularity may be for the cosmological flow to connect early and late time de Sitter spacetimes. In a general sense, our solution is an example of such a smooth flow, albeit the division of the early spacetime between $dS_4$ and $S^2$. In light of the proposed $dS/CFT$ correspondence [19, 20, 21, 22, 23], the cosmological evolution that we presently study corresponds to an RG flow from a three-dimensional Euclidean CFT to a five-dimensional Euclidean CFT with two compact dimensions on $S^2$.

Since the reduction ansatz from type IIA$^*$ is consistent, it allows us to lift these supergravity solutions back to $D = 10$. In particular, we find two smooth embeddings of $dS_4$ in massive type IIA$^*$, where the internal space is either $H^4 \times S^2$ or an $H^4$ bundle over $S^2$.

We also obtain the analogous $D = 5$ and $D = 4$ de Sitter supergravity solutions. An interesting feature of these solutions is that, although they are time-dependent, they arise from first-order equations via a superpotential construction. These solutions are supersymmetric from the point of view of * theories. A superpotential has also been used to construct the analogous solutions in AdS supergravity which radially interpolate between $AdS_{D-2} \times S^2$ and an $AdS_D$-type geometry with the boundary $M_{D-3} \times S^2$, where $M_d$ is $d$-dimensional Minkowski spacetime [24, 14].
We also demonstrate that there is a larger class of non-supersymmetric cosmological solutions that arise from the second-order equations of motion. For appropriate choices of parameters, the solution can describe an expanding universe whose expansion rate is significantly larger in the past than in the future, providing a realistic inflationary model, with no singularity. Solutions with such a property can also arise from first-order system when appropriate matter fields are coupled.

This paper is organized as follows. In section 2, we consider $D$-dimensional de Sitter Einstein-Maxwell gravity as a toy model. We give a detailed presentation of the construction of a cosmological solution using a superpotential approach and discuss the properties of the solutions. In section 3, we consider massive type IIA$^*$ theory in $D = 10$. We perform a Kaluza-Klein hyperbolic reduction to obtain $D = 6$, $N = (1, 1)$ de Sitter supergravity, obtain a cosmological solution and analyse its properties. In section 4, we obtain the analogous solutions in $D = 5$ and $D = 4$. We conclude our paper in section 5.

2 Cosmological solution in Einstein-Maxwell de Sitter gravity

The Lagrangian for $D$-dimensional Einstein-Maxwell de Sitter gravity is given by

$$e^{-1} \mathcal{L} = R - \frac{1}{4} \eta F_{(2)}^2 - (D - 1)(D - 2) g^2,$$

where $\eta = 1$ for the standard kinetic term and $\eta = -1$ for the kinetic term with opposite sign. Kinetic terms with “wrong” signs occur in type IIA$^*$, type IIB$^*$ and M$^*$-theories, as we have discussed in the introduction. We consider a cosmological solution with the ansatz

$$ds^2 = -d\tau^2 + a^2 dx^i dx^i + b^2 d\Omega_2^2,$$

$$F_{(2)} = \lambda \Omega_{(2)}^2,$$

where $d\Omega_2^2$ is the metric on a unit 2-sphere $S^2$, 2-torus $T^2$ or hyperbolic 2-plane $H^2$, and $\Omega_{(2)}$ is the corresponding volume 2-form. The functions $a$ and $b$ depend only on the co-moving time coordinate $\tau$. The equations of motion are given by

$$\frac{2\ddot{b}}{b} + \frac{(D - 3) \ddot{a}}{a} = (D - 1) g^2 - \frac{\eta \lambda^2}{2(D - 2) b^4},$$
\[\begin{align*}
\frac{\ddot{a}}{a} + \frac{2\dot{a}\dot{b}}{ab} + \frac{(D - 4)\dot{a}^2}{a^2} &= (D - 1)g^2 - \frac{\eta \lambda^2}{2(D - 2)b^4}, \\
\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{(D - 3)\dot{a}\dot{b}}{ab} + \frac{\epsilon}{b^2} &= (D - 1)g^2 + \frac{(D - 3)\eta \lambda^2}{2(D - 2)b^4},
\end{align*}\]

where a dot denotes a derivative with respect to \( \tau \) and \( \epsilon = 1, 0, -1 \) for \( S^2 \), \( T^2 \) and \( H^2 \), respectively. Defining \( a = e^{\varphi_1} \) and \( b = e^{\varphi_2} \), we find that the equations (2.3) can be obtained from a Hamiltonian \( \mathcal{H} = T + U = 0 \), where

\[
T = \frac{1}{2} g_{\alpha\beta} (\varphi^{(\alpha)})' (\varphi^{(\beta)})', \quad \text{with} \quad g_{\alpha\beta} = \begin{pmatrix} -2(D - 3)(D - 4) & -4(D - 3) \\ -4(D - 3) & -4 \end{pmatrix},
\]

and a prime denotes a derivative with respect to \( t \), defined by \( d\tau = b^2 a^{D-3} dt \). The potential \( V \) is given by

\[
U = a^{2(D-3)} \left( \frac{1}{2} \eta \lambda^2 - 2\epsilon b^2 + (D - 2)(D - 1)g^2 b^4 \right).
\]

We find that the potential \( U \) can be expressed in terms of a superpotential \( W \) by

\[
U = -\frac{1}{2} g^{\alpha\beta} \frac{\partial W}{\partial \varphi^{(\alpha)}} \frac{\partial W}{\partial \varphi^{(\beta)}},
\]

provided that the following constraint is satisfied

\[
\lambda^2 = -\frac{2\epsilon^2}{\eta (D - 2)(D - 3) g^2}.
\]

The superpotential \( W \) is given by

\[
W = a^{D-3} \left( \frac{2\epsilon}{(D - 3) g} - 2(D - 2)g^2 b^2 \right).
\]

The second-order equations (2.3) are satisfied if the first-order equations \( (\varphi^{(\alpha)})' = \pm g^{\alpha\beta} \partial_{\beta} W \) are satisfied. Thus, we arrive at the first-order equations

\[
\frac{\dot{a}}{a} = g + \frac{\epsilon}{(D - 2)(D - 3)g b^2}, \quad \frac{\dot{b}}{b} = g - \frac{\epsilon}{(D - 2)g b^2}.
\]

The equations can be solved explicitly, giving a cosmological solution

\[
\begin{align*}
\frac{ds^2}{(g t)^2 H^2} &= -\frac{dt^2}{(g t)^2} + (g t)^2 H^{D-2} \ dx^i \ dx^i + t^2 \ d\Omega^2_2, \\
F_2 &= \lambda \Omega_2, \quad H = 1 - \frac{\epsilon}{(D - 2)(g t)^2},
\end{align*}
\]

(2.10)
where the charge $\lambda$ is given by
\[
\lambda = \frac{e}{g \sqrt{-\frac{1}{D} \eta (D-2)(D-3)}}. \tag{2.11}
\]

The solution can also be obtained from the analytical continuation of the magnetic brane solutions constructed in [25, 26]. It is clear that the reality of $\lambda$ requires that $\eta = -1$, implying that the kinetic term for $F_{(2)}$ must have the “wrong” sign \(^1\). As we have mentioned in the introduction, the * theories introduced by time-like T-duality precisely have the opposite signs for the kinetic terms of the R-R fields. This implies that the aforementioned type of cosmological solution might arise naturally from a * theory.

The solution can also be analytically expressed in terms of the co-moving coordinate $\tau$ as
\[
ds^2 = -d\tau^2 + e^{\frac{2(D-2)g}{D-3}\tau} \left[ e^{2g\tau} + \frac{e}{(D-2)g^2} \right]^{-\frac{1}{D-3}} dx^i dx^i + \left( e^{2g\tau} + \frac{e}{(D-2)g^2} \right) d\Omega_2^2. \tag{2.12}
\]

For $\epsilon = 0$, the solution is nothing but de Sitter spacetime in $D$-dimensions. For $\epsilon = -1$, the solution approaches a dS\(_D\)-type spacetime in the future but has a naked singularity at a certain time in the past. We are interested in the solution with $\epsilon = 1$. In this case, the solution is regular everywhere and time runs from an infinite past, which is dS\(_{D-2}\) × $S^2$ with Hubble constant $H = \frac{D-2}{D-3}g$, to an infinite future, which is a dS\(_D\)-type spacetime with Hubble constant $H = g$. Thus we see that the Hubble constant at infinite past is of the same magnitude as that at the infinite future. The value decreases by $\frac{100}{D-2}\%$. The larger value of $H$ in the past is due to the contribution from the 2-form flux.

Note that for $\epsilon = 1$, the dS\(_1\) × $S^2$ of the infinite past is itself a solution. It can be obtained by taking $\tau \to \tau + \tau_0$ and sending $\tau_0$ to $-\infty$. With an appropriate rescaling of $x^i$, the metric is given by
\[
ds^2 = -d\tau^2 + e^{\frac{2(D-2)g}{D-3}\tau} dx^i dx^i + \frac{1}{(D-2)g^2} d\Omega_2^2 \tag{2.13}
\]

\(^1\)In contrast to these solutions, de Sitter black holes require the standard sign for the $F_{(2)}$ kinetic term [6]. As we will see, certain hyperbolic reductions from * theories contain kinetic terms of both signs and can thereby accommodate both types of solutions.
It is worth mentioning that the second-order equations (2.3) admit a larger class of cosmological solutions that run from dS$_{D-2}$ in the past to dS$_D$ in the future, regardless of the sign of $\epsilon$ and $\eta$. To see this, we first consider the “fixed-point” solution dS$_{D-2} \times S^2$ or dS$_{D-2} \times H^2$, where $b$ is a constant. The solution is given by

$$
\begin{align*}
 ds^2 &= -d\tau^2 + e^{2\gamma \tau} dx^i dx^i + b^2 d\Omega_2^2, \\
 b^2 &= \frac{1}{2(D-1)g^2} \left( \epsilon \pm \sqrt{\epsilon^2 - \frac{2(D-1)(D-3)g^2 \eta \lambda^2}{D-2}} \right), \\
 \gamma^2 &= \frac{(D-1)(D-2)g^2}{(D-3)^2} - \frac{\epsilon}{(D-3)^2 b^2}.
\end{align*}
$$

(2.14)

Thus for $\eta = 1$, the value of $\lambda$ is restricted to satisfy $g^2 \lambda^2 \leq (D-2)/(2(D-1)(D-3))$. For $\eta = -1$, there is no such restriction; in this case, with $\epsilon = 1$, when $\lambda$ satisfies the constraint (2.7), the solution becomes the one that can arise from the first-order system, as we discussed earlier. It is straightforward to demonstrate using numerical methods that there exist cosmological solutions that run from (2.14) in the infinite past to a dS$_D$-like spacetime in the infinite future.

In order to construct a realistic model, we would like to have the effective cosmological constant much larger in the past than in the future. This can be achieved by the choice of $\epsilon = -1$ and $\eta = -1$, with $g^2 \lambda^2 << 1$. The resulting dS$_{D-2} \times H^2$ metric in the infinite past is given by (2.14), but with $b$ and $\gamma$ given by

$$
\begin{align*}
 \gamma^2 &= \frac{(D-1)(D-2)g^2}{(D-3)^2} + \frac{2(D-2)}{(D-3)^3 \lambda^2}, \\
 b^2 &= \frac{(D-3)\lambda^2}{2(D-2)}.
\end{align*}
$$

(2.15)

Thus the Hubble constant $H \sim 1/\lambda$ in the past for small $\lambda$ can be significantly larger than $H = g$, the Hubble constant in the future. This is analogous to the recent observation that a vacuum solution dimensionally-reduced on a compact hyperbolic manifold of time-varying volume might yield an inflationary cosmology [27], which is a special case of an S-brane solution [16, 28].

Clearly, the charge $\lambda$ is restricted to take the value defined in (2.7) in solution that arises from the first-order system. Thus the Hubble constant in the future and in the past are of the same orders of magnitude. In the general solutions from the second-order equations, such a restriction no longer applies, and therefore we can adjust parameters in such a way that the expansion rate of the universe in the past is significantly larger than that in the future, in line with observational data. In fact, as
we shall show in the next section, when appropriate matter fields are coupled, even the solutions arising from the first-order system associated with supersymmetry can have expansion rate significantly larger in the past than in the future.

Although here we considered the general de Sitter Einstein-Maxwell supergravity, only the \( D = 4 \) case can be embedded in the \( * \) theories. In other dimensions, the de Sitter supergravity coming from the reduction of \( * \) theories involves additional fields such as a dilaton and antisymmetric tensor fields. We shall discuss this with further detail in subsequent sections. Nevertheless, the solution (2.12) captures the essence of all the cosmological solutions we obtain in this section, namely the metric is \( \text{dS}_{D-2} \times S^2 \) in the infinite past and of \( \text{dS}_D \)-type in the infinite future. The contributions of scalars or additional field strengths might alter the Hubble constant in the past but only in a mild way.

\section{D = 6 cosmological solution}

\subsection{D = 6 de Sitter supergravity from massive type IIA*}

Massive type IIA supergravity was constructed in [29]. The bosonic Lagrangian for the massive type IIA* supergravity can be obtained by changing the sign of the kinetic terms for the R-R fields, namely the 2-form and 4-form field strengths, together with a sign change for the cosmological term; it is given by

\[ \mathcal{L}_{10} = \hat{R} \ast 1 - \frac{1}{2} \hat{e}^\phi d\hat{\phi} \wedge d\hat{\phi} + \frac{1}{2} e^{\frac{5}{2}\hat{\phi}} \ast \hat{F} \wedge \hat{F} - \frac{1}{2} e^{-\hat{\phi}} \ast \hat{\phi} \wedge \hat{F} \wedge \hat{F} + \frac{1}{2} e^{\frac{5}{2}\hat{\phi}} \ast \hat{F} \wedge \hat{F} \wedge \hat{F} \]

\[ + \frac{1}{2} d\hat{A} \wedge d\hat{A} \wedge \hat{A} + \frac{1}{6} m d\hat{A} \wedge (\hat{A})^3 + \frac{1}{40} m^2 (\hat{A})^5 + \frac{1}{2} m^2 e^{\frac{5}{2}\hat{\phi}} \ast 1, \tag{3.1} \]

where the field strengths are given in terms of potentials by

\[ \hat{F} \wedge = d\hat{A} + m \hat{A} \]

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Here we present the Lagrangian in the language of differential forms, as in [30]. The theory admits a solution in the form of a warped product of \( \text{dS}_6 \times H^4 \), given by

\[ ds^2_{10} = (\cosh \xi)^{\frac{1}{12}} \left[ ds^2_{6} + 2 g^{-2} \left( d\xi^2 + \frac{1}{4} \sinh^2 \xi \left( \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) \right) \right], \tag{3.3} \]

\[ \hat{F} \wedge = \frac{5\hat{\nabla}}{\hat{6}} g^{-3}(\cosh \xi)^{\frac{1}{3}} \sinh \xi d\xi \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3 \wedge e^\phi = (\cosh \xi)^{-5/6}, \]
where \( \sigma_i \) are \( SU(2) \) left-invariant 1-forms which satisfy \( d\sigma_i = -\frac{1}{2} \epsilon_{ijk} \sigma^j \wedge \sigma^k \). In other words, the metric for the hyperbolic \( H^4 \) is written as a foliation by \( S^3 \). This solution is an analytical continuation of the warped product of AdS\(_6\) and \( S^4 \) obtained in [31]. It is of interest to note that, while the warped product of AdS\(_6\) and \( S^4 \) is singular at the equator of the \( S^4 \), our solution (3.3) is regular everywhere.

We can now consider the Kaluza-Klein hyperbolic reduction of the massive type IIA\(^*\) theory. The \( S^4 \) reduction of the usual massive type IIA supergravity was obtained in [32]. Here we need only to perform an analytic continuation of the reduction ansätze obtained in [32], which is then given by

\[
\begin{align*}
\hat{d}s_{10}^2 &= (\cosh \xi)^{\frac{1}{2}} X^\frac{3}{8} \left[ \Delta^\frac{3}{8} ds_6^2 + 2g^{-2} \Delta^\frac{3}{8} X^2 d\xi^2 + \frac{1}{2}g^{-2} \Delta^{-\frac{5}{8}} X^{-1} \sinh^2 \xi \sum_{i=1}^3 (h^i)^2 \right], \\
\hat{F}_{(4)} &= -\sqrt{\frac{2}{3}} g^{-3} c^{1/3} s^3 \Delta^{-2} U d\xi \wedge \epsilon_{(3)} - \sqrt{2}g^{-3} c^{4/3} s^4 \Delta^{-2} X^{-3} dX \wedge \epsilon_{(3)} \\
&\quad + \frac{1}{\sqrt{2}} g^{-2} c^{1/3} s F_{(2)}^i \hat{h}^i \wedge d\xi - \frac{1}{\sqrt{2}} g^{-2} c^{4/3} s^2 \Delta^{-1} X^{-3} F_{(2)}^i \wedge h^i \wedge h^k \epsilon_{ijk}, \\
\hat{F}_{(2)} &= \frac{1}{\sqrt{2}} c^{2/3} F_{(2)} \wedge d\xi, \\
\hat{F}_{(2)}^i &= c^{-5/6} \Delta^{1/4} X^{-5/4},
\end{align*}
\]

where \( X \) is related to the dilaton \( \phi \) by \( X = e^{-\frac{1}{2\pi} \hat{\phi}} \) and

\[
\begin{align*}
\Delta &\equiv -X \sinh^2 \xi + X^{-3} \cosh^2 \xi, \\
U &\equiv X^{-6} c^2 + 3X^2 s^2 - 4X^{-2} s^2 - 6X^{-2}.
\end{align*}
\]

We have defined \( h^i \equiv \sigma^i + gA_{(1)}^i, \epsilon_{(3)} \equiv h^1 \wedge h^2 \wedge h^3, s = \sin \xi \) and \( c = \cos \xi \). Also, \( * \) is the six-dimensional Hodge dual. (There should be no confusion between Hodge dual \( * \) and \( * \) theories.) The gauge coupling constant \( g \) is related to the mass parameter \( m \) of the massive type IIA theory by \( m = \frac{\sqrt{2}}{3} g \). The resulting six-dimensional theory is given by

\[
\begin{align*}
\mathcal{L}_6 &= R * \mathbb{I} - \frac{1}{2} * d\phi \wedge d\phi + g^2 \left( \frac{2}{9} e^{\frac{3}{2} \phi} - \frac{8}{3} e^{\frac{1}{2} \phi} - 2 e^{-\frac{1}{2} \phi} \right) * \mathbb{I} \\
&\quad - \frac{1}{2} e^{-\sqrt{2} \phi} * F_{(2)} \wedge F_{(2)} + \frac{1}{2} e^{\sqrt{2} \phi} \left( F_{(2)} \wedge F_{(2)}^i + F_{(2)}^i \wedge F_{(2)}^i \right) \\
&\quad + A_{(2)} \wedge (\frac{1}{2} dA_{(1)} \wedge dA_{(1)} + \frac{1}{3} g A_{(2)} \wedge dA_{(1)} + \frac{2}{27} g^2 A_{(2)} \wedge A_{(2)} + \frac{1}{4} F_{(2)}^i \wedge F_{(2)}^i),
\end{align*}
\]

where \( F_{(3)} = dA_{(2)}, F_{(2)} = dA_{(1)} + \frac{2}{3} g A_{(2)} \) and \( F_{(2)}^i = dA_{(1)}^i - \frac{1}{2} g \epsilon_{ijk} A_{(1)}^j \wedge A_{(1)}^k \).
The de Sitter gauged supergravity in $D = 6$ which we have obtained from the hyperbolic reduction of the massive type IIA* theory was also constructed in $D = 6$ directly in [34]. It has an $SU(2) \times U(1)$ gauge symmetry. This is the same as in the AdS gauged supergravity in $D = 6$. The difference between the theories is that the kinetic terms for the gauge fields have opposite signs. Furthermore, the cosmological constant is positive instead of negative. However, as we have shown in section 2, these features are necessary in order to construct the “supersymmetric” cosmological solution which is presented in the next subsection.

### 3.2 Cosmological solution

We use the metric, the dilaton and a 2-form field strength to construct a cosmological solution. The relevant Lagrangian is given by

$$e^{-1} \mathcal{L}_6 = R - \frac{1}{2} (\partial \phi)^2 + \frac{1}{4} e^\phi F_{(2)}^2 + g^2 \left( \frac{2}{9} e^{\frac{3}{2} \phi} - \frac{8}{3} e^{\phi} - 2 e^{-\frac{1}{2} \phi} \right). \quad (3.7)$$

We consider the same ansatz for the cosmological solution in (2.2). The system admits the following first-order equations

$$\begin{align*}
\dot{\phi} &= \sqrt{2} \left( \frac{\sqrt{2}}{4} \epsilon g^{-1} e^{\frac{1}{2} \sqrt{2} \phi} b^{-2} + \frac{dW}{d\phi} \right), \\
\frac{\dot{b}}{b} &= -\frac{1}{4\sqrt{2}} \left( 3\epsilon g^{-1} e^{\frac{1}{2} \sqrt{2} \phi} b^{-2} + W \right), \\
\frac{\dot{a}}{a} &= \frac{1}{4\sqrt{2}} \left( \epsilon g^{-1} e^{\frac{1}{2} \sqrt{2} \phi} b^{-2} - W \right), \quad (3.8)
\end{align*}$$

provided that the charge $\lambda$ is fixed to be $\lambda = \epsilon/g$. The superpotential $W$ is given by

$$W = -2g \left( e^{-\frac{1}{2} \sqrt{2} \phi} + \frac{1}{3} e^{\frac{3}{2} \sqrt{2} \phi} \right). \quad (3.9)$$

For all $\epsilon$, the solution runs to a dS$_6$-type solution in the infinite future, with the Hubble constant being $H = \sqrt{\frac{2}{3}} g$. For $\epsilon = -1$ and 0, the solution has a naked singularity at a particular time in the finite past. We are interested in $\epsilon = 1$, for which there is a fixed-point solution, namely

$$\begin{align*}
ds_6^2 &= -d\tau^2 + e^{\frac{2}{3} \sqrt{2} g \tau} \left( dx_1^2 + dx_2^2 + dx_3^2 \right) + \sqrt{\frac{3}{2}} g^{-2} d\Omega_2^2, \\
e^{\sqrt{2} \phi} &= \frac{3}{2}, \quad F_{(2)} = g^{-1} \Omega_{(2)}. \quad (3.10)
\end{align*}$$
It describes a direct product of $dS_4 \times S^2$. Using a numerical approach, it is straightforward to demonstrate that there exists a solution whose co-moving time runs from infinite past, with the above $dS_4 \times S^2$ with the Hubble constant $6^{-1/4} \, g$, to an infinite future, with a $dS_6$-type metric having boundary $R^2 \times S^2$, and a Hubble constant $\frac{\sqrt{2}}{3} \, g$. Again, the two Hubble constants are of the same orders of magnitude but the one in the infinite future is slightly smaller than the one in the infinite past.

As we have discussed in section 2, the reason for the Hubble constant to take the values of the same magnitude in the past and in the future is that the first-order equations arising from supersymmetry require that the charge carried by $F_{(2)}$ take a specific value. There also exists a more general class of solutions of the second-order equations of motion, where this restriction does not apply. It is rather straightforward to show, as in section 2, that by adjusting the parameters appropriately, we can obtain a realistic inflationary model whose expansion rate in the past is significantly larger than that in the future.

### 3.3 Smooth embedding of $dS_4$ in massive type IIA

Owing to the consistency of the reduction ansatz, the solutions can all be lifted back to $D = 10$. As we have seen earlier, the $dS_6$ becomes a smooth warped product of $dS_6$ and $H^4$. Here we consider the oxidation of the $dS_4 \times S^2$ solution, given in (3.10). There are two possible endpoints in $D = 10$. If the 2-form field strength is taken to be in the $U(1)$ factor in the $SU(2) \times U(1)$ gauge group, then there is no non-trivial bundle structure in $D = 10$, and the metric is given by

$$
\begin{align*}
\frac{ds^2}{10} &= \left(\frac{2}{3}\right)^{1/2} (cosh(\xi))^{1/2} \Delta^{3/8} \left[ ds^2_{dS_4} + \sqrt{2} \, g^{-2} \, d\Omega_2^2 \right. \\
&\quad \quad \left. + g^{-2} \left( \left(\frac{8}{3}\right)^{1/2} \, d\xi^2 + \frac{1}{2} \left(\frac{3}{2}\right)^{1/4} \Delta^{-1} \sinh^2(\xi) \, d\Omega_3^2 \right) \right],
\end{align*}
$$

where

$$
\Delta = \left(\frac{3}{2}\right)^{3/4} \cosh^2(\xi) - \left(\frac{2}{3}\right)^{1/4} \sinh^2(\xi) > 0.
$$

The $dS_4$ metric is given by

$$
ds^2_{dS_4} = -d\tau^2 + e^{6 \, \xi} \, g^\tau \left( dx_1^2 + dx_2^2 + dx_3^2 \right).
$$

The resulting ten-dimensional metric is also completely smooth.
An alternative possibility is that the 2-form field strength in (3.10) corresponds to a $U(1)$ subgroup of the $SU(2)$ gauge group. If we lift this solution back to $D = 10$, then there is a non-trivial bundle structure in the metric, which is given by

$$ds^2_{10} = \left(\frac{2}{3}\right)^{\frac{1}{12}} (\cosh \xi)^{\frac{1}{12}} \Delta^{\frac{3}{8}} \left[ ds^2_{dS_4} + \sqrt{\frac{3}{2}} g^{-2} d\Omega^2_2 \\
+ g^{-2} \left( \left(\frac{\pi}{2}\right)^{\frac{1}{2}} d\xi^2 + \frac{1}{2} \left(\frac{3}{2}\right)^{\frac{1}{2}} \Delta^{-\frac{1}{4}} \sinh^2 \xi \left( d\tilde{\Omega}^2_2 + (d\psi + \tilde{A}_1)^2 \right) \right) \right], \quad (3.14)$$

where $\Delta$ is given by (3.12) and

$$d\tilde{A}_{(1)} = \Omega_{(2)} + \tilde{\Omega}_{(2)}. \quad (3.15)$$

The internal six-dimensional space can be viewed as an $H^4$ bundle over $S^2$.

Thus, we have obtained two smooth embeddings of $dS_4$ in massive type IIA*, one of which has $H^4 \times S^2$ as its internal space, whilst the other is an $H^4$ bundle over $S^2$. If we lift the cosmological solution back to $D = 10$, then we obtain ten-dimensional smooth cosmological solutions, which in the infinite past are either (3.11) or (3.14) and they approach (3.3) in the infinite future.

### 3.4 Matter-coupled de Sitter supergravity

So far we have considered only the pure $\mathcal{N} = (1, 1)$ de Sitter supergravity. It is also possible to add matter multiplets. Here we are interested in a vector-tensor multiplet, since the theory then allows us to truncate to $U(1)^2$, with the Lagrangian given by\(^2\)

$$\hat{e}^{-1} \mathcal{L}_6 = \hat{R} - \frac{1}{2} (\partial \phi_1)^2 - \frac{1}{2} (\partial \phi_2)^2 - \hat{V} + \frac{1}{4} \sum_{i=1}^2 X_i^{-2} (\hat{F}_{(2)}^i)^2, \quad (3.16)$$

where $X_i = e^{\frac{1}{2} \tilde{a}_i \cdot \tilde{\phi}}$ with

$$\tilde{a}_1 = (\sqrt{2}, \frac{1}{\sqrt{2}}), \quad \tilde{a}_2 = (-\sqrt{2}, \frac{1}{\sqrt{2}}). \quad (3.17)$$

The scalar potential is given by

$$\hat{V} = -\frac{4}{9} g^2 \left( X_0^2 - 9X_1 X_2 - 6X_0 X_1 - 6X_0 X_2 \right), \quad (3.18)$$

\(^2\)Though in general not the case, the truncation is consistent for the present purpose of constructing cosmological solutions.
where $X_0 = (X_1 X_2)^{-3/2}$. The cosmological solution can be obtained from the following first-order equations

$$
\dot{\phi} = \sqrt{2} \left( - \frac{\epsilon}{2\sqrt{2} g} (q_1 a_1 X_1^{-1} + q_2 a_2 X_2^{-1}) b^{-2} + \frac{dW}{d\phi} \right),
$$

$$
\dot{b} = - \frac{1}{4\sqrt{2}} \left( \frac{3}{\sqrt{2} g} \epsilon (q_1 X_1^{-1} + q_2 X_2^{-1}) b^{-2} + W \right),
$$

$$
\dot{a} = \frac{1}{4\sqrt{2}} \left( \frac{\epsilon}{\sqrt{2} g} (q_1 X_1^{-1} + q_2 X_2^{-1}) b^{-2} - W \right),
$$

(3.19)

provided that the two $U(1)$ charges are given by $\lambda_i = \epsilon g^{-1} q_i$ with $q_1 + q_2 = 1$. Note that the ($\phi, b$) fields form a closed system. The superpotential $W$ is given by

$$
W = \frac{g}{\sqrt{2}} \left( \frac{4}{3} X_0 + 2X_1 + 2X_2 \right). 
$$

(3.20)

The case with $q_1 = q_2 = \frac{1}{2}$ reduces to the previous example of pure de Sitter gauged supergravity. For general $q_i$, the solution can be obtained by analytical continuation of the magnetic brane solutions in [24, 14]. In particular, if $q_1 = 1$ and $q_2 = 0$, then we can consistently set $\phi_1 = 2\phi_2$ and (3.19) can be solved explicitly by making the coordinate transformation $d\tau = e^{3\sqrt{2} \phi_2} dy$. Defining

$$
F \equiv e^{-3\sqrt{2} \phi_2}, \quad G \equiv e^{\frac{1}{\sqrt{2}} \phi_2} b^2, 
$$

(3.21)

we find that the first two equations in (3.19) yield

$$
G' - \frac{\epsilon}{g} + \frac{4}{3} g G = 0. 
$$

(3.22)

The solution is

$$
G = g^{-2} \left( e^{-\frac{4}{3} gy} + \frac{3}{4} \epsilon \right). 
$$

(3.23)

We have absorbed a trivial integration constant by a constant shift in the coordinate $y$. The dilaton equation of motion gives

$$
\frac{F'}{F} = - \frac{\epsilon}{g G} + 2g (1 - F^{-1}). 
$$

(3.24)

Plugging in $G$, we find that

$$
F = e^{-\frac{4}{3} gy} + \frac{3}{4} \epsilon + c_1 e^{\frac{2}{3} gy} = e^{-\sqrt{2} \phi_2},
$$

$$
a^2 = c_2 e^{-\frac{4}{3} gy} e^{-\frac{1}{\sqrt{2} \phi_2}}, \quad b^2 = g^{-2} (e^{-\frac{4}{3} gy} + \frac{3}{4} \epsilon) e^{-\frac{1}{\sqrt{2} \phi_2}}.
$$

(3.25)
The metric of the solution can be expressed, using the new coordinate $t = g^{-1} e^{-\frac{2}{3}g y}$, as 

$$ds_6^2 = -\frac{9}{4} H^\frac{3}{2} \frac{dt^2}{(g t)^2} + (g t)^2 H^{-\frac{1}{4}} \left( dx_i^2 + (1 + \frac{3}{4} \frac{\epsilon}{(g t)^2}) g^{-2} d\Omega_2^2 \right),$$

(3.26)

where the function $H$ and the dilaton are given by

$$e^{\sqrt{5} \phi^2} = H = \frac{1 + \frac{3}{4} \frac{\epsilon}{(g t)^2}}{1 + \frac{\epsilon}{4} \frac{1}{(g t)^2} + \frac{c_1}{(g t)^4}}.$$ (3.27)

As shown previously for the case of $dS_4 \times S^2$, the above cosmological solution can be lifted to type IIA* theory.

To understand the solution better, it is instructive to write the function $H = \tilde{H}/W$, where the definitions of $\tilde{H}$ and $W$ can be straightforwardly read off from (3.27). Then the metric can be expressed as

$$ds_6^2 = \tilde{H}^{-\frac{1}{4}} W^{-\frac{1}{4}} \left[ -\frac{9}{4} \tilde{H} W^{-1} \frac{dt^2}{(g t)^2} + (g t)^2 \left( dx_i^2 + \tilde{H} g^{-2} d\Omega_2^2 \right) \right].$$ (3.28)

Thus, the solution can be viewed as an intersection of a spatial domain wall, characterized by the function $H$, and a brane with a three-dimensional Euclidean worldvolume\(^3\), characterized by the function $\tilde{H}$.

In the infinite future, $t \to \infty$, $H \to 1$ and the metric behaves as

$$ds_6^2 = -\frac{9}{4} \frac{dt^2}{(g t)^2} + (g t)^2 \left( dx_i^2 + g^{-2} d\Omega_2^2 \right).$$ (3.29)

If $\epsilon = 0$, in which case $d\Omega_2^2$ is the metric on a 2-torus, then the above metric describes locally $dS_6$ spacetime. The full solution with $\epsilon = 0$ describes a spatial domain wall. This can be viewed as a distribution of S-branes from the ten or eleven-dimensional point of view.

For $\epsilon = \pm 1$, the above metric can be viewed as a spatial domain wall wrapped on $\Omega^2$. For $\epsilon = 1$, the metric is regular everywhere provided that the constant $c_1 = 0$. The corresponding metric interpolates between $dS_4 \times S^2$ in the infinite past to the $dS_6$-type metric (3.29) in the infinite future. When $c_1$ is non-vanishing, the $c_1$ term can dominate at early time and hence the metric can become singular.

If we set $c_1 = 0$ and rescale the coordinates as

$$g t \to k g t, \quad x_i \to \frac{1}{k} x_i,$$ (3.30)

\(^3\)This is known as an S2-brane [16, 33].
then for \( k \to 0 \) we obtain the geometry \( dS_4 \times S^2 \) with a constant dilaton. If we instead take \( c_1 = k \tilde{c}_1 \frac{m}{\gamma} \), then after sending \( k \to 0 \) we obtain the solution

\[
ds_6^2 = -\frac{9}{4} H^{3/4} \frac{dt^2}{(gt)^2} + \frac{(gt)^2}{H^{1/4}} dx_i^2 + \frac{3}{4} \frac{1}{g^2 H^{1/4}} d\Omega_2^2,
\]

\[
H^{-1} = 3 - \frac{\tilde{c}_1}{gt},
\]

(3.31)

which approaches \( dS_4 \times S^2 \) in the asymptotic region, where \( dS_4 \) approaches the boundary. The early time behavior is dependent on \( \tilde{c}_1 \), but always singular. Thus, the \( dS_4 \times S^2 \) solution can exist in either the infinite past or future. For \( \epsilon = -1 \), the solution is also always singular at early times.

### 3.5 Large past-future Hubble constant ratio

For general values of \( q_1 + q_2 = 1 \), the first-order equations (3.19) cannot be solved analytically. However, it is straightforward to find the fixed-point solution of \( dS_4 \times S^2 \) or \( dS_4 \times H^2 \) where \( b \) and \( \phi \) are constants. The solution is given by

\[
e^{\sqrt{2} \phi_1} = \frac{3}{2} \left( 1 - q \pm \sqrt{(1 - q)^2 + \frac{4}{9} q} \right), \quad e^{-\sqrt{2} \phi_2} = \frac{3}{2} \cosh(\phi_1 / \sqrt{2}),
\]

\[
b^2 = \frac{q_1 + q_2 e^{\sqrt{2} \phi_1}}{4 \epsilon g^2}, \quad a^2 = \text{Exp} \left( \frac{1}{3} g e^{-\sqrt{8} \phi_2} \tau \right),
\]

(3.32)

where \( q = \lambda_1 / \lambda_2 \). Thus, it is clear that if \( q \in [0, \infty) \), we have \( \epsilon = 1 \) corresponding to \( S^2 \), and for \( q \in (-\infty, 0) \), we have \( \epsilon = -1 \) corresponding to \( H^2 \). Using a numerical method, we can verify that there are cosmological solutions which are (3.32) in the infinite past and are of \( ds_6 \)-type in the future. The ratio of the Hubble constant in the infinite past and future can be straightforwardly calculated. In particular, for \( -q >> 1 \), the ratio is given by

\[
\frac{H_{\text{past}}}{H_{\text{future}}} \sim \frac{\sqrt{2}}{16} \left( \frac{3}{2} \sqrt{-3q} \right)^{3/4},
\]

(3.33)

which can be arbitrarily large. It is surprising that we can get a somewhat realistic cosmological model from a first-order system, which implies supersymmetry from the point of view of * theories.
4 Further cosmological solutions

4.1 $D = 5$

De Sitter supergravity from a hyperbolic 5-space reduction of type IIB* theory was obtained in [6]. Let us now consider the minimal gauged supergravity in $D = 5$ coupled to two vector multiplets. The Lagrangian is given by

$$e^{-1}L_5 = R - \frac{1}{2} (\partial \phi_1)^2 - \frac{1}{2} (\partial \phi_2)^2 - \frac{1}{4} \sum_{i=1}^{3} X_i^{-2} \eta_i (F_{i(2)})^2 - V + \frac{1}{4} e^{-1} \epsilon_{\mu \nu \rho \sigma \lambda} F_{\mu \nu}^1 F_{\rho \sigma}^2 A_{\lambda}^3,$$

with the scalar potential

$$V = 4g^2 \sum_{i=1}^{3} X_i^{-1}. \quad (4.1)$$

The quantities $X_i$ are given by

$$X_i = e^{\frac{1}{2} \vec{a}_i \cdot \vec{\phi}}, \quad \vec{a}_1 = (\sqrt{2}, \frac{2}{\sqrt{6}}), \quad \vec{a}_2 = (-\sqrt{2}, \frac{2}{\sqrt{6}}), \quad \vec{a}_3 = (0, -\frac{4}{\sqrt{6}}). \quad (4.3)$$

Note that $\eta_i = \{-, -, +\} [6]$. Thus, the reality condition does not permit one to construct pure de Sitter Einstein-Maxwell supergravity. For the present purpose of constructing a cosmological solution, we select the field strengths $F_{i(2)}^1$ and $F_{i(2)}^2$, whose kinetic terms have the “wrong” sign. With the metric ansatz of the form (2.2), we find that the system admits the following first-order equations

$$\dot{\vec{\phi}} = \sqrt{2} \left( \frac{\epsilon}{2 \sqrt{2} g} (q_1 \vec{a}_1 X_1^{-1} + q_2 \vec{a}_2 X_2^{-1}) b^{-2} + \frac{dW}{d\phi} \right),$$

$$\dot{b} = -\frac{1}{3 \sqrt{2}} \left( -\sqrt{2} \epsilon (q_1 X_1^{-1} + q_2 X_2^{-1}) b^{-2} + W \right),$$

$$\dot{a} = \frac{1}{3 \sqrt{2}} \left( -\frac{\epsilon}{\sqrt{2}} (q_1 X_1^{-1} + m_2 X_2^{-1}) b^{-2} - W \right), \quad (4.4)$$

provided that the charges of $F_{(2)}^1$ and $F_{(2)}^2$ are given by $\lambda_i = \epsilon g^{-1} q_i$, with $q_1 + q_2 = 1$. Here, the superpotential $W$ is given by

$$W = \sqrt{2} g \sum_i X_i. \quad (4.5)$$
In order to have a fixed-point solution of $dS_3 \times S^2$, it is necessary to have $q_1 = q_2 = \frac{1}{2}$, in which case the solution is given by
\[
ds^2 = -d\tau^2 + e^2 \frac{2^{2/3}}{} g\tau (dx_1^2 + dx_2^2) + \frac{1}{2^{1/3} g^2} d\Omega_2^2,
\]
\[X_1 = X_2 = X_3^{-1/2} = (\frac{1}{2})^{1/3}.\quad (4.6)
\]
It is straightforward, using numerical methods, to show that there exists a smooth solution that runs from $dS_3 \times S^2$ in the infinite past to an infinite future which has a $dS_5$-type metric with the boundary being $R^2 \times S^2$.

### 4.2 $D = 4$

$M^*$-theory is a $(2,9)$ theory that admits a $dS_4 \times \text{AdS}_7$ vacuum solution. Kaluza-Klein reduction on $\text{AdS}_7$ gives rise to a de Sitter gauged supergravity in $D = 4$ [6]. For the truncation to the $U(1)^4$ subsector, the bosonic Lagrangian is given by
\[
e^{-1} L_4 = R - \frac{1}{2} (\partial \phi_1)^2 - \frac{1}{2} (\partial \phi_2)^2 - \frac{1}{2} (\partial \phi_3)^2 + \frac{1}{4} \sum_{i=1}^4 X_i^{-2} (F_i^4)^2 - \hat{V},
\]
with the scalar potential
\[
\hat{V} = 4g^2 \sum_{i<j} X_i X_j.
\]

The quantities $X_i$ are given by
\[
X_i = e^{\frac{1}{2} \vec{a}_i \cdot \vec{\phi}},
\]
\[
\vec{a}_1 = (1, 1, 1), \quad \vec{a}_2 = (1, -1, -1), \quad \vec{a}_3 = (-1, 1, -1), \quad \vec{a}_4 = (-1, -1, 1). \quad (4.9)
\]

Note that all four of the above kinetic terms for the field strengths $F_i^4$ have the “wrong” sign [6]. Thus, this theory can be truncated to pure de Sitter Einstein-Maxwell supergravity. The cosmological solution ansatz admits the first-order equations
\[
\dot{\phi} = \sqrt{2} \left( \frac{\epsilon}{2\sqrt{2} g} \sum_{i=1}^4 q_i \vec{a}_i X_i^{-1} b^{-2} + \frac{dW}{d\phi} \right),
\]
\[
\frac{\dot{b}}{b} = -\frac{1}{2\sqrt{2}} \left( -\frac{\epsilon}{\sqrt{2} g} \sum_{i=1}^4 q_i X_i^{-1} b^{-2} + W \right),
\]
\[
\frac{\dot{a}}{a} = \frac{1}{2\sqrt{2}} \left( -\frac{\epsilon}{\sqrt{2} g} \sum_{i=1}^4 q_i X_i^{-1} b^{-2} - W \right),
\]
\[(4.10)\]
provided that \( \lambda_i = \epsilon g^{-1} q_i \), with \( q_1 + q_2 + q_3 + q_4 = 1 \). Here, the superpotential \( W \) is given by

\[
W = \sqrt{2} g \sum_{i=1}^{4} X_i. \tag{4.11}
\]

If all of the \( q_i \) are equal, the scalars can be consistently set to zero and the solution reduces to the previous four-dimensional solution of section 2. In general, with an appropriate choice of \( q_i \), it is straightforward to find solutions that run from \( dS_2 \times S^2 \) in the infinite past to a \( dS_4 \)-type geometry in the infinite future.

\section{Conclusions}

We have considered massive type IIA* theory in which the kinetic terms for the R-R fields have the “wrong” sign. We find that the theory admits a smooth solution as a warped product of \( dS_6 \) and \( H^4 \). This enables us to perform a hyperbolic reduction to \( D = 6, \mathcal{N} = (1,1) \) pure gauged supergravity which admits \( dS_6 \) spacetime as its vacuum solution. The gauge group is \( SU(2) \times U(1) \), which is the same as the \( D = 6 \) AdS gauged supergravity. However, the kinetic terms for the gauge fields have the “wrong” sign. This apparently undesirable feature enables us to construct a cosmological solution that runs smoothly from the infinite past, which is \( dS_4 \times S^2 \), to the infinite future, with a \( dS_6 \)-type metric having an \( R^3 \times S^2 \) boundary. One interesting feature of the solution is that although it is time-dependent, it arises from a first-order system \textit{via} a superpotential construction. The solution provides an effective four-dimensional cosmological model that incorporates de Sitter spacetime and also two compact extra dimensions forming an \( S^2 \). Of course since the solution comes directly from string theory, the cosmological constant is clearly too large in comparison to the observational data. It is nevertheless interesting to observe that the * theories can provide “supersymmetric” time-dependent cosmological solutions.

We applied the same analysis to \( D = 5 \) and \( D = 4 \), where the theories are reductions of type IIB* and M*-theory, respectively. We also provided the cosmological solutions for general de Sitter Einstein-Maxwell theories. It is of interest to note that although AdS_7 gauged supergravity exists, the corresponding dS_7 theory does not.

We also demonstrated that there is a larger class of non-supersymmetric cosmolog-
ical solutions arising from the second-order equations of motion. For an appropriate choice of parameters, the solution can describe an expanding universe whose expansion rate is significantly larger in the past than in the future, providing a realistic inflationary model, with no singularity. In fact, with some appropriate matter coupling, we show that solutions with such a property can also arise from first-order system.

All of these cosmological solutions can be lifted to ten or eleven dimensions. In particular, we obtained two ways of smoothly embedding $dS_4$ in massive type IIA*, with the internal space being either $H^4 \times S^2$ or an $H^4$ bundle over $S^2$.

Although the * theories are necessary from the point of view of time-like T-duality, they suffer from an instability due to the ghost-like nature of the supergravity fields. It is of interest, therefore, to study further whether the stability is protected by the time-like T-duality or the “supersymmetry,” or whether the time scale of the instability is large enough to nevertheless validate the cosmological solutions.

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