Study of tunable resonances in laser beam divergence and beam deflection

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ABSTRACT
New, fundamental resonant properties of laser resonators are theoretically predicted and experimentally demonstrated. These resonances occur either in the time dependence of the beam width and that of beam radius of curvature of the wavefront or in the time dependent pointing and position stability of the output light beam of a laser resonator. The resonant frequency can be tuned continuously from zero to the round-trip frequency in the first case; and from zero to the half of the round-trip frequency in the second case, by for example, moving one of the mirrors of the resonator. In both cases besides a resonant frequency its complementary frequency to the round-trip frequency is also resonant, and their shifted frequencies by multiples of the round-trip frequency are also resonant. In our experimental demonstration we measured the radiofrequency noise spectrum of the output laser beam, that was partially blocked by a knife-edge. We observed increased noise at the theoretically predicted frequencies. Similar resonances are predicted either in the time dependent pulse-width and phase modulation or time jitter and the central frequency of the ultrashort light pulses of the mode-locked lasers because of the analogy between the space description of the light beams and the time-description of the light pulses.

Keywords: Tunable radiofrequency resonances, noise of lasers, beam-width fluctuation, beam-deviation fluctuation, Gouy-phase.

1. INTRODUCTION
Investigating laser noise is of high interest, both in the context of fundamental physics and for a variety of laser applications. The obtained understanding has also brought significant benefits for the further development of lasers. The noise of lasers can also provide us useful information about the laser alignment as in the case of CEO-frequency measurement via heterodyning different harmonics of the mode-locked laser spectrum. The knowledge of the fluctuations, noises in laser resonators are also important for understanding the build up of the pulsed modes in lasers.

In this paper we study radiofrequency resonances of lasers. We predict and experimentally demonstrate two tunable resonances of continuous-wave laser beams. The properties of these resonances are derived from the paraxial beam propagation equations assuming real ABCD round-trip matrices in our calculations for the sake of simplicity.

We think that the introduced resonances either have not been observed, or have been observed but their nature was not recognized or have been observed only in second order effects when the fluctuations affect also the power of the output laser beam.

The self-consistent equation for the complex radius of curvature of light beams was perturbed in chapter 21 of Siegman’s book. The perturbed complex radius of curvature after a round trip becomes perturbed with the same extent of deviation and the complex phase of the deviation changes by a phase angle that depends only on the half of the trace of the round-trip matrix. The resultant oscillations are said to damp out due to diffraction filtering in sufficient number of round trips. We repeat this short deduction in Sec. just for completeness. We show, however, in Sec. that, because of that phase shift, laser resonators show resonant sensitivity to perturbations from the surroundings and from the pump laser. This tunable sensitivity cause the beam width

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and the radial curvature of the phase front of the light beam of a laser to fluctuate with a frequency that depends only on the trace of the overall round-trip ABCD matrix of the laser resonator.

We show in Sec. 3 that if the direction of propagation of the self-consistent light beam of a laser resonator is perturbed then there is an also complex parameter which describes, depending on its complex phase, propagation direction and/or transversal position deviation. It turns out in chapter 3 that the complex phase of this complex parameter is changed by a phase angle that depends also on trace of the overall round-trip ABCD matrix of the laser resonator. In chapter 4 the general deduction is applicable also in this case. There is a tunable resonance of the fluctuation of the deviation angle and transversal position of the laser beam.

In Sec. 4 we also point out that there are two similar resonances in the noise of the time domain parameters of ultrashort laser pulses because of the analogy between the spatial description of the propagation of light beams and the temporal description of the propagation of light pulses.

In Sec. 5 we report on our experimental demonstration of the resonances of light beams. We built a longitudinally pumped astigmatically compensated Ti:sapphire laser resonator. The output laser beam was partially blocked by a knife edge. We found tunable resonances in the noise of the power of the surviving laser beam. The observed properties of the resonances correspond to the theoretically predicted attributes.

2. BEAM-WIDTH PERTURBATION IN LASER RESONATORS

In this section we will consider a light beam in a laser resonator using a paraxial approximation where the complex radius of curvature of the beam is perturbed from its self-consistent value. The topic of this section is practically known from Ref. 1. Laser resonators may be astigmatic, planar resonators, but for the sake of simplicity, we deal here only with one complex radius of curvature describing the behaviour of the light beam in either the meridional or in the sagittal direction. We deal here only with real ABCD matrices because our main conclusions may be drawn from this simplified picture also. We show that if the perturbation of the complex radius of curvature is small, only the complex phase of the perturbation changes after a round trip in the first order of perturbation calculation. The value of the phase change depends only on the trace of the round-trip ABCD matrix.

The self-consistent complex radius of curvature of the laser beam in a resonator is determined from the following equation:

\[
q = \frac{Aq + B}{Cq + D},
\]

where \( q \) is a complex radius of curvature of the light-beam at a position inside the laser resonator; \( A, B, C \) and \( D \) are the round-trip ABCD matrix elements. The solution of the equation (1) can be written as

\[
q = \frac{A - D}{2C} \pm j\sqrt{1 - \left(\frac{A + D}{2C}\right)^2},
\]

where \( j \) is the imaginary unit \((j^2 = -1)\); the alternative sign should be chosen to ensure the imaginary part of \( q \) to be positive. It can be achieved only if

\[
|m| < 1, \quad m = \frac{A + D}{2},
\]

Hereafter in this paper \( m \) will be called as a stability parameter of the resonator.

We suppose that a slightly perturbed light beam starts propagating around the resonator. The deviation of the complex radius of curvature of the light beam starting from the self-consistent value is denoted by \( y_1 \). The deviation of the complex radius of curvature of the light beam after a round trip is \( y_2 \):

\[
q + y_2 = \frac{A}{C} \frac{(q + y_1) + B}{(q + y_1) + D}.
\]
In the first order of perturbation calculation we obtain

\[ y_2 = \frac{A - Cq}{Cq + D} y_1 \]  
(5)

After using equation (2) and the \( m \) parameter defined in (3) and we get:

\[ y_2 = \left( m \mp j \sqrt{1 - m^2} \right)^2 y = e^{j \varphi(m)} y_1, \]  
(6)

where \( \varphi(m) \) is a complex phase-angle rotation of one round trip (see Fig. 1).

It may be interesting to note that the value of the phase shift (see equation (6)) is equal to the double of the generalised Gouy-phase shift of cylindrically symmetric light beams.

We will show in Sec. 4 that the phase shift determined by the equation (6) implies a tunable resonant behaviour of the laser resonator.

3. BEAM-DEFLECTION IN LASER RESONATORS

In this section we will consider a light beam with small angular deflection in a laser resonator using paraxial approximation and first order of the perturbation calculation. The propagation of a light beam through some optical system is described by the generalised Fresnel-integral involving the elements of the overall ABCD matrix of the given optical system. Similarly to the treatment in Sec. 2 we deal only with real ABCD matrices of one dimensional beam description.

We start from the complex scalar wave function of the self-consistent laser beam at a position inside the laser resonator given on a transversal plane:

\[ u(x_1) = E_1 \exp\left( -\frac{j \pi x_1^2}{\lambda q} \right), \]  
(7)

where \( x_1 \) is one of the principal transversal directions of the paraxial laser beam, \( E_1 \) gives the amplitude and phase of the beam, \( j \) is the imaginary unit, \( \lambda \) is the wavelength of the light in vacuum, and \( q \) is the self-consistent complex radius of curvature of the light beam.

An angularly deviated light beam can be described on the same transversal plane as in equation (7) as follows:

\[ u(x_1) = E_1 \exp\left( -\frac{j \pi x_1^2}{\lambda q} + j 2 \beta_1 x_1 \right), \]  
(8)
where $\beta_1$ describes the angular deviation and it is supposed to have a small value.

After a round trip we can obtained the resultant light beam by the generalised Fresnel-diffraction integral

$$u (x_2) = e^{-j k L} \int_{-\infty}^{+\infty} K (x_2, x_1) u (x_1) \, dx_1,$$

where $L$ is the effective path length of the laser resonator and the kernel of the integral is

$$K (x_2, x_1) = \sqrt{\frac{j}{B \lambda}} \exp \left[ - \frac{j \pi}{B \lambda} (A x_1^2 - 2 x_1 x_2 + D x_2^2) \right],$$

where $A$, $B$ and $D$ are the appropriate elements of an ABCD matrix of the round-trip propagation. Using the Siegman-lemma

$$\int_{-\infty}^{+\infty} \exp (-a x^2 - 2 b x) \, dx = \sqrt{\frac{\pi}{a}} \exp \left( b^2 \frac{1}{a} \right),$$

in the first order of the perturbation calculation we get

$$u (x_2) = E_2 \exp \left( - \frac{j \pi x_2^2}{\lambda q_2} + j 2 \beta_2 x_2 \right),$$

with the following three new notations

$$q_2 = \frac{A q + B}{C q + D},$$

$$\beta_2 = \beta_1 \frac{-q}{A q + B},$$

$$E_2 = E_1 e^{-j k L} \sqrt{\frac{q}{A q + B}}.$$

If the complex radius of curvature of the original beam given in equation (8) corresponds self-consistent beam of the resonator in the starting position and if the ABCD matrix is the round-trip matrix of the resonator then

$$q_2 = q \text{ (see Eq. (10))} \text{ just like in equation (2).}$$

Using equations (10) and (2) we can obtain from Eq. (11)

$$\beta_2 = \left( m \mp j \sqrt{1 - m^2} \right) \cdot \beta_1 = e^{j \psi (m)} \cdot \beta_1,$$

where $\psi (m)$ is a complex phase-angle rotation of one round trip (see Fig. 2).

In equation (8) $\beta_1$ was treated as a real variable (just to describe angular deviation), but $\beta_2$ in equation (9) turned out to be a complex quantity generally (see equation (13)). For the next round trip $\beta_2$ is the starting value describing some complexized angular deviation. We think that $\beta_1$ and $\beta_2$ should be considered generally as a complex parameter. Depending on its complex phase angle it can describe angular deviation and/or transversal displacement.

It is interesting to note that the value of the phase shift (see equation (13)) is equal to the negative value of the generalised Gouy-phase shift for cylindrically symmetric light beams.

From equation (13) we can conclude that after a round trip the complex angular displacement parameter of the light beam maintain its amplitude and changes its complex phase angle by an amount depending on the stability parameter ($m$) of the resonator (see equation (3)) in the first order perturbation calculation. That is
4. RESONANT BEHAVIOUR

In this section we show that a light beam in a laser resonator (or in any optical system with feedback) has a resonant noise sensitivity if a complex beam parameter gets a complex phase shift per round trip.

Let us consider a complex-valued beam parameter $y$ excited by some noise $n$. The value of $y$ at a time $t$ is the result of an evolution from the previous round-trip as follows: 1. the parameter suffers attenuation as multiplication by $(1 - \alpha)$; 2.a the parameter propagates through the feedback loop which formally appears as multiplication by a complex phase factor $e^{j\varphi}$; 2.b as part of the propagation, the parameter is delayed by time interval $T$; 3. the noise added at a time $t$ (see Fig. 3):

$$n(t) + (1 - \alpha) e^{j\varphi} y(t - T) = y(t) ,$$

where $\varphi$ is a constant phase value, $(1 - \alpha)$ is an attenuation factor $(0 < \alpha < 1)$ and $T$ is a constant time delay.

After taking the Fourier-transform of equation (14) we get

$$N(f) + (1 - \alpha) e^{j\varphi} Y(f) e^{2\pi f T} = Y(f) ,$$
The amplitude-square spectrum of the investigated beam parameter (a) and a real valued i.e. measurable signal (b) where \( f \) is a frequency value, \( N(f) \) and \( Y(f) \) are the Fourier-transform of the noise \( n(t) \) and the beam parameter \( y(t) \) time functions, respectively.

\[
|Y(f)|^2 = \frac{|N(f)|^2}{1 + (1 - \alpha)^2 - 2(1 - \alpha) \cos (\varphi - 2\pi f/f_0)} , \tag{16}
\]

where \( f_0 = 1/T \).

The amplitude-square spectrum \(|Y(f)|^2\) shows a resonant behaviour. If we suppose the spectrum of noise to be constant (white noise) then we get simple resonance curves (see Fig. 4a).

The resonance in the amplitude of \(|Y(f)|\) occurs when the denominator is minimal in equation (16), that is

\[
f_{\text{res}} = f_0 \left( \frac{\varphi}{2\pi} + k \right) , \tag{17}
\]

where \( k \) is an arbitrary integer value.

When we measure, naturally, real quantity that depends on the complex beam parameter \( y \) we would observe resonances not only at positive frequencies given by equation (17) but also at the absolute value of the negative frequencies also given by equation (17). If a frequency is resonant then the complementary frequency to the round-trip frequency is also resonant. Finally we obtain noise spectra similar to the curve given by Fig. 4b instead of the curve given by Fig. 4a as it was checked with simple simulations.

According to our analysis of this section we can plot the resonant behaviour in the beam width and angular deviation in Fig. 5.
There is an analogy between the spatial description of light beams and the temporal description of ultrashort light pulses: their evolution is described by appropriate two-by-two matrices. In a time domain, dispersion and self-phase modulation correspond to spatial distance and focusing in the spatial description, respectively. The calculation presented in Sec. re-derived in time domain would result a tunable resonance of the duration and the phase modulation of the laser pulses, while the calculation presented in Sec. re-derived in time domain would result a tunable resonance of the timing jitter and the central frequency of the laser pulses. In these cases the tuning can be achieved by varying the dispersion and/or the self-phase modulation in the laser resonator. We expect these resonances to be detectable for example, in the noise of the second harmonic signal of the output laser beam. We think that these resonances have been detected but has been misinterpreted. The authors of the cited works labelled some resonances as “spurious” and interpreted them as being generated by nonlinear electronic mixing processes. We suppose those resonances to be the type here being investigated because of their occurrence in pairs just as in Fig. 4.b.

The CEO (carrier-envelope offset) phase of an ultrashort light pulse in a laser resonator is also a complex parameter that changes a given amount per round trip. That is why the CEO-frequency also has the property described by equation.

5. EXPERIMENT

We have built a longitudinally pumped astigmatically compensated Ti:sapphire ring laser (see fig. and table 1) to demonstrate the predicted resonances of light beams. We chose ring-resonator arrangement to realize the model described in Secs. and the light beam in the resonator was perturbed in the laser crystal only once a round-trip. The parameters of our laser resonator can be seen in table 1. Moreover this laser configuration has the beneficial property that the stability parameter \( m \) introduced in Sec. is linearly scaled with the \( d_4 \) parameter of the laser resonator (Fig. 6). This property was used to scan the stability range of the laser.

| \( \lambda_0 = 0.532 \mu \text{m} \) | \( w_0 = 1.15 \text{mm} \) | \( f = 84 \text{mm} \) |
|-----|-----|-----|
| \( \varphi_1 = 5.0^\circ \) | \( \varphi_2 = 10.9^\circ \) | \( R_2 = R_1 = 75 \text{mm} \) |
| \( d_0 = 3.46 \text{ mm} \) | \( \alpha = 0.45 \text{ mm}^{-1} \) | \( n = 1.752 \) |
| \( \lambda = 800 \text{ nm} \) | \( d_2 = 1375 \text{ mm} \) | \( \text{Ref}l_{OC} = 92\% \) |

Table 1. The parameters of the astigmatically compensated laser-resonator whose schematic layout can be seen on Fig. 6.

The resonator is not a special resonator, it is optimised just for maximal output power at 4 W pump power. The mirrors in the resonator are simple quarter-wave stack mirrors. The laser was pumped by a Verdi G5 laser (Coherent Inc.). The parameters of the laser resonator can be seen in table 1.
The focusing lens (FL), the laser-crystal (LC) and the $M_2$ mirror were mounted on translator stages with micrometer screws. The position of these elements originally were optimised for minimal pump power, but during the measurements the position of the focusing lens and the laser crystal were left unmoved. The direction of the movement of the $M_2$ mirror was not perfectly parallel to the direction of the light beams, that is why the $M_3$, $M_4$ and $M_5$ mirrors were adjusted for minimal pump power each time the $M_2$ mirror was moved. At pump laser power of 4 W we could achieve laser action at the micrometer position of $M_2$ mirror between 4.00 mm and 7.80 mm values.

To detect the resonances described in Secs. 2-4. we set a knife-edge on a micrometer positioner with a vertical edge into the beam path of the output laser beam at a distance of 95 cm from the output coupling mirror ($M_4$). Behind the knife edge at a distance of 15 cm a fast photodetector (Newport model 818-BB-22) was measuring the power of the residual light beam. The residual light beam was regularly checked to fall on the sensitive surface of the photodetector. The signal of the photodetector was measured by an Agilent DSO 9254 oscilloscope. We used external 50 $\Omega$ terminator resistor. The noise was measured at a sensitivity of 1 mV/scaling at a 2 GSample/s sampling rate measuring 32768 points a scan using AC coupling. The data of the scans was Fourier-transformed and later averaged by the plug-ins of the oscilloscope. For adjustment we used 64 time averaging, for measuring 1024 time averaging. The background noise of the detecting system was between -95 dBm and -97 dBm values in the investigated 30-250 MHz frequency range besides some background resonances of the surroundings. (The values given in dBm units means the ratio of the given electric signal’s power on 50 $\Omega$ resistor relative and the 1 mW value given in dB units. We checked this law width a signal generator.)

The shape of the laser beam was measured by the knife-edge method (see Fig. 7). To preclude saturation of our detection system we used neutral grey-filter before the photodetector.

Two pairs of resonances can be observed on Fig. 8 which shows a typical measured spectrum of the signal of the photodetector above the background noise level. When the $M_2$ mirror was moved the resonances were sweeping in the frequency range. The resonances were occurring always in pairs: there was in the middle position the half of the round-trip frequency just the way that was predicted in Sec. 4.

Pump power dependency of the resonant frequencies are measured at a fixed mechanical alignment (see Fig. 9). The change of the pump power changes the effective focal length of the thermal lensing and the gain guiding lensing. These changes deviate elements of the round-trip matrix from the values calculated from a bare resonator theory. These deviations keep us from being able to precisely determine the mirror positions from the measured resonant frequency values. But from these deviations we may determine the strength of self focusing and thermal lensing.
We measured the dependency of the frequencies of the resonances as a function of the $M_2$ mirror position (see Fig. 6). The relative frequency means here the ratio of the measured frequency of the resonance and the round-trip frequency of the laser resonator. The frequencies denoted by red circles are connected to the beam-width fluctuation ($W$-meas) and the frequencies denoted by blue rectangles are connected to the beam-tilt deviation ($T$-meas). The lines on the figure show fitted calculated dependencies of the resonator frequencies (see Fig. 5). We measured similar resonances in the sagittal direction (with horizontal knife edge), the observed frequencies corresponds well to the shifted effective resonator.

We checked that there is no resonance in the noise of the power of the entire output beam at the same setup as in the case of Fig. 8. To avoid saturation of our measuring system we attenuated the beam by a reflection from a glass surface.

The resonances connected to the fluctuation of the beam width variation was always observed at the edge of the light beam (corresponding to high beam-stop ratio), where the DC signal was high enough (without any attenuation of the light beam other than the effect of the knife edge).

The resonances connected to the fluctuation of the beam propagation direction could have been observed at every position of the knife edge provided that the measuring system did not get saturated (see Fig. 11). To avoid saturation we applied a neutral grey-filter before the photodetector. The amplitude of the fluctuation is proportional to the amplitude of the light intensity at a given position of the knife edge (see Fig. 11), as it can be expected from the theory.

We found the same kind of resonances in the sagittal direction also, that is, when the knife with vertical edge was moved in horizontal, transversal direction. The frequencies of the resonances was shifted according to the shifted stability ranges in the meridional and in the sagittal directions.
Figure 9. Pump power dependence of the resonant frequencies. Resonance of the beam tilt angle (a) and the beam width (b) variation (see Fig. 8)

Figure 10. Measured frequencies of resonances as a function of the $M_2$ mirror position

6. SUMMARY

We theoretically predicted and experimentally demonstrated two new, fundamental resonances of laser resonators. These radiofrequency resonances occur in the power noise in the output laser beam partially blocked by a knife edge. There is a tunable resonance of the beam-width and the radius of curvature of the output light beam, and there is another resonance of the angular- and the transversal positional deviation of the light beams. The resonant frequencies can be tuned with, for example, changing the position of one of the mirrors of the resonator.

The resonant behaviour was experimentally demonstrated with a longitudinally pumped astigmatically compensated Ti:sapphire ring laser. Using a knife-edge to stop most of the output beam and only its small part was measured with a fast linear detector. The signal was observed by a digital oscilloscope. A Fourier-transform was taken of the sample points. The Fourier-spectrum was averaged many times to reduce its noise. We observed resonant behaviour in the noise of the detected signal. The observed resonances was 3-10 dB above the noise-background of the measuring system. If we observe resonance at a frequency value then we also observe resonance at the complement frequency up to the round-trip frequency. The observed dependencies of the frequencies of the resonances on the stability parameter of the laser resonator was in good agreement with the theory. One of the possible applications of these resonances are an approximate mapping tool of the stability region.

Because of the analogy between the description of light beams in the spatial domain and the description of light pulses in the time-domain we expect two type of tunable resonances in the noise of the second harmonic of the output laser pulse train. We expect a tunable resonance of the pulse-duration and the phase modulation of the output light pulse train, and we expect an another resonance of the time-jitter and the central frequency deviation of the pulse train.
Figure 11. Measuring the beam deviation resonance with changing the position of the knife edge (the position of the $M_2$ mirror was 4.60 mm). a) The amplitude of the resonance and the beam intensity as a function of the position of the knife edge. b) The shape of the resonances at different positions of the knife edge.

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