D-branes on a Deformation of $SU(2)$

Stefan Förste

Physikalisches Institut, Universität Bonn
Nussallee 12, D-53115 Bonn, Germany

Abstract

We discuss D-branes on a line of conformal field theories connected by an exact marginal deformation. The line contains an $SU(2)$ WZW model and two mutually T-dual $SU(2)/U(1)$ cosets times a free boson. We find the D-branes preserving a $U(1)$ isometry, an $F$-flux quantization condition and conformal invariance. Away from the $SU(2)$ point a $U(1) \times U(1)$ symmetry is broken to $U(1) \times \mathbb{Z}_k$, i.e. continuous rotations of branes are accompanied by rotations along the branes. Requiring decoupling of the cosets from the free boson at the endpoints of the deformation breaks the continuous rotation of branes to $\mathbb{Z}_k$. At the $SU(2)$ point the full $U(1) \times U(1)$ symmetry is restored. This suggests the occurrence of phase transitions for branes at angles in the coset model, at a semiclassical level. We also discuss briefly the orientifold planes along the deformation line.
1 Introduction

In the present article we consider D-branes on a family of conformal field theories connecting an $SU(2)$ WZW model with two $SU(2)/U(1)$ coset models differing by the way the $U(1)$ is embedded into $SU(2)$. For the undeformed WZW model and the coset models possible D-brane setups have been discussed before. Geometrically the $SU(2)$ is a three dimensional sphere $S^3$. Hence, the isometries form an $SO(4)$ group. Wrapping D2 branes on two dimensional spheres in $S^3$ one can set up D-brane configurations preserving an $SO(3)$ subgroup of the isometry transformations. The corresponding boundary conditions where given in [1] but not interpreted as belonging to D2-branes. This point was clarified in [3] where the general solutions to the boundary conditions were given. It was also pointed out that the position of the D2 brane can take only discrete values due to a topological argument.

Because a two dimensional sphere is contractable on a three dimensional sphere, there is a potential stability problem with the above discussed D2 branes. This puzzle was solved in [4, 5] and it was pointed out that the quantization of the D2-brane position corresponds to an $F$-flux quantization condition, where $F$ is the fieldstrength of a $U(1)$ gauge field living on the brane.

The $S^3$ geometry can be deformed to a class of geometries with $U(1) \times U(1)$ isometry. In the conformal field theory this corresponds to an exact marginal deformation [6]. Therefore, a natural way to deform the D2-branes on $S^3$ is given by the prescription that they should preserve a $U(1)$ isometry. By imposing this condition we will find deformed branes satisfying an $F$-flux quantization condition and preserving conformal invariance.

At the end of the deformation line the geometry degenerates into an $SU(2)/U(1)$ coset times a decoupled $U(1)$ of vanishing size. D-branes on the cosets were discussed in [8], [9], [10], [11], [12], [13], [14]. An important information is that the $U(1)$ isometry of rotations in the coset geometry is broken to $Z_k$, because the corresponding modulus couples as a ‘theta angle’ in the gauged WZW model [8]. This stabilizes D-branes which would be unstable if continuous rotations were allowed. Observing that the line of marginally deformed models can be viewed as an $SU(2) \times U(1)/U(1)$ model [14], we find that a $U(1) \times U(1)$ isometry is broken to $U(1) \times Z_k$ along the line of deformed models except for the point corresponding to the undeformed case. The reason is that for the undeformed case the size of the extra $U(1)$ becomes a modulus and continuous shifts in the ‘theta’ angle can be absorbed in a rescaling of the gauge group size. Hence, phase transitions can be triggered by rotating branes in the undeformed model.

The paper is organized as follows. In section 2 we review the $SU(2)$ preserving D-branes in the WZW model. This is done on an explicit level. We also discuss the $F$-flux quantization condition. Section 3 briefly recalls the family of conformal field theories obtained by exact marginal deformations of the closed string on $SU(2)$. In the fourth section we introduce D-branes corresponding to the deformed $SU(2)$ preserving D2 branes. We check that conformal invariance and the $F$-flux quantization condition are preserved. Section 5 is devoted to an investigation of the moduli spaces along the deformation line. By imposing decoupling of

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\footnote{For a ‘pre-brane-era’ discussion of open strings on the $SU(2)$ manifold see [2].}
the free boson at the ends of the deformation line, we argue that at one end of the line branes can be rotated only by discrete angles whereas at the other end of the deformation line Wilson lines can be shifted only by discrete values. At the $SU(2)$ point those moduli become continuous. In section 6 we discuss the A- and B-branes along the deformation line and suggest that phase transitions might by associated with the rotation of branes at the $SU(2)$ point. In a seventh section the possible orientifold fixed planes are briefly described. We summarize our results in a concluding section 8.

2 Recap of D-branes on an $SU(2)$ WZW model - coordinate dependent description

Let us first discuss the WZW model for worldsheets without boundaries. The action is given by

$$S = S^{\text{kin}} + S^{\text{WZW}} = \frac{k}{2\pi} \left[ \int_{\Sigma} d^2 z L^{\text{kin}} + \int_{\mathcal{B}} \omega^{\text{WZW}} \right], \quad (1)$$

where $k$ is the level of the WZW model. The worldsheet $\Sigma$ is parameterized by

$$z \pm = \frac{\tau \pm \sigma}{2} \quad (2)$$

and $\mathcal{B}$ is a three dimensional manifold whose boundary is $\Sigma$. The requirement of the theory to be independent of the particular choice of $\mathcal{B}$ leads to the quantization condition that the level $k$ should be integer valued\footnote{Reference}. The Lagrangian for the kinetic term reads

$$L^{\text{kin}} = tr \left( \partial_+ g \partial_- g^{-1} \right) \quad (3)$$

whereas the integrand in the Wess Zumino term is given as

$$\omega^{\text{WZW}} = \frac{1}{3} tr \left[ (g^{-1} dg)^3 \right], \quad (4)$$

where the power of three is understood with respect to the wedge product of differentials.

We parameterize the $SU(2)$ group element $g$ with “spherical coordinates”, i.e.

$$g = \cos \chi + i \sin \chi \cos \vartheta \sigma^1 + i \sin \chi \sin \vartheta \cos \varphi \sigma^2 + i \sin \chi \sin \vartheta \sin \varphi \sigma^3, \quad (5)$$

where $\sigma^i$ are the Pauli matrices and the first term is understood to be multiplied by the identity matrix. The parameter ranges are

$$\chi = 0 \ldots \pi , \ \vartheta = 0 \ldots \pi , \ \varphi = 0 \ldots 2\pi. \quad (6)$$

For later use let us give the following differential on the group manifold

$$g^{-1} dg = \{ \cos \vartheta d\chi - \sin \chi \cos \chi \sin \vartheta d\vartheta + \sin^2 \chi \sin^2 \vartheta d\varphi \} i\sigma^1 + \{ \sin \vartheta \cos \varphi d\chi + (\sin \chi \cos \chi \cos \vartheta \cos \varphi - \sin^2 \chi \sin \varphi) d\vartheta - (\sin^2 \chi \sin \vartheta \cos \vartheta \cos \varphi + \sin \chi \cos \chi \sin \vartheta \sin \varphi) d\varphi \} i\sigma^2 + \{ \sin \vartheta \sin \varphi d\chi + (\sin \chi \cos \chi \cos \vartheta \sin \varphi + \sin^2 \chi \cos \varphi) d\vartheta + (\sin \chi \cos \chi \sin \vartheta \cos \varphi - \sin^2 \chi \sin \vartheta \cos \varphi \sin \varphi) d\varphi \} i\sigma^3 \quad (7)$$
The differential $dgg^{-1}$ can be obtained from (7) by multiplication with $-1$ and replacing $\chi$ by $-\chi$. Hence, from the expression (7) one can read off the explicit form of the Kac-Moody currents

$$J = -\partial_+ gg^{-1} \equiv J_\mu \partial_+ X^\mu,$$

$$\bar{J} = g^{-1} \partial_- g \equiv \bar{J}_\mu \partial_- X^\mu,$$

(8)

where $X^\mu$ denotes the three targetspace coordinates $\chi, \vartheta, \varphi$.

For the action of the closed string on the $SU(2)$ manifold we find

$$S_{WZW} = \frac{k}{4\pi} \int d^2 \sigma \left\{ \partial_\alpha \chi \partial^\alpha \chi + \sin^2 \chi \partial_\alpha \vartheta \partial^\alpha \vartheta + \sin^2 \chi \sin^2 \vartheta \partial_\alpha \varphi \partial^\alpha \varphi 
- 2 \left( \chi - \frac{\sin 2\chi}{2} \right) \sin \vartheta \left( \partial_\tau \vartheta \partial_\sigma \varphi - \partial_\sigma \vartheta \partial_\tau \varphi \right) \right\}.$$  

(10)

The worldsheet indices $\alpha, \beta = 0, 1$ are raised and lowered with the Minkowski metric $\text{diag}(1, -1)$. Note, that we are free to add terms to the Lagrangian which vanish upon integration over the compact worldsheet $\Sigma$, i.e. we can add terms corresponding to an antisymmetric tensor field (a $B$-field) which is pure gauge. Our $B$-field reads

$$B = k\alpha' \left( \chi - \frac{\sin 2\chi}{2} \right) \sin \vartheta d\vartheta \wedge d\varphi.$$  

(11)

This choice is taken from[4] and motivated by the observation that the Aharanov-Bohm phase of a fundamental string wrapping around this potential is an integer multiple of $2\pi$ due to the quantization condition on $k$.

After we have written the action of the WZW model in terms of integrals over the worldsheet only, we can try to describe open strings on $SU(2)$ via just replacing the closed worldsheet in (10) by a worldsheet with boundary (the upper half plane). Then we have to impose boundary conditions which do not spoil the conformal invariance, i.e. the Dirac-Born-Infeld (DBI) action on the corresponding D-brane should be minimal[17]. We do not explore directly all possible boundary conditions but require instead that out of the $SU(2) \times SU(2)$ symmetry of the WZW model the boundary conditions preserve a residual $SU(2)$ symmetry. Afterwards we will check that conformal invariance is preserved.

Let us now work out explicitly the possible boundary conditions imposed by the requirement that they preserve a residual $SU(2)$ symmetry. The corresponding gluing conditions in the closed string channel have been worked out in[1]. In the open string channel this results in the following boundary conditions for the Kac-Moody currents[18, 3, 19],

$$(J_\mu - \bar{J}_\mu) \partial_\tau X^\mu + (J_\mu + \bar{J}_\mu) \partial_\sigma X^\mu = 0.$$  

(12)

Plugging in our explicit parameterization of the $SU(2)$ elements leads to the three equations
(corresponding to the factors in front of the three Pauli matrices)

\begin{align*}
0 &= -\cos \vartheta \partial_\tau \chi + \sin \chi \cos \chi \sin \vartheta \partial_\tau \vartheta + \sin^2 \chi \sin^2 \vartheta \partial_{\sigma} \varphi, \\
0 &= -\sin \vartheta \cos \varphi \partial_\tau \chi - \sin \chi \cos \chi \cos \varphi \partial_\tau \vartheta + \sin \chi \cos \chi \sin \vartheta \cos \varphi \partial_\tau \varphi \\
&\quad - \sin^2 \chi \sin \varphi \partial_\sigma \vartheta - \sin^2 \chi \sin \vartheta \cos \varphi \partial_\sigma \varphi, \\
0 &= -\sin \vartheta \sin \varphi \partial_\tau \chi - \sin \chi \cos \chi \sin \varphi \partial_\tau \vartheta - \cos \chi \sin \chi \sin \vartheta \cos \varphi \partial_\tau \varphi \\
&\quad + \sin^2 \chi \cos \varphi \partial_\sigma \vartheta - \sin^2 \chi \sin \vartheta \cos \varphi \partial_\sigma \varphi. 
\end{align*}

\begin{align}
\chi_{|\sigma=0} &= \chi_0. 
\end{align}

We observe that the coordinate \( \chi \) does not enter the boundary conditions with its transverse derivative \( \partial_\sigma \chi \). Therefore, we choose Dirichlet boundary conditions fixing \( \chi \) to some constant value

\begin{align}
\chi_{|\sigma=0} &= \chi_0. 
\end{align}

The rest of the equations \((13)-(15)\) yields inhomogeneous Neumann conditions for \( \vartheta \) and \( \varphi \)

\begin{align}
\frac{1}{k\alpha'} G_{\vartheta \vartheta} \partial_\sigma \vartheta &= \frac{\sin 2\chi_0}{2} \sin \vartheta \partial_\tau \varphi, \\
\frac{1}{k\alpha'} G_{\varphi \varphi} \partial_\sigma \varphi &= -\frac{\sin 2\chi_0}{2} \sin \vartheta \partial_\tau \vartheta, 
\end{align}

where \( G_{\varphi \varphi} = \sin^2 \vartheta G_{\varphi \vartheta} = k\alpha' \sin^2 \chi_0 \sin^2 \vartheta \) are the metric components of the target space. Note, that the resulting boundary value of the group element can be written as

\begin{align}
g(\tau) = k(\tau) e^{i\chi_0\sigma^3} k^{-1}(\tau),
\end{align}

with

\begin{align}
k(\tau) = e^{i(\frac{\chi_0}{4} - \frac{\varphi(\tau)}{4})\sigma^3} e^{i(\frac{\varphi(\tau)}{4} - \frac{\chi_0}{4})\sigma^2}.
\end{align}

Thus, the statement that the D-brane is the conjugacy class of a fixed group element is verified explicitly. In addition the expression \((13)\) allows us to read off the allowed quantized values of \( \chi_0 \) which are known from CFT analysis.

\begin{align}
\chi_0 \in \frac{\pi \mathbb{Z}}{k}.
\end{align}

As it stands our boundary conditions cannot be derived from \((10)\) (with the integration region being the upper half plane) by the Hamiltonian principle. In order to achieve consistency we

\footnote{At least in the semiclassical treatment, one obtains zero dimensional branes at \( \chi = 0, \pi \). Since the coordinates degenerate there, it is problematic to differ between D0 branes (Dirichlet conditions in all directions) and collapsed higher dimensional branes. A coordinate independent way to see this problem is given by the observation that the conjugacy classes of plus or minus the identity are plus or minus the identity. We will exclude the zero dimensional branes from our discussion, mostly.}
have to add a boundary term such that the action for the open string on the WZW model finally reads
\[ S_{\text{open}}^{\text{WZW}} = \frac{k}{4\pi} \int d^2\sigma \left\{ \partial_\alpha \chi \partial^\alpha \chi + \sin^2 \chi \partial_\alpha \varphi \partial^\alpha \varphi + \sin^2 \chi \sin^2 \vartheta \partial_\alpha \varphi \partial^\alpha \vartheta \\
-2 \left( \chi - \chi_0 - \frac{\sin 2\chi}{2} \right) \sin \vartheta \left( \partial_{r_\sigma} \partial_\alpha \varphi - \partial_{r_\varphi} \partial_\alpha \vartheta \right) \right\} . \] (22)

The term with the explicit dependence on the position \( \chi_0 \) of the D-brane can be written as an integral over the real line. Thus, the additional term corresponds to an \( F \)-flux on the worldvolume of the D-brane given by\[ F = -\frac{k}{2\pi} \chi_0 \sin \vartheta d\vartheta \wedge d\varphi. \] (23)

Comparison with (21) shows that the flux \( F \) is quantized. The flux quantization condition looks like a gauge \( (B \rightarrow B + d\Lambda) \) dependent condition. However, gauges changing the \( F \)-flux are not single valued and multi valued gauge transformations shift the flux by an integer amount\[ \text{[4]} \]. Moreover, it is easy to check that the DBI action is at a stationary point and thus conformal invariance is preserved\[ \text{[17]} \]. (We will present a more detailed discussion of this point when introducing branes in the deformed models, later.)

In summary, we have reviewed the possible boundary conditions preserving an \( SU(2) \) symmetry and satisfying a quantization condition of the \( F \)-flux on the D-brane. This will give us some guide for finding possible D-branes on a deformed \( SU(2) \) manifold later.

3 The closed string on a deformed \( SU(2) \)

In this section we review the results of an exact marginal deformation of the WZW model\[ \text{[6]} \]. We closely follow the presentation in \[ \text{[7]} \]. The exact marginal deformation is obtained by integrating an infinitesimal perturbation with an operator of conformal dimension \( (1, 1) \). The integration of the perturbation is much easier in different coordinates. Therefore, we parameterize the group element as follows
\[ g = \cos x \cos \tilde{\theta} - i\sigma^1 \sin x \sin \theta + i\sigma^2 \sin x \cos \theta + i\sigma^3 \cos x \sin \tilde{\theta}, \] (24)

with the parameter ranges
\[ x = 0 \ldots \frac{\pi}{2}, \quad \theta = -\pi \ldots \pi, \quad \tilde{\theta} = -\pi \ldots \pi. \] (25)

In this parameterization the action of the closed string on the group manifold reads
\[ S_{\text{WZW}} = \frac{k}{2\pi} \int d^2 z \left\{ \partial_+ x \partial_- x + \sin^2 x \partial_+ \theta \partial_- \theta + \cos^2 x \partial_+ \tilde{\theta} \partial_- \tilde{\theta} \\
+ \cos^2 x \left( \partial_+ \theta \partial_- \tilde{\theta} - \partial_+ \tilde{\theta} \partial_- \theta \right) \right\} . \] (26)
The advantage of the above parameterization is that a chiral and an anti-chiral current are manifest in (26). These are

\[ J = k \left( \sin^2 x \partial_+ \theta - \cos^2 x \partial_+ \tilde{\theta} \right), \]
\[ \bar{J} = k \left( \sin^2 x \partial_- \theta + \cos^2 x \partial_- \tilde{\theta} \right). \]

Hence, a good candidate for a marginal deformation is the product \( J \bar{J} \). Indeed, such a perturbation can be integrated to finite deformations with the resulting action

\[ S^R = \frac{k}{2\pi} \int d^2 z \left\{ \partial_+ x \partial_- x + \frac{\sin^2 x}{\cos^2 x + R^2 \sin^2 x} \partial_+ \theta \partial_- \theta \\
+ \frac{R^2 \cos^2 x}{\cos^2 x + R^2 \sin^2 x} \partial_+ \tilde{\theta} \partial_- \tilde{\theta} + \frac{\cos^2 x}{\cos^2 x + R^2 \sin^2 x} \left( \partial_+ \theta \partial_- \tilde{\theta} - \partial_+ \tilde{\theta} \partial_- \theta \right) \right\}. \]

In addition a nontrivial dilaton \( \Phi \) is generated by the deformation according to

\[ e^{-2\Phi(R=1)} \sqrt{G(R=1)} = e^{-2\Phi(R)} \sqrt{G(R)}, \]

where \( R = 1 \) corresponds to the undeformed case with a constant dilaton, and \( G \) is the determinant of the target space metric.

The deformation breaks the original \( SU(2) \times SU(2) \) symmetry to a \( U(1) \times U(1) \) symmetry. The corresponding chiral and anti-chiral currents are manifest in the present parameterization and read

\[ J(R) = \frac{J(R=1)}{\cos^2 x + R^2 \sin^2 x}, \]
\[ \bar{J}(R) = \frac{\bar{J}(R=1)}{\cos^2 x + R^2 \sin^2 x}. \]

That these currents are conserved follows from a combination of the \( \theta \) and \( \tilde{\theta} \) equation of motion and hence is not sensitive to the \( x \) dependent dilaton.

At the endpoints of the marginal deformation at \( R = 0 \) and \( R = \infty \) the geometry factorizes into a two dimensional manifold times a circle (with vanishing radius). Hence, we obtain the picture drawn in figure 1. The geometries at the endpoints of the deformation are related by T-duality with respect to \( \theta (\tilde{\theta}) \) (and renaming \( \theta \leftrightarrow \tilde{\theta} \) afterwards).

To close this section we should note that the \( B \) field in (26) does not coincide with (11) transformed to the coordinates \( \theta, \tilde{\theta}, x \). In these coordinates one obtains a rather lengthy expression for (11) and we did not attempt to find the additional gauge transformation connecting (11) with the \( B \) field in (26). This gauge transformation should not be single valued[4]. Therefore, we cannot expect to find the \( F \)-flux quantization condition (23) without performing a multi valued gauge transformation.
\[ R = 0 \quad R = 1 \quad R = \infty \]

Figure 1: The “middle” and end points of the deformed model. At the endpoints there is an additional circle with vanishing radius.

4 The open string on a deformed \( SU(2) \)

In this section we would like to investigate the question whether and where we can add D-branes to the deformed geometry discussed in the previous section. As a guideline we take first the requirement that the \( U(1) \times U(1) \) symmetry of the deformed model is broken by the boundary conditions to a residual \( U(1) \) symmetry. This provides gluing conditions of the chiral (31) with the anti-chiral (32) currents. We choose these gluing conditions such that in the undeformed case they are solved by our previous boundary conditions. Performing a coordinate transformation one can verify that a condition satisfying our criteria is given by (9) with \( J, \bar{J} \) as given in (31), (32), (27), (28). Hence, the gluing conditions are independent of the deformation parameter \( R \). Expressed in the coordinates \( x, \theta, \bar{\theta} \) the gluing condition reads

\[ -\cos^2 x \partial_x \bar{\theta} + \sin^2 x \partial_x \theta = 0. \quad (33) \]

As it stands, this is one condition for three coordinates. However, our knowledge of the undeformed model leads us to impose in addition the Dirichlet condition

\[ \cos \chi_0 = \cos x \cos \bar{\theta}. \quad (34) \]

Now, it is natural to read (33) as inhomogeneous Neumann condition on \( \theta \)

\[
\frac{1}{k\alpha'} G_{\theta\theta} \partial_\sigma \theta = \frac{\cos^2 x}{\cos^2 x + R^2 \sin^2 x} \partial_x \bar{\theta} \\
= -\frac{\sin x \cos x \cos \chi_0}{(\cos^2 x + R^2 \sin^2 x) \sqrt{\cos^2 x - \cos^2 \chi_0}} \partial_x x,
\]

where \( G_{\mu\nu} \) denotes again the target space metric and in the second step we have used (34) to eliminate \( \bar{\theta} \). The boundary term picked up in the \( \theta \) variation of (29) gives the same boundary condition as in (33). Thus, we do not need to add \( F \)-flux to the boundary in contrast to

3Expressed in the coordinates of the previous section, \( \chi \) is the only direction which does not enter (33) with its \( \sigma \) derivative. The particular boundary value for \( \chi \) could depend on the deformation parameter \( R \). We will argue below that this is prohibited by the quantization condition on the \( F \)-flux.
section 2. (This confirms our earlier statement that the model in [4] and [7] are connected
by a multi valued gauge transformation.) In addition to (35) we find a Neumann boundary
condition for the second direction along the D2 brane,
\[
\partial_x x - \frac{R^2 \sin x \cos x \cos \chi_0}{(\cos^2 x + R^2 \sin^2 x) \sqrt{\cos^2 x - \cos^2 \chi_0}} \partial_x \tilde{\theta} =
\]
\[
= \frac{\sin x \cos x \cos \chi_0}{(\cos^2 x + R^2 \sin^2 x) \sqrt{\cos^2 x - \cos^2 \chi_0}} \partial_x \theta.
\]
(36)

For consistency, we should check that the DBI action on the D-brane is minimized by
our boundary conditions. The DBI action reads (for vanishing $F$-fieldstrength)
\[
S_{DBI} = T(2) \int_{D^2} d^2 \xi e^{-\Phi + \Phi_0} \sqrt{\det \hat{G}},
\]
(37)

where we identify $\Phi_0$ with the constant dilaton at $R = 1$, and $\hat{G}$ is the value of $G + B$
on the brane. The action (37) is invariant under reparameterizations of the D-brane. We fix
this invariance by the “static” gauge
\[
x = \xi^1, \quad \theta = \xi^2, \quad \tilde{\theta} = \tilde{\theta}(x).
\]
(38)

Here, we consider only boundary conditions preserving the invariance under constant shifts
of $\theta$. The metric induced on the brane is
\[
ds_{D^2}^2 = k \alpha' \left(1 + \frac{R^2 \cos^2 x \left(\partial_x \tilde{\theta}\right)^2}{\cos^2 x + R^2 \sin^2 x}\right) dx^2 + \frac{k \alpha' \sin^2 x}{\cos^2 x + R^2 \sin^2 x} d\theta^2,
\]
(39)

the induced $B$-field reads
\[
B_{D^2} = k \alpha' \frac{\cos^2 x}{\cos^2 x + R^2 \sin^2 x} \partial_x \tilde{\theta} dxd\theta,
\]
(40)

and the dilaton is given by (see (31))
\[
e^{\Phi_0 - \Phi} = \sqrt{\frac{\cos^2 x + R^2 \sin^2 x}{R}}.
\]
(41)

Plugging this into (37) yields
\[
S_{DBI} = \frac{k \alpha'}{R} T(2) \int dx d\theta \sqrt{\sin^2 x + \cos^2 x \left(\partial_x \tilde{\theta}\right)^2}.
\]
(42)

The corresponding equation of motion
\[
\partial_x \left(\frac{\cos^2 x \partial_x \tilde{\theta}}{\sqrt{\sin^2 x + \cos^2 x \left(\partial_x \tilde{\theta}\right)^2}}\right) = 0
\]
(43)
is satisfied by our boundary condition (34). Hence, we have placed the D-brane in a conformal invariance preserving way.

Multi valued gauge transformations (on \(B\)) can shift the \(F\) flux only by integer values and hence our result of vanishing \(F\)-flux agrees with a quantization condition on the \(F\)-flux. To see this more explicitly let us perform the following gauge transformation (first in the closed string case)

\[
\delta B = 2\pi \alpha' d\Lambda, \tag{44}
\]

with

\[
\Lambda = \frac{k}{2\pi} \tilde{\theta} \, d\theta, \tag{45}
\]

being a multi valued function. The fact that the integral

\[
\frac{1}{2\pi \alpha'} \int \delta B = 2\pi k \tag{46}
\]

is an integer multiple of \(2\pi\) ensures that the wave function of a fundamental string wrapping around \(\delta B\) picks up an invisible Aharanov-Bohm phase for integer \(k\). For the open string we have to combine (44) with a shift in the gauge fieldstrength taking us from the previously vanishing \(F\) field to

\[
F = -d\Lambda = -\frac{k}{2\pi} d\tilde{\theta} \wedge d\theta, \tag{47}
\]

where (34) should be imposed. The corresponding (integrated) \(F\)-flux is\footnote{Our computation assumes \(\chi_0 \leq \frac{\pi}{2}\). For \(\chi_0 \geq \frac{\pi}{2}\) we just need to replace \(\chi_0\) by \(\pi - \chi_0\).}

\[
\int_{D^2} F = -k \int_{-\chi_0}^{\chi_0} d\tilde{\theta} = -2k\chi_0, \tag{48}
\]

which agrees with the flux obtained by integrating (23). This derivation of the quantization condition for the \(F\)-flux is completely independent of the value of the deformation parameter \(R\), and confirms our earlier statement that the position of the D-brane should be given by (34) for all \(R\).

To summarize, we first discuss the D-branes for the model at \(R = \infty\). The \(\theta\) direction decouples there and the D-brane becomes a D1-brane on the remaining two dimensional surface. The range for \(x\) is restricted to \(0 \ldots \chi_0\) (for \(\chi_0 \leq \frac{\pi}{2}\)) or \(0 \ldots \pi - \chi_0\) (for \(\chi_0 \geq \frac{\pi}{2}\)). At \(x = 0\) the radius of the \(\tilde{\theta}\) circle diverges. The boundary value for \(\tilde{\theta}\) is given by \(\cos \tilde{\theta} = \cos \chi_0\) there. When \(x\) takes its maximal size \(\tilde{\theta}\) must be either 0 or \(\pi\) depending on the sign of \(\cos \chi_0\). Thus the D-brane connects the points \(x = 0, \tilde{\theta} = \pm \chi_0\) along a geodesic through the two dimensional surface.

At \(R = 0\) the \(\tilde{\theta}\) direction decouples and the D2-brane remains a two dimensional surface on the remaining geometry. The maximal value for \(x\) is again \(\chi_0\) (or \(\pi - \chi_0\)). Hence, we obtain the picture drawn in figure 2 (for \(k = 3\)) Note, also that the D-branes fit into the
Figure 2: The “middle” and end points of the deformed model with the possible D-branes for $k = 3$.

picture that the two endpoints of the deformation are related by T-duality. Performing a T-duality with respect to $\tilde{\theta}$ of the $R = \infty$ model the position of the D-brane becomes a gauge field component $A_\theta$ in the $R = 0$ model$[21],[22]$ (see also section 2.3.3.2 of$[23]$). Computing the corresponding fieldstrength one finds agreement with the deformed model at $R = 0$.

5 Rotating branes and shifting Wilson lines

In the class of models (parameterized by the deformation parameter $R$), we are going to discuss the two following moduli. Firstly, we can shift $\tilde{\theta}$ by a constant and secondly we can shift Wilson lines (constant values of $A_\theta$). Shifting $\tilde{\theta}$ by a constant corresponds to rotating the branes in the zero-three plane of the embedding $\mathbb{R}^4$ (see$[24]$). Therefore, one could for example construct configurations with more branes by adding to our previous setup rotated branes. However, strings stretched between branes and rotated branes will pull those branes on top of each other. Below, we are going to argue that the shift symmetry in $\tilde{\theta}$ is broken to $\mathbb{Z}_k$ for $R \to \infty$. This means that branes and rotated branes cannot be pulled on top of each other in a continuous way and hence we expect new stable configurations.

Our argument that the $U(1)$ symmetries are broken to $\mathbb{Z}_k$ is very similar to the one given in$$[8]$. We consider the case with only closed strings. Our deformed models can be viewed as a class of $SU(2) \times U(1)/U(1)$ gauged models, where the deformation parameter corresponds to the way the gauged $U(1)$ is embedded in $SU(2) \times U(1)$$[13]$. To be explicit, we start with an $SU(2) \times U(1)$ WZW model,

$$S = \frac{k}{2\pi} \int d^2 z \left\{ \partial_+ x \partial_- x + \sin^2 x \partial_+ \theta' \partial_- \theta' + \cos^2 x \partial_+ \tilde{\theta} \partial_- \tilde{\theta} + \cos^2 x \left( \partial_+ \theta \partial_- \tilde{\theta} - \partial_+ \tilde{\theta} \partial_- \theta \right) + \partial_+ y \partial_- y \right\}. \quad (49)$$

Now, we gauge a combination of constant shifts in $y$ and $\theta'$, promoting the corresponding symmetry to a local one. This is done by replacing partial derivatives with the following covariant derivatives ($\alpha$ is a worldsheet index)

$$\partial_\alpha \theta' \to \partial_\alpha \theta' + \sqrt{R^2 - 1} A_\alpha, \quad (50)$$

$$\partial_\alpha y \to \partial_\alpha y + A_\alpha \quad (51)$$
Under local shifts $\delta \theta'$ the worldsheet gauge field transforms as

$$
\delta A_\alpha = -\frac{1}{\sqrt{R^2 - 1}} \partial_\alpha \delta \theta'
$$

and hence the size of the gauge group is $2\pi/\sqrt{R^2 - 1}$. The action is invariant if we simultaneously shift $y$ by

$$
\delta y = \frac{1}{\sqrt{R^2 - 1}} \delta \theta'.
$$

This in turn fixes the size of the $U(1)$ parameterized by $y$. The gauge field $A_\alpha$ appears without derivatives in the action and can be integrated out by replacing it with its classical solution or solving the Gaussian integral. The resulting action is (29) with $	heta = \theta' - \sqrt{R^2 - 1} y$. The other combination $\theta' + \sqrt{R^2 - 1} y$ drops out. Its integration cancels the integration over gauge equivalent fields $A_\alpha$ up to a Jacobian which can be taken into account by redefining the dilaton according to (30). Thus, for $R > 1$ our deformed model is equivalent to the gauged $SU(2) \times U(1)$ model described above. Performing the multi valued transformation (44) with (45) (and $\theta$ replaced by $\theta'$)\(^5\) generates the following term in the gauged $SU(2) \times U(1)$ model,

$$
\delta S_{\text{gauged}} = \frac{k \sqrt{R^2 - 1}}{2\pi} \int d^2 z \tilde{\theta} F_{+-},
$$

where $F_{+-}$ is the fieldstrength corresponding to $A_\alpha$. Recalling that the size of the gauge group is $2\pi/\sqrt{R^2 - 1}$ one might conclude that the shift symmetry in $\tilde{\theta}$ is broken to $\mathbb{Z}_k$ (i.e. only rotations about integer multiples of $2\pi/k$ are allowed). However, performing another multi valued gauge transformation one can also generate discrete values of a $B_{y\theta'}$ component, e.g.

$$
B = k \alpha' \sqrt{R^2 - 1} d\theta' \wedge dy.
$$

After gauging, the associated terms result for example in the following coupling

$$
\delta S_{\text{gauged}} = \frac{k \sqrt{R^2 - 1}}{2\pi} \int d^2 z \theta F_{+-}.
$$

Thus the global $U(1) \times U(1)$ transformations of shifting $\tilde{\theta}$ and $\theta$ are broken to a $U(1) \times \mathbb{Z}_k$. By shifting $B$ field components by discrete amounts the embedding of the $U(1)$ into the $U(1) \times U(1)$ can be changed. In the limit $R \to \infty$ one gauges only the shift symmetry in $\theta$ and obtains the quantization conditions on shifts in $\tilde{\theta}$\[^4\]. The connection to our discussion is obtained by imposing decoupling of the $y$ direction for $R \to \infty$. This prohibits a $B_{y\theta'}$ term and hence the coupling (56) cannot be generated. One could be tempted to conclude

\[^5\]Alternatively one can switch on a constant $B_{\tilde{\theta} \theta'}$ component, whose value is quantized by the requirement that the corresponding Aharanov-Bohm phase for the wave function of a string wrapped around this $B$-field is an integer multiple of $2\pi$. This will result in the same quantization condition.
that a similar argument provides a quantization condition in the other limit \( R = 1 \). In this limit one would gauge only shifts in the \( y \) direction and thus the size of the additional \( U(1) \) would be a modulus. Hence, no quantization condition on the constant \( \tilde{\theta} \) shifts arises in the undeformed model.

By gauging a suitable combination of shifts in \( \tilde{\theta} \) and shifts in \( y \) in (49) one can also relate the models with \( R < 1 \) to gauged \( SU(2) \times U(1) \) models. At \( R = 0 \) one finds that shifts in \( \theta \) are broken to a discrete \( \mathbb{Z}_k \) symmetry, if one imposes decoupling of the \( y \) direction in the limit \( R \to 0 \). Wilson lines \( C_\theta \) can be written as

\[
iC_\theta = e^{-iC_\theta \partial_\theta} e^{iC_\theta \theta}
\]

where \( \theta \) is the center of mass position of the open string ending on the \( D2 \) brane, and \( C_\theta \) is a pure gauge field only for integer \( C_\theta \), because we have to require periodicity under \( 2\pi \) shifts in \( \theta \) for gauge transformations. Hence, there is a one-to-one correspondence between the moduli space of Wilson lines and the space of center of mass positions of the open string. If the space of center of mass positions of the open string is broken from \( U(1) \) to \( \mathbb{Z}_k \) the definition (57) does not make sense because the \( \theta \) derivative is ill defined. We assume that the moduli space of Wilson lines is broken to \( \mathbb{Z}_k \). This assumption is supported by the requirement that the two endpoints of the deformation line are related by T-duality and for \( R = \infty \) the branes can be rotated only by an integer times \( \frac{2\pi}{k} \).

6 The A- and B-branes

So far, we have considered only the deformation of branes leaving an \( SU(2) \) invariant in the undeformed model. These are the branes wrapped on “parallel” \( S^2 \) submanifolds of \( S^3 \), i.e. the \( S^2 \)'s have an identical rotation axis. In such a setup the \( SO(4) \) (with universal covering \( SU(2) \times SU(2) \)) isometry is broken to an \( SO(3) \) (with universal covering \( SU(2) \)) subgroup. If we rotate the rotation axis of one brane with respect to the other, each brane leaves a different subgroup of \( SO(4) \) invariant and hence the isometry is completely broken. In the undeformed case such a setup should not be stable, since open strings stretching between the two branes will ‘pull’ them to the symmetric setup. However, for \( R \to \infty \) the \( U(1) \) rotation group is broken to \( \mathbb{Z}_k \) and hence there can be new stable setups. These setups we are going to discuss in the following.

Let us restrict to the case \( R > 1 \). (For \( R < 1 \) relative angles between branes should be replaced by relative angles between Wilson lines.) We start with two \( SU(2) \) preserving \( D2 \)-branes (at coinciding or different locations). Then we rotate one of the \( D2 \) branes until they touch each other in one point and form a new \( D2 \) brane. For \( R \to \infty \) we cannot rotate the branes back into a “parallel” position in a continuous way. At that point the \( S^3 \) transforms into a geometry which is topologically a disc times a circle. On the disc our brane setup takes the form depicted in figure 3. This corresponds to the A-branes discussed in §6.

In the following we are going to investigate the A-branes along the line \( R > 1 \). Let us restrict to the setup obtained from two \( D2 \)-branes with originally coinciding positions. After
the rotation the position of the second brane becomes

$$\cos \chi_0 = \cos x \cos \left( \tilde{\theta} + \Delta \tilde{\theta} \right),$$

(58)

where

$$\Delta \tilde{\theta} = \frac{\pi}{k}.$$  

(59)

For $R > 1$ we have D2-branes. These should satisfy the $F$-flux quantization condition. The total $F$-flux of the A-brane is computed to

$$\int_{D2} F = -k \int_{-\chi_0}^{\chi_0} d\tilde{\theta} - k \int_{-\chi_0 + \Delta \tilde{\theta}}^{\chi_0 - \Delta \tilde{\theta}} d\tilde{\theta} = -2k \left( 2\chi_0 - \Delta \tilde{\theta} \right) \equiv -2k \chi_0',$$

(60)

where in the last step we have written the $F$-flux in the form of (18). Since $\chi_0' = 2\chi_0 - \pi/k$ satisfies the quantization condition (21) the considered A-brane satisfies the $F$-flux quantization condition. For our particular example, the $F$-flux coincides with the $F$-flux of a single D2-brane located at $\chi_0$. This is consistent with the statement in [8] that the ‘elementary’ A-branes consist out of straight lines connecting pairs of the $k$ points on the disc and D0 branes located at the $Z_k$ points. (Since for a single brane in $SU(2)$ the rotation is irrelevant, we can identify each deformed single $SU(2)$ brane with a straight line connecting two of the $k$ points. However, a set of $SU(2)$ preserving branes with different locations leads to parallel lines on the disc.) Extending our analysis to more general A-branes in a straightforward way one can show that all possible A-branes satisfy the $F$-flux quantization condition.

Again, we assume $\chi_0 \leq \frac{\pi}{2}$. If this is not the case we replace $\chi_0$ by $\pi - \chi_0$ in the result.
Figure 4: A typical B-brane setup at \( R = \infty \) for \( k = 6 \). The triangle drawn in softer lines corresponds to \( \chi_0 \to \pi - \chi_0 \). Adding this configuration leads to a \( \mathbb{Z}_k \) invariant picture.

One can also start with more than two D2-branes and rotate the second brane such that it touches the first one in a point and the third brane such that it touches the second one in a point and so on. Then the value of \( \chi'_0 \) in (60) becomes

\[
\chi'_0 = \sum_{i=1}^{N} \chi_i - \sum_{i=1}^{N-1} \Delta \tilde{\theta}_i \tag{61}
\]

where \( \chi_i \) are the original positions of the branes and \( \Delta \tilde{\theta}_i \) is the relative angle between the first and the \( i + 1 \)st brane.

Now, let us consider the following configuration at \( R = 1 \). We start (for even \( k \)) with \( k/2N \) D2-branes located at \( \chi_0 = \frac{2N\pi}{k} \), where \( N \) is a positive integer such that \( k/2N \) is an integer. Next, we rotate the second brane by an angle \( \frac{2N\pi}{k} \), the third brane by \( \frac{4N\pi}{k} \) and so on. In the \( R \to \infty \) limit this yields the setup drawn in figure 4 (for \( k = 6 \)). This is the T-dual version of the B-branes discussed in[8]. At \( R = \infty \) it is not stabilized by an \( F \)-flux, since (after T-dualizing the decoupled \( \theta \) direction) the D2-branes become D1-branes which do not support an \( F \)-flux. Another way, to stabilize branes could be given by the \( \mathbb{Z}_k \) quantization of shifts in \( \tilde{\theta} \). However, the B-branes form closed loops and can decay by shrinking to zero branes at the center of the disc[8]. (In the T-dual description the B-branes correspond to D2 branes covering only a part of the disc. Those D2 branes can collapse to zero branes.) For the B-brane setup at finite \( R \) we learn from (61) that the \( F \)-flux generically vanishes (modulo \( 2\pi k \)). Therefore, we expect that the B-branes are not protected by an \( F \)-flux quantization condition. Since they form closed loops also for finite \( R \geq 1 \) they can collapse in a \( \mathbb{Z}_k \) invariant way also along the deformation line. An exception is given for \( \chi_0 = \frac{\pi}{2} \). In this case a \( \mathbb{Z}_k \) invariant setup is formed by \( k/2 \) branes connecting antipodal points with relative angles \( 2\pi/k \). These do not form closed loops and are protected from decay by the \( \mathbb{Z}_k \) quantization condition of shifts in \( \tilde{\theta} \). At \( R < \infty \) this quantization condition is lifted and
the B-branes can continuously rotate. This will change the $F$-flux and the system becomes unstable. (In the conformal field theory prescription this should correspond to a relevant boundary perturbation. Such perturbations are studied in \cite{24,25}.) The $F$-flux is preserved if the number of D2 branes decreases and D0 branes might be created. (Since the D0 branes do not carry $F$-flux we cannot determine their existence within our present approach. A description of spherical D2 branes in terms of (noncommuting) D0 branes is given in \cite{27}, see also \cite{25} for the $SU(2)$ WZW model. In particular, the creation of D0 branes can be attributed to a conservation of brane charges taking values in $K_H^*(SU(2)) = \mathbb{Z}_k$ \cite{26,25}.) Deformed back to the $R = \infty$ point the rotated B-brane will be seen as a $\mathbb{Z}_k$ symmetric setup with less D1 branes stretching between antipodal points on the disc, and the missing D1 branes are replaced by D0 branes. Since the $F$-flux of the $\mathbb{Z}_k$ symmetric setup vanishes one can obtain a system with D0 branes, only. The description of B-branes in terms of D0-branes is given in \cite{8}. Our view on the phase transition of B-branes is similar to the way phase transitions among stable non BPS D-branes can be connected to marginal deformations (for reviews see e.g. \cite{28}, \cite{29}, \cite{30}).

### 7 Orientifold fixed planes

In this section we will briefly describe the location of orientifold fixed planes in the models along the line of marginal deformations. Early investigations of orientifolds of the $SU(2)$ models can be found in \cite{31}, \cite{32}, \cite{33}. More recent thorough studies are performed in \cite{34}, \cite{35}, \cite{36}. Here, we just give the fixed point sets under symmetries containing a worldsheet parity reversal, i.e. the O-planes. In the WZW model worldsheet parity transformations leave the action invariant provided that we combine them with replacing the group element $g$ by $\pm g^{-1}$. For the plus sign this gives the antipodal points $g = \pm 1$ as fixed planes whereas for the minus sign the equatorial $S^2$ is left invariant. If we parameterize the group element as in \cite{24} the possible orientifold transformations read

\begin{equation}
\begin{aligned}
x(z_+, z_-) \rightarrow -x(z_-, z_+), \quad \tilde{\theta}(z_+, z_-) \rightarrow -\tilde{\theta}(z_-, z_+), \quad \theta(z_+, z_-) \rightarrow \theta(z_-, z_+),
\end{aligned}
\end{equation}

and

\begin{equation}
\begin{aligned}
x(z_+, z_-) \rightarrow x(z_-, z_+), \quad \tilde{\theta}(z_+, z_-) \rightarrow \pi - \tilde{\theta}(z_-, z_+), \quad \theta(z_+, z_-) \rightarrow \theta(z_-, z_+).
\end{aligned}
\end{equation}

It is easy to see that also the deformed model \cite{29} is invariant under these orientifold transformations. Hence the corresponding O-planes can be followed along the deformation line. Figure 5 shows the possible O-planes in the undeformed model and for the two endpoints of the deformation line. Again the models at the end of the deformation line are related by T-duality if we view the O-zero-plane at the left tip of the left picture as a degenerated O-two-plane.
8 Conclusions

In the present article we gave a semiclassical description for D-branes along a line of marginally deformed $SU(2)$ WZW models. At the ends of the line one obtains $SU(2)/U(1)$ coset models. At the $SU(2)$ point the moduli space of rotating branes and Wilson lines is $U(1) \times U(1)$ whereas it is broken to $U(1) \times \mathbb{Z}_k$ away from the $SU(2)$ point. This suggests a mechanism for phase transitions in brane configurations of coset models. One can marginally deform the coset configuration to the $SU(2)$ point where phase transitions can be triggered by rotating the branes such that the $F$-flux quantization condition is violated. Since the $F$-flux quantization is not directly connected to conformal invariance we are not sure whether the phase transition can be associated to a relevant perturbation of the CFT description. Assuming that this is the case, we can use known results$^{[25]}$ to identify the phase transition as a transition between D2 and D0 branes. The geometric picture suggested by our analysis is, that at the $SU(2)$ point D2 branes at angles transform into less D2 branes and D0 branes. (The term ‘at angles’ refers to an angle between the rotation axes of the D2 branes through the center of $S^3$.) The transformed setup can be marginally deformed to the coset point where it can be associated to a phase transition in the coset model.

It should be interesting to extend our discussion to a more general analysis in deformed WZW models and beyond the semiclassical level. A CFT treatment is the subject of ongoing work$^{[37]}$ and preliminary results agree with the presented discussions. Another open question is to give an effective field theory description of the presented models. Using the observation that each point along the line of deformations can be identified with an $SU(2) \times U(1)/U(1)$ coset this should be a straightforward application of the results presented in$^{[12]}$. New features might arise for non compact groups. In non compact groups the Cartan-Killing metric is indefinite and therefore qualitatively different deformations are possible (into space-like, time-like and null-directions)$^{[38]}$.

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