Superconductivity and charge density wave under a time-dependent periodic field in the one-dimensional attractive Hubbard model

Ryo Fujiuchi\textsuperscript{1}, Tatsuya Kaneko\textsuperscript{2}, Koudai Sugimoto\textsuperscript{3}, Seiji Yunoki\textsuperscript{4,5,6}, and Yukinori Ohta\textsuperscript{1}

\textsuperscript{1}Department of Physics, Chiba University, Chiba 263-8522, Japan
\textsuperscript{2}Department of Physics, Columbia University, New York, New York 10027, USA
\textsuperscript{3}Department of Physics, Keio University, Yokohama 223-8522, Japan
\textsuperscript{4}Computational Condensed Matter Physics Laboratory, RIKEN Cluster for Pioneering Research (CPR), Wako, Saitama 351-0198, Japan
\textsuperscript{5}Computational Quantum Matter Research Team, RIKEN Center for Emergent Matter Science (CEMS), Wako, Saitama 351-0198, Japan
\textsuperscript{6}Computational Materials Science Research Team, RIKEN Center for Computational Science (R-CCS), Kobe, Hyogo 650-0047, Japan

(Dated: January 30, 2020)

We investigate the competition between superconductivity (SC) and charge density wave (CDW) under a time-dependent periodic field in the attractive Hubbard model. By employing the time-dependent exact diagonalization method, we show that the driving frequency and amplitude of the external field can control the enhancement of either superconducting pair or CDW correlations, which are degenerate in the ground state of the attractive Hubbard model in the absence of the field. In the strong-coupling limit of the attractive Hubbard interaction, the controllability is characterized by the anisotropic interaction of the effective model. The anisotropy is induced by the external field and lifts the degeneracy of SC and CDW. We find that the enhancement or suppression of the superconducting pair and CDW correlations in the periodically-driven attractive Hubbard model can be well interpreted by the quench dynamics of the effective model derived in the strong-coupling limit.

I. INTRODUCTION

Field driven nonequilibrium systems have attracted much attention as a platform of new states of matter [1–3]. In these systems, light control and detection of intriguing electronic and structural properties are implemented by the ultrafast pump-probe spectroscopy [4]. One striking example of recent experimental observations is the light induced superconducting like properties in some high-$T_c$ cuprates [5–8] and alkali-doped fullerides [9, 10], which has stimulated many theoretical investigations [11–18]. On the other hand, quantum systems under a time-dependent periodic field are interpreted with the Floquet formalism [19], which is also employed to design new quantum materials [20].

Here, we address how superconductivity (SC) and charge density wave (CDW) are influenced under a time-dependent periodic field. For this purpose, we consider the attractive Hubbard model at half-filling, which is a minimal model hosting SC and CDW as the ground state [21], with a time-dependent periodic electric field introduced via the Peierls substitution [22, 23]. In the weak-coupling regime of the attractive Hubbard interaction, the previous mean-field analysis reveals that CDW (SC) is enhanced (suppressed) when $\omega_p < 2\Delta_0$ (the field frequency $\omega_p$ is smaller than the single-particle energy gap $2\Delta_0$), while SC (CDW) is enhanced (suppressed) when $\omega_p > 2\Delta_0$ [22]. In the strong-coupling regime, introducing the effective model for doublons, the strong-coupling expansion with the Floquet formalism has shown that $\eta$-pairing [24] can possibly be induced due to the sign inversion of the pair hopping amplitude in the effective model [23].

In this paper, in order to explore the dynamics of the model in the entire driving regime, we employ the time-dependent exact diagonalization (ED) method and investigate the superconducting pair and CDW correlations in the periodically-driven one-dimensional (1D) attractive Hubbard model at half-filling. We show how the superconducting pairing and CDW correlations are modified in a wide range of the control parameters, including the field amplitude and frequency. When the external field is small, the behavior of the enhancement of SC and CDW shows good qualitative correspondence with the results in the weak-coupling mean-field analysis [22]. With the strong attractive Hubbard interaction $U$, the CDW (superconducting pair) correlation is enhanced (suppressed) when $\omega_p < U$, while the superconducting pair (CDW) correlation is enhanced (suppressed) when $\omega_p > U$. We can interpret the mechanism on the basis of the anisotropic effective Heisenberg model derived by the strong-coupling expansion in the Floquet formalism. When the external field is strong, the modification of the superconducting pair and CDW correlations shows the complex parameter dependence, which is not simply interpreted by the ground-state phase diagram of the effective model in equilibrium. We find that these behaviors can be understood from the nonequilibrium dynamics after a quench of the effective interactions in the anisotropic effective Heisenberg model.

The rest of this paper is organized as follows. In Sec. II, we introduce the model and briefly explain the method to study the time evolution of the pair and charge density correlations under the time-dependent periodic field. In Sec. III, we provide the numerical results for the at-
tractive Hubbard model and interpret these behaviors in
terms of the equilibrium ground-state phase diagram of
the strong-coupling effective model as well as the quench
dynamics in the strong-coupling effective model. Sum-
mary is provided in Sec. IV.

II. MODEL AND METHOD

A. Attractive Hubbard model

Here, we consider the 1D attractive Hubbard model
defined by the following Hamiltonian:

$$\hat{H} = -t_h \sum_{j=1}^{L} \sum_{\sigma} \left( \hat{c}_{j,\sigma}^\dagger \hat{c}_{j+1,\sigma} + H.c. \right) - U \sum_{j=1}^{L} \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow},$$

(1)

where $\hat{c}_{j,\sigma}(\hat{c}_{j,\sigma}^\dagger)$ is the annihilation (creation) operator
of an electron at site $j$ with spin $\sigma (=\uparrow, \downarrow)$, and $\hat{n}_{j,\sigma} = \hat{c}_{j,\sigma}^\dagger \hat{c}_{j,\sigma}$. $t_h$ is the hopping integral between the nearest-
neighboring sites and $U (> 0)$ is the on-site attractive
interaction. The number of sites $L$ is taken to be even
and we consider the half-filled case with the same number
of up and down electrons, i.e., $N_\uparrow = N_\downarrow = L/2$.

In the strong-coupling limit $U \gg t_h$, up and down
electrons tend to form an on-site pair and no single occu-
pied sites are favored. Neglecting singly occupied sites,
the low-energy effective Hamiltonian $\hat{H}_{\text{eff}}$ in the strong-
coupling limit is described by

$$\hat{H}_{\text{eff}} = -\frac{J_0}{2} \sum_{j=1}^{L} \left( \hat{c}_{j,\uparrow}^{\dagger} \hat{c}_{j+1,\uparrow} + \hat{c}_{j,\downarrow}^{\dagger} \hat{c}_{j+1,\downarrow} + H.c. \right) + V_0 \sum_{j=1}^{L} \hat{n}_{j,d} \hat{n}_{j+1,d},$$

(2)

with $J_0 = V_0 = 4t_h^2/U$, where $J_0$ is the pair hopping
amplitude and $V_0$ is the nearest-neighbor pair repul-
sion [23, 25]. Here, $\hat{n}_{j,d} = \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow}$ is the number of
doublets (doubly occupied electrons) at site $j$.

The effective Hamiltonian $\hat{H}_{\text{eff}}$ in Eq. (2) can be ex-
pressed as the notion of pseudospin operators. If the
lattice is bipartite, one can define pseudospin operators
via

$$\hat{n}_{j,\uparrow} = \hat{n}_{j,\uparrow} + i\hat{n}_{j,\downarrow} = (-1)^j \hat{c}_{j,\uparrow}^{\dagger} \hat{c}_{j,\downarrow}^{\dagger},$$
$$\hat{n}_{j,\downarrow} = \hat{n}_{j,\uparrow} - i\hat{n}_{j,\downarrow} = (-1)^j \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow},$$
$$\hat{\eta}_{j}^z = \frac{1}{2} (\hat{n}_{j,\uparrow} + \hat{n}_{j,\downarrow} - 1).$$

(3)

These operators are called $\eta$-spin (or $\eta$-pairing) op-
erators, which satisfy SU(2) algebra [26, 27]. Note that $\hat{\eta}_{j}^z$
plays the same role with $\hat{n}_{j,\sigma} - 1/2$ when there is no singly
occupied site in this strong-coupling model. It is easy to
show that the effective Hamiltonian $\hat{H}_{\text{eff}}$ in Eq. (2) can
be mapped onto the isotropic (i.e., $J_0 = V_0$) Heisenberg
model with these $\eta$ operators:

$$\hat{H}_{\text{eff}} = J_0 \sum_{j=1}^{L} (\hat{\eta}_{j}^x \hat{\eta}_{j+1}^x + \hat{\eta}_{j}^y \hat{\eta}_{j+1}^y) + V_0 \sum_{j=1}^{L} \hat{\eta}_{j}^z \hat{\eta}_{j+1}^z.$$  

(4)

This pseudospin Hamiltonian is equivalent to the spin-
1/2 isotropic Heisenberg Hamiltonian under the Shiba
transformation [28, 29]. The $xy$ and $z$ components of
the antiferromagnetism (AF) in this effective model cor-
respond to the SC and CDW in the original attractive
Hubbard model, respectively. They are degenerate be-
cause of the SU(2) symmetry ($J_0 = V_0$).

B. External field

The time-dependent external field is introduced in the
hopping term in Eq. (1) via the Peierls substitution

$$t_h \hat{c}_{j,\sigma}^{\dagger} \hat{c}_{j+1,\sigma} \rightarrow t_h e^{i A(t) (\hat{c}_{j,\sigma}^{\dagger} \hat{c}_{j+1,\sigma})},$$

(5)

with the time-dependent vector potential $A(t)$. Here, the
velocity of light $c$, elementary charge $e$, Planck constant
$h$, and the lattice constant are all set to 1. In this paper,
we consider the periodic driving external field given as

$$A(t) = \begin{cases} A_0 e^{-((t-t_0)/2\sigma)^2} \cos[\omega_p(t-t_0)] & (t \leq t_0) \\ A_0 \cos[\omega_p(t-t_0)] & (t > t_0) \end{cases}$$

(6)

with the amplitude $A_0$ and frequency $\omega_p$. Corresponding
to a semi-infinite ac field [30], this external field is intro-
duced with the width $\sigma_p$ and becomes time periodic for
t > $t_0$.

C. Method and correlation functions

In the presence of the external field $A(t)$, the Hamil-
tonian is time dependent, $\hat{H} \rightarrow \hat{H}(t)$, and hence we have to
solve the time dependent Schrödinger equation to evolve the
state $|\Psi(t)\rangle$ in time. For this purpose, we employ the
time dependent ED method based on the Lanczos
algorithm, where the time evolution with a short time
step $\delta t$ is calculated in the corresponding Krylov sub-
space generated by $M_L$ Lanczos iterations [31, 32]. In
our calculation, we use the finite-size clusters of $L$ sites
with periodic boundary conditions (PBC). As the initial
condition, we assume $|\Psi(t = 0)\rangle = |\psi_0\rangle$, where $|\psi_0\rangle$ is the
ground state of $\hat{H}$ without the external field. We adopt
$\delta t = 0.01/t_h$ and $M_L = 15$ for the time evolution.

In order to estimate the superconducting pair corre-
lation, we calculate the time-dependent pair structure
factor

$$P(g, t) = \frac{1}{L} \sum_{i,j} e^{ig \cdot (\mathbf{R}_i - \mathbf{R}_j)} \langle \Psi(t) (\hat{\Delta}_i^\dagger \hat{\Delta}_j + \text{c.c.}) | \Psi(t) \rangle,$$

(7)
where $\hat{\Delta}_i = \hat{c}_{i,\uparrow} \hat{c}_{i,\downarrow}$ is the on-site pairing operator and $R_j$ is the position of site $j$. We also calculate the charge structure factor

$$C(q, t) = \frac{1}{L} \sum_{i,j} e^{i\mathbf{q} \cdot (R_j - R_i)} \langle \hat{\Psi}(t) | (\hat{\rho}_i - \rho)(\hat{\rho}_j - \rho) | \hat{\Psi}(t) \rangle,$$

(8)

where $\hat{\rho}_i = \hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow}$ is the charge density operator and $\rho$ is the average density, which is 1 at half-filling. These correlation functions satisfy $P(q = 0, t) = C(q = \pi, t)$ at $t = 0$ since SC and CDW are degenerate in the ground (initial) state at half-filling. We indicate the time-averaged value of a structure factor $P(q, t)$ (e.g. $P(q, t)$ and $C(q, t)$) as

$$\overline{F}(q) = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} dt F(q, t),$$

(9)

where $t_i$ and $t_f$ are the lower and upper limit of the time average, respectively. In order to examine the enhancement or suppression of the superconducting pair and CDW correlations, we calculate the difference between the time averaged value and the initial value given by

$$\Delta F(q) = \overline{F}(q) - F(q, t = 0).$$

(10)

III. RESULTS

A. Attractive Hubbard model

We first discuss the numerical results in the attractive Hubbard model. Figure 1 shows the time evolution of the superconducting pair correlation $P(q = 0, t)$ and the CDW correlation $C(q = \pi, t)$. These structure factors

$P(q = 0, t)$ and $C(q = \pi, t)$ are indeed degenerate in the initial state at $t = 0$. As shown in Fig. 1(a), when the frequency $\omega_p$ is smaller than the attractive interaction, $\omega_p < U$, we find an enhancement of the CDW correlation $C(q = \pi, t)$ and a suppression of the superconducting pair correlation $P(q = 0, t)$. In contrast, when $\omega_p > U$, $P(q = 0, t)$ is enhanced, while $C(q = \pi, t)$ is suppressed, as compared to the initial value [see Fig. 1(b)]. Although we take the large value of $U$ in Fig. 1, these behaviors of the enhancement and suppression of the superconducting pair and CDW correlations are consistent with the results of the mean-field theory in the weak-coupling region [22].

Figure 2 shows time averaged $\overline{P}(q)$ and $\overline{C}(q)$ under the periodic driving field. As shown in Figs. 2(a) and 2(b), when $A_0$ is small, $\overline{C}(q = \pi)$ is enhanced for $\omega_p < U$, while $\overline{P}(q = 0)$ is enhanced for $\omega_p > U$, corresponding to the results in Fig. 1. On the other hand, when $A_0$ is relatively large, e.g., $A_0 = 2.5$ in Figs. 2(c) and 2(d), $\overline{P}(q = 0)$ and $\overline{C}(q = \pi)$ are both suppressed from the initial value at $t = 0$. It is also observed in Fig. 2 that, while the $\eta$-pairing correlation $P(q = \pi, t)$ is strongly enhanced by the optical pulse in the case of the repulsive model [33–35], $P(q, t)$ does not exhibit a sharp peak at $q = \pi$ in the attractive model with the periodic driving field $A(t)$ in Eq. (6).

In order to explore the parameter dependence of the su-

![FIG. 1. Time evolution of the superconducting pair structure factor $P(q, t)$ at $q = 0$ and the charge structure factor $C(q, t)$ at $q = \pi$ with (a) $\omega_p/U = 0.15$ and $A_0 = 1$, and (b) $\omega_p/U = 1.5$ and $A_0 = 1$. Dashed lines indicate $\overline{P}(q = 0)$ (blue) and $\overline{C}(q = \pi)$ (orange) averaged from $t_i = 0$ to $t_f = 300/t_h$. Dotted black line indicates $P(q = 0, t = 0)$ and $C(q = \pi, t = 0)$, which are degenerate in the initial state. The results are calculated by the ED method for $L = 12$ (PBC) at $U = 20t_h$ with $\sigma_p = 2/t_h$ and $t_0 = 10/t_h$ in $A(t)$.](image)

![FIG. 2. Superconducting pair structure factor $\overline{P}(q)$ (blue) and charge structure factor $\overline{C}(q)$ (orange) averaged from $t_i = 0/t_h$ to $t_f = 100/t_h$ with (a) $\omega_p/U = 0.15$ and $A_0 = 1$, (b) $\omega_p/U = 1.5$ and $A_0 = 1$, (c) $\omega_p/U = 0.15$ and $A_0 = 2.5$, and (d) $\omega_p/U = 1.5$ and $A_0 = 2.5$. Dotted line indicates $P(q = 0, t = 0)$ and $C(q = \pi, t = 0)$, which are degenerate in the initial state. The results are calculated by the ED method for $L = 12$ (PBC) at $U = 20t_h$ with $\sigma_p = 2/t_h$ and $t_0 = 10/t_h$ in $A(t)$.](image)
perconducting pair and CDW correlations, Fig. 3 shows $\Delta P(q = 0)$ and $\Delta C(q = \pi)$ with different values of $A_0$ and $\omega_p$. In the small $A_0$ region, the CDW correlation $C(q = \pi)$ is enhanced for $\omega_p < U$, while the superconducting pair correlation $P(q = 0)$ is enhanced for $\omega_p > U$. These results are in good qualitative accordance with the previous study using the mean-field theory [22]. However, in the large $A_0$ region, the parameter dependence of these correlations is not simple. For example, in the region around $2 < A_0 < 3$, the superconducting pair correlation is suppressed even for $\omega_p > U$ but it is enhanced for $U/2 < \omega_p < U$ [see Fig. 3(a)]. This behavior is opposite to the results found in the small $A_0$ region. In addition, we notice rather steep suppressions of the correlation functions around the parameters at $\omega_p = U/m$ ($m$: integer). This complex behavior in the large $A_0$ region is not simply interpreted by the mean-field picture with a small external field [22].

B. Effective model in the strong-coupling limit

To interpret the behavior of $P(q = 0, t)$ and $C(q = \pi, t)$ in the wide parameter space, we now introduce the effective model derived by the strong-coupling expansion in the Floquet formalism [23]. Under the periodic driving field $A(t) = A_0 \cos(\omega_p t)$, the effective model for the attractive Hubbard model with a large $U$ is given by

$$\hat{H}_{\text{eff}} = J_{\text{eff}} \sum_{j=1}^{L} (\hat{n}_j^x \hat{n}_{j+1}^x + \hat{n}_j^y \hat{n}_{j+1}^y) + V_{\text{eff}} \sum_{j=1}^{L} \hat{n}_j^z \hat{n}_{j+1}^z,$$

with the effective interactions

$$J_{\text{eff}} = \sum_{m=-\infty}^{\infty} (-1)^m \frac{4t_0^2 J_m(A_0)^2}{U + m\omega_p},$$

$$V_{\text{eff}} = \sum_{m=-\infty}^{\infty} \frac{4t_0^2 J_m(A_0)^2}{U + m\omega_p},$$

where $J_m(x)$ is the $m$th Bessel function [23]. Notice that this effective model corresponds to an anisotropic Heisenberg (XXZ) model and the effective interactions $J_{\text{eff}}$ and $V_{\text{eff}}$ vary in different manners, which is the manifestation of the broken $\eta$-SU(2) symmetry due to the external field $A(t)$. Therefore, the degeneracy of SC and CDW is lifted by the external field $A(t)$ and the anisotropy of $J_{\text{eff}}$ and $V_{\text{eff}}$ gives rise to the enhancement or suppression of the superconducting pair and CDW correlations. This should be contrasted with the strong-coupling expansion in the repulsive Hubbard model, for which the effective model is spin SU(2) symmetric (i.e., isotropic for the spin degrees of freedom) even in the presence of a time-dependent periodic electric field [36]. As shown in Eqs. (12) and (13), $J_{\text{eff}}$ and $V_{\text{eff}}$ diverge at $\omega_p = U/m$, which explains the observation of the rapid change in the correlation functions at $\omega_p = U/m$ shown in Fig. 3.

In the small $A_0$ region, the enhancement or suppression of the superconducting pair and CDW correlations can be understood by the anisotropic effective interactions $J_{\text{eff}}$ and $V_{\text{eff}}$. When $A_0 \ll 1$, $J_{\text{eff}}$ and $V_{\text{eff}}$ are given by

$$J_{\text{eff}} \approx \frac{4t_0^2}{U} \left( 1 - \frac{A_0^2}{2} \right) + \frac{2U t_0^2}{\omega_p - U/2} A_0^2,$$

$$V_{\text{eff}} \approx \frac{4t_0^2}{U} \left( 1 - \frac{A_0^2}{2} \right) - \frac{2U t_0^2}{\omega_p - U/2} A_0^2,$$

Therefore, when $\omega_p > U$, $J_{\text{eff}} > V_{\text{eff}}$ and thus the superconducting pair correlation is enhanced, while when $\omega_p < U$, $V_{\text{eff}} > J_{\text{eff}}$ and hence the CDW correlation is enhanced.

However, in the large $A_0$ region, the enhancement or suppression of $\Delta P(q = 0)$ and $\Delta C(q = \pi)$ in Fig. 3 is not simply interpreted by the ground-state phase diagram of the effective model $\hat{H}_{\text{eff}}$ in Eq. (11). For instance, although $\eta$-pairing is anticipated when $J_{\text{eff}} < 0$ in the
shows the time evolution of the $\eta$-spin correlation functions, $S_\pm(q = \pi, t)$ and $S_z(q = \pi, t)$, respectively, with (a) $\omega_p/U = 0.15$ and $A_0 = 1$, and (b) $\omega_p/U = 1.5$ and $A_0 = 1$. We assume $J_{\text{eff}}$ and $V_{\text{eff}}$ at $U = 20t_h$. Dashed lines indicate $S_{\pm}(q = \pi)$ (blue) and $S_z(q = \pi)$ (orange) averaged from $t_i = 0$ to $t_f = 300/t_h$. Dotted black line indicates $S_\pm(q = \pi, t = 0)$ and $S_z(q = \pi, t = 0)$, which are degenerate in the initial state. The results are calculated in the anisotropic Heisenberg (XXZ) model for $L = 18$ (PBC).

The ground state of the effective model, $P(q, t)$ does not show a sharp peak at $q = \pi$ in the corresponding region [see, e.g., Fig. 2(c)]. This is because the time-evolved state under the external field $A(t)$ retains the memory of the initial state $|\psi_0\rangle$ and the system may not necessarily relax to the ground state of the effective model. This may be interpreted by the dynamical instability of the effective Hamiltonian discussed in Ref. [23]. Therefore, as shown below, the memory effect of the initial state has to be incorporated to understand the behavior of $P(q = 0, t)$ and $C(q = \pi, t)$ in the wide parameter region.

C. Quench dynamics of the effective model

To address this issue described above, here we investigate the nonequilibrium dynamics after a quench of the exchange coupling in the XXZ model $K_{\text{eff}}$ in Eq. (11). We set as the initial state the ground state of the isotropic Heisenberg model with $J_0 = V_0$ in Eq. (4), and change the parameters to the effective values $J_{\text{eff}}$ and $V_{\text{eff}}$, given in Eqs. (12) and (13), abruptly at time $t = 0$. To examine the quench dynamics in the XXZ model, we calculate the time evolution of the $xy$ and $z$ components of the $\eta$-spin structure factors

$$S_\pm(q, t) = \frac{1}{2} \sum_{i,j} e^{iH(R_i - R_j)} \langle \Psi(t) | \hat{n}_i^\pm \hat{n}_j^- + \hat{n}_i^- \hat{n}_j^\pm | \Psi(t) \rangle,$$

$$S_z(q, t) = \frac{4}{L} \sum_{i,j} e^{iH(R_i - R_j)} \langle \Psi(t) | \hat{n}_i^z \hat{n}_j^z | \Psi(t) \rangle,$$

(16)

(17)

corresponding to the pair and charge structure factors $P(q, t)$ and $C(q, t)$ in the attractive Hubbard model, respectively. Note that the $xy$ component of AF correlation $S_\pm(q = \pi, t)$ in the XXZ model corresponds to the superconducting pair correlation $P(q = 0, t)$ in the attractive Hubbard model.

Figure 4 shows the time evolution of the $xy$ and $z$ components of the $\eta$-spin correlations, $S_\pm(q = \pi, t)$ and $S_z(q = \pi, t)$, respectively, after the parameter quench $(J_0, V_0) \rightarrow (J_{\text{eff}}, V_{\text{eff}})$ in the parameter space of $\omega_p$ and $A_0$. $\Delta S_\pm(q = \pi)$ and $\Delta S_z(q = \pi)$ are averaged from $t_i = 0$ to $t_f = 100/t_h$. We assume $J_{\text{eff}}$ and $V_{\text{eff}}$ at $U = 20t_h$. The results are calculated in the anisotropic Heisenberg (XXZ) model for $L = 18$ (PBC).

FIG. 5. Contour plots of (a) the $xy$-component of the $\eta$-spin correlation function $\Delta S_\pm(q = \pi)$ and (b) the $z$-component of the $\eta$-spin correlation function $\Delta S_z(q = \pi)$ after the parameter quench $(J_0, V_0) \rightarrow (J_{\text{eff}}, V_{\text{eff}})$ in the parameter space of $\omega_p$ and $A_0$. $\Delta S_\pm(q = \pi)$ and $\Delta S_z(q = \pi)$ are averaged from $t_i = 0$ to $t_f = 100/t_h$. We assume $J_{\text{eff}}$ and $V_{\text{eff}}$ at $U = 20t_h$. The results are calculated in the anisotropic Heisenberg (XXZ) model for $L = 18$ (PBC).
We have investigated the change of the superconducting pair and charge correlations in the 1D periodically-driven attractive Hubbard model in the strong-coupling regime. When the external field is small, the CDW (superconducting pair) correlation is enhanced (suppressed) for $\omega_p < U$, while the superconducting pair (CDW) correlation is enhanced (suppressed) for $\omega_p > U$. This mechanism is well interpreted by the change of the effective interactions in the effective anisotropic Heisenberg (XXZ) model derived by the strong-coupling expansion in the Floquet formalism. When the external field is strong, the parameter dependence of the enhancement or suppression of the correlations is more complex and is not simply interpreted by the ground-state phase diagram of the effective model. We have shown that these behaviors can be understood from the nonequilibrium dynamics after a quench of the effective interactions in the effective model.

**ACKNOWLEDGMENTS**

The authors acknowledge H. Aoki, S. Miyakoshi, Y. Murakami, K. Seki, and T. Shirakawa for fruitful discussion. This work was supported in part by Grants-in-Aid for Scientific Research from JSPS (Projects No. JP17K05530, No. JP18H01183, No. JP18K13509, No. JP19J02768, and No. JP19K14644) of Japan and Keio University Academic Development Funds for Individual Research. R.F. acknowledges support from the JSPS Research Fellowship for Young Scientists. T.K. was supported by the JSPS Overseas Research Fellowship.

[1] J. Zhang and R. Averitt, Annu. Rev. Mater. Res. **44**, 19 (2014).
[2] D. N. Basov, R. D. Averitt, and D. Hsieh, Nat. Mater. **16**, 1077 (2017).
[3] S. Ishihara, J. Phys. Soc. Jpn. **88**, 072001 (2019).
[4] C. Giannetti, M. Capone, D. Fausti, M. Fabrizio, F. Parmigiani, and D. Mihailovic, Adv. Phys. **65**, 58 (2016).
[5] D. Fausti, R. I. Tobey, N. Dean, S. Kaiser, A. Dienst, M. C. Hoffmann, S. Pyon, T. Takayama, H. Takagi, and A. Cavalleri, Science **331**, 189 (2011).
[6] W. Hu, S. Kaiser, D. Nicoletti, C. R. Hunt, I. Gierz, M. C. Hoffmann, M. Le Tacon, T. Loew, B. Keimer, and A. Cavalleri, Nat. Mater. **13**, 705 (2014).
[7] S. Kaiser, C. R. Hunt, D. Nicoletti, W. Hu, I. Gierz, H. Y. Liu, M. Le Tacon, T. Loew, D. Haug, B. Keimer, and A. Cavalleri, Phys. Rev. B **89**, 184516 (2014).
[8] D. Nicoletti, E. Casandruce, Y. Laplace, V. Khanna, C. R. Hunt, S. Kaiser, S. S. Dhesi, G. D. Gu, J. P. Hill, and A. Cavalleri, Phys. Rev. B **90**, 100503(R) (2014).
[9] M. Mitrano, A. Cantaluppi, D. Nicoletti, S. Kaiser, A. Perucchi, S. Lupi, P. Di Pietro, D. Pootirol, M. Ricciò, S. R. Clark, D. Jaksch, and A. Cavalleri, Nature (London) 530, 461 (2016).

[10] A. Cantaluppi, M. Buzzi, G. Jotzu, D. Nicoletti, M. Mitrano, D. Pontiroli, M. Ricciò, A. Perucchi, P. Di Pietro, and A. Cavalleri, Nat. Phys. 14, 837 (2018).

[11] M. A. Sentef, A. F. Kemper, A. Georges, and C. Kollath, Phys. Rev. B 93, 144506 (2016).

[12] A. A. Patel and A. Eberlein, Phys. Rev. B 93, 195139 (2016).

[13] M. Knap, M. Babadi, G. Refael, I. Martin, and E. Demler, Phys. Rev. B 94, 214504 (2016).

[14] D. M. Kennes, E. Y. Wilner, D. R. Reichman, and A. J. Millis, Nat. Phys. 13, 479 (2017).

[15] M. A. Sentef, Phys. Rev. B 95, 205111 (2017).

[16] M. Babadi, M. Knap, I. Martin, G. Refael, and E. Demler, Phys. Rev. B 96, 045125 (2017).

[17] Y. Murakami, N. Tsuji, M. Eckstein, and P. Werner, Phys. Rev. B 96, 045125 (2017).

[18] G. Mazza and A. Georges, Phys. Rev. B 96, 064515 (2017).

[19] G. Floquet, Ann. Sci. École Norm. Sup. 12, 47 (1883).

[20] T. Oka and S. Kitamura, Annu. Rev. Condens. Matter Phys. 10, 387 (2019).

[21] R. Micnas, J. Ranninger, and S. Robaszkiewicz, Rev. Mod. Phys. 62, 113 (1990).

[22] M. A. Sentef, A. Tokuno, A. Georges, and C. Kollath, Phys. Rev. Lett. 118, 087002 (2017).

[23] S. Kitamura and H. Aoki, Phys. Rev. B 94, 174503 (2016).

[24] C. N. Yang, Phys. Rev. Lett. 63, 2144 (1989).

[25] A. Rosch, D. Rasch, B. Binz, and M. Vojta, Phys. Rev. Lett. 101, 265301 (2008).

[26] C. N. Yang and S. Zhang, Mod. Phys. Lett. B 04, 759 (1990).

[27] F. H. Essler, H. Frahm, F. Göhmann, A. Klümper, and V. E. Korepin, The One-Dimensional Hubbard Model (Cambridge University Press, Cambridge, 2005).

[28] H. Shiba, Prog. Theor. Phys. 48, 2171 (1972).

[29] V. J. Emery, Phys. Rev. B 14, 2989 (1976).

[30] A. Ono, H. Hashimoto, and S. Ishihara, Phys. Rev. B 95, 085123 (2017).

[31] N. Mohankumar and S. M. Auerbach, Comput. Phys. Commun. 175, 473 (2006).

[32] T. J. Park and J. Light, J. Chem. Phys. 85, 5870 (1986).

[33] T. Kaneko, T. Shirakawa, S. Sorella, and S. Yunoki, Phys. Rev. Lett. 122, 077002 (2019).

[34] R. Fujchi, T. Kaneko, Y. Ohta, and S. Yunoki, Phys. Rev. B 100, 045121 (2019).

[35] T. Kaneko, S. Yunoki, and A. J. Millis, arXiv:1910.11229.

[36] J. H. Mentink, K. Balzer, and M. Eckstein, Nat. Commun. 6, 6708 (2015).