We present a summary of recent studies on CP violation with the Belle experiment using the final data sample of $7.72 \times 10^6 B\bar{B}$ pairs produced at the $\Upsilon(4S)$ resonance at the KEK asymmetric $e^+e^-$ collider. We discuss preliminary measurements of the branching fraction, the polarization and the CP asymmetries of the decay $B^0 \rightarrow \rho^+\rho^-$ and an updated constraint on the CKM angle $\phi_2$ from the $B \rightarrow \rho\rho$ system. Being also related to $\phi_2$, we present a preliminary measurement of the branching fraction of $B^0 \rightarrow \pi^0\pi^0$ decays. Last, a preliminary model independent Dalitz plot analysis of the decay $B^0 \rightarrow D^0[K^0_S\pi^+\pi^-]K^0$ is presented and its sensitivity to the CKM angle $\phi_3$ is discussed.
1. Introduction

Testing the predictions of the Cabibbo-Kobayashi-Maskawa (CKM) mechanism for violation of the combined charge-parity (CP) symmetry [1, 2] is one of the major precision tests [3–5] of the flavor sector of the Standard-Model (SM). The Belle experiment at KEK significantly contributed to the validation of the CKM scheme and to constraining the unitarity triangle for B decays to its current precision. Up-to-date, the SM describes the data impressively well, but there is still room for a deviation from unitarity, which clearly would point towards physics beyond the SM. These proceedings give a summary of the experimental status of measurements related to two of the CKM angles defined from the CKM matrix elements as: \( \phi_2 \equiv \arg(V_{ub}V_{ub}^*)/(V_{ub}V_{ub}^*) \) and \( \phi_3 \equiv \arg(V_{ud}V_{ub}^*)/(V_{cd}V_{ub}^*) \). All measurements presented here are based on Belle’s final data set of 772 \times 10^6 \( B\bar{B} \) pairs.

2. The CKM Angle \( \phi_2 \)

The CKM angle \( \phi_2 \) can be determined by measuring the time-dependent asymmetry between \( B^0 \) and \( \bar{B}^0 \) decays into a common CP eigenstate [6] made out of unflavored quarks (\( b \rightarrow u\bar{u}d\bar{d} \) quark transitions). Examples are the decays \( B^0 \rightarrow \pi\pi, \rho\pi, \rho\rho \) and \( a_1(1260)\pi \) [7–14]. In the decay sequence, \( Y(4S) \rightarrow B_{CP}B_{Tag} \rightarrow f_{CP}f_{Tag} \), where one of the \( B \) mesons decays into a CP eigenstate \( f_{CP} \) at a time \( t_{CP} \) and the other decays into a flavor specific final state \( f_{Tag} \) at a time \( t_{Tag} \), the time-dependent decay rate is given by

\[
P(\Delta t, q) = e^{-|\Delta t|/\tau_{\bar{B}^0}} \left[ 1 + q(A_{CP} \cos \Delta m_d \Delta t + S_{CP} \sin \Delta m_d \Delta t) \right],
\]

where \( \Delta t \equiv t_{CP} - t_{Tag} \) is the lifetime difference between the two \( B \) mesons, \( \Delta m_d \) is the mass difference between the mass eigenstates \( B_H \) and \( B_L \) and \( q = +1(-1) \) for \( B_{Tag} = B^0(\bar{B}^0) \). The CP asymmetry is given by

\[
\frac{N(B \rightarrow f_{CP}) - N(B \rightarrow f_{CP})}{N(B \rightarrow f_{CP}) + N(B \rightarrow f_{CP})},
\]

where \( N(B \rightarrow f_{CP}) \) is the number of events of a \( B(\bar{B}) \) decaying to \( f_{CP} \), the asymmetry can be time-dependent. The parameters \( A_{CP} \) and \( S_{CP} \) describe direct and mixing-induced CP violation, respectively.\(^1\)

At tree level one expects \( A_{CP} = 0 \) and \( S_{CP} = \sin 2\phi_2 \) for the above mentioned decays sensitive to \( \phi_2 \). Possible penguin contributions can give rise of direct CP violation, \( A_{CP} \neq 0 \) and also pollute the measurement of \( \phi_2 \). \( S_{CP} = \sqrt{1 - A_{CP}^2 \sin^2(2\phi_2^{eff})} \) where the observed \( \phi_2^{eff} \equiv \phi_2 - \Delta \phi_2 \) is shifted by \( \Delta \phi_2 \) due to different weak and strong phases from additional non-leading contributions. Despite this, it is possible to determine \( \Delta \phi_2 \) in \( B^0 \rightarrow h^+h^- \) with an SU(2) isospin analysis by considering the set of three \( B \rightarrow hh \) decays where the \( hhs \) are either two pions or two longitudinally polarized \( \rho s \), related via isospin symmetry [15]. The \( B \rightarrow h^+h^- \) amplitudes \( A_{ij} \) obey the triangle relations,

\[
A_{1+0} = \frac{1}{\sqrt{2}}(A_{+-} + A_{00}), \quad \bar{A}_{-0} = \frac{1}{\sqrt{2}}(\bar{A}_{+-} + \bar{A}_{00}).
\]

\(^1\)There exists an alternate notation where \( C_{CP} = -A_{CP} \).
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Isospin arguments demonstrate that $B^+ \rightarrow h^+ h^0$ is a pure first-order mode in the limit of neglecting electroweak penguins, thus these triangles share the same base, $A_{+0} = \bar{A}_{-0}$, see Fig. 2 for an illustration. $\Delta \phi_2$ can then be determined from the difference between the two triangles. This method has an inherent four-fold discrete ambiguity in the determination of $\sin(2\phi_2)$.

Figure 1: a) leading order and b) penguin Feynman diagrams for color-allowed $b \rightarrow u\bar{u}d$ transitions.

2.1 The Decay $B^0 \rightarrow \rho^+ \rho^-$

Having a decay into two vector particles, an angular analysis is performed to separate the CP-even states from the CP-odd states for the isospin analysis. Longitudinal polarized states correspond to pure CP-even states and their fraction, $f_L$, is obtained from a fit to the cosine of helicity angles, $\Theta^{\pm}_H$, which are defined as sketched in Fig. 3. Previous measurements show that $f_L$ is consistent with one [10–12], consequently the isospin analysis is performed for longitudinal polarization only. In addition to combinatorial background, the presence of multiple, largely unknown backgrounds with the same four-pion final state as $B^0 \rightarrow \rho^+ \rho^-$ make this decay quite difficult to isolate and interference between the various $4\pi$ modes needs to be considered. Besides updating to the full data set, the branching fraction, the fraction of longitudinally polarized $\rho$ mesons on this decay and the CP violating parameters are obtained simultaneously from the fit to the data. In the fit, Belle uses the missing energy ($\Delta E \equiv E_{B}^{*} - E_{beam}^{*}$), the beam-constraint $B$ mass ($M_{bc} = \sqrt{E_{beam}^{*2} - p_{B}^{*2}}$), the masses and helicity angles of the two reconstructed $\rho^\pm$ mesons, a fisher discriminant to separate the jet-like $e^+ e^- \rightarrow q\bar{q}$, ($q = u, d, s, c$) background from the spherical $B\bar{B}$ decays and the $\Delta t$ distribution for the two flavors of $B_{Tag}$. They obtain
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Figure 3: The helicity angles $\cos \Theta^\pm_H$; each one is defined in its $\rho$ rest frame.

$$\mathcal{B}(B^0 \rightarrow \rho^+ \rho^-) = (28.3 \pm 1.5 \text{ (stat)} \pm 1.4 \text{ (syst)}) \times 10^{-6},$$

$$f_L = 0.988 \pm 0.012 \text{ (stat)} \pm 0.023 \text{ (syst)},$$

$$S_{CP} = -0.13 \pm 0.15 \text{ (stat)} \pm 0.05 \text{ (syst)} \text{ and}$$

$$A_{CP} = 0.00 \pm 0.10 \text{ (stat)} \pm 0.06 \text{ (syst)}.$$  

This is currently the most precise measurement of the branching fraction and polarization of $B \rightarrow \rho^+ \rho^-$ decays as well as the tightest constraint on $CP$ violation in this mode. Fig. 4 shows the projections onto $\Delta E$, $\cos \Theta_H$ and onto $\Delta t$ for the two flavor of $B_{Tag}$, each with the fit result on top. The results from this measurement are used in an isospin analysis together with other Belle measurements [13,16] (longitudinal polarization only). Fig. 5 b) shows the $\phi_2$ scan from the isospin analysis, the constraint most consistent with other measurements of the CKM triangle is $\phi_2 = (93.7 \pm 10.6)^\circ$ and the penguin pollution is consistent with zero: $\Delta \phi_2 = (0.0 \pm 9.6)^\circ$. In the $B \rightarrow \rho \rho$ system, the relatively small amplitude of $B^0 \rightarrow \rho^0 \rho^0$ makes the isospin triangles flat and therefore the isospin analysis has no ambiguity.
2.2 The Decay $B^0 \rightarrow \pi^0\pi^0$

This decay is an important input for the isospin analysis in the $B \rightarrow \pi\pi$ system. Being reconstructed from four $\gamma$s makes this measurement experimentally quite challenging. A fit to $\Delta E$, $M_{bc}$ and a fisher discriminant $T_C$ is performed and a preliminary branching fraction of

$$\mathcal{B}(B \rightarrow \pi^0\pi^0) = (0.9 \pm 0.12 \text{ stat} \pm 0.10 \text{ syst}) \times 10^{-6}$$

is obtained. Signal enhanced projections are shown in Fig. 6. It is planned to supersede this currently most precise measurement of the branching fraction with one including the determination of the direct CP violation in this mode.

The upcoming Belle 2 experiment will make a time-dependent analysis possible, as the accumulated data will provides enough data to use converted photons to determine the $B$ vertex. Including the mixing-induced CP-violation parameter of $B \rightarrow \pi^0\pi^0$ in the isospin analysis might remove the four-fold ambiguity from the isospin analysis being currently present in the $B \rightarrow \pi\pi$ system.

3. The Decay $B^0 \rightarrow D^0[K^0_S\pi^+\pi^-]K^{*0}$ and the CKM Angle $\phi_3$

A model independent dalitz plot analysis [17,18] of the decay $B^0 \rightarrow D^0[K^0_S\pi^+\pi^-]K^{*0}$ has been performed for the first time. The flavor-specific decay, $K^{*0} \rightarrow K^+\pi^-$, allows to determine the flavor of the $B$ meson. The number of events in different bins of the dalitz plot of the $D$ meson coming from either a $B^0 (N_i^+)$ or a $\bar{B}^0 (N_i^-)$ is given by

$$N_i^\pm = h_B[K_{\pm i} + r_i^2K_{\mp i} + 2k\sqrt{K_{i}K_{-i}}(x_{\pm i}c_i \pm y_{\pm i}s_i)],$$

(3.1)

where $h_B$ is a normalization constant, $K_{\pm i}$ are the entries in the $D$ (±) or $\bar{D}$ (±) dalitz plot, $k = 0.95 \pm 0.03$ [20] accounts for interference effects in the $D$ decay, and $c_i$ and $s_i$ include information on the average of the phase variation within a dalitz plot bin. All information on the $D$ dalitz plot is
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Figure 6: Signal enhanced distributions of (a) $\Delta E$, (b) $\cos \Theta_H$ and (c) $\Delta t$ with the fit result on top. Contributions from signal, continuum, $\rho \pi$ and other rare B decays are shown by blue, green, red and cyan respectively.

provided by measurements from the CLEO collaboration [19]. The observables $x_\pm \equiv r_s \cos (\delta_S \pm \phi_3)$ and $y_\pm \equiv r_s \sin (\delta_S \pm \phi_3)$ from the interference term in Equ. 5.1 allow an extraction of the CKM angle $\phi_3$ in general, where the ratio between the cabbibo-allowed and double-cabbibo-suppressed amplitudes, $r_S \equiv \frac{A_{(B^0 \to D^{(*)} K^{(*)})}}{A_{(B^0 \to D^{(*)} K^{(*)})}}$, indicates the sensitivity to $\phi_3$. Belle obtains

$$x_+ = +0.4^{+0.7+0.0}_{-0.4-0.1} \pm 0.1, \quad y_+ = +0.3^{+0.5+0.0}_{-0.8-0.1} \pm 0.1,$$
$$x_- = +0.4^{+0.5+0.0}_{-0.8-0.1} \pm 0.0, \quad y_- = -0.6^{+0.8+0.1}_{-1.0-0.0} \pm 0.1,$$

and uses the result to obtain an upper limit of

$$r_S < 0.87$$

at the one $\sigma$ level. The (dalitz plot integrated) fit results are shown in Fig. 7 and the $r_S$ scan is shown in Fig. 8. This mode is still statistically limited but will give additional insights on $\phi_3$ when the Belle 2 data will be available.

Figure 7: Signal enhanced distributions of (a) $\Delta E$, (b) $\cos \Theta_H$ and (c) $\Delta t$ with the fit result on top. The signal contribution is shown in red.

4. Summary

We have presented three recent and preliminary measurements from Belle sensitive to the CKM phases $\phi_2$ and $\phi_3$ using the full data set of 772 million $B\bar{B}$ pairs. Measurements of the
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branching fraction, the polarization and the CP asymmetries in $B \to \rho^+ \rho^-$ were used to update the $\phi_2$ isospin constraint from Belle. The branching fraction measurement of $B \to \pi^0 \pi^0$ has been presented and the importance for this mode the isospin analysis in the $B \to \pi \pi$ system has been discussed. The current world averages of $\phi_2$ as computed by the CKMfitter [21] and UTfit [22] collaborations are $\phi_2 = (87.6^{+3.5}_{-3.3})^{\circ}$ and $\phi_2 = (88/6 \pm 3.3)\circ$, respectively. Furthermore we presented a preliminary measurement of $B^0 \to D^0[K^0_S \pi^+ \pi^-]K^0$ decays and discussed the sensitivity to $\phi_3$. All shown results are in good agreement with other SM based constraints on the CKM triangle. With Belle 2 being built [23] and the LHCb operating, the next generation of $B$ physics experiments are expected to further reduce the uncertainty of the CKM observables and might reveal new phenomena.

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