Study on the Best Shape and Stability of Sand Castle

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Abstract. This paper analyzes the factors that influence the stability of the sand castle, and sandcastle stability of different shapes are care, by modeling concluded. Beach sand castle by both tidal and wave interaction. Through some will be converted to calculate the impact of the two together, we simulate water flow after the start ashore process to Shah’s fort, through to find reasonable assumptions speed of the water and the water depth and leave as a function of the distance between the coast, while the unified regulations the volume of the sand castle is 8 m$^3$, and the distance from the bottom of the sand castle to the coast is 2 m to 4 m to facilitate the comparison of the influence of water flow. Firstly, we have determined that cones are the best basic 3D geometric foundation of sandcastle. This model is a simple static model, because we want to avoid the situation that force exerted by seawater can directly destroy our foundation of sandcastle. Secondly, we made starting coulombic stars increases the shear strength is necessary to increase the internal friction angle, and then we made derived experimental data of the relationship between the angle of internal friction and the ratio of water and sediment. In the role of rainwater from this San Ge analyzed angle that is, the rain will affect the flow of water into the sand castle of speed and depth, while the rain continued the process, make sandcastles water content increases large, shadow rang its strength, finally, rain the shock effect, the presence of momentum, will destroy the strength of the sand castle.

1. Introduction
People all over the world enjoy bathing in the sun on the beach. But for children, they enjoy making sandcastles more. Using tools, mixing sand and water, choosing shapes, using their imagination. Making sandcastles is not an easy task for children.

In terms of how to make a perfect sandcastle, the major problems is that you need to choose a proper 3-dimensional geometric shape and an optimal sand-to-water mixture proportion for your base, except for your fantastic ideas about the shape of the sandcastle. Due to tides and waves will erode the sand castle, but that is the case is the same distance from the sea, as the scale of the construction of the castle, the castle erosion by the waves of different knots fruit different. Therefore, it is speculated whether there is a best three-dimensional geometric foundation for constructing sand castles.

2. A Static Water Hitting Model
Consider the force exerted by water may directly destroy the sandcastle, our team put forward a simple static model that can describe the 3D geometric foundation. The first thing is getting the velocity field of waves through simplifying the Navier – Stokes equations and continuity equations in fluid mechanics:
\[ \rho \frac{d \vec{v}}{dt} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{v} \]

\[ \nabla \cdot \rho \vec{v} + \frac{\partial \rho}{\partial t} = 0 \]

Where \( \rho \) is the density of water, \( V \) is velocity field, \( \mu \) is dynamic viscosity, \( p \) is local thermodynamics pressure and \( g \) is acceleration of gravity. By assuming the 2-D motion (\( \vec{z} = 0 \)), steady state (\( \nabla \cdot \vec{v} = 0 \)), incompressible fluid (\( \rho \) is constant) with fully development in x direction (\( \frac{\partial \vec{v}}{\partial t} = 0 \)). The above equations can be simplified as:

\[ \mu \frac{\partial^2 u}{\partial y^2} = 0 \]

Where the boundary conditions are \( y=0, u=0, y=H, u=u_\infty \) where \( u \) is the velocity in x direction and \( u_\infty \) is assumed to be the free stream velocity of seawater. Solve the differential equation and get the velocity profile of wave:

\[ u = \frac{u_\infty}{H} y \quad 0 \leq y \leq H \]

Now we need to establish the connection between the velocity and statics. For this point, we take a control volume with a very small thickness on the contact surface between sandcastle and waves, and then use the momentum equation for control volume.

\[ \vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t} \int_{CV} \vec{v} \rho dV + \int_{CS} \vec{v} \rho \vec{v} dA \]

To figure out force acted on the control volume, and then convert it to the uniform surface load. It is noted that we ignore the body force of waves \( F_B \), because compared to the surface one \( F_S \), it is really small. Also, the load only acted on the side of 3D foundation facing the sea. To get this load, we do the analysis for three common structures of sandcastles’ foundation: column, frustum and cone. With using the same amount of sand, which means their volumes are the same, we can get the result for the uniform surface load:

\[ \vec{L}_\text{column} = \frac{1}{3} u_\infty^2 \rho \sin \alpha \vec{i} \]

\[ \vec{L}_\text{cone} = \frac{1}{6} u_\infty^2 \rho \sin \alpha \vec{i} \]

\[ \vec{L}_\text{frustum} = \frac{3k + 1}{6(k + 1)} u_\infty^2 \rho \sin \alpha \vec{i} \]

Where the unit of \( L \) is \( N/m^2 \), \( \alpha \) is the angle of repose, which is the angle between the horizontal level and the inclined plane facing the sea, and \( k \) is the ratio of the upper surface area and lower surface area of a common frustum, so it will satisfy the relationship:

\[ \text{Supper} = k \text{Slower} \quad 0 \leq k \leq 1 \]

It is indicated that when \( k=1 \), it becomes a column, and when \( k=0 \), it becomes a cone. According to above results, we can get a basic structure from these three structures.

According to figure 1, the upper and lower limit of frustum is column and cone respectively that fits our inference. In this figure we can get a preliminary conclusion that only consider the 3D geometric structure, and make other condition be given, the cone is under the smallest load. Now we can reduce our range from three common structures (column, cone and frustum) to just cone. We still don’t know the what kind of cones and what the angle of repose is optimal, because the curves for these structures are monotonous without information about the specific cones.
Figure 1. The uniform surface load respect to the repose angle for three common structures

3. Relationship Analysis
The capillary water between the particles shrink inwardly formed surface tension of curved liquid surface, the surface tension of the direction tangential to the meniscus, which refers to the direction of force to the contact surface as shown, so that the sand show some cohesiveness. For sand relative sliding occurs between, not only to overcome the friction between the sand and the sand rubbing force, but also to overcome the cohesion between the sand and the sand moisture content which is from 0 to 15, from 29 internal friction angle.

Figure 2. Relationship between internal friction angle and water content

Water lubricated as by friction between the particles is reduced to make. Water lubricates because shear deformation capacity to resist water is zero, the water adsorbed on the contact surface of the isolation ground contact surface. This is the moisture content increased from 15% to 20%. According to our data of five points, draw a continuous curve, so that more intuitive to see between the moisture content and the internal friction angle relationship. Thereby verifying the accuracy of the experiment.
4. Impact of Rain
Sandcastle stability mainly from three areas affected by the impact of: the speed of water flow and water depth, sand castle moisture content, as well as sand castle suffered the force acting. Therefore, the impact of rain can be from San Ge analyzed angle that is, the rain will affect the flow of water into the sand castle of speed and depth, while the rain continued the process, make sandcastles water content increases large, affecting its strength, finally the impact of rain, there is momentum, which will destroy the strength of the sand castle.

4.1. Ater flow rate, depth of change
In the process of rainwater fall, the water content between the surface layers of the beach will inevitably increase, forming a model similar to a “water pad”, which increases the pore water between the sand particles, making the water flowing through the beach more difficult to penetrate, thereby reducing the frictional resistance of water flow, so flow to the sandcastle speed will increase, the same time by the impact of rain, will make the flow of traffic increases, the depth of the water also will increase, which is in the water The impact of the flow aspect on the sand castle.

After experience can be drawn: Sand castle water front side of the flow velocity of \( V = 0.2 \text{ m/s} \) becomes \( V = 0.5 \text{ m/s} \), depth from the previous of \( H = 0.2 \text{ m} \) becomes \( H = 0.4 \text{ m} \), the water flow rate for sandcastle rear of \( V = 0.1 \text{ m/s} \) becomes \( V = 0.4 \text{ m/s} \), depth from the previous of \( H = 0.1 \text{ m} \) becomes \( H = 0.2 \text{ m} \).

4.2. Changes in the moisture content of the sand castle
Drop rain process, rainwater flowing sand castle, water will penetrate into the interior of a sand castle, it is bound to make sandcastles moisture content increases large. Previous experiments, we are in the case where the optimum water content, and therefore, the water content continued to increase the time, so that the resistance capacity to continue flushing is reduced, Here, we introduce a coefficient \( C \), represents the water content increases due to Changes in scour effect. The through experience can be \( C = 1.2 \), therefore, the water content is increased rainwater case where the erosion that the volume of the original 1.2 -fold, i.e., \( L' = 1.2V L \).

4.3. Conical impact force calculation
For sand castle is being from the rain and under the impact role. But the impact of the rain is not easy to calculate directly, we are here to convert, the effect of the shock effect of rain and water flow contact. We assume that Yi Tian within the rain continued to decline, the role of the unit time rain effect is the effect of water flow 0.8 times, due to the flow of action period is 8 H, and rain is sustained decline in a single day, therefore two by the period ratio of 1:3, therefore, the combined effect of rain equivalent to 2 flow. 4 times. Therefore, in the case of rain, the total of the impact effect of the original 3 under the impact of the individual effects of water .4 times.
4.4. The rainfall different stability analysis
A comprehensive comparative analysis is obtained by combining the above factors, and the results of
the analysis are shown here.
Analysis calculated results: precipitation under the action, the influence is the effect of three different,
from the flow velocity, depth perspective, the influence of the three is the same, does not cause the three
stability vary; the water content point of view, since the volume of the three are identical, but the contact
area with rain different, and therefore affect the infiltration of rain intensity, so that the water content is
different; Similarly, for the impact of rain impact, due to the different contact area with rain, so that
three different subjected to an impact force, and therefore, the final result mainly three is the surface
area and the projected area of a different result.

Cube flushing times after rainwater number = 577; the number of days after rainwater adhere to is:
0.2000. The number of scours after the cylinder was exposed to rainwater = 628; the number of days of
persistence after rainwater was applied: 0.2177
Cone after rainwater impulse frequency and a brush = 1396; Number of days after rainwater adhere
to: 0.4844.
In summary comparison it can be drawn raining when, conical shape of the sand castle foundation
still is the best three-dimensional geometric shapes.

5. Conclusion
This paper aims to design the longest lasting sandcastle with the best sandto-water ratio. It is noted that
even though the sandcastle is built roughly the same size and at roughly the same distance from the sea,
it still responds to wave differently. We think there are two main reason. First of all, our static model
regards the sandcastle as a whole structure, but it is not actually. So we need to consider the sand effect,
which means when a sandpile reaches a critical state, one more sand particle can make the whole
sandpile collapse. The second one is stress concentration, due to the wave, rain, even a breeze, there is
a possibility that a small crack will appear in the surface of the sandcastle. This small crack will lead to
stress concentration that make the local stress become extremely large and then the failure will appear
rapidly.

References
[1] Effect of Cement Content on Shear Strength of Modified Expansive Soil LI Yong biao1 DING
San bao1 SHEN Hui1 JIANG Xiao qing21 School of Civil EngineeringAnhui architecture
UniversityHefei 230601China; 2 College of Open EducationAnhui Radio and Television
UniversityHefei 230601China
[2] Journal of Chang an University (Natural science Edition)
[3] Philip J. Pritchard. Fox and McDonald’s Introduction to Fluid Mechanics [M] New York, John
Wiley&Sons, Inc, 2011.
[4] WilliamF.Riley, LeroyD.Sturges, DonH.MorrisStaticandMechanicsofMaterials [M] New York,
John Wiley&Sons, Inc 2002.
[5] T. G. Mason, A. J. Levine, D. Ertas, T. C. Halsey Critical Angle of Wet Sandpiles. [J] Rapid
Communications Physical Review E Volume 60, Number 5. 1999.
[6] Richard G. Budynas, J. Keith Nisbett. Shigley’s Mechanical Engineering Design [M] New York,
McGraw-Hill Education, 2013.