Research and Application of Xin'anjiang Model Parameters Uncertainty Based on GLUE Algorithm

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Abstract. Uncertainty study of hydrological model is an important subject in hydrological science research. Uncertainty analysis of model parameters is one of the important contents of hydrological model uncertainty research. This paper takes Hongjiata watershed as the research object, uses GLUE algorithm to analyze the uncertainty of parameters of Xin'anjiang model, and determines the sensitivity and range of parameters through the distribution law of qualified parameters. The conclusion shows that the GLUE algorithm can improve the calibration efficiency of Xin'anjiang model parameters and narrow the range of values, which can provide reference for model application in other areas.

1. Introduction

Basin hydrological model is a mathematical model which simulates the whole development process from precipitation to runoff on the basis of physical laws and hydrological knowledge. So far, hydrological experts at home and abroad have put forward various hydrological models, which have been widely used in flood forecasting and water resources management.[1]. With the development of computer and remote sensing technology, hydrological models are more diversified, but because of the complexity of model structure and model parameters, the sources of uncertainty of models are also increasing. Therefore, the uncertainty analysis of the model is an indispensable link in the study of hydrological model. The uncertainty of hydrological model includes three aspects[2][3]:

- Input uncertainty
- Uncertainty of hydrological model parameters
- Uncertainty of hydrological model structure

Model parameters are one of the most important components of hydrological models, and their values have a decisive impact on the accuracy of runoff simulation results. Generally, the model parameters are calibrated and verified by actual observation data before they are used for runoff simulation. In 1992, Beven and Binley proposed the Generalized Likelihood Uncertainty Estimation(GLUE) to measure the uncertainty of model parameters[4]. It has been widely used because of its simplicity and efficiency. Shu Chang et al. used GLUE algorithm to analyze the uncertainty of Xin'anjiang model parameters, and classified the parameters based on runoff simulation
of different hydrological characteristics basins. Dai Jiannan et al. used GLUE algorithm based on Bayesian theory to analyze and evaluate the uncertainty of Xin'anjiang model parameters.

This paper chooses Xin'anjiang daily model and uses GLUE algorithm to analyze the rationality of parameters, their sensitivity to model parameters and the impact of uncertainty on runoff simulation, which can provide reference for hydrological simulation of river basins.

2. Research on watershed and data

2.1. Overview of the research area
Hongjiata River Basin is located in Ninghai County, Zhejiang Province, with an area of 151 km². Affected by subtropical monsoon, it belongs to the coastal wet area, with an average annual rainfall of 1737 mm. In the basin, the potential is low, the dimension is low, the annual average temperature is high, and the evaporation is large. The annual average evaporation is 829 mm. Hongjiata Hydrological Station is located at the outlet of Hongjiata River Basin. It has the ability to measure rainfall, evaporation and runoff. The average runoff depth measured by this station is 1053 mm for many years.

2.2. Hydrological data in river basins
There are four rainfall stations and one hydrological station in Hongjiata basin, namely Dacai, Lilan, Rigakeng, Shenzhen Rainfall Station and Hongjiata Hydrological Station. The data types and series lengths of these stations are shown in Table 1.

| Station name | Station type       | Data type          | Data length(year)   |
|--------------|--------------------|--------------------|---------------------|
| Dacai        | Rainfall station   | rainfall           | 1966-2018           |
| Li’ao         | Rainfall station   | rainfall           | 1959-2018           |
| Lijiakeng    | Rainfall station   | rainfall           | 1966-2018           |
| Shenzhen     | Rainfall station   | rainfall           | 1962-1994           |
| Hjiata       | Hydrological Station | Rainfall, evaporation and runoff | 1957-2018 |

3. Research method

3.1. Xin’anjiang Model
Xin’anjiang model is a conceptual watershed hydrological model suitable for humid and semi-humid areas. The main feature of the model is to assume that the runoff generation mode in wet areas is full storage, and the curve of water storage capacity is the core of the model. At present, the Xin’anjiang model with three sources of water, which is divided into surface runoff, soil runoff and underground runoff, is widely used[5]. The structural framework of the three-source model is shown in Figure 1 and the parameters are explained in Table 2.

| Parameters, variables, input and output | Physical meaning                        | units |
|-----------------------------------------|-----------------------------------------|-------|
| Evapotranspiration                      |                                         |       |
| $W_M$                                   | Averaged soil moisture storage capacity | mm    |
| $W_{UM}$                                | Averaged soil moisture storage capacity of the upper layer | mm |
| $W_{LM}$                                | Averaged soil moisture storage capacity of the lower layer | mm |
| $W_{DM}$                                | Averaged soil moisture storage capacity of the deep layer | mm |
| Symbol | Description | Unit |
|--------|-------------|------|
| $K$    | Conversion coefficient of Evapotranspiration | - |
| $C$    | Coefficient of the deep layer | - |
| $B$    | Exponential of the distribution to tension water capacity | - |
| $I_{MP}$ | Percentage of imprevious and saturated areas in the catchment | % |
| $S_M$  | Areal mean free water capacity of the surface soil layer | mm |
| $E_X$  | Exponent of the free water capacity curve influencing the development of the saturated area | - |
| $K_G$  | Outflow coefficients of the free water storage to groundwater relationships | - |
| $K_{SS}$ | Outflow coefficients of the free water storage to interflow relationships | - |
| $KK_G$ | Recession constants of the groundwater storage | - |
| $KK_{SS}$ | Recession constants of the lower interflow storage | - |
| $C_R$  | Recession constant in the lag and route method | - |
| $R_S$  | Surface runoff | m³/s |
| $R_I$  | Interflow runoff | m³/s |
| $R_G$  | Groundwater runoff | m³/s |
| $F_R$  | The runoff producing area proportion | – |
| $W$    | Area mean tension water storage | mm |
| $W_U$  | Areal mean tension water storage in the upper layer | mm |
| $W_L$  | Areal mean tension water storage in the lower layer | mm |
| $W_D$  | Areal mean tension water storage in the deepest layer | mm |
| $S$    | Areal mean free water storage | mm |
| $Q_S$  | Surface runoff | m³/s |
| $Q_I$  | Interflow | m³/s |
| $Q_G$  | Groundwater | m³/s |
| $P$    | The measured areal mean rainfall | mm |
| $E_M$  | The measured pan evaporation | mm |
| $E$    | The actual evapotranspiration from the whole basin | mm |
| $E_U$  | The evapotranspiration from the upper soil layer | mm |
| $E_L$  | The evapotranspiration from the lower soil layer | mm |
| $E_D$  | The evapotranspiration from the deepest soil layer | mm |
| $Q$    | The total sub-basin inflow to the channel network | m³/s |
Xin'anjiang model can simulate flood and runoff processes. This paper is used to study long series of runoff simulation with time step of 1 day.

3.2. Multi-objective GLUE algorithm

GLUE (Generalized Likelihood Uncertainty Estimation) algorithm is an empirical frequency method combining Bayesian theory to estimate parameter uncertainty. The basic steps of GLUE method are as follows[7][8].

**Step 1.** Choose the likelihood objective function to judge the simulation results. In this paper, the relative errors (L1) and deterministic coefficients(L2) of total runoff are selected as the likelihood objective functions. L1 ranges from [0, 100%], 0 represents the best result of the model, L2 ranges from [0, 1], and 1 represents the best result of the model. Reference to the hydrological information specification (GB/T 22482-2008), the thresholds of the functions are L1 < 10%, L2 > 0.7. The expression of each likelihood function is:

\[
L_1[M(\theta_k)] = \left| \frac{\sum_{i=1}^{N} q_i - \sum_{i=1}^{N} q'_{ki}}{\sum_{i=1}^{N} q_i} \right| \times 100\%
\]

\[
L_2[M(\theta_k)] = 1 - \frac{\sum_{i=1}^{N} (q_i - q'_{ki})^2}{\sum_{i=1}^{N} (q_i - \bar{q})^2}
\]

\(M(\theta_k)\) is the output of the model given the parameters \(\theta_k\) of group K.It is composed of \(q'_{k1}, q'_{k2}, q'_{k3} \ldots q'_{kN}\), \(q'_{ki}\) is the forecast of runoff on day i when \(q'_{ki}\) is a given parameter of Group K, \(q_i\) is the measured runoff for day i, \(\bar{q}\) is the measured average runoff.

**Step 2.** Random sampling was carried out within the initial range of model parameters to determine the simulated samples. Generally, the initial range of model parameters is determined according to their physical characteristics, and the prior distribution is expressed in the form of uniform distribution.
Step 3. Analyses the uncertainty of model parameters. Each group of randomly generated parameters needs to be evaluated by likelihood function. Sampling is divided into effective solution and invalid solution according to the threshold satisfying the objective function. For the effective solution, the posterior distribution of parameters is drawn, and the uncertainty of parameters is analyzed. Then, according to the ranking of the calculated objective function values, the uncertainty range of the model prediction with a certain confidence level is estimated.

4. Result and discussion

4.1. Equivalence of parameters
An important manifestation of hydrological model uncertainty is isoparametric phenomenon. The simulation results show a large number of isoparametric phenomenon, namely the equivalence of parameter groups. This phenomenon validates the important point of view of GLUE method: the key to the simulation results is the parameter group of the model, not the single parameter [9]. Thus, a set of optimal parameters calibrated by the automatic optimization algorithm is unreliable and uncertain. In Xin'anjiang model, according to a large number of application practices, this paper chooses six typical parameters, $K, C, W_M, S_M, K_G, K_{SS}$, to specifically analyze the impact of these six parameters on the uncertainty of the model for the study basin. Table 3 lists the ranges of parameters and table 4 lists the equivalent values of six selected parameters in Hongjiata watershed.

| parameter  | Range of values | parameter  | Range of values |
|-----------|-----------------|-----------|-----------------|
| $K$       | 0.7-1.2         | $S_M$     | 2-40            |
| $C$       | 0.05-0.3        | $K_G$     | 0.1-0.9         |
| $W_M$     | 80-200          | $K_{SS}$  | 0-0.9           |

Table 4. Value table of equivalent parameter group

| parameter | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| $K$       | 0.87| 0.95| 0.93| 0.71| 1.01| 1.14| 0.73| 1.18|
| $C$       | 0.15| 0.13| 0.21| 0.18| 0.11| 0.08| 0.1 | 0.24|
| $W_M$     | 121.37| 135.88| 151.83| 99.82| 148.61| 127.39| 88.25| 169.87|
| $S_M$     | 10.21| 14.53| 2.68| 15.31| 20.17| 19.83| 5.13| 29.33|
| $K_G$     | 0.33| 0.34| 0.55| 0.27| 0.49| 0.47| 0.62| 0.25|
| $K_{SS}$  | 0.72| 0.52| 0.9 | 0.42| 0.81| 0.62| 0.41| 0.64|
| relative error | 5.24%| 5.24%| 5.24%| 5.24%| 5.24%| 5.24%| 5.24%| 5.24%|
| $R^2$     | 0.84| 0.84| 0.84| 0.84| 0.84| 0.84| 0.84| 0.84|

4.2. Uncertainty analysis of parameters
Taking the 52-year measured data of Hongjiata watershed from 1966 to 2018 as the control, the Monte Carlo random sampling method was used to randomly extract 1 million sets of sample parameters within the range of parameters, and the objective functions of each group of samples were calculated respectively. According to the threshold value of the objective functions, the random parameters were screened, and 9375 sets of effective parameters were obtained, and the distribution of the parameters in the effective parameters was improved. Line statistics, get the distribution map, see Figure 2. Figure 2 shows that $K_{SS}$ distributes uniformly in its range and has a large range of uncertainties. There is no better range of $K_{SS}$, which makes it difficult to optimize the parameters. It further illustrates that the solution of the model is unstable when calibrating the parameters of Xin'anjiang model. However, $K, C, W_M, S_M, K_G$, approximate normal distribution, with the optimal parameter value as the center, the uncertainty range is small, and the parameter value region is more concentrated than the previous empirical range.
5. Conclusion
Taking Hongjiata watershed as the research object, this paper simulates the long series runoff process of the watershed by using the Xin'anjiang daily model, and analyses the uncertainty of the model parameters through GLUE algorithm. The following conclusions are drawn:

- For Hongjiata basin, GLUE method is used to analyse model parameters, and the results are satisfactory. Among the sensitive parameters, $K, C, W_M, S_M, K_G$ have a smaller range of central uncertainty with the optimal parameter value, and obtain a more concentrated parameter value region than the previous empirical range.

- In this paper, two objective functions are used as likelihood functions, but the effect of setting different objective functions on the results remains to be further explored.
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