Charged Excitons of Composite Fermions in the Fractional Quantum Hall Effect

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Charged excitons of composite fermions (CFs) are considered in the fractional quantum Hall effect. Energies of the charged CF excitons are computed at $\nu = 1/3$ as a function of the size of exciton. We show that the charged CF exciton with size of roton is lower in energy than the unbound state of a neutral roton and a lone charge. Therefore we propose that the lowest-energy excitation of the fractional quantum Hall effect is in fact due to the charged excitons of composite fermions composed of two CF-particles and one CF-hole. We believe that charged excitons of composite fermions shed new light on interpreting the resonant inelastic light scattering experiments.

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Two-dimensional electron systems (2DES) exhibit spectacular phenomena such as the fractional quantum Hall effect \cite{1} (FQHE) when subjected to an intense, perpendicular magnetic field. Excitations of the fractional quantum Hall effect have attracted considerable interest because of their key role in the existence of the fractional quantum Hall effect. Now there has been much progress in understanding the excitations of FQHE in terms of the composite fermion (CF) theory \cite{2–7}. Excitation gaps at a general momentum as well as the transport gap have been described in terms of a single pair of CF-particle and CF-hole, and shown to be in excellent agreement with numerous experiments and numerical studies. Also, excitations with more complex structure than a simple, neutral exciton have proven to be very important in the long-wavelength limit \cite{8}. Originally, the question of whether the lowest-energy excitation at long wavelengths was described by a single exciton or was instead a two-roton excitation was raised by Girvin, MacDonald, and Platzman \cite{9}, and later was considered in numerical studies of finite systems \cite{10}. More recently, a conclusive evidence has been obtained from the composite fermion theory in which the two-roton state was explicitly constructed and was shown to be lower in energy than a single CF exciton in the long wavelength limit \cite{8}.

However, there still exist possibilities for yet another excitation partly because of the variational nature of the study. But more importantly, when isolated quasi-particles or quasi-holes are already present in the system, it is possible that an exciton takes advantage of them to form a bound state. Among the best candidates for the new excitation is the charged exciton state of composite fermions which is composed of two CF-particles and one CF-hole (denoted as $\text{CF}^{-}$), or one CF-particle and two CF-holes (denoted as $\text{CF}^{+}$). Figure 2(a) shows a schematic diagram for the physical situation of interest in this article. A lone charge is combined with a nearby, neutral exciton of opposite momentum to form a stationary, charged exciton of composite fermion. (For clear physical picture, consider that the lone charge is moving under the Landau gauge where each basis describes a plane wave along, say, $x$-direction.)

In fact, there has been considerable interest in the charged excitons of electrons in two spatial dimension because of the increased binding energy due to the reduced dimensionality \cite{11–13}. Even the charged excitons in strong magnetic fields have been studied by a number of authors, all of whose works, however, concentrated on the excitons between electrons in the conduction band and holes in the valence band \cite{14–17}. On the contrary, the charged excitons studied in this article are formed by quasi-particles and quasi-holes of composite fermions which are in the same layer, band and electronic Landau level. We wish to show that these charged excitons of composite fermions are resposible for the lowest excitations of FQHE.

A composite fermion (CF) is the bound state of an electron and an even number of magnetic flux quanta (a flux quantum is defined as $\phi_0 = \hbar c/e$), formed when electrons confined to two dimensions are exposed to a strong magnetic field \cite{2–4}. According to this theory, the interacting electrons at the Landau level (LL) filling factor $\nu = n/(2p \pm 1)$, $n$ and $p$ being integers, transform into weakly interacting composite fermions at an effective filling $\nu^* = n$; the ground state corresponds to $n$ filled CF Landau levels (CF-LLs) and it is natural to expect that the neutral excitations correspond to a particle-hole pair of composite fermions, called the (neutral) CF exciton (denoted as $\text{CF}^0$). At the minimum in its dispersion, the CF exciton is called the roton, borrowing the terminology from the $^4\text{He}$ literature \cite{9}. Addition to the remarkable success of the composite fermion theory for the FQHE states at the filling factors $\nu = n/(2p \pm 1)$, it is equally amazing that the composite fermion theory continues to provide excellent states even away from those special filling factors. This broad accuracy enables us to utilize the composite fermion theory in order to study the systems with additional particles or holes on top of the filled CF-Landau level, the excited states of which are described by the charged excitons of composite fermions.
We will use the spherical geometry \( \mathbb{S} \) below, which considers \( N \) electrons on the surface of a sphere in the presence of a radial magnetic field emanating from a magnetic monopole of strength \( Q \), which corresponds to a total flux of \( 2Q \phi_0 \) through the surface of the sphere. The wave function for the CF state at \( Q \), denoted by \( \Psi_{2Q} \), is constructed by mapping to the wave function of the corresponding electron states at \( q \), denoted by \( \Phi_{2q} \):

\[
\Psi_{2Q} = \mathcal{P}_{LLL} \Phi_{2q}^{2p}(N-1) \Phi_{2q}
\]

Here \( \Phi_{N-1} = \prod_{j<k}(u_j v_k - u_k v_j) \) is the wavefunction of the fully occupied lowest Landau level with monopole strength equal to \( (N-1)/2 \), where \( u_j = \cos(\theta_j/2) \exp(-i\phi_j/2) \) and \( v_j = \sin(\theta_j/2) \exp(i\phi_j/2) \). \( \mathcal{P}_{LLL} \) denotes projection of the wave function into the lowest Landau level (LLL). The monopole strengths for \( \Phi_{2q} \) and \( \Psi_{2Q} \), \( q \) and \( Q \), respectively, are related by \( Q = q + p(N-1) \). It is crucial to note that formally the mapping between \( \Psi_{2Q} \) and \( \Phi_{2q} \) can be defined regardless of whether or not they are incompressible. The accuracy of \( \Psi_{2Q} \) against the exact state, however, is dependent on whether \( \Phi_{2q} \) is robust when subjected to perturbations such as residual interactions between composite fermions. In particular, for the ground state and the single exciton state of the system with completely filled CF-Landau levels, the wave functions \( \Phi_{2q} \) are completely determined by symmetry (i.e., by fixing the total orbital angular momentum \( L \), which is preserved in going from \( \Phi_{2q} \) to \( \Psi_{2Q} \) according to Eq. (1)), giving parameter-free wave functions \( \Psi_{2Q} \) for the ground and excited states of interacting electrons. These have been found to be extremely accurate in tests against exact diagonalization results available for small systems [2,13], establishing the essential validity of the CF exciton description of the neutral mode of the FQHE.

Figure 2(b) shows a schematic diagram for the construction of the negatively charged CF exciton. Unlike the previous case of completely filled CF Landau levels, there are in general multiple states to be constructed for a given value of angular momentum \( L \) according to the angular-momentum addition rule and the Fermi statistics. In fact, the distinction between the lone charge and the constituent charge from the neutral exciton is arbitrary because their contribution to the final charged exciton state must be identical. But whenever there is a unique state for \( \Phi_{2q} \), determined only by symmetry, the choice of intermediate route for constructing the wavefunction does not affect the final state of the charged CF exciton. More importantly, its uniqueness suggests the robustness of \( \Phi_{2q} \), and therefore \( \Psi_{2Q} \). However, when there are multiple states for \( \Phi_{2q} \) in the same angular momentum channel, it is necessary to diagonalize the Hamiltonian in the restricted Hilbert space, the basis functions of which are provided by the composite fermion theory. In this case, the final composite fermion wavefunction is more susceptible to the form of interactions between particles.

Therefore it is very convenient that the \( L = 0 \) state of charged CF exciton is uniquely determined, which, in turn, guarantees that correlations between CF-particles and CF-holes are automatically taken into account because of the exactly same reason as for the neutral excitons, \( CFX^0 \). Figure 2 shows the comparison between the composite fermion states and the exact states for the system of number of electrons \( N = 8 \) and the monopole strength \( Q = 10 \), where the ground state describes a lone, negatively charged composite fermion at \( \nu = 1/3 \). Addition to the \( L = 0 \) case mentioned above, a unique choice is obtained for the charged CF exciton state at \( L = 1, 9, \) and 10. (Note that the Fermi statistics is important to determine the number of states in each angular-momentum channel since two particles are in the same CF Landau level.) Energies per particle for the charged CF exciton state at \( L = 0, 1, 9, \) and 10 are respectively \(-0.433257(63)\), \(-0.429878(60)\), \(-0.429184(40)\), and \(-0.428253(57)\) in units of \( e^2/\ell a_0 \), which agree with the exact energies, \(-0.433592\), \(-0.429188\), \(-0.429676\), and \(-0.428982\), within less than 0.2\%. Also, the energy of the ground state at \( L = 4 \) in the composite fermion theory, \(-0.440415(61)\), is in excellent agreement with the exact ground state energy, \(-0.440764\).

Since it is established that the composite fermion theory provides accurate states for the charged exciton state, we next turn to the energetics. But, before we go into the detail, several comments are in order. First, though the formalism of constructing the charged CF exciton state can be applied to general filling factors, we restrict our attention to the charged CF excitons at \( \nu = 1/3 \). Second, it is assumed that the Zeeman splitting energy is large enough to suppress the spin-flip excitations. Third, we only consider the negatively charged CF exciton with \( L = 0 \) which is, in other words, a stationary exciton. Note that in general a non-zero momentum-transfer is required to form a charged exciton since the ground state of a lone charge is not a uniform state. And finally, regarding the technical aspect of Monte Carlo simulation, the efficient determinant-updating technique in Ref. [6] has been used because of the large number of Slater determinants required in constructing the charged CF exciton state.

Now, the energy cost of creating a stationary, charged CF exciton, is defined as follows:

\[
\Delta E_{CFX} = E_{CFX} - (L = 0) - E_{lone CF} (L = L_{gr})
\]

where \( L_{gr} \) is the angular momentum of the ground state containing the lone composite fermion. For example, \( L_{gr} = 4 \) for the system of \( N = 8 \) and \( Q = 10 \). And \( L_{ex} \) is the angular momentum of the constituent single exciton, which is identical to \( L_{gr} \) since we are only interested in the stationary charged exciton, i.e. \( L = 0 \). Note that, as
usual, the size of constituent single exciton, and therefore the charged exciton, is proportional to \(k_{eex}l_0^3\) where \(k_{eex} = L_{ex}/R\) and \(R = \sqrt{Ql_0}\). However, the above relationship between the angular momentum in the spherical geometry and the linear momentum in planar geometry is valid only for the neutral exciton. Also note that it is not possible to increase the number of electrons in order to reach the thermodynamic limit for a fixed value of \(k_{eex}l_0\) because of the nature of the spherical geometry. So \(k_{eex}l_0\) is also changed upon increasing the system size. Of course, it is possible in principle to construct a charged CF exciton for an arbitrary number of electrons by using the diagonalization within the restricted CF Hilbert space. However, this procedure is very hard to be carried out in practice.

The energy cost of excitation far away from the lone charge, or \(CFX^0 + CF\), is identical to that of neutral excitation from the filled CF shell:

\[
\Delta E_{CFX^0 + CF}(L_{ex}) = E_{CFX^0}(L = L_{ex}) - E_{filled\ shell}(L = 0) \tag{3}
\]

Therefore it is natural to define the binding energy of the charged CF exciton, \(\Delta_{CFX^-}\), as follows:

\[
\Delta_{CFX^-}(L_{ex}) = \Delta E_{CFX^0 + CF}(L_{ex}) - \Delta E_{CFX^-}(L_{ex}) \tag{4}
\]

Figure 3 shows the Coulomb energies of the charged CF exciton as a function of \(k_{eex}l_0\), compared with those of the neutral excitons. As shown in Fig. 3, the charged CF exciton is quite lower in energy than the single exciton state around \(k_{eex}l_0 = 1.5\). In other words, the charged CF exciton has the lowest energy when created by \textit{combining a lone charge and a roton}. The binding energy of the charged CF exciton is estimated to be roughly \(0.02e^2/\epsilon l_0\). As expected, for large \(k_{eex}l_0\), the energy of charged CF exciton is equal to that of single exciton since the charged exciton with large \(k_{eex}l_0\) means the far-separated, independent collection of quasi-particles and quasi-holes.

Now let us turn to the relevance of this work to the resonant inelastic light scattering experiments. A conventional interpretation of the excitations measured in those experiments is that they are neutral excitons. However, since there are always some lone charges in the real system of experiments, it is plausible that the light-scattering gives rise to the charged CF excitons. As shown in the above, the charged CF exciton can have a lower energy than the single, neutral roton. Moreover, by using the analogy to the charged excitons of electrons \[20\], the charged CF exciton can decrease its energy further relative to the neutral excitons because it can be bound to a donor. The distance between 2DES and the donor layer is not very far when measured in units of magnetic length: typically less than 10 times magnetic length. Therefore the interaction between the charged exciton and a donor can lower the energy significantly, bringing theory and experiments closer.

However, a more interesting consequence is that the energy reduction due to the interaction with donors is constant in laboratory units such as \(MeV\), but is increasing in units of \(e^2/\epsilon l_0\) as the electron density becomes smaller. This may explain the recent, curious experimental observation \[8\] that the energies of the roton as well as the long-wavelength excitation is almost constant as a function of electron density whereas the CF theory combined with the local density approximation \[21\] predicts the increase of energies as the electron density decreases because of the smaller role of the finite thickness effect. The energy reduction due to the interaction with donors is bigger for a smaller density, and therefore bring down the excitation energies more. Finally, it is speculated that the long-wavelength excitation can be composed of two charged CF excitons at their minimum energy, which is analogous to the two-roton state.

In conclusion, we have shown that the charged CF exciton composed of two CF-particles and one CF-hole is lower in energy than the single roton, when created by combining a lone charge and a roton, and its binding energy is estimated to be around \(0.02e^2/\epsilon l_0\). Therefore it is proposed that the lowest excitation of the FQHE state at \(\nu = 1/3\), is in fact due to the charged exciton of composite fermions. Study of the relevance of the charged CF exciton at other filling factors is in progress.

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\[ \Psi_{\text{CFX}}(L,M=0) = \sum_{m,m'} (-1)^m \left( \begin{array}{ccc} l_{\text{ex}} & l + 1 & L \\ m & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} l + 1 & l_{\text{ex}} & m' - m \\ m - m & m & m \end{array} \right) \]

\[ X[ \begin{array}{cccc} m & m' & m' - m & m \\ \vdots & \vdots & \vdots & \vdots \end{array} ] \]

FIG. 1. (a) Schematic diagram for the creation of a negatively charged exciton of composite fermions ($\text{CFX}^-$) with $L = 0$. (b) Schematic diagram for the wavefunction of $\text{CFX}^-$, with well defined $L$ and $M$ quantum numbers. (We have chosen $M = 0$ with no loss of generality, since the energy is independent of $M$.) The figure in square brackets shows schematically the Slater determinant obtained by promoting one composite fermion from the topmost fully occupied CF Landau level to the next higher CF Landau level in single CF basis states indicated. (Note that the single CF basis states for this Slater determinant are the CF correlated basis functions which contains correlations from all other particles.) The topmost fully occupied LL corresponds to the angular momentum $l$ shell in the spherical geometry; other Landau level shells are not shown for simplicity. The Wigner 3-j symbols are used in order to make a definite angular-momentum eigenstate. The relative signs of the various terms in the sum follow from the antisymmetry requirement. For a given $L$, there are in general multiple states to be constructed according to the angular-momentum addition rule between the angular momentum of single composite fermion ($l + 1$) and that of neutral CF exciton ($l_{\text{ex}}$).
FIG. 2. Comparison between the energies of the ground state (isolated charge) at $L = 4$ and the lowest lying excitations (charged CF excitons) at $L = 0, 1, 9,$ and $10$ in the composite fermion theory, and those of the exact diagonalization study for the system of the number of electrons $N = 8$ and the monopole strength $Q = 10$. Energies from the composite fermion theory are denoted as solid circles while the exact energies are indicated by short horizontal lines. Note that the Coulomb interaction is taken for the interaction between electrons. Also, note that the statistical uncertainty from the Monte Carlo simulation is smaller than the size of symbols.

FIG. 3. Energy of the charged CF excitons as a function of the momentum ($k_{ex} l_0$) of the constituent single exciton. Note that $k_{ex} l_0^2$ is proportional to the distance between the CF-particle and the CF-hole of the constituent single exciton, and therefore it measures the size of the charged CF exciton. As shown from the comparison with the dispersion curve of neutral exciton, the energy of the charged CF exciton is quite lower than that of single exciton around $k_{ex} l_0 = 1.5$, i.e. by combining a lone charge and a single roton.