Role of thermal plumes on particle dispersion in a turbulent Rayleigh-Bénard cell

V. Lavezzo¹, H.J.H. Clercx¹,² and F. Toschi¹,³,⁴

¹ Department of Applied Physics and J.M. Burgers Centre for Fluid Dynamics, Eindhoven University of Technology, Den Dolech, 2, 5612 AZ Eindhoven, The Netherlands
² Department of Applied Mathematics, University of Twente, Post Office Box 217, 7500 AE Enschede, The Netherlands
³ Department of Mathematics and Computer Science and International Collaboration for Turbulence Research, Eindhoven University of Technology, Den Dolech, 2, 5612 AZ Eindhoven, The Netherlands
⁴ CNR, Istituto per le Applicazioni del Calcolo, Via dei Taurini 19, 00185 Rome, Italy

E-mail: v.lavezzo@tue.nl

Abstract. A high resolution numerical technique coupled with Lagrangian particle tracking is employed to investigate the behaviour of inertial particles in a periodic turbulent Rayleigh-Bénard convection cell. In particular, we focus on the relation between thermal structures and particle re-suspension. Different particle Stokes and Froude numbers are considered to evaluate the influence of inertia and gravity on particle clustering and consequently, on the dispersion of particles in the domain. Statistics on particle and fluid velocity and temperature fields are also presented.

1. Introduction

The dispersion of inertial particles in turbulent convection has direct relevance for many industrial and environmental applications, where the fluid heat transfer can be modified by the presence and deposition of particles at the walls (e.g. nuclear power plants, petrochemical multiphase reactors, cooling systems for electronic devices, pollutant dispersion in the atmospheric boundary layer, aerosol deposition, etc.). Over the past years, turbulent convection has been the subject of extensive studies (see e.g. Ahlers et al. (2009); Kunnen et al. (2009); Lohse & Xia (2010)), which attempted to determine the main flow features and the relation between the different parameters which characterize the fluid velocity and temperature fields, but only few of them focused on Lagrangian statistics. A Lagrangian tracking of tracer particles has been employed in Schumacher (2009, 2008) to study the properties of the mixing zone, the zone close to the wall dominated by rising and falling thermal plumes. The behaviour of heavy and light particles in turbulence has been the subject of many previous works (see Toschi & Bodenschatz (2009) for a review), which focused on the effects of inertia, preferential sampling of specific regions of the flow, underlying turbulence fluctuations, flow stratification, etc. on particle dispersion in different geometries (see among others Bec et al. (2006), Bec et al. (2010) and van Aartrijk & Clercx (2008)). A similar approach is used, in this work, to evaluate the effect of thermal plumes on inertial particles tracked in a Lagrangian frame of reference in a
turbulent Rayleigh-Bénard cell. A Lattice Boltzmann Method (LBM) is used to obtain the fluid velocity and temperature fields in which particles have been dispersed. By following in time particles initially released on a horizontal plane close to the bottom wall, a possible correlation between particle position and the fluid structures responsible for their suspension has been shown. It has been observed that particles tend to sample regions where a high correlation between vertical velocity and temperature fluctuations was found. These regions correspond to the sheetlike plumes, as described in Schumacher (2009). The morphological evolution of these structures into mushroom-like plumes drives, depending on their inertia, particle re-suspension.

2. Description of numerical method

The Lattice Boltzmann method is a well established tool based on the Boltzmann equation (see e.g. Succi (2001) and Benzi et al. (1992) for details), which, under the BGK simplification, can be written as follows:

$$f_i(x + \Delta tc_i, t + \Delta t) - f_i(x, t) = -\frac{\Delta t}{\tau}[f_i(x, t) - f_i^{eq}(x, t)] + F_i \Delta t,$$

where $f_i(x, t) = f_i(x, v = c_i, t), i = 1, ..., n$ is the probability of finding a particle at lattice site $x$ at time $t$, moving along the lattice direction defined by the discrete speed $c_i$ during a time step $\Delta t$. The left-hand side of the equation represents the so-called streaming term, which corresponds to the evolution in time of the probability function $f$. The RHS describes the collision via a simple relaxation towards local equilibrium $f_i^{eq}$ (a local Maxwellian distribution expanded to second order in the fluid speed) in a time lapse $\tau$. This relaxation time fixes the fluid kinematic viscosity as $\nu = c_s^2(\tau - 1/2)$, where $c_s$ is the speed of sound of the lattice fluid, $(c_s^2 = 1/3$ in the present work). Finally, $F_i$ represents the effects of external forces. The set of discrete speeds must be chosen such that rotational symmetry is fulfilled to the required level of accuracy. The fluid density $\rho = \sum_i f_i$, and speed $u = \sum_i f_i c_i / \rho$ can be shown to evolve according to the (quasi-incompressible) Navier-Stokes equations of fluid dynamics. In this paper, we will refer to the nineteen speed D3Q19 model shown in Figure 1a. The temperature population $g_i$ is evolved on the lattice according to the same formulation of the Boltzmann equation used for the fluid momentum. The temperature is modeled as a scalar $T = \sum_i g_i$, and it provides an external forcing in the Navier-Stokes equations via a buoyancy term, which is taken into account by introducing a suitable shift of the velocity and temperature fields entering the local equilibrium distributions.

The reference geometry, shown schematically in Figure 1b, is a Cartesian slab bounded by two horizontal walls having dimensions equal to the size of the computational grid employed for the calculations and, therefore, equal to $256 \times 256 \times 128$ lattice units in the streamwise ($x$), spanwise ($y$) and wall-normal ($z$) direction, respectively.

The aspect ratio $\Gamma$ is then equal to 2 and the grid is equispaced in all directions. Periodic boundary conditions are imposed on both the velocity and the temperature field in the two homogeneous directions. No-slip boundary conditions are enforced at the walls for the velocity field whereas constant temperature is considered for the temperature field. The temperature field is decomposed into a mean field and fluctuations around the mean as: $T = \langle T \rangle + T'$. As the mean value can be recovered by the analytical expression given by the conductive profile in absence of convection, just the evolution of the temperature fluctuations is taken into account during the simulation. The mean profile can be written as: $\Delta T/H$ where $H$ is the cell height and $\Delta T$ the temperature difference between the two walls which is equal to 1 in the present case.

Parameters characterizing the temperature field are: the Prandtl number defined as the ratio of the kinematic viscosity to the thermal diffusivity of the fluid as $Pr = \nu/\kappa$ set equal to 1.
and the Rayleigh number equal to $2.5 \cdot 10^6$ and defined as $Ra = (\alpha g \Delta T H^3)/\nu \kappa$ where $g$ is the gravitational acceleration, $\alpha$ the coefficient of thermal expansion, $\Delta T$ the temperature difference between the two walls, $\kappa$ the thermal diffusivity and $\nu$ is the fluid kinematic viscosity. This choice of parameters was made also for its possibility of a validation of the numerical method against other works in similar geometries as the one from Kunnen et al. (2009).

A Lagrangian approach has been used to track particles dispersed in the domain. Swarms of $6.4 \cdot 10^4$ particles characterized by different inertia and gravitational force are simulated simultaneously, resulting in a polydispersed distribution of $1.6 \cdot 10^6$ particles in total. Particles are then sorted out when calculating statistics. Particle inertia is defined in terms of the particle Stokes number, which can be defined as the ratio between the particle response time $\tau_p$ and the Kolmogorov time scale $\tau_\eta$, $St = \frac{\tau_p}{\tau_\eta}$. As in other wall-bounded flows the Kolmogorov time scale is varying with respect to the distance from the wall, as visible in Figure 2. An average estimation of the Kolmogorov time scale was proposed by Shraiman & Siggia (1990) and can be used, in this case, as fixed reference value to express the importance of particle inertia with respect to the viscous forces. $\tau_\eta$ was estimated from the energy dissipation rate as:

$$\langle \epsilon \rangle = \frac{\nu^3 (Nu - 1) Ra}{Pr^2 H^4},$$

$$\tau_\eta = \left( \frac{\nu}{\langle \epsilon \rangle} \right)^{1/2} = \frac{Pr H^2}{\nu((Nu - 1) Ra)^{1/2}},$$

where $\nu$ is the kinematic viscosity of the fluid and $\tau_p$, the particle response time, is equal to: $\tau_p = (\rho_p d_p^2)/18 \mu$, being $d_p$ the particle diameter, $\rho_p$ the particle density and $\mu$ the dynamic viscosity of the fluid. In the present case $\tau_\eta$ was found to be equal to 504 lattice units. The simulated Stokes numbers are: 0 (tracers), 0.6, 1, 5 and 20. Only results of Stokes 1 and 20 will be reported here as these are considered the most representative of particle behaviour.

Particles are initially released at random positions on a plane placed close to the bottom wall in an already developed and statistically steady turbulent flow field. Specifically, the position $H_0$ of the plane is 0.03125 non-dimensional units from the bottom wall. This particular configuration has been chosen to capture the effects of the plumes on particle re-suspension at the early stages of the simulation, when hot plumes start to re-suspend particles from at the bottom plate and before they settle down again towards the wall due to gravitational effects.

As we consider particles with density much higher than the surrounding fluid, the particle equation of motion reduces to a balance of the Stokes drag, gravity forces and can be written
as:
\[
\frac{du}{dt} = \frac{v - u}{\tau_p} + \frac{\rho_f - \rho_p}{\rho_p} g, \tag{4}
\]
where \(v\) is the fluid velocity at the particle position, \(u\) the particle velocity, \(\rho_f\) the fluid density and \(\rho_p\) the particle density and \(g\) the gravitational acceleration. The second term in the RHS of the equation is considered as one single parameter entering the simulation and will be referred to as \(g' = |g'|\) in the rest of this paper.

The particle temperature evolves according to the following equation:
\[
\frac{dT}{dt} = \frac{(T_f - T_p)}{\tau_{p,\theta}}, \tag{5}
\]
where \(T_f\) is the temperature of the fluid at the particle position, \(T_p\) is the particle temperature and \(\tau_{p,\theta}\) the particle thermal response time defined as: \(\tau_{p,\theta} = \frac{c_p \rho_p d_p^2}{12 \kappa}\). In the present work the thermal response time and the velocity one are considered to be the same.

An explicit Adams-Bashforth scheme is used for the time integration of the equation of particle motion and a linear interpolation scheme is used to obtain both the fluid velocity and temperature at the point of the particle.

Another important parameter which allows to understand the interplay between inertial forces and buoyancy forces experienced by a particle along its trajectory is the Froude number of a particle defined as:
\[
Fr = \frac{\langle u_{rms} \rangle V}{\tau_p g'}, \tag{6}
\]
where \(g'\) refers to the second term on the RHS of the equation of particle motion and \(\langle u_{rms} \rangle V\) is the volume averaged fluid rms velocity. High value of Froude means that the gravity force is low and the motion of the particles is driven by the buoyancy of the fluid whereas low values of Froude number correspond to high values of \(g'\), so particle behaviour is mainly governed by this force.
Figure 3. Benchmark between the LBM code (symbols) and the work by Kunnen et al. (2009) (solid black line). (a) Mean temperature profile varying with the cell height with, in the inset, geometrical extrapolation of the thermal boundary layer thickness and (b) RMS of the temperature varying with the cell height with zoom in of the region where sharp temperature gradients are present.

3. Results

3.1. Fluid velocity and temperature field

To understand the importance of fluid plumes on particle re-suspension it is necessary to first characterize the flow field from a statistical view point.

In Figure 3a the mean fluid temperature varying with respect to the cell height is plotted. Results are compared to those obtained by Kunnen et al. (2009) in a similar geometry using a finite-difference code. This choice was made for its relatively low Rayleigh number, the use of grid refinement close to the wall according to a log-normal distribution of the grid nodes and for the realistic choice of no-slip boundary conditions. A broader validation of the employed methodology at different Rayleigh numbers has been presented in an earlier paper, see Lavezzo et al. (2011), therefore here only results relative to this particular geometry are reported.

From Figure 3a, it is possible to notice that the curves are overlapping on top of each other, thus suggesting that mean quantities are not sensitive to both the numerical scheme and the employed grid resolution. The LBM, despite the regular grid, is capable to correctly capture the thermal boundary layer thickness $\delta_{th}$ in its geometrical extrapolation (see the inset of Figure 3a) when compared to the finite-difference code result. A value of about 0.045 in non-dimensional units is found. This value is consistent with the position of the peak in the temperature RMS profile visible in Figure 3b.

To quantify the total (conductive and convective) turbulent heat transport inside the domain, the volume and time averaged Nusselt number, normalized by the conductive heat flux obtained in absence of convection, has been calculated as:

$$ Nu = \frac{1}{H(t-t_0)} \int_0^H \int_{t_0}^t Nu(z,t)dzdt = 1 + \frac{H}{\kappa \Delta T} \langle u_z T \rangle_{V,t}, \quad \text{with} \quad (7) $$

$$ Nu(z,t) = \frac{\langle u_z T \rangle_{A,t} - \kappa \partial_z \langle T \rangle_{A,t}}{\kappa \Delta T / H}. \quad (8) $$
A Nusselt number of $11.16 \pm 1.15$ has been obtained. This value compares well with the results of Kunnen et al. (2009) (see also Kunnen et al. (2006)) where a value of 11.08 was found. This result confirms the good agreement between the two codes and gives an introduced error less than 1%. Using the value for the Nusselt number the boundary layer thickness can be calculated as:

$$\delta_{th} = \frac{H}{2Nu}.$$  \hspace{1cm} (9)

A value of $0.045 \pm 0.005$ has been found, thus confirming the previous result obtained with the geometrical extrapolation.

3.2. Particle tracking

In this section preliminary results on particle tracking are reported. The sheetlike plumes close to the wall are responsible, in their morphological evolution, for the formation of mushroom-like plumes in a region confined to few $\delta_{th}$ (where $\delta_{th}$ is the boundary layer thickness) Shishkina & Wagner (2008); Lohse & Xia (2010). These three dimensional plumes are accountable, depending on their temperature, for the upward or downward particle transport. For this reason, the plane of release for the particles was chosen to be located within the boundary layer, with $H_0 < \delta_{th}$ ($\delta_{th} \approx 0.045$, $H_0 \approx 0.031$, see also Section 2).

Low-inertia particles are easily transported upwards by the fluid structures, whereas particles with higher Stokes number tend to first collect at the edges of sheet-like plumes and to leave the wall at later times, as visible in Figure 4, where the z-coordinate of the particle center of mass varying with time is plotted for two Stokes numbers: $St = 1$ (left) and $St = 20$ (right). The different curves correspond to different Froude numbers obtained by varying the value of the gravitational acceleration of the particles $g'$ and by keeping the particle response time constant. The $St = 1$ particles are moving quickly away from the wall, but after some time, if the gravitational acceleration is high enough, they are subject to the descending plumes, which transport them downward toward the wall. $St = 20$ particles, on the contrary, are less prompt in adapting to the fluid velocity changes and less sensitive to the presence of rising or descending plumes. For this reason, both the suspension and the deposition take longer time and, therefore, $St = 20$ particles tend to remain dispersed in a larger portion of the domain with respect to the $St = 1$ ones. The dashed red curve in Figure 4a refers to the volume averaged turbulent kinetic energy of the fluid, which is constant throughout the simulation as particles are released in an already developed turbulent flow.

The difference in particle behaviour is more noticeable while considering the qualitative picture given in Figure 5. Instantaneous snapshots of particle position taken at the same instant

![Figure 4](image-url)
for different particle inertia coloured with particle temperature fluctuations, is reported. The temperature fluctuations for the particles are obtained in the same way as for the fluid i.e. by subtracting the mean (conductive) profile from the total temperature. From left to right the position of tracers, $St = 1$ and $St = 20$ particles at Froude number equal to 3 are plotted at the early stages of the simulation (after about half large-scale circulation i.e. 4000 time steps). It is possible to notice that while tracers are already resuspended from the plane of release, $St = 1$ and 20 are, at this early stage, still collected in the boundary layer. The different inertia brings particles with Stokes number 1 at the edges of sheet-like vortices whereas $St = 20$ are still almost homogeneously dispersed on the plane of release as they are less sensitive to fluid changes.

To understand the interplay between fluid structures and the subsequent particle re-suspension it is necessary to correlate particle position with the fluid properties. In particular, it is known that rising and falling plumes can be identified by a positive product of vertical velocity and temperature fluctuation, $u_z T' > 0$ (see e.g. Gasteuil et al. (2007)). In Figure 6 instantaneous isocontours of $T'$ (top), $u_z$ (center) and $u_z T'$ (bottom), taken on a $z-x$ plane at $y = 250$ lattice units, are plotted against particle ($St = 1$) position coloured with their temperature fluctuations. Particles are brought upwards by rising plumes, characterized by positive and high vertical velocities, as visible in the central panel. In the same region positive temperature fluctuations surrounded by negative (blue) fluctuations are found (see top panel). These regions correspond to the mushroom like type of structures and confirm their importance in particle re-suspension. This is also visible in the lower panel in which the product $u_z T'$ is reported against particle scatter plot. If the energy of the plumes is not enough to move particles upwards, the gravitational force dominates particle behaviour by driving particles again towards the wall, as visible in the lower left corner of the panels.

4. Conclusions

A Lattice Boltzmann approach, coupled with Lagrangian particle tracking of inertial particles, is used in this work to understand the effects of the fluid structures on particle behaviour in a Rayleigh-Bénard cell. A good agreement in the temperature and heat transfer statistics has been found while comparing the presents results with those obtained by Kunnen et al. (2009), thus validating our methodology. By following in time particles initially released on a horizontal plane close to the bottom wall, it has been possible to show, in a qualitative sense, the importance of
Figure 6. Instantaneous isoconturs of $T'$ (top), $u_z$ (center) and $u_z T'$ (bottom) taken on a $z - x$ plane at $y = 250$ lattice units, is plotted against particle position coloured with their temperature fluctuations.

the mushroom like and sheet-like type of structures in particle re-suspension. Particle behaviour changes with changing particle inertia, as particles are sampling different regions of the flow.

5. Acknowledgments

Authors wish to thank Dr. Rudie Kunnen for making his data accessible to us and for the useful discussions. This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organization for Scientific Research (NWO). The computational resources made available by SARA supercomputing center in Amsterdam under the DCCP project SH-176-10 and project SH-004-10 are gratefully acknowledged. The European COST Action MP0806 “Particles in Turbulence” is also acknowledged.

References

van Aartrijk, M. & Clercx, H.J.H. 2008 Preferential concentration of heavy particles in stably stratified turbulence. Phys Rev Lett 100, 254501.

Ahlers, G., Grossmann, S. & Lohse, D. 2009 Heat transfer and large scale dynamics in turbulent rayleigh-benard convection. Rev. Mod. Phys. 81, 503–537.
Bec, J., Biferale, L., Boffetta, G., Celani, A., Cencini, M., Lanotte, A. & Musacchio, S. 2006 Acceleration statistics of heavy particles in turbulence. J. Fluid Mech. 550, 349–358.

Bec, J., Biferale, L., Lanotte, A., Scagliarini, A. & Toschi, F. 2010 Turbulent pair dispersion of inertial particles. J. Fluid Mech. 645, 497–528.

Benzi, R., Succi, S. & Vergassola, M. 1992 The lattice boltzmann equation: theory and applications. Phys. Rep. 222, 145–197.

Gasteuil, Y., Shew, W., Gibert, M., Chilla, F., Castaing, B. & Pinton, J.-F. 2007 Lagrangian temperature, velocity, and local heat flux measurement in rayleigh-bénard convection. Phys. Rev. Lett. 99 (23), 234302.

Kunnen, R.P.J., Clercx, H.J.H & Geurts, B.J. 2006 Heat flux intensification by vortical flow localization in rotating convection. Phys Rev E 74, 056306.

Kunnen, R.P.J., Geurts, B.J. & Clercx, H.J.H. 2009 Turbulence statistics and energy budget in rotating rayleigh–bénard convection. Eur. J. of Mech. B/Fluids 28, 578–589.

Lavezzo, V., Toschi, F. & Clercx, H.J.H. 2011 Rayleigh-bénard convection via lattice boltzmann method: code validation and grid resolution effects. submitted to Journal of Physics: Conference series.

Lohse, D. & Xia, K-Q. 2010 Small-scale properties of turbulent rayleigh-bénard convection. Annu. Rev. Fluid Mech. 42, 335–364.

Schumacher, J. 2008 Lagrangian dispersion and heat transport in convective turbulence. Phys. Rev. Lett. 100, 134502.

Schumacher, J. 2009 Lagrangian studies in convective turbulence. Phys. Rev. E 79, 056301.

Shishkina, O. & Wagner, C. 2008 Analysis of sheet-like thermal plumes in turbulent rayleigh-benard convection. J. Fluid Mech. 599, 383–404.

Shraiman, B. & Siggia, E.D. 1990 Heat transport in high-rayleigh-number convection. Phys. Rev. A 42, 3650–3653.

Succi, S. 2001 Oxford Science Publications, Oxford.

Toschi, F. & Bodenschatz, E. 2009 Lagrangian properties of particles in turbulence. Ann. Rev. Fluid Mech. 41, 375–404.