1 Introduction

Supersymmetry (SUSY) has many well known attractions, especially in the context of Grand Unified Theories (GUTs). SUSY stabilizes scalar mass corrections (the hierarchy problem), greatly reduces the number of free parameters, facilitates gauge coupling unification, and provides a plausible candidate for cosmological dark matter. In this conference report we survey some recent examples of progress in SUSY-GUT applications.

2 Gauge coupling unification

As the renormalization mass scale $\mu$ is changed, the evolution of couplings is governed by the Renormalization Group Equations (RGE). For the gauge group SU(3)×SU(2)×U(1), with corresponding gauge couplings $g_3(=g_s), g_2(=g), g_1(=\sqrt{5/3}g')$, the RGE can be written

$$\frac{dg_i}{dt} = \frac{g_i}{16\pi^2} \left[ b_i g_i^2 + \frac{1}{16\pi^2} \left( \sum_{j=1}^{3} b_{ij} g_j^2 g_i^2 - \sum_{j=1}^{3} a_{ij} g_i^2 \lambda_j^2 \right) \right],$$

where $t = \ln(\mu/M_G)$ and $M_G$ is the GUT scale. The first term on the right is the one-loop approximation; the second and third terms contain two-loop effects, involving other gauge couplings $g_j$ and Yukawa couplings $\lambda_j$. The coefficients $b_i$, $b_{ij}$ and $a_{ij}$ are determined at given scale $\mu$ by the content of active particles (those with mass < $\mu$). If there are no thresholds (i.e. no changes of particle content) between $\mu$ and $M_G$, then the coefficients are constants through this range and the one-loop solution is

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_G) - \frac{t b_i}{(2\pi)},$$

where $\alpha_i = g_i^2/(4\pi)$; thus $\alpha_i^{-1}$ evolves linearly with $\ln \mu$ at one-loop order. If there are no new physics thresholds between $\mu = M_Z \approx m_t$ and $M_G$, as in the basic Standard

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Model (SM), then equations of this kind should evolve the observed couplings at the electroweak scale

\[
\begin{align*}
\alpha_1(M_Z)^{-1} & = 58.89 \pm 0.11, \\
\alpha_2(M_Z)^{-1} & = 29.75 \pm 0.11, \\
\alpha_3(M_Z) & = 0.118 \pm 0.007, 
\end{align*}
\]

(3) to converge to a common value at some large scale. Figure 1(a) shows that such a SM extrapolation does NOT converge; this figure actually includes two-loop effects but the evolution is still approximately linear versus \( \ln \mu \), as at one-loop order. GUTs do not work, if we assume just SM particles plus a desert up to \( M_G \).

But if we increase the particle content to the minimal SUSY model (MSSM), with a threshold not too far above \( M_Z \), then GUT-type convergence can happen. Figure 1(b) shows an example with SUSY threshold \( M_{\text{SUSY}} = 1 \) TeV \(^2 \). The evolved couplings are consistent with a common intersection at \( M_G \sim 10^{16} \) GeV; GUTs are plainly more successful with MSSM than with SM. Henceforth we assume MSSM. In fact a precise single-point intersection is not strictly necessary; the exotic GUT gauge, fermion and scalar particles may not be quite degenerate, giving several non-degenerate thresholds near \( M_G \), to be passed through on the way to GUT unification.

Fig. 1. Gauge coupling evolution: (a) in the SM; (b) in a SUSY-GUT example \(^3 \).

### 3 Yukawa coupling evolution

The Yukawa couplings also evolve. Typical evolution equations are \(^4 \)

\[
\begin{align*}
\frac{d\lambda_i}{dt} & = \frac{\lambda_i}{16\pi^2} \left[ - \sum c_i g_i^2 + 6\lambda_i^2 + \lambda_1^2 + \text{2-loop terms} \right], \\
\frac{d\lambda_b}{dt} & = \frac{\lambda_b}{16\pi^2} \left[ - \sum c'_i g_i^2 + \lambda_2^2 + 6\lambda_b^2 + \lambda_2^2 + \text{2-loop} \right], \\
\frac{d\lambda_\tau}{dt} & = \frac{\lambda_\tau}{16\pi^2} \left[ - \sum c''_i g_i^2 + 3\lambda_b^2 + 4\lambda_\tau^2 + \text{2-loop} \right], 
\end{align*}
\]

(6a) (6b) (6c)

with \( c_i = (13/15, 3, 16/3) \), \( c'_i = (7/15, 3, 16/3) \), \( c''_i = (9/5, 3, 0) \), and hence

\[
\frac{d(\lambda_b/\lambda_\tau)}{dt} = \frac{(\lambda_b/\lambda_\tau)}{16\pi^2} \left[ - \sum d_i g_i^2 + \lambda_t^2 + 3\lambda_b^2 - 3\lambda_\tau^2 + \text{2-loop terms} \right],
\]

(7)

with \( d_i = (-4/3, 0, 16/3) \). Evolution is mainly driven by the largest couplings \( g_3, \lambda_t, \lambda_b, \lambda_\tau \). The low-energy values at \( \mu = m_t \) are

\[
\begin{align*}
\lambda_b(m_t) & = \frac{\sqrt{2} m_b(m_b)}{\eta_0 v \cos \beta}, \\
\lambda_\tau(m_t) & = \frac{\sqrt{2} m_\tau(m_\tau)}{\eta_\tau v \cos \beta}, \\
\lambda_t(m_t) & = \frac{\sqrt{2} m_t(m_t)}{v \sin \beta},
\end{align*}
\]

(8)
where \( \eta_f = m_f(m_f)/m_f(m_t) \) gives the running of the masses below \( \mu = m_t \), obtained from 3-loop QCD and 1-loop QED evolution, for heavy flavors \( f = t, b, c, \tau \). For light flavors \( f = s, u, d, e, \mu \) we stop at \( \mu = 1 \) GeV and define \( \eta_f = m_f(1 \) GeV)/\( m_f(m_t) \). The \( \eta_f \) values depend principally on the value of \( \alpha_3(M_Z) \); for \( \alpha_3(M_Z) = 0.118 \), \( \eta_b \approx 1.5 \), \( \eta_c \approx 2.1 \), \( \eta_s = \eta_u = \eta_d \approx 2.4 \). The running mass values are \( m_b(m_b) = 4.25 \pm 0.15 \) GeV, \( m_{\tau}(m_{\tau}) = 1.777 \) GeV, \( m_c(m_c) \approx 1.2 \) GeV, \( m_s(1 \) GeV) \( \approx 0.175 \) GeV, \( m_u(1 \) GeV) \( \approx 0.006 \) GeV, \( m_d(1 \) GeV) \( \approx 0.008 \) GeV. The denominator factors in Eq. (8) arise in the MSSM from the two Higgs vevs \( v_1 = v \cos \beta \) and \( v_2 = v \sin \beta \); they are related to the SM vev \( v = 246 \) GeV by \( v_1^2 + v_2^2 = v^2 \), while \( \tan \beta = v_2/v_1 \) measures their ratio.

The above RGE allow the Yukawa couplings to be evolved from \( m_t \) up to the GUT scale. Figure 2 illustrates the scaling factors \( S_i = \lambda_i(M_G)/\lambda_i(m_t) \) for the case \( M_{\text{SUSY}} = m_t(m_t) = 150 \) GeV and \( \alpha_3(M_Z) = 0.118 \). It shows that the scaling factors are sensitive to \( \tan \beta \) when the latter is very large or very small. Large and small \( \tan \beta \) correspond to the regions in which the Yukawa couplings become large; between these regions and if \( m_t \) is not too large (\( \lesssim 170 \) GeV), the gauge couplings dominate the evolution giving rise to scaling factors less than one.

Figure 3 compares the values of \( \lambda_i(M_G) \) corresponding to the scaling factors in Figure 2. (caution: the input \( b, c, s, d, u \) mass values here have substantial uncertainties). Extrapolations of this kind allow us to test various postulated GUT relations such as

\[
\begin{align*}
\lambda_b(M_G) & \approx \lambda_r(M_G), \quad \text{Ref.}[8], \tag{9a} \\
\lambda_\mu(M_G) & \approx 3\lambda_s(M_G), \quad \text{Ref.}[9], \tag{9b} \\
\lambda_e(M_G) & \approx \frac{1}{3}\lambda_d(M_G), \quad \text{Ref.}[10], \tag{9c} \\
\lambda_t(M_G) & \approx \lambda_b(M_G) \approx \lambda_{\tau}(M_G), \quad \text{Ref.}[11]. \tag{9d}
\end{align*}
\]

In the rest of this section we discuss various aspects of Yukawa coupling evolution.

The theoretical requirement that Yukawa couplings remain perturbative throughout their evolution up to \( M_G \) places constraints on \( \tan \beta \). If we require that the ratio of 2-loop/1-loop contributions in the RGE remains less than 1/4, then

\[
0.6 \lesssim \tan \beta \lesssim 65. \tag{10}
\]

There is also an indirect perturbative constraint on the input parameter \( \alpha_3(M_Z) \), if we wish to have \( b-\tau \) Yukawa unification (Eq.9c). Since the \( g_3^2 \) and \( \lambda_t^2 \) terms enter the
\[ \frac{\lambda_b}{\lambda_r} \text{ RGE with opposite signs, an increase in } g_3 \text{ requires a compensating increase in } \lambda_t \text{ to maintain unification; see Fig. 4. To keep } \lambda_t \text{ perturbative requires}\]

\[ \alpha_3(M_Z) \lesssim 0.13. \quad (11) \]

As \( \mu \rightarrow m_t \), \( \lambda_t \) rapidly approaches an infrared fixed point \[13\] as shown in Figure 5. An approximate fixed-point solution for \( m_t \) is given by the vanishing of the one-loop terms on the right of Eq. (6a)

\[ -\sum c_i g_i^2 + 6\lambda_t^2 + \lambda_b^2 = 0. \quad (12) \]

Neglecting \( g_1, g_2 \) and \( \lambda_b, m_t \) is then predicted in terms of \( \alpha_s(m_t) \) and \( \beta \): \[7, 11, 12, 14, 15, 16\]

\[ m_t(m_t) \approx \frac{4}{3} \sqrt{2\pi \alpha_3(m_t)} \frac{v}{\sqrt{2}} \sin \beta \approx (192 \text{ GeV}) \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}}. \quad (13) \]

Thus the scale of the top-quark mass is naturally large in SUSY-GUT models but depends on \( \tan \beta \). Note that the propagator-pole mass is related to this running mass by

\[ m_t(\text{pole}) = m_t(m_t) \left[ 1 + \frac{4}{3\pi} \alpha_3(m_t) + \cdots \right]. \quad (14) \]

An exact numerical solution for the relation between \( m_t \) and \( \tan \beta \), obtained from the 2-loop RGEs for \( \lambda_t \) and \( \lambda_b/\lambda_r \), with \( \lambda_b(M_G) = \lambda_r(M_G) \) unification, is shown in Fig. 6 taking \( M_{\text{SUSY}} = m_t \). At large \( \tan \beta \), \( \lambda_b \) becomes large and the above fixed-point solution no longer applies. In fact, the solutions become non-perturbative at large \( \tan \beta \); our perturbative requirement \( (2\text{-loop})/(1\text{-loop}) \leq 1/4 \) leads to \( \lambda_t(M_G) \leq 3.3, \lambda_b(M_G) \leq 3.1 \) and \( \tan \beta \lesssim 65 \). For most \( m_t \) values there are two possible solutions for \( \tan \beta \); the lower solution is controlled by the \( \lambda_t \) fixed point, following Eqs. \[13\], \[14\]:

\[ \sin \beta \approx m_t(\text{pole})/(200 \text{ GeV}). \quad (15) \]

An upper limit \( m_t(\text{pole}) \lesssim 200 \text{ GeV} \) is found with these RGE solutions.

Fig. 4: Qualitative dependence of \( \lambda_t \) at the GUT scale on \( \alpha_3(M_Z) \) \[3\].

Fig. 5: The Yukawa coupling \( \lambda_t \) approaches a fixed point at the electroweak scale \[3\].

Fig. 6. Contours of constant \( m_b(m_b) \) in the \( (m_t(m_t), \tan \beta) \) plane \[3\].
Figures 3 and 6 show that there is the possibility of \( \lambda_t = \lambda_b = \lambda_\tau \) unification at \( M_G \); \( m_t \) and tan \( \beta \) must then be large.

In the presence of GUT threshold corrections, there may effectively be corrections to GUT relations like Eq. 9 \[12, 17, 18\]. Figure 7 shows the effects of small deviations of \( \lambda_b/\lambda_\tau \) from unity at \( \mu = M_G \). Large threshold corrections are only possible in the case that \( \lambda_b < \lambda_\tau \) due to the proximity of the Landau pole.

Fig. 7: GUT threshold corrections to Yukawa coupling unification \[\text{[7]}\].

4 Evolution of the CKM matrix

The CKM matrix comes from the mismatch between transformations that diagonalize the up-type and down-type quark mass matrices, arising from the matrices of Yukawa couplings. We can therefore define a running CKM matrix, with its own RGE, by diagonalizing the running mass matrices. The RGE become especially simple if we keep only the leading terms in the experimentally observed mass and CKM hierarchies, i.e. if we neglect \( \lambda_c/\lambda_t, \lambda_u/\lambda_c, \lambda_d/\lambda_b, |V_{ub}|^2 \) and \( |V_{cb}|^2 \) \[19, 20, 21\]. Then the only off-diagonal CKM elements that evolve are those connected to the third generation, i.e. \( V_{ub}, V_{cb}, V_{td}, V_{ts} \), and these all have the same RGE (to all loops):

\[
\frac{d|V_{Qq}|}{dt} = -\frac{|V_{Qq}|}{16\pi^2} \left[ \lambda_t^2 + \lambda_b^2 \right] \text{2-loop}. \tag{16}
\]

All other off-diagonal matrix elements have \( d|V_{Qq}|/dt = 0 \), while the diagonal elements remain \( \simeq 1 \) by unitarity. Hence the moduli of CKM elements have the scaling behaviour\[19, 20, 21\]

\[
|V_{CKM}|(\mu = M_G) = \left( \begin{array}{ccc} |V_{ud}| & |V_{us}| & \sqrt{S}|V_{ub}| \\ |V_{cd}| & |V_{cs}| & \sqrt{S}|V_{cb}| \\ \sqrt{S}|V_{td}| & \sqrt{S}|V_{ts}| & |V_{tb}| \end{array} \right)_{\mu = m_t} \tag{17}
\]

where \( S \) is a universal scaling factor. A small unitarity violation here is of sub-leading order in the mass/CKM hierarchy. Similarly, it can be shown that the rephase-invariant CP-violation parameter \( J = \text{Im}(V_{ud}V_{cs}V_{us}^*V_{cd}^*) \) scales as \[19, 20, 21\]

\[
J(\mu = M_G) = S J(\mu = m_t). \tag{18}
\]

Figure 8 shows how the universal scaling factor depends on tan \( \beta \) in typical cases. This approximate scaling property offers a quick and simple way to test GUT-scale hypotheses about mass- and CKM-matrices.

Fig. 8: Typical CKM scaling factor \( \sqrt{S} \) versus tan \( \beta \) \[20\].
GUT-scale Yukawa hypotheses usually take the form of assumed parametric forms, called “textures” for the quark and lepton mass matrices at $M_G$. For example, the SU(5) SUSY-GUT model of Ref.[9] postulates up-quark, down-quark and charged lepton mass matrices (Yukawa coupling matrices) of the forms

$$U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, \quad D = \begin{pmatrix} 0 & F & 0 \\ 0 & F' & E \\ 0 & 0 & D \end{pmatrix}, \quad E = \begin{pmatrix} 0 & F & 0 \\ 0 & F' & -3E \\ 0 & 0 & D \end{pmatrix}. \quad (19)$$

These immediately imply the relation\[9, 22, 23\]

$$|V_{cb}| = \sqrt{\lambda_c/\lambda_t}, \quad (20)$$

since $V_{cb}$ originates entirely from the $U$-matrix, and also

$$\lambda_\tau \simeq \lambda_b, \quad \lambda_\mu = 3\lambda_s, \quad \lambda_e = \frac{1}{3}\lambda_d, \quad \text{(all at } \mu = M_G). \quad (21)$$

The DHR model [15, 22] introduces changes by putting

$$U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, \quad D = \begin{pmatrix} 0 & F e^{i\phi} & 0 \\ F e^{-i\phi} & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad E = \begin{pmatrix} 0 & F & 0 \\ 0 & F & -3E \\ 0 & 0 & D \end{pmatrix}, \quad (22)$$

when the left and right down quarks and charged leptons appear in the same multiplet such as the 16 of SO(10). Other phases can be rotated away. The ADHRS models [24]

$$U = \begin{pmatrix} 0 & \frac{1}{27}C & x_u B \\ \frac{1}{27}C & 0 & x_u B \\ 0 & x_u B & A \end{pmatrix}, \quad D = \begin{pmatrix} 0 & C & 0 \\ C & E e^{i\phi} & x_d B \\ 0 & x_d B & A \end{pmatrix}, \quad E = \begin{pmatrix} 0 & C & 0 \\ 0 & 3 E e^{i\phi} & x_e B \\ 0 & x_e B & A \end{pmatrix}, \quad (23)$$

have fewer zeros in $D$ and $E$ but retain SO(10)-type relations $|V_{cb}| = \chi \sqrt{\lambda_c/\lambda_t}$ at $\mu = M_G$, where $\chi$ is a (discrete) Clebsch factor. These models are more predictive since they relate the up-quark Yukawa matrix to the down-quark Yukawa matrix resulting in two fewer continuous parameters.

In order to test the prediction $|V_{cb}| = \sqrt{\lambda_c/\lambda_t}$ at $\mu = M_G$, we can proceed as follows.

(i) Start with low-energy input, $m_t(m_t), m_b(m_b), m_c(m_c)$;

(ii) evolve Yukawa couplings up to $\mu = M_G$;

(iii) when $\lambda_b(M_G) = \lambda_\tau(M_G)$ is satisfied, construct $|V_{cb}(M_G)|$ and evolve it down to $|V_{cb}(m_t)|$, and compare with experiment.

6
Figure 9 shows contours of $|V_{cb}(m_t)|$ in the $(m_t, \tan \beta)$ plane, for MSSM GUT solutions with the $b$-$\tau$ Yukawa unification constraint and various $m_c(m_c)$ input choices. The region of $b$-$\tau$-$t$ Yukawa unification is indicated. The relation Eq. (20) leads to a lower bound\[3\]

$$|V_{cb}(m_t)| \geq 0.043(200 \text{ GeV}/m_{t\text{pole}})^{1/2}. \quad (24)$$

Fig. 9: Typical contours of $|V_{cb}(m_t)|$ \[3\].

Figure 10 shows the effects of threshold corrections and/or a group theory Clebsch factor on the GUT scale relation. These effects are parametrized by the factor $X$ in the revised unification criteria $|V_{cb}| = X \sqrt{\lambda_c/\lambda_t}$ at $M_G$.

Fig. 10: Effects of threshold corrections or a Clebsch factor\[24\] on $|V_{cb}| = \sqrt{\lambda_c/\lambda_t}$ unification \[3\].

Some relations between quark masses and CKM mixing angles are satisfied under more general assumptions. For example, the GUT scale relationships $|V_{ud}/V_{ud}| = \sqrt{m_u/m_c}$ and $|V_{td}/V_{ud}| = \sqrt{m_d/m_s}$ have been shown to pertain to a whole class of unification scenarios\[25\].

5 Implications for SUSY Higgs searches

The $\lambda_t$ fixed-point solutions have interesting implications for the phenomenology of Higgs bosons in the MSSM. Recall that there are 5 Higgs bosons in this model\[20\]: neutral CP-even $h$ and $H$ ($m_h < m_H$), neutral CP-odd $A$, and charged $H^\pm$. At tree level there are just two parameters, usually taken to be $m_A$ and $\tan \beta$ and a mass bound $m_h < M_Z$, but large one-loop radiative corrections (principally depending on $m_t$ and the mean $t$-squark mass $m_{t\tilde{t}}$) affect the Higgs masses and couplings and push up the $m_h$ bound to

$$m_h^2 < M_Z^2 + \frac{6G_F m_t^4}{\sqrt{2}\pi^2} \ln(m_t/m_{\text{pole}}). \quad (25)$$

Studies of MSSM Higgses usually refer to the $(m_A, \tan \beta)$ parameter plane, taking a range of $m_t$ and assuming $m_{\tilde{t}} \sim 1$ TeV.

Several groups\[27, 28, 29, 30\] have systematically discussed the potential of present and future colliders to discover one or more MSSM Higgses. LEP I has already excluded part of the $(m_A, \tan \beta)$ plane; LEP II will cover more but not all. SSC/LHC offer new search possibilities, but there remains a region $m_A \sim 100–150$ GeV, $\tan \beta \gtrsim 5$ where apparently no MSSM Higgs signals whatever would be detectable (unless high-performance $b$-tagging\[31\] and rapidity gap searches\[32\] can succeed).
It is therefore very interesting to find any theoretical arguments why this inaccessible region may be forbidden. The $\lambda_t$ fixed-point solutions provide a possible argument if $m_t \lesssim 160$ GeV, since these solutions are then constrained to a range of small $\tan \beta$ as shown in Fig. 6, with $m_t$ and $\tan \beta$ directly correlated via Eq. (15) \cite{7}. With this correlation, the $(m_A, \tan \beta)$ region excluded by 1992 LEP I searches is shown in Fig. 11(a) (assuming $m_t \simeq 1$ TeV). The corresponding region in the $(m_h, \tan \beta)$ plane is shown in Fig. 11(b), where the left-hand boundary comes from LEP I data and the right-hand boundary comes from internal MSSM constraints with one-loop corrections. We see that the lower limit on $m_h$ is about 60 GeV (close to the 61 GeV SM Higgs limit with these data), while the input assumption $m_t < 160$ GeV implies that $m_h \lesssim 85$ GeV — within reach of LEP II searches. The other Higgses are $m_A \gtrsim 70$ GeV, $m_{H^\pm} \gtrsim 105$ GeV, $m_H \gtrsim 140$ GeV. In principle, $A$ too might be discoverable at LEP II via $e^+e^- \to Ah$ production, but in fact there is only a small parameter region where the cross section would be big enough; the other production channels $AH, ZH, H^+H^-$ are kinematically inaccessible.

Fig. 11: $\lambda_t$-fixed-point solution regions allowed by the LEP I data: (a) in the $(m_A, \tan \beta)$ plane, (b) in the $(m_h, \tan \beta)$ plane. The top quark masses are $m_t$(pole), correlated to $\tan \beta$ by Eq. (15) \cite{7}.

Thus this range of $\lambda_t$ fixed-point solutions implies that we shall not have to wait for SSC/LHC to discover a MSSM Higgs boson. What more will be detectable there? Figure 12 shows the limits of detectability for the principal SSC/LHC signals, in the $h \to \gamma\gamma$, $H \to \ell\ell\ell\ell$, $A \to \gamma\gamma$ and $H^\pm \to \tau\nu$ channels. Depending on $m_A$ and $\tan \beta$, we see there could be good chances to discover one or more additional Higgses, though not all of them at once; but there also exists a parameter region where no Higgs signals whatever would be expected at SSC/LHC \cite{7}.

Fig. 12: SSC/LHC signal detectability regions, compared with the LEP I allowed region of $\lambda_t$-fixed-point solutions from Fig. 11(a) and the probable reach of LEP II. The top quark masses are $m_t$(pole) \cite{7}.

Possible future $e^+e^-$ linear colliders with energies above LEP II offer interesting further possibilities, however. The principal neutral-Higgs production channels are

$$e^+e^- \to Zh, Ah, ZH, AH$$

$$e^+e^- \to \nu\nu h, \nu\nu H, e^+e^-, e^+e^- H.$$

Here the two-body cross sections fall with $1/s$ while the others (WW and ZZ fusion) rise logarithmically. Now the $Z^* \to ZH, Ah$ plus $WW, ZZ \to H$ rates are all suppressed by a factor $\cos^2(\beta - \alpha)$, where $\alpha$ is a CP-even mixing angle; in the $\lambda_t$ fixed-point solutions, $\cos^2(\beta - \alpha) < 0.3$ (0.05) for $m_t < 160$ GeV (145 GeV). However, the
remaining $Z \to Zh, AH$ plus $WW, ZZ \to h$ rates contain the complementary factor $\sin^2(\beta - \alpha)$ and are unsuppressed, while the charged-Higgs process

$$e^+e^- \to H^+H^-$$

has no such factors. Copious $h$ production is therefore guaranteed, with $H, A, H^\pm$ too if they are not too heavy.

6 Summary

a) The success of SUSY GUTS in gauge coupling unification is tantalizing.

b) Yukawa coupling possibilities ($\lambda_b \simeq \lambda_\tau$, etc) are equally attractive.

c) The constraint $\lambda_b(M_G) = \lambda_\tau(M_G)$ leads to a narrow corridor in the plane of $\tan \beta$ and $m_t^{\text{pole}}$.

d) $\lambda_t$ fixed-point solutions with $\alpha_s(M_Z) = 0.118$ predict $\sin \beta \simeq m_t^{\text{pole}}/(200 \text{ GeV})$ or $\tan \beta$ large.

e) Perturbativity at the GUT scale implies several constraints: $m_t^{\text{pole}} \lesssim 200 \text{ GeV}$ (for $\alpha_s(M_Z) = 0.118$), $\alpha_s(M_Z) \lesssim 0.13$, $\tan \beta \lesssim 65$.

f) A simple scaling law connects CKM matrix elements at $\mu = m_t$ and $M_G$.

g) GUT textures give interesting low-energy predictions; e.g. $|V_{cb}(M_G)| = \sqrt{\lambda_c(M_G)/\lambda_t(M_G)}$ gives $|V_{cb}(m_t)| > 0.043(200 \text{ GeV}/m_t^{\text{pole}})^{1/2}$.

h) Threshold effects at the GUT scale may not be negligible.

i) $\lambda_t$ fixed-point solutions imply that $m_t \gtrsim 130 \text{ GeV}$ and the lightest MSSM Higgs mass $m_h \gtrsim 60 \text{GeV}$; if in fact $m_t \lesssim 160 \text{ GeV}$, then $m_h \lesssim 85 \text{ GeV}$ and $h$ will be discoverable at LEP.

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