Students' Mathematical Representation Ability in Learning Algebraic Expression using Realistic Mathematics Education

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Abstract. Mathematical representation ability (MRA) is crucial in solving mathematical problems, particularly in the topic of algebraic expressions. However, some students struggle to grasp algebraic forms in real-world circumstances. Learning through realistic mathematics education using context is one of the learning approaches that could help improve students’ MRA. This study aims to analyze students' MRA in learning algebraic expressions through realistic mathematics education. This research employed mixed-method research with a sequential explanatory design. The subjects of this study were 27 seventh-grade students from a junior high school in Banda Aceh, Indonesia. Data on students' MRA were gathered through written tests and task-based interviews. The descriptive analysis of written test data revealed that students' average score is in a low category. The majority of students (n=24) satisfied the visual indicators of MRA as they successfully drew the composite shape. Whereas the most challenging MRA aspect for students was symbolic representation, only a few students (n=6) could solve the problem related to the multiplication of algebraic expressions. Students’ learning loss in the prerequisites due to online learning during the Covid-19 outbreak was one cause of the students' low performance on MRA.

Keywords: mathematical representation, algebraic expressions, realistic mathematics education.

Introduction

Algebra is one of the mathematics topics important to everyday life since it significantly impacts decision-making (Usiskin, 1995). Furthermore, because algebra is viewed as a gateway to abstract thinking (Witzel, Mercer, & Miller, 2003), algebraic knowledge and comprehension are essential for many fields of mathematics.

The algebra topic begins with simple arithmetic and progresses to more abstract algebraic processes, making it difficult for students (Baroudi, 2006; Sarımanoğlu, 2019). Several studies have found that students with low algebraic understanding experience major difficulties when they begin to study mathematics at a higher level (Brandell, Hemmi, & Thunberg, 2008; Hiebert et al., 2005). Furthermore, according to the Trends in International Mathematics and Science Study (TIMSS) data, algebra is challenging for students in many nations (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2014). Most students have difficulty understanding basic algebraic forms, particularly the function of variables contained in algebraic expressions (Rudyanto,
Marsigit, Wangid, & Gembong, 2019). Panasuk and Beyranevand (2011) refer to conceptual understanding in algebra as recognizing functional linkages between known and unknown variables, independent and dependent variables, and differentiating and interpreting various representations of algebraic concepts.

Students use representations to support their mathematical understanding by constructing abstract ideas such as algebraic concepts into concrete ideas using logical thinking (Goldin, 2020); therefore, representation is essential in mathematics learning (Cai & Lester Jr, 2005). Mathematical representations themselves can be portrayed in both visual (charts, tables, sketches/drawings, and diagrams) and non-visual (mathematical equations and models) forms (Minarni, Napitupulu, & Husein, 2016; Thompson & Chappell, 2007). Therefore it could help teachers and students in comprehending abstract mathematical concepts (Roubiček, 2006), particularly in expressing mathematical ideas (Bal, 2015) as well as comprehending and interpreting mathematical concepts (Pape & Tchoshanov, 2001). Mathematical concepts should be represented in a basic form so that people may comprehend and use them (NCTM, 2000). Therefore, representation is a tool for communicating ideas about mathematical problem-solving. Additionally, representations can help and promote judgments (Pape & Tchoshanov, 2001). Hence, students' mathematical representation ability (MRA) must be developed.

Through their research, Fauzan, Musdi, and Yani (2018) revealed that realistic mathematics education (RME) positively influences students’ MRA on the ratio topic. In line with Freudenthal's idea that mathematics is human activity (Freudenthal, 1991), RME is a theory about mathematics learning oriented to the mathematization process of daily activities. RME starts with real context from students' perspectives (Gravemeijer, 1994). Thus, students should learn mathematics from realistic situations while also mathematizing their own mathematical activities (Rasmussen & King, 2000). The principal purpose of RME is to make mathematics learning more enjoyable and valuable for students by providing real problems (Laurens, Batlolona, Batlolona, & Leasa, 2018). RME is also part of a classroom learning activities sequence to increase students' ability to perceive topics they have not learned by involving them in the learning process (Palinussa, 2020).

Several studies on students' MRA have been conducted, including the research of Fitrianna, Dinia, Mayasari, and Nurhaffifah (2018). They investigated students' difficulties in solving mathematical representation problems based on students' mathematical dispositions. Further, the research of Supandi, Waluya, Rochmad, Suyitno and Dewi (2018) examined the effect of the think-talk-write strategy and the expository strategy on students' MRA to comprehend the relationship between mathematical representation and self-efficacy. However, research on students' mathematical representation abilities and students’ errors in the topic of
algebraic expressions using the RME approach is limited. Therefore, the research question in this study is “to what extent are the students’ mathematical representation abilities in learning algebraic expressions through realistic mathematics education?”

**Method**

The present study employed a mixed-method approach with a sequential explanatory design (Creswell & Creswell, 2018) because the qualitative data obtained through task-based interviews explores and enriches quantitative data obtained from the written test. The participants of this research were 27 seven-grade students from a junior high school in Banda Aceh, Indonesia.

The learning activities to develop students’ mathematical representation abilities were organized into six meetings, i.e., five meetings of 2×30 minutes and one meeting of 2×40 minutes. This research was carried out when the Covid-19 pandemic was still ongoing. Therefore, the learning activities in the first five meetings were carried out using a shift method; half of the students in one class learned mathematics one day while the other half learned another day because they have to keep social distance and the maximum room capacity was 50%.

Thus, each student only studied mathematics for 2 × 30 minutes a week. The learning design is presented in Table 1.

| Topics                               | Meeting No | Students Activities |
|--------------------------------------|------------|---------------------|
| Introduction to algebraic expressions| 1          | Classroom activities |
|                                      |            | 1. Given the information about the age gap between a father and his son. Students were asked to identify the possible combinations of father's and son's ages. |
|                                      |            | 2. Play math games about weight scales. |
|                                      |            | Group activities |
|                                      |            | 3. Solve problems related to weight scales. |
|                                      |            | 4. Identify possible combinations of integers added up to 10. |
|                                      | 2          | 5. Find the possible total number of balls inside closed different-size baskets. |
|                                      |            | 6. Solve problems related to dividing rectangular land with unknown size |
|                                      |            | 7. Find the total number of balls inside closed different-size baskets if x represents the number of balls in a small basket and y represents the number of balls in a large basket. |
|                                      |            | 8. Find the area of rectangles with size involves variables and numbers |
|                                      |            | 9. Write down the meaning of variables and give other coefficients and constant examples. |
| Addition and subtraction of algebraic expressions | 3          | 1. Given two boxes consisted of several jars of cookies. Students were asked to determine the total number of cookies in the boxes using variables as the unknown number of cookies in large and small jars. |
|                                      |            | 2. Using two strategies, find the total area of composite shapes that size involves variables and numbers. |
3. Using two strategies, find the total area of more complex composite shapes whose size involves variables only.
4. Sketch a composite shape with unknown sizes and then determine its total area in variables.
5. Add bricks that represent variables.
6. Practice addition and subtraction of algebraic expressions using algebraic pyramids.
7. Explore the difference between \(2 + y\) and \(2y\) also \(2x + y\) and \(2xy\) using visual representation.
8. Solve formal problems (without visual representation) involving addition and subtraction of algebraic expressions.
9. Conclude the rules to simplify algebraic expressions involving addition and subtraction.

Multiplication of algebraic expressions

1. Given three boxes consisting of several jars of an unknown number of cookies (the number of jars and cookies is similar). Students were asked to determine the total number of cookies in the boxes using the concept of multiplication as repeated addition.
2. Find the total area of shapes that size involves adding variables and numbers, such as \(y + 3\) and \(x\).
3. Find the area of the shaded region of composite shapes using two strategies involving subtraction of algebraic expression.
4. Rewrite the answers obtained from the second and the third activities in a table.
5. Find the area of composite shapes with the help of multiplication tables.
6. Draw rectangles that represent the multiplication of one term and two terms or two terms and two terms of algebraic expression.
7. Solve formal problems involving the multiplication of algebraic expressions.
8. Conclude the rules to multiply algebraic expressions.

Note: A more detailed explanation of the learning design of the introduction of algebraic expressions has been published in Khairunnisak, Johar, Yuhasriati, Zubainur, Suhartati, and Sasalia (2021)

The three principles of RME introduced by Gravemeijer (1994), i.e. guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed model, were implemented during the lesson. To understand the abstract concept of the algebraic expression, the learning activities were designed in stages through the situational, model-of, model-for, and formal knowledge levels (Gravemeijer, 1994). For example, as presented in Table 1, the third meeting employed the activity to find the area of shapes at the situational level. This activity was a "real problem" for the students because they had already been taught it in elementary school and the previous semester. In the model-of level, the students were then invited to use their own model to represent the addition and subtraction of the algebraic expressions. In the fourth meeting, the model-for level, students were encouraged to explore the different meanings of \(2 + y\), \(2y\), \(2x + y\), and \(2xy\) using visual representation. This activity aimed to overcome the common misconception concerning the addition of algebraic expressions. In the last level, the formal
knowledge level, the students were expected to comprehend the addition of algebraic expressions and to understand the rule to simplify algebraic expressions involving addition and subtraction. An iceberg visualizing the second and third learning designs is displayed in Figure 1.

Figure 1. Iceberg of the second and third meetings of learning algebraic expression
After six meetings of learning activities using RME on algebraic expression topics, the participants of this study took an MRA test. The researcher then interviewed several selected students based on their test scores and their errors.

The instruments in this study were written test and interview guideline designed by the research team. The test, which intended to assess students' algebraic representation skills after studying with the RME approach, had been classified as valid by eight validators (five mathematics education lecturers and three junior high school mathematics teachers), with an average score of 4.54. The indicators of mathematical representation used in the test were: 1) using numbers and symbols to solve everyday life problems; 2) creating and using representations in various visual forms such as graphs, tables, and diagrams; and 3) selecting, applying, and interpreting mathematical representations in problem-solving or, in other words, verbal representation ability (Mainali, 2021). The problems in the MRA test are presented in Table 2.

Table 2. Problems in the MRA test

| Item No | Aspects of MRA | Questions |
|---------|----------------|-----------|
| 1       | Visual - Symbolic | Find the total area of the following shape using two strategies. |
|         | ![Shape Diagram] |           |
|         | a. Strategy 1: … |           |
|         | b. Strategy 2: … |           |
| 2       | Verbal - Visual | Sketch a composite shape whose sizes are partially written in variables. |
|         | Visual - Symbolic | Find its total area |
| 3       | Verbal - Symbolic | Statement: "The sum of two natural numbers is 8" Rewrite the statement using variable(s). |
| 4       | Symbolic - Symbolic | Solve the following problem. |
|         | ![Multiplication Diagram] |           |
|         | a. \(9x + 6 - 2x + 3y = \ldots\) |           |
|         | b. \(6(4+y) = \ldots\) |           |
|         | c. \((x+1)(x+2) = \ldots\) |           |

The data were analyzed descriptively using the scoring rubric presented in Table 3. After computing the average score of student learning outcomes, the student's answers were then categorized based on indicators of mathematical representation. Furthermore, an interview involving several students with unique test answers was conducted to identify the students' challenges in learning the algebraic expressions with the RME approach.
Table 3. MRA test scoring guidelines

| Item No | Aspects of MRA       | Student answers                            | Score |
|---------|----------------------|--------------------------------------------|-------|
| 1a & 1b | Visual - Symbolic    | Complete and correct answer                | 1     |
|         |                      | Correct solution steps but incorrect answer| 0.5   |
|         |                      | Incorrect solution steps and incorrect answer| 0     |
| 2a      | Verbal - Visual      | Complete drawing the composite shapes      | 1     |
|         |                      | Draw only one shape                        | 0.5   |
|         |                      | No answer                                   | 0     |
| 2b      | Visual - Symbolic    | Complete and correct answer                | 1     |
|         |                      | Correct solution steps but incorrect answer| 0.5   |
|         |                      | Incorrect solution steps and incorrect answer| 0     |
| 3       | Verbal - Symbolic    | Correct answer                              | 1     |
|         |                      | Incorrect answer                            | 0     |
| 4a, 4b, 4c | Symbolic - Symbolic | correct answer                             | 1     |
|         |                      | Incorrect answer                            | 0     |

Results and Discussion

The MRA test was administered after six meetings of learning algebraic expressions utilizing the RME approach. Table 4 summarizes the students' test results.

Table 4. Students' scores on the MRA test

| No. | Subject Code | Item No | Total Score |
|-----|--------------|---------|-------------|
|     |              | 1       | 2           | 3           | 4           |         |
|     |              | a       | b           | a           | b           | c         |         |
| 1   | S1           | 1       | 1           | 1           | 1           | 1         | 1         | 0         | 0         | 6         |         |
| 2   | S2           | 0       | 0           | 0           | 0           | 0           | 0         | 0         | 0         | 0         |         |
| 3   | S3           | 0       | 0           | 1           | 1           | 0           | 1         | 0         | 0         | 3         |         |
| 4   | S4           | 0       | 0           | 0           | 1           | 0           | 1         | 0         | 0         | 2         |         |
| 5   | S5           | 0       | 0           | 1           | 0           | 1           | 0         | 0         | 1         | 3         |         |
| 6   | S6           | 0       | 0           | 1           | 0           | 0           | 0         | 0         | 0         | 1         |         |
| 7   | S7           | 1       | 1           | 1           | 1           | 0           | 1         | 0         | 1         | 6         |         |
| 8   | S8           | 0.5     | 0.5         | 1           | 1           | 1           | 1         | 0         | 1         | 6         |         |
| 9   | S9           | 0       | 0           | 1           | 1           | 0           | 0         | 0         | 1         | 3         |         |
| 10  | S10          | 0       | 0           | 1           | 1           | 0           | 1         | 0         | 0         | 3         |         |
| 11  | S11          | 0       | 0           | 1           | 0           | 0           | 0         | 0         | 0         | 1         |         |
| 12  | S12          | 1       | 1           | 1           | 1           | 1           | 1         | 1         | 8         |         |
| 13  | S13          | 0       | 0           | 1           | 0           | 0           | 1         | 0         | 1         | 3         |         |
| 14  | S14          | 0       | 0.5         | 1           | 0           | 0           | 1         | 1         | 1         | 4.5       |         |
| 15  | S15          | 0       | 0           | 0.5         | 0           | 0           | 0         | 0         | 0         | 0.5       |         |
| 16  | S16          | 0       | 0           | 1           | 0           | 0           | 0         | 0         | 0         | 1         |         |
| 17  | S17          | 1       | 1           | 1           | 1           | 1           | 0         | 0         | 1         | 6         |         |
| 18  | S18          | 1       | 0           | 1           | 1           | 0           | 1         | 0         | 0         | 4         |         |
| 19  | S19          | 0       | 0           | 1           | 1           | 0           | 0         | 0         | 2         |         |
| 20  | S20          | 0       | 0           | 1           | 1           | 1           | 1         | 1         | 6         |         |
| 21  | S21          | 1       | 1           | 1           | 1           | 1           | 1         | 1         | 8         |         |
| 22  | S22          | 0       | 0           | 1           | 0           | 0           | 0         | 0         | 0         | 1         |         |
| 23  | S23          | 0       | 0           | 1           | 1           | 1           | 0         | 0         | 0         | 3         |         |
| 24  | S24          | 0       | 0           | 1           | 0           | 0           | 1         | 0         | 1         | 3         |         |
| 25  | S25          | 0       | 0           | 1           | 1           | 1           | 1         | 1         | 6         |         |
| 26  | S26          | 0.5     | 1           | 1           | 1           | 0           | 0         | 1         | 0         | 4.5       |         |
| 27  | S27          | 1       | 0           | 0.5         | 0           | 0           | 0         | 0         | 0         | 1.5       |         |

Average Score 3.6

Note: Ideal score is 8
Table 4 reveals that the average score of the students is 3.6, which is regarded as low compared to the ideal score of 8. Only two students (7.4%) received the full score of 8, while 22.2% of students had a score of 6, and the majority of students (more than 70%) received a score of less than 6. Furthermore, students’ MRA test scores for each problem are illustrated in Figure 2.

As depicted in Figure 2, the problem that most students could solve was problem number 2a, which assessed the visual aspect of MRA, where 24 out of 27 students (88%) were able to sketch a composite image, which size was partially written in the variables. On the other hand, only six students (22.2%) could correctly answer problems 1b and 4b. It indicated that the symbolic aspect of MRA was the least comprehended by the students.

**Students’ verbal-visual representation ability**

Problem 2a asked students to draw a composite shape, which is a combination of some 2D shapes. S15's response (see Figure 3) revealed that S15 did not understand the meaning of the composite shape; rather than drawing a composite shape that combined more than one shape, S15 drew a rectangle. Furthermore, the other drawing given by the students (as provided by S12, S27, S6, and S14) depicted a composite shape, as well as the number and variables involved, that was relatively identical to the problem given in the student worksheet or the MRA test. The constituent shapes could be observed through the pictures drawn by S12, S6, and S14. On the other hand, S27 sketched a shape in which we could not detect the constituent shapes immediately. During the interview, S27 stated that the shape is made up of the top and bottom shapes.
The students’ response to problem number 2a suggests that they had good visual representation ability, even though the representation, numbers, and symbols utilized were almost identical to prior shapes provided during the learning process and the MRA test. Furthermore, as earlier mentioned, the visual representation was mastered by more students than the other two aspects of mathematical representation. However, when students were asked to determine the area of the composite shape they had drawn, the other case happened.

**Students’ visual-symbolic representation ability**

Among the 24 students who provided the correct visual representation of the composite shape, only 15 students could correctly determine its area. Some of the students' answers in determining the area of their composite shape are portrayed in Figure 2. S12 was one student who could represent the composite shape area he had drawn correctly. However, even though the shape consisted of three rectangular, S12 considered it two shapes: the top and the bottom. Thus, he found the area of the composite shape by adding the area of the two shapes, $(y)(2) + (3)(x)$.

On the other hand, S6 failed to represent the area of his drawing correctly. During the interview, he said, "Here [in his drawing], there are 2, 3, and x. Figure number one also like
that” S6 considered his composite shape similar to the composite shape in the problem number 1 in the MRA test; thus, the answer also should be similar. Another student, S15, drew a rectangle, and although he knew the area formula for a rectangle is “length times width,” S15 failed to represent the shape correctly. Rather than writing the answer as $8x$, S15 wrote it as $8x + y$. The interview revealed that S15 added the variable $y$ because question number 1 consists of two variables, $x$ and $y$. Thus, she also wrote the area of his drawing as $8x + y$. S14’s composite shape also formed a rectangle, and he also knew the area formula of a rectangle. However, even though S14 knew that the rectangle's length and width are $x$ and $y$, respectively, S14 said that the area of his drawing was $2x + xy$ because, in his shape, he had $2$, $x$, and $y$. Further, S14 thought $2x + xy$ was different from $(2.x) + (x.y)$ as he wrote them as two different representations of the shape’s area.

A smaller number (less than 25%) of students could answer the other problem assessed visual-symbolic representation ability, problem number 1. Some of the students’ answers are presented in Figure 4.

It shows that S12 could calculate the area of the given composite shape by dividing it into two shapes, the upper and the bottom, and then adding the area of the two shapes.

S24 wrote that the area is the addition of $2x$ and $3y$. According to the interview, she just noticed the numbers and variables listed on the shape. However, S14 wrote $(2 + x)(3 + y)$.

S14 explained her answer during the interview as follows.

\[ R : \text{In question number 1, how many shapes are there?} \]
\[ S14 : \text{Three.} \]
\[ R : \text{What shapes formed the composite shape?} \]
\[ S14 : \text{The first is a rectangle, the second is a square, and the third is a rectangle.} \]
\[ R : \text{Okay. The question is about the area. What is the formula for calculating a rectangle's area?} \]
\[ S14 : \text{Length times width.} \]
\[ R : \text{How about the area of a square?} \]
\[ S14 : \text{Side times side.} \]
\[ R : \text{Then, where did the } 2 + x \text{ and } 3 + y \text{ come from?} \]
\[ S14 : \text{I calculated the area of all the objects; the length is } 2, \text{ plus } x, \text{ then multiplied by the width } 3 + y. \]
\[ R : \text{What is the difference with this picture [the following figure]?} \]

```
\[ \begin{array}{c}
\text{2} \\
\hline
\text{x} \\
\hline
\text{3} \\
\text{y}
\end{array} \]
```

\[ S14 : \text{Oh, yes. It is different. However, I do not know how to write the area.} \]

The above interview indicated that S14 realized that the composite shape consists of three shapes, a rectangle, a square, and another rectangle. However, S14 made a mistake in determining
its area. According to S14, the area of the composite shape is \((2 + x)(3 + y)\) because the length is \((2 + x)\) and the width is \((3 + y)\). It should be \((2 + x)(3 + y) - x \cdot y\). S14 just realized that her answer was incorrect when the researcher showed another composite shape, which area is \((2 + x)(3 + y)\). However, S14 still couldn’t determine the correct area.

![Figure 4. Students’ answer to problem number 1a](image)

In contrast, although S5 also considered the area of the composite shape to be the sum of the area of its compound shapes, S5 provided a different result. S5 wrote that the total area of the composite shape should be calculated by adding \(3x\) and \(2y\), as depicted in the following excerpt.

\[
\begin{align*}
R &: \text{In question number 1, how many shapes are there?} \\
S5 &: \text{Three.} \\
R &: \text{Could you tell me the three shapes?} \\
S5 &: \text{Quadrilateral, parallelogram. The first is a square, the second is also a square, and the third is a square.} \\
R &: \text{The question was to find the area. Where did } 3x + 2y \text{ come from?} \\
S5 &: \text{The area.} \\
R &: \text{What area?} \\
&\quad \text{[Pointing at the figure on the MRA test]} \\
S5 &: \text{So, do you mean the total area of the figure is } 3x + 2y? \\
R &: \text{Yes.}
\end{align*}
\]

**Students’ verbal-symbolic representation ability**

Problem number 3 of the MRA test expected students to convert the language "*The sum of two natural numbers is 8*" into a mathematical sentence using algebraic expression. However, as shown in Figure 2, less than half of the students \((n=10, 37\%)\) could provide the correct response; they could not provide a symbolic representation of the word statement. Figure 5 displays some of the inaccurate answers given by students to problem number 3.

As presented in Figure 5, S15 represented the statement as \(8y + 9\). In the interview, S15 said that it should be \(8 + 6\). S15 said that she could not understand the statement. Further, when the researcher asked about two numbers adding up to 8, S15 replied \(6 + 2\), but she stated that she did not remember the meaning of a variable. Another student, S22, understood the meaning of sum as "+" and utilized \(x\) as the variable. However, he had a misunderstanding regarding the operation of algebraic expressions because he wrote \(8 + x = 8x\). He considered the (+) symbol
as an invitation to do something (Chow & Treagust, 2013). In addition, he wrote $3x + 5x = 8x$, which was a correct operation but an incorrect expression for the statement required since he was confused about the variables. Students showed that they do not understand the meaning of symbols and variables on the topic of algebraic expressions themselves (Davidenko, 1997; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005).

Another student, S2, misinterpreted the terms "sum up" and "multiplied". This ambiguity, however, was cleared up throughout the interview. When questioned, "Are you certain about your response? Try to grasp it well. Did you respond with multiplication or addition?" the student revised the answer to $a + b = 8$.

**Students’ symbolic-symbolic representation ability**

The fourth problem in the MRA test involved the operations of algebraic expressions. Among the three items of problem number 4, item number 4b was the easiest problem; almost half of the students ($n=21$) could solve the problem. Figure 6 depicts some of the students’ answers.

As presented in Figure 6, the answer given by S5 and S18 were almost similar. They were correctly multiplying the 6 with the 4; however, their understanding was incomplete. According to Yasseen, Yew, & Meng (2020), both S5 and S18 showed conceptual errors. The following interview excerpt shows S5 conceptual errors.

| R     | How do you get $24y$?                |
|-------|--------------------------------------|
| S5    | I multiplied 6 by 4, then put $y$. Thus, I got $24y$ |
| R     | Then, what is the result of 6$y$?   |
| S5    | $10y$                                    |
| R     | How about the result of $6y + 4y$?  |
| S5    | It is $10y^2$, isn’t it?               |
Regarding problem 4a, the number of students who provided correct answers \((n=13)\) was nearly equal to that of students who provided incorrect answers \((n=14)\). Figure 7 shows some of the students’ answers to problem 4a of the MRA test.

![Figure 7. Students’ answer to problem number 4a](image)

As exposed by Figure 7, S5 gave an inaccurate response by writing the result of \(9x - 2x\) as \(-7x\) instead of \(7x\). S5 claimed that because the number 2 had a negative sign in front of it, the answer should include one as well, as shown in the following excerpt.

\[
\begin{align*}
R &: \text{ How do you get } 6 - 7x + 3x? \text{ Where did } -7x \text{ come from?} \\
S5 &: \text{ } 9x - 2x = 7x, \text{ and I added minus because there is the negative symbol in front of the number 2} \\
R &: \text{ So, the result of } 9x - 2x \text{ is } 7x. \text{ Why did you write the minus sign in front of } 7x? \\
S5 &: \text{ Yes, the result is } 7x, \text{ but because there is the negative symbol in front of } 2, \text{ the result becomes } -7x.
\end{align*}
\]

Further, another student, S19, also provided an incorrect answer. In the interview, S19 stated that he calculated all of the numbers, 9 plus 6 minus 2 plus 3, and then merged them with the variables. According to Thomas & Tall (1991) and Jupri et al. (2014), S19 had a parsing obstacle as he could not decompose the process order of the algebraic expressions.

Problem number 4c consisted of a more complex operation of algebraic expressions, and more students provided the incorrect answer \((n=15)\) than those who gave the correct answer (\(n=12\)). Some of the students’ incorrect answers are presented in Figure 8 showing students’ conception errors in the multiplication of algebraic expressions.

![Figure 8. Students’ answers to problem number 4c](image)

The current study intends to examine students’ MRA in learning algebraic expressions through the RME approach. Following the findings of Fitrianna et al. (2018) and Khairunnisak et al. (2021), the participants in this study were more likely to be able to answer problems incorporating visual representation than the other two aspects of MRA. The problem of assessing symbolic representation, on the other hand, was the most difficult for the participants to solve (Aziz, Pramudiani, & Purnomo, 2017; Sari, Darhim, & Rosjanuardi, 2018), mainly when the problem involved the multiplication of algebraic expressions. Students’ performance on the
problem that required them to represent verbal language into mathematical sentences was also poor. The participants of Marpa's study uttered that translating mathematical sentences into mathematical symbols or equations is very difficult because it requires comprehension and analysis (Marpa, 2019).

As previously described, the average score of students' MRA test was deficient, which corresponds to the statement of Greenes and Rubenstein (Greenes & Rubenstein, 2008). Many algebraic problems are challenging for students because solving them may require understanding the conceptual aspects of fractions, decimals, negative numbers, equivalence, ratios, percentages, or rates (Norton & Irvin, 2007; Stacey & Chick, 2004; Stacey & MacGregor, 1994).

The students who participated in this study learned algebraic expression through the RME approach. It was argued that the RME approach would provide students with numerous opportunities to improve their mathematical representation (Fauzan et al., 2018). Following the RME approach, this study provided real situations related to objects on weighing scales and in closed baskets, (see the learning design in Khairunnisak et al., 2021), to introduce the “variable” term. The lesson also aligned with the intertwinement principles of RME (Treffers, 1987), which linked the algebraic topic to geometry, particularly in the area of shapes.

However, in contrast to the RME positive influences on learning algebra resulting from previous studies (Fauzan et al., 2018; Palinussa, 2020), the participants of this study still faced many obstacles. These obstacles included students' misunderstanding regarding the operation of algebraic expressions. For example, a student wrote $8 + x = 8x$, which according to Chow and Treagust (2013), happened because the student considered the (+) sign as an invitation to do something. Another constraint was students' misunderstanding of the meaning of symbols and variables on the topic of algebraic expressions themselves (Davidenko, 1997; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005); for example, when a student wrote $3x + 5x = 8x$, which was a correct operation but an incorrect expression for the statement required. They failed to understand the process order of simplifying algebraic expressions. Further, some students also showed a lack of understanding of the operation of integers as they considered $9 - 7 = -2$.

Researchers and instructors then discussed the cause of poor average scores experienced by students in this study. One of the causes was that the MRA test consisted of more problems requiring students to use their symbolic representation ability, which is more challenging than the other aspects of MRA (Fitrianna et al., 2018; Johar, Zubainur, & Khairunnisak, 2017). Further, it was assumed that another issue lay in the students' inability to do integer operations which should have been mastered in the elementary school or the preceding semester. Whereas, according to Egodawatte (2011), to develop students' understanding of algebra, they must understand fundamental mathematical concepts such as the addition of integers. Therefore, an additional test
related to integer operation was then administered to the students, and it yielded an average score of 1.01 out of an ideal score of 8. Figure 9 depicts some of the students’ performance in the additional test. The test result confirmed the assumption concerning students’ lack of proficiency in integer operations.

![Figure 9. Students’ answers on the additional test related to operations of integers](image)

One cause of the students’ lack of understanding of the prerequisite topics is learning loss. A poll conducted by the Ministry of Education and Culture revealed that 20% of schools nationwide agree that some students were hampered in their learning process; they suffered from learning loss (Putra, 2021). Indonesian Ministry of Education and Culture (Kemendikbud, 2021) also stated that students did not fulfill the anticipated competency standards during studying at home. Therefore, it was presumed that the participants of this study also suffered from learning loss due to online learning during the implementation of the Covid-19 health protocol.

Based on interviews with a teacher in this research, it was known that before implementing the shift method, students learned online through asynchronous learning for three semesters. During the early stages of the Covid-19 pandemic, March to June 2020, students interacted with teachers through WhatsApp Groups (WAG). The teacher sent materials to be studied and the assignments via WAG and then at the end of every week the students delivered their works to school and the teacher gave feedback. For students who did not understand the task would be directly guided by the teacher at the school. In the next semester, July to December 2020 and January to June 2021, the Banda Aceh government had prepared e-learning platform where teachers should upload files about explanations of mathematics topics and asynchronous videos from youtube along with assignments. At the end of every month students delivered their works to school and the teacher gave feedback. The teacher said that the weakness of this asynchronous system learning was that the teacher could not detect the originality of student works and she could not cover all suggestions through her feedbacks. This is in accordance with Hamilton et al., (2020) stated that in the spring of 2020 during learning amid the covid pandemic, only 12% of teachers could teach material as demanded by the curriculum to students. Further, according to
Diliberti and Kaufman (in Hamilton & Ercikan, 2022), most teachers rarely provide verbal feedback on student assignments, teachers lack interaction with students and teachers cannot reach all students.

In addition, it was assumed that the shift model of learning also became one of the reasons because of the reducing time to study. Students normally learn mathematics twice a week, \(2 \times 40\) minutes and \(3 \times 40\) minutes. However, during the implementation of the shift model, students only learn for \(2 \times 30\) minutes each week, causing learning activities to be terminated before students completely comprehend the content. A week between meetings is also considered to be long enough for pupils to forget what they learned before.

**Conclusion**

The students' average scores on the MRA test were 3.6, which was considered low compared to the ideal score of 8. Furthermore, most students met the visual aspect of the MRA; they could sketch composite shapes whose sizes are partially written in variables. Whereas the minor aspect of MRA reached by the students is the symbolic aspect, students found it difficult to multiply the algebraic forms.

This study has several limitations to be addressed in future research. First, the reduced time allocated for the learning process was indicated to be one of the causes of the low MRA test scores. Further research should be conducted to examine the effect of RME learning on students' MRA. Furthermore, a discussion with the teacher indicated that another reason was students' lack in solving operation of integers, which should have been mastered in the elementary school and the previous semester. It indicates that students faced learning loss during online learning because of the health protocol of the Covid-19 pandemic. Hence, further research about students' learning loss would be meaningful since the current data concerning that topic is inadequate.

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