Neutrino oscillations in a $3\nu_L + 3\nu_R$ framework with five light neutrinos

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Abstract
We propose a neutrino mass matrix model in which five neutrino species remain light through the seesaw mechanism within a supersymmetric $3\nu_L + 3\nu_R$ framework. We construct such a model based on the nonrenormalizable terms in the superpotential constrained by the discrete symmetry which may be expected in the models at the high energy scale such as superstring. We study the possible oscillation phenomena by fixing mass parameters so as to explain the solar and atmospheric neutrino deficits and also include a candidate of the suitable dark matter. We also discuss the charged lepton mass matrix based on this neutrino model. LSND results may be consistently explained within this model.

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1 Introduction

Neutrino mass is one of unsolved problems in the present particle physics, although it is a very important issue which can be a clue to go beyond the standard model. Experimentally, in both particle physics and astrophysics there are a lot of indications for massive neutrinos[1]. Analyses of observations of the solar neutrino[2] and the atmospheric neutrino[3] suggest the existence of neutrino oscillations. Especially, a recent SuperKamiokande observation of the Zenith angle dependence of the atmospheric $\nu_\mu$ strongly suggests that $\nu_\mu$ oscillates to $\nu_\tau$[4]. An interesting feature in these indications is that they may require wide range mixing angles, in particular, a maximal mixing among different neutrinos in addition to hierarchically small mass eigenvalues or closely degenerating mass eigenvalues. Although this smallness of masses may be considered to be explained by the seesaw mechanism[5] in general, the hierarchy of masses and the mixing structure will be completely dependent on models.

On the other hand, it has also been suggested that neutrinos with suitable masses could be a dark matter candidate for the explanation of astrophysical observations related to the structure formation of the universe. As such examples, we may list an active neutrino with $O(10)$ eV mass in the hot dark matter scenario (HDM)[6], a sterile neutrino with $O(10^{-2})$ eV mass in the warm dark matter scenario (WDM)[7] and neutrinos with $O(1)$ eV mass in the cold + hot dark matter scenario (CHDM)[8, 9]. When we consider the neutrino models, we may need accommodate this feature to the models. If we impose such a requirement, we can restrict the models in a rather severe way.

Now various experiments using reactors, accelerators and underground facilities are proceeding and planned. In the near future these experimental results will be presented to inform us details of the neutrino sector and then the predictions for various neutrino phenomena including oscillations on the basis of possible neutrino models will also be very useful. Under this situation it seems to be a worthy and interesting subject to consider various types of model which can explain both of these neutrino deficits consistently and present a dark matter candidate.

The introduction of a sterile neutrino is one way to this direction and a lot of works have been done by now[10, 11, 12, 13]. It has also been suggested that sterile neutrinos may play important roles in various phenomena[14, 15, 16]. Although the existence of sterile neutrinos is an interesting possibility, it seems to be rather difficult to find its
natural candidate in particle physics models\[1\]. One of such reasons is that it is not so easy to introduce a substantial mixing between active neutrinos and light sterile neutrinos in the natural way \[18\].

The aim of this paper is to propose such a candidate and analyze neutrino oscillation phenomena on the basis of it. In this study the neutrino sector is extended into $3\nu_L + 3\nu_R$ species, among which only five neutrinos remain light through the seesaw mechanism based on the mixings with a heavy right-handed neutrino ($N_R$). Two light right-handed neutrinos behave as sterile neutrinos. This seems to be natural from a viewpoint of the generation structure of other quarks and leptons, although the seesaw mechanism works in a different way from the one in the ordinary grand unified models such as SO(10). To realize the substantial mixing between these sterile neutrinos and active neutrinos we will consider the nonrenormalizable interactions coming from the fundamental theory in a high energy region. This may be considered as an example of the scheme proposed in Ref. \[18\]. Phenomenologically, as partly discussed in Refs. \[12, 13\], this model can give a framework to explain both deficits of the solar and atmospheric neutrinos consistently and also to present a candidate of the dark matter with a suitable mass for a certain kind of dark matter scenario.

This paper is organized as follows. In section 2 we give a brief review on the oscillation parameters to fix our notation. In section 3 we introduce our model and discuss its theoretical background. A possibility to realize this model in the supersymmetric framework in the basis of nonrenormalizable terms in the superpotential is studied. Its detailed phenomenological analyses are presented in section 4. We show two typical parameter settings which have different features. Other oscillation processes than the ones related to the solar and atmospheric neutrino deficits are also discussed in each case. LSND results reported in Ref. \[19\] may be simultaneously explained in this model. Section 5 is devoted to the summary.

### 2 Oscillation parameters

At first we briefly review a basic formula for the neutrino oscillation. We define a mixing matrix $V$ as

$$\nu_f = \sum_\alpha V_{\alpha f} \tilde{\nu}_\alpha$$

where $(V^\dagger)_{f\alpha} = V_{\alpha f}^*$ and $\tilde{\nu}_\alpha$ ($\alpha = \bar{1}, \bar{2}, \bar{3}, \cdots$) is a mass

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\[1\] For example, a candidate other than the right-handed neutrino has been proposed in Ref. \[17\].
eigenstate. $\nu_f (f = e, \mu, \tau, \cdots)$ is a weak eigenstate chosen in a way that both leptonic charged currents and a charged lepton mass matrix are diagonal. Thus $V$ can be written as $V = U^{(\nu)} U^{(l)\dagger}$ by using diagonalization matrices $U^{(l)}$ and $U^{(\nu)}$ of the charged lepton and neutrino mass matrices $M^{(l)}$ and $M^{(\nu)}$,

$$M^{(l)2}_{\text{diag}} = U^{(l)} M^{(l)\dagger} M^{(l)} U^{(l)\dagger}, \quad M^{(\nu)}_{\text{diag}} = U^{(\nu)} M^{(\nu)\dagger} U^{(\nu)\dagger}. \quad (1)$$

In the following discussion, $V$ is assumed to be real, for simplicity. Then $V$ satisfies an orthogonality condition

$$\sum_f V_{\alpha f} V_{\beta f} = \delta_{\alpha \beta}, \quad \sum_{\alpha} V_{\alpha f} V_{\beta f} = \delta_{\alpha \beta}. \quad (2)$$

Using mass eigenstates, a time evolution equation of neutrinos in a vacuum is given as

$$i \frac{d}{dt} \tilde{\nu}_\alpha = \frac{M^2_{\alpha}}{2E} \tilde{\nu}_\alpha, \quad (3)$$

where $E$ is energy of neutrinos and $M_\alpha$ stands for an $\alpha$-th neutrino mass eigenvalue. This equation can be easily solved as

$$\tilde{\nu}_\alpha(t) = \exp \left( -i \frac{M^2_{\alpha}}{2E} t \right) \tilde{\nu}_\alpha(0). \quad (4)$$

By transforming this into a solution in terms of the weak eigenstates, the transition probability for $\nu_f \rightarrow \nu_{f'}$ during the time interval $t$ is expressed as

$$P_{\nu_f \rightarrow \nu_{f'}(t)} = |\langle \nu_{f'}(t) | \nu_f(0) \rangle|^2$$

$$= \sum_\alpha V_{\alpha f} V_{\alpha f'}^2 + 2 \sum_\alpha \sum_{\beta(>\alpha)} V_{\alpha f} V_{\alpha f'} V_{\beta f} V_{\beta f'} \cos \left( \frac{\Delta M^2_{\alpha \beta} t}{2E} \right), \quad (5)$$

where $\Delta M^2_{\alpha \beta} \equiv |M^2_{\alpha} - M^2_{\beta}|$. If we use a following relation derived from the orthogonality of $V$:

$$\sum_\alpha V_{\alpha f} V_{\alpha f'}^2 = -2 \sum_\alpha \sum_{\beta(>\alpha)} V_{\alpha f} V_{\alpha f'} V_{\beta f} V_{\beta f'}, \quad (6)$$

we can obtain

$$P_{\nu_f \rightarrow \nu_{f'}(t)} = -4 \sum_\alpha \sum_{\beta(>\alpha)} V_{\alpha f} V_{\alpha f'} V_{\beta f} V_{\beta f'} \sin^2 \left( \frac{\Delta M^2_{\alpha \beta} t}{4E} \right). \quad (7)$$

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2 We neglect CP phase here. In a model studied in this paper there are three charged lepton flavors and five light neutrino flavors. When we define $V$, it is necessary to extend $U^{(l)}$ into a $5 \times 5$ matrix by adding formally 1’s as diagonal elements.
Here we note that a contribution from a $\alpha\beta$-sector to the $\nu_f \leftrightarrow \nu_{f'}$ oscillation is represented by parameters $(\Delta M^2_{\alpha\beta}, -4V_{\alpha f}V_{\alpha f'}V_{\beta f}V_{\beta f'})$. In a sector where $V_{\alpha f}$ and $V_{\beta f'}$ correspond to the diagonal elements the amplitude (or the mixing factor) reduces to $-4V_{\alpha f}V_{\beta f}$, as long as the mixings with other neutrino flavors are sufficiently small and $V_{\alpha f} \approx V_{\beta f'} \approx 1$ is satisfied. In such a case these parameters can be understood as the usual two flavor oscillation parameters $(\Delta m^2, \sin^2 2\theta)$. On the other hand, if the mixings with other flavors are not so small in the sectors specified by the off-diagonal elements $V_{\alpha f}$ and $V_{\beta f'}$, there appear new contributions which are induced due to the existence of many neutrino flavors. They may be understood in such a way that an additional mixing between $\nu_{f_1}$ and $\nu_{f_2}$ induces the $\nu_f \leftrightarrow \nu_{f'}$ oscillation through the flavor mixings $\nu_f - \nu_{f_1}$ and $\nu_{f'} - \nu_{f_2}$. Thus these processes are generally recognized as higher order effects in comparison with the direct two flavor oscillation concerning the mixing factors. Anyway it is necessary to be careful when we apply Eq. (7) to the results on the oscillation parameters obtained by the two flavor analysis of the solar and atmospheric neutrino problems. In particular, matter effects on the oscillations will not be observed in a simple way through an analytical study in the cases with many neutrino flavors.

3 A model with sterile and active neutrino mixings

3.1 Basic framework

We consider a model containing three generation left-handed neutrinos $\nu_{f_L}$ ($f_L = e, \mu, \tau$), and right-handed neutrinos $N_{f_R}$ ($f_R = A, B, C$). As a guiding principle to construct a neutrino mass matrix in this kind of models phenomenologically and reduce the free parameters systematically, we take a viewpoint that the realization of the mixing angle required in the solar and atmospheric neutrino problems is the most important clue. Along this line we first prepare mass terms which can produce such a mixing structure among the five light neutrinos through the seesaw mechanism. After this procedure we introduce mass correction to resolve the mass degeneracy in a consistent way with this mixing structure.

Following this strategy, we require that six neutrino species have mass terms which
are written as
\[ -\mathcal{L}_{\text{mass}} = \sum_{f,f'} m_{ff'} \psi_f \psi_{f'} + \sum_{f=1}^5 m_f \psi_f \bar{N}_C + \frac{1}{2} M \bar{N}_C N_C + \text{h.c.}, \]  
(8)
where \( \psi_f \) represents \( \nu_{fL} \) and \( \bar{N}_{fR} \) (\( f_R = A, B \)). Although the state identification is still not done at this stage, hierarchies among the above mass parameters are assumed as\(^3\)
\[ m_{ff'} \ll m_1 \ll m_2 \ll m_3 \sim m_4 \ll m_5 \ll M. \]  
(9)
Since \( m_{ff'} \) is small enough compared with others, we can neglect it for a while. As a result of the seesaw mechanism, a heavy right-handed neutrino \( N_C \) decouples from other neutrinos and a mass matrix for five light states \( \psi_f \) becomes\(^4\) [12]
\[
\mathcal{M}_0 = M \begin{pmatrix}
\mu_1 
\mu_2 
\mu_3 
\mu_4 
\mu_5 \\
\mu_1 \mu_2 
\mu_1 \mu_3 
\mu_2 \mu_3 
\mu_2 \mu_4 
\mu_2 \mu_5 \\
\mu_1 \mu_3 
\mu_2 \mu_3 
\mu_2 \mu_4 
\mu_3 \mu_4 
\mu_3 \mu_5 \\
\mu_1 \mu_4 
\mu_2 \mu_4 
\mu_3 \mu_4 
\mu_4 \mu_5 
\mu_4 \mu_5 \\
\mu_1 \mu_5 
\mu_2 \mu_5 
\mu_3 \mu_5 
\mu_4 \mu_5 
\mu_5 
\end{pmatrix},
\]  
(10)
where \( \mu_f = m_f / M \). As is easily checked, \( \mathcal{M}_0 \) is a matrix with a rank one and diagonalized as \( U^{(\nu)} \mathcal{M}_0 U^{(\nu)T} \) by using the matrix
\[
U^{(\nu)} = \begin{pmatrix}
\mathcal{O} & 0 \\
0 & 1 
\end{pmatrix} \begin{pmatrix}
\xi_1 & -\xi_1 & 0 & 0 & 0 \\
\xi_2 & -\xi_2 & 0 & 0 & 0 \\
\xi_3 & -\xi_3 & 0 & 0 & 0 \\
\xi_4 & -\xi_4 & 0 & 0 & 0 \\
\xi_5 & -\xi_5 & 0 & 0 & 0 
\end{pmatrix},
\]  
(11)
where \( \xi_n^2 = \sum_{f=1}^{n+1} \mu_f^2 \). \( \mathcal{O} \) is an undetermined \( 4 \times 4 \) matrix which should be introduced because of the mass degeneracy.

\(^3\) The inequality between \( m_1 \) and \( m_2 \) is reversed from the one in Ref. [12]. The present one should be used to make the MSW mechanism applicable in the \( (\psi_1, \psi_2) \) when the state identification is assumed such as \( \psi_1 \equiv \bar{\nu}_{s_1} \) and \( \psi_2 \equiv \nu_e \) which will be used in this paper. Some related equations in Ref. [12] should be replaced by the ones presented in this paper.

\(^4\) It should be noted that we approximately call these states as \( \psi_f \). They are not pure \( \psi_f \)'s but have a small mixture component from \( N_C \).
To resolve this mass degeneracy and fix $U^{(\nu)}$ completely, we now switch on rather complicated mass corrections $m_{ff'}$ to the five light neutrinos in order to yield the hierarchical mass eigenvalues without disturbing the mixing structure $U^{(\nu)}$. As such masses we consider the simplest example as

$$
\mathcal{M}_{\text{per}} \simeq \begin{pmatrix}
A\mu_1^2 & B\mu_1\mu_2 & D\mu_1\mu_3 & E\mu_1\mu_4 & F\mu_1\mu_5 \\
B\mu_1\mu_2 & C\mu_2^2 & D\mu_2\mu_3 & E\mu_2\mu_4 & F\mu_2\mu_5 \\
D\mu_1\mu_3 & D\mu_2\mu_3 & D\mu_3^2 & E\mu_3\mu_4 & F\mu_3\mu_5 \\
E\mu_1\mu_4 & E\mu_2\mu_4 & E\mu_3\mu_4 & D\mu_4^2 & F\mu_4\mu_5 \\
F\mu_1\mu_5 & F\mu_2\mu_5 & F\mu_3\mu_5 & F\mu_4\mu_5 & G\mu_5^2
\end{pmatrix},
$$

(12)

where $A \sim F$ are parameters which satisfy

$$
\frac{A - D}{B - D} = \frac{B - D}{C - D} = -\frac{\mu_2^2}{\mu_1^2}, \quad \frac{F}{D + E} = -\frac{\mu_4^2}{\mu_5^2}.
$$

(13)

This matrix can also be checked to be diagonalized by $U^{(\nu)}$ and the mass eigenvalues of $\mathcal{M}_0 + \mathcal{M}_{\text{per}}$ is obtained as

$$
M_1 = (A - D)\mu_1^2, \quad M_2 = 0, \quad M_3 = (D - E)\mu_3^2, \quad M_4 = (D + E)\mu_4^2, \quad M_5 = M\mu_5^2.
$$

(14)

Hereafter we use numerical indices with a bar to specify the mass eigenstates. A 55-element of $\mathcal{M}_{\text{per}}$ can take its value in a range like $|G| \lesssim O(|F|)$ to guarantee the approximate diagonalization by $U^{(\nu)}$ because the fifth mass eigenvalue $M_5$ coming from $\mathcal{M}_0$ is rather large. After the introduction of this correction the mass degeneracy is completely resolved and $\mathcal{O}$ in Eq. (11) is determined as $\mathcal{O} = 1$.

Here it is useful to note how many number of parameters are included in this model. As easily found from Eqs. (11) and (14), there are nine free parameters. They may be taken as $\mu_f(f = 1 \sim 5), M, C, D$ and $E$. It is convenient for the later study to present concrete expressions of oscillation parameters in Eq. (7), in particular, the amplitude $-4V_{\alpha f}^*V_{\alpha f'}V_{\beta f'}V_{\beta f}$ by using these parameters.\footnote{For a while, we confine our attention to $U^{(\nu)}$ assuming that a charged lepton mass matrix is diagonal so that $U^{(l)} = 1$. In section 4.3 we extend our analysis to more general cases.} For the $\psi_f \leftrightarrow \psi_{f'}$ oscillation their expressions due to the $\alpha\beta$-sector are presented in Table 1. Although there are small negative contributions to $\mathcal{P}_{\psi_f \rightarrow \psi_{f'}}$ from some $\alpha\beta$-sectors, we omit them from Table 1. The detailed explanation of numerical values listed in the columns (I) and (II) of Table 1 is given in the next section.
3.2 Construction based on nonrenormalizable terms

In the previous subsection we introduced our mass matrix $M_0 + M_{\text{per}}$ in the phenomenological way so as to realize the substantial mixing between sterile and active neutrinos with nondegenerating masses. However, we can also construct it under a certain theoretical background. We present such an example in this subsection. To make our argument definite we consider a certain type of supersymmetric nonanomalous extra U(1) models with a D-flat direction for the extra U(1), which comes from $E_6$ unification group [20]. This kind of models may be expected to be induced from the perturbative superstring. The following argument will be straightforwardly applied to other type of models, for example, anomalous U(1) models.

This example contains three right-handed neutrinos $\bar{N}_{fR}$ and pairs of singlets $(S_\ell, \bar{S}_\ell)$ ($\ell = 1, 2$), where $\bar{N}_{fR}$ and $S_\ell$ belong to 27 chiral superfield of $E_6$ and a conjugate partner $\bar{S}_\ell$ belongs to $27^*$ chiral superfield. The extension to the case with $\ell \geq 3$ is straightforward. $\bar{N}_{fR}$, $S_\ell$ and $\bar{S}_\ell$ are singlets under the gauge groups of the standard model but have the extra U(1) charge. $\bar{N}_{fR}$ and $S_\ell$ are assumed to have the same charge of this extra U(1).

The gauge invariant relevant terms in the superpotential to the present investigation are composed by two parts,

\begin{equation}
W_0 = \sum_{\ell=1,2} \frac{k_{K_\ell}}{M_G^{2K_\ell-3}} (S_\ell \bar{S}_\ell)^{K_\ell},
\end{equation}

\begin{equation}
W_1 = \sum_{fR,fR'} \frac{k_{S}^{fRfR'}}{M_G^{2P_{fRfR'}-3}} (S_2 \bar{S}_2)^{P_{fRfR'}} (S_1 \bar{S}_1)^{P_{fRfR'}-P_{fRfR'}} \bar{S}_1^2 \bar{N}_{fR} \bar{N}_{fR'}
+ \sum_{fL,fL'} \frac{k_{D}^{fLfL'}}{M_G^{2Q_{fLfL'}}} (S_2 \bar{S}_2)^{Q_{fLfL'}} (S_1 \bar{S}_1)^{Q_{fLfL'}-Q_{fLfL'}} L_{fL} H_2 \bar{N}_{fR}
+ \sum_{fL,fL'} \frac{k_{M}^{fLfL'}}{M_G^{2R_{fLfL'}+3}} (S_2 \bar{S}_2)^{R_{fLfL'}} (S_1 \bar{S}_1)^{R_{fLfL'}-R_{fLfL'}} S_1^2 H_2^2 L_{fL} L_{fL'},
\end{equation}

where the parameters determining the power structure of each term are integers and satisfy

\begin{align*}
K_\ell & \geq 2, & P_{fRfR'} - 2 & \geq P_{fRfR'} \geq 0, & Q_{fLfL'} & \geq Q_{fLfL'} \geq 0, & R_{fLfL'} & \geq R_{fLfL'} \geq 0.
\end{align*}

$M_G$ is a suitable scale such as a string scale or a Planck scale. The higher order terms can be neglected in comparison with these lowest dimensional terms. The scalar potential for
written by using the parameters in Eq. (16) as 

\[ V = \frac{g^2}{2} \sum_{\ell=1,2} Q_{S_\ell}^2 \left( |S_{\ell}|^2 - |\bar{S}_{\ell}|^2 \right)^2 + \sum_{\ell=1,2} \left\{ \frac{k_{K_\ell} K_{K_\ell} S_{\ell}^{K_{K_\ell} - 1} \bar{S}_{\ell}^{K_{K_\ell} - 1}}{M_G^{2K_{K_\ell} - 3}} \right\}^2 - m^2 \left( |S_{\ell}|^2 + |\bar{S}_{\ell}|^2 \right). \] (17)

The first line is a D-term contribution of the extra U(1) and the second line represents an F-term contribution coming from \( W_0 \) and soft supersymmetry breaking scalar masses which we assumed as \( m_{S_{\ell}}^2 = m_{\bar{S}_{\ell}}^2 = m^2 \), for simplicity. Clearly this scalar potential has D-flat directions \( |S_{\ell}| = |\bar{S}_{\ell}| \equiv u_{\ell} \) along which the potential minimum is realized. After soft supersymmetry breaking masses are introduced as in Eq. (17), the magnitude of the intermediate scale \( u_{\ell} \) is determined by the nonrenormalizable F-term contribution in the second line as

\[ u_{\ell} = \frac{1}{(2K_{\ell} - 1)^{1/2}k_{K_{\ell}}K_{\ell}} \frac{mM_G^{2K_{K_{\ell} - 3}}}{2m^2}, \] (18)

where the value of \( u_{\ell} \) is crucially dependent on an integer \( K_{\ell} \). Moreover, as easily seen in Eq. (16), depending on the values of powers in the nonrenormalizable terms such as \( P_{f_{\ell}f'_{\ell}} \), \( Q_{f_{\ell}f'_{\ell}} \) and \( R_{f_{\ell}f'_{\ell}} \), various types of neutrino mass terms can be induced through the effective couplings controled by the power of \( u_{\ell}/M_G \) after the vacuum expectation value \( u_{\ell} \) becomes nonzero.

Since there is no restriction on the right-handed neutrino mass terms due to the gauge symmetry unlike the usual Grand Unified models, this framework can contain light sterile neutrinos in addition to the ordinary three active light neutrinos. If we take the basis \( \psi_{f} \) in Eq. (8) as \( \psi_{f} = (\tilde{N}_{A} (\equiv \nu_{s_1}), \nu_{f_L}, \tilde{N}_{B} (\equiv \nu_{s_2})) \), the mass parameters in Eq. (8) can be written by using the parameters in Eq. (16) as

\[ m_1 = k_{S}^{AC} \epsilon^{2P_{AC} - 3} \delta_{2p_{AC}} u_1, \quad m_{f_L} = k_{D}^{LC} \epsilon^{2Q_{f_{LC}} - 3} \delta_{2f_{LC}} v_2, \]

\[ m_5 = k_{S}^{BC} \epsilon^{2P_{BC} - 3} \delta_{2p_{BC}} u_1, \quad M = k_{S}^{CC} \epsilon^{2P_{CC} - 3} \delta_{2p_{CC}} u_1, \] (19)

where we use the definitions \( \epsilon \equiv u_1/M_G \) and \( \delta \equiv u_2/u_1 \). In Eq. (19) the indices \( f_{\ell} = e, \mu, \tau \) should be interpreted to correspond to the numerical indices 2, 3 and 4 in Eq. (8). Then the parameters \( \mu_{f} \) are also expressed as

\[ \mu_{1} = \frac{k_{S}^{AC}}{k_{S}^{CC}} \epsilon^{2(P_{AC} - P_{CC})} \delta_{2(p_{AC} - p_{CC})}, \quad \mu_{f_L} = \frac{k_{D}^{LC}}{k_{S}^{CC}} \frac{v_2}{u_1} \epsilon^{2(Q_{f_{LC}} - C_{CC}) + 3} \delta_{2(q_{f_{LC}} - p_{CC})}, \]

6The phenomenological validity of this identification will be discussed in the next section.
\[ \mu_5 = \frac{k_{BC}^S}{k_{CC}^S} \epsilon^{2(P_{BC} - P_{CC})} \delta^{2(P_{BC} - P_{CC})}. \]  

Moreover, based on the superpotential \( W_1 \) we can write the neutrino mass matrix \( M_{\text{per}} \) under the same basis as follows,

\[
M_{\text{per}} = \begin{pmatrix}
 k_{AA}^S \epsilon^{2P_{AA}} \delta^{2P_{AA}^2} u_1 & k_{SL}^A \epsilon^{2Q_{SL}^A} \delta^{2Q_{SL}^A} v_2 & k_{LH}^A \epsilon^{2P_{LH}^A} \delta^{2P_{LH}^A} u_1 \\
 k_{D}^{L} \epsilon^{2Q_{DL}^A} \delta^{2Q_{DL}^A} v_2 & M_{SL} & k_{DL}^{L} \epsilon^{2Q_{DL}^A} \delta^{2Q_{DL}^A} v_2 \\
 k_{D}^{L} \epsilon^{2Q_{DL}^A} \delta^{2Q_{DL}^A} v_2 & k_{D}^{L} \epsilon^{2Q_{DL}^A} \delta^{2Q_{DL}^A} v_2 & k_{LH}^{L} \epsilon^{2P_{LH}^A} \delta^{2P_{LH}^A} u_1
\end{pmatrix},
\]

where \( v_2 \) is a VEV of the doublet Higgs scalar \( H_2 \). Majorana masses \( M_{fLf'}_L \) can be caused by the last terms in Eq. (16). However, even if \( R_{fLf'}_L = 0 \), these Majorana masses are \( \sim 10^{-7} \) eV and then generally too small for the explanation of the solar and atmospheric neutrino problem \[18\]. Within the interactions included in Eq. (16) \( M_{fLf'}_L \) can be also induced as the mixing terms composed by the first and the second terms of Eq. (16). But it is difficult to arrange in the way to satisfy the condition given by Eq. (13) and guarantee the sufficient largeness of eigenvalues. Finally we are forced to introduce new interaction terms. Here we introduce a triplet Higgs scalar \( \Phi \) and consider the following interaction terms in the superpotential \[7\] :

\[
W_2 = \frac{k_{T}^{fL f'}_L}{M_G^{2T_{fL f'}_L}} (S_2 \bar{S}_2)^T f_L f'_L (S_1 \bar{S}_1)^T f_L f'_L \Phi L_{fL} L_{f'_L}.
\]

This gives the Majorana masses

\[
M_{fLf'}_L = k_T^{fL f'}_L \epsilon^{2T_{fL f'}_L} \delta^{T_{fL f'}_L} v_T,
\]

where \( v_T \) is a VEV of \( \Phi \). There is a constraint on \( v_T \) from an electroweak \( \rho \) parameter. To satisfy its constraint it will be necessary to take it as \( v_T \lesssim 1 \) GeV.

What kind of the mass matrix \( M_{\text{per}} \) is induced in this model is completely dependent on the values of the lowest powers and couplings of each terms in the superpotential shown by Eqs. (15), (16) and (22). In order to constrain the superpotential, it is necessary to be able to introduce a certain kind of symmetry which can forbid the lower dimensional terms consistently. In many works \[21\] an Abelian horizontal symmetry has been used to constrain the nonrenormalizable superpotential, which can induce the small neutrino
masses with a favorable texture after the breakdown of this horizontal symmetry due to the VEV of some scalar fields. In our model the discrete symmetry may play the same role. To realize the superpotential starting from high dimensional terms, we need rather complicated higher order discrete symmetry. Here we assume $Z_9 \times Z_9$ as a concrete example of such a discrete symmetry and take the charge assignments for the relevant fields under this symmetry as the ones shown in Table 2. The charge assignment for this discrete symmetry allows the lowest terms with $K_1 = K_2 = 9$ in $W_0$. This results in $u_1 \sim u_2 \sim 10^{17}$ GeV and then $\epsilon \sim 0.1$ and $\delta \sim 1$. We should note here that in this case $\epsilon$ is not so small that the extremely high dimensional terms are necessary to realize the desirable neutrino masses. If we adopt a smaller $K_\ell$ and make $\epsilon$ small enough, we do not need such high dimensional terms but additional fine tunings for coupling coefficients are required to induce a necessary mass pattern. In such a case the main feature of the mass pattern is determined by the tunings of the couplings. Thus we do not take this way here.

Terms in $W_1$ and $W_2$ are also constrained by this discrete symmetry and the following values for the powers in $W_1$ and $W_2$ are allowed as the lowest ones:

$$P_{f_Rf'_{R}} = \begin{pmatrix} 18 & 17 & 11 \\ 17 & 16 & 10 \\ 11 & 10 & 4 \end{pmatrix}, \quad Q_{f_Lf_R} = \begin{pmatrix} 9 & 8 & 2 \\ 8 & 7 & 1 \end{pmatrix}, \quad T_{f_Lf'_L} = \begin{pmatrix} 8 & 7 & 7 \\ 7 & 6 & 6 \end{pmatrix},$$

$$p_{f_Rf'_R} = \begin{pmatrix} 8 & 8 & 4 \\ 8 & 8 & 4 \\ 4 & 4 & 0 \end{pmatrix}, \quad q_{f_Lf_R} = \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix}, \quad t_{f_Lf'_L} = \begin{pmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \end{pmatrix}. \quad (24)$$

The present examples may not be considered to be realistic because of their extremely large dimensions of the necessary nonrenormalizable terms. Although this feature seems to be general in this type of models, the situation may be made mild to some extent by considering a type of extra U(1) models in which a role of $S_\ell\bar{S}_\ell$ is replaced by suitable elementary singlet fields with D-flat directions. Anyway this example shows the direction how we can constrain the nonrenormalizable terms in the superpotential.

Next we study the neutrino mass matrix induced by this superpotential. If we take $v_2 \sim \epsilon^{15}u_1$, $v_T \sim 1$ GeV, these values give $\mu_f$ and $M_{\text{per}}$ as follows:

$$\mu_1 = \tilde{k}_S^{AC} \epsilon^{14}, \quad \mu_2 = \tilde{k}_S^{\sigma C} \epsilon^{14}, \quad \mu_3 = \tilde{k}_D^{\mu C} \epsilon^{12}, \quad \mu_4 = \tilde{k}_D^{\tau C} \epsilon^{12}, \quad \mu_5 = \tilde{k}_S^{BC} \epsilon^{12},$$

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where \( \tilde{k}_x^{ff'} \equiv k_x^{ff'}/k_S^{CC} \). To obtain the phenomenologically favorable hierarchy among \( \mu_f \) \((f = 1 \sim 5)\) and realize the mass matrix form defined by Eqs. (12) and (13), we need additional hierarchical structure in the coupling constants \( \tilde{k}_x^{ff'} \). Among these tunings of coupling constants we introduce the following useful parameters expressing the freedom which can not be determined by the present experimental results in our state identification used later:

\[
a = 2 - \log_{10}\left( \frac{k_{BC}^{C}}{k_{D}^{C}} \right), \quad b = - \log_{10}\left( \frac{k_{BC}^{C}}{k_{S}^{C}} \right).
\]

(26)

Using these parameters and if we make the suitable assumptions on \( \tilde{k}_x^{ff'} \), \( \mu_f \) can have the following hierarchy:

\[
\mu_1 \sim \epsilon^{16}, \quad \mu_2 \sim \epsilon^{12+a}, \quad \mu_3 \sim \epsilon^{12}, \quad \mu_4 \sim \epsilon^{10+b}, \quad \mu_5 \sim \epsilon^{10}.
\]

(27)

Although there are many possibilities for \( \mathcal{M}_{\text{per}} \) depending on the assumption on the coupling constants, it will be useful to present examples to see what kind of tunings of the coupling constants are required to construct the interesting \( \mathcal{M}_{\text{per}} \). Here we give two examples:

\[
(1): \quad \mathcal{M}_{\text{per}} \sim M \begin{pmatrix}
\epsilon^{-6.3} \mu_1^2 & -\epsilon^{-3.5} \mu_1 \mu_2 & \epsilon^{-1.9} \mu_1 \mu_3 & \epsilon^{-1.9} \mu_1 \mu_4 & -\epsilon^{2.2} \mu_1 \mu_5 \\
-\epsilon^{-3.5} \mu_1 \mu_2 & \epsilon^{-1.9} \mu_1 \mu_2 & \epsilon^{-1.9} \mu_2 \mu_3 & \epsilon^{-1.9} \mu_2 \mu_4 & -\epsilon^{2.2} \mu_2 \mu_5 \\
\epsilon^{-1.9} \mu_1 \mu_3 & \epsilon^{-1.9} \mu_2 \mu_3 & \epsilon^{-1.9} \mu_3 \mu_3 & \epsilon^{-1.9} \mu_3 \mu_4 & -\epsilon^{2.2} \mu_3 \mu_5 \\
\epsilon^{-1.9} \mu_1 \mu_4 & \epsilon^{-1.9} \mu_2 \mu_4 & \epsilon^{-1.9} \mu_3 \mu_4 & \epsilon^{-1.9} \mu_4 \mu_4 & -\epsilon^{2.2} \mu_4 \mu_5 \\
-\epsilon^{2.2} \mu_1 \mu_5 & -\epsilon^{2.2} \mu_2 \mu_5 & -\epsilon^{2.2} \mu_3 \mu_5 & -\epsilon^{2.2} \mu_4 \mu_5 & \epsilon^{2.2} \mu_5^2
\end{pmatrix}
\]

(25)
The diagonalization matrix \( M_{\text{per}} \sim M \)
\[
\begin{pmatrix}
\epsilon^{-6.3} \mu_1^2 & -\epsilon^{-3.5} \mu_1 \mu_2 & \epsilon^{-3.4} \mu_1 \mu_3 & \epsilon^{0.4} \mu_1 \mu_4 & -\epsilon \mu_1 \mu_5 \\
-\epsilon^{-3.5} \mu_1 \mu_2 & \epsilon^{-3.4} \mu_2^2 & \epsilon^{-3.4} \mu_2 \mu_3 & \epsilon^{0.4} \mu_2 \mu_4 & -\epsilon \mu_2 \mu_5 \\
\epsilon^{-3.4} \mu_1 \mu_3 & \epsilon^{-3.4} \mu_2 \mu_3 & \epsilon^{-3.4} \mu_3^2 & \epsilon^{0.4} \mu_3 \mu_4 & -\epsilon \mu_3 \mu_5 \\
\epsilon \mu_1 \mu_4 & \epsilon \mu_2 \mu_4 & \epsilon \mu_3 \mu_4 & \epsilon^{-3.4} \mu_4^2 & -\epsilon \mu_4 \mu_5 \\
-\epsilon \mu_1 \mu_5 & -\epsilon \mu_2 \mu_5 & -\epsilon \mu_3 \mu_5 & -\epsilon \mu_4 \mu_5 & \epsilon_5^2
\end{pmatrix}
\]
\( (28) \)

These examples have the interesting penomenological features as explained in the next section. Although we need additional tunings for the coupling constants to satisfy the condition shown by Eq. (13) as found from these examples, such tunings seem not to be so hard but rather mild as we can find by comparing Eqs. (25) and (28). We believe that it does not spoil the interesting feature of the present model.

For the charged lepton sector the same discrete symmetry can also determine the structure of the nonrenormalizable terms in the superpotential
\[
W_3 = \sum_{f_L,f_R} \frac{k_{lf_R}}{2x_{lf_R}} (S_2 \bar{S}_2)^{x_{lf_R}} (S_1 \bar{S}_1)^{x_{lf_R}} - x_{lf_R} L_L H_1 \bar{E}_{f_R}
\]
(29)

following the charge assignment in Table 2. By the use of this symmetry we can write down the charged lepton mass matrix \( M^{(e)} \) as follows:
\[
(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix}
k_{\bar{e}e} v_1 \epsilon_1^6 & k_{\bar{e}\mu} v_1 \epsilon_1^4 & k_{\bar{e}\tau} v_1 \epsilon_1^4 \\
k_{\bar{\mu}e} v_1 \epsilon_1^4 & k_{\bar{\mu}\mu} v_1 \epsilon_1^2 & k_{\bar{\mu}\tau} v_1 \epsilon_1^2 \\
k_{\bar{\tau}e} v_1 \epsilon_1^4 & k_{\bar{\tau}\mu} v_1 \epsilon_1^2 & k_{\bar{\tau}\tau} v_1 \epsilon_1^2
\end{pmatrix}\begin{pmatrix}
e_L \\
\mu_L \\
\tau_L
\end{pmatrix},
\]
(30)

where coupling constants again need to be tuned to realize the desirable charged lepton mass eigenvalues. Here we assume that \( k_{\bar{\mu}e} < k_{\bar{\mu}\tau} \) and also the off-diagonal couplings are smaller than these diagonal couplings. Then this mass matrix can be diagonalized by the bi-unitary transformation and results in the eigenvalues
\[
m_e \sim k_{\bar{e}e} v_1 \epsilon_1^6, \quad m_\mu \sim k_{\bar{\mu}e} v_1 \epsilon_1^4, \quad m_\tau \sim k_{\bar{\tau}e} v_1 \epsilon_1^2.
\]
(31)

The diagonalization matrix \( U^{(e)} \) is almost diagonal and can be written as
\[
U^{(e)} \sim \begin{pmatrix}
1 & -(k_{\bar{\mu}e} / k_{\bar{e}e}) \epsilon_1^2 & -(k_{\bar{\mu}\tau} / k_{\bar{e}\tau}) \epsilon_1^2 \\
(k_{\bar{\mu}e} / k_{\bar{e}e}) \epsilon_1^2 & 1 & -(k_{\bar{\mu}\tau} / k_{\bar{e}\tau}) \epsilon_1^2 \\
(k_{\bar{\mu}\tau} / k_{\bar{e}\tau}) \epsilon_1^2 & -(k_{\bar{\mu}\tau} / k_{\bar{e}\tau}) \epsilon_1^2 & 1
\end{pmatrix}
\]
(32)

A possibility of the mixings among sterile neutrinos and active neutrinos has already been proposed based on the nonrenormalizable interactions in the superpotential in the
context of superstring inspired models in Ref. [18]. Our model may be considered as a concrete example of its realization. Although our scheme may need some tunings at least for the coupling constants in the superpotential to make the mass matrix the required form defined by Eqs. (12) and (13) in the precise way, it is interesting that only the above values of $P_{fR}'$, $Q_{fL}$ and $T_{fL}'$ can approximately make it close to the required form. If we assume $u_1 \neq u_2$, more available freedom may be applicable. This kind of models may be recognized as one of many candidates for the possible neutrino mass matrix realized in the promising supersymmetric models inspired by perturbative superstring.

4 Analysis of various oscillations

We apply our model to the analysis of neutrino oscillations. Deficiencies of the solar neutrinos [2] and the atmospheric neutrinos [3, 4] have been suggested to be explained by $\nu_e \leftrightarrow \nu_x$ and $\nu_\mu \leftrightarrow \nu_y$ oscillations, respectively. Within a two flavor oscillation framework the neutrino squared mass differences and mixing angles predicted from these observations are for the solar neutrino problem [5, 6, 22],

$$\Delta m_{\nu_x\nu_e}^2 \sim (0.3 - 1.2) \times 10^{-5} \text{eV}^2, \quad \sin^2 2\theta \sim (0.4 - 1.5) \times 10^{-2},$$  

and for the atmospheric neutrino problem [3, 4],

$$\Delta m_{\nu_y\nu_\mu}^2 \sim (4 - 6) \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta \gtrsim 0.85.$$  

We also take account here that the existence of one neutrino species with mass such as $1 \sim 10^2 \text{ eV}$ has an interesting relevance to the astrophysical observations for the large scale structure of the universe.

In order to present the consistent explanation for the neutrino mixings shown in Eqs. (33) and (34), we need to identify the five light states $\psi_f$ with physical neutrino states. In this consideration the constraint from the standard big bang nucleosynthesis (BBN) [23] may be useful because it can constrain a sterile neutrino sector. The BBN predicts that the effective neutrino species during the primordial nucleosynthesis should be less than 3.3. This fact severely constrains the mixing angle $\theta$ and the squared mass difference $\Delta m^2$ between a sterile neutrino ($\nu_s$) and left-handed active neutrinos which mix with it.

\[8 \text{ There are also large mixing solutions. However, we do not consider these solutions in this paper.}\]
As long as we do not assume the large lepton number asymmetry at the BBN epoch, these constraints rule out the large mixing MSW solution of the solar neutrino problem due to $\nu_e \rightarrow \nu_s$ and also the explanation of the atmospheric neutrino problem by $\nu_\mu \rightarrow \nu_s$. Taking account of these facts, we concentrate our study on a possibility such that $\psi_1$ and $\psi_5$ are right-handed sterile neutrinos $\nu_{s_1}$ and $\nu_{s_2}$, and $\psi_2$, $\psi_3$ and $\psi_4$ are active neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$. In this identification the solar and atmospheric neutrino deficits are considered to be explained by the small mixing MSW solution due to $\nu_e \rightarrow \nu_{s_1}$ and the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation in the vacuum, respectively. In Table 1 this identification has been assumed.

Now we impose that the two flavors oscillation scheme is good enough for these oscillation processes. And then we can determine some of the mixing parameters numerically based on both oscillations in the ($\nu_{s_1}, \nu_e$) sector with $\Delta M_{12}^2$ and in the ($\nu_\mu, \nu_\tau$) sector through a mode with $\alpha = 3$ and $\beta = 4$ which are shown in Table 1. As $U^{(0)} = 1$ is assumed here, the desired mixing angles in Eqs. (33) and (34) can be realized by setting

$$16 \lesssim \frac{\mu_2}{\mu_1} \lesssim 32, \quad 0.44 \lesssim \frac{\mu_3}{\mu_4} \lesssim 2.3.$$  \hspace{1cm} (35)

In the following discussion we take these values as $\mu_2/\mu_1 \simeq 25$ and $\mu_3/\mu_4 \simeq 1$, for simplicity. To investigate other oscillation processes it is convenient to use the parameters $a$ and $b$ introduced in Eq. (26) which gives their physical meanings as the ratio of coupling constants. They can be also expressed as $\mu_2/\mu_3 \equiv 10^{-a}$ and $\mu_4/\mu_5 \equiv 10^{-b}$ and parametrize the mixing among different neutrino species. There seem to be no quantitative constraints on these parameters at the present stage. From the viewpoint that the solar and atmospheric neutrino deficits are explained by the two flavors scheme for the oscillation processes, it is enough for them to be sufficiently large.

However, if we take account of the BBN constraint in the more quantitative way, the $a$ and $b$ dependence of the mixing parameters seems to allow us to restrict the region of $a$ and $b$. As mentioned earlier, the restriction on the number of the effective neutrino species during the primordial nucleosynthesis gives the condition on the $\nu_{s_2}-\nu_{e,\mu,\tau}$ sector. Here it is sufficient to consider the most stringent one. In the two flavors oscillation scheme when
$m_{\nu_{s2}}^2 > m_{\nu_{e,\mu,\tau}}^2$ is satisfied, it can be formulated as \cite{24, 25},

$$\Delta m^2 \sin^4 2\theta \lesssim 3 \times 10^{-6} \text{eV}^2 \quad \text{for} \quad (\nu_{\mu,\tau}, \nu_s). \quad (36)$$

If we can apply this constraint to the ($\nu_\mu, \nu_{s2}$) and ($\nu_\tau, \nu_{s2}$) sectors in Table 1, nontrivial constraints on $b$ can be obtained.\footnote{We should be careful in this application since this constraint has been derived in the two flavors oscillation scheme. The BBN constraints are crucially affected by the interaction with the plasma at finite temperature so that the situation may be changed in many flavors case from the one of two flavor oscillation scheme. Although we need a numerical calculation for the correct analysis of this aspect, such a study is beyond the scope of this paper. However, this kind of consideration may be useful to show the importance of the BBN constraint in the model building.} This condition may be written as

$$10^{-4b}(M_5^2 - M_4^2) \lesssim 10^{-6.7}. \quad (37)$$

If we require that the heavier right-handed neutrino $\nu_{s2}$ can be a dark matter, a value of $M_5$ should be fixed so as to be a suitable value\footnote{This value should be changed depending on what kind of dark matter scenario is considered. Here we take a conservative value not far from required values in the various models \cite{1, 2, 3}.} and then the parameter $b$ should satisfy the following conditions:

$$b \gtrsim 1.6 \quad (M_5 \sim 1 \text{eV}), \quad b \gtrsim 2.1 \quad (M_5 \sim 10 \text{eV}), \quad b \gtrsim 2.6 \quad (M_5 \sim 10^2 \text{eV}). \quad (38)$$

If we consider the ($\nu_{s1}, \nu_{e,\mu,\tau}$) sector with $m_{\nu_{s1}}^2 < m_{\nu_{e,\mu,\tau}}^2$, through the BBN constraint given in Refs. \cite{24, 25} there appears no condition on $a$ for the case of $\Delta M^2 \sim 10^{-2.4}$ but $a \gtrsim 1$ should be satisfied for the case of $\Delta M^2 \sim 1$. The situation completely depends on the squared mass difference among $\nu_{s1}$ and $\nu_{e,\mu,\tau}$. Of course, if we assume the existence of the large lepton asymmetry at the BBN epoch, these constraints can disappear \cite{26}. However, a lower bound on the parameter $a$ can be always obtained from the condition on the amplitude in Eq. (7) such as $-4V_{\alpha f}^* V_{\alpha f} V_{\beta f}^* V_{\beta f} \leq 1$. Using Table 1, a weak constraint on $a$ is brought as $a \gtrsim 0.15$ from this condition.

In order to realize the desired squared mass differences in Eqs. (33) and (34), we need to require

$$M_1 \sim 10^{-2.5} \text{eV}, \quad |M_3^2 - M_4^2| \sim 10^{-2.4} \text{eV}^2. \quad (39)$$

If we take $M_5 \sim 10 \text{eV}$ as an example and use the first one in Eq. (39), the parameters in our model defined by Eq. (8) should be settled by using Eqs. (14) and (39) as,

$$m_1 \sim 10^{-5.4-(a+b)} M_1^2, \quad m_2 \sim 10^{-4-(a+b)} M_1^2,$$
where we use a GeV unit, and $a$ and $b$ should satisfy the constraint given by Eq. (38) and $a \gtrsim 0.15$. To make $|M_3^2 - M_4^2|$ a value presented in Eq. (39), there can be many possibilities for the values of $M_3$ and $M_4$. Here we consider the following typical two cases: (I) $M_3 < M_4 \sim 10^{-1.2}$ eV, and (II) $M_3 \simeq M_4 (\gg 10^{-1.2}$ eV). In the case (II) we take two eigenvalues as $M_3 \simeq M_4 \sim 1$ eV which has been studied in the various works as an interesting example, although such a choice requires a rather strict degeneracy between the third and fourth mass eigenvalues. Numerical expressions of the oscillation parameters for each of these two cases are given in the columns (I) and (II) of Table 1. We should also note that these cases with certain values of $a$ and $b$ can be realized as the two models (I) and (II) presented in the previous section.

If we observe Table 1 taking account of Eq. (38) and $a \gtrsim 0.15$, we immediately find that the very restricted oscillation modes can effectively occur and others are negligible because of the small amplitudes (mixing angles). There are two $a$-independent processes

$$(\nu_s, \nu_e) \text{ with } \Delta M_{12}^2, \quad (\nu_{\mu}, \nu_{\tau}) \text{ with } \Delta M_{34}^2,$$

(41)

and also as the $a$-dependent but non-negligible interesting oscillation modes, we have

$$(\nu_e, \nu_{\mu}) \text{ with } \Delta M_{23}^2, \Delta M_{24}^2, \quad (\nu_e, \nu_{\tau}) \text{ with } \Delta M_{34}^2.$$

(42)

As already mentioned, two processes given in Eq. (41) can be treated within the two flavors oscillation scheme, as long as the parameter $a$ takes a suitable value which can guarantee such a treatment. In that case they can be used for the explanation of the solar and atmospheric neutrino problems in both cases of (I) and (II). On the other hand, although some of the processes listed in Eq. (42) come out as the effects due to many flavors existence, they may bring about the important contributions to the $\psi_f \leftrightarrow \psi_{f'}$ oscillation according to the value of $a$. Next we examine both cases (I) and (II) in more detail and also discuss these processes in each case.

4.1 Case I : $M_3 < M_4 \sim 10^{-1.2}$ eV

In this case, from Eqs. (13), (14), (39) and (40), we should take the parameters as

$$A \sim 10^{2(\alpha + b) - 0.7} M, \quad B \sim -10^{2(\alpha + b) - 3.5} M,$$

$$C \sim D \sim E \sim 10^{2b - 2.5} M, \quad F \sim -10^{-2.2} M,$$

(43)
where $M_5 \sim 10$ eV and $a > 0.5$ are assumed. The allowed range of $a$ can determine the
sign of $\mathcal{B}$, since $\mathcal{B} < 0$ requires $(\mathcal{B} - \mathcal{D})/\mathcal{D} < -1$ and then $a > 0.5$. $\mathcal{D} \neq \mathcal{E}$ should be
reminded in order to resolve the mass degeneracy $M_2 = M_3 = 0$. The consistency of
this model requires that the largest mass parameter $A$ should be smaller than $M_{pl}$. This
requirement brings about an additional constraint on the parameters $a$ and $b$,
\begin{equation}
    a + b \lesssim 0.35 + \log_{10} \left( \frac{M_{pl}}{M} \right) \frac{1}{2}.
\end{equation}
If we take $M \sim 10^{12}$ GeV which is realized in the examples defined by Eq. (24) and use
Eq. (38), we can constrain the value of $a$, for example as follows:
\begin{equation}
    a \lesssim 1.75 \quad \text{for} \quad b \gtrsim 2.1 \quad (M_5 \sim 10 \text{ eV}).
\end{equation}
It should be also noted that we can judge the phenomenological validity and consistency
of the constructed models based on the discrete symmetry such as the ones shown in Eq.
(28) by comparing it with Eq. (43) and studying whether $a$ and $b$ satisfy the conditions
given by Eqs. (37) and (44). In the model defined by (I) of Eq. (28) we obtain $a \sim 1.5$
and $b \sim 2.2$, which satisfy these conditions.

In the basis of these knowledge we can readily investigate the processes shown in Eq.
(42). As easily found in Table 1, if we take $a \sim 1.25$ and $M_3 \sim 10^{-2.5}$, the oscillation
parameters of the first process with $\Delta M_{23}^2$ in Eq. (42) also seems to take appropriate
values for the small mixing MSW solution of the solar neutrino problem. In this case
the light sterile neutrino may not be necessary for the explanation of the solar neutrino
deficit. For smaller $a$, anyway, this process with larger $M_3$, $(\nu_e, \nu_\mu)$ with $\Delta M_{23}^2$ and $(\nu_e, \nu_\tau)$
with $\Delta M_{34}^2$ may be good targets of long baseline experiments. However, these processes
in the interesting regions of $M_3$ and $a$ does not seem to be the ones suitable for the
two flavors treatment. Especially, in these processes the matter effect seems not to be
analytically estimated in the precise way and we may not be able to apply the condition
given in Eq. (33) to this case naively. The numerical analysis for the oscillations among
many flavors will be indispensable for more detailed study. If we want to guarantee the
validity of the two flavor oscillation analysis for both the solar neutrino and atmospheric
neutrino problems due to $\nu_{s_1} \leftrightarrow \nu_e$ and $\nu_\mu \leftrightarrow \nu_\tau$, we need to require $a \gtrsim 1.3$ and then
$M \lesssim 10^{13}$ GeV, which can be satisfied in our model defined by Eq. (24). In such a case
all other oscillation processes than $\nu_{s_1} \leftrightarrow \nu_e$ and $\nu_\mu \leftrightarrow \nu_\tau$ listed in Eq. (41) unfortunately
seem to be inaccessible experimentally.
In this case all active neutrinos are too light to be a hot dark matter but $\nu_{s2}$ may be a warm dark matter with $m_{\nu_{s2}} = O(10 - 10^2)$ eV and $\Omega_{\nu_{s2}} \sim 1$ \cite{7}. The main problem is how it is sufficiently produced at the early universe. There seem to be two possibilities for its production. If $\nu_{s2}$ has an interaction with other light fields to be in the thermal equilibrium and then decouples as a relativistic particles, there is a relic \cite{27}

$$\Omega_{\nu_{s2}}h^2 = \frac{1}{g_*} \frac{m_{\nu_{s2}}}{8.5 \text{ eV}},$$

(46)

where $h = H_0/(100\text{km/sec/Mpc})$ and $H_0$ is the present Hubble constant. $g_*$ is the effective degrees of freedom of the light fields at the $\nu_{s2}$ decoupling time. In the present model $\nu_{s2}$ has the interaction with other fields due to the extra $U(1)$ gauge symmetry which breaks down at a very high energy scale $u_e$. The above formula is applicable to $\nu_{s2}$. If we assume $h \sim 0.5$ and $g_* \gtrsim 300$ which is usually expected for this type of supersymmetric models at the decoupling epoch in the very high energy scale, we have $\Omega_{\nu_{s2}} \sim 1$ for $M_5 \sim 10^2$ eV. If inflation occurs after the decoupling of $\nu_{s2}$, however, this possibility cannot be applied.

Another possibility is the production through the $\nu_{\mu,\tau}$-$\nu_{s2}$ oscillation as suggested in Ref. \cite{7}. In this case $\nu_{\mu,\tau}$ are at first in the thermal equilibrium and then decouples satisfying the same relation between $\Omega_{\nu}$ and $m_{\nu}$ given by Eq. (46). During this period $\nu_{s2}$ is considered to be produced from $\nu_{\mu,\tau}$ through the oscillation process $\nu_{\mu,\tau} \leftrightarrow \nu_{s2}$. When we take this possibility, the constraint on $M_5$ comes from the requirement for both of the sufficient abundance and the consistency with the BBN \cite{25}. If we follow Ref. \cite{7}, the ratio of the distribution functions $f_s$ and $f_A$ of sterile and active neutrinos can be estimated in our model as

$$\frac{f_s}{f_A} = \frac{6.0}{g_5^{1/2}} \left(\frac{M_{\mu_3\mu_5}}{\text{eV}}\right)^2 \left(\frac{\text{keV}}{M_{\mu_5}}\right), \quad \frac{\Omega_{\nu_{s2}}}{\Omega_A} = \frac{M_5 f_s}{M_4 f_A}.\tag{47}$$

By applying the hot relic relation given by Eq. (46) to these formulas and remembering $M_5 = M_{\mu_5}^2$, we can derive the following relation:

$$M_5 \frac{\mu_3}{\mu_5} = 0.22 h \Omega_{\nu_{s2}}^{1/2} \text{ eV}.$$\tag{48}

On the other hand, the BBN constraint requires $f_s/f_A \lesssim 0.4$ and if we impose this on the latter of Eq. (47) and use Eq. (46), we can obtain

$$M_5 \gtrsim 230 h^2 \Omega_{\nu_{s2}} \text{ eV}.\tag{49}$$
In this case we should take \( g_\ast \sim 10.8 \) and then the required values for \( M_5 \) and \( b = \ln(\mu_5/\mu_3) \) are \( M_5 \gtrsim 58 \) eV and \( b \gtrsim 2.7 \) for \( \Omega_{\nu_{s_2}} \sim 1 \) (WDM) and \( M_5 \gtrsim 17 \) eV and \( b \gtrsim 2.4 \) for \( \Omega_{\nu_{s_2}} \sim 0.3 \) (CHDM) \[13\]. This value of \( M_5 \) for the case of CDHM is so large that the free streaming length of \( \nu_{s_2} \) becomes too short and then seems not to contribute the structure formation at the supercluster scale \[9\]. Thus \( \nu_{s_2} \) can be expected only to play the role as the warm dark matter. In that case \( a \) cannot take large value and \( a \lesssim 1.1 \).

**4.2 Case II :** \( M_3 \simeq M_4 \sim 1 \) eV

In this case we need a rather strict fine tuning like \( M_4 - M_3 \sim 10^{-2.7} \) eV. If we assume such a fine tuning, the parameters can be settled as

\[
\begin{align*}
\mathcal{A} & \sim 10^{(a+b)-0.7} M, & \mathcal{B} & \sim -10^{(a+b)-3.5} M, \\
\mathcal{C} & \sim \mathcal{D} \sim 10^{2b-1} M, & \mathcal{E} & \sim 10^{2b-4} M, & \mathcal{F} & \sim -10^{-1} M
\end{align*}
\]

(50)

where \( M_5 \sim 10 \) eV and \( a > 1.3 \) is assumed. The condition given by Eq. (44) should be satisfied also in this case. If we try to realize this case in terms of the model defined by (II) of Eq. (28), we obtain \( a \sim 1.5 \) and \( b \sim 2.2 \) and the required conditions for \( a \) and \( b \) are fulfilled. The difference from the case (I) concerning the oscillation phenomena is that the rather large squared mass difference such as \( O(1) \) eV\(^2\) can appear between \( \nu_e \) and \( \nu_{\mu,\tau} \).

For such a squared mass difference, we can expect that the processes in Eq. (42) become interesting ones from the experimental viewpoint. If we apply the results of BNL E776 \[28\] and KARMEN \[29\] experiments for the \( \nu_e \) appearance through \( \nu_\mu \rightarrow \nu_e \) to the first process in Eq. (42), we can obtain a new lower bound on \( a \). Two flavors oscillation analysis of the data obtained by these puts the most stringent bound on a mixing angle among \( \nu_e \) and \( \nu_\mu \) such as \( \sin^2 2\theta \lesssim 7 \times 10^{-3} \) for \( \Delta m^2 = O(1) \) eV\(^2\). Using Table 1, we can obtain \( a \gtrsim 1.23 \) from this bound. This also satisfies the constraint from the \( \nu_e \rightarrow \nu_\mu \) transition from Burgey \[30\] which requires \( a \gtrsim 1.0 \) for the first one in Eq. (42) and also the constraint from the BBN which was mentioned before.

In this context the interesting experimental results are the ones of LSND \[19\]. The evidences for the oscillations \( \nu_\mu \rightarrow \nu_e \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) have been reported there. One of nice features in this case is that these LSND results seem to be explained in the present model in terms of the first two processes in Eq. (42). In fact, as we take \( \Delta M_{23}^2, \Delta M_{24}^2 \sim 1 \) eV\(^2\),
for this squared mass difference the LSND results require that the mixing angle should be 
\[3 \times 10^{-3} \lesssim \sin^2 2\theta \lesssim 1.5 \times 10^{-2}\] for \(\bar{\nu}_\mu \to \bar{\nu}_e\) and \(1.5 \times 10^{-3} \lesssim \sin^2 2\theta \lesssim 1.5 \times 10^{-1}\) for \(\nu_\mu \to \nu_e\).

These can be realized by taking \(1.1 \lesssim a \lesssim 1.4\) and \(0.6 \lesssim a \lesssim 1.6\), respectively. If these are combined with the above results of BNL E776 and KARMEN, \(a\) can be constrained to a very narrow region \(1.23 \lesssim a \lesssim 1.4\). If \(a\) takes a small value which cannot explain the LSND results, the second process in Eq. (42) may become a very interesting target in the future experiments. An interesting aspect of this case is that the above region of \(a\) relevant to the LSND results can put the constraint on the value of \(M_5\) which is the mass of a candidate of the dark matter. In fact, if we assume \(a \sim 1.4\) and \(M \sim 10^{12} \text{ GeV}\) as an example, we obtain the bound on \(M_5\) as \(M_5 \lesssim 36 \text{ eV}\) by combining Eq. (44) and the BBN constraint given by Eq. (37).

Anyway this case in our framework corresponds to an interesting realization of the model which has been pointed out by various authors [10], in which the solar and atmospheric neutrino deficits and the LSND results can be simultaneously explained and additionally \(\nu_\mu\) and \(\mu_\tau\) can be the hot dark matter candidates in the CHDM scenario. In this case we may naturally ask the physical role of \(\nu_{s2}\). The first interest is whether \(\nu_{s2}\) can have some affection for the structure formation or not. We can estimate this by using Eqs. (48) and (49) as in the previous case. For example, if we assume \(\Omega_{\nu_{s2}} \sim 0.1\), we have \(M_5 \gtrsim 5.8 \text{ eV}\) and also \(b \gtrsim 2.2\) which is included in the allowed region presented by Eq. (37). This suggests that \(\nu_{s2}\) may have some affection on the structure formation as a part of hot dark matter in the CHDM scenario. Another interesting possibility of the physical role of \(\nu_{s2}\) may be an effect on the leptogenesis discussed in Ref. [10]. We do not study it here but it may be an interesting aspect of our model.

4.3 Relation to the charged lepton sector

It may be useful to comment on the constraint on the charged lepton mass matrix in the present framework. Although we have assumed that the charged lepton mass matrix is diagonal up to now, it is also related to the neutrino oscillation phenomena through the mixing matrix \(V\) as shown in Eq. (7). If we once fix the mixing matrix of the neutrino sector as Eq. (11), the neutrino oscillation data can constrain the structure of the charged lepton mass matrix.

Here we consider two typical examples as the charged lepton mass matrix. The first
example (A) is represented by Eq. (30) which is obtained in the basis of the discrete symmetry. Using the mixing matrix $U(l)$ given in Eq. (32), we can obtain the mixing matrix elements $V_{af}$ defined in section 2 as follows,

\begin{align*}
V_{2e} & \simeq 1, \quad V_{2\mu} \simeq -e^{i\sigma} \frac{\mu_2}{\mu_3}, \quad V_{2\tau} \simeq -e^{i\sigma} \frac{\mu_2 k E_{\tau}}{\mu_3 k E_{\tau}}, \\
V_{3e} & \simeq \frac{1}{\sqrt{2}} \frac{\mu_2}{\mu_3}, \quad V_{3\mu} \simeq \frac{e^{i\sigma}}{\sqrt{2}}, \quad V_{3\tau} \simeq -\frac{e^{i\tau}}{\sqrt{2}}, \\
V_{4e} & \simeq \frac{1}{\sqrt{2}} \frac{\mu_2}{\mu_3}, \quad V_{4\mu} \simeq \frac{e^{i\sigma}}{\sqrt{2}}, \quad V_{4\tau} \simeq \frac{e^{i\tau}}{\sqrt{2}},
\end{align*}

(51)

where we have introduced phases $\sigma$ and $\tau$ for completeness.

As another typical phenomenological example (B), we adopt a Fritzsch mass matrix [31] for the charged lepton sector. Although it is not relevant to our construction of the neutrino sector based on the discrete symmetry given in Table 2, it will be useful to find the feature of the mixing matrix presented in Eq. (11). Using a well-known formula in the diagonalization of the Fritzsch mass matrix as $U(l)$, we can get the mixing matrix elements for the lepton sector as

\begin{align*}
V_{2e} & \simeq 1, \quad V_{2\mu} \simeq -\left(\frac{m_e}{m_\mu} - e^{i\sigma} \frac{\mu_2}{\mu_3}\right) \frac{m_\mu}{\sqrt{m_\mu m_\tau}}, \quad V_{2\tau} \simeq -e^{i\sigma} \frac{\mu_2 k E_{\tau}}{\mu_3 k E_{\tau}}, \\
V_{3e} & \simeq \frac{1}{\sqrt{2}} \left(e^{i\sigma} + e^{i\tau} \frac{m_\mu}{m_\tau}\right), \quad V_{3\mu} \simeq \frac{1}{\sqrt{2}} \left(e^{i\sigma} \frac{m_\mu}{m_\tau} - e^{i\tau}\right), \\
V_{4e} & \simeq \frac{1}{\sqrt{2}} \left(e^{i\sigma} - e^{i\tau} \frac{m_\mu}{m_\tau}\right), \quad V_{4\mu} \simeq \frac{1}{\sqrt{2}} \left(e^{i\sigma} \frac{m_\mu}{m_\tau} + e^{i\tau}\right), \\
V_{5e} & \simeq \frac{1}{\sqrt{2}} \frac{\mu_2}{\mu_3} + e^{i\sigma} \frac{m_e}{\sqrt{2}} \frac{1}{m_\mu} + e^{i\tau} \frac{m_e}{\sqrt{2}} \frac{m_\mu}{m_\tau}, \quad V_{4e} \simeq \frac{1}{\sqrt{2}} \frac{\mu_2}{\mu_3} + e^{i\sigma} \frac{m_e}{\sqrt{2}} \frac{m_\mu}{m_\tau} - e^{i\tau} \frac{m_e}{\sqrt{2}} \frac{1}{m_\mu},
\end{align*}

(52)

where $m_e, m_\mu$ and $m_\tau$ are charged lepton mass eigenvalues.

In both cases $V$ is found to have the similar form except that the latter example has extra contributions to the off-diagonal part compared to the former one. We present numerical values of the representative oscillation parameters for suitable settings of $a$ and $b$ in Table 3. From this table we find that both solar and atmospheric neutrino deficits can be simultaneously explained for these charged lepton mass matrices. Related to this point it may be useful to note that these charged lepton mass matrices have no effect on $V_{1e}$ and $V_{2s1}$, which are the elements of the above mixing matrix relevant to the $\nu_e \leftrightarrow \nu_{s1}$

\[\text{These expressions are somehow different from the ones given in Ref. [12] since we take the mass hierarchy given by Eq. (9) which should be assumed for the explanation of the solar neutrino problem.}\]
oscillation as found from Eq. (7). This feature is very different from the models in which the similar mass matrices are assumed for both the charged lepton and neutrino sectors. In those models, the mixings $V_{3\mu}$ and $V_{3\tau}$ has a tendency to become too large due to the contribution from the charged lepton sector without assuming suitable values of phases to explain the solar neutrino deficit by the small mixing MSW solution for $\nu_e \rightarrow \nu_\mu$ if we keep $V_{3\tau}$ and $V_{3\mu}$ to be suitable for the explanation of the atmospheric $\nu_\mu$ deficit due to $\nu_\mu \rightarrow \nu_\tau$ [32, 33]. This aspect also appears in the $(\nu_e, \nu_\mu)$ mixing of the case (B) in Table 3. The present model does not suffer from this problem as a direct result that the solar neutrino deficit is explained by the $\nu_e \leftrightarrow \nu_{s1}$ oscillation due to the introduction of a sterile neutrino. These two examples show that as long as the $U_{\ell}$ is approximately diagonal in the similar way to these examples, our scenario is always expected to be applicable independently of the details of the charged lepton mass matrix.

When we adopt these charged lepton mass matrices, the LSND results can be also explained in the case (II) in the same way as discussed in the previous part as long as we take $a$ in a suitable region around $a \sim 1.3$. The situation on the consistency with the BBN constraint is also similar to the case of $U_{\ell} = 1$ since the charged lepton sector has no important effect in the $(\nu_{s2}, \nu_\tau)$ sector. No contradiction happens against the BBN constraint through the oscillations in the $(\nu_{s2}, \nu_\tau)$ sector if $b$ and $M_{5\ell}$ are in the suitable region shown in Eq. (38). This feature can be seen in Table 3. For other processes the same discussions presented in the present section are also valid in the present case.

5 Summary

We proposed the neutrino mass matrix in the $3\nu_L + 3\nu_R$ framework, which could be constructed using the nonrenormalizable terms in the superpotential constrained by the suitable discrete symmetry. We showed that it could explain the solar and atmospheric neutrino deficits and give a dark matter candidate. We also discussed that there could be two typical parameter settings for Yukawa coupling constants, which brought about rather different phenomenological features. An interesting aspect of this model is that one of these parameter settings can also realize the mass and mixing pattern which has been known to explain the LSND results, simultaneously. It may be considered as another interesting feature of our mass matrix that it can explain both deficits of the solar and
atmospheric neutrinos without severely constraining the charged lepton mass matrix as long as it has no large off-diagonal elements. Although these features simply come from the extension of the parameter space due to the introduction of the new light sterile neutrino species, this kind of investigation can be considered to have a sufficient meaning to show the way for the extension of the neutrino sector.

The introduction of the light sterile neutrinos is usually considered to be artificial. However, their appearance seems to be not so unnatural as shown in this paper if we take account of the generation structure of quarks and charged leptons and also assume the constrained nonrenormalizable superpotential. One of such simple and promising candidates may be the extra U(1) models coming from the $E_6$ models inspired by the perturbative superstring, in which the group theoretical constraints on the Yukawa couplings are very weak. In such a case all but one or two of the right-handed neutrinos can be generally very light by the cooperation of both the superpotential constrained by the discrete symmetry and the extra U(1) D-flat direction. They can play the important role in the neutrino physics such as neutrino oscillations. Although this scheme seems to be successful, it is generally not so easy to yield small neutrino masses and to induce the neutrino oscillations without bringing other phenomenological difficulties like proton decay in this framework \[20, 34\]. The simultaneous explanation of them will be the next step to build the realistic models in this direction. Anyway we believe that it will be worthy to proceed the further investigation of this kind of possibilities.

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| $(\psi_f, \psi'_{f^\prime})$ | $\Delta M^2_{\alpha\beta}$ | $-4V_{\alpha f'}V^{*}_{\alpha f}V_{\beta f'}V^{*}_{\beta f}$ |
|-----------------|-----------------|-----------------|
| | (I) | (II) | (I), (II) |
| $(\nu_{s1}, \nu_e)$ | $\Delta M^2_{21}$ | $10^{-5}$ | $10^{-5}$ | $4\left(\frac{\mu}{\nu_2}\right)^2$ | $10^{-2.2}$ |
| | $\Delta M^2_{13}$ | $10^{-5}$ | 1 | $2\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a-2.5}$ |
| | $\Delta M^2_{14}$ | $10^{-2.4}$ | 1 | $2\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a-2.5}$ |
| | $\Delta M^2_{15}$ | $M^2_5$ | $M^2_5$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)-2.2}$ |
| $(\nu_{s1}, \nu_\mu)$ | $\Delta M^2_{23}$ | $10^{-5}$ | 1 | $2\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a-2.5}$ |
| | $\Delta M^2_{24}$ | $10^{-2.4}$ | 1 | $2\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a-2.5}$ |
| | $\Delta M^2_{25}$ | $M^2_5$ | $M^2_5$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)-2.2}$ |
| $(\nu_{s1}, \nu_\tau)$ | $\Delta M^2_{31}$ | $10^{-2.4}$ | $10^{-2.4}$ | $\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a-2.8}$ |
| | $\Delta M^2_{35}$ | $M^2_5$ | $M^2_5 - 1$ | $2\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)-2.5}$ |
| $(\nu_{s1}, \nu_{s2})$ | $\Delta M^2_{45}$ | $M^2_5$ | $M^2_5 - 1$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)-2.2}$ |
| $(\nu_e, \nu_\mu)$ | $\Delta M^2_{23}$ | $< 10^{-2.4}$ | 1 | $2\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a+0.3}$ |
| | $\Delta M^2_{24}$ | $10^{-2.4}$ | 1 | $2\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a+0.3}$ |
| | $\Delta M^2_{25}$ | $M^2_5$ | $M^2_5$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)+0.6}$ |
| $(\nu_e, \nu_\tau)$ | $\Delta M^2_{31}$ | $10^{-2.4}$ | $10^{-2.4}$ | $\left(\frac{\mu}{\nu_3}\right)^2$ | $10^{-2a}$ |
| | $\Delta M^2_{35}$ | $M^2_5$ | $M^2_5 - 1$ | $2\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)+0.3}$ |
| $(\nu_e, \nu_{s2})$ | $\Delta M^2_{45}$ | $M^2_5$ | $M^2_5 - 1$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2(a+b)+0.6}$ |
| $(\nu_\mu, \nu_\tau)$ | $\Delta M^2_{31}$ | $10^{-2.4}$ | $10^{-2.4}$ | $4\left(\frac{\mu}{\nu_3}\right)^2\left(1 + \left(\frac{\mu}{\nu_4}\right)^2\right)^{-2}$ | 1 |
| | $\Delta M^2_{35}$ | $M^2_5$ | $M^2_5 - 1$ | $2\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2b+0.3}$ |
| | $\Delta M^2_{45}$ | $M^2_5$ | $M^2_5 - 1$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2b+0.6}$ |
| $(\nu_\tau, \nu_{s2})$ | $\Delta M^2_{45}$ | $M^2_5$ | $M^2_5 - 1$ | $4\left(\frac{\mu}{\nu_5}\right)^2$ | $10^{-2b+0.6}$ |

**Table 1** Oscillation parameters for $\psi_f \leftrightarrow \psi'_{f^\prime}$. The positive contribution to the oscillation probability does not come from other combinations of $\alpha$ and $\beta$. In this table we assume $U^{(0)} = 1$. The state identification $\psi_f = (\bar{N}_A(\equiv s_1), \nu_{fL}, \bar{N}_B(\equiv s_2))$ is assumed.
\[
\begin{array}{c|cc|c|cc|c}
\text{Fields} & Z_9 & Z_9 & \text{Fields} & Z_9 & Z_9 \\
\hline
S_1 & (1 - \xi_1)/9 & 0 & L_e & -(1 + \xi_3)/9 & 0 \\
\bar{S}_1 & \xi_1/9 & 0 & L_\mu & -\xi_3/9 & 0 \\
S_2 & 0 & (1 - \xi_2)/9 & L_\tau & -\xi_3/9 & 0 \\
\bar{S}_2 & 0 & \xi_2/9 & \bar{E}_e & -(2 + \xi_1)/9 & 0 \\
\bar{N}_A & -(4 + \xi_1)/9 & -4/9 & \bar{E}_\mu & -(1 + \xi_1)/9 & 0 \\
\bar{N}_B & -(3 + \xi_1)/9 & -4/9 & \bar{E}_\tau & -(1 + \xi_1)/9 & 0 \\
\bar{N}_C & -(1 + \xi_1)/9 & 0 & \Phi & -2\xi_3/9 & -6/9 \\
H_{1,2} & (\xi_1 + \xi_3)/9 & 0 & \\
\end{array}
\]

Table 2  Charge assignments under $Z_9 \times Z_9$ discrete symmetry for lepton and Higgs sectors. $L_{f_L}$ and $E_{f_L}$ are the SU(2)$_L$ doublet and singlet lepton chiral superfields, respectively. $H_{1,2}$ and $\Phi$ are the usual doublet Higgs and triplet Higgs chiral superfields. $\xi_i$ ($i = 1, 2, 3$) are the integers which can give the nontrivial charges to each fields and satisfy $1 \leq \xi_i \leq 8$. 
\begin{tabular}{|c|ccc|c|}
\hline
\((\psi_f, \psi_{f'})\) & \(\Delta M^2_{\alpha\beta}\) & \(-4V_{\alpha f'}V_{\alpha f}V_{\beta f'}V_{\beta f}\) & Relevant Phenomena & \\
\hline
\((\nu_{s1}, \nu_e)\) & \(\Delta M^2_{21}\) & 6.4 \times 10^{-3} & 6.4 \times 10^{-3} & Solar Neutrino Deficit \\
\((\nu_\mu, \nu_\tau)\) & \(\Delta M^2_{34}\) & 1 & 0.9 & Atmospheric Neutrino Deficit \\
\((\nu_e, \nu_\mu)\) & \(\Delta M^2_{23}\) & 5.1 \times 10^{-3} & 4.1 \times 10^{-2} & LSND \\
\((\nu_e, \nu_\mu)\) & \(\Delta M^2_{24}\) & 5.1 \times 10^{-3} & 1.9 \times 10^{-2} & LSND \\
\((\nu_e, \nu_\tau)\) & \(\Delta M^2_{34}\) & 2.6 \times 10^{-3} & 1.3 \times 10^{-2} & Prediction of the model \\
\((\nu_e, \nu_\tau)\) & \(\Delta M^2_{24}\) & 5.1 \times 10^{-4} & 3.1 \times 10^{-3} & Prediction of the model \\
\((\nu_\mu, \nu_\tau)\) & \(\Delta M^2_{34}\) & 5.0 \times 10^{-4} & 2.7 \times 10^{-3} & Prediction of the model \\
\((\nu_{s1}, \nu_\mu)\) & \(\Delta M^2_{23}\) & 8.0 \times 10^{-6} & 2.4 \times 10^{-5} & BBN Constraint \\
\((\nu_{s1}, \nu_\tau)\) & \(\Delta M^2_{24}\) & 8.0 \times 10^{-6} & 2.4 \times 10^{-6} & BBN Constraint \\
\((\nu_{s2}, \nu_e)\) & \(\Delta M^2_{45}\) & 1.6 \times 10^{-7} & 4.1 \times 10^{-8} & BBN Constraint \\
\((\nu_{s2}, \nu_\mu)\) & \(\Delta M^2_{45}\) & 6.4 \times 10^{-5} & 2.3 \times 10^{-4} & BBN Constraint \\
\((\nu_{s2}, \nu_\tau)\) & \(\Delta M^2_{45}\) & 6.4 \times 10^{-5} & 6.0 \times 10^{-4} & BBN Constraint \\
\hline
\end{tabular}

\begin{table}
\caption{Numerical values of neutrino oscillation parameters for the charged lepton mass matrices (A) and (B). We take parameters as \(a \sim 1.3\), \(b \sim 2.4\), \(\sigma \sim \tau \sim 0\) and \(k_{E}^{\mu\tau}/k_{E}^{\tau\tau} = 0.1\). Assuming the mass differences which can explain the solar and atmospheric neutrino deficits, only the dominant contributions in each \((\psi_f, \psi_{f'})\) sector are presented.}
\end{table}