Candidates for Inflaton in Quiver Gauge Theory

Paul H. Frampton and Tomo Takahashi

Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255, USA

Abstract

The inflaton necessary to implement the mechanism of inflationary cosmology has natural candidates in quiver gauge theory. We discuss the dimensionless coefficients of quartic couplings and enumerate scalars which are singlet under the low-energy gauge group. The inflaton mass is generally predicted to be in the TeV region, close to 4 TeV for one specific unified model. A quartic inflaton potential, and a mutated hybrid inflation, are discussed. They can give adequate inflation and appropriate fluctuations but different spectral indices.
1 Introduction

The unadorned Big Bang model has well-known shortcomings, particularly the extreme fine-tuning of initial conditions necessary to accommodate the flatness and horizon issues. Inflation [1] is the leading candidate to augment the Big Bang and it is hardly necessary to justify here its successes, not only in solving the flatness and horizon problems but also in providing a plausible origin of the primordial fluctuations which lead to structure formation.

Inflation, however, requires the presence of one or more scalar fields whose potential underlies the mechanism for early era of super-rapid cosmic expansion. In the minimal standard model (SM) of particle phenomenology the only scalar is the Higgs boson which cannot be the inflaton for the obvious reason that it would not respect the gauge invariance of the SM in the early universe. Usually, therefore, one or more additional scalar fields are postulated to set up the inflaton potential in a way unconnected to the SM. Some partially successful efforts #1 have been made to identify the inflaton within supersymmetry theory e.g. [3] and in superstring theory e.g. [4].

In this article we study how quiver gauge theories which have been suggested as attractive extensions of the SM up to TeV energies may at the same time provide natural candidates for the scalars necessary for the inflationary scenario in the early universe.

It will turn out that there is a plethora of possible scalars to play the inflaton role and the quartic potential describing them has a lot of flexibility in its coefficients. Here we merely make a preliminary exploration of the types of inflaton potential which are suggested by the quiver gauge theory.

2 Quiver Gauge Theories

It has been argued that the appropriate way to extend the standard model to higher energies in the TeV regime, as an alternative to either supersymmetry or technicolor, is to embed it in a nonsupersymmetric quiver gauge theory. It is conjectured that such a theory achieves naturalness in the sense of absence of quadratic divergences [5,6]

Such theories have been described in detail elsewhere so suffice it to characterize two types of abelian quiver $\mathcal{N} = 0$ theories, based on $Z_7$ and $Z_{12}$ respectively.

The theories are specified by the embedding of the abelian finite group $\Gamma = Z_p$ in the $SU(4)$ global symmetry of the ancestral $\mathcal{N} = 4$ $U(N)$ super Yang-Mills (SYM) model.

The embedding is defined by the 4 of SU(4) but for the scalar sector it is adequate to cite the corresponding 3 in the $6 = 3 + 3^*$. Thus for the three viable $Z_7$ theories of [7] we have respectively:

$a_i = (2, 3, 3)$ (Model 7B), $a_i = (1, 1, 3)$ (Model 7D) and $a_i = (1, 2, 2)$ (Model 7E).

#1For a comprehensive review, see [2]
The corresponding labeling of the heptagonal quiver nodes are:

(7B) - C - W - H - W - H - H - H -
(7D) - C - W - H - H - W - H -
(7E) - W - H - H - W - W - H -
where the C, W, H refer to the three gauge groups in $SU(3)_C \times SU(3)_W \times SU(3)_H$ trinification. The two ends of the quivers are in each case identified.

In the $Z_{12}$ model of [8] the embedding is $3 = (3, 4, 5)$ and the node identification on the dodecahedral quiver is

- C - W - H - C - W - H -

In this $Z_{12}$ model the conformal scale is about 4 TeV (more precisely 3.8 TeV). In the $Z_7$ models it also must be no too far above the weak scale to allow the correct value of the electroweak mixing angle $\sin^2 \theta$ to survive, so here again it is at the TeV scale.

3 Standard Model Singlets

The inflaton fields must necessarily be singlets under the SM gauge group to avoid unwanted symmetry breaking above the weak scale if the inflation involves scalar values which are at higher energy. The bifundamental scalars of the $Z_p$ quiver model all contain eighteen real fields and transform under a pair of the gauge groups in the trinification $SU(3)_C \times SU(3)_W \times SU(3)_H$.

We analyze each of the six possible pairs from C, W, H in turn:

(C,C)

This bifundamental transforms under color $SU(3)_C$ as $2(1 + 8)$ and carries no $SU(2) \times U(1)$ quantum numbers.

(C, C) contains two 321-singlets.

(C,W) + (W,C)

Using the weak hypercharge formula [9]

$$Y = \frac{2}{\sqrt{3}} (T_{sw} - T_{sh})$$  \hspace{1cm} (1)

with $(2/\sqrt{3})T_8 = (1/3) \text{ diag}(1, 1, -2)$ we see that all the components have non-zero $Y$.

(C, W) + (W, C) contains no 321-singlet.
(W,W)
This corresponds to 2(8 + 1) under SU(2)_L, singlet under SU(3)_C and U(1)_Y.

(W,W) contains two 321-singlets.

(C,H)+(H,C)
Similarly to (C, W) + (W, C), all components have non-vanishing Y, and triplet color.

(C, H) + (H, C) contains no 321-singlet.

(W,H)+(H,W)
There are several ways to see that this has two 321-singlets. Think of the 27 of E(6) which contains 27 = 10 + 5 + (5 + 5) + 1 + 1 under SU(5). One can confirm this in [10].

(W, H) + (H, W) contains two 321-singlets.

(H,H)
This is 2(8 + 1) and has no color or weak isospin. The 1 has vanishing Y.

(H, H) contains two 321-singlets.

4 Counting Candidate for Inflatons

Let us first consider a general model which contains the SM. The node identification on
the quiver is some pattern of C, W, and H and the scalar embedding is \( a_i = (a_1, a_2, a_3) \).
If we consider the scalar bifundamentals emanating from the node \( p \) there are six of them
\( (i = 1, 2, 3) \):

\[
\left[(3_p, 3^*_{p+a_1}), (3_{p-a_1}, 3^*_p)\right]
\]

(2)

and we can label these six fields as \( \Phi_i^{(p)}, \Phi_i^{(p)'} \) respectively. We now define linear combina-
tions:

\[
\Psi_k^{(p)} = \sum_i \left[ \lambda_{ki}^{(p)} \Phi_i + \lambda_{ki}^{(p)'} \Phi_i^{(p)'} \right]
\]

(3)

From [11] the relevant potential is summing \( p \) from 1 to 7:

\[
V = 2g_0^2 \sum_{p=1}^7 \left[ \left( \sum_{i=3}^3 |(3_p, 3^*_{p+a_1})|^2 \right)^2 - \sum_{i,j=1}^3 |(3_{p-a_1}, 3^*_p)|^2 |(3_p, 3^*_{3+a_1})|^2 \right]
\]

(4)
together with other terms not relevant to this discussion.

By studying the linear combinations in Eq. (3) and asking that in Eq. (4) the coefficient of $|\Psi_k|^4$ be arbitrarily adjustable, it can be shown that for each fixed node number $p$ there are exactly five independent $\Psi_k$ which correspond to the desirable directions.

However, a second requirement for a candidate inflaton is that it be a 321-singlet. This uses the analysis of the previous Section and is model dependent.

Let us first examine the $Z_7$ models in [7]. For the case (7B), all node except the one C node generate five candidate directions so the total number is $6 \times 5 = 30$. The same number of such directions is available for the others $Z_7$ cases (7D) and (7E).

In the $Z_{12}$ model, the two C nodes do not generate appropriate directions. By looking at 321-singlets, there is also a reduction by one direction for each of one W and one H node which bisect the two C nodes on the opposite side of the quiver. The total number of candidate inflatons for $Z_{12}$ is therefore $10 \times 5 - 2 = 48$.

For the scalars so enumerated, linear combinations can give arbitrary coefficients in the resultant quartic inflaton potential.

5 Inflaton Potential

Now we consider an inflaton potential arising from the quiver gauge theory. Since there are many quartic terms in the scalar potential, considerations in the previous sections lead initially to the following simplest possibility for the inflaton potential:

$$V(\Phi) = V_0 + \frac{1}{4} \lambda \Phi^4$$

where $V_0 = M_{CSB}^4$. $M_{CSB}$ is the scale at which the conformal symmetry breaks down. Since the energy scale at which conformality is broken lies between the unification scale $\Lambda \simeq 4$ TeV and the (reduced) Planck scale $M_{\text{Planck}} \simeq 2.4 \times 10^{18}$ GeV, we assume $\Lambda < M_{CSB} < M_{\text{Planck}}$ in the following arguments. Furthermore, the scalar fields which are inflaton candidates must not develop vacuum values $< \Phi >$ which are greater than $\Lambda$ during inflation to avoid breaking the $SU(N)^p$ gauge symmetry. Thus such values are very much smaller than the inflation scale $< \Phi > \ll M_{CSB}$.

Having these consideration in mind, now we look into the inflation with the potential given by Eq. (5). In the slow-roll approximation, the number of e-foldings $N$ during inflation is given by [2]:

$$N \simeq \frac{1}{M_{\text{Planck}}^2} \int_{\Phi_{\text{end}}}^{\Phi} d\Phi \frac{V}{V'}$$

(6)
where a prime denotes a derivative with respect to $\Phi$. For the potential Eq. (5) $N$ becomes

$$N = \frac{V_0}{2\lambda M_{\text{Planck}}^2} \left( \frac{1}{\Phi_{\text{end}}^2} - 1 \right) \simeq \frac{1}{2\lambda} \left( \frac{M_{\text{CSB}}}{\Phi_{\text{end}}} \right)^2 \left( \frac{M_{\text{CSB}}}{M_{\text{Planck}}} \right)^2$$

where we assumed $\Phi_{\text{end}} \ll \Phi$.

If we put a coupling $\lambda \sim 1$ then for scalar values below TeV scale and the conformality symmetry breaking scale $M_{\text{CSB}} \gtrsim \Lambda$, we can have the sufficient amount of inflation which is needed to solve the horizon and flatness problems. Now we consider the density perturbation. One of the major virtues of the inflation is that it can generate the density fluctuation which is the origin of cosmic microwave background anisotropy and large scale structure\(^{\text{*2}}\). The normalization\(^{\text{*3}}\) of the spectrum at the COBE scale is given by $\delta_H \sim 2 \times 10^{-5}$ where $\delta_H$ can be written as

$$\delta_H = \frac{2}{5} P_R = \frac{1}{\sqrt{75\pi^2}} \frac{V^{3/2}}{M_{\text{Planck}}^3 V'}$$

With the potential Eq. (5),

$$\frac{V^{3/2}}{M_{\text{Planck}}^3 V'} \simeq \frac{M_{\text{CSB}}^6}{\lambda \Phi_{\text{COBE}}^3 M_{\text{Planck}}^3}$$

which should be $5 \times 10^{-4}$.

Since $\Phi_{\text{end}} \leq \Phi_{\text{COBE}}$ in this model, from Eq. (7),

$$\frac{V^{3/2}}{M_{\text{Planck}}^3 V'} \geq \sqrt{8\lambda N^3}$$

For $\lambda \sim 1$, $\Phi \leq 1$ TeV, and using\(^{\text{*4}}\) $N \simeq 50$, the fluctuation will be too large as seen from Eq. (10). However we can make use of defining linear combinations of fields which appear in the theory, and can use different values (e.g. $\lambda \sim 10^{-13}$, $M_{\text{CSB}} \sim 10^4$ TeV). With

\(^{\text{*2}}\)Other mechanisms to generate the density perturbation have been proposed such as the curvaton mechanism [12] and the inhomogeneous reheating scenario [13]. However we do not consider such scenarios here.

\(^{\text{*3}}\)Although we do not need the precise value of $\delta_H$ for the purpose of this paper, we give the value from WMAP [14]. The observation of WMAP gives $(25/4)\delta_H^2 = 2.95 \times 10^{-9}$ with $A = 0.9 \pm 0.1$ at $k = 0.05\text{Mpc}^{-1}$ for a power-law $\Lambda$CDM model.

\(^{\text{*4}}\)Actually, the number $N$ depends on the thermal history of the universe as [2]

$$N = 62 - \ln \left( \frac{k}{a_0 H_0} \right) - \ln \left( \frac{10^{16}\text{GeV}}{V_k^{1/4}} \right) + \ln \left( \frac{V_k^{1/4}}{V_{\text{end}}^{1/4}} \right) - \frac{1}{3} \ln \left( \frac{V_{\text{end}}^{1/4}}{\rho_{\text{reh}}} \right),$$

where the subscripts “$k$,” “end” and “reh” indicate the value of quantities when $k = a H$, the inflation ends and after reheating respectively. For the purpose of this article, we can take $N = 50$.\(^{\text{5}}\)
these parameter values, we can have the right amount of fluctuation. In this potential, the slow-roll parameters are

$$
\epsilon = \frac{1}{2} \lambda^2 \left( \frac{M_{\text{Planck}}}{M_{\text{CSB}}} \right)^2 \left( \frac{\Phi}{M_{\text{CSB}}} \right)^6,
$$

$$
\eta = 3\lambda \left( \frac{M_{\text{Planck}}}{M_{\text{CSB}}} \right)^2 \left( \frac{\Phi}{M_{\text{CSB}}} \right)^2.
$$

(12) (13)

Since $\epsilon$ can be neglected compared to $\eta$, we have

$$
n - 1 \simeq 2\eta = 6 \lambda \left( \frac{M_{\text{Planck}}}{M_{\text{CSB}}} \right)^2 \left( \frac{\Phi}{M_{\text{CSB}}} \right)^2
$$

(14)

which is a slightly blue-tilted spectrum. For example, if we choose the parameters as $\lambda \sim 10^{-13}$, $M_{\text{CSB}} \sim 10^4$ TeV and $\Phi \sim 100$ GeV, we get $n - 1 \sim 0.03$.

We can have other fields in the general scalar potential. Thus, next we consider the possibility of an inflaton potential which involves two scalar fields.

*Mutated Hybrid Inflation*

From the consideration in the previous section, we can have a more general potential involving two scalars which can be written as

$$
V(\Phi, \Psi) = V_0 - \frac{1}{2} m^2 \Psi^2 + \frac{1}{3} \lambda \Psi^3 \Phi
$$

(15)

An inflation model with this kind of potential is known as “mutated hybrid inflation” [2]. In the potential above, for some fixed $\Phi$, there exists a minimum for $\Psi_c = -m^2/\lambda \Phi$. Substituting $\Psi_c$ into the potential, $V$ can be written as

$$
V = V_0 (1 - \mu \Phi^{-2})
$$

(16)

where $\mu = m^6/(6\lambda^2 V_0)$. Now we consider the number of e-foldings and density perturbation from this potential. Using Eq. (16), The number of e-foldings is given by:

$$
N \approx \frac{\Phi^4}{8 M_{\text{Planck}}^2 \mu}
$$

(17)

With this equation, the value of $\Phi$ during inflation can be written with $N$. The density perturbation generated by this potential is, Using Eqs. (16) and (17),

$$
\frac{V_0^{3/2}}{M_{\text{Planck}}^3 V'} = \frac{\sqrt{V_0} \Phi^3}{2 \mu M_{\text{Planck}}^3} = \left( \frac{M_{\text{CSB}}}{M_{\text{Planck}}} \right)^2 \mu^{-1/4} \left( 32 M_{\text{Planck}}^2 N^3 \right)^{1/4} \sim 5 \times 10^{-4}.
$$

(18)
From Eq. (17), assuming $\Phi \sim 1$ TeV and using $N \simeq 50$, we can fix the value of $\mu$. Putting this value into Eq. (18), we find $M_{\text{CSB}} \simeq 10^5$ TeV which is reasonable value for $M_{\text{CSB}}$. In other words, choosing the value of $\mu$ appropriately, this model gives the sufficient e-folding number as well as right amount of the density perturbation.

To see the spectral index of the scalar perturbation, first we write down the slow-roll parameters

$$
\epsilon = \frac{2M_{\text{Planck}}^2 \mu^2}{\Phi^6} = \frac{1}{32N^2} \left( \frac{\Phi}{M_{\text{Planck}}} \right)^2,
$$

$$
\eta = -\frac{6M_{\text{Planck}}^2 \mu}{\Phi^4} = -\frac{3}{4N}.
$$

The spectral index is

$$
n - 1 = 2\eta - 6\epsilon \simeq -\frac{3}{2N}
$$

where we neglect $\epsilon$ since it is negligible for $\Phi \lesssim \Lambda$. Hence this model predicts a slightly red-tilted spectrum which is consistent with current observations. Furthermore, because of the low inflation scale in this model, the contribution of gravitational waves is negligible, which is also consistent with observations.

6 Discussion

Here we have made an attempt to connect particle phenomenology to the inflationary era in the early universe.

In CFT there is no shortage of candidates for the scalar fields to be involved in the inflaton potential. By taking linear combinations of scalar fields which are ubiquitous in the theory thereof we can arrive at a quartic potential with quite general coefficients with arbitrary sign.

Taking the simplest single-inflaton potential\(^5\) we arrived at sufficient inflation by assuming the conformal symmetry breaking scale is much higher than the TeV scale and that the scalar values are around the TeV scale. In this case, primordial fluctuation with a slightly blue-tilted spectrum is predicted.

By taking a more general mutated hybrid potential involving two scalar fields, not only an adequate amount of inflation was achieved but also the correct size for the primordial fluctuations with a slightly red-tilted spectrum. Again this assumed that the conformal symmetry breaking scale is at about $10^5$ TeV while the scalar values remain at or below a TeV scale.

\(^5\)We have provisionally assumed that 1-loop contributions to $V$ are negligible although this assumption depends on the details of conformal symmetry breaking as well as renormalization properties of quiver gauge theories which are currently under study.
Thus we conclude that quiver gauge theory does contain natural candidate(s) for inflaton(s).

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