Contribution of $b \to sgg$ through the QCD anomaly in exclusive decays $B \to (\eta', \eta)(K, K^*)$

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The charm content in $\eta'$ is estimated using the QCD $U(1)$ axial anomaly applied to $b \to sgg \to ^{c}\eta' s$. We find $f_{^{c}\eta'} \approx -3.1$ MeV for $m_c = 1.3$ GeV. Our estimate agrees with other independent methods by Feldmann and Kroll, and Araki, Musakhanov and Toki. The resulting branching ratios for $B \to \eta K$ in the generalized factorization model are marginally consistent with the CLEO data, although they are lower than the data by $\sim 2$.

1 Introduction

In the year of 1997, the CLEO collaboration reported measurements in a number of exclusive two-body non-leptonic decays of the type $B \to h_1 h_2$, where $h_1$ and $h_2$ are light mesons and the inclusive decay $B^\pm \to \eta' X_s$. In particular, large branching ratios into the final states including $\eta'$ are reported:

\begin{align}
BR(B^\pm \to \eta' + X_s) &= (6.2 \pm 1.6 \pm 1.3) \times 10^{-4} \\
&\quad \text{for } 2.0 \text{ GeV} \leq p_{\eta'} \leq 2.7 \text{ GeV}, \\
BR(B^\pm \to \eta' + K^\pm) &= (7.1^{+2.5}_{-2.1} \pm 1.0) \times 10^{-5}, \\
BR(B^0 \to \eta' K^0) &= (5.3^{+2.8}_{-2.2} \pm 1.2) \times 10^{-5}.
\end{align}

They did not observe any decay involving the $(\eta K)$ or $(\eta, \eta')K^*$ modes. These measurements have stimulated a lot of theoretical activity, both in the inclusive and exclusive decays involving $\eta'$ mesons.

Such unexpectedly large branching ratios of $B$ decays into the final states with an $\eta'$ meson led to an idea that the charm content in $\eta'$ meson might be very large. The amplitude for $b \to s(^{c}\bar{c}c) \to s(\eta', \eta)$ can be parametrized as

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\[ \langle \eta' | c_{\mu} \gamma_5 | c \rangle(0) = -i f^c_{\eta'} g_{\mu}, \text{ and similarly for the } \eta \text{ case. The relevant quantity } f^c_{\eta'} \text{ is often referred to as the charm content of the } \eta^{(')} \text{. This quantity is } a \text{ priori not known, but it was claimed that the data (1)-(3) can be accomodated if } f^c_{\eta'} \text{ is as large as } (50 - 180) \text{ MeV.} \]

However, this is uncomfortably large compared to the typical meson decay constants, e.g., \( f_\pi = 132 \text{ MeV.} \) Later it turned out that they could be determined in a number of different ways, also including the \( B \)-decays combined with the QCD \( U_A(1) \) anomaly being discussed here.

In this talk, I’d like to present another method for computing the contribution of the amplitudes \( b \to s(gg) \to s(\eta', \eta) \). This method is based on calculating the amplitude for the chromomagnetic penguin process \( b \to sgg \), followed by the transitions \( gg \to (\eta', \eta) \) which are calculated using the QCD anomaly, determining both the sign and magnitude of these contributions. The branching ratios for \( B^\pm \to (\eta', \eta)(K^\pm, K^{*\pm}) \) and \( B^0 \to (\eta', \eta)(K^0, K^{*0}) \) based on the QCD-anomaly method are calculated in this letter and compared with the present CLEO measurements and with the ones in Ref. [5]. We find that the theoretical branching ratios for \( B^\pm \to \eta' K^\pm \) and \( B^0 \to \eta' K^0 \) are almost equal and both are in the range \((2 - 4) \times 10^{-5}\), in agreement with the estimates in Ref. [5].

### 2 Estimate of \( b \to (\eta, \eta')s \) via QCD anomaly

Let us consider the charm-anticharm pair into two gluons, followed by the transition \( gg \to \eta' \) (see Fig. 1). The first part of this two-step process, i.e. \( b \to s(\bar{c}c) \to g(k_1)g(k_2) \) which amounts to calculating the charm-quark-loop from which two gluons are emitted, has been worked out by Simma and Wyler in the context of a calculation in the full theory. Their result is readily translated to our effective theory approach and can be compactly written as a new (induced) effective Hamiltonian \( H^{gg}_{\text{eff}} \).

\[
H^{gg}_{\text{eff}} = -\frac{\alpha_s}{2\pi} \left( C_2^{\text{eff}} + \frac{C_1^{\text{eff}}}{N_c} \right) \frac{G_F}{\sqrt{2}} V_{cb} V^{*}_{cs} \Delta i_5 \left( \frac{q^2}{m_c^2} \right) \frac{1}{k_1 \cdot k_2} \times G_{a\beta} (D_{\beta} \tilde{G}_{\alpha\mu}) a \bar{s} \gamma^\mu (1 - \gamma_5)b,
\]

with \( \tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu
u\alpha\beta} G^{\alpha\beta} (\epsilon_{0123} = +1) \). The \( C_{1,2}^{\text{eff}} \) are modified Wilson coefficients for the current-current operators as described in Ref. [5]. In this formula, which holds for on-shell gluons \( (q^2 = (k_1 + k_2)^2 = 2k_1 \cdot k_2) \), the sum over color indices is understood. The function \( \Delta i_5(q^2/m_c^2) \) is defined as

\[
\Delta i_5(z) = -1 + \frac{1}{z} \left[ \pi - 2 \arctan \left( \frac{4}{z} - 1 \right)^{1/2} \right]^2, \text{ for } 0 < z < 4.
\]

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Note that the $b \to sgg$ calculation brings in an explicit factor of $\alpha_s$. However, this explicit $\alpha_s$ factor gets absorbed into the matrix element of the operator resulting from the anomaly. So, to the order that we are working, we use the coefficients $C_1^{\text{eff}}$ and $C_2^{\text{eff}}$ in Eq. (4). The $u\bar{u}$ contribution in Fig. 1 is suppressed due to the unfavourable CKM factors. The $t\bar{t}$ contribution is included in the effective Hamiltonian via the $bsgg$ piece present in the operator $O_g$. However, in the factorization framework, the $bsgg$ term in $O_g$ does not contribute to the decays discussed. So, the $c\bar{c}$ contribution in Fig. 1 is the only one that survives.

Working out the hadronic matrix element of Eq. (4) using factorization, we now need to evaluate the matrix elements:

$$\langle \eta | G^{\alpha \beta}_a (D_\beta \tilde{G}_{\alpha \mu, a}) | 0 \rangle,$$

which can be written as

$$G^{\alpha \beta}_a (D_\beta \tilde{G}_{\alpha \mu, a}) = \partial_\beta (G^{\alpha \beta}_a \tilde{G}_{\alpha \mu, a}) - (D_\beta G^{\alpha \beta}_a) \tilde{G}_{\alpha \mu, a}.$$  \hspace{1cm} (7)

Now we can discard the second term since it is suppressed by an additional power of $g_s$ which follows on using the equation of motion, and furthermore, the first term is enhanced by $N_c$ in the large $N_c$ limit. The matrix elements of $\partial_\beta (G^{\alpha \beta}_a \tilde{G}_{\alpha \mu, a})$ are related to those of $G_{\tilde{G}}$:

$$\partial_\beta \langle \eta | G^{\alpha \beta}_a \tilde{G}_{\alpha \mu, a} | 0 \rangle = \frac{i q_\mu}{4} \langle \eta | G^{\alpha \beta}_a \tilde{G}_{\alpha \beta, a} | 0 \rangle.$$  \hspace{1cm} (8)

The conversion of the gluons into $\eta$ and $\eta'$ is described by an amplitude which is fixed by the $SU(3)$ symmetry and the axial $U(1)$ current triangle anomaly. The matrix elements for $G_{\tilde{G}}$ can be written as

$$\langle \eta | G^{\alpha \beta}_a \tilde{G}_{\alpha \beta, a} | 0 \rangle = m_{\eta, \eta'}^2 f_{\eta, \eta'}^a.$$  \hspace{1cm} (9)

In Eq. (9) the decay constants $f_{\eta, \eta'}^a$ read

$$f_{\eta}^a = \frac{f_8}{\sqrt{6}} \cos \theta_8 - \frac{f_0}{\sqrt{3}} \sin \theta_0, \quad f_{\eta'}^a = \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0,$$

where the coupling constants $f_8$, $f_0$ and the mixing angles $\theta_8$ and $\theta_0$ have been introduced earlier. We follow here the two-angle $(\eta, \eta')$ mixing formalism of Ref. [21], where the mass eigenstates $|\eta\rangle$ and $|\eta'\rangle$ have the following decompositions:

$$|\eta\rangle = \cos \theta_8 |\eta_8\rangle - \sin \theta_0 |\eta_0\rangle,$$

$$|\eta'\rangle = \sin \theta_8 |\eta_8\rangle + \cos \theta_0 |\eta_0\rangle.$$  \hspace{1cm} (11)
Collecting the individual steps, the matrix elements in Eq. (6) can be written as

$$\langle \eta'(q)|\frac{\alpha_s}{4\pi}G_{\mu}^{\alpha}(D_{\beta}\tilde{G}_{\alpha\mu})|0\rangle = i q_{\mu} \frac{m_{\eta'}/4}{f_{\eta'}(\gamma)} .$$

(12)

One would have naively expected that the gluonic matrix elements are small since they contain an extra factor of $\alpha_s$. However, as shown by Eq. (12), this is obviously not the case and the gluon operator with $\alpha_s$ as a whole is responsible for the invariant mass of the $\eta'$ mesons. Also, the combination entering in Eq. (12) involving the product of $\alpha_s$ and the gluon field operators is independent of the renormalization scale.

Before closing this section, let me comment on the alternative method to estimate $f_{\eta'}^{c}$ using the $\eta_c - \eta'$ mixing (Fig. 2 (a)):

$$J/\psi \rightarrow \eta_c^* + \gamma$$

(13)

For example, in Ref. [5], these quantities were determined from the decays $J/\psi \rightarrow (\eta,\eta',\eta_c)\gamma$, extending the usual $(\eta,\eta')$-mixing formalism to the $(\eta_c,\eta',\eta)$ system. Using the measured decay widths for the decays $J/\psi \rightarrow (\eta,\eta',\eta_c)\gamma$ and $(\eta_c,\eta',\eta)\rightarrow \gamma\gamma$ yields $|f_{\eta'}^{c}| \simeq 5.8$ MeV and $|f_{\eta}^{c}| \simeq 2.3$ MeV. However, in such a scheme, the intermediate $\eta_c$ is far off-shell, and one has to know the form factor for $J/\psi - \eta_c - \gamma$ vertex. Moreover, this picture is not consistent with electromagnetic gauge invariance, since at the quark level, the above transition occurs through $c\bar{c} \rightarrow \gamma + gg$ followed by $gg \rightarrow \eta'$ (Fig. 2 (b)). Note that there is a diagram in which the photon is emitted in the middle of two gluons. Such diagram cannot be represented by the simple $\eta_c - \eta$ mixing picture. One cannot simply neglect this diagram either, since it would violate electromagnetic gauge invariance. Therefore, the usual picture of $\eta_c - \eta'$ mixing for $J/\psi \rightarrow \eta^* \gamma$ may not be a good one, although it is widely used in the literature.

3 Estimate of the decay rate for $B \rightarrow (\eta', \eta)(K, K^*)$

In order to compute the complete amplitude for the exclusive decays, one has to combine the contribution from the decay $b \rightarrow s(c\bar{c}) \rightarrow s(gg) \rightarrow s\eta'$ discussed in the previous section with all the others arising from the four-quark and chromomagnetic operators, as detailed in Ref. [5]. The resulting amplitudes in the form factor approximation are listed in Ref. [18]. By comparing two expressions in Refs. [5] and [18], we can make the following identification:

$$- \Delta i_5 (m_{\eta}^2 / m_{\eta'}^2) f_{\eta'}^{c} \rightarrow f_{\eta'}^{c}, \quad - \Delta i_5 (m_{\eta}^2 / m_{\eta'}^2) f_{\eta}^{u} \rightarrow f_{\eta}^{u} .$$

(14)
Therefore, we have a simple relation between the decay constants $f_{\eta'}$, $f_\eta$, introduced in the intrinsic charm content method, and the form factor $\Delta f_{c}$ entering via the operator in Eq. (4). The idea of intrinsic charm quark content of $\eta'$ and $\eta$ and the contribution of the operator in Eq. (4) are related since this operator comes from the charm quark loop. Using the best-fit values of the ($\eta, \eta'$)-mixing parameters from Ref. [23], yielding $\theta = -22.2^\circ, \theta_0 = -9.1^\circ, f_8 = 168$ MeV, $f_0 = 157$ MeV, which in turn yields $f_{\eta'} = 63.6$ MeV and $f_\eta = 77.8$ MeV, the relations in Eq. (4) give $f_{\eta'} \sim -3.1$ MeV ($-2.3$ MeV) and $f_\eta \sim -1.2$ MeV ($-0.9$ MeV), with $m_c$ having the value 1.3 GeV (1.5 GeV). In our approach the relative signs of the contributions from $O_1^c$ and $O_2^c$ to the other contributions are determined; we obtain the negative-$f_{\eta'}$ (and $f_\eta$) solution of the two possible ones. Our estimate of $f_{\eta'}$ is consistent with the results obtained by Feldmann, Kroll and Stech,\cite{24} $f_{\eta'} = -(6.3 \pm 0.6)$ MeV, $f_\eta = -(2.4 \pm 0.2)$ MeV, \quad (15)
which is based on a new scheme for the flavor mixing and the axial anomaly. Also there appeared Ref. [25] by Araki \textit{et al.}, who did an independent calculation in the same way as Haperin and Zhitnitsky and got
$$f_{\eta'}^{(c)} = -(12.3 - 18.4) \text{ MeV,} \quad (16)$$
after correcting some mistakes in the original calculations in Ref. [7]. By now it is amusing that three independent methods for estimating $f_{\eta'}^{(c)}$ agree on its sign and the size within a factor of 2–3. So one can say that the issue of the charm content in $\eta'$ is now settled with reasonable confidence.

We plot the resulting branching ratios $BR(B^{\pm} \to \eta' K^{0\mp})$ and $BR(B^0 \to \eta' K^0)$ as functions of the parameter $\xi$ in Figs. 3 and 4 for three different sets of CKM elements $\rho$ and $\eta$ in the Wolfenstein parametrization: \cite{26}
$$\ (\rho, \eta) = (0.05, 0.36), \ (0.30, 0.42), \text{ and } (0, 0.22) \quad (17)$$
The branching ratios for the neutral $B$-meson decays are averages with the corresponding charged conjugated decays in the figures. The numerical results for the branching ratios depend on various input parameters, the details of which can be found in Ref. [18]. Let us discuss the sensitivity on various parameters in brief. The branching ratios show a mild dependence (of order 10%) on the CKM parameters. However the branching ratios depend on the $s$-quark mass very sensitively, because the amplitude for $B \to \eta' K$ contains terms with the $1/m_s$ factor. In this work, we used $\Pi_s(2.5 \text{ GeV}) = 122$ MeV. However, if the present high values of the branching ratios for $B^{0(\pm)} \to \eta' K^{0(\pm)}$ continue
to persist, one might have to consider smaller values of \( m_s \). We also expect progress in calculating quark masses on the lattice, sharpening the theoretical estimates presented here. Dependence on \( \xi \) amounts to between 20% and 35% depending on the other parameters if one varies \( \xi \) in the range \( 0 \leq \xi \leq 0.5 \). In all cases, the branching ratios are larger for \( \xi = 0 \). Taking into account the parametric dependences just discussed, we note that the theoretical branching ratio \( BR(B^0(\pm) \to \eta' K^0(\pm)) \) are uncertain by a factor 2. For the ratio of the branching ratios \( BR(B^\pm \to \eta' K^\pm)/BR(B^0 \to \eta' K^0) \), which is useful quantity since it is practically independent of the form factors and most input parameters, the residual uncertainty is due to the CKM-parameter dependence of this ratio. It is estimated as about 10%. We get (for \( 0 \leq \xi \leq 0.5 \))

\[
\frac{BR(B^\pm \to \eta' K^\pm)}{BR(B^0 \to \eta' K^0)} = 0.9 - 1.02. \tag{18}
\]

The present experimental value of this ratio as calculated by adding the experimental errors in the numerator and denominator in quadrature is 1.34 ± 0.85. One can also study other modes involving \( K^* \) instead of \( K \). Summarizing the results of Ref. [18], the branching ratios for the decay modes \( B^\pm \to \eta K^\pm \) and \( B^0 \to \eta K^0 \) are smaller compared to their \( \eta' \)-counterparts by at least an order of magnitude, namely, \( BR(B^\pm \to \eta K^\pm) = (1 - 2) \times 10^{-6} \) and a similar value for the neutral B decay mode. On the other hand, the branching ratios for the decay modes \( B^\pm \to \eta(K^\pm, K^{*\pm}) \) and \( B^0 \to \eta(K^0, K^{*0}) \) are all comparable to each other somewhere in the range \((1 - 3) \times 10^{-6}\).

4 Concluding Remarks

In this talk, we have presented an independent estimate of the charm content of \( \eta' \) meson using the process \( b \to s(\bar{c}c) \to s(gg) \to s(\eta', \eta) \) and QCD anomaly. We could fix its sign as well as the size. Our results are in good agreement with other results based on independent methods, and thus we can be confident by now that the charm content in \( \eta' \) meson is indeed small. One could further study the branching ratios in \( B \to (\eta', \eta)(K, K^+) \) in the generalized factorization approximation. Our result is marginally consistent with the CLEO data, considering various theoretical uncertainties in this kind of game (Figs. 3 and 4). Also our result is drastically different from the ones which follow in other scenarios. Hence, ongoing and future experiments will be able to test the predictions of the present approach as well as of the competing ones, such as models based on the dominance of the intrinsic charm contributions in \( \eta' \), as suggested in Refs. [7,8], or models in which dominant role is attributed to the soft-gluon-fusion process to form an \( \eta \) or \( \eta' \).
Figure 1: Feynman diagram contributing to the processes $b \to c(c\bar{c}) \to s(gg) \to s\eta^\prime$ in the full and effective theory. The lower vertex in the diagram on the right is calculated with the insertion of the operators $O^c_{c1}$ and $O^c_{c2}$ in the effective Hamiltonian approach; the upper vertex in both the full and effective theory is determined by the QCD triangle anomaly.

Figure 2: Feynman diagram contributing to the processes $J/\psi \to \gamma\eta^\prime$ (a) in the $\eta_c - \text{eta}'$ mixing picture, and (b) in the parton picture.
Figure 3: The branching ratio $BR(B^\pm \rightarrow \eta'K^\pm)$ plotted against the parameter $\xi$. The lower three curves correspond to the value $m_s(2.5 \text{ GeV}) = 122 \text{ MeV}$ and the three choices of the CKM parameters: $\rho = 0.05, \eta = 0.36$ (solid curve); $\rho = 0.30, \eta = 0.42$ (dashed curve); $\rho = 0, \eta = 0.22$ (dashed-dotted curve). The upper two curves correspond to the value $m_s(2.5 \text{ GeV}) = 100 \text{ MeV}$, $\rho = 0.05, \eta = 0.36$ and $f_{\eta'} = -2.3 \text{ MeV}$ from the QCD-anomaly method (dotted curve) and $f_{\eta'} = -5.8 \text{ MeV}$ from (long-short dashed curve). The horizontal thick solid lines represent the present CLEO measurements (with $\pm 1\sigma$ errors).

Figure 4: The branching ratio $BR(B^0 \rightarrow \eta'K^0)$ plotted against the parameter $\xi$. The legends are the same as in Fig. 3, and the horizontal thick solid lines represent the present CLEO measurements (with $\pm 1\sigma$ errors).
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