Magnetothermoelectric transport in modulated and unmodulated graphene

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Abstract

We draw motivation from recent experimental studies and present a comprehensive study of magnetothermoelectric transport in a graphene monolayer within the linear response regime. We employ the modified Kubo formalism developed for thermal transport in a magnetic field. Thermopower as well as thermal conductivity as a function of the gate voltage of a graphene monolayer in the presence of a magnetic field perpendicular to the graphene plane is determined for low magnetic fields (∼1 T) as well as high fields (∼8 T). We include the effects of screened charged impurities on thermal transport. We find good qualitative and quantitative agreement with recent experimental work on the subject. In addition, in order to analyze the effects of modulation, which can be induced by various means, on the thermal transport in graphene, we evaluate the thermal transport coefficients for a graphene monolayer subjected to a periodic electric modulation in a magnetic field. The results are presented as a function of the magnetic field and the gate voltage.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Graphene exhibits remarkable thermal properties. The measured values of thermal conductivity of graphene reach as high as several thousand watts per meter Kelvin [1–4], and these are among the highest values of known materials. Heat transport measures the energy carried by both electrons and phonons and is fundamental to understanding a material, its ground states, excitations and scattering mechanisms. If the dream of carbon-based electronics is to be realized, it is essential to study how and how fast heat is dissipated across graphene devices. This requires systematic measurements of thermal conductivity and thermopower over a broad temperature range (1.5–300 K) under various external conditions. Therefore, recently there has been considerable interest, both experimental [5–8] and theoretical [9–16], in the study of thermoelectric and magnetothermoelectric transport in graphene. This is partly due to the realization that the information provided by thermoelectric transport is complementary to electrical transport. Furthermore, thermoelectric and magnetothermoelectric transport studies are extremely useful in providing insight into the scattering mechanism involved in transport. Fundamentally related to the electrical conductivity, the thermal conductivity and thermoelectric coefficients can be determined by the band structure and scattering mechanisms. The thermoelectric coefficients involve the energy derivatives of their electrical transport counterparts such as the conductivity \( \sigma \). Recent measurements of thermoelectric power (TEP) on graphene samples in zero and non-zero magnetic fields have shown a linear temperature dependence of TEP which suggests that the dominant contribution is that of diffusive thermopower \( S_d \). A comparison between the measured TEP and that predicted by the Mott formula shows general agreement, particularly at lower temperatures \( T < 50 \text{ K} \) [10]. However, at higher temperatures deviations from the Mott relation have been reported [6, 7]. In theoretical work, Yan et al. [15] have determined the TEP of Dirac fermions in graphene within the self-consistent Born approximation. Also, Hwang et al. [9], in their calculation of TEP, incorporate the energy dependence of various transport scattering rates and show that the dominant contribution is from the screened charged impurities in the graphene’s environment. Further, Vaidya et al. [10] used Boltzmann transport theory to calculate \( S_d \).
in graphene after considering contributions of optical phonon and surface roughness scattering.

Application of a magnetic field in addition to a thermal gradient has profound effects on the thermal transport in a system and serves as an additional probe. When a magnetic field is applied perpendicular to the $x$–$y$ plane of a sample, the diffusing charge carriers experience a Lorentz force. This results in developing a transverse electric field $E_y$ in addition to the longitudinal field $E_x$. The thermopower is determined from the thermal gradient $\nabla T$ and the induced voltage $\nabla V$ as $S_{xy} = -\frac{V_y}{V_T}$ (also known as the Seebeck coefficient) and $S_{xx} = \frac{V_x}{V_T}$ (the Nernst coefficient). They are measures of the magnitudes of the longitudinal and transverse voltages generated in response to an applied temperature gradient. They are very sensitive in graphene due to its semimetal nature [12].

The quantum magnetic oscillations in electrical and thermal transport were investigated theoretically earlier by Gusynin and Sharapov [17] and they obtained analytical results for the longitudinal thermal conductivity and the Nernst coefficient. However, they assumed a scattering rate that is constant in energy, independent of magnetic field and temperature. Hence the self energy used is not self-consistent. Moreover, they evaluated the longitudinal thermal conductivity as a function of the magnetic field at different temperatures but at fixed chemical potential and constant impurity broadening. Further, they determined the Nernst coefficient (signal) without recourse to the modified Kubo formalism appropriate for thermal transport in a magnetic field. They neglected the dependence of $\Gamma$ on the chemical potential/cARRIER concentration. Dora and Thalmeier extended the work presented in [17] and studied the electric and thermal response of two-dimensional Dirac fermions in a quantizing magnetic field in the presence of localized disorder [18]. They evaluated the Seebeck coefficient and the corresponding thermal conductivity as a function of the chemical potential and the magnetic field. They did not determine the Nernst coefficient and the transverse thermal conductivity.

What distinguishes our work on unmodulated graphene from the aforementioned previous papers is that we employ the modified Kubo formalism required to study thermal transport in a magnetic field. As has been discussed earlier, the usual Kubo formula for thermal response functions is invalid in a magnetic field and needs to be modified when calculating the transverse (Hall) thermal conductivity and the Nernst coefficient [19, 20]. We use the phenomenological transport equations obtained from the modified Kubo formalism [20, 21]. Further, in the scattering rate and the impurity broadening of the Landau levels the effects of the carrier concentration that can be varied by the gate voltage are taken into account. In the first stage, we determine the components of magnetoelectrothermal (MET) power and MET conductivity of an unmodulated graphene monolayer in the presence of randomly distributed charged impurities. The results are presented as a function of the gate voltage for small and large magnetic fields applied perpendicular to the graphene sheet. We determine both the Nernst and Seebeck coefficients as well as the longitudinal and transverse thermal conductivity. These results are then compared with experimental work. In addition, we have also carried out a detailed investigation of the MET transport properties of a graphene monolayer which is modulated by a weak one-dimensional periodic potential in the presence of a perpendicular magnetic field. The motivation for this has arisen from recent work, experimental and theoretical, that has shown that interaction with a substrate can lead to weak periodic modulation of the graphene spectrum. Furthermore, applying a patterned gate voltage or placing graphene on a pre-patterned substrate can also lead to modulated graphene [22–24]. Placing impurities or adatom deposition can do the same. In a previous work, we have computed the electric transport coefficients of electrically modulated graphene [25]. It was shown that modulation turns the sharp Landau levels into bands whose widths oscillate periodically with the magnetic field. This affects the magnetoelectric transport coefficients which exhibit commensurability (Weiss) oscillations. The origin of these Weiss oscillations is the commensurability of the two characteristic length scales of the system: the cyclotron diameter at the Fermi energy and the period of the modulation [26]. An interesting feature of electronic conduction in the modulated system is the opening of the diffusive (band) transport channel in addition to hopping (collisional) transport. Both these contributions to MET transport are taken into account in this work.

In section 2, we present the general formulation of the magnetoelectrothermal transport problem and perform the calculation of the thermopower and the thermal conductivity of unmodulated graphene as well as graphene subjected to one-dimensional (1D) weak periodic modulation. The results for the transport coefficients as a function of gate voltage ($V_g$) for unmodulated graphene are discussed in section 3, where we also make a comparison with experimental results. Following this, in section 4, the results for modulated graphene as a function of the gate voltage and the external magnetic field are presented. The present paper ends with a summary and conclusions.

### 2. Thermal magnetotransport coefficients

As mentioned in the introduction, corrections to the usual Kubo formula for transport have to be made when studying thermal transport in a magnetic field. This was carried out by Luttinger, Smerka, Streda and Oji [21, 20]. We employ the modified Kubo formalism to determine the thermal transport coefficients from the electrical $J_e$ and thermal (energy) current densities $J_Q$

$$J_{Q\mu} = L_{\mu\nu}^{(0)} \left[ -\frac{1}{e} \langle \nabla_v \tilde{n} \rangle \right] + L_{\mu\nu}^{(1)} \left[ T \nabla_v \left( \frac{1}{T} \right) \right] \quad (1)$$

$$J_{Q\mu} = L_{\mu\nu}^{(1)} \left[ -\frac{1}{e} \langle \nabla_v \tilde{n} \rangle \right] + L_{\mu\nu}^{(2)} \left[ T \nabla_v \left( \frac{1}{T} \right) \right]. \quad (2)$$

Here $\tilde{n} = n_e - e\phi$ with $n_e$ the chemical potential, $\phi$ the scalar potential, $e$ the electronic charge and $T$ the temperature of the system. The electrical and thermal transport coefficients,
the electrical conductivity $\sigma$, thermopower $S$ and the thermal conductivity $\kappa$, can be obtained from the above expressions, following [20, 21, 27–29], as

$$\sigma_{\mu\nu} = L_{\mu\nu}^{(0)},$$

$$S_{\mu\nu} = \frac{1}{eT} \left( (L_{\mu\nu}^{(0)})^{-1} L_{\mu\nu}^{(1)} \right),$$

$$\kappa_{\mu\nu} = \frac{1}{eT} \left[ L_{\mu\nu}^{(2)} - eT (L_{\mu\nu}^{(1)} S_{\mu\nu}) \right],$$

with

$$L_{\mu\nu}^{(0)}(\alpha = 0, 1, 2)$$

are, in general, tensors where $\mu, \nu = x, y$. These phenomenological transport coefficients satisfy the Onsager relation [21, 27] $L_{\mu\nu}^{(0)}(B) = L_{\mu\nu}^{(0)}(-B)$. $\sigma_{\mu\nu}(E)$ is the zero-temperature conductivity and $f(E) = \exp(E - \mu)/k_B T + 1$ is the Fermi Dirac distribution function with the chemical potential. The quantity $\rho_{\mu\nu} = (L_{\mu\nu}^{(0)})_{\mu\nu}$ is the resistivity tensor whose components are $\rho_{xx} = \sigma_{xx}/\Lambda$, $\rho_{xy} = \sigma_{xx}/\Lambda$, $\rho_{yx} = -\rho_{xx} = \sigma_{yy}/\Lambda = \sigma_{xy} - \sigma_{yx}$. In order to calculate the thermal transport coefficients for graphene, we consider a graphene monolayer in the $xy$-plane subjected to a magnetic field $B$ along the $z$-direction. In the Landau gauge, the unperturbed single particle Dirac-like Hamiltonian may be written as

$$H_0 = v_F \mathbf{\sigma} \cdot (-i\hbar \nabla + e\mathbf{A}).$$

Here, $\sigma = \{\sigma_x, \sigma_y\}$ are the Pauli matrices and $v_F = 10^6$ m s$^{-1}$ characterizes the electron velocity with $\Lambda = (0, Bx, 0)$ the vector potential. The normalized eigenfunctions of the Hamiltonian given in equation (7) are

$$\Psi_{n,k_x} = \frac{e^{i k_x y}}{\sqrt{2L_y I}} \left[ \frac{\phi_n}{\phi_{n-1}} \right] \left[ \frac{(x + x_0)/l}{(x + x_0)/l} \right],$$

where $\phi_n(x)$ and $\phi_{n-1}(x)$ are the harmonic oscillator wavefunctions centered at $x_0 = \frac{1}{2} l k_x$. $I$ is the Landau-level index, $l = \sqrt{\frac{\hbar}{e B}}$ is the magnetic length and $L_y$ is the length of the 2D graphene system in the $y$-direction. The corresponding eigenvalue is $E_n = \hbar \omega_0 \sqrt{n}$, where $\omega_0 = v_F \sqrt{2eB}/\hbar = v_F \sqrt{2}/l$ is the cyclotron frequency of the Dirac electrons in graphene.

In order to investigate the effects of modulation, we express the Hamiltonian in the presence of modulation as $H = H_0 + U(x)$. Here, $U(x)$ is the one-dimensional periodic modulation potential along the $x$-axis. It is given by $U(x) = V_e \cos K x$ such that $K = \frac{2\pi}{a}$, $a$ is the period of modulation and $V_e$ is the constant modulation amplitude. To account for weak modulation, we take $V_e$ to be an order of magnitude smaller than the Fermi energy $E_F = v_F \hbar k_F$, where $k_F = \sqrt{3\pi/4 \hbar}$ is the magnitude of the Fermi wave vector with $\hbar$ the density of electrons. This allows us to apply standard first order perturbation theory to determine the energy eigenvalues in the presence of modulation. Thus, the energy eigenvalues for weak modulation ($V_e < E_F$) are $E_{n,k_x} = E_n + F_{n,B} \cos K x$.

Here, $F_{n,B} = \frac{\hbar}{2m} \exp(-\frac{\hbar}{2})(L_n(u) + L_{n-1}(u))$, $u = \frac{K^2}{2}$ and $L_n(u)$ and $L_{n-1}(u)$ are Laguerre polynomials.

In the presence of a periodic modulation, there are two contributions to magnetoconductivity: the collisional (hopping) contribution and the diffusive (band) contribution. The former is the localized state contribution which carries the effects of Shubnikov–de Hass (SDH) oscillations that are modified by periodic modulation. The diffusive contribution is the extended state contribution and arises due to the finite drift velocity acquired by the charge carriers in the presence of modulation. In the linear response regime, the conductivity tensor is a sum of a diagonal and a nondiagonal part: $\sigma_{\mu\nu}(\omega) = \sigma_{\mu\nu}^d(\omega) + \sigma_{\mu\nu}^{ud}(\omega)$, $\mu, \nu = x, y$. In general, $\sigma_{\mu\nu}^d(\omega) = \sigma_{\mu\nu}^{diff}(\omega) + \sigma_{\mu\nu}^{col}(\omega)$, accounting for both diffusive and collisional contributions, whereas $\sigma_{\mu\nu}^{ud}(\omega)$ is the Hall contribution. Here, $\sigma_{xx} = \sigma_{xx}^{col}$ and $\sigma_{yy} = \sigma_{xx}^{col} + \sigma_{xy}^{diff}$. Similarly to the conductivity tensors, the diagonal components of the thermal transport coefficients are determined by the following expressions:

$$\sigma_{xx}^{(0)} = L_{xx}^{(0)} = L_{yy}^{(0)},$$

$$\sigma_{xy}^{(0)} = L_{xy}^{(0)} + \epsilon L_{xx}^{(0)} + L_{yy}^{(0)},$$

with

$$\sigma_{xx}^{(0)} = \frac{e^2 \tau}{\hbar \epsilon^2} \sum_{n=0}^{\infty} \left[ F_{n,B}^2 \right] (E - \eta)^2 \left[ -\frac{\partial f(E)}{\partial E} \right]_{E=\eta},$$

$$\sigma_{yy}^{(0)} = \frac{e^2 \tau}{\hbar \epsilon^2} \sum_{n=0}^{\infty} \left[ F_{n,B}^2 \right] (E - \eta)^2 \left[ -\frac{\partial f(E)}{\partial E} \right]_{E=\eta},$$

where $\tau$ is the scattering time. Here, we have taken the scattering time to be independent of the Landau-level index $n$. The components of the thermopower are given by the following equations:

$$S_{xx} = \frac{1}{eT} \left( \frac{\sigma_{yy}}{\sigma_{xx}} \right) L_{xx}^{(1)} + \left( \frac{1}{\sigma_{yx}} \right) L_{yx}^{(1)}.$$
the sample \[30\]. The components of thermopower (\(S\)) through the relationship 
\[\tau e\]

electron charge is constant for graphene on a SiO\(_2\). The number density 
as a function of the gate voltage are presented in this section. 
Employing these, the components of thermopower and transport coefficients

\[\sigma\]

(non\-modulated graphene 
conductive of unmodulated graphene 
3. Magnetothermopower and magnetothermal conductivity of unmodulated graphene 

The number density \(n_e\) is related to the gate voltage \(V_g\) through the relationship 
\[n_e = e\epsilon\epsilon_0V_g/\epsilon\]
where \(\epsilon_0\) and \(\epsilon = 3.9\) are the permittivities for free space and the dielectric constant for graphene on a SiO\(_2\) substrate, respectively. The electron charge is \(e\) and \(t \approx 300\) nm is the thickness of the sample [30]. The components of thermopower \(S_{xx}\) and thermal conductivity \(\kappa_{xx}\), as the system moves away from the charge neutral point on the electron side on changing the gate voltage, are shown in figure 1 at a magnetic field of 1 T. The lattice temperature of 10 K and mobility of \(\mu = 20\) m\(^2\) V\(^{-1}\) s\(^{-1}\) [31] are chosen. The scattering time is related to the mobility as 
\[\tau = e^2/\sigma\epsilon\]
in a graphene monolayer [32]. The impurity broadening \(\Gamma\) can be expressed in terms of the self energy \(\Sigma^- (E)\), \(\Gamma \equiv \Gamma (E) = 2 \text{Im} \Sigma^- (E)\) and also \(\Gamma (E) = \hbar/\tau\) [33]. We use the expression for \(\text{Im} \Sigma^- (E)\) derived in [25] to find 
\[\Gamma = \sqrt{\hbar (\hbar \omega_k)^2/(4\pi e E_F)}\]
The electron number density is \(n_e = 7.19V_g \times 10^{14}\) m\(^{-2}\) and the Fermi energy is \(E_F = \hbar^2k^2/2m_e\). The impurity density is related to \(\Gamma\) through \(N_i = \pi \Gamma^2/\hbar^2\) [34]. The scattering time of \(\tau = 3.133\sqrt{\bar{B}/(\mu V_g)}\) ms and impurity 

density of \(N_i = 2.46 \times 10^{14}\) m\(^{-2}\) were employed in this work [31–37]. Moreover, the same study is carried out at a higher magnetic field of 8.8 T for graphene with mobilities of \(\mu = 1\) m\(^2\) V\(^{-1}\) s\(^{-1}\) and \(\mu = 20\) m\(^2\) V\(^{-1}\) s\(^{-1}\) respectively and the results are shown in figure 2. Since \(S_{xx}\) and \(S_{yy}\) are identical only \(S_{xx}\) is depicted in these figures. The longitudinal coefficient of thermopower \(S_{xx}\) is equivalent to the Seebeck coefficient and our results provide a qualitative as well as quantitative understanding of the overall behavior of the observed \(S_{xx}(V_g)\). \(S_{xx}\) can have either sign and it is negative in our case since the charge carriers are electrons in this range of \(V_g\). The transverse component of thermopower \(S_{yy}\) is also known as the Nernst signal and it arises due to the presence of the perpendicular magnetic field as the Lorentz force bends the trajectories of the thermally diffusing carriers. It can be seen from figures 1(a) and 2(a) that \(S_{xx}\) follows \(1/\sqrt{V_g}\) (with \(V_g \propto n_e\)). Similar behavior of \(S_{xx}\) is observed in experiments [5–7]. Notice that we have presented results for diffusive thermopower and we have ignored the phonon contribution to the thermopower due to the weak electron–phonon coupling in graphene [6, 9]. \(S_{xx}\) and \(S_{yy}\) show Shubnikov–de Haas (SdH) type oscillations in the Landau quantizing magnetic field. At the lower magnetic field of 1 T (figure 1) the oscillations are more closely spaced since the separation between the Landau levels, which is proportional to the magnetic field strength, is smaller compared to the results for the higher magnetic field of 8.8 T, figure 2. Moreover, we observe in figures 1(a) and 2(a) that \(S_{xx}\) approaches its minimum value at those values of \(V_g\) where there are boundaries of Landau levels...
conductivity $\kappa$ is significant. However, the transverse component of thermal conductivity increases, where the Landau quantization effects become less apparent in higher magnetic fields. Splitting of the peaks in the longitudinal and transverse components of electrical conductivity follows that of the corresponding components of electrical conductivity. At higher magnetic fields, quantum Hall steps have begun to appear. The behavior of the longitudinal thermal conductivity $\kappa_{xx}$ is shown in figure 2(b), which was also observed in [18] where it was shown that the splitting occurs in such a way that it produces antiphase oscillations with respect to the electric ones and leads to violation of the Wiedemann–Franz law. For the unmodulated case, $\sigma_{xy} = \sigma_{xx, \alpha} = \epsilon_{\alpha}$ and using equation (14)–(21) we find that $\kappa_{xx} \approx \kappa_{yy}$ and $\kappa_{xy} = -\kappa_{yx}$. Therefore, only $\kappa_{xx}$ and $\kappa_{yx}$ are shown in the figures. We find that the results for the magnetothermal power obtained in our work at $B = 8.8$ T with $T = 10$ K are in good agreement, both qualitatively and quantitatively, with the experimental results obtained in [6, 7], see figure 3 of [6]. These results indicate that scattering from screened charged impurities is the dominant scattering mechanism required to explain the experimental results. We must add that our quantitative results for $\kappa_{xx}$ depend strongly on the mobility of the graphene system. The heights of the peaks of the Seebeck coefficient increase with the increase in impurity in the sample. However, from equation (14) we can see that the major contribution to the thermopower comes from the second term as $\sigma_{xy} L_{xx}^{(1)} \approx \sigma_{xx} L_{xx}^{(1)}$ and at low temperatures the heights of the peak values of thermopower approach the quantized value of $\hbar e / L$. However, this universal result is only obtained for charge carriers in the absence of impurity scattering. In fact, in the presence of impurity scattering we find a higher value of $n S_{xx}$ for $\kappa_{xx}$ is shown in these figures. The amplitude of oscillations in $S_{xx}$ and $S_{xy}$ increase with increasing magnetic field strength (see figures 1(a) and 2(a)). In these figures, we also present the thermal conductivity as a function of the gate voltage. The longitudinal thermal conductivity $\kappa_{xx}$ shows oscillating behavior which damps out as $V_g$ increases, where the Landau quantization effects become less significant. However, the transverse component of thermal conductivity $\kappa_{xy}$ rises monotonically with $V_g$ as shown in figures 1(b) and 2(b). At the higher magnetic field, quantum Hall steps have begun to appear. The behavior of the longitudinal and transverse thermal conductivity follows that of the corresponding components of electrical conductivity.

4. Magnetothermopower and magnetothermal conductivity of periodically modulated graphene

Now we consider the effects of modulation. The 1D modulation broadens the sharp Landau levels into bands and gives rise to an additional diffusive (or band) contribution to transport. This additional contribution is absent without modulation. We now focus on the modulation induced changes in the thermal magnetotransport coefficients of graphene. Therefore, in the first part we present the thermopower and thermal conductivity of modulated graphene with a mobility of $20$ m$^2$/V·s as a function of the gate voltage. These are shown in figures 3 and 5 respectively. The results are for a constant external magnetic field of $B = 1$ T applied perpendicular to the graphene sheet, with electric modulation of strength $V_g = 3$ meV applied in the $x$-direction at a temperature of $T = 10$ K. In this case $\Gamma = \frac{1}{\sqrt{V_g}} \approx 36.3$ meV, such that $\Gamma \ll V_g \ll h_0/e$ to satisfy the requirements of weak modulation. The period of modulation is $a = 382$ nm. The results for $S_{xx}$ and $S_{yy}$ are identical, so only $S_{xx}$ is shown in these figures. The amplitude of oscillations in $S_{xx}$ ($\Delta S_{xx}$) is greater than that of $S_{xy}$ ($\Delta S_{xy}$) which damps out with increasing gate voltage ($V_g$). Both $S_{xx}$ and $S_{xy}$ show SdH-type oscillations and this verifies that the system is Landau quantized. The modulation effects are apparent in $S_{xx}$ and $S_{xy}$ which show modulation of SdH-type oscillations and $\Delta S_{xx} \gg \Delta S_{xy}$, figure 3. $\kappa_{xx}$ is greater than $\kappa_{xx}$ and $\kappa_{xy}$ as shown in figure 5. These modulation induced...
effects on thermal transport coefficients can be highlighted by calculating the difference between the modulated case and the unmodulated case. The contributions of modulation to thermopower $S_{\mu\nu}(V_e)$ and thermal conductivity $\kappa_{\mu\nu}(V_g)$ are shown in figures 4 and 6 respectively. These figures clearly show the modulation of SdH oscillations in both the thermopower and the thermal conductivity. For an unmodulated case $\kappa_{xx} = \kappa_{yy}$; however, for modulated graphene $\kappa_{xx} \neq \kappa_{yy}$ and this expected behavior is seen in figure 6 where $\Delta \kappa_{xx} \neq \Delta \kappa_{yy}$. The 1D modulation gives a positive contribution to $\Delta \kappa_{yy}$ while $\Delta \kappa_{xx}$ and $\Delta \kappa_{xy}$ oscillate around zero. $\Delta \kappa_{yy} \gg \Delta \kappa_{xx}$, which is a consequence of the fact that $\Delta \kappa_{xx}$ has only a collisional contribution, whereas $\Delta \kappa_{yy}$, in addition to the collisional part, has a large contribution from band conduction. Comparing figures 1(a), 3 and 4, we can see that the peak values of $S_{\mu\nu}$ of an unmodulated system increase with the application of modulation on it; this is discussed in detail below.

We also show the results when the magnetic field is varied and the electron density is fixed at $n_e = 3.16 \times 10^{15}$ m$^{-2}$, which corresponds to a gate voltage of $V_g = 4.39$ V. The Fermi energy of the system is $E_F = h v_F \sqrt{\pi n_e} \approx 65.66$ meV. We have taken the mobility of $20$ m$^2$ V$^{-1}$ s$^{-1}$ [31] and hence the scattering time is taken to be $\tau = 1.3 \times 10^{-12}$ s. An impurity broadening of $\Gamma = 0.895 \sqrt{B}$ meV and impurity density of $N_I = 1.23 \times 10^{13}$ m$^{-2}$ were employed in this part of the work. The strength of the electrical modulation is taken to be $V_e = 2$ meV and 3 meV respectively with period $a = 382$ nm and temperature $T = 10$ K. The difference between the modulated case and the unmodulated case highlights the modulation induced effects in these thermoelectric quantities.
The thermopower and the change in thermopower due to the modulation $\Delta S_{xx}(B)$ are shown in figure 7 as a function of the magnetic field in units of $-kT/e$. When $B$ is less than 0.2 T Weiss oscillations are observed whereas SdH-type oscillations dominate at higher magnetic fields. It is also seen that these oscillations in $S_{xx}$ are 90° out of phase with those in $S_{yy}$. The amplitude of the oscillations in $\Delta S_{xx}$ is much greater than in $\Delta S_{yy}$ and they are 90° out of phase. Again for $B$ greater than 0.2 T the oscillations appear as envelopes of SdH oscillations. The different components of the thermal conductivity tensor and the correction to it due to 1D modulation are shown in figure 8. The magnetic field dependence of the thermal conductivity tensor is similar to that of the electrical conductivity tensor obtained in [25].

Electric modulation acting on the system results in a positive contribution to $\Delta \kappa_{yy}$ whereas $\Delta \kappa_{xx}$ and $\Delta \kappa_{xy}$oscillate around zero. In figure 8 we see that $\Delta \kappa_{yx} \gg \Delta \kappa_{yy} \gg \Delta \kappa_{xx}$, such that $\Delta \kappa_{xx}$ and $\Delta \kappa_{xy}$ are 180° out of phase with each other. We find that $\Delta \kappa_{xx}$ is greater than $\Delta \kappa_{yx}$, which is a consequence of the fact that $\Delta \kappa_{xx}$ has only a collisional contribution, while $\Delta \kappa_{yx}$, in addition to the collisional part, has contributions due to band conduction. The flat band condition for electrically modulated graphene is $\frac{\pi}{2} = i - \frac{1}{4}$, with $i = 1, 2, \ldots$, where $R_c = \ell k_F$ and $k_F = \sqrt{\pi n_c}$ is the cyclotron orbit [25]. From figures 7 (b) and 8(b) we observe that the zeros in $\Delta S_{yy}(B)$ and $\Delta \kappa_{xx}(B)$ are in close agreement with the values of magnetic field predicted by the flat band condition. With the increase in modulation strength $V_g$ at a fixed gate voltage, the modulation of SdH oscillations increases. The amplitude of the oscillations in $S_{xx}$ ($\Delta S_{xx}$) increases with increase in modulation whereas the amplitude of the oscillations in $S_{xy}$ is weakly dependent on the modulation strength. Similarly the amplitudes of the oscillations in $\Delta \kappa_{xx}$ and $\Delta \kappa_{yy}$ increase with increase in modulation. However $\Delta \kappa_{yx}$ is weakly dependent on the modulation strength as compared to $\Delta \kappa_{yy}$ and $\Delta \kappa_{xy}$. In order to clearly observe the effects of modulation we have to consider higher mobilities or use a higher amplitude of modulation $V_c$. In this regard, we have to bear in mind the condition $\Gamma \ll V_c \ll h \omega_F$ for weak modulation. Although the mobility of $20 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ used for the modulated system is still only achievable for suspended graphene where the primary scattering mechanism is short range scattering due to sharp defects and not screened charged impurity scattering, the results obtained are qualitatively correct. As mentioned above, we can consider lower mobility for our work but that would require higher modulation amplitude in order to observe its effects; qualitatively the results would be the same.

To conclude, in this work we have studied magnetothermoelectric transport in graphene in the linear response regime using the modified Kubo formalism appropriate for thermal transport in a magnetic field. Results are presented for both unmodulated graphene as well as graphene that is weakly modulated by an electric modulation. We take into account scattering from screened charged impurities and our results indicate that these provide the most dominant scattering mechanism at low temperatures. The thermopower, the
Figure 8. (a) The components of thermal conductivity as a function of magnetic field at a gate voltage of $V_g = 4.39$ V and temperature of $T = 10$ K for $V_c = 2$ meV (solid curve) and $V_c = 3$ meV (broken curve) respectively. The correction to the thermal conductivity due to 1D electric modulation is shown in (b).

Seebeck coefficient and the Nernst coefficient are determined as functions of the gate voltage. Furthermore, we also determine the magnetothermal conductivity tensor, and both the longitudinal and the transverse (Hall) components. For unmodulated graphene we were able to make a comparison of the thermopower with experimental results and we find that they are in good agreement, both qualitatively and quantitatively, with experimental results. In the case of modulated graphene, we focus on the modulation induced effects that appear as commensurability (Weiss-type) oscillations in the magnetothermoelectric coefficients. The results are presented as functions of both the gate voltage and the magnetic field.

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