Form Factors of $B_{u,d,s}$ Decays into P-Wave Axial-Vector Mesons in the Light-Cone Sum Rule Approach

Kwei-Chou Yang
Department of Physics, Chung Yuan Christian University
Chung-Li 320, Taiwan

Abstract

We calculate the vector and axial-vector form factors of $B_{u,d,s}$ decays into P-wave axial-vector mesons in the light-cone sum rule approach. For the sum rule results, we have included corrections of order $m_A/m_b$, where $m_A$ is the mass of the axial-vector meson $A$. The results are relevant to the light-cone distribution amplitudes of the axial-vector mesons. It is important to note that, owing to the $G$-parity, the chiral-even two-parton light-cone distribution amplitudes of the $^3P_1$ ($^1P_1$) mesons are symmetric (antisymmetric) under the exchange of quark and anti-quark momentum fractions in the SU(3) limit. For chiral-odd light-cone distribution amplitudes, it is the other way around. The predictions for decay rates of $B_{u,d,s} \rightarrow A\nu\bar{\nu}$ are also presented.
I. INTRODUCTION

The inclusive and exclusive \( B \) decays provide potentially stringent test of the Standard Model. Although the inclusive rare decays are theoretically clean, they are a challenge for measurements at \( B \) factories. The exclusive processes may easily be accessible for experiments, but knowledge of form factors is required. The production of the axial-vector mesons has been seen in charmful \( B \) decays: \( B \rightarrow J/\psi K_1(1270) \) and \( B \rightarrow Da_1(1260) \) \cite{1}. As for charmless hadronic \( B \) decays, \( B^0 \rightarrow a_1^+(1260)\pi^\mp \) are the first modes measured by BABAR and Belle \cite{2,3,4}. Information for weak phase \( \alpha \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*) \) can be extracted from their time-dependent measurement or by relating these decays with corresponding \( \Delta S = 1 \) decays. BABAR has further reported the observation of the decays \( B^0 \rightarrow b_1^+\pi^\mp, b_1^+K^-, K_1^-(1270)\pi^+, K_1^-(1400)\pi^+, a_1^+K^- \) and \( B^- \rightarrow b_1^0\pi^-, b_1^0K^-, a_1^0\pi^-a_1^0K^0, f_1(1285)K^-, f_1(1420)K^- \) \cite{5,6}. Very recently, \( B^- \rightarrow K_1^-(1270)\phi, K_1^-(1400)\phi \) have been observed by BABAR \cite{7}. Using the QCD factorization approach, we have studied charmless two-body \( B \) decays involving one or two axial-vector meson(s) in the final state \cite{8,9,10}.

In the quark model, two lowest nonets of \( J^P = 1^+ \) axial-vector mesons are expected as the orbitally excited \( q\bar{q}' \) states. In terms of the spectroscopic notation \( n2S+1L_J \), where the radial excitation is denoted by the principle number \( n \), there are two types of the lowest \( p \)-wave mesons, namely, \( 1^3P_1 \) and \( 1^1P_1 \). These two nonets have distinctive \( C \) quantum numbers, \( C = + \) and \( C = - \), respectively. Experimentally, the \( J^{PC} = 1^{++} \) nonet consists of \( a_1(1260), f_1(1285), f_1(1420) \) and \( K_{1A} \), while the \( 1^{+-} \) nonet contains \( b_1(1235), h_1(1170), h_1(1380) \) and \( K_{1B} \). The physical mass eigenstates \( K_1(1270) \) and \( K_1(1400) \) are mixtures of \( K_{1A} \) and \( K_{1B} \) states owing to the mass difference of the strange and non-strange light quarks.

In QCD language a real hadron should be described in terms of a set of Fock states for which each state has the same quantum number as the hadron. Because of G-parity, the decay constant for the local axial-vector (local tensor) current coupling to the \( 1P_1 \) (\( 3P_1 \)) state vanishes in the SU(3) limit. However the constituent partons within a hadron are actually non-localized. Projecting the axial-vector meson along the light-cone, due to the G-parity the chiral-even light-cone distribution amplitudes (LCDAs) of a \( 1^3P_1 \) (\( 1^1P_1 \)) meson defined by the nonlocal axial-vector current is antisymmetric (symmetric) under the exchange of quark and anti-quark momentum fractions in the SU(3) limit, whereas the chiral-odd LCDAs defined by the non-local tensor current are symmetric (antisymmetric) \cite{11,12}. The large magnitude of the first Gegenbauer moment of the mentioned antisymmetric LCDAs can have large impact on \( B \) decays involving a \( 1^3P_1 \) or/and \( 1^1P_1 \) meson(s). The related phenomenologies are thus interesting \cite{8,9,10}.

In this paper we present the first complete analysis for the form factors of the \( B_{u,d,s} \) decays into light axial-vector mesons (\( A \)) via the vector/axial-vector current in the light-cone sum rule (LCSR) approach, where \( A \) is the light \( P \)-wave meson which can be the \( 1^3P_1, 1^1P_1 \) or their mixture state. The method of light-cone sum rules has been widely used in the studies of nonperturbative processes, including weak baryon decays \cite{14}, heavy meson decays \cite{15}, and heavy to light transition form factors \cite{16,17,18}. Using the traditional QCD sum rule approach \cite{19}, where the three-point correlation function is considered, \( B \rightarrow a_1 \) form factors were calculated in Ref. \cite{20}. The BABAR
measurement of $B^0 \rightarrow a_1^+ \pi^-$ favors $V_0^{Ba_1}(0) \approx 0.30$, which is in good agreement with the LCSR result that we obtain here. The value given in Ref. [20] is a little small but still consistent with the data within the errors. $B \rightarrow A$ form factors were studied in Ref. [21] by using the light-front quark model. Nevertheless, it is found to be $V_0^{Ba_1}(0) = 0.13$ in the light-front quark model calculation. It is interesting to note that very recently Wang [22] used the $B$ meson light-cone sum rule approach to calculate $B \rightarrow a_1$ form factors.

The rest of the paper is organized as follows. The definitions of decay constants and $B_q \rightarrow A$ form factors are given in Sec. II. In Sec. III we derive the light-cone sum rules for the relevant form factors. The numerical results for form factors are given in Sec. IV, where we also give the predictions for decay rates of $B_{u,d,s} \rightarrow A e \nu_e$. A brief summary is given in Sec. V. The relevant expressions for two-parton and three-parton LCDAs are collected in Appendix A, an alternative definition for the form factors is given in Appendix B and the formula for semileptonic $B_{u,d,s} \rightarrow A e \nu_e$ decays is presented in Appendix C.

II. DEFINITIONS OF DECAY CONSTANTS AND FORM FACTORS

The G-parity$^1$ conserving decay constants of the axial-vector mesons are defined as

$$\langle 1^3P_1(P,\lambda)\bar{q}_1(0)\gamma_\mu\gamma_5q_2(0)|0\rangle = if_{3P_1}m_{3P_1}e^*_\mu^{(\lambda)},$$

$$\langle 1^1P_1(P,\lambda)\bar{q}_1(0)\sigma_{\mu\nu}\gamma_5q_2(0)|0\rangle = f_{1P_1}^{1+}(\epsilon_\mu^{(\lambda)}P_\nu - \epsilon_\nu^{(\lambda)}P_\mu),$$

where $f_{3P_1}$ is scale-independent, but $f_{1P_1}$ is scale-dependent \[11,12\]. On the other hand, we define the G-parity violating decay constants to be

$$\langle 1^3P_1(P,\lambda)\bar{q}_1(0)\sigma_{\mu\nu}\gamma_5q_2(0)|0\rangle = f_{3P_1}a_{0}^{1,3P_1}(\epsilon_\mu^{(\lambda)}P_\nu - \epsilon_\nu^{(\lambda)}P_\mu),$$

$$\langle 1^1P_1(P,\lambda)\bar{q}_1(0)\gamma_\mu\gamma_5q_2(0)|0\rangle = if_{1P_1}^{1+}(1\text{ GeV})a_{0}^{1,1P_1}m_{1P_1}e^*_\mu^{(\lambda)},$$

where $a_{0}^{1,3P_1}$ and $a_{0}^{1,1P_1}$, which are respectively the zeroth Gegenbauer moments of $\Phi_{\perp}^{1P_1}$ and $\Phi_{\parallel}^{1P_1}$, are zero in the SU(3) limit. The definitions for the LCDAs $\Phi_{\perp}^{1P_1}$ and $\Phi_{\parallel}^{1P_1}$ are collected in Appendix A. We further define $f_{1P_1}^{1+} = f_{3P_1}$ and $f_{1P_1} = f_{1P_1}^{1+}(\mu = 1\text{ GeV})$ in the present study as did in Ref. [11]. In the present work, the G-parity violating parameters, e.g. $a_{0}^{1,K_{1A}}, a_{0}^{2,K_{1A}}, a_{0}^{1,K_{1B}}$ and $a_{0,2}^{1,K_{1B}}$, are considered for mesons containing a strange quark. Note that for G-parity violating quantities, their signs have to be flipped from mesons to anti-mesons.

The semileptonic form factors for the $\mathcal{B}_q \rightarrow A$ transition are defined as

$$\langle A(P,\lambda)|A_\mu|\mathcal{B}_q(p_B)\rangle = if_{3P_1}m_{B_q} + m_A\epsilon_{\mu\nu\alpha\beta}^{(\lambda)}e_{\nu}^{*\alpha}P_B^{\alpha}P^{\beta}A_{B_0}^{B_qA}(q^2),$$

$$\langle A(P,\lambda)|V_\mu|\mathcal{B}_q(p_B)\rangle = -\left\{ (m_{B_q} + m_A)e_\mu^{(\lambda)*}V_1^{B_qA}(q^2) - (e_\mu^{(\lambda)*} \cdot p_B)(p_B + P)_\mu \frac{V_2^{B_qA}(q^2)}{m_{B_q} + m_A} \right\} - 2m_A\frac{e_\mu^{(\lambda)*} \cdot p_B}{q^2}q^\mu\left[ V_3^{B_qA}(q^2) - V_0^{B_qA}(q^2) \right].$$

\[1\] Here the idea of G-parity is extended to U-spin and V-spin multiplets.
where \( q = p_B - P \), \( V_{3B_q}^A(0) = V_0^{B_qA}(0) \),

\[ V_{3B_q}^A(q^2) = \frac{m_B + m_A}{2m_A} V_1^{B_qA}(q^2) - \frac{m_B - m_A}{2m_A} V_2^{B_qA}(q^2), \tag{2.6} \]

and we adopt the convention \( \epsilon^{0123} = -1 \). An alternative definition for the form factors is given in Appendix B.

### III. THE LIGHT-CONE SUM RULES

We consider the following two-point correlation function, which is sandwiched between the vacuum and transversely polarized \( A \) meson (in this section \( A \equiv \) a pure \( 1^3P_1 \) or \( 1^1P_1 \) state), to calculate the form factors:

\[
i \int d^4x e^{iqx} \langle A(P, \perp)|T[\bar{q}_1(x)\gamma_\mu(1 - \gamma_5)b(x) j_{Bq_2}^\dagger(0)]|0 \rangle
\]

\[
= -V_1(q^2)\epsilon_\mu^{(\perp)} + V_2(q^2)\epsilon_\mu^{(\perp)}(q)(2P + q)_\mu + V(q^2)\epsilon_\mu^{(\perp)}q_\mu
\]

\[-iA(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu}_{(\perp)}q^\rho P^\sigma, \tag{3.1} \]

where \( p_B^2 = (P + q)^2 \), \( P \) is the momentum of the \( A \) meson, and \( j_B = i\bar{q}_2\gamma_5b \) (with \( q_{1(2)} \equiv u, d \) or \( s \) is the interpolating current for the \( B_{q_2} \) meson, so that

\[
\langle 0|j_{Bq_2}(0)|\bar{B}_{q_2}(p_B) \rangle = \frac{f_{B_{q_2}} m_{B_{q_2}}^2}{m_b + m_{q_2}}. \tag{3.2} \]

In the region of sufficiently large virtualities: \( m_b^2 - p_B^2 \gg \Lambda_{\text{QCD}}m_b \) with \( q^2 \) being small and positive, the operator product expansion is applicable in Eq. (3.1), so that for an energetic \( A \) meson the correlation function in Eq. (3.1) can be represented in terms of the LCDAs of the \( A \) meson:

\[
i \int d^4x e^{iqx} \langle A(P, \perp)|T[\bar{q}_1(x)\gamma_\mu(1 - \gamma_5)b(x) j_{B}^\dagger(0)]|0 \rangle
\]

\[
= \int_0^1 du \left( \frac{-i}{q + k} \right)^2 - m_b^2 \text{Tr} \left[ \gamma_\mu(1 - \gamma_5)(\not{q} + \not{k} + m_b)\gamma_5 M_\perp^A \right]_{k \rightarrow uE\not{n}_-}
\]

\[+ O\left( \frac{m_b^2}{E^2} \right), \tag{3.3} \]

where \( E = |\vec{P}| \), \( P^\mu = En^\mu + m_A n_+^\mu/(4E) \simeq En^\mu \) with two light-like vectors \( n_\mu = (1, 0, 0, -1) \) and \( n_+^\mu = (1, 0, 0, 1) \) satisfying \( n_- n_+ = 2 \) and \( n_+^2 = n_+^2 = 0 \). Here, \( E \sim m_b \), and we have assigned the momentum of the \( q_1 \)-quark in the \( A \) meson to be

\[
k^\mu = uE n^\mu + k_\perp + \frac{k_\perp^2}{4uE} n_+^\mu, \tag{3.4} \]

where \( k_\perp \) is of order \( \Lambda_{\text{QCD}} \). Here \( u \) is the momentum fraction carried by the quark in the axial-vector meson. In Eq. (3.3), to calculate contributions in the momentum space, we have used the following substitution

\[
x^\mu \rightarrow -i \frac{\partial}{\partial k^\mu} \simeq -i \left( \frac{n_+^\mu}{2E} \frac{\partial}{\partial u} + \frac{\partial}{\partial k_{\perp\mu}} \right), \tag{3.5} \]

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to the Fourier transform for
\[
\langle A(P, \lambda) | \bar{q}_\alpha(x) q_2(0) | 0 \rangle = -\frac{i}{4} \int_0^1 du e^{iu P x} \times \left\{ f_A m_A \left[ P \gamma_5 \frac{\epsilon^{(\lambda)} x}{P x} \left( \Phi_{\parallel}(u) + \frac{m_A^2 x^2}{16 A_{\perp}^2} \right) + \left( \epsilon^{\ast}_{\lambda} - P \frac{\epsilon^{(\lambda)} x}{P x} \right) \gamma_5 g_{\perp}(u) \right] \\
- \frac{1}{2} \gamma_5 \frac{\epsilon^{(\lambda)} x}{P x} m_A^2 \bar{g}_3(u) + \epsilon_{\mu \nu \rho \sigma} \epsilon^{\ast}_{\lambda} p^\mu x^\sigma \gamma^\mu \frac{g_{\parallel}(u)}{4} \right] \right\} \delta_{\alpha} \right.,
\]
where \( x^2 \neq 0 \),
\[
\bar{g}_3(u) = g_3(u) + \Phi_{\parallel} - 2g_{\perp}(u), \\
\bar{h}_{\parallel}^{(t)} = h_{\parallel}^{(t)} - \frac{1}{2} \Phi_{\perp} - \frac{1}{2} h_{\perp}(u), \\
\bar{h}_3(u) = h_3(u) - \Phi_{\perp}(u).
\]
The detailed definitions for the relevant two-parton LCDAs are collected in Appendix A. In Eq. (3.5) the term of order \( k_{\perp}^2 \) is omitted in the calculation. Consequently, we can obtain the light-cone projection operator of the \( A \) meson in the momentum space,
\[
M_{\delta \alpha}^A = M_{\delta \alpha}^A + M_{\delta \alpha}^A \perp,
\]
where \( M_{\delta \alpha}^A \) and \( M_{\delta \alpha}^A \perp \) are the longitudinal and transverse projectors, respectively. The longitudinal projector which projects the longitudinal component of the axial-vector meson is given by
\[
M_{\parallel}^A = -\frac{i f_A}{4} \frac{m_A \epsilon^{\ast}_{\parallel}(n_+) x}{2} \left\{ \epsilon_{\lambda} \gamma_5 \Phi_{\parallel}(u) - \frac{f_A^+ m_A}{E} \left[ -\frac{i}{2} \sigma_{\mu \nu} \gamma_5 n_\mu n_\nu h_{\parallel}^{(t)}(u) \right. \\
- i E \int_0^u dv (\Phi_{\parallel}(v) - h_{\parallel}^{(t)}(v)) \sigma_{\mu \nu} \gamma_5 n_\mu \frac{\partial}{\partial k_{\perp} \nu} + \gamma_5 \frac{h_{\parallel}^{(t)}(u)}{2} \left. \right|_{k=up} + O \left( \frac{m_A^2}{E^2} \right) \right\},
\]
and the transverse projector reads
\[
M_{\perp}^A = i \frac{f_A^+}{4} E \left\{ \epsilon^{(\lambda)}_{\perp} \epsilon_{\lambda} \gamma_5 \Phi_{\perp}(u) \\
- \frac{f_A^+ m_A}{f_A^+ E} \left[ \epsilon^{(\lambda)}_{\perp} \gamma_5 g_{\perp}(u) - E \int_0^u dv (\Phi_{\parallel}(v) - g_{\perp}(v)) \epsilon_{\lambda} \gamma_5 \epsilon^{\ast}_{\perp} \frac{\partial}{\partial k_{\perp} \mu} \right. \\
+ i \epsilon_{\mu \nu \rho \sigma} \gamma^\mu \epsilon_{\perp}^{\ast}(n_\mu \frac{g_{\perp}^{(v)}(u)}{8} - E \frac{g_{\parallel}^{(v)}(u)}{4} \frac{\partial}{\partial k_{\perp} \sigma} \left. \right|_{k=up} + O \left( \frac{m_A^2}{E^2} \right) \right\},
\]
where the *exactly* longitudinal and transverse polarization vectors of the axial-vector meson, independent of the coordinate variable $x$, are defined as

$$
\epsilon^{(0)\mu} = \frac{E}{m_A} \left( 1 - \frac{m_A^2}{4E^2} \right) n_\mu - \frac{m_A^2}{4E^2} n_\perp, \quad \epsilon^{(\lambda)\mu} = \left( \epsilon^{(\lambda)\mu} - \frac{\epsilon^{(\lambda)\perp}}{2} n_\perp - \frac{\epsilon^{(\lambda)n}}{2} n_\parallel \right) \delta_{\lambda,\pm 1} .
$$

Here we have assumed that the meson moves along the $n_\perp$ direction. A similar discussion about the projection operator for the vector meson can be found in Ref. [23]. From the expansion of the transverse projection operator, one can find that contributions arising from the projection operator for the vector meson can be found in Ref. [23]. From the expansion of the

\[2\pi \delta_{\lambda,\pm 1}\]

the derivative with respect to the transverse momentum acts on the hard scattering amplitude before the collinear approximation is taken. Note also that applying equations of motion, the twist-three two-parton LCDAs can be expressed in terms of leading-twist and twist-three three-parton light-cone distribution amplitudes. One can thus understand from Eqs. (3.9) and (3.10) that the contributions due to the lowest-lying $A$ meson and $g_{1}(v)$ are suppressed by $m_A/E$ as compared with the term with $\Phi_{\perp}$. Note that in Eq. (3.3) the derivative with respect to the transverse momentum acts on the hard scattering amplitude before the collinear approximation is taken. Note also that applying equations of motion, the twist-three two-parton LCDAs can be expressed in terms of leading-twist and twist-three three-parton light-cone distribution amplitudes. One can thus understand from Eqs. (3.9) and (3.10) that the expansion parameter in the light-cone sum rules should be $m_A/m_b$, instead of twist.

At the quark-gluon level, after performing the integration of Eq. (3.3), the results up to $O(m_A/m_b)$ read

\[
V_{1}^{QCD} = -\frac{m_{B_2}f_{A}}{2} \int_{0}^{1} \frac{du}{u} \left\{ \frac{1}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \left[ \frac{m_{B}^{2} - q^{2}}{m_{B}^{2}} \Phi_{\perp}(u) - \left( \frac{m_{A}f_{A}}{m_{b}f_{A}} \right) 2ug_{1}(v)(u) \right] \right\},
\]

\[
V_{2}^{QCD} = -\frac{m_{B_2}f_{A}}{2} \int_{0}^{1} \frac{du}{u} \left\{ \frac{1}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \left[ \frac{1}{m_{B}^{2}} \Phi_{\perp}(u) - \frac{m_{A}f_{A}}{m_{b}f_{A}} \left( \frac{2\Phi_{a}(u)}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \right) \right] \right\},
\]

\[
V^{QCD} = \frac{q^{2}m_{B_2}f_{A}}{2} \int_{0}^{1} \frac{du}{u} \left\{ \frac{1}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \left[ \frac{1}{m_{B}^{2}} \Phi_{\perp}(u) - \frac{m_{A}f_{A}}{m_{b}f_{A}} \left( \frac{2\Phi_{a}(u)}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \right) \right] \right\},
\]

\[
A^{QCD} = -m_{B_2}f_{A} \int_{0}^{1} \frac{du}{u} \left\{ \frac{1}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \left[ \frac{1}{m_{B}^{2}} \Phi_{\perp}(u) - \frac{m_{A}f_{A}}{m_{b}f_{A}} \left( \frac{g_{1}(v)(u)/2}{m_{B}^{2} - uP_{B}^{2} - \bar{u}q^{2}} \right) \right] \right\},
\]

where $\Phi_{a}(u) \equiv \int_{0}^{u} dv \left( \Phi_{\parallel}(v) - g_{1}^{(v)}(v) \right)$ and $\bar{u} \equiv 1 - u$. Note that here the contributions due to the \textit{explicit} three-parton LCDAs are suppressed by $O(m_{A}/m_{B})$ as compared with the term involving $\Phi_{\perp}$.

We have given the results for $V_{1}, V_{2}$ and $A$ from the hadron and quark-gluon points of view. Thus, for instance, the form factor $V_{1}$ the contribution due to the lowest-lying $A$ meson can be further approximated with the help of quark-hadron duality:

\[
V_{1}(q^{2}) \cdot \frac{m_{B_2} + m_{A}}{m_{B_2}^{2} - P_{B}^{2}} \cdot \frac{m_{B_2}^{2}}{m_{b} + m_{q_2}} = \frac{1}{\pi} \int_{s_{0}^{2}}^{s_{0}^{2}} \frac{\text{Im}V_{1}^{QCD}(s, q^{2})}{s - P_{B}^{2}} ds,
\]

where $s_{0}$ is the excited state threshold. After applying the Borel transform $P_{B}^{2} \to M^{2}$ to the above equation, we obtain

\[
V_{1}(q^{2}) = \left( \frac{m_{B_2} + m_{A}}{m_{B_2}^{2}f_{B}} \right) e^{-m_{B_2}/M^{2}} \frac{1}{\pi} \int_{m_{B}^{2}}^{s_{0}^{2}} e^{s/M^{2}} \text{Im}V_{1}^{QCD}(s, q^{2}) ds.
\]
We obtain the light-cone sum rule results:

\[ V_{B_q}^{A}(q^2) = -\frac{(m_b + m_{q_2})m_B^2 f_A^+}{(m_{B_{q_2}} + m_A)} e^{(m_{B_{q_2}} - m_B^2)/M^2} \int_0^1 du \left\{ \frac{1}{u} e^{i(q^2 - m_b^2)/(uM^2)} \left[ \Phi^\perp(u) \theta \right] \right\}, \]

where \( V_{B_q}^{A}(q^2) \) is given by Eq. (2.6), and \( c(u, s_0) = us_0 - m_b^2 + (1 - u)q^2 \).

\[ V_{B_q}^{A}(q^2) = -\frac{(m_b + m_{q_2})m_B^2 f_A^+}{2m_{B_{q_2}} f_{B_{q_2}}} e^{(m_{B_{q_2}} - m_B^2)/M^2} \int_0^1 du \left\{ \frac{1}{u} e^{i(q^2 - m_b^2)/(uM^2)} \left[ \Phi^\perp(u) \theta \right] \right\}, \]

\[ V_{B_q}^{A}(q^2) = -\frac{(m_b + m_{q_2})m_{B_{q_2}} m_B}{2m_{B_{q_2}} f_{B_{q_2}}} \Phi^\perp(u) \left[ \theta + uM^2 \delta \right] \]

where \( V_{B_q}^{A}(q^2) \) is given by Eq. (2.6), and \( c(u, s_0) = us_0 - m_b^2 + (1 - u)q^2 \).

\[ A^{B_q}^{A}(q^2) = -\frac{(m_b + m_{q_2})m_B^2 f_A^+}{2m_{B_{q_2}} f_{B_{q_2}}} e^{(m_{B_{q_2}} - m_B^2)/M^2} \int_0^1 du \left\{ \frac{1}{u} e^{i(q^2 - m_b^2)/(uM^2)} \left[ \Phi^\perp(u) \theta \right] \right\}, \]

\[ A^{B_q}^{A}(q^2) = -\frac{(m_b + m_{q_2})m_{B_{q_2}} m_B}{2m_{B_{q_2}} f_{B_{q_2}}} \Phi^\perp(u) \left[ \theta + uM^2 \delta \right] \]

\[ \text{IV. RESULTS} \]

A. Input parameters

In this subsection we shall briefly summarize the relevant input parameters, which are collected in Tables I, II and III. The masses of \( u \) and \( d \) quarks are neglected.

The physical states \( K_1(1270) \) and \( K_1(1400) \) are the mixtures of the \( K_{1A} \) and \( K_{1B} \). \( K_{1A} \) and \( K_{1B} \) are not mass eigenstates, and they can be mixed together due to the strange and nonstrange light quark mass difference. Their relations can be written as

\[ |K_1(1270)\rangle = |K_{1A}\rangle \sin \theta_{K_1} + |K_{1B}\rangle \cos \theta_{K_1}, \]

\[ |K_1(1400)\rangle = |K_{1A}\rangle \cos \theta_{K_1} - |K_{1B}\rangle \sin \theta_{K_1}. \]

The sign ambiguity for \( \theta_{K_1} \) is due to the fact that one can add arbitrary phases to \( |K_{1A}\rangle \) and \( |K_{1B}\rangle \). This sign ambiguity can be removed by fixing the signs for \( f_{K_{1A}} \) and \( f_{K_{1B}} \), which do not vanish in the SU(3) limit. Following Ref. [11], we adopt the convention: \( f_{K_{1A}} > 0 \), \( f_{K_{1B}} > 0 \). Combining the analyses for the data of the decays \( B \rightarrow K_{1\gamma} \) and \( \tau^- \rightarrow K_{1\gamma}^- (1270) \nu_{\tau} \) [1, 24], the mixing angle was found to be \( \theta_{K_1} = -(34 \pm 13)\degree \) [13].
Analogous to the $\eta - \eta'$ mixing in the pseudoscalar sector, the $1^3P_1$ states, $f_1(1285)$ and $f_1(1420)$, have mixing via

$$|f_1(1285)\rangle = |f_1\rangle \cos \theta_{3P_1} + |f_8\rangle \sin \theta_{3P_1}, \quad |f_1(1420)\rangle = -|f_1\rangle \sin \theta_{3P_1} + |f_8\rangle \cos \theta_{3P_1},$$

and likewise the $1^1P_1$ states, $h_1(1170)$ and $h_1(1380)$, can be mixed in terms of the pure octet $h_8$ and singlet $h_1$,

$$|h_1(1170)\rangle = |h_1\rangle \cos \theta_{1P_1} + |h_8\rangle \sin \theta_{1P_1}, \quad |h_1(1380)\rangle = -|h_1\rangle \sin \theta_{1P_1} + |h_8\rangle \cos \theta_{1P_1}.$$  \hfill (4.3)

Using the Gell-Mann-Okubo mass formula \cite{11}, we obtain the mixing angles $\theta_{1P_1}$ and $\theta_{3P_1}$ to be

$$\theta_{3P_1} = (23.6^{+17}_{-23.6})^\circ, \quad \theta_{1P_1} = (28.1^{+9.8}_{-17.2})^\circ, \quad \text{for } \theta_{K_1} = -(34 \pm 13)^\circ.$$ \hfill (4.4)

For $3^P_1$ states the decay constants $f^q_{f_1(1285)}$ and $f^q_{f_1(1420)}$ are defined by

$$\langle 0|\bar{q}\gamma_\mu \gamma_5 q|f_1(1285)(P, \lambda)\rangle = -im f_{f_1(1285)} f^q_{f_1(1285)} \epsilon^{(\lambda)}_\mu,$$ \hfill (4.5)

$$\langle 0|\bar{q}\gamma_\mu \gamma_5 q|f_1(1420)(P, \lambda)\rangle = -im f_{f_1(1420)} f^q_{f_1(1420)} \epsilon^{(\lambda)}_\mu,$$

and for $1^1P_1$ states the tensor decay constants are

$$\langle 0|\bar{q}\sigma_{\mu\nu} q|h_1(1170)(P, \lambda)\rangle = if^{\perp q}_{h_1(1170)} \epsilon_{\mu\nu\alpha\beta} \epsilon^{(\lambda)}_\alpha P^\beta,$$ \hfill (4.6)

$$\langle 0|\bar{q}\sigma_{\mu\nu} q|h_1(1380)(P, \lambda)\rangle = if^{\perp q}_{h_1(1380)} \epsilon_{\mu\nu\alpha\beta} \epsilon^{(\lambda)}_\alpha P^\beta.$$

The reader is referred to \cite{8,11} for details.

**TABLE I**: Masses and decay constants for $1^3P_1$ and $1^1P_1$ states obtained in the QCD sum rule calculation \cite{11}.

| State   | Mass [GeV] | Decay constant $f_{3(1)P_1}$ [MeV] |
|---------|------------|-------------------------------------|
| $a_1(1260)$ | 1.23 ± 0.06 | 238 ± 10 |
| $f_1(1^3P_1)$ | 1.28 ± 0.06 | 245 ± 13 |
| $f_8(1^3P_1)$ | 1.29 ± 0.05 | 239 ± 13 |
| $K_{1A}$ | 1.31 ± 0.06 | 250 ± 13 |
| $b_1(1235)$ | 1.21 ± 0.07 | 180 ± 8 |
| $h_1(1^1P_1)$ | 1.23 ± 0.07 | 180 ± 12 |
| $h_8(1^1P_1)$ | 1.37 ± 0.07 | 190 ± 10 |
| $K_{1B}$ | 1.34 ± 0.08 | 190 ± 10 |
TABLE II: Gegenbauer moments of $\Phi_\perp$ and $\Phi_\parallel$ for $1^3P_1$ and $1^1P_1$ mesons, respectively, where $a_0^{\perp,K_1A}$ and $a_0^{\parallel,K_1B}$ are updated from the $B \to K_1\gamma$ analysis, and $a_1^{\parallel,K_1A}$, $a_2^{\perp,K_1A}$, $a_0^{\parallel,K_1B}$, $a_1^{\perp,K_1B}$ are then obtained from Eq. (141) in Ref. [11].

| $\mu$ | $a_2^{\parallel,a_1(1260)}$ | $a_2^{\parallel,f_1^{3P_1}}$ | $a_2^{\parallel,f_1^{3P_2}}$ | $a_2^{\parallel,K_1A}$ | $a_1^{\parallel,K_1A}$ |
|-------|------------------|------------------|------------------|------------------|------------------|
| 1 GeV | −0.02 ± 0.02    | −0.04 ± 0.03    | −0.07 ± 0.04    | −0.05 ± 0.03    | −0.30^{+0.00}_{−0.20} |
| 2.2 GeV | −0.01 ± 0.01 | −0.03 ± 0.02 | −0.05 ± 0.03 | −0.04 ± 0.02 | −0.25^{+0.00}_{−0.17} |

| $\mu$ | $a_1^{\perp,a_1(1260)}$ | $a_1^{\perp,f_1^{3P_1}}$ | $a_1^{\perp,f_1^{3P_2}}$ | $a_1^{\perp,K_1A}$ | $a_0^{\perp,K_1A}$ | $a_2^{\perp,K_1A}$ |
|-------|------------------|------------------|------------------|------------------|------------------|------------------|
| 1 GeV | −1.04 ± 0.34    | −1.06 ± 0.36    | −1.11 ± 0.31    | −1.08 ± 0.48    | 0.27^{+0.03}_{−0.17} | 0.02 ± 0.21 |
| 2.2 GeV | −0.85 ± 0.28 | −0.86 ± 0.29 | −0.90 ± 0.25 | −0.88 ± 0.39 | 0.25^{+0.03}_{−0.16} | 0.01 ± 0.15 |

| $\mu$ | $a_1^{\parallel,h_1^{3P_1}}$ | $a_1^{\parallel,h_1^{3P_2}}$ | $a_1^{\parallel,K_1B}$ | $a_0^{\parallel,K_1B}$ | $a_2^{\parallel,K_1B}$ |
|-------|------------------|------------------|------------------|------------------|------------------|
| 1 GeV | −1.95 ± 0.35    | −2.00 ± 0.35    | −1.95 ± 0.35    | −1.95 ± 0.45    | −0.19 ± 0.07    | 0.10^{+0.15}_{−0.19} |
| 2.2 GeV | −1.61 ± 0.29 | −1.65 ± 0.29 | −1.61 ± 0.29 | −1.57 ± 0.37 | −0.19 ± 0.07 | 0.07^{+0.11}_{−0.14} |

| $\mu$ | $a_2^{\perp,h_1^{3P_1}}$ | $a_2^{\perp,h_1^{3P_2}}$ | $a_2^{\perp,K_1B}$ | $a_2^{\perp,K_1B}$ |
|-------|------------------|------------------|------------------|------------------|
| 1 GeV | 0.03 ± 0.19    | 0.18 ± 0.22    | 0.14 ± 0.22    | −0.02 ± 0.22    | 0.30^{+0.00}_{−0.33} |
| 2.2 GeV | 0.02 ± 0.15 | 0.14 ± 0.17 | 0.11 ± 0.17 | −0.02 ± 0.17 | 0.24^{+0.00}_{−0.27} |

**B. Numerical results for the form factors**

We numerically analyze the light-cone sum rules for the transition form factors, where the pole $b$ quark mass is adopted in the calculation. The parameters appearing in the sum rules are evaluated at the factorization scale $\mu_f = \sqrt{m_{B_0}^2 - m_{b,pole}^2}$.

We find that, for $s_0 \simeq (34 \sim 37)\text{ GeV}^2$, the $V_1$ sum rule can be stable within the Borel mass range $6.0\text{ GeV}^2 < M^2 < 12.0\text{ GeV}^2$. Therefore we choose the Borel windows to be $6.0\text{ GeV}^2 < M^2 < 12.0\text{ GeV}^2$, $(6.0 + \delta_{v_2})\text{ GeV}^2 < M^2 < (12.0 + \delta_{v_2})\text{ GeV}^2$, and $(6.0 + \delta_{A})\text{ GeV}^2 < M^2 < (12.0 + \delta_{A})\text{ GeV}^2$ for $V_1, V_2$, and $A$, respectively, where the correction originating from higher resonance states amounts to 8% to 20%. $V_0(0)$ equals to $V_3(0)$, where the latter can be obtained from $V_1(0)$ and $V_2(0)$. As for $q^2 \neq 0$, the Borel windows for $V_3(q^2) - V_0(q^2)$ is $(6.0 + \delta_{v})\text{ GeV}^2 < M^2 < (12.0 + \delta_{v})\text{ GeV}^2$. The excited threshold $s_0$ is determined when the most stable plateau
TABLE III: Input parameters for quark masses, CKM matrix element, and effective $B_{(s)}$ decay constants, and for twist-3 3-parton LCDAs of the $K_{1A}$ and $K_{1B}$ states [1,11]. The G-parity violating parameters are updated due to new values for $a_0^{\perp}K_{1A}$ and $a_0^{||}K_{1B}$ given in Ref. [13].

| Parameter | Value |
|-----------|-------|
| $m_s(2\,\text{GeV})$ | $0.09 \pm 0.01$ |
| $m_b(2\,\text{GeV})$ | $4.85 \pm 0.05$ |
| $\alpha_s(1\,\text{GeV})$ | $0.495$ |
| $\alpha_s(2.2\,\text{GeV})$ | $0.287$ |

The CKM matrix element and the effective $B_{(s)}$ decay constants

| Parameter | Value |
|-----------|-------|
| $|V_{ub}|$ | $(4.31 \pm 0.30) \times 10^{-3}$ |
| $f_B(\alpha_s = 0)$ | $145 \pm 10$ |
| $f_B(\alpha_s = 0)$ | $165 \pm 10$ |

G-parity conserving parameters of twist-3 3-parton LCDAs at the scale 2.2 GeV

| Parameter | Value |
|-----------|-------|
| $f^V_{3,3P_1}$ | $0.0036 \pm 0.0018$ |
| $\omega^V_{3P_1}$ | $-2.9 \pm 0.9$ |
| $f^A_{3,3P_1}$ | $0.0034 \pm 0.0018$ |
| $\omega^A_{3P_1}$ | $-3.1 \pm 1.1$ |

G-parity violating parameters of twist-3 3-parton LCDAs of the $K_{1A}$ at $\mu = 2.2\,\text{GeV}$

| Parameter | Value |
|-----------|-------|
| $\sigma^V_{K_{1A}}$ | $0.01 \pm 0.04$ |
| $\lambda^K_{K_{1A}}$ | $-0.12 \pm 0.22$ |
| $\sigma^K_{K_{1A}}$ | $-1.9 \pm 1.1$ |

TABLE IV: Parameters (in units of $\text{GeV}^2$) relative to the excited-state thresholds of the light-cone sum rules, where $m_{m_b,\text{pole}} = 4.85\,\text{GeV}$ is used. Here parameters correspond to $q^2 = 0$.

| Parameter | Value |
|-----------|-------|
| $s_0$ | 34.18 |
| $\delta_{V_2}$ | 3.05 |
| $\delta_V$ | 3.05 |
| $\delta_A$ | 1.61 |

| Parameter | Value |
|-----------|-------|
| $b_1$ | $-0.0036 \pm 0.0014$ |
| $h_1$ | $-0.0033 \pm 0.0014$ |
| $h_8$ | $-0.0035 \pm 0.0014$ |
| $K_{1B}$ | $-0.0041 \pm 0.0018$ |

| Parameter | Value |
|-----------|-------|
| $|V_{ub}|$ | $(4.31 \pm 0.30) \times 10^{-3}$ |
| $f_B(\alpha_s = 0)$ | $145 \pm 10$ |
| $f_B(\alpha_s = 0)$ | $165 \pm 10$ |

G-parity violating parameters of twist-3 3-parton LCDAs of the $K_{1B}$ at $\mu = 2.2\,\text{GeV}$

| Parameter | Value |
|-----------|-------|
| $\lambda^K_{K_{1B}}$ | $-0.23 \pm 0.18$ |
| $\sigma^K_{K_{1B}}$ | $1.3 \pm 0.8$ |
| $\sigma^K_{K_{1B}}$ | $0.03 \pm 0.03$ |
of the \( V_1 \) sum rule result is obtained within the Borel window. Using the same \( s_0 \), we can then determine \( \delta V_1, \delta A, \) and \( \delta V \), so that the sum rule results for \( V_2, A \), and \( V_3 - V_0 \) are stable within the Borel windows. In Table \textbf{IV} we show that, for \( q^2 = 0 \) and \( m_{b,pole} = 4.85 \) GeV, the corresponding \( s_0 \)'s lie in the interval 34 – 35 GeV\(^2\). The values of \( \delta V_{2,A,V} \), corresponding to \( m_{b,pole} = 4.85 \) GeV, are also collected in Table \textbf{IV}. In the present study, the excited thresholds change slightly for larger \( q^2 \). However, for simplicity the values of \( s_0 \) and \( \delta V_{2,A,V} \) are chosen to be independent of \( q^2 \).

In the numerical analysis, we use the effective \( B \) decay constant \( f_B(\alpha_s = 0) = 145 \pm 10 \) MeV, which is in agreement with the QCD sum rule result without radiative corrections \cite{25}. We have checked that, using this value of \( f_B \) and \( m_b = 4.85 \) GeV in the light-cone sum rules of \( B \to \rho \) transition form factors of the same order of \( \alpha_s \) and \( m_\rho/m_b \), we can get results: \( A_1^{B\rho}(0) \simeq 0.23, A_2^{B\rho}(0) \simeq 0.22, \) and \( V^{B\rho}(0) \simeq 0.32 \), in good agreement with that given in Ref. \cite{18}, where the radiative corrections are included. In the literature, it was found that the contributions due to radiative corrections in the form factor sum rules can be canceled if one adopts the \( f_B \) sum rule result with the same order of \( \alpha_s \)-corrections in the calculation \cite{18,26}. Therefore, radiative corrections might be negligible in the present analysis.

Including the terms up to order of \( m_A/m_b \) in the light-cone expansion, the three-parton distribution amplitudes do not contribute directly to the sum rules, but they enter the sum rules since \( g_+(^{(a)}_b) \) and \( g_+(^{(v)}_b) \) can be represented in terms of the leading-twist (two-parton) and twist-3 three-parton LCDAs. To estimate the theoretical uncertainties of the sum rule results due to higher-twist effects, we put all parameters related to twist-3 three-parton LCDAs to be zero and find that the changes of the resulting form factors are less than 3%. We thus conclude that the higher-twist effects might be negligible.

The form factors results in the light-cone sum rule calculation are exhibited in Table \textbf{V} where the momentum dependence is parameterized in the three-parameter form:

\[
F^{B_A}(q^2) = \frac{F^{B_A}(0)}{1 - a(q^2/m_{B_A}^2) + b(q^2/m_{B_A}^2)^2},
\]

with \( F^{B_A} \equiv V_{0,1,2}^{B_A} \) or \( A^{B_A} \). For simplicity, we do not show the theoretical errors for the parameters \( a \) and \( b \). As \( q^2 \gtrsim 10 \) GeV\(^2\), the sum rule results become less stable. To get reliable estimate for the \( q^2 \)-dependence of the form factors, the present results are fitted in the range \( 0 \leq q^2 \leq 6 \) GeV\(^2\). The theoretical errors for \( F(0) \) are due to variation of the Borel mass, the Gegenbauer moments, the decay constants, the strange quark mass, and the pole \( b \) quark mass, which are then added in quadrature. The errors are dominated by variation of the pole \( b \) quark mass. It should be stressed that in the convention of the present work, the decay constants of \( 1^P_1 \) and \( 1^P_3 \) axial-vector mesons are of the same sign, so that the form factors for \( B \to 1^P_1 \) and \( B \to 1^P_3 \) transitions have opposite signs. The sign convention is the other way around in the light-front quark model \cite{21} and perturbative QCD \cite{27} calculations.

For the numerical analysis of \( B_s \to \) form factors, we adopt the effective decay constant \( f_{B_s}(\alpha_s = 0) \simeq 1.14 \times f_B(\alpha_s = 0) \simeq 165 \pm 11 \) MeV, which is estimated by using the relevant QCD sum rule result. Finally, we obtain the relations (with \( F \equiv V_{0,1,2} \) or \( A \)):

\[
\frac{F^{B_{K^*}A}(q^2)}{F_{B_sK^*A}(q^2)} \simeq \frac{F^{B_{P}A}(q^2)}{F_{B_sP}(q^2)} \simeq -\frac{2F^{B_s}(q^2)}{F_{B_s}(q^2)}
\]
TABLE V: Form factors for $B_{u,d} \to a_1, b_1, K_{1A}, K_{1B}, f_1, f_8, h_1, h_8$ transitions obtained in the light-cone sum rule calculation are fitted to the 3-parameter form in Eq. (4.7). Here, because the decay constants, $f_3 P_1$ and $f_1^+ P_1$, which are defined in Eqs. (2.1) and (2.2), are of the same sign, the form factors for $B \to 1^3 P_1$ and $B \to 1^1 P_1$ transitions have opposite signs.

| $F$ | $F(0)$ | $a$ | $b$ | $F$ | $F(0)$ | $a$ | $b$ |
|-----|--------|-----|-----|-----|--------|-----|-----|
| $V_{1B_{a1}}$ | $0.37 \pm 0.07$ | 0.645 | 0.250 | $V_{1B_{b1}}$ | $-0.20 \pm 0.04$ | 0.748 | 0.063 |
| $V_{2B_{a1}}$ | $0.42 \pm 0.08$ | 1.48 | 1.00 | $V_{2B_{b1}}$ | $-0.09 \pm 0.02$ | 0.539 | 1.76 |
| $V_{0B_{a1}}$ | $0.30 \pm 0.05$ | 1.77 | 0.926 | $V_{0B_{b1}}$ | $-0.39 \pm 0.07$ | 1.22 | 0.426 |
| $A_{B_{a1}}$ | $0.48 \pm 0.09$ | 1.64 | 0.986 | $A_{B_{b1}}$ | $-0.25 \pm 0.05$ | 1.69 | 0.910 |
| $V_{1BK_{1A}}$ | $0.34 \pm 0.07$ | 0.635 | 0.211 | $V_{1BK_{1B}}$ | $-0.29^{+0.08}_{-0.05}$ | 0.729 | 0.074 |
| $V_{2BK_{1A}}$ | $0.41 \pm 0.08$ | 1.51 | 1.18 | $V_{2BK_{1B}}$ | $-0.17^{+0.05}_{-0.03}$ | 0.919 | 0.855 |
| $V_{0BK_{1A}}$ | $0.22 \pm 0.04$ | 2.40 | 1.78 | $V_{0BK_{1B}}$ | $-0.45^{+0.12}_{-0.08}$ | 1.34 | 0.690 |
| $A_{BK_{1A}}$ | $0.45 \pm 0.09$ | 1.60 | 0.974 | $A_{BK_{1B}}$ | $-0.37^{+0.10}_{-0.06}$ | 1.72 | 0.912 |
| $V_{1B_{f1}}$ | $0.23 \pm 0.04$ | 0.640 | 0.153 | $V_{1B_{h1}}$ | $-0.13 \pm 0.03$ | 0.612 | 0.078 |
| $V_{2B_{f1}}$ | $0.26 \pm 0.05$ | 1.47 | 0.956 | $V_{2B_{h1}}$ | $-0.07 \pm 0.02$ | 0.500 | 1.63 |
| $V_{0B_{f1}}$ | $0.18 \pm 0.03$ | 1.81 | 0.880 | $V_{0B_{h1}}$ | $-0.24 \pm 0.04$ | 1.16 | 0.294 |
| $A_{B_{f1}}$ | $0.30 \pm 0.05$ | 1.63 | 0.900 | $A_{B_{h1}}$ | $-0.17 \pm 0.03$ | 1.54 | 0.848 |
| $V_{1B_{f8}}$ | $0.16 \pm 0.03$ | 0.644 | 0.209 | $V_{1B_{h8}}$ | $-0.11 \pm 0.02$ | 0.623 | 0.094 |
| $V_{2B_{f8}}$ | $0.19 \pm 0.03$ | 1.49 | 1.09 | $V_{2B_{h8}}$ | $-0.06 \pm 0.01$ | 0.529 | 1.53 |
| $V_{0B_{f8}}$ | $0.12 \pm 0.02$ | 1.84 | 0.749 | $V_{0B_{h8}}$ | $-0.18 \pm 0.03$ | 1.22 | 0.609 |
| $A_{B_{f8}}$ | $0.22 \pm 0.04$ | 1.64 | 0.919 | $A_{B_{h8}}$ | $-0.13 \pm 0.02$ | 1.56 | 0.827 |

\[ \frac{F_{BK_{1B}}^B(q^2)}{F_{BK_{1B}}^F(q^2)} \simeq \frac{F_{B_{h1}}^F(q^2)}{F_{B_{h1}}^B(q^2)} \simeq -\frac{2F_{B_{h8}}^F(q^2)}{F_{B_{h8}}^B(q^2)} \simeq 1.14. \]  

(4.8)

In Table VI we show the form factor results at the maximum recoil (i.e., at $q^2 = 0$).

C. Branching ratios

Our results for the semileptonic decay rates $B_{u,d,s} \to A e \bar{\nu}_e$ are listed in Table VII. Most branching ratios are of order $10^{-4}$. For $B_{u,d}$ decays involving the $a_1$ or $f_1(1285)$ we obtain $\Gamma_L/\Gamma_T \simeq 0.6$, whereas for decays containing $1^3 P_1$ mesons we find that $\Gamma_L/\Gamma_T$ is close to 2. In short, the polarization fractions follow the relations: $\Gamma_- > \Gamma_L \gg \Gamma_+$ for the former, and $\Gamma_L > \Gamma_- \gg \Gamma_+$ for the latter. On the other hand, we have $\Gamma_L/\Gamma_T \sim 0.6 - 0.7, 1.1, 1.4 - 1.8$ for the semileptonic $B_s$ decays involving the $f_1, h_1$ and $K_1$, respectively. These results are sensitive to the values of form factors. Moreover, we have the salient patterns
\[ \mathcal{B}^u[a_1(1260)] > \mathcal{B}^u[b_1(1235)] > \mathcal{B}^u[f_1(1285)] > \mathcal{B}^u[h_1(1170)] \gg \mathcal{B}^u[f_1(1420)] \gg \mathcal{B}^u[h_1(1380)], \]

(4.9)
\[ \mathcal{B}^s[K_1(1270)] \gg \mathcal{B}^s[K_1(1400)] > \mathcal{B}^s[f_1(1420)] > \mathcal{B}^s[h_1(1380)] \gg \mathcal{B}^s[f_1(1285)] \gg \mathcal{B}^s[h_1(1170)], \]

(4.10)

where $\mathcal{B}^q[a_1(1260)] \equiv \mathcal{B}(B_q \to A e \bar{\nu}_e)$. These patterns are sensitive to the mixing angles and thus can offer deeper insights for the quark contents of the $P$-wave mesons.
TABLE VI: Form factors for $B_s \to K_{1A}, K_{1B}, f_1, f_8, h_1, h_8$ transitions obtained in the light-cone sum rule calculation are fitted to the 3-parameter form in Eq. (4.7). Here, because the decay constants, $f_{3P_1}$ and $f_{1P_1}^\perp$, are of the same sign, the form factors for $B_s \to 1^1P_1$ and $B_s \to 1^3P_1$ transitions have opposite signs.

| $F$ | $F(0)$ | $a$ | $b$ |
|-----|--------|-----|-----|
| $V^B_{1, K_{1A}}$ | $0.30 \pm 0.06$ | 0.635 | 0.211 |
| $V^B_{2, K_{1A}}$ | $0.36 \pm 0.07$ | 1.51 | 1.18 |
| $V^B_{0, K_{1A}}$ | $0.19 \pm 0.04$ | 2.40 | 1.78 |
| $A^B_{1, K_{1A}}$ | $0.40 \pm 0.08$ | 1.60 | 0.974 |
| $V^B_{1, f_1}$ | $0.20 \pm 0.04$ | 0.640 | 0.153 |
| $V^B_{2, f_1}$ | $0.23 \pm 0.04$ | 1.47 | 0.956 |
| $V^B_{0, f_1}$ | $0.16 \pm 0.03$ | 1.81 | 0.880 |
| $A^B_{1, f_1}$ | $0.26 \pm 0.04$ | 1.63 | 0.900 |
| $V^B_{1, f_8}$ | $-0.28 \pm 0.05$ | 0.644 | 0.209 |
| $V^B_{2, f_8}$ | $0.33 \pm 0.05$ | 1.49 | 1.09 |
| $V^B_{0, f_8}$ | $0.21 \pm 0.04$ | 1.84 | 0.749 |
| $A^B_{1, f_8}$ | $-0.39 \pm 0.07$ | 1.64 | 0.919 |

| $F$ | $F(0)$ | $a$ | $b$ |
|-----|--------|-----|-----|
| $V^B_{1, K_{1B}}$ | $-0.25^{+0.07}_{-0.04}$ | 0.729 | 0.074 |
| $V^B_{2, K_{1B}}$ | $-0.15^{+0.04}_{-0.03}$ | 0.919 | 0.855 |
| $V^B_{0, K_{1B}}$ | $-0.40^{+0.11}_{-0.07}$ | 1.34 | 0.690 |
| $A^B_{1, K_{1B}}$ | $-0.33^{+0.09}_{-0.05}$ | 1.72 | 0.912 |
| $V^B_{1, h_1}$ | $0.11 \pm 0.03$ | 0.612 | 0.078 |
| $V^B_{2, h_1}$ | $-0.06 \pm 0.02$ | 0.500 | 1.63 |
| $V^B_{0, h_1}$ | $0.21 \pm 0.04$ | 1.16 | 0.294 |
| $A^B_{1, h_1}$ | $-0.15 \pm 0.03$ | 1.54 | 0.848 |
| $V^B_{1, h_8}$ | $0.19 \pm 0.04$ | 0.623 | 0.094 |
| $V^B_{2, h_8}$ | $0.11 \pm 0.02$ | 0.529 | 1.53 |
| $V^B_{0, h_8}$ | $0.32 \pm 0.05$ | 1.22 | 0.609 |
| $A^B_{1, h_8}$ | $0.23 \pm 0.04$ | 1.56 | 0.827 |

V. SUMMARY

We have calculated the vector and axial-vector form factors of $B$ decays into $P$-wave axial-vector mesons in the light-cone sum rule approach. Owing to the $G$-parity, the chiral-even two-parton light-cone distribution amplitudes of the $1^3P_1$ and $1^1P_1$ mesons are respectively symmetric and antisymmetric under the exchange of quark and anti-quark momentum fractions in the SU(3) limit. For chiral-odd light-cone distribution amplitudes, it is the other way around. The sum rule results for form factors are sensitive to the light-cone distribution amplitudes of the axial-vector mesons. To extract the relevant form factors, the polarization of the axial-vector meson is chosen to be transversely polarized in the light-cone sum rule calculation. For the resulting sum rules, we have included the terms up to order of $m_A/m_b$ in the light-cone expansion. The numerical impact of $1/m_b$ corrections is under control. As discussed in Sec. III, it should be stressed that the expansion parameter in the light-cone sum rules is $m_A/m_b$, instead of twist. In the present study, because the decay constants, $f_{3P_1}$ and $f_{1P_1}^\perp$, which are defined in Eqs. (2.1) and (2.2), are of the same sign, the form factors for $B \to 1^1P_1$ and $B \to 1^3P_1$ transitions have opposite signs. The sum rule results could be improved in the future by including $O(\alpha_s)$ corrections to the sum rules and by improving the input parameters describing the light-meson distribution amplitudes, for instance from lattice calculations. We have presented the results for the semileptonic decay rates of $B_{u,d,s} \to Ae\bar{\nu}_e$. Theses will allow further tests of our form factor results and of the mixing angles in the future.
TABLE VII: Decay rates of $B_{u,d,s} \to A e\bar{\nu}_e$ obtained in this work, where $\Gamma_{\pm,L}$ are in units of $10^6\text{s}^{-1}$, and the branching ratios are in units of $10^{-4}$. $\Gamma_L$ stands for the portion of the rate with a longitudinal polarization $A$, $\Gamma_+$ with a positive helicity $A$, and $\Gamma_-$ with a negative helicity $A$. $\Gamma_T = \Gamma_+ + \Gamma_-$. Here we use $\theta_{K_1} = -(34 \pm 13)^\circ$, $\theta_{3P_3} = (23.6^{+17.0}_{-23.0})^\circ$, and $\theta_{1P_1} = (28.1^{+9.8}_{-17.2})^\circ$. The first error comes from the variation of form factors, and the second from the mixing angles.

| $A$   | $\Gamma_+$ | $\Gamma_-$ | $\Gamma_L$ | $\Gamma_L/\Gamma_T$ | $\mathcal{B}(\bar{B}^0 \to A^+ e^- \bar{\nu}_e)$ | $\mathcal{B}(B^- \to A^0 e^- \bar{\nu}_e)$ |
|-------|------------|------------|------------|---------------------|---------------------------------|---------------------------------|
| $a_1(1260)$ | 7.6$^{+3.2}_{-2.6}$ | 115$^{+48}_{-39}$ | 74.8$^{+30.7}_{-25.4}$ | 0.61$^{+0.00}_{-0.00}$ | 3.02$^{+1.03}_{-1.03}$ | 3.24$^{+1.33}_{-1.13}$ |
| $f_1(1285)$ | 4.9$^{+1.7+0.0}_{-1.4-1.1}$ | 62.1$^{+23.6+2.1}_{-19.8-18.9}$ | 40.4$^{+14.9+0.5}_{-12.5-11.3}$ | 0.61$^{+0.01+0.02}_{-0.00-0.01}$ | 1.63$^{+0.60+0.04}_{-0.51-0.48}$ | 1.75$^{+0.65+0.04}_{-0.55-0.52}$ |
| $f_1(1420)$ | 0.1$^{+0.1+0.2}_{-0.0-0.0}$ | 2.2$^{+10.1+15.7}_{-9.0-10.3}$ | 1.1$^{+0.10+0.60}_{-0.14-0.00}$ | 0.48$^{+0.05+0.42}_{-0.05-0.03}$ | 0.06$^{+0.03+0.44}_{-0.03-0.04}$ |
| $h_1(1235)$ | 3.3$^{+1.4}_{-1.2}$ | 37.7$^{+16.6}_{-13.5}$ | 85.5$^{+36.5}_{-30.2}$ | 2.08$^{+0.03}_{-0.01}$ | 1.93$^{+0.84}_{-0.68}$ | 2.07$^{+0.90}_{-0.73}$ |
| $h_1(1170)$ | 1.9$^{+1.2+0.1}_{-0.9-0.4}$ | 24.7$^{+10.8+1.0}_{-8.8-0.8}$ | 54.7$^{+24.8+2.4}_{-20.2-11.2}$ | 2.05$^{+0.01+0.01}_{-0.01-0.01}$ | 1.24$^{+0.57+0.06}_{-0.45-0.25}$ | 1.33$^{+0.60+0.06}_{-0.49-0.27}$ |
| $h_1(1380)$ | 0.1$^{+0.2+0.5}_{-0.0-0.1}$ | 0.8$^{+0.1+0.3}_{-0.2-0.8}$ | 1.8$^{+0.5+0.78}_{-0.5-1.7}$ | 1.97$^{+0.03+0.12}_{-0.14-0.07}$ | 0.04$^{+0.01+0.18}_{-0.01-0.00}$ | 0.04$^{+0.02+0.20}_{-0.01-0.00}$ |

| $A$   | $\Gamma_+$ | $\Gamma_-$ | $\Gamma_L$ | $\Gamma_L/\Gamma_T$ | $\mathcal{B}(\bar{B}^0 \to A^+ e^- \bar{\nu}_e)$ | $\mathcal{B}(B^- \to A^0 e^- \bar{\nu}_e)$ |
|-------|------------|------------|------------|---------------------|---------------------------------|---------------------------------|
| $f_1(1285)$ | 0.4$^{+1.0+0.2}_{-0.2-0.3}$ | 4.4$^{+9.0+32.8}_{-4.2-3.3}$ | 3.3$^{+8.2+20.1}_{-3.2-2.9}$ | 0.68$^{+0.07+0.00}_{-0.42-0.36}$ | 0.12$^{+0.27+0.00}_{-0.12-0.10}$ |
| $f_1(1420)$ | 0.4$^{+0.9+0.4}_{-0.8-2.1}$ | 90.8$^{+17.9+2.8}_{-16.3-27.3}$ | 57.6$^{+12.3+2.6}_{-11.2-18.3}$ | 0.59$^{+0.01+0.02}_{-0.01-0.00}$ | 2.27$^{+0.45+0.08}_{-0.42-0.70}$ |
| $h_1(1170)$ | 0.1$^{+0.1+0.2}_{-0.0-0.0}$ | 0.1$^{+2.0+5.7}_{-0.1-0.0}$ | 0.2$^{+0.3+11.6}_{-0.2-2.0}$ | 1.37$^{+0.02+0.71}_{-0.48-0.00}$ | 0.01$^{+0.00+0.25}_{-0.00-0.00}$ |
| $h_1(1380)$ | 3.7$^{+2.2+0.0}_{-1.7-0.3}$ | 37.6$^{+17.4+9.0}_{-14.1-4.4}$ | 74.2$^{+37.7+0.0}_{-30.0-8.2}$ | 1.80$^{+0.04+0.00}_{-0.06-0.01}$ | 1.69$^{+0.84+0.00}_{-0.67-0.19}$ |
| $K_1(1270)$ | 9.3$^{+3.7+0.6}_{-4.1-1.4}$ | 141$^{+54+9}_{-61-22}$ | 159$^{+57+0}_{-72-10}$ | 1.06$^{+0.00+0.15}_{-0.05-0.12}$ | 4.53$^{+0.16+0.00}_{-0.10-0.00}$ |
| $K_1(1400)$ | 9.4$^{+5.4+0.6}_{-4.1-1.5}$ | 199$^{+54+8}_{-51-19}$ | 135$^{+49+0}_{-61-7}$ | 1.05$^{+0.00+0.13}_{-0.04-0.11}$ | 3.86$^{+1.43+0.03}_{-1.70-0.40}$ |

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APPENDIX A: TWO-PARTON DISTRIBUTION AMPLITUDES

The chiral-even LCDAs are given by

$$\langle A(P, \lambda)|\bar{q}_1(x)\gamma_\mu\gamma_5q_2(0)|0\rangle = if_A m_A \int_0^1 du e^{iu px} \left\{\frac{e^{(\lambda)}_x}{px} \Phi_\parallel(u) + e^{(\lambda)}_{\perp\mu} g^{(a)}_\perp(u) \right.$$ \[ \left. - \frac{1}{2} x^\mu \frac{e^{(\lambda)}_x}{px} x^\nu x^\sigma x^\rho g_3(u) + O(x^2) \right\}, \] (A1)

$$\langle A(P, \lambda)|\bar{q}_1(x)\gamma_\mu q_2(0)|0\rangle = -if_A m_A e^{\mu\rho\sigma} e^{(\lambda)}_x P^\rho x^\sigma \int_0^1 du e^{iu px} \left(\frac{g^{(v)}_\perp(u)}{4} + O(x^2) \right), \] (A2)

where $u$ and $\bar{u} \equiv 1 - u$ are the momentum fractions carried by the $q_1$ and $\bar{q}_2$ quarks, respectively, in the axial-vector meson. The chiral-odd LCDAs are defined by

$$\langle A(P, \lambda)|\bar{q}_1(x)\sigma_\mu\gamma_5q_2(0)|0\rangle = f_A \int_0^1 du e^{iu px} \left\{e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu \right\} \Phi_\perp(u).$$
\[ + \frac{m^2_A}{(p x)^2} (p_\mu x_\nu - p_\nu x_\mu) h^{(t)}_\mu(x) \]
\[ + \frac{1}{2} (\varepsilon^{*(\lambda)}_{\perp} x_\nu - \varepsilon^{*(\lambda)}_{\perp} x_\mu) \frac{m^2_A}{p x} h_3(x) + O(x^2) \} \right. \right) , \quad \varepsilon^{*(\lambda)}_{\mu} = \varepsilon^{*(\lambda)}_{\mu} - \varepsilon^{*(\lambda)}_{\parallel} \right. \right). \quad \text{(A5)}

where \( p_\mu = P_\mu - m^2_A x_\mu/(2px) \). Here, \( \Phi_{\parallel}, \Phi_{\perp} \) are of twist-2, \( g^{(a)}_{\perp}, g^{(v)}_{\perp}, h^{(t)}_{\parallel}, h^{(p)}_{\parallel} \) of twist-3, and \( g_3, h_3 \) of twist-4. Note that in the definitions of LCDAs, the longitudinal and transverse projections of polarization vectors \( \varepsilon^{*(\lambda)}_{\mu} \) along the \( x \)-direction for the axial-vector meson are given by

\[ \varepsilon^{*(\lambda)}_{\parallel} = \frac{\varepsilon^{*(\lambda)}_{\mu} x_\mu}{p x} \left( p_\mu - \frac{m^2_A}{2px} x_\mu \right) \]  
\[ \varepsilon^{*(\lambda)}_{\perp} = \varepsilon^{*(\lambda)}_{\mu} - \varepsilon^{*(\lambda)}_{\parallel} \right. \right). \quad \text{(A5)}

One should distinguish the above projections from the \textit{exactly} longitudinal (\( \varepsilon^{*(0)\mu} \)) and transverse (\( \varepsilon^{*(\lambda)\mu} \)) polarization vectors of the axial-vector meson, given in Eq. (2.11) where the results are independent of the coordinate variable \( x \).

In SU(3) limit, due to \( G \)-parity, \( \Phi_{\parallel}, g^{(a)}_{\perp}, g^{(v)}_{\perp} \), and \( g_3 \) are symmetric (antisymmetric) under the replacement \( u \rightarrow 1 - u \) for the \( 1^3P_1 \) \((1^1P_1) \) states, whereas \( \Phi_{\perp}, h^{(t)}_{\parallel}, h^{(p)}_{\parallel} \), and \( h_3 \) are antisymmetric (symmetric). In other words, in the SU(3) limit it follows that

\[ \int_0^1 du \Phi_{\perp}(u) = \int_0^1 du h^{(t)}_{\parallel}(u) = \int_0^1 du h^{(p)}_{\parallel}(u) = \int_0^1 du h_3(u) = 0 \]  
\[ \int_0^1 du \Phi_{\parallel}(u) = \int_0^1 du g^{(a)}_{\parallel}(u) = \int_0^1 du g^{(v)}_{\parallel}(u) = \int_0^1 du g_3(u) = 0 \]  
\[ \text{(A6)} \]

for \( 1^3P_1 \) states, but becomes

\[ \int_0^1 du \Phi^{iP_1}_{\parallel}(u) = \int_0^1 du g^{(a)}_{\parallel}(u) = \int_0^1 du g^{(v)}_{\parallel}(u) = \int_0^1 du g_3(u) = 0 \]  
\[ \int_0^1 du \Phi^{iP_1}_{\perp}(u) = 1 \]  
\[ \text{(A7)} \]

for \( 1^1P_1 \) states. The above integrals are not zero if \( m_{q_1} \neq m_{q_2} \), but becomes zero if \( m_{q_1} = m_{q_2} \). For convenience, we therefore normalize the distribution amplitudes of the \( 1^3P_1 \) and \( 1^1P_1 \) states to be subject to

\[ \int_0^1 du \Phi^{iP_1}_{\parallel}(u) = 1 \]  
\[ \int_0^1 du \Phi^{iP_1}_{\perp}(u) = 1 \]  
\[ \text{(A8)} \]

We set \( f^{iP_1}_{\parallel} = f_{3P_1} \) and \( f^{iP_1}_{\perp} = f_{3P_3} \) (\( \mu = 1 \text{ GeV} \)) in the study, such that we have

\[ \langle 1^3P_1(P,\lambda)|\vec{q}_1(0)\sigma_{\mu\nu}\gamma_5q_2(0)|0\rangle = f_{3P_1} a^{i3P_1}_{\parallel}(\varepsilon^{*(\lambda)}_{\mu} P_\nu - \varepsilon^{*(\lambda)}_{\mu} P_\mu) , \quad \text{(A9)} \]
\[ \langle 1^1P_1(P,\lambda)|\vec{q}_1(0)\gamma_\mu\gamma_5q_2(0)|0\rangle = i f_{1P_1} (1 \text{ GeV}) a_{1P_1}^{i3P_1} m_{1P_1} \varepsilon^{*(\lambda)}_{\mu} , \quad \text{(A10)} \]

where \( a^{i3P_1}_{\parallel} \) and \( a^{i3P_1}_{\perp} \) are the Gegenbauer zeroth moments and vanish in the SU(3) limit.

We take into account the approximate forms of twist-2 distributions for the \( 1^3P_1 \) mesons to be

\[ \Phi_{\parallel}(u) = 6u\vec{u} \left[ 1 + 3a_0^i \xi + a_2^i \frac{3}{2}(5\xi^2 - 1) \right] \]  
\[ \Phi_{\perp}(u) = 6u\vec{u} \left[ a_0^i + 3a_2^i \xi + a_2^i \frac{3}{2}(5\xi^2 - 1) \right] \]  
\[ \text{(A11)} \]
and for the $1^1P_1$ mesons to be

$$
\Phi_\parallel (u) = 6u\bar{u} \left[ a_0^\parallel + 3a_1^\parallel \xi + a_2^\parallel \frac{3}{2} (5\xi^2 - 1) \right], \quad (A13)
$$
$$
\Phi_\perp (u) = 6u\bar{u} \left[ 1 + 3a_1^\perp \xi + a_2^\perp \frac{3}{2} (5\xi^2 - 1) \right], \quad (A14)
$$

where $\xi = 2u - 1$.

For the relevant two-parton twist-3 chiral-even LCDAs, we take the approximate expressions up to conformal spin $9/2$ and $\mathcal{O}(m_s)$ [11]:

$$
g_\perp^{(a)}(u) = \frac{3}{4} (1 + \xi^2) + \frac{3}{2} a_1^\parallel \xi^3 + \left( \frac{3}{7} a_2^\parallel + 5\zeta^V_{3,3} P_1 \right) (3\xi^2 - 1) + \left( \frac{9}{112} a_2^\parallel + \frac{105}{16} \zeta^A_{3,3} P_1 - \frac{15}{64} \zeta^V_{3,3} P_1 \omega^V_{3,3} P_1 \right) (35\xi^4 - 30\xi^2 + 3)
$$

$$
+ 5 \left[ \frac{21}{4} \zeta^V_{3,3} P_1 \sigma^V_{3,3} P_1 + \zeta^A_{3,3} P_1 \left( \lambda^A_{3,3} P_1 - \frac{3}{16} \zeta^A_{3,3} P_1 \right) \right] \xi (5\xi^2 - 3)
$$

$$
- \frac{9}{2} a_1^\perp \delta_+ \left( \frac{3}{2} + \frac{3}{2} \xi^2 + \ln u + \ln \bar{u} \right) - \frac{9}{2} a_1^\perp \delta_- (3\xi + \ln \bar{u} - \ln u), \quad (A15)
$$

$$
g_\perp^{(v)}(u) = \frac{3}{4} u\bar{u} \left[ 1 + \left( \frac{3}{2} a_1^\parallel + \frac{20}{3} \zeta^A_{3,3} P_1 \lambda^A_{3,3} P_1 \right) \xi \right]
$$

$$
+ \left[ \frac{1}{4} a_2^\parallel + \frac{5}{3} \zeta^V_{3,3} P_1 \left( 1 - \frac{3}{16} \omega^V_{3,3} P_1 \right) + \frac{35}{4} \zeta^A_{3,3} P_1 \right] (5\xi^2 - 1)
$$

$$
+ \frac{35}{4} \left( \zeta^V_{3,3} P_1 \sigma^V_{3,3} P_1 - \frac{1}{28} \zeta^A_{3,3} P_1 \omega^V_{3,3} P_1 \right) (7\xi^2 - 3)
$$

$$
- 18 a_1^\perp \delta_+ (3u\bar{u} + \bar{u} \ln \bar{u} + u \ln u) - 18 a_1^\perp \delta_- (u\bar{u} \xi + \bar{u} \ln \bar{u} - u \ln u), \quad (A16)
$$

for the $1^3P_1$ states, and

$$
g_\perp^{(a)}(u) = \frac{3}{4} u\bar{u} (1 + \xi^2) + \frac{3}{2} a_1^\parallel \xi^3 + 5 \left[ \frac{21}{4} \zeta^V_{3,3} P_1 + \zeta^A_{3,3} P_1 \left( 1 - \frac{3}{16} \omega^A_{3,3} P_1 \right) \right] \xi (3\xi^2 - 1)
$$

$$
+ \frac{3}{16} a_2^\parallel \left( 15\xi^4 - 6\xi^2 - 2 \right) + 5 \zeta^V_{3,3} P_1 (3\xi^2 - 1)
$$

$$
+ \frac{105}{16} \zeta^A_{3,3} P_1 \sigma^A_{3,3} P_1 - \frac{1}{28} \zeta^V_{3,3} P_1 \omega^V_{3,3} P_1 \right) (35\xi^4 - 30\xi^2 + 3)
$$

$$
- 15 a_2^\perp \delta_+ \xi^3 + \frac{3}{2} \delta_- (3\xi^2 - 1)
$$

$$
- \frac{3}{2} \delta_+ (2\xi + \ln \bar{u} - \ln u) + \delta_- (2 + \ln u + \ln \bar{u}) (1 + 6a_2^\perp), \quad (A17)
$$

$$
g_\perp^{(v)}(u) = 6u\bar{u} \left[ a_0^\parallel + a_1^\parallel \xi + \left[ \frac{1}{4} a_2^\parallel + \frac{5}{3} \zeta^V_{3,3} P_1 \left( \lambda^V_{3,3} P_1 - \frac{3}{16} \sigma^V_{3,3} P_1 \right) + \frac{35}{4} \zeta^A_{3,3} P_1 \sigma^A_{3,3} P_1 \right] (5\xi^2 - 1)
$$

$$
+ \frac{20}{3} \xi \zeta^A_{3,3} P_1 + \frac{21}{16} \zeta^V_{3,3} P_1 \left( \lambda^V_{3,3} P_1 - \frac{1}{28} \zeta^A_{3,3} P_1 \omega^V_{3,3} P_1 \right) (7\xi^2 - 3)
$$

$$
- 5 a_2^\perp \left( 2\partial_+ \xi + \partial_- (1 + \xi^2) \right)
$$

$$
- 6 \left[ \partial_+ (u\bar{u} \ln \bar{u} - u \ln u) + \partial_- (2u\bar{u} + \bar{u} \ln \bar{u} + u \ln u) \right] (1 + 6a_2^\perp), \quad (A18)
$$

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for the $1^1P_1$ states, where
\[
\bar{\delta}_\pm = \frac{f_A^+}{f_A} \frac{m_q \pm m_A}{m_A}, \quad \zeta_V^{(A)} = \frac{f_A^{V(A)}}{f_A m_A}.
\]  

APPENDIX B: AN ALTERNATIVE DEFINITION OF FORM FACTORS

Following Ref. [21], the semileptonic form factors for the $B \to A$ transition are alternatively defined in the following way
\[
\langle A(p,\lambda)|A_\mu|B_q(p_B)\rangle = i \frac{2}{m_{B_q} - m_A} \epsilon_{\mu\nu\alpha\beta} (q^\nu P^\alpha - q^\alpha P^\nu) V_B^\beta A(q^2),
\]
\[
\langle A(p,\lambda)|V_\mu|B_q(p_B)\rangle = -\left\{ (m_{B_q} - m_A) \epsilon_\mu^{(\lambda)*} V_1^{B_q A}(q^2) - (\epsilon_\lambda^{(\lambda)*} \cdot p_B)(p_B + p)_\mu V_2^{B_q A}(q^2) \\
-2m_A \epsilon_\mu^{(\lambda)*} \cdot p_B q^\mu \left[ V_3^{B_q A}(q^2) - V_0^{B_q A}(q^2) \right] \right\},
\]
where $q = p_B - p$, $V_3^{B_q A}(0) = V_0^{B_q A}(0)$ and
\[
V_3^{B_q A}(q^2) = \frac{m_{B_q} - m_A}{2m_A} V_1^{B_q A}(q^2) - \frac{m_{B_q} + m_A}{2m_A} V_2^{B_q A}(q^2),
\]
\[
\langle A|\partial_\mu V^\mu|B_q\rangle = 2im_A (\epsilon^* p_B)V_0^{B_q A}(q^2).
\]

Following the above parametrization, the numerical results of the light-cone sum rules for the form factors are listed in Table VIII.

APPENDIX C: DECAY AMPLITUDES

The differential decay rates for $\bar{B}_q^0 \to A^+ e^- \bar{\nu}_e$ are given by
\[
\frac{d\Gamma(\bar{B}_q^0 \to A^+ e^- \bar{\nu}_e)}{dE_e dq^2} = \frac{G_F^2}{128\pi^3} |V_{CKM}|^2 \frac{q^2}{m_B^2} \times [(1 - \cos \theta)^2 H_+^2 + (1 + \cos \theta)^2 H_0] + 2(1 - \cos^2 \theta) H_0^2,
\]
with the helicity amplitudes being
\[
H_\pm = \frac{1}{m_{B_q} + m_A} \left[ (m_{B_q}^2 - m_A^2 - q^2)^2 (m_{B_q} + m_A) V_1(q^2) - \frac{\lambda^{1/2}}{m_{B_q} + m_A} V_2(q^2) \right],
\]
\[
H_0 = \frac{1}{2m_A(q^2)^{1/2}} \left[ (m_{B_q}^2 - m_A^2 - q^2)(m_{B_q} + m_A) V_1(q^2) - \frac{\lambda^{1/2}}{m_{B_q} + m_A} V_2(q^2) \right].
\]
Here $E_e$ is the electron energy in the $B_q$ rest system. $\theta$ is the polar angle between the $A$ and $e^-$ in the $(e^-, \bar{\nu}_e)$ system, and is given by
\[
\cos \theta = \frac{(m_{B_q}^2 - m_A^2 + q^2) - 4m_{B_q} E_e}{\lambda^{1/2}},
\]
TABLE VIII: Following the parametrization given in Eq. (B1), form factors for $B_{u,d,s} \to A$ transitions obtained in the light-cone sum rule calculation are fitted to the 3-parameter form in Eq. (17). Here, because the decay constants, $f_{3P_1}$ and $f_{1P_1}$, are of the same sign, the form factors for $B_{(s)} \to 1^1P_1$ and $B_{(s)} \to 1^3P_1$ transitions have opposite signs.

| $F$         | $F(0)$ | $a$ | $b$ | $F$          | $F(0)$ | $a$ | $b$ |
|-------------|--------|-----|-----|--------------|--------|-----|-----|
| $V_{1}^{B_{B_{I}}} $ | 0.60 ± 0.11 | 0.645 | 0.250 | $V_{1}^{B_{B_{I}}} $ | −0.32 ± 0.06 | 0.748 | 0.063 |
| $V_{2}^{B_{B_{I}}} $ | 0.26 ± 0.05 | 1.48 | 1.00 | $V_{2}^{B_{B_{I}}} $ | −0.06 ± 0.01 | 0.539 | 1.76 |
| $V_{0}^{B_{B_{I}}} $ | 0.30 ± 0.05 | 1.77 | 0.926 | $V_{0}^{B_{B_{I}}} $ | −0.39 ± 0.07 | 1.22 | 0.426 |
| $A_{B_{B_{I}}} $ | 0.30 ± 0.05 | 1.64 | 0.986 | $A_{B_{B_{I}}} $ | −0.16 ± 0.03 | 1.69 | 0.910 |
| $V_{1}^{B_{B_{K_{I}}}} $ | 0.56 ± 0.11 | 0.635 | 0.211 | $V_{1}^{B_{B_{K_{I}}}} $ | −0.48 ± 0.13 | 0.729 | 0.074 |
| $V_{2}^{B_{B_{K_{I}}}} $ | 0.25 ± 0.05 | 1.51 | 1.18 | $V_{2}^{B_{B_{K_{I}}}} $ | −0.10 ± 0.02 | 0.919 | 0.855 |
| $V_{0}^{B_{B_{K_{I}}}} $ | 0.22 ± 0.04 | 2.40 | 1.78 | $V_{0}^{B_{B_{K_{I}}}} $ | −0.45 ± 0.12 | 1.34 | 0.690 |
| $A_{B_{B_{K_{I}}}} $ | 0.27 ± 0.05 | 1.60 | 0.974 | $A_{B_{B_{K_{I}}}} $ | −0.22 ± 0.06 | 1.72 | 0.912 |
| $V_{1}^{B_{B_{F_{I}}}} $ | 0.37 ± 0.07 | 0.640 | 0.153 | $V_{1}^{B_{B_{F_{I}}}} $ | −0.21 ± 0.04 | 0.612 | 0.078 |
| $V_{2}^{B_{B_{F_{I}}}} $ | 0.16 ± 0.03 | 1.47 | 0.956 | $V_{2}^{B_{B_{F_{I}}}} $ | −0.04 ± 0.01 | 0.500 | 1.63 |
| $V_{0}^{B_{B_{F_{I}}}} $ | 0.18 ± 0.03 | 1.81 | 0.880 | $V_{0}^{B_{B_{F_{I}}}} $ | −0.24 ± 0.04 | 1.16 | 0.294 |
| $A_{B_{B_{F_{I}}}} $ | 0.18 ± 0.03 | 1.63 | 0.900 | $A_{B_{B_{F_{I}}}} $ | −0.10 ± 0.02 | 1.54 | 0.848 |
| $V_{1}^{B_{B_{F_{S}}}} $ | 0.26 ± 0.05 | 0.644 | 0.209 | $V_{1}^{B_{B_{F_{S}}}} $ | −0.18 ± 0.03 | 0.623 | 0.094 |
| $V_{2}^{B_{B_{F_{S}}}} $ | 0.11 ± 0.02 | 1.49 | 1.09 | $V_{2}^{B_{B_{F_{S}}}} $ | −0.03 ± 0.01 | 0.529 | 1.53 |
| $V_{0}^{B_{B_{F_{S}}}} $ | 0.12 ± 0.02 | 1.84 | 0.749 | $V_{0}^{B_{B_{F_{S}}}} $ | −0.18 ± 0.03 | 1.22 | 0.609 |
| $A_{B_{B_{F_{S}}}} $ | 0.13 ± 0.02 | 1.64 | 0.919 | $A_{B_{B_{F_{S}}}} $ | −0.08 ± 0.02 | 1.56 | 0.827 |
| $V_{1}^{B_{B_{K_{I}}}} $ | 0.49 ± 0.10 | 0.635 | 0.211 | $V_{1}^{B_{B_{K_{I}}}} $ | −0.42 ± 0.11 | 0.729 | 0.074 |
| $V_{2}^{B_{B_{K_{I}}}} $ | 0.22 ± 0.04 | 1.51 | 1.18 | $V_{2}^{B_{B_{K_{I}}}} $ | −0.09 ± 0.03 | 0.919 | 0.855 |
| $V_{0}^{B_{B_{K_{I}}}} $ | 0.19 ± 0.04 | 2.40 | 1.78 | $V_{0}^{B_{B_{K_{I}}}} $ | −0.40 ± 0.05 | 1.34 | 0.690 |
| $A_{B_{B_{K_{I}}}} $ | 0.24 ± 0.04 | 1.60 | 0.974 | $A_{B_{B_{K_{I}}}} $ | −0.19 ± 0.03 | 1.72 | 0.912 |
| $V_{1}^{B_{B_{F_{I}}}} $ | 0.33 ± 0.06 | 0.640 | 0.153 | $V_{1}^{B_{B_{F_{I}}}} $ | −0.18 ± 0.04 | 0.612 | 0.078 |
| $V_{2}^{B_{B_{F_{I}}}} $ | 0.14 ± 0.03 | 1.47 | 0.956 | $V_{2}^{B_{B_{F_{I}}}} $ | −0.04 ± 0.01 | 0.500 | 1.63 |
| $V_{0}^{B_{B_{F_{I}}}} $ | 0.16 ± 0.03 | 1.81 | 0.880 | $V_{0}^{B_{B_{F_{I}}}} $ | −0.21 ± 0.04 | 1.16 | 0.294 |
| $A_{B_{B_{F_{I}}}} $ | 0.16 ± 0.03 | 1.63 | 0.900 | $A_{B_{B_{F_{I}}}} $ | −0.09 ± 0.02 | 1.54 | 0.848 |
| $V_{1}^{B_{B_{F_{S}}}} $ | −0.46 ± 0.09 | 0.644 | 0.209 | $V_{1}^{B_{B_{F_{S}}}} $ | 0.32 ± 0.05 | 0.623 | 0.094 |
| $V_{2}^{B_{B_{F_{S}}}} $ | −0.19 ± 0.03 | 1.49 | 1.09 | $V_{2}^{B_{B_{F_{S}}}} $ | 0.05 ± 0.02 | 0.529 | 1.53 |
| $V_{0}^{B_{B_{F_{S}}}} $ | −0.21 ± 0.04 | 1.84 | 0.749 | $V_{0}^{B_{B_{F_{S}}}} $ | 0.32 ± 0.05 | 1.22 | 0.609 |
| $A_{B_{B_{F_{S}}}} $ | −0.23 ± 0.04 | 1.64 | 0.919 | $A_{B_{B_{F_{S}}}} $ | 0.14 ± 0.03 | 1.56 | 0.827 |

with $\lambda = (m_{B_{q}}^2 + m_{A}^2 - q^2)^2 - 4m_{B_{q}}^2 m_{A}^2$. For a fixed electron energy, $q^2$ varies over the region $0 \leq q^2 \leq q_{\text{max}}^2$, where

$$q_{\text{max}}^2 = \frac{2E_{e}(m_{B_{q}}^2 - m_{A}^2 - 2m_{B_{q}}E_{e})}{m_{B_{q}} - 2E_{e}}.$$  (C4)
and the related range of $E_e$ is: $0 \leq E_e \leq (m^2_{B_q} - m^2_A)/(2m_{B_q})$.

[1] Particle Data Group, Y.M. Yao et al., J. Phys. G 33, 1 (2006).
[2] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 97, 051802 (2006).
[3] K. Abe et al. (Belle Collaboration), arXiv:0706.3279 [hep-ex].
[4] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 98, 181803 (2007).
[5] B. Aubert et al. [The BABAR Collaboration], Phys. Rev. Lett. 99, 241803 (2007) arXiv:0707.4561 [hep-ex].
[6] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 99, 261801 (2007) arXiv:0708.0050 [hep-ex].
[7] B. Aubert et al. [The BABAR Collaboration], arXiv:0806.4419 [hep-ex].
[8] H. Y. Cheng and K. C. Yang, Phys. Rev. D 76, 114020 (2007) arXiv:0709.0137 [hep-ph].
[9] K. C. Yang, Phys. Rev. D 76, 094002 (2007) arXiv:0705.4029 [hep-ph].
[10] H. Y. Cheng and K. C. Yang, arXiv:0805.0329 [hep-ph].
[11] K. C. Yang, Nucl. Phys. B 776, 187 (2007) arXiv:0705.0692 [hep-ph].
[12] K. C. Yang, JHEP 0510, 108 (2005) arXiv:hep-ph/0509337.
[13] H. Hatanaka and K. C. Yang, Phys. Rev. D 77, 094023 (2008) arXiv:0804.3198 [hep-ph].
[14] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989).
[15] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B 345 (1990) 137.
[16] V. M. Belyaev, A. Khodjamirian and R. Ruckl, Z. Phys. C 60, 349 (1993) arXiv:hep-ph/9305348.
[17] P. Ball and V. M. Braun, Phys. Rev. D 55, 5561 (1997) arXiv:hep-ph/9701238.
[18] See, for example, P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005) arXiv:hep-ph/0412079, and references therein.
[19] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448, 519.
[20] T. M. Aliev and M. Savci, Phys. Lett. B 456, 256 (1999) arXiv:hep-ph/9901395.
[21] H.Y. Cheng, C.K. Chua, and C.W. Hwang, Phys. Rev. D 69, 074025 (2004).
[22] Z. G. Wang, arXiv:0804.0907 [hep-ph].
[23] M. Beneke and T. Feldmann, Nucl. Phys. B 592, 3 (2001) arXiv:hep-ph/0008255.
[24] K. Abe et al. [BELLE Collaboration], arXiv:hep-ex/0408138; H. Yang et al., Phys. Rev. Lett. 94, 111802 (2005) arXiv:hep-ex/0412039; Heavy Flavor Averaging Group, E. Barberio et al., arXiv:0704.3575 [hep-ex] and online update at http://www.slac.stanford.edu/xorg/hfag.
[25] A. Khodjamirian and R. Ruckl, Adv. Ser. Direct. High Energy Phys. 15, 345 (1998) arXiv:hep-ph/9801443.
[26] P. Ball and V. M. Braun, Phys. Rev. D 58, 094016 (1998) arXiv:hep-ph/9805422.
[27] W. Wang, R.H. Li, and C.D. Lü, arXiv:0711.0432 [hep-ph].