Research Article
A Numerical Model of Vapour Transfer and Phase Change in Unsaturated Freezing Soils

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1. Introduction

Many engineering problems, including cracking of pavements, damage to the foundation of structures, and fracture of pipelines, are caused by the freezing and thawing process in cold regions. [1–3]. Previous studies mostly focused on the problem of frost heave caused by the liquid water transfer from warm to cold side in freezing soils [4–7]. In recent studies, it is shown that vapour transfer can result in ice accumulation in unsaturated freezing soil and, thus, cause frost damage to infrastructures. For example, Eigenbrod and Kenneppohl [8] contended that vapour flow led to water accumulation at the base of pavements in the northern parts of Canada and the United States. Severe frost heave problems were reported to occur at an airport in Northwestern China with 20 m-deep groundwater table and limited annual rainfall. The specific phenomenon was named as the canopy effect by Zhang et al. [9]. The researchers attributed the water accumulation beneath the impervious cover to the vapour transfer in unsaturated freezing soils. Niu et al. [10] performed field monitoring of soil temperature, water content, and deformation in the railway embankment and concluded that water vapour diffusion might be an essential influence factor to the formation of near-surface ice.

The study on frozen soil models began in the 1970s, which aims to solve the coupled heat and mass conservative equation, for example, the works of Harlan [11] and Talyor and Luthin [12]. In the earlier studies, vapour flow is usually neglected comparing to the liquid water transfer in freezing soil. This assumption is made on the basis of the cognition that (1) neglecting the vapour flow could largely simplify the numerical computation and (2) it is truly insignificant for the vapour flow in the soil of having a continuous liquid phase. Considering that more evidence has revealed the complexity in soil freezing process, a large number of sophisticated models or parameterizations have been presented during the following several decades [13–17]. Some of these studies take the contribution of vapour flow and its
phase change into account, while others choose to neglect yet. The proposed models have laid a good foundation for understanding the soil freezing process.

In the most popular models for frozen soils, it is common to set up two governing equations with four undetermined variables, i.e., soil temperature, matric suction of soil, liquid water content, and ice content. In order to numerically solve the governing equations, another two relations are required. The one is a relationship between matric suction and liquid water content called the Soil Water Characteristic Curve (SWCC), such as the VG model [18] and BC model [19]. The other is a formula that relates liquid water content and ice content to temperature, which is referred to in the Soil Freezing Characteristic Curve (SFCC) [20, 21]. The differences in the existing models are the form of governing equations and the functions for the SWCC and SFCC. It is noted that the studies that take vapour flow into account usually relate soil vapour content (or vapour density) to temperature and matric suction on the basis of a local equilibrium theory, which was initially proposed by Philip and de Vries [22]. But, here, the characterising parameter for vapour flow is a nonindependent variable.

When applying the previous models to analyse the frost heave in unsaturated soils, it is found they are incapable to reveal the mechanism of vapour flow, due to the following reasons. Firstly, when soil water content is relatively low, specifically near to the residual water content, it is inaccurate to describe liquid water flow by using the SWCC and the derived unsaturated permeability function [23, 24]. Besides, liquid water flow may be smaller several magnitude orders than vapour flow at this stage. The vapour flow may dominate the mass transfer when the soil is relatively dry [25]. Secondly, the theory proposed by Philip and de Vries [22] assumes that vapour concentration in air is always in equilibrium with liquid water, i.e., the vapour concentration is determined by the curved liquid/vapour interface. But, the recent study suggests that the equilibrium assumption works at the low water content in soil because the equilibrium establishment is not instantaneous at that time [26–28]. Moreover, when the soil temperature reaches to the freezing point, the formation of ice phase in the soil will change the liquid/vapour interface. Thus, the local equilibrium theory would be challenged in case of being used to describe the moisture transfer in frozen soil. Thirdly, the mechanism of multiphase transfer in unsaturated freezing soils has been less studied. They remain unknown, for example, the phase changes among pore ice, vapour, and liquid water and the influence of ice phase on vapour and liquid water flow [29].

The one-dimensional governing equation for vapour flow in the unsaturated freezing soil is given by the following mass conservation equation:

\[
\frac{\partial (n(1 - S)\rho_v)}{\partial t} = \frac{\partial q_v}{\partial z} \frac{\partial (nS\rho_v)}{\partial t}
\]

where \(z(m)\) and \(t(s)\) represent the position and time, respectively, \(n\) is the porosity of soil (unitless), and \(\rho\) is the density in soil (kg m\(^{-3}\)). The subscripts \(w\), \(v\), and \(i\) denote the liquid water, vapour, and pore ice, respectively. \(q_v\) is vapour flux (kg m\(^{-2}\) s\(^{-1}\)), and \(S\) is the total saturation of liquid water and pore ice (unitless); here, \(S = S_w + S_i\rho_v/\rho_w\).
The vapour movement in unsaturated freezing soil is recognised to be driven by both the pressure gradient and vapour concentration gradient [33]. The vapour mass flux in this region according to Darcy’s law and Fick’s law is

\[ q_v = -\frac{\rho_v K \partial P}{\mu_v} - \frac{(1 - S)n}{\tau} D \frac{\partial \rho_v}{\partial z} \]  

(2)

where \( K \) is the air permeability of soil (m\(^2\)), \( \mu_v \) is the dynamic viscosity of vapour (kg m\(^{-1}\)s\(^{-1}\)), \( P \) is the vapour pressure (kPa), \( D \) is the vapour diffusivity in soil (m\(^2\)s\(^{-1}\)), and \( \tau \) is the tortuosity factor (unitless). The parameter \( \tau \) is the ratio of real length of transfer path to apparent length, which is assigned to be 1.2 in this study [34].

The vapour phase in unsaturated freezing soil can be considered as an ideal gas. It follows the ideal gas law, \( P = \rho_v RT \). Substituting this equation into equation (2) leads to

\[ q_v = -K_T \frac{\partial T}{\partial z} - K_s \frac{\partial S}{\partial z} \]  

(3a)

\[ K_T = \frac{K}{\mu_v} R \rho_v^2 \]  

(3b)

\[ K_s = \left[ \frac{K}{\mu_v} R \rho_v T + \frac{(1 - S)n}{\tau} D \right] \frac{\partial \rho_v}{\partial z} \]  

(3c)

where \( K_s \) and \( K_T \) are the effective diffusivity by the saturation gradient (kg m\(^{-1}\) s\(^{-1}\)) and effective mass conductivity by the temperature gradient (kg m\(^{-1}\) s\(^{-1}\) K\(^{-1}\)), respectively. \( R \) is the specific gas constant of water vapour (461.89 J kg\(^{-1}\) K\(^{-1}\)).

The mass transfer equation can be obtained by substituting equation (3a) into equation (1):

\[ \frac{\partial (n(1 - S) \rho_v)}{\partial t} + \frac{\partial (n \rho_w \rho_v)}{\partial z} = -\frac{\partial}{\partial z} \left( K_T \frac{\partial T}{\partial z} + K_S \frac{\partial S}{\partial z} \right) \]  

(4)

It has been revealed that the heat conduction is two orders of magnitude larger than the sensible heat of vapour flow [35]. Thus, the heat conservative equation neglects the term of the sensible heat of vapour flow in the soil and is written as

\[ \rho C_T \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( \lambda + K_T L_{v,v} \right) \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( K_s L_{s,v} \frac{\partial S}{\partial z} - L_{i,w} \frac{\partial (n \rho_w \rho_v)}{\partial t} \right) \]  

(5a)

\[ \rho C = (1 - n) \rho_v C_v + n S \rho_i C_i + n S \rho_w C_w \]  

(5b)

\[ \lambda = (1 - n) \lambda_v + n S \lambda_i + n S \lambda_w + n (1 - S) \lambda_v \]  

(5c)

where \( \rho \) is the total density of soil and \( C \) is the volumetric heat capacity and is given in equation (5b). \( \lambda \) is the thermal conductivity (W m\(^{-1}\) K\(^{-1}\)) that is defined in equation (5c). \( L \) donates the latent heat between different phases. The
subscripts s in the abovementioned equations indicate the solid phase of soil particles.

The saturation of the unfrozen water is an empirical function of temperature in the unsaturated freezing soil. Anderson and Tice [36] and Anderson and Morgenstern [37] found a reasonable approximate equation with a power law:

\[
S_{uw} = \frac{\rho_s}{\rho_w} \left(1 - n \right) \alpha (T_0 - T)^\beta, \quad T \leq T_0,
\]

where \( T_0 = 273.15 \) K, the freezing point of free water, and \( \alpha \) and \( \beta \) are empirical fitting parameters that relate to the specific surface area of soil. Anderson and Tice [36]; Anderson and Morgenstern [37]; and Blanchard and Fremond [38] tested the empirical formulas of parameters \( \alpha \) and \( \beta \) for different types of soils and listed their recommended values. Although the power relationship as equation (6) has been proposed several decades ago, it is still commonly given in reference texts as a valid approach to estimating the unfrozen water content in freezing soils [39, 40].

\( S_{uw} \) in equation (6) presents the maximum liquid water content that will not freeze at a subzero temperature \( T \). Based on the computed values of \( S \) and \( S_{uw} \), the criteria for determining the saturation of pore ice and liquid water can be obtained [7], as follows:

\[
\begin{align*}
S_i &= \begin{cases} 
0, & T > T_0, \\
0, & T \leq T_0 \text{ and } S \leq S_{uw}, \\
\frac{\rho_w (S - S_{uw})}{\rho_i}, & T \leq T_0 \text{ and } S > S_{uw}, 
\end{cases} \\
S_w &= S - S_i \frac{\rho_i}{\rho_w}. 
\end{align*}
\] (7)

In the hygroscopic porous material, the three phases of water, i.e., unfrozen liquid water, ice, and vapour coexist in the freezing material pores. The presence of unfrozen liquid water complicates the process of phase change. In this case, the thermodynamic equilibrium relationship (the Clapeyron equation) fails to describe the truth because the Clapeyron equation is a way only for characterising the discontinuous phase transition between two phases of matter of a single constituent [41, 42]. It has been recognised from chemical engineering that an adsorption-desorption equilibrium relationship should be replaced to describe the desublimation or sublimation process of a hygroscopic porous media with bound moisture [43, 44]. Such a relation is used to describe the equilibrium state among vapour, liquid water, and pore ice in unsaturated freezing soil. Wang et al. [45] tested several kinds of adsorption-desorption equilibrium relations in the literature and found that Kelvin’s style in the exponential form proposed by Rajniak and Yang [46] could generate a better performance. The equation is expressed as

\[
P(T, S) = \frac{P_{vs}(T)}{P_{vs}(T_0)} = e^{(\frac{\gamma (1 - S)}{SRT})},
\]

where \( P_{vs}(T) \) is the saturated vapour pressure at temperature \( T \) (Pa), given in Table 1. \( P(T, S) \) is the vapour pressure in unsaturated freezing soil (Pa), which is a function of temperature \( T \) and saturation \( S \). \( \gamma \) is an empirical parameter, a suggested value is 5000 according to Wang et al. [45].

The simulated model consists of mass conservation equation (4) and energy balance equation (5), which are highly nonlinear and coupled. There are four unknown variables in the two equations, \( S_w, S_i, T, \) and \( \rho_w \). Equations (6)–(8) provide the other two additional relations by adding the saturation of unfrozen water \( S_{uw} \), such that the simultaneous equations can be solved mathematically.

2.2. Governing Equations for the Ice Lens Zone. It is assumed that there is no moisture movement in the ice lens zone, i.e., the density of the frost layer is uniform. The mass balance at this region can be expressed by

\[
\rho = \rho_i, \quad 0 < z < z_{fs}(t),
\]

where \( z_{fs}(t) \) represents the position of the frost surface of the ice lens, which is a function of time.

The governing equation for heat transfer in the ice lens zone can be written as

\[
\rho_i C_i \frac{\partial T}{\partial t} = \lambda_i \frac{\partial^2 T}{\partial z^2}, \quad 0 < z < z_{fs}(t),
\]

where \( C_i \) is the specific heat of ice lens (J kg\(^{-1}\) K\(^{-1}\)) and \( \lambda_i \) is the effective thermal conductivity of the frost layer (W m\(^{-1}\) K\(^{-1}\)).

2.3. Heat and Moisture Balance at the Frost Surface. The heat and moisture mass balance at the moving front can be expressed by

\[
\lambda_i \left( \frac{\partial T}{\partial z} \right)_{z=z_{fs}} - \lambda \left( \frac{\partial T}{\partial z} \right)_{z=z_{fs}} = (-L_e q_a)_{z=z_{fs}}.
\]

The water vapour on the frost surface is assumed to be saturated.

\[
\rho_v = \rho_{vs} = \frac{P_{vs}(T_{fs})}{RT_{fs}}, \quad \text{at} \quad z = z_{fs}.
\] (12)

Two new parameters are added into the model, \( z_{fs} \) at the frost surface and \( T \) at the ice lens zone, and \( \rho_{vs} \) is the saturated vapour density, which is a function of temperature [53].

2.4. Soil Hydraulic Properties. In complete dry soil, the air permeability \( K \) is equal to inherent permeability \( k_c \). As for the unsaturated freezing soil, an impedance factor is added to the air permeability function [9, 12, 19].
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Table 1: The parameters used in computation.

| Symbol          | Value and equations            | Reference |
|-----------------|--------------------------------|-----------|
| \( \rho_i \) (kg m\(^{-3}\)) | 1000                           | [47]      |
| \( \rho_f \) (kg m\(^{-3}\))  | 913                            | [48]      |
| \( C_i \) (J kg\(^{-1}\)K\(^{-1}\)) | 1930                           | [48]      |
| \( C_f \) (J kg\(^{-1}\)K\(^{-1}\)) | 800                             | [49]      |
| \( C_w \) (J kg\(^{-1}\)K\(^{-1}\)) | 1886                           | [48]      |
| \( \lambda_i \) (W m\(^{-1}\)K\(^{-1}\))  | 4180                           | [25]      |
| \( \lambda_f \) (W m\(^{-1}\)K\(^{-1}\)) | 2.22                           | [48]      |
| \( \lambda_w \) (W m\(^{-1}\)K\(^{-1}\)) | 2.68                           | [25]      |
| \( \lambda_\| \) (W m\(^{-1}\)K\(^{-1}\))  | 0.022                          | [48]      |
| \( \lambda_\|_w \) (W m\(^{-1}\)K\(^{-1}\)) | 0.54                           | [25]      |
| \( \mu_\| \) (kg m\(^{-1}\)s\(^{-1}\)) | 0.011 \times (T/273.15)\(^{1.5}\)/(T + 961) | [50]      |
| \( L_i,v \) (J kg\(^{-1}\))  | 2.839 \times 10^3             | [51]      |
| \( L_i,w \) (J kg\(^{-1}\))  | 0.334 \times 10^8             | [15]      |
| \( P_{v,T} \) (Pa)  | \( \ln P_{v,T} = 9.55 - 5723.265/T + 3.53\ln(T) - 0.007283 \times T \) | [52]      |
| \( D_m \) (m\(^2\) s\(^{-1}\))  | \( D_m = 2.12 \times 10^{-3}/(T/273.15)^2 \) | [49]      |

Equations (4), (5a), (5b), and (5c) are highly nonlinear since primary variables vary with time and primary variables. The finite element method for spatial discretization and the finite difference method for temporal discretization are used to numerically solve the governing equations, which is performed in the COMSOL Multiphysics package. A detailed description for the numerical approach can be found in the work of Teng et al. [32].

In order to achieve a numerical stability of the solution, the following strategies are adopted: the domain is divided into 10000 elements, and the total time duration is divided into 12000 steps.

3. Application to a Laboratory Freezing Experiment

3.1. Testing Program. A series of laboratory tests were carried out on the basis of a specifically designed new device. A detailed illustration can be found in the work of Teng et al. [32]. The temperatures of the top and bottom ends of the specimen are controlled accurately by a pair of plates. The top end of the specimen is sealed to prevent moisture source or sink. A Mariotte bottle is used to supply distilled water, leaving a void approximately 1 cm in height between the bottom end of the specimen and water surface, such that only vapour can enter the specimen.

A kind of silica sand is applied in this study, which has a particle size range from 0.5 mm to 1 mm. The physical and hydraulic properties of the sample are presented in Table 2. The measured Soil Freezing Characteristic Curve is shown in Figure 2, where the curve is fitted by equation (6). The soil specimen is placed into the cylinder with a controlled dry density of 1.40 g/cm\(^3\). The final height of the sample is 13.5 cm. Three Time-Domain-Reflectometer transducers and seven thermistors are inserted into the cylinder to measure the transit temperatures and water contents. In addition, the specimen is divided into 1 cm-high columns to check water contents at different depths after the completion of the test.

Five experimental cases are designed as listed in Table 3. In all the 5 cases, the top end of the sample is subjected to a

\[
K(S) = 10^{-cs} k_s (1 - S)^2 (1 - S^{((2b/b) - b)}), \quad (13)
\]

where \( b \) and \( c \) are empirical parameters and the impedance factor \( 10^{-cs} \) denotes the obstruction of pore ice to vapour flows. When the soil temperature is greater than the freezing point, \( S \) becomes 0, and equation (13) reverts to the formula as defined by Brooks and Corey [19].

The vapour transfer in the porous medium described by Fick’s law can be divided into two types, molecular diffusion and Knudsen diffusion. Molecular diffusion is caused by the collisions with the gas molecules, while Knudsen diffusion is caused by the collisions between gas molecules and the wall of the transport channel. Molecular diffusion dominates the process when the mean free path of the molecules is smaller than the average pore radius. Otherwise, Knudsen diffusion does [54]. The harmonic averaging method is adopted here to compute the vapour diffusivity \( D \) based on the molecular diffusion coefficient \( D_m \) and the Knudsen diffusion coefficient \( D_k \) [55]:

\[
D = \frac{1}{(1/D_m) + (1/D_k)}, \quad (14)
\]

where \( D_m \) can be written as follows [56]:

\[
D_m = 0.66n(1 - S)D_a, \quad (15)
\]

where \( D_a \) is the diffusivity of water vapour in air (m\(^2\) s\(^{-1}\)), as given in Table 1. The expression of \( D_k \) was given by Geankoplis [57] as follows:

\[
D_k = 97.0 \frac{d}{2 \sqrt{m_w}}, \quad (16)
\]

where \( d \) is the averaged soil pore diameter (m) and \( m_w \) is the molecular weight of water (kg mol\(^{-1}\)). The values of related parameters are presented in Table 1.

2.5. Numerical Implementation. Equations (4), (5a), (5b), and (5c) are highly nonlinear since primary variables vary
subfreezing temperature, while the bottom end is subjected to a superfreezing different temperature. Case 1 and case 2 correspond to two different initial water contents. Case 3 to case 5 change the test period in order to study the transient total water content profile in time.

3.2. Simulation Result. The computed and measured water content profiles for cases 1 and 2 are shown in Figure 3. As for the result of case 1, it shows that the peak water content appears at the top surface and the freezing front. A large amount of ice is accumulated at the top surface, which is the ice lens zone. The simulated result has a good general agreement with the measured data and the simulated saturation of ice and water at the top surface, and the freezing front is quite close to the measured result. A minor disagreement can be observed in the unfrozen zone, which may attribute to the deviation of adsorption-desorption equilibrium relations. The simulated result has a quite good match with the tested data, where only one peak value at the top surface can be observed.

Figure 4 shows a comparison between the measured and computed result for cases 3, 4, and 5. In both the simulated and measured results, the peak values can be observed at both the top surface and the freezing front where the ice content increases with the test period. At the same time, the water content in the unfrozen zone seems to be independent of the test period. It is apparent that liquid water goes downward within the first day of freezing due to gravity, which implies that the formation of ice is mainly induced by vapour flow.

### 4. Application to a High-Speed Railway Subgrade

4.1. Problem Description. In order to evaluate the numerical stability of the proposed model and to illustrate its application to high-speed railway subgrade, this section will perform a long-term test of the model by Niu et al. [10] who performed in situ tests to measure the ground temperatures, moisture content, and frost heave of the subgrade of the Harbin-Dalian Passenger Dedicated Line. The subgrade at one site (K977) is built on the undisturbed ground surface, while the other is in a cut section (K1004). The measured data start from Nov 1, 2013, to Oct 17, 2014, that last for 351 days. The upper layer of the subgrade structure is a well-graded gravel with cement (thickness is 70 cm). The materials in the second layer are group A/B fills (thickness is 230 cm). The lower layer is a well-graded crushed stone and sand (thickness is 50 cm). In order to simplify the numerical simulation, the subgrade is considered as a 300 cm-high embankment with the 70 cm cemented gravel at the top position, and the geomembrane is not considered.

The main differences between the in situ measurement and the laboratory column experiment are that the soil is subject to freezing, as well as thawing, and the temperature variation in situ is much higher. The measured near-surface temperature is shown in Figure 5, where a fitting function is obtained as the input upper boundary. The lower temperature of the soil specimen is kept constant 3°C. It is assumed that there is no flow at both the upper and lower boundaries. The thermal properties of soil can be found in Table 1. The other parameters are the same as those for silica sand, which can be found in Zhang et al. [9]. The hydraulic properties of the soil are the same as those in the work of Teng et al. [32].

4.2. Freezing-Thawing Simulations. Figure 6 presents the predicted frost and thawed depth variation in the 160 days, which indicates one freezing and thawing cycle. The frost and thawed depth are usually defined as the depth where the soil temperature is zero. Thus, the predicted frost and thawed depth are obtained from the simulated transient temperature profiles. It can be observed that the frost depths increase roughly linearly and reach a peak at about 90 days and 80 days, respectively, for the site of K977 and K1004, and then, the depth gradually decreases with time. The thawed depth shows a linear increase tendency. A quite good agreement between the measured and predicted frost and thawed depth can be observed, which indicates that the proposed numerical code performs well in predicting the temperature field.

Figure 7 presents measured and predicted water content profiles in the subgrades of the two sites during the
Table 3: Test conditions.

| Case | Initial saturation (%) | Bottom temperature (°C) | Top temperature (°C) | Water supply pattern and test period |
|------|------------------------|-------------------------|----------------------|-------------------------------------|
| 1    | 31.81                  | 10                      | −10                  | Control test, vapour, 7d            |
| 2    | 0                      | 10                      | −10                  | Vapour, 7d                          |
| 3    | 31.81                  | 10                      | −10                  | Vapour, 1d                          |
| 4    | 31.81                  | 10                      | −10                  | Vapour, 3d                          |
| 5    | 31.81                  | 10                      | −10                  | Vapour, 5d                          |

Figure 3: Comparison between the measured and simulated saturation of water and ice for cases 1 and 2.

Figure 4: Comparison between the measured and simulated saturation of water and ice for cases 3, 4, and 5. The symbols are the measured result, while the solid lines are the numerical simulation.

Figure 5: The measured temperature for the K977 site and K1004 site. The solid lines are the best-fit function, which is the input of the upper boundary.

Figure 6: The predicted and measured frozen/thawed depth versus time.
onset of freezing conditions and rapid warming period. It
can be observed that the water content near the top position
of the subgrade at both sites decreases rapidly as the
freezing condition develops, which is caused by the ice
accumulation as the freezing front progresses downward
and the upward migration of vapour into the frozen zone.
The predicted result can well capture the features of
moisture profile. During the thawing period, the moisture
in soil drains downward and the soil moisture becomes
lower comparing to the freezing condition. The position
of the peak moisture content during the thawing condition is
relatively lower than that during the freezing condition.
The proposed result can agree reasonably with the mea-
sured data. It is emphasized that a subgrade of the layered
structure has much lower water contents in the upper part.
The measured distributions may be different from the
hypothetical condition. This case is hard to be modelled in a
numerical code [15], while the proposed model can rea-
sonably reproduce the tendency.

5. Conclusions

The mechanism of vapour migration and phase transfor-
mation in unsaturated freezing soils has been less under-
stood. In order to avoid the great uncertainty caused by the
phase change term of vapour-water-ice in the numerical
iteration process, this study presents numerical solution
schemes for coupled vapour and heat transfer at above-zero
and subzero temperature conditions, and hence freezing-
thawing cycles. The new model is solved numerically by the
finite element method, which is, then, validated by com-
paring to the laboratory measurement and in situ obser-
vation. The main findings are as follows:

(1) The new model avoids using the local equilibrium
assumption that accounts for water-vapour-ice
phase change, and an adsorption-desorption
equilibrium relation is proposed to direct describe
phase change between vapour and ice phase, which
provides a novel way for modelling the water-heat
coupling movement problem.

(2) The numerical model has a close match with the
results of laboratory freezing experiment, which
shows a good performance in numerical instability
and mass or heat conservation.

(3) The numerical solutions performed quite well for the
field application that considers rapidly varying
surface temperatures. The proposed model can
generate a reasonable result for the frost or thawed
depth and soil moisture profiles.

(4) It should be noted that the new model should be
tested eventually against a more realistic and com-
plete field data set, which can provide detailed in-
formation on soil properties. The new expressions
used for vapour transfer and the adsorption-de-
 sorption equilibrium relations are applicable to both
frozen and unfrozen conditions. The new expres-
sions can replenish the existing models, which
should be tested for more kinds of soils.

Notations

$b$: Empirical parameter of the BC model, unitless
$c$: Empirical parameter of air permeability in frozen soil, unitless
$C$: Effective volumetric heat capacity, $\text{J m}^{-3} \text{K}^{-1}$
$C_i$: Volumetric heat capacity of ice, $\text{J m}^{-3} \text{K}^{-1}$
$C_s$: Volumetric heat capacity of soil particle, $\text{J m}^{-3} \text{K}^{-1}$
$C_v$: Volumetric heat capacity of vapour, $\text{J m}^{-3} \text{K}^{-1}$
$C_w$: Volumetric heat capacity of water, $\text{J m}^{-3} \text{K}^{-1}$
$d$: Average pore diameter, m
$D$: Effective vapour diffusivity, $\text{m}^2 \text{s}^{-1}$
$D_v$: Diffusivity of water vapour in air, $\text{m}^2 \text{s}^{-1}$
$D_k$: Knudsen diffusion coefficient, $m^2 \cdot s^{-1}$
$D_m$: Molecular diffusion coefficient, $m^2 \cdot s^{-1}$
$K$: Air permeability of soil, $m^2$
$K_{G}$: Effective mass conductivity by the saturation gradient, $kg \cdot m^{-1} \cdot s^{-1}$
$K_{T}$: Effective mass conductivity by the temperature gradient, $kg \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$
$k_i$: Inherent permeability, $m^2$
$L_i$: Latent heat between ice and vapour, $J \cdot kg^{-1}$
$L_{w,i}$: Latent heat between ice and water, $J \cdot kg^{-1}$
$m_w$: Molecular weight of water, $kg \cdot mol^{-1}$
$n$: Porosity, unitless
$P$: Vapour pressure, kPa
$q_v$: Vapour flux, $kg \cdot m^{-1} \cdot s^{-1}$
$R$: Specific gas constant of water vapour, $461.89 \ J \cdot kg^{-1} \cdot K^{-1}$
$S$: Saturation, unitless
$S_w$: Saturation of water, unitless
$S_i$: Saturation of ice, unitless
$S_{uw}$: Saturation of unfrozen water, unitless
$T$: Temperature, K
$t$: Time, s
$T_0$: Freezing point of water, 273.15 K
$z$: Soil depth, m
$z_0$: The position of the frost surface of ice lens, m.

**Greek Letters**

$\alpha$: Empirical parameter of unfrozen water, unitless
$\beta$: Empirical parameter of unfrozen water, unitless
$\gamma$: Empirical parameter of vapour pressure, unitless
$\tau$: The tortuosity factor, unitless
$\mu_v$: Dynamic viscosity of vapour, $kg \cdot m^{-1} \cdot s^{-1}$
$\lambda$: Effective thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
$\lambda_i$: Effective thermal conductivity of ice, $W \cdot m^{-1} \cdot K^{-1}$
$\lambda_s$: Effective thermal conductivity of soil particle, $W \cdot m^{-1} \cdot K^{-1}$
$\lambda_{uw}$: Effective thermal conductivity of water, $W \cdot m^{-1} \cdot K^{-1}$
$\rho$: Total density of soil, $kg \cdot m^{-3}$
$\rho_i$: Density of ice, $kg \cdot m^{-3}$
$\rho_s$: Density of soil particle, $kg \cdot m^{-3}$
$\rho_v$: Density of vapour, $kg \cdot m^{-3}$
$\rho_w$: Density of water, $kg \cdot m^{-3}$
$\rho_{uw}$: Saturated vapour density, $kg \cdot m^{-3}$.

**Data Availability**

A theoretical model is established to formulate the coupled thermal and hydrological process by COMSOL, where the vapour flow governs the mass transfer process. A series of laboratory test results, which are published on Geotechnique [17], are used to validate the numerical formulations/codes.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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