Computation of certain integral formulas involving generalized Wright function

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Abstract

The aim of the paper is to derive certain formulas involving integral transforms and a family of generalized Wright functions, expressed in terms of the generalized Wright hypergeometric function and in terms of the generalized hypergeometric function as well. Based on the main results, some integral formulas involving different special functions connected with the generalized Wright function are explicitly established as special cases for different values of the parameters. Moreover, a Gaussian quadrature formula has been used to compute the integrals and compare with the main results by using graphical representations.

Keywords: Generalized Wright function; Wright hypergeometric function; Mittag-Leffler function; Lavoie–Trottier integral formula; Generalized hypergeometric function; Gamma function

1 Introduction and preliminaries

The research on integral transforms involving special functions (see, e.g., [1–4, 6, 8–12, 19]) has received a considerable attention of the research community primarily because their application has made prominent contributions in several domains of mathematics, engineering and their applications in mathematical physics (see, e.g., [7, 15, 21, 22, 27–30]). Among these special functions of mathematical physics, the Wright, the Bessel, and similar functions are of central importance and are fairly useful in the theory of integral and fractional calculus. To this end, a very little or no work on integral transforms involving Wright function has been done so far. In the main section, our focal point is to derive two essential theorems concerning integral transforms which will be used further, followed by related corollaries. In the last section, we specifically evaluate the Wright type auxiliary functions and some other deducible functions to derive a few results as special cases.

Now we recall some useful definitions of functions that are essential for the present investigation.

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Definition 1.1 Wright introduced the classical Wright function $W_{\alpha, \beta}(t)$ \cite{13, 20}, defined by the series representation

$$W_{\alpha, \beta}(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\alpha n + \beta) n!} \quad (\beta \in \mathbb{C}, \alpha > -1),$$

(1)

\forall \alpha \in \mathbb{R} and $\beta, \gamma, \delta \in \mathbb{C}; \alpha > -1, \delta \neq 0, -1, -2, \ldots$, with $t \in \mathbb{C}$ and $|t| < 1$ with $\alpha = -1$, a further generalization of the Wright function (see, for details, [18]) was introduced as

$$W_{\gamma, \delta}^{\alpha, \beta}(t) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{(\delta)_n \Gamma(\alpha n + \beta)} \frac{t^n}{n!}$$

(2)

where $(\gamma)_n$ is the Pochhammer symbol (see [21] p. 2 and pp. 4–6) and $\Gamma(\cdot)$ is the gamma function (see [21] Sect. 1.1), with the two auxiliary functions

$$M_{\alpha, \beta}^{\gamma, \delta}(t) = W_{-\alpha, 1-\alpha}^{\gamma, \delta}(t) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{(1-\alpha - \alpha n)(\delta)_n} \frac{(-t)^n}{n!}$$

(3)

and

$$F_{\alpha, \beta}^{\gamma, \delta}(t) = W_{-\alpha, 0}^{\gamma, \delta}(t) = \sum_{n=1}^{\infty} \frac{(\gamma)_n}{(\alpha n)(\delta)_n} \frac{(-t)^n}{n!}$$

(4)

Wright \cite{24–26} studied the generalized Wright hypergeometric function defined as

$$pF_q \left[ \left( \alpha_1 A_1, \ldots, \alpha_p A_p; \beta_1 B_1, \ldots, \beta_q B_q \right); t \right] = \sum_{m=0}^{\infty} \prod_{j=1}^{p} \Gamma(\alpha_j + A_j m) \prod_{j=1}^{q} \Gamma(\beta_j + B_j m) \frac{t^m}{m!}$$

(5)

where the coefficients $(\alpha_1, \ldots, \alpha_p; B_1, \ldots, B_q) \in \mathbb{R}^+$ obey

$$1 - \sum_{j=1}^{p} A_j + \sum_{j=1}^{q} B_j \geq 0.$$

It is easily seen that (5) is the generalization of the famous generalized hypergeometric series $pF_q$ defined by

$$pF_q \left[ \left( \alpha_1 A_1, \ldots, \alpha_p A_p; \beta_1 B_1, \ldots, \beta_q B_q \right); t \right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_p)_n}{(\beta_1)_n \cdots (\beta_q)_n} \frac{t^n}{n!}$$

(6)

The Fox H-function \cite{16}, a generalization of the Fox–Wright function, is defined in terms of the Mellin–Barnes integral as

$$H_{\alpha, \beta}^{\gamma, \delta} \left[ x^\gamma \left( \alpha_1 A_1, \ldots, \alpha_A A_A; \beta_1 B_1, \ldots, \beta_B B_B \right) \right] = \frac{1}{2\pi i} \int_L \Theta(s) x^{-s} \, ds,$$

(7)

where $i = (-1)^{\frac{1}{2}}$, $x \neq 0$ and $x^{-s} = \exp[-\text{sgn}\,|x|\,i \arg x]$ and

$$\Theta(s) = \frac{\prod_{j=1}^{p} \Gamma(1-\alpha_j - A_j \beta) \prod_{j=1}^{q} \Gamma(b_j + B_j s) \prod_{i=\sigma+1}^{\rho} \Gamma(a_j + A_j \beta)}{\prod_{j=\mu+1}^{\rho} \Gamma(1-b_j - B_j \beta) \prod_{i=\sigma+1}^{\rho} \Gamma(a_j + A_j \beta)}.$$
The Meijer G-function [5] introduced by Meijer is defined as
\[
G^{\mu,\sigma}_{\alpha,\beta}(x|a_1,a_2,\ldots,a_\alpha; c_1,c_2,\ldots,c_\beta) = H^{\mu,\sigma}_{\alpha,\beta}[x|a_1,a_2,\ldots,a_\alpha; c_1,c_2,\ldots,c_\beta] = \sum_{k=1}^{\mu} \prod_{j=1}^{\mu} \Gamma(c_j - a_j) \prod_{j=1}^{\sigma} \Gamma(1 + c_k - a_j) \Gamma(1 + a_j - c_k) \\
\times pFq(1 + c_k - a_1,\ldots,1 + c_k - a_\alpha; 1 + c_k - c_1,\ldots,1 + c_k - c_\beta; (-1)^{\alpha-\mu-\sigma}x),
\] (8)
where \(1 \leq \mu \leq \beta, 0 \leq \sigma \leq \alpha \leq \beta - 1\).

The classical Mittag-Leffler function [17, 23] is defined as
\[
E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(k\alpha + \beta)} \quad (\alpha > 0, \beta \in \mathbb{C}).
\] (9)

We also note that \(W_{\alpha,\beta}^{\gamma,\delta}\) immediately reduces to the above-mentioned functions as follows:
1. On replacing \(\alpha\) by \(-\alpha\), \(t\) by \(-t\), setting \(\beta = 0\) in (2) and with the help of (4), we get
\[
W_{-\alpha,0}^{\gamma,\delta}(-t) = F_{\alpha}^{\gamma,\delta}(t).
\] (10)
2. On replacing \(\alpha\) by \(-\alpha\), \(t\) by \(-t\), setting \(\beta = 1 - \alpha\) in (2) and with the help of (3), we get
\[
W_{-\alpha,1-\alpha}^{\gamma,\delta}(-t) = M_{\alpha}^{\gamma,\delta}(t).
\] (11)
3. On replacing \(t\) by \(-t\) in (2) and with the help of (7), we can easily relate the generalized Wright function with the Fox H-function as
\[
\frac{\Gamma(\gamma)}{\Gamma(\delta)} W_{\alpha,\beta}^{\gamma,\delta}(t) = H^{1 \frac{1}{1 \frac{1}{3}}}[0,1),(1-\gamma,1) \mid 1 - \gamma, 0, 0, 1 - \beta, 1 - \delta, 1].
\] (12)
4. On replacing \(t\) by \(-t\), \(\alpha = 1\) in (2) and with the help of (8), we can easily relate the generalized Wright function with the Meijer G-function as
\[
\frac{\Gamma(\gamma)}{\Gamma(\delta)} W_{1,\beta}^{\gamma,\delta}(t) = G^{1 \frac{1}{1 \frac{1}{3}}}[0,1), 1 - \gamma, 0, 0, 1 - \beta, 1 - \delta].
\] (13)
5. On setting \(\alpha = 0, \gamma = 1\) in (2) and with the help of (9), we get
\[
\frac{\Gamma(\beta)}{\Gamma(\delta)} W_{0,\beta}^{1,\delta}(t) = E_{1,\delta}(t).
\] (14)

**Definition 1.2** The following result is also essential for the present investigation. For \(\Re(u) > 0\) and \(\Re(v) > 0\), the Lavoie–Trottier [14] integral formula is defined as
\[
\int_{0}^{1} x^{\alpha-1}(1-x)^{\beta-1} \left(1 - x \frac{2^{\nu-1}}{\frac{2}{3}} \right)^{2^{\nu-1}} \left(1 - x \frac{1}{4} \right)^{\nu-1} dx = \left(\frac{2}{3}\right)^{2\nu} \frac{\Gamma(u)\Gamma(v)}{\Gamma(u + v)}.
\] (15)
Using (15) and interchanging the order of integration and summation, we obtain

\[ \int_0^1 x^{2u+1} (1-x)^{2v+1} \left( 1 - \frac{x}{3} \right)^{u-1} \left( 1 - \frac{x}{4} \right)^{v-1} W_{a,b}^{\gamma,\delta} \left( \left( 1 - \frac{x}{3} \right)^2 \right) \, dx \]

\[ = \left( \frac{2}{3} \right)^{2u+2v} \Gamma(u+v) \Gamma(\delta) \Gamma(\gamma) \frac{t^u}{2 \psi_3} \left[ \begin{array}{c} (u,1), (\gamma,1); \\ (\delta,1), (\beta,\alpha), (2u+v,1); \end{array} \right]. \]  

(16)

**Proof** Applying the definition (2) in the left-hand side of (16) and denoting it by \( I \), we write

\[ I = \sum_{n=0}^{\infty} t^n (\frac{\Gamma(\gamma)}{\Gamma(\delta)} n!) \int_0^1 x^{u+v} (1-x)^{2u+2v} \left( 1 - \frac{x}{3} \right)^{u-1} \left( 1 - \frac{x}{4} \right)^{v-1} \, dx. \]  

(17)

Using (15) and interchanging the order of integration and summation, we obtain

\[ I = \left( \frac{2}{3} \right)^{2u+2v} \Gamma(u+v) \Gamma(\delta) \Gamma(\gamma) \sum_{n=0}^{\infty} \frac{\Gamma(u+n) \Gamma(\gamma+n) \Gamma(\delta+n) \Gamma(\alpha+n) \Gamma(\beta+n) \Gamma(2u+v+n)}{n!} \frac{t^n}{n!} \]  

(18)

which on using (5), yields the required assertion (16) of Theorem 1.

**Theorem 2** With the conditions already mentioned in Theorem 1, the following formula holds:

\[ \int_0^1 x^{u-1} (1-x)^{2u+2v-1} \left( 1 - \frac{x}{3} \right)^{u-1} \left( 1 - \frac{x}{4} \right)^{v-1} W_{a,b}^{\gamma,\delta} \left( 1 - \frac{x}{3} \right)^2 \, dx \]

\[ = \left( \frac{2}{3} \right)^{2u+2v} \Gamma(u+v) \Gamma(\delta) \Gamma(\gamma) \frac{t^u}{2 \psi_3} \left[ \begin{array}{c} (u,1), (\gamma,1); \\ (\delta,1), (\beta,\alpha), (2u+v,1); \end{array} \right]. \]  

(19)

**Proof** The result in (19) can be derived with ease by the same procedure as followed in the establishment of (16).

Based on the previous theorems, at least two corollaries immediately follow which exploit the use of the hypergeometric function in (6).

**Corollary 2.1** Let \( u+v \in \mathbb{C} \setminus \mathbb{Z}_\alpha \) and with all conditions of Theorem 1, the following integral formula holds true:

\[ \int_0^1 \frac{1-x}{x} x^{u} (1-x)^{2u+2v-1} \left( 1 - \frac{x}{3} \right)^{u-1} \left( 1 - \frac{x}{4} \right)^{v-1} W_{a,b}^{\gamma,\delta} \left( 1 - \frac{x}{3} \right)^2 \, dx \]

\[ = \left( \frac{2}{3} \right)^{2u+2v} \Gamma(u+v) \Gamma(\delta) \Gamma(\gamma) \frac{t^u}{2 \psi_3} \left[ \begin{array}{c} (u,1), (\gamma,1); \\ (\delta,1), (\beta,\alpha), (2u+v,1); \end{array} \right]. \]  

(20)
Corollary 2.2 Let \( u + v \in \mathbb{C} \setminus \mathbb{Z}_0 \) and with all conditions of Theorem 1, the following integral formula holds true:

\[
\int_0^1 (1 - x)^{2u + 2v - 1} x^{\mu - 1} \left( 1 - \frac{x}{3} \right)^{2u - 1} \left( 1 - \frac{x}{4} \right)^{\frac{u - v - 1}{2}} W_{\gamma, \delta} \left\{ t x \left( 1 - \frac{x}{3} \right) \right\} dx
\]

\[
= \left( \frac{2}{3} \right)^{2u + 2v} \Gamma(\mu + \sigma) \Gamma(\delta) \frac{F_{\alpha + 2}}{\Gamma(2 \mu + \sigma) \Gamma(2 \mu + \gamma)} \left[ (\gamma, 1), (u + v, \sigma), (\frac{u - v - 1}{2}, \frac{u - v - 1}{2}) \left| \frac{4t}{9 \alpha^2} \right. \right].
\]

Proof Making use of the result (for \( n \in \mathbb{N}_0 \))

\[
(\lambda)_{2n} = \left( \frac{1}{2} \right)^n 2^{2n} \left( \frac{\lambda + 1}{2} \right),
\]

in (18) and with the help of (5), we can easily establish (20). A similar approach will establish (21).

3 Special cases

According to the procedure we have employed, we can appreciate the importance of the special cases mentioned in (10)–(14) to establish some new results. We have indeed the following result.

Corollary 3.1 \( \forall u, v, b \in \mathbb{C} \) with \( \Re(u) > \frac{b}{2}, \Re(u + v) > 0, \Re(2u + v) > \frac{b}{2}, \alpha \in (0, 1) \) and \( x > 0 \), let

\[
\int_0^1 (1 - x)^{2u + 2v - 1} x^{\mu - 1} \left( 1 - \frac{x}{3} \right)^{2u - 1} \left( 1 - \frac{x}{4} \right)^{u - v - 1} F_{\gamma, \delta} \left\{ t x \left( 1 - \frac{x}{3} \right) \right\} dx
\]

\[
= \left( \frac{2}{3} \right)^{2u + 2v} \Gamma(\mu + \sigma) \Gamma(\delta) \frac{F_{\alpha + 2}}{\Gamma(2 \mu + \sigma) \Gamma(2 \mu + \gamma)} \left[ (\gamma, 1), (u + v, \sigma), (\frac{u - v - 1}{2}, \frac{u - v - 1}{2}) \left| \frac{4t}{9 \alpha^2} \right. \right].
\]

Corollary 3.2 Allowing for the conditions already stated in (22), we have

\[
\int_0^1 (1 - x)^{2u + 2v - 1} x^{\mu - 1} \left( 1 - \frac{x}{3} \right)^{2u - 1} \left( 1 - \frac{x}{4} \right)^{u - v - 1} M_{\gamma, \delta} \left\{ t x \left( 1 - \frac{x}{3} \right) \right\} dx
\]

\[
= \left( \frac{2}{3} \right)^{2u + 2v} \Gamma(\mu + \sigma) \Gamma(\delta) \frac{M_{\alpha + 2}}{\Gamma(2 \mu + \sigma) \Gamma(2 \mu + \gamma)} \left[ (\gamma, 1), (u + v, \sigma), (\frac{u - v - 1}{2}, \frac{u - v - 1}{2}) \left| \frac{4t}{9 \alpha^2} \right. \right].
\]

The proof of (22) and (23) is very similar to that of Theorem 1 and Theorem 2. It can be easily established by setting \( \beta = 0, \alpha = -\alpha \), and replacing \( t \) by \(-t\) and then using (10).

Corollary 3.3 Allowing for the conditions already stated in (22), we get

\[
\int_0^1 (1 - x)^{2u + 2v - 1} x^{\mu - 1} \left( 1 - \frac{x}{3} \right)^{2u - 1} \left( 1 - \frac{x}{4} \right)^{u - v - 1} M_{\gamma, \delta} \left\{ t x \left( 1 - \frac{x}{3} \right) \right\} dx
\]

\[
= \left( \frac{2}{3} \right)^{2(u + v)} \Gamma(\mu + \sigma) \Gamma(\delta) \frac{M_{\alpha + 2}}{\Gamma(2 \mu + \sigma) \Gamma(2 \mu + \gamma)} \left[ (\gamma, 1), (u + v, \sigma), (\frac{u - v - 1}{2}, \frac{u - v - 1}{2}) \left| \frac{4t}{9 \alpha^2} \right. \right].
\]
Corollary 3.4 Allowing for the conditions already stated in (22), we get
\[
\int_0^1 (1-x)2u+2v-1x^{u-\frac{b}{2}-1} \left(1 - \frac{x}{3}\right)^{2u-b-1} \left(1 - \frac{x}{4}\right)^{u+v-1} M_{\gamma,\delta} \left[ t x \left(1 - \frac{x}{3}\right)^2 \right] dx \\
= \left(\frac{2}{3}\right) 2^{u-b} \frac{\Gamma(u+v)\Gamma(\delta)}{\Gamma(\gamma)} \left[ (\gamma, 1), (u - \frac{b}{2}, 1); -\frac{4t}{9} \right].
\]

Corollary 3.5 Allowing for the conditions already stated in (22), we get
\[
\int_0^1 (1-x)2u-b-1x^{u+v-1} \left(1 - \frac{x}{3}\right)^{2u+2v-1} \left(1 - \frac{x}{4}\right)^{u-\frac{b}{2}-1} \\
\times H_{1,3}^{1,1} \left[ \left(1 - \frac{x}{4}\right)(1-x)^2 t \left\{ (1 - \gamma, 1) \\
(0,1), (1-\beta, \alpha), (1-\delta, 1) \right\} \right] dx \\
= \left(\frac{2}{3}\right) 2^{u+v} \Gamma(u+v+\psi_3) \left[ (\gamma, 1), (u - \frac{b}{2}, 1); \\
(\delta, 1), (\beta, \alpha), (2u + v - \frac{b}{2}, 1); -\frac{4t}{9} \right].
\]

Corollary 3.6 Allowing for the conditions already stated in (22), we get
\[
\int_0^1 x^{u-\frac{b}{2}-1}(1-x)^{2u(v+1)-1} \left(1 - \frac{x}{3}\right)^{2u-b-1} \left(1 - \frac{x}{4}\right)^{u+v-1} \\
\times G_{1,3}^{1,1} \left[ x t \left(1 - \frac{x}{3}\right)^2 t \left\{ 1 - \gamma \right\} \\
(0,1), (1-\beta, 1-\delta) \right] dx \\
= \left(\frac{2}{3}\right) 2^{(u+v)} \Gamma(u+v+\psi_3) \left[ (\gamma, 1), (u - \frac{b}{2}, 1); \\
(\delta, 1), (\beta, 1), (2u + v - \frac{b}{2}, 1); -\frac{4t}{9} \right].
\]

Corollary 3.7 Allowing for the conditions already stated in (22), the following integral formula holds true:
\[
\int_0^1 (1-x)2u+2v-1x^{u-\frac{b}{2}-1} \left(1 - \frac{x}{3}\right)^{2u-b-1} \left(1 - \frac{x}{4}\right)^{u+v-1} \\
\times G_{1,3}^{1,1} \left[ x t \left(1 - \frac{x}{3}\right)^2 t \left\{ (1 - \gamma, 1) \\
(0,1), (1-\beta, \alpha), (1-\delta, 1) \right\} \right] dx \\
= \left(\frac{2}{3}\right) 2^{u-b} \Gamma(u+v+\psi_3) \left[ (\gamma, 1), (u - \frac{b}{2}, 1); \\
(\delta, 1), (\beta, 1), (2u + v - \frac{b}{2}, 1); -\frac{4t}{9} \right].
\]
Corollary 3.9 Allowing for the conditions already stated in (22), we get

\[
\int_0^1 (1-x)^{2u-2b-1} x^{u+v-1} \left( 1 - \frac{x}{3} \right)^{2u+2v-1} \left( 1 - \frac{x}{4} \right)^{u+\frac{1}{2}-1} E_1,\delta \left\{ t \left( 1 - \frac{x}{4} \right)^2 \right\} dx \\
= \left( \frac{2}{3} \right)^{2(u+v)} \Gamma(u+v)_{2\psi_2} \left[ (1, t), (u - b, 1); (\delta, 1), (2u + v - b, 1); t \right].
\]

(30)

Corollary 3.10 Allowing for the conditions already stated in (22), we get

\[
\int_0^1 (1-x)^{2u+2v-1} x^{u+\frac{1}{2}-1} \left( 1 - \frac{x}{3} \right)^{2u-b-1} \left( 1 - \frac{x}{4} \right)^{u+v-1} E_1,\alpha \left\{ t \left( 1 - \frac{x}{3} \right)^2 \right\} dx \\
= \left( \frac{2}{3} \right)^{2u-b} \Gamma(u+v)_{2\psi_2} \left[ (1, 1), (u - b, 1); (\delta, 1), (2u + v - b, 1); \frac{4t}{9} \right].
\]

(31)

4 Graphical interpretation

In this section, we illustrate the solutions (16), (19), (22), (23), (24) and (25) using a graphical representation in terms of the parameter \( t \). We use the numerical method named the Gaussian quadrature method to evaluate the integral of these equations and compare with the main results. To get the specific results, we take \( n = 8 \) and \( k = 5 \) (see Figs. 1–6).
5 Conclusion

In these final remarks, it is worth stressing that the integral formulas computed in this paper involving a generalized Wright function are amenable for further research and generalizations. It is natural to note that the generalized Wright function depicts a close connection with several important special functions mentioned in the paper. As a consequence, we have attempted to compute the integrals of the above-mentioned functions in the form of generalized Wright function by some suitable parametric replacement, linking different families of special functions. We have also used the Gaussian quadrature formula to
Figure 6 Solution of (24) with the parameters used in the simulation $u = 6, v = 3, \delta = 10, \alpha = 0.5, \gamma = 12$ and $b = 4$. Solution of (25) with the parameters used in the simulation $u = 6, v = 3, \delta = 5, \alpha = 0.5, \gamma = 5$ and $b = -6$.

compare our main results graphically using Matlab. The conclusion we may draw is that the generalized Wright function has a wide range of applications in different domains; therefore, the results obtained provide a significant step which can yield some potential applications in the field of classical and applied mathematics.

Acknowledgements
This work was supported by the Science and Engineering Research Board (SERB), DST, Government of India (GoI), for the project under the Mathematical Research Impact Centric Support (MATRICES) with reference no. MTR/2017/000821.

Funding
No funding sources to be declared.

Availability of data and materials
Please contact the authors for data requests.

Competing interests
Authors declare they have no competing interests regarding the publication of the article.

Authors’ contributions
The authors contributed equally in writing the present paper and performed all the steps of the proofs in this research. All authors read and approved the final manuscript.

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Publisher’s Note
Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 6 July 2020 Accepted: 6 September 2020 Published online: 15 September 2020

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