Determination of SSUT of HSC Beams with WASKAS Method

Gopal Charan Behera, Subhasri Panda
Department of Civil Engineering, Government College of Engineering, Kalahandi, Bhawanipatna, Odisha, India

E-mail: beheragb@gmail.com

Abstract. Distressed beams should be retrofitted rather than demolishing. Nowadays there are so many retrofitting materials available. Retrofitting is to be done with materials those are easy to apply, economic and less time consuming. Engineers are more interested in ultimate strength of a structure. Strength is related to stiffness. Finding secant stiffness at ultimate torque of a suitable wrapping material for distressed concrete beams that is economic, long lasting, sustainable and appropriate to the common people is the main focus of this investigation. The paper progresses with search of different wrapping strategies and different materials available. Then the paper aims exploring sustainability options for torsional retrofitting with ferrocement. Aim of the paper is to evaluate SSUT of “U” wrapped beam with different methods. This study shows there is reduction of stiffness at ultimate stage of wrapped beams. Experimental, analytical and Soft computing method MARS and WASPAS were discussed here to predict the value of SSUT. Torsionally over reinforced sections are found with more stiffness at ultimate than all other types of beams. The predicted values are well within the acceptable limits.

1. Introduction
Degradation by climate, change in functional requirements or excess load during earthquake, has necessitated reinforced Concrete (RC) structures to be structurally upgraded. Poor maintenance, inferior workmanship or due to codal updates, RC structures may be upgraded rather than demolished [1-2]. Fiber reinforced polymer (FRP) is the commonly used wrapping material for strengthening of structures in present era.

Each Repair technique differs from one to other. Building reoccupancy time limit, cost of repair, amount and type of labour decide type of repair [3]. Retrofitting of deficient structures by FRP is easy. Retrofitting by FRP against bending and shear has attained a good pace, for torsion it is in back seat [4-5]. Enhancement of strength has been observed with FRP wrapped structures against shear, torsion and flexure [6]. Wrap if it is possible may be in all four sides or it may be on three sides leaving the top face due to existing structures. Many investigation are carried out for full with FRP [7-10] investigated with “U” wrap. Requirement of skilled labour and high cost keeps FRP wrapping familiar in developed countries [11,12], may not be suitable for developing countries. Better torsional capacity is observed in full wrap than “U” wraps [13]. Torsional stiffness is a key parameter. Stiffness of a structure depends on its size, materials used and its properties [14-16]. Stiffness is the other way of measuring ductility. Here determination of the stiffness of beams is the main aim of this investigation. Analytical method is developed to supplement the work. The objective is to strengthen distraught structure to enhance the torsional secant stiffness at ultimate torque (SSUT) by an economical material such as ferrocement. Destruction of prototype structure for experimental work and consumption of time in analytical method
push both the methods in back seat. Soft computing procedure like MARS and WASPAS have been employed to evaluate the stiffness of different high strength concrete beams with reinforcement in core and ferrocement “U” wrap on periphery

1.1. Substitute for FRP and Evaluation Processes for Torsional Parameters
Ferrocement is a low cost material and easy to apply. Torsional strength is found to increasing in distressed structures with wrap of ferrocement [17]. Construction on ferrocement is easy. It has low permeability and strength in tension is high [18]. Ferrocement is a combination of welded wire mesh with rich grade of mortar. Ferrocement is a good repairing material, increases ductility and strength of distressed members when wrapped with them. Ferrocement is easy to apply as it has the flowable property along with resistance to sulphate and corrosion. When a distressed structure is wrapped with ferrocement, failure mode shifts to ductile from shear. Studies were performed on “U” jacketing for enhancement of strength on distressed structures. Values obtained from test values shows that there is enough enhancement of strength with ferrocement wrapping. The analytical models were also predicting the enhancement of stiffness and dynamic characteristics [19]. Three methods were adopted to evaluate the torsional parameters. (1)Experimental Evaluation (2) Analytical Method ad (3) Soft computing.

2. Different Evaluation Methods
The different methods adopted here for evaluation of secant stiffness at ultimate are described here.

2.1. Experimental Method
For evaluation of SSUT, high strength beams were prepared and tested respectively. Different quantity of reinforcement was used in beams to make different states of torsion. Shear stress is induced due torsion which creates shear cracks with 450 angle to longitudinal direction. To accommodate two spirals of crack pattern a length of 2mt. was taken with width 125 mm and depth 250 mm. The concrete strength was kept 60 MPa while mortar was used of 55 MPa. The two end zones with 250 mm each were heavily reinforced to avoid failure due to gripping. To make different states of torsional beams, reinforcement was varied in longitudinal and transverse direction. Reinforcement in both directions is less in under reinforced beam than balance reinforcement. For over reinforced in longitudinal direction, higher quantity of reinforcement than that is required for a balance section in the longitudinal direction. To make over reinforced in transverse direction, less quantity of steel in longitudinal and more steel in transverse direction are kept in high strength concrete. For completely over reinforced sections higher amount reinforcement in longitudinal and transverse directions is kept. Numbers of beams prepared and tested are presented in Table 1. A sketch of section of a beam presented in Figure 1 and details are covered in [20-21].

Figure 1. Details of the beams
Table 1. Details of the beam cast

| Designation | Amount of steel | Reinforcement in core concrete | Ferrocement |
|-------------|----------------|-------------------------------|-------------|
|             |                | Reinforcement Longitudinal direction | Reinforcement Transverse direction |               |
|             |                | No. of bars, Diameter | Young’s modulus (MPa) | Spacing, Diameter | Young’s modulus (MPa) | Dia. 0.72 mm No. of mesh layers |
| BOH         |                | 4                          | 4            |                |
| BO4H        | 6 nos.,12 mm   | 440                        | 70 mm c/c, 10mm | 445           | 4            |
| T4H         | 6 nos.,6 mm    | 350                        | 70 mm c/c, 6mm | 350           | 4            |
| U4H         | 6 nos.,12 mm   | 440                        | 70 mm c/c, 6mm | 350           | 4            |
| Lo4H        | 6 nos.,12 mm   | 440                        | 70 mm c/c, 10mm | 445          | 4            |
| To4H        | 6 mm, 6 nos.   | 350                        | 70 mm c/c, 10mm | 445          | 4            |
| Co4H        | 6 nos.,12 mm   | 440                        | 70 mm c/c, 10mm | 445          | 4            |

2.2. Analytical Model
To find out torsional stiffness, experimental method is one. This method is a time consuming process and destruction of specimens makes the process uneconomical as well as wastage of material. From the experimental data, prediction of torsional parameters with change of dimension, material and material properties could not evaluated. To get rid of this situation, analytical method is developed by use of Softened truss model of Hsu [22]. As there is asymmetry in material, soften truss model is taken with some changes. The results predicted are found to be well within acceptable limit. Formulation of analytical model is described in [20—21]. Table 2 shows the values of SSUT.

2.3. Soft Computing Method
The analytical model developed for the evaluation of the torsional parameters, which solves a set of basic equations by iterative process. This does not provide any basic equation to evaluate the parameters taking consideration into the size aspect ratio, material type and their properties. So, soft computing method is adopted to evaluate the parameters.

2.3.1 Multivariate Adaptive Regression Spline (MARS)
In MARS method, some experimental results are taken for fitting and others are used for testing. Equations are developed in simple form to calculate the values for this MARS method. There is no need for any suppositions to be taken for this method. Finally, it provides simple equations to calculate the parameter for which some are calling it white box method. The final equation fetches the relation between various parameters. Friedman (1991) [23] presented this valuable theory. The method is very flexible. MARS predicts the value of secant stiffness at ultimate torque as presented below.

Secant stiffness at Ultimate Torque= maximum of [0, Fly-350] * 4.6414 + maximum of [0, 350- Fly] * 0.9407 +169.9223-maximum of [0, Amount of Longitudinal Steel] * 1.1151 + maximum of [0, Mortar strength-40]* 5.8969 + maximum [0, 40 - Mortar strength] * 14.2836 + maximum of [0, 0.921- Ultimate twist] * 3928.1784+ maximum of [0, 350- Fly] *0.5248
2.3.2. Method of Soft Computing: (WASPAS)

Another method of soft computing such as weighted aggregated sum product assessment (WASPAS) was used to solve multi-criteria decision-making (MCDM) problems. Zavadskas [24] proposed this. This is developed by the combination of two methods such weighted product method (WPM) and weighted sum method (WSM). The different steps required to solve the problem by WASPAS method are presented in [25-26].

1st Step: Decision matrix is to be initialized.

2nd Step: Decision matrix is to be normalized by use of the equations given below. Maximize or minimize as per the equation.

\[ \tilde{x}_{ij} = \frac{x_{ij}}{\max x_{ij}} \quad (1) \]

\[ \tilde{x}_{ij} = \min_i x_{ij} \quad (2) \]

Where \( x_{ij} \) is the assessment value of the \( i^{th} \) alternative with respect to the \( j^{th} \) criterion.

3rd Step: It involves for calculation of the total relative importance of \( i^{th} \) value alternative. It is based on weighted sum method (WSM) with use of this values.

\[ Q_i^{(1)} = \sum_{j=1}^{n} \tilde{x}_{ij} \cdot w_j \quad (3) \]

4th Step: Again the estimation of the total relative importance value of \( i^{th} \) alternative is to be done. It depends upon (WPM) weighted product method. This involves the below mentioned formulae.

\[ Q_i^{(2)} = \prod_{j}^{n} \tilde{x}_{ij}^{w_j} \quad (4) \]

5th Step: In this step, importance is given for accuracy and effectiveness. In this method of WASPAS, decision-making process was given more importance. To determine total relative importance, the following equation was employed.

\[ Q_i = \lambda \cdot Q_i^{(1)} + (1 - \lambda) \cdot Q_i^{(2)} \quad (5) \]

Where \( \lambda \) can take the values either 0 or 1 or it can take the value 0.1. The usual value of \( \lambda \) is 0.5. When the values of \( \lambda \) is changed, there is change in the value of rankings of alternatives and also values of total relative importance of alternatives gets altered.

| Beam | SSUT (kNm²)       | WASPAS  | MARS   | Softened Truss model | Experimental |
|------|-------------------|---------|--------|----------------------|--------------|
| BH   | 1362.43           | 1605.00 |        |                      | 1605.00      |
| B4H  | 1362.43           | 1111.66 | 1145.00| 1145.00              |              |
| L4H  | 558.41            | 869.40  |        |                      | 869.40       |
| T4H  | 940.41            | 927.42  |        |                      | 915.28       |
| U4H  | 37.10             | 69.13   | 38.00  |                      | 38.00        |
| Lo4H | 65.29             | 60.93   | 66.00  |                      | 68.00        |
| To4H | 64.20             | 69.13   | 72.75  |                      | 72.00        |
| Co4H | 136.39            | 130.00  | 129.00 |                      | 130.00       |

3. Discussion on Torsional Parameter SSUT

SSUT values were taken by experimental results. The same has been derived from analytical and soft computing methods. The variation of the SSUT values with states of torsion is presented here. Errors found in the predicted values by these three methods are calculated to find out suitability of these methods.
3.1. SSUT of High Strength Beams

Torsional stiffness of a beam depends on the strength of core material. Stiffness of a high strength beam is not same as normal strength beam. Properties such as softening co-efficient and elastic modulus change with change in strength of concrete. High strength beams with and without reinforcement and “U” wrap were cast and analyzed. All the beams have aspect ratio 2, mortar grade M55 and concrete of grade M60. Here focus is to evaluate SSUT. SSUT depends on the core and outer periphery material and their properties. During loading, structure behaves elastically at initial stage, then micro-cracking stage initiates, beyond that cracking takes place. After cracking, ultimate stage takes place. Beyond cracking, reinforcement plays a vital role. In the tested beams, designation BH refers to a beam in which there is no reinforcement in core concrete and no wrap. Beam B4H refers to a beam without core reinforcement and with “U” wrap. All other six beams are reinforced beams with “U” wrap covering six states of torsion.

The beam BH was found with highest SSUT value of 1605 kNm² and U4H was with lowest value of 38 kNm². Just after cracking, BH failed without any increase in ultimate torque. This proves unwrapped beam fails after cracking. The unwrapped beams are unable to sustain any load beyond cracking. The other beams B4H, L4H, T4H, Lo4H, To4H and Co4H were found with SSUT values of 1145 kNm², 869.40 kNm², 915.28 kNm², 68 kNm², 72 kNm² and 130 kNm² respectively. Beams when reinforced in both directions of core concrete, difference in SSCT values was very less. SSUT values by experimental and predicted were presented in Table 2. Softened truss model could not predict SSUT of beams of without reinforcement or with singly type of reinforcement. Figure 2 presents the value of SSUT of all beams.

![Figure 2. Experimental and predicted values of SSUT and percentage of error in predicted values.](image)

3.2. Percentage of Error in Predicted Values

The percentage of errors in the predicted values was also presented in the same figure. The errors in percentage were found as -2.94, 1.04 and -0.77 respectively for longitudinally over reinforced, transversely over reinforced and completely over reinforced. Other values are found same with experimental results. The percentage of error in MARS was found to be 0.0, -2.91, 0.0, 1.33, 81.92, -10.39, -3.98, 0.0 for plain beam without wrap, B4H with wrap, only longitudinal reinforcement with wrap, T4H, under reinforced with wrap, Lo4H, transversely over reinforced with wrap and Co4H respectively. Only under reinforced beam was found with some error more than 10 % by MARS. The percentage of error in WASPAS was found to be -15.11, 18.99, -35.77, 2.75, -2.37, -3.99, -10.83 and 4.92 for BH, plain beam with wrap, L4H, only transverse reinforcement with wrap, U4H, longitudinally over reinforced with wrap, To4H and completely over reinforced with wrap respectively. The percentage of error is very less except control specimens and beam with only longitudinal reinforcement. So,
softened truss model, soft computing models MARS and WASPAS can be employed for prediction of secant stiffness at cracking torque (SSUT).

### 3.3. Percentage of decrease in SSCT value in comparison to BH and B4H

SSUT values are found to be decreased in comparison to their control specimen BH. It has been noticed that percentage of reduction was found to be 97.63 for U4H, which is the maximum for all beams. The beams with “U” wrap and reinforcement in core concrete in both directions are found to have percentage of reduction more than 90. The percentage decrease of SSUT value of all reinforced beams with respect to control “U” wrap beam B4H was found to be up to 90 and for U4H, percentage decrease is 96.68. This proves under reinforced beams have undergone maximum twist. The same has been plotted in Figure 3. Stiffness at ultimate torque is controlled by the reinforcement present in the core. Completely reinforced beams are provided with maximum reinforcement. So, more SSUT was noticed for this beam.

![Figure 3](image_url)

**Figure 3.** Experimental values of SSUT and percentage of decrease of SSUT with respect to BH and B4H.

### 4. Conclusion

The experimental study was conducted for calculation of SSCT values of different beams. The secant stiffness at ultimate torque was predicted by other three methods. Main conclusions highlighting the novelty of the work is briefly mentioned in this section.

- The results of SSUT is better predicted by analytical model than soft computing methods such as MARS and WASPAS.
- A significant decrease in SSUT is found with “U” ferrocement wrapped under reinforced high strength concrete beams over the beams without any reinforcement. “U” wrapped beam has 97.63 % reduction of SSCT value over their control specimen.
- SSUT is reliant upon the core concrete reinforcement.
- Under reinforced concrete, beams showed overall decrease in SSUT.
- The experimental values, predicted values from soft computing model and analytical model reveal that the SSCT of a ferrocement “U” wrap beam is more influenced by the state of torsion.

The percentage of error in predicted values for most of the beams are within 10%. So, all these methods; analytical, MARS and WASPAS can be used to predict the secant stiffness at ultimate torque of ferrocement “U” wrap high strength beams.

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