Compaction Characteristics and Minimum Void Ratio Prediction Model for Gap-Graded Soil-Rock Mixture

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Featured Application: Prediction of the minimum void ratio of gap-graded soil-rock mixture.

Abstract: Gap-graded soil-rock mixtures (SRMs), composed of coarse-grained rocks and fine-grained soils particles, are very inhomogeneous materials and widely encountered in geoengineering. In geoengineering applications, it is necessary to know the compaction characteristics in order to estimate the minimum void ratio of gap-graded SRMs. In this paper, the void ratios of compacted SRMs as well as the particle breakage during vibrating compaction were investigated through a series of vibrating compaction tests. The test results show that gap-graded SRMs may reach a smaller void ratio than the SRM with a continuous gradation under some circumstances. When the particles in a gap interval play the role of filling components, the absence of them will increase the void ratio of the SRM. The particle breakage of gap-graded SRMs is more prominent than the SRM with continuous gradation on the whole, especially at the gap interval of 5–20 mm. Based on the test results, a minimum void ratio prediction model incorporating particle breakage during compaction is proposed. The developed model is evaluated by the compaction test results and its validation is discussed.

Keywords: soil-rock mixtures; compaction; gap gradation; void ratio; particle breakage

1. Introduction

Soil-rock mixtures (SRMs), which are composed of coarse-grained rocks and fine-grained soils particles, are very inhomogeneous materials and widely encountered in geoengineering [1,2]. Due to the inhomogeneous characteristics of SRMs, gap-graded SRMs are commonly found in geoengineering applications [3–5], such as waste rock and tailings from mining [6], clay-aggregate mixtures in rockfill dams [7–9] and glacial tills [10].

Gap-graded SRMs have a range of missing particles [11,12] in which fine particles may more easily be washed out of the matrix of the coarse particles by seepage forces. Therefore, great attention has been paid to the internal erosion of gap-graded SRMs [13–16]. However, little research has been done on the compaction characteristics of gap-graded SRMs. The compaction characteristics significantly influence the density that SRMs can reach under a certain compaction effort, and in turn influence the mechanical behaviour of SRMs [17]. Therefore, it is important to investigate the compaction characteristics of gap-graded SRMs, especially when used in projects where settlement has to be strictly controlled, such as highway and airport foundations [18,19].

On the other hand, a suitable model for estimating the compacted void ratio of gap-graded SRMs is needed for their compaction quality control. Studies show that the void ratio is closely related to the engineering properties of gap-graded materials, such as compressibility [20], shear strength [21] and permeability [22]. The existing void ratio prediction models are built mainly for sand-silt mixture...
within a gap interval is transferred into the remaining particles in proportion to their mass ratios. In a gap gradation, particles are divided into two groups at two sides of the gap interval, fine-grained group and coarse-grained group. The mass content of particles in the fine-grained group is defined as $f_c$. The basic gradation is characterized using the fractal dimension $D$, defined as the gradient in a double logarithmic plot of mass of particles against particle size [25].

$$\frac{M(d < R)}{M_T} = \left(\frac{R}{R_L}\right)^{3-D}$$  \hspace{1cm} (1)

in which $M(d < R)$ is the mass of particles with diameters finer than $R$, $M_T$ is the total mass, $R_L$ is the maximum diameter. In this test, $R_L = 60$ mm.

The lithology of the test SRM is slightly-weathered dolomite, with a saturated uniaxial compression strength of 30~60 MPa (medium hard rock), which was taken from a rockfill dam site in China. The specific gravity of the SRM is measured at 2.65 and its shape is subangular. Five basic gradations with different fractal dimension $D$ were tested. For each basic gradation, four gap gradations were generated, with gap intervals of 5–20 mm, 2–5 mm, 0.5–2 mm and 0.25–0.5 mm, which correspond to medium gravel, fine gravel, coarse sand and medium sand, respectively, according to Chinese standards on soil classification. Furthermore, for the gap gradation with the gap interval of 5–20 mm, seven different values of $f_c$ were considered. The detailed test program is given in Table 1.
From these tests, we attempt to explore the effects of different basic gradations, gap intervals and mass contents of the fine-grained group ($f_c$) on the compaction characteristics of SRMs.

| Table 1. Program of vibrating compaction tests. |
|-----------------------------------------------|
| Fractal dimension $D$ of basic gradation       |
| Gap interval (mm)                             |
| $f_c$ for the gap interval of 5–20 mm          |
| 1.9, 2.1, 2.3, 2.5, 2.7                       |
| 5–20, 2–5, 0.5–2, 0.25–0.5                    |
| 0, 0.2, 0.4, 0.5, 0.6, 0.8, 1.0               |

The vibrating compaction tests were carried out on a steel-made compaction cylinder with an inner diameter of 300 mm and a height of 500 mm. Each SRM sample has 75 kg in mass and was divided into three equal parts. Then, each part was dropped carefully into the cylinder. As shown in Figure 2, a vibrating compactor (Frequency: 45 Hz; Exciting force: 5.4 kN) was placed on the top of the sample and compacted the sample for eight minutes. It was found that the height of the sample would hardly change after a five-minute vibrating compaction. Therefore, the sample was considered to be in the densest state after the eight-minute vibrating compaction. After each test, the total height of the sample was measured, and then its void ratio was calculated. Also, grain-size analysis was performed and the particle breakage during vibrating compaction was analyzed.

![Diagram of the vibrating compaction on soil-rock mixtures.](image)

Figure 2. Diagram of the vibrating compaction on soil-rock mixtures.

2.2. Results and Discussion

2.2.1. Influence of gap intervals

Figure 3 shows the change of the void ratios of the SRMs with different gap gradations, generated from the basic gradation $D = 2.3$, with gap intervals. The test results demonstrate that the influence of the absence of particles in different gap intervals on void ratios of SRMs is different. If the particles within 5–20 mm are absent, the void ratio of the SRM is smaller than that of the basic gradation, contrary to the conventional belief that gap-graded SRMs are less compactible. This is because the particles within 5–20 mm are relatively coarse and difficult to fill into the voids of the skeleton particles. The absence of the particles within 5–20 mm avoids the probability of enlarging the voids of the skeleton particles. The particles smaller than 5 mm usually act as the filling components in SRMs. The absence of these particles means that voids of the skeleton particles cannot be fully filled during vibrating compaction, so the void ratio of the SRMs increases. As the mass content of the particles within 0.25–0.5 mm is relatively fine, the absence of these particles affects slightly on the void ratio of the SRMs.
Figure 3. Void ratios of the soil-rock mixture (SRM) with basic gradation $D = 2.3$ at different gap intervals.

Figure 4 presents the changes of non-uniform coefficients $C_u (C_u = d_{60} / d_{10})$ and coefficients of curvature $C_c (C_c = d_{30}^2 / (d_{10}d_{60}))$ of the SRMs before and after the tests at different gap intervals. It is seen that after vibrating compaction, $C_u$ increases, while $C_c$ decreases, especially in the absence of particles within 5–20 mm. This means the SRM gradations have changed during the vibrating compaction, resulting from the particle breakage that will be discussed later.

![Graph showing changes of non-uniform coefficients and coefficients of curvature](image)

**Figure 4.** Changes of non-uniform coefficients $C_u$ and coefficients of curvature $C_c$ after vibrating compaction.

2.2.2. Influence of basic gradations

Figure 5 gives the changes of the void ratios of the SRMs with gap intervals under the basic gradations with different fractal dimension $D$. It can be seen that the changing trend of the curves is similar when $D = 2.3, 2.5$ and 2.7 (as shown in Figure 3), but it is different from that of $D = 1.9$ and 2.1. From the definition of $D$ in Equation (1), the particle size in an assembly increases with the decreasing $D$. When $D = 1.9$ and 2.1, the particles within 5–20 mm may act as filling components in SRMs. The absence of these particles leads to insufficient filling of the voids among the skeleton particles, resulting in the bigger void ratios of the SRMs than those of the basic gradations when $D = 1.9$ and 2.1. Figure 5 also demonstrates that there are the vital grain size groups acting as filling components in SRMs. The vital grain size group is 0.5–2 mm when $D = 2.3, 2.5, 2.7$ and it is 2–5 mm when $D = 1.9, 2.1$. The absence of the vital grain size group will significantly increase the void ratio of the SRM, which should be taken seriously in engineering practice.
When the mean particle size difference between the two groups is coarse, we use the ratio of 
\( \frac{d_{B15}}{d_{50}} \) to represent the degree of particle size difference between the fine-grained group and coarse-grained group, where \( d_{50} \) and \( d'_{50} \) are the mean diameters of the fine-grained group and coarse-grained group, respectively. The values of \( \frac{d'_{50}}{d_{50}} \) at different gap intervals are plotted against \( D \) in Figure 6. It can be seen that the ratios of \( \frac{d'_{50}}{d_{50}} \) increase with \( D \) irrespective of gap intervals. According to Mcgeary’s finding, it is understandable that the void ratio \( e \) of the SRM decreases with the increasing \( D \) of the basic gradation.

**Figure 5.** Void ratios of the SRMs with gap intervals under different basic gradations.

At the same gap interval, the void ratio \( e \) of the SRM decreases with the increasing \( D \) of the basic gradation. For example, at the gap interval of 5–20 mm, \( e \) decreases from 0.36 to 0.11 when \( D \) increases from 1.9 to 2.7. Mcgeary [26] found that the mixture of two grain size groups would have less void ratio irrespective of gap intervals. Commonly, the degree of particle breakage is quantitatively evaluated using three indices: \( B_{15} \) [27], \( B_{S} \) [28] and \( B_{r} \) [29], where \( B_{15} \) is the ratio of the diameter before test and after test corresponding to 15% finer passing in the gradation curve; \( B_{S} \) is the maximum distance between the original gradation curve and gradation curve after test; \( B_{r} \) is the ratio of \( B_{r} \) to \( B_{p} \), where \( B_{r} \) is the area between original gradation curves and gradation curve after test, \( B_{p} \) is area between original gradation curve and the line \( d = 0.074 \) mm. Figure 7 shows the particle breakage of SRMs with gap intervals under the basic gradations with different fractal dimension \( D \). The test results demonstrate that the particle breakage of

**Figure 6.** Changes of \( \frac{d'_{50}}{d_{50}} \) with fractal dimension \( D \) at different gap intervals.
gap-graded SRMs is more prominent than the SRM with continuous gradation on the whole, especially at the gap interval of 5–20 mm. The degree of particle breakage decreases gradually with the increasing fractal dimension $D$ of the basic gradation. This is because the particle sizes in SRM decrease with the increasing $D$, and consequently less particle breakage will happen during the vibrating compaction.

![Graphs showing particle breakage of SRMs with fractal dimension $D$ at different gap intervals.](image)

**Figure 7.** Particle breakage of SRMs with fractal dimension $D$ at different gap intervals.

### 2.2.3. Influence of mass content in fine-grained group ($f_c$)

In engineering practice, the gap intervals and the gradations of the fine-grained group and the coarse-grained group of SRMs are known, and they are dependent upon the available materials. It is
necessary to study the mass content $f_c$ of the fine-grained group in an SRM to achieve the minimum void ratio of the SRM. Figure 8 presents the relationship between the void ratios $e$ of the SRMs with different basic gradations and the mass contents $f_c$ at the gap interval of 5–20 mm.

![Figure 8. Changes of void ratios of the SRMs with $f_c$ under different basic gradations.](image1)

The relationship is approximately bilinear, and the turning point corresponds to the minimum void ratio of the SRM, which can be explained with the help of Figure 9. Figure 9a illustrates coarse-grained particles enclosing a void space in the case of $f_c = 0$. When $f_c$ increases gradually, the fine-grained particles enter into the voids as filling components, leading to the decrease of the void ratio of the SRM (Figure 9b). Figure 9c shows the state when all voids are fully occupied by the fine-grained particles and the minimum void ratio of the SRM is achieved. The corresponding $f_c$ is called the optimal mass contents of the fine-grained group. If $f_c$ continues to increase, coarse-grained particles will be separated by the fine-grained particles and “float” in the matrix of fine-grained particles, as shown in Figure 9d,e, resulting in the increase of the void ratio of the SRM. As $f_c$ increases up to 1.0, the coarse-grained particles disappear and only fine-grained particles exist in the mixture (Figure 9f), where the maximum void ratio is reached.

![Figure 9. Illustration of the mixture of coarse-grained and fine-grained particles.](image2)

The optimal mass content $f_c$ and the minimum void ratio of gap-graded SRMs are important parameters in engineering practice. In the next part, we attempt to build a model for predicting the optimal mass content $f_c$ and minimum void ratio of gap-graded SRMs.
3. A New Model for Predicting Minimum Void Ratio of Gap-Graded SRMs Incorporating Particle Breakage

In this part, the existing models for predicting the minimum void ratio of gap-graded SRMs by Vallejo [5] and Chang et al. [18] are evaluated by comparing the measured and predicted minimum void ratios from compaction tests. Deficiencies of the existing models are identified and a new model for predicting minimum void ratio of gap-graded SRM is proposed.

3.1. Existing Model

Vallejo [7] measured porosities of sand-clay mixtures. He found that the minimum porosity of the mixture occurs when all the void space in the sand is completely filled by the bulk volume of clay. He also proposed an equation for estimating the minimum porosity of the binary mixtures, as Equation (2).

\[ n_{mix-min} = n_f n_c \]  

(2)

where \( n_{mix-min} \) is the minimum porosity of the sand-clay mixture, \( n_c \) is the porosity of pure sand, \( n_f \) is the porosity of pure clay.

However, the ideal condition in which all the void space of coarse-grained particles is completely filled by the bulk volume of fine-grained particles can hardly be achieved. During the process of achieving minimum void ratio of the mixture, mutual interference between the coarse-grained particles and fine-grained particles will happen, that is, the so-called “wall effect” and “loosening effect” [30], as shown in Figure 10a. Wall effect means that when some isolated coarse-grained particles are immersed in a sea of fine-grained particles (which are dominant), there is a further amount of voids in the packing of fine-grained particles located in the interface vicinity. Loosening effect refers to when fine-grained particles are inserted in the porosity of a coarse-grain packing (coarse particles dominant), and if it is no longer able to fit in a void, there is locally a decrease of volume of the coarse-grained particles.

Chang et al. [23] considered the mutual inferences between fine-grained and coarse-grained particles when studying the minimum void ratio of sand-silt mixtures. They found that the added fine-grained particles would inevitably distort the structure of coarse-grained particles and cause a change of total void volume. They assumed that the change of void volume is proportional to the amount of fine-grained particles added to the mixture. Similarly, for a fine-grained particles dominant structure, the added coarse-grained particles will distort the structure of fine-grained particles and cause a change of total void volume, which is proportional to the amount of coarse particles added to the mixture. They gave the optimal \( f_c \) corresponding to the minimum void ratio as Equation (3).

\[ f_{c-optimum} = \frac{e_c - \tilde{b}}{1 + e_c + e_f - \tilde{a} - \tilde{b}} \]  

(3)

where \( e_c \) is the void ratio of pure coarse-grained particles, \( e_f \) is the void ratio of pure fine-grained particles, \( \tilde{a} \) and \( \tilde{b} \) are material constants.

Chang et al. [23] also proposed an equation for estimating the minimum porosity of the binary mixtures, as Equation (4).

\[ n_{mix-min} = e_c f_{c-optimum} + e_f (1 - f_{c-optimum}) - b_{12} e_c f_{c-optimum} \]  

(4)

where \( b_{12} \) is material constant.
Figure 10. The two parts constituting the change of void volume $\Delta V$: (a) Mutual interference between coarse-grained and fine-grained particles $\Delta V_A$, (b) Particle breakage $\Delta V_B$.

The minimum void ratio of gap-graded SRMs computed using Vallejo and Chang et al. models are shown in Figure 11 and compared with measured values from compaction tests. The comparison shows that the predictability of the model for sand-clay or sand-silt mixtures is not suitable for gap-graded SRMs. It is because, unlike sand-clay or sand-silt mixtures, gap-graded SRM will undergo more particle breakage during vibrating compaction, as the test results show in Section 2.2.1. The particle breakage during the process of compaction will generate some much finer particles, and these finer particles will further fill the voids of the mixture (shown in Figure 10b). Therefore, the effect of particle breakage should be taken into consideration when building models for predicting minimum void ratio of gap-graded SRMs.

Figure 11. Cont.
Figure 11. Comparison of measured minimum void ratios of gap-graded SRM and predicted void ratios using models from Vallejo and Chang et al.

3.2. A Model for Predicting Minimum Void Ratio of Gap-Graded SRMs Incorporating Particle Breakage

A gap-graded SRM consists of two grain size particles: Fine-grained and coarse-grained. The volume of the two grain size particles is denoted as \( V_f \) and \( V_c \). The void ratio of the pure two grain size groups is \( e_f \) and \( e_c \). Our objective is estimating the minimum void ratio of the gap-graded SRM.

Firstly, we consider the coarse-grained particles as the dominant materials. The phase diagram of pure coarse-grained particles is shown in Figure 12a. The solid volume of coarse particles is \( V_c \), and the void among coarse-grained particles is \( V_{vc} \). Then, as shown in Figure 12b, fine-grained particles are added into the pure coarse-grained particles. In a limiting case, all the added fine-grained particles fill into the voids among coarse-grained particles without altering the structure of coarse-grained particles. Therefore, the solid volume of fine particles \( (V_f) \) occupies a space in the void volume \( (V_{vc}) \) and the total volume remains constant. However, as stated above, the limiting case can hardly be achieved, and the total void volume will change due to the particle breakage and altering of the structure of coarse-grained particles. The change of total volume is \( \Delta V \), and it is caused by two parts, one is caused by the altering of the structure of coarse-grained particles \( \Delta V_A \), the other is caused by particle breakage \( \Delta V_B \), \( \Delta V = \Delta V_A + \Delta V_B \).
(1) Calculation of $\Delta V_A$: According to Chang et al. (2015), the change of void volume $\Delta V_A$ caused by the added fine-grained particles is proportional to the amount of fine-grained particles added to the mixture. Therefore, $\Delta V_A$ can be calculated using Equation (5).

$$\Delta V_A = aV_f$$ (5)

where $a$ is material constant.

(2) Calculation of $\Delta V_B$: The data of particle breakage index $B_g$, which is measured in the compaction tests, is plotted versus $f_c$ in Figure 13. The particle breakage index $B_g$ is found to be linearly related to $f_c$. Here, we only show the cases of $D = 2.3$ and 2.5 for the limitation of the paper length. Therefore, $B_g$ can be represented with Equation (6):

$$B_g = k_1f_c + b_1$$ (6)

where $k_1$ and $b_1$ are material constants.

![Figure 12. Phase diagrams for coarse-grained particles dominant: (a) pure coarse particles (before fine-grained particles added), (b) mixture (limiting case), (c) mixture (general case).](image)

Figure 12. Phase diagrams for coarse-grained particles dominant: (a) pure coarse particles (before fine-grained particles added), (b) mixture (limiting case), (c) mixture (general case).

![Figure 13. Changes of particle breakage index $B_g$ with $f_c$ under fractal dimension $D = 2.3$ and 2.5.](image)

Figure 13. Changes of particle breakage index $B_g$ with $f_c$ under fractal dimension $D = 2.3$ and 2.5.
$B_g$ reflects the degree of particle breakage of coarse-grained and fine-grained particles. The more particle breakage occurs, the larger $B_g$ is, and thus the void volume change compared with the pure coarse-grained particles (Figure 10a) due to particle breakage $\Delta V_B$ is larger. We assume $\Delta V_B$ is proportional to the product of solid volume ($V_c + V_f$) and $(B_g - B_g|_{f_c=0})$, thus:

$$\Delta V_B = k_2(V_f + V_c)(B_g - B_g|_{f_c=0})$$

(7)

where $k_2$ is material constant.

$$B_g = k_1f_c + b_1, \quad B_g|_{f_c=0} = b_1$$

and thus:

$$\Delta V_B = k_2(V_f + V_c)k_1f_c$$

(8)

Combining Equations (5) and (8):

$$\Delta V = \Delta V_A + \Delta V_B = aV_f + k_1k_2(V_f + V_c)f_c$$

(9)

According to the phase diagrams of coarse-grained particles dominant structure, the minimum void ratio of the mixture can be expressed as Equation (10)

$$e_M = \frac{V_{VC} - V_f + \Delta V}{V_c + V_f}$$

(10)

According to the definition of $f_c$:

$$f_c = \frac{m_f}{m_c + m_f} = \frac{\gamma_fV_f}{\gamma_cV_c + \gamma_fV_f}$$

(11)

where $m_c$ is the mass of coarse-grained particles, $m_f$ is the mass of fine-grained particles, $\gamma_c$ is the unit weight of coarse-grained particles, $\gamma_f$ is the unit weight of fine-grained particles.

Because the coarse-grained particles and fine-grained particles used in this paper have the same lithology, their unit weights are almost the same, $\gamma_c = \gamma_f$, and thus:

$$f_c = \frac{V_f}{V_c + V_f}$$

(12)

Substituting Equations (9) and (12) for Equation (10), and together with $V_{VC} = e_cV_C$, the minimum void ratio of the mixture can be expressed as:

$$e_M = (a + k_1k_2 - 1 - e_c)f_c + e_c$$

(13)

Let $k = a + k_1k_2 - 1$, thus Equation (13) can be rewritten as:

$$e_M = (k - e_c)f_c + e_c$$

(14)

where $k$ is a material constant.

It can be seen from Equation (14) that the minimum void ratio of gap-graded SRM is linearly related to the fine particles content, which is consistent with the compaction test results shown in Figure 8.

In the case of fine-grained particle dominant, the phase diagram is shown in Figure 14. The solid volume of fine-grained particles is $V_f$, and the void among coarse-grained particles is $V_{vf}$. Then, as shown in Figure 14b, coarse-grained particles are added to the pure fine-grained particles. In a limiting case, all the added coarse-grained particles are embedded in the sea of fine-grained particles without altering the structure of fine-grained particles. The total void volume $V_{vf}$ remains constant.
However, in a general sense, the total void volume will change due to the particle breakage and altering the structure of fine-grained particles. The change of total volume is $\Delta V$, and it is caused by two parts: One is caused by the altering the structure of fine-grained particles $\Delta V_A$, the other is caused by particle breakage $\Delta V_B$. $\Delta V = \Delta V_A + \Delta V_B$. $\Delta V_A$ is proportional to the amount of coarse-grained particles added to the mixture. $\Delta V_B$ is assumed to be the product of solid volume ($V_c + V_f$) and $(B_g - B_g|_{f_c=1})$. Similar to the derivation of Equation (14), the minimum void ratio of gap-graded SRM with fine-grain dominant can be expressed as:

\[
e^M = (e_f + k')f_c - k'
\]  \hspace{1cm} (15)

where $k'$ is material constant.

**Figure 14.** Phase diagrams for fine-grained particles dominant: (a) pure fine-grained particles (before coarse-grained particles added), (b) mixture (limiting case), (c) mixture (general case).

For a given mass content in fine-grained group $f_c$, two values of minimum void ratio $e^M$ can be calculated: One is from Equation (14), the other is from Equation (15). The greater of the two void ratio values is likely to be achieved, because it requires less energy to reach the state. Therefore, the greater of the two values is considered to be the solution.

The linear relationship between the minimum void ratio of gap-graded SRM and $f_c$ is shown in Figure 15. The line represented by Equation (14) is shown in Figure 15 as the line AP. When $f_c = 0$, the minimum void ratio is $e_c$. The line represented by Equation (15) is shown in Figure 15 as the line PB. When $f_c = 1$, the minimum void ratio is $e_f$. The value of $f_c$ corresponding to point P can be solved from Equations (14) and (15), and this value is the optimal mass content of fine-grained particles.

\[
f_c-\text{optimum} = \frac{e_c + k'}{e_f + k' - k}
\]  \hspace{1cm} (16)

The model for estimating minimum void ratio of gap-graded SRM employs 4 parameters, that is, $e_c$, $e_f$, $k$ and $k'$. $e_c$ and $e_f$ are the void ratio of pure coarse-grain particles and pure fine-grained particles, which can be easily obtained by conducting the compaction tests on pure coarse-grained and fine-grained particles. $k$ and $k'$ are material constants. Data of $k$ and $k'$ are plotted versus fractal dimension $D$ of basic gradation in Figure 16. It can be found that the relationship between $k$, $k'$ and $D$ can be well fitted by a quadratic polynomial function, which is given by $k$ or $k' = AD^2 + BD + C$. $A$, $B$ and $C$ are material constants irrespective of gap intervals. Therefore, if the parameters of $D$, $e_c$, $e_f$ of a gap-graded SRM are given, we only need to conduct compaction tests of one gap interval to obtain the parameters of $e_c$, $e_f$, $A$, $B$, $C$, and we can predict the minimum void ratio of gap-graded SRM with other gap-intervals. It is noted that we can obtain the parameters $k$ and $k'$ by conducting some compaction tests if the gradation information is not given.
with gap intervals of 2–5 mm, 0.5–2 mm, and 0.25–0.5 mm using Equations (14) and (15). The model parameters are listed in Table 2. The comparison of predicted minimum void ratio and measured void ratio by vibrating compaction tests is shown in Figure 17. It can be seen from Figure 16 that the predicted minimum void ratio is close to the measured values, which shows the validation of the model.

3.3. Validation of the Model

In order to evaluate the model, we use the vibrating compaction test data of gap-graded SRMs with gap interval of 5–20 mm to obtain the model parameters like $k$ and $k'$ by fitting $k$ and $k'$ with $D$. Together with the data of $e_c$ and $e_f$, we can predict the minimum void ratio of gap-graded SRM with gap intervals of 2–5 mm, 0.5–2 mm, and 0.25–0.5 mm using Equations (14) and (15). The model parameters are listed in Table 2. The comparison of predicted minimum void ratio and measured minimum void ratio by vibrating compaction tests is shown in Figure 17. It can be seen from Figure 16 that the predicted minimum void ratio is close to the measured values, which shows the validation of the model.

### Table 2. Model parameters for the prediction of void ratio of gap-graded SRM with gap intervals of 2–5 mm, 0.5–2 mm and 0.25–0.5 mm.

| Gap Interval | $e_c$ | $e_f$ | $k$ | $k'$ |
|--------------|------|------|-----|-----|
| 2–5 mm       | 0.433| 0.442|     |     |
| 0.5–2 mm     | 0.413| 0.467| $k = 0.094D^2 - 0.44D + 0.49$ | $k' = -0.24D^2 + 1.11D - 1.13$ |
| 0.25–0.5 mm  | 0.403| 0.472|     |     |

Figure 15. Changes of minimum void ratio of gap-graded SRM with $f_c$.

Figure 16. Fitting the material constants $k$ and $k'$ with a quadratic polynomial function of fractal dimension $D$. 

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| 2–5 mm       | 0.433| 0.442|     |     |
| 0.5–2 mm     | 0.413| 0.467| $k = 0.094D^2 - 0.44D + 0.49$ | $k' = -0.24D^2 + 1.11D - 1.13$ |
| 0.25–0.5 mm  | 0.403| 0.472|     |     |
4. Conclusions

A series of vibrating compaction tests were conducted on SRMs with different basic gradations, gap intervals and mass contents of fine-grained group. It was found that gap-graded SRM may reach a smaller void ratio than the SRM with a continuous gradation under some circumstances, for example, the gap-graded SRMs without 5–20 mm particles have a smaller void ratio than the SRMs with basic gradation, in the case of $D = 2.3$, $2.5$ and $2.7$. If the particles in a gap interval play the role of filling components, the absence of them will increase the void ratio of the SRM. Among gap intervals for a certain basic gradation, there is a vital grain size group that dominates the compaction of the SRM. The particle breakage of gap-graded SRMs is more prominent than the SRM with continuous gradation on the whole, especially at the gap interval of 5–20 mm among the selected gap intervals.

A new model is proposed for better predicting the minimum void ratio of gap-graded SRMs. The model requires three parameters, $e_v$, $e_f$ and $D$, for prediction of gap-graded SRMs with different mass contents of fine-grained particles. Comparison between the predicted minimum void ratios measured void ratios proves the validation of this model. It should be noted that the new model is only valid for the lithology and the shape of the particles studied in this work.

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