A Critical String Theory in (3+1)+4 Dimensions for the Standard Model with Three Generations of Fermions and Ultra-Heavy Sterile Right-Handed Neutrinos

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Abstract

Redefining the vacuum state of a free two-fold $N = 1$ covariant supersymmetric string action as the one with all the excited states of world-sheet fermions occupied, makes the theory anomaly free in (3+1)+4 dimensions. While in the $NS$ sector the spectrum resembles the same for the standard $N = 1$ superstring theory with one of the $N$ species in the background, in the $R$ sector both the species of fermions are required to describe the relevant spin operators to describe the fermion spectra. A crucial difference from the $D = 10, N = 1$ theory is that the fermions states are Dirac particles instead of Majorana-Weyl. Even though the full spectrum of the theory contains both bosons and fermions of various spin, there is no space-time supersymmetry due to obvious lack of triality. The four coordinates of the 4 dimensional Euclidean space describes a $CP^2$ to define the confined gauge symmetry $SU(3)_C$. The bosonization of their supersymmetric partners yields additional two coordinates to describe a $S^2$ that defines the $SU(2)_L$ of the Standard Model. There should be three types of left-handed neutrinos. The right-handed neutrino becomes sterile. Spinors and gauge bosons derive their masses through coupling to
the tachyons present in the spectrum via SSB and strings become point-particles below the critical temperature.
Careful computation of the commutators of fermionic currents in a quantum field theory reveals that they do not always have the form anticipated from naive manipulations. Additional terms usually called the Schwinger terms (ST) are to be expected in all current algebras [1-4]. Though apparently incompatible results were reported sometimes, the existence of two solutions for the ST different in sign only, seems to be a distinct possibility [5-19].

These STs arise due to the short distance singularities of the current-current correlation functions and can be computed in many ways as discussed in the literature. The oldest among them is the canonical method. This is what we pursued in [20] and demonstrated explicitly with massless fermions in two dimensions that if the positive energy states of fermions were filled instead of the negative energy states, the Schwinger terms in the corresponding current algebras including the Virasoro algebra [21] would change sign. Because, the anomaly term involves only the odd powers of the Fourier indices (m) and replacing a vacant state with an occupied state, reverses the roles of the creation and annihilation operators of the fermions (m → −m). This corresponds to anti-normal ordering of the current-current commutators. Creation operators of the fermions are brought to the right of the annihilation operators in the anti-normal ordered expression for the current-current commutators. The infinite Fermi Sea where all positive energy states are filled up with exactly one fermion per level according to Pauli principle is a unique state too like the canonical vacuum and all states built from this would only contain de-excitations (obtained by action of positive fourier modes) of the fermion. The scenario is analogous to Dirac’s original hole theory [22], where the canonical vacuum was defined to be the one with all negative energy states filled up.

We consider open strings moving in a (3+1)+4 dimensional space. There are two \( N = 1 \) local supersymmetries. One acts on four world-sheet scalar fields and four world-sheet Majorana spinors that are coordinates and components of vectors in the (3+1) dimensional Minkowski spacetime. The other supersymmetry acts on another set of world-sheet scalar fields and Majorana spinors that are coordinates and components of vectors in the four dimensional Euclidean space. It is important to note that the signatures of the target spaces are also the one suggested by the norm of the ghost Hilbert space. This is the same criterion that in the other string theories dictates the number of time-like directions [23,24].

\[ 1 \]

Since the Fermi energy \( E_F = \infty \) for massless fermions, all positive energy states of fermions will be filled at absolute zero temperature.
world-sheet action is \[\text{(25)}\]

\[
S = -\frac{1}{2\pi} \int d^2 \sigma e^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\mu - i \bar{\psi}^{\alpha} \gamma^\mu \partial_\mu \psi_{\alpha}
\]

\[
+ 2 \chi^{1,2}_1 \gamma^{\alpha} \gamma^\mu \partial_\mu X^\alpha + \frac{1}{2} \bar{\psi}^{\alpha} \gamma^\mu \chi^1_1 \gamma^\alpha \chi^1_1
\]

\[
+ 2 \chi^{2,2}_2 \gamma^{\alpha} \gamma^\mu \partial_\mu X^A + \frac{1}{4} \bar{\psi}^A \gamma^\alpha \gamma^{\alpha} \chi^{2,2}
\]

Where \(\chi^1_1\) and \(\chi^2_2\) are the two world-sheet gravitinos corresponding to the two local supersymmetries:

\[
\delta X^\mu, A = \epsilon^{1,2} A^\mu, \delta \psi^{\mu, A} = -i \gamma^\alpha \epsilon^{1,2} \left( \partial_\alpha X^\mu - \bar{\psi}^{\mu} \chi^1_1 \right)
\]

\[
\delta e^a_\alpha = -2 i \epsilon^{1,2} A^a \chi^{1,2}_1, \delta \chi^{1,2}_2 = \nabla^a \epsilon^{1,2}
\]

Here \(\mu\) runs over the values \((0, 1, 2, 3)\) and \(A\) runs over the values \((1, 2, 3, 4)\). The local Weyl transformation that leaves the action invariant is

\[
\delta X^\mu, A = 0, \delta \psi^{\mu, A} = -\frac{1}{2} \Lambda \psi^{\mu, A}
\]

\[
\delta e^a_\alpha = \Lambda e^a_\alpha, \delta \chi^{1,2}_1 = \frac{1}{2} \Lambda \chi^{1,2}_1
\]

There is also another local fermionic symmetry given by

\[
\delta \chi^{1,2}_1 = i \gamma^\alpha \eta^{1,2}
\]

\[
\delta e^a_\alpha = \delta \psi^{\mu, A} = \delta X^\mu, A = 0
\]

There are altogether four local bosonic symmetries: two world-sheet reparametrizations, one local Lorentz and one local Weyl scaling. Locally they can be used to gauge the four components of the zweibein into the standard form \(e^a_\alpha = \delta^a_\alpha\) \((h_{++} = h_{--} = 0)\). Similarly, the four supersymmetries \((\epsilon^1, \epsilon^2)\) and the four superconformal symmetries \((\eta^1, \eta^2)\) can be used to set the eight components of \(\chi^1\) and \(\chi^2\) to zero. The Faddeev-Popov determinant for the residual reparametrization \((\sigma \rightarrow \sigma + \xi)\) symmetry:

\[
\delta h_{++} = \nabla_+ \xi_+ = 0 \text{ and } \delta h_{--} = \nabla_- \xi_- = 0 \text{ yields the pair of reparametrization ghosts and anti-ghosts } (c, b).
\]

Similarly the residual supersymmetries:

\[
\delta \chi^{1,2}_1 = \nabla_+ \epsilon^{1,2}_1 = 0 \text{ yield two pairs of superconformal ghosts and anti-ghosts } (\gamma^1, \beta^1) \text{ and } (\gamma^2, \beta^2).
\]

Thus, the gauge-fixed world-
sheet action becomes

$$ S = \frac{1}{\pi} \int d^2 \sigma \{ c^+ \partial_- b_+ + c^- \partial_+ b_- + \partial_+ X^\mu \partial_- X_\mu \} + \partial_- X^\mu \partial_+ X_\mu + \psi_+^\mu \partial_- \psi_+ + \psi_-^\mu \partial_+ \psi_- + \beta_+^1 3/2 \partial_- \gamma_+^{1/2} + \beta_-^1 3/2 \partial_+ \gamma_-^{1/2} \} + \{ \partial_+ X^A \partial_- X^A + \partial_- X^A \partial_+ X^A + \psi_+^A \partial_- \psi_+^A + \psi_-^A \partial_+ \psi_-^A + \beta_+^2 3/2 \partial_- \gamma_+^{2/12} + \beta_-^2 3/2 \partial_+ \gamma_-^{2/12} \} \} $$

If we anti-normal order the contributions of the $\psi$ to the Virasoro algebra and normal order the rest, the Virasoro anomaly of the full theory will cancel, because $2[4(1 - \frac{1}{8}) + 11] - 26 = 0$, where the central charge for the $\psi$ is $-\frac{1}{2}$ instead of $\frac{1}{2}$ in this case.

We write the Virasoro algebra for each component in the bosonic sector of the RNS Formulation as

$$ [L_0^+ A, L_{-m}^+ A] = 2m : L_0^+ A : + \frac{m^3 - m}{12} \cdot m $$

$$ [L_0^-, L_{-m}^-] = 2m : L_0^- : - \frac{2m^3 - 2m}{12} \cdot m $$

$$ [L_1^{1,2}, L_{-m}^{1,2}] = 2m : L_0^{1,2} : + \frac{11m^3 + m}{12} \cdot m $$

$$ [L_m^+ A, L_{-m}^- A] = 2m : L_0^+ A : - \frac{m^3 - m}{24} \cdot m $$

Where

$$ X^\mu = x^\mu + p^\mu \tau + \sum_{n \neq 0} \frac{1}{n} \alpha_+^\mu e^{-in \tau} \cos n \sigma \cdot n $$

$$ X^A = x^A + q^A \tau + \sum_{n \neq 0} \frac{1}{n} \alpha_+^A e^{-in \tau} \cos n \sigma \cdot n $$

$$ \psi_+ = \sum_{r=2}^{+\frac{1}{2}} b^\mu_r e^{-i(r \pm \sigma)} \cdot n $$

$$ \psi_- = \sum_{r=+\frac{1}{2}}^{-\infty} c_n e^{-2in(r \pm \sigma)} \cdot n $$

$$ b_{+-} = \sum_{n=-\infty}^{+\infty} b_{n} e^{-2in(r \pm \sigma)} \cdot n $$

$$ \gamma_{-} = \sum_{n=-\infty}^{+\infty} \gamma_n e^{-2in(r \pm \sigma)} \cdot n $$

Here :: is the symbol for anti-normal ordering.

One might wonder how the anomaly, which is basically a c-number central extension of an operator algebra, would have a different form with
what looks like a partly upside down situation of the original Fock space (i.e. only as far as the fermions are concerned). The reason behind such difference is the fact that the normalization coefficient of the Fermi Sea is ill-defined with respect to the canonical vacuum because of infinite number of fermions that defines the former. One should note here that the two $\mathcal{N} = 1$ world-sheet supersymmetries that are manifest in the action are not quite reflected in the usual fashion in the states. The bosonic state created by the creation operator of a boson field on the vacuum will transform into a “hole” state of the fermions obtained by the action of the destruction operator of the fermion on the Fermi vacuum. In that sense it can be termed as anti-supersymmetry.

Combining all the contributions of the individual components, we get

$$[L_m, L_{-m}] = 2m(\sum_{\alpha,0}^\alpha L_0^\alpha : + :: + : L_0^\alpha : + : L_0^\alpha : ) \quad (14)$$

in the bosonic sector. Here $L_0^\alpha = \sum_{\mu, A} L_0^{\mu, A}, L_0^b = \sum_{\mu, A} L_0^{b, A}, L_0^c = \sum_i L_0^c_i$.

Now

$$L_0^b =: L_0^b : - a_b = : L_0^b : + a_b \quad (15)$$

The normal-ordering constant $a_b$ is ill-defined before cancellation of anomaly, because the canonical commutation relations are themselves ill-defined. After cancellation of anomaly they can be defined without ambiguity and the normal ordering constant can be regularized with a $\zeta$-function [28] to yield

$$:: L_0^b :: = : L_0^b : - 2.2.4.\frac{1}{48} \quad (16)$$

We assume that a physical state is annihilated by all the modes of the anti-ghost $b$. It is a state that contains no ghosts, but is completely filled with anti-ghosts. Therefore, it can’t be annihilated by any modes of the ghost $c$. The BRST operator [29][31] can be written as

$$Q = c^i K_i - \frac{1}{2} f_{ij}^k c^i c^j b_k \quad (17)$$

If we assume that the covariant indices are positive integers and contravariant indices are negative integers, then the condition:

$$Q |\psi\rangle = 0 \quad (18)$$

yields

\[ K_i |\chi\rangle = L_m |\chi\rangle \]  
\[ = 0 \]  

We shall also assume that a physical state is annihilated by \( L_0 \) and not \( L_0^\dagger \). We bosonize a \((\beta, \gamma)\) system to get the action

\[ S = \int d^2\sqrt{h}(\frac{1}{2\pi} \partial^\alpha \varphi \partial_\alpha \varphi - i\frac{K_\varphi}{4\pi} R \varphi + \frac{1}{2\pi} \partial^\alpha \chi \partial_\alpha \chi - i\frac{K_\chi}{4\pi} R \chi) \]  

Where \( K_\varphi = 2 \) and \( K_\chi = -1 \). Let \( \varphi = i\varphi' \) and \( K_\varphi = -iK_\varphi' \). Hence

\[ S = \int d^2\sqrt{h}(\frac{1}{2\pi} \partial^\alpha \varphi' \partial_\alpha \varphi' - i\frac{K_\varphi'}{4\pi} R \varphi' + \frac{1}{2\pi} \partial^\alpha \chi \partial_\alpha \chi - i\frac{K_\chi}{4\pi} R \chi) \]  

We fermionize \( \varphi' \) and \( \chi \) to get two Dirac spinors, one satisfying periodic boundary conditions and the other satisfying anti-periodic boundary conditions to yield the correct value of the zero-point energy \([32-36]\).

We note that the Virasoro anomaly for the \((\varphi', \chi)\) system is \( 1 - 3K_\varphi'^2 + 1 - 3K_\chi^2 = 11 \) to check on consistency. Drawing an analogy with the \((b,c)\) system, we can then say that a physical state annihilated by \( L_0 \) should be filled completely with the resultant anti-fermions, but contains no fermions (absolute vacuum and not the Dirac Sea).

Therefore, we should write \( L_0^\gamma \) instead of \( L_0^\dagger \) in \([14]\), but

\[ L_0^\gamma = L_0^\dagger = L_0^\dagger + \frac{1}{12} \]  

Substituting \([16], [22] \) and \([23]\) in \([14]\) we get

\[ [L_m, L_{-m}] = 2m(L_0^\gamma : + : L_0^\dagger : + L_0^\dagger : + L_0^\gamma : - \frac{1}{2}) \]  

Where

\[ L_0 = L_0^\dagger : + : L_0^\dagger : + L_0^\gamma \]  

and \( H = L_0 - \frac{1}{2} \) is the effective Hamiltonian of the system.

In the fermionic sector \([27]\)

\[ \psi_\pm = \sum_{-\infty}^{\infty} d^\mu_\pm e^{-i\eta_x (\tau \pm \sigma)} \]  

So

\[ [L_m^{\mu, A}, L_{-m}^{\mu, A}] = 2m(L_0^{\mu, A} : + : L_0^{\mu, A} : + L_0 : + L_0^\gamma : - \frac{m^2 + 2m}{24}) \]  

and

\[ [L_m, L_{-m}] = 2m(L_0^\gamma : + : L_0^\gamma : + L_0^\gamma : + : L_0^\dagger : + L_0^\dagger : + : L_0^\dagger : : - \frac{1}{2}) \]
Now,

\[ \cd : L^d_0 : = : L^d_0 : + 2.4 \frac{1}{24} \]

\[ = : L^d_0 : + \frac{2}{3} \]

Substituting (22), (23) and (28) in (27) we get

\[ [L_{m}, L_{-m}] = 2mL_{0} \]  

(29)

Where

\[ L_{0} =: L^a_{0} : + : L^d_{0} : + L^c_{0} + L^\gamma_{0} \]

We assume that the modes of the superconformal ghosts are half-integral in both the bosonic and the fermionic sectors. Integral modes in the fermionic sector would have led to infinite degeneracy of the ground state due to the presence of the zero modes of the superconformal ghosts.

If we replace \( \sigma^a \) by \( -\sigma^a \) the canonical vacuum transforms into the Fermi Sea [20]. It has the occupation number \( \sum_{n=1}^{\infty} 1 = \zeta(0) = -\frac{1}{2} \) with respect to the canonical vacuum. So bosons become fermions and vice versa. Equations (24) and (29) indicate that our theory is very similar to the \( D = 10, N = 1 \) superstring theory in the light-cone gauge. It identifies \( \sigma \) with \( -\sigma \). So a previously un-oriented string becomes oriented.

Since ghosts do not contribute, we can’t discard the longitudinal and time-like excitations without altering the zero-point energy. Physical states are admixtures of both, but their total contribution to the norms vanishes [37, 38].

Let us concentrate on the bosonic sector first. Since the ghosts do not contribute, physical state conditions in this case can be written as

\[ (L_{m}^{1, \text{matter}} + L_{m}^{2, \text{matter}} - \frac{1}{2} \delta_{m}) |\phi\rangle = 0 \]  

(30)

for \( m \geq 0 \). We refer to \( (\psi^{\mu}, X^{\mu}) \) collectively as \( (1, \text{matter}) \) and \( (\psi^{A}, X^{A}) \) collectively as \( (2, \text{matter}) \). We set

\[ (L_{m}^{1, \text{matter}} - \frac{1}{2} \delta_{m}) |\phi\rangle = 0 \]  

(31)

to get massless vectors (gauge bosons) in the physical \((3 + 1)\) dimensional spacetime. Hence

\[ L_{m}^{2, \text{matter}} |\phi\rangle = 0 \]  

(32)

2World-sheet supersymmetry and residual reparametrization invariance are sufficient to gauge away the \( + \) components of all nonzero mode oscillators of \( X \) and allows to set \( \psi^{+} |\phi\rangle = 0 \), where \( |\phi\rangle \) is a physical state. So physical states will contain equal number of quanta (either 0 or 1) of longitudinal and time like excitations of \( \psi^{+} \) for a particular frequency.
From (32)
\[ L^{1,2,\text{matter}}_{2r} |\phi\rangle = (G^{1,2,\text{matter}}_{r})^2 |\phi\rangle = 0 \]
for \( r > 0 \). Therefore, we can safely write
\[ G^{1,2,\text{matter}}_{r} |\phi\rangle = 0 \]
(34)
Where \( G^{1,2,\text{matter}}_{r} \) are the half-integral modes of the supercurrents \( G^{1,2,\text{matter}} = \psi_{\mu}^A \partial_+ X_{\mu,A}^- \). From (32) and (31)
\[ L^{1,\text{matter}}_0 = \alpha' p^2 + N_1 \]
(35)
\[ = \frac{1}{2} \]
and
\[ L^{2,\text{matter}}_0 = \alpha' q^2 + N_2 \]
(36)
\[ = 0 \]
Where \( N_1 \) and \( N_2 \) are the number operators for the systems \( (\psi_{\mu}, X^\mu) \) and \( (\psi^A, X^A) \). Therefore,
\[ q^A = 0 \]
(37)
and
\[ N_2 = 0 \]
(38)
The vertex operator for emission of the ground state tachyon will be
\[ V_0(\tau) = e^{i\phi(\tau)} e^{ip X(\tau)} \]
(39)
Here \( p^2 = 1 \). For \( N_1 = \frac{1}{2} \), the first excited state will be a massless vector boson. The vertex operator for emission of this state will be
\[ V_1(\tau) = e^{i\phi(\tau)} \psi_\mu(\tau) \zeta^\mu e^{ip X(\tau)} \]
(40)
Here \( \phi \) belongs to the \( (\gamma^1, \beta^1) \) system. The string propagator \( (L_0^1 - \frac{1}{2} + L_0^2)^{-1} \) in the expression for tree amplitudes can be expanded in powers of \( \frac{L_0^2}{L_0^1} \). The \( L_0^2 \) can then be brought past the subsequent vertices and propagators until it annihilates against the physical state at the right end of the tree. Here \( L_0^2 \) includes contributions of \( (\psi^A, X^A) \) and the superconformal ghosts \( (\gamma^2, \beta^2) \) to \( L_0 \) and \( L_0^1 \) includes contributions of \( (\psi^\mu, X^\mu) \), the superconformal ghosts \( (\gamma^1, \beta^1) \) and the reparametrization ghosts \( (c,b) \) to \( L_0 \). Tree unitarity is ensured by decoupling of ghosts as usual [25].
In the fermionic sector

\[(L^{1,\text{matter}}_m + L^{2,\text{matter}}_m) |\phi\rangle = 0 \] (41)

for \(m \geq 0\). So, we can write

\[L^{1,\text{matter}}_m |\phi\rangle = 0 \] (42)

and

\[L^{2,\text{matter}}_m |\phi\rangle = 0 \] (43)

Therefore,

\[q^A = 0 \] (44)

and

\[N_2 = 0 \] (45)

again. In analogy with (34) we can write

\[F^{1,2,\text{matter}}_m |\phi\rangle = 0 \] (46)

Where \(F^{1,2,\text{matter}}_m\) are the integral modes of \(G^{1,2,\text{matter}}\) in the fermionic sector. For the ground state, the condition \(F^{1,\text{matter}}_0 |\phi\rangle = 0\) yields the Dirac equation describing a massless spinor. The vertex operator for emission of this state will essentially be

\[V_s(\tau) = e^{i\theta_1} \Theta^1_{0,1,\pm}(\tau) \Theta^{2,3,\pm}_{1,2,\pm}(\tau) \Theta^{3,4,\pm}_{3,1,\pm} e^{ip \cdot X(\tau)} \] (47)

The zero-mode condition \(F^{2,\text{matter}}_0 |\phi\rangle = 0\) is trivial in this case. Here \(\Theta^1_{0,1,\pm} = e^{\pm i \frac{\theta_1}{2}}\) are the spin operators for the pair \((\psi^{\mu=0}, \psi^{\mu=1})\), where \(\psi^{\mu=0} = i \psi^{\mu=1} = e^{\pm i \theta_1}\) and \(\Theta^{2,3,\pm}_{1,2,\pm} = e^{\pm i \frac{\theta_2}{2}}\) are the spin operators for the pairs \((\psi^{\mu=2}, \psi^{\mu=3})\). Similarly we can define \(\Theta^{2,3,\pm}_{3,1,\pm} = e^{\pm i \frac{\theta_3}{2}}\) and \(\Theta^{3,4,\pm}_{3,4,\pm} = e^{\pm i \frac{\theta_4}{2}}\) as the spin operators for the pair \((\psi^{A=1}, \psi^{A=2})\) and \((\psi^{A=3}, \psi^{A=4})\) [39][42]. Where \(\Theta\) is the product of all the four spin operators and is a spinor representation of \(SO(7,1)\). Since \(N_2 = 0\) the corresponding vector boson doesn’t exist. The product \(\Theta^1_{0,1,\pm} \Theta^{2,3,\pm}_{1,2,\pm}\) describes the fourfold degenerate massless Dirac spinor in the \((3+1)\) dimensional Minkowski space. We insert the picture-changing operators \(\Gamma_m = e^{-in\varphi}G\partial G...\partial^{n-1}G\) at suitable points on an open string tree to compensate the ghost-number anomaly.\[3\] The present description is called the \(F_2\) picture. We also insert the fermionic field \(e^{-i\chi}\) on the tree to take into consideration of the spin-statistics theorem. These operators have zero conformal dimensions.

\[3\]The ghost-number anomaly for the superconformal ghosts and anti-ghosts of the \(N = 2\) theory is twice that of the \(N = 1\) theory.
We write the in-states of a tachyon, a massless vector and a massless spinor as

\[ |0^p; p\rangle = \lim_{\tau \to \infty} e^{-iH\tau} V_0(\tau) |0\rangle \]

\[ = \lim_{\tau \to \infty} e^{-i(L_0 - \frac{1}{2})\tau} V_0(\tau) |0\rangle \]

\[ = \lim_{\tau \to \infty} V_0(\tau) |0\rangle \]

\[ = e^{i\varphi_0} e^{ip.x} |0\rangle \]

\[ |0^v; v\rangle = \lim_{\tau \to \infty} e^{i\varphi_0} e^{i\chi_0} e^{i\varphi_2} e^{ip.x} |0\rangle \]

\[ |0^s; s\rangle = \lim_{\tau \to \infty} V_0(\tau) |0\rangle \]

\[ = e^{i\varphi_0} e^{i\chi_0} e^{i\varphi_2} e^{ip.x} |0\rangle \]

We set \( p_\varphi |0\rangle = 0 \) in the above equations.

We begin with the vertices for three on-shell particles. For tachyons we simply have

\[ g \langle 0; p_1 | e^{-i\chi_0} V_0(0, 2) |0^p; p_2 \rangle \]

\[ = g \langle 0^p; p_1 | e^{i\varphi_0} e^{i\chi_0} e^{i\varphi_2} e^{ip.x} |0; p_3 \rangle \]

We henceforth drop the superscripts \('' \) and \('' \) from the vacuum states for simplicity and write: \( e^{i\varphi_0} \approx e^{i\chi_0} \approx 1 \). The ghost number anomaly will cancel, if we replace \( \varphi_2 \) and \( \chi_2 \) by \(-\varphi_2\) and \(-\chi_2\) in the bosonized action \((20)\) for the second set of superconformal ghosts and anti-ghosts. At first sight, it is rather peculiar to find two solutions for the ghost number anomaly, but on reflection, this may not be so surprising. The choice between different solutions for the ghost number anomaly is analogous to the choice between different pictures of the RNS model \([25]\).

Tachyons can couple to other states also. For two tachyons and one
massless vector we have

$$ g \langle 0; p_1 | \Gamma_1(0) V_v(0, 2) | 0; p_3 \rangle = g \zeta^{(2)} \cdot p_3 $$  (52)

For the Yukawa interaction

$$ g \langle 0; s; p_1 | e^{-i\chi_0} V_0(0, 2) | 0; s; p_3 \rangle = g $$  (53)

For two massless spinors and one massless vector we have

$$ g \langle 0; s_1; p_1 | e^{-i\chi_0} V_v(0, 2) | 0; s_2; p_3 \rangle = g \langle s_1; p_1 | \gamma \cdot \zeta^{(2)} | s_2; p_3 \rangle $$  (54)

The tree level amplitude for Compton scattering \[43\] after \( SL(2, R) \) gauge fixing can be written as

$$ A = g^2 \langle s_1; p_1 | e^{-i\chi_0} V_v(0, 2) \frac{1}{F_0} \Gamma_1(0) V_v(0, 3) | s_2; p_4 \rangle $$  (55)

$$ = g^2 \langle s_1; p_1 | e^{-i\chi_0} V_v(0, 2) \frac{p_1}{F_0} V_v(0, 3) | s_2; p_4 \rangle $$

$$ = g^2 \langle s_1; p_1 | \psi(0) \zeta^{(2)} e^{ip_2 \cdot X^{(0)}} \frac{1}{F_0} \psi(0) \zeta^{(3)} e^{ip_3 \cdot X^{(0)}} | s_2; p_4 \rangle $$

$$ = g^2 \langle s_1; p_1 | \gamma \cdot \zeta^{(2)} \frac{1}{(y_5 + p_1)} \gamma \cdot \zeta^{(3)} | s_2; p_4 \rangle + O(\alpha') $$

Since Dirac spinors appear in the spectrum, the strings are oriented and the gauge groups should be unitary. We assume that \( X^A \) describes a \( CP^2 \) to define the gauge symmetry \( SU(3)_C \) over it \[44\]. Equation \[44\] then states that physical states are color singlet. The four \( \psi^A \)'s can be bosonized for the two coordinates \( X^A' \) on a \( S^2 \) to define the gauge symmetry \( SU(2)_L \) on it. To complete the Standard Model \[45\], an extra coordinate compactified to a circle will be needed for the definition of the \( U(1)_{Y_W} \) gauge symmetry \[46, 47\]. This coordinate merely add weak hypercharge \( Y_W \) but no new dynamics to the string.

\[4\] In Kaluza-Klein theory, the Dirac action for fermions is written as

$$ S_D = \int d^2x \bar{\Psi} \gamma^a f^a_\mu \gamma^\mu \Psi $$

$$ = \int d^2x (2\pi R) \bar{\Psi} \gamma^m (p_m - p_4 f^4_m) \Psi $$

$$ = \int d^2x (2\pi R) \bar{\Psi} \gamma^m (p_m - \frac{1}{R} A_m) \Psi $$

Where \( f^a_m \) is the inverse vielbein. Here \( \alpha \) and \( a \) run from 0 to 4 and \( m \) runs from 0 to 3. Therefore, \( g = 2\pi R. \frac{1}{\pi} = 2\pi \).
The $Y$ coordinate of the string corresponding to the $U(1)$ symmetry should be written as $Y = y + Y_W \tau$ without any oscillator terms. Since $:L_0 := \frac{1}{2}$ in the Fermi sector,

$$\alpha' (p^2 + q^2 + Y_W^2) = \frac{1}{2}$$

(56)

for the Fermi vacuum. This gives

$$Y_W^2 = 1$$

(57)

The same is true for the ground state boson also. We can verify it by choosing the $F_1$ picture, because $:L_0 := 1$ in this case.

However, these solutions suffer from the difficulty that $CP^2$ does not admit spinors. One-Way round the problem is to put a spin$^c$ structure on $CP^2$ [48–50]. Another solution to (30) is

$$L_{0,\text{matter}} = 0$$

(58)

$$L_{0,\text{matter}}^2 = \alpha' q^2 + N_2$$

(59)

Hence if $q^2 = 0$, $N_2 = \frac{1}{2}$. The gauge bosons required to define spin$^c$ structure on $CP^2$ can thus be generated by the action of $b_{-1}^1$ on the canonical vacuum. If the internal manifold has the topology $CP^2 \otimes M$, the chirality content of the total particle spectrum could well be different from that of the spinors on the $CP^2$ only [51]. $CP^2 \otimes S^2$ is a six dimensional internal manifold with Euler characteristic

$$\chi = 3 \times 2$$

(60)

$$= 6$$

Hence, the number of families will be $\frac{5}{2} = 3$ [52,53]. There will be one left-handed neutrino per generation. The right-handed neutrino becomes sterile [54]. The four states corresponding to $\Theta_{1,2,\pm}^2 \otimes \Theta_{3,4,\pm}^2$ in (47) are the four states of a left-handed Weyl spinor of the six dimensional internal manifold and should be identified as the two isospinors of the Standard

---

$^5$Betti Hodge numbers for $S^2$ or $CP^1$ are given by the matrix

$$
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
$$

and those for $CP^2$ are given by the matrix

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

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model and not the gauginos. Since one isospinor represents a particle and 
the other isospinor represents the corresponding antiparticle, they should 
have opposite charges $Y_W = \mp 1$. Therefore, the isospinors should involve 
baryons and not quarks. They are
\[
\begin{pmatrix}
\nu \\
e \\
p \\
n
\end{pmatrix}
\] (61)
for the first generation. It reaffirms the principle of color confinement.

Open strings with matching Chan-Paton factors [55] can join ends to 
form closed strings. Since Chan-Paton factors of bosons and fermions 
in the open string sector don’t match, gravitinos and for that matter 
the entire $NS - R$ sector of the close string should be absent from the 
spectrum.

If $\psi^A$ is bosonic instead of fermionic in the fermionic sector
\[
L_{0,\text{matter}}^2 |\phi\rangle = \alpha' q^2 + N_2
\]
\[
= \frac{1}{4}
\] (62)
Therefore, $N_2 = 0$ and $q^2 = \frac{1}{4}$. Now two of the four $X^A$s should be able 
to be fermionized with $q = \pm \frac{1}{2}$ to get four $\psi$s satisfying periodic boundary 
conditions. This together with the two remaining $X^A$s and the two $X'^{A'}$s 
with integral momenta yield the same spectrum.

In the low energy limit, the string theory effectively reduces to a point 
particle field theory [56]. It should be the Standard Model in our case.
Below a critical temperature, self-interaction of tachyons lowers the en-
ergy of the actual vacuum. If the universe is supercooled, it stays in a false 
vacuum at $\phi_H = 0$ before rolling down to the stable vacuum that is asym-
metric under the $SU(2)$ of the Standard Model (Spontaneous Symmetry 
Breaking) [57–61]. Here $\phi_H$ is the tachyon field.

Therefore, for $CP^2 \otimes CP^1$ they are given by the matrix
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
This after proper addition of cells becomes
\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
1 & 0 & 0 & 1
\end{pmatrix}
\]
It is the Hodge diamond of a Calabi-Yau Manifold. So there will be three families.
To study the entire process in the context of string theory, we consider
the action for the nonlinear sigma model that we regularize by dimensional
regularization to yield

\[
S = -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma \sqrt{\eta^{\alpha\beta} \partial_{\alpha}X \partial_{\beta}X} (63)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma e^{\phi} \partial X \partial X (64)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma e^{\phi} g_{\mu\nu} \partial X^\mu \partial X^\nu (65)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma e^{\phi} (\eta_{\mu\nu} - R_{\mu\rho\nu\sigma} x^\rho x^\sigma) \partial X^\mu \partial X^\nu (66)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma \left[\partial X \partial X - \frac{1}{2\epsilon} \alpha' R_{\mu\nu}\right] (1 + \epsilon\phi) \partial X^\mu \partial X^\nu (67)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma \left[\partial X \partial X - \frac{1}{2\epsilon} \alpha' \lambda \eta_{\mu\nu}\right] (1 + \epsilon\phi) \partial X^\mu \partial X^\nu (68)
\]

\[
= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+\epsilon)} \sigma \left[(1 - \frac{\alpha' \lambda}{2\epsilon}) \partial X \partial X - \frac{\alpha' \lambda}{2\epsilon} \phi \partial X \partial X\right] (69)
\]

Reparametrization invariance of the world-sheet action will allow us to
set \(h^{\alpha\beta} = \eta^{\alpha\beta} e^\phi\) in (64). We used Riemann normal coordinates to write
\(g_{\mu\nu} = g_{\mu\nu}(X) = \eta_{\mu\nu} - R_{\mu\rho\nu\sigma}(X_0)x^\rho x^\sigma\) in (66) \((X = X_0 + x\) are locally in-
tertial coordinates at \(X_0)\) and put the logarithmically divergent contraction
\(\lim_{\sigma \to \sigma'} < x^\rho(\sigma)x^\sigma(\sigma') >= \frac{\alpha' \eta^{\rho\sigma}}{\epsilon} \) in (66). We used the lambda-vacuum
solution to the Einstein field equation [64] to write \(R_{\mu\nu} = \lambda \eta_{\mu\nu}\) in (65),
where \(\lambda\) is proportional to the vacuum energy (latent heat of the super-
cooled state) per unit volume.

Since the conformal invariance of the world-sheet action [69] is lost,
we can only define the subalgebra \(OSp(1|2)\) of the super-Virasoro algebra
consistently. So physical states will have spin \(\leq 1\) in the matter and
gauge sectors. The size of the strings will be extremely small below the
critical temperature, because the effective Regge slope \(\alpha'' \equiv \lim_{\epsilon \to 0} \frac{\alpha' \lambda}{2\epsilon} \phi \partial X \partial X\)
vanishes in the limit \(\epsilon \to 0\) for finite \(\lambda\).\(^6\)

\(^6\)From the vacuum energy density, we can estimate the scale of CP violation in particle
physics (see A. D. Sakharov 1967, JETP Lett. 5, p. 24). As the latent heat of the false vacuum
is liberated, the zero level of energy sinks and innumerable fermions (and not antifermions)
come out of the Dirac Sea. The process is very similar to Hawking radiation (see S. W.
Hawking 1974, Nature 248, p. 30). It results in a huge matter-antimatter asymmetry. The
number density of fermions thus created is \(\approx (10^{16})^3 \text{cm}^{-3}\) on the electroweak scale. If the
cosmic scale factor increase by a factor \(10^{18}\) since the electroweak symmetry breaking, the
current density should be \(\approx 10^{-6}\text{cm}^{-3}\).
The $\phi$ dependent term in (69) will change the zero-point energy by an amount proportional to $\lambda\phi_0$. Therefore, the $U(1)$ charge will change.

The sterile right-handed neutrino can only interact via gravity and derives a Majorana mass \[65\] from a coupling to the ground state tachyon in the gravity sector through the Spontaneous Symmetry Breaking.

We can obtain the quartic Higgs self-coupling $\lambda_H$ from the low energy limit $t \to 0$ of the four-tachyon amplitude. It is the same as the Veneziano amplitude \[66\]. (We can prove it quite easily by going into the $F_1$ picture again.) Therefore, $\lambda_H \approx 1$. The vacuum expectation value of the tachyon field

$$< \phi_H > = \sqrt{\frac{\mu^2}{\lambda_H}} \approx 1$$

where $\mu$ is the mass of the tachyon. Hence the Dirac mass of a lepton or a baryon derived through the Yukawa coupling $\bar{\psi}\psi\phi_H$ is also of the order of unity. The Majorana mass of a right-handed neutrino derived through its coupling $[\bar{\nu}_R\nu_R^c + \bar{\nu}_R^c(\nu_R)^c]\phi_G$ to the tachyon field $\phi_G$ in the gravity sector should be very large. Because the quartic coupling constant obtained from the low energy limit $t \to 0$ of the four-tachyon amplitude $A_4^c \approx \sin\frac{\pi t}{8}$ in the closed string sector \[67\] should be vanishingly small.

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[1] J. Schwinger, Phys. Rev. Lett. 3(1959)296
[2] T. Goto and T. Imamura, Prog. Theor. Phys. 14(1955)396
[3] R. Jackiw, "Current Algebras and Anomalies", eds. S. B. Treiman (Princeton Univ. Press, Princeton, 1985)
[4] P. Jordan, Z. Phys. 93(1935)464
[5] D.C. Mattis, E.H. Lieb, J. Math. Phys. 6(1965)304
[6] S. Jo, Nucl. Phys. B259(1985)616 ; Phys. Lett. B163(1985)353
[7] L.D. Faddeev, Phys. Lett. B145(1984)81
[8] A.J. Niemi and G.W. Semenoff, Phys. Rev. Lett. 55(1985) 927; (1985) 2627; 56(1986)1019
[9] M. Kobayashi, K. Seo and A. Sugamoto, Nucl. Phys. B273(1986)607
[10] H. Sonoda, Nucl. Phys. B266(1986)410; Phys. Lett. B156(1986)220
[11] L.D. Faddeev and S.L. Shatashvili, Phys. Lett. B167(1986)225
[12] S. Hosono and K. Seo, Phys. Rev. D38(1988)1296
[13] M. Stone and W. Goff, Nucl. Phys. B295[FS21](1988)243
[14] I. Tsutsui, Phys. Lett. B229(1989)51
[15] G. Kelhofer, J. Math. Phys. 34(1993)3901
[16] C. Adam, C. Ekstrand and T. Sykora, Phys. Rev. D62(105033), 15 (2000) arXiv:hep-th/0005019v2 14 May 2000; C. Ekstrand, J. Math. Phys. 41(2000)7294; Ph.D. thesis, Royal Institute of Technology, Stockholm, April 1999
[17] K. Isler, C. Schmid and C.A. Trugenberger, Nucl. Phys. B301(1988)327
[18] A. A. Vladimirov, J. Phys. A23(1990)87
[19] C. Adam, Annals Phys. 265(1998)198
[20] J.S. Bhattacharyya, arXiv:1302.7080[hep-th]
[21] M.A. Virasoro, Phys. Rev. D1(1970)2933
[22] P.A.M. Dirac, Principles of Quantum Mechanics, Cambridge University press, 1935
[23] I.B. Frenkel, H. Garland, G.J. Zuckerman, Proc. Natl. Acad. Sci. 83(1986)8442
[24] H. Ooguri, C. Vafa, Nucl. Phys. B361(1991)469
[25] M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Volume 1, Cambridge University Press 1987
[26] A. Neveu and J.H. Schwarz, Nucl. Phys. B31(1971)86
[27] P. Ramond, Phys. Rev. D3(1971)2415
[28] L. Vanzo, S. Zerbini (Trento U.), Phys. Lett. B214(1988)51
[29] C. Becchi, A.Rouet and R.Stora, Phys. Lett. B52(1974)344
[30] C. Becchi, A.Rouet and R.Stora, Ann. Phys. 98(1976)287
[31] M. Kato and K.Ogawa, Nucl. Phys. B212(1983)443
[32] S. Coleman, Phys. Rev. D11(1975)2088
[33] A. Luther, Phys. Rev. B12(1975)3908
[34] S. Mandelstam, Phys. Rev. D11(1975)3026
[35] T.H.R. Skyrme, Proc. Roy. Soc. A262(1961)237
[36] R.F. Streater and I.F. Wilde, Nucl. Phys. B24(1970)561
[37] S. Gupta, Proc. Phys. Soc., 63A(1950)681
[38] K. Bleuler, "Eine neue Methode zur Behandlung der longitudinalen und skalaren Photonen", Helv. Phys. Acta 23(1950)567
[39] D. Friedan, S. Shenker, E. Martinec, Phys. Lett. B160(1985)55
[40] D. Friedan, S. Shenker, E. Martinec, Nucl. Phys. B271(1986)93
[41] V.G. Knizhnik, Phys. Lett. B160(1985)403
[42] D. Polyakov, Nucl. Phys. B449 (1995)159
[43] A.H. Compton, Phys. Rev. 21(1923)483
[44] G. Khanna, S. Mukhopadhyay, R. Simon, and N. Mukunda, Ann. Phys.253(1997)55
[45] R. Oerter (2006). The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics (Kindle ed.). Penguin Group. p. 2
[46] Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, Math: Phys. K1(1921)966
[47] O. Klein, Z. Phys., 37(1926)89
[48] S. W. Hawking and C. N. Pope, Phys. Lett. B73(1978)42
[49] T. Eguchi, P. B. Gilkey and A. J. Hanson, Phys. Rep. 66(1980)213
[50] A. Trautman, Intern. J. of theor. Phys. 16(1977)561
[51] B. Dolan, Annales de l'I.H.P. Physique théorique, Volume 42 (1985) no. 4, p. 375-382
[52] M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Volume 2, Cambridge University Press 1987
[53] S.P. Misra, Introduction to Supersymmetry and Supergravity, SERC School Series, Department of Science and Technology, Wiley Eastern Limited 1992
[54] M. Drewes, Int. J. Mod. Phys. E, 22, 1330019 (2013)
[55] H. M. Chan, J. E. Paton, Nucl. Phys. B10, 516 (1969)
[56] J. Scherk, Nucl. Phys. B31(1971)222
[57] D.A. Kirzhnits and A.D. Linde, Phys. Lett. B42(1972)471
[58] J. Goldstone, Nuovo Cemento 19(1961)154
[59] P. W. Higgs, Physics. Lett. 12(1964)132
[60] S. Weinberg, Phys. Rev. Lett. 19(1967)1264
[61] A. Salam, Weak and electro magnetic interactions. In N. Svartholm, ed., Elementary Particle Physics, p. 367 (Stockholm: Almquist and Wiksells, 1968)
[62] A.M. Polyakov, Phys. Lett. B103(1981)207
[63] C. Lovelace, Phys. Lett. B135(1984)75
[64] A. Einstein, Sitz. Preuss. Akad. d. Wiss. Phys.-Math 142(1917)

[65] T.P. Cheng, L.F. Li, Gauge Theory of Elementary Particle Physics, Oxford University Press 1983

[66] G. Veneziano, Nuovo Cimento A. 57 (1968)190

[67] M.A, Virasoro, Phys. Rev. 177(1969)2309