Optimal Aero-Structural Design of an Adaptive Surface for Boundary Layer Motivation Using an Auxetic Lattice Skin

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Abstract
The aero-structural design of an adaptive vortex generator for repeatable, elastic, deployment and retraction from an aerodynamically clean surface is presented. A multidisciplinary objective function, containing geometrically nonlinear finite element analysis and large eddy simulation, is used to derive the optimal adaptive geometry for increasing the momentum of the near wall fluid. It is found that the rapid increase of in-plane membrane stress with deflection is a significant limitation on achievable deformation of a continuous skin with uniform section. Use of a 2D auxetic lattice structure in place of the continuous skin allows significantly larger deformations and thus a significant improvement in performance. The optimal deformed geometry is replicated statically and the effect on the boundary layer is validated in a wind tunnel experiment. The lattice structure is then manufactured and actuated. The deformed geometry is shown to compare well with the FEA predictions. The surface is re-examined post actuation and shown to return to the initial position, demonstrating the deformation is elastic and hence repeatable.

Keywords
Adaptive Systems, Design Optimisation, Morphing Structures

Introduction
This paper presents the optimal design of an adaptive vortex generator for delaying the separation of a fluid boundary layer from an aerodynamic surface. Boundary layer separation is a significant limitation in the peak performance of high-lift aerofoil configurations such as those used for take-off and landing. Due to the impact of separation on aircraft safety and performance, a significant body of research exists on the application of static structures termed vortex generators on wings for this purpose. However, as will be described, the application of static vortex generators is accompanied by a loss of system efficiency at the off-design conditions which account for the majority of operational use (Ashill et al. 2001). This variation in requirement is typical of systems which benefit from an adaptive solution.

For the majority of the upper surface of an aerofoil, the fluid passing over the surface is experiencing an adverse pressure gradient. This acts to retard the flow next to the surface until the streamwise velocity gradient perpendicular to the surface reaches a local minimum. At this point the flow streamlines, which had been following the profile of the aerofoil, depart the surface and separation occurs. This is associated with a loss of lift and a rapid increase in drag.

The most generally used method to prevent flow separation is the static vortex generator (VG). A multitude of designs and layouts of VG exist in literature. Common variants of VG are illustrated in Fig. 1 and include: inclined vanes (Taylor and Hoadley 1948); wedges (Kuethe 1971); and wishbones (Wheeler 1991). Although the topology and positioning of these devices change, the underlying concept remains the same.

The application of VG in a boundary layer introduces a three-dimensional variation in the wetted geometry and so deflects the fluid from its original path. This produces a spanwise variation in pressure which, in conjunction with the viscous effects near the wall, results in a quantity of streamwise vorticity

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within the boundary layer (Lighthill 1963). The trailing edge of these devices is typically sharp, resulting in a region of high pressure gradient which causes the boundary layer to separate. The streamwise vorticity produced by the VG is then shed into the flow where it forms a bound vortex as shown in Fig. 1a. The induced flow perpendicular to the wall provides a momentum transport mechanism through the boundary layer. This accelerates the flow near to the wing surface and counteracts the effect of the adverse pressure gradient, delaying separation at the expense of an increase in skin friction drag.

As VG are traditionally static, this mixing effect occurs at all flight conditions and so during cruise, when the flow is highly resistant to separation, the increase in near wall momentum serves only to decrease system efficiency. Mitigation of the adverse off-design effects of VG have been investigated by: using low profile VG to contain the mixing within the boundary layer (Kuethe 1971); mechanised deployment of VG through breaks in the aircraft skin (Joubert et al. 2013); and using synthetic air jets to provide the mixing and so having no physical device in the boundary layer (Glezer and Amitay 2002). While these devices are effective, they still require gaps or breaks in the aircraft skin which are sources of parasitic drag.

An important challenge in commercial aircraft design is to increase the efficiency over the flight envelope. Of particular interest is the production of wings which maintain laminar airflow over a large proportion of the aerodynamic surface. Such aircraft would still require the critical stall prevention and performance modification that is currently provided by vortex generators. However, the presence of static mixing devices or breaks in the aerodynamic surface through which dynamic devices could be deployed, introduces a disturbance in the fluid boundary which promotes the onset of turbulent transition (Duncan et al. 2013). Boundary layer transition is an effective method of increasing the natural mixing of the boundary layer momentum and so preventing flow separation of an initially laminar boundary layer. The use of discreet roughness elements, very small excrescences from the wetted surface, have been shown to be effective at promoting transition and so reducing laminar separation (Simeus and Gunog 2014). However, as static features, the effect again remains regardless of the current need for flow control. The development of an adaptive surface which produces a similar mixing effect but which can be deployed and retracted from an aerodynamically clean surface as required, and thus only influencing the flow when and where necessary, is therefore of particular interest.

Under traditional rigid body design, the geometry of a wing is a compromise between the highly efficient thin, low camber, aerofoil optimal for cruise conditions and the high lifting performance of a thick, high camber, aerofoil necessary for take-off and landing. Typically the adaptation between these two conditions is achieved through the use of mechanisms to deploy discrete, geometry-altering, components such as flaps and slats from the wing (Torenbeek 1979). These mechanisms allow the aircraft to deliver the desired performance at multiple points in the flight envelope from the same airframe.

The use of adaptive structures on aircraft aims to achieve a greater degree of flexibility by providing a continuous, rather than discretised, variation in geometry. This allows the aircraft to achieve a better solution to the current set of flight conditions than is possible with discrete components (Weisshaar 2006). Additionally, in an adaptive design, the underlying structure provides the required geometry variation. This means that the breaks, gaps and cavities in the aircraft skin which are necessary in a mechanical solution are no longer needed.

A large body of research has been conducted on the production of morphing wings with precise applications determining the specific design requirements. A comprehensive review of morphing designs and actuation methods on aircraft has been produced by Barbarino et al. (2011) and for unmanned aerial vehicles by Gomez and Garcia (2011). Of these adaptive structures the majority deal with large-scale ‘global’ alterations to modify the wing performance by varying the whole wing geometry. Variable span wings have been developed which can be expanded or retracted to provide both optimal adjustment to
system performance and roll authority (Ajaj et al. 2013). Structures with variable torsional stiffness have been used as wing supports which, when coupled with aerodynamic loads, provide a degree of twist control (Griffin and Hopkins 1997) and Chen et al. (2000)).

The most prevalent application of morphing on aircraft is that of aerofoil profile variation. These methods can be broadly split into two groups: those which apply load directly to the aerofoil skin, and those which use topologically designed wing ribs to distribute an actuation load to the variable region. Applications where the actuation is applied directly to the skin include transonic shock control bumps (Jinks et al. 2016) and stall prevention through an oscillating suction surface (Jones et al. 2015). In these examples the variation in geometry is localised to the upper surface of the aerofoil, an area of low curvature, and the overall variation in geometry is small. For larger variations, the alteration of a section of the profile’s rib support allows a much larger change in geometry. Examples of such adaptive wing profiles include a variable geometry trailing edge flap (Kota et al. 2003), leading edge camber alteration (Santer and Pellegrino 2009) and whole aerofoil variation (Woods and Friswell 2012).

Not all flow control devices can be achieved with a global surface change and devices such as shock control bumps, winglets, and vortex generators require a local geometry change which is finite in all directions. Examples of three-dimensional geometry change are rare, with the notable exceptions of Arrieta et al. (2013) and Rhodes (2012).

In their work on adaptive winglets, Arrieta and co-workers produced a bistable wingtip structure which could be selectively actuated between two configurations: that of a thin wing section, and a winglet for reducing induced drag. While this produces a large 3D variation in geometry, it requires the device to be unrestrained on three sides, which is not suitable for local actuation from a wing surface. In his work on adaptive shock control bumps Rhodes (2012) produced a demonstration model of a three dimensional adaptive approximation based on experimentally-derived geometry. In this work, Rhodes used an optimisation algorithm to determine the actuation and boundary parameters to deform the geometry from a 0.4 mm thick aluminium skin using a finite element model and then produced an experimental demonstration. The finite element model closely matched the final morph but was only able to achieve $\approx 40\%$ of the peak displacement levels in the target geometry. This represents the current state of the art and underlines the challenges faced in this field.

In this paper the development of a compliant surface which can be repeatedly actuated to increase the resistance to separation of a turbulent boundary layer is presented. Initially, the aero-structural design optimisation process is described and then applied to a continuous surface of uniform section. A rapid increase of membrane stress is exhibited by this surface when producing the required geometry change which has non-zero curvature in both principle directions. This increase, coupled with the requirement for elastic deformation found with any repeatable morphing solution, severely constrains the design space and so limits the efficacy of the solution. In order to overcome this limitation a novel application of a 2D lattice is proposed to replace the uniform surface. The optimisation process is applied to the lattice structure and the resulting deployable compliant geometry is presented. The static deformed geometry is produced and tested experimentally to validate the computational fluid simulations and the lattice structure is produced to verify the finite element simulations.

### Aero-Structural Design Methodology

The design of any adaptive structure is a complex problem with many design variables. In the current work this is compounded by the highly nonlinear response of a boundary layer to geometry change and so a computationally expensive objective function must be used to quantify fitness (Garland et al. 2015). It is therefore infeasible to conduct a systematic search of the entire design space and instead, the optimisation process described in Fig. 2 is used.

In this process, the design parameters, which define the surface and actuation magnitude, are selected using a constrained gradient-based optimisation algorithm. From an initial, user-selected, point in the design space an estimate of the local Hessian matrix is made using a quasi-Newton method. Once the local gradient of the objective function is known, a sequential quadratic programming SQP method is used to identify the next feasible point in the design space. In the current work, this is implemented in the function fmincon from the MATLAB Optimisation toolbox (Mathworks Inc. 2015).

The deformed surface is then computed and the resulting geometry is used as the boundary for a computational fluid simulation. The flow field resulting from this analysis is reduced to a scalar quantity which may be used to describe separation resistance and on which the optimisation algorithm acts. Each step in this process is presented in further detail in the following sections.
Figure 2. Flowchart of design optimisation process

Figure 3. Spatial design parametrisation for an adaptive surface. The adaptive domain is defined by vertices 1 and 2 within the exterior domain (—) and is actuated at vertex A.

Structural Analysis

In order to limit the problem complexity, the structural domain is prescribed as a quadrilateral of variable size but which is symmetrical around the x-axis as shown in Fig. 3. The adaptive domain is therefore described by four design variables \((x_1,y_1)\) and \((x_2,y_2)\). These vertices are varied during the optimisation within a larger domain, shown with dashed lines, which is representative of the maximum size of physical model which can be produced.

To calculate the deformed geometry, the commercial finite element package Abaqus version 6.13-2 (Dassault Systemes 2013) is used. The deforming domain is discretised using a mesh of quadrilateral shell elements with 6 degrees of freedom at each node and linear interpolation of variables (S4). Due to the high levels of in-plane strain reported by Rhodes (2012) in his work, all analyses conducted account for geometrical nonlinearity. The coupled strain energy equations are solved using a Newton method and convergence is taken when residuals of all variables are below \(1 \times 10^{-4}\).

The edges of the model are restrained in all degrees of freedom, maintaining continuity in both position and surface gradient with the surrounding material, as required for a compliant structure. The mesh is actuated out-of-plane through a displacement boundary condition acting perpendicular to the surface which is imposed at a chosen actuation location, \((x_A)\) and height \((y_A)\).

Aerodynamic Analysis

Once computed, the effect of the resulting geometry on the boundary layer is required. To accomplish this, the deformation field is incorporated into the base of structured hexahedral mesh for use in the open-source CFD toolbox OpenFOAM. To map between the nodes of the finite element solution and the required regular Cartesian spacing of the CFD mesh, a Delaunay triangulation is used to linearly interpolate the data. The fluid domain in this work represents turbulent flow over a plate and is described by the boundaries

\[
0 \leq x \leq 0.22 \quad \text{(streamwise)}
\]
\[
0 \leq y \leq 0.025 \quad \text{(surface normal)}
\]
\[
-0.05 \leq z \leq 0.05 \quad \text{(spanwise)}
\]

The mesh has \(220 \times 34 \times 280\) \((x,y,z)\) grid points with the nodes in \(x\) and \(z\) spaced evenly across the domain. The first \(y\) coordinate for each point in the \(x-z\) grid is located according to the interpolation above and a constant expansion ratio is applied to the \(y\) coordinates. The first wall-normal grid point is located with non-dimensional wall distance \(y^+ < 1\) across the domain ensuring sufficient resolution to capture the boundary layer. The non-dimensional streamwise and spanwise grid spacings are \(\Delta x^+ \approx 100\) and \(\Delta z^+ \approx 35\) respectively, in line with accepted empirical requirements (Tucker 2014).

In order to capture the behaviour of the separating turbulent boundary layer with sufficient accuracy, a Large Eddy Simulation LES has been selected.
provides a necessary compromise between resolving all of the turbulent structures, and their influence in the boundary layer, generated by the modified geometry and the limitation in both mesh density and simulation run-time due to the available computational capability.

The fluid simulation consists of two steps, initially a Reynolds Averaged Navier-Stokes (RANS) solution is found using the Spalart-Allmaras turbulence model (Spalart and Allmaras 1992). This simulation is used to produce an approximate result which in turn is used to initialise a Large Eddy Simulation (LES) using the Spalart-Allmaras sub-grid model. The simulation is continued until the boundary layer metric, described below, converges to a mean determined using a rolling average of 500 samples. A typical convergence plot for the objective function is given in Fig. 4a.

In order to improve the efficacy of the adaptive structure using optimisation it is necessary to choose a suitable metric to reduce the three dimensional data from the LES to a scalar value. The boundary layer shape factor $H$ is the non-dimensional ratio between two measures of boundary layer thickness, displacement thickness $\delta^*$ and momentum thickness $\theta$, and is used to identify the distribution of momentum through the boundary layer. These values are defined as

$$\delta^* = \int_0^\delta \left( 1 - \frac{u(y)}{U_e} \right) dy$$

$$\theta = \int_0^\delta \frac{u(y)}{U_e} \left( 1 - \frac{u(y)}{U_e} \right) dy$$

$$H = \frac{\delta^*}{\theta}$$

where $u(y)$ is the streamwise velocity and $U_e$ is the velocity at the edge of the boundary layer at distance $y = \delta$ from the wall. A high value of $H$ corresponds to a flow such as that found in an adverse pressure gradient (Schlichting and Gersten 2000) with little momentum near the wall. Conversely, a low value represents a highly turbulent flow which is resistant to separation and therefore provides the basis for a suitable objective function to be minimised by `fmincon`.

**Optimisation with Structural Constraints**

In addition to maximising the mixing effect of the adaptive device, it is also necessary to ensure that the final deformation remains elastic. This will enable the device to be retracted when not required, removing its influence on the flow in off-design conditions. To achieve this an external penalty function has been used which artificially penalises the objective function (Smith et al. 1997), by the ratio by which the yield criterion is breached multiplied by a tunable coefficient $a$, if the peak von Mises stress $\sigma_{\text{max}}$ is above the material yield stress $\sigma_{\text{yield}}$. The optimisation problem can hence be formally described as

$$\min \left( S_H(x) \right)$$

where:

$$S_H = \begin{cases} H(x) & \sigma_{\text{max}} \leq \sigma_{\text{yield}} \\ H(x) + a \left( \frac{\sigma_{\text{max}}}{\sigma_{\text{yield}}} - 1 \right) & \sigma_{\text{max}} > \sigma_{\text{yield}} \end{cases}$$

s.t. $x_{LB} \leq x \leq x_{UB}$

where $S_H$ is the objective function and the vectors $x$, $x_{LB}$, and $x_{UB}$ contain the design variables and their lower and upper bounds respectively.

The area of interest for the increase in boundary layer momentum is fixed as a plane 150 mm from the leading edge. To quantify the efficacy of the geometry, the shape factor is therefore calculated at 11 uniformly distributed locations across a spanwise region $-5 \text{ mm} \leq z \leq 5 \text{ mm}$. The mean of these values is used as the final objective function to be minimised.

### Optimal Deformation of a Surface with Uniform Section

We first consider the case when the surface has a uniform section and thickness $t = 0.1 \text{ mm}$. The design parameters under investigation in this study, which form the vector $x$, along with their constraints are given in Table 1. The base material selected for this study is fully hardened stainless steel alloy 1.4310 which has a Young’s Modulus $E = 193 \text{ GPa}$ and yield stress $\sigma_{\text{yield}} = 1.0 \text{ GPa}$. This material has a similar yield strain to traditional aerospace grade aluminium but is readily available as shim in a large range of thickness. The streamwise location of the actuation $x_A$ is defined by

$$x_A = x_2 + P_A(x_1 - x_2)$$

where $x_1$ and $x_2$ are the streamwise boundary locations and $P_A$ is the ratio between streamwise actuation location and adaptive domain length. It is necessary to define $x_A$ in this way to prevent the situation where actuation is prescribed outside of the adaptive domain, as could occur if $x_A$ were to vary independently of $x_1$ and $x_2$.

Figure 4b shows a rapid improvement of the shape factor during the optimisation with convergence to the identified minimum after 4 iterations. The peak von Mises stress within the surface is plotted alongside the shape factor. As described above, the simulations used for the objective function are geometrically nonlinear, however, the peak linear stress, negating in-plane loads,
is plotted for comparison. Streamwise and spanwise sections of the final geometry are shown in Fig. 5. The computed boundary layer velocity profiles are presented in Fig. 6 and the associated shape factors are shown in Table 2. The results from a static forwards wedge vortex generator of height 2 mm are also computed in the same position to provide a comparison of the values achievable without the constraints imposed by morphing.

The morphed surface decreases the computed shape factor and thus increases the separation resistance of the boundary layer. However, the improvement in shape factor is only half of that achieved by the static vortex generator. In Fig. 4b it can be seen that, as the optimisation progresses, the peak stress in the surface increases to the threshold of the penalty function at which point no further significant improvement in objective is possible. When the in-plane membrane stress is neglected, in a linear solution, a similar trend is seen but the increase is not as great and, critically, the result remains significantly below yield. It is therefore clear that the in-plane loads are acting as a significant limitation in the design space for this case.

In order to develop a solution to the problems faced in this type of adaptive surface, the geometric differences between the profile morphs described previously and a local 3D geometry change will be investigated in the following section.

### Table 1. Optimisation design variables $x$ and their constraints $x_{LB}$ and $x_{UB}$

| Description                        | Symbol | Lower Bound | Upper Bound |
|------------------------------------|--------|-------------|-------------|
| Boundary Vertex (mm)               | $x_1$  | 72.5        | 106.0       |
| Boundary Vertex (mm)               | $z_1$  | 10.0        | 50.0        |
| Boundary Vertex (mm)               | $x_2$  | 19.0        | 52.5        |
| Boundary Vertex (mm)               | $z_2$  | 10.0        | 50.0        |
| Actuation Location Parameter       | $P_A$  | 0.1         | 0.9         |
| Actuation Height (mm)              | $y_A$  | 1.0         | 10.0        |
| Yield Stress (GPa)                 | $\sigma_{yield}$ | — | 1.0 |

### Table 2. Comparison of boundary layer shape factors

| Base Condition                     | Shape Factor $H$ |
|------------------------------------|------------------|
| Forwards Wedge VG                  | 1.81             |
| Uniform Adaptive Surface           | 1.31             |
|                                    | 1.54             |

### Figure 4. Typical convergence plots of the objective function

(a) Convergence of LES objective function

(b) Plot of optimisation function convergence ($\circ$) and corresponding peak von Mises stress for nonlinear ($\times$) and linear ($\triangledown$) cases.

### Figure 5. Deformed geometry resulting from the converged optimisation function using a shell section.

(a) X-Y section at $z = 0$ m (mid-plane)

(b) Z-Y section at $x = 0.045$ m (peak deformation)
The Generation of Membrane Stress due to Changes in Gaussian Curvature

In the profile morphing structures developed by previous authors a large elastic out-of-plane deformation from a uniform plate is achieved through a combination of a change in curvature \( \kappa(x,y) \) and in-plane strain \( \epsilon(x,y) \). The material yield stress provides a limitation on the achievable elastic deformation and is found by the superposition of bending \( \sigma_y \) and membrane (in-plane) \( \sigma_{Mz} \) stresses. For the principle stress in the \( x \) direction these are found by

Peak stress: \( \sigma_x = \sigma_y + \sigma_{Mz} \) \( (4a) \)

Bending stress: \( \sigma_y = \frac{E \epsilon_y}{2 (1 - \nu^2)} (\delta x + \nu \delta z) \) \( (4b) \)

Membrane stress: \( \sigma_{Mz} = \frac{E \epsilon_z}{(1 - \nu^2)} (\epsilon_x + \nu \epsilon_z) \) \( (4c) \)

in which \( E \) is the Young’s modulus, \( \nu \) is the Poisson’s ratio of the material, and \( \epsilon \) is strain in the principle dimensions \( x \) and \( z \). The distribution of curvature change and material elongation produced by a surface under load is a complicated problem frequently simplified through the assumption of geometric linearity. This is not valid for situations with large displacement, however, an examination of surface curvature change provides an initial understanding of the required magnitude of each component.

For any surface there exist two principle curvatures \( \kappa_1, \kappa_2 \) which are the maximum and minimum curvatures respectively. The Gaussian curvature \( \kappa \) of the surface can then be expressed as the product of these two principle curvatures

\[ \kappa = \kappa_1 \kappa_2 \] \( (5) \)

For the special case of a shell with constant cross-section, such as a wing, while \( \kappa_1 \) may vary with position or with actuation, \( \kappa_2 = 0 \) and so \( \kappa = 0 \) at all locations (Fig. 7a). For all other shapes however, \( \kappa \) on a surface may be either positive, such as a bowl shaped surface, or negative, which corresponds to a saddle shape.

Calladine (1983) develops the relationship between a change in Gaussian curvature \( \Delta \kappa \) and the generation of non-uniform in-plane strain

\[ \Delta \kappa = -\frac{\delta^2 \epsilon_y}{\delta x^2} + \frac{\delta^2 \gamma_{xy}}{\delta x \delta y} - \frac{\delta^2 \epsilon_x}{\delta y^2} \] \( (6) \)

where \( \gamma \) is the twist in the surface and all other terms are as previously defined. In the global morphing applications outlined previously, although a change in chordwise curvature is produced to alter the aerofoil profile, the spanwise curvature remains negligible and so, despite a significant change in geometry, \( \kappa_2 \) and hence \( \kappa \) are negligible in each configuration (Fig. 7b). This result means that, in such ‘2.5D’ morphing structures, \( \Delta \kappa \) is negligible and so the morph can be effected through bending alone, with little or no in-plane elongation.

In order for an adaptive surface to generate a spanwise variation in the flow, such as that found downstream of a vortex generator, it is clear that a constant sectional change to the structure is not sufficient. Instead, in the actuated configuration, the device must be developed from the surface and yet be finite in all directions. The developed geometry must therefore also contain areas where both principle curvatures are finite and thus must result in a change in Gaussian curvature (Fig. 7c). From Eq. 6 it can be seen that this must be accompanied by the generation of in-plane membrane strains. Equation 4c shows that the resulting stresses from membrane strains do not scale with thickness, as is the case with bending stresses, and so cannot be mitigated through the selection of a thinner material. The resulting limitation depends solely on the change in arc length over the surface, and so severely constrains the achievable elastic deformation.

In addition, the maximum out-of-plane deformation will be achieved when the contribution to peak stress has the minimum possible component due to bending and this occurs as \( t \to 0 \). If the original geometry has initial distributed curvature, such as is found towards the leading edge of an aerofoil, then under point actuation the structure will favour concentrating curvature rather than material extension to meet the new boundary conditions (Garland et al. 2015).

This ‘tenting’ occurs due to the much larger levels of strain energy required to extend the material rather than reduce the curvature in order to meet the constraints. Due to the inherent coupling between
material thickness and bending stiffness in a shell, the chosen parameters must be a compromise between the deformation levels required and the loss of fidelity of the original geometry. It is therefore desirable to develop a method to decouple the bending and membrane properties of the surface such that a low in-plane stiffness can co-exist with a high out-of-plane bending stiffness.

The limitation on shape change due to membrane stress has been encountered previously in profile morphing applications. In order to achieve more extreme elastic variations in cross section, dedicated anisotropic materials, such as the corrugated structures proposed by Yokozeki et al. (2006), have been used to allow elongation in one direction while maintaining high bending strength in the other. These expand the range of available profile morphs but again rely on a 2.5D geometry change. An alternative method to increase the range of available morphs, which has been demonstrated in both profile and span morphing applications, is to replace the wing skin with a two-dimensional lattice structure. These structures are formed when a unit cell of slender beams and constant height is tessellated in-plane to form a surface. Under load, the principle deformation mechanism of the structure is either the bending or stretching of the constituent spars.

In order to distinguish between these two types of lattice, Deshpande et al. (2001) proposed that the rigid joints of the structure be replaced with pin-joints and the resulting system analysed for kinematic determinacy. If this version of the lattice is kinematically determinate then the problem is stretching dominated and the membrane stress remains significant. If, however, the pin-jointed lattice is kinematically indeterminate then mechanisms, regions where displacements are possible without strain, are present. In the rigid structure the moments which cause rotations in the pin-jointed system instead cause the beams to bend, which has been shown by Gibson and Ashby (1999) to result in a similar global deformation. An example of a bending-dominated structure is shown in Fig. 8. The in-plane load, which would normally result in-plane strain and thus be limited to a small elastic deformation, causes a moment in the nodes. As the beams are slender these bending stresses are much smaller than the equivalent membrane stresses within a stretching dominated lattice and so a larger global elastic deformation is possible. Unlike the corrugated surface, the bending of a lattice occurs in-plane and throughout the surface allowing for deformations in both principle directions.

Lattice structures, with $\nu = 0$ have been used by Olympio and Gandhi (2007) and Bubert et al. (2010) to provide a high yield strain skin for profile morphing applications. This allows the surface to strain in the chordwise direction without inducing the detrimental spanwise loading due to Poisson’s effects associated

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Gaussian Curvature distributions on three wing segments: (a) Unmodified geometry; (b) the same section following a camber variation (zero Gaussian curvature); and (c) the original section with a local, out-of-plane, geometry change (non-zero Gaussian curvature).} \label{fig:gaussian_curvature}
\end{figure}
in traditional materials while the bending-dominated lattice is able to produce in-plane strains far in excess of those achievable through traditional shells.

**Hex-Chiral Lattice Surface**

Referring to Eqs. (4) and (5), for an out-of-plane deformation, the use of a negative Poisson’s ratio auxetic material reduces the stress induced in the formation of a geometry with \( \kappa > 0 \) from a clamped plate with \( \kappa = 0 \). This reduction, when combined with the decoupling of bending and membrane stiffness through the use of a bending-dominated lattice, extends the range of achievable deformation geometries. One such lattice geometry is that of a hex-chiral lattice (Fig. 9). This design, proposed by Prall and Lakes (1997), has a theoretical bulk Poisson’s ratio \( \nu = -1 \) while the other in-plane bulk properties can be varied by the adjustment of the unit-cell topological parameters.

**Design Parameters and Constraints**

The location of boundary constraints remain design variables, as in the study above, however, additional variables are required to fully define the lattice topology and allow the adjustment of all bulk material properties during the optimisation. These parameters, shown in Fig. 9b, are spar length \( l \), width \( w \), and node radius \( r \). In addition, for the consideration of out-of-plane geometry change, material thickness \( t \) is an important parameter which can be varied independently of lattice topology. Additional constraints are therefore also required to bound the unit cell topology of the lattice. The bounds on cell size are predominantly determined from manufacturing limitations, however, the upper bound on core radius is determined from the limits of kinematic determinacy in the structure. At the limit \( r/l = 1.0 \) the structure is no longer kinematically determinate and so the lattice becomes a stretching dominated lattice. All design parameters controlled by the optimisation algorithm, and their constraints, are given in Table 3.

**Structural Analysis**

The analysis is again conducted in Abaqus over the quadrilateral domain with design parameters described previously. The unit-cell topology is produced with Timoshenko beam elements with 6 degrees of freedom at each node and which capture both shear and bending deformation (B31). Each spar in the unit cell is split into 7 elements and the initial node is centered at \( x_A \) to provide a viable location at which to apply actuation. The unit cell is then tessellated over the adaptive domain to provide a continuous surface. Any nodes which fall outside of the domain are removed. The free nodes, which are connected to only one element, are constrained in all degrees of freedom.

Once the analysis has converged, the resulting nodal displacement is used to form the fluid boundary in
Table 3. Constraints on Optimisation Design Variables

| Description                  | Symbol | Lower Bound | Upper Bound |
|------------------------------|--------|-------------|-------------|
| Spar Length (mm)             | $\ell$ | 2.5         | 10.0        |
| Core Radius (ratio)          | $r/\ell$ | 0.1        | 0.45        |
| Spar Width (mm)              | $w$    | 0.05        | 1.0         |
| Thickness (mm)               | $t$    | 0.1         | 1.0         |
| Boundary Vertex (mm)         | $x_1$  | 72.5        | 106.0       |
| Boundary Vertex (mm)         | $x_2$  | 19.0        | 52.5        |
| Boundary Vertex (mm)         | $z_1$  | 10.0        | 50.0        |
| Boundary Vertex (mm)         | $z_2$  | 10.0        | 50.0        |
| Actuation Location Parameter | $P_A$  | 0.1         | 0.9         |
| Actuation Height (mm)        | $y_A$  | 1.0         | 10.0        |
| Yield Stress (MPa)           | $\sigma_{yield}$ | —    | 1000        |

The aerodynamic analysis and the remainder of the optimisation process continues as described previously.

**Optimal Deformation of a Lattice Based Surface**

The optimisation was again run for a number of iterations and checked for convergence. The optimisation history shown in Fig. 10 shows a rapid improvement over the initial iterations followed by limited further improvement with each subsequent iteration, indicating convergence to the identified minimum point. It can be seen that, for all iterations, the peak stress remains significantly below yield. The penalty function is therefore not constraining the design and hence the selection of design parameters is based solely on geometry.

Section views of the deformed geometry are shown in Figs. 11a and 11b alongside the shell results from the previous study. The peak deformation using a lattice based surface has increased significantly when compared to the original solution from the surface with uniform section presented previously due to the increased in-plane yield strain of the lattice structure. The increased strain is accompanied by a decreased stiffness of the aerodynamic surface. To ensure the morphed surface maintains its geometry, under the induced aerodynamic loading, the mean pressure field calculated during the simulation is applied to the deformed lattice. The mean nodal displacement due to aerodynamic loading is $1.048 \times 10^{-4}$ m and the mid-plane section of the loaded geometry is shown in Fig. 12. The additional deformation is equivalent to 2.07% of the peak deflection and is therefore considered sufficiently small to justify the use of the static version of the actuated shape in these experiments.

**Experimental Validation**

Once the optimised design had been produced, the results were validated in separate wind tunnel and structural experiments. These elements were tested separately to ensure that unexpected influences such as manufacturing irregularities did not influence the results.
Boundary Layer Velocity Profiles

In order to test the effect of the final geometry on the boundary layer, a static version of the surface was reproduced using a Connex Objet 350 3D printer with spatial accuracy of $\pm 2.1 \times 10^{-5}$ m in the $x$ and $z$ directions, and $\pm 7.9 \times 10^{-6}$ m in the $y$ direction. This was fixed to the base geometry which was a plate, fitted with an elliptical leading edge with major radius 60 mm, positioned in the freestream. The inclination of the plate was adjusted relative to the freestream to minimise the pressure gradient. The surface tested was therefore representative of the analytical morphed surface under the same conditions. The achieved centreline pressure gradient was recorded during the baseline measurement using an FCO310 micromanometer as $\frac{dp}{dz} \approx 0.2$ Pa/m.

Stereo Particle Image Velocimetry sPIV was used to provide details of the flow field downstream of the manipulated geometry. The illuminating light sheet was produced using a Litron LDY-304 high-speed laser operating at 1 kHz. The image pairs were captured using two Phantom Miro M310 cameras at either side of the wind tunnel with Scheimpflug mounts and 135 mm objective lenses with an a fixed aperture of f2.0. The results presented were processed using LaVision DaVis software using a square interrogation window size of $16 \times 16$ pixels with 75% overlap. Each window is therefore a spatially filtered result with a filter size of $0.194 \text{ mm} \times 0.194 \text{ mm}$. A schematic of the validation experiment is shown in Fig. 13. To capture the full wake of the structure, the PIV plane is orientated normal to both the mean flow direction and the plate. The through plane component of a stereo PIV field is associated with the highest potential error due to the required vector reconstruction. The data are therefore taken from over 20,000 vector fields and the statistical uncertainty, based on a 99% confidence interval and uniformly distributed error, is presented for each result.

Table 4. Comparison of boundary layer shape factors

| Shape Factor $H$          | Computational | Experimental |
|---------------------------|---------------|--------------|
| Base Condition            | 1.81          | 2.06         |
| Uniform Section           | 1.54          | –            |
| Forwards Wedge VG         | 1.31          | 1.12         |
| Adaptive Geometry         | 1.42          | 1.16         |

The boundary layer profiles from the LES used in the optimisation process are compared to those of the ‘clean’ surface and the experimental results in Fig. 14. Assuming a normally distributed measurement error, the 99% confidence intervals for the experimental velocity profiles are $Z_{99} = 0.95\%$, 1.22\%, and 1.08\% for the base condition, discrete vortex generator, and adaptive geometry respectively.
From the shape factors in Table 4, it is clear that both the surface representing the adaptive geometry, and the traditional VG are effective at transporting momentum towards the wall within the boundary layer. Comparing the velocity profiles it can be seen that the discrete vortex generator is slightly better than the adaptive structure. This is to be expected as the requirement for surface and gradient continuity to accomplish an elastic adaptive structure is in direct opposition to the formation of tightly bound vortices, which requires sharp discontinuities at the device trailing edge.

The experimental data from the base geometry agree well with the computational results, however, a small overestimation of the velocity is visible close to the wall in the CFD result. The transfer of momentum by the adaptive geometry is qualitatively captured by the LES simulation. However, the velocity discrepancy between experiment and simulation is larger with the increased surface complexity. This is believed to be due to the limitations of grid density in the LES simulation used as part of the optimisation routine. While locally increased mesh density downstream of the device could improve the result accuracy by capturing more of the small scale motion, the increased computational resource required is prohibitive. Based on both the computational and experimental results, it has been possible to produce an adaptive geometry to replace the function of a static vortex generator for transfer of momentum towards the wall in a boundary layer.

**Physical Model of Adaptive Vortex Generator**

Once the effect of the modified geometry on the boundary layer was validated experimentally, the parametrized lattice was manufactured from stainless alloy 1.4310 shim steel. The lattice geometry was cut from this shim using an Oxford Lasers Compact Micromachining System. The geometry was inspected under a calibrated AxioVision measurement system and Zeiss Imager.M2M microscope. The dimensional accuracy of the cut geometry was found to be within 1.2 µm and a magnified typical lattice node with inspected dimensions is shown in Fig. 15.

To form an impermeable boundary, the lattice was encapsulated at the mid-plane of a 1.0 mm elastomer skin formed of EcoFlex silicone 00:50 rubber. This membrane was not included in the finite element simulation as, with a Young’s modulus $E \approx 29.5$ kPa (Hollenstein 2005), the skin stiffness is approximately 3 orders of magnitude less than the lattice and thus has a negligible effect on the deformed geometry.

The encapsulated lattice structure is fixed to a plastic housing for measurement as shown in Fig. 16a.

Actuation of the structure requires a sustained force over a large, precise, stroke. This is implemented using a Haydon Kirk G4 19000 stepper motor driven linear actuator which has a step angle of 7.5°/step and a linear travel of 12.5 µm/step. The motor is driven with a MSD415 micro-stepping motor driver which further subdivides each step into 64 increments giving a theoretical positional increment of 0.20 µm.
The surface of the encapsulated lattice was coated in a thin layer of powdered titanium dioxide and measured using a Faro measurement arm and 3D laser scanning head (Fig. 16b) with a spatial accuracy of ±2.5 µm. The position of the structure was recorded in the base condition before, during, and after actuation and the measured position in each case is shown in Fig. 17. The deformed surface compares well with the finite element prediction and so demonstrates that the adaptive structure is able to produce the geometry which has already been shown to be effective. The post-actuation case shows that the structure returns to the base geometry. The deformation is therefore purely elastic allowing a non-permanent alteration of geometry, and therefore effective boundary layer control.

Conclusion

This paper examines the production of a locally finite morph from a continuous surface of uniform section for the prevention of boundary layer separation. A gradient-based optimisation algorithm with a multidisciplinary objective function allows the identification of an optimal set of design parameters for increasing the near-wall momentum of the flow with an elastically adaptive surface. A limitation on the design space is encountered due to the rapid increase in in-plane ‘membrane’ strains. This severe constraint is attributed to the variation of the surface Gaussian curvature and hence is not seen in 2D profile geometry changes.

To overcome this issue, the use of a bending-dominated hex-chiral lattice has been proposed with Poisson’s ratio $\nu < 0$. The use of a bending-dominated lattice structure transforms a high-strain in-plane stretching problem into the distributed bending of multiple slender spars, reducing the peak stress within the system. The re-entrant cells present in the chiral structure reduce the in-plane forces further by the reversal of the Poisson’s effect found in a surface of traditional aerospace material with uniform section. The geometry of the lattice unit-cell can be configured to alter the bulk surface properties and so allows the relative bending and membrane stiffnesses to be altered independently.

The resulting elastic deformation geometry was replicated and tested experimentally. The adaptive structure significantly decreased the measured downstream shape factor, indicating a transfer of momentum towards the wall and an increase in separation resistance. The downstream effects of the adaptive surface were also compared to that of a static forwards-wedge vortex generator. The static device provides better mixing than that found with the adaptive structure and this is due to the fixed separation at discontinuities in surface gradient found on the vortex generator which are not possible with an adaptive surface.

Once the effect of the static morphed geometry was validated, the lattice was manufactured in stainless steel to the parameters resulting from the optimisation. The model was then deformed and
measured using a 3D laser scanning instrument. The actuated surface compares well with the results from the FE simulation and hence the geometry used in the experimental validation conducted in the wind tunnel. Critically, the surface returned to its original position when the actuation was removed, demonstrating the deformation was achieved elastically. It has therefore been shown that, by using an auxetic lattice structure it has been possible to produce an adaptive boundary layer control device which may deployed and retracted from an aerodynamically clean surface as required. This is an improved design compared to traditional static vortex generators and implies that the technique may be applied to time dependent surface changes.

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