OBSERVABLE CONSEQUENCES OF A SCALAR BOSON COUPLED ONLY TO NEUTRINOS

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Abstract

We have examined the consequences of assuming the existence of a light scalar boson, weakly coupled to neutrinos, and not coupled to any other light fermions. For a range of parameters, we find that this hypothesis leads to the development of neutrino clusters which form in the early Universe and which provide gravitational fluctuations on scales small compared to a parsec (i.e., the scale of solar systems). The existence of such clustering produces an effect which would appear as a negative mass squared for the electron neutrino in Tritium beta decay, without conflicting with other experiments. The neutrino masses arising in unified gauge theories would then be very much larger than the masses extracted from experiments within the solar system.

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In spite of the great success of the standard model for the description of (electrically) charged fermions\textsuperscript{1}, there is still substantial experimental ambiguity about the properties of neutrinos. Experiments designed to measure or limit neutrino masses appear to give negative values for the mass squared\textsuperscript{2−7}; neutrinoless double beta decay, which would confirm the expected Majorana nature of the neutrino\textsuperscript{8}, has not been observed\textsuperscript{9}; and, if the neutrinos are Dirac particles, nothing definite is known about the interactions of the right handed neutrinos. Theoretical extensions of the standard model to larger gauge groups are, however, primarily constrained by interactions among the charged fermions.

We have investigated the possibility that these and other experimental data are evidence for an interaction mediated by a boson which couples only to neutrinos and not to other light (electrically charged) fermions\textsuperscript{10}. Were this boson a vector, the need to cancel anomalies arising from the coupling to one such vector and two $Z^0$'s would require either coupling to charged fermions or the existence of other new fermions designed for this purpose only. We have, therefore, studied the case of a scalar boson. The theory of a system of relativistic fermions interacting via scalar exchange is a special case of Quantum Hadrodynamics\textsuperscript{11} (QHD, developed for the study of nuclear matter) some results of which we use here. For clarity of presentation, we shall concentrate on electron neutrinos, making occasional reference to extensions to other flavors.

In this Letter we shall show that, for values of the scalar mass and coupling constant which are compatible with known experimental data, the coherent attraction arising from scalar exchange drives clustering of neutrinos in the early Universe. This causes them to decouple from the general expansion at about the epoch of recombination and to provide a source of gravitational fluctuations on a scale small compared to a parsec yet large enough to influence stellar formation. The existence of such clustering, persisting to the present epoch, could provide sufficient neutrino capture events\textsuperscript{2−6} to modify the beta spectrum in Tritium beta decay at the end point and thus explain the data. A simple extension to include the interactions of muon neutrinos with the background neutrinos, however, does not predict a sufficient modification
of the muon energy in pion decay to account for the experimental observations\textsuperscript{7}. Finally, we shall point out that the effective masses which enter into the description of experiments performed in the presence of such a background of clustered neutrinos (i.e. within the solar system) are severely modified from the masses that would be observed in the interstellar vacuum and which should correspond to the mass parameters arising in gauge theories. Derivations of the required formulas are deferred to a longer paper\textsuperscript{12}.

Given a scalar boson, $\phi$, coupled to neutrinos, the interaction Lagrangian density may be written in the form

$$\mathcal{L}_I = g \overline{\Psi}_\nu \left( \cos \xi + i \gamma^5 \sin \xi \right) \phi \Psi_\nu$$

where $g$ is the coupling constant and the angle $\xi$ fixes the Lorentz properties of the coupling. $\xi = 0$ yields a scalar interaction, $\xi = \frac{\pi}{2}$ gives a pseudo-scalar coupling, and angles in between lead to CP violation. For this paper, we only consider $\xi = 0$.

Following QHD, we express the effect of the medium on a single neutrino of momentum $k$ in terms of an effective mass,

$$m_i^*(k) = m_i^0 - \left( \frac{g_i}{2\pi m_\phi} \right) \frac{m_i^*(k)}{E_i^*(k)} \sum_j \left( \frac{g_j}{2\pi m_\phi} \right) \frac{1}{\pi} \int d^3k' \frac{m_j^*(k')}{E_j^*(k')} F(k')$$

where $m^0$ is the vacuum value of the neutrino mass which includes all field theoretic corrections except for the finite density effect, $g$ is the coupling constant in (1), $m_\phi$ is the mass of the scalar, $m^*(k)$ is the effective neutrino mass, $E^* = \sqrt{m^*^2 + k^2}$ is the effective energy and $F(k)$ is the Fermi-Dirac distribution function for the appropriate temperature. (We use units with $\hbar = c = 1$.) The factors of $m^*/E^*$ arise because the effective mass corresponds to forward scattering which must preserve helicity of both the subject neutrino and the background neutrino, while the scalar coupling requires a chirality flip. We differ slightly from Ref. 11 by defining the momentum-dependent mass as the matrix element of the scalar self-energy operator at momentum $k$, rather than the momentum-independent value at $k = 0$. This affords a clearer comparison with the covariant interaction energy as discussed by MacRae and Riegert\textsuperscript{13}. The
subscripts $i$ and $j$ refer to different flavors; for the bulk of the Letter we consider $i$ to refer to the electron flavor and suppress the sum.

The physical solutions of (2) are restricted to $0 < m^* < m^o$. Writing

$$A = \left( \frac{1}{m^o} \right) \frac{g^2}{4\pi^2 m^o_\phi^2} \int d^3k' \frac{m^*(k')}{E^*(k')} F(k')$$

we will show below that, for $A$ very large which is the case of interest, $m^*(k)/m^o << 1$ for values of $k/m^o < A$. This implies that, for a large range of $k$, the neutrinos obey relativistic kinematics and $E^* \simeq k$. Hence neutrinos with $k < m^o$ do not have enough energy to propagate freely outside the medium and are bound to the system. These remarks are illustrated in Fig. 1.

Consider now the evolution of the neutrino gas in the early Universe from the point at which the weak interactions cease to be effective in coupling neutrinos to other forms of matter. As long as the temperature is large compared to $m^o$, the difference between $m^*$ and $m^o$ is irrelevant and the neutrino gas is subject to the general expansion. When, however, the temperature drops to the order of $m^o$, a large fraction of the neutrinos form a self-bound Fermi gas and will nucleate into clusters whose size scale is set by $1/m_\phi$. These clusters, being self-bound, will retain their size while they separate from each other due to the Universal expansion, although some coalescence may occur. Some of the most energetic neutrinos might well escape from these clusters, providing a lower density of free relic neutrinos than given by the standard model. However, we estimate that the energy loss rate due to bremsstrahlung of scalars from forward scattering is sufficiently fast to cool much of the system, as will be seen from the parameters below.

So far the discussion has been general and the scenario will develop at some temperature and density, determined by $g$, $m^o$ and $m_\phi$. To learn if the scenario can be realized for physically relevant parameters, we need to be guided by data.

We begin with Tritium beta decay. There are five modern experiments $2^{-6}$, done by a variety of techniques and with various source configurations, which study the end point of the electron spectrum to search for evidence of non-zero electron anti-neutrino mass. All analyze the data under the assumption that the only extension of
the standard model is the existence of such a mass and that the signature would be a suppression of the counting rate at and just below the end point. All five report a barely significant negative value for the best fit to the square of the mass. If real, this corresponds to an increase in the counting rate in the vicinity of the end point. In fact, the Los Alamos group reported\textsuperscript{6} that an equally good fit could be obtained by assuming the existence of counts just at the end point with a $10^{-9}$ branching ratio. This could occur by the capture of massless relic neutrinos through the normal weak interaction, providing the density is about $5 \times 10^{15}/cm^3$. Such a density of cold neutrinos would correspond to a Fermi momentum ($k_F$) of $13\,eV/c$; hence, as discussed below, the assignment of all the counts to the endpoint is not justified. A reanalysis, taking into account the fact that $13\,eV$ is not negligible on the scale of the experiment and that such a density, if extended to anti-neutrinos, provides additional distortion of the spectrum, is in progress\textsuperscript{14}.

Assuming that $A$ is large enough that $m^*$ is negligible for all $k < k_F$, the argument goes as follows. The density of final states is calculated including the potential well that is represented by $m^*$ and gives, to observable accuracy, the same spectrum, with the same end point $E_o$, as for free decay with zero neutrino mass. However, the well is filled with anti-neutrinos (right-handed Majorana neutrinos) so Pauli blocking truncates the spectrum a distance $E^* = k_F$ below the end point. Since the well is also filled with neutrinos (left-handed Majorana neutrinos) with a density proportional to $k^2$, they will be captured and the beta spectrum will grow quadratically from $E_o$ up to $E_o + k_F$ where it cuts off sharply. The Kurie plot is shown in Fig. 2. Note that the interpretation of this differs from that presented by Weinberg\textsuperscript{15} and by Bergkvist\textsuperscript{16} due to the existence of the potential well.

To satisfy the condition that $m^*$ is negligible below $k_F$, we need only require that $A \gg 1$. To limit the parameters further, we must consider other data. One might be concerned that the existence of the extra scalar degree of freedom, in thermal equilibrium with the neutrino gas, would have an adverse effect on primordial nucleosynthesis. This question was studied by Kolb, Turner, and Walker\textsuperscript{17}, who showed that, while a vector coupled either to neutrinos or photons, or a scalar coupled to
photons, would have serious effects, the case at hand corresponds to adding an additional half family of neutrinos which is not definitely ruled out by standard analyses. Furthermore, the efficacy of $\mu$- and $\tau$-neutrinos may well be affected by this interaction, depending on the details of the coupling to other flavors. This question requires further study. However, since the scalar does couple left- and right-handed chiralities, if the neutrinos were Dirac particles, one would expect twice the number of light neutrino species. This would conventionally cause difficulties with the calculated primordial abundances of nuclear species. Furthermore, the right-handed neutrinos and left-handed anti-neutrinos would not interact with matter in a supernova and would provide too efficient a cooling mechanism. These considerations add further credence to the assumption that the neutrinos are Majorana particles.

The Lagrangian density (1) does lead to the annihilation of pairs of neutrinos (with the same chirality) into pairs of scalars. In order that background neutrinos have an effect on Tritium beta decay, they must persist until the present epoch. This puts a very severe constraint on the size of the coupling. Writing $\tilde{\alpha} = g^2/4\pi$, and averaging over a zero temperature Fermi gas, we obtain the mean annihilation rate per neutrino, given by

$$< \omega > = \frac{3}{8} \tilde{\alpha}^2 k_F \left( \ln \left( \frac{k_F}{m_\phi} \right) - \frac{3}{8} \right).$$  (4)

Requiring $< \omega >$ to be less than $10^{-18}$/s constrains $\tilde{\alpha} < 10^{-18}$ upon iterating with $m_\phi$ as determined below. (The functional form of this rate is generic for all two body processes involving the scalars or neutrinos.)

A further constraint comes from the non-observation of neutrinoless double beta decay. From Tritium, we expect $m^\alpha$ to be of the order of $13 eV/c^2$ and, from the arguments above, that the neutrino is a Majorana particle. Neutrinoless double beta decay can arise from the exchange of virtual Majorana neutrinos between the affected quarks in a nucleus with the matrix element proportional to the effective mass$^{18}$. Of course, the effective mass must be evaluated at momenta appropriate to the size of the nucleus, or several hundred $MeV/c$. To safely suppress the zero-neutrino mode,
$m^*$ should be kept negligible at values of $k/m^o$ up to $10^8$ which demands $A > 10^9$, as we now show.

Writing $m_\phi/m^o = \mu$, $y = m^*/m^o$, and $x = k/m^o$, we may rewrite (2) as

\[
y = 1 - \frac{\tilde{\alpha}}{\pi^2 \mu^2} \frac{y}{\sqrt{x^2 + y^2}} \int d^3 x' \frac{y'}{\sqrt{x'^2 + y'^2}} F(x') = 1 - \frac{y}{\sqrt{x^2 + y^2}} A \tag{5}
\]

or, solving for $x^2$,

\[
x^2 = y^2 \left[ \frac{A^2}{(1-y)^2} - 1 \right] \sim \left( \frac{y}{1-y} \right)^2 A^2 \tag{6}
\]

Thus to ensure $y$ does not approach 1 for $x < 10^8$, we must have $A > 10^9$.

We now are in a position to bound the value of $m_\phi$. With $A > 10^9$, $y = x/A$, to an extremely good approximation over all of the range of integration supported by $F(x)$. When the temperature equals $m^o$, (3) may be written

\[
A \approx \frac{4\tilde{\alpha}}{\pi \mu^2} \int_0^\infty x^2 dx \cdot \frac{1}{\sqrt{A^2 + 1(e^x + 1)}} \tag{7}
\]

This gives

\[
\frac{\tilde{\alpha}}{\mu^2} \approx \frac{\pi A^2}{6\zeta(3)} \tag{8}
\]

where $\zeta$ is the Riemann zeta function, $\zeta(3) = 1.20206$. From the bound on $A$,

\[
\frac{\tilde{\alpha}}{\mu^2} > 10^{18} \tag{9}
\]

Thus, $\mu < 10^{-18}$, or $m_\phi < 10^{-17} \text{eV}/\text{c}^2$. This implies a range greater than $10^{12} \text{cm}$, consistent with the notion that the neutrino clustering can provide seeds for the formation of stars. Clusters with large density and small extent (less than a parsec) will behave as mass points (as do stars) for considerations of galaxy formation and rotation curves. The fraction of neutrinos that is not in clusters may well play a role in the required dark matter.

If there is a large density of relic neutrinos in the vicinity of the Sun (and, of course, of the Earth), the scattering, by the background, of neutrinos which are observed in
experiments must not be too large, providing further constraints on the parameters. In the chirality basis, the total neutrino-neutrino scattering cross sections are given by

\[
\sigma_{LL} = \sigma_{RR} = \frac{\pi^2}{s} \left[ 4 + \frac{2(2m^2 - m_\phi^2)}{m_\phi^2(s - 4m^2 + m_\phi^2)} \right]
\]

\[
+ 4 \left\{ \frac{(2m^2 - m_\phi^2)}{(s - 4m^2)} - \frac{(2m^4 + m_\phi^2(s - 4m^2) + m_\phi^4)}{(s - 4m^2)(s - 4m^2 + 2m_\phi^2)} \right\} \ln \left( \frac{s - 4m^2 + m_\phi^2}{m_\phi^2} \right) \]

(10)

and

\[
\sigma_{LR} = \frac{\pi^2}{s} \left[ 1 + \frac{(2m^2 - m_\phi^2)^2}{m_\phi^2(s - 4m^2 + m_\phi^2)} \right]
\]

\[
+ \frac{2(2m^2 - m_\phi^2)}{(s - 4m^2)} \ln \left( \frac{s - 4m^2 + m_\phi^2}{m_\phi^2} \right) \]

(11),

where \(m\) is the appropriate neutrino mass and \(s\) is the total c. m. energy squared.

Those neutrinos which have been observed experimentally have laboratory energies in excess of 1 MeV, so the dependence on \(s\) allows the possibility of both the strong effects needed for large \(A\) and a sufficient mean free path to observe reactor or accelerator neutrinos over kilometers, and solar neutrinos over an Astronomical Unit. The worst case is presumably given by the observation of neutrinos from supernova SN1987a. To obtain the strongest constraint, we use the most conservative assumption.

Assume that all the neutrinos created in the early Universe survive to the present epoch and are collected in clusters around the stars, proportional to the baryon number associated with the stellar system. Since the ratio of neutrinos to baryons is essentially the same as that of photons to baryons \((10^9)\) the number of neutrinos associated with the solar system would be \(10^{66}\). At a uniform density of \(5 \times 10^{15}/cm^3\), they would fill a sphere of radius \(3.6 \times 10^{16} cm\), or about .01 parsec. To achieve a mean free path greater than \(4 \times 10^{16} cm\), the cross section must be less than \(5 \times 10^{-33} cm^2\).
The neutrinos observed from SN1987a had an energy of about $10 \, MeV$ and the effective energy of the relic neutrinos is about $10 \, eV$, so the average $s$ is $10^8 eV^2$. The dominant pieces of equations (10) and (11) then give

$$\sigma_{tot} \simeq \frac{5 \pi \alpha^2}{s}$$

which easily satisfies the cross-section bound if the annihilation rate bound discussed above has been satisfied.

We note that, under these assumptions, the total energy of the neutrinosphere approaches 10 solar mass equivalents. We expect such a mass concentration to be efficient at driving baryonic clustering. Even if only a fraction of the neutrinos cluster, which eases the cross-section constraint, the effect on baryon clustering should still be strong.

If the predominant cluster size favors the formation of smaller baryon agglomerations than a typical solar system, i.e., -“Jupiters”, then much of the baryonic matter in the Universe will not be visible, and the nucleosynthesis bound on the baryonic contribution to the universal deceleration parameter may be saturated. In that case, the neutrino contribution to the universal energy density being an order of magnitude larger from our estimate, the deceleration parameter may well equal unity on large scales.

We should return here to the questions of annihilation and cooling. The structure of the annihilation cross-section is similar to (12). We used that together with the flux of essentially massless neutrinos in the cluster to make the estimate (4) for the annihilation rate. The cooling depends on bremsstrahlung of scalars and their ability to subsequently escape the neutrinosphere without rescattering. Again, the scattering rate of scalars on neutrinos is negligible because the cross-section is similar to (12). Only coherent scattering, which dominates the bremsstrahlung process, can be different.

In making the cooling rate estimate, we use a Weizsacker-Williams approximation to estimate the probability of $\phi$-bremsstrahlung as $P \sim \left(\frac{g^2}{8\pi^2}\right) \ln \left(\frac{E_{\max}}{E_{\min}}\right)$, per neutrino scattering. The neutrinos bound in a cluster of size $1/m_\phi$ form a coherent
potential for which a naive estimate of the neutrino scattering cross section would exceed the unitarity bound. We therefore use a geometrical cross section and take the flux to be that given by a density of order the local density with each neutrino moving relativistically. The energy loss rate is then given by

$$dE/dt \sim \frac{1}{\pi m_\phi^2} \cdot P \cdot \text{flux} \cdot <\text{energy loss per scatter}>$$  \hspace{1cm} (13)$$

The momentum transfer involved must be less than or of order $m_\phi$ for coherence to obtain over the entire cluster. For forward inelastic scattering, one may estimate straightforwardly that the neutrino-scalar pair mass in the c. m. of the cluster and a neutrino (and so, the maximum energy of the scalar) is limited to $A m_\phi$. Using this, we estimate that the energy lost to $\phi$ emission approaches the binding energy of a neutrino essentially instantaneously. The decoupled scalars then red-shift the energy away in the general expansion of the Universe. The eventual result will be that a significant fraction of the primordial neutrinos will form cold, self-bound Fermi gas clusters of dimensions comparable to the original clusters.

The scalar boson under consideration is, in many respects, like the Majoron proposed many years ago. Limits on $\tilde{\alpha}$ can be obtained from the bremsstrahlung of the $\phi$ in elementary processes, such as $K \rightarrow \mu + \nu$, or a study of the two electron spectrum in double beta decay, as in analyses for Majorons. The strongest reading of these limits requires that $\tilde{\alpha} < 10^{-12}$, which again is not constraining.

A constraint on the entire scenario, rather than on certain parameters, may eventually be obtained from solar system dynamics. For a cold, self bound relativistic Fermi gas, the average energy per particle is $3k_Fc/4$. Assuming $k_F = 10 \ eV/c$ and a density of $5 \times 10^{15}/cm^3$, one obtains an energy density of $4 \times 10^{16} eV/cm^3 = 7 \times 10^{-17} g/cm^3$. The orbit of Jupiter is $7.8 \times 10^{13} cm$, so the extra mass interior to the orbit is $1.3 \times 10^{26} g = 10^{-7} M_\odot$. To the best of our knowledge, this is not ruled out by existing data. Future studies of the motion of interplanetary probes might be able to determine the presence of the neutrinosphere.

Finally, a comment on the muon neutrino is in order. Note that the parameters for the electron neutrino do not constrain the specific coupling of the scalar to other
flavors. For example, if the vacuum masses were proportional to the \( g_i \), the effective masses would be in the ratio of the vacuum masses. Nonetheless, generally, the effect of an attractive well on the muon neutrino is such that a neutrino emitted in pion decay must have a larger momentum than it would outside the neutrinosphere. To balance momentum and conserve energy, the muon momentum will be increased, with a concomitant increase in the muon energy. However, independent of how large the vacuum mass value is, this can not lead to a negative effective value of the square of the neutrino mass\(^1\), but can only reduce it to a negligibly small value. Thus the scenario described here does not account for the results of Ref. 7. Large vacuum masses for the non-electron neutrino flavors would, of course, cause severe difficulty for our understanding of the present expansion of the Universe unless those neutrinos annihilate into scalars or convert into electron neutrinos sufficiently rapidly, which could place lower limits on their coupling to scalars. While such extensions are very interesting, and could ultimately demonstrate the feasibility, or lack thereof, of this entire scenario, they are beyond the scope of the present Letter.

In summary, we have shown that the assumption of a very light scalar boson, coupled to neutrinos and to no other light fermions, can be accommodated within existing experimental data. The existence of such a scalar can have serious implications for the evolution of matter concentrations in the early Universe since it provides fluctuations on a scale much smaller than those observed by COBE\(^{24}\) (which presumably drive the large scale structure). These include the possibility that neutrino clustering seeds stellar formation and so influences the stellar mass distribution. Furthermore, the clustering of neutrinos driven by such a scalar interaction can provide an explanation of the negative mass-squared found in modern Tritium beta decay experiments.

A disturbing consequence for model builders is the fact that experiments performed within the solar system will yield values of the effective neutrino masses, not the vacuum masses that must arise from an eventual unified theory. In particular, it should be noted that the \( 13 \text{eV}/c^2 \) vacuum value of the electron neutrino mass discussed above in no way obviates currently popular interpretations of the observed value of the solar neutrino flux.
Taking the Tritium experiments as a datum, the clustering would be manifest at a temperature of about $13\, eV$, or near the time of electron-ion recombination. Accommodating the lack of neutrinoless double beta decay leads to a minimum range of the scalar interaction which is a reasonable fraction of an Astronomical Unit and allows a range of the order of, or larger than, planetary orbits. This suggests that the primordial neutrino distribution could evolve into droplets with a scale commensurate with solar systems, which would then provide a seed mechanism for the formation of stars at all times from recombination to the present epoch.

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REFERENCES

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1. T. Goldman in Adv. Nucl. Phys. (J. N. Negele, and Erich Vogt, eds.), Vol. 18, p. 315 (1987) Plenum Press, NY.

2. Ch. Weinheimer, M. Przyrembel, H. Backe, H. Barth, J. Bonn, B. Degen, Th. Edling, H. Fischer, L. Fleischmann, J. U. Grooss, R. Haid, A. Hermanni, G. Kube, P. Leiderer, Th. Loeken, A. Molz, R. B. Moore, A. Osipowicz, E. W. Otten, A. Ricard, M. Schrader, and M. Steininger, Phys. Lett. B300, 210 (1993).

3. E. Holzschuh, M. Fritschi, and W. Kundig, Phys. Lett. B287, 381 (1992).

4. H. Kawami, S. Kato, T. Ohshima, S. Shibata, K. Ukai, N. Morikawa, N. Nogowa, K. Haga, T. Nagafuchi, M. Shigeta, Y. Fukushima and T. Taniguchi, Phys. Lett. B256, 105 (1991).

5. W. Stoeffl, Bull. Am. Phys. Soc. Ser ll 37, 1286 (1992).

6. R. G. H. Robertson, T. J. Bowles, G. J. Stephenson, Jr., D. L. Wark, J. F. Wilkerson and D. A. Knapp, Phys. Rev. Lett. 67, 957 (1991).

7. R. Abela, M. Daum, G. H. Eaton, R. Frosch, B. Jost, P.R. Kettle, and E. Steiner, Phys. Lett. 146B, 431 (1984); B. Jeckelmann, T. Nakada, W. Beer, G. de Chambrier, O. Elsenhans, K. L. Giovanetti, P. F. A. Goudsmit, H. J. Leisi, O. Piller, and W. Schwitz, Phys. Rev. Lett. 56, 1444 (1986).

8. H. Primakoff and S. P. Rosen, Ann. Rev. Nucl. Part. Sci. 31, 195 (1981); and references therein.

9. M. K. Moe, U. C. Irvine preprint, UCI-Neutrino 93-1; T. Bernatowicz, J. Brannon, R. Brazzle, R. Cowsik, C. Hohenberg, and F. Pobseh, Phys. Rev. Lett. 69, 2341 (1992).
10. If, as is argued in the text, the neutrinos are Majorana fermions, \( \Gamma = i\gamma^0\gamma^2 \) will achieve this result.

11. Brian D. Serot and John Dirk Walecka in Adv. Nucl. Phys. (J. N. Negele, and Erich Vogt, eds.), Vol. 16, p. 1 (1986) Plenum Press, NY.

12. G. J. Stephenson Jr. and T. Goldman, in preparation.

13. K. I. MacRae and R. J. Riegert, Nucl. Phys. B244, 513 (1984).

14. R. G. H. Robertson, G. J. Stephenson Jr., J. F. Wilkerson, and D. A. Knapp, in preparation.

15. Steven Weinberg, Phys. Rev. 128, 1457 (1962).

16. Karl-Erik Bergkvist, Nucl. Phys. B39, 317 (1972).

17. Edward W. Kolb, Michael S. Turner, and Terrance P. Walker, Phys. Rev. D34, 2197 (1986).

18. W. C. Haxton and G. J. Stephenson Jr., in Progress in Particle and Nuclear Physics (Sir Denys Wilkinson, ed.), Vol. 12, 409 (Pergamon Press, New York, 1984), and references therein.

19. For a discussion of the effect of non-interacting neutrinos on rotation curves, and a guide to earlier literature, see John P. Ralston and Lesley L. Smith, Ap. J. 367, 54 (1991).

20. Peter J. Quinn, private communication.

21. Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. 98B, 265 (1981); G. B. Gelmini and M. Roncadelli, Phys. Lett. 99B, 411 (1981); H. M. Georgi, S. L. Gashow, and S. Nussimov, Nucl. Phys. B193, 297 (1981).

22. V. Barger, W. Y. Keung, and S. Pakvasa, Phys. Rev. D25, 907 (1982); T. Goldman, Edward W. Kolb, and G. J. Stephenson, Jr., Phys. Rev. D26, 2503 (1982).
23. A. Piepke et al., U. of Heidelberg preprint, 1993; J. C. Vuilleumier, J. Busto, J. Farine, V. Jörgens, L. W. Mitchell, M. Treichel, J. L. Vuilleumier, H. T. Wong, F. Boehm, P. Fisher, H. E. Henrikson, D. A. Imel, M. Z. Igbal, B. M. O’Callaghan-Hay, J. Thomas, and K. Gabathuler, Phys. Rev. D48, 1009 (1993).

24. G. F. Smoot et al., Ap. J. 396, L1 (1992).

Figure Captions

Figure 1. Ratio of $E^*/m^0$ and $m^*/m^0$ as a function of $k/m^0$. Note the logarithmic scale for $k/m^0$, and that $E^*/m^0$ goes through 1 for $k/m^0 < 1$.

Figure 2. Modified Kurie plot, showing the square root of the counting rate versus the energy of the electron. The cut off is due to the Pauli principle suppression by the filled anti-neutrino well; the additional counts beyond the end point are due to the capture of neutrinos bound in the well.
