Conformal Invariance of the D-Particle Effective Action

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Abstract

It is shown that the effective theory of D-particles has conformal symmetry with field-dependent parameters. This is a consequence of the supersymmetry. The string coupling constant is not transformed in contrast with the recent proposal of generalized conformal symmetry by Jevicki et al.[5][6]. This conformal symmetry can also be generalized to other Dp-brane systems.

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1 Introduction

Duality between string theory in Anti-de Sitter (AdS) background space-time and conformal field theory was conjectured in [1]. This correspondence was further elaborated in [2]. Especially near-horizon geometry of nearly coincident D3-branes [3] is $AdS_5 \times S^5$ and its isometry is the conformal symmetry. On the other hand four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory is conformally invariant. It was shown in [4] that there is a correspondence between the conformal symmetries on both sides.

Near-horizon geometry of other Dp-branes is not AdS space-time and this conjecture does not simply apply. The corresponding SYM theory is not apparently conformally invariant because the coupling constant is dimensionful. Recently, however, it was pointed out by Jevicki et al [5] [6] that the near-horizon geometry of Dp-branes can also be interpreted as conformally invariant, if one varies the string coupling constant together with other backgrounds. Then they claimed that by regarding the coupling constant of SYM theory as a background field and transforming this background appropriately with other fields, they can also make SYM theory conformally invariant. They also argued that the 2 conformal transformations on the SYM and supergravity sides are related by some coordinate transformations [6].

We will not adopt this proposal to regard the coupling constant as a background, because a coordinate-dependent coupling constant breaks the special conformal symmetry as well as supersymmetry (SUSY). (See sec 4.) The coupling constant must be kept constant. In this paper we will specialize to the D-particle system and show that this system has conformal symmetry. For this purpose we will use the SUSY transformation to induce a variation of the functional measure which is equivalent to effectively changing the coupling constant. Additional BRS transformation needs to be also performed. The ordinary conformal transformation combined with the SUSY and BRS transformation yields the desired, complete conformal transformation.

In sec 2 we will show that by introducing an auxiliary field the SUSY transformation can be extended to an off-shell-closed symmetry, with respect to whose generator the D-particle action can be rewritten as an exact form. This is the T-dual version of the nil-potent symmetry studied in [7]. In sec 3 this symmetry is used to effectively change the string coupling constant in the action by transformation of the field variables. The parameter of the transformation is chosen to be field-dependent and non-local. In order to cancel the variations of the gauge-fixing and ghost action BRS transforma-
tion with a field-dependent parameter needs to be also performed. In sec 4 these results are used to show that D-particle effective theory has conformal symmetry. Discussions will be given in sec 5.

2 Q-symmetry of the D-particle effective action

The action which describes the low-energy dynamics of D-particles is given by

\[ S = \int dt \frac{1}{g} \text{Tr} \left( \frac{1}{2} \sum_{i=1}^{9} (DX^i)^2 + \frac{1}{4} \sum_{i,j=1}^{9} [X^i, X^j]^2 - \frac{i}{2} \psi^T D\psi - \frac{1}{2} \sum_{i=1}^{9} \psi^T \gamma^i [X^i, \psi] \right), \]

where \( D = \partial_t - iA^0 \) is a gauge covariant derivative and \( X^i, A^0 \) and \( \psi \) are \( N \times N \) hermitian matrices. \( \psi \) is also a sixteen-component spinor. \( g \) is the string coupling constant. \( T \) on \( \psi \) stands for a transposition. Some conventions for \( \gamma \) matrices and the spinor \( \psi \) are given in Appendix. In what follows summation symbols will be omitted. Indices \( i, j, \ldots \) run from 1 to 9, and \( a, b, \ldots \) from 1 to 8.

Action (1) is invariant under the extended SUSY transformation.

\[ \delta X^i = -i \epsilon^T \gamma^i \psi, \quad \delta A^0 = i \epsilon^T \psi, \]
\[ \delta \psi = -(DX^i) \gamma^i \epsilon - \frac{i}{2} [X^i, X^j] \gamma^{ij} \epsilon + \epsilon' \]

(2)

Let us pick up a particular transformation with \( \epsilon' = 0, \epsilon = \epsilon \zeta \) and \( \zeta = (\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, \cdots, 0)^T \). \( \epsilon \) is a Grassmann odd constant. Generality will not be lost by this choice of \( \epsilon \), because any \( \epsilon \) can be put in this form by SO(9) rotation. By substituting the representation of \( \gamma \)'s (34), (35) and that of \( \psi \) (36), (37) into (2), we explicitly obtain the SUSY transformation of the component fields.

\[ \delta X^a = -i \epsilon \psi_a \quad (a = 1, \cdots, 8), \quad \delta X^9 = -i \epsilon \eta, \]
\[ \delta A^0 = i \epsilon \eta, \quad \delta \psi_a = i \epsilon ([X^9, X^a] + iDX^a) \quad (a = 1, \cdots, 8), \]
\[ \delta \bar{\chi} = -\frac{1}{2} \epsilon \bar{E}, \quad \delta \eta = -\epsilon DX^9 \]

(3)

Here \( \psi_a, \bar{\chi} = (\chi^1, \cdots, \chi^7) \) and \( \eta \) are components of \( \psi \) and \( \bar{E} \) a seven-vector function of \( X^a \). These are defined in Appendix.
We could achieve the goal of this paper by working directly with transformation (3). It is, however, more instructive to relate it to the nilpotent transformation of [7]. Let us introduce an auxiliary seven-vector \( \vec{H} \) by adding a term \( \int dt (2g)^{-1} \text{Tr}(\vec{H} + \frac{1}{2} \vec{E})^2 \) to (1). \( \vec{H} \) will coincide with \( -\frac{1}{2} \vec{E} \) by the equations of motion. The transformation rule of \( \vec{\chi} \) is modified to \( \delta \vec{\chi} = \varepsilon \vec{H} \). \( \delta \vec{H} \) is defined to be \( \varepsilon ([X^0, \vec{\chi}] + i D \vec{\chi}) \). Now the square of this new transformation turns out to be equivalent to a time translation up to a gauge transformation: \([Q^2, \vec{\chi}] = i \partial_t \vec{\chi} + [X^0 + A^0, \vec{\chi}], \text{ etc.}\) Here \( Q \) is the generator of this transformation. (\( \delta = \varepsilon Q \)) We will hereafter call this a Q transformation. Importantly, action (1) can be rewritten into a Q-exact form.

\[
S_{coh} = \int dt \frac{1}{g} \left\{ Q, \text{Tr} \left( -\frac{1}{2} \eta DX^0 + \frac{1}{2} \vec{\chi} \cdot \vec{E} + \frac{1}{2} \vec{H} \cdot \vec{H} + i \frac{1}{2} \psi_a [X^a, X^9] - \frac{1}{2} \psi_a DX^a \right) \right\}
\] (4)

This action is the T-dual version of the cohomological action for D-instantons considered in [7].

To quantize this model we will choose the background field gauge. We will decompose \( X^i \) into a background field \( B^i \) and a quantum fluctuation \( Y^i \)

\[
X^i = B^i + Y^i
\] (5)

and choose a gauge function [8]

\[
G = \partial_t A^0 + i [B^i, Y^i].
\] (6)

The gauge fixing and ghost actions are given by

\[
S_{gf} = \int dt \frac{1}{2g} \text{Tr} C^2,
\] (7)

\[
S_{gh} = \int dt i \text{Tr} \left\{ C \left( \partial_t DC + [B^i, [X^i, C]] \right) \right\}.
\] (8)

The total action \( S_{tot} = S_{coh} + S_{gf} + S_{gh} \) is invariant under the BRS transformation

\[
\begin{align*}
\delta_B X^i &= -\lambda [C, X^i], & \delta_B A^0 &= i \lambda DC, \\
\delta_B \psi &= -\lambda \{C, \psi\}, & \delta_B \vec{H} &= -\lambda [C, \vec{H}], \\
\delta_B C &= -\lambda C^2, & \delta_B \bar{C} &= \lambda \frac{1}{g} G.
\end{align*}
\] (9)

Here \( \lambda \) is a Grassmann odd parameter.

1 The algebra of transformation (3) closes only on shell, i.e. when equations of motion are used. By the introduction of \( \vec{H} \), Q part of the SUSY algebra closes off shell.
3 Transformation with a field-dependent parameter

Because of the Q-exact nature of action (III) the string coupling constant \( g \) in (III) can be effectively changed by a Q transformation of fields without varying \( g \) explicitly. The variation \( \delta g \) may even be a function of \( t \). We will carry out this program in two steps.

We first perform a Q transformation

\[
\begin{align*}
\delta_Q X^a &= -i\varepsilon\psi_a, \\
\delta_Q A^0 &= i\varepsilon\eta, \\
\delta_Q \bar{\chi} &= \varepsilon\bar{H}, \\
\delta_Q C &= 0, \\
\delta_Q \bar{C} &= 0,
\end{align*}
\]

with the following parameter.

\[
\varepsilon = \int dt \frac{i}{g^2} \delta g(t) \text{Tr} \left( -\frac{1}{2} \eta DX^9 - \frac{1}{2} \psi_a DX^a + \frac{1}{2} \bar{\chi} \cdot \bar{E} + \frac{1}{2} \bar{\chi} \cdot \bar{H} + \frac{i}{2} \psi_a [X^a, X^9] \right)
\]

This is a field-dependent and non-local transformation. Action(III) is invariant, while the gauge fixing and ghost actions are changed.

\[
\begin{align*}
\delta_Q S_{gf} &= -\varepsilon \int dt \frac{1}{g} \text{Tr} \left\{ G \left( i\partial_t \eta + [B^a, \psi_a] + [B^9, \eta] \right) \right\}, \\
\delta_Q S_{gh} &= -\varepsilon \int dt \text{Tr} \left\{ \bar{C} \left( i\partial_t \{ \eta, C \} + [B^a, \{ \psi_a, C \}] + [B^9, \{ \eta, C \}] \right) \right\}
\end{align*}
\]

The functional measure is not invariant, either, because the transformation parameter depends on the fields. Calculation of the superjacobian is straightforward and we obtain\(^2\)

\[
(1 + \delta_Q) \mathcal{D}(\text{Fields}) =
\]

\[
\mathcal{D}(\text{Fields}) \exp \left\{ i \int dt \frac{-\delta g(t)}{g^2} \text{Tr} \left( -\frac{i}{2} \psi_a D\psi_a + \frac{1}{2} \psi_a [X^9, \psi_a] - \frac{1}{2} \bar{\chi} \cdot [Q, \bar{E}] \\
- \frac{1}{2} \bar{\chi} \cdot [X^9, \bar{\chi}] - \frac{i}{2} \bar{\chi} \cdot D\bar{\chi} + \frac{1}{2} [X^9, X^a]^2 + \frac{1}{2} (DX^a)^2 + \frac{1}{2} (DX^9)^2 \\
- \frac{i}{2} \eta D\eta - \eta [X^a, \psi_a] - \frac{1}{2} \eta [X^9, \eta] + \frac{1}{2} \bar{H}^2 + \frac{1}{2} \bar{H} \cdot \bar{E} \right) \right\}.
\]

\(^2\) Similar calculation of the superjacobian for 4d bosonic gauge theory in a different context was performed in [10]. See also [13].
The exponent on RHS is equal to the difference $i\{S_{coh}(g + \delta g) - S_{coh}(g)\}$. Because $S_{gh}$ does not depend on $g$, and the variation of $S_{gf}$ with respect to $g$ is BRS exact and does not contribute to the path integral, the effect of the transformation is just to change $g$ to $g + \delta g(t)$ in $S_{tot}$.

Secondly, to cancel the variations we perform the BRS transformation with the parameter

$$\lambda = \varepsilon \int dt \text{Tr} \left\{ \left( \partial_t \eta - i[B^a, \psi_a] - i[B^9, \eta] \right) \bar{C} \right\}. \quad (15)$$

While the total action is invariant, the functional measure changes as in (14). It can be checked that this change cancels out (12) and (13) exactly.

To summarize, the combination

$$\delta = \delta_Q + \delta_B \quad (16)$$

has the same effect as changing $g$ into $g + \delta g(t)$. It is important to notice that this infinitesimal transformation cannot be repeated to generate a non-constant $g(t)$, because $Q$ symmetry is broken if $g$ is not a constant.

It is straightforward to eliminate $\vec{H}$ from (16) by using

$$\langle \vec{H} \rangle = -\frac{1}{2} \vec{E}$$

and

$$\langle H_{a\beta}^A(t) H_{\gamma\delta}^B(t') \rangle = ig\delta^{AB}\delta_{a\beta}\delta_{\gamma\delta}(t-t') + \frac{1}{4} E_{a\beta}^A(t) E_{\gamma\delta}^B(t').$$

Here $A, B = 1, \cdots, 7$ are vector indices, respectively. The results are the same as (16) except for replacement of $\varepsilon$ (11) and $\lambda$ (15) by

$$\tilde{\varepsilon} = \int dt \frac{i}{g^2} \delta g(t) \text{Tr} \left( -\frac{1}{2} \eta DX^9 - \frac{1}{2} \psi_a DX^a + \frac{1}{4} \bar{\chi} \cdot \vec{E} + \frac{i}{2} \psi_a [X^a, X^9] \right)$$

and

$$\tilde{\lambda} = \tilde{\varepsilon} \int dt \text{Tr} \left\{ \left( \partial_t \eta - i[B^a, \psi_a] - i[B^9, \eta] \right) \bar{C} \right\}, \quad (17)$$

respectively, and

$$\delta \bar{\chi} = -\frac{1}{2} \tilde{\varepsilon} \vec{E} - \tilde{\lambda} \{ \bar{C}, \bar{\chi} \} - \frac{\delta g(t)}{2g} \bar{\chi}, \quad (19)$$

where the last term comes from the contraction of $2 \vec{H}$’s.

4 Conformal symmetry
It was claimed in [6] that Dp-brane action has conformal symmetry provided one regards $g$ as a background field $g(x)$ and makes it transformed appropriately together with other fields. They called this symmetry a generalized conformal one.

The conformal group is generated by translation, dilatation and special conformal transformation (SCT). We will specialize to the D-particle case. These 3 transformations on $t$ are defined by

$$\delta t = -a \quad \text{(translation)},$$

$$\delta t = -at \quad \text{(dilatation)},$$

$$\delta t = -at^2 \quad \text{(SCT)}.$$  (22)

Here $a$ is an infinitesimal parameter. A field $F(t)$ of a scale dimension $w$ is then transformed as

$$\delta_n F(t) = a(nwt^{n-1} + t^n \partial_t)F(t)$$  (23)

for translation ($n = 0$), dilatation ($n = 1$) and SCT ($n = 2$).

Actions $S$ and $S_{coh}$ are invariant under these transformations only if the string coupling constant $g$ is also transformed like

$$\delta_n g = 3ant^{n-1}g.$$  (24)

The scale dimensions $w$ of the fields are 1, 1, $\frac{3}{2}$, 2 for $X^i$, $A^0$, $\psi$ and $\vec{H}$, respectively. Because (24) for SCT ($n = 2$) depends on $t$ explicitly, the authors of [5] [6] proposed to regard $g$ itself as a function of the coordinate $t$ and assumed a transformation rule

$$\delta_n g(t) = a(3nt^{n-1} + t^n \partial_t)g(t).$$  (25)

As for $S_{gf}$ and $S_{gh}$ these are not invariant under SCT even if $C$ and $\bar{C}$ are assigned scale dimensions 0 and $-1$. By using the formalism of [10] the authors of [6] then showed that the variations of these actions can be cancelled by a non-local BRS transformation (9) with $\lambda = \nu_n,$

$$\nu_n = -n(n - 1)ia \int dt Tr \bar{C} A^0$$  (26)

We do not adopt their proposal to regard $g$ as a $t$-dependent background field $g(t)$ because of the following reasons.

\[\text{Here } n = 2 \text{ for SCT. An index } n \text{ is introduced for later convenience.}\]
• Once $g$ is promoted to a function of $t$, then action (1) is no longer invariant under SCT because the integration by parts is used in the proof of invariance. After a simple calculation we indeed obtain

$$\delta_2 S = a \int dt \frac{1}{g(t)} \partial_t \text{Tr}(X^i)^2.$$  (27)

• If $g$ is a function of $t$, $S$ is not invariant under SUSY transformation (2) due to the same reason as above. This symmetry is important and cannot be abandoned.

For these reasons we will leave $g$ a constant and will not change it. In the previous sections we showed that the same effect as changing $g$ can be accomplished by transforming the field variables. By combining (16) and the BRS transformation (9) with the parameter (26), we obtain the 3 conformal transformations ($n = 0, 1, 2$)

$$\Delta_n X^i = a(nt^{-1} + t^n \partial t)X^i - i \bar{\bar{\epsilon}}_n \zeta^\gamma \gamma^i \psi - (\bar{\bar{\lambda}}_n + \nu_n)[C, X^i],$$
$$\Delta_n A^0 = a(nt^{-1} + t^n \partial t)A^0 + i \bar{\bar{\epsilon}}_n \zeta^T \psi + i(\bar{\bar{\lambda}}_n + \nu_n)DC,$$
$$\Delta_n \psi_a = a(\frac{3}{2} nt^{-1} + t^n \partial t) \psi_a + i \bar{\bar{\epsilon}}_n([X^9, X^a] + i DX^a) - (\bar{\bar{\lambda}}_n + \nu_n)\{C, \psi_a\},$$
$$\Delta_n \bar{\chi} = a(\frac{3}{2} nt^{-1} + t^n \partial t) \bar{\chi} - \frac{1}{2} \bar{\bar{\epsilon}}_n \bar{E} - \frac{\delta g}{2g} \bar{\chi} - (\bar{\bar{\lambda}}_n + \nu_n)\{C, \bar{\chi}\},$$
$$\Delta_n \eta = a(\frac{3}{2} nt^{-1} + t^n \partial t) \eta - \bar{\bar{\epsilon}}_n DX^9 - (\bar{\bar{\lambda}}_n + \nu_n)\{C, \eta\},$$
$$\Delta_n C = at^n \partial t C - (\bar{\bar{\lambda}}_n + \nu_n)C^2,$$
$$\Delta_n \bar{C} = a(-nt^{-1} + t^n \partial t) \bar{C} + (\bar{\bar{\lambda}}_n + \nu_n)\frac{1}{g}G.$$  (28)

Here the first 2 lines are simplified by using $\zeta$, $\bar{\bar{\epsilon}}_n$ and $\bar{\bar{\lambda}}_n$ are (17) and (18), respectively, with $\delta g$ replaced by $\delta_n g$ (24).

In the rest of this section we will compute the expectation value $\langle \Delta_n X^i \rangle$, which will become a symmetry transformation $\Delta_n B^i$ of the effective action $\Gamma[B^i]$. This may be performed in perturbative expansions in $g$. Because $\bar{\bar{\epsilon}}_n$ (17) and $\bar{\bar{\lambda}}_n$ (18) are of order $O(g^{-1})$, it turns out that in order to obtain $O(g^m)$ result we have to perform $(m + 1)$-loop calculation. Here we will present only the result of one-loop calculation ($O(g^0)$). The result and details of two-loop calculation will be reported in a forthcoming paper [12].

4In fact we do not change $g$. Instead we use transformation (14) to cancel the effective variation of $g$ due to the conformal transformation. Thus $g$ remains constant. This also enables us to repeat infinitesimal transformations to obtain a finite one.
The background field $B^i(t)$ is diagonal and linear in $t$. We found up to first order in $\dot{B} = \partial_t B$ that $\langle -\dot{\lambda}_n[C,X^i]\rangle$ is of order $\mathcal{O}(g^1)$ and
\[
\langle -i\tilde{\epsilon}_n\xi^T\gamma^i\psi \rangle_{\alpha\beta} = -\frac{1}{2}\delta_{\alpha\beta} \int dt' \frac{\delta_n g(t')}{g} \theta(t-t') \dot{B}^i_\alpha(t').
\] (29)

Here $B^i_\alpha$ is the $\alpha$-th diagonal component of $B^i$ and $\theta(t-t')$ is a step function. $\langle -\nu_n[C,X^i]\rangle$ was obtained in [6] and is also of order $\mathcal{O}(g^1)$. We thus obtain the ‘quantum’ transformation rule
\[
\Delta_1 B^i_\alpha(t) = a(1 + t\partial_t)B^i_\alpha(t) - \frac{3}{2}aB^i_\alpha(t) = a\left(-\frac{1}{2} + t\partial_t\right)B^i_\alpha(t) \tag{30}
\]
for dilatation and
\[
\Delta_2 B^i_\alpha(t) = a(2t + t^2\partial_t)B^i_\alpha(t) - 3a\int^t dt' t' \dot{B}^i_\alpha(t') = a\left(-t + t^2\partial_t\right)B^i_\alpha(t) + 3a\int^t dt' B^i_\alpha(t') \tag{31}
\]
for SCT. It is easy to verify that the tree-level effective action
\[
\Gamma_0[B] = \int dt \frac{1}{2g} (\dot{B}^i)^2 \tag{32}
\]
is left invariant under (30) and (31). We note that the scale dimension $w$ of the background $B^i_\alpha$ has shifted from 1 to $-\frac{1}{2}$ due to quantum effects. We also find that SCT rule (31) acquired an extra non-local term. Nonetheless we can check that (30), (31) and $\Delta_0 B^i_\alpha = a\partial_t B^i_\alpha$ generate the conformal algebra.

5 Discussion

We found that the low-energy effective theory of D-particles is invariant under conformal transformation (28). The transformation rule is obtained explicitly, although the parameters depend on the field variables in a non-local way. Here we stress the point that $g$ is not changed under the transformation. We also found that the scale dimension of $X^i$ changed from 1 to $-\frac{1}{2}$ due to quantum effects.

Because the actions of Dp-branes are related by T-dual transformation [11], the procedure adopted in this paper is easily generalized to all Dp-branes. It can be verified that all Dp-brane theories have similar conformal symmetry. This conformal symmetry will put strong constraints on the correlation functions. Some lower-point functions may be obtained by solving
conformal Ward-Takahashi identities. Action (1) also defines the Matrix theory [9]. Study of the conformal W-T identities may also shed some light on the understanding of M theory.

Another important issue is the AdS/CFT correspondence. Because the near-horizon geometry of Dp-branes \( p \neq 3 \) is not an AdS space-time and the conformal group is not its isometry, this correspondence does not simply apply. Because Dp-brane system has turned out to have conformal symmetry, however, it is expected that its near-horizon geometry may also have the corresponding ‘conformal symmetry’. If such a symmetry is found, Dp-brane effective theory may also be interpreted as a boundary ‘conformal field theory’ in the classical background of the Dp-branes. It will play an important rôle in determining the effective action for the radial distance of a probe Dp-brane in the background field of N coincident Dp-branes placed at the origin.[1]

The non-local nature of transformation (31), however, makes the problem difficult and the realization of such symmetry on the supergravity side is not yet clear. The calculation of \( \langle \Delta_n X^i \rangle \) to higher orders may elucidate this point. The result will be reported elsewhere[12].

Finally, generalization of the present analysis to the superconformal transformation will be straightforward and may be useful in putting further constraints on the Dp-brane dynamics.

Note added

If \( g(t) \) is set a constant after SCT, the variation of \( S \) (27) vanishes. We were informed by T. Yoneya that this is the generalized conformal symmetry of [5][6]. We were also informed that he could extend this conformal symmetry to that for a non-constant \( g \) by using the SO(2,1) orbit of \( g \). We thank T. Yoneya for comments and discussions.

Appendix

\( \gamma^i, \quad i = 1, \cdots, 9 \) are 16 \( \times \) 16 real symmetric matrices and satisfy Clifford algebra

\[
\{ \gamma^i, \gamma^j \} = 2 \delta^{ij}.
\] (33)

We choose the following special representation.

\[
\gamma^i = i \sigma_2 \otimes \mu^i = \begin{pmatrix} 0 & \mu^i \\ -\mu^i & 0 \end{pmatrix} \quad (i = 1, \cdots, 7),
\]
\[ \gamma^8 = \sigma_1 \otimes 1_8 = \begin{pmatrix} 0 & 1_8 \\ 1_8 & 0 \end{pmatrix}, \]
\[ \gamma^9 = \sigma_3 \otimes 1_8 = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix} \] (34)

8 \times 8 matrices \( \mu^i \) are given by
\[
\begin{align*}
\mu^1 &= i\sigma_2 \otimes i\sigma_2 \otimes i\sigma_2, & \mu^2 &= 1_2 \otimes \sigma_1 \otimes i\sigma_2, & \mu^3 &= 1_2 \otimes \sigma_3 \otimes i\sigma_2, \\
\mu^4 &= \sigma_1 \otimes i\sigma_2 \otimes 1_2, & \mu^5 &= \sigma_3 \otimes i\sigma_2 \otimes 1_2, & \mu^6 &= i\sigma_2 \otimes 1_2 \otimes \sigma_1, \\
\mu^7 &= i\sigma_2 \otimes 1_2 \otimes \sigma_3. & & & & & (35)
\end{align*}
\]

The spinor \( \psi \) is decomposed into 2 eight-component spinors \( \psi^{(i)} \):
\[
\psi = \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \end{pmatrix}
\] (36)

Their components are given by
\[
\begin{align*}
\psi^{(1)} &= \frac{1}{\sqrt{2}} (\eta + \chi^7, -\chi^2 - \chi^4, \chi^2 - \chi^4, \eta - \chi^7, \chi^1 - \chi^6, -\chi^3 - \chi^5, -\chi^3 + \chi^5, -\chi^1 - \chi^6)^T, \\
\psi^{(2)} &= \frac{1}{\sqrt{2}} (-\psi_2 + \psi_8, \psi_3 - \psi_5, \psi_3 + \psi_5, \psi_2 + \psi_8, \psi_1 + \psi_7, -\psi_4 + \psi_6, \psi_4 + \psi_6, \psi_1 - \psi_7)^T.
\end{align*}
\] (37)

The seven-vector \( \vec{E} = (E^1, \ldots, E^7) \) is defined by
\[
\begin{align*}
E^1 &= 2i(-[X^1, X^2] - [X^3, X^4] - [X^5, X^6] - [X^7, X^8]), \\
E^2 &= 2i([X^1, X^4] + [X^2, X^3] - [X^5, X^8] - [X^6, X^7]), \\
E^3 &= 2i([-X^1, X^3] + [X^2, X^4] - [X^5, X^7] + [X^6, X^8]), \\
E^4 &= 2i([X^1, X^6] + [X^2, X^5] + [X^3, X^8] + [X^4, X^7]), \\
E^5 &= 2i([-X^1, X^5] + [X^2, X^6] + [X^3, X^7] - [X^4, X^8]), \\
E^6 &= 2i([X^1, X^8] + [X^2, X^7] - [X^3, X^6] - [X^4, X^5]), \\
E^7 &= 2i([-X^1, X^7] + [X^2, X^8] - [X^3, X^5] + [X^4, X^6]).
\end{align*}
\] (38)
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