SUSY-Induced CP Violation in $t$ Decays at $e^-e^+$ Colliders

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Abstract

CP violation in the decays $t \rightarrow \ell^+\nu b$ and $\bar{t} \rightarrow \ell^-\bar{\nu}\bar{b}$ from the production process $e^-e^+ \rightarrow t\bar{t}$ is discussed. Since the asymmetry proposed as a measure of CP violation vanishes even at the one-loop level in the Standard Model (SM), it may be a useful tool to search for sources of CP violation outside of the SM. As an illustration the asymmetry has been computed within supersymmetric extensions of the SM. Prospects for its measurement at future high-energy linear colliders are discussed.
1 Introduction

Collider physics can prove to be an important area, complementary to the low-energy kaon and B-meson physics, in searching for CP violation [1, 2]. In particular, future linear high-energy $e^-e^+\rightarrow t\bar{t}$ colliders can provide very useful laboratories to study CP violation in the top-quark system. The top quark being very heavy can offer a few relevant advantages for the study of CP violation:

- If $m_t > 130$ GeV, it would decay before it can form a bound state [3]; therefore the perturbative description is much more reliable.
- For the same reason, spin effects would not be diluted by hadronization effects — which provides a very useful tool in searching for CP-violating effects.
- Again, because of its large mass, the top quark properties are sensitive to interactions mediated by Higgs bosons [4].
- The Kobayashi-Maskawa [5] mechanism of CP violation is strongly suppressed for the top quark since its mixing with other generations is very weak; therefore it is sensitive to non-conventional sources of CP violation.

Simultaneously one should bear in mind that in spite of spectacular successes obtained in experimental high-energy physics (e.g. precision tests of the Standard Model) the origin of CP violation is still a mystery. The standard theory of Kobayashi and Maskawa [5] provides explicit CP violation through phases of Yukawa couplings; however, many other attractive schemes of CP violation may emerge in extensions of the Standard Model (SM). Those unconventional sources of CP violation can be tested in $t\bar{t}$ production at future linear $e^-e^+$ colliders. For instance, CP violation induced by the neutral scalar sector in the two-Higgs-doublet standard model [4] (2HDM) may lead to observable effects [6] in $e^-e^+\rightarrow t\bar{t}$, provided sufficient luminosity can be obtained. The supersymmetric extension of the Standard Model (SSM) can also produce CP violation at the one-loop level both in $t\bar{t}$ production and in its decay [11, 12] at $e^-e^+$ colliders. One can also imagine some unknown high-scale theory, which induces effective, low-energy, CP-violating interactions. Here, we will generalize the method of observation of CP violation, proposed originally [7] for $W-g$ fusion at a $pp$ collider, to the $e^-e^+$ environment, and calculate the CP-violating asymmetry within a supersymmetric extension of the Standard Model (SSM). We will emphasize that the asymmetry we define is (at least) at the leading order of perturbation expansion a measure of CP violation in the top quark decay and that it is not affected by any CP violation taking place at $t\bar{t}$ production.

2 The Strategy

We will consider production of $t\bar{t}$ pairs at future high-energy linear colliders. We assume that the production mechanism can be described by the $\gamma$ and $Z$ exchange where CP violation may enter at $t\bar{t}$ production vertex. The production is then followed by the decays $t \rightarrow W^+b$ and $\bar{t} \rightarrow W^-\bar{b}$, and eventually by $W^+ \rightarrow \ell^+\nu$ and $W^- \rightarrow \ell^-\bar{\nu}$. Again,
CP violation may be present at $t \bar{t}$ decays; however we assume that $W^{+}$ and $W^{-}$ decay conventionally, i.e. without CP violation.

In order to describe $t$ and $\bar{t}$ decays we will use the most general 4-form factor parametrization of $t \rightarrow W^{+}b$ and $\bar{t} \rightarrow W^{-}\bar{b}$ decay vertices:

$$
\Gamma^{\mu} = \frac{-igV_{tb}^{KM}}{\sqrt{2}} \bar{u}(p_b) \left[ \gamma^{\mu}(f_{1}^{L}P_{L} + f_{1}^{R}P_{R}) - \frac{i\sigma^{\mu\nu}k_{\nu}}{m_{W}}(f_{2}^{L}P_{L} + f_{2}^{R}P_{R}) \right] u(p_t),
$$

$$
\bar{\Gamma}^{\mu} = \frac{-igV_{tb}^{KM*}}{\sqrt{2}} \bar{v}(p_{\bar{t}}) \left[ \gamma^{\mu}(\bar{f}_{1}^{L}P_{L} + \bar{f}_{1}^{R}P_{R}) - \frac{i\sigma^{\mu\nu}k_{\nu}}{m_{W}}(\bar{f}_{2}^{L}P_{L} + \bar{f}_{2}^{R}P_{R}) \right] v(p_{\bar{b}}),
$$

where $P_{R/L}$ are projection operators, $k$ is the $W$ momentum, $V^{KM}$ is the Kobayashi–Maskawa matrix and $g$ is the $SU(2)$ gauge coupling constant. Since $W$ decays into massless fermions, two other, in principle present, form factors do not contribute.

It is easy to show that [2, 7]:

$$
f_{1}^{L/R} = \pm \bar{f}_{1}^{L/R}, \quad f_{2}^{L/R} = \pm \bar{f}_{2}^{L/R},
$$

where the upper (lower) signs are those for contributions induced by CP-conserving (- violating) interactions.

The general form of the phase-space element for the process $e^{-}e^{+} \rightarrow t\bar{t} \rightarrow W^{+}W^{-}\bar{b}b \rightarrow \ell^{+}\ell^{-}\nu\bar{\nu}bb$,

$$
d\Phi = (2\pi)^{4} \prod_{i=1}^{6} \frac{d^{3}p_{i}}{(2\pi)^{3}2E_{i}} \delta^{(4)}(P_{m} - \sum_{i=1}^{6} p_{i}),
$$

could be written as:

$$
d\Phi = (2\pi)^{-4}ds_{t}ds_{\bar{t}}ds_{W^{+}}ds_{W^{-}}d\Phi(e^{-}e^{+} \rightarrow t\bar{t}) \times 
\text{d} \Phi(t \rightarrow W^{+}b) \text{d} \Phi(W^{+} \rightarrow \ell^{+}\nu) \text{d} \Phi(\bar{t} \rightarrow W^{-}\bar{b}) \text{d} \Phi(W^{-} \rightarrow \ell^{-}\bar{\nu}),
$$

where $s_{x}$ denotes the corresponding invariant mass of the decaying particles. The appropriate matrix element for a given helicity final state reads:

$$
(h_{b}, h_{\bar{b}}, h_{\ell^{+}}, h_{\ell^{-}}, h_{\nu}, h_{\bar{\nu}}) = D_{t}(s_{t})D_{\bar{t}}(s_{\bar{t}})D_{W^{+}}(s_{W^{+}})D_{W^{-}}(s_{W^{-}}) \sum_{h_{t}, h_{\bar{t}}} (h_{t}, h_{\bar{t}})
\times \sum_{\lambda_{W^{+}}} (h_{t}, \lambda_{W^{+}}, h_{b})(\lambda_{W^{+}}, h_{\ell^{+}}, h_{\nu}) \sum_{\lambda_{W^{-}}} (h_{\bar{t}}, \lambda_{W^{-}}, h_{\bar{b}})(\lambda_{W^{-}}, h_{\ell^{-}}, h_{\bar{\nu}}),
$$

where $h_{x}$ denotes the $x$-helicity and the helicities of the initial $e^{-}e^{+}$ have been omitted in the amplitudes for $e^{-}e^{+} \rightarrow t\bar{t}$: $(h_{t}, h_{\bar{t}})$; $D_{x}$ are the propagators of decaying $t$, $\bar{t}$, $W^{+}$ and $W^{-}$. Since we are interested in decays of real particles we will use for $D_{x}(s_{x})$ the Breit-Wigner parametrization:

$$
D_{x}(s_{x}) = \frac{1}{s_{x} - m_{x}^{2} + im_{x}\Gamma_{x}}.
$$

The absolute value of the amplitude squared can be written as:

$$
|\langle \cdot \cdot \rangle|^{2} = |D_{t}(s_{t})|^{2}|D_{\bar{t}}(s_{\bar{t}})|^{2}|D_{W^{+}}(s_{W^{+}})|^{2}|D_{W^{-}}(s_{W^{-}})|^{2} \sum_{h_{t}, h_{\ell^{+}}, h_{\ell^{-}, h_{\nu}, h_{\bar{\nu}}} \lambda_{W^{+}}} (h_{t}, h_{\ell^{+}}, h_{\ell^{-}}, h_{\nu}) (h_{\bar{t}}, h_{\ell^{-}}, h_{\bar{\nu}}) \times
$$
\[
\sum_{\lambda_{W^+}} (h_t, \lambda_{W^+}, h_b)(\lambda_{W^+}, h_{\ell^+}, h_\nu) \sum_{\lambda_{W^+}'} (h'_t, \lambda_{W^+'}, h'_b)(\lambda_{W^+'}, h'_{\ell^+}, h'_{\nu}) \times (8)
\]

Within the narrow-width approximation [8] one can easily perform phase-space integrations over invariant masses of decaying \( t, \bar{t}, W^+ \) and \( W^- \) since:

\[
|D_x(s_x)|^2 \simeq \frac{\pi}{m_x \Gamma_x} \delta(s_x - m_x^2).
\]

We concentrate on the decay products of the top quark and therefore we shall integrate over \( d\Phi(\bar{t} \rightarrow W^- \bar{b}) \) and \( d\Phi(W^- \rightarrow \ell^- \bar{\nu}) \):

\[
\int d\Phi(\bar{t} \rightarrow W^- \bar{b}) d\Phi(W^- \rightarrow \ell^- \bar{\nu}) \sum_{\lambda_{W^-}} (h_{\bar{t}}, \lambda_{W^-}, h_{\bar{b}})(\lambda_{W^-}, h_{\ell^-}, h_{\bar{\nu}}) \times (10)
\]

\[
\sum_{\lambda_{W^-}'} (h'_{\bar{t}}, \lambda_{W^-'}, h'_{\bar{b}})(\lambda_{W^-'}, h'_{\ell^-}, h'_{\bar{\nu}}) = \delta_{h_{\bar{t}}h'_{\bar{t}}} 2m_t \Gamma(\bar{t} \rightarrow W^- \bar{b}) 2m_W \Gamma(W^- \rightarrow \ell^- \bar{\nu}),
\]

where a summation over the final-state helicities has been implicitly assumed. It will be useful to define normalized \((\text{Tr } \rho = 1)\) top-quark density matrix:

\[
\sum_{h_{\bar{t}}}(h_{\bar{t}}, h_{\bar{t}}) (h'_{\bar{t}}, h'_{\bar{t}})^* = \rho_{h_{\bar{t}}h'_{\bar{t}}} \sum_{h_{\bar{t}}h'_{\bar{t}}} |(h_{\bar{t}}, h_{\bar{t}})|^2,
\]

again summed over initial \( e^- e^+ \) helicities.

The differential cross-section for the total decay sequence could now be written as:

\[
\frac{d\sigma_{\text{tot}}}{d\Phi(t \rightarrow W^+ b) \ d\Phi(W^+ \rightarrow \ell^+ \nu)} = \frac{BR(\bar{t} \rightarrow W^- \bar{b})}{2m_t \Gamma_t} \frac{BR(W^- \rightarrow \ell^- \bar{\nu})}{2m_W \Gamma_W} d\sigma(e^- e^+ \rightarrow t\bar{t}) \times (12)
\]

\[
\sum_{h_{\bar{t}}h'_{\bar{t}}} \rho_{h_{\bar{t}}h'_{\bar{t}}} \sum_{\lambda_{W^+}} (h_{\bar{t}}, \lambda_{W^+}, h_{\bar{b}})(\lambda_{W^+}, h_{\ell^+}, h_\nu) \left[ \sum_{\lambda_{W^+}'} (h'_{\bar{t}}, \lambda_{W^+'}, h'_{\bar{b}})(\lambda_{W^+'}, h'_{\ell^+}, h'_{\nu}) \right]^*.
\]

The factor \( BR(W^- \rightarrow \ell^- \bar{\nu}) \) will become the branching fraction of \( W^- \) into hadrons if we use the more abundant non-leptonic mode. Now, we are in a position to make use of the asymmetry defined in Ref. [9] by the integration over the total top-quark decay phase space \( d\Phi(t \rightarrow W^+ b) \) and a restricted one for \((\ell^+ \nu)\):

\[
\mathcal{A}^t \equiv \frac{N_t}{D_t},
\]

where

\[
N^t \sim \int d\Phi(t \rightarrow W^+ b) \int_1^{1} d\cos(\theta_{\ell^+}) \left[ \int_0^{\pi} - \int_{-\pi}^{0} \right] d\phi_{\ell^+} \frac{d\sigma_{\text{tot}}}{d\Phi(t \rightarrow W^+ b) \ d\Phi(W^+ \rightarrow \ell^+ \nu)}
\]

\[
D^t \sim \int d\Phi(t \rightarrow W^+ b) \int_1^{1} d\cos(\theta_{\ell^+}) \left[ \int_0^{\pi} + \int_{-\pi}^{0} \right] d\phi_{\ell^+} \frac{d\sigma_{\text{tot}}}{d\Phi(t \rightarrow W^+ b) \ d\Phi(W^+ \rightarrow \ell^+ \nu)};
\]
\( \theta_{e^+} \) and \( \phi_{e^+} \) are the polar and azimuthal angles of the \( \ell^+ \) defined in the \( W^+ \) rest frame \[4\]. For the density matrix we shall adopt the following parametrization \[9\]:

\[
P^t = \frac{1}{2} \left[ \frac{1 + P_{\parallel}^t}{P_{\perp}^t e^{-i\alpha}} \right] \]

(14)

where \( P_{\perp}^t \) and \( P_{\parallel}^t \) describe the polarization of the top quark \[3\]:

\[
s^t = P_{\perp}^t(0, \hat{s}) + \frac{P_{\parallel}^t}{m_t}(|p_t|, E_t, \hat{p}_t).
\]

(15)

In the above equation, \( E_t \) and \( \hat{p}_t \) denote the top-quark energy and the direction of its 3-momentum, respectively; \( \hat{s} \) is a unit 3-vector perpendicular to \( \hat{p}_t \) and the angle \( \alpha \) specifies the direction of \( \hat{s} \). A direct calculation leads to the following result:

\[
\mathcal{A}^t = h(m_t) \text{Im}(f_1^t f_2^{R*}) \frac{d\sigma(e^- e^+ \rightarrow t\bar{t}) P_{\parallel}^t(\theta_{t\bar{t}})}{d\sigma(e^- e^+ \rightarrow t\bar{t})},
\]

(16)

where

\[
h(m) = \frac{3\pi}{8} \frac{m^2 - m_W^2}{m^2 + 2m_W^2}.
\]

Form factors \( f_1^t \) and \( f_2^R \) are defined in Eq. \[1\]. The tree-level value \( f_1^t = 1 \) will be adopted hereafter. \[4\] As we can see, only parallel-spin components are relevant for our considerations. Under \( \phi_{e^+} \), integration all non-diagonal elements of the density matrix cancelled. It is worth-while to remember that the longitudinal polarization \( P_{\parallel}^t \) is a function of the \( t\bar{t} \) production angle \( \theta_{t\bar{t}} \).

Imaginary parts of \( f_2^R \) may be induced not only by CP-violating interactions, but also by final-state CP-conserving interactions like \( \gamma \) or \( Z \) exchange between final \( b \) and \( W^+ \). In order to cancel all such CP-conserving contributions \[9\] to \( \mathcal{A}^t \), we define a new asymmetry \( \mathcal{A} \), adding to \( \mathcal{A}^t \) its analogue for \( \bar{t} \), \( \mathcal{A}^\ell \):

\[
\mathcal{A} \equiv \mathcal{A}^t + \mathcal{A}^\ell.
\]

(17)

Let us split CP-conserving (CPC) and CP-violating (CPV) contributions to \( f_2^R \):

\[
f_2^R = f_2^{R, CPC} + f_2^{R, CPV}.
\]

Using relations, see Eq. \[3\], between form factors for top and antitop decays one can write \( \mathcal{A} \) as:

\[
\mathcal{A} = -h(m_t) \frac{\text{Im}(f_2^{R, CPC})(P_{\parallel}^t + P_{\parallel}^\ell) + \text{Im}(f_2^{R, CPV})(P_{\parallel}^t - P_{\parallel}^\ell))}{d\sigma(e^- e^+ \rightarrow t\bar{t})}.
\]

(18)

However, since \( (P_{\parallel}^t + P_{\parallel}^\ell) \) vanishes at the tree-level approximation to the production mechanism \[4\], it is seen that \( \mathcal{A} \) is defined in such a way that all CP-conserving contributions to \( \mathcal{A}^t \) and \( \mathcal{A}^\ell \) cancel in leading order:

\[
\mathcal{A} = -2h(m_t) \text{Im}(f_2^{R, CPV}) \frac{P_{\parallel}^t(\theta_{t\bar{t}}) d\sigma(e^- e^+ \rightarrow t\bar{t})}{d\sigma(e^- e^+ \rightarrow t\bar{t})}.
\]

(19)

\[3\] Notice that \( f_2^{R, CPV} \) vanishes at the tree level. It is easy to see that \( f_2^{R, CPV} \) is zero in the SM, even at the one-loop level; therefore, \( \mathcal{A} \) is very sensitive to non-standard sources of CP violation.
The above shows that our asymmetry is not sensitive to possible CP violation in the $t\bar{t}$ production mechanism, since the production entered at the tree level through $P_t^t$. The longitudinal polarization is given by the following formula [4]:

$$P_{\parallel}(\theta_{t\bar{t}}) = \frac{|(++)|^2 + |(+−)|^2 − |(−+)|^2 − |(−−)|^2}{|(+−)|^2 + |(+−)|^2 + |(−+)|^2 + |(−−)|^2}, \tag{20}$$

In order to calculate $P_{\parallel}$ we will assume that the production process $e^−e^+ → t\bar{t}$ can be correctly described by $\gamma$ and $Z$ $s$-channel exchange. We start by writing down the general form factors of the $Vt\bar{t}$ interaction ($V = \gamma, Z$). The vertex amplitude $ie\Gamma^V$ for the virtual $V$ can be parametrized by the following expression:

$$\Gamma^V = c_v\gamma_\mu + c_a\gamma_\mu\gamma_5 + c_d^V i\gamma_5 \frac{p_\mu - p_\mu}{2m_t} + \cdots. \tag{21}$$

We use the tree-level values for $c_v$ and $c_a$. They are

$$c_v^2 = 2/3, \quad c_a^2 = 0,$$

$$c_v^2 = \frac{1/2 - 2/(3s_W^2)}{\sqrt{s_W^2(1 - s_W^2)}}, \tag{22}$$

$$c_a^2 = -1/[4\sqrt{s_W^2(1 - s_W^2)}],$$

where $s_W \equiv \sin(\theta_W)$.

The helicity amplitudes $(h_t, h_{\bar{t}})$ for the process $e^−e^+ → t\bar{t}$ at the scattering angle $\theta_{t\bar{t}}$ have been given in the literature [3, 4]. For the initial $e^−e^+$ helicity configuration of $(−+)$, we have

$$(-+) = e^2[c_v^2 + r_Lc_v^2 − \beta r_Lc_a^2](1 + \cos \theta_{t\bar{t}})$$

$$(+−) = e^2[c_v^2 + r_Lc_v^2 + \beta r_Lc_a^2](1 − \cos \theta_{t\bar{t}})$$

$$(-−) = e^2[2t(c_v^2 + r_Lc_v^2) − (i/2)(c_d^V + r_Lc_a^2)\beta/t] \sin \theta_{t\bar{t}}$$

$$(++) = e^2[2t(c_v^2 + r_Lc_v^2) + (i/2)(c_d^V + r_Lc_a^2)\beta/t] \sin \theta_{t\bar{t}}. \tag{23}$$

The dimensionless variables are defined by $t = m_t/\sqrt{s}$, $z = m_Z/\sqrt{s}$, $\beta^2 = 1 − 4t^2$. The $Z$ propagator and its coupling to the left handed electron gives $−e\gamma\beta/s$, with

$$r_L = (1/2 − s_W^2)/[(1 − z^2)\sqrt{s_W^2(1 − s_W^2)}]. \tag{24}$$

Similarly, we obtain formulas for the initial $e^−e^+$ configuration $(−+)$ with $r_L$ replaced by $r_R$,

$$r_R = −s_W^2/[(1 − z^2)\sqrt{s_W^2(1 − s_W^2)}], \tag{25}$$

and $\cos \theta_{t\bar{t}}$ by $−\cos \theta_{t\bar{t}}$ and $\sin \theta_{t\bar{t}}$ by $−\sin \theta_{t\bar{t}}$ in Eq. (23).

Using the above formulas $P_{\parallel}(\cos \theta_{t\bar{t}})$ can be calculated. We plot it in Fig. [4].

The $\mathcal{A}$ could also be rewritten as:

$$\mathcal{A} = −2h(t)m_{t}\text{Im}(f^R_{CPV}) \sum_{h_{\bar{t}}=+,-} \left\{ \frac{dσ(e^−e^+ → t\bar{t})_{(+,h_{\bar{t}})}}{dσ(e^−e^+ → t\bar{t})} − \frac{dσ(e^−e^+ → t\bar{t})_{(-,h_{\bar{t}})}}{dσ(e^−e^+ → t\bar{t})} \right\}. \tag{26}$$
The longitudinal polarization of top quarks produced in $e^−e^+ \rightarrow t\bar{t}$, $P^t_∥$ as a function of the $\cos \theta_{t\bar{t}}$ for several top-quark masses, $m_t = 100$, 150 and 200 GeV.

Here the subscripts ($\pm, h_t$) specify the spin configuration of the $t\bar{t}$ system. In order to maximize the number of available events we will present also an integrated form of the asymmetry $A$ obtained by integration over $t\bar{t}$ production phase space:

$$A_{int} = -2h(m_t)\text{Im}(f^R_{2\,CPV}) \frac{\int P^t_∥(\theta_{t\bar{t}})d\sigma(e^−e^+ \rightarrow t\bar{t})}{\sigma(e^−e^+ \rightarrow t\bar{t})}.$$  \hspace{1cm} (27)

As one can see from Fig.1, $P^t_∥(\cos \theta_{t\bar{t}})$ switches its sign between 0 and $\pi$, therefore $A_{int}$ integrated over the full $t\bar{t}$ phase space suffers from strong cancellation. However, if we restrict the region of $\cos \theta_{t\bar{t}}$ in such a way that the differential asymmetry $A$ is either positive or negative, we can effectively enhance the integrated asymmetry.

3 Illustration: Supersymmetric Standard Models

In Ref.[7] the asymmetry $A$ has been calculated in the 2HDM, where CP violation appears through mixing of CP-even and CP-odd states in the mass matrix of neutral scalars. CP
violation within this model has been extensively discussed in the recent literature [6, 7, 10]. Many papers have been devoted to possible tests of CP violation induced in a SSM at colliders [11, 12, 13]. Here we will calculate our $A$ also within a SSM.

The asymmetry $A$ cannot be produced by any renormalizable interactions at the tree level of perturbation expansion. Therefore it may appear either as an artefact of some unknown high-scale theory in an effective low-energy Lagrangian or it can be generated at a loop level in the SM or its extensions. However, in the SM, $f_{2CPV}^R$ vanishes, even at the one-loop level; we therefore illustrate the above general consideration by the one-loop-generated $f_{2CPV}^R$ in the SSM. We write down the relevant interaction:

$$\mathcal{L} = \frac{i\sqrt{2}g_s}{\sqrt{2}}[\bar{t}_L^a T^a(\bar{\lambda}^a t_L) + \bar{t}_R^a T^a(\bar{\lambda}^a t_R)] + (t \leftrightarrow b)$$

$$- \frac{i}{\sqrt{2}} V_{tb}^{KM} \lambda \bar{\tilde{t}}_L W^{-\mu} + h.c.,$$

(28)

where $g_s$ is the QCD coupling constant. For our purpose the most relevant new source of CP violation, which appears in the SSM, would be the phase in the $\tilde{t}_L$–$\tilde{t}_R$ mixing. The stop quarks of different handedness are related to the stop-quark mass eigenstates $\tilde{t}_+ , \tilde{t}_-$ through the following transformation:

$$\tilde{t}_L = \cos \alpha_t \tilde{t}_- - e^{i\phi_t} \sin \alpha_t \tilde{t}_+$$

$$\tilde{t}_R = e^{-i\phi_t} \sin \alpha_t \tilde{t}_- + \cos \alpha_t \tilde{t}_+.$$  

(29)

The only one-loop diagram responsible for the generation of $f_{2}^R$ is shown in Fig. 2.

![Figure 2: The one-loop diagram, which generates $f_{2}^R$ within a SSM.](image)

The $b_L$–$b_R$ mixing, which may also provide the necessary phase, has the same structure as the one for the top sector with the substitutions: $\phi_t \rightarrow \phi_b$ and $\alpha_t \rightarrow \alpha_b$. However, if we assume that the scalar $b$-quarks are almost degenerate, their mixing effect can be neglected. The generalization is obvious. It is interesting to note that, if we add phases
\(e^{i\phi}\) and \(e^{-i\phi}\) to the terms \((\hat{\lambda}^a t_L)\) and \((\hat{\lambda}^a t_R)\) in Eq. (28), owing to the complex gluino mass, their effect can be absorbed into \(\phi_i(\rightarrow \phi_i - 2\phi_\lambda)\) and \(\phi_b(\rightarrow \phi_b - 2\phi_\lambda)\). Since the same interactions generate the neutron’s electric dipole moment (NEDM), we have to take into account the limits originating from this measurement. However, direct restrictions on \(\phi_{t/b}\) from the NEDM turn out not to be very reliable \[14\] and therefore will not be applied here. Indirect bounds may be obtained within the supergravity-induced SSM. However, as showed in Ref. \[13\], even assuming the same phase for all quark families, the model allows for maximal CP-violating phases for sufficiently heavy up- and down-squarks; therefore it is legitimate to assume maximal CP violation. It should be stressed here that we are not restricting ourselves to the minimal supergravity-induced models.

The result \[1\] for the CP-violating part of \(f_2^R\) due to the \(t_L-t_R\) mixing can be written as:

\[
\text{Im}(f_2^{R\text{CPV}}) = \frac{\alpha_s}{3\pi} \sin(2\alpha_t) \sin(\phi_t) \frac{m_\lambda m_W}{(m_t^2 - m_W^2)^2} \text{Re} \left[ \mathcal{I}(m_{t+}^2) - \mathcal{I}(m_{t-}^2) \right]
\]

\[\mathcal{I}(m_{t_i}^2) = \int_0^1 \int_0^{1-\alpha} \frac{d\beta d\alpha (m_{t_i}^2 - m_W^2)^2(1 - \alpha - \beta)}{\alpha m_{t_i}^2 + \beta m_b^2 + (1 - \alpha - \beta) m_{\lambda}^2 + \alpha \beta (m_{t_i}^2 - m_W^2) - \alpha(1 - \alpha) m_{t_i}^2} \]

\[= (m_W^2 - m_{t_i}^2) B_0(-p_b, m_{\lambda}^2, m_b^2) + (m_W^2 + m_{t_i}^2) B_0(-p_t, m_{\lambda}^2, m_b^2) - 2m_W^2 B_0(p_v, m_{t_i}^2, m_b^2) + C_0(-p_t, m_W^2, m_{t_i}^2, m_b^2) \times \]

\[\left[ (m_W^2 - m_{t_i}^2)^2 - (m_t^2 - m_\lambda^2 - m_t^2)(m_W^2 - m_t^2) - (m_b^2 - m_\lambda^2)(m_t^2 + m_W^2) \right]
\]

where \(m_{t_i}^2\) stands for the top-squark masses, \(m_b^2\) denotes degenerated bottom squark mass and \(m_\lambda\) is the gluino mass. The functions \(B_0(\cdots)\) and \(C_0(\cdots)\) are defined as:

\[
\frac{i}{16\pi^2} B_0(k; m_{t_1}^2, m_{t_2}^2) \equiv \mu^{4-n} \int \frac{d^d r}{(2\pi)^n} \frac{1}{[r^2 - m_{t_1}^2 + i\varepsilon][(r + k)^2 - m_W^2 + i\varepsilon]}
\]

\[
-\frac{i}{16\pi^2} C_0(k; p; m_{t_1}^2, m_{t_2}^2, m_{t_3}^2) \equiv
\]

\[
\mu^{4-n} \int \frac{d^d r}{(2\pi)^n} \frac{1}{[r^2 - m_{t_1}^2 + i\varepsilon][(r + k)^2 - m_W^2 + i\varepsilon][(r + k + p)^2 - m_{t_2}^2 + i\varepsilon]}
\]

where \(\mu\) is the regularization scale. \(B_0\) and \(C_0\) can be expressed in the standard manner through logarithms and Spence functions \[16\]. The result, Eq. (30), is of course, UV finite. In order to obtain effects as big as possible we will assume for illustration \(\sin(2\alpha_t) \sin(\phi_t) = 1\); for the lighter stop-quark mass, we take the lower bound \[17\] \(m_{t+} = 50\) GeV, whereas for the heavier one and for degenerated bottom squarks we use \(m_{t+} = m_b = 150\) GeV.

Figure 3 shows \(\text{Im}(f_2^{R\text{CPV}})\), where we have fixed \(\alpha_s = 0.1\) and assumed \(m_{t-} = \infty\). Because of the unitarity form in Eq. (30), \(\text{Im}(f_2^{R\text{CPV}})\) can be obtained for the general case of finite \(m_{t+}\) and \(m_{t-}\), by taking the corresponding difference of two curves in Fig. 3.

In Fig. 4 we plot results for the asymmetry \(\mathcal{A}\) as a function of \(\cos \theta_{tL}\).

In Fig. 5 we show both \(\mathcal{A}_{int}^+\) and \(\mathcal{A}_{int}^-\) defined similarly as \(\mathcal{A}_{int}\), but with integration (in the numerator) over \(\cos \theta_{tL}\) restricted to regions of positive and negative differential asymmetry, respectively.
Figure 3: The CP-violating contribution to $\text{Im}(f_2^R)$ as a function of $m_{\tilde{b}} = m_\lambda$.

For the most optimistic proposal of a 500 GeV linear collider \cite{18}, the total integrated luminosity is $8.5 \times 10^4 \text{ pb}^{-1} \text{yr}^{-1}$; therefore the SM cross-section for $t\bar{t}$ production (0.66 pb at $m_t = 150 \text{ GeV}$) \cite{18} predicts about $N \simeq 50000$ events per year. This means that the smallest possible measurable asymmetry is about $1/\sqrt{B \ell N} \simeq 1\%$. One should however have in mind that the above estimates do not include any cuts and, of course, some number of events must be lost because of non-perfect efficiency. The results that we have obtained here presumed the narrow-width approximation, where all possible interference effects between production and decay are neglected. In order to justify this we must in addition assume that the final ($Wb$) mass resolution is sufficiently good to be sure that $W$ and $b$ are coming from on-shell top quarks. Since the largest asymmetry $A_{int}$ is about $10^{-4} \sim 10^{-3}$, it seems very hard to find CP violation of the $t$ decay in the SSM.

\footnote{After completing this paper we have received a preprint by W. Bernreuther and P. Overmann \cite{12} where $f_2^R$ has been also calculated within SSM. We agree with their results.}
Figure 4: The asymmetry $A$ as a function of the $\cos \theta_{t\bar{t}}$ for top-quark masses $m_t = 100$, 150 and 200 GeV.

4 Measurement of the Asymmetry

Let us define the 4-momentum of the $b$ quark and $W^+$ from the $t$ decay in the rest frame of the $t$ quark as (we shall neglect $m_b$ in our kinematics):

$$p_{b\text{rest}}^t = E_b(1, -\sin \theta_{W^+}, 0, -\cos \theta_{W^+}) \quad (34)$$

$$p_{W^+\text{rest}}^t = (E_{W^+}, E_b \sin \theta_{W^+}, 0, E_b \cos \theta_{W^+}), \quad (35)$$

where $\theta_{W^+}$ is the polar angle defined in the top-quark rest frame, using for the $z$ axis the direction of the top seen from the $e^-e^+$ centre-of-mass frame, the $(x, z)$ plane is defined by the top and $W^+$ momentum (having $p_{W^+} > 0$), and the $y$ axis is provided by the right hand rule. (From simple kinematics we know that in the $t$ rest frame $E_{W^+} = (m_t^2 + m_{W^+}^2)/(2m_t)$ and $E_b = (m_t^2 - m_{W^+}^2)/(2m_t)$.) The axes of the $W^+$ rest frame are defined by the sequence of transformations used to go from this to the $t$ rest frame; a detailed general description has been presented elsewhere [7]. The choice of the $(x, z)$ plane in the top-quark centre of mass allows for a substantial simplification; it turns out that besides the obvious boost, the $W^+$ rest frame is obtained from the top-quark rest frame by a simple rotation about
Figure 5: The integrated versions of the asymmetry $A$, $A_{int}^+$ and $A_{int}^-$ defined in the text, as a function of the top quark mass.

their common $y$ axis by the angle $\theta_{W^+}$. Therefore the $\phi_{\ell^+}$, which must be measured for the asymmetry determination, is simply the angle between the planes defined by $W^+ - t$ and $W^+ - \ell^+$, see Fig. 6.

The easiest way to measure $A$ is offered by events where the top quark decays semileptonically whereas the antitop decays purely hadronically. In this case, in the $e^- e^+$ centre of mass (CM), the top quark momentum $p_t^{CM}$ can be determined by measurement of the antitop momentum. Since we need to know the $W^+$ momentum, we will assume hereafter that the $b$ and $\ell^+$ momentum are measured. This is enough to reconstruct the $W^+$ rest frame and measure $\phi_{\ell^+}$. However, one can show the following useful formula:

$$p_{\ell^+}^{CM} \cdot (p_b^{CM} \times p_{W^+}^{CM}) = E_{\ell^+} E_b \sin(\phi_{\ell^+}) \sin(\theta_{W^+}) \sin(\theta_{\ell^+}) \sinh(y_t) e^{y_W},$$  \hspace{1cm} (36)

where $E_{\ell^+} = m_W/2$ is the $W^+$ rest frame $\ell^+$ energy, and $y_t, y_W$ are the boost parameters for transformations from the CM frame to the $t$ rest frame and from this frame to the $W$ rest frame, respectively. Since, $\theta_{W^+}$ and $\theta_{\ell^+}$ both lie between 0 and $\pi$, we find that the sign of $\sin \phi_{\ell^+}$ is given by the sign of $p_{\ell^+}^{CM} \cdot (p_b^{CM} \times p_{W^+}^{CM})$. 
5 Summary

We have discussed CP violation in the decays $t \to \ell^+ \nu b$ and $\bar{t} \to \ell^- \bar{\nu} \bar{b}$ from the production $e^- e^+ \to t \bar{t}$. The asymmetry $A_{int}$ defined by Eq. (27) turned out to be a useful observable in searching for non-standard sources of CP violation, since it vanishes even at the one-loop level in the SM. We can conclude that for the planned luminosity at future high-energy linear colliders the observation of $A_{int}$ predicted within the SSM ($10^{-4} \sim 10^{-3}$) looks very difficult.

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