Large corrections to asymptotic $F_{\eta_c \gamma}$ and $F_{\eta_b \gamma}$ in the light-cone perturbative QCD

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The large-$Q^2$ behavior of $\eta_c-\gamma$ and $\eta_b-\gamma$ transition form factors, $F_{\eta_c \gamma}(Q^2)$ and $F_{\eta_b \gamma}(Q^2)$ are analyzed in the framework of light-cone perturbative QCD with the heavy quark ($c$ and $b$) mass effect, the parton’s transverse momentum dependence and the higher helicity components in the light-cone wave function are respected. It is pointed out that the quark mass effect brings significant modifications to the asymptotic predictions of the transition form factors in a rather broad energy region, and this modification is much severer for $F_{\eta_b \gamma}(Q^2)$ than that for $F_{\eta_c \gamma}(Q^2)$ due to the $b$-quark being heavier than the $c$-quark. The parton’s transverse momentum and the higher helicity components are another two factors which decrease the perturbative predictions. For the transition form factor $F_{\eta_c \gamma}(Q^2)$, they bring sizable corrections in the present experimentally accessible energy region ($Q^2 \leq 10$ GeV$^2$). For the transition form factor $F_{\eta_b \gamma}(Q^2)$, the corrections coming from these two factors are negligible since the $b$-quark mass is much larger than the parton’s average transverse momentum. The coming $e^+e^-$ collider (LEP2) will provide the opportunity to examine these theoretical predictions.

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I. INTRODUCTION

Among a large number of exclusive processes, neutral meson production in two-photon collision, $\gamma^*\gamma \rightarrow P$ ($P$ being $\pi^0, \eta, \eta', \eta_c, \eta_b, ...$) is the simplest one since two photons and one meson are involved in the initial and final states, respectively. Only one form factor named meson-photon transition form factor ($F_{P\gamma}$) is necessary to describe this class of processes. Studying $F_{P\gamma}$ provides a rather simple and rigorous way to the test of QCD and the determination of the meson wave function (non-perturbative physics) \[1\]. Experimentally, a lot of collaborations (TPC/Two-Gamma \[2\], CELLO \[3\], CLEO \[4\] and L3 \[5\], etc.) have measured the form factors $F_{\pi\gamma}(Q^2)$, $F_{\eta\gamma}(Q^2)$, and $F_{\eta'\gamma}(Q^2)$ in the $Q^2$ region up to 9, 20 and 30 GeV$^{-2}$, respectively, where $Q^2$ is the virtuality of the virtual photon. Although with poor statistics, the $c\bar{c}$ states ($\eta_c, \chi_{c0}$ and $\chi_{c2}$) productions have been observed \[1\]. In LEP2, the dominant process is $e^+e^- \rightarrow e^+e^- + X$ ($\gamma\gamma \rightarrow X$). Considering the higher energy (the center of mass energy will reach 100 GeV) and the higher luminosity (the cross section of this process grows like $(\ln s/m_c^2)^2$ with $s$ being the invariant energy square of the incoming $e^+e^-$ pair, whereas the annihilation cross section decrease like $s^{-1}$), LEP2 will be a good factory for the production of the heavy quarkonium production ($c\bar{c}$ and $b\bar{b}$), and will greatly stimulate theoretical studies on these processes. At present, it seems a measurement of $F_{\eta\gamma}$ up to about 10 GeV$^2$ is possible \[1\]. Theoretically, there are also a lot of studies on these form factors \[1\]–\[4\]. In the large-$Q^2$ region, perturbative QCD can be employed as a powerful tool. The large-$Q^2$ behavior of form factors $F_{\pi\gamma}$, $F_{\eta\gamma}$ and $F_{\eta'\gamma}$ have been studied in some detail by several authors \[4\]–\[6\]. Recently, the form factor $F_{\eta\gamma}$ has also been analyzed in the invariant perturbative theory by adopting the Breit reference frame \[6\]. In this note, we present a theoretical study on the $F_{\eta\gamma}$ and $F_{\eta'\gamma}$ in the framework of light-cone perturbative QCD (LCPQCD). It is pointed out that in the LCPQCD calculations of $F_{\eta\gamma}$ and $F_{\eta'\gamma}$, there are two differences from that in the case of $F_{\pi\gamma}$, $F_{\eta\gamma}$ and $F_{\eta'\gamma}$. First, compared with the light quark ($u$, $d$ and $s$) masses, the $c$- and $b$-quark masses should not be neglected in evaluating the hard scattering amplitude, while the quark masses involved in the calculation of $F_{\pi\gamma}$, $F_{\eta\gamma}$ and $F_{\eta'\gamma}$ can be neglected reasonably. Second, considering the Wigner-Melosh rotation and $c$- and $b$-quark masses being large, one finds that there are contributions coming from the higher helicity components in the light-cone wave functions besides that come from the ordinary helicity components. For the $\pi$, $\eta$ and $\eta'$ mesons, the contributions from the higher helicity components can also be neglected in the limit of vanishing the quark masses.

II. LIGHT-CONE FORMALISM AND LIGHT-CONE WAVE FUNCTION

The light-cone (LC) formalism \[15\] provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and the application of perturbative QCD to exclusive processes has mainly been developed in this formalism (light-cone perturbative QCD) \[16\]. In this formalism, the quantization is chosen at a particular light-cone time $\tau = t + z$. Thereby, several characters arise in this formalism: i) The hadronic wave function which describes the hadronic composite state at a particular $\tau$ is expressed in terms of a series of light-cone wave functions in Fock-state basis, for example,

$$|\pi\rangle = \sum |q\bar{q}\rangle \psi_{q\bar{q}/\pi} + \sum |q\bar{q}\bar{q}\bar{g}\rangle \psi_{q\bar{q}\bar{g}/\pi} + \cdots, \quad (1)$$

and the temporal evolution of the state is generated by the light-cone Hamiltonian $H_{LC} = P^- - P^0 = P^3$; ii) The vacuum is very simple. The zero-particle state is the only one which has zero total $P^+$, since all quanta must have positive light-cone momentum $k_i^+$, and $P^+ = \sum_i k_i^+$. The zero-particle state can’t mix with the other states which contain a certain number of particles. Hence the vacuum state in the light-cone Fock basis (Eq. (3)) is an exact eigenstate of the full Hamiltonian $H_{LC}$, and all bare quanta in a hadronic Fock state are parts of the hadron. This point does very differ from that in the equal-$t$ perturbative theory in which the quantization is performed at a given time $t$. In the equal-$t$ quantization, it is possible to make up zero-momentum state which contains some particles, since the momentum of each particle may be positive or negative, and the momentum of a composite state is the sum of the momentum of each participant particle. Thus the zero-particle state may mix with some zero-momentum states which contain particles to build up the ground state, which makes the vacuum become complex. iii) The contributions coming from higher Fock states are suppressed by $1/Q^2$, therefore one can employ only the valence state to the leading order in the large-$Q^2$ region. Light-cone perturbative QCD is very convenient for light-cone dominated processes. For the detail calculation rules we refer to literatures \[16\]–\[17\].

The essential feature of light-cone PQCD applying to exclusive processes is that the amplitudes for these processes can be written as a convolution of hadron light-cone wave functions (LCWF) (or quark distribution amplitudes, DA) for each hadron involved in the process with a hard-scattering amplitude $T_H$. Both LCWF and the $T_H$ are the basic blocks for the LCPQCD calculation. It has been pointed out that the Wigner-Melosh \[18\] rotation should be taken
we have

where

Here \( \varphi(x, k_{\perp}) \) is the momentum space wave function in the light-cone formalism. The coefficients \( C^F_0(x, k_{\perp}, \lambda_1, \lambda_2) \) which result from the considering of the Wigner-Melosh rotation turn out to be \[19\]

\[
C^F_0(x, k_{\perp}, \uparrow, \downarrow) = \frac{m}{[2(m^2 + k^2_{\perp})^{1/2}]};
\]

\[
C^F_0(x, k_{\perp}, \downarrow, \uparrow) = -\frac{m}{[2(m^2 + k^2_{\perp})^{1/2}]};
\]

\[
C^F_0(x, k_{\perp}, \uparrow, \uparrow) = -\frac{(k_1 - ik_2)}{[2(m^2 + k^2_{\perp})^{1/2}]};
\]

\[
C^F_0(x, k_{\perp}, \downarrow, \downarrow) = -\frac{(k_1 + ik_2)}{[2(m^2 + k^2_{\perp})^{1/2}]};
\]

where \( m \) is the c- (b-) quark mass for \( \eta_c (\eta_b) \), and \( k_{\perp} \) is the quark transverse momentum. \( C^F_0 \) satisfy the relation

\[
\sum_{\lambda_1, \lambda_2} C^F_0(x, k_{\perp}, \lambda_1, \lambda_2) C^F_0(x, k_{\perp}, \lambda_1, \lambda_2) = 1.
\]

One character of the light-cone wave function is that there are higher helicity \( (\lambda_1 + \lambda_2 = \pm 1) \) components besides the ordinary helicity \( (\lambda_1 + \lambda_2 = 0) \) components, while the instant-form wave function has only the ordinary helicity components. The above result means that the light-cone spin of a composite particle is not directly the sum of its constituents’ light-cone spins but the sum of Wigner rotated light-cone spins of the individual constituents. A natural consequence is that in light-cone formalism a hadron’s helicity is not necessarily equal to the sum of the quark’s helicities, i.e., \( \lambda_H \neq \sum_i \lambda_i \). This result has been employed in the studies of several processes: the proton “spin puzzle” \[20\], proton’s structure, the ratio \( F_2^p/F_2^n \), the proton, neutron, and deuteron polarization asymmetries, \( A_1^p, A_1^n, A_1^d \) etc. \[21\].

### III. The Meson-Photom Transition Form Factors \( F_{\eta_c \gamma} \) and \( F_{\eta_b \gamma} \)

In the following, we first analyze the \( \eta_c \gamma \) transition form factor \( F_{\eta_c \gamma} \). The analysis for \( F_{\eta_b \gamma} \) can be obtained in a similar way. The \( \eta_c \gamma \) transition form factor \( F_{\eta_c \gamma} \) is extracted from the \( \eta_c \gamma^{*} \) vertex,

\[
\Gamma_{\mu} = -ie^2 F_{\eta_c \gamma} \epsilon_{\mu \nu \alpha \beta} p^\nu_{\eta_c} \epsilon^\alpha q^\beta,
\]

where \( p_{\eta_c} \) and \( q \) are the momenta of the \( \eta_c \) meson and the virtual photon respectively, and \( \epsilon \) is the polarization vector of the on-shell photon. In the standard “infinite-momentum” frame \[1\], the momentum assignment can been written as

\[
p_{\eta_c} = (p^+, p^-, p_{\perp}) = (1, m_{\eta_c}^2, 0_{\perp}),
\]

\[
q = (0, q^2_{\perp}, m_{\eta_c}, q_{\perp}),
\]

\[
q' = (1, q^2_{\perp} + m_{\eta_c}^2, q_{\perp}),
\]

where \( p^+ \) is arbitrary, and \( q' \) is the momentum of the final (on-shell) photon. For simplicity we choose \( p^+ = 1 \), and we have \( q^2 = -q^2_{\perp} = -Q^2 \). Then the \( F_{\eta_c \gamma} \) is given by

\[
F_{\eta_c \gamma}(Q^2) = \frac{\Gamma^+}{-ie(\epsilon_{\perp} \times q_{\perp})},
\]
where \( \epsilon = (0, 0, \epsilon_\perp) \) and \( \epsilon \cdot q_\perp = 0 \) is chosen.

The contribution coming from the ordinary helicity components \((\lambda_1 + \lambda_2 = 0)\) turns out to be

\[
F_{\eta_c \gamma}^{(\lambda_1 + \lambda_2 = 0)}(Q^2) = \frac{\sqrt{n_c e_c^2}}{i(\epsilon_\perp \times q_\perp)} \int_0^1 [dx] \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \frac{m_c}{\sqrt{m_c^2 + k_\perp^2}} \psi(x_i, k_\perp) \\
\times \left[ \bar{v}_i(x_2, -k_\perp) \gamma^\mu \left( \frac{u_\tau(x_1, k_\perp + q_\perp) + u_\tau(x_1, k_\perp - q_\perp)}{\sqrt{x_1}} \right) \gamma^\nu \left( \frac{u_\tau(x_1, k_\perp)}{\sqrt{x_1}} \right) \frac{1}{D} + (1 \leftrightarrow 2) \right],
\]

where \([dx] = dx_1 dx_2 \delta(1 - x_1 - x_2)\), \(e_c\) is the c-quark charge in unit of \(e\), and \(D\) is the “energy-denominator”,

\[
D = q_1^2 - \frac{(q_1 + k_\perp)^2 + m_c^2 - k_\perp^2 + m_c^2}{x_1} \\
- \frac{(q_2 q_\perp + k_\perp)^2 + m_c^2}{x_2}.
\]

Being different from the case of the light meson such as \(\pi, \eta\) and \(\eta'\), the present of the large quark mass \((m_c \simeq 1.5\ GeV)\) always prevent \(1/D\) from the singular point \(D \to 0\), i.e. the partons in the intermediate state are always far off energy-shell. This means that even at the low \(Q^2\) region, the LCPQCD calculation may be still available. Employing the LCPQCD calculation rules Eq. (9) becomes \([11,16]\),

\[
F_{\eta_c \gamma}^{(\lambda_1 + \lambda_2 = 0)}(Q^2) = 2 \sqrt{2} \sqrt{n_c e_c^2} \int_0^1 [dx] \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \frac{m_c}{\sqrt{m_c^2 + k_\perp^2}} \psi(x_i, k_\perp) \\
\times \left[ \frac{q_\perp \cdot (q_2 q_\perp + k_\perp)}{Q_\perp^2 ((q_2 q_\perp + k_\perp)^2 + m_c^2)} + (1 \leftrightarrow 2) \right].
\]

Similarly, one can obtain the contribution coming from the higher helicity components,

\[
F_{\eta_c \gamma}^{(\lambda_1 + \lambda_2 = \pm 1)}(Q^2) = \frac{\sqrt{n_c e_c^2}}{i(\epsilon_\perp \times q_\perp)} \int_0^1 [dx] \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \frac{m_c}{\sqrt{m_c^2 + k_\perp^2}} \psi(x_i, k_\perp) \\
\times \left[ \bar{v}_i(x_2, -k_\perp) \gamma^\mu \left( \frac{u_\tau(x_1, k_\perp + q_\perp) + u_\tau(x_1, k_\perp - q_\perp)}{\sqrt{x_1}} \right) \gamma^\nu \left( \frac{u_\tau(x_1, k_\perp)}{\sqrt{x_1}} \right) \frac{1}{D} + (1 \leftrightarrow 2) \right] \\
= 2 \sqrt{2} \sqrt{n_c e_c^2} \int_0^1 [dx] \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \frac{m_c}{\sqrt{m_c^2 + k_\perp^2}} \psi(x_i, k_\perp) \\
\times \left[ \frac{q_\perp \cdot k_\perp}{Q_\perp^2 ((q_2 q_\perp + k_\perp)^2 + m_c^2)} + (1 \leftrightarrow 2) \right].
\]

Once again, a non-zero quark mass, \(m_c\), plays an important role in the calculation of \(F_{\eta_c \gamma}^{(\pm 1)}\), since in the \(m_c \to 0\) limit the matrix \(\bar{v}_i(x_2, -k_\perp) u_\tau(x_1, q_\perp \pm k_\perp)\) will goes to zero. Therefore, for the light meson such as \(\pi, \eta\) and \(\eta'\) neglecting the contributions coming from higher helicity components should be a good approximation. Combining this matrix with the coefficients \(C_0(x, k_{\perp, \uparrow, \downarrow})\) and \(C_0(x, k_{\perp, \downarrow, \uparrow})\), one arrives the second expression in Eq. (12). The full result is obtained by summing up the contributions from the ordinary helicity components (Eq. (11)) and that from the higher helicity components (Eq. (12)),

\[
F_{\eta_c \gamma}(Q^2) = F_{\eta_c \gamma}^{(\lambda_1 + \lambda_2 = 0)}(Q^2) + F_{\eta_c \gamma}^{(\lambda_1 + \lambda_2 = \pm 1)}(Q^2).
\]

Neglecting \(k_\perp\) and \(m_c\) relative to \(x_2 q_\perp\) in Eqs. (11) and (12), and employing the asymptotic form distribution amplitude

\[
\phi(x) = \sqrt{3/2} f_{\eta_c} x_1 x_2,
\]

where \(f_{\eta_c}\) is the decay constant, one can obtain the asymptotic prediction for the \(\eta_c - \gamma\) transition form factor,

\[\text{Any meson distribution amplitude should evolve into the asymptotic form in the } Q^2 \to \infty \text{ limit.} \]
Corrections to the asymptotic prediction (Eq. (15)) come from $c$-quark mass, the $k_{\perp}$-dependence and the higher helicity components (see Eqs. (11), (12) and (13)). All of these corrections are suppressed by the factor $1/Q^2$ at the large-$Q^2$ region. But in the present experimentally available energy region, these contributions may be important and should be taken into account.

In order to study the $c$-quark mass effect, one may first neglect the $k_{\perp}$-dependence in the hard-scattering amplitude of Eq. (11), then one can obtain,

$$F_{\eta_2}(Q^2 \to \infty) = \frac{8f_{\eta}}{3Q^2},$$  \hspace{1cm} (15)

where $\phi(x)$ is the distribution amplitude of the $\eta_c$-meson,

$$\phi(x) = \int_0^1 \frac{d^2k_{\perp}}{16\pi^3} \frac{m_c}{m_c^2 + k_{\perp}^2} \psi(x,k_{\perp}).$$  \hspace{1cm} (17)

Because of the $c$-quark mass being large, Eq. (16) will approach to the asymptotic prediction (Eq. (15)) in a rather slow way, that is, the corrections coming from $c$-quark mass effect are large in a rather broad energy region. The effects of the $k_{\perp}$-dependence and higher helicity components can be studied by comparing the results obtained from Eqs. (11), (12), (13) and (16). Also, it is interesting to notice that the correction coming from the higher helicity components is the same as that from the $k_{\perp}$-dependence in the ordinary helicity components (The right hand side in Eq. (12) is the same as the second term in the right hand side of Eq. (11)). In the low and medium $Q^2$ region, these corrections may provide sizable contributions which should be taken into account.

We point out that the above analysis for $F_{\eta_2}$ is applicable to the form factor $F_{\eta\gamma}$ with the physics quantities corresponding to the $c$ quark ($m_c$ and $m_b$) and decay constant $f_{\eta_2}$ being replaced by the ones corresponding to the $b$ quark ($m_b$ and $m_b$) and $f_{\eta_b}$, respectively. The differences resulting from the $b$-quark being much heavier than the $c$-quark are as follows: First, the modification coming from $b$-quark mass effect become much severer, i.e. the perturbative calculation with $m_b$ effect being respected approaches to the asymptotic prediction more slowly. Second, the corrections coming from the transverse momentum dependence and the higher helicity components of the light-cone wave function may become rather mild because the $b$-quark mass is much larger than the parton’s average transverse momentum.

**IV. NUMERICAL CALCULATIONS**

We employ the Brodsky-Huang-Lepage (BHL) model [17] for the $\eta_c$ ($\eta_b$) meson light-cone wave functions,

$$\psi^{BHL}(x,k_{\perp}) = A \exp \left[ -\frac{k_{\perp}^2 + m^2}{8\beta^2x(1-x)} \right].$$  \hspace{1cm} (18)

In this model, the light-cone wave function is obtained from the instant-form wave function by demanding the off-shell energies being equal in the two reference frames. The parameters $A$ and $\beta$ are determined by the following two constraints:

$$\int_0^1 [dx] \int \frac{d^2k_{\perp}}{16\pi^3} \frac{m_q}{m_q^2 + k_{\perp}^2} \psi(x,k_{\perp}) = \frac{f_{\eta}}{\sqrt{6}},$$  \hspace{1cm} (19)

$$\int_0^1 [dx] \int \frac{d^2k_{\perp}}{16\pi^3} |\psi(x,k_{\perp})|^2 = P_{q\bar{q}/\eta},$$  \hspace{1cm} (20)

where $f_{\eta_q}$ ($q = c, b$) is the decay constant of the $\eta_c$ ($\eta_b$) meson corresponding to $f_x = 131$ MeV, and $P_{q\bar{q}/\eta}$ is the probability of finding $|\bar{c}c\rangle$ ($|\bar{b}b\rangle$) Fock state in the $\eta_c$ ($\eta_b$) meson. Because of the lack of experimental information,
one often evaluates \( f_{\eta_c} \) through various theoretical approaches. Employing the Van Royen-Weisskopf formula\(^3\) for the decay constant

\[
f_M = \sqrt{\frac{12}{m_M}} |\psi_M(0)|
\]

(21)

where \( m_M \) and \( \psi_M(0) \) are the mass and wave function at the origin of the meson respectively, one can obtain that the decay constant of the pseudoscalar meson is almost the same as that of the vector meson, i.e., \( f_p = f_V \). Although the hyperfine splitting Hamiltonian may destroy this relation\(^2\)\(^3\), the consideration of the difference coming from the mock meson spin structure may rescue it\(^2\)\(^4\)\(^5\). Hence, we adopt\(^2\)\(^4\)\(^5\)

\[
f_{\eta_c} \simeq f_{f/\psi} \simeq 420 \text{ MeV}, \quad f_{\eta_b} \simeq f_{\Upsilon} \simeq 705 \text{ MeV}.
\]

(22)

As well known, with the increasing of the constitute quark mass the valence Fock state occupies a bigger fraction in the hadron, and in the nonrelativistic limit the probability of finding the valence Fock state is going to approach unity. So one can expect that LEP2 may examine all of these theoretical predictions in the near future.

\(^3\)The decay constants of the pseudoscalar and vector mesons are defined by \( \langle 0|\bar{Q}\gamma^\mu\gamma_5Q|\bar{M}_\nu(K) \rangle = f_P K^\mu \) and \( \langle 0|\bar{Q}\gamma^\mu Q|\bar{M}_\nu(K, \varepsilon) \rangle = f_V m_V \varepsilon^\mu \), respectively, where \( \varepsilon \) is the polarization vector of the vector meson, and \( K \) is the meson momentum.
V. SUMMARY

In summary, the meson photon transition form factors $F_{P\gamma}(Q^2)$ ($P$ being $\pi^0$, $\eta$, $\eta'$, $\eta_c$, $\eta_b$ ...) extracted from the two photon collision are the simple exclusive processes which can provide a rather simple and rigorous way to the test of QCD and the determination of the meson wave function (non-perturbative physics). Many experimental collaboration such as TPC/Two-Gamma, CELLO, CLEO and L3 etc. have studied these processes. A measurement for the $F_{\eta\gamma}(Q^2)$ is very likely to be feasible in LEP2. In this note, we analyze the $\eta_c$- and $\eta_b$-photon transition form factors in the light-cone perturbative theory with the quark mass effect, the parton’s transverse momentum dependence and the higher helicity components of the light cone wave function are respected. It is pointed out that due to $c$- ($b$-) quark being heavy, considering the quark mass effect brings significant modifications to the perturbative predictions in a rather broad energy region. This effect is much severer for the $F_{\eta\gamma}$ than that for the $F_{b\gamma}$ because of the $b$-quark being heavier than $c$-quark. Also it is found that, for the $F_{b\gamma}$, the parton’s transverse momentum and higher helicity components bring sizable corrections in the present experimentally accessible energy region ($Q^2 \leq 10 \sim 20$ GeV$^2$), while these corrections are negligible in the perturbative calculation of $F_{b\gamma}$. We conclude that the coming $e^+e^-$ collider LEP2 will provide the opportunity to examine all of these theoretical predictions.

Acknowledgments

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Fig. 1 The lowest order diagrams contributing to $F_{\eta_c\gamma}$ and $F_{\eta_b\gamma}$ in the light-cone perturbative QCD. The momenta are expressed in the light-cone variables $(+,\perp)$.

Fig. 2(a) The $\eta_c\gamma$ transition form factor given in $Q^2 F_{\eta_c\gamma}(Q^2)$.

Fig. 2(b) The $\eta_c\gamma$ transition form factor given in $F_{\eta_c\gamma}(Q^2)$.

Fig. 3(a) The $\eta_b\gamma$ transition form factor given in $Q^2 F_{\eta_b\gamma}(Q^2)$.

Fig. 3(b) The $\eta_b\gamma$ transition form factor given in $F_{\eta_b\gamma}(Q^2)$.
$\eta_\gamma(Q^2) \ (\text{GeV}^{-1})$

$F_{\eta_b^2}(Q^2) \ (\text{GeV}^2)$

$Q^2 \ (\text{GeV}^2)$

Fig. 3(b)
Fig. 3(a)
Fig. 2(b)
Fig. 2(a)
