A Novel Approach to the Cosmological Constant Problem

Alberto Iglesias\(^1\) and Zurab Kakushadze\(^2\)

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Abstract

We propose a novel infinite-volume brane world scenario where we live on a non-inflating spherical 3-brane, whose radius is somewhat larger than the present Hubble size, embedded in higher dimensional bulk. Once we include higher curvature terms in the bulk, we find completely smooth solutions with the property that the 3-brane world-volume is non-inflating for a continuous range of positive values of the brane tension, that is, without fine-tuning. In particular, our solution, which is a near-BPS background with supersymmetry broken on the brane around TeV, is controlled by a single integration constant.

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\(\text{\textcopyright E-mail: iglesias@insti.physics.sunysb.edu} \)

\(\text{\textcopyright E-mail: zurab.kakushadze@rbccm.com} \)

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I. INTRODUCTION AND SUMMARY

As was proposed in [1,2], one can reproduce 4-dimensional gravity on a 3-brane embedded in higher dimensional infinite-volume bulk if one includes the Einstein-Hilbert term on the brane. Gravity becomes higher dimensional above some cross-over scale, which can be larger than the present Hubble size. As was pointed out in [1,3,2], infinite-volume scenarios offer a new arena for addressing the cosmological constant problem. In particular, since at large distances gravity is no longer 4-dimensional, it is possible that the usual four-dimensional arguments for why fine-tuning is unavoidable are no longer valid. What one needs in this context, however, is to construct explicit solutions where the 3-brane tension is large yet the 4-dimensional cosmological constant is vanishing (or small) without fine-tuning.

The goal is to construct infinite-volume solutions where the brane is non-inflating (or very slowly inflating) with the property that such solutions exist for a continuous range of positive values of the 3-brane tension. One approach is to look for solutions where the 3-brane world-volume is flat. Such solutions do not exist for codimension-1 branes [1]. In the codimension-2 case such solutions do exist, but the brane tension has an upper bound such that it does not improve the experimental bound on the 4-dimensional cosmological constant [4]. One is therefore compelled to go to codimension-3 and higher setups.

Here one encounters the presence of naked singularities [5] unless one includes higher curvature terms in the bulk action [6,7]. However, inclusion of higher curvature terms in the bulk does indeed cure the singularity problem. Thus, [7] constructed explicit non-singular higher codimension solutions with the property that the brane world-volume contains the 4-dimensional Minkowski space\(^1\), and such solutions exist for a continuous range of positive values of the brane tension. That is, the solutions of [7] provide explicit examples of infinite-volume brane world solutions where vanishing of the brane cosmological constant does not require fine-tuning. One of the key ingredients in [7] is that the brane in these solutions is not just a flat 3-brane. Rather, following the proposal of [9], the brane geometry is that of the 4-dimensional Minkowski space times a small \(n\)-sphere (\(n \geq 2\)).

In this paper we take various observations of [7] even further. Our main idea here is that a priori there is no reason why the 4-dimensional part of the brane world-volume should be flat. Thus, consider a 3-brane whose world-volume is a 3-sphere of a huge radius (somewhat larger than the present Hubble size). This is perfectly consistent with observation. We can then imagine a scenario where this brane is embedded in, say, 5-dimensional bulk\(^2\). To avoid singularities we must include higher curvature terms in the bulk action. Now we can ask whether we can find non-singular solutions where the 3-brane is non-inflating for a continuous range of large positive values of the brane tension. This resembles the setup

\(^1\)Since non-inflating smooth solutions constructed in [7] do not require fine-tuning, we expect that consistent slowly inflating solutions should also exist (explicitly constructing such solutions is, however, technically challenging) without fine-tuning. In [8] it was proposed that considering slowly inflating solutions without higher curvature terms in the bulk might solve the singularity problem. It would be interesting to find explicit solutions with this property.

\(^2\)Higher dimensional generalizations can also be considered - see below.
of [7], but the key difference here is that the 3-brane itself is a sphere (and the only flat direction is time) of a huge radius.

In this paper we present explicit non-singular solutions of this type. To make the problem computationally tractable, we choose the higher curvature terms in the bulk to be the quadratic (in curvature) Gauss-Bonnet combination\(^3\) (we expect that such solutions should exist for generic higher curvature terms as well, albeit we do not have a proof of this statement). In our solutions, which are spherically symmetric, the space is asymptotically flat in both \(r \to 0\) and \(r \to +\infty\) limits, but it is curved for finite \(r\). There are no singularities whatsoever, the volume of the extra space is infinite, and the brane tension takes values in a continuous range, so we do not need any fine-tuning. In particular, our solution, which is a near-BPS background with supersymmetry broken on the brane around TeV, is controlled by a single integration constant.

Finally, let us point out that, in the 5-dimensional setup we focus on in this paper, the bulk Planck scale should be of order of the 4-dimensional Planck scale to have: supersymmetry breaking around TeV; the radius of the 3-brane above the present Hubble size; no need for fine-tuning the brane tension. Then, even in the presence of the Einstein-Hilbert term on the brane, gravity on the brane is actually 5-dimensional. This is because there is no hierarchy between the bulk and brane Planck scales while the volume of the extra dimension is infinite. One approach to circumvent this, which we comment on in the last section, would be to consider higher dimensional setups where a spherical 3-brane is embedded in higher-than-five dimensional bulk. At any rate, our results suggest that it might be worthwhile to study string theory backgrounds in the presence of large closed (spherical) D3-branes (and, more generally, \(p\)-branes).

### II. SETUP

The brane world model we study in this paper is described by the following action:

\[
S = \tilde{M}_P^{D-3} \int_\Sigma d^{D-1} x \sqrt{-\tilde{G}} \left[ \tilde{R} - \tilde{\Lambda} \right] + M_P^{D-2} \int d^D x \sqrt{-G} \left[ R + \xi \left( R^2 - 4 R_{MN}^2 + R_{MNP}^2 \right) \right]. \tag{1}
\]

Here \(M_P\) is the (reduced) \(D\)-dimensional Planck mass, while \(\tilde{M}_P\) is the (reduced) \((D-1)\)-dimensional Planck mass; \(\Sigma\) is a source brane, whose geometry is given by the product \(\mathbb{R}^{D-d,1} \times S^{d-1}_\epsilon\), where \(\mathbb{R}^{D-d,1}\) is the \((D-d)\)-dimensional Minkowski space, and \(S^{d-1}_\epsilon\) is a \((d-1)\)-sphere of radius \(\epsilon\) (in the following we will assume that \(d \geq 3\)). The quantity \(\tilde{M}_P^{D-1} \tilde{\Lambda}\) plays the role of the tension of the brane \(\Sigma\). Also,

\[
\tilde{G}_{mn} \equiv \delta_m^M \delta_n^N G_{MN}\big|_\Sigma,
\]

\(^3\)In fact, when the higher curvature terms are of the Gauss-Bonnet type, the problem turns out to be solvable analytically, so all of the following statements can be verified via explicit calculations.
where \( x^m \) are the \((D - 1)\) coordinates along the brane (the \(D\)-dimensional coordinates are given by \( x^M = (x^m, r) \), where \( r \geq 0 \) is a non-compact radial coordinate transverse to the brane, and the signature of the \(D\)-dimensional metric is \((-+, \ldots, +)\)); finally, the \((D - 1)\)-dimensional Ricci scalar \( \bar{R} \) is constructed from the \((D - 1)\)-dimensional metric \( \bar{G}_{\mu\nu} \). In the following we will use the notation \( x^i = (x^\alpha, r) \), where \( x^\alpha \) are the \((d - 1)\) angular coordinates on the sphere \( S_{\alpha}^{d-1} \). Moreover, the metric for the coordinates \( x^i \) will be (conformally) flat:

\[
\delta_{ij} \, dx^i dx^j = dr^2 + r^2 \gamma_{\alpha\beta} \, dx^\alpha dx^\beta , \tag{3}
\]

where \( \gamma_{\alpha\beta} \) is the metric on a unit \((d - 1)\)-sphere. Also, we will denote the \((D-d)\) Minkowski coordinates on \( \mathbb{R}^{D-d+1} \) via \( x^\mu \) (note that \( x^m = (x^\mu, x^\alpha) \)).

The equations of motion read

\[
R_{MN} - \frac{1}{2} G_{MN} R + \xi \left[ 2 \Xi_{MN} - \frac{1}{2} G_{MN} \Xi \right] + \frac{\sqrt{-\bar{G}}}{\sqrt{G}} \delta^m_N \delta^N_\mu \left[ \bar{R}_{mn} - \frac{1}{2} \bar{G}_{mn} (\bar{R} - \bar{\Lambda}) \right] \bar{L} \delta(r - \epsilon) = 0 , \tag{4}
\]

where \( \Xi \equiv \Xi_M^M \), and

\[
\Xi_{MN} \equiv R R_{MN} - 2 R_{MS} R^S_N + R_{MRST} R^R_N R^ST - 2 R^{RS} R_{MRNS} , \tag{5}
\]

\[
\bar{L} \equiv \frac{M_p^{D-3}}{M_p^{D-2}} . \tag{6}
\]

Here we are interested in solutions with vanishing \((D - d)\)-dimensional cosmological constant, which, at the same time, are radially symmetric in the extra \(d\) dimensions. The corresponding ansatz for the background metric reads:

\[
ds^2 = \exp(2A) \, \eta_{\mu\nu} \, dx^\mu dx^\nu + \exp(2B) \, \delta_{ij} \, dx^i dx^j , \tag{7}
\]

where \( A \) and \( B \) are functions of \( r \) but are independent of \( x^\mu \) and \( x^\alpha \). We then have (here prime denotes derivative w.r.t. \( r \)):

\[
\bar{R}_{\mu\nu} = 0 \, , \tag{8}
\]

\[
\bar{R}_{\alpha\beta} = \lambda \, \bar{G}_{\alpha\beta} \, , \tag{9}
\]

\[
\bar{R} = (d - 1) \, \lambda \, , \tag{10}
\]

\[
R_{\mu\nu} = -\eta_{\mu\nu} e^{2(A-B)} \left[ A'' + (d - 1)\frac{1}{r} A' + (D - d)(A')^2 + (d - 2)A'B' \right] , \tag{11}
\]

\[
R_{rr} = - (d - 1) \left[ B'' + \frac{1}{r} B' + (D - d) \left[ A'B' - (A')^2 - A'' \right] \right] , \tag{12}
\]

\[
R_{\alpha\beta} = -r^2 \gamma_{\alpha\beta} \left[ B'' + (2d - 3)\frac{1}{r} B' + (d - 2)(B')^2 + (D - d)A'B' + (D - d)\frac{1}{r} A' \right] , \tag{13}
\]

\[
R = -e^{-2B} \left[ 2(d - 1)B'' + 2(d - 1)^2 \frac{1}{r} B' + 2(D - d)A'' + 2(D - d)(d - 1)\frac{1}{r} A' + (d - 1)(d - 2)(B')^2 + (D - d)(D - d + 1)(A')^2 + 2(D - d)(d - 2)A'B' \right] . \tag{14}
\]

where
\[ \lambda \equiv \frac{d - 2}{\varepsilon^2} e^{-2B(\varepsilon)}. \]  

The equations of motion then read:

\[ \frac{1}{2} \ddot{d}(d - 1)(A')^2 + \ddot{d}(d - 1)N + \frac{1}{2}(d - 1)(d - 2)S - \]

\[ \xi e^{-2B} \left\{ \frac{1}{2} \ddot{d}(d - 1)(\ddot{d} - 2)(\ddot{d} - 3)(A')^4 + 2\ddot{d}(\ddot{d} - 1)(d - 1)(d - 2)N^2 + \right. \]

\[ \frac{1}{2}(d - 1)(d - 2)(d - 3)(d - 4)S^2 + 2\ddot{d}(\ddot{d} - 1)(\ddot{d} - 2)(\ddot{d} - 3)(A')^2M + \]

\[ 2(\ddot{d} - 1)(\ddot{d} - 2)(\ddot{d} - 3)(d - 1)(A')^2N + 2\ddot{d}(\ddot{d} - 1)(\ddot{d} - 2)(d - 1)(A')^2P + \]

\[ (\ddot{d} - 1)(\ddot{d} - 2)(d - 1)(d - 2)(A')^2S + 4\ddot{d}(\ddot{d} - 1)(\ddot{d} - 2)(d - 1)MN + \]

\[ 2(\ddot{d} - 1)(d - 1)(d - 2)MS + 4\ddot{d}(\ddot{d} - 1)(d - 1)(d - 2)NP + \]

\[ 2(\ddot{d} - 1)(d - 1)(d - 2)(d - 3)NS + 2(d - 1)(d - 2)(d - 3)PS \right\} + \]

\[ \frac{1}{2} e^{B} \left[ A - (d - 1)\lambda \right] \ddot{L}\delta(r - \ell) = 0, \]  

\[ \frac{1}{2} \ddot{d}(d - 1)(A')^2 + \ddot{d}M + \ddot{d}(d - 2)N + (d - 2)P + \frac{1}{2}(d - 2)(d - 3)S - \]

\[ \xi e^{-2B} \left\{ \frac{1}{2} \ddot{d}(d - 1)(\ddot{d} - 2)(\ddot{d} - 3)(A')^4 + 2\ddot{d}(\ddot{d} - 1)(d - 2)(d - 3)N^2 + \right. \]

\[ \frac{1}{2}(d - 2)(d - 3)(d - 4)(d - 5)S^2 + 2\ddot{d}(\ddot{d} - 1)(\ddot{d} - 2)(A')^2M + \]

\[ 2\ddot{d}(\ddot{d} - 1)(\ddot{d} - 2)(d - 2)(A')^2N + 2\ddot{d}(\ddot{d} - 1)(d - 2)(A')^2P + \]

\[ \ddot{d}(\ddot{d} - 1)(d - 2)(d - 3)(A')^2S + 4\ddot{d}(\ddot{d} - 1)(d - 2)MN + 2\ddot{d}(d - 2)(d - 3)MS + \]

\[ 4\ddot{d}(d - 2)(d - 3)NP + 2\ddot{d}(d - 2)(d - 3)(d - 4)NS + 2(d - 2)(d - 3)(d - 4)PS \right\} + \]

\[ \frac{1}{2} e^{B} \left[ A - (d - 3)\lambda \right] \ddot{L}\delta(r - \ell) = 0, \]

where we have defined

\[ M \equiv \Lambda'' + (A')^2 - A'B', \quad N \equiv A' \left( B' + \frac{1}{r} \right), \quad P \equiv B'' + \frac{B'}{r}, \quad S \equiv (B')^2 + 2\frac{B'}{r}, \quad \tilde{d} \equiv D - d. \]

Above the third equation is the \((\alpha\beta)\) equation, the second equation is the \((\mu\nu)\) equation, while the first equation is the \((rr)\) equation. Note that the latter does not contain second
derivatives of $A$ and $B$. Also note that the other two equations do not contain third and fourth derivatives of $A$ and $B$ (this is a special property of the Gauss-Bonnet combination).

The equations of motion (16)-(18) are highly non-linear and difficult to solve in the general case. However, in this paper we focus on the case $d = 1$ and $d = 4$. That is, our brane has the geometry of $\mathbb{R}^{0,1} \times S^3$ (the first factor corresponds to the time direction), and it is embedded in a 5-dimensional bulk. As we will see in the next section, we can then solve these equations analytically.

III. SPHERICAL 3-BRANE IN 5D BULK

In the $d = 1$ and $d = 4$ case the equations of motion simplify substantially:

\begin{align}
N + S - 4\xi e^{-2B}NS &= 0 , \\
P + S - 4\xi e^{-2B}PS + \frac{1}{6}e^{B} \left[ \bar{\Lambda} - 3\lambda \right] \bar{\delta}(r - \epsilon) &= 0 , \\
M + 2N + 2P + S - 4\xi e^{-2B} [MS + 2NP] + \frac{1}{2}e^{B} \left[ \bar{\Lambda} - \lambda \right] \bar{\delta}(r - \epsilon) &= 0 .
\end{align}

Note that $P$ and $S$ do not contain $A$, so the second of the above equations is a (non-linear) ordinary differential equation for $B$. We can actually solve it analytically.

To begin with, we will solve this equation without the $\delta$-function source term. Then we will find appropriate solutions by imposing the matching conditions. Let us introduce the following simplifying notations:

\begin{align}
& r_0^2 = 4\xi , \\
& C \equiv B + \ln(r/r_0) , \\
& z \equiv \ln(r/r_*),
\end{align}

where $r_*$ is an arbitrary positive parameter (which, as will become clear in the following, is just an integration constant). Then (20) reads (without the source term):

\begin{equation}
C_{zz} = \frac{1 - (C_z)^2}{1 + [1 - (C_z)^2] e^{-2C}} ,
\end{equation}

where subscript $z$ denotes derivative w.r.t. $z$. This equation can be solved as follows. Let

\begin{equation}
1 - (C_z)^2 \equiv (f(C) - 1)e^{2C} ,
\end{equation}

where $f(C)$ is a function of $C$ only. Then we have the following first order differential equation for $f(C)$:

\begin{equation}
f_C = -2(f^2 - 1)/f ,
\end{equation}

whose solution is given by

\begin{equation}
f(C) = \eta \sqrt{1 + \omega e^{-4C}} ,
\end{equation}

\textsuperscript{6}
where $\eta = \pm 1$ corresponds to two distinct branches, and $\omega$ is the integration constant ($\omega \in \mathbb{R}$). The solution for $C$ is then given by:

$$C(z) = F(\zeta z, \eta, \omega), \quad (29)$$

where $F(x, \eta, \omega)$ is a solution of the following differential equation (note that this solution depends on another integration constant)

$$\frac{dF}{dx} = \sqrt{1 + e^{2F} - \eta \sqrt{\omega + e^{4F}}}, \quad (30)$$

and $\zeta = \pm 1$ corresponds to two distinct branches.

A. Non-singular Solutions without the Source Brane

Here we would like to see if there are non-singular solutions (in the absence of the source terms - if we find such solutions, we can then see if we can construct solutions with the source terms with desirable properties). Such a solution indeed exists. Thus, consider the case where $\eta = -1$, $\zeta = +1$ and $\omega = 0$. We then have (in this solution $-\infty < z < z_1$):

$$C(z) = -\ln \left[-\sqrt{2} \sinh (z - z_1)\right], \quad (31)$$

where $z_1$ is the remaining integration constant (note that we have set the first integration constant $\omega = 0$). From (19) it follows that

$$A = \ln |C_z| + \text{constant}. \quad (32)$$

This implies that in the above solution we have

$$A(z) = -\ln [-\tanh (z - z_1)] + \text{constant}. \quad (33)$$

The asymptotic behavior of this solution is given by:

$$C(z \to -\infty) = z + \mathcal{O}(1), \quad (34)$$
$$A(z \to -\infty) = \text{constant}, \quad (35)$$
$$C(z \to z_1-) = -\ln (z_1 - z) + \mathcal{O}(1), \quad (36)$$
$$A(z \to z_1-) = -\ln (z_1 - z) + \mathcal{O}(1). \quad (37)$$

Thus, when $r \to 0$ we have

$$B(r \to 0) = \text{constant}, \quad (38)$$
$$A(r \to 0) = \text{constant}, \quad (39)$$

that is, in this limit the space is asymptotically flat.

What about $z \to z_1-$ limit? The Ricci scalar is given by:

$$R = -\frac{2}{r_0^2} e^{-2C} \left\{ 3 \left[ C_{zz} + (C_z)^2 - 1 \right] + A_{zz} + 2A_z C_z + (A_z)^2 \right\}. \quad (40)$$
In the above limit the Ricci scalar actually goes to a constant:

\[ R \rightarrow -\frac{40}{r_0^2}. \]  

(41)

Note, however, that the space is not asymptotically AdS$_5$ in this limit (in particular, our solution is spherically symmetric, while AdS$_5$ is not). Moreover, consider the volume outside of a 3-sphere of radius $b$ (where $b < r_1 \equiv r_\ast e^{z_1}$):

\[ v(b) = v_3 \int_b^{r_1} dr \, r^3 \, e^{A+4B} = v_3 \, r_0^4 \int_{z_b}^{z_1} dz \, e^{A+4C}, \]  

(42)

where $v_3$ is the volume of a unit 3-sphere, and $z_b \equiv \ln(b/r_\ast) < z_1$. The volume $v(b)$ is actually infinite. Thus, as $z \rightarrow z_1$ we have a coordinate singularity (and not a true singularity). In particular, all geodesics are complete in this limit.

Finally, let us mention that we also have the $\zeta = -1$ counterpart of the above solution, that is, the solution with $\eta = -1$, $\zeta = -1$ and $\omega = 0$, which is also non-singular. This solution is given by (in this solution $z_2 < z < +\infty$):

\[ C(z) = -\ln \left[ \sqrt{2} \sinh (z - z_2) \right], \]  

(43)

\[ A(z) = -\ln \left[ \tanh (z - z_2) \right] + \text{constant}, \]  

(44)

where $z_2$ is an integration constant. In constructing the full solution with the source term we will use the $\zeta = +1$ solution. Also, in the full solution we will not even have a coordinate singularity (as the brane will be located to the left of this would-be coordinate singularity, while to the right of the brane we will have another solution, so that the resulting full solution is completely non-singular).

**B. Matching Conditions**

Note that (19) contains only first derivatives of $A$ and $B$, and it does not contain a $\delta$-function source term. This equation, therefore, is satisfied by the above solutions. However, the other two equations (20) and (21) contain second derivatives and source terms, and produce non-trivial matching conditions. We must therefore make sure that they are satisfied. In the $z$-coordinate frame these three equations read:

\[ A_z C_z \left\{ 1 - \left[ 1 - (C_z)^2 \right] e^{-2C} \right\} = 1 - (C_z)^2, \]  

(45)

\[ C_{zz} \left\{ 1 + \left[ 1 - (C_z)^2 \right] e^{-2C} \right\} = \left[ 1 - (C_z)^2 \right] e^C \left[ \tilde{\Lambda} - 3\lambda \right] r_0 \tilde{L} \delta(z - z_\varepsilon), \]  

(46)

\[ \left[ A_{zz} + (A_z)^2 \right] \left\{ 1 + \left[ 1 - (C_z)^2 \right] e^{-2C} \right\} + 2C_{zz} \left\{ 1 - A_z C_z e^{-2C} \right\} = 2 \left[ 1 - (C_z)^2 - A_z C_z \right] - \frac{1}{2} e^C \left[ \tilde{\Lambda} - \lambda \right] r_0 \tilde{L} \delta(z - z_\varepsilon), \]  

(47)

where

\[ z_\varepsilon \equiv \ln(\epsilon/r_\ast), \]  

(48)
and in arriving at (47) we have used (19).

The third equation (47) looks cumbersome. We can simplify it as follows. Note that $C_z$ is always non-vanishing. We can therefore rewrite (45) as follows:

$$A_z \left\{ 1 + \left[ 1 - (C_z)^2 \right] e^{-2C} \right\} + C_z - 1/C_z = 0 .$$  (49)

Differentiating this equation w.r.t. $z$ we obtain:

$$A_{zz} \left\{ 1 + \left[ 1 - (C_z)^2 \right] e^{-2C} \right\} - 2A_z C_z \left\{ C_{zz} + \left[ 1 - (C_z)^2 \right] \right\} e^{-2C} + C_{zz} \left[ 1 + 1/(C_z)^2 \right] = 0 .$$  (50)

Equation (47) can then be rewritten as follows:

$$[C_{zz} - A_z C_z] \left[ 1 - 1/(C_z)^2 \right] = -\frac{1}{2} e^C \left[ \bar{\Lambda} - \lambda \right] r_0 \tilde{L} \delta(z - z_\epsilon) ,$$  (51)

which is much nicer than the original equation (47).

Note that (51) gives (32) away from the brane, and we obtained (32) away from the brane from the first two equations (19) and (20). This is, as usual, a consequence of the diffeomorphism invariance - away from the brane only two out of the three equations are independent (which is just as well for we have only two warp factors $A$ and $B$). However, since the brane is a source brane (as opposed to a solitonic one), at the brane the diffeomorphism invariance is not all intact (more precisely, it is partially reduced by an implicit gauge choice). As a result, (51) does produce a non-trivial matching condition at the brane (that is, at $z = z_\epsilon$), and so does (46).

Thus, we have the following two matching conditions from (46) and (32), respectively:

$$C_z \left\{ 1 + \left[ 1 - \frac{1}{3} (C_z)^2 \right] e^{-2C} \right\} \bigg|_{z_\epsilon^+}^{z_\epsilon^-} = -\frac{1}{6} e^{C(z_\epsilon)} r_0 \tilde{L} \left[ \bar{\Lambda} - 3\lambda \right] ,$$  (52)

$$[C_z + 1/C_z] \bigg|_{z_\epsilon^-}^{z_\epsilon^+} = -\frac{1}{2} e^{C(z_\epsilon)} r_0 \tilde{L} \left[ \bar{\Lambda} - \lambda \right] .$$  (53)

Let us simplify these conditions as follows. First, let

$$\kappa = \bar{\Lambda}/\lambda .$$  (54)

In the following we will set the corresponding integration constant so that

$$A(z = z_\epsilon) = 0 .$$  (55)

Next, note that the radius of the 3-sphere actually is not $\epsilon$, rather, it is given by

$$\bar{R} \equiv \epsilon e^{B(r=\epsilon)} = r_0 e^{C(z_\epsilon)} .$$  (56)

Finally, recall that

$$\lambda = \frac{2}{\epsilon^2} e^{-2B(r=\epsilon)} = \frac{2}{\bar{R}^2} .$$  (57)

With these notations we then have:
\[ C_z \left( 1 + \frac{r^2_0}{R^2} \left[ 1 - \frac{1}{3} (C_z)^2 \right] \right)^{z_{c^+}^-} = -\frac{\kappa - 3}{3} \frac{L}{R}, \tag{58} \]
\[ [C_z + 1/C_z]^{z_{c^+}^-} = -(\kappa - 1) \frac{L}{R}. \tag{59} \]

What we would like to see is if these two equations have a solution for \( C_z \) and \( \kappa \). More precisely, we need two things. First, to successfully address the cosmological constant problem, solutions must exist for a continuous range of values of \( \kappa \) (or else \( \tilde{\Lambda} \) would have to be fine tuned). This is actually guaranteed if we find one solution - indeed, the size of the 3-sphere \( \tilde{R} \) is not a parameter of the theory, it contains a free integration constant via \( C(z_{\epsilon}) \) (it is clear that \( \kappa \) does not scale exactly as \( \tilde{R}^2 \)). Second, we must make sure that solutions exist for a phenomenologically interesting range of parameters. In the next subsection we give explicit solutions of the aforementioned type.

C. Explicit Non-Singular Solutions with the Source Brane

We are interested in constructing a solution such that the space is at least asymptotically flat for \( r > \epsilon \), and the volume in the extra dimension is infinite. With the aforementioned phenomenological constraints, we are then led to consider a solution of the following type (we have chosen the integration constant in \( A(z) \) such that \( A(z_{\epsilon}) = 0 \); moreover, we have absorbed the integration constant \( z_1 \) from above into the definition of \( r_+ \)):

\[ z > z_{\epsilon} : \quad C(z) = \Phi(z, \omega), \tag{60} \]
\[ A(z) = \ln \left( \frac{\Phi_z(z, \omega)}{\Phi_z(z_{\epsilon}, \omega)} \right), \tag{61} \]
\[ z < z_{\epsilon} : \quad C(z) = -\ln \left[ -\sqrt{2} \sinh(z) \right], \tag{62} \]
\[ A(z) = -\ln \left[ \frac{\tanh(z)}{\tanh(z_{\epsilon})} \right], \tag{63} \]

where \( \Phi(z, \omega) \) is the solution of the following differential equation

\[ \Phi_z = \sqrt{1 + e^{2\Phi} - \sqrt{\omega + e^{4\Phi}}} \tag{64} \]

subject to the initial condition

\[ \Phi(z_{\epsilon}, \omega) = -\ln \left[ -\sqrt{2} \sinh(z_{\epsilon}) \right]. \tag{65} \]

Here \( \omega \) is an integration constant. It is, however, constrained as follows (with this constraint the above full solution is non-singular for \( z \in \mathbb{R} \)):

\[ \frac{1}{4 \sinh^4(z_{\epsilon})} < \omega < \coth^2(z_{\epsilon}). \tag{66} \]

Note that \( \epsilon < r_+ \) so that \( z_{\epsilon} < 0 \). Moreover, the solution is asymptotically flat in both \( z \to \pm \infty \) limits, and the volume of the fifth dimension is infinite.
In the following it will be more convenient to parametrize $\omega$ as follows:

$$\omega \equiv \nu \coth^2(z_\epsilon) \, ,$$

(67)

where

$$-\frac{1}{\sinh^2(2z_\epsilon)} < \nu < 1 \, .$$

(68)

Note that

$$-\sinh(z_\epsilon) = \frac{r_0}{\sqrt{2R}} \ll 1 \, .$$

(69)

That is, $-z_\epsilon \ll 1$, and the brane is located very close to the would-be coordinate singularity (at $r = r_*$):

$$\frac{\epsilon}{r_*} - 1 \approx -\frac{r_0}{\sqrt{2R}} \, .$$

(70)

We then have (for the reasons that will become clear in the following, here we are assuming $|\nu| \sim 1)$:

$$C_z(z_-) = -1/z_\epsilon + \mathcal{O}(z_\epsilon) \, ,$$

(71)

$$C_z(z_+) = \sqrt{1-\nu} + \mathcal{O}(z_\epsilon^2) \, .$$

(72)

The matching conditions (58) and (59) then read:

$$\sqrt{2} \tilde{L} r_0 = \frac{1}{z_\epsilon} \frac{2\nu - 1}{2\sqrt{1-\nu}} - \frac{3}{2} + \mathcal{O}(z_\epsilon) \, ,$$

(73)

$$\sqrt{2\kappa} \tilde{L} r_0 = \frac{1}{z_\epsilon^2} + \frac{1}{z_\epsilon} \frac{3}{2\sqrt{1-\nu}} + \mathcal{O}(1) \, .$$

(74)

Let

$$\tilde{\nu} \equiv \frac{1}{2} - \nu \, .$$

(75)

Then we have (the fact that $\tilde{\nu}$ must be small will become clear momentarily):

$$0 < \tilde{\nu} \ll 1 \, ,$$

(76)

$$z_\epsilon \approx -\tilde{\nu} \left[ \frac{\tilde{L}}{r_0} + \frac{3}{2\sqrt{2}} \right]^{-1} \, ,$$

(77)

$$\tilde{R} \approx \frac{1}{\sqrt{2}\tilde{\nu}} \left[ \tilde{L} + \frac{3}{2\sqrt{2}} r_0 \right] \, ,$$

(78)

$$\kappa \approx \frac{\sqrt{2R^2}}{Lr_0} \left[ 1 - \frac{3r_0}{2\tilde{R}} \right] \, ,$$

(79)

$$\tilde{T} = \tilde{M}_P^2 \tilde{\Lambda} \approx 2\sqrt{2} M_P^4 (r_0 M_P)^{-1} \left[ 1 - \frac{3r_0}{2\tilde{R}} \right] \, .$$

(80)
Note that the entire solution is controlled by one integration constant $\tilde{\nu}$. In particular, $r_*$ is related to $\tilde{\nu}$ via

$$r_* \approx \epsilon \left\{ 1 + \tilde{\nu} \left[ \frac{\tilde{L}}{r_0} + \frac{3}{2\sqrt{2}} \right]^{-1} \right\} . \quad (81)$$

Moreover, the solution exists for a continuous range of positive values of the brane tension also controlled by $\tilde{\nu}$ via $\tilde{R}$. Note, however, that we have three phenomenological constraints: $\tilde{R}$ must be large; $\tilde{T}$ cannot be smaller than $(\text{TeV})^4$ (supersymmetry breaking considerations); the range of values of $\tilde{T}$ cannot be smaller than $(\text{TeV})^4$ (or else this would imply fine-tuning of the brane tension). As we will see below, we can satisfy these conditions, and this is precisely what requires that $\tilde{\nu}$ be small.

Thus, naturalness considerations suggest that (unless the bulk theory is either very strongly or very weakly coupled)

$$\beta \equiv r_0 M_P \sim 1 . \quad (82)$$

The brane tension can take values in the following window:

$$\tilde{T}_{\text{min}} < \tilde{T} < \tilde{T}_{\text{max}} , \quad (83)$$

where

$$\tilde{T}_{\text{min}} \approx \tilde{T}_{\text{max}} - 3\sqrt{2} \frac{M_P^3}{\tilde{R}_{\text{min}}} , \quad (84)$$

$$\tilde{T}_{\text{max}} = \frac{2\sqrt{2}}{\beta} M_P^4 , \quad (85)$$

where $\tilde{R}_{\text{min}}$ must be somewhat larger than the present Hubble size. Regardless of the value of $\tilde{T}_{\text{max}}$, the width of the aforementioned window should not be smaller than $(\text{TeV})^4$ (assuming that the supersymmetry breaking scale is around TeV) or else this would amount to fine-tuning of the brane tension. This then implies that the 5-dimensional Planck scale $M_P$ cannot be much different from the 4-dimensional Planck scale $\tilde{M}_P$ (this simply follows from the fact that we expect $\tilde{R}_{\text{min}}$ to be around $10^{30}$ mm, while TeV $\sim 10^{-15}$ mm$^{-1}$, and $\tilde{M}_P \sim 10^{15}$ TeV $\sim 10^{30}$ mm$^{-1}$):

$$M_P \sim \tilde{M}_P . \quad (86)$$

Note that here various numerical factors might be only roughly of order 1. Thus, the window for the brane tension is of order $(\text{TeV})^4$, but the brane tension itself is of order $\tilde{M}_P^4 \sim 10^{60}$ (TeV)$^4$. At first this might seem as gross fine-tuning. However, this is not necessarily the case. It is perfectly consistent to have brane tension which is much larger than (the fourth power of) the supersymmetry breaking scale. Thus, consider a supersymmetric BPS brane whose tension is of order $\tilde{M}_P^4$. Suppose supersymmetry is dynamically broken on the brane, and the supersymmetry breaking scale is of order $M_{\text{SUSY}} \ll \tilde{M}_P$. It is clear that, after supersymmetry breaking (assuming it is spontaneous), the brane tension remains of order $\tilde{M}_P^4$. The contribution to the brane tension due to the supersymmetry breaking, on
the other hand, is expected to be of order $M_{\text{SUSY}}^4$. As we will see in the next subsection, this is exactly what happens in our solution.

Before we end this subsection, let us give the above solution in terms of $A(r)$ and $B(r)$:

\begin{align}
  r > \epsilon : & \quad B(r) = Q(r) , \\
  & \quad A(r) = \ln \left( \frac{rQ'(r)}{\epsilon Q'(\epsilon)} \right) , \tag{88} \\
  r < \epsilon : & \quad B(r) = \ln \left( \frac{\sqrt{2r_0}}{r_*} \right) - \ln \left[ 1 - \frac{r^2}{r_*^2} \right] , \\
  & \quad A(r) = \ln \left( \frac{r_*^2 + r^2}{r_*^2 + \epsilon^2} \right) - \ln \left( \frac{r_*^2 - r^2}{r_*^2 - \epsilon^2} \right) , \tag{90}
\end{align}

where $Q(r)$ is the solution to the following differential equation

\begin{equation}
  rQ' = \sqrt{1 + e^{2Q} - \nu \left( \frac{r_*^2 + 2\epsilon^2}{r_*^2 - \epsilon^2} \right)^2 + e^{4Q}} , \tag{91}
\end{equation}

subject to the initial condition

\begin{equation}
  Q(\epsilon) = \ln \left( \frac{\sqrt{2\epsilon} r_*}{r_*^2 - \epsilon^2} \right) . \tag{92}
\end{equation}

Note that $r_*$ ($r_* > \epsilon$) is not independent but is related to $\nu$ (note that $\nu$ lies in a finite interval - see above). This relation comes from the matching conditions, and its approximate form is given by (81).

### D. The BPS Solution

The 5-dimensional bulk action we started with (the Einstein-Hilbert plus Gauss-Bonnet terms) can be supersymmetrized\(^4\). We will not give details here, but one can show that for $\tilde{\nu} > 0$, that is, finite (but large) $\tilde{R}$ the solution of the previous subsection does not preserve any supersymmetries. In particular, there are no non-trivial Killing spinors in this background.

However, if we consider the $\tilde{R} \to \infty$ limit, we have a BPS solution. In this limit the brane is flat\(^5\) and has the geometry of $\mathbb{R}^{3,1}$. We have only one warp factor $A(z)$, where $z$ is the coordinate transverse to the brane, and the background metric is conformally flat.

\(^4\)In doing so we will have fields other than the metric. In the following we will focus on backgrounds where all bosonic fields other than the metric have vanishing expectation values. Then the background we presented in the previous subsection is also a solution to the equations of motion in the supersymmetric case.

\(^5\)Note that one must appropriately rescale coordinates in this limit.
ds^2 = \exp(2A) \eta_{MN} dx^M dx^N. \quad (93)

The equations of motion read (note that $z$ here has dimension of length):

\begin{align*}
(A_z)^2 \left[ 1 - 2\xi (A_z)^2 e^{-2A} \right] &= 0, \\
\left[ A_{zz} - (A_z)^2 \right] \left[ 1 - 4\xi (A_z)^2 e^{-2A} \right] + \frac{1}{6} \tilde{\Lambda} \tilde{L} \delta(z - z_0) &= 0, \quad (94, 95)
\end{align*}

where $z_0$ is the location of the brane. As before, let $r_0^2 \equiv 4\xi$. Then we have the following solution (this is the solution corresponding to the $\tilde{R} \to \infty$ limit of the solution of the previous subsection):

\begin{align*}
A(z) &= -\ln \left[ \frac{\sqrt{2} \theta(z_0 - z)(z_0 - z)}{r_0} + 1 \right], \quad (96)
\end{align*}

where $\theta(x)$ is the Heavyside step-function. The matching condition

\begin{align*}
A_z \left[ 1 - \frac{r_0^2}{3} (A_z)^2 e^{-2A} \right] \bigg|_{z_0^+}^{z_0^-} + \frac{1}{6} \tilde{\Lambda} \tilde{L} = 0 \quad (97)
\end{align*}

implies that this solution exists for the following value of the 3-brane tension:

\begin{align*}
\tilde{\Lambda} &= \frac{2\sqrt{2}}{L r_0}, \quad (98) \\
\tilde{T} &= \tilde{M}_P^2 \tilde{\Lambda} = 2\sqrt{2} M^3_P (r_0 M_P)^{-1}. \quad (99)
\end{align*}

Note that this is precisely the critical value $\tilde{T}_{\text{max}}$ of the brane tension we found in the previous section (recall that $\tilde{T}_{\text{max}}$ precisely corresponds to the $\tilde{R} \to \infty$ limit).

In a bosonic theory the fact that this solution exists only for the above special value of the brane tension would be interpreted as fine-tuning. However, in a supersymmetric theory this is not necessarily the case. Indeed, this value could simply be the BPS saturated value for the brane tension. In fact, in a supersymmetric version of this model the above solution is indeed a BPS solution (preserving 1/2 of the supersymmetries). To see this, let us study Killing spinors in the above background:

\begin{align*}
\mathcal{D}_M \varepsilon &= 0. \quad (100)
\end{align*}

Here $\mathcal{D}_M$ is a generalized covariant derivative:

\begin{align*}
\mathcal{D}_M &= D_M - \frac{1}{2} W \Gamma_M, \quad (101)
\end{align*}

where $D_M$ is the usual covariant derivative containing the spin connection, and $W$ is interpreted as the superpotential (we will give its value below). The $D$-dimensional gamma matrices $\Gamma_M$ are given by

\begin{align*}
\Gamma_M &= \exp(A) \tilde{\Gamma}_M, \quad (102)
\end{align*}
where \( \hat{\Gamma}_M \) are the corresponding Minkowski gamma matrices (which are independent of coordinates):

\[
\{ \hat{\Gamma}_M, \hat{\Gamma}_N \} = 2\eta_{MN} .
\]

(103)

In the following we will use the notation \( x^M = (x^m, z) \), and \( \hat{\Gamma}_z \) will be the gamma matrix corresponding to the \( z \)-direction.

Thus, we have

\[
0 = \mathcal{D}_z \varepsilon = \partial_z \varepsilon - \frac{1}{2} W \exp(A) \hat{\Gamma}_z \varepsilon ,
\]

(104)

\[
0 = \mathcal{D}_m \varepsilon = \partial_m \varepsilon + \frac{1}{2} \hat{\Gamma}_m \left[ A_z \hat{\Gamma}_z - W \exp(A) \right] \varepsilon .
\]

(105)

Before solving the Killing spinor equations, let us note that to define an unbroken supercharge for a given Killing spinor we must make sure that the global integrability conditions

\[
[\mathcal{D}_M, \mathcal{D}_N] \varepsilon = 0
\]

(106)

are also satisfied. In the component form these conditions read:

\[
0 = [\mathcal{D}_m, \mathcal{D}_n] \varepsilon = \frac{1}{4} \left[ W^2 \exp(2A) - (A_z)^2 \right] [\hat{\Gamma}_m, \hat{\Gamma}_n] \varepsilon ,
\]

(107)

\[
0 = [\mathcal{D}_m, \mathcal{D}_z] \varepsilon = \frac{1}{2} \hat{\Gamma}_m \left( \exp(A) \partial_z W + \left[ W^2 \exp(2A) - A_{zz} \right] \hat{\Gamma}_z \right) \varepsilon .
\]

(108)

Since in the solution (96) \( A_z \) is discontinuous, to satisfy the last condition \( W \) must be discontinuous as well. Then only 1/2 of supersymmetries can be preserved, and the corresponding Killing spinor has a definite helicity w.r.t. \( \hat{\Gamma}_z \):

\[
\hat{\Gamma}_z \varepsilon = \eta \varepsilon ,
\]

(109)

where \( \eta \) is either +1 or −1. We therefore have the following BPS equation:

\[
A_z = \eta W \exp(A) .
\]

(110)

This equation together with the solution (96) then implies that

\[
W = \frac{\sqrt{2} \eta}{r_0} \theta(z_0 - z) .
\]

(111)

The Killing spinor is then given by

\[
\varepsilon = \exp \left[ \frac{1}{2} A \right] \varepsilon_0 ,
\]

(112)

where \( \varepsilon_0 \) is a constant spinor with helicity \( \eta \):

\[
\hat{\Gamma}_z \varepsilon_0 = \eta \varepsilon_0 .
\]

(113)

Thus, as we see, the solution (96) is a BPS solution preserving 1/2 of supersymmetries.
At first it might appear strange that we have non-vanishing superpotential for \( z < z_0 \). Note, however, that this is just as well. First of all, the space for \( z < z_0 \) is (locally) AdS\(_5\), while for \( z > z_0 \) it is flat. Secondly, as was shown in [10], the scalar potential in the presence of the Gauss-Bonnet term is given by\(^6\):

\[
V = \frac{3M_P^3}{2\xi} \left[ (1 - 4\xi W^2)^2 - 1 \right]. \tag{114}
\]

Note that in our setup the bulk scalar potential is vanishing. The values of the superpotential consistent with vanishing \( V \) are \( W = 0 \) and \( W = \pm \sqrt{2}/r_0 \), which is precisely what we have in (111).

Thus, as we see, in the limit where the 3-brane is flat we have a BPS solution (with \( \mathcal{N} = 1 \) supersymmetry in four dimensions). On the other hand, when the brane is a 3-sphere of a large finite radius \( \tilde{R} \) the solution is no longer BPS saturated, rather it is a near-BPS solution. Here we would like to comment on some important properties of these solutions.

Thus, since we have a BPS solution, we can make the discussion of the previous subsection precise. We can populate the BPS brane with an \( \mathcal{N} = 1 \) supersymmetric theory. Let part of this theory contain a chiral superfield \( \phi \) with a vanishing tree-level superpotential. Let us assume that there is a dynamically generated superpotential in this theory. Supersymmetry dictates that the the 3-brane action has the following form (here we give only the terms relevant for our discussion):

\[
S_\Sigma = \int_\Sigma d^4x \left\{ \sqrt{-G} \left[ \tilde{M}_P^2 \tilde{R} - \tilde{T}_{\text{BPS}} - \tilde{V}(\phi, \phi^*) \right] \right\}, \tag{115}
\]

where the scalar potential is invariably given by

\[
\tilde{V}(\phi, \phi^*) = e^{K/\tilde{M}_P^2} \left[ K_{\phi \phi}^{-1} |F_\phi|^2 - \frac{3}{\tilde{M}_P^2} \tilde{W}^2 \right]. \tag{116}
\]

Here \( K(\phi, \phi^*) \) is the Kähler potential, \( \tilde{W}(\phi) \) is the superpotential, and the F-term is given by:

\[
F_\phi = \tilde{W}_\phi + K_{\phi \phi} \tilde{W} / \tilde{M}_P^2. \tag{117}
\]

The scale \( \tilde{M}_P \) is the 4-dimensional (reduced) Planck scale\(^7\). Also, note the \( \tilde{T}_{\text{BPS}} \) is the critical value of the brane tension (that is, \( \tilde{T}_{\text{BPS}} = \tilde{T}_{\text{max}} \)). Note that if \( \tilde{V} \) is non-zero, it contributes to the brane tension. Here we would like to emphasize that supersymmetry precludes this contribution to be of any form other than that given above.

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\(^6\)This is the 5-dimensional version of the general \( D \)-dimensional formula derived in [10]. In fact, the formula of [10] also contains a scalar field, but here \( W \) is independent of any such field.

\(^7\)Up to now we identified this scale with \( \tilde{M}_P \). However, \( \tilde{M}_P \) is not exactly equal \( \tilde{M}_P \). Rather, \( \tilde{M}_P^2 \) receives an additional contribution from the higher curvature terms in the bulk as the second derivatives \( A_{zz} \) and \( C_{zz} \) have \( \delta \)-function-like behavior at the location of the brane. At any rate, in the present context \( \tilde{M}_P \sim \tilde{M}_P \).
Thus, let us assume that at the tree-level $\tilde{W} = 0$. Due to the non-renormalization theorem, non-trivial $\tilde{W}$ can only be generated non-perturbatively. Let us assume that we indeed have such a dynamically generated superpotential. Here we would like to show that we can find non-supersymmetric solutions (with supersymmetry broken around TeV) such that the brane is non-inflating and we do not need any fine-tuning. Thus, all we need here is to consider a theory where the F-term $|F_\phi| \sim (\text{TeV})^2$ with the scalar potential taking a negative value $-\tilde{V} \sim (\text{TeV})^4$. This, for instance, can be achieved by taking the minimal Kähler potential $K(\phi, \phi^*) = \phi \phi^* \equiv \rho \hat{M}^2_P$, and a non-vanishing constant superpotential $\tilde{W}_\phi = 0$. We can obtain the latter via, say, gaugino condensation in a non-Abelian supersymmetric gauge theory without matter. Let the gauge group $G$ be simple. Then we have:

$$\tilde{W} = h_G \Lambda^3_G ,$$

where $h_G$ is the dual Coxeter number of the group $G$, and $\Lambda_G$ is the dynamically generated scale. We then have:

$$\tilde{V}(\rho) = \frac{|\tilde{W}|^2}{\hat{M}^2_P} e^\rho (\rho - 3) .$$

This scalar potential has a non-supersymmetric minimum at $\rho = 2$, where the F-term $|F_\phi| = \sqrt{2} |\tilde{W}|/\hat{M}_P$ and the scalar potential $V = -e^\rho |\tilde{W}|^2/\hat{M}^2_P$. With an appropriate choice of $\Lambda_G$ we can have the F-term of order (TeV)$^2$ (which implies that the supersymmetry breaking scale $M_{\text{SUSY}} \sim \text{TeV}$), and $-V \sim (\text{TeV})^4$.

Before we end this subsection, we would like to address the following question. Thus, at first it might appear strange that going from a flat brane to a spherical one with large radius is consistent with supersymmetry breaking on the brane around TeV. In particular, note that there are no covariantly constant spinors on a 3-sphere, so we expect supersymmetry to be broken, but one might naively expect that, since the shift in the brane tension should be of order $\hat{M}^2_P/\hat{R}^2$ (the curvature of the 3-sphere is of order 1/\(\hat{R}^2\)), the supersymmetry breaking scale should be (at most) of order $\sqrt{\hat{M}_P/\hat{R}} \sim 1/\text{mm}$. Such naive thinking, however, is not quite correct. Thus, note that the brane tension in our solution is given by:

$$\tilde{T}(\hat{R}) = \tilde{T}_{\text{max}} - 3 \sqrt{2} \frac{\hat{M}_P^3}{\hat{R}} + O \left( \frac{\hat{M}_P^3}{\hat{R}^2} \right) ,$$

where we are assuming that $1/r_0 \sim M_P \sim \hat{M}_P$. So we do have the aforementioned naive contribution of order $\hat{M}^3_P/\hat{R}^2$ to the brane tension. However, the next-to-leading contribution is not this one, rather it is of order $M^3_P/\hat{R}$ (which is related to the fact that we can have supersymmetry breaking around TeV). The correct way of thinking about this question is the following. We have a BPS solution with a flat brane whose tension is of order $M^4_P$. Now we break supersymmetry at $M_{\text{SUSY}}$. The shift in the brane tension is of order $M^4_{\text{SUSY}}$. Nonetheless, in the corresponding non-inflating solution the radius of the brane, which is

\[8\] Here we are assuming that the dilaton (as well as any other fields controlling the gauge coupling are stabilized via, say, the mechanism of [11].
now a sphere, is *not* of order \( M_P/M_{\text{SUSY}}^2 \) (which would be of order millimeter if we take \( M_{\text{SUSY}} \sim \text{TeV} \)), rather it is much larger, namely, of order \( M_P^3/M_{\text{SUSY}}^4 \). And this is possible due to the fact that the space in the vicinity of the brane is highly *curved*, that is, the warp factor \( C(z) \) near the brane is very large. In turn, this is possible due to the fact that the brane tension is very large compared with \( M_{\text{SUSY}}^4 \). Put it another way, the effect of the supersymmetry breaking on the brane is *diluted* because of huge curvature near the brane, so in the resulting non-inflating solution the radius of the brane is huge!

### IV. COMMENTS

In the previous section we constructed a solution where the 3-brane is a 3-sphere of large radius \( \tilde{R} \) (which is somewhat larger than the present Hubble size). This solution is completely non-singular, which is due to the presence of higher curvature terms in the 5-dimensional bulk action. We chose these terms to be the quadratic (in curvature) Gauss-Bonnet combination. However, we did this only to make the problem computationally tractable. In particular, albeit we do not have a proof of this, we expect that solutions with all the qualitative features of our solution should also exist in the presence of generic higher curvature terms in the bulk.

A key property of our solution is that the 3-brane is non-inflating, and the solution exists for a continuous range of values of the 3-brane tension. That is, we do not need any fine-tuning for addressing the cosmological constant problem in our scenario. Instead, the brane tension is controlled by an integration constant in this solution. Moreover, we can have large brane tension. Let us also mention that, since our solution does not require fine-tuning, we should also be able to accommodate small cosmological constant on the brane, that is, there should also exist solutions with slowly inflating brane, and such solutions are not expected to require any fine-tuning either.

Since our brane is a non-zero tension 3-sphere, one might worry about it collapsing, that is, there might be instability due to a tachyonic radion mode. Note, however, that since the radius of the 3-sphere is huge, the (negative) mass squared of this tachyonic mode would be tiny, and the collapse would take a very long time. We can estimate this time by noting that the sphere cannot be shrinking faster than the speed of light. Then if we take \( \tilde{R} \) even a few orders of magnitude larger than the present Hubble size, it would take the sphere many times the age of the universe to shrink.

Finally, let us comment on the most important phenomenological issue in our scenario. Since in our solution the bulk Planck scale \( M_P \) is of order the brane Planck scale \( \tilde{M}_P \), gravity on the brane is actually 5-dimensional. Thus, for generic values of \( M_P \), we expect the cross-over scale \( r_c \sim \tilde{L} \) - below \( r_c \) gravity is 4-dimensional (that is, if \( r_c \gg M_P \)), while above \( r_c \) gravity is 5-dimensional. However, in our case \( r_c \sim 1/\tilde{M}_P \), so gravity is always 5-dimensional. Here we would like to discuss possible ways of circumventing this.

To begin with, note that we could *a priori* take \( M_P \ll \tilde{M}_P \) such that the cross-over scale would be of order or somewhat larger than the present Hubble size thus satisfying phenomenological constraints [12]. However, this would require having \( M_P \sim 10 - 100 \text{ MeV} \ll M_{\text{SUSY}} \). Since the BPS brane tension \( \tilde{T}_{\text{BPS}} \sim M_P^3/r_0 \), this would require \( r_0 \ll 1/M_P \) (this can in principle be the case if the underlying fundamental theory such as string theory is strongly coupled in the bulk) to have the BPS brane tension at least of order (TeV)\(^4\). Note,
however, that the window for the brane tension is of order $M_P^3/{\tilde{R}}_{\text{min}}$, which would be much smaller than (TeV)$^4$ unless we allow $R_{\text{min}}$ to be much smaller than the present Hubble size, namely, $R_{\text{min}} \sim \text{mm}$. This way we could in principle avoid fine-tuning the brane tension, but in reality we would be disguising it as another fine-tuning. Indeed, for generic contributions to the brane tension coming from supersymmetry breaking the size of the 3-brane would be of order millimeter. To have $R$ above the present Hubble size, we would need to fine-tune this contribution with the accuracy of $10^{-60}$. This approach, therefore, does not solve this issue.

To circumvent this, one could consider infinite-volume scenarios where a spherical 3-brane is embedded in higher-than-five dimensional bulk. In the $R \to \infty$ limit we then have a codimension-2 or higher brane. In the limit of a $\delta$-function-like brane we then do not have cross-over to higher-dimensional gravity - gravity on the brane is essentially always 4-dimensional [2]. More precisely, the cross-over scale depends on the thickness of the brane and goes to infinity when the latter goes to zero. However, in this limit the graviton propagator on the brane is actually singular, and once we include higher derivative terms, the cross over scale is expected to be $r_c \sim M_P^2/M_P^2$, which would require $M_P \sim \text{mm}^{-1}$ [13]. More precisely, this is the case when higher derivative terms are suppressed by powers of $1/M_P$. If, however, we have a greater suppression factor (this can, for instance, happen if the underlying fundamental theory is strongly coupled in the bulk), then $r_c$ could a priori be much higher. More work needs to be done to see whether non-fine-tuned solutions with 4-dimensional gravity on the brane and large cross-over scale can be constructed in this context. A study in these directions will be reported elsewhere [14].

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