Implications of space-momentum correlations and geometric fluctuations in heavy-ion collisions

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Abstract. The standard picture of heavy-ion collisions includes a collective expansion. If the initial energy density in the collisions is lumpy, then the expansion can convert the spatial lumpiness into correlations between final-state particles. Correlations in heavy-ion collisions show prominent features not present in p+p collisions. I argue that many features of these correlations are related to the transference of over-densities from the initial overlap region into momentum-space during the QGP phase of the expansion. I show results from a toy Monte-Carlo to illustrate the consequences of lumpy initial conditions and a collective expansion.

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1. Introduction

Heavy ionized nuclei are collided at high energies in laboratory experiments to recreate temperatures and densities similar to those in the early universe at approximately one microsecond after the big bang [1]. At that time, the universe was filled with deconfined quarks and gluons, a phase of matter known as the quark-gluon plasma. Studying the quark-gluon plasma in the laboratory involves observing hadrons emitted from the fireball created in the collision of ultrarelativistic heavy nuclei. The fireball is $10^{-14}$ meters across and expands for approximately $5 \times 10^{-23}$ seconds before hadrons form, re-scattering ceases, and the stable particles stream away from the collision zone to eventually be detected by particle detectors many of orders of magnitude larger than the speck of quark-gluon-plasma created in the collisions. Understanding the evolution of the matter created in those collisions by observing the hadrons that stream into the detectors is a challenge.

In this talk I discuss how fluctuations in the geometry of the initial overlap zone [2] can manifest as two-particle correlations. These correlations may therefore provide information about the expansion phase that converts the over-densities in the initial overlap zone into momentum space correlations in the final-state. I use a toy Monte-Carlo model to illustrate the relevance of $v_n$ fluctuations where $v_n$ are the coefficients
in a Fourier expansion of the particle multiplicity with respect to the reaction plane. It’s important to consider the odd values of $n$ particularly since some of the interesting features of recent two particle correlations data from RHIC can be readily understood in terms of $v_3$ fluctuations. An analysis of $\sqrt{v_n^2}$ to study the expansion of heavy-ion collisions was first proposed in Ref. [3]. $v_2$ fluctuations and di-hadron correlations are discussed in Ref. [4] where $v_n$ fluctuations for $n > 2$ are argued to be zero and the remaining structures in correlations data are argued to arise from jets.

2. The Correlation Landscape at RHIC

Correlations and fluctuations have long been considered a good signature for Quark Gluon Plasma (QGP) formation in heavy-ion collisions [5]. Early proposals for QGP searches suggested searching for a non-monotonic dependence of fluctuations on variables related to the energy systems density (e.g. center-of-mass energy or collision centrality) the expectation being that above some energy density threshold, a phase transition to QGP would occur. Data from the experiments at RHIC indeed reveal interesting features in the two-particle correlation landscape [6, 7] but that data does not seem to support a picture based on correlations arising from a phase transition. Correlation structures unique to Nucleus-Nucleus collisions are found but their longitudinal width demonstrates that they come from the earliest moments of the collisions [8]. While two-particle correlations in p+p and d+Au collisions show a jet-peak narrow in relative azimuth $\Delta \phi$ and pseudo-rapidity $\Delta \eta$, the near-side peak in Au+Au collisions broadens substantially in the longitudinal direction and narrows in azimuth [6].

An analysis of the width of the peak for particles of all $p_T$ finds the correlation extends across nearly 2 units of pseudo-rapidity $\eta$ [6, 7]. When triggering on higher momentum particles ($p_T > 2$ GeV/c for example), the correlation extends beyond the acceptance of the STAR detector ($\Delta \eta < 2$) and perhaps as far as $\Delta \eta = 4$ as indicated by PHOBOS data [7]. It has been proposed that the ridge-like correlations are related to either non-perturbative multi-quark or gluon effects on mini-jets in Au+Au collisions [9], to soft gluons radiated by hard partons traversing the overlap region [10], to initial spatial correlations in the system converted to momentum-space correlations by a radial Hubble expansion [11], to beam-jets also boosted by the radial expansion [12], or to viscous broadening [13].

The modifications to the correlations are not limited to small $\Delta \phi$. When at least one of the particles used in the analysis has $p_T > 2$ GeV/c (a selection made to attempt to increase sensitivity to jets), the correlation structure at $\Delta \phi > \pi/2$ (away-side) is also highly modified in comparison to $p + p$ collisions [14]. Instead of a narrow peak with a maximum at $\Delta \phi = \pi$ corresponding to an away-side jet, a peak shifted away from $\Delta \phi = \pi$ is observed. It has been proposed that the correlations on the away-side may be the result of a mach-cone induced by the fast moving away-side parton as it traverses the medium [15]. Other proposals suggest the off-axis peak is due to Cerenkov radiation from the away-side jet [16] or deflection of jets in the medium [17].
In this talk, I argue that the ridge and away-side “cone” may be a manifestation of \( v_n \) fluctuations; the distinction between two- or few-particle correlations and \( v_n \) fluctuations being that \( v_n \) fluctuations are a property of the bulk-medium. The ridge and cone might not be related to a single hard or semi-hard scattered parton, but instead might be due to acoustics in the expansion of the quark-gluon plasma initiated from a highly textured initial energy density; one having hot-spots and regions of over- and under-densities. This hypothesis will need to be further developed to test for consistency with the full set of heavy-ion data including the \( \Delta \eta, \) charge, and particle type dependence of correlations.

3. A Toy Monte-Carlo

A hydrodynamic expansion leads to strong space-momentum correlations: spatial density gradients and interactions lead to boosts which are largest in the direction of the largest gradient. Information about the spatial gradient is therefore transferred into momentum space via the boost driven by the pressure. A system that starts from lumpy initial conditions and then undergoes a hydrodynamic expansion [18] should contain non-trivial two-particle correlations in the final momentum space [19]. The conversion of the initial eccentricity \( \varepsilon = \frac{\langle y-x \rangle}{\langle y+x \rangle} \) into \( v_2 \) has been discussed extensively in the literature; see Ref. [20] and references therein. The manifestation of smaller scale spatial fluctuations into \( v_n \) fluctuations, however, is a novel topic. I show results from a toy model to illustrate the connection of \( v_n \) fluctuations and azimuthal correlations to fluctuations in the initial overlap geometry.

The toy model assumes that particles in an event are generated from a finite number of boosted sources. These boosted sources could be ”hot-spots” left over from high-density regions in the initial overlap area. The boost velocity depends on the radial and azimuthal position of the source. The number of sources is taken to be \( N_{\text{part}} \) from a Monte Carlo Glauber model [21]. The coordinates of the sources are also taken from the Monte Carlo Glauber model. A blast-wave source function [22] is sampled enough times for each source point so that the total multiplicity produced in our toy model matches the \( \sqrt{s_{NN}} = 200 \text{ GeV Au+Au data} [23] \). We simulate particles within \( |\eta| < 1 \). The details of the model will be published in an upcoming publication.

Applying a pure Hubble boost to sources distributed according to the distribution of participants in a Glauber model will lead to negative \( v_2 \) values. Typically the blast-wave model is used to model particle emission from a freeze-out surface which can be highly deformed from the initial geometry. The deformation is assumed to arise from rescattering and flow. This toy model is constructed to study how correlations in the initial conditions can be reflected in correlations and fluctuations in the produced particles. Since in our model, the freeze-out geometry is fixed by the initial distributions, we must apply a boost with a large azimuthal asymmetry in order to obtain reasonable \( v_2 \) values. We do not attempt to model the expansion or to evolve the hot spots with a dynamic model. Such a calculation has been carried out elsewhere [24].
Figure 1. A representation of the space-momentum correlations induced in this toy Monte Carlo. The arrows represent the mean transverse momentum vectors of particles emitted from $x, y$ points in the collision overlap region. The four panels show four different impact parameters: $b = 0, 4, 8, \text{and} 12 \text{ fm}$.

Figure 1 shows how the boost employed in our model influences the momentum of the produced particles. The arrows in the figure show the average transverse momentum vector for particles emitted from sources at $x$ and $y$. This model clearly leads to the desired space momentum correlations since particles emitted from positive $x$ have a preference for having momentum in the same direction. The boost also leads to the desired $v_2$ with most arrows pointing preferentially in the direction of the shortest axis as anticipated for a pressure driven expansion. The red points show the locations of participants in one event. The points have been shifted so that $\langle x \rangle$ and $\langle y \rangle$ are centered at 0,0. This shift has a major effect on $v_1$ fluctuations, reducing the value of $\langle v_1^2 \rangle$ significantly. The uneven distribution of participants in a single event indicates that, in the case that space-momentum correlations develop in the expansion of the system, the produced particles should possess non-trivial correlations.

4. Model Results

The model parameters have been tuned so that the integrated $v_2$ matches RHIC data [23]. The odd $\langle v_n \rangle$ terms from the model are zero at mid-rapidity but we will see that the odd $\langle v_n^2 \rangle$ terms can be finite. First, in fig. 2 I show the variance of $v_2$ relative to the mean $v_2$. This ratio has been used in comparisons of data to model calculations for eccentricity fluctuations [25]. The data in this figure should be compared with data using $v_2$ with respect to the reaction plane and not the participant plane. These model
Figure 2. The ratio of $v_2$ fluctuations over the mean as a function of impact parameter.

results are similar to what has been observed in preliminary data. We do not compare to published STAR data [23] because that data was calculated by integrating $v_2\{4\}(p_T)$ weighting by the spectra which is not equivalent to what is shown in this figure. The ratio is very large in central and peripheral collisions where the denominator approaches zero. In the intermediate region, the ratio has a minimum of about 0.5 at $b = 7$ fm.

4.1. Azimuthal Correlations

Two-particle azimuthal correlations for all unique pairs of particles from our model with $b = 6$ fm are shown in Fig. 3. The correlation function $C(\Delta \phi) = \rho / \rho_{ref}$ where $\rho$ is the pair density and the reference $\rho_{ref} = \frac{N_{pairs}}{N_{events}} \frac{2}{M(M-1)}$ ($M$ is the average multiplicity). In the left panel, the correlations are fit to a Gaussian peak centered at $\Delta \phi = 0$, a $\cos(\Delta \phi)$, a $\cos(2\Delta \phi)$ term, and a constant offset (Gaussian fit). In the bottom panel, the same model results are fit with $A(1 + \sum_n 2a_n \cos(n\Delta \phi))$ for $n = 1, 2, 3$ and 4 (Cosine fit). $a_n$ denotes the fit parameters which should be equal to $\langle v_n^2 \rangle + \delta_n$. Both functions have the same number of fit parameters and give equally good descriptions of the correlation function. Fits similar to the Gaussian fit have been used to parametrize data in terms of a near-side mini-jet peak plus two cosine terms for momentum conservation and for elliptic flow [4].

Neither of the fits returns the correct value for $\langle v_2^2 \rangle$. The Cosine fit comes closest but overestimates the value by 4.6% while the Gaussian fit underestimates the value by 18%. The Gaussian fit underestimates $\langle v_2^2 \rangle$ because part of $v_1^2$ and $v_2^2$ along with all of the higher order $v_n^2$ terms are subsumed into the Gaussian peak which overestimates the contribution from non-flow in our model. We use the true values of $v_n^2$ in our model to extract the true nonflow that arise from correlations between particles emitted from the same participant pair. The true nonflow peak (not shown) is well described by an offset and a narrow Gaussian with a width of 0.54 radians. The $n = 2$ component of the the true nonflow accounts exactly for the differences between the
known $\langle v_n^2 \rangle = 1.451e-03$ and the parameter $a_2 = 1.517e-03$ in the Cosine fit. This exercise demonstrates that the parameters extracted from a fit to a correlation function are not easily related to $\langle v_n^2 \rangle$. The results of this simulation contradict the conclusion in Ref. [26] that $\langle v_n^2 \rangle$ is typically over-estimated in di-hadron analyses and that a Gaussian type fit can be used to extract $\langle v_n^2 \rangle$ with minimal systematic error. Even when that fit uses the $\Delta \eta$ dependence, it requires the assumption that $v_n(\eta_1)v_n(\eta_2)$ has no covariance: $E(v_n(\eta)v_n(\eta+\Delta \eta)) = E(v_n(\eta)v_n(\eta))$. Such an assumption can mimic a Gaussian dependence for the nonflow in $\Delta \eta$. In this simulation, the Gaussian fit underestimates $\langle v_n^2 \rangle$ and overestimates the nonflow correlations. It is best for model builders to compare their models to measured correlation functions and experiments should publish unmanipulated correlation functions.

![Figure 3](image.png)

**Figure 3.** Both panels: the correlation function for all particles produced in the Monte-Carlo with b=6 fm. Left panel: Monte-Carlo data fit with a five parameter fitting function including a Gaussian, an offset, and two cosine terms. Right panel: data fit with a five parameter fitting function including only an offset and cosine terms.

4.2. $v_n$ Fluctuations

Fig. 4 shows $v_n$ fluctuations as a function of $n$ where $\sigma_{v_n} = \sqrt{\langle v_n^2 \rangle - \langle v_n \rangle^2}$. Results are shown for pions and protons produced from events with impact parameters of 0 and 6 fm. As anticipated, both the even and the odd terms of $\sigma_{v_n}$ are non-zero. Except for $n = 1$, the values of $\sigma_{v_n}$ generally drop with $n$. The $n = 1$ term is suppressed because we applied a shift to the $x$ and $y$ of the participants in our model so that $\langle x \rangle = 0$ and $\langle y \rangle = 0$ (re-centering the events). This significantly reduces $\sigma_{v_1}$ by constraining the geometric fluctuations, so that for most cases $\sigma_{v_2} > \sigma_{v_1}$. The $\sigma_{v_n}$ values are larger for protons than for pions. This comes about in our model because geometry fluctuations drive the $v_n$ fluctuations and heavier particles carry more information about the geometry (space-momentum correlations are larger when the ratio of the particle mass to freeze out temperature $m/T$ is larger).
The non-zero values of $\sigma_{v_n}$ for odd terms and the falling trend of $\sigma_{v_n}$ with $n$ implies that $v_3$ fluctuations should not be neglected. In this model, the $v_3$ fluctuations are particularly prominent for protons and at higher momentum where the space-momentum correlations are strongest. $v_3$ fluctuations provide a ready explanation for the double-hump structure on the away-side of di-hadron correlations. $v_n$ fluctuations also provide a ready explanation for why correlations on the near- and away-side of di-hadron correlations both increase rapidly together with centrality [6]; they are actually the same correlation function arising from the bulk but artificially divided into components.

![Figure 4](image.png)

*Figure 4.* The variance or RMS of the $v_n$ distributions for $n$ from 1 to 9. Results are shown for protons and pions at impact parameter $b = 0$ and 6 fm.

### 4.3. Relationship to Strangeness and Conclusion

At this conference, several speakers have discussed how correlations and fluctuations affect $p_T$ spectra. Fits using Tsallis statistics have proved successful for fitting the spectra of strange and heavy flavor hadrons [27] (the focus of this conference). The same lumpy initial energy density that leads to the $v_n$ fluctuations presented here, can also be the source of the temperature fluctuations implied by the Tsallis fits. If this is true, one should be able to construct a model that describes all these phenomena at once. That model of bulk QCD matter should transition smoothly to a pQCD picture when enough energy is focused within a small enough region.

In this talk, I’ve shown results from a toy Monte-Carlo model that illustrate the potential importance of $v_n$ fluctuations in understanding the initial conditions and expansion of heavy-ion collisions. I showed that lumpy initial conditions coupled with space-momentum correlations developed in an expansion phase, leads to correlations and fluctuations similar to those seen in RHIC data.

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