Effects of oscillatory motion of a visco-elastic dusty fluid (blood) through arteries under the influence of magnetic field with porous medium

M Chitra1, R Bhaskaran2 and S Parthasarathy3

1Associate Professor, 2Research Scholar, Department of Mathematics, Thiruvalluvar University, Vellore-632 115, Tamilnadu, India
3Associate Professor, Department of Statistics, Annamalai University, Annamalai Nagar, Chidambaram-608 002, Tamilnadu, India
Email: 1chitratvu@gmail.com, 2bhasswathi@yahoo.com, 3statsarathy@yahoo.co.in

Abstract. In this paper, we study oscillatory flow of Visco-elastic dusty fluid in blood flow through an artery. Solid and spherical non-conducting dust particles (the cells) are assumed to be the micro particles which are structural components of cells (RBC,WBC,Platelets) present in the blood( fluid) are distributed symmetrically. Governing equations are solved analytically by using Bessel function and using different values of parameters on the axial velocity(ub,vd), volumetric flow rate (Qb,Qd) and wall shear stress (τwb,τwd) for both the blood flow and dust particles through human artery are discussed through graphs. The results revealed that the velocity of blood,flow rate and shear stress increases when increasing the value of Visco-elastic parameter and it decreases as magnetic parameter value increases.

1. Introduction

During few decades considerable attention has been given to the study of dusty Visco-elastic fluid in various fields like field of engineering, environmental science and blood flow in arteries. The fluid such as oil, honey and blood with viscous and elastic properties such type of fluid refers as Visco-elastic fluid. The dust particles (the cells) are considered to be micro particles present in the blood( fluid). The first systematic vascular model has been developed by Womersley (1989). His model is based on Navier –stokes equations which connect forces involved in a viscous fluid motion. He analysed the laminar flow of a Newtonian fluid in rigid, elastic and finally, Visco-elastic cylindrical types.

Figure 1. Visco-elastic fluid contains the cells in the blood vessel
Blood circulates throughout our body with four basic components which are plasma (liquid), red blood cells (RBC), white blood cells (WBC) and platelets. Plasma represents 55% in the blood which is 92% of water. It carries water and nutrients to tissues in our body. RBC represent 40%-45% of our blood volume which is generated from our bone marrow from our body and it carries oxygen to the tissues. White blood cells amount of 1% in the blood which fight infections, illness and disease. Platelets are the smallest cell blood cells which control bleeding. Blood transport metabolic waste products like carbon dioxide, urea, and acid away from those same cells. The fluid is removing all waste products and also providing nutrition and oxygen through micro particles.

Many researchers have analyzed the dusty Visco-elastic fluid in blood flow characteristics. Ashok Kumar et al. [1] introduced effect of oscillatory motion of a Visco-elastic dusty fluid passes through a porous medium under the presence of magnetic field. Gabriela Varuara et al. [2] investigated the modelling of the blood two phase dusty fluid flow and also the effects of biological parameter on volumetric flow rate are shown graphically. Das [3] studied slow unsteady flow of an incompressible dusty viscous fluid through a circular tube with axial roughness. Saffman [4] have been derived on the stability of laminar flow of a dusty gas. Ruchi Chaturvedi et al. [5] developed the numerical analysis of MHD flow of dusty Visco-elastic first order oldroyd fluid when passing through a porous rectangular channel. Devasish et al. [6] analysed oscillatory dusty flow with volume fraction and pressure gradient in rotating channel. Om Prakash et al. [7] have considered heat transfer to MHD oscillatory dusty Visco-elastic fluid flow in a channel filled with porous medium Chang and Atabek et al. [8] analysed the inlet length for oscillatory flow and its effects on the determination of the rate of flow in arteries. Dey et al. [9] worked on the free convective dusty fluid passes through a porous media in the presence of inclined magnetic field and Manjunatha et al. [10] have obtained an unsteady flow and heat transfer of a rotating dusty fluid with radiation effects. S. Eswaramoorthi et al. [11] analysed effect of radiation on MHD convective flow and heat transfer of a Visco-elastic fluid over a stretching surface. Effect of viscous dissipation and Newtonian heating on a second grade fluid over a stretching surface was analytically and numerically studied by Eswaramoorthi et al. [12].

The objective of this work is the analysis of oscillatory flow of Visco-elastic dusty fluid in blood flow through artery filled with porous medium. The effects of various parameters on flow characteristics such as velocity profile, flow rate and wall shear stress for the flow of blood and micro dust particles through human artery are analysed through graphs.

2. Mathematical formulation

We consider oscillatory flow of Visco-elastic of blood in human carrying micro particles with porous medium formulated by the Saffman (1962). Let $u^*$ be the velocity of blood and $v^*$ be velocity of micro dust particles in the human artery. 

The governing equation of the motion of the circular tube of the artery is

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial u^*}{\partial t} = -\frac{1}{\rho} \frac{\partial P^*}{\partial z} + \gamma \left[1+\lambda_1 \frac{\partial}{\partial t}\right] \left[\frac{\partial^2 u^*}{\partial r^2} + \frac{1}{r} \frac{\partial u^*}{\partial r}\right] + \frac{kN_t}{\rho} \left[1+\lambda_1 \frac{\partial}{\partial t}\right] (v^*-u^*) - M^* u^* - \frac{u^*}{K}

(1)$$

$$M \frac{\partial v^*}{\partial t} = k (u^* - v^*)

(2)$$
Boundary conditions are

\[
(\text{i}) \quad \frac{\partial u}{\partial r} = 0 \text{ and } \frac{\partial v}{\partial r} = 0 \text{ at } r = 0 \\
(\text{ii}) \quad u = u^* \text{ and } v = U^*(s) \text{ at } r^* = R_0
\] (3)

Where \( t^* \) is the time, \( P^* \) is the gradient of pressure, \( \rho \) is the density, \( \gamma \) is the kinematic viscosity, \( \lambda \) is the elastic viscous parameter, \( k \) is the stokes drag coefficient, \( K \) is the permeability constant, \( M \) is the magnetic parameter and \( N_0 \) is the number density and \( z^* \) is the direction of axis.

Let us take the following non-dimensional quantities are

\[
\begin{align*}
  u &= \frac{u^*}{u_0}, \quad v = \frac{v^*}{u_0}, \quad r = \frac{r^*}{R_0}, \quad z = \frac{z^*}{u_0}, \quad \omega = \frac{M \gamma}{k u_0^2} \\
  u_s &= \frac{u_s^*}{u_0}, \quad U(s) = \frac{v^*}{u_0}, \quad P = \frac{P^*}{u_0}, \quad B^2 = \frac{M^2 u_0^2}{\gamma} \\
  T &= \frac{\gamma k}{u_0}, \quad I = \frac{MN_0}{\rho}, \quad \tau = \frac{\tau^*}{u_0}, \quad \lambda_z = \frac{\gamma \lambda_2}{u_0^2}
\end{align*}
\] (4)

Where \( B \) is the magnetic parameter and \( \lambda_z \) is the Visco-elastic parameter.

Using above non-dimensional quantities in equation (1) and (2) as follows

\[
\frac{\partial u}{\partial t} + \frac{u^2}{u_0} \frac{\partial^2 u}{\partial z^2} = -\frac{\gamma}{u_0} \frac{\partial P}{\partial z} + \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial^2 u}{\partial r} + \frac{\partial^2 u}{\partial \tau^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \tau r} \right] + \frac{I}{\omega} (v - u) + \frac{I}{\omega} \lambda_z \frac{\partial}{\partial \tau} (v - u) - B^2 u - \frac{u}{T}
\] (5)

\[
\omega \frac{\partial v}{\partial t} = u - v
\] (6)

where \( \omega = \frac{M \gamma}{k u_0^2}, \quad I = \frac{k u_0^2 N_0}{\gamma \rho}, \quad B^2 = \frac{M^2 u_0^2}{\gamma} \) and \( T = \frac{\gamma k}{u_0^2} \)

The non-dimensional boundary condition from equation (3) are

\[
(\text{i}) \quad \frac{\partial u}{\partial r} = 0 \text{ and } \frac{\partial v}{\partial r} = 0 \text{ at } r = 0 \\
(\text{ii}) \quad u = u^* \text{ and } v = U^*(s) \text{ at } r = 1
\] (7)

**3. Method of Solution**

Let us consider the solution of \( u \) and \( v \) are

\[
u(r,t) = u(r)e^{i\omega t} \text{ and } v(r,t) = v(r)e^{i\omega t}\] (8)
By using equations (6) and (8) becomes

The solution of the equation (6) is

\[ v(r) = \frac{u(r)}{1 + in \omega} \]  

(9)

Let \(-\frac{\partial P}{\partial \zeta} = Pe^{int}\)  

(10)

By using equation (9) and (10) in equation (5), we get

\[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - inu - \frac{B^2 u}{1 + in \omega} - \frac{u}{T (1 + in \lambda_2)} = \frac{-P \gamma}{u_0 (1 + in \lambda_2)} \]

\[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - k_i^2 u = -c \]

Where \(c = \frac{P \gamma}{u_0 (1 + in \lambda_2)}\), \(k_i^2 = \frac{(1 + in \lambda_2) \left( in + \frac{1}{1 + in \omega} \right) + B^2 + \frac{1}{T}}{(1 + in \lambda_2)}\)

Let \(u = \frac{-c}{k_i^2} + y\) and hence \(\frac{d^2 y}{dr^2} + \frac{1}{r} \frac{dy}{dr} - k_i^2 y = 0\)  

(11)

By using Bessel function, we get \(y = AJ_0(ik_i r)\)  

(12)

Using boundary condition equation (7) in the equation (12), we get

\[ u(r) = \frac{-c}{k_i^2} + \left[ \frac{u_0 + \frac{c}{k_i^2}}{J_0 (ik_i)} \right] J_0 (ik_i r) \]  

(13)

The velocity of the Visco–elastic fluid of blood through the human artery is

\[ u_b(r, t) = u(r, t) = \left[ \frac{-c}{k_i^2} + \left( \frac{u_0 + \frac{c}{k_i^2}}{J_0 (ik_i)} \right) J_0 (ik_i r) \right] e^{int} \]  

(14)

The velocity of the Visco–elastic fluid of micro particles through the human artery is
\[ v_d(r, t) = v(r, t) = \frac{1}{1 + in \omega} \left[ -c \left( \frac{u_s + c}{k_i^2} \right) J_0 \left( i k_i r \right) \right] e^{int} \]  

(15)

Volumetric flow rate for the Visco -elastic fluid of blood through the human artery is

\[ Q_b = 2\pi \int_0^{R_b} \bar{u} r^2 dr' = 2\pi u_0 R_b^2 \int_0^1 \bar{u} r dr \]

\[ Q_b = 2\pi u_0 R_b^2 \left[ \left\{ \frac{-c}{2k_i^2} + \left( \frac{u_s + c}{k_i^2} \right) \right\} \right] e^{int} \]  

(16)

Volumetric flow rate for the Visco elastic fluid of micro particles through the human artery is

\[ Q_d = \frac{2\pi u_0 R_b^2}{1 + in \omega} \left[ \left\{ \frac{-c}{2k_i^2} + \left( \frac{u_s + c}{k_i^2} \right) \right\} \right] e^{int} \]  

(17)

Wall shear stress for the Visco –elastic fluid of blood through the human artery is

\[ \tau_{eb} = -\mu \left[ \frac{\partial \bar{u}'}{\partial r} \right]_{at \ r=R_b} = -\mu \frac{u_0}{R_0} \left[ \frac{\partial \bar{u}'}{\partial r} \right]_{at \ r=1} \]

\[ \tau_{eb} = \frac{\mu i k_i u_0 \left( \frac{u_s + c}{k_i^2} \right)}{R_0 J_0 \left( i k_i \right)} J_1 \left( i k_i \right) e^{int} \]  

(18)

Wall shear stress for the Visco –elastic fluid of micro particles through human artery is

\[ \tau_{ed} = \frac{\mu i k_i u_0 \left( \frac{u_s + c}{k_i^2} \right)}{R_0 J_0 \left( i k_i \right) \left( 1 + in \omega \right)} e^{int} \]

(19)

4. Result and Discussion

The objective of the Present study is to understand and bring out the effects of Elastic-Viscous dusty fluid through artery on flow quantities of blood flow.
Figure 2 & Figure 3 depicts the axial velocity profile of blood ($u_b$) increases and micro particles ($u_d$) decreases as the increasing the values of Visco-elastic parameter. The decreased micro particles which affect to carry oxygen slowly to the tissues.

Figure 4 & Figure 5 illustrates that axial velocity profile of blood decreases and micro particles increases as increasing the values of the magnetic parameter.

From, Figure 6 & Figure 7 it is observed that the volumetric flow rate of blood ($Q_b$) and micro particles ($Q_d$) increases as the increasing the values of Visco-elastic parameter.

Figure 8 & Figure 9 noted that the volumetric flow rate of blood and micro particles decreases as the increasing the magnetic parameter. The flow rate of micro particles in the blood are decreased which affect intercellular communication.

From, Figure 10 & Figure 11 shows that the wall shear stress of blood ($\tau_{wb}$) and micro particles ($\tau_{wd}$) increases as the increasing the values of Visco-elastic parameter.

Figure 12 & Figure 13 depicts that the wall shear stress of blood and micro particles decreases as the increasing the magnetic parameter values.

**Figure 2.** Velocity of the blood versus Radius of artery for different values of Visco-elastic parameter $I = 0.4, u_k = 0.5, \gamma = 0.4, P = 0.2, n = 4, B = 5, T = 5, \omega = 1$

**Figure 3.** Velocity of the micro particles versus Radius of artery for different values of Visco-elastic parameter $I = 0.4, u_k = 0.5, \gamma = 0.4, P = 0.2, n = 4, B = 5, T = 5, \omega = 1$
Figure 4. Velocity of the blood versus Radius of artery for different values of Magnetic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, n = 4, \lambda_2 = 5, T = 5, \omega = 1$

Figure 5. Velocity of the micro particles versus Radius of artery for different values of Magnetic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, n = 4, \lambda_2 = 5, T = 5, \omega = 1$

Figure 6. Volumetric flow rate of the blood verses time for different values of Visco-elastic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, t = 0.2, n = 4, B = 5, T = 5, \omega = 1$

Figure 7. Volumetric flow rate of the micro particles verses time for different values of Visco-elastic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, t = 0.2, n = 4, B = 5, T = 5, \omega = 1$
Figure 8. Volumetric flow rate of the blood versus time for different values of Magnetic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 2, t = 0.2, n = 4, \lambda_2 = 5, T = 1, \omega = 1$

Figure 9. Volumetric flow rate of the micro particles versus time for different values of Magnetic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, t = 0.2, n = 4, \lambda_2 = 5 = 5, T = 5, \omega = 1$

Figure 10. Shear stress of the blood versus time for different values of Visco-elastic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, t = 0.2, n = 4, B = 5, T = 5, \omega = 1$

Figure 11. Shear stress of the micro particles versus time for different values of Visco-elastic parameter with $l = 0.4, u_s = 0.5, \gamma = 0.4, P = 0.2, t = 0.2, n = 4, B = 5, T = 5, \omega = 1$
5. Conclusion

The purpose of finding the velocity profile, volumetric flow rate and shear stress and relevant computational work have been performed for the various parameters encountered in the present analysis. From the above discussion, the following observations have been made:

- The velocity distribution increases for blood and decreases for micro particles as the value of Visco-elastic parameter increases.
- The velocity distribution decreases for blood and increases for micro particles as the values of magnetic parameter increases.
- The volumetric flow rate increases for both blood and micro particles as the value of Visco-elastic parameter increases.
- The volumetric flow rate decreases for both blood and micro particles as the value of magnetic parameter increases.
- The wall shear stress increases for both blood and micro particles as the Visco-elastic parameter increases.
- The shear stress decreases for both blood and micro particles as the magnetic parameter increases.

It is concluded that micro particles are reduced in blood flow when the Visco-elastic parameter increased which affect the capacity of the blood to transport oxygen to the tissues in our body. The micro particles are increased in the blood flow when the magnetic parameter increased which play an essential role in intercellular communication and coagulation events in blood flow through arteries that contribute to over all homeostasis of the body.
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