Avalanche dynamics of an idealized neuron function in the brain on uncorrelated random scale-free network

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Abstract

We study a simple model for a neuron function in a collective brain system. The neural network is composed of uncorrelated random scale-free network for eliminating the degree correlation of dynamical processes. The interaction of neurons is supposed to be isotropic and idealized. This neuron dynamics is similar to biological evolution in extremal dynamics with isotropic locally interaction but has different time scale. The evolution of neuron spike takes place according to punctuated patterns similar to the avalanche dynamics. We find that the evolutionary dynamics of this neuron function exhibit self-organized criticality which shows power-law behavior of the avalanche sizes. For a given network, the avalanche dynamic behavior is not changed with different degree exponents of networks, \( \gamma \geq 2.4 \) and refractory periods correspondent to the memory effect, \( T_r \). In addition, the avalanche size distributions exhibit the power-law behavior in a single scaling region in contrast to other networks. However, the return time distributions displaying spatiotemporal complexity have three characteristic time scaling regimes.

Key words: neuron functioning, uncorrelated scale-free network, return time distribution, avalanche dynamics, self-organized criticality

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1 Introduction

A human being is most complex organism in the nature. A human brain is extremely complicate in the organs of human. That is the reason that it is exceedingly difficult to understand brain through individual and classical

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The nervous tissue of the human brain contains many billions of neurons, large complex cells that conduct nerve impulses from one part of the body to another part. Scientists have investigated neural network in the brain through various methods [1,2,3,4,5]. Recently, neural network was reported to be like the scale-free network, which is characterized by displaying power law distribution of the degree [6]. An well known scale-free network is Barabási-Albert(BA) network characterized by evolving and preferential attachment [7,8]. Goh et al. introduce a static scale-free network with no degree-degree correlation [9]. The firing pattern of the neuron is similar to the avalanche pattern of the self-organized criticality(SOC) [10,11,12,13]. The propagative size of neuron functioning exhibits the power-law distribution. Thus, the punctuated pattern and power-law behavior occur without fine-tuning parameter. L. da Silva et al. offered a simple model for brain functioning similar to the Bak-Sneppen(BS) model [11] with the memory effect on the lattice [14]. The same model is investigated on small-world network [15] by Lin and Chen [16]. We consider an avalanche dynamics of an idealized neuron function on a uncorrelated random scale-free network. The structure and the dynamics of neurons describe as follows. The anatomical unit of the nervous system is the neuron. The brain possesses about $10^{10} - 10^{12}$ neurons. Neurons are quite complex, but each of these is made up of dendrite, a cell body, and an axon. A dendrite conducts signals toward the cell body. The cell body is the part of a neuron that contains the nucleus and other organelles. An axon conducts nerve impulses away from the cell body. Dendrites and axons collectively are called neuron fibers. A nerve impulse is the way a neuron transmits information. When an axon is not conducting a nerve impulse, the resting potential indicates that the inside of an axon is negative compared to the outside. In contrast, if an axon is conducting a nerve impulse, an action potential (i.e., electrochemical change) travels along a neuron. As an axon is stimulated by an electric shock, threshold may be reached for an action potential. A fiber can conduct a volley of nerve impulses because only a small number of ions are exchanged with each impulse. As soon as an impulse has passed by each successive portion of a fiber, it undergoes a refractory period during which it is unable to conduct an impulse. This ensures a one-way direction of the impulse. During a refractory period, the sodium gate cannot yet open. Every axon branches into $10^3 - 10^4$ fine terminal branches called a synaptic bulb. Each bulb lies very close to the dendrite (or the cell body) of another neuron. This region of close proximity is called a synapse. At a synapse, the membrane of the first neuron is called the presynaptic membrane, and the membrane of the next neuron is called the postsynaptic membrane. Transmission of the nerve impulse from one neuron to another takes place across a synapse. In humans, synaptic vesicles release a chemical, known as a neurotransmitter to receptors in the postsynaptic membrane increases the chance of a nerve impulse (stimulation) or decreases the chance of a nerve impulse (inhibition) in the next neuron [17]. The brain as a whole is a system capable of auto-regulations [14].
Fig. 1. The log-log plot of the probability distribution function \( P(s) \) of the \( B_0(\gamma) \) avalanche size as a function of the avalanche size \( s \) at the critical fitness on UCM for \( \gamma = 2.1, 2.5, 2.9, 3.0, 4.0, \) and 5.0 with \( N = 10000, m = 3, \) and \( T_r = 1 \). The critical threshold barriers where \( B_0 = 0.039, 0.080, 0.121, 0.130, 0.212, \) and 0.260 respectively.

2 Model and Simulation Method

To make a modeling of this neuron dynamics, we make networks. The networks is composed of uncorrelated random scale-free network. Here, the uncorrelated random scale-free network is called by the uncorrelated configuration model (UCM) [18]. We generate \( N \)-cells by the static method. Let’s make \( N_k \)-cells of the degree \( k \) satisfying the degree distribution \( P(k) \sim k^{-\gamma} \). Select two nodes randomly and connect them if they are not connected before. We exclude duplicated connections and self-connection. The network generated by the UCM is not only fully connected and have but also no degree-degree correlation \( D_{nn}(k) \), defined as the average degree of the nearest neighbors (NN) of the nodes with degree \( k \) and clustering correlation \( C(k) \), mean by the probability that a node of degree \( k \) form loop with two NN [18]. The number of minimal degree is fixed at \( m=3 \) to prevent alteration of dynamics from dangling node. Now, we introduce the evolution rule of neuron dynamics. Each node of the UCM represents a neuron and a link between two nodes represents a synapse. The uniform random numbers are distributed over each node. This random number is called by a barrier, which is the possibility of the firing of a neuron’s
Fig. 2. Dependence of the avalanche size exponents as a function of network degree exponents $\gamma$ with $N = 10000$, $m = 3$, and $T_r = 1$.

spike. The lower barrier has a higher potentialities to fire [14]. To simplify the firing is occurred at an neuron with lowest barrier sequentially. And directly connected neurons also is fired because they are enough stimulated to fire. The selected neuron with the lowest barrier can not fire if the elapsed time after the firing is less than the refractory period $T_r$. In fact, neuron’s spike transfer directly through their synapse. However, the evolution of the signal is just considered to spread out to all of nearest neighbors. If this process is iterated the system reaches to critical stationary state, which all the barriers are above the $B_c$ barrier so-called a critical threshold. That is, neural system is self organized without well tuning parameter to stationary state. And the brain functioning is occurred abruptly underlying avalanche dynamics. The purpose of this research is to investigate how the avalanche dynamics is changed according to varying degree exponent $\gamma$ of networks with different refractory periods $T_r$. For investigating spatiotemporal correlation of avalanche dynamics, we examined first and all return time distribution(RTDs).

3 Results

In stationary state, we consider the branching process of a avalanche dynamics to be unrestricted from network size corrections [19]. The avalanche is always
started from the hub neuron and all of the neurons with $B_i > B_0$, where $B_0$ is an auxiliary parameter, are treated as inactive neurons. A $B_0$ avalanche size $S$ is defined as the number of the firing less than $B_0$. As $B_0 \to B_c$, the avalanche size distribution follow a power-law behavior $P(S) \sim S^{-\tau}$ with an exponential cutoff. Figure 1 shows avalanche size distribution for the $B_0(\gamma)$ avalanche with the degree exponents of the networks, $\gamma = 2.1, 2.5, 2.9, 3.0, 4.0,$ and $5.0$. The avalanche size distribution follows power-law behavior, $P(S) \sim S^{-\tau(\gamma)}$ extending over more larger regime than on other scale-free networks such the BA network [7] and the static model introduced by Goh et al. [9,20]. The more interesting thing is that the power law behaviors of the avalanche size distribution do not exhibit the crossover between two different scaling regimes. The avalanche size distribution shows a short intermediate regime and follows exponential decay at the cut-off regime. The absence of the two regimes in the avalanche size distribution may be able to explain by following two reasons. One of the reasons is that an average degree $\langle k \rangle$ is not fixed with different degree exponent $\gamma$ as compared with another static models by the Goh’s algorithm [9] and in addition is increased as $\gamma$ gets smaller. Another reason is the absence of clustering correlations $\bar{C}(k)$ as well as degree-degree correlations $\bar{D}_{nn}(k)$ [18]. Figure 2 presents the basic critical exponents $\tau$, the so called avalanche size exponent from different $\gamma$, where the increment is 0.1.
Fig. 4. Dependence of the avalanche size exponents as a function of refractory periods $T_r$ with $N = 10000$, $m = 3$, and $\gamma = 3$.

in $2 < \gamma < 3$ and 0.5 in $\gamma > 3$, with $T_r = 1$. As we can observe, the critical avalanche size exponent is the same as mean field result i.e, $\tau \simeq 1.5$ for $\gamma \geq 2.4$. It is difficult to compare the avalanche size critical exponent $\tau$ in the UCM with the BA network because the avalanche size distribution on the BA network shows different power-law behavior with two regimes [21,22]. Occasionally, the critical thresholds are very close value each other, $f_c = 0.086 \pm 1$ (on UCM) and $f_c = 0.089 \pm 2$ (on BA) within error bar. Even though it is not universal value, $f_c$ seem to a criterion distinguishing different avalanche dynamics of $\gamma < 2.4$ from mean field result for $\gamma \geq 2.4$. In the lattice [14] and small-world network [16] with low rewiring probability $\phi = 0.01$, the lévy-flight exponents and the avalanche size exponents increase according to the increment of the refractory period $T_r$, respectively. Figure 3 illustrates the probability distribution for the avalanche size on UCM with different $T_r$. At the same time, in Fig.4, we demonstrate the dependence of the exponent $\tau$ from 1 to 10 with $B_0 = 0.13$. All $\tau$ are not changed according to varying $T_r$ unlike the results on lattice [14] or small-world network [16]. In the lattice, the avalanche dynamics of the firing is propagated further far away from first update neuron because the firing is rejected during $S < T_r$ as refractory period $T_r$ increase. But in scale-free network, the active neuron returns soon to the hub and the direct linked node in stationary state because a hub neuron has many nearest neighbor’s neurons. Accordingly, the firing neuron is not evolved far from hub quickly with refractory period. For that reason, the memory effects of refractory time
Fig. 5. The log-log plot of the probability distribution function $P(s)$ of the first return time as a function of the time $t$ at the critical fitness on UCM with $N = 10000$, $m = 3$ at $T_r = 1$, and $\gamma = 3$. The solid symbol presents the histogram, using the exponential bin plot.

vanish as the existence of hub is growing larger. In case of small-world network, the rewiring probability approaches a threshold to eliminate memory effect, the exponent $\tau$ also follows the mean-field value for different $T_r$.

The probability distribution of first and all return time is valuable quantities for investigating the spatiotemporal correlation and punctuated pattern [11,12,13,19]. The definition of first return time with the size $t$ is a separating intervals which activated subsequently from a given active neuron. In Fig.5, we present first return time distribution (FRTd) for $\gamma = 3$, $T_r = 1$. The first return time distribution do not satisfy power-law behavior contrary to the lattice case. The power-law behavior of the early return time region is mostly affected by the dynamics of a hub node as the hub and nearest neighbors frequently fire in the ratio of $k_{hub} + 1$, which $k_{hub}$ is the degree of the hub and $\langle k_{hub} \rangle = 45$ in Fig.5, that is, the hub has a probability of the firing as $1/(k_{hub} + 1)$. An interval of power-law regime in early return time increases with the degree of the hub. The intermediate return time distribution become almost constant for all nodes as $N \to \infty$ because each node has a same probability to be active again since UCM do not have the degree correlations. Finally, long return time has a long exponential decay regime with cut-off for $t \to L \xi$ by the finite size effect of the dangling node because the diameter of scale-free network is
Fig. 6. The log-log plot of the probability distribution function $P(s)$ of the all return time as a function of the time $t$ at the critical fitness on UCM with $N = 10000$, $m = 3$ at $T_r = 1$, and $\gamma = 3$. The solid symbol presents the histogram, using the exponential bin plot.

very small. All return times with the size $t$ is the elapsed time steps to time $t$ regardless of the intermediate firing since a given neuron fired at time $t_0$. In Fig. 6, we plot all return time distribution for $\gamma = 3$, and $T_r = 1$. The all return time distribution (ARTd) also is divided up three characteristic time scaling regimes. Furthermore, the slope of each regime is small in early return time regime like the lattice model although the general scaling relation is not satisfied.

4 Conclusion

We have studied the simple model of the neuron function in the brain on the uncorrelated scale-free network. Our model show the avalanche dynamics with different degree exponent $\gamma$ of the networks and different refractory periods $T_r$. The avalanche size exponent, $\tau$ is not changed according to varying $\gamma$, for $\gamma \geq 2.4$ as well as $T_r$. We measured first and all return time distribution (RTDs). The RTDs do not follow power-law behaviors consistently and show the three characteristic regions. In future work, the simple model for the SOC system on the scale-free network that memory effects contribute would be inquired and
a kind of scale-free network more neural system-like should be investigated.

Acknowledgments

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References

[1] S. H. Strogatz, Nature 410 (2001) 268.
[2] O. Sporns, G. Tononi, Complexity 7 (2002) 28.
[3] X.-W. Zhao, T.-L. Chen, Commun. Theor. Phys. 40 (2003) 363.
[4] S. Bornholdt, T. Röhl, Phys. Rev. E. 67 (2003) 363.
[5] C. Haldeman, J. M. Beggs, Phys. Rev. Lett. 94 (2005) 05810.
[6] V. M. Eguíluz, D. R. Chialvo, G. A. Cecchi, M. Baliki, A. V. Apkarian, Phys. Rev. Lett. 94 (2005) 018102.
[7] A.-L. Barabási, R. Albert, Science 286 (1999) 509.
[8] R. Albert, Barabási, Rev. Mod. Phys. 74 (2002) 47.
[9] K.-I. Goh, B. Kahng, D. Kim, Phys. Rev. Lett. 87 (2001) 278701.
[10] P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. Lett. 59 (1987) 381.
[11] P. Bak, K. Sneppen, Phys. Rev. Lett. 71 (1993) 4083.
[12] H. J. Jensen, Self-organized criticality: Emergent complex behavior in physical and biological systems, Cambridge University Press, Cambridge, 1998.
[13] P. Bak, How nature works: the science of self-organized criticality, Springer-Verlag, New York, 1999.
[14] L. da Silva, A. R. R. Papa, A. M. C de Souza, Phys. Lett. A 242 (1998) 343.
[15] D. J. Watts, S. H. Strogatz, Nature 393 (1998) 440.
[16] M. Lin, T.-L. Chen, Phys. Rev. E 71 (2005) 016133.
[17] S. S. Mader, Human Biology, McGraw-hill, New York, 1998.
[18] M. Catanzaro, M. Boguñá, R. Pastor-Satorras, Phys. Rev. E 71 (2005) 027103.
[19] M. Paczuski, S. Maslov, P. Bak, Phys. Rev. E 53 (1996) 414.
[20] D.-S. Lee, K.-I. Goh, B. Kahng, D. Kim, Nucl. Phys. B 696 (2004) 351.
[21] S. Lee, Y. Kim, Phys. Rev. E 71 (2005) 057102.
[22] N. Masuda, K.-I. Goh, B. Kahng, cond-mat/0508623.