Compressed sensing model for vibration signals of mechanical faults based on modulated wideband converters

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Abstract. Based on Compressive Sampling, the Modulated Wideband Converter (MWC) sampling method can implement sampling at a rate lower than the required by Nyquist theory. This paper designs a spectrum reconstruction system, based on MWC, which can reconstruct the spectrum of Sparse-Frequency signal at the rate lower than Nyquist-Frequency, and verify through the simulation signals and the measured fault bearing vibration signals.

1. Introduction
Mechanical vibration signal is the main means of machinery fault diagnosis. Its sampling process has always been based on the classical Nyquist sampling theorem. The theory points out that it is possible to reconstruct the original signal accurately from the sampled signal only when the sampling rate is at least twice the signal bandwidth[1]. However, with the continuous development of mechanical equipment, the demand for sampling rate of signals is getting higher and higher. There are a series of problems in high-speed sampling system, such as transmission, storage and calculation of massive data, which greatly increase the complexity[2]. It also brings great pressure to the digital signal processing at the back end[3]. In order to realize the low-speed sampling of useful signal information, Candès, Romberg, Tao and Donoho put forward the theory of compressive sensing[4-8]. The core of the theory is that when the signal is sparse in a transform domain, the original signal can be accurately recovered from a small part of the linear and non-adaptive measurements of the signal. At present, the main methods of compression sampling are random demodulator, multi-coset and modulated wideband converter (MWC)[9-10]. Random demodulation is mainly used for spectral line restoration. Although multi-coset can reduce the sampling rate, it is difficult to accurately achieve the delay of each channel. Modulated wideband converter has no precise delay problem similar to multi-coset requirement, and it is different from random demodulation only used for spectral line recovery[11]. This paper mainly verifies the application of modulated wideband converter model in fault feature extraction of mechanical vibration signals.

2. Sparse signal model in frequency domain
MWC can modulate high frequency signals, move high frequency lines to low frequency domain, and collect them in low frequency domain. Because of the sparsity of signal frequency domain, refactoring can be done. The signal model of sparse frequency domain is as follows.
Let \( x(t) \) be a continuous time signal. Its frequency domain range is \( F = [ -f_{NYQ}/2, f_{NYQ}/2 ] \). Its nyquist sampling rate is \( f_{NYQ} = 1/T_{NYQ} \). If there are only \( N \) intermixed sub bands in frequency spectrum \( X(f) \) of \( x(t) \), and the bandwidth of each subband is no more than \( B \) Hz, it can be called sparse multi-
band signal. Figure 1 depicts the simplest spectrum of typical sparse multiband signals, where the number of bands is N=2.

Fig.1 Spectrum of sparse frequency-band signal

3. MWC sampling system model

MWC technology originates from communication theory. Through the front-end periodic signal mixer, the original signal spectrum is moved to the low frequency domain and sampled in the low frequency domain. The sampled signal contains all the spectrum information of the original signal, and can theoretically restore the frequency band of the original signal.

The block diagram of the MWC sampling system is shown in Figure 2. Generally it includes random mixer, low pass filter, low speed ADC, etc. After sampling, reconstruction algorithm is needed to reconstruct the original signal.

The signal $x(t)$ is sampled simultaneously by $M$ path sampling channel. In any channel, signal $x(t)$ are first mixed with different pseudo-random sequences $p_i(t)$ ($i$ is channel labels) of the same period $T_p$. After mixing, the low-pass filter $H(f)$ with cut-off frequency of $1/2T$ is used for filtering. Finally, the $M$ group low frequency sampling sequence $y_i(n)$ is obtained by ADC with a sampling rate of $1/T$. Theoretically, the mixing function $p_i(t)$ can be any pseudo-random sequence with the same frequency $F_p$. Its frequency spectrum is $P_i(f) = \sum n f_p$. In this paper, the mixing function is $M$ sequences. Its expression is:

$$p_i(t) = a_k \cdot \frac{T_p}{M} \leq t \leq (k+1) \frac{T_p}{M}, 0 \leq k \leq M - 1$$

(1)

Where $a_k \in \{-1, 1\}$, $p_i(t + n T_p) = p_i(t)$.

Not each $a_k$ need to be strictly $\pm1$, As long as each cycle remains unchanged. But it should be reflected in calculation of $a_{ik}$.

After mixing, $z_i(t)$ through the lowpass filter $H(f)$. Filtering performance of $H(f)$ has great influence on signal reconstruction quality, and should be approach the ideal filter as far as possible. After filtering, the next is A/D sampling by rate of $1/T_s$. And then, the sub Nyquist sampling of low sampling rate ADC for high frequency signal is realized.

Fig.2 Block diagram of MWC sampling system
4. MWC sampling principle analysis
The signal reconstruction of MWC sampling system is based on the spectrum modulation of the signal. This section will analyze the MWC sampling system in frequency domain.

For the i channel:
Because \( p_i(t) \) is pseudo-random periodic signal, its spectrum consists of several equal interval bands and its Fourier series is:

\[
p_i(t) = \sum_{l=-\infty}^{\infty} c_{il} e^{j2\pi l T_i p_t}
\]

(2)

After mixing, the signal \( x_i(t) = x(t) p_i(t) \) fourier transform is:

\[
\tilde{X}_i(f) = \sum_{l=-\infty}^{\infty} e^{j2\pi l f} X(f - lf_p)
\]

(3)

After \( x_i(t) \) though lowpass filter \( H(f) \), only the signals in the frequency domain \([-f_s / 2, +f_s / 2]\) are preserved. Therefore, the discrete time Fourier transform (DTFT) of the sampling sequence \( y_i[n] \) is:

\[
Y_i(f) = \sum_{n=-\infty}^{\infty} y_i[n] e^{-j2\pi n f} = \sum_{n=n_0}^{n_0+L} c_{il} X(f - lf_p)
\]

\[
-\frac{f_s}{2} + (L_0 + 1) f_p \geq \frac{f_{nyq}}{2} \rightarrow L_0 = \left[ \frac{f_{nyq} + f_s}{2 f_p} \right] - 1
\]

(4)

(5)

From formula (4) we can find that the spectrum \( Y_i(f) \) of sampling sequence \( y_i[n] \) is composed of linear combination of the spectrum components \( X(f - lf_p) \), which obtained after the spectrum \( X(f) \) of signal \( x(t) \) is moved by \( lf_p \). This is an important basis for signal reconstruction.

We can transform formula (4) to be matrix:

\[
y(f) = A z(f), \quad f \in [-f_s / 2, +f_s / 2]
\]

(6)

Where \( y(f) = [Y_1(f), \ldots, Y_L(f)]^T \), \( z(f) = [X(f - l_0 f_p), \ldots, X(f + L_0 f_p)]^T \).

The observation matrix A is a \( m \times L \) dimensional matrix. And \( A_{il} = c_{il} \).

After make inverse DTFT transform at opposite ends of formula (6), we can get the relationship of the unknown spectrum-after-moving signal \( z(n) \) and the sampling signal \( y(n) \):

\[
y(n) = A \cdot z(n)
\]

(7)

Where \( y(n) = [y(n), \ldots, y_{n_0}]^T \), \( z(n) = [z(n), \ldots, z_{L_0 + 1}]^T \).

The formula (6) is the expression of the spectrum relationship between the sampled signal and the original signal, and formula (7) is the time-domain relationship between the sampled signal and the theoretical signal modulated by the original signal. According to this, the original signal can be reconstructed by using the reconstruction algorithm.

5. MWC reconstruction after sampling
In formula (7), dimension m of \( y(n) \) is less than dimension \( L = 2L_0 + 1 \) of \( z(n) \), so the equations are underdetermined. We can’t directly obtain the unique solution by inverse. But because of the sparsity of \( z(n) \), it can simplify, what means we can get the unique dilute solution by optimization. To solve this problem, document [13] proposed a CTF (Continuous to Finite) block. Its structure is that:
In this block \( Q = \sum \gamma[n]y[n] \), construct matrix \( V \) to satisfy \( Q = VV^H \). Matrix \( V \) can be obtained by solving the eigenvalues and eigenvectors of \( Q \). Support set \( S \) is a set of records of location information of non zero elements in sparse vectors \( z(f) \), set \( S = \text{supp}(z(f)) \). The solution of \( S \) can be obtained by OMP and other reconstruction algorithms. After obtaining the support set \( S \) of \( z(f) \), we can use the theoretical model 7 to reconstruct the signal. \( z(n) \) can be expressed as:

\[
\begin{align*}
z_s[n] &= A_s^t y[n] \\
z_0[n] &= 0, i \notin S
\end{align*}
\]

(8)

Where \( A_s^t = (A_s^H A_s)^{-1} A_s^H \) is Moore-Penrose generalized inverse matrix of matrix \( A_s \). Document [10] proves that as long as the rank is full, \( z(n) \) can be reconstructed by formula (8). And the sampling rate of the reconstructed sequence \( z(n) \) is \( f_s \). By interpolation filtering, increase the sampling rate to Nyquist frequency sequence \( z_e[k] \). Finally, make time-domain modulation by formula (6), move the signals recovered on the baseband to the original frequency spectrum to obtain the original sampling data and complete reconstruction.

\[
\tilde{x}[n] = \sum_{i \in S} \tilde{z}[\tilde{n}] e^{j2\pi f_i n \tau}
\]

(9)

6. Fault signal compressed sensing based on MWC

6.1. Simulation signal verification

To sample and reconstruct single frequency band signal, the original signal is:

\[
x(t) = \sqrt{E} B \text{sinc}(B(t - \tau)) \cos(2\pi f(t - \tau))
\]

(10)

Where carrier frequency \( f = 40 \text{KHz} \), bandwidth \( B = 200 \text{Hz} \), energy coefficient \( E = 100 \), the signal has \( N=2 \) bands (Positive and negative frequency symmetry). \( \tau \) is time shift coefficient. \( F_{\text{NyQ}} = 80K \), sampling rate \( f_s = 10KHz \) , chose 8 order M sequences for modulation sequences, \( M = 255 \), \( f_p = 3 \text{KHz} \), \( L_0 = 14 \), \( L = 29 \), number of sampling channels is \( m = 8 \).

This paper simulates hardware acquisition system based on Simulink. It includes signal generator module, M sequence generation module, multiplication mixer, filter and sampling module. The orthogonal matching pursuit algorithm (OMP) is used to solve the support set. Original signal and reconstructed signal spectrum are shown in Fig.4.
As shown in Fig.4, MWC system can restore the original signal spectrum. But due to the filtering distortion in the process of sampling and reconstruction, and approximate data processing and algorithm error in simulation, there will be the side band and small errors in signal spectrum.

6.2 Verification of measured fault signals

We use the bearing fault signal from Case Western Reserve University website [14], which is generated by a bearing experimental platform. We selects a set of outer ring fault signal of driving shaft end bearing. Bearing parameters and outer ring fault parameters are as follows:

Table 1.1 bearing parameters of driving shaft end of experimental platform

| Inner ring diameter /in | Outer ring diameter /in | Bearing thickness /in | Rolling body diameter /in | Nodal diameter /in | Manufacturer |
|------------------------|------------------------|----------------------|--------------------------|-------------------|--------------|
| 0.9843                 | 2.0472                 | 0.5906               | 0.3126                   | 1.537             | SKF          |

Table 1.2 failure parameters of outer bearing ring

| Damage diameter /in | Depth of damage /in | Motor load | speed |
|---------------------|---------------------|------------|-------|
| 0.021               | 0.011               | 2HP        | 1746 RPM |

According to the above parameters, Case Western Reserve University given the characteristic frequency of bearing outer ring fault, which is 29Hz and its frequency doubling.

First, the measured signal of external ring bearing pass the low-pass filtering. Set the filter frequency to 20Hz to 100Hz. That is to retain three fault feature lines of 29Hz, 58Hz and 87Hz. Because the amplitude of the fault characteristic band is small, and there are external factors such as noise, so there are many other frequency bands in the passband pass band, besides the fault characteristic frequency. They generate the sparsity of the spectrum together, as shown in Fig.5.

The maximum frequency of the fault signal after filtering is 100Hz. The sampling rate of traditional Nyquist sampling is greater than 200Hz. Well we set the compressed sensing sample rate \( f_s = 30Hz \), what equivalent to 1/20 of Nyquist's sampling rate. Other parameters of the model are set as follows:

The model frequency of the modulated wideband converter is equal to the sampling frequency of the measured signal, so \( f_{SIM} = 12kHz \). The sampling channel number is set to 25. The mixing function is taken as the 9 order M sequence, and the M sequence length \( W = 2^9 - 1 = 511 \). Mixer frequency \( f_p = f_{SIM}/W \approx 23.5Hz \). Set sampling frequency \( f_s = 30Hz > f_p \). After calculated, the effective spectrum components of spectrum after moving \( z(f) \) is \( L = 9 \), \( L_0 = 4 \). After filtering, the measured signal of bearing outer ring fault and the reconstructed signal after delayed sampling and amplitude compensation are obtained as follows:

Fig.5 frequency spectrum of Measured and reconstructed bearing fault signal after filtering

The reconstructed signal retains three characteristic spectral lines of 29 Hz, 58 Hz and 87 Hz, and reconstructs other noise signal spectral lines as well. But there is a certain error in the spectrum.
amplitude and the original signal spectrum. The main reason is that the frequency of the mixing function is too large, and the distinction between the non-sparse frequency bands is not enough. However, if the frequency of mixing function is further reduced, the sampling frequency will be reduced accordingly, which will bring difficulties to the design of digital filters. And this problem only exists in simulation experiments, and has little effect on the realization of analog filters in practice.

7. Conclusions
Compressed sensing theory is different from traditional signal compression theory. By utilizing the sparse characteristic of the signal, the limitation of the traditional Nyquist sampling theorem on the minimum sampling rate of the signal can be overcome, and the sampling rate can be greatly reduced. However, there are still many limitations in practical application. such as the original signal spectrum must be sparsity, but the presence of noise will affect the sparsity of the spectrum. Although there is still a long way to go for practicality, the compressed sensing theory avoids high-speed sampling, which proves that the sampling and processing of signals can be carried out at very low rates. It provides a new idea for significantly reducing the cost of data sampling and transmission.

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