Comparison of normalized and non-normalized block caving comminution models

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Synopsis

In block cave mining, rock fragmentation is a key parameter that influences the production level design and mine planning. Fragmentation occurs mainly by natural breakage during the caving process and in the draw column. The breakage that occurs within the column is known as secondary fragmentation. Secondary fragmentation has been successfully described using the block caving comminution model, which replicates the fragmentation mechanics between particles under drawn and vertical loads in a draw column. This model is based on a kinetic and population balance approach, in which non-normalized and normalized assumptions can be used depending on material and comminution system behaviour. In this paper, the non-normalized and normalized approaches are applied and compared to laboratory data to determine which assumption should be used for secondary breakage in block caving. Approaches are compared graphically, with the mean square error and the Fisher test with a false-rejection probability of 0.05. Based on a statistical analysis, the results show that the normalized model can be applied to all the rock types tested.

Keywords

block caving, breakage, comminution, mass balance, secondary fragmentation, modelling.

Introduction

The block caving method has been widely used in the mining industry for over 30 years. In this process, three stages can be identified: in-situ fragmentation, and primary and secondary fragmentation. The rock size obtained after secondary fragmentation has a strong influence on design stages, ore extraction and haulage, and mineral processing (Laubscher, 1994, 2000; Eadie, 2003; Brown, 2004). Particularly, in cave mining, the process is also influenced by the geometry of draw zones, which defines the drawpoint spacing, draw control, rate of draw, and secondary breakage (Laubscher 1994; Kurniawan and Setyoko, 2008). During ore draw, the rock fragment size can cause operational problems such as hangups, oversized material, fine material migration, inrush of fines, and mudrush events. These are some of the main reasons to study and estimate the rock fragmentation in block caving mines.

During block caving, fragmentation of the ore column occurs mainly by natural breakage induced by mining or by blasting applied in the base of the ore column from the undercut level or production level (Paredes et al., 2020). Additionally, preconditioning can be used to create new fractures or expand existing ones by hydraulic fracturing, confined blasting, or both. However, the fragmentation of the ore occurs mainly during caving and ore draw.

Fragmentation takes place in three stages. in-situ fragmentation is the natural fragmentation in the rock mass prior to mining activity, and which can be modified using preconditioning (Brzovic et al., 2016). Primary fragmentation occurs during caving, where the rock around the cave back is fragmented and then falls. Secondary fragmentation occurs in the broken column during draw and is the last stage of fragmentation before extraction at the drawpoint. In this stage the rock fragmentation is due mainly to compression and abrasion (splitting and attrition). Some authors also include impact fragmentation, which is highly influenced by the air gap height, as a mechanism of secondary fragmentation (Laubscher, 2000; Brown 2004; Pierce, Weatherley, and Kojovic, 2010; Dorador, 2016).

The breakage of the granular material under stress depends on various factors such as particle size distribution (PSD), particle shape, the state and trajectory of effective stresses, void ratio, material strength, presence of water, and extraction rate (Hardin, 1985; Brown, 2004; Liu et al., 2018). Among these factors, strength is one of the most relevant. Low rock strength implies early breakage of the...
Comparison of normalized and non-normalized block caving comminution models

rock block under a stress arch (Laubscher, 2000). Conversely, increasing strength decreases the amount of fragment breakage under a given stress (Lade, Yamamouchi, and Bopp, 1996).

Similarly, for an individual block, it is well known that strength decreases as size increases (Brady and Brown, 2004; Cundal, Pierce, and Mas Ivars, 2008). Particularly in block caving, where large volumes of rock are common, rock strength is a consequence of the intact rock strength, the presence of discontinuities, and the deformation capacity (Brady and Brown, 2004; Dorador, 2016). This behaviour is explained by the load concentration in small areas of the particles, increasing the stress and facilitating fracture (Lade, Yamamouchi, and Bopp, 1996). Another possible explanation is that angular particles fracture preferentially by abrasion (breaking of the edges), while rounded ones fracture by attrition model (Bridgewater et al., 2003), Srinkant and Rachmandl (2004) combined CoreFrag2 (to estimate primary fragmentation) and BCF (for secondary fragmentation), obtaining good results in medium and coarse sizes ($d_0$ and $d_0$). Dorador (2016) divided the broken column into a far field, characterized by plug flow, and a near field, characterized by mixed flow, to estimate secondary fragmentation. In the Block Caving Comminution Model (Gómez et al., 2017), the rock flow is simulated considering compression and abrasion, the main fragmentation mechanisms during secondary fragmentation, and is validated through experiments. In this work we extend the model by Gómez et al. (2017) by studying a hypothesis about the model and performing experiments with different types of rock.

An example in which the extended model is applied to real data is given in the Appendix.

Block Caving Comminution Model

Block caving is widely used in massive underground mining because of the high production rates and low operational costs. With this method, rock fragmentation occurs naturally during ore extraction while caving propagates to the surface. Thus, fragmentation is determined mainly by rock mass properties, stress fields, and draw policies. Here we focus our analysis on a model used to estimate the secondary fragmentation presented as the Block Caving Comminution Model (BCCM) (Gómez et al., 2017).

The BCCM in its batch version is determined by the following mass conservation equation:

$$\frac{df_i(t)}{dt} = -S_i f_i(t) + \sum_{j=1}^{i} b_{ij} S_j$$

where $f_i$ is the solid mass fraction of size interval $i$, $S_i$ is breakage velocity of the $i$th interval, and $b_{ij}$ is the breakage function and represents the fraction of the primary breakage product of material in the $i$th interval, which appears in the $j$th interval after fragmentation. Extraction in block caving is a continuous process that is usually modelled by plug flow. Then, incorporating this plug flow assumption in Equation [1] and using the Reid solution for the batch population balance (Reid, 1965), we obtain

$$m_i = \sum_{j=1}^{i} A_{ij} e^{-S_j \tau}$$

where $m_i$ is the mass in the $i$th interval, $\tau$ is the residence time and $A_{ij}$ is given by

$$A_{ij} = \begin{cases} f_{ij} & \text{if } i < j \\ 0 & \text{if } i = j \\ b_{ij} S_i - S_j A_{kj} & \text{if } i > j \end{cases}$$

where $f_{ij}$ is the initial particle size distribution.

In order to compute Equation [2] it is necessary to know the values of the functions $B_i$ and $S_i$. The common way to determine these functions is by assuming a constitutive parametric form.
Comparison of normalized and non-normalized block caving comminution models

and performing a back-calculation from experimental data (Klimpel and Austin, 1977). In this work we consider for \( S_j \) the parametric expression proposed by Herbst and Fuertenau (1980) and later modified by Gómez et al. (2017):

\[
S_j = S_{1j} \exp \left( Z_1 \ln \left( \frac{d_1}{d_j} \right) + Z_2 \left( \ln \left( \frac{d_1}{d_j} \right) \right)^2 \right) \sigma_v \tag{4}
\]

Here \( Z_1 \) and \( Z_2 \) are parameters depending on the material properties, and \( a_j \) is a variable that represents the mean vertical load. It is common to use the cumulative fraction of breakage product \( Bij = \sum_{x_j} b_{ij} \) (instead of \( b_j \)) as \( b_j = Bij = B_{ij} - B_{i,j-1} \), the so-called normalized expression for \( B_{ij} \) is given by

\[
B_{ij} = a_1 \left( \frac{x_j}{x_1} \right)^{a_2} + (1-a_1) \left( \frac{x_j}{x_2} \right)^{a_3} \tag{5}
\]

where \( a_1, a_2, \) and \( a_3 \) are model parameters (\( 0 < a_1 < 1; a_2 < a_3 \)), is the mesh size for the \( i \)th interval, and \( x_j \), which corresponds to the second mesh size interval, is the normalizing parameter. We remark that in the normalized expression Equation [5], it is assumed that all particles have a rupture distribution with dimensional similarity.

The non-normalized model raises this hypothesis, which is given by the expression (Austin and Luckie, 1972):

\[
B_{ij} = a_1 \left( \frac{x_j}{x_1} \right)^{a_2} + (1-a_1) \left( \frac{x_j}{x_2} \right)^{a_3} \tag{6}
\]

where

\[
a_j = a_1 \left( \frac{x_j}{x_1} \right)^{-\delta} \tag{7}
\]

and \( \delta \) is a new parameter to be adjusted (\( \delta > 0 \)) for \( 0 < a_j \) < 1. For \( I > j \), it must be noted that the normalized expression presented in Equation [5] is a simplified case of Equation [6] when \( \delta = 0 \) and \( x_1 = x_2 \). According to the constitutive expressions selected in this work, the back-calculation method yields a nonlinear optimization problem with six parameters \( S_i, Z_1, Z_2, a_1, \delta, \) \( a_2 \) in the normalized framework and seven parameters \( S_i, Z_1, Z_2, a_1, a_2, a_3, \delta \) in the non-normalized framework.

In milling processes, normal models are typically suitable (Austin and Concha, 1994), but in the block caving process the normalization of the model has not been studied. The aim of this paper to compare the normalized and non-normalized models using statistical analysis of a variety of rock types to identify which approach should be used with block caving methods.

Methodology

Laboratory set-up

The confined flow model set-up consisted of a steel cylinder able to retain rock fragments while vertical pressure was applied through a hydraulic press machine. The model geometry consisted of an inside diameter of 0.34 m, height of 0.70 m, and steel width contour of 0.03 m. Fragmented material was drawn from a drawbell at the bottom, emulating a block caving mine extraction system using a scaled LHD to draw the material. A constant vertical pressure was applied in each test.

Material was drawn from the bottom until the flow reached the top of the fragmented material (around 10% of the total fragmented material in over two hours). Then all material was homogenized, and the fragment size distribution was measured by sieving.
Comparison of normalized and non-normalized block caving comminution models

In order to evaluate whether the addition of the parameter $\delta$ is relevant, an F-test was performed. The null hypothesis is the normalized model with $\delta = 0$ nearly equal to the non-normalized model with $\delta > 0$. The F-value was calculated using Equation [9]:

$$ F = \frac{\frac{\text{MSE}_{\delta = 0} - \text{MSE}_{\delta > 0}}{k}}{\frac{\text{MSE}_{\delta > 0}}{N - M}} $$

where MSE is the mean square error, $k$ is the number of conditions (here $\delta = 0$), $N$ is the number of data points, and $M$ is the number of parameters of the model $i$.

Results

In this section we present the results of the parameter identification for the normalized and non-normalized approaches, the selection and breakage functions, and the Fisher test. The complete experimental data can be found in the Appendix.

Fitted parameters

The non-normalized model parameters obtained for each rock type are summarized in Table II. In all cases the residence time $\tau$ is kept constant and equals the average residence time of the tests (approx. 0.1 day).

In general, the non-normalized and normalized model parameters do not present significant differences: for example, rock types A and C present the same $a_1$. These similarities between the models are also observed in the selection and breakage functions as shown in Figures 2 and 3.

Selection and breakage functions

Figure 2 shows the rate of breakage ($S_i^1$), and Figure 3 shows the breakage functions ($B_{ij}$) of rock types A, B, C, and D.

The influence of fragment sizes on the rate of breakage when $\delta = 0$ is not significant. Thus, $\alpha_i$ (Equation [7]) could be assumed constant, depending on fragment size. This is also noticed in the breakage function in Figure 3 for $\delta > 0$, where for each rock type data-point, the breakage functions follow the same resulting fragment size distribution, independent of initial fragment size (as modelled by the normalized BCCM presented in Gómez et al. 2017).

Fisher test

Using a false-rejection probability of 0.05, the critical values of the F-distribution are presented in Table III. Here, all the F-values are lower than the respective critical values; therefore, the use of the simplified normalized model is recommended in these cases.

Table II

| Model      | Rock type | $\tau$ | $S_i^1$ | $Z_i^1$ | $Z_i^2$ | $a_1$ | $a_2$ | $a_3$ | $\delta$ |
|------------|-----------|--------|---------|---------|---------|-------|-------|-------|---------|
| Normalized | A         | 0.1    | 0.002   | -19.521 | -13.637 | 0.810 | 0.998 | 1.780 | 0.000   |
|            | B         | 0.1    | 0.744   | -3.139  | -2.310  | 0.098 | 0.569 | 0.779 | 0.000   |
|            | C         | 0.2    | 2.518   | -1.762  | -3.290  | 0.048 | 0.739 | 0.828 | 0.000   |
|            | D         | 0.1    | 0.190   | -10.373 | -14.080 | 0.859 | 0.588 | 0.598 | 0.000   |
| Non-normalized | A | 0.1    | 0.002   | -19.517 | -13.635 | 0.810 | 1.050 | 1.060 | 0.079   |
|              | B         | 0.1    | 0.744   | -3.139  | -2.310  | 0.467 | 0.779 | 0.830 | 0.000   |
|              | C         | 0.2    | 2.518   | -1.763  | -3.290  | 0.048 | 0.680 | 0.690 | 0.000   |
|              | D         | 0.1    | 0.182   | -10.567 | -14.265 | 0.683 | 0.588 | 0.598 | 0.102   |

Table III

| Rock type | MSE $\delta = 0$ (%) | MSE $\delta > 0$ (%) | F critical | F |
|-----------|----------------------|----------------------|------------|---|
| A         | 10.62                | 10.35                | 3.97       | 2.01|
| B         | 2.49                 | 2.46                 | 18.51      | 0.03|
| C         | 1.03                 | 0.96                 | 6.61       | 0.27|
| D         | 3.44                 | 3.43                 | 3.98       | 0.22|

Conclusions

The literature indicates that in milling systems the normalized model (Equation [5]) can be applied in almost all cases. However, this assumption cannot be considered a priori in the block caving method since the mechanism of breakage differs from...
that in the milling process. The non-normalized model implies that in the breakage function, $B_{ij}$, the resulting fragment size distribution depends on the original fragment size. According to the experimental results and parameter fitting, we found that the fragment size distribution is independent of the initial fragment sizes. Thus, for the materials and confined flow system examined here, a simplified normalized model should be used, independent of the material.

It must be noted that abrasion and compression are the fragmentation mechanisms observed in the experiments, but compression is probably the dominant mechanism. Compression generates a narrow particle size distribution with relatively low quantities of fine and large particles, which we speculate explains the results of this work. It is well known that the stress over the rock depends on the height at which it is located. In the present model, this stress is supposed to be constant throughout the column, which can lead to differences with respect to experimental data. In a later work, we will extend the present model by incorporating this dependence into the governing equations.

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Conflict of Interest
The authors declare that they have no conflict of interest.

Appendix A: BCCM applications
First, it is necessary to determine the parameters of the model $(S, Z_i, Z_2, a_1, a_2, a_3)$ to apply the BCCM, by making an adjustment from experimental laboratory tests. Subsequently, it is possible to solve the governing equations of the model if we know the input curve (primary fragmentation), physical characteristics of the column, and the operational conditions.

The model was evaluated for five reported mine cases. The first fragmentation reported is considered as the primary fragmentation. The laboratory calibration parameters of rock type A (Table IV), which has a strength similar to the selected mine cases, were used. The mean vertical stress in the broken column was estimated through the Janssen equation (Nedderman, 1992).

The fragmentation rate is scaled with respect to laboratory experiments so that

$$\tau = \frac{H_{Ap}}{D_r}$$

The residence time for all cases was estimated using

$$\tau_{S_i} = \tau_{S_i}^{(mine)} \text{constant} \quad [10]$$

The results obtained using the residence time calculated by the previous equation show that the model overestimates the rock fracture. This behaviour had already been reported in Gómez et al. (2017). These differences can be ascribed to three reasons: (i) the material suffers the greatest fractures at the beginning of the process; (ii) the vertical forces are usually low close to the movement zones, and (iii) the draw rate and vertical force are dependent on the height of the column, which changes over time. Figure 4 shows the results considering a residence time between 1.5 and 3.7% of that calculated by Equation [11]. In this case, the data provided by the model fit well to the real data of the cases studied.

Figure 5 shows the fragmentation results using the model considering the real residence time (Equation [11], blue points in Figure 5) and the adjusted residence time (red squares in Figure

| Mine cases          | Intact rock strength (MPa) | Vertical load (MPa) | Draw height primary frag |
|---------------------|---------------------------|---------------------|-------------------------|
| DOZ-diorite         | 111                       | 3.2                 | Unknown                 |
| Esmeralda-B2        | 100–130                   | 2.2                 | 0–20 m                  |
| Reno-HW             | 115                       | 2.5                 | 0–50 m                  |
| Teniente 4 Sur      | 125                       | 4.4                 | 0–50 m                  |
| Ridgeway (volcanic) | 130                       | 3.5                 | 0–25 m                  |
Comparison of normalized and non-normalized block caving comminution models

5). Importantly, we need to use a shorter residence time to obtain a good fitting. Also, the effect of the greater extraction height of the same mine can be observed in Figures 5C and D, and Figures 5E and F.

Appendix B: Experimental data

Experimental data used to calibrate both models is presented here.
### Table V
Comparison of normalized and non-normalized block caving comminution models

| Rock type A | % passing |
|-------------|-----------|
| Size (mm)   | Initial | Test 0.8 MPa | Test 1.5 MPa | Test 3 MPa | Test 5 MPa |
| 28.58       | 100.0%  | 100.0%        | 100.0%       | 100.0%     | 100.0%     |
| 25.40       | 97.5%   | 97.5%         | 98.0%        | 98.0%      | 98.0%      |
| 22.23       | 95.7%   | 95.6%         | 96.6%        | 96.6%      | 96.1%      |
| 19.05       | 92.4%   | 93.3%         | 93.0%        | 93.7%      | 93.7%      |
| 12.70       | 68.8%   | 75.5%         | 78.5%        | 80.2%      | 82.7%      |
| 9.53        | 39.1%   | 55.4%         | 60.7%        | 61.8%      | 62.4%      |
| 6.35        | 12.2%   | 21.3%         | 24.5%        | 26.9%      | 30.3%      |
| 4.75        | 5.0%    | 8.4%          | 10.1%        | 14.9%      | 16.2%      |
| 3.35        | 2.2%    | 3.5%          | 5.2%         | 6.4%       | 8.1%       |
| 2.36        | 1.2%    | 1.7%          | 2.8%         | 3.0%       | 4.1%       |
| 2.00        | 1.1%    | 1.1%          | 2.0%         | 1.6%       | 1.9%       |
| 25.40       | 100.0%  | 100.0%        | 100.0%       | 100.0%     | 100.0%     |

### Table VI
Fragmentation results obtained from confined flow test, rock type B

| Rock type B | % passing |
|-------------|-----------|
| Size (mm)   | Initial | Test 2 MPa |
| 28.58       | 100.0%  | 100.0%     |
| 22.23       | 97.7%   | 98.3%      |
| 15.88       | 91.0%   | 93.4%      |
| 14.29       | 86.1%   | 90.5%      |
| 12.70       | 78.0%   | 85.7%      |
| 9.53        | 51.1%   | 66.8%      |
| 6.35        | 22.0%   | 41.8%      |
| 3.35        | 5.0%    | 19.9%      |
| 1.20        | 0.0%    | 6.3%       |

### Table VII
Fragmentation results obtained from confined flow test, rock type C

| Rock type C | % passing |
|-------------|-----------|
| Size (mm)   | Initial | Test 1 MPa | Test 2 MPa |
| 28.58       | 100.0%  | 100.0%     | 100.0%     |
| 15.88       | 91.0%   | 92.7%      | 94.0%      |
| 12.70       | 78.0%   | 81.6%      | 86.3%      |
| 6.35        | 22.0%   | 29.1%      | 37.3%      |
| 3.35        | 5.0%    | 9.4%       | 15.2%      |
| 1.20        | 0.0%    | 2.7%       | 4.0%       |

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Comparison of normalized and non-normalized block caving comminution models

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