The Nuclear Response in the Isoscalar Channel

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ABSTRACT
The nuclear response is evaluated in the frame of the bosonic loop expansion in a purely nucleonic dynamical scheme, which seems to be reliable in handling those channels where a direct excitation of a $\Delta$-resonance is not allowed. It is shown that the response strongly depends upon the effective interaction in the spin-transverse isovector channel. New experiments at CEBAF on parity-violating electron scattering could further constrain the form of the effective interaction.

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In previous papers [1, 2] the nuclear charge-longitudinal response function was examined in detail within the theoretical frame of the bosonic loop expansion, developed in refs. [3, 4, 5, 6]. The same scheme was also successfully applied to the study of the energy and momentum dependence of the effective interaction in the \( S = 1, T = 1 \) channels [7].

The key issue of this approach is that a well-behaved expansion can be constructed by gathering together all those diagrams which have the same number of RPA-dressed bosonic loops. To properly determine the order of any given Feynman diagram one should shrink to a point each fermionic line in the graph, after having RPA-dressed all its interaction lines: the number of loops so obtained fixes its order. The bosonic loop expansion for the nuclear response amounts, at the 1-loop level, to evaluate the diagrams of fig. 1. We will specify later the interaction we are going to use, but, from the beginning, we limit ourselves in the present work to nucleonic degrees of freedom only. Thus the \( \Delta \)'s in the ground state are neglected and the probe cannot allow any \( N-\Delta \) transition. The channels expected to be reasonably well described in this context are then \( T = 0 \ S = 0 \), \( T = 0 \ S = 1 \) and \( T = 1 \ S = 0 \).

We expect instead that the \( N-\Delta \) transition induced by the external probe significantly alters the \( T = 1 \ S = 1 \) channel, as some preliminary results
already show.

The dynamics considered in [2] was the exchange of $\pi, \rho$ and the transverse component of the $\omega$ plus a residual interaction in the two first cases.

The longitudinal propagation of the $\omega$, as well as the $\sigma$-meson exchange, are instead neglected. This deserves a few comments because we know that both provide large contributions in the frame of a mesonic theory. In different dynamical schemes the effects associated with them can be alternatively viewed as coming from other mechanisms, like for instance - in a quark model - the quark exchange effect implied by the antisymmetrization of a 6–quark bag. However the overall effect inside a nuclear medium is roughly negligible independently of the specific dynamical mechanism: this can be motivated, for instance, by the phenomenological analyses of Speth et al. [8], leading to the conclusion that the Landau parameter in the scalar-isoscalar channel – where both mesons propagate – is compatible with 0, thus implying a large cancellation between them. Alternatively, within a quark model, the inclusion of the 6–quark configurations requires, to avoid double-counting, to neglect the longitudinal $\omega$ propagation as well[3, 10]. In a mesonic frame, where the $\sigma$ is explained in terms of box diagrams[7, 12] (exchange of two isovector mesons with simultaneous excitation of one or two nucleons to $\Delta$)
the cancellation translates into an analogous one between the longitudinal \( \omega \) exchange and the box diagrams. Since we are limiting ourselves for the moment to a purely nucleonic dynamics, it seemed to us coherent to neglect, together with the \( \Delta \)'s, those mesons which are in this way associated to them. Of course a less crude parametrization of the residual interaction in the scalar-isoscalar channel must be pursued, but it cannot be directly linked to an existing meson (the \( \omega \)) until we neglect the presence of \( \Delta \)'s in nuclear matter. Moreover it should be noted that, in the present dynamical model, some part of the missing short range repulsion is effectively described both by the values that the “Landau parameters” \( g'_{L,T} \) actually assume and by the momentum dependence we phenomenologically ascribe to the effective interaction through the cutoffs \( q_{cL,T} \).

Still a little inconsistency survives in our approach: we included indeed the possibility of \( \Delta \)-h propagation in the RPA-dressing of \( \pi \) and \( \rho \). This is the price we pay to remain in closer contact with the phenomenology of the pion propagation in the nuclear medium, which must not allow a pion condensation pole. On the other hand, as shown in ref [2] (and as will be discussed in a subsequent work), the \( f' \) sum rule is well satisfied in our approach: this entails that the quoted inconsistency is truly small.
Coming to the results of ref. [2], the comparison with experimental data is in our opinion good, apart from an overall shift of the data, whose origin was there described in detail.

However, some problems still survive to fix the input parameters of the model. An explicit calculation shows that the depletion of the peak is mainly ruled by the correlated $\rho$-meson exchange from diags. (a-c) of fig. 1.

This is not surprising: in fact, the major contribution comes from the short range correlations in that channel, summarized by $g'_T$ – the true $\rho$-meson exchange being suppressed by its high mass – and in the Landau limit we know that both $g'_T$ and $g'_L$ (the corresponding effective interaction in the pion channel) must coincide. Thus the spatial parts of our diagrams receive more or less the same contribution from the $\pi$ and $\rho$ channels, the isospin traces are the same, but the traces over the spin matrices in the $\rho$-channel are multiplied by a factor 2 with respect to the pionic one.

The conclusion of the above simple discussion is that the dominant contribution to the response comes from the less known dynamics. In fact, in ref. [3], we introduced a parameter ($q_{ct}$) describing the $\rho$-exchange effective potential in the intermediate range (for small momenta it must tend to $g'_0$ and at higher momenta it is expected to vanish). To be explicit, the effective
interaction will be written in the form

\[ V_\rho(q) = \frac{f_{\pi NN}^2}{m_\pi^2} \left\{ g'_T(q) - C_\rho \frac{q^2}{q^2 + m_\rho^2} \right\} v_\rho^2(q^2) \tag{1} \]

with

\[ g'_T(q) = 1 + (g'_0 - 1) \left[ \frac{q_{cT}^2}{q_{cT}^2 + q^2} \right]^2 \tag{2} \]

\( v_\rho \) being chosen in dipole form with a cutoff of 2500 MeV/c. For other details of the calculation we refer the reader to [2].

The behaviour of the potential \( V_\rho \) is shown in fig. 2 for \( q_{cT} \) ranging from 500 to 2000 MeV/c. We see that the term \( g'_T \) is repulsive, while the true \( \rho \) exchange is attractive, the former being dominant at low \( q \) and the latter at high momenta. For \( q \to \infty \) they must exactly cancel, but low values of \( q_{cT} \) mean that the repulsive part increases quickly and that the interaction remains always repulsive, while very high values of \( q_{cT} \) can allow a change of sign of the interaction at a finite value of the momentum.

Now, and this is the central point, different values of \( q_{cT} \), ranging from \( q_{cT} = 1.1 \) GeV/c to 1.4 GeV/c can nevertheless satisfactorily reproduce the charge-longitudinal response. In fig. 3 we compare indeed the results of our calculations (according to [2]) for different values of the parameter \( q_{cT} \) with the experimental data of Meziani et al. [13, 14] on \(^{12}\text{C}\). Here the free
quasi-elastic peak is evaluated both in a non relativistic and in a relativistic scheme. The latter, in particular, seems to provide a better agreement with the data, but is somehow inconsistent with the non relativistic formalism used for the 1-loop corrections. On the other hand the difficulties involved in a fully relativistic calculations seem to be, at present, prohibitive and moreover they are expected to produce a few percent change on the 1-loop corrections, which are by themselves a 20 % of the total result. The origin of the instability at the edges of the response region has been widely explained in ref. [2].

We note that the different curves display sizable differences but they are not decisive - also in view of the corrections stemming from the terms with $\Delta$-resonances both in the ground or intermediate states - to definitely rule out one of the values of the parameters.

Let us anticipate, before going on, that the corrections quoted above are in any case small. They will be discussed in a subsequent paper.

However, when we try to separate the isoscalar and isovector part of the charge-longitudinal response the dependence upon $q_{cr}$ is emphasized: the separated responses are shown in fig. [3] for the two extreme cases of $q_{cr} = 1100$ MeV/c and $q_{cr} = 1400$ MeV/c. The different behaviour between the
$S = 0 \; T = 0$ and $S = 0 \; T = 1$ channels is quite evident. The reason resides now in the isospin coefficients. While in fact the behaviour of the self-energy terms diags. 1b) and 1c) is exactly the same in both channel and the correlation terms 1d) and 1e) do not differ so much, the coefficient of the exchange diagram 1a) is $3/2$ for the isoscalar part and $-1/2$ for the isovector one. This diagram was largely responsible\cite{2} for the cancellation of the too high attraction coming from the self-energy (the total result in the charge-longitudinal channel being 1). Once the two channels are separated the isoscalar one is even more enhanced, while in the isovector no compensation arise to the self-energy, the net result being a strong depletion.

Clearly the amount of the latter depends upon the details of the model. It is however clear, and largely model-independent, that the isovector channel is expected to be depleted and that this depletion depends upon the strength of the interaction. Remarkably the smoother isovector response corresponds to a potential in the $\rho$-channel having some attraction in the intermediate momentum range. Noteworthy the potential used by Oset and coworkers [see, e.g., \cite{15, 16}] displays exactly this kind of behaviour.

The physical point, however, is that a separation between isoscalar and isovector contributions could give us valuable information on the shape of
the effective interaction in the $\rho$-channel. The same conclusions hold true also when looking to the $S = 1$, $T = 0$ responses, in the spin-longitudinal and transverse channels.

We display in fig. 5 the response functions to an isoscalar probe, separating the spin-longitudinal and spin-transverse responses. The difference between the vector-isoscalar channel calculation and the scalar one is again mainly in the spin-isospin coefficients; if case also some vertex functions could be different, a fact which may only slightly alter the net results.

We again carried out the calculation for $q_{cr} = 1100, 1400$ MeV/c. Our results display in both cases a hardening of the responses, its amount being again ruled by the form of the interaction in the $\rho$-channel. Our outcomes display a sizable difference between isoscalar spin longitudinal and transverse response functions, the former being more depleted in the 1p- 1h response region.

Notably, the calculations of ref. [17], performed in the variational frame of the Fermi Hyper Netted Chain expansion and the Correlated Basis Function perturbation theory, are in good agreement with those of the present work.

An experience at SATURNE has been recently carried out [18, 19] where polarized deuterons are scattered off complex nuclei. By selecting the deuteron
polarization the experimentalists are able to select the $S = 1$, $T = 0$
channel and, further, to separate the spin-longitudinal and spin-transverse
responses.

Preliminary analyses indicate that the ratio $R_L/R_T$ for the isoscalar
response at $q = 500$ MeV/c is significantly larger than 1 in the region inside
the quasielastic peak, in contradiction both with the present work and the
one of Fabrocini, and even with older continuum-RPA calculations.

These experimental results are however still preliminary and not yet free
from uncertainties on the deuteron form factor and on the distortion of its
wave function. It is further conceivable, as stressed by the experimentalists
themselves, that the experimental results could be dominated by the surface
response. Only when the analysis will be completed and one is able to dis-
 criminate between surface and volume effects, the information coming from
this experiment would allow a direct comparison with the present calculation
and help us in selecting between the various dynamics which, in our model,
provide the same charge- longitudinal response, but different effects in the
$S = 0$ $T = 0$ and $S = 1$ $T = 0$ channels.

On the ground of the present experimental situation the determination
of the volume contribution to the previously quoted responses seems to be
a hard task. We believe however that a larger systematic on a variety of nuclei, ranging from carbon to lead, could provide some hints toward the desired answer.

Another chance could arise in the future to separate out, instead, the two isospin contributions to the $S = 0$ channel by means of high precision polarized electron scattering off nuclei at CEBAF. Could the experimentalists be able to perform a Rosenbluth-like separation on the parity violating part of the cross section, then the electro-weak longitudinal response would become available. This can be expressed, like the e.m. one, as a combination of the $T = 0$ and $T = 1$ contribution but with different coefficients. More precisely the parity-violating electro-weak responses in the longitudinal channel display an almost complete cancellation between isoscalar and isovector contributions at least at the mean field level[22], so that only correlations are observed. Having a model able to provide both the parity-conserving (present work) and the parity violating (work in preparation [23]) response, and knowing, as it is, the coefficients of the isoscalar and isovector contributions, both could be determined.

This would thus offer the opportunity of getting a good knowledge of the nuclear effective interaction in the otherwise elusive spin-transverse channel.
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Figure 1: The Feynman diagrams considered in present work: a): exchange, b) and c): self-energy, d) and e): correlation
Figure 2: The potential $V_\rho$ for different values of $q_{\text{cp}}$: from the top to the bottom $q_{\text{cp}} = 500 \text{ MeV/c}, 1000 \text{ MeV/c}, 1500 \text{ MeV/c}, 2000 \text{ MeV/c}$. 
Figure 3: The charge-longitudinal response evaluated at different $q_{ct}$: 1100 MeV/c (dotted line), 1200 MeV/c (dashed line), 1300 MeV/c (dash-dotted) and 1400 MeV/c (solid line). Calculation is performed with $k_F=1.2$ fm$^{-1}$ and $q=350$ MeV/c. The dash-dotted-dotted line is the free peak, within a non relativistic and a relativistic kinematics on the left and on the right, respectively.
Figure 4: The isoscalar (dashed line) and isovector (dash-dotted line) contribution to the charge-longitudinal response for $q_{cT} = 1400$ MeV/c (on the left) and $q_{cT} = 1100$ MeV/c (on the right). The solid line is the non-relativistic free Fermi gas and the dotted line is 1/2 the sum of the isoscalar and isovector responses. Calculations are performed with $k_F = 1.2$ fm$^{-1}$ and $q = 350$ MeV/c.
Figure 5: The isoscalar response. Solid line represent the Free Fermi Gas, the dash-dotted line the isoscalar spin-longitudinal response, the dashed line the isoscalar spin-transverse response. Calculations are performed with $q_{ex} = 1400\,\text{MeV/c}$ (left figure) and $q_{ex} = 1100\,\text{MeV/c}$ (right figure), $k_F=1.2\,\text{fm}^{-1}$ and $q=350\,\text{MeV/c}$. 
$R_{\text{e.m.}}^L \times 1000 \text{ (MeV}^{-1} \text{)}$
