Baryon interactions in lattice QCD: the direct method vs. the HAL QCD potential method

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Ref. TI for HAL Coll., “Mirage in Temporal Correlation functions for Baryon-Baryon Interactions in Lattice QCD”, [arXiv:1607.06371], PoS(Lattice2015) 089, [arXiv:1511.05246].

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1. Baryon interactions from lattice QCD

2. Direct measurement vs HAL QCD method
   - Formalisms
   - Direct Measurement
   - HAL QCD Measurement

3. Origin of Fake Signal in Direct Method

4. Summary
1. Baryon interactions from lattice QCD

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2 Methods for Hadron Interaction from Lattice QCD

QCD ▶ Hadron Interaction ▶ Nuclear Physics

1 Lüscher’s finite volume method — Lüscher ’86, ’91
   energy shift of two-particle in “box” ▶ phase shift

\[ \Delta E_L = 2\sqrt{k^2 + m^2} - 2m \quad \Rightarrow \quad k \cot \delta(k) = \frac{1}{\pi L} \sum_{n \in \mathbb{Z}^3} \frac{1}{|n|^2 - (kL/2\pi)^2} \]

2 HAL QCD method — Ishii-Aoki-Hatsuda ’07
   NBS wave function ▶ potential ▶ phase shift
NN Interactions from Lattice QCD

|                  | Lüscher | HAL QCD | phys. point |
|------------------|---------|---------|-------------|
| dineutron ($^1S_0$) | bound   | unbound | unbound     |
| deuteron ($^3S_1$) | bound   | unbound | bound       |

⇒ inconsistencies between two methods, which is correct?

▶ Today we will clarify the origin of this puzzle
Baryon interactions from lattice QCD

Direct measurement vs HAL QCD method
- Formalisms
- Direct Measurement
- HAL QCD Measurement

Origin of Fake Signal in Direct Method

Summary
Lüscher’s Finite Volume Method

- **“energy shift” in finite box** $L^3$

\[
\Delta E_L = E_{BB} - 2m_B = 2\sqrt{k^2 + m_B^2} - 2m_B
\]

⇒ **phase shift** $\delta(k)$

\[
k \cot \delta(k) = \frac{1}{\pi L} \sum_{n \in \mathbb{Z}^3} \frac{1}{|n|^2 - (kL/2\pi)^2}
\]

↑ **THEORY**

↓ **PRACTICE — “Direct Method”**

- **measure**: plateau in **effective mass**

\[
\Delta E_{\text{eff}}(t) = \log \frac{R(t)}{R(t+1)} \rightarrow \Delta E_L
\]

\[
R(t) = \frac{G_{BB}(t)}{\{G_B(t)\}^2} \rightarrow \exp \left[-(E_{BB} - 2m_B) t\right]
\]

with $G_{BB}(t)(G_B(t))$: BB(B) correlators

- **NN($^1S_0$)** (Yamazaki et al. '12)
Time-dependent HAL QCD Method

Nambu-Bethe-Salpeter wave function

\[
R(\vec{r}, t) \equiv \frac{\langle 0|T\{B(\vec{x} + \vec{r}, t)B(\vec{x}, t)\}\bar{J}(0)|0\rangle}{\{G_B(t)\}^2}
= \sum_n A_n \psi_n(\vec{r}) e^{-(E_n - 2m_B)t} + O(e^{-(E_{\text{th}} - 2m_B)t})
\]

with elastic saturation \( R(r, t) \) satisfies

\[
\left[ \frac{1}{4m_B} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\vec{r}, t) = \int d\vec{r}' U(\vec{r}, \vec{r}') R(\vec{r}', t)
\]

“potential” using velocity expansion \( U(r, r') \approx V(r)\delta(r - r') \)

\[
V(\vec{r}) = \frac{1}{4m_B} \frac{(\partial/\partial t)^2 R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{(\partial/\partial t) R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{H_0 R(\vec{r}, t)}{R(\vec{r}, t)}
\]

This method does not require the ground state saturation.
Difficulties in Multi-Baryons

- Lüscher’s method requires **ground state saturation**

\[ G_{NN}(t) = c_0 \exp(-E_0^{(NN)}t) + c_1 \exp(-E_1^{(NN)}t) + \cdots \simeq c_0 \exp(-E_0^{(NN)}t) \]

- **S/N problem:** [mass number \( A \)] \( \times \) [light quark] \( \times \) [\( t \to \infty \)]

\[ S/N \sim \exp[-A \times (m_N - (3/2)m_\pi) \times t] \]

- **smaller gap of scattering state:** \( \Delta E \sim \vec{p}^2/m \sim \mathcal{O}(1/L^2) \)

\[ L = L_0 \quad \quad L = 2 \times L_0 \quad \quad L = \infty \]

Elastic

Inelastic

\( NN + \pi \)

\( NN \)
Contamination of Scattering State and Fake Plateau

Example

\[ R(t) = b_0 e^{-\Delta E_{BB} t} + b_1 e^{-\delta E_{el} t} + c_0 e^{-\delta E_{inel} t} \]

with \( \delta E_{el} - \Delta E_{BB} = 50 \text{ MeV} \sim \mathcal{O}(1/L^2) \), \( \delta E_{inel} - \Delta E_{BB} = 500 \text{ MeV} \sim \mathcal{O}(\Lambda_{QCD}) \)

- g.s. saturation
  \( \Delta E_{BB}^{\text{eff}}(t) - \Delta E_{BB} \rightarrow 0 \)

- elastic saturation \( t \sim 1 \text{ fm} \)

- few % of contamination
  \( \Rightarrow \) “mirage” of plateau
  around \( t \sim 1 - 1.5 \text{ fm} \)
  much larger \( t \) for true g.s.

\( \Rightarrow \) a true ground state can be checked by quark source dependence

\[ \text{HAL QCD} - \] scattering state are not noises, but signals
Lattice Setup: Wall Source and Smeared Source

- **interaction** from both direct and HAL QCD methods

- **CHECK 2 quark sources** — mixture of excited states are different

- **wall source**
  - standard of HAL QCD

- **smeared source**
  - standard of direct method†

- setup — 2 + 1 improved Wilson + Iwasaki gauge†
  - lattice spacing: \( a = 0.08995(40) \) fm, \( a^{-1} = 2.194(10) \) GeV
  - lattice volume: \( 32^3 \times 48, 40^3 \times 48, 48^3 \times 48, \) and \( 64^3 \times 64 \)
  - \( m_\pi = 0.51 \) GeV, \( m_N = 1.32 \) GeV, \( m_K = 0.62 \) GeV, \( m_\Xi = 1.46 \) GeV

† Yamazaki-Ishikawa-Kuramashi-Ukawa, arXiv:1207.4277.
Energy Shift of $\Xi\Xi$: Smeared Src. vs. Wall Src.

$\Delta E_{L}^{\text{eff}}(t) \longrightarrow \Delta E_{L}$ — depends on quark source (smeared or wall)

$\Xi\Xi(^1S_0)$ at $48^3 \times 48$

$\Xi\Xi(^3S_1)$ at $48^3 \times 48$

- source dependence suggests these plateaux are "fake" signal

| $L \rightarrow \infty$ | Smeared src. $\Delta E_{\Xi\Xi}(^1S_0)$ | Wall source $\Delta E_{\Xi\Xi}(^3S_1)$ |
|-----------------------|---------------------------------|-----------------
|                       | $< 0$ bound                      | $\simeq 0$ unbound |
|                       | $> 0$ unphysical                 | $\simeq 0$ unbound |

cf. $\Delta E < 0 \Rightarrow$ binding or $\Delta E = 0 \Rightarrow$ scattering
Generalized Sink Operator

\[
C^{(g)}_{\Xi\Xi}(t) = \sum_{\vec{r}} g(|\vec{r}|) \sum_{\vec{R}} \langle \Xi(\vec{R} + \vec{r}, t) \Xi(\vec{R}, t) J_{\Xi\Xi}(t = 0) \rangle \rightarrow \exp(-E_{\Xi\Xi}t)
\]

\[\Rightarrow \text{g.s. energy does not depend on } g(r)\]

- \(g(r) = 1\): standard sink operator
- \(g(r) = 1 + A \exp(-B r)\): exp-type projection

**Smeared Src.**

one can make any "fake plateau"

**Wall Src.**

"stable"

\[
\Delta E_{\Xi\Xi}^{\text{eff}}(t) [\text{MeV}]
\]

\[
\Delta E_{\Xi\Xi}^{\text{eff}}(t) [\text{MeV}]
\]
HAL: Potential of $\Xi\Xi(^1S_0)$ Smeared Src. vs Wall Src.

NBS wavefunction: $R^{\text{smear}}(r, t)$ or $R^{\text{wall}}(r, t)$

\[ V_c(r) = \frac{1}{4m} \left( \frac{\partial^2}{\partial t^2} R(r, t) \right) - \frac{\partial}{\partial t} \frac{R(r, t)}{R(r, t)} - \frac{H_0 R(r, t)}{R(r, t)} \]

**smeared src. $t$-depend**

**wall src. $t$-stable**
HAL: Potential of $\Xi\Xi(1S_0)$ Smeared Src. vs Wall Src.

- **wall src.** — good convergence
- **smeared src.** — $t$-dep.
- **smeared src.** $\rightarrow$ **wall src.** for large $t$
Residual Diff. of Pot.: Next Leading Order Correction

Derivative expansion: \( U(r, r') = \{ V_0(r) + V_1(r) \nabla^2 \} \delta(r - r') \) (for \(^1S_0\))

\[
\left[ \frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(r, t) = \int d^3r' U(r, r') R(r', t) \\
\simeq V_0(r) R(r, t) + V_1(r) \nabla^2 R(r, t) + \cdots
\]

\( R^{\text{smear}} \) and \( R^{\text{wall}} \Rightarrow V_0(r) \) and \( V_1(r) \)

▶ HAL method works — quark src. independent w/o g.s. saturation

□ Leading order approximation

□ Next leading order correction
HAL meets Lüscher: Energy Shift from Potential

- HAL QCD works well \textit{w/o g.s. saturation problem}
  use potential $\implies$ true “energy shift” in finite volume

$\uparrow$ Eigenequation in finite box $L^3$ with HAL QCD potential $V(\vec{r})$

\[ [H_0 + V] \psi = \Delta E \psi \]

$\square$ eigenvalue $\Delta E_0 \propto 1/L^3 \longrightarrow 0 \implies$ scattering by Lüscher’s formula

- potential $V(r)$

\begin{itemize}
  \item $\xi(1S_0)$ potential [MeV]
  \item $\alpha/L^3$-fit
  \item eigenvalue
\end{itemize}
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4 Summary
Wavefunction, Potential, Eigenvalues and Eigenfunctions

**NBS wave function**

- **smear**
- **HAL method**
- **wall**
- **feed back**
- decomposition
- projection

**Potential**

- **Solve** $[H_0 + V] \psi = E \psi$
- ground state & excited states (elastic scattering)

**Eigenvalues & Eigenfunctions**

- $\psi_n(r,t=12)$
- $4th \ A1$, $3rd \ A1$, $2nd \ A1$, $1st \ g.s.$
Excited States in Wavefunction

- $R$-corr. decomposition by energy eigenmodes

$$R^{\text{wall/smear}}(\vec{r}, t) = \sum_n a_n^{\text{wall/smear}} \Psi_n(\vec{r}, t) \exp(-\Delta E_n t)$$

$$\therefore R(p = 0, t) = \sum_r R(\vec{r}, t) = \sum_n b_n^{\text{wall/smear}} e^{-\Delta E_n t}$$

\[ \Box \text{ex. 1st excited state} \]

- **wall source**
  - $b_1/b_0 \ll 0.01$

- **smeared source**
  - $b_1/b_0 \simeq -0.1$

- with energy gap
  - $E_1 - E_0 \simeq 50 \text{ MeV}$
  - for $L^3 = 48^3$

\[ |b_n/b_0| \Xi \Xi(1S_0) \text{ at } t = 14 \]

\[ \Delta E_n [\text{MeV}] \]

“contamination” of excited states $b_n/b_0$
Origin of Fake Plateau — Contamination of Excited States

\[ \Delta E_{\text{eff}}(t) \equiv \log \frac{R(p = 0, t)}{R(p = 0, t + 1)} = \log \frac{\sum_n b_n \exp(-\Delta E_n t)}{\sum_n b_n \exp(-\Delta E_n (t + 1))} \]

**“direct measurement” — reproduced by low-lying modes**

\[ b_{\text{wall}}^{n=0,\ldots,2} \text{ at } t = 14 \]
\[ b_{\text{smeared}}^{n=0,\ldots,2} \text{ at } t = 14 \]

wall src. \( \Xi \Xi (1S_0) \)

smeared src. \( \Xi \Xi (1S_0) \)

† eigenvalues \( \Delta E_n \), coefficients \( b_{\text{smeared/wall}}^n \) for \( n = 0, 1, 2 \), at \( t = 14 \).
\[ \Delta E_{\text{eff}}(t) \equiv \log \frac{R(p = 0, t)}{R(p = 0, t + 1)} = \log \frac{\sum_{n} b_{n} \exp (-\Delta E_{n} t)}{\sum_{n} b_{n} \exp (-\Delta E_{n} (t + 1))} \]

“direct measurement” — reproduced by low-lying modes\(^{\dagger}\)

\[ \square \text{g.s. saturation} \text{ of smeared source} \text{ — 100 lattice units } \sim \text{ 10 fm} !!! \]

\(^{\dagger}\) eigenvalues \(\Delta E_{n}\), coefficients \(b_{n}^{\text{smear/wall}}\) for \(n = 0, 1, 2,\) at \(t = 14\).
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Summary: Lüscher Direct vs HAL QCD

- “Direct method” — **ground state saturation** is extremely difficult
  - scattering states $\Rightarrow$ “fake plateau” $\implies$ **Wrong Conclusion!**
  - much smaller gap & larger noise @ phys. pt. $\Rightarrow$ almost impossible

- HAL QCD works well **without g.s. saturation**
  - HAL QCD $\Rightarrow$ “correct” $\Delta E_L$ and input of Lüscher’s formula
- **NBS corr. + “potential”** $\Rightarrow$ excited states contamination and origin of fake plateau.

- *(even if you do not trust HAL QCD method)*
  - fake plateau can be checked by Lüscher’s formula $\implies$ Aoki’s Talk

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**Pot. with wall src.** $\uparrow\downarrow$ **NLO pot. corr.**

**Pot. with smear src.** $\downarrow\uparrow$

**Direct with wall src.**

$\implies$ explain $\Delta E_{eff}(t)$

**Direct with smear src.**

**Fake plateaux**

**Conflict**
Demo: Contamination of Scattering State

Mock up data

\[ R(t) = b_0 e^{-\Delta E_{BB} t} + b_1 e^{-\delta E_{el} t} + c_0 e^{-\delta E_{inel} t} \]

with \( \delta E_{el} - \Delta E_{BB} = 50 \text{ MeV} \sim \mathcal{O}(1/L^2) \), \( \delta E_{inel} - \Delta E_{BB} = 500 \text{ MeV} \sim \mathcal{O}(\Lambda_{QCD}) \).

- g.s. saturation around \( t \rightarrow 10 \text{ fm} \)
- fake plateau around \( t \sim 1 \text{ fm} \)
\( \Xi \Xi (^{1}S_{0}) \)

Graphs showing the change in energy \( \Delta E \) vs. time \( t \) and \( 1/L^3 \) for relativistic and non-relativistic operations. The graphs indicate the behavior of smeared and wall source scenarios.
\[ \Xi \Xi \left( ^3S_1 \right) \]

![Graphs showing\( \Delta E_{\Xi \Xi}(t) \) and \( \Delta E_{\Xi \Xi}(\frac{1}{L^3}) \) for different \( L \) values.]
NN($^1S_0$)

![Graphs showing $\Delta E_{\text{eff}}(t)$ and $\Delta E_{\text{NN}}$ vs. $1/L^3$ for different $L$ values and sources.](image)

relativistic op. and non-rela. op. (NR)
$NN^{(3S_1)}$
Triton

\[ \Delta E_{3\text{He}}(t) \text{ [MeV]} \]

\[ L = 32 \]

\[ L = 40 \]

\[ L = 48 \]

\[ L = 64 \]

\[ \Delta E_{3\text{He}} \text{ [MeV]} \]

relativistic op. and non-rela. op. (NR)

\[ 1/L^3 [a^{-3} \times 10^{-5}] \]
Helium

\[ \Delta E_{\text{eff}}(t) [\text{MeV}] \]

\[ L = 32 \text{ sm. are d src. } 4\text{He} \]

\[ L = 40 \text{ wall src. 4He} \]

\[ L = 48 \text{ wall src. 4He} \]

\[ L = 64 \text{ wall src. 4He} \]

\[ \Delta E_{\text{eff}} (t) [\text{MeV}] \]

\[ \frac{1}{L^3} [a^{-3} \times 10^{-5}] \]

relativistic op. and non-rela. op. (NR)
$\Delta E_{\text{eff}}(t) = E_{\Xi \Xi}^{\text{eff}}(t) - 2m_{\Xi}^{\text{eff}}(t)$: Smeared Src. vs. Wall Src.
$\Xi \Xi (^1S_0)$ is Unbound at $m_\pi = 510$ MeV

\[ k \cot \delta(k) = \frac{1}{\pi L} \sum_{n \in \mathbb{Z}^3} \frac{1}{|n|^2 - (kL/2\pi)^2}, \]

\[ \Delta E = 2\sqrt{m^2 + k^2 - 2m} \]

- **Volume dep. of $\Delta E_0$**
  - $\Delta E_0$ [MeV] vs. $1/L^3$ [$a^{-3} \times 10^{-5}$]
  - $\alpha/L^3$-fit
  - Eigenvalue

- **Phase shift $\delta$**
  - $E_{\text{CM}}$ [MeV] vs. $\delta$ [deg.]
  - $40^3 \times 48$ at $t = 12$
  - $48^3 \times 48$ at $t = 12$
  - $64^3 \times 64$ at $t = 12$
  - Luscher formula
$t$-depenence of Potential

$t$-dependence of Wall Src. potential is stable

$64^3 \ t = 12 - 17$

$\Xi(1S_0)$ potential [MeV]

$2 \times m_{\Xi}^{\text{eff}}$ or $E_{\Xi \Xi}^{\text{eff}}$ [MeV] $L = 64$

wall src. $\Xi$

wall src. $\Xi \Xi(1S_0)$
Time-dependent HAL QCD Method

- space-time correlation function

\[ R(\vec{r}, t) \equiv \langle 0 | T \{ B(\vec{x} + \vec{r}, t) B(\vec{x}, t) \} \bar{J}(0) | 0 \rangle / \{ G_B(t) \}^2 \]

\[ = \sum_n A_n \psi_n(\vec{r}) e^{-(E_n - 2m_B)t} + O(e^{-(E_{th} - 2m_B)t}) \]

\[ \square \text{ each } \psi_n(\vec{r}) e^{-E_n t} \equiv \langle 0 | T \{ B(\vec{x} + \vec{r}, t) B(\vec{x}, t) \} | 2B, n \rangle \text{ satisfies } \]

\[ \left[ \frac{k_n^2}{m_B} - H_0 \right] \psi_n(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}') \]

with non-local interaction kernel \( U(\vec{r}, \vec{r}') \)

- \( R \)-corr. satisfies \( t \)-dep. Schrödinger-like eq. with \textbf{elastic} saturation

\[ \left[ \frac{1}{4m_B} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(\vec{r}, t) = \int d\vec{r}' U(\vec{r}, \vec{r}') R(\vec{r}', t) \]

\[ \text{“potential” using velocity expansion } U(r, r') \simeq V(r) \delta(r - r') \]

\[ V(\vec{r}) = \frac{1}{4m_B} \frac{(\partial/\partial t)^2 R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{(\partial/\partial t) R(\vec{r}, t)}{R(\vec{r}, t)} - \frac{H_0 R(\vec{r}, t)}{R(\vec{r}, t)} \]

\[ \text{This method does not require the ground state saturation.} \]
HAL: Wave Function and $\Xi\bar{\Xi}(^{1}S_{0})$ Potential $V_{c}(\vec{r})$

- **wall src.** — weak $t$-dep.
- **smeared src.** — strong $t$-dep.
- Contribution of excited states
- Time-dep. HAL method works well
- $\mathcal{O}(100)$ MeV of cancellation

$$V_{c}(\vec{r}) = -\frac{H_{0}R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$
Next Leading Order of Derivative Expansion

Derivative expansion: \( U(r, r') = \{V_0(r) + V_1(r)\nabla^2\} \delta(r - r') \) (for \(^1S_0\))

\[
\left[ \frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right] R(r, t) = \int d^3r' U(r, r') R(r', t)
\]

\[
\begin{align*}
\frac{1}{4m} \frac{\partial^2}{\partial t^2} R & - \frac{\partial}{\partial t} R - \frac{H_0 R}{R} = V_0(r) + V_1(r) \frac{\nabla^2 R(r, t)}{R(r, t)} \equiv \tilde{V}_{\text{eff}}(r, t) \\
\end{align*}
\]

\( R^{\text{smeear}} \) and \( R^{\text{wall}} \)

\[
\begin{align*}
V_0(r) + V_1(r) \nabla^2 R^{\text{smeear}} / R^{\text{smeear}} &= \tilde{V}^{\text{smeear}}_{\text{eff}} (r, t^{\text{smeear}}) \\
V_0(r) + V_1(r) \nabla^2 R^{\text{wall}} / R^{\text{wall}} &= \tilde{V}^{\text{wall}}_{\text{eff}} (r, t^{\text{wall}}),
\end{align*}
\]

\( \tilde{V}_{\text{eff}}^{\text{smeear}} (r, t^{\text{smeear}}) \) and \( \tilde{V}_{\text{eff}}^{\text{wall}} (r, t^{\text{wall}}) \) potentials are given by

\[
V_1(r) = \frac{\tilde{V}^{\text{smeear}}_{\text{eff}} (r, t^{\text{smeear}}) - \tilde{V}^{\text{wall}}_{\text{eff}} (r, t^{\text{wall}})}{\nabla^2 R^{\text{smeear}} / R^{\text{smeear}} - \nabla^2 R^{\text{wall}} / R^{\text{wall}}}
\]

\[
V_0(r) = \tilde{V}^{\text{smeear}}_{\text{eff}} (r, t^{\text{smeear}}) - V_1(r) \frac{\nabla^2 R^{\text{smeear}}}{R^{\text{smeear}}}.
\]