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ABSTRACT
The properties of electromagnetic waves propagation through a collisional, magnetized and inhomogeneous plasma with obliquely incident angles are investigated in this paper. The reflection, transmission, and attenuation ratios are calculated first with the scattering matrix method. Based on the derived formulas, we then analyze the effects of different parameters on the propagation properties under oblique incidence. Simulation results indicate that a larger incident angle causes a longer wave transmission distance in the plasma and is more likely to impede the waves propagation. Under an obliquely incident angle, both the transmission and attenuation ratios are greatly affected by other parameters, such as the effective plasma collisional frequency, background magnetic flux density, plasma thickness and electron density, additionally. These conclusions are of guiding significance to the applications of the interaction between electromagnetic waves and plasma.

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I. INTRODUCTION
Plasma has numerous applications in aircraft re-entry communication, plasma antennas, controllable nuclear fusion and other fields.1–2 However, plasma tends to act as an electromagnetic (EM) wave absorber and greatly impedes the wave propagation when interacting with EM waves, such as communications on high-speed aircraft in plasma sheaths and communications between planets wrapped in dusty plasma. Considering the above defects, it is vital to study the properties of EM wave propagation in a plasma.

Recently, many researchers have conducted a lot of ingenious research and achieved fruitful results around this subject. Wang et al. established a numerical model with a one-dimensional assumption to investigate the propagation properties of the EM waves in plasma slabs, then discussed the influence of wave frequencies and collisional frequencies on the amplitude of the transmitted waves.3 Yang et al. studied the interaction mechanisms between the plasma sheath and EM waves by numerically solving ray tracing equations to observe the EM wave propagation characteristics in the plasma sheath.4 The detailed research methods for EM wave propagation in a plasma include numerical solutions, such as the Finite-Difference Time-Domain method (FDTD) and Green’s Function Integral Equation method (GFIE), and analytical solutions such as the Scattering-Matrix Method (SMM). FDTD is a granulation-based method and requires the whole target plasma to be divided into large quantities of dense and small cells. The total propagation properties can be obtained by calculating the interactions between adjacent plasma cells. Because cells can be combined to form any kind of plasma, FDTD has universality in dealing with EM waves propagation problems, and is becoming a widely employed method. Chaudhury et al. used FDTD to analyze EM waves reflection on a radar cross-section wrapped by a plasma shroud with an Epstein variation.
in the Z-direction and a Gaussian variation in the X-Y plane, showing the optimal plasma frequency point of EM waves reflection. In addition, Cerri et al. established a temperature-based model for EM waves propagation in a plasma and obtained the particle velocity distribution function and propagation properties with FDTD. Despite the advantages that FDTD has, calculating all the interactions of plasma cells means a massive amount of calculation resulting in a tremendous time consumption. To reduce that, an integral equation-based method, GFIE, is proposed. By contrast with FDTD, GFIE has the advantages of high efficiency and precision. Soltanmoradi et al. studied EM waves propagation in gas plasma for the case of cold, collisional and inhomogeneous plasma slabs with GFIE.

In their study, the absorbance, reflectance and transmittance were investigated, considering different electron densities, collisional frequencies and plasma thicknesses. Their plasma model yielded satisfactory results in quantitative analyses of the effects of plasma parameters. However, when considering the fact that the plasma is usually applied with a magnetic field to obtain the EM transmission window, the defects of neglecting magnetic fields in their plasma model are already apparent. For a magnetized plasma, the scattering matrix method (SMM), a stratification-based method, was proposed by Hu et al. In its scattering matrices, the SMM considers all the boundary conditions when EM waves penetrate in each plasma layer, and all the matrices are multiplied to obtain total reflection and transmission ratios. Besides, phase shift information is still kept after solving the cascade scattering matrix equations. SMM was applied by Hu et al. in their analyses of propagation properties as functions of collisional frequency, electron gyration frequency (determined by the background magnetic field) and electron density. Chen et al. proposed an improved scattering matrix method (ISMM) based on traditional SMM for solving the propagation properties of a metal substrate plasma. Within the ISMM, the phenomena of “sunken regions” and “phase jumps” are further investigated and were certified to affect wave propagation during the re-entry process.

According to the above analysis, plenty of splendid studies and brilliant achievements have been obtained with the assumption of wave vertical incidence. However, waves usually enter obliquely into a plasma in real propagation scenarios and studies of waves oblique incidence into a plasma have rarely been conducted. Considering the complexity of wave incidence and the advantages of SMM, we used SMM to calculate the propagation properties of obliquely incident waves in a plasma. In this study, the incident angle is taken as a key parameter in discussion of the reflection, transmission and attenuation ratio. The influence of parameters such as the effective collisional frequency, background magnetic flux density, plasma thickness and electron density are discussed under the circumstances of oblique incidence. Also, the plasma is assumed to be cold, weakly ionized, stable, collisional and inhomogeneous and is handled with Maxwell’s equations as a fluid model to use SMM. EM waves are set in a right-handed polarization and transverse electric field for simplification.

The rest of this paper is organized as follows. The physical model of the plasma is established first in Section II. Then, the attenuation, transmission and reflection ratios are derived. In Section III, the effects of different incident angles, collisional frequencies, background magnetic flux densities, plasma thicknesses and electron densities on the transmission and attenuation ratios are shown and analyzed. Conclusions are eventually summarized in Section IV.

II. PHYSICAL MODEL AND FORMULATIONS

As described in Section I, the incident angle of the EM waves plays a key role in the propagation properties. Therefore, the reflection ratio, R, transmission ratio, T, and absorption ratio, A, of the plasma slabs with various incident angles are derived with SMM in this section.

The multilayer plasma model is established by dividing it into several sub-layers. Note that each layer is assumed to be very thin and the corresponding electron density has a uniform distribution. The total electron density profile throughout all plasma layers obeys a certain distribution function. The schematic diagram of EM waves propagation with oblique incidence in a magnetized plasma and the distribution function that the electron density obeys are represented in Fig. 1 (a) and (b), respectively. As shown in Fig. 1 (a), the left and right side of the plasma layers are assumed as vacuum. The total plasma thickness is L, and that inhomogeneous plasma is divided into N sub-layers. In this study, we assumed that the background magnetic field, B, transmits parallel to the Z-axis. In addition, the EM wave enters plasma layers with a slant angle θ0. Note that the incident wave is considered to be a plane transverse electric (TE) wave along the X-Z plane and its electric field is perpendicular to the X-Z plane. Fig. 1 (b) describes the electron density profile, N,.
along the plasma layer as in the form of a sinusoidal function of the
distance $z$ from the origin point as \(^{15}\)

$$N_e = N_0 \left[ 1.535 + 1.5 \cos \left( 18 \pi z + \frac{\pi}{6} \right) \right] \quad (0 \leq z \leq L) \quad (1)$$

where $N_0$ denotes the magnitude of electron density. As can be seen
in Fig. 1 (b), the maximum electron density is $3.035 \times N_0$ m\(^{-3}\),
and the minimum is $0.035 \times N_0$ m\(^{-3}\).

The complex dielectric constant of the i-th plasma layer $\varepsilon_i$ can be obtained by Appleton’s formula \(^{16}\) as

$$\varepsilon_i = 1 - \left( \frac{\omega_{pe, i}}{\omega} \right)^2 \begin{pmatrix} 1 - j \frac{\omega_m}{\omega} - \left( \frac{\omega_{ce, i}}{\omega} \right)^2 \sin^2 \theta_i \left( \cos \theta_i - 1 \right) \right) \left( 1 - \sin \theta_i \left( \sin \theta_i - 1 \right) \right)^{1/2} \right]$$

$$= \left[ \frac{(\omega_{ce, i})^4 \sin^4 \theta_i + (\omega_{pe, i})^2 \cos^2 \theta_i}{4 \left( 1 - \sin \theta_i \left( \sin \theta_i - 1 \right) \right)^2} \right]^{1/2} \quad (2)$$

Here, the “±” symbol indicates the EM wave in left- or right-hand polarization. In this study, the “−” symbol is selected for sim- plification. $\nu_m$ is the effective collisional frequency of the plasma and is a set number in this study. $\omega_{ce, i}$ and $\omega_{pe, i}$ are the electron gyration frequency under external magnetic field and the plasma frequency of the i-th layer, which are expressed as $\omega_{ce, i} = \frac{eB_i}{m_e}$ and $\omega_{pe, i} = \sqrt{\frac{\sqrt{4 \pi} N_i}{m_e N_e}}$, respectively. $c$, $m_e$, $j$, and $\varepsilon_0$ are constants correspondingly for electron charge, electron mass, imaginary unit and vacuum permittivity. Particularly, $\varepsilon_i(0)$ is defined as the relative permittivity of vacuum. $\theta_i$ represents the angle between the EM wave propagation direction and the x-axis in the i-th plasma layer, which can be obtained by Snell’s law \(^{16}\)

$$1 \times \sin \theta_0 = \sqrt{\varepsilon_i(0)} \sin \theta = \sqrt{\varepsilon(N)} \sin \theta_N. \quad (3)$$

With the boundary conditions that the tangential components of electric and magnetic fields on reflecting surfaces are continuous, the reflection coefficient for TE waves on the i-th reflecting surface $\Gamma_i$ can be obtained as \(^{6}\)

$$\Gamma_i = \sqrt{\varepsilon_i(0)} \cos \theta_{i-1} - \sqrt{\varepsilon_i(1)} \cos \theta_i \quad (4)$$

$$\sqrt{\varepsilon_i(0)} \cos \theta_{i-1} + \sqrt{\varepsilon_i(1)} \cos \theta_i$$

Besides, the attenuation coefficient for waves passing through the i-th plasma layer can be written as

$$\alpha_i = \text{real} \left( \frac{\omega}{c} \sqrt{-\varepsilon_i(0)} \right) \quad (5)$$

where real(x) means to take the real part of imaginary number $x$ and $c$ is the speed of light in free space. Note that the reflection coefficient we calculated in Eq. 4 is used to describe the extent of electric field amplitude of the wave which is reflected by a certain interface between plasma layers. While the reflection ratio to be obtained in the following calculation is adopted to describe the energy reflection.

Let the power of the incidence wave be $P_i$, and the length of each plasma layer be $d$ ($d = L/N$). Then the total reflection ratio, $R$, transmission ratio, $T$, and attenuation ratio, $A$, can eventually be calculated by the following derivations.

An EM wave obliquely enters from vacuum into the plasma layer with a slant angle $\theta_3$ and is reflected at the first interface. According to Eq. (4), the reflection coefficient of the first reflection surface, $\Gamma_1$, can be obtained. After that, the power reflected into the vacuum and the power penetrating the first reflection surface are

$$\begin{align*}
\Gamma_1 &= P_1 |\Gamma_1|^2 \\
\Gamma_2 &= P_2 (1 - |\Gamma_1|^2) \quad (6)
\end{align*}$$

Then, the EM wave propagates through the first plasma layer with a slant angle of $\theta_3$, during which part of the power is absorbed by the plasma layer as shown in Fig. 1 (a). The power of the wave which arrives at the second reflecting surface can be calculated as

$$P_1 = P_p \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right) = P_1 (1 - |\Gamma_1|^2) \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right). \quad (7)$$

When the EM wave arrives at the second reflecting surface, the reflected power is $P_2 |\Gamma_2|^2$. Before the reflected EM wave can go back to vacuum, some power will be absorbed by the first plasma layer again. Also, multiple reflections are neglected because a high proportion of the energy of EM waves is transmitted when reflecting and waves reflected two or more times are extremely weak and can be overlooked. This simplification brings high efficiency without sacrificing accuracy. \(^{17,18}\) From the analyses and discussions above, the final power reflected into the vacuum is

$$P_{r2} = P_2 |\Gamma_2|^2 \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right) = P_2 (1 - |\Gamma_2|^2) \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right) |\Gamma_2|^2. \quad (8)$$

And the penetrating power can be obtained as

$$P_{t2} = P_2 (1 - |\Gamma_2|^2) = P_2 (1 - |\Gamma_2|^2) (1 - |\Gamma_2|^2) \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right). \quad (9)$$

After being absorbed by the second plasma layer, the power that reaches the third reflecting surface is

$$P_{r3} = P_{t2} \exp \left( -\frac{2 \alpha d}{\cos \theta_2} \right) = P_2 (1 - |\Gamma_2|^2) \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right) \exp \left( -\frac{2 \alpha d}{\cos \theta_2} \right) \exp \left( -\frac{2 \alpha d}{\cos \theta_2} \right) \exp \left( -\frac{2 \alpha d}{\cos \theta_2} \right). \quad (10)$$

Similarly, the power reflected by the third reflecting surface is then propagated back into vacuum and the power that arrives at the fourth reflecting surface is

$$P_{r4} = P_2 (1 - |\Gamma_2|^2) (1 - |\Gamma_3|^2) \exp \left( -\frac{2 \alpha d}{\cos \theta_1} \right) \exp \left( -\frac{2 \alpha d}{\cos \theta_2} \right) \exp \left( -\frac{2 \alpha d}{\cos \theta_3} \right) \exp \left( -\frac{2 \alpha d}{\cos \theta_4} \right). \quad (11)$$

$$P_{t3} = P_2 (1 - |\Gamma_2|^2) (1 - |\Gamma_3|^2) (1 - |\Gamma_4|^2) \quad (12)$$
Repeating the reflection and attenuation process, the entire expressions for the total reflection ratio, $R$, and transmission ratio, $T$, are concluded as

$$R = \sum_{k=1}^{N} \frac{P_k}{P_i} = |\Gamma_{1}|^2 + \sum_{k=2}^{N} \left( |\Gamma_k|^2 \prod_{i=1}^{k-1} \left( 1 - |\Gamma_i|^2 \right) \exp \left( -\frac{4\alpha_k d}{\cos \theta_i} \right) \right)$$

(13)

$$T = \frac{P_{N}}{P_i} = \prod_{k=1}^{N} \left( 1 - |\Gamma_k|^2 \right) \exp \left( -\frac{2\alpha_k d}{\cos \theta_i} \right).$$

(14)

where $d$ is the thickness of each layer, $d = L/N$. Note that the reflected wave in the rightmost interface induces very little contribution to the total reflection ratio due to the multi-reflection and absorption in the sub-layers. Therefore, it is reasonable for us to ignore the effect of the rightmost interface when calculating the reflection ratio.

Eventually, the attenuation ratio, $A$, is therefore calculated from

$$A = 1 - R - T.\quad (15)$$

According to the above theoretical analyses and formula derivations, we can directly find that our formulas (13), (14) and (15) for the EM waves propagation in a plasma are similar to the results derived from the traditional SMM. However, the effect of the incident angle is taken into consideration in this study, which is usually ignored in the above method to the best of our knowledge. In fact, EM waves normally propagate into a plasma at various slant angles in actual applications, such as radar detection, plasma stealth and re-entry communication blackout. Therefore, it is reasonable to consider the case when the EM waves propagate in a plasma slab with different incident angles. In the following section, we will first discuss the influence of the incident angle on the properties of EM wave propagation, and then investigate the impact of the other parameters under various incident angles.

III. RESULTS AND DISCUSSION

In this section, the influence of different parameters such as the incident angle, plasma collisional frequency, background magnetic flux density, plasma thickness and electron density on the reflection ratio, transmission ratio and attenuation ratio are investigated in sequence through the change of transmission and attenuation ratios with the incident wave frequency. Our investigation clearly identifies the effect of each parameter, since only one parameter is varied at a time. For precision, the plasma is divided into 120 pieces in the calculations ($N = 120$).

A. Effects of incident angles

We first investigate the effects of the incident angles on the transmission and attenuation ratios as shown in Fig. 2. The incident angle is set as $\theta_i = \pi/8, \pi/6, \pi/4$. In addition, we also select $\theta_0 = 0$ (vertical incidence) for comparison. Parameters other than the incident angle are taken as $\nu_{\text{ion}} = 0.1$ THz, $B = 0.3$ T, $L = 0.1$ m and $N_0 = 1 \times 10^{19}$ m$^{-3}$. As shown in Fig. 2, the transmission ratio grows and finally reaches its maximum value, and the attenuation ratio falls correspondingly, as the wave frequency uniformly increases from 0.1 THz to 1.0 THz. This tendency can be explained by the fact that the electric field of the wave changes so fast that the electrons in the plasma have no time to respond to absorb energy as the wave frequency increases. As a result, the transmitted EM wave energy increases, leading to the growth of the transmission ratio and the decline of the attenuation ratio.

Fig. 2 also shows negligible differences in both the transmission and attenuation ratio when the incident angles are small (e.g., $\theta_i < \pi/6$). However, when the incident angles are larger (e.g., $\theta_i > \pi/6$), the differences become obvious and cannot be ignored. This trend can also be concluded from Eqs. (2), (13) and (14). From another aspect, the transmission ratio decreases and the attenuation ratio increases with the increase of the incident angle. This can be attributed to the fact that the larger the incident angle, the longer the transmission distance. According to Fig. 3, the total transmission distance can be achieved as $\sum_{i=1}^{N} \frac{d}{\cos \theta_i}$. Intuitively, the transmission distance increases along with the increase of the incident angle, which means the wave suffers more absorption and reflection during its propagation in the plasma slab. Therefore, the incident angle is a crucial parameter during the calculation of the reflection ratio, transmission ratio and attenuation ratio.

To be clear, we further define the reflection ratio in decibel scale $R_{dB}$ as $R_{dB} = -20 \times \log_{10} R$, where $R$ is the normalized reflection ratio. Fig. 3 depicts the variation of $R_{dB}$ along the wave frequency under different angles. Note that $R_{dB}$ of the oblique incidence wave with

![FIG. 2. Transmission (a) and attenuation (b) ratios versus wave frequency under four different incident angles.](image-url)
θ_0 = π/8, π/6, π/4, decreases with the increase of the wave frequency, which means the normalized reflection ratio, \( R \), is rising, while vertical the incident waves do just the reverse. Fig. 3 also demonstrates that a larger incident angle results in smaller \( R_{dB} \), and induces a larger reflection ratio. Therefore, we can conclude that the large incident angle impedes the waves propagation in the plasma.

**B. Effects of collisional frequencies**

To better show the tendency of the propagation properties, the three-dimension graphs of the transmission and attenuation ratios versus the collisional frequency \( \nu_{en} \) from 10 to 500 GHz and wave frequency \( f \) from 0.1 to 1.0 THz are presented in Fig. 4. The other simulation parameters are set as \( \theta_0 = \pi/18, B = 0.3 \text{ T}, L = 0.1 \text{ m} \) and \( N_0 = 1 \times 10^{19} \text{ m}^{-3} \). We find that the transmission ratio and attenuation ratio in Fig. 4 are the same as the trend in Fig. 2, and that a higher wave frequency leads to a high transmission ratio and a low attenuation ratio. Notably, the trend is universally true in Figs. 5, 6, and 7. Figs. 4 (a), (c) and (b), (d) indicate that the transmission ratio decreases and attenuation ratio increases, respectively, with the increase of the collisional frequency at a fixed wave frequency. The above phenomenon can be explained as the electrons in the plasma slab with high collisional frequencies being more likely to collide with neutral particles, thus passing more energy to neutral particles through collision than those with low collisional frequencies. Therefore, the EM wave transmission ratio decreases and the attenuation ratio increases. In addition, the attenuation ratio in Figs. 4 (b) and (d) increases to its peak value first and then decreases along with the increase of the wave frequency. The maximum value of the attenuation ratio for right-polarized waves is located where the
wave frequency is close to the collisional frequency. This is reasonable due to the resonance effect of the plasma. According to the above analyses and discussion, a plasma with a low collisional frequency will be beneficial for terahertz waves propagation and finally promote the communication performance in many application scenarios.

C. Effects of magnetic flux densities

Fig. 5 shows the transmission and attenuation ratios for EM waves propagation in plasma slabs with continuously changeable background magnetic flux density $B = 0.1 \sim 5$ T, when $\theta_0 = \pi/18$, $v_{en} = 0.1$ THz, $L = 0.1$ m, and $N_0 = 1 \times 10^{19}$ m$^{-3}$. As shown in
Figs. 5 (a) and (b), the EM waves can transmit entirely through the plasma with a very high transmission ratio and a very low attenuation ratio when 0.3 THz < \( f < 1 \) THz. Note that there is a transmission valley and a high absorption peak correspondingly due to the cyclotron resonance of the magnetic field in the frequency band \( f = 0.1 \sim 0.3 \) THz. From Figs. 5 (b) and (d) we know that the increasing external magnetic field will not only broaden the absorption peak, but move the absorption peaks to higher frequency regions as well. In view of the direct relationship between the magnetic flux density and absorption peak region, a locating property of the magnetic field on the attenuation peak is revealed for engineering application. Therefore, we can employ a magnetic field with a proper flux density to avoid the wide attenuation band so as to promote the communication performance in many communication scenarios such as EM wave propagation during the re-entry process.

**D. Effects of plasma thicknesses**

According to Eqs. (13) and (14), the thickness of the plasma slab, \( L \), also plays a crucial role in the propagation properties and is analyzed in this subsection. We choose \( L = 0.1, 0.3, 0.5, 0.7 \) m. Besides, the other parameters are set as \( \theta_0 = \pi/18, \nu_{en} = 0.1 \) THz, \( B = 0.3 \) T, and \( N_0 = 1 \times 10^{10} \) m\(^{-3} \). The transmission ratio and attenuation ratio are shown in Fig. 6. As depicted in Figs. 6 (a) and (b), the transmission ratio is lower and the attenuation ratio is higher when the plasma slab is thicker. Thicker plasma increases the interaction distance between the waves and plasma slabs, hence more energy is absorbed when EM waves pass, which leads to a lower transmission ratio and a higher attenuation ratio. Consequently, decreasing the plasma thickness is an alternative way to improve the communication performance for EM wave propagation in the plasma slab, especially during the re-entry process.

**E. Effects of electron densities**

The effects of different magnitudes of electron densities, \( N_0 = 1 \times 10^{10}, 1 \times 10^{18}, 1 \times 10^{19}, 1 \times 10^{20} \) m\(^{-3} \), on the transmission and attenuation ratio are illustrated in Fig. 7. The other specific simulation parameters are set as \( \theta_0 = \pi/18, \nu_{en} = 0.1 \) THz, \( B = 0.3 \) T and \( L = 0.1 \) m. Obviously in Figs. 7 (a) and (b), with the rise of electron density, the transmission ratio decreases and the attenuation ratio increases. This can be explained the fact that higher electron densities mean more electrons and more electron collisions, inducing more energy to be absorbed through electron collisions and then released in the form of heat, which leads to a lower transmission ratio and a higher attenuation ratio. From Fig. 7 (b) we can see that large electron densities can both broaden the attenuation peak and increase the attenuation peak value. Moreover, the absorption peak will occupy a higher frequency. In addition, when the electron density is \( N_0 = 1 \times 10^{20} \) m\(^{-3} \) and the wave frequency is 0.1 THz, a small sunken region appears. This is caused by the reflection of the interface between the plasma and vacuum (the first reflecting surface), whose parameters are of great significance. In summary, decreasing the electron density can be an alternative and effective scheme to enlarge the transmission ratio and decrease the attenuation ratio of EM wave propagation in a plasma slab.

**IV. CONCLUSION**

In this study, we investigated the propagation properties of electromagnetic (EM) waves oblique propagation in a plasma with the scattering matrix method. The impacts of the EM wave incident angle, plasma collisional frequency, background magnetic flux density, plasma thickness and plasma electron density were analyzed, respectively. Simulation results intuitively indicated the impediment of the incident angles on EM wave propagation in a plasma, and the impediment will increase as the incident angle increases. The effects of other parameters on the propagation properties were also investigated under the circumstances of oblique incidence. In this system, the transmission ratio and attenuation ratio curves can be shifted linearly by different plasma thicknesses and electron densities within certain ranges. However, both transmission valleys and attenuation peaks may be adjusted by tuning the background magnetic field and collisional frequency. Our theoretical research can offer a fundamental basis in the real application involving the interaction between EM waves and plasma, such as re-entry blackout, plasma stealth, and deep space communication.

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