PARTICLE PHYSICS, ASTROPHYSICS AND COSMOLOGY WITH FORBIDDEN NEUTRINOS

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Astrophysical and cosmological consequences of a standard $\nu_\tau$ of $(15 \pm 3) \text{ keV}/c^2$ mass are discussed in the light of the recent results of the solar, atmospheric and LSND neutrino experiments and theoretical prejudices.

1 An exotic neutrino scenario

Let us assume that the LSND experiment$^1$ is correct and its parameters $\delta m^2_{\mu e} = m^2_{\nu_\mu} - m^2_{\nu_e} \approx 1 \text{ eV}^2/c^4$ and $\sin^2 2\theta_{\mu e} \approx 10^{-2}$ can be interpreted as $m_{\nu_\mu} \approx 1 \text{ eV}/c^2 \gg m_{\nu_e}$, in spite of the fact that the KARMEN collaboration$^2$ has not observed $\nu_\mu \rightarrow \nu_e$ oscillations in at least part of the parameter space advocated by LSND. Let us further assume that the original seesaw mechanism based on the up, charm and top quark masses$^3$ is the correct explanation for the smallness of the neutrino masses, i.e. $m_{\nu_q} = m_q^2/M$ with $q = u, c, t$. Inserting the experimental quark masses $m_u \approx 5 \text{ MeV}/c^2$, $m_c \approx 1.5 \text{ GeV}/c^2$ and $m_t \approx 180 \text{ GeV}/c^2$ $^4$, we conclude that the $\nu_e$ and $\nu_\tau$ masses are $m_{\nu_e} \approx 11.1 \mu \text{eV}/c^2$ and $m_{\nu_\tau} \approx 14.4 \text{ keV}/c^2$, respectively. As the $\nu_\tau$ mass lies in the cosmologically forbidden region between $93 \hbar^2 \text{eV}/c^2$ ($0.5 \leq \hbar \leq 0.8$) and $4 \text{ GeV}/c^2$, we will investigate its cosmological consequences below. Note also that the see-saw Majorana mass $M \approx 2.25 \cdot 10^9 \text{ GeV}$ is much smaller than the GUT scale $M_{\text{GUT}} \approx 10^{16} \text{ GeV}$ in this scenario.

In order to make sure that our choice of neutrino masses does not contradict the successful neutrino oscillation interpretation of the solar and atmospheric neutrino deficits$^5$ and the $Z^0$ width$^4$, there must be at least two, but most probably three sterile neutrinos $\nu'_e, \nu'_\mu$ and $\nu'_\tau$. Active-sterile vacuum oscillations could then account for the solar and atmospheric neutrino deficits with $\delta m^2_{\nu_e e'} = m^2_{\nu_e} - m^2_{\nu_{e'}} \approx 10^{-10} \text{ eV}^2/c^4$, $\delta m^2_{\nu_\mu \mu'} = m^2_{\nu_\mu} - m^2_{\nu_{\mu'}} \approx 3 \cdot 10^{-3} \text{ eV}^2/c^4$ and maximal mixing angles for both $\nu_e \rightarrow \nu'_e$ and $\nu_\mu \rightarrow \nu'_\mu$ mixing. At this stage it is perhaps important to stress the fundamental difference between the maximal mixing angles which appear in active-sterile mixing and the small flavour mixing $\sin^2 2\theta_{\mu e} \approx 10^{-2}$ for oscillations between second and first generation. In fact, if neutrino flavour mixing behaves similar to that of the quark sector, we expect third generation mixing angles of about $\sin^2 2\theta_{\tau \mu} \approx 10^{-5}$ and $\sin^2 2\theta_{\tau e}$.
\( \approx 10^{-9} \), respectively. With such a small mixing angle between third and first generation, the \( \nu_\tau \) is clearly not observable as a kink in the \( \beta \)-decay spectrum. Moreover, a Dirac mass of \( m_{\nu_\tau} < 30 \text{ keV}/c^2 \) is not in conflict with the duration of the neutrino burst of SN 1987A\(^{25} \). A \( \nu_\tau \) mass of 14.4 keV/c\(^2 \) would, however, pose serious problems in neutrinoless \( \beta \beta \)-decay if the \( \nu_\tau \) was indeed a Majorana neutrino, as required by the see-saw mechanism. We will thus address this problem below. Of course, one needs to explain why large angle \( \nu_\mu \to \nu'_\mu \) and \( \nu_\tau \to \nu'_\tau \) oscillations would not spoil the success of Big Bang nucleosynthesis. The effective number of neutrinos present during nucleosynthesis could be kept close to \( N_{\text{eff}} \approx 3 \) through matter-enhanced oscillations with large neutrino-antineutrino asymmetries just before nucleosynthesis\(^6 \). Alternatively, a phase transition at temperatures around \( T \approx 1 \text{ MeV}/k \), associated with the generation of mass differences between active and sterile neutrinos, could prevent excessive oscillations into the sterile sector\(^6 \).

In order to guarantee that the \( \nu'_e \) and \( \nu'_\mu \) masses are small and nearly degenerate with the \( \nu_e \) and \( \nu_\mu \), respectively, one requires an extension of the see-saw mechanism to the sterile sector based the same vacuum expectation value\(^6 \). Apart from gravitational interactions and possible neutrino oscillations, sterile neutrinos have no interactions in common with particles of the active sector; they might, however, have interactions with other sterile particles in which the particles of the active sector cannot take part. It is not so easy to design a consistent model for the sterile particles with interactions that are renormalisable and anomaly free. In fact, the simplest way to get around this problem, is to double the number of particles and assume that there is a Standard Model of Particle Physics operating in the sterile sector, as well, as e.g. in \( E_8 \times E_8' \) superstring theory. The quarks, charged leptons and intermediate bosons of the sterile sector would be degenerate with those of the active sector, since they are all governed by the same vacuum expectation value and Yukawa coupling constants which are presumably fixed by a unified theory at higher energy. Thus, one of the attractive features of this scenario is that parity is conserved in both the particle spectrum and the interactions, with the exception of the neutrino sector. Indeed, since we actually observe neutrino oscillations, there must be some small breaking of parity symmetry between the active and sterile neutrino sectors.

Assuming the minimal Higgs sector of one ordinary active Higgs doublet and its sterile partner, and restricting ourselves to one generation, the most general neutrino mass matrix\(^6 \) can be written in the basis of the maximally mixed
parity eigenstates $\nu_L^\pm = 2^{-\frac{1}{2}} (\nu_L \pm (N_R)^c)$ and $\nu_R^\pm = 2^{-\frac{1}{4}} (\nu_R \pm (N_L)^c)$ as

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & m_+ \\ 0 & 0 & m_- & 0 \\ 0 & m_- & M_- & 0 \\ m_+ & 0 & 0 & M_+ \end{pmatrix}$$

with $m_\pm = m_1 \pm m_2$ and $M_\pm = M_1 \pm M_2$. In the limit $M_\pm \gg m_\pm$, the eigenvalues of this mass matrix are given by $m_\pm^2/M_+, m_-^2/M_-, M_+$ and $M_-$ for the maximally mixed mass and parity eigenstates $\nu_L^\pm, \nu_L^-, \nu_R^+ \text{ and } \nu_R^-$, respectively. Specializing to $M_+ = M_- = M$ and parametrizing $m_+ = m_q$ and $m_- = m_q - \mu$, the difference of the neutrino mass squared is

$$\delta m^2_{\ell\ell'} = \frac{m^4_q}{M^2} \left( 1 - \left[ 1 - \frac{\mu}{m_q} \right]^4 \right).$$

(2)

Assuming that $\mu$ does not depend on the generation because, similar to $M$, it is a Majorana mass, we obtain for $\mu = 1 \text{ MeV}/c^2$ reasonable values for the parameters of the vacuum oscillation interpretation of the solar and atmospheric neutrino deficits and a prediction for $\nu_\tau$ oscillations

$$\delta m^2_{ee'} = 0.73 \cdot 10^{-10} \text{ eV}^2/c^4 \quad \sin^2 2\theta_{ee'} = 1$$
$$\delta m^2_{\mu\mu'} = 2.7 \cdot 10^{-3} \text{ eV}^2/c^4 \quad \sin^2 2\theta_{\mu\mu'} = 1$$
$$\delta m^2_{\tau\tau'} = 4.6 \cdot 10^3 \text{ eV}^2/c^4 \quad \sin^2 2\theta_{\tau\tau'} = 1.$$  

(3)

As $\delta m^2_{\ell\ell'}/m^2_{\nu\ell}$ decreases rapidly with increasing neutrino mass, the neutrino eigenstates corresponding to low mass eigenvalues may be combined to form quasi-Dirac spinors, and this would explain why the massive $\nu_\mu$ and $\nu_\tau$ do not make an observable contribution to neutrinoless $\beta\beta$-decay. It is interesting to note that the Majorana mass $\mu$ is approximately equal to the expected scale of the active-sterile phase transition which we mentioned earlier.

2 Astrophysical implications

If a suitable dissipation mechanism exists, massive neutrinos and antineutrinos will form supermassive compact dark objects in which the degeneracy pressure of the neutrinos and antineutrinos balances their gravitational attraction. Mass
and radius of the most massive object that can be formed in this way are determined by the Oppenheimer-Volkoff (OV) limit\(^9\)

\[
M_{OV} = 0.54195 \left( \frac{\hbar c}{G} \right)^{3/2} m_{\nu}^{-2} g_{\nu}^{-1/2}, \quad R_{OV} = 4.4466 R_{SOV}^S, \quad \text{(4)}
\]

where \(g_{\nu}\) denotes the spin degeneracy factor of the neutrinos and antineutrinos and \(R_{SOV}^S = 2GM_{OV}/c^2\) is the Schwarzschild radius of the mass \(M_{OV}\). There will be equal amounts of right-handed and left-handed neutrinos in the neutrino ball, as these are mixed by the gravitational interaction. There is little difference between a supermassive black hole mass and a neutrino ball at the OV-limit, a few Schwarzschild radii away from the object, as the radius of the last stable orbit around a black hole of the same mass is anyway \(3 R_{SOV}^S\). Supermassive neutrino balls could, therefore, mimic the role of the supermassive black holes which are purported to exist at the centres of a large number of galaxies, including our own, with masses ranging from \(10^{6.5} M_{\odot}\) to \(10^{9.5} M_{\odot}\).

For instance, if we want to interpret the presumably most massive and violent compact dark object in our vicinity, which is located at the centre of M 87 15 Mpc away and has a mass of \(M = (3.2 \pm 0.9) \cdot 10^9 M_{\odot}\), as a neutrino ball at the OV limit, the neutrino mass is constrained by\(^9\)

\[
12.4 \text{ keV}/c^2 \leq m_{\nu} \leq 16.5 \text{ keV}/c^2 \text{ for } g_{\nu} = 2
\]

\[
10.4 \text{ keV}/c^2 \leq m_{\nu} \leq 13.9 \text{ keV}/c^2 \text{ for } g_{\nu} = 4
\]

which fits our earlier estimate \(m_{\nu} \approx 14.4 \text{ keV}\) rather well for \(g_{\nu} = 2\). A neutrino ball at the OV-limit with a mass \(M_{OV} = 3 \cdot 10^9 M_{\odot}\) would have a radius of \(R_{OV} = 1.52 \text{ ld}\). Of course, nonrelativistic neutrino balls which are well below the OV limit, i.e. \(M \ll M_{OV}\), have a size much larger than black holes, although they are still dark and much more compact than any known baryonic object of the same mass. Mass and radius of nonrelativistic neutrino balls scale as\(^{19,20,21}\)

\[
MR^3 = \frac{91.869 \hbar^6}{G^3 m_{\nu}^8} \left( \frac{2}{g_{\nu}} \right)^2.
\]

As the gravitational potential of such an extended neutrino ball is much shallower, significantly less energy is being dissipated through accreting matter than in the case of a black hole of the same mass. In fact, there is a compact dark object at the centre of our galaxy which is with \(M = (2.6 \pm 0.2) \cdot 10^6 M_{\odot}\) probably at the lower end of the mass spectrum for such objects. Its mass is
concentrated within a radius smaller than 0.015 pc or 18 ld\textsuperscript{13}, as determined from the motion of stars in the vicinity of the strong radio source SgrA\textsuperscript{*}. Interpreting this supermassive compact dark object in terms of a neutrino ball, the upper limit for the size of the object provides us with a lower limit for the neutrino mass, i.e. $m_\nu \geq 15.9 \text{ keV}/c^2$ for $g_\nu = 2$, and $m_\nu \geq 13.4 \text{ keV}/c^2$ for $g_\nu = 4$, both corresponding to $M = 2.6 \cdot 10^6 M_\odot$ and $R = 22.4 \text{ ld}\textsuperscript{10,11,22}$. In this context, it is important to note that one can, in standard accretion disc theory\textsuperscript{14,23}, explain the enigmatic radio and infrared emission spectrum of SgrA\textsuperscript{*} from $\lambda \approx 0.3 \text{ cm}$ to $\lambda \approx 10^{-3} \text{ cm}$ much better in the neutrino ball scenario than in the black hole scenario. For a fit of the spectrum with a neutrino ball, the neutrino mass should be between 12 keV/$c^2$ and 18 keV/$c^2$\textsuperscript{14,23} for $g_\nu = 2$. We thus conclude that there are compelling astrophysical arguments for the existence of a neutral weakly interacting fermion of mass $m_\nu = (15 \pm 3) \text{ keV}/c^2$ which could be the $\nu_\tau$.

3 Cosmological implications

To be specific, we now assume that the $\nu_\tau$ is a standard Dirac neutrino with mass $m_\nu = 14.4 \text{ keV}/c^2$. Such a neutrino is quasi-stable, as its lifetime is many orders of magnitude larger than the age of the universe\textsuperscript{21}. The $\nu_\tau$ will be non-relativistic 54 min after the Big Bang at a photon temperature $T_{NR}^\gamma = 20 \text{ keV}/k$, long after nucleosynthesis. The universe will become heavy neutrino matter dominated 22 d after the Big Bang at $T_E^\gamma = 1 \text{ keV}/k$. From now on the evolution of the universe will differ substantially from that of the Standard Model of Cosmology\textsuperscript{24}. In fact, based on the Thomas-Fermi model at finite temperature, it has been shown\textsuperscript{15,16,17,18} that some time during the heavy neutrino matter dominated epoch, the universe will undergo a gravitational phase transition, yielding a condensed phase that consists of quasi-degenerate supermassive neutrino balls with masses close to the OV-limit $M_{OV} = 3 \cdot 10^9 M_\odot$. Of course, for this phase transition to happen, one would need an efficient dissipation mechanism which could be based on nonstandard bosons associated with the 1 MeV phase transition mentioned earlier. The time at which the first-order gravitational phase transition starts, depends on the detailed model, but it will typically begin 1 to 2 yr after the Big Bang. The first neutrino balls will be formed about 10 yr after Big Bang, which allows the neutrinos to condense in the neutrino balls in the free-fall time. Only a fraction of $10^{-3}$ of the neutrinos are estimated to be in the gaseous phase after this phase transition leading to a neutrino dominated critical universe today. The latent heat released is about 3.6% of the rest mass of the neutrino balls\textsuperscript{9}. Soon after the formation of the neutrino balls, annihilation of the heavy neutrinos into nonstandard light
Figure 1: Contributions of the various particle species to the critical density as a function of time, for a Hubble parameter $h = 0.5$, age of the universe $t_0 = 11.7$ Gyr, formation time $t_f = 10$ yr and annihilation time scale $t_a = 50$ yr.

Bosons will take place efficiently in the dense interior of the neutrino balls\(^{21}\) reducing the neutrino number density. Due to e.g. S-wave annihilation the mass of the neutrino balls will decrease as\(^ {21}\) $M(t) = M_0 \left[ t_a / (t_a + t - t_f) \right]^{2}$, where the annihilation time $t_a$ is determined by the annihilation rate and $t_f$ is the formation time of the neutrino balls. In Fig.1 we have plotted for a critical universe the fraction of the total energy density that the various particles make up as a function of time. The annihilation products will start to dominate the energy density of the universe around 650 yr. Recombination will take place in this radiation dominated phase around 31 kyr or $z \approx 1100$. Once the $\nu_\tau$ or $\bar{\nu}_\tau$ in the neutrino balls have annihilated below a certain level, and the energy density of the annihilation products has cooled sufficiently, the dispersed $\nu_\tau$ and $\bar{\nu}_\tau$ that escaped the phase transition, will start dominating the universe around 1 Gyr again, thereby igniting the quasars through accretion. Of course, annihilation of the $\nu_\tau$ and $\bar{\nu}_\tau$ in the neutrino balls will stop as soon as either the $\bar{\nu}_\tau$ or the $\nu_\tau$ are depleted. A $\nu_\tau - \bar{\nu}_\tau$ asymmetry of about $10^{-3}$ would be consistent with masses of the neutrino balls today. In summary, it is refreshing to see that
the desert of the Standard Model of Cosmology between nucleosynthesis and recombination is being revived in this scenario.

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