T-dual RR couplings on D-branes 
from S-matrix elements

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Abstract

Using the linear T-dual ward identity associated with the NSNS gauge transformations, some RR couplings on D\_p-branes have been found at order \( O(\alpha'^2) \). We examine the \( C^{(p-1)} \) couplings with the S-matrix elements of one RR, one graviton and one antisymmetric B-field vertex operators. We find the consistency of T-dual S-matrix elements and explicit results of scattering string amplitude and show that the string amplitude reproduces these couplings as well as some other couplings. This illustration is found for \( C^{(p-3)} \) couplings in the literature which is extended to the \( C^{(p-1)} \) couplings in this paper.

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1 Introduction

The dynamics of the D-branes of type II superstring theories at the lowest order in $\alpha'$ is given by the world-volume theory which is the sum of Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions \[1, 2, 3, 4\]. The bosonic part of this action is

\[
S = -T_p \int d^{p+1}x \, e^{-\phi} \sqrt{-\det (G_{ab} + B_{ab})} + T_p \int_{M^{p+1}} e^B C + \cdots
\]

(1)

where dots refer to the terms at higher order of $\alpha'$. In the Chern-Simons part, $M^{p+1}$ and $C$, represents the world volume of the $D_p$-brane and the sum over all RR potential forms, respectively. The multiplication rule is wedge product. The closed string fields $G_{ab}$ and $B_{ab}$ are the pulled back of the bulk fields $G_{\mu\nu}$ and $B_{\mu\nu}$ onto the world-volume of D-brane.

The action to be invariant under the B-field gauge transformation by adding the abelian gauge field as $B \rightarrow B + 2\pi \alpha' f$. The Chern-Simons part is invariant under the RR gauge transformations $\delta C = d\Lambda + H \wedge \Lambda$ in which $H$ is the field strength of $B$ and $\Lambda = \sum_{n=0}^{7} \Lambda^{(n)}$.

The couplings of D-brane at order $\alpha'^2$ involving the linear RR field strengths $F^{(p)}$, $F^{(p+2)}$ and $F^{(p+4)}$ have been found in [6]. It has been shown that these couplings are invariant under the linear T-duality and the B-field gauge transformations.

These couplings for $F^{(p+2)}$ in the string frame are [10]:

\[
\frac{2\pi^2 \alpha'^2 T_p}{p!} \int d^{p+1}x \, e^{a_0\cdots a_p} \left( \frac{1}{2!} \tilde{F}^{(p+2)}_{ia_1\cdots a_{p+1}} \mathcal{R}^a_{a_0} \, ^{ij} \mathcal{R}^{ij} - \frac{1}{p+1} \tilde{F}^{(p+2)}_{a_0\cdots a_{p+1}} \hat{\mathcal{R}}^{ij}_{ij} \right)
\]

(2)

where $F^{(n)} = dC^{(n-1)}$, $\mathcal{R}$ is the linear curvature tensor, $\hat{\mathcal{R}}_{ij} = \frac{1}{2} (\mathcal{R}_{ia} a_j - \mathcal{R}_{ik} k_j)$ and commas denote partial differentiation.

These quadratic couplings can be extended to higher order terms by making them to be covariant under the coordinate transformations and invariant under the RR gauge transformations. That is, the partial derivatives in that couplings should be replaced by the covariant derivatives, the closed string tensors should be extended to the pull-back of the bulk fields onto the world-volume of D-brane, the linear RR gauge field strength should be extended to the nonlinear field strength $\tilde{F}^{(n)}$ that is given as

\[
\tilde{F}^{(n)} = dC^{(n-1)} + H \wedge C^{(n-3)}
\]

(3)

and the linear curvature tensor should be extended to the nonlinear curvature tensor $\mathcal{R}$, i.e.,

\[
\frac{2\pi^2 \alpha'^2 T_p}{p!} \int d^{p+1}x \, e^{a_0\cdots a_p} \left( \frac{1}{2!} \tilde{F}^{(p+2)}_{ia_1\cdots a_{p+1}} R_{a_0} \, ^{ij} R^{ij} + \frac{1}{p+1} \tilde{F}^{(p+2)}_{a_0\cdots a_{p+1}} (R_{ia} a_j - \phi^{ij}) \right)
\]

(4)

where the semicolons are used to denote the covariant differentiation.
The $\alpha'^2$ corrections to the D-brane action for one RR, one NSNS and one open string NS states have been found from the low energy of the string amplitude [9, 13]. These corrections for the RR field strength $F^{(p)}$ are:

$$\begin{align*}
\pi^2 \alpha'^2 T_p & \frac{1}{2!(p-2)!} \int d^{p+1}x \epsilon^{a_0 \ldots a_p} F^{(p)}_{ij} a_3 \ldots a_p \left( R_{a_0 a_0}^{ij} a \left( 2\pi \alpha' f_{a_1 a_2} \right) - H_{a_0 a_1 a_2}^{ij} \left( 2\pi \alpha' f_{a_1 a_2} \right) \right).
\end{align*}$$

where it has been extended $2\pi \alpha' f$ to gauge invariant $B + 2\pi \alpha' f$ combination[11].

The second term which involve the transverse scalar field, are exactly reproduced by the pull-back operator in (4) in the static gauge [13]. The first terms should be added to the D-brane action at order $\alpha'^2$. Extending the first coupling above to be covariant under the coordinate transformations and invariant under the B-field and RR gauge transformations, one finds the following nonlinear couplings at order $\alpha'^2$:

$$\begin{align*}
\pi^2 \alpha'^2 T_p & \frac{1}{2!(p-2)!} \int d^{p+1}x \epsilon^{a_0 \ldots a_p} F^{(p)}_{ij} a_3 \ldots a_p \left[ 2 R_{a_0 a_0}^{ij} a \left( B_{a_1 a_2} + 2\pi \alpha' f_{a_1 a_2} \right) \right].
\end{align*}$$

We calculate the field theory amplitudes by using the above couplings for the scattering of one RR $(p-1)$-form with three and two transverse index and two NSNS states at order $\alpha'^2$. We then compare the results with low energy limit of string amplitudes at order $\alpha'^2$ and corresponding T-dual multiplets that has been found in [13]. Using S-matrix method in details, the amplitude of one RR $(p-3)$-form with one transverse index, and two arbitrary B-fields has been studied in [11]. Here we perform similar computation for the amplitude of RR $(p-1)$-form in details and show that all structures of this amplitude, regardless the overall factor, can be produced by T-duality that has been found in [13].

It has been shown in [13] that the T-duality at linear order should appear in the amplitudes through the associated Ward identities. Scattering amplitude at any loop order should satisfy the T-dual Ward identity. This classifies the tree-level amplitudes into T-dual multiplets. Each multiplet includes the scattering amplitudes which interchange under the linear T-duality transformations.

An outline of the paper is as follows: In section 2.1, we examine the calculation of string amplitude of one RR and two NSNS vertex operators in superstring theory. In section 2.2.1, we perform the calculation in full details for $C^{(p-1)}_{ijk}$ and expand the amplitude at low energy. Regarding the low energy expansion of integral term in amplitude, we show that there is neither contact term nor massless open string pole at order $O(\alpha'^2)$ which is consistent with the couplings [4]. In section 2.2.2, we perform the same calculation for $C^{(p-1)}_{ij}$. We find that the contact terms and the massless open string pole of the field theory, which have been found in [13], are reproduced exactly by this amplitude at order $O(\alpha'^2)$. In fact, by using the standard S-matrix method, we find all elements of S-matrix in proposal case that found by T-duality in [13].

2
2 String scattering amplitude

The S-matrix method is a very important tool in superstring theory to discover new couplings between strings. It has been shown that the corrections in field theory might have been obtained in $\alpha'$ by standard S-matrix method. A great deal of effort for the understanding of scattering amplitudes has been made [5].

The couplings in [1] can be confirmed by the scattering amplitude of one RR and two NSNS states that has been found by the T-duality in [13]. We reproduce these coupling by explicit calculations of S-matrix.

Some of the couplings resulting from the consistency of the Chern-Simons action at order $O(\alpha'^2)$ with the linear T-duality transformations confirm by the calculation of three closed string scattering amplitudes [8 11 12]. In this work, we are interested in finding all couplings that string theory produces for the RR potential $C^{(p-1)}$ which carries any world volume indices.

To calculate a S-matrix element, one needs to choose the picture of the vertex operators appropriately. The sum of the superghost charge must be -2 for the disk level amplitude. The tree level scattering amplitude of one RR and two NSNS states on the world-volume of a $D_p$-brane has been studied in different vertex pictures [8 9 11 12]. We are interested to evaluate the amplitude of one RR, one graviton and one B-field in the picture that the symmetry between two NSNS states is manifest from the beginning. Hence, we work in $(-1/2, -3/2)$ picture for RR state [15] and the two NSNS vertex operators in $(0, 0)$ picture. It has been shown that the final result is independent of the choice of the picture.

We work in the RNS worldsheet formalism, with the closed string vertex operators being constructed out of bosons $X^\mu(z), X^\mu(\bar{z})$ and fermions $\psi^\mu(z), \psi^\mu(\bar{z})$ as well as the picture ghosts $\phi(z)$ and $\phi(\bar{z})$. We will now perform the computation for interested distribution of picture.

The string scattering amplitude is given by the following correlation function:

$$ A \sim < V_{RR}^{(-1/2,-3/2)}(z_1, p_1) V_{NSNS}^{(n,0)}(z_2, p_2) V_{NSNS}^{(0,0)}(z_3, p_3) > $$

(6)

Using the doubling trick [6], explicitly we have:

$$ V_{RR}^{(-1/2,-3/2)} = (P_- H_1(n) M_p)^{AB} \int d^2 z_1 : e^{-\phi(z_1)/2 S_A(z_1)} e^{ip_1 \cdot X} : e^{-\phi(z_1)/2 S_B(z_1)} e^{ip_1 \cdot D \cdot X} : $$

$$ V_{NSNS}^{(n,0)} = (\varepsilon_2 \cdot D)_{\mu_3 \mu_4} \int d^2 z_2 : (\partial X^{\mu_3} + ip_2 \cdot \psi_3^{\mu_3}) e^{ip_2 \cdot X} : (\partial X^{\mu_4} + ip_2 \cdot D \cdot \psi_3^{\mu_4}) e^{ip_2 \cdot D \cdot X} : $$

$$ V_{NSNS}^{(0,0)} = (\varepsilon_3 \cdot D)_{\mu_5 \mu_6} \int d^2 z_3 : (\partial X^{\mu_5} + ip_3 \cdot \psi_3^{\mu_5}) e^{ip_3 \cdot X} : (\partial X^{\mu_6} + ip_3 \cdot D \cdot \psi_3^{\mu_6}) e^{ip_3 \cdot D \cdot X} : $$

Our conversions set $\alpha' = 2$ in the string theory calculations.
where the NSNS vertex operators include polarizations $\varepsilon_i$ and momenta $p_i, i = 2, 3$. The RR vertex operator has momentum $p_1$ and antisymmetric polarization $\varepsilon_1$. The indices $A, B, \cdots$ are the Dirac spinor indices and $P_+ = \frac{1}{2}(1 - \gamma_{11})$ is the chiral projection operator, and

\[
H_{1(n)} = \frac{1}{n!} \varepsilon_{\mu_1 \cdots \mu_n} \gamma^{\mu_1} \cdots \gamma^{\mu_n}
\]
\[
M_p = \frac{\pm 1}{(p + 1)!} \varepsilon_{\alpha_0 \cdots \alpha_p} \gamma^{\alpha_0} \cdots \gamma^{\alpha_p}
\]

where $\varepsilon$ is the volume $(p + 1)$-form of the $D_p$-brane. Here the matrix $D_{\mu\nu}$ is a diagonal matrix that agrees with $\eta_{\mu\nu}$ in directions along the brane (Neumann boundary conditions) and with $-\eta_{\mu\nu}$ in directions normal to the brane (Dirichlet boundary conditions). In this notation, $D_i = -\delta_i^j, D_{\alpha\beta} = \delta_{\alpha\beta}$ and thus $D_{\mu\nu} = V_{\mu\nu} - N_{\mu\nu}$.

After performing the correlators, we will remove the volume of $SL(2, R)$ group which is the conformal symmetry of the upper half $z$-plane $\mathbb{H}$. The amplitude (6) can be written as

\[
A \sim \frac{1}{2} (H_{1(n)} M_p)^{AB} (\varepsilon_2 \cdot D)_{\mu_4 \mu_6} (\varepsilon_3 \cdot D)_{\mu_5 \mu_6} \int d^2 z_1 d^2 z_2 d^2 z_3 (z_{11})^{-3/4} \sum_{i=1}^{16} b_i^{\mu_4 \mu_5 \mu_6} \delta^{p+1}(p_1^a + p_2^b + p_3^c) + (2 \leftrightarrow 3)
\]

where we use the standard world-sheet propagators. $z_{ij} = z_i - \bar{z}_j$, and

\[
\begin{align*}
(b_1)_{AB}^{\mu_4 \mu_5 \mu_6} &= < S_A(z_1) : S_B(\bar{z}_1) : g_1^{\mu_4 \mu_5 \mu_6} \\
(b_2)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} : g_2^{\mu_4 \mu_5 \mu_6} \\
(b_3)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_4} : g_3^{\mu_4 \mu_5 \mu_6} \\
(b_4)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_5} : g_4^{\mu_4 \mu_5 \mu_6} \\
(b_5)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_6} : g_5^{\mu_4 \mu_5 \mu_6} \\
(b_6)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} \psi^{\mu_4} \psi^{\mu_5} : g_6^{\mu_4 \mu_5 \mu_6} \\
(b_7)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} \psi^{\mu_5} : g_7^{\mu_4 \mu_5 \mu_6} \\
(b_8)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_4} \psi^{\mu_5} : g_8^{\mu_4 \mu_5 \mu_6} \\
(b_9)_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_6} : g_9^{\mu_4 \mu_5 \mu_6} \\
(b_{10})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_4} \psi^{\mu_5} : g_{10}^{\mu_4 \mu_5 \mu_6} \\
(b_{11})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_3 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_6} : g_{11}^{\mu_4 \mu_5 \mu_6} \\
(b_{12})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_4} \psi^{\mu_5} : g_{12}^{\mu_4 \mu_5 \mu_6} \\
(b_{13})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_6} : g_{13}^{\mu_4 \mu_5 \mu_6} \\
(b_{14})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} \psi^{\mu_4} \psi^{\mu_5} : g_{14}^{\mu_4 \mu_5 \mu_6} \\
(b_{15})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} \psi^{\mu_5} : g_{15}^{\mu_4 \mu_5 \mu_6} \\
(b_{16})_{AB}^{\mu_4 \mu_5 \mu_6} &= < ip_2 \cdot D \delta_1 : S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} \psi^{\mu_4} \psi^{\mu_5} : g_{16}^{\mu_4 \mu_5 \mu_6} \times < S_A : S_B : \psi^{\beta_1 \beta_2} \psi^{\mu_3} \psi^{\mu_4} \psi^{\mu_5}
where $g$’s are the correlators of $X$’s which can easily be performed using the standard world-sheet propagators, and the correlator of $\psi$ can be calculated using the Wick-like rule [16].

If the symmetry of two NSNS polarizations are similar then ten independent $b_i$ contribute to the amplitude as [11, 12]. Two NSNS polarizations in this work have different symmetry and sixteen independent $b_i$ contribute to the amplitude. By considering the relations (7) and (9) in amplitude (8), one can find that the scattering amplitude involves the following trace of gamma matrices:

$$T(n, p, m) = \frac{1}{n!(p + 1)!} \varepsilon_{\nu_1 \cdots \nu_n} \varepsilon_{a_0 \cdots a_p} A_{[\alpha_1 \cdots \alpha_m]} \text{Tr}(\gamma^{\nu_1} \cdots \gamma^{\nu_n} \gamma^{a_0} \cdots \gamma^{a_p} \gamma^{\alpha_1 \cdots \alpha_m})$$ (10)

where $A_{[\alpha_1 \cdots \alpha_m]}$ is an antisymmetric combination of the momenta and/or the polarizations of the NSNS states[11, 12].

It could be verified that the trace (10) is non-zero only for $n = p - 3$, $n = p - 1$, $n = p + 1$, $n = p + 3$, $n = p + 5$. The case $n = p - 3$ is studied in [11] and [12] where the RR potential carries transverse indices and world volume indices, respectively.

We are interested in the case $n = p - 1$. In this case the only RR potentials that lead to non zero amplitudes are that carried zero, one, two and three transvers indices. In this paper, we consider $(\varepsilon_1^{(p - 1)})^{ijk}$, $(\varepsilon_1^{(p - 1)})^{ij}$ and $(\varepsilon_1^{(p - 1)})^i$. The result can easily be extended to the RR $n$-form by contracting its indices with the world volume form. For the cases that we will evaluate, the trace relation (10) gives non-zero result only for the following $T(n, p, m)$:

$$(3, 4, 8) \quad (2, 3, 6) \quad (1, 2, 4)$$

So it could be concluded easily that $b_1$, $b_2$ and $b_3$ in (9) have no contribution to the amplitude. Let us begin with the RR potential $(\varepsilon_1^{(p - 1)})^{ijk}$.

2.1 The amplitude of one RR $(p - 1)$-form with three transvers indices and two NSNS.

For this RR potential $n = 3$, and from the relation $n = p - 1$ one gets $p = 4$. The trace (10) is non-zero only for $m = 8$. It becomes

$$T(3, 4, 8) = 32 \frac{8!}{3! 5!} \varepsilon_1^{ij k} \varepsilon^{a_0 \cdots a_4} A_{[ij ka_0 \cdots a_4]}$$ (11)

where 32 is the trace of the $32 \times 32$ identity matrix. Since $m = 8$, only the $\psi$ correlator in $b_{16}$ have non-zero contribution to the amplitude (6). The $X$ correlator in $b_{16}$ is

$$g_{16} = |z_{12}|^2 p_1 \cdot p_2 |z_{13}|^2 p_1 \cdot p_3 |z_{32}|^2 p_2 \cdot p_3 |z_{12} |^2 p_1 \cdot D \cdot p_2 |z_{13} |^2 p_1 \cdot D \cdot p_3 |z_{23} |^2 p_2 \cdot D \cdot p_3 \times (z_{11})^{p_1 \cdot D \cdot p_1} (z_{22})^{p_2 \cdot D \cdot p_2} (z_{33})^{p_3 \cdot D \cdot p_3} (i)^{p_1 \cdot D \cdot p_1 + p_2 \cdot D \cdot p_2 + p_3 \cdot D \cdot p_3}$$ (12)
one can easily check that the integrand is invariant under the $SL(2, R)$ transformation. So we can map the results to disk with unit radius. To fix this symmetry, we then set $z_1 = 0$. The correlator $b_{16}$ then becomes

$$|z_2|^{2p_1 \cdot p_2} |z_3|^{2p_1 \cdot p_3} \left(1 - |z_2|^2 \right)^{p_2 \cdot D \cdot p_2} \left(1 - |z_3|^2 \right)^{p_3 \cdot D \cdot p_3} \left|z_2 - z_3\right|^{2p_2 \cdot p_3} \left|1 - z_2 \bar{z}_3\right|^{2p_2 \cdot D \cdot p_3} \equiv K. \quad (13)$$

Replacing in (8) the above $X$-correlator and the $\psi$-correlator from Wick-like rule, one finds

$$A \sim 2(\varepsilon_1^{(p-1)})_{ij} a^5 \cdots a^p \epsilon_{a_0 \cdots a_6} \varepsilon_2^{a_3 a_4} \varepsilon_3^{a_5 a_6} \left(\varepsilon^{S}_{2} a^{3} k (\varepsilon^{A}_{3} a^{4} a^{2} + (2 \leftrightarrow 3))\right) \times \mathcal{I}_1 \delta(p_1^a + p_2^a + p_3^a) \quad (14)$$

The amplitude for two graviton or two $B$-field is zero. This amplitude has been found as the second component of a T-dual multiplet (equation (15) in [13]).

The conservation of momentum is understood in all amplitudes in this paper. The integral in the above amplitude is

$$\mathcal{I}_1 = \int_{|z_i| \leq 1} d^2 z_2 d^2 z_3 \frac{K}{|z_2|^2 |z_3|^2} \quad (15)$$

To fix completely the $SL(2, R)$ symmetry, we then set $z_i = r_i e^{i \theta_i}, \quad i = 2, 3$. Since the integrand depends only on $\theta_2 - \theta_3$, one of the integrals can be explicitly performed. To study the low energy limit of the amplitude (14), one has to expand $\mathcal{I}_1$ at the low energy. We Taylor expand the integral in $r_2$ and $r_3$ using

$$\frac{1}{(1 - x)^m} = \sum_{n=0}^{\infty} \binom{m + n - 1}{n} x^n \quad (16)$$

The integration over $\theta_2 - \theta_3$ produce the Kronecker delta function and the integrals over the radial coordinates are a set of infinite sums

$$\sum_{n_i=0}^{\infty} \left(\frac{1}{s + n_1 + n_3 + n_5} + \frac{1}{t + n_2 + n_3 + n_5}\right) \frac{\delta_{n_3-n_4+n_5-n_6}}{s + t + u + n_1 + n_2 + n_5 + n_6} \quad (17)$$

where we have used the following definitions for the mandelstam variables:

$$s = p_1 \cdot p_2 \quad ; \quad t = p_1 \cdot p_3 \quad ; \quad u = p_2 \cdot p_3$$
\( p = p_2 \cdot D \cdot p_2 \quad ; \quad q = p_3 \cdot D \cdot p_3 \quad ; \quad v = p_2 \cdot D \cdot p_3. \) \hspace{1cm} (18)

Using (17) it is possible to show that asymptotically in the region of small momenta the following expansions hold up to terms quadratic in momenta:

\[ I_1 = \frac{\pi^2}{t} \left( \frac{1}{s + t + u} - \frac{\pi^2}{6} p \right) + \frac{\pi^2}{s} \left( \frac{1}{s + t + u} - \frac{\pi^2}{6} q \right). \] \hspace{1cm} (19)

The expansion of this integral is found in [11] in a restricted kinematic setup. Since the amplitude considered in this paper has no massless pole in the \( u \) and \( v \)-channel and also the kinematic factor \( \varepsilon_1^{ij} \epsilon^{a_0 \cdots a_5} A_{[ij]a_0 \cdots a_5} \) does not have any term proportional to \( p_2 \cdot p_3 \) and \( p_2 \cdot D \cdot p_3 \). So the integral in (14) has no \( u \) and \( v \)-channel poles. Hence, it is safe to restrict the Mandelstam variables in (13) to \( u = v = 0 \). By these consideration the expansion (19) is exactly equal to the results in [11].

All terms in the above expansion are closed string poles. This is consistent with the field theory calculation in the previous section that there is no massless open string channel for \( \varepsilon_1^{(p-1)} \).

### 2.2 The amplitude of one RR \((p-1)\)-form with two transverse indices and two NSNS.

In this case, \( n = 2 \) and from the relation \( n = p - 1 \), it could be found that \( p = 3 \). Since the index of the RR potential is transverse and the indices of the volume form are the world-volume indices, one finds that the trace relation (10) is non-zero only for \( m = 6 \). The trace in this case becomes

\[ T(2, 3, 6) = 32 \frac{6!}{2!4!} \varepsilon_1^{ij} \epsilon^{a_0 \cdots a_3} A_{[ij]a_0 \cdots a_3} \] \hspace{1cm} (20)

The \( \psi \) correlators in \( b_{12}, b_{13}, b_{14}, b_{15} \) and \( b_{16} \) have non-zero contributions. Using the on-shell condition \( \varepsilon_2 \cdot p_2 = \varepsilon_3 \cdot p_3 = 0 \) and conservation of momentum along the world volume, one can find the \( X \) correlators corresponding to these \( b_i \)'s, i.e.,

\[
g_{12}^{\mu_6} = iK \left( \frac{p_1^{\mu_6} z_{31}}{z_{33}} + \frac{p_2^{\mu_6} z_{32}}{z_{33}} + \frac{(p_1 \cdot D)^{\mu_6} z_{31}}{z_{13}} + \frac{(p_2 \cdot D)^{\mu_6} z_{32}}{z_{23}} \right)
\]

\[
g_{13}^{\mu_5} = iK \left( \frac{p_1^{\mu_5} z_{31}}{z_{33}} + \frac{p_2^{\mu_5} z_{32}}{z_{33}} + \frac{(p_1 \cdot D)^{\mu_5} z_{31}}{z_{13}} + \frac{(p_2 \cdot D)^{\mu_5} z_{32}}{z_{23}} \right)
\]

\[^2\text{In scattering theory, the number of momenta are related to the degree of } \alpha', \text{ so one can expand the integrals up to arbitrary degree of } \alpha'.\]
\[
g_{14} = i K z_{22} \left( \frac{p_1^{\mu 4} z_{21}}{z_{12}} + \frac{p_3^{\mu 4} z_{23}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu 4} z_{21}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu 4} z_{23}}{z_{32}} \right) \\
g_{15} = i K z_{22} \left( \frac{p_1^{\mu 3} z_{21}}{z_{12}} + \frac{p_3^{\mu 3} z_{23}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu 3} z_{21}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu 3} z_{23}}{z_{32}} \right) \\
g_{16} = K z_{22} \left( \frac{p_1^{\mu 3} z_{21}}{z_{12}} + \frac{p_3^{\mu 3} z_{23}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu 3} z_{21}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu 3} z_{23}}{z_{32}} \right)
\]

where \( K \) is given in \([13]\).

We note that all terms behave similarly under the \( SL(2, R) \) transformation. Writing the sub-amplitudes \( A_i \) in \([8]\) corresponding to \( b_i \), the sub-amplitudes corresponding to \( b_{12}, b_{13}, b_{14}, b_{15} \) are

\[
A_{12} \sim 16 \varepsilon_1^{ij} \epsilon^{a_0 \ldots a_3} (D \cdot \varepsilon_3^T)_{\mu \nu a_1} (\varepsilon_2)_{a_2 j} (p_2)_{i} (p_2)_{a_3} (p_3)_{a_0} \\
\int d^2 z_1 d^2 z_2 d^2 z_3 \left( \frac{z_{11} K}{z_{21}^2 z_{21}^2 z_{31} z_{33}} \right) \left( \frac{p_1^{\mu a} z_{31}}{z_{12}} + \frac{p_3^{\mu a} z_{32}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu a} z_{31}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu a} z_{32}}{z_{32}} \right)
\]

\[
A_{13} \sim -16 \varepsilon_1^{ij} \epsilon^{a_0 \ldots a_3} (\varepsilon_3)_{a_0 a_1} (\varepsilon_2)_{a_2 j} (p_2)_{i} (p_2)_{a_3} (p_3)_{a_0} \\
\int d^2 z_1 d^2 z_2 d^2 z_3 \left( \frac{z_{11} K}{z_{21}^2 z_{21}^2 z_{31} z_{33}} \right) \left( \frac{p_1^{\mu a} z_{31}}{z_{12}} + \frac{p_3^{\mu a} z_{32}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu a} z_{31}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu a} z_{32}}{z_{32}} \right)
\]

\[
A_{14} \sim 8 \varepsilon_1^{ij} \epsilon^{a_0 \ldots a_3} (p_3)_{j} (p_3)_{a_1} \left[ (D \cdot \varepsilon_2^T)_{\mu a_1} (p_2)_{a_3} - (D \cdot \varepsilon_2^T)_{\mu a_3} (p_2)_{a_1} \right] \\
\int d^2 z_1 d^2 z_2 d^2 z_3 \left( \frac{z_{11} K}{z_{31}^2 z_{31}^2 z_{21} z_{22}} \right) \left( \frac{p_1^{\mu 4} z_{21}}{z_{12}} + \frac{p_3^{\mu 4} z_{23}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu 4} z_{21}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu 4} z_{23}}{z_{32}} \right)
\]

\[
A_{15} \sim 8 \varepsilon_1^{ij} \epsilon^{a_0 \ldots a_3} (p_3)_{a_1} (p_3)_{j} \left[ (\varepsilon_2)_{\mu a_1} (p_2)_{a_3} - (\varepsilon_2)_{\mu a_3} (p_2)_{a_1} \right] \\
\int d^2 z_1 d^2 z_2 d^2 z_3 \left( \frac{z_{11} K}{z_{31}^2 z_{31}^2 z_{21} z_{22}} \right) \left( \frac{p_1^{\mu 3} z_{21}}{z_{12}} + \frac{p_3^{\mu 3} z_{23}}{z_{32}} + \frac{(p_1 \cdot D)^{\mu 3} z_{21}}{z_{12}} + \frac{(p_3 \cdot D)^{\mu 3} z_{23}}{z_{32}} \right)
\]

It is obvious that the above amplitudes are non-zero for symmetric polarization tensor \( \varepsilon_2^S \) and antisymmetric polarization tensor \( \varepsilon_3^A \). There are similar sub-amplitudes as above in the \((2 \leftrightarrow 3)\) part of the amplitude \([8]\) where the polarization tensors are \( \varepsilon_2^A \) and \( \varepsilon_3^S \).

The above amplitudes produce structures which contain the contraction of \( p_1 \) and \( NSNS \) polarizations. In the world volume contraction of \( p_1 \), the conservation of momentum along the brane could be used to write \( p_1 \cdot V \cdot \varepsilon \) in terms of \( p_2 \cdot V \cdot \varepsilon \) and \( p_3 \cdot V \cdot \varepsilon \).

By using this consideration and on-shell condition, it could be found that the contraction of momenta with corresponding \( NSNS \) polarizations in the transvers direction are not independent structures, \( i.e., \)

\[
p_2 \cdot N \cdot \varepsilon_2 = -p_2 \cdot V \cdot \varepsilon_2 \\
p_3 \cdot N \cdot \varepsilon_3 = -p_3 \cdot V \cdot \varepsilon_3
\]
Therefor, the independent structures that contain the contraction of momentum and polarization are

\[ p_1 \cdot N \cdot \varepsilon_2, \; p_3 \cdot N \cdot \varepsilon_2, \; p_2 \cdot V \cdot \varepsilon_2, \; p_3 \cdot V \cdot \varepsilon_2, \; p_1 \cdot N \cdot \varepsilon_3, \; p_2 \cdot N \cdot \varepsilon_3, \; p_2 \cdot V \cdot \varepsilon_3, \; p_3 \cdot V \cdot \varepsilon_3 \]

using this fact that there is no momentum conservation in the transverse subspace.

We will see that there are some other contributions as \( pp \) and \( \varepsilon \varepsilon \) from \( b_{16} \). Let us evaluate the only structures in the sub-amplitude \( \mathcal{A}_{16} \) in (8) in details.

We finally will collect all terms come from all subamplitudes and will write them in form of a final amplitude.

The contractions which produce structure \( p\varepsilon_3 \) give the following contribution to the amplitude \( \mathcal{A}_{16} \):

\[ \mathcal{A}_{16}(p\varepsilon_3) \sim -8\varepsilon_{1}^{ij}\varepsilon^{a_0\ldots a_3} \int d^2z_1d^2z_2d^2z_3 \frac{z_{11}^2K}{|z_{21}|^2|z_{31}|^2|z_{21}|^2|z_{31}|^2} \]

\[ \left( \mathcal{P}(z_2, z_3)A_1[ija_0\ldots a_3] + \mathcal{P}(\bar{z}_2, z_3)A_2[ija_0\ldots a_3] + \mathcal{P}(z_3, \bar{z}_3)A_3[ija_0\ldots a_3] \right. \]

\[ + \mathcal{P}(z_2, \bar{z}_3)A_4[ija_0\ldots a_3] + \mathcal{P}(z_3, \bar{z}_3)A_5[ija_0\ldots a_3] + \mathcal{P}(\bar{z}_2, \bar{z}_3)A_6[ija_0\ldots a_3] \]

where \( \mathcal{P}(z_i, z_j) \) is given by the Wick-like contraction

\[ \mathcal{P}(z_i, z_j)\eta^{\mu\nu} = [\psi^\mu(z_i), \psi^\nu(z_j)] = \eta^{\mu\nu} \frac{(z_i - z_1)(z_j - \bar{z}_1) + (z_j - z_1)(z_i - \bar{z}_1)}{(z_i - z_j)(z_1 - \bar{z}_1)} \]

Using the fact that above kinematic factors \( (A_i, \; i = 1, \ldots, 6) \) contract with \( \varepsilon_{1}^{ij}\varepsilon^{a_0\ldots a_3} \), one observes that there are 15 different terms in each case, however, 11 of them are zero and the other four terms are equal. They are simplified as

\[ A_1[ija_0\ldots a_3] = \frac{4}{15}(p_2 \cdot \varepsilon_3^A)_{a_3}(\varepsilon_3^S)_{ia_1}(p_2)_{a_2}(p_3)_{j}(p_3)_{a_0} \]

\[ A_2[ija_0\ldots a_3] = -\frac{4}{15}(p_2 \cdot D \cdot \varepsilon_3^A)_{a_3}(\varepsilon_2^S)_{ia_1}(p_2)_{a_2}(p_3)_{a_0}(p_3)_{j} \]

\[ A_3[ija_0\ldots a_3] = \frac{4}{15}(p_3 \cdot D \cdot \varepsilon_3^A)_{a_3}(\varepsilon_2^S)_{ia_1}(p_2)_{j}(p_2)_{a_2}(p_3)_{a_0} \]

\[ A_4[ija_0\ldots a_3] = -\frac{4}{15}(p_2 \cdot D \cdot (\varepsilon_3^A)^T)_{a_3}(\varepsilon_2^S)_{ia_1}(p_2)_{a_2}(p_3)_{a_0}(p_3)_{j} \]

\[ A_5[ija_0\ldots a_3] = -\frac{4}{15}(p_3 \cdot D \cdot (\varepsilon_3^A)^T)_{a_3}(\varepsilon_2^S)_{ia_1}(p_2)_{j}(p_2)_{a_2}(p_3)_{a_0} \]

\[ A_6[ija_0\ldots a_3] = \frac{4}{15}(p_2 \cdot (\varepsilon_3^A)^T)_{a_3}(\varepsilon_2^S)_{ia_1}(p_2)_{a_2}(p_3)_{a_0}(p_3)_{j} \]

These factors are zero for \( \varepsilon_2^A \) and \( \varepsilon_3^S \).
The contractions which produce structure \( p \varepsilon_2^S \) give the following contribution to the amplitude \( \mathcal{A}_{16} \):

\[
\mathcal{A}_{16}(p \varepsilon_2^S) \sim -4 \varepsilon_{1}^{ij} \varepsilon_{a}^{a_0\ldots a_3} \int d^2 z_1 d^2 z_2 d^2 z_3 \frac{z_{12}^2 K}{|z_{21}|^2 |z_{31}|^2 |z_{21}|^2 |z_{31}|^2} \tag{26}
\]

\[
\left( \mathcal{P}(z_2, z_3) A'_{1[ija_0\ldots a_3]} + \mathcal{P}(z_2, z_3) A'_{2[ija_0\ldots a_3]} + \mathcal{P}(z_3, z_2) A'_{3[ija_0\ldots a_3]}
\right.
\]

\[
+ \mathcal{P}(z_2, \bar{z}_3) A'_{4[ija_0\ldots a_3]} + \mathcal{P}(z_3, \bar{z}_3) A'_{5[ija_0\ldots a_3]} + \mathcal{P}(\bar{z}_2, z_3) A'_{6[ija_0\ldots a_3]}
\]

where

\[
A'_{1[ija_0\ldots a_3]} = \frac{2}{15} [(p_3 \cdot \varepsilon_2^S)_i (p_3)_a - (p_3 \cdot \varepsilon_2^S)_a (p_3)_i] (\varepsilon_3^A)_{a_0 a_1} (p_2)_a (p_2)_j
\]

\[
A'_{2[ija_0\ldots a_3]} = \frac{2}{15} [(p_3 \cdot D \cdot (\varepsilon_2^S)^T)_i (p_3)_a + (p_3 \cdot D \cdot (\varepsilon_2^S)^T)_a (p_3)_i] (\varepsilon_3^A)_{a_0 a_1} (p_2)_a (p_2)_j
\]

\[
A'_{3[ija_0\ldots a_3]} = \frac{2}{15} [(p_2 \cdot D \cdot \varepsilon_2^S)_i (p_2)_a + (p_2 \cdot D \cdot \varepsilon_2^S)_a (p_2)_i] (\varepsilon_3^A)_{a_0 a_1} (p_3)_a (p_3)_j
\]

\[
A'_{4[ija_0\ldots a_3]} = \frac{2}{15} [(p_3 \cdot D \cdot (\varepsilon_2^S)^T)_i (p_3)_a + (p_3 \cdot D \cdot (\varepsilon_2^S)^T)_a (p_3)_i] (\varepsilon_3^A)_{a_0 a_1} (p_3)_a (p_3)_j
\]

\[
A'_{5[ija_0\ldots a_3]} = \frac{2}{15} [(p_3 \cdot D \cdot (\varepsilon_2^S)^T)_i (p_3)_a - (p_3 \cdot D \cdot (\varepsilon_2^S)^T)_a (p_3)_i] (\varepsilon_3^A)_{a_0 a_1} (p_2)_a (p_2)_j
\]

It is easy to verify that the above amplitude is zero, when NSNS polarization tensors are \( \varepsilon_2^A \) and \( \varepsilon_3^S \).

The \( pp \) structures in \( \mathcal{A}_{16} \) are found in terms of \( p_2 \cdot D \cdot p_3, p_2 \cdot D \cdot p_2, p_3 \cdot D \cdot p_3 \) and \( p_2 \cdot p_3 \).

\[
\mathcal{A}_{16}(pp) \sim \frac{6!}{4!} \varepsilon_{1}^{ij} \varepsilon_{a}^{a_0\ldots a_3} \int d^2 z_1 d^2 z_2 d^2 z_3 \frac{z_{12}^2 K}{|z_{21}|^2 |z_{31}|^2 |z_{21}|^2 |z_{31}|^2} \tag{27}
\]

\[
\left( \mathcal{P}(z_2, z_3) A_{7[ija_0\ldots a_3]} + \mathcal{P}(z_2, \bar{z}_3) A_{8[ija_0\ldots a_3]} + \mathcal{P}(\bar{z}_2, z_3) A_{9[ija_0\ldots a_3]}
\right.
\]

\[
+ \mathcal{P}(z_2, \bar{z}_3) A_{10[ija_0\ldots a_3]} + \mathcal{P}(\bar{z}_2, \bar{z}_3) A_{11[ija_0\ldots a_3]}
\]

When both NSNS polarization tensors are symmetric or antisymmetric, some non-zero terms could be found. However, the integrand for those terms are pure imaginary which are zero after integration. When the above kinematic factors \( A_i, i = 7, \ldots, 12 \) contain \( \varepsilon_2^S \) and \( \varepsilon_3^A \), they are

\[
A_{7[ija_0\ldots a_3]} = -A_{12[ija_0\ldots a_3]} = \frac{2}{15} [(p_2 \cdot p_3)_i (p_2)_a (p_3)_j - (p_2)_j (p_3)_a] (\varepsilon_2^S)_{ia_1} (\varepsilon_3^A)_{a_0 a_3}
\]

\[
A_{8[ija_0\ldots a_3]} = -\frac{4}{15} (p_2 \cdot D \cdot p_2) (p_3)_a (p_2)_j (\varepsilon_2^S)_{ia_1} (\varepsilon_3^A)_{a_0 a_3}
\]

\[
A_{9[ija_0\ldots a_3]} = -A_{10[ija_0\ldots a_3]} = -\frac{2}{15} [(p_2 \cdot D \cdot p_3)_i (p_2)_a (p_3)_j + (p_2)_j (p_3)_a] (\varepsilon_2^S)_{ia_1} (\varepsilon_3^A)_{a_0 a_3}
\]

\[
A_{11[ija_0\ldots a_3]} = -\frac{4}{15} (p_3 \cdot D \cdot p_3) (p_2)_a (p_2)_j (\varepsilon_2^S)_{ia_1} (\varepsilon_3^A)_{a_0 a_3}
\]
The other structure that contribute to the amplitude $A_{16}$ is $\varepsilon_2\varepsilon_3$:

$$A_{16}(\varepsilon_2\varepsilon_3^A) \sim 8\varepsilon_1^{ij}\varepsilon^{a_0...a_3} \int d^2z_1 d^2z_2 d^2z_3 \frac{z_2^2 z_1^4 K}{|z_{21}|^2|z_{31}|^2|z_{21}|^2|z_{31}|^2} \left( \mathcal{P}(z_2, z_3) A'_7[ija_0...a_3] + \mathcal{P}(\bar{z}_2, z_3) A'_8[ija_0...a_3] + \mathcal{P}(z_3, \bar{z}_3) A'_9[ija_0...a_3] 
+ \mathcal{P}(z_2, \bar{z}_3) A'_{10}[ija_0...a_3] + \mathcal{P}(\bar{z}_2, \bar{z}_3) A'_{11}[ija_0...a_3] \right)$$

(28)

where

$$A'_7[ija_0...a_3] = -A'_11[ija_0...a_3] = \frac{4}{15} (p_2)_{a_2} (p_3)_{a_0} (p_2)_i (p_3)_j (\varepsilon_2^S \cdot \varepsilon_3^A)_{a_1a_3}$$

$$A'_8[ija_0...a_3] = -A'_10[ija_0...a_3] = -\frac{4}{15} (p_2)_{a_2} (p_3)_{a_0} (p_2)_i (p_3)_j (\varepsilon_2^S \cdot D \cdot \varepsilon_3^A)_{a_1a_3}$$

$$A'_9[ija_0...a_3] = 0$$

There are the contribution to the amplitude $A_{16}$ in which contain structures with any contraction. In fact, all momenta and both NSNS polarizations contract with RR polarization and volume $(p+1)$-form. In this structure the symmetric polarization tensor appears in the form of a trace.

$$A_{16}(\text{no} - \text{contraction}) \sim 8\varepsilon_1^{ij}\varepsilon^{a_0...a_3} \int d^2z_1 d^2z_2 d^2z_3 \frac{z_2^2 z_1^4 K}{|z_{21}|^2|z_{31}|^2|z_{21}|^2|z_{31}|^2} \mathcal{P}(z_2, \bar{z}_2) 
\times \left( \frac{4}{15} \text{Tr}(\varepsilon_2^S \cdot D)(p_2)_{a_3} (p_3)_{a_0} (p_2)_i (p_3)_j (\varepsilon_3^A)_{a_1a_2} \right)$$

(29)

These two latter amplitudes are also zero for antisymmetric polarization tensor $\varepsilon_2$ and symmetric polarization tensor $\varepsilon_3$.

We perform this calculation to find all structures that contribute to the nonzero subamplitudes $A_{12}$, $A_{13}$, $A_{14}$ and $A_{15}$. One can observe when both NSNS polarization tensors are symmetric or antisymmetric, the result is zero. The result for one graviton and one B-field is

$$A \sim (\varepsilon_1^{(p-1)})^{ij} \epsilon^{a_4...a_p} \epsilon_{a_0...a_p} p_2^{a_3} \left[ 2p_2^2 p_3^{a_0} \left( 2(\varepsilon_2^S)^{a_2j}(p_1 \cdot N \cdot \varepsilon_3^A)^{a_1} + (\varepsilon_2^A)^{a_1a_2}(p_1 \cdot N \cdot \varepsilon_3^S)^j \right) \mathcal{I}_1 \right]$$

$$-p_2^2 p_3^{a_0} \left( 2(\varepsilon_2^S)^{a_2j}(p_2 \cdot N \cdot \varepsilon_3^A)^{a_1} + (\varepsilon_2^A)^{a_1a_2}(p_2 \cdot N \cdot \varepsilon_3^S)^j \right) \mathcal{I}_2$$

$$-p_3^2 p_3^{a_0} \left( 2(\varepsilon_2^S)^{a_2j}(p_2 \cdot V \cdot \varepsilon_3^A)^{a_1} + (\varepsilon_2^A)^{a_1a_2}(p_2 \cdot V \cdot \varepsilon_3^S)^j \right) \mathcal{I}_2$$

$$+p_3^2 p_2 \cdot V \cdot p_3 \left( (\varepsilon_2^S)^{a_2j}(\varepsilon_3^A)^{a_0a_1} + (2 \leftrightarrow 3) \right) \mathcal{I}_2$$

$$+p_2^2 p_3^{a_0} \left( 2(\varepsilon_2^S)^{a_2j}(p_2 \cdot V \cdot \varepsilon_3^A)^{a_1} + (\varepsilon_2^A)^{a_1a_2}(p_2 \cdot V \cdot \varepsilon_3^S)^j \right) \mathcal{I}_3$$

(30)
where the function $\mathcal{I}_1$ is the one appears in (15) and $\mathcal{I}_2, \mathcal{I}_4$ are

\[
\mathcal{I}_2 = \int_{|z_2| \leq 1} d^2z_2 d^2z_3 \frac{1}{|z_2|^2 |z_3|^2} \left( \frac{1 - |z_2|^2 |z_3|^2}{|1 - z_2 \bar{z}_3|^2} - \frac{|z_2|^2 - |z_3|^2}{|z_2 - z_3|^2} \right) \mathcal{K}
\]

\[
\mathcal{I}_4 = - \int_{|z_1| \leq 1} d^2z_2 d^2z_3 \frac{1 + |z_3|^2}{|z_2|^2 |z_3|^2} \left( 1 - |z_3|^2 \right) \mathcal{K}
\]

(31)

The function $\mathcal{I}_3$ is the same as the function $\mathcal{I}_2$ in which the momentum labels 2 and 3 are exchanged. These functions satisfy the following relations:

\[
-2p_1 \cdot N \cdot p_3 \mathcal{I}_1 + 2p_3 \cdot V \cdot p_3 \mathcal{I}_4 + p_2 \cdot N \cdot p_3 \mathcal{I}_2 - p_2 \cdot V \cdot p_3 \mathcal{I}_3 = 0
\]

\[
-2p_1 \cdot N \cdot p_2 \mathcal{I}_1 + 2p_2 \cdot V \cdot p_2 \mathcal{I}_7 + p_2 \cdot N \cdot p_3 \mathcal{I}_3 - p_2 \cdot V \cdot p_3 \mathcal{I}_2 = 0
\]

(32)

where the function $\mathcal{I}_7$ is the same as the function $\mathcal{I}_4$, in which the momentum labels 2 and 3 are exchanged.

The amplitude (30) satisfies the Ward identities associated with the symmetric and anti-symmetric NSNS gauge transformations. It would satisfy the Ward identity associated with the RR gauge transformations when it combines with the amplitude (14), the amplitude of the RR $(p - 1)$-form with one transverse index (see appendix) and RR $(p - 1)$-form with any transverse index in which we are not interested in this work. This amplitude has been found in (13) as the sum of the second and the first components of the T-dual multiplets $\mathcal{A}_2$ and $\mathcal{A}_3'$ respectively (equation (34) in [13]). The amplitude has been written in terms of $H$, i.e.,

\[
\mathcal{A} \sim 2(\varepsilon_1^{(p-1)})_{ij} a_1 \cdots a_p \epsilon_{a_0 \cdots a_p} \left[ - H_3^{a_0 a_1 a_2} p_3^j \left( 2p_2^i (p_1 \cdot N \cdot \varepsilon_2^S)^{a_3} \right) \mathcal{I}_1 + p_2^i (p_3 \cdot U \cdot \varepsilon_2^S)^{a_3} \mathcal{I}_3 \right]
\]
\[ 
\begin{align*}
+ [4p_2^{a3}(p_2 \cdot V \cdot \varepsilon_2^{S}i) - p_2^{a3}p_2^i \text{Tr}(\varepsilon_2^{S} \cdot D) - 2p_2 \cdot V \cdot p_2(\varepsilon_2^{S}a_3)^i] \mathcal{I}_7 & \\
- 3p_2p_2^{a3}(\varepsilon_2^{S}a_2j)(2(p_1 \cdot N \cdot H_3)^{a0a1} \mathcal{I}_1 + (p_2 \cdot W \cdot H_3)^{a0a1} - 4(p_3 \cdot V \cdot H_3)^{a0a1} \mathcal{I}_4) & \\
+ 3p_3p_2^{a3}\left( - (\varepsilon_2^{S}a_2i)(p_2 \cdot U \cdot H_3)^{a0a1} + p_2^i(\varepsilon_2^{S} \cdot U \cdot H_3)^{a0a1a2} \right) & \\
+ H_3^{a0a1a2j}(\varepsilon_2^{S}a_3)(p_2 \cdot W \cdot p_3 - p_2^{a3}(p_3 \cdot W \cdot \varepsilon_2^{S}i)\right)] + (2 \leftrightarrow 3) (33) 
\end{align*} 
\]

where \( H_i \) is the field strength of the \( B_i \) polarization tensor, e.g.,

\[
H_i^{\mu \alpha} = i[(\varepsilon_i^A)^{\mu \nu}p_i^\nu + (\varepsilon_i^A)^{\alpha \mu}p_i^\mu + (\varepsilon_i^A)^{\nu \alpha}H_i^\mu] \tag{34}
\]

\( U \) and \( W \) have been defined in (13) as \( U \equiv V\mathcal{I}_2 - N\mathcal{I}_3, W \equiv V\mathcal{I}_3 - N\mathcal{I}_2 \). Note that we have used the identity (32) to write the above amplitude in terms of field strength \( H \). The amplitude does not satisfy the Ward identity corresponding to the graviton unless one rewrite the field strength \( H \) in terms of \( \varepsilon^A \). Therefore, one can not write the amplitude in terms of field strengths \( H \) and the curvature \( R \). However, some of them can be written as \( RH \). Because the metric appears in the effective action in the curvature tensor as well as in contracting the indices and in the definition of the covariant derivatives, one can not expect that all terms to be rewritten as \( RH \).

To study the low energy limit of the amplitude, one has to expand \( \mathcal{I}_2 \) and \( \mathcal{I}_4 \) at the low energy as well as \( \mathcal{I}_1 \) in (19). (Mapping the integrand in each of the integrals \( \mathcal{I}_2 \) and \( \mathcal{I}_4 \) to unit disk and fixing the \( SL(2,R) \) symmetry as in section 2.2.1). Taylor expansion of integrals \( \mathcal{I}_4 \) in \( r_2 \) and \( r_3 \) using (16) and then the integrals over the radial coordinates contain following infinite sums

\[
\sum_{n_1=0}^{\infty} \frac{1}{(s + n_1 + n_3 + n_5)} \frac{1}{t + n_2 + n_3 + n_5} \frac{1}{s + t + u + n_1 + n_2 + n_5 + n_6} + \\
\frac{1}{(s + 1 + n_1 + n_3 + n_5)} \frac{1}{t + n_2 + n_3 + n_5} \frac{1}{s + t + u + 1 + n_1 + n_2 + n_5 + n_6} \\
\left( \frac{-p + n_1}{n_1} \right) \left( \frac{-q - 1 + n_2}{n_2} \right) \left( \frac{-u - 1 + n_3}{n_3} \right) \\
\left( \frac{-u - 1 + n_4}{n_4} \right) \left( \frac{-v - 1 + n_5}{n_5} \right) \left( \frac{-v - 1 + n_6}{n_6} \right) \delta_{n_3-n_4+n_5-n_6,0} \tag{35}
\]

One can find such infinite sums for the integral \( \mathcal{I}_2 \). Using these sums, in the region of small momenta the following expansions can be found for these integrals:

\[
\mathcal{I}_2 = \frac{2\pi^2}{s(s+t)} - \frac{\pi^4q}{3s} + ... \\
\mathcal{I}_4 = \frac{\pi^4p}{3q} + \frac{\pi^4p}{6t} + \frac{\pi^4t}{3s} + \frac{\pi^4q}{6s} - \frac{2\pi^2}{qs} - \frac{\pi^2}{s(s+t)} - \frac{\pi^2}{t(s+t)} + .... \tag{36}
\]
the expansions hold up to terms quadratic in momenta. It is obvious from the above expansions and the expansion (19) that \( \mathcal{I}_1, \mathcal{I}_2 \) and \( \mathcal{I}_3 \) have only closed string poles while the integrals \( \mathcal{I}_4 \) and \( \mathcal{I}_7 \) have both open and closed poles. To find the open pole amplitude one has to consider the first two terms in the second line and the last term in the third line of (33). Hence, the amplitude (33) has the following massless open string pole at order \( O(\alpha'^2) \):

\[
\mathcal{A}^\text{pole}_{L-E} \sim -4\pi^4 \frac{p_3 \cdot V \cdot p_3}{p_2 \cdot V \cdot p_2} \epsilon_{a_0 \cdots a_3} p_2^{a_2} p_3^{a_3} \left[ -2(p_2 \cdot V \cdot \epsilon_2^A)^a_1 (p_3 \cdot N \cdot \epsilon_1 \cdot N \cdot \epsilon_3^S)^a_0 \right. \\
\left. + \left( p_2 \cdot N \cdot \epsilon_1 \cdot N \cdot p_3 Tr(\epsilon_2^S \cdot V) + 2 p_2 \cdot V \cdot \epsilon_2^S \cdot N \cdot \epsilon_1 \cdot N \cdot p_3 \right) (\epsilon_3^A)^{a_0 a_1} \right] + (2 \leftrightarrow 3). (37)
\]

We will compare this amplitude with the corresponding field theory amplitude in the next section. The part of the third term in the third line and the third term in the second line of (33) produce following contact term:

\[
4\pi^4 \epsilon_{a_0 \cdots a_3} p_3^{a_3} p_3 \left[ (p_3 \cdot N \cdot \epsilon_1 \cdot N \cdot \epsilon_3^S)^0 \epsilon_2^A)^a_1 a_2 + (p_3 \cdot N \cdot \epsilon_1 \cdot N \cdot \epsilon_2) (\epsilon_3^A)^{a_0 a_2} \right] + (2 \leftrightarrow 3).
\]

These contact terms contribute to the couplings (4) and (5).

### 3 Field theory amplitude

The scattering amplitude of one RR, one graviton and one B-field in the field theory is given by the following two Feynman amplitudes:

\[
\mathcal{A}^{A-pole} = V_a(\epsilon_2^A, A)G_{ab}(A)V_b(A, \epsilon_3^S, \epsilon_1) + (2 \leftrightarrow 3) \quad (38)
\]

where \( A \) is the gauge field on the \( D_p \)-brane and

\[
\mathcal{A}^{\Phi-pole} = V_m(\epsilon_2^S, \Phi)G_{mn}(\Phi)V_n(\Phi, \epsilon_3^A, \epsilon_1) + (2 \leftrightarrow 3) \quad (39)
\]

where \( \Phi \) is the scalar field on the \( D_p \)-brane. In these amplitudes and in all amplitudes in this paper, the polarization of the RR field is given by \( \epsilon_1 \) and the polarizations of the graviton and B-field are given by \( \epsilon^S, \epsilon^A \) respectively.

We assume that the RR potentials carry no world volume indices. Then the only non-zero vertex \( V_b(A, \epsilon^S, \epsilon_1) \) is given by the second term of (5) for \( p = 3 \) and the vertex \( V_n(\Phi, \epsilon^A, \epsilon_1) \) arises from the second term in (5):

\[
V_b(A, \epsilon_3^S, \epsilon_1) = \frac{4(\pi\alpha')^3 T_3}{3!} \epsilon_{a_0 a_1 a_2 b} \epsilon_1^i (\epsilon_3^S)^{a_0 i} (p_3 \cdot V \cdot p_3) p_3^{a_1} p_2^{a_2}
\]

\[
V_n(\Phi, \epsilon_3^A, \epsilon_1) = -\frac{(\pi\alpha')^2 T_3}{2} \epsilon_{a_0 \cdots a_3} (\epsilon_1)^{ij} p_2^{a_2} p_3^{a_3} p_3^i (p_3 \cdot V \cdot p_3) (\epsilon_3^A)^{a_0 a_1}
\]
Spacetime vectors project into transverse and parallel subspace to the $D_p$-brane by the matrices $N_{\mu\nu}$ and $V_{\mu\nu}$, respectively. $T_p$ is the tension of $D_p$-brane in the string frame

$$T_p = \frac{1}{g_s (2\pi \alpha')^{(p+1)/2}}.$$ 

The gauge field propagator and the vertex $V_a(\varepsilon^A, A)$ can be read from the DBI action (1), i.e.,

$$G_{ab}(A) = \left(\frac{-i}{T_3 (2\pi \alpha')^2}\right) \frac{\eta_{ab}}{p_2 \cdot V \cdot p_2},$$

$$V_a(\varepsilon^A_2, A) = (2\pi \alpha') T_3 (p_2 \cdot V \cdot \varepsilon^A_2)_a.$$ 

The scalar field propagator and the vertex $V_a(\Phi, \varepsilon^S)$ arises from the pull-back and the Taylor expansion of the linear graviton in the DBI action, i.e.,

$$G_{mn}(\Phi) = \frac{-i}{T_3 p_2 \cdot V \cdot p_2},$$

$$V_m(\varepsilon^S_2, \Phi) = -2T_3 \left(p_2^i Tr(\varepsilon^S_2 \cdot V) + (p_2 \cdot V \cdot \varepsilon^S_2)^i\right).$$ 

The pole amplitude then becomes

$$A^{pole} = A^{A-pole} + A^{\Phi-pole}$$

$$= -i \left(\frac{(\pi \alpha')^2 T_3}{3}\right) \frac{p_3 \cdot V \cdot p_3}{p_2 \cdot V \cdot p_2} \epsilon_{a_0 \cdots a_3} (\varepsilon^A_{1})_{ij} p_2^{a_2} p_3^{a_3} p_3^i$$

$$\times \left[(p_2 \cdot V \cdot \varepsilon^A_2)^{a_1} (\varepsilon^S_3)^{a_{0i}} + (3p_2^3 Tr(\varepsilon^S_2 \cdot V) + 3(p_2 \cdot V \cdot \varepsilon^S_2)^i) (\varepsilon^A_{a_0a_1}) + (2 \leftrightarrow 3)\right].$$

This amplitude has six momenta in the numerator and two momenta in the denominator ($O(\alpha'^2)$).

The first term in (1) and the first term in (4) create contact terms at the order of $O(\alpha'^2)$ that are simplified to

$$A^{contact} = -i \left(\frac{(\pi \alpha')^2 T_3}{2 \times 3!}\right) \epsilon_{a_0 \cdots a_3} (\varepsilon^A_{1})_{ij} (\varepsilon^A_3)^{a_2a_3} R_2^{a_1a_0ij} p_2 \cdot V \cdot p_2 + (2 \leftrightarrow 3).$$

The amplitude corresponding to the sum of $A^{pole}$ and $A^{contact}$ is the following:

$$A = -i \left(\frac{(\pi \alpha')^2 T_3}{6}\right) \frac{p_3 \cdot V \cdot p_3}{p_2 \cdot V \cdot p_2} \epsilon_{a_0 \cdots a_3} (\varepsilon^A_{1})_{ij} p_3^j$$

$$\times \left[p_3^{a_1} (\varepsilon^S_3)^{a_{0i}} (p_2 \cdot V \cdot H_2)^{a_2a_3} + 2p_2^{a_2} \left(3p_2^3 Tr(\varepsilon^S_2 \cdot V) + (p_2 \cdot V \cdot \varepsilon^S_2)^i\right) H_3^{a_0a_1a_2}\right] + (2 \leftrightarrow 3).$$
The above field theory open amplitude is exactly the string low energy result (41) provided that the normalization factor of the string amplitude (33) is fixed at \((3i\alpha'^2 T_p)/(4\pi^2)\).

Therefore, by using the explicit calculation of string theory disk amplitude and compare with corresponding field theory evaluation, we examine the S-matrix of one RR \((p-1)\)-form (with three and two transvers indices) and two NSNS states and also their corresponding couplings. We illustrate the consistency between T-duality and explicit calculation of string scattering. At first, we show that the S-matrix that produced by T-dual ward identity is reproduced by three point amplitude and then we show this consistency between corresponding couplings. We have performed the same steps for the case that the RR \((p-1)\)-form carry one transvers index. We present this result in the appendix.

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A The amplitude of one $RR\ (p-1)$-form with one transvers index and two $NSNS$

In this appendix we calculate three-point amplitude $\mathcal{A}$ for one RR potential $(C^{p-1})^i$ and two NSNS states. This scattering amplitude is zero for two graviton or two B-field vertex operators. Using the same steps as in sections 2.2.1 and 2.2.2, we find this result by explicitly calculating in $(-3/2, -1/2)$-picture. For this case, we have $n = 1$ and $p = 2$. The trace $\text{III}$ is non-zero only for $T(1,2,4)$. The $\psi$ correlators in $b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}$ and $b_{16}$ have non-zero contribution to the amplitude $\mathcal{A}$.

Using the explicit calculation, one can find that the polarization of $RR\ (p-1)$-form with one transvers index appears in amplitude in the form of following RR structures:

$$
(p \cdot N \cdot \varepsilon_1)^\mu, \quad (\varepsilon^S \cdot N \cdot \varepsilon_1)^\mu
$$

$$
(p \cdot V \cdot \varepsilon^S \cdot N \cdot \varepsilon_1)^\mu, \quad (p \cdot N \cdot \varepsilon^S \cdot N \cdot \varepsilon_1)^\mu
$$

$$
(p \cdot N \cdot \varepsilon^A \cdot N \cdot \varepsilon_1)^\mu, \quad (p \cdot V \cdot \varepsilon^A \cdot N \cdot \varepsilon_1)^\mu
$$

$$
(\varepsilon_1 \cdot N \cdot \varepsilon^S \cdot V \cdot \varepsilon^A)^\mu, \quad (\varepsilon_1 \cdot N \cdot \varepsilon^S \cdot N \cdot \varepsilon^A)^\mu
$$

$$
(\varepsilon_1 \cdot N \cdot \varepsilon^A \cdot V \cdot \varepsilon^S)^\mu, \quad (\varepsilon_1 \cdot N \cdot \varepsilon^A \cdot N \cdot \varepsilon^S)^\mu
$$

where $\mu$ and $\nu$ are the world volume indices that contract with the volume $(p+1)$-form $\epsilon$ and $\mu \neq \nu$. We find the amplitude for one graviton and one B-field in terms of above RR structures:

$$
\mathcal{A} \sim \epsilon_{a_0 a_1 a_2} \left[ (p_2 \cdot N \cdot \varepsilon_1)^{a_3} \mathcal{M}_1^{a_0 a_1 a_2} + (\varepsilon^S_3 \cdot N \cdot \varepsilon_1)^{a_2 a_3} \mathcal{M}_2^{a_0 a_1 a_2} + (p_2 \cdot V \cdot \varepsilon^S_3 \cdot N \cdot \varepsilon_1)^{a_3} \mathcal{M}_3^{a_0 a_1 a_2} 
\right. \\
+ (p_2 \cdot N \cdot \varepsilon^S_3 \cdot N \cdot \varepsilon_1)^{a_3} \mathcal{M}_4^{a_0 a_1 a_2} + (p_3 \cdot V \cdot \varepsilon^S_3 \cdot N \cdot \varepsilon_1)^{a_3} \mathcal{M}_5^{a_0 a_1 a_2} \\
+ (p_1 \cdot N \cdot \varepsilon^S_3 \cdot N \cdot \varepsilon_1)^{a_3} \mathcal{M}_6^{a_0 a_1 a_2} + (\varepsilon_1 \cdot N \cdot \varepsilon^S_3 \cdot V \cdot \varepsilon^A_2)^{a_2 a_3} \mathcal{M}_7^{a_0 a_1 a_2} \\
+ (\varepsilon_1 \cdot N \cdot \varepsilon^S_3 \cdot N \cdot \varepsilon^A_2)^{a_2 a_3} \mathcal{M}_8^{a_0 a_1 a_2} + (\varepsilon_1 \cdot N \cdot \varepsilon^A_2 \cdot V \cdot \varepsilon^S_3)^{a_2 a_3} \mathcal{M}_9^{a_0 a_1 a_2} \\
+ (\varepsilon_1 \cdot N \cdot \varepsilon^A_2 \cdot N \cdot \varepsilon^S_3)^{a_2 a_3} \mathcal{M}_{10}^{a_0 a_1 a_2} + (p_3 \cdot N \cdot \varepsilon^A_2 \cdot N \cdot \varepsilon_1)^{a_0} \mathcal{M}_{11}^{a_0 a_1 a_2} \\
\left. + (p_3 \cdot V \cdot \varepsilon^A_2 \cdot N \cdot \varepsilon_1)^{a_0} \mathcal{M}_{12}^{a_0 a_1 a_2} \right) + (2 \leftrightarrow 3) \quad (42)
$$

where

$$
\mathcal{M}_1^{a_0 a_1 a_2} = \frac{1}{4} \left[ (p_2 \cdot V \cdot \varepsilon^S_3 \cdot N \cdot p_1 (\varepsilon^A_2)^{a_1 a_2} - 2(p_3 \cdot V \cdot \varepsilon^A_2)^{a_2} (p_1 \cdot N \cdot \varepsilon^S_3)^{a_1} + (2 \leftrightarrow 3) p_2^{a_0} \mathcal{T}_3 
\right. \\
+ (p_2 \cdot N \cdot \varepsilon^S_3 \cdot N \cdot p_1 (\varepsilon^A_2)^{a_1 a_2} - 2(p_3 \cdot N \cdot \varepsilon^S_3)^{a_2} (p_1 \cdot N \cdot \varepsilon^A_2)^{a_1} + (2 \leftrightarrow 3) p_3^{a_0} \mathcal{T}_2 
\left. - 4(p_3 \cdot V \cdot \varepsilon^S_3 \cdot N \cdot p_1 \mathcal{T}_4 - 2p_1 \cdot N \cdot \varepsilon^S_3 \cdot N \cdot p_1 \mathcal{T}_1) p_2^{a_0} (\varepsilon^A_2)^{a_1 a_2} 
\right. \\
+ (8(p_3 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{T}_4 - 2(p_1 \cdot N \cdot \varepsilon^A_3)^{a_2} \mathcal{T}_1) p_2^{a_0} (p_1 \cdot N \cdot \varepsilon^S_3)^{a_1} 
\left. - \left( (p_1 \cdot N \cdot \varepsilon^S_3)^{a_0} (\varepsilon^A_2)^{a_1 a_2} + (2 \leftrightarrow 3) \right) (2p_3 \cdot V \cdot p_3 \mathcal{T}_4 + p_2 \cdot V \cdot p_3 \mathcal{T}_3 - p_2 \cdot N \cdot p_3 \mathcal{T}_2) \right]
$$

where
\[ \mathcal{M}^{a_0a_1}_2 = \frac{1}{4} \left[ 2(p_3 \cdot V \cdot \varepsilon^A_2)^{a_1} \left( p_1 \cdot N \cdot p_2 p_0^a \mathcal{I}_3 + (2 \leftrightarrow 3) \right) + 2(p_3 \cdot N \cdot \varepsilon^A_2)^{a_1} \left( p_1 \cdot N \cdot p_3 p_0^a \mathcal{I}_3 + (2 \leftrightarrow 3) \right) - 8(p_2 \cdot V \cdot \varepsilon^A_2)^{a_1} p_1 \cdot N \cdot p_3 p_0^a \mathcal{I}_7 + 4(p_1 \cdot N \cdot \varepsilon^A_2)^{a_1} p_1 \cdot N \cdot p_3 p_0^a \mathcal{I}_1 - (\varepsilon^A_2)^{a_0a_1} \left( 2p_1 \cdot N \cdot p_2 p_3 \cdot V \cdot p_3 \mathcal{I}_4 + p_1 \cdot N \cdot p_3 (p_2 \cdot V \cdot p_3 \mathcal{I}_2 - p_2 \cdot N \cdot p_3 \mathcal{I}_3) \right) - p_2^a \left( p_3^a \cdot V \cdot \varepsilon^A_2 \cdot V \cdot p_2 \mathcal{J}_1 - \frac{1}{2} p_3 \cdot V \cdot \varepsilon^A_2 \cdot N \cdot p_1 \mathcal{I}_3 + p_2 \cdot V \cdot \varepsilon^A_2 \cdot N \cdot p_3 \mathcal{J}_2 - p_3 \cdot V \cdot \varepsilon^A_2 \cdot N \cdot p_3 (\mathcal{J} + \mathcal{J}_5) + \frac{1}{2} p_3 \cdot N \cdot \varepsilon^A_2 \cdot N \cdot p_3 \mathcal{I}_2 \right) + \frac{1}{2} (p_3 \cdot V \cdot \varepsilon^A_2)^{a_1} (p_2 \cdot V \cdot p_2 \mathcal{J}_1 + p_3 \cdot V \cdot p_3 \mathcal{J}_4 - 2p_2 \cdot N \cdot p_3 \mathcal{J} - 2(p_2 \cdot V \cdot \varepsilon^A_2)^{a_1} p_3 \cdot V \cdot p_3 \mathcal{J}_3 + \frac{1}{2} (p_3 \cdot N \cdot \varepsilon^A_2)^{a_1} (p_2 \cdot N \cdot p_3 \mathcal{J}_{15} + p_2 \cdot V \cdot p_3 (\mathcal{J}_{16} - 2 \mathcal{J}) + (p_1 \cdot N \cdot \varepsilon^A_2)^{a_1} p_3 \cdot V \cdot p_3 \mathcal{I}_4) \right] \]

\[ \mathcal{M}^{a_0a_1a_2}_3 = -\frac{1}{4} \left[ (\varepsilon^A_2)^{a_1a_2} \left( p_1 \cdot N \cdot p_2 p_0^a \mathcal{I}_3 + (2 \leftrightarrow 3) \right) + p_2^a p_3^a \left( \frac{1}{2} (p_1 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_3 + (p_2 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_1 - (p_3 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_5 \right) + \frac{1}{4} p_1^a (\varepsilon^A_2)^{a_1a_2} \left( p_2 \cdot V \cdot p_2 \mathcal{J}_1 - p_3 \cdot V \cdot p_3 \mathcal{J}_4 + 2p_2 \cdot N \cdot p_3 \mathcal{J} \right) \right] \]

\[ \mathcal{M}^{a_0a_1a_2}_4 = -\frac{1}{4} \left[ (\varepsilon^A_2)^{a_1a_2} \left( p_1 \cdot N \cdot p_3 p_0^a \mathcal{I}_3 + (2 \leftrightarrow 3) \right) - p_2^a p_3^a \left( \frac{1}{2} (p_1 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_2 + p_3 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_5 - (p_2 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_2 \right) + \frac{1}{4} p_1^a (\varepsilon^A_2)^{a_1a_2} \left( p_2 \cdot N \cdot p_3 \mathcal{J}_{15} - p_2 \cdot V \cdot p_3 (\mathcal{J}_{16} - 2 \mathcal{J}) \right) \right] \]

\[ \mathcal{M}^{a_0a_1a_2}_5 = \frac{1}{4} \left[ 4(\varepsilon^A_2)^{a_1a_2} p_1 \cdot N \cdot p_2 p_0^a \mathcal{I}_4 + p_2^a p_3^a \left( 2(p_1 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_1 - 4(p_2 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_3 \right) - (p_3 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_{12} + (p_3 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{J}_4 \right] + p_1^a (\varepsilon^A_2)^{a_1a_2} p_2 \cdot V \cdot p_2 \mathcal{J}_3 \right] \]

\[ \mathcal{M}^{a_0a_1a_2}_6 = \frac{1}{2} \left[ (\varepsilon^A_2)^{a_1a_2} p_1 \cdot N \cdot p_2 p_0^a \mathcal{I}_1 + 2p_2^a p_3^a \left( (p_1 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_1 - \frac{1}{2} (p_3 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_2 + \frac{1}{2} (p_3 \cdot N \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_3 + 2(p_2 \cdot V \cdot \varepsilon^A_2)^{a_2} \mathcal{I}_7 \right) + p_1^a (\varepsilon^A_2)^{a_1a_2} p_2 \cdot V \cdot p_2 \mathcal{I}_7 \right] \]
\[ M_7^{a_0a_1} = M_8^{a_0a_1} = p_2^a p_3^a p_2 \cdot N \cdot p_3 J \]

\[ M_9^{a_0a_1} = M_{10}^{a_0a_1} = p_2^a p_3^a p_2 \cdot V \cdot p_3 J \]

\[ M_{11}^{a_0a_1a_2} = p_2^a p_3^a (p_2 \cdot N \cdot \varepsilon_3^S)^a_2 J \]

\[ M_{12}^{a_0a_1a_2} = p_2^a p_3^a (p_2 \cdot V \cdot \varepsilon_3^S)^a_2 J \]

where \( J, J_1, J_2, J_3, J_4, J_5, J_{12}, J_{15} \) and \( J_{16} \) are new integrals which appear in this case. The explicit form of these integrals has been found in [12, 14]. The operator \( G \), which conspicuous in \( M_{12}^{a_0a_1a_2} \), is defined as [14]:

\[
G(\varepsilon_n^A \cdot V \cdot \varepsilon_m^S)^{\mu \nu} \rightarrow (\varepsilon_n^A \cdot V \cdot \varepsilon_m^S)^{\mu \nu} - (\varepsilon_n^S \cdot N \cdot \varepsilon_m^A)^{\mu \nu}
\]

\[
G(\varepsilon_n^S \cdot V \cdot \varepsilon_m^A)^{\mu \nu} \rightarrow (\varepsilon_n^S \cdot V \cdot \varepsilon_m^A)^{\mu \nu} - (\varepsilon_n^A \cdot N \cdot \varepsilon_m^S)^{\mu \nu}
\]

\[
G(\varepsilon_n^A \cdot N \cdot \varepsilon_m^S)^{\mu \nu} \rightarrow (\varepsilon_n^A \cdot N \cdot \varepsilon_m^S)^{\mu \nu} - (\varepsilon_n^S \cdot V \cdot \varepsilon_m^A)^{\mu \nu}
\]

\[
G(\varepsilon_n^S \cdot N \cdot \varepsilon_m^A)^{\mu \nu} \rightarrow (\varepsilon_n^S \cdot N \cdot \varepsilon_m^A)^{\mu \nu} - (\varepsilon_n^A \cdot V \cdot \varepsilon_m^S)^{\mu \nu}
\]

\[
G(p \cdot \varepsilon_n^A \cdot V \cdot \varepsilon_m^S)^{\mu} \rightarrow (p \cdot \varepsilon_n^A \cdot V \cdot \varepsilon_m^S)^{\mu} - (p \cdot \varepsilon_n^S \cdot N \cdot \varepsilon_m^A)^{\mu}
\]  

where \( n \) and \( m \) are the particle labels of the polarization tensors.

Using the \( SL(2, R) \) symmetry fixing in [5], one finds the low energy expansion of the integrals \( J, J_1, J_2, J_3, J_4, J_5, J_{12}, J_{15} \) and \( J_{16} \), for the general setup. Considering the expansion of integrals \( I_1, I_2, I_3, I_4, I_7 \) in the sections 2.2.1 and 2.2.2, it is clear that only the integrals \( I_4, I_7, J_1, J_4, J_3 \) have massless open pole as well as closed pole. The other integrals have only closed pole. Using the open pole terms in the low energy expansion of these integrals, one can find the string open pole amplitude at low energy.

We find the exact consistency between this amplitude and corresponding field theory amplitude. The amplitude [12] satisfies the NSNS ward identity; however, it does not satisfy the RR ward identity. It can easily be extended to the RR invariant amplitude by including the amplitude of the RR \((p - 1)\)-form with three, two and no transvers indices, which the first two cases has been found in sections 2.2.1 and 2.2.2. In this work, we are not interested in the case that the RR potential carry no transvers index. We then check that this amplitude is exactly equal to the result of T-dual ward identity in [14].
References

[1] R. G. Leigh, Mod. Phys. Lett. A 4, 2767 (1989).
[2] C. Bachas, Phys. Lett. B 374, 37 (1996) arXiv:hep-th/9511043.
[3] J. Polchinski, Phys. Rev. Lett. 75, 4724 (1995) arXiv:hep-th/9510017.
[4] M. R. Douglas, arXiv:hep-th/9512077.
[5] M. R. Garousi and R. C. Myers, Nucl. Phys. B 542, 73 (1999) arXiv:hep-th/9809100.
[6] M. R. Garousi and R. C. Myers, Nucl. Phys. B 475, 193 (1996) arXiv:hep-th/9603194.
[7] B. Craps and F. Roose, Phys. Lett. B 445, 150 (1998) arXiv:hep-th/9808074.
[8] K. Becker, G. Guo and D. Robbins, JHEP 1009, 029 (2010) arXiv:1007.0441 [hep-th].
[9] K. Becker, G. Guo and D. Robbins, JHEP 12, 050 (2011) arXiv:1110.3831[hep-th].
[10] M. R. Garousi, JHEP 1003, 126 (2010) arXiv:1002.0903 [hep-th].
[11] M. R. Garousi and M. Mir, JHEP 1102, 008 (2011) arXiv:1012.2747 [hep-th].
[12] M. R. Garousi and M. Mir, JHEP 1102, 066 (2011) arXiv:1102.5510 [hep-th].
[13] K. B. Velni, M. R. Garousi, Nucl. Phys. B 869, 216 (2013)
[14] K. B. Velni, M. R. Garousi, Phys. Rev. D 89, 106002 (2014)
[15] M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys. B 526, 199 (1998) arXiv:hep-th/9802088.
[16] H. Liu and J. Michelson, Nucl. Phys. B 614, 330 (2001) arXiv:hep-th/0107172.
[17] M. R. Garousi , Phys. Rev. D 87, 025006 (2013) arXiv:1210.4379 [hep-th].
[18] O. Hohm, S. K. Kwak, B. Zwiebach, Phys. Rev. Lett 107, 171603 (2011) arXiv:1106.5452 [hep-th] ; O. Hohm, S. K. Kwak, B. Zwiebach, JHEP 1007, 013 (2011) arXiv:1107.0008 [hep-th] ; O. Hohm, A. Sen, B. Zwiebach, JHEP 1007, 079 (2015) arXiv:1411.5696 [hep-th]; D. Marques, C. A. Nunez, JHEP 1007, 084 (2015) arXiv:1507.00652 [hep-th].