Power transformer life analysis based on Lambert W function

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Abstract—As the core equipment in many construction power fields, the reasonable life management of power transformer plays an important role in ensuring the safe production and economic benefits of the public and enterprises. Based on lambert-w function, the exact solution is obtained by calculating the total annual allocated cost. The variable values that may have errors in the function model are deeply analyzed. Combined with the inherent characteristics of power transformer, the factors affecting the economic life in the results are analyzed. The reliability of the method is verified by an example. Compared with the traditional tabular method, this method is convenient to further analyze the influencing factors and change laws of economic life. At the same time, through the analysis of specific problems, this method can also be used to study the economic life of other equipment. It has popularization value.

1. Introduction
Transformers are widely used in almost all electronic products, especially power transformers in modern industrial enterprises. As one of the core equipment of the enterprise, the transformer needs to be fully guaranteed in terms of equipment installation, stable operation, safety and reliability. The research on the economic life of power transformer is a powerful guarantee for these factors. By studying the economic life and formulating an economic and reasonable maintenance and renewal plan, it will more effectively promote the utilization of enterprise core assets and bring greater economic benefits to the enterprise.

There is no specific mandatory standard for the service life of the transformer. At present, most studies at home and abroad focus on economic evaluation, and there is no unified definition of the economic life of transformer. At the same time, there is little research on the operation and management of power equipment in China, and there is a lack of sufficient information to form a scientific and reliable retirement decision-making criterion. In recent years, CiteSpace has little analysis and Research on the economic life and reference frequency of transformers. Literature[3] comprehensively considers the impact of the life cycle cost of active transformer, the economic performance of the selected reference transformer and the random fuzzy uncertainty of parameters on the economic life of active transformer, but there are few quantitative data and more qualitative components; The economic life defined in reference[4] refers to the year in which the net benefit value obtained by comprehensive economic indicators such as depreciation, energy conservation and loans is the largest; Literature[5] is a comparison of maintenance and replacement based on the principle of maximum annual average income; Reference[6] considers the investment, maintenance and fault loss of transformer in the whole life cycle; Reference[7] puts forward the economic life model of power transformer based on the lowest equivalent cost of transformer life year; The above documents do not
provide accurate solutions for economic life, nor do they combine sufficient influencing factors. Reference\cite{9} proposes an economic life solution for general equipment, but it is not completely applicable to any equipment. The power transformer needs to be analyzed in detail.

Combined with the advantages of the above literature, the exact solution of the economic life of power transformer is derived by using Lambert W function. By analyzing the factors affecting the economic life of transformer and the variation law of equipment economic life, it is of great significance to improve the economic life and economic benefits of transformer.

2. Mathematical Model of Economic Life of Power Transformer

The method for calculating the economic feasibility of energy transformer shall consider the procurement cost, maintenance cost and residual value of equipment. Considering these factors, there are other reasons why the machine cannot be reused before the end of the service life of the power transformer. This is the loss caused by excessive damage caused by frequent use of equipment in the later stage of the transformer. With the increase of operation period, the annual investment will decrease, but on the contrary, the cost of maintaining and repairing equipment will increase in the opposite direction, resulting in a sharp increase in the annual allocation of the lowest total cost.

Therefore, in order to reduce the cost as much as possible, according to the annual cost method, the economic feasibility of the equipment is the operation period with the lowest average cost per unit time, and the lowest value is determined.

This document describes the idea of establishing the model used in literature\cite{9}. Combined with the characteristics of transformer, the original model is improved, and the model is reconstructed to improve the accuracy of the model.

It is assumed that the annual allocation of procurement cost and residual value is $C_1$. $C_1$ is the purchase cost multiplied by the present value of the pension minus the residual value multiplied by the final value of the pension. Experience has shown that the residual value of transformer closure is usually 30% to 40% of the original value. Only 30% here.

$$C_1 = K(A / P, i, n) - 0.3K(A / F, i, n)$$

In formula (1), $K$ represents the purchase fee, the equivalent annual value of the fund, $P$ represents the capital limit, $f$ represents the final value of the fund, $I$ represents the annual interest rate, and $N$ represents the year.

If the annual allocation of control and maintenance costs is $C_2$, you can get:

$$C_2 = \left\{ \begin{array}{l} C(1)(P/F, i, 1) + C(2)(P/F, i, 2) \\ + \cdots + C(n)(P/F, i, n) \end{array} \right.$$

$$A / P, i, n) = \frac{i}{(1 + i)^n - 1} \sum_{j=1}^{n} C(j)(1 + i)^{n-j}$$

In formula (2), $j$ is one year. It can be assumed that the annual growth of control and maintenance costs is, and $C_2$ can be changed to

$$C_2 = \frac{i}{(1 + i)^n - 1} \sum_{j=1}^{n} \left[C(1) + (j-1)\Delta C \right](1+i)^{n-j}$$

Through simplification, the final term $C_2$ can be as follows:

$$C_2 = \Delta C \frac{i}{(1 + i)^n - 1} \frac{(1 + i)^n - ni - 1}{i^2} + C(1)$$
Assuming that the total annual cost is allocated as CT, we can get

\[ C_T(n) = C_1 + C_2 = K_i \left(1 + \frac{n}{i}\right)^{11} - 0.3 \left(1 + \frac{n}{i}\right)^{11} - 1 \]  

\[ +\Delta C \left(1 + \frac{n}{i}\right)^{11} \left(1 + \frac{n}{i}\right)^{11} - n + 1 \]  

\[ + C(1) \]  

Make

\[ \frac{dC_T(n)}{dn} = 0 \]  

\[ \Delta C - \frac{\Delta C \left(1 + \frac{n}{i}\right)^{11} \left(0.7K_i - n\Delta C\right) \ln \left(1 + \frac{n}{i}\right)}{(1 + \left(1 + \frac{n}{i}\right)^{11})^2} = 0 \]  

As can be seen from formula (7), if the denominator on the left side of the equation is equal to 0, we can get

\[ n \ln \left(1 + \frac{n}{i}\right) - 1 - \frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C} = \frac{1}{\left(1 + \frac{n}{i}\right)^{11}} \]  

Multiply both sides at the same time

\[ e^{n \ln \left(1 + \frac{n}{i}\right) - 1 - \frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C}} \]  

Obtain

\[ \left\{ n \ln \left(1 + \frac{n}{i}\right) - 1 - \frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C} \right\} e^{n \ln \left(1 + \frac{n}{i}\right) - 1 - \frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C}} \]  

\[ = -\frac{1}{\left(1 + \frac{n}{i}\right)^{11}} e^{-\frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C}} \]  

\[ = -e^{-\frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C}} \]  

You can see that formula (10) is in the form of Lambert W function and can be described as follows

\[ W(z) = n \ln \left(1 + \frac{n}{i}\right) - 1 - \frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C} \]  

\[ z = -e^{-\frac{0.7K_i \ln \left(1 + \frac{n}{i}\right)}{\Delta C}} \]  

The exact solution of the economic life is as follows:

\[ n^* = \frac{0.7K_i \ln \left(1 + \frac{n}{i}\right) + 1}{\Delta C + \ln \left(1 + \frac{n}{i}\right)} \]  

In formula (11), the cost and annual percentage I of purchasing K are greater than 0, and the Z range in the function Lambert W should be (-e-1, 0). According to the Lambert W function, it can be
seen from this property that in (z), there must be a solution of the actual number, and the exact solution of the Lambert W function can be obtained by Mathematica. Mathematics draws a curve, as shown below.

Fig 1. Lambert W function graph

As shown in Figure 1, the horizontal axis represents Z and the vertical axis represents Z. W (z) increases with the increase of E, and Z increases monotonically (- e-1,0).

3. Analysis of Factors Affecting Economic Life of Power Transformer

It is included in the cost allocation model of the original model, but does not consider the accurate economic solution. Therefore, its value does not affect the determination of economic life of power transformer. According to the value of Lambert W function of the actual solution, the function image increases monotonically. With its growth, Z will decrease and in W(z) will decrease accordingly. At the same time, it will also decrease, resulting in the reduction of economic life. Therefore, with the increase of annual control and maintenance costs, the economic life of energy transformer will be shortened. In the case of purchase fee K, the increase of purchase fee K will increase Z, and in W(z) will increase accordingly. At the same time,. Therefore, the economic life of transformer increases with the increase of order cost.

For the annual percentage I, literature[^9] mentioned that the higher the annual percentage I, the greater the annual value of procurement cost allocation. As the annual percentage I increases, the annual value of control and maintenance cost allocation also increases. Therefore, the impact of annual percentage I on the economic life of energy transformer can not be proved only by equating the economic life with the capital recovery formula. The influence of I value on the economic life of transformer is affected by K / H ratio. It is necessary to use professional mathematical software to evaluate variable control methods. The following figure shows that the economic life change trend of annual percentage energy transformer I is 0-0.3 when the remaining variables remain unchanged.

Fig 2. Economic life curve of power transformer with annual interest rate I in the range (0-0.3)
As shown in Figure 2, with the increase of annual percentage I, the economic life of energy transformer is still declining, which shows that the annual common living cost is an important part in solving its own economic problems. This also confirms that with the increase of business period, the annual allocation of control and maintenance costs continues to increase, while the annual allocation of contracts becomes less and less.

In the process of solving the procurement cost K and annual percentage I, it is easy to obtain, but in terms of value, it is usually difficult to obtain the control and maintenance cost that increases almost every year. Therefore, C2 needs further analysis.

The annual allocation of control and maintenance costs shall be:

\[
C_2(t) = \frac{r(t)C_{MAL}(t) - (1 + r(t))C_{MAL}(t)}{(1 + \delta)^t}
\]  

(14)

In equation (14), \(C_2(t)\) represents the control and maintenance cost in year t, \(r(t)\) represents the function of power transformer fault coefficient. Power transformers installed in different years have different fault coefficient functions, \(C_{MAL}(t)\) represents the loss and maintenance cost caused by power transformer fault this year. \(C_{MAL}(t)\) represents the temporary control fee and \(\delta\) represents the discount rate for the current year.

According to this formula, if the maintenance cost must be amortized and increased every year, the maintenance cost must be deducted to achieve annual growth, and the best solution can be found by analyzing the relationship.

Based on the analysis of the above influencing factors, we should start with these influencing factors in order to improve the economic life of energy transformer. The annual percentage rate cannot be controlled. For energy transformers that have been purchased and put into use, the purchase fee K cannot be changed, i.e. The annual allocation of control and maintenance costs should be reduced. Economical and reliable maintenance methods can be adopted to reduce maintenance costs. For example, the main internal components of a transformer are iron core, winding and transformer oil. If there is a problem, replacing these parts will greatly increase the maintenance cost. When repairing the iron core, first remove the oil stain on the iron core to ensure that dust and impurities will not enter the transformer oil tank during disassembly and assembly, and then check the shape and position of the iron core (mainly including color, purity, connection tightness, grounding, dye foil, etc.). If there is a problem, it shall be adjusted at any time. The initial steps of repairing moisture are the same as those of repairing iron core, flushing, controlling appearance position and aging pressure. If the pressure is very fine, the insulating packaging must be replaced. The number of transformers and oil in power transformers is relatively large. Considering the high oil cost, the most economical method is to absorb the oil and pollutants at the bottom of the transformer through the oil absorption device.

Although the control and maintenance costs of energy transformers are minimized, considering that the design durability of energy transformers defined in the national standard is 20 years, few energy transformers can operate beyond this limit, and the increase of control and maintenance costs is only taking a batch of thin insulated aluminum tube transformers produced around 1976 as an example. You has issued documents on early elimination, Therefore, innovation in infrastructure and technology continues to have a significant impact.

4. Case analysis

The analysis and calculation shall be taken as an example of substation power transformer. The voltage level of the energy transformer is 1000% of the voltage used in the 1980s. The annual percentage rate is 5%, the discount rate is 5%, the average error cost of the first transformer is 1000 yuan / time, and the conventional control cost is 1500 yuan. Try to analyze the economic life of power transformer (the error coefficient function is% / unit 1833; year).

Pictures showing losses and maintenance costs caused by transformer failure.
As shown in Figure 3, the horizontal axis represents the year and the vertical axis represents the loss and maintenance costs. Although the annual loss and maintenance cost increase with the increase of operation period, the curve in the figure shows an approximate linear increase. Well, the trend is about 5000 yuan.

Enter this value into the Lambert W function and analyze it using the product logarithm template.

$$\text{ProductLog} \left[ -e^{-1 \cdot \frac{0.7 \times 80000 \times \ln(1.05)}{5000}} \right] = -0.2825$$

Which is

$$W \left[ -e^{-1 \cdot \frac{0.7 \times 80000 \times \ln(1.05)}{5000}} \right] = -0.2825$$

Then, the economic life of the transformer is accurately solved as

$$n^* = \frac{0.7 \times 80000 \times 0.05}{5000} + \frac{-0.2825 + 1}{\ln(1 + 0.05)} = 15.26 \text{ years}$$

The economic life is much shorter than the design life. The best economic life does not mean that the longer the economic life, the better. In fact, as long as the maximum annual average net income is lower than its control and maintenance costs until it reaches positive income, the power transformer can continue to be used when it reaches its economic life. In the design process, it is difficult to realize the feasibility of power transformer. Different reasons may reduce the economic feasibility of the transformer during use, thus shortening the service life.

In this example, based on the comprehensive analysis of the operation state of the transformer, it is recommended to complete the optimal operation years of the transformer within 15-18 years.

5. Conclusion

In this paper, the Lambert W function is used to establish the analysis model to solve the economic life problem of energy transformer. Although the original model is based on the same conditions as the annual growth of control and maintenance costs, through a thorough analysis of the annual breakdown of control and maintenance costs, the average annual growth of control and maintenance costs is obtained. The design of the complete model is persuasive and accurate. The results obtained by lambert-w function can be applied to practice. This is a new method to calculate the economic life of energy transformer, not just the economic life of transformer. It has high reference value for planning other types of equipment and even substations in the future.
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