LETTER TO THE EDITOR

Thermally induced instability of a doubly quantized vortex in a Bose–Einstein condensate

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Abstract

We study the instability of a doubly quantized vortex topologically imprinted on ²³Na condensate, as reported in recent experiment (Shin et al 2004 Phys. Rev. Lett. 93 160406). We have performed numerical simulations using three-dimensional Gross–Pitaevskii equation with classical thermal noise. Splitting of a doubly quantized vortex turns out to be a process that is very sensitive to the presence of thermal atoms. We observe that even very small thermal fluctuations, corresponding to 10–15% of thermal atoms, cause the decay of doubly quantized vortex into two singly quantized vortices in tens of milliseconds. As in the experiment, the lifetime of doubly quantized vortex is a monotonic function of the interaction strength.

Experimental studies of vortices in Bose–Einstein condensates have revealed their peculiar properties related to the quantized circulation [1], originally predicted by Onsager and Feynman in the context of rotating superfluid ⁴He [2] and further explored in [3, 4]. The quantization of the circulation is a manifestation of the existence of a macroscopic wavefunction. Direct 2π-change of the phase of the condensate wavefunction when going around the vortex core was experimentally demonstrated by using the interferometric technique [5]. The quantized vortex cannot just disappear, it can leave the condensate or annihilate with a vortex having the opposite circulation. For vortices that have multiple topological charge, there exists another possibility—such vortices can also split into several vortices having smaller charges.

In recent experiments [6, 7], doubly quantized vortices have been imprinted in Bose–Einstein condensate of ²³Na atoms. A novel approach allowing to create vortices with multiple topological charge was implemented [8]. In this new method, as opposed to dynamical phase-imprinting techniques like rotating the atomic cloud in an anisotropic trap or stirring the condensate with the help of a laser beam, vortices are generated by adiabatically reversing
the magnetic bias field along the trap axis. This topological phase-imprinting technique
leads to vortices displaying winding numbers 2 or 4 depending on the hyperfine state the
sodium condensate was prepared in \((|1, -1\rangle \text{ and } |2, +2\rangle, \text{ respectively}) [6]. In [7], the authors
investigate the evolution of doubly quantized vortices using a tomographic imaging method
[9]. They observe the decay of doubly quantized vortex into singly quantized vortices and
suggest the possible explanation of this splitting as being a result of dynamical instability,
however, not specifying the character of the perturbation seeding the instability.

Existing theoretical explanation of decay of doubly quantized vortices involves the
analysis of stability of the vortex at zero temperature in terms of eigenmode spectrum of
the Bogoliubov equations in two- and three-dimensional geometry as well as the numerical
solution of the Gross–Pitaevskii equation in slightly anisotropic trap [10]. Two-dimensional
Bogoliubov eigenvalue spectrum shows complex frequencies for certain values of the
interaction strength \([10, 11]\). In fact, quasiperiodic behaviour is found—the stability
windows are followed by the instability regions. Neither the anisotropy of the trap nor
the phenomenologically introduced dissipation is able to force the decay of doubly quantized
vortex when there are no complex eigenvalues (i.e., the stability window conditions are
fulfilled) [10]. However, in the experiment the monotonic increase of the lifetime of the
vortex with the strength of the nonlinearity is observed [7]. It turns out that such behaviour
is attributed to the three-dimensional geometry of the system and its origin could be due to
the presence of trap anisotropy (as suggested in [10]) or the thermal noise (as claimed by this
letter).

In this letter we show that only thermal fluctuations, not other disturbances (as, for
example, due to the confinement anisotropy), lead to the decay times comparable to that
reported in the experiment [7]. To this end, we have performed numerical simulations using
the three-dimensional Gross–Pitaevskii equation in the version described in [12], i.e., with a
classical thermal noise. We find that even extremely small thermal fluctuations dramatically
accelerate the decay of doubly quantized vortex into two singly quantized vortices. Increasing
the number of thermal (uncondensed) atoms already to 10–15% reduces the lifetime of the
vortex below 100 ms. Therefore, we argue that although the authors of [7] report that the
experiment is performed under the condition of no discernible thermal atoms presence,
the decay of the vortex is triggered by thermal rather than quantum fluctuations.

We describe the system of degenerate bosonic atoms in terms of the classical field that
is the complex function representing all atoms not only those within the condensed fraction.
At zero temperature, all atoms are condensed and the classical field becomes the condensate
wavefunction which satisfies the Gross–Pitaevskii equation. However, as it was argued in
[12], the same equation is fulfilled by the classical field at finite temperatures, although in
this case it must be interpreted in a different way. The high energy solution of the Gross–
Pitaevskii equation describes both the condensed and thermal atoms. To get the correct
physical interpretation of the classical field, one has to perform averaging over time or space
of corresponding single-particle density matrix. The condensate wavefunction is just the
dominantly populated eigenmode of the averaged single-particle density matrix. Other modes
represent thermal atoms.

We start our simulations with the wavefunction of the cigar-shaped condensate with a
vortex imprinted along the symmetry axis. The topological charge of the vortex equals 2.
The trap parameters are the same as in the experiment of [7], i.e., the radial (axial) trap
frequency equals 220 Hz (3 Hz). The number of atoms is changed in such a way that
the values of the control parameter defined as \(a n_z\), where \(a\) is the scattering length and
\(n_z = \int |\psi(x, y, z = 0)|^2 dx \, dy\) is the linear atom density along the symmetry axis taken at
the centre of the trap, vary up to about 14 (see [7]). To find the condensate wavefunction with
Figure 1. Decay time of doubly quantized vortex as a function of the energy per particle. The successive curves correspond to different values of the control parameter defined as \( a n_z \), where \( a \) is the scattering length and \( n_z \) is an axial density at the centre of the trap. From top to bottom, \( a n_z \) equals 12.2, 10.0, 7.2, 5.9, 3.9 and 2.3 respectively.

In figure 1, we plot the time needed to split the doubly quantized vortex as a function of the energy pumped into the system while introducing the thermal noise. For each curve (i.e., the value of the interaction strength), the considered energies range from the zero-temperature energy to an amount approximately twice larger. We have checked that this corresponds at most to 20% condensate depletion. The basic observation is that the decay time is extremely sensitive to the presence of the noise. The rise of the energy by less than 1% (in comparison with the zero-temperature energy) already results in a dramatic decrease of the decay time. For example, it drops from 200 ms to 87 ms for \( a n_z = 5.9 \) (the case of zero-temperature energy per particle equal to 250\( \hbar \omega_z \)). Rigorously speaking, at zero temperature and without any disturbances that might initialize the decay (like the trap anisotropy or any deficiencies related to the process of phase imprinting), the lifetime of the vortex should be infinite. Indeed, curves in figure 1 show well visible asymptotes when approaching zero-temperature energies (although the top-lying end of each curve is a zero-temperature decay time that is finite due to the presence of numerical noise). Further increase of the amount of the thermal noise causes additional, although slower, decrease of the decay time. Figure 1 shows that the decay time becomes comparable with the experimentally measured values (tens of milliseconds) only when the thermal noise at the appropriate level is included in the dynamics.
Even at zero temperature the vortex decays (see [10]). The decay process sets in independently of the density as opposed to what is predicted based on two-dimensional spectral analysis of the Bogoliubov eigenmodes. According to this analysis, the stability windows with respect to the interaction strength exist, the first one appearing approximately for $3 < an_z < 12$. They are defined by the requirement that all eigenfrequencies of all excitation modes are real. Within the stability window no decay of the doubly quantized vortex is expected. Outside the window, i.e., when there is a complex eigenfrequency, the corresponding excitation mode grows exponentially in time resulting in the splitting of the vortex. Adding the third dimension changes the results dramatically as it is shown in figure 1. Splitting of the vortex in three-dimensional condensate need not be a simple and uniform process. In fact, it starts in the region where the local value of $an_z$ is in the two-dimensional instability window (see [10]). So, the vortex splitting begins at the ends of the condensate (assuming $an_z < 12$) and propagates towards the centre of the trap. In figure 1, we plot the lifetime of the doubly quantized vortex understood just as the time needed to split the vortex and disentangle it along the whole condensate.

In figure 2, we compare the decay time of a doubly quantized vortex for different levels of the introduced thermal noise. The pictures are obtained numerically according to the tomographic imaging technique as described in [7]. In this method, atoms within a 30 $\mu$m thick central slice of the condensate are transferred to a different hyperfine state and then imaged by using another resonant laser pulse. Following this prescription, each frame in figure 2 shows the contour plots of the density integrated axially within the slice of 30 $\mu$m thickness located at the centre of the trap. The first two rows demonstrate how important is the thermal noise. Introducing the thermal noise on a very low level (the energy of the
system is increased by less than 1%) already decreases the time needed to split the vortex by half. Rising the level of the thermal noise causes further lowering of the decay time which is necessary to get an agreement with the experiment. In the bottom panel, we plot the separation between two singly quantized vortices as a function of time. The case when the energy of the system is increased by approximately 50% in comparison with the zero-temperature energy is denoted by triangles. Although the population of the condensate is now about 94%, the distance between the two cores (∼3 µm) almost does not change over the period of 90 ms.

Finally, in figure 3 we present the data regarding the time of the decay of doubly quantized vortices which are intended to reproduce the main result of the experimental work [7]. The authors of [7] only say that the experiment was performed at the lowest possible temperature. Since the temperature of the system is determined based on the expansion of the thermal cloud, this statement puts on, in fact, the constraint on the number of thermal atoms. To our knowledge, the presence of less than 15% of thermal atoms cannot be detected by using currently available experimental techniques. Therefore, in figure 3 we plot three sets of data, each of them corresponding to different levels of thermal noise. More precisely, the relative increase of the energy of the system is constant for each data set. It is clear from figure 3 that the decay time gets larger when the interaction strength increases what remains in agreement with the experiment. For larger density, however, the decay time is getting saturated.

To answer the question whether other sources of instability are able to reproduce the experimental results of [7], we investigated the influence of the trap anisotropy (as high as 2.3% according to [14]) caused by the gravitational sag as well as imprinting deficiencies related to the rapid change of the magnetic field producing the vortex and resulting in radial squeeze and vertical kick of the condensate [7, 14]. We checked that the trap anisotropy makes the lifetime of a doubly quantized vortex finite but still bigger than the experimental values. For example, for \( an_z = 10.0 \) the decay time equals 170 ms whereas for \( an_z = 3.9 \) it is 76 ms. Moreover, since the gravitational field does not fluctuate, the trap anisotropy cannot explain the huge scatter of data in figure 3 of [7]. Shifting the whole condensate off the centre of the trap also does not help. It makes the total energy higher by increasing the potential energy not the kinetic one. Therefore, the number of uncondensed atoms remains constant and the lifetime of the vortex equals that for unshifted condensate. Finally, we considered the dynamical phase imprinting (for the details of this way of generating vortices see [15]) instead of the topological one. In this case, the energy pumped into the system goes to the kinetic energy and results in a production of uncondensed atoms. The decay time is comparable with

![Figure 3. Decay time of doubly quantized vortex as a function of the density. Three sets of points correspond to different values of the energy brought in the system while introducing the thermal noise. From top to bottom, the relative increase of the energy equals 40% (squares), 85% (triangles) and 135% (stars). The condensate depletion amounts to 6%, 13% and 20%, respectively.](image-url)
the experimental values revealing in this way the basic role of the uncondensed atoms in the process of decaying of doubly quantized vortices.

Another interesting phenomenon is discovered while tracing the dynamics of the splitting and subsequent evolution of two singly quantized vortices for longer times. It turns out that additional vortices enter the condensate. In figure 4, we show the density cuts at the plane $z = 0$ in the case when the number of uncondensed atoms is equal to 6% and the interaction strength $a n_z = 3.9$. We observe that two singly quantized vortices (as numerically checked by using the interference technique as described in [15]) enter the condensate at the same time and a lattice (in a rotating frame of reference) is formed like in the first scenario reported in [16]. Later on, however, one vortex is lost and the system settles into the lattice consisting of three vortices. What happens here resembles the early experiments on condensate stirred by blue-detuned laser beams that led to the generation of vortex arrays. The role of the laser beams is taken by the singly quantized vortices and their motion is efficient enough to produce the array consisted of three vortices.

In conclusion, we have addressed the issue concerning the lifetime of a doubly quantized vortex raised by a recent experiment [7]. We show that the decay of a doubly quantized vortex is driven rather by the thermal fluctuations than other kinds of perturbation (quantum fluctuations since we need 10–15% of thermal atoms to achieve the agreement with experiment [7] or the trap anisotropy). The uncondensed atoms are the only reason, we find, able to decrease the time needed to split the vortex and make it comparable to the experimental values. As a result, the decay time is a monotonic function of the interaction strength. Our stressing of the role of thermal noise does not contradict the statement of the authors of [7] saying that the experiment was performed at the lowest possible temperature.

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