Underwater Target Tracking Using Unscented Kalman Filter

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Article Info

ABSTRACT

Unlike conventional active sonar, that transmits the sound signals and revealing their presence and position to enemy forces. The probable advantage of passive sonar is that it detects the signals emitted by the target, leads to improve localization, target tracking, and categorization. The challenging aspect is to estimate the true bearing and frequency measurements from the noisy measurements of the target. Here in this paper, it is recommended for the Unscented Kalman Filter (UKF) to track the target by using these noisy measurements. The Target Motion Analysis (TMA), which is the way to find the target’s trajectory by using frequency and bearing measurements, is explored. This method provides a tactical advantage over the classical bearing only tracking target motion analysis. It makes the observer maneuver unnecessary.

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1. INTRODUCTION

This project concerns with an underwater Target Motion Analysis (TMA) in the passive mode. Here the target is assumed to be moving with a uniform velocity. The sound signals from the target are obtained at the ownship’s sensor. These sound signals are processed by the ownship in order to find out the Target Motion Parameters (TMP) which are bearing, range, speed and course of the target.

Passive target tracking (PTT) is a way to find the path of the target exclusively from measurements (signals) originating from the target. Anybody under the water will generate acoustic turbulence or signal from self generated noise or while in motion. These noise signals from the target are observed by the ownship’s solar sensor. The information that is acquired at the ownship sensor is the noisy bearings and frequency measurements of the target.

Thus, the radiated spectrum comprises of continues spectrum of noise with peaks (tonals) at certain frequencies which can be used for further classification. When the harmonic components are emitted by the source, these harmonic components will experience a Doppler shifts at the ownship. Due to these frequency measurements the accuracy of the estimated target motion parameters will increases. The use of both bearing angles and Doppler shifts to analyze a moving target is termed as Doppler Bearing Tracking (DBT). DBT has an advantage over BOT, i.e., DBT does not involve ownship’s maneuvering to obtain target motion parameters [1].

TMA for DBT, so far carried out by two groups- recursive instrumental variables method [1] and batch processing methods such as MLE [2]. The work carried out in [2] is not suitable for real time applications as solution is established by using search technique. So the batch processing like Maximum Likelihood Estimator cannot be implemented in the real scenario. In practice, it is required to estimate each sample on every arrival of recent measurement. The recursive instrumental variable method [1] is based on
pseudo linear formation. In their work, the strong bias generated by pseudo linear formation is minimised by using the estimated bearing in the place of measured bearing. The estimation accuracy of the instrumental variable technique can attain the Cramer-Rao lower bound for Gaussian noise at moderate noise levels. Recently “constrained least squares minimization” with sequential processing [3] is proposed. This is proven to be asymptotically unbiased. If the measurements are linear then the estimation of parameters can be done by Kalman filter. But in real world, no linear systems will exists. Thought the Extended Kalman filter is used for nonlinear application in contrast only achieves first order accuracy. So, EKF is not used for high nonlinearity systems [4].

So, here in this paper the Unscented Kalman Filter is used to track the target by estimating the TMP from the noisy bearing and frequency measurements and this algorithm is named as Doppler Bearing Unscented Kalman Filter (DBUKF). Thus, no bias will be produced.

Here, assuming that the measurements are available from the towed array of the ownship. The sonar hydrophones may be towed behind the ownship in order to reduce the effect of noise generated by the ownship itself.

In this the Doppler shift in the frequency measurement is described in terms of the speed of the propagation of sound in water and in ownship, target speed components. The zero mean Gaussian noise is assumed to be present in the measurements and the noise in the bearing measurement is not correlated with that of frequency measurement. It is also assumed that the measurements are continuously available at every second. Here the sum of the tonals is taken as a state variable in the state vector. The concept of constant state vector formulation [1], that the dimension of state vector does not increase with the number of frequency tonals is followed.

2. MATHEMATICAL MODELING
2.1. Motion of The Ownship (O)

The ownship motion is carried out as shown in the fig.1. Assuming that the ownship (observer) is moving with a velocity $v_0$, the distance of the ownship from x-coordinate is $x_0$, the distance of the ownship from y-coordinate is $y_0$ and OCR is the angle made by the ownship w.r.t true north.

$$x_0 = \sin(ocr) * v_0$$

$$y_0 = \cos(ocr) * v_0$$

After every second, the change in X and Y component of the ownship are added to the previous X and Y components of the ownship position.

For $t=1$ sec

$$dx_0 = v_0 * \sin(ocr) * t$$

$$dy_0 = v_0 * \cos(ocr) * t$$
Where, change in X component of the ownship in 1 sec is \( dX_0 \) and change in y direction of the ownship in 1 sec is \( dY_0 \). \( V_0 \) is the velocity of ownship. \( \theta_c \) is the ownship course, \((x_0, y_0)\) is the ownship position.

\[
\begin{align*}
X_0 &= X_0 + dX_0 \\
Y_0 &= Y_0 + dY_0
\end{align*}
\]

(5)

### 2.2. Initial Position of the Target

The target (T) is assumed to be moving with a constant velocity as follows

![Figure 2. Target’s Position in The Coordinate Axis](image)

where,

- \( O \) is the observer or ownship,
- \( T \) is the target,
- \( R \) is range,
- \( B \) is bearing i.e, angle between ownship (O) and target (T).

\[
\begin{align*}
X_t &= R \times \sin (B) \\
Y_t &= R \times \cos (B)
\end{align*}
\]

(6)

(7)

Where, \((X_0, Y_0)\) is the position of the target with respect to ownship as origin.

### 2.3. Modeling for States :

The set of measured data consists of frequency and bearing measurements. The measurement vector \( z(i) \) is given by

\[
z(i) = \begin{bmatrix} B_m(i) \\ F_m(i) \end{bmatrix}
\]

(8)

The measured bearing is given as

\[
B_m(i) = B(i) + \gamma_B(i)
\]

(9)

Where, \( B_m(i) \) is the measured bearing, relative to the Y-axis at \( i^{th} \) instant \((i = 1, 2,..., n)\).

\( B(i) \) is the actual bearing and \( \gamma_B(i) \) is a Gaussian Random Variable (GRV) with zero mean and variance \( \sigma^2_B(i) \). \( B(i) \) is given by

\[
\tan B(i) = \frac{R_y(i)}{R_x(i)}
\]

(10)
In sonar, a broadband noise is generally accompanied by one or more tonals. These tonals are constant in frequency and because of Doppler shift; a particular frequency measured by the ownship is given by

\[ f_{m}^{(i)}(t) = f_{i}^{(i)}(t) \left[ 1 + \frac{R_{i}(i) \sin \theta(i) + R_{i}(i) \cos \theta(i)}{C} \right] + \gamma^{(i)}(t) \]

(11)

Where, the \( j^{th} \) \( (j = 1, 2, 3, \ldots, n) \) frequency measured by the ownship at \( i^{th} \) instant is \( f_{m}^{(i)}(i) \), the \( j^{th} \) unknown constant source frequency is \( f_{i}^{(j)} \), the speed of propagation of the signal is \( C \), \( \gamma^{(j)}(i) \) is the zero mean Gaussian random frequency measurement error with variance \( \sigma_{j}^{2} \). \( \bar{R}_{x} \) and \( \bar{R}_{y} \) are components of relative velocity between the target and the ownship.

### 2.4. Algorithm for Unscented Kalman Filter

Unscented transformation is a way to approximate how the mean and covariance of a random variable change when the random variable undergoes a nonlinear transformation. Consider a random variable \( x \) (dimension \( L_{x} \)) propagating through a nonlinear function, \( y = \sigma(x) \). Assume \( x \) has mean \( \bar{x} \) and covariance \( P \). To calculate the statistics of \( y \), a matrix \( X \) of \( 2L_{x} + 1 \) sigma vectors \( x_{j} \) (with corresponding weights \( w_{j} \)), is formed according to the following equation [4]:

\[
\begin{align*}
X_{0} &= \bar{x} \\
X_{j} &= \bar{x} + \left( \sqrt{L_{x} + \lambda} \right) \xi_{j}, \quad j = 1, \ldots, L_{x} \\
X_{j}^{(m)} &= \bar{x} - \left( \sqrt{L_{x} + \lambda} \right) \xi_{j}, \quad j = L_{x} + 1, \ldots, 2L_{x} \\
W_{0}^{(m)} &= \lambda / \left( L_{x} + \lambda \right) \\
W_{j}^{(c)} &= \lambda / \left( L_{x} + \lambda \right) + \left( 1 - \sigma^{2} + \xi \right) \\
W_{j}^{(m)} &= W_{j}^{(c)} = 1 / 2 \left( L_{x} + \lambda \right) \quad j = 1, \ldots, 2L_{x}
\end{align*}
\]

(19)

where \( \lambda = \sigma^{2} \left( L_{x} + \kappa \right) - L_{x} \) is a scaling parameter. \( \sigma \) determines the spread of the sigma points around \( \bar{x} \) and is usually set to a small positive value, \( \kappa \) is a secondary scaling parameter which is usually set to 0 and \( \xi = 2 \) is optimal. \( \sqrt{L_{x} + \lambda} \) is the \( j^{th} \) row of the matrix square root. \( W_{0}^{(m)}, W_{j}^{(c)}, W_{j}^{(m)} \) are the weights of initialized target state vector, covariance matrix of initialized target state vector, target state sigma point vector and covariance matrix of target state sigma point vector respectively. These sigma vectors are propagated through the nonlinear function

\[ y_{j} = \sigma(x_{j}) \quad j = 1, \ldots, 2L_{x} \]

(20)

The mean and covariance are approximated using a weighted sample mean and covariance of the posterior sigma points [4].

UKF is a straightforward extension of the unscented transformation to the recursive estimation. In UKF, the state random variable is redefined as the concatenation of the original state and noise variables. The unscented transformation sigma point selection scheme is applied to this new augmented state random variable to calculate the corresponding sigma matrix.

The standard UKF implementation consists of the following steps:

a. By using sigma points starting from the initial conditions \( (2n + 1) \) state vectors are calculated, where \( n \) is dimension of target state vector

\[
X(i) = \begin{bmatrix}
X_{0}(i) & X_{0}(i) + \sqrt{(n + \lambda)P(i)} & X_{0}(i) - \sqrt{(n + \lambda)P(i)}
\end{bmatrix}
\]

(21)
b. Conversion of these sigma points through the process model using eqn. (16).

c. The prediction of the state estimate at time \( i + 1 \) with measurements up to time \( i \) is given as

\[
X_i(i+1) = \sum_{j=0}^{2n} W_j^{(m)} X_i(j, i+1)
\]  

(22)

d. As the process noise is additive and independent, the predicted covariance is given as

\[
P(i+1) = \sum_{j=0}^{2n} W_j^{(m)} [X_i(j, i+1) - X_i(i+1)] X_i(j, i+1)^T
\]

\[
+ [X_i(i, i+1) - X_i(i+1)]^T + Q(i)
\]

(23)

e. Updating of the sigma points with the predicted mean and covariance. The updated sigma points are given as

\[
\hat{X}(i+1) = \sum_{j=0}^{2n} W_j^{(m)} Y(i+1, i)
\]

(25)

where

\[
Y(i+1, i) = h(X_i(i+1, i))
\]

(26)

f. Transformation of each of the predicted points through measurement model eqn. (22)

g. Prediction of measurement given as

\[
\hat{z}(i+1, i) = \sum_{j=0}^{2n} W_j^{(m)} Y(i+1, i)
\]

(28)

i. The cross covariance is given as

\[
P_x = \sum_{j=0}^{2n} W_j^{(m)} [X_i(j, i+1) - \hat{X}(i+1)] [Y(j, i+1) - \hat{z}(i+1)]^T + \sigma_e^2
\]

(27)

j. Kalman gain is calculated as

\[
G(i+1) = P_{x} P_{yy}^{-1}
\]

(29)

k. The estimated state is given as

\[
\hat{X}(i+1, i) = X(i+1, i) + G(i+1) (\hat{z}(i+1, i) - \hat{z}(i+1, i))
\]

(30)

where \( \hat{z}(i+1) \) is measurement vector.

l. Estimated error covariance is given as

\[
P(i+1, i+1) = P(i+1, i) - G(i+1) P_{xy} G(i+1)
\]

(31)
3. SIMULATION AND RESULTS

Using number of geometries the performance of this algorithm is evaluated. The period of simulation is 1800 s and the measurement interval has been considered as 1 s. Additive zero mean Gaussian noise corrupts all the raw frequency and bearing measurements with a maximum level of 0.9 Hz and 1° respectively. Here all angles are calculated with respect to Y-axis, 0-360° and clockwise positive.

Here in this paper, the result of one scenario is implemented by using the Initial Range as 2800m, Initial Bearing as 3deg, Target Speed as 5m/s, Target Course as 200 deg, Sum of Source Frequencies as 500 Hz, Ownship Speed as 5m/s and Ownship Course as 225 deg.

The results of one scenario are shown in figures. The position of the target and ownship is shown in the Figure 1. The estimated range error, estimated speed error and the estimated course error are shown in Figure 2a, Figure 2b and Figure 2c respectively. In underwater applications the acceptable errors in estimated speed, course and range are less than or equal to 20%, 5° and 10% respectively. As per the required accuracies the entire solution of the estimated parameters speed, course and range are obtained at 79th, 64th and 183rd s for the scenario values given above.

4. CONCLUSION

Now a days there are many methods for obtaining the target trajectory by using passive sonar which is installed in the ownship without manoeuvring. It is very difficult to carry out the process until the required accuracy is achieved in the estimated target motion parameters. So, one can use DBT for obtaining the target motion parameters without the need of manoeuvring the ownship. This method is very easy to implement and adopt in underwater application for passive target tracking.

In this paper unscented Kalman filter is proposed to estimate target motion parameters without the need for manoeuvring the ownship for passive target tracking.
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