Improved Naturalness and the Two Higgs Doublet Model

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Abstract

The natural cutoff scale for the quadratically divergent top quark contribution to the Higgs mass parameter can be significantly raised above the surprisingly low standard model value, with important consequences for the LHC: the physics that cancels the top quark divergence may be out of reach, while an electroweak sector with “improved naturalness” may be discovered. Such a sector, consistent with electroweak precision tests, arises in the two Higgs doublet model with heavy Higgs and top quark interactions that approach strong coupling.
1 Improved Naturalness

The Standard Model (SM) is inadequate as a fundamental theory because quadratic divergences in the Higgs mass parameter lead to a high sensitivity of electroweak physics to large energy scales. The dominant quadratic divergence arises from virtual top quarks, requiring new physics to appear at or below the scale

\[ \Lambda_t \lesssim 400 \text{ GeV} \left( \frac{m_h}{115 \text{ GeV}} \right) D_t^{1/2}, \]  

(1)

where \( D_t \) is the sensitivity of the Higgs mass to \( \Lambda_t \); or equivalently, the amount of fine tuning necessary to make \( \Lambda_t \) large is 1 in \( D_t \). Since electroweak precision tests (EWPT) indicate that the Higgs boson is light, \( m_h < m_{EW} = 285 \text{ GeV} \) at 95% CL \( [1] \) with a central value close to the lower bound of 115 GeV from direct searches, the new physics at \( \Lambda_t \) should be accessible to the LHC. What is this new physics? The most ambitious attempt at a complete theory incorporates supersymmetry at the weak scale to understand the hierarchy problem, with the top squark cancelling the quadratic divergence of the top quark. While the simplest model requires some fine tuning, supersymmetry remains, perhaps, the leading candidate. A less ambitious approach is to delay the need for fine tuning up to 5—10 TeV, yielding a little hierarchy that at least explains why higher dimension operators from this cutoff scale do not upset the success of a light SM Higgs with EWPT. Such theories have their own answer for the new physics that cancels the top quark quadratic divergence; for example, in Little Higgs models the LHC will discover new vector-like quarks. The simplest theories, however, require some amount of fine tuning to agree with EWPT\(^1\).

In this letter we pursue an alternative idea, of limited scope, that nevertheless has crucial implications for the LHC. Is it possible to construct theories where \( \Lambda_t \) is modestly increased above \( \Lambda_t \), for example by a factor of 3—5, so that the new physics associated with this scale is inaccessible to LHC? If so, what is the effective field theory below \( \Lambda_t \)—certainly not the SM—how is it consistent with EWPT, and what signals does it give at the LHC? While the “LEP paradox” \( [2] \) may not be solved, it is nevertheless ameliorated, and the implications for the LHC are immediate. Instead of focussing on the new physics that cancels the top quark divergence (squarks, vector quarks, ...) one must study the physics of the modified electroweak theory below \( \Lambda_t \). In a previous paper it was shown that such a theory can result from mixing the Higgs with a Higgs of a mirror world \( [3, 4] \). In this paper we demonstrate that Higgs mixing in the two Higgs doublet model provides a conceptually simpler example.

2 The Two Higgs Doublet Model

The scalar potential for the most general two Higgs doublet model that satisfies natural flavor conservation \( [6] \) is \( [7] \)

\[ V = -\mu_1^2 H_1^\dagger H_1 - \mu_2^2 H_2^\dagger H_2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \]
\[ + \lambda_4 H_1^\dagger H_2 H_2^\dagger H_1 + \lambda_5 [(H_1^\dagger H_2)^2 + (H_2^\dagger H_1)^2]. \]

(2)

(3)

\(^1\)For a recent analysis see Ref. \( [2] \) and references therein.
A discrete symmetry acts on $H_2$ so that it alone couples to the up quarks—in particular to the top quark—while the down quark masses can arise from either an interaction with $H_1$ or $H_2$. In a phase where both Higgs doublets acquire vevs, $H_i = (0, v_i + h_i)$ with $v_{1,2} \neq 0$ and real, the mass matrix for the two neutral Higgs bosons is

$$V_2 = (h_1, h_2) \begin{pmatrix} 4\lambda_1 v_1^2 & 2\delta v_1 v_2 \\ 2\delta v_1 v_2 & 4\lambda_2 v_2^2 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix},$$  \hspace{1cm} (4)$$

where $\delta = \lambda_3 + \lambda_4 + 2\lambda_5$. Motivated by the possibility of raising $\Lambda_t$ in a way consistent with EWPT, we assume that the 22 entry is the largest. Thus the heaviest Higgs boson is $h_+ = \cos \alpha h_2 + \sin \alpha h_1$, with mass

$$m_+^2 \approx 4\lambda_2 v_2^2,$$  \hspace{1cm} (5)$$

and the Higgs mixing angle is small

$$\alpha \approx \frac{\delta}{2\lambda_2} \frac{v_1}{v_2}.$$  \hspace{1cm} (6)$$

This contrasts with the minimal supersymmetric standard model, where it is the lightest Higgs boson that couples dominantly to the top quark. Of the seven parameters of the potential, five ($\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \delta$) can be specified as $v = \sqrt{v_1^2 + v_2^2}$, $\tan \beta = v_2/v_1$, $m_+^2$ and $\alpha$. The remaining two parameters $\lambda_4, 5$ can be traded for the charged scalar mass $m_{H^{-}}$ and the pseudoscalar mass $m_A$.

### 3 The Scale $\Lambda_t$ and Electroweak Precision Tests

The quadratic divergence induced at 1 loop by virtual top quarks appears only in the parameter $\mu_2^2$ and is cutoff at some scale $\Lambda_t$

$$\mu_2^2 = \mu_0^2 + a_t \Lambda_t^2,$$  \hspace{1cm} (7)$$

where $\mu_0^2$ is the bare parameter, $a_t = 3\lambda_t^2/8\pi^2$ and $\lambda_t = m_t/(v \sin \beta)$. The sensitivity of the Higgs masses, $m_{\pm}^2$, to the scale $\Lambda_t$ is given by $D_{\pm}^l \equiv \partial \ln m_{\pm}^2 / \partial \ln \Lambda_t^2$, which can be inverted to give

$$\Lambda_t = \left( \frac{2\pi v m_{\pm}}{\sqrt{3} m_t} \right) \frac{\sin \beta \sqrt{D_{-}^l}}{\cos \beta \sqrt{D_{+}^l} \sqrt{2\lambda_1/\delta}}.$$  \hspace{1cm} (8)$$

The usual SM result \cite{ref1} is given by the first parenthesis with the replacement $m_+ \to m_h$. In the SM $m_h$ is limited by EWPT, so that a crucial question becomes how EWPT limit $m_+$. We assume that neither $\sin \beta$ nor $\cos \beta \sqrt{2\lambda_1/\delta}$ are small, so that a substantial increase in $m_+ > m_h$ guarantees an increase in $\Lambda_t$ above the SM value.

The quantity $\cos \beta \sqrt{2\lambda_1/\delta}$ can be rewritten as $m_{-} / \sqrt{2\lambda_3 v^2 - m_{H^{-}}^2}$. If $m_{H^{-}}$ is relatively light, for example 200 GeV, this factor can be naturally of order unity. Such a light charged Higgs contributes radiative corrections to the $Z\bar{b}b$ vertex $g_L$, contributing an amount to $R_b$ that, for $\tan \beta = 1$, corresponds to about 1 standard deviation in the measured value. While formally this can be used to place limits on $m_{H^{-}}$ \cite{ref9}, such limits may prove untrustworthy as $A_b$ lies nearly three standard deviations from the SM prediction \cite{ref10}. For example, if some new physics provides a contribution to $g_R$ to account for $A_b$, then a contribution to $g_L$ from a light charged Higgs would...
be welcome. Nevertheless, this radiative correction to $g_L$ is proportional to $\cot^2 \beta$, so that $v_2$ should not be much less than $v_1$. For $\tan \beta$ close to 1, there is a limit of about 200—250 GeV on $m_{H^-}$ from $b \to s \gamma$ \[10, 11\]. Even if the charged Higgs boson is much heavier than this, a large $\Lambda_t$ is still possible provided there is a cancellation between $m_{H^-}^2$ and $2\lambda_3 v^2$.

Do EWPT allow large values for $m_+$? In the two Higgs doublet model with $m_{H^-} = m_A$, the contributions of the scalars to the $S$ and $T$ parameters take the form

$$S = \cos^2 \beta \, S_{SM}(m_-) + \sin^2 \beta \, S_{SM}(m_+) + \Delta S$$

$$T = \cos^2 \beta \, T_{SM}(m_-) + \sin^2 \beta \, T_{SM}(m_+),$$

where $S_{SM}(m_h)$ and $T_{SM}(m_h)$ are the contributions in the SM, and we have approximated $\beta - \alpha$ by $\beta$ because $\alpha$ is small. Consider first the approximation that $S_{SM}(m_h)$ and $T_{SM}(m_h)$ both have the form $A + B \ln m_h$ and that $\Delta S$ can be neglected. In this case the scalar contributions to $S$ and $T$ in the two Higgs doublet model are obtained from those in the SM by the replacement

$$m_- \cos^2 \beta \, m_+ \sin^2 \beta < m_{EW} \quad \text{or} \quad m_+ < m_- \left( \frac{m_{EW}}{m_-} \right)^{\frac{1}{\sin^2 \beta}}.$$  \(11\)

This shows that for $m_- < m_{EW}$ the bound on $m_+$ gets exponentially relaxed as $\sin \beta$ is reduced. Thus we are led to consider low values for $m_-$ and $v_2 \ll v_1$. The factor of $\sin \beta$ in the upper line of \[8\] is sub-dominant to the exponential behaviour of $m_+$. The point is very simple: as $v_2$ is reduced so the state $h_+$, which is mainly $h_2$, decouples from electroweak breaking phenomena. Furthermore, because the bound on $m_+$ has an exponential dependence on $\sin \beta$, $v_2$ need not be much less than $v_1$. Indeed, $v_2$ cannot be reduced too much as we have assumed that $4\lambda_2 v_2^2$ is the largest term in the Higgs boson mass matrix, so that reducing $v_2$ leads to strong coupling in $\lambda_2$. The perturbativity limit on $m_+$ is $4\pi v \sin \beta$. For $m_-$ close to the direct search limit of 115 GeV, a value of $\sin \beta = 0.6—0.7$ is sufficient to raise $m_+$ to near a TeV. The cutoff scale $\Lambda_t$ is raised above the SM value by a factor of 5 to 2 TeV.

In the above analysis we have ignored $\Delta S$, which is not justified since it involves a large logarithm as $m_+$ is made large. We find that at 1 loop order

$$\Delta S = \frac{\cos^2 \beta}{6\pi} \left( \ln \frac{m_+}{m_A} \right)$$

for $m_+ \geq m_{H^-} = m_A \geq m_-$, up to a small non-logarithmic term. This positive contribution to $S$ gives a final result that is well within the 90% C.L. region of fits to electroweak data. The reason why this data excludes the SM for large $m_h$ while allowing the two Higgs doublet model with large $m_+$ is that the large logarithm making a negative contribution to $T$ in the SM has a coefficient reduced by $\sin^2 \beta$ in the two Higgs doublet theory.

\footnote{The explicit expression for the radiative corrections in a general two Higgs doublet model can be found in Ref. \[12\].}
4 Summary

The leading question of electroweak symmetry breaking is often taken to be: what is the physics that cancels the quadratic divergence of the top quark, and what signals does it have at the LHC? We have pointed out that this might not be the right question as far as LHC physics is concerned. A theory with improved naturalness compared to the SM may place the physics of top-cancellation beyond the reach of the LHC, while leaving a non-SM Higgs sector to explore. We have shown that the two Higgs doublet model is able to accomplish this, with a light Higgs \( m_- \approx 115—150 \) GeV, a heavy Higgs \( m_+ \approx 500 \) GeV—1,000 GeV, relatively small Higgs mixing \( \alpha \lesssim 1/3 \) and \( \tan \beta \approx 0.8—1 \). These ranges are only meant as a rough guide, but the underlying physics is that one should think of the theory as having two sectors: the \((t, h_2)\) sector which is approaching strong coupling, and the perturbative \((W, h_1)\) sector with slightly more of the \(W\) mass coming from \(v_1\) than \(v_2\).

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References

[1] The ALEPH, DELPHI, L3, OPAL and SLD Collaborations and the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, \texttt{arXiv:hep-ex/0509008}.

[2] G. Marandella, C. Schappacher and A. Strumia, Phys. Rev. D \textbf{72}, 035014 (2005) \texttt{arXiv:hep-ph/0502096}.

[3] R. Barbieri and A. Strumia, \texttt{arXiv:hep-ph/0007265}.

[4] R. Barbieri, T. Gregoire and L.J. Hall, \texttt{arXiv:hep-ph/0509242}.

[5] Z. Chacko, H. S. Goh and R. Harnik, \texttt{arXiv:hep-ph/0506256}.

[6] S. L. Glashow and S. Weinberg, Phys. Rev. D \textbf{15}, 1958 (1977).

[7] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, SCIPP-89/13.

[8] J. F. Gunion and H. E. Haber, Phys. Rev. D \textbf{67}, 075019 (2003) \texttt{arXiv:hep-ph/0207010}.

[9] H. E. Haber and H. E. Logan, Phys. Rev. D \textbf{62}, 015011 (2000) \texttt{arXiv:hep-ph/9909335}.

[10] M. Neubert, Eur.Phys.J. C40 (2005) 165-186, \texttt{arXiv:hep-ph/0408179}.

[11] P. Gambino and M. Misiak, Nucl. Phys. B \textbf{611}, 338 (2001) \texttt{arXiv:hep-ph/0104034}.

[12] K. Hagiwara, S. Matsumoto, D. Haidt and C. S. Kim, Z. Phys. C \textbf{64}, 559 (1994) [Erratum-ibid. C \textbf{68}, 352 (1995)] \texttt{arXiv:hep-ph/9409380}.