Dynamics of Liquid Crystals in Variable External Fields

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Abstract. Behavior and dynamic properties of liquid crystals in variable magnetic and electric fields of different types (oscillating, pulsing, rotating, conical) are investigated. The results and prospects of their experimental study are discussed.

1. Introduction
Hydrodynamics of nematic liquid crystals was developed based on the conservation laws in the works by Leslie and Ericksen [1, 2]. Later dozens of versions of such theory (both linear and nonlinear ones) were suggested. However, as a rule hydrodynamics by Leslie – Ericksen or Forster – Martin – Parodi – Pershan is used for solving certain practical problems and describing experimental results. The majority of the theories give similar or equivalent results.

In addition, the experimental and theoretical research of dynamics of nematics in external magnetic and electric fields varied in value or direction is of practical interest due to the determination of magnetic and dielectric susceptibilities anisotropy, acoustic parameters, orientational relaxation times, viscosity coefficients [3–5].

Measurements of time dependences of optical, dielectric or acoustic anisotropic parameters allow to monitor the director motion in liquid crystal.

Behavior of liquid crystals in variable external fields of different types (pulsing, oscillating, rotating, conical) is discussed in this paper. Orientational effect of fast-oscillating electric field \(E\) and constant magnetic field \(H\) is similar. One can replace \(\mu_0 \chi_a H^2\) on \(\frac{1}{2} \varepsilon_0 \varepsilon_a E^2\) in the formula, where \(\mu_0\) – magnetic permeability of vacuum, \(\varepsilon_0\) – electrical permittivity of vacuum, \(\chi_a\) – diamagnetic susceptibility anisotropy, \(\varepsilon_a\) – dielectric permeability anisotropy.

Experimentally, it is more suitable to use an electric field as an external field orthogonal to the layer and a magnetic field as a rotating one.

2. Oscillating and pulsing fields
The fields of this type stay collinear (e.g. to axis 3) and therefore they are of interest, for example, for the study of the Fredericks effect dynamics in thin layers of a nematic or for the study of the fast orientation processes when the field is switched on and slow relaxation to nonoriented state (misorientation when the field is switched off) of the volume sample (when nematic size is much greater then magnetic coherence length). In these cases the boundary conditions, the appearing flows as well as the generation and motion of orientation defects play...
a significant part. Such fields are relatively easy implemented in an experiment. However, the results of measurements in such an experiment are substantially more difficult to analyze and use for the determination of the nematic parameters. External fields varying their direction are more perspective for the experiment.

3. Rotating and conical fields

Let us consider a magnetic field with an intensity vector \( \mathbf{H} = H \mathbf{h} \) which circumscribes in space a cone with an axis along axis 3 and a vertex angle \( 2\beta \) at rotation frequency \( \omega \). Vector index 0 means their 12 plane projection. (Instead of \( H_3 \) component we may to use oscillating electric field in this direction.)

At typical frequencies of the field variation (lower than SFH range) in the absence of flow the motion equation of the director in magnetic field \( \mathbf{H} \) in the Leslie – Ericksen hydrodynamics is given by:

\[
\gamma_1 \ddot{n} + \mu_0 \chi_a \mathbf{n} \cdot \mathbf{H} \dot{H} = \lambda n,
\]

where \( \lambda = \mu_0 \chi_a (\mathbf{n} \cdot \mathbf{H})^2 \) – Lagrange factor, \( \gamma_1 \) – rotational viscosity coefficient, the dot above means a time derivative. For magnetic and viscous torque densities we have:

\[
\Gamma = \mu_0 \chi_a (\mathbf{n} \cdot \mathbf{H}) [\mathbf{n} \times \mathbf{H}] - \gamma_1 [\mathbf{n} \times \dot{\mathbf{n}}] = 0.
\]

In this case from the motion equation of the director (1) for the director polar angle cotangent \( \frac{d}{\tan \alpha} = \frac{n_3}{n_0} \) and retardation \( \varphi = \omega t - \varphi_n \) of azimuth angle \( \varphi_n \) we obtain the following autonomous nonlinear equations system:

\[
\dot{\varphi} + 2\omega_c h_0 \sin \varphi (h_0 \cos \varphi + h_3 d) - \omega = 0, \tag{3}
\]

\[
\dot{d} + 2\omega_c (h_0 \cos \varphi + h_3 d) (h_0 \sin \varphi - h_3) = 0, \tag{4}
\]

where \( \omega_c = \mu_0 \chi_a H^2 / 2\gamma_1 \) – critical frequency.

Its stationary solution (synchronous mode) is given by:

\[
2 \sin \varphi_s \cos \varphi_s \left( h_0^2 + \frac{h_3^2}{\cos^2 \varphi_s} \right) = \frac{\omega}{\omega_c}, \tag{5}
\]

\[
d_c = \frac{h_3}{h_0 \cos \varphi_s}. \tag{6}
\]

The linearization of the system (3), (4) in the vicinity of solutions (5), (6) gives:

\[
\frac{1}{2\omega_c} \delta \dot{\varphi} = (2h_0^2 \sin^2 \varphi_s - 1) \delta \varphi - h_0 h_3 \sin \varphi_s \delta d, \tag{7}
\]

\[
\frac{1}{2\omega_c} \delta \dot{d} = h_0 h_3 \sin \varphi_s \left( 1 + \frac{h_3^2}{h_0^2 \cos^2 \varphi_s} \right) \delta \varphi + (h_0^2 \sin^2 \varphi_s - 1) \delta d, \tag{8}
\]

From where we find that for the steady-state solution stability the trace of the matrix of the system (7), (8) is required to be negative

\[
3h_0^2 \sin^2 \varphi_s - 2 < 0, \tag{9}
\]

and its determinant is required to be positive

\[
2h_0^4 \cos^4 \varphi_s + h_0^2 (2h_3^2 - h_0^2) \cos^2 \varphi_s + \frac{h_3^4}{\cos^2 \varphi_s} > 0. \tag{10}
\]
The family of solutions of equation (5) on a phase diagram (rotation frequency via angle, fig.1) has a typical critical point $h_3 = 1/3$ and $\omega/\omega_c = 2/\sqrt{3}$ at which the lines of maximum and minimum curves, stability boundaries and different stationary points converge. At this point we have $\tan \varphi_s = \sqrt{3}$ and $\varphi_s = \pi/3$. At $h_3 < 1/3$ equation (5) at a certain frequency range has three solutions whose stability is defined by inequality (10). The minimum angle (focus) corresponds to the stable solution and the others (saddle and focus) correspond to unstable solutions. At lower frequencies the only solution (node) is stable and at high frequencies (focus) it is unstable. At $h_3 \geq 1/3$ at any frequencies equation (5) has the only solution (focus). At $1/9 \leq h_3^2 \leq 1/3$ the solutions are stable if the inequality (9) is true. At $h_3^2 \geq 1/3$ the solution is stable at any frequencies and, therefore, the asynchronous mode is absent.

This result means that the conical magnetic field with angle $\beta < 54.74^\circ$, unlike the classical rotating field, allows to measure high values of rotating viscosity in the synchronous mode. For instance, it is possible in the vicinity of phase transition from nematic to smectic phase or in smectic phase C.

It should be noted that in the absence of the synchronous mode or in case of its instability a liquid crystal goes to the asynchronous mode with a complicated movement of the director. Due to the degeneracy of this state a slow process of liquid crystal disorientation begins. As a result the liquid crystal goes to a new stationary state with incomplete orientation and multiple orientational defects (disclinations). The anisotropy of this state reduced with the increase of the rotation frequency and (as well as the duration of the process) is determined by the dynamics of the defects.

![Figure 1](image.png)

**Figure 1.** Phase diagram of the dependence the retardation of azimuth angle of the director rotation $\varphi_s$ on the relative rotation frequency of a magnetic field $\omega/\omega_c$. For the values $h_3^2$ equal: 0 – 0; 1/90 – 1; 2/45 – 2; 1/9 – 3; 7/45 – 4; 2/9 – 5; 4/15 – 6; 3/10 – 7; 1/3 – 8; 1/2 – 9; 7/10 – 10; 9/10 – 11; 1 – 12. Solid lines correspond to stable solutions; dotted lines correspond to unstable solutions.
It is obvious that in a particular case of a rotating field equation (5) turns into a classical Tsvetkov equation and the frequency area of stable aligned synchronous mode is limited above by a critical frequency \( \omega_c \) (fig. 1, curve 0).

Conical and rotating magnetic fields of high frequency (\( \omega >> \omega_c \)) are advisable to be used for orientation of discotics with a negative diamagnetic susceptibility anisotropy.

Lately, new liquid-crystal phases have been detected, for example, biaxial nematics and twist-bend (TB) nematics [6, 7], which are also oriented by a magnetic field. In this case cone fields will be a useful tool to study their anisotropic properties and orientational structure, as well.

4. Conclusions

Thus, the study of dynamic properties of liquid crystals in variable external fields is an effective method of experimental research of liquid crystal state and it can give valuable information for verification of theory.

A cone magnetic field or the combination of a rotating field with variable (oscillating) electric field appears to be the most perspective.

Ultrasonic and dielectric spectrosocopies in a magnetic field are the most suitable for the study of volume liquid crystal samples which are more free of distortions of their orientational structure by the boundary constraints. (At that, a special treatment of the surfaces of a measuring chamber is desirable to ensure the boundary conditions of a weak orientational anchoring to reduce the influence of the disclinations generation process).

An alternative line is connected with the measurements in the asynchronous mode and with the construction of a dynamic theory of orientational defects movement for the description of experimental data in nonaligned (or partly misorientated in fast-changing fields) liquid crystals. However, this approach is still underdeveloped and its practical prospects stay unclear. In this case in order to analyze the experimental data one has to use Fokker–Planck simplified models or rotating diffusion for ”swarms”.

References

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