Robustness analysis of enhanced adaptive feed-forward cancellation

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Abstract

This paper presents a robustness analysis of an enhanced adaptive feed-forward cancellation (AFC) function for a control system. The AFC is known as an adaptive control method, and the adaptive algorithm can estimate a periodic disturbance. In a previous study, an enhanced AFC was developed to compensate for non-periodic disturbances. The effectiveness of the enhanced AFC there was shown with only simulation results. In this study, the stable robustness of the enhanced AFC is analyzed for a one degree of freedom system. When the enhanced AFC is implemented to around a resonant frequency, the variations in the mechanical characteristics may cause instabilities in the control system, because the performance of the enhanced AFC depends on the phase condition of the mechanical characteristics. The experimental results show that variations in the resonant frequency may cause oscillation when the enhanced AFC design does not consider this kind of variation. The study confirms that the enhanced AFC must be designed considering variations in the resonant frequency.

Key words: Disturbance compensation, Adaptive feed-forward cancellation, Robustness, Vibration control

Introduction

Disturbance compensation is critical in control systems. For example, increasing the recording density of a hard disk drive (HDD) requires compensation for disturbances in the head positioning control system and producability of the XY-stage depends on the positioning accuracy of the control system. The positioning accuracy is negatively affected by disturbances to the control system. Therefore, a control system must be able to compensate for disturbances, and a number of disturbance compensation methods have been reported in previous studies.

Adaptive feed-forward cancellation (AFC) is one such disturbance compensation method (Bodson et al. 1992; Sacks et al. 1993). Here the adaptive algorithm adjusts the coefficients of the AFC to compensate for disturbances. The AFC has also been developed to compensate for periodic disturbances (Wu and Tomizuka, 2006). Further, the range of applications of AFC has also been expanded by a feature to compensate for non-periodic disturbances called an enhanced AFC (Yabui et al. 2013). This enhanced AFC was applied to compensate for disturbances in an HDD. The AFC involves a number of optional parameters that are decided by the design, and the performance of the AFC depends on these optional parameters. In previous studies, an optimal design method for AFC was also introduced (Okuyama et al. 2011; Yabui et al. 2012). The performance of the AFC has been optimized by using loop shaping techniques based on a vector locus in the frequency domain (Messner and Bodson, 1994; Atsumi et al. 2007; Messner, 2008). The effectiveness of this approach was verified in simulations and experimental results. However, the performance of the disturbance compensation is evaluated in the control systems, and there are no reports of study of the stable robustness.
In this paper, an analysis of stable robustness of the enhanced AFC is introduced. The analysis is conducted for a one degree of freedom system, as a case study. The robustness performance was evaluated with the sensitivity function gain and vector locus. As a result, when the enhanced AFC is implemented to around a resonant frequency, the mechanical characteristics of the variation may give rise to instabilities in the control system. Because the enhanced AFC’s performance depends on the phase of the mechanical characteristics and a resonant mode provides a 180 degree phase shift. A large phase variation in the mechanical characteristics may then be a cause of degradations in the performance of the enhanced AFC. The experiments with a one degree of freedom system were conducted to demonstrate the robustness of the enhanced AFC. The experimental results show that the variations in the resonant frequency may cause oscillations. This study confirms that the enhanced AFC must be designed with careful consideration of the variations in the resonant frequency.

1. The Control Systems with an Enhanced AFC

This chapter introduces adaptive algorithms of a traditional AFC and an enhanced AFC. The traditional AFC can compensate for periodic disturbances, and the enhanced AFC can compensate for both periodic and non-periodic disturbances.

1.1. Adaptive algorithm of a traditional AFC

Firstly, Fig. 1 shows the block diagram of the control system with AFC, where $P$ is the controlled object and $C$ is the stabilizing controller; $u$ is the AFC outputs. The AFC can compensate for a periodic disturbance by the adaptive algorithm with the form of the adaptive algorithm expressed as follows.

$$u(k) = p(k-1) \cos(\omega T k) + q(k-1) \sin(\omega T k)$$  \hspace{1cm} (1)

$$p(k) = p(k-1) + \lambda \ e(k) \cos(\omega T k + \theta)$$  \hspace{1cm} (2)

$$q(k) = q(k-1) + \lambda \ e(k) \sin(\omega T k + \theta)$$  \hspace{1cm} (3)

Equation (1) indicates the AFC output; (2) and (3) indicate adaptive laws. In these equations, $p$ and $q$ are adaptive parameters, $T$ is a sampling time, $k$ is a sample number, $e(k)$ is position error signal, and $d(k)$ is disturbance. The $\omega$ is a natural frequency that is the target frequency for disturbance compensation; $\lambda$ is the learning rate of the algorithm, $\theta$ is a phase parameter of the AFC output: $\lambda$ and $\theta$ are defined by the designer, in advance. The enhanced AFC is a control technique for disturbance attenuation that is based on comparing an error signal $e(k)$ to an estimated signal, which is adjusted continuously to the error $e(k)$ asymptotically towards zero. Convergence of the error to zero ensures that the estimated $p(k)$ and $q(k)$ parameters converge to the true values.
1.2. Adaptive algorithm of an enhanced AFC

The enhanced AFC here is expanded with a feature to compensate for non-periodic disturbances. Figure 2 is a block diagram of the control system with the enhanced AFC. 

\[ u(k) = \text{output of enhanced AFC} \]

\[ u(k) = p(k - 1) \cos(\sqrt{1 - \zeta^2 \omega T k}) + q(k - 1) \sin(\sqrt{1 - \zeta^2 \omega T k}). \]  

(4)

Equation (4) indicates the output of the enhanced AFC; the adaptive parameters \( p(k) \) and \( q(k) \) are updated by adaptive laws as in the following equations.

\[ p(k) = e^{-\zeta \omega T k} p(k - 1) + \lambda e(k) \cos(\sqrt{1 - \zeta^2 \omega T k} + \theta), \]

(5)

\[ q(k) = e^{-\zeta \omega T k} q(k - 1) + \lambda e(k) \sin(\sqrt{1 - \zeta^2 \omega T k} + \theta). \]

(6)

In these equations, \( T \) is the sampling time; \( \omega \) is the natural frequency that is the target frequency for the disturbance compensation, \( \lambda \) is the learning rate of the algorithm, \( \zeta \) is a forgetting factor of the algorithm and \( \theta \) is a phase parameter for the AFC output. In the recurrence formula of the enhanced AFC, the adaptive algorithm has a damping function as a forgetting factor, \( e^{-\zeta \omega T k} \) (e is Napier’s constant). If \( \zeta \) is equal to 0, the adaptive algorithm is equal to the traditional AFC.

2. Theoretical Study of Stable Robustness for the Enhanced AFC

In the previous chapters, the enhanced AFC is introduced. The enhanced AFC can compensate periodic and non-periodic disturbances, similarly. In this chapter, the design method of the enhanced AFC is introduced.

2.1. Linear Time-Independent (LTI) model of the enhanced AFC

In general, the disturbance compensation performance is evaluated in a frequency domain. To design the parameters, the adaptive algorithm of the enhanced AFC was converted to an LTI model, and the adaptive algorithm Eqs. (5), (6) may be rewritten as

\[ p(k) = \sum_{a=1}^{k} e^{-\zeta \omega T (k-a)} \lambda e(a) \cos(\Omega T a + \theta), \]

(7)

\[ q(k) = \sum_{a=1}^{k} e^{-\zeta \omega T (k-a)} \lambda e(a) \sin(\Omega T a + \theta). \]

(8)

Where, \( \sqrt{1 - \zeta^2 \omega} = \Omega \). Equations (4), (5) and (6) provide,

\[ u(k) = \sum_{a=0}^{k} e^{-\zeta \omega T (k-a)} \lambda e(a) \cos(\Omega T a + \theta) \cos(\Omega T k) + \]
\[ \sum_{a=0}^{k} e^{-\xi aT(k-a)} \lambda e(a) \sin(\Omega T a + \theta) \sin(\Omega Tk) \]  \hspace{1cm} (9)

The trigonometric function can be transformed as

\[
\cos(\Omega T a + \theta) \cos(\Omega Tk) + \sin(\Omega T a + \theta) \sin(\Omega Tk) = \cos(\theta) \cos(\Omega T(k - a)) + \sin(\theta) \sin(\Omega T(k - a)). \hspace{1cm} (10)
\]

By using matched Z-transformation and convolution theory, the transfer function from \(e(k)\) to \(u(k)\) can be described as

\[
F_{AFC}(z) = \mathcal{Z}\left[ e^{-\xi aT} \lambda \cos(\theta) \cos(\Omega Tk) \right] + \mathcal{Z}\left[ e^{-\xi aT} \lambda \sin(\theta) \sin(\Omega Tk) \right] = \lambda z^{-2} \cos(\theta) + e^{-\xi aT} z^{-1} \cos(\Omega T) \cos(\theta) + e^{-2\xi aT} z^{-2} \cos(\Omega Tk) + e^{-2\xi aT} \sin(\theta)
\]

Equation (11) is the LTI model of the enhanced AFC. The LTI model is equal to the resonant model, and the design parameter \(\theta\) decides the zero, \(\lambda\) decides the gain of the LTI model.

### 2.2. Optimization of enhanced AFC parameters

The optimal \(\theta\) is decided by using the vector locus. Here, the coordinates \([a(\omega), b(\omega)]\) are the points of \(P(j\omega)C(j\omega)\) on the Nyquist diagram. In Fig.3, \(a(\omega)\) and \(b(\omega)\) can be given as

\[
a(\omega) = \text{Re}[P(j\omega)C(j\omega)]
\]

\[
b(\omega) = \text{Im}[P(j\omega)C(j\omega)]
\]

The vector locus of the enhanced AFC is described as a circle in the Nyquist chart. To suppress disturbances, the vector locus should recede from the critical point \([-1, 0]\) on the Nyquist diagram \((7,8)\). The sensitivity function gain is the reciprocal of the distance from an open loop characteristic to the critical point \([-1, 0]\) on the Nyquist diagram. The angle \(\theta\) in Fig.3 indicates the opposite side of \([-1, 0]\] from \([a(\omega), b(\omega)]\) and \(\theta\) can be given as

\[
\theta = \arctan\left( \frac{b(\omega)}{a(\omega) + 1} \right) - \angle P(j\omega) \hspace{1cm} (12)
\]

In Fig.3, the line \(m\) is a tangent to the circle that is the vector locus of the enhanced AFC. Line \(l\) passes through the critical point and \([a(\omega), b(\omega)]\). When the line \(m\) is perpendicular to line \(l\), the distance between the vector locus and the critical point is the maximum on the Nyquist diagram. The angle between line \(m\) and line \(l\) depends on \(\theta\). To recede from the critical point \([-1, 0]\), the proposed method sets the parameters of \(\theta\) on the Nyquist chart. Tuning \(\theta\) is simple by using the proposed method because it is not necessary to assume a mathematical model, the \(\lambda\) is decided to a value that does not cause closed loop system instability.
3. Robustness Analysis of the Enhanced AFC in Simulation

The previous chapter detailed the design method for the enhanced AFC. In the design method an optimal phase parameter $\theta$ for the compensation performance is set. However, when the enhanced AFC is implemented in a control system, the robustness of the control system is important, and the mechanical characteristics of the variation should be considered for the robustness. In this chapter, the following introduces the robustness analysis of the enhanced AFC.

3.1. Design of the enhanced AFC for a nominal model

In this section, the design results for a nominal model are introduced. Figure 4 outlines the block diagram of the control system. In the simulation model, $x(k)$ is the displacement of the plant $P$. Figure 5 shows the frequency response of the plant $P$ that is a second order model. Assuming the frequency variation from the nominal value for the resonant frequency, the four variation models are also indicated in Fig.5. The transfer function of the plant is described as,

$$ P = \frac{K_p}{s^2 + 2\zeta\omega_p + \omega_p^2}. $$

Table 1 shows the parameters of the plant with details of the four perturbation models.
Table 1 Parameters of $P(s)$ from the HDD Benchmark Problem

|                | $\omega_n$ [rad/s] | $\zeta_n$ [%] | $K_p$ |
|----------------|--------------------|---------------|-------|
| Nominal        | $2 \times \pi \times 81$ | 0.001        | 2000  |
| Perturbation 1 | $2 \times \pi \times 80.2$ | 0.001        | 2000  |
| Perturbation 2 | $2 \times \pi \times 79.8$ | 0.001        | 2000  |
| Perturbation 3 | $2 \times \pi \times 79.5$ | 0.001        | 2000  |
| Perturbation 4 | $2 \times \pi \times 79$  | 0.001        | 2000  |

The enhanced AFC was designed to compensate the sinusoidal disturbance at 80Hz for the nominal plant. Firstly, the simulation results of the time responses are indicated. Figure 6 shows the time response of the displacement $x(k)$ for the nominal model. Figure 7 shows the time response of the adaptive parameters $p(k)$ and $q(k)$. Figure 6 confirms that the enhanced AFC can compensate the disturbance, and Fig.7 indicate that the adaptive parameters are converging.

However, the time responses are sensitive to mechanical variations, and Fig.8 shows the time response of displacement $x(k)$ for the Perturbation 4 model. Figure 9 shows the time response of the adaptive parameters $p(k)$ and $q(k)$. Figure 8 confirms that the enhanced AFC cannot compensate the disturbance, and Fig.9 indicate that the adaptive parameters are divergent. Secondly, the simulation results of the frequency responses will now be detailed. Figure 10 shows the Nyquist diagram and Fig. 11 shows the sensitivity function for the four perturbations in Table 3. The sensitivity functions have a peak around 80Hz for two of the variation models, especially sharp for Perturbation 4. These vector loci are very close to the critical point [-1, 0] for the variation models, and it may be hypothesized that the peak is responsible for the instability of the control system. That is, the control system is not sufficiently robust to deal with the variations here. The results here shows the need for the enhanced AFC to be designed with consideration of the actual variation.
3.2. Design of the enhanced AFC with consideration for mechanical variations

In the above, the control system with the enhanced AFC was shown to suffer from potential instabilities arising from variations in the resonant mode, and the robustness of the control system with the enhanced AFC will be analyzed next. The optimal \( \theta \) depends on the phase of the mechanical characteristics, and variations in the mechanical characteristics may be a cause of degradation of the stability. Especially, when the enhanced AFC is implemented at a resonant frequency, the variation in the mechanical characteristics may cause instability in the control system, because the resonant mode is subject to a 180 degree phase shift. If the frequency of the enhanced AFC is set within the range of variation of the resonance frequency, the optimal phase condition can be changed by this variation.

The sensitivity function gain is commonly evaluated for stability, here the maximum gain is at the point closest to \([-1, 0]\) in the Nyquist chart. Further, when the enhanced AFC is implemented at around a resonant frequency, the closest point varies greatly depending on the mechanical characteristics. Figure 12 shows the point of 80Hz in the Nyquist chart for nominal model and Perturbation 4. Therefore, \( \theta \) should be designed to be the point closest to \([-1, 0]\). The \( \theta \) was next designed to be the closest point. The 80Hz point of Perturbation 4 is closest to the critical point in this simulation. Table 2 shows the parameters of the enhanced AFC in this case. The criterion for stability was set as a sensitivity functions gain of less than 10dB. The sensitivity function gain is the reciprocal of the distance from the critical point \([-1, 0]\) on the Nyquist diagram. The distance must be more than 0.316 on the Nyquist diagram (\(20 \log_{10}(0.316) = 10\)). Figure 13 shows the Nyquist diagram with this condition, and Fig. 14 shows the resulting sensitivity function. The vector locus is further from the critical point for all the models. Here the sensitivity function gain is less than 10dB for all models.

The responses were simulated, and Fig.15 shows the time response of displacement \(x(k)\) for the nominal model, with Fig.16 the time response of the adaptive parameters \(p(k)\) and \(q(k)\).

| Without consideration of variation | With consideration of variation |
|-----------------------------------|---------------------------------|
| \(\omega\) [rad/s] | \(2 \times \pi \times 80\) | \(2 \times \pi \times 80\) |
| \(\zeta\) [%] | 0.002 | 0.002 |
| \(\theta\) [deg] | -70 | -83 |
| \(\lambda\) | 30 | 65 |
Figure 17 confirms that the enhanced AFC can compensate the disturbance for the nominal model, and indicates that the adaptive parameters are converging with the nominal model. Figure 17 shows the time response of displacement $x(k)$ for Perturbation 4. Figure 18 shows the time response of adaptive parameters $p(k)$ and $q(k)$. The enhanced AFC can also compensate the disturbance for the model of Perturbation 4, and the adaptive parameters also converge with Perturbation 4. The enhanced AFC should be designed at closest to the critical point considering the variations for the robustness.
4. Verification the Robustness Analysis of the Enhanced AFC in Experiments

Figure 19 shows the experimental setup used to verify the robustness of the enhanced AFC discussed so far. The experiments with a one degree of freedom arrangement (Yahagi et al. 2012) with a plane 442mm long, 100mm wide, and 10mm thick. The two short sides of the plate are fixed. A proof mass actuator with a voice coil motor (VCM) is installed at the center of the plate. A proof mass actuator with a voice coil motor (VCM) is installed on the center of the plate. An additional mass is attached to the opposite surface, under the plate. A vibration exciter inputs vibrations to the plane plate as disturbances, and the vibrations are measured by a load cell. The velocity of the plane plate is measured by a laser Doppler velocimetry (LDV). The velocity of the actuator is also measured by a LDV. The displacements are calculated from the integral of the velocity with respect to time. In the experimental setup, the control objective is vibration suppression of the plane plate. The vibration is suppressed by the actuator output with the actuator input the displacement.
Disturbance compensation was investigated with two mass: mass 1 is 3.78kg, mass 2 is 4.16kg. The additional mass is attached to control the resonant frequency. Figure 21 shows the frequency responses from the input of the vibration exciter to induce displacement to the plane plate. The resonant frequency of the plane plate depends on the mass: 81.85Hz for mass 1, 79.50Hz for mass 2. The vibration exciter inputs an 80Hz sinusoidal signal to the plane plate. Firstly, the enhanced AFC is designed to compensate the sinusoidal signal, and implemented to the plane plate with mass 1 with Fig.22 showing the time response of the displacement, showing that the enhanced AFC is able to compensate the sinusoidal signal.

Next, the enhanced AFC is applied to compensate the sinusoidal signal for the plane plate with mass 2. The enhanced AFC is optimized to fit the plane plate with mass 1 (without considering the variations). Figure 23 shows the Nyquist diagram for the enhanced AFC. Figure 23 shows that the vector locus for the AFC designed to fit mass 2 is very close to the critical point and Fig.24 shows that the rejection gain has a peak around 80Hz. To avoid the problem, we optimized the second enhanced AFCs to consider the variations. The second enhanced AFC’s parameter $\theta$ is designed to be at the point closest to the critical point in Nyquist diagram. Figure 25 shows the Nyquist diagram and Fig.26 shows the sensitivity function for the enhanced AFC considering for the mechanical variation. The vector locus has moved away from critical point in Fig.25 and the sensitivity function gain is less than 10dB in Fig.26.
Figure 27 shows the time response of the displacement of the plane plate with mass 2. The enhanced AFC with considering for the variations is able to compensate for the sinusoidal signal, the enhanced AFC not considering the variations is not able to compensate for the sinusoidal signal. Figure 28 shows the time response of the output of the enhanced AFC. It is observed from Fig. 28 that the enhanced AFC’s output without consideration for the variation causes an excess of oscillation, as the actuator’s output is limited to 0.5V in the experiment. This result clearly shows the critical importance of considering the mechanical variation in the design of the enhanced AFC.
5. Conclusion

This paper presents a robustness analysis of an enhanced AFC for a one degree of freedom system. The optimal parameter of the enhanced AFC depends on the phase of the mechanical characteristics. Especially, when the enhanced AFC is implemented to around a resonant frequency, the mechanical characteristics of a variation may cause instability of the control system. Because a resonant mode may give rise to a 180 degree phase shift. The experimental results show that the variation of the resonant frequency can cause oscillation, and the enhanced AFC should be designed with consideration for the variations in the resonant frequency. The analysis is helpful to enable designing of an enhanced AFC for a real system, like a HDD.

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