Pentaquark components in low-lying baryon resonances

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We study pentaquark states of both light \( q^3 \bar{q} \) and hidden heavy \( q^3 \bar{Q} \) (\( q = u,d,s \) quark in SU(3) flavor symmetry; \( Q = c, b \) quark) systems with a general group theory approach in the constituent quark model, and the spectrum of light baryon resonances in the ansatz that the \( l = 1 \) baryon states may consist of the \( q^3 \) as well as \( q^3 \bar{q} \) pentaquark component. The model is fitted to ground state baryons and light baryon resonances which are believed to be normal three-quark states. The work reveals that the \( N(1535)1/2^- \) and \( N(1520)3/2^- \) may consist of a large \( q^3 \bar{q} \) component while the \( N(1895)1/2^- \) and \( N(1875)3/2^- \) are respectively their partners, and the \( N^+(1685) \) might be a \( q^3 \bar{q} \) state. By the way, a new set of color-spin-flavor-spatial wave function for \( q^3 \bar{Q} \) systems in the compact pentaquark picture are constructed systematically for studying hidden charm pentaquark states.

I. INTRODUCTION

Baryon resonance spectrum has been studied over decades, but theoretical results are still largely inconsistent with experimental data. Except for the ground state baryons, even the low-lying resonances, for example, the Roper resonance \( N(1440) \), \( N(1520) \) and \( N(1535) \) have been of an ordering problem. Theoretical works in the three-quark picture always predict a larger mass for the lowest positive-parity state \( N(1440) \) than for the lowest negative-parity states \( N(1520) \) and \( N(1535) \) [1]. Since the discoveries of \( N(1895)1/2^- \), \( N(1875)3/2^- \), \( \Delta(1900)1/2^- \), and \( \Delta(1940)3/2^- \) [2], these states and other baryon resonance states near 1900 MeV have not been well explained in conventional constituent quark models [3–6].

By applying the new approaches of photoproduction and electroproduction experiments, more baryon resonances have been discovered and confirmed [2–7] and the internal structures of some resonance states have been revealed by the properties including Breit-Wigner amplitudes, transitions amplitudes, and form factors [2, 3, 11]. The Roper electroproduction amplitudes [8] has proven us that it is mainly the nucleon first radial excitation as interpreted in the review paper [9]. That the decay width of \( \Gamma_{N(1535)\rightarrow N\eta} \) (65 ± 25 MeV) is as large as \( \Gamma_{N(1535)\rightarrow N\pi} \) (67.5 ± 19 MeV) [12] indicates that \( N(1535) \) may couple to the \( \eta \) meson much more strongly than predicted by flavor symmetry [10]. The strangeness component in \( N(1535)1/2^- \) is shown to account for the mass ordering of \( N(1440) \) and \( N(1535) \) [13], and it is claimed that \( s\bar{s} \) pair contribution is important to the properties of the nucleon in Ref. [14]. As for the other lowest orbital excited state \( N(1520)3/2^- \), the branching ratio of \( \Gamma_{\eta N}/\Gamma_{tot} \) is less than 1% [12] which reveals that there is little strange component contribution. It is also stated that \( \gamma N \rightarrow N(1520) \) form factors are dominated by the meson cloud contributions which means \( N(1520) \) may not be pure \( q^3 \) state but include the extrinsic \( qq \) pair contribution in the form of \( q^3 \bar{q} \) components [11]. And the baryon states including pentaquark components have also been studied in the light quark sectors for Roper resonance [15] and \( N(1535) \) [16–18] to give a better explanation of the experimental results like transitions amplitudes and form factors.

In this work we study the role of pentaquark components in low-lying baryon resonance states. The constructions of light pentaquark wave functions in the Yamaguchi technique have been formulated in the previous works [11, 22]. As a consequence, the light baryon resonance spectrum is newly reproduced by mixing three quark and pentaquark components. And we extend the group theory approach to hidden heavy pentaquarks in the SU(3) flavor symmetry, where the pentaquark wave functions for the \( q^3 \bar{Q} \) systems are systematically constructed in the harmonic oscillator interaction and applied as complete bases to evaluate hidden charm and bottom pentaquark mass spectra for all possible quark configurations and interactions of other types.

The paper is organized as follows: We briefly review in Sec. II the constituent quark model extensively described in our previous work [22], and predetermined all the model parameters by comparing the theoretical and experimental masses of all the ground state baryons and low-lying \( q^3 \) baryon resonance states. The baryon masses in the \( q^3 \) picture are also presented in Sec. II. In Sec. III we derive the mass spectra of light \( q^3 \bar{q} \) pentaquark states, and to reproduce the negative-parity nucleon and \( \Delta \) resonances below 2 GeV by introducing light pentaquark components in three-quark baryon states. The wave functions of \( q^3 \bar{Q} \) systems are constructed in the harmonic oscillator interaction for all possible quark configurations and applied as complete bases to evaluate hidden heavy pentaquark mass spectra in Sec. IV. A summary is
The details of $q^3$ wave functions as well as the construction of $q^3QQ$ pentaquark wave functions are shown in Appendices.

II. THEORETICAL MODEL

A group theory approach to construct the wave functions for baryon and pentaquark states has been described in Refs. [19–22], and we refer the readers to those works for details. Here, we just present the general Hamiltonian for multiquark systems,

$$H = H_0 + H_{hyp}^{OGE},$$

$$H_0 = \sum_{k=1}^{N} \left( m_k + \frac{p_k^2}{2m_k} \right) + \sum_{i<j}^{N} \left( -\frac{3}{8} \lambda_i^C \cdot \lambda_j^C (A_{ij} r_{ij} - B_{ij} \frac{1}{r_{ij}}) \right),$$

$$H_{hyp}^{OGE} = -C_{OGE} \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \tilde{\sigma}_i \cdot \tilde{\sigma}_j,$$

where $A_{ij}$ and $B_{ij}$ are mass-dependent coupling parameters, taking the form

$$A_{ij} = a \sqrt{\frac{m_{ij}}{m_u}}, \quad B_{ij} = b \sqrt{\frac{m_u}{m_{ij}}},$$

with $m_{ij}$ being the reduced mass of $i$th and $j$th quarks, defined as $m_{ij} = \frac{m_i m_j}{m_i + m_j}$. The hyperfine interaction, $H_{hyp}^{OGE}$ includes only one-gluon exchange contribution, where $C_{OGE} = C_m m_u^2$, with $m_u$ being the constituent $u$ quark mass and $C_m$ a constant. $X_i^C$ in the above equations are the generators of color $SU(3)$ group.

The model parameters are determined by fitting the theoretical results to the experimental data of the mass of all the ground state baryons, namely, eight light baryon isospin states, seven charm baryon states, and six bottom baryon states as well as light baryon resonances of energy level $N \leq 2$, including the first radial excitation state $N(1440)$ with mass at 1.5 GeV and a number of orbital excited $l=1$ and $l=2$ baryons. All these baryons are believed to be mainly $3\bar{q}$ states whose masses were taken from Particle Data Group [12]. The least squares method is applied to minimize the weighted squared distance $\delta^2$,

$$\delta^2 = \sum_{i=1}^{N} \omega_i \left( \frac{M^{exp} - M^{cal}}{M^{exp}} \right)^2,$$

where $\omega_i$ are weights being 1 for all the states except for $N(939)$ and $\Delta(1232)$ which are set to be 100. $M^{exp}$ and $M^{cal}$ are respectively the experimental and theoretical masses. Listed in Tables I, II, III and IV are the theoretical masses which are calculated in the Hamiltonian in Eq. (1) in the $q^3$ picture and fitted to the experimental data. Possible assignments of the theoretical results of excited nucleon and $\Delta$ resonances below 2.2 GeV to all the known baryon states are presented in Tables II, III and IV following the $SU(6)_{SF}$ representations. The orbital-spin-flavor wave functions of $q^3$ baryon states are listed in Appendix A.

The 3 model coupling constants and 4 constituent quark masses are fitted,

$$m_u = m_d = 327 \text{ MeV}, \quad m_s = 498 \text{ MeV},$$

$$m_c = 1642 \text{ MeV}, \quad m_b = 4960 \text{ MeV},$$

$$C_m = 18.3 \text{ MeV}, \quad a = 49500 \text{ MeV}^2, \quad b = 0.75$$

(Similar model parameters were obtained in the previous work [22]. The parameters fixed in the work are slightly different from the preliminary ones since charm and bottom baryons are included and more accurate method is used for the model fixing. And the u and d constituent quark masses are taken from PDG [12].)

In Table I, the ground state baryons are listed. The least squares method is employed to fit the experimental and theoretical mean values, $D = 100 \cdot (M^{exp} - M^{cal})/M^{exp}$. $M^{exp}$ and $M^{cal}$ are taken from PDG [12].

| Baryon     | $M^{exp}$(MeV) | $M^{cal}$(MeV) | $D$ (%) |
|------------|----------------|----------------|---------|
| N(939)    | 939            | 939            | 0       |
| $\Delta$(1232) | 1232           | 1232           | 0       |
| $\Lambda$(1116) | 1116           | 1129           | -1.16   |
| $\Sigma$(1193) | 1193           | 1163           | 2.56    |
| $\Sigma^*$ (1385) | 1385           | 1372           | 0.97    |
| $\Xi$(1318) | 1318           | 1329           | -0.83   |
| $\Xi^*$ (1530) | 1533           | 1510           | 1.49    |
| $\Omega$(1672) | 1672           | 1662           | 0.62    |
| $\Lambda_c$(2286) | 2286           | 2272           | 0.62    |
| $\Sigma_c$(2455) | 2454           | 2428           | 1.06    |
| $\Sigma_c^*$ (2520) | 2518           | 2486           | 1.26    |
| $\Xi_c$(2470) | 2469           | 2489           | -0.82   |
| $\Xi_c^*$ (2645) | 2646           | 2633           | 0.47    |
| $\Omega_c$(2695) | 2695           | 2751           | -2.07   |
| $\Omega_c^*$ (2770) | 2766           | 2789           | -0.84   |
| $\Lambda_b$(5620) | 5620           | 5599           | 0.37    |
| $\Sigma_b$(5811) | 5811           | 5781           | 0.51    |
| $\Sigma_b^*$ (5832) | 5832           | 5801           | 0.54    |
| $\Xi_b$(5792) | 5792           | 5819           | -0.47   |
| $\Xi_b^*$ (5945) | 5950           | 5953           | -0.05   |
| $\Omega_b$(6046) | 6046           | 6097           | -0.84   |

TABLE I. Ground state baryons applied to fit the model parameters. The last column shows the deviation between the experimental and theoretical mean values, $D = 100 \cdot (M^{exp} - M^{cal})/M^{exp}$. $M^{exp}$ and $M^{cal}$ are taken from PDG [12].
TABLE II. Nucleon resonances of positive parity applied to fit the model parameters.

| \((\Gamma, 2^{+}I, D, N, L^F)\) | Status | \(J^P\) | \(M^{exp}\) (MeV) | \(M^{cal}\) (MeV) |
|--------------------------|--------|--------|----------------|----------------|
| \((N(56,4s, 0, 0^+))\)  | ****  | \(\frac{1}{2}^+\) | 939            | 939            |
| \((N(56,4s, 2, 0^+))\)  | ****  | \(\frac{1}{2}^+\) | 1499           |                |
| \((N(56,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1655           |                |
| \((N(56,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1655           |                |
| \((N(20,4s, 2, 1^+))\)  | ***    | \(\frac{1}{2}^+\) | 1749           |                |
| \((N(20,4s, 2, 1^+))\)  | -      | \(\frac{3}{2}^+\) | missing        | 1749           |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1631           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1924           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | **     | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ***    | \(\frac{3}{2}^+\) | 1994           |                |
| \((N(70,4s, 2, 2^+))\)  | *      | \(\frac{3}{2}^+\) | 1994           |                |
| \((N(70,4s, 2, 2^+))\)  | **     | \(\frac{3}{2}^+\) | 1994           |                |

TABLE III. Resonances of negative parity applied to fit the model parameters.

| \((\Gamma, 2^{+}I, D, N, L^F)\) | Status | \(J^P\) | \(M^{exp}\) (MeV) | \(M^{cal}\) (MeV) |
|--------------------------|--------|--------|----------------|----------------|
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |
| \((N(70,4s, 2, 2^+))\)  | ****  | \(\frac{3}{2}^+\) | 1702           |                |

The Roper resonance just as other predictions of the conventional constituent quark models. We assume that the lowest negative-parity baryon resonances may consist of the \(q^3\) component as well as the \(q^4\) pentaquark component. The spin-orbit interactions are not included in this work, so the states in the same spatial-spin-flavor configuration as shown in Appendix A have the same mass value. Except for the two missing \(\Delta(70,2^+, 2^{+})\) states and the two missing nucleon states \(N(20,2^+, 1^{+})\) and \(N(70,2^+, 2^{+})\), most positive-parity states are reasonably reproducible.

### III. LIGHT QUARK SPECTRUM

#### A. Mass of \(q^4\) pentaquark states

The mass spectra of the ground state \(q^4\) and \(q^4\) pentaquarks are evaluated in the Hamiltonian in Eq. (1), by applying the complete bases of the pentaquark wave functions derived in our previous work [22]. Listed in Tables V and VI are the theoretical results, with the model parameters fixed in the previous section. Comparing to other works [24, 25] for \(q^4\) and \(q^3\) hidden strange pentaquark states, the model here employs much less model parameters and predict relatively higher mass spectra. It is predicted in the calculation that the pentaquark state with the \([31]\_{FS}[22]_{F}[31]_{S}\) configuration and the quantum numbers \(I(J^P) = \frac{1}{2}(\frac{1}{2}^+)\) has the lowest mass, 1683 MeV which is quite close to the mass of the isospin-1/2 narrow resonance \(N^+(1685)\). One may make a bold guess that this \(N^+(1685)\) resonance could be the lowest pentaquark state.

#### B. Mass of \(q^3\) pentaquark states

The mass spectra of the ground state \(q^3\) pentaquarks are evaluated in the Hamiltonian in Eq. (1), by applying the complete bases of the pentaquark wave functions derived in our previous work [22]. Listed in Tables V and VI are the theoretical results, with the model parameters fixed in the previous section. Comparing to other works [24, 25] for \(q^4\) and \(q^3\) hidden strange pentaquark states, the model here employs much less model parameters and predict relatively higher mass spectra. It is predicted in the calculation that the pentaquark state with the \([31]\_{FS}[22]_{F}[31]_{S}\) configuration and the quantum numbers \(I(J^P) = \frac{1}{2}(\frac{1}{2}^+)\) has the lowest mass, 1683 MeV which is quite close to the mass of the isospin-1/2 narrow resonance \(N^+(1685)\). One may make a bold guess that this \(N^+(1685)\) resonance could be the lowest pentaquark state.
B. Possible mixtures of $q^3$ and $q^4\bar{q}$ states

Ground state pentaquarks always have a negative parity, thus only $l = 1$ nucleon and $\Delta$ orbitally excited states could mix with ground state pentaquarks. Considering the low theoretical masses for the $N(1535)$ and $N(1520)$ resonances in the $q^3$ picture and their quantum numbers, it is natural to assume that the two baryon resonances may include both the $q^3$ and $q^4\bar{q}$ pentaquark component contributions. Based on this assumption the wave function of baryon resonances may be represented as a mixed state,

$$|N\rangle = A_3|q^3\rangle + B_3|q^4\bar{q}\rangle \quad (5)$$

The $q^3$ and $q^4\bar{q}$ pentaquark components may couple as

$$\psi_1 = \cos \theta |q^3\rangle - \sin \theta |q^4\bar{q}\rangle,$$
$$\psi_2 = \sin \theta |q^3\rangle + \cos \theta |q^4\bar{q}\rangle. \quad (6)$$

where $\theta$ is the mixing angle between the $q^3$ and $q^4\bar{q}$ components of the corresponding states. Taking the pentaquark states which share the same quantum number and parity of $N(1535)$, $N(1520)$, $\Delta(1620)$, and $\Delta(1700)$ and have the relatively lowest mass, one gets mixed states as shown in Table VII.



| Resonance | Status | $J^P$ | $M^{exp}$ (MeV) | $M^{cal}$ (MeV) |
|-----------|--------|-------|-----------------|-----------------|
| $N(1520)$ | ****   | $\frac{3}{2}^-$ | 1510-1520 | 1520 |
| $N(1535)$ | ****   | $\frac{1}{2}^-$ | 1525-1545 | 1535 |
| $N(1650)$ | ****   | $\frac{1}{2}^-$ | 1645-1670 | 1672 |
| $N(1675)$ | ****   | $\frac{3}{2}^-$ | 1670-1680 | 1672 |
| $N(1685)$ | *       | $\frac{1}{2}^-$ | 1665-1675 | 1683 |
| $N(1700)$ | ***     | $\frac{3}{2}^-$ | 1650-1750 | 1672 |
| $N(1875)$ | ***     | $\frac{5}{2}^-$ | 1850-1950 | 1947 |
| $N(1895)$ | ***     | $\frac{7}{2}^-$ | 1890-1930 | 1918 |
| $\Delta(1620)$ | ****   | $\frac{3}{2}^+$ | 1590-1610 | 1600 |
| $\Delta(1700)$ | ****   | $\frac{5}{2}^+$ | 1640-1690 | 1665 |
| $\Delta(1900)$ | ***     | $\frac{1}{2}^+$ | 1830-1900 | 1856 |
| $\Delta(1940)$ | **      | $\frac{1}{2}^+$ | 1850-2050 | 2040 |

TABLE VII. The mixture of $q^3$ and $q^4\bar{q}$ components. All four $q^3$ states take the same mass, 1380 MeV. The chosen pentaquark states and masses are listed as $q^4\bar{q}$ config. and $q^4\bar{q}$ Mass (in MeV) from Tables I and VIII.

| $\psi_1$ State | $J^P$ | $\theta$ | $\psi_2$ State | $q^4\bar{q}$ config. | $q^4\bar{q}$ Mass |
|----------------|-------|---------|----------------|---------------------|-----------------|
| 1535           | $\frac{3}{2}^-$ | 26.88°   | 1918           | $q^4\bar{q}_{[21]F[31]}_S$ | 2032 |
| 1520           | $\frac{1}{2}^-$ | 24.9°     | 1947           | $q^4\bar{q}_{[22]F[31]}_S$ | 2049 |
| 1600           | $\frac{1}{2}^-$ | 33.1°     | 1856           | $q^4\bar{q}_{[21]F[22]}_S$ | 2025 |
| 1665           | $\frac{1}{2}^-$ | 31.8°     | 2040           | $q^4\bar{q}_{[21]F[31]}_S$ | 2243 |

IV. $q^3\bar{QQ}$ PENTAQUARK SPECTRUM

Motivated by the hidden-charm pentaquark candidates recently found by the LHCb Collaboration [29] we also calculate the mass spectra of hidden heavy pentaquarks of $q^3\bar{QQ}$ systems. The quark configurations and wave functions of the $q^3\bar{QQ}$ systems are derived in Appendix B. The spatial wave functions, which are derived in the harmonic oscillator quark-quark interaction and grouped in Appendix B according to the permutation symmetry, are employed as complete bases to study the $q^3\bar{QQ}$ systems described with the color dependent Hamiltonian in Eq. (1). The mass spectra of the hidden charm and hidden bottom pentaquarks of the $q^3$ color octet configuration are presented in Tables IX and X separately.

It’s noted that the hidden-charm pentaquark mass spectra in this work is slightly higher than the three narrow pentaquark-like states, $P_c(3412)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ measured by LHCb. The predicted values of 4438 and 4495 MeV for the lowest hidden-charm pentaquark in the $[21]cc[21]_{FS} [21]F[21]_S$ configuration are close to the experimental values of 4440 and 4457 MeV, but still about 100-200 MeV higher than the $P_c(3412)^+$ state. For the hidden-bottom pentaquarks, the work predicts the mass of the ground states to be 10.9-11.2 GeV,
lying below the threshold of a single bottom baryon and $B(B^*)$ mesons, which is consistent with other work [40].

The newly observed $P_c$ states by the LHCb collaboration have been largely interpreted as hadronic molecule states since there are abundant charmed meson and charmed baryon thresholds available [26, 40]. Within the molecular scenario, the mass spectrum [26, 40] and dynamical properties [25, 40] have been successfully explained in various methods. The compact pentaquark interpretation works well [41, 44] when the parameters are fixed to both baryons and mesons. With the limited experimental results, the nature of $P_c$ states will keep as an open question in the near future.

V. SUMMARY

The masses of low-lying $q^3$ states and ground $q^3\bar{q}$ states are evaluated, where all model parameters are predetermined by fitting the theoretical masses to the experimental data for the baryons which are believed to be mainly $3q$ states. In the work we have assumed that the Roper resonance is the first radial excitation state of nucleon.

It is interesting that the theoretical work predicts the pentaquark state with the $[31]_{FS}[22]_F[31]_S$ configuration and the quantum numbers $I(J^P) = \frac{1}{2}(\frac{3}{2}^-)$ has the lowest mass, about 1680 MeV. One may make a bold guess that this $q^3\bar{q}$ pentaquark state could be the isospin-1/2 narrow resonance $N^+(1685)$ which can not be accommodated as a $q^3$ particle.

The work shows that the ordering problem of the $N(1440), N(1520)$ and $N(1535)$ may be solved by introducing the $q^3\bar{q}$ contribution. The same calculation leads to that the $N(1895)\frac{1}{2}^-, N(1875)\frac{3}{2}^-, \Delta(1900)\frac{1}{2}^-$, and $\Delta(1940)\frac{3}{2}^-$ resonances may pair respectively with the $N(1535)\frac{1}{2}^-, N(1520)\frac{3}{2}^-, \Delta(1620)\frac{1}{2}^-$, and $\Delta(1700)\frac{3}{2}^-$ in the $q^3$ and $q^3\bar{q}$ interpretation.

The mass spectra of ground hidden heavy pentaquark states $q^3\bar{Q}$ are accurately evaluated using the same predetermined model parameters. It is found that the hidden charm pentaquark states with the $[21]_{CS}[21]_{FS}[21]_F[21]_S$ configuration have the lowest masses which are slightly larger than the LHCb results. In this communication, however, the work can not draw any conclusion about the nature of $P_c$ states.

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Appendix A: Explicit $q^3$ wave functions

In this Appendix the $q^3$ color-orbital-spin-flavor wave functions with the principle quantum number $N \leq 2$ are listed in Table XI where $X_i, \Phi_j, \phi^S_{P,M,Y}$ are the spin, flavor, and spatial wave functions, respectively. The SU(3)$_F$ singlet states are excluded since only nucleon and $\Delta$ resonances are discussed.

Appendix B: Construction of pentaquark wave functions for $q^3\bar{Q}$ system

The construction of the $q^3\bar{Q}$ pentaquark state follows the rule that $q^3\bar{Q}$ state must be a color singlet and the $q^3\bar{Q}$ wave function should be antisymmetric under any permutation between identical quarks. Requiring the $q^3\bar{Q}$ pentaquark to be a color singlet demands that the color part of the $q^3$ and $Q$ must form a $[222]_i$ singlet state, there are two possible color configurations: the color part of the $q^3$ is a $[111]$ singlet and the $Q$ is also a singlet and the color part of the $q^3$ is a $[21]$ octet and $Q$ is also an octet. The pentaquark state in the $q^3\bar{Q}$ system with the $q^3$ color singlet configuration corresponds to the hadronic molecular pentaquark state which is not confined in our Hamiltonian. And the $q^3\bar{Q}$ system in the compact pentaquark picture takes the $q^3$ color octet configuration. Requiring the wave function of the three-quark configuration to be antisymmetric, the spatial-spin-flavor part of $q^3$ is required to be $[21]$ state by conjugation, and directly couples with the spatial-spin-flavor part of $\bar{Q}$. Firstly we study the total antisymmetric wave function for the $q^3$ color octet configuration,

$$
\psi_{[3]_A} = \frac{1}{\sqrt{2}} \left( \psi_{[21]_A} \psi^{osf}_{[21]_A} - \psi_{[21]_A} \psi^{osf}_{[21]_A} \right)
$$

with

$$
\psi^{osf}_{[21]_A} = \sum_{i,j=S,p,\lambda} b_{ij} \psi^{s}_{[X_i]} \psi^{f}_{[Y_j]},
$$

$$
\psi^{s}_{[X]} = \sum_{i,j=S,p,\lambda} c_{ij} \psi^{s}_{[X_i]} \psi^{f}_{[y_j]},
$$

TABLE X. Ground hidden-bottom pentaquark $q^3\bar{b}\bar{b}$ mass spectrum, where the $q^3$ and $Q\bar{Q}$ components are in the color octet states.

| $q^3\bar{Q}$ configurations | $J^P$ | $M(q^3\bar{b}\bar{b})(\text{MeV})$ |
|-----------------------------|-------|----------------------------------|
| $\Psi_{[21]_{CS}[21]_{FS}[21]_F[21]_S}(q^3\bar{b}\bar{b})$ | $\frac{3}{2}^-$ | 10964, 10968 |
| $\Psi_{[21]_{CS}[21]_{FS}[3]_{FS}[21]_S}(q^3\bar{b}\bar{b})$ | $\frac{5}{2}^-$ | 11183, 11183 |
| $\Psi_{[21]_{CS}[21]_{FS}[21]_F[1]_S}(q^3\bar{b}\bar{b})$ | $\frac{5}{2}^-$ | 11037, 11051 |
TABLE XI. Explicit $q^3$ color-orbital-spin-flavor wave functions.

| N | $SU(6)_{SF}$ Res. | $t^P$ | $SU(6)_{SF} \times O(3)$ wave functions | $SU(3)_{SF}$ decuplet |
|---|---|---|---|---|
| 0 | 56 | 0$^+$ | $J^P = \frac{1}{2}^+$ | $J^P = \frac{3}{2}^+$ |
| & | | | $\frac{1}{\sqrt{2}} \psi_{[1]}^{16} \phi_{000} \Phi_{\lambda \chi \rho} \Phi_{\lambda \chi \lambda}$ | $\psi_{[1]}^{s} \phi_{000} \Phi_{\lambda \chi \rho}$ |
| 1 | 70 | 1$^-$ | | $J^P = \frac{3}{2}^+$ |
| & | | | $\frac{1}{\sqrt{2}} \psi_{[1]}^{16} \phi_{000} \Phi_{\lambda \chi \rho} \Phi_{\lambda \chi \lambda}$ | $\frac{1}{\sqrt{2}} \psi_{[1]}^{16} \phi_{000} \Phi_{\lambda \chi \rho}$ |
| 2 | 56 | 0$^+$ | $J^P = \frac{3}{2}^+$ | $J^P = \frac{3}{2}^+$ |
| & | | | $\frac{1}{\sqrt{2}} \psi_{[1]}^{16} \phi_{000} \Phi_{\lambda \chi \rho} \Phi_{\lambda \chi \lambda}$ | $\psi_{[1]}^{3} \phi_{000} \Phi_{\lambda \chi \rho}$ |
| 2 | 20 | 1$^+$ | $J^P = \frac{3}{2}^+$ | $J^P = \frac{3}{2}^+$ |
| & | | | $\psi_{[1]}^{3} \phi_{000} \Phi_{\lambda \chi \rho} \Phi_{\lambda \chi \lambda}$ | $\psi_{[1]}^{3} \phi_{000} \Phi_{\lambda \chi \rho}$ |
| 2 | 56 | 2$^+$ | $J^P = \frac{3}{2}^+$ | $J^P = \frac{3}{2}^+$ |
| & | | | $\frac{1}{\sqrt{2}} \psi_{[1]}^{16} \phi_{000} \Phi_{\lambda \chi \rho} \Phi_{\lambda \chi \lambda}$ | $\psi_{[1]}^{3} \phi_{000} \Phi_{\lambda \chi \rho}$ |
| 2 | 70 | 2$^+$ | $J^P = \frac{3}{2}^+$ | $J^P = \frac{3}{2}^+$ |
| & | | | $\frac{1}{\sqrt{2}} \psi_{[1]}^{16} \phi_{000} \Phi_{\lambda \chi \rho} \Phi_{\lambda \chi \lambda}$ | $\psi_{[1]}^{3} \phi_{000} \Phi_{\lambda \chi \rho}$ |

\[
\psi_{[1]}^{s} = \{ \psi_{[1]}^{s} \}, \quad \psi_{[1]}^{f} = \{ \psi_{[1]}^{f} \}
\]

The total color wave function for $q^3Q\bar{Q}$ pentaquark state takes the form,

\[
\Psi_{[21]}^{c} = \frac{1}{\sqrt{8}} \sum_{i} \psi_{[21]}^{c} (q^{3}) \psi_{[21]}^{c} (Q\bar{Q})
\]

where the $\rho$ and $\lambda$ stand for the types of $[21]$ color octet configuration in Eq. $[B11]$. The detailed color wave function for both color singlet and color octet states for the $q^3$ and $Q\bar{Q}$ are listed in Table $[XI]$. We construct the spatial wave functions of $q^3Q\bar{Q}$ systems in the harmonic oscillator potential for the quark-quark interaction. A new set of relative Jacobi coordinates was introduced for the $q^3Q\bar{Q}$ system, different from the ones in our previous work $[22]$ for $q^4$ system, the Hamiltonian for the harmonic oscillator potential is written as

\[
H_{q^3Q\bar{Q}} = \frac{p_{\rho}^2}{2m} + \frac{p_{\rho}^2}{2M} + \frac{p_{\rho}^2}{2m} + 5C(\mathcal{A}^2 + \mathcal{B}^2 + \mathcal{C}^2)
\]
where $\vec{p}_i$ and $\vec{r}_i$ are the momenta and coordinate of $i$th quark, the antiquark is assigned the coordinate $\vec{r}_5$, the fourth and fifth quark form the third Jacobi coordinate $\sigma$ and the centers of first three quarks and the last two heavy quarks form the fourth Jacobi coordinate $\chi$. The permutation symmetry of pentaquarks is simply represented by the $q^3$ cluster since the $\psi_{n_\sigma,l_\sigma}(\vec{\sigma})$ and $\psi_{n_\chi,l_\chi}(\vec{\chi})$ is fully symmetric for any permutation between quarks. The total spatial wave function of pentaquarks may take the form,

$$\Psi_{NLM}^{[\sigma]|y} = \psi_{N'|L'|M'}^{q^3}\psi_{n_\sigma,l_\sigma}(\vec{\sigma})\psi_{n_\chi,l_\chi}(\vec{\chi})$$

which is simply the product of the $q^3$ spatial wave function shown in Table XIII and the harmonic oscillator wave functions $\psi_{n_\sigma,l_\sigma}(\vec{\sigma})$ and $\psi_{n_\chi,l_\chi}(\vec{\chi})$ for the Jacobi coordinate $\sigma$ and $\chi$. $[\sigma]_y$ stands for all possible permutation symmetries of the $q^3$ cluster, where, $[\sigma]_y = \{[3]_S, [21]_{\rho,\lambda}, [111]_A\}$. $N$, $L$, and $M$ are respectively the total principle quantum number, total angular momentum, and magnetic quantum number of the pentaquark ($l_\sigma = 0$, $l_\chi = 0$), with

$$N = 2n_\rho + l_\rho + 2n_\lambda + l_\lambda + 2n_\sigma + l_\sigma + 2n_\chi + l_\chi$$

The spatial wave functions of the $q^3$ subsystem of $q^3Q\bar{Q}$ pentaquarks with the permutation symmetries $[3]_S$ are listed in Table XIII up to $N' = 22$, where $l_\rho$, $l_\lambda$, and $l_\sigma$ are limited to 0, 1 and 2 only. To save space, we show only the symmetric spatial wave function while the spatial wave function for other possible permutation symmetries $[21]_{\rho,\lambda}$ and $[111]_A$ will not be specified here. Note that we have set $M' = 0$ and used the abbreviation,

$$\psi_{n_\sigma,l_\sigma,m_\sigma}^{q^3}(\vec{\sigma}) = \frac{1}{\sqrt{N_\sigma}}\sum_{\{n,\sigma\}} C_{n,p,l_\rho,n_\sigma,l_\sigma,m_\sigma}(\vec{\sigma})\psi_{n_\sigma,l_\sigma,m_\sigma}(\vec{\chi})$$

The spatial wave functions of pentaquarks with the $q^3Q\bar{Q}$ symmetry $[5]_S$ are listed in the Table XIV (Up to $N = 14$ energy level is sufficient for the numerical calculations), where $\psi_{N'|L'|M'}^{q^3}(L' = M' = 0)$ and $\psi_{n_\sigma,l_\sigma}(\vec{\sigma})$ ($l_\sigma = 0$), $\psi_{n_\chi,l_\chi}(\vec{\chi})$ ($l_\chi = 0$) are the spatial wave functions of the $q^3$ subsystem and the harmonic oscillator wave function for the $\vec{\sigma}$ and $\vec{\chi}$ coordinates, respectively. Without any limitation for $n_\sigma$ and $n_\chi$, all degenerate states of each pentaquark energy level up to $N = 14$ served as a complete basis. $N \leq 8$ states are listed below, the higher ones follow the rule that $N = N'|q^3 + 2(n_\sigma + n_\chi)$.

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TABLE XIII. Normalized $q^2$ spatial wave functions with quantum number, $N' = 2n$ and $L' = M' = 0$.

| Wave Function | Quantum Number | Wave Function |
|---------------|----------------|--------------|
| 000[3S]       | (0, 0, 0)      |              |
| 200[3S]       | $\sqrt{2}(1, 0, 0)$, $\frac{1}{\sqrt{2}}(0, 0, 1)$ |              |
| 400[3S]       | $\sqrt{2}(2, 0, 0)$, $\sqrt{2}(1, 0, 1)$, $\sqrt{2}(0, 0, 2)$ |              |
| 600[3S]       | $\sqrt{2}(3, 0, 0)$, $\sqrt{2}(2, 0, 1)$, $\sqrt{2}(1, 0, 2)$, $\sqrt{2}(0, 0, 3)$ |              |
| 800[3S]       | $\sqrt{2}(4, 0, 0)$, $\sqrt{2}(3, 0, 1)$, $\sqrt{2}(2, 0, 2)$, $\sqrt{2}(1, 0, 3)$, $\sqrt{2}(0, 0, 4)$ |              |
| 1000[3S]      | $\sqrt{2}(5, 0, 0)$, $\sqrt{2}(4, 0, 1)$, $\sqrt{2}(3, 0, 2)$, $\sqrt{2}(2, 0, 3)$, $\sqrt{2}(1, 0, 4)$, $\sqrt{2}(0, 0, 5)$ |              |
| 1200[3S]      | $\sqrt{2}(6, 0, 0)$, $\sqrt{2}(5, 0, 1)$, $\sqrt{2}(4, 0, 2)$, $\sqrt{2}(3, 0, 3)$, $\sqrt{2}(2, 0, 4)$, $\sqrt{2}(1, 0, 5)$, $\sqrt{2}(0, 0, 6)$ |              |
| 1400[3S]      | $\sqrt{2}(7, 0, 0)$, $\sqrt{2}(6, 0, 1)$, $\sqrt{2}(5, 0, 2)$, $\sqrt{2}(4, 0, 3)$, $\sqrt{2}(3, 0, 4)$, $\sqrt{2}(2, 0, 5)$, $\sqrt{2}(1, 0, 6)$, $\sqrt{2}(0, 0, 7)$ |              |
| 1600[3S]      | $\sqrt{2}(8, 0, 0)$, $\sqrt{2}(7, 0, 1)$, $\sqrt{2}(6, 0, 2)$, $\sqrt{2}(5, 0, 3)$, $\sqrt{2}(4, 0, 4)$, $\sqrt{2}(3, 0, 5)$, $\sqrt{2}(2, 0, 6)$, $\sqrt{2}(1, 0, 7)$, $\sqrt{2}(0, 0, 8)$ |              |
| 1800[3S]      | $\sqrt{2}(9, 0, 0)$, $\sqrt{2}(8, 0, 1)$, $\sqrt{2}(7, 0, 2)$, $\sqrt{2}(6, 0, 3)$, $\sqrt{2}(5, 0, 4)$, $\sqrt{2}(4, 0, 5)$, $\sqrt{2}(3, 0, 6)$, $\sqrt{2}(2, 0, 7)$, $\sqrt{2}(1, 0, 8)$, $\sqrt{2}(0, 0, 9)$ |              |
| 2000[3S]      | $\sqrt{2}(10, 0, 0)$, $\sqrt{2}(9, 0, 1)$, $\sqrt{2}(8, 0, 2)$, $\sqrt{2}(7, 0, 3)$, $\sqrt{2}(6, 0, 4)$, $\sqrt{2}(5, 0, 5)$, $\sqrt{2}(4, 0, 6)$, $\sqrt{2}(3, 0, 7)$, $\sqrt{2}(2, 0, 8)$, $\sqrt{2}(1, 0, 9)$, $\sqrt{2}(0, 0, 10)$ |              |
| 2200[3S]      | $\sqrt{2}(11, 0, 0)$, $\sqrt{2}(10, 0, 1)$, $\sqrt{2}(9, 0, 2)$, $\sqrt{2}(8, 0, 3)$, $\sqrt{2}(7, 0, 4)$, $\sqrt{2}(6, 0, 5)$, $\sqrt{2}(5, 0, 6)$, $\sqrt{2}(4, 0, 7)$, $\sqrt{2}(3, 0, 8)$, $\sqrt{2}(2, 0, 9)$, $\sqrt{2}(1, 0, 10)$, $\sqrt{2}(0, 0, 11)$ |              |

TABLE XIV. $q^2 Q\bar{Q}$ pentaquark spatial wave functions of symmetric type.

| Wave Function | $q^2 Q\bar{Q}$ |
|---------------|---------------|
| $\Psi_{000}^{3S}$ | $\psi_{000}^{3S}$, $\bar{\psi}_{000}^{3S}$, $\psi_{000}^{3S} \bar{\psi}_{000}^{3S}$ |
| $\Psi_{200}^{3S}$ | $\psi_{200}^{3S}$, $\bar{\psi}_{200}^{3S}$, $\psi_{200}^{3S} \bar{\psi}_{200}^{3S}$ |
| $\Psi_{400}^{3S}$ | $\psi_{400}^{3S}$, $\bar{\psi}_{400}^{3S}$, $\psi_{400}^{3S} \bar{\psi}_{400}^{3S}$ |
| $\Psi_{600}^{3S}$ | $\psi_{600}^{3S}$, $\bar{\psi}_{600}^{3S}$, $\psi_{600}^{3S} \bar{\psi}_{600}^{3S}$ |

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