Research on voltage restoration of island microgrid based on multi-agent fixed-time algorithm

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Abstract. This paper presents a distributed, robust and fixed-time secondary control for voltage restoration of an islanded microgrid with droop-controlled inverter-based distributed generators (DGs). The network inverter of AC microgrid is set as a multi-agent cooperating system, and voltage recovery can be regarded as a synchronization problem. Different from the existing distributed methods, the method in this paper can ignore the dependence on the initial conditions and the distributed controller based on consensus can restore the voltage magnitude of the island microgrid to the standard reference value in a fixed-time. In order to improve the stability, a fixed-time second-order convergent sliding mode surface is proposed. This method can quickly reach a consensus and has better anti-interference ability and robustness. Finally, the performance of the proposed control strategy is verified.

1. Introduction
With the increasing popularity of DGs, the problems of coordinated control between distributed power sources and low power quality of microgrids have become increasingly prominent [1-3]. When the microgrid is in an isolated state, the microgrid is completely disconnected from the main grid, and the microgrid usually needs to rely on the DG local controller to maintain operation. In the island microgrid, droop control is selected as the voltage controller for regulation. Droop control [4] is divided into two types: centralized and distributed. The centralized control method uses a central controller to obtain system global information and provide control commands. The single node of the central controller has the disadvantages of heavy communication and calculation burden, poor robustness, and poor scalability. In this paper, distributed control strategy is adopted to overcome the problem of centralized single point fault in traditional droop control. This distributed sparse network makes the microgrid structure more robust and reliable. However, the droop control will cause the voltage to deviate from the rated value and cannot accurately track the system reference value. Literature [5] uses feedback linearization to suppress disturbances, but it is not comprehensive. Literature [6] has improved and proposed a finite-time consensus control method based on multi-agent, but it is still not stable enough. Literature [7] proposed a sliding mode control method based on finite-time control. This method has good anti-disturbance performance, but it relies too much on initial conditions. Therefore, the concept of fixed-time [8-9] is proposed in this paper. The fixed-time control technique overcomes the problem of over-dependence on initial conditions in finite-time and has faster convergence speed and better stability.
This paper proposes a distributed continuous sliding mode control, which uses a fixed-time sliding mode convergence surface [10] to make the system design reasonable control input and effectively ensure the robustness of the system.

1.1 Graph Theory
Consider a weighted graph $G = (\mathcal{V}, \mathcal{E}, \lambda)$ with a set of $N$ nodes $\mathcal{V} = \{1, 2, \ldots, N\}$, a set of edges $(i, j) \in \mathcal{E}$ formed by ordered pairs of vertices $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and the associated adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$. Inverters are modelled as graph nodes and communication links are mapped to edges connecting nodes. If, for all $(i, j) \in \mathcal{E}$, $(j, i) \in \mathcal{E}$,then $G$ is undirected. $a_{ij}$ is the weight of edge $(i, j)$, for an undirected graph $G$. $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$, otherwise $a_{ij} = a_{ji} = 0$. The set of neighbors of node $i$ is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}, j \neq i\}$. The in-degree matrix is defined as $D = \text{diag}(d_i) \in \mathbb{R}^{N \times N}$ with $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The Laplacian matrix is defined as $L = D - A$. In this paper, both the physical network and the communication graph are modelled as an undirected connected graph $G$.

1.2 Fixed-time theorem
$\text{sig}(\cdot)$ is an odd function and can be written as

$$\text{sig}(z)^\alpha = \left[|z_1|^\alpha \text{sgn}(z_1) \ldots |z_n|^\alpha \text{sgn}(z_n)\right]^T$$

(1)

with $\alpha > 0$, where $\text{sgn}$ is the signum function and can be defined as

$$\text{sgn}(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

(2)

Consider the following nonlinear system:

$$\dot{\sigma}(t) = f(\sigma(t))$$

(3)

where $\sigma \in \mathbb{R}^n$ and $f : \mathbb{R}^n$ is a nonlinear function such that $f(0) = 0$, that is $\sigma = 0$, the origin $\sigma = 0$ is an equilibrium point (2).

If the system is globally stable with a finite-time and the fixed-time is bounded, that means

$$\exists T_{\text{max}} > 0 : \forall \sigma_0 \in \mathbb{R}^n$$

In the case of fixed-time stable equilibrium, if the time of convergence is bounded by some finite-time $T_{\text{max}}$ irrespective of any initial condition, it is also bounded by $\lambda T_{\text{max}}$ for all $\lambda \geq 1$, where $\lambda \in \mathbb{R}$. 

2. Dynamic Modelling of AC Microgrid

2.1 Large-signal dynamic model of inverter-based DG

Fig. 1 shows the internal flow chart of an inverter-based DG in islanded-mode operation. The DG is represented by the primary energy source, the voltage source converter (VSC), the series LCL filter, the output connector, and the power, voltage, and current control loops. The control loops use a PI controller to regulate the VSC’s output voltage. The primary controller provides the voltage references for the VSC. The voltage droop characteristics for the \( i \) DG are presented by the following equations:

\[
odi = E_i - D_{Q_i} Q_i
\]  

(4)

where \( odv_i \) represent the output voltage of inverter, \( E_i \) represent the voltage set point for primary control provide by secondary control of inverter, \( D_{Q_i} \) are the droop coefficients.

2.2 Non-linear mathematical model of DGs

The following formula is the nonlinear mathematical model of DGs:

\[
\begin{align*}
\dot{x}_i &= f_i(x_i) + k_i(x_i) D_i + g_i(x_i) u_i \\
y_i &= h_i(x_i)
\end{align*}
\]  

(5)

where \( x_i = \begin{bmatrix} a_i, P_i, Q_i, l_{di}, l_{qdi}, l_{dqi}, l_{qqi}, v_{odi}, v_{odi}, l_{odi}, l_{odi} \end{bmatrix}^T \); \( f_i(x_i), k_i(x_i), g_i(x_i) \) are smooth nonlinear functions; \( u_i = E_i \); \( y_i = v_{odi} \); \( D_i = \begin{bmatrix} w_{com}, v_{bd}, v_{bq} \end{bmatrix}^T \) are considered the outputs, inputs, and known disturbance. The voltage provided by the main control can maintain the system operation, but it will cause the system deviation. Therefore, the secondary control is used to restore the system deviation value to the reference value, which is \( v_{odi} \rightarrow v_{ref} \).

3. Distributed fixed-time robust secondary voltage control

In order to achieve voltage synchronization, the feedback linearization method is used to derive the output \( y_i \) of each DG in the system:
\[ \dot{y}_i = L_{x_i}h_i + L_{\mu_i}h_i u_i + w_i \quad (6) \]

where \( F_i(x_i) = f_i(x_i) + k_i(x_i)D_i \), \( L_{\mu_i}h_i = \frac{\partial h_i}{\partial x_i} F_i \) represent derived function.

In feedback linearization, a direct relationship between the dynamics of the output \( y_i \) and the control input \( u_i \) is generated by repetitively differentiating with respect to time.

Take the second derivative of the equation:
\[ \ddot{y}_i = L_{\mu_i} \dot{h}_i + L_{\mu_i}^2 h_i u_i + \ddot{w}_i \quad (7) \]
\[ \dot{\mu}_i = W_i(y_i, \mu_i), \forall i \]

where \( \mu_i \in \mathbb{R}^{n_i} \) represents the set of internal dynamics; \( L_{\mu_i}^2 F_i = \frac{\partial (L_{\mu_i} h_i)}{\partial x_i} F_i \) represents the Lie derivative function.

In the island microgrid, secondary control voltage recovery is equivalent to the second-order multi-agent consistency problem. and the second-order multi-agent expression is given:
\[ \begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = f_i(x_i) + g_i(x_i)u_i + d_i(x, v, t) \end{cases} \quad (8) \]

where \( d_i(x, v, t) \) represents the input disturbance.

The principle of voltage recovery is to design the control input for the agent DG, regardless of the initial conditions, it can always reach a consensus within a fixed-time. Such that \( v_i \rightarrow v_{ref} \).

4. Fixed-time convergent sliding surface

4.1 Design convergence sliding surface
\[ s_i = f_i(x_i) + g_i(x_i)u_i(t) + d_i(x, v, t) - \dot{u}_i \quad (9) \]

When the system realizes sliding, the second-order multi-agent system can be expressed as:
\[ \begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = \ddot{w}_i \end{cases} \quad (10) \]

where \( \dot{u}_i = \sum_{j=1}^{N} a_{ij} \left[ \phi_1(\text{sign}(e_{i,j} - e_{j,i})) + \phi_2(\text{sign}(e_{i,j} - e_{j,i})) \right] \); \( \alpha > 0, \beta > 1 \) and are odd function;

4.2 Set fixed-time error
\[ \begin{cases} e_{i,j} = e_{i,j} \\ \dot{e}_{i,j} = \ddot{u}_i - \ddot{u} \end{cases} \quad (11) \]

where \( e_{i,j} = [e_{i,1}, e_{i,2}, \ldots, e_{i,N}]' \) is equivalent to \( v_i - \left( \frac{1}{N} \sum_{j=1}^{N} v_j \right) \)
\[ e_{i,j} = [e_{i,1}, e_{i,2}, \ldots, e_{i,N}]' ; \]
\[ \ddot{u} = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \left[ \phi_1(\text{sign}(e_{i,j} - e_{j,i})) + \phi_2(\text{sign}(e_{i,j} - e_{j,i})) \right] \]
When \( e_{ix} - e_{yx} \to 0 \)
\( e_{vy} - e_{yx} \to 0 \) That is \( t \to \infty \) when
\[
\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0
\]
\[
\lim_{t \to \infty} \|y_i(t) - v_j(t)\| = 0
\]
, the sliding surface designed in a fixed-time can achieve a second-order consensus.

4.3 Design control input:

\[
u_i(t) = g_i^{-1}(x_i, v_i, t) \left( \dot{u}_i - \int o_i \sigma_i(t) + k_i \text{sgn}(\sigma_i(t))dt - f(x_i, v_i, t) \right)
\]

(12)

where \( o_i \sigma_i(t) \) is a filter to smooth interference, \( o_i > 0 \), \( k_i > \varepsilon + \eta \). If the disturbance \( d_i(x, v, t) \) is smooth, it exists
\[
\|\dot{d}_i(x, v, t)\| \leq \varepsilon, \varepsilon < 0, \eta < 0
\]

When the trajectory reaches the surface \( \sigma_i \), a consensus can be reached within a fixed-time. Construct the Lyapunov function:

\[
V = \frac{1}{2} \sum_{i=1}^{n} \dot{\gamma}_i(t)^T \gamma_i(t)
\]

(13)

\[
\dot{V} = \sum_{i=1}^{n} \dot{\gamma}_i(t)^T [d_i(x, v, t) + \int_0^t k_i \text{sgn}(\sigma_i(t))dv] \leq m_i \sqrt{V} \leq 0
\]

After verification, it proves that the system has good robustness.

5. Example analysis

![Microgrid Test Framework Diagram](image)

Fig. 2 the microgrid test framework diagram

According to the communication topology shown in Fig 2. Disturbance is represented by \( d_i = \sin 30t \).

Table I provides the electrical parameters, \( K_{PV}, K_{IV}, K_{PC}, K_{IC} \) is the internal parameter of the system voltage controller, \( \alpha = 0.7, \beta = 1.2 \) and the control gain \( k_i > 32 \).

| DGs | DGs 1 and 2 | DGs 3 and 4 |
|-----|-------------|-------------|
| \( n_Q \) | 1.3×10^{-4} | 1.5×10^{-2} |
| \( K_{PV} \) | 0.1 | 0.05 |
| \( K_{IV} \) | 420 | 390 |
| \( K_{PC} \) | 15 | 10.5 |
| \( K_{IC} \) | 2×10^{4} | 1.6×10^{4} |
First, analyze and compare the error trajectories of different DGs. \((x_i - \bar{x})_{\text{min}} = -7.449\), \((x_i - \bar{x})_{\text{max}} = 6.2537\). As shown in Fig. 3, regardless of the initial conditions, the tested DGs will reach agreement within about 15s after being disturbed.

![Fig. 3 The error trajectory of DG under the influence of disturbance.](image)

To verify the effectiveness of fixed-time distributed secondary control, we perform simulation analysis. First, when \(t=0\)s, only the main control is activated; when \(t=1.5\)s, it is connected to the proposed distributed secondary control of DG at fixed-time. Fig. 4 shows the output voltage magnitude of the distributed power supply. As shown in the figure, after the secondary control is turned on, the voltage will quickly return to the reference value within a fixed-time after being disturbed by the disturbance. The above results show that the fixed-time distributed-level coordinated control strategy has good stability and robustness.

![Fig. 4 DGs output (a) voltage magnitude](image)

6. Conclusions
This paper studies the voltage recovery problem of island microgrid, and uses distributed sparse network for information communication, which greatly reduces the communication cost. Using the second-order fixed-time consistency method based on multi-agents, the voltage can be restored to all reference values stably and efficiently within a fixed-time. It does not rely on the initial conditions of the system while...
ensuring fast system convergence and strong anti-interference ability. Therefore, the offline pre-judgment and other prior operations can be satisfied. The sliding mode control technology is introduced to smooth the disturbance, which can effectively solve the influence of parameter uncertainty and strong disturbance on the system in the microgrid. Lyapunov proved that the system has good robustness. Finally, we conducted a simulation, and the results showed that the fixed-time scheme improves the synchronization speed and has better stability.

Acknowledgment

This work is supported by Liaoning Revitalization Talents Program (XLYC1907138), the Key R&D Program of Liaoning Province (2020JH2/10300101), the Doctoral Scientific Research Foundation of Liaoning Province (2020-BS-181), the Natural Science Foundation of Liaoning Province (2019-MS-239) and the Technology Innovation Talent Fund of Shenyang (RC190360, RC200252).

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