Emergence of classical theories from quantum mechanics

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Abstract. Three problems stand in the way of deriving classical theories from quantum mechanics: those of realist interpretation, of classical properties and of quantum measurement. Recently, we have identified some tacit assumptions that lie at the roots of these problems. Thus, a realist interpretation is hindered by the assumption that the only properties of quantum systems are values of observables. If one simply postulates the properties to be objective that are uniquely defined by preparation then all difficulties disappear. As for classical properties, the wrong assumption is that there are arbitrarily sharp classical trajectories. It turns out that fuzzy classical trajectories can be obtained from quantum mechanics by taking the limit of high entropy. Finally, standard quantum mechanics implies that any registration on a quantum system is disturbed by all quantum systems of the same kind existing somewhere in the universe. If one works out systematically how quantum mechanics must be corrected so that there is no such disturbance, one finds a new interpretation of von Neumann’s “first kind of dynamics”, and so a new way to a solution of the quantum measurement problem. The present paper gives a very short review of this work.

1. Introduction
All systems investigated by classical physics seem to consist of quantum particles and their dependent fields. Then, they seem to be also quantum systems and their classical properties ought to be derivable from quantum mechanics. More precisely, we expect that one can find quantum models of all classical properties. There are three independent problems that stand in the way of a satisfactory completion of this project.

First, classical systems and their properties are objective at least in the sense that the assumption of their real existence, independent of measurements, does not lead to any contradictions. Exactly this does not seem to hold for quantum systems and their properties, which is called Problem of Realist Interpretation of quantum mechanics. Second, classical properties such as a position do not allow linear superposition. Nobody has ever seen a table to be in a linear superposition of being simultaneously in the kitchen as well as in the bedroom. Moreover, observation of the table does not shift it while quantum registration changes the state of the registered system. Let us call this Problem of Classical Properties. Finally, evidence suggests the assumption that the registration apparatus always is in a well-defined classical state at the end of any quantum measurement indicating just one value of the registered
observable. This is called objectification requirement \[1\]. However, if the registered system is in a linear superposition of different eigenvectors of the observable, then the linearity of Schrödinger equation implies that the apparatus is also in a linear superposition eigenstates of its pointer observable. Let us call this Problem of Quantum Measurement.

Our work during the recent five years has lead to a surprising discovery: The problems are considerably aggravated by some inveterate tacit assumptions about quantum mechanics that are clearly wrong if one states them explicitly \[2, 3, 4, 5, 6, 7\]. Quantum theory purified from these errors has been called The Reformed Quantum Mechanics. The present paper gives a very short review of our main ideas. The paper starts with the Problem of Quantum Measurement because only this requires changes in the conceptual and mathematical structure of quantum mechanics, proceeds to the Problem of Realist Interpretation and finishes by the Problem of Classical Properties.

2. Problem of Quantum Measurement

In classical physics, any measurement can be done, at least in principle, so that the system on which we measure is not disturbed by the measurement. We assume, as did the co-called Copenhagen interpretation, that this is not true for quantum measurement. A registration of any observable on quantum system \(S\) needs a special device \(A\), the so-called registration apparatus, and the registration is always an interaction between \(S\) and \(A\) that changes the state of the \(S\) except for some special cases. Moreover, to speak of the state of \(S\), there must be a preparation. Thus each measurement on microsystems can be split into preparation and registration.

The errors that cause the confusion in the theory of quantum measurement are two: 1) neglecting the disturbance of registration due to identical particles and 2) disregarding the role and structure of detectors.

2.1. Disturbance of registration due to identical particles

Let us consider two experiments.

**Experiment I**: State \(\psi(\vec{x}_1)\) of particle \(S_1\) is prepared in our laboratory.

**Experiment II**: State \(\psi(\vec{x}_1)\) is prepared as in Experiment I and state \(\phi(\vec{x}_2)\) of particle \(S_2\) of the same type is prepared simultaneously in a remote laboratory.

If our laboratory does not know about the second one, it believes that the state of \(S_1\) is \(\psi(\vec{x}_1)\). If it does then it believes that the state is

\[
2^{-1/2} \left( \psi(\vec{x}_1) \phi(\vec{x}_2) \pm \phi(\vec{x}_1) \psi(\vec{x}_2) \right) . \tag{1}
\]

Thus, it seems that the notions of preparation and of state are ambiguous. Has this ambiguity any observable consequences? To answer this question, let us first consider Experiment I supplemented by a registration corresponding to the observable of \(S_1\) with kernel \(a(\vec{x}_1; \vec{x}_1')\), and let the registration be made in our laboratory. Measurements of this kind lead to average value

\[
\int d^3x_1 d^3x_1' a(\vec{x}_1; \vec{x}_1') \psi^*(\vec{x}_1) \psi(\vec{x}_1') . \tag{2}
\]

Second, perform Experiment II supplemented by the registration by the same apparatus in our laboratory as above. Because the apparatus cannot distinguish between the contributions by the two particles, the correct observable corresponding to this registration now is:

\[
a(\vec{x}_1; \vec{x}_1') \delta(\vec{x}_2 - \vec{x}_2') + \delta(\vec{x}_1 - \vec{x}_1') a(\vec{x}_2; \vec{x}_2') . \tag{3}
\]
Such measurements lead to the average value defined by Eqs. (1) and (3):
\[
\int d^3x_1 d^3x'_1 a(\vec{x}_1; \vec{x}'_1) \psi^*(\vec{x}_1) \psi(\vec{x}'_1) + \int d^3x_1 d^3x'_1 a(\vec{x}_1; \vec{x}'_1) \phi^*(\vec{x}_1) \phi(\vec{x}'_1). \tag{4}
\]
Expression (4) appreciably differs from (2) for all standard observables such as position, momentum, spin, angular momentum energy etc.

We conclude that the registrations of the standard quantum observables on a system is always disturbed by all other systems of the same kind existing somewhere in the universe.

\subsection*{2.2. Separation status}
Cluster separability principle is a kind of locality assumption that has been fruitful in several branches of quantum physics, see e.g. [8] and Chap. 4 of [9]. An application of the principle to identical particles can be found for instance in [10] p. 128. We formulate it as follows:

**Cluster Separability Principle** No quantum experiment with a system in a local laboratory is affected by the mere presence of an identical system in remote parts of the universe.

This is violated by results similar to those of the previous section but confirmed by experience. We must therefore correct quantum mechanics so that it satisfies the Cluster Separability Principle.

We introduce an important locality property of observables [4] (for generalisation to composite systems, see [5]; a similar local condition on observables has been introduced in [11, 12]):

**Definition 1** Let \( D \subset \mathbb{R}^3 \) be open. Operator with kernel \( a(\vec{x}_1; \vec{x}'_1) \) is \( D \)-local if
\[
\int d^3x'_1 a(\vec{x}_1; \vec{x}'_1)f(\vec{x}'_1) = \int d^3x_1 a(\vec{x}_1; \vec{x}'_1)f(\vec{x}_1) = 0,
\]
for any test function \( f \) that vanishes in \( D \).

Now assume for Experiment II, to keep everything simple, that
(i) our laboratory is inside open set \( D \subset \mathbb{R}^3 \),
(ii) \( \text{supp} \phi \cap D = \emptyset \).

Then, the second term in (4) vanishes for all \( D \)-local observables and Eqs. (2) and (4) agree in this case (for a more general theorem see [5]). This suggests the following approach.

(i) We introduce the key notion of our theory:

**Definition 2** Let \( S \) be a particle and \( D \subset \mathbb{R}^3 \) an open set satisfying the conditions:
- Registrations of any \( D \)-local observable \( A \) of \( S \) lead to average \( \langle \psi(\vec{x}) \vert A \psi(\vec{x}) \rangle \) for all states \( \psi(\vec{x}) \) of \( S \).
- \( S \) is prepared in state \( \phi(\vec{x}) \) such that \( \text{supp} \phi \cap D \neq \emptyset \).

In such a case, we say that \( S \) has separation status \( D \).

For generalisation to composite systems and non-vector states, see [5].

(ii) We assume: Any preparation of \( S \) must give it a non-trivial separation status \( D \neq \emptyset \). Then \( D \)-local observables are individually registrable on \( S \) but only these are.

\footnote{Further generalisation is possible. We are working on two: First, to replace the condition \( \text{supp} \phi \cap D \neq \emptyset \) by \( \int_D d^3x \vert \phi \vert^2 < \epsilon \) and that on the registered averages to the averages lying in intervals \( \langle \psi(\vec{x}) \vert A \psi(\vec{x}) \rangle - \epsilon', \langle \psi(\vec{x}) \vert A \psi(\vec{x}) \rangle + \epsilon' \), the epsilons being related to the finite sensitivity of registration apparatuses. Second, to impose conditions on the ranges of momenta defined by the sensitivity of detectors in addition to those on coordinates.}
2.3. Registration
Preparation transfers a trivial into a non-trivial separation status. Thus, the separation status changes during a preparation. What is the relation of registrations to separation status change?

An important assumption of our theory of measurement is:

**Pointer hypothesis** *Any registration apparatus for microsystems must contain at least one detector and every ‘reading of a pointer value’ (see e.g. [1]) is a macroscopic signal from a detector.*

We take the definition of detector from experimentalists [13, 14]. Then, during registration, the system must enter the sensitive matter of a detector and its non-trivial separation status changes into a trivial one [4, 5].

In this way, preparation and registration have even more importance in our theory than in Copenhagen interpretation: they must include changes of separation status. Clearly, any change of separation status of a system $S$ is also a change of its observable algebra, i.e., change of the kinematics of $S$ [6].

What has been said up to now shows that textbook quantum mechanics is incomplete in the following sense:

(i) It accepts and knows only two separation statuses:

(a) that of isolated systems, $D = \mathbb{R}^3$, with all self-adjoint operators as observables, and
(b) that of a member of a system of identical particles, $D = \emptyset$, with no individual observables of its own.

(ii) It provides no rules for changes of separation status.

Hence, quantum mechanics must be supplemented by a theory of general separation status [4], a correct theory of observables [4] and by new rules that govern processes in which separation status changes [7].

In [7], a general rule for interaction between microsystem $S$ and macrosystem $A$ has been motivated and discussed. Formally, the process is decomposed into three steps. First, there is a change of kinematics and second, the unitary evolution of the state of $S + A$ in the framework of the new kinematics. Finally, the state of $S + A$ given by the unitary evolution must be corrected by a state reduction at the time of any detector signal in a way that is uniquely determined by the signals. The decomposition into the three steps is not a description of how things proceed in time: the rule gives only the end state.

The state reduction itself consists of two changes. First, some non-diagonal elements of the state operator of the total isolated system are erased. Second, the resulting mixture is postulated to be a proper one (a gemenge, see [4]). Something similar to the first change can also be achieved for an open subsystem of the total isolated system within standard quantum mechanics by methods such as quantum decoherence [1]. However, the second change can only be accomplished (or circumvented) with help of new assumptions that must be added to quantum mechanics such as the Everett interpretation in the case of decoherence, see [1, 15]. For a discussion of super-selection methods such as Wan’s [11], see [7].

The Reformed Quantum Mechanics thus returns to von Neumann’s “two kinds of dynamics” (see also [4]) but its notion of state reduction differs from von Neumann’s in two points. First, it is less ad hoc because it is justified by the argument of separation status change, which is logically independent from the proper quantum measurement problem and, second, it is more specific because it happens only in a detector and its form is determined by objective processes inside the detector sensitive matter including approach to thermal equilibrium.
3. The problem of realist interpretation

The very subject of quantum mechanics is usually formulated very cautiously, e.g., [10], p. 13:

\[ \ldots \text{quantum theory is a set of rules allowing the computation of probabilities for the outcomes of tests [registrations] which follow specific preparations.} \]

This can be contrasted with naive expectation that quantum theory studies properties of existing microscopic objects.

The cause of such reserve is an apparent lack of objective properties of quantum systems. For example, one cannot ascribe values of observables to quantum systems prior to registrations, or else there will be nasty contradictions such as those due to contextuality [10], p. 187. An existing object must have enough objective properties. Thus, the very existence of quantum systems is uncertain, see, e.g., [10], p. 13.

The origin of this difficulty is the surprisingly inveterate tacit assumption that the values of observables of a quantum system \( S \) are a kind of basic properties of \( S \) so that no other properties can be accepted in their place.

In our opinion, every value \( a \) of observable \( A \) is only created during registration of \( A \) by an apparatus \( A \) and it is an objective property of composite \( S + A \), not \( S \). It contains only some indirect information about \( S \). Then, all the enormous effort to justify values of observables of \( A \) as properties of \( S \) is futile and must be rejected. Instead, we postulate [2, 5]:

**Basic Ontological Hypothesis of Quantum Mechanics** A property is objective if its value is uniquely determined by a preparation according to the rules of quantum mechanics. The “value” is the value of the mathematical expression that describes the property and it may be more general than just a real number. No registration is necessary to establish such a property but correct registrations cannot disprove its value; in most cases, registrations can confirm the value.

Thus, we associate objective properties with preparations instead of registrations. Let us give some obvious examples of objective properties. First, the so-called structural properties: mass, charge, spin, structure of Hamiltonian for isolated systems, etc. Second, the so-called dynamical properties: state operator (not just wave function), average value and variance of an observable, etc.

It may be helpful to recall that preparations include more general processes than just manipulations in a human laboratory. For example, solar neutrinos are prepared at the centre of the Sun. But even if a property is defined by such a manipulation, the assumption that it can be ascribed to the prepared system is non-trivial and helpful. In fact, this is also assumed in classical physics for systems created in laboratory. In any case, a preparation is defined by objective (classical) conditions on \( S \), which determine the statistics of subsequent registrations on \( S \). More about the relation between objective and subjective aspects of probability theory can be found in [5].

We have shown in [5] that all properties determined by preparations form a Boolean lattice similarly to all objective properties of a classical system. Hence, there is no need for an extra logic in quantum mechanics.

Moreover, one can completely describe the dynamical situation of any quantum system by these objective properties. Hence, the condition on a system to be a physical object are satisfied. Such an object—a real quantum system—is theoretically modelled by an occupied quantum state [5]. In all considerations about quantum systems, it is then possible to refer directly to their objective properties, e.g., to speak of registration devices instead of observers. “Unspeakable” becomes “speakable” in quantum mechanics and it is a surprising liberation.
4. The problem of classical properties

Quantum mechanics leads to linear superposition of different states of macroscopic systems but classical theories such as Newtonian mechanics and Maxwellian electrodynamics do not allow such superpositions. How can this feature of the theories be derived from quantum mechanics?

The origin of all difficulties to obtain such derivation is a common inveterate assumption that a sharp trajectory is the key objective property of classical systems. Hence, quantum models of classical systems usually start from states with minimum uncertainty. These are pure and can easily be linearly superposed.

However, a completely sharp classical trajectory is a figment of imagination. Real states of all classical theories always have a non-zero variance, which is always much larger than the minimum quantum uncertainty. This is an old idea, cf. [16, 17]. It is, therefore, sufficient that quantum models of classical systems explain such fuzzy states. Instead of sharp trajectories, we assume:

**Basic classicality hypothesis** To construct models of classical systems, high-entropy states of macroscopic quantum systems must be used.

This seems to be in accord with state reduction being associated with approach to the thermal equilibrium of a detector (Sec. 2.3). Moreover, high-entropy states are mixed and cannot be linearly superposed. Of course, it is possible (but difficult) to prepare other states of macroscopic systems: strong laser beams, large EBC, etc. This simply means that not every state of a macroscopic system must be classical.

An example of suitable states modelling Newton mechanics of mass centre are the so-called maximum entropy (ME) packets: they maximise entropy for fixed averages and variances of coordinates and momenta [3]:

**Theorem 1** Let \( S \) be a quantum system with coordinates \( q_k \) and momenta \( p_k \), \( k = 1, \cdots, n \). Then, the state operator of the ME packet with averages and variances \( Q_k, \Delta Q_k \) of coordinates and \( P_k, \Delta P_k \) of momenta is

\[
T(Q,P,\Delta Q,\Delta P) = \prod_{k=1}^{n} \left[ \frac{2}{\nu_k^2 - 1} \exp \left( -\frac{1}{\hbar} \ln \frac{\nu_k + 1}{\nu_k - 1} K_k \right) \right],
\]

where

\[
K_k = \frac{1}{2} \frac{\Delta P_k}{\Delta Q_k} (q_k - Q_k)^2 + \frac{1}{2} \frac{\Delta Q_k}{\Delta P_k} (p_k - P_k)^2
\]

and

\[
\nu_k = \frac{2 \Delta P_k \Delta Q_k}{\hbar} \in (1, \infty).
\]

Here, \( K_k \) is not the Hamiltonian of \( S \): the Hamiltonian can be arbitrary. A similar theorem can be shown for classical EM packets [3].

Von Neumann entropy of \( T(Q,P,\Delta Q,\Delta P) \) is

\[
S(T(Q,P,\Delta Q,\Delta P)) = tr[T(Q,P,\Delta Q,\Delta P) \ln T(Q,P,\Delta Q,\Delta P)] = \sum_{k=1}^{n} \left( \frac{\nu_k + 1}{2} \ln(\nu_k + 1) - \frac{\nu_k - 1}{2} \ln(\nu_k - 1) - \ln 2 \right)
\]

and can be shown to be an increasing function of \( \nu_k \). Limit

\[
\nu_k \to 1
\]
for all \( k \) gives

\[
S(T(Q, P, \Delta Q, \Delta P)) \to 0, \quad T(Q, P, \Delta Q, \Delta P) \to |Q, P, \Delta Q, \Delta P\rangle \langle Q, P, \Delta Q, \Delta P|,
\]

where \( |Q, P, \Delta Q, \Delta P\rangle \) is the Gaussian wave packet for the averages and variances \( Q, P, \Delta Q, \Delta P \).

The time dependence of the averages and variances can be calculated from ME packets as initial states using the Hamiltonian of \( S \) in both Newtonian and quantum mechanics. Comparison of function \( Q_k(t), P_k(t), \Delta Q_k(t) \) and \( \Delta P_k(t) \) in the two theories shows full agreement for all Hamiltonians (potential functions) if

\[
\nu_k \to \infty
\]

for all \( k \). Thus, classical limit is just the opposite of the textbook one: it is not the Gaussian wave packet but a high entropy limit.

Observe that classical properties \( Q_k(t), P_k(t), \Delta Q_k(t) \) and \( \Delta P_k(t) \) are uniquely determined by the preparation process and are therefore objective. More discussion can be found in [2] and [5].

We close this section by a few words on semi-classical (or WKB) approximation. It is an approximation method within quantum mechanics, usually defined as the limit \( \hbar \to 0 \) in some quantum expressions [10]. The resulting equations may be similar to the corresponding classical equations. Limit \( \nu \to \infty \) also results from \( \hbar \to 0 \), if the variances are kept constant. However, the semi-classical approximation alone is not sufficient for a derivation of complete classical theories [10].

5. Conclusion

(i) Quantum mechanics is as objective as any other theory of physics.

(ii) Classical objects can be modelled by high-entropy states of macroscopic quantum systems. Classical limit is a suitably taken high-entropy limit. Coherent states have nothing to do with classical limit.

(iii) The theory of observables and of quantum measurement must be corrected to remove the disturbance due to identical particles. This leads to a solution of quantum measurement problem based on a state reduction. Our state reduction, however, is less ad hoc than von Neumann’s because it is justified by a change of separation status, and it is more specific because it is associated with objective processes in the sensitive matter of a detector.

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