Extreme wind turbine response during operation

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Abstract. Estimation of extreme response values is very important for structural design of wind turbines. Due to the influence of control system and nonlinear structural behavior the extreme response is usually assessed based on simulation of turbulence time series. In this paper the problem of statistical load extrapolation is considered using techniques from structural reliability theory. Different simulation techniques to estimate extreme response characteristics are described and compared, including crude Monte Carlo simulation, Importance Sampling, and splitting methods such as the Russian Roulette and the Double and Clump algorithm. A statistically consistent technique is described for including statistical uncertainty and assessing the extreme 50-year response using simulated time series and conditioned on the model parameters. The peak over threshold method together with the Maximum Likelihood Method provides a tool to obtain consistent estimates incl. the statistical uncertainty. An illustrative example indicates that the statistical uncertainty is important compared to the coefficient of variation of the extreme response when the number of 10 minutes simulations at each mean wind speed is limited to 10.

1. Introduction

Structural design of wind turbines for extreme load is normally based computer simulated response time series, based on which the extreme 50 years design load is estimated. Each response series is for a given mean wind and turbulence, which then is weighted with the long-term distribution of these. The wind turbine structural system is modeled by a non-linear finite element model and time series of the turbulence are generally simulated based on the models in design standard IEC61400-1 [1].

However, estimation of the extreme response is complicated among other things due to the advanced control mechanisms used in modern wind turbines, which result in non-linear behaviour of the response. Further, also the non-Gaussian character of the turbulence distribution in the tail region complicates the estimation. The problem of extreme response estimation has been subject of many papers, see e.g. [2] and [3]. Further, a method is described in the design standard IEC61400-1 [1]. Unfortunately, the different methods do not result in consistent results. Both the number of simulations required how to extract extreme data for statistical analysis and the underlying distribution type for statistical extrapolation are subject to discussion.

In this paper the problem of statistical load extrapolation is considered using techniques from structural reliability theory and turbulence theory. Different simulation techniques are considered, where realizations of turbulence time series are assumed to be generated using a spectral presentation of a frozen turbulence field, which is convected with the mean wind velocity into the rotor according to the Taylor’s hypothesis. The underlying homogeneous and anisotropic presumes constant mean
shear throughout the rotor area, see [4]. The structural analysis can be based on a multibody analysis of the turbine, where each wing is modelled in a local frame of reference following the substructure, but not attached to this, see e.g. [5].

In order to assess the uncertainty related to response quantities the following uncertainty sources have to be considered:

- Physical uncertainty (aleatory): e.g. maximum annual mean wind speed and turbulence intensity given mean wind speed;
- Statistical uncertainty (epistemic): due to limited number of data;
- Model uncertainty (epistemic): choice of statistical model (incl. distribution type), neglected effect of control system for extreme response, mathematical idealisation of wind turbine in computer model as compared to real behaviour of wind turbine.

It is noted that epistemic uncertainty can be decreased (removed) by more data, better models, more stochastic variables, etc. whereas physical uncertainty cannot be removed. This is important in reliability assessment for wind turbines. For wind turbines it could be a question of effort (cost) of more simulations, better numerical modeling of wind turbine behavior incl. control system, etc. against the benefits of a design which could be expected to be less expensive and not the less with less uncertainty related to expected extreme response behavior.

In section 2 different simulation techniques to estimate extreme response characteristics are described and compared, including crude Monte Carlo simulation, Importance Sampling, and splitting methods such as the Russian Roulette and the Double and Clump algorithm. These simulation techniques are discussed in relation to the physical uncertainties associated with the wind field and the model uncertainties with the parameters in the control algorithm, undamped eigenfrequencies, damping ratios at stand still etc.

Physical uncertainty can be assessed by statistical analysis of historical environmental data and/or data samples of e.g. strengths. Model uncertainty can for wind turbines be assessed by measurements on prototype wind turbines and/or subjective information using Bayesian statistical techniques, see [6].

The aspect of statistical uncertainty is considered in section 3. If a set of time series is available e.g. obtained by Monte Carlo simulation, then the peak over threshold method can be used to estimate extreme values when combined with a suitable distribution type. In [7] different extreme distribution types are investigated, and statistical uncertainty assessed by bootstrapping. In section 3 is described consistent technique is described to assess the extreme 50-year response using simulated time series and conditioned on the model parameters. The peak over threshold method together with the Maximum Likelihood Method provides a tool to obtain consistent estimates incl. the statistical uncertainty. This procedure is illustrated in section 4.

2. Simulation techniques for estimation of extreme response

Wind turbines are complex dynamic systems where the dynamic response is uncertain partly because the randomness of the external wind loading and partly because of uncertainty of the effectiveness of the control mechanisms of the yaw, tilt and rotational speed of the rotor and the pitch of the blades. The evolution with time of stochastic structural response quantities such as mean values, variances, first passage time distributions and extreme values are determined by the structural equations of motion. However, due to the complexity of these equations the said response quantities cannot be determined by analytical methods. Hence, the determination of any stochastic response must be based on numerical simulation, e.g. Monte Carlo simulation.

The wind load is specified by a stochastic model in time and space for the undisturbed fluid velocity at the rotor position. Typically, a Gaussian model is assumed, which demands for a model for the mean wind field and for the three-dimensional correlation structure of the turbulence. The mean wind field includes parameters describing the surface irregularities, and the direction and magnitude of the time-averaged mean wind (typically during a 10 minutes period) at a referential height. The turbulence field is usually modelled by a homogeneous, non-isotropic frozen Gaussian field, which is convected into the rotor according to Taylor's hypothesis with the mean wind speed, which is assumed...
to be constant over the rotor area. Anisotropy due to the shear of the mean value field may be introduced in the model by rapid distortion theory, [4], [8]. Typically, this modelling requires a parameter specifying the average mean shear velocity gradient over the rotor, a length scale parameter and a variance parameter. The length scale specifies the correlation structure and the variance parameter indicates the component variance of the underlying isotropic and homogeneous frozen turbulence field. Finally, a parameter is introduced specifying the time length of non-isotropic vortices before the energy of these has been transferred to smaller and isotropic vortices via the Richardson cascade process, when left to themselves.

Figure 1. One-sided auto-spectral density function of along-wind turbulence. Fixed frame of reference: -----. Rotating frame of reference: -----. The control algorithms contain parameters specifying the sensor and activator dynamics. Malfunctioning of the control mechanisms during operation means that these parameters have changed, leading to dangerous situations may. As an example the auto-spectra of the turbulence components contain spectral peaks at the frequencies $\Omega$, $2\Omega$, ... when observed in a frame of reference fixed to a rotating blade. Whenever these peaks approach the undamped fixed bay circular eigenfrequencies $\omega_1$, $\omega_2$, ..., large resonance blade or edge-wise vibrations may occur. Similarly, if $3\Omega$ comes close to the so-called tower frequency (i.e. the fundamental circular eigenfrequency of the wind turbine at stand-still) resonance of the whole structure may occur. Another critical situation occurs if the pitch actuators are malfunctioning, causing stall of the blades, [9]. This has as its main effect that most or all of the aerodynamic damping is lost causing highly increased structural responses. The indicated situations may be modelled stochastically by considering the parameters of the control algorithms as random variables. These parameters, along with structural modal parameters such as undamped eigenfrequencies, damping ratios at stand still etc., are assembled in an $m$-dimensional random vector $Y$.

The degrees of freedom specifying the structural performance are assembled in the $n$-dimensional vector process $X(t)$. Notice, that $X(t) = X(Y,t)$ is random both because of the exposure from the stochastic turbulence field, and because of the uncertain structural and control parameters. If the turbulence field is parameterized, either by means of the basic variables used in the generation of samples of the frozen field, or by means of some kind of stochastic reduction scheme such as stochastic finite elements, polynomial chaos etc., these parameters may be included into the parameter vector $Y$, which in this case becomes of very high dimension. At first this representation is assumed.

Let $X(t)$ denote any response quantity included in or derived from the response vector $X(t)$, and let $X_{\text{max}}$ be the maximum value registered in the referential interval $T$. Failure of the considered response quantity relative to the barrier $x^*$ implies the event $\{X_{\text{max}} \geq x^*\}$. Notice that this is merely a component limit state, which provides a lower bound of the system failure, where joint failure events need to be considered. Assume that $N$ independent realizations $x_1(t), x_2(t), \ldots, x_N(t)$ of $X(t)$ have been generated by numerical time integration of the equations of motion, and let $x_{\text{max},i}$ denote the
corresponding samples of $X_{max}$. The failure probability may then be calculated from the unbiased estimator

$$P_j = \int_S f_Y(y)dy = P(X_{max} \geq x^*) \approx \frac{1}{N} \sum_{j=1}^{N} I_j$$

where $S$ denotes the safe domain in the parameter space, $f_Y(y)$ is the joint probability density of the basic variables, and $I_j$ is an indicator function, which is equal to 1 or 0, depending on whether a failure event has occurred or not. General for all Monte Carlo methods is that the variance of the estimator of the failure probability decrease as $\frac{1}{N}$ with the number of sample curves. Dynamical systems under service must be reliable, which means that failure events should be rare. This means that the indicated so-called crude Monte Carlo algorithm is not efficient, because only a small fraction of realizations contribute to the statistical information. Since each realization requires a numerical structural dynamical analysis one has been looking at so-called variance reduction Monte Carlo methods such as Importance Sampling, Antithetic Variates, Stratified Sampling, Latin Hypercube in order to reduce the estimation error for a given number of realizations, see e.g. [10] and [11]. Importance sampling implies that the sample values of the basic variables $y_j$ are not drawn from their original joint probability density function $f_y(y)$, but rather from a so-called simulation density $g_y(y)$. In this case (1) is replaced by the following un-biased estimator

$$P_j = \int_S f_Y(y)g_Y(y)dy = P(X_{max} \geq x^*) \approx \frac{1}{N} \sum_{j=1}^{N} I_j \frac{f_Y(y_j)}{g_Y(y_j)}$$

where $I_j$ is the same indicator function as in (1). $g_y(y)$ is chosen such that it is provides a higher failure compared to the expected one, i.e. the probability mass of the simulation density should be close to the limit state of the system. In the examples above the probability mass should be concentrated at samples of the structural and control random variables making the described failure scenarios likely.

The applications of Importance Sampling has been questioned for fully parameterized complex dynamic systems such as a wind turbine exposed to stochastic turbulent wind because of the difficulty in establishing the simulation density for the highly dimensional parameter vector. At least the specification of the simulation density for the random parameters describing the frozen turbulence model requires a very profound understanding of the evolution of dynamic system. The pragmatic compromise is to apply Importance Sampling to the basic stochastic variables modelling the structure, the control algorithms, the mean wind field, the correlation length, the isotropic variance and the life length of anisotropic vortices, whereas the amplitudes and phases of the frozen field conditioned on the previous mentioned variables are simulated by crude Monte Carlo simulation.

Alternatively to Importance Sampling, and in an attempt to overcome the problem with the specification of the sampling density, so-called splitting methods such as the Russian Roulette and Splitting algorithm and the Double and Clump algorithm have been proposed, [12], [13]. These methods require a separation of the safe domain into 'important' and 'unimportant' subsets, which is a less demanding knowledge of the safe domain than required for Importance Sampling. Whenever a sample passes from the unimportant to the important subset it splits into two, i.e. one more sample curve is initiated which is classified as unimportant. Unimportant samples are checked in subsequent time steps by a procedure to either discard or preserve them. In this way focus in the time integration process is always on a rather limited number of important sample curves.

The methods described above which take into account uncertainty in the parameters related to the control algorithm, undamped eigenfrequencies, damping ratios at stand still etc. (in random vector $Y$) are not at present used in practical applications. Instead best estimates or conservative values are used.
for these parameters, and only the randomness related to the wind field is taken into account. The next section describes a method for statistical assessment of extreme response conditioned on specified realizations of the random vector $Y$.

3. Statistical assessment based on Crude Monte Carlo response simulations

This section describes a consistent technique for estimation of quantiles of extreme response taking into account statistical uncertainty. The technique is based on crude Monte Carlo simulation of time series of the response quantity of interest with given values of the random vector $Y$. Similar methods have been used to estimate extreme wave heights and wave loads for oil & gas offshore structures, see e.g. [14], here based on measured time series.

In accordance with the method described in IEC 61400-1 [1] the probability distribution function for the extreme response, $X_{\text{max}}$ during operation with a reference period $T$ (usually = 10 minutes) is estimated from

$$
F_{X_{\text{max}}}(x) = 1 - P(x) = 1 - P(X_{\text{max}} \geq x|T) = 1 - \int_{v_{\text{in}}}^{v_{\text{out}}} P(X_{\text{max}} \geq x|T, v) f_v(v) dv
$$

(3)

where $V$ is the mean wind speed with density function $f_v$. $V_{\text{in}}$ and $V_{\text{out}}$ are the cut-in and cut-out wind velocities. The probability $P(X_{\text{max}} \geq x|T, v)$ conditioned on mean wind speed $V=v$ is obtained by simulation. Assuming independence between successive 10 minutes periods the $T_R=50$-year max response is obtained from

$$
P(X_{\text{max}} \geq x|T) = P(x) = \frac{T}{T_R} = 3.8 \cdot 10^{-7}
$$

(4)

The following methods are used:

- Peak Over Threshold (POT) method is used to extract extreme response values. The largest 30 independent extremes are selected for each wind speed bin.
- Maximum Likelihood Method (MLM) is used to determine the statistical parameters and to estimate the uncertainty related to these parameters.
- Estimation of extreme quantiles (e.g. 50 year)
  - Without statistical uncertainty and
  - With statistical uncertainty using e.g. FORM (First Order Reliability Method) techniques, see [15]

The following aspects have to be considered:

- The datasets to be analyzed should come from a statistical homogeneous population – that means that if different physical / control aspects control the extreme response then the response should be separated in different populations with same controlling physical / control aspects.
- The result of the statistical evaluation of extreme response data is the Quantile value – e.g. with 50 year return period, and it is the COV (Coefficient Of Variation) which is important since partial safety factors depend on the COV, see e.g. ISO 2394 [16].

Based on 10 minute simulation results for mean wind bins and turbulence the peak over threshold method is used to extract extreme response values. Different extreme distribution probabilistic models are evaluated with the statistical parameters estimated using the Maximum Likelihood method which implies that also the statistical uncertainty can be estimated, see [17]. As an example the statistical parameters, $\alpha$ and $\beta$ in a Weibull distribution, can be determined using the Maximum-Likelihood method where the Log-Likelihood function is written:
\[
\ln L(\alpha, \beta) = \ln \left( \prod_{i=1}^{n} f_X(x_i) \right) = \sum_{i=1}^{n} \ln \left( \frac{\alpha}{\beta} \left( \frac{x_i - \gamma}{\beta} \right)^{a-1} \exp \left( -\frac{(x_i - \gamma)^a}{\beta} \right) \right)
\]

(5)

where \( f_X(x) \) is the density function and \( x_i, i=1,n \) are the \( n \) data available. The optimization problem \( \max_{\alpha, \beta} \ln L(\alpha, \beta) \) is solved using a standard nonlinear optimizer, e.g. the NLPQL algorithm, see [18]. In this example \( \gamma \) is assumed fixed, but can easily be included as optimization parameter. Because the parameters \( \alpha \) and \( \beta \) are determined using a limited number of data they are subject to statistical uncertainty. Since the parameters are estimated by the Maximum Likelihood technique they become asymptotically Normally distributed stochastic variables with expected values equal to the Maximum Likelihood estimators and covariance matrix equal to, see e.g. [17]

\[
C_{\alpha, \beta} = \left[ -H_{\alpha \beta} \right]^{-1} = \begin{bmatrix} \sigma^2_\alpha & \rho_{\alpha \beta} \sigma_\alpha \sigma_\beta \\ \rho_{\alpha \beta} \sigma_\alpha \sigma_\beta & \sigma^2_\beta \end{bmatrix}
\]

(6)

where \( H_{\alpha \beta} \) is the Hessian matrix with second order derivatives of the log-Likelihood function. \( \sigma_\alpha \) and \( \sigma_\beta \) denote the standard deviations of \( \alpha \) and \( \beta \), respectively. \( \rho_{\alpha \beta} \) is the correlation coefficient between \( \alpha \) and \( \beta \). The Hessian matrix is estimated by numerical differentiation.

The result is a consistent probabilistic model for the extreme response which can be used not only to estimate the 50-year extreme response, but also the design extreme response values and the coefficient of variation for the extreme response, all incl. statistical uncertainty.

4. Example - Crude Monte Carlo response simulations

In this section is shown an example of estimating the extreme response quantiles using the techniques described in section 3. Data from the IEC 61-400 maintenance group provided by NREL are used. The response quantities 'Flap bending moment' and 'Tower base downwind moment' are considered. 10 simulations at each wind speed bin are used. Table 1 shows an example for each wind speed bin the estimated Weibull parameters, the 50-year value (for each bin isolated) and the statistical uncertainties. It is seen that the statistical uncertainty (related to the parameter \( \beta \)) is relatively high for the mean wind speeds 15 and 17 m/s resulting in the largest response values. The uncertainty related to the \( \alpha \) parameter is seen to be relatively small. The correlation coefficient is approximately 0.3 indicating a small positive correlation between the two statistical parameters.

| \( V_j [m/s] \) | \( \gamma \) [MNm] | \( \alpha \) [MNm] | \( \beta \) [MNm] | \( X_{50} \) [MNm] | \( \sigma_\alpha \) [MNm] | \( \sigma_\beta \) [MNm] | \( \rho_{\alpha \beta} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 3              | 24.5           | 1.08           | 22.6           | 57.0           | 0.055          | 10.4           | 0.31           |
| 5              | 32.1           | 1.09           | 51.7           | 43.2           | 0.008          | 7.3            | 0.30           |
| 7              | 47.9           | 1.35           | 51.6           | 77.7           | 0.015          | 7.1            | 0.31           |
| 9              | 70.5           | 1.12           | 1131           | 82.5           | 0.004          | 15.2           | 0.29           |
| 11             | 76.8           | 1.09           | 132.6          | 87.4           | 0.003          | 17.2           | 0.28           |
| 13             | 79.9           | 1.11           | 107.7          | 98.4           | 0.005          | 14.5           | 0.30           |
| 15             | 78.3           | 1.18           | 105.0          | 112.6          | 0.030          | 30.0           | 0.31           |
| 17             | 74.3           | 1.07           | 102.8          | 112.7          | 0.015          | 19.7           | 0.31           |
| 19             | 59.6           | 1.29           | 66.3           | 109.3          | 0.018          | 9.7            | 0.30           |
| 21             | 58.4           | 1.05           | 62.2           | 106.5          | 0.014          | 9.6            | 0.32           |
| 23             | 54.9           | 1.19           | 65.2           | 88.1           | 0.012          | 9.1            | 0.31           |
| 25             | 54.9           | 1.12           | 83.0           | 77.0           | 0.009          | 11.8           | 0.30           |
Using the results from each mean wind speed, probability of exceedence curves are obtained for the normalised response by integration over all operational mean wind speeds, see figures 2 and 3. Statistical uncertainty is seen to be quite important. The 50 year response values are determined to (without and with statistical uncertainty):

- Flap bending moment: \(0.98\) (without) \(1.05\) (with)
- Tower base downwind moment: \(1.09\) (without) \(1.16\) (with)

It is seen that the 50 year response in both cases is increased by approximately 7% when statistical uncertainty is included. It is noted that if more simulations are used at each bin, then this effect of statistical uncertainty is decreased.

If the design value method, see [16], is used to determine the response design value, and an annual reliability index equal to 3.5 is used then the ratio between the design value and the 50-year value is approximately \(1.03\) indicating that the statistical uncertainty is important. The ratio 1.03 depends mainly on the coefficient of variation of the response and the reliability level. It is noted that other uncertainties, e.g. wind climate also should be accounted for when determining the partial safety factor to be used.

**Figure 2.** Probability of exceedence \(P(X_{\text{max}} \geq x | T) = P(x)\) for normalized flap bending moment. Dashed line: without statistical uncertainty; full line: with statistical uncertainty.

**Figure 3.** Probability of exceedence \(P(X_{\text{max}} \geq x | T) = P(x)\) for normalized tower base downwind moment. Dashed line: without statistical uncertainty; full line: with statistical uncertainty.

**5. Conclusions**

Estimation of extreme response values is very important for structural design of wind turbines. Due to the influence of control system, nonlinear structural behavior and non-Gaussian extreme turbulence...
the extreme response is usually assessed based on simulation of turbulence time series. In this paper the problem of statistical load extrapolation is considered using techniques from structural reliability theory. Different simulation techniques to estimate extreme response characteristics are described and compared, including crude Monte Carlo simulation, Importance Sampling, and splitting methods such as the Russian Roulette and the Double and Clump algorithm. These simulation techniques are discussed in relation to the physical and model uncertainties.

The different types of uncertainty relevant for extreme response are described: physical uncertainty (aleatory), statistical uncertainty (epistemic) and model uncertainty (epistemic). It is noted that epistemic uncertainty can be decreased (removed) by more data, better models, more stochastic variables, etc. whereas physical uncertainty cannot be removed. This is important in reliability assessment for wind turbines.

A statistically consistent technique is described for including statistical uncertainty and assessing the extreme 50-year response using simulated time series and conditioned on the model parameters. The peak over threshold method together with the Maximum Likelihood Method provides a tool to obtain consistent estimates incl. the statistical uncertainty. An illustrative example indicates that the statistical uncertainty is important compared to the coefficient of variation of the extreme response when the number of 10 minutes simulations at each mean wind speed is limited to 10.

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