SOME RECENT DEVELOPMENTS IN CHIRAL PERTURBATION THEORY*

Ulf-G. Meißner†
HISKP, Universität Bonn, D-53115 Bonn, Germany
and
IKP, Forschungszentrum Jülich, D-52425 Jülich, Germany
†E-mail: meissner@itkp.uni-bonn.de

In this talk, I address some recent developments in chiral perturbation theory at unphysical and physical quark masses.

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1. Introduction I: Remarks on chiral extrapolations

The first part of this talk concerns the application of chiral perturbation theory (CHPT) at unphysical quark masses. More precisely, lattice QCD (LQCD) allows one in principle to calculate hadronic matrix elements ab initio using capability computing on a discretized space-time. To connect to the real world, various extrapolations are necessary: LQCD operates at a finite volume $V$, at a finite lattice spacing $a$ and at large (unphysical) quark masses $m_q$. All these effects can be treated in suitably tailored effective field theories (EFTs) (for a recent review, see\(^1\)). Here, I consider the quark mass expansion of certain baryon observables. Various nucleon (baryon) observables have already been computed on the lattice, like e.g. masses of ground and excited states, nucleon electromagnetic radii, the nucleon axial-vector coupling, and so on. CHPT in principle provides extrapolation functions for all these observables, parameterized in terms of a number of low-energy constants (LECs). These LECs relate many observables, they are not dependent on the process one considers. Given the present situation with only a few lattice results at reasonably small quark masses available, it is mandatory to incorporate as much phenomenological

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input as is available for the LECs from studies of pion-nucleon scattering, pion production and so on. Also, since many LECs appear in various observables, a true check of our understanding of the chiral symmetry breaking of QCD requires global fits at sufficiently small quark masses. It is absolutely necessary for such extrapolations to make sense that one is in a regime where higher order terms stay sufficiently small. Consequently, results must be independent of the regularization scheme, these can differ by higher order terms (e.g. comparing results based on heavy baryon CHPT to ones obtained employing e.g. infrared regularization). For this interplay of CHPT and LQCD to make sense, the lattice “data” should be in the true chiral regime. I will illustrate these issues for two very different examples: the axial-vector coupling $g_A$ and the Roper mass $m_R$.

2. Application I: The nucleon axial-vector coupling

The nucleon axial-vector coupling $g_A$ is a fundamental quantity in hadron physics as it appears prominently in the Goldberger-Treiman relation. Lattice results for $g_A$ obtained by various collaborations for pion masses between 300 and 1000 MeV show a very flat quark (pion) mass dependence. On the other hand, it is known since long that the one–loop representation of $g_A$ is not converging well and is dominated by the $M_\pi^2$ term with increasing pion mass; thus $g_A(M_\pi)$ rises steeply as the pion mass increases. The large coefficient of this term is, however, understood in terms of the large values of the dimension two LECs $c_3$ and $c_4$ combined with some large numerical prefactors. In Ref. we have therefore worked out the two–loop representation of $g_A$,

$$g_A = g_0 \left\{ 1 + \left( \frac{\alpha_2}{4\pi F} \ln \frac{M_\pi}{\lambda} + \beta_2 \right) M_\pi^2 + \alpha_3 M_\pi^3 \\
+ \left( \frac{\alpha_4}{(4\pi F)^2} \ln^2 \frac{M_\pi}{\lambda} + \frac{\gamma_4}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_4 \right) M_\pi^4 + \alpha_5 M_\pi^5 \right\} + \mathcal{O}(M_\pi^6),$$

with $g_0$ the chiral limit value of $g_A$, $\lambda$ is the scale of dimensional regularization, and the coefficients $\alpha_{2,3}, \beta_2$ encode the one-loop result. Further, $F$ denotes the chiral limit value of the pion decay constant, $F \simeq 86$ MeV, and $\Delta^{(n)}$ collects the corrections $\sim M_\pi^n$. At two-loop order, one has corrections of fourth and fifth order in the pion mass, given in terms of the coefficients $\alpha_{4,5}, \beta_4, \gamma_4$ (note that there is also a $M_\pi \ln M_\pi^2$ term whose contribution we have absorbed in the uncertainty of $\alpha_5$). The LEC $\alpha_4$ can be analyzed...
Fig. 1. The axial-vector coupling $g_A$ as a function of the pion mass. The solid (red), the dot-dashed (black) and the dashed (green) line correspond to various input values for the LECs (see\cite{3} for details). The (magenta) circle denotes the physical value of $g_A$ at the physical pion mass, the triangles are the lowest mass data from Ref.\cite{8} and the inverted triangles are recent results from QCDSF.\cite{9}

Using the renormalization group (as stressed long ago by Weinberg\cite{4}) and is entirely given in terms of the dimension three coefficients of the one–loop generating functional. We have performed this calculation based on two existing versions of the dimension three pion-nucleon chiral Lagrangian without and with equation of motion terms (see\cite{5} and,\cite{6} respectively) and obtained

$$\alpha_4 = -\frac{16}{3} - \frac{11}{3} g_0^2 + 16g_0^4. \quad (2)$$

In\cite{3} the dominant contributions to the LECs $\alpha_5, \beta_4, \gamma_4$ from $1/m_N$ corrections to dimension two and three insertions (with $m_N$ the nucleon mass) as well as from the pion mass expansion of $F_\pi$ were also worked out, for details see that paper. Setting the remaining contributions to zero, we find (in the notation of Eq. (1)) $g_A = g_0(1 - 0.15 + 0.26 - 0.06 - 0.001)$ and $g_0 = 1.21$, using $g_A = 1.267$ and central values for the LECs $c_3, c_4, d_{16}$, see e.g.\cite{7}. This shows that for the physical pion mass, the higher order corrections are small and one thus has a convergent representation. Varying the LECs $\alpha_5, \beta_4, \gamma_4$ within bounds given by naturalness, one finds that the pion mass dependence of $g_A$ stays flat for $M_\pi \lesssim 350$ MeV, see Fig. 1. From this figure one also sees that there is just a little overlap between the lattice re-
results and the chiral representation, where it can be applied with a tolerable uncertainty. We remark that another solution to this problem was offered in Ref.\textsuperscript{10} where an effective field theory with explicit delta degrees of freedom at leading one–loop order could lead to a flat pion mass dependence of $g_A$, requiring, however, a fine tuning of certain low–energy constants. For a recent update, see \textsuperscript{11} It should also be noted that most of the lattice results analyzed in these papers are far outside the range of applicability of that particular EFT evaluated only to leading one-loop order – that such a representation works at such large pion masses is an interesting observation but certainly does not support claims of a controlled and precise determination of $g_A$ from LQCD.

3. Application II: Chiral corrections to the Roper mass

Understanding the (ir)regularities of the light quark baryon spectrum poses an important challenge for lattice QCD. In particular, the first even-parity excited state of the nucleon, the Roper $N^*(1440)$ (from here on called the Roper) is very intriguing—it is lighter than the first odd-parity nucleon excitation, the $S_{11}(1535)$, and also has a significant branching ratio into two pions. Recent lattice studies have not offered a clear picture about the nucleon resonance spectrum. In particular, in Ref.\textsuperscript{12} an indication of a rapid cross over of the first positive and negative excited nucleon states close to the chiral limit was reported – so far not seen in other simulations at higher quark masses. Note also that so far very simple chiral extrapolation functions have been employed in most approaches, e.g., a linear extrapolation in the quark masses, thus $\sim M^2$, was applied in.\textsuperscript{13} It is therefore important to provide the lattice practitioners with improved chiral extrapolation functions. A complete one–loop representation for the pion mass dependence of the Roper mass was recently given in.\textsuperscript{14} Since the Roper is the first even-parity excited state of the nucleon, the construction of the chiral SU(2) effective Lagrangian follows standard procedures, see e.g.\textsuperscript{15} The effective Lagrangian relevant for our calculation is (working in the isospin limit $m_u = m_d$ and neglecting electromagnetism)

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_R + \mathcal{L}_{NR} ,$$

$$\mathcal{L}_0 = i \bar{N} \gamma_{\mu} D^\mu N - m_N \bar{N} N + i \bar{R} \gamma_{\mu} D^\mu R - m_R \bar{R} R ,$$

$$\mathcal{L}_R^{(1)} = \frac{1}{2} g_R \bar{R} \gamma_{\mu} \gamma_5 u^\mu R ,$$

$$\mathcal{L}_{NR}^{(1)} = \frac{g_{NR}}{2} \bar{R} \gamma_{\mu} \gamma_5 u^\mu N + \text{h.c.} ,$$

$$\mathcal{L}_R^{(2)} = e_1^* \langle \chi_+ \rangle \bar{R} R - \frac{e_2^*}{8m_R} \bar{R} (\langle u_\mu u_\nu \rangle \{D^\mu , D^\nu \} + \text{h.c.}) R + \frac{e_3^*}{2} (u_\mu u^\mu) \bar{R} R ,$$
\[ \mathcal{L}_R^{(4)} = -\frac{c_i^*}{16} (\chi_+)^2 \bar{RR}, \]  

where \( N, R \) are nucleon and Roper fields, respectively, and \( m_N, m_R \) the corresponding baryon masses in the chiral limit. The pion fields are collected in \( u_\mu = -\partial_\mu \pi/F_\pi + \mathcal{O}(\pi^3) \). \( D_\mu \) is the chiral covariant derivative, for our purpose we can set \( D_\mu = \partial_\mu \), see e.g. \(^{15}\) for definitions. Further, \( \chi_+ \) is proportional to the pion mass and induces explicit chiral symmetry breaking, and \( \langle \rangle \) denotes the trace in flavor space. The dimension two and four LECs \( c_i^* \) and \( e_i^* \) correspond to the \( c_i \) and \( e_i \) of the effective chiral pion–nucleon Lagrangian. The pion-Roper coupling is given to leading chiral order by \( \mathcal{L}_R^{(1)} \), with a coupling \( g_R \). This coupling is bounded by the nucleon axial coupling, \( |g_R| < |g_A| \), in what follows we use \( g_R = 1 \). The leading interaction piece between nucleons and the Roper is given by \( \mathcal{L}_N R^{(1)} \). The coupling \( g_{NR} \) can be determined from the strong decays of the Roper resonance, its actual value is \( g_{NR} = 0.35 \) using the Roper width extracted from the speed plot (and not from a Breit-Wigner fit). Further pion-Roper couplings are encoded in \( \mathcal{L}_R^{(2)} \) and \( \mathcal{L}_R^{(4)} \). To analyze the real part of the Roper self-energy, one has to calculate a) tree graphs with insertion \( c_1^*, e_1^* \), self-energy diagrams with intermediate b) nucleon and c) Roper states and d) tadpoles with vertices from \( \mathcal{L}_R^{(2)} \). In fact, the graphs of type b) require a modification of the regularization scheme due to the appearance of the two large mass scales \( m_N \) and \( m_R \). The solution to this problem – assuming \( m_N^2/m_R^2 < 1 \) (in nature, this ratio is \( \approx 1/2.4 \)) – is described in.\(^{14}\) As discussed in that paper, the LECs \( c_i^* \) and \( e_i^* \) can be bounded assuming naturalness and by direct comparison with the corresponding pion-nucleon couplings: \( |c_1^*| \lesssim 0.5 \text{ GeV}^{-1} \), \( |c_{2,3}^*| \lesssim 1.0 \text{ GeV}^{-1} \) and \( |e_i^*| \lesssim 0.5 \text{ GeV}^{-3} \). In Fig. 2 an estimated range for the pion mass dependence of the Roper mass is presented by taking the extreme values for \( c_{2,3}^* \) and \( e_1^* \), while keeping \( c_1^* = -0.5 \text{ GeV}^{-1} \), \( g_{NR} = 0.35 \), \( g_R = 1 \) fixed. The masses of the baryons in the chiral limit are taken to be \( m_N = 0.885 \text{ GeV}^{16} \) and \( m_R = 1.4 \text{ GeV} \), respectively. The dash-dotted curve is obtained by setting the couplings \( c_{2,3}^*, e_1^* \) all to zero, and exhibits up to an offset a similar quark mass dependence as the nucleon result (dotted curve, taken from Ref.\(^{16}\)). It should be emphasized, however, that the one-loop formula cannot be trusted for pion masses much beyond 350 MeV. No sharp decrease of the Roper mass for small pion masses is observed for natural values of the couplings. Note that the important \( \Delta \pi \) and \( N\pi\pi \) channels are effectively included through the dimension two and four contact interactions, still it would be worthwhile to extend these considerations including the delta explicitly. Note further
that the formalism developed in\textsuperscript{14} is in general suited to study systems with two heavy mass scales in addition to a light mass scale. In this sense, it can be applied to other resonances as well, such as the $S_{11}(1535)$. In this case, however, an SU(3) calculation is necessary due to the important $\eta N$ decay channel.

4. Introduction II: Hadronic atoms

Let us come back to the real world of physical quark masses. In what follows, I will discuss a spectacular effect of isospin violation in pionic deuterium, which is one particular hadronic atom. More generally, such atoms are made of certain hadrons bound by the static Coulomb force. There exist many species, e.g. $\pi^+\pi^-, \pi^\pm K^\mp, \pi^- p, \pi^- d, K^- p, \text{or } K^- d$. In these systems, the Bohr radii are much larger than any typical scale of strong interactions (QCD), so that their effects can be treated as perturbations. These are the energy shift $\Delta E$ from the Coulomb value and the decay width $\Gamma$ (often combined in the complex valued energy shift). Since the average momenta in such systems are very small compared to any hadronic scale, hadronic atoms give access to scattering at zero energy and thus the pertinent S-wave scattering length(s). As it is well known, these scattering lengths are...
very sensitive to the chiral and isospin symmetry breaking in QCD. In fact, hadronic atoms offer may be the most precise method of determining these fundamental parameters since theory and experiment can in some cases be driven to an accuracy of one or a few percent. On the theoretical side, hadronic atoms can be analyzed systematically and consistently in the framework of non-relativistic low-energy EFT including virtual photons, see e.g.\textsuperscript{17}

5. Isospin violation in pionic deuterium

Combined measurements of the energy shift and decay width of pionic hydrogen and the energy shift of pionic deuterium offer an excellent test of isospin symmetry and its breaking because these three quantities are expressed in terms of two scattering lengths, $\Delta E(\pi^- p) \sim a^+ + a^-$, $\Gamma(\pi^- p) \sim a^-$, and $\text{Re} \Delta E(\pi^- d) \sim a^+ + \ldots$, where the ellipses denotes three-body effects such as multiple scattering within the deuteron. Here, $a^+$ and $a^-$ are the isoscalar and the isovector S-wave $\pi N$ scattering lengths, respectively. The Bern group has championed the EFT treatment of pionic hydrogen, the calculations including strong and electromagnetic isospin violation can be found in\textsuperscript{18} and\textsuperscript{19} for the ground state energy and the width, respectively. Using this formalism to analyze the data from PSI (as reviewed in\textsuperscript{20}) and combining these with the EFT treatment of pionic deuterium in the isospin limit\textsuperscript{21} (compare the bands denoted hydrogen energy, isospin breaking and hydrogen width, isospin breaking and deuteron, no isospin breaking in Fig. 3) one faces a problem - these bands do not intersect. Since the analysis of pionic deuterium in Ref.\textsuperscript{21} was done in the isospin limit, the question naturally arises whether this is the source of the trouble? In principle, isospin violation (IV) leads to corrections in the bound-state as well as in the pertinent scattering amplitude. From experience with pionium, pionic hydrogen and $\pi K$ atoms (see e.g. Ref.\textsuperscript{22}), such bound-state corrections are expected to be small. On the other hand, already in 1977 Weinberg pointed out that IV effects can be unnaturally large if the isospin-conserving (IC) contribution is chirally suppressed.\textsuperscript{23} In particular, such an effect is very pronounced in neutral pion scattering off nucleons (for an update, see\textsuperscript{24}), but it is very hard to observe. On the other hand, the leading order contribution to $\pi d$ scattering is chirally suppressed, $\text{Re} a_{\pi d} \sim (a_{\pi^- p} + a_{\pi^- n}) = \mathcal{O}(p^2)$, despite the fact that $a_{\pi^- p}$ and $a_{\pi^- n}$ are individually of $\mathcal{O}(p)$. Here, $p$ denotes collectively the small parameters of CHPT. While this is well-known, nobody has ever systematically investigated IV in $\pi^- d$. The leading order IV in pionic deuterium
was only analyzed recently in Ref. 25. To be specific, consider the threshold pion-deuteron scattering amplitude:

$$\text{Re } a_{\pi d}^{th} = \text{Re } a_{\pi d}^{(0)} + \Delta a_{\pi d},$$

where the IV piece $\sim \Delta a_{\pi d}$ appears at the same order as the leading IC piece $\sim \text{Re } a_{\pi d}^{(0)}$. Also, to this order one has no dependence on the deuteron structure. The explicit calculation leads to

$$\Delta a_{\pi d}^{LO} = (4\pi(1 + \mu/2))^{-1}(\delta T_p + \delta T_n).$$

Here, $\mu = M_{\pi^+}/m_\rho$ and $T_{p,n}$ are the leading isospin breaking corrections to the $\pi^- p$ and $\pi^- n$ threshold scattering amplitudes. These are given by

$$\delta T_p = \frac{4(M_{\pi^+}^2 - M_{\pi^0}^2)}{F_{\pi}^2} c_1 - \frac{e^2}{2} (4f_1 + f_2) + \mathcal{O}(p^3),$$

$$\delta T_n = \frac{4(M_{\pi^+}^2 - M_{\pi^0}^2)}{F_{\pi}^2} c_1 - \frac{e^2}{2} (4f_1 - f_2) + \mathcal{O}(p^3),$$

where $F_{\pi} = 92.4$ MeV is the pion decay constant, $g_A = 1.27$ denotes the axial-vector charge of the nucleon and $c_1$ is a strong and $f_1, f_2$ are electromagnetic $\mathcal{O}(p^2)$ LECs, respectively. Note also that at lowest order in CHPT $c_1$ is directly related to the value of the pion-nucleon $\sigma$-term and $f_2$ to the proton-neutron mass difference. In the numerical calculations we take $c_1 = -0.9^{+0.5}_{-0.2}$ GeV$^{-1}$, $f_2 = -(0.97 \pm 0.38)$ GeV$^{-1}$. Note that the errors on the LEC $c_1$ are most conservative. The largest uncertainty in the results is introduced by the constant $f_1$, whose value at present is unknown and for which the dimensional estimate $|f_1| \leq 1.4$ GeV$^{-1}$ has been used. Note also, that the hydrogen energy band, which is shown in Fig. 3 corresponds to the new value of $c_1$ given above. From this, the leading order IV contribution to pionic deuterium follows as:

$$\Delta a_{\pi d}^{LO} = (4\pi(1 + \mu/2))^{-1}(\delta T_p + \delta T_n)$$

$$= \frac{1}{4\pi(1 + \mu/2)} \left\{ \frac{8\Delta M_{\pi}^2}{F_{\pi}^2} c_1 - 4e^2 f_1 \right\} + \mathcal{O}(p^3),$$

with $\Delta M_{\pi}^2 = M_{\pi^+}^2 - M_{\pi^0}^2$ the squared charged-to-neutral pion mass difference. Substituting numerical values for the various low-energy constants, which were specified above, one obtains that the correction at $\mathcal{O}(p^2)$ is extremely large

$$\Delta a_{\pi d}^{LO} = -(0.0110^{+0.0081}_{-0.0058}) M_\pi^{-1},$$

that is $\Delta a_{\pi d}^{LO}/\text{Re } a_{\pi d}^{exp} = 0.42$ (central values), using the experimental value $\text{Re } a_{\pi d}^{exp} = -(0.0261 \pm 0.0005) M_\pi^{-1}$. Moreover, one can immediately see
that the correction moves the deuteron band in Fig. 3 in the right direction: the isospin-breaking corrections amount for the bulk of the discrepancy between the experimental data on pionic hydrogen and deuterium. Including the corrections $\Delta a_{\pi d}^{LQ}$, all bands now have a common intersection area in the $a^+, a^-\text{-plane}$, see Fig. 3. The resulting values for the $\pi N$ scattering lengths are:

\[
\begin{align*}
a^+ &= (0.0015 \pm 0.0022) M_{\pi}^{-1},  \\
a^- &= (0.0852 \pm 0.0018) M_{\pi}^{-1}.
\end{align*}
\]

Further, using the hydrogen energy shift to estimate the LEC $f_1$, we obtain

\[
f_1 = -2.1^{+3.2}_{-1.2} \text{ GeV}^{-1},
\]

which is consistent with the dimensional analysis and a recent evaluation based on a quark model.\(^{28}\) Note that the error displayed here does not include the uncertainty coming from the higher orders in CHPT and should thus be considered preliminary.
As we see, the presence of the $O(p^2)$ LECs in the expressions for the isospin-breaking corrections leads to a sizeable increase of the uncertainty in the output. In order to gain precision, in the fit one might also use those particular linear combination(s) of the experimental observables that do not contain $f_1$ and $c_1$. However, it should be pointed out that such a fit imposes much more severe constraints on the data than the uncorrelated fit considered above. In fact, applying isospin-breaking corrections only at $O(p^2)$, we find that the data are still over-constrained in the combined fit. From this we finally conclude that to carry out such a combined analysis with the required precision, one would have e.g. first to evaluate the isospin-breaking corrections with a better accuracy.

Up to now, we have restricted ourselves to the leading-order isospin-breaking correction in CHPT. Calculations at $O(p^3)$ exist only for the hydrogen energy shift and yield $\delta_e = (-7.2 \pm 2.9) \cdot 10^{-218}$ (using $c_1 = (-0.93 \pm 0.07)\ \text{GeV}^{-1}$). The corrections to $O(p^2)$ result are sizable (the energy band in Fig. 3 will be shifted further upwards), but the uncertainty, which is almost completely determined by the $O(p^2)$ LECs, remains practically the same. On the other hand, consistent studies at $O(p^3)$ imply the treatment of the scattering process in the three-body system in the effective field theory with virtual photons. To the best of our knowledge, such investigations have not been yet carried out, although certain three-body contributions at $O(p^3)$ were calculated in the past.\textsuperscript{29} It is natural to expect that generally three-body terms at $O(p^3)$ should not depend on the additional LECs from the two-nucleon sector and hence the extraction of the $\pi N$ scattering lengths at a high precision is still possible. Of course, these arguments can not be a substitute for a rigorous proof in the framework of EFT, which in the light of the above discussion, is urgently called for.

It was also shown in Ref.\textsuperscript{25} that the $O(p^4)$ correction which emerges from the double-scattering term in the multiple-scattering series is very small, $\Delta a_{\pi d}^{\text{double scat.}} = 0.003 \, \text{Re} \, a_{\pi d}^{\exp}$, using the scattering lengths from Eq. (9) as input. Of course, such a partial result can only be considered indicative. Evidently, the systematic analysis of all $O(p^3)$ (and eventually $O(p^4)$) corrections should be carried out.

6. Summary and outlook

In the first part of this talk, I have considered aspects related to baryon CHPT for light quark masses above their physical values. It should be stressed that baryon CHPT is a mature field in the up and down quark sector and provides unambiguous extrapolation functions for LQCD – certainly
more work is needed for the three-flavor case. To carry out the required chiral extrapolations, one should keep in mind that different observables are linked by general operator structures and the appearing low-energy constants (LECs) are universal, which means that they are independent of the process considered. Given the present status of LQCD, it appears mandatory to perform global fits to observables and constrain the appearing LECs by input from phenomenology, whenever available. I have discussed two specific examples of the interplay between CHPT and LQCD. A chiral extrapolation for $g_A$ exists now at two–loop accuracy. For pion masses \( \lessapprox 350 \text{ MeV} \), the theoretical uncertainty related to it is reasonably small. I have also provided a chiral extrapolation function for the Roper mass - certainly much more work is needed for such excited states from both CHPT and LQCD. Evidently, we need more lattice “data” at low quark masses to really perform precision studies.

In the second part of this talk, I returned to the real world (physical quark masses) and considered hadronic atoms. These can be systematically analyzed in non-relativistic effective field theory including virtual photons. We have found a very large isospin-violating effect in pionic deuterium at leading order. That there is such an effect is not so surprising because the leading isospin-conserving contribution is chirally suppressed. What is surprising, however, is the actual size of the effect and that it was only found recently. Combining this with the information obtained from the analysis of the energy shift and width in pionic hydrogen, one is led to a consistent extraction of the S-wave $\pi N$ scattering lengths and can furthermore determine the electromagnetic LEC $f_1$. Clearly, higher order calculations are necessary to reduce the theoretical uncertainty. In this context it is also important to stress that it was recently shown that there are only tiny dispersive corrections to Re $a_{\pi d}$, see$^{30}$ and Hanhart’s talk at this conference.

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