Spinodal instability of baryon-rich quark matter

Feng Li\(^1\) and Che Ming Ko\(^2\)

\(^1\) Frankfurt Institute for Advanced Studies, Ruth-Moufang Str. 1, Frankfurt 60438, Germany

\(^2\) Cyclotron Institute and Department of Physics, Texas A&M University, College Station, Texas 77843-3366, USA

E-mail: fengli@fias.uni-frankfurt.de

Abstract. The spinodal instabilities of both confined and expanding baryon-rich quark matters are studied in a transport model derived from the Nambu-Jona-Lasino model. Appreciable higher-order density moments are seen as a result of the first-order phase transition in both cases. The skewness of the quark number event-by-event distribution in a small sub-volume of the system becomes appreciable for the confined quark matter. For the expanding quark matter, the density fluctuations lead to enhanced anisotropic flows and dilepton yield.

1. Introduction

The beam energy scan (BES) experiments [1, 2, 3] at the Relativistic Heavy Ion Collider (RHIC) are designed for the search of the critical end point (CEP) in the baryon-rich region of the QCD phase diagram. At CEP, the first-order phase transition between the hadronic matter and the quark-gluon-plasma (QGP) changes to a second-order one. To understand the properties of this phase transition, both schematic models and hydrodynamic approach [4, 5, 6] have been used to study how the spinodal instability of a QGP leads to the self-amplified deviation from its equilibrium state during the phase transition. Using the test particle method, the spinodal instabilities of both the confined and expanding quark matters have been studied in a transport model that is derived from the Nambu-Jona-Lasino (NJL) model [7]. By varying the coupling strength of the vector interaction \(G_V\) in the NJL model, different equations of state are obtained for a baryon-rich quark matter. A first-order phase transition occurs if \(G_V = 0\) with the CEP at about \(T_c \approx 70\) MeV and \(n_c \approx 0.9\) fm\(^{-3}\) [8], and it is absent if \(G_V = G_S\). The density fluctuations induced by the spinodal instability can be quantified by the scaled density moments \(\langle \rho^N \rangle / \langle \rho \rangle^N\) [6], where \(\langle \rho^N \rangle \equiv \int d^3r \rho(r)^{N+1} / \int d^3r \rho(r)\). They are all equal to one for a uniform density distribution but become larger if the density is nonuniform. Also, density fluctuations can affect the anisotropic flows and the dilepton yield in heavy ion collisions. Results on these observables for the two cases of with and without a first-order phase transition are compared in the following for both the confined and the expanding quark matter.

2. Quark matter in a box

Shown in the left window of Fig. 1 by the dotted, dashed, and solid lines are the time evolution of scaled density moments for \(N = 2, 4\) and 6, respectively, during the first-order phase transition of a quark matter that is confined in a box with an initially uniform net quark density \(n_q = 0.5\) fm\(^{-3}\) and temperature \(T = 20\) MeV. They are obtained by averaged over 1,000 events. The scaled moments are seen to increase during the phase separation and reach their saturated values at...
Figure 1. Time evolution of scaled density moments during a first-order phase transition for a quark matter confined in a box with an initially uniform net quark density $n_q = 0.5 \, \text{fm}^{-3}$ and temperature $T = 20 \, \text{MeV}$ (left window), and for an expanding quark matter with a spherical Wood-Saxon distribution of an initial net quark density $n_q = 1.5 \, \text{fm}^{-3}$ and temperature $T_0 = 70 \, \text{MeV}$ at the center (right window). Red lines in the right panel are for an expanding quark matter with same initial conditions but without a first-order phase transition. About $t = 40 \, \text{fm}/c$, when the phase separation almost ends. Also, moments with larger $N$ increase faster and saturate at larger values.

Figure 2. Time evolution of the event-by-event distribution of the number of quarks in a sub-volume of size $0.6 \, \text{fm}^3$ (left window) and $30 \, \text{fm}^3$ (right window) for a quark matter of temperature $T = 20 \, \text{MeV}$ and average net quark density $n_q = 0.5 \, \text{fm}^{-3}$ inside the spinodal region. The total number of events is 1000.

The number of quarks inside a sub-volume also varies drastically from event to event during a first-order phase transition as shown in Fig. 2 for the central cell from 1000 events at $t = 0$, 20, and $40 \, \text{fm}/c$ by the solid, dashed and dotted lines, respectively, for the two cases of sub-volume of size $0.6 \, \text{fm}^3$ (left window) and $30 \, \text{fm}^3$ (right window). The left panel of Fig. 2 clearly shows that the distribution for the small sub-volume becomes asymmetric as time increases, starting
with an initial skewness of 0.11 and increasing to 0.60 at 20 fm/c and 0.75 at 40 fm/c, agreeing with the prediction in Ref. [9]. This feature is absent in the right panel of Fig. 2 for the larger sub-volume, where the distribution remains essentially symmetric with increasing time.

3. Expanding quark matter

For an expanding quark matter as in relativistic heavy ion collisions, the blast wave initial conditions of a spherical Wood-Saxon distribution with radius $R = 5$ fm, skin thickness $a = 0.5$ fm, and central density $\rho_0 = 1.5$ fm$^3$, and temperature $T_0 = 70$ MeV are used in Ref. [8] for two different equations of state with $(G_V = 0)$ and without $(G_V = G_S)$ a first-order phase transition. The phase trajectory of the central part of the quark matter is shown to traverse through the spinodal instability region in the case of $G_V = 0$.

Shown in the right window of Fig. 1 by the dotted, dashed, and solid lines are the time evolution of the density moments for $N = 2$, 4, and 6, respectively. The black and red lines correspond to the two cases with and without a first-order phase transition, respectively. In both cases, the scaled density moments first increase and then decrease with time. Although the scaled density moments in the case without a first-order phase transition are larger than one, those in the case with a first-order phase transition are even larger, reflecting the effect due to density clumps that distribute randomly inside the expanding quark matter.

![Figure 3](image-url)  

**Figure 3.** Event distributions of final anisotropic flows $v_2$ (left window) and $v_4$ (right window) for 100 events of an expanding quark matter with the blast wave initial conditions.

Since density fluctuations can lead to spatial anisotropy even in central heavy ion collisions, it can affect the anisotropic flows in the transverse plane of heavy ion collisions [11, 12]. The latter are characterized by the coefficients $v_n$ in the Fourier expansion of the azimuthal angle distribution of particle transverse momenta and can be calculated by the two particle cumulant method [13], namely, $v_n \{2\} = \sqrt{\langle \cos(n\Delta \phi) \rangle}$ by averaging over all pairs of particles in an event. The event distributions of final $v_2 \{2\}$ (left window) and $v_4 \{2\}$ (right window) of freeze-out partons for 100 events are shown in Fig. 3, with the solid and dashed lines for the cases with and without first order phase transition, respectively. Both distributions peak at a larger value for the case with a first-order phase transition, particularly the $v_4$, so enhanced anisotropic flows provide a plausible signal for the first-order phase transition in the quark matter produced in relativistic heavy ion collisions.

Because the rate of the dilepton production through quark-antiquark annihilation is proportional to the product of their densities, it will also be enhanced as the density fluctuations...
become large. The dilepton invariant mass spectrum obtained by averaging over 100 events are plotted in Fig. 4 by the solid and dashed lines for the cases with and without a first-order phase transition, respectively. As expected, more dileptions are produced from a quark matter with a first-order phase transition.

4. Summary
The spinodal instability of a baryon-rich quark matter has been studied in the transport model that is based on from the NJL model. Dense clumps are observed in both confined and expanding quark matters, and they lead to the enhancement of higher-order scaled density moments. The skewness of the quark number event-by-event distribution in a small sub-volume also increases during the first-order phase transition in the confined quark matter, but this feature disappears if the sub-volume is large. The density fluctuations further affect the anisotropy in the momentum space and enhance the dilepton production through quark-antiquark annihilation in the expanding quark matter.

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Figure 4. Dilepton yield as a function of the invariant mass \( \sqrt{s} \) for the cases with (solid line) and without (dashed line) a first-order phase transition in an expanding quark matter with the blast wave initial conditions.