ON USING THE WMAP DISTANCE INFORMATION IN CONSTRAINING THE TIME-EVOLVING EQUATION OF STATE OF DARK ENERGY

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ABSTRACT

Recently, the WMAP group published their 5 year data and considered the constraints on the time-evolving equation of state of dark energy for the first time from the WMAP distance information. In this Letter, we study the effectiveness of the usage of this distance information and find that the compressed CMB information can give similar constraints on dark energy parameters compared with the full CMB power spectrum if dark energy perturbations are included; however, if the dark energy perturbations are incorrectly neglected, the difference of the results is sizable.

Subject headings: cosmic microwave background — cosmological parameters — cosmology: theory

Online material: color figures

1. INTRODUCTION

The newly released Wilkinson Microwave Anisotropy Probe 5 year data (WMAP5) (Hinshaw et al. 2008; Nolta et al. 2008; Dunkley et al. 2008; Komatsu et al. 2008a) detecting the cosmic microwave background (CMB) to an unprecedented precision make it possible to improve the constraints on almost all the cosmological parameters, including the equation of state (EoS) of dark energy using the w of the unknown energy budget, dark energy. Defined as the ratio of pressure over energy density, , the EoS can be used to classify various dark energy models, such as quintessence (Wetterich 1995; Ratra & Peebles 1988; Peebles & Ratra 1988; Wetterich 1988), phantom (Caldwell 2002), k-essence (Armendariz-Picon et al. 2000, 2001), etc., which are of great theoretical significance to unveil the mystery of dark energy. Therefore, trying to study the evolution history of the EoS of dark energy plays a crucial role in modern observational cosmology (Huterer & Starkman 2003; Wang & Tegmark 2005; Zhao et al. 2007a). Simply put, one can choose an arbitrary parameterization of w and constrain the introduced dark energy parameters from the astronomical observational data, including CMB, Type Ia supernovae (SNe Ia), large-scale structure, and so forth (Wang & Mukherjee 2007; Wright 2007; Zhao et al. 2007b, 2007c).

Recently, the WMAP group released their 5 year data and for the first time considered the constraints on the time-evolving EoS of dark energy using the WMAP distance information. This method has the advantage of reducing computation time by orders of magnitude, yet the effectiveness and the level of approximation compared with the full CMB power spectrum computation remain unclear. In this Letter, we make a thorough test of this simplified method to investigate whether it is safe to constrain dark energy with the time-evolving EoS. Our Letter is structured as follows: In § 2 we describe the method and the data; in § 3 we present our main results; finally we present our conclusions in § 4.

2. METHOD AND DATA

To study the dynamical behavior of dark energy, we choose the parameterization of the time-evolving EoS of dark energy given by Chevallier & Polarski (2001), Linder (2003), and Komatsu et al. (2008a):

\[ w(a) = w_0 + w_a (1 - a), \]  

where \( a = 1/(1 + z) \) is the scale factor and \( w_a = -dw/da \) characterizes the “running” of the EoS (RunW henceforth). For the \( \Lambda \)CDM model, \( w_0 = -1 \) and \( w_a = 0 \).

When using the Markov chain Monte Carlo (MCMC) global fitting strategy to constrain cosmological parameters, it is crucial to include dark energy perturbations, especially for the time-evolving EoS of dark energy models. This issue has been realized by many researchers including the WMAP group (Weller & Lewis 2003; Yeche et al. 2006; Zhao et al. 2005; Xia et al. 2006; Spergel et al. 2007). However, when the parameterized EoS crosses \(-1\), one cannot handle the dark energy perturbations based on quintessence, phantom, k-essence, and other noncrossing models. By virtue of quintom, the perturbations at the crossing points are continuous; thus, we have proposed a technique to treat dark energy perturbations in the whole parameter space. For details of this method, we refer the readers to our previous papers (Zhao et al. 2005; Xia et al. 2006).

In this study, we have modified the publicly available MCMC package CosmoMC (Lewis & Bridle 2002) to include the dark energy perturbations with the EoS across \(-1\). Furthermore, we assume purely adiabatic initial conditions and a flat universe. Our most general parameter space is

\[ P = (\omega_b, \omega_c, \Omega_b, \tau, w_0, w_a, n_s, \ln 10^{10} A_s), \]  

where \( \omega_b \equiv \Omega_b h^2 \) and \( \omega_c \equiv \Omega_c h^2 \), \( \Omega_b \) and \( \Omega_c \) are the baryon and cold dark matter densities relative to the critical density, \( \Omega_b \) is the ratio (multiplied by 100) of the sound horizon at decoupling to the angular diameter distance to the last scattering surface, \( \tau \) is the optical depth to reionization, and \( A_s \) and \( n_s \) are the amplitude and the tilt of the power spectrum of primordial scalar perturbations. For the pivot scale of the primordial spectrum, we set \( k_0 = 0.05 \) Mpc\(^{-1}\).

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4 Find CosmoMC at http://cosmologist.info/cosmomc/.
The WMAP distance information used by the WMAP group includes the “shift parameter” $R$, the “acoustic scale” $l_A$, and the photon decoupling epoch $z_*$. $R$ and $l_A$ correspond to the ratio of angular diameter distance to the decoupling era over the Hubble horizon and the sound horizon at decoupling, respectively, given by

$$ R(z_*) = \sqrt{\Omega_m H_0^2} \chi(z_*), \quad (3) $$

$$ l_A(z_*) = \frac{\pi \chi(z_*)}{\chi(z_*)}, \quad (4) $$

where $\chi(z_*)$ and $\chi(z_*)$ denote the comoving distance to $z_*$ and the comoving sound horizon at $z_*$, respectively. The decoupling epoch $z_*$ is given by Eisenstein & Hu (1998):

$$ z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}] [1 + g_1(\Omega_m h^2)^{2z}], \quad (5) $$

where

$$ g_1 = 0.0783(\Omega_b h^2)^{-0.238} \quad \text{and} \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_m h^2)^{1.81}}. \quad (6) $$

The WMAP distance information is encoded in part of the CMB information and can constrain cosmological parameters to some extent. It is worth carefully investigating the effectiveness of the constraints from the distance information compared with the full CMB power spectrum computation. To do this, we follow the procedure shown in the flow chart:

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MCMC
Full WMAP5 Data → w_0, w_a
MCMC ↓
l_A, R, z_* → w_0, w_a
MCMC

(7)
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which are detailed as follows:

1. Making a global fitting with the MCMC method to constrain $w_0, w_a$, and also $l_A, R,$ and $z_*$ using the full WMAP5 power spectrum. In this step, we have done two types of calculations, one with and the other without dark energy perturbations.
2. Using the resultant $l_A, R,$ and $z_*$ to constrain dark energy parameters $w_0$ and $w_a$.
3. Comparing the results of the constraints on $w_0$ and $w_a$ obtained from steps 1 and 2.

In step 1, we calculate the likelihood of the CMB power spectrum using the routine supplied by the WMAP group. In step 2, we calculate the likelihood of the WMAP distance information as follows (Komatsu et al. 2008a):

$$ \chi^2 \equiv -2 \ln L = (x^b - x^{data})(x^b - x^{data})^T (C^{-1})_b, $$

where $x = (l_A, R, z_*)$ is the parameter vector and $(C^{-1})_b$ is the inverse covariance matrix for the WMAP distance information.

Since the purpose of this Letter is not to make a global analysis, in order to see the effects of the other cosmological

![Fig. 1.—One-dimensional posterior distributions of $l_A, R,$ and $z_*$ with the WMAP5 data for different cosmological models. In the upper panels, the black solid line represents the standard flat $\Lambda$CDM model, while the red dashed line, the blue dash-dotted line, the purple solid line, and the green dotted line represent $\Lambda$CDM with nonzero $\Omega_c$ and flat $\Lambda$CDM with $f_a$, with $\alpha_*$, and with $r$, respectively. In the lower panels, the black solid line is still from the standard $\Lambda$CDM model, while the red dashed lines and the blue dash-dotted lines represent the dark energy model with the time-evolving EoS (RunW model) with and (incorrectly) without dark energy perturbations, respectively. [See the electronic edition of the Journal for a color version of this figure.]](image)
3. RESULTS

The WMAP distance information is extracted from the full WMAP5 power spectrum by assuming a certain cosmological model, and it should be model dependent. In Figure 1 we present the one-dimensional distributions of the WMAP distance information for different cosmological models.

In the upper three panels of Figure 1 we show the distributions of ̅, , and for five cosmological models: the flat ΛCDM model; the ΛCDM with curvature; and the flat ΛCDM model with massive neutrinos, with running of spectral index, and with tensor perturbations, respectively. We find that the distributions of , and are quite different in these five cases, while the acoustic scale ̅ does not change significantly. These results indicate that when using the distance information to constrain cosmological parameters, one should be clear about the assumed cosmological model. In Table 1 we also list the median 1σ constraints on the WMAP distance information from the full WMAP5 data for different cosmological models.

In the lower three panels of Figure 1 we show the results for three flat models with different dark energy properties: the ΛCDM model and the RunW model, with and (incorrectly) without dark energy perturbations. These results do not show significant differences in the WMAP distance information among different dark energy models. We also compare the results obtained with and (incorrectly) without dark energy perturbations and find that simply switching off dark energy perturbations does not bias the results much at this stage.6 In the following calculations, we use the WMAP distance information obtained from the RunW model with dark energy perturbations included. The corresponding inverse covariance matrix is shown in Table 2.

In Figure 2 we compare the constraints on and obtained from the full WMAP5 power spectrum with the one obtained from WMAP distance information given in Tables 1 and 2. From this plot we can see that the WMAP distance information and the full WMAP5 power spectrum with dark energy perturbations included can give quite similar constraints on and . However, when the dark energy perturbations are incorrectly switched off (black dash-dotted lines in Fig. 2), the results between the two methods are quite different.

The WMAP distance information mainly includes the information on the oscillatory structures of the CMB power spectrum, which come from the small angular scale (large ) of the power spectrum. On the other hand, for the full CMB power spectrum, they combine more information than the distance information, especially at large angular scale (small ). At large angular scale, they are affected by the late integrated Sachs-Wolfe (ISW) effects, which are dark energy dependent. Thus, tighter constraints on ( , ) are anticipated by using the full CMB spectrum than those from using the distance priors only. This is clearly demonstrated in Figure 2 (dashed contours vs. dash dotted one). It is noted that the dash-dotted contours are calculated without including dark energy perturbations, and thus the constraining power of the late ISW effect on dark

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6 The distance information is determined by the background parameters and not affected by dark energy perturbations significantly.

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TABLE 1

| Models         | ̅ | | |
|----------------|---|---|---|
| ΛCDM           | 301.5 ± 0.842 | 1.71 ± 0.021 | 1090.92 ± 0.969 |
| ΛCDM + Ω,       | 302.32 ± 0.899 | 1.72 ± 0.021 | 1091.26 ± 1.004 |
| ΛCDM + m,       | 302.40 ± 0.873 | 1.75 ± 0.031 | 1091.98 ± 1.244 |
| ΛCDM + α,       | 302.36 ± 0.878 | 1.74 ± 0.031 | 1092.72 ± 1.817 |
| ΛCDM + r,       | 301.76 ± 0.944 | 1.69 ± 0.027 | 1089.72 ± 1.366 |
| RunW with perturbations | 302.20 ± 0.865 | 1.72 ± 0.021 | 1091.10 ± 0.991 |
| RunW without perturbations | 301.14 ± 0.875 | 1.71 ± 0.021 | 1090.97 ± 0.985 |

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TABLE 2

| Parameter | ̅ | | |
|-----------|---|---|---|
| ̅(| ) | 1.795 | 31.596 | −1.146 |
| (| ) | 5409.68 | −94.58 |
| ( | ) | 2.891 |

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FIG. 2—68% and 95% confidence levels constraints on ( , ) from full WMAP5 data and WMAP distance information, respectively. Red solid lines are obtained from the full WMAP5 data including dark energy perturbations, black dash-dotted lines are from the full WMAP5 data incorrectly neglecting dark energy perturbations, and blue dashed lines are from WMAP distance information. [See the electronic edition of the Journal for a color version of this figure.]
energy parameters is fully realized. However, when including the dark energy perturbations, which are mainly effective at small l, the constraints on dark energy parameters from the late ISW effects are significantly reduced, resulting in similar contours shown by the dashed and solid lines in Figure 27 (E. Komatsu 2008, private communication). The differences between the solid and the dash-dotted contours also show how biased results can be obtained if the dark energy perturbations are incorrectly neglected in the full CMB data analysis (Xia et al. 2006; Spergel et al. 2007).

In Figure 3, we give the constraints on dark energy parameters by adding the SN Ia data. We can see that the constraints on dark energy parameters are tightened and the differences between the results obtained from “full WMAP5 power spectrum + SN Ia” and from “WMAP distance information + SN Ia” become insignificant.

4. SUMMARY

In this Letter, we have studied the effectiveness of the WMAP distance information on constraining the dark energy parameters, by comparison with the full WMAP5 power spectrum analysis. We first present the level of the model dependence of the distance information in different cosmological models. We further clarify that by properly taking into account dark energy perturbations, the WMAP distances can give unbiased information on dark energy parameters relative to the full CMB analysis.

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In our analysis we use a specific parameterization called the “RunW” model. We expect that our results hold qualitatively for other dark energy parameterizations. Quantitatively, however, the specific results are dependent on the detailed calculations on different dark energy parameterizations that are used.

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