Interaction between scalar field and ideal fluid with inhomogeneous equation of state

Writambhara Chakraborty and Ujjal Debnath

1 Department of Mathematics, New Alipore College, New Alipore, Kolkata- 700 053, India.
2 Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.

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In this letter we study a model of interaction between the scalar field and an inhomogeneous ideal fluid. We have considered two forms of the ideal fluid and a power law expansion for the scale factor. We have solved the equations for the energy densities. Also we show that besides being a dark energy model to explain the cosmic acceleration, this model shows a decaying nature of the scalar field potential and the interaction parameter.

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Recent observations of type Ia Supernovae indicate that Universe is expanding with acceleration [1-5] and lead to the search for a new type of matter which violates the strong energy condition, i.e., $\rho + 3p < 0$. In Einstein’s general relativity, an energy component with large negative pressure has to be introduced in the total energy density of the Universe in order to explain this cosmic acceleration. This energy component is known as dark energy [6 - 8]. There are many candidates supporting this behaviour [9], scalar field or quintessence [10] being one of the most favoured candidates as it has a decaying potential term which dominates over the kinetic term thus generating enough pressure to drive acceleration.

Presently we live in an epoch where the densities of the dark energy and the dark matter are comparable. It becomes difficult to solve this coincidence problem without a suitable interaction. Generally interacting dark energy models are studied to explain the cosmic coincidence problem [11, 12]. Also the transition from matter domination to dark energy domination can be explained through an appropriate energy exchange rate. Therefore, to obtain a suitable evolution of the Universe an interaction is assumed and the decay rate should be proportional to the present value of the Hubble parameter for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data [11]. A variety of interacting dark energy models have been proposed and studied for this purpose [11-14].

Although a lot of models have been proposed to examine the nature of the dark energy, it is not known what is the fundamental nature of the dark energy. Usually models mentioned above are considered for producing the present day acceleration. Also there is modified gravity theories where the EOS depends on geometry, such as Hubble parameter. It is therefore interesting to investigate models that involve EOS different from the usual ones, and whether these EOS is able to give rise to cosmological models meeting the present day dark energy problem. In this letter, we consider model of interaction between scalar field and an ideal fluid with inhomogeneous equation of state (EOS), through a phenomenological interaction which describes the energy flow between them. Ideal fluids with inhomogeneous EOS were introduced in [15-17]. Here we have considered two exotic kind of equation of states which were studied in [18-20] with a linear inhomogeneous EOS. Here we take the inhomogeneous EOS to be in polynomial form to generalize the case. Also, the ideal fluid present here behaves more like dark matter dominated by the scalar field so that the total energy density and pressure of the Universe decreases with time. Also the potential corresponding to the scalar field shows a decaying nature. Here we have considered a power law expansion of the scale factor, so that we always get a non-decelerated expansion of the Universe for the power being greater than or equal to unity. We have solved the energy densities of both the scalar field and ideal fluid and the potential of the scalar field. Also a decaying nature of the interaction parameter is shown.

The metric of a spatially flat isotropic and homogeneous Universe in FRW model is,

$$ds^2 = dt^2 - a^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$$  (1)
where \(a(t)\) is the scale factor.

The Einstein field equations are (choosing \(8\pi G = c = 1\))

\[
3\frac{\dot{a}^2}{a^2} = \rho_{tot} \quad (2)
\]

and

\[
6\frac{\ddot{a}}{a} = -(\rho_{tot} + 3p_{tot}) \quad (3)
\]

The energy conservation equation \((T^\nu_\mu; \nu = 0)\) is

\[
\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + p_{tot}) = 0 \quad (4)
\]

where, \(\rho_{tot}\) and \(p_{tot}\) are the total energy density and the pressure of the Universe, given by,

\[
\rho_{tot} = \rho_\phi + \rho_d \quad (5)
\]

and

\[
p_{tot} = p_\phi + p_d \quad (6)
\]

with \(\rho_\phi\) and \(p_\phi\) are respectively the energy density and pressure due to the scalar field given by,

\[
\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \quad (7)
\]

and

\[
p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \quad (8)
\]

where, \(V(\phi)\) is the relevant potential for the scalar field \(\phi\).

Also, \(\rho_d\) and \(p_d\) are the energy density and the pressure corresponding to the ideal fluid with an inhomogeneous EOS,

\[
p_d = \omega(t)\rho_d + \omega_1 f(H, t) \quad (9)
\]

where, \(\omega(t)\) is a function of \(t\) and \(f(H, t)\) is a function of \(H\) and \(t\) (\(H\) is the Hubble parameter = \(\dot{a}/a\)).

Now we consider the scalar field interacting with the ideal fluid with inhomogeneous EOS through an energy exchange between them. The equations of motion of the scalar field and the ideal fluid can be written as,

\[
\dot{\rho}_d + 3H(\rho_d + p_d) = -3H\rho_d\delta \quad (10)
\]

and

\[
\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 3H\rho_d\delta \quad (11)
\]

where \(\delta\) is a constant.

Taking into account the recent cosmological considerations of variations of fundamental constants, one may start from the case that the pressure depends on the time \(t\) [18]. Unlike the EOS studied in [19] where the parameters involved in EOS are linear in \(t\), we consider rather a polynomial form. First, we choose the EOS of the ideal fluid to be,

\[
p_d = a_1 t^{-\alpha} \rho_d - ct^{-\beta} \quad (12)
\]
where, \(a_1, c, \alpha, \beta\) are constants.

Here, we see that initially the pressure is very large and as time increases pressure falls down, which is very much compatible with the recent observational data.

We consider a Universe with power law expansion
\[
a(t) = t^n
\]
so as to get a non-decelerated expansion for \(n \geq 1\), as the deceleration parameter reduces to
\[
q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1-n}{n} < 0.
\]

Now equation (10) together with (12) and (13) gives the solution for \(\rho_d\) to be,
\[
\rho_d = t^{-3n(1+\delta)}e^{\frac{3na_1}{\alpha}t^{-\alpha}}\left(\frac{3na_1}{\alpha}\right)^{\frac{3n(1+\delta)+\alpha-\beta}{\alpha}}\frac{c}{a_1}\Gamma\left(\frac{\beta-3n(1+\delta)}{\alpha}, \frac{3na_1t^{-\alpha}}{\alpha}\right)
\]
where, \(\Gamma(a, x)\) is upper incomplete Gamma function.

Further substitution in the above equations give the solution for \(\rho_\phi\), \(\dot{\phi}^2\) and \(V(\phi)\) to be,
\[
\rho_\phi = 3\frac{n^2}{t^2} - \rho_d
\]
\[
\dot{\phi}^2 = 2\frac{n}{t^2} - \left[(1 + a_1t^{-\alpha})\rho_d - ct^{-\beta}\right]
\]
such that,
\[
\phi = \phi_0 + \int \sqrt{2\frac{n}{t^2} - \left[(1 + a_1t^{-\alpha})\rho_d - ct^{-\beta}\right]} \, dt
\]
and
\[
V = \frac{3n^2 - n}{t^2} + \frac{(-1 + a_1t^{-\alpha})\rho_d}{2} - \frac{ct^{-\beta}}{2}
\]

Since we have considered a power law expansion of the scale factor so we can see from the above expressions that \(\rho_d\) and \(\rho_\phi\) are decreasing functions of time so that the total energy density as well as pressure decreases with time. The evolution of the Universe therefore can be explained without any singularity. Normalizing the parameters, we get the variation of \(V(\phi)\) against \(\phi\) in figure 3. Equation (18) shows that, for \(\beta < 2\), the potential being positive initially, may not retain this as \(t \to \infty\) (as the 3rd term dominates over first term and the third and second term being negative for large values); for \(\beta = 2\) the potential can be positive depending on the value of \((3n^2 - n - \frac{c}{2})\) and for, \(\beta > 2\) the potential can be either positive depending on the choices of the constants, but always decreases with time. Hence \(\beta\) is completely arbitrary and depending on various values of \(\beta\) and the other constants, potential to be positive, although it is always decreasing with time. Fig 3 shows the nature of the potential for arbitrarily chosen values of the constants. Also if we consider \(w_d = \frac{p_d}{\rho_d}, w_\phi = \frac{p_\phi}{\rho_\phi}, w_{tot} = \frac{p_{tot}}{\rho_{tot}},\) and plot them (figure 1) against time, we see this represents an XCDM model and therefore it makes a positive contribution to \(\ddot{a}/a\).

Inhomogeneous dark energy EOS coming from geometry, for example, \(H\) can yield cosmological models which can avoid shortcomings coming from coincidence problem and a fine-tuned sudden evolution of the Universe from the early phase of deceleration driven by dark matter to the present phase of acceleration driven by dark energy. Furthermore, such models allow to recover also early accelerated regimes with the meaning of inflationary behaviors [20]. The following model is often referred to as Increased Matter Model where the pressure depends on energy density and \(H\). A detailed discussion of this kind of EOS can be found in ref. [20].

Now we choose the EOS of the ideal fluid to be,
\[
p_d = A\rho_d + BH^2
\]
Fig. 1 and 2 show the variation of $1 + 3w$ where $w = w_d, w_\phi, w_{\text{tot}}$ against time normalizing the parameters as mentioned above.

where, $A$ and $B$ are constants.

Considering the power law expansion (13) and using (10) and (19), we get the solution for $\rho_d$ to be,

$$\rho_d = C_0 t^{-3n(1+A+\delta)} - \frac{3n^3B}{3n(1+A+\delta) - 2} t^{-2}$$  \hspace{1cm} (20)

Further substitution in the related equations yields the solution for $\rho_\phi, \phi, V(\phi)$ to be,

$$\rho_\phi = \frac{3n^2}{t^2} - \rho_d$$  \hspace{1cm} (21)

$$\phi = \phi_0 + \frac{2}{2 - K_1} \left[ \sqrt{K_1 + K_2 t^2 - K_3} - \sqrt{K_1} \sinh^{-1} \left( \sqrt{\frac{K_1}{K_2}} x \right) \right]$$  \hspace{1cm} (22)

where, $x = t^{K_3 - 1}, K_2 = -C_0(1 + A), K_1 = \frac{6n^2(1+A+\delta) - 4n - 3Bn^3 + 2n^2}{K_3 - 2}, K_3 = 3n(1 + A + \delta)$

and

$$V = \frac{3n^2}{t^2} - n - \frac{A - 1}{2} \rho_d + \frac{Bn^2}{2t^2}$$  \hspace{1cm} (23)

Equation (22) shows that $K_1$ must be positive and hence $K_2$ also must be positive for a valid expression. Also equation (20) says that $C_0$ must be positive, otherwise $\rho_d$ becomes negative initially. Therefore expression of $K_2$ says that $A$ must be negative, in fact, $A < -1$, such that depending on the value of $B$ pressure can be positive or negative. Normalizing the parameters, we get the variation of $V$ against $\phi$ in figure 4. The figure shows a decaying nature of the potential. Also if we consider $w_d = \frac{\rho_d}{\rho_{\text{tot}}}, w_\phi = \frac{\rho_\phi}{\rho_{\text{tot}}}, w_{\text{tot}} = \frac{\rho_{\text{tot}}}{\rho_{\text{tot}}}$, and plot them (figure 2) against time, like the previous case, we see this represents an XCDM model and therefore it makes a positive contribution to $\ddot{a}/a$.

In this letter we study a cosmological model of the Universe in which the scalar field has an interaction with an ideal fluid with inhomogeneous EOS. The interaction is introduced phenomenologically by considering term parameterized by the product of the Hubble parameter, the energy density of the ideal fluid and a coupling constant in the equations of motion of the fluid and the scalar field. This type of phenomenological interaction term has been investigated in [12]. This describes an energy flow between the scalar field and the ideal fluid. Also we consider a power law form of the scale factor $a(t)$ to keep the recent observational support of cosmic acceleration. For the first model putting $c = 0, \alpha = 0$ we get the results for barotropic fluid. Here for $\alpha$ and
\( n = 2, \alpha = 1, \beta = 2, a_1 = .1, c = 1, \delta = .01 \)

\( A = \frac{1}{4}, B = -2, \phi_0 = 1, \delta = .1, n = 2 \)

Fig. 3 and 4 show the variation of \( V \) against \( \phi \) normalizing the parameters as mentioned above.

\( \beta \) to be positive, the ideal fluid and the scalar field behave as dark energy. Also we see that the interaction term decreases with time showing strong interaction at the earlier stage and weak interaction later. Also the potential corresponding to the scalar field is positive and shows a decaying nature. In the second model where \( p_d \) is a function of \( \rho_d \) and the Hubble parameter \( H \), we see that the energy density and the pressure of the ideal fluid and that of the scalar field always decreases with time. From figures 3 and 4, we see that, the potential function \( V \) decreases for both decelerating \((n < 1)\) and accelerating phase \((n > 1)\). Also from the values of density and pressure terms, it can be shown that the individual fluids and their mixtures satisfy strong energy condition for \( n < 1 \) and violate for \( n > 1 \). A detailed discussion of the potential of a scalar field can be found in ref. [21]. We see that the coupling parameter shows a decaying nature in both the cases implying strong interaction at the early times and weak interaction later. Thus following the recipe provided in ref. [22] we can establish a model which can be a suitable alternative to dark energy explaining the decaying energy flow between the scalar field and the fluid and giving rise to a decaying potential. As a scalar field with potential to drive acceleration is a common practice in cosmology [22], the potential presented here can reproduce enough acceleration together with the ideal fluid, thus explaining the evolution of the Universe. Also we have considered inhomogeneous EOS interacting with the scalar field which can represent an alternative to the usual dark energy model. However, stability analysis and spatial inhomogeneity analysis [10] are more complicated for our investigation, since we are considering the ideal fluid with two types of equation of states and are analysing whether they can be considered as an alternative to dark energy. Also we have seen that the equations of motion (10) together with the given form of the pressure (12) and (19) are difficult to solve unless we consider the power law form (13). Once the power law form is considered, we can easily find exact solution of \( \rho_d \) (from eq.(10)) and hence \( \rho_\phi \) (from eq.(2)), which lead to the given expression for the potential \( V(\phi) \) [from eqs. (7), (8)] analytically. Though this is the backward approach, but otherwise if we start from \( V(\phi) \) i.e., say \( V(\phi) = V_0 Exp(-k\phi) \), we cannot find any exact solution of \( \rho_d, \rho_\phi, p_d, p_\phi, \phi, a \). So we can only draw conclusions graphically, not analytically. For example, Ellis et al [23] have discussed for the model with radiation and scalar field and found exact solutions in the backward approach.

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