\( \Xi \)-Nucleus Potential and \((K^-, K^+)\) Inclusive Spectrum at \( \Xi^- \) Production Threshold Region

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The \((K^-, K^+)\) \( \Xi^- \) production inclusive spectrum is reinvestigated in view of the very weak \( \Xi^- \)-nucleus potential predicted by microscopic calculations with the \( SU_6 \) quark-model baryon-baryon interaction. The inclusive spectrum is evaluated by the semiclassical distorted wave (SCDW) method. The explicit comparison of the strength function with that of the Green-function method demonstrates the quantitative reliability of the SCDW approximation. It is presumed that the presently available data at the \( \Xi^- \) production threshold region does not necessarily imply the attractive strength of about 15 MeV for the \( \Xi^- \)-nucleus potential in a conventional Woods-Saxon form. Instead, an almost zero potential is preferable.

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§1. Introduction

The study of the baryon-baryon interactions has made steady progress toward the \( S = -2 \) sector. The construction of a \( \Lambda N \) interaction model is almost under control, based on the experimental data of \( \Lambda \) hyper nuclei. The \( \Sigma N \) interaction was elusive until 1990s because there was no clear \( \Sigma \) bound state observed in \( \Sigma \) formation spectra, except for the \( ^4\Sigma \)He due to the attraction in the specific isospin \( T = 1/2 \) channel. Now it seems to be established that the \( \Sigma \)-nucleus potential is repulsive with the weak attraction at the nuclear surface. Such property was suggested first by the analyses of the \( \Sigma^- \) atomic level shifts by Batty, Friedman, and Gal.\(^1\) The overall repulsive nature of the \( \Sigma \)-nucleus potential was indicated by the \((\pi^-, K^+)\) \( \Sigma^- \) inclusive spectra at KEK.\(^2\) This feature of the \( \Sigma \)-nucleus interaction is predicted by the microscopic calculations\(^3\) starting from the \( SU_6 \) quark-model interactions.\(^4\) Then, the interactions in the \( S = -2 \) sector, either \( \Lambda \Lambda \) or \( \Xi N \), is the forefront of the experimental studies of the strangeness nuclear physics. This knowledge is naturally invaluable for the quantitative predictions for various aspects of neutron star matter.

There have been a few experimental data for the properties of the \( \Xi \) hyperon in the nuclear medium. The old emulsion data was used by Dover and Gal\(^5\) to conclude that the depth of the \( \Xi \)-nucleus potential was more than 20 MeV. At present, there are only a few sets of experimental cross sections for the \( \Xi^- \) production by \((K^-, K^+)\) reactions. One is the experiment at KEK by Iijima et al.,\(^6\) that covers a wide range of the outgoing \( K^+ \) energy. The other is the data by Fukuda et al.\(^7\) and Khaustov et al.,\(^8\) in which the chief motivation is the search of the \( \Xi \) bound
state and thus the measurements were concentrated on the energy region around the \( \Xi \) production threshold. Both data sets are insufficient to establish the \( \Xi \)-nucleus interaction, because of the energy and angle resolutions as well as the limited number of counts. Nevertheless, these data around the \( \Xi \) production threshold were analyzed to indicate that the depth is 16 MeV and 14 MeV, respectively. This magnitude seems to be canonical at present for the \( \Xi \)-nucleus potential. The estimation relies solely on analyses by means of distorted wave impulse approximation (DWIA) reported in Refs. 7) and 8). Those theoretical calculations of the \( \Xi \) formation inclusive spectra, however, contain various simplifying approximations and uncertainties, which hinders the reliability of discussing absolute value of the cross sections. It is necessary to reanalyze the data by an independent calculational framework.

The microscopic calculations in Ref. 9) with the \( SU_6 \) quark-model baryon-baryon interactions fss2^1) predict that the localized \( \Xi \)-nucleus potential in finite nuclei fluctuates around 0 inside a nucleus with some weak attraction at the nuclear surface region. Observing that the \( \Sigma \)-nucleus potential calculated microscopically in Ref. 9) shows a good correspondence with the empirical character without adjustments, the prediction for the \( \Xi \) is also credible. It is interesting to investigate whether such a weak \( \Xi \)-nucleus potential provides \( \Xi \) formation spectra consistent with the experimental data.

In this paper, we examine the data of Ref. 8) by employing the SCDW method^10) for evaluating the spectrum in which the energy and angle dependences of the elementary cross section are respected.\(^{11}\) The nuclear Fermi motion is properly taken into account by the Wigner transformation of the target s.p. wave functions. The distorted waves of the incoming and outgoing kaons are described by the Klein-Gordon equation. Actually we reported results of the SCDW calculation for \((K^-,K^+)\) inclusive spectra in Ref. 12). Although the whole spectrum including the transition to possible bound states was presented in that paper, the reliability of the SCDW method at the threshold region was not a priori certain, because the semiclassical approximation had to be used over a large distance. Therefore we mainly referred to the data by Iijima et al.\(^6\) Here, we show in §2 by an explicit numerical comparison of the SCDW method and the Green-function method that the SCDW method is reliable to discuss quantitatively the cross section even at the threshold region. On this basis, the attention in this paper is focused on the data around the \( \Xi \) production threshold. Calculated results are compared with the data by Khaustov et al.\(^8\) in §3. A summary is given in §4.

§2. Comparison of the SCDW method with the Green-function method

A Green-function method has been widely used for analyzing various hadron production inclusive spectra. The use of the Green function for treating final state interactions in inclusive reactions was presented, for example, in Ref. 13) for the inclusive \((e,e')\) reactions. The application to hyperon production processes was initiated by the study of the production of \( \Sigma \)-hypernuclear states in \((K^-,\pi^+)\) reaction by Morimatsu and Yazaki.\(^{14}\) The calculation for inferring the \( \Xi \)-nucleus potential depth to be about 14 MeV on the basis of the \((K^-,K^+)\) events on carbon\(^8\) essen-
Note that this expression can be extended to the case that the \( v \) amplitude otherwise practical calculations are hard to be carried out. Namely, the elementary \( a \) Lorentz-type distribution function.

In this section we compare our SCDW method with the Green-function method in numerical detail. The basic formula of the double differential cross section for the inclusive \((K^-, K^+)\) \( \Xi^- \) production reaction in a distorted wave impulse approximation is

\[
\frac{d^2\sigma}{dWd\Omega} = \frac{\omega_i,\text{red}\omega_f,\text{red}}{(2\pi)^2} \frac{pf}{p_i} \sum_{p,h} \frac{1}{4\omega_i\omega_f} |\langle \chi_f^{(-)}|v_{f,p,i,h}\rangle|^{2}\delta(W - \epsilon_p + \epsilon_h), \quad (2.1)
\]

where \( \chi_i^{(+)} \) and \( \chi_f^{(-)} \) represent the incident \( K^- \) and final \( K^+ \) wave functions with energies \( \omega_i \) and \( \omega_f \), respectively, and \( W = \omega_i - \omega_f \) is the energy transfer. The corresponding momenta are represented by \( p_i \) and \( p_f \). The reduced energy with respect to the target nucleus (residual hyper nucleus) is denoted by \( \omega_i,\text{red} \) (\( \omega_f,\text{red} \)). The formula describes the process in which the nucleon in the occupied single-particle state \( h \) is converted to the unobserved outgoing \( \Xi \) hyperon state \( p \). The elementary amplitude of the process \( K^- + p \to K^+ + \Xi^- \) is denoted by \( v_{f,p,i,h} \), which depends on the energy and momentum of the particles in the reaction.

The summation \( \sum_p \) is taken over the complete set of the unobserved final \( \Xi \) hyperon states. This summation with the energy-conserving \( \delta \)-function can be written by the Green function \( G(r,r';W) \).

\[
\sum_p \delta(W - \epsilon_p + \epsilon_h)\langle r|\phi_p^{(-)}\rangle\langle \phi_p^{(-)}|r'\rangle = -\frac{1}{\pi} \text{Im} \sum_p \frac{\langle r|\phi_p^{(-)}\rangle\langle \phi_p^{(-)}|r'\rangle}{W - \epsilon_p + \epsilon_h + i\epsilon} = -\frac{1}{\pi} \text{Im} G(r,r';W). \quad (2.2)
\]

Note that this expression can be extended to the case that the \( \Xi \) hyperon is described by a complex optical model potential to take care of the decaying processes to other channels. In our SCDW method, \( \phi_p^{(-)} \) is described by a real potential. Effects of the inelastic channels are taken into account by convoluting the calculated spectrum by a Lorentz-type distribution function.

The Green-function method commonly introduces a factorization approximation; otherwise practical calculations are hard to be carried out. Namely, the elementary amplitude \( v_{f,p,i,h} \) is replaced by some average and taken out of the integration and the summation. In this case, the differential cross section becomes

\[
\frac{d^2\sigma}{dWd\Omega} = \beta \left( \frac{d\sigma}{d\Omega} \right)_{av} S_{\text{GF}}(W), \quad (2.3)
\]

introducing the factor \( \beta \) which represents the difference of the kinematics in the two-body and \( A \)-body systems. In actual calculations, a recoil correction is incorporated. Denoting \( \zeta \equiv \frac{A-1}{A} \) with \( A \) being the mass number of the target nucleus, the strength
function $S_{GF}(W)$ is defined by

$$S_{GF}(W) = -\frac{1}{\pi} \text{Im} \langle \chi_{f}^{(-)*}(\zeta r) \chi_{i}^{(+)}(\zeta r) \phi_{h}(r) | G(r, r'; W) | \chi_{f}^{(-)*}(\zeta r') \chi_{i}^{(+)}(\zeta r') \phi_{h}(r') \rangle. \quad (2.4)$$

The determination of $\beta$ and $(\frac{d\sigma}{dW})_{av}$ admits ambiguities, although there have been some attempts to introduce more elaborate approximation for $\beta(\frac{d\sigma}{dW})_{av}$ than the naive frozen approximation, such as the optimal Fermi-averaging by Harada and Hirabayashi\textsuperscript{16} and the local optimal Fermi-averaging by Maekawa et al.\textsuperscript{17} for describing hyperon production spectra.

The SCDW model does not introduce the factorization approximation. Instead, the wave function between two points $r$ and $r'$ is approximated by a plane wave with the local classical momentum $k(R)$:

$$\chi_{i,f}(r) \chi_{i,f}^{(+)*}(r') = \chi_{i,f}(R + \frac{1}{2}s) \chi_{i,f}^{(+)*}(R - \frac{1}{2}s) \simeq |\chi_{i,f}(R)|^2 e^{i k(R) \cdot s}. \quad (2.5)$$

The direction of $k(R)$ is calculated by the quantum mechanical momentum density $k_q(R)$

$$k_q(R) = \frac{\text{Re}\{\chi^{(+)*}(R)(-i)\nabla \chi^{(\pm)}(R)\}}{|\chi^{(\pm)}(R)|^2}, \quad (2.6)$$

and the magnitude of $k(R)$ is determined by the energy-momentum relation at $R$

$$m_K^2 + k^2(R) + 2\omega_{i,f}(U_R(R) + V_{\text{Coul}}(R)) - V_{\text{Coul}}^2(R) = \omega_{i,f}, \quad (2.7)$$

where $m_K$ is the kaon mass, $V_{\text{Coul}}$ is the Coulomb potential, and $U_R(R)$ is the real part of the optical potential for $\chi_{i,f}$ with the energy $\omega_{i,f}$. The expression of the double differential cross section in this approximation is explained in Ref. 11\textsuperscript{11} and the inclusion of recoil corrections is specified in Ref. 12\textsuperscript{12}. The final expression reads

$$\frac{d^2\sigma}{dWd\Omega} = \frac{\omega_{i,f} p_f}{(2\pi)^2 p_i} \xi^6 \int \int dRdK \sum_p \frac{1}{4\omega_{i,f}} |\chi_{f}^{(-)}(R)|^2 |\chi_{i}^{(+)}(R)|^2 \times|\phi_p^{(-)}(\xi R)|^2 |v_{f,p,i,h}|^2 \frac{(2\pi)^3}{\xi^3} \sum_h \Phi_h \left( \xi R, \frac{1}{\xi} K \right) \times \delta(K + k_i(R) - k_f(R) - k_p(R)) \delta(W - \epsilon_p + \epsilon_h), \quad (2.8)$$

where $\xi = \frac{1}{\xi} = \frac{A}{A+1}$ appears by taking care of the recoil effects and $\Phi_h$ is the Wigner transformation of the density matrix of the nucleon hole state wave function. If we introduce the factorization approximation, we can define the strength function $S_{SCDW}(W)$ in the SCDW method.

$$S_{SCDW}(W) = \xi^6 \int \int dRdK \sum_p |\chi_{f}^{(-)}(R)|^2 |\chi_{i}^{(+)}(R)|^2 |\phi_p^{(-)}(\xi R)|^2 \frac{(2\pi)^3}{\xi^3} \times \sum_h \Phi_h \left( \xi R, \frac{1}{\xi} K \right) \delta(K + k_i(R) - k_f(R) - k_p(R)) \delta(W - \epsilon_p + \epsilon_h). \quad (2.9)$$
While the energy and angle dependences of the elementary amplitude $|v_{f,p,i,h}|^2$ are, in the SCDW method, treated explicitly inside the integration of Eq. (2.8), though the on-shell approximation has to be used, the approximation of Eq. (2.5) brings about certain uncertainties. The SCDW approximation has been successful in quantitatively describing intermediate energy $(p, p'x)$ and $(p, p'n)$ inclusive reactions. However, we cannot expect a priori that the replacement of Eq. (2.5) is always reliable. As noted above, if we set $|v_{f,p,i,h}|^2 = 1$, we obtain the strength function in the SCDW treatment, which can be directly compared with the exact strength function $S_{GF}(W)$ in the Green-function method. It is useful to assess the reliability of the SCDW method by comparing $S_{SCDW}(W)$ with $S_{GF}(W)$ by numerical calculations.

We show, in Fig. 1, the strength functions $S_{SCDW}(W)$ and $S_{GF}(W)$ for the $(K^-, K^+) \Xi^-$ production at $p_{K^-} = 1.8$ GeV/c which are obtained from the three choices of the strength of the $\Xi$-nucleus potential, $U_0^\Xi = -20, -5, +10$ MeV, in a standard Woods-Saxon form ($r_0 = 1.2 \times A^{1/3}$ fm and $a = 0.65$ fm). The density-dependent Hartree-Fock wave functions of Campi and Sprung are used for the hole states of $^{12}$C. The $K^-$ and $K^+$ distorted waves are provided by solving the Klein-Gordon equation with the optical potential parameters given in Ref. 12). There are bound states in $s$ and $p$ orbits for the case of $U_0^\Xi = -20$ MeV, but those contributions are not shown in Fig. 1. The Coulomb potential regularized by the finite charge distribution is included, which provides non-zero strength of the calculated strength function at the threshold. The $S_{SCDW}(W)$ agrees well with the $S_{GF}(W)$ at all energies. The difference is seen at most 10%. On the basis of this correspondence, we are assured to use the SCDW method to include the energy and angle dependences of the elementary process in the nuclear medium together with the explicit treatment of the nucleon Fermi motion.

Note that if we multiply the strength function by $\beta \left( \frac{d\sigma}{d\Omega} \right)_{av}$, we readily obtain

![Fig. 1. Strength functions in the case of the $(K^-, K^+) \Xi^-$ production inclusive reaction on $^{12}$C at $p_{K^-} = 1.8$ GeV/c. The solid, dashed, and dotted curves correspond to the three choices of the strength of the $\Xi$-nucleus potential in a standard Woods-Saxon form ($r_0 = 1.2 \times A^{1/3}$ fm and $a = 0.65$ fm); $U_0^\Xi = -20, -5, +10$ MeV, respectively. The left panel shows the results of the SCDW method, $S_{SCDW}(W)$, and the right panel those of the GF method, $S_{GF}(W)$.](https://academic.oup.com/ptp/article-abstract/123/1/157/1917808)
double differential cross sections in a factorization approximation, although the energy dependence of the multiplicative factor is not simple to determine when discussing the yield over the wide range of excitation energies.

§3. SCDW-Model calculations of \((K^-, K^+)\) \(\Xi^-\) production inclusive spectrum on carbon

We show, in Fig. 2, the calculated spectra by the SCDW model for \((K^-, K^+)\) \(\Xi^-\) production inclusive reactions on \(^{12}\text{C}\) at \(p_{K^-} = 1.8\text{ GeV/c}\), corresponding to the experiment by Khaustov et al.\(^8\)) at KEK. The threshold region is magnified in Fig. 3. Because the experimental data is the average cross section between the angles of \(0^\circ\) and \(8^\circ\), the calculations carried out at \(\theta_{K^+} = 5.5^\circ\) are shown. This angle is the mean value in the following meaning. The angle average of the cross section \(\sigma(\theta)\) over the angle between \(\theta_{\text{min}}\) and \(\theta_{\text{max}}\) is given by

\[
\sigma^{\text{av}}(\theta_{\text{min}}, \theta_{\text{max}}) = \frac{\int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \sigma(\theta) \sin \theta d\theta}{\int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \sin \theta d\theta}. \quad (3.1)
\]

Because the scattering angle is limited to the forward region, it is sufficient to adopt the approximation \(\sin \theta \sim \theta\) and \(\sigma(\theta) \sim \sigma(0) + \sigma'(0)\theta + \frac{1}{2}\sigma''(0)\theta^2\). The mean angle \(\theta_{\text{av}}\) that gives \(\sigma(\theta_{\text{av}}) = \sigma^{\text{av}}(0^\circ, 8^\circ)\) is in the vicinity of \(5.5^\circ\), irrespective of \(\sigma(0)\), \(\sigma'(0)\), and \(\sigma''(0)\).

![Fig. 2. \((K^-, K^+)\) \(\Xi^-\) production inclusive spectra on \(^{12}\text{C}\) at \(p_{K^-} = 1.8\text{ GeV/c}\). The solid curve is the calculated result at \(\theta_{K^+} = 5.5^\circ\) for the \(\Xi\)-nucleus potential suggested by the quark-model \(\Xi N\) interaction fss2. Other curves are the results at \(\theta_{K^+} = 5.5^\circ\) for potentials in a single Woods-Saxon form with the depths of +10, 0, −10, and −20 MeV, respectively, assuming standard geometry parameters of \(r_0 = 1.2 \times 12^1/3\) fm and \(a = 0.65\) fm. The data by Khaustov et al.\(^8\)) is shown by filled circles. Note that the experimental data is the average of the cross sections over the angles between \(0^\circ\) and \(8^\circ\) and the original cross section in Ref. 8) is given as a histogram with a 2 MeV step. The data by Iijima et al.\(^6\)) at the threshold region is also shown by a filled square, though the incident \(K^-\) momentum is 1.65 GeV/c.](https://academic.oup.com/ptp/article-abstract/123/1/157/1917808)
In the SCDW method, the wave function of the unobserved $\Xi$ hyperon $\phi_p^(-)$ is described by a real optical model potential. The effects of inelastic channels are incorporated by convoluting the calculated spectrum with a Lorentz-type distribution function. We assign 2 MeV to the half width $\Gamma/2$, which is consistent with the empirical indication by the $\Xi^-p$ elastic and inelastic cross section measurements at low energy by Ahn et al.\textsuperscript{20} and also the weak imaginary part of the $\Xi$-nucleus potential obtained from the microscopic calculation with the quark-model potential.\textsuperscript{9} In addition, we take into account the experimental resolution of $\Delta E = 6.1 \text{ MeV}^8$ by smearing the spectrum with a Gaussian function:

$$f(E, \Delta E) = \frac{1}{\Delta E} \sqrt{\frac{\log 2}{\pi}} e^{-\log 2(E/\Delta E)^2}.$$  

Calculations are carried out for the $\Xi^{-12}C$ potential parametrized in a sum of two Woods-Saxon forms on the basis of the microscopic calculations in Ref. 9). The parameters are tabulated in Table I. We also evaluate, for the purpose of reference, the spectrum with potentials described by a single Woods-Saxon form in a standard geometry of $r_0 = 1.2 \times 12^{1/3} \text{ fm}$ and $a = 0.65 \text{ fm}$ with $U^0_\Xi = +10$, 0, $-10$, and $-20 \text{ MeV}$, respectively. The $K^- + p \rightarrow K^+ + \Xi^-$ elementary differential cross section is taken from the parametrization by Nara et al.\textsuperscript{21} We show, in Fig. 4, the angle dependence of the elementary cross section of this parametrization in the laboratory frame for two incident $p_{K^-}$ momenta of $1.8 \text{ GeV}/c$ and $1.65 \text{ GeV}/c$. Note that although the differential cross section in the center-of-mass frame is backward peaking, in the laboratory frame the cross section in the forward angles is larger.

The potential based on the quark-model potential fss2 gives a very similar result to the case of $U^0_\Xi = 0 \text{ MeV}$. Calculations with different strengths of the $\Xi$-nucleus potential show that the peak position and the width of the spectrum change systematically. An attractive potential, even a weak one as $U^0_\Xi = -10 \text{ MeV}$, is seen to

![Fig. 3. Magnification of the threshold region of Fig. 2.](https://academic.oup.com/ptp/article-abstract/123/1/157/1917808)
Table I. Strength and geometry parameters of the Woods-Saxon form $f_i(r) = U_i^0/[1 + \exp((r - r_{0,i})/a_i)]$ fitted to the real part of the $\Xi$ single-particle in $^{12}\text{C}$ calculated microscopically$^9$ with the quark-model potential $fss2$.

| $i$ | $U_i^0$ (MeV) | $r_{0,i}$ (fm) | $a_i$ (fm) |
|-----|---------------|----------------|------------|
| 1   | $-4.139$      | $3.569$        | $0.5291$   |
| 2   | $10.53$       | $2.130$        | $0.3032$   |

Fig. 4. Angle dependence of the $K^- + p \rightarrow K^+ + \Xi^-$ elementary cross section parametrized by Nara et al.$^{21}$ in the laboratory frame at $p_{K^-} = 1.65$ GeV/c and $p_{K^-} = 1.8$ GeV/c, respectively.

...predict larger cross sections at the $\Xi$ production threshold region than the experimental data. Thus, an almost zero potential is preferable to account for data by Khaustov et al.,$^8$ even when the uncertainty of about 10% of the SCDW treatment is taken into account.

The same data was analyzed in Ref. 8) to judge that the depth of the $\Xi$-nucleus potential is 14 MeV or less. Because the details of the theoretical calculation are not found in Ref. 8), it is hard to figure out the cause of the difference from our result. In Fig. 5, we compare the spectrum before folding the experimental resolution by the SCDW model calculation with that given by Eq. (2.3) of the Green-function method for the two cases of the $\Xi$-nucleus potential in a Woods-Saxon form. In this case, the strength function $S_{GF}(W)$ is evaluated with including the imaginary potential. The absorptive strength in the same Woods-Saxon form as the real part is set to be 4 MeV. Considering surface effects, this strength is regarded to correspond to $\Gamma = 2$ MeV for smearing the spectrum of the SCDW model. In Ref. 7) for the $(K^-,K^+)$ reaction at $p_{K^-} = 1.65$ GeV/c, $\beta \left( \frac{d\sigma}{d\Omega} \right)_{av}$ in Eq. (2.3) is set as $0.73 \times (35 \pm 5) \sim 26 \mu b/sr$. The average differential cross section at forward angles of $35 \pm 5 \mu b/sr$ is consistent with the cross section shown in Fig. 4. However, if we use $\beta = 0.73$, the spectrum overestimates the experimental data. To obtain the similar strength as the SCDW cross sections around the $\Xi$ production threshold, we need $\beta \left( \frac{d\sigma}{d\Omega} \right)_{av} = 0.33 \times 35 \mu b/sr$. It is seen in Fig. 5 that if the multiplicative average cross section is assumed...
to be energy independent, the spectrum of the Green-function method tends to underestimate the cross section at around the peak position. The importance of taking into account the energy dependence of the Fermi-averaging is also pointed out by Harada and Hirabayashi.\(^{16}\)

As for the other \((K^-, K^+)\) \(\Xi^-\) production data by Iijima et al.,\(^6\) the SCDW-model calculation accounts only the half of the experimental magnitude, as reported in Ref. 12). Other theoretical calculations so far reported show similar underestimation. The result on carbon target from the intra-nuclear cascade model calculation by Nara et al.\(^{21}\) is similar to our spectrum. The DWIA calculation with the Green-function method by Tadokoro et al.\(^{22}\) seemingly agrees well with the experimental data. However, they present only the spectrum at \(\theta_{K^+} = 0^\circ\). Although they commented that the angle dependence was negligibly small, this is questionable because our calculations both in the SCDW method and the Green-function method as well as the calculation in Ref. 8) indicate strong angle dependence of the \((K^-, K^+)\) \(\Xi^-\) production spectrum. If we consider the spectrum at \(\theta_{K^+} = 0^\circ\), our SCDW result is close to that of Tadokoro et al.

The change of the strength of the \(\Xi\)-nucleus potential shifts the peak position and alters the width of the spectrum, but it cannot increase the height of the spectrum by a factor of 2. If the magnitude of the elementary amplitude is reliable, we presume that there is inconsistency between the data at \(p_{K^-} = 1.65\ GeV/c\) by Iijima et al.\(^6\) which covers a wide range of the \(\Xi\) excitation energy and the data at \(p_{K^-} = 1.8\ GeV/c\) which is compared with that of the Green-function method without including the experimental resolution. The latter spectrum is given by Eq. (2.3) with \(\beta (d\sigma/d\Omega)_av = 0.33 \times 35\ \mu b\). \(S_{\text{GF}}(W)\) is evaluated with including the absorptive potential of the strength of 4 MeV. The number of \(\beta = 0.33\) is chosen to match with the cross sections of the SCDW method at the threshold region. Note that \(\beta = 0.73\) is used in Ref. 8) for \(p_{K^-} = 1.65\ GeV/c\).
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GeV/c by Khaustov et al.\textsuperscript{8}) which is limited to the energy region around the $\Xi^-$ production threshold.

§4. Summary

We have examined whether the weak $\Xi$-nucleus potential suggested by microscopic calculations\textsuperscript{9}) using the $SU_6$ quark-model baryon-baryon interaction fss2\textsuperscript{4}) provides a reasonable description for $\Xi$ formation spectra available at present. We employ the SCDW model to calculate the $(K^-, K^+)\Xi^-$ production spectrum. This method was developed\textsuperscript{11}) for evaluating various inclusive cross sections, respecting the energy and angle dependences of the elementary process together with the nuclear Fermi motion. The semiclassical local momentum approximation is demonstrated, in this paper, to be sufficiently reliable to discuss quantitatively cross sections by explicitly comparing the SCDW strength function and the precise strength function calculated in the Green-function method.

In the present calculation we use the $\Xi$-nucleus potential as parameterized in a sum of two Woods-Saxon forms on the basis of the microscopic calculation\textsuperscript{9}) in $^{12}$C. We do not claim that the potential is definitely accurate and reliable, but regard it as the typical weak potential with non-monotonic radial dependence which does not support any $\Xi$ nuclear bound state. The microscopic calculation of the $\Xi$-nucleus potential with the quark-model $\Xi N$ interaction fss2 indicates that the imaginary strength is rather small, which is consistent with the experimental data.\textsuperscript{20}) We employ the half width of 2 MeV to smear the SCDW spectrum using the Lorentz-type distribution function. To compare the calculated result with the data, we fold, in addition, the experimental resolution of 6.1 MeV\textsuperscript{8}) by a Gauss functional form. We also calculate the spectrum with the $\Xi$-nucleus potentials, $U_\Xi^0 = +10, 0, -10,$ and $-20$ MeV, in a standard Woods-Saxon form to examine the potential dependence of the $\Xi$ production cross section.

Our conclusion is that the weak $\Xi$-nucleus potential yields a right order of magnitude of the experimental $\Xi$ production cross section available at present. If we use a standard Woods-Saxon form, the strength of the $\Xi$-nucleus potential should be almost zero. This result does not agree with the speculation by the DWIA analyses in Ref. 8) that the well depth is about 14 MeV. We have to bear in mind, however, that this experimental data is based on the small number of counts that are a few tens or fewer and thus may not be accurate enough to conclude the strength of the $\Xi$-nucleus potential. More experimental data with a better resolution and statistics will be obtained from the $(K^-, K^+)$ experiment being prepared in the J-PARC project.\textsuperscript{23}) On the theoretical side, efforts to reduce ambiguities due to $K^\pm$ distorted waves and possible multi-step contributions have to be made. Our calculations show that even if the experiment finds little $\Xi$ production strength in the $\Xi$ bound state region, we are able to infer the strength of the $\Xi$-nucleus potential. This will promote our understanding of the baryon-baryon interactions in the $S = -2$ sector and improve theoretical models of them.
References

1) C. J. Batty, E. Friedman and A. Gal, Phys. Lett. B 335 (1994), 273; Phys. Rep. 287 (1997), 385.
2) H. Noumi et al., Phys. Rev. Lett. 89 (2002), 072301.
3) M. Kohno, Y. Fujiwara, T. Fujita, C. Nakamoto and Y. Suzuki, Nucl. Phys. A 674 (2006), 229.
4) Y. Fujiwara, Y. Suzuki and C. Nakamoto, Prog. Part. Nucl. Phys. 58 (2007), 439.
5) C. B. Dover and A. Gal, Ann. of Phys. 146 (1983), 309.
6) T. Iijima et al., Nucl. Phys. A 546 (1992), 588.
7) T. Fukuda et al., Phys. Rev. C 58 (1998), 1306.
8) P. Khaustov et al., Phys. Rev. C 61 (2000), 054603.
9) M. Kohno and Y. Fujiwara, Phys. Rev. C 79 (2009), 054318.
10) Y. L. Luo and M. Kawai, Phys. Lett. B 235 (1990), 211; Phys. Rev. C 43 (1991), 2367.
11) M. Kohno, Y. Fujiwara, Y. Watanabe, K. Ogata and M. Kawai, Phys. Rev. C 74 (2006), 064613.
12) S. Hashimoto, M. Kohno, K. Ogata and M. Kawai, Prog. Theor. Phys. 119 (2008), 1005.
13) Y. Horikawa, F. Lenz and N. C. Mukhopadhyay, Phys. Rev. C 22 (1980), 1680.
14) O. Morimatsu and K. Yazaki, Nucl. Phys. A 483 (1988), 493.
15) T. Motoba, H. Bando, R. Wünsch and J. Zofka, Phys. Rev. C 38 (1988), 1322.
16) H. Harada and Y. Hirabayashi, Nucl. Phys. A 759 (2005), 143.
17) H. Maeda, K. Tsubakihara and A. Ohnishi, Eur. Phys. J. A 33 (2007), 269.
18) K. Ogata, M. Kawai, Y. Watanabe, Sun Weili and M. Kohno, Phys. Rev. C 60 (1999), 054605 [Errata: 63 (2000), 019902].
19) X. Campi and D. W. Sprung, Nucl. Phys. A 194 (1972), 401.
20) J. K. Ahn et al., Phys. Lett. B 633 (2006), 214.
21) Y. Nara, A. Ohnishi, T. Harada and A. Engel, Nucl. Phys. A 614 (1997), 433.
22) S. Tadokoro, H. Kobayashi and Y. Akaishi, Phys. Rev. C 51 (1995), 2656.
23) T. Nagae, Nucl. Phys. A 805 (2008), 486c.