Nernst-Ettingshausen effect in two-component electronic liquids

A. A. Varlamov\textsuperscript{1(a)} and A. V. Kavokin\textsuperscript{2,3}

\textsuperscript{1} COHERENTIA-INFM, CNR - Viale del Politecnico 1, I-00133 Rome, Italy, EU
\textsuperscript{2} Dipartimento di Fisica, Università “Tor Vergata” - Via della Ricerca Scientifica, I-00133 Rome, Italy, EU
\textsuperscript{3} Physics and Astronomy School, University of Southampton - Highfield, Southampton, SO171BJ, UK, EU

received 26 March 2009; accepted in final form 4 May 2009
published online 4 June 2009

PACS 74.40.+k – Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.)
PACS 72.15.Jf – Thermoelectric and thermomagnetic effects
PACS 74.25.Fy – Transport properties (electric and thermal conductivity, thermoelectric effects, etc.)

Abstract – A simple model describing the Nernst-Ettingshausen effect (NEE) in two-component electronic liquids is formulated. The examples considered include graphite, where the normal and Dirac fermions coexist, superconductor in fluctuating regime, with coexisting Cooper pairs and normal electrons, and the inter-stellar plasma of electrons and protons. We give a general expression for the Nernst coefficient and show that the origin of a giant NEE is in the strong dependence of the chemical potential on temperature in all cases.

The Nernst-Ettingshausen effect (NEE)\footnote{\textsuperscript{(a)}E-mail: varlamov@ing.uniroma2.it} consists in induction of a stationary electric field $E_y$ by the crossed magnetic field $H$ and temperature gradient $\nabla T$ (see fig. 1) in the absence of electric current. The Nernst constant $N = \left(-\frac{E_y}{H \nabla T}\right)$ varies drastically in different solid-state systems, being extremely small ($N \sim 0.01 \text{–} 1 \mu \text{V} \cdot \text{K}^{-1} \cdot \text{T}^{-1}$) in metals\footnote{Fig. 1: Geometry of the Nernst-Ettingshausen effect.}, but quite large in semimetals and semiconductors ($N$ approaching $7000 \mu \text{V} \cdot \text{K}^{-1} \cdot \text{T}^{-1}$ in Bi\footnote{Temperature derivative and longitudinal conductivity. This approach uses the concept of compensation of the drift current induced by crossed electric and magnetic fields $j^{(t)}_i = qneE_0/H$, by a thermal current induced by the temperature gradient. We apply it for description of two-component electronic systems including fluctuating superconductors, graphene/graphite, bismuth systems, and the interstellar electron-proton plasma. Besides giving the general formula for the Nernst coefficient in two-component electronic liquids we predict the appearance of the carriers concentration gradient along the temperature gradient and the longitudinal Nernst effect for graphene.}), very noticeable in the pseudogap state of high-temperature superconductors (HTSC) ($N \sim 1 \mu \text{V} \cdot \text{K}^{-1} \cdot \text{T}^{-1}$)\footnote{Let us consider a conductor, placed in the magnetic field $H$ oriented along the $z$-axis, and subjected to the temperature gradient $\nabla T$ applied along the $x$-axis.}, and, as it was recently discovered, it is even larger in the fluctuation regime of a conventional superconductor $\text{Nb}_{0.15}\text{Sn}_{0.85}$ ($N \sim 15 \mu \text{V} \cdot \text{K}^{-1} \cdot \text{T}^{-1}$)\footnote{In this letter, basing on the detailed current balance condition, we propose a simple model where the Nernst coefficient is expressed in terms of the chemical potential}. The existing theories of the Nernst effect are mainly based on the semiclassical transport theory\footnote{This model is based on the detailed current balance condition, which states that the drift current $j^{(t)}_i$ induced by the magnetic field $H$ and temperature gradient $\nabla T$ is compensated by the thermal current $j^{(t)}_T$ induced by the temperature gradient.} or Kubo formalism\footnote{In this letter, we propose a simple model where the Nernst coefficient is expressed in terms of the chemical potential}. We stress that the specific form of the transport equation differs strongly from one system to another. Also the form of the heat transfer operator, fundamental for the Kubo approach, strongly depends on the character of interactions between the carriers. Moreover, in the existing theories a single-component electronic liquid has been addressed, which is clearly not the case in semimetals\\footnote{In this letter, we propose a simple model where the Nernst coefficient is expressed in terms of the chemical potential}, superconductors above critical temperature\footnote{In this letter, we propose a simple model where the Nernst coefficient is expressed in terms of the chemical potential}, and many other systems.

In this letter, basing on the detailed current balance condition, we propose a simple model where the Nernst coefficient is expressed in terms of the chemical potential.
There is no electric currents flowing in the system, \( j_x = j_y = 0 \). Zero-current condition in the temperature gradient direction formally yields:

\[
\sigma_{xx} E_x + \sigma_{xy} E_y = 0. \tag{1}
\]

For an isotropic single-component system with the carriers charge \( q \) and concentration \( n \) the transversal (Hall) component of conductivity is \( \sigma_{xy} = -\sigma_{yx} = qne/H \). Using the constancy of the electrochemical potential \( \mu(n, T) = qE_{x}x \) along the \( x \)-direction one can link the electric field component \( E_x \) to the temperature-induced variation of the chemical potential: \( qE_x = \nabla \mu(n, T) \). The electroneutrality requirement in the case of a one-component system provides the constancy of the charge concentration, so that \( qE_x = (d \mu/dT) \nabla T \). Note that while the thermoelectric term is not explicitly included in eq. (1), it appears when substituting \( E_x \). This allows to obtain for the Nernst coefficient:

\[
N = \left( \frac{d \mu}{dT} \right) \frac{\sigma_{xx}}{q^2 nc}. \tag{2}
\]

Equation (2) has been proposed for the first time in the recent paper [11]. Here we generalize it for the two-component electronic systems and analyze the particular cases of graphene/graphite, fluctuating superconductor and interstellar plasma. Being extremely simple, eq. (2) works fairly well. For instance, applying it to the degenerate 3D electron gas in metals where \( \sigma_{xx} = ne^2 \tau/m \) [12], while \( \tau \) is the electron mean free path time, \( m \) is the electron effective mass, and \( \mu(T) = E_F - \pi^2 k_B^2 T^2/(12E_F) \) with \( E_F \) the Fermi energy and \( k_B \) the Boltzmann constant, one can obtain the familiar expression:

\[
N_e = -\frac{\pi^2}{6} \left( \frac{k_B^2 T}{E_F} \right) \frac{\tau}{me^2},
\]

which can be easily derived from the Sondheimer formula [7].

As was mentioned by Obraztsov [13] in some cases the Nernst coefficient can be considerably renormalized by the effect of magnetization currents developed in the sample when a magnetic field is applied. For instance, this effect is negligible in the case of a normal metal in non-quantizing fields (by the parameter \((a/\ell)^2\), where \( a \) is the lattice constant and \( \ell \) is the electron mean free path) but affects significantly the value of NEE coefficient in a fluctuating superconductor [14].

Let us stress that eq. (2) sheds light on the physical origin of the strong variation of the Nernst constant in different solid-state systems. Roughly speaking, the NEE is strong in those systems where the chemical potential strongly varies with temperature.

**Two-component systems.** In this case eq. (1) needs to be applied for each type of carriers separately. Indeed, in the stationary regime, there is no current in the \( x \)-direction for each particular type of the carriers because of the absence of the circuit in the NEE geometry (see fig. 1). This can be referred to as the detailed current balance condition. Having in mind that the chemical potential for each type of carriers depends on temperature and carrier concentration, so that

\[
\nabla \mu_{1,2}(T, n) = \left( \frac{\partial \mu_{1,2}}{\partial T} \right) \nabla T + \left( \frac{\partial \mu_{1,2}}{\partial n_{1,2}} \right) \nabla n_{1,2}, \tag{3}
\]

from the detailed current balance condition one readily obtains

\[
q_1^2 n_1 q_{yy} E_y = \sigma_{xx} \left[ \left( \frac{\partial \mu_{1,2}}{\partial T} \right) \nabla T + \left( \frac{\partial \mu_{1,2}}{\partial n_{1,2}} \right) \nabla n_{1,2} \right]. \tag{4}
\]

The total charge density remains constant which yields:

\[
q_1 \nabla n_1 + q_2 \nabla n_2 = 0. \tag{5}
\]

Finally, from eqs. (4), (5) one can express the Nernst constant:

\[
N = \frac{\left[ q_1 \left( \frac{\partial \mu_1}{\partial T} \right) \left( \frac{\partial n_1}{\partial T} \right) + q_2 \left( \frac{\partial \mu_2}{\partial T} \right) \left( \frac{\partial n_2}{\partial T} \right) \right]}{c \left( \sigma_{xx}^{-1} \frac{\partial \mu_1}{\partial n_1} \right) + q_2 \left( \sigma_{xx}^{-1} \frac{\partial \mu_2}{\partial n_2} \right) q_1^3 n_1}. \tag{6}
\]

In the case of a single-component electronic liquid eq. (6) naturally reduces to eq. (2) obtained above. Note also that the developed formalism applies both for bosonic and fermionic charge carriers. In the rest of this letter we apply the general eq. (6) to the specific cases of semimetals, graphene, fluctuating superconductor and interstellar plasma.

An interesting peculiarity of two-component electronic systems consists in the appearance of the gradient of carrier concentration along the temperature gradient. From eq. (4) one readily obtains

\[
\nabla n_{1,2} = - \left( \frac{\partial \mu_{1,2}}{\partial n_{1,2}} \right)^{-1} \nabla T \left[ \frac{q_1^2 n_1 q_{yy}}{\sigma_{xx}^{-1}} N + \left( \frac{\partial \mu_{1,2}}{\partial T} \right) \right].
\]

As it was mentioned above, in single-component systems the concentration gradient is forbidden by the electric neutrality condition (5). On the other hand, in two-component systems like narrow-gap semiconductors or semimetals the concentration gradient may be quite important.

**Semimetals and graphene.** In quasi-2D highly oriented pyrolytic graphite, the 3D electrons having a parabolic spectrum coexist with 2D holes having a linear (Dirac) energy spectrum [15,16]: \( E(p) = \beta p \) (here \( p \) is the 2D quasi-momentum and \( \beta \) is the effective light speed). The same situation is most likely realized in Bismuth which exhibits electronic characteristics quite similar to...
derivatives of the chemical potential for the Boltzmann equation reduces in this case to electrons having a Dirac spectrum \[19\]. Equation (7) of the chemical potential for the Dirac NEE in bismuth is governed by the temperature dependence of the parameters \(a=3.35 \text{Å}\) and \(l\) being the mean free path length for Dirac fermions, \(n_h(2D) = a m_h\). In the case of graphite, \(a=3.35 \text{Å}\) is the distance between neighboring graphene planes. The derivatives of the chemical potential for the Boltzmann holes read as

\[
\frac{\partial \mu_h}{\partial n_h} = e \hbar c a \frac{l}{\sqrt{2 \pi n_h (2D)}}, \quad \frac{\partial \mu_h}{\partial T} \approx -2 k_B. 
\]

Considering the 3D electrons as a degenerate Fermi gas, we obtain:

\[
\frac{\partial \mu_e}{\partial n_e} = \frac{e^2}{m_e (3n_e)^{2/3}}, \quad \frac{\partial \mu_e}{\partial T} = -\frac{e^2 k_B^2 T}{2 e F}. 
\]

Substituting the eqs. (8), (9) into eq. (7) and taking the values of the parameters \(\tau = 10^8 \text{cm/s}, m_e = 10^{-3} m_0\), from \[17\] with \(n_0\) being the free electron mass, \(n_e = 10^{18} \text{cm}^{-3}\), \(n_h(2D) = 10^{11} \text{cm}^{-2}\), \(l = 10^{-4} \text{cm}, \tau = 1 \text{ps}\), we obtain \(N_{(sm)} = -7.2 \text{mV} \cdot \text{K}^{-1} \cdot \text{T}^{-1}\), at the liquid-helium temperature, which is orders of magnitude larger than the Nernst constant in metals. This value is very close to the value of \(N\) reported in \[4\] for bismuth. The strength of NEE in bismuth is governed by the temperature dependence of the chemical potential for the Dirac fermions.

In graphene there is only one type of charge carriers: electrons having a Dirac spectrum \[19\]. Equation (7) reduces in this case to

\[
N_{(gr)} = -\frac{2 k_B l}{\pi \hbar c} \sqrt{\frac{2 \pi}{n_e (2D)}}. 
\]

We expect the Nernst constant in graphene be of the same order as in graphite and much larger than in conventional metals. This is consistent with the recently published theory of thermoelectric and thermomagnetic effects in graphene in the framework of the relativistic hydrodynamics \[20\] and with very recent experimental results \[21\].

Very interestingly, the conventional transverse Nernst effect we described so far in graphite may be accompanied by an unconventional longitudinal NEE, if the temperature gradient exceeds the critical value \(\nabla T_c = N_{(gr)}^- c\).

This critical gradient corresponds to the drift current \(j^{(x)}_{\text{dr}} = -e n_e \tau c\), which is the strongest drift current allowed in the system of Dirac fermions having a fixed group velocity \(\tau\). If \(\nabla T > \nabla T_c\), the drift current of Dirac fermions in the \(x\)-direction cannot fully compensate the thermo-current induced by the temperature gradient \(j^{(x)}_{\text{th}} = e \sigma_{xx}(\frac{\partial \mu_e}{\partial T}) \nabla T\). In this regime, an additional electric field parallel to the temperature gradient appears \(E_x = \frac{1}{e} (\frac{\partial \mu_e}{\partial T})(\nabla T - \nabla T_c)\) which leads to the appearance of the longitudinal NEE.

Figure 2 shows schematically the transverse and longitudinal electric field behavior as a function of the temperature gradient for different values of magnetic field \((H_1 < H_2 < H_3)\) in graphene. The vertical dashed lines show the critical temperature gradient in graphene.

**Fluctuating superconductor.** – As it is well known, the NEE cannot be observed in type-I superconductors due to the Meissner effect, which does not allow magnetic field to penetrate in the bulk. On the other hand, it is strong in type-II superconductors in the absence of pinning \[22\] due to the specific mechanism of the entropy transport in process of vortex motion. Special attention has been attracted recently by the giant NEE observed in the pseudogap state of the underdoped phases of HTSC \[5\], which motivated speculations \[10\] about the possibility of existence of some specific vortices and antivortices there or the special role of the phase fluctuations \[23\]. Finally, very recently the giant NEE was found also in the wide range of temperatures in a conventional disordered superconductor Nb\(_3\)Si\(_{1-x}\) \[6\] what has been successfully explained in the frameworks of both phenomenological and microscopic fluctuation theories \[11,14,24\].
The approach, based on eq. (2), allows not only to get in a simple way the correct temperature dependence of the fluctuation NEE coefficient but also to catch the reason of its giant magnitude.

In a superconductor being in the fluctuating regime Cooper pairs (ep) coexist with normal electrons (e). Having in mind the double charge of the Cooper pair, eq. (6) becomes

\[ N_{sc} = \frac{1}{ce^2} \left[ \left( \frac{\partial \mu_p}{\partial T} \right) \frac{\partial n_{cp}}{\partial \mu_p} + 2 \left( \frac{\partial \mu_p}{\partial T} \right) \frac{\partial n_{cp}}{\partial \mu_p} \right] \left( \frac{\partial n_{cp}}{\partial \mu_p} \right) \left( \frac{\partial \mu_p}{\partial T} \right). \]

First of all let us mention that the value of chemical potential of the gas of fluctuating Cooper pairs is defined by their “binding energy” [25] taken with the opposite sign: \( \mu_p = -k_B(T - T_c) \). Hence the terms containing \( \partial n_{cp}/\partial \mu_p = 0 \) disappear.

In order to analyze both 2D and 3D cases simultaneously one can evaluate the value of fluctuation contribution to the Nernst coefficient on the example of a layered superconductor in the vicinity of critical temperature and in the limit of weak magnetic fields. The values of paraconductivity \( \sigma^{(ep)}_{xx} = e^2/(16\hbar s\sqrt{\epsilon(\epsilon + \epsilon_T)}) \) and fluctuating Cooper pairs concentration (up to the logarithmic accuracy) \( n_{cp} \sim k_BT_c/2\pi Dsh \) in this case are available in [25] (here \( \epsilon \) is reduced temperature \( (T - T_c)/T_c \), \( \epsilon_T \) is its crossover between 2D and 3D regimes value, \( s \) is the interlayer distance, and \( D \) is the in-plane diffusion coefficient). In accordance with [26] we find

\[ N_{ep} = \frac{\partial \mu_p}{\partial T} \frac{\sigma^{(ep)}_{xx}}{4n_{cp}ce^2} \sim \frac{\pi D}{32cT_c} \frac{1}{\sqrt{\epsilon(\epsilon + \epsilon_T)}}. \]  

The effect of fluctuation diamagnetic currents in this regime results in the reduction of the Nernst constant by a factor of 3, according to [14], so that

\[ \tilde{N}_p \sim -\frac{\pi D}{96cT_c} \begin{cases} \frac{T_c}{T_c - T}, & D = 2, \\ \frac{1}{\epsilon_T} \sqrt{\frac{T_c}{T_c - T}}, & D = 3. \end{cases} \]

This formula in its 2D limit can be applied for description of the experimental results obtained in thin films of the Nb_{0.15}Si_{0.85} [6]. Substituting \( T_c = 0.38 \text{ K} \) and \( D = 0.143 \text{ cm}^2/\text{s} \) we obtain \( \tilde{N}_p^{(2)} \sim -T_c/(T - T_c) \mu\text{V} \cdot \text{K}^{-1} \), which corresponds well to the experimental findings and is three orders of magnitude more than the value of the Nernst constant in typical metals.

Hence one can conclude that the giant Nernst effect in fluctuating superconductors comes from the strong dependence of the fluctuation Cooper pairs chemical potential on temperature.

\[ 1 \text{ We stress that the factor 3 of the Nernst coefficient reduction is valid only in the considered regime. A detailed discussion on the magnetization current effects in fluctuating superconductors can be found in ref. [11].} \]

**Interstellar plasma.** – Consider the intergalactic medium which is mostly ionized hydrogen, i.e., a plasma consisting of equal numbers of electrons and protons, which exists at a density of 10 to 100 times the average density of the Universe (10 to 100 hydrogen atoms per cubic meter) [27]. Both types of carriers can be described as classical gases obeying the Boltzmann statistics. Since the plasma is electrically neutral, one readily obtains from eq. (6)

\[ N_{ap} = -3k_B\tau_p m_p m_e \approx \frac{3k_B\tau_p}{2cm_p}, \]

where \( m_e \) and \( m_p \) are electron and proton masses, respectively. The plasma temperature is thought to be quite high by terrestrial standards: it heats up to \( 10^5 \text{ K} \) to \( 10^6 \text{ K} \), which is high enough for the bound electrons to escape from the hydrogen nuclei upon collisions and corresponds to speeds of the order of \( 10^7 \text{ cm/s} \). Using the Rutherford formula for the scattering cross-section with these data one can evaluate the scattering time as \( \tau_p = 10^{-3} \text{ s} \), which results in the huge NEE coefficient of the order \( N_{ap} = 10^{-6} \mu\text{V} \cdot \text{K}^{-1} \). Observation of a giant NEE in the interstellar plasma could be an important challenge for the experimental astrophysics.

In conclusion, we have formulated a unified approach to the Nernst-Etttingshausen effect allowing for comparative analysis of the Nernst constant in a wide range of systems. We expect the giant NEE to take place in the charged systems where the chemical potential strongly depends on temperature. We predict the huge values of the Nernst constant in graphene, graphite and the electron-proton plasma and confirm the values of the Nernst constant in metals, bismuth and superconductors above the critical temperature. The proposed approach could be the key for understanding of the giant NEE in pseudogap phase of HTSC. We predict an existence of the critical temperature gradient in graphene above which the unconventional longitudinal NEE develops.

***

We are grateful to B. L. Altshuler, L. Falkovski, Y. Kopelevich and I. Luk’yanchuk for valuable discussions.

REFERENCES

[1] ETTINGSHAUSEN A. V. and NERNST W., Wied. Ann., 29 (1886) 343.
[2] BEI R. et al., Phys. Rev. Lett., 92 (2004) 217002.
[3] SHEIKIN I. et al., Phys. Rev. Lett., 96 (2006) 077207.
[4] BEHNIA K., MEASSON M.-A. and KOPELEVICH Y., Phys. Rev. Lett., 98 (2007) 076603.
[5] XU Z. A. et al., Nature, 406 (2000) 486.
[6] FOURET A. et al., Phys. Rev. Lett., 96 (2006) 176402.
[7] SONDHEIMER E. H., Proc. R. Soc. London, Ser. A, 193 (1948) 484.
Nernst-Ettingshausen effect in two-component electronic liquids

[8] Behnia K., *J. Phys.: Condens. Matter*, **21** (2009) 113101.
[9] Oganesyan V. and Ussishkin I., *Phys. Rev. B*, **70** (2004) 054503.
[10] Anderson P. W., *Nat. Phys.*, **3** (2007) 160.
[11] Serbyn M. N. et al., *Phys. Rev. Lett.*, **102** (2009) 067001.
[12] Abrikosov A. A., *Fundamentals of Metal Theory* (Elsevier) 1989.
[13] Obraztsov Yu. N., *Sov. Phys. Solid State*, **6** (1964) 331.
[14] Ussishkin I., Sondhi S. L. and Huse D. A., *Phys. Rev. Lett.*, **89** (2002) 287001.
[15] Zhou S. Y. et al., *Nat. Phys.*, **2** (2006) 595.
[16] Lukyanchuk I. A. and Kopelevich Y., *Phys. Rev. Lett.*, **97** (2006) 256801.
[17] Kopelevich Y. et al., *Phys. Rev. B*, **78** (2008) 115419.
[18] Nomura K. and MacDonald A. H., *Phys. Rev. Lett.*, **96** (2006) 256602.
[19] Note, however, that a two-liquid model may also be applied to graphene if the interband electron-hole generation is efficient, see Foster M. S. and Aleiner I. L., *Phys. Rev. B*, **79** (2009) 085415.
[20] Muller M. and Sachdev S., *Phys. Rev. B*, **73** (2006) 165128.
[21] Zuev Yuri M., Chang W. and Kim P., arXiv:cond-mat/0812.1393; Wei P., Bao W., Pu Y., Lau Ch. N. and Shi J., arXiv:cond-mat/0812.1411; Checkelsky J. G. and Ong N. P., arXiv:cond-mat/0812.2866.
[22] Mukerjee S. and Huse D. A., *Phys. Rev. B*, **70** (2004) 014506.
[23] Podolsky D., Raghu S. and Vishwanath A., *Phys. Rev. Lett.*, **99** (2007) 117004.
[24] Michaeli K. and Finkel’stein A., arXiv:cond-mat/0812.4268.
[25] Larkin A. I. and Varlamov A. A., *Theory of Fluctuations in Superconductors* (Oxford University Press) 2005.
[26] Ullah S. and Dorsey A., *Phys. Rev. B*, **44** (1991) 262.
[27] Gurnett Donald A., *Introduction to Plasma Physics: With Space and Laboratory Applications* (Cambridge University Press) 2005.