Weight efficiency analysis of a ship carbon shaft manufactured by winding with various winding angles in layers

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Abstract. The task of the optimal winding parameters selection for the ship composite intermediate shaft has been formulated and the solution algorithm has been considered. A method for the shaft mechanical characteristics estimation on the base of the layer-by-layer method has been proposed. The shaft weight efficiency analysis for the various winding angles in the winding program has been carried out.

1. Introduction
Metal shafting has a number of irremediable disadvantages: high weight, high noise level during operation, corrosibility, complexity of installation. The use of composite materials is an alternative for the shafting elements production. There is a positive experience in the composite intermediate shafts creation in world practice, as has been evidenced by the product catalogs of CENTA, VULKAN firms. Nowadays the interest to this problem is being formed in Russia. The author largely used the proposals of A.N. Polilov [1] on the optimal design of composite driveshafts during this work.

2. Problem statement
The designed intermediate shaft must transmit the maximum torque $M_{\text{max}}$ specified accounting the safety factor at the operating rotation speed $\omega$. According to the installation conditions, the considered shaft section does not perceive longitudinal force. According to technical requirements, the shaft length $L$ is considered to be predetermined, its outer diameter cannot exceed the value $D$. The lowest natural frequency $p_1$ of the shaft bending vibrations must be tuned to the operating frequency. The tuning coefficient of the first eigenfrequency $k = p_1/\omega$ is introduced, the value of which must be no less than a given value $k_{\text{min}}$.

Helical cross winding technology is used for the composite shaft creation. An orthotropic structure is formed by winding pairs of consecutive monolayers with reinforcement angles $\pm \phi$. An orthotropic pair is taken as a layer. There are variants of shaft design with one corner winding layers and with a specific sequence of several winding angles. The shaft design is determined by the winding parameters, depending on the number of $N$ different reinforcement angles:

\[ \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N] - \text{winding angles.} \]

\[ n = [n_1, n_2, \ldots, n_N] - \text{the number of layer pairs for corresponding sets.} \]
The task is to determine the winding parameters that provide the lowest weight of the composite shaft to achieve a clear advantage in weight over a metal analogue equivalent in critical moment, and to reduce technological risks.

3. The optimal winding parameters determination

The shafting scaled mass is set as an objective function

$$ f(n) = \frac{W}{W_0} $$

where $W_0$ – the weight of the shaft, which is hypothetically completely filled with composite material.

Accounting the requirements is carried out by adding the terms in the form of penalty functions to the objective function. The penalty functions in the form, proposed in [2], have been used to solve this problem. The modified objective function for the constraints on the critical moment $M_{\text{limit}} > M_{\text{max}}$ and the detuning $k > k_{\text{min}}$ has the form (2).

$$ f_1(a, n) = f(n) + R_i G_i(a, n) + R_j G_j(a, n) $$

where

$$ G_i(a, n) = \begin{cases} \frac{1}{g_i(a, n)}, & \text{if } g_i(a, n) \geq \varepsilon \\ \frac{2\varepsilon - g_i(a, n)}{\varepsilon^2}, & \text{if } g_i(a, n) < \varepsilon \end{cases} $$

$$ g_i(a, n) = \frac{M_{\text{limit}}}{M_{\text{max}}} - 1, \quad g_j(a, n) = \frac{k}{k_{\text{min}}} - 1 $$

The penalty parameters $R_i$, $\varepsilon$ have been selected in such way to achieve a minimum solutions dispersion while maintaining the algorithm reliability. The MATLAB package, namely, the Optimization Toolbox, has been used to find the absolute minimum of the objective function.

4. Mechanical characteristics estimation

The penalty functions require a determination of the shaft mechanical characteristics. Composite shaft made by winding has a close to layered structure. Layer-by-layer methods (the so-called theory of laminates) may be applied for analysis. The stiffness characteristics, obtained experimentally on samples which are close to the real structure of the shaft – on thick-walled tubes, are the most reliable. However, an estimation based on the theory of laminates is justified in the presence of only the mechanical characteristics of the monolayer.

The following characteristics have been used in the design: technical stiffnesses of unidirectional material – $E_1$, $E_2$, $G_{12}$, $v_{12}$, strength characteristics – $\sigma_{11}$, $\sigma_{22}$, $\sigma_{12}$, $\sigma_{21}$, $\tau_{12}$ (1 - along the fiber, 2 - across, $t$ - tension, $c$ - compression).

For a composite shaft, as for a beam, stiffnesses, for example, $K_{xy}$ - torsional stiffness, $K_{xy}$ - bending stiffness, etc are distinguished. One can estimate these quantities if the winding program in known. To do this, the technical constants for each orthotropic pair of layers in the shaft coordinate system (x - along the shaft axis, y - circumferential coordinate) should be determined firstly (see [3]).

$$ E_1^i, E_2^i, G_{12}^i, v_{12}^i \rightarrow E_x^i, E_y^i, G_{xy}^i, V_{xy}^i $$

Next, the corresponding shaft stiffness is calculated (integral stiffness, for example, torsional):

$$ K_{\text{torsoonal}} = GJ_p = \frac{M_{\text{torsoonal}}}{\theta} = \sum_{i=1}^{n} G_{xy}^i \cdot J_p^i $$

For a thick-walled shaft, the loss of bearing capacity occurs mainly due to the destruction of the shaft material under the torque. Failure occurs when shear stresses reach ultimate values.
\[ M_{str} = \min \left( \frac{\tau_{\text{limit}}^{\text{xy}}(\phi)}{\tau_{\text{xy}}^i} \right) \cdot M^1 \]  

(7)

where \( \tau_{\text{limit}}^{\text{xy}}(\phi) \) – ultimate shear stress for material, \( \tau_{\text{xy}}^i \) – shear stress in the \( i \)th layer from a unit moment \( M^1 = 1 \text{ N*m} \).

The ultimate characteristic has been evaluated by the Hoffman criterion [4]. The accepted hypothesis of straight radius during torsion guarantees a linear distribution of strains; only shear strain \( \gamma_{\text{xy}} \) occurs during torsion in a monolayer.

The stress components appearing in the Hoffman criterion in the axes of the monolayer (\( \sigma_1, \sigma_2, \tau_{12} \)) are written through \( \gamma_{\text{xy}} \) and, as a result, the criterion is reduced to the following quadratic equation:

\[ a(\gamma_{\text{xy}}^\text{str})^2 + b\gamma_{\text{xy}}^\text{str} - 1 = 0 \]  

(8)

The critical stresses of the monolayer:

\[ \tau_{\text{xy}}^\text{str}(\phi) = G_{\gamma}(\phi) \cdot \gamma_{\text{xy}}^{\text{str}}(\phi) \]  

(9)

The strength of a pair of layers is determined by the minimum value of ultimate stresses for paired monolayers.

\[ \tau_{\text{xy}}^\text{str}(\pm \phi) = \min \{ \tau_{\text{xy}}^\text{str}(\phi), \tau_{\text{xy}}^\text{str}(-\phi) \} \]  

(10)

With high ultimate stress, a shaft design that satisfies only the strength requirement may appear to be thin-walled. A thin hollow shaft can become unstable like a shell. So it is necessary to evaluate the ultimate load. For long shafts, such an estimation is possible by the following approximate expression [5],

\[ M_{\text{buckl}} = 0.272 \cdot (2\pi r^2 t) \left( \frac{E_1 E_2}{E_3} \right) \left( \frac{t}{r} \right)^2 \]  

(11)

where

\[ r = (D + d)/2, t = (D - d)/2 \]  

(12)

In total, the critical moment:

\[ M_{\text{limit}} = \min \{ M_{\text{str}}, M_{\text{buckl}} \} \]  

(13)

The first natural frequency of the beam bending vibrations required to determine the detuning:

\[ p_1 = \frac{\alpha^2 \pi^2}{L^2} \sqrt{\frac{K_{\text{bending}}}{\rho A}} \]  

(14)

where \( \rho \) – density, \( A \) – section area, \( \alpha \) – coefficient taking into account the type of beam fixing (for a pin supported beam \( \alpha = 1 \)).

5. Results for numerical example

The calculation of the optimal winding parameters has been carried out for three types of structures, differing in the number of reinforcement angles \( N = 1, 2, 3 \).

The initial conditions for analysis:

\( D = 450 \text{ mm}, t = 0.5 \text{ mm} - \text{pair thickness}, L = 5 \text{ m} - \text{the shaft length} \).

Technical constants of elasticity and strength of carbon fiber, from a monograph [4]:

\( E_1 = 207 \text{ GPa}, E_2 = 5 \text{ GPa}, G_{12} = 2.6 \text{ GPa}, \nu_{12} = 0.25; \sigma_{1t} = 1035 \text{ MPa}, \sigma_{1c} = -689 \text{ MPa}, \sigma_{2t} = -41 \text{ MPa}, \sigma_{2c} = -117 \text{ MPa}, \tau_{12} = 69 \text{ MPa} \).
\( \varphi = 1500 \text{ kg/m}^3 \) – characteristic density of carbon fiber.

Requirements:
\( M_{\text{max}} = 1000 \text{ kN*m}; \ k_{\text{min}} = 10 \) (for \( \omega = 320 \text{ rpm} \)).

**Table 1.** Shaft design results.

| N=1 \( \theta \) | N=2 \( \theta \) | N=3 \( \theta \) | N=1 \text{ layers} | N=2 \text{ layers} | N=3 \text{ layers} |
|-----------------|-----------------|-----------------|-------------------|-------------------|-------------------|
| Shaft mass, kg  | 144             | 144             | 134              | 231              | 160              |
| Winding angles, ° | 28             | 29.27           | 42              | 37.9             | 16               |
| The number of layers | 28             | 14.14           | 8.14           | 46               | 22.9             |

\( \theta \) indicates the option found without taking into account the possibility of buckling during torsion.

\( ^1 \) respectively, with the possibility of buckling.

6. **Analysis of various options effectiveness**

The optimization problems have been solved for shafts with different strength requirements to analyze the efficiency of using a shaft, which has been wound with \( N \) reinforcement angles.

The term «efficiency» means how many percent of the weight of shaft, which has been wound in layers with different angles, is less than the weight of shaft with one winding angle. Shafts, which are equivalent by established requirements, have been compared. The results are presented in figure 1.

![Figure 1](image)

**Figure 1.** Efficiency based on requirements.

The greatest efficiency of use appears when the imposed requirements lead to design options that contradict each other. As an example, it is necessary to increase the ring stiffness \( E_y \), to increase torsional stability, therefore, the winding angle should be shifted to a value of 90°, and on the contrary, it is necessary to wind layers with angles close to 0° to satisfy the requirement for natural frequency tuning. The shaft of one winding angle is not able to satisfy both requirements optimally. A contradiction arises for the thin shafts, so winding with different angles may be applied. The designed shaft becomes thick-walled with higher requirements for the transmitted moment, for which buckling during torsion is impossible, therefore it is permissible to use one winding angled shaft.
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