Range dependence of interlayer exchange coupling

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We have considered the effects of non magnetic impurities and interface roughness on the interlayer coupling between magnetic layers in metallic multilayers. The two types of defects alter the interlayer coupling in quite different ways. Elastic electron scattering by impurities in the non magnetic spacer layers between magnetic layers produces an exponential decay of the coupling with a characteristic decay length that is considerably longer than the “global” transport mean free path for the spacer layer with its surrounding interfaces. Interfacial roughness leads to an attenuation of the coupling that is related to the width of the roughness in relation to the Fermi wavelength; roughness does not alter the range dependence of the coupling. For certain types of electrical transport, e.g., for current perpendicular to the plane of the layers, the scattering from interface roughness and impurities in the spacer layers contribute on an equal footing to the exponential decay of the electron propagators, i.e., global mean free path. We show that interface roughness and impurities in the spacer layer affect the interlayer coupling differently.

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I. INTRODUCTION

The range dependence of the indirect coupling between magnetic ions mediated by conduction electrons, i.e., the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction, in the presence of scattering by non magnetic impurities has been debated over the past 35 years. Recently this question has been resolved; it is now clear that scattering by non magnetic impurities does not alter the range dependence of the coupling between two magnetic ions; it only introduces a phase factor that depends on the specific distribution of impurities between the two ions. As long as one does not average over different impurity configurations the coupling between a pair of magnetic ions remains undamped. Only when one looks at properties that require an average of the coupling between two ions for different realizations of impurity distributions, and therefore by taking an average over the phase factor produced by the impurity scattering, does one find that the coupling is damped.

The recent interest in the interlayer exchange coupling in magnetic multilayers raises the questions whether scattering in the non magnetic spacer layers damps the coupling between magnetic layers, and what effect the roughness of interfaces will have on the range of the coupling. Here we show that for the coupling between two sheets of spins one does average over different realizations of impurity distributions in the intervening spacer, and the coupling is exponentially damped as a function of the distance between planes, i.e., magnetic layers. The damping is proportional to the strength of the impurity scattering in the spacer; it has nothing to do with the scattering due to roughness at the interfaces. While this characteristic decay distance of the interlayer coupling and the transport mean free path in the spacer layer are both due to impurity scattering, they are in no way simply related to one another. Interfacial roughness attenuates the coupling in proportion to the size of the inter diffused region relative to the Fermi wavelength of the conduction electrons providing the coupling. However, this decrease does not alter the range dependence of the coupling. For this reason it would be completely erroneous to combine the effects of the scattering from impurities in the spacer and interface roughness into one decay coefficient to produce an exponential decay of the interlayer coupling. While this procedure is correct for certain types of electrical transport in magnetic multilayers, e.g., for current perpendicular to the plane of the layers (CPP), interlayer coupling is not in this class of situations. Therefore the range of the interlayer coupling is considerably longer (characteristic decay of the coupling is considerably slower) than what one anticipates from the transport properties (resistivity) of a magnetic multilayer.

In the following section we first derive the RKKY coupling between a pair of magnetic ions in the presence of non magnetic impurities. Next we show how averaging...
over the phase induced by the impurity scattering produces a coupling that decays with the distance between the ions, and the extent to which this can be described by an exponential of the distance. We use the approximation of representing the coupling between two magnetic layers as that due to the two planes of magnetic ions that interface with the nonmagnetic spacer layer and we show that when one averages over the magnetic ions in these planes that this coupling has a slowly decaying exponential component as a function of the thickness of the spacer layer. Finally we take into account of effect of the roughness of the interfaces and show how this attenuates the coupling.

II. TWO MAGNETIC IONS

To calculate the coupling between two magnetic ions when the conduction electrons are scattered by impurities we use the approach adopted by Bulaevskii and Panyukov. It consists of using a semiclassical form for the electron propagator (Green’s function) connecting the positions of the two ions, and taking account of the scattering by impurities through the phase of the propagator. The coupling between two magnetic impurities located, respectively, at \( \mathbf{r} \) and \( \mathbf{r}' \) is given by

\[
J(\mathbf{r}, \mathbf{r}') \sim \int_{\varepsilon_F - i\infty}^{\varepsilon_F + i\infty} d\varepsilon \text{Tr} \left[ G(\mathbf{r}, \mathbf{r}', \varepsilon) G(\mathbf{r}', \mathbf{r}, \varepsilon) \right].
\]

Here, \( G \) is the Green’s function corresponding to a particular configuration of impurities. A typical Feynmann diagram contributing to the above expression, is a “bubble” diagramm with an electron line going from \( \mathbf{r} \) to \( \mathbf{r}' \) over a given set of impurities, and then back from \( \mathbf{r}' \) to \( \mathbf{r} \) over a (generally) different set of impurities; this contribution to Eq. \( (1) \) contains a phase factor

\[
\exp \left\{ i k(z) \left[ L(\mathbf{r} \rightarrow \mathbf{r}') + L(\mathbf{r}' \rightarrow \mathbf{r}) \right] \right\},
\]

where \( k(z) \) is the (complex) wavevector corresponding to the complex energy \( z \), and where \( L(\mathbf{r} \rightarrow \mathbf{r}') \) (\( L(\mathbf{r}' \rightarrow \mathbf{r}) \)) is the length of the path from \( \mathbf{r} \) to \( \mathbf{r}' \) (\( \mathbf{r}' \) to \( \mathbf{r} \)) over the corresponding set of impurities.

For points \( \mathbf{r} \) and \( \mathbf{r}' \) not very close to each other, when summing over all diagrams (i.e., over all paths over impurities), the above factor oscillates rapidly, leading to a strong cancellation. Thus, as pointed out by Bulaevskii and Panyukov, the only significant terms are due to the paths going through impurities lying on a straight line between \( \mathbf{r} \) and \( \mathbf{r}' \) (the impurities being passed sequentially, without back tracking); for all such paths, the phase factor is

\[
\exp \left[ 2i k(z)|\mathbf{r} - \mathbf{r}'| \right].
\]

We wish to stress that the present situation is completely different from the one encountered when computing the two-point conductivity \( \sigma(\mathbf{r}, \mathbf{r}') \), which involves the product \( G(\mathbf{r}, \mathbf{r}'; \varepsilon_F + i0^+)G(\mathbf{r}', \mathbf{r}; \varepsilon_F - i0^+) \). This yields an oscillatory factor

\[
\exp \left\{ i k_F \left[ L(\mathbf{r} \rightarrow \mathbf{r}') - L(\mathbf{r}' \rightarrow \mathbf{r}) \right] \right\}.
\]

Because of the minus sign in the above expression, there is a significant contribution from all diagrams such that the path \( \mathbf{r}' \rightarrow \mathbf{r} \) is the reverse of the path \( \mathbf{r} \rightarrow \mathbf{r}' \), so that \( L(\mathbf{r} \rightarrow \mathbf{r}') = L(\mathbf{r}' \rightarrow \mathbf{r}) \); these are the ladder diagrams characteristic of a diffusive process, which give the leading contribution to \( \sigma(\mathbf{r}, \mathbf{r}') \).

Following Bulaevskii and Panyukov, we obtain the expression of the coupling:

\[
J(\mathbf{r}, \mathbf{r}') \sim \frac{\cos[2k_F|\mathbf{r} - \mathbf{r}'| + \phi(\mathbf{r}, \mathbf{r}')]}{|\mathbf{r} - \mathbf{r}'|^3};
\]

the phase shift due to impurity scattering is

\[
\phi(\mathbf{r}, \mathbf{r}') = \frac{-2}{\hbar v_F} \int_0^{|\mathbf{r} - \mathbf{r}'|} ds U(\mathbf{r} + \mathbf{n}s),
\]

where \( \mathbf{n} \) is a unit vector of the \( (\mathbf{r}, \mathbf{r}') \) axis and \( U(\mathbf{r}) \) is the impurity scattering potential. Equations \( (5) \) and \( (6) \) are valid if the perturbation potential \( U(\mathbf{r}) \) is small compared to the Fermi energy. As there is a specific distribution of impurities between a pair of ions there is a definite phase; the coupling is phase shifted but it is not damped by impurity scattering (its decay law is as \( |\mathbf{r} - \mathbf{r}'|^{-3} \) like in the pure system).

The distribution of \( J(\mathbf{r}, \mathbf{r}') \) is thus determined by the distribution of the phases \( \phi(\mathbf{r}, \mathbf{r}') \). To ascertain the distribution of phase angles we fill the space between two spins with cubes of length \( a \), and rewrite the phase integral Eq. \( (5) \) as a sum over the \( N \) cells \( (N \equiv |\mathbf{r} - \mathbf{r}'|/a) \) crossed by the trajectory

\[
\phi(\mathbf{r}, \mathbf{r}') = \sum_{i=1}^N \phi_i;
\]

\[
\phi_i = \begin{cases} -2Ua/\hbar v_F & \text{(if impurity in cell } i) \\ 0 & \text{(if no impurity in cell } i) \end{cases}
\]

If an impurity is in a cell it yields a contribution \( \kappa = -2Ua/\hbar v_F = -(3\pi)^{1/3} U/E_F \) to the phase; otherwise it gives zero. For a given concentration of impurities \( c \) there is a probability \( c \) at each site of their being an impurity and therefore of picking up a phase of \( \kappa \), and a probability of \( 1-c \) of picking up zero. Therefore the phase in Eq. \( (5) \) is just a binomial distribution of \( N \) events.

\[ J^\mu(R) \equiv \langle J^\mu(\mathbf{r}, \mathbf{r}') \rangle,
\]

where the angular brackets indicate that we average over all possible configurations of impurities. The averaging restores the translational and rotational invariance, therefore the moments \( J^\mu \) depend only on the distance
\[ R \equiv |\mathbf{r} - \mathbf{r}'| \] between the two spins. As was emphasized by various authors, the first moment \( \mathcal{J}(R) \) bears little physical significance in the case of magnetic ions embedded in a disordered nonmagnetic host; for instance, the transition temperature of spin glasses is determined by the second moment \( \mathcal{F}(R) \), not by the first moment. To illustrate this point, we compute now the first two moments, \( \mathcal{J}(R) \) and \( \mathcal{F}(R) \).

The configuration averaged exchange interaction (first moment) is given by

\[
\mathcal{J}(R) \sim \text{Re} \left( e^{2ikFR} \left\langle e^{i\phi(r,r')} \right\rangle \right). \tag{9}
\]

The characteristic function of the phase \( \phi \) for the binomial distribution is given as

\[
\left\langle e^{i\phi(r,r')} \right\rangle = \left[(1-c) + ce^{i\kappa}\right]^{R/a}, \tag{10}
\]

where we have used \( R = Na \).

By taking the logarithm of the characteristic function for the binomial distribution we find in the limit of low impurity concentrations

\[
\left\langle e^{i\phi(r,r')} \right\rangle \approx \exp \left[ (1-c) - 1 \right] R/a,
\approx e^{ic} R/a e^{-c(1-c) R/a}.
\tag{11}
\]

Thus, we obtain an exponential decay of the exchange interaction with a decay length \( \lambda \) given by

\[
\lambda^{-1} = \frac{c}{a} (1 - \cos \kappa) \approx \frac{c}{2a} \left( \frac{U}{\varepsilon_F} \right)^2 \tag{12}
\]

and a shift in the Fermi wavevector

\[
\delta k = \frac{c \sin \kappa}{2a} \approx -\frac{c}{2a} \left( \frac{U}{\varepsilon_F} \right). \tag{13}
\]

The wavevector shift \( \delta k \) is simply due to the shift in the average value of the potential; in the following it will be incorporated into a redefinition of the Fermi wavevector \( k_F \).

In this way we find that the average RKKY coupling is

\[
\mathcal{J}(R) \sim \frac{\cos(2k_FR) e^{-R/\lambda}}{R^3}. \tag{14}
\]

This result was first obtained by de Gennes.

The second moment is easily calculated directly from Eq. (14) and we obtain

\[
\mathcal{F}(R) \sim \frac{1}{2} \frac{1}{R^6}. \tag{15}
\]

Thus it appears clearly that

\[
\frac{\mathcal{F}(R) - \mathcal{J}^2(R)}{\mathcal{F}(R)} \sim e^{2R/\lambda}, \tag{16}
\]

i.e., that the exchange interaction between a pair of magnetic ions is not a self-averaging quantity (in the sense of Kohn and Luttinger).

In the case of a system with a non-spherical Fermi surface, the configuration averaged exchange interaction takes a form similar to Eq. (14), but the wavevector of oscillations and the decay length both depend on the direction \( \mathbf{n} \). We stress that the decay length of the exchange interaction corresponding to a particular direction generally has no simple relation with the transport mean free path, because the later results from averaging over all directions.

### III. TWO SHEETS OF SPINS

#### A. Perfectly flat layers

We now consider the interlayer exchange coupling between two ferromagnetics layers, \( F_1 \) and \( F_2 \). These are modelled by taking two sheets of magnetic ions of normal coordinates \( r_{\perp 1} \) and \( r_{\perp 2} \), respectively. Within a given sheet, we assume that all the magnetic moments are maintained parallel to each other by some intralayer exchange coupling (which we do not describe explicitly here); thus, the only variable is the angle between the magnetizations of the two sheets.

Following Yafet, we express the coupling between \( F_1 \) and \( F_2 \), as the sum over the pairs of magnetic ions \( (r_1, r_2) \) (divided by the total area \( S \)), with \( r_1 \equiv (r_{\parallel 1}, r_{\perp 1}) \) belonging to \( F_1 \) and \( r_2 \equiv (r_{\parallel 2}, r_{\perp 2}) \) belonging to \( F_2 \):

\[
I(r_{\perp 1}, r_{\perp 2}) \equiv \frac{1}{S} \int d^2r_{\parallel 1} \int d^2r_{\parallel 2} J(r_1, r_2). \tag{17}
\]

To compute this, we first sum over all pairs \( (r_1, r_2) \) with \( r_1 - r_2 \) parallel to a given direction, i.e., we rewrite the above equation as

\[
I(r_{\perp 1}, r_{\perp 2}) = \int d^2 \mathbf{r}_\parallel K(r_{\perp 1}, r_{\perp 2}; \mathbf{r}_\parallel), \tag{18}
\]

with

\[
K(r_{\perp 1}, r_{\perp 2}; \mathbf{r}_\parallel) = \frac{1}{S} \int d^2 \mathbf{r}_\parallel J \left( (\mathbf{r}_\parallel, r_{\perp 1}), (\mathbf{r}_\parallel + \mathbf{r}_\parallel, r_{\perp 2}) \right). \tag{19}
\]

It is easy to see that, when summing over \( \mathbf{r}_\parallel \), all configurations of impurities between \( r_1 = (r_{\parallel 1}, r_{\perp 1}) \) and \( r_2 = (r_{\parallel 2}, r_{\perp 2}) \) are encountered, thus this is equivalent to performing a configuration average, i.e.,

\[
K(r_{\perp 1}, r_{\perp 2}; \mathbf{r}_\parallel) = \mathcal{J}(R_{1,2}), \tag{20}
\]

where

\[
R_{1,2} \equiv \sqrt{\rho_\parallel^2 + D^2}. \tag{21}
\]
is the distance between the spins for the pairs considered, and $D \equiv |r_{1,1} - r_{1,2}|$ is the distance between the two sheets of spins.

This implies that, in contrast to the exchange interaction between two magnetic ions, the exchange coupling between two sheets of spins is self-averaging in the sense of Kohn and Luttinger.  Thus, it is only a function of the distance $D$ between the two sheets, and not of $r_{1,1}$ and $r_{1,2}$ separately.

The remaining integration over $\rho_||$ is then easily calculated by using Yafet’s method and we find

$$ I(D) \sim 2\pi \text{Re} \left[ -\frac{1}{\gamma} e^{\gamma D} \frac{D}{D^2} \right], \quad (22) $$

with $\gamma = 2k_F + i\lambda^{-1}$; finally, we obtain

$$ I(D) \sim \frac{\pi}{k_F} \sin \left( \frac{2k_F D + \varphi}{D^2} \right) e^{-D/\lambda}, \quad (23) $$

with

$$ \varphi \equiv \arctan \left( -\frac{1}{2k_F \lambda} \right) \approx -\frac{1}{2k_F \lambda} \quad (24) $$

Thus, the presence of impurities in the non-magnetic spacer layer leads to an exponential decay of the interlayer exchange coupling with the distance between magnetic layers; this result is in contrast to the one obtained for the exchange interaction between magnetic ions in the previous section. It is traced back to the self-averaging character of the interaction between planes. Another effect of impurity scattering is the phase shift $\varphi$ [Eq. (24)].

The results of the present subsection are in full agreement with the ones obtained previously by Bruno et al. by using first-principles calculations together with the “vertex cancellation theorem.” Moreover, this previous study allows us to generalize the above result to the case of a spacer material with non-spherical Fermi surface; in this case, one obtains

$$ I(D) \sim \sum_\alpha I_\alpha \sin \left( \frac{q_\perp D + \phi_\alpha}{D^2} \right) e^{-D/\lambda_\alpha}. \quad (25) $$

In the above equation, $q_\perp$ and $\lambda_\alpha^{-1}$ are the real and imaginary parts of stationary spanning vectors of the complex Fermi surface of the alloy spacer material (since there may be several such vectors, they are labeled by the index $\alpha$); $I_\alpha$ and $\phi_\alpha$ are the corresponding amplitude and phase.

The model calculation presented here provides a simple physical explanation for the “vertex cancellation theorem.” As explained at the beginning of Sec. II, only the paths going in straight line between two ions contribute significantly to the exchange interaction between them; this, together with the self-averaging property for the coupling between layers, forms the physical basis of the “vertex cancellation theorem.”

### B. Effect of interface roughness

Next we discuss the effect of interface roughness on the interlayer exchange coupling. To be specific, we consider the case where the normal coordinates $r_{1,1}$ and $r_{1,2}$ characterizing $F_1$ and $F_2$ vary with the in-plane coordinates $r_{1,1}^\parallel$ and $r_{1,2}^\parallel$. The roughness is characterized by at least two parameters: the average amplitude of the fluctuations of $r_{1,1}$ and $r_{1,2}$, and the lateral correlation length, $\xi$.

The simplest approach to the effect of roughness consists in calculating the effective interlayer exchange coupling by averaging over thickness fluctuations. In order to be allowed to do so however, some conditions must be satisfied. The first condition is that the lateral correlation length of the roughness, $\xi$, should be large enough for the interlayer exchange coupling to be locally well defined; typically this requires that $\xi > D$. On the other hand, we wish to consider that the sheets of spin are uniformly magnetized; but local fluctuations of spacer layer thickness induce local interlayer coupling fluctuations, that tend to produce local fluctuations of the magnetization direction in the magnetic layers. Thus, in order to keep the magnetization direction constant in the magnetic layers, the intralayer exchange coupling must be large enough, and the correlation length $\xi$ small enough; in practice, this condition $\xi$ is not very restrictive, and we shall not consider it further.

Within the above conditions, the effective interlayer exchange coupling $\tilde{T}$ is given by

$$ \tilde{T} = \int dD \ P(D) \ I(D), \quad (26) $$

where $P(D)$ is the distribution function of spacer thicknesses. Thus, we have

$$ \tilde{T} \sim 2\pi \text{Re} \left[ -\frac{1}{\gamma} \int dD \ P(D) \ e^{\gamma D} \frac{D}{D^2} \right]; \quad (27) $$

if the width of the distribution of thicknesses is small compared to the average thickness $D$, then the above equation becomes

$$ \tilde{T} \sim 2\pi \text{Re} \left[ -\frac{1}{\gamma} A(\gamma) \ e^{\gamma D} \frac{D}{D^2} \right]; \quad (28) $$

where

$$ A(\gamma) \equiv \int dD \ P(D) \ e^{\gamma (D - \overline{D})} \quad (29) $$

is the form factor for the roughness. For a Gaussian distribution of width $\sigma$, i.e.,

$$ P(D) = \frac{1}{\sqrt{2\pi}\sigma} \ e^{-\left(\frac{D - \overline{D}}{2\sigma^2}\right)^2}, \quad (30) $$

one has
\[ A(\gamma) \approx e^{-2k_F^2 \sigma^2} e^{i2k_F \sigma^2/\lambda}. \]  

Thus, the effective coupling becomes
\[ \mathcal{T} \sim \frac{\pi}{k_F} e^{-2k_F^2 \sigma^2} \sin \left( \frac{2k_F D + \varphi + \psi}{\lambda} \right) e^{-\gamma/\lambda} \]  

where
\[ \psi = \frac{2k_F \sigma^2}{\lambda}. \]  

The effect of interface roughness is essentially to attenuate the oscillatory coupling by the factor \( \exp \left( -2k_F^2 \sigma^2 \right) \). In contrast to the attenuation due to impurities in the spacer layer, this effect is independent of the thickness of the spacer layer, \( D \). The phase shift \( \psi \) in Eq. (33) is a combined effect of the roughness and of the impurities. An alternative approach for treating the effect of roughness is also presented in the Appendix.

In the more general case where the interlayer exchange coupling comprises several oscillatory components, the effect of the roughness is to strongly attenuate those oscillatory components which have a period of the order of (or smaller than) the amplitude of roughness, \( \sigma \).

\[ II \]

\section*{IV. CONCLUSION}

By using a semi-classical approach, we have shown how the various kinds of defects, i.e., impurity scattering in the bulk of the layers and interface roughness, modify the exchange coupling between a pair of magnetic ions, or between two magnetic sheets. Our approach emphasizes the intrinsic self-averaging character of the coupling between layers, by contrast to the interaction between ions.

We stress that although the same type of defects (impurities, roughness) play an essential role both in the interlayer exchange coupling and in the transport properties of magnetic multilayers, one cannot draw any simple relationship between their influence in the two cases. For example, in the absence of impurity scattering, it has been shown by Zhang and Levy that the effect of interface roughness on the perpendicular transport can be described in terms of an effective mean free path; but, as the above discussion has shown, the interlayer exchange coupling is not exponentially damped by the roughness alone, and it would be wrong to believe that this effective mean free path is of any significance for the interlayer exchange coupling.

Here, we wish to comment on the limitations and extensions of the results obtained in this paper. For the sake of clarity and simplicity, we have restricted ourselves here to a very simple model (magnetic layers of infinitesimal thickness), and to a perturbative approach (expressing the exchange interaction via a susceptibility). In order to treat more realistic systems, with magnetic layers of finite (or infinite) thickness, a more sophisticated approach, such as the one developed by Bruno, should be used; however, we expect that the conclusions obtained here would still hold. The effect of impurity scattering is described in terms of complex wavevectors and complex Fermi surface. Impurity scattering in the spacer layer gives rise to an exponential damping of the coupling, but not impurity scattering in the magnetic layers (in this case, only the amplitudes and phases are affected), which agrees with what one would expect intuitively. Interface roughness modifies not only the spacer layer thickness, but also the thickness of the magnetic layers. It is known that the interlayer exchange coupling varies not only with the spacer thickness, but also with the thickness of the magnetic layer; however, the latter is a secondary effect and can be neglected here. Thus, we expect that the effect of roughness would be essentially the same as described within the simple approach of the present paper.

Finally, we have considered only the limit case of “geometrical” roughness. An opposite limit case is the one of an interdiffusion of the magnetic material and spacer material near the interface. In such a situation, it is completely inappropriate to discuss the effect of roughness in terms of fluctuations of the spacer thickness. Rather, as done by Kudrnovský et al., one can consider that there is a thin layer of disordered magnetic alloy in the interface region, whose magnetization remains parallel to the magnetization of the magnetic layer nearby. In such a case, as one would expect intuitively, one finds that the interdiffusion modifies the amplitude(s) and phase(s) of oscillatory coupling, but does not alter the period(s) nor the \( D^{-2} \) decay.

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\section*{APPENDIX:

As we have shown in Sec. \[ II \] interface roughness makes it necessary for us to average the interlayer coupling over a distribution of spacer thicknesses; see Eq. (26). Here, we present an alternate way of arriving at the attenuation factor due to roughness (thickness fluctuations) by using the canonical transformation introduced by Tešanović, Jarić and Maekawa to transform a film with a
rough boundary to one with smooth ones and extended to films with two rough boundaries by Meyerovich and Stepaniants. In doing this, the transformation induces a perturbation (scattering potential) into an otherwise impurity-free layer. It follows that we can replace the average over spacer layer thicknesses by a fluctuating phase shift that is induced by the canonical transformation that replaces the spacer with rough boundaries (interface roughness) by one with smooth boundaries.

By following Refs. or Trivedi and Ashcroft the perturbation due to the surface roughness is

\[ V_{\text{surface}}(\mathbf{r}) = \frac{i}{2\hbar} \eta(\mathbf{r}_\parallel) \left\{ [r_\perp p_\perp + p_\perp r_\perp] H_0 - c.c. \right\} + 2i\eta(\mathbf{r}_\parallel) H_0 \]  

(A1)

where

\[ H_0 = \frac{p_\perp^2}{2m} + V_0(\perp) + \frac{p_\|^2}{2m}, \]  

(A2)

\[ \eta(\mathbf{r}_\parallel) \equiv \frac{\delta D(\mathbf{r}_\parallel)}{D}, \]  

(A3)

and \( V_0(\mathbf{r}_\perp) \) is the confining potential, \( \delta D(\mathbf{r}_\parallel) \) is the variation in thickness over the surface, and \( D \) is the average thickness of the layer.

As our treatment up till now has neglected the confining potential, or equivalently in the limit of large thickness over the surface, and \( D \) is the average thickness of the layer.

As our treatment up till now has neglected the confining potential, or equivalently in the limit of large \( D \), the distribution of eigenvalues is quasi-continuous. The matrix element of the perturbation, Eq. (A1), between states of \( H_0 \) at the Fermi level is

\[ \langle \mathbf{k}_\parallel, n | V_{\text{surface}} | \mathbf{k}_\parallel + \mathbf{q}_\parallel, n \rangle = 2\tilde{\eta}(\mathbf{q}_\parallel) \varepsilon_F \]  

(A4)

where \( n \) labels the states referring to energy levels in the \( \mathbf{r}_\perp \) direction, and \( \tilde{\eta}(\mathbf{q}_\parallel) \) is the two-dimensional Fourier transform of \( \eta(\mathbf{r}_\parallel) \).

By placing this scattering potential in the semiclassical expression for the Green’s function, we find the exchange interaction between two spins located on \( F_1 \) and \( F_2 \) is given again by Eq. (B), but with a fluctuating phase shift, Eq. (B), of

\[ \phi(\mathbf{r}_1, \mathbf{r}_2) = 2k_F R_{1,2} \eta(\mathbf{r}_1 - \mathbf{r}_2). \]  

(A5)

By proceeding as in Secs. and we obtain an expression of the interlayer coupling averaged over thickness fluctuations:

\[ T \sim 2\pi \text{Re} \left\{ \frac{-i}{2k_F} A'(2k_F) e^{i2k_F D} \right\}, \]  

(A6)

with

\[ A'(2k_F) \equiv \langle e^{i\phi(\mathbf{r}_1)} \rangle = \langle e^{i2k_F \delta D(\mathbf{r}_\parallel)} \rangle. \]  

(A7)

As in Eq. (13) we include a shift \( \delta k \) by redefining \( k_F \). Comparison with Eq. (20) shows that \( A'(2k_F) \) is equal to \( A(2k_F) \). Thus, one obtains results that are equivalent to those obtained in Sec. when no impurities are present, i.e., \( \lambda^{-1} = 0. \)