Direct quantitative measurements of Doppler effects for sound sources with gravitational acceleration

Kenichiro Aoki$^{1*}$, Takahisa Mitsui$^1$ and Yuki Yamamoto$^{1,2}$

$^1$Research and Education Center for Natural Sciences, and Dept. of Physics, Keio University, 4—1—1 Hiyoshi, Kouhoku-ku, Yokohama 223-8521, Japan

$^2$Tohoku University of Community Service and Science, Imoriyama 3-5-1, Sakata 998-8580, Japan

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We explain simple laboratory experiments for making quantitative measurements of the Doppler effect from sources with acceleration. We analyze the spectra and clarify the conditions for the Doppler effect to be experimentally measurable, which turn out to be non-trivial when acceleration is involved. The experiments use sources with gravitational acceleration, in free fall and in motion as a pendulum, so that the results can be checked against fundamental physics principles. The experiments can be easily set up from “off the shelf” components only. The experiments are suitable for a wide range of students, including undergraduates not majoring in science or engineering.

I. INTRODUCTION

The Doppler effect is a fundamental aspect of wave phenomena important in almost all areas of physics. The effect can be experienced in everyday life from fast moving cars or trains passing by and is also used as a practical method for measuring the velocity of objects such as baseballs and automobiles. It is also of crucial importance in astrophysics, where the receding or approaching velocity of an object is determined using the Doppler effect in light. The results are used as a basis for such fundamental aspects as Hubble’s Law and the existence of black holes. We believe that the Doppler effect, given its importance and practicality, is a quite suitable theme as a basic physics experiment for all types of students.

In this work, we explain concretely experiments for quantitatively measuring the Doppler effect due to the motion of the wave source with gravitational acceleration. Our purpose is to design experiments for undergraduate students including non-science majors. As such, we need an experiment that can be easily set up and safely conducted in an ordinary laboratory situation. Further, data acquisition needs to be robust and not sensitive to the environment. The students should be able to clearly understand what is going on and the results should be quantitative. Since we have a wide variety of students in mind, the analysis should not necessitate “higher” mathematics and desirably, the setup should use inexpensive components which are readily available. As explained below, we have been able to achieve all these goals. Since practical considerations are of import, we provide the details of the experiment, such as the parameters and the equipment used. Due to its significance as a fundamental physics phenomenon, various forms of Doppler effect experiments have been proposed and conducted for some time: Doppler effect due to a wave source on a toy car, on an air track, in circular motion, carried by a student and in free fall as well as the effect due to the motion of the receiver have been studied. Demonstrations of Doppler effects from real cars have also been performed. However, the majority of the past experiments seem either rather difficult to conduct, use ultrasonic frequencies, not quantitative or require specialized equipment or circuitry.

We explain the basic principle and the setup of the experiments in section II. Section III contains the details and the results from Doppler effect measurements of a freely falling source. We analyze the properties of Doppler effects from accelerating sources in section IV. In the process, we clarify some non-trivial conditions and limitations in measuring Doppler effects. We explain the Doppler effect measurements of a pendulum source in section V.

II. EXPERIMENTAL PRINCIPLE AND SETUP

In this work, we directly measure the Doppler shift due to the motion of the source, since this approach is most often used in various areas of fundamental physics and in practical measurements. The principle of the experiments is to measure the velocity of a moving wave source using the Doppler effect relation, $f / f_s = 1 / (1 - v/c)$. The measured frequency is $f_s$ when the source is stationary and $f$ when it is approaching the observer with velocity $v$. $c$ is the speed of sound. From the time dependence of $v$, we can obtain the acceleration. While this principle can indeed be made to work, the simplicity is somewhat deceptive, even in principle. To measure a frequency at any given instant, a finite

* E-mail addresses: ken@phys-h.keio.ac.jp, mitsui@hc.keio.ac.jp, yamamoto@koeki-u.ac.jp.
amount of time $T$ is necessary and the instantaneous velocity picture ceases to hold for large accelerations or for long measurement times, as explained in section IV ($T$ is different from the time of observation, $T_{\text{obs}}$, during which we make multiple measurements of the frequency.)

While the idea is simple, one main obstacle is that $v$ is at most few % of $c$, due to practical considerations, including safety. Consequently, to measure $v$ to 10% accuracy, for instance, we need 0.1% level accuracy in the measurement of $f$, which is not trivial. A theoretical limitation exists that to obtain a frequency resolution $\Delta f$, time $1/\Delta f$ is necessary for the measurement. To obtain the desired precision, we use a relatively high frequency sound $\sim 4\, \text{kHz}$ and require $\Delta f \lesssim 10\, \text{Hz}$. Consequently, the amount of time required for the measurement is $T \gtrsim 0.1\, \text{s}$. A free fall from the height of 2 m takes 0.64 s, for instance, which is not that much larger than $T$. Increasing $T$ to decrease $\Delta f$ can conflict with measuring Doppler effects from accelerating sources, as explained in section IV.

Let $T = N/R$ so that $\Delta f = 1/T = R/N$, where $R$ is the sampling rate and $N$ is the number of samples. Any $R$ can be used as long as $R$ is higher than twice the frequency of interest. $N$ is chosen appropriately depending on the situation at hand. Since $\Delta f$ is fixed by $T$, larger $f_s$ can achieve higher relative accuracy. For the clarity of exposition to students and since generic components we utilize are not guaranteed to function properly outside the audible range, we use an audible sound source. All the experiments below were conducted with $f_s = 3.520\, \text{kHz}$, a clearly audible sound while not being too unpleasant.

In the experiments, we need a simple way to measure frequencies in a short period of time with adequate accuracy. For this we use a microphone attached to a personal computer (PC) and a spectrum analyzer software. While most microphones work, one with high directivity facilitates the experiment. For spectral analysis, we used the GNU Octave software\(^\text{14}\) and an explicit example script for extracting spectra from a sound file is provided in Appendix A.

An important technical point is to find an object that emits a sound at a distinct frequency steadily and a digital voice recorder is suitable for this task. It has the additional advantage that the sound can be synthesized on a PC, transferred to it, then played back. Synthesizing such a sound file can be accomplished by using GNU Octave\(^\text{14}\) or SoX\(^\text{15}\), for instance. A generic recorder with sufficient sound volume can perform the task and a means to directly transfer the sound from a PC is convenient. We also tested various simple buzzers for this purpose, but none of those tested could emit a sound with a steady enough frequency, especially when in motion. Instead of a digital voice recorder, in some cases, a speaker attached to the same PC can be used to replay the synthesized sound, which can be somewhat cheaper. However, in this case, enough volume needs to be attainable and we need to use a software that does not emit the sound as it is recording, since it will otherwise interfere. Having an independent playback device such as a digital recorder, we find, is versatile, convenient and facilitates the experiment. The components we used are an inexpensive PC, Eee PC (ASUS, $\sim$ $300), the microphone Audio Technica AT815b ($\sim$ $200) and the voice recorder, Olympus DS-51 ($\sim$ $100). All these components are generic and, in our experience, other similar components also suffice.

### III. A FREE FALL DOPPLER EXPERIMENT

![Graph of frequency vs. time](image)

**FIG. 1:** Examples of the measured sound spectrum, from the source at rest (solid) and when approaching the detector during free fall (dashed). The peak is Doppler shifted in the latter case. The subdominant peak corresponds to the reflection off the ceiling and is Doppler shifted as a receding source. The overall scale of $|F(f)|^2$ is arbitrary and Hann window is used.

In this experiment, we let a sound source fall freely towards a microphone and measure the time dependence of the frequency. The experiment is similar in spirit to some previous experiments, though more quantitative. To obtain
the experimental results, we start recording on the PC and let the voice recorder drop (attached to a line to avoid it dropping all the way). The spectrum is analyzed at every $\Delta t$ and the peak frequencies are obtained. Example spectra from a stationary and a freely falling source approaching the observer are shown in Fig. 1. Here, we used $N = 2^{15}$, $R = 192$ kHz taking into consideration the constraints of section IV. Consequently, $\Delta f = 6$ Hz with an error $\Delta v = 0.6$ m/s in $v$. An example of the time dependence of $f$ is shown in Fig. 2 (left), where we can see the Doppler effect from a source falling repeatedly, after being pulled back up by the line.

![Graph of peak frequencies vs time](image1)

**FIG. 2:** Doppler shifts measured from a freely falling sound source. (Left) Measured peak frequencies with $\Delta t = 0.01$ s. After $4.2$ s, the line is stretched to maximum and the source is pulled back up and falls again, which repeats. The frequency resolution $\Delta f = 6$ Hz is clearly visible. (Right) An example of an analysis performed in a student experiment. The same data as the previous plot, in the region $t = 3.5$ to $t = 4.2$ with $\Delta t = 0.05$ s. A linear fit to the Doppler shifted frequencies is shown and the slope corresponds to $g = 9.4 \pm 0.7$ m/s$^2$ (ambient temperature $17^\circ$C).

In the student experiments, we let the students read off the peak spectrum frequencies with $\Delta t = 0.05$ s or 0.1 s. Then the students plot these values against time and obtain the slope (Fig. 2 (right)). Using $f = f_s/(1-at/c) \approx f_s(1+at/c)$ ($t = 0$ shifted here), they obtain the gravitational acceleration, $g$, from the slope. The students often discover the “bouncing effect” in Fig. 2 (left) during the analysis and it is educational for them to realize what they are observing. By using free fall for this experiment, the students can also clearly observe the constant acceleration due to gravity, which can contribute to the understanding of the gravitational force as well as the Doppler effect. $g$ is measured within 10% accuracy, enough to convince the student of the mechanism and the effectiveness of the Doppler shift measurements. The point here is confirm the understanding of the Doppler effect by obtaining $g$ and its precision of $g$ is not the main objective. Indeed, if measuring $g$ were the primary objective, we can do so to 0.1% accuracy using a simple pendulum.

## IV. DOPPLER EFFECT FROM ACCELERATING SOURCES

In this section, we derive and explain the spectral properties of the Doppler effect from accelerating sources briefly. The practical problem is the following: Consider an accelerating monotonic source. We might expect the measured spectrum to have a peak Doppler shifted by the instantaneous velocity of the source, but a finite time is needed to make the measurement so no “instantaneous” velocity is measured. We provide quantitative criteria for observing a simple Doppler shifted spectrum and clarify what exactly can be measured.

When a source approaching the observer with a constant acceleration, $a$, emits a signal at time $t_s$, it is received by the observer at time $t$, and their relations are $c(t - t_s) = L - at^2/2$. Here, without loss in generality, the initial distance between the source and the observer is $L$ and the source is stationary when $t_s = 0$. If the signal has a definite frequency $f_s$, it can be reexpressed in terms of the observer time as

$$\sin(2\pi f_s t_s) = \sin \left[ 2\pi f_s \left( t - \frac{L}{c} \right) \left( 1 + \frac{a}{2c} \left( t - \frac{L}{c} \right) \right) \right].$$

(1)

Here, we assumed the source velocity $v$ is such that $v \ll c$ and have kept the leading order terms. The signal is measured
by the observer from time $t_o$ to $t_o + T$ and its power spectrum can be obtained from the Fourier transformed signal,

$$F(f) = \int_{t_o - L/c}^{t_o + T - L/c} dt \sin \left[ 2\pi f_s \left( 1 + \frac{\gamma f t}{2} \right) t \right] e^{2\pi i f t} \frac{1}{2\pi f_s} \left\{ \tilde{C}(f) \cos \frac{\pi}{\gamma} \left( 1 + \frac{f}{f_s} \right)^2 + \tilde{S}(f) \sin \frac{\pi}{\gamma} \left( 1 + \frac{f}{f_s} \right)^2 \right\}$$

$$- \tilde{C}(-f) \cos \frac{\pi}{\gamma} \left( 1 - \frac{f}{f_s} \right)^2 - \tilde{S}(-f) \sin \frac{\pi}{\gamma} \left( 1 - \frac{f}{f_s} \right)^2 + i \left\{ -\tilde{C}(f) \sin \frac{\pi}{\gamma} \left( 1 + \frac{f}{f_s} \right)^2 + \tilde{S}(f) \cos \frac{\pi}{\gamma} \left( 1 + \frac{f}{f_s} \right)^2 \right\} .$$

Here we defined a dimensionless parameter, $\gamma \equiv a/(c f_s)$. $\gamma$ is the relative change in the velocity in one period of the sound oscillation so that $\gamma \ll 1$. We also defined $\tilde{C}(f) \equiv C(x_\pm, y_o), \tilde{S}(f) \equiv S(x_\pm, y_o)$ where $x_\pm \equiv \sqrt{2\gamma} f_s (t_o - L/c + \sqrt{2/\gamma} (1 \pm f/f_s)), y_o \equiv \sqrt{2\gamma} f_s T$. These functions are related to the Fresnel integrals and their necessary properties are explained in Appendix B. The power spectrum is

$$|F(f)|^2 = \frac{1}{8\pi f_s^2} (G_0 + G_1 + G_2), \ G_0 \equiv \tilde{C}(f)^2 + \tilde{S}(f)^2$$

$$G_1 \equiv -2 \left[ \tilde{C}(f)\tilde{C}(-f) - \tilde{S}(f)\tilde{S}(-f) \right] \cos \frac{\pi}{2} \phi_o - 2 \left[ \tilde{C}(f)\tilde{S}(-f) + \tilde{S}(f)\tilde{C}(-f) \right] \sin \frac{\pi}{2} \phi_o G_2 \equiv \tilde{C}(f)^2 + \tilde{S}(f)^2 ,$$

where we defined $\phi_o \equiv 4(1 + f^2/f_s^2)/\gamma$. The spectrum is involved, but we first note that when $f \simeq f_s$, $x_+ = \mathcal{O} (\gamma^{-1/2}) \gg 1$. Therefore, using the asymptotics of the Fresnel functions in Eq. (B3), $G_1/G_0 = \mathcal{O} (\gamma^{1/2}), G_2/G_0 = \mathcal{O} (\gamma)$ and both are much smaller than 1, when $f \sim f_s$. We now analyze the dominant term $G_0$ and then show this suffices. As explained in Appendix B, $G_0$ has a well defined peak when

$$y_o = \sqrt{2\gamma} f_s T < C_0 \Rightarrow f_s T^2 < \frac{c}{2a} C_0^2 ,$$

where $C_0 = 3.073490$, numerically. $y_o$ is the product of (the square root of) the acceleration relative to the sound velocity and $T$, both measured in the time scale $1/f_s$. Heuristically, this condition is reasonable; $v$ is changing so that a simple peak does not exist when $a$ is too large or $T$ is too long. However, since both $\gamma$ and $f_s T$ are dimensionless, it is a priori not clear why this particular combination is the relevant criteria and this concrete computation enabled us to clarify the point. The condition puts an upper bound on $T$ which competes with the precision $\Delta f = 1/T$, so that judicious balancing is needed. In all our experiments, $T$ was chosen taking these considerations into account.

The peak, when it exists, is at $f_{peak}/f_s - 1 = a/c(t_o - L/c + T/2)$. This Doppler shift corresponds to $v$ for which the signal is received at the central time of the measurement, as expected. When $\sqrt{2\gamma} f_s T > C_0$, the spectrum has multiple peaks and the determination of the “peak” becomes ambiguous.

We now analyze the effect of the subleading terms, $G_{1,2}$, to $f_{peak}$. In general, when a function $g_0(f)$ is peaked at $f_0$, the peak for $g_0(f) + \epsilon g_1(f)$ occurs at $f_0 - \epsilon g_1'(f_0)/g_0''(f_0)$, to leading order in $\epsilon$. Therefore, we need to know the size of $f dG_{1,2}(f)/df$. Their individual terms are $\mathcal{O}(\gamma^{-1/2})$ which can give rise to shifts comparable to the Doppler shift, so that a finer estimate of these terms is necessary. After some non-trivial manipulations, we obtain

$$f_s^2 \left. \frac{dG_1(f)}{df} \right|_{f = f_{peak}} = -2 \sqrt{\frac{2}{\gamma}} \tilde{C}(-f) \left\{ \cos \frac{\pi}{2} \left[ (x_+ + y_o)^2 - \phi_o \right] - \cos \frac{\pi}{2} \left[ (x_+ - y_o)^2 - \phi_o \right] \right\} \bigg|_{f = f_{peak}}$$

$$+ \pi (x_+ - x_-) \left[ \tilde{S}(f) \cos \frac{\pi}{2} \phi_o - \tilde{C}(f) \sin \frac{\pi}{2} \phi_o \right] \bigg|_{f = f_{peak}}$$

$$+ 2 \sqrt{\frac{2}{\gamma}} \tilde{S}(f) \left\{ \sin \frac{\pi}{2} \left[ (x_+ + y_o)^2 - \phi_o \right] - \sin \frac{\pi}{2} \left[ (x_+ - y_o)^2 - \phi_o \right] \right\}$$

$$- \pi (x_+ - x_-) \left[ \tilde{C}(f) \cos \frac{\pi}{2} \phi_o + \tilde{S}(f) \sin \frac{\pi}{2} \phi_o \right] \bigg|_{f = f_{peak}}$$

$$f_s^2 \left. \frac{dG_2(f)}{df} \right|_{f = f_{peak}} = 2 \sqrt{\frac{2}{\gamma}} \left\{ \tilde{C}(f) \left[ \cos \frac{\pi}{2} (x_+ + y_o)^2 - \cos \frac{\pi}{2} x_+^2 \right] + \tilde{S}(f) \left[ \sin \frac{\pi}{2} (x_+ + y_o)^2 - \sin \frac{\pi}{2} x_+^2 \right] \right\} .$$

The leading order terms in $\gamma$ cancel and $f_s^2 dG_1(f_{peak})/df = \mathcal{O}(y_o), f_s^2 dG_2(f_{peak})/df = \mathcal{O}(\gamma^{1/2})$, using the asymptotic behaviors Eq. (B3). Consequently, the contributions from $G_{1,2}$ to the relative shift in $f_{peak}$ are $\mathcal{O}(\gamma y_o), \mathcal{O}(\gamma^{3/2})$, respectively. Since the relative shift in the frequency due to the Doppler effect is $f_{peak}/f_s - 1 \geq \sqrt{2\gamma}/y_o/2$, the
relative error in the shift due to $G_{1,2}$ is at most $O(\gamma^{1/2})$. We note that $y_o$ is at most of order one from Eq. (3) and typically not much smaller than 1 due to the required frequency resolution.

We now illustrate these points in the free fall experiment in section III. The condition for the existence of an unambiguous peak, Eq. (3), corresponds to $f_sT^2 < 166$ s. In Fig. 3, we compare the cases when this condition is and is not satisfied both in simulations and in the analyses of the same experimental data. When condition Eq. (3) is satisfied, a distinct peak exists in the spectrum. However, when the condition is not satisfied, a simple peak no longer exists in theory and it is impossible to ascertain the peak frequency in the experimental situation. Condition Eq. (3) necessarily depends on the acceleration and for small acceleration or for constant speeds, we can use longer measurement times. For instance, if we use $N = 2^{17} = 131072$ (which is not appropriate for measuring $g$), the error is $\Delta f = 1$ Hz, corresponding to $\Delta v = 0.1 \text{ m/s}$. Errors $\Delta f, \Delta v$ are inversely proportional to $N, T$. However, due to Eq. (3), $\Delta f, \Delta v$ can be decreased only as $\sqrt{a}$.

Let us summarize the situation: When measuring constant velocity, we can reduce the error in $v$ by using longer $T$. However, there will usually be practical limitations, such as one can not let the object move at constant velocity for an arbitrary amount of time nor can we pick up the sound for a long time. When $a = 0$, $T$ needs to be small enough to satisfy Eq. (3) for the Doppler shift to be unambiguously observable. Further, if we want to measure $a$ during time $T_{\text{obs}}$, we have an additional requirement that the Doppler shift needs to be larger than the frequency resolution. Letting $\eta(< 1)$ be the relative precision with which we measure $a$, $aT_{\text{obs}}/c > 1/\eta c_0 \sqrt{2a/cf_s}$, equivalent to $T_{\text{obs}}/T > 2(\eta c_0 y_o)^{-1}$. For larger $a$, $T_{\text{obs}}$ can be smaller, yet the attained velocity difference is larger.

V. DOPPLER EFFECT MEASUREMENTS OF A PENDULUM SOURCE

An experiment which provides more challenge in its analysis is the measurement of Doppler effects from a source used as a pendulum. We used the voice recorder strung by a line to a high beam as the pendulum and the configuration
of the experiment is shown in Fig. 4. The strategy follows that used in the previous sections, exemplifying the ease with which we can adapt our methods to various Doppler shift measurements. The measurement is easier if relatively large velocities are attained. Consequently, we would like to a pendulum with a long length, $l$, and a large initial angle. Using typical values for a laboratory situation, $l = 2 \text{ m}$, $\theta_{\text{init}} = \pi/4$, the maximum velocity is 3.5 m/s which is quite measurable, as seen in the previous sections. The experiment is not as simple as it may seem for the following reasons. First, if we use a large initial angle, the harmonic approximation to the oscillation is no longer quite valid. Furthermore, larger oscillations tend to die down rapidly. Smaller oscillations can be observed for a longer period of time, yet the velocities involved are smaller and the analysis becomes technically more delicate.

Several approaches to analyzing this experiment are apparent. One simple approach is to take various time periods of the oscillation and compare them to theoretical predictions, as shown in Fig. 5. In these examples, $l = 2.32 \pm 0.01 \text{ m}$, the ambient temperature is 23°C. A lower sampling rate $R = 48 \text{ kHz}$ was used to reduce the data size and $N = 2^{14}$ was used to satisfy Eq. (3). The observed oscillation frequency agrees with the standard formula, $\sqrt{g/l}/(2\pi)$. We can easily obtain $g$ to 1% accuracy or less and $g = 9.8 \pm 0.1 \text{ m/s}^2$ in this example. If the objective is solely to measure $g$, there is no distinct advantage over timing measurements of a simple pendulum. For our purposes, the pendulum source provides a variety of Doppler effects that can be quantitatively measured, including approaching, receding and accelerating sources all in one experiment, that can be checked against basic physics. Another advantage is that the time dependence of $v$ can be obtained more globally than that in simple timing measurements. A precise global description of the data, which is not attempted here, needs to include the modeling of the decay in the oscillations and the anharmonic effects in the pendulum. As a Doppler effect measurement, we are quite satisfied that the oscillations can be seen clearly and agree so well with the theory. There are a number of aspects to this experiment and more intricate analysis can be performed if desired. For instance, the change in the period due to anharmonicity can be observed, as can be seen in the results in Fig. 6. This kind of experiment might be appropriate for more advanced students or for student projects.

VI. DISCUSSION

Considering the prevalence and the importance of the Doppler effect in basic sciences, along with its practical usefulness, we believe that easily realizable experiments in which the students can quantitatively measure the Doppler effect are highly desirable. In this work, we explained two such experiments, which are suitable for undergraduate students majoring in any field and require only generic, off the shelf components. In the experiments, the source undergoes gravitational acceleration so that the students can check the measured results against fundamental physics principles. The experiments are not difficult for students to perform and one can visibly confirm the induced shift in the peak of the audible sound. The setup we explained, as seen above, is quite flexible and the experiment can be conducted with the sound source attached to various objects in motion. It might be interesting to measure Doppler shifts of thrown objects, swung bats/rackets, and so on, using this method.

In all our experiments, we picked out the highest peaks in the spectrum and no attempt was made to identify the “correct” peak, yet the Doppler effect could be measured quantitatively. The main reason for the non-ideal spectra and the occurrence of multiple peaks is the existence of reflections which inevitable occur in ordinary student laboratories. In our experience, objects that are approaching and therefore have higher peak frequencies tend to be easier objects for measurement, since the interference effects do not affect the highest frequency peak substantially. Our objective is to design student experiments which can be performed in ordinary laboratory situations and for such purposes, we designed the experiments with the data extraction that is robust and clear cut.

The Doppler effect is a phenomenon that can be experienced in everyday life and is not hard for students to understand. One possible concern might be that students, particularly those not majoring in science or engineering, find spectral methods hard to comprehend. However, they are exposed to music and pitch in everyday life and most students understand the concept of frequency and harmonics, though not necessarily fully conscious of their spectral context. Therefore, the shift in the peak frequency caused by the Doppler effect is not a difficult concept for students to grasp. We believe, rather, that experiments with sounds allow students to pick up the idea of the spectrum and Fourier analysis intuitively and serves as an excellent introduction to spectral methods. In our experiments, we combine the spectral analysis of sounds from musical instruments with the free fall Doppler experiment in section III in order for the students to first gain a good grasp of the relation between sound, harmonics and the frequency
APPENDIX A: EXTRACTING SPECTRA FROM A SOUND FILE

We provide an example script for GNU Octave (for Windows, Linux and other Unix variants) that extracts spectra from a sound file at predetermined time slices. This example script plots the spectrum at times 0, Δt, 2Δt, · · · , and displays the time (at the beginning of each measurement) and the peak frequency on the terminal, given a sound file. It can be modified to look at a particular region in time and window functions can also be incorporated.

```octave
#!/usr/bin/octave -qf
[data,R]=wavread('freefall.wav'); # data, sampling rate from file
dt=0.05; # time between data points
N=2^15; # number of samples
fmin=3400; fmax=3700; # freq. plot range
df=R/N; # frequency resolution
range=round((fmin/df):(fmax/df)); # x axis
printf("# t[s]\nPeak[Hz]\n");
for i=round((0:floor(length(data)-N)/R/dt)*R*dt);
    spectrum=power(abs(fft(data(i+1:i+N))),2);
```

spectrum. Undergraduate students majoring in humanities and social sciences at Keio University currently perform these experiments in class smoothly and we are quite satisfied with the outcome.
APPENDIX B: SOME PROPERTIES RELATED TO FRESNEL INTEGRALS

We define the following functions

\[ C(x, y) \equiv C(x + y) - C(x) \quad \text{and} \quad S(x, y) \equiv S(x + y) - S(x) \quad . \]  

Here \( C(x), S(x) \) are the Fresnel functions\(^{16}\).

\[ C(x) = \int_{0}^{x} dt \cos \left( \frac{\pi t^2}{2} \right) \quad \text{and} \quad S(x) = \int_{0}^{x} dt \sin \left( \frac{\pi t^2}{2} \right) \quad . \]  

We need the asymptotic behavior of the Fresnel functions which are, to leading order\(^{16}\),

\[ C(x) = \frac{1}{2} + \frac{\sin \left( \frac{\pi x^2}{2} \right)}{\pi x} - \frac{\cos \left( \frac{\pi x^2}{2} \right)}{\pi x^3} + O \left( x^{-5} \right) \quad , \quad S(x) = \frac{1}{2} - \frac{\cos \left( \frac{\pi x^2}{2} \right)}{\pi x} - \frac{\sin \left( \frac{\pi x^2}{2} \right)}{\pi x^3} + O \left( x^{-5} \right) . \]  

It is straightforward to find that the extrema of \( C(x, y) \) are at

\[ x = -\frac{y}{2} + \frac{2n}{y}, \quad -\frac{y \pm \sqrt{8m - y^2}}{2} \quad (m \geq y^2/8) \]  

and similarly for \( S(x, y) \),

\[ x = -\frac{y}{2} + \frac{2n}{y}, \quad -\frac{y \pm \sqrt{4(2m - 1) - y^2}}{2} \quad (m \geq y^2/8 + 1/2) \quad . \]

Here, \( m, n \) are integers. The function \( C(x, y)^2 + S(x, y)^2 \) has a maximum at \( x = -y/2 \) when \( y < C_0 = 3.073490 \) as seen in Fig. \( B \) where the value of \( C_0 \) was determined numerically.

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FIG. 6: $C(x, y)^2$ (dashes), $S(x, y)^2$ (dots) and $C(x, y)^2 + S(x, y)^2$ (solid) for $y = 1$ (left top), $y = 3.0735$ (right top) and $y = 4$ (left bottom). The maximum at $x = -y/2$ disappears for $y > C_0$. 