Stellar bars in counter-rotating dark matter haloes: the role of halo orbit reversals

Angela Collier,1 Isaac Shlosman2,3 and Clayton Heller3

1Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506-0055, USA
2Theoretical Astrophysics, Graduate School of Science, Osaka University, Osaka 560-0043, Japan
3Department of Physics and Astronomy, Georgia Southern University, Statesboro, GA 30460, USA

Accepted 2019 August 13. Received 2019 August 13; in original form 2019 June 10

ABSTRACT
Disc galaxies can exchange angular momentum and baryons with their host dark matter (DM) haloes. These haloes possess internal spin, \( \lambda \), which is insignificant rotationally but does affect interactions between the baryonic and DM components. While statistics of prograde and retrograde spinning haloes in galaxies is not available at present, the existence of such haloes is important for galaxy evolution. In the previous works, we analysed dynamical and secular evolution of stellar bars in prograde spinning haloes and the DM response to the bar perturbation, and found that it is modified by the resonant interactions between the bar and the DM halo orbits. In this work, we follow the evolution of stellar bars in retrograde haloes. We find that this evolution differs substantially from evolution in rigid unresponsive haloes, discussed in the literature. First, we confirm that the bar instability is delayed progressively along the retrograde \( \lambda \) sequence. Secondly, the bar evolution in the retrograde haloes differs also from that in the prograde haloes, in that the bars continue to grow substantially over the simulation time of 10 Gyr. The DM response is also substantially weaker compared to this response in the prograde haloes. Thirdly, using orbital spectral analysis of the DM orbital structure, we find a phenomenon we call the orbit reversal – when retrograde DM orbits interact with the stellar bar, reverse their streaming and precession, and become prograde. This process dominates the inner halo region adjacent to the bar and allows these orbits to be trapped by the bar, thus increasing efficiency of angular momentum transfer by the inner Lindblad resonance. We demonstrate this reversal process explicitly in a number of examples.

Key words: methods: numerical – galaxies: evolution – galaxies: formation – galaxies: haloes – galaxies: interactions – galaxies: kinematic and dynamics.

1 INTRODUCTION
In the current paradigm of galactic structure, the baryonic component is deeply embedded in the massive dark matter (DM) haloes. Numerical simulations of DM structure formation in the universe have shown that haloes exhibit internal spins (e.g. Peebles 1969). The spin parameter can be defined as \( \lambda \equiv J/J_{\text{max}} \), where \( J \) is the halo’s angular momentum, \( J_{\text{max}} \) is its Keplerian maximum, and its range is practically limited to \( \lambda \sim 0–0.1 \) (e.g. Bullock et al. 2001). The halo spin distribution can be fit by a lognormal distribution

\[
P(\lambda) = \frac{1}{\lambda(2\pi \sigma)^{1/2}} \exp \left[ -\frac{\ln^{2}(\lambda/\lambda_{0})}{2\sigma^{2}} \right],
\]

where \( \lambda_{0} = 0.035 \pm 0.005 \) and \( \sigma = 0.5 \pm 0.03 \) are the fitting parameters (Bullock et al. 2001). While the spin is insignificant rotationally, the majority of DM haloes must be spinning to some extent.

Owing to a complex assembly history of individual haloes, their angular momentum vectors can vary with respect to the embedded galactic discs. The halo can consist of multiple kinematically distinct DM and baryonic components, e.g. streamers, subhaloes, and disc contributions, while overall being virialized (e.g. Romano-Díaz et al. 2009).

Galactic discs can host single and double bars (e.g. review by Shlosman (2013)). Barred galactic discs have been shown to lose their angular momenta to the host haloes (e.g. Sellwood & Debattista & Sellwood 2000; Athanassoula 2003; Martinez-Valpuesta, Shlosman & Heller 2006; Berentzen et al. 2007; Villa-Vargas, Shlosman & Heller 2009, 2010), and do it mainly by resonance interactions (Athanassoula 2003; Martinez-Valpuesta, Shlosman & Heller 2006; Dubinski, Berentzen & Shlosman 2009), as first derived by Lynden-Bell & Kalnajs (1972) and applied to barred discs by Tremaine & Weinberg (1984). However, these works

* E-mail: shlosman@pa.uky.edu
have analysed interactions with non-rotating host haloes, clearly a tiny minority among the DM halo population. Alternatively, cosmological simulations that produce haloes with various $\lambda$ still lack the necessary resolution to account for the resonant interactions.

Recent modelling has indicated that the bar instability time-scale shortens in disc galaxies with increased $\lambda$ (Saha & Naab 2013; Long, Shlosman & Heller 2014), confirming the theoretical prediction (Weinberg 1985). Moreover, Collier, Shlosman & Heller (2018, 2019) have investigated both dynamical and secular evolution of stellar bars in prograde\(^1\) spinning haloes, in the range of $\lambda \approx 0$–0.09, and found substantial differences between the non-rotating and prograde spherical, oblate, and prolate haloes.

In this work, we have extended our analysis to the retrograde DM haloes, i.e. when the haloes counter-rotate with respect to the underlying barred stellar discs. We present a suite of models based on the same initial conditions, range of $\lambda$ up to $-0.09$, and perform a careful orbit analysis of these models to delineate the role of the resonances in the angular momentum redistribution.

Observations, theory, and numerical simulations have demonstrated that galactic discs reside in massive, responsive, DM haloes. The tidal torque theory predicts that these haloes gain most of their angular momentum by the action of gravitational torques during the phase of a maximal expansion (e.g. Doroshkevich 1970; White 1978; Fall & Efstathiou 1980). It is less clear whether during the phase of a maximal expansion (e.g. Doroshkevich 1970; White 1978; Fall & Efstathiou 1980), their angular momentum by the action of gravitational torques may be modified during the subsequent evolution of the halo systems and often compared to rigid haloes (Christodoulou, Shlosman & Tohline 1995 and references therein). However, this conclusion is based on assumption that the orbital structure of retrograde haloes does not evolve. We find that certain aspects of this problem must be modified, and address this issue in this work.

Saha & Naab (2013) have simulated a counter-rotating halo of $\lambda = 0.05$ and found that such a halo delays the bar formation in the stellar disc. The stellar bar instability was slowed down compared to the non-spinning or prograde spinning haloes. However, these results are limited in scope in that they only analysed model in the dynamical phase of bar evolution and only for one value of $\lambda$. The secular evolution of stellar bars in retrograde haloes remains unknown.

Does increasing the number of retrograde orbits in counter-rotating DM haloes along the negative $\lambda$ sequence completely cut off all or almost all angular momentum transfer from the barred stellar disc to the host halo? The near or complete absence of prograde orbits in the halo should limit the resonant coupling between DM and baryonic components. In this case, the small amount of material in the outer disc makes it more difficult for the inner disc (which is bar-unstable) to transfer large amount of angular momentum, perhaps limiting the disc expansion. We ask, if the retrograde halo orbits cannot resonate with the stellar disc, does this halo behave similarly to a rigid unresponsive halo and slows down the bar formation and growth?

This paper is organized as follows. Section 2 presents the numerical aspects of our simulations, including initial conditions and the orbital spectral analysis method. Section 3 introduces our results on evolution of stellar bars in retrograde haloes, and Section 4 discusses our results. The last section summarizes our conclusions.

2 NUMERICS

We model stellar discs inside spherical Navarro, Frenk & White (1996, hereafter NFW) haloes using the $N$-body part of the tree-particle-mesh smoothed particle hydrodynamics (SPH/N-body) code GIZMO (Hopkins 2015), which is a modified version of GADGET (Springel 2005). Our code units for mass, distance, and time are $10^{10} M_\odot$, 1 kpc, and 1 Gyr.

The DM halo contains $7.2 \times 10^6$ particles and the stellar disc has $0.8 \times 10^6$ particles. The halo mass is $M_h = 6.3 \times 10^{11} M_\odot$ and the disc mass is $M_d = 6.3 \times 10^{10} M_\odot$. Hence, the ratio of DM to stellar particle mass is near unity. For convergence analysis, we doubled the particle number in some models to create models with higher resolution, which resulted in quantitatively similar evolution as those discussed here.

The opening angle, $\theta$, of the tree code and gravitational softening parameter are set to 0.4 and 25 pc, respectively. Models presented...
here conserve energy at the level of 0.05 per cent and angular momentum at 0.03 per cent, for the length of the 10 Gyr runs.

2.1 Initial conditions

The initial conditions of the models follow the prescription of Collier et al. (2018), and are briefly restated here. The halo density is given by the NFW profile

\[ \rho_h(r) = \frac{\rho_s}{\left(1 + r/r_s\right)^2}, \]

where \( \rho(r) \) is the DM density in spherical coordinates, \( \rho_s \) is the fitting density parameter, \( r_s = 9 \) kpc is the characteristic radius, where the power-law slope is \(-2\), and \( r_s \) is a central density core where \( r_s = 1.4 \) kpc. The Gaussian cut-off is applied at \( r_c = 86 \) kpc for the halo.

The stellar disc is an exponential and we ignore the bulge potential. Its volume density is

\[ \rho_d(R, z) = \frac{M_d}{4\pi h^2\sigma_0} \exp\left(-\frac{R}{h}\right)\text{sech}^2\left(\frac{z}{\sigma_0}\right), \]

where \( M_d \) is the disc mass, \( h = 2.85 \) kpc is its radial scale length, and \( \sigma_0 = 0.6 \) kpc is the scale height. \( R \) and \( z \) represent the cylindrical coordinates. The Gaussian cut-off is applied at \( R_t = 6h \sim 17 \) kpc. Using these initial inputs, the halo-to-disc mass ratio within \( R_t \) is about 2. To initialize the halo velocities, we freeze the disc potential and use a modified version of the iterative method from Rodionov & Sotnikova (2006); see also Rodionov, Athanassoula & Sotnikova (2009). For a detailed description of technique applied, see Collier et al. (2018). A short introduction to the iterative method follows.

We allow the halo to adjust to equilibrium velocities in the presence of the frozen disc potential by allowing the DM particles to evolve from their initial positions and zero velocities for 0.3 Gyr. Next, we use a nearest-neighbour program to find the evolved particle that is closest to the position of an original particle at the start of the iteration. The original positioned particle is given the new velocity. We repeat the iterations until the halo velocities converge and obtain halo in virial and velocity equilibrium. The iteration routine required about 50 iterations to create a spherical NFW halo in equilibrium. To test the equilibrium, we ran the haloes for an additional 3 Gyr to verify that it is indeed in equilibrium.

The disc velocity profile depends on halo and disc mass distributions. We calculate the disc rotational and dispersion velocities. The radial and vertical dispersion velocities assigned to the disc are

\[ \sigma_R(R) = \sigma_{R,0}(R)\exp(-R/2h), \]

\[ \sigma_z(R) = \sigma_{z,0}(R)\exp(-R/2h), \]

where \( \sigma_{R,0} = 120 \) km s\(^{-1}\) and \( \sigma_{z,0} = 100 \) km s\(^{-1}\). Toomre’s parameter was calculated to have a minimum of \( Q \sim 1.6 \) at \( R \sim 2.4h \). As expected, \( Q \) increases towards the centre and the outer disc.

The aforementioned procedure creates a halo with cosmological spin parameter \( \lambda = 0 \). To spin-up the halo in the retrograde direction, we have reversed the tangential velocities of a fraction, \( f \), of prograde particles (with respect to the rotation of the disc), \( f \) is increased to create haloes of increasingly negative \( \lambda \). Here, we present haloes with a range of \( \lambda \sim 0.09 \). The new velocity distributions maintain the solution to the Boltzmann equation and do not alter the velocity profile (Lynden-Bell 1960; Weinberg 1985; Long et al. 2014; Collier et al. 2018). For axisymmetric systems, the invariance with velocity reversals is a direct consequence of the Jeans (1919) theorem (see also Binney & Tremaine 2008).

Therefore, we have produced a suite of disc–halo models that differ only in the spin, \( \lambda \). Following the notation of Collier et al. (2018, 2019), the models are labelled as \( P \) if they are prograde and \( R \) if they are retrograde, and then multiplied by 1000. For example, the standard model, \( P00 \), is the non-spinning halo with \( \lambda = 0 \), and \( R60 \) is the halo with retrograde rotation and \( \lambda = 0.06 \).

2.2 Orbital spectral analysis method

We analyse the orbital structure of our disc–halo models and examine the role of resonant angular momentum transfer by using the orbital spectral analysis method (Binney & Spergel 1982; Athanassoula 2003; Martinez-Valpuesta et al. 2006; Dubinski et al. 2009; Collier et al. 2019). Using Fourier analysis, we determine the angular velocity, \( \Omega \), and the radial epicyclic frequency, \( \kappa \), for stellar and DM orbits that resonate with the stellar bar pattern speed, \( \Omega_b \). This is performed in the frozen total potential. In the bar frame, we construct the dimensionless frequency \( \nu = (\Omega - \Omega_b)/\kappa \), and plot the distribution of orbits with \( \nu \). Each stellar or DM orbit has been evolved for 30–50 orbits. For more information, see Collier et al. (2019).

3 RESULTS

We present our results dealing separately with the stellar and DM components in the counter-rotating DM haloes. We start with tracking the stellar bar evolution and follow up with the DM response in these models.

3.1 Retrograde stellar model evolution

We measure the stellar bar strength amplitude, \( A_2 \), the bar length, \( R_b \), the pattern speed, \( \Omega_b \), and finally the vertical buckling amplitude, \( A_{1z} \). Fig. 1(a) exhibits the evolution of the stellar bar amplitudes in retrograde models. The P00 model evolution has been added for comparison. A number of conclusions can be drawn by comparing this figure with behaviour of \( A_2 \) in fig. 1 of Collier et al. (2018). First, arranging the prograde and retrograde models from largest positive \( \lambda \) to the most negative one, the bar instability time-scale increases monotonically.

Next, while stellar bars in prograde models display approximately the same maximal pre-buckling amplitude in the pre-buckling stage, as Fig. 1(a) shows, there is a gradual decrease in this amplitude for retrograde models.

Thirdly, all stellar bars buckle and reduce their amplitude abruptly in both the prograde and retrograde models. The prograde models display progressively lower minimum in \( A_2 \) with \( \lambda \), and this trend continues in the retrograde models. However, the drop in \( A_2 \), i.e. \( \Delta A_2 \), is much less dramatic in the retrograde models.

Fourthly, in the secular stage of evolution, the amplitudes of bars in retrograde models experience a monotonic growth, with rather minor differences in the bar strength. Contrary to this, the prograde models show a much more complex behaviour, which includes essentially the bar dissolution for larger prograde (i.e. positive) \( \lambda \) (Collier et al. 2018, 2019). This constitutes probably the largest and most profound difference in the evolution of the prograde and retrograde models. During the secular phase, the stellar bars experience a healthy growth in the retrograde models, but those in the prograde models do not grow after the buckling, and appear nearly dissolved.
The stellar bar sizes have been determined from extension of the major axes of the $x_1$ orbits. These orbits are the main orbits that support the bar and are elongated along the bar. They populate the region between the co-rotation resonance (CR) and the inner Lindblad resonance (ILR). Because finding these orbits can be time consuming, we have confirmed the bar length using an alternative method, by measuring ellipticity profiles of their isodensity contours in the $xy$-plane. The bar has been assumed to extend to the point where ellipticity has decreased by 15 per cent from its maximum (e.g. Martinez-Valpuesta et al. 2006; Collier et al. 2018). We plot evolution of the stellar bar length in Fig. 1(b).

The continued stellar bar growth is indicative of its braking against the halo and transferring its angular momentum to the halo and outer disc. Each stellar bar in retrograde haloes grows in size for the entire run. During the secular stage, we observe an increasing difference between the bar sizes. The $R_{90}$ stellar bar grows more slowly than the $P00$ stellar bar, and at $t = 10$ Gyr, this bar is about 20 per cent shorter than the $P00$ bar.

Evolution of the bar pattern speeds in all retrograde models and in $P00$ is given in Fig. 1(c). We observe that angular momentum transport, which is facilitated by the bar braking against the outer disc and the halo, is not inhibited by an increase in the retrograde halo spin. In each model, the stellar bar slows down over the entire simulation. We can compare this evolution to that of the stellar bars in the prograde models where increasing $\lambda$ leads to a decrease in the bar amplitude and the near dissolution of the bar, leading to a much weaker bar braking and slowdown.

We plot the Fourier amplitude of the vertical buckling, $A_{1z}$, in the $rz$-plane in Fig. 1(d). The $A_{1z}$ is normalized by the monopole term, $A_0$. The maximum amplitude of the buckling instability does not depend on the $\lambda$, but the time of buckling does depend, as is also evident from evolution of $A_2$.

### 3.2 DM response in retrograde models: orbital reversals

The DM response to the stellar bar perturbation is shown in Fig. 2. For a comparison, two models have been added – the $\lambda = 0$ model, $P00$, and the $\lambda = 0.09$ prograde model, $P90$. Note that, as usually, the Fourier amplitude of $m = 2$ mode in the DM is much weaker than the stellar amplitude. Avoiding the semantic discussion pertaining to what $A_2$ value defines a ‘DM bar’, we refer to the DM response in all models as a DM bar. We do note that the DM response in retrograde haloes is distinctively weaker than its response in the prograde models. It tumbles in the direction of the stellar bar. In other words, the DM response follows the stellar bar, thus propagating against the spin of the DM halo. Furthermore, as we show later, the DM response lags the stellar bar by almost $90^\circ$.

During the dynamical phase of the bar instability, we observe a substantial difference in the DM response. The time-scale of the bar instability becomes more prolonged with decreasing $\lambda$ from 0.09 to $-0.09$. Hence, in pre-buckling evolution of prograde and retrograde
models, we observe a clear hierarchy, from P90 to P00, and to R30, followed by R60 and R90. However, what is most interesting is the behaviour of the amplitude, $A_2$, of the DM response, whose pre-buckling maximum decreases from 0.08 for P90 to 0.024 for P00 and to 0.008 for R90 – overall by a factor of 10. In comparison, the stellar bar pre-buckling amplitudes stay about the same for $\lambda = 0.09$, and display a small decrease for the retrograde models, down to $\lambda = -0.09$, as shown by fig. 1 of Collier et al. (2018).

During the secular phase of evolution, the DM response in retrograde haloes appears much weaker than in the prograde models. An important point to be emphasized now and to which we shall return in the subsequent sections is the break in the monotonic evolution of $A_2$ DM amplitudes in the secular phase. This is especially noticeable in the amplitude for the R90 model. It has been naively expected to be the weakest in Fig. 2, based on the evolution of other retrograde models, yet it evolved and became the strongest instead. The explanation for this phenomenon is not a trivial one, and will be addressed later.

To emphasize the difference between the DM response in prograde and retrograde models, we display the projected DM density on to the $xy$-plane in Fig. 3. Note the dramatic change in the morphology of this response between $\lambda = -0.09$, 0, and +0.09. The prograde model displays a response aligned with the stellar bar, while the retrograde model shows response that is nearly $90^\circ$ trailing the stellar bar. This difference underscores the importance of the CR in the prograde models versus the ILR in the retrograde models. We discuss the importance of these resonances in this and the following sections.

A more careful study of Fig. 2 reveals a more complex behaviour of DM bar in the R90 model compared to other retrograde models. It displays a faster growth of $A_2$ after $t \sim 6$ Gyr and associated stronger braking at the same time. The DM bar in the R60 model shows a weaker version of this evolution, when its DM bar strength increases to match the rival R30. Analysing this behaviour, we came across an unexpected process, which was not discussed in the literature so far – this process sheds a new light on stellar bar evolution and DM response in retrograde models. It involves angular momentum exchange between the stellar bar and the retrograde DM orbits. As a result of this interaction, the DM orbit gains angular momentum and reverses its direction of streaming and precession. We term this process as orbit reversal. To understand the dissentic evolution of the R90 model, and to a lesser degree of all the retrograde models, we correlate it with the DM orbit reversal. During this process, the DM orbit and the halo as a whole absorb angular momentum from the disc. We first demonstrate that the orbit reversals do occur.

For each orbit, we start by choosing three characteristic times. The first one is close to the starting time of the run, at $t = 0.3$ Gyr, when the disc is axisymmetric, and we integrate it until 1 Gyr. The next time period for the first orbit is picked when the stellar bar is close to its maximal strength, i.e. $t = 4.5$ Gyr, and we integrate it till $t \sim 5.2$ Gyr. Lastly, we choose the time close to the end of the run, during the secular phase, at $t = 8$ Gyr, and integrate it to $t \sim 8.6$ Gyr. At each time period, we follow this orbit for about three revolutions in the live potential of the system. The second example displays a reversal DM orbit at the time periods of $t \sim 0.3–1$, 8–8.8, and 8.9–9.9 Gyr.

The left frames of Fig. 4 display the orbital precession, the rosette, in the rest frame, for both orbits. These rosettes have streaming and precession in the same direction. Their angular momenta, $J_z$, being negative, are shown in Fig. 5 with the associated colours in Fig. 4. The middle frames, which are in the reference frame of the stellar bar (being horizontal), show the same orbits being trapped by the bar and already reversed their direction of streaming and precession. Their $J_z$ is positive at this time. The right frames show the orbits being trapped by the bar towards the end of the run.

Fig. 5 reveals that the first orbit was captured early and briefly released by the bar into the disc after buckling and captured again. The second orbit was not captured by the stellar bar until late in the evolution. However, it interacted strongly with the bar beforehand.

![Figure 3. Projected density response of DM haloes in retrograde (R90) versus prograde (P90) DM haloes. The response is displayed in the pre-buckling phase, when the stellar bars are close to their maximal strength, i.e. at $t \sim 5$ Gyr (R90) and $\sim 1.8$ Gyr (P90). Shown is the ratio of the projected DM density on the $xy$-plane within $|z| < 3$ kpc over the same at $t = 0$, and subtracting unity from the ratio. The contour levels are given in the colour palette. Positions of stellar bars are delineated by the straight horizontal line. The P90 model is from Collier et al. (2019). The positive contours are black solid lines and the negative ones are dashed lines. The outline of density perturbation amplitude of the DM response vary with $\lambda$.](https://academic.oup.com/mnras/article/489/3/3102/5553488)
Figure 4. The randomly chosen two DM orbits of R90 model, initially counter-rotating with the disc (solid lines) and finally co-rotating (dashed lines) with it. Both orbits have been evolved for three revolutions in the live potential in each of the three frames (time periods are shown): before the stellar bar appears (left frames), during the strong bar before its buckling (upper middle frame), and when the stellar bar strengthens during secular evolution (lower middle frame). Orbits remain captured towards the end of the run, during secular evolution (right frames). The middle and right frames are shown in the stellar bar frame of reference, i.e. with the bar pattern speed subtracted. The stellar bar is positioned horizontally. The upper left frame curled arrow shows the direction of precession of both orbits before the bar instability. This direction is reversed when the orbits are captured. The orbits remain trapped by the stellar bar in the last two frames.

Figure 5. Evolution of the angular momenta, $J_z$, of two DM orbits, projected on the $xy$-plane, which are mapped in Fig. 4. The colours of individual orbits in these figures are matched. The orbits start as retrograde, i.e. having negative angular momentum with respect to the stellar disc and bar rotation. Both orbits switched their streaming and precession directions when trapped by the stellar bar, either before buckling or during the secular phase of evolution. Both orbits are briefly released back to the disc and captured again. The units of angular momentum on the $y$-axis are $M_\odot \, \text{km} \, \text{s}^{-1} \, \text{kpc}^{-1}$. As its $J_z$ oscillated widely. Acquiring positive angular momentum does not assure that the orbit will remain prograde.

The frequency and importance of these DM orbital reversals can be quantified. For this purpose, we calculated the fraction of retrograde DM orbits as a function of time for each of the retrograde models, and for the P00 model for comparison. All DM orbits within the region of $R < 20 \, \text{kpc}$ and $|z| < 10 \, \text{kpc}$ have been counted.

Fig. 6 presents evolution of the ratio of prograde to retrograde orbits, $\beta(t)$, in the inner halo of $R < 20 \, \text{kpc}$ and $|z| < 10 \, \text{kpc}$. This region contains DM orbits that can possibly interact and be trapped by the stellar bar. All the curves in this figure are relatively flat before the stellar bar acquires its strength, indicating that the orbit reversals in the DM halo are due to the stellar bar and not due to the instability in the DM halo. In fact, we have previously run discless spinning haloes and found them to be completely stable and their density and velocity distributions show no evolution (Collier et al. 2019).

During the buckling and the subsequent secular evolution of stellar bars in retrograde haloes, we observe that $\beta(t)$ increases with time (Fig. 6). This increase is more substantial with $\lambda$ becoming more negative. For the R90 model, the initial $\beta$ is 0.12, and it increases to 0.31 after the buckling. The comparison P00 model displays a minimal change – from 1.0 to 1.08 only. The sequence of retrograde models shows a monotonic increase of orbital reversals, from P00 to R90. In fact, $\beta$ at 10 Gyr in R90 is equal to the initial $\beta$ of these orbits in the R60 model. Hence, the difference between
Figure 6. The ratio of prograde to retrograde DM orbits, \( \beta(t) \), within \( R < 20 \text{kpc} \) and \( |z| < 10 \text{kpc} \) for retrograde models and for P00 in comparison. While the fraction of prograde orbits in P00 increases little during the evolution, more negative \( \lambda \) leads to the increase in \( \beta \) with time, due to the retrograde DM orbit reversals (see the text for more information).

Figure 7. The orbital spectral analysis of disc and halo orbital structure: the negative \( \lambda \) sequence of resonance trapping for DM haloes (top) and stellar discs (bottom). The x-axis gives the normalized frequency, \( \nu \equiv (\Omega - \Omega_b)/\kappa \) (see definition in the text). The y-axis is the fraction of orbits trapped at each of the major resonances, the ILR \((\nu = 0.5)\), the CR \((\nu = 0.0)\), and the OLR \((\nu = -0.5)\). The frequencies are binned in \( \Delta \nu = 0.01 \). The chosen time for spectral orbital analysis is \( t = 8 \text{Gyr} \) for all models.

3.3 Orbital spectral analysis during the secular evolution

For a more detailed look at the behaviour of retrograde models, we perform the orbital spectral analysis to determine the fraction of orbits trapped at each resonance, and to clarify the role of the orbital reversals in this process along the negative \( \lambda \) sequence. The chosen time for this analysis is identical for all models, at \( t = 8 \text{Gyr} \). This time is long after buckling, i.e. during the secular growth phase of stellar bars. For comparison, Collier et al. (2019) performed this analysis for the prograde models before buckling.

The reason for this was that for prograde models, increasing \( \lambda \) essentially destroys the stellar bars for \( \lambda \gtrsim 0.06 \). Contrary to this, the retrograde models presented here experience a strong growth of stellar bars after the buckling.

The result of orbital spectral analysis for each model is shown in Fig. 7. We plot the fraction of trapped orbits on the y-axis versus dimensionless frequency, \( \nu \equiv (\Omega - \Omega_b)/\kappa \), on the x-axis. The peaks correspond to frequencies where stellar and/or DM particles are trapped by the resonances. The bottom frames show the stellar discs and the top frames show the DM haloes with increasingly negative \( \lambda \) sequence. In all models, the highest spikes correspond to the familiar resonances: the ILR at \( \nu = 0.5 \), the CR at \( \nu = 0.0 \), and the outer Lindblad resonance (OLR) at \( \nu = -0.5 \). The stellar bar strength at this time is approximately the same in all retrograde models.

The bottom row of Fig. 7 shows that the efficiency of disc orbit trapping at the ILR does not change with \( \lambda \). This resonance resides deep inside the stellar bar and is the dominant resonance that ‘emits’ the angular momentum by the disc. On the other hand, as we move along the \( \lambda \) sequence, a monotonic decline in the trapping ability of the OLR can be observed. For example, for R90, the OLR appears to trap about half of the orbits compared to the OLR in the P00 model. There is also a smaller reduction in the trapping efficiency of the CR,
with increasingly negative $\lambda$. In addition, a peak develops between the CR and the ILR of the disc, at $v \sim 0.25$ – the ultra-harmonic 1:4 resonance (UHR). This result implies that increasingly retrograde $\lambda$ gradually weakens the trapping ability of the outer disc resonances. The top row of Fig. 7 displays an opposite trend in the DM haloes to that observed in the stellar discs. In the P00 halo, we see that the OLR is weak and the ILR is very weak. The ILR increases the efficiency of the DM orbit trapping, while the CR shows a decrease, with increasingly negative $\lambda$. The OLR exhibits a moderate increase in its trapping ability. In the R60 and R90 models, for example, the ILR and the OLR rival the trapping efficiency of the CR. The bottom row shows unchanged activity in the ILR and the CR, and decreased trapping efficiency by the OLR.

In a way, the stellar bar displays the trend of preferentially trapping the DM halo orbits rather than stellar orbits in the outer disc. The P00 model shows a strong DM peak at the CR, which is thought to be the most important resonance for absorbing the angular momentum by the halo. With increasingly negative $\lambda$, this resonance becomes less important compared to the OLR and the ILR. We find that this decrease in the CR trapping is associated with the increase in the retrograde particle fraction with $\lambda$ in the DM halo.

Though the importance of each resonance in the halo changes with $\lambda$, we note that the total fraction of trapped orbits remains similar, $\sim 20$ per cent, within the sampled region. This trapping process distinguishes retrograde haloes from the rigid unresponsive haloes and shows that this system is more complex than previously thought. Rigid haloes do not respond to the torques of stellar bars at all, while we find that the disc is adept at trapping orbits even in retrograde haloes. For prograde haloes, this was pointed out by Athanassoula (2002), using DM haloes that were initially non-rotating.

This varying efficiency of resonance trapping for the disc and halo orbits with the retrograde $\lambda$ sequence demonstrates that the details of angular momentum transfer in these models must vary as well. We take a closer look at the angular momentum transfer between discs and haloes in the following section.

### 3.4 Rates of angular momentum transfer

An alternative way of analysing the interactions between retrograde haloes and embedded discs, without referring to the resonances, is to visualize the flow of angular momentum in a galaxy using the method prescribed in Villa-Vargas et al. (2009), and implemented elsewhere (e.g. Long et al. 2014; Collier et al. 2018, 2019). The halo and disc are binned into cylindrical shells of $\Delta R = 1$ kpc. We create 2D maps of the rate of change of $J$ in each shell as a function of $R$ and time. These $J$ maps are then assigned a colour palette, where a gain of angular momentum is given in red and a loss of angular momentum in blue. The colour palette has been normalized separately for the disc and the halo.

In the top row of Fig. 8, $\dot{J}$ – the rate of the angular momentum transfer – has been calculated from and to the DM halo, for models along the retrograde $\lambda$ sequence. The P00 halo displays a nearly pure absorption of $J$ by the halo. The weak emission is related to the UHR. Three resonances are clearly seen in this frame – the ILR, CR, and OLR, which also appear in the $\dot{J}$ map of the P00 disc. In the P00 model, the highest $J$ transfer happens close to the time of buckling, where the stellar bar is the strongest, and where we see the deepest emission and absorption.

Increasing the retrograde $\lambda$, along the top row in Fig. 8, a stark contrast appears between the halo models. In the inner $R < 10$ kpc of models with larger (negative) $\lambda$, two deep absorption features are seen. The first one corresponds to the buckling of the stellar bar associated with high $J$ transfer to the halo. The second deep absorption feature appears in R60 and increases in strength to R90. Note that this feature appears after $t \sim 6$ Gyr, and so can be associated with the reversal of DM orbits discussed earlier. The version of this figure for the prograde spinning haloes is shown in Collier et al. (2019). We note that increasing $\lambda$ in the retrograde direction shows that the DM haloes only absorb the angular momentum. The emission features are absent in the top row of Fig. 8. However, these emission regions can be seen prominently in haloes rotating in the prograde direction, e.g. fig. 10 of Collier et al. (2019).

The second and third rows of Fig. 8 display the rate of transfer of angular momentum for the prograde and retrograde DM orbits separately. The P00 halo has 50 per cent of orbits rotating with the disc, and these prograde orbits gain and lose $J$, clearly following the resonances produced by the stellar bar. In the retrograde orbit plot for the P00 model, only absorption can be seen. For different models with increasing (negative) $\lambda$, the fraction of prograde orbits decreases, and only absorption is visible for the prograde orbits. The ILR appears more important to prograde orbits in these haloes, and the gain of angular momentum increases with retrograde $\lambda$.

The double peaks in $J$ are clearly visible in the prograde DM $J$ maps, and the second peak appears after $t \sim 6$ Gyr.

The third row of Fig. 8 displays only the retrograde orbits, where a gain of angular momentum is pronounced in all models. Notably, this gain in angular momentum increases with $\lambda$, which corresponds to the increase in prograde orbit fraction, as seen in Fig. 6. The second maximum in $J$ is weaker here and happens slightly early in time than for the prograde orbits.

The final row of Fig. 8 displays the rate of angular momentum transfer for all orbits found in the stellar disc. In the P00 disc, the importance of the resonances can be clearly observed, and the $J$ transfer reaches larger $R$, as the bar grows in size and slows down. As we shall see later, the size of the disc correlates with the $J$ transfer as well. The stellar bars differ in length by not more than 20 per cent (Fig. 1b), so they do not differ dramatically from each other. However, the $J$ transfer appears quite different when mapped using this method.

The OLR in the P00 disc extends to 25 kpc, as seen in the colour map of Fig. 8. For comparison, along the retrograde $\lambda$ sequence, the OLR is stunted and stops well before 20 kpc in R60 and R90 models. Stellar discs in prograde models are shown in fig. 6 of Collier et al. (2018) and analysed in Collier et al. (2018, 2019).

The second deep absorption feature in the DM haloes, which gets stronger with increasing $\lambda$, coincides with the emission feature within central $R < 5$ kpc in the disc, which gets stronger with $\lambda$. We shall return to this issue in the following section.

### 4 DISCUSSION

We have analysed the evolution of stellar discs embedded in counter-rotating DM haloes over time period of 10 Gyr. We focused on the dynamical and secular evolution of stellar bars in these systems, on the DM response to the stellar bar perturbation, and on the flow of angular momentum between the haloes and the embedded discs. The range of the counter-rotating DM halo spin used is $\lambda = 0 \sim 0.09$. Finally, we have compared the evolution in the prograde and retrograde haloes. After summarizing our results, we discuss their corollaries and additional questions they bring.
Our main results are as follows. First, we find that the maximum strength and size of stellar bars is only moderately affected by the retrograde halo spin during dynamical and secular phases of evolution. This is in stark contrast with the prograde sequence (Collier et al. 2018, 2019). The largest difference comes from larger $\lambda$ – prograde or retrograde. While the prograde models are characterized with dissolution of stellar bars in the secular stage of the evolution, no such trend is found for the retrograde models – all bars here show a healthy growth until the end of the simulations.

Secondly, the stellar bar amplitudes during the bar instability in the retrograde haloes form an extension to the sequence of bar evolution in prograde haloes by delaying the bar instability. However, the amplitude peaks of the retrograde models appear more crowded in time – the delay in prolonging the bar instability saturates.

Thirdly, we have performed the orbital spectral analysis on retrograde models in order to quantify the overall trapping efficiency of DM orbits by the stellar bar. We find that trapping is not affected along the counter-rotating $\lambda$ sequence, i.e. stellar bars trap $\sim 20\%$ of the DM halo particles in the sampled region of the inner haloes, despite the increasing fraction of retrograde orbits in the initial conditions. Again, this is contrary to the prograde $\lambda$ sequence of disc–halo models that exhibits a strong effect of the halo spin on stellar bar evolution.

Fourthly, although the overall trapping ability of the DM orbits is not affected along the retrograde $\lambda$ sequence, the trapping by individual resonances does vary. For example, the ILR and the OLR in the DM haloes become progressively more important with increasing $\lambda$, while the stellar disc shows a decrease in trapping ability of stellar orbits by the OLR. The CR and the ILR do not change. The UHR appears in the disc and becomes stronger with $\lambda$. We also have measured the angular momentum, $J$, absorbed or emitted by each of the resonances during the secular evolution regime and discuss it in this section, together with the corollaries of this process on the evolution of the disc size.

Fifthly, we analyse the importance of the prograde and retrograde orbits in the DM halo, and find that they both contribute to the angular momentum transfer in the system. Interestingly enough, we find that the retrograde DM orbits trapped by the stellar bar can…
reverse their angular momentum, $J$, and become prograde, being trapped by the stellar bar. They contribute to the fraction of prograde orbits that can resonate with the stellar bar and acts to increase the DM bar strength. We can observe this by tracking the evolution of a fraction of retrograde orbits (Fig. 9) and comparing the strength of the DM bar (Fig. 2). We elaborate on this interesting process in the next section.

4.1 Stellar bar growth in retrograde models and DM orbit reversals

We start the discussion by addressing the growth of stellar bars during the secular phase of evolution in the retrograde DM haloes. At the face value, such a growth is surprising. Along the negative $\lambda$ sequence, the fraction of the prograde DM orbits is decreasing, based on the initial conditions. Such a decrease should reduce the efficiency of angular momentum transfer between the disc and the halo because less DM orbits can resonate with the stellar bar. However, we do not detect this trend that should show up during the secular evolution of stellar bars. What is the reason?

Collier et al. (2019) found that the fraction of angular momentum moved by the resonances in each prograde model was on par with the fraction of prograde orbits found in the DM halo. P90 has 88 per cent of orbits rotating with the disc. About 88 per cent of $J$ lost by the disc in this model was transferred by the resonances.

This is not the case with the retrograde models. For example, the R90 model that started with only 12 per cent of DM orbits rotating with the disc exhibited 53 per cent of $J$ loss by the disc, which was moved through the resonances at the time of application of the spectral analysis. In all of our models, prograde and retrograde, the stellar bar moves at least $\sim 40$ per cent of $J$ by means of the resonant angular momentum transfer.

In Section 3.2, we found that some of the DM orbits, which initially are counter-rotating against the bar tumbling, exchange their angular momentum with the stellar bar, gain $J_\ell$, and reverse their direction of rotation. Figs 4 and 5 display two examples of such reversals and trapping by the stellar bar. The angular momenta of these orbits, which are negative originally, became positive with the trapping.

Moreover, Fig. 6 provides a quantitative measure of importance of this process in the counter-rotating model R90. Though R90 halo is comprised of a majority of retrograde orbits, the disc finds a way to resonate with this DM halo by trapping low-$J$ retrograde DM halo orbits and turning them into prograde orbits precessing in the opposite direction to the original one. They are converted into the low-$J$ prograde DM halo orbits that remain trapped in the bar. This accounts for the increase in the DM bar strength in R90 in Fig. 2 at later times.

We now wish to confirm that the DM reversals take place indeed when the orbit is trapped by the stellar bar. Fig. 9 shows the radial profiles of the prograde-to-retrograde DM orbit ratios, $\beta(R)$, in the inner haloes, i.e. $R < 10$ kpc and $|z| < 10$ kpc. The profiles, $\beta(R)$, have been calculated at $t = 0$ and $t = 10$ Gyr. We observe that initially $\beta$ is flat with $R$ by construction, and at the end of the runs it is peaked at smaller $R$.

For larger retrograde $\lambda$, $\beta$ increases, mostly in the central region, $R < 10$ kpc. Here, it even reaches unity, which corresponds to the P00 model with $\lambda = 0$. This is a substantial modification with respect to the initial conditions.

This increase in the number of prograde orbits can be noticed in the second row of Fig. 8 as well, where the prograde halo orbits gain more angular momentum for larger retrograde $\lambda$, creating a second local maximum in $J$ after $t \sim 6$ Gyr. The stellar bar is able to resonate with an increased number of prograde orbits and trap them. Indirectly, this is confirmed by the behaviour of the ILR in retrograde haloes, where this resonance becomes very prominent (Fig. 7). We return to this point later on with Fig. 12 – in the R90 halo, the ILR is the most important resonance, with the largest value of $\Delta J$, and is positioned deep inside the bar. In the disc, the CR and the OLR becoming less important and instead of moving $J$ to the outer disc, the ILR is preferentially moving $J$ to the DM halo by reversing orbits.

Together with Fig. 6, which shows the ratio of prograde to retrograde orbits, $\beta(t)$, in the same region of the inner halo, as a function of time, Fig. 8 reveals the rate of angular momentum absorption by the halo peaks exactly when the stellar bar amplitude, $A_2$, accelerates its growth (see Fig. 1a). It confirms that the majority of reversals occurred after buckling, and that the number of reversals increases with retrograde $\lambda$. Both figures confirm that reversals compensate for the initially smaller fraction of prograde orbits in retrograde haloes. In principle, the increased number of reversals can allow the stellar bar to maintain its constant trapping efficiency in retrograde haloes.
Evolution of disc radius containing 97 per cent of the stellar disc mass for each model. The P00 disc grows substantially, which is associated with expanding spiral arms in the outer disc. Contrary to this, the discs within the retrograde haloes grow progressively slower and have smaller sizes by the end of the run, despite that their bars continue to grow until the end of the simulations, as seen in Fig. 1(a).

Note that the increase in the fraction of the prograde DM orbits due to the reversals has little effect on the value of \( \lambda \), which is a global property of DM haloes. For example, in R90, we observe the largest number of orbit reversals and the largest gain of \( J \) over 10 Gyr for any model. However, its retrograde \( \lambda \) decreases only from \( \lambda = 0.0903 \) at \( t = 0 \) Gyr to \( \lambda = 0.0902 \) at \( t = 10 \) Gyr, which is negligible.

We can summarize that models with retrograde haloes do not behave similarly to models with rigid unresponsive DM haloes. We point out two main differences between the unresponsive and live haloes. First, along the retrograde \( \lambda \) sequence, the ratio of prograde to retrograde orbits is not negligible. It varies between 1.0 in the \( \lambda = 0 \) model and 0.12 in the \( \lambda = 0.09 \) model at \( t = 0 \). Secondly, the orbit reversals in DM haloes act to increase the ratio of prograde to retrograde orbits, and this effect is much more pronounced in the central region that contains the inner resonance, the ILR. The CR and the UHR can be affected as well, but to a lesser degree.

### 4.2 Angular momentum redistribution and the disc size

Next, we look into corollaries of the angular momentum redistribution in the models, and most importantly how this affects the disc size evolution. Some relation between the angular momentum transfer to the outer disc and the disc size is expected because the stellar orbits in the outer disc are nearly circular, and the only way they can absorb more angular momentum is by increasing their radii, i.e. by disc expansion in the radial direction. The only possible alternative to this evolution is if the angular momentum is directed to the DM halo rather than to the outer disc. If the spiral arms are excited in the outer disc, this is a clear indication that at least some of the angular momentum is absorbed by the region.

Fig. 10 displays the evolution of radii that encompass 97 per cent of the stellar disc masses. Such a measurement is comparable to an observational measurement of the 25th magnitude isophote of the galactic disc, \( R_{25} \), the Holmberg radius. We observe that a clear hierarchy in disc sizes has developed after the buckling, with the radius of P00 disc growing linearly with time, the R30 disc growing slower, while all other retrograde models nearly saturating after buckling, i.e. \( t \sim 6 \) Gyr. Note that the original size of the disc that contains 97 per cent of its mass is 16.5 kpc for all prograde and retrograde models. Hence, models with increasingly retrograde \( \lambda \) exhibit progressively smaller sizes at the end of the simulations.

We found the same trend along the prograde \( \lambda \) sequence – disc size decreases with increasing \( \lambda \). This can be seen, for example, in fig. 6 of Collier et al. (2018). In this case, the reason for this behaviour is directly related to the stellar bar evolution. Bars within faster spinning prograde haloes decay after the buckling. At the end of the runs, the higher \( \lambda \) models have substantially lower amplitude, so less angular momentum is transferred from the underlying discs to the host haloes.

However, this explanation does not apply for the retrograde models, because the stellar bars remain strong at the end of the run, although differ in strength by about 20 per cent among themselves. The spread in the stellar bar sizes by the end of the runs is also about 20 per cent, with the P00 bar being the longest and R90 the shortest, e.g. Fig. 1(b).

Hence, it appears that the most substantial growth of the stellar disc occurs in the P00 model, and decreases with \( \lambda \) along the prograde and retrograde sequences, resulting in measurably smaller discs. These results agree well with the orbital spectral analysis, e.g. Fig. 7. Fewer stellar orbits are trapped at the OLR with increasing retrograde \( \lambda \), because this resonance wanders close to the disc edge as the stellar bar brakes. Little mass resides in this region that can absorb the angular momentum. What is the reason for such an evolution of stellar discs in retrograde spinning haloes? We shall tackle this issue now.

First, we look for an indication that the spiral activity in the outer stellar discs indeed confirms our claim that in its presence the outer disc absorbs \( J \) from the bar region and drives the material out, thus increasing the disc size. Fig. 11 exhibits the surface density of the discs at the end of the runs in retrograde models. In models R60 and R90, we observe only weak outer spirals after the buckling. These decay completely by the end of the runs. On the other hand, the P00 model displays an on-going activity in the spiral arms that increases the disc size beyond 25 kpc. R30 behaves similarly, but the spiral arm generation is less vigorous, and the disc increases less than that in P00.

The spiral arm activity of the outer disc can be verified by the rate of angular momentum transfer and by spectral orbit analysis. In the bottom row of Fig. 8, the angular momentum transfer is not visible at larger radii in the disc, for larger retrograde \( \lambda \). The OLR in the P00 disc extends to nearly \( R \sim 25 \) kpc, while there is no action of angular momentum transfer in the R90 disc beyond \( R \sim 20 \) kpc. This confirms the overall picture discussed earlier that the outer disc does not absorb \( J \) in the high \( \lambda \) of retrograde haloes emitted from the bar region.

The orbital spectral analysis performed in Section 3.3 displays the decreasing efficiency of stellar orbit trapping by the outer resonance, the OLR, and associated declining trapping of DM orbits by the main halo resonance, the CR (for the \( \lambda = 0 \) model) (Fig. 7). However, note the increased trapping efficiency of DM orbits by the halo’s ILR and OLR. We have calculated the angular momentum, \( \Delta J \), absorbed and emitted by all the disc and halo main resonances during secular evolution in Fig. 12.

The main loss of \( J \) in the disc is by the ILR. This resonance is strongest in R90, even compared to the P00 model. Additional resonance that appears in the disc is the UHR, which also emits \( J \). The disc CR exhibits absorption of \( J \), but its \( \Delta J \) decreases along the
retrograde $\lambda$ sequence. Importantly, the disc OLR shows very little absorption of $J$ and looks completely insignificant in R90.

The main absorption of $J$ in the DM haloes switches from the CR in P00 to ILR in R90. In prograde models, we have observed the increased absorption by the halo ILR as well, but all prograde models have been dominated by absorption of $J$ by the CR (see fig. 11 in Collier et al. (2019)). So, the dominant role of the halo ILR is a clear signature of the faster spinning retrograde halo.

We plot the 1D overall $J$ loss with time by the stellar discs (Fig. 13), which is simpler than the 2D map in Fig. 8. It provides complementary information that helps to distinguish both similarities and differences in the total $J$ lost by the discs. To calculate the fraction of angular momentum lost by the resonant and nonresonant interactions in the disc–halo system in our models, we use the orbital spectral analysis shown in Fig. 12 and measure $\Delta J$ lost by each of the resonances.

We can now quantify the flow of angular momentum in our retrograde models and compare it with P00. We group the resonances into outer ones, which include the CR and the OLR, and the inner one, the ILR, comparing directly the two extreme models, P00 and R90. The amount of angular momentum absorbed by the disc CR and OLR in P00 is $\sim 91 M_\odot \text{km s}^{-1} \text{kpc}^{-1}$. The total amount $\Delta J$ lost by the disc during the same time is $\sim 250 M_\odot \text{km s}^{-1} \text{kpc}^{-1}$. Hence, about 36 per cent of this $\Delta J$ has been absorbed by the disc OLR and CR.

On the other hand, $\Delta J$ absorbed by the disc CR and OLR in R90 is $\sim 50 M_\odot \text{km s}^{-1} \text{kpc}^{-1}$. The total amount $\Delta J$ lost by the disc during the same time is $\sim 725 M_\odot \text{km s}^{-1} \text{kpc}^{-1}$. This means that only $\sim 7$ per cent of this $\Delta J$ has been absorbed by the disc OLR and CR. This is about five times less than that in P00, and appears to be the reason why the disc in R90 expands much less than that in P00.
To summarize, the disc size growth depends on the fraction of angular momentum that is transferred to the outer disc, in the retrograde models. We find that the stellar discs inside the faster retrograde haloes preferentially communicate with the inner halo rather than with the outer disc. The lack of the outer spiral arms is observable, and a clear imprint of the parent retrograde halo.

5 Conclusions

We have performed a high-resolution study of stellar bars and associated DM response in retrograde spinning DM haloes, with $\lambda \sim 0.09$. We find that evolution of stellar bars in these haloes differs substantially from that in the prograde haloes studied in Collier et al. (2018, 2019). Moreover, it differs from evolution of stellar bars in the frozen unresponsive haloes. The DM response to the stellar bar perturbation in retrograde haloes is modified from their prograde counterparts. We have analysed the orbital structure and the angular momentum transfer in these disc–halo systems, and focused on the role of the resonances in transferring the angular momentum.

While statistics of retrograde galactic DM haloes or their components is not available at present, numerical and theoretical modelling point to their possibility, and observations reveal individual cases of counter-rotating components in the DM and stars. We summarize our results here.

First, we find that the bar instability is slowed down with increasingly counter-rotating $\lambda$ – a trend first noticed by Saha & Naab (2013) for a single model with $\lambda = -0.05$. Together with the prograde models, the retrograde models form a monotonic sequence of prolonging the characteristic time-scale of bar instability.

Secondly, and probably, the most dramatic difference between the prograde and retrograde models lies in the secular evolution of the stellar bars. While increasing prograde $\lambda$ ultimately damps the stellar bars, which basically dissolve leaving a weak oval distortion, increasing the retrograde halo spin has only a minor effect on the secular evolution of the stellar bars. Their amplitudes and sizes differ by $\sim 20$ per cent at the end of the runs. We have quantified the rate of the angular momentum transfer from the stellar disc to the DM halo, and analysed the role of the prograde and retrograde orbits in this process.

Thirdly, the DM response to the stellar bar is much weaker in all retrograde models, substantially weaker than that in the prograde ones – the $m = 2$ mode Fourier amplitudes for this DM response are a factor of a few weaker during the bar instability, the dynamical phase of the evolution. Moreover, while in prograde models the DM response is aligned with stellar bar, in retrograde models it is nearly orthogonal to the bar. This emphasizes the increased importance of the ILR in the $J$ transfer for the latter models.

The DM response to the stellar bar perturbation during the secular phase includes trapping of DM orbits and angular momentum transfer by the main resonances. We find that both the trapping of DM orbits and $J$ transfer from the disc to the halo at this stage do not depend on the retrograde spin $\lambda$, in stark difference with the prograde models. Yet, the contribution of individual resonances does change along the retrograde sequence, as the orbital spectral analysis shows. The efficiency of the resonance trapping remains at $\sim 20$ per cent of the sampled orbits. This percentage includes the main resonances, the ILR, the CR, and the OLR. In the non-rotating P00 halo model, the most important resonance for the angular momentum absorption by the DM halo is the CR followed by the OLR (Athanassoula 2003; Martinez-Valpuesta et al. 2006; Dubinski et al. 2009). Our results show that the ILR replaces the CR in trapping the DM orbits and angular momentum absorption in retrograde haloes, which has not been seen earlier.

Finally, the strength of the DM response and DM orbit trapping by the resonances during the secular phase of evolution is regulated by two factors. One is trivially related to the initial conditions – the fraction of prograde DM orbits decreases with $\lambda$ by construction. Another one is much more interesting – ability of trapped DM orbits to reverse their angular momentum. These reversals have modified substantially the fraction of prograde DM orbits, and this change is heavily weighted towards the central regions of about few kpc. Not surprisingly, the reversals are dominated by the low-$J$ DM orbits. The fraction of prograde DM orbits in this regions has increased to about unity. Both the streaming along the orbits and its precession have been reversed. This process has never been discussed in the context of disc–halo interactions. Reversals of DM orbits have been responsible for increased trapping and angular momentum losses by the inner disc, thus resulting in increased strength of stellar bars during secular evolution phase.

Reversals of DM orbits have led to a number of corollaries, of which we point out the most important one – evolution of the disc size. We find that stellar discs with low retrograde $\lambda$ pump a substantial fraction of angular momentum into the outer disc via the OLR. This leads to expansion of the outer disc and on-going spiral activity there. However, with increasing $\lambda$, the fraction of prograde stellar orbits in the outer disc decreases, by construction. The angular momentum transfer to the outer disc decreases by a factor of $\sim 5$ from $\lambda = 0$ to $\lambda = -0.09$. Instead, this angular momentum is diverted to the inner halo, which absorbs it mostly via the ILR, and by the CR to a lesser degree.

Stellar discs in these haloes do not show spiral arm activity in the outer region and do not grow with time. When comparing the disc sizes, both prograde and retrograde models of larger $\lambda$ show smaller discs. In fact, the P00 model with $\lambda = 0$ has the largest disc and most prominent spiral arms at the end of the simulation. In prograde models, this is explained by the bar dissolution after buckling that prevents the movement of angular momentum to the outer disc. In the retrograde models, it follows from reduced activity of the disc OLR and CR – the disc does not expand, being much less efficient in absorbing the angular momentum by its outermost
part. The angular momentum is absorbed instead by the increased efficiency of the halo OLR.

In summary, as stellar bars are the prime internal factor that drives the galaxy evolution, they warrant a very careful numerical study. The steady interest in the stellar bar evolution over the last few decades is due to their ability to link the DM halo to the stellar disc through angular momentum transfer. Studying dynamics of the disc–halo systems consistently reveals new effects that are important for galaxy evolution. Through careful study, we begin to understand the observational corollaries of stellar bars’ action and its effect on the dynamics of the inner DM halo.

ACKNOWLEDGEMENTS

The authors acknowledge helpful discussions with Scott Tremaine and Alar Toomre, and are grateful to Jorge Villa-Vargas and Emilio Romano-Díaz for the help with numerical issues. This work was partially supported by the HST/STScI Theory grant AR-14584, and by JSPS KAKENHI grant #16H02163 (to IS). IS is grateful for support from International Joint Research Promotion Program at Osaka University. The STScI is operated by the AURA, Inc., under NASA contract NAS5-26555. Simulations have been performed on the University of Kentucky DLX Cluster.

REFERENCES

Algorry D. G., Navarro J. F., Abadi M. G., Sales L. V., Steinmetz M., Piontek F., 2014, MNRAS, 437, 3596

Athanassoula E., 2002, ApJ, 569, L83

Athanassoula E., 2003, MNRAS, 341, 1179

Berentzen I., Shlosman I., Martinez-Valpuesta I., Heller C., 2007, ApJ, 666, 189

Binney J., Spergel D., 1982, ApJ, 252, 308

Binney J., Tremaine S., 2008, Galactic Dynamics. Princeton Univ. Press, Princeton, NJ

Bournaud F., Combes F., Semelin B., 2005, MNRAS, 364, L18

Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R., 2001, ApJ, 555, 240

Christodoulou D. M., Shlosman I., Tohline J. E., 1995, ApJ, 443, 551

Collier A., Shlosman I., Heller C., 2018, MNRAS, 476, 1331

Collier A., Shlosman I., Heller C., 2019, MNRAS, 488, 5788

Corsini E. M., Méndez-Abreu J., Pastorello N., Dalla Bontà E., Morelli L., Beifiori A., Pizzella A., Bertola F., 2012, MNRAS, 423, L79

Davis T. A. et al., 2011, MNRAS, 417, 882

Debattista V. P., Sellwood J. A., 2000, ApJ, 543, 704

Dekel A., Shlosman I., 1983, in Athanassoula E., ed., Proc. IAU Symp. 100, Internal Kinematics and Dynamics of Galaxies. Reidel, Dordrecht, p. 187

Doroshkevich A. G., 1970, Astrofizika, 6, 581

Dubinski J., Berentzen I., Shlosman I., 2009, ApJ, 697, 293

Fall S. M., Efstathiou G., 1980, MNRAS, 193, 189

Hopkins P. E., 2015, MNRAS, 450, 53

Jeans J. H., 1919, Problems of Cosmogony and Stellar Dynamics. Cambridge Univ. Press, Cambridge

Kannappan S. J., Fabricant D. G., 2001, AJ, 121, 140

Kruijken K., Merrifield M. R., 1993, MNRAS, 264, 712

Lockman F. J., 2003, ApJ, 591, L33

Long S., Shlosman I., Heller C., 2014, ApJ, 783, L18

Lynden-Bell D., 1960, MNRAS, 120, 204

Lynden-Bell D., Kalnajs A. J., 1972, MNRAS, 157, 1

Martinez-Valpuesta I., Shlosman I., Heller C., 2006, ApJ, 637, 214

Navarro J. F., Frenk C. S., White S. D. M., 1996, ApJ, 462, 563 (NFW)

Pawlovski M. S., Kroupa P., de Boer K. S., 2011, A&A, 532, A118

Peebles P. J. A., 1969, ApJ, 155, 393

Pizzella A., Morelli L., Corsini E. M., Dalla Bontà E., Coccato L., Sanjana G., 2014, A&A, 570, A79

Porciani C., Dekel A., Hoffman Y., 2002, MNRAS, 332, 325

Prada F., Gutierrez C. M., Peletier R. F., McKeith C. D., 1996, ApJ, 463, L9

Rodionov S. A., Sotnikova N. Ya., 2006, Astron. Rep., 50, 983

Rodionov S. A., Athanassoula E., Sotnikova N. Ya., 2009, MNRAS, 392, 904

Romano-Díaz E., Hoffman Y., Heller C., Faltenbacher A., Jones D., Shlosman I., 2007, ApJ, 657, 56

Romano-Díaz E., Shlosman I., Heller C., Hoffman Y., 2009, ApJ, 702, 1250

Romano-Díaz E., Shlosman I., Heller C., Hoffman Y., 2010, ApJ, 716, 1095

Saha K., Naab T., 2013, MNRAS, 434, 1287

Sellwood J. A., 1980, A&A, 89, 2968

Shlosman I., 2013, in Falcon-Barroso J., Knapen J. H., eds, Secular Evolution of Galaxies. Cambridge Univ. Press, Cambridge, p. 555

Springel V., 2005, MNRAS, 364, 11055

Tremaine S., Weinberg M. D., 1984, MNRAS, 209, 729

Villa-Vargas J., Shlosman I., Heller C. H., 2009, ApJ, 707, 218

Villa-Vargas J., Shlosman I., Heller C. H., 2010, ApJ, 790, 1470

Weinberg M. D., 1985, MNRAS, 213, 451

White S. D. M., 1978, MNRAS, 183, 341

This paper has been typeset from a TeX/LATEX file prepared by the author.