Inflation and Monopoles in Supersymmetric
\( SU(4)_c \times SU(2)_L \times SU(2)_R \)

R. Jeannerot\(^1\), S. Khalil\(^2,3\), G. Lazarides\(^4\) and Q. Shafi\(^5\)

\(^1\)The Abdus Salam International Center for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy.

\(^2\)Departamento de Fisica Teórica, C.XI, Universidad Autónoma de Madrid, 28049 Cantoblanco, Madrid, Spain.

\(^3\)Ain Shams University, Faculty of Science, Cairo 11566, Egypt.

\(^4\)Physics Division, School of Technology, Aristotle University of Thessaloniki, Thessaloniki 540 06, Greece.

\(^5\)Bartol Research Institute, University of Delaware Newark, DE 19716, USA.

Abstract

We show how hybrid inflation can be successfully realized in a supersymmetric model with gauge group \( G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R \). By including a non-renormalizable superpotential term, we generate an inflationary valley along which \( G_{PS} \) is broken to the standard model gauge group. Thus, catastrophic production of the doubly charged magnetic monopoles, which are predicted by the model, cannot occur at the end of inflation. The results of the cosmic background explorer can be reproduced with natural values (of order \( 10^{-3} \)) of the relevant coupling constant, and symmetry breaking scale of \( G_{PS} \) of the order of \( 10^{16} \) GeV. The spectral index of density perturbations lies between unity and 0.9. Moreover, the \( \mu \)-term is generated via a Peccei-Quinn symmetry and proton is practically stable. Baryogenesis in the universe takes place via leptogenesis. The low deuterium abundance constraint on the baryon asymmetry, the gravitino limit on the reheat temperature and the requirement of almost maximal \( \nu_\mu - \nu_\tau \) mixing from SuperKamiokande can be simultaneously met with \( m_{\nu_\mu}, m_{\nu_\tau} \) and heaviest Dirac neutrino mass determined from the large angle MSW resolution of the solar neutrino problem, the SuperKamiokande results and \( SU(4)_c \) symmetry respectively.
1 Introduction

After the recent discovery of neutrino oscillations by the SuperKamiokande experiment [1], supersymmetric (SUSY) models with left-right symmetric gauge groups have attracted a great deal of attention. These models provide a natural framework for implementing the seesaw mechanism [2] which explains the existence of the small neutrino masses. The implications of these models have been considered in Ref.[3], in the case of the gauge group $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and in Ref.[4] for the SUSY Pati-Salam (PS) model based on the gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. It was shown that they lead to a constraint version of the minimal supersymmetric standard model (MSSM).

Recently, it was demonstrated [6] that the N=1 SUSY PS model can emerge as the effective four dimensional limit of brane models from type I string theory. This result provides further motivation for considering the phenomenological and cosmological implications of this model. Also, it is known [7, 8] that the gauge symmetry $G_{PS}$ can arise from the weakly coupled heterotic string as well.

Hybrid inflation [9] has been extensively studied [10, 11, 12] in the case of the SUSY model based on the gauge group $G_{LR}$. However, in trying to extend this scheme to $G_{PS}$, we encounter the following difficulty. The spontaneous symmetry breaking of $G_{PS}$ to the standard model gauge group $G_{SM}$ leads to the existence of topologically stable magnetic monopole solutions. This is due to the fact that the second homotopy group of the vacuum manifold $\pi_2(G_{PS}/G_{SM})$ is non-trivial and equal to the set of integers $\mathbb{Z}$. These monopoles carry two units of Dirac magnetic charge [13]. Inflation is terminated abruptly when the system reaches a critical point (instability) on the inflationary trajectory and is followed by a ‘waterfall’ regime during which the spontaneous breaking of $G_{PS}$ occurs. The appropriate Higgs fields develop their non-zero vacuum expectation values (vevs) starting from zero and they can end up at any point of the vacuum manifold with equal probability. As a consequence, magnetic monopoles are copiously produced [14] by the Kibble mechanism [15] leading to a cosmological disaster.

In this paper, we propose a specific SUSY model based on $G_{PS}$ which avoids this cosmological catastrophe. This is achieved by including a non-renormalizable term in the part of the superpotential involving the inflaton system and causing the breaking of
It is worth mentioning that an analogous non-renormalizable term was also used in Ref. [14] for the same purpose. In that case, however, the leading renormalizable term was eliminated by imposing a discrete symmetry. Here, we keep this leading term along with the non-renormalizable contribution. The picture that emerges turns out to be considerably different. In particular, there exists a non-trivial (classically) flat direction along which $G_{PS}$ is spontaneously broken with the appropriate Higgs fields acquiring constant values. This direction can be used as inflationary trajectory with the necessary inclination obtained from one-loop radiative corrections [16] in contrast to the model of Ref. [14], where a classical inclination was present. Another difference is that here the termination of inflation is abrupt (as in the original hybrid inflationary scenario) and not smooth as in Ref. [14]. Nevertheless, no magnetic monopoles are formed in this transition since $G_{PS}$ is already broken during inflation.

We show that, for a certain range of parameters, the system always passes from the above mentioned inflationary trajectory before falling into the SUSY vacuum. Thus, the magnetic monopole problem is solved for all initial conditions. It is interesting to note that the idea of breaking the gauge symmetry before (or during) inflation in order to avoid monopoles was also employed in Ref. [17]. However, the monopole problem was solved only for a certain (wide) class of initial values of the fields.

The constraints on the quadrupole anisotropy of the cosmic microwave background radiation from the cosmic background explorer (COBE) [18] measurements can be easily met with natural values (of order $10^{-3}$) of the relevant coupling constant and a grand unification theory (GUT) scale $M_{GUT}$ close to (or somewhat smaller than) the SUSY GUT scale. Note that the mass scale in the model of Ref. [19], which uses only renormalizable couplings in the inflationary superpotential, is considerably smaller. Our model possesses a number of other interesting features too. The $\mu$-problem of MSSM is solved [19] via a Peccei-Quinn (PQ) symmetry which also solves the strong CP problem. Although the baryon ($B$) and lepton ($L$) numbers are explicitly violated, the proton life time is considerably higher than the present experimental limits. Light neutrinos acquire masses by the seesaw mechanism and the baryon asymmetry of the universe can be generated through a primordial leptogenesis [20]. The gravitino constraint [21] on the reheat temperature, the low deuterium abundance limits [22] on the baryon asymmetry of the universe and the requirement of almost maximal $\nu_\mu - \nu_\tau$ mixing from SuperKamiokande [1] can be met.
for $\mu$- and $\tau$-neutrino masses restricted by SuperKamiokande and the large angle MSW solution of the solar neutrino puzzle respectively. The required values of the relevant coupling constants are more or less natural.

The plan of the paper is as follows. In Sec.2, we introduce our SUSY model which is based on the gauge group $G_{PS}$ and motivate the inclusion of a non-renormalizable coupling in the inflaton sector of the theory. The full superpotential and its global symmetries are then discussed together with the solution of the $\mu$-problem via the PQ symmetry of the model. In Sec.3, the hybrid inflationary scenario is studied in detail in this model. The structure of the potential is carefully analyzed. We calculate the one-loop radiative corrections along the inflationary trajectory by first deriving (see Appendix) the mass spectrum of the theory during inflation. The parameters of the model are then restricted by employing COBE results. In Sec.4, we discuss the reheating process following inflation, neutrino masses and mixing and baryogenesis via leptogenesis. We show that all the relevant constraints can be satisfied with natural values of the coupling constants. We summarize our conclusions in Sec.5.

2 A SUSY $SU(4)_c \times SU(2)_L \times SU(2)_R$ model

In the SUSY PS model, the left-handed quark and lepton superfields are accommodated in the following representations:

$$F_i = (4, 2, 1) \equiv \begin{pmatrix} u_i & u_i & u_i & \nu_i \\ d_i & d_i & d_i & e_i \end{pmatrix},$$

$$F_i^c = (\bar{4}, 1, 2) \equiv \begin{pmatrix} u^c_i & u^c_i & u^c_i & \nu^c_i \\ d^c_i & d^c_i & d^c_i & e^c_i \end{pmatrix},$$

(1)

where the subscript $i = 1, 2, 3$ denotes the family index \[\Box\]. The $G_{PS}$ gauge symmetry can be spontaneously broken to $G_{SM}$ by a pair of Higgs superfields

$$H^c = (\bar{4}, 1, 2) \equiv \begin{pmatrix} u^c_H & u^c_H & u^c_H & \nu^c_H \\ d^c_H & d^c_H & d^c_H & e^c_H \end{pmatrix},$$

$$\bar{H}^c = (4, 1, 2) \equiv \begin{pmatrix} \bar{u}^c_H & \bar{u}^c_H & \bar{u}^c_H & \bar{\nu}^c_H \\ \bar{d}^c_H & \bar{d}^c_H & \bar{d}^c_H & \bar{e}^c_H \end{pmatrix},$$

(2)

acquiring non-vanishing vevs in the right-handed neutrino direction, $|\langle \nu^c_H \rangle|, |\langle \bar{\nu}^c_H \rangle| \neq 0$. 


The two low energy Higgs doublets of the MSSM are contained in the following representation:

\[
h = (1, 2, 2) \equiv \begin{pmatrix} h^+_2 & h^0_1 \\ h^0_2 & h^-_1 \end{pmatrix}.
\]  

(3)

After \(G_{PS}\) breaking, the bidoublet Higgs field \(h\) splits into two Higgs doublets \(h_1, h_2\), whose neutral components subsequently develop weak vevs \(\langle h^0_1 \rangle = v_1\) and \(\langle h^0_2 \rangle = v_2\) with \(\tan \beta = v_2/v_1\).

The breaking of \(G_{PS}\) can be achieved by introducing a gauge singlet superfield \(S\), which has a trilinear (renormalizable) coupling to \(H^c, \bar{H}^c\). The resulting scalar potential automatically possesses an in-built (classically) flat direction along which inflation can take place [23] with the system driven by an inclination from one-loop radiative corrections [16]. The \(G_{PS}\) gauge symmetry is restored along this trajectory and breaks spontaneously only at the end of inflation when the system falls towards the SUSY vacua. This transition leads to a cosmologically unacceptable copious production of doubly charged magnetic monopoles [14]. One way to resolve this problem is to use, for inflation, another flat direction in which \(G_{PS}\) is already broken. Such a trajectory naturally appears if we include the next order non-renormalizable superpotential coupling of \(S\) to \(H^c, \bar{H}^c\) too. We find that, together with the usual flat direction with unbroken \(G_{PS}\), an extra flat trajectory along which \(G_{PS}\) is spontaneously broken to \(G_{SM}\) emerges. The termination of inflation can then take place with \(G_{PS}\) already broken and no monopoles being produced.

An important issue is the generation of the \(\mu\)-term of MSSM. This could be easily achieved [24] by coupling \(S\) to the electroweak Higgs superfields and using the fact that \(S\), after gravity-mediated SUSY breaking, develops a vev. However, this is not totally satisfactory since the inflaton decays into electroweak Higgs superfields via an unsuppressed (renormalizable) coupling. As a consequence, the gravitino constraint [21] on the reheat temperature implies [25] unnaturally small values for the relevant coupling constants (of order \(10^{-6}\) or so). We thus prefer to follow here Ref. [19] and impose a PQ symmetry on the superpotential by introducing a pair of gauge singlet superfields \(N, \bar{N}\). The PQ breaking occurs at an intermediate scale by the vevs of \(N, \bar{N}\) and the \(\mu\)-term is generated via a non-renormalizable coupling of \(N\) and \(h\). The inflaton can be made to decay into right-handed neutrinos by introducing into the scheme quartic (non-renormalizable) superpotential couplings of \(\bar{H}^c\) to \(F^c_i\) and the gravitino constraint can be satisfied with more
natural values of the parameters. It should be noted that the presence of these quartic terms is anyway necessary for generating intermediate scale masses for the right-handed neutrinos and, thus, masses for the light neutrinos via the seesaw mechanism. Finally, in order to give superheavy masses to $d_H^c$ and $\bar{d}_H^c$, we introduce an $SU(4)_c$ 6-plet superfield $G = (6, 1, 1)$ which, under $G_{SM}$, splits into $g^c = (\bar{3}, 1, 1/3)$ and $\bar{g}^c = (3, 1, -1/3)$.

The superpotential of the model, which incorporates all the above couplings, is

$$W = \kappa S(H^c\bar{H}^c - M^2) - \beta S \frac{(H^c\bar{H}^c)^2}{M_S^2} + \lambda_1 \frac{N^2h^2}{M_S} + \lambda_2 \frac{N^2\bar{N}^2}{M_S} + \lambda_{ij} F_i^c F_j \bar{h} + \gamma_i \frac{\bar{H}^c \bar{H}^c}{M_S} F_i^c F_j^c + aG H^c \bar{H}^c + bG \bar{H}^c \bar{H}^c,$$ (4)

where $M_S \sim 5 \times 10^{17}$ GeV is a superheavy string scale. Also, $M$, $\kappa$, $\lambda_{1,2}$, $\gamma_i$, $a$ and $b$ can be made positive by field redefinitions, while $\beta$ is chosen positive for simplicity (it could be genuinely complex). Here, we are in the basis where $\gamma$’s are diagonal.

Note that the existence of the third (non-renormalizable) coupling in the right hand side of Eq.(4) is an automatic consequence of the first two couplings which constitute the standard superpotential for hybrid inflation. Indeed, the operator $\bar{H}^c H^c$ is neutral under all the R and non-R symmetries of this standard superpotential and, thus, the above coupling, which is crucial for our inflationary scheme, cannot be forbidden. Our only assumption is then that the dimensionless coefficient $\beta$ of this non-renormalizable coupling is of order unity so that it can be comparable to the trilinear coupling in the standard superpotential whose coefficient $\kappa$ is typically relatively small ($\sim 10^{-3}$). This non-renormalizable coupling can then play an important role in the inflationary scenario. The non-renormalizable couplings $S(H^c\bar{H}^c)^n/M_S^{2(n-1)}$ with $n \geq 3$ are also allowed. They are, however, subdominant to the leading non-renormalizable coupling (with $n = 2$) in the relevant region of the field space even if their coefficients are of order one. The R symmetry (see below) of the standard superpotential, extended to higher orders, is needed for avoiding terms non-linear in $S$ which can destroy inflation. The couplings, then, permitted between $S$, $H^c$, $\bar{H}^c$ are the ones already mentioned modulo arbitrary multiplications by $(H^c)^4$, $(\bar{H}^c)^4$. The new couplings do not affect the potential for inflation.

In addition to $G_{PS}$, the superpotential in Eq.(4) possesses two global anomalous symmetries, a R symmetry $U(1)_R$ and a PQ symmetry $U(1)_{PQ}$. The R and PQ charges of
the superfields are assigned as follows:

\[
R : \quad H^c(0), \bar{H}^c(0), S(1), G(1), F(1/2), F^c(1/2), N(1/2), \bar{N}(0), h(0);
\]

\[
PQ : \quad H^c(0), \bar{H}^c(0), S(0), G(0), F(-1), F^c(0), N(-1), \bar{N}(1), h(1). \tag{5}
\]

To avoid undesirable mixing of \( F \) and \( h \) or \( F^c \) and \( H^c \), we also impose a discrete \( Z_2^m \) symmetry (known as ‘matter parity’), under which \( F \) and \( F^c \) change sign.

Additional superpotential terms allowed by the symmetries of the model are

\[
FFH^cH^c\bar{N}^2, \quad FFH^cH^c hh, \quad FF\bar{H}^c\bar{H}^c\bar{N}^2, \quad FF\bar{H}^c\bar{H}^c hh, \quad F^cF^cH^cH^c,
\]

modulo arbitrary multiplications by non-negative powers of the combinations \( H^c\bar{H}^c \), \( (H^c)^4 \), \( (\bar{H}^c)^4 \) (this applies to the terms in Eq. (3) too). Note that the \( SU(4)_c \) indices in all couplings except the last three in Eq. (3) and the combinations \( (H^c)^4 \), \( (\bar{H}^c)^4 \) are contracted between 4’s and 4’s, while in these terms and combinations the four \( SU(4)_c \) indices of the 4’s or \( \bar{4} \)’s are contracted with an \( \epsilon_{ijkl} \). The soft SUSY breaking and instanton effects explicitly break \( U(1)_R \) to \( Z_2 \), under which \( N \rightarrow -N \), and \( U(1)_{PQ} \) to \( Z_6 \). These two discrete symmetries are spontaneously broken by the vevs of \( N, \bar{N} \) and would create a domain wall problem if the PQ transition took place after inflation. When \( H^c, \bar{H}^c, N \) and \( \bar{N} \) acquire non-vanishing vevs, the symmetry which is left unbroken is \( G_{SM} \times Z_2^m \).

We can assign baryon number \( 1/3(-1/3) \) to all color triplets (antitriplets). Recall that there are (anti)triplets not only in \( F, F^c \) but also in \( H^c, \bar{H}^c, G \). Lepton number is then defined via \( B - L \). \( B \) (and \( L \)) violation comes from the last three terms in Eq. (3) (and the combinations \( (H^c)^4 \), \( (\bar{H}^c)^4 \) which give couplings like \( u^c d^c \bar{d}^c_H \nu_H^c \) (or \( u^c \bar{d}^c u_H^c e_H^c \)), \( u d \bar{d}_H \nu_H^c \) (or \( u d \bar{e}_H^c e_H^c \)) with appropriate coefficients. Also, the terms \( GH^cH^c \) and \( G\bar{H}^c\bar{H}^c \) give rise to the \( B \) (and \( L \)) violating couplings \( g^c u_H^c \bar{d}_H \bar{e}_H^c \), \( g^c \bar{d}_H \bar{e}_H^c \bar{d}_H \). All other combinations are \( B \) (and \( L \)) conserving since 4’s are contracted with \( \bar{4} \)’s.

The dominant contribution to proton decay comes from effective dimension five operators generated by one-loop diagrams with two of the \( u_H^c, \bar{d}_H^c \) or one of the \( u_H^c, \bar{d}_H^c \) and one of the \( \nu_H^c, e_H^c \) circulating in the loop. The amplitudes corresponding to these operators are estimated to be at most of order \( m_{3/2}M_{GUT}/M_S^3 \lesssim 10^{-34} \text{ GeV}^{-1} \) (\( m_{3/2} \) is the gravitino mass). This makes the proton practically stable. Furthermore, the dominant contribution to the Majorana mass term of light neutrinos comes from \( FFH^cH^c hh \) and is utterly small. So the seesaw mechanism is the only source of light neutrino masses.
The \( \mu \)-term is generated, as mentioned, by a non-renormalizable superpotential coupling which contains the electroweak Higgs and the \( N \) superfield after the breaking of \( U(1)_{PQ} \) by \( \langle N \rangle, \langle \bar{N} \rangle \). The relevant part of the scalar potential for the PQ breaking is given by [13]

\[
V_{PQ} = 2|N|^2 m_{3/2}^2 \left( 4\lambda_2^2 \frac{|N|^4}{m_{3/2}^2 M_S^2} - |A|\lambda_2 \frac{|N|^2}{m_{3/2}^2 M_S} + 1 \right),
\]

(7)

where \( A \) is the dimensionless coefficient of the soft SUSY breaking term corresponding to the \( N^2\bar{N}^2 \) term in Eq.(4). Here, the phases \( \epsilon, \theta \) and \( \bar{\theta} \) of \( A, N \) and \( \bar{N} \) are taken to satisfy the relation \( \epsilon + 2\theta + 2\bar{\theta} = \pi \) and \( |N|, |\bar{N}| \) are assumed equal which minimizes the potential. For \( |A| > 4 \), the absolute minimum of this potential is given by [19]

\[
|\langle N\rangle| = |\langle \bar{N}\rangle| = (m_{3/2}^2 M_S)^{1/2} \left( \frac{|A| + \sqrt{|A|^2 - 12}}{12\lambda_2} \right)^{1/2}.
\]

(8)

Hence the PQ symmetry breaking scale is of order \( \sqrt{m_{3/2}^2 M_S} \simeq 10^{10} - 10^{11} \) GeV and the \( \mu \)-term of the MSSM is \( \sim m_{3/2}^2 \) as desired.

Note that the zero temperature PQ potential (in Eq.(7)), shown in Fig.1, possesses two local minima, the trivial one at \( |N| = 0 \) and the PQ minimum which, for \( |A| > 4 \), is the absolute minimum. These minima are separated by a sizable potential barrier which prevents a successful transition from the trivial to the PQ vacuum. Taking the one-loop temperature corrections [20] to the potential into account, one can show that the PQ vacuum remains the absolute minimum at least for temperatures below the reheat temperature \( T_r \sim 10^9 \) GeV. The trivial vacuum is still protected by a potential barrier. We, thus, conclude that if, after inflation, the system emerges in the trivial vacuum the completion of the PQ transition will be practically impossible. We are obliged to assume that the PQ symmetry is already broken before or during inflation. The PQ vacuum then remains stable after inflation and reheating. There is yet another reason which disfavors a PQ transition after inflation. The vevs of \( N, \bar{N} \) break spontaneously the \( Z_2 \) symmetry \( (N \rightarrow -N) \) and the \( Z_6 \) subgroup of \( U(1)_{PQ} \) which is left unbroken by instantons. This would lead to disastrous domain walls in the universe.

In concluding this section, we emphasize that the extra couplings which we incorporated into our basic inflationary superpotential (see Eq.(3)) are simple, quite general, and well-motivated. In particular, the couplings \( N^2\bar{N}^2, N^2h^2 \) were included in order to
solve the $\mu$-problem. However, they also provide a solution to the strong CP problem. Moreover, the presence of $\bar{H}^c H^c F_i^c F_i^c$ is necessary for the generation of the right-handed neutrino masses. It is an extra benefit, though, that, via these same couplings, the inflaton decays to right-handed neutrinos, thereby leading to a successful and ‘natural’ (no tiny couplings) reheating of the universe with the observed baryon asymmetry generated via leptogenesis (see below). Finally, $G H^c H^c$, $G \bar{H}^c \bar{H}^c$ were added for merely giving superheavy masses to $d^c_H$, $\bar{d}^c_H$. It is worth mentioning that the emerging theory is ‘natural’ in the sense that all couplings which are consistent with a simple and general set of global symmetries ($U(1)_R$, $U(1)_{PQ}$ and $Z_{2MP}$) can be allowed without jeopardizing our inflationary scenario or leading into trouble with other requirements such as proton stability.

3 The inflationary scenario

The part of the superpotential in Eq.(4) which is relevant for inflation is given by

$$\delta W = \kappa S (\bar{H}^c H^c - M^2) - \beta S (\bar{H}^c H^c)^2 \over M_S^2.$$

(9)

where $M$ is a superheavy mass scale close to the GUT scale. Note that the rest of the superpotential in Eq.(4) does not affect the inflationary dynamics. The scalar potential obtained from $\delta W$ is given by

$$V = \left[ \kappa (\bar{H}^c H^c - M^2) - \beta (\bar{H}^c H^c)^2 \right]^2 + \kappa^2 |S|^2 (|H^c|^2 + |\bar{H}^c|^2) \left[ 1 - \frac{2\beta}{\kappa M_S^2} \bar{H}^c H^c \right]^2 + \text{D-terms},$$

(10)

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. Vanishing of the D-terms is achieved with $|\bar{H}^c| = |H^c|$ ($\bar{H}^c$ ($H^c$) lies in the $\bar{\nu}_R^c$ ($\nu_R^c$) direction). Restricting ourselves to this direction and performing an appropriate R transformation, we can bring the complex field $S$ to the real axis, $S = \sigma/\sqrt{2}$, where $\sigma$ is a normalized real scalar field. An $|S|$-independent flat trajectory suitable for hybrid inflation driven by radiative corrections is obtained in the direction $\text{arg}(H^c) + \text{arg}(\bar{H}^c) = 0$ along which the potential takes the form

$$V = \left[ \kappa (|H^c|^2 - M^2) - \beta |H^c|^4 \right]^2 + \kappa^2 \sigma^2 |H^c|^2 \left[ 1 - \frac{2\beta}{\kappa M_S^2} |H^c|^2 \right]^2 .$$

(11)
We can now rewrite this potential in terms of the dimensionless variables

\[ w = \frac{|S|}{M}, \quad y = \frac{|H^c|}{M}. \]

We get

\[ \tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2y^2(1 - 2\xi y^2)^2, \quad (12) \]

where \( \xi = \frac{\beta M^2/\kappa M^2}{S} \). This potential is a simple extension of the standard potential for SUSY hybrid inflation (obtained from Eq.(12) by putting \( \xi = 0 \)) and appears generically in a wide class of models which incorporate the leading non-renormalizable correction to the standard hybrid inflationary superpotential. (Recall that this correction is naturally present as we explained.) It is, thus, interesting to study in some detail the structure of this potential and the new inflationary possibilities which typically emerge.

For constant \( w^2 \), the potential in Eq.(12) has the following extrema

\[ y_1 = 0, \]

\[ y_2 = \sqrt{\frac{1}{2\xi}}, \]

\[ y_{3\pm} = \frac{1}{2\xi} \left[ (1 - 6\xi w^2) \pm \sqrt{(1 - 6\xi w^2)^2 - 4\xi(1 - w^2)} \right]. \]

Note that the first two extrema are \( \sigma \)-independent. As it turns out, \( y_1 \) is a local minimum (maximum) if \( w > (<) 1 \), while \( y_2 \) is a local minimum (maximum) if \( w^2 > (<) \rho_0 = 1/8\xi - 1/2 \). Inflation will take place when the system is trapped along the \( y_2 \) minimum.

We restrict ourselves to \( \xi < 1/4 \) since, in this case, the inflationary trajectory (at \( y_2 \)) is destabilized in the direction of the real part of \( H^c \overline{H^c} \) before \( w \) reaches zero. Inflation can then be terminated with the system falling towards the SUSY vacua (see below) following the direction \( \arg(H^c) + \arg(\overline{H^c}) = 0 \), where the potential is given by Eq.(12).

The potential at \( y_2 \) is \( \tilde{V}_2 = (1/4\xi - 1)^2 \), while at \( y_1 \) is \( \tilde{V}_1 = 1 \). So that, for \( \xi > (<) 1/8 \), the extremum at \( y_1 \) lies higher (lower) than the one at \( y_2 \).

For \( 1/4 > \xi > 1/7.2 \), the discriminant \( \Delta \) appearing under the square root in Eq.(15) is negative if \( w^2 \) lies between the positive numbers \( \rho_\pm = (2 \pm \sqrt{36\xi - 5})/18\xi \) and non-negative otherwise. So, for \( \rho_- < w^2 < \rho_+ \), the extrema at \( y_{3\pm} \) do not exist. Note, however, that \( \Delta \geq 0 \) does not necessarily guarantee the presence of these extrema. This requires that, in addition, the right hand side of Eq.(15) is non-negative. An important ingredient on which this requirement depends is the sign of the expression \( 1 - 6\xi w^2 \) which
is positive (negative) for \( w^2 < (>) \rho_1 = 1/6 \xi \). One can show that, for \( 1/4 > \xi > 1/7.2 \), \( 0 < \rho_0 < \rho_- < \rho_+ \leq 1 \) (equality holds for \( \xi = 1/6 \)) and \( \rho_0 < \rho_1 \). Also, \( \rho_+ > (>) \rho_1 \) for \( \xi > (>) 1/6 \).

For \( \xi \leq 1/7.2 \), we always have \( \Delta \geq 0 \). In addition, note that \( \rho_0 < (>)1 \) for \( \xi > (>) 1/12 \), \( \rho_1 < (>)1 \) for \( \xi > (>) 1/6 \) and \( 0 < \rho_0 < \rho_1 \) always. It is interesting to point out that, at \( w^2 = \rho_0 \), above which the extremum at \( y_2 \) turns into a local minimum, \( y_{3+} \) coincides with \( y_2 \). The minima at \( y_{3\pm} \), for \( w^2 = 0 \), become supersymmetric, i.e., \( \bar{V}(w^2 = 0, y = y_{3\pm}) = 0 \). For reasons to become obvious later, we consider the \( y_{3-} \) minimum at \( w^2 = 0 \) as the relevant SUSY vacuum of the theory.

Taking into account these facts, we can distinguish five cases with qualitatively different structure of the potential:

(i) For \( 1/4 > \xi > 1/6 \), we have \( 0 < \rho_0 < \rho_- < \rho_1 < \rho_+ < 1 \). One can then show that, for fixed \( w^2 > 1 \), there exist two local minima at \( y_1 \) and \( y_2 \) (the interesting inflationary trajectory) and a local maximum at \( y_{3+} \) between them (see Fig.2). For \( w^2 \) between \( 1 \) and \( \rho_- \), the trivial extremum at \( y_1 \) becomes a local maximum and the extremum at \( y_{3+} \) disappears (see Fig.3). In this range of \( w^2 \), the system can freely fall into the desirable (inflationary) minimum at \( y_2 \) even if it was initially along the trivial trajectory at \( y_1 \) (remember that the extremum at \( y_2 \) lies lower than the one at \( y_1 \) in this case). As we further decrease \( w^2 \) to become smaller than \( \rho_- \), a pair of two new extrema, a local minimum at \( y_{3-} \) and a local maximum at \( y_{3+} \), are created between \( y_1 \) and \( y_2 \). As \( w^2 \) crosses \( \rho_0 \), the local maximum at \( y_{3+} \) crosses \( y_2 \) becoming a local minimum (see Fig.4). At the same time, the local minimum at \( y_2 \) turns into a local maximum and inflation is terminated with the system falling into the local minimum at \( y_{3-} \) which at \( w^2 = 0 \) develops into a SUSY vacuum (see below).

(ii) For \( 1/6 > \xi > 1/7.2 \), we have \( 0 < \rho_0 < \rho_- < \rho_+ < 1 < \rho_1 \). The situation for \( w^2 > 1 \) and \( w^2 < \rho_- \) is similar to the previous case. For \( w^2 \) between \( 1 \) and \( \rho_+ \), the \( y_1 \) extremum becomes a local maximum and a local minimum at \( y_{3-} \) appears between \( y_1 \) and \( y_{3+} \). As \( w^2 \) decreases below \( \rho_+ \), the extrema at \( y_{3\pm} \) disappear and there exists no obstacle for the system to fall to \( y_2 \) even if it was initially at \( y_1 \). The extrema at \( y_{3\pm} \) reappear as \( w^2 \) becomes smaller than \( \rho_- \).

(iii) For \( 1/7.2 > \xi > 1/8 \), \( 0 < \rho_0 < 1 < \rho_1 \). The behavior of the potential for \( w^2 > 1 \) and \( w^2 < \rho_0 \) is similar to the previous cases. For \( 1 > w^2 > \rho_0 \), however, the extremum at
$y_1$ becomes a local maximum and a local minimum at $y_{3-}$ appears between $y_1$ and $y_{3+}$. Notice that, in this case, although the extremum at $y_2$ lies lower than the one at $y_1$, there is no range of $w^2$ where the system can fall into $y_2$ if it was initially at $y_1$. Instead, it ends up directly in $y_{3-}$ from $y_1$ and monopoles can be copiously produced. Of course, if the system happens to be at $y_2$ from the beginning, there is no production of monopoles.

(iv) For $1/8 > \xi > 1/12$, the situation is exactly as in case (iii) with the only difference that the extremum at $y_2$ now lies higher than the one at $y_1$.

(v) For $1/12 > \xi$, we have $0 < 1 < \rho_0 < \rho_1$. It turns out that, for $w^2 > \rho_0$, the local minima at $y_1$ and $y_2$ (which lies higher) are again separated by a local maximum at $y_{3+}$. As $w^2$ crosses $\rho_0$, the $y_{3+}$ local maximum turns into minimum and crosses $y_2$ which becomes a local maximum. There is then no obstacle to keep the system from falling into $y_1$ even if it was at $y_2$. Subsequently, when $w^2$ becomes smaller than 1, $y_1$ turns into a local maximum and the system falls into $y_{3-}$ with a copious production of magnetic monopoles.

We will restrict ourselves here to the first two cases above ($1/4 > \xi > 1/7.2$). We saw that, in these cases, even if the system starts along the trivial valley at $y_1$, it always falls into the (classically) flat direction at $y_2$. The relevant part of inflation can then take place along this trajectory with the inflaton being driven by radiative corrections [16]. So, $G_{PS}$ is already broken during inflation and there is no production of magnetic monopoles at the end of inflation where the system falls into the $y_{3-}$ minimum. Case (iii) could also solve the monopole problem provided the system starts at $y_2$. Case (iv), although quite similar to case (iii), is more tricky requiring further study. The reason is that, since $y_2$ lies higher than $y_1$, the system oscillates over the local maximum at $y_1$ after falling from $y_2$. Finally, case (v) is always unacceptable since the system, for all initial conditions, falls to $y_{3-}$ from $y_1$ and the copious production of monopoles is unavoidable.

As we already mentioned, after inflation ends, the system falls into the minimum at $y_{3-}$ which, at $w^2 = 0$, develops into the final SUSY vacuum of the theory. However, the system could, in principle, fall into the minimum at $y_{3+}$ which appears only after the instability of the inflationary trajectory at $w^2 = \rho_0$ is reached. (The minimum at $y_{3+}$ also develops into a SUSY vacuum at $w^2 = 0$.) We will argue that this does not happen. For the values of the parameters used here, the potential barrier separating the inflationary path at $y_2$ and the minimum at $y_{3-}$ is considerably reduced in the last e-folding or so. (The peak of this
barrier coincides with the maximum at \( y_{3+} \).) As a consequence, the rate per unit volume and time of forming bubbles of the \( y_{3-} \) minimum ceases to be exponentially suppressed. An order of magnitude estimate then shows that the decay of the false vacuum at \( y_2 \) to the minimum at \( y_{3-} \) is completed within a fraction of one e-folding. This happens before the appearance of the minimum at \( y_{3+} \), i.e., before the system reaches the critical point at \( w^2 = \rho_0 \) (but very close to this point). Moreover, in the last stages of inflation, the above barrier is small enough to be overcome by the inflationary density perturbations. This can also accelerate the completion of this transition.

To avoid confusion we should mention here that \( \xi \) is not an extra free parameter. It depends on the coupling \( \kappa \) and the superheavy mass scale \( M \) (we put \( \beta = 1 \)). The values of \( \kappa \) and \( M \) will be related by calculating the quadrupole anisotropy of the cosmic microwave background radiation \( (\delta T/T)_Q \) as a function of the number of e-foldings of our present horizon \( N_Q \) and compare it with the measurements of COBE [18]. The parameter \( \xi \) then becomes a function of the basic coupling constant of the scheme \( \kappa \). So, searching for solutions with \( \xi \) in the desirable range and also satisfying all the other requirements which we will discuss below is a highly non-trivial task.

As already mentioned, the interesting part of inflation takes place when the system is trapped along the trajectory at \( y_2 \). Inflation is driven by the constant classical energy density on this trajectory which also breaks SUSY. This breaking gives rise to non-trivial radiative corrections [16, 27] which lift the (classical) flatness of this trajectory producing a necessary inclination for driving the inflaton towards the SUSY vacua. As will be seen later, the slow-roll conditions [28] are satisfied and inflation continues essentially till \( w^2 \) reaches \( \rho_0 \), where the inflationary trajectory is destabilized. To calculate the one-loop radiative corrections at \( y_2 \) we need to construct the mass spectrum of the theory on this path where both \( G_{PS} \) and SUSY are broken. Details of the calculation can be found in the Appendix. We summarize the results in Table 1.

Using this spectrum, we can now calculate the one-loop radiative correction to the potential along the inflationary trajectory from the Coleman-Weinberg formula [29]

\[
\Delta V = \frac{1}{64\pi^2} \sum_i (-)^{F_i} M_i^4 \ln \frac{M_i^2}{\Lambda^2},
\]

where the sum extends over all helicity states \( i \), \( F_i \) and \( M_i^2 \) are the fermion number and squared mass of the \( i \)th state and \( \Lambda \) is a renormalization mass scale. We find that the
| Fields              | Squared Masses                                      |
|--------------------|-----------------------------------------------------|
| 2 real scalars     | $4\kappa^2|S|^2 + 2\kappa^2M^2\left(\frac{1}{4\xi} - 1\right)$ |
| 1 Majorana fermion  | $4\kappa^2|S|^2$                                    |
| 1 real scalar      | $5g^2v^2/2$                                         |
| 1 gauge boson       | $5g^2v^2/2$                                         |
| 1 Dirac fermion     | $5g^2v^2/2$                                         |
| 8 real scalars      | $g^2v^2$                                            |
| 8 gauge bosons      | $g^2v^2$                                            |
| 8 Dirac fermions    | $g^2v^2$                                            |
| 6 complex scalars   | $4a^2v^2$                                           |
| 3 Dirac fermions    | $4a^2v^2$                                           |
| 6 complex scalars   | $4b^2v^2$                                           |
| 3 Dirac fermions    | $4b^2v^2$                                           |

Table 1: The mass spectrum of the model as the system moves along the inflationary trajectory at $y_2$. The parameter $v = (\kappa M_S^2/2\beta)^{1/2}$ is the vev of $\nu^c_H, \bar{\nu}^c_H$ on this trajectory and $g$ is the $G_{PS}$ gauge coupling constant.

The inflationary effective potential is given by

$$V_{\text{eff}}^{\text{infl}} = \kappa^2m^4\left(1 + \frac{\kappa^2}{16\pi^2}\left[2\ln\frac{2\kappa^2\sigma^2}{\Lambda^2} + (z + 1)^2\ln(1 + z^{-1}) + (z - 1)^2\ln(1 - z^{-1})\right]\right), \quad (17)$$

where $m^2 = M^2(1/4\xi - 1)$ and $z = \sigma^2/m^2$. We see that the only non-zero contributions to the effective potential come from the $|S|$-dependent part of the spectrum (in the first two lines of Table 1). This is a consequence of the fact that, along the inflationary trajectory, SUSY breaking, due to the presence of non-zero vacuum energy density, occurs only in the inflaton sector. In particular, there is mass splitting only in the supermultiplet which contains the complex scalar field $\theta = (\delta\nu^c_H + \delta\bar{\nu}^c_H)/\sqrt{2}$ (see Appendix).

Note that radiative corrections lift the (classical) flatness of the inflationary trajectory providing the necessary inclination for driving the inflaton field $S$ towards zero. It is important to observe that although the effective potential in Eq.(17) does depend on the unknown scale $\Lambda$, its inclination (derivative with respect to $\sigma$) is $\Lambda$-independent. This is due to the fact that the supertrace of $M^4$ ($M^2$ being the mass squared matrix) appearing in Eq.(16) is, as one can readily deduce using the spectrum in Table 1, $\sigma$-independent.
This is an important property since otherwise \( (\delta T/T)_Q \) and \( N_Q \) would depend on the unknown mass parameter \( \Lambda \).

Inflation is terminated only very close to the critical point \( \sigma = m \) \( (z = 1 \) or \( w^2 = \rho_0) \) after which the inflationary path is destabilized and the system falls into \( y_{3–} \). This can be checked, for all relevant values of the coupling constants, by employing the slow-roll parameters \( \epsilon \) and \( \eta^{28} \). It turns out that \( \epsilon \ll 1 \) for \( z \geq 1 \), while \( |\eta| \) exceeds unity only for \( z \)’s extremely close to 1.

The quadrupole anisotropy of the cosmic microwave background radiation can be calculated to be \[ 12 \]:

\[
\left( \frac{\delta T}{T} \right)_Q \simeq \pi \left( \frac{32N_Q}{45} \right)^{1/2} \left( \frac{m}{M_P} \right)^2 x_Q^{-1} y_Q^{-1} \Lambda (x_Q^2)^{-1}, \tag{18}
\]

where \( M_P = 1.22 \times 10^{19} \) GeV is the Planck scale,

\[
\Lambda(z) = (z + 1) \ln(1 + z^{-1}) + (z - 1) \ln(1 - z^{-1}), \tag{19}
\]

and

\[
y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z} \Lambda^{-1}(z), y_Q \geq 0, \tag{20}
\]

with \( x_Q = |\sigma_Q|/m, \sigma_Q \) being the value of \( \sigma \) when our present horizon scale crossed outside the inflationary horizon. The coupling constant \( \kappa \) can be evaluated from \[ 12 \]

\[
\kappa = \frac{4\pi^{3/2} m}{\sqrt{N_Q M_P} y_Q}. \tag{21}
\]

Now, using the COBE constraint, \( (\delta T/T)_Q = 6.6 \times 10^{-6} \) \[ 18 \], taking \( N_Q = 55 \) and eliminating \( x_Q \) between Eqs.\( (18) \) and \( (21) \), we can obtain \( m \) and, consequently, \( \xi \) and \( M \) as functions of \( \kappa \) (we put \( \beta = 1 \) and \( M_S = 5 \times 10^{17} \) GeV). The inflationary scale \( v_{\text{inf}} = \kappa^{1/2} m \) and the spectral index of density perturbations \( n = 1 - 6\epsilon + 2\eta \) are then also found as functions of \( \kappa \) and are depicted in Figs.5 and 6 respectively.

The SUSY minimum can be obtained from \( y_{3–} \) in Eq.\( (13) \) by putting \( w = 0 \). The common vev \( v_0 = |\langle \nu_{\bar{H}}^c \rangle| = |\langle \bar{\nu}_H^c \rangle| \) of \( H^c \) and \( \bar{H}^c \) at this minimum is then given by

\[
\left( \frac{v_0}{M} \right)^2 = \frac{1}{2\xi} (1 - \sqrt{1 - 4\xi}), \tag{22}
\]

and is shown in Fig.7 as a function of \( \kappa \).
4 Neutrino masses and lepton asymmetry

A complete inflationary scenario should be followed by a successful reheating which can generate the observed baryon asymmetry of the universe. We will now turn to the discussion of this reheating process in our model. The inflaton consists of the two complex scalar fields $S$ and $\theta$, which have equal masses given by

$$m_{\text{infl}}^2 = 2\kappa^2 v_0^2 (1 - \frac{2\xi v_0^2}{M^2})^2.$$

At the end of inflation, the two fields $S$ and $\theta$ oscillate about the SUSY minimum and decay into a pair of right-handed sneutrinos ($\nu^c_i$) and neutrinos ($\psi_{\nu^c_i}$) respectively. The masses of these (s)neutrinos are generated, after the breaking of $G_{PS}$, by the superpotential coupling $\gamma_i \bar{H}^c \tilde{H}^c F_i^c \tilde{F}_i^c / M_S$ in Eq.(4) and turn out to be

$$M_i = 2\gamma_i \frac{v_0^2}{M_S}.$$  \hspace{1cm} (24)

This same coupling together with the terms in Eq.(4) constitute the part of the superpotential which is relevant for the decay of the inflaton. We obtain the Lagrangian terms

$$L_{\text{decay}}^S = -\sqrt{2} \gamma_i \frac{v_0}{M_S} S^* \nu^c_i \nu^c_i \frac{m_{\text{infl}}^2}{M^2} + h.c.$$  \hspace{1cm} (25)

for the decay of $S$ and

$$L_{\text{decay}}^\theta = -\sqrt{2} \gamma_i \frac{v_0}{M_S} \theta \psi_{\nu^c_i} \psi_{\nu^c_i} + h.c.$$  \hspace{1cm} (26)

for the decay of $\theta$ respectively. From Eqs.(25) and (26), we deduce that $S$ and $\theta$ have equal decay widths given by

$$\Gamma_{S \to \nu^c_i \nu^c_i} = \Gamma_{\theta \to \psi_{\nu^c_i} \psi_{\nu^c_i}} \equiv \Gamma = \frac{1}{8\pi} \left( \frac{M_i}{v_0} \right)^2 m_{\text{infl}},$$  \hspace{1cm} (27)

provided that $M_i < m_{\text{infl}}/2$. To minimize the number of small couplings we assume that

$$M_2 < m_{\text{infl}}/2 \leq M_3,$$  \hspace{1cm} (28)

so that the coupling $\gamma_3$ can be of order one. The inflaton then decays into the second heaviest right-handed neutrino superfield with mass $M_2$. (There always exist $\gamma_3$’s smaller than one which satisfy the second inequality in this equation for all relevant values of
the other parameters.) Thus the reheat temperature $T_r$ after inflation, for the MSSM spectrum, is given by \[ T_r = \frac{1}{7}(\Gamma M_P)^{1/2} = \frac{1}{7} \left( \frac{M_P m_{\text{infl}}}{8\pi} \right)^{1/2} \frac{M_2}{v_0}. \] (29)

The gravitino constraint [21] gives an upper bound on $T_r$ of about $10^9$ GeV for gravity-mediated SUSY breaking with universal boundary conditions. To maximize the naturalness of the model, we take the maximal value of $M_2$ (and thus $\gamma_2$) allowed by the gravitino constraint. Note that these values of $M_2$ turn out to be about two orders of magnitude lower than the corresponding values of $m_{\text{infl}}/2$ and, thus, the first inequality in Eq. (28) is well satisfied.

Analysis [30] of the CHOOZ experiment [31] shows that the solar and atmospheric neutrino oscillations decouple, allowing us to concentrate on the two heaviest families. The light neutrino mass matrix is then given by

$$m_\nu = -\tilde{M}^D \frac{1}{M R} M^D,$$  

(30)

where $M^D$ is the Dirac neutrino mass matrix with positive eigenvalues $m_{2,3}^D$ ($m_2^D \leq m_3^D$), and $M^R$ the Majorana mass matrix of the right-handed neutrinos with positive eigenvalues $M_{2,3}$ ($M_2 \leq M_3$) given in Eq. (24). The two positive eigenvalues of $m_\nu$ will be denoted by $m_2$ (or $m_{\nu_2}$) and $m_3$ (or $m_{\nu_3}$), with $m_2 \leq m_3$. The determinant and trace invariance of $m_\nu^\dagger m_\nu$ provide us with two constraints [32] on the mass parameters $m_{2,3}$, $m_{2,3}^D$, $M_{2,3}$ and the angle $\theta$ and phase $\delta$ of the rotation matrix which diagonalizes the right-handed neutrino mass matrix $M^R$ in the basis where $M^D$ is diagonal.

The bounds on $m_{\nu_\mu}$ from the small or large angle MSW solution of the solar neutrino puzzle are respectively $2 \times 10^{-3}$ eV $ \leq m_{\nu_\mu} \leq 3.2 \times 10^{-3}$ eV or $3.6 \times 10^{-3}$eV $ \leq m_{\nu_\mu} \leq 1.3 \times 10^{-2}$ eV [33]. As we will see below, the latter solution is favored in our model. The $\tau$-neutrino mass is restricted in the range $3 \times 10^{-2}$ eV $ \leq m_{\nu_\tau} \leq 11 \times 10^{-2}$ eV from the results of SuperKamiokande [1] which also imply almost maximal $\nu_\mu - \nu_\tau$ mixing, i.e., $\sin^2 2\theta_{\mu\tau} > 0.8$. Assuming that the Dirac mixing angle $\theta^D$ (i.e., the mixing angle in the absence of right-handed neutrino Majorana masses) is negligible, we find [32] $\theta_{\mu\tau} \simeq \varphi$, where $\varphi$ is the rotation angle which diagonalizes $m_\nu$.

An important constraint comes from the baryon asymmetry of the universe. In this model, a primordial lepton asymmetry is generated [24] by the decay of the superfield
ν^c_2 which emerges as the decay product of the inflaton. (This lepton asymmetry is subsequently partially converted into baryon asymmetry by electroweak sphalerons.) The superfield ν^c_2 decays into electroweak Higgs and (anti)lepton superfields. The resulting lepton asymmetry is \[ \frac{n_L}{s} \simeq 1.33 \frac{9 T_r}{16 \pi m_{\text{infl}}} \frac{M_2}{M_3} \frac{c^2 s^2 \sin 2\delta (m_{3}^{D^2} - m_{2}^{D^2})^2}{v_2^2 (m_{3}^{D^2} s^2 + m_{2}^{D^2} c^2)}, \] (31)

where \( s = \sin \theta \) and \( c = \cos \theta \). This is related to the baryon asymmetry \( n_B/s \) by \( n_L/s = -(79/28)(n_B/s) \) for the spectrum of the MSSM. Thus, the low deuterium abundance constraint \( 0.017 \leq \Omega_B h^2 \leq 0.021 \) gives \( 1.8 \times 10^{-10} \leq -n_L/s \leq 2.3 \times 10^{-10} \).

Due to the presence of \( SU(4)_c \) in \( G_{PS} \), the Dirac mass parameter \( m_3^D \) coincides with the asymptotic value of the top quark mass. Taking renormalization effects into account, in the context of the MSSM with large \( \tan \beta \), we find \( m_3^D = 110 - 120 \) GeV.

For each value of \( \kappa \), the Majorana masses \( M_{2,3} \) are fixed. Taking \( m_{2,3} \) and \( m_{3}^{D} \) also fixed in their allowed ranges, we are left with only three undetermined parameters \( \delta, \theta \) and \( m_2^D \) which are further restricted by four constraints: almost maximal \( \nu_\mu - \nu_\tau \) mixing (\( \sin^2 2\theta_{\mu\tau} > 0.8 \)), the leptogenesis restriction (\( 1.8 \times 10^{-10} \leq -n_L/s \leq 2.3 \times 10^{-10} \)) and the constraints from the trace and determinant invariance of \( m^\dagger m \). It is highly non-trivial that solutions satisfying all the above requirements can be found with natural values of \( \kappa \) (of order \( 10^{-3} \)) and \( m_2^D \) of order 1 GeV. Typical solutions can be constructed, for instance, for \( \kappa = 4 \times 10^{-3} \), which corresponds to \( \xi \simeq 0.2, \; v_0 \simeq 1.7 \times 10^{16} \) GeV, \( m_{\text{infl}} \simeq 4.1 \times 10^{13} \) GeV, \( M_2 \simeq 5.9 \times 10^{10} \) GeV and \( M_3 \simeq 1.1 \times 10^{15} \) GeV (\( \gamma_3 = 0.5 \)). (Remember \( \beta = 1, \; M_S = 5 \times 10^{17} \) GeV and \( T_r = 10^9 \) GeV.) Taking, for example, \( m_{\mu\mu} = 7.6 \times 10^{-3} \) eV, \( m_{\mu\tau} = 8 \times 10^{-2} \) eV and \( m_3^D = 120 \) GeV, we find \( m_2^D \simeq 1.2 \) GeV, \( \sin^2 2\theta_{\mu\tau} \simeq 0.87 \), \( n_L/s \simeq -1.8 \times 10^{-10} \) and \( \theta \simeq 0.016 \) for \( \delta \simeq -\pi/3 \).

It is interesting to note that the mass scale \( v_0 \) is of order \( 10^{16} \) GeV which is consistent with the unification of the gauge couplings of the MSSM. Also, the values of the \( \mu \)-neutrino mass, for which solutions are found, turn out to be consistent with the large rather than the small angle MSW mechanism.

In summary, we found that, within the framework of our inflationary scheme, a number of cosmological and phenomenological constraints could be easily satisfied. These
constraints include the gravitino bound on the reheat temperature, successful baryogenesis in the universe, and consistency of neutrino masses and mixing with the solar and atmospheric neutrino oscillations. It should be pointed out that, for this success of our model, no extra structure was needed to be added to it. Actually, the possibility of accounting for these requirements is more or less automatic in this inflationary scheme and can, in principle, remain in gauge groups other than $G_{PS}$ too.

5 Conclusions

We have constructed a SUSY GUT model based on the $G_{PS}$ gauge symmetry group. This model is consistent with all particle physics and cosmological requirements. The $\mu$-problem is solved by introducing a global anomalous PQ symmetry $U(1)_{PQ}$, which also solves the strong CP problem. Although baryon and lepton numbers are violated in the superpotential, the proton turns out to be practically stable. SUSY hybrid inflation is ‘naturally’ and successfully incorporated in this model but in an unconventional way. In the standard realizations of SUSY hybrid inflation, the superpotential involves only renormalizable couplings of the GUT Higgs superfields and a gauge singlet. We have modified this picture by including the next order non-renormalizable coupling too. In contrast to the usual case, inflation now takes place along a classically flat direction where the gauge symmetry ($G_{PS}$) is spontaneously broken to $G_{SM}$. As a consequence, after inflation ends, there is absolutely no production of doubly charged magnetic monopoles, which are associated with the breaking of $G_{PS}$. Thus, the cosmological catastrophe one would encounter by employing the usual inflationary scheme in the SUSY PS theory is avoided. Our mechanism is crucial for the viability of any model containing cosmologically disastrous topological defects such as magnetic monopoles or domain walls and leads to complete absence of such objects. It is interesting to point out that, although the usual trajectory with unbroken $G_{PS}$ also exists, there is a range of parameters for which the system finally inflates along the non-trivial path before falling into the SUSY vacua. Thus, the monopole problem can be solved for all possible initial conditions.

The classical flatness of the inflationary valley is lifted by one-loop radiative corrections which produce an inclination for driving the inflaton towards the SUSY vacua. The measurements of COBE can be easily reproduced with natural values (of order $10^{-3}$) of the relevant coupling constant. The GUT mass scale comes out equal to (or somewhat
smaller than) the SUSY GUT scale and can certainly be much closer to it than in the standard SUSY inflationary scheme. The spectral index of density perturbations ranges between about 1 and 0.9.

After inflation ends, the inflaton oscillates about the SUSY vacuum and decays into the second heaviest right-handed neutrino superfield thereby reheating the universe. The subsequent decay of these right-handed neutrinos to lepton and electroweak Higgs superfields generates a lepton asymmetry which is then partially converted to baryon asymmetry by the electroweak instantons. We require that the so obtained baryon asymmetry of the universe is consistent with the low deuterium abundance constraint. We also take almost maximal $\nu_\mu - \nu_\tau$ mixing as indicated by SuperKamiokande. The $\mu$- and $\tau$-neutrino masses are restricted by the MSW resolution of the solar neutrino puzzle and the heaviest Dirac neutrino mass by $SU(4)_c$ symmetry. We find that all these requirements can be met with natural values (of order $10^{-3}$) of the relevant coupling constant. Note that the second heaviest Dirac neutrino mass turns out to be of order 1 GeV and masses of $\nu_\mu$ consistent with the large rather than the small angle MSW mechanism are favored.

Finally, we would like to point out that, although we restricted our discussion to the PS gauge group, this new SUSY hybrid inflationary scenario is of much wider applicability. The reason for choosing this particular framework for our presentation is that $G_{PS}$ is one of the simplest gauge groups with magnetic monopoles. The scenario can be readily extended to other semi-simple gauge groups such as the trinification group $SU(3)_c \times SU(3)_L \times SU(3)_R$ which emerges from string theory. Of course, the superfields $H^c$ and $\bar{H}^c$ should be replaced by the appropriate pair of superfields $(1, 3, 3)$ and $(1, 3, \bar{3})$ from the $E_6$ 27-plet and $\bar{27}$-plet which cause the breaking of the trinification group to $G_{LR}$. Further breaking of $G_{LR}$ to $G_{SM}$ is achieved by a similar pair of superfields with vevs, though, in the right-handed neutrino direction. Our scheme could also be extended to simple gauge groups such as $SO(10)$. The breaking of $SO(10)$ can be achieved by including, among other representations, a pair of $16$, $16$ Higgs superfields acquiring vevs in the right-handed neutrino direction. Inflation and reheating are expected to be quite similar to the ones discussed here with the only complication that more Higgs superfield representations such as 54 and 45 will be involved for gauge symmetry breaking to $G_{SM}$. The magnetic monopole problem can then be solved only if some of these fields are non-zero too on the inflationary trajectory.
Acknowledgement

This work was supported by European Union under the TMR contract No. ERBFMRX-CT96-0090. S. K. is supported by a Ministerio de Educacion y Cultura research grant. Also, Q. S. would like to acknowledge the DOE support under grant number DE-FG02-91ER40626. Finally, R. J. and S. K. would like to thank G. Senjanović for discussions.

Appendix: Derivation of the mass spectrum during inflation

In this Appendix, we sketch the derivation of the mass spectrum of the model when the system is trapped along the inflationary trajectory at $y_2$. During inflation, the fields $H^c$, $\bar{H}^c$ acquire vevs in the $\nu^c_H$, $\bar{\nu}^c_H$ direction which break the gauge symmetry $G_{PS}$ down to $G_{SM}$. These vevs are given by $\langle \nu^c_H \rangle = \langle \bar{\nu}^c_H \rangle = v = (\kappa M_S^2/2\beta)^{1/2}$ and we can write $\nu^c_H = v + \delta \nu^c_H$ and $\bar{\nu}^c_H = v + \delta \bar{\nu}^c_H$. One can show that the scalar potential in Eq.(10) does not generate masses for the scalar components of the superfield $H^c$ ($\bar{H}^c$) in the directions $u^c_H$, $d^c_H$, $e^c_H$ ($\bar{u}^c_H$, $\bar{d}^c_H$, $\bar{e}^c_H$). On the contrary, a simple calculation yields that the normalized real scalar fields $\text{Re}(\delta \nu^c_H + \delta \bar{\nu}^c_H)$ and $\text{Im}(\delta \nu^c_H + \delta \bar{\nu}^c_H)$ acquire non-zero masses given by

$$m^2_{\pm} = 4\kappa^2|S|^2 \mp 2\kappa^2 M^2(\frac{1}{4\xi} - 1)$$

respectively. The superpotential in Eq.(9) gives rise to just one massive Majorana fermion with $m^2 = 4\kappa^2|S|^2$ corresponding to the direction $(\nu^c_H + \bar{\nu}^c_H)/\sqrt{2}$. We see that the SUSY breaking along the inflationary trajectory, which is due to the non-zero vacuum energy density $\kappa^2 M^4(1/4\xi - 1)^2$, produces a mass splitting in the $\nu^c_H$, $\bar{\nu}^c_H$ supermultiplets. Actually, as we will show, this is the only place where such a mass splitting appears.

The D-term contribution to the scalar masses can be found from

$$\frac{1}{2}g^2 \sum_a (\bar{H}^c T^a H^c + H^c T^a \bar{H}^c)^2,$$

where $g$ is the $G_{PS}$ gauge coupling constant and the sum extends over all the generators $T^a$ of $G_{PS}$. The part of this sum over the generators $T^{15} = (1/2\sqrt{6}) \text{diag}(1,1,1,-3)$ of $SU(4)_c$ and $T^3 = (1/2) \text{diag}(1,-1)$ of $SU(2)_R$ gives rise to a mass term for the normalized real scalar field $\text{Re}(\delta \nu^c_H + \delta \bar{\nu}^c_H)$ with $m^2 = 5g^2 v^2/2$ as one can show by using the above expansion of $\nu^c_H$, $\bar{\nu}^c_H$. 

21
The gauge bosons $A^a$ can acquire masses from the Lagrangian terms

$$g^2(|\sum_a A^a T^a H_c|^2 + |\sum_a A^a T^a H^c|^2).$$

(34)

Taking the contribution of $T^{15}$ and $T^3$ again, we obtain a mass term for the normalized gauge field

$$A^\perp = -\sqrt{\frac{3}{5}} A^{15} + \sqrt{\frac{2}{5}} A^3$$

with $m^2 = 5g^2v^2/2$ (the real field $\text{Im}(\delta \nu^c_H - \delta \bar{\nu}^c_H)$, which is so far left massless, is absorbed by this gauge boson).

Fermion masses get also contributions from the Lagrangian terms

$$i\sqrt{2}g \sum_a \lambda^a (\bar{\psi} \cdot T^a \psi + H^c \cdot T^a \psi_H + h.c.),$$

(35)

where $\lambda^a$ is the gaugino corresponding to $T^a$ and $\psi_H, \psi_{\bar{H}}$ represent the chiral fermions belonging to the superfields $\bar{H}^c, H^c$ respectively. Concentrating again on $T^{15}, T^3$, we obtain a Dirac mass term between the chiral fermion in the superfield $(\nu^c_H - \bar{\nu}^c_H)/\sqrt{2}$ and the gaugino $-i\lambda^\perp$ with $m^2 = 5g^2v^2/2$. This completes the analysis of the $\nu^c_H, \bar{\nu}^c_H$ sector together with the gauge supermultiplet in the $T^{\perp}$ direction.

The eight normalized real scalar fields $\text{Re}(u^c_H - \bar{u}^c_H), \text{Im}(u^c_H - \bar{u}^c_H)$ (three colors), $\text{Re}(e^c_H - \bar{e}^c_H), \text{Im}(e^c_H - \bar{e}^c_H)$ acquire mass terms from the D-term contribution in Eq.(33) with $m^2 = g^2v^2$. Indeed, the part of the sum in this equation over the generators

$$T^1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(36)

of $SU(2)_R$ gives

$$\frac{1}{2}g^2v^2(\text{Re}(e^c_H - \bar{e}^c_H))^2 + \frac{1}{2}g^2v^2(\text{Im}(e^c_H - \bar{e}^c_H))^2.$$  

(37)

Similarly, the sum over the $SU(4)_c$ generators $T^1_i$ and $T^2_i$ ($i = 1, 2, 3$) with $1/2$ ($1/2$) and $-i/2$ ($i/2$) in the $i4$ ($4i$) entries respectively and zero everywhere else generates the masses of $\text{Re}(u^c_H - \bar{u}^c_H), \text{Im}(u^c_H - \bar{u}^c_H)$ (three colors). Using Eq.(34), one can show that the eight gauge bosons $A^1, A^2, A^1_i, A^2_i$ ($i = 1, 2, 3$) become massive with $m^2 = g^2v^2$ (they absorb the real fields $\text{Re}(e^c_H + \bar{e}^c_H), \text{Im}(e^c_H + \bar{e}^c_H)$, and $\text{Re}(u^c_H + \bar{u}^c_H), \text{Im}(u^c_H + \bar{u}^c_H)$ (three colors)).
The chiral fermions $\psi_{\bar{e}_c H}$ and $\psi_{e_c H}$ combine with the gauginos $\lambda^+ = (\lambda_1 + i\lambda_2)/\sqrt{2}$ and $\lambda^- = (\lambda_1 - i\lambda_2)/\sqrt{2}$ respectively to form two Dirac fermion states with $m^2 = g^2 v^2$ as one deduces from Eq.(35). Similarly, $\psi_{\bar{u}_c H}$ and $\psi_{u_c H}$ (three colors) together with $\lambda^+_i = (\lambda^+_1 + i\lambda^+_2)/\sqrt{2}$ and $\lambda^-_i = (\lambda^-_1 - i\lambda^-_2)/\sqrt{2}$ give six more Dirac fermions with the same mass squared.

The only superfields left are the $d^c_H$, $\bar{d}^c_H$, $g^c$, $\bar{g}^c$. They do not mix with the rest of the spectrum, as one can easily show, and acquire masses from the last two superpotential terms in Eq.(36). These terms can be explicitly written as

\[
\begin{align*}
a G H^c H^c & = 2a (-d^c_H \nu^c_H + u^c_H e^c_H) \bar{g}^c + 2a u^c_H d^c_H g^c, \\
b G \bar{H}^c \bar{H}^c & = 2b (-d^c_H \nu^c_H + \bar{u}^c_H \bar{e}^c_H) g^c + 2b \bar{u}^c_H \bar{d}^c_H \bar{g}^c. \\
\end{align*}
\]  

The scalar potential then contains the terms

\[
4a^2 v^2 (|d^c_H|^2 + |\bar{g}^c|^2) + 4b^2 v^2 (|d^c_H|^2 + |g^c|^2)
\]

and we obtain six complex scalars $(d^c_H, \bar{g}^c)$ with $m^2 = 4a^2 v^2$ and six complex scalars $(\bar{d}^c_H, g^c)$ with $m^2 = 4b^2 v^2$. Also, the chiral fermions $\psi_{d^c_H}$ and $\psi_{\bar{g}^c}$ combine to give three Dirac fermions with $m^2 = 4a^2 v^2$, while $\psi_{\bar{d}^c_H}$ and $\psi_{g^c}$ give three Dirac fermions with $m^2 = 4b^2 v^2$.

We see that all the fields acquire non-zero masses but SUSY is broken only in the sector of the two real scalar fields with masses given in Eq.(32) and the Majorana fermion with $m^2 = 4\kappa^2 |S|^2$. In all other supermultiplets, the fermionic and bosonic components have equal masses. As a consequence, these supermultiplets give zero contribution to the supertraces $\text{STr} M^{2n}$ for any integer $n \geq 0$ (and in general to the supertrace of any function of the mass squared matrix $M^2$). Thus, in calculating any such supertrace, we only have to consider the two real scalars and the Majorana fermion mentioned above. Note that, in particular, their contribution to $\text{STr} M^2$ is zero.

References

[1] T. Kajita, talk given at XVIIIth International Conference on Neutrino Physics and Astrophysics (Neutrino’98), Takayama, Japan, 4-9 June, 1998.
[2] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Freemann (North Holland, Amsterdam, 1979) p. 315;
T. Yanagida, Prog. Theor. Phys. 64 (1980) 1103;
R. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.

[3] K. Babu, B. Dutta and R. Mohapatra, Phys. Rev. D 60 (1999) 095004;
Phys. Lett. B 458 (1999) 93; Phys. Rev. D 61 (2000) 091701.

[4] S. Khalil and Q. Shafi, Phys. Rev. D 61 (2000) 035003;
S. King and Q. Shafi, Phys. Lett. B 442 (1998) 135.

[5] J. C. Pati and A. Salam, Phys. Rev. D 10 (1974) 275.

[6] G. Shiu and S.-H. H. Tye, Phys. Rev. D 58 (1998) 106007.

[7] I. Antoniadis and G. K. Leontaris, Phys. Lett. B 216 (1989) 333.

[8] I. Antoniadis, G. K. Leontaris and J. Rizos, Phys. Lett. B 245 (1990) 161.

[9] A. D. Linde, Phys. Rev. D 49 (1994) 748.

[10] G. Lazarides, R. Schaefer and Q. Shafi, Phys. Rev. D 56 (1997) 1324.

[11] G. Lazarides and N. D. Vlachos, Phys. Lett. B 459 (1999) 482.

[12] G. Lazarides, PRHEP-trieste99/008 [hep-ph/9905450].

[13] G. Lazarides, M. Magg and Q. Shafi, Phys. Lett. B 97 (1980) 87.

[14] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D 52 (1995) 559.

[15] T. W. B. Kibble, J. Phys. A 9 (1976) 387.

[16] G. Dvali, R. Schaefer and Q. Shafi, Phys. Rev. Lett. 73 (1994) 1886.

[17] L. Covi, G. Mangano, A. Masiero and G. Miele, Phys. Lett. B 424 (1998) 253.

[18] G. F. Smoot et al., Astrophys. J. Lett. 396 (1992) L1; C. L. Bennett et al., Astrophys.
J. Lett. 464 (1996) 1.

[19] G. Lazarides and Q. Shafi, Phys. Rev. D 58 (1998) 071702.
[20] M. Fukugita and T. Yanagita, Phys. Lett. B 174 (1986) 45; W. Buchmüller and M. Plümacher, Phys. Lett. B 389 (1996) 73; G. Lazarides and Q. Shafi, Phys. Lett. B 258 (1991) 305. For a recent review in the context of SUSY hybrid inflation see G. Lazarides, Sprin. Trac. in Mod. Phys. 163 (2000) 227 [hep-ph/9904428].

[21] M. Yu. Khlopov and A. D. Linde, Phys. Lett. B 138 (1984) 265; J. Ellis, J. E. Kim and D. Nanopoulos, Phys. Lett. B 145 (1984) 181.

[22] S. Burles and D. Tytler, Astrophys. Journal 499 (1998) 699; ibid. 507 (1998) 732.

[23] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D 49 (1994) 6410.

[24] G. Dvali, G. Lazarides and Q. Shafi, Phys. Lett. B 424 (1998) 259.

[25] G. Lazarides and N. D. Vlachos, Phys. Lett. B 441 (1998) 46.

[26] L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320; S. Weinberg, Phys. Rev. D 9 (1974) 3357.

[27] R. Jeannerot and J. Lesgourgues, Phys. Rev. D 62 (2000) 023514.

[28] A. R. Liddle and D. H. Lyth, Phys. Rep. 231 (1993) 1.

[29] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.

[30] C. Giunti, [hep-ph/9802201].

[31] M. Apollonio et al., Phys. Lett. B 420 (1998) 397.

[32] G. Lazarides, Q. Shafi and N. D. Vlachos, Phys. Lett. B 427 (1998) 53.

[33] J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, Phys. Rev. D 58 (1998) 096016; ibid. D 60 (1999) 093001.

[34] L. E. Ibáñez and F. Quevedo, Phys. Lett. B 283 (1992) 261.
Figure 1: The (dimensionless) zero temperature potential $\tilde{V}_{PQ} = V_{PQ} / (m_{3/2} M_S)^2$ with $V_{PQ}$ given in Eq.(7) versus $|\tilde{N}| = |N| / (m_{3/2} M_S)^{1/2}$, for $|A| = 5, \lambda_1 = 0.3, \lambda_2 = 0.1, m_{3/2} = 300 \text{ GeV}$ and $M_S = 5 \times 10^{17} \text{ GeV} (\mu = 600 \text{ GeV})$. 
Figure 2: The (dimensionless) potential $\tilde{V}$, given in Eq.(12), versus $y$ for $w = 2, \xi = 1/5$.

Figure 3: The (dimensionless) potential $\tilde{V}$, given in Eq.(12), versus $y$ for $w = 0.7, \xi = 1/5$. 

27
Figure 4: The (dimensionless) potential $\tilde{V}$, given in Eq.(12), versus $y$ for $w = 0.2$, $\xi = 1/5$.

Figure 5: The inflationary scale $v_{\text{infl}}$ as a function of the coupling constant $\kappa$. 
Figure 6: The spectral index $n$ as a function of the coupling constant $\kappa$.

Figure 7: The common vev $v_0$ of $\bar{H}^c$, $H^c$ at the SUSY minimum as a function of the coupling constant $\kappa$. 