Cumulative structure function in terms of nucleonic wave function of the nucleus

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Abstract

The structure function of the nucleus in the cumulative region $x > 1$ is studied in terms of nucleon degrees of freedom. At high $Q^2$ the resulting expressions are presented as a sum of contributions from few-nucleon correlations. Two-nucleon correlations are studied in some detail. Spin variables are averaged out. In the region $1 < x < 2$ the structure functions are calculated for the relativistic interaction proposed by F.Gross et al. They are found to fall with $x$ faster than the exponential. For Carbon at $x = 1.05$, where the method is not rigorously applicable, they turn out to be roughly twice larger than the experimental data.

1 Introduction

The cumulative phenomena, i.e. processes with nuclei in the kinematical region prohibited for the colliding free nucleons, are of considerable interest both from the practical and theoretical points of view. On the practical side they give a possibility to effectively raise the energy of the colliding particles, allowing for subthreshold production of high mass particles. On the theoretical side they allow to study small internucleon distances and thus study the nuclear matter at high densities and possibly its quark structure. One may speculate that in these phenomena the cold quark-gluon plasma is formed in the region of many overlapping nucleons.

Most of the experimental data on the cumulative phenomena refer to particle production in the region outside the free nucleon kinematics. Obtained more than twenty years ago, they were actively discussed at that time. Many models were proposed to explain the data. However no general consensus was reached. Even the basic question, whether the quark rather than nucleon degrees of freedom were necessary to describe the experimental results remained unsolved. The reason for this lies in the complexity of the production process. It inevitably involves soft interaction of the colliding particles, the description of which is

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highly model dependent and thus introduces uncontrollable elements into the predictions for cumulative particle production rates.

From the theoretical point of view a much cleaner observable is the structure function of the nucleus in the cumulative region, that is, at the values of $x$ greater than unity. Unfortunately the relevant experimental information is quite scarce, limited to data on the deuteron [1] and carbon [2] at relatively low values of $x < 1.5$. However even these modest data present a clear possibility for theoretical analysis and testing of different approaches. A description of the cumulative structure function of the nucleus based on the QCD approach and quark structure of the nucleus was proposed in [3]. Potentially this approach can relate the observed magnitude and behaviour of the cumulative structure function to the fundamental QCD parameter $\Lambda$. However the results obtained in [3] strictly speaking refer to the extreme cumulative kinematical limits. They also result infrared unstable and require introduction of an infrared cutoff ("quark mass") as a parameter. With this single parameter both the slope and the magnitude of the cumulative structure function were described reasonably well in [3].

For all that, it remains unclear whether the description of the nuclear structure function at moderate cumulativity, that is for $x \sim 2 \div 3$ necessarily requires using the quark language and thus throwing away most of the information accumulated from the study of the nucleus in terms of hadrons. It is worth mentioning that many years ago Frankfurt and Strikman strongly advocated the thesis that the cumulative phenomena could be well described in terms of nucleons, provided the relevant wave functions be appropriately relativized [4].

In this paper we pursue exactly this approach. Exploiting the fact that forms for the relativistic internucleon interaction have lately been proposed, which lead to excellent agreement with all low energy data, we use this interaction to calculate the cumulative nuclear structure function. It is important that the cumulative structure function is directly related to the internucleon interaction, unlike most of other properties which rather involve all sort of averages of the interaction. Thus our results constitute a stringent test on the proposed relativistic interaction. In principle our approach allows to find the cumulative structure function at all values of $x < A$. However calculational difficulties grow very fast with $x$. For that reason in this paper, apart from presenting the general formalism, we calculate the nuclear structure function only in the interval $1 < x < 2$ from the contribution which is known as "two-nucleon correlation" [4], leaving the three-(and more-)nucleon correlations for future studies.

The paper is organized as follows. In Secs.2 and 3 we present our formalism and separate contributions from correlations of several nucleons. In Sec. 4 we analyze the two-nucleon correlation first for the simplified scalar case and then for the relaistic spinor case. Sec. 5 presents our numerical results. Some conclusions are drawn in Sec. 6. In the Appendices the parameters of the used relativistic interaction are presented and also some calculational details.
2 Formalism

At high values of $Q^2$ the nuclear structure function $F^{(A)}(x,Q^2)$ is given by the “impulse approximation” diagram shown in Fig. 1, corresponding to the leading twist contribution. Our notations are clear from Fig. 1. We define in the standard manner $q^2 = -Q^2 < 0$ and

$$x = \frac{Q^2}{2qP}$$

Note that $x$ is defined respective to the collision with a nucleon. The mass of the nucleus is $M = A(m - \epsilon)$, so that

$$p^2 = m^2 - 2m\epsilon \equiv m^2 - \Delta^2$$

In future we shall neglect the binding energies wherever it is possible. The physical region for the reaction is evidently

$$(q + Ap)^2 \geq M^2 \text{ or } 0 \leq x \leq A$$

All the region

$$1 \leq x \leq A$$

is cumulative.

The separation of the structure function from the diagram shown in Fig.1 is standard. Let $W_{\mu\nu}^{(A)}$ be imaginary part of the amplitude (the discontinuity in $s$ divided by $2i$). Then one presents

$$W_{\mu\nu}^{(A)} = \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right)F_1^{(A)}(x,Q^2) + \frac{1}{qP} \left(P_{\mu} - q_{\mu} \frac{qP}{q^2}\right) \left(P_{\nu} - q_{\nu} \frac{qP}{q^2}\right) F_2^{(A)}(x,Q^2)$$

(2)

with $P = Ap$ the nucleus 4-momentum. Functions $F_{1,2}^{(A)}$ are the nuclear structure functions. In the following we concentrate on $F_2^{(A)}$ and suppress subindex 2. It is convenient to choose a system in which $q_+ = 0$. Then taking the $++$ component of (2) one gets

$$W_{++}^{(A)} = \frac{p^2}{qP} F^{(A)}(x,Q^2)$$

(3)

This relation serves to read the structure function directly from the diagram shown in Fig. 1.

To move further for simplicity we assume nucleons to be spinless particles. However the final results will also be valid for realistic nucleons after averaging over spins. Some details concerning inclusion of spins will be presented in Section 4. We present $W_{\mu\nu}^{(A)}$ as an integral over the momentum $k$ of the active nucleon

$$W_{\mu\nu}^{(A)} = \int \frac{d^4k}{(2\pi)^4} W_{\mu\nu}^{(N)}(q,k) \ 2 \text{ Im } \Phi(p,k)$$

(4)

where $\Phi$ denotes the lower blob in Fig. 1 with the propagators of the active nucleon included and $W_{\mu\nu}^{(N)}$ is the imaginary part of the upper blob. The latter can be presented as in (2),
with the off-shell structure functions $F_{1,2}^{(N)}(x', Q^2, k^2)$ where $x' = Q^2/(2qk)$. At high $Q^2$ at lower twist these structure functions do not depend on $k^2$ and coincide with the observable structure functions of the nucleon. At high $Q^2$ in the system $q_+ = 0$ we have $q_+ \sim Q^2$, $q_\perp \sim Q << q_+ = k_+$. So if the scaling variable of the active nucleon is $z = k_+/p_+$, then $x' = x/z$. With these relations we find from (3) and (4):

$$F^{(N)}(x, Q^2) = \frac{1}{A} \int_x^A dz \int \frac{dz d^2 k_\perp}{(2\pi)^2} 2 \text{Im} \Phi(p, k)$$

(5)

where we used the fact that the structure function of the nucleon depends only on the scaling variable of the active nucleon.

The integral over $k_-, k_\perp$ of the $\text{Im} \Phi$ can be related to the distribution in $x$ of the nucleons in the target nucleus. To do this consider the baryonic form-factor $B_\mu(q)$ at $q = 0$ described by the diagram shown in Fig. 2. In it the lower blob is the same forward scattering amplitude $\Phi(p, k)$ as in Fig. 1 for the structure function, only non-cut. Blob $\Phi$ is a function of two variables: the active nucleon virtuality $m^2 - k^2$ and the square of the c.m. energy of the recoil particles $s = (A p - k)^2$. Neglecting the dependence of the nucleon form-factor on the nucleon virtuality we have:

$$2 P_\mu B^{(A)}(0) = \int \frac{d^4 k}{(2\pi)^4 i} 2 k_\mu \Phi(p, k) = \int \frac{dz d^2 k_\perp ds}{2(2\pi)^4 i (A - z)} 2 k_\mu \Phi(s, k^2)$$

(6)

In terms of integration variables in the second integral

$$k^2 = \frac{z}{A} M_A^2 - \frac{z}{A - z} s - \frac{A}{A - z} k_\perp^2$$

(7)

and the integration over $s$ goes along the Feynman path.

In the complex $s$ plane the singularities of $\Phi$ come, first, from the standard unitarity righthand cut and, second, from the singularities in $k^2$. These lie along the positive values of $k^2$. However due to a coefficient in (7) they transform into a lefthand or righthand cut in $s$ depending on whether the ratio $z/(A - z)$ is positive or negative. If the cut coming from the singularities in $k^2$ lies to the right, we shall get zero, since the integration contour in $s$ then can be closed in the upper hemisphere with no singularities inside. So, as expected, a non-zero result only follows if

$$0 < z < A$$

(8)

when the cut from the singularities in $k^2$ lies to the left, and the result can be obtained by closing the contour around the unitarity cut. Taking the "+" component of (6) we get

$$\int \frac{d^2 k_\perp}{(2\pi)^2} \int_0^A dz \frac{dz}{2\pi (A - z)} \int_{s_0}^\infty \frac{ds}{2\pi} \text{Im} \Phi(s, k^2) = A^2$$

(9)

This relation can be rewritten as a normalization condition for the relativistic nuclear density in $z$ and $k_\perp$

$$\int_0^A \frac{dz}{z} \int \frac{d^2 k_\perp}{(2\pi)^2} \rho(z, k^2) = 1$$

(10)
where the density \( \rho(z, k^2) \) is defined as an integral over the "-" component of \( k \):

\[
\rho(z, k^2) = \frac{z^2}{A^2 2\pi (A - z)} \int \frac{ds}{2\pi} \text{Im} \Phi(s, k^2) = \frac{z^2}{A^2} \int \frac{p_+ dk_-}{(2\pi)^2} 2 \text{Im} \Phi(p, k)
\]

(11)

Note that on the formal level,

\[
2 \text{Im} \Phi(s, k^2) = \int d^4 r e^{-ikr} \langle A|N^\dagger(r)N(0)|A \rangle = \sum_\alpha (2\pi)^2 \delta^4(P_\alpha + k - P) \langle A|N^\dagger(0)|A - 1, \alpha \rangle \langle A - 1, \alpha|N(0)|A \rangle
\]

(12)

where \( |A\rangle \) is the state of the nucleus at rest, \( |A - 1, \alpha\rangle \) are the recoiling nuclear states with \( A - 1 \) nucleons, including states with \( A - 1 \) free nucleons, and \( N(x) \) is the nucleon field operator.

The amplitudes \( \langle A - 1, \alpha|N(0)|A \rangle \) describe splitting of the nucleus into a virtual nucleon and recoil states. The summation over all states in (12) is restricted by the energy conservation law (the conservation of "-" component of the momentum in light-cone variables). Integration over \( k_- \) lifts this restriction and, as is evident from (12), puts the operators \( N \) and \( N^\dagger \) at equal "time" \( x_+ = 0 \). This allows to demonstrate that in the light-cone formalism with a conserved number of nucleons \( \rho(x, k^2) \) is indeed the relativistic nuclear density

\[
\rho(x, k^2) = \frac{1}{4\pi} \int \prod_{j=2}^A dx_j \frac{d^2k_{j\perp}}{2\pi (2\pi)^3} \delta \left( x + \sum_{j=2}^A x_j - A \right) \delta^2 \left( k_\perp + \sum_{j=2}^A k_{j\perp} \right) \langle \psi(x, k_\perp; x_2, k_{2\perp}; \ldots; x_A, k_{A\perp}) \rangle^2
\]

(13)

where \( \psi \) is the standard light-cone wave function of the nucleus, symmetric in all the nucleons and normalized according to

\[
\int \prod_{j=1}^A dx_j \frac{d^2k_{j\perp}}{2\pi (2\pi)^3} \delta \left( \sum_{j=1}^A x_j - A \right) \delta^2 \left( \sum_{j=1}^A k_{j\perp} \right) |\psi(x_1, k_1\perp; x_2, k_{2\perp}; \ldots; x_A, k_{A\perp})|^2 = 1
\]

(14)

The derivation is presented in Appendix 1.

The light-cone wave function \( \psi \) can be related to the non-relativistic wave function by taking the non-relativistic limit in the normalization condition (14), which corresponds to \( x_j \to 1 \). In this limit (14) gives

\[
\int \prod_{j=2}^A d^3k_j |\psi(x, k_\perp; x_2, k_{2\perp}; \ldots; x_A, k_{A\perp})|^2 = (16\pi^3)^A m^{A - 1}
\]

(15)

where we introduced \( k_{jz} = m(x_j - 1) \) for \( j > 2 \). From this we conclude that in this limit

\[
\psi(x, k_\perp; x_2, k_{2\perp}; \ldots; x_A, k_{A\perp}) \simeq (16\pi^3)^{A/2} m^{(A - 1)/2} \phi(k_1, k_2, \ldots, k_A)
\]

(16)

where \( \phi \) is the non-relativistic wave function of the nucleus at rest. On the right-hand side \( k_j \) denote the 3-dimensional momenta, their sum equal to zero.
Returning to the structure function we find from (5)

\[ F^{(A)}(x, Q^2) = A \int_x^A \frac{dz}{z} F^{(N)} \left( \frac{x}{z}, Q^2 \right) \rho(z) \]  

(17)

where the distribution in \( z \) is just

\[ \rho(z) = \int \frac{d^2 k_\perp}{(2\pi)^2} \rho(z, k_\perp^2) \]  

(18)

normalized according to

\[ \int_0^A \frac{dz}{z} \rho(z) = 1 \]  

(19)

3 Cumulative vs. non-cumulative regions. Correlations

As indicated, blob \( \Phi(p, k) \) includes two external propagators for the active nucleon and so has a double pole at \( k^2 = m^2 \). If we neglect the binding, then with all recoil particle at rest we shall have \( k = p \) and so \( k^2 = m^2 \), which implies that the distributions \( \rho(z, k_\perp^2) \) and \( \rho(z) \) are strongly peaked at \( z = 1 \) and \( k_\perp = 0 \). These values correspond to the non-relativistic domain, where the density \( \rho \) is more or less known from the existing nuclear data at relatively small transferred momenta.

Going back to the nuclear structure function we observe from Eq. (17) that one can distinguish between two different kinematical situations. If \( x \leq 1 \) (non-cumulative region) then the integration region includes the vicinity of \( z = 1 \), whereas as we have just seen, \( \rho(z) \) has a sharp maximum. Then, as a first approximation, one can take \( F(x', Q^2) \) out of the integral at this point (i.e. at \( x' = x \)) and integrate the rest using the normalization condition (19), which gives

\[ F^{(A)}(x, Q^2) = AF^{(N)}(x, Q^2) \]  

(20)

One can improve this result by taking some form for \( \rho(z) \) and integrating over \( z \) in a proper way. Then one will get corrections to (20) (the EMC effect). It is worth noting that it is always possible to match the experimental data by appropriately choosing \( \rho \) at finite (relativistic) values of \( |z - 1| \).

However in the cumulative region \( x > 1 \) the point \( z = 1 \) stays outside the integration region in (17). This means that in this interval of \( x \) values of \( \rho \) are important which come from \( \Phi(p, k) \) with the active nucleon far off-shell, well outside the non-relativistic region. From the point of view of the latter, it is the far relativistic asymptotics of the nuclear dynamics which now matters.

The technique to study this asymptotics is in fact standard. Let us consider a simple example of the deuteron, for which \( \Phi(p, k) \) reduces to the square of the Bethe-Salpiter wave function \( \psi^{(d)}(k, 2p - k) \) satisfying the equation

\[ (m^2 - k_1^2)(m^2 - k_2^2)\psi^{(d)}(k_1, k_2) = \int \frac{d^2 k'_1}{(2\pi)^2} V(k_1, k_2; k'_1, k'_2)\psi^{(d)}(k'_1, k'_2) \]  

(21)
where $k_1 + k_2 = k'_1 + k'_2 = 2p$. In the vicinity of the mass shell $k_1^2 = k_2^2 = m^2$ this equation reduces to the ordinary Schrödinger equation and correspondingly $\psi^{(d)}$ reduces to its nonrelativistic limit $\psi^{(d)}_{NR}$. Now suppose we want to study the asymptotics of $\psi^{(d)}$ far off shell. A simple way do to it is to rewrite (22) as

$$\psi^{(d)}(k_1, k_2) = \frac{1}{(m^2 - k_1^2)(m^2 - k_2^2)} \int \frac{d^2 k'_1}{(2\pi)^4} V(k_1, k_2; k'_1, k'_2) \psi^{(d)}(k'_1, k'_2)$$

(22)

and take into account that the integration is dominated by the close to on-shell values of the momenta, that is, by the non-relativistic region. So inside the integral one can substitute $\psi^{(d)}$ by its non-relativistic limit and moreover take the interaction $V$ outside the integral at small values of the 3-momenta $k'_{1,2}$ corresponding to the nonrelativistic region. In this way one gets the desired asymptotics as

$$\psi^{(d)}(k_1, k_2) \sim \frac{1}{(m^2 - k_1^2)(m^2 - k_2^2)} V(k_1, k_2; p, p) \int \frac{d^2 k'_1}{(2\pi)^4} \psi^{(d)}_{NR}(k'_1, k'_2)$$

(23)

where $p = (m - \epsilon, \vec{0})$. In terms of diagrams this procedure corresponds to transforming the NND vertex as shown in Fig. 3, where it is assumed that the internal lines correspond to the non-relativistic deuteron and the external ones correspond to the relativistic deuteron.

For a nucleus consisting of more than two nucleons this procedure leads to the diagrams shown in Fig. 4a. If more than two nucleons are relativistic, it can be repeated, leading to diagrams like Fig. 4b. Following [4] one speaks of two- (Fig. 4a) or three- (Fig. 4b) or more-body correlations in the nucleus to describe two, three or more relativistic nucleons.

From the pure kinematical considerations one immediately concludes that the $k$-fold correlation gives a contribution to the distribution $\rho(x)$ in the region

$$0 \leq x \leq k$$

Equivalently, choosing a particular interval of $x$

$$n - 1 \leq x \leq n$$

(24)

the contribution to the nuclear structure function in this region comes from correlations of $k$ nucleons with $k \geq n$.

At large virtualities, with interaction and propagators falling like powers, one finds that the contribution from the $k$-fold correlations diminishes fast with the growth of $k$. The bulk of the contribution in the region (24) will then come from precisely the $n$-fold correlation, the ones with $k > n$ giving only a small correction. So, in the first approximation, to study the structure function in the region (24) one has to take into account only the $n$-fold correlation.

Up to now the cumulative nuclear structure function has been studied experimentally only in the region

$$1 < x < 2$$

(25)

This gives motivation to start from the pair correlations in the nucleus, which will be the subject of the next sections.


4 Two-body correlations

4.1 Scalar nucleons

To clearly formulate our approach we continue with the scalar nucleons case. As explained in the previous section, the contribution of the 2-body correlations to function $\Phi(p,k_1)$ corresponds to the diagram shown in Fig. 5 in which high- and low-momentum particles are indicated with thick and thin lines respectively. Explicitly the contribution of the diagram in Fig. 5 can be written as

$$2 \text{Im} \Phi(p,k_1) = \frac{1}{(m^2 - k^2_1)^2} \int \prod_{j=1}^2 \frac{d^4k'_j d^4k''_j}{(2\pi)^8} (2\pi)^4 \delta^4(k'_1 + k'_2 - k''_1 - k''_2)$$

$$2\pi \delta(k^2_2 - m^2) V(k_1, k_2|k'_1, k'_2)V(k_1, k_2|k''_1, k''_2) 2 \text{Im} \Phi_2(p; k'_1, k'_2; k''_1, k''_2)$$

(26)

Here $V$ is the relativistic (Bethe-Salpeter) potential and $\Phi_2$ corresponds to the lower blob with all intermediate states of $A-2$ nucleons.

The high-momentum part includes the first factor, potentials and the $\delta$-function coming from the real nucleon number 2. We can take the latter factors out of the integral at the point which corresponds to the initial nucleons at rest, that is at $k'_j = k''_j = p$, $j = 1, 2$ and $p = (m - \epsilon, \vec{0})$. Then the high-momentum part separates as a factor, so that we get

$$2 \text{Im} \Phi(p, k_1) = 2\pi \delta((k_1 - 2p)^2 - m^2) A^2(A-1) w_2 H(p, k_1).$$

(27)

Here the high-momentum factor $H$ is

$$H(p, k_1) = H(k^2_1) = \frac{V^2(k_1, 2p - k_1|p, p)}{(m^2 - k^2_1)^2}$$

(28)

($H$ depends only on $k^2_1$ since the product $pk_1$ can be related to it from the condition $k^2_2 = (2p - k_1)^2 = m^2$). Factor $w_2$ is just a constant which accumulates all the information from the nucleus:

$$A^2(A-1) w_2 = \int \prod_{j=1}^2 \frac{d^4k'_j d^4k''_j}{(2\pi)^8} (2\pi)^4 \delta^4(k'_1 + k'_2 - k''_1 - k''_2) 2 \text{Im} \Phi_2(p; k'_1, k'_2; k''_1, k''_2)$$

(29)

The calculation of $w_2$, is more or less standard. Similarly to Im $\Phi$, the imaginary part of $\Phi_2$ can be presented in the form

$$2 \text{Im} \Phi_2(p; k_1, k_2; \tilde{k}_1, \tilde{k}_2) = \int d^4r d^4\tilde{r} e^{i R K + i R (k_1 - K/2) - i R (\tilde{k}_1 - K/2)}$$

$$\langle A|T\{N^\dagger(R + r/2)N^\dagger(R - r/2)\}T\{N(\tilde{r}/2)N(-\tilde{r}/2)\}|A \rangle$$

(30)

where $K = k_1 + k_2 = \tilde{k}_1 + \tilde{k}_2$ and, as before, $N(N^\dagger)$ are the annihilation (creation) operators of the nucleon field and $|A\rangle$ is the nucleus state at rest. Putting this representation into (29) we find that the momentum integration puts all coordinates to zero:

$$A^2(A-1) w_2 = \langle A|(N^\dagger(0))^2 N^2(0)|A \rangle$$

(31)
To relate this expression with the standardly defined quantities, we note that all particles may be considered non-relativistic. So we may pass to non-relativistic states and operators.

\[ |A⟩ = \sqrt{2Am}|A_{NR}⟩, \quad N^\uparrow(r) = \frac{1}{\sqrt{2m}}N_{NR}^\uparrow(N_{NR})(r) \]  

Thus in terms of non-relativistic quantities we get (suppressing subindex NR)

\[ A^2(A - 1)w_2 = \frac{A}{2m}(A|N^\uparrow(0)|^2N^\downarrow(0)) = \frac{A^2(A - 1)}{2m} \int \prod_{i=3}^A d^3r_i |\psi^{(A)}(0,0,r_3,...r_A)|^2 \]  

where \( \psi^{(A)} \) is the non-relativistic wave function of the target nucleus. It appears at zero distance between the 1st and 2nd nucleons, which is physically natural and corresponds to a correlation between them. To express (33) via better known quantities, one may take into account the translational invariance by presenting

\[ |\psi^{(A)}(r_1,r_2,r_3,...r_A)|^2 = \int d^3r \rho^{(A)}(r_1 + r, r_2 + r, ...r_A + r) \]  

with the nuclear \( \rho \)-matrix normalized as

\[ \int \prod_{i=1}^A d^3r_i \rho^{(A)}(r_1,...r_A) = 1 \]  

In terms of this \( \rho \)-matrix we find from (33)

\[ w_2 = \frac{1}{2m} \int d^3r \rho^{(A)}(r) \]  

where

\[ \rho(r_1,r_2) = \int \prod_{i=3}^A d^3r_i \rho^{(A)}(r_1,r_2,r_3,...r_A) \]  

and is normalized to unity after integration over \( r_1 \) and \( r_2 \). The form (34) admits presenting \( \rho^{(A)} \) as a product of single-nucleon distributions. In this approximation

\[ w_2 = \frac{1}{2m} \int d^3r (\rho^{(A)}(r))^2 \]  

and thus is directly expressed via the nuclear density.

4.2 Distributions in \( x \) and \( k_z^2 \)

According to (11), to find the distributions in \( z \) and \( k_z^2 \) we have to integrate the obtained function \( \Phi(p,k) \) over \( k_− \). This integration is trivial due to the \( \delta \)-function in (27). It fixes the value of \( k^2 \) to be

\[ k^2 = zm^2 \frac{3 - 2z}{2 - z} - k_z^2 \frac{2}{2 - z} - \Delta \frac{2z}{2 - z} \]  

(39)
One finds that $\rho(z, k^2_\perp)$ is different from zero in the region (see Appendix 2)

$$k^2_\perp \geq \Delta^2 \left(2 - z - \frac{2z}{A - 2}\right) - m^2(z - 1)^2$$  \hspace{1cm} (40)

This condition is operative only if we take into account the binding energy $\Delta^2/m$. Otherwise it is satisfied at all $k_\perp$ and $1 < z < 2$.

The internucleon relativistic potential can be taken as a sum of one-boson exchange contributions

$$V(k_1, k_1|k'_1, k'_2) = \sum_i \frac{g^2_i}{\mu^2_i - q^2}$$  \hspace{1cm} (41)

where $q = k_1 - k'_1$ is the momentum transfer. In our case $k'_1 = k'_2 = p$, $k_1 = k$ and $k_2 = 2p - k$. So one easily finds

$$q^2 = \frac{1}{2}(k^2_1 - m^2) + \Delta^2$$  \hspace{1cm} (42)

The distribution in $z$ and $k^2_\perp$ is obtained directly from (11):

$$\rho(z, k^2_\perp) = w_2 \frac{z}{4\pi(2 - z)}H(k^2) = w_2 \frac{z}{4\pi(2 - z)} \frac{1}{(m^2 - k^2)^2} \sum_i \frac{g^2_i g^2_i}{\mu^2_i - q^2(\mu^2_i - q^2)}$$  \hspace{1cm} (43)

where $k^2$ and $q^2$ are expressed via $z$ and $k^2_\perp$ by Eqs. (39) and (42) and $w_2$ is given by (38).

The remaining task is to integrate over $k^2_\perp$ to obtain the final distribution in $z$, which is straightforward. Rather than discuss it we proceed to the realistic spinor case.

### 4.3 Spinor nucleons

A rigorous treatment of the spinor nucleon case requires knowledge both of the spinor structure of the virtual photon-nucleon amplitude and the nucleus wave function, which is far beyond present (and future) possibilities. For this reason it is natural to use spin-averaged quantities for both quantities. More concretely we separate the integration over the virtual nucleon momentum and present the contribution from the diagram in Fig. 1 (right-hand side of Eq. (4)) in the form

$$D = \int d^4k \text{Sp}\{\rho + \hat{k})U(q, k)(\rho + \hat{k})L(p, k)\}$$  \hspace{1cm} (44)

where $U(p, k)$ is the upper blob (virtual $\gamma$-N amplitude) and $L(p, k)$ is the lower blob (with factor $1/(m^2 - k^2)^2$ included). Our approximation is then to take

$$\text{Sp}\{(\rho + \hat{k})U(q, k)(\rho + \hat{k})L(p, k)\} \simeq \frac{1}{2}\text{Sp}\{(\rho + \hat{k})U(p, k)\}\text{Sp}\{(\rho + \hat{k})L(p, k)\}$$  \hspace{1cm} (45)

which amounts to taking spin averaged values multiplied by the number of the spin states (two). After this averaging is made, all our formulas derived for the scalar case remain valid also for spinor nucleons.
Let us apply this recipe for the contribution from the two-nucleon correlation, corresponding to Fig. 5. In the spinor case instead of Eq. (26) we find for the function $2 \text{Im} \Phi(p, k_1)$:

$$2 \text{Im} \Phi(p, k_1) = \frac{1}{(m^2 - k_1^2)^2} \int \prod_{j=1}^{2} \frac{d^4k'_j d^4k''_j}{(2\pi)^8} \frac{1}{2\pi}\delta^4(k'_1 + k'_2 - k''_1 - k''_2)2\pi\delta(k_2^2 - m^2)$$

$$\text{Sp} \left\{ 2 \text{Im} \Phi_2(p; k'_1, k'_2; k''_1, k''_2)(m + \hat{k}_1)(m + \hat{k}_2) \right\} (C(k_1, k_2) | k'_1, k'_2; k''_1, k''_2)$$

where $C$ is the two-nucleon correlation blob:

$$C(k_1, k_2 | k'_1, k'_2; k''_1, k''_2) = V(k'_1, k'_2 | k_1, k_2) (m + \hat{k}_1)(m + \hat{k}_2)V(k'_1, k'_2 | k''_1, k''_2)$$

and $V$ is the relativistic interaction. Both $C$ and $V$ are matrices in spinor indices of both nucleons. Operators $(m + \hat{k})_{1(2)}$ act on the spinor indices of 1st (2nd) nucleon. The trace is to be taken over spinor indices of both nucleons. In deriving (46) we have applied (45) to average over the spin states of the active nucleon (momentum $k_1$). Note that the factor $1/2$ is to be included in the definition of the spin-averaged nucleon structure function $F$ in Eq. (5).

Now we use the fact that the momenta $k'_1, k'_2, k''_1$ and $k''_2$ are non-relativistic and also apply the recipe (45) two more times to sum over spin states of the nucleons 1 and 2 before the left interaction and after the right interaction in Fig. 5 to obtain a representation identical to (27) where now

$$H(k_1^2) = \frac{1}{4} \text{Sp} \left\{ (m + \hat{p})_1 (m + \hat{p})_2 C(k_1, 2p - k_1 | p, p, p) \right\},$$

and

$$A^2(A - 1)w_2 = \int \prod_{j=1}^{2} \frac{d^4k'_j d^4k''_j}{(2\pi)^8} \frac{1}{2\pi}\delta^4(k'_1 + k'_2 - k''_1 - k''_2) \text{Sp} \left\{ 2 \text{Im} \Phi_2(p; k'_1, k'_2; k''_1, k''_2)(m + \hat{p})_1 (m + \hat{p})_2 \right\}$$

In the latter expression in the non-relativistic approximation we may write

$$\text{Sp} \left\{ 2 \text{Im} \Phi_2(p; k'_1, k'_2; k''_1, k''_2)(m + \hat{p})_1 (m + \hat{p})_2 \right\} \approx \frac{1}{4m^2} \text{Sp} \left\{ (m + \hat{k}_1)(m + \hat{k}_2) 2 \text{Im} \Phi_2(p; k'_1, k'_2; k''_1, k''_2)(m + \hat{k}_1)(m + \hat{k}_2) \right\}$$

$$= \int d^4r d^4\tilde{r} e^{iRK + i\tilde{r}(k'_1 - K/2) - i\tilde{r}(k''_1 - K/2)} \text{Sp} \left\{ A | T \{ N^\dagger (R + r/2) N^\dagger (R - r/2) \} T \{ N(\tilde{r}/2) N(-\tilde{r}/2) \} | A \right\}$$

where now $N(N^\dagger)$ are 4-spinor operators of the nucleon field. In the non-relativistic limit their first two components reduce to non-relativistic spinor fields for the annihilation (creation) of a nucleon with a given spin component at a given point, the second two reducing to zero. As
in (30), the integration over the momenta in (50) puts all coordinates to zero and we obtain similarly to (33)

\[ A^2 (A - 1) w_2 = \frac{A}{2m} \sum_{\sigma, \sigma_2 = 1}^2 \langle A | (N_{\sigma_1}^\dagger (0) N_{\sigma_2}^\dagger (0) N_{\sigma_1} (0) N_{\sigma_2} (0)) \rangle \]

\[ = \frac{A^2 (A - 1)}{2m} \int \prod_{i=3}^A d^3 r_i \prod_{j=1}^A \sum_{\sigma_j = 1}^2 |\psi_{\sigma_1, \sigma_2, ..., \sigma_A} (0, 0, r_3, ... r_A)|^2 \quad (51) \]

where \(\sigma\)'s denote non-relativistic spin projections of the nucleons. Obviously the integral on the right-hand side has the same meaning as in the scalar case: the total probability to have the two active nucleons at the origin irrespective of their spins. Presenting the spin-summed square modulus of the nuclear wave function in the form (34) one arrives at the same formulas (36) or (38) as for the scalar case.

So in the end the only difference from the scalar case is the expression (48) for the function \(H\) as a spin average over the two-nucleon correlation contribution given by Eq. (47) in terms of the relativistic internucleon interaction.

## 5 Numerical results

Calculation of the cumulative nuclear structure function clearly splits into three steps. First the density \(\rho(x, k)\) has to be determined from the given relativistic internucleon potential \(V\). Second this density has to be integrated over the transverse momenta to find the resulting distribution in \(x\). Finally one should convolute this distribution with the known structure function of the nucleon.

To determine \(\rho(x, k)\) one first should calculate the two-nucleon correlation factor \(H\) according to Eqs. (47) and (48). We borrow the relativistic internucleon potential \(V\) from [5], where it is represented by a sum of \(\pi, \eta, \sigma, \omega\) and \(\rho\) exchanges, modified by form-factors to cut the high momentum contribution and adapted to off-shell nucleons. In fact four different parametrizations of the potential are presented in [5]. They are given in Appendix 1 together with the concrete form of various contributions. As we shall see different parametrizations give practically the same results for the cumulative densities and structure functions.

The calculation of \(H\) is rather tedious due to many different terms and relativistic traces. Some intermediate formulas are given in Appendix 2.

The nuclear factor \(w_2\) was calculated from the standard Woods-Saxon nuclear density. Obviously it is of the order \(1/A\) and has a dimension of mass squared. Correspondingly in Fig. 6 we show its values multiplied by \((A - 1)/m^2\) for different nuclei. The resulting values steadily grow with \(A\). Their \(A\)-dependence does not correspond to \(A^\alpha\): one finds that in this parametrization \(\alpha\) goes down from 0.3 to 0.15 as \(A\) rises from 20 to 200.

Integrating the found \(\rho(x, k)\) over the transverse momenta we found the distribution \(\rho(x)\). In Fig. 7 we show its values for Carbon with two potential models from [5]: IA and IIA. One observes that up to \(x \sim 1.3\) both potentials gives the same \(\rho(x)\). At higher \(x\) model IA gives values for \(\rho(x)\) higher by factor \(1.5 \div 3\), which is quite a small difference on the cumulative scale.
The resulting cumulative structure function of Carbon are presented in Figs. 8 and 9 at $Q^2 = 3, 30$ and $1000$ (Gev/c)$^2$ for potential models IA and IIA respectively. For the nucleon structure function $F(x,Q^2)$ the GRV LO parametrization has been taken from [6]. The results are quite similar for the two potential models, both in form and magnitude. One observes that the cumulative structure function turns out to be rather weakly dependent on $Q^2$: at moderate $x$ it falls with $Q^2$ by a factor of the order 4, as $Q^2$ rises from 3 to 1000 (Gev/c)$^2$. As to the $x$ dependence it is evidently non-exponential, although universal in $Q^2$, the slope of $F(x,A)$ changing from 10 to 40 as $x$ rises from 1.2 to 1.7.

Comparison with the experimental data from [2] is presented in Table 1. One observes that at $x = 1.05$ and 1.15 our results lie above the data by roughly 100% and 50% respectively. This disagreement is not so dramatic considering the steep fall of the structure function beyond $x = 1$. One has also take into account that these values of $x$ are too close to unity for our formalism to work well.

Table 1. Structure Function of Carbon ($x 10^6$) in different potential models of [5]

| $x$ | $Q^2$ (Gev/c)$^2$ | IA  | IB  | IIA | IIIB | Exp  |
|-----|------------------|-----|-----|-----|------|------|
| 1.05| 61.0             | 162.0 | 153.0 | 126.0 | 153.0 | 64.1±7.4 |
|     | 85.0             | 150.0 | 142.0 | 116.0 | 142.0 | 47.8±5.2 |
|     | 150.0            | 133.0 | 126.0 | 103.0 | 125.0 | 45.8±5.1 |
| 1.15| 61.0             | 39.5  | 40.7  | 30.9  | 39.7  | <27.0 |
|     | 85.0             | 36.4  | 37.6  | 28.5  | 36.7  | <15.1 |
|     | 150.0            | 32.0  | 33.1  | 25.0  | 32.3  | <22.3 |
| 1.30| 61.0             | 3.65  | 4.15  | 2.71  | 3.89  | <6.9  |
|     | 85.0             | 3.35  | 3.81  | 2.48  | 3.56  | <5.4  |
|     | 150.0            | 2.90  | 3.30  | 2.15  | 3.09  | <3.8  |

6 Conclusions

We have developed a method which allows to calculate the nuclear structure function at $x > 1$ in terms of nucleonic degrees of freedom with full relativistic kinematics. The method avoids solving the complete relativistic many-body problem and relates the structure function directly to the high-momentum asymptotics of the relativistic potentials. While for the 2-body correlations it can be considered as a simplification of the solution of the Bethe-Salpeter or Gross equations, for the correlations involving more nucleons it seems to be the only realistic approach.

The results of our calculations confirm that the nuclear structure function at $x > 1$ depends on $Q^2$ only weakly. Its $x$-behaviour cannot be well desribed by an exponential. Comparison to (scarce) experimental data for C at $x = 1.05$ and 1.15 shows that our results have the same order of magnitude, lying above the experimental data by 100% and 50% respectively. This disagreement may be completely explained by the comparatively low values
of $x$ where our formalism is not supposed to work well. One should wait for more data at higher $x$ to draw final conclusions as to the validity of the chosen picture and the potentials.

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8 References

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9 Figure captions

Fig. 1. The impulse approximation for the nuclear structure function.
Fig. 2. The impulse approximation for the nuclear form-factor.
Fig. 3. The asymptotics of the Bethe-Salpeter function from an iteration of the acting potential.
Fig. 4. The two-nucleon (a) and three-nucleon (b) correlations in the nucleus.
Fig. 5. The contribution from the two-nucleon correlation (Eqs. (26) and (46)).
Fig. 6. The nuclear factor $(A - 1)w_2/m^2$ as a function of $A$.
Fig. 7. The distribution $\rho(x)$ for $^{12}$C in the cumulative region for potential models IA (upper curve) and IIA.
Fig. 8. The cumulative structure function of $^{12}$C with the potential model IA. Curves from top to bottom correspond to $Q^2 = 3, 10, 30, 100$ and 1000 (GeV/c)$^2$.
Fig. 9. Same as Fig. 8 with a potential model IIA.
Fig. 3
Fig. 4
Fig. 1
Fig. 5
Potential Models IA and IIA

CARBON
$F_{2A}(x, Q^2)$

Potential Model IA

CARBON

$Q^2 = 3 \ (GeV/c)^2$

$Q^2 = 1000 \ (GeV/c)^2$
$F_{2A}(x,Q^2)$

- $Q^2 = 3 \text{ (GeV/c)}^2$
- $Q^2 = 1000 \text{ (GeV/c)}^2$

CARBON

Potential Model IIA

$x$

$F_{2A}(x,Q^2)$

$Q^2 = 1000 \text{ (GeV/c)}^2$

$Q^2 = 3 \text{ (GeV/c)}^2$