Common-Resolution Convolution Kernels for Space- and Ground-Based Telescopes

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Received 2011 June 28; accepted 2011 August 10; published 2011 September 30

ABSTRACT. Multiwavelength study of extended astronomical objects requires combining images from instruments with differing point-spread functions (PSFs). We describe the construction of convolution kernels that allow one to generate (multiwavelength) images with a common PSF, thus preserving the colors of the astronomical sources. We generate convolution kernels for the cameras of Spitzer, Herschel Space Observatory, Galaxy Evolution Explorer (GALEX), Wide-field Infrared Survey Explorer (WISE), ground-based optical telescopes (Moffat functions and sum of Gaussians), and Gaussian PSFs. Kernels for other telescopes including IRAS, AKARI, and Planck, are currently being constructed. These kernels allow the study of the spectral energy distribution (SED) of extended objects, preserving the characteristic SED in each pixel. The convolution kernels and the IDL packages used to construct and use them are made publicly available.

Online material: color figures

1. INTRODUCTION

Spectral energy distribution studies of astronomical objects provide insight into the ongoing physical processes. In order to achieve a wide range of wavelengths, it is often necessary to combine observations from cameras with very different point-spread functions (PSFs), in some cases with full width at half-maximum (FWHM) differing by factors of 100. Direct comparison (e.g., ratios) of images with structures on multiple angular/spatial scales obtained with different PSFs can result in unphysical intensity ratios (i.e., colors). To preserve colors, intensity ratios should be calculated from images with a common PSF. We therefore require convolution kernels that will transform the images taken with several instruments into a common PSF, so we can generate image cubes (i.e., a collection of images expressed in the same sky coordinates grid) in which each pixel corresponds to the same sky region for all the cameras used.

By “camera” we refer to the combination of telescope optics and physical detector, including the effect of atmospheric “seeing” if applicable. The PSF \( \Psi_j(x, y, x', y') \) of a camera \( j \) gives the measured intensity at \( (x, y) \) produced by a point source with unit flux at the point \( (x', y') \), where we use the Cartesian coordinates \( (x, y) \) to denote positions in a small region of the sky. With this definition, the PSF has normalization

\[
\iint \Psi_j(x, y, x', y') dx dy = 1, \tag{1}
\]

for any source position \( (x', y') \).

It is often possible to approximate the PSF (denoted as \( \Psi \) from now on) as constant across the useful field of view of the camera, so \( \Psi_j(x, y, x', y') = \Psi_j(x - x', y - y') \). The observed image \( I_j(x, y) \) will then be the convolution of the source \( S(x, y) \) with the PSF \( \Psi \):

\[
I_j(x, y) = \iint S(x', y') \Psi_j(x - x', y - y') dx' dy'. \tag{2}
\]

Clearly, given two cameras \( A \) and \( B \), with (different) PSFs \( \Psi_A \) and \( \Psi_B \), the images obtained of an astronomical object will be different, even if the spectral response of the cameras were identical. A convolution kernel is a tool that transforms the image observed by one camera into an image corresponding to the PSF of another camera.

The convolution kernel \( K \{ A \Rightarrow B \} \) from camera \( A \) to camera \( B \) should satisfy

\[
I_B(x, y) = \iint I_A(x', y') K \{ A \Rightarrow B \}(x - x', y - y') dx' dy'. \tag{3}
\]

where \( I_A \) and \( I_B \) are the observed images by the cameras \( A \) and \( B \), respectively.

The actual PSF of an instrument will show variations with the source color, variations along the field of view, changes over time, and deviations from rotational symmetry. Full 2D characterization of a PSF is extremely challenging, and its extended wings are often not well determined. To take into account

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deviations of PSFs from rotational symmetry would require separate kernels $K\{ A \Rightarrow B \}(\gamma)$ for each relative orientation $\gamma$ of cameras $A$ and $B$. In the present study, the actual PSFs are close enough of having rotational symmetry that such additional complication is not justified (see § 3 for a detailed study). The current work assumes that $\Psi_A$ and $\Psi_B$ can be approximated by rotationally symmetric functions.

Using a different technique, Alard & Lupton (1998) presented a method for finding optimal kernels to convolve (to a common resolution) images of a sky region taken with a single camera under different seeing conditions. Using techniques similar to those used here, Gordon et al. (2008) created kernels for the Infrared Array Camera (IRAC) and Multiband Imaging Photometer for Spitzer (MIPS) cameras of Spitzer, and Sandstrom et al. (2009) created kernels for the Infrared Spectrograph (IRS) of Spitzer. In the present work, we use the latest characterization of the PSFs of the cameras on Spitzer (IRAC and MIPS), Herschel Space Observatory (Photocamera Array Camera and Spectrometer for Herschel [PACS] and Spectral and Photometric Imaging Receiver for Herschel [SPIRE]), Galaxy Evolution Explorer (GALEX) (FUV, NUV), and the Wide-field Infrared Survey Explorer (WISE) (W1–W4). Additional PSFs (including those characterizing the IRS spectrograph on board Spitzer; the PACS spectrographs on board the Herschel Space Observatory; and all-sky images produced by IRAS, AKARI, and Planck) are currently being constructed and will be added to the kernel library. We also include a family of analytical PSF profiles that are commonly used to model the PSFs for ground-based telescopes. We construct the set of kernels to transform from different instrumental PSFs into Gaussian PSFs, another form that is widely used. We find an optimal Gaussian PSF for each camera: i.e., a Gaussian PSF that is sharp enough to capture the angular resolution of the camera and wide enough to be robust against image artifacts.

Additional image processing (i.e., co-adding images) or different data reduction schemes (i.e., the Scanamorphos pipeline [Roussel 2011] for the Herschel images) will alter the PSF, and thus new kernels should be constructed using the effective PSF.

This article is organized as follows. In § 2 we describe the generation of convolution kernels, in § 3 we describe the PSF used, and in § 4 we describe the kernel construction strategy. The performance of the kernels is examined in § 5, and in § 6 we describe a set of Gaussian kernels that are compatible with the different instruments. In § 7 we describe the kernel usage and show their performance on NGC 1097, and we summarize in § 8.

All the kernels, IDL routines to use the kernels, and IDL routines to make new kernels, along with detailed analysis of the generated kernels, are publicly available.

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2. CONVOLUTION KERNELS

Given two cameras $A$ and $B$, with PSFs $\Psi_A$ and $\Psi_B$, we seek $K\{ A \Rightarrow B \}$ that fulfills equation (3) for any astronomical source. Thus,

$$\langle S \ast \Psi_B \rangle = I_B = (I_A \ast K\{ A \Rightarrow B \}) = \langle S \ast \Psi_A \ast K\{ A \Rightarrow B \} \rangle$$

(4)

for any astronomical source $S$, so the convolution kernel must satisfy

$$\Psi_B = (\Psi_A \ast K\{ A \Rightarrow B \}).$$

(5)

With the normalization condition given by equation (1), the kernel must have

$$\iint K\{ A \Rightarrow B \}(x,y)dxdy = 1.$$  

(6)

We can easily invert equation (5) in Fourier space; taking the two-dimensional Fourier transform (FT) of equation (5)

$$\text{FT}(\Psi_B) = \text{FT}(\Psi_A \ast \Psi_B \ast K\{ A \Rightarrow B \})$$

$$= \text{FT}(\Psi_A) \times \text{FT}(K\{ A \Rightarrow B \}).$$

(7)

This can be inverted to obtain

$$K\{ A \Rightarrow B \} = \text{FT}^{-1} \left( \frac{\text{FT}(\Psi_B)}{\text{FT}(\Psi_A)} \right).$$

(8)

where FT and FT$^{-1}$ stand for the Fourier transform and its inverse transformation, respectively.

Equation (8) provides a condition for the existence of such a kernel and a practical way of computing it. We can see that a condition for the existence of a kernel is that the Fourier components for which FT($\Psi_A$) = 0 should satisfy FT($\Psi_B$) = 0. Informally speaking, this means that the PSF of camera $A$ must be narrower than the PSF of camera $B$.

For each camera $A$, we identify a high-frequency cutoff $k_{H,A}$ as the highest spatial frequency for which FT($\Psi_A$) is appreciable by setting

$$\text{FT}(\Psi_A)(k_{H,A}) = 5 \times 10^{-3} \times \max[\text{FT}(\Psi_A)].$$

(9)

The cutoff frequency $k_{H,A}$ can be normalized as $k_{H,A} = \kappa_A \times 2\pi$/FWHM$_A$. The values of $\kappa_A$ are in the range of 1.08–1.46 and are given in Table 1.

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4 See http://www.astro.princeton.edu/~draine/Kernels.html. Kernels for additional cameras and updates will be included when new PSF characterizations become available.
Equation (8) also provides insight of a possible problem in computing kernels. PSF Fourier transforms do not have significant power at (spatial) frequencies above the $2\pi$/FWHM. The high-frequency components of the FT will be small, introducing large uncertainties when inverted.

A possible way of avoiding problems in the high-frequency components of the kernel is to introduce a filter $f_A$ in the kernel construction:

$$K\{A \Rightarrow B\} = \text{FT}^{-1}\left(\text{FT}(\Psi_B) \times \frac{1}{\text{FT}(\Psi_A)} \times f_A\right), \quad (10)$$

where $f_A$ is a suitable low-pass filter. Because this differs from equation (8), it is clear that a kernel satisfying equation (10) will, in general, not be an exact solution to equation (8).

However, we can expect that if the filter $f_A$ does not remove significant power from the high-frequency components of either $\text{FT}(\Psi_A)$ or $\text{FT}(\Psi_B)$, then the kernel computed will be a good approximate solution to equation (8).

We use a filter $f_A$ of the form

$$f_A(k) = \begin{cases} 
1 & \text{for } k \leq k_{L,A} \\
\frac{1}{2} \times \left[1 + \cos\left(\pi \times \frac{k - k_{L,A}}{k_{H,A} - k_{L,A}}\right)\right] & \text{for } k_{L,A} \leq k \leq k_{H,A}, \\
0 & \text{for } k_{H,A} \leq k 
\end{cases} \quad (11)$$

where we set $k_{L,A} = 0.7 \times k_{H,A}$. Note that the cutoff frequency of our filter $f_A$ depends only on camera $A$, since small values of $\text{FT}(\Psi_B)$ have no negative impact in the kernel construction. In

| Camera | Rayleigh diff. limit * | Measured FWHM | 99% of energy radius | Asymmetry |
|--------|------------------------|--------------|---------------------|-----------|
| IRAC 3.6 μm | 1.04 | 1.90 | 62.52 | 1.29 | 0.16 |
| IRAC 4.5 μm | 1.31 | 1.81 | 64.46 | 1.26 | 0.17 |
| IRAC 5.8 μm | 1.68 | 2.11 | 133.55 | 1.20 | 0.19 |
| IRAC 8.0 μm | 2.30 | 2.82 | 114.20 | 1.19 | 0.18 |
| MIPS 24 μm | 6.93 | 6.43 | 224.53 | 1.05 | 0.08 |
| MIPS 70 μm | 20.90 | 18.74 | 461.44 | 1.12 | 0.05 |
| MIPS 160 μm | 45.62 | 38.78 | 678.77 | 1.10 | 0.05 |
| PACS 70 μm | 5.11 | 5.67 | 249.81 | 1.23 | 0.20 |
| PACS 100 μm | 7.28 | 7.04 | 350.63 | 1.19 | 0.20 |
| PACS 160 μm | 11.70 | 11.18 | 417.36 | 1.21 | 0.20 |
| SPIRE 250 μm | 17.93 | 18.15 | 205.07 | 1.16 | 0.19 |
| SPIRE 350 μm | 25.16 | 24.88 | 192.47 | 1.15 | 0.18 |
| SPIRE 500 μm | 36.22 | 36.09 | 198.43 | 1.16 | 0.19 |
| GALEX FUV | 0.08 | 4.48 | 50.28 | 1.26 | 0.07 |
| GALEX NUV | 0.11 | 5.05 | 39.56 | 1.32 | 0.05 |
| WISE 3.35 μm | 2.11 | 5.79 | 19.08 | 1.33 | 0.13 |
| WISE 4.60 μm | 2.89 | 6.37 | 19.08 | 1.33 | 0.13 |
| WISE 11.56 μm | 7.27 | 6.60 | 19.56 | 1.23 | 0.12 |
| WISE 22.1 μm | 13.90 | 11.89 | 35.15 | 1.05 | 0.07 |
| Gauss 12° | ... | 12.00 | 15.41 | 1.33 | 0.0 |
| Gauss 20° | ... | 20.00 | 25.68 | 1.33 | 0.0 |
| Gauss 23° | ... | 23.00 | 29.53 | 1.33 | 0.0 |
| Gauss 28° | ... | 28.00 | 35.95 | 1.33 | 0.0 |
| Gauss 40° | ... | 40.00 | 51.33 | 1.33 | 0.0 |
| Gauss 50° | ... | 50.00 | 64.05 | 1.33 | 0.0 |
| BiGauss 0.5° | ... | 0.50 | 0.90 | 1.37 | 0.0 |
| BiGauss 1.0° | ... | 1.00 | 1.79 | 1.37 | 0.0 |
| BiGauss 1.5° | ... | 1.50 | 2.69 | 1.37 | 0.0 |
| BiGauss 2.0° | ... | 2.00 | 3.57 | 1.37 | 0.0 |
| BiGauss 2.5° | ... | 2.50 | 4.44 | 1.37 | 0.0 |
| Moffat 0.5° | ... | 0.50 | 1.39 | 1.46 | 0.0 |
| Moffat 1.0° | ... | 1.00 | 2.77 | 1.46 | 0.0 |
| Moffat 1.5° | ... | 1.50 | 4.12 | 1.46 | 0.0 |
| Moffat 2.0° | ... | 2.00 | 5.45 | 1.46 | 0.0 |
| Moffat 2.5° | ... | 2.50 | 6.74 | 1.46 | 0.0 |

* We take the Rayleigh diffraction limit as $1.22 \times \text{central} \lambda / \text{telescope diameter}$.  
* We define $\kappa$ as $(k_H \times \text{FWHM}_A)/(2\pi)$, where $k_H$ is the high-frequency cutoff (see text for details).  
* The parameter $g$ is a measure of the departure of a PSF from rotational symmetry (see eq. [12]).
principle, any smoothly varying function that is close to 1 in the low-frequency range and goes to zero in frequencies larger than \( k_{H,A} \) should work as well. We have experimented using several smoothing functions and find that the particular form of \( f_A(k) \) given by equation (11), with these particular choices of \( k_{L,A} \) and \( k_{H,A} \), gives excellent results. More details of the smoothing function effects can be found in § 5.

If the cutoff frequencies associated with cameras \( A \) and \( B \) are such that \( k_{H,B} \leq k_{H,A} \times 0.7 \), then the filter \( f_A \) has little effect and the resulting kernel from equation (10) will satisfy equation (3) to a very good approximation. In the regime \( k_{H,A} \times 0.7 \leq k_{H,B} \leq k_{H,A} \) the kernel \( K\{A \Rightarrow B\} \) will depend on the exact form of the filter \( f_A \) used. Most of the kernels in this regime have good performance; the performance of such “filtered” kernels will be evaluated in § 5. A limiting case is a kernel \( K\{A \Rightarrow A\} \) that transforms a PSF into itself; it is the Fourier transform of the filter \( f_A \).

In the cases of \( k_{H,A} < k_{H,B} \) (convolving into narrower PSFs), use of the filter \( f_A \) allows one to compute convolution kernels, but their performance can be poor. This will be further discussed subsequently.

3. INSTRUMENTAL POINT-SPREAD FUNCTIONS

We generate appropriate kernels for the measured PSFs of the cameras on Spitzer, Herschel, GALEX, and WISE and for certain analytical PSFs that are used in the literature:

IRAC.—The Infrared Array Camera (Fazio et al. 2004) has four infrared bands, centered at 3.6 \( \mu \)m, 5.0 \( \mu \)m, 5.8 \( \mu \)m, and 8.0 \( \mu \)m. We use the in-flight extended point response functions (PRFs).\(^7\) The core portion of the PSF was measured with 300 observations of a calibration, with different exposure time, combined into a high dynamic range image. Observations of the stars Vega, \( \epsilon \) Eridani, Fomalhaut, \( \epsilon \) Indi, and Sirius were used in the construction of the extended wings of the PSF.\(^8\)

MIPS.—The Multiband Imaging Photometer for Spitzer (Rieke et al. 2004) has three photometric infrared bands, centered at 24 \( \mu \)m, 70 \( \mu \)m, and 160 \( \mu \)m. Following Engelbracht et al. (2007), Gordon et al. (2007), and Stansberry et al. (2007), we generate theoretical PSFs for the MIPS cameras, assuming a blackbody source at \( T = 25 \) K. The PSFs are generated using the software STinyTim\(^9\) in a 0.5” grid, and they are smoothed with a square kernel with sides of 4.5”, 13.5”, and 25.5” for the bands at 70 \( \mu \)m, 100 \( \mu \)m, and 160 \( \mu \)m respectively. The smoothing sizes correspond to 1.6, 1.35, and 1.8 times the camera detector pixel size, and they should cause the core of the theoretical PSF to be in close agreement with the calibration point-source images.

PACS.—The Photocamera Array Camera and Spectrometer for Herschel (Poglitsch et al. 2010) has three photometric infrared bands, centered at 70 \( \mu \)m, 100 \( \mu \)m, and 160 \( \mu \)m. We use the in-flight PSF (Geis & Lutz 2010; Lutz 2010; Müller 2010); the core was defined by observations of the star \( \alpha \) Tau and the asteroid Vesta, with extended wings reconstructed from (saturated) observations of Mars, Neptune, IK Tau, and the Red Rectangle. The azimuthally averaged encircled energy fraction of the PSFs out to 1000” were obtained from the HCSS/HiPE software.

SPIRE.—The Spectral and Photometric Imaging Receiver for Herschel (Griffin et al. 2010) has three photometric far-infrared bands, centered at 250 \( \mu \)m, 350 \( \mu \)m, and 500 \( \mu \)m.\(^10\)

GALEX.—The Galaxy Evolution Explorer (Martin et al. 2005) has two ultraviolet bands, FUV (1350–1750 Å) and NUV (1750–2800 Å).\(^11\)

WISE.—The Wide-field Infrared Survey Explorer (Wright et al. 2010) has four photometric infrared bands, centered at 3.4 \( \mu \)m, 4.6 \( \mu \)m, 12 \( \mu \)m, and 22 \( \mu \)m. The PSF shape varies significantly over the focal plane due to distortion from the telescope optics; so a library of PSFs corresponding to a 9 \( \times \) 9 grid of locations on the focal plane was determined. For each camera, we average the 81 different PSFs to generate a single PSF per camera.

As a way of measuring the departure of a PSF \( \Psi_j \) from rotational symmetry, we define an asymmetry parameter:

\[
g_j = \int |\Psi_j - C[\Psi_j]| dxdy, \quad (12)
\]

where \( C[\Psi_j] \) is the azimuthally averaged version of \( \Psi_j \) (i.e., \( C[\Psi_j] \) is a PSF with rotational symmetry and the same radial profile as \( \Psi_j \)). The PSFs used have \( g \lesssim 0.2 \).

In Table 1 we have a summary of the properties of the different PSFs: the camera Rayleigh diffraction angle, the PSF FWHM, the radius containing 99% of the PSF energy, the (normalized) high-frequency cutoff \( \kappa_A \) used in the filter \( f_A \), and the anisotropy parameter \( g \). The PSF radial profiles [out to \( \Psi(\theta) \approx 10^{-5}\Psi(0) \)] and enclosed power are plotted in Figures 1 and 2.

For each PSF the radii containing 25%, 50%, 65%, 80%, 90%, 95%, 98%, 99%, 99.5%, and 99.9% of the total power are given in Table 2. Table 3 gives the enclosed power for selected radii.

For convenience, we add several families of PSFs that are often used for ground-based optical and radio telescopes. For

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\(^7\)In the current work we always use the full response of the optical systems including the camera effects, and for simplicity we will not make any further distinction between PSFs and PRFs.

\(^8\)The PSFs are available at http://ssc.spitzer.caltech.edu/.

\(^9\)STinyTim is available at http://irsa.ipac.caltech.edu/data/SPITZER/docs/dataanalysistools/tools/contributed/general/STinyTim/.

\(^10\)We use the in-flight 1.0” PSF maps from ftp://ftp.ipsi.esa.int/pub/hsc-calibration/SPIRE/.

\(^11\)In-flight PSFs are available at http://www.galex.caltech.edu/researcher/techdoc-ch5.html.
FIG. 1.—PSF radial profiles for the 20 cameras considered here (see text). All PSFs are shown out to $\Psi(\theta) \approx 10^{-5}\Psi(0)$, with the exception of WISE (W1–W4), for which we have data only down to $\approx 0.002\Psi(0)$. See the electronic edition of the PASP for a color version of this figure.
FIG. 2.—Fraction of the power enclosed by a circle of radius $\theta$ for the 20 cameras considered (see text). See the electronic edition of the PASP for a color version of this figure.
each analytical profile, we generate the kernels for a range of FWHM values. We consider the following analytical profiles:

**Gaussians.**—We use a Gaussian PSF of the form

$$
\Psi(\theta) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{\theta^2}{2\sigma^2} \right),
$$

where the FWHM = \(2 \times \sigma\sqrt{2\ln2}\). We generate kernels with 5\(^\prime\) < FWHM < 65\(^\prime\).

**Optical (SDSS).**—In the SDSS survey, it is found that a good approximation to the telescope PSF is given by the sum of two Gaussians. The two components have relative weights of 0.9 and 0.1, and the FWHM of the second component is twice that of the first. We use a family of PSFs of the form

$$
\Psi(\theta) = 0.9 \frac{1}{2\pi\sigma^2} \exp \left( -\frac{\theta^2}{2\sigma^2} \right) + 0.1 \frac{1}{2\pi(2\sigma)^2} \exp \left( -\frac{\theta^2}{2(2\sigma)^2} \right),
$$

where the FWHM = 1.01354 \(\times 2 \times \sigma\sqrt{2\ln2}\). We generate kernels with FWHM = 0.5, 1.0, 1.5, 2.0, and 2.5\(^\prime\).

**Optical (General).**—Moffat (1969) proposed PSFs of the form

$$
M_\beta(\theta) = \frac{(\beta-1)(2^{1/\beta}-1)}{\pi\theta_0^\beta} \left[ 1 + (2^{1/\beta}-1)\left( \frac{\theta}{\theta_0} \right)^2 \right]^{-\beta},
$$

where \(\beta\) is a parameter, and FWHM = 2\(\theta_0\). Following Racine (1996) we use PSFs of the form

$$
\Psi(r) = 0.8 \times M_1(\theta) + 0.2 \times M_2(\theta),
$$

where the FWHM = 2\(\theta_0\).
where the same $\theta_0$ is used in $M_7$ and $M_2$. We generate kernels with FWHM = $2\theta_0 = 0.5$, 1.0, 1.5, 2.0, and 2.5°.

### 4. KERNEL GENERATION

Generation of the convolution kernels was accomplished as follows.

#### 4.1. Input the PSF and Correct for Missing Data

When an input PSF image has missing data pixels (like the current SPIRE PSFs), we iteratively estimate the missing values. We start by replacing the missing data pixels by a value of 0 in the original image. We compute a smoothed image by convolving it with a (normalized) Gaussian kernel $\propto \exp[-(\theta/2\theta_0)^2]$, with $\theta_0$ equal to 2 pixels. We replace the original image missing data pixels by the value they have in the convolved image (the original data are not altered). We iterate the convolution and replacement steps 5 times. The resulting image has all the missing data points replaced by a smooth interpolation. This technique produces robust results, even if we have missing data in a patch of a few contiguous pixels.

#### 4.2. Resample the PSFs

Each PSF comes in a grid of different pixel size. We transform each PSF into a grid of a common pixel size of 0.20° using the IDL procedure `congrid`, using the cubic convolution interpolation method with a parameter of $-0.5$. The 0.20° pixel size captures all the details on the instrumental and Gaussian PSFs. We also pad with 0 the resulting images into an odd-sized square array if needed.

We use a grid of a common pixel size of 0.10° to regenerate the kernels from optical PSFs into IRAC cameras.

#### TABLE 3

| Camera     | 25   | 50   | 65   | 80   | 90   | 95   | 98   | 99   | 99.5 | 99.9 |
|------------|------|------|------|------|------|------|------|------|------|------|
| IRAC 3.6 μm | 0.74 | 1.19 | 1.60 | 2.44 | 5.83 | 12.9 | 36.0 | 62.5 | 88.1 | 132  |
| IRAC 4.5 μm | 0.75 | 1.24 | 1.73 | 2.68 | 7.06 | 14.7 | 37.9 | 64.5 | 90.8 | 134  |
| IRAC 5.8 μm | 0.99 | 1.97 | 3.17 | 4.60 | 42.3 | 75.3 | 114  | 134  | 145  | 154  |
| IRAC 8.0 μm | 1.16 | 2.21 | 3.32 | 6.70 | 18.9 | 39.5 | 82.9 | 114  | 134  | 152  |
| MIPS 24 μm  | 2.73 | 4.93 | 9.18 | 12.2 | 29.4 | 60.4 | 130  | 225  | 368  | 738  |
| MIPS 70 μm  | 8.02 | 14.4 | 25.5 | 35.6 | 80.2 | 158  | 318  | 461  | 628  | 894  |
| MIPS 160 μm | 16.3 | 29.0 | 52.8 | 71.9 | 155  | 287  | 518  | 802  | 950  |      |
| PACS 70 μm  | 2.41 | 4.23 | 6.47 | 12.5 | 37.5 | 85.3 | 165  | 250  | 378  | 712  |
| PACS 100 μm | 2.83 | 4.87 | 7.50 | 14.1 | 43.5 | 105  | 237  | 351  | 477  | 711  |
| PACS 160 μm | 4.50 | 7.66 | 11.3 | 20.8 | 53.3 | 133  | 294  | 417  | 524  | 712  |
| SPIRE 250 μm| 6.39 | 10.2 | 13.1 | 20.8 | 36.6 | 66.1 | 138  | 205  | 382  |       |
| SPIRE 350 μm| 8.49 | 13.4 | 17.0 | 23.7 | 43.9 | 75.0 | 122  | 192  | 397  | 499  |
| SPIRE 500 μm| 12.4 | 19.5 | 24.6 | 33.5 | 60.3 | 85.9 | 137  | 198  | 411  | 511  |

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4.3. Center the PSFs

To determine the image center, we smooth the image with a 5 pixel radius circular kernel and locate the image maximum. In some PSFs the maximum value is achieved over a (small) ring. To avoid possible misidentification of the real image center, we take the centroid of all the pixels that satisfy

\[
\frac{\max[\Psi] - \Psi(x, y)}{\max[\Psi]} \leq 5 \times 10^{-4}.
\]  

(17)

4.4. Circularize the PSFs

In order to make a rotationally symmetric PSF, we average over \(2^{14}\) rotations of the image through angles \(\beta_n = n \times 360^\circ / 2^{14}\) for \(n = 1, 2, 3, \ldots, 2^{14} = 16,384\), producing a PSF image that is invariant under rotations of any angle that is a multiple of \(360^\circ / 2^{14} = 0.022^\circ\) (i.e., is as rotationally symmetric as we can numerically expect).

Computing \(2^{14}\) rotations naively would be extremely time-consuming, but the final result can in fact be computed performing only 14 rotations, as follows.

We start by rotating \(\Psi\) by an angle \(\theta_1 = 360^\circ / 2^{1} = 180^\circ\), producing an image \(R_1\), and computing their average \(\bar{R}_1 = \frac{1}{2} \times [\Psi + R_1]\). Clearly, \(\bar{R}_1\) is now invariant under rotations of \(\theta_1 = 360^\circ / 2^{1} = 180^\circ\).

We continue this procedure iteratively. We rotate \(R_1\) by an angle \(\theta_2 = 360^\circ / 2^{2} = 90^\circ\), producing an image \(R_2\), and set \(R_2 = \frac{1}{2} \times [R_1 + R_2]\). \(R_2\) is invariant under rotations of \(\theta_1 = 360^\circ / 2^{1} = 180^\circ\) and \(\theta_2 = 360^\circ / 2^{2} = 90^\circ\); i.e., it is invariant under rotations of any angle that is a multiple of \(360^\circ / 2^{2} = 90^\circ\).

We iterate this procedure 14 times; the last average calculated image \(\bar{R}_{14}\) is the desired rotationally symmetric PSF.

We further set to 0 all the pixels that lie outside the largest circle included in the square image, since those regions would correspond to areas with partial image coverage. If there are pixels with (very small) negative values (due to the noise in the original PSFs) we set them to 0.

In order to have a more stable algorithm, the previous rotations are performed in reverse order (i.e., starting with the smallest angles).

All the remaining steps in the kernel construction should preserve the rotational symmetry in the images. A way of estimating the noise induced by some steps (e.g., computing Fourier transform) is to compute the difference from rotational symmetry in the resulting image. Circularizing helps to reduce numerical noise and will be performed after every step that can potentially decrease the image quality. When a circularization is performed to a rotationally symmetric image, the asymmetry parameter \(g\) of the resulting image is smaller than 0.0008 (i.e., the circularization procedure itself induces very small departures from rotational symmetry).

4.5. Resize the PSFs

We trim (or pad with 0) all the PSFs into a common grid, to be able to compute all the convolution kernels using only one Fourier transform per PSF. We choose a grid size that is large enough to contain most of the power in each PSF. We also optimize its size to make the FFT algorithm as efficient as possible (minimal sum of prime factors). The adopted grid size is \(729^\prime \times 729^\prime\), giving an image size of \(3645^\prime \times 3645^\prime\).

4.6. Compute the Fourier Transform of the PSF FT(\(\Psi\))

We use an efficient FFT algorithm. Since the PSFs are invariant under reflections, \(\bar{f} \leftrightarrow -\bar{f}\), their Fourier transforms should be real. We impose this real condition to reduce the numerical noise introduced by the double-precision FFT algorithm.

4.7. Circularize the FT(\(\Psi\))

Using the procedure as before, we circularize the FT(\(\Psi\)). In principle, they should already be rotationally symmetric, but numerical noise in the FFT algorithm makes them slightly non-rotationally symmetric.

4.8. Filter the FT(\(\Psi\))

We filter the highest frequencies in each FT(\(\Psi\)). We use a filter \(\phi\) of the form

\[
\phi(k) = \begin{cases} 
1 & \text{for } k \leq k_\alpha \\
\exp \left[ -\left( \frac{1.8249 \times (k - k_\alpha)}{k_\beta - k_\alpha} \right)^4 \right] & \text{for } k_\alpha \leq k \leq k_\beta \\
0 & \text{for } k_\beta \leq k
\end{cases}
\]

(18)

where we set \(k_\alpha = 0.9 \times k_\beta\). The factor 1.8249 is chosen so that \(\phi(0.5 \times (k_\beta + k_\alpha)) = 0.5\). For each camera we choose \(k_\beta = 4 \times (2\pi / \text{FWHM})\). We tested several filter functions and found that the particular form given by equation (19) provided the best results. Each PSF has structure at spatial wavelengths comparable with the FWHM, so the Fourier components with frequencies much higher than this cannot be important. The Fourier components removed by this filter were mainly introduced by the original image resizing algorithm. In the following discussion, we let \(\text{FT}_\phi(\Psi) = \phi \times \text{FT}(\Psi)\).

4.9. Invert the FT(\(\Psi\))

We evaluate \(1 / \text{FT}_\phi(\Psi)\) at the points where \(\text{FT}_\phi(\Psi) \neq 0\). We set \(1 / \text{FT}_\phi(\Psi) = 0\) at the remaining points (which will be filtered soon).

4.10. Compute the FT of the Filtered Kernel

We compute the FT of the filtered kernel using the filter \(f_A\) from equation (11):
for all the appropriate combinations \((A, B)\).

\[
\text{FT}(K\{A \Rightarrow B\}) = \text{FT}_\phi(\Psi_B) \times \frac{f_A}{\text{FT}_\phi(\Psi_A)} \quad (19)
\]

4.11. Compute the Kernels

We compute the inverse Fourier transform to the previously calculated \(\text{FT}(K)\). We again impose the condition that \(K\) must be real.

4.12. Circularize the Kernels

Using the procedure as before, we circularize the kernels. Again, they should already be rotationally symmetric. Numerical noise in the inverse FFT algorithm makes them slightly asymmetric, but this is easily corrected.

4.13. Resample the Kernels

All the computed kernels are given in a grid of a common pixel size of 0.20\ '', but will be needed in grids of different pixel sizes. Again, we resample the kernels using the IDL procedure \texttt{congrid}, using the cubic convolution interpolation method with a parameter of \(-0.5\).

4.14. Final Trim of the Kernels

We trim each kernel to a smaller size (to speed up further convolution) such that it contains 99.9\% of the total kernel energy. Moreover, we use a square grid with an odd number of pixels so that the kernel peaks in a single central pixel.

4.15. Circularize the Final Kernels

We finish the kernels by circularizing them again, to remove the small noise introduced in the resampling process. All previous calculations were done in double precision to reduce numerical noise.

5. KERNEL PERFORMANCE

For each generated kernel, we compute \(\Psi_A \ast K\{A \Rightarrow B\}\), the convolution of \(\Psi_A\) and \(K\{A \Rightarrow B\}\), and compare it with \(\Psi_B\).\(^{12}\) For a perfect kernel, both quantities should coincide at all radii.

Figure 3 shows the analysis of \(K\{M24 \Rightarrow S250\}\).\(^{13}\) This kernel shows good behavior: it transforms from a camera with FWHM\(_{M24} = 6.5\ ''\) into a camera with FWHM\(_{S250} = 18.2\''\). This kernel essentially spreads the energy of the core of MIPS 24 \(\mu\)m into a larger area. The filter \(f_{M24}\) has no effect on the construction of this kernel, because \(\text{FT}(\Psi_B) \approx 0\) for \(k > 0.7 \times k_{H, A}\).

The left panel of Figure 3 compares the integrated power of the PSFs. It includes \(\Psi_{M24}\) (dot-dashed line), \(\Psi_{S250}\) (solid line), and \(\Psi_{M24} \ast K\{M24 \Rightarrow S250\}\) (dashed line). For an ideal kernel the solid line and the dashed line should coincide. The departures in this case are very small.

\(^{12}\) It can be easily shown that the radial profile of \(\langle \Psi_A \ast K\{A \Rightarrow B\} \rangle\) is the same whether \(\Psi_A\) is rotationally symmetric or not, so for simplicity, we use the circularized version of \(\Psi_A\) in the comparison.

\(^{13}\) In our PSF and kernel notations, we will abbreviate IRAC, MIPS, PACS, SPIRE, GALEX, and WISE as I, M, P, S, GAL, and W, respectively, and will omit the \(\mu\)m symbol.
The power per unit radius is proportional to $\theta \Psi(\theta)$, or $\theta K(\theta)$. The right panel of Figure 3 shows $\theta \Psi$ and $\theta K$ (normalized to the maximum value). The lower part of the right panel includes four traces: $\theta \times \Psi_{250}^M$ (dot-dashed line), $\theta \times \Psi_{250}^S$ (solid line), $\theta \times (\Psi_{24} \ast K\{ M24 \Rightarrow S250 \})$ (dashed line) and $\theta \times K\{ M24 \Rightarrow S250 \}$ (dotted line) for visualization of the kernel behavior. For an ideal kernel the solid line and the dashed line should coincide. The upper part of the right panel is a plot of the difference between $\theta \times \Psi_{250}^M$ and $\theta \times (\Psi_{24} \ast K\{ M24 \Rightarrow S250 \})$. For an ideal kernel this graph should be 0. For this example, $\Psi_{250}^M$ is not recomputed numerically. Due to finite grid resolution the numerical values may be off by a few percent in some cases. This can be computed numerically. Due to finite grid resolution the numerical values may be off by a few percent in some cases. This can be seen from the $W_-$ values computed for the self-kernels. The self-kernels are simply the Fourier transform of the filter $f_A$, and $W_-$ should therefore be the same (1.15) in all cases. However, in Table 5 the $W_-$ for the smallest PSFs are larger than 1.15 by as much as 0.03 (e.g., 1.18 for PACS 70 $\mu$m).

In Figures 4 and 5 we analyze the kernels $K\{ M70 \Rightarrow S250 \}$ and $K\{ S250 \Rightarrow M70 \}$. Their construction is more challenging since both cameras have similar FWHM: FWHM$_{M70} = 18.7''$ and FWHM$_{S250} = 18.2''$. These kernels have to redistribute the energy within the core and Airy rings of the PSFs. The plotted quantities are similar to those in the right panel of Figure 3 for $K\{ M24 \Rightarrow S250 \}$. In this case, the kernels still perform well, but they have large areas with negative values.

One measure of kernel performance is its accuracy in redistribution of PSF power. We define

$$D = \iint |\Psi_B - K\{ A \Rightarrow B \} \ast \Psi_A| \, dx \, dy. \quad (20)$$

A kernel with perfect performance will have $D = 0$, and normalization of the PSFs requires $D \leq 2$. $D$ measures how much flux has not been redistributed correctly. Good kernels have small $D$ values: $D(K\{ M24 \Rightarrow S250 \}) = 0.011$. In Table 4 we give $D$ for the kernels constructed.

A second quantitative measure of kernel performance is obtained by studying its negative values. We define

$$W_\pm = \frac{1}{2} \iint \left| (K\{ A \Rightarrow B \}) + K\{ A \Rightarrow B \} \right| \, dx \, dy. \quad (21)$$

Flux conservation requires that $W_+ = 1 + W_-$. In general, kernels will have $W_- \geq 0$. Well-behaved kernels have small $W_-$ values: $W_-(K\{ M24 \Rightarrow SP250 \}) = 0.07$. The integral of $|K\{ A \Rightarrow B \}|$ is $|1 + 2W_-|$, so a kernel with a large value of $W_-$ could potentially amplify image artifacts. Additionally, a kernel with large $W_-$ can generate areas of negative flux near point sources if nonlinearities are present or if the background levels were subtracted incorrectly. Table 5 lists the $W_-$ values for the kernels constructed. The values in Table 5 were computed numerically. Due to finite grid resolution the numerical values may be off by a few percent in some cases. This can be seen from the $W_-$ values computed for the self-kernels. The self-kernels are simply the Fourier transform of the filter $f_A$, and $W_-$ should therefore be the same (1.15) in all cases. However, in Table 5 the $W_-$ for the smallest PSFs are larger than 1.15 by as much as 0.03 (e.g., 1.18 for PACS 70 $\mu$m).

Essentially, there are two sources of $W_-$: oscillations due to the filter $f_A$ and the need to remove energy from some region to relocate to another region (when the target $\Psi_B$ is narrower than $\Psi_A$ or has less energy in some annuli).

The kernels between cameras with similar FWHM also have oscillations. Using a softer filter $f_A$ would reduce the oscillatory

Fig. 4.—Performance of the kernel $K\{ M70 \Rightarrow S250 \}$. Because of the large value $W_- = 2.14$, convolution MIPS70 $\mu$m $\Rightarrow$ SPIRE250 $\mu$m is not recommended. See the electronic edition of the PASP for a color version of this figure.

Fig. 5.—Performance of the kernel $K\{ S250 \Rightarrow M70 \}$. With $W_- = 0.61$, this kernel is safe to use. $\Psi_{250}^S \ast K\{ S250 \Rightarrow M70 \}$ deviates from $\Psi_{250}^S$ less than 2%. See the electronic edition of the PASP for a color version of this figure.
| To       | M 24 | M 70 | M 160 | P 70 | P 100 | P 160 | S 250 | S 350 | S 500 | GAL NUV | GAL NUV | W 3.4 | W 4.6 | W 11.6 | W 22.1 |
|----------|------|------|--------|------|-------|-------|-------|-------|-------|--------|--------|-------|------|--------|-------|
| I3.6     | 0.065| 0.029| 0.037  | 0.051| 0.047 | 0.031 | 0.025 | 0.022 | 0.018 | 0.006  | 0.005  | 0.005 | 0.005 | 0.007  | 0.008 |
| I4.5     | 0.065| 0.029| 0.037  | 0.051| 0.047 | 0.031 | 0.025 | 0.022 | 0.018 | 0.006  | 0.005  | 0.005 | 0.005 | 0.007  | 0.008 |
| I5.8     | 0.064| 0.031| 0.048  | 0.047| 0.045 | 0.031 | 0.029 | 0.027 | 0.018 | 0.012  | 0.010  | 0.012 | 0.013 | 0.012  |       |
| I8.0     | 0.064| 0.030| 0.047  | 0.047| 0.046 | 0.032 | 0.026 | 0.024 | 0.018 | 0.012  | 0.007  | 0.007 | 0.007 | 0.009  | 0.009 |
| M24      | 0.091| 0.018| 0.048  | 0.213| 0.068 | 0.015 | 0.011 | 0.013 | 0.017 | NC     | 0.352  | 0.225 | 0.122 | 0.098  | 0.027 |
| M70      | NC   | 0.055| 0.038  | NC   | NC   | 0.102 | 0.015 | 0.014 | NC   | NC     | NC     | NC    | NC    | NC     | NC    |
| M160     | NC   | NC   | 0.064  | NC   | NC   | NC   | NC   | NC   | NC   | NC     | NC     | NC    | NC    | NC     | NC    |
| P70      | 0.006| 0.019| 0.049  | 0.045| 0.010 | 0.014 | 0.011 | 0.014 | 0.017 | 0.227  | 0.113  | 0.064 | 0.045 | 0.027  | 0.027 |
| P100     | 0.072| 0.014| 0.045  | 0.188| 0.050 | 0.010 | 0.011 | 0.014 | 0.015 | NC     | NC     | 0.202 | 0.111 | 0.087  | 0.028 |
| P160     | NC   | 0.009| 0.040  | NC   | NC   | 0.044 | 0.013 | 0.014 | 0.015 | NC     | NC     | NC    | NC    | NC     | 0.032 |
| S250     | NC   | 0.037| 0.045  | NC   | NC   | 0.058 | 0.009 | 0.014 | NC   | NC     | NC     | NC    | NC    | NC     | NC    |
| S350     | NC   | 0.300| 0.043  | NC   | NC   | NC   | 0.060 | 0.012 | NC   | NC     | NC     | NC    | NC    | NC     | NC    |
| S500     | NC   | NC   | 0.042  | NC   | NC   | NC   | NC   | NC   | NC   | NC     | NC     | NC    | NC    | NC     | NC    |
| FUV      | 0.065| 0.030| 0.036  | 0.049| 0.046 | 0.032 | 0.025 | 0.024 | 0.018 | 0.046  | 0.017  | 0.007 | 0.006 | 0.007  | 0.007 |
| NUV      | 0.065| 0.030| 0.037  | 0.051| 0.046 | 0.032 | 0.027 | 0.024 | 0.018 | 0.071  | 0.028  | 0.008 | 0.006 | 0.006  | 0.007 |
| W 3.4    | 0.066| 0.029| 0.037  | 0.080| 0.048 | 0.032 | 0.026 | 0.023 | 0.018 | 0.252  | 0.122  | 0.053 | 0.034 | 0.017  | 0.007 |
| W 4.6    | 0.066| 0.029| 0.037  | 0.079| 0.047 | 0.032 | 0.026 | 0.023 | 0.018 | NC     | 0.118  | 0.051 | 0.033 | 0.016  | 0.006 |
| W 11.6   | 0.074| 0.028| 0.037  | 0.117| 0.056 | 0.031 | 0.026 | 0.023 | 0.018 | NC     | 0.206  | 0.109 | 0.063 | 0.040  | 0.005 |
| W 22.1   | NC   | 0.028| 0.036  | NC   | 0.163 | 0.028 | 0.023 | 0.018 | NC   | NC     | NC     | NC    | NC    | 0.094  |       |

**NOTES.**—D is the integral of the absolute value of the difference between a target PSF and the PSF reproduced by the kernel (see eq. [20]).

We are abbreviating IRAC, MIPS, PACS, SPIRE, GALEX, and WISE as I, M, P, S, GAL, and W, respectively. NC stands for not computed: the kernel performance would be too poor.
### TABLE 5

| From | M24 | M70 | M160 | P70 | P100 | P160 | S250 | S350 | S500 | FUV | NUV | W3.4 | W4.6 | W11.6 | W22.1 |
|------|-----|-----|------|-----|------|------|------|------|------|-----|-----|------|------|-------|-------|
| To   | M   | M   | M    | P    | P    | S    | S    | GAL  | GAL  | FUV | NUV | 3.4  | 4.6  | 11.6  | 22.1  |
| 3.6  | 0.00| 0.00| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.03 | 0.05| 0.05| 0.05 | 0.05 | 0.05  | 0.02  |
| 4.5  | 0.01| 0.00| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.03| 0.05| 0.05 | 0.05 | 0.05  | 0.02  |
| 8.0  | 0.11| 0.00| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31| 0.31| 0.31 | 0.31 | 0.31  | 0.31  |
| M24  | 1.17| 0.02| 0.00 | 2.85 | 0.75 | 0.16 | 0.06 | 0.00 | 0.23| 0.14| 0.14| 0.14 | 0.14 | 0.14  | 0.06  |
| M70  | NC  | 1.15| 0.20 | NC   | NC   | NC   | 2.14 | 0.66 | 0.34 | NC  | NC  | NC   | NC   | NC    | NC    |
| M160 | NC  | NC  | 1.14 | NC   | NC   | NC   | NC   | NC   | NC   | NC  | NC  | NC   | NC   | NC    | NC    |
| P70  | 0.83| 0.01| 0.01 | 1.18 | 0.32 | 0.00 | 0.05 | 0.05 | 0.02| 5.74| 5.74| 5.74 | 5.74 | 5.74  | 5.74  |
| P100 | 2.16| 0.01| 0.01 | 4.29 | 1.17 | 0.04 | 0.07 | 0.00 | 0.00 | 2.68| 2.68| 2.68 | 2.68 | 2.68  | 2.68  |
| P160 | NC  | 0.11| 0.01 | NC   | NC   | NC   | 1.16 | 0.24 | 0.14 | 0.06| NC  | NC  | NC   | NC   | NC    | NC    |
| S250 | NC  | 0.61| 0.02 | NC   | NC   | NC   | NC   | 1.15 | 0.21 | 0.06| NC  | NC  | NC   | NC   | NC    | NC    |
| S350 | NC  | 6.08| 0.08 | NC   | NC   | NC   | 1.15 | 0.14 | 0.06 | NC  | NC  | NC   | NC   | NC    | NC    |
| S500 | NC  | NC  | NC   | 0.67 | NC   | NC   | NC   | NC   | NC   | NC  | NC  | NC   | NC   | NC    | NC    |
| FUV  | 0.15| 0.00| 0.00 | 0.10 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30| 0.30| 0.30 | 0.30 | 0.30  | 0.30  |
| NUV  | 0.33| 0.00| 0.00 | 0.47 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | 1.18| 1.18| 1.18 | 1.18 | 1.18  | 1.18  |
| W3.4 | 0.60| 0.00| 0.00 | 1.05 | 0.21 | 0.00 | 0.01 | 0.00 | 0.00 | 5.02| 5.02| 5.02 | 5.02 | 5.02  | 5.02  |
| W4.6 | 1.27| 0.00| 0.00 | 2.19 | 0.56 | 0.00 | 0.01 | 0.00 | 0.00 | 4.29| 4.29| 4.29 | 4.29 | 4.29  | 4.29  |
| W11.6| 2.06| 0.02| 0.00 | 3.14 | 1.14 | 0.22 | 0.07 | 0.02 | 0.00 | 5.70| 5.70| 5.70 | 5.70 | 5.70  | 5.70  |
| W22.1| NC  | 0.39| 0.02 | NC   | NC   | NC   | 1.65 | 0.66 | 0.28 | 0.04| NC  | NC  | NC   | NC   | NC    | NC    |

**Notes.**—$W_-$ is the integral of the negative values for each kernel (see eq. [21]).

We are abbreviating IRAC, MIPS, PACS, SPIRE, GALEX, and WISE as I, M, P, S, GAL, and W, respectively. NC stands for not computed: the kernel performance would be too poor.
behavior, giving smaller values of $W_-$, but would also produce worse matched PSFs (larger values of $D$). The particular form of filter $f_A$ used in the present work is a good compromise between having good PSF matching and moderate oscillatory behavior.

In Figure 6 we analyze a kernel of particular interest: $K\{M160 \Rightarrow S500\}$. Despite both cameras having similar FWHM (FWHM$_{M160}$/FWHM$_{S500} = 1.07$), their extended wings are very different. The kernel $K\{M160 \Rightarrow S500\}$ is particularly badly behaved, with large negative excursions, having $W_- = 2.81$ and $D = 0.17$. In a convolution of MIPS 160 $\mu$m images of NGC 6946, some bright point sources generated regions with negative flux around them. We do not recommend using the kernel $K\{M160 \Rightarrow S500\}$; if MIPS 160 $\mu$m and SPIRE 500 $\mu$m images need to be combined, we recommend using the PSF of MIPS 160 $\mu$m ($K\{S500 \Rightarrow M160\}$ has $W_- \approx 0.47$ and $D \approx 0.042$) or some Gaussian PSF compatible with MIPS 160 $\mu$m, such as a Gaussian with FWHM = 64" (see § 6).

Finally, in Figure 7 we analyze the kernel $K\{M70 \Rightarrow M70\}$. This kernel is essentially FT($f_{M70}$), and it illustrates the effect of all the kernel construction steps. All the kernels of the form $K\{A \Rightarrow A\}$ are scaled versions of $K\{M70 \Rightarrow M70\}$, aside from small differences due to finite grids. All of the $K\{A \Rightarrow A\}$ kernels have $W_- \approx 1.15$ and $D \approx 0.06$.

Even though kernels $K\{A \Rightarrow A\}$ have $W_- = 1.15$ and $W_- = 2.15$, they do not amplify the noise that arises from astronomical sources. The image of an astronomical point source will be $\Psi_A$. When we convolve the camera with a kernel of the form $K\{A \Rightarrow A\}$, the image of a point source will still be very close to $\Psi_A$, since $D(K\{A \Rightarrow A\}) \approx 0.06$. This implies that the

![Figure 6](image1.png)

Fig. 6.—Performance of the kernel $K\{M160 \Rightarrow S500\}$. With $W_- = 2.81$, convolution of MIPS 160 $\mu$m images into SPIRE 500 $\mu$m resolution is risky and not recommended. The convolved PSF differs from the target PSF by up to 6%. See the electronic edition of the PASP for a color version of this figure.

![Figure 7](image2.png)

Fig. 7.—Performance of the self-kernel $K\{M70 \Rightarrow M70\}$. All of the self-kernels have $W_- = 1.16$ and reproduce the original PSF to within 3%. See the electronic edition of the PASP for a color version of this figure.

## Table 6

| Camera       | Actual FWHM ($^\circ$) | Aggressive Gaussian with $W_- \approx 1.0$ | Moderate Gaussian with $W_- \approx 0.5$ | Very safe Gaussian with $W_- \approx 0.3$ |
|--------------|------------------------|------------------------------------------|----------------------------------------|----------------------------------------|
|             |                        | FWHM ($^\circ$)                          | W_-                                    | FWHM ($^\circ$)                          | W_-                                    | FWHM ($^\circ$)                          | W_-                                    |
| MIPS 24 $\mu$m | 6.5                    | 8.0                                      | 1.00                                   | 11.0                                    | 0.49                                   | 13.0                                    | 0.30                                   |
| MIPS 70 $\mu$m | 18.7                   | 22.0                                     | 1.01                                   | 30.0                                    | 0.51                                   | 37.0                                    | 0.30                                   |
| MIPS 160 $\mu$m | 38.8                   | 46.0                                     | 1.01                                   | 64.0                                    | 0.50                                   | 76.0                                    | 0.30                                   |
| PACS 70 $\mu$m | 5.8                    | 6.5                                      | 0.84                                   | 8.0                                     | 0.48                                   | 10.5                                    | 0.31                                   |
| PACS 100 $\mu$m | 7.1                    | 7.5                                      | 1.10                                   | 9.0                                     | 0.52                                   | 12.5                                    | 0.31                                   |
| PACS 160 $\mu$m | 11.2                   | 12.0                                     | 1.05                                   | 14.0                                    | 0.50                                   | 18.0                                    | 0.33                                   |
| SPIRE 250 $\mu$m | 18.2                   | 19.0                                     | 1.05                                   | 21.0                                    | 0.44                                   | 22.0                                    | 0.30                                   |
| SPIRE 350 $\mu$m | 25.0                   | 26.0                                     | 0.98                                   | 28.0                                    | 0.50                                   | 30.0                                    | 0.27                                   |
| SPIRE 500 $\mu$m | 36.4                   | 38.0                                     | 0.96                                   | 41.0                                    | 0.48                                   | 43.0                                    | 0.30                                   |
Fig. 8.—Performance of the kernels $K\{P160 \Rightarrow \text{Gaussian}\}$ (left) and their FT (right). We show the kernel Fourier transform $\text{FT}(K)$ (eq. [19]) and also the unfiltered kernel Fourier transform $\equiv \text{FT}_\phi(\Psi_B)/\text{FT}_\phi(\Psi_A)$. The filter $f_{P160}$ has little impact on $K\{P160 \Rightarrow \text{Gaussian}18''\}$ ($W_\ast = 0.33$), moderate effect on $K\{P160 \Rightarrow \text{Gaussian}14''\}$ ($W_\ast = 0.50$), and large effect on $K\{P160 \Rightarrow \text{Gaussian}12''\}$ ($W_\ast = 1.05$). See the electronic edition of the PASP for a color version of this figure.
noise coming from a field of unresolved astronomical background sources will not change significantly when we convolve the image with $K_f A \Rightarrow A$. To verify this reasoning, we generate an image $S$ having independent Gaussian noise in each pixel in a very fine (0.2″) grid. We convolve $S$ with $\Psi_{M70}$ to have a simulated observed image of the noise: $O = S \ast \Psi_{M70}$. We further convolve $O$ with $K\{M70 \Rightarrow M70\}$: $C = O \ast K\{M70 \Rightarrow M70\}$. We found that $|1 - \langle C^2 \rangle/\langle O^2 \rangle| \approx 10^{-3}$. While uncorrelated noise is not amplified by a self-kernel $K\{A \Rightarrow A\}$, imprecise characterization of the PSFs and camera artifacts can be amplified by kernels with large $W_-$ values.

There is no single number that captures all of the characteristics of a convolution kernel, but we find that $W_-$ serves as a good figure of merit. Based on experimentation with various kernels, we regard kernels with $W_- \leq 0.3$ to be very safe. Kernels with $W_- \approx 0.5$ also appear to be quite safe. A kernel with $W_- \approx 1$ is somewhat aggressive in moving power around, but remember that self-kernels $K\{A \Rightarrow A\}$ also have $W_- \approx 1.15$. We consider kernels with $W_- \approx 1$ to be reasonable.
to use, with inspection of the before and after images recommended in regions with large gradients or bright sources. We recommend against using convolution kernels with $W/C_0 > 1/2$, as these are, in effect, attempts to deconvolve the image to higher resolution, with attendant risks.

6. GAUSSIAN PSFS

Gaussian PSFs are commonly used in radio astronomy. It is desirable for radio telescopes to have PSFs without extended structure to avoid sidelobe contamination. The illumination pattern of a single dish is often designed to return an approximately Gaussian PSF. A Gaussian PSF lacks extended wings; the fraction of the power outside radius $\theta$ is $\exp(-\theta^2/2\sigma^2)$. Because real instrumental PSFs do not fall off so rapidly, a convolution kernel $K$ going from a real $\Psi_A$ to a Gaussian $\Psi_B$ with similar FWHM must “move” power from the wings of $\Psi_A$ to the core of $\Psi_B$.

For a given instrumental PSF $\Psi_A$, the optimal Gaussian PSF $\Psi_B$ will be such that the FWHM $B$ will be close to FWHM $A$, with only mild filtering by the function $f_A$ and with $W/C_0$ not too large. In order to determine an optimal Gaussian FWHM for a camera $A$, we compute convolution kernels from $A$ into Gaussian PSFs with FWHM in a range of possible values. For each candidate FWHM, we evaluate $W/C_0$. We provide three possible FWHM. The first FWHM is obtained by requiring that $W/C_0 \sim 0.3$, giving a conservative (very safe) kernel that does not seek to move too much energy from the wings into the main Gaussian core, at the cost of having a larger FWHM (i.e., lower resolution). The second FWHM has $W/C_0 \sim 0.5$, and we consider it to be a good (moderate) Gaussian FWHM to use. The third FWHM has $W/C_0 \sim 1.0$. Because this kernel is somewhat “aggressive” in moving energy from the PSF wings into the Gaussian core, it should be used with care, inspecting that the convolved images do not have any induced artifact. Table 6 gives the FWHM for three such Gaussian target PSFs for the MIPS, PACS, and SPIRE cameras.

As an example, Figure 8 shows the performance of the kernels for the PACS 160 $\mu$m camera going into Gaussian

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14 Interferometric arrays have complicated sidelobe responses and would not resemble Gaussian PSFs.
PSFs with FWHM = 12″ (aggressive, $W_\ast = 1.05$, $D = 0.05$), 14″ (moderate, $W_\ast = 0.50$, $D = 0.02$), and 18″ (very safe, $W_\ast = 0.33$, $D = 0.01$). The left panels are the performance analyses, similar to those of Figures 4.7. The right panels show the kernel Fourier transform FT($K$) eq. [19]. In the right panel, we include the unfiltered Fourier transform ($= FT_g(\Psi_{Gauss})/ FT_g(\Psi_{Pauli}))$ for comparison (dotted line). We observe that the filtering is only important in the Gaussian kernels with (small) FWHM = 12″ and 14″.

7. USAGE OF THE KERNELS

The kernels $K\{A \Rightarrow B\}$ computed here are given on a 0.20′′ grid. Before performing an image convolution, the kernel $K\{A \Rightarrow B\}$ should be resampled onto a grid with the same pixel size as the original image $I_A$. The resampled kernels should be centered (to avoid shifts in the image) and normalized so that $\int K\{A \Rightarrow B\}(x, y)dx\,d y = 1$ to ensure flux conservation. The flux in the image to be convolved should be in surface brightness units. After convolving the image $I_A$ with the kernel $K\{A \Rightarrow B\}$, the resulting image will be expressed in the original image grid and original surface brightness units, but with PSF $\Psi_B$.

Table 5 also summarizes the kernels available. For each camera $A$ we construct all the kernels $K\{A \Rightarrow B\}$ with FWHM$_B \geq$ FWHM$_A/1.35$ (i.e., the kernels that degrade the resolution or sharpen it up to 35%) plus the self-kernels $K_{A \Rightarrow A}$. Kernels with FWHM$_A \geq$ FWHM$_B$ (that have larger $W_\ast$ values) tend to perform poorly and should be used with care. We do not recommend using any kernel with $W_\ast \gtrsim 1.2$.

As an example of the performance of the kernels applied to real (noisy) images, in Figure 9 we show the result of convolving images of the barred spiral galaxy NGC 1097 (after subtraction of a “tilted-plane” background from each image) into the SPIRE 250 μm PSF. The PACS images have been reduced using the Scammanorphos pipeline (Roussel 2011). Visual inspection of the images in Figure 9 shows them to be very similar in morphology; the convolution does not appear to have introduced any noticeable artifact.

Figure 10 shows the results of convolving the SPIRE 250 μm image into different PSFs. The top row (left) shows the original image, (center) the image convolved with $K\{S250 \Rightarrow S250\}$, and (right) the image convolved with a very aggressive kernel into a Gaussian PSF with FWHM = 18″ ($W_\ast = 1.47$). The bottom row shows the image convolved to the recommended Gaussian PSFs: (left) FWHM = 19″ ($W_\ast = 1.05$), (center) FWHM = 21″ ($W_\ast = 0.44$), and (right) FWHM = 22″ ($W_\ast = 0.30$). As discussed in § 5, we recommend against using kernels with $W_\ast > 1.2$. Visual inspection of the upper-right image in fact shows artifacts where the kernel (with $W_\ast = 1.47$) has moved too much power out of some interarm pixels, which have been brought down to unreasonably low intensities. In the convolutions to the suitable Gaussian PSFs (bottom row in Fig. 10) energy is moved from the interarm regions into the bright nucleus and spiral arms, but the intensity levels in the interarm regions seem reasonable. The power that is removed from the interarm regions was presumably originally power from the arms that was transferred by the wings of the SPIRE 250 μm PSF.

Using the kernels described in the current work, Aniano et al. (2011, in preparation) studied resolved dust modeling for NGC 628 and NGC 6946, two galaxies in the KINGFISH galaxy sample (Kennicutt et al. 2011, in preparation), using images obtained with Spitzer and Herschel Space Observatory.

8. SUMMARY

We present the construction and analysis of convolution kernels for transforming images into a common PSF. They allow generation of a multiwavelength image cube with a common PSF, preserving the colors of the regions imaged.

We generate and make available a library of convolution kernels for the cameras of Spitzer, Herschel Space Observatory, GALEX, WISE, ground-based telescopes, and Gaussian PSFs. All the kernels are constructed with the best PSF characterizations available, approximated by rotationally symmetric functions. Deviations of the actual PSF from circular symmetry are characterized by an asymmetry parameter $g$, given in Table 1. Table 5 summarizes the kernels available and their negative integral $W_\ast$, a measure of their performance. We recommend using only kernels with $W_\ast \leq 1.2$. In Table 6 we give a set of optimal Gaussian FWHM that are compatible with MIPS, PACS, and SPIRE cameras.

All the kernels and their individual performance analyses, the IDL routines to make new kernels, and IDL routines to use them are publicly available.16

We thank Roc Cutri and Edward Wright for their help providing the Wide-field Infrared Survey Explorer point-spread functions (PSFs); Markus Nielbock for his help providing the Photocamera Array Camera and Spectrometer for Herschel PSFs; and Richard Bamler, James Gunn, and Robert Lupton for helpful suggestions. This research was supported in part by NSF grant AST-1008570 and JPL grant 1373687.

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15By “original” image we refer to the SPIRE 250 μm image delivered by the HIPE pipeline, with subsequent subtraction of a tilted-plane background. This is the original image that is convolved to produce the other images in Fig. 10.

16See http://www.astro.princeton.edu/~draine/Kernels.html. Kernels for additional cameras and updates will be included when new PSF characterizations become available.
REFERENCES

Alard, C., & Lupton, R. H. 1998, ApJ, 503, 325
Engelbracht, C. W., et al. 2007, PASP, 119, 994
Fazio, G. G., et al. 2004, ApJS, 154, 10
Geis, N., & Lutz, D. 2010, PACS ICC Document, PICC-ME-TN-029 v2.0
Gordon, K. D., Engelbracht, C. W., Rieke, G. H., Misselt, K. A., Smith, J., & Kennicutt, R. C., Jr. 2008, ApJ, 682, 336
Gordon, K. D., et al. 2007, PASP, 119, 1019
Griffin, M. J., et al. 2010, A&A, 518, L3
Lutz, D. 2010, PACS ICC Document, PICC-ME-TN-033
Martin, D. C., et al. 2005, ApJ, 619, L1
Moffat, A. F. J. 1969, A&A, 3, 455
Müller, T. 2010, PACS ICC Document, PICC-ME-TN-036 v2.0
Poglitsch, A., et al. 2010, A&A, 518, L2
Racine, R. 1996, PASP, 108, 699
Rieke, G. H., et al. 2004, ApJS, 154, 25
Roussel, H. 2011, A&A, submitted
Sandstrom, K. M., Bolatto, A. D., Stanimirović, S., van Loon, J. T., & Smith, J. D. T. 2009, ApJ, 696, 2138
Stansberry, J. A., et al. 2007, PASP, 119, 1038
Wright, E. L., et al. 2010, AJ, 140, 1868