Gravity-driven Transport along Cylindrical Topological Defects: Possible Dark Matter and Nearly Frictionless States

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Abstract

The gravity-driven flow along an annular topological defect (string) with transversely corrugations is investigated by using the verified transition-rate model and boundary perturbation method. We found that for certain activation volume and energy there exists possible frictionless states which might be associated with the missing momentum of inertia or dark matter.

Keywords: Activation energy, cosmic string, shear, boundary perturbation

1 Introduction

Quite recently Vitelli et al. suggested that topological defects in the cosmic shear can be used as a probe of the gravitational potential generated by the lensing mass fluctuations on large scales [1] based on the facts that shear fields due to weak gravitational lensing have characteristic coherent patterns. They described the topological defects in shear fields in terms of the curvature of the surface described by the lensing potential. In that paper they explored the connection between the theory of topological defects and the spatial patterns of shear fields due to weak gravitational lensing.

The starting point of their approach rests on an analogy between gravitational lensing shear fields, as a probe of structure formation on cosmological scales, and the anisotropic optical or mechanical response of materials, as a probe of their inhomogeneous structure on microscopic scales. As an illustration, the topological defects in the local shear field of an elastic medium reflect the external deformations applied to the solid. Similarly for thin liquid crystal films confined on a curved substrate, the density of topological defects depends on the inhomogeneous curvature of the underlying surface [1]. This may also allow them to infer how the dark matter is concentrated around galaxies and galaxy clusters, as well as providing a testing ground for dark energy and modified gravity theories [2,3].

Above mentioned or borrowed analogy is one motivation for our present study. The other motivation is related to the possible dark matter associated with possible superfluidity formation after shear-thinning. While superflow in a state of matter possessing a shear modulus might initially seem untenable, experimental claims for precisely this phenomenon in solid $^4$He now abound [4]. Reported in the experiments of Kim and Chan [5] was a dramatic change below 200
mK in the period of a torsional oscillator containing solid $^4$He. Because superfluids come out of equilibrium and detach from the walls of the rotated container, they are expected to give rise to a period shift in such a geometry, assuming, of course, the rotation velocity is less than the critical velocity to create a vortex. The result is a missing moment of inertia (MMI) \cite{4} and hence the period of oscillation decreases. The magnitude of the MMI is a direct measure of the superfluid fraction. However, the present author likes to link this MMI which occurs as there is formation of superfluidity with the possible formation of dark matter.

Meanwhile, researchers have been interested in the question of how matter responds to an external mechanical load. External loads cause transport, in Newtonian or various types of non-Newtonian ways. Amorphous matter, composed of polymers, metals, or ceramics, can deform under mechanical loads, and the nature of the response to loads often dictates the choice of matter in various applications. The nature of all of these responses depends on both the temperature and loading rate.

To the best knowledge of the author, the simplest model that makes a prediction for the rate and temperature dependence of shear yielding is the rate-state model of stress-biased thermal activation \cite{6-8}. Structural rearrangement is associated with a single energy barrier $E$ that is lowered or raised linearly by an applied stress $\sigma$: $R_{\pm} = \nu_0 \exp[-E/(k_B T)] \exp[\pm \sigma V^*/(k_B T)]$, where $k_B$ is the Boltzmann constant, $\nu_0$ is an attempt frequency and $V^*$ is a constant called the 'activation volume'. In amorphous matter, the transition rates are negligible at zero stress. Thus, at finite stress one needs to consider only the rate $R_+$ of transitions in the direction aided by stress.

The linear dependence will always correctly describe small changes in the barrier height, since it is simply the first term in the Taylor expansion of the barrier height as a function of load. It is thus appropriate when the barrier height changes only slightly before the system escapes the local energy minimum. This situation occurs at higher temperatures; for example, Newtonian transport is obtained in the rate-state model in the limit where the system experiences only small changes in the barrier height before thermally escaping the energy minimum. As the temperature decreases, larger changes in the barrier height occur before the system escapes the energy minimum (giving rise to, for example, non-Newtonian transport). In this regime, the linear dependence is not necessarily appropriate, and can lead to inaccurate modeling. To be precise, at low shear rates ($\dot{\gamma} \leq \dot{\gamma}_c$), the system behaves as a power law shear-thinning material while, at high shear rates, the stress varies affinely with the shear rate. These two regimes correspond to two stable branches of stationary states, for which data obtained by imposing either $\sigma$ or $\dot{\gamma}$ exactly superpose.

In this short paper, motivated by the analogy used in \cite{1}, we shall adopt the verified transition-rate-state model \cite{6-8} to study the gravity-driven transport of cosmic textures (presumed to be amorphous) within a corrugated annular (cosmic) string. The possible nearly frictionless states due to strong shear-thinning will be relevant to the dark matter formation as mentioned above.
(considering the MMI [4]). To obtain the law of shear-thinning matter for explaining the too rapid annealing at the earliest time, because the relaxation at the beginning was steeper than could be explained by the bimolecular law, a hyperbolic sine law between the shear (strain) rate $\dot{\gamma}$ and shear stress $\tau$ was proposed and the close agreement with experimental data was obtained. This model has sound physical foundation from the thermal activation process [6-8] (a kind of (quantum) tunneling which relates to the matter rearranging by surmounting a potential energy barrier was discussed therein). With this model we can associate the (shear-thinning) fluid with the momentum transfer between neighboring atomic clusters on the microscopic scale and reveals the atomic interaction in the relaxation of flow with dissipation (the momentum transfer depends on the activation (shear) volume $V^* \equiv V_h$ which is associated with the center distance between atoms and is equal to $k_B T/\tau_0$ ($T$ is temperature in Kelvin, and $\tau_0$ a constant with the dimension of stress).

To consider the more realistic but complicated boundary conditions in the walls of the annular (cosmic) string, however, we will use the boundary perturbation technique [10] to handle the presumed wavy-roughness along the walls of the annular (cosmic) string. To obtain the analytical and approximate solutions, here, the roughness is only introduced in the radial or transverse direction. The relevant boundary conditions along the wavy-rough surfaces will be prescribed below. We shall describe our approach after this section: Introduction with the focus upon the boundary perturbation method. The approximate expression of the transport is then demonstrated at the end. Finally, we will illustrate our results into two figures and give discussions therein.

2 Theoretical Formulations

We shall consider a steady transport of the (shear-thinning) amorphous matter in a wavy-rough annular (cosmic) string of $r_1$ (mean-averaged inner radius) with the inner wall being a fixed wavy-rough surface: $r = r_1 + \epsilon \sin(k\theta + \beta)$ and $r_2$ (mean-averaged outer radius) with the outer wall being a fixed wavy-rough surface: $r = r_2 + \epsilon \sin(k\theta)$, where $\epsilon$ is the amplitude of the (wavy) roughness, $\beta$ is the phase shift between two walls, and the roughness wave number: $k = 2\pi/L$ ($L$ is the wavelength of the surface modulation in transverse direction).

Firstly, this amorphous matter (composed of cosmic textures) can be expressed as [6-8] $\dot{\gamma} = \dot{\gamma}_0 \sinh(\tau/\tau_0)$, where $\dot{\gamma}$ is the shear rate, $\tau$ is the shear stress, and $\dot{\gamma}_0 (\equiv C_k k_B T \exp(-\Delta E/k_B T)/h)$ is with the dimension of the shear rate; here $C_k \equiv 2V_h/V_m$ is a constant relating rate of strain to the jump frequency ($V_h = \lambda_2 \lambda_3 \lambda$, $V_m = \lambda_2 \lambda_3 \lambda_1$, $\lambda_2 \lambda_3$ is the cross-section of the transport unit on which the shear stress acts, $\lambda$ is the distance jumped on each relaxation, $\lambda_1$ is the perpendicular distance between two neighboring layers of particles sliding past each other), accounting for the interchain co-operation required, $h$ is the Planck constant, $\Delta E$ is the activation energy. In fact, the force balance gives the shear stress at a radius $r$ as $\tau = -(r \delta G)/2$ [6-8]. $\delta G$ is the
net effective gravity forcing along the transport (or tube-axis: z-axis) direction (considering \(dz\) element).

Introducing the forcing parameter \(\Phi = -(r_2/2\tau_0)\delta G\) then we have \(\dot{\gamma} = \dot{\gamma}_0 \sinh(\Phi r/r_2)\). As \(\dot{\gamma} = -du/dr\) (\(u\) is the velocity of the transport in the longitudinal (z-)direction of the annular (cosmic) string), after integration, we obtain

\[
u = u_s + \frac{\dot{\gamma}_0 r_2}{\Phi} \left[ \cosh \Phi - \cosh \left( \frac{\Phi r}{r_2} \right) \right],
\]

(1)

here, \(u_s(\equiv u_{slip})\) is the velocity over the (inner or outer) surface of the annular (cosmic) string, which is determined by the boundary condition. We noticed that a general boundary condition for transport over a solid surface [9] was

\[
\delta u = L_s^0 \dot{\gamma} \left( 1 - \frac{\dot{\gamma}}{\dot{\gamma}_c} \right)^{-1/2},
\]

(2)

where \(\delta u\) is the velocity jump over the solid surface, \(L_s^0\) is a constant slip length, \(\dot{\gamma}_c\) is the critical shear rate at which the slip length diverges. The slip (velocity) boundary condition above (related to the slip length) is closely linked to the mean free path of the particles together with a geometry-dependent factor (it is the quantum-mechanical scattering of Bogoliubov quasiparticles which is responsible for the loss of transverse momentum transfer to the container walls [10]). The value of \(\dot{\gamma}_c\) is a function of the corrugation of interfacial energy.

With the slip boundary condition [9], we can derive the velocity fields and transport rates along the wavy-rough annular (cosmic) string below using the verified boundary perturbation technique [11] and dimensionless analysis. We firstly select \(L_s^0\) to be the characteristic length scale and set \(r' = r/L_s^0\), \(R_1 = r_1/L_s^0\), \(R_2 = r_2/L_s^0\), \(\epsilon' = \epsilon / L_s^0\). After this, for simplicity, we drop all the primes. It means, now, \(r, R_1, R_2\) and \(\epsilon\) become dimensionless (\(\Phi\) and \(\dot{\gamma}\) also follow).

The wavy boundaries are prescribed as \(r = R_2 + \epsilon \sin(k\theta)\) and \(r = R_1 + \epsilon \sin(k\theta + \beta)\) and the presumed steady transport is along the z-direction (microannulus-axis direction).

2.1 Boundary Perturbation

Along the outer boundary (the same treatment below could also be applied to the inner boundary), we have \(\dot{\gamma} = (du)/(dn)_{on walls}\). Here, \(n\) means the normal. Let \(u\) be expanded in \(\epsilon\) :

\[
u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \cdots,
\]

and on the boundary, we expand \(u(r_0 + \epsilon dr, \theta(= \theta_0))\) into

\[
u(r, \theta)\big|_{(r_0 + \epsilon dr, \theta_0)} = u(r_0, \theta) + \epsilon [dr u_r(r_0, \theta)] + \epsilon^2 \left[ \frac{dr^2}{2} u_{rr}(r_0, \theta) \right] + \cdots =
\]

\[
\{u_{slip} + \frac{\dot{\gamma} R_2}{\Phi} \left[ \cosh \frac{\Phi r}{R_2} - \cosh \left( \frac{\Phi r}{R_2} \right) \right] \}_{on walls}, \quad r_0 = R_1, R_2;
\]

(3)

where

\[
u_{slip}_{on walls} = L_s^0 \{\dot{\gamma} [\left( 1 - \frac{\dot{\gamma}}{\dot{\gamma}_c} \right)^{-1/2}] \}_{on walls};
\]

(4)
Now, on the outer wall (cf. [11])

\[ \dot{\gamma} = \frac{du}{dn} = \nabla u \cdot \nabla (r - R_2 - \epsilon \sin(k\theta)) \bigg| \frac{|\nabla (r - R_2 - \epsilon \sin(k\theta))|}{\nabla (r - R_2 - \epsilon \sin(k\theta))} = [1 + \epsilon^2 \frac{k^2}{r^2} \cos^2(k\theta)]^{-\frac{1}{2}} [u_r(R_2 + \epsilon \theta) - u_r]\]

\[ \frac{k}{r^2} \cos(k\theta) u_0 |(R_2 + \epsilon \theta)| = u_0 |R_2 + \epsilon |u_1, |R_2 + u_{0rr} |R_2 \sin(k\theta) - \]

\[ \frac{k}{r^2} u_0 |R_2 \cos(k\theta)| + \epsilon^2 [\frac{1}{2} \frac{k^2}{r^2} \cos^2(k\theta) u_0 |R_2 + u_2, |R_2 + u_{1rr} |R_2 \sin(k\theta) + \]

\[ \frac{1}{2} u_{0rr}, |R_2 \sin^2(k\theta) - \frac{k}{r^2} \cos(k\theta)(u_{1r} |R_2 + u_{0r}, |R_2 \sin(k\theta))] + O(\epsilon^3). \tag{5} \]

Considering \( L^0_s \sim R_1, R_2 \gg \epsilon \) case, we also presume \( \sinh \Phi \ll \dot{\gamma}_c/\gamma_0 \). With equations (1) and (5), using the definition of \( \dot{\gamma} \), we can derive the velocity field \( u \) up to the second order:

\[ u(r, \theta) = -(R_2 \dot{\gamma}_0/\Phi) \{ \cosh(\Phi r/R_2) - \cosh \Phi [1 + \epsilon^2 \Phi^2 \sin^2(k\theta)/(2R_2^2)] + \]

\[ \epsilon \Phi \sinh \Phi \sin(k\theta)/R_2 \} + u_{\text{slip}}|r=R_2+\epsilon \sin(k\theta). \]

The key point is to firstly obtain the slip velocity along the boundaries or surfaces. After lengthy mathematical manipulations, we obtain the velocity fields (up to the second order) and then we can integrate them with respect to the cross-section to get the transport (volume flow) rate \( Q \), also up to the second order here:

\[ Q = \int_0^{\theta_p} \int_{R_1+\epsilon \sin(k\theta+\beta)}^{R_2+\epsilon \sin(k\theta)} u(r, \theta) r dr d\theta = Q_0 + \epsilon Q_{p0} + \epsilon^2 Q_{p2}. \]

In fact, the approximate (up to the second order) net transport (volume flow) rate reads:

\[ Q = \pi \dot{\gamma}_0 \{ L^0_s (R_2 - R_1^2) \sinh \Phi (1 - \frac{\sinh \Phi}{\dot{\gamma}_c/\gamma_0})^{-1/2} + \frac{R_2}{\Phi} [(R_2^2 - R_1^2) \cosh \Phi - \frac{2}{\Phi} (R_2^2 \sinh \Phi - \]

\[ R_1 R_2 \sinh(\Phi \frac{R_1}{R_2}) + \frac{2R_2^2}{\Phi^2} (\cosh \Phi - \cosh(\Phi \frac{R_1}{R_2})] + \epsilon^2 \{ \frac{\pi}{2} u_{\text{slip}_0} (R_2^2 - R_1^2) + \]

\[ L^0_s \frac{\pi}{4} \dot{\gamma}_0 \sinh(\Phi (1 + \frac{\sinh \Phi}{\dot{\gamma}_c/\gamma_0})(-k^2 + \Phi^2)[1 - (\frac{R_1}{R_2})^2] + \frac{\pi}{2} \dot{\gamma}_0 [R_1 \sinh(\frac{R_1}{R_2} \Phi) - R_2 \sinh \Phi - \]

\[ \frac{1}{2} \dot{\gamma}_0 \frac{R_2}{\Phi} [\cosh \Phi - \cosh(\Phi \frac{R_1}{R_2})] + \frac{\pi}{4} \dot{\gamma}_0 \Phi \sinh \Phi [R_2 - \frac{R_2^2}{R_2}] + \]

\[ \pi \dot{\gamma}_0 \{ [\sinh \Phi + L^0_s \cosh \Phi (1 + \frac{\sinh \Phi}{\dot{\gamma}_c/\gamma_0})] (R_2 - R_1 \cos \beta) \} + \frac{\pi}{2} \dot{\gamma}_0 \frac{R_2}{\Phi} \cosh \Phi + \]

\[ L^0_s \frac{\pi}{4} \Phi^2 \dot{\gamma}_0^2 \frac{\sinh \Phi}{\dot{\gamma}_0^2} \frac{\gamma_c}{\dot{\gamma}_0} [1 - (\frac{R_1}{R_2})^2] \} \cosh \Phi. \tag{6} \]

Here,

\[ u_{\text{slip}_0} = L^0_s \dot{\gamma}_0 [\sinh \Phi (1 - \frac{\sinh \Phi}{\dot{\gamma}_c/\gamma_0})^{-1/2}], \tag{7} \]
3 Results and Discussions

We firstly check the roughness effect (or combination of curvature and confinement effects [12-13]) upon the gravity-driven transport via strongly shearing because there are no available experimental data and numerical simulations for the same geometric configuration (annular (cosmic) string with wavy corrugations in transverse direction). With a series of forcings (due to imposed gravity forcings): \( \Phi \equiv R_2(\delta G)/(2\tau_0) \), we can determine the enhanced shear rates \((d\gamma/dt)\) due to gravity forcings. From equation (5), we have (up to the first order)

\[
\frac{d\gamma}{dt} = \frac{d\gamma_0}{dt}[\sinh \Phi + \epsilon \sin(k\theta) \frac{\Phi}{R_2} \cosh \Phi].
\] (8)

The parameters are fixed below (the orientation effect : \( \sin(k\theta) \) is fixed here). \( r_2 \) (the mean outer radius) is selected as the same as the slip length \( L_0^s \). The amplitude of wavy roughness can be tuned easily. The effect of wavy-roughness is significant once the forcing (\( \Phi \)) is rather large (the maximum is of the order of magnitude of \( \epsilon[\Phi \tanh(\Phi)/R_2] \)).

If we select a (fixed) temperature, then from the expression of \( \tau_0 \), we can obtain the shear stress \( \tau \) corresponding to above gravity forcings (\( \Phi \)):

\[
\tau = \tau_0 \sinh^{-1}[\sinh(\Phi) + \epsilon \sin(k\theta) \frac{\Phi}{R_2} \cosh(\Phi)].
\] (9)

There is no doubt that the orientation effect (\( \theta \)) is also present for the amorphous matter. For illustration below, we only consider the maximum case: \( |\sin(k\theta)| = 1 \). We shall demonstrate our transport results below. The wave number of roughness in transverse direction is fixed to be 10 (presumed to be the same for both walls of the annular (cosmic) string) here.

Now, we start to examine the temperature effect. We fix the forcing \( \Phi \) to be 1 as its effect is of the order \( O(1) \) for the shear rate (cf. Fig. 2). As the gravity forcing (\( \delta G \)) might depend on the temperature (\( \delta G = 2\tau_0 \Phi/R_2 \), \( \tau_0 \equiv \tau_0(T) \)), \( V^*(\equiv V_h) \) is presumed to be temperature independent here for simplicity). Note that, according to [7], \( V^* = 3V\delta\gamma/2 \) for certain matter during an activation event [7], where \( V \) is the deformation volume, \( \delta\gamma \) is the increment of shear strain.

As the primary interest of present study is related to the possible phase transition [14-16] or formation of superfluidity (presumed to be relevant to the formation of dark matter mentioned in Introduction) due to strong shearing, we shall present our main results in the following. We performed intensive calculations or manipulations of related physical and geometric parameters, considering a hot big-bang universe [14] and examine what happens as it expands and cools through the transition temperature \( T_c \). The selected temperature range and the activation energy follows this reasoning. Note that in unified models of weak and electromagnetic interactions \( T_c \) is of the order of the square root of the Fermi coupling constant [14], \( G_F^{1/2} \), i.e. a few hundred GeV. Thus the transition occurs when the universe is aged between \( 10^{-10} \) and \( 10^{-12} \) seconds and far above nuclear densities [14,17]. One possible superfluidity formation regime is demonstrated in Fig. 1. The activation energy (\( \Delta E \)) is \( 10^{-10} \) Joule (\( \sim 1 \) GeV) and the activation volume is \( 10^{-9} \) m\(^3\). In fact, all the results shown in this figure depend on \( \dot{\gamma}_0 \) and are thus very sensitive
to $\Delta E$. Here $C_k = 2$ and the sudden jump of the shear stress (directly linked to the friction) occurring around $T \sim 10^{11}$ °K could be the transition temperature for the selected $\Delta E$ and $C_k$. There is a sudden friction drop around two orders of magnitude below $T \sim 10^{11}$ °K and it is almost frictionless below $T \sim 10^{10}$ °K. If we borrow the analogy from the MMI [4] (the rotational properties of a superfluid as well as a supersolid in which some of the particles remain still while the rest of them rotate with the container) we can identify the formation of superfluidity as the possible dark matter formation as the mass is missing within this regime.

The possible reasoning for this formation can be illustrated in Fig. 2. It could be due to the strong shearing driven by larger gravity forcings along a confined (cosmic) string or topological defect. The shear-thinning (the viscosity diminishes with increasing shear rate) reduces the viscosity significantly. One possible outcome for almost vanishing viscosity is the nearly frictionless transport. It seems to us that the formation of dark matter is a genuinely dynamic effect. As to the question whether the internal structure of defects lead to persistent currents in their cores [17] is yet open. We shall investigate the evolution of the (cosmic) string [14,17] in the future.

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Fig. 1. Comparison of calculated (shear) stresses using an activation energy $10^{-10}$ J or $\sim 1$ GeV. There is a sharp decrease of shear stress around $T \sim 10^{11}$ K. Below $10^{10}$ K, the transport of amorphous matter is nearly frictionless.
Fig. 2. Increasing shear causes a local energy minimum to flatten until it disappears (energy barrier removal or quantum-like tunneling). The structural contribution to the shear stress is referred to shear-thinning.