Entangled-Photon Imaging of a Pure Phase Object

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Abstract

We demonstrate experimentally and theoretically that a coherent image of a pure phase object may be obtained by use of a spatially incoherent illumination beam. This is accomplished by employing a two-beam source of entangled photons generated by spontaneous parametric down-conversion. Though each of the beams is, in and of itself, spatially incoherent, the pair of beams exhibits higher-order inter-beam coherence. One of the beams probes the phase object while the other is scanned. The image is recorded by measuring the photon coincidence rate using a photon-counting detector in each beam. Using a reflection configuration, we successfully imaged a phase object implemented by a MEMS micro-mirror array. The experimental results are in accord with theoretical predictions.

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**Introduction.**—It is well known in conventional optics that a pure phase object cannot be imaged by illumination with a spatially incoherent optical field. “Coherence,” in this case, refers to coherence in the second-order. The question we ask is the following: Can an optical field lacking second-order spatial coherence, but endowed with higher-order coherence, be used to measure the spatial distribution of a pure phase object? Clearly, this is not possible by making use of conventional detectors, which are sensitive to the optical intensity. Measurement of the fourth-order coherence function has been used for imaging, as in the Hanbury-Brown–Twiss interferometer, but the imaging of a pure phase object cannot be carried out using such a configuration if a conventional thermal source of light is used.

In this paper we demonstrate that the imaging of a phase object can, in fact, be achieved by making use of a special light source that is incoherent in second-order, but exhibits higher-order coherence properties. Perhaps the most celebrated example of an optical field exhibiting fourth-order coherence, in combination with second-order incoherence, is the twin-beam light emitted by the process of spontaneous parametric down-conversion (SPDC) in a non-linear optical crystal. Each of the emitted beams lacks second-order spatial coherence (as well as temporal coherence), but the two beams are endowed with inter-beam higher-order spatial (and temporal) coherence, which is exhibited via enhanced photon coincidences at directions for which photon momentum is conserved. Such sources exhibit unique quantum-correlation features that have generated considerable interest, particularly in the past few years. Since photons are emitted in pairs, in an entangled quantum state, imaging using such a light source has been referred to as entangled-photon imaging. In this imaging technique, only one of the beams interacts with the object; the photon coincidence rate is measured much the same way as it is in the Hanbury-Brown–Twiss interferometer.

Several experimental demonstrations and theoretical studies have been directed at dissecting the capabilities and limitations of entangled-photon imaging. On the theoretical side, it has been shown that such a two-photon imaging system requires entanglement in order to function as a coherent imaging system (i.e., as a system capable of imaging a phase object). On the experimental side, the class of objects that has been utilized in such demonstrations has been limited to that of amplitude objects, e.g., objects with a spatially varying amplitude transmittance. Such objects have usually taken the form of
amplitude-modulating transparencies inserted in a transmission configuration.

We proceed to provide an experimental demonstration of entangled-photon imaging of a pure phase object in a reflection configuration. The phase object is illuminated by one of the (spatially incoherent) beams and the photon-coincidence rate for detectors placed at both beams is measured. This allows the formation of a coherent image of the phase object. Phase objects are of special interest in quantum information processing since they introduce a unitary operation that is reversible (this is in contrast to amplitude masks of any form).

Imaging configuration.—The arrangement used in our experiment is shown schematically in Fig. 1(a). The source of light is spontaneous parametric down-conversion generated in a nonlinear crystal (NLC) pumped by a laser beam (LASER). The downconverted photons are emitted in pairs in two spatially separated beams (each of which is spatially incoherent, as indicated above). One of the beams (PROBE) impinges on the object, in our case a MEMS micro-mirror array configured so as to modulate the phase of the impinging wavefront. Upon reflection from the object, the probe beam is detected by a single detector D1 fitted with a fixed small pinhole P1. The other beam (REFERENCE) does not interact with the object, and is simply directed to a small pinhole P2 and a detector D2. The terms “PROBE” and “REFERENCE” are used since they are more descriptive in this case than the usual terminology: signal and idler. The coincidence rate of the photons detected by the two detectors is measured as detector D2 (together with pinhole P2) is scanned. As will be shown theoretically, and confirmed experimentally, if the intervening optical system is appropriately designed, a coherent image of the phase object may be obtained.

The system is configured such that the probe beam cannot, by itself, generate an image of the phase object. First, detector D1 is a single fixed detector lacking spatial resolution. Second, the probe beam lacks second-order spatial coherence. Third, the distance \( d_b \) between the object and the detector D1 is deliberately chosen such that D1 does not lie in the far-field, thus precluding the formation of a far-field diffraction pattern of the phase object, which could provide information about the phase spatial distribution were the beam coherent.

The pure phase object we consider is characterized by a spatially varying unimodular complex amplitude transmittance or reflectance of the form \( \exp[i\theta(r)] \), where \( \theta(r) \) is a real function of the position \( r \) in the object plane. Such an object could not be imaged directly by using a conventional single-lens imaging system satisfying the imaging condition and a conventional intensity-sensitive detector. Since such a system ideally provides a geometric
mapping between each point in the object plane and a corresponding point in the image plane, it yields no phase information. In the context of entangled-photon imaging, such a system was implemented in Ref. [10] for imaging the intensity transmittance of an object.

In conventional coherent optics, a phase object is typically imaged either interferometrically or by use of an optical spatial Fourier-transform system, which converts the phase distribution of the optical wave front at the object plane into a spatially varying amplitude distribution that is detectable by an intensity-sensitive detector. Two examples of Fourier-transform systems are (1) a single lens 2–f system, and (2) free-space propagation in the Fraunhofer regime, commonly known as the far-field [18]. In our entangled-photon imaging system, we have implemented a Fourier-transform configuration based on lensless propagation to the far-field, a system similar to that used in Ref. [19]. The exact Fourier transform is achieved only in the Fraunhofer-diffraction region, which necessitates traveling a prohibitively long distance [18]. The distances used in our experiment actually place the image in the Fresnel-diffraction region.

Theory.—A theoretical expression for the photon coincidence rate in the above imaging configuration is obtained by using the formalism developed in Ref. [20]. The coincidence rate of photon pairs at points \( x_1 \) and \( x_2 \), in the planes of detectors \( d_1 \) and \( d_2 \), is proportional to the fourth-order coherence function of the optical fields

\[
G^{(2)}(x_1, x_2) = \left| \int \int d\phi(x, x') \ h_1(x_1, x) \ h_2(x_2, x') \right|^2.
\]

(1)

Here \( h_1 \) and \( h_2 \) are the impulse response functions describing the optical systems that the probe and reference photons traverse from the crystal to \( d_1 \) and \( d_2 \), respectively. In the arrangement outlined here, both \( h_1 \) and \( h_2 \) represent free-space propagation, and \( h_1 \) includes the phase object. The quantum state of light produced in the process of SPDC is represented by the state vector

\[
|\Psi\rangle = \int \int d\phi(x, x') \ |1_x, 1_{x'}\rangle;
\]

(2)

the state function \( \phi \) is normalized so that \( \int \int d\phi(x, x') |\phi(x, x')|^2 = 1 \). The state function is related to the physical parameters of the SPDC source through the relation

\[
\phi(x, x') = \int dy \ E_p(y) \ \xi(x - y, x' - y),
\]

(3)

where \( E_p(x) \) is the pump field spatial profile, \( \xi(x, x') \) is the Fourier transform of the phase-matching function \( \tilde{\xi}(q_1, q_2) = \text{sinc}(\frac{l}{2\pi} \Delta) \ exp(-i \frac{l}{2} \Delta) \); \( l \) is the thickness of the nonlinear
crystal; \(q_1\) and \(q_2\) are the transverse momenta of the probe and reference beams; and 
\(\Delta(q_1, q_2)\) is the mismatch in longitudinal momenta of the pump, probe, and reference beams. 
These formulas all assume that only a narrow band of wavelengths, centered around the degenerate wavelength, is allowed, as is usually imposed by the use of interference filters in the optical arrangement.

Coherent image formation in this configuration may be understood by noting that the left hand side of Eq. (1) is mathematically identical to the coherent image at \(x_2\) of a point source at \(x_1\) illuminating an optical system composed of a cascade of systems with impulse response functions \(h_1(x, x_1)\), \(\phi(x', x)\), and \(h_2(x_2, x')\). This is physically identical to a point source located at the point detector \(D_1\) emitting light traveling backward through \(h_1\) towards the nonlinear crystal, which serves as a special mirror reflecting the wave through \(h_2\), and creating a coherent image in the plane of detector \(D_2\). This advanced-wave interpretation, pioneered by D. N. Klyshko [21], provides a rationale for the formation of a Fourier-transform image of the phase object in the far field.

The effect of the finite size of \(P_1\) may be accommodated by integrating the variable \(x_1\) over the area of \(P_1\),

\[
C(x_2) = \int_{P_1} dx_1 G^{(2)}(x_1, x_2).
\]

The quantity \(C(x_2)\) is the coincidence rate measured when \(P_2\) is scanned, while \(P_1\) is held fixed, and corresponds to the data collected in the experiment. In view of the advanced-wave interpretation, the finite size of \(P_1\) introduces partial coherence into the imaging system and may render the system effectively incoherent [20]. The size of the pinhole \(P_2\), on the other hand, sets a limit on the resolution of the scanned coherent image.

Calculating \(C(x_2)\) is difficult when all the physical parameters of the configuration are considered. To simplify the calculation we assumed that the nonlinear crystal is thin \((l \to 0)\) and that the pump is a plane wave incident normally on the crystal. This zeroth-order approximation underestimates the width of the produced far-field image, since an infinite spatial spectrum is implicit in the thin-crystal approximation. We have therefore approximately accommodated the finite width of the pump and finite crystal thickness by multiplying the calculated \(C(x_2)\) by the measured conventional far-field image of the reference obtained from single-photon rates at \(D_2\), which is independent of the object and depends only on the parameters of the pump beam and the nonlinear crystal.

Experiment.—The pure phase object used in our experiments comprised a 12×12 array
of gold-plated micro-mirrors, each of dimension $300 \times 300 \mu m^2$. The height of each mirror, with respect to a fixed datum, is altered via an electrostatic potential. If one micro-mirror is pulled down from the datum a distance $d$, the portion of the wavefront impinging on this region accumulates a phase $2\pi (2d/\lambda)$ relative to the datum (which is taken to be phase 0), after reflection from the micro-mirror and double traversal of the distance $d$. For example, light at $\lambda = 812$ nm, such as that in our experiment, when reflected from a micro-mirror pulled back a distance $d = 200$ nm, accumulates a phase of approximately $\pi$ radians.

We conducted experiments using three distinct phase distributions: (a) zero phase everywhere (flat mirrors); (b) a single line of micro-mirrors pulled down to implement a phase of $\pi$; and (c) two lines of micro-mirrors pulled down to implement a phase of $\pi$ separated by an undisturbed line of phase zero. All three distributions are independent of one dimension of the array, and are thus effectively one-dimensional distributions ($r \rightarrow x$). This is helpful since it enables us to integrate along the uniform direction at $D_1$ and $D_2$.

A more detailed view of the experimental arrangement is illustrated in Fig. 2. The pump was the 406-nm line of a cw Kr-ion laser with a power of 30 mW. The nonlinear crystal was a 1.5-mm-thick BBO crystal cut for collinear (probe and reference emitted into the same beam), degenerate, type-I (probe- and reference have the same polarization) SPDC. We chose a type-I collinear configuration, rather than a type-II (probe- and reference with orthogonal polarizations) collinear configuration such as that used in Refs. [10] and [11], or the type-I non-collinear configuration used in Ref. [5]. The advantage of the type-I collinear configuration is that the two down-converted beams are emitted in the same circularly symmetric spatial mode. This is useful for carrying out imaging experiments since artifacts arising from differences between the spatial distributions of the two beams, as well as peculiarities of beam shape, are eliminated. However, half of the photon-pair flux is lost in this configuration, as will be elaborated upon below.

The pump (extraordinary polarization) was separated from the down-converted photons (ordinary polarization) by means of a pair of Glan-Laser polarizing beam splitters ($GL_1$ and $GL_2$), placed before and after the nonlinear crystal, respectively. A long-pass colored glass filter (CGF) of cutoff wavelength 560 nm was used to further separate away the pump. The photon pairs then permitted to impinge on a nonpolarizing sheet beam splitter (BS). As a result, half of the pairs are separated into the two output beams whereas the other half of the pairs emerge together at the same output port and thus fail to contribute to coincidences;
this accounts for the 50% reduction of photon flux mentioned above.

The beam reflected from the beam splitter impinges on the phase object and from there is reflected to a single-photon-detector module (fiber-coupled EG&G SPCM-AQR-15). The detector is preceded by a vertical slit $p_1$ of width 1.4 mm and an interference filter $f_1$ (centered at 800 nm with a bandwidth of 66 nm) to eliminate any remaining pump photons. The detected photons are integrated along the direction parallel to the slit. The beam transmitted through the beam splitter, which does not interact with the object, is directed to an identical detection unit (vertical slit $p_2$ and filter $f_2$). However this detector is mounted on a computer-controlled stage that permits scanning of the beam. The electrical pulses from the two detectors are sent to a coincidence circuit ($\otimes$) with a 2-nsec timing window.

The distance from the NLC through the BS to the micro-mirrors is $d_a = 1.17$ m; the distance from the micro-mirrors to the detector $D_1$ is $d_b = 1.98$ m; the distance from the NLC, reflecting from the BS, to $D_2$ is $d_2 = 3.96$ m. For this configuration, the diffraction pattern is formed at a distance $d_2 + d_a$ from the mirrors: from the mirrors to the BS to the NLC, back to the BS and then on to $D_2$.

**Results.**—The results are displayed in Fig. 3 for the three phase objects on which we conducted imaging experiments. In each case, the coincidence counts measured by detectors $D_1$ and $D_2$ is plotted as a function of the scanned position $x_2$ of detector $D_2$. The experimental results (open circles) are seen to match the theoretical predictions based on the approximate model (solid curves) reasonably well, although the theoretical profiles are slightly wider. We have also recorded the single-photon counts collected from $D_2$, independently of the counts recorded from $D_1$. This measure is, of course, completely independent of the object; it represents the far-field pattern of the reference beam and thus depends only on the physical parameters of the pump and nonlinear crystal. The first object has a uniform phase distribution, $\theta(x) = 0$, which represents a highly reflecting, uniform mirror of finite aperture. The coincidence profile shown in Fig. 3(a) is simply the Fourier transform of this object, which is the diffraction pattern of the object aperture.

The second object has a phase distribution in the form of a single strip of phase $\pi$ in a uniform background of phase zero. We call this object a phase slit. The measured coincidence image presented in Fig. 3(b) has a double-peaked profile that is the Fresnel transform (approximately the Fourier transform) of the object phase distribution. This profile is dramatically different from that associated with the usual amplitude slit, which has
a single central peak and smaller side lobes.

The third object is a double phase slit. The measured coincidence profile displayed in Fig. 3(c) is qualitatively similar to that for the single phase slit illustrated in Fig. 3(b), but has a wider and deeper dip, and also a lower peak value. There are two reasons for this: (1) The diffraction pattern for the double phase slit is more spread out, and since no photons are absorbed by the phase object, the height of the distribution must be lower; (2) The rate of single photons detected by $d_1$ is smaller. The pattern reflected towards $d_1$ is wider, and since the pinhole has the same width, fewer photons are collected for the double phase slit, thereby resulting in a further reduction of coincidence counts for this case. We have recorded the single-photon rate at both detectors in the two cases. It is gratifying that the incorporation of this additional reduction factor into the theoretical calculation leads to a ratio of heights of the theoretical patterns that match experiment.

The diffraction profile of a double-phase-slit object is dramatically different from that of a conventional double-slit amplitude object; the latter exhibits a single central peak with smaller side lobes, rather than a double-peaked distribution. For the same configuration and dimensions, the amplitude object produces a substantially wider diffraction pattern. Note, however, that a “double-strip” amplitude object (an aperture that transmits light everywhere except at two parallel strips) has a diffraction pattern similar to that of a double phase slit of the same dimensions, but it has far less visibility (it is well known in phase lithography that the maximum contrast produced by slit modulation is obtained for $\pi$-phase-shift slits).

**Conclusion.**—We have experimentally demonstrated that a coherent image of a reflective pure phase object may be obtained by using a spatially incoherent probe beam. This is accomplished by use of an auxiliary reference beam that does not interact with the object. Generated by spontaneous parametric down-conversion, each of the two beams is spatially incoherent but together they exhibit inter-beam higher-order coherence, by virtue of the fact that the source emits photon pairs in an entangled state. By measuring the photon coincidence rate, using a single-photon detector in each of the two beams, we observe the fourth-order cross-coherence function for the reflected probe beam and the reference beam. This contains an image of the phase object identical to that measurable with coherent light. The experiments we carried out include measurements of the Fresnel (approximately Fourier) transform of simple phase objects, but other coherent images may be similarly measured. This includes phase-contrast imaging, which may be realized by splitting the probe beam
into two laterally displaced probe beams transmitted through the object. It also includes holography, which may be realized by splitting the probe beam into a reference and an object beam as in conventional holography \[22\].

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FIG. 1: (a) Schematic of the experimental arrangement for the quantum imaging of a reflective pure phase object (MEMS). NLC represents a nonlinear crystal, $p_1$ and $p_2$ are pinholes, $d_1$ and $d_2$ are detectors, $\otimes$ represents an electronic coincidence circuit, and the $d_{ij}$ are distances.

FIG. 2: (a) Actual experimental arrangement for the quantum imaging of a reflective pure phase object (MEMS). NLC represents the nonlinear crystal, CGF stands for a colored glass filter, GL$_1$ and GL$_2$ are two orthogonally oriented Glan-Laser polarizing beam splitters, BS is a nonpolarizing beam splitter, $p_1$ and $p_2$ are vertical slits, $f_1$ and $f_2$ are interference filters, $d_1$ and $d_2$ are single-photon detectors, and $\otimes$ represents an electronic coincidence circuit.

FIG. 3: Experimental coincidence counts (open circles) measured from $d_1$ and $d_2$ as the position $x_2$ of detector $d_2$ is scanned. The collection time is 80 sec per point. The solid curves are the theoretical predictions. (a) Object with uniform phase zero; (b) Single-slit object with phase $\pi$; (c) Double-slit object with phase $\pi$ separated by a line of phase 0.
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