Coverage in Heterogeneous Downlink Millimeter Wave Cellular Networks

Esma Turgut and M. Cenk Gursoy
Department of Electrical Engineering and Computer Science
Syracuse University, Syracuse, NY 13244
Email: eturgut@syr.edu, mcgursoy@syr.edu

Abstract—In this paper, we provide an analytical framework to analyze heterogeneous downlink mmWave cellular networks consisting of $K$ tiers of randomly located base stations (BSs) where each tier operates in a mmWave frequency band. Signal-to-interference-plus-noise ratio (SINR) coverage probability is derived for the entire network using tools from stochastic geometry. The distinguishing features of mmWave communications such as directional beamforming and having different path loss laws for line-of-sight (LOS) and non-line-of-sight (NLOS) links are incorporated into the coverage analysis by assuming averaged biased-received power association and Nakagami-$m$ fading. By using the noise-limited assumption for mmWave networks, a simpler expression requiring the computation of only one numerical integral for coverage probability is obtained. Finally, effect of beamforming alignment errors on the coverage probability analysis is investigated to get insight on the performance in practical scenarios.

I. INTRODUCTION

There has been an exponential growth in mobile data and traffic in recent years due to, e.g., ever increasing use of smartphones, portable devices, and data-hungry multimedia applications. Limited available spectrum in microwave (μWave) bands does not seem to be capable of meeting this demand in the near future, motivating the move to new frequency bands. Therefore, the use of large-bandwidth at millimeter wave (mmWave) frequency bands becomes a good candidate for fifth generation (5G) cellular networks and has attracted considerable attention recently [1]–[5].

Despite the great potential of mmWave bands, they have been considered attractive only for short-range indoor communication due to increase in free-space path loss with increasing frequency, and poor penetration through solid materials. However, these high frequencies may also be used for outdoor communication over a transmission range of about 150-200 meters as demonstrated by recent channel measurements [1], [2], [5]. Also, comparable coverage area and much higher data rates than μWave networks can be achieved provided that the base station density is sufficiently high and highly directional antennas are used [6]. With the employment of directional antennas, mmWave cellular networks can be considered as noise-limited rather than interference-limited [3], [7], [8], [9], [10]. Also, another key feature of mmWave cellular networks is expected to be heterogeneity to have higher data rates and expanded coverage [4].

A general model for heterogeneous cellular networks is described as a combination of $K$ spatially and spectrally coexisting tiers which are distinguished by their transmit powers, spatial densities, blockage models [11], [12]. For example, high-power and low-density large cell base stations (BSs) may coexist with denser but lower power small cell BSs. Small cell BSs help the congested large cell BSs by offloading some percentage of their user equipments (UEs), which results in a better quality of service per UE [13]. Moreover, to provide more relief to the large cell network, cell range expansion technique which is enabled through cell biasing for load balancing was considered e.g., in [12], [14], [15].

Several recent studies have also addressed heterogeneous mmWave cellular networks. In [16], authors consider two different types of heterogeneity in mmWave cellular networks: spectrum heterogeneity and deployment heterogeneity. A hybrid cellular network scenario is considered in [7] for characterizing uplink-downlink coverage and rate distribution of self-backhauled mmWave cellular networks, and in [17] for the analysis of downlink-uplink decoupling. In both papers, mmWave small cells are opportunistically used and UEs are offloaded to the μwave network when it is not possible to establish a mmWave connection. A more general mathematical framework to analyze the multi-tier mmWave cellular networks is provided in [10]. In [18], benefits of BS cooperation in the downlink of a heterogeneous mmWave cellular system are analyzed. Contrary to the hybrid scenario, each tier is assumed to be operate in a mmWave frequency band in both [10] and [18]. Similarly, in this paper we consider a cellular network operating exclusively with mmWave cells, while an extension to a hybrid scenario can be addressed and a similar analytical framework can be employed by eliminating the unique properties of mmWave transmissions in the analysis of the μWave tier.

Since the path loss and blockage models for mmWave communications are significantly different from μWave communications, three different states namely line-of-sight (LOS), non-line-of-sight (NLOS) and outage state are considered for mmWave frequencies [9], [10]. For analytical tractability, equivalent LOS ball model was proposed in [6]. In [7], authors considered probabilistic LOS ball model, which is more flexible than the LOS ball model to capture the effect of different realistic settings. In [10], probabilistic LOS ball model is generalized to a two-ball model, which is based on path loss intensity matching algorithm.

II. SYSTEM MODEL

In this section, a $K$-tier heterogeneous downlink mmWave cellular network is modeled where the BSs in the $k$th tier
are distributed according to a homogeneous PPP $\Phi_k$ of density $\lambda_k$ on the Euclidean plane for $k = 1, 2, \ldots, K$. BSs in all tiers are assumed to be transmitting in a mmWave frequency band where the $k$th tier of BSs is distinguished by its transmit power $P_k$, biasing factor $B_k$, and blockage model parameters. The UEs are also spatially distributed according to an independent homogeneous PPP $\Phi_u$ of density $\lambda_u$. Without loss of generality, a typical UE is assumed to be located at the origin according to Slivnyak’s theorem [19], and it is associated with the tier providing the maximum located at the origin according to Slivnyak’s theorem [19], and it is associated with the tier providing the maximum

In this setting, we have the following assumptions regarding the system model of the $K$-tier heterogeneous downlink mmWave cellular network:

**Assumption 1 (Directional beamforming):** Antenna arrays at the BSs of all tiers and UEs are assumed to perform directional beamforming where the main lobe is directed towards the dominant propagation path while smaller side lobes direct energy in other directions. For tractability in the analysis and similar to [6], [7], [10], [20], [21], [22], antenna arrays are approximated by a sectored antenna model, in which the array gains are assumed to be constant $M$ for all angles in the main lobe and another smaller constant $m$ in the side lobes [23]. Initially, perfect beam alignment is assumed in between UE and its serving BS, leading to an overall antenna gain

In other words, maximum directivity gain can be achieved for the intended link by assuming serving BS and UE can adjust their antenna steering orientation using the estimated angles of arrivals. Also, beam direction of the interfering links is modeled as a uniform random variable on $[0, 2\pi]$. Therefore, the effective antenna gain between an interfering BS and UE is a discrete random variable (RV) described by

$$G = \begin{cases} MM & \text{with prob. } p_{MM} = (\frac{\theta}{2\pi})^2 \\ Mm & \text{with prob. } p_{Mm} = 2\frac{\theta}{2\pi} - \frac{\theta}{2\pi} \\ mm & \text{with prob. } p_{mm} = (\frac{2\pi - \theta}{2\pi})^2, \end{cases}$$

where $\theta$ is the beam width of the main lobe, and $p_G$ is the probability of having an antenna gain of $G$.

**Assumption 2 (Path loss model and blockage modeling):** Link between a BS and a typical UE can be either a line-of-sight (LOS) or non-line-of-sight (NLOS) link. However, according to recent results on mmWave channel modeling, an additional outage state can also be included to represent link conditions. Therefore, a link can be in a LOS, NLOS or in an outage state [9]. In a LOS state, BS should be visible to UE, i.e., there is no blockage in the link. On the other hand, in a NLOS state, blockage occurs in the link, and if this blockage causes a very high path loss, an outage state occurs, i.e., no link is established between the BS and the UE.

Consider an arbitrary link of length $r$, and define the LOS probability function $p(r)$ as the probability that the link is LOS. Using field measurements and stochastic blockage models, $p(r)$ can be modeled as $e^{-\gamma r}$ where decay rate $\gamma$ depends on the building parameter and density [24]. For simplicity, LOS probability function $p(r)$ can be approximated by step functions. In this approach, the irregular geometry of the LOS region is replaced with its equivalent

$^2$Subsequently, beamsteering errors are also addressed.

Different path loss laws are applied to LOS and NLOS links. Thus, the path-loss on each link in the $k$th tier can be expressed as follows:

$$L_k(r) = \begin{cases} \frac{d_\text{LOS}}{R_k} & \text{w.p. } \beta_{k1} \\ \frac{d_\text{NLOS}}{R_k} & \text{w.p. } (1 - \beta_{k1}) \end{cases}$$

where $\beta_{k1}$ and $\beta_{k2}$ are the LOS and NLOS path loss exponents, respectively, and $R_{k1}$ and $R_{k2}$ are the radii of the inner and outer balls for $k$th tier, respectively.

**A. Statistical Characterization of the Path Loss**

Let $N_k = \{L_k(r)\}_{r \in d_k}$ denote the point process of the path loss between the typical UE and BSs in the $k$th tier. The characteristics of the typical UE which depend on the path loss can be determined by the distribution of $N_k$ [25]. Therefore, in Lemma 1 and Lemma 2 below, characterization of the complementary cumulative distribution function (CCDF) and the probability density function (PDF) of the path loss are provided.

**Lemma 1:** The CCDF of the path loss from a typical UE to the BS in the $k$th tier can be formulated as $F_{L_k}(x) = \mathbb{P}(L_k(r) > x) = \exp(-\Lambda_k((0,x)))$ for $k = 1, 2, \ldots, K$ by applying the void probability theorem of PPPs [25] with $\Lambda_k((0,x))$ defined at the top of the next page in (3) where $1(\cdot)$ is the indicator function.

**Proof:** See [26, Appendix A].

**Lemma 2:** The CCDF of the path loss from the typical UE to the LOS/NLOS BS in the $k$th tier can be formulated as $F_{L_k}(x) = \mathbb{P}(L_k(r) > x) = \exp(-\Lambda_{k,L}((0,x)))$ for $k = 1, 2, \ldots, K$ and $s \in \{\text{LOS, NLOS}\}$ with $\Lambda_{k,s}((0,x))$ defined at the top of the next page for LOS and NLOS in (4) and (5), respectively.
λ_k([0, x]) = \pi \lambda_k \left[ (\beta_1 k_2 x^{2/\alpha_k L} + (1 - \beta_1 k_2) x^{2/\alpha_k N}) 1(x < R_{k1}^{\alpha_k L}) + (\beta_1 k_2 R_{k1}^2 + (1 - \beta_1 k_2) x^{2/\alpha_k N} + \beta_2 k_2 (x^{2/\alpha_k L} - R_{k1}^2)) 1(R_{k1}^{\alpha_k L} < x < R_{k2}^{\alpha_k L}) + \right.
\left. (\beta_2 k_2 R_{k2}^2 + (1 - \beta_2 k_2) x^{2/\alpha_k N}) 1(R_{k1}^{\alpha_k N} < x < R_{k2}^{\alpha_k N}) + (R_{k2}^2) 1(x > R_{k2}^{\alpha_k N}) \right],
(3)

λ_{k,LOS}([0, x]) = \pi \lambda_k \left[ ((1 - \beta_1 k_2) x^{2/\alpha_k L}) 1(x < R_{k1}^{\alpha_k L}) + (\beta_2 k_2 (x^{2/\alpha_k N} - R_{k1}^2)) 1(R_{k1}^{\alpha_k L} < x < R_{k2}^{\alpha_k L}) + \right.
\left. (\beta_2 k_2 (x^{2/\alpha_k L} - R_{k1}^2) + (1 - \beta_2 k_2)(x^{2/\alpha_k N} - R_{k1}^2)) 1(x > R_{k2}^{\alpha_k N}) \right]
(4)

λ_{k,NLOS}([0, x]) = \pi \lambda_k \left[ ((1 - \beta_1 k_2) x^{2/\alpha_k N}) 1(x < R_{k1}^{\alpha_k N}) + (\beta_1 k_2 (x^{2/\alpha_k L} - R_{k1}^2)) 1(R_{k1}^{\alpha_k N} < x < R_{k2}^{\alpha_k N}) + \right.
\left. (\beta_1 k_2 (x^{2/\alpha_k L} - R_{k1}^2) + (1 - \beta_1 k_2)(x^{2/\alpha_k N} - R_{k1}^2)) 1(x > R_{k2}^{\alpha_k N}) \right]
(5)

Proof: We can compute the intensities, \( \lambda_{k,LOS}(\cdot) \) and \( \lambda_{k,NLOS}(\cdot) \) of \( \Phi_{LOS} \) and \( \Phi_{NLOS} \), respectively, by following similar steps as in the proof of Lemma 1.

Also, the PDF of \( L_{k,s}(r) \), denoted by \( f_{L_{k,s}} \), which will be used in the following section is given as follows:

\[
f_{L_{k,s}}(x) = -\frac{dF_{L_{k,s}}(x)}{dx} = \Lambda_{k,s}((0, x)) \exp(-\Lambda_{k,s}((0, x)))
\]
(6)

where \( \Lambda_{k,s}((0, x)) \) is given at the top of the next page in (7).

B. Cell Association

In this work, a flexible cell association scheme similarly as in [12] is considered. In this scheme, UEs are assumed to be associated with the BS offering the strongest long-term averaged biased-received power. In other words, a typical UE is associated with a BS at tier-\( k \) for \( k = 1, 2, \ldots, K \) if

\[
P_k B_k L_k(r)^{-1} \geq P_j B_j L_{min,j}(r)^{-1}, \text{ for all } j = 1, 2, \ldots, K, j \neq k
\]
(8)

where \( P_k \) and \( B_k \) denote the transmission power and biasing factor in the corresponding tier (indicated by the index in the subscript), \( L_k(r) \) is the path loss in the \( k \)th tier as formulated in (2), and \( L_{min,j}(r) \) is the minimum path loss of the typical UE from a BS in the \( j \)th tier. Although the analysis is done according to averaged biased-received power association, other association schemes like smallest path loss and highest average received power can be considered as well because they are special cases of biased association. When \( B_k = 1/P_k \) for \( k = 1, 2, \ldots, K \), biased association becomes the same as the smallest path loss association while \( B_k = 1 \) for \( k = 1, 2, \ldots, K \) corresponds to highest average received power association. In the following lemma, we provide the association probabilities with a BS in the \( k \)th tier using the result of Lemma 1.

Lemma 3: The probability that a typical UE is associated with a LOS/NLOS BS at tier-\( k \) for \( k = 1, 2, \ldots, K \) is

\[
A_{k,s} = \int_{0}^{\infty} \Lambda_{k,s}((0, l_k)) e^{-\sum_{j=1}^{K} \Lambda_j((0, l_k/R_{k,j}^L))} dl_k
\]
(9)

where \( \Lambda_j((0, x)) \) and \( \Lambda_{k,s}((0, x)) \) are given in (3) and (7), respectively.

Proof: See [26, Appendix B].

III. SINR COVERAGE ANALYSIS

In this section, we develop a theoretical framework to analyze the downlink SINR coverage probability for a typical UE using stochastic geometry. Although an averaged biased-received power association scheme is considered for tier selection, the developed framework can also be applied to different tier association schemes.

A. Signal-to-Interference-plus-Noise Ratio (SINR)

The SINR experienced at a typical UE at a random distance \( r \) from its associated BS in the \( k \)th tier can be written as

\[
\text{SINR}_k = \frac{P_k G_0 h_{k,0} L_c^{-1}(r)}{\sigma_k^2 + \sum_{j=1}^{K} \sum_{i \in \Phi_j \backslash B_{k,0}} P_j G_{j,i} h_{j,i} L_c^{-1}(r)}
\]
(10)

where \( G_0 \) is the effective antenna gain of the link between the serving BS and UE which is assumed to be equal to \( MM \), \( h_{k,0} \) is the small-scale fading gains from the serving BS, \( \sigma_k^2 \) is the variance of the additive white Gaussian noise component. Interference has two components: intracell and intercell interference, where the first one is from the active BSs operating in the same cell with the serving BS, and the second one is from the BSs in the other cells. A similar notation is used for interfering links, but note that the effective antenna gains \( G_{j,i} \) are different for different interfering links as described in (1). Since the small-scale fading at mmWave is less severe than the conventional systems due to deployment of directional antennas, all links are assumed to be subject to independent Nakagami fading (i.e., small-scale fading gains have a gamma distribution). Parameters of Nakagami fading are \( N_{LOS} \) and \( N_{NLOS} \) for LOS and NLOS links, respectively, and they are assumed to be positive integers for simplicity. When \( N_{LOS} = N_{NLOS} = 1 \), the Nakagami fading specializes to Rayleigh fading.

B. SINR Coverage Probability

The SINR coverage probability \( P_{C_k}(T_k) \) is defined as the probability that the received SINR is larger than a certain threshold \( T_k > 0 \) when the typical UE is associated with a BS from the \( k \)th tier, i.e., \( P_{C_k}(T_k) = \mathbb{P}(\text{SINR}_k > T_k; t = k) \) where \( t \) indicates the associated tier. Moreover, homogeneous PPPs describing the spatial distribution of the BSs in each tier can be decomposed into two independent non-homogeneous PPPs: the LOS BS process \( \Phi_{k,LOS} \) and NLOS BS process...
\( \Lambda_{k,s}([0,x]) = \begin{cases} \frac{2\pi\lambda_s e^{-\frac{x^2}{\alpha_s^2}}}{\alpha_s^2} \sum_{k=1}^{K} \phi_k \left( \frac{x}{\alpha_s} \right) & \text{for } s=\text{LOS} \\ \frac{2\pi\lambda_s e^{-\frac{x^2}{\alpha_s^2}}}{\alpha_s^2} \sum_{k=1}^{K} \phi_k \left( \frac{x}{\alpha_s} \right) & \text{for } s=\text{NLOS} \end{cases} \) for \( 0 \leq x \leq R_{k_s} \)

\( \Phi_{k,NLOS} \). Therefore, the total SINR coverage probability \( P_C \) of the network can be computed using the law of total probability as follows:

\[
P_C = \sum_{k=1}^{K} \left[ P_{C,k,LOS}^k (\Gamma_k) A_{k,LOS} + P_{C,k,NLOS}^k (\Gamma_k) A_{k,NLOS} \right],
\]

where \( s \in \{\text{LOS}, \text{NLOS}\} \), \( P_{C,k,s}^k \) is the conditional coverage probability given that the UE is associated with a BS in \( \Phi_{k,s} \) and \( A_{k,s} \) is the association probability with a BS in \( \Phi_{k,s} \), which is given in Lemma 3. In the next theorem, we provide the main result for the total network coverage.

**Theorem 1.** The total SINR coverage probability of a heterogeneous mmWave cellular network with Nakagami fading is given by

\[
P_C \approx \sum_{k=1}^{K} \sum_{s=\{\text{LOS}, \text{NLOS}\}} \int_0^\infty \left( \sum_{n=1}^{N_s} \left( -1 \right)^{n+1} \left( \frac{N_s}{n} \right) e^{-\frac{n\gamma_k l_k k_s \sigma^2_{\text{BE}}}{P_k g_0 t N_L}} \right) \Lambda_{k,L,LOS}(dt)
\]

where

\[
A = \sum_{G \in \{M,M,m,m\}} p_G \int_0^\infty \Psi \left( N_{LOS}, \frac{m_{\text{LOS}}^k P_k G_{k,L,s}}{P_k g_0 t N_L} \right) \Lambda_{j,LOS}(dt)
\]

and

\[
B = \sum_{G \in \{M,M,m,m\}} p_G \int_0^\infty \Psi \left( N_{NLOS}, \frac{m_{\text{NLOS}}^k P_k G_{k,L,s}}{P_k g_0 t N_N} \right) \Lambda_{j,NLOS}(dt)
\]

for \( s = \text{LOS}, \text{NLOS} \). In the following corollary, coverage probability expression is provided assuming a noise-limited cellular network.

**Corollary 1:** When there is no interference, coverage probability of the network is given by

\[
P_C \approx \sum_{k=1}^{K} \sum_{s=\{\text{LOS}, \text{NLOS}\}} \int_0^\infty \left( \sum_{n=1}^{N_s} \left( -1 \right)^{n+1} \left( \frac{N_s}{n} \right) e^{-\frac{n\gamma_k l_k k_s \sigma^2_{\text{BE}}}{P_k g_0 t N_L}} \right) \Lambda_{k,L,LOS}(dt)
\]

\( D. \) **SINR Coverage Probability Analysis In the Presence of Beamsteering Errors**

In Section III-B and the preceding analysis, antenna arrays at the serving BS and the typical UE are assumed to be aligned perfectly and downlink SINR coverage probability is calculated in the absence of beamsteering errors. However, in practice, it may not be easy to have perfect alignment. Therefore, in this section, we investigate the effect of beamforming alignment errors on the coverage probability analysis. We employ an error model similar to that in [22]. Let \( \epsilon \) be the random absolute beamsteering error of the transmitting node toward the receiving node with zero-mean and bounded absolute error \( |\epsilon|_{\text{max}} \leq \pi \). Due to symmetry in the gain \( G_0 \), it is appropriate to consider the absolute beamsteering error. The PDF of the effective antenna gain \( G_0 \) with alignment error can be explicitly written as [10]

\[
f_{G_0}(|\epsilon|) = \frac{\theta}{2} \delta(g - M M) + 2 |\epsilon| \left( \frac{\theta}{2} \right) \left( 1 - F_{|\epsilon|} \left( \frac{\theta}{2} \right) \right) \times \delta(g - M m) + \left( 1 - F_{|\epsilon|} \left( \frac{\theta}{2} \right) \right) \delta(g - mm),
\]

where \( \delta(\cdot) \) is the Kronecker’s delta function, \( F_{|\epsilon|}(x) \) is the CDF of the misalignment error and (16) follows from the definition of CDF, i.e., \( F_{|\epsilon|}(x) = \mathbb{P}[|\epsilon| \leq x] \). Assume that the error \( \epsilon \) is Gaussian distributed, and therefore the absolute error \( |\epsilon| \) follows a half-normal distribution with \( F_{|\epsilon|}(x) = \text{erf}(\frac{x}{\sqrt{2} \sigma_{\text{BE}}}) \), where \( \text{erf}(\cdot) \) again denotes the error function and \( \sigma_{\text{BE}} \) is the standard deviation of the Gaussian error \( \epsilon \).

It is clear that total SINR coverage probability expression in (15) depends on the effective antenna gain \( G_0 \) between the typical UE and the serving BS in each tier. Thus, total SINR coverage probability \( P_C \) can be calculated by averaging over the distribution of \( G_0 \), \( f_{G_0}(|\epsilon|) \), as follows:

\[
P_C = \int_0^\infty P_C(g) f_{G_0}(|\epsilon|) dg
= (F_{|\epsilon|}(\theta/2))^2 P_C(M M) + 2(F_{|\epsilon|}(\theta/2)) F_{|\epsilon|}(\theta/2) P_C(M m) + F_{|\epsilon|}(\theta/2)^2 P_C(m m),
\]

where we define \( F_{|\epsilon|}(\theta/2) = 1 - F_{|\epsilon|}(\theta/2) \).

**IV. NUMERICAL RESULTS**

In this section, we evaluate the theoretical expressions numerically. Simulation results are also provided to validate the accuracy of the proposed model for the heterogeneous downlink mmWave cellular network. In the numerical evaluations and simulations, unless otherwise stated, a 3-tier heterogeneous network is considered and the parameter values are listed in Table I. For this 3-tier scenario, \( k = 1, k = 2 \) and \( k = 3 \) correspond to the microcell, picocell, and femtocell, respectively. In other words, a relatively high-
First, we investigate the noise-limited assumption of the mmWave cellular networks. In Fig. 2, we plot the SINR and SNR coverage probabilities for three different number of tiers. When only microcell exists, since the interference is only from the same tier (i.e., microcell BSs), SINR and SNR coverage probabilities match with each other almost perfectly. As the number of tiers increases, the difference between SINR and SNR coverage probabilities becomes noticeable for higher values of the threshold because in a multi-tier scenario, interference is arising from BSs from different type of cells in different tiers as well. However, this performance gap is still negligible and heterogeneous mmWave cellular networks can be assumed to be noise-limited. Also, note that as more tiers are added to the network, coverage probability increases significantly. Specifically, multi-tier network outperforms that with a single tier especially for small values of the threshold.

Since in Fig. 2 we show that the difference between SINR and SNR coverage probabilities are negligible even in multi-tier network scenarios, we henceforth consider the SNR coverage probabilities in the remaining simulation and numerical results. Next, we compare the SNR coverage probabilities for different values of the antenna main lobe gain $M$. As expected, better SNR coverage is achieved with increasing main lobe gain as shown in Fig. 3. Also note that, in both Fig. 2 and Fig. 3, there are break points at certain points of the curves after which coverage probability degrades faster. In Fig. 3, for example, break points occur at approximately 70% of SNR coverage probability. These break points are occurring due to the assumption of the LOS ball model. Finally, we also observe that simulation results very closely match the analytical results.

In Fig. 4, we analyze the effect of biasing factor on the SNR coverage performance. We use the same biasing factor for picocells and femtocells, and no biasing for microcells. As the biasing factor increases, number of UEs associated with smaller cells increases resulting in an increase in coverage probabilities for picocells and femtocells while causing a degradation in the coverage performance of the microcell. This result is quite intuitive because with positive biasing, more UEs are encouraged to connect with the smaller cells. On the other hand, with biasing, UEs are associated with the BS not offering the strongest average received power.

TABLE I: System Parameters

| Parameters | Values |
|------------|--------|
| $\alpha_k, \lambda_k, \eta_k, N_k \forall k$ | 2, 4 |
| $N_{LOS} - N_{NLOS}$ | 3, 2 |
| $M, m, \theta$ | $10$dB, $-10$dB, $30^\circ$ |
| $\lambda_1, \lambda_2, \lambda_3$ | $10^{-3}, 10^{-4}, 5 \times 10^{-4} (1/m^2)$ |
| $B_0, B_1, B_2$ | 1, 1, 1 |
| $[R_{11} R_{12}], [R_{12} R_{22}]$ | $[50, 200], [0.8, 0.2]$ |
| $[R_{13} R_{23}], [R_{31} R_{32}]$ | $[20, 40], [1, 0]$ |
| $1, \sigma_k$ | 0dB, $-7$dBm |
| $P_T, P_f, P_0$ | 53dBm, 53dBm, 25dBm |
| Carrier frequency ($f_c$) | 28 GHz |

Fig. 2: Coverage Probability as a function of the threshold in dB comparison between SINR and SNR.

Fig. 3: SNR Coverage Probability as a function of the threshold in dB for different values of antenna main lobe gain $M$. The curves in Fig. 3 have break points at certain threshold values after which the coverage probability degrades faster. These break points are due to the assumption of the LOS ball model. Also, note that, the simulation results very closely match the analytical results.

Fig. 4: SNR Coverage Probability as a function of the biasing factor of picocells and femtocells in dB ($B_1 = 0$dB).
probability diminishes with the increase in alignment error standard deviation, and this deterioration becomes evident after $\sigma_{BE} = 7^\circ$.

### V. CONCLUSION

In this paper, we have provided a general analytical framework to compute the SNR coverage probability of heterogeneous downlink mmWave cellular networks composed of $K$ tiers. Directional beamforming with sectored antenna model and two-ball approximation for blockage model have been considered in the analysis. BSs of each tier and UEs are assumed to be distributed according to independent PPPs, and UEs are assumed to be connected to the tier providing the maximum average biased-received power. Numerical results show that mmWave cellular networks are noise limited rather than being interference limited. We have also shown that increasing main lobe gain results in higher SNR coverage. Moreover, increase in the biasing factor of smaller cells has led to better coverage probability of smaller cells because of the higher number of UEs connected to them, while the overall network coverage probability has slightly diminished due to association with the BS not offering the strongest average received power. Finally, the effect of alignment error on coverage probability is quantified. Investigating the effect of using different cell association techniques remains as future work.

### REFERENCES

[1] T. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi, and F. Gutierrez, “Millimeter wave mobile communications for 5G cellular: It will work!,” IEEE Access, vol. 1, pp. 335-349, May 2013.

[2] W. Roh et al., “Millimeter-wave beamforming as an enabling technology for 5G cellular communications: Theoretical feasibility and prototype results,” IEEE Commun. Mag., vol. 52, no. 2, pp. 106-113, Feb. 2014.

[3] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, “What will 5G be?,” IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1065-1082, Jun. 2014.

[4] S. Rangan, T.S. Rappaport, and E. Erkip, “Millimeter-wave cellular wireless networks: Potentials and challenges,” Proc. of the IEEE, vol. 102, no. 3, pp. 366-385, Mar. 2014.

[5] A. Ghosh, T. A. Thomas, M. C. Cudak, R. Ratasuk, P. Moonr, F. W. Vook, T. S. Rappaport, G. R. MacCartney, S. Sun, and S. Nie, “Millimeter wave enhanced local area systems: A high data rate approach for future wireless networks,” IEEE Journal on Sel. Areas in Commun., Special Issue on 5G, Jul. 2014.

[6] T. Bai, and R. W. Heath, “Coverage and rate analysis for millimeter wave cellular networks,” IEEE Trans. Wireless Commun., vol. 14, no. 2, pp. 1100-1114, Feb. 2015.

[7] S. Singh, M. Kulkarni, A. Ghosh, and J. Andrews, “Tractable model for rate in self-backhauling millimeter wave cellular networks,” IEEE Journal on Selected Areas in Communications, vol. 33, no. 10, pp. 2196-2211, Oct. 2015.

[8] M. N. Kulkarni, S. Singh, and J. G. Andrews “Coverage and rate trends in dense urban mmWave cellular networks,” IEEE Global Communications Conference (GLOBECOM) pp. 3809-3814, Dec. 2014.

[9] A. Akdeniz, Y. Liu, M. Samimi, S. Sun, S. Rangan, T. Rappaport, and E. Erkip, “Millimeter wave channel modeling and cellular capacity evaluation,” IEEE Journal on Selected Areas in Communications, vol. 32, no. 6, pp. 1164-1179, Jun. 2014.

[10] M. Di Renzo, “Stochastic geometry modeling and analysis of multi-tier millimeter wave cellular networks,” IEEE Trans. Commun., vol. 14, no. 9, pp. 5658-5677, Sep. 2015.

[11] H. S. Dhillion, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-Tier downlink heterogeneous cellular networks,” IEEE Journal on Selected Areas in Communications, vol. 30, no.3, pp. 550-560, Apr. 2012.

[12] H.-S. Jo, Y. I. Sang, P. Xia, and J. G. Andrews, “Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis,” IEEE Wireless Commun., vol. 11, no. 10, pp. 3484-3495, Oct. 2012.

[13] H. ElSawy, E. Hossain, and M. Haenggi, “Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey,” IEEE Commun. Surveys Tutorials, vol. 15, no. 3, pp. 996-1019, 2013.

[14] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, and D. Malladi, “A survey on 3GPP heterogeneous networks,” IEEE Wireless Commun. Mag., vol. 18, no. 3, pp. 10-21, Jun. 2011.

[15] M. Di Renzo, A. Guidotti, and G. E. Corazza, “Average rate of downlink heterogeneous cellular networks over generalized fading channels: A stochastic geometry approach,” IEEE Trans. on Commun., vol. 61, no. 7, pp. 3050-3071, Jul. 2013.

[16] H. Shokri-Ghadikolaei, C. Fischione, G. Fodor, P. Popovski, and M. Zorzi, “Millimeter wave cellular networks: A MAC layer perspective,” IEEE Transactions on Communications, vol. 63, no. 10, pp. 3437-3458, Oct. 2015.

[17] H. Elshaer, M. N. Kulkarni, F. Boccardi, J. G. Andrews, and M. Dohler, “Downlink and uplink cell association with traditional macrocells and millimeter wave small cells,” 2016. Submitted, available at http://arxiv.org/abs/1601.05281.

[18] D. Maamari, N. Devroye, and D. Tuninetti, “Coverage in mmWave cellular networks with base station cooperation,” IEEE Trans. Wireless Commun., vol. 15, no. 4, pp. 2981-2994, Apr. 2016.

[19] F. Baccelli and B. Blaszczyszyn, “Stochastic geometry and wireless networks, Part I: Theory, Part II: Applications,” NOW: Foundations and Trends in Networking, 2010.

[20] A. Thorburn, T. Bai, and R. W. Heath, “Performance analysis of mmWave ad hoc networks,” IEEE Trans. Signal Process., 64, no. vol. 15. pp. 4065-4079, Aug. 2016.

[21] Tianyang Bai and Robert Heath Jr, “ Coverage in dense millimeter wave cellular networks,” Asilomar Conference on Signals, Systems and Computers, pp. 2062-2066, Nov. 2013.

[22] J. Wildman, P. H. J. Nardeciti, M. Latva-aho, and Steven Weber, “On the joint impact of beamwidth and orientation error on throughput in directional wireless Poisson networks,” IEEE Trans. Wireless Commun., vol. 13, no. 12, pp. 7072-7085, Dec. 2014.

[23] A. Hunter, J. Andrews, and S. Weber, “Transmission capacity of ad hoc networks with spatial diversity,” IEEE Trans. Wireless Commun., vol. 7, no. 12, pp. 5058-5071, Dec. 2008.

[24] T. Bai and R. Vaze and R. W. Heath Jr, “Analysis of blockage effects on urban cellular networks,” IEEE Trans. Wireless Commun., vol. 13, no. 9, pp. 5070-5083, Sept. 2014.

[25] B. Blaszczyszyn, M. K. Karray, and H.P. Keeler, “Using Poisson processes to model lattice cellular networks,” IEEE International Conference on Computer Communications, pp. 773-781, Apr. 2013.

[26] E. Turgut and M. C. Gursoy, “Coverage in heterogeneous downlink millimeter wave cellular networks,” 2016. Submitted, available at http://arxiv.org/pdf/1608.01790v1.pdf.