Entropic Pressure of Cosmic Gas

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Entropic Pressure of Cosmic Gas

By Martin Reid Johnson

Abstract

Under the low-density conditions found in most of the Universe, the energy released by adiabatic expansion of its gases exceeds the amount required to offset the energy lost to decreased density. This excess is found locally as radial kinetic energy and more broadly manifest as a reduction in universal density over time.

Introduction

The phenomenon of “dark energy”, wherein a mysterious force of unknown origin seems to be pushing apart distant galaxies from us and each other, has long been the subject of speculation. In cosmology, interstellar gas is treated as perfect fluid with constant internal energy. This paper contradicts that assumption: The internal energy of cosmic gas isn’t constant. It decreases over time, the decrease fuels the expansion of the universe, and is in fact dark energy.

One might imagine anything is possible with an infinite universe, but when it comes to gases, in some respects the opposite is true. For one thing, all processes are adiabatic. There is no mass with which to exchange heat that is not already there. For another, there is no free expansion in the classic sense. There is no place “outside” the universe where the gas can go. Cosmic free expansion, however, can and does occur, because empty space is created. This means more volume to accommodate the same amount of gas, just like classic free expansion. The result: a less dense universe. Cosmic free expansion increases entropy, just like classic free expansion. Unlike its classic cousin, however, the gas loses internal energy, to gravity. Cosmic isoentropic expansion is also different from classic isoentropic (i.e., “reversible”) expansion. Work is performed, against gravity, but it isn’t equal to the internal energy lost, as we will demonstrate.

The Two-Step Model with 100% Helium

Hydrogen and helium still dominate as components of ordinary cosmic matter, and most of it still lies in the intergalactic void. We use helium. It’s an abundant gas, accounting for some 25% of the atoms in the universe, and about half the thermodynamic mass. It’s hard to ionize (24.6 eV) and inert to chemical reaction. Helium is the most ideal of any common gas and is the best choice when using the ideal and adiabatic gas laws. If one is interested in cosmic gas behavior, helium is a good place to start.

We won’t be looking at stars or galaxies. We will look at the intergalactic void, where mass density is very low. Newtonian laws are well followed, and Minkowski space similarly is well

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1 “Classic free expansion” retains the textbook definition. The terms “free” and “freely” now refer to cosmic free expansion, unless “classic” is prepended. We use the terms “isoentropic” and “reversible” interchangeably in this paper.
approximated by Euclidean. Central to our model is the assumption that gas behavior at low density is the same as at higher densities, especially with respect to internal energy changes.

Our universe will be made of 100% helium at its natural isotopic abundance, at the critical density $\delta_u$.\(^2\) It will be Euclidean, Newtonian, and unbounded: infinitely large and massive. Consider a finite sphere within our model universe. It is one of an infinite number of identical spheres, be they touching, overlapping, or distant. What happens in this sphere, happens everywhere else. When this sphere gets bigger, all the spheres get bigger, and the universe gets less dense. When this sphere gets colder, the whole universe gets colder, and so forth. Our sphere is a proxy, acting in tandem with and describing the behavior of the universe as a whole, so anything this sphere does, it does adiabatically. This sphere is in thermal equilibrium, a major departure from real world conditions. Nonequilibrium thermodynamics must be set aside so that the underlying transfer of conserved energy is more clearly described.

We will look at the consequences of the energy released by expanding helium vs. energy absorbed by the expanded sphere’s gravity. We have four independent variables: mass ($M$), density ($\delta$), temperature ($T$), and size ($r$). We start with a sphere of radius about Earth size, $6.3781 \times 10^6$ m, at 7 K, which at $\delta_u$ has 9.37 milligrams, or 1.3 atoms of helium per cubic meter.

The sphere’s gravitational potential energy ($U_1$),\(^3\) which is the amount required to explode it to infinity, is negative and given by:

$$U = -\frac{3GM^2}{5r}$$ \hspace{1cm} (1)

Where $G$ is the gravitational constant ($6.67408 \times 10^{-11}$ m$^3$kg$^{-1}$sec$^{-2}$). We find $U_1 = -2.45 \times 10^{-26}$ J.

The ideal gas law is given as:

$$PV = nRT$$ \hspace{1cm} (2)

Where $R$ is the universal gas constant ($=8.31446$ kg-m$^2$sec$^{-2}$mole$^{-1}$K$^{-1}$).

The number of moles of helium ($n$) is given by the mass divided by its atomic weight ($\mathcal{M}_{He} = 4.00260 \times 10^{-3}$ kg/mole):

$$n = \frac{M}{\mathcal{M}_{He}}$$ \hspace{1cm} (3)

The volume of a sphere is:

\(^2\) The critical density $\delta_u = 8.62 \times 10^{-27}$ kg/m$^3$ is the mass density above which, the physics community concludes, the universe would collapse from gravity.

\(^3\) The term $U$ is also commonly used to denote the internal energy of a gas. A difference in $U$ values, for example $U_2-U_1$, does in fact describe an internal energy change. In this paper the numeric subscripts $U_1$, $U_2$, and $U_3$ mean gravitational potential energy and the letter subscripts $U_r$ and $U_f$ mean an internal energy loss.
\[ V = \frac{4}{3} \pi r^3 \]  

When (2), (3), and (4) are combined and rearranged we get the internal pressure:

\[ P_1 = \frac{nRT}{V} = \frac{3nRT}{4\pi r_1^3} = \frac{3MRT}{4K_{He} \pi r_1^3} \]  

Entering our values for \( M \), \( T \), and \( r \), we obtain \( P_1 = 1.25 \times 10^{-22} \) Pa. This is the pressure everywhere in this universe and the sphere experiences no net force on its surface. We will also suppose that the sphere isn’t getting any bigger over time. It is, as we will see, but for now we will say it isn’t. Let’s examine what happens as we increase the sphere’s radius by \( \sqrt[3]{1.01} \), adding a “shell” and giving a volume increase of one percent. The sphere absorbs energy to offset its less-negative gravitational potential, and the helium inside the sphere releases energy from reversible expansion.

The internal energy of the gas which is lost to gravity from reversible expansion (\( U_r \)) is:

\[ U_r = U_1 - U_2 = \frac{-3GM^2}{5} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]  

Where \( U_1 \) and \( U_2 \) are the gravitational potential energies at radii \( r_1 \) and \( r_2 \) respectively. Entering the values for \( M \), \( r_1 \) and \( r_2 \) we find that \( U_r = 1.82 \times 10^{-30} \) J. We define the increment as \( \frac{r_2 - r_1}{r_1} = \frac{\Delta r}{r} \). Since the gas is expanding adiabatically, work is performed. The expression for work performed by adiabatic expansion of a gas is:\footnote{The derivation of all adiabatic expressions used herein can be found at \( \text{https://en.wikipedia.org/wiki/Adiabatic_process} \)}

\[ W = E = -\alpha P_1 V_1 \left( \frac{V_2}{V_1} \right)^{\frac{1}{\gamma} - 1} \]  

Where \( W \) has the classic meaning of work performed and is included for perspective, \( E \) is the energy released by the expansion, \( P_1 \) is the pressure before expansion, \( V_1 \) and \( V_2 \) are the before and after volumes of the sphere respectively, and \( 2\alpha \) is the number of degrees of freedom in the molecule. Helium, being monatomic, has three translational degrees of freedom only, so \( \alpha = 3/2 \). The term \( \gamma \) is related to \( \alpha \) by:

\[ \gamma = (\alpha + 1)/\alpha \]  

Entering the value for helium into (8) and (7) we get:

\[ E = -\left( \frac{3}{2} \right) P_1 V_1 \left( \frac{V_2}{V_1} \right)^{-\frac{2}{3}} - 1 \]
For helium, $V$ and $P$ can be calculated from (4) and (5). Entering these into (9) gives $E = 1.35 \times 10^{-3}$ J, or 144 J/kg. This is $10^{25}$ times as much energy released as absorbed. The excess ($E_k'$) is now outward, radial kinetic energy:

$$E_k' = E + U_r = E + (U_1 - U_2) = \frac{3GM^2}{5} \left(\frac{1}{r_2} - \frac{1}{r_1}\right) - \left(\frac{3}{2}\right) P_1 V_1 \left(\frac{V_2}{V_1}\right)^2 - \frac{2}{3} P_1 \left(\frac{V_2}{V_1}\right)^3$$  \(10\)

Gravity loss is negligible and $E_k' = E = 1.35 \times 10^{-3}$ J. The internal pressure drops to a new value, $P_2$. The general expression for reversible pressure change is:

$$P_2 = P_1 \left(\frac{V_2}{V_1}\right)^\gamma$$  \(11\)

This gives for helium:

$$P_2 = P_1 \left(\frac{V_2}{V_1}\right)^\frac{5}{3}$$  \(12\)

Eq. (12) gives $P_2 = 1.23 \times 10^{-22}$ Pa. Dividing $E_k'$ by $V_2$ gives us the entropic pressure ($P_s$):

$$P_s = \frac{E_k' - E_k'}{V_2}$$  \(13\)

Our sphere was static to start so $E_k' = 0$. Our expanded sphere has $P_s = 1.23 \times 10^{-24}$ Pa, or 1% of $P_2$. It’s important to emphasize that entropic pressure does not add to the internal pressure of the sphere.\(^5\) Entropic pressure is a vector quantity which results in outward expansion only: an entirely separate, time-dependent entity. Entropic pressure already existed in the sphere. We could have started with a smaller static sphere to get to our Earth-sized sphere, which would then have entropic pressure. The size of the sphere is relevant, as we will see.

Now we look at the rate of expansion over time ($t$). The general expression for kinetic energy of mass in motion is given as:

$$E = \frac{1}{2} M v^2$$  \(14\)

The linear expansion rate, or radial velocity of the sphere as a whole ($v_s$) is thus:

$$v_s = v_{s_1} + \sqrt{\frac{2E_k'}{M}}$$  \(15\)

Where $v_{s_1}$ is the initial radial velocity of the sphere. Our sphere was static to start so $v_{s_1} = 0$. From (15), the radial velocity is $v_s = 16.98$ m/sec, giving an expansion time:

\(^5\) Since one of our assumptions is thermal equilibrium, $P$ remains a state function.
\[
\Delta t = t_2 - t_1 = \frac{r_2 - r_1}{v_s} = \frac{\Delta r}{v_s}
\]  

(16)

For \( t_1 = 0, \ t_2 = \Delta t \). The value \( \Delta r \) is the shell thickness. Entering our values for \( v_s, r_1, \) and \( r_2 \) we get \( \Delta t = 1248 \) seconds, or about 20.8 minutes.

We have only calculated the isoentropic expansion. The actual gain in entropy from “entropic” pressure lies in the free expansion of the sphere, which must be included in order to properly calculate the sphere’s increase from both sources. This interplay between free and reversible expansion, both performing work adiabatically against gravity, is a complex issue unknown in classic thermodynamics, and awaits an analytic solution. For now, we approximate with a “two-step model”. We expand the sphere twice: once reversibly to an inner shell, then once freely to an outer shell. The inner shell creates entropic pressure and the outer shell reduces it, in a stepwise approach to what is really a single event. We set the shells at equal thickness.\(^6\) To preserve the 1% volume increase, we need to drop the increment to 0.001661142,\(^7\) applied once for reversible expansion and once for free expansion. We now have three increasing radii to consider: \( r_1, r_2, \) and \( r_3, \) and two gravity loss terms: reversible inner \( U_r \) (6), and free outer \( U_f \) (17):

\[
U_f = U_2 - U_3 = \frac{-3GM^2}{5} \left( \frac{1}{r_2} - \frac{1}{r_3} \right)
\]

(17)

The radial kinetic energy, adjusted down for free loss, is:

\[
E_k = E_{k'} + U_f = E + U_1 - U_3 = \frac{3GM^2}{5} \left( \frac{1}{r_3} - \frac{1}{r_1} \right) - \left( \frac{3}{2} \right) P_1 V_1 \left( \frac{V_2}{V_1} \right)^{\frac{2}{3}} - 1
\]

(18)

There is a new internal pressure, \( P_3 \), less than \( P_2 \) due to two factors: a) increased volume, which follows the ideal gas law (2);\(^8\) and b) loss of internal energy to gravity (\( U_f \)). In a sphere this small, the gravity terms can be neglected, so \( E_k = E \) as before. Entering values for \( E \) and \( M \) into (15) gives \( v_s = 12.0 \) m/sec for the sphere, and a \( \Delta t \) of 1762 seconds, or about 29.4 minutes. The entropic pressure before free expansion is \( 6.20 \times 10^{-25} \) Pa, then drops slightly afterward to \( 6.17 \times 10^{-25} \) Pa.

The temperature drop can be found from (21):\(^9\)

\[
T_3 \leq T_2 = T_1 \left[ \left( \frac{P_2}{P_1} \right) \right]^{2/5}
\]

(19)

\(^6\) In a classic free expansion they would be equal, moving at the same speed for the same time as the first expansion.

\(^7\) Obtained manually.

\(^8\) While the math is the same as for classic free expansion, the underlying principle is quite different. The movement of the atoms is vectored. Each atom is moving in a straight line away from the center, like a bunch of tiny rockets blasting away from their despoiled planet. There is nothing to alter or impede their movement, save gravity.

\(^9\) The \( \leq \) can be replaced with = when \( U_f \ll E_k' \).
The temperature drops from 7 to 6.98 K. This is a big drop for thirty minutes of elapsed time and an artifact of the shell thickness. The true expansion time is much longer than this.

Temperature has more effect: Raising $T$ to 50 K gives $v_s = 32.1$ m/sec; to 100 K, 45.3 m/sec.

Cold Dark Matter (CDM)

CDM, whose existence is inferred from the rotation of the arms of galaxies around their center, is about 85% of all mass in the Universe, and doesn’t act as a gas. Its only influence is gravitational.\(^{10}\) This is included in the model by dividing the gas mass by 0.15. Eq (1) is now (20):

$$U = -\frac{3\cdot G \left( \frac{M}{0.15} \right)^2}{5r}$$

At Earth radius, $U_f$ remains negligible ($E/10^{22}$). The gas thermodynamics dominate the energy budget, and $E_k = E$, a function of the gas properties alone.

Entropic Pressure is a Local Phenomenon

We define a quantity which is important to follow, the cutoff ratio ($X'$):

$$X' = \frac{-U_f}{E_k'} = \frac{U_3 - U_2}{E_k'} \leq 0.001$$

We also define the enthalpic ratio ($X$):

$$X = \frac{U_3 - U_1}{E}$$

In plain language, $X'$ is the ratio of the gas’s internal energy lost to gravity during free expansion divided by the radial kinetic energy gained from reversible expansion, and $X$ is energy lost divided by energy gained. Our model’s accuracy depends on neglecting $U_f$.\(^{11}\) We set 0.1% as the upper limit for $X'$, below which we deem our calculations to be accurate. $X'$ has a positive correlation with the radius $r$, the density $\delta$, and a negative correlation with $\Delta r/r$. It must be monitored as $r$ or $\delta$ are increased, or $\Delta r/r$ reduced, for the model to remain valid.\(^{12}\) We call any sphere with $X' < 0.001$ a “small sphere”.

As we increase $r_f$ at $\delta_u$, the mass rises, $X$ and $X'$ grow larger and the free shell shrinks until the endpoint ($r_e$) is reached, where $X = 1$.\(^{13}\) For helium at $\delta_u$ and 7 K, $r_e = 1.8423 \times 10^{19}$ m, or about

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\(^{10}\) The weight $M$ of gas, or thermodynamic mass, remains unchanged, and is used to find $E$. There’s no stellar mass, but if there was, it would lie far outside the sphere, evenly distributed.

\(^{11}\) We cannot properly find $U_f$, because we need the internal energy of helium. This is experimentally obtained and known, but at pressures much higher than ours. We work at radii where $X' < 0.001$, and $U_f$ can be neglected.

\(^{12}\) $X'$ has a negative correlation with increasing temperature.

\(^{13}\) Any further expansion requires energy input, and there is no source. There’s more to be said on the subject but it’s outside the scope of the paper.
195 light-years (ly). The cutoff radius \( r_c \) is much lower: 8.23 \( \times 10^{17} \) m, or about 87 ly. Our model is accurate only to about 45% of \( r_e \), but the sphere still produces entropic pressure all the way out to \( r_e \). The endpoint \( r_e \) is independent of \( \Delta r/r \), always giving 1.84 \( \times 10^{19} \) m, as long as \( v_I = 0 \) and \( \delta \) and \( T \) are held at \( \delta_u \) and 7 K respectively. This means that while entropic pressure may be universal, it’s a local phenomenon, and not some sort of all-pervasive force field.

To summarize: In the interstellar void, every atom is surrounded by a finite sphere containing more atoms, with outward radial force exerted on those atoms.

Surface Radial Velocity

The surface radial velocity is given by (23):

\[
v_r = \frac{d r_3}{d t} = \frac{d r_3}{d v} \frac{d v}{d t} = \frac{(dV/dt)}{4\pi r_3^2} \approx v_{r_1} + \frac{(v_3-v_1)}{(t_3-t_1)4\pi r_3^2} = v_{r_1} + \frac{v_s(v_3-v_1)}{4\pi r_3^2(r_3-r_1)}
\]

Eq. (23) gives both the differential and finite expressions for \( v_r \). For the finite expression an initial value \( v_{r_1} \) is needed. The sphere is expanding at the same rate as its surface, so \( v_r \) and \( v_s \) should be equal, and they are close. However, \( v_s \) appears more reliable than \( v_r \). Using (15) and (23) at 10 ly / 7K / \( \delta_u \) with \( v_1 = 0 \), we decrement \( \Delta r/r \) by decades on a spreadsheet from \( 10^{-5} \) to \( 10^{-12} \), and find that \( v_r/v_s \) fluctuates around 1 at the fourth decimal place. At \( \Delta r/r = 0.1 \), \( v_r/v_s \) is only 0.84. The \( v_s \) term gives a more consistent profile.

Any map of a small sphere with log \( [v_s] \) vs. log \( [\Delta r/r] \) gives a line with the same slope and no x intercept to the left, meaning \( \frac{d r_3}{d t} = v_{r_0} \). If this sphere isn’t already expanding, it won’t, showing the time-dependent nature of entropic pressure in the model. Since the sphere is expanding, it will generate entropic pressure, but a differential can’t be found to measure the increase with accuracy. We can make the increment very small and perform a stepwise addition of shells, but even an optimal increment, where \( v_r = v_s \), will only give a result with a large cumulative error. The finite element approach of the model, combined with the local nature of \( P_s \), impedes any direct meaningful comparison with either the Hubble function or cosmic redshift values. In this context, it’s worth mentioning the unchanging slope of the line, log \( [v_s] \) vs. log \( [\Delta r/r] \). For small spheres, it remains nearly constant over a wide range of decades, temperatures,

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\footnote{14 \( \Delta r/r < .001 \). Larger \( \Delta r/r \) shows a slight upward variance. \( \Delta r/r = 0.1 \) gives \( r_e = 1.87 \times 10^{19} \) m.}

\footnote{15 Outside this range the variance is larger. The author attributes the fluctuation at the low end to rounding errors in the spreadsheet, but other interpretations are possible.}

\footnote{16 For a sphere of radius 1 ly at \( \delta_u \) and 7K, the map (log \( [v_s] \) versus log \( [\Delta r/r] \)) from \( \Delta r/r = 10^{-1} \) to \( 10^{-15} \) gives \( y = C_0 + C_1x + C_2x^2; C_0 = 2.45, C_1 = 0.497, C_2 = -0.00008 \); correlation coefficient is 0.99999. The plot appears quite linear to the eye. Downward deviation from linearity in log \( [v_s] \) vs log \( [\Delta r/r] \) is observed as \( \Delta r/r \) gets large, starting at 0.1, and the curve flattens out around \( \Delta r/r = 20 \).}

\footnote{17 Ever since the Big Bang.}
densities, and sizes. A similar result is obtained when an initial radial velocity \( v_I \) is added. This slope may prove useful.

Model with Hydrogen and Helium

We’ve shown that helium alone causes entropic pressure. A better model includes the effect of hydrogen, which comprises about 75 atom % of universal thermodynamic mass. This is a brief overview. Hydrogen has two cosmic forms, monatomic \( H \) and diatomic \( H_2 \), and a low-abundance isotope, deuterium, which we neglect. The local mole ratio of interstellar \( H_2/[H+H_2] \) has been estimated at 0.196±3.9%\(^1\); we will use a value of 0.2. This results in a mole distribution of about 1/4 \( He \), 3/20 \( H_2 \), and 3/5 \( H \), and a mass distribution of 10/19 \( He \), 3/19 \( H_2 \), and 6/19 \( H \) after adjusting for molecular weight.\(^2\) The internal pressure is:

\[
P_1 = P_H + P_{H_2} + P_{He} = \frac{3RT}{4\pi r_1^3} (n_H + n_{H_2} + n_{He}) = \frac{3MRT}{4\pi r_1^3} \left( \frac{6}{19\mathcal{K}_H} + \frac{3}{19\mathcal{K}_{H_2}} + \frac{10}{19\mathcal{K}_{He}} \right)
\]

We treat monatomic hydrogen as an ideal gas. Hydrogen, a diatomic gas, has \( \alpha = 5/2 \). We combine a few terms and get:

\[
E_k = \frac{3GM^2}{5} \left( \frac{1}{r_3} - \frac{1}{r_1} \right) - \left( \frac{3}{2} \right) \frac{3MRT}{4\pi r_1^3} \left( \frac{6}{19\mathcal{K}_H} + \frac{10}{19\mathcal{K}_{H_2}} \right) V_1 \left( \frac{V_2}{V_1} \right)^{-\frac{2}{3}} - 1 \right) - \left( \frac{5}{2} \right) \frac{9MRT}{76\pi r_1^3\mathcal{K}_{H_2}} (V_1 \left( \frac{V_2}{V_1} \right)^{-\frac{2}{3}} - 1) \right)
\]

The author recommends (25) as a more accurate description of \( E_k \).

Supplementary information: A spreadsheet is available at:
https://k2s.cc/file/94f8f4a166f0d/helium_workbook%20rev2.xlsx

Competing Interests: The author declares no competing financial interest.

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\(^1\) The slope is 10\(^0.495\) = 3.12 m/sec/log[\( \Delta r/r \)].

\(^2\) Fletcher, T. J.; Saintonge, A.; Soares, P. S.; Pontzen, A. *Monthly Notices of the Royal Astronomical Society* **2020**, *501*, 411-418.

\(^2\) The mole fraction of helium is now 25.1%, not 25%, because helium is not exactly four times as massive as hydrogen. The mole budget still adds to 100%.