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COMPUTER SIMULATION SYSTEM FOR THE NUMERICAL SOLUTION OF THE HEAT EQUATION WITH A POWER-LAW NONLINEARITY BY MESHLESS METHOD

ORCID ID: 0000-0002-6500-6775  Emiliia Usatenko
Student of the Education and Research Institute of Computer Physics and Energy
V. N. Karazin Kharkiv National University

ACADEMIC ADVISER:

ORCID ID: 0000-0002-4630-9519  Iryna Hariachevska
Candidate of Engineering Sciences, Associate Professor of the Department of
Information Technologies in Physical and Power Systems
V. N. Karazin Kharkiv National University

UKRAINE

The nonlinear parabolic partial differential equations of the second order are the basis of many mathematical models used in physics, mechanics, biology, chemistry, and ecology. For example, the nonlinear heat equation describes the processes of electron and ion thermal conductivity in a plasma, of adiabatic filtration of gases and liquids in porous media, blood flow in capillaries, diffusion of neutrons and alpha particles in reactor materials, chemical kinetics and biological activity.

Nonlinear heat conduction processes were first studied by Zel'dovich and Kompaneets [1]. The authors considered the process of heat transfer using the mechanism of radiative heat conduction from an instantaneous point source for a one-dimensional problem. The solution to this problem is obtained in an analytical form.

The heat equation with a power-law nonlinearity is especially common among the equations of this type. The universal character of this equation makes it possible to assert that the numerical solution of boundary-value problems, which are described by the heat equation with a power-law nonlinearity, is a relevant research.

Authors have developed the computer simulation system for the numerical solution of the one-dimensional nonstationary heat equation with a power-law nonlinearity by meshless method [2].

Consider the one-dimensional nonstationary heat equation with a power-law nonlinearity:

\[ \rho c_p \frac{\partial u(x, t)}{\partial t} = k \frac{\partial}{\partial x} \left( u^\sigma(x, t) \frac{\partial u(x, t)}{\partial x} \right) \quad (1) \]

where:
\( x \in [a, b], \ t \in [0, T], \ \rho - \text{density}, \ c_p - \text{specific heat at constant pressure}, \ k - \text{coefficient of thermal conductivity}, \ \sigma > 0 - \text{known coefficient}. \]
The initial condition is 
\[ u(x, 0) = u_0(x), \ x \in [a, b] \]

The boundary conditions are 
\[ \begin{cases} u(a, t) = g_1(t) \\ u(b, t) = g_2(t), \ t \in [0, T] \end{cases} \]

By substitution \( u(x, t) = v^{1/\sigma}(x, t) \) in equation (1), we obtain the equation
\[ \rho c_p \frac{\partial v(x, t)}{\partial t} = \frac{k}{\sigma} \left( \frac{\partial v(x, t)}{\partial x} \right)^2 + k v(x, t) \frac{\partial^2 v(x, t)}{\partial x^2} \] (2)
with initial condition
\[ v(x, 0) = v_0(x), \ x \in [a, b] \] (3)
and boundary conditions:
\[ \begin{cases} v(a, t) = h_1(t) \\ v(b, t) = h_2(t), \ t \in [0, T] \end{cases} \]

Using the forward Euler method for time discretization [3], we obtain
\[ v^{n+1} = v^n + \tau k \frac{1}{\rho c_p} \left( \frac{1}{\sigma} (\nabla v^n)^2 + v^n \Delta v^n \right) \] (4)
where:
\[ v^n = v(x, t^n), \ t^n = n \tau, n = \text{iteration number}, \tau = \text{the time step size}, \nabla = \frac{\partial}{\partial x}, \Delta = \frac{\partial^2}{\partial x^2}. \]

We solve equation (4) step by step, starting with the initial condition (3). Approximate solution of equation (4) can be constructed using a linear combination of radial basis functions \( \varphi(r_{ij}) \):
\[ v^n(x_i) \approx \sum_{j=1}^{N} \alpha^n_j \varphi(r_{ij}) \] (5)
where:
\( N = \text{number of collocation points}, r_{ij} = \| x_i - x_j \| - \text{Euclidean distance between points}, \alpha^n_j = \text{unknown coefficients to be determined} \).

Substituting (5) into (4), we obtain the system of linear algebraic equations, which can be solved for unknown coefficients \( \alpha^n_j \). By substituting the values of \( \alpha^n_j \) in (5), approximate solution of the equation (2) at \( n \)-th step can be obtained.

This iterative method was implemented in the created computer simulation system.

The interface of the computer simulation system is shown in Fig. 1.
For detailed information on the computer simulation system, please refer to the references.

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