Dynamics Of Proton Spin : Role Of 
$qqq$ Force

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Abstract

The analytic structure of the $qqq$ wave function, obtained recently in the high momentum regime of QCD, is employed for the formulation of baryonic transition amplitudes via quark loops. A new aspect of this study is the role of a direct (Y-shaped, Mercedes-Benz type) $qqq$ force in generating the $qqq$ wave function. The dynamics is that of a Salpeter-like equation (3D support for the kernel) formulated covariantly on the light front, à la Markov-Yukawa Transversality Principle (MYTP) which warrants a 2-way interconnection between the 3D and 4D Bethe-Salpeter (BSE) forms for 2 as well as 3 fermion quarks. The dynamics of this 3-body force shows up through a characteristic singularity in the hypergeometric differential equation for the 3D wave function $\phi$, corresponding to a negative eigenvalue of the spin operator $i\sigma_1\cdot\sigma_2 \times \sigma_3$ which is an integral part of the $qqq$ force. As a first application of this wave function to the problem of the proton spin anomaly, the two-gluon contribution to the anomaly yields an estimate of the right sign, although somewhat smaller in magnitude.

Keywords: 3bodyforce; proton-spin ; 2gluon anomaly ; fractional correction $\theta$

1 Introduction

The concept of a fundamental 3-body force (on par with a 2-body force) is hard to realize in physics, leaving aside certain ad hoc representations

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of higher order effects, for example those of $\Delta, N^*$ resonances in hadron physics. At the deeper quark-gluon level on the other hand, a truly 3-body $qqq$ force shows up as a folding of a $ggg$ vertex (a genuine part of the gluon Lagrangian in QCD) with 3 distinct $\bar{q}gg$ vertices, so as to form a $Y$-shaped diagram. Indeed a 3-body $qqq$ force of this type, albeit for ‘scalar’ gluons, was first suggested by Ernest Ma [1], when QCD was still in its infancy. [ A similar representation is also possible for $NNN$ interaction via $\rho\rho\rho$ or $\sigma\sigma\sigma$ vertices, but was never in fashion in the literature [2]]. We note in passing that a $Y$-shaped (Mercedes-Benz type) picture [3] was once considered in the context of a preon model for quarks and leptons.

In the context of QCD as a Yang-Mills field, a $ggg$ vertex has a momentum representation of the form [4]

$$W_{ggg} = -ig_s f_{abc}[(k_1 - k_2)_\lambda \delta_{\mu\nu} + (k_2 - k_3)_\mu \delta_{\nu\lambda} + (k_3 - k_1)_\nu \delta_{\lambda\mu}] \tag{1.1}$$

where the 4-momenta emanating from the $ggg$ vertex satisfy $k_1 + k_2 + k_3 = 0$, and $f_{abc}$ is the color factor. When this vertex is folded into 3 $\bar{q}gg$ vertices of the respective forms $g_s \bar{u}(p'_1)i\gamma^\mu\{\lambda_1^a/2\}u(p_1)$, and two similar terms, the resultant $qqq$ interaction matrix (suppressing the Dirac spinors for the 3 quarks) becomes [5]

$$V_{qqq} = \frac{g_s^4}{24}[i\gamma^{(1)}(k_2 - k_3)\gamma^{(2)}\gamma^{(3)} + \{2\} + \{3\}\{\lambda_1\lambda_2\lambda_3\}/\{k_1^2k_2^2k_3^2\}] \tag{1.2}$$

where $k_i = p_i - p'_i$; $\lambda_i$ are the color matrices which get contracted into the corresponding scalar triple products in an obvious notation. [Note that the flavour indices are absent here since the quark gluon interaction is flavour blind].

This interaction will be considered in conjunction with 3 pairs of $qq$ forces within the framework of a Bethe-Salpeter type dynamics to be specified below. Before proceeding further, a possible motivation for the use of a direct $qqq$ force, apart from its intrinsic beauty, comes from the issue of ”proton spin” which, after making headlines about two decades ago, has come to the fore once again, thanks to the progress of experimental techniques in polarized deep inelastic scattering off polarized protons, and their variations thereof, which allow for an experimental determination of certain key QCD parameters by relating them to certain observable quantities emanating from external probes ; ( see a recent review [6] for references and other details). On the other hand it is also of considerable theoretical interest to determine
these very quantities directly from the intrinsic premises of QCD provided one has a "good" $qqq$ wave function to play with. Such a plea would have sounded rather utopian in the early days of QCD when phenomenology was the order of the day. Today however many aspects of QCD are understood well enough to make such studies worthwhile by hindsight, with possible ramifications beyond their educational value. For simplicity we work in the experimentally accessible regime of valence quarks. and make free use of the results of a recent paper [5] for many details on the specific effect of the 3-body force (1.2) on the analytical structure of the $qqq$ wave function, while giving more emphasis on the formalism relating to the loop diagrams towards the determination of proton spin with appropriate 4D BS normalization.

1.1 Theoretical Ingredients

In the valence quark regime, we need to consider a $qqq$ system governed by pairwise $qq$ forces as well as a direct 3-quark force of the type (1.2). A further simplification occurs in the high momentum regime where the effect of confining forces may be neglected, so that only coulombic forces are relevant. As explained in [5], we shall take the dynamics of a $qqq$ hadron in the high – momentum regime to be governed by the Salpeter equation [7] formulated in a covariant manner, which has the remarkable property of 3D-4D interlinkage (see [5] for a detailed picture). A covariant formulation of the Salpeter Equation in turn, is centered around the hadron 4-momentum $P_\mu$ in accordance with the Markov-Yukawa Transversality Principle (MYTP) [8, 9], which is a ‘gauge principle’ in disguise [10], and ensures that the interactions among the constituents be transverse to the direction of $P_\mu$. In the high momentum regime to be considered here, the confining interaction has been ignored for simplicity [5], which leaves the 3D form of the BS dynamics inadequate for mass spectral determination, yet its dynamical effect on the spin-structure of the wave function should be realistic enough for dealing with the hadron spin in the high momentum limit.

A further ingredient concerns the use of Dirac’s light-front form of dynamics [11] which has a bigger \{7\} stability group than the more conventional instant form whose stability group is only \{6\}. All this can be covariantly formulated; see [5] and [12] for details.
1.2 Plan of the Paper

The plan of the paper, which is based on an interlinked 3D-4D BSE formalism characterized by a Lorentz-covariant 3D support for its kernel a la MYTP \[8, 9\], adapted to the light front (LF) \[12\] is as follows. In Section 2 we summarise the principal results of ref \[5\] on both the 3D (φ) and the full-fledged 4D(Ψ) forms of the qqq wave function, so as to give a basically self-contained picture omitting the non essential details from \[5\]. Section 3 outlines the construction of the normalized 4D wave function after assessing the possible options on BS normalization for the same. (As a check, some of the conventional results are reproduced). Section 4 is devoted to the principal result of this investigation, viz., the construction of the two-gluon coupling to the axial operator $i\gamma_\mu\gamma_5$ and its insertion into the quark lines involved in the two types (self-energy and exchange) of possible baryonic transition diagrams for such coupling. Section 5 concludes with a short discussion of the results obtained vis-a-vis experiment.

2 Structure of the Full BS Wave Function $\Psi$

In this Section we collect the principal results of ref \[5\] on the full structure of the BS wave function in both the 3D (φ) and 4D (Ψ) forms.

2.1 Instant vs LF Representations of Momenta

We first record the correspondence between the instant and LF forms of the dynamics, starting with some definitions \[12\] for the LF quantities $p_\pm = p_0 \pm p_3$ defined covariantly as $p_+ = n.p\sqrt{2}$ and $p_- = -\tilde{n}.p\sqrt{2}$, while the perpendicular components continue to be denoted by $p_\perp$ in both notations. For a typical internal momentum $q_\mu$, the parallel component $P.qP/P^2$ of the instant form translates in the LF form as $q_\perp = zP.n$, where $P.n = \tilde{P}.\tilde{n}$, and $z = n.q/n.P$. As a check, $\hat{q}^2 = q_\perp^2 + z^2M^2$ which shows that $zM$ plays the role of the third component of $\hat{q}$ on LF. Next, we collect some of the more important definitions / results of the LF formalism \[12\]

\[
\begin{align*}
q_\perp &= q - q.n; \hat{q} = q_\perp + zP.n; z = q.n/P.n; q.n = q.\tilde{n}; \\
P.n &= \tilde{P}.\tilde{n}; P.q = P.nq.n + P.nq.n; \tilde{q}.\tilde{n} = P_\perp.q_\perp = 0; \\
P.\hat{q} &= P.nq.n; \hat{q}^2 = q_\perp^2 + M^2z^2; P^2 = -M^2
\end{align*}
\]
For a \(qqq\) baryon, there are two internal momenta, each separately satisfying the relations (2.1). Note that for any 4-vector \(A\), \(A\cdot n\) and \(-A\cdot n\) correspond to \(1/\sqrt{2}\) times the usual light front quantities \(A_{\pm} = A_0 \pm A_z\) respectively. But since a physical amplitude must not depend on the orientation \(n\), a simple device termed Lorentz–completion via the collinear trick [12] yields a Lorentz-invariant amplitude for a transition process with three external lines \(P, P', P''(= P + P')\) as explained in [12, 5]. And for ready reference, the precise correspondence between the instant and LF definitions of the ‘parallel (\(z\))’ and ‘time-like (0) ’ components of the various 4-momenta for a \(qqq\) baryon (\(i = 1,2,3\)) [13]:

\[
p_{iz}; p_i^0 = \frac{M p_i^+}{P^+}; \frac{M p_i^-}{2P^-}; \hat{p}_i \equiv \{p_{i\perp}, p_{iz}\}
\]  

(2.2)

The last part of Eq.(2.2) defines a covariant 3-vector on the LF that will frequently appear as arguments of 3D wave function \(\phi\) for the \(qqq\) proton.

### 2.2 From \(\Psi\) to \(\Phi\) via Gordon Reduction

The full wave function for three fermion quarks complete with all internal d.o.f.’s, satisfies the following Master equation whose kernel includes both \(qq\) and direct \(qqq\) forces [14]:

\[
\Psi(p_1p_2p_3) = \sum_1^3 S_F(p_1)S_F(p_2)g_s^2 \int \frac{d^4q_{12}}{(2\pi)^4} \gamma_\mu^{(1)} \gamma_\nu^{(2)} D_{\mu\nu}(k_{12}) \Psi(p_1', p_2', p_3) + S_F(p_1)S_F(p_2)S_F(p_3) \int \frac{d^4q_{12}^{(1)}d^4q_{12}^{(2)}}{(2\pi)^8} V_{qqq} \Psi(p_1'^{(1)}p_2'^{(1)}p_3)
\]

(2.3)

where the definitions for the various momenta, and the phase conventions for the quark propagators are those of [14], while the direct 3-quark interaction \(V_{qqq}\) in the last term is given by (1.2). Here the internal variables must be defined in a pre-assigned basis, say indexed by \#3 as [13]

\[
\sqrt{2}\xi_3 = p_1 - p_2; \sqrt{6}\eta_3 = -2p_3 + p_1 + p_2; \quad P = p_1 + p_2 + p_3
\]

(2.4)

where the time-like and space-like parts of each are given by (2.2), and the corresponding 3-vector defined as \(\hat{p}_i \equiv \{p_{i\perp}, p_{iz}\}\). (Two identical sets of momentum pairs \(\xi_1, \eta_1\) and \(\xi_2, \eta_2\) are similarly defined, but can be expressed in terms of the set (2.4) via permutation symmetry). The solution of this
Master equation (2.3) was then achieved in three steps (A, B, C). Step A consists in defining an auxiliary scalar function $\Phi$ related to the actual BS wave function $\Psi$ by

$$
\Psi = \Pi_{123}^{-1}(\xi)\Phi(p_1p_2p_3)W(P)
$$

(2.5)

where the quantity $W(P)$ is independent of the internal momenta but includes the spin-cum-flavour wave functions $\chi, \phi$ of the 3 quarks involved (see [15] for notation and other details):

$$
W(P) = [\chi^\prime \phi + \chi^\prime\prime \phi^\prime]/\sqrt{2}
$$

(2.6)

The quantities $\phi^\prime, \phi^\prime\prime$ are the standard flavour functions of mixed symmetry [16] [not to be confused with the 3D wave function $\phi$!], and $\chi^\prime, \chi^\prime\prime$ are the corresponding relativistic spin functions. The latter may be defined either in terms of the quark # indices as in Eqs (1.2) or (2.3), or sometimes more conveniently in a common Dirac matrix space as [15] [17]

$$
|\chi^\prime >; |\chi^\prime\prime > = \left[ \frac{M - i\gamma, P}{2M} [i\gamma_5; i\gamma^\mu/\sqrt{3}|C/\sqrt{2}]_{\beta\gamma} \otimes [1; \gamma_5\gamma^\mu]u(P) \right]_{\alpha}
$$

(2.7)

where the first factor is the $\beta\gamma$-element of a 4 x 4 matrix in the joint spin space of the quark #s 1, 2 [17], and the second factor the $\alpha$ element of a 4 x 1 spinor in the spin space of quark # 3; $C$ is a charge conjugation matrix with the properties [18]

$$
-\tilde{\gamma}_\mu = C^{-1}\gamma_\mu C; \tilde{\gamma}_5 = C^{-1}\gamma_5 C;
$$

and $\tilde{\gamma}_\mu$ is the component of $\gamma_\mu$ orthogonal to $P_\mu$. Finally, the representations of the flavour functions $\phi^\prime, \phi^\prime\prime$ satisfy the following relations in the "3" basis [19]

$$
<\phi''|1; \tilde{\pi}^{(3)}|\phi'' > = <\phi'|1; -\frac{1}{3}\tilde{\pi}^{(3)}|\phi'>
$$

(2.8)

Step B now consists in recasting Eq.(2.3) in terms of the scalar quantity $\Phi$ a la Eq.(2.5) with a simultaneous use of Gordon reduction on the pairwise kernels $V(\xi_i\xi_i)$ and the 3-body kernel $V_{qqq}$, as described in [4] following the original treatment of [20]. This has the effect of eliminating the Dirac matrices in favour of the Pauli matrices $\sigma_{\mu\nu}$. [We skip these details which may be found in [3].]
2.3 3D-4D Interlinkage by Green’s Function Method

The next step (Step C) now consists in a reduction of the 4D BSE for the quantity \( \Phi \) defined above to one for a 3D scalar \( \phi \) by the standard method of elimination of the time-like variables, and a reconstruction of the 4D quantity \( \Phi \), thus establishing a 3D-4D interconnection between these two wave functions. This last is facilitated by the Green’s function approach \([13]\) adapted to the LF formalism, as described in \([5]\). Calling the 4D Green’s functions associated with \( \Psi \) and \( \Phi \) by \( G_F \) and \( G_S \) respectively, the connection between them, analogously to Eq.(2.5), may be written as

\[
G_F(\xi;\eta;\xi';\eta') = W(P) \otimes \Pi_{123} S_{F_i}^{-1}(-p_i) G_S(\xi;\eta;\xi';\eta') \Pi_{123} S_{F_i}^{-1}(-p'_i) \bar{W}(P') \quad (2.9)
\]

where we have indicated the 4-momentum arguments of the Green’s functions involved, in a common \( S_3 \) basis \((\xi, \eta)\), and expressed the spin-flavour dependence of \( G_F \) as a matrix product implied by the notation \( W(P) \otimes \bar{W}(P') \).

It was shown in \([5]\) how the 3D-4D interconnection is first achieved at the level of the ‘scalar’ Green’s functions whose 4D and 3D forms are labelled by \( G_S \) and \( g_s \) respectively, and thence to the corresponding wave functions \( \Phi \) and \( \phi \) by the method of spectral representations. Finally the connection to the 4D spinor wave function \( \Psi \) is established via Eq. (2.5). We skip these steps which are given in sufficient details in \([5]\). The final result for \( \Psi \) in terms of \( \phi \) is

\[
\Psi(\xi, \eta) = \Pi_{123} S_F(p_i) D_{123} \sum_{123} [\phi(\hat{\xi}, \hat{\eta})] \frac{1}{(2\pi i)^2} \times W(P) \quad (2.10)
\]

where the structure of \( D_{123} \) is expressed by a double integral over two time-like momenta:

\[
\frac{1}{D_{123}} = \int \frac{P^2 dq_{12} dp_3}{4M^2(2i\pi)^2 \Delta_1 \Delta_2 \Delta_3} \quad (2.11)
\]

and the 3D wave function \( \phi \) satisfies the equation

\[
(2\pi)^3 D_{123} \phi(\hat{\xi}, \hat{\eta}) = \sum_{123} \frac{P_{3z}}{\sqrt{2}} \int d^3 \xi_3 V_{qq3} \phi(\hat{\xi}_3, \hat{\eta}_3) + \frac{1}{3\sqrt{3}(2\pi)^3} \int d^3 \xi'' d^3 \eta'' V_{qqq} \phi(\hat{\xi}'', \hat{\eta}'') \quad (2.12)
\]

The solution of this equation has been obtained in \([5]\) in coordinate space, using combinations analogous to (2.4), viz.,

\[
\sqrt{2}s_3 = r_1 - r_2; \quad \sqrt{6}t_3 = -2r_3 + r_1 + r_2 \quad (2.13)
\]
The final result for $\phi$ in coordinate space (see [5] for details) is

$$\phi = F(a, b|3|x); \quad a + b = 2; \quad ab = \beta/\sqrt{2}; \quad \beta \approx 0.058 \quad (2.14)$$

where $F$ is a standard hypergeometric function of its arguments, and has a particularly convenient representation for $a + b = 2$ [21]

$$F(a, b; 3; x) = \int_0^1 dy y^{a-1} \frac{(1-y)^a}{(1-xy)^a}; \quad a \approx 2 - \beta/\sqrt{2} \quad (2.15)$$

where $x = R^2/R_0^2$, $R^2 = s^2 + t^2$, and $x = 1$ corresponds to the point $R = R_0$. This completes our summary of the full structure of the 4D $qqq$ wave function $\Psi$ in terms of the 3D quantity $\phi$ a la [5].

### 3 Proton Spin Formalism

As a first application of this wave function, we shall determine the baryon spin, together with its corrections, in a general enough manner involving loop diagrams. To that end a key ingredient is the baryon normalisation within the Bethe Salpeter formalism, for which the appropriate diagram is Fig 1 with the spin operator $i\gamma_\mu\gamma_5$ replaced by an appropriate one signifying conservation of charge, mass or probability with corresponding operators $e(1/6 + \tau/2)i\gamma_\mu$, $M^2$ or 1 respectively. We adopt the last one (probability) in preference to the others in view of its simplicity as well as universal appeal as an even operator.

#### 3.1 BS Normalization of $qqq$ Wave Function

Consider Fig 1 where the 3 internal quark lines (1, 2, 3) are labelled by momenta $p_1$, $p_2$ and $p_3$ respectively. and the operator $i\gamma_\mu\gamma_5$ is temporarily replaced by 1 to signify probability conservation for a BS normalization calculation. This exercise is patterned closely on the lines of [15], albeit in a suitably corrected form in which the matrix elements are not factored into two parts (as done erroneously in [15]), but otherwise maintaining its mixed symmetric [m’ m”] notation for the matrix elements for each separate d.o.f. (spin and flavour). Keeping track of the indices (1, 2, 3), the two spin matrix
Figure 1: Schematic baryon spin diagram, with internal quark momenta 
\( p_1, p_2, p_3 \); basic spin operator \( i\gamma_\mu\gamma_5 \) is inserted in line \( p_1 \).

Elements \( N', N'' \) for BS normalization (taken between the functions (2.7)) may be written in an obvious notation as,

\[
N' = N'_{1;23} + N'_{1;32} + N'_{2;31} + N'_{3;12} + N'_{3;21} \\
N'' = N''_{1;23} + N''_{1;32} + N''_{2;31} + N''_{3;12} + N''_{3;21}
\] (3.1)

and should be multiplied by the corresponding flavour matrix elements (2.8) in accordance with the structure of the function \( W(P) \) of (2.6). The individual terms in Eq (3.1) are related by permutation symmetry, and two typical elements are given by

\[
N'_{1;23} = \bar{u}(P)P_s S_F(p_1) \{ 1 \} S_F(p_1) P_E \frac{\gamma_5 C}{\sqrt{2}} S_F(p_2) \frac{C^{-1}\gamma_5}{\sqrt{2}} P_E S_F(p_3) P_s u(P)
\] (3.2)

\[
N''_{1;23} = \bar{u}(P)P_s \gamma^\rho \gamma_5 S_F(p_1) \{ 1 \} S_F(p_1) P_E \frac{\gamma^\rho C}{\sqrt{6}} S_F(p_2) \frac{C^{-1}\gamma_5}{\sqrt{6}} P_E S_F(p_3) \gamma_5 \gamma^\rho P_s u(P)
\] (3.3)

where \( P \) is the baryon 4-momentum with mass \( M \) ( \( P^2 = -M^2 \)), and

\[
P_s = (1 + i\gamma.s\gamma_5)/2; \quad P_E = (M - i\gamma.P)/2
\] (3.4)

and the normalization condition is (c.f., [15])

\[
2 = \int d\tau [N' < \phi' | 1 | \phi' > + N'' < \phi'' | 1 | \phi'' >] = \int d\tau [N' + N''] \equiv 2N
\] (3.5)
where the flavour functions $\phi', \phi''$ are defined in (2.8) and $d\tau$ is the full measure of the internal integration variables defined by (2.4)

$$d\tau \equiv d^4\xi d^4\eta[D_{123}(\hat{\xi}, \hat{\eta})]^2$$

(3.6)

and the 3D wave function $\phi$ and the associated denominator function $D_{123}$ are as defined in Eq (2.10). Note that the time-like variables $\xi_0$ and $\eta_0$ of Eq.(2.2) do not appear in the factors $D_{123}\phi$ on the rhs of (3.6). We may now use the same pattern for the evaluation of some standard physical quantities which may serve as checks on the self-consistency of this formalism. Thus for the nucleon charge, the probability operator 1 employed for BS normalization above should be replaced by

$$1 \Rightarrow e\gamma_\mu[1/6 + \tau_3/2]$$

(3.7)

and the corresponding matrix elements $Q'$ and $Q''$ may be written down in the same notation and phase convention as for $N'$ and $N''$ above, and then divided by the total BS normalizer $N$ for correct overall normalization. The final result for the nucleon charge, after evaluating the flavour matrix elements a la Eq.(2.8) is

$$2QN = \int d\tau [Q'(1/6 + \tau_3/2) + Q''(1/6 - \tau_3/6)]$$

(3.8)

where $Q'$ and $Q''$ are given by Eqs (3.2) and (3.3) respectively, except for the replacement of $\{1\}$ by $i\gamma_\mu$, and $\tau_3$ has the values $\pm 1$ for proton / neutron. The momentum integrals are involved, but if terms of order $(\xi^2, \eta^2)/M^2$ are ignored compared to unity in the integrands concerned, some remarkable simplifications bring out the full flavour of $SU(6)$ symmetry, albeit in a relativistic manner. Thus as a first check on the self-consistency of the formalism, the proton / neutron charges work out as $e$ and 0 respectively.

### 3.2 Spin Matrix Elements in Lowest Order

We now employ this formalism for the determination of nucleon spin in lowest order, for which the basic spin operator is $i\gamma_m u\gamma_5$, (as in Fig 1), multiplied by appropriate flavour matrices. It is simplest to speak of the ‘axial charges’ whose proportionality to the spin vector $s_\mu$ comes out from analogous equations to (2) and (3) of ref. [6], with the substitution of $\{1\}$ by $i\gamma_\mu\gamma_5$ in Eqs (3.2-3) above. The flavour dependent axial charges $g^{(3)}_A$, $g^{(8)}_A$ and $g^{(0)}_A$ of
are then reproduced by the multiplication of this spin operator with the successive Gell-Mann matrices $\lambda_{3,8,0}$ respectively, and taking their matrix elements between the states defined by (2.8). Now the spin anomaly occurs mainly with respect to $g_A^{(0)}$, while the other two parameters remain almost unaffected. In the lowest order, i.e., neglecting terms of order $(\xi^2, \eta^2)/M^2$, these quantities may be worked out in the same normalization as defined in Section 3.1 above, to yield the values

$$ g_A^{(3)} = 10/9; \quad g_A^{(8)} = 2/3; \quad g_A^{(0)} = 2/3 \quad (3.9) $$

Comparison with Eq.(9) of ref.[6] reveals a difference of a factor of $2/3$ between the two results. This is due to the BS normalization employed here, viz., a relativistic one normalizing direct to unit probability which does not distinguish between the proton and the neutron , instead of to the charge which does, as in ref [15]. The latter agrees with the standard non-relativistic value cited in ref.[6], but the former indicates a welcome alternative possibility to ensure better with experiment without relativistic corrections. Further, it is only the last one, $g_A^{(0)}$, that is subject to anomaly corrections arising mainly from two-gluon effects that we consider next.

## 4 Spin correction from Two-gluon Anomaly

### 4.1 Two-gluon Anomaly Operator

The 2-gluon anomaly operator $\Delta_{\mu\nu\lambda}$ appears in Fig 2 as a ‘crossed box’ represented by a sum of two triangle diagrams, the second one being merely the effect of exchanging the two gluon lines connected to the triangle loop. In this Section we indicate its evaluation in a general manner in preparation for its insertion in the internal quark lines (Fig 3) for obtaining the gluon anomaly corrections to $g_A^{(0)}$. The 2-gluon anomaly operator, with gluon momenta $k_1 = k$ (entering) and $k_2 = k$ (leaving ) may be expressed in the form

$$ \Delta_{\mu\nu\lambda}(k) = \frac{ig}{(2\pi)^4} Tr \left[ \int d^4q i\gamma_\nu S_F(q + k_1)i\gamma_\mu\gamma_5 S_F(q + k_2)i\gamma_\lambda S_F(q) \right] \quad (4.1) $$

A second one is obtained by the simultaneous interchanges $k \to -k$ and $\nu \to \lambda$. The calculation is straightforward and will be mostly skipped except for a quick indication of how to incorporate gauge invariance. While
the modern method is that of dimensional regularization, it should be adequate to follow an old-fashioned (simpler) method due to Rosenberg [22], which effectively amounts to subtracting out the non-gauge-invariant terms at the integrand itself, so as to ensure separate conservation of currents at the two vertices $\nu$ and $\lambda$. After the trace evaluation in (3.9), this procedure leaves a numerator proportional to $q$ in the integrand. This needs at least an extra power of $q$ arising from an expansion of the propagator denominators in powers of $q.k/(q^2 + k^2)$. In the lowest order in $k$, the integral over $q^2$ becomes convergent, and after standard $q$ integration via the Feynman auxiliary variable $u$, reduces to an integral over $u$

$$\Delta_{\mu\nu\lambda} \approx \frac{2\alpha_s}{\pi} \int_0^1 du u^2 / [m_q^2 + k^2 u]$$

which for small $m_q^2$ further reduces to a very simple form:

$$\Delta_{\mu\nu\lambda} \approx \alpha_s \epsilon_{\mu\nu\lambda\sigma} k_\sigma; \quad m_q^2 << k^2 \quad (4.2)$$

### 4.2 2-gluon anomaly correction to spin amplitude

The operator $\Delta_{\mu\nu\lambda}$ is now ready for insertion in the internal quark lines of Fig. 3 signifying the forward scattering amplitude of the baryon. The insertion
Figure 3: Two-gluon operator, fig (2), inserted in the internal quark lines of the baryon: (a) ‘self-energy’ like insertion in line $p_1$; (b) ‘exchange-like’ insertion connecting lines $p_2$ and $p_3$

can be done in two different ways: self-energy like insertion in line $p_1$ a la Fig.3 (a); and exchange like insertion connecting two quark lines $p_1$ and $p_2$, as in Fig 3(b). We designate these contributions by $\Sigma'$, $\Sigma''$; and $V'$, $V''$ respectively, in accordance with the two types of spin matrix elements a la Eq.(2.7). These contributions are further indexed by the subscripts 1; 23, etc since three such diagrams for each type must be added up like in Eq.(3.1).

The master expressions for these matrix elements are as follows.

$$\Sigma'_{1;23} = \frac{2g_s^2}{3(2\pi)^4} \int d^4k \bar{u}(P) P_s S_F(p_1) \Delta_{\mu\nu\lambda} \gamma_\nu S_F(p_1 - k) i\gamma_\lambda D^2(k)$$

$$S_F(p_1) P_E \frac{\gamma_5}{\sqrt{2}} S_F(-p_2) \frac{C^{-1}\gamma_5}{\sqrt{2}} P_E S_F(p_3) P_s u(P) + \text{conj} \quad (4.3)$$
These quantities, when integrated over $\int d\tau$, Eq. (3.6), and divided by the normalizer $N$, Eq.(3.5), qualify directly as 2-gluon anomaly corrections (in the same relative normalization) to the spin matrix element $g_A^{(0)}$ listed in (3.9). The result for the fractional correction to $g_A^{(0)}$ may be expressed in the form

$$\delta g_A = \theta \left[ \frac{\alpha_s}{\pi} \right]^2 g_A^{(0)}.$$

where the dimensionless quantity $\theta$ may be termed the ‘reduced fractional 2-gluon anomaly correction’.

The calculation of $\theta$ - a long and elaborate process - involves two distinct steps : (a) integration over $d^4k$ (b) integration over $d\tau$. While step (a) is necessarily a dynamic correction, step (b) may be further divided into two parts, i) ‘kinematic’ and ii) ‘dynamic’, according as the effects of the internal momenta ($\xi, \eta$) are neglected or included respectively. The reason for this break-up is that only the latter involves an interplay of the the 3D wave function $|\phi|^2$, appearing via the integration measure $d\tau$, with the internal momenta.
which are copiously present in the large number of propagators which make up the integrands of the types (4.3 - 4.6), while the ‘kinematical’ part almost entirely suppresses this contribution by dropping the effects of these internal momenta from the said propagators. [Note that the hypergeometric form (2.15) of $\phi$ which appears through the integral measure $d\tau$, carries the dynamical signature of the ‘spin-part’ of the 3-body force!]. In this paper we are able to give only the results of the ‘kinematical’ part, while the calculation of the more difficult ‘dynamical’ part is in progress. To that end, the ‘kinematical’ part is calculable on closely analogous lines to the spin matrix elements in lowest order (see Sect. 3.2), using the normalization of Sect (3.1). The essential steps are very briefly indicated below.

4.3 ‘Kinematical’ Part of the Spin Correction

First, to incorporate the operator $\Delta_{\mu \nu \lambda}$ of Eq. (3.10), the following results are useful:

$$\gamma^\nu \gamma^\lambda \gamma^\sigma \epsilon_{\mu \nu \lambda \sigma} = 6 \gamma^\mu \gamma^5; \quad \gamma^\lambda \gamma^\sigma \epsilon_{\mu \nu \lambda \sigma} = 2 \gamma^\nu \gamma^\mu \gamma^5$$

Next, the (logarithmic) divergence of the $k$-integration requires the standard process of dimensional regularization [23], with a typical result of the form [24]

$$\int \frac{d^4k}{i} \int_0^1 du 2(1-u) \frac{k^2}{(k^2 + \Lambda_u)^3} = -\pi^2 [\gamma - 1 + \ln \pi \Delta_1]$$

where

$$\Lambda_u = u\Delta_1 + m_q^2 (1-u); \quad \Delta_1 = m_q^2 + p^2_1$$

After the $k$-integration (step (a)), the $d\tau$ integration (step (b)) involves some drastic approximations effectively involving the replacement of the 4-momenta $p_i$ of the various propagators by their ‘central’ values. At the end of this exercise, the effect of the factor $\phi^2$ in $d\tau$ almost ‘decouples’ from that of the various propagators involved in step (b), and the integrations can be performed without much further ado. Omitting these steps, the two contributions $\theta_1$ and $\theta_2$ from the ‘self-energy’ and ‘exchange’ effects respectively become the following:

$$\theta_1 \approx -0.5; \quad \theta_2 \approx -1.5$$

resulting in a total effect ‘kinematical’ contribution

$$\theta \approx -2.0$$
which with $\alpha \approx 0.39$ in (4.7), amounts to a tiny correction to the spin anomaly, albeit of the right sign.

## 5 Summary and Conclusion

To summarise, we have presented a first application of a new form of dynamics within the framework of QCD in the high momentum limit, viz., the role of a direct $qqq$ force which has been shown [5] to produce an additional singularity in the structure $\phi$ of the 3D $qqq$ wave function. The application is intended to address the issue of the proton spin anomaly in terms of a two-gluon anomaly effect. To that end, a good part of the paper has been devoted to a fairly general formulation of baryonic transition amplitudes, looked upon as $qqq$ systems in terms of Feynman amplitudes involving appropriate quark loops. The Bethe-Salpeter normalization has been attuned to the total probability which maintains a symmetry between the proton and the neutron, instead of to the total charge which does not. This relativistic formulation has the advantage that the axial charges $g_A^{(i)}$, $(i = 0, 8, 3)$, are already $2/3$ times the corresponding non-relativistic quantities [6], thus obviating major `relativistic corrections' [6] for them. Thus calibrated, the formalism is applied to the evaluation of two-gluon anomaly corrections [self-energy and exchange] to $g_A^{(0)}$, by inserting the anomaly operator $\Delta_{\mu\nu\lambda}$ into the internal quark lines, so as to produce a fractional correction of the general form (4.7), in which the dimensionless quantity $\theta$ is a measure of the correction. Unfortunately we have so far been able to calculate only the `kinematical' correction which corresponds to the neglect of the internal momenta $(\xi, \eta)$ in the integrands of the amplitudes involved. The resulting value of $\theta$ is $-2.0$ which has the right sign, but a rather small magnitude. This still leaves open the possibilities of `dynamical' corrections which involve an interplay of the internal momenta, mostly arising from the various propagators, with the 3D wave function $\phi$ whose hypergeometric form (2.14) reflects the dynamics of the 3-body force, namely the negative eigenvalue of the associated spin operator. The `correct' (negative) sign of $\theta$ is an encouraging sign for the vast scope for the role of this crucial dynamics yet to be included in its derivation. This calculation is currently in progress.

The author is grateful to the organizers of THEOPHYS-07 for an opportunity to present these preliminary results at this Conference.
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