Attitude determination in integrated GNSS-inertial navigation systems in information and measurement insufficiency conditions

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Abstract. The article presents the attitude determination algorithm for multi-antenna global navigation satellite and inertial integrated navigation system operating under information and measurement insufficiency conditions. The information and measurement insufficiency conditions are understood as the impossibility of the inertial system alignment on a fixed base and the absence of a redundant number of satellites to resolve the carrier phase integer ambiguity. The ability of the navigation system to operate in such conditions is achieved by combining several existing attitude determination methods and using the new method of attitude determination using ambiguous satellite phase measurements and absolute angular rate measurements. The hardware-in-the-loop simulation results using a 2-axis motion simulator and microelectromechanical systems inertial sensors are presented. The results confirm the efficiency of the proposed approach to the design of the navigation system algorithm.

1. Introduction
Integrated Global Navigation Satellite and Inertial Navigation Systems are widely used in many applications for control of moving objects of any type [1-5]. Coupling of strapdown inertial navigation system (SINS) and global navigation satellite systems (GNSS) receiver in an integrated navigation system significantly improves the performance of a position, velocity and attitude determination except heading [6]. The problem of low heading determination accuracy is especially critical for the low-cost inertial measurement unit (IMU). This problem of low heading accuracy is eliminated by adding one or more additional GNSS receivers to the integrated system [3-5]. In this case, GNSS antennas are mounted on the moving object at a certain distance from each other. Attitude is determined using carrier phase GNSS differenced measurements. This system is called GNSS attitude determination system or GNSS compass [7,8].

The following conditions must be met for the multiple-antenna GNSS-inertial navigation system to function successfully: strapdown inertial navigation system must be initialized (coordinates, velocity, and attitude initial values must be entered or determined during the initial alignment process); GNSS carrier phase integer ambiguities must be resolved.

In turn, the autonomous SINS alignment requires the following conditions [6,9]: the object must be fixed, the alignment process should take a relatively long time (usually about 10 minutes), gyroscopes should have relatively high accuracy for heading alignment. On the other hand, successful GNSS carrier
phase integer ambiguities resolution using search techniques (least squares ambiguity decorrelation adjustment (LAMBDA) or other) is possible with the redundant number of visible satellites [7].

In some cases, the above conditions cannot be met. An example is the launch of an unmanned aerial vehicle from a moving carrier in the absence of external attitude information and limited navigation satellites visibility (for example, in urban areas). Conceptually, the inability to fix the object, the lack of external attitude information, the SINS alignment time limit, the insufficient accuracy of the gyroscopes, the limited satellite visibility creates the information and measurement insufficiency conditions for an integrated system.

Various approaches to solving the formulated problem of information and measurement insufficiency are known. Some of the methods are based on using the angular object rotation [3] or the angular rotation of the integrated system onboard the object [4]. Such methods are often called motion-based methods. Other approaches are based on the SINS attitude errors estimation by the optimal Kalman filter (OKF) using position/velocity GNSS and SINS measurements [6,9]. Even a coarse attitude estimation can significantly reduce the ambiguities search space. At the same time, to improve the attitude estimation capability, it is often necessary to move the object along a special type of trajectory [9]. Such a trajectory should provide the variability of the measured specific force and absolute angular rates values.

Feasibility investigation of attitude determination in integrated GNSS-inertial navigation systems in information and measurement insufficiency conditions is considered in the paper. The attitude determination algorithm based on a combination of the above-mentioned approaches is proposed. The proposed algorithm performance was evaluated using computer simulation and processing of inertial measurements obtained on a motion simulator.

2. Methodology

As mentioned above, the proposed attitude determination algorithm is based on combining motion-based ambiguity resolution methods and methods of SINS and GNSS integration using position/velocity measurements. The key problem of the integrated system operating in the conditions of information and measurement insufficiency is the initial alignment. The flowchart of the initial alignment algorithm is shown in figure 1.

The initial alignment procedure begins with a SINS coarse alignment based on available external attitude information (magnetic compass, attitude and heading reference system (AHRS), GNSS receiver track angle). If there is no external data, zero values are entered. After a coarse alignment, parallel execution of motion-based attitude determination algorithm and attitude determination algorithm based on optimal Kalman filter using position/velocity measurements begins. The new method of attitude determination using ambiguous GNSS phase measurements and absolute angular rate measurements is selected as the motion-based method [3]. At the end of each algorithm execution step, the pitch and roll data source is selected. The selection criterion is based on the analysis of the coefficient matrix condition number formed by the selected motion-based method [3]. If the condition number is higher than a certain threshold, the accuracy of the position/velocity OKF in determining pitch and roll is usually higher. Otherwise, the opposite is true. The threshold value is selected depending on the trajectory of the moving object and other measurement conditions. The study results of coefficient matrix condition number dependence on the type of trajectory and other parameters are presented in [3]. For example, the pitch and roll determination accuracy reaches a sub-degree level with a coefficient matrix condition number of less than 40 when using 4 satellites and changing the heading and roll angles with angular rates of 5 deg/s [3]. However, the heading estimation by position/velocity OKF is either significantly delayed or impossible. Therefore, the source of heading information is the only motion-based algorithm. The integer phase ambiguities difference search space is formed based on the values of attitude angles and their errors. Each potential ambiguity value is checked using the baseline length test and the ratio test [7,10]. After passing the baseline length test and the ratio test, the ambiguities are considered resolved and the execution of attitude determination algorithm based on optimal Kalman filter (OKF) SINS errors estimation using position/velocity/attitude GNSS and SINS measurements begin. The position/velocity/attitude OKF attitude determination algorithm is considered the main one and the
system switches to the main mode. The attitude determination algorithms mentioned in the flowchart are described below.

![Flowchart of attitude determination algorithms](image)

**Figure 1.** Multiple-antenna integrated GNSS-inertial navigation system alignment algorithm.

### 2.1. Motion-based attitude determination in the multi-antenna GNSS-inertial navigation system

The selected motion-based attitude determination algorithm is based on the interferometric model of GNSS measurements [3]. Using this model, the GNSS carrier phase double difference $\nabla \Delta F_{a,b}^{i,j}(k)$ measured by antennas $a$ and $b$ from satellites $j$ and $i$ can be written as follows (1):

$$\nabla \Delta F_{a,b}^{i,j}(k) = -\left[\bar{T}_{a,b}^{T} B_{j} \right]_{\theta_{a,b}} B_{j} C_{B_{i}B_{j}}^{L_{k}} C_{L_{k}L_{j}}^{T} \left[\nabla \bar{e}_{i,j}^{k}(k)\right]_{L_{k}} + \nabla \Delta N_{a,b}^{i,j}$$  \hspace{1cm} (1)

where $\nabla \Delta F_{a,b}^{i,j}(k)$ is the carrier phase double difference for the epoch $k$; $\bar{T}_{a,b}$ is the baseline vector for $a$ and $b$ antenna; $C_{B_{i}B_{j}}^{L_{k}}$ is the transformation matrix between the body frame for the epoch $\theta$ and the body frame for the epoch $k$; $C_{L_{k}L_{j}}^{T_{0}}$ is the transformation matrix between the local level coordinate system and the body frame for the epoch $\theta$; $C_{L_{k}L_{j}}^{T_{0}}$ is the transformation matrix between the local level coordinate system for the epoch $k$ and $\theta$; $\nabla \bar{e}_{i,j}^{k}(k)$ is the satellite $i$ and $j$ unit vector difference for the epoch $k$; $\nabla \Delta N_{a,b}^{i,j}$ - carrier phase integer ambiguities double difference for antennas $a$ and $b$ and for satellites $j$ and $i$; the subscript $B_{k}$ for a vector means that the vector is written in projections to the body frame.

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**References**

[3] Interferometric GNSS model and its application in attitude determination.
coordinate system for the epoch $k$; the subscript $L_k$ for a vector means that the vector is written in projections to the local level coordinate system for the epoch $k$.

The use of double differences in equation (1) is dictated by the possibility of using original equipment manufacturer (OEM) receivers without a common oscillator. It should be noted that the integer ambiguities double difference will have the same value for all epochs in the absence of cycle slip.

Thus, the known values in equation (1) are $\nabla \Delta F^{i,j}_{a,b} (k)$ (measured by GNSS receivers), $\overline{T}_{a,b}$ (measured during antennas installation), $C^{i,j}_{a,b}$ (calculated using GNSS receiver coordinates determined using pseudoranges; this matrix can be considered an identity matrix at time intervals of several tens of minutes), $\nabla \overline{C}^{i,j}_{a,b} (k)$ (calculated using GNSS receiver coordinates determined using pseudoranges and GNSS satellites ephemeris), $C^{0}_{B,C}$ (calculated using Poisson matrix equation and gyroscopes absolute angular rate measurements [3]). The unknowns in the equation (1) are $C^{i,j}_{B,C}$ and $\Delta \nabla N^{i,j}_{a,b}$. Note that the matrix $C^{i,j}_{B,C}$ depends on heading, pitch and roll.

Several equations (1) for different satellites, different bases (if available), and for different epochs form a system of linear equations. This system can be solved by the least squares method. As a result of solving the system of linear equations (1), the attitude for the first epoch will be determined and ambiguities will be resolved. The successful attitude determination and ambiguity resolution require an angular rotation of the object and the accumulation of measurements in time. The accuracy of unknown parameters calculating will depend on the coefficient matrix condition number of the system of linear equations (1).

2.2. Attitude determination algorithm based on optimal Kalman filter

Attitude determination algorithm based on loosely-coupled approach. Thus the optimal Kalman filter estimates the SINS errors using the measurement containing the difference between SINS and GNSS position/velocity/attitude values. The scheme of multiple-antenna GNSS and SINS integration is shown in figure 2.

The state-space model of the system has the following form (2):

$$\dot{\overline{X}} = F\overline{X} + B\overline{U} + G\overline{W}$$

(2)

where $\overline{X}$ is the state vector, which models the SINS errors; $F$ is the system dynamics matrix; $B$ is the control matrix; $\overline{U}$ is the control vector; $G$ is the system noises matrix; $\overline{W}$ is the system noises vector.

The measurement model is (3):

$$\overline{Z} = H\overline{X} + \overline{V}$$

(3)

where $\overline{Z}$ is the measurement vector (SINS and GNSS position/velocity/attitude differences); $H$ is the measurement matrix (provides the linear connection between the states and the observations); $\overline{V}$ is the measurement noise. The state vector is (4):

$$\overline{X} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad \alpha \quad \beta \quad \gamma \quad \Delta \Omega_1 \quad \Delta \Omega_2 \quad \Delta \Omega_3 \quad \Delta n_1 \quad \Delta n_2 \quad \Delta n_3]^T$$

(4)

where $x_1$, $x_2$ are the SINS coordinates errors; $x_3$, $x_4$ are the SINS velocities errors; $\alpha$, $\beta$, $\gamma$ are the SINS attitude errors (angles between the SINS calculated local level frame relative to true local level frame in SINS calculated position); $\Delta \Omega_1$, $\Delta \Omega_2$, $\Delta \Omega_3$ - gyroscopes biases; $\Delta n_1$, $\Delta n_2$, $\Delta n_3$ - accelerometers biases.
Figure 2. Multiple-antenna GNSS and SINS integration.

The system dynamics matrix has the form (5):

\[
F = \begin{bmatrix}
0_{2 \times 2} & E_{2 \times 2} & 0_{2 \times 3} & 0_{2 \times 6} \\
F_{21} & F_{22} & F_{23} & F_{24} \\
0_{3 \times 2} & 0_{3 \times 2} & F_{33} & F_{34} \\
0 & 0 & \mathbf{0}_{6 \times 13}
\end{bmatrix}
\]

where 

\[
F_{21} = \begin{bmatrix}
(\Omega^2 + \Omega_y^2 - \omega_0^2) & (\dot{\Omega}_z - \Omega_x \Omega_y) \\
- (\dot{\Omega}_z + \Omega_x \Omega_y) & (\Omega_x^2 + \Omega_y^2 - \omega_0^2)
\end{bmatrix},
\]

\[
F_{22} = \begin{bmatrix}
0 & 2\Omega_z \\
-2\Omega_z & 0
\end{bmatrix},
\]

\[
F_{23} = \begin{bmatrix}
0 & n_z & -n_y \\
-n_z & 0 & n_x
\end{bmatrix},
\]

\[
F_{24} = \begin{bmatrix}
0_{2 \times 3} & C_{2 \times 3}
\end{bmatrix},
\]

\[
F_{33} = \begin{bmatrix}
0 & \Omega_z & -\Omega_y \\
-\Omega_z & 0 & \Omega_x \\
\Omega_y & -\Omega_x & 0
\end{bmatrix},
\]

\[
F_{34} = \begin{bmatrix}
C_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix},
\]

\[
E \text{ is the identity matrix of the specified dimension; } 0 \text{ is the zero matrix of the specified dimension; } \Omega_x, \Omega_y, \Omega_z, \dot{\Omega}_z \text{ are the local level coordinate system angular rate projections and its derivatives; } \omega_0 \text{ is the Shuler frequency; } n_x, n_y, n_z \text{ are the “specific force” acceleration local level coordinate system projections; } C_{3 \times 3} \text{ is the transformation matrix between body frame and local level coordinate system; } C_{2 \times 3} \text{ is the matrix, containing two first rows of the matrix } C_{3 \times 3} \text{.}
\]

The control vector is (6):

\[
\bar{U} = \begin{bmatrix}
h_{\text{ext}} & \dot{h}_{\text{ext}}
\end{bmatrix}^T
\]

where \( h_{\text{ext}}, \dot{h}_{\text{ext}} \) are the altitude and the vertical velocity obtained from external sensor (GNSS, baroaltimeter etc.).

The control matrix has the form (7):
The system noise vector contains noise error components of gyroscopes \( \delta \Omega_1, \delta \Omega_2, \delta \Omega_3 \) and accelerometers \( \delta n_1, \delta n_2, \delta n_3 \) (8):

\[
\mathbf{\bar{W}} = \begin{bmatrix} \delta \Omega_1 & \delta \Omega_2 & \delta \Omega_3 & \delta n_1 & \delta n_2 & \delta n_3 \end{bmatrix}^T
\]

The system noise matrix is (9):

\[
\mathbf{G} = \begin{bmatrix}
0_{2 \times 3} & 0_{2 \times 3} \\
0_{2 \times 3} & C_{2 \times 3} \\
C_{3 \times 3} & 0_{3 \times 3} \\
0_{6 \times 3} & 0_{6 \times 3}
\end{bmatrix}
\]

The key difference between the position/velocity OKF algorithm and the position/velocity/attitude algorithm (figure 1) is the composition of the measurement vector. In case of position/velocity OKF algorithm measurement vector has the form (10):

\[
\mathbf{Z}_{\text{POS/VEL}} = \begin{bmatrix}
\mathbf{\phi}_{\text{SINS}} - \mathbf{\phi}_{\text{GNSS}} \\
\mathbf{\lambda}_{\text{SINS}} - \mathbf{\lambda}_{\text{GNSS}} \\
\mathbf{U}^E_{\text{SINS}} - \mathbf{U}^E_{\text{GNSS}} \\
\mathbf{U}^N_{\text{SINS}} - \mathbf{U}^N_{\text{GNSS}}
\end{bmatrix}^T
\]

where \( \mathbf{\phi}_{\text{SINS}}, \mathbf{\lambda}_{\text{SINS}}, \mathbf{U}^E_{\text{SINS}}, \mathbf{U}^N_{\text{SINS}} \) are the latitude, longitude, east and north velocity projections determined by SINS; \( \mathbf{\phi}_{\text{GNSS}}, \mathbf{\lambda}_{\text{GNSS}}, \mathbf{U}^E_{\text{GNSS}}, \mathbf{U}^N_{\text{GNSS}} \) are the latitude, the longitude, the east and the north velocity projections determined by GNSS; \( \delta \mathbf{\phi}_{\text{SINS}}, \delta \mathbf{\lambda}_{\text{SINS}}, \delta \mathbf{U}^E_{\text{SINS}}, \delta \mathbf{U}^N_{\text{SINS}} \) are the latitude, longitude, east and north velocity projections SINS determination errors; \( \delta \mathbf{\phi}_{\text{GNSS}}, \delta \mathbf{\lambda}_{\text{GNSS}}, \delta \mathbf{U}^E_{\text{GNSS}}, \delta \mathbf{U}^N_{\text{GNSS}} \) are the latitude, the longitude, the east and the north velocity projections GNSS determination errors.

At the same time position/velocity/attitude OKF algorithm measurement vector has the form (11):

\[
\mathbf{Z}_{\text{POS/VEL/ATT}} = \begin{bmatrix}
\mathbf{\phi}_{\text{SINS}} - \mathbf{\phi}_{\text{GNSS}} \\
\mathbf{\lambda}_{\text{SINS}} - \mathbf{\lambda}_{\text{GNSS}} \\
\mathbf{U}^E_{\text{SINS}} - \mathbf{U}^E_{\text{GNSS}} \\
\mathbf{U}^N_{\text{SINS}} - \mathbf{U}^N_{\text{GNSS}}
\end{bmatrix}^T
\]

\[
= \begin{bmatrix}
\delta \mathbf{\phi}_{\text{SINS}} - \delta \mathbf{\phi}_{\text{GNSS}} \\
\delta \mathbf{\lambda}_{\text{SINS}} - \delta \mathbf{\lambda}_{\text{GNSS}} \\
\delta \mathbf{U}^E_{\text{SINS}} - \delta \mathbf{U}^E_{\text{GNSS}} \\
\delta \mathbf{U}^N_{\text{SINS}} - \delta \mathbf{U}^N_{\text{GNSS}}
\end{bmatrix}^T
\]

where \( \mathbf{\phi}_{\text{SINS}}, \mathbf{\lambda}_{\text{SINS}}, \mathbf{U}^E_{\text{SINS}}, \mathbf{U}^N_{\text{SINS}} \) are the latitude, longitude, east and north velocity projections determined by SINS; \( \mathbf{\phi}_{\text{GNSS}}, \mathbf{\lambda}_{\text{GNSS}}, \mathbf{U}^E_{\text{GNSS}}, \mathbf{U}^N_{\text{GNSS}} \) are the latitude, the longitude, the east and the north velocity projections determined by GNSS; \( \delta \mathbf{\phi}_{\text{SINS}}, \delta \mathbf{\lambda}_{\text{SINS}}, \delta \mathbf{U}^E_{\text{SINS}}, \delta \mathbf{U}^N_{\text{SINS}} \) are the latitude, longitude, east and north velocity projections SINS determination errors; \( \delta \mathbf{\phi}_{\text{GNSS}}, \delta \mathbf{\lambda}_{\text{GNSS}}, \delta \mathbf{U}^E_{\text{GNSS}}, \delta \mathbf{U}^N_{\text{GNSS}} \) are the latitude, the longitude, the east and the north velocity projections GNSS determination errors.
where $\psi^{SINS}$, $\delta^{SINS}$, $\chi^{SINS}$ are the heading, the pitch and the roll determined by SINS; $\psi^{GNSS}$, $\theta^{GNSS}$, $\chi^{GNSS}$ are the heading, the pitch and the roll determined by GNSS; $\delta \psi^{SINS}$, $\delta \theta^{SINS}$, $\delta \chi^{SINS}$ are the heading, the pitch and the roll SINS determination errors; $\delta \psi^{GNSS}$, $\delta \theta^{GNSS}$, $\delta \chi^{GNSS}$ are the heading, the pitch and the roll GNSS determination errors. Note that one of the attitude angles (usually roll) cannot be calculated by GNSS with two antennas mounted on the moving object.

The measurement matrix and measurement noise vector are formed depending on the composition of the measurement vector. In case of $Z_{POS/VEL}$ selection measurement matrix and measurement noise vector are (12) and (13) respectively:

\[
H_{POS/VEL} = \begin{bmatrix}
H_1 & 0_{2\times11} \\
H_2 & E_{2\times2} & 0_{2\times9}
\end{bmatrix}
\]

\[
\bar{V}_{POS/VEL} = \begin{bmatrix}
\delta \phi^{GNSS} & \delta \lambda^{GNSS} & \delta U^E_{GNSS} & \delta U^N_{GNSS}
\end{bmatrix}^T
\]

where $H_1 = \begin{bmatrix} 0 & 1 / \rho_1 \cos \varphi^{GNSS} \\ 1 / \rho_2 \cos \varphi^{GNSS} & 0 \end{bmatrix}$; $H_2 = \begin{bmatrix} -\Omega_x \tan \phi^{SINS} + U^SINS_Z / \rho_2 & -\Omega_z \\ 0 & -U^SINS_Z / \rho_1 \end{bmatrix}$.

$\rho_1$, $\rho_2$ are correspondent ellipsoid curvature radiuses.

Measurement matrix and measurement noise vector for $Z_{POS/VEL/ATT}$ selection have the form (14) and (15) respectively:

\[
H_{POS/VEL/ATT} = \begin{bmatrix}
H_1 & 0_{2\times11} \\
H_2 & E_{2\times2} & 0_{2\times9} \\
H_3 & 0_{3\times2} & H_4 & 0_{3\times3}
\end{bmatrix}
\]

\[
\bar{V}_{POS/VEL/ATT} = \begin{bmatrix}
\delta \phi^{GNSS} & \delta \lambda^{GNSS} & \delta U^E_{GNSS} & \delta U^N_{GNSS} & \delta \psi^{GNSS} & \delta \theta^{GNSS} & \delta \chi^{GNSS}
\end{bmatrix}^T
\]

\[
H_3 = \begin{bmatrix}
(\cos \psi^{SINS} \tan \theta^{SINS} - \tan \psi^{SINS}) / \rho_2 & -\sin \psi^{SINS} \tan \theta^{SINS} / \rho_1 & -\cos \psi^{SINS} / \rho_1 \\
-\sin \psi^{SINS} / \rho_2 & \cos \psi^{SINS} \tan \theta^{SINS} - \tan \psi^{SINS} & 1
\end{bmatrix}
\]

where, $H_4 = \begin{bmatrix} \sin \psi^{SINS} \tan \theta^{SINS} & \cos \psi^{SINS} \sec \theta^{SINS} & -1 \\
\cos \psi^{SINS} \sec \theta^{SINS} & -\sin \psi^{SINS} \sec \theta^{SINS} & 0 \\
\sin \psi^{SINS} \tan \theta^{SINS} & \cos \psi^{SINS} \sec \theta^{SINS} & 0
\end{bmatrix}$.

The state vector estimation is performed by the optimal Kalman filter [6].

3. Results and discussion

The above algorithm performance was evaluated using HIL simulation. Inertial measurements for the SINS algorithm were obtained on the 2-axis motion simulator Actidyn ‘ST2356C’ [11] (figure 3). GNSS measurements were generated by the software simulator using Global positioning system (GPS) ephemeris and the angular motion parameters obtained from Actidyn ‘ST2356C’.

The 2-axis motion simulator angular trajectory was S-turn with a motion in the heading with the magnitude of 30 degrees and the roll with the magnitude of 15 degrees. Attitude angular rate was in the range of 5 to 10 deg/sec. Silicon Sensing DMU-11 [12] was used as the inertial measurement unit.
The number of antennas in the GNSS measurements simulation was 3, baseline length was 1 m. The number of navigation satellites in the GPS constellation was 4 and PDOP (position dilution of precision) was 1.87. GPS range and Doppler measurements were simulated as a white noise with intensity: pseudorange – 10 m, carrier phase – 0.002 m, Doppler – 0.1 m/s.

Figure 3. HIL simulation using 2-axis motion simulator Actidyn ‘ST2356C’. Multiple-antenna Integrated Navigation System on the simulator inner table (a) inside the climatic chamber (b).

Execution of the multiple-antenna integrated GNSS-inertial navigation system algorithm (figure 1) began immediately after the start of motion simulator rotation. The initial attitude values were set with an errors in coarse SINS alignment stage. Heading initial alignment error was 60 degrees. Pitch and roll initial alignment error were 30 degrees.

The results of attitude determination using ambiguous GNSS phase measurements and absolute angular rate measurements are presented in table 1. The results are presented for the different measurement accumulation times (5, 10, 20, 40 sec.). For each measurement accumulation time, the values of the coefficient matrix condition number, floating carrier phase ambiguities error (percentage of the GPS L1 wavelength), heading, pitch and roll errors are given. The coefficient matrix condition number decreases from 76 to 17 with increasing measurement accumulation time as expected. The attitude determination errors limit the range of the heading, pitch, roll possible values (uncertainty areas in the attitude domain). Therefore, given the length of the base 1m, attitude determination accuracy after 5 seconds allows reducing the ambiguities search space to several set of values. In addition, the accuracy of the floating ambiguity calculation is also at a relatively high level after 5 seconds (24% of the wavelength). But after 10 seconds, the accuracy reaches 9% and decreases to 3% by 40 seconds. Taking also into account the attitude determination accuracy after 10 seconds, the ambiguities can be practically resolved without additional operations.

| Parameter                  | 5 sec | 10 sec | 20 sec | 40 sec |
|----------------------------|-------|--------|--------|--------|
| Condition number           | 76    | 47     | 24     | 17     |
| \( \Delta N^a \), %       | 24    | 9      | 5      | 3      |
| Heading error, deg.        | 5.1   | 0.9    | 0.5    | 0.2    |
| Pitch error, deg.          | 6.5   | 1.3    | 0.6    | 0.3    |
| Roll error, deg.           | 4.9   | 1.1    | 0.4    | 0.2    |

\(^a\)Floating carrier phase ambiguities error (percentage of the GPS L1 wavelength)

In accordance with the flowchart (figure 1), the attitude determination algorithm based on optimal Kalman filter using position/velocity measurements was run in parallel with the motion-based algorithm.
Pitch and roll errors estimation errors and RMS (root mean square) using position/velocity OKF algorithm are shown in figures 4 and 5 respectively. The graphs are cropped at the top for easy results analysis. The pitch determination error (after SINS correction using its error estimation results) is less than 0.2 degrees after 10 seconds and less than 0.1 degrees after 20 seconds. At the same time, the roll determination error is less than 0.1 degrees after 10 seconds already.

Figure 4. Pitch error estimation error and RMS using position/velocity OKF algorithm.  
Figure 5. Roll error estimation error and RMS using position/velocity OKF algorithm.

Heading error estimation error and RMS using position/velocity OKF algorithm are shown in figure 6. The results confirm the poor heading estimation capability. Practically the heading determination using this algorithm is impossible.

Thus, the results of the SINS correction using the position/velocity OKF algorithm allow us to determine with relatively acceptable accuracy (tenths of a degree) only the pitch and roll and do not actually allow us to determinate the heading. However, reducing the area of the attitude uncertainty even to the above values (heading, pitch and roll errors) allows you to significantly reduce ambiguities search space.

By reducing the search space, baseline length test and ratio test were passed after 5 seconds for the HIL-simulation scenario described above. After the ambiguity was resolved, the measurement vector was expanded by attitude angles (figure 2) and position/velocity/attitude OKF algorithm was run (figure 1).

Pitch, roll and heading errors estimation errors and RMS using position/velocity/attitude OKF algorithm are shown in figures 7-9 respectively.

Figure 6. Heading error estimation error and RMS using position/velocity OKF algorithm.
Figure 7. Pitch error estimation error and RMS using position/velocity/attitude OKF algorithm.

Figure 8. Roll error estimation error and RMS using position/velocity/attitude OKF algorithm.

All three attitude angle errors (after SINS correction using its error estimation results) are less than 0.02 degrees after a few seconds of estimation. A significant increase in pitch and roll determination accuracy is due to the inclusion of the differences of attitude angles determined by the SINS and GNSS in the measurement vector. In this case, the GNSS, after resolving the ambiguities, calculates the attitude angles using carrier phase measurements (carrier phase range error is 0.002 m in simulation). Besides, after the expansion of the measurement vector, the system became able to calculate the heading. Moreover, the accuracy of heading determination is also at a very high level (less than 0.02 degrees).

Thus, by combining several attitude determination methods in the multiple-antenna GNSS-inertial integrated system algorithm, it is possible to maintain the system's performance under the conditions of information and measurement insufficiency.

Figure 9. Heading error estimation error and RMS using position/velocity/attitude OKF algorithm.

4. Conclusion
The investigation results allow to evaluate the feasibility of attitude determination in integrated GNSS-inertial navigation systems in information and measurement insufficiency conditions. The experimental sample based on MEMS sensors demonstrated the possibility of reaching the expected level of accuracy within a few seconds after power supply. This result was achieved without a preliminary SINS static alignment, with only four visible GPS satellites and moving along the S-turn trajectory.
Such a result would be impossible to obtain using the applied attitude determination methods separately. On the one hand, without a preliminary SINS static alignment, in the absence of a redundant number of satellites, it would be impossible to resolve the ambiguities of the carrier phase measurements using search techniques (LAMBDA or other) and run the position/velocity/attitude OKF algorithm. On the other hand, using only the position/velocity OKF algorithm, it would be impossible to determine the pitch and roll with such accuracy, and, most importantly, it would be impossible to determine the heading at all. In addition, if the trajectory of a moving object meets the requirements described above (movement with a change in attitude angles), then the use of a new method of attitude determination using ambiguous GNSS phase measurements and absolute angular rate measurements helps to resolve the ambiguity as soon as possible.

In the future, it is necessary to test the system on a mobile object using real measurements. One of the important elements of the system described in the article is the GNSS integrity monitoring algorithm, including carrier phase measurements monitoring. Such a block is even included in the multiple-antenna GNSS and SINS integration scheme in figure 2. The further direction of work on improving the characteristics of the system should be the development and research of GNSS integrity monitoring algorithm using SINS data.

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References

[1] Antonov D, Kolganov L, Savkin A, Chekhov E and Ryabinkin M 2020 Navigation and motion control systems of the autonomous underwater vehicle. EUREKA: Physics and Engineering 4 38 https://doi.org/10.21303/2461-4262.2020.001361
[2] Chernomorsky A, Lelkov K and Kuris E 2020 About One way to increase the accuracy of navigation system for ground wheeled robot used in aircraft parking. Smart Science 8(4) 219 doi: 10.1080/23080477.2020.1824055
[3] Zharkov M V, Veremeenko K K, Antonov D A and Kuznetsov I M 2018 Attitude determination using ambiguous GNSS phase measurements and absolute angular rate measurements. Gyroscopy and Navigation 9(4) 277 https://doi.org/10.1134/S2075108718040090
[4] Emel’yantsev G, Stepanov O, Stepanov A, Blazhnov B, Dranitsyna E, Evtishchev M, Eliseev D and Volynskiy D 2020 Integrated GNSS/IMU-Gyrocompass with rotating IMU. Development and test results. Remote Sens-Basel. 12(22) 3736 https://doi.org/10.3390/rs12223736
[5] Emel’yantsev G I, Blazhnov B A and Stepanov A P 2018 Specific features of constructing a dual-mode GNSS gyrocompass as a tightly-coupled integrated system. Gyroscopy and Navigation 9(2) 97 https://doi.org/10.1134/S2075108718020049
[6] Noureldin A, Karamat T and Georgy J 2013 Fundamentals Of Inertial Navigation, Satellite-Based Positioning And Their Integration (Berlin: Springer, Springer-Verlag Berlin Heidelberg) p 314
[7] Teunissen P 2010 Integer least-squares theory for the GNSS compass. J. Geod. 84(7) 433 https://doi.org/10.1007/s00190-010-0380-8
[8] Goh S T and Low K-S 2017 Survey of global-positioning-system-based attitude determination algorithms. J. Guid. Control Dynam. 40(6) 1 https://doi.org/10.2514/1.G002504
[9] Titterton D and Weston J 2004 Strapdown Inertial Navigation Technology. IEE Radar, Sonar, Navigations and Avionics (Stevenage: IET) p 558
[10] Teunissen P and Verhagen S 2009 The GNSS ambiguity ratio-test revisited: a better way of using it. Surv. Rev. 41(312) 138 doi: 10.1179/003962609X390058
[11] ST 2356 Serie. Dual Axis Motion Simulator, Actidyn Systemes SA, available at: https://www.actidyn.com/sites/default/files/2020-04/st2356.pdf
[12] DMU11 Technical Datasheet, Silicon Sensing Systems Limited, available at: https://www.siliconsensing.com/media/30957/dmu11-00-0100-132-rev5.pdf

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