Dwarf Galaxy Sized Monopoles as Dark Matter?

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Abstract

We propose a model of dark matter: galaxy-sized 't Hooft-Polyakov magnetic monopoles in a new, extraordinarily weakly coupled SU(2) gauge sector with an adjoint Higgs field and two flavors of fundamental fermions. We fit the parameters by asserting that the dark matter halos of the lightest dwarf spheroidal (dSph) galaxies consist of a single charge \(Q = 1\) monopole. Lensing and wide binary bounds are then easily satisfied and the monopoles form in time to help with CMB fluctuations. In this model dSph and low surface brightness (LSB) halos automatically have (1) A minimum mass - Dirac quantization solves the missing satellite problem, (2) A constant density core \((r < r_1)\), (3) An intermediate regime \((r_1 < r < r_2)\) with density \(\rho \sim 1/r^2\). The model predicts that (A) \(r_1\) is proportional to the stellar rotational/dispersion velocities at \(r_1 < r < r_2\), (B) \(r_2\) is reasonably \(Q\) independent and so dSph halos extend at least ten times farther than their half-light and tidal radii, (C) The minimal stellar dispersion is \(1/\sqrt{2}\) times the next-smallest allowed value. A serious potential problem with our proposal is that non-BPS monopoles are repulsive. The Jackiw-Rebbi mechanism yields four species of monopoles, and we assume that, for some choice of Yukawa couplings, one species is light and serves only to screen the repulsive interactions of another.

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1 Introduction

1.1 Motivation

Dwarf spheroidal (dSph) and low surface brightness (LSB) galaxies provide the purest known samples of dark matter in the Universe, and so provide a unique laboratory for studying its properties. The CMB indicates that the Universe was much more homogeneous before recombination than it is today, thus dark matter must be made of something which is able to clump together in a reasonably short amount of time (13 billion years). This has led to a wide acceptance that dark matter consists of particles which are so massive that they were already moving at nonrelativistic speeds during recombination, such particles are referred to as cold dark matter (CDM).

Simulations of structure formation in the presence of cold dark matter generally agree on two predictions. First, there should be hundreds if not tens of thousands of dwarf satellite galaxies in our local group (Klypin et al., 1999; Moore et al., 1999a) with masses of under $10^6 M_\odot$. Second, the density profiles of these galaxies should be qualitatively of the form proposed by Navarro, Frenk and White (Navarro et al., 1996), in particular they should have cusps at small radii $r$, diverging at least as $1/r$, with more recent studies suggesting $1/r^{3/2}$ (Moore et al., 1999b). As we now review, neither of these predictions agrees with observations.

Recently the number of known dwarf galaxies has been increasing quite rapidly, but various studies still place it between 4 and 400 times below the cold dark matter prediction in our local group (Mateo, 1998) and beyond (Strigari & Wechsler, 2011). On the contrary, galactic masses appear to have a lower bound. More specifically, the lightest satellite galaxies of the Milky Way all appear to have about $10^7 M_\odot$ within 300 pc of their center (Strigari et al., 2008) and between $10^7$ and $10^8 M_\odot$ within 600 pc (Strigari et al., 2007). Thus the light dark matter halos predicted by CDM simulations have not been observed.

This is not in itself a contradiction. It is a logical possibility that halos exist with all of the masses predicted by CDM, but those lighter than this minimum mass do not have deep enough gravitational potentials to have led to star formation, as a consequence of gas being lost to radiation during reionization (Couchman & Rees, 1986; Efstathiou, 1992), supernova feedback (Larson, 1974) or cosmic ray pressure (Wadepuhl & Springel, 2010), as has been supported for

1 Only two dwarf galaxies (Coma Berenices (Simon & Geha, 2007) and Segue I (Geha et al., 2009)) have been observed with masses compatible with $10^6 M_\odot$ or less. However, due to their close proximities to the Milky way, these dSphs have half-light radii of about 60 and 30 pc respectively (Belokurov et al., 2007). The measured values of the mass reflect only the mass inside of this radius, and so are very consistent with $10^7 M_\odot$ within 300 pc.
example by the Aquarius simulations in Font et al. (2011). The problem (Boylan-Kolchin et al., 2011a,b) is that the 6 halos simulated by the Aquarius project (Springel et al., 2008) as well as the halo simulated by Via Lactea II (Diemand et al., 2008) also each have at least 10 halos in the mass range between that of dSphs and irregular dwarfs, whereas no dwarfs in this mass range have been found in our local group. While it is plausible that the missing light satellites have not been seen because they simply lack stars, no explanation has been proposed for the missing heavy satellites.

In Navarro et al. (1996) it was already shown that density profiles of dwarf galaxies do not exhibit divergent cusps, on the contrary they are consistent with constant density cores. In the case of dSphs, a core with a reasonably constant density is required both by the long lifetimes of globular clusters embedded in dwarfs (Klevna et al., 2003; Goerdt et al., 2006; Gilmore et al., 2007) and also by analyses of chemically distinct stellar components (Walker & Penarrubia, 2011). Such a constant density core appears in the King model (King, 1966) of a steady state system, however dSph galaxies have not had time to settle into such a steady state (Gilmore et al., 2007) and also the model poorly captures the observed stellar kinematics (Wu, 2007).

Beyond the core, dark matter dominated galaxies exhibit an intermediate radius regime $r_1 < r < r_2$ with density $\rho \propto 1/r^2$. This can be seen clearly in the HI (the 21 cm 1S hyperfine transition) rotation curves for example for the 15 fairly large dwarf galaxies and LSBs presented in Swaters et al. (2003). The corresponding Hα ($n = 3$ to $n = 2$ transition) curves probe the central regions of these galaxies and are consistent with constant density cores. In Walker et al. (2009), stellar velocity dispersions in the intermediate regimes of 8 dSphs are plotted. These dispersions are radius-independent, and so the Jeans equation again supports a $1/r^2$ density profile. If the intermediate region extended to infinite radius, the halo mass would be linearly divergent and so the finite mass implies the existence of a third regime $r > r_2$ in which the density falls faster than $1/r^3$.

### 1.2 Monopoles

Summarizing, dark matter dominated galaxies, in contradiction with CDM simulations, have a minimum mass, a constant density core, an intermediate regime with $1/r^2$ density and an external regime in which the density falls faster. While particulate models of dark matter may fail to reproduce these facts, in this note we claim that they are reproduced by a new model of dark matter in which each galactic halo consists of a single, extended particle called a ’t Hooft-Polyakov monopole (’t Hooft, 1974; Polyakov, 1974). These particles are classical field theory
solutions. They are stable, and as a result of Dirac quantization they are automatically quantized, meaning that there is a lightest solution. We will identify the lightest solution with the dark matter halo of the lightest dSph. Therefore in our model, the missing (light) satellite problem is automatically solved by the fact that any lighter halo would violate the Dirac quantization condition, and so not correspond to any finite energy solution of our quantum field theory. We will review the fact that the density profiles of these solutions have just the same structure as those observed in dark matter halos.

Our model will not require any radically new physics. We just add to the standard model a new sector which is roughly the model of electroweak interactions proposed in Georgi & Glashow (1972). More precisely, we consider a new SU(2) gauge theory with an adjoint scalar field and two fundamental spinor fields. We consider a Higgs potential for the scalar field with quartic coupling larger than the square of the gauge coupling and so the resulting monopoles will be highly non-BPS.

The bosonic sector of the model, which is all that will be relevant for the smallest dSphs and for galactic cores, has only 3 free parameters: the gauge coupling, the tachyonic mass of the Higgs field and its quartic coupling. These three parameters need to reproduce the rotation curves of all dark matter dominated galaxies, and so the model is overconstrained and very falsifiable. Also, given these three parameters, one can determine when the monopoles formed via the Higgs mechanism and also whether the gauge interactions dominate over gravitational interactions inside of the halos. Consistency with CMB perturbations requires that the monopoles nucleated before the last scattering surface, and the relevance of the gauge theory solutions requires gauge interactions to dominate over gravity inside of the halo. We will see that the three parameters determined using galactic rotation curves fall within the reasonably narrow window that satisfies both of these conditions, and thus the model surprisingly passes two nontrivial consistency tests.

Our monopoles are far heavier than the upper bounds on MACHO masses excluded by gravitational lensing searches (Tisserand et al., 2007). Traditional MACHO models introduce inhomogeneities at distance scales of parsecs or less and so are ruled out by studies of wide binaries (Yoo et al., 2004), but our monopoles are so spatially extended and homogeneous at these scales that they survive these tests as well.

\footnote{We would like to emphasize that this SU(2) gauge group has nothing to do with the SU(2)_L of the electroweak theory of the standard model. It is a new gauge group and the new matter fields are neutral with respect to the standard model gauge symmetries. This dark sector interacts with the standard model gravitationally, and we leave open the possibility that standard model fields may be charged under the dark gauge symmetry.}

\footnote{We thank Malcolm Fairbairn for stressing this point.}
1.3 Screening

Despite this list of nice properties, there is one major problem with a model of galactic dark matter halos using non-BPS 't Hooft-Polyakov monopoles of various charges $Q$. The problem is that for charges $Q$ greater than 1, the monopoles are unstable, decaying into monopoles of charge 1. The reason is that the scalar field mediates an attractive force, but for all positively charged monopoles the $U(1)$ magnetic fields repel. As the scalar is massive and the photon is massless, the magnetic fields win at large distances and the galaxies repel, and even explode. This is certainly inconsistent with observations.

This is similar to a world made only of protons. Perhaps protons near enough to each other could form nuclei, due to the short distance attractions of the strong force (in this case the scalar field and gravity), but at large distances there would be a larger and larger electric field leading to repulsion and even Olber’s paradox. The solution to this problem is that the Universe has as many electrons as protons. The electrons screen the protons, as they have opposite charges. Yet the protons and electrons do not annihilate each other, as they possess distinct conserved charges. If the electrons are free, these configurations are plasmas or jellium. If the electrons are bound, the configurations are atoms. If the electrons are light enough, they may condense and the screening will resemble that of a superconductor. In any case, the magnetic repulsion between distant protons is screened, and perhaps the repulsion between nearby protons is tempered enough to allow the attractive forces to win.

How can this screening be realized in the case of magnetic monopoles? We need magnetic monopoles with different conserved charges, and opposite magnetic charges. We will consider the case in which the differing charges correspond to global symmetries, to minimize the new interactions that need to be introduced. There is only one known mechanism that allows magnetic monopoles to acquire global charges, the Jackiw-Rebbi mechanism (Jackiw & Rebbi, 1976). For each flavor of fundamental fermions in the theory, the magnetic monopoles acquire a fermion number of $\pm 1/2$ with respect to that flavor. Thus in the case of 2 flavors there will be 2 binary choices of charges, and so 4 possible charges corresponding to 4 kinds of monopoles and their antimonopoles. We will consider monopoles of one flavor and an equal number of antimonopoles of another, so that we have an equal number of positive and negative charges.

The fermion wave functions are stabilized by the Yukawa couplings. In [Jackiw & Rebbi](1976)

4A crucial difference is that in the case of our monopoles, the repulsion results solely from the field configuration far from the monopole, therefore a screening mechanism in this outer region may suppress the unwanted repulsion while leaving the inner and intermediate layers intact and well described by the bosonic sector of the field theory.
the authors considered Yukawa couplings of order the gauge coupling $g$. In this case the fermionic wave functions are subdominant by a factor of $g$, which we will see is extraordinary small in our models and so the fermionic contribution is irrelevant. The monopoles will therefore all have similar masses, and our monopoles and antimonopoles may annihilate leaving behind the much lighter fermions. Different species of galaxies which annihilate into particles have not been observed, and so this cannot be the right approach. Instead we will consider Yukawa couplings of $O(1)$, leading to fermionic wave functions of the same order as the bosonic wave functions. This opens the possibility that one of the fermionic flavors is light. Perhaps, as in the $r$-vacua of supersymmetric gauge theories (Seiberg & Witten, 1994; Carlino et al., 2000), this means that only monopoles of particular flavors condense, or at least only one becomes light. Once one of the flavors is light, like the electron, whether it condenses or not, it will not attract enough stars to yield a new flavor of galaxy and so it may provide a reasonable screening candidate. Hopefully in this case the interflavor monopole annihilation cross section is suppressed. Of course, the fermions will also yield an $O(1)$ correction to the heavy monopole wave function. Thus there will be systematic errors of $O(1)$ in this entire note. A next logical step would be to make a concrete choice of fermion couplings, adjusting them to make one monopole light, and to attempt to understand the resulting corrections on the second monopole. There will be further corrections to the profile, resulting from the screening itself, in the outer regions of the massive monopoles $r \gtrsim r_2$. Fortunately we will be able to learn a lot about our model from smaller radii where the effect of screening can be safely neglected, which is also the region in which the dark matter profile is most constrained by observations.

We begin in Sec. 2 with a summary of the key predictions of this model. In Sec. 3 we introduce the gauge theory and the monopole Ansatz. We will ignore the fermions entirely, considering only the massive monopole flavor. We will review a combination of analytic and numerical results on non-BPS charge $Q = 1$ monopoles, in particular displaying the density profiles in the 3 regions described above. In Sec. 4 we provide formulae for the parameters of the gauge theory in terms of observables of these lightest galaxies and estimate their numerical values. Next in Sec. 5 we provide a rough scaling for the $Q$-dependence of the monopole solutions, ignoring the intermonopole repulsion and screening. We check to see whether the parameters of the theory, already fixed in Sec. 4, are able to reproduce the rotation curves of higher $Q$ galaxies. In Sec. 6 the Newtonian gravitational potential is calculated and the single ($Q = 1$) monopole theory is also coupled to general relativity. In both cases gravitational effects are seen to be negligible. Indeed in this model the $1/r^2$ density profile results not from gravity, but from the topology of the Higgs field.
2 Predictions

Needless to say, our model suggests that direct and indirect WIMP searches will not find cosmologically significant quantities of dark matter. It also yields at least three astrophysical predictions. First, the core radius is proportional to the flat velocity dispersion and rotational velocities in the intermediate range, which we will see are both proportional to the square root of the charge $r_1 \propto \text{vel} \propto \sqrt{Q}$. Second, just as Dirac quantization solves the missing satellite problem by implying that no halos exist with masses between those of globular clusters ($Q = 0$) and the smallest dSphs ($Q = 1$), it also implies that the minimal velocity dispersion for a dSph is smaller than the second minimal value by a factor of $\sqrt{2}$. For example the 6 km/s average dispersion (Walker et al., 2009) in Sextans and Carina may correspond to $Q = 1$, and the 9 km/s of the Ursa Minor dwarf to $Q = 2$. An improvement in these measurements by a factor of 3 or 4 may already be enough to confirm or refute this prediction and therefore the model.

Finally, we will see that a consistent screening mechanism, in which galaxies only interact with each other gravitationally, requires $r_2$ to be reasonably independent of $Q$. This implies that dark matter halos of dSphs extend more than an order of magnitude beyond both their tidal radii and their half-light radii. Extending beyond their tidal radii is not problematic, as the halos are held together not by gravity but by the interactions in the new gauge sector. How would one detect such an extension of the halo radius? First of all, one would expect stars beyond the tidal radius to be stripped away more slowly, leaving more matter beyond the tidal radius than would otherwise be expected, agreeing with the observations of Hayashi et al. (2003). There have been numerous studies of the stars beyond the tidal radius of the Fornax dwarf, with some suggesting that they result from simple accretion (Battaglia et al., 2006) and some suggesting that it provides evidence for such a large dark matter halo (Walker et al., 2006). It would also affect the structure of objects created by tidal forces such as the Sagittarius dwarf’s large tail (Majewski et al., 2003) or the change in slope of stellar density distributions at the tidal radius measured in Komiyama et al. (2007). Effects of the halo size on the evolution of the stellar dispersions can be used to estimate the mass of the halo during the period in which it formed. Such an estimate, using the Jeans equation at a radius at which the effects of anisotropy are minimal, was provided in Walker et al. (2009); Wolf et al. (2010). They found a favored value of order $3 \times 10^9 \, M_\odot$ for the original masses of dSph halos, which fits the current halo mass that would be predicted in our model to within a factor of two.

If the intermediate regions of dSphs are indeed an order of magnitude larger than expected, then one would expect them to contribute an order of magnitude more to gravitational lensing
than expected, which may explain the results of Kochanek & Dalal (2004); Mao et al. (2004) which appear to find much more lensing caused by dwarfs than can be accounted for from using their known abundances and masses within their half-light radii.

While all dwarf galaxies that have been observed appear to have a minimum mass, our model predicts much more. All dark matter halos, even those not containing stars, must also contain this minimum mass, in sharp contradiction with CDM predictions. Furthermore, while large scale structure is known to form hierarchically from smaller components, this minimum halo mass must be respected during all times since the formations of the halos themselves, in other words this model would be falsified by a single lower mass halo even at high redshift. Gravitational lensing may be used to search both for halos which contain no stars and also for halos at high redshift, and thus provides a powerful tool for falsifying this model. In particular, the technique of Vegetti & Koopmans (2009) can be used to detect dwarf galaxies an order of magnitude lighter than the lightest observed, and at least two dwarfs have so far been observed using this technique, one at redshift $z = .22$ (Vegetti et al., 2010) and another at redshift $z = .81$ (Vegetti et al., 2012). The measurement of these masses depends strongly on the profile template used, and needless to say the profile used is quite different from the halo profile suggested here, however in both cases the mass measured within 600 pc of the core is above the minimum dSph mass observed in our local group, and so is consistent with the absolute minimum mass required by our model. A similar technique with an identical template was used to find one or two dwarf galaxies at $z = .46$ in Fadely & Keeton (2012). The second dwarf, whose existence is quite uncertain, has a mass which is compatible with that of the lightest dwarfs but is also compatible with a lower mass. Therefore a more precise measurement of this galaxy can potentially lead to a time-dependent minimum mass, which would falsify our model. If this technique could be applied to determine the size of the halo, and if this size exceeds the tidal radius, it would provide a smoking gun for our nongravitationally bound halo proposal.

3 The gauge theory and its monopole solutions

We will briefly review the ’t Hooft-Polyakov monopole solution in an SU(2) gauge theory with an adjoint matter field $\Phi$. We would like to stress again that this gauge group is new and is not the same as the SU(2)$_L$ gauge symmetry of the standard model. Furthermore, the fields of the model are neutral with respect to the standard model charges and so likely only interact with
the standard model particles gravitationally. The model is described by the Lagrangian density

\[ L^M = \text{Tr} \left[ -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} - D_\mu \Phi D^\mu \Phi \right] - \frac{\lambda}{4} \left( v^2 - 2 \text{Tr} [\Phi^2] \right)^2, \] (3.1)

where \( \Phi = \Phi^a t^a \) is an \( \mathfrak{su}(2) \) algebra-valued adjoint Higgs field, \( g \) is the gauge coupling, \( \lambda > 0 \) is the scalar coupling, \( D_\mu \Phi = \partial_\mu \Phi + i [A_\mu, \Phi] \) and finally \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu] \) is the Yang-Mills field strength tensor. We use the convention \( \text{Tr}[t^a t^b] = \delta^{ab}/2 \).

The (hedgehog) radial Ansatz for a single (charge \( Q = 1 \)) regular 't Hooft-Polyakov monopole reads

\[ \Phi = \frac{1}{r} v h(r) x^a t^a, \quad A_i = -\frac{1}{r^2} (1 - k(r)) \epsilon_{ija} x^j t^a, \] (3.3)

where \( r \) is the (spatial) radial coordinate. The functions \( h(r) \) and \( k(r) \) parametrize the Higgs field and gauge field respectively and must solve the equations of motion

\[ h'' + \frac{2}{gvr} h' = \frac{2}{(gv^r)^2} k^2 h - \frac{\lambda}{g^2} \left( 1 - h^2 \right) h, \] (3.4)

\[ k'' = \frac{1}{(gv^r)^2} \left( k^2 - 1 \right) k + h^2 k, \] (3.5)

where \( \prime \equiv \frac{d}{dr} \) is a rescaled radial derivative. The reason for writing the equations of motion in this form is that it is easy to identify the independent variables; i.e. \( 1/(gv) \) sets the overall length scale and \( \lambda/g^2 \) characterizes the non-BPS-ness of the system. \( \lambda = 0 \) is the BPS case and \( \lambda/g^2 \gg 1 \) is strongly non-BPS.

In a static configuration the energy density is

\[ \mathcal{H} = \frac{1}{g^2} \left( \frac{k'}{r} \right)^2 + \frac{1}{2g^2 r^4} (1 - k^2)^2 + \frac{1}{2} \left( vh' \right)^2 + \left( \frac{vkh}{r} \right)^2 + \frac{\lambda v^4}{4} \left( 1 - h^2 \right)^2. \] (3.6)

We are interested in configurations with a finite total energy. Finiteness at small and large radii implies the boundary conditions

\[ h(0) = 0, \quad k(0) = 1, \quad h(\infty) = 1, \quad k(\infty) = 0. \] (3.7)

The magnetic charge can be calculated using Gauss’ law for the unbroken U(1)

\[ Q = \frac{1}{2\pi v} \int_{\mathbb{R}^3} \text{Tr} [B_i D_i \Phi] = \int_0^\infty dr \frac{d}{dr} \left( h (1 - k^2) \right) = 1, \] (3.8)
with \( B_i \equiv \frac{1}{2} \epsilon_{ijk} F_{jk} \). The second equality follows from the hedgehog Ansatz (3.3), while the third equality is found using the boundary conditions (3.7). This charge is the topological monopole charge, which is valued in the second homotopy group of the gauge orbits of the space of Higgs vacua \( \pi_2(SU(2)/U(1)) \simeq \pi_2(S^2) = \mathbb{Z} \).

Perhaps the simplest and most studied case is that in which the coupling \( \lambda \) is sent to zero while the symmetry breaking is kept by fixing the VEV of \( \Phi \) equal to \( v \). The corresponding solution is the Bogomol'nyi-Prasad-Sommerfield (BPS) monopole (Bogomol'nyi, 1976; Prasad & Sommerfield, 1975) which satisfies the BPS equations

\[
\frac{1}{gv} h' = \frac{1}{(gvr)^2} (1 - k^2) , \quad \frac{1}{gv} k' = -hk .
\]

(3.9)

These equations can be integrated, yielding the analytic solution

\[
h = \coth(gvr) - \frac{1}{gvr} , \quad k = \frac{gvr}{\sinh(gvr)} ,
\]

(3.10)

which has a mass

\[
M_{\text{BPS}} = \frac{4\pi v}{g} .
\]

(3.11)

The BPS monopole has no intermediate region in which the density scales as \( 1/r^2 \), and so does not resemble a dark matter halo. We will therefore consider a monopole with \( \lambda/g^2 \gg 1 \). The monopole mass is a function of the parameter \( \lambda/g^2 \)

\[
M(\lambda/g^2) \simeq \frac{4\pi v}{g} f(\lambda/g^2) ,
\]

(3.12)

where \( f(\lambda/g^2) \in [1, 1.79] \) is a smooth and monotonically increasing function with the limiting values \( f(0) = 1 \) and \( f(\infty) = 1.79 \) as shown in Kirkman & Zachos (1981).

4 The monopole halo for the minimal dSph

4.1 The approximate density profile

We now identify the basic \( Q = 1 \) magnetic monopole described above with the dark matter halo of the lightest dwarf galaxies. We may express the parameters of the bosonic sector (3.1) of our theory in terms of observable properties of these galaxies. However we stress that at radii \( r \gtrsim r_2 \) the screening mechanism is necessarily important, and thus calculations of the behavior in this regime and consequently even of the value of \( r_2 \) itself are dependent on the specific model
describing this screening. As screening is not necessary for the stability of the \( Q = 1 \) monopole, we will ignore it in this section, simply recalling that the value of \( r_2 \) so derived cannot be trusted.

In order to parametrize the structure of the basic monopole, we will estimate the energy density profile. We will consider the following approximation

\[
h(r) = \begin{cases} \frac{r}{r_1}, & r \in [0, r_1], \\ 1, & r \in (r_1, \infty) \end{cases}, \quad k(r) = \begin{cases} 1, & r \in [0, r_2], \\ 0, & r \in (r_2, \infty) \end{cases}.
\] (4.1)

In particular we neglect the exponential tail of \( k(r) \) at \( r > r_2 \), which we expect to be dominated by the model-dependent screening.

Substituting this estimate into the energy density (3.6) and integrating over all space we find

\[
M = 4\pi v \left[ \frac{1}{2gv r_2} + gv \left( r_2 - \frac{1}{2} r_1 \right) + \frac{2\lambda}{105g^2 (gv r_1)^3} \right].
\] (4.2)

Now, if we minimize this expression with respect to \( r_{1,2} \) we obtain the following characteristic radii for the single monopole

\[
r_1 = \frac{1}{2v} \sqrt{\frac{35}{\lambda}}, \quad r_2 = \frac{1}{\sqrt{2g}v}.
\] (4.3)

Intuitively these two formulae express the fact that the scalar and \( W \) boson masses are of order \( v\sqrt{\lambda} \) and \( gv \) respectively, and \( r_1 \) and \( r_2 \) are their corresponding de Broglie wavelengths. When we estimate these quantities, we will see that these masses are extremely small, roughly \( 10^{-25} \) eV and \( 10^{-28} \) eV, respectively. In a subsequent publication we intend to study the compatibility of such light particles with cosmological bounds.

The ratio of the radii is

\[
\frac{r_2}{r_1} = \frac{2\lambda}{35g^2}.
\] (4.4)

This implies that the relative size of the intermediate region with respect to the core is proportional to \( \sqrt{\lambda/g^2} \). The fact that galactic rotation curves exhibit a large flat region then implies that \( \lambda/g^2 \) is large and so our monopoles are very non-BPS.

We can now find the scaling behavior of the density profile in the various regions using Eq. (4.1) and the energy density (3.6)

\[
H(r) = \begin{cases} \frac{3v^2}{2r_1^2} + \frac{\lambda v^4}{4} \left[ 1 - \left( \frac{r}{r_1} \right)^2 \right]^2, & r \in [0, r_1], \\ \frac{v^2}{r^2}, & r \in (r_1, r_2], \\ \frac{1}{2g^2 r_1^4}, & r \in (r_2, \infty). \end{cases}
\] (4.5)
Note that this approximate energy density is not continuous, which the real solution of course is. It is however quite clear from this small calculation that there are three regimes inside of the monopole: the core which has a quite high and roughly constant energy density, an intermediate regime where the energy density drops as $1/r^2$ and finally a tail where the energy density drops as $1/r^4$, although this last estimate is very sensitive to the screening.

Substituting the radii $r_{1,2}$ (4.3) into the mass (4.2), we find

$$M = \frac{4\pi v}{g} \left[ \sqrt{2} - \frac{1}{6} \sqrt{\frac{35}{\xi}} \right],$$

which corresponds to $f(\infty) \sim \sqrt{2} = 1.44$ - not that far from the precise numerical result $f(\infty) \sim 1.79$. We can also estimate the mass in each region of the monopole. In the core $r < r_1$

$$M_{r<r_1} = \frac{4\pi v}{g} \frac{1}{3} \sqrt{\frac{35}{\xi}},$$

while in the intermediate region $r_1 < r < r_2$ it is

$$M_{r_1<r<r_2} = \frac{4\pi v}{g} \left[ \frac{1}{\sqrt{2}} - \frac{1}{2} \sqrt{\frac{35}{\xi}} \right],$$

and finally for $r > r_2$, where we hope that the screening will dramatically alter the profile

$$M_{r>r_2} = \frac{4\pi v}{g} \frac{1}{\sqrt{2}}.$$

In Fig. 1 we compare the energy density of this estimate with that of a numerical solution to the equations of motion of the $Q = 1$ monopole with $\lambda/g^2 = 5 \times 10^6$.

### 4.2 Fitting the two scalar parameters of the model

The two parameters of the scalar sector of the model, $v$ and $\lambda$, can be determined in terms of observable features of the minimal dwarf at radii much smaller than $r_2$, so that screening may be neglected and this determination will be independent of the particular screening model. The gauge sector is not relevant in the core of a dark matter halo, as all components of the gauge field are so light that their de Broglie wavelength is much bigger than the core.

More precisely, the core density (3.6) is dominated by the Higgs potential and the contribution from the covariant derivative (see Eq. (4.5))

$$\rho_{r<r_1} = \frac{699}{2240} \lambda v^4 \sim \frac{1}{3} \lambda v^4,$$

(4.10)
Fig. 1: The energy density of the $Q = 1$ monopole solution with $\lambda/g^2 = 5 \times 10^6$ which corresponds to $f \sim 1.74$. Here we have taken $r_1 = 60$ pc and $r_2 = 30$ kpc.

where we have approximated the mean value of the core density by setting $r = r_1/2$ in Eq. (4.5).

The core radius $r_1$ is given by Eq. (4.3). These two equations can easily be solved to yield the scalar sector parameters in terms of observables

$$v \simeq 0.61 \times r_1 \sqrt{\rho_{r<r_1}}, \quad \lambda \simeq \frac{23.9}{r_1^4 \rho_{r<r_1}} . \quad (4.11)$$

As a check, which is more or less guaranteed to work by dimensional analysis, one can use the Virial theorem in the intermediate region to verify that the stellar dispersion velocity $u$ is correctly reproduced by these parameters

$$v \sqrt{2\pi G_N} = u\sqrt{3} , \quad (4.12)$$

where $G_N$ is Newton’s constant.

Segue I is the darkest known dSph (Geha et al., 2009) and the only galaxy whose half-light radius is much less than $r_1$. This means that the corresponding velocity dispersion can be used to estimate its halo density $\rho$ at $r < r_1$. The dispersion has been calculated in Simon et al. (2011) to within about a factor of 2. We will estimate $\rho_{r<r_1}$ to be

$$\rho_{r<r_1} = 30 \text{ GeV/cm}^3 , \quad (4.13)$$

which is compatible with both the measurements of Segue I and the dSphs in Gilmore et al. (2007). Furthermore we will estimate

$$r_1 = 60 \text{ pc} , \quad (4.14)$$
which is compatible with Gilmore et al. (2007) and the curves of Walker et al. (2009). Using these estimates we find the scalar parameters

\[ v = 9 \times 10^{13} \text{ GeV}, \quad \lambda = 10^{-95}. \]  

(4.15)

It is perhaps remarkable that these galactic scale inputs, \( r_1 \) and \( \rho_{r<r_1} \), conspire so as to give a particle physics scale output. Indeed \( v \) is about the GUT scale. In the intermediate region, if we estimate the smallest dSph stellar dispersion relation to be

\[ u = 6 \text{ km/s}, \]  

(4.16)

based on, for example, Walker et al. (2009) then it can be seen that the check Eq. (4.12) is satisfied to within a factor of 2.

### 4.3 Estimating \( v \) using dSph masses within 300 pc

Now that we have fixed both parameters of the scalar sector of the theory, we can attempt to rederive them from different data, as a consistency check. In this subsection we will rederive the scalar \( v \) using the observation in Strigari et al. (2008) that the smallest dwarf galaxies contain \( 10^7 \) solar masses within their innermost 300 pc.

This derivation is not entirely independent of the previous derivation, as the Virial theorem already relates the density and stellar velocity dispersion. However it has the advantage that it does not require an estimate of \( r_1 \), which is difficult to determine as the solutions of the Jeans equation for the core velocities are degenerate if a deviation from spherical symmetry is allowed (Gilmore et al., 2007). We will use the fact, from the velocity curves, that 300 pc is much larger than \( r_1 \), however this will not greatly affect our result. Using the scaling relations derived in Sec. 5, the fact that the masses vary by about a factor of three suggests that the charge \( Q \) of these galaxies varies from \( Q = 1 \) to \( Q = 3 \). If we imagine that the mass \( 10^7 \, M_\odot \) corresponds to \( Q = 2 \), then the corresponding mass for \( Q = 1 \) halos will be about \( 5 \times 10^6 \, M_\odot \).

First, note that at distances beyond \( r_1 \) the mass per radius is reasonably constant. This region yields the largest contribution to the mass. Thus if \( M(r) \) is the mass out to a distance \( r \), we will make the approximation

\[ \frac{dM(r)}{dr} \sim \frac{M(r)}{r}, \]  

(4.17)

in this region. Using the figures quoted above, we then find

\[ \frac{dM(r)}{dr} \sim \frac{5 \times 10^6 \, M_\odot}{300 \, \text{pc}} = 10^{29} \text{ GeV}^2. \]  

(4.18)
In this regime the gauge field is negligible and the scalar is at its minimum. The vast majority of the energy comes from the winding of the scalar field, which contributes to the kinetic energy. For a $Q = 1$ minimal dwarf galaxy, it winds once around the 2-sphere. In each winding the VEV moves a distance $2\pi v$, over a physical distance $2\pi r$. Therefore the norm of the derivative of $\Phi$ is simply $v/r$, yielding a kinetic energy density of

$$\frac{dM(r)}{dr} \sim 4\pi r^2 \left| \frac{\partial \Phi}{\partial x} \right|^2 = 4\pi v^2.$$  \hspace{1cm} (4.19)

Combining Eqs. (4.18) and (4.19) we may now determine the VEV $v$

$$v \sim 10^{14} \text{GeV},$$  \hspace{1cm} (4.20)

again yielding a VEV which is roughly the GUT scale, in agreement with the value quoted in Eq. (4.15).

4.4 Estimating $r_2$ and $g$

Estimating the gauge field $g$ is much more difficult, as the gauge field only becomes relevant at distances of order $r_2$, where the screening mechanism may also be relevant. We will estimate it as follows. At large distances observations dictate that galaxies interact largely gravitationally. Therefore, however the screening works, it must be subdominant to gravity at large distances. This implies that the energy of a galaxy must be more or less linear with respect to its charge, so that not too much binding energy or repulsive energy exists. The energy density in the intermediate region is proportional to $1/r^2$, therefore integrating over the angular directions it is constant. This means that the total energy of the intermediate region, which dominates over the core energy, is proportional to $r_2$ times the density. We will argue below using a simple topological argument that the density is proportional to $Q$. The total energy is then proportional to the product of $r_2$ and $Q$ which, in order to eliminate long range gauge interactions between galaxies, must be proportional to $Q$. This means that $r_2$ must be roughly $Q$ independent.

This is one of the most robust and surprising predictions of our model, and may well be key to its eventual falsification. It implies that the dark matter halos of dwarf galaxies have the same radii as dark matter halos of LSBs, that there is a universal halo radius for dark matter dominated galaxies. Thus dSph dark matter halos extend an order of magnitude beyond their half-light and tidal radii.

So just how big is $r_2$? Since there are so few stars at these distances, it is difficult to estimate. In the case of much larger galaxies, gravitational lensing gives some estimate of the total mass
which can be used to more or less determine the extent of the dark matter halo. Such estimates are hard to come by for dark matter dominated galaxies, but for the sake of concreteness one may take an order of magnitude estimate

\[ r_2 = 30 \text{ kpc}. \]  

(4.21)

Then Eq. (4.3) yields

\[ g = 2 \times 10^{-51}. \]  

(4.22)

4.5 The temperature at which the symmetry breaks

The gauge symmetry breaks at approximately the temperature equal to the VEV of the Higgs field, \( T \approx v \approx 10^{14} \text{ GeV} \). This is because when the temperature gives an effective mass to the Higgs of this order, the symmetry is restored.

In a future publication we will attempt to study the time dependence of the temperature of this model, assuming that it was initially in equilibrium, due perhaps to gravitational interactions, with the standard model fields. For now we simply note that the fact that the energy scale is so much higher than an eV implies that the symmetry broke long before the recombination, and so in time to help amplify CMB fluctuations. The monopoles themselves could not yet form when the gauge symmetry broke because their radius is larger than the Hubble radius at the time. The Hubble radius only became larger than \( r_2 \) shortly before the last scattering surface, and so it is unlikely that fully formed monopoles could have yet existed. However it surpassed \( r_1 \) much earlier, thus in these models one could have expected the gauge symmetry breaking to have yielded some structure in the primordial plasma. Thus in our model it may be important that the size of galactic cores today is smaller than the Hubble radius of the last scattering surface and crucially also smaller than the Hubble radius at the epoch of matter-radiation equality.

CMB fluctuations reliably describe the primordial plasma at multipole numbers up to about \( l = 1,500 \) corresponding to fluctuations which were about 5 kpc across at the time of recombination. Smaller sized features affect larger multipole numbers where they are drowned out by silk damping and the integrated Sachs-Wolfe effect. As this size is much larger than \( r_1 \), it is possible that many monopole nuclei, up to a million, may have lied inside of each such region. As the CMB power spectrum is not sensitive to small enough scales (\( l \gg 1,500 \)) to detect the internal structures of these monopoles, and as they indeed have a negligible speed of sound due to their high mass, it is likely that the monopole contribution to CMB fluctuations is indistinguishable.
from that of CDM particles. Furthermore, as they may form when the Universe is only of order 100 years old, and as they move very slowly, their distance from the baryon perturbations at recombination would be more or less equal to that of CDM and thus one could expect the same successful production of the 150 Mpc scale of large scale structure from baryon acoustic oscillations.

Note that while the various uncertainties in the parameters are only of about 1 or 2 orders of magnitude, the model satisfies several reasonably tight constraints. We have remarked that it is important that the Higgs VEV is much greater than 1 eV in order to help amplify perturbations in the primordial plasma. Also it is essential that the VEV be less than the Planck scale, because otherwise gravitational interactions would dominate over gauge interactions and so destroy the density profiles of these monopoles, leaving instead a hairy black hole (Lee et al., 1992a,b; Lue & Weinberg, 1999; Breitenlohner et al., 1992, 1995). It is remarkable that the value of $v$ determined just by the dimensions of galaxies happens to lie in this apparently unrelated window.

5 Large $Q$

So far we have considered only the basic monopole $Q = 1$, which corresponds in our model to the dark matter halo of the smallest allowed dSphs. The analysis of monopoles at higher $Q$ is complicated by the fact that, without the fermionic sector, they are unstable. We will begin in Subsec. 5.1 by providing scaling arguments which describe these unstable solutions in the absence of screening. We will see that the cores of these solutions are stable, and so it is not necessary that the stabilizing screening mechanism affect the solutions at $r \ll r_2$. However, without screening, we find that $r_2 \propto \sqrt{Q}$ whereas stability implies that $M \propto Q$ which implies that $r_2$ is reasonably $Q$ independent. Therefore we conclude that the screening shifts the location of the outer radius $r_2$ dramatically. This in turn implies that the energy density beyond $r_2$ is proportional not to $Q^2$ as in the unscreened case, but to $Q$. Thus the screening cancels the effective charge $Q$ except for a residual charge of order $\sqrt{Q}$, as might be expected for example from a Gaussian distribution of screening antimonopoles.

5.1 Scaling arguments

We will assume spherical symmetry in the following argument and also that the field strength tensor scales proportionally to $Q$. Furthermore, we will consider the case in which $Q$ is large and
\(\lambda/g^2\) is large but finite. It is known that no monopoles of charge \(Q > 1\) are truly spherically symmetric (Weinberg & Guth, 1976), for example monopoles of charge 2 have only axial symmetry (Ward, 1981). However, we expect that spherical symmetry will be recovered in the limit \(Q \to \infty\). This is reasonable in light of the restoration of spherical symmetry in the BPS case seen in Manton (2011); Evslin & Gudnason (2011).

Relying on the spherical symmetry we can now assume that the scalar profile function will scale as \(h \sim r^{\eta(Q) \to 0}\), in the region \(r < r_1\) as \(Q \to \infty\), where \(\eta(Q)\) is a monotonically increasing function (Bolognesi, 2006). As the magnetic charge goes to infinity, we can neglect the transitional regime near \(r_1\) where \(h\) passes from 0 to 1. All that matters is the size of the different regions. We do however need to take the angular derivatives into consideration in this argument, because the norm of the Higgs VEV will be \(v\) in the region from \(r_1 < r < r_2\) and its direction will wind \(Q\) times around the vacuum manifold \(S^2\). This winding will of course be canceled by the gauge field at distances larger than \(r > r_2\). To a good approximation these angular derivatives contribute \(Q(vkh/r)^2\) to the kinetic energy. As will be explained in Subsec. 5.2, this energy density is proportional to \(Q\), instead of \((Q^2 + 1)\) as in the axial symmetric case, as a result of the approximate spherical symmetry at large \(Q\). Neglecting the radial derivatives we arrive at the following crude estimate for the energy density

\[
\mathcal{H} = \frac{Q^2}{2g^2r^4} (1 - k^2)^2 + Q \left(\frac{vkh}{r}\right)^2 + \frac{\lambda v^4}{4} (1 - h^2)^2. \tag{5.1}
\]

We will estimate the functions \(h, k\) to be

\[
h(r) = \begin{cases} 
0, & r \in [0, r_1], \\
1, & r \in (r_1, \infty), 
\end{cases} \quad k(r) = \begin{cases} 
1, & r \in [0, r_2], \\
0, & r \in (r_2, \infty). 
\end{cases} \tag{5.2}
\]

Substituting the above estimates into the energy density (5.1) gives us

\[
M = \frac{4\pi v}{g} \left[ \frac{Q^2}{2gvr_2} + Qgv(r_2 - r_1) + \frac{\lambda gv^3}{12} r_1^3 \right]. \tag{5.3}
\]

Minimization with respect to \(r_1, r_2\) then yields

\[
r_1 = \frac{2}{v} \sqrt{\frac{Q}{\lambda}}, \quad r_2 = \frac{1}{gv} \sqrt{\frac{Q}{2}}, \tag{5.4}
\]

which we reinsert into Eq. (5.3) to obtain

\[
M = \frac{4\pi v}{g} \left[ \sqrt{2} - \frac{4g}{3\sqrt{\lambda}} \right] Q^{\frac{3}{2}}. \tag{5.5}
\]

Note that, as the exponent of the magnetic charge is \(3/2 > 1\), the monopoles will repel each other and the bound states will be unstable in the absence of screening and gravitational attraction.
As described in Subsec. 4.4, if screening leads to a $Q$-independent value of $r_2$ then the mass will be proportional to $Q$, and so the long distance gauge interactions of halos will be negligible as desired. Let us now summarize our estimate of the energy density profile

$$H(r) = \begin{cases} \frac{\lambda v^4}{4}, & r \in [0, r_1], \\ \frac{Qv^2}{r}, & r \in (r_1, r_2], \\ \frac{Q^2}{2g^2r^3}, & r \in (r_2, \infty). \end{cases}$$

(5.6)

Again this approximate energy density is not continuous, but it captures the scaling in the core and intermediate regimes, although stability implies that the scaling in the outer regime will need to be dramatically altered by the screening.

While an instability resulting from an energy surplus in the outer regions of the solution is worrying, implying the necessity of a rather arbitrary screening mechanism, an energy surplus in the core would be fatal. If the core of the solution is itself unstable, then no additional physics could stabilize these solutions without qualitatively changing them everywhere.

In order to make a crude estimate of the stability of the core, we use the energy density (5.1) to calculate a mass up to a certain distance $r' > r_1$

$$M(r') = \frac{4\pi v}{g} \left[ Qr' - \frac{4gQ^2}{3\sqrt{\lambda}} \right].$$

(5.7)

The last term in this mass formula can be interpreted as a binding energy. As this formula is only valid for $r > r_1$, the mass is always strictly positive.

Recall that we have assumed that the screening mechanism imposes that $r_2$ be $Q$ independent. Then we can determine whether the interior is stable by simply taking the second derivative with respect to the charge $Q$ of the mass at $r < r_2$

$$\frac{d^2M(r')}{dQ^2} = -\frac{4\pi v}{g} \frac{g}{\sqrt{\lambda}Q}.$$

(5.8)

The negativity of this second derivative implies that the core of the monopole is stable and in fact even slightly bound. The instability found above comes instead from the outer regions, where we postulate that it will be remedied by screening.

It is clear from Eq. (5.7) that our halo profiles are at best reliable up to some maximum value of $Q$. Consider a mass scaling corrected by a screening mechanism

$$M = \frac{4\pi v}{g} \left[ \sqrt{2} - \frac{4g}{3\sqrt{\lambda}} \sqrt{Q} \right] Q,$$

(5.9)

$^5$It would be interesting to determine whether this binding has observable consequences for galactic mergers and elliptical galaxies.
which is monotonically increasing up to the maximum charge

\[ Q_{\text{max}} \sim \frac{\lambda}{2g^2} \sim 2 \times 10^6. \]  

(5.10)

This charge corresponds to a mass of order \(10^{15} M_\odot\), greater than that of any known galaxy and much greater than that of any dark matter dominated galaxy.

### 5.2 A consistency check using larger dwarfs

So far we have essentially used three pieces of kinetic information, the sizes \(r_1\) and \(r_2\) together with the core density, to determine 3 unknowns: \(g\), \(\lambda\) and \(v\). All of the other data that we used could be determined from the continuity of the density function together with the \(1/r^2\) density scaling in the intermediate region. Thus, while our model nontrivially gave the correct scalings and satisfied some necessary inequalities, the kinetic data itself consisted of the same number of data points and unknowns and so the existence of a solution was reasonably trivial.

However, now that we have determined all three parameters in the bosonic sector of our model, which we have argued is all that is relevant at \(r < r_2\), any new kinetic data will provide a nontrivial check of our model. In this subsection we will consider the intermediate region of large dwarfs, with \(Q \gg 1\). First we will derive the scaling of the energy density in this region with respect to \(Q\). Then we will use the data from Navarro et al. (1996) to determine the stellar velocity rotation curves and radii \(r_1\) of such a dwarf galaxy. The velocity dispersion will be used to determine \(Q\). Then we can use the fact that \(r_1 \propto \sqrt{Q}\) and the value of \(r_1\) for a minimal dwarf to compare the value of \(r_1\) predicted by our model with the measured value.

\(Q\) is easily determined from the rotational velocity by generalizing Eq. (4.19). For large \(Q\), 't Hooft-Polyakov monopoles are approximately spherically symmetric. The Higgs field winds \(Q\) times around the color \(S^2 = SU(2)/U(1)\), and so the determinant of its derivative matrix is equal to \(Q\) times that of the \(Q = 1\) monopole. The derivative matrix is \(2 \times 2\), therefore isotropy implies that each derivative in an angular direction is enhanced by a factor of \(\sqrt{Q}\). Thus the kinetic term and so the mass is proportional to \(Q\). Using the Newtonian formula

\[ \frac{G_N M(r)}{r^2} = \frac{u^2}{r}, \]  

(5.11)

where \(G_N\) is Newton’s constant, the fact that the left hand side is proportional to \(Q\) implies that the rotational velocity \(u\) is proportional \(\sqrt{Q}\). In Subsec. 5.1 we have already used this result to conclude that \(r_1 \propto \sqrt{Q}\). Therefore we arrive at a prediction of our model: the core radius \(r_1\) is proportional to the stellar rotational velocity in the intermediate region.
For concreteness, we consider the rotation curve of the dwarf galaxy DD0168 in Navarro et al. (1996), although that of the other dwarf galaxies in this reference would give a similar result. In the intermediate region $r_1 < r < r_2$ the stars in this galaxy rotate at about 60 km/s, and the inner radius is about

$$r_1 \sim 600 \text{ pc} .$$

(5.12)

In the case of a $Q = 1$ galaxy, we have claimed that Eq. (5.11) would give a velocity equal to the dispersion velocity of the smallest dSph, about 6 km/s. Since DD0168 rotates 10 times faster, its dark matter halo consists of a monopole with charge $Q \sim 100$. As $r_1 \sim \sqrt{Q}$, this implies that the radius $r_1$ of the core of DD0168 should be about 10 times larger than that of a minimal dwarf galaxy $Q = 1$. Above we have very roughly estimated that such dwarfs have a core radius of 60 pc, therefore we conclude that the core radius of DD0168 is 600 pc, in agreement with the measured value (5.12). Note that had our model predicted that $r_1$ scales as another power of $Q$, for example were it $Q$ independent or proportional to $Q$, then there would have been an order of magnitude discrepancy in $r_1$. Thus our model not only correctly produces the density scaling as a function of radius, but also the $Q$ dependence of the structure and density passes a nontrivial check.

One may be tempted to push this relation yet further, using the Milky Way. The core of the Milky Way is not dark matter dominated, and so a degree of caution is needed. The rotational velocity is about 220 km/s, suggesting $Q = 1,350$. Therefore one expects that $r_1 = 2$ kpc. This is difficult to check, the core of our Milky way is thought to be an elliptical bar. 2 kpc indeed may well be between lengths of the semi-minor and semi-major axes, and so again this is consistent. Were $r_1$ $Q$-independent, the resulting core radius of 60 pc would be strongly excluded, as would the 80 kpc result were $r_1$ proportional to $Q$. Thus the proportionality of $r_1$ and the stellar rotational or dispersion velocity works quite well over this large range of values of $Q$, whereas any other integral exponent is very strongly excluded.

## 6 Gravitating monopole

't Hooft-Polyakov monopoles have been extensively studied in the Einstein-Yang Mills-Higgs theory (Bais & Russell, 1975; Cho & Freund, 1975; Van Nieuwenhuizen et al., 1976). It was found that, when the order parameter $v$ exceeds a scale roughly equal to the Planck energy, the monopoles become hairy black holes (Lee et al., 1992a,b; Lue & Weinberg, 1999; Breitenlohner et al., 1992, 1993). The exact threshold and the direction of the intermonopole force depend on the
ratio $\lambda/g^2$ (Hartmann et al., 2001, 2002) and the charge $Q$ (Bolognesi, 2010). We have seen that $v$ is several orders of magnitude below the Planck scale, and thus we may expect magnetic interactions to dominate over gravitational interactions, except for intermonopole interactions in which the former are screened.

Thus, at least for $r < r_2$, we do not expect gravitational corrections to our profiles to be significant. As a check, in the $Q = 1$ case, we have used our nongravitational solutions to create a classical Newtonian gravitational potential

$$V(r') = -G_N \int_{\mathbb{R}^3} \frac{\mathcal{H}(r)}{|r' - r|} d^3r,$$

where $G_N = 6.7 \times 10^{-39}$ GeV$^{-2}$. Since $\mathcal{H}(r)$ contains a factor of $v^4$, it is clearly important that the factor $G_N v^2 \ll 1$ or equivalently that $v \ll M_{\text{Planck}}$. Thus the potential will not affect our flat space solution significantly.

As a more rigid check of the influence of the gravity on our monopole solution, we will consider the coupling of general relativity with our gauge theory. The Lagrangian is (Van Nieuwenhuizen et al., 1976)

$$\mathcal{L} = \mathcal{L}^E + \mathcal{L}^M,$$

$$\mathcal{L}^E = -\frac{1}{16\pi G_N} \sqrt{-g} R,$$

$$\mathcal{L}^M = -\sqrt{-g} \operatorname{Tr} \left[ \frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi D^{\mu} \Phi \right] - \frac{\lambda}{16\pi} \sqrt{-g} \left( v^2 - 2 \operatorname{Tr} [\Phi]^2 \right),$$

where $g$ is the determinant of the metric while the Lorentz indices are raised and lowered with the metric. For the spherical case $Q = 1$, we can assume that the metric is also spherically symmetric and hence choose

$$g_{\mu\nu} = \text{diag} \left( -e^\alpha, e^\beta, r^2, r^2 \sin^2 \theta \right),$$

with $\mu, \nu = t, r, \theta, \phi$. We will use the same Ansatz for the gauge field and change the variables of the metric to the following gravitational variables $x \equiv (\alpha - \beta)/2$ and $y \equiv (\alpha + \beta)/2$.

In terms of the dimensionless radial coordinate $\eta \equiv gvr$, the equations of motion read

$$y' = \Lambda \eta U_1,$$

$$[\eta (e^y - e^x)]' = \Lambda \eta^2 e^y (U_1 + U_2),$$

$$(k'e^x)' = e^y \left( \frac{1}{\eta^2} (k^2 - 1) k + h^2 k \right),$$

$$(\eta^2 h'e^x)' = e^y \left( 2k^2 h - \frac{\lambda \eta^2 (1 - h^2) h}{g^2} \right),$$

21
where \( \prime \) is the derivative with respect to \( \eta \) and we have defined \( \Lambda \equiv 8\pi G_N v^2 \) as well as the following two functionals

\[
U_1 \equiv \left( \frac{k'}{\eta} \right)^2 + \frac{1}{2} (h')^2,
\]

\[
U_2 \equiv \frac{1}{2\eta^4} (1 - k^2)^2 + \left( \frac{k h}{\eta} \right)^2 + \frac{\lambda}{4g^2} (1 - h^2)^2.
\]

The asymptotic boundary conditions are

\[
h(\infty) = v, \quad k(\infty) = 0, \quad y(\infty) = 0, \quad x(\infty) = 0,
\]

while at \( r \to 0 \) they are

\[
\eta (e^y - e^x) = 0, \quad k' e^x = 0, \quad \eta^2 h' e^x = 0.
\]

The energy density including gravity is

\[
\mathcal{H} = g^2 v^4 e^y [U_1 + U_2].
\]

Glancing at the differential equations (6.6)-(6.7) it is observed that the factor \( \Lambda \sim 3.3 \times 10^{-9} \) and hence the value of \( y \) will always be within the numerical error of 0. Hence we can already conclude that gravity does not affect the shape of our monopole solution. A numerical solution leads to the same conclusion.

Of course a more detailed analysis of the core at various values of \( Q \) may be interesting, in case one may find that general relativistic corrections indeed lead to supermassive black hole (SMBH) formation. It may well be that in this model SMBHs are formed primarily not by accreting stars, but rather are an integral part of the stationary solution for the dark sector. This would mean that they are formed by dark forces, which as \( v \) is smaller than the Planck scale are much stronger than gravity in this regime. This could explain how it is that SMBHs have already grown to be as large as they are, which is quite a challenge in CDM dark matter models. It may also help to explain why the sizes of galactic center black holes obey so many universal relations to other galactic characteristics, such as the bulge mass.

7 Discussion and outlook

We have proposed a new model of dark matter halos as galactic-scale quantized solitons. The idea that Dirac quantization yields very large minimal dark matter profiles of course is quite old, appearing in the earliest cosmic string literature (Kibble, 1976) and in more modern proposals.
for dark matter halos (Lee & Lim, 2010; Mielczarek et al., 2010) as classical solutions of a scalar field (Sin, 1994; Matos & Guzman, 2000; Robles & Mateos, 2012; Magana et al., 2012). If the scalar is in thermal equilibrium, as may be expected due to the similarity of galactic rotation curves, then a fit to the parameters of these rotation curves yields a dark matter scattering cross section which is higher than the limits placed by the bullet cluster (Randall et al., 2008) and so in general these models are ruled out (Slepian & Goodman, 2011). One might already suspect that due to our extremely low couplings $\lambda$ (4.15) and $g$ (4.22) our model has sufficiently little self-interaction in order to satisfy such bounds. Indeed, the estimate in Slepian & Goodman (2011) suggests that it is sufficient for our fields to have masses of less than about $10^{-5}$ eV. Our dark sector masses are about equal to the inverse radii $1/r_1$ and $1/r_2$, and so are about 20 orders of magnitude within this bound.

Our model contains two ingredients not present in interacting scalar models. First of all, the $1/r^2$ density dependence of the intermediate region results from the winding of the vacuum expectation value $v$ around the vacuum manifold $SU(2)/U(1) = S^2$. This winding is already present in a purely scalar field configuration called the global monopole, which was postulated as a dark matter candidate in Nucamendi et al. (2000). However in such cases the total mass of a monopole is generally divergent. We cure this divergence by introducing new gauge fields which carry dark forces, inspired by the dark force model of Arkani-Hamed et al. (2009). Dark forces have been applied to the cusp problem in Loeb & Wiener (2011); Vogelsberger et al. (2011) and a model with a similar particle content to ours has been used to generate the observed baryon asymmetry in Agashe et al. (2010); Walker (2012). This leads one to wonder whether an extension of our model may also contribute to baryon asymmetry.

In this note we propose that the Dirac quantization of monopoles may explain the apparent minimum mass of dark matter halos. Indeed, large globular clusters and small galaxies often have equivalent stellar content, but there is a large gap between the two sets of solutions (Gilmore et al., 2007), and we propose that this gap is the result of Dirac quantization of a winding number of a new $S^2$-valued condensate about the galaxy.

These solutions unfortunately repel, and so we are forced to add a new species of monopole, which we hope screens this repulsion at large distances. Clearly this hope, that the $O(1)$ Yukawa couplings render one monopole light and reduce the monopole-antimonopole annihilation cross-section of monopoles of different species while not qualitatively affecting the heavy monopole, is

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$^6$Our masses are below the lower mass limit which Slepian & Goodman (2011) claim is necessary to reproduce the intermediate range density profile. This is consistent because our intermediate range density is determined not by collisions, but by the nontrivial topology of the scalar condensate.
the weakest link of the proposal. While it is difficult to verify using semiclassical methods, it is
easy to falsify, and this will be the subject of a future publication.

The claim leads to a lot of unexpected and pleasant features. ’t Hooft-Polyakov monopoles,
unlike cold dark matter, lead to cored density profiles of galaxies, which seem to be favored
observationally. The parameters of this model can be determined from observations of galaxies,
and despite the galactic inputs, the outputs are of the correct scales demanded by particle physics.
In particular, the symmetry breaking temperature is of order of the VEV of the dark Higgs field
which is about $10^{14}$ GeV, i.e. near the GUT scale. As it is lower than the Planck scale, the
gauge theory and scalar interactions dominate over gravity, allowing the monopole profile to
survive gravitational corrections. Yet it is close enough to the Planck scale that with just a bit
of screening, just outside of the Milky Way, the magnetic forces do not cause the Milky Way
to repel its satellites and neighbors, and so this construction is consistent with the existence
of the local group and with clusters of galaxies. It would be interesting to compare results on
monopole scattering, albeit with significant and difficult to quantify corrections from screening,
with observations from collisions of clusters such as the Bullet cluster and MACS J0025.4-1222.
Another pleasant feature is the $1/r^2$ fall-off of the density in an intermediate region, reproducing
the familiar result that stars at intermediate distances tend to have constant rotational speeds,
or in the case of dSphs constant velocity dispersions.

Despite all of these pleasant features, the monopole model of galactic dark matter halos is
easily falsifiable and probably can be falsified with the data currently available. As long distance
interactions continue to be gravity dominated and these monopoles are much smaller than the
basic units used in structure formation simulations, this model does not obviously lead to any
new predictions for large scale structure. However, it may be possible to predict the abundancy
of galaxies from a simple model of symmetry breaking along the lines of that in Kibble (1976)
and compare it with the actual abundance. Of course, such an effort will be hampered by the
fact that the agglomeration of galaxies into larger galaxies depends heavily on the screening
mechanism. In addition the very light fields introduced in this model, despite their equally weak
couplings, may lead to any number of cosmological problems.

Of course a model of dark matter must do more than reproduce the halo profiles of dark
matter dominated galaxies, it must for example also reproduce those of spiral galaxies. While
the pure dark matter halos that we have found contain central core densities which are essentially
independent of the size of the halo, the core density is very sensitive to the baryon density. In
fact, the inclusion of baryons can already be seen to dramatically affect the core density in the
isothermal fits of dwarfs and LSBs in Swaters et al. (2003). In spiral galaxies, whose cores are
baryon dominated, one therefore expects the core densities to diverge dramatically from the pure dark matter value presented here. For example in [Donato et al. (2009); Salucci et al. (2011)] it is claimed that the central density is inversely proportional to the core radius. Thus the dark matter galaxies of the cores of larger galaxies are appreciably less dense than the pure dark matter cores. While in principle it is possible that this reduction in density results from the outward gravitational pull of baryons that are, for example, ejected from supernova, it seems quite plausible that it implies that the dark sector also interacts nongravitationally with the standard model particles. It remains to be seen whether such interactions may be strong enough to explain the reduced core density and yet weak enough to satisfy the Bullet cluster bounds of [Randall et al. (2008)].

One very falsifiable prediction of this proposal is the discrete galaxy spectrum. The main strength of the proposal is the distinction between charges $Q = 0$ and $Q = 1$, corresponding to globular clusters and minimal dwarf galaxies. $Q = 2$ is an interesting case. Monopole solutions with spherical symmetry do not exist when $Q > 1$, and when $Q = 2$ they are quite elliptical. This leads to an energy which is higher than it would be in the spherical case, a naive estimate using the scaling arguments presented above yields an energy in the intermediate regions which is about $\sqrt{5}/2$ times higher than that of two separated $Q = 1$ cores, and so may suggest that $Q = 2$ halos will be unstable with any amount of screening. Thus the next lightest galaxies, after $Q = 2$ galaxies, may well have $Q = 4$ or higher. In conclusion, one expects a gap in the stellar velocity dispersions of dSphs of at least a factor of about $\sqrt{2}$. This prediction is already on the verge of being falsified by the Sloan Digital Sky Survey data of [Strigari et al. (2007)], and hopefully will soon lead to a falsification of the entire model.

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