U GEMINORUM: A TEST CASE FOR ORBITAL PARAMETER DETERMINATION

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Received 2007 January 31; accepted 2007 March 28

ABSTRACT

High-resolution spectroscopy of U Gem was obtained during quiescence. We did not find a hot spot or gas stream around the outer boundaries of the accretion disk. Instead, we detected a strong narrow emission region near the location of the secondary star. We measured the radial velocity curve from the wings of the double-peaked Hα emission line and obtained a semiamplitude value that is in excellent agreement with that obtained from observations in the ultraviolet spectral region by Sion et al. We also present a new method to obtain $K_2$, which enhances the detection of absorption or emission features arising in the late-type companion. Our results are compared with published values derived from the near-infrared Na I line doublet. From a comparison of the TiO band with those of late-type M stars, we find that a best fit is obtained for an M6 V star, contributing 5% of the total light at that spectral region. Assuming that the radial velocity semiamplitudes accurately reflect the motion of the binary components, then from our results $K_{\text{em}} = 107 \pm 2$ km s$^{-1}$ and $K_{\text{abs}} = 310 \pm 5$ km s$^{-1}$; using the inclination angle given by Zhang & Robinson, $i = 69.7^\circ \pm 0.7^\circ$, the system parameters become $M_{\text{WD}} = 1.20 \pm 0.05$ $M_\odot$, $M_{\text{RD}} = 0.42 \pm 0.04$ $M_\odot$, and $a = 1.55 \pm 0.02$ $R_\odot$. Based on the separation of the double emission peaks, we calculate an outer disk radius of $R_{\text{out}}/a \sim 0.61$, close to the distance of the inner Lagrangian point L1/a $\sim 0.63$. Therefore, we suggest that, at the time of observations, the accretion disk was filling the Roche lobe of the primary and the matter leaving the L1 point was colliding with the disk directly, producing the hot spot at this location.

Key words: binaries: close — novae, cataclysmic variables — stars: individual (U Geminorum)

Online material: color figures

1. INTRODUCTION

Discovered by Hind (1856), U Geminorum is the prototype of a subclass of dwarf novae, a descriptive term suggested by Payne-Gaposchkin & Gaposchkin (1938) due to the small-scale similarity of the outbursts in these objects to those of novae. After the work of Kraft (1962), who found U Gem to be a single-lined spectroscopic binary with an orbital period around 4.25 hr, and the studies by Kremski (1965), who established the eclipsing nature of this binary, Warner & Nather (1971) and Smak (1971) established the classical model for cataclysmic variable stars. The model includes a white dwarf primary surrounded by a disk accreted from a Roche lobe—filling late-type secondary star. The stream of material coming through the L1 point intersects the edge of the disk producing a bright spot which can contribute a large fraction of the visual flux. The bright spot is observed as a strong hump in the light curves of U Gem and precedes a partial eclipse of the accretion disk and bright spot themselves (the white dwarf is not eclipsed in this object).

A mean recurrence time for U Gem outbursts of $\approx 118$ days, with $\Delta m_V = 5$ and an outburst width of 12 days, was first found by Szkody & Mattei (1984). However, recent analysis shows that the object has a complex outburst behavior (Cook 1987; Mattei et al. 1987; Cannizzo et al. 2002). Smak (2004), using American Association of Variable Star Observers (AAVSO) data on the 1985 outburst, has discovered the presence of superhumps, a fact that challenges the current theories of superoutbursts and superhumps for long-period systems with mass ratios above 1/3. The latter author also pointed out the fact that calculations of the radius of the disk, obtained from the separation of the emission peaks (R. P. Kraft 1975, private communication to J. Smak [1976]) in quiescence, are in disagreement with the calculations of the disk radii obtained from the photometric eclipse data (Smak 2001).

Several radial velocity studies have been conducted since the first results published by Kraft (1962). In the visible spectral range, where the secondary star was not detected, their results are mainly based on spectroscopic radial velocity analysis of the emission lines arising from the accretion disk (Kraft 1962; Smak 1976; Stover 1981; Unda-Sanzana et al. 2006). At other wave-lengths, there are works based on individual absorption lines: in the near-infrared, on the Na I doublet from the secondary star (Wade 1981; Friend et al. 1990; Naylor et al. 2005), and in the ultraviolet, on lines coming from the white dwarf itself (Sion et al. 1998; Long & Gilliland 1999).

Although the research work on U Gem has been of paramount importance in our understanding of cataclysmic variables, the fact that it is a partially eclipsed and, in the visual range, a single-lined spectroscopic binary makes the determination of its physical parameters difficult to achieve through precise measurements of the semiamplitudes $K_{1,2}$ and of the inclination angle $i$ of the orbit. Spectroscopic results for $K_{1,2}$ differ in the ultraviolet, visual, and infrared ranges. Therefore, auxiliary assumptions have been used to derive its more fundamental parameters (Smak 2001). In this paper we present a value of $K_1$ obtained from our high-dispersion echelle spectra that is in agreement with the ultraviolet results, and a value of $K_2$ from a new method applicable to optical spectroscopy. By chance, the system was observed at a peculiar low state, when the classical hot spot was absent.

2. OBSERVATIONS

U Gem was observed on 1999 January 15 with the echelle spectrograph at the f/7.5 Cassegrain focus of the 2.1 m telescope
peaks, second by using a method based on a cross-correlating technique, and third by using the standard double-Gaussian technique designed to measure only the wings of the line. In the case of the secondary star, we were unable to detect any single absorption line in the individual spectra; therefore, it was not possible to use any standard method. However, here we propose and use a new method based on a co-adding technique to derive the semi-amplitude of the orbital radial velocity of the companion star. In this section we compare our results with published values for both components in the binary. We first discuss the basic mathematical method used here to derive the orbital parameters and its limitations in the context of cataclysmic variables, then we present our results for the orbital parameters calculated from the different methods, and finally, we discuss an improved ephemeris for U Gem.

3.1. Orbital Parameter Calculations

To find the orbital parameters of the components in a cataclysmic variable— in which no eccentricity is expected (Zahn 1966; Warner 1995) — we use an equation of the form

\[ V(t) = \gamma + K_{\text{abs}} \sin \left[ \frac{2\pi (t - HJD_0)}{P_{\text{orb}}} \right], \]

where \( V(t) \) are the observed radial velocities as measured from the emission lines in the accretion disk or from the absorption lines of the red star, \( \gamma \) is the systemic velocity, \( K_{\text{abs}} \) are the corresponding semiamplitudes derived from the radial velocity curve, \( HJD_0 \) is the heliocentric Julian Date of the inferior conjunction of the companion, and \( P_{\text{orb}} \) is the orbital period of the binary.

A minimum least-squares sinusoidal fit is found that uses initial values for the four \( (P_{\text{orb}}, \gamma, K_{\text{em}, \text{abs}}, HJD_{0, \text{abs}}) \) orbital parameters. The program allows for one or more of these variables to be fixed; i.e., they can be set to constant values in the initial parameters file.

If the orbital period is not previously known, a frequency search, using a variety of methods for evenly or unevenly sampled time-series data (Schwarzenberg-Czerny 1999), may be applied to the measured radial velocities in order to obtain an initial value for \( P_{\text{orb}} \), which is then used in the minimum least-squares sinusoidal fit. If the time coverage of the observations is not sufficient or is uneven, period aliases may appear, and their values have to be considered in the least-squares fits. A tentative orbital period is selected by comparing the quality of each result. In these cases, additional radial velocity observations should be sought until the true orbital period is found unequivocally. Time-series photometric observations are usually helpful for finding orbital modulations and are definitely important in establishing the orbital period of eclipsing binaries. In the case of U Gem, the presence of eclipses and the ample photometric coverage since the early work of Kremsinski (1965) has permitted us to establish its orbital period with a high degree of accuracy (Marsh et al. 1990). Although in eclipsing binaries a zero phase is also usually determined, in the case of U Gem the variable positions of the hot spot and stream cause the zero point to oscillate, as mentioned by the latter authors. Accurate spectroscopic observations are necessary to correctly establish the time when the secondary star is closest to Earth, i.e., in inferior conjunction. Further discussion on this subject is given in §4.

To obtain the real semiamplitudes of the binary, i.e., \( K_{\text{em}, \text{abs}} = K_{1,2} \), some reasonable auxiliary assumptions are made: first, that the measurements of the emission lines produced in the accretion disk are free from distortions and accurately follow the

### Table 1: Measured Hα Radial Velocities

| HJD (2,400,000+) | \( \phi^a \) | Peaks\(^b\) (km s\(^{-1}\)) | \( f_{\text{cc}}^c \) (km s\(^{-1}\)) | Wings\(^d\) (km s\(^{-1}\)) |
|-----------------|---------|-----------------|--------|--------|
| 51,193.67651... | 0.68    | 166.1           | 139.1  | 121.1  |
| 51,193.68997... | 0.75    | 183.4           | 130.0  | 133.8  |
| 51,193.69679... | 0.80    | 181.9           | 125.0  | 126.9  |
| 51,193.70723... | 0.86    | 167.9           | 102.0  | 101.1  |
| 51,193.71744... | 0.92    | 137.1           | 81.7   | 90.9   |
| 51,193.72726... | 0.97    | 90.0            | 46.8   | 41.7   |
| 51,193.73581... | 0.02    | 14.0            | -17.9  | 6.9    |
| 51,193.74700... | 0.09    | -47.9           | -48.1  | -27.1  |
| 51,193.75691... | 0.14    | -67.1           | -66.7  | -48.2  |
| 51,193.76743... | 0.20    | -99.6           | -84.6  | -79.3  |
| 51,193.77738... | 0.26    | -132.3          | -86.1  | -75.7  |
| 51,193.78900... | 0.32    | -152.6          | -60.2  | -48.8  |
| 51,193.80174... | 0.39    | -77.9           | -32.9  | -33.6  |
| 51,193.81211... | 0.45    | 9.0             | 10.9   | 14.5   |
| 51,193.82196... | 0.51    | 104.5           | 59.2   | 65.1   |
| 51,193.83176... | 0.56    | 134.6           | 113.7  | 107.0  |
| 51,193.84175... | 0.62    | 141.0           | 142.8  | 124.9  |
| 51,193.85156... | 0.67    | 159.3           | 158.6  | 147.6  |
| 51,193.86133... | 0.73    | 165.6           | 148.0  | 131.7  |
| 51,193.87101... | 0.79    | 192.9           | 142.8  | 130.3  |
| 51,193.88116... | 0.84    | 175.0           | 120.7  | 110.6  |
| 51,193.88306... | 0.91    | 154.6           | 106.5  | 91.1   |
| 51,193.90530... | 0.98    | 90.6            | 32.3   | 31.9   |
| 51,193.91751... | 0.05    | -70.5           | 8.0    | -23.1  |
| 51,193.93029... | 0.12    | -88.5           | -71.8  | -51.6  |
| 51,193.94259... | 0.19    | -97.1           | -79.0  | -66.7  |
| 51,193.95483... | 0.26    | -114.4          | -88.7  | -75.6  |
| 51,193.95955... | 0.29    | -142.2          | -70.9  | -67.9  |

\(^a\) Orbital phases derived from the ephemeris given in §4.
\(^b\) Velocities derived as described in §3.2.1.
\(^c\) Velocities derived as described in §3.2.2.
\(^d\) Velocities derived as described in §3.2.3.
Orbital Parameters Derived from Several Radial Velocity Calculations of the Hα Emission Line

| Orbital Parameters | Peaksa | βc | Wingsb |
|--------------------|----------------|-------|-------|
| (1)                | (2)          | (3)   | (4)   |
| γ (km s⁻¹)         | 38 ± 5       | 35 ± 3 | 34 ± 2 |
| K (km s⁻¹)         | 162 ± 7      | 119 ± 3 | 107 ± 2 |
| HJD( +2,437,638 days) | 0.8259(2) | 0.8246(6) | 0.82152(9) |
| Porb (days)        | d            | d     | d     |
| d                 | 25.2         | 12.2  | 9.1   |

a Derived from measurements of the double peaks.
b Derived from cross-correlation methods.
c Results from the fitting of fixed double-Gaussian profiles to the wings.
d Period fixed, P = 0.1769061911 days.

3.2. The Primary Star

In this section we compare three methods for determining the radial velocity of the primary star, based on measurements of the Hα emission line. Although, as we see in the next subsections, the last method results in far better accuracy and agrees with the ultraviolet results, we have included all of them here because the first method essentially provides an accurate way to determine the separations of the blue and red peaks, which is an indicator of the outer radius of the disk (Smak 2001), and the second yields a Kem value much closer to that obtained from UV results than any other published method. This cross-correlation method might be worthwhile to consider for its use in other objects. Furthermore, as we see in the discussion, all three methods yield a consistent value of the systemic velocity, which is essential to the understanding of other parameters in the binary system.

To match the signal-to-noise ratio of the first 21 spectra, we have co-added, in pairs, the 13.5 minute exposures. The last three spectra were added to form two different spectra, in order to avoid losing the last single spectrum. A handicap to this approach is that, due to the large read-out time of the Thomson CCD, we are effectively smearing the phase coverage of the co-added spectra to nearly 900 s. However, the mean heliocentric time was accordingly corrected for each sum. This adds to a total sample of 2600 s spectra.

3.2.1. Measurements from the Double Peaks

We have measured the positions of the peaks using a double-Gaussian fit, with their separation, width, and position as free parameters. The results yield a mean half-peak separation Vout of about 460 km s⁻¹. The average values of the velocities of the red and blue peaks, for each spectrum, are shown in column (3) of Table 1. We then applied our nonlinear least-squares fit to these radial velocities. The obtained orbital parameters are shown in column (2) of Table 2. The numbers in parentheses after the zero-point results are the evaluated errors of the last digit. We use this notation for large numbers throughout the paper. The radial velocities are also shown in Figure 1, folded with the orbital period and the time of inferior conjunction adopted in § 3.1. The solid lines in this figure correspond to sinusoidal fits using the derived parameters in our program. Although we have not independently tabulated the measured velocities of the blue and red peaks, they are shown in Figure 1 together with their average. The semiamplitudes of the plotted curves are 154 and 167 km s⁻¹ for the blue and red peaks, respectively.

3.2.2. Cross-Correlation Using a Template

We have also cross-correlated the Hα line in our spectra with a template constructed as follows. First, we selected a spectrum from the observed orbital cycle close to phase 0.02 when, in the case of our observations, we should expect a minimum distortion in the double-peaked line due to asymmetric components (see § 5). The blue peak in this spectrum is slightly stronger than the red one. This is probably caused by the hot spot near the L1 point (see § 3.3.2), which might be visible at this phase due to the fact that the binary has an inclination angle smaller than 70°. The half-separation of the peaks is 470 km s⁻¹, a value similar to that measured in a spectrum taken during the same orbital phase in next cycle. The chosen spectrum was then highly smoothed to minimize high-frequency correlations. The resulting template is shown in Figure 2. A radial velocity for the template was derived from the wavelength measured at the dip between the two peaks.
and corrected to give a heliocentric velocity. The IRAF fsc task was then used to derive the radial velocities, which are shown in column (4) of Table 1. As in § 3.2.1, we have fitted the radial velocities with our nonlinear least-squares fit algorithm. The resulting orbital parameters are given in column (3) of Table 2. In Figure 3 the obtained velocities and the corresponding sinusoidal fit (solid line) are plotted.

3.2.3. Measurements from the Wings and Diagnostic Diagrams

The Hα emission line was also measured using the standard double-Gaussian technique and its diagnostic diagrams, as described in Shafter et al. (1986). We refer to that paper for the details on the interpretation of our results. We have used the comovese routine from the IRAF rsvao package, kindly made available to us by J. R. Thorstensen (2000, private communication). The double-peaked Hα emission line, with a separation of about 20 Å, shows broad wings reaching up to 40 Å from the line center. Unlike the case of low-resolution spectra, where for oversampled data the fitting is made with individual Gaussian curves having a FWHM of about 1 resolution element, in our spectra, with resolution ≈0.34 Å, such Gaussian profiles would be inadequately narrow, as they would cover only a very small region in the wings. To measure the wings appropriately and, at the same time, avoid possible low-velocity asymmetric features, we must select a σ-value that fits the line regions corresponding to disk velocities from about 700 to 1000 km s⁻¹.

As a first step, we evaluated the width of the Gaussian profiles by setting this as a free parameter from 10 to 40 pixels and for a wide range of Gaussian separations (between 180 and 280 pixels). For each run, we applied a nonlinear least-squares fit of the computed radial velocities to sinusoids of the form described in § 3.1. The results are shown in Figure 4, in particular for three different Gaussian separations, a = 180, 230, and 280 pixels. These correspond to the lower and upper limits, as well as to the value for a preferred solution, all of which are self-consistent with the second step (see below). In the bottom panel of the figure we have plotted the overall rms value for each least-squares fit, as this parameter is very sensitive to the selected Gaussian separations. As expected at this high spectral resolution, the parameters in the diagram change rapidly for low values of σ, and there are even cases for which no solution is found. At low values of a (crosses) there are no solutions for widths narrower than 20 pixels. The rms values increase rapidly with width, while the σ(K)/K, γ, and phase-shift values...
differ strongly from the other cases. For higher values of $a$ (open circles) we obtain lower values for $\sigma(K)/K$, but the rms results are still large, in particular for intermediate values of the width of the Gaussian curves. For the middle solution (filled circles) the results are comparable with those for large $a$-values, but the rms is much lower. Similar results were found for other intermediate values of $a$, and they all converge to a minimum rms for a width of 26 pixels at $a = 230$ pixels.

For the second step we fixed the width to a value of 26 Å and ran the double-Gaussian program for a range of $a$ separations, from about 60 to 120 Å. The results obtained are shown in Figure 5.

If only an asymmetric low-velocity component is present, the semiamplitude should decrease asymptotically as $a$ increases, until $K_1$ reaches the correct value. Here we observe such behavior, although for larger values of $a$, there is a $K_1$ increase for values of $a$ up to 40 Å before it decreases strongly with high values of $a$. This behavior might be due to the fact that we are observing a narrow hot spot near the L1 point (see §5). On the other hand, as expected, the $\sigma(K)/K$ versus $a$ curve has a change in slope at a value of $a$ for which the individual Gaussian curves have reached the velocity width of the line at the continuum. For larger values of $a$ the velocity measurements become dominated by noise. For low values of $a$, the phase shift usually gives spurious results, although in our case it approaches a stable value around 0.015. We believe this value reflects the difference between the eclipse ephemeris, which is based mainly on the eclipse of the hot spot, and the true inferior conjunction of the secondary star. This problem is further discussed in §5. Finally, we must point out that the systemic velocity smoothly increases up to a maximum of about 40 km s$^{-1}$ at Gaussian separation of nearly 42 Å, while the best results, as seen from the figure, are obtained for $a = 31$ Å. This discrepancy may also be related to the narrow hot spot near the L1 point and might be due to the phase shift between the hot spot eclipse and the true inferior conjunction. This problem is also addressed in §4. The radial velocities corresponding to the adopted solution are shown in column (5) of Table 1 and plotted in Figure 6, while the corresponding orbital parameters, obtained from the nonlinear least-squares fit, are given in column (4) of Table 2.

3.3. The Secondary Star

We were unable to detect single features from the secondary star in any individual spectra, after careful correction for telluric lines. In particular we found no radial velocity results using a standard cross-correlation technique near the Na I $\lambda\lambda 8183.3, 8194.8$ doublet. As we see below, this doublet was very weak compared with previous observations (Wade 1981; Friend et al. 1990; Naylor et al. 2005). We have been able, however, to detect the Na I doublet and the TiO band head around 7050 Å with a new technique that enables us to derive the semiamplitude $K_2$ of the secondary-star velocity curve. We first present here the general method for deriving the semiamplitude and then apply it to U Gem, using not only the absorption features but the Hα emission as well.

3.3.1. A New Method to Determine $K_2$

In many cataclysmic variables the secondary star is poorly visible, or even absent, in the optical spectral range. Consequently, no $V(t)$ measurements are feasible for this component. Among these systems are dwarf novae with orbital periods under 0.25 days, for which it is thought that the disk luminosity dominates over the luminosity of the Roche lobe—filling secondary, whose brightness depends on the orbital period of the binary (Echevarría & Jones 1984). For such binaries, the orbital parameters have been derived only for the white dwarf accretion disk system in a way similar to that described in §3.1.

In order to determine a value of $K_{\text{abs}}$ from a set of spectra from a cataclysmic variable, for which the orbital period and time

![Fig. 5.—Diagnostic diagram 2. The best estimate of the semiamplitude of the white dwarf is 107 km s$^{-1}$, corresponding to $a \approx 34$ Å.](image1)

![Fig. 6.—Radial velocities for U Gem. The open circles correspond to the measurements of the first 21 single spectra, while the filled circles correspond to those of the co-added spectra (see §3.2). The solid line close to the symbols corresponds to the solution with $K_{\text{em}} = 107$ km s$^{-1}$ (see text), while the large-amplitude line corresponds to the solution found for $K_2$ (see §3.3). [See the electronic edition of the Journal for a color version of this figure.]](image2)
of inferior conjunction have been already determined from the emission lines, we propose to reverse the process: derive \( V(t)_{\text{obs}} \) using \( K_{pr} \) as the initial value for the semiamplitude and set the values of \( P_{\text{orb}} \) and \( \text{HJD} \), derived from the emission lines, as constants. The initial value for the systemic velocity is set to zero, and its final value may be calculated later (see below). The individual spectra are then co-added in the frame of reference of the secondary star, i.e., by Doppler shifting the spectra using the calculated \( V(t)_{\text{calc}} \) from the equation given in § 3.1 and then adding them together. Hereafter we refer to this procedure as the “cophasing process.” Ideally, as the proposed \( K_{pr} \) is changed through a range of possible values, there will be one for which the cophased spectral features associated with the absorption spectrum will have an optimal signal-to-noise ratio.

In fact, this will also be the case for any emission-line features associated with the red star, if present. In a way, this process works similarly to the double-Gaussian fitting used in § 3.2.3, provided that adequate criteria are set in order to select the best value for \( K_{pr} \). We propose three criteria or tests that, for late-type stars, may be used with this method. The first one consists of analyzing the behavior of the measured depths or widths of a well-identified absorption line in the cophased spectra, as a function of the proposed \( K_{pr} \); one would expect that the width of the line will show a minimum and its depth a maximum value at the optimal solution. This method could be particularly useful for K-type stars that have strong single metallic lines like Ca \(_i\) and Fe \(_i\). The second criterion is based on measurements of the slope of band heads, such as that of TiO at 7050 Å. It should be relevant to short-period systems, with low-mass M-type secondaries with spectra featuring strong molecular bands. In this case one could expect that the slope of the band head will be a function of \( K_{pr} \) and will have a maximum negative value at the best solution. A third test is to measure the strength of a narrow emission arising from the secondary. This emission, if present, would be particularly visible in the cophased spectrum and would have minimum width and maximum height at the best selected semiamplitude \( K_{pr} \).

We have tested these three methods by means of an artificial spectrum with simulated narrow absorption lines, a TiO-like band head, and a narrow emission line. The spectrum with these artificial features was then Doppler shifted using the pre-established inferior conjunction phase and orbital period to produce a series of test spectra. An amount of random Gaussian noise was added to each Doppler-shifted spectrum, sufficient to mask the artificial features. We then proceeded to apply the cophasing process to recover our predetermined orbital values. All three criteria reproduced the original set of values, as long as the random noise amplitude was of the same order of magnitude as the strength of the clean artificial features.

### 3.3.2. Determination of \( K_2 \) for U Gem

We have applied the above-mentioned criteria to U Gem. The time of the inferior conjunction of the secondary and the orbital period were taken from § 4. To attain the best signal-to-noise ratio we have used all 28 observed spectra. Although they span slightly more than 1.5 orbital periods, any departure from a real \( K_2 \) value will not depend on selecting data in exact multiples of the orbital period, as any possible deviation from the real semiamplitude will already be present in one complete orbital period and will depend mainly on the intrinsic intensity distribution of the selected feature around the secondary itself (also, see below the results for \( \gamma \)).

Figure 7 shows the application of the first test to the Na \(_i\) doublet \( \lambda \lambda 8183, 8195 \). The spectra were cophased, varying \( K_{pr} \) between 250 and 450 km s\(^{-1}\). The line depth of the blue (stars) and red (open circles) components of the doublet, as well as their mean value (filled circles), are shown in the diagram. We find the best solution for \( K_2 = 310 \pm 5 \) km s\(^{-1}\). The error has been estimated from the intrinsic modulation of the solution curve. As it approaches its maximum value, the line-depth value oscillates slightly, but in the same way for both lines. A similar behavior was present when low signal-to-noise features were used on the artificial spectra process described above. Figure 8 shows the cophased spectrum of the Na \(_i\) doublet of our best solution for \( K_2 \). These lines appear very weak compared to those reported by Friend et al. (1990) and Naylor et al. (2005). We have also measured the \( \gamma \)-velocity from the cophased spectrum by fitting a double-Gaussian profile to the Na \(_i\) doublet (Fig. 9, dotted line) and find a mean value \( \gamma = 69 \pm 10 \) km s\(^{-1}\) (corrected to the heliocentric standard of motion). We did a similar calculation for \( \gamma \) by cophasing the selected spectra used in § 5, covering a full cycle only. The results were very similar to those obtained by using all spectra.
The second test, to measure the slope that the TiO band has at 7050 Å, was not successful. The solution curve oscillates strongly near values between 250 and 350 km s\(^{-1}\). We believe that the signal-to-noise ratio in our spectra is too poor for this test, and that more observations, accumulated during several orbital cycles, have to be obtained in order to attain a reliable result using this method.

However, we have cophased our spectra for \(K_p = 310\) km s\(^{-1}\), with the results shown in Figure 9. The TiO band is clearly seen, while the noise is prominent, particularly along the slope of the band head. We have used this co-added spectrum to compare it with that of several late-type M stars extracted from the published data by Montes et al. (1997) fitted to our cophased spectrum. A gray continuum has been added to the comparison spectra in order to compensate for the fill-in effect arising from the other light sources in the system so as to obtain the best fit. In particular, we show in the same figure the fits when two close candidates, GJ 406 (M6 V, top) and GJ 402 (M4 V, bottom), are used. The best fit is obtained for the M6 V star, to which we have added a 95% continuum. For the M4–5 V star the fit is poor, as we observe a flux excess around 7000 Å and a stronger TiO band head. Increasing the gray flux contribution will fit the TiO band head, but will result in a larger excess at the 7000 Å region. On the other hand, the fit with the M6 V star is much better all along the spectral interval.

There are a number of publications that assign to U Gem spectral types M4 (Harrison et al. 2000), M5 (Wade 1981), and possibly as far as M5.5 (Berriman et al. 1983). Even in the case that the spectral type of the secondary star is variable, its spectral classification is still incompatible with its mass determination (Echevarría 1983).

For the third test, we have selected the region around H\(\alpha\), as in the individual spectra we see evidence of a narrow spot, which is very well defined in our spectrum near orbital phase 0.5. In this test we have cophased the spectra as before and have adopted as the test parameter the peak intensity around the emission line.

The results are shown in Figure 10. A clear and smooth maximum is obtained for \(K_p = 310 \pm 3\) km s\(^{-1}\). The cophased spectrum obtained from this solution is shown in Figure 11. The double-peak structure has been completely smeared, as expected.
when co-adding in the reference frame of the secondary star, as opposed to that of the primary star, and instead we observe a narrow and strong peak at the center of the line. We have also fitted the peak to find the radial velocity of the spot. We find $\gamma = 33 \pm 10$ km s$^{-1}$, compatible with the $\gamma$-velocity derived from the radial velocity analysis of the emission line, $\gamma = 34 \pm 2$ km s$^{-1}$ (see § 3.2.3). This is a key result for the determination of the true systemic velocity and can be compared with the values derived from the secondary star (see § 7).

4. IMPROVED EPHEMERIS OF U GEM

As mentioned in § 3.1, the presence of eclipses in U Gem and an ample photometric coverage over 30 years has permitted us to establish, with a high degree of accuracy, the value of the orbital period. This has been discussed in detail by Marsh et al. (1990). However, as pointed out by these authors, this object shows erratic variations in the timing of the photometric mid-eclipse that may be caused by either orbital period changes, or variations in the position of the hot spot, or they may even be the consequence of the different methods of measuring of the eclipse phases. A variation in the position and intensity of the gas stream will also contribute to such changes. A date for the zero phase determined independently from spectroscopic measurements would evidently be desirable. Marsh et al. (1990) discussed two spectroscopic measurements by Marsh & Horne (1988) and Wade (1981) and concluded that the spectroscopic inferior conjunction of the secondary star occurs about 0.016 in phase prior to the mean photometric zero phase. There are two published spectroscopic studies (Honeycutt et al. 1987; Stover 1981), as well as one in this paper, that could be used to confirm this result. Unfortunately, there is no radial velocity analysis in the former paper nor in the excellent Doppler imaging paper by Marsh et al. (1990) based on their original observations. However, the results by Stover (1981) are of particular interest since he found the spectroscopic conjunction in agreement with the time of the eclipse when using the photometric ephemerides by Wade (1981) taken from Arnold et al. (1976). The latter authors introduce a small quadratic term that is consistent with the $O-C$ oscillations shown in Marsh et al. (1990).

It is difficult to compare results derived from emission lines to those obtained from absorption lines, especially if they are based on different ephemerides. Furthermore, the contamination in the timing of the spectroscopic conjunction, caused either by a hot spot, a gas stream, or irradiation on the secondary, has not been properly evaluated. However, since our observations were made at a time when the hot spot was absent (or, at least, was along the line between the two components in the binary) and the disk was very symmetric (see § 5), we can safely assume that in our case the photometric and spectroscopic phases must coincide. If we then take the orbital period derived by Marsh et al. (1990) and use the zero-point value derived from our measurements of the H$\alpha$ wings (§ 3.2.3), we can improve the ephemeris,

$$HJD = 2,437,638.82566(4) + 0.1769061911(28)E,$$

for the inferior conjunction of the secondary star. This ephemeris is used throughout this paper for all our phase-folded diagrams and Doppler tomography.

5. DOPPLER TOMOGRAPHY

Doppler tomography is a useful and powerful tool for studying the material orbiting the white dwarf, including the gas stream coming from the secondary star, as well as emission regions arising from the companion itself. It uses the emission-line profiles observed as a function of the orbital phase to reconstruct a two-dimensional velocity map of the emitting material. A detailed formulation of this technique can be found in Marsh & Horne (1988). A careful interpretation of these velocity maps has to be made, as the main assumption invoked by tomography is that all the observed material is in the orbital plane and is visible at all times.

The Doppler tomography, derived here from the H$\alpha$ emission line in U Gem, was constructed using the code developed by Spruit (1998). Our observations of the object cover 1.5 orbital cycles. Consequently, to avoid disparities in the intensity of the trailed and reconstructed spectra, as well as in the tomographic map, we have carefully selected spectra covering a full cycle only. For this purpose we discarded the first three spectra (which have the largest air mass) and used only 18 spectra out of the first 21 of the 600 s exposures, starting with the spectrum at orbital phase 0.88 and ending with the one at phase 0.86 (see Table 1). In addition, in generating the tomography map we have excluded the spectra taken during the partial eclipse of the accretion disk (phases between 0.95 and 0.05). The original and reconstructed trailed spectra are shown in Figure 12. They show the sinusoidal variation of the blue and red peaks, which are strong at all phases. The typical S-wave is also seen showing the same simple sinusoidal variation, but shifted by 0.5 in orbital phase with respect to the double peaks. The Doppler tomogram is shown in Figure 13; as customary, the oval represents the Roche lobe of the secondary and the solid lines the Keplerian ($upper$) and ballistic ($lower$) trajectories. The tomogram reveals a disk reaching to the distance of the inner Lagrangian point in most phases. Compact and strong emission is seen close to the center of velocities of the secondary star. A zoom-in of this region is shown in Figure 14. Both maps have been constructed using the parameters shown at the top of the diagrams and a $\gamma$-velocity of 34 km s$^{-1}$. The velocity resolution of the map near the secondary star is about 10 km s$^{-1}$. The $F(x, y)$ position of the hot spot (in km s$^{-1}$) is ($-50, 305$), within the uncertainties.

The tomography shown in Figure 13 is very different from what we expected to find and from what has been observed by other authors. We find a very symmetric full disk, reaching close to the inner Lagrangian point and a compact bright spot also close to the L1 point, instead of a complex system like that observed by Unda-Sanzana et al. (2006), who found U Gem at a stage when the Doppler tomographs show emission at low velocity close to the center of mass a transient narrow absorption in the Balmer...
Fig. 12.—Trailed spectra of the H$\alpha$ emission line. Left, original data; right, reconstructed data. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 13.—Doppler tomography of U Gem. The various features are discussed in the text. The $v_x$- and $v_y$-axes are in km s$^{-1}$. A compact hot spot close to the inner Lagrangian point is detected instead of the usual bright spot and/or broad stream, where the material, following a Keplerian or ballistic trajectory, strikes the disk. The tomogram reveals a full disk whose outer edge is very close to the L1 point (see text).

Fig. 14.—Zoom-in of the region around the hot spot. Note that this feature is slightly ahead of the center of mass of the secondary star. Since this is a velocity map and not a geometric one, its physical position in the binary is carefully discussed in the text.
lines, as well as two distinct spots: one very narrow and close in velocity to the accretion disk near the impact region and another much broader, located between the ballistic and Keplerian trajectories. They also presented tentative evidence of a weak spiral structure, which has been seen as strong spiral shocks during an outburst observed by Groot (2001). Our results also differ from those of Marsh et al. (1990), who also found that the bulk of the bright spot arising from the Balmer, He I, and He II emission comes from a region between the ballistic and Keplerian trajectories. We interpret the difference between our results and previous studies simply by the fact that we have observed the system at a peculiar low state not detected before (see §§ 1 and 7). This should not be at all surprising because, although U Gem is a well-observed object, it is also a very unusual and variable system.

Figure 14 shows a zoom-in of the region around the secondary star. The bright spot is shown close to the center of mass of the late-type star, located slightly toward the leading hemisphere. Since this is a velocity map and not a geometric one, there are two possible interpretations of the position in space of the bright spot (assuming that the observed material is in the orbital plane). The first one is that the emission is produced at the surface of the secondary, i.e., still attached to its gravitational field. The second is that the emission is the result of a direct shock front with the accretion disk and that the compact spot is starting to gain velocity toward the Keplerian trajectory. We believe that the second explanation is more plausible, as it is consistent with the well-accepted mechanism to produce a bright spot. On the other hand, at this peculiar low state it is difficult to invoke an external source strong enough to produce a back-illuminated secondary and, especially, a bright and compact spot on its leading hemisphere.

6. BASIC SYSTEM PARAMETERS

Assuming that the radial velocity semi-amplitudes reflect accurately the motion of the binary components, then from our results, \( K_{\text{em}} = K_1 = 107 \pm 2 \text{ km s}^{-1} \) and \( K_{\text{abs}} = K_2 = 310 \pm 5 \text{ km s}^{-1} \); adopting \( P = 0.176906191 \) we obtain

\[
q = \frac{K_1}{K_2} = \frac{M_2}{M_1} = 0.35 \pm 0.05,
\]

\[
M_1 \sin^2 i \frac{P}{2\pi G} = \frac{PK_2(K_1 + K_2)^2}{2G} = 0.99 \pm 0.03 \, M_\odot,
\]

\[
M_2 \sin^2 i \frac{P}{2\pi G} = \frac{PK_1(K_1 + K_2)^2}{2G} = 0.35 \pm 0.02 \, M_\odot,
\]

\[
a \sin i = \frac{P(K_1 + K_2)}{2\pi} = 1.46 \pm 0.02 \, R_\odot.
\]

Using the inclination angle derived by Zhang & Robinson (1987), \( i = 69.7^\circ \pm 0.7^\circ \), the system parameters become \( M_{\text{WD}} = 1.20 \pm 0.05 \, M_\odot, M_{\text{RD}} = 0.42 \pm 0.04 \, M_\odot, \) and \( a = 1.55 \pm 0.02 \, R_\odot \).

6.1. The Inner and Outer Size of the Disk

A first-order estimate of the dimensions of the disk, the inner and outer radius, can be made from the observed Balmer emission line. Its peak-to-peak velocity separation is related to the outer radius of the accreted material, while the wings of the line, coming from the high-velocity regions of the disk, can give an estimate of the inner radius (Smak 2001). The peak-to-peak velocity separations of the 31 individual spectra were measured (see § 3.2.1), as well as the velocities of the blue and red wings of H\( \alpha \) at 10% of the continuum level. From these measurements we derive mean values of \( V_{\text{out}} = 460 \text{ km s}^{-1} \) and \( V_{\text{in}} = 1200 \text{ km s}^{-1} \).

These velocities can be related to the disk radii from numerical disk simulations, tidal limitations, and analytical approximations (see Warner 1995 and references therein). If we assume the material in the disk at radius \( r \) is moving with Keplerian rotational velocity \( V_\text{K}(r) \), then the radius in units of the binary separation is given by (Horne et al. 1986),

\[
r/a = (K_{\text{em}} + K_{\text{abs}})K_{\text{abs}}/V_\text{K}(r)^2.
\]

The observed maximum intensity of the double-peak emission in a Keplerian disk occurs close to the velocity of its outer radius (Smak 1981). From the observed \( V_{\text{out}} \) and \( V_{\text{in}} \) values we obtain an outer radius of \( R_{\text{out}}/a = 0.61 \) and an inner radius of \( R_{\text{in}}/a = 0.09 \). If we take \( a = 1.55 \pm 0.02 \, R_\odot \) from the last section we obtain an inner radius of the disk \( R_{\text{in}} = 0.1395 \, R_\odot \), equivalent to about 97,000 km. This is about 25 times larger than the expected radius of the white dwarf (see § 7). On the other hand, the distance from the center of the primary to the inner Lagrangian point, \( R_{L1}/a \), is

\[
R_{L1}/a = 1 - w/3 + w^2 + 1/9w^3,
\]

where \( w^3 = q/3(1 + q) \) (Kopal 1959). Using \( q = 0.35 \) we obtain \( R_{L1}/a = 0.63 \). The disk, therefore, appears to be large, almost filling the Roche lobe of the primary, with the matter that leaves the secondary component through the L1 point colliding with the disk directly and producing the hot spot near this location.

7. DISCUSSION

For the first time, a radial velocity semi-amplitude of the primary component of U Gem has been obtained in the visual spectral region, which agrees with the value obtained from ultraviolet observations by Sion et al. (1998) and Long & Gilliland (1999). In a recent paper, Unda-Sanzana et al. (2006) presented high-resolution spectroscopy around H\( \alpha \) and H\( \beta \) and concluded that they could not recover the ultraviolet value for \( K_1 \) to better than about 20% by any method. Although the spectral resolution at H\( \alpha \) of the instrument they used is only a factor of 2 smaller than that of the one we used, the diagnostic diagrams they obtain show a completely different behavior compared to those we present here, with best values for \( K_1 \) of about 95 km s\(^{-1}\) from H\( \alpha \) and 150 km s\(^{-1}\) from H\( \beta \) (see their Figs. 13 and 14, respectively). We believe that the disagreement with our result lies not in the quality of the data or the measuring method, but in the distortion of the emission lines due to the presence of a complex accretion disk at the time of their observations, as the authors themselves suggest. Their Doppler tomograms show emission at low velocity close to the center of mass at two distinct spots, a narrow component close to the L1 point and a broader, larger one between the Keplerian and the ballistic trajectories. There is even evidence of a weak spiral structure. In contrast, we have observed U Gem during a favorable stage, one in which the disk was fully symmetric, and the hot spot was narrow and near the inner Lagrangian point. This allowed us to measure the real motion of the white dwarf by means of the time-resolved behavior of the H\( \alpha \) emission line.

Our highly consistent results for the systemic velocity derived from the H\( \alpha \) spot (\( \gamma = 33 \pm 10 \) km s\(^{-1}\)) and those found from the different methods used for the radial velocity analysis of the emission arising from the accretion disk (see § 3.2 and Table 2) give strong support to our adopting a true systemic velocity of \( \gamma = 34 \pm 2 \) km s\(^{-1}\). If we are indeed detecting the true motion of the white dwarf, we can use this adopted value to make an independent check on the mass of the primary. The observed total
redshift of the white dwarf (gravitational plus systemic), found by Long & Gilliland (1999), is 172 km s⁻¹, from which, after subtraction of the adopted systemic velocity, we derive a gravitational shift of the white dwarf of 138 km s⁻¹. From the mass-radius relationship for white dwarfs (Anderson 1988), we obtain consistent results for $M_{\text{WD}} = 1.23 M_\odot$ and $R_{\text{WD}} = 3900$ km (see Fig. 7 in Long & Gilliland 1999). This mass is in excellent agreement with that obtained in this paper from the radial velocity analysis.

From our new method to determine the radial velocity curve of the secondary (§ 3.3.2), we obtain a value for the semiamplitude close to 310 km s⁻¹. Three previous papers have determinations of the radial velocity curves from the observed Na i doublet in the near-infrared. In order to evaluate whether our method is valid, we here compare our result with these direct determinations. The published values are $K_{RD} = 283 ± 15$ km s⁻¹ (Wade 1981), $K_{RD} = 309 ± 3$ km s⁻¹ (before correction for irradiation effects; Friend et al. 1990), and $K_{RD} = 300$ km s⁻¹ (Naylor et al. 2005). Wade (1981) noted that an elliptical orbit ($e = 0.086$) may better fit his data, as the velocity extremum near phase 0.25 appears somewhat shallower than that near phase 0.75 (see Fig. 3). However, he also found a very large systemic velocity, $\gamma = 85$ km s⁻¹, much larger than the values found by Kraft (1962; $\gamma = 42$ km s⁻¹) and Smak (1976; $\gamma = 40 ± 6$ km s⁻¹), both obtained from the emission lines. Since the discrepancy with the results of these two authors was large, Wade (1981) deferred this discussion to further confirmation of his results. Instead, and more importantly, that author discussed two scenarios that may significantly alter the real value of $K_{2}$, the nonsphericity and the back-illumination of the secondary. In the latter effect, each particular absorption line may move further away from, or closer to, the center of mass of the binary. He estimated the magnitude of this effect and concluded that the deviation of the photocenter would probably be much less than 0.1 radii. Friend et al. (1990) further discussed the circumstances that might cause the photocenter to deviate and concluded that their observed value for the semiamplitude should be corrected down by 3.5%, to yield $K_{2} = 298 ± 9$ km s⁻¹. Although they discussed the results by Martin (1988), which indicate that the relatively small heating effects in quiescent dwarf novae always lead to a decrease in the measured $K_{RD}$ for the Na i lines, they argued that line quenching, produced by ionization of the same lines, may also be important and result in an increased $K_{RD}$. Another disturbing effect, considered by the same authors, is line contamination by the presence of weak disk features, such as the Paschen lines. In this respect we point out here that a poor correction for telluric lines will function as an anchor, reducing also the amplitude of the radial velocity measurements. Friend et al. (1990) also found an observed systemic velocity of $\gamma = 43 ± 6$ km s⁻¹ and a small eccentricity of $e = 0.027$. Naylor et al. (2005) also discussed the distortion effects on the Na i lines and, based on their fit residuals, argued in favor of a depletion of the doublet in the leading hemisphere of the secondary, around phases 0.4 and 0.6, since removing flux from the blueward wing of the lines results in an apparent redshift, which would explain the observed residuals. However, they also found that fitting the data to an eccentric orbit, with $e = 0.024$, results in a significant decrease in the residuals caused by this depletion and concluded that it may be unnecessary to further correct the radial velocity curve. We must point out that a depletion of the blueward wing of the Na i lines will result in a contraction of the observed radial velocity curves, as the measured velocities, especially around phases 0.25 and 0.75, will be pulled toward the systemic velocity. Naylor et al. (2005) presented their results derived from the Na i doublet and the K i/TiO region (around 7550—7750 Å) compared with several spectral standards, all giving values between 289 and 305 km s⁻¹ (no errors are quoted). Based on the radial velocity measurements for Na i obtained by these authors in 2001 January (115 spectra), and using GJ 213 as a template (see their Table 1), we have recalculated the circular orbital parameters through our nonlinear least-squares fit. We find $K_{2} = 300 ± 1$ km s⁻¹, in close agreement with their published value.

It would be advisable to establish a link between the observed $\gamma$-velocity of the secondary and the semiamplitude $K_{2}$, under the assumption that its value may be distorted by heating effects. We take as a reference our results from the radial velocity analysis of the broad Hα line and the hot spot from the secondary, which support a true systemic velocity of 34 km s⁻¹. However, we find no positive correlation in the available results derived from the Na i lines, either between different authors or even among one data set. In the case of Naylor et al. (2005) the $\gamma$-values show a range between 11 and 43 km s⁻¹, depending on the standard star used as a template, for $K_{2}$ velocities in the range 289—305 km s⁻¹. Wade (1981) found $\gamma = 85 ± 10$ km s⁻¹ for a low $K_{2}$ value of 283 km s⁻¹, while Friend et al. (1990) found $\gamma = 43 ± 6$ km s⁻¹ for $K_{2}$ about 309 km s⁻¹, and we obtain a large $\gamma$-velocity of about 69 km s⁻¹ for a $K_{2}$ value of 310 km s⁻¹. We believe that further and more specific spectroscopic observations of the secondary star should be conducted in order to understand the possible distortion effects on lines like the Na i doublet and their implications for the derived semiamplitude and systemic velocity values.

E. d. l. F. wishes to thank Andrés Rodríguez for his useful computer help. The Thomson detector, used in our observations, was obtained through PACIME–CONACYT project F325-E9211.

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