Cosmology with varying scales and couplings

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Abstract

The time variation of fundamental mass scales can have profound cosmological implications. We investigate a particular model of crossover quintessence which is compatible with all present cosmological observations. This model can also reconcile the reported time variation of the fine structure constant from quasar absorption lines with the bounds from archeo-nuclear physics and tests of the equivalence principle.

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Dynamical mass scales

The recent confirmation of dark energy by the high precision KMAP data [1] challenges for an answer to the question if dark energy is constituted by a static cosmological constant or by dynamical quintessence [2]. A dynamical dark energy may shed light on the profound question of the role and origin of mass scales in physics. It may also lead to a cosmological time variation of dimensionless fundamental parameters like the electromagnetic fine structure constant.

In quantum field theory or statistical physics the dimensionless couplings depend on length scales [3]. This effect is perhaps most striking in QCD where the running gauge coupling generates the mass scale for the hadrons. The modern view of the renormalization group [4] associates this running with an “integrating out” of fluctuations as one looks at the effective physics on larger and larger length scales. The fluctuation effects can be treated in a conceptual simple and elegant form in the context of an exact functional renormalization group equation for the effective average action [5]. It describes for both renormalizable and nonrenormalizable theories the dependence of the fluctuation effects on an effective infrared cutoff. The physical length scale which cuts off the fluctuations in the infrared may correspond to the expectation value of a scalar field [6] and therefore become dynamical. We explore here the hypothesis that the time evolution of scales persists until the present epoch in the universe [2]. This would have striking effects for cosmology, the time variation of fundamental “constants” and tests of the equivalence principle.

For the renormalization of the standard model of particle physics one usually assumes implicitly the existence of an ultraviolet scale - typically the Planck scale or a grand unification scale. Similarly, for solids the ultraviolet scale is given by the lattice distance or for gases and liquids by the molecular size. In contrast, we explore here the concept of a unified theory where the action does not involve any explicit fundamental mass scale. Even without knowing the fundamental unified theory the experience with quantum physics lets us expect two types of possible mass scales in such a scenario. First, some intrinsic mass scales may be generated by the running of dimensionless couplings, similar to the hadron masses in QCD. Second, the expectation values of scalar fields induce dynamical mass scales. This second type of scales can change in the course of the cosmological evolution.

We explore here the scenario that the Planck mass or Newtons constant is dynamical. In contrast to the early formulation of this idea by Dirac and Jordan [7], [8] we postulate [9], [2] that the particle masses are dynamical as well, such that in a first approximation the ratio between the nucleon mass and the Planck mass, $m_n/M_p$, remains fixed, and similar for the weak scale $M_W/M_p$, the electron mass $m_e/M_p$ etc. In this first approximation all mass scales are proportional to a scalar field $\chi$, i.e. $M_p = \chi$, $m_n = h_n \chi$ or $m_e = h_e \chi$. This changes profoundly both the

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In string theories the apparent mass scale corresponding to the string tension can be replaced by the expectation value of the dilaton, thereby making this scale dynamical as well.
conceptual status and the observational consequences as compared to scenarios with fixed particle masses [8, 10].

In fact, in absence of intrinsic mass scales generated by the running dimensionless couplings the effective action (after integrating out all quantum fluctuations) would exhibit an exact dilatation symmetry, corresponding to a multiplicative rescaling $\chi \rightarrow \alpha \chi$. If we neglect in a first approximation the intrinsic mass scales, any nonzero value of the scalar field $\chi$ would break the exact global dilatation symmetry. In consequence of the spontaneous breaking of scale symmetry most particles would acquire a mass $\sim \chi$. Another consequence would be a Goldstone boson $\sim \ln \chi$ which has only derivative interactions - the dilaton. In this approximation of an exact spontaneously broken dilatation symmetry no interesting consequences for late cosmology are expected [9, 2]. However, the impact of the dilaton on cosmology changes dramatically if the effect of the possible intrinsic mass scales is taken into account. In terms of the dilatation symmetry the intrinsic mass scales reflect an “anomaly” induced by the quantum fluctuations. The scalar field $\chi$ can now constitute a dynamical dark energy or quintessence [2] which is relevant for the present universe. Because of its important cosmological consequences the “pseudo-dilaton” $\ln \chi$ has been named the cosmon.

We present here a particular version of crossover quintessence [11] with two intrinsic mass scales generated by running dimensionless couplings. The first mass scale $\chi_c$ indicates a crossover behavior in the derivative interactions of the pseudo Goldstone boson, without introducing a potential. More concretely, it affects the kinetic term of the cosmon and indirectly the location of an ultraviolet fixed point for the running grand unified gauge coupling. The second scale $m$ is analogous to the hadron scale $\Lambda_{QCD}$ in QCD. It is an infrared scale where the running grand unified gauge coupling grows large. This scale $m$ induces a mass term for the cosmon field $\sim m^2 \chi^2$. Similar to the ratio $\Lambda_{QCD}/\bar{M}_p$ the ratio $m/\chi_c$ can be exponentially small.

We will see that for $\ln(m/\chi_c) \approx 1/138$ a realistic cosmology arises, with quintessence responsible for about 70% of the present energy density of the universe. Our model turns out to be compatible with all present cosmological tests.

Furthermore, this model predicts that the fundamental “constants” depend on time [9, 2]. In fact, the running of a typical dimensionless coupling like the fine structure constant $\alpha_{em}$ can be written in the functional form $\alpha_{em}(q^2/\chi^2, \chi^2/\mu^2)$ where $\mu$ is an arbitrary renormalization scale. For fixed $\chi$ the dependence on the squared momentum $q^2$ is governed by the usual $\beta$-function of the standard model. On the other hand, for fixed $q^2/\chi^2$ the variation of $\chi$ is governed by the running of the grand unified gauge coupling $\alpha_X$. The cosmological time dependence of $\chi$ results in a time dependence of the fine structure constant! We show that for reasonable $\beta$-functions for the running grand unified gauge coupling our model can reconcile the claimed time variation from the observation of quasar absorption lines [12] with severe bounds from archeo-nuclear physics [13, 14] and tests of the equivalence principle [15].
Crossover Quintessence

Crossover quintessence (CQ) can be characterized by a recent increase of the fraction in homogeneous dark energy $\Omega_h = \rho_h/\rho_{cr}$ from a small but not negligible value for early cosmology to a value $\Omega_h \approx 0.7$ today. A small early value (say $\Omega_h = 0.01$) typically results from a cosmic attractor solution [2, 16, 17, 18] (“tracker solution”) independently of the detailed initial conditions. For such models of “early quintessence” the dark energy decreases in early cosmology at the same pace as radiation. Dark energy and radiation have therefore always been of a comparable magnitude and excessive “fine tuning” can be avoided [2, 19]. The amount of early quintessence is accessible to observation by nucleosynthesis [2][20], the cosmic microwave background (CMB) [21] or structure formation [22, 23]. The recent crossover to a domination of the universe by dark energy ($\Omega_h \approx 0.7$) may be related to properties of the underlying model \footnote{This typically requires a modest “tuning” of parameters at the level somewhat below 1%} or triggered by matter domination [24] or structure formation [25]. On the level of the equation of state CQ corresponds to a crossover from a value $w_h \approx 1/3$ during the radiation dominated universe to a substantially negative value for $z < 0.5$.

In this note we elaborate on a recent proposal [11] that CQ is connected to the existence of a conformal fixed point in a fundamental theory. This will serve as a natural starting point for a discussion of the variation of couplings. We assume that for low momenta ($q^2 \ll \chi^2$) the yet unknown unified theory results in an effective grand unified model in four dimensions. There the coupling of the cosmon field $\chi$ to gravity and the gauge fields is described by an effective action

$$S = \int d^4x\sqrt{g} \left\{ -\frac{1}{2}\chi^2 R + \frac{1}{2}(\delta(\chi) - 6)\partial^\mu\chi\partial_\mu\chi \
 + m^2\chi^2 + \frac{Z_F(\chi)}{4}F^{\mu\nu}F_{\mu\nu} \right\}. \quad (1)$$

The running of the effective grand unified gauge coupling $g^2_X = \tilde{g}^2/Z_F$ is determined by the $\chi$-dependence of $Z_F$ and we assume a fixed point behavior for $\alpha_X = g^2_X/4\pi$

$$\frac{\partial\alpha_X}{\partial \ln\chi} = b_2\alpha_X - b_4\alpha_X^2 - b_6\alpha_X^3 S(\delta). \quad (2)$$

We take here a small value of $|b_6|$ such that the last term only leads to a small shift in the fixed point value

$$\alpha_{X,*} \approx \frac{b_2}{b_4} \approx \frac{1}{40}. \quad (3)$$

(We choose positive $b_2$ and $b_4$ and $S(\delta)$ will be specified below.) For $\alpha_X(\chi) > \alpha_{X,*}$ the grand unified gauge coupling increases with decreasing $\chi$. Similar to QCD it grows large at some nonperturbative scale (“confinement scale”) that we associate
with \( m \). Our model therefore has an “infrared scale” \( m \) which is generated by dimensional transmutation from the running of the gauge coupling. We assume that \( m \) determines the mass of the cosmon. Neglecting the effects of the dilatation anomaly reflected by \( m \) the cosmon potential would vanish.

For \( \chi \gg m \) the gauge coupling runs towards its fixed point (3) as \( \chi \) increases. At the fixed point the effective action would have an effective dilatation symmetry if the running of \( \delta(\chi) \) could be neglected. We will assume, however, that the running of \( \delta \) is instable towards large \( \chi \) \((E > 0)\) and postulate

\[
\frac{\partial \delta}{\partial \ln \chi} = \beta_\delta(\delta) = E \delta^2. \tag{4}
\]

For small enough \( \delta_i \) \(= \delta(\chi = m) \) the dimensionless coupling \( \delta \) stays small for a large range \( m < \chi \leq \chi_c \). For \( \chi \gg m \) we may neglect the small mass \( m \) for the evolution of the dimensionless couplings. The evolution in this range is then governed by the vicinity to a conformal fixed point \([11]\) at \( \delta = \delta_* = 0 \) , \(\alpha_X = \alpha_{X,*} \). In the same approximation our model corresponds to a flat direction \(^4\) in the effective potential, consistent with conformal symmetry. All cosmon interactions are therefore given by its derivative coupling to the graviton and gauge fields. We note that the fixed point behavior bears a certain resemblance with the “runaway dilaton”. \([26]\)

Nevertheless, as \( \chi \) increases further an effective “ultraviolet scale” \( \chi_c \) characterizes the region where \( \delta \) grows large according to the solution

\[
\delta(\chi) = \frac{1}{E \ln(\chi_c/\chi)}. \tag{5}
\]

The ratio between the ultraviolet and infrared scales turns out exponentially huge

\[
\frac{\chi_c}{m} = \exp\left(\frac{1}{E \delta_i}\right). \tag{6}
\]

At the crossover scale \( \chi_c \) the flow of the couplings witnesses a crossover from the vicinity of the conformal fixed point (small \( \delta \)) to another regime (perhaps a different fixed point) for large \( \delta \). We will see that the crossover behavior in the present epoch of the cosmological evolution precisely corresponds to the crossover from small to large \( \delta \) for \( \chi \) in the vicinity of \( \chi_c \). Realistic cosmology obtains for \( E \delta_i \approx 1/138 \).

The precise behavior of \( \beta_\delta \) in the region of large \( \delta \) will only be of secondary relevance for past and present cosmology provided \( \delta \) grows large enough near \( \chi_c \). For the present note we investigate first a scenario where the increase of \( \delta \) with \( \chi \) is unbounded while \( \delta(\chi) \) is defined for all \( \chi \), namely

\[
\beta_\delta = \frac{E \delta^2}{1 + J_\delta \delta}. \tag{7}
\]

\(^4\)In string theories such flat directions are often called “moduli”. In presence of additional matter fields, e. g. scalar and fermions, the flat direction in the space of scalar fields corresponds to constant ratios of all masses \([11]\).
(We take $J_\delta = 0.05$ and observe that cosmology shows no strong dependence on the value of $J_\delta$. This holds provided that $J_\delta$ is small enough such that the difference between eqs. (4) and (7) matters only in the region of large $\delta$. At the end of this note we will compare this scenario with an alternative where $\beta_\delta$ exhibits a fixed point for large $\delta$.) For very large $\delta$ in the region $\chi \gg \chi_c$ the solution obeys now 

$$\frac{\delta(\chi)}{\delta(\chi_c)} \sim (\chi/\chi_c)^{E/1J_\delta}.$$ 

We observe that for large $\delta$ a standard scalar kinetic term obtains for a rescaled scalar field $\tilde{\chi} \sim \chi^{1/E(2J_\delta)}$. In this language the coupling to gravity vanishes for $\tilde{\chi} \to \infty$ according to $\chi^2 R \sim \delta^{-1} \chi^2 R$. Our first scenario for the behavior at large $\delta$ can therefore also be viewed as a fixed point in the cosmon-gravity coupling $\delta^{-1}$ at $\delta^{-1}_c = 0$.

In the sense of a renormalization group running our first scenario describes a crossover between two fixed points at $1/\delta = 0$ and $\delta = 0$. The scale $\chi$ corresponds to a dynamical infrared scale of the unknown fundamental theory. As it is lowered the flow of the couplings switches from the fixed point at $\delta^{-1}_c = 0$ to the range of attraction of the conformal fixed point at $\delta = 0$. However, this latter fixed point is not stable with respect to the running of the gauge coupling which finally grows large and produces a new infrared scale $m$. This non-perturbative scale lifts the degeneracy in the flat cosmon direction. This type of behavior is common in statistical physics when the crossover between two fixed points plays a role.

**Dark energy**

The cosmological dynamics of CQ is described in [19, 32] and best expressed after Weyl scaling with a constant reduced Planck mass $\bar{M}^2 = M_p^2/8\pi$ and a rescaled dimensionless scalar field $\Phi = \varphi/\bar{M} = 2 \ln(\chi/m)$. In terms of the rescaled metric and cosmon field the effective action (20) reads [19]

$$S = \int d^4x \sqrt{\hat{g}} \left\{ \frac{\bar{M}^2}{2} \left[ k^2(\phi) \partial^\mu \phi \partial_\mu \phi - R \right] + \bar{M}^4 \exp(-\phi) + \frac{Z_F(\phi)}{4} F_{\mu\nu} F^{\mu\nu} \right\}$$  \hspace{1cm} (8)

with $5 \ k^2(\phi) = \delta/4$. The cosmon potential $V = \bar{M}^4 \exp(-\phi)$ vanishes for $\phi \to \infty$. In this language it is easy to see that the asymptotic behavior for increasing time corresponds to increasing $\phi$ and $\chi$.

The cosmological evolution equations are best expressed in terms of the logarithm of the scale factor

$$x = \ln a = -\ln(1 + z) \ , \ \dot{x} = H.$$ \hspace{1cm} (9)

The first equation expresses the change of the energy density of homogeneous quintessence, $\rho_h$, in terms of its equation of state $w_h = p_h/\rho_h$.

$$\dot{\rho}_h + 3(1 + w_h)H \rho_h = 0 \ , \ \frac{d\ln \rho_h}{dx} = -3(1 + w_h).$$ \hspace{1cm} (10)

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5Eq. (4) corresponds to $k^{-2} = 2E(\phi_c - \phi)$. Similar to eq. (7) this is a form of “leaping kinetic term quintessence” [19].

6We have neglected the effects of the cosmon coupling to matter and radiation which are assumed to be very small in the present model.
Using \( \rho_{cr} = 3M^2H^2 = \rho_h + \rho_m + \rho_r \), \( d\ln \rho_m/dx = -3 \), \( d\ln \rho_r/dx = -4 \), \( \rho_r = (a_{eq}/a)\rho_m \), eq. (10) can be translated into an evolution equation for \( \Omega_h = \rho_h/\rho_{cr} \), namely

\[
\frac{d\Omega_h}{dx} = -\Omega_h(1-\Omega_h) \left\{ 3w_h - \left( \frac{1 + e^x}{a_{eq}} \right)^{-1} \right\}. \tag{11}
\]

We recover the fixed points for \( w_h \approx 1/3 \) if \( a = e^x \ll a_{eq} \), and \( w_h \approx 0 \) if \( e^x \gg a_{eq} \). They are relevant for the cosmic attractors in the radiation and matter dominated epochs.

As a second equation we need the time evolution of \( w_h \) for which we find \(^7\)

\[
\frac{dw_h}{dx} = (1 - w_h) \left\{ k^{-1}(\phi) \sqrt{3\Omega_h(1+w_h) - 3(1+w_h)} \right\}. \tag{12}
\]

The dynamics of the particular model enters through \( k^{-1}(\phi) = 2/\sqrt{\delta} \). Our scaling of \( \phi \) is chosen such that at all times the value of \( \phi \) is directly connected with the value of the cosmon potential and therefore also with \( H^2 \),

\[
\frac{H^2}{M^2} = \frac{2}{3} e^{-\phi}[\Omega_h(1-w_h)]^{-1}. \tag{13}
\]

For given present values of the Hubble parameter \( H_0 \) and the energy fractions in matter, radiation, and quintessence, \( \Omega_m^{(0)}, \Omega_r^{(0)} = a_{eq}\Omega_m^{(0)}, \Omega_h^{(0)} = 1 - \Omega_m^{(0)} - \Omega_r^{(0)} \) this expresses \( \phi \) in terms of \( x, \Omega_h \) and \( w_h \)

\[
\phi = 4x - 2\ln(H_0/M) - \ln \left[ \frac{3}{2}(a_{eq} + e^x)\Omega_m^{(0)} \right] - \ln \left( \frac{1-w_h}{1-\Omega_h} \right). \tag{14}
\]

The system of the two differential equations (11)(12) is therefore closed and can easily be solved numerically.

For given \( E \) and \( J_\delta = 0.05 \) we solve the flow equation (7). The “initial value” \( \delta_i = \delta(\chi = m) \) is tuned such that at present \( \Omega_h^{(0)} = 0.73 \). We start for \( \chi(t = 0) = m \) with rather arbitrary values of \( \Omega_h \) and \( w_h \). The late time behavior will not depend on this. In table 1 we present a few characteristic cosmological quantities, namely the present equation of state, \( w_h^{(0)} \), the age of the universe, \( t^{(0)} \), the fraction of dark energy at last scattering, \( \Omega_h^{(ls)} \), the location of the third peak in the CMB unisotropies in angular momentum space, \( l_3 \), as well as the normalization of the spectrum of density fluctuations, \( \sigma_8 \), divided by the value which would obtain for a cosmological constant with the same \( \Omega_h^{(0)} = 0.73 \). Here the “present” values are characterized by the Hubble parameter reaching its present value with \( h = 0.71 \). We recall that smaller \( h \) and larger \( \Omega_h^{(0)} \) shift \( l_3 \) to larger values.

| \( E \) | \( w_h^{(0)} \) | \( t^{(0)}/10^{10} \text{yr} \) | \( \Omega_h^{(ls)} \) | \( l_3 \) | \( \sigma_8/\sigma_8^{(\Lambda)} \) |
|---|---|---|---|---|---|
| 5 | -0.94 | 1.31 | 0.019 | 779 | 0.70 |
| 12 | -0.99 | 1.35 | 0.0083 | 794 | 0.85 |

\(^7\)We use \( d\phi/dx = k^{-1}(6\Omega_h(1-V/\rho_h))^{1/2} \) and \( V/\rho_h = (1-w_h)/2 \).
Table 1: Characteristic cosmological quantities for two CQ models.

We have chosen the cosmological parameters $\Omega_h^{(0)}$ and $\Omega_h^{(l)}$ in accordance with the best fit of the KMAP-data [1] for the case of a cosmological constant ($w_h^{(0)} = -1$). For $E = 12$ the values $w_h^{(0)} = -0.99$ and $\Omega_h^{(ls)} = 0.008$ imply that for the CMB spectrum there is barely any difference as compared to the case of a cosmological constant with the same amount of dark energy [21]. We emphasize, however, that the value $\sigma_8/\sigma_8^\Lambda < 1$ indicates that more power on large scales can be tolerated in order to achieve consistency with the galaxy surveys. This may make the KMAP data more compatible with a $k$-independent spectral index. We have not tried to optimize the choice of $\Omega_h^{(0)}$ and $h$. For $E = 5$ a better agreement with the KMAP-data may be achieved by varying these parameters. We conclude that for large enough $E$ our model is consistent with present cosmological observations.

Cosmological phenomenology of crossover quintessence

For a detailed comparison with cosmological observation one is interested in the dependence of the Hubble parameter on redshift, $H(z)$, and related quantities. Rather than testing each particular model - as characterized here by the function $k(\phi)$ - one wants to describe the function $H(z)$ by a few parameters whose values may be fixed by future cosmological precision tests. We propose to concentrate for this purpose on the average equation of state ($x = -\ln(1+z)$)

$$\bar{w}_h(x) = -\frac{1}{x} \int_x^0 dx' w_h(x').$$

(15)

This function is linked very directly with cosmological observables and admits a simple parameterization. We believe that for the next generation of observations a quadratic formula will be sufficient (for $z < 10^4$)

$$\bar{w}_h(x) = w_h^{(0)} + (\bar{w}_h^{(ls)} - w_h^{(0)}) \frac{x}{x_{ls}} + Ax(x - x_{ls}).$$

(16)

Here the three parameters are the equation of state today, $w_h^{(0)}$, the averaged equation of state at last scattering, $\bar{w}_h^{(ls)}$ with $x_{ls} = -\ln(1100)$, and $A$. (In the future, one wants to measure the whole function $\bar{w}_h(x)$ as precisely as possible - it contains the complete information about the dynamics of dark energy.)

Indeed, from $\bar{w}_h(x)$ one can easily construct the time history of $\Omega_h$ according to

$$\frac{\Omega_h(x)}{1 - \Omega_h(x)} = \frac{\Omega_h^{(0)} (1 + a_{eq}) \exp \left(-3x \bar{w}_h(x)\right)}{1 - \Omega_h^{(0)}} \frac{1 + a_{eq} \exp(-x)}{1 + a_{eq} \exp(-x)}$$

(17)

Adding to $\bar{w}_h(x)$ a term linear in $\delta_i$ would further suppress $\Omega_h$ at early time and bring the phenomenology even closer to the one for a cosmological constant (for given $E$). The prize to pay are unnaturally small values of $\delta_i$, similar to quintessence with an inverse power law potential [16].
with \( a_{eq} \) the scale factor at matter-radiation equality. We can also obtain the equation of state at any time

\[
w_h(x) = \frac{d}{dx}(x \bar{w}_h(x)).
\]

(18)

The function \( H(z) \) is related to \( \bar{w}_h(x) \) by

\[
\frac{H^2(z)}{H_0^2} = \Omega_h^{(0)}(1 + z)^3(1 + \bar{w}_h(x)) + \Omega_m^{(0)}[(1 + z)^3 + a_{eq}(1 + z)^4]
\]

(19)

where \( \Omega_m^{(0)} = (1 - \Omega_h^{(0)})/(1 + a_{eq}) \). We note that \( \bar{w}_h^{(ls)} \) in eq. (16) is directly related to the fraction of dark energy at last scattering, \( \Omega_h^{(ls)} \), via eq. (17). This quantity can be tested by the CMB-unisotropies [21]. For a given model of quintessence we determine \( A \) by equating the fit formula (16) with the model-computation of \( \bar{w}_h \) at \( x = x_{ls}/2 \). For the models displayed in table 1 this yields reasonable fits for the range \( x_{ls} \leq x \leq 0 \), with \( \bar{w}_h^{(ls)} = -0.221, A = -0.016 \) for \( E = 5 \) and \( \bar{w}_h^{(ls)} = -0.261, A = -0.013 \) for \( E = 12 \). We emphasize that the polynomial fit should not be used too far outside this range (e.g. for nucleosynthesis) since CQ typically results in a crossover from \( \bar{w}_h \) near 1/3 for large negative \( x \) to a constant negative value of \( \bar{w}_h \) for large positive \( x \). However, for all observations of recent cosmology from the time of last scattering until now the proposed fit will do reasonably well. The fact that we choose \( \bar{w}_h \) and a polynomial in \( \ln(1 + z) \) permits us to cover the whole range between last scattering and now. Supernovae observations at \( z \approx 1 \) can therefore be linked to structure formation and CMB within a simple parameterization. This would be impossible with a Taylor expansion of \( w_h(z) \) in \( z \) around \( z = 0 \).

**Cosmological history of fundamental couplings**

Under quite general circumstances quintessence or a time varying dark energy can be associated with a scalar field whose cosmological expectation value varies during the recent history of the universe [2]. Generically, this scalar field - the cosmon - may also couple to matter and radiation. As a consequence, the values of “fundamental constants” like the fine structure constant or the ratio between the nucleon mass and the Planck mass depend on the value of the cosmon field and therefore on cosmological time [9, 2, 17, 27, 28]. Recently, a time variation of the fine structure constant has been reported [12] from the observation of quasar absorption lines (QSO) at redshift \( z \approx 2 \). A typical value corresponds to

\[
\frac{\Delta \alpha_{em}(z = 2)}{\alpha_{em}} = -7 \cdot 10^{-6}.
\]

(20)

Similar observations infer \(^9\) a substantially smaller value of \( |\Delta \alpha_{em}| \) at lower redshift \( z \leq 0.7 \). From the Oklo natural nuclear reactor one obtains a typical bound [13];

\(^9\)A recent collection of results on \( \Delta \alpha_{em} \) as well as the time variation of other fundamental constants can be found in [29].
and analysis of the decay rate $Re^{187} \to Os^{187}$ restricts [14]

$$\frac{|\Delta \alpha_{em}(z = 0.13)|}{\alpha_{em}} < 10^{-7} \tag{21}$$

As an obvious question one may ask if an increase of $|\Delta \alpha_{em}|$ by almost two orders of magnitude between $z = 0.13$ and $z = 2$ can be reasonably explained by quintessence models. In this note we demonstrate that this can indeed be the case in a class of models of “crossover quintessence” proposed recently [11]. This contrasts to constant rates $\partial \Delta \alpha_{em}/\partial z = \text{const.}$ or $\partial \Delta \alpha_{em}/\partial t = \text{const.}$ for which the values and bounds (20)-(22) clearly are in discrepancy. The deviation from constant rates also affects strongly the comparison of eq. (20) with bounds from nuclear synthesis [30],[31] and CBM [31].

We are aware that the bound from the Oklo natural reactor is subject to substantial uncertainties due to possible cancellations between effects from the variation of $\alpha_{em}$ and other “fundamental constants” like the mass ratio between pion and nucleon mass $m_{\pi}/m_n$ or the weak interaction rates. Also the QSO observations need further verification and investigation of systematics. Nevertheless, it is well conceivable that we can get a reasonable “measurement” of the function $\Delta \alpha_{em}(z)$ in a not too distant future. Furthermore, the derivative $\partial \Delta \alpha_{em}(z)/\partial z$ at $z = 0$ can be related to precision tests of the equivalence principle [15]. The aim of this note is to present a sample computation how the knowledge of $\Delta \alpha_{em}(z)$ can be used to constrain models of quintessence. For this purpose we will use the value (20) as a “benchmark value” which fixes the strength of the cosmon coupling to matter and radiation. The time history $\Delta \alpha_{em}(z)$ can then be related to the time history of quintessence.

In particular, we may investigate the ratio

$$R = \frac{\Delta \alpha_{em}(z = 0.13)}{\Delta \alpha_{em}(z = 2)} \tag{23}$$

A bound $R < 1/70$ strongly favors [32] quintessence with a time varying equation of state $w_h = p_h/\rho_h$, where the value of $(1 + w_h)$ at present is substantially smaller than for $z = 2$. A logarithmic dependence of $\Delta \alpha_{em}$ on the cosmon field $\chi$ leads to a bound for the present equation of state [32]

$$w_h^{(0)} < -0.9. \tag{24}$$

In this note we investigate how the detailed dependence of $\alpha_{em}$ on $\chi$ influences the time history of $\Delta \alpha_{em}(z)$. We argue that reasonable $\beta$-functions for the “running” $\alpha_{em}(z)$...
of a grand unified coupling with \( \ln \chi \) may lead to nonlinearities in \( \alpha_{em}(\ln \chi) \). Those are quantitatively important, influencing \( R \) within a factor two. The overall picture remains rather solid, however: a small value of \( |R| \) requires a quintessence model where the evolution of the cosmon field has considerably slowed down in the recent history as compared to a redshift \( z \approx 2 \). This feature is characteristic for crossover quintessence and will not be shared by arbitrary quintessence models. In particular, a constant equation of state with \( w_h \) independent of \( z \) over a range \( 0 < z < 3 \) will have severe difficulties to explain \( R < 0.1 \). This demonstrates how a measurement of \( \Delta \alpha_{em}(z) \) could become an important ingredient for the determination of the nature of dark energy.

**Running couplings**

In a grand unified theory the value of the electromagnetic fine structure constant depends on the gauge coupling at the unification scale, \( \alpha_X \), the ratio between the weak scale and the unification scale, \( M_W/M_X \), and particular particle mass ratios like \( m_n/M_W \) or \( m_e/M_W \) for the nucleons and electrons. Since the nucleon mass is determined by the running of the strong gauge coupling similar dependencies arise for the ratio \( m_n/M_X \). Combining the relevant one loop formula and assuming for simplicity \( \chi \)-independent ratios \( m_b/M_W \) etc. for the heavy quarks and \( m_\mu/m_e \) etc. for the leptons one finds \(^{11}[32] \)

\[
\frac{1}{\alpha_{em}(z)} - \frac{1}{\alpha_{em}(0)} = \frac{22}{7} \left( \frac{1}{\alpha_X(z)} - \frac{1}{\alpha_X(0)} \right)
- \frac{17}{21\pi} \ln \left( \frac{M_W/m_n(z)}{M_W/m_n(0)} \right) + \frac{2}{\pi} \ln \left( \frac{M_W/m_e(z)}{M_W/m_e(0)} \right)
\]

(25)

and

\[
\ln \left( \frac{m_n/M}{m_n/M(0)} \right) = -\frac{\pi}{11} \left( \frac{1}{\alpha_{em}(z)} - \frac{1}{\alpha_{em}(0)} \right)
+ \frac{7}{33} \ln \left( \frac{M_W/m_n(z)}{M_W/m_n(0)} \right)
+ \frac{2}{11} \ln \left( \frac{M_W/m_e(z)}{M_W/m_e(0)} \right)
+ \ln \left( \frac{M_X/M}{M_X/M(0)} \right).
\]

(26)

We will make here the simplifying assumptions that the effects due to the variation of \( M_W/m_n \) and \( M_W/m_e \) are subleading, as suggested by the more complete investigations in \(^{32}\) (in the linear approximation).

In the approximation of time-independent \( M_W/m_n \) and \( M_W/m_e \) the dependence of the fine structure constant on redshift can be related directly to the \( \beta \)-function for the grand unified gauge coupling (2). For \( \Delta \alpha_{em}(z) = \alpha_{em}(z) - \alpha_{em}(0) \) one finds (\( x = -\ln(1 + z) \))

\[
\frac{\Delta \alpha_{em}(z)}{\alpha_{em}} = \frac{22\alpha_{em}}{7\alpha_X^2} \Delta \alpha_X(x)
\]

(27)

\(^{11}\)We have eliminated the dependence of \( \alpha_{em} \) on \( M_X/M_W \) in favor of \( M_W/m_n \).
where \( \alpha_X(x) \) obtains as a solution of

\[
\frac{\partial \alpha_X}{\partial x} = \beta_\alpha \frac{\partial \ln \chi}{\partial x} = \frac{\beta_\alpha(\alpha_X, \delta)}{\sqrt{\delta}} \sqrt{3 \Omega_h(1 + w_h)}.
\]  

(28)

One observes that the \( z \)-dependence of \( \Delta \alpha_{em} \) is particularly weak in the region of large \( \delta \) and \( w_h \) close to \(-1\). This will be our explanation why \( R = \Delta \alpha_{em}(z = 0.13)/\Delta \alpha_{em}(z = 2) \) is considerably smaller than expected from a simple extrapolation linear in \( z \). Crossover quintessence is precisely characterized by large \( \delta \) and \( w_h \approx -1 \) in the region of small \( z \), whereas \( \sqrt{1 + w_h}/\sqrt{\delta} \) is considerably larger for intermediate \( z \approx 1.5 - 3 \). One also observes that for constant \( (\Omega_h(1 + w_h)/\delta)^{1/2} \) and \( \eta = \beta_\alpha/\alpha \) the dependence of \( \Delta \alpha_{em}(z) \) is logarithmic in \( 1 + z \). This is crucial for large \( z \) as for nucleosynthesis, where \( \Delta \alpha_{em} \) turns always out to be much smaller than expected from a linear extrapolation in redshift or time.

A quantitative estimate needs information about \( \beta_\alpha \) - the form of the latter may induce further nonlinearities. From our association of the scale \( m \) with the scale where \( \alpha_X \) grows large the \( \beta \)-function (2) would lead to a decrease of \( \alpha_X \) for decreasing \( z \) if the last term \( \sim S(\delta) \) is neglected. This would result in a positive \( \Delta \alpha_{em}(z = 2) \), in contrast to the negative QSO value (20). Starting with large \( \alpha_X(\chi = m) \) (we take \( \alpha_X(\chi = m) = 100 \) for definiteness) one would actually find values of \( |\Delta \alpha_{em}(z = 2)| \) much smaller than the QSO value if \( b_2 \) exceeds 0.1. This demonstrates that a fixed point behavior for the running of \( \alpha_X \) (and similar for other couplings) provides a powerful explanation why the time variation of “fundamental constants” is actually a small effect!

There is, however, no reason why the fixed point value \( \alpha^*_X \) should be precisely independent of \( \delta \). We parameterize a possible \( \delta \)-dependence by adding the last term in eq. (2) with

\[
S(\delta) = -\frac{\delta}{1 + J_\alpha \delta} + 6.
\]

(29)

This leads \(^{12}\) to a crossover of the effective fixed point \( \alpha^*_X(\delta) \) between two values for small and large \( \delta \). For \( b_6 > 0 \), \( J_\alpha > 0 \) the value of \( \alpha^*_X(\delta) \) increases with increasing \( \delta \) and this will induce \( \Delta \alpha_{em}(z = 2) < 0 \). At this stage the form of \( \beta_\alpha \) is parameterized by four parameters \( b_2, b_4, B_6, J_\alpha \). Since the influence of the term \( S(\delta) \) on the location of the fixed point is small the ratio \( b_2/b_4 \) is fixed by eq. (3).

**Time variation of the fine structure constant**

At this point our model is fixed and we can now compute the detailed cosmological evolution of the fine structure constant. The overall normalization of the time variation strongly depends on \( b_6 \). As a benchmark we use eq. (20). Therefore, for given \( b_2 \) and \( J_\alpha \) we fix \( b_6 \) such that \( \Delta \alpha_{em}(z = 2) = -7 \cdot 10^{-6} \). The shape of \( \Delta \alpha_{em}(z) \)

\(^{12}\)We choose \( S(\delta = 0) > 0 \) such that for \( b_6 > 0 \) no new fixed point for \( \alpha_X \) is produced in the region of large \( \alpha_X \). The value \( S(\delta = 0) = 6 \) is unimportant.
depends\textsuperscript{13} now on $b_2$ and $J_α$ as well as on on $E$. In table 2 we show the values of $Δα_{em}$ at $z = 0.13, 0.45, 1100$ and $10^{10}$ for several values of $(b_2, J_α, E)$.

| model | $z = 0.13$ | $z = 0.45$ | $z = 1100$ | $z = 10^{10}$ | $R$  | $η$  |
|-------|------------|------------|------------|-------------|------|------|
| A     | $-6 \cdot 10^{-8}$ | $-2.6 \cdot 10^{-7}$ | $-6.5 \cdot 10^{-5}$ | $-7.8 \cdot 10^{-5}$ | 0.0085 | $5.1 \cdot 10^{-14}$ |
| B     | $-6.2 \cdot 10^{-8}$ | $-2.7 \cdot 10^{-7}$ | $1.3 \cdot 10^{-4}$ | $3.1 \cdot 10^{-5}$ | 0.0088 | $4.9 \cdot 10^{-14}$ |
| C     | $-7.8 \cdot 10^{-8}$ | $-3.5 \cdot 10^{-7}$ | $-2.5 \cdot 10^{-5}$ | $-2.8 \cdot 10^{-5}$ | 0.011 | $8.9 \cdot 10^{-14}$ |

Table 2a: Time variation of the fine structure constant $Δα_{em}(z)/α_{em}$ for various redshifts $z$, for $E = 12$. The models for $β_α$ are (A): $J_α = 6, b_2 = 0.2, b_6 = 0.84$, (B): $J_α = 6, b_2 = 0.05, b_6 = 1.12$, (C): $J_α = 1, b_2 = 0.2, b_6 = 0.15$. For all models $Δα_{em}(z = 2)/α_{em} = -7 \cdot 10^{-6}$. We also show the differential acceleration $η$.

| model | $z = 0.13$ | $z = 0.45$ | $z = 1100$ | $z = 10^{10}$ | $R$  | $η$  |
|-------|------------|------------|------------|-------------|------|------|
| A     | $-1.4 \cdot 10^{-7}$ | $-7.1 \cdot 10^{-7}$ | $-4.4 \cdot 10^{-5}$ | $-5.7 \cdot 10^{-5}$ | 0.020 | $4.3 \cdot 10^{-14}$ |
| B     | $-2 \cdot 10^{-7}$ | $-9.3 \cdot 10^{-7}$ | $1.7 \cdot 10^{-4}$ | $3.6 \cdot 10^{-3}$ | 0.028 | $6.2 \cdot 10^{-14}$ |
| C     | $-2 \cdot 10^{-7}$ | $-9.9 \cdot 10^{-7}$ | $-2.1 \cdot 10^{-5}$ | $-2.3 \cdot 10^{-5}$ | 0.029 | $8.7 \cdot 10^{-14}$ |

Table 2b: Same as table 2a, for $E = 5$. The models for $β_α$ are (A): $J_α = 6, b_2 = 0.2, b_6 = 0.385$, (B): $J_α = 6, b_2 = 0.05, b_6 = 0.783$, (C): $J_α = 1, b_2 = 0.2, b_6 = 0.068$.

Consistency with the Oklo bound (21) and the Re-decay bound (22) is found for large $E$, in particular for large $J_α$. It becomes clear that a precise knowledge of $Δα_{em}(z)$ could be used as a probe for the dynamics of quintessence. For example, it seems very difficult to achieve $R = |Δα_m(z = 0.13)/Δα_{em}(z = 2)| \leq 1/70$ for small values of $E$, quite independently of the precise form of $β_α$. The present investigation confirms that a small value of $R$ requires $(1 + w_h^{(0)}) \ll 1$. This property of the equation of state seems to be favored by the cosmological tests (cf. table 1). We conclude that the bound (24) can be justified by our nonlinear analysis for CQ.

The generic result that $w_h^{(0)}$ should be close to $-1$ does not depend on the precise values of $b_2$ and $J_α$. Increasing $b_2$ beyond $b_2 = 0.2$ (say $b_2 = 1$) induces almost no change. On the other hand, for $b_2$ substantially smaller than 0.05 we come to a region where the fixed point $α_{X,*}$ has not yet been reached with high precision and $Δα_{em}(z = 2)$ is positive. For $b_2 = 0.05$ we have the interesting effect that $Δα_{em}(z)$ reaches a minimum near $z = 2$, changing sign at somewhat higher redshift. Only for the large value $E = 12$ the functional dependence of $Δα_{em}(z)$ deviates substantially from a linear behavior in $z$ in the range $0.5 < z < 2$, as seems to be required by observation. For the models (A) (B) (cf. table 2) one finds a strong “jump” of $Δα_{em}$ by a factor of 5 between $z = 1$ and $z = 2$.

Let us next turn to the high values $z \approx 10^{10}$ characteristic for nucleosynthesis and ask if the values for $Δα_{em}$ quoted in table 2 are consistent with observation.\textsuperscript{13}

\textsuperscript{13}In our case an increase of $|β_α|$ for small $z$ partially cancels the decrease of $(Ω_h(1 + w_h)/δ)^{1/2}$. 

12
For nucleosynthesis, a change in $m_n/\bar{M}$ modifies the clock, i.e. the rate of decrease of temperature in time units $\sim m_n^{-1}$. Furthermore, a change in $\alpha_{em}$ affects the proton to neutron mass difference. Let us assume that the time-variation of $m_n/\bar{M}$ is dominated by $\beta_\alpha$ via the change of the strong gauge coupling (and therefore the ratio $\Lambda_{QCD}/M_X$). Including only the first term in eq. (26) yields

$$\Delta(m_n/\bar{M}) (m_n/\bar{M}) = \frac{\pi}{11\alpha_{em}} \frac{\Delta\alpha_{em}}{\alpha_{em}} = 39.1 \frac{\Delta\alpha_{em}}{\alpha_{em}}. \quad (30)$$

A decrease of $m_n$ relative to $\bar{M}$ is equivalent to an increase of $\bar{M}$ at fixed $m_n$ (and thereby to a decrease of Newton’s constant). In turn, this decreases the Hubble parameter for a fixed temperature $T$ (measured in units of $m_n$). The net effect is equivalent to the substraction of a (fractional) number of neutrino species. We can thereby turn the limits on the effective number of neutrino species $\bar{N}_\nu$ into a limit on $\Delta\alpha_{em}/\alpha_{em}$ at $z \approx 10^{10}$. Assuming that the primordial helium abundance $Y_p$ is in agreement with the value for time independent couplings within an uncertainty $|\Delta Y_p/Y_p| < 8 \cdot 10^{-3}$ and using $\Delta \ln Y_p = \frac{1}{3} \Delta \ln (m_n/\bar{M})$, yields the bounds

$$|\Delta \ln (m_n/\bar{M})| < 0.025 \quad (31)$$

and

$$\left| \frac{\Delta\alpha_{em}(z = 10^{10})}{\alpha_{em}} \right| < 6.4 \cdot 10^{-4}. \quad (32)$$

We emphasize that an effective neutrino number $\bar{N}_\nu < 3$ would be a clear signal for a time variation of couplings! We also observe that bounds on $m_n/\bar{M}$ as (31) can be interpreted equivalently as bounds on the time variation of Newton’s constant (with $m_n$ fixed).

We may compare the effect of the change of $m_n/\bar{M}$ with the effect of the change in the proton-neutron mass difference $\delta m = m_n - m_p$ which contributes $\Delta \ln Y_p \approx \frac{1}{3} \Delta \ln (m_n/\bar{M}) - \Delta \ln \delta m$. Keeping the ratios of up-and down quark masses to $m_n$ fixed one has [34] $\Delta \ln \delta m \approx -0.6 \Delta \ln \alpha_{em}$. The contribution to $\Delta \ln Y_p$ is far less than the contribution from $\Delta \ln (m_n/\bar{M})$. This explains why the bound (32) is more restrictive than several previous bounds [30], [31] concentrating on the effect from $\Delta \ln \delta m$. We are aware that the variation of $\Delta \ln (m_n/\bar{M})$ may be affected by the neglected terms in eq. (26). If there are no particular cancellations, however, the bound (32) should remain of the same order of magnitude.

The size of $\Delta\alpha_{em}$ at nucleosynthesis and even the sign depend critically on $b_2$ (cf. table 2). If we adopt the bound

$$\left| \frac{\Delta\alpha_{em}(z = 10^{10})}{\alpha_{em}} \right| < 10^{-3} \quad (33)$$

\footnote{This relation differs from [33] where $M_W/M_X$ is kept fixed instead of fixed $m_n/M_W$ in the present work. The quantitative difference is not very important, however.}
we infer that $b_2$ should be larger than 0.05. It is striking that the values of $\Delta \alpha_{em}/\alpha_{em}$ at nucleosynthesis are very far from an extrapolation with $\partial \ln \alpha_{em}/\partial t = const.$ or even from $\partial \ln \alpha_{em}/\partial \ln a = const..$ The nonlinearity in $\partial \ln \alpha_{em}/\partial \ln a$ is mainly due to the attraction of $\alpha_X$ to a $\delta$-dependent fixed point. Both for $E = 5, 12$ the models with sufficiently large $b_2$ meet the bounds (32) or (33) (model A). They are also consistent with bounds from the CMB unisotropies [31].

Precision experiments with atomic clocks measure the variation $\Delta \alpha_{em}/\alpha_{em}$ per year

$$\frac{\dot{\alpha}_{em}}{\alpha_{em}}[yr^{-1}] = -H[yr^{-1}] \frac{\partial \ln \alpha_{em}}{\partial \ln(1+z)}$$

$$= -5.44 \cdot 10^{-10} \frac{\Delta \alpha_{em}(z = 0.13)}{\alpha_{em}} yr^{-1}. \quad (34)$$

Here the Hubble parameter is expressed in units of $yr^{-1}$ and we use the observation that for our models $\partial \ln \alpha_{em}/\partial x$ is essentially constant in the range $0 < z < 0.13$. Measuring the values quoted in table 2 would need a considerable improvement of the present accuracy [35] $\dot{\alpha}_{em}/\alpha_{em} = (4.2 \pm 6.9) \cdot 10^{-15} yr^{-1}$. Using eq. (30) we obtain a similar formula for the variation of Newton’s constant in the range of low redshift,

$$\frac{\dot{G}}{G} = 78.2 \frac{\dot{\alpha}_{em}}{\alpha_{em}} = -4.25 \cdot 10^{-8} \frac{\Delta \alpha_{em}(z = 0.13)}{\alpha_{em}}, \quad (35)$$

safely below the present bounds.

In our approximation where the dominant field dependence of the various couplings arises from $\beta_\alpha$ we can also estimate the size of the differential acceleration $\eta$ between two test bodies with equal mass but different composition. One finds [32]

$$\eta = 5 \cdot 10^{-2} \left( \frac{\beta_\alpha}{\alpha_X} \right)^2 \delta^{-1} \Delta R_Z \quad (36)$$

where for typical experimental tests of the equivalence principle $\Delta R_Z = \Delta Z/(Z + N) \approx 0.1$. Here $\beta_\alpha/\alpha_X$ and $\delta$ have to be evaluated at $z = 0$. Results for $\eta$ are also displayed in table 2. We conclude that our CQ-model is compatible with the present experimental bounds [15] $|\eta| < 3 \cdot 10^{-13}$ for most choices of $\beta_\alpha$. Nevertheless, the models with $J_\alpha = 1$ show that already now the tests of the equivalence principle can help to discriminate between different models! It is obvious that further improvements of the accuracy of tests of the equivalence principle should either confirm or reject the interpretation of the QSO-results within our CQ-model.

In summary, we have found models of crossover quintessence that can explain the QSO value for the time dependence of the fine structure constant and are nevertheless compatible with all observational bounds on the time variation of couplings and tests of the equivalence principle. This explanation does not involve any particular cancellation of effects from the variation of different couplings. The parameter set $E = 12, b_2 = 0.2, J_\alpha = 6$ may serve as an illustration. The same models do also very well with all present cosmological tests!
Variation of electron-proton mass ratio

Let us finally ask if our assumption that $\Delta \alpha_{em}$ is dominated by the change of $\alpha_X$ can be justified. In addition to the arguments presented in [32] we may also have a look at the recently reported result [36] on the variation of $m_e/m_p$ in the range $z = 2.3 - 3$

$$\Delta \left( \frac{m_e}{m_p} \right) / \left( \frac{m_e}{m_p} \right) = (-5.7 \pm 3.8) \cdot 10^{-5}. \quad (37)$$

The deviation from zero is statistically not very significant. Nevertheless, it is interesting to know if such an effect would influence the time history of the fine structure constant and the tests of the equivalence principle. We therefore investigate what would be the effect on $\Delta \alpha_{em}$ of a variation $\Delta \ln(\frac{m_e}{m_n})(z = 2) = -6 \cdot 10^{-5}$. Keeping $M_W/m_n$ fixed would yield a contribution (cf. eq. (25))

$$\frac{\Delta \alpha_{em}(z = 2)}{\alpha_{em}} = \frac{2\alpha_{em}}{\pi} \Delta \ln \left( \frac{m_e}{m_n} \right) \approx -2.8 \cdot 10^{-7}. \quad (38)$$

This amounts only to 4% of the QSO value (20). Similarly, the contribution (cf. eq. (26)) to

$$\ln \left( \frac{m_n/\bar{M}}{(m_n/\bar{M})(z = 0)} \right) \approx -\frac{2}{11} \Delta \ln \left( \frac{m_e}{m_n} \right) \approx 1.1 \cdot 10^{-5} \quad (39)$$

is only 4% of the effect due to $\Delta \alpha_{em}$, i.e.

$$\ln \left( \frac{m_n/\bar{M}}{(m_n/\bar{M})(z = 0)} \right) = \frac{\pi}{11\alpha_{em}} \frac{\Delta \alpha_{em}(z = 2)}{\alpha_{em}} \approx -2.7 \cdot 10^{-4}. \quad (40)$$

Neglecting those contributions seems therefore well justified. Comparison of eqs. (40) and (37) incidentally shows that the value (37) corresponds roughly to the size of the effect that would be expected in absence of particular cancellations - with $\ln(\frac{m_e/\bar{M}}{(m_e/\bar{M})(z = 0)} \approx -3.3 \cdot 10^{-4}$. The effect of $\Delta \ln(\frac{m_e}{m_n})$ on the tests of the equivalence principle is more sizeable. Adding this effect to the differential acceleration multiplies $\eta$ in eq. (36) by a factor $1 + \tilde{Q}$ [32]

$$\tilde{Q} \approx 10 \frac{d(\frac{m_e}{m_n})}{d\alpha_{em}} \bigg|_{z=0} \approx 10 \frac{\Delta(\frac{m_e}{m_n})}{\Delta\alpha_{em}} (z = 2) \approx 6, \quad (41)$$

where the quantitative estimate neglects a possible $z$-dependence of $\tilde{Q}$. Multiplication of the $\eta$-values in table 2 by a factor 7 shows that for part of the models a value $\Delta \ln(\frac{m_e}{m_p})(z = 2) = -6 \cdot 10^{-5}$ enters in conflict with the tests of the equivalence principle. The models with $E = 12$ and $J_{\alpha} = 6$ may be considered as borderline in view of the possibility of partial cancellations with effects from the variation of other mass ratios.
The future universe

If we live today a crossover period of our universe, to which future will the transition lead? In our model, the answer depends in important aspects on the behavior of $\beta_\delta$ for large $\delta$. The scenario that we have discussed first is characterized by ever increasing $\delta$ (or the approach to a fixed point $(1/\delta)_* = 0$). In this case $\Omega_h$ will grow towards one and $w_h$ approaches $-1$ in future times. In these respects the universe will resemble asymptotically a universe characterized by a cosmological constant. Nevertheless, if $k(\phi)$ remains finite for all finite $\phi$, the dark energy density $\rho_h$ will not approach a constant since $w_h + 1$ will always remain positive, approaching zero only for $\phi \to \infty$. An asymptotic behavior $\beta_\delta = D\delta$ for large $\delta$ corresponds to $k(\phi) = k_C \exp \left\{ \frac{D}{4}(\phi - \phi_c) \right\}$. This is the asymptotic behavior of inverse power law quintessence [16]: rescaling the cosmon kinetic term leads to a potential $V(\tilde{\phi}) \sim \tilde{\phi}^{-\alpha}$, $\alpha = 4/D$. In leading order one finds $1 + w_h = 1/(3k^2)$. The potential energy decreases logarithmically with $a$, $V \sim (\ln a)^{-2/D}$ and we infer the asymptotic behavior $\Omega_m \sim (\ln a)^{2/D}/a^3$, $\Omega_h = 1 - \Omega_m$.

As a second scenario we discuss the alternative that $\beta_\delta$ has a zero at some finite value $\delta_*$. As a natural choice we take $\delta_* = 6$, noting that for $\delta = 6$ the kinetic term for $\chi$ in eq. (1) vanishes, and consider

$$\beta_\delta = E\delta^2 \left( 1 - \frac{\delta}{6} \right). \quad (42)$$

Our cosmological model describes now a crossover between the two fixed points $\delta_* = 0$ and $\delta_* = 6$, being relevant for the early and late universe, respectively. The solution $\delta(\chi)$ to eq. (42) obeys

$$\frac{\delta}{1 + \frac{4}{6} \ln \left( \frac{\tilde{\phi}}{3} - \frac{1}{6} \right)} = \frac{1}{E \ln(\chi_c/\chi)} \quad (43)$$

and approaches the solutions of eqs. (4) or (7) for small $\delta$. The asymptotic cosmology for large $t$ is different, however. It corresponds to “power law inflation” where the equation of state approaches the asymptotic value $w_h^\infty = -7/9$, with

$$V = \frac{24M^2}{t^2}, \quad T = \frac{M^2}{2} \phi^2 = \frac{3M^2}{t^2}, \quad H = \frac{3}{t},$$

$$\delta = 6 - \frac{36}{e} \exp[-3E(\phi - \phi_c)]. \quad (44)$$

The Hubble parameter decreases $\sim t^{-1}$, with $a \sim t^3$. Comparing $\rho_h \sim a^{-2/3}, \rho_m \sim a^{-3}$ we conclude that $\Omega_h \to 1, \Omega_m \to a^{-7/3}$. As for the first scenario, the future universe will be completely dominated by dark energy. However, the dynamics of the dominant quintessence component differs between the two scenarios.

\[\text{For our ansatz (7) with } D = E/J_\delta \text{ we find very small } \alpha, \text{ i.e. for } E = 5, J_\delta = 0.05 \text{ follows } \alpha = 0.04.\]
We have displayed the characteristic cosmological observables in table 3. In order to demonstrate the spread in this type of models we have chosen $h = 0.65$ and $\Omega_h^{(0)} = 0.7$ and also display smaller values of $E$ as compared to table 1.

| $E$ | $w_h^{(0)}$ | $t_0/10^{10}$ yr | $\Omega_{ls}^{(0)}$ | $l_3$ | $\sigma_8/\sigma_8^{(A)}$ |
|-----|-------------|-----------------|-----------------|-----|-----------------|
| 2   | -0.65       | 1.31            | 0.045           | 768 | 0.46            |
| 5   | -0.81       | 1.37            | 0.019           | 795 | 0.69            |

Table 3: Characteristic cosmological quantities for CQ-models with fixed point at $\delta_s = 6$.

We also show in table 4 the time variation of $\alpha_{em}$, using in eqs. (2)(29) $J_\alpha = 0$ and tuning $b_6$ such that $\Delta \alpha_{em}(z = 2)/\alpha_{em} = -7 \cdot 10^{-6}$.

| $E$ | $b_2$ | $b_6$ | $\Delta \alpha_{em}(z = 0.13)$ |
|-----|-------|-------|-------------------------------|
| 2   | 0.2   | 0.011 | $-1.1 \cdot 10^{-6}$ |
| 5   | 0.06  | 0.022 | $-1.8 \cdot 10^{-6}$ |
| 5   | 0.2   | 0.01  | $-9.5 \cdot 10^{-7}$ |

Table 4: Variation of $\Delta \alpha_{em}(z = 0.13)/\alpha_{em}$ for fixed $\Delta \alpha_{em}(z = 2)/\alpha_{em} = -7 \cdot 10^{-6}$. The models for $\beta_\alpha$ use $J_\alpha = 0$.

It becomes obvious that the bound (21) is not obeyed. Consistency with the Oklo-natural reactor results would therefore require a substantial cancellation between effects from the time variation of different couplings.

**Testing the model**

Within a rather wide class of quintessence models we have seen how a combination of the QSO-result (20) with the bounds on $\dot{\alpha}_{em}$ at lower redshift (21) (22) restricts the allowed parameter space. The present status of observations favors a rather sharp crossover for quintessence, where the equation of state has shifted from a moderately negative value at $z \approx 2$ to a value very close to $-1$ for the recent cosmological epoch. An example is given by the effective action (1) with running couplings determined by the $\beta$-functions (7)(2)(29), and parameters $E = 12, J_5 = 0.05, b_2 = 0.2, b_4 = 8, b_6 = 0.84, J_\alpha = 6$ (first line in table 1, model A in table 2a). Without the need of excessive tuning of parameters or invoking cancellation effects from the time variation of different couplings, this demonstrates the existence of a field theoretical model that is consistent with all data on the cosmological evolution and the time variation of couplings.

What will be the tests of this model? First of all, the QSO result should be confirmed by new independent data. Hopefully this will allow for more precise restrictions on the shape of the function $\Delta \ln \alpha_{em}(z)$ which can be directly compared.
with the prediction of this model. Second, an improvement of the accuracy of the tests of the equivalence principle by an order of magnitude should lead to a direct detection of the new interaction mediated by the cosmon field. In view of its important role in present cosmology we may call this fifth force (besides gravity electromagnetism, weak and strong interactions) “cosmo-dynamics”. Third, cosmological tests should distinguish crossover quintessence from a cosmological constant. The main signature in this respect is probably not the equation of state of dark energy in the most recent epoch (say for $0 < z < 0.5$). The value of $w_h^{(0)}$ in table 1 shows that this quantity may be quite near to the value for a cosmological constant (i.e. $w_h^{(0)} = -1$). If the QSO-results on the time variation of the fine structure constant are to be explained by quintessence this requires a substantial evolution of the cosmon field in the epoch near $z = 2$ (as compared to the rate of evolution today). We therefore expect that $w_h(z)$ increases with increasing $z$, leading to $\Omega_h(z)$ substantially larger than for a cosmological constant during structure formation and last scattering. Test of structure formation and precision tests on the CMB may be able to measure the value of $\sigma_8/\sigma_8^{(A)}$ and $\Omega_{h}^{(s)}$ in table 1. Finally, an improvement of the accuracy of atomic clocks by three orders of magnitude would replace the bounds (21)(22) within an experimentally well controlled setting.

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