The Method of Maximum Power Transmission Efficiency for the Design of Antenna Arrays

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ABSTRACT Various methods have been proposed to control the electromagnetic fields in the near- or far-field region, one of which is to use antenna arrays. This paper reviews a technique, called the method of maximum power transmission efficiency (MMPTE), for the design of antenna arrays and wireless power transmission (WPT) systems. The method is motivated by the fact that all wireless systems aim to maximize the power transmission efficiency (PTE) between the transmitter and receiver and the PTE can thus be considered as a performance index for the design of WPT system and antenna as well. The MMPTE makes use of the PTE as the performance index to be optimized, and reduces a field synthesis problem into a circuit analysis problem, and is therefore easy to master and implement. It overcomes all the challenges associated with conventional array design methods, and is applicable to the design of any type of antenna array in any environment. This paper summarizes the basic theory of MMPTE and demonstrates MMPTE through a number of near- and far-field applications, including focused antennas, smart antennas, shaped beam antennas, end-fire antennas, multi-beam antennas, polarization-reconfigurable antennas, and wireless power transmission systems.

INDEX TERMS Antenna array, wireless power transmission, pattern synthesis, near-field region, far-field region.

I. INTRODUCTION

In microwave frequency regime, antenna arrays are often used to realize desired radiation patterns in the far-field region [1], [2]. The conventional methods for antenna pattern synthesis are problem specific and rely on certain simplifications in order to reduce the complexity of the optimization process. Antenna engineers have been facing a number of challenges associated with array designs. For example, the pattern multiplication rule fails and conventional array analysis based on the antenna array factor is no longer valid when the array elements are not identical or its surrounding environment is too complicated or the inter-element spacing is very small. In view of the fact that numerous antenna arrays are to be deployed in next-generation wireless communication systems, a method that can solve the existing challenges associated with conventional array design methods is urgently required.

Some progresses have been made in recent years in order to overcome the above challenges. For example, a field synthesis method based on a prescribed field distribution on a closed surface enclosing the antenna array has been provided in [3]. By minimizing the active power flow of the error field, an optimal or suboptimal distribution of excitations can be obtained. In this paper, we present a systematic review of an optimization technique for the design of antenna arrays and wireless power transmission (WPT) systems which can overcome the aforementioned challenges, called the Method of Maximum Power Transmission Efficiency (MMPTE). In order to achieve a desired field pattern in the near- or far-field region, a performance index (target function) is usually introduced and optimized during the design of antenna. For the design of a wireless system intended either for the transmission of information or power, a natural performance index is the power transmission efficiency (PTE) between the transmitting (Tx)
and receiving (Rx) antennas, which is defined as the ratio of 
the power delivered to the load of the receiver to the input 
power of the transmitter. To attain the best possible quality 
of wireless communication or power transfer, the PTE has 
to be maximized. Motivated by the fact that all antennas are 
designed to enhance the PTE for a wireless system, the PTE 
can thus be considered as a performance index for the design 
of antennas.

The MMPTE for multiple antenna system was first 
proposed by the author in 2010 [4], and has evolved into 
a powerful optimization method that can be used to design 
a antenna arrays to achieve various field patterns, either in the 
near- or far-field region and in any environment [5]–[31]. 

Conventional methods of antenna synthesis largely rely on 
field theory, while the MMPTE reduces the field synthe-
sis problem into a circuit analysis problem so that the 
circuit theory can be applied to solve the original field 
problem. This feature of MMPTE makes the design pro-
cess of antenna array more accessible for those who are 
not very familiar with electromagnetic (EM) field theory. 
The circuit parameters can be acquired either by simula-
tion or by measurement, and for this reason, the MMPTE is 
applicable to any complicated problem. Whenever the design 
problem cannot be handled by a state-of-the-art computer, 
one can resort to measurement to obtain the circuit param-
ters. Another important feature of the MMPTE is that it 
contains the information of the environment between Tx and 
Rx arrays, and therefore can be made adaptive to complicated 
environments, guaranteeing the best possible performance of 
the antenna array. The MMPTE has been demonstrated to 
be superior to most existing array design methods in terms 
of simplicity, applicability, generality and design accuracy. 

It generates an optimized distribution of excitation (ODE) 
for the antenna array to assure that the gain of the array 
is maximized for a fixed array configuration and is equally 
applicable for both near- and far-field synthesis problems.

Applications of MMPTE and its latest developments are 
scattered throughout various publications, and its unique 
value has not been fully recognized by many researchers. 
The main purpose of this paper is to give a detailed review 
of the MMPTE covering the basic theory and typical appli-
cations. The paper is organized as follows. A comprehen-
sive description of the theory of power transmission between Tx and Rx arrays of a generic WPT system is presented in 
Section II. The most general expression of the PTE for the 
WPT system is derived and then optimized, which yields an 
expression for determining the ODE of the Tx antenna array is 
obtained. Section IV demonstrates the MMPTE through 
a number of practical designs in both near- and far-field 
regions, including focused antennas, smart antennas, shaped 
beam antennas, end-fire antennas, multi-beam antennas, and 
polarization-reconfigurable antennas as well as their appli-
cations in mobile devices, base stations, implanted devices, 
microwave hyperthermia, RFID systems, energy harvesting, 
etc. Section V shows how the MMPTE is applied to the 
design of WPT systems. Finally, the main features of the 
MMPTE are summarized in Section VI.

II. THEORY OF MMPTE

The theory of power transmission between two antennas has 
been investigated by many authors [32]–[39]. For a power 
transmission system consisting of antenna 1 and 2, the 
PTE is defined as the ratio of the power received by 
antenna 2 to the input power of antenna 1, and can be 
expressed by [4], [5], [32]

\[
\text{PTE} = \frac{\left| \int_{S_1} (E_1 \times H_2 - E_2 \times H_1) \cdot u_n dS \right|^2}{4 \text{Re} \int_{S_1} (E_1 \times H_1) \cdot u_n dS \int_{S_2} (E_2 \times H_2) \cdot u_n dS} \tag{1}
\]

where \( E_i \) and \( H_i \) stand for the fields generated by antenna 
\( i \) when antenna \( j(i \neq i) \) is receiving; \( S_i \) \((i = 1, 2) \) denotes 
the closed surface that encloses antenna \( i \) only; the over-
bar represents the complex conjugate operation; and \( u_n \) is 
unit outward normal to the surface \( S_i \). The PTE gets maxi-
mized if the fields satisfy the conjugate matching conditions 
\( E_1 = \bar{E}_2 \) and \( H_1 = -\bar{H}_2 \) on the closed surface \( S_1 \) or \( S_2 \). 
Determining \( E_i \) and \( H_i \) with antenna \( j(i \neq i) \) in place is not 
easy. When the two antennas are in the Fresnel region or far-
field region of each other, the problem can be simplified by 

neglecting the reflections between the antennas. In this case, 
the fields \( E_i \) and \( H_i \) can be obtained by removing the antenna 
\( j(i \neq i) \). For a WPT system consisting of two planar ap-
tures, the optimization of (1) yields an eigenvalue equation. 
The maximum eigenvalue of the eigenvalue equation gives 
the maximum PTE and the corresponding eigenvector is 
the optimized aperture field distribution [34]–[39]. Once the 
PTE, the transmission distance and the operating frequency 
are specified, one can increase the aperture size of antenna to 
meet the specifications. When the Tx and Rx antennas are 
in the Fresnel region of each other, the optimized aperture 
field distributions for both Tx and Rx antennas are found to 

obey a spherical distribution in phase and a Gaussian-like 
distribution in amplitude. Physically this implies that the Tx 
and Rx antennas must focus to each other [37]. 

Realizing the optimized aperture distribution via a sin-
gle aperture is impossible in most cases. In addition, the 
optimization of PTE based on (1) gets very complicated 
when two antennas are in close proximity. To overcome 
these difficulties, one can consider the power transmis-
sion between two antennas from the circuit point of view 

instead. The circuit theory of power transmission between
two antenna arrays was first investigated in [4]. Consider a generic WPT system located in an arbitrary scattering environment, as illustrated by Fig. 1. The system consists of \( m \) Tx antennas and \( n \) Rx antennas, and each Rx antenna is terminated by a load of reflection coefficient \( \Gamma_i (i = m + 1, m + 2, \ldots, m + n) \). The EM waves emanated from the Tx antenna array may bump into a cluster of obstacles and scatterers in the propagation medium and get reflected, refracted and diffracted before they reach the Rx antenna array. The separation between the Tx and Rx antenna arrays is assumed to be arbitrary. The system forms an \((m+n)\)-port network and may be characterized by scattering parameters. The normalized reflective waves and incident waves at the ports are related by

\[
\begin{bmatrix}
[b_l] \\
[b_r]
\end{bmatrix} = \begin{bmatrix}
[S_{rl}] & [S_{rr}]
\end{bmatrix} \begin{bmatrix}
[a_l] \\
[a_r]
\end{bmatrix},
\]

(2)

where

\[
[a_l] = [a_1, a_2, \ldots, a_m]^T, \quad [b_l] = [b_1, b_2, \ldots, b_m]^T
\]

are respectively the normalized incident and reflected waves for the Tx array and

\[
[a_r] = [a_{m+1}, a_{m+2}, \ldots, a_{m+n}]^T, \quad [b_r] = [b_{m+1}, b_{m+2}, \ldots, b_{m+n}]^T
\]

are respectively the normalized incident and reflected waves for the Rx array. The superscript \( T \) denotes the transpose operation. For the Rx array, the normalized incident and reflective waves are related by the condition imposed by the loads

\[
[a_r] = [\Gamma_L][b_r],
\]

(3)

where

\[
[\Gamma_L] = \text{diag}[\Gamma_{m+1}, \Gamma_{m+2}, \ldots, \Gamma_{m+n}]
\]

is the load reflection coefficient matrix. Combining (2) and (3) yields

\[
[b_l] = [S_{rl}][a_l] + [S_{rr}][\Gamma_L][b_r],
\]

\[
[b_r] = [S_{rl}][a_l] + [S_{rr}][\Gamma_L][b_r].
\]

By eliminating \([b_r]\) from the first equation, the normalized reflected waves can be expressed in terms of the incident waves \([a_r]\) as follows

\[
[b_l] = [\Gamma_{in}][a_l], \quad [b_r] = [T][a_r],
\]

(4)

where \([\Gamma_{in}]\) and \([T]\) are the input reflection coefficient matrix and the transmission coefficient matrix of the system, respectively given by

\[
[\Gamma_{in}] = [S_{rl}] + [S_{rr}][\Gamma_L][T],
\]

\[
[T] = ([1] - [S_{rr}][\Gamma_L])^{-1}[S_{rl}],
\]

(5)

and \([1]\) denotes the identity matrix of dimension \( n \). The input power of the WPT system is given by

\[
P_{in} = \frac{1}{2} (||a_l||^2 - ||b_l||^2) = \frac{1}{2} [a_l]^H [B] [a_l],
\]

where the superscript \( H \) denotes the conjugate transpose of matrix, and

\[
[B] = [1] - [\Gamma_{in}]^H [\Gamma_{in}],
\]

(6)

where \([1]\) denotes the identity matrix of dimension \( m \). The received power of the WPT system can be obtained from (3) and the second equation of (4)

\[
P_{rec} = \frac{1}{2} (||b_r||^2 - ||a_r||^2) = \frac{1}{2} [a_l]^H ([1] - [\Gamma_L]^H [\Gamma_L]) [b_r]
\]

\[
= \frac{1}{2} [a_l]^H [T]^H ([1] - [\Gamma_L]^H [\Gamma_L]) [T][a_l]
\]

\[
= \frac{1}{2} [a_l]^H [A][a_l],
\]

where

\[
[A] = [T]^H ([1] - [\Gamma_L]^H [\Gamma_L])[T].
\]

The PTE for the WPT system can then be expressed by

\[
P_{rec}
\]

The matrices \([A]\) and \([B]\) are solely determined by the scattering parameters and the load reflection coefficients of the WPT system, and the latter can be obtained either by simulation or by measurement. This flexibility offers great convenience when the system gets too complicated to be modeled by a computer. It is noted that the above derivation is applicable to any WPT system. In other words, the surrounding environment of the system, the separation between the Tx and Rx arrays, the array elements, and the array configuration are all assumed to be arbitrary.
A. UNCONSTRAINED MMPTE
To optimize the PTE expressed by the Rayleigh quotient (7), one can use the variational method. If the quotient (7) is required to be stationary at $[a_i]$, we may obtain the following generalized algebraic eigenvalue equation [4], [5]

$$[A][a_i] = PTE[B][a_i].$$  \hspace{1cm} (8)

Since both the matrix $[A]$ and $[B]$ are positive, the eigenvalues in (8) are all non-negative. In view of the fact that the number of positive eigenvalues of (8) is equal to the rank of the matrix $[A]$ as well as the following inequality [40]

$$\text{rank}[A] \leq \text{rank}[S_{rt}] \leq \min\{m, n\},$$

the number of positive eigenvalues of (8) is less than or equal to $\min\{m, n\}$. The maximum eigenvalue gives the maximum PTE and the corresponding eigenvector gives the ODE for the Tx array which generates a null in the direction of the Rx array elements. To this end, the UMMPTE may be modified by introducing a weighting function defined by

$$\rho_{W} = \frac{|b_{m+1}|^2}{|b_{m+1}|^2 - |b_m|^2} = \ldots = |b_{m+n}|^2.$$  \hspace{1cm} (11)

Accordingly (7) becomes

$$\rho_{W} \text{PTE} = \frac{\left(|[b'_i]|^2 - |[a_i]|^2\right)/2}{\left(|[a_i]|^2 - |[b_i]|^2\right)/2} = \frac{([A']^H[a_i], [a_i])}{([B][a_i], [a_i])},$$

where

$$[A'] = [T']^H \left([1] - [\Gamma_L]^H[\Gamma_L]\right)[T']$$

with

$$[T'] = [W][T].$$

Equation (8) is then modified to

$$[A'][a_i] = \rho_{W}PTE[B][a_i].$$  \hspace{1cm} (12)

The MMPTE imposed with the weighting matrix is called weighted MMPTE (WMMPTE). The above procedure not only achieves the specific distribution of received power but also guarantees that the received power of the Rx array is maximized for fixed input power.

B. WEIGHTED MMPTE
Sometimes constraints have to be imposed to the design of WPT systems to realize a specific distribution of the received power among the Rx array elements. To this end, the UMMPTE may be modified by introducing a weighting matrix

$$[W] = \text{diag}[w_1, w_2, \ldots, w_{m+n}]$$

for the received power $[b_i]$

$$[b'_i] = [W][b_i] = [w_{m+1}b_{m+1}, w_{m+2}b_{m+2}, \ldots, w_{m+n}b_{m+n}]^T.$$  \hspace{1cm} (10)

Accordingly (7) becomes

$$\text{PTE} = \left(|[b'_i]|^2 - |[a_i]|^2\right)/2 = \frac{([A']^H[a_i], [a_i])}{([B][a_i], [a_i])},$$

where

$$[A'] = [T']^H \left([1] - [\Gamma_L]^H[\Gamma_L]\right)[T']$$

with

$$[T'] = [W][T].$$

The extremum point $(a_i, \lambda)$ of (14) satisfies the following Lagrangian equations

$$\frac{\delta L(a_i, \lambda)}{\delta [a_i]} = 2[A][a_i] - [S_{rt}]^H[\lambda] = 0,$$

$$\frac{\delta L(a_i, \lambda)}{\delta \lambda} = [S_{rt}][a_i] - [c] = 0.$$  \hspace{1cm} (15)

where $\delta / \delta [a_i]$ and $\delta / \delta \lambda$ denote the functional derivatives with respect to $[a_i]$ and $[\lambda]$ respectively. It follows from (15) that the optimized solution of (14) is given by

$$[a_i] = [A]^{-1}[S_{rt}]^H([S_{rt}][A]^{-1}[S_{rt}]^H)^{-1}[c],$$

$$[\lambda] = 2([S_{rt}][A]^{-1}[S_{rt}]^H)^{-1}[c].$$  \hspace{1cm} (16)
If the Rx array is located in the near-field region of the Tx array, one cannot ignore the phase differences among the Rx elements. If one still follows the above procedure in this case, the distribution of the received power along the Rx array will no longer be flat. Instead, it will oscillate around a constant (the average distribution) with peaks and troughs. To avoid solving the QCQP problem directly, a weighting matrix \( W \) as defined by (10) may be introduced to bring the peaks and troughs back to their average. The weighting matrix can be determined by simulations. The weighting coefficients must be less than one for the Rx elements at the peaks and larger than one for the troughs, as illustrated in Fig. 2. A few iterations may be needed in order to finalize the weighting matrix \( W \). We may replace the constant vector \( c \) in (14) with a new weighted vector \( W[c] \) to solve a new weighted LCQP problem expressed by

\[
\begin{align*}
\max \; [a_l]^H[A][a_l] \\
\text{s.t.} \; [S_{rt}][a_l] = [W][c].
\end{align*}
\]  

(17)

The solution of (17) is then given by

\[
[a_l] = [A]^{-1}[S_{rt}]^H([S_{rt}][A]^{-1}[S_{rt}]^H)^{-1}[W][c].
\]

\[
[\lambda] = 2([S_{rt}][A]^{-1}[S_{rt}]^H)^{-1}[W][c].
\]

(18)

The MMPTE imposed with the linear constraints is called constrained MMPTE (CMMPTE). Compared with the UMMPTE and WMMPTE, the optimal solution from the CMMPTE can be determined analytically.

III. EXTENDED MMPTE

The MMPTE discussed in previous Section is based on the circuit theory for a multport network where both Tx and Rx arrays are involved. One can also get rid of the Rx arrays and introduce other performance indices to achieve various field patterns.

A. ANTENNA ARRAY WITH SPECIFIED ENERGY DISTRIBUTION

If the transmitting antenna is required to focus the EM field energy to multiple regions designated by \( \Omega_p(p = 1, 2, \ldots, n) \) as illustrated in Fig. 3, we may introduce the ratio of the weighted sum of the radiated energies in the regions \( \Omega_p(p = 1, 2, \ldots, n) \) over the total input power into the transmitting array as the performance index

\[
PTE = \frac{\sum_{p=1}^{n} \int_{\Omega_p} W_p(r)|\mathbf{E}(r)|^2 d\Omega(r)}{P_{in}}.
\]

(19)

where \( W_p(p = 1, 2, \ldots, n) \) are weighting functions that regulate the energy distribution among the designated regions \( \Omega_p(p = 1, 2, \ldots, n) \). It is noted that one can make the ratio (19) dimensionless by properly selecting the weighting functions although this may not be necessary. Assuming that the transmitting array is well matched, the fields radiated from the transmitting antenna array can then be written as

\[
\begin{align*}
\mathbf{E}(r) &= \sum_{j=1}^{m} a_j \mathbf{E}_j(r), \\
\mathbf{H}(r) &= \sum_{j=1}^{m} a_j \mathbf{H}_j(r),
\end{align*}
\]

(20)

where \( \mathbf{E}_j(r) \) and \( \mathbf{H}_j(r) \) are the fields generated by the \( j \)th antenna element of the transmitting array when the \( j \)th element is excited by \( a_j = 1 \) and the rest are terminated in a matched load, i.e., \( a_i = 0 (i \neq j) \). Thus we may write

\[
\int_{\Omega_p} W_p(r)|\mathbf{E}(r)|^2 d\Omega(r) = \sum_{j=1}^{m} \bar{a}_j \sum_{j=1}^{m} a_j \int_{\Omega_p} W_p(r)\mathbf{E}_j(r) \cdot \mathbf{E}_j(r) d\Omega(r)
\]

\[
= \left([A^p][a_1], [a_2], \ldots, [a_n]\right)\]

where \( [A^p] \) is an \( m \times m \) matrix with the \((i,j)\) elements given by

\[
A^p_{ij} = \int_{\Omega_p} W_p(r)\mathbf{E}_j(r) \cdot \mathbf{E}_j(r) d\Omega(r).
\]

(21)

If the transmitting antenna elements are all matched, we have \( P_{in} = (|a_i|, |a_i|)/2 \), and (19) can thus be written as

\[
PTE = \frac{\sum_{p=1}^{n} (|A^p||a_1|, |a_2|) \cdot (|A||a_1|, |a_1|)}{(|a_1|, |a_1|)/2} = \frac{(|A||a_1|, |a_1|)}{(|a_1|, |a_1|)},
\]

(22)

with \( [A] = 2 \sum_{p=1}^{n} [A^p] \). Similarly the optimized solution of (22) for the transmitting antenna array can be determined by the eigenvalue equation (8) with \([B]\) set to the identity matrix. The transmitting antenna array excited by the ODE obtained from maximizing (22) focuses the EM field energy to the multiple target regions designated by \( \Omega_p(p = 1, 2, \ldots, n) \) with a desired energy distribution controlled by the weighting functions. Especially if the weighting functions are selected as \( W_p = w_p \delta(r - r_p) \) \((p = 1, 2, \ldots, n)\), the optimization of (22) gives the ODE
for the transmitting antenna array focused on multiple points \(r_p (p=1,2,\ldots,n)\). The above strategy can be used to design focused antenna array, smart antenna array, and multi-beam antenna array.

One can also introduce the ratio of the total energy absorbed in the region \(\sum_{l=1}^{l<n} \Omega_p (l<n)\) over that absorbed in the region \(\sum_{l=1}^{l=n} \Omega_p (l<n)\) as the performance index

\[
PTE = \frac{\sum_{p=1}^{l} \int \omega \omega W_p(r) |E(r)|^2 d\Omega(r)}{\sum_{p=1}^{n} \int \omega \omega W_p(r) |E(r)|^2 d\Omega(r)}.
\]

Inserting (20) into (23) leads to

\[
PTE = \frac{\langle [A][a_l], [a_l] \rangle}{\langle [B][a_l], [a_l] \rangle}.
\]

where

\[
[A] = \sum_{p=1}^{l} [A^p], \quad [B] = \sum_{p=1}^{n} [A^p].
\]

The optimized solution of (24) can be determined by the eigenvalue equation (8). An optimization criterion similar to (23) was introduced to find the optimal excitations for the multi-antenna applicators in regional hyperthermia [44], [45].

**B. ANTENNA ARRAYS WITH SPECIFIED POWER DISTRIBUTION**

Let \(S_p (p=1,2,\ldots,n)\) denote the surface elements located in various directions as illustrated in Fig. 4. One can introduce the performance index

\[
PTE = \frac{\sum_{p=1}^{n} \int \omega \omega W_p(r) \frac{1}{2} \text{Re} \{E(r) \times \bar{H}(r) \cdot u_n dS(r)\}}{P_{in}},
\]

which is the ratio of the weighted sum of the radiated power in the directions specified by \(S_p (p=1,2,\ldots,n)\) over the total input power into the transmitting antenna array. In the above, \(W_p(r) (p=1,2,\ldots,n)\) are the weighting functions that regulate the power distribution in the directions designated by \(S_p (p=1,2,\ldots,n)\) and \(u_n\) is the unit normal vector to the surface element. Taking (20) into account, we may write

\[
\int \omega \omega S_p \frac{1}{2} \text{Re} \{E(r) \times \bar{H}(r) \cdot u_n dS(r)\}
\]

\[
= \frac{1}{2} \text{Re} \sum_{i=1}^{m} \bar{a}_i \sum_{j=1}^{m} a_j \int \omega \omega S_p \{ E_i(r) \times \bar{H}_i(r) \cdot u_n dS(r)\}
\]

\[
= \frac{1}{2} \text{Re} \{ [A^p] \cdot [a_l], [a_l] \},
\]

where \([A^p]\) is an \(m \times m\) matrix with the elements given by

\[
A^p_{ij} = \int \omega \omega S_p \{ E_j(r) \times \bar{H}_j(r) \cdot u_n dS(r)\}.\quad (26)
\]

If the transmitting antenna elements are all matched, (25) can thus be expressed by

\[
PTE = \frac{\text{Re} \{ [A][a_l], [a_l] \} \langle [B][a_l], [a_l] \rangle}{\langle [A][a_l], [a_l] \rangle \langle [B][a_l], [a_l] \rangle},
\]

with \([A] = \sum_{p=1}^{n} [A^p]\) and \([A_c] = \frac{1}{2} \{ [A] + [A]^H \}\).

The above strategy can be applied to the design of focused antenna array, smart antenna array, and multi-beam antenna array.

Similar to (23), one can also introduce the ratio of the total power radiated in the directions \(S_1, S_2,\ldots, S_l\) over that radiated in the directions \(S_{l+1}, S_{l+2},\ldots, S_n\) as the performance index

\[
PTE = \frac{\sum_{p=1}^{l} \int \omega \omega S_p \frac{1}{2} \text{Re} \{E(r) \times \bar{H}(r) \cdot u_n dS(r)\}}{\sum_{p=l+1}^{n} \int \omega \omega S_p \frac{1}{2} \text{Re} \{E(r) \times \bar{H}(r) \cdot u_n dS(r)\}}.
\]

Introducing (20) into (28) yields

\[
PTE = \frac{\text{Re} \{ [A][a_l], [a_l] \}}{\text{Re} \{ [B][a_l], [a_l] \}} \langle [B][a_l], [a_l] \rangle = \frac{\langle [A_c][a_l], [a_l] \rangle}{\langle [B_c][a_l], [a_l] \rangle},
\]

where

\[
[A] = \sum_{p=1}^{l} [A^p], \quad [A_c] = \frac{1}{2} \{ [A] + [A]^H \},
\]

\[
[B] = \sum_{p=l+1}^{n} [A^p], \quad [B_c] = \frac{1}{2} \{ [B] + [B]^H \},
\]

and the matrix elements of \([A^p]\) are given by (26).

For most applications, such as the design of focused antenna array, smart antenna array and multi-beam antenna, it has been verified that the extended MMPTE without using the test receiving antennas yields results comparable with MMPTE. The extended MMPTE is based on a pure field formulation, whose application studies are still ongoing and will be published in the future. For the above reasons, we will concentrate on the MMPTE and its applications in the following.
IV. OPTIMAL DESIGN OF ANTENNA ARRAYS FOR NEAR- AND FAR-FIELD APPLICATIONS

Since the final goal of antenna design is to maximize the PTE between the Tx and Rx antennas, the MMPTE can also be applied to the design of antenna arrays. In fact, the PTE may be used as a performance index or an objective function to be optimized for all antenna designs. For this purpose, the array under design may be set as the Tx and a test array may be introduced and set as the Rx so that they form a WPT system, as illustrated in Fig. 5. Therefore, by properly introducing a test Rx array, the optimal design of an antenna array is transformed into the optimal design of a WPT system and the best possible antenna performance is guaranteed. Different from the design of WPT system, the Rx array introduced in the design of an antenna array is a virtual tool set for determining the scattering parameters and achieving the prescribed field pattern. There is no need to fabricate it eventually.

The design of antenna array for applications in far- or near-field regions can be accomplished in terms of MMPTE by following the same procedures listed below:

1) Set up the WPT system: A WPT system may be set up by assuming that the array under design is used as Tx and a test array is introduced as Rx. The type and the number of the test Rx elements and their configuration depend on the field pattern to be realized on the Rx side. The polarization of the test Rx array elements must match that of the Tx array elements. For example, the design of multi-beam antennas requires a test Rx array with its elements being positioned in the directions where the beam needs to be generated.

2) Determine the scattering parameters of the WPT system: The scattering parameters of the WPT system can be acquired by simulation or by measurement if the surrounding environment is too complicated or unknown.

3) Find the ODE for the array: The ODE can be obtained from (8), (12), (16) or (18), depending on which formulation of MMPTE is used.

4) Design the feeding network: Based on the ODE, a feeding network can be designed by the theory of transmission line. The power dividers in the feeding networks control the amplitude distribution of the excitations while the lengths of the feeding lines control the phase distribution of excitations. The feeding network can also be realized by attenuators and phase shifters available in the market. The excitations with negligible amplitudes in the ODE can be ignored to reduce the number of array elements and simplify the design of the feeding network.

The scattering parameters in MMPTE contain information about the environment between the Tx and Rx arrays and can be obtained by measurement in real time. This is convenient for the design of antennas in a changing environment. For example, the targeted hyperthermia treatment requires accurate knowledge of the electrical properties of the human body, which are usually acquired at the diagnostic stage through medical imaging. In hyperthermia treatment, the patient and properties of the body tissues may change. In these cases, an adaptive approach in real time for hyperthermia treatment is in high demand.

Typically the MMPTE can be applied to the design of focused antenna arrays, smart (beam steering) antenna arrays, end-fire antenna arrays, multi-beam antenna arrays, and the shaped beam antenna arrays. By introducing a single test Rx antenna in the direction where the radiation intensity needs to be maximized or minimized, the UMMMPTE (where only one positive eigenvalue is involved) can be used to design the focused antennas [11]–[17], smart antennas [18]–[21], end-fire antennas [24], [25], and polarization-reconfigurable antennas. By introducing a test Rx array, the CMMPTE and WMMPTE (where multiple positive eigenvalues are involved) can be used to shape beam patterns [22], [23], and design antenna arrays with multiple focal points [17], and multiple beam antennas [26], [27].

In recent years, near-field communication (NFC) has found wide application. Understanding the behavior of fields near a radiator and being able to control them are extremely helpful in designing NFC devices. The MMPTE is applicable to the design of antenna arrays in the near- or far-field region and the design procedures are exactly the same.

A. FOCUSED ANTENNA ARRAYS

EM field focusing is required in areas where a high power density must be applied, such as noncontact microwave sensing [46], radiometric temperature sensors [47], microwave hyperthermia [48]–[51], and radio frequency identification (RFID) [52]–[55]. Conventional design methods for the focused antenna array assume a quadratic phase distribution with constant amplitude distribution, which may yield high sidelobes at the focal plane [56]–[57]. Quadratic phase distribution combined with Dolph-Chebyshev amplitude distribution can be used to improve the sidelobes [58]. Another interesting method for focusing EM wave is based on the time reversal (TR) technique [59]–[61], which was first used in acoustics [62].
Physically focusing the EM energy to a spot (the focal point) by an antenna array is equivalent to delivering the maximum possible power from the array to a test antenna located at the focal point so that the radiated energies from all the array elements converge to the focal point. For this reason, the PTE is an ideal performance index for the design of focused antenna array. For an antenna array with multiple focal points, a testing Rx antenna array may be introduced with its elements placed at the focal points. The eigenvector corresponding to the maximum PTE, obtained from equation (8) or (18), represents the ODE for the focused antenna array and the best focusing performance is guaranteed. Based on the above procedure, different types of focused antenna arrays have been designed for various applications.

1) LINEARLY-POLARIZED FOCUSED ANTENNA ARRAYS [11], [12]

The focused antennas have been investigated for a long time and mainly concentrated on linearly-polarized antenna arrays. A 16-element array operating at 2.45GHz, using a rectangular microstrip patch element with an inset-feed, is designed by UMMPTE in [11]. The array is built on FR4 substrate with relative dielectric constant 4.4, loss tangent 0.02 and thickness of 3mm, as shown in Fig. 6(a). The feeding network is designed in terms of the ODE obtained from (8) with a linearly-polarized testing antenna placed 150mm away from the array along its center axis. It is found that the ODE has a spherical phase distribution and a Gaussian-like amplitude distribution, which is similar to the optimized solution for a continuous aperture [37]. The ODE ensures that the fields generated by the array elements add in phase at the focal point, and the electric fields at the focal plane have no sidelobes. The contour plot of the electric field at the focal plane indicates that the $-3\text{dB}$ beam contour has good circular symmetry, indicating that most of the power is concentrated in the middle of the focal plane (see Fig. 6(b)).

The antenna array and the feeding network can be built on the different sides of substrate by introducing a common ground plane in between the array and the feeding network [12]. The latter arrangement is better suited for large-scale arrays and is less vulnerable to the environments.

2) CIRCULARLY-POLARIZED FOCUSED ANTENNA ARRAYS [13], [14]

A circularly-polarized (CP) wave has outstanding capability of anti-interference, which motivates the study of CP focused antenna arrays. The design of CP focused antenna array seem more challenging since the distribution of excitation for the antenna array not only determines the focusing properties but also affects the axial ratio (AR) of the antenna array. Two different CP focused antenna arrays, one in a square shape and the other in a ring shape, operating at 2.45GHz, are designed with UMMPTE in [13], [14], and they are built on FR4 substrate. The array element is a rectangular patch with an L-shaped slot for creating a pair of orthogonal modes and two triangles removed from the two corners to further improve the CP performance. A CP patch is used as the test Rx antenna and placed 100 mm away from the array. Again, no sidelobes occur at the focal plane, which indicates that the radiated energy from the array is concentrated around the focal point as illustrated in Fig. 7.

3) FOCUSED ANTENNA ARRAYS FOR MEDICAL APPLICATIONS [15], [16]

The MMPTE can be applied to the design of focused antennas in more complicated environments, such as for microwave hyperthermia, where the influence of human body tissues must be taken into account in the design of antenna arrays [15], [16]. An 18-element focused hexagonal patch array operating at 433MHz, is designed under the load of a body-mimicking phantom by UMMPTE in [16], as illustrated in Fig. 8. The array is built on the FR4 substrate. To reduce the size of the array, a pair of slots are introduced in the rectangular patch element. The patch array and the feeding network are separated by a common ground plane. The body-mimicking phantom and the matching material are prepared and are placed in a cylindrical glass tank. The focal position can be adjusted in real time in the body-mimicking phantom by changing the feeding schemes determined by the ODEs.
Another interesting application of focused antenna is to wirelessly power implantable devices, which is often accomplished by near-field inductive coupling or non-radiating WPT [63]–[65]. The UMMPTE has been successfully applied to design a focused antenna to power an implantable device in the radiating near-field region [30].

4) ANTENNA ARRAYS WITH MULTIPLE FOCAL POINTS [17]
In some circumstances, a prescribed power distribution among multiple focal points may be required to achieve, for example, simultaneous wireless power transmission (SWPT) to different devices with different input power levels. The proper feeding schemes for SWPT may be obtained by various optimization algorithms such as the Levenberg-Marquardt algorithm [66], [67] and artificial neural networks [68], which seem to be very time-consuming if the number of antenna elements is large. An alternative to realize SWPT is to use the TR method [69]–[71]. Several TR focusing fields inside a closed metal cavity have been achieved with the synthesized distributions of excitations [72]. For most SWPT scenarios, the transmitters and receivers are deployed in an open space, instead of in a closed metal cavity. An algorithm for the synthesis of the near-field pattern in open space is proposed in [73], in which the phase distribution is realized by a coding metasurface, but this algorithm requires huge computational time due to massive source phases.

Two antenna arrays operating at 2.45GHz have been designed in [17] by CMMPTE to focus the EM field energy to multiple targets in both closed and open regions, as shown in Fig. 9(a) and (b). In the design process, the test Rx array elements are divided into two sets. One set denotes the test elements whose received power must be maximized and the other set the test elements whose received power must be minimized. The number of focal points and the power intensity at each focal point are all manageable by regulating the weighting matrix. As a demonstration, the X-shaped, Y-shaped and L-shaped focusing field patterns are achieved in both closed and open regions and are shown in Fig. 9(c).

Compared with other existing methods for the design of focused antenna arrays, the MMPTE features that the field distribution at the focal plane has no sidelobes as the exact theory for a continuous aperture predicts [37]. In addition, the realized focal point is closer to the targeted one.

B. SMART ANTENNAS (BEAM-STEERING ANTENNAS)
Smart antennas have been used in radar system for many years [74], [75], and have recently received increased interest in mobile communication systems for their capability in improving channel capacity and spectrum efficiency, extending coverage, enhancing data rate, steering multiple beams to track many mobiles, and improving service quality [76], [77].

Smart antennas can be categorized as either switched-beam array system or adaptive array system. A switched-beam array system selects one of the predefined beam patterns and changes one beam to another as the mobile user moves around. An adaptive array system optimizes its beam patterns adaptively in response to the signal environment, directs the main beam towards the signal of interest and suppresses the radiation in the direction of interference. Moreover, it largely relies on the signal processing algorithms such as the multiple signal classification algorithm for the estimation of the direction of arrival of the signal and the least mean square algorithm for beamforming.

The MMPTE can be used to determine the distribution of excitations for a smart antenna array to steer the main beam or a null in the desired directions and to ensure that the array gain is optimized. A test Rx antenna may be introduced in the desired direction and placed in the far-field region of the smart array to be designed. The smart array and the test Rx antenna constitute a WPT system. By optimizing the PTE of the WPT system, the ODEs for the smart array can be
determined and be used to generate the main beam or a null in the desired directions.

1) SMART ANTENNAS FOR HANDHELD APPLICATIONS [18], [19]

The smart antenna design for handheld devices has been a challenge for years due to its small size and the complexity of the mobile environments. The MMPTE is capable of solving this problem [18], [19].

A new MIMO (multi-input and multi-output) beam-steering antenna array for a compact and thin handheld device is demonstrated in [18], in which the beam-steering function is enabled for transmitting while the MIMO function for receiving. The MIMO beam-steering antenna array consists of eight printed planar inverted-F elements operating at GSM1900 and LTE2300, as illustrated in Fig. 10(a). A test Rx antenna is introduced in the desired direction, and the UMMPTE is then applied to the WPT system consisting of the 8-element array and the test Rx antenna to find the ODE. The simulation and experiments show that the radiation of the antenna array is greatly enhanced in the desired directions and the maximum gain reaches 7.7 dBi.

In practice, the environments, such as the LCD, battery, and other components inside the mobile handsets, have significant impact on the antenna performance. A practical design must consider all these factors, which is difficult to handle even with a state-of-the-art computer. Therefore, the measurement has to be used to attack the problem. A human body phantom is introduced to show how the influence of environments can be determined in [18], as illustrated in Fig. 10(b). A measurement system is set up with a human phantom in place to simulate the phone operation in talking position. The eight-element antenna array is held by a hand model and placed near a head model and is set as the Tx antenna while a horn antenna is set as the Rx antenna of the WPT system. The scattering parameters of the WPT system are acquired by a network analyzer which connects the Tx array and the Rx horn placed in the direction in which the radiation intensity is to be enhanced. Once the scattering parameters in the desired direction are determined, the ODE can be obtained from (8). Due to the influence of the phantom, the realized gain is 2 to 3 dB lower than in free space. Fig. 10(c) and (d) show the beam patterns in the desired directions at 2.45GHz.

2) SMART ANTENNAS FOR BASE STATION APPLICATIONS [20], [21]

Customer-premises equipment (CPE) refers to equipment that is located at a subscriber’s premises and is connected with a carrier’s telecommunication channel. In many situations, the CPE must be “smart” enough to be able to steer the beam towards the desired direction for enhancing the performance of wireless communications. Smart antenna with two radiation modes for CPE applications must have compact size, low cost and beam steering capability in the azimuth plane [78]–[80]. One radiation mode has omni-directional coverage in the azimuth plane to search for the sources while the other mode has the directional radiation to communicate with high gain and high data-rate.

A 4-element folded monopole antenna array, operating at 2.45GHz, for CPE application is designed by UMMPTE in [20]. The folded monopole and the sleeve ground form a quasi-balanced radiating structure, as illustrated in Fig. 11(a). When the amplitudes and phases at the four feeding ports are uniform, the radiation pattern is omni-directional and the measured gain for the omni-directional pattern is 1.7 dBi. The directional radiation patterns in the desired directions are formed by the ODEs obtained from UMMPTE. Fig. 11(b) is the feeding circuit board designed to realize the ODEs. The measured gain for directional pattern is 6.7dBi.

To further reduce the height and improve the front-to-back ratio in the directional mode, two novel dual-mode beamsteerable antenna arrays using arc-shaped dipoles are studied by UMMPTE in [21]. Both arrays operate at 2.45GHz and have 360° beam scanning capability in azimuth when
operating in the directional mode. The first array consists of a circular ring reflector and four arc-shaped half-wavelength dipoles with omnidirectional gain of 1.5 dBi, and the peak gain and front-to-back ratio being 7.4 dBi and 15.5 dB respectively in the directional mode. The second array uses the first array as the antenna subarray. Two subarrays form an eight-element array separated by a distance of 0.4 wavelength and placed coaxially with one rotated 45° around the center axis. The second array has a higher omnidirectional gain of 2.9 dBi, and more uniform gain distribution in azimuth in the directional mode, with a peak gain of 8.4 dBi and a front-to-back ratio of 12.5 dB.

It is noted that the signal models used in conventional beamforming algorithms often ignore the mutual couplings among the array elements and the complicated environments [76]. The MMPTE has however incorporated all these factors in the formulation and guarantees that the highest possible gain is achieved in the desired direction.

C. SHAPED BEAM ANTENNA ARRAYS
Radiation patterns with desired shapes can be realized by controlling the distribution of excitations for the antenna arrays. Various optimization methods have been proposed to determine the amplitude and phase distributions for the antenna arrays to achieve a prescribed field pattern in the far-field region [81]–[98]. Various performance indices can be used in pattern synthesis, such as pattern shape, sidelobe levels, front-to-back ratio and gain. It is noted that the desired performance may be unattainable for fixed antenna geometries [99]. The traditional methods of Dolph-Chebyshev, Taylor-Kaiser, Schelkunoff, Fourier transform, and Woodward-Lawson [100], [101], as well as other global optimization methods such as particle swarm optimization [102] and genetic algorithm [103], [104] have found applications in far-field pattern synthesis problems. All these optimization methods are based on the array factor, and they fail whenever the array factor is not available.

In contrast to the sophisticated synthesis methods in the far-field region, only a few methods are available to synthesize a desired pattern in the near-field region [105]–[109]. Most studies are computationally cumbersome since the desired radiation pattern is synthesized on the basis of the radiations of array elements, which needs to be done element-by-element. They are not only complicated, but also inaccurate when the antenna is required to operate in a complicated environment such as inside a metal enclosure.

The MMPTE solves all the challenges associated with the conventional pattern synthesis methods, and is superior to them in many ways. The MMPTE avoids the long searching process in conventional optimization algorithms and can be used to shape beam patterns in both near- and far-field regions as well as in material bodies. According to the beam shape to be realized, a test Rx array may be introduced with its elements properly positioned (see Fig. 12) and weighted as described in WMMPT and CMMPTE.
Fig. 14(c) and (d) are the 2-D and 3-D shaped radiation patterns. It can be seen that field drops quickly off the flat top region and the measured side lobe levels are below −30dB. In order to generate the desired square flat top radiation pattern, nine test receiving antennas are used and placed on a sphere in the far-field region and arranged as a square shape, as illustrated in Fig. 12.

2) BEAM SHAPING IN A BOOKSHELF ENVIRONMENT [22]
Most reader antennas for library applications are designed in free space without considering the influences of the surrounding books. The CMMPTE has been applied to the design of a 5-element RFID reader antenna array, operating at 922.5 MHz, for a smart bookshelf of 730 mm in width [22]. The dimensions of the five-element reader antenna array are 730 mm × 250 mm × 5 mm. In order to achieve a flat field pattern across the bookshelf, eleven test Rx antennas are positioned in a row with equal spacing inside a book model of dimensions 850 mm × 176 mm × 250 mm whose dielectric constant and loss tangent change with frequency, as illustrated in Fig. 15(a). The five-element reader antenna array and the eleven test Rx antennas constitute a sixteen-port network. The ODE for the reader antenna array is obtained from (16) through properly selecting the weighting matrix to generate a flat top radiation pattern inside the book model with the coverage measured at 3 dB being exactly 730 mm. The electric field intensity drops abruptly on both left and right side of the bookshelf, which is enabled by placing the outermost test Rx antennas right above the edge of the reader antenna. The rapid cut-off of the field intensity is helpful in reducing the probability of misreading the books on the adjacent bookshelf. Fig. 15(b) is the experimental testing system for the reader antenna array.

Many previously reported RFID reader antennas for near-field applications suffer from large fluctuations in field distribution. As a result, the input power may not meet the demands in the lowest electric field areas and may thus cause reading failures. In contrast, the reader antenna designed by the CMMPTE guarantees the reading accuracy for its wide and flat field distribution which is also optimally enhanced in the reading area.

3) BEAM SHAPING IN A METAL CABINET ENVIRONMENT [23]
A cabinet-level RFID solution for the management of communication servers is proposed in [23]. The reader antenna monitors the servers in a metal cabinet by the tags attached to the servers. The tags are arranged vertically along a straight scanning line, which is 200 mm away from the array and 140 mm away from the front side of the metal cabinet as illustrated in Fig. 16(a). The reader antenna is placed along the edge of the left side of the cabinet for reducing the space occupied by the antenna. The dimensions of the cabinet are 600 mm × 600 mm × 900 mm. The communication servers are placed longitudinally in the cabinet with maximum extension height less than 700 mm. Therefore, it is required that the tracking range of the antenna in the vertical direction of the cabinet is not less than 700 mm.

A four-element patch array, operating at 900MHz, arranged in the vertical direction is introduced to meet the coverage requirement of the cabinet in the near-field region.
The antenna elements are printed on top of the first substrate of dimensions 650 mm × 120 mm × 3 mm, while the feeding network is printed on the bottom side of the second substrate of dimensions 650 mm × 120 mm × 1.6 mm, both sharing the same ground. To ensure that the array has no dead zone in the identification area, the electric field distribution in the target identification area must be uniform. To this end, four test Rx dipole antennas positioned along the scanning line are introduced. The four-element patch array and the four test dipoles constitute a WPT system. The four-element array excited with the ODE from CMMPTE has a uniform field distribution with less than 1.5dBi variation across the scanning line with the length of 710 mm, which is significantly lower than previously reported.

Fig. 16(b) shows the RFID testing system.

**FIGURE 17.** Generalized Yagi-Uda dipole antenna.

**FIGURE 18.** (a) 8-element multi-beam antenna array. (b) Radiation pattern with eight beams.

A printed dipole array with end-fire radiation is designed in terms of UMMPTE in [25], which achieves the end-fire radiation through the ODE for all the array elements. In this way, neither reflectors nor directors are involved.

**E. MULTI-BEAM ANTENNA ARRAYS [26], [27]**

Multi-beam antennas simultaneously generate multiple beams and have found applications in satellite and mobile communications [119]–[122]. The WMMPTE and CMMPTE can be applied to the array design for multi-beam applications. By introducing multiple test Rx antennas in the directions where the beams are to be generated, the WMMPTE and CMMPTE produce the ODE to achieve multiple beams in the desired directions.

A four-element printed bidirectional dipole array is designed in terms of WMMPTE in [26]. Compared with the conventional designs using dipole elements, the optimized dipole array can achieve a desired distribution of end-fire gains with reduced size and enhanced gain. The simulated and measured results show that the end-fire gain in one direction varies from 1.5 to 10.2 dBi at 2.45 GHz as the ratio of weights changes from 0 to 1. The end-fire gain reaches 9.3 dBi when the radiation pattern is equally shaped in both directions.

A four-element rectenna antenna array for RF power harvesting applications is presented in [27], in which the beamwidth of the array is enhanced by introducing two test Rx antennas in UMMPTE so that two beams can be formed in the desired directions. It is shown that the enhanced beamwidth increases the output DC power significantly in comparison with the same antenna array fed by the uniform distribution of excitation.

A low-profile multi-beam antenna array consisting of eight E-shaped patch elements has been designed with WMMPTE, whose configuration, the feeding circuit and radiation patterns are shown in Fig. 18. The antenna array operates at 5.8 GHz and can generate eight squint beams in different directions simultaneously with a simple feeding circuit. The peak gains of the beams vary from 8.2 to 8.9 dBi. The multi-beam antenna array can be used as an indoor wireless base station installed on the ceiling or wall. In comparison with the similar applications in [123], the feeding network...
obtained from MMPTE is much simpler, and the highest gain for each beam is guaranteed.

F. POLARIZATION-RECONFIGURABLE ANTENNA ARRAYS

The polarization-reconfigurable antenna arrays are usually realized by diodes, switches, and multi-ports feeding networks through controlling the operating modes of antenna elements [124], [125]. The MMPTE can also be applied to design of polarization-reconfigurable antennas. The array under design and a test receiving antenna form a wireless power transmission (WPT) system. The PTE is increased if the Tx and Rx antennas are polarization matched. For this reason, the polarization and the beam direction of the array under design can be simultaneously controlled by the polarization state and the position of the test receiving antenna through imposing that the PTE is maximized. The test receiving antenna can be chosen as a linearly polarized, a circularly polarized or a dual linearly polarized antenna placed in the desired beam direction, depending on the polarization to be achieved. As long as the array has the physical potential of realizing a designated polarization state, the MMPTE always produces an ODE for the array to realize the designated polarization state in the desired direction.

Fig. 19(a) shows a polarization-reconfigurable array working at 2.45GHz, which consists of 12 identical linearly polarized patch elements. The patch elements are purposely arranged so that the radiated electric field from the array has both x and y components. As a result, the array has the potential to realize multiple polarization states, such as x or y-polarized, dual linearly polarized, and circularly polarized, and the ODEs for achieving the polarization states can be obtained from UMMPTE by introducing a test receiving antenna with desired polarization state. Fig. 19(b) shows the simulated and measured axial ratios for left-hand circular polarization (LHCP) state and right-hand circular polarization (RHCP) state.

Compared with other existing design methods, the polarization-reconfigurable antenna array designed by MMPTE has the advantages of high gain and low side lobes for a fixed array configuration as guaranteed by the MMPTE.

V. OPTIMAL DESIGN OF WPT SYSTEMS

A WPT system generally consists of a transmitting array and a receiving array, and the power transmission efficiency naturally becomes the performance index for the design of antenna arrays. Much progress on the design of WPT systems has been made in recent decades [126]–[144], with the solar power satellite being considered the highest-potential application of WPT technology [126]–[131]. WPT technology has gradually been applied to other fields, such as wireless power distribution system in buildings, microwave hyperthermia, implantable devices and the vehicles powered by microwaves [132]. Various methods have been tried to improve the performances, efficiency, and robustness of WPT systems by dealing with the design of Tx arrays [133]–[142]. Moreover the power reaction factor can also be used as a performance index to design a WPT system [143], [144].

The application of MMPTE to the design of WPT system is straightforward. The PTE is a key performance index for the design of WPT system, and the MMPTE maximizes the PTE and therefore provides the best possible system performance. There are three application scenarios for the WPT systems, respectively corresponding to UMMPTE, WMMPTE, and CMMPTE.

A. DESIGN OF WPT SYSTEMS BY UMMPTE [28]–[31]

The UMMPTE corresponds to the scenario where no constraint is involved in the design of a WPT system. Two WPT systems operating at 5.8 GHz in free space are designed in terms of UMMPTE in [29]. The first system is shown in Fig. 20 and uses a 36-element patch array as the Tx and a 16-element patch array as the Rx, separated by a distance of 40 cm so that the Tx and Rx arrays are in the Fresnel region of each other. The inset-fed patch is used as the array element. The feeding networks for both Tx and Rx arrays are designed according to the theory of transmission lines in terms of ODEs determined from (8) and (9) respectively. The antenna elements and the feeding network are printed on the same side of a 1.524 mm thick Rogers 4003 substrate with a relative dielectric constant of 3.55 and loss tangent of 0.0027. The measured PTE of the WPT system is 39.4%,
GEYI: METHOD OF MAXIMUM POWER TRANSMISSION EFFICIENCY FOR DESIGN OF ANTENNA ARRAYS

which is significantly higher than a similar design in [139] in view of the fact that the element number of the Tx array in the design with UMMPTE is only half of that used in [139].

A WPT system operating at 1.9GHz for powering an implantable device, which is embedded in a human head model with relative dielectric constant of 39.9 and conductivity of 1.42 S/m and located in the radiative near field of the Tx array, is presented in [30]. The system is optimized by UMMPTE. Compared to a reference WPT system with the Tx array fed by the uniform amplitude and phase, the PTE of the optimized system is about 1.3 times that of the reference one.

The MMPTTE can also be applied to the design of WPT system in an unknown electromagnetic environment [31]. Whenever the environment is unknown or too complicated to be modeled by a computer, the scattering parameters of the WPT system can be acquired by measurement, and the MMPTTE is still applicable. The maximum PTE can be reached and the array can be optimized without identifying the electrical properties of the unknown environment.

B. DESIGN OF WPT SYSTEMS BY WMMPTE AND CMMPTE

Both WMMPTE and CMMPTE can be applied to achieve a specified distribution of received power among the Rx array elements. The WMMPTE is most suitable for the situation where the power levels are required to be different among the Rx array elements (e.g., different electronic devices are simultaneously powered). The CMMPTE is most applicable to the scenario where the distribution of received power must be flat along the Rx array elements (e.g., several closely spaced identical electronic devices are wirelessly powered). The CMMPTE is most applicable to the scenario where the distribution of received power must be flat along the Rx array elements (e.g., several closely spaced identical electronic devices are wirelessly powered). The CMMPTE is still applicable. The maximum PTE can be reached and the array can be optimized without identifying the electrical properties of the unknown environment.

VI. CONCLUSION

In summary, one can make use of the PTE as an objective function to be maximized to design various WPT systems, and such an optimization design technique is referred to as MMPTTE. By introducing a test Rx array, the optimal design of an antenna array can be converted to the optimal design of a WPT system. The ODE obtained from MMPTTE for the antenna array guarantees that the gain of the array is maximized for a fixed array configuration.

There are three different formulations of MMPTTE, respectively referred to as UMMPTE, WMMPTE and CMMPTE, and they can be extended to different versions by replacing the test receiving antennas with selected volumes or surfaces. The PTE for the extended MMPTTE is defined by the ratio of the weighted sum of the radiated energies in the volumes or the radiated power through the surfaces over the total input power into the Tx antenna array.

The MMPTTE exhibits some remarkable features:

1) It breaks through the limitations that lie in the conventional array design methods when the pattern multiplication rule fails, and is applicable to the design of antenna array of any type or configuration.
2) The MMPTTE reduces the field synthesis problem into a circuit analysis problem, and is thus easy to master and implement, and is more accessible for those who are not very familiar with the EM field theory.
3) The MMPTTE can incorporate the information about the environments around the Tx and Rx arrays, and therefore it can be made adaptive to any complicated environment.
4) The MMPTTE always provides an optimal solution to the array design problem wherever such a solution physically exists for a fixed configuration of the array.

Like the other array design methods, the computational time of the MMPTTE grows as the size of the array increases. Whenever the computational resources are not available, or the system is too complicated to be handled with a state-of-the-art computer, one can make use of measurement to determine the scattering parameters in the MMPTTE formulation.

FIGURE 21. Simulation model for WTP system.
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