Modelling dimensional change of machined parts and optimizing replacement time and initial position of cutting tool

Renyan Jiang
School of Science and Technology, Changsha University, Changsha, China
Email: jiang@csust.edu.cn

Abstract. In a manufacturing process, a key quality characteristic such as a certain dimension of machined parts can have a systematic shift toward the upper or lower specification limit due to the cutting tool wear. That is, the quality characteristic is a stochastic process with trend. As a result, the process capability and quality loss vary with time and depend on the initial position of the cutting tool. In this setting, decision problems that need to be solved are optimization of replacement time and initial position of the cutting tool. This paper proposes a non-homogenous Wiener process to model the dimensional change of machined parts; and the above-mentioned decision problems are solved through combining this model with a cost model that involves Taguchi quality loss. A real-world example is included to illustrate the appropriateness of the proposed models. The results are useful for both reliability researchers and manufacturing engineers.

1. Introduction

In a manufacturing process, the wear of a cutting tool can lead to itself failure and other problems such as nonconforming geometry and poor surface finish [1-3]. Thus, the preventive replacement of cutting tools becomes necessary for improving manufacturing productivity and quality [4].

There are two kinds of approaches to model the wear process of the cutting tool. In the first kind of approaches, the underlying variable is wear amount, which can be measured in a direct way (e.g., optical or vision system) or in an indirect way (e.g., cutting force, vibration and acoustic emission) [5]. In this case, the wear limit is determined based on the standard [6]. In the second kind of approaches, the underlying variable is a quality characteristic such as a certain dimension of produced parts [7-8]. This characteristic is highly related to the tool wear and hence reflects the consequence of the tool wear [9]. In this case, the quality failure threshold can be taken as the upper specification limit (USL) or lower specification limit (LSL) of the quality characteristic.

The tool wear results in a systematic shift of the quality characteristic toward its USL or LSL; and this shift cannot be physically removed from the process unless the cutting tool is adjusted or replaced [7]. As a result, the quality characteristic is a stochastic process with trend; and the process capability and quality loss vary with time and depend on the initial position of the cutting tool, which is described by the initial value of the characteristic [1]. In this setting, two basic decision problems are (a) optimization of preventive replacement time of the cutting tool and (b) optimization of the initial position of the cutting tool.
The first problem has attracted wide attention; and the decision models include cost models, process capability models and their combination. The cost models are usually associated with a known tool life distribution and an age or a block replacement policy [1, 10]. In a replacement cycle, periodic adjustment, resetting or grinding actions may be performed. Thus, the involved costs include replacement cost [1], resetting, adjustment or grinding cost [1, 9, 11], production capacity loss and quality loss [2, 12]. The quality loss is usually described by the Taguchi quadratic loss function [13]. As an exception, Xu and Cao present a sectional model to represent the percentage of quality failures and the quality loss is proportional to this percentage [2]. This approach actually assumes that any conforming item does not have quality loss, which is different from the Taguchi’s idea.

The process capability index (PCI) depends on process mean function and process variance of the quality characteristic. The process variance is the sum of the non-random variance caused by the trend term and the random variance after removing the trend term. As a result, the PCI varies with time, and the cutting tool is replaced when the PCI drops blow a critical point [7]. Though it looks reasonable, the PCI-based replacement may be too conservative, as shown later. Deng et al. optimize the tool replacement time based on a cost model subject to a PCI constraint [11].

For the optimization problem of initial position of the cutting tool, no decision model is found. This gap needs to be filled. In addition, it is noted that the current cost models are associated with a given distribution of the tool failure time. It appears more natural to use a stochastic process model for representing the change of the quality characteristic with time. From such a model, the distribution of time to quality failure can be derived and the PCI can be evaluated. This paper aims to address the above two problems. A non-homogenous Wiener process is proposed to model the change of the quality characteristic and the decision problems are solved through combining this model with a cost model. A real-world example is included to illustrate the appropriateness and usefulness of the proposed models.

The paper is organized as follows. The proposed models are presented in Section 2 and illustrated in Section 3. The paper is concluded in Section 4.

2. Proposed models
Let \( X \) denote a quality characteristic of a machined part in a manufacturing process and \( x \) is its observation. The LSL and USL of \( X \) are known. Let \( T \) denote its target and it is usually taken as \( T = (\text{LSL} + \text{USL})/2 \). Since the LSL is usually larger than zero so that the initial value of \( X \) (denoted as \( X_0 \)) is larger than zero.

A sample with size \( n \) is drawn at discrete time points \( t_i \) (\( i = 1, ..., m \)) and let \( x_{ij} \) (\( j = 1, ..., n \)) denote their values. Usually, \( t_i = t_{i-1} + \Delta \), where \( \Delta \) is called the sampling interval. Clearly, \( X \) changes with time \( t \) and hence is a stochastic process. Let \( X(t) \) denote this process, and \( \mu(t) \) and \( \sigma^2(t) \) denote the mean and variance (relative to the mean) of the process, respectively.

The process mean has an increasing or decreasing trend due to the tool wear. Without loss of generality, the increasing trend is assumed in the remainder of the paper. The cumulative wear curve of the cutting tool is often inverse S-shaped, as shown in figure 1, implying that the wear rate is bathtub-shaped (e.g., see [14-17]).

The process mean should have a similar trend as the wear curve of the cutting tool. Thus, the models for \( \mu(t) \) need to be developed. The process variance, \( \sigma^2(t) \), is the variance of residual \( X(t) - \mu(t) \), and can be a constant or a function of \( t \). For the latter case, the models for \( \sigma(t) \) need to be developed.

The residual is usually assumed to be normally distributed, and hence \( X(t) \) is a non-homogenous Wiener process. The proposed stochastic process model and decision model are outlined as follows.
2.1. Stochastic process model of the quality characteristic

Three optional models for $\mu(t)$ are shown in the second row of Table 1. It is noted that all the three models have common constant and linear terms. For a given model, $a_0$ can be viewed as the initial position of the cutting tool. In a condition-based maintenance setting, $a_0$ can be viewed as an individual-oriented parameter and $a_1$ to $a_3$ as population-oriented parameters. The coefficients can be easily obtained through regression. Two regression statistics to assist model selection are $R^2$ (denoted as $R^2$) and Standard Error (denoted as $\sigma_e$). The best model has the smallest value of $\rho = \sigma_e/R^2$.

The plots of Models 1-3 are all inverse S-shaped. The third row shows the horizontal coordinates of the inflection points.

The wear rate is defined as $r(t) = \mu'(t)$. It is clear that the wear rate is symmetry for Model 1 and non-symmetry for Models 2 and 3. The asymptotic wear rates are shown in the fourth and fifth rows. As seen, the models have considerably different trends for small and large $t$.

Three optional models for $\sigma(t)$ are shown in the last row of Table 1. It is noted that the third option is the usual assumption for the Wiener and gamma processes. For a given process mean function, the plot of residuals can provide a clue to determine which is better, and the model selection can be made based on the goodness-of-fitting such as the sum of squared errors or the like.

| Model | $\mu(t)$ | $\sigma(t)$ |
|-------|----------|-------------|
| Model 1 | $\sum_{k=0}^{3} a_k t^k$ | $\alpha \sigma(t)$ |
| Model 2 | $\sum_{k=0}^{3} a_k t^{k/2}$ | $\alpha \sigma(t)$ |
| Model 3 | $\sum_{k=0}^{3} a_k t^{k/3}$ | $\alpha [\mu(t) - a_0]^{0.5}$ |

All the model parameters can be jointly estimated using the maximum likelihood method (MLM). The log-likelihood function is given by

$$\ln(L) = \sum_{j=1}^{n} \sum_{i=1}^{m} \ln \{f(x_{ij}; \mu(t_j), \sigma(t_j))\}$$

Figure 1. A wear curve at a cutting speed of 81.64 mm/min in high-speed milling [17].
where \( \phi(.) \) is the normal density function. The maximum likelihood estimates (MLEs) of the parameters can be obtained through maximizing \( \ln(L) \). This can be done using Solver of Microsoft Excel.

The distribution of time to the USL is given by

\[
F(t) = 1 - \Phi[\text{USL} - \mu(t), \sigma(t)]
\]

(2)

It is not the distribution of time to the hard failure of the cutting tool and can be viewed as the distribution of time to the quality failure.

### 2.2. Process capability index

Pearn and Hsu define the process capability index (CPI) as below [7]

\[
C_{\text{pmk}}(t) = \frac{d - |T - \mu(t)|}{\sqrt{3(T - \mu(t))^2 + \sigma^2(t)}}
\]

(3)

where \( d = (\text{USL} - \text{LSL})/2 \).

For the proposed mean models with \( \text{LSL} < a_0 < T \), the CPI at \( t = 0 \) is given by

\[
C_{\text{pmk}}(0) = \frac{d - |T - a_0|}{3(T - a_0)} = \frac{d}{3(T - a_0)} - 1/3
\]

(4)

This implies that \( C_{\text{pmk}}(0) \) increases as \( a_0 \) increases. Let \( t_T \) denote the solution of \( \mu(t) = T \). When \( t \approx t_T \), we have

\[
C_{\text{pmk}}(t_T) = \frac{d}{3\sigma(t_T)} = 2 \times \frac{d}{6\sigma(t_T)}
\]

(5)

Generally, \( d / [6\sigma(t_T)] > 1 \) and hence \( C_{\text{pmk}}(t_T) \) is generally larger than 2. Let \( t_U \) denote the solution of \( \mu(t) = \text{USL} \). When \( t = t_U \), we have

\[
C_{\text{pmk}}(t_U) = \frac{d/2}{3(d/2)^2 + \sigma^2(t_U)} < \frac{d/2}{3d/2} = 1/3
\]

(6)

According to the above analysis, the CPI is a unimodal function of time \( t \); and the process must be stopped at a time point within \( (t_T, t_U) \).

### 2.3. Age replacement policy and cost model

The age replacement policy is appropriate for the cutting tool since it is relatively expensive [18]. Three cost elements are considered: preventive replacement cost \( c_p \) with a probability of \( 1 - F(t) \), failure replacement cost \( c_f \) with a probability of \( F(t) \) and a Taguchi quality loss, which is given by

\[
l(t) = KV(t) = K\{[T - \mu(t)]^2 + \sigma^2(t)\}
\]

(7)

Let \( c_q \) denote the quality loss when \( x = \text{USL} \). Using this to Equation (7) yields

\[
l(t) = KV(t) = c_q\{[T - \mu(t)]^2 + \sigma^2(t)\}/d^2
\]

(8)

Let \( p \) denote that the production rate, i.e., the number of produced parts per unit time. The total quality loss over \( (0, t) \) is given by

\[
L(t) = \int_0^t l(x) pdx \approx 0.5p \sum_{k=1}^{N(t)} [l(k\Delta t) + l(k\Delta t - \Delta t)]\Delta t
\]

(9)

where \( \Delta t \) is a small time interval and \( N(t) = t/\Delta t \).

The cost rate model is given by [18]

\[
J(t) = [c_f F(t) + c_f R(t) + L(t)]/ \int_0^t R(x) dx
\]

(10)

The optimal replacement age corresponds to the minimum of \( J(t) \). Let \( t^* \) denote the optimal replacement age and \( J^* \) denote the corresponding cost rate.
2.4. Optimal initial position

It is noted that $t^*$ and $J^*$ are functions of $a_0$. That is, $J^*$ can be written as

$$J^*(a_0) = \phi(a_0)$$ (11)

The optimal initial position of the cutting tool corresponds to the minimum of $J^*(a_0)$. For a set of given values of $a_0$, compute the corresponding values of $J^*$. Fitting these points into a smooth curve yields $\phi(a_0)$, and the optimal value of $a_0$ can be obtained from the fitted curve.

3. Illustration

3.1. Data and earlier results

Pearn and Hsu present a case study, which deals with a particular type of aluminium lid product produced on a press\[7\]. The key characteristic is lid height, and its USL, LSL and $T$ are 68.4 mm, 64.65 mm and 66.525 mm, respectively. The data collected over one-week period are shown in table 2 in terms of values of $x_{ij}$ ($i = 1, 2, \ldots, 7; j = 1, 2, \ldots, 15$).

| $j$ \ $t_i$ | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|------------|------|------|------|------|------|------|------|
| 1          | 66.100 | 66.335 | 66.470 | 66.542 | 66.670 | 66.722 | 66.872 |
| 2          | 66.261 | 66.295 | 66.387 | 66.551 | 66.665 | 66.722 | 66.931 |
| 3          | 66.147 | 66.335 | 66.456 | 66.501 | 66.684 | 66.715 | 66.860 |
| 4          | 66.214 | 66.361 | 66.402 | 66.504 | 66.644 | 66.777 | 66.836 |
| 5          | 66.133 | 66.314 | 66.468 | 66.568 | 66.689 | 66.724 | 66.922 |
| 6          | 66.223 | 66.335 | 66.430 | 66.546 | 66.715 | 66.770 | 66.943 |
| 7          | 66.216 | 66.428 | 66.480 | 66.470 | 66.695 | 66.803 | 66.907 |
| 8          | 66.288 | 66.337 | 66.428 | 66.572 | 66.732 | 66.770 | 66.900 |
| 9          | 66.159 | 66.397 | 66.413 | 66.618 | 66.665 | 66.753 | 66.929 |
| 10         | 66.252 | 66.337 | 66.499 | 66.625 | 66.606 | 66.789 | 66.919 |
| 11         | 66.288 | 66.418 | 66.387 | 66.599 | 66.717 | 66.758 | 66.862 |
| 12         | 66.242 | 66.416 | 66.504 | 66.656 | 66.675 | 66.805 | 66.922 |
| 13         | 66.297 | 66.423 | 66.432 | 66.596 | 66.727 | 66.774 | 66.836 |
| 14         | 66.304 | 66.361 | 66.516 | 66.594 | 66.708 | 66.800 | 66.929 |
| 15         | 66.221 | 66.435 | 66.546 | 66.665 | 66.739 | 66.781 | 66.950 |

Figure 2 shows the plot of data. As seen, $X(t)$ has an increasing trend due to the tool wear. Therefore, some minimum level of process capability such as $C_{pmu} > 1$ can be maintained through monitoring the PCI.
Pearn and Hsu derive the distribution of $C_{pmk}$ [7]. According to the derived distribution, the critical value of $C_{pmk}$ associated with a type-I error of 0.05 and sample size $n = 15$ is 1.55. When the estimated process capability is smaller than the critical value, the process is stopped and the tool is reset or replaced. For the current example, $C_{pmk}$ is computed using the sample mean and sample variance [7]. The results are displayed in figure 3. Since $C_{pmk}$ is smaller than the critical value at $t = 7$, Pearn and Hsu recommend that the tool should be replaced at $t = 7$. This replacement time is too conservative as illustrated later.

![Figure 3. Plot of Cpk](image)

**3.2. Results obtained from the proposed models**

**3.2.1. Process mean function.** Figure 4 shows the plot of process mean function (i.e., those dots), which looks linear. Considering the fact that the wear rate of the cutting tool is bathtub-shaped, it is more reasonable to assume that the process mean function is inverse S-shaped.

![Figure 4. Plot of empirical mean degradation function](image)

The regression coefficients of the optional models and the corresponding values of $\rho$ are shown in table 3. In terms of $\rho$, Model 3 is slightly better than the other three models. This will be further confirmed in Section 3.2.3.
Table 3. Regression parameters and performances of the process mean functions.

|  | Linear | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| $a_0$ | 66.131 | 66.067 | 65.770 | 65.051 |
| $a_1$ | 0.1093 | 0.1742 | 0.6221 | 2.2242 |
| $a_2$ | -0.01733 | -0.2251 | -1.4806 |
| $a_3$ | 0.001341 | 0.05698 | 0.4298 |
| $\rho$ | 0.049969 | 0.049068 | 0.049032 | 0.049016 |

3.2.2. Process variance function. Let $\sigma_i$ denote the sample standard deviation at $t_i$. The empirical wear rate is computed using difference method given by

$$r(t_{i-1} + \Delta t / 2) = [x(t_i) - x(t_{i-1})] / (t_i - t_{i-1})$$  \hspace{1cm} (12)

where $\Delta t = 1$. Define

$$x(t_{i-1} + \Delta t / 2) = [x(t_i) + x(t_{i-1})] / 2$$  \hspace{1cm} (13)

Thus, using (12) and (13), one can compute correlation coefficient between the sample standard deviation and the empirical wear rate. Similarly, the correlation coefficients between the sample standard deviation and $[x(t) - a_0]^q$ with $q = 10^{-5}$ and 0.5 can be computed. Here, $q = 10^{-5}$ corresponds to Model 1 and $q = 0.5$ corresponds to Model 3. The best model has the largest correlation coefficient. For the current example, Table 4 shows the correlation coefficients of Models 1-3. As seen, Model 2 is the best, implying that the standard deviation function is approximately proportional to the wear rate.

Table 4. Correlation coefficients between $\sigma(t)$, $r(t)$ and $[\mu(t)-a_0]^q$.

| Model 1 ($q = 10^{-5}$) | Model 2 | Model 3 ($q = 0.5$) |
|---|---|---|
| -0.8061 | 0.3900 | -0.8028 |

3.2.3. Maximum likelihood estimates of model parameters.

For the standard deviation model given by Model 2, table 5 shows the MLEs of parameters of the fitted Wiener process model.

Table 5. Maximum likelihood estimates of the Wiener process model parameters.

| $a_0$ | $a_1$ | $a_2$ | $a_3$ | $\alpha$ |
|---|---|---|---|---|
| 65.934 | 0.3031 | -0.0332 | 0.02085 | 0.4075 |

3.2.4. Distribution of time to the USL. The distribution of time to the USL is given by equation (2), which can be well approximated by a normal distribution with parameters $\mu = t_U = 19.36$ and $\sigma = 0.4075$. Clearly, the distribution is highly concentrated.

3.2.5. Process capability index. Figure 5 shows the plot of PCI. As seen, the maximum of PCI occurs at $t = 3.54$, which corresponds to $\mu(t) = t$. When $t = 7.95$, $C_{pmk} = 1$. It seems that the cutting tool should be replaced at $t = 8$. However, the value of F(8) is nearly equal to zero, implying that replacing the cutting tool at $t = 8$ is too early. In other words, the PCI-based replacement decision is too conservative.

It should be noted that the third process mean model has $r(0) = \infty$ so that $\sigma(0) = \infty$ according to the second standard deviation model. As a result, $C_{pmk}(0) = 0$, which appears unreasonable. Therefore, the value of $C_{pmk}(t)$ inferred from the fitted model for small $t$ is generally unreliable.
3.2.6. Cost model for optimizing tool replacement time. Assume that production rate per unit time is $p = 1$ and cost parameters are $(c_p, c_l, c_o) = (1, 4, 0.5)$. Since the distribution is highly concentrated, it is reasonable to assume that the optimal replacement time $t^*$ is much smaller than $\mu - 3\sigma$ so that $F(t^*) \approx 0$. In this case, Equation (10) can be simplified as

\[
J(t) = \frac{c_p + L(t)}{t}, \quad t \leq \mu - 3\sigma
\]  

Figure 6 shows the plot of $J(t)$. As seen, the optimal solution is $t^* = 11.81 (< \mu - 3\sigma = 18.97)$, which is much larger than the PCI-based solution ($= 7.95$). This confirms the conclusion that the PCI-based replacement decision is too conservative.

It is noted that the cost rate curve is flat in the adjacent region of $t^*$ so that the relative error of the cost rate within $(10.87, 12.80)$ is smaller than 1%. This interval can be used as the opportunistic replacement interval.

3.2.7. Optimal initial position of the cutting tool. As mentioned earlier, parameter $a_0$ represents the initial position of the cutting tool. Different value of $a_0$ leads to different values of $t^*$ and $J^*$. Figure 7
shows the plot of $J^*$ vs. $a_0$. As seen, there exists an optimal value of $a_0$, where $J^*$ achieves its minimum. For the current example, the optimal value of $a_0$ is 65.229.

Table 6 shows the optimal solutions before and after the optimization of initial position. As seen, through the optimization of initial position, the useful life of cutting tool increases by 27.13% and the cost rate decreases by 12.60%. The effect of the optimization is very obvious.

|                | $a_0$  | $t^*$ | $J(t^*)$ |
|----------------|--------|-------|----------|
| Before optimization | 65.655 | 11.81 | 0.1138   |
| After optimization  | 65.229 | 15.01 | 0.0994   |
| Relative error, %   | 27.13  | 12.60 |          |

4. Conclusions
Dimensions of a machined part change with time due to the cutting tool wear. The dimension can be modelled by a stochastic process and the resulting model can be used to optimize the replacement time and initial position of the cutting tool. The main contributions of the paper have been:

- the Wiener process has been proposed to model the dimensional change in a manufacturing process; three process mean models and three process variance models have been proposed, and the method for model selection has been developed;
- a method to optimize the initial position of the cutting tool has been proposed and its effectiveness has been illustrated; and
- the CPI-based replacement decision is found to be too conservative. As a result, the paper is useful for both reliability researchers and manufacturing engineers.

Two topics for future research are:
- to develop the inverse S-shaped process mean models, whose initial wear rates are finite, and
- to develop the cost models that include the possible resetting, adjustment and/or grinding costs.

Acknowledgment
The research was supported by the National Natural Science Foundation of China (No. 71771029).
References
[1] Sheik A K, Raouf A, Sekerdey U A and Younas M 1999 Optimal tool replacement and resetting strategies in automated manufacturing systems Int. J. Prod. Res. 4 917–37.
[2] Xu W and Cao L 2015 Optimal tool replacement with product quality deterioration and random tool failure Int. J. Prod. Res. 53 1736-45.
[3] Tian W, Wells L J and Camelio J A 2016 ASME 2016 11th International Manufacturing Science and Engineering Conference pp 1-10.
[4] Li Z, Liu R and Wu D 2019 Data-driven smart manufacturing: tool wear monitoring with audio signals and machine learning J. Manuf. Processes pp 66-76.
[5] Ambhore N, Kamble D, Chinchankar S and Wayal V 2015 Tool condition monitoring system: A review Materials Today: Proceedings 2 3419-28.
[6] ISO 1993 ISO3685: Tool-life testing with single-point turning tools (International Standard)
[7] Pearn W L and Hsu Y C 2007 Optimal Tool replacement for processes with low fraction defective European Journal of Operational Research 180 1116-29.
[8] Zhang M, He Z and Liu Z 2007 2007 2nd IEEE Conference on Industrial Electronics and Applications pp 516–21.
[9] Hsu B M, and Shu M H 2010 Reliability assessment and replacement for machine tools under wear deterioration Int. J. Adv. Manuf. Technol. pp 355-65.
[10] Vagnorius Z, Rausand M and Sorby K 2010 Determining optimal replacement time for metal cutting tools European Journal of Operational Research pp 407-16.
[11] Deng Y, Zhu H, Zhang G and Yin H 2012 Optimal tool replacement decision method based on cost and process capability Mechanical Engineering and Technology (Advances in Intelligent and Soft Computing vol 125) ed Zhang T (Berlin: Springer Berlin Heidelberg) pp 9-14.
[12] Cao L, Xu W, Li H, Chen J and Chen P 2014 Tool replacement decision-making method for automatic production line Int. J. Mech. Eng. Educ. 50 pp 131-5.
[13] Genichi T 1986 Introduction to quality engineering: designing quality into products and processes (New York: White Plains).
[14] Sheik A K and Ahmad M 1987 A reliability model for a nonlinear damage process Reliability Engineering 18 pp 73-99.
[15] Prasad K N and Ramamoorthy B 2001 Tool wear evaluation by stereo vision and prediction by artificial neural network J. Mater. Process. Technol. 112 pp 43-52.
[16] Su J C, Huang C K and Tarng Y S 2006 An automated flank wear measurement of microdrills using machine vision J. Mater. Process. Technol. 180 pp 328-35.
[17] Lei X, Liao W, Xie F, Zheng K and Zhao J 2013 Predict on cutter wear and life in high-speed milling of pre-sintered zirconia used for denture Journal of Nanjing University of Science and Technology 37 pp 567-72.
[18] Murthy D N P and Jiang R 2008 Maintenance: decision models for management (Beijing: Science Press).