Elliptic flow from partially thermalized heavy-ion collisions

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We study to what extent the measured elliptic flow at RHIC constrains viscous deviations from ideal hydrodynamics. We solve a toy model where only transverse momenta are thermalized while the system undergoes longitudinal free-streaming. We show that RHIC data exclude such a model and thus require fast 3-dimensional thermalization.

1. MOTIVATION

One of the most important discoveries at RHIC so far has been the strong elliptic flow generated in non-central collisions [1, 2, 3]. For not too peripheral collisions the measured elliptic flow coefficient $v_2(p_\perp)$ at midrapidity almost exhausts the hydrodynamic limit [4, 5, 6, 7] up to transverse momenta $p_\perp \leq 2$ GeV, and in this entire domain the dependence of $v_2$ on the mass of the emitted particles [4, 8] accurately follows the hydrodynamically predicted pattern [3]. This means that the distribution of the momenta of well over 99% of the emitted particles is accurately described by (ideal) hydrodynamics.

Why is this so important? The initial transverse momentum distribution of the particles generated by the colliding nuclei is locally isotropic. Only their spatial distribution in the transverse plane is initially deformed (for $b \neq 0$). Interactions among the produced quanta are required to transfer this spatial anisotropy to momentum space. A non-vanishing $v_2$ is thus an unambiguous signature for reinteractions in the produced matter, and the observed large $v_2$ values prove that the initial parton liberation process is separated from the experimentally observed final state by a violently interacting stage of dynamical evolution. Consequently, there is a priori little reason to expect that calculable properties of the early matter formed in the reaction zone immediately after nuclear impact (such as, for example, the central rapidity density and the shapes of the rapidity and transverse momentum distributions of the produced gluons [3]) have any direct relationship with the corresponding experimentally observed values. The evolution of spectral shapes due to the strong rescattering must be taken into account, and even though adiabatic cooling and the build-up of collective flow move the spectral slopes in opposite directions it is highly unlikely that these effects cancel completely. Elliptic flow itself is an example for a qualitative change of the spectra between particle formation and decoupling.

Microscopic studies [10, 11] show that $v_2(p_\perp)$ is a monotonic function of the mean free path, $\lambda \sim (\sigma\rho)^{-1}$; the hydrodynamic limit is approached from below as $\lambda \to 0$. The observation that at RHIC $v_2(p_\perp)$ almost exhausts the hydrodynamic limit thus appears to force the conclusion [1, 3, 6, 7, 12] that the fireballs formed in Au+Au collisions at

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RHIC thermalize fast and efficiently. Since the creation of elliptic flow is driven by the spatial deformation of the reaction zone which quickly decreases either spontaneously via free-streaming [5] or (more rapidly) as a result of the flow anisotropy itself [13], $v_2$ is sensitive to the very early collision stage. Within hydrodynamics the RHIC $v_2$ data can only be reproduced by assuming thermalization at times $\tau_{th} \leq 1 \text{ fm/c}$ (we use $\tau_{th} = 0.6 \text{ fm/c}$ [3, 6, 12]). As the hydrodynamic limit of $v_2$ can only be reached from below and the data almost exhaust it, we can use the hydrodynamic model to estimate the energy density at thermalization. One finds values [6] at least an order of magnitude above the critical one for hadronization, $\epsilon \approx 1 \text{ GeV/fm}^3$. The thermalized state formed $\sim 1 \text{ fm/c}$ after nuclear impact must therefore have been a quark-gluon plasma which, according to the hydrodynamical simulations, lives for several fm/c before hadrons first appear. Unless the presented chain of arguments leading to the conclusion of early thermalization can be broken, the implication is unavoidable that at RHIC a well-developed quark-gluon plasma has been created.

The purpose of the work reported here [14] is to try to poke holes into the early thermalization argument. It was already pointed out by Ollitrault [4] that $v_2$ is sensitive to the stiffness of the equation of state (EOS) of the thermalized matter and increases monotonically with the sound velocity $c_s^2 = \partial P/\partial \epsilon$. Might it be possible to trade off thermalization against a stiffer EOS? The concrete idea studied by us [14] was that perhaps the thermalization of the (on average much larger) longitudinal momenta of the liberated partons takes longer than transverse thermalization, resulting in a smaller longitudinal than transverse thermal pressure. As shown by Teaney at this conference, this is similar to the expected effects from shear viscosity on the longitudinal hydrodynamic expansion [15]. If we exaggerate a bit and idealize the model by assuming collisionless free-streaming with boost-invariant initial conditions in the longitudinal direction coupled with complete local thermalization of the transverse momenta, we can write down an analytic expression for the phase-space distribution function in terms of macroscopic parameters for which we can analytically derive macroscopic equations of motion which generalize the usual ideal hydrodynamic equations. In this idealization there is no longitudinal pressure at all; all the hydrodynamic work goes in the transverse direction. Can we generate larger elliptic flow in this way? If yes, the RHIC data would no longer saturate the theoretical limit and the fast thermalization argument would break down. We’ll see that it doesn’t.

2. A TRANSVERSALLY THERMALIZED MODEL (TTHM)

With boost-invariant initial conditions, assuming that all particles originate at $t = 0$ from $z = 0$ (i.e. infinitely Lorentz contracted colliding nuclei), an appropriate ansatz for the phase-space distribution of massless gluons is

$$f(x, k, t) = \frac{\tau_0}{\tau} \frac{\delta(y - \eta)}{\gamma_\perp e^{k \cdot u/T} - 1}.$$  \hspace{1cm} (1)

Here $\tau = \sqrt{t^2 - z^2}$, $\eta = \frac{1}{2} \ln \frac{t + z}{t - z}$, and $y = \frac{1}{2} \ln \frac{k_0 + k_\perp}{k_0 - k_\perp}$. We use longitudinal boost-invariance to parametrize the local flow as $u^\mu = \gamma_\perp (\cosh \eta, v_\perp, \sinh \eta)$. The transverse flow velocity $v_\perp$ with $\gamma_\perp = 1/\sqrt{1 - v_\perp^2}$ and the “transverse temperature” $T$ are functions of the longitudinal proper time $\tau$ and the transverse position $x_\perp$, but independent of space-time rapidity $\eta$. 
Due to the free-streaming constraint $\delta(y-\eta)$ the exponent of the Bose distribution reduces to $k \cdot u = \gamma_\perp k_\perp (1 - k_\perp \cdot v_\perp)$, showing thermalization of only the transverse momenta.

Inserting this ansatz into the kinetic definition of the gluon energy momentum tensor, $T^{\mu\nu}(x, t) = \nu_g \int \frac{d^3k}{(2\pi)^3} \frac{k^\mu k^\nu}{E} f(x, k, t)$ ($\nu_g = 16$ is the gluon spin-color degeneracy), one finds

$$T^{\mu\nu} = \left( e + P_\perp \right) u^\mu u^\nu - P_\perp \left[ g^{\mu\nu} + n^\mu n^\nu + v_\perp (u^\mu m^\nu + m^\mu u^\nu) \right] \tag{2}$$

with the equation of state (EOS) $e(x_\perp, \tau) = 2 P_\perp(x_\perp, \tau) = \nu_g (\tau_0/\tau)(\pi^2 T^4(x_\perp, \tau)/60)$. The difference between longitudinal and transverse momenta requires two additional vectors to decompose $T^{\mu\nu}$, $n^\mu = (\sinh \eta, 0, 0, \cosh \eta)$ and $m^\mu = \gamma_\perp (v_\perp \cosh \eta, \hat{v}_\perp, v_\perp \sinh \eta)$ with $n \cdot m = n \cdot u = m \cdot u = 0$. The resulting extra terms relative to the ideal fluid decomposition can be viewed as viscous corrections \[15\]. The EOS is consistent with a vanishing trace of $T^{\mu\nu}$ for massless particles and the absence of longitudinal pressure; the corresponding sound velocity $c_s = 1/\sqrt{2}$ is larger than for an ideal gluon gas, i.e. the EOS is stiffer.

With the decomposition (2) the macroscopic equations of motion $\partial_\mu T^{\mu\nu} = 0$ become

$$\frac{\partial T^{0\nu}}{\partial \tau} + \frac{T^{0\nu}}{\tau} + \nabla^j T^{j0\nu} = 0, \quad \nu = 0, 1, 2, 3, \quad j = 1, 2, 3. \tag{3}$$

The only difference from the analogous ideal hydrodynamic equations \[8\] is the absence of a term $-P/\tau$ on the right hand side of the $\nu=0$ equation, reflecting work done by the longitudinal pressure. Standard hydrodynamic codes for the transverse expansion of systems with longitudinal boost invariance can thus be used to solve these equations.

### 3. RESULTS

Lacking longitudinal pressure, the energy density in TTHM decreases more slowly with $\tau$ than in ideal hydrodynamics (HDM). However, the larger transverse pressure performs more work in the transverse directions, resulting in stronger radial and elliptic flow. As a consequence, initial conditions which in HDM lead to a consistent description of the data produce in TTHM much too flat transverse momentum spectra. The reproduction of the central collision data via TTHM thus requires retuned initial conditions. This retuning is facilitated by the numerical observation \[14\] that for massless particles the TTHM dynamics leads to completely time-independent transverse momentum spectra; transverse flow buildup thus exactly compensates for cooling! We have no analytical explanation for this fact. It implies that the final slope of the transverse momentum spectrum is given by the initial temperature; steeper final spectra thus require a reduced initial temperature. To maintain the measured normalization $dN/dy$ one then must also increase the initial longitudinal volume, by increasing the starting time $\tau_0$ for the TTHM dynamics. This implies late transverse thermalization (and, of course, an even later longitudinal one).

To obtain the same final spectra and radial flow in TTHM as in HDM tuned to RHIC data, we find that we should start the transverse expansion about 10 times later with a $\approx 15 - 20$ times lower initial energy density. The total remaining time until freeze-out is thereby shortened considerably, and even though we do not take into account the decrease of the spatial deformation of the reaction zone by transverse free-streaming prior to the onset of transverse TTHM dynamics, we find that now considerably less elliptic flow (only about half as much as in HDM) is generated (Fig. 1). As the data almost
exhaust the HDM values, TTHM underpredicts them by about a factor two, both for the $p_{\perp}$-integrated elliptic flow $v_2$ (Fig. 1a) and for the $p_{\perp}$-slope of the differential elliptic flow $v_2(p_{\perp})$ (Fig. 1b). The TTHM model is thus experimentally excluded; only a model such as HDM with a considerable degree of early longitudinal thermalization can describe the RHIC measurements.

![Figure 1](image.png)

Figure 1. $v_2$ as a function of impact parameter $b$ (left) and of $p_{\perp}$ for two values of $b$ (right) from TTHM (solid) and HDM (dashed). For details see Ref. [14].

In closing we note that for massless bosons $v_2(p_{\perp} \to 0)$ approaches a non-zero value (Fig. 1b). This reflects the singular nature of the Bose distribution at $p_{\perp} = 0$ [14]. We suggest to use this as a test for thermalization of small-$p_{\perp}$ gluons in parton cascades and of direct photons in experiment.

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