Estimate of the long-distance contribution through $b \rightarrow s\psi$ to the $B_s \rightarrow \gamma\gamma$ decay rate

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Abstract

The $B_s \rightarrow \phi\psi$ decay is modeled through the inclusive $b \rightarrow s\psi$ decay. Using the Vector Meson Dominance model, the amplitude for the chain process $B_s \rightarrow \phi\psi \rightarrow \phi\gamma \rightarrow \gamma\gamma$ is estimated and it is found to be at most 4% of the corresponding amplitude from the $O_7$ type LD contribution. The intermediate amplitude for the process $B_s \rightarrow \phi\gamma$ is compared with the corresponding one obtained by a different approach based on the interaction of the virtual charm quark loop with soft gluons [7]. Both amplitudes are found to agree within 10%. Further the influence on the branching ratio $B(B_s \rightarrow \gamma\gamma)_{SD+LD_{O7}}$ from inclusive $b \rightarrow s\psi$ is estimated as less than 1%.

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1 Introduction

Rare B-meson decays are one of the main research fields in recent particle physics. Since they occur only at loop level in common flavour changing neutral current (FCNC)-forbidden models, the theoretical and experimental investigations provide precise tests of the Standard Model (SM) and possible new physics beyond. An interesting candidate for rare decays is $B_s \to \gamma\gamma$, which has a rich final state since the two photon system can be in a CP-odd or a CP-even state. In the literature, $B_s \to \gamma\gamma$ has been studied in the lowest order in the framework of the constituent quark model [2] and found to consist in the effective Hamiltonian theory [1], which is obtained by integrating out heavy particles (top quark and $W$ boson in the SM) from the full theory, of contributions from $O_7$ and four-quark operators. The calculation for $B_s \to \gamma\gamma$ decay with leading logarithmic QCD corrections including $O_7$ type long distance (LD) effects and in an approach based on heavy quark effective theory to model the bound state was recently done in [3].

In the present work we continue and estimate the additional LD effect due to the dominant four-quark operators $O_1$ and $O_2$ (see eq. (2)) through the $B_s \to \phi\psi \to \phi\gamma \to \gamma\gamma$ chain decay. We use at quark level $b \to s\psi$ followed by the $b \to s\gamma$ decay [4] and we pass to the hadronic level using the transition form factor $F_1(0)$ from the amplitude $A(B_s \to \phi)$ [3,4]. For both the conversions $\psi \to \gamma$ and $\phi \to \gamma$ we impose the Vector Meson Dominance (VMD) model [4]. The conversion $\psi \to \gamma$ needs further manipulation because of the strong contribution of the longitudinal part of the $\psi$ meson. We extract the transverse part using the Golowich-Pakvasa procedure [4,5]. Further we calculate the $O_{1,2}$ type LD effect to the $B_s \to \phi\gamma$ decay using the method given in [7], namely, by taking into account the virtual c-quark loop instead of the hadronization of the $\bar{c}c$ pair. This procedure was originally applied to estimate the LD effect in $B \to K^*\gamma$ decay and uses operator product expansion and QCD sum rule techniques. We compare the amplitudes obtained by these two different methods and show that they are in good agreement within the errors of the calculation. Finally we estimate the influence of the $O_{1,2}$ type LD contribution on the branching ratio for $B_s \to \gamma\gamma$ decay.
The paper is organized as follows: In section 2, we calculate the $O_{1,2}$ type LD contribution to $B_s \to \gamma\gamma$ due to the chain process $B_s \to \phi\psi \to \phi\gamma \to \gamma\gamma$ using the Gordon decomposition and further the VMD model. We compare our amplitude for the decay $B_s \to \phi\psi \to \phi\gamma$ with the $O_{1,2}$ type one for $B_s \to \phi\gamma$, calculated by virtual charm loop-soft gluon interaction \cite{7}. Section 3 contains a discussion of the branching ratio $B(B_s \to \gamma\gamma)_{SD+LD_{O7}+LD_{O2}}$ where the subscripts $SD$, $LD_{O7}$, $LD_{O2}$ denote QCD short distance (SD) contributions and LD ones due to $B_s \to \phi\gamma \to \gamma\gamma$ and $B_s \to \phi\psi \to \phi\gamma \to \gamma\gamma$ decays, respectively.

2 The chain process $B_s \to \phi\psi \to \phi\gamma \to \gamma\gamma$

We first consider the additional contribution to $b \to s\gamma$ from $b \to s\psi \to s\gamma$, where $\psi \chi$ are all $\bar{c}c$ $J = 1$ bound states, see fig. 1. The relevant part of the effective Hamiltonian describing this process is given as

$$H_{eff} = 4\frac{G_F}{\sqrt{2}}V_{cs}^*V_{cb}(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)), \quad (1)$$

with the dominant four-Fermi operators

$$O_1 = \bar{s}_\alpha \gamma_\mu \frac{1 - \gamma_5}{2} c_\beta \bar{c}_{\beta}' \gamma_\mu \frac{1 - \gamma_5}{2} b_\alpha,$$

$$O_2 = \bar{s}_\mu \gamma_\mu \frac{1 - \gamma_5}{2} c_\mu \bar{c}_{\mu}' \gamma_\mu \frac{1 - \gamma_5}{2} b. \quad (2)$$

Here $\alpha, \beta$ are $SU(3)$ colour indices and $V_{ij}^{(*)}$ are the relevant elements of the quark mixing matrix. The initial values of the corresponding Wilson coefficients are $C_1(m_W) = 0$ and $C_2(m_W) = 1$. To include leading logarithmic QCD corrections we evaluate $C_{1,2}(\mu)$ at the relevant scale, $\mu \approx m_b$ for $B$-decays, and this takes into account short distance effects from single gluon exchange. The analytical expressions can be found in \cite{1}.

Further we have used the unitarity of the CKM matrix $V_{cs}^*V_{cb} = -V_{ts}^*V_{tb} - V_{us}^*V_{ub}$ and have neglected the contribution due to an internal u-quark, since $V_{ts}^*V_{tb} \ll V_{ts}^*V_{tb} = \lambda_t$.

Using factorization, we obtain the inclusive decay amplitude for the process $b \to s\psi$ \cite{4} as

$$A(b \to s\psi(k_1, \epsilon_\psi)) = -iC_f\psi(m_\psi^2)m_\psi \bar{s}_\mu \gamma^\mu(1 - \gamma_5)b_\mu \epsilon_\psi. \quad (3)$$
Here

$$C = -\frac{G_F}{\sqrt{2}} \lambda_t a_2(\mu)$$

(4)

with, assuming naive factorization,

$$a_2(\mu) = C_1(\mu) + \frac{C_2(\mu)}{N_C} ,$$

(5)

where $N_C = 3$ in colour $SU(3)$ and $k_1, \epsilon^\psi$ are the momentum and the polarization vector of the $\psi$, respectively. In eq. (3) we used the matrix element

$$< 0|\bar{c}\gamma_\mu c|\psi(k_1, \epsilon^\psi) >= f_\psi(m^2_\psi) m_\psi \epsilon^\psi_\mu .$$

(6)

At this stage there is a critical remark about factorization in order, concerning the value of $a_2(\mu)$ used. The decay under consideration is a class II decay following the classification of [8]. In general eq. (3) is written as

$$a_2^{eff} = (C_1(\mu) + \frac{C_2(\mu)}{N_C}) [1 + \epsilon_1(\mu)] + C_2(\mu)\epsilon_2(\mu) ,$$

(7)

where $\epsilon_1(\mu)$ and $\epsilon_2(\mu)$ parametrize the non-factorizable contributions to the hadronic matrix elements. $a_2^{eff}$ takes into account all contributions of the matrix elements in contrast to $a_2(\mu)$, which assumes naive factorization $\epsilon_1(\mu) = \epsilon_2(\mu) = 0$. Especially $\epsilon_2(\mu)$, which is the colour octet piece, has sizable contributions to naive factorization in class II decays [8]. Furthermore, the additional problem is not to know the correct factorization scale. In order to include the

Figure 1: The diagram contributing to $B_s \rightarrow \phi\psi \rightarrow \phi\gamma$.
non-factorizable corrections we use in the definition of $C$ in eq. (4) the effective coefficient $a_2^{\text{eff}}$, which is determined experimentally from the world average branching ratio of $\bar{B} \to \bar{K}^{(*)}\psi$ as

$$a_2^{\text{eff}} = 0.21.$$  \hspace{1cm} (8)

This choice restores the correct scale and is $\mu$ independent. Writing the Ansatz

$$a_2^{\text{eff}} = C_1(m_b) + \xi C_2(m_b),$$  \hspace{1cm} (9)

it follows that $\xi \approx 0.41$ with $C_1(m_b) = -0.25$ and $C_2(m_b) = 1.11$ for the input values given in Table 1. For comparison, naive factorization would give $a_2(m_b) = 0.12$.

| Parameter | Value |
|-----------|-------|
| $m_b$     | 4.8 (GeV) |
| $m_c$     | 1.4 (GeV) |
| $a_{em}^{-1}$ | 129 |
| $\lambda_t$ | 0.04 |
| $m_t$     | 175 (GeV) |
| $m_W$     | 80.26 (GeV) |
| $m_Z$     | 91.19 (GeV) |
| $\Lambda_{QCD}^{(5)}$ | 0.214 (GeV) |
| $\alpha_s(m_Z)$ | 0.117 |

Table 1: Values of the input parameters used in the numerical calculations.

Now our aim is to replace the $\psi$ meson with the photon $\gamma$ and to construct a gauge invariant amplitude. This can be done by killing the longitudinal component of the meson $\psi$ so, that $\epsilon_\mu$ is changed to the polarization vector $\epsilon_\mu^\gamma$ of the photon $\gamma$. For this we use the Golowich-Pakvasa procedure with the help of the Gordon identity, namely $\gamma_\mu \gamma_\alpha = g_{\mu\alpha} - i \sigma_{\mu\alpha}$. We start with the vertex $\bar{s} \gamma_\mu (1 - \gamma_5) b$ and using the equation of motion $\not\!\!p b = m_b b$ and momentum conservation $p = p' + k_1$, we get

$$\bar{s} \gamma_\mu (1 - \gamma_5) b = \frac{1}{m_b} \{ \bar{s} \gamma_\mu \not\!\!p' (1 + \gamma_5) b + \bar{s} \gamma_\mu \not\!k_1 (1 + \gamma_5) b \},$$  \hspace{1cm} (10)

where $p, p'$ are the momenta of the $b$ and $s$ quark, respectively. We neglect the first term in eq. (10) since $\frac{m_s}{m_b} \ll 1$ and $p'^\mu \epsilon_\mu^T = 0$, which follows from $\epsilon_\mu^T p'^\mu = 0$ in the rest frame of the $b$
quark and the transversality condition \( \epsilon^T \kappa_1^\mu = 0 \), where \( \epsilon^T \) is the transversal polarization vector of the \( \psi \) meson [4]. The second term can be written as

\[
\frac{1}{m_b} \bar{s} \gamma_\mu k_1 (1 + \gamma_5) b = \frac{1}{m_b} \{ \bar{s}(1 + \gamma_5) k_1 \mu b - i \bar{s} \sigma_{\mu\alpha} k_1^\alpha (1 + \gamma_5) b \}. \tag{11}
\]

Only the \( \sigma_{\mu\alpha} \) term in eq. (11) couples to the transversal component of the \( \psi \) and we obtain the corresponding amplitude as

\[
\mathcal{A}(b \to s \psi^T) = -2C f_{\psi}(m^2_\psi) \frac{m_{\psi}}{m_b} \bar{s} \sigma_{\mu\alpha} k_1^\alpha R b \epsilon^T_\mu, \tag{12}
\]

where \( R = \frac{1 + \gamma_5}{2} \) denotes the chiral right projection. Note that the coupling structure is the same as due to a direct use of \( O_7 = \frac{e}{16\pi^2} \bar{s} \sigma_{\mu\nu} m_b R b F^{\mu\nu} \) [1] with the photon field strength tensor \( F^{\mu\nu} \) and \( m_s = 0 \). For the \( \psi^T \to \gamma \) conversion following the VMD mechanism we have

\[
<0| J_{\mu,el} | \psi^T(k_1, \epsilon^T) 0> = e Q_c f_{\psi}(0) m_\psi \epsilon^T_\mu, \tag{13}
\]

where \( Q_c = 2/3 \) and \( f_{\psi}(0) \) is the coupling at \( k_1^2 = 0 \), see eq. (17). Using the intermediate propagator of the \( \psi \) meson at \( k_1^2 = 0 \), we get

\[
\mathcal{A}(b \to s \psi^T \to s \gamma) = 2C f^2_{\psi_i}(0) \frac{e Q_c}{m_b} \bar{s} \sigma_{\mu\alpha} k_1^\alpha R b \epsilon^T_\mu. \tag{14}
\]

The expression for the amplitude eq. (14) can be completed by summing over all \( \bar{c}c \) resonant states \( \psi(1S), \psi(2S), \psi(3770), \psi(4040), \psi(4160) \) and \( \psi(4415) \)

\[
\mathcal{A}(b \to s \psi^T_i \to s \gamma) = 2C \sum_i f^2_{\psi_i}(0) \frac{e Q_c}{m_b} \bar{s} \sigma_{\mu\alpha} k_1^\alpha R b \epsilon^T_\mu. \tag{15}
\]

The various decay couplings \( f_{\psi_i} = f_{\psi_i}(m^2_{\psi_i}) \) are calculated using

\[
f^2_{\psi_i} = \Gamma(\psi_i \to e^+ e^-) \frac{3m_{\psi_i}}{Q_c^2 4\pi \alpha^2_{em}} \tag{16}
\]

and the measured widths from [10] and given in Table 2.

Here we need to extrapolate the coupling \( f_{\psi_i}(k_1^2 = m^2_{\psi_i}) \) to \( f_{\psi_i}(0) \). We used the suppression factor [4]

\[
\kappa = f^2_{\psi(1S)}(0)/f^2_{\psi(1S)}(m^2_{\psi}) = 0.12 \tag{17}
\]
| $\psi_i$ | $f_{\psi_i} \text{[GeV]}$ |
|---------|----------------|
| $f_{\psi(1S)}$ | 0.405 |
| $f_{\psi(2S)}$ | 0.282 |
| $f_{\psi(3770)}$ | 0.099 |
| $f_{\psi(4040)}$ | 0.175 |
| $f_{\psi(4160)}$ | 0.180 |
| $f_{\psi(4415)}$ | 0.145 |

Table 2: Vector meson coupling constants used in the numerical calculations.

obtained from data on the photoproduction of the $\psi$ and taking the factor $\kappa$ as universal for the other resonances. \[1\] We now use eq. (15) to find the matrix element of $B_s \to \phi \gamma$ through the $b \to s \bar{\psi} T \to s \gamma$ transition at quark level. The matrix element \[3\] is given as

$$<\phi(p')|\bar{s}\sigma_{\mu\alpha} R k_1^\alpha b|B_s(p)> = i\epsilon_{\mu\nu\rho\sigma} p^{\rho} p'^{\sigma} F_1(k_1^2)$$

$$+ \epsilon^{\phi}_{\mu} p.k_1 - p_{\mu} k_1. \epsilon^{\phi} G(k_1^2),$$

and we get the amplitude

$$A(B_s \to \phi \gamma) = 2C \epsilon^{\phi}_{\mu} \sum_{i} \frac{f_{\psi_i}^2(0)}{m_b} eQ_c \{i\epsilon_{\mu\nu\rho\sigma} k_1^{\rho} p'^{\sigma}$$

$$+ \frac{g_{\mu\nu} m_{B_s}^2 - m_{\phi}^2}{2} \} F_1(0),$$

where $\epsilon_{\mu}$, $\epsilon^{\phi}_{\nu}$ are the polarization vectors and $k_1$, $p'$ are the momenta of the photon and $\phi$ meson, respectively. We used $G(k_1^2 = 0) = F_1(k_1^2 = 0)$ \[3\]. Note, that the form factors introduced above are in general functions of two variables $k_1^2$ and $p'^2$. We abbreviated here $F_1(k_1^2) \equiv F_1(k_1^2, p'^2 = m_{\phi}^2)$ and took $F_1(0) = 0.24 \pm 0.02$ from \[3\].

Now we want to compare our result for $A(B_s \to \phi \gamma)$ eq. (19) with the same amplitude calculated by the method worked out in \[7\]. This method is based on the new effective quark-gluon operator obtained by the interaction of the virtual charm quark loop with soft gluons, in contrast to a phenomenological description in terms of $\psi$ resonances converting into a photon, as we used. In this approach, the operator $O_1$ does not give any contribution to the matrix element of $B_s \to \phi \gamma$ for an on-shell photon. The Fierz transformation of the operator $O_2$ reads

$$O_2 = 1/N_C O_1 + 1/2 O_{octet},$$

\[1\] This is consistent with $\kappa = 0.11$ \[11\] based on a dispersion relation calculation.
where

\[ O_{\text{octet}} = 4(\bar{c}\gamma_{\mu}\frac{1 - \gamma_5 \lambda_a}{2} c)(\bar{s}\gamma_\mu\frac{1 - \gamma_5 \lambda_b}{2} b), \]  

(21)

and \( \lambda^a/2 \) are the \( SU(3) \) colour generators. Then the only contribution comes from the colour octet part \( O_{\text{octet}} \). Using the operator \( O_{\text{octet}} \) as a vertex of the virtual charm quark loop, which emits a real photon, and taking into account the c-quark-soft gluon interaction, a new effective operator is obtained. The matrix element of this operator between \( B_s \) and \( \phi \) meson states gives the long distance amplitude of \( B_s \to \phi \gamma \) decay due to the \( O_{1,2} \) operators and it is written as (see [7] for details; there the amplitude for the decay \( B \to K^* \gamma \) is given)

\[ \mathcal{A}'(B_s \to \phi \gamma) = 2C'e^{\mu}_{1}e^{\phi}_{\nu}\{i\epsilon_{\mu\nu\rho\sigma}k^{\rho}p^{\sigma}L + \frac{m_{B_s}^2 - m_{\phi}^2}{2}g_{\mu\nu}\tilde{L}\}, \]

(22)

where \( C' = \frac{g_{\pi}f_{\phi}C_{\phi}(\mu)}{8\sqrt{2}\pi^3 m_{\pi}^3} \). The form factors \( L \) and \( \tilde{L} \) are calculated using QCD sum rules [7],

\[
L = \frac{m_b}{m_\phi m_{B_s}^2 f_{B_s} f_{\phi}} \exp\left(\frac{m_{B_s}^2}{M^2} + \frac{m_{\phi}^2}{M'^2}\right) \\
\cdot \left\{ \frac{m_b}{48} \left[ \frac{\alpha_s}{\pi} < G^2 > \right] \int \frac{m_{M^2}}{s} ds \frac{e^{-s}}{s} M^4 - \frac{m_{B_s}^2}{M^2} \left(1 + \frac{M'^2}{sM^2}\right) \right\} \\
- \left[ \frac{m_{B_s}^2}{12} - \frac{4\pi\alpha_s}{27} \right] \left(1 + \frac{m_{\phi}^2}{M'^2}\right) \exp\left(-\frac{m_{\phi}^2}{M'^2}\right),
\]

\[
\tilde{L} = \frac{m_b}{m_\phi m_{B_s}^2 f_{B_s} f_{\phi}} \exp\left(\frac{m_{B_s}^2}{M^2} + \frac{m_{\phi}^2}{M'^2}\right) \\
\cdot \left\{ \frac{m_b}{48} \left[ \frac{\alpha_s}{\pi} < G^2 > \right] \int \frac{m_{M^2}}{s} ds \frac{e^{-s}}{s} M^4 - \frac{m_{B_s}^2}{M^2} \left(1 + \frac{M'^2}{sM^2}\right) \right\} \\
- \left[ \frac{m_{B_s}^2}{12} - \frac{16\pi\alpha_s}{27} \right] \left(1 + \frac{m_{\phi}^2}{M'^2}\right) \exp\left(-\frac{m_{\phi}^2}{M'^2}\right). 
\]

(23)

The Borel parameters \( M \) and \( M' \) are varied to find the stability region for \( L \) and \( \tilde{L} \). We used the following input parameters in the sum rules: \( m_0^2 = 0.8 \text{ GeV}^2, < \bar{s}s > = -0.011 \text{ GeV}^3, \frac{\alpha_s}{\pi} < G^2 > = 0.012 \text{ GeV}^4, f_\phi = 0.23 \text{ GeV}, f_{B_s} = 0.2 \text{ GeV}, m_\phi = 1.019 \text{ GeV} \) and \( m_{B_s} = 5.369 \text{ GeV} \).

The stability region is reached for \( 6 \text{ GeV}^2 \leq M^2 \leq 9 \text{ GeV}^2 \) and \( 2 \text{ GeV}^2 \leq M'^2 \leq 4 \text{ GeV}^2 \) and we get

\[
L = (0.30 \pm 0.05) \text{ GeV}^3,
\]

\[
\tilde{L} = (0.35 \pm 0.05) \text{ GeV}^3.
\]

(24)
Writing the amplitude for $B_s \to \phi \gamma$ as

$$\mathcal{A}^{(t)}(B_s \to \phi \gamma) = \epsilon_1^\mu \epsilon_2^\nu (i\epsilon_{\mu\nu\rho\sigma} k_1^\rho p_2^\sigma A^{-} + g_{\mu\nu} A^{+}) ,$$

(25)

and using eq. (19), (22) and (24), we can compare the coefficients obtained by two different methods and get

$$\frac{|A^- - A'^-|}{A^-} = 10\% ,$$

$$\frac{|A^+ - A'^+|}{A^+} = 5\% .$$

(26)

This means, that the amplitudes agree within 10%.

In our approach, the structure for the transition $b \to s \psi^T \to s \gamma$ is proportional to $\sigma_{\mu\alpha} \frac{1+\gamma_5}{2} k_1^\alpha$ (see eq. (13)), since the longitudinal part of the $\psi$ meson is disregarded to convert into a photon. Further, the form factors $F_1(k_1^2)$ and $G(k_1^2)$ in eq. (18) are related for a real photon ($k_1^2 = 0$), $F_1(0) = G(0)$. Therefore, in the amplitude $\mathcal{A}(B_s \to \phi \gamma)$ appears only one form factor, which is $F_1(0)$ in eq. (19). However, the form factors $L$ and $\tilde{L}$ in $\mathcal{A}'(B_s \to \phi \gamma)$ given in eq. (22) are not related. They are calculated separately using QCD sum rules and this causes the difference between the ratios in eq. (26). In spite of the fact that the amplitudes $A^\pm$ and $A'^\pm$ are different from each other, they coincide within the given approximation and theoretical uncertainties lying in both methods.

We can now present the amplitude for $B_s \to \gamma \gamma$ due to the chain reaction $B_s \to \phi \psi \to \phi \gamma \to \gamma \gamma$ using the intermediate propagator at zero momentum transfer and the $\phi \to \gamma$ conversion vertex from the VMD model,

$$< 0 | J_{\mu \epsilon_i} | \phi(p', \epsilon^\phi) > = e Q_s f_\phi(0) m_\phi \epsilon^\phi_{\mu},$$

(27)

where the polarization vector $\epsilon^\phi_{\mu}$ is treated as transversal. To apply the VMD mechanism to the amplitude eq. (13), we have to know the form factor at $F_1(k_1^2 = 0, p'^2 = 0)$. We took the extrapolated value $\bar{F}_1(0) \equiv F_1(0, 0) = 0.16 \pm 0.02$ from [3]. Then the amplitude can be written with $p' \to k_2$, $\epsilon^\phi \to \epsilon_2$ as

$$\mathcal{A}(B_s \to \phi \psi \to \phi \gamma \to \gamma \gamma) = \epsilon_1^\mu(k_1) \epsilon_2^\nu(k_2) [g_{\mu\nu} A^{+}_{LD,2} + i\epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta A^{-}_{LD,2}]$$

(28)
with the CP-even \( A^+ \) and CP-odd \( A^- \) parts

\[
A_{LD_{O_2}}^+ = 4\chi \frac{C}{m_b} \bar{f}_1(0) \sum_i f_{\psi_i}^2(0) e Q_c \frac{m_{B_s}^2 - m_{\phi}^2}{2},
\]

\[
A_{LD_{O_2}}^- = 4\chi \frac{C}{m_b} \bar{f}_1(0) \sum_i f_{\psi_i}^2(0) e Q_c ,
\]

(29)

where \( C \) is defined in eq. (4) and the conversion factor \( \chi \) is defined as \( \chi = -e Q_s \frac{f_\phi(0)}{m_\phi} \). Here \( f_\phi(0) = 0.18 \text{ GeV} \) \[1\] and \( Q_s = -1/3 \). The extra factor 2 comes from the addition of the diagram with interchanged photons.

The decay width for \( B_s \rightarrow \gamma \gamma \) is obtained by adding the short distance amplitudes, the \( O_7 \) type LD effects \[3\] and the LD effects due to the \( B_s \rightarrow \phi \psi \rightarrow \phi \gamma \rightarrow \gamma \gamma \) transition, resulting in

\[
\Gamma(B_s \rightarrow \gamma \gamma) = \frac{1}{32\pi m_{B_s}} (4|A^+ + A_{LD_{O_7}}^+ + A_{LD_{O_2}}^+|^2
\]

\[
+ \frac{m_{B_s}^4}{2}|A^- + A_{LD_{O_7}}^- + A_{LD_{O_2}}^-|^2) .
\]

(30)

3 Numerical estimates and discussion

We presented a VMD model based calculation of the LD contribution to CP-even \( A^+ \) and CP-odd \( A^- \) decay amplitudes for \( B_s \rightarrow \gamma \gamma \) decay due to the inclusive process \( b \rightarrow s \psi \). The conversions to photons from both the \( \psi_i \) resonances and the \( \phi \) meson lead to two suppressions and make the amplitudes in eq. (29) smaller compared to the ones from the LD effect of the operator \( O_7 \) \[3\].

We estimated the ratio

\[
\rho = \left| \frac{A_{LD_{O_2}}^{+(-)}(B_s \rightarrow \phi \psi \rightarrow \phi \gamma \rightarrow \gamma \gamma)}{A_{LD_{O_7}}^{+(-)}(B_s \rightarrow \phi \gamma \rightarrow \gamma \gamma)} \right| = 4\pi^2 Q_c \frac{a_{2}^{eff}}{|C_{7}^{eff}(\mu)|} \sum_i f_{\psi_i}^2(0) \frac{m_{B_s}^2 - m_{\psi_i}^2}{m_b^2}
\]

(31)

and found

\[
2\% \leq \rho \leq 4\%
\]

(32)

while varying \( \frac{m_b}{2} \leq \mu \leq 2m_b \) and allowing \( a_{2}^{eff} \) to have a theoretical error of 25\% as stated in \[4\]. The analytical expression of the ”effective” coefficient \( C_{7}^{eff}(\mu) \) of the operator \( O_7 \) can be found in \[1\].
As a conclusion, we investigated the LD-contribution to the $B_s \to \gamma\gamma$ decay resulting from intermediate $\psi_i$ production and compared our result with the one obtained by the interaction of the virtual charm loop with soft gluons \cite{7}. We see that both amplitudes are in good agreement within the errors of the calculation. The new LD contribution resulting from the four-quark operators $O_1$ and $O_2$ is smaller compared to the one coming from the $B_s \to \phi\gamma \to \gamma\gamma$ chain decay \cite{3} and effects our old estimate \cite{3} for the branching ratio $\mathcal{B}(B_s \to \gamma\gamma)_{SD+LD_{O_7}}$ not more than 1%.

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