Rapidity dependent momentum anisotropy at RHIC

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Abstract. In Au+Au collisions at RHIC, elliptic flow was found to rapidly decrease as a function of rapidity. We argue that the origin of this phenomenon is incomplete thermalization of the initial fireball outside the midrapidity region and show that it can be quantitatively related to the analogous effect at midrapidity in peripheral collisions and in collisions at lower beam energies.

1. Introduction and overview

Semicentral Au+Au collisions at RHIC exhibit very strong elliptic flow \( v_2 \) which exhausts the prediction from ideal fluid dynamics (for a review see [1]). This has been interpreted as strong evidence for early thermalization of the collision fireball, on a timescale of \(< 1 \text{ fm}/c\) and at energy densities of \(\sim 10 - 20 \text{ times the critical value for quark-gluon plasma formation} \) [2]. However, in more peripheral Au+Au collisions at RHIC the measured elliptic flow remains increasingly below the hydrodynamic value, and in lower energy Pb+Pb and Au+Au collisions at the SPS and AGS it does not reach the hydrodynamic limit even in central collisions [3, 4]. The STAR and NA49 Collaborations have suggested (see Fig. 22 in [3] and Fig. 25 in [4]) that the discrepancy between the hydrodynamically predicted and measured elliptic flow at midrapidity scales with \((1/S) dN_{\text{ch}}/dy\) where \(dN_{\text{ch}}/dy\) is the measured charged multiplicity density at midrapidity and \(S\) is the initial transverse overlap area between the colliding nuclei.

When multiplied with the average transverse mass \(\langle m_\perp \rangle\) per hadron and divided by \(\tau_0\), this scaling variable is just the Bjorken energy density at proper time \(\tau_0\) [5]. This suggests that the degree of thermalization reached early in the collision (as measured by the fraction \(v_2^{\text{meas}}/v_2^{\text{hydro}}\) of the hydrodynamically predicted elliptic flow achieved in the experiment) is controlled by the initial energy or particle density. This makes sense since the collision rate is proportional to the density of scatterers.

The PHOBOS Collaboration first observed [6] that \(v_2\) shows a strong pseudorapidity dependence, with a shape that seems to roughly follow the charged particle pseudorapidity distribution. This has now been confirmed by STAR [7]. BRAHMS and PHOBOS also reported that the pseudorapidity distributions of both \(N_{\text{ch}}\) [8, 9] and \(v_2\) [10] have very similar shapes for all collision centralities. The similarity in shape between the rapidity distributions of \(v_2\) and \(N_{\text{ch}}\) becomes even stronger if the Jacobian between rapidity \(y\) and pseudorapidity \(\eta\) (which affects \(v_2(\eta)\) and \(dN_{\text{ch}}/d\eta\) mostly near midrapidity and with opposite signs [11]) is taken out.

Hydrodynamic models can reproduce the shape of \(dN_{\text{ch}}/d\eta\) but not that of \(v_2(\eta)\) [12]. Instead of falling off with increasing rapidity, the hydrodynamically predicted \(v_2(\eta)\) actually first increases with \(|\eta|\) before eventually dropping steeply near the
projectile and beam rapidities \[^{12, 13}\]. We find two reasons for this behaviour: (i) The initial transverse spatial deformation \(\varepsilon_z\) of the nuclear overlap region, which causes the anisotropic pressure gradients driving the buildup of elliptic flow, slightly increases with space-time rapidity \(|\eta_s| = \frac{1}{2} \ln \frac{s}{s_0^2}\), favoring larger \(v_2\) at \(\eta \neq 0\). (ii) The initial transversally averaged energy density \(e(\eta_s; \tau_0) = \langle e(\mathbf{r}, \eta_s; \tau_0)\rangle\) decreases with \(|\eta_s|\). This reduces the softening effects on the flow buildup from the quark-hadron phase transition and again results in larger \(v_2\). The same effect leads to larger \(v_2\) at midrapidity when one reduces the beam energy from RHIC to SPS and AGS energies \[^{14}\]. \(v_2\) falls to zero only near the projectile and target rapidities since there the initial energy density becomes so small that no time is left for flow buildup before freeze-out.

Hirano interpreted the observation that for \(\eta \neq 0\) the measured \(v_2\) in Au+Au at RHIC is significantly below the hydrodynamically predicted one as evidence that local thermal equilibrium in the initial state of the collision is only achieved at midrapidity \[^{12}\]. We here support this conclusion by showing that the ratio \(v_2^{\text{beam}}/v_2^{\text{hydro}}\) scales with the initial transversally averaged energy density as a function of space-time resp. pseudo-rapidity in the same way as previously observed by STAR and NA49 as a function of centrality at midrapidity \[^{13, 14}\]. Both effects indicate that the initial thermalization of the system and the validity of hydrodynamics are largely controlled by the initial energy or particle density achieved in the collision.

2. Initialization and longitudinal structure of the reaction zone

Understanding the rapidity dependence of elliptic flow requires a model which correlates the initial transverse distribution with longitudinal position, i.e. with space-time rapidity \(\eta_s\). We use Bjorken scaling \(\eta_s = y\) \[^{15}\] during the particle formation stage to relate initial longitudinal position to initial longitudinal momentum by \(z = v_L t\) on a surface \(\tau_0 = (t^2 - z^2)^{1/2} = \text{const}\). The average longitudinal momentum is correlated with transverse position \(r\) by overlap geometry coupled with momentum conservation \[^{15}\]: At transverse position \(r\) two cylinders of matter with thicknesses \(t_{1,2}(r; b) = T_A(r \pm b/2)\) hit each other, carrying total energy \(\gamma_{\text{beam}}(t_1 + t_2)\) and net \(z\)-momentum \(\gamma_{\text{beam}} v_{\text{beam}}(t_1 - t_2)\). In a collision with impact parameter \(b\), the produced matter thus has a rapidity distribution centered at the \(r\)-dependent average rapidity \[^{15}\]

\[
Y(r; b) = \frac{1}{2} \ln \left( \frac{t_1(r; b) + t_2(r; b)}{t_1(r; b) + t_2(r; b) - v_{\text{beam}}(t_1(r; b) - t_2(r; b))} \right).
\]

In coordinate space the center of mass of the matter produced at transverse position \(r\) is located at \(\eta_s^{\text{cm}}(r) = Y(r)\). We distribute the initial matter symmetrically around this mean space-time rapidity as follows: Noting that 1+1 and 3+1 dimensional hydrodynamical calculations \[^{16, 17}\] show very little dynamical evolution in rapidity direction, we simply identify the shape of the initial entropy distribution \(dS/d\eta_s\) with that of the measured final charged particle rapidity distribution \(dN_{\text{ch}}/dy\). We parametrize the latter by a double Gaussian,

\[
dN_{\text{ch}}/dy \propto e^{-\frac{(y-\eta_0)^2}{2\sigma^2}} + e^{-\frac{(y+a)^2}{2\sigma^2}},
\]

with a free width parameter \(a\). Combining Eqs. \[^{12}\] and noting that the normalization of the rapidity densities at midrapidity scales roughly with the number of wounded

\[^{4}\] \(T_A(r) = \int_{-z}^{z} dz \rho_A(r, z)\) is the thickness function of a nucleus of mass \(A\), calculated from a Woods-Saxon profile \(\rho_A(r) = \rho_0/[\exp((r - R_A)/\delta) + 1]\), with \(R_A = 6.37\) fm and \(\delta = 0.54\) fm.

\[^{9}\]
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3. Time evolution and anisotropic flow observables

For each value of $\eta_s$, we evolve these initial conditions with a locally boost-invariant hydrodynamic code [14] with the equation state of [18] to simulate the transverse evolution of the reaction zone. Here we represent minimum bias Au+Au data by a fixed average impact parameter of $b = 6.8$ fm. The evolution is started at $\tau_0 = 0.6$ fm/c and stopped at decoupling energy density $\epsilon_{\text{dec}} = 0.075$ GeV/fm$^3$, both independent of rapidity. As noted by Hirano [17], the smaller transverse size of the fireball at forward rapidities should lead to an earlier decoupling at higher temperature and smaller transverse flow, causing steeper $p_\perp$ spectra and smaller $p_\perp$-integrated elliptic flow at forward rapidities. The same point was made previously by Teaney for the smaller collision systems formed in peripheral collisions at midrapidity [19]. This effect is, however, not enough to explain the strong rapidity dependence of the measured $v_2$ [17], and it is not included in our present calculations.

In the right panel of Fig. 1 we show the time evolution of the momentum anisotropy $\epsilon_p = \langle T_{xx} - T_{yy} \rangle / \langle T_{xx} + T_{yy} \rangle$ at various rapidities $\eta_s$. As $\eta_s$ increases, the initial energy density

\[ \epsilon_{\text{dec}} = 0.075 \text{ GeV/fm}^3 \]

Beyond $\tau - \tau_0 \approx 11 - 12$ fm/c our numerical results for $\epsilon_p$ are affected by matter leaving our finite grid area and tend to be too large. From [14] we know that $\epsilon_p$ saturates beyond $\tau - \tau_0 \approx 12$ fm/c.
at $\eta_s$ decreases, and the time evolution of $\epsilon_p$ follows the same pattern as previously observed at midrapidity when reducing the collision energy (see Fig. 7 in [14]).

At forward rapidities the transverse overlap region becomes asymmetric and is shifted sidewards in the $x$ (or impact parameter) direction. This turns out to give rise to a non-zero directed flow signal $v_1(p_\perp)$ which increases with $|\eta_s|$ (left panel in Fig. 2). Of course, since the colliding matter receives no overall transverse kick, the $p_\perp$-integrated directed flow is zero.

The hydrodynamically calculated elliptic flow $v_2(\eta)$ has the same general shape as previously obtained by Hirano with a fully (3+1)-dimensional code. We now correct this hydrodynamic behaviour with a “thermalization coefficient” $F(x)$ which is fitted to midrapidity data in peripheral and lower-energy collisions [3, 4]. $F$ depends on the initial transversally averaged energy density at rapidity $y=\eta_s$ through the ratio $x(\eta) = \langle e(\eta_s) \rangle / e_0$ (where $e_0=9.5 \text{ GeV/fm}^3$ is the average initial energy density in central Au+Au collisions at 130 $\text{A} \text{GeV}$). As discussed in the Introduction, this scaling variable is, up to a multiplicative constant, identical with the variable $(1/S) dN/dy$ found by STAR and NA49 to control the magnitude of $v_2$ at midrapidity [5, 6]. We parametrize the behavior shown in Fig. 25 of [4] with a simple linear function $F(x) \equiv \frac{v_2^{\text{meas}}}{v_2^{\text{hydro}}} = 0.15 + 0.85 x$ for $x \leq 1$ while $F(x) = 1$ for $x > 1$. ($x = 1$ corresponds in Fig. 25 of [4] to $(1/S) dN_{\text{ch}}/dy = 25 \text{ fm}^{-2}$.) The corrected $v_2^{\text{meas}}(\eta) = F(x(\eta)) \cdot v_2^{\text{hydro}}(\eta)$ for $b=6.8 \text{ fm}$ is shown by the full circles in the right panel of Fig. 2 together with minimum bias data from PHOBOS and STAR. Even if our $v_2$ values are still a bit high at $|\eta| > 2$, we see good qualitative agreement with the data. We conclude that the same incomplete thermalization effects previously seen at midrapidity in peripheral and lower-energy collisions also describe qualitatively the rapid decrease of $v_2$ at non-zero rapidity in minimum bias collisions at RHIC. Local thermalization seems to be driven by the local initial energy density reached in the collision.
Our analysis suggests that the critical initial energy density, which is required for reaching sufficiently complete local equilibrium during the early collision stages to validate a hydrodynamic description, has for the first time been reached at midrapidity in almost central Au+Au collisions at top RHIC energy. At even higher collision energies the elliptic flow is therefore expected to follow the hydrodynamic predictions over a wider range of rapidities and centralities. However, there is no need to wait for the LHC to verify this prediction. One way to test this already at RHIC energies would be to explore fully central collisions (“zero spectators”) between Uranium nuclei which provide slightly larger energy densities than central Au+Au collisions and a significant transverse deformation and elliptic flow signal even at zero impact parameter, due to the deformation of the projectiles [14]. Such full-overlap U+U collisions yield very large, transversally deformed fireballs which would also provide a testbed for studying more efficiently the predicted nonlinear pathlength dependence of jet energy loss (by studying jets as a function of their emission angle relative to the reaction plane as extracted from $v_2$) and exploring this energy loss out to larger $p_T$ values than presently possible with the much smaller deformed fireballs created in non-central Au+Au collisions.

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