Dipoles in blackbody radiation: momentum fluctuations, decoherence, and drag force

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Abstract
An expression is derived for the momentum diffusion constant due to photons scattering of a small polarizable particle in blackbody radiation, and is shown to be related to the long-wavelength collisional decoherence rate for such a particle in a thermal environment. We show how this diffusion constant appears in the steady-state photon emission rate of two dipoles induced by blackbody radiation. We consider in addition the Einstein–Hopf drag force on a small polarizable particle moving in a blackbody field, and derive its relativistic form from the Lorentz transformation of forces. We obtain an expression for the rate of change of the field energy density associated with changes in the particles’ kinetic energies and relate it to the Kompaneets equation for the case of Compton scattering by thermalized electrons.

Keywords: quantum optics, decoherence, blackbody radiation, thermal drag

(Some figures may appear in colour only in the online journal)

1. Introduction

One of the ways in which the quantized nature of the electromagnetic field is manifested is in the recoil of particles in scattering, absorption, and emission processes [1]. In spontaneous emission, for example, a classical treatment of the field would not account for the recoil of an atom because the field from the atom would have zero linear momentum, a consequence of the inversion symmetry of the field about the atom. Einstein [2] showed that atoms in a homogeneous and isotropic field with spectral energy density \( \rho(\omega) \) undergo a mean-square momentum gain resulting from the recoil momentum of magnitude \( \hbar \omega / c \) in each emission and absorption event. Assuming the atoms have a mean kinetic energy \( \langle \frac{1}{2}mv^2 \rangle = \frac{1}{2}k_B T \) in equilibrium, this mean-square momentum increase is compensated by a drag force in just such a way as to yield the Planck distribution for \( \rho(\omega) \). In earlier work, Einstein and Hopf [3, 4] obtained classical expressions for the variance of the fluctuating momentum and the drag force on a small polarizable particle in blackbody radiation and used these expressions to derive the Rayleigh–Jeans spectrum. In this paper we derive a simple quantum-mechanical generalization of their expression for the momentum variance and diffusion constant. A similar generalization of the Einstein–Hopf formula for the drag force was obtained earlier [5]. We consider the extent to which these two generalizations can be used to extend Einstein’s analysis for atoms [2] to any small polarizable particle in thermal equilibrium with radiation. The momentum diffusion constant is obtained in the Heisenberg picture as an ensemble average over the momentum kicks imparted by the thermal field. We show that this momentum diffusion constant appears also in the rate of photon emission from two localized dipoles in blackbody radiation, and has the same form as the center-of-mass decoherence rate of a particle in a thermal photon environment. Such blackbody-induced decoherence was previously analyzed in a scattering-theory framework [6], wherein which-path information carried by the thermal photons scattering off the particle causes the particle’s center of
mass to decohere and localize in the position basis. If the characteristic thermal wavelength $\lambda_n$ of the electromagnetic environment is larger than the coherence length $\Delta x$, each scattered thermal photon only gains partial information about the particle’s position (‘long-wavelength limit’); for shorter wavelengths a single scatterer has sufficient information to resolve a coherent superposition (‘short-wavelength limit’). It will be shown that the momentum diffusion constant is related specifically to the long-wavelength limit of the blackbody-induced center-of-mass decoherence rate.

The rest of the paper is organized as follows. In section 2 we calculate the momentum variance and diffusion constant based first on a model in which the particle experiences random, instantaneous, and statistically independent momentum kicks. This is followed by a more rigorous calculation which is simplified by treating the blackbody field at first as a classical stochastic field as in the original Einstein–Hopf theory [3, 4]. Following Einstein and Hopf we treat the electric field and its spatial derivative as independent stochastic processes; a proof of this independence is given in appendix A. In section 3 we calculate the photon emission rate for two dipoles in blackbody radiation. We show how the reduction with the dipole separation of an interference term in this rate involves the diffusion constant. In section 4 we derive the relativistic form of the drag force on a particle immersed in blackbody radiation directly from the Lorentz transformation of forces. We obtain in section 5 an expression for the rate of change of the energy density of a homogeneous and isotropic field as it induces changes in the particles’ kinetic energies. For Compton scattering by thermalized electrons, this is shown to be consistent with the Kompaneets equation. We discuss in section 6 some aspects of our results as they relate to the Rayleigh–Jeans, Wien, and Planck spectra, and conclude with a brief discussion of their applicability in equilibrium with radiation. Appendix B gives a brief derivation of the momentum diffusion constant due to scattering of air molecules and the corresponding center-of-mass decoherence rate.

2. Momentum fluctuations

2.1. Momentum kicks

We begin by assuming that the momentum diffusion of a particle results from statistically independent momentum kicks of magnitude $\hbar \omega/c$ due to photons of frequency $\omega$ (see figure 1). This implies, for the cumulative change in the $i$th Cartesian component of the linear momentum,

$$ p_i(t) + \beta p_i(t) = \sum_{t_j} \frac{\hbar \omega}{c} \delta(t - t_j) \left( \hat{k} - \hat{k}' \right) \cdot \hat{e}_i, \quad (1) $$

where $\hat{e}_i$ is the unit vector in the $i$-direction, $-\beta p_i(t)$ is the frictional force acting on the particle, and $t_j$ is the (random) time at which the $j$th instantaneous momentum kick occurs. $\hat{k}$ and $\hat{k}'$ are the unit vectors for the directions of incoming and scattered photons, respectively. Thus, for times $t \gg 1/\beta$,

$$ p_i(t) = \sum_{t_j} \frac{\hbar \omega}{c} G(t - t_j) \left( \hat{k} - \hat{k}' \right) \cdot \hat{e}_i, \quad (2) $$

where $G(t) \equiv e^{-\beta t}$ is the response to a unit impulse.

The field energy density in the interval $[\omega, \omega + d\omega]$ is given by $\rho(\omega)d\omega$. The contribution of photons with energy between $[\omega, \omega + d\omega]$ to the steady-state, mean-square momentum of the particle is given, according to Campbell’s theorem [7, 8], by

$$ d\langle \Delta p_i^2 \rangle = \left( \frac{\hbar \omega}{c} \right)^2 \left( \left( \hat{k} - \hat{k}' \right) \cdot \hat{e}_i \right)^2 \times \text{(average photon rate } \alpha(\omega) \text{)} \times G^2(t) \text{d}t. \quad (3) $$

For $\alpha(\omega) \text{d}\omega$ we use the photon rate per unit area per unit time, $c\rho(\omega) \text{d}\omega / \hbar \omega$, times the Rayleigh cross section

$$ \sigma(\omega) = \frac{8\pi}{3} \left( \frac{\omega}{c} \right)^4 |\alpha(\omega)|^2, \quad (4) $$

where $\alpha(\omega)$ is complex polarizability of the particle [9].

We integrate equation (3) over all field frequencies, using the relation [10]

$$ \rho(\omega) = \frac{\hbar \omega^3}{\pi^2 c^3} n(\omega) \quad (5) $$

between the free-space spectral energy density $\rho(\omega)$ and the average number $n(\omega)$ of photons at frequency $\omega$, to obtain

$$ \langle \Delta p_i^2 \rangle = \frac{8\hbar^2}{3\pi c^3} \int d\Omega_k \int d\Omega_{k'} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 n(\omega) \times \frac{1}{\beta} \left( \left( \hat{k} - \hat{k}' \right) \cdot \hat{e}_i \right)^2. \quad (6) $$

where $d\Omega_k$ and $d\Omega_{k'}$ refer to the differential solid angles about $\hat{k}$ and $\hat{k}'$ respectively, $2/\beta$ is the rate at which $\langle \Delta p_i^2 \rangle$ decreases due to the drag force, and in steady state the rate $\langle \Delta p_i^2 \rangle / \Delta t$ at which $\langle \Delta p_i^2 \rangle$ increases must equal $2/\beta \langle \Delta p_i^2 \rangle$. Thus the frictional force and the momentum diffusion of the particle balance each other on average in equilibrium, although of
course the momentum \( p(t) \) itself is a time-dependent random process. It follows that
\[
\frac{\langle \Delta p_i^2 \rangle}{\Delta t} = \frac{8\hbar^2}{3\pi^2} \int \frac{d\Omega_k}{V} \int \frac{d\Omega_k'}{V} \left. \frac{\partial}{\partial \omega} \right|_{\omega = \omega_0} n(\omega) \left( (\mathbf{k} - \mathbf{k}') \cdot \hat{\mathbf{e}} \right)^2.
\]
(7)

Note that in terms of the scattering angle \( \theta \), \( |\mathbf{k} - \mathbf{k}'|^2 = 2(1 - \cos \theta) \). Averaging equation (7) over all scattering angles \( \theta \) and directions \( i \) yields a factor of \( 2/3 \). The momentum diffusion constant becomes
\[
\frac{\langle \Delta p_i^2 \rangle}{\Delta t} = \frac{16\hbar^2}{9\pi^2} \int_0^\infty d\omega \omega^6 |n(\omega)|^2 n(\omega).
\]
(8)

This derivation of the momentum diffusion constant \( \langle \Delta p_i^2 \rangle/\Delta t \) based on Campbell’s theorem is simpler and more intuitive than the more rigorous derivation given below. However, it does not account for the Bose–Einstein statistics of blackbody photons, the effect of which is to replace \( n(\omega) \) in equation (8) by \( n(\omega)[n(\omega) + 1] \). The application of Campbell’s theorem requires that successive impulses are statistically independent, whereas the ‘photon bunching’ effect in blackbody radiation implies that successive momentum kicks are not, in fact, independent. The effect of the Bose–Einstein factor \( n(\omega) + 1 \) is considered for two examples in the following section. For these examples the effect of the Bose–Einstein factor is negligible and Campbell’s theorem provides an accurate estimate of the momentum diffusion constant.

It might be worth noting that while Campbell’s theorem does not account for Bose–Einstein statistics, it can be used, as shown in appendix B, to describe momentum diffusion from other environmental scatterers such as air molecules.

2.2. Calculation of the momentum diffusion constant from the force on an induced dipole

We start from the classical expression for the force on an electric dipole moment \( \mathbf{d}_i \) in an electromagnetic field:
\[
\mathbf{F} = (\mathbf{d}_i \cdot \nabla)\mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{d}_i}{\partial t} \times \mathbf{B} = (\mathbf{d}_i \cdot \nabla)\mathbf{E} + \frac{1}{c} \frac{\partial}{\partial t} (\epsilon_{ijk} \mathbf{d}_j B_k),
\]
\( i \neq j \)
\( (9) \)

where we have used the Maxwell equation \( \nabla \times \mathbf{E} = -\nabla \times \mathbf{B} \), where \( \epsilon_{ijk} \) denotes the Levi-Civita symbol. The impulse experienced by the dipole along \( i \) in a time interval from \( t = 0 \) to \( t = \Delta t \) is
\[
\Delta p_i = \int_0^{\Delta t} dt (\mathbf{d}_i \cdot \partial_i \mathbf{E}_j) + \frac{1}{c} (\epsilon_{ijk} \mathbf{d}_j B_k)_i|_0^{\Delta t}.
\]
(10)

The second term vanishes in the steady state, leading to
\[
\Delta p_i = \int_0^{\Delta t} dt \mathbf{d}_i \partial_i \mathbf{E}_j.
\]
(11)

The electric field at the position of the dipole is written as
\[
E_j(t) = i \sum_{k \lambda} \left( \frac{2\pi\hbar \omega_j}{V} \right)^{1/2} \hat{a}_{k\lambda} e^{-i\omega t} - \hat{a}_{k\lambda}^+ e^{i\omega t} \right] c_{k\lambda j}.
\]
(12)

in the conventional notation \([11]\), except that in the classical approach we have taken thus far we regard \( \hat{a}_{k\lambda} \) and \( \hat{a}_{k\lambda}^+ \) as classical stochastic variables rather than photon annihilation and creation operators. Similarly,
\[
\partial_i E_j = - \sum_{k \lambda} \left( \frac{2\pi\hbar \omega_j}{V} \right)^{1/2} \left[ \hat{a}_{k\lambda} e^{-i\omega t} + \hat{a}_{k\lambda}^+ e^{i\omega t} \right] c_{k\lambda j}.
\]
(13)

The dipole moment induced by the EM field along \( j \) is given as
\[
d_{ij} = i \sum_{k \lambda} \left( \frac{2\pi\hbar \omega_j}{V} \right)^{1/2} \left[ \hat{a}_{k\lambda} e^{-i\omega t} - \hat{a}_{k\lambda}^+ e^{i\omega t} \right] c_{k\lambda j},
\]
(14)

assuming a scalar polarizability tensor such that \( \alpha_{mn}(\omega) = \alpha(\omega) \delta_{mn} \).

Using equations (13) and (14) in equation (11), and performing the integral over time, one obtains
\[
\Delta p_i = -2i \sum_{k_1 \lambda_1 k_2 \lambda_2} \left( \frac{2\pi\hbar \omega_1}{V} \right)^{1/2} \left( \frac{2\pi\hbar \omega_2}{V} \right)^{1/2} \times k_1 \sin \left( \frac{1}{2}(\omega_1 - \omega_2) \Delta t \right)
\]
\[
\times \left[ \alpha(\omega_1) \tilde{a}_{k_1 \lambda_1} e^{-i\omega_1 \Delta t} - \alpha(\omega_2) \tilde{a}_{k_2 \lambda_2} e^{i\omega_2 \Delta t} \right] c_{k_1 \lambda_1 j} c_{k_2 \lambda_2 j}
\]
\[
= \sum_{k_1 \lambda_1 k_2 \lambda_2} \left( \frac{2\pi\hbar \omega_1}{V} \right)^{1/2} \left( \frac{2\pi\hbar \omega_2}{V} \right)^{1/2} \times A(\omega_1) A(\omega_2) \alpha(\omega_1) \alpha(\omega_2) \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} c_{k_1 \lambda_1 j} c_{k_2 \lambda_2 j},
\]
(15)

where we neglect non-resonant terms varying as \( 1/(\omega_1 + \omega_2) \). The terms denoted by subscripts 1 and 2 in equation (15) derive from \( \partial_i E_j \) and \( E_j \), respectively. Einstein and Hopf [3] showed that these terms are statistically independent. We give a simplified proof of this independence, which significantly simplifies the calculation of \( \langle \Delta p_i^2 \rangle \), in appendix A. From this independence it follows from equation (15) that
\[
\langle \Delta p_i^2 \rangle = \frac{4}{k_i \lambda_i} \sum_{k_1 \lambda_1 k_2 \lambda_2} \left( \frac{2\pi\hbar \omega_1}{V} \right)^{1/2} \left( \frac{2\pi\hbar \omega_2}{V} \right)^{1/2} \sin^2 \left( \frac{1}{2}(\omega_1 - \omega_2) \Delta t \right)
\]
\[
\times \left[ \alpha(\omega_1) \alpha(\omega_2) \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} \right]
\]
\[
+ \left( \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} \right) \left( \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} \right) c_{k_1 \lambda_1 j} c_{k_2 \lambda_2 j} + \left( \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} \right) \left( \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} \right) c_{k_2 \lambda_2 j} c_{k_2 \lambda_2 j}
\]
\[
= \sum_{k_1 \lambda_1 k_2 \lambda_2} \left( \frac{2\pi\hbar \omega_1}{V} \right)^{1/2} \left( \frac{2\pi\hbar \omega_2}{V} \right)^{1/2} \sin^2 \left( \frac{1}{2}(\omega_1 - \omega_2) \Delta t \right)
\]
\[
\times A(\omega_1) A(\omega_2) \alpha(\omega_1) \alpha(\omega_2) \tilde{a}_{k_1 \lambda_1} \tilde{a}_{k_2 \lambda_2} c_{k_1 \lambda_1 j} c_{k_2 \lambda_2 j},
\]
(16)

where we have assumed the ensemble averages
\[
\langle \tilde{a}_{k\lambda} \tilde{a}_{k'\lambda'}^\dagger \rangle = A(\omega) \delta_{k,k'}^3 \delta_{\lambda,\lambda'} \}, \quad \omega = |\mathbf{k}|c,
\]
\[ \langle \hat{a}_{k_1} a_{k_1} \rangle = 0, \]  

(17)

for the classical stochastic variables describing blackbody radiation. Here \( A(\omega) \) is a positive number defined by the first line of equation (17).

To account for all three components of the induced momentum we write \( \langle \Delta p^2 \rangle = \frac{1}{3} \sum_{\lambda_1=\lambda_2=\lambda_3} \langle \Delta p^2 \rangle \). Taking \( V \to \infty \) in the usual fashion,

\[
\langle \Delta p^2 \rangle = \frac{1}{3} \times \frac{8(2\pi)^2}{(8\pi^3)^2} \int_0^\infty d\omega_1 \omega_1^4 \\
\times \int_0^\infty d\omega_2 \omega_2^3 \sin^2 \left( \frac{(\omega_1 - \omega_2) \Delta t}{(\omega_1 - \omega_2)^2} \right) \\
\times A(\omega_1) A(\omega_2) |\alpha(\omega_2)|^2 \left( \sum_{\lambda_1} e_{k_1\lambda_1}^2 \right) \left( \sum_{\lambda_2} e_{k_2\lambda_2}^2 \right),
\]

(18)

where the factors of \( \frac{4\pi}{3} \) and \( 4\pi \) result from the angular integration over \( k_1 \) and \( k_2 \), respectively, in equation (16).

Considering that each \( e_{k\lambda} \) is a unit vector and the polarization sum \( \lambda \) accounts for the two orthonormal directions to \( k \), we replace the polarization sums by

\[
\sum_{\lambda_1} \left[ e_{k_1\lambda_1 x}^2 + e_{k_1\lambda_1 y}^2 + e_{k_1\lambda_1 z}^2 \right] \sum_{\lambda_2} \left[ e_{k_2\lambda_2 x}^2 + e_{k_2\lambda_2 y}^2 + e_{k_2\lambda_2 z}^2 \right] = \sum_{\lambda_1} e_{k_1\lambda_1}^2 \sum_{\lambda_2} e_{k_2\lambda_2}^2 = 4.
\]

(19)

Thus equation (18) for the momentum variance of the particle becomes

\[
\langle \Delta p^2 \rangle = \frac{32\hbar^2}{9\pi^2 c^3} \int_0^\infty d\omega_1 \omega_1^4 \int_0^\infty d\omega_2 \omega_2^3 \\
\times \sin^2 \left( \frac{(\omega_1 - \omega_2) \Delta t}{(\omega_1 - \omega_2)^2} \right) A(\omega_1) A(\omega_2) |\alpha(\omega_2)|^2.
\]

(20)

The term involving \( \omega_1 - \omega_2 \) in equation (20) is sharply peaked at \( \omega_1 = \omega_2 \) compared with the remaining factors in the integrand. We therefore evaluate these other factors at \( \omega_2 = \omega_1 \) and use

\[
\int_0^\infty d\omega_2 \sin^2 \left( \frac{(\omega_2 - \omega_1) \Delta t}{(\omega_2 - \omega_1)^2} \right) \leq \frac{1}{2} \int_0^\infty \frac{\sin^2 x \Delta t}{x^2} = \pi \Delta t/2
\]

(21)

in the remaining part of the \( \omega_2 \) integral to obtain

\[
\langle \Delta p^2 \rangle \frac{1}{\Delta t} = \frac{16\hbar^2}{9\pi^2 c} \int_0^\infty d\omega \omega^8 A^2(\omega) |\alpha(\omega)|^2,
\]

(22)

for the momentum diffusion constant.

The expression equation (22) was obtained from a classical stochastic treatment of the field. In the quantum treatment of the field \( \tilde{a}_{k_1\lambda_1} \) and \( \tilde{a}_{k_1\lambda_1}^\dagger \) in equation (16) are replaced by photon annihilation and creation operators \( a_{k_1\lambda_1} \) and \( a_{k_1\lambda_1}^\dagger \), respectively. Then the quantized-field expression for the momentum diffusion constant can be obtained by replacing classical ensemble averages by quantum expectation values:

\[
\langle \tilde{a}_{k_1\lambda_1} \tilde{a}_{k_1\lambda_1}^\dagger \rangle \to \langle a_{k_1\lambda_1} a_{k_1\lambda_1}^\dagger \rangle = n(\omega_1) + 1,
\]

\[
\langle \tilde{a}_{k_1\lambda_1} \tilde{a}_{k_2\lambda_2} \rangle \to \langle a_{k_1\lambda_1} a_{k_2\lambda_2} \rangle = n(\omega_1),
\]

\[
\langle \tilde{a}_{k_1\lambda_1} \rangle \to \langle a_{k_1\lambda_1} \rangle = n(\omega_1), \]

\[
\langle \tilde{a}_{k_1\lambda_1} \tilde{a}_{k_2\lambda_2} \rangle \to \langle a_{k_1\lambda_1} a_{k_2\lambda_2} \rangle = n(\omega_2) + 1.
\]

(23)

Then, following essentially the same arguments that led from equations (16) to (22), we obtain the quantized-field expression for the momentum diffusion constant:

\[
\langle \Delta p^2 \rangle \frac{1}{\Delta t} = \frac{16\hbar^2}{9\pi^2 c^3} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 [n^2(\omega) + n(\omega)],
\]

(24)

which differs from equation (8) by the Bose–Einstein factor \( n(\omega) + 1 \). To the best of our knowledge equation (24) is a new and general expression for the momentum diffusion constant due to photon scattering of a small particle in blackbody radiation. We note that, with a bit more algebra, it follows directly of course from a fully quantized-field calculation; this involves the symmetrization of \( \Delta p^2 \) in equation (15) to make it manifestly Hermitian, and taking \( \tilde{a}_{k_1\lambda_1} \) and \( \tilde{a}_{k_2\lambda_2}^\dagger \) to be the operators \( a_{k_1\lambda_1} \) and \( a_{k_2\lambda_2}^\dagger \), respectively. The physical significance of the \( n^2(\omega) \) and \( n(\omega) \) contributions to the momentum diffusion constant is discussed further in section 6.

Defining a wave number \( K = p/\hbar \) associated with a de Broglie wavelength \( \lambda_{db} = 2\pi / K \), we can express equation (24) as

\[
\langle \Delta K^2 \rangle \frac{1}{\Delta t} = \frac{16}{9\pi^2 c^3} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 [n^2(\omega) + n(\omega)].
\]

(25)

As discussed below, \( \langle \Delta K^2 \rangle \Delta n \) is closely related to \( \Delta n \), the measure of the rate at which interference, or coherent superposition, is lost between two localized wave packets separated by a distance \( \Delta x \). A here is the ‘scattering constant’ appearing in the theory of collisional decoherence due to thermal photon scattering.

2.2.1. Electron in blackbody radiation. Consider as an example an electron in blackbody radiation. From the classical Abraham–Lorentz equation of motion

\[
m_e \ddot{\mathbf{r}} - m_e \tau_e \dot{\mathbf{r}} = \mathbf{E},
\]

(26)

we deduce a ‘polarizability’ from the linear response of the electron to the external field:

\[
\alpha(\omega) = -\frac{e^2}{m_e} \frac{1}{\omega^2 + \tau_e^2 \omega^2}.
\]

(27)

(Here and throughout we treat the internal dynamics of the particles non-relativistically.) Since \( \tau_e = 2e^2 / 3mc^3 \approx 6.3 \times 10^{-24} \, s \), we approximate \( |\alpha(\omega)|^2 \) by

\[
|\alpha(\omega)|^2 = \frac{e^4}{m_e^2 \omega^4}.
\]

(28)
Then, from equation (24) with $n(\omega) = (e^{\omega h\nu/k_BT} - 1)^{-1}$,
\[
\frac{\langle \Delta p_x^2 \rangle}{\Delta t} = \frac{16\hbar^2}{9\pi^2 c^2} \left( \frac{e^2}{m_e c^2} \right)^2 \int_0^\infty d\omega \omega^4 \frac{e^{\omega h\nu/k_BT}}{(e^{\omega h\nu/k_BT} - 1)^2} = \frac{64\pi^3}{135} \left( \frac{e^2}{m_e c^2} \right)^2 \frac{(k_B T)^8}{\hbar^2 c^4},
\]
(29)

exactly as obtained by Oxenius [12] in a different approach based directly on Thomson scattering. With $T$ the absolute temperature (K),
\[
\frac{1}{2m_e} \langle \Delta \mathbf{p}^2 \rangle_{\Delta t} \approx 3.4 \times 10^{-38} T^5 \text{ erg s}^{-1}.
\]
(30)

$\langle \Delta K^2 \rangle / \Delta t = \langle \Delta p_x^2 \rangle / (\hbar^2 \Delta t)$, aside from a numerical prefactor due to a different averaging procedure, has the same form as the decoherence rate for free electrons obtained by Joos and Zeh [6].

The role of the Bose–Einstein factor in this example turns out to be very small. If we replace $n^2(\omega) + n(\omega)$ by $n(\omega)$ in equation (24) we obtain
\[
\frac{\langle \Delta p_x^2 \rangle}{\Delta t} \approx \frac{128}{3\pi} \langle 5 \rangle \left( \frac{e^2}{m_e c^2} \right)^2 \frac{(k_B T)^8}{\hbar^2 c^4},
\]
(31)

with $\langle 5 \rangle \approx 1.0369$, which is about 96% of the complete result with both terms retained. $n^2(\omega)$ is much larger than $n(\omega)$ at low frequencies, but this dominance is suppressed by the $\omega^5$ factor in the integrand of equation (24); this is due in part to the fact that the field mode density decreases rapidly with decreasing frequency. The suppression of the $n^2(\omega)$ contribution is even more pronounced in the following example.

2.2.2. Dielectric sphere in blackbody radiation.

For a small dielectric sphere of radius $a$,
\[
\alpha(\omega) = \left( \frac{\epsilon - 1}{\epsilon + 2} \right) a^3,
\]
(32)

and, if $n(\omega) = (e^{\omega h\nu/k_BT} - 1)^{-1}$ and the variation of the permittivity $\epsilon$ with $\omega$ is negligible, the momentum diffusion is obtained as
\[
\frac{\langle \Delta p_x^2 \rangle}{\Delta t} = \frac{16\hbar^2}{9\pi^2 c^2} \left( \frac{e^2}{m_e c^2} \right)^2 \int_0^\infty d\omega \omega^4 \frac{e^{\omega h\nu/k_BT}}{(e^{\omega h\nu/k_BT} - 1)^2} = \frac{1024\pi}{135} \frac{a^6 c^2 h^2}{\epsilon^2} \left( \frac{e^{\omega h\nu/k_BT}}{\epsilon + 2} \right)^2 \left( \frac{k_B T}{\hbar c} \right)^9.
\]
(33)

For a sphere of radius $a = 50$ nm, and density $\rho_s = 2$ g cm$^{-3}$ one obtains $\frac{1}{2m_e} \langle \Delta \mathbf{p}^2 \rangle / \Delta t = 2.7 \times 10^{-31} \left( \frac{c}{\epsilon + 2} \right)^2 T^6$ erg s$^{-1}$, where $m_e = 4\pi a^3 \rho_s / 3$ refers to the mass of the sphere.

Defining $\frac{\langle \Delta K^2 \rangle}{\Delta t} = \langle \Delta p_x^2 \rangle / (\hbar^2 \Delta t)$, and retaining only the term $n(\omega)$ in equation (25), we obtain, independently of the temperature, 99.8% of the full result, i.e.,
\[
\frac{\langle \Delta K^2 \rangle}{\Delta t} \approx \frac{16}{9\pi^2} \left( \frac{e^2}{m_e c^2} \right)^2 \int_0^\infty d\omega \omega^4 \left( \frac{e^{\omega h\nu/k_BT}}{e^{\omega h\nu/k_BT} - 1} - 1 \right)
\]

Figure 2. Schematic representation of two dipoles located at positions $x_1$ and $x_2$, interacting with a thermal field. We consider the rate $R$ of the average number of photons emitted from the dipole moments $d_1$ and $d_2$ induced by the blackbody field.

\[
R = \frac{16}{9\pi} \zeta(9) \rho_c e \left( \frac{\epsilon - 1}{\epsilon + 2} \right) \left( \frac{k_B T}{\hbar c} \right)^9 \equiv 2\Lambda.
\]
(34)

$\zeta(9) \equiv 1.002$. For this example $\Lambda$ as defined by equation (34) is the scattering constant derived, for example, in the book by Schlosshauer [6, 13], where numerical estimates of $\Lambda$ for both free electrons and dielectric spheres are given.

The momentum diffusion rate we calculated is a mean-square ensemble average over momentum fluctuations and makes no reference to entanglement with the environment that is generally understood to be the hallmark of decoherence [13]. $\Lambda$ as defined in our equation (34) is nevertheless consistent with rigorous calculations of the decoherence rate [13–16]. In a similar vein Adler [16] has compared a diffusion rate with results of ‘decoherence-based’ calculations of collisional decoherence to provide more generally an ‘independent cross-check’ of those calculations. Such a correspondence between the center-of-mass decoherence rate and the momentum diffusion has also been deduced from the variance of momentum as governed by the position-localization-decoherence master equation [17].

3. Field interference and decoherence.

Consider two point dipoles $d_1$ and $d_2$ at fixed positions $x_1$ and $x_2$ (figure2). The interaction Hamiltonian may be taken for our purposes to be
\[
H_I = -d_1 \cdot E(x_1, t) - d_2 \cdot E(x_2, t).
\]
(35)

The electric field operator at a point $x$ may be expanded in plane-wave mode functions as in equation (12), but here $a_{k\lambda}(t)$ and $a_{k\lambda}^\dagger(t)$ are photon annihilation and creation operators:
\[
E(r, t) = \sum_{k\lambda} \left( \frac{2\pi \hbar c}{V} \right)^{1/2} \left[ a_{k\lambda}(t) e^{ik \cdot x} - a_{k\lambda}^\dagger(t) e^{-ik \cdot x} \right] \mathbf{e}_{k\lambda}.
\]
(36)

From the Heisenberg equation of motion
\[
i\hbar \dot{a}_{k\lambda} = [a_{k\lambda}, H_F] + [a_{k\lambda}, H_I]
\]
(37)

with $H_F = \sum_{k\lambda} a_{k\lambda}^\dagger a_{k\lambda}$ and the commutation relations for the photon operators, it follows that
\[ a_{k\lambda}(t) = a_{k\lambda}(0)e^{-i\omega t} - \left(\frac{2\pi \omega}{hV}\right)^{1/2} \times \sum_i e^{-i k x_i} \int_0^t df' [\mathbf{d}(f') \cdot \mathbf{e}_{k\lambda}] e^{i\omega(t-f')} \]  

(38)

We define a rate of change of the average number of photons radiated by the dipole moments induced at \( x_1 \) and \( x_2 \):

\[ R = \frac{d}{dt} \sum_{k\lambda} \left\{ \left[ a_{k\lambda}(t) - a_{k\lambda}(0)e^{-i\omega t} \right] \left[ a_{k\lambda}(t) - a_{k\lambda}(0)e^{-i\omega t} \right] \right\} . \]  

(39)

From equation (38),

\[ R = \frac{2\pi}{hv} \times 2Re \sum_{k} \sum_{\lambda} \omega e^{i k(x_2-x_1)} \int_0^t df'e^{i\omega(t-f')} \times \sum_{\lambda} \left\{ \left[ \mathbf{d}(f') \cdot \mathbf{e}_{k\lambda} \right] \left[ \mathbf{d}(f') \cdot \mathbf{e}_{k\lambda} \right] \right\} . \]  

(40)

Consider first

\[ X_{ij} = \int_0^t df' e^{i\omega(t-f')} \sum_{\lambda} \left\{ \left[ \mathbf{d}(f') \cdot \mathbf{e}_{k\lambda} \right] \left[ \mathbf{d}(f') \cdot \mathbf{e}_{k\lambda} \right] \right\} , \]  

(41)

where \( \mathbf{d}(t) \) is the dipole moment induced by the blackbody field at \( x_i \):

\[ \mathbf{d}(t) \cdot \mathbf{e}_{k\lambda} = \sum_{k'\lambda'} \left( \frac{2\pi \hbar \omega'}{V} \right)^{1/2} \left[ \alpha(\omega') a_{k'\lambda'}(0)e^{-i\omega't} e^{ikx_i} \right. \]

\[ \left. - \alpha(\omega') a_{k'\lambda'}(0)e^{i\omega't} e^{-ikx_i} \right] \times \mathbf{e}_{k'\lambda'}. \mathbf{e}_{k\lambda}. \]  

(42)

For \( X_{ij} \) we obtain

\[ X_{ij} = \sum_{k'\lambda'} \left( \frac{2\pi \hbar \omega'}{V} \right)^{1/2} \left( \alpha(\omega') a_{k'\lambda'}(0) a_{k'\lambda'}(0) \right) \times e^{-i k'(x_2-x_1)} \int_0^t df' e^{i(\omega'-\omega)(t-f')} \left( a_{k'\lambda'}(0) a_{k'\lambda'}(0) \right) \]

\[ \times e^{i k'(x_2-x_1)} \int_0^t df' e^{i(\omega+\omega')(t-f')} \left| \mathbf{e}_{k\lambda'} \cdot \mathbf{e}_{k\lambda} \right|^2 . \]  

(43)

We have used the expectation values

\[ \langle a_{k'\lambda'}(0) a_{k'\lambda'}(0) \rangle = n_{k'\lambda'} \delta_{k'k} \delta_{\lambda'\lambda} = n_{k'} \delta_{k'k} \delta_{\lambda'\lambda} , \]

\[ \langle a_{k\lambda}(0) a_{k\lambda}(0) \rangle = 0 . \]  

(44)

for the blackbody field. From the form of the time integrals in equation (43) it is seen that only the normally ordered expectation value will contribute to the final result, and therefore we write

\[ X_{ij} = \frac{2\pi \hbar}{V} \sum_k \omega' |\alpha(\omega')|^2 n_k e^{-i k(x_2-x_1)} \times \sum_{\lambda} \sum_{\lambda'} |\mathbf{e}_{k'\lambda'} \cdot \mathbf{e}_{k\lambda}|^2 \int_0^t df' e^{i(\omega-\omega')(t-f')} . \]  

(45)

or, since

\[ \sum_{\lambda} \sum_{\lambda'} |\mathbf{e}_{k'\lambda'} \cdot \mathbf{e}_{k\lambda}|^2 = 1 + (k \cdot k')^2/(k'k)^2 , \]  

(46)

\[ X_{ij} = \frac{2\pi \hbar}{V} \left( \frac{V}{8\pi} \right) \int d^3 k' \omega' |\alpha(\omega')|^2 n_k e^{-i k(x_2-x_1)} \times \left[ 1 + (k \cdot k')^2/(k'k)^2 \right] \int_0^t df' e^{i(\omega-\omega')(t-f')} . \]  

(47)

in the mode continuum limit. Therefore,

\[ R = \frac{4\pi \hbar}{V} \left( \frac{V}{8\pi} \right) \sum_{i,j} \int d^3 k d^3 k' |\omega(\omega')|^2 n_k \times e^{i k(x_2-x_1)} \left[ 1 + (k \cdot k')^2/(k'k)^2 \right] \times \int_0^t df' \cos[(\omega-\omega')(t-f')] . \]  

(48)

For \( t \rightarrow \infty \),

\[ \int_0^t df' \cos[(\omega-\omega')(t-f')] \rightarrow \pi \delta(\omega-\omega') = \frac{\pi}{c} \delta(k-k') , \]  

(49)

and

\[ R = \sum_{i,j} \left( \frac{c}{8\pi^3} \right) \int d^3 k d^3 k' |\omega(\omega')|^2 n_k e^{i k(x_2-x_1)} \times \left[ 1 + (k \cdot k')^2/(k'k)^2 \right] \delta(k-k') \]

\[ = \sum_{i,j} \left( \frac{c}{8\pi^3} \right) \int d^3 k d^3 k' |\omega(\omega')|^2 n_k \int d\Omega_k \]

\[ \times \int d\Omega_{k'} e^{i \delta(k-k')(x_2-x_1)}(1 + \cos^2 \theta) = \sum_{i,j} R_{ij} . \]  

(50)

where \( \mathbf{k} \) is a unit vector in the direction of \( \mathbf{k} \), \( \theta \) is the angle between \( \mathbf{k} \) and \( \mathbf{k}' \), and \( d\Omega_{k'} \) is a differential element of solid angle about \( \mathbf{k} \) \( \int d^3 k = \int d^3 k' \int d\Omega_{k'} \). We rewrite

\[ R_{12} = \left( \frac{c}{4\pi^3} \right) \int d^3 k d^3 k' |\omega(\omega')|^2 n_k \int d\Omega_k \]

\[ \times \int d\Omega_{k'} e^{i \delta(k-k')(x_2-x_1)}(1 + \cos^2 \theta) \]  

(51)

using \( n_k = (\epsilon_{kk}/h\nu - 1)^{-1} \) and the differential cross section for Rayleigh scattering:

\[ |f(k, k')|^2 = \frac{d\sigma}{d\Omega} = k'^3 |\alpha(\omega)|^2 \frac{1}{2}(1 + \cos^2 \theta) . \]  

(52)

Thus

\[ R_{12} = \left( \frac{c}{4\pi^3} \right) \int d\nu \int d^2 k \left( \frac{k^2}{c \epsilon_{kk}/h\nu - 1} \right) \int d\Omega_k \]

\[ \int d\Omega_{k'} e^{i \delta(k-k')(x_2-x_1)}|f(k, k')|^2 . \]  

(53)
We express this in terms of \( q = \hbar k, v(q) = c, \) and
\[
dq \rho(k) = \frac{k^2 dk/\pi^2}{\hbar e^{ikx/y - t}}.
\]
and, to compare with reference [13], for instance, we also change notation, writing \( x_1 - x_2 = x - x', \) \( d\Omega_k d\Omega_{k'} = d^n n', \) and replacing \( f(k, k, k) \) by \( f(q n, q n') \). Then
\[
R_{12} = \int dq(q)\nu(q) \left( \frac{d^n n'}{4\pi} e^{i(\hat{n} - \hat{n}')(x-x'/\hbar)} \right)^2 f(q n, q n').
\]
and similarly
\[
R_{11} = \int dq(q)\nu(q) \left( \frac{d^n n'}{4\pi} \right)^2 |f(q n, q n')|^2.
\]
The photon scattering rate \( R_{12} \) as opposed to \( R_{11} \) involves interference between the fields of the two dipoles. The difference \( R_{11} - R_{12} \) is therefore a measure of the rate at which this interference decreases with increasing separation of the dipoles. In other words, it characterizes the loss of spatial coherence between the two dipole fields. In fact
\[
F(x - x') = R_{11} - R_{12}
\]
has exactly the form of the more general collisional decoherence rate [13–16]. In the long-wavelength limit the decay rate of the reduced density matrix for a dielectric sphere reduces to \(-\Lambda(x - x')^2\), where \( \Lambda \) is defined in equation (34). The ‘decoherence factor’ \( F(x - x') \), as it appears in the theory of collisional decoherence [13–16], represents the overlap between the states of environmental scatterers that scatter off the positions \( x \) and \( x' \) of the particle’s center of mass, denoting their distinguishability. Though, it is to be noted that the scattering model of decoherence considers two points \( x \) and \( x' \) located within the same particle whose center of mass is spatially delocalized; in the above calculation \( x \) and \( x' \) correspond to the location of two distinct dipoles, that are not necessarily constrained to constitute the same particle.

4. Drag force

As noted in the introduction, Einstein and Hopf [3, 4] and later Einstein [2] showed that there is a drag force \( F = -\xi v \) on a polarizable particle moving with velocity \( v \) \((v \ll c)\) with respect to a frame in which there is a homogeneous and isotropic (blackbody) field of spectral energy density \( \rho(\omega) \) [18]. A more general expression for \( \xi \) was derived much later [5], and more recently the drag force has been derived for relativistic particles [19–22]. We now derive the relativistic form of the drag force following a different and arguably more direct approach. The particle is assumed to have a uniform velocity \( v \) along the \( x \) axis in the laboratory frame \( S \), and we are interested in the force \( F_x \) on the particle in this frame. The Lorentz transformations relating the spacetime coordinates and the momenta \((p_x, p'_x)\) and energies \((E, E')\) in the laboratory frame and the particle’s rest frame \( S' \) are
\[
x = \gamma(x' + vt),
\]
\[
t = \gamma \left( t' + \frac{v^2}{c^2} \right),
\]
\[
p_x = \gamma \left( p'_x + \frac{v^2}{c^2} E' \right),
\]
\[
E = \gamma \left( E' + v E' \right),
\]
where as usual \( \gamma = (1 - v^2/c^2)^{-1/2} \). These transformations relate the force \( F_x \) in \( S \) to the force \( F'_x \) in \( S' \):
\[
F_x = \frac{dp_x}{dt} = \gamma \left( \frac{dp'_x}{dt} + \frac{v}{c^2} \frac{dE'}{dr} \right) = F'_x + \frac{v}{c^2} \frac{dE'}{dr}.
\]
The force on the particle has two contributions, \( F_{\text{ind}} \) resulting from the dipole moment induced by the fluctuating blackbody field, and \( F_{\text{ad}} \) resulting from the dipole’s own radiation field and fluctuations. \( F_{\text{ind}} \) is the force originally considered non-relativistically by Einstein and Hopf [2–4], while \( F_{\text{ad}} \) is needed to obtain the fully relativistic expression for the force that has been of interest in recent work [19–22].

4.1. Calculation of the force \( F_{\text{ind}} \)

In section 4.2 we calculated the force when the electric field is along a single direction \( z \) and then exploited the isotropy of the field to include the contributions from all three components of the field. The situation is more complicated for a particle in motion because in its reference frame \( (S') \) the fields are different in different directions and are related to those in \( S \) by
\[
E'_x(x', t') = E_x(x, t),
\]
\[
E'_z(x', t') = \gamma \left[ E_z(x, t) - \frac{v}{c} B_r(x, t) \right],
\]
\[
E'_x(x', t') = \gamma \left[ E_x(x, t) + \frac{v}{c} B_r(x, t) \right],
\]
along with the corresponding transformations for the magnetic fields. The force on the dipole along the \( x \) direction is
\[
F_x = \frac{dp_x}{dt} + d_y \frac{dE_y}{dx} + d_z \frac{dE_z}{dx},
\]
as is easily shown. For the quantum calculation it will be convenient to consider first a single field mode polarized along the \( x \) direction:
\[
E'(x', t') = E_x(x, t) = i \left( \frac{2\pi \hbar \omega}{V} \right)^{1/2} \left[ a_{k\lambda} e^{-i(\omega t - k x)} - a_{k\lambda}^\dagger e^{i(\omega t + k x)} \right],
\]
\[
E'(x', t') = E_x(x, t) = i \left( \frac{2\pi \hbar \omega}{V} \right)^{1/2} \left[ a_{k\lambda} e^{-i(\omega t - k x')} + a_{k\lambda}^\dagger e^{i(\omega t' + k x')} \right],
\]
\[
E'(x', t') = E_x(x, t) = i \left( \frac{2\pi \hbar \omega}{V} \right)^{1/2} \left[ a_{k\lambda} e^{-i(\omega t' - k x')} - a_{k\lambda}^\dagger e^{i(\omega t + k x')} \right].
\]
We have used the fact that $e^{i(k'\cdot k)x} = e^{i(k'\cdot k'x)}$ since we will require $\partial E'_x/\partial x'$. The primed and unprimed frequencies and wave vectors are related by

$$\omega' = \gamma(\omega - vk_x),$$
$$k'_x = \gamma(k_x - \frac{v}{c} \omega), \quad k'_y = k_y, \quad k'_z = k_z. \quad (66)$$

The induced dipole moment $d'_i(x', t')$ is given by

$$d'_i(x', t') = i\left(\frac{2\pi \hbar \omega}{V}\right)^{1/2} \left[\alpha(\omega') a_{k\lambda} e^{-i(k'\cdot k'x)} - \alpha^*(\omega') a_{k\lambda}^\dagger e^{i(k'\cdot k'x)}\right] e_{\lambda\alpha}, \quad (67)$$

since $\alpha(-\omega') = \alpha^*(\omega')$.

Consider first the force

$$[F'_{\text{ind}}]_{xx} = \frac{\hbar c}{2\pi^2} \int d^3 k \left[4\pi \hbar \omega \right]^{1/2} \left[\alpha(\omega') a_{k\lambda} \gamma(k') \right]^2 \left[\alpha(\omega') a_{k\lambda} \gamma(k') \right] e_{\lambda\alpha}, \quad (68)$$

corresponding quantum mechanically to the first term in equation (64). The expectation value is over the thermal equilibrium state of the field. It follows from equations (65) and (67) that

$$[F'_{\text{ind}}]_{xx} = \frac{4\pi \hbar \omega}{V} k'_x \alpha(\omega') \left[n(\omega) + 1\right] \frac{1}{2} e_{\lambda\alpha}, \quad (69)$$

where we have defined $n(\omega) = \langle a_{k\lambda}^\dagger a_{k\lambda}\rangle$ for the (unpolarized) blackbody field and used the fact that $\langle a_{k\lambda}^\dagger a_{k\lambda}\rangle = 0$ for this field. Next we add up the contributions to $F'_{\text{ind}}$ from all field modes:

$$[F'_{\text{ind}}]_{xx} \to \frac{V}{8\pi^3} \int d^3 k \left(4\pi \hbar \omega \right)^{1/2} \left[k'_x \alpha(\omega') \right]^2 \left[n(\omega) + 1\right] \frac{1}{2} e_{\lambda\alpha}, \quad (70)$$

To compare with previous work we wish to write this force as an integral over the primed variables. To this end we note from the Jacobian that follows from equation (66) that

$$d^3 k = \gamma \left(1 + \frac{v}{c} \frac{k'_x}{k'}\right) d^3 k', \quad (71)$$

and that, again from equation (66),

$$1 - \frac{k_x^2}{k'^2} = \frac{1}{\gamma^2} \left(1 + \frac{v}{c} \frac{k'_x}{k'}\right)^2, \quad (72)$$

where $n(\omega) = \langle a_{k\lambda}^\dagger a_{k\lambda}\rangle$ and $T$ is the temperature in the laboratory frame. The contribution to $F'_{\text{ind}}$ from photon momentum $\hbar \omega$ along the $x$ direction varies with the number of photons at the Doppler-shifted frequency $\gamma(\omega + vk_x)$. Since $n$ decreases with increasing frequency, the force acts in a
direction opposite to that of the particle velocity. Note that we are assuming that \( \alpha_f(\omega) > 0 \) at all frequencies.

### 4.2. Calculation of the force \( F_{dd} \)

The force \( F_{dd} \) is the force associated with the dipole’s radiation reaction and therefore radiation, and is given by the second term in equation (62). Thus \( F_{dd} = (v/c^2) dE'/dr' \), where the rate of change \( dE'/dr' \) of the particle’s energy in \( S' \) is minus the power radiated by the fluctuating electric dipole of the particle at the temperature \( T' \) in the particle frame:

\[
\frac{dE'}{dr'} = -\frac{2}{3c^2} \langle \hat{d}^2(t) \rangle = -\frac{2}{3c^2} \int_{0}^{\infty} d\omega \omega^3 \alpha_f(\omega) \langle |\hat{d}(\omega)|^2 \rangle.
\]  
(78)

Using the fluctuation–dissipation relation [23]

\[
\langle |\hat{d}(\omega)|^2 \rangle = \frac{h}{\pi} \alpha_f(\omega) \coth(\hbar\omega/2k_B T')
\]  
(79)

for each Cartesian component of \( \hat{d}(\omega) \), we obtain

\[
\frac{dE'}{dr'} = \frac{2h}{\pi c^2} \int_{0}^{\infty} d\omega \omega^3 \alpha_f(\omega) \coth(\hbar\omega/2k_B T')
\]  
(80)

and therefore

\[
F_{dd} = -\frac{2h v}{\pi c^2} \int_{0}^{\infty} d\omega \omega^3 \alpha_f(\omega) \coth(\hbar\omega/2k_B T')
\]  
(81)

We note that a force of the form \( (v/c^2) dE'/dr' \) is consistent with the insightful analysis by Sonneleiter et al [24] of a ‘vacuum friction force’ for an excited atom moving with uniform velocity \( v \) in a vacuum. For example, for a two-level atom in its excited state at \( t = 0 \),

\[
F_\alpha = (v/c^2) \frac{dE}{dr} [\hbar \omega_\alpha e^{-\gamma t}] = -\frac{v}{c^2} \hbar \omega_\alpha T e^{-\gamma t}
\]  
(82)

to order \( v/c \), where \( \omega_\alpha \) and \( T \) are respectively the atom’s transition frequency and spontaneous emission rate in \( S' \) [25].

### 4.3. Total force

The total force \( F_\alpha = F_{ind} + F_{dd} \) in the laboratory frame follows from equations (62), (76) and (81). After some straightforward algebra we obtain

\[
F_\alpha = \frac{2h}{\pi^2 c^2 v^2} \int_{0}^{\infty} d\omega \omega^3 \alpha_f(\omega) \int_{-\infty}^{\infty} dy \left( y - \frac{1}{\gamma} \right) \times \left[ \frac{1}{e^{\hbar\omega_\alpha/k_BT} - 1} - \frac{1}{e^{\hbar\omega_\alpha/k_BT} - 1} \right]
\]

\[
= -\frac{h}{\pi^2 c^2 v^2} \int_{0}^{\infty} d\omega \omega^4 \alpha_f(\omega) \int_{-\infty}^{\infty} dy \left( y - \frac{1}{\gamma} \right) \times \left[ \coth(h\omega_\alpha/2k_BT') - \coth(h\omega_\alpha/2k_BT) \right],
\]  
(83)

where \( u_+ = \sqrt{1 + \frac{\omega_\alpha}{c^2 \gamma}} \), \( u_- = \sqrt{1 - \frac{\omega_\alpha}{c^2 \gamma}} \) [22]. This result, which we have obtained by the direct transformation of force from one frame to another, is identical to that obtained by Milton et al in their elegant theory based on Green functions [22].

### 4.4. Non-relativistic force

In the non-relativistic limit \( v \ll c \) the force equation (83) reduces to [22]

\[
F_\alpha = \frac{4\hbar}{3\pi c^5} v \int_{0}^{\infty} d\omega \omega^3 \alpha_f(\omega) \frac{\partial n}{\partial \omega}
\]

\[
= -\frac{4\pi}{c^2} v \int_{0}^{\infty} d\omega \omega \alpha_f(\omega) \left[ \rho(\omega) - \frac{\omega}{3} \frac{d\rho}{d\omega} \right] = -\xi mv.
\]  
(84)

In this limit this is the total force in the laboratory frame. It has the basic form obtained by Einstein and Hopf [3] and is equivalent to the more general expression obtained by Mkrtchian et al [5].

Using equation (5) and \( n(\omega) = (e^{\hbar\omega/k_BT} - 1)^{-1} \) we can express \( \xi \) from equation (84) alternatively as

\[
\xi = -\frac{4\hbar}{3\pi mc^5} \int_{0}^{\infty} d\omega \omega^3 \alpha_f(\omega) \frac{\partial n}{\partial \omega}
\]

\[
= -\frac{4\hbar^2}{3\pi mc^3 k_BT} \int_{0}^{\infty} d\omega \omega^2 \alpha_f(\omega) \left( e^{\hbar\omega/k_BT} - 1 \right)^3.
\]  
(85)

One can consider the thermal drag for the previous examples of an electron and a dielectric sphere. The non-relativistic thermal drag coefficient for an electron in blackbody radiation can be obtained from equation (85) with \( \alpha(\omega) \) given by equation (27):

\[
\alpha_f(\omega) \equiv \frac{e^2 \tau e}{m_e c^2} = \frac{c\sigma_T}{4\pi \omega},
\]  
(86)

where \( \sigma_T = (8\pi/3)(e^2/m_e c^2)^2 \) is the Thomson cross section. Thus

\[
\xi_e = \frac{32\pi^3 h}{135m_e} \left( \frac{e^2}{m_e c^2} \right)^2 \left( \frac{k_BT}{h\omega} \right)^4 ,
\]  
(87)

which is related to the momentum diffusion of an electron in blackbody radiation (equation (29)) via the fluctuation–dissipation relation

\[
\frac{\langle \Delta p^2 \rangle}{\Delta t} = 2m_e \xi_e T.
\]  
(88)

With \( T \) the absolute temperature (K),

\[
\xi_e = 2.5 \times 10^{-22} T^4 \text{ s}^{-1},
\]  
(89)

and

\[
F_{ind} = -\xi_e m_e v = -1.2 \times 10^{-31} v T^4 \text{ dyne},
\]  
(90)

with \( v \) in units of cm s\(^{-1}\).

Similarly, for the non-relativistic thermal drag on a small dielectric sphere, we obtain from equations (85) and (32)

\[
\xi_s = \frac{512\pi^3 h}{135m_s} \left( \frac{e^2}{m_e c^2} \right)^2 \left( \frac{k_BT}{h\epsilon} \right)^8 ,
\]  
(91)

where \( m_s \) is the mass of the particle. For example, for a nanoparticle of radius 50 nm and density 2 g cm\(^{-3}\),

\[
\xi_s \approx 2.4 \times 10^{-35} \left( \frac{\epsilon - 1}{\epsilon + 2} \right)^2 T^8 \text{ s}^{-1}.
\]  
(92)
We note that the above drag force is related to the momentum diffusion constant of the sphere (equation (33)) via the fluctuation–dissipation relation (equation (88)); the momentum diffusion constant in turn is related to the rate of decoherence of the particle via equation (34). Such a connection between decoherence rate, momentum diffusion and thermal drag is pertinent to deducing decoherence rate of dielectric nanospheres in the quantum regime, as was done in [26].

5. Power balance

The polarizabilities in the formulas we have derived for the momentum fluctuations for a particle in blackbody radiation make no assumptions about the physical processes contributing to $\alpha(\omega)$, and allow for scattering, absorption, and radiation by the particle. We do assume, as in previous work we have cited on the drag force, that the response of the dipole particles to the field is linear and passive, and in particular that there is no stimulated emission. We address the question of stimulated emission briefly in the following section.

The drag force on a particle is associated with an increase in the field energy, and similarly the momentum fluctuations are associated with a decrease in field energy. We consider now the implications of energy conservation when, as in the original model of Einstein and Hopf [3, 4], we assume that the polarizability accounts for only radiation reaction and scattering of radiation. In particular, in this model there is no accounting for absorption. Then, as we now show, the requirement of energy conservation imposes a condition on the form of the polarizability. Some remarks pertaining to absorption (and stimulated emission) are included in the following section.

Consider a medium consisting of a uniform distribution, on average, of $N$ polarizable particles per unit volume. The drag forces $F_{x,y,z} = -mv_{x,y,z}$ on the particles of mass $m$ imply that the average energy density $W$ of the (homogeneous and isotropic) field increases at a rate

$$\frac{dW}{dt} = \frac{-4\pi\hbar k_BT}{mc^2} \int_0^\infty d\omega \omega^5 \alpha(\omega) \frac{\partial n}{\partial \omega},$$

where we use equation (84) for the (non-relativistic) drag and assume that the particles’ average kinetic energy is $\frac{1}{2} k_BT$. Similarly the rate of decrease of the field energy density associated with the momentum fluctuations the field induces in the particles is

$$\frac{dW}{dt} = \frac{-3N}{2m} \frac{1}{\Delta t} \langle \Delta p^2 \rangle = \frac{-8\pi\hbar k_BT}{3mc^2} \int_0^\infty d\omega \omega^8 |\alpha(\omega)|^2 \left[ n^2(\omega) + n(\omega) \right],$$

where we have used equation (24) and multiplied by 3 to account for the fluctuations in particle momenta along all three directions in the isotropic field. The rate of change of the field energy density attributable to drag forces and momentum fluctuations is therefore

$$\frac{dW}{dt} = \frac{-4\pi\hbar^2}{mc^2} \int_0^\infty d\omega \omega^5 \left[ \frac{2\omega^3}{3c^2} |\alpha(\omega)|^2 - \alpha_f(\omega) \right]$$

$$\times \left[ n^2(\omega) + n(\omega) \right],$$

But $n(\omega) = (e^{\hbar\omega/k_BT} - 1)^{-1}$ implies

$$\alpha_f(\omega) = \frac{2\omega^3}{3c^2} |\alpha(\omega)|^2,$$

and therefore

$$\frac{dW}{dt} = \frac{-4\pi\hbar^2}{mc^2} \int_0^\infty d\omega \omega^5 \left[ \frac{2\omega^3}{3c^2} |\alpha(\omega)|^2 \right]$$

or, for $\langle dW/dt \rangle = 0$,

$$\alpha_f(\omega) = \frac{2\omega^3}{3c^2} |\alpha(\omega)|^2,$$

which is just the optical theorem expressing energy conservation in Rayleigh scattering [10].

Alternatively, we could have assumed this condition for energy conservation in equation (95) to deduce equation (96) and therefore the Planck spectrum. This is what was done by Einstein and Hopf [3, 4]. It might be worth noting that they did not invoke the optical theorem (equation (98)), but worked with a model of a dipole oscillator whose polarizability does satisfy the optical theorem. The same was true in later considerations of the Einstein–Hopf model [27, 28]. We further remark that for a polarizable particle obeying the optical theorem, the momentum diffusion constant given by equation (24) and the non-relativistic thermal drag given by equation (85) are related via the fluctuation–dissipation theorem (equation (88)).

Let us return to equation (95), but now using equation (98) and leaving $n(\omega)$ unspecified:

$$\frac{dW}{dt} = \frac{-4\pi\hbar^2}{mc^2} \int_0^\infty d\omega \omega^5 \alpha_f(\omega)$$

$$\times \left[ n^2(\omega) + n(\omega) + \frac{k_BT}{\hbar} \frac{\partial n}{\partial \omega} \right]$$

$$= -\int_0^\infty d\omega \omega^5 |\alpha(\omega)|^2.$$

This expresses the rate of change of field energy as the field induces drag forces and momentum fluctuations in a rarefied gas in thermal equilibrium at temperature $T$. The physical requirement that there be no high-frequency divergence implies that $\lim_{\omega \to \infty} |\alpha(\omega)|^2 = 0$. Then we obtain from equation (99), after an integration by parts,

$$\frac{dW}{dt} = \int_0^\infty d\omega \frac{\partial}{\partial \omega} \left[ \omega^5 |\alpha(\omega)|^2 \right].$$

For the isotropic field under consideration there are $(\omega^2/\pi^2 c^3) d\omega$ modes per unit volume in the frequency
interval $[\omega, \omega + d\omega]$, each having an average energy $\hbar \omega n(\omega)$, and so
\[\langle \frac{dW}{dt} \rangle = \frac{\hbar}{\pi e^2} \int_0^\infty d\omega \omega^3 \frac{\partial n}{\partial t}, \tag{101}\]
and therefore
\[\int_0^\infty d\omega \left( \omega^3 \frac{\partial n}{\partial t} - \frac{\pi e^2 c^4}{\hbar} \frac{1}{\omega^3} \frac{\partial}{\partial \omega} \left[ \omega^5 y(\omega) \right] \right) = 0. \tag{102}\]
If we assume that this energy conservation requirement holds for any frequency interval, i.e.,
\[\frac{\partial n}{\partial t} = \frac{\pi^2 e^2 c^4}{\hbar} \frac{1}{\omega^3} \frac{\partial}{\partial \omega} \left[ \omega^5 y(\omega) \right], \tag{103}\]
it follows that the total number density of photons is conserved:
\[\frac{\partial}{\partial t} \left[ \frac{\pi^2 e^2 c^4}{\hbar} \int_0^\infty d\omega \omega^3 n \right] = 0. \tag{104}\]
Suppose equation (103) describes the evolution of the photon distribution when the field with some initial distribution $n(\omega, 0)$ is brought into interaction with a rarefied electron gas which is assumed to be in thermal equilibrium at the temperature $T$ and to remain so. Since the electrons do not absorb radiation, our assumption of no absorption leading to equation (103) is valid and, in the approximation equation (86), $n(\omega, t)$ satisfies the Kompaneet equation [29–34]:
\[\frac{\partial n}{\partial t} = \frac{N_e h c}{m_e c} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[ \omega^4 \left( n^2 + c_{\nu T} \frac{\partial n}{\partial \omega} \right) \right]. \tag{105}\]
Because it plays such an important role in the theory of radiation–plasma interactions in astrophysics, this equation for the evolution of the photon spectrum due to Compton scattering by electrons has been discussed in many publications, especially in more recent years in connection with the observed distortions in the spectrum of the cosmic microwave background radiation [31, 32, 35, 36]. Here we only note that the formulas we have obtained for the drag and momentum diffusion coefficients, and the rate of change of field energy density, are entirely consistent with rigorous derivations of the Kompaneets that start from a Boltzmann transport equation.

6. Discussion

The average photon number $n(\omega)$ in thermal equilibrium appears differently in the momentum diffusion constant, the decoherence rate, and the drag force. This is because the momentum diffusion constant has a fourth-order dependence on the electric field, whereas the decoherence rate and drag force have a second-order dependence.

The momentum diffusion constant we have obtained for a small polarizable particle in blackbody radiation, for example, involves the factor $n^2(\omega) + n(\omega)$. The two contributions correspond respectively to the ‘wave’ and ‘particle’ contributions to the Einstein fluctuation formula for blackbody radiation [10].

But in the two examples we considered—a free electron and a dielectric sphere—the $n^2(\omega)$ contribution is negligible and the momentum diffusion constant has effectively only a ‘particle’ contribution from the field. It then has the same form as the collisional decoherence rate $\Lambda$.

In the case of momentum diffusion and decoherence of a particle due to collisions with gas molecules [14, 16], in contrast, no ‘wave’ or Bose–Einstein factor appears because the de Broglie wavelengths of the gas molecules are so small that quantum statistics are irrelevant.

Of course the $n^2(\omega)$ and $n(\omega)$ terms in the momentum variance and the drag force are both essential in determining the equilibrium photon spectrum. Consider the condition for thermal equilibrium implied by equation (95) when the optical theorem equation (98) is assumed for $\alpha(\omega)$:
\[\frac{\partial n}{\partial \omega} = -\frac{\hbar}{k_B T} (n^2 + n). \tag{106}\]
If we assume the form of the momentum diffusion constant obtained in section 4.1, the $n^2$ term is absent and we obtain
\[\frac{\partial n}{\partial \omega} = -\frac{\hbar}{k_B T} n \tag{107}\]
instead of equation (106), with the solution
\[n(\omega) = (\text{const.}) \times e^{-\hbar \omega/k_B T}. \tag{108}\]
Together with equation (5), this implies the Wien form of the spectrum. If instead we assume the momentum diffusion constant $\propto n^2$ obtained with classical stochastic wave theory by Einstein and Hopf, we obtain
\[\frac{\partial n}{\partial \omega} = -\frac{\hbar}{k_B T} n^2 \tag{109}\]
instead of equation (106), with the solution
\[n(\omega) = k_B T / \hbar \omega. \tag{110}\]
when a constant of integration is chosen to give the Rayleigh–Jean spectrum. The solution of equation (106) is of course
\[n(\omega) = (e^{(\hbar \omega/k_B T)} - 1)^{-1}, \tag{111}\]
where in general we must include the chemical potential $\mu$. Expressed in terms of $\rho(\omega)$, equation (106) is the equation obtained by Einstein [2].

The fluctuation–dissipation relation (equation (88)), can be used in the Fokker–Planck equation for the velocity distribution function $f(v, t)$ to obtain the Maxwell–Boltzmann distribution. That is, the Fokker–Planck equation with the drift and diffusion terms satisfying equation (88),
\[\frac{\partial f}{\partial t} = -\frac{\xi}{\sqrt{m}} \frac{\partial}{\partial v} (v f) + \frac{\xi k_B T}{m} \frac{\partial^2 f}{\partial v^2}, \tag{112}\]
has the solution
\[f(v, t) = \left( \frac{m}{2 \pi k_B T} \right)^{1/2} e^{-m v^2 / 2 k_B T} \tag{113}\]
for $\xi t \gg 1$. The condition that the momentum diffusion and the drag force together determine the blackbody spectrum, subject to the assumption $\langle \frac{1}{2} m v^2 \rangle = \frac{1}{2} k_B T$ for each translational
degree of freedom, therefore implies the Maxwell–Boltzmann distribution for the particle velocities [37].

In the preceding section, to simplify considerations of power balance, we made the simplifying assumption of Einstein and Hopf that the particles scatter but do not absorb radiation. More generally our expressions for the drag force are applicable when \( \alpha(\omega) \) includes effects of absorption. Consider, for example, the effect of absorption on the drag force. For this purpose it suffices to assume a single absorption frequency \( \omega_0 \) and the form of the polarizability given by the classical Lorentz electron-oscillator model:

\[
\alpha(\omega) = \frac{e^2/m}{\omega_0^2 - \omega^2 - 2ie^2\omega^3/3mc^3 - i\beta\omega},
\]

where the two purely imaginary terms in the denominator are due to radiation reaction and absorption, respectively, and \( \beta \) is essentially just the homogeneous linewidth of the absorption line. Then the drag force (equation (84)) becomes

\[
F_x = \frac{4h\nu}{3\pi e^2} \int_0^\infty d\omega \omega^5 \alpha_T(\omega) \frac{\partial n}{\partial \omega}
\]

\[
= \frac{4h\nu}{3\pi e^2} \int_0^\infty d\omega \omega^5 \left[ \alpha_T(\omega) + \alpha_T^{abs}(\omega) \right] \frac{\partial n}{\partial \omega},
\]

where

\[
\alpha_T(\omega) = \frac{2\omega^3}{3\epsilon_0^3} |\alpha(\omega)|^2
\]

and

\[
\alpha_T^{abs}(\omega) = \frac{e^2 \beta \omega/m}{(\omega_0^2 - \omega^2)^2 + (2e^2\omega^3/3mc^3 + \beta\omega)^2}
\]

are due to scattering and absorption, respectively. Using the optical theorem expression (equation (116)), we can write equation (115) as

\[
F_x = \frac{8h\nu}{9\pi e^2} \int_0^\infty d\omega \omega^5 |\alpha(\omega)|^2 \frac{\partial n}{\partial \omega}
\]

\[
+ \frac{4h\nu}{3\pi e^2} \int_0^\infty d\omega \omega^5 \alpha_T^{abs}(\omega) \frac{\partial n}{\partial \omega}
\]

This is consistent with equation (15.83) of reference [38] when we identify \( \alpha(\omega) \) in that equation with our \( \alpha_T^{abs}(\omega) \). The point here is simply to illustrate the generality of the form equation (84) for the drag force.

The Lorentz oscillator model above can model absorption but not stimulated emission. Stimulated emission introduces additional features, especially for momentum fluctuations, that require us to go beyond our assumption that the response of the dipole particles to the field is linear and passive. This is not difficult to do under some simplifying approximations. We conclude with a simple model accounting for stimulated emission—a two-level atom with purely radiative line broadening—and apply equation (84) for the drag force. In the rotating-wave approximation for a two-level atom, assuming a radiative frequency shift has been included in the definition of the transition frequency [39],

\[
\alpha(\omega) = \frac{\mu^2}{3h\omega_0} - \frac{P_1 - P_2}{2\omega^3/3hc^3},
\]

\[
\alpha_T^{st}(\omega) = \frac{\mu^2}{3h\omega_0} - \frac{(P_1 - P_2)\omega}{2(2\omega^3/3hc^3)^2},
\]

where \( \omega_0 \) and \( \mu \) are the transition frequency and transition dipole moment, respectively, and we have included the lower- and upper-state occupation probabilities \( p_1 \) and \( p_2 \), respectively.

Then, from equation (84),

\[
F_x \approx \frac{4\pi\omega_0\nu}{c^2} \left[ \frac{\omega_0}{3} \frac{d\rho(\omega_0)}{d\omega_0} \right] \int_0^\infty d\omega \omega^5 \alpha_T^{st}(\omega)
\]

\[
\approx \frac{\hbar\omega_0}{c^2} (p_1 - p_2) B \left[ \frac{\rho(\omega_0)}{3} \frac{d\rho(\omega_0)}{d\omega_0} \right] \nu
\]

\[
= \frac{\mu^2}{3hc^3} A p_1 - p_2 \frac{e^\hbar\omega_0/k_B T}{(e^\hbar\omega_0/k_B T - 1)^2},
\]

where \( A = 4\mu^2\omega_0^3/3hc^3 \) and \( B = 4\pi \mu^2/3hc^2 \) are the Einstein \( A \) and \( B \) coefficients. This is equivalent to the expression obtained by Einstein for the drag force when we assume \( p_2 = p_1 e^{-\hbar\omega_0/k_B T} \) and \( p_1 + p_2 = 1 \). For example, for the sodium \( D_2 \) line (1/\( A \approx 16 \) ns, \( \lambda \approx 589 \) nm) and \( T = 5000 \) K, \( F_x \approx 9 \times 10^{-27} \) dyne (\( \nu \) in cm s\(^{-1} \)).

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**Data availability statement**

No new data were created or analysed in this study.

**Appendix A. Statistical independence of \( E_j \) and \( \partial_j E_j \)**

Consider first the following elementary example. Let \( X \) and \( Y \) be two classical Gaussian random processes such that \( \langle X \rangle = \langle Y \rangle = 0 \) and \( \langle XY \rangle = 0 \). Then \( X \) and \( Y \) are statistically independent, i.e., the joint probability distribution \( p_{XY}(X,Y) = p_X(X)p_Y(Y) \). The simple proof of this goes as follows. Consider the characteristic function

\[
C_{XY}(\xi, \eta) = \langle \exp[i(\xi X + \eta Y)] \rangle
\]

whose Fourier transform is \( p_{XY}(X,Y) \). Since \( X \) and \( Y \) are Gaussian, so are linear combinations of \( X \) and \( Y \). So \( Z = \xi X + \eta Y \)
is Gaussian and \((Z) = 0\). Then
\[
\langle \exp(iZ) \rangle = 1 - \frac{1}{2} \langle Z^2 \rangle + \frac{1}{24} \langle Z^2 \rangle^2 - \cdots = \exp(-\langle Z^2 \rangle/2)
\]
since \(\langle XY \rangle = 0\) by assumption. We have used the fact that for Gaussian statistics \((Z)^n = 0\) for \(n\) an odd positive integer and
\[
\langle Z^{2n} \rangle = 1 \cdot 3 \cdots (2n - 1) \langle Z^2 \rangle^n.
\]

**Appendix B. Momentum diffusion and decoherence from scattering by air molecules**

According to Campbell’s theorem the momentum variance of dielectric particles due to momentum kicks from air molecules is
\[
d(\Delta p^2) \approx dq(q^2) \rho(q) \frac{q}{m_{air}} \sigma_{air},
\]
where \(q = m_{air}v\) is the momentum of an air molecule, \(\sigma_{air} = 2\pi a^2/3\) is the scattering cross section, and
\[
\rho(q) = \frac{N_{air}}{V} 4\pi q^2 \left( \frac{1}{2\pi m_{air} k_B T} \right)^{3/2} e^{-q^2/(2m_{air} k_B T)}
\]
corresponds to the Maxwell–Boltzmann distribution for the air molecules. Integrating over the velocities and using similar arguments as in the case of blackbody radiation, we obtain
\[
\frac{\langle \Delta p^2 \rangle}{\Delta t} \approx \frac{8}{\pi} \frac{8N_{air}}{m_{air} V} \left( \frac{1}{2\pi m_{air} k_B T} \right)^{3/2} \left( \frac{1}{\sqrt{2m_{air} k_B T}} \right)^3\int_0^\infty dq q^5 e^{-q^2/(2m_{air} k_B T)}
\]
which is twice the center-of-mass decoherence rate for scattering of air molecules in the long-wavelength limit [13], similar to the case of thermal photon scattering.

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[8] See, for instance, Rice S O 1944 Mathematical analysis of random noise *Bell System Tech. J.* 23 282 reprinted in Selected Papers on Noise and Stochastic Processes ed N Wax (New York: Dover)

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[10] See, for instance, Milonni P W 2019 *An Introduction to Quantum Optics and Quantum Fluctuations* (New York: Oxford University Press)

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