BLAZAR FLARES FROM COMPTON DRAGGED SHELLS

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ABSTRACT

We compute the dynamics and emission of dissipative shells that are subject to a strong Compton drag, under simplifying assumptions about the dissipation mechanism. We show that under conditions prevailing in blazars, substantial deceleration is anticipated on sub-parsec and parsec scales in cases of rapid dissipation. Such episodes may be the origin of some of the flaring activity occasionally observed in gamma-ray blazars. The shape of the light curves thereby produced reflects the geometry of the emitting surface if the deceleration is very rapid, or the dynamics of the shell if the deceleration is delayed, or initially more gradual, owing, e.g., to continuous injection of energy and momentum.

Key words: galaxies: active – quasars: general – radiation mechanisms: non-thermal – X-rays: galaxies

1. INTRODUCTION

The broadband spectrum observed in blazars is dominated by beamed emission produced in relativistic jets that emanate from the central black hole. These jets propagate through a dense radiative environment, and interact with seed photons that are supplied by extended radiation sources, notably the accretion disk around the black hole, gaseous clouds in the broad line region (BLR), and a dusty molecular torus located at larger scales (see, e.g., Joshi et al. 2014 for a recent account). This interaction should affect the dynamics and emission of the jet. The primary concern of most previous works (e.g., Dermer & Schlickeiser 1993; Sikora et al. 1994; Blandford & Levinson 1995; Ghisellini & Madau 1996 see also Levinson 2006 and references therein) has been the effect of this interaction on the observed spectrum, and simple emission models that ignore dynamical effects and complex structures have been constructed for this purpose, although some recent works incorporate more realistic models for the dynamics of the emitting plasma (e.g., Joshi & Bottcher 2011; Joshi et al. 2014 and references therein). In ERC models, the high-energy component of the spectral energy distribution (SED) is attributed to inverse Compton scattering of ambient seed photons by non-thermal electrons accelerated in dissipative regions inside the jet. This process, unlike synchrotron emission, gives rise to a loss of linear momentum of emitting fluid elements and a consequent radiative drag that tends to decelerate the bulk flow. In certain circumstances, explored below, this can lead to rapid, large amplitude flares.

Rapid variability over the entire electromagnetic spectrum is a characteristic property of blazars. Episodic gamma-ray emission with flares durations of hours to weeks is quite typical to many gamma-ray blazars, with the most extreme activity recorded in the sub-class of TeV active galactic nuclei, e.g., Mrk 421, Mrk 501, PKS 2155-304. The observed variability time imposes a stringent constraint on the maximum size of the emission region, \( \Delta r \), which in extreme cases is inferred to be of the order of the gravitational radius of the putative black hole. The naïve expectation is that the large amplitude short duration flares seen in gamma-ray blazars originate from small radii, as the fraction of jet energy that can be tapped for production of \(\gamma\)-rays scales as \( \eta \simeq (\Delta r / \theta_j r_{\text{em}})^2 \), where \( \theta_j \) is opening angle of the jet and \( r_{\text{em}} \) the emission radius, and is small for \( r_{\text{em}} \gg \Delta r \). Some of the variable gamma-ray flux may be attributed to a sparking gap at the base of the jet, as proposed for M87 (Levinson 2000; Neronov & Aharonian 2007; Levinson & Rieger 2011), however, in powerful blazars the spectrum emitted from the gap is unlikely to extend beyond a few GeV, owing to the large pair-production opacity. This, and the indications of correlated emission at much lower energies (radio-to-X-rays) strongly suggest that the jets are important sources of episodic gamma-ray emission. It has been argued that in some cases the observations favor models in which the variable gamma-ray emission originates from small regions locate at large radii (e.g., Sikora et al. 2008; Agudo et al. 2011). Such events may be produced by a converging shock in a reconfinement nozzle (Bromberg & Levinson 2009) or magnetic reconnection in minijets (Giannios 2013). However, recent analysis (Nalewajko et al. 2014) challenges the far dissipation scenarios, indicating typical emission radii in the range 0.1–1 pc. As shown below, on these scales the radiative drag can be substantial.

In this paper we consider the effect of Compton drag on the dynamics and emission of dissipative fluid shells. We show that if dissipation of the bulk energy commences not too far out, the blob experiences strong deceleration that leads to large variation of the observed flux emitted from the blob by virtue of the change in its bulk Lorentz factor. The effect of Compton drag on the dynamics of a relativistic jet has been considered earlier by several authors under different assumptions (e.g., Phinney et al. 1987; Li et al. 1992; Sikora et al. 1996). Our method is similar to that presented in Sikora et al. (1996); however, we focus on short events that may lead to rapid flares, and compute the Lorentz factor profiles and the resulting gamma-ray light curves for a range of conditions. For typical ambient luminosities, the duration of flares produced by this mechanism is on the order of the characteristic size of the emitting blob. Thus, flare durations as short as the dynamical time of the central engine can naturally be accounted for in this model, provided that the bulk energy can be dissipated at a large enough rate. A preliminary account of the decelerating shell model is given in Levinson (2007). Here, we present an elaborated analysis of the dynamics of the flow, and also compute the resulting light curves. The construction of this model was originally motivated by the apparent discrepancy inferred in TeV blazars between the relatively large Doppler
factors, $\delta_D \sim 30–50$, required to avoid strong attenuation of the VHE flux emitted from the inner regions, and the much lower values, $\delta_D \sim 1$, inferred from superluminal motions and source statistics (Georganopoulos & Kazanas 2003, Levinson 2007). However, the question of how the Compton drag affects the dynamics and emission of dissipative outflows is of general interest, and is relevant essentially to all blazars.

2. THE MODEL
In the simplified model invoked here, a blob is ejected from a central engine of size $r_i = 2GM/c^2$ and accelerated to a Lorentz factor $\Gamma_0$. When it reaches some radius $r_d \gtrsim 10^{17}r_sM_0$ cm, dissipation suddenly starts, e.g., via formation of shocks or explosive conversion of magnetic energy, leading to continuous particle acceleration. Inverse Compton scattering of ambient photons then leads to a strong radiative drag that tends to decelerate the blob. In certain circumstances, this drag may be compensated by injection of energy and momentum into the blob by some external agent, delaying the deceleration of the emitting plasma. The deceleration may be delayed also in cases where the dissipation commences very close to the central engine, where the radiation field is dominated by direct illumination from the disk and is highly anisotropic, as in this zone the blob may propagate at the equilibrium Lorentz factor until reaching scales where the external radiation field is roughly isotropic (e.g., Sikora et al. 1996, Vuillaume et al. 2014). In the present model we shall refer to this case as delayed deceleration. In reality, the structure and velocity of the emitting plasma are expected to be non-uniform by virtue of confined dissipation and rapid cooling. In what follows we ignore such complications and compute the dynamics and emission of the blob under simplifying assumptions about the microphysics of dissipation.

The energy and momentum fluxes of the emitting fluid are given by

$$ T^{0r} = \left( w + B^2/4\pi \right) \Gamma^2 \beta_1 \equiv u_j' \Gamma^2 \beta_1, $$

$$ T^{rr} = \left( w + B^2/4\pi \right) \Gamma^2\beta_1^2 + p + B^2/8\pi, $$

where $w = \rho + e + p$ is the proper specific enthalpy, $\rho$, $e$ and $p$ are the pressure, proper density and proper internal energy, respectively, $B$ is the proper magnetic field, and $\Gamma = (1 - \beta_1^2)^{-1/2}$ is the bulk Lorentz factor. In terms of the total power, $P_j = c T^{0r} \theta_j^2 r^2$, where $\theta_j$ is the half opening angle of the flow, we have

$$ u_j' = \frac{P_j}{\pi \theta_j^2 r^2 \Gamma^2} = 0.1 P_{44} \theta_j \Gamma \eta_j^{-2} \text{ erg cm}^{-3}, $$

for $\beta_1 = 1$, $r = 10^{17} \eta_7$ cm and $P_j = 10^{44} P_{44}$ erg s$^{-1}$.

For simplicity, we assume that the ambient radiation field is roughly isotropic in the frame of the central engine, and has a luminosity $L_\gamma$. This quasi-isotropic radiation field is contributed by scattering and reprocessing of the central UV radiation by gas in the BLR, and by emission from a dusty torus (e.g., Joshi et al. 2014). Here, $L_\gamma$ represents the sum of these components, and is a fraction of the total luminosity of the continuum source. The radial profile of the ambient intensity intercepted by the jet depends on the geometries of the BLR and the dusty torus (e.g., Joshi et al. 2014). Denoting $x = r/r_d$, we express it as

$$ I_\gamma(x, \mu, r) = \kappa_\gamma(r) F(\epsilon_\gamma), \quad \kappa_\gamma(r) = \frac{L_\gamma}{16\pi^2 r_d^2 c} f_\gamma(x), $$

with the normalization $\int_0^\infty F(\epsilon_\gamma) d\epsilon_\gamma = 1$, so that the total energy density at radius $r$ is $u_\gamma(r) = 4\pi \kappa_\gamma(r)$. For central emission, $f_\gamma(x) = x^{-2} = (r_d/r)^2$. For a flat profile within some scale $n > r_d$, $f_\gamma(x) = (x/n)^2$ at $x < n/r_d$. For the spectrum of the ambient radiation field we adopt

$$ F(\epsilon_\gamma) \propto \begin{cases} \epsilon_\gamma^{1/2} & 10^{-8} < \epsilon_\gamma/m_e c^2 < 10^{-4}, \\ \epsilon_\gamma^{-3/2} & 10^{-4} < \epsilon_\gamma/m_e c^2 < 10^{-1}, \\ 0 & \text{else} \end{cases} $$

that mimics a typical soft spectrum.

In the rest frame of the blob the intensity is given by

$$ I'_\gamma(\epsilon'_\gamma, \mu', r) = \frac{\kappa_\gamma(r)}{\Gamma_3 (1 + \beta_1 \mu')^3} F\left( \epsilon'_\gamma \Gamma (1 + \beta_1 \mu') \right), $$

and the comoving energy density by

$$ u'_\gamma = 2\pi \int I'_\gamma d\epsilon'_\gamma d\mu' = 2\pi \kappa_\gamma(r) \int_{-1}^{1} \frac{d\mu'}{\Gamma^4 (1 + \beta_1 \mu')^4} = u_\gamma \Gamma^2 \left( 1 + \beta_1^2 / 3 \right). $$

From Equation (6) it is seen that the comoving intensity peaks sharply around the direction opposite to the blob velocity, viz., $\mu' = -1$. To simplify our calculations we approximate the comoving intensity as a beam moving in the direction $\mu' = -1$:

$$ I'_\gamma(\epsilon'_\gamma, \mu', r) = \eta_\gamma(r) F\left( \epsilon'_\gamma \Gamma (1 + \beta_1 \mu') \right) \delta(1 + \mu'), $$

with $\eta_\gamma(r) = \kappa_\gamma(r) 2\Gamma (1 + \beta_1)^{-1} (1 + \beta_1^2 / 3) \approx 4\Gamma \kappa_\gamma(r)/3$. It can be readily verified that with this choice the comoving energy density satisfies Equation (7).

We further suppose that the electron distribution function is isotropic in the fluid rest frame and can be approximated as a power law: $d^2 n'_e / d\gamma d\gamma = \kappa_e(r) \gamma^{-q}; \gamma_1 < \gamma < \gamma_2$, where $m_e c^2 \gamma_1$ is the corresponding electron energy, as measured in the comoving frame. We define $\xi_e(r)$ to be the fraction of the total internal energy carried by the relativistic electrons, that is, $\xi_e = u'_e / u'_\gamma$. From $u'_e = \int m_e c^2 \gamma n'_e d\gamma$, subject to the boundary condition $\xi_e(r \leq r_d) = 0$. In terms of this parameter $\kappa_e = \xi_e (u'_e / m_e c^2) (2 - q) / (\gamma_2^2 - \gamma_1^2)$, and at the same time $\xi_e = \int_0^{r_\gamma} m_e c^2 / (\gamma_2^2 - \gamma_1^2) d\gamma$, and we note that $\gamma_1^2 = \kappa_e (u'_e / m_e c^2) (2 - q) / (\gamma_2^2 - \gamma_1^2)$, and we note that $\gamma_2^2 = \kappa_e (u'_e / m_e c^2) (2 - q) = \ln(\gamma_2/\gamma_1)$ in the limit $q \to 2$. We suppose that $\xi_e$ reaches a maximum value over some characteristic length scale that depends on the microphysics of the specific dissipation mechanism. This length scale is typically of the order of the gyroradius of accelerated electrons, which is shorter than the cooling time of the highest energy electrons. Thus, we take $\xi_e$ to be constant at $r > r_d$.

The maximum Lorentz factor of the electron distribution, $\gamma_2$, is likely to be limited by cooling. In the limit of Bohm diffusion the acceleration time is $t_{acc} = (\gamma m_e c^2) / (\eta_\gamma c E B)$. Equating with the Compton cooling time, $t'_c = \gamma m_e c^2 / P_{\text{com}} = m_e c / (\gamma \sigma_T u'_e)$,
and adopting \( \eta_{\text{sec}} = 0.1 \), we estimate

\[
\Gamma \gamma_2 \simeq 3 \times 10^7 (B/B_0)^{1/2} \xi_{B-1}^{1/4} P_{104}^{1/4} J_{44}^{-1/2}
\times \left( \Gamma_0 \beta \right)^{-1/2} R_{14}^{1/2} r^{-1/2}
\tag{9}
\]

in terms of the fraction \( \xi = 0.1 \xi_{B-1} = u'_e/u' \), \( u'_b = B^2/8\pi \).

Here \( B_0 = B(r = r_d) \) is the magnetic field at the onset of dissipation.

2.1. Dynamics

We consider only situations in which dissipation is rapid enough to produce high enthalpy inside the blob on a timescale comparable to the radiative time. Then, one can adopt a simple prescription in which the dissipation is formally treated as an initial condition \((r = r_d)\) of the flow equations. The magnetization of the flow is then expected to be low, so that the effect of the magnetic field on the dynamics of the system can be neglected. This may not apply to cases where the dissipation time is much longer than the light crossing time of the shell.

We suppose that the dissipated energy is redistributed in a way that a fraction \( \xi \) of the total dissipation energy is uniformly injected in the form of a power law electron distribution inside the blob. For clarity, we ignore here the contribution of the thermal population to the Compton drag. As shown below, this is justified when \( \xi \gamma_2 m_e c^2 \) is larger than the thermal energy, here \( \gamma_2 \) is the upper cutoff of the electron distribution given in Equation (9), and \( \chi \) is defined below.

The inclusion of the thermal electrons in the source terms does not alter our results significantly, and at any rate, if the dissipation produces a population of relativistically hot electrons with a thermal energy \( \gamma T \, m_e c^2 \) rather than a power law distribution, it can be readily accounted for by taking \( \chi = 1 \). \( \gamma_2 = \gamma_\tau \) in Equation (19) below.

We find it convenient to use the parametrization \( \gamma^2 = \gamma_2 \), where \( \langle \gamma \rangle = \gamma_2 \gamma_0 \), \( \gamma_0 = \gamma_\tau \). For clarity, we adopt henceforth, is \( \gamma_0 = m_e / m_e c^2 \). A reasonable choice for the minimum electron energy, adopted henceforth, is \( \gamma_0 = \gamma_\tau \). We further define the fiducial coordinate \( x = ct/r_2 \), and write \( u_0 = u_0 f_0(x)/f_{\gamma_0} \), denoting \( f_{\gamma_0} = f_\gamma(x = 1) \).

Equations (11) and (12) with \( \partial \partial_p \Gamma = 0 \) can then be re-expressed as

\[
\frac{d}{dx} \ln \left( \Gamma^2 x^2 w \right) = -6 \alpha (\chi/\chi_0) \left( \gamma_2/\gamma_0 \right)
\times \left( \Gamma/\Gamma_0 \right) (f_\gamma/f_{\gamma_0}) (4e/3w),
\tag{17}
\]

\[
d \ln \left\{ \frac{w x^2}{\Gamma} \right\} - 3 \frac{p}{w} d \ln p = 0,
\tag{18}
\]

in terms of the constant

\[
\alpha = \frac{r_a \sigma_\gamma c \chi_0 \xi_{B-1} \Gamma \gamma_\tau u_{\gamma_0} / 3 m_e c^2}
= 3 \times 10^4 \chi_{0.5} \xi_{B-1}^{1/4} P_{104}^{1/4} J_{44}^{-1/2} f_{\gamma_0}^{-1/2}.
\tag{19}
\]

The solution of these coupled equations depend on the equation of state and the assumptions about \( \gamma_2 \). Approximate analytic solutions can be obtained in the case \( q \leq 2 \) for which \( \chi \approx \chi_0 \) is a good approximation. For a relativistically hot blob, we adopt the equation of state \( e = 3p \). We then obtain

\[
\frac{d}{dx} \ln (\Gamma/\chi) = -\alpha (\gamma_2/\gamma_0) (\Gamma/\Gamma_0) (f_\gamma/f_{\gamma_0}).
\tag{20}
\]

As a first example, let us take the maximum electron energy to be constant during the deceleration phase, that is, \( \gamma_2 = \gamma_0 \).
\[ \frac{d\gamma}{dt} = \frac{-\gamma}{\Gamma_\ell} = \frac{-4\sigma_T u_\ell}{3mc^2} \gamma_T^2. \]  

To compute the shock structure one needs to solve Equations (11)–(12) coupled to Equation (23). Such treatment is beyond the scope of this paper. To illustrate the effect of radiative drag on the shock we invoke the uniform shell approximation, that is, keep only the Lagrangian derivatives in Equations (11)–(12). Then, adopting \( \langle \gamma^2 \rangle \approx m_e c^2 n_{\ell}^{\prime}/w = 4\gamma_T \) and \( w = 4\rho \) we obtain the rate of change of the bulk Lorentz factor:

\[ \frac{d}{dt} \ln(\Gamma/x) = \frac{-16\sigma_T u_\ell}{9m_e c} \Gamma \gamma_T. \]  

With \( u_\ell = u_{\ell 0} f_\ell(x)/f_{\ell 0} \), the solution of the coupled Equations (23) and (24) reads:

\[ \Gamma(x) = \frac{\Gamma_0}{\left[1 + \alpha T g_T(x)\right]^{4/3}}, \]  

here \( g_T(x) = \int_1^x x^{7/4} f_\ell(x)/f_{\ell 0} \, dx' \) and

\[ \alpha_T = \frac{28\sigma_T \Gamma_0 \gamma_T u_{\ell 0}}{9m_e c^2} \approx 4\Gamma_0 \alpha_{\ell 0} L_{44}^{1/3} r_{10}^{-1/3}. \]

Since \( \Gamma_0 \alpha_{\ell 0} \approx 1 \) significant deceleration is expected on sub-parsec scales. For instance, assuming a flat intensity profile below the radius \( r = 10^{18} \text{ cm} \), with \( u_{\ell 0} = 10^{-3} \text{ erg cm}^{-3} \) at \( r < n_\ell \), we obtain \( \alpha_{\ell 0} = 0.2\Gamma_0 \alpha_{\ell 0} L_{44}^{1/3} \). Thus, colliding shells having a Lorentz factor \( \Gamma_0 > 10 \) will experience substantial deceleration. Now, the deceleration of the downstream plasma should lead to a gradual strengthening of the reverse shock and a weakening of the forward shock. Our preliminary calculations indicate substantial over-compression of the reverse shock already at \( \alpha_T \sim a \). In the frame of the central engine this translates to a deceleration of the entire shocked shell, as described qualitatively by the simple blob model outlined in the preceding section. The strengthening of the reverse shock should lead to enhanced dissipation rate, whereby the bulk energy of the unshocked shell is ultimately radiated away with high efficiency. Consequently, as long as the beaming cone of emission is narrower than the angular extent of the shell, the total flux observed will remain roughly constant. Thus, the delayed-deceleration model is relevant for the evolution of the total flux. However, the change in the Doppler factor resulting from the deceleration of the shocked shell leads to a change in
the observed energy of scattered photons and this, in turn, can significantly alter the evolution of the SED. In particular, we anticipate different durations and times of peak emission of flares observed in different energy bands. The non-uniformity of the emitting plasma downstream of the shock adds complexity.

A comprehensive analysis of emission from internal shocks in blazars is given in Joshi & Bottcher (2011) and Joshi et al. (2014) ignoring the effect of Compton drag on the dynamics of the shock. The decay of the emission in their model is due to cooling of the emitting electrons following shock crossing. This situation is well represented by the delayed deceleration model we adopt below. The neglect of Compton drag is justified only at radii where $\alpha_T < 1$ in Equation (26). As explained above, the inclusion of Compton drag and non-uniform particle acceleration should have a profound effect on the light curves. A complete treatment of the dynamics and emission of internal shocks that are subject to a strong Compton drag will be presented in a future publication.

2.2. Inverse Compton Emission

As will be shown below, the shape of the light curves reflects the geometry and dynamics of the emitting material. In particular, time retardation associated with the curvature of the emitting surface sets a limit on the rise and decay times of the flare. Furthermore, as mentioned above certain situations can be described by delayed deceleration of the blob, that can have an important effect. To illustrate such effects, we shall consider also cases in which the Lorentz factor remains constant at its initial value $\Gamma_0$ up to some radius $r_d < r_{dec} < r_d + \Delta/(1-\beta_T)$, and only then deceleration commences. In reality, the structure of the emitting zone is expected to be non-uniform in those cases, depending on the specific model. Those details may affect the resulting emission. Our purpose here is merely to illustrate how dynamical effects are imprinted in the light curves. For this purpose our simple treatment of delayed deceleration is sufficient. Moreover, this prescription also describes situations in which the radiative drag is too small to affect the dynamics of the shell, and the decay of the emission at the end of the dissipation phase is due to cooling of the emitting electrons.

Since we are mainly interested here in the high-energy emission from the blob, we consider only the contribution of Inverse Compton scattering. The spectral evolution measured by a distant observer is computed as follows: for a given choice of parameters we first solve Equations (17)–(18) numerically to obtain the Lorentz factor profile $\Gamma(r)$ in the frame of the black hole. Adopting the beam approximation, the intensity of the background radiation at any radius $r$ is then transformed into the rest frame of blob using $\Gamma(r)$ in Equation (8). The comoving approximation thereby obtained is used to compute the comoving emissivity of the scattered radiation, $j_{sc}(\epsilon', \mu', r)$. The emissivity in the black hole frame is given by $j_{sc}(\epsilon', \mu', r) = \int_0^{\infty} j_{sc}(\epsilon', \mu', r) dy$ where $\mu = (\mu' + \beta_\gamma)/(1 + \beta_\gamma \mu')$ and $\epsilon = (1 + \beta_\gamma \mu') \epsilon'$. The scattered intensity emitted by the blob is obtained upon integrating the emissivity across the blob, taking into account the time delay between emission of photons from different locations:

$$I_{sc}(\epsilon, \mu, r) = \int_0^{\Delta r} j_{sc}(\epsilon, \mu, r-y) dy$$  \hspace{1cm} (27)$$

where the distance $\Delta r$ is related to the blob’s length $\Delta$ through

$$\Delta = \int_{r-r_{\Delta}}^{r} [1 - \beta_T(r') \mu] dr'$$  \hspace{1cm} (28)$$

and is a function of $r$. In deriving Equation (27) we assumed that the emissivity is uniform inside the blob and vanishes outside it. Note that

$$t_{ob}(r) = \int_0^r [1 - \beta_T(r') \mu] dr'/c$$  \hspace{1cm} (29)$$

is the time measured by a distant observer viewing the blob at an angle $\theta = \arccos(\mu)$ relative to its direction of motion, so that formally $\Delta/c = t_{ob}(r) - t_{ob}(r-\Delta r)$.

High energy photons emitted by the blob will be attenuated by pair production on background photons. Here we model this attenuation by an exponential cutoff at the corresponding pair production optical depth $\tau_{\gamma\gamma}(\epsilon, r)$. The latter is computed using Equations (3.1)–(3.3) in Blandford & Levinson (1995), and plotted in Figure 2. The spectral flux measured by a distant observer viewing the blob at time $t_{obs}$ at an angle $\theta$ satisfies

$$F_{\infty}(\epsilon, \mu, t_{ob}) \propto \int_0^{t_{ob}} dy \int_{\Sigma(r-y)} j_{sc}(\epsilon, \mu, r-y) \times e^{-\tau_{\gamma\gamma}(\epsilon, r-y)} d\Sigma'$$  \hspace{1cm} (30)$$

where the radius $r$ is computed at the observed time $t_{ob}$ using Equation (29), that is $r = r(t_{ob})$, and the inner integration is over the emitting surface of the blob at the retarded time $(r-y)/c$.

In the immediate deceleration case, the rise and decay times of the observed flux are dominated by temporal delays associated with the curvature of the emitting surface. For a conically expanding shell the emitting surface is spherical, and Equation (30) reduces to

$$F_{\infty}(\epsilon, \mu, t_{ob}) \propto \int_0^{t_{ob}} dy \int_{\mu_1(r-y)} j_{sc}(\epsilon, \mu, r-y) \times (r-y)^2 e^{-\tau_{\gamma\gamma}(\epsilon, r-y)} \mu^{-3} d\mu,$$  \hspace{1cm} (31)$$

\hspace{1cm} Figure 2. Pair-production optical depth as a function of photon energy.
Figure 3. Spectral energy distribution of ERC radiation emitted from a decelerating blob following onset of dissipation, for a non-delayed deceleration (left panel) and delayed deceleration with $r_{\text{dec}} - r_t = \Delta/(1 - \beta_t)$ (right panel).

Figure 4. Sample light curves for non-delayed deceleration (upper panels) and delayed deceleration with $r_{\text{dec}} - r_t = \Delta/(1 - \beta_t)$ (lower panels). The left panel in each case exhibits the (normalized) flux at a photon energy of 1 GeV measured by a distant observer as a function of observer time, and the right panel the corresponding emissivity as a function of radius $r = ct$. The two curves in the upper left panel correspond to different jet opening angles, as indicated.

with

$$\mu_2(r, y) = \max \left[ \cos \theta_j, \frac{1}{1 - y} \frac{1}{r - y} \int_{r-y}^{r} \beta_{T}^{-1}(y') - 1 \, dy' \right],$$

$$\mu_1(r, y) = \min \left[ 1, 1 - \frac{(1 - \mu_2)(r - y) - \Delta}{r - y - \Delta} \right],$$

where $\theta_j$ is the opening angle of the flow, and $y_0$ is determined from the condition $\mu_1(r, y_0) = \cos \theta_j$.

The evolution of the SED is shown in Figure 3 in the case of immediate deceleration (left panel) and delayed deceleration with $r_{\text{dec}} - r_t = \Delta/(1 - \beta_t)$ (right panel). The corresponding light curves are displayed in Figure 4 at a photon energy $\epsilon = 1$ GeV. The right pannels exhibit the emissivity, and the left pannels the observed flux computed for these emissivities using Equation (31). The effect of the jet opening angle on the shape of the lughtcurve is more prominent in the non-delayed case, as seen in the upper left panel of Figure 4; in the delayed case this
effect is essentially negligible. These light curves are rather
typical in the regime where the deceleration time of the blob,
t_\text{dec}/(c\Delta), is not much larger than its light crossing time
\Delta/(1 - \beta_1). In this regime, the duration of the flare measured
by a distant observer viewing the source at an angle smaller
than the opening angle of the flow and the overall shape of the
light curve depend on details, as seen in Figure 4.

In the case of immediate deceleration the light curve is
asymmetrical, with the rise and decay times determined by the
curvature of the emitting surface. For a sufficiently large
Lorentz factor, such that the beaming angle is smaller than the
opening angle of the flow (i.e., \Gamma_0 \theta_j > 1), these times, as
measured by a distant observer, satisfy \( t_\text{rise} \sim t_\text{decay} \sim r_j/\Gamma_0^2 \).

Those times might be comparable to the light crossing time of
the blob if dissipation commences at a radius \( r_d \sim \Gamma_0^2 \Delta \), as in
the case shown in Figures 4 and 5. In the case of delayed
deceleration the rise and decay times are determined by the
delay: \( t_\text{rise} \sim t_\text{decay} \sim (t_\text{dec} - r_d)(1 - \beta_1) \). The light curve
tends to be more symmetric in this case (see Figure 5 for a
comparison). The change in the emissivity during the coasting
(pre-deceleration) phase, as seen in the lower right panel in
Figure 4, is due to the dependence of the intensity of target
photons and the density of emitting electrons on \( r \sim j_\text{c} \propto r^{-4} \)
for the conical flow considered in the above examples, with
intensity profile \( j_\text{c} \propto x^{-2} \) in Equation (4). This dependence
may slightly affect the shape of the light curve.

The shape of the light curve displayed in the lower left panel of
Figure 4 is in qualitative agreement with those computed in
Joshi et al. (2014), and is quite typical to observed gamma-ray
flares. The rough symmetry of the light curve is due to the
assumed uniformity of the emissivity. In cases where the
acceleration of the electrons is confined to a small region inside
the shell, e.g., acceleration in shock fronts, we anticipate a
faster rise and a slower decay, particularly at very high
energies, at which the cooling time is much shorter than the
light crossing time of the shell.

3. DISCUSSION

We considered the dynamics of a dissipative shell in the
presence of a strong radiative drag. Our analysis indicates that
for rapid dissipation substantial deceleration is anticipated in
blazars on sub-parsec scales (and even parsec scales for
sufficiently luminous sources), that should give rise to rapid,
large amplitude variability owing to changes in the beaming
factor. It is worth noting that even modest changes in the
Lorentz factor can lead to large amplitude variations of the
emitted flux, owing to its sensitive dependence on the Doppler
factor. In principle, this mechanism can produce flares with
durations as short as the dynamical time of the central engine,
at a very high efficiency. The radiative drag exerted on the
thermal electrons alone should lead to rapid deceleration at
radii \( r < 4 \times 10^{17} \Gamma_0 L_\Delta \) cm, where \( \Gamma_0 \) is the bulk Lorentz
factor at the onset of dissipation and \( L_\Delta = 10^{44} L_{\Delta,44} \) erg s\(^{-1}\) is
the luminosity of the ambient radiation field intercepted by the
flow, provided the dissipation is rapid enough to keep the
specific enthalpy large (that is, \( h \gg 1 \)). This is, for instance,
the situation in internal shocks, as shown in Section 2.1.2, but
may also occur in other cases, e.g., effective magnetic field
dissipation in Poynting flux dominated jets (Sikora et al. 1996).

Scattering of ambient photons by non-thermal electrons
accelerated in situ will dominate the radiative force if the
proper scale over which electrons are accelerated exceeds
t_\text{c}/\xi_c, where \( t_\text{c} \) is the cooling time of the non-thermal
electrons, as measured in the rest frame of the flow, and \( \xi_c \)
is the fraction of total energy carried by the non-thermal
population.

In general, the effect of radiative drag is expected to be more
prominent in FSRQs than in BL Lacs. Nonetheless, Equation (19)
indicates that deceleration of dissipative shells may also be relevant to low luminosity sources, provided that electrons can be effectively accelerated to the cooling cutoff.

This seems to be case in TeV blazars. It has been shown
elsewhere (Levinson 2007) that if the TeV spectrum emitted
during strong flares extends to energies at which the pair
production opacity exceeds unity deceleration should be
effective.

It is naively anticipated that the gamma-ray emission
produced through IC scattering of ambient photons by the
nonthermal electrons accelerated in the blob will be correlated
with lower energy emission generated by synchrotron cooling
of the same electrons. However, synchrotron self-absorption
may give rise to a strong suppression of the radio emission in
cases where the deceleration occurs well below the radio core.

In that case, the gamma-ray flare will either precede the
ejection of a superluminal component, or not be accompanied
by one at all, depending on the asymptotic bulk Lorentz factor
and the synchrotron cooling time. On the other hand, the onset
of dissipation depends on the duty cycle (the time interval
between ejections of consecutive shells in the case of internal
shocks), and is expected to occur over a range of scales, even
in an individual object. Blobs that dissipate their energy at large
enough radii will not experience strong deceleration. As a
consequence, a variety in the behavior of flares is expected, as
indeed revealed by recent multi-waveband studies (e.g.,
Marscher et al. 2011).

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APPENDIX A
DERIVATION OF THE SOURCE TERMS

The general expression for the source term associated with
Compton drag is (Phinney 1982; Sikora et al. 1996; Van Putten
Now, in the black hole frame the radiation electron distribution $f_e$ is isotropic, and it is convenient to compute the source terms there, and then to transform. We have

\[
\frac{p'_e k'^e}{p^0} = -\frac{p^0 k^0}{(1 - \beta \mu)} + \frac{p' \cdot k'}{m_e}
\]

and

\[
\frac{p'_e k'^e}{p^0} = -\frac{p^0 k^0}{(1 - \beta \mu)} + \frac{p' \cdot k'}{m_e}
\]

where $\mu$ is the cosine of the angle between the photon and electron directions, and $p^0/m_e = \gamma, p^1/m_e = \gamma \beta$. The zeroth component reads:

\[
S_{c0} = -\sigma T \int \frac{d^3 p'}{p^0} \int \frac{d^3 k'}{k^0} f_{e c} f_{e} \frac{p'_e k'^e}{p^0} \left[ k^{00} + \frac{(p'_e k'^e) p^0}{m_e^2} \right]
\]

(36)

Now, $n'_e = \int dp f_{e} = 2 \int dp f_{e}^2$, and using $\int dp (1 - \beta \mu)^2 = 2(1 + \beta^2/3)$ one has

\[
S_{c0} = -\frac{4}{3} \sigma T \int \frac{d^3 k'}{k^0} f_{e c} \int d^3 p f_{e} \gamma^2 \beta^2
\]

\[
= -\frac{4}{3} \sigma T n'_e < \gamma^2 \beta^2 > u_e'.
\]

(39)

The spatial component reads:

\[
S_{c}^{i} = -\sigma T \int \frac{d^3 p'}{p^0} \int \frac{d^3 k'}{k^0} f_{e c} f_{e} \frac{p'_e k'^e}{p^0} \left[ k^{ii} + \frac{(p'_e k'^e) p^0}{m_e^2} \right]
\]

(40)

\[
= \sigma T \int \frac{d^3 k'}{k^0} f_{e c} \int d^3 p f_{e} \left[ (1 - \beta \mu) - \gamma^2 \beta (1 - \beta \mu)^2 \right]
\]

\[
= \sigma T n'_e < \gamma^2 \beta^2 > u_e'.
\]

(41)

where

\[
T^{i\mu} = \int \frac{d^3 k'}{k^0} k'^{i\mu} f_{e c} f_{e}.
\]

(43)

Now, in the black hole frame the radiation field is isotropic and we have $T^{00} = u_T, T^{ij} = (u_T/\beta) \delta_{ij}$, and

\[
T^{00} = \Lambda_0 n'_e T^{00} = -\frac{4}{3} \Gamma^2 \beta u_T.
\]

(44)

Thus, we finally have for the source terms in the comoving frame:

\[
S_{c}^{0} = -\frac{4}{3} \sigma T n'_e < \gamma^2 \beta^2 > u_T \Gamma^2 (1 + \frac{\beta^2}{3}).
\]

(45)

To obtain the source term in the Lab frame we perform a Lorentz transformation:

\[
S_{c}^{0} = \Gamma S_{c}^{0} + \Gamma \beta_\gamma \beta T S_{c}^{\gamma} = -\frac{4}{3} \sigma T n'_e \Gamma^3 u_T
\]

\[
\times \left[ < \gamma^2 \beta^2 > \left( 1 + \beta^2 \right) + \beta^2 \right].
\]

(46)

\[
S_{c}^{i} = \Gamma \beta_T S_{c}^{0} + \Gamma S_{c}^{\gamma} = -\frac{4}{3} \sigma T n'_e \Gamma^3 \beta_T u_T
\]

\[
\times \left[ 1 + (5 + \beta^2) < \gamma^2 \beta^2 > \right].
\]

(47)

For cold electrons $\langle \gamma^2 \beta^2 \rangle = 0$, and $S_{c}^{0} = 0$, as required, since in the Thomson limit invoked here the scattering is fully elastic and there should be no energy loss in the rest frame of the blob if the electrons are cold. In the Lab frame we then recover the result $S_{c}^{0} = -\frac{4}{3} \sigma T n'_e \Gamma^3 \beta_T u_T$.

APPENDIX B

FLOW EQUATIONS

In spherical coordinates, the radial expansion of a hydrodynamic shell is governed by the equations

\[
\partial_t (w G^2 - p) + \frac{1}{r^2} \partial_r (r^2 w G^2 \beta_T) = S_{c}^{0},
\]

(49)

\[
\partial_t (w G^2 \beta_T) + \frac{1}{r} \partial_r (r^2 w G^2 \beta_T^2) + \partial_r p = S_{c}^{i},
\]

(50)

where $S_{c}^{0}$ and $S_{c}^{i}$ are source terms that account, respectively, for energy and momentum losses by the radiative drag. The above equations can be combined to yield

\[
w T^2 \frac{d \beta_T}{dt} + \beta_T \partial_r p + \partial_r p = S_{c}^{0} - \beta_T S_{c}^{i},
\]

(51)

in terms of the Lagrangian derivative $d/\rho = \partial_t + \beta_T \partial_r$. We suppose that at time $t$ the dissipative shell is contained between the radii $n(t)$ and $r(t) = n(t) + \Delta(t)$. For simplicity we suppose that the length of the shell $\Delta$ is constant in the black hole frame. The velocity of the shell is then uniform with $\beta_T(t) = \Delta/dt$. Transforming to the coordinates $\tau(r, t) = r - t$, $\zeta(r, t) = r(t) - r$, we have

\[
\partial_t = -\partial_\zeta,
\]

(52)

\[
\partial_t = \partial_r + \beta_T \partial_\zeta.
\]

(53)

\[
\frac{d}{dt} = \partial_t + \beta_T \partial_r = \partial_r.
\]

(54)

Using the relation $\beta_T d\beta_T = \Gamma^{-3} d\Gamma$, Equation (51) gives

\[
w T^2 \frac{d \beta_T}{dt} \ln \Gamma + \frac{dp}{dt} - \Gamma^{-2} \partial_\zeta p = S_{c}^{0} - \beta_T S_{c}^{i},
\]

(55)

to order $O(\Gamma^{-2})$. Equation (49) yields

\[
\frac{d}{dt} (w G^2 - p) + \frac{2}{r} w T^2 \beta_T - \partial_\zeta p - w \partial_\zeta \ln \Gamma = S_{c}^{0}.
\]

(56)
To order $O(\Delta/r_2)$ we have $2\beta_1/r = (2/r_2)dr_2/dt = r_2^{-2}$
$dr_2^2/dt$, and the latter equation reduces to
$$\frac{1}{r_2^2} \frac{d}{dt} \left( r_2^2 \omega^2 \right) - \frac{d}{dt} \omega - \partial_\xi \omega_{0} = S_0.$$

(57)

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