Measurements of $\phi_2$ and $\phi_3$ at Belle

Atsuko Kibayashi for the Belle Collaboration
High Energy Accelerator Research Organization (KEK), Tsukuba, Japan
E-mail: atsuko@post.kek.jp

Abstract. We report recent measurements of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix angles $\phi_2$ and $\phi_3$ based on a large data sample of $B\bar{B}$ pairs collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider operating at the $\Upsilon(4S)$ resonance. We employ the time-dependent $CP$ violation in $B \to \pi\pi, \rho\pi$ and $\rho\rho$ decays to determine $\phi_2$, and $CP$ violation in the interferences between $b \to c$ and $b \to u$ transitions to extract $\phi_3$.

1. Introduction
In the standard model (SM), $CP$ violation is attributed to an irreducible complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) weak interaction quark-mixing matrix [1]. The unitarity of the CKM matrix implies the relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ in the complex plane, and measurements of the angles of the triangle, $\phi_2 \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$ and $\phi_3 \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$, give crucial tests of the CKM unitarity.

We present recent measurements of the angles $\phi_2$ and $\phi_3$ based on a large data sample of $B\bar{B}$ pairs collected with the Belle detector [2] at the KEKB $e^+e^-$ asymmetric-energy (3.5 on 8 GeV) collider [3] operating at the $\Upsilon(4S)$ resonance.

2. $\phi_2$ Measurements
The CKM angle $\phi_2$ is determined from the time-dependent $CP$ asymmetry in $B^0$ decays into $\pi^+\pi^-$, $(\rho\pi)^0$, $\rho^+\rho^-$ and $\omega^+\omega^-$ [4].

In the $\Upsilon(4S) \to B^0\bar{B}^0$ decay chain, one $B^0$ decays into a $CP$ eigenstate $f_{CP}$ at time $t_{CP}$, while the other decays into a flavor specific final state $f_{tag}$ at time $t_{tag}$. The time-dependent decay rate is [5]

$$P^q(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}[1 + q\{S_{f_{CP}}\sin(\Delta m_d\Delta t) + A_{f_{CP}}\cos(\Delta m_d\Delta t)]}],$$

(1)

where $\Delta t = t_{CP} - t_{tag}$, $\tau_{B^0}$ is the $B^0$ lifetime, $\Delta m_d$ is the mass difference between the two $B$ mass eigenstates, and $q = +1(-1)$ when $f_{tag} = B^0(\bar{B}^0)$. $S_{f_{CP}}$ and $A_{f_{CP}}$ are the mixing-induced and direct $CP$-violating parameters, respectively. For $f_{CP} = \pi^+\pi^-$ or $\rho^+\rho^-$, $S_{f_{CP}} = \sqrt{1 - A_{f_{CP}}^2}\sin(2\phi_2 + \kappa)$, where $\kappa$ is a decay mode dependent extra phase caused by the $b \to d$ “penguin” pollution and can be determined using isospin relations [6].

We measure the $CP$-violating parameters in $B^0 \to \pi^+\pi^-$ decays using 1464±65 signal events extracted from a data sample of $535 \times 10^6 B\bar{B}$ pairs: $A_{\pi\pi} = +0.55 \pm 0.08$(stat) $\pm 0.05$(syst) and
\[ S_{\pi\pi} = -0.61 \pm 0.10 \text{(stat)} \pm 0.04 \text{(syst)}. \] We observe large direct \( CP \) violation with the significance of 5.5 standard deviations (\( \sigma \)) and measure \( \phi_2 = (97 \pm 11)^\circ \) [7].

We measure the \( CP \)-violating parameters in \( B^0 \rightarrow \rho^+\rho^- \) decays: \( A = +0.16 \pm 0.21 \text{(stat)} \pm 0.07 \text{(syst)} \) and \( S = +0.19 \pm 0.30 \text{(stat)} \pm 0.07 \text{(syst)} \) using the same data sample. We find two “flat-top” \( \phi_2 \) solutions; the solution consistent with the SM is \( 61^\circ < \phi_2 < 107^\circ \) at the 68% confidence level [8].

Another technique to extract \( \phi_2 \) is to use a time-dependent Dalitz plot analysis of the decay \( B^0 \rightarrow (\rho\pi)^0 \rightarrow \pi^+\pi^-\pi^0 \). This method provides \( \phi_2 \) without discrete ambiguities [9]. We perform the full Dalitz and isospin analysis, and obtain a constraint on the \( \phi_2 \), \( 68^\circ < \phi_2 < 95^\circ \), at the 68.3% confidence interval for the solution consistent with the SM using a data sample of \( 449 \times 10^6 \) \( B\bar{B} \) pairs [10].

The \( B^0 \rightarrow a_1^+\pi^- \) decay is also sensitive to \( \phi_2 \). A method is proposed to extract the angle from this decay [11]. We measure the branching fraction with a data sample of \( 353 \times 10^6 \) \( B\bar{B} \) pairs, \( \mathcal{B}(B^0 \rightarrow a_1^+\pi^-) \times \mathcal{B}(a_1^+ \rightarrow \pi^+\pi^-\pi^-) = (14.9 \pm 1.6 \pm 2.3) \times 10^{-6} \), where the first and second errors are statistical and systematic, respectively [12].

3. \( \phi_3 \) Measurements

We make use of the interferences between \( b \rightarrow c\bar{u}s \) and \( b \rightarrow u\bar{c}s \) transitions in the decay of \( B^- \rightarrow DK^+ \) to extract the CKM angle \( \phi_3 \).

Gronau, London and Wyler (GLW) proposed to use \( D \) decays into \( CP \) eigenstates: \( D_1 = K^+K^- \), \( \pi^+\pi^- \) (\( CP \)-even) and \( D_2 = K_S^0\pi^0 \), \( K_S^0\omega \), \( K_S^0\phi \) (\( CP \)-odd) [13]. We measure

\[
A_{1,2} = \frac{BR(B^- \rightarrow D_{1,2}K^-) - BR(B^+ \rightarrow D_{1,2}K^+)}{BR(B^- \rightarrow D_{1,2}K^-) + BR(B^+ \rightarrow D_{1,2}K^+)}
\]

\[
= \frac{2r_B \cos \delta \sin \phi_3}{1 + r_B^2 + 2r_B \cos \delta' \cos \phi_3},
\]

where \( \delta' = \delta_B (\delta_B + \pi) \) for \( D_1 \) (\( D_2 \)), and \( \delta_B \) is the strong phase difference. Using a data sample of \( 275 \times 10^6 \) \( B\bar{B} \) pairs, we obtain \( A_1 = 0.06 \pm 0.14 \text{(stat)} \pm 0.05 \text{(syst)} \) and \( A_2 = -0.12 \pm 0.14 \text{(stat)} \pm 0.05 \text{(syst)} \) [14]. These measurements can be used to constrain \( \phi_3 \) through a global fit [15].

Atwood, Dunietz and Soni (ADS) [16] pointed out that \( CP \) violation effects are enhanced if the final state is chosen such that the interfering amplitudes are comparable. In the decay of \( B^- \rightarrow D(\rightarrow K^+\pi^-)K^- \), two decay transitions are involved: the color-allowed \( B \) decay followed by the doubly Cabibbo-suppressed \( D \) decay and color-suppressed \( B \) decay followed by the Cabibbo-allowed \( D \) decay. The interference term between the two amplitudes is sensitive to \( \phi_3 \). We calculate a ratio of branching fractions

\[
R_{DK} = \frac{BR(B^- \rightarrow D(\rightarrow K^+\pi^-)K^-)}{BR(B^- \rightarrow D(\rightarrow K^-\pi^-)K^-)} = r_B^2 + r_D^2 + r_Br_D \cos \phi_3 \cos \delta,
\]

where \( r_B = |A(D^0 \rightarrow K^+\pi^-)/A(D^0 \rightarrow K^-\pi^+)| = 0.060 \pm 0.003 \), \( \delta = \delta_B + \delta_D \), and \( \delta_D \) is the strong phase difference between the two \( D \) decay amplitudes. With a data sample of \( 386 \times 10^6 \) \( B\bar{B} \) pairs no significant signal for \( B^- \rightarrow D(\rightarrow K^+\pi^-)K^- \) is found; we measure \( R_{DK} = (0.0^{+8.4}_{-7.9} \pm 1.0) \times 10^{-3} \), where the first (second) error is statistical (systematic), and we set a limit \( r_B < 0.18 \) at the 90% confidence level [17].

One of the promising modes for the extraction of \( \phi_3 \) is the three body states such as \( D^0 \) and \( \bar{D}^0 \) to \( K_S^0\pi^+\pi^- \). The decays of \( B^- \rightarrow D^0K^- \) and \( \bar{D}^0 \) \( K^- \) have the same final state so that the two amplitudes interfere [18]. Assuming no \( CP \) violation in neutral \( D \) decays, the \( B^+ \) and \( B^- \)
The combination of factorization and SU(3) symmetry assumptions, and lattice QCD calculations. We obtain the partially reconstructed decay amplitudes are

\[ M(B^+) = f(m_{B^+}^2, m_{\pi}^2) + r_B e^{i\phi_3 + i\delta} f(m_{B^+}^2, m_{\pi}^2), \]

\[ M(B^-) = f(m_{B^-}^2, m_{\pi}^2) + r_B e^{-i\phi_3 + i\delta} f(m_{B^-}^2, m_{\pi}^2), \]

where \( m_{B^+}^2 \) (\( m_{B^-}^2 \)) is the invariant mass of \( K^0 \pi^+ (K^0 \pi^-) \) decay, obtained using a huge number of \( e^+ e^- \rightarrow \overline{c}\bar{c} \) continuum events, \( r_B = |A(B^- \rightarrow D^0 K^-)/A(B^- \rightarrow D^0 K^0)| \), and \( \delta \) is the strong phase difference between them. The value \( r_B \) is given by the ratio \( |V_{ub} V_{cs}/V_{ub} V_{us}| \sim 0.38 \) and the color suppression factor, and is estimated to be in the range 0.1 \( \sim 0.2 \). Fitting the Dalitz plot distributions in the decays of \( B^- \rightarrow D^{(*)} K^{(*)-} \) simultaneously, the parameters \( \phi_3 \), \( r_B \) and \( \delta \) are obtained. From the combination of \( B^- \rightarrow D K^- \), \( B^- \rightarrow D^* K^- \) with \( D^* \rightarrow D \pi^0 \) and \( B^- \rightarrow DK^{*-} \) with \( K^{*-} \rightarrow K^0 \pi^- \) modes, we obtain \( \phi_3 = 53^\circ \pm 15^\circ \) (stat) \( \pm 3^\circ \) (syst) \( \pm 9^\circ \) (model) in a data sample of \( 386 \times 10^6 B\overline{B} \) pairs [19].

A theoretically clean technique to extract \( \sin(2\phi_1 + \phi_3) \) is to measure the time-dependent decay rate of \( B^0 \rightarrow D^{(*)+} \pi^\pm \) [20], which can be mediated by both Cabibbo-favored decay (CFD) and doubly-Cabibbo-suppressed decay (DCSD) amplitudes, \( V_{ub}^* V_{ud} \) and \( V_{ub}^* V_{us} \), having a relative weak phase \( \phi_3 \). The mixing-induced CP-violating parameter is \( S^+ = 2(1-L)R \sin(2\phi_1 + \phi_3 \pm \delta)/(1+R^2) \), where \( L = 0 \) (1) for the \( D \pi \) (\( D^* \pi \)) decay, \( R \sim 0.02 \) is the ratio of magnitude of DCSD to CFD, \( \delta \) is their strong phase difference, and \( S^+ \) (\( S^- \)) measures the CP-violating parameter in \( B^0 \) decays into \( D^{(*)+} \pi^- \) (\( D^{(*)-} \pi^+ \)). We find an indication of CP violation in \( B^0 \rightarrow D^- \pi^+ \) and \( B^0 \rightarrow D^\star+ \pi^- \) decays at 2.2\( \sigma \) and 2.5\( \sigma \) levels, using fully reconstructed \( D^{(*)+} \) events and partially reconstructed \( D^\star \) events from a data sample of \( 386 \times 10^6 B\overline{B} \) pairs, respectively [21]. To constrain \( \phi_3 \), we need to use the measured branching fractions of \( B \rightarrow D^{(*)+} \pi \), a combination of factorization and SU(3) symmetry assumptions, and lattice QCD calculations. We obtain the lower limit on \( \sin(2\phi_1 + \phi_3) \) of 0.52 (0.44) for \( D \pi \) (\( D^* \pi \)) modes at the 68\% confidence level.

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[2] A. Abashian et al. (Belle Collaboration), Nucl. Instr. and Meth. A 479, 117 (2002); A. Kibayashi, Nucl. Instr. and Meth. A 569, 1 (2006).
[3] S. Kurokawa and E. Kitakami, Nucl. Instr. and Meth. A 499, 1 (2003), and other papers included in this volume.
[4] Throughout this paper, the inclusion of the charge conjugate mode decay is implied unless otherwise stated.
[5] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. 45, 952 (1980); A. B. Carter and A. I. Sanda, Phys. Rev. D 23, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. 193, 85 (1981); M. Gronau, Phys. Rev. Lett. 63, 1451 (1989).
[6] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).
[7] H. Ishino et al. (Belle Collaboration), Phys. Rev. Lett. 98, 211801 (2007).
[8] A. Somov, A.J. Schwartz et al. (Belle Collaboration), Phys. Rev. D 76, 011104(R) (2007).
[9] A.E. Snyder and H.R. Quinn, Phys. Rev. D 48, 2139 (1993).
[10] A. Kusaka, C.C. Wang, H. Ishino et al. (Belle Collaboration), Phys. Rev. Lett. 98, 221602 (2007).
[11] M. Gronau and J. Zupan, Phys. Rev. D 73, 057502 (2006).
[12] K. Abe et al. (Belle Collaboration), arXiv:0706.3279v3 [hep-ex]
[13] M. Gronau and D. London, Phys. Lett. B 253, 483 (1991); M. Gronau and D. Wyler, Phys. Lett. B 265, 172 (1991).
[14] K. Abe et al. (Belle Collaboration), Phys. Rev. D 73, 051106(R) (2006).
[15] J. Charles et al. (CKMfitter group), Eur. Phys. J. C 41, 1 (2005); M. Bona et al. (UTfit group), JHEP 0507, 028 (2005).
[16] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3257 (1997); Phys. Rev. D 63, 036005 (2001).
[17] K. Abe et al. (Belle Collaboration), arXiv:hep-ex/0508048.
[18] A. Girz, Y. Grossman, A. Soffer, J. Zupan, Phys. Rev. D 68, 054018 (2003).
[19] A. Poluektov et al. (Belle Collaboration), Phys. Rev. D 73, 112009 (2006).
[20] I. Dunietz and R.G. Sachs, Phys. Rev. D 37, 3186 (1988); 39, 3515(E) (1989); I. Dunietz, Phys. Lett. B 427, 179 (1998).
[21] F.J. Ronga, T.R. Sarangi et al. (Belle Collaboration), Phys. Rev. D 73, 092003 (2006).