Quantum entanglement in plasmonic waveguides with near-zero mode indices

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We investigate the quantum entanglement between two quantum dots in a plasmonic waveguide with near-zero mode index, considering the dependence of concurrence on interdot distance, quantum dot-waveguide frequency detuning and coupling strength ratio. High concurrence is achieved for a wide range of interdot distance due to the near-zero mode index, which largely relaxes the strict requirement of interdot distance in conventional dielectric waveguides or metal nanowires. The proposed quantum dot-waveguide system with near-zero phase variation along the waveguide near the mode cutoff frequency shows very promising potential in quantum optics and quantum information processing.

Highly entangled quantum states play important roles in quantum information science, such as schemes for quantum cryptography, quantum teleportation, and quantum computation [1–4]. Among different physical realizations, scalable solid-state quantum entanglement is the most promising one, and the recent successes of quantum entanglement have been obtained with quantum dots (QDs) or diamond nitrogen vacancy centers [5–8] in the visible frequency range. For long-distance entanglement, the correlation between two spatially separated qubits is usually mediated by photons. However, instead of photon, surface plasmon [9] has been attracted much attention since it reveals strong analogy to light propagation in conventional dielectric optical components. Plasmonic waveguides and resonators can be used to confine light to subwavelength dimensions below the diffraction limit for achieving photonic circuit miniaturization [10] and furthermore strongly interact with quantum emitters for the applications of detectors, transistors and quantum information processing [11–14]. For one-dimensional plasmonic waveguide, the scattering properties of surface plasmon interacting with QDs have been studied widely [15–20]. Recently, Chen et al. [21] and Gonzalez-Tudela et al. [22] have reported quantum entanglement generation between two separated QDs mediated by a plasmonic waveguide. Highly entangled state between QDs can be achieved only when the interdot distance is controlled with specific values, due to the sinusoidal phase variation of the propagating surface plasmon mode in the waveguide.

In this work, we examine plasmonic waveguides with near-zero mode indices, and investigate the quantum entanglement between two QDs simultaneously interacting with the waveguide mode. High concurrence of the entangled state can be obtained for wide ranges of interdot distance $d$ and QD-waveguide coupling strength ratio $g_2/g_1$, showing great advantage of relaxing the strict requirements of QDs positions in comparison to previous schemes. The physical mechanism of the interdot distance flexibility is the vanishing phase variation between two arbitrary positions along the plasmonic waveguide with near-zero mode index. With the pioneering experimental verification of $n = 0$ structures for visible light by Vesseure et al. [23], the proposed QD-waveguide platform is convinced to be a promising experimental platform for realizing highly entangled quantum states.

![Fig. 1](image.png)

**Fig. 1.** (Color online) (a) Schematic of the cross section of a SiO$_2$ waveguide with a thick silver cladding. $D$ is the diameter of SiO$_2$ core. 3 µm-long waveguide is used in the model. (b) Two two-level QDs separated by distance $d$ interacting with the waveguide mode. $\delta(\Delta)$ is the frequency detuning between QD$_1$ (QD$_2$) transition and the incident waveguide mode.

Fig. 1(a) shows a schematic of one SiO$_2$/Ag waveguide engineered to exhibit near-zero mode index in the visible frequency range. The diameter of the SiO$_2$ core is $D$, which is fully surrounded by a thick silver cladding. The permittivity of silver is described by the Drude model, with the plasmon frequency $\omega_p$ of $1.37 \times 10^{16}$ rad/s and the collision frequency $\gamma$ of $8.5 \times 10^{13}$ rad/s at room temperature. The refractive index of SiO$_2$ is 1.46. A pair of QDs, each of which has one excited state $|e\rangle$ and one ground state $|g\rangle$, are embedded in the SiO$_2$/Ag waveguide as illustrated in Fig. 1(b). $\delta(\Delta) = \omega_k - \omega_{1(2)}$ is the frequency detuning between the incident waveguide mode and the QD exciton transition. $d$ is the interdot distance between the two separated QDs.

The mode indices $n$ of SiO$_2$/Ag plasmonic waveguides are plotted in Fig. 2(a) where the loss of Ag is neglected. Near-zero mode index can be reached around the cut-off frequency of the waveguide mode, which can be designed by varying the waveguide diameter. The working wavelength with near-zero mode index can be controlled from visible 685 nm to near-infrared 920 nm when the wave-
uide diameter \( D \) changes from 100 nm to 150 nm. Regarding the experimental condition at cryogenic temperature for the interaction between visible QDs and waveguide mode, silver cladding layer with 0.1\( \gamma \) damping rate and waveguide of \( D = 110 \text{ nm} \) are considered in the following analysis. Fig. 2(b) shows the corresponding mode index \( n \) and group velocity \( v_g \), where the mode index gradually approaches to a vanishing small number and group velocity slows down to \( c/42 \) at the wavelength of 728.6 nm. Group velocity \( v_g \) is calculated according to \( v_g = c/(n + \omega (dn/d\omega)) \), where \( c \) is the speed of light in vacuum. Figs. 2(c) and 2(d) show electric field distributions at the wavelengths of 600 nm and 725 nm for the SiO\(_2\)/Ag waveguide calculated in Fig. 2(b). The corresponding mode indices at the wavelengths of 600 nm and 725 nm are 0.962 and 0.164, respectively. For the waveguide mode with near-zero index, light can propagate along the waveguide with a spatially uniform phase, near infinity phase velocity and slow group velocity.

\[ H = \int dx \left\{ -iv_g \dot{c}_R^\dagger(x) \frac{\partial}{\partial x} c_R(x) + iv_g \dot{c}_L^\dagger(x) \frac{\partial}{\partial x} c_L(x) \right\} + \sum_{j=1}^2 g_j \delta[x - (j - 1)d] [c_R^\dagger(x) \sigma_-^j + c_R(x) \sigma_+^j] + c_L^\dagger(x) \sigma_-^j + c_L(x) \sigma_+^j \right\} + \sum_{j=1}^2 (\omega_j - i\Gamma/2) \sigma_{e_j, e_j}, \]  

where \( c_R^\dagger(x) \) (\( c_R(x) \)) is the bosonic operator creating a right-going (left-going) surface plasmon at the position \( x \), and \( g_j \) (\( j = 1, 2 \)) is the coupling strength between individual QDs and the waveguide mode. Here, the dipole moments of the two QDs have the same orientation and both dipole moments are parallel to the polarization direction of the waveguide mode. \( \sigma_{e_j, e_j} = |e_j \rangle \langle e_j| \) represents the diagonal element of the \( j \)th QD operator and \( \sigma_{L} = |e_{j_l} \rangle \langle g_j| \) (\( \sigma_{R} = |g_j \rangle \langle e_j| \)) represents the raising (lowering) operator. \( \omega_j \) is the transition frequency of the \( j \)th QD and \( \Gamma \) is the total dissipation including the exciton decay to free space, ohmic loss and other dissipative channels. The eigenstate of the system can be written as

\[ |E_k\rangle = \int dx [\phi_{k,R}^\dagger(x) c_R^\dagger(x) \\
\phi_{k,L}^\dagger(x) c_L^\dagger(x)] |g_1, g_2 \rangle |0\rangle_{sp} + \xi_k |e_1, g_2 \rangle |0\rangle_{sp} + \xi_k |g_1, e_2 \rangle |0\rangle_{sp}, \]

where \( \xi_k \) is the probability amplitude that QD\(_1 \) absorbs the waveguide mode and jumps to its excited state. Supposing a surface plasmon incident from the left, the scattering amplitudes can be written as

\[ \phi_{k,R}^\dagger(x) = e^{ikx} [\theta(-x) + a\theta(x) \theta(d - x) + t\theta(x - d)] \] and

\[ \phi_{k,L}^\dagger(x) = e^{-ikx} [r\theta(-x) + b\theta(x) \theta(d - x)], \]

\( \theta(x) \) is the unit step function, which equals unity when \( x \geq 0 \) and zero when \( x < 0 \). \( a \) and \( b \) are the probability amplitudes of the field between the two QDs at \( x = 0 \) and \( x = d \). \( t \) and \( r \) are the transmission and reflection amplitudes, respectively. By solving the eigenvalue equation \( H |E_k\rangle = E_k |E_k\rangle \), one can obtain the following relations for the coefficients:

\[ g_1(1 + a + r + b) = (\delta + i\Gamma/2) \xi_{k_1}, \]

\[ g_2(2ae^{ikd} + 2be^{-ikd}) = (\Delta + i\Gamma/2) \xi_{k_2}, \]

\[ a = 1 + \frac{g_1\xi_{k_1}}{iv_g}, \quad t = 1 + \frac{1}{iv_g} (g_1 \xi_{k_1} + g_2 \xi_{k_2} e^{-ikd}), \]

\[ b = \frac{g_2\xi_{k_2}}{iv_g} e^{ikd}, \quad r = \frac{1}{iv_g} (g_1 \xi_{k_1} + g_2 \xi_{k_2} e^{ikd}). \]

Through solving the Eq. (3), \( \xi_{k_1} \) and \( \xi_{k_2} \) can be obtained.

Here, we consider the incident waveguide mode with energy \( E_k = v_g k \) interacting simultaneously with two QDs, where \( k \) is the wave vector of the incident mode. Thus the real-space Hamiltonian can be written as (as-
as follows:

\[\xi_{k_1} = \frac{-i4g_1[-4(-1 + e^{2i\Delta})J_2 + (\Gamma - 2i\delta)]}{\eta},\]

\[\xi_{k_2} = \frac{-i4g_2e^{i\Delta}(\Gamma - 2i\Delta)}{\eta},\]  

\[\eta = -16(-1 + e^{2i\Delta})J_1^2 J_2^2 + 4J_1(\Gamma - 2i\delta) + J_2(\Gamma - 2i\Delta) + (\Gamma - 2i\delta)(\Gamma - 2i\Delta),\]  

where \(J_1 = g_1^2/v_\nu\) and \(J_2 = g_2^2/v_\nu\). If there is no transmission and reflection observed by detectors at the two ends of the waveguide, the state of system is projected to \(\xi_{k_1}|e_1, g_2\rangle|0\rangle_{sp} + \xi_{k_2}|g_1, e_2\rangle|0\rangle_{sp}\), which means that the entangled state between the two QDs has been generated.

The degree of entanglement can be measured by the concurrence \(C = \max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)\). \(\lambda_i(i = 1, 2, 3, 4)\) are the square roots of the eigenvalues of the matrix \(R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\), where \(\rho\) is the density matrix of the system and \(\sigma_y\) is the Pauli matrix. For the system of two QDs, concurrence \(C\) takes the form of \(C = \frac{2\xi_{k_1}\xi_{k_2}}{||\xi_{k_1}||^2 + ||\xi_{k_2}||^2}\). Maximum entanglement can be created when amplitude \(|\xi_{k_1}|\) is equal to \(|\xi_{k_2}|\).

FIG. 3. (Color online) Dependence of concurrence \(C\) on the plasmonic waveguide mode index \(n\) and the interdot distance \(d\) for (a) on-resonance case \(\Delta = 0\) and (b) off-resonance case \(\Delta = 0.5J\). Here \(\Gamma = 0.01225J\) is used in the calculations.

First, we examine the dependence of the concurrence on waveguide mode index, interdot distance and QD-waveguide coupling strengths in the model. Fig. 3 shows the concurrence \(C\) as a function of the interdot distance \(d\) and the plasmonic waveguide mode index \(n\) when the coupling strength \(g_1 = g_2\). We take the same detuning \(\Delta = \delta\) between two QDs and the waveguide mode (thus \(J = J_1 = J_2\)) for simplicity in the following discussion. Clearly, in Fig. 3(b) when the incident waveguide mode is off-resonance with QDs (\(\Delta = 0.5J\)), the concurrence \(C\) is higher than the on-resonant case shown in Fig. 3(a) over wide ranges of mode indices and interdot distance. For conventional waveguide modes with non-zero mode indices, maximum entanglement can be achieved if the two QDs are placed at the right locations where the interdot distance \(d\) is equal to a multiple half-wavelength of the waveguide mode. However, when the plasmonic waveguide mode index \(n\) gets close to zero, the concurrence maintains above 0.9 over a wide range of interdot distance. The relaxed distance requirement strongly overcomes the challenges of precise QD position control in practical situations.

FIG. 4. (Color online) Dependence of the concurrence \(C\) on the \(g_2/g_1\) ratio and the interdot distance \(d\) when the plasmonic waveguide mode index \(n\) is equal to (a) \(n = 0\) and (b) \(n = 0.1\). Here \(\Gamma = 0.01225J\) is used in the calculations.

When the coupling strengths between QDs and the waveguide mode are not identical, the dependence of the concurrence \(C\) as a function of the ratio \(g_2/g_1\) is plotted in Fig. 4 when \(\Delta = 0.5J\). In Fig. 4(a) when the mode index \(n = 0\), maximum concurrence \(C\) around unity can be created for any arbitrary interdot distance when \(g_2 = g_1\) and the concurrence decreases when the difference between \(g_2\) and \(g_1\) increases. This phenomenon can be understood that the amplitudes \(\xi_{k_1}\) and \(\xi_{k_2}\) always have the same value in the case of \(g_1 = g_2\) and \(n = 0\), which results in the concurrence \(C = 1\). Even when the mode index cannot reach exact zero in practical situations, for example \(n = 0.1\), high concurrence \(C\) can still be achieved within relative broad ranges of interdot distance \(d\) and \(g_2/g_1\) ratio. The dark red region in Fig. 4(b) highlights that high concurrence can be obtained as long as \(d < 100\) nm and \(1 < g_2/g_1 < 4\), which significantly relaxes the strict conditions required in conventional waveguides.

Next, we discuss concurrence of the entanglement state between two QDs in a practical \(D = 110\) nm SiO\(_2\)/Ag waveguide with dispersion illustrated in Fig. 2(b). Fig. 5(a) shows the concurrence \(C\) when the waveguide mode index \(n = 0.022, 0.164, 0.462, \) and \(0.962\). Near-zero mode index can be chosen by working near the cutoff wavelength of the waveguide mode. The corresponding group velocity \(v_g\) are 0.82 \(\times\) \(10^7\) m/s, 1.65 \(\times\) \(10^7\) m/s, 3.44 \(\times\) \(10^7\) m/s, and 9.55 \(\times\) \(10^7\) m/s, respectively. For
FIG. 5. (Color online) (a) Dependence of the concurrence $C$ on the interdot distance $d$ when $\Delta = 0.5J$ for different mode index $n$ at 0.022 (red solid line), 0.164 (blue dashed line), 0.462 (black dotted line), and 0.962 (green dash-dotted line). (b) Dependence of the concurrence $C$ on the interdot distance $d$ when the mode index $n = 0.022$ for different QD-waveguide detuning $\Delta$ at 0.5J (red solid line), 0.4J (blue dashed line), 0.3J (black dotted line), 0.2J (green dash-dotted line), and 0.1J (magenta dash-dot-dot line).

other parameters of the QD-waveguide device, coupling strength $g_1 = g_2 = 35$ GHz, detuning $\Delta = 0.5J$ with $J = J_1 = J_2$, and total dissipation $\Gamma = 500$ GHz. When $n = 0.962$, the maximum value of concurrence occurs at two locations (green dash-dotted line in Fig. 5(a)) where the interdot distance $d = 288$ nm and $d = 312$ nm. The special locations for creating high concurrence generally satisfy $\Delta = -(\frac{\Gamma}{4J^2}) \tan(kd)$ and $kd = m\pi$ (m is an integer), which results in $|\xi_{k_1}| = |\xi_{k_2}|$. However, when $n$ is 0.022, high concurrence can be maintained for any interdot distance $d$ over several hundreds of nanometers. This is due to the fact that phase variation along the waveguide is very small for the plasmonic waveguide mode with near-zero index. Moreover, in Fig. 5(b), the concurrence $C$ for various $\Delta$ from 0.1J to 0.5J is shown when the mode index $n$ is 0.022. Clearly, the high concurrence $C$ can be created for a wide range of the interdot distance $d$ when $\Delta$ is 0.5J. This means that certain amount of QD-waveguide detuning in experiment will not be detrimental to the high concurrence across large interdot distance as long as the QD-waveguide coupling strength is maintained.

In conclusion, we have examined quantum entanglement in SiO$_2$/Ag plasmonic waveguides with considerations of QD-waveguide detunings, asymmetric coupling strengths and dissipations. The waveguides can be designed to possess near-zero mode indices around the QDs transitions, and high concurrence can be achieved between two QDs interacting with the plasmonic waveguide modes. A wide range of interdot distance is allowed for achieving high concurrence due to the near-zero phase variation along the waveguide, which shows advantages over the schemes implemented by dielectric waveguides or metal nanowires where specific interdot distances are required. The plasmonic waveguide with near-zero mode indices serve as a great platform for solid-state quantum optics and quantum information processing.

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