Bulk viscosity in quasi particle models

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Abstract

We discuss transport properties of dynamical fluid composed of quasi-particles whose masses depend on temperature and charge chemical potentials. Based on the relativistic kinetic theory formulated under the relaxation time approximation, we derive a general expression for the bulk viscosity in the quasi-particle medium. We show that dynamically generated particle masses imply an essential modification of the fluid compressibility. As an application of our results we consider a class of quasi-particle models with the chiral phase transition belonging to $O(4)$ and $Z(2)$ universality class. Based on the Ginzburg-Landau and the scaling theory we study the critical properties of the bulk viscosity $\zeta$ near the phase transition. We show that under the relaxation time approximation the $\zeta$ is not expected to show singular behavior near the $O(4)$ and $Z(2)$ critical point through static critical exponents.

1 Introduction

In the hydrodynamical evolution of fluid leading dissipative processes can be quantified by the transport coefficients, the shear $\eta$ and bulk $\zeta$ viscosities. Their values and properties are not only carrying information on how far the system appears from an ideal hydrodynamics but can also provide relevant insight into the fluid dynamics and its critical phenomena \cite{12345678}. For certain materials, e.g. helium, nitrogen or water, the shear viscosity to entropy ratio $\eta/s$ is known experimentally to show a minimum at the phase transition \cite{3}. On the other hand the bulk viscosity $\zeta/s$ was argued to be large or even divergent at the critical point \cite{5678}. The recent Lattice Gauge Theory calculations seem to be consistent with the expectation of decreasing $\eta/s$ and increasing $\zeta/s$ toward the QCD phase transition from above \cite{910}.
Thus, transport coefficients are of particular interest to quantify the properties of strongly interacting relativistic fluid and its phase transition \cite{3}.

In modeling strongly interacting media near equilibrium, the interactions usually lead to a quasi-particle description with its mass depending on thermal parameters \cite{11}. The thermodynamics of perturbative QCD is also to large extent quantified by thermal quark and gluon masses which are $T$ and $\mu$ dependent \cite{12}. In the QCD-like chiral models where the phase transition is governed by an effective mass generated through the dynamics, the quasi-particle mass plays a role of an order parameter and thus is sensitive to change in thermal parameters.

In this paper we discuss the bulk and shear viscosities of the dynamical fluid composed of quasi-particles with $T$ and $\mu$ dependent masses. Our calculations are based on the kinetic theory in the relaxation time approximation. We derive a general expression for the viscosities in the quasi-particle medium which has a broad spectrum of applications. At vanishing chemical potential or for fixed particle masses our results are consistent with that formulated in \cite{13} and \cite{14} respectively.

Extrapolating the calculated bulk viscosity to the phase transition region we discuss its critical properties. Based on the Ginzburg-Landau and the scaling theory we show that under the relaxation time approximation the $\zeta$ is not expected to show a singular behavior near the $O(4)$ and $Z(2)$ critical point through the static critical exponents.

## 2 Shear and bulk viscosities from transport theory

The transport parameters, the shear $\eta$ and the bulk $\zeta$ viscosities, are defined as coefficients of the space-space component of a deviation of the energy momentum tensor from equilibrium. In a medium composed of bosons and/or fermions with the momentum distribution function $f(p, x)$ for a particle and $\bar{f}(p, x)$ for an anti-particle the energy momentum tensor is defined as

$$T^{\mu\nu} = \int d\Gamma \frac{p^\mu p^\nu}{E} \left[ f + \bar{f} \right], \quad (1)$$

where $d\Gamma = g d^3p/(2\pi)^3$ is the integration measure in the momentum space with the degeneracy factor $g$ associated with the particle quantum numbers. The four momentum $p^\mu = (E, \vec{p})$ with $E(\vec{p}) = \sqrt{\vec{p}^2 + M^2}$ and $M$ being a
particle mass.\footnote{In general, it is not necessary to specify this dispersion relation. The following derivation is valid for any $E(\vec{p})$}

Assuming that the medium appears near equilibrium we introduce the particle momentum distribution $f(p, x)$ in the following form

$$f = \left( e^{(E - \vec{p} \cdot \vec{u} \mp \mu)/T} \pm 1 \right)^{-1},$$

(2)

where $\vec{u}$ is the flow velocity and $\mu$ is the chemical potential related with any conserved charges. The $\pm 1$ corresponds to fermion and boson statistics whereas $\mp \mu$ to particle and antiparticle contributions, respectively.

In our further discussion we assume that $M$ is not necessarily a bare particle mass but rather a dynamical quasi-particle mass which depends on temperature and chemical potentials, thus $M = M(T, \mu)$ in Eq. \(2\).

A deviation of the system from equilibrium is quantified by the corresponding change of distribution functions $\delta f = f - f_0$ with $f_0$ being the equilibrium particle momentum distribution. In the relaxation time approximation \cite{15}, the $\delta f$ is obtained from

$$p^\mu \partial_\mu f_0 = - \frac{p \cdot u}{\tau} \delta f,$$

(3)

where the collision time

$$\tau^{-1} = n_f(T, \mu) \langle v\sigma(T, \mu) \rangle,$$

(4)

is determined by the thermal-averaged total scattering cross section $\langle v\sigma \rangle$, with the relative velocity of two colliding particles $v$ and the particle density $n_f$ in equilibrium. The $\delta f$ results in the corresponding change in the energy momentum tensor

$$\delta T^{\mu\nu} = - \int d\Gamma \frac{p^\mu p^\nu}{E^2} p^\alpha \partial_\alpha \left[ \tau f_0 + \tau \bar{f}_0 \right].$$

(5)

The thermodynamic quantities are time dependent through thermal parameters. The time dependence of $T$ and $\mu$ are obtained from the charge number density $\partial_0 j^0 = 0$ and the energy density $\partial_0 T^{00} = 0$ conservations. The charged current $j^\mu = (j^0, \vec{j})$ is defined by
\[ j^\mu = \int d\Gamma \frac{p^\mu}{E} \left[ f - \bar{f} \right], \]  \tag{6}

The energy and charge density conservations can be expressed in terms of the thermodynamic quantities as

\[ \frac{\partial \epsilon}{\partial t} = -(\epsilon + P) \nabla \cdot \vec{u} = - \left( T \frac{\partial P}{\partial T} + \mu \frac{\partial P}{\partial \mu} \right) \nabla \cdot \vec{u}, \]  \tag{7}

\[ \frac{\partial n}{\partial t} = -n \nabla \cdot \vec{u} = - \frac{\partial P}{\partial \mu} \nabla \cdot \vec{u}, \]  \tag{8}

where the energy \( \epsilon \) and charge \( n \) densities are related with the pressure \( P \) through the thermodynamic relations: \( \epsilon = T \frac{\partial P}{\partial T} - P + \mu \frac{\partial P}{\partial \mu} \). From the chain rules

\[ \frac{\partial P}{\partial t} = \frac{\partial P}{\partial \epsilon} \frac{\partial \epsilon}{\partial t} + \frac{\partial P}{\partial n} \frac{\partial n}{\partial t}, \]  \tag{9}

and from Eq. (8) one gets

\[ \frac{\partial P}{\partial t} = - \left[ \frac{\partial P}{\partial \epsilon} \left( T \frac{\partial P}{\partial T} + \mu \frac{\partial P}{\partial \mu} \right) + \frac{\partial P}{\partial n} \frac{\partial P}{\partial \mu} \right] \nabla \cdot \vec{u}. \]  \tag{10}

The pressure \( P \) is a function of \( T \) and \( \mu \), thus

\[ \frac{\partial P}{\partial t} = \frac{\partial P}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial P}{\partial \mu} \frac{\partial \mu}{\partial t}, \]  \tag{11}

therefore one arrives at the following equations for the time dependence of \( T \) and \( \mu \):

\[ \frac{\partial T}{\partial t} = -T \left( \frac{\partial P}{\partial \epsilon} \right)_n \nabla \cdot \vec{u}, \]

\[ \frac{\partial \mu}{\partial t} = - \left[ \mu \left( \frac{\partial P}{\partial \epsilon} \right)_n + \left( \frac{\partial P}{\partial n} \right) \right] \nabla \cdot \vec{u}. \]  \tag{12}

Consequently, the change of the energy momentum tensor \( \delta T^{\mu\nu} \) becomes

\[ \delta T^{\mu\nu} = \delta T^{\mu\nu}_f + \delta T^{\mu\nu}_{\bar{f}}, \]

\[ \delta T^{\mu\nu}_f = \int d\Gamma \frac{p^\mu p^\nu}{E} f_0(1 \pm f_0) q_f(\vec{p}; T, \mu), \]  \tag{13}

where we have introduced
\[ q_{f,j}(\vec{p}; T, \mu) = \left[ -\frac{\vec{p}^2}{3E} + \left( \frac{\partial P}{\partial \epsilon} \right)_n \left( E - T \frac{\partial E}{\partial T} - \mu \frac{\partial E}{\partial \mu} \right) - \left( \frac{\partial P}{\partial n} \right)_\epsilon \left( \frac{\partial E}{\partial \mu} \mp 1 \right) \right] \partial_k u_i \delta^{kl} - \frac{p_k p_l}{2E} W^{kl}, \]  

(14)

with the following tensor decomposition

\[ \partial_k u^l = \frac{1}{2} \left( \partial_k u^l + \partial_k u^k - \frac{2}{3} \delta_{kl} \partial_l u^i \right) + \frac{1}{3} \delta_{kl} \partial_i u^i \]

\[ \equiv \frac{1}{2} W_{kl} + \frac{1}{3} \delta_{kl} \partial_i u^i. \]  

(15)

With the above decomposition, the change of the tensor \( T^{ij} \) can be written as a sum of traceless \( W_{ij} \) and scalar part

\[ \delta T^{ij} = -\zeta \delta_{ij} \partial_k u^k - \eta W_{ij}, \]  

(16)

which defines the transport coefficients, the bulk \( \zeta \) and the shear \( \eta \) viscosities respectively. Implementing in Eq. (13) the energy conservation

\[ \int d\Gamma \left[ \tau f_0 (1 \pm f_0) q_f + \bar{\tau} \bar{f}_0 (1 \pm \bar{f}_0) q_{\bar{f}} \right] = 0 \]  

(17)

leads to the final expression for the transport coefficients obtained under the relaxation time approximation in the quasi-particle model with \( T \) and \( \mu \) dependent masses. The bulk \( \zeta = \zeta_f + \zeta_{\bar{f}} \) and the shear \( \eta = \eta_f + \eta_{\bar{f}} \) viscosities in a medium composed of one type of particle/antiparticle is obtained as

\[ \eta = \frac{1}{15T} \int \frac{d^3 p}{(2\pi)^3} \frac{\vec{p}^4}{E^2} \left[ \frac{g \tau f_0 (1 \pm f_0) + g \bar{\tau} \bar{f}_0 (1 \pm \bar{f}_0)}{M^2} \right], \]  

(18)

and

\[ \zeta = -\frac{1}{3T} \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{M^2}{E} \left( g \tau f_0 (1 \pm f_0) + g \bar{\tau} \bar{f}_0 (1 \pm \bar{f}_0) \right) \times \left( \frac{\vec{p}^2}{3E} - \left( \frac{\partial P}{\partial \epsilon} \right)_n \left( E - T \frac{\partial E}{\partial T} - \mu \frac{\partial E}{\partial \mu} \right) + \left( \frac{\partial P}{\partial n} \right)_\epsilon \frac{\partial E}{\partial \mu} \right) \right. \]

\[ \left. - \frac{M^2}{E} \left( g \tau f_0 (1 \pm f_0) - g \bar{\tau} \bar{f}_0 (1 \pm \bar{f}_0) \right) \left( \frac{\partial P}{\partial n} \right)_\epsilon \right], \]  

(19)

respectively. The above results can be obviously generalized to any systems composed of different particle species by summing up their contributions in Eqs. (18) and (19).
The derivatives of pressure \( \partial P / \partial \epsilon \big|_n \) and \( \partial P / \partial n \big|_\epsilon \) entering in Eq. (19) can be expressed in terms of the net particle number density \( n \), the entropy density \( s \) and different susceptibilities \( \chi_{xy} \) which are defined by

\[
n = \frac{\partial P}{\partial \mu}, \quad s = \frac{\partial P}{\partial T}, \quad \chi_{xy} = \frac{\partial^2 P}{\partial x \partial y}.
\]  

(20)

Applying the Jacobian methods to the above derivatives of \( P \) one finds

\[
\left( \frac{\partial P}{\partial \epsilon} \right)_n = \frac{1}{C_V \chi_{\mu\mu}} (s \chi_{\mu\mu} - n \chi_{\mu T}) \quad \text{and} \quad \left( \frac{\partial P}{\partial n} \right)_\epsilon = \frac{1}{C_V \chi_{\mu\mu}} \left[ nT \chi_{TT} + (n \mu - sT) \chi_{\mu T} - s \mu \chi_{\mu\mu} \right],
\]

(21) (22)

with \( C_V \) being the specific heat at constant volume and at constant \( s/n \). The \( C_V \) can be expressed through different susceptibilities as

\[
C_V = T \left( \frac{\partial s}{\partial T} \right)_V = T \left[ \chi_{TT} - \frac{\chi_{\mu T}^2}{\chi_{\mu\mu}} \right].
\]

(23)

The shear viscosity \( \eta \) has the same form as previously obtained in \cite{14,16} where the thermal modification of particle dispersion relations was not included. Thus, the shear viscosity is not directly affected by the quasi-particle dynamics. However, the bulk viscosity is essentially modified by the terms \( \partial E / \partial T \) and \( \partial E / \partial \mu \) that appear only when there is an explicit \( (T, \mu) \)-dependence of the particle mass. For \( \dot{M}(T, \mu) = \text{const.} \), Eq. (19) is reduced to the result obtained in \cite{14}. At vanishing chemical potential Eq. (19) coincides with the expression recently formulated in \cite{13} for an interacting quark-gluon plasma.

3 Transport coefficients near the phase transition

To quantify the transport properties of thermodynamic systems characterized by viscosity coefficients one would need to formulate a specific model for particle interactions. However, there are some generic properties of \( \zeta \) which can be discussed in a model-independent way through the universality arguments. In this context, of particular interest is the specific behavior of \( \zeta \) near the phase transition. For the bare particle mass, the dynamics is entering only through the collision time. Thus, information on phase transition can only be contained in the change of the thermal averaged cross section. However, in the quasi-particle picture due to dynamical particle masses, further information
on the phase transition appears in $\zeta$ through an explicit contribution of different observables which are sensitive to critical phenomena. From Eq. (19) it is clear that such observables are the specific heat $C_V$ and different susceptibilities. In addition, in the model where the particle mass $M(T, \mu)$ is related with an order parameter, the thermal derivatives of $M$ can be as well singular at the phase transition. This is e.g. the case when considering effective models related with the chiral phase transition [5][17].

To study the sensitivity of the bulk viscosity to the phase transition we separate from Eq. (19) the term $\zeta^{(\text{der})}$ that can be singular near the critical point,

$$\zeta^{(\text{der})} = -\frac{1}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{M^2}{E} \left[ (g\tau f_0(1 \pm f_0) + \bar{g}\bar{\tau} \bar{f}_0(1 \pm \bar{f}_0)) - \left( -\left( \frac{\partial P}{\partial \epsilon} \right)_n \left( E - T \frac{\partial E}{\partial T} - \mu \frac{\partial E}{\partial \mu} \right) + \left( \frac{\partial P}{\partial n} \right)_\epsilon \frac{\partial E}{\partial \mu} \right) - \left( g\tau f_0(1 \pm f_0) - \bar{g}\bar{\tau} \bar{f}_0(1 \pm \bar{f}_0) \right) \left( \frac{\partial P}{\partial n} \right)_\epsilon \right].$$

(24)

Hereafter, assuming a validity of the relaxation time approximation we discuss the critical behavior of $\zeta^{(\text{der})}$ in different models with a phase transition. In particular, based on the Ginzburg-Landau as well as on the scaling theory we analyze the scaling of $\zeta$ in the $O(4)$ and $Z(2)$ universality classes which are expected in the QCD chiral phase transition.

3.1 Mean field scaling of the bulk viscosity in the Ginzburg-Landau model

According to the Ginzburg-Landau theory, close to the phase boundary the thermodynamic potential may be expanded in a power series of the order parameter $M$, which plays the role of the dynamical particle mass

$$\Omega(T, \mu) \sim \Omega_0(T, \mu; M = 0) + \frac{a(T, \mu)}{2} M^2 + \frac{b(T, \mu)}{4} M^4 - mM. \quad (25)$$

The order parameter is determined in such a way that the free energy is minimized. Thus, $M$ is obtained as the solution of the gap equation

$$M = \sqrt{-\frac{a}{b}}. \quad (26)$$
The fluctuations of \( M \) are defined through the susceptibility
\[
\chi = \frac{\partial M}{\partial m} \bigg|_{m=0} = \frac{1}{a + 3bM^2}.
\] (27)

With the particular choice of the parameters, \( a = 0 \) for \( b > 0 \) the thermodynamic potential describes the second order phase transition. On the other hand for \( a = b = 0 \) the system exhibits the tricritical point (TCP). Thus, the Ginzburg-Landau model has generic critical properties expected in the QCD chiral phase transition.

In the vicinity of the phase transition the temperature and chemical potential dependence of the coefficients \( a(T, \mu) \) and \( b(T, \mu) \) can be parameterized as
\[
a(T, \mu) = \alpha |T - T_c| + \beta |\mu - \mu_c|,
\] (28)

with constant \( \alpha \) and \( \beta \).

From the thermodynamic potential (25) one gets all relevant thermodynamic quantities near the phase transition. In particular, near the TCP the singular part of susceptibilities
\[
\chi_{\mu\mu} \sim \frac{\beta^2}{b}, \quad \chi_{\mu T} \sim \frac{\alpha \beta}{b}, \quad \chi_{TT} \sim \frac{\alpha^2}{b}.
\] (29)

and derivatives of the dynamical mass
\[
\frac{\partial M}{\partial T} \sim -\frac{1}{M} \frac{\alpha}{b}, \quad \frac{\partial M}{\partial \mu} \sim -\frac{1}{M} \frac{\beta}{b}.
\] (30)

With the above scaling relations and from Eqs. (21) and (23) one finds, that near the TCP the pressure derivatives and the specific heat are finite and that the potentially singular part \( \zeta^{(\text{der})} \) behaves as
\[
\zeta^{(\text{der})} \sim -\frac{M^3}{C_V} \left( s - n \frac{\alpha}{\beta} \right) \left( \frac{\partial M}{\partial T} - \frac{\alpha}{\beta} \frac{\partial M}{\partial \mu} \right).\] (31)

Consequently, due to the vanishing coefficient of the second bracket in Eq. (31) together with the scaling (17)
\[
\frac{M^2}{b} \sim M^4 \chi \sim t^0,
\] (32)

the singularities of the susceptibilities do not show up in \( \zeta \) near TCP.
The singular part of $\zeta^{\text{der}}$ also vanishes at the second order transition since there $\zeta^{\text{der}} \sim M^2 \to 0$. In addition, if the explicit chiral symmetry breaking is absent then also the regular part of $\zeta$ vanishes. Thus the $\zeta$ is precisely zero at the critical point.

The above example shows that within the mean field dynamics the bulk viscosity is non-singular at the second order phase transition and at the TCP.

3.2 Scaling of the bulk viscosity in the $O(4)$ and $Z(2)$ universality class

In order to verify the singular behavior of $\zeta$ along the second order line and at the TCP that belongs to the $O(4)$ and $Z(2)$ universality class respectively, one needs to go beyond the mean field approximation. The scaling behavior of different observables in the vicinity of phase transition emerges from the scaling function of the singular part of the free energy.

Along the $O(4)$ transition line and for $\mu/T << 1$, the singular part of the free energy can be parameterized as \[ F_s(T, \mu) \simeq t^{2-\alpha} f_s(1, t^{-\beta} h) \] \[ (33) \]

where

\[ t = \bar{t} + A \mu^2, \] \[ (34) \]

with $\bar{t} = |T - T_c|/T_c$, $\bar{\mu} = \mu/T_c$ and $h$ being an external field. The $\alpha$, $\beta$ and $\delta$ are the critical exponents in the $O(4)$ universality class.

From the scaling function (33), by taking derivatives, one finds the properties of all relevant quantities required in Eq. (19) to find the behavior of the bulk viscosity near the $O(4)$ line. In the following we consider only the transition point at $\mu = 0$. From Eq. (33) one gets

\[ \chi_{\mu\mu} \sim t^{1-\alpha}, \quad \chi_{TT} \sim t^{-\alpha}, \quad C_V \sim t^{-\alpha}, \quad M \sim t^\beta, \quad \partial M/\partial T \sim t^{\beta-1}. \] \[ (35) \]

Substituting the above scaling to Eq. (19) one finds that near the $O(4)$ transition point the singular part of the bulk viscosity $\zeta^{\text{der}}$ scales as

\[ \zeta^{\text{der}} \sim t^{\alpha+4\beta-1}. \] \[ (36) \]

Thus, with the $O(4)$ critical exponents: $\alpha \simeq -0.24$ and $\beta \simeq 0.38$, the singular part of $\zeta$ vanishes at the critical point. Consequently, in this approach, there is no singularity in the bulk viscosity along the $O(4)$ line.
The above scaling of $\zeta$ can be different near the tricritical point, because the TCP belongs to the $Z(2)$ universality class of the three dimensional Ising model. To match the spin system parameters, the reduced temperature $t$ and the external field $h$, to those in the QCD chiral models we follow the discussion of [19] and replace: $t \rightarrow a_t \bar{t} + b_t \bar{\mu}$ and $h \rightarrow a_h \bar{t} + b_h \bar{\mu}$. The singular part of the free energy in the $Z(2)$ universality class is parameterized as:

$$F_s(t, \mu) \sim h^{1+\delta} f_s(h^{-1/\delta} t, 1).$$

(37)

This scaling form of the free energy leads to the following critical behavior of the thermodynamic quantities near TCP at $t = 0$:

$$\chi_{\mu\mu, TT, \mu T} \sim h^{-\gamma/\beta\delta}, \ C_V \sim h^{-\gamma/\beta\delta}, \ M \sim h^{1/\delta}, \ \partial M / \partial T \sim h^{-\gamma/\beta\delta}.$$  (38)

Consequently, the singular part of the bulk viscosity (19) in the $Z(2)$ universality class scales as

$$\zeta^{(\text{der})} \sim h^{\gamma/\beta\delta+4/\delta-1}.$$  (39)

A similar scaling of $\zeta$ can be also found at a finite quark mass by adding in the $h$ scaling field the particle mass. Substituting into Eq. (39) the $Z(2)$ exponents: $\beta \simeq 0.31, \delta \simeq 5.2$ and $\gamma \simeq 1.25$ one finds $\zeta \sim h^{0.54}$. Thus, similarly as in the $O(4)$ universality class the bulk viscosity is non-singular near the TCP.

The above scalings of $\zeta$, obtained under relaxation time approximation, are different from that recently found in [8]. There, it was argued that the critical behavior of the bulk viscosity is governed by the critical exponent of the specific heat since $\zeta \sim C_V$. Consequently, the bulk viscosity was shown to diverge at the TCP or CEP in the $Z(2)$ universality class. In our approach, under the relaxation time approximation, the scaling of $\zeta$ is determined by the product

$$\zeta^{(\text{der})} \sim \frac{M^3}{C_V} \left( \frac{\partial M}{\partial T} + C \frac{\partial M}{\partial \mu} \right).$$  (40)

Consequently, $\zeta$ is proportional to $C_V^{-1}$ rather than to $C_V$. The same, negative power of $C_V$ is also reported in [20].

From the above discussion, valid under relaxation time approximation, one sees that $\zeta$ is not expected to show divergent behavior near the critical end point through the static critical exponents. However, an extension of the relaxation time approximation, that allows to incorporate properly a long-range
fluctuations of soft modes, can result in divergence of $\zeta$ at the TCP as well as at the second order $O(4)$ transition [20]. In this case, divergence of the bulk and shear viscosities is governed by the dynamic rather than static critical exponents [21,22].

4 Conclusions

We have studied the non-equilibrium properties of a quasi-particle medium at finite temperature and density using the kinetic theory in the relaxation time approximation. Assuming, that the quasi-particle masses are temperature and chemical potential dependent, we have derived a consistent expression for the bulk viscosity coefficient $\zeta$. We have shown that in the presence of dynamical mass $M$ the fluid compressibility is essentially modified. Our result for $\zeta$ is valid in different physical systems where interactions yield some modification in particle dispersion relations through explicit variation of $M$ with thermal parameters.

We have applied our results to a class of effective chiral models where the dynamical mass is identified as the order parameter for the chiral phase transition. We extrapolated the bulk viscosity obtained in the relaxation time approximation to the phase transition and studied the influence of critical fluctuations on $\zeta$. We have shown that under the mean field dynamics as well as in the presence of quantum fluctuations implemented through the scaling functions, the bulk viscosity is not sensitive to the chiral phase transition. This is because, the singularities generated from the susceptibilities are totally canceled in $\zeta$ at the $O(4)$ as well as at the $Z(2)$ critical point. A possible modification of our conclusions due to a proper treatment of soft modes at the phase transition is not excluded. Nevertheless, the scaling properties given in this paper indicate a tendency of the transport coefficients when approaching the critical point. The phenomenological relevance of soft modes at the phase transition strongly depends on how large the critical region is. If the critical region where the description through the static critical exponents is not valid anymore is very narrow in thermal parameters, it may be rather hard to observe the singularity of bulk viscosity around the critical point.

\[\text{The dynamic universality class of the QCD critical point is that of the model H} \ [23].\]
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