Dynamics of interdependent multidimensional opinions

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Abstract. In opinion dynamics, individuals in a group can change their opinions on issues as a result of connections to other individuals. Motivated by the DeGroot model, we proposed an extended DeGroot model which allows agents to revise their opinions on multiple interdependent topics. We introduce a coupling function which indicates couplings between opinions of multiple interdependent topics. This extended model allows us to choose a specific mathematical function to represent a specific situation. A model is presented which describes the dynamics of the opinions of individuals in a group on one topic by pooling their neighbors’ opinions and the influences of their own opinions on multiple topics. Different cases of coupling between agents opinions are investigated. Numerical results are provided that demonstrate how couplings between a number of issue specific opinions influence the convergence of opinions of agents in the network.

1. Introduction

A social network is a graph used to model social interaction among individuals in a group. Dynamics and convergence of opinions of individuals, normally called agents, in the network are one of the important aspects of social networks analysis. We use the term opinion to refer to an individual’s state value. In opinion dynamics, agents in a group can change their opinions on some particular issues as a result of connections to other agents. A stubborn node is a node that influences others but does not change its opinion. The opinion dynamics model can represent the effect of mass media, stubborn agent, on people’s opinions.

The DeGroot model [1] describes how a group of n agents might reach a consensus by pooling the agent opinions:

\[ x(t + 1) = Wx(t), \]

where \( x_i(t) \), the ith component of vector \( x(t) \), is the opinion of agent \( i \) at time \( t \), \( w_{ij} \geq 0 \), the \( i,j \)th entry of \( n \times n \) matrix \( w \) is the weight that agent \( i \) assigns to the opinion of agent \( j \), and \( \sum_{j=1}^{n} w_{ij} = 1 \) for all \( i \). A consensus is reached if and only if there is a constant \( x^* \) such that \( \lim_{t \to \infty} x_i(t) = x^* \) for \( i = 1, \ldots, n \). The iterative process can also be written as

\[ x(t + 1) = Wx(t) = W^{t+1}x(0). \]

Hence if there exists a vector \( \lambda = (\lambda_1, \ldots, \lambda_n) \) such that, \( \lim_{t \to \infty} [w^t]_{ij} = \lambda_i \) for \( i = 1, \ldots, n \) and \( j = 1, \ldots, n \). A consensus is reached and the opinion of all agents is \( \sum_{i=1}^{n} \lambda_i x_i(0) \).
The Friedkin-Johnsen (FJ) model [2–4] is a DeGroot model with prejudice whereby each agent never forgets their initial opinion:

\[ \mathbf{x}(t + 1) = \mathbf{A} \mathbf{W}(t) + (\mathbf{I} - \mathbf{A})\mathbf{x}(0), \]

where \( \mathbf{A} \) is an \( n \times n \) diagonal matrix of agent susceptibilities to interpersonal influence such that \( 0 \leq A_{ii} \leq 1 \), and \( \mathbf{I} \) is the identity matrix. In the FJ model, there are many situations where the agents do not reach a consensus but form several clusters.

Recent research has focused on filling gaps in the well-known models but most of them deal with single (scalar) opinions [5]. In reality, people have opinions on several topics and these are referred to as vector-valued opinions [6]. The work of [6] study multidimensional continuous opinion dynamics, where each opinion is non-negative vector which components sum up to one, eg. budget plan of a fixed amount of money to various projects. The simplest case when opinions on different topics are independent were suggest. The multidimensional extension of FJ model [7] assumed that updated opinions on one topic are not influenced by opinions held on the other topics then the opinions on each topic evolve independently. The opinion dynamics of independent issues evolve just like it is a group of scalar opinion networks with the same topology that don’t interact with each other.

The opinion a person has on one topic can depend on their opinion on other topics, eg. a committee who makes decisions on multiple related topics. A multidimensional opinion dynamics model of interdependent topics for FJ models [8] has also been suggested as follows:

\[ \mathbf{x}_i(t + 1) = A_{ii} \mathbf{C} \sum_{j=1}^{n} w_{ij} \mathbf{x}_j(t) + (1 - A_{ii})\mathbf{x}_i(0). \]

For \( m \) interdependent topics, the difference of the extended FJ model with the FJ model is the presence of a constant coupling matrix, \( \mathbf{C} \in \mathbb{R}^{m \times m} \), called the matrix of multi-issues dependence structure. When \( \mathbf{C} = \mathbf{I}_m \) the extended model reduces to the regular FJ model. However, this model assumes that interdependent opinions have constant coupling in time.

In this paper, we formulate a multidimensional extension of the DeGroot model. The model describes the dynamics of multidimensional opinions on interdependent topics. Motivated from the original DeGroot model and the multidimensional opinion dynamics model of interdependent topics [8], we propose a modified model that allows the coupling between topics to change depending on the value of the updated opinions on each of the topics. For example, a group of people or committee deciding about two business plans and suppose a person that agree with the idea of plan A is more likely to be disagree with plan B. In this particular example, we can use step function to describe the coupling between agents’ opinions about the two plans.

2. The model

Consider an interacting group of \( n \) agents, and suppose that each of \( n \) individuals can have opinions on some topics. The agents’ opinion with the same coupling function converge to the different value. For the case of independent topic, agents’ opinion in each topic converge to the initial of agent 1 who is a stubborn agent. Let \( x_1, \ldots, x_n \in \mathbb{R} \) denote the agents’ vector opinions and each vector \( \mathbf{x}_i(t) = (x_1^i(t), \ldots, x_m^i(t)) \) gives the opinions at time \( t \) of agent \( i \) on \( m \) different topics. Let the initial state of figure 1 equals

\[ \mathbf{x}(0) = \begin{pmatrix} (1, -1), (1, 1), (-1, -1), (1, -1), (-1, 1) \end{pmatrix} \]

\[ x_1(0) \quad x_2(0) \quad x_3(0) \quad x_4(0) \quad x_5(0) \]

The updated opinion of agent \( i \) on issue \( p \) is calculated by the addition of two terms. The first term is the weighted average opinion of its own and its neighbours’ opinion on issue \( p \). The
second term is a function that depends on the weighted average opinion of agent $i$ of topic $q$. Then

$$x^p_i(t + 1) = c_{pp} \sum_{j=1}^{n} w_{ij} x^p_j(t) + \sum_{q \neq p} c_{pq} F(x^q_i(t)) \quad (1)$$

where $c_{pq}$ are chosen so that $\sum_{q=1}^{m} c_{pq} = 1$. The coupling function $F(x)$ can be any function that can describe the desired situation and $-1 \leq F(x) \leq 1; \forall F(x) \in \mathbb{R}$. We insist that $-1 \leq x^p_i(0) \leq 1$ for all $i$ and $p$. The rules on the values of $w_{ij}$, $c_{pq}$, and $F$ then ensure that $-1 \leq x^p_i(t) \leq 1$ for all $t$.

When the two topics are dependent on each other, the leader may change their opinion on topic 1 by an influence of its opinion in topic 2 and vice versa. If $c_{12}$ is larger than $c_{21}$, it means the influence that topic 2 has over topic 1 is greater than the influence of topic 1 over topic 2 and the opinions of topic 2 will dominate the opinion of topic 1 and it also works the other way around.

3. Results and discussion
In figure 2 the left plot shows the dynamics of opinions governed by (1) on two interdependent topics with $F(x) = x$. All agent opinions converge to the same value after a few interactions. In the right plot of figure 2 where $F(x) = \sin x$, after a few interactions the opinions of all agents converge to the same value but after some time the consensus value of each topic eventually moves to another value. The presence of a coupling function allows the opinion dynamics network to exhibit some interesting results.

According to the last plot of figure 3 when $c_{12} = c_{21}$, the network can not reach a consensus
Figure 2. The dynamics of opinions of $n = 200$ agents on interdependent topics with coupling function.

Figure 3. The dynamics of opinions of $n = 5$ agents (figure 1) on interdependent topics with coupling function $F(x) = 1$ when $x > 0$ and $F(x) = -1$ when $x \leq 0$.

but reach a disagreement instead. There are many parameter that effect this phenomena such as the topology of the network, the weight that each agent assign to itself and the initial state.
of agents.

From the example network in figure 1, this model can represent a group of 5 agents discussing two topics. Suppose the two interdependent topics are two business plans, project A and project B. The coupling function that is a step function imply that if a person have a positive (negative) impression about project A that person will have a positive (negative) impression about project B and vice versa. The initial state of agent 2, $x_2(0) = (1, 1)$, tells that agent 2 would love the idea of project A and has a positive impression of project B. On the other hand, the leader, node 1, loves project A but does not like project B. When $c_{12} < c_{21}$, the influence of project A is higher than that of project B, the opinion of the leader of project B converge to 1 because of an influence of her own opinion of the project A. Eventually, they are all agree with the idea of both two projects.

There are also many possibilities that groups of individuals may exhibit disagreement and clustering.

![Figure 4](image)

**Figure 4.** $F(x) = \cos(x)$ means that if an agent have an extreme opinion about topic 1(or 2), then it will have negative influence of topic 2(or 1). Whereas an agent that have neutral opinion about topic 1(or 2) will be willing to change its opinion of topic 2(or 1) to be more positive. In this particular example, the opinions do not reach a consensus but form several clusters.

4. Conclusions

In the extended model that has been presented, there is no outside information about the value of opinions of agents. The main goal of modeling opinion dynamics model in general is to find mathematical model for social interaction. We can not tell weather the value of opinions of one agent in the network is closer to the true value than that of another. The initial value of opinions of agents in the network is the only information available. It is also assumed that each individual distributes the weights to its neighbors at the beginning and will continues to use these weights throughout the whole process.

In this work, we have introduced an extended DeGroot model, which allow agents to revise their opinions toward interdependent issues. The coupling between issues is a function depending on opinions of others issues. A step function is one of the coupling function that we have investigated. Compared with the result without coupling between two topics, the proposed model has a different dynamics. The model presented here differs from the multidimensional model [8], constant coupling, in that different coupling functions can be chosen to fit specific situations.

Besides coupling function, there are many conditions of modeling an opinion dynamics network remain to be considered.
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