Three-terminal spin filter based on spin-orbit coupling in the presence of anti-dot

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Abstract. We have numerically investigated the spin-dependent electronic transport in a multi-terminal conductor with an anti-dot at the junction in the presence of spin-orbit coupling. It is shown that the direction of spin polarization can be flipped by changing the Fermi energy with respect to the height of anti-dot potential in a three-terminal conductor. This is understood by the interplay between spin-orbit coupling and the curvature of the potential. The spin Hall conductance in a four-terminal conductor with an anti-dot is also investigated.

1. Introduction
Recently, the spin Hall effect (SHE) induced by spin-orbit coupling (SOC) has been attracted a lot of interests. There are two types of SHE depending on the origin of SOC, namely, the intrinsic and extrinsic SHEs. In two-dimensional gases (2DEG), the intrinsic SHE induced by Rashba SOC vanishes at steady state [1]. On the other hand, the extrinsic SHE due to the SOC induced by impurities has been observed experimentally [2].

Although this SHE is induced by many impurities, a single potential in the presence of SOC is basically sufficient to generate a spin-polarized electric current [3, 4, 5, 6]. In the present paper, we numerically investigate the spin-dependent multi-channel transport in the presence of an anti-dot potential. It is found that a spin polarization can be obtained and that the direction of the spin polarization can be flipped by changing the Fermi energy. This can be understood by the interplay between SOC and the curvature of a potential [7].

2. Model and method
In two-dimensional electron gases (2DEG), the Hamiltonian including SOC is given by
\begin{equation}
\mathcal{H} = \frac{p^2}{2m^*} + U(x, y) + \frac{\alpha_{46}}{e\hbar} \sigma_z \left( p_x \frac{\partial U(x, y)}{\partial y} - p_y \frac{\partial U(x, y)}{\partial x} \right) + \frac{\alpha}{\hbar} (\sigma_x p_y - \sigma_y p_x),
\end{equation}
where $m^*$ denotes the effective mass and $U(x, y)$ the in-plane potential. The parameter $\alpha_{46}$ is the material-dependent strength of SOC [8]. The last term is well known as Rashba spin-orbit coupling [9].
In the discrete lattice model, this Hamiltonian can be written as

$$\mathcal{H} = 4V_0 + \sum_{i,j,\sigma} U_{ij} c_{ij\sigma}^\dagger c_{ij\sigma} - \sum_{i,j,\sigma'} V_{\sigma,i\pm \hat{x}\sigma';j} c_{ij\sigma}^\dagger c_{i\pm \hat{x}j\sigma'} - \sum_{i,j,\sigma'} V_{i;\sigma,j;\sigma'} c_{ij\sigma'}^\dagger c_{i\sigma} + \text{c.c.},$$  \tag{2}

where $c_{ij\sigma}^\dagger$ denotes the creation (annihilation) of an electron with a spin $\sigma$ at the site $(i,j)$, $U_{ij}$ the on-site potential, $V_{\sigma,i\pm \hat{x}\sigma';j} V_{i;\sigma,j;\sigma'}$ the spin-dependent hopping term in the $x$- ($y$-) direction, and “c.c.” the complex conjugate. We assume that there is no Rashba SOC. The unit of energy is given by $V_0 = \hbar^2/2m^*a^2$ where $a$ denotes the lattice spacing. In the following, energies and lengths are given in units of $V_0$ and $a$.

The spin-dependent 2 × 2 hopping terms in the $x$- and $y$-directions are

$$V_{\sigma,i\pm \hat{x}\sigma';j} = \left\{ 1 + i\chi\gamma(U_{i+\hat{x},j+\hat{y}} - U_{i+\hat{x},j-\hat{y}}) \right\} \delta_{\sigma\sigma'},$$  \tag{3}

$$V_{i;\sigma,j;\sigma'} = \left\{ 1 - i\chi\gamma(U_{i+\hat{x},j+\hat{y}} - U_{i-\hat{x},j+\hat{y}}) \right\} \delta_{\sigma\sigma'},$$  \tag{4}

with $\chi = (+/-)$ for $\sigma = \uparrow (\downarrow)$. Here $\gamma$ denotes the strength of SOC. The potentials at the middle point between two sites are assumed to be $U_{i+\hat{x},j/2} = (U_{ij} + U_{i+\hat{x},j})/2$ and $U_{i, j+\hat{y}/2} = (U_{ij} + U_{i,j+\hat{y}})/2$. The strength of SOC, $\gamma$, is related to $a_{46}$ in (1) as $\gamma = a_{46}/4ea^2$. The anti-dot is assumed by the Gaussian potential,

$$U_{ij} = U_0 \exp\left(-\frac{|r - r_0|^2}{\Delta r^2}\right),$$  \tag{5}

with $r = (i,j)$. Here $U_0$ and $\Delta r$ denote the height and width of the potential, respectively.

The spin-dependent conductance between reservoirs $J$ and $I$ is given by the Landauer formula [10],

$$G_{\sigma\sigma'}^{IJ} = G_0 \sum_{\mu} |t_{\mu\sigma\sigma'}^{IJ}|^2,$$  \tag{6}

with $G_0 \equiv e^2/h$. The transmission coefficient $t_{\mu\sigma\sigma'}^{IJ}$ from channel $\mu$ with spin $\sigma'$ in the reservoir $J$ to channel $\mu$ with spin $\sigma$ in the reservoir $I$ can be calculated from the Green function [11]. The total and spin-polarized conductances are given by

$$G^{IJ} = \sum_{\sigma'} G_{\sigma\sigma'}^{IJ} \quad \text{and} \quad G_{Pz}^{IJ} = \sum_{\sigma} \left(G_{\sigma\sigma}^{IJ} - \frac{1}{2} G_{1\sigma}^{IJ}\right),$$  \tag{7}

respectively. The spin polarization is $P^{IJ}_{Pz} = G_{Pz}^{IJ}/G^{IJ}$. In this paper, we focus on the spin-polarized conductance $G_{Pz}^{IJ}$ instead of the spin polarization $P^{IJ}_{Pz}$. Since $G_{Pz}^{IJ}$ is the quantity which can be measured in experiments.

3. Three-terminal conductor

Firstly, we investigate the spin polarized conductance in a T-shape conductor with an anti-dot at the junction shown in Fig 1. The $(N_w+1) \times (N_w+1)$ junction region is attached to three reservoirs via semi-infinite ideal wires. Electrons are injected from the reservoir 1 and outgoing into reservoirs 2 and 3. The spins of the injected electrons are assumed to be unpolarized. The voltages at the reservoirs 2 and 3 are assumed to be equal.

In the following, we assume $N_w = 49, r_0 = ((N_w + 1)/2, (N_w + 1)/2)$, $\Delta r = 5$ and $\gamma = 0.1$. We focus on the transport from the reservoir 1 to 2 and omit the superscripts 21 of $G^{21}$ and $G_{Pz}^{21}$. 

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Figure 2 shows the energy dependence of the total conductance $G$ and the spin-polarized conductance $G_{P_z}$ for different potential heights $U_0$. The inset shows the spin polarization $P_z = G_{P_z}/G$. It is shown that the sign of the spin polarized conductance $G_{P_z}$ changes when the Fermi energy $E_F$ becomes larger than the potential height $U_0$. This can be easily explained by considering the interplay between SOC and the curvature of a potential [7].

In our previous work [7], we assume $N_w = 29$ and $\Delta r = 5$ so that the Gaussian potential is broaden all over the junction region. In this work, the Gaussian potential is rather sharp and localized at the center of junction. In latter case, the spin polarization is enhanced for $E_F < U_0$ but suppressed for $E_F > U_0$.

**Figure 1.** Scheme of the T-shape conductor. The shaded circular area at the junction represents an anti-dot.

**Figure 2.** The total conductance $G$ (upper panel) and spin-polarized conductance $G_{P_z}$ (lower panel) as a function of the Fermi energy $E_F$ for different potential heights $U_0$. The inset of the lower panel shows the spin polarization $P_z = G_{P_z}/G$.

4. **Four-terminal conductor**

Secondly, we investigate the spin Hall conductance $G_{sH}$ in a four-terminal conductor as shown in Fig 3. In a symmetric conductor, the spin Hall conductance is simply proportional to the spin polarized conductance from the terminal 1 to 2, $G_{sH} = (h/2e) G^2_{P_z}$ [12]. This is natural since the electrons coming back from the side reservoirs (2 and 3) are spin unpolarized. In order to compare results to those obtained in the three-terminal conductor, we again focus on the the spin polarized conductance from the terminal 1 to 2.

Figure 4 shows the energy dependence of the total conductance $G$ and the spin-polarized conductance $G_{P_z}$ for different potential heights $U_0$. In contrast to the T-shape conductor, the spin polarized conductance vanishes when the Fermi energy is getting larger than the height of anti-dot except the band center ($E_F \simeq 4$). This is simply because electrons are passing through the junction into the terminal 4. Figure 5 shows the charge and spin distribution for $E_F = 1.9$ and $U_0 = 3$. It is clearly shown that the spin-polarized current is generated at the junction.

5. **Summary**

In this paper, the spin polarized conductance in multi-terminal conductors in the presence of spin-orbit coupling is numerically investigated. We consider the three- and four-terminal conductors with an anti-dot potential at the junction. In the three-terminal conductor, the
direction of spin polarization can be controlled by changing the Fermi energy. In the four-terminal conductor, the spin Hall conductance suddenly disappears when the Fermi energy becomes larger than the height of anti-dot potential. These properties may be applicable for switching devices in the field of spintronics.

Figure 3. Scheme of the four-terminal conductor. The shaded circular area at the junction represents an anti-dot.

Figure 4. The total conductance $G$ and spin-polarized conductance $G_{Pz}$ as a function of the Fermi energy $E_F$.

Figure 5. Charge distribution (left) and spin distribution (right) for $E_F = 1.9$ and $U_0 = 3$.

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