Investigation of influence of the outlet channel length of extruder die hydrodynamics of extrusion process

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Abstract. Influence of the outlet channel length of an extruder on a steady creeping outflow of viscoelastic fluid is investigated. Fluid motion is described by equations of conservation of mass and momentum supplemented by Giesekus’s determinative rheological constitutive equation of the medium. On the basis of finite element method a numerical algorithm for solving the problem was developed. Distribution pattern of fluid velocity in an outflowing jet, pressure and stresses is analyzed; results of dependency of swelling degree of polymer fluid on die geometry and parameters of a rheological model are investigated.

1. Introduction
In the process of flowing of polymer melts and solutions through the forming channels of processing equipment different specific effects such as Barrus effect occur. This effect is of great importance in the process of product forming of polymer materials by extrusion method and represents an increase of outflowing jet diameter $D_e$ extruded from the die compared to die diameter $D_w$. In technological practice it is estimated by a coefficient $h_f$ which is equal to the ratio $D_e/D_w$ [1-5]. Narrowing of outflowing jet of Newtonian fluid at high flow velocities is shown in work [1] and for low velocities (at Reynolds numbers less than 16) widening of outflowing jet is approximately 1.1 [2]. Application of numerical methods to obtain the flow structure and the form of outflowing jet is discussed in works [3, 4]. Severity of exposure of various factors on the extrusion character of polymer melt described by the Oldroyd rheological relation is discussed in work [5]. It is shown that the swelling during extrusion from flat channels exceeds similar values for axisymmetric channels for the same values of average velocities of the polymer at the inlet.

2. Governing equations
This work considers outflowing of a viscoelastic fluid from cylindrical stepped channel its scheme is presented in figure 1. In the figure $h$ denotes a radius of the narrow channel of the jet outlet part, $4h$ – radius at the tube inlet, $h_1 = 0.5h$ – size of the rounded part, $L$ – length of the die outlet channel. For calculations values of $L$ are chosen to equal to $L = 0, 2h, 4h, 10h$. In this case we analyze the influence of outlet channel length on the structure of the output flow of the polymer fluid in the extrusion
process as well as investigate the influence of rheological parameters of the Giesekus constitutive relation characterizing the level of viscoelastic properties of the polymer material.

![Figure 1. Scheme of outflowing of the viscoelastic fluid from the cylindrical stepped channel.](image)

In a cylindrical coordinate system creeping flow of a viscoelastic fluid in the absence of gravity is described by the system equations conservation of mass and momentum which are closed by the Giesekus determinative rheological constitutive equation [6, 7]:

\[
\rho \left( \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial \sigma_{ij}}{\partial x_j}, \quad \sigma_{ij} = -P \delta_{ij} + \tau_{ij}, \quad \frac{\partial \tau_{ij}}{\partial x_i} = 0, \quad (1)
\]

\[
\tau_{ij} = \tau^v_{ij} + 2 \eta N D_{ij}, \quad \delta \tau^v_{ij} = \frac{\partial \tau^v_{ij}}{\partial t} + u^k \frac{\partial \tau^v_{ij}}{\partial x_k} - \tau^v_{ik} \frac{\partial u_i}{\partial x_j} - \tau^v_{kj} \frac{\partial u_k}{\partial x_i}, \quad (2)
\]

\[
\tau^v_{ij} + \lambda \frac{\delta \tau^v_{ij}}{\partial t} + \frac{Z_G \lambda}{\eta_V} \tau^v_{ij} = 2 \eta_V D_{ij}, \quad D_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (3)
\]

In the system of equations (1) – (3) \( u_i \) – fluid velocity components, \( P \) – pressure, \( \sigma_{ij} \) – total stress, \( \tau_{ij} \) – stress deviator, \( \delta_{ij} \) – unit tensor, \( \rho \) – density, \( \eta_V, \eta_N \) – dynamic viscosities of polymer and solvent respectively, \( \lambda \) – stress relaxation time of a polymer fluid, \( \chi_G \) – dimensionless parameter of Giesekus rheological model.

For a free surface described by the equation \( F(x, t) = 0 \) kinematic relation is satisfied [8]

\[
\frac{\partial F}{\partial t} + u_i \nabla F = 0 \quad (4)
\]

Approximation of the equations (1) - (4) and calculations are carried out by finite element method of second order with linear and quadratic approximations of the unknown functions [2].

### 3. Results

Steady extrusion of polymer fluid from the stepped cylindrical die with the following physical properties: \( \rho = 924 \text{ kg/m}^3 \), \( \eta_V = 1.2 \cdot 10^4 \text{ Pa·s} \), \( \eta_N = 0.9 \cdot 10^3 \text{ Pa·s} \) – shear viscosity at zero shear rate is considered. Channel radius of a narrow part of the die \( h = 10^{-2} \text{ m} \), average velocity of a supplied fluid at the inlet \( U_0 = 4.0 \cdot 10^{-1} \text{ m/s} \). Relaxation time characterizing the viscoelastic properties of a fluid was varied in the range of \( \lambda = 0.1-10 \text{ seconds} \). For such values of selected parameters the following values of dimensionless criteria are obtained: Reynolds number determined as \( \text{Re} = \rho U_0 L / (\eta_V + \eta_N) = 10^{-4} \), Weissenberg number \( \text{We} = \lambda U_0 / L \), varying in the range of 0.1-5. As \( U_0 \) an average fluid velocity at the inlet is considered, \( L \) – characteristic dimension, which is equal to the radius of a narrow part of the channel \( h \).

Paper [5] presents experimental data on the degree of swelling of a viscoelastic fluid outflowing from a cylindrical channel in case when polystyrene was used as a polymer fluid. For comparison with experimental data fluid Oldroyd-B is selected as rheological model its constitutive equation is...
described by equations (1) – (3). Figure 2 shows comparison of calculated and experimental data \( h_f \) [5] in the range of \( We \) numbers from 0.1 to 5.

Figure 2. Dependency of \( h_f \) on \( We \) for the fluid Oldroyd-B: curve 1 – calculations, curve 2 – experiment [5].

Quite satisfactory agreement of the numerical predictions for different grids with experimental data indicates the adequacy of the developed numerical methods for calculating flows of Newtonian and viscoelastic fluids with free surfaces. Figure 3a shows a dependency of swelling degree \( h_f \) of outflowing jet of a polymer fluid on the value of \( We \) number for different dies. Forms of the dies differ with length of narrow outlet part \( L \) varying from 0 to 10\( h \). Results show that swelling degree \( h_f \) of the jet increases with decreasing of the length of outlet section. It can be seen that the values \( h_f \) depend on the kinematics of the preceding deformation - shear rate and length of the output part of the capillary. This can be explained by the fact that post-extrusion swelling in the processing of polymers is caused by the release of elastic energy stored during the previous flow in a channel. Viscoelastic fluids are the mediums possessing a “hereditary” memory. A hereditary fluid remembers prehistory, i.e. the change of the velocity field in the previous times. Duration of memory is characterized by time of relaxation processes \( \lambda \), which is determined by the parameter \( We \). It can be seen that with increasing the \( We \) number the swelling degree increases for all dies. The investigation results of influence of a rheological parameter of the Giesekus model \( \chi_G \) on a swelling process under an isothermal flow is

Figure 3. Degree of Barrus effect \( h_f \) depending on \( We \) number a) for various dies: 1 – \( L = 10h \), 2 – \( L = 4h \), 3 – \( L = 2h \), 4 – \( L = 0 \); b) for various values of parameter \( \chi_G \) for the die with length \( L = 0 \), 1 – \( \chi_G = 0.86 \), 2 – \( \chi_G = 0.33 \), 3 – \( \chi_G = 0.1 \).
shown in figure 3b, which shows the dependency of a swelling degree of the outflowing jet of a polymer fluid on the value of We number for the die with the length \( L = 0 \) when increasing the parameter \( \chi_G \) of the Giesekus model. The increase of this parameter for viscoelastic fluids means that they possess a greater viscosity anomaly with increasing the shear rate. Fluids which does not possess a viscosity anomaly are described by Oldroyd rheological equation [5] this corresponds to \( \chi_G = 0 \) in Giesekus model. It can be seen that for viscoelastic fluids showing great viscosity anomaly the swelling degree decreases during extruding from the die. This is explained by the fact that the elastic properties of a fluid play a significant role in increasing the diameter of the output jet. For low values of stress relaxation time (respectively We) behavior of the polymer mixture is close to a Newtonian fluid when the value \( h_f \) does not exceed 1.18. Figure 4 shows, as an example, a flow pattern for the die with the size of the output channel equal to \( L = 2h_f \) in the form of the streamlines (figure 4a) and contours of the first difference of principal stresses (figure 4b) at \( We = 3.0 \). From the presented data it can be seen that as soon as the polymer fluid passes the outlet section and enters the zone of a free flow pressure and stresses immediately begin to relax.

![Image of Figure 4](image)

**Figure 4.** Streamlines \( \psi \) (a); contours \( N_1 \) (b) when \( We = 3.0 \) for the die \( L = 0 \).

The relaxation process takes place until the accumulated highly elastic deformation falls below a value corresponding to the elastic deformation of the polymer in a state of steady flow. We distinguish two main reasons for the change of the jet cross-section:

- reconstruction of a velocity profile from a parabolic corresponding to the flow in a channel, to a rectangular corresponding to the jet movement as a continuous solid rod;
- relaxation of the highly elastic tensile strains accumulated in the jet material during its passage through the channel.

Figure 5 presents fluid velocity profiles for the five cross-sections for the die of length \( L = 0 \). These data show the influence of the viscoelastic properties of the fluid on the character of their outflow from the die. For low values of We number the behavior of polymer fluids due to a slight appearance of viscoelastic properties is close to Newtonian (figure 5a). The figure also shows that redistribution of the velocity profile from parabolic to one-dimensional occurs at the channel outlet with increasing the distance. In this case influence of viscoelastic properties of the fluid on the swelling degree and change of the character of velocity redistribution in the outlet flow becomes apparent. The general trend is reduction of the velocity along the central axis with increasing distance and the difference is that for viscoelastic fluids, because of the greater swelling the steady speed in the output flow is significantly less. One of the essential nonlinear distinctions is the difference in the velocity distribution along the surface of the output flow. If for a Newtonian fluid a velocity along the surface increases monotonically up to a steady value, for viscoelastic fluid velocity on the surface increases and then decreases.
Figure 5. Velocity profiles in transverse cross-section in the output jet for $1 - z/h = 0.0, 2 - 0.50, 3 - 0.75, 4 - 1.0, 5 - 3.0$ for the die $L = 0$ when $We = 0.1$ (a) and $We = 3.0$ (b).

Figure 5b shows that steady-state value of the velocity at the border for the cross section $z/h = 3.0$ (curve – 5) is less than the values for $z/h = 0.75, 1.0$ (curves 3 and 4). Overspeeding at the border is not so significant compared with the Newtonian case. More significant difference appears in the character of the velocity distribution across the flow. For a Newtonian fluid minimum velocity value is at the jet border which then monotonously increases toward the centre of the flow. Influence of viscoelastic fluid properties reflected in the fact that the velocity in cross-section is minimal not on the flow border, but is located on a certain deepening from the surface. Velocity profile across the flow is nonmonotonic, the maximum velocity at the central axis of the flow decreases as it approaches the border, but then near the surface it begins to increase again. Thus the greatest gradients of the velocity change are observed in the immediate vicinity of the channel outlet, and the minimum value of the velocity in the cross section begins to increase with increasing the distance from the outlet. Also as a significant difference it should be noted that a nonmonotonicity of the velocity in cross section is observed along the entire length of velocity establishment and reconstruction up to a homogeneous flow. The results of numerical modeling of the outflowing of a viscoelastic fluid from the die based on the Giesekus fluid model showed that for simulating the effect of fluid swelling it is necessary to choose a non-linear viscoelastic models that take into account the first and second difference of normal stress in shear flow. The existence of normal stresses is one of the reasons for the display of Barrus effect.

Presented results of the calculations of the extrusion process of a viscoelastic fluid from the stepped forming dies with different length of the output part showed that the swelling degree of the output jet increases for the short dies. These results support the conclusion that one of the main factors influencing the swelling degree of the hereditary viscoelastic mediums is a stress relaxation time.

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