FORMATION RATES OF POPULATION III STARS AND CHEMICAL ENRICHMENT OF HALOS DURING THE REIONIZATION ERA

Michele Trenti\textsuperscript{1,2} and Massimo Stiavelli\textsuperscript{2}

\textsuperscript{1} University of Colorado, Center for Astrophysics and Space Astronomy, 389-UCB, Boulder, CO 80309, USA; trenti@colorado.edu
\textsuperscript{2} Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, USA

Received 2008 October 16; accepted 2008 December 30; published 2009 March 20

ABSTRACT

The first stars in the universe formed out of pristine primordial gas clouds that were radiatively cooled to a few hundreds of degrees Kelvin either via molecular or atomic (Lyman-\(\alpha\)) hydrogen lines. This primordial mode of star formation was eventually quenched once radiative and/or chemical (metal enrichment) feedbacks marked the transition to Population II stars. In this paper, we present a model for the formation rate of Population III stars based on Press–Schechter modeling coupled with analytical recipes for gas cooling and radiative feedback. Our model also includes a novel treatment for metal pollution based on self-enrichment due to a previous episode of Population III star formation in progenitor halos. With this model, we derive the star formation history of Population III stars, their contribution to the reionization of the universe and the time of the transition from Population III star formation in minihalos (\(M \approx 10^6\ M_\odot\), cooled via molecular hydrogen) to that in more massive halos (\(M \gtrsim 2 \times 10^7\ M_\odot\), where atomic hydrogen cooling is also possible). We consider a grid of models highlighting the impact of varying the values for the free parameters used, such as star formation and feedback efficiency. The most critical factor is the assumption that only one Population III star is formed in a halo. In this scenario, metal-free stars contribute only to a minor fraction of the total number of photons required to reionize the universe. In addition, metal-free star formation is primarily located in minihalos, and chemically enriched halos become the dominant locus of star formation very early in the life of the universe—at redshift \(\approx 25\)—even assuming a modest fraction (\(0.5\%\)) of enriched gas converted in stars. If instead multiple metal-free stars are allowed to form out of a single halo, then there is an overall boost of Population III star formation, with a consequent significant contribution to the reionizing radiation budget. In addition, the bulk of metal-free stars are produced in halos with \(M \gtrsim 2 \times 10^7\ M_\odot\).

Key words: early universe – cosmology: theory – galaxies: high-redshift – ISM: evolution – stars: formation

Online-only material: color figures

1. INTRODUCTION

Population III stars are considered to be the first luminous objects formed during the dark ages of the universe, when the hydrogen was in a neutral state (e.g., see Bromm & Larson 2004). The first generation of stars, formed out of pristine primordial gas, had a top-heavy initial mass function, with a typical mass scale of order of \(\approx 100\ M_\odot\) and most probably just one star per halo (e.g., see Abel et al. 2002; O’Shea & Norman 2007). These stars started forming after about 30–40 million years from the big bang at redshift \(z \approx 55–65\) (Naoz et al. 2006; Trenti & Stiavelli 2007; see also Gao et al. 2005) and, given their high mass, they live only a few million years ending with their high mass, they live only a few million years ending with a black hole (Heger et al. 2003).

Population III stars thus initiate the chemical enrichment of the universe and open the way to more normal modes of star formation, namely Population II (e.g., see Ostriker & Gnedin 1996; Furlanetto & Loeb 2003). In fact, the metals released into the intergalactic medium (IGM) after a pair-instability supernova explosion can travel outside the parent dark matter halo that hosts the Population III star. Calculations by Bromm et al. (2001) found that a region containing up to about \(10^6\ M_\odot\) can be enriched to a critical metallicity \(Z_{\text{crit}} \gtrsim 10^{-4}\ Z_\odot\) by the most massive pair-instability supernovae. More typical explosions may instead enrich significantly less gas (\(\approx 10^5\ M_\odot\)) although at a correspondingly higher metallicity (see Bromm et al. 2003; Kitayama & Yoshida 2005; Greif et al. 2007; Whalen et al. 2008). Even in the latter case, a halo of mass \(\lesssim 10^8\ M_\odot\) that had one of its progenitors hosting a pair-instability supernova is still likely to be enriched to an average metallicity of \(\gtrsim 10^{-4}\ Z_\odot\), thanks to violent relaxation mixing (Lynden-Bell 1967) during its hierarchical build-up.

Population III stars are also the sources that start to reionize the universe, creating ionized islands within the neutral hydrogen interstellar and intergalactic medium. Ionizing photons are emitted with an enhanced efficiency compared to Population II stars due to the high effective temperatures of massive metal-free stars (Tumlinson & Shull 2000; Schaerer 2002), and these sources could be responsible for a significant fraction of the Thompson optical depth to reionization deriving from \(z > 7\) (Shull & Venkatesan 2008). Another hint suggesting that Population III stars contribute significantly to the reionization of hydrogen can also be inferred by the rapid evolution of the galaxy luminosity function at \(z > 6\), which implies that observed galaxies alone do not seem capable of reionizing the universe (e.g., see Oesch et al. 2009; Bouwens & Illingworth 2006).

Two main modes of Population III star formation have been proposed: either in minihalos with virial temperature of \(T_{\text{vir}} \approx 10^3\ K\), where the gas is cooled via molecular hydrogen (H\(_2\)), or in more massive, rarer, halos with \(T_{\text{vir}} \approx 10^4\ K\), where cooling through atomic hydrogen (Lyman-\(\alpha\)) lines becomes possible (e.g., see Bromm & Larson 2004). H\(_2\) is formed during the initial collapse of the gas within the minihalo, but it is sensitive to photo-dissociating radiation in the Lyman–Werner...
formation of quasi-stars (Begelman et al. 2006) at \( z \approx 15 \) which have been proposed as progenitors of the supermassive black holes present after the end of reionization. Quasi-stars are in fact able to form only if the gas is not polluted by metals (Omukai et al. 2008). Our novel approach to self-enrichment is based on the properties of the Gaussian random field of the primordial density fluctuations, which allow us to derive a closed form for the probability that a dark matter halo of mass \( M_1 \) at redshift \( z_1 \) had at redshift \( z_2 > z_1 \) a progenitor of mass \( M_2 > M_1 \) (Trenti & Stiavelli 2007). We then combine our results on self-enrichment with the probability of wind pollution derived by Furlanetto & Loeb (2003) to infer the overall likeliness of collapse of pristine gas in halos with \( T_{\text{vir}} \approx 10^4 \) K.

This paper is organized as follows. In Section 2, we introduce our model for Population III star formation, including radiative and self-enrichment feedback. The model is applied in Section 3 to derive the global Population III star formation rate and in Section 4 to obtain the enrichment probability of \( T_{\text{vir}} \approx 10^4 \) K dark matter halos. Section 5 discusses the implications in terms of contributions to reionization from Population III stars and Section 6 concludes.

2. POPULATION III STAR FORMATION MODEL

To derive the star formation rate of Population III stars, we combine the dark matter halo formation rate with an analytical model to populate halos with stars. In this paper, we assume a flat concordance \( \Lambda \)CDM cosmology, with the cosmological parameters given by the WMAP \( \Lambda \)CDM best-fitting parameters (Komatsu et al. 2009): \( \Omega_\Lambda = 0.72, \Omega_m = 0.28, \Omega_b = 0.0462, \sigma_8 = 0.817, n_s = 0.96, h = 0.7 \). We also assume a primordial helium mass fraction \( Y = 0.2477 \) (Peimbert et al. 2006).

2.1. Minimum Minihalo Mass for Population III Formation

The minimum dark matter halo mass \( M \) capable of cooling by molecular hydrogen at redshift \( z \) is estimated by requiring the cooling time \( \tau_{\text{cool}}(M, z) \) to be no larger than the local Hubble time \( t_H(z) \) (e.g., see Couchman & Rees 1986). We write the cooling time as

\[
\tau_{\text{cool}}(M, z) = \frac{3k_B T_{\text{vir}}(M, z)}{2\Lambda(T_{\text{vir}}, n_H) f_{\text{H}_2}},
\]

where \( k_B \) is Boltzmann’s constant and \( T_{\text{vir}}(M, z) \) is the virial temperature of the halo, \( \Lambda \) is the cooling function per H\(_2\) molecule, which depends on the temperature and the hydrogen number density \( n_H \), and \( f_{\text{H}_2} \) is the molecular to atomic hydrogen fraction. We write \( T_{\text{vir}} \) following Tegmark et al. (1997)

\[
T_{\text{vir}}(M, z) \approx 2554 K \left( \frac{M}{10^6 M_\odot} \right)^{2/3} \left( \frac{1 + z}{31} \right)^{2/3}.
\]

For the molecular hydrogen cooling function \( \Lambda \), we use the form derived by Galli & Palla (1998), which we approximate between the temperatures of 120 K and 6400 K (the range we are interested in for cooling in minihalos) with

\[
\Lambda(T, n_H) \approx 10^{-31.6} \times \left( \frac{T}{100K} \right)^{3.4} \times \left( \frac{n_H}{10^{-4} \text{ cm}^{-3}} \right) \text{ erg s}^{-1} \text{ cm}^{3} \text{ molecule}^{-1} \text{ s}^{-1}.
\]

We estimate the hydrogen number density in a halo from its virial density (e.g., Equation (22) of Tegmark et al. 1997) to find

\[
n_H \approx 1.01 \left( \frac{1 + z}{31} \right)^{3} \text{ cm}^{-3}.
\]
Replacing Equations (2)–(4) in Equation (1), we find
\[
\tau_{\text{cool}} \simeq 3.46 \times 10^{10} s \left( \frac{M}{10^{6} M_{\odot}} \right)^{-1.6} \left( \frac{1+z}{31} \right)^{-5.4} f_{H_{2}}^{-1}.
\]
We can then obtain the molecular hydrogen fraction required for cooling by equating the cooling time given by Equation (5) to the local Hubble time \( t_{H} \), approximated at \( z \gg 1 \) as
\[
t_{H}(z) \simeq \frac{1.191 \times 10^{15} s}{h \sqrt{\Omega_{m}}} \left( \frac{1+z}{31} \right)^{-3/2} \simeq 3.216 \times 10^{15} s \left( \frac{1+z}{31} \right)^{-3/2}.
\]
This gives us
\[
f_{H_{2}} \simeq 1.09 \times 10^{-5} \left( \frac{M}{10^{6} M_{\odot}} \right)^{-1.6} \left( \frac{1+z}{31} \right)^{-3.9}.
\]
(Tegmark et al. (1997) determine the (maximum) molecular hydrogen fraction capable of forming in a halo as
\[
f_{H_{2,\text{max}}} \simeq 3.5 \times 10^{-4} \left( \frac{T}{1000 K} \right)^{1.52}.
\]
Equating the required molecular hydrogen fraction for cooling within a Hubble time given by Equation (7) with the maximum that can form (assuming \( T = T_{\text{vir}} \)), we find the minimum mass for a minihalo in order to cool within a Hubble time \( (M_{H_{2}-\text{cool}}) \), namely
\[
M_{H_{2}-\text{cool}} \simeq 1.54 \times 10^{5} M_{\odot} \left( \frac{1+z}{31} \right)^{-2.074}.
\]
\[2.2. Minimum H_{2} Cooling Mass in the Presence of Radiative Feedback\]
To compute the effect of a radiative flux in the LW band on the formation rate of molecular hydrogen and on the cooling of a minihalo, we resort to an approach based on Machacek et al. (2003). We obtain the minimum halo mass capable of cooling via \( H_{2} \) in the presence of an LW background by equating the timescale for photodissociation of molecular hydrogen \( (\tau_{\text{diss}}) \) to its formation timescale \( (\tau_{\text{form}}) \).

In presence of an LW flux \( f_{LW} = 4\pi J_{21} \times 10^{-21} \text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \) (whose calculation is presented in Section 2.7), the dissociation timescale can be written as (Machacek et al. 2003)
\[
\tau_{\text{diss}} \simeq \frac{7.16 \times 10^{11} s}{J_{21}}.
\]
The \( H_{2} \) formation timescale is given by
\[
\tau_{\text{form}} = \frac{n_{H_{2}}}{k_{\gamma} n_{H} n_{e}} \simeq \frac{f_{H_{2}}}{k_{\gamma} n_{e}}.
\]
where \( k_{\gamma} \approx 1.8 \times 10^{-18} T^{-0.88} \text{cm}^{-3} \text{s}^{-1} \) is the \( H^{-} \) formation rate which dominates the formation of molecular hydrogen and \( n_{e} \) is the electron density. Here, \( n_{e} \) is obtained assuming a residual ionizing fraction \( 2 \times 10^{-4} \) (Peebles 1993). Imposing the equilibrium molecular hydrogen function to be the minimum needed for collapse as given by Equation (7), we finally find
\[
M_{H_{2}-\text{cool}} \simeq 6.44 \times 10^{6} M_{\odot} \left( \frac{1+z}{31} \right)^{-3.557}.
\]
We note that Equation (12) is in good agreement with the results by Machacek et al. (2003) at \( z \approx 30 \). Our derivation does, however, include an explicit redshift dependence. Equation (12) also compares well with the results of numerical simulations by O'Shea & Norman (2008), which include LW background of varying intensities. The redshift dependence which we find increases the minimum mass required for cooling with respect to the formula by Machacek et al. (2003) at \( z < 30 \), and this gives us a better agreement with the numerical results obtained for \( z \lesssim 25 \) (see Figure 3 in O'Shea & Norman 2008).

In conclusion, for a dark matter halo to be able to cool via \( H_{2} \), its mass must be above both the limits set by Equations (9) and (12), that is
\[
M_{\text{min}} = \max(M_{H_{2}-\text{cool}}; M_{H_{2}-\text{cool}}).
\]

\[2.3. Cooling in Halos with T_{vir} \gtrsim 10^{4} K\]

Pristine halos with a virial temperature above \( T_{\text{vir}} \simeq 10^{4} K \) can cool irrespective of the LW background intensity (O'Shea & Norman 2008). In fact, not only atomic hydrogen cooling becomes in principle available, but also cosmological simulations by O'Shea & Norman (2008) have shown that a small fraction of \( H_{2} \) can still be produced at the center of the halo, thanks to the high density and self-shielding of the surrounding gas. Once the gas temperature starts to decrease, further cooling and collapse will proceed progressively faster via molecular hydrogen as the halo temperature is initially high enough to enhance the abundance of \( H^{-} \), a precursor for \( H_{2} \) production (Lepp & Shull 1984). In our model, we thus consider that all halos above the \( M_{T=10^{4} K} \) limit will cool efficiently,
\[
M_{T=10^{4} K} = 7.75 \times 10^{6} M_{\odot} \left( \frac{1+z}{31} \right)^{-3/2}.
\]

\[2.4. Forming Population III Stars\]

There is of course a delay between the virialization of a dark matter halo potentially able to cool via \( H_{2} \), that is of mass \( M > M_{\text{min}} \) (Equation (13)), and the actual formation of a Population III star. We estimate this delay considering two contributions: (1) the actual time needed to cool down to a few hundred degrees kelvin and (2) the free-fall time for the gravitational collapse once cooling has triggered the Jeans instability.

The \( H_{2} \) cooling time can be obtained by Equations (5) and (8),
\[
\tau_{\text{cool}} = 2.38 \times 10^{13} s \left( \frac{M}{10^{6} M_{\odot}} \right)^{-2.627} \left( \frac{1+z}{31} \right)^{-6.94},
\]
while the free-fall time can be obtained from the Jeans instability timescale, taking into account that during the cooling phase the density of the gas has increased by about a factor 7,
\[
t_{ff} = 2.77 \times 10^{14} s \left( \frac{1+z}{31} \right)^{-3/2}.
\]

Therefore, a Population III star originated in dark halo virialized at redshift \( z_{\text{vir}} \) will be formed at redshift \( z_{\text{form}} < z_{\text{vir}} \) such that
\[
\tau_{\text{cool}} + t_{ff} = 3.21 \times 10^{15} s \left[ \left( \frac{1+z_{\text{form}}}{31} \right)^{-3/2} - \left( \frac{1+z_{\text{vir}}}{31} \right)^{-3/2} \right],
\]
where the right side of the equation simply derives from the age of the universe at $z \gg 1$ (Equation (6)).

2.5. Metal Enrichment Probability

To account for previous episodes of Population III star formation in a progenitor of a halo of mass $M$ at redshift $z_1$, we resort to the method presented in Trenti & Stiavelli (2007), based on the linear growth of density perturbation in the context of spherical collapse. We start by assuming that a newly virialized halo has an average linear overdensity $\delta(z) = 1.69$ as estimated by a top-hat filter on a scale $R = (3M/4\pi \rho_0)^{1/3}$. Then, for a progenitor mass $M_{\text{prog}} < M$, we compute the extra variance in the density power spectrum $\sigma^2_{\text{add}} = \sigma^2(M_{\text{prog}}) - \sigma^2(M)$ and then the refinement factor $N_{\text{ref}} = M/M_{\text{prog}}$. With these ingredients, we can write the probability distribution for the maximum of $N_{\text{ref}}$ Gaussian random numbers with variance $\sigma^2_{\text{add}}$ as the derivative of the $N_{\text{ref}}$ power of the partition function for a normal distribution with zero mean and variance $\sigma^2_{\text{add}}$. In the context of spherical collapse this translates to a probability distribution for the formation redshift of the first progenitor of a halo $M$ virialized at $z_1$.

For every progenitor mass $M_{\text{prog}}$, we compute the delay time $t_{\text{ff}} + t_{\text{cool}}$ needed to form a Population III star in the parent halo and from this we derive the minimum redshift $(z_{\text{min,seed}})$ at which such a halo must form in order to preseed the descendant halo. Of course, for some values of $M_{\text{prog}}$ the delay time might be longer than the Hubble time, this simply means that no preseeding is possible from parent halos of mass $M \leq M_{\text{prog}}$. We then integrate the probability distribution for the formation time of the parent of mass $M_{\text{prog}}$ for $z > z_{\text{min,seed}}$ to obtain the preseeding probability from a progenitor at this mass scale. The overall probability of preseeding is the maximum preseeding probability computed over all the possible progenitor masses.

2.6. From Dark Matter to Stars

The dark matter halo formation rate is derived in our reference model using the Sheth & Tormen (1999) mass function. The Sheth & Tormen (1999) mass function is in better agreement with N-body simulations than the Press & Schechter (1974) mass function at $z \lesssim 30$ (Heitmann et al. 2006; Reed et al. 2006). Note that differences of $\approx 20\%$ have been observed between the measurements from cosmological simulations and the Sheth & Tormen (1999) mass function and that the Warren et al. (2006) mass function appears a better match to the numerical results (Lukić et al. 2007). However, the Sheth & Tormen (1999) and the Warren et al. (2006) formulae give very similar results in the range of halo masses of interest for Population III star formation ($M \lesssim 10^8 M_\odot$—see Figure 3 in Lukić et al. 2007), thus we keep the Sheth & Tormen (1999) model as our reference. We then compute the formation rate of H$_2$ Population III stars by integrating between $M_{\text{min}}(z)$ and $M_{T=10^4 K}(z)$ the number of halos per unit mass per unit redshift $dN(M,z)/dM dz$, convolved with the probability that such halos are pristine (see Section 2.5).

The characteristic mass of Population III stars and the form of their initial mass function are highly uncertain, even though they are likely very massive—of the order of $O(100 M_\odot)$ (e.g., see Bromm & Larson 2004). This expectation is based on theoretical models and numerical simulations (Abel et al. 2002; Omukai & Yoshii 2003; Yoshida et al. 2006; Gao et al. 2007), but there is some tension with the abundance patterns observed in the most metal-poor Milky Way stars, which are better explained under the assumption that their progenitor Population III stars were only moderately massive—$8 M_\odot \lesssim M < 42 M_\odot$—(Tumlinson 2006). There is however no guarantee that the progenitors of the extremely metal poor stars considered by Tumlinson (2006) are Population III stars formed before the reionization of the universe: if their progenitors formed instead in presence of a strong UV background within a reionized bubble, then their expected mass is about $\approx 40 M_\odot$ fully consistent with the inference from the observations (Yoshida et al. 2007). Within this complex scenario, we choose to adopt conventionally one Population III per minihalo (O'Shea & Norman 2007) and we consider a Salpeter (1955) mass function in the range $[50 : 300] M_\odot$ (average mass $100 M_\odot$), as suggested by the theoretical investigations. A modification in the initial mass function used in our model primarily affects the enrichment history of the IGM and thus the transition toward Population II star formation. If Population III stars are less massive than we assume, then the efficiency of metal pollution may be reduced as core collapse supernovae explosions are not as energetic as pair instability ones (Heger et al. 2003). A mass function more biased toward very massive stars with $M > 270 M_\odot$ would also reduce the efficiency of metal pollution, because these stars directly collapse into black holes without an explosive phase (Heger et al. 2003).

The formation rate for Population III stars in halos with $T > 10^4 K$ is similarly computed from the dark matter halo mass function for $M > M_{T=10^4 K}$, again after convolution with the probability that the gas forming new halos has not been contaminated by metals. As no sign of fragmentation has been found during the collapse of metal-free halos with masses up to $2 \times 10^5 M_\odot$ (O'Shea & Norman 2008), we adopt the same initial mass function as for H$_2$ cooled Population IIIs (one star per halo, Salpeter in $[50 : 300] M_\odot$). The efficiency of star formation is however uncertain, and thus we explore different models where multiple Population III stars in a single halo are allowed, adopting a reference star formation efficiency (star-to-gas mass ratio) of $\epsilon_{\text{PopIII}} = 0.005$ and $\epsilon_{\text{PopII}} = 0.05$.

Finally, the star formation rate in enriched gas (Population II stars) is computed by convolving the dark matter halo mass function with the preseeding probability (the complement of the pristine probability) and assuming a star formation efficiency of $\epsilon_{\text{PopII}} = 0.005$ or $\epsilon_{\text{PopII}} = 0.05$ (to explore the uncertainties in this parameter), a Salpeter mass function in the range $[1 : 100] M_\odot$ (average mass $\approx 3 M_\odot$) and an average metallicity $10^{-4} Z_\odot$. Our choice of the IMF for metal-enriched gas reflects the expectation that the typical mass of stars was higher at higher redshift (Tumlinson 2007). In our model, the IMF of metal-enriched star formation impacts the radiative LW feedback, but only in a minor way due to its self-regulating nature (see Section 3).

2.7. Flux in the LW Band

We compute the LW flux that enters Equation (10) by two different approaches: (1) self-consistently from our model, based on the star formation rate and (2) adding a reference number of LW photons to those derived self-consistently in order to take into account Population II formation not included in our model. The first method is most suitable at $z \gtrsim 20$, when Population III are most likely the dominant sources of radiation. At a lower redshift protogalaxies become more and more common, the universe starts becoming reionized and our simple model for star formation does not capture all the
Population II formation that is available, and therefore using a reference LW photons production provides a good check on the validity of our assumed LW flux.

For the self-consistent LW flux calculation, we obtain that Population III stars following our assumed IMF emit an LW flux that is 7.5% of the ionizing flux (Schaerer 2003). Metal-enriched stars have instead a higher ration of LW to ionizing photons, because these stars have a lower effective temperature, but their LW photon yield per unit mass is also lower. We assume the following photon yields over the star lifetime (based on Schaerer 2003):

1. Population III: \(8 \times 10^{60} \text{M}^{-1}\);  
2. Population II: \(8 \times 10^{69} \text{M}^{-1}\).

The comoving LW photon density \(n_{\text{LW}}\) is then associated with a flux

\[
J_{21} = 1.6 \times 10^{-65} \left( \frac{n_{\text{LW}}}{1 \text{Mpc}^{-3}} \right) \left( \frac{1 + z}{31} \right)^3 \text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}. \tag{18}
\]

To compute \(n_{\text{LW}}(z)\), we only consider star formation that has happened within a redshift interval such that the photons have not been redshifted out of the LW band on average. This means that the upper limit, expressed in terms of the redshift, is \(z_{\text{up}} = 12.39/11.18(1 + z) - 1\). We also take into account the screen provided by primordial H\(_2\) present in the IGM outside virialized halos, which can absorb LW photons. Following Trenti & Stiavelli (2007), a flux of \(J_{21} = 1.58 \times 10^{-3}\) is needed to photodissociate a molecular hydrogen density of \(10^{-6}\) times the neutral hydrogen density. Of course, a (very) small residual fraction of H\(_2\) will still be present but the re-formation of H\(_2\) outside virialized structures is strongly suppressed, not only by the radiative feedback but also by the decreased average density of the universe compared to that at the time of primordial H\(_2\) formation (\(z \approx 200\); see Peebles 1993).

For the LW flux based on a fixed reionization history of the universe, we assume that the number of LW photons that produced self-consistently by Population III plus a contribution from other sources which reaches \(n_{\text{LW}} = 7 \times 10^{66}\) at \(z = 10\) (corresponding to \(J_{21} \approx 4.9\)). As a model for the redshift dependence of \(n_{\text{LW}}\), we take inspiration from the rapid growth of the fraction of mass in collapsed dark matter halos that can host stars and write

\[
n_{\text{LW}}(z) = 7 \times 10^{66} \times 10^{3.3 - 3.3(1+z)/11}. \tag{19}\]

The flux in the LW band approximately matches the reionizing flux given for a stellar population with a Salpeter IMF in the range [1 : 100] \(M_{\odot}\) and metallicity \(Z \approx 10^{-3} Z_{\odot}\) (Schaerer 2003). Thus, Equation (19) implies that about one ionizing photon per hydrogen atom is emitted at redshift 10, a budget still well short of what is required for reionization after considering the clumpiness of the IGM and recombination.

3. POPULATION III STAR FORMATION RATES

From our fiducial model, which has one first star per halo (see Table 1 for a summary of the main parameters), it is immediate to note that Population III stars initiate the chemical enrichment of the universe well before redshift \(z = 50\) (see Figure 1). This result derives from the relatively small mass required at very high redshift to be able to cool via molecular hydrogen. Such mass is in fact as low as \(M \approx 4 \times 10^4 M_{\odot}\) at \(z = 60\). At lower redshift, the minimum mass for cooling progressively increases but halos capable of cooling become more and more common, so the Population III star formation rate steadily increases. Eventually, around redshift \(z \approx 35\), radiative feedback in the LW band starts to significantly increase the mass required for cooling and the star formation rate of this class of Population III stars levels off at \(\approx 10^{-3} M_{\odot} \text{Mpc}^{-3} \text{yr}^{-1}\), that is about one star in a comoving Mpc\(^3\) formed per unit redshift. In our reference model, the self-shielding mass becomes larger than that of a halo made entirely of pristine gas is able to grow independently of the LW background intensity.

Forming Population III stars in these more massive halos is possible only if there were no previous episode of star formation within their progenitors. Thus, as long as cooling can be efficiently achieved via H\(_2\), halos with \(T_{\text{vir}} > 10^4\) K are most likely chemically enriched (see the bottom right panel of Figure 1) and Population III stars in these halos are very rare compared to their counterparts in minihalos (see the upper left panel of the same figure). Their star formation rate becomes higher than that in minihalos only at \(z \lesssim 14\), when the LW feedback strongly suppresses H\(_2\) cooling, and thus it is more likely that a halo made entirely of pristine gas is able to grow via mergers to reach \(T_{\text{vir}} > 10^4\) K before having the possibility to cool.

The majority of halos with \(T_{\text{vir}} > 10^4\) K at \(z \gtrsim 16\) have been instead chemically enriched and Population II star formation grows rapidly in time. By \(z \approx 26\), it becomes the dominant factor in the global star formation rate, despite our conservative
assumption that only 0.5% of the gas is converted into stars (but note that this is still larger than the efficiency in Population III stars at $z \lesssim 30$ if only one per halo is formed). This early rise of Population II is a novel result which derives from our detailed treatment of self-enrichment. When metal transport is instead modeled via winds, then transition toward Population II stars is predicted significantly later, at $10 \lesssim z \lesssim 20$ (Furlanetto & Loeb 2003, 2005).

The qualitative picture described in our standard model holds even if we vary the free parameters that regulate radiative feedback. In Figure 2, we show the star formation history when LW feedback is strongly suppressed. Population III stars are formed at a higher rate at $z \lesssim 30$ compared to the standard model, but the difference is only a factor of a few (here the formation rate goes up to $\approx 5 \times 10^{-5} M_\odot \text{Mpc}^{-3} \text{yr}^{-1}$). This is about one-fourth of the peak-formation rate without any radiative feedback (see Figure 3). The main difference from the standard scenario is that Population III stars formed in halos with $T_{\text{vir}} \geq 10^4$ K are now suppressed even at $z \lesssim 15$, because the enrichment probability of massive halos remains high (see the bottom right panel in Figure 2). The formation of Population II stars is instead essentially as in the standard model. Note that the picture does not deviate significantly from the standard model if we vary the efficiency of star formation in Population II stars, except of course for a corresponding proportional variation of their star formation rate. In fact, Population III stars in minihalos are the main agents of the radiative feedback that leads to their suppression at $z \gtrsim 20$ (see the bottom left panel in Figures 1 and 2). Therefore, their formation is self-regulated and tends to reach an equilibrium level. Similar results hold even when we add a stronger radiative feedback based on the fixed LW background given by Equation (19; see Figure 4). Note that in

Figure 1. Star formation during the reionization epoch predicted by our standard model for Population III stars. Upper left panel: star formation rate vs. redshift for Population III stars in minihalos (solid black line), Population III stars in more massive halos, with $T_{\text{vir}} \geq 10^4$ K (long-dashed blue line), and for Population II stars formed out of metal-enriched gas (short-dashed red line). Upper right panel: minimum dark matter halo mass required to form a Population III star via $H_2$ cooling (solid black line) and in halos with $T_{\text{vir}} \geq 10^4$ K (long-dashed blue line) in function of redshift. Lower left panel: $J_{21}$ flux in function of redshift with contribution from Population III stars (black shaded area) and from Population II stars (red shaded area). For $z \gtrsim 22$ Population III stars are the main source of radiative feedback. Lower right panel: probability of metal enrichment via progenitor pollution for a halo with $T_{\text{vir}} = 10^4$ K. The results have been obtained using a WMAP5 cosmology, a Sheth & Tormen (1999) halo mass function and our model includes cooling and $J_{21}$ feedback. The star formation rate is one star per halo for Population III stars and $5 \times 10^{-3}$ for Population II stars.

(A color version of this figure is available in the online journal.)
this case the background radiation greatly exceeds that created by Population III stars at $z \lesssim 20$, and thus the suppression of H$_2$ cooling is even sharper.

Figure 5 shows a model with our standard parameters except for the use of the Press & Schechter (1974) mass function rather than the Sheth & Tormen (1999) formula. The predictions for the Population III formation rate are very different at $z > 40$, but once the self-regulated feedback phase starts the two models converge together. The strong difference at $z > 40$ originates from the fact that Population III stars are hosted in very rare peaks at such early times: in the Press & Schechter (1974) formalism these halos have $v = \delta_c^2/\sigma^2(M) \gtrsim 30$ and the ratio of the Sheth–Tormen to Press–Schechter mass function is proportional to $\exp(-0.707v)/\exp(-v)$ in the limit of very large $v$. An interesting open question is the form of the mass function for such rare peaks, which are expected to be progressively more spherical as they become rarer (Bardeen et al. 1986). Fortunately, the difference in the star formation rate does not propagate significantly below $z \lesssim 30$.

Figure 2. As in Figure 1 but for our model with a reduced escape fraction, which implies a less efficient LW feedback. In this case, the formation rate of H$_2$ Population III stars is increased, while Population III stars in halos with $T_{\text{vir}} \gtrsim 10^4$ K are partially quenched by chemical enrichment compared to Figure 1. (A color version of this figure is available in the online journal.)

A major qualitative change in the star formation history during the Dark Ages arises if we allow for multiple Population III stars in a single halo (see Figure 6). In this case, the metal-free star formation rate is comparable to that of second generation of stars (Population II) down to $z \approx 10$. From the star formation rate, we can identify two eras: an early period dominated by Population III in minihalos up to $z \approx 18$ and a later period where larger halos are still able to form metal-free stars. This happens because by allowing multiple Population III stars in minihalos their formation rate is significantly enhanced at later redshift over the assumption of a single star per halo. In fact, when a significant LW background is present, a single minihalo can have up to $10^6 M_\odot$ of gas; thus an $\epsilon_{\text{PopIII}} = 5 \times 10^{-3}$ corresponds to an SFR 50 times higher than the one obtained for a single metal-free star per halo. In this scenario, metal-free stars formed in $T_{\text{vir}} = 10^4$ K halos appear to be several orders of magnitude more common than in our standard scenario, again because the star formation efficiency is increased by more than two orders of magnitude compared to our standard model. They are therefore
Figure 3. As in Figure 1 but for our model with no LW feedback. Note that in this model, $J_{21}$ does not influence star formation, hence it is not shown. In this model, quenching of Population III stars in the more massive $T_{\text{vir}} \geq 10^4$ K halos is further enhanced compared to Figure 2.

4. CONSEQUENCES FOR THE CHEMICAL ENRICHMENT OF $T_{\text{vir}} = 10^4$ K HALOS

The first generation of Population III stars, formed in mini-halos, releases metals into the IGM and opens the way to metal-enriched star formation when the gas recollapses later as part of a larger halo. When the second-generation halo has a virial temperature $T_{\text{vir}} \approx 10^4$ K, the metallicity of its gas is expected to have been enriched to $Z \gtrsim 10^{-2} Z_\odot$, high enough to mark a transition toward a different mode of star formation, especially if some dust is present (Schneider et al. 2006; Clark et al. 2008). Our model allows us to quantify the likelihood of this self-enrichment scenario as a function of the redshift of formation of a $T_{\text{vir}} = 10^4$ K halo (see the bottom right panel in Figure 1). Interestingly, the probability of having a pristine halo large enough to reach $T_{\text{vir}} = 10^4$ K is small at very high redshift ($z \approx 30$) and progressively increases as the redshift decreases.

This apparently surprising result can be understood in terms of the difference in halo mass required to cool via molecular and atomic hydrogen. At $z \gtrsim 30$, the difference in the two masses is large (see the upper right panel in Figure 1); hence, it is highly likely that in the merger tree of the $T_{\text{vir}} = 10^4$ K halo there has been a progenitor halo that was able to form a Population III star via H$_2$ cooling at an earlier redshift. However, as the redshift decreases, the LW self-shielding mass grows and it becomes progressively more unlikely that one of the progenitor halos hosted a star. Therefore, halos with $T_{\text{vir}} \approx 10^4$ K are more likely to be metal-free at lower redshift. However, our model does not include metal pollution by winds, which becomes progressively more important as the redshift decreases. In fact, the wind pollution model by Furlanetto & Loeb (2005) predicts a sharp drop in the probability of forming pristine halos around $z \approx 15$. Therefore, if we combine our reference scenario results with the Furlanetto & Loeb (2005) model for the wind enrichment (e.g., see their Figure 2), the overall picture is that the pollution probability for such halos remains high at all redshifts, probably presenting a minimum (with enrichment probability down to $\approx 50\%$) between $z = 15$ and $z = 20$. This further strengthens...
Figure 4. As in Figure 1 but for our model with a strong external $J_21$ radiative field (see Equation (19)). This model yields similar results to our standard model in Figure 1 because once the LW feedback is above the critical threshold necessary to quench star formation in minihalos, its further increase has a modest additional effect.

(A color version of this figure is available in the online journal.)

our conclusion that Population III star formation in halos with $T_{\text{vir}} \geq 10^4$ K is expected to be subdominant compared to formation in minihalos if primordial stars are formed in isolation. This conclusion becomes stronger as the LW feedback efficiency is decreased or set to zero (see Figures 2 and 3).

A different possibility for star formation in massive metal-free halos is the creation of a quasi-star, that is a black hole formed via direct collapse accreting inside a massive envelope (Begelman et al. 2008). This scenario has been proposed to explain the rapid growth of supermassive black holes, but a critical requirement for its viability is the absence of metals in the gas (Omukai et al. 2008). Our results on chemical enrichment suggest that quasi-stars are likely to be formed only in a redshift window between $z \approx 20$ and $z \approx 10$. At the lower end of this redshift window, quasi-stars might only live in regions that lack primordial galaxies, and thus have a relative suppression of structure formation compared to the average over the whole universe. The environment where the progenitors of bright high-redshift quasars live is unlikely to qualify as one of such regions (e.g., see, Trenti et al. 2008). If we assume that a bright, rare $z = 6$ QSO is located at the center of a dark matter halo of mass $M = 5 \times 10^{12} M_\odot$ (Springel et al. 2005), then we derive that its first progenitor with $T_{\text{vir}} > 10^4$ K has been formed at $z > 26$ with a confidence level greater than 0.9999. Such progenitor halo has a probability of being metal enriched greater than 98%, so it only has a small chance to host a quasi-star. Note, however, that quasi-stars can still lead to the formation of a bright $z \approx 6$ QSO if they are first formed in a relatively void region at $z \approx 15$ and then merge by $z \approx 10$ into the gas-rich environment of the main QSO progenitor halo. This scenario allows the black hole seed from the quasi-star to grow with maximal efficiency. In fact, the black hole starts with a large mass from the initial direct collapse and then it enters a gas-rich region where it can grow continuously at near Eddington limit. As a $z = 6$ bright QSO halo consists of material originating in a sphere with comoving radius larger than 3 Mpc, this is not unlikely. In fact, a wind traveling at speeds of 30 km s$^{-1}$ only covers a fraction of this distance in half a billion years. However, a detailed modeling
can be obtained only through the study of QSO merger trees that include information on the spatial distribution of the progenitors, such as those given by cosmological simulations.

5. POPULATION III STARS AND THE REIONIZATION OF THE UNIVERSE

The role of Population III stars in the reionization of the universe has been much debated in the last years, especially after the first year WMAP data release, which included a high optical depth to reionization $\tau_e = 0.17 \pm 0.03$ (Spergel et al. 2003). Several models had been proposed (Venkatesan et al. 2003; Cen 2003; Wyithe & Loeb 2003; Hui & Haiman 2003; Stiavelli et al. 2004) and many of these included a significant contribution from first stars to $\tau_e$. With the latest WMAP data release, the optical depth to reionization is rather low ($\tau_e = 0.084 \pm 0.016$ implying an instantaneous reionization redshift $z_{\text{reion}} \approx 10.0$; see Komatsu et al. 2009), and its major contribution comes from complete ionization after $z = 6$ (Shull & Venkatesan 2008). The contribution from higher redshift is limited to $\Delta \tau_e = 0.03 \pm 0.02$, providing an upper limit to the luminosity of primordial galaxies (Shull & Venkatesan 2008).

From our study it appears that the contribution of Population III stars to the total budget of reionizing photons is limited if only one star per halo is formed, even neglecting negative radiative feedback. Despite the fact that first stars are more than one order of magnitude more efficient at producing ionizing photons per unit mass than Population II stars (Tumlinson & Shull 2000; Schaerer 2003), their overall star formation rate in our standard model is significantly lower for $z \lesssim 20$. Based on our standard model and assuming a Population III formation rate of $10^{-5} M_\odot \text{Mpc}^{-3} \text{yr}^{-1}$ from $z = 35$ to $z = 10$, we obtain that about $n \approx 4 \times 10^{60} \text{Mpc}^{-3}$ ionizing photons are emitted by metal-free stars. This falls short of the number density of hydrogen atoms $n_H \approx 7 \times 10^{66} \text{Mpc}^{-3}$. Thus, after taking into account the effect of clumpiness of the IGM and of recombination, it is clear that Population III stars can reionize only a minor fraction of the hydrogen atoms, even if the escape fraction is near unity. A large escape fraction is in fact possible in minihalos ($M \lesssim 10^6 M_\odot$), but is likely significantly smaller.
in larger halos \( (M \lesssim 10^6 M_\odot) \), where the \( \text{H}\,\text{II} \) region may remain confined well within the virial radius of the host halo (see Whalen et al. 2004; Kitayama et al. 2004). The number of ionizing photons produced is smaller than the number of hydrogen atoms even in our model with no negative feedback (see Figure 3). In order for Population III stars to be a significant agent of reionization, multiple PopIII stars must be formed in a single halo. For our model with \( \epsilon_{\text{PopIII}} = 0.005 \), we obtain a cumulative ionizing photon production of \( \approx 4 \times 10^{67} \) down to \( z = 10 \). Such number of photons starts to become sufficient to contribute to reionization even for a relatively low escape fraction \( (f_{\text{esc}} \sim 0.1) \). Certainly, Population III stars are major agents of reionization if their star formation efficiency goes up one order of magnitude to \( \epsilon_{\text{PopIII}} = 0.05 \) (see Figure 7). Note that in both these scenarios with multiple metal-free stars per halo, the main sources of reionization are primordial galaxies in halos with \( T_{\text{vir}} > 10^4 \) K. In fact, the reionizing photon budget from Population III remains significant even when only one star per minihalo is formed, but clusters of metal-free stars are allowed in larger halos (see Figure 8; see also Haiman & Bryan 2006).

6. CONCLUSIONS

In this paper, we present a model for the star formation rate of metal-free (Population III) and second-generation (Population II) stars during the dark ages of the universe at \( z \geq 10 \). The model relies on dark matter halo mass function coupled with analytical prescription for cooling and collapse of gas clouds. Our model includes radiative Lyman–Werner feedback, which can suppress star formation in minihalos, and self-enrichment feedback, which marks the transition from metal free to Population II stars.

Thanks to our novel treatment of chemical enrichment, based on the formalism developed in Trenti & Stiavelli (2007), we show that halos with a virial temperature \( T_{\text{vir}} \geq 10^4 \) K are most likely to host a second generation of stars, formed from gas enriched to a metallicity \( Z \geq 10^{-4} Z_\odot \) by a progenitor Population III star in a minihalo at a higher redshift. Metal-free stars can form in halos with \( T_{\text{vir}} \geq 10^4 \) K only once the cooling of gas in minihalos is strongly suppressed by radiative Lyman–Werner feedback, which in our reference model happens at \( z \lesssim 20 \). If only one Population III star forms per dark matter halo, then their number is dominated by those formed in minihalos.
with a peak star formation rate of $\approx 10^{-5} M_\odot$ Mpc$^{-3}$ yr$^{-1}$ at $z \approx 20$. This prediction is robust and does not depend on the detail of the model. In fact, the negative radiative LW feedback acts as a self-regulator of star formation in minihalos, keeping variations of the star formation rate limited when the feedback efficiency or the halo mass function is changed.

The metal enrichment also leads to an early rise of the star formation rate of Population II stars. By redshift $z \lesssim 26$, their SFR is higher than that of Population III stars and steadily rises as the redshift decreases. In our model, we do not include positive radiative feedback that can promote H$_2$ formation in the neighborhood of a first star (Ricotti et al. 2001). If this is the case, then the transition to metal-enriched stars in halos with $T_{\text{vir}} > 10^4$ K is expected to be even more solid, because multiple Population III stars in clustered minihalos can pollute to a higher metallicity the gas that later constitutes a $T_{\text{vir}} > 10^4$ K halo. The metal enrichment probability from minihalo pollution decreases once a strong LW background is in place, so at redshift $z \lesssim 15$ halos containing pristine gas with a mass $M \gtrsim 2 \times 10^7 M_\odot$ are possible, provided that winds from protogalaxies, absent in our model, are not too efficient in polluting the IGM. Redshift $z \approx 15$ might thus be the most favorable period for the formation of quasi-stars (Begelman et al. 2008), which are however expected to reside preferentially in underdense environments, where winds are more unlikely to be present and pollute the IGM.

For this standard scenario, the contribution to reionization given by Population III stars is only indirect (they enrich the IGM and allow Population II to form). In fact, the cumulative number of ionizing photons they produce falls shorter of the number of hydrogen atoms. Metal-enriched stars in the first galaxies are thus expected to be the main agents of reionization, even though the rapid decrease of the galaxy luminosity function at $z > 6$ (Bouwens & Illingworth 2006; Oesch et al. 2009) casts some doubts on this scenario.

A main change in the Population III star formation rate, which increases their contribution to reionization, can be introduced if one releases the assumption that only a single metal-free star is formed per halo: by converting a fixed fraction of gas into primordial stars in minihalos, the feedback mechanism is less efficient because as the critical gas mass needed for cooling increases so does the number of stars produced per
halo. Therefore, in this scenario there is a constant growth of the Population III star formation rate in minihalos until this formation channel is suddenly inhibited because the minimum mass required to self-shield molecular hydrogen in the halo corresponds to a virial temperature above $10^4$ K (see Figure 6). Under this scenario, Population III stars can easily produce a significant amount of ionizing photons and could account for a significant fraction of the optical depth to reionization originating from $z > 7$. One issue to be addressed if multiple Population III stars are formed in a single halo is however the impact of local radiative feedback. The first massive star formed in a minihalo emits enough energy to completely photodissociate the gas in the halo, thus multiple stars can be formed only if there is a single star formation burst of limited time duration and very high efficiency. This is a crucial issue that can be properly addressed only once cosmological simulations of Population III star formation will be able to go past the formation of the first protostellar core and follow multiple episodes of star formation (see Wise & Abel 2008 for promising progress in this direction).

This work was supported in part by NASA JWST IDS grant NAG5-12458. Michele acknowledges support from the University of Colorado Astrophysics Theory Program through grants NASA ATP NNX07AG77G and NSF AST 0707474. We thank Britton Smith, Mike Santos, and Ravi Sheth for useful suggestions and discussions and the referee for a thorough and constructive report.

REFERENCES
Abel, T., Bryan, G. L., & Norman, M. L. 2002, Science, 295, 93
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Begelman, M. C., Rossi, E. M., & Armitage, P. J. 2008, MNRAS, 387, 1649
Begelman, M. C., Volonteri, M., & Rees, M. J. 2006, MNRAS, 370, 289
Bouwens, R. J., & Illingworth, G. D. 2006, Nature, 443, 189
Bromm, V., Ferrara, A., Coppi, P. S., & Larson, R. B. 2001, MNRAS, 328, 969
Bromm, V., & Larson, R. B. 2004, ARA&A, 42, 79
Bromm, V., Yoshida, N., & Hernquist, L. 2003, ApJ, 596, 135
Cen, R. 2003, ApJ, 591, 5
Ciardi, B., Ferrara, A., & Abel, T. 2000, ApJ, 533, 594
Clark, P. C., Glover, S. C. O., & Klessen, R. S. 2008, ApJ, 672, 757
