Axial-Vector Form Factor of Nucleons in the Isospin Medium from the Hard-Wall AdS/QCD Model

Ibrahim Atayev1 · Shahin Mamedov1,2

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Abstract
We study the axial-vector form factor of the nucleons in a constant and homogenous isospin chemical potential using holographic QCD. Nucleon mass splitting in such an isospin medium is taken into account. According to AdS/CFT correspondence, the critical value of the isospin chemical potential equals to UV- boundary value of the time component of the bulk gauge field. We calculate the axial-vector form factor of the proton and neutron in such background in the framework of the hard-wall model and plot the form factors for the isospin chemical potential critical value.

Keywords Axial-vector form factor · AdS/QCD · Hard-wall model · Isospin medium · Pion condensation

1 Introduction

In recent years, it has been found the correspondence between the classical gravitational theory in a higher dimensional space-time and conformal field theory on the boundary of this space-time. Anti-de Sitter /Conformal Field Theory (AdS/CFT) correspondence [1–3] allows a solving those problems, which cannot be solved within the perturbation theory in the quantum chromodynamics (QCD) at low energies. QCD models, which were based on direct application of the AdS/CFT correspondence, were called holographic QCD or AdS/QCD models. These are mainly two models, which are called the hard-wall and soft-wall models [4–11], and were applied to the calculation of mass spectra, couplings, decay constants, form factors, and other quantities [12–20], which can be measured in the experiments. Hard- and soft-wall models widely applied for nuclear medium studies as well, which are arise in the collision of heavy ions or protons at high energies. Depending on the energy of colliding particles, this nuclear medium can be in confinement or deconfinement (or quark-gluon plasma) phases. In the holographic QCD models, each phase is described

* Shahin Mamedov
sh.mamedov62@gmail.com

1 Institute of Physics, Azerbaijan National Academy of Sciences, H. Javid 131, AZ-1143 Baku, Azerbaijan

2 Institute for Physical Problems, Baku State University, Z. Khalilov 23, AZ-1148 Baku, Azerbaijan
by specific gravitational background. In the confined phase of the nuclear matter, the nucleons of the medium are still whole and the quarks and gluons inside them are confined.

This phase in the dual gravity theory is described by the thermal AdS space (tAdS) [21, 22]. Dual gravity for the confinement phase containing the quark fields as well was found in [22] and the background geometry was named the thermal charged AdS space (tcAdS). To study analytically (or even numerically) the effects and phenomena in a nuclear matter are highly complicated and different simplifications of the medium are available. The most simplified model for the nuclear medium is the isospin medium, where the temperature and quark number density quantities are turned off. The only quantity describing the medium is the isospin chemical potential. Such a model enables us to separate and qualitatively study the effects, which are caused by an isospin interaction of the particles with the medium.

In the holographic hard-wall model the background geometry for the isospin medium is reduced from the tcAdS space to the ordinary AdS space. Isospin medium was applied for the meson mass splitting due to the isospin interaction study in the Refs. [23–26]. Nucleons in the constant isospin background were considered in Ref. [24, 25] and the chiral symmetry breaking in the isospin field was studied in Ref. [23]. Splitting of meson decay constants because of interaction with the medium’s isospin was determined in Ref. [25].

Within the hard- and soft-wall models framework in the Refs. [14, 17, 27–29] it was investigated the nucleon’s axial-vector form factor $G_A(Q^2)$ in the vacuum. The temperature dependence of this form factor was studied in Ref. [30]. The main advantage of this model is that there are no limitations on the transferred momentum square $Q^2$ and therefore, the $G_A(Q^2)$ form factor was calculated for all values of this variable, while non-holographic models are applicable only at small or only at large values $Q^2$. The isospin medium affects the $G_A(Q^2)$ form factor as well since the nucleons have a splitting due to the isospin interaction with the medium. As a result of this interaction, this form factor should be different for the neutron and proton states, since these states have different values of the isospin projection. Here we aim to investigate the splitting of the $G_A(Q^2)$ form factor in the result of isospin interaction using the hard-wall model.

## 2 Isospin Medium Setup

We introduce the isospin medium within the holographic hard-wall model following earlier works [23–26, 31]. Since this medium is described by the $SU(2)$ group, the bulk flavor gauge group should be chosen as $SU(2)_L \times SU(2)_R$ which then is broken to $SU(2)_V$. In the bulk of AdS space there are two gauge fields $A_L$ and $A_R$ transforming under the $(1,0)$ and $(0,1)$ representations of the flavor symmetry group, respectively. The field strength tensor for these fields is

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig \left[ A_M , A_N \right],$$

with $A_M = A^\mu_M T^\mu$, where the $T^\mu$ generators are the Pauli matrices $\sigma^\mu/2$. $A_\pm$ components of the gauge fields are fixed by the axial gauge condition $A_\pm = 0$.

Complete action for the model is the sum of actions for the gravity and gauge fields $-S_V$, spinor fields $-S_F$, scalar field $-S_X$ and interaction between spinor, scalar and axial- vector fields $-S_{int}^4$.

$$S = S_V + S_F + S_X + S_{int}^4.$$

Here we do not consider an interaction with the vector field and do not include $S_{int}^V$ action into the complete action. The action for the gravity and gauge field parts will be written as:
where $\Lambda = -6/R^2$ is the cosmological constant. The AdS/CFT correspondence relates the constants $k^2$, $g^2$, the number of colors $N_c$, and the radius $R$ of space-time by the relations: $1/g^2 = N_c/(4\pi^2R)$ and $1/(2k^2) = N_c^2/(8\pi^2R^3)$.

The gauge fields $A_{M}^{(L),(R)}$ have the background $L_M$, $R_M$ and fluctuation $l_M$, $r_M$ (parts)

$$A_{M}^{(L)} = L_M + l_M, \quad A_{M}^{(R)} = R_M + r_M.$$ 

In the holographic approach, a model for the nuclear matter is constructed by the background fields $L_M$ and $R_M$, while the fluctuations $l_M$, $r_M$ are necessary for description of the vector and axial-vector mesons in the boundary theory. The homogenous and isotropic matter is described by the $L_M$ and $R_M$ fields not depending on $x_\mu$ coordinates. Moreover, only time components $L^3_t$ and $R^3_t$ are taken non-zero, because the only flavor diagonal element of them ($\alpha = 3$) corresponds to the physical quantity of the boundary matter. From these components we compose the vector and axial vector fields:

$$V^3_i = \frac{1}{2}(L^3_i + R^3_i), \quad A^3_i = \frac{1}{2}(L^3_i - R^3_i).$$

The boundary value of $V^3_i$ maps to the isospin chemical potential of the medium $u$ and $d$-quarks (or nucleons) in the deconfinement (or confinement) phase Ref. [31]. Following Refs. [24, 26] we take $L^3_t = R^3_t$, which means a Lagrangian invariance under the change of the left and right flavor subgroups in the $SU(2)_L \times SU(2)_R$ symmetry group of the model.

It is obvious $A^3_t = 0$ for this case. The $L \leftrightarrow R$ symmetry in the bulk means that in the dual boundary theory the medium nucleons are in the parity-even states. As is known, the ground states of the nucleons are parity-even ones, and the first excited state of the nucleon can be parity-even or parity-odd state. In parity-even state the nucleons have less energy than in parity-odd one [18, 32].

The field strength tensor for $V^3_M$ also is a flavor diagonal matrix and hence, it gets the following form for an Abelian field:

$$F^3_{MN} = \partial_M V^3_N - \partial_N V^3_M.$$ 

In such a way the $SU(2)$ symmetry of the $V^\mu_M$ part of gauge fields is broken down to two $U(1)$ symmetries. As a result the $V^3_M$ part of the action (1) gets the form:

$$S_1 = \int d^3x \sqrt{G} \left[ \frac{1}{2k^2}(R - 2\Lambda) - \frac{1}{4g^2} F^3_{MN} F^3_{MN} \right] + S_A + S_V.$$  (2)

$S_A$ and $S_V$ are the parts of the action corresponding to the axial-vector and vector fields. The holographic dual of the $A^3_{(\mu,\nu)} = \pm \frac{i}{\sqrt{2}} V^3_i$ fields will be the $u$ - and $d$-quarks of the boundary medium. Imposing the hard-wall cut-off on the bulk radial coordinate $z$ makes these quarks confined ones in the boundary theory.

It should be noted that the number densities of quarks (isoquarks) thus defined are zero (see [24]).

In the nuclear matter case the bulk geometry is a thermal charged AdS space (tcAdS) with radius $R$, which is the non-black brane solution of the Einstein-Maxwell system of equations obtained from (2) [23]:

$$S_1 = \int d^3x \sqrt{G} \left[ \frac{1}{2k^2}(R - 2\Lambda) - \frac{1}{4g^2} F^3_{MN} F^3_{MN} \right] + S_A + S_V.$$  (2)
Here

\[ f(z) = f_{\alpha}(z) = 1 + (q_u^2 + q_d^2)z^6. \]

The constants \( q_\alpha \) are related to the number densities \( Q_\alpha \) of the medium \( u \) and \( d \) quarks

\[ q_\alpha = \sqrt{2kQ_\alpha/\sqrt{3gR}}. \]

In the isospin medium case the number densities \( Q_\alpha \) are taken zero and consequently, the metric (3) returns into the metric of ordinary AdS space:

\[ ds^2 = \frac{R^2}{z^2} \left( -f(z) dt^2 + \frac{1}{f(z)} d\vec{x}^2 + dz^2 \right). \]  

Here

\[ \eta_{\mu\nu} = (+1, -1, -1, -1). \]

The radial coordinate \( z \) ranges in the limited area \( 0 < z \leq z_m \) due to the hard-wall cut-off. Solutions to the Maxwell equations obtained from the action (2) have the form:

\[ A_0^a = A_\alpha(z), A_i^a = 0 (\alpha = u, d; i = 1, 2, 3), A_\alpha(z) = 2\pi^2 \mu_\alpha - z^2. \]

In the isospin medium case \( (Q_\alpha = 0) \) the vector potential \( A_0^a \) is constant:

\[ A_0^a = 2\pi^2 \mu_\alpha. \]

Basic particles of the medium model are the nucleons, which are just confined \( u \) and \( d \) quarks. So, the \( A_0^a \) solution should be expressed in terms of the chemical potentials of the nucleons. Taking into account the quark content of nucleons as in [24] the isospin chemical potentials of nucleons may be defined as a sum of quark chemical potentials \( \mu_p = 2\mu_u + \mu_d \) and \( \mu_n = \mu_u + 2\mu_d \). For isospin matter with two flavors the number densities of the nucleons are zero and the \( V_3^3 \) equals \( \sqrt{2\pi^2 (\mu_p - \mu_n)} \) in the case of deconfinement phase and in the confinement phase one. Thus, \( V_3^3 \) in the dual boundary theory describes the homogenous and constant isospin background field of medium which is made of isonucleons.

\[ V_3^3 = \sqrt{2\pi^2 (\mu_p - \mu_n)} \]

When we turn on the isospin chemical potential \( \mu_I \) at zero baryon number density the pion condensation is expected to occur at a critical point. Son and Stephanov showed that, using the chiral Lagrangian at \( O(p^2) \) the phase transition to the pion condensation phase is the second order, and critical value of \( \mu_I \) is equal to the pion mass \( \mu_I = m_\pi [33-35] \).

3 Nucleons in the Isospin Medium

Nucleons in AdS/QCD are introduced by means of two bulk spinor \( \psi_1 \) and \( \psi_2 \), which are necessary for the description of the left and right components of this particles [32, 36, 37]. Action for the Dirac field in the 5 D-dimensional AdS space has a form:
where $G$ denotes $G = \det g_{MN}(M,N=0,1,2,3,5)$. For the AdS space (4) an inverse vielbein is $e^M_a = z\delta^M_a$. The covariant derivative for this field is $D_M = \partial_M + \frac{1}{8} \omega^A_{\mu AB} \gamma^5 \Gamma^A \gamma^\mu - iV_M$. Non-zero components of the spin connection are $\omega^5_A = -\omega^A_5 = \frac{i}{2} \delta^A$. The 5D-dimensional $\Gamma^A$ matrices are defined as $\Gamma^A = (\gamma^\mu, -i\delta^5)$ and obey anti-commutation relations $\{ \Gamma^A, \Gamma^B \} = 2\eta^{AB}$. For consistency with the chirality of dual boundary operator the sign of the mass is chosen positive ($m_1 > 0$) for the $\psi_1$ and negative ($m_2 < 0$) for the $\psi_2$. This chirality is related to the chirality of the boundary fermion, i.e. proton and neutron [32, 36]. Following the AdS/CFT relation one should take $m_1 = \frac{5}{2}$ and at the same time $m_2 = -\frac{5}{2}$. Dual composite fermionic operators $\psi_{1,2}$ have conformal dimension $\Delta = \frac{9}{2}$.

\[
(m_{1,2})^2 = (\Delta - 2)^2.
\]

From the action (3) one obtains the equation of motion for the bulk Dirac fields:

\[
\left[ i e^A_a \Gamma^A D_M - m_{1,2} \right] \psi^{1,2} = 0.
\]

Fourier transform for the fermion is:

\[
\psi^{1,2}_{L,R}(x, z) = \int \psi^{1,2}_{L,R}(p) \gamma^{1,2}_{L,R}(p, z) e^{-ipx} d^4 p
\]

and in the Weyl spinor representation with $\psi^{1,2}_L = \gamma^5 \psi^{1,2}_L$ and $\psi^{1,2}_R = -\gamma^5 \psi^{1,2}_R$

\[
\psi^{1,2}(p) = \begin{pmatrix} \psi^{1,2}_L(p) \\ \psi^{1,2}_R(p) \end{pmatrix}.
\]

Here the subscripts $L$ and $R$ denote the 4-dimensional chirality. The boundary fields $\psi^{(1,2)}_{L,R}(p)$ are related to each other by the 4-dimensional Dirac equation:

\[
\gamma^\mu p_\mu \psi^{1,2}_{L,R}(p) = \mp \left| p \right| \psi^{1,2}_{L,R}(p).
\]

If one takes the normalizable modes as $f^1_L$ and $f^1_R$ for $\psi^1_L$ and $\psi^1_R$ respectively, the chirality of the $SU(2)_L \times SU(2)_R$ flavor group can be associated with the 4-dimensional chirality. In terms of the Weyl fermions, the equations in (6) are reduced to the following ones:

\[
\begin{pmatrix} \partial_z - \frac{\Delta}{z} & 0 \\ 0 & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f^1_L \\ f^1_R \end{pmatrix} = \left( \left| p \right| - V_t \right) \begin{pmatrix} f^1_L \\ f^1_R \end{pmatrix},
\]

and

\[
\begin{pmatrix} \partial_z - \frac{\Delta}{z} & 0 \\ 0 & \partial_z - \frac{4-\Delta}{z} \end{pmatrix} \begin{pmatrix} f^2_L \\ f^2_R \end{pmatrix} = \left( \left| p \right| - V_t \right) \begin{pmatrix} f^2_L \\ f^2_R \end{pmatrix}.
\]

These matrix equations can be further reduced to the symmetric and anti-symmetric combinations, which describe the parity-even and parity-odd excitations under $1 \leftrightarrow 2$ and simultaneously $L \leftrightarrow R$ change. As a result, the lowest nucleon spectra corresponding to the proton and neutron, which are parity-even states, are described by the lowest excitation of the symmetric combination $f^1_L + f^2_R$ together with $f^1_R - f^2_L$. On the other hand, the parity-odd states are represented as $f^1_L - f^2_R$ and $f^1_R + f^2_L$. In order to investigate the parity-even mass
spectra, one can impose \( f^1_L = f^2_R \) and \( f^1_R = -f^2_L \), then the above two matrix Eqs. (10) and (11), reduce to the same matrix equation

\[
\begin{pmatrix} \partial \zeta - \Delta / \zeta & 0 \\ 0 & \partial \zeta - 4\Delta / \zeta \end{pmatrix} \begin{pmatrix} f^1_L \\ f^1_R \end{pmatrix} = \begin{pmatrix} -(|p| - V_t) & 0 \\ 0 & |p| - V_t \end{pmatrix} \begin{pmatrix} f^1_L \\ f^1_R \end{pmatrix}.
\]

Furthermore, since the nucleon has an isospin charge, it can interact with the background isospin matter. If the \( n \)-th excited mode of \( f^1_{L,R} \) to denote by \( f^1_{L,R}^{(n,\pm)} \) where the first and second sign imply the parity and isospin quantum numbers respectively, the parity-even state satisfying (11) can be further decomposed, depending on the isospin charge, into

\[
\begin{pmatrix} \partial \zeta - \Delta / \zeta & 0 \\ 0 & \partial \zeta - 4\Delta / \zeta \end{pmatrix} \begin{pmatrix} f^1_L^{(n,+)} \\ f^1_R^{(n,+)} \end{pmatrix} = \begin{pmatrix} -(|p| - V^3 / 2) & 0 \\ 0 & |p| - V^3 / 2 \end{pmatrix} \begin{pmatrix} f^1_L^{(n,+)} \\ f^1_R^{(n,+)} \end{pmatrix},
\]

(13)

\[
\begin{pmatrix} \partial \zeta - \Delta / \zeta & 0 \\ 0 & \partial \zeta - 4\Delta / \zeta \end{pmatrix} \begin{pmatrix} f^1_L^{(n,-)} \\ f^1_R^{(n,-)} \end{pmatrix} = \begin{pmatrix} -(|p| + V^3 / 2) & 0 \\ 0 & |p| + V^3 / 2 \end{pmatrix} \begin{pmatrix} f^1_L^{(n,-)} \\ f^1_R^{(n,-)} \end{pmatrix}.
\]

(14)

On the other hand, the parity-odd states are governed by following equations obtained from the Eqs. (13) and (14).

\[
\begin{pmatrix} \partial \zeta - \Delta / \zeta & 0 \\ 0 & \partial \zeta - 4\Delta / \zeta \end{pmatrix} \begin{pmatrix} f^1_L^{(n,-)} \\ f^1_R^{(n,-)} \end{pmatrix} = \begin{pmatrix} -(|p| - V^3 / 2) & 0 \\ 0 & |p| - V^3 / 2 \end{pmatrix} \begin{pmatrix} f^1_L^{(n,-)} \\ f^1_R^{(n,-)} \end{pmatrix}.
\]

(15)

The lowest excitation modes, when \( n = 1 \), the proton and neutron, can be represented by the \( f^1_{L,R}^{(1,+,+)} \) and \( f^1_{L,R}^{(1,+,+)} \) profile functions. Since the lowest excitations have only the parity-even states, there is no the \( f^1_{L,R}^{(1,\pm,\pm)} \) profiles. For the higher resonances, they can have even and odd parity states.

From the above matrix equation, one can easily derive two second order differential equations for \( f^1_L^{(n,\pm,\pm)} \) isospin states:

\[
\left( \partial^2 \zeta - \frac{4 \Delta}{\zeta} \partial \zeta + \frac{(5 - \Delta)\Delta}{\zeta^2} \right) f^1_L^{(1,\pm,+)} = -\left( |p| - V^3 / 2 \right)^2 f^1_L^{(1,\pm,+)} \text{ for proton,}
\]

(16)

\[
\left( \partial^2 \zeta - \frac{4 \Delta}{\zeta} \partial \zeta + \frac{(5 - \Delta)\Delta}{\zeta^2} \right) f^1_L^{(1,\pm,+)} = -\left( |p| + V^3 / 2 \right)^2 f^1_L^{(1,\pm,+)} \text{ for neutron.}
\]

(17)

Similarly, the differential equations for the \( f^1_R^{(n,\pm,\pm)} \) profiles are the following ones:

\[
\left( \partial^2 \zeta - \frac{4 \Delta}{\zeta} \partial \zeta + \frac{(5 - \Delta)\Delta}{\zeta^2} \right) f^1_R^{(1,\pm,+)} = -\left( |p| - V^3 / 2 \right)^2 f^1_R^{(1,\pm,+)} \text{ for proton,}
\]

(18)
\[
\left( \frac{d^2}{dz^2} - \frac{4}{z} \frac{d}{dz} + \frac{(5 - \Delta)\Delta}{z^2} \right) f^{l(1,\pm,\pm)}_R = - \left( |p| + \frac{V^3}{2} \right) z f^{l(1,\pm,\pm)}_R \quad \text{for neutron}. \tag{19}
\]

The mass of nucleon is given by \( p \) and is find from the following two boundary conditions on profile functions:

\[
f^{l(1,\pm,\pm)}_L(0) = 0 \quad \text{and} \quad f^{l(1,\pm,\pm)}_R(z_m) = 0. \tag{20}
\]

Now, let us take into account the mass splitting of nucleons in the isospin medium. The nucleon mass is given by \( p_0 \) at \( V^3_i = 0 \) is the same for the proton and neutron. In the isospin medium with nonzero \( V^3_i \), the proton and neutron masses are shifted due to the isospin interaction. Since the isospin medium has a constant \( V^3_i \), the boundary conditions (20) lead to \( p_0 = p_p - \frac{V^3_i}{2} \) for proton and \( p_0 = p_n + \frac{V^3_i}{2} \) for neutron. In order to study the form factor splitting in the isospin medium we should fix value of \( V^3_i \). As is known [33–35], at the isospin chemical potential value \( \mu_I = m_x \) it occurs pion condensation. So, taking \( V^3_i = m_x \), we can study the form factor splitting in the pion condensation state. Proton and neutron masses in this isospin medium are given by:

\[
p_p = p_0 + \frac{m_x}{2}, \quad p_n = p_0 - \frac{m_x}{2} \tag{21}
\]

Solutions of the (16)–(19) equations obeying the UV boundary conditions are the Bessel functions of first kind:

\[
f_{LR} = C_{1,2} z^\frac{5}{2} J_{M-n/2}(|p_N|, z), \tag{22}
\]

where \( C_{1,2} \) are normalization constants and \( p_N \) is the \( p_{|p_N|} \). The value of \( m_x \) can be found from the relation \( M = \Delta_1 - 2 \), where scaling dimension \( \Delta_1 \) for the composite baryon operator is \( \Delta_1 = \frac{9}{2} \) [18, 32, 36, 38] and \( |M| = \frac{5}{2} \). Consequently, for the \( \psi^{1,2} \) spinors the \( M \) mass have the values \( M = \pm \frac{5}{2} \) correspondingly. Thus, the \( f^1_{LR} \) and \( f^2_{LR} \) profile functions are given by

\[
\begin{align*}
  f^1_L &= C_1 z^\frac{3}{2} J_2(|p_N|, z), \\
  f^1_R &= C_2 z^\frac{3}{2} J_3(|p_N|, z), \\
  f^2_L &= C_2 z^\frac{3}{2} J_3(|p_N|, z), \\
  f^2_R &= C_1 z^\frac{3}{2} J_2(|p_N|, z).
\end{align*}
\tag{23}
\]

As seen as from (23) \( f^1_{LR} \) and \( f^2_{LR} \) are related one with another:

\[
f^1_L = f^2_R, \quad f^1_R = -f^2_L \tag{24}
\]

The normalization constants \( C_{1,2} \) in (23) are equal for the \( n \)-th excited state were found in [17]:

\[
C^{n}_{1,2} = C^n = \frac{\sqrt{2}}{z_m J_2(M_n z_m)} \tag{25}
\]

\( M_n \) is the Kaluza-Klein mass spectrum of exited states and is expressed in terms of zeros \( \alpha_n^{(3)} \) of the Bessel function \( J_3 \):

\[
M_n = \frac{\alpha_n^{(3)}}{z_m}.
\]
4 Axial-Vector Current in QCD

Isovector axial-vector current of nucleons is defined as following [38]:

\[ j^{\mu, a}(x) = \bar{\psi}(x)\gamma^\mu \gamma^5 \frac{\tau_a}{2} \psi(x). \] (26)

Here \( \psi(x) \) denotes the doublet of \( u \) and \( d \) quarks \( \psi = \begin{pmatrix} u \\ d \end{pmatrix} \) and \( \tau^a \) are the Pauli matrices describing isospin. A matrix element of the isovector current (26) between one-nucleon states is defined in terms of two form factors \( G_A(q^2) \) and \( G_P(q^2) \) [38]:

\[ \langle N(p')|j^{\mu, a}(0)|N(p)\rangle = \bar{u}(p') \left[ \gamma^\mu \gamma^5 G_A(q^2) + \frac{q^2}{2m_N} \gamma^5 G_P(q^2) \right] \frac{\tau^a}{2} u(p). \] (27)

Here \( m_N \) is nucleon mass, \( q_\mu = p'_\mu - p_\mu \) total momentum in the interaction vertex. \( G_A(q^2) \) and \( G_P(q^2) \) are called the axial-vector and pseudoscalar form factors respectively.

5 Axial-Vector Field in the AdS Space

In addition to (2) we have the kinetic term of the axial-vector field in the action:

\[ S_A = \int d^5x \sqrt{g} Tr \left( -\frac{F^2_A}{2g_s^2} \right). \] (28)

where \( F^A_{MN} = \partial_M A_N - \partial_N A_M, g_s^2 = \frac{12\pi^2}{N_c} \). The transverse part of the axial-vector field can be written as \( A_\mu(q, z) = A(q, z) A^0(0) \). Near the UV boundary an equation of motion for this field coincide with one for a vector field and in the \( A_z = 0 \) gauge it has a form [18–22]:

\[ z\partial_z \left( \frac{1}{z} \partial_z A(q, z) \right) + q^2 A(q, z) = 0. \] (29)

UV and IR boundary conditions on this solution are \( A(q, z = \epsilon) = 1 \) at \( \epsilon \to 0 \) and \( \partial_z A(q, z = z_m) = 0 \) correspondingly. Solution of the Eq. (29) is expressed via the first and second kind Bessel functions \( J_m \) and \( Y_m \) [4, 19, 20]:

\[ A(q, z) = \frac{\pi}{2} \left[ \frac{Y_0(qz_m)}{J_0(qz_m)} J_1(qz) - Y_1(qz) \right]. \]

6 The Chiral Symmetry Breaking by Scalar Field

Action for the scalar \( X \) field is the usual one for the scalar field in the five-dimesional AdS space:

\[ S_X = - \int d^5x \sqrt{G} Tr \left[ |DX|^2 + 3|X|^2 \right]. \] (30)
The covariant derivative $D_M$ includes the interaction of this field with the $A_L$ and $A_R$ gauge fields:

$$D_M X = \partial_M X - i (A_L)_M X + iX (A_R)_M.$$  

An interaction of this field with the spinor fields will be written in separate terms in the interaction Lagrangian. The $X$ field is written in the form:

$$X = \frac{1}{\sqrt{2}} \frac{N_C}{2\pi} m_q z + \frac{1}{\sqrt{N_C}} \sigma z^3,$$

where $m_q$ is the mass of bare light quarks and $\sigma$ is the value of the quark condensate and $N_C = 3$.

### 7 Axial-Vector Isovector Form Factor of Nucleons

The $G_A$ form factor in the AdS/QCD models framework will be extracted from the 5D action integral for the interaction between the axial-vector, scalar, and fermion fields in the bulk of AdS space [14]:

$$S^A_{\text{int}} = \int d^5 x \sqrt{G} \mathcal{L}^A_{\text{int}}(x, z).$$

Here the interaction Lagrangian $\mathcal{L}^A_{\text{int}}(x, z)$ describes several kind of interactions between $A_M X$ and $\psi_{1,2}$ fields in the bulk and consists of corresponding terms in the AdS space [14–18]. Let us list the bulk interactions, that contribute to the $G_A(q^2)$ form factor:

a. Minimal coupling term:

$$L = \bar{\psi}_1 \Gamma^M (A_L)_M \psi_1 - \bar{\psi}_2 \Gamma^M (A_R)_M \psi_2 = \frac{1}{2} \left( \bar{\psi}_1 \Gamma^M A_M \psi_1 - \bar{\psi}_2 \Gamma^M A_M \psi_2 \right).$$

b. Magnetic gauge coupling term:

$$L = i k_1 \left\{ \bar{\psi}_1 \Gamma^{MN} (F_L)_{MN} \psi_1 - \bar{\psi}_2 \Gamma^{MN} (F_R)_{MN} \psi_2 \right\} = k_1 \left\{ \bar{\psi}_1 \Gamma^{MN} F_{MN} \psi_1 + \bar{\psi}_2 \Gamma^{MN} F_{MN} \psi_2 \right\}.$$ 

(34)

c. Triple interaction term, which was introduced in [14]:

$$L = \frac{g}{2} \left[ \bar{\psi}_1 X \Gamma^M (A_L)_M \psi_2 - \bar{\psi}_2 X^+ \Gamma^M (A_R)_M \psi_1 + \text{h.c} \right] = g Y \left[ \bar{\psi}_1 X \Gamma^M A_M \psi_2 + \bar{\psi}_2 X^+ \Gamma^M A_M \psi_1 \right].$$

(35)

Having an interaction Lagrangian in the bulk we can derive a holographic expression for the $G_A$ form factor. The AdS/CFT correspondence in our case matches the axial-vector current of the bulk fermions with the axial-vector current of nucleons in the boundary QCD and at the same time it relates the bulk axial-vector field with the axial-vector meson in this
boundary theory. According to AdS/CFT correspondence, the generating functional $Z_{AdS}$ which is defined as an exponent of the classical bulk action

$$Z_{AdS} = e^{S_{cl}}$$

is equal to generating function $Z_{QCD}$ of the boundary QCD:

$$Z_{AdS} = Z_{QCD}.$$  \hspace{1cm} (37)

The above statement allows us to calculate the vacuum expectation value of the nucleons axial-vector current in the boundary QCD by taking variation from the gravity functional $Z_{AdS}$:

$$\langle J_\mu \rangle_{QCD} = -i \frac{\delta Z_{AdS}}{\delta A_\mu} \bigg|_{A_\mu = 0}. \hspace{1cm} (38)$$

Formula (38) produces the axial-vector current $J^\mu(p', p) = G_A(q^2) \bar{u}(p') \gamma_\mu \gamma_5 \frac{ie}{2} u(p)$, where $G_A(q^2)$ denotes the integral over the $z$ coordinate and is accepted as the axial-vector form factor of nucleons due to the holographic correspondence.

Using (33), (34), (35) we can calculate $S^{(a)}_{int}$ in the momentum space. This gives us the following results:

$$c^{(a)} = \frac{1}{2} \int d^4x \int_0^{\infty} dz \sqrt{g} \left( \Gamma^\alpha A_\alpha \psi_1 - \overline{\psi}_2 \Gamma^\alpha A_\alpha \psi_2 \right) = \frac{1}{2} \int d^4x' d^4p J^{\mu}(p', \mu) A_\mu^{(a)}(q) \int_0^{\infty} dz \frac{1}{z^4} A(q, z) \left[ F^{2 \mu}_{1R}(p_{P, n}, z) - F^{2 \mu}_{1L}(p_{P, n}, z) \right]$$ \hspace{1cm} (39)

$$c^{(b)} = \frac{1}{4} k_1 \int d^4x \int_0^{\infty} dz \sqrt{g} \left( \Gamma^\alpha A_\alpha \psi_1 + \overline{\psi}_2 \Gamma^\alpha A_\alpha \psi_2 \right) \psi_1 = k_1 \int d^4x' d^4p J^{\mu}(p', \mu) A_\mu^{(b)}(q) \int_0^{\infty} dz \frac{1}{z^4} \partial_z A(q, z) \left[ F^{2 \mu}_{1R}(p_{P, n}, z) + F^{2 \mu}_{1L}(p_{P, n}, z) \right]$$ \hspace{1cm} (40)

$$c^{(c)} = g_Y \int d^4x \int_0^{\infty} dz \sqrt{g} \left( \Gamma^\alpha X^\alpha A_\alpha \psi_1 + \overline{\psi}_2 X^\alpha \Gamma^\alpha A_\alpha \psi_2 \right) = 2 g_Y \int d^4x' d^4p J^{\mu}(p', \mu) A_\mu^{(c)}(q) \int_0^{\infty} dz \frac{1}{z^4} A(q, z) \partial_z A^1 \left[ F^{2 \mu}_{1R}(p_{P, n}, z) F^{2 \mu}_{1L}(p_{P, n}, z) \right]$$ \hspace{1cm} (41)

According to the holographic formula (38) the total action $S = S^{(a)} + S^{(b)} + S^{(c)}$ will produce the axial-vector form factor $G_A(q^2)$. Taking derivatives over $A_\mu^{(a)}(q)$ from the $S^{(a)}$ action terms we shall get the $G_A^{(a)}(q^2)$ contributions of these terms into the axial-vector form factor $G_A(q^2)$:

$$G_A^{(a)}(q^2) = \frac{1}{2} \int_0^{\infty} dz \frac{1}{z^4} A(q, z) \left[ (f_R^1)^2(p_{P, n}, z) - (f_L^1)^2(p_{P, n}, z) \right]$$ \hspace{1cm} (42)

$$G_A^{(b)}(q^2) = \frac{k_1}{2} \int_0^{\infty} dz \frac{1}{z^3} \partial_z A(q, z) \left[ (f_R^1)^2(p_{P, n}, z) + (f_L^1)^2(p_{P, n}, z) \right]$$ \hspace{1cm} (43)

$$G_A^{(c)}(q^2) = 2 g_Y \int_0^{\infty} dz \frac{1}{z^4} A(q, z) \partial_z A^1 \left[ f_L^1(p_{P, n}, z) \right]$$ \hspace{1cm} (44)
So, making a numerical integration, we can plot the $G_A(q^2)$ form factor, which is defined as the sum of $G_A^{(a)}(q^2)$ terms: $G_A = G_A^{(a)} + G_A^{(b)} + G_A^{(c)}$, for the $Q^2 = -q^2$ domain. In terms of $Q$ dependence the profile function $A(q, z)$ for axial-vector field gets the form below:

$$A(Q, z) = \frac{\pi}{2} Q z \left[ \frac{K_0(Q z m)}{J_0(Q z m)} J_1(Q z) + K_1(Q z) \right].$$  \hspace{1cm} (45)

8 Numerical Analysis

In order to perform the numerical integration of the $G_A(Q^2)$ form factor, the value of light quark mass $m_q$ and the value of quark condensate $\sigma$ were taken $m_q = 0.00234 \text{ GeV}$ and $(\sigma)^{\frac{1}{3}} = 0.311 \text{ GeV}$ correspondingly [36, 37]. The constant $k_1 = -0.98$ was taken from [36]. The value $g_Y = 9.182$ was taken from [37], which was found at establishing the correct nucleon mass within the hard-wall model having fixed parameters $m_q = 0.00234 \text{ GeV}$, $(\sigma)^{\frac{1}{3}} = 0.311 \text{ GeV}$ and $z_m = (0.330 \text{ GeV})^{-1}$. We use the (31) expression for $v(z)$, which is included into the $G_A(Q^2)$ form factor expressions (42), (43), (44). Nucleons were taken in the ground state.

9 Summary

In this work we studied the splitting of axial-vector form factor of the nucleons in the constant isospin background. Value of the background isospin field was chosen the value, at which the pion condensation occur. We plot the $G_A(Q^2)$ form factor in this background for the proton and neutron. Axial-vector form factor graph for the proton is shifted up, while the form factor for the neutron is shifted down from the graph of the $G_A(Q^2)$ dependence at zero isospin case (Fig. 1). The splitting of the axial-vector form factor dependencies occurs due to nucleon mass splitting in the isospin background. Comparison of our result for zero isospin case with the experimental data described in Figs. 2 and 3 [40, 41], shows that our results are close to the data.

![Fig. 1 $G_A$ form factor splitting in the isospin medium](image)
Fig. 2 Experimental data for the normalized $G_A$ form factor [40]

![Graph of $G_A(Q^2)/G_A(0)$ vs. $Q^2 [GeV^2]$](image)

Fig. 3 CLAS collaboration data for the $G_A$ form factor [41]

![Graph of $G_L/G_S$ and $G_A$ vs. $Q^2 [GeV^2]$](image)

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