One target classification method based on target feature distribution fusion

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Abstract. Target classification technology is an important method to improve the operational performance of radar in complex electromagnetic environment, as well as an important means to reduce radar false alarm. In this paper, according to the randomness of target plot parameters, a feature prior model is established to fuse the features distribution of true/false targets based on Bayesian estimation so as to achieve intra-feature information fusion. Under the guidance of the classification hyper-plane, feature dimension reduction is conducted to multi-dimension Gauss distribution after fusion, so as to achieve inter-feature information fusion and obtain 1-D Gauss probability distribution of target features. The target probability is obtained according to the vector distance integral of the target, thus the target classification is completed according to the probability threshold. This paper makes analysis, calculation and simulation to target classification algorithm under the features of 1-D, 2-D, 3-D and multi-dimension so as to validate the effectiveness of the proposed method.

1. Introduction
Classification technique is an important method to effectively decrease radar false alarm, increase radar clutter suppression and ECCM performance so as to improve radar operational performance in complex E.M. environment. Faced with complex background consisting of ground & sea clutter and active/passive jamming, radar needs to conduct effective extraction to high threat targets including aircraft, UAV and missile, etc. Since background echo and target echo have separate random features, when radar conducts parameter extraction for target plot including range, azimuth, amplitude and Doppler, etc[1-3], each feature has its own unique random distribution characteristics as well as mean value and variance, so that the prior distribution function of true/false target features can be established. This paper establishes feature right-angle coordinates based on feature true/false prior probability, makes fusion to true/false target targets distribution through Bayesian estimation so as to obtain target true/false targets Gauss distribution function, under the guidance of true/false classification vector, solving of classification interface and fusion dimension reduction of multi-dimension Gauss distribution are conducted in vector space so as to obtain the classification Gauss distribution curve. Target probability value can be obtained by vector distance integral to that curve and target true/false property can be determined according to probability threshold. This paper
makes derivation, calculation and simulation to classification formula of 1-D, 2-D, 3-D and multi-dimension features of true/false targets and validates the effective of the proposed method.

2. Modeling of target features

The true targets for radar detection including aircraft, missile and ship, etc. The plot parameters of radar target have separate randomness due to radar beam characteristics, measurement error and Swerling RCS fluctuation characteristics of target. The false targets include ground/sea clutter, jamming and cloud/rain climate, etc., which also has separate plot parameter randomness. The separate randomness of radar plot parameters including distance, azimuth, amplitude and Doppler information, etc. can be described by mean value and variance. The true/false distribution is different for each feature as follows.

| Feature 1 X | Feature 2 Y |
|-------------|-------------|
| True        | True        |
| False       | False       |

Employ T to represent true target and F to represent false target. The feature random satisfies Gauss distribution, then separate probability density distribution is as follows.

\[
f_T(x) = \frac{1}{\sqrt{2\pi\sigma_{XT}}} e^{-\frac{(x-u_{XT})^2}{2\sigma_{XT}^2}}
\]

\[
f_T(y) = \frac{1}{\sqrt{2\pi\sigma_{YT}}} e^{-\frac{(y-u_{YT})^2}{2\sigma_{YT}^2}}
\]

The function of multi-dimension associated distribution probability density is as follows.

\[
N(x,u,\Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(x-u)^T \Sigma^{-1} (x-u)}
\]

Where x is the random vector of D-dimension, \(\Sigma\) is the covariance matrix of random vector and \(u\) is mean value.

For 2-dimension situation, the 2 features to describe target are mutually independent and the 2-dimension probability is the product of their probability density. So the associated distribution probability density function of 2 features is as follows.

\[
f_{TF}(x,y) = \frac{1}{2\pi\sigma_{XT}\sigma_{YT}} e^{-\frac{1}{2} \left( \begin{pmatrix} x-u_{XT} \\ y-u_{YT} \end{pmatrix} ^T \begin{pmatrix} \sigma_{YT}^2 & 0 \\ 0 & \sigma_{YT}^2 \end{pmatrix} ^{-1} \begin{pmatrix} x-u_{XT} \\ y-u_{YT} \end{pmatrix} \right)}
\]

For false target, there are separate mean value \(u_{XF}\) and \(u_{YF}\) and variance \(\sigma_{XF}\) and \(\sigma_{YF}\), then separate probability density distribution is as follows.

\[
f_F(x) = \frac{1}{\sqrt{2\pi\sigma_{XF}^2}} e^{-\frac{(x-u_{XF})^2}{2\sigma_{XF}^2}}
\]

\[
f_F(y) = \frac{1}{\sqrt{2\pi\sigma_{YF}^2}} e^{-\frac{(y-u_{YF})^2}{2\sigma_{YF}^2}}
\]

The associated probability density distribution is as follows.
Establish the feature coordinates consisting of feature 1 and feature 2 as 2 axis. According to the provided distribution feature parameters, generate the target point satisfying the distribution and deploy the generated target points into the coordinates as follows. (red for true and blue for false).

\[
f_F(x, y) = \frac{1}{2\pi\sigma_{xf}\sigma_{yf}} e^{-\frac{1}{2} \left( \frac{(x-u_{xf})^2}{\sigma_{xf}^2} + \frac{(y-u_{yf})^2}{\sigma_{yf}^2} \right)}
\]  

Figure 1. 2-D feature distribution of true/false targets

3. 2-D feature distribution

From Figure 1, we can see that the distribution of true/false targets in feature 1 and feature 2 are 2 different patterns which can be divided. We can draw a straight line between the true/false distribution patterns as the boundary to divide the true/false targets as follows[4-7].

Figure 2. Sketch map of classification line of true/false targets

As drawn in Figure 2, any of the lines \( k_1, k_2 \) and \( k_3 \) can be the boundary line. But the line \( k_2 \) is the best option because it is at the same distance to the distributions of true/false targets. The line \( k_2 \) will be determined as follows.

3.1 Position of classification center

At first, we will determine the start point of the boundary line and whether there is a central point that the boundary line must pass through. The boundary line of true/false targets represents the distribution center which is difficult to distinguish true/false targets. True/false distribution of target features is the target description in probability domain. The distribution center which is difficult to distinguish true/false is the center of target true/false distribution fusion. The qualitative description is that the
distribution fusion indicates the true/false ambiguity, i.e. that point is true with false property and that point is false with true characteristics. The quantitative description by probability is that how much is the false probability when that point is true and vice versa. According to principle of Bayesian Estimation[3], there is

\[ F_A(x) = f_T(x)f_F(x|x_T) \quad (6) \]

Since the probability distributions of true/false features are mutually independent, so the Bayesian condition distribution feature is the product of true/false feature distribution function (1a) and (4a), i.e.

\[ F_A(x) = \frac{1}{2\pi\sigma_{XF}\sigma_{XT}} e^{-\frac{1}{2}\left(\frac{(x-u_{XT})^2}{\sigma_{XT}^2} + \frac{(x-u_{XF})^2}{\sigma_{XF}^2}\right)} \quad (7) \]

After normalization, the obtained probability density function is as follows.

\[ f_A(x) = \frac{1}{\sqrt{2\pi}\sigma_{XA}} e^{-\frac{(x-u_{XA})^2}{2\sigma_{XA}^2}} \quad (8) \]

Where the mean value (center) is \( u_{XA} \):

\[ u_{XA} = \frac{u_{XT}\sigma_{XF}^2 + u_{XF}\sigma_{XT}^2}{\sigma_{XF}^2 + \sigma_{XT}^2} \quad (9) \]

The variance is \( \sigma_{XA} \).

\[ \sigma_{XA} = \frac{\sigma_{XF}\sigma_{XT}}{\sqrt{\sigma_{XF}^2 + \sigma_{XT}^2}} \quad (10) \]

We can see that true/false targets still satisfy Gauss distribution after Gauss distribution fusion. The distribution characteristics of true/false distribution fusion of target feature \( X \) is the black curve in following diagram. The true distribution is red and the false distribution is blue.

**Figure 3.** Target true/false and fusion distribution diagram

This step conducts intra-feature information fusion, i.e. in one same feature, true/false probability density distribution of target is fused into 1D Gauss distribution. That distribution is also the classification probability distribution of 1D feature. The distribution center is at \( u_{XA} \) and variance is \( \sigma_{XA} \). The classification of 1D feature is to substitute the position \( x \) of true/false targets into formula(8).
The target distribution is the difference between target position \( x \) and \( u_{xa} \). The target probability can be obtained by probability density integral to formula (8) from left to right and the result is the probability of that point. True/false target is determined according to probability threshold.

In 2D distribution, after true/false distribution is fused for Y feature similarly, the mean value and variance are as follows.

The mean value (center) is \( u_y \)

\[
u_{ya} = \frac{u_{yt} \cdot \sigma_{yt}^2 + u_{ya} \cdot \sigma_{yt}^2}{\sigma_{yt}^2 + \sigma_{yt}^2}
\]

(11)

The variance \( \sigma_y \) is

\[
\sigma_{ya} = \sqrt{\frac{\sigma_{yt} \cdot \sigma_{yt}}{\sigma_{yt}^2 + \sigma_{yt}^2}}
\]

(12)

The distributions of target feature 1, feature 2 and true/false fusion distribution characteristics are simulated as follows. The green is true/false fusion distribution characteristics.

\[
f_a(x, y) = \frac{1}{2\pi \sigma_{xa} \cdot \sigma_{ya}} e^{-\frac{1}{2} \left( \frac{(x-u_{xa})^2}{\sigma_{xa}^2} + \frac{(y-u_{ya})^2}{\sigma_{ya}^2} \right)}
\]

(13)

Figure 4. 2D true/false and fusion target distribution

In the whole coordinates, distribution features of false target are blue and distribution features of true target are red, and the fusion area distribution which is difficult to distinguish true/false is green. Thus the true/false distribution patterns are fused into one pattern through the Bayesian Estimation method. That area indicates the difficulty degree to distinguish true/false targets in feature 1 and feature 2. But we need to distinguish the compatibility of true/false i.e. the result of Bayesian Estimation.

3.2 Determination of boundary line

After the determination of central point of true/false fusion distribution, i.e. the point that boundary line \( k_2 \) must pass through, the boundary line can be determined after the slope of line \( k_2 \) is obtained.

The true/false distribution points T and F indicate true/false maximum likelihood. Connection line of F and T is line \( k_1 \). Fuse center of true/false distribution is point A. Then how to decide the best boundary line among the radiation lines taking point A as the center? That boundary line should be the one with maximum sum of distance from point T and distance from point F. According to geometric
principle, the line $k_2$ vertical to $k_1$ is the best option. Otherwise if $k_3$ is the best option, then the sum of vertical lines from point F and point T to $k_2$ is segment FT, while the sum of vertical lines from point F and point T to $k_3$ is ET+FD which is less than FT, so the best classification line is $k_2$ [4-7].

Figure 5. Selection of 2D feature classification line

In order to obtain line $k_2$, let us obtain line $k_1$ first. Slope of $k_1$ is as follows.

$$k_1 = \frac{u_{YT}-u_{YF}}{u_{XT}-u_{XF}}$$

(14)

The boundary line is the center point (mean value point) after true/false distribution fusion. It is the vertical line of connection line of true/false distribution centers and its scope is $-1/k_1$.

$$k_2 = -\frac{1}{k_1}$$

(15)

Formula of line $k_2$ crossing point $(X_A,Y_A)$ is as follows.

$$\frac{Y-Y_A}{X-X_A} = k_2 = -\frac{1}{k_1}$$

(16)

i.e.

$$Y - k_2X - Y_A + k_2X_A = 0$$

(17)

Distance from any point $(X_T,Y_T)$ inside the plane to that line is as follows.

$$d_T = \frac{Y_T - k_2X_T - Y_A + k_2X_A}{\sqrt{1+k_2^2}}$$

(18)

In the example,

If $Y_T - k_2X_T - Y_A + k_2X_A > 0$, then that point is above the boundary line which is false.

If $Y_T - k_2X_T - Y_A + k_2X_A < 0$, then that point is below the boundary line which is true.

We can see that $d_T$ is with positive/negative sign which is convenient for succeeding calculation of true/false fusion probability. Since the range is with positive/negative sign indicating the direction, so $d_T$ is the vector range.

This step achieves the location and calculation of boundary line of true/false targets classification.

### 3.3 Calculation of distribution variance

After classification line $k_3$ is determined, with increase of distance from the boundary, the target classification is more obvious. How to describe the classification degree indicated by target distance
from line $k_2$. Obviously we can draw the lines $k_3$ and $k_4$ parallel to $k_2$ so as to generate a distribution line family. The target point is on one of the lines. Each line indicates local target probability. The probability value is relevant to distance from target to $k_2$. All line family will fill the whole 2D coordinates system so as to fuse the 2D feature target distribution to one 1D distribution about line family. Shown as following diagram, point T and F are center of true/false target distribution separately indicating maximum likelihood. The point A is the true/false feature fusion distribution center indicating the maximum likelihood as well. All of them satisfy 2D Gauss distribution as follows.

![Figure 6. Sketch map of 2D feature classification probability distribution](image)

If we observe the fusion distribution area A from axis Y, the 2D distribution will be projected to axis X and the distribution variance is $\sigma_{XA}$. If we observe fusion distribution area A from axis X, the 2D distribution will be projected to axis Y and the distribution variance is $\sigma_{YA}$. If we observe area A from $k_2$ direction, the observed target 2D distribution will be compressed to projection in $\overrightarrow{FT}$ direction as 1D Gauss distribution. What is the variance for observing area A from $k_2$ direction?

In order to research on variance distribution, the exponential item of formula (13) describes the relation between random variable and mean value & variance. Let it equal to 1, i.e.

$$\frac{(x-u_{XA})^2}{\sigma_{XA}^2} + \frac{(y-u_{YA})^2}{\sigma_{YA}^2} = 1$$

The above formula expresses the oval motion trajectory of variance distribution whose center is fusion average value and long/short axis is feature variance. Thus we observe area A from $k_2$ direction, variance is the oval radius along FT direction as oval eccentric angle, i.e. the product of feature variance $\sigma_{XA}$ of axis X and feature variance $\sigma_{YA}$ of axis Y with cosine of $\overrightarrow{FT}$ direction generates new vector, which indicates variance contribution of the feature to $\overrightarrow{FT}$ direction. The vector mode is variance, i.e.

$$\sigma = \sqrt{(\sigma_{XA}\cos(\theta_x))^2 + (\sigma_{YA}\cos(\theta_y))^2}$$

Where $\theta_x = \arctan(k_1), \theta_y + \theta_x = \pi / 2 \text{ in 2D.}$

From the geometric aspect, the classification line distribution is to draw a line parallel to true/false likelihood center vector $\overrightarrow{FT}$ crossing fusion likelihood center point A. That line of oval eccentric angle intersects with variance oval to obtain the oval radius, which is the variance of classification line family.
This step achieves inter-feature information fusion. From above derivation we can see that line probability is employed to replace target point probability in 2D feature distribution, so that 2D Gauss distribution is reduced to 1D Gauss distribution.

### 3.4 Calculation of classification probability

In previous section, we obtained that the classification line distribution is a 1D Gauss distribution. The integral to that Gauss probability distribution is the target probability. Thus we obtain the classification line Gauss distribution function whose center is line $k_2$ with distribution variance $\sigma_A$, i.e.

$$f(k) = \frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{d^2}{2\sigma^2_A}}$$

Where $d_T$ is target vector distance of formula (18).

That expression indicates the target probability distribution characteristics. The integral from left to right along $FT$ indicates the target probability value and expresses the true/false degree of target in that area. The curve of Gauss distribution probability density is as follow consisting of the distribution feature of $\sigma$, $2\sigma$ and $3\sigma$ distribution probability.

![Distribution Curve of Gauss Probability Density](image)

That integral result is indicated by a special function named error function as follows.

$$P_T = \frac{1}{2}(1 + erf\left(\frac{d_T}{\sqrt{2} \cdot \sigma_A}\right))$$

The integral of Gauss probability distribution is one point indicating target probability. The probability distribution is obtained according to $d_T$ as follows.
So we can obtain the target probability value $P_T$ of any point in the plane. Determine one probability threshold $P_{TH}$ and if $P_T > P_{TH}$ then the target is true, otherwise it is false. Thus true/false targets classification judgment is conducted based on probability threshold criterion.

During the above whole demonstration procedure, we can see that the vector generated by the connection line of true/false distribution center is of great importance. It represents the steering vector of target classification and can be expressed as follows.

$$\overrightarrow{FT} = (u_{XT} - u_{XP})i + (u_{YT} - u_{YP})j$$  \hspace{1cm} (23)

After true/false distribution fusion of one same feature through Bayesian principle, the 2D Gauss distribution of true/false fusion is generated and intra-feature information fusion is conducted, thus the true distribution and false distribution is unified into one distribution for discussion, which is beneficial to convert the probability problem into algebra problem and geometric problem for solution. The center (mean value) and variance of fusion distribution area can be described as following vectors. The fusion center vector (i.e. mean value) is

$$\overline{u}_A = u_{x_Ai} + u_{y_Aj}$$  \hspace{1cm} (24)

The variance vector is

$$\overline{\sigma}_A = \sigma_{x_Ai} + \sigma_{y_Aj}$$  \hspace{1cm} (25)

From fusion center $\overrightarrow{OA}$ draw a line vertical to the connection line FT of true/false distribution centers, which is the boundary line. Select any point P on the boundary line, then the equation of line vertical to FT crossing A is as follows.

$$(\overrightarrow{OA} - \overrightarrow{OP}) \cdot \overrightarrow{FT} = 0$$  \hspace{1cm} (26)

The distance from any point Q in 2D space to boundary line is as follows[8].

$$d_T = \frac{\left| \overrightarrow{OA} - \overrightarrow{OQ} \right| \cdot \overrightarrow{FT}}{|\overrightarrow{FT}|}$$  \hspace{1cm} (27)

Where $\cdot$ is inner product of vector, $||$ is vector mode.

Take boundary line $k_2$ as view direction to observe the fusion area A, the variance is oval radius intersected by variance oval and the line taking fusion area point A as center and parallel to FT. The direction cosine vector expression of vector $\overrightarrow{FT}$ is as follows.

Figure 8. Distribution Curve of Gauss Probability
(28)

Where $\theta_X, \theta_Y$ is the included angle of $\overrightarrow{FT}$ with axis X and axis Y. While the product of each component of 2D feature variance vector with direction cosine symbolizes the vector contribution of each feature to 1D fusion distribution, and new vector is generated which is the intercepted oval radius vector.

(29)

After fusion, the intercepted oval variance is the mode of above vector, which is also the variance of line 1D Gauss distribution and the oval radius length of variance oval parallel to $\overrightarrow{FT}$ as oval eccentric angle direction. The mode is

(30)

That mode is the variance of 1D Gauss distribution. So the target probability value can be obtained through $d_r$ integral to 1D Gauss distribution and the true/false judgment can be acquired by comparison with the probability threshold. Thus the above derivation procedure is expressed through vector which satisfies the characteristics of vector space and can provide convenience for succeeding analysis to 3D and multi-dimension distributions.

### 3.5 2D distribution simulation

The simulation conditions are as follows. 2 features are simulated and the parameters see the following table. 2000 true points and 2000 false points are generated by simulation and the feature distributions are as follows.

| Feature | Attribute | Mean value | Variance |
|---------|-----------|------------|----------|
| Feature 1 X | True | 3.5 | 0.5 |
| | False | 2.0 | 0.2 |
| Feature 2 Y | True | 1.8 | 0.3 |
| | False | 2.6 | 0.5 |

According to 2D feature distribution, the result is as follows.

![Figure 9. 2D true/false target distribution and fusion distribution](image)

According to above equation, true/false plot classification probability distribution is as follows.
From the above 2D simulation result, we can see that most of true/false targets are classified effectively. According to statistics, if the detection probability threshold is set as 40%, there are 4 true targets below threshold i.e. the missing alarm rate is 0.2%. There are 169 false targets beyond threshold i.e. the false alarm rate is 8.45%. 2D classification achieves certain effect.

4. Classification of 3D features

4.1 Distribution of 3D features

The 2D feature distribution is analyzed in plane right-angle coordinates system. 3D feature distribution describes target true/false classification in 3D cubic space. Similar to employment of one boundary line to distinguish true/false targets in 2D space, one plane is employed to distinguish true/false targets in 3D space. The method to classify true/false targets by 3 features will be analyzed as follows. One feature dimension z is added based on Section 3. Suppose its true/false feature satisfy following table.

| Feature | Attribute | Mean value | Variance |
|---------|-----------|------------|----------|
| Feature 3 Z | True | $u_{ZT}$ | $\sigma_{ZT}$ |
|          | False | $u_{ZF}$ | $\sigma_{ZF}$ |

Then true/false targets distribution in 3D features see the following figure. The true targets are red and false targets are blue.
Figure 11. 3D true/false targets distribution

4.2 Fusion of 3D feature true/false distribution

Same as 2D analysis, Bayesian Estimation principle is employed to conduct fusion to 3rd feature coordinates. The mean value and variance of true/false fusion distribution of feature 3 are as follows. The mean value $u_{ZA}$ is as follows.

$$u_{ZA} = \frac{u_{ZT}\cdot\sigma_{ZF}^2 + u_{ZF}\cdot\sigma_{ZT}^2}{\sigma_{ZF}^2 + \sigma_{ZT}^2}$$ (31)

The variance $\sigma_{ZA}$ is as follows.

$$\sigma_{ZA} = \frac{\sqrt{\sigma_{ZF}^2 \cdot \sigma_{ZT}^2}}{\sqrt{\sigma_{ZF}^2 + \sigma_{ZT}^2}}$$ (32)

The true/false fusion probability distribution of target feature 1, 2 and 3 is 3D Gauss distribution whose expression is as follows.

$$f_d(x, y, z) = \frac{1}{(2\pi)^{1/2}} \frac{1}{\sigma_{x} \cdot \sigma_{y} \cdot \sigma_{z}} \cdot e^{-\frac{1}{2} \left(\frac{(x-u_x)^2}{\sigma_{x}^2} + \frac{(y-u_y)^2}{\sigma_{y}^2} + \frac{(z-u_z)^2}{\sigma_{z}^2}\right)}$$ (33)

The central vector (mean value) is as follows.

$$\overline{u_d} = u_{x_d}i + u_{y_d}j + u_{z_d}k$$ (34)

The variance vector is as follows.

$$\overline{\sigma_d} = \sigma_{x_d}i + \sigma_{y_d}j + \sigma_{z_d}k$$ (35)

The simulation to 3D feature and true/false fusion distribution is as follows. The green part is feature true/false distribution fusion.
Figure 12. 3D feature fusion distribution

In the above figure, the green ellipsoid symbolizes the fusion distribution area of true/false 3D features in Bayesian Estimation to conduct intra-feature information fusion.

4.3 Calculation of 3D feature distribution probability

3D feature space distribution satisfies the characteristics of vector space. For convenience of research, $k_1$ symbolizes straight line while $k_2$ and $k_3$ are planes. The true targets distribution center is point T and false targets distribution center is point F. The connection line is the steering vector of true/false classification in the figure expressed as follows.

$$
\overrightarrow{FT} = (u_{XT} - u_{XF})i + (u_{YT} - u_{YF})j + (u_{ZT} - u_{ZF})k
$$

In 3D space, the vertical plane $k_2$ from the true/false fusion distribution center A to steering vector $\overrightarrow{FT}$ is the best boundary plane. Otherwise, if we suppose plane $k_3$ is the best boundary plane, the vertical lines to the plane crossing point T and point F are FD and TE separately, obviously $TE < TC$ and $FD < FC$, then

$$
TE + FD < TC + FC = TF
$$
That proves the vertical plane $k_2$ from point A to steering vector is the classification boundary plane. So draw a plane vertical to connection line of true/false center at fusion center $\overrightarrow{OA}$, whose normal is the steering vector $\overrightarrow{TF}$ of true/false distribution. $P$ is any point of that plane, so the equation of plane vertical to $TF$ is as follows (8):

$$\overrightarrow{(OA-\Omega P)} \cdot \overrightarrow{TF} = 0$$

(38)

The vector distance from any point $Q$ in 3D space to classification plane is as follows.

$$d_r = \frac{\overrightarrow{(OA-\Omega Q)} \cdot \overrightarrow{TF}}{\vert \overrightarrow{FT} \vert}$$

(39)

We can see that the whole 3D feature space is filled with the plane family. If we regard the plane as random variable, the 3D space is reduced to 1D space since the plane is 2D. That compresses the 3D Gauss distribution to 1D Gauss distribution in the maximum likelihood plane $AFT$. The variance is the radius intersected with variance ellipsoid by the parallel line of $FT$ as ellipsoid eccentric angle direction whose center is point A.

According to formula (33), let exponential item equal to 1, then the equation of 3D variance ellipsoid is as follows.

$$\frac{(x-u_x)^2}{\sigma_{x_{\lambda}}^2} + \frac{(y-u_y)^2}{\sigma_{y_{\lambda}}^2} + \frac{(z-u_z)^2}{\sigma_{z_{\lambda}}^2} = 1$$

(40)

Draw a line parallel with $FT$ as ellipsoid eccentric angle direction from fusion center point A and the radius of its intersection point with ellipsoid is the desired variance. The direction cosine of steering vector $\overrightarrow{FT}$ is needed at first. The directional cosine vector expression of vector $\overrightarrow{FT}$ is as follows.

$$\overrightarrow{FT} = \cos \theta_x \cdot i + \cos \theta_y \cdot j + \cos \theta_z \cdot k$$

(41)

Where $\theta_x, \theta_y, \theta_z$ are the included angle between FT vector and axis X, Y and Z.

So the fusion ellipsoid variance vector in FT direction is as follows.

$$\overrightarrow{Ar} = \sigma_{x_{\lambda}} \cdot \cos \theta_x \cdot i + \sigma_{y_{\lambda}} \cdot \cos \theta_y \cdot j + \sigma_{z_{\lambda}} \cdot \cos \theta_z \cdot k$$

(42)

The vector mode is variance, then we obtain the radius of ellipsoid as follows.

$$\sigma_A = \vert \overrightarrow{Ar} \vert$$

(43)

That mode is the variance of 1D Gauss distribution. The procedure to obtain variance in mathematics is to employ the true/false feature fusion variance as long & short axis of ellipsoid and substitute the direction cosine of steering vector into the ellipsoid equation, so as to obtain the ellipsoid radius. Thus inter-feature information fusion is conducted and classification 1D Gauss distribution is as follows.

$$f(d_T) = \frac{1}{\sqrt{2\pi} \sigma_A} e^{-\frac{d_T^2}{2\sigma_A^2}}$$

(44)

Make integral to that equation and obtain probability value $P_T$ according to formula (22). Compare $P_T$ with the probability threshold $P_{TH}$ so as to obtain the conclusion of whether the target is true or false.

From the above derivation procedure, we can see that 2D plane probability is employed to replace target point probability in 3D feature distribution, so that 3D Gauss distribution is reduced to 1D Gauss distribution.
4.4 3D classification simulation
After component of feature 3 is added based on Section 3, 2000 true targets and 2000 false targets are simulated. The parameters of 3rd dimension feature is as follows.

**Table 4. 3rd Dimension feature parameter value**

| Feature  3 Z | Attribute | Mean value | Variance |
|-------------|-----------|------------|----------|
| True        | 2.5       |            | 0.5      |
| False       | 1.7       |            | 0.3      |

The simulated result of classification probability distribution is as follows.

![3D true/false targets probability distribution](image_url)

**Figure 14.** 3D true/false targets probability distribution

From the above 3D simulation result, we can see that most of true/false targets are classified effectively. According to statistics, if the detection probability threshold is set as 40%, there are 5 true targets below threshold i.e. the missing alarm rate is 0.25%. There are 121 false targets beyond threshold i.e. the false alarm rate is 6.05%. Compared with 2D simulation result, one additional dimension feature can decrease false alarm with better classification effect. The better effect is because the points that can not be distinguished in 2D space is divided in 3D space after additional dimension information is added.

5. High-dimension feature classification

5.1 Theory analysis to high-dimension classification

The feature space over 3D belongs to high-dimension feature space. There is a hyper-plane in high-dimension space to discriminate the true/false targets in high-dimension features. The solving equation in high-dimension vector space is similar to that in 3D space as follows.
In multi-dimension space, the false targets are distributed in multi-dimension Gauss distribution whose center is F and the true targets are distributed in multi-dimension Gauss distribution whose center is T. True/false distribution is fused by Bayesian Estimation into true/false fusion high-dimension Gauss distribution area whose center is point A. Employ the multi-dimension classification steering vector as normal to draw a hyper-plane from point A vertical to FT, which is the true/false classification plane i.e. maximum likelihood classification hyper-plane. The target distribution is the vector distance between target and hyper-plane. The targets are distributed in the hyper-planes parallel to hyper-plane so the hyper-planes fill the whole multi-dimension distribution space. Each hyper-plane symbolizes the probability of local target. In the maximum likelihood hyper-plane AFT, AD is vertical to TF. So when the high-dimension Gauss distribution intercepted by hyper-plane is observed from DA direction, 1D Gauss distribution will be generated whose variance is the ellipsoid radius of ellipsoid plane of true/false fusion distribution variance in TF as ellipsoid eccentric angle direction. It is also the product of component of multi-dimension true/false fusion variance vector and direction cosine of classification steering vector, indicating the function of feature components in variance. The product of each component generates ellipsoid variance vector whose mode is the variance value indicating 1D Gauss distribution degree. Actually, maximum likelihood hyper-plane AFT is employed to intercept the ellipsoid A of high-dimension Gauss distribution variance and one 1D Gauss distribution is generated inside plane AFT whose steering is vertical to TF. Employ the ellipsoid radius parallel with TF as ellipsoid eccentric angle direction crossing point A as variance and conduct vector distance integral of target along the classification steering vector to obtain the target probability, which is compared with probability threshold to obtain the conclusion whether the target is true or false.

From the above derivation procedure, we can see that hyper-plane probability is employed to replace target point probability in Gauss feature distribution, so that high-dimension Gauss distribution is reduced to 1D Gauss distribution. The target probability is obtained by vector distance integral of target through 1D Gauss distribution. The obtained probability value is compared with probability threshold so as to obtain the conclusion whether the target is true or false.

5.2 Simulation of High-dimension classification

Now the simulation to classification results of true/false classifier of 4D feature space. One feature dimension w is added based on Section 4. Suppose its true/false feature satisfies the following table.

![Figure 15. Sketch map of multi-dimension classification distribution](image-url)
Table 5. 4th Dimension feature parameter

| Feature 4 W | Attribute | Mean value | Variance |
|------------|-----------|------------|----------|
| True       | $u_{WT}$  | $\sigma_{WT}$ |
| False      | $u_{WF}$  | $\sigma_{WF}$ |

The feature value is as follows.

Table 6. 4th Dimension feature parameter value

| Feature 4 W | Attribute | Mean value | Variance |
|------------|-----------|------------|----------|
| True       | 3.0       | 0.6        |
| False      | 2.1       | 0.3        |

The high-dimension space over 3D is difficult to visualize, so the display of multi-dimension feature distribution simulation is omitted. 2000 true targets and 2000 false targets are generated separately. The true/false targets probability distribution in 4D features is simulated as follows.

Figure 16. 4D true/false target probability distribution

From the above 4D simulation results, we can see that true targets and false targets are effectively classified. The true targets tend to 1 and false targets tend to 0. According to statistics, if the detection probability threshold is set as 40%, there are 6 true targets below threshold i.e. the missing alarm rate is 0.3%. There are 22 false targets beyond threshold i.e. the false alarm rate is 1.1%. Compared with 3D simulation results, 4D space classification is more effective for comprehensive judgment to missing alarm and false alarm.

So from the above simulation results of 2D, 3D and 4D feature distribution diagram, we can see that the classification effect is more obvious and the discrimination is higher with increase of dimension quantity. In the target probability distribution diagram, the plot density in the middle is decreased and 4D classification result is greatly improved.

6. Conclusion

This paper utilizes the randomness of radar target echo parameters, establishes true/false priori random model of target plot parameters. The coordinates system is established with features as axis. Each axis employs Bayesian Estimation concept to express the true/false target characteristics fusion in one same feature by one same 1D Gauss distribution, so as to achieve intra-feature and inter-feature information fusion. So the multi-dimension space is reduced to 1D space and multi-dimension vector is mapped to real number within [0-1] i.e. probability, so as to accomplish the judgment to true/false target based on probability threshold criterion. The theory analysis indicates that target feature point is mapped to 1D Gauss distribution finally. The conclusion is that 1D, 2D, 3D and multi-dimension mapping is point, line, plane and hyper-plane separately, so that feature dimension reduction processing is conducted. The classification effect is more obvious with increase of target feature
dimension quantity because high-dimension features separate the true/false targets in high-dimension probability space which is beneficial for target classification. The effectiveness of proposed method is validated by simulation.

This paper makes illustration based on radar platform. The conclusion can also be employed for other platforms as long as the target feature has the true/false Gaussian distribution characteristics.

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