Slope reliability analysis based on PSO-RBF neural network

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Abstract. In this paper, a reliability analysis method based on ABAQUS and particle swarm optimization radial basis function neural network is proposed. The strength reduction method based on ABAQUS is used to calculate the safety factor corresponding to the selected random variable. The data fitting function of the radial basis function neural network is used to establish the model and map the relationship between the safety factor and the random variable to construct the response surface function. A large number of random samples generated by Monte Carlo are substituted into the function to obtain the corresponding safety coefficient, in order to calculate the instability probability and reliability index of the slope.

1. Introduction

At present, the analysis method of slope stability can be divided into deterministic analysis method and uncertainty analysis method. The deterministic analysis method is mainly based on the limit equilibrium theory, but the variability and randomness of the actual slope calculation parameters[1], it is difficult to reflect the actual working state of the slope. The most representative of the uncertainty analysis is the reliability analysis method, which estimates the probability that the slope will complete the predetermined function at the specified time and condition[2] according to the known statistical parameters of the random variable and the probability distribution model and the given function. The probability that the function cannot be completed is called the failure probability.

Common methods for calculating slope reliability include a second-order moment method and Monte Carlo simulation method. The accuracy of the second-order moment method is lower, and the Monte Carlo method is more computationally intensive. With the development of nonlinear theory, the use of alternative models to approximate the constructor[3], the complex slope stability implicit function is displayed, to improve work efficiency. Common models are including intelligent algorithms such as neural networks and support vector machines, which open up new avenues for slope stability analysis. Aiming at the practical problem, this paper proposes a slope reliability analysis method based on particle swarm optimization (PSO) to optimize radial basis function (RBF) neural network. The strength reduction method is used to calculate the cohesion and internal friction angle of rock and soil. Safety factor; the obtained data is trained as a training sample into the RBF network model, and the relationship between the safety factor and the safety factor is mapped, the PSO algorithm is further optimized to construct the response surface function; Combined with Monte Carlo method, a large number of generations are generated. The random number sample is simulated to solve the slope instability probability.
2. Reliability Analysis Based on RBF Network

2.1. ABAQUS strength reduction method

ABAQUS can be used to solve nonlinear problems, and can simulate various complex material constitutive relations, also can adapt to various complex boundary problem. The strength reduction method was first proposed by Zienkiewicz et al., and later adopted by the scholars and proposed the concept of shear strength reduction factor (SSRF), which is defined as: assuming that the external load remains unchanged, the soil within the slope can provide the maximum shear stress to the actual shear stress generated by the external load in the slope. When the shear strength of all soils in the slope is the same, the shear strength coefficient is equivalent to the stability safety factor $F_s$. The reduced shear strength parameters can be expressed as:

$$c_m = c / F_r,$$

$$\varphi_m = \arctan(tan\varphi / F_r),$$

In the formula, $c$ and $\varphi$ is the shear strength parameter that the soil can provide; $c_m$ and $\varphi_m$ is the shear strength parameter required to maintain equilibrium; $F_r$ is the strength reduction factor.

2.2. RBF neural network

The RBF neural network is a three-layer feed forward neural network consisting of an input layer, an implicit layer and an output layer. It can approximate any continuous function with a given precision[4] under the premise of proper parameter selection. It uses the radial basis function as the excitation function. Since the most commonly used basis function is a Gaussian function, the activation function defining the $i$-th implicit unit is:

$$\phi_i(x) = \exp(-\frac{1}{2\sigma_i^2} \|x - \mu_i\|^2), \quad i = 1, 2, \cdots H$$

The function center $\mu_i$ is the clustering center obtained by the K-means algorithm on the training samples, and $\sigma_i$ is the width parameter of the function, $\sigma_i = d_{max} / \sqrt{2H}$, $d_{max}$ is the maximum distance of the cluster center (between two samples).

The hidden layer to output layer maps to: $y(x) = \sum_{i=1}^{H} \omega_i \phi_i(x)$

2.3. PSO algorithm

The PSO algorithm is that each particle in the group constantly changes its own velocity vector and displacement vector during the iterative process, to find the global optimal position. The iterative process of the particle satisfies:

$$v_i^{t+1} = \omega' v_i^t + c_1 r_1 [p_{best}^t - x_i^t] + c_2 r_2 [g_{best}^t - x_i^t],$$

$$x_i^{t+1} = x_i^t + v_i^{t+1},$$

In the formula, $v_i^{t+1}$ and $x_i^{t+1}$ represent the velocity vector and displacement vector of the $i$-th particle after iterations $t+1$ times, $p_{best}^t$ is the optimal position of the particles found by the $i$-th particles after iteration $t$ times, $g_{best}^t$ is the optimal position of the population found by the particle group after iterations $t$ times, $r_1$ and $r_2$ take the random number between $[0, 1]$; $c_1 = c_2 = 2$; $\omega'$ is the weight of the $t$-th iteration, and the expression is as in equation : $\omega_t = \omega - t(\omega - \omega_1)/T^*$.

In the formula, $\omega_i$ and $\omega_2$ represent the final and initial iteration rights, respectively heavy, $t$ is the current number of iterations, and $T$ is the maximum number of iterations.

The PSO optimization RBF model[5] steps are mainly including:
First, model initialization, population size, number of iterations, and weight; Calculate the fitness of each particle as follows:

\[ f(x_i) = \frac{1}{K} \sum_{j=1}^{K} \left| f_j - y_j \right|, \]

(6)

In the formula, \( y_j \) and \( f_j \) represent the measured and predicted values.

Second, taking the particle with the least fitness in the population as the initial value of \( g_{\text{best}} \), taking the current position of the particle as the optimal \( p_{\text{best}} \), finding the position of the particle with the optimal fitness value as \( p_{\text{best}} \).

Third, compare the optimal solution fitness with individuals and populations, and smaller as \( g_{\text{best}} \). update the velocity and position of the particle and the weight of the iteration until the number of iterations satisfies the end condition, the particle corresponding to \( g_{\text{best}} \) is taken as the parameter of the RBF.

2.4. Monte Carlo simulation method

Monte Carlo is also known as stochastic simulation method or statistical test method. Because of its limited limitation and simple idea, it has been widely used\(^6\). It can be assumed that the function of the structure is known and the probability distribution of the basic random variable. When a number of selected samples are large enough, the probability of the event actually occurring can be obtained by frequency approximation.

Firstly determine the parameter statistics and probability distribution of the state variables, and determine the structural function of the slope according to the response surface function of the RBF fit: \( F = g(x_1, x_2, \ldots, x_n) \). The limit state equation can be expressed as \( Z = F - 1 \), so it is repeated \( N \) times to obtain sample values \( Z_1, Z_2, \ldots, Z_N \). If \( Z < 0 \) is a landslide failure event, then \( M \) times appear in \( N \) samples, then Bernoulli’s large number theorem. It can be seen that the probability of failure is:

\[ P_f = P(Z < 0) = P(F < 1) = \frac{M}{N}, \]

(7)

The above formula is the failure probability calculated by Monte Carlo method. For the obtained \( N \) groups of \( Z \), the mean and standard deviation are:

\[ \mu_Z = \frac{1}{N} \sum_{i=1}^{N} Z_i, \]

(8)

\[ \sigma_Z = \left( \frac{1}{N-1} \sum_{i=1}^{N} (Z_i - \mu_Z)^2 \right)^{1/2}, \]

(9)

Using \( \beta \) to represent the reliability indicator, then \( \beta \) can be expressed as: \( \beta = \frac{\mu_Z}{\sigma_Z} \).

3. Case analysis

The Banling Landslide is located in the Lower Banling Natural Village, Huangshawa Town, Suichang County, Lishui City, Zhejiang Province. In this experiment, the cohesive force \( c \) values and internal friction angle \( \phi \) values of the geotechnical parameters obtained from multiple experiments were calculated, and the mean and standard deviation were calculated.

3.1. Solving the slope safety factor

The mean value of the parameters in the saturated state is taken as input: cohesion \( c = 13.38 \text{kPa} \), internal friction angle \( \phi = 12.26^{\circ} \), gravity \( \gamma = 20 \text{kN/m}^2 \), Young’s modulus of elasticity \( E = 20 \text{MPa} \), coefficient of variation \( \delta = 0.315 \), Poisson’s ratio \( \nu = 0.3 \).
First, build the components in the ABAQUS software and analyze the results. When the $c \cdot \phi$ value takes the saturation state mean, FV1 and U1 are selected as the output variables. It can be seen from Figure 1 that the displacement inflection point is used as the evaluation criterion for slope stability, the safety factor is 0.81; Convergence was used as the evaluation criterion and the corresponding FV1 was 0.83. These two values are relatively close to the $F_S = 0.82$ calculated by the limit equilibrium method, indicating that the method is feasible.

**Figure 1. Plastic strain incremental**

**Figure 2. Prediction results**

3.2. Generate training test samples

In this experiment, a total of 48 parameters were selected to solve the safety factor by ABAQUS software, and the data was divided into 7:3 ratios as a sample for network model training and testing, as shown in Table 1.

| Numbering | Cohesion $(c/\text{kPa})$ | Internal friction angle $\phi/(\circ)$ | Stability factor |
|-----------|--------------------------|----------------------------------------|-----------------|
| 1         | 12.24                    | 11.59                                  | 0.78            |
| 2         | 18.33                    | 14.80                                  | 0.92            |
| 3         | 13.66                    | 11.87                                  | 0.80            |
| ...       | ...                      | ...                                    | ...             |
| 46        | 12.07                    | 11.41                                  | 0.78            |
| 47        | 15.76                    | 17.26                                  | 0.90            |
| 48        | 30.53                    | 16.53                                  | 1.19            |

3.3. Analysis of training results

After many calculations, the particle swarm size is 20, the particle dimension is 12, the maximum number of iterations is 250, the initial weight is 0.9, $c_1 = c_2 = 1.49$, and when the number of network hidden nodes is about 10, the network has the most predictive performance. Well, the decision coefficient is above 99%, the simulation error is no more than 0.01; The relative error of direct prediction using RBF network is about 0.05, and the accuracy is significantly lower than the result of PSO-RBF prediction. The result is shown in Figure 2. It can be seen that the PSO optimized RBF neural network is suitable for the research in this paper. The response surface function $S = f(c, \phi)$ fitted to the RBF is shown in Figure 3.

3.4. Monte Carlo simulation calculation

The positive distribution usually obeyed by the random variable is a non-standard normal distribution $N(0, \sigma^2)$, which can be obtained by linear transformation using the random variable $N(0,1)$ of the standard normal distribution $x'$:
\[ X = \mu + \alpha \sigma' \] (10)

The random variable \( X \) obeys the lognormal distribution\([7]\) and can be transformed by the formula \( Y = \ln X \), then \( Y \) obeys the positive distribution.

According to the literature\([8]\), the cohesion force obeys the lognormal distribution, and the internal friction angle obeys the normal distribution. The probability density distribution obtained by randomly acquiring 15000 sets of data using function of matlab is shown in figure 3.

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The generated 15000 sets of data are brought into the trained model, and the corresponding safety factors can be obtained in turn. According to formulas (7)-(9), the natural and saturated instability probability is 0.1398 and 0.4127, respectively. It is 1.755 and 0.225.

The number of simulations is increased from small to large. When the number of simulations is less than 2000, the probability of instability is relatively large. When the number of simulations exceeds 6000, the probability of instability gradually becomes stable. Finally, the simulation reaches 15000 times and the simulation times are counted. As shown in figure 5.

3.5. Comparative analysis of different methods

Comparison of related literature, the direct Monte Carlo method is used, and the reliability analysis method based on BP and RBF and the RBF network method optimized by PSO are used to calculate the reliability of the two slope conditions. The direct Monte Carlo method is used as the comparison object. The error of this method is 0.79% and 2.59%; the error of BP method is 5.77% and 9.52%; The error of RBF method is 2.33% and 7.36%. In summary, the error of the calculation results of this method is small, which proves the feasibility of the method.

4. Conclusions and prospects

In this paper, the feasibility of the slope reliability analysis method based on ABAQUS and RBF neural network is verified by theoretical analysis combined with specific experiments. Taking the specific slope as an example, the parameters obtained by the geotechnical experiment and the ABAQUS strength reduction method are used to construct the sample data, and the function is fitted by the RSO neural network optimized by PSO; Then the instability probability of the slope is obtained by Monte Carlo simulation. The accuracy of the model is more than 99%. The method is similar to the calculation results of other related methods, but it uses the fast convergence of learning, the fitting
precision is better, and the calculation efficiency is higher. The optimization accuracy of the network makes the calculation accuracy relatively simple. The RBF network method has been further improved, and the reliability analysis method is introduced, which avoid some shortcomings of the traditional method that also play a key role in the risk assessment of the future landslide. It has certain practical value in practical engineering research. Later research can further increase the training data, and further optimize the network to achieve better results.

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