Plasmoid statistics in relativistic magnetic reconnection

M. Petropoulou\textsuperscript{1}, I. M. Christie\textsuperscript{1}, L. Sironi\textsuperscript{2}, D. Giannios\textsuperscript{1}

\textsuperscript{1}Department of Physics, Purdue University, 525 Northwestern Avenue, West Lafayette, IN, 47907, USA
\textsuperscript{2}Department of Astronomy, Columbia University, 550 W 120th St, New York, NY 10027, USA

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ABSTRACT

Plasmoids, quasi-spherical regions of plasma containing magnetic fields and high-energy particles, are a self-consistent by-product of the reconnection process in the relativistic regime. Recent two-dimensional particle-in-cell (PIC) simulations have shown that plasmoids can undergo a variety of processes (e.g. mergers, bulk acceleration, growth, and advection) within the reconnection layer. We developed a Monte Carlo (MC) code, benchmarked with the recent PIC simulations, to examine the effects of these processes on the steady-state size and momentum distributions of the plasmoid chain. The differential plasmoid size distribution is shown to be a power law, $N(w) \propto w^{-\chi}$, ranging from a few plasma skin depths to $\sim 0.1$ of the reconnection layer’s length. We demonstrate numerically and analytically that the power law slope $\chi$ is linearly dependent upon the ratio of the plasmoid acceleration and growth rates and that it slightly decreases (from $\sim 2$ to $\sim 1.3$) with increasing plasma magnetization (from 3 to 50). We perform a detailed comparison of our results with those of recent PIC simulations and briefly discuss the astrophysical implications of our findings through the representative case of flaring events from blazar jets.

Key words: accretion discs – galaxies: jets – gamma-ray burst: general – magnetic reconnection – stars: pulsar winds

1 INTRODUCTION

Magnetic fields are ubiquitous in a variety of astrophysical sources. How their energy is transferred to the plasma to be later radiated away and power the observed emission remains a fundamental question in modern high-energy astrophysics. The topological rearrangement of magnetic field lines with opposite polarity – the so-called magnetic reconnection process – is a mechanism that can transfer the magnetic energy to plasma resulting in heating and particle acceleration.

The reconnection process has been studied extensively and is an excellent candidate for producing variable, high-energy radiation in many astrophysical environments such as pulsar wind nebulae (PWNe; e.g., Lyubarsky & Kirk 2001; Lyubarsky 2003; Kirk & Skjæraasen 2003; Péri & Lyubarsky 2007; Sironi & Spitkovsky 2011; Cerutti et al. 2013, 2014; Lyutikov et al. 2016), jets from active galactic nuclei (AGNs; e.g., Romanova & Lovelace 1992; Giannios et al. 2009, 2010; Giannios 2013), gamma-ray bursts (GRBs; e.g., Thompson 1994, 2006; Usov 1994; Spruit et al. 2001; Drenkhahn & Spruit 2002; Lyutikov & Blandford 2003; Giannios 2008), accreting black holes (Galeev et al. 1979; Haardt & Maraschi 1991; Beloborodov 2017), and solar flares (Lin et al. 2005; Hassanin & Kliem 2016).

Magnetic reconnection is a highly dynamical process for the plasma conditions present in the aforementioned astrophysical environments. The reconnection layer is prone to tearing instabilities (e.g. Furth et al. 1963; Loureiro et al. 2007; Uzdensky & Loureiro 2016; Comisso et al. 2016) and can undergo fragmentation into magnetic islands, the so-called plasmoids. These are, in turn, separated by secondary current sheets that are shorter and still subject to the same instabilities. This fragmentation process recurs at progressively smaller scales that exhibit an approximate self-similarity in their properties (Shibata & Tanuma 2001; Huang & Bhattacharjee 2010). The fractal-like appearance of the reconnection layer may also be related to the generation of power laws (Schroeder 1991) that allow the extrapolation and prediction over a wide range of scales.

Recent analytical calculations and magnetohydronamic simulations of magnetic reconnection in highly conducting plasmas have been used to determine the statistical properties of the resulting plasmoid chain. The plasmoid magnetic flux distribution was shown to obey a power-law of slope $-2$ (Uzdensky et al. 2010; Loureiro et al. 2012) or $-1$ (Huang & Bhattacharjee 2012, 2013), depending on the underlying assumptions made in each case.

In the collisionless regime of reconnection, which is applicable to most astrophysical environments, the most fundamental way to capture the interplay between particles and fields is by means of kinetic particle-in-cell (PIC) simula-
tions. PIC simulations of reconnection have been recently extended to the so-called relativistic regime, where the magnetic energy density is much larger than the rest-mass energy density of the unconnected plasma, or the plasma magnetization $\sigma \gtrsim 1$ (Zenitani & Hoshino 2001; Guo et al. 2014; Sironi & Spitkovsky 2014; Nalewajko et al. 2015; Sironi et al. 2015; Kagan et al. 2016; Werner et al. 2016). This regime is most likely relevant to AGN jets, pulsar winds, and GRBs. Sironi et al. (2016) (hereafter, SGP16) performed two-dimensional (2D) PIC simulations of antiparallel reconnection (i.e., in the absence of a guide field) in electron-positron plasmas for three different plasma magnetizations. The simulations revealed the rich and complex dynamics of the reconnection layer. Being extended to unprecedentedly long time scales and length scales, they also allowed the investigation of the properties of the plasmoid chain (e.g., the geometry, the momentum, the particle and magnetic energy content of individual plasmoids) as a function of time and system size.

The aim of this study is to shed light into the processes that shape the plasmoid size and momentum distributions in light of recent PIC simulations of relativistic magnetic reconnection. To achieve our goal, we develop a Monte Carlo (MC) code that follows the evolution of individual plasmoids as they grow, accelerate, merge or leave the layer. We calibrate the MC code using some findings of the SGP16 PIC simulations (e.g., plasmoid acceleration rate), but we also make predictions (e.g., distribution of plasmoid four-velocities) that compare well with PIC results.

Although the MC approach cannot replace first-principle PIC simulations of the reconnection layer (e.g., it carries no information on particle acceleration), it offers several advantages over them:

(i) it is computationally cheaper, thus allowing the realization of many numerical experiments for the same physical conditions of the system (e.g., plasma magnetization). The repetition of numerical experiments is particularly important in studies focusing on the statistical properties of a system (here, the plasmoid chain) whose evolution is governed by some intrinsically random process (e.g., mergers).

(ii) it facilitates the study of the system over long temporal and spatial scales without the need of extensive computational resources. In fact, the extreme separation between the microscopic plasma scales that PIC simulations need to resolve and the large physical scales where the emission takes place (e.g., nine orders of magnitude in AGN jets) usually hinders a direct application of PIC findings to astrophysical observations.

(iii) it allows us to isolate the role of individual physical processes, such as plasmoid acceleration and mergers, on the statistical properties of the plasmoid chain. This is achieved by artificially turning off individual processes in the MC code. This approach is not possible in PIC simulations.

Using the MC code we have developed, we find that the differential plasmoid size distribution forms a power law, $N(w) \propto w^{-\chi}$, beyond a characteristic plasmoid size of several plasma skin depths\(^1\). We find that the power-law slope $\chi$ decreases from $2$ to $1.3$ as the magnetization $\sigma$ increases from $3$ to $50$. The slope of the power law is proportional to the ratio of the plasmoid acceleration rate to the plasmoid growth rate ($\chi \propto \beta_a/\beta_g$), a result we also derive analytically, and is not altered by mergers. We show that the formation of a power law in sizes is the result of the interplay between plasmoid acceleration and growth. The power law extends from small sizes (i.e., several plasma skin depths) to large sizes that are a significant fraction (i.e., up to $\sim 10$ percent) of the reconnection layer’s length. As the system becomes larger, so does the extent of the power law. The cutoff of the distribution at large sizes is populated by a few plasmoids or “monster” plasmoids (Uzdensky et al. 2010). The cutoff shifts towards larger sizes almost linearly with time until the monster plasmoids exit the system. We also measure the size distribution of plasmoids at the moment they exit the layer and find that it is similar to the position-integrated distribution of sizes, as expected in a steady state system. We demonstrate that the differential distribution of plasmoid momenta becomes softer at higher $\sigma$, in agreement with PIC results, and does not depend on the system’s size. Our main results can also be extended to high magnetization ($\sigma \gtrsim 3$) electron-proton reconnection, since the system behaves similarly to the electron-positron case (Sironi et al. 2015).

This paper is structured as follows. In Sect. 2 we summarize the basic findings of the PIC simulations presented in SGP16. These were used in the development of the MC code, which is outlined in Sect. 3. The results of our MC simulations of the reconnection layer are presented in Sect. 4. An application of our results to blazar variability is presented in Sect. 5. We conclude in Sect. 6 with a short discussion and summary of our results.

2 SUMMARY OF PIC RESULTS

SGP16 employed large-scale 2D PIC simulations in electron-positron plasmas for three different magnetizations ($\sigma = 3, 10, 50$). The simulations were extended to unprecedentedly large spatial scales (i.e., thousands of electron skin depths). SGP16 were able to follow the system evolution for several light crossing times of the layer, due to the use of outflow boundary conditions instead of the commonly employed periodic boundary conditions. The simulations were able to capture the dynamics of the reconnection layer at times when the system was no longer affected by the initial setup. SGP16 showed that plasmoids are continuously generated as a self-consistent by-product of the reconnection process and highlighted the dynamic nature of the layer.

Here, we summarize the main findings of PIC simulations that are relevant to the calculations presented in this paper (see Sect. 3) and we refer the reader to SGP16 for more details.

2.1 Plasmoid birth

Plasmoids are constantly generated due to the fragmentation of secondary current sheets. The initial plasmoid size

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\(^1\) The electron skin depth is defined as $c/\omega_p \equiv c\sqrt{\mu_e/(4\pi n e^2)}$. 

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is a few electron skin depths and, thus, a small fraction of the layer’s length. The separation of neighboring plasmoids at birth is on average ten times larger than their initial size. Plasmoids are uniformly formed in the available free spaces of the layer at any time. We indicate the plasmoid location at birth with $x_0$. The separation of plasmoids born close to the center of the layer and on opposite sides with respect to it quickly increases, as the tension force of the field lines drags them in opposite directions. Thus, as time progresses the available locations for additional plasmoid generation become concentrated close to the center of the layer. This results in a distribution of birth locations that peaks at $x_0 \approx 0$ at later times. The initial plasmoid momentum, $p_0$, is found to correlate with the birth location. This is exemplified in Fig. 1, where $p_0$ is plotted against $x_0$ for all the plasmoids formed in the layer in the course of the PIC simulation with $\sigma = 10$. The average $p_0$ of the plasmoid population is described well by the relation

$$\langle p_0 \rangle = 0.66 \sqrt{\sigma} \tanh \left( \frac{x_0}{\ell_0} \right),$$

where $\sqrt{\sigma}$ is the asymptotic bulk four-velocity in the reconnection layer (Lyubarsky 2005). In fact, the distribution of the differences $\langle p_0 \rangle - p_0$ (residuals) is approximately Gaussian with zero mean and standard deviation of $0.3 \sqrt{\sigma}$ (Andræ et al. 2010). The characteristic length scale, $\ell_0$, also depends on the plasma magnetization as:

$$\ell_0 \approx 0.25 L \left( \frac{\sigma}{10} \right),$$

where $L$ is the half length of the layer. This implies that at higher magnetizations few plasmoids are born with momenta as large as $\sqrt{\sigma}$. As a consequence, few plasmoids will reach the terminal momentum $\sqrt{\sigma}$ in the course of their evolution (see also SGP16).

2.2 Plasmoid growth
Secondary plasmoids grow mostly via the accretion of smaller plasmoids. SGP16 demonstrated that during the most active phase of growth the plasmoid size increases as:

$$\frac{dw}{dt} = \beta_g c \gamma$$

where time is measured in the layer’s frame, $\gamma = \sqrt{1 + p^2}$ is the Lorentz factor of the plasmoid and $\beta_g$ is the growth rate as measured in the co-moving frame of the plasmoid. SGP16 demonstrated that $\beta_g$ is about half of the reconnection inflow rate which, in turn, depends only weakly on $\sigma$ (see Table 1). Although the equation above provides an overall good description of the average plasmoid growth, it over-predicts the growth of the fastest plasmoids – see Fig. 8 in SGP16 and Appendix A for more details. We therefore introduce a “suppression factor” of the plasmoid growth, which should multiply the right-hand side of eqn. (3). This depends upon the plasmoid four-velocity $p$ and is given by:

$$f_{\text{sup}}(p) = \frac{1}{\sqrt{\sigma}} \left( 1 - \tanh \left( \frac{|p| - A}{B} \right) \right)$$

where $A, B$ are parameters to be determined (see Appendix A). For the values of $\sigma$ explored here, $A/B \gg 1$ (see Table 1), so that $f_{\text{sup}} \to 1$ when $|p|/\sqrt{\sigma} \ll 1$ (i.e., no suppression of the growth at non-relativistic speeds).

2.3 Plasmoid acceleration
Secondary plasmoids are accelerated to the Alfvén speed ($v_A/c = \sqrt{\sigma/(1 + \sigma)}$) by the tension force of the reconnected magnetic field. The momentum $p$ of an individual plasmoid at time $t$ is found to depend upon the position of its center in the layer $x$ and its size $w$ as (see Fig. 10 in SGP16):

$$p(t) \approx \sqrt{\sigma} \tanh \left( \frac{\beta_s x(t) - x_0}{w(t)} \right) + p_0,$$

where $t$ is measured in the layer’s frame and $\beta_s$ is the acceleration rate of the plasmoid, which is only weakly dependent upon $\sigma$ (see Table 1).

3 MONTE CARLO CODE
Here, we present the main ingredients of the MC code and its calibration using the SGP16 simulation results.

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2 In the PIC simulations of SGP16, reconnection is triggered at the center of the layer, a choice that eventually determines the geometry of the field lines in steady state, see Fig. 1 in SGP16.

3 Henceforth, we use the term momentum and dimensionless four-velocity interchangeably.
All plasmoids are born with transverse size of several $c/\omega_p$, which is typically a small fraction of the layer’s half-length $L$. For most of our results and for their direct comparison with the PIC simulations (Sect. 4), we set the plasmoid size at birth $w_0 = 10^{-3} L$. In Sect. 4.7 we explore how our results change for smaller values of $w_0/L$, effectively corresponding to larger system sizes. The transverse size refers to the width of the plasmoid in the direction perpendicular to its motion and is Lorentz invariant. The longitudinal size as measured in the co-moving frame of the plasmoid is $w_1 \simeq (3/2) w_0$ at all times (see Fig. 5 in SGP16).

The separation distance between two newly born neighboring plasmoids is $\delta x \sim 10 w_0$, as found in PIC simulations. At the beginning of the MC simulation, the total number of plasmoids is, therefore, $N_0 = 2L/\delta x$. At later times, new plasmoids are formed in regions of the layer where there is unoccupied space. The new plasmoids are born at random locations $x_0$ within the available free space, with the same size and separation distance as the initial ones.

The plasmoid momentum at birth is drawn from a Gaussian distribution of random numbers. The mean of the distribution is given by eqn. (1) and its standard deviation is 0.3$\sqrt{\sigma}$. Although the number of plasmoids with $|p_0| > \sqrt{\sigma}$ is small (i.e., a few percent of the total number), we impose a hard upper limit of $1.5\sqrt{\sigma}$ on the initial four-velocity. Any plasmoid that initially does not satisfy this condition is rejected and another random $p_0$ value is drawn from the distribution.

At every time step, we identify the plasmoids that terminate their lives due to advection beyond the layer by checking if their inner boundary is located outside the layer at time $t$, namely:

$$x_i(t) \equiv x(t) - \frac{1}{2} \frac{w_i(t)}{\sqrt{1 + p(t)^2}} > L, \quad x(t) > 0$$

$$x_c(t) \equiv x(t) + \frac{1}{2} \frac{w_c(t)}{\sqrt{1 + p(t)^2}} < -L, \quad x(t) < 0,$$

where $x$ is the location of the plasmoid’s center at time $t$. Mergers are another way of terminating the life of a plasmoid. At every time step, we check for mergers between existing plasmoids. As in Fig. 2, let us consider the case $x_i(t) < x_c^{\delta t}(t)$ and assume that the two plasmoids move to the right. A merger is defined based on one of the following criteria:

(i) $x_i(t) + \delta t > x_c^{\delta t}(t + \delta t)$. This suggests that an intersection of the outer boundaries of the two plasmoids took place.

(ii) $x_i(t + \delta t) > x_c^{\delta t}(t + \delta t)$. This suggests that an intersection of the plasmoids’ centers took place.

(iii) $x_c(t + \delta t) > x_c^{\delta t}(t + \delta t)$ or $x_i(t + \delta t) > x_i^{\delta t}(t + \delta t)$. This suggests that an intersection of the two plasmoids took place.

Case (a) shown in Fig. 2 would be identified as a merger only using criterion (iii), whereas case (b) would be recognized as a merger by criteria (ii) and (iii). All the above criteria would identify case (c) as merger. For the results presented in the next section, we use criterion (i), but we demonstrate how different criteria affect our results in Sect. 4.5. The smaller plasmoid of the merging pair is removed from the simulation after the merger. The plasmoid surviving the merger is assumed to have the same size as the larger plasmoid of the merging pair, since the average growth rate $\beta_k$ already includes the effect of mergers.

At every time step, we identify all the plasmoids that are still present in the layer and update their transverse size and momentum. We first advance the position of a plasmoid according to $x(t) \rightarrow x(t) + v(t)\delta t$, where $v$ is the plasmoid velocity and $\delta t$ is the time step of integration. In order to capture the evolution of the fastest plasmoids in the layer, we use $\delta t < \delta x/v_s$. The transverse size is then updated according to eqn. (3) including the suppression factor. Finally, the plasmoid momentum is updated according to eqn. (5).

4 RESULTS

In this section, we compare the results of the MC code described in Sect. 3 against PIC simulations of relativistic
magnetic reconnection with respect to the statistics of the plasmoind chain. We show that the basic properties of the plasmoind chain seen in detailed PIC simulations of the reconnection layer can be recovered by the MC code. We then compare our results with those of previous studies of the plasmoind chain (Uzdensky et al. 2010; Huang & Bhattacharjee 2012). Finally, we study the role of individual processes, such as plasmoind acceleration and merging, on the shape of the plasmoind size distribution.

### 4.1 Comparison of MC with PIC results

The distributions of plasmoind sizes ($dN/d\log w$) and momenta ($dN/d|p|$) are useful diagnostics of the plasmoind chain statistics. Henceforth, we adopt the notation $N(X) \equiv dN/dX$ to refer to the differential distribution of plasmoinds with respect to quantity $X$. The distributions are position- and time-integrated, unless stated otherwise. The distribution of magnetic fluxes $\Psi$ is expected to approximately follow the size distribution, since $\Psi \propto w$ for $w \gtrsim 10\omega_a$ (see Fig. 5 in SGP16). The parameters of our MC code have been benchmarked with the PIC simulations of SGP16 and are presented in Table 1.

We simulated the formation of the plasmoind chain using the MC code for $\sigma = 3, 10, \text{and } 50$. We let the system evolve for two light crossing times ($2L/c$). This period is comparable to the time interval during which secondary plasmoind generation occurs in PIC simulations along the whole layer (for details, see SGP16). A comparison of the distributions obtained from our MC code and the PIC simulations of SGP16 is presented in Figs. 3-4. The results from the SGP16 simulations are plotted with black symbols, while coloured lines show the results of 20 MC realizations. The size distributions can be approximated by a broken power law (for $\sigma = 3$ and 10) or a single power law (for $\sigma = 50$) that cuts off at sizes that are a significant fraction of the layer’s length (here, at $w \sim 0.1L$). More details about the shape of the size distribution can be found in Sect. 4.3.

The scatter observed in the MC results stems from the intrinsic randomness of the MC approach and is typically much larger than the errors associated with the number of counts per bin. There is an overall good agreement between the MC results (coloured lines) and PIC results (black symbols) for the plasmoind size distribution (Fig. 3). This is not unexpected, since we used the plasmoind size distribution in order to determine the values of the parameters $A$ and $B$ appearing in the growth suppression factor (for details, see Appendix A).

Still, it is not obvious a priori if the MC code can reproduce the distribution of plasmoind momenta, $dN/d(|p|/\sqrt{\sigma})$, which is an independent diagnostic. As shown in Fig. 4, there is an overall good agreement between the MC and PIC results. The momentum distribution obtained from PIC simulations extends beyond $\sqrt{\sigma}$, most likely due to the acceleration of a few plasmoinds born in the vicinity of larger plasmoinds where the local Alfvén velocity is higher. This effect is not included in our general description for plasmoind momentum in eqn. (5), which was benchmarked using moderately large plasmoinds from PIC simulations. Thus, a deviation between the MC and PIC results is expected for the highest four-velocities. We also find an excess of plasmoinds with $|p| > 0.7\sqrt{\sigma}$ with respect to the PIC results for the $\sigma = 50$ case. This discrepancy implies that our chosen general prescription for the evolution of the plasmoind momentum with time cannot capture in full detail the plasmoind dynamics for the $\sigma = 50$ case.

Let us take a closer look at the properties of plasmoinds that undergo mergers (see Fig. 5). Quantities with subscript “f” correspond to the plasmoind that survives the merger. Similarly, the properties of the plasmoind that will be absorbed by the larger one during the merger are denoted with the subscript “i”. These properties in the MC code are measured just before a merger. However, in PIC simulations this is not always possible due to the limited time sampling in output data that will be used for the post-processing. For example, two plasmoinds (especially those with small sizes) may be born and merge within a time window during which no PIC data have been recorded. These plasmoinds will not be counted as a merging pair but as one plasmoind, thus producing a bias towards a smaller number of merging pairs at small sizes. Hence, we consider only plasmoinds with $\omega_i > 3 \omega_a$ which had some time to grow and can be confidently identified when post-processing the PIC data.

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**Table 1. Parameters of the plasmoind chain**

| $\sigma$ | 3 | 10 | 50 |
|----------|---|----|----|
| $\beta_a$ | 0.10 | 0.12 | 0.13 |
| $\beta_k$ | 0.06 | 0.08 | 0.10 |
| $\ell_a/L$ | 0.075 | 0.25 | 1.25 |
| $A$ | 0.86 | 0.77 | 0.63 |
| $B$ | 0.019 | 0.033 | 0.024 |

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**Figure 4.** Histograms of $|p|/\sqrt{\sigma}$ normalized to the total number of plasmoinds $N_{isl}$ for the three magnetizations considered in the text. Symbols and lines have the same meaning as in Fig. 3.
Figure 5 shows a density map of the relative four-velocity and relative size of merging pairs obtained in one of the SGP16 simulations (top panel) and in one of our MC realizations for $\sigma = 10$ (bottom panel). To facilitate the comparison of the two, we randomly selected a sub-sample of pairs from the MC code that matches the total number of merging pairs from PIC. A similar plot is presented in Fig. 6 with the additional information of the plasmoid size after the merger, $w_f$ (see colour bar). A few things that are worth mentioning follow:

- The density of merging pairs in the MC realization peaks at $w_f - w_i \approx 0$ and $|p_f - p_i| \lesssim 0.5 \sqrt{\sigma}$, in agreement with PIC results (Fig. 5). Most of the merging plasmoids have similar sizes of the order of $w_f$ (i.e., $w_i \approx 3 w_f$, see Fig. 6).
- The MC code also predicts a population of plasmoids with large size difference and $|p_f - p_i| \sim 0.5 \sqrt{\sigma}$ at merger. This is in excellent agreement with PIC results (Figs. 5-6).
- The MC code produces a few merging pairs with $|p_f - p_i| \approx \sqrt{\sigma}$ and $w_f - w_i \approx 0$ that are absent in the PIC simulation. These are plasmoids that did not have time to grow ($w_f \approx w_i$) and quickly merged because of their opposite motions at initialization. Thus, this discrepancy with the PIC results is related to the initialization of the plasmoid momentum in the MC code. The difference would become less prominent, if a smaller scatter around $p_0$ (see eqn. 1) were to be used.
- The differences in the properties of the merging pairs between PIC simulations and MC calculations do not seem to strongly affect the size and momenta distributions (see Figs. 3-4).
- The distribution of merging pairs in Figs. 5 and 6 is subject to the randomness of the reconnection process. We verified this by creating the same plots for different MC realizations. This does not alter the aforementioned results, though.

4.2 Comparison with other studies

The statistics of the plasmoid chain in magnetic reconnection has been discussed in detail by Uzdensky et al. (2010) and Huang & Bhattacharjee (2012). Using heuristic arguments Uzdensky et al. (2010) showed that $dN/dw = N(w) \propto w^{-2}$ for a range of plasmoid sizes where the merging rate approximately equals the growth rate. On the contrary, Huang & Bhattacharjee (2012) suggested that $N(w) \propto w^{-1}$. The main difference between the two approaches lies in the assumptions made about the relative velocity between merging plasmoids. More specifically, Uzdensky et al. (2010) assumed that all plasmoids move with the Alfvén speed, whereas Huang & Bhattacharjee (2012) used a size-dependent relative velocity of the merging plasmoids, as dictated by resistive magnetohydrodynamic simulations of non-relativistic reconnection (see also Huang & Bhattacharjee 2013).

In order to compare our results with those presented by Uzdensky et al. (2010) we (i) fix the plasmoid velocity in our MC code to the Alfvén speed for $\sigma = 10$, (ii) assume a constant growth rate in the co-moving frame of the plasmoids, (iii) set the separation distance of newly born plasmoids to be a multiple of the initial plasmoid size $w_f$, and let the system evolve for two light crossing times. By choosing $\beta_\delta = 0.4$ we ensure that plasmoids may grow at most up to $\sim \beta_\delta L/\sqrt{\sigma} \sim 0.1 L$ for $\sigma = 10^3$. This results in a dynamic range of sizes that is sufficient to explore the formation of a power law, as demonstrated by Uzdensky et al. (2010).

Our results are presented in Fig. 7 for three choices of the separation distance $\delta x$ as indicated on the plot. The dashed line has slope $-1$ and corresponds to $N(w) \propto w^{-2}$. For each choice of $\delta x$ the size distribution exhibits two power law segments (i.e., $N(w) \propto w^{-1}$ at small sizes and $N(w) \propto w^{-2}$ at larger sizes). Figure 5. Density map of the relative four-velocity($\Delta p \equiv p_f - p_i$) and size ($\Delta w \equiv w_f - w_i$) of 356 merging plasmoid pairs with $w_i > 0.003 L$ and $\sigma = 10$. The results of a PIC simulation and a MC realization are shown in the top and bottom panels, respectively.
$N(w) \propto w^{-\chi}$ with $\chi \sim 1.7 - 1.9$ at intermediate sizes) followed by a steep cutoff. Our findings are in agreement with those presented by Uzdensky et al. (2010). Because all plasmoids in this scenario move with the same speed along the layer, mergers between neighboring plasmoids occur only if the plasmoids grow to a size comparable to their separation distance. As a consequence, the position of the break in the size distribution appears at $w = \delta x$, as illustrated in Fig. 7. Growth and advection from the layer are the two main processes that determine the evolution of plasmoids with $w \ll \delta x$ and, in this regime, $N(w) \propto \text{const}$ as mentioned by Uzdensky et al. (2010) and Huang & Bhattacharjee (2012). At sizes $w \gg \delta x$, the competition of plasmoid

**Figure 6.** Relative four-velocity ($\Delta p \equiv p_f - p_i$) and size ($\Delta w \equiv w_f - w_i$) of 356 merging plasmoid pairs with $w_i > 0.003L$ and $\sigma = 10$. The size of the plasmoid that survives the merger is colour coded, as shown in the colour bar at the top. The results of a PIC simulation and a MC realization are shown in the top and bottom panels, respectively.

**Figure 7.** Histogram of $\log(w/L)$ normalized to the total number of plasmoids, $N_{isl}$, for different choices of the initial plasmoid separation distance $\delta x$, as indicated on the plot. All plasmoids move with the Alfvén speed for $\sigma = 10$, undergo mergers, and grow at a constant rate $\beta_g = 0.4$. The dashed, dash-dotted, and dash triple-dotted lines correspond to the scalings $N(w) \propto w^{-2}$, $N(w) \propto w^{-1}$, and $N(w) \propto \text{const}$ respectively.

**Figure 8.** Normalized histogram of $\log(w/L)$ for different choices of the initial plasmoid separation distance $\delta x$. Plasmoids move with constant speed drawn from a Gaussian distribution with zero mean and standard deviation equal to $v_A/2$. Plasmoids undergo mergers, and grow at a constant rate $\beta_g = 0.08$. Black lines have the same meaning as in Fig. 7.
growth and mergers leads to a steeper power law whose index is in rough agreement with the findings by Uzdensky et al. (2010).

We next illustrate the effect of the plasmoid velocity distribution on the size distribution and comment on the scaling $N(w) \propto w^{-3}$ reported by Huang & Bhattacharjee (2012). To do so, we assume that plasmoids are born with velocity drawn from a Gaussian distribution with zero mean and standard deviation equal to $v_\chi/2$. As long as the adopted distribution has an extent of the order of the Alfvén speed, the exact functional form does not affect the size distribution. Furthermore, the plasmoid velocity remains constant in time and does not depend on the birth location in the layer. Our numerical setup is similar but not identical to that used in Huang & Bhattacharjee (2012). In their kinetic equation approach, the Gaussian profile referred to the relative velocity of merging plasmoids. This cannot be set as an independent parameter in our MC code, since it is automatically determined by the histories of the merging plasmoids. Additionally, the advection velocity of plasmoids from the layer was set equal to $v_\chi$ by (Huang & Bhattacharjee 2012). In our MC description of the plasmoid chain, the advection velocity is a parameter that is being determined by the histories of individual plasmoids.

Figure 8 shows the histogram of $\log w$ as obtained from our MC code for different choices of the initial separation distance and $\beta_\chi = 0.08$. The size distributions are qualitatively different from those shown in Fig. 7, thus highlighting the role of the plasmoids’ relative motion in shaping the size distributions. The relative motion between plasmoids leads to more frequent mergers than in the scenario where all plasmoids move with the same speed. For $\delta x \gtrsim 10w_\chi$, we find that $N(w) \propto w^{-\chi}$ with $\chi \sim 1.1 - 1.3$ (see black and magenta symbols). Given the fact that we cannot make the exact same assumptions as Huang & Bhattacharjee (2012), our result is in rough agreement with the scaling $\propto w^{-3}$ reported therein. However, the distribution tends to become softer for a given growth rate, if the initial separation distance gets smaller (blue and green symbols in Fig. 8). The slope of the differential size distribution lies between $-2$ and $-1$, i.e. between the values reported by Uzdensky et al. (2010) and Huang & Bhattacharjee (2012), respectively. In this regime, plasmoids merge not only due to their relative motion but also due to their growth. Even if two neighboring plasmoids move with the same speed, they will eventually overlap because of their growth.

Interestingly, our results support a scenario where $N(w) \propto w^{-2}$ for $\delta x \approx w_\chi$, and $N(w) \propto w^{-1}$ for $\delta x \gg w_\chi$, within the common simplifying assumptions made by Uzdensky et al. (2010) and Huang & Bhattacharjee (2012) (i.e., constant plasmoid speed and constant growth rate).

4.3 Analysis of individual processes

The focus of this section is the plasmoid acceleration and merging as well as their role in shaping the plasmoid size distribution. Armed with the MC code described in the previous sections, we may study in more detail individual processes. Readers who are not interested in the interplay of the various physical processes can skip this section and move directly to Sect. 5.
produce the second power-law segment and not the plasmoid mergers.

Using analytical arguments, one can show that a power law \((N(w) \propto w^{-\beta_a/\beta_g})\) is expected for plasmoids with \(w > \beta_a L/\sqrt{\sigma} \approx w_{max}^{\beta_a/\beta_g}\) (see Appendix B). These plasmoids are advected from the layer before reaching their terminal velocity. The size distribution obtained from the MC code (magenta symbols) is softer by 0.5 than the analytical prediction. This discrepancy is mainly caused by one of the simplifying assumptions made in our analytical approach, namely the omission of the term \(p_0\) in eqn. (B15). Indeed, if we modify the MC code so that \(|p_0| \ll \sqrt{\sigma}\) everywhere in the layer, we recover the power law index of \(-\beta_a/\beta_g\) predicted by our analytical study.

Despite this discrepancy, we were able to verify that the slope of the second power law segment depends linearly on the ratio \(\beta_a/\beta_g\), as predicted analytically. Figure 10 shows the size distribution obtained for different choices of \(\beta_a\) and \(\beta_g\), which, however, retain the same ratio \(\beta_a/\beta_g\). Indeed, the slope of the second power law segment \(\chi\) is the same for all three cases (here, \(\chi \approx 2\)). In addition, we find that the break in the size distribution is \(\propto \beta_g\). This supports our previous interpretation that the break occurs at \(\approx \beta_g L/\sqrt{\sigma}\) (see also Fig. 9). Overall, the second power law segment appears shifted towards larger sizes as long as the growth and acceleration rates increase by the same amount.

We reach similar conclusions for the other plasma magnetizations. SGP16 pointed out that the size distributions obtained from their PIC simulations for different magnetizations are similar (see also Fig. 3). A posteriori this is not unexpected, since the ratio \(\beta_a/\beta_g\) does not vary much for the values of \(\sigma\) explored therein (see Table 1).
cal green line) that scales with the system size $L$. Mergers do not affect the broken power-law shape of the distribution, but they push the position of the break to smaller sizes (magenta symbols). As we show in Sect. 4.7, the position of the break (henceforth noted as $w_m$) is a multiple of $w_*$ and, in contrast to $w_{m\text{,max}}$ (the monster size), does not depend on the system’s size. Fig. 11 also demonstrates that $w_m \to w_*$ when $\sigma = 50$, thus making the distribution look like a single power law (see also Fig. 3).

The results presented in Fig. 11 suggest that mergers do not affect the slope of the second power law segment (i.e., at $w \gtrsim w_m$). We demonstrated that the latter depends on the ratio $\beta_g/\beta_a$ (Sect. 4.4). To investigate if the power law slope of the distribution is affected by mergers, we varied the acceleration rate and computed the corresponding size distributions. Our results are presented in Fig. 12. It is evident that the slope of the second power law segment (for plasmoids smaller than the monster ones) changes with $\beta_a$, as discussed in Sect. 4.4, and that it is not affected by the coalescence of plasmoids.

So far, the identification of a merger in the MC code was based on the relative location of the plasmoids outer boundaries (for different merging criteria, see Sect. 3). Our results, however, are not sensitive to this choice, as demonstrated in Fig. 13, where the size distribution is shown for three different prescriptions for the plasmoid mergers (for details, see Sect. 3).

4.6 The role of growth suppression

The results of the SGP16 PIC simulations suggest that the growth rate of plasmoids moving with velocities that approach the Alfvén speed is suppressed compared to the average growth rate of slower plasmoids in the chain (see also

Figure 12. Same as Fig. 11 but for three fiducial values of the acceleration rate $\beta_a$ that are indicated on the plot.

Figure 13. Normalized histogram of $\log w$ obtained with our MC code for $\sigma = 10$ and three different prescriptions for plasmoid merging, as defined in Sect. 3: (a) intersection of outer boundaries, (b) intersection of centers, and (c) intersection of inner or outer boundaries. We let the system evolve for $6L/c$. Black lines have the same meaning as in Fig. 7.

Figure 14. Normalized histogram of $\log w$ obtained with our MC code for a duration of $6L/c$ with (black filled symbols) and without (magenta open symbols) growth suppression, for $\sigma = 3$ to 50 (from top to bottom). The magenta coloured histograms are the same as those displayed in Fig. 11. For both magenta and black symbols, plasmoids accelerate according to eqn. (5), merge, or leave the layer. For the adopted values of $\beta_g$ and $\beta_a$, see Table 1. Black lines have the same meaning as in Fig. 7.
Appendix A). The suppression of the growth is typically relevant for the smallest plasmoids, as these have $|p| \approx \sqrt{\sigma}$.

Fig. 14 shows the size distribution obtained with (black filled symbols) and without (magenta open symbols) growth suppression for $\sigma = 3$ to 50 (from top to bottom). Growth suppression does not seem to affect the size distribution for $\sigma = 50$ (bottom panel), since the majority of the plasmoids moves with $v < v_A$, as shown in Fig. 4 and Fig. 8 in SGP16. In particular, it remains true that most of the plasmoids for $\sigma = 50$ lie at the smallest sizes and the distribution resembles a single power law. For $\sigma = 3$ and 10, we find that growth suppression affects the distribution of the smallest plasmoids (i.e., $w \lesssim w_m$) since these are, in general, the fastest plasmoids in the layer. Growth suppression affects the size distribution in two ways. Firstly, the majority of the plasmoids has $w \approx w_s$, since all newly formed plasmoids with $w \sim w_s$ and $v \approx v_A$ cannot grow much. Secondly, the characteristic break size of the distribution shifts towards smaller sizes, i.e. $w_s \approx 4w_s \lesssim w_m$ (see also the following subsection).

In summary, when growth suppression is taken into account, the size distribution at $w \gtrsim 4w_s$ is a single power law for all the magnetizations we explore, with power-law index that decreases from $\sim 2$ to $\sim 1.3$ as $\sigma$ increases from 3 to 50 (see also Sect. 4.4).

We showed that the suppression can be phenomenologically described by eqn. (4), with the parameter $A$ being essentially a threshold of the plasmoid’s four-velocity. Its dependence on $\sigma$ is weak, as shown in Table 1, and can be modelled as $A(\sigma) = \sigma^{-z}$ where $z \approx 0.11$. The plasmoid four-velocity at this threshold is then $v_{th} \equiv A\sqrt{\sigma}$. This corresponds to a dimensionless plasmoid velocity of $v_{th} \equiv p_{th}c/\sqrt{p_{th}^2 + 1}$ that is, in good approximation, a constant fraction of the Alfvén speed (i.e., $\sim 0.96 - 0.98 v_A$) for $\sigma = 3 - 50$. This suggests that the accretion of trailing fast plasmoids, which is responsible for most of the plasmoid’s growth at low speeds, is quenched when the growing plasmoid is also fast with $v \rightarrow v_A$.

### 4.7 Effects of the system’s size

The length of the reconnection layer can widely vary depending upon the astrophysical environment. SGP16 demonstrated with large scale PIC simulations of relativistic reconnection that the plasmoid growth rate $\beta_k$ and acceleration rate $\beta_\alpha$ do not depend on the system’s length. Similarly, the fluid properties of the plasmoids, such as plasma density and magnetic energy density, were found to be the same in systems with different sizes. In this section, we discuss the effects of the system’s size on the statistical properties of the plasmoid chain.

Let us consider systems with different sizes and let $L$ denote the half-length of the layer. Then, the ratio $w_s/L$ is a measure of the system’s size, since the plasmoid size at birth is, in all cases, equal to a few electron skin depths. Fig. 15 shows the size distribution of plasmoids for $\sigma = 10$ and systems with $w_s/L = 10^{-3}$ (circles) or $10^{-4}$ (diamonds).

#### Figure 15.

Normalized histogram of $\log w$ obtained with our MC code for $\sigma = 10$ and two values of the ratio $w_s/L$, namely $10^{-3}$ (circles) and $10^{-4}$ (diamonds). Black filled symbols show the distribution when all processes are included. Histograms plotted with magenta open symbols show the distribution when growth suppression is neglected. Histograms plotted with circles are the same as in Fig. 14. The rates $\beta_k$ and $\beta_\alpha$ are fixed to their nominal values (Table 1) and black lines have the same meaning as in Fig. 7.

In both cases, growth suppression affects the size distribution the same way (for details, see Sect. 4.6). Our results demonstrate the formation of a power law whose slope does not depend on the growth suppression. Its dynamic range becomes wider for larger systems. On the one hand, the power law cuts off at a maximum plasmoid size that is a fraction (10-30 per cent) of the layer’s length. On the other hand, the power law develops above a characteristic size $w_{nr}$ that is a constant multiple of the plasmoid’s size at birth. The break size $w_{nr}$ can identified as $w_m$ in the absence of growth suppression (see Sect. 4.5) or $w_{\lesssim} \lesssim w_m$ when growth suppression is taken into account (see Sect. 4.6). We also find no difference between the momentum distributions (not shown), since the prescriptions for the plasmoid acceleration (eqn. (5)) and initial momentum (eqn. (1)) do not depend on the system’s size. Similar calculations can be performed for astrophysical systems where the separation between the plasma scales and the system’s size is $\gtrsim 10^5$ (e.g., flaring PWNe).

#### 4.8 Distributions of exiting plasmoids

So far, our analysis focused on the time- and position-integrated statistical properties of the plasmoid chain. Yet, only a fraction of the plasmoids that form in the layer at any time will be able to exit before merging. How does the distribution of plasmoid sizes and momenta at the time of their exit from the layer compare to the integrated distributions? In a steady state system the two distributions should be identical provided that the properties of the plasmoid chain are uniform across the layer. However, the PIC results recently presented by SGP16 suggest that the properties of...
individual plasmoids may depend on their location in the layer (see also Sect. 2).

Figure 16 shows the integrated size and momenta distributions (black symbols in top and bottom panels, respectively) of the plasmoid chain from a MC simulation with \( \sigma = 10 \). The distributions of plasmoid sizes and momenta at the time of their exit from the layer is also shown for comparison (magenta symbols). We find that the two size distributions match for small and intermediate plasmoid sizes (i.e., \( w \lesssim 0.03L \)). The integrated size distribution shows an excess for \( w \gtrsim 0.03L \), which is caused by the presence of slow moving plasmoids in the layer. These have not yet reached the edge of the layer and are, thus, not counted in the magenta coloured histogram. The exiting momentum distribution is qualitatively different from the integrated one, as presented in the bottom panel of Fig. 16. Plasmoids that have low four-velocities are typically born close to the center of the layer (see Fig. 1). Those that will eventually exit the layer will have been accelerated to higher four-velocities by that time. Additionally, plasmoids are born fast, if their birth location is closer to the edge of the layer (see Fig. 1).

5 BLAZAR VARIABILITY

Although blazars constitute a small subclass of AGN, they are discovered in increasingly large numbers by surveys at microwave wavelengths and \( \gamma \)-ray energies (e.g. Giommi et al. 2009; Abdo et al. 2010; Giommi & et al 2012; Ackermann et al. 2015). Blazars also represent the most abundant population of extragalactic sources at TeV energies\(^6\) (e.g. Holder 2014; de Naurois 2015), suggesting particle acceleration up to very high energies. The extreme observational properties of blazars, such as continuum emission over the entire electromagnetic spectrum, rapid and large-amplitude variability, make them to stand out among other AGN. The blazar broadband emission, from radio up to very high energy \( \gamma \)-rays (>100 GeV), is believed to originate from a relativistic jet that is nearly aligned with the observer’s line of sight and emerges from the central supermassive black hole (Blandford & Rees 1978; Urry & Padovani 1995).

Jets are likely to be launched as Poynting-flux dominated flows (e.g. Blandford & Znajek 1977; Blandford & Payne 1982) and remain such after their acceleration and collimation (e.g. Spruit 1996; Vlahakis & Königl 2004). In regions where the magnetic field changes polarity, energy can be dissipated and eventually transferred to radiating particles via magnetic reconnection.

Sironi et al. (2015) have recently demonstrated that magnetic reconnection can satisfy all the basic conditions for the blazar emission: efficient dissipation, extended (in energy) particle distributions, and rough energy equipartition between particles and magnetic fields in the plasmoids, which are identified as the emitting regions in blazar jets. Plasmoids may also naturally reproduce the extreme energetics and timescales of the observed flaring episodes in blazars, as proposed by Giannios (2013).

Using the PIC results of SGP16, Petropoulou et al. (2016) (henceforth, PGS16) derived analytical estimates for two fundamental observables of flares powered by plasmoids, namely the flux-doubling timescale \( \Delta t_{1/2} \) and the peak luminosity \( \mathcal{L}_{pk} \). Both quantities are expressed in terms of the plasmoid’s size \( (\omega t) \) and Doppler factor \( (\delta t) \) at the end of its lifetime, as shown below:

\[
\Delta t_{1/2} \approx \frac{\omega_{i} t_{i}^{2} \beta_{i}}{\delta t_{i}^{2} c} \quad (8)
\]

and

\[
\mathcal{L}_{pk} \approx \frac{f_{\text{rec}} L_{i}}{8 \pi \omega_{i}^{2} c^{3} \Gamma_{j}^{2}} \beta_{i} c \omega_{i}^{2} \delta_{i}^{2} \quad (9)
\]

where \( L_{i} \) is the absolute jet power, \( \Gamma_{j} = (1 - \beta_{j}^{2})^{-1/2} \) is the jet’s bulk Lorentz factor, \( \omega \) is the jet’s cross sectional radius, and \( f_{\text{rec}} \) is the fraction of energy that is transferred to radiating particles by reconnection. Before applying these analytical results to the plasmoid chain derived from our MC simulations, we summarize a few caveats entering the calculation by PGS16:

- The suppression of the growth was not taken into account.
- The expression for the peak bolometric luminosity was derived under the assumption that the radiating particles are fast cooling, which is, in general, true for the largest plasmoids in the layer. For smaller plasmoids, expression (9) gives an upper limit to the peak flare luminosity.
- Both expressions were derived assuming that the peak flux is always reached at the end of a plasmoid’s lifetime.

\(^6\) http://tevcat.uchicago.edu/

\(^7\) For the definition of the plasmoid’s Doppler factor, see eqn. (8) in PGS16.
when its velocity is also maximum. This assumption is always valid for non-relativistic plasmoids. For the smaller and faster (relativistic) plasmoids though, our assumption holds as long as there is optimal orientation between the direction of the plasmoid’s motion and the observer’s line of sight. For any other alignment, the emission produced by the smaller and faster plasmoids may peak before they reach their final size.

Figure 17 shows the density map (top panel) and the scatter plot (bottom panel) of the peak bolometric luminosity and flux-doubling timescale of flares powered by the plasmoid chain from a MC simulation with $\sigma = 10$. Other parameters used are: $L_j = 10^{46}$ erg s$^{-1}$, $\Gamma_j = 10$, $f_{rec} = 0.5$, and $\varpi = 10^{16}$ cm. The half-length of the layer is taken to be equal to $\varpi$. The average luminosity of the reconnection event can be also estimated as:

$$L_{av} \approx \frac{1}{T} \sum_{i} L_{pk,i} \Delta t_{1/2,i}$$

(10)

where $T$ is the observed duration of the event and is given by $L_j/\beta_{rec} c \delta_j \approx 2.5$ days, where $\delta_j \approx 2\Gamma_j$ is the jet’s doppler factor and $\beta_{rec} \simeq 0.08$ is the reconnection rate. Overplotted in the top and bottom panels of Fig. 17 with white and black dashed lines, respectively, is $L_{av}/3$. Using eqs. (8) and (9) one can also show that the peak luminosity of flares powered by plasmoids of fixed $w_i$ depends on the flux-doubling timescale as $L_{pk} \propto \Delta t_{1/2}^{1/2}$ – see red dash-dotted line in the bottom panel of Fig. 17. Plasmoids with fixed final Doppler factor but different $w_i$ power flares with $L_{pk} \propto \Delta t_{1/2}^{2}$ (green dash-dotted line in bottom panel in Fig. 17).

Most of the flares powered by the plasmoid chain have short durations and peak luminosities below $L_{av}/3$ (see yellow and red coloured regions in the top panel of Fig. 17) and only a small fraction of flares ($\sim 8\%$) has $L_{pk} > L_{av}/3$. These will most likely be resolved as individual short-duration flares, whereas the superposition of many and less luminous flares will form an envelope of longer duration (see also Giannios 2013). Detailed light curves from the plasmoid chain will be presented elsewhere (Christie et al., in prep.).

In this example, the single reconnection event lasts for several days and results in many bright ($L_{pk} > 10^{46}$ erg s$^{-1}$) flares with durations ranging from ~5 min to several hours. We find that long duration flares (here with $\Delta t_{1/2} \gtrsim 10^4$ s) are also luminous ($L_{pk} \gtrsim L_{av}/3$) and are produced by the largest plasmoids of the chain ($w_i \gtrsim 0.03 L$), in agreement with PGS16. Plasmoids of intermediate sizes ($0.003 L \lesssim w_i \lesssim 0.03 L$) produce shorter duration flares with a wide range of luminosities, while the smallest plasmoids ($w < 0.003 L$) power flares that are typically less luminous (i.e., $L_{pk} < L_{av}/3$).

6 SUMMARY & DISCUSSION

Magnetic reconnection is a highly dynamical process which leads to the formation of self-similar structures containing magnetic fields and energetic particles, the so-called plasmoids. These can merge with each other, grow in size, accelerate due to magnetic tension forces, and advect out of the layer.

We have developed a MC code to study the effects of these physical processes on the size and momentum distributions of plasmoids, in light of recent results from large-scale PIC simulations of relativistic magnetic reconnection.

We showed that the differential plasmoid size distribution forms a power law, $N(w) \propto w^{-\chi}$, beyond a characteristic size $w_*$ that is a constant multiple of the plasmoid’s size at birth $w_*$. The break size shifts closer to $w_*$ for higher
magnetizations, thus making the size distribution for $\sigma = 50$ to appear like a single power law. In general, we find that the power law segment above $w_0$ extends from small sizes (i.e., few to tens of plasma skin depths) up to large sizes (i.e., a few percent of the reconnection layer’s length). The slope of the power law $\chi$ above $w_0$ decreases from $\sim 2$ to $\sim 1.3$ as $\sigma$ increases from 3 to 50. We demonstrated numerically and analytically that the slope depends linearly on the ratio of the plasmoid acceleration and growth rates. The differential distribution of plasmoid momenta becomes softer at higher $\sigma$, in agreement with PIC results, and does not depend on the system’s size.

The MC code we presented in this paper was developed based on the findings of PIC simulations of relativistic magnetic reconnection. In principle, a similar analysis could be performed for the non-relativistic case with application to, e.g., accretion discs. The MC code is also flexible, for it can be easily modified to account for additional forces acting upon the plasmoids, such as the inverse Compton drag force induced by strong radiation fields. These could be either external to the reconnection layer or produced by the plasmoid chain itself (see e.g. Beloborodov 2017).

The MC approach facilitates also the study of the plasmoid chain in large spatial domains. This is important for bridging the separation of the microscopic plasma scales with the macroscopic astrophysical scales and making meaningful predictions for the temporal properties of non-thermal radiation. Not requiring the use of computationally heavy PIC simulations, the results of the MC approach can be directly mapped to the statistical properties of flaring astrophysical sources, such as blazars and PWNe.

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Figure A1. Cumulative distribution function (CDF) of the ratio of plasmoids’ final sizes as obtained from PIC data $w_1$ over that obtained using eqn. (3), $w_2$. The CDF was calculated for two sub-samples of the data. The sample was divided according to the plasmoids’ final size (magenta coloured lines) or momentum (blue coloured lines).

**APPENDIX A: GROWTH SUPPRESSION IN PIC AND MC SIMULATIONS**

The requirement for a growth suppression factor, briefly discussed in Sect. 2, arises from the overestimate of the plasmoids’ sizes, if we were to adopt our first order approximation for the plasmoid growth (see eqn. (3)). This is exemplified in Fig. 8 in SGP16. The co-moving growth rate of the largest and longest-living plasmoids can be accurately considered to be constant. These appear as straight lines with a slope equal to $\beta_2$ in a plot of the plasmoid size versus co-moving time (Fig. 8 in SGP16). On the other hand, the small plasmoids, which are typically born with high momenta, do not follow this trend and quickly deviate away from a linear growth.

This discrepancy can also be seen in Fig. A1. Here, we plot the cumulative distribution function (CDF) of the ratio of a plasmoid’s final size as determined in PIC, $w_1$, to the final size predicted from eq. (3), $w_2$, for a constant growth rate $\beta_2 = 0.08$. For those plasmoids who have either low final momenta or a large final size, we find the distribution is shifted below unity suggesting that eqn. (3) is over predicting the plasmoid’s growth. Our choice for the suppression factor is given by eqn. (4). In the following section, we discuss our method of best determining its free parameters as a function of plasma magnetization.

**A1 Calibration of the suppression factor**

As a next step, we calibrate the suppression factor, namely we determine the free parameters $A, B$. To do so, we make a quantitative comparison of the plasmoid size distributions obtained from the PIC simulations of SGP16 and our MC code. For each plasma magnetization we:

(i) create a $20 \times 20$ grid of $A, B$ values.
(ii) run the MC code for every pair of $A, B$ values from the grid.

(iii) compute the frequency histograms of log $w$ for all plasmoids formed in our MC and PIC simulations using the same number of bins $N$.

(iv) introduce a measure of the goodness of fit. The test statistic is defined as:

$$\chi^2 = \sum_{i=1}^{N} \frac{(h_i^{(MC)} - h_i^{(PIC)})^2}{h_i^{(PIC)}},$$  \hspace{1cm} (A1)

where $h_i^{(PIC)}$ and $h_i^{(MC)}$ are the values of the frequency histograms computed for the PIC and MC simulations, respectively. The sum is over the bins in log $w$. We caution the reader that the absolute $\chi^2$ values calculated by eqn. (A1) should not be taken at face value, since systematic uncertainties in both experiments (PIC and MC) are poorly defined. The distribution of $\chi^2$ values for different choices of the $A, B$ parameters in the MC simulations is what will be used in our analysis. We also compared the distributions using a generalization of the classical $\chi^2$ test (eqn. 5 in Gagnushvili 2010) and we obtained the same qualitative results as those presented in Fig. A2.

(v) create a map of the $\chi^2$ values and repeat steps (ii)-(iv) for 10 different MC realizations. This is necessary, since the test statistic is subjected to the randomness of the MC and PIC simulations.

(vi) compute the median and the standard deviation of $\chi^2$ values obtained from the multiple MC realization for every pair of $A, B$ values. The median $\chi^2$ map is expected to be smoother, since features related to by-chance bad fits can be canceled out. Regions of the $A – B$ parameter space with a large standard deviation in the $\chi^2$ values are more variable due to the randomness of the process.

(vii) calibrate our MC code using the $A, B$ values that correspond to the minimum $\chi^2$ value of the median map.

Our results are presented in Fig. A2 for $\sigma = 3, 10$, and 50 (from top to bottom). In all cases, we find an extended diagonal region of lower $\chi^2$ values. This indicates a correlation between $A$ and $B$, which can be understood as follows: parameter $A$ sets a threshold in plasmoid momentum above which the suppression of the plasmoid growth becomes significant, while parameter $B$ determines the degree of the suppression; lower $B$ values lead to larger suppression factors. Although our MC code can reproduce satisfactorily the results of PIC simulations for $A, B$ values drawn from the deep red coloured regions of the maps, we will calibrate our code using the $A, B$ as indicated on the plot with blue stars (see Sect. 4).

**APPENDIX B: ANALYTICAL EXPRESSIONS FOR THE PLASMOID SIZE DISTRIBUTION**

We wish to obtain an analytical solution for the steady-state plasmoid distribution $N(w) \equiv dN/dw$. The governing equation for the size- and position-dependent plasmoid distribution, $n(w, x; x_0)$, is a partial differential equation (PDE) of the general form:

$$\frac{dn}{dt} = Q + L,$$  \hspace{1cm} (B1)

where $Q$ and $L$ represent source and sink terms, respectively. The left-hand side of the eqn. can be expanded as the fol-
where $\partial_0 n = 0$ in steady state and $v \equiv dx/dt$ is the plasmoid velocity, which can be written as:

$$v(w, x; x_0) = \frac{p(w, x; x_0) c}{\sqrt{1 + p(w, x; x_0)^2}}.$$  

(B3)

Here, $x_0$ is the plasmoid’s birth location and $p \equiv \beta \gamma$ is its dimensionless four-velocity (or, momentum) that also depends implicitly on time $t$ through $w$ and $x$ (see eqn. 5). Equation (B2) can be written as:

$$\frac{dn}{dt} = v \left( \partial_x n + \frac{\beta_\gamma}{p_{\sup}} \partial_w n \right),$$  

(B4)

where we made use of $dw/dt = v (dw/dx)$ and $dw/dx = \beta_\gamma / (p_{\sup})$ (see also eqn. (4) in Petropoulou et al. 2016). The production rate of new plasmoids ($\bar{Q}$) is given by:

$$\bar{Q} = q_0 \delta (w - w_\star) S(x; 0, L),$$  

(B5)

where $q_0$ is a normalization constant and $S(y; y_1, y_2)$ is the unit box function. The source term as defined above describes a uniform generation of plasmoids with size $w_\star$ across the layer. The loss of plasmoids occurs either by the advection from the reconnection layer ($L_{\text{esc}}$) or through the coalescence of two plasmoids ($L_{\text{m}}$). The advection term is defined as $L_{\text{esc}} = -n/t_{\text{esc}}$, where the escape time is defined as

$$t_{\text{esc}}(w, x; x_0) = \epsilon \int_s^L \frac{dy}{v(w, y; x_0)}, \quad \epsilon \geq 1.$$  

(B6)

A choice of $\epsilon \gg 1$ ensures that $t_{\text{esc}} \gg t_\star \approx (L - x)/v_A$. Thus, plasmoids leave the layer only when they are located close to the edge, as it is observed in PIC simulations. Replacing the injection and advection terms with their explicit expressions, eqn. (B1) becomes

$$\partial_x n + \frac{\beta_\gamma}{p_{\sup}} \partial_w n = \frac{q_0 \delta (w - w_\star) S(x; 0, L)}{v} - \frac{n}{v_{\text{esc}}} + \bar{L}_{\text{m}}.$$  

(B7)

Equation B7 can be solved by the method of characteristics to obtain a family of solutions $n(w, x; x_0)$ for different values of $x_0$. The plasmoid distribution $N(w, x)$ can be then expressed as the weighted average of $n(w, x; x_0)$ over $x_0$, namely

$$N(w, x) = \int_0^L dx_0 n(w, x; x_0) z(x_0),$$  

(B8)

where $z(x_0)$ is a weighting function; in the simplest scenario, $z(x_0) = 1$. The differential size distribution is then obtained as $N(w) = \int dx N(w, x)$.

The first characteristic equation of the PDE is

$$\frac{dw}{dx} = \frac{\beta_\gamma}{p(w, x; x_0) f_{\sup}(|p|)}.$$  

(B9)

This can be solved only numerically in the most general case where $p$ and $f_{\sup} \neq 1$ are given respectively by eqns (5) and (4).

We thus derive analytical solutions of eqn. (B7) under the following simplifying assumptions:

(i) the growth rate is constant (e.g., $\beta_\gamma = 0.1 \beta_\gamma_{\text{a} - 1}$) and there is no growth suppression ($f_{\sup} = 1$).
ii) plasmoids are lost from the layer only due to advection, i.e., \( \dot{L}_m = 0 \).

iii) the plasmoid momentum is constant and equal to its asymptotic value, i.e., \( p = \sqrt{\sigma} > 1 \), or the plasmoid momentum is non-relativistic and increases linearly with \( x/w \), namely \( p \approx \beta_a(x - x_0)/w \) (see eqn. (5)).

\[ \text{Statistics of the plasmoid chain} \]

\[ \text{B1 Plasmoids with terminal momentum} \]

The solution to the characteristic eqn. (B9) is \( w - w' = A_k(x - x') \), where \( A_k = \beta_g/\sqrt{\sigma} \) and \( w, w' \) are initial values. The solution to eqn. (B7) is then calculated as

\[ n(w; x; x_0) = \frac{q_0}{v_A A_k} \int_{x_1}^{x_2} dx' \delta(w' - w_s) \left( \frac{L - x}{L - x + \frac{w - w_s}{A_k}} \right)^{1/s} \]

where \( w_1 = w - A_k x \) and \( w_2 = w - A_k(x - L) \). For \( w_1 < w_s < w_2 \) or, equivalently, \( w_s - A_k(L - x) < w < w_s + A_k x \), the above equation results in

\[ N(w; x) = \frac{q_0}{v_A A_k} \left( \frac{L - x}{L - x + \frac{w - w_s}{A_k}} \right)^{1/s} \]

(B11)

At \( x \ll L \) most of the plasmoids have \( w \gtrsim w_s \), whereas plasmoids with larger sizes exist at larger distances from the center of the layer. The reason is that an individual plasmoid grows in size with a constant rate \( \beta_\lambda \) as it moves along the layer \( x \). The maximum size of the plasmoid chain is

\[ w_{\text{max}}^{\sqrt{\sigma}} = w_s + A_k L \]

(B12)

We find the differential size distribution by integrating \( N(w; x) \) along the layer:

\[ N(w) = \frac{q_0}{v_A A_k} \int_{x_1}^{x_2} dx \left( \frac{L - x}{L - x + \frac{w - w_s}{A_k}} \right)^{1/s} \]

(B13)

\[ N(w) \gtrsim 1 \quad Q_0 \beta_a A_k \left( \frac{L - w - w_s}{L + \frac{w - w_s}{A_k}} \right)^{1/s} \]

(B14)

where \( Q_0 = q_0/v_A x_1, x_1 = \max[0, (w - w_s)/A_k] = (w - w_s)/A_k \) and \( x_2 = \min[L, L + (w - w_s)/A_k] = L \). For \( \epsilon > 1 \) plasmoids can leave the layer only when they are located close to the edge, as it is observed in PIC simulations. We find that \( N(w) \propto \sqrt{\sigma} \), while it decreases fast as \( w \rightarrow w_{\text{max}}^{\sqrt{\sigma}} \).

\[ \text{B2 Accelerating plasmoids} \]

Let us solve eqn. (B7) in the limit where

\[ p \approx \frac{\beta_a(x - x_0)}{w} \]

and \( p_0 \approx 0 \). This approximation is valid when following condition is satisfied:

\[ \frac{\beta_a(x - x_0)}{w\sqrt{\sigma}} < 1. \]

(B15)

The characteristic equation of the PDE now reads \( dw/dx = \beta_k w/[\beta_a(x - x_0)] \) and its solution is given by

\[ w' \left( \frac{x - x_0}{w' - x_0} \right)^{1/s} \]

(B17)

where

\[ s \equiv \frac{\beta_k}{\beta_a}. \]

(B18)

The solution to the eqn. (B7) can be then calculated as

\[ n(x; w; x_0) = \frac{q_0}{\sqrt{\sigma}} \int_0^{w_1} dw' \frac{\delta(w' - w_s)}{v(x', w'; x_0)} \]

(B19)

\[ = \frac{q_0 \beta_s}{\beta_a} \int_{w_1}^{w_2} dw' \left( \frac{x'(x - x_0)}{w'v(x', w'; x_0)} \right) \]

where the integration limits are:

\[ w_1 = w \left( \frac{x_0}{x - x_0} \right)^s \]

(B20)

\[ w_2 = w \left( \frac{L - x_0}{x - x_0} \right)^s \]

(B21)

Substitution of eqn. (B17) to eqn. (B19) and performance of the integral leads to

\[ n(x; w; x_0) = \frac{q_0}{\sqrt{\sigma}} \left[ 1 + \frac{\beta_s(x - x_0)}{w_s} \frac{w_s^{1/s}}{w} \right]^2 \]

(B22)

for plasmoid sizes satisfying the following conditions:

\[ w_s \left( \frac{x - x_0}{L - x_0} \right)^s < w < w_s \left( \frac{x - x_0}{w_s} \right) \]

(B23)

\[ w > \frac{\beta_s(x - x_0)}{\sqrt{\sigma}} \]

(B24)

In the limit where the approximation of momentum is valid (see eqn. (B16)) and for plasmoids with \( w/w_s > \sqrt{\sigma}^{(1/s)} \), the square root in eqn. (B22) can be approximated by \( \sqrt{1 + w^2} \approx 1 + w^2/2 \). Integration of eqn. (B22) with respect to \( x \) results in:

\[ n(x; w_0; x_0) = \frac{q_0}{\sqrt{\sigma}} \int x + \frac{\beta_s^2(x - x_0)^3}{6w^2} \frac{w_w^{2/s}}{w} \left( \frac{x}{x_0} \right)^{1/s} dx \]

(B25)

where \( x_1 = x_0 + x_0(w/w_s)^{1/s} \), \( x_2 = \min[L, x_0 + w_{\text{max}}^{\sqrt{\sigma}}] \), and \( w_{\text{max}}^{\sqrt{\sigma}} \equiv \beta_s L/\sqrt{\sigma} \). This is the typical size of plasmoids that accelerate and exit the layer with \( p \approx \sqrt{\sigma} \), while even larger plasmoids are, in general, slower (see also Petropoulou et al. 2016).

The upper integration limit is \( x_2 = L \) when \( x_0 > L - \sqrt{\sigma} w_{\text{max}}^{\sqrt{\sigma}} = L(1 - w/w_{\text{max}}^{\sqrt{\sigma}}) \). Additionally, \( x_2 > x_1 \) only if \( x_0 < L/[1 + \zeta^{1/s}] \) where \( \zeta = w/w_s \). The lower and upper limits for integration over \( x_0 \) (see eqn. (B25)) are respectively \( x_{0,1} = \max[0, L(1 - w/w_{\text{max}}^{\sqrt{\sigma}})] \) and \( x_{0,2} = L/[1 + \zeta^{1/s}] \). We are interested in the plasmoid distribution of plasmoids with \( w \approx w_{\text{max}}^{\sqrt{\sigma}} \). For \( w > w_{\text{max}}^{\sqrt{\sigma}} \), the plasmoid distribution is given by:

\[ N(w) = \frac{q_0 L}{256 e c} \left[ \frac{1}{1 + \zeta^{1/s}} + \frac{\beta_k^2 L^2}{12 w^2 \zeta^{2/s}} \left( 1 - \frac{\zeta^{1/s}}{(1 + \zeta^{1/s})} \right) \right] \]

(B26)

where for \( w > w_s \) simplifies in:

\[ N(w) \approx \frac{q_0 L}{256 e c} \zeta^{-1/s} \]

(B27)

Thus, the differential size distribution of plasmoids with a position- and size-dependent momentum which is \( \zeta \approx \sqrt{\sigma} \) can be described by a power law with slope \( -\beta_k/\sqrt{\sigma} \). For plasmoids with \( w > w_{\text{max}}^{\sqrt{\sigma}} \) mergers are less frequent compared to the smaller plasmoids (see Fig. 6). It is therefore safe
to ignore the merger term in eqn. (B7). Furthermore, their growth rate is not suppressed, for they do not move with the terminal momentum.