Optimization of Sensor-Placement on Vehicles using Quantum-Classical Hybrid Methods

Sayantan Pramanik1∗, Vishnu Vaidya1, Gajendra Malviya2, Sudhir Sinha2, Shripad Salsingikar2∗, M Girish Chandra2∗, C V Sridhar1, Godfrey Mathais1, Vidyut Navelkar1

1TCS Incubation, 2TCS Research, TATA Consultancy Services, India

{sayantan.pramanik, vaidya.vishnu, gajendra.malviya, sudhir.sinha, shripad.salsingikar, m.gchandra, sridhar.cv, godfrey.mathais, vidyut.navelkar}@tcs.com

Abstract—The paper presents a quantum method to optimize the placement of sensors on the surface of a vehicle. The problem, as posted in the BMW Quantum Challenge 2021, is to arrive at the optimal positions and configurations (type and orientation) of the sensors on the vehicle surface that maximizes coverage of the Region of Interest (RoI), while minimizing the total cost of the selected sensors. The dataset contains approximately 100,000 points in the RoI, with defined measures of criticality (ranging between 0 to 1), distributed over a volume of approximately 40,000 cubic metres around the vehicle. The types of sensors, their coverage parameters and costs are inputs to the problem.

The optimization problem is decomposed and solved individually for each side of the vehicle, and the results for the entire vehicle are collated and presented in the paper. The approach considers a quadratic approximation to the maximum set-coverage formulation with the advantage of obtaining the optimal number of sensors as an output from the model itself. The resultant quadratic program is solved using quantum methods, as well as a standard, state-of-the-art Integer Quadratic Program (IQP) solver, respectively. The quadratic approximation consumes qubits that are linear in the number of total configurations and significantly reduces the number of decision variables required in an Ising formulation of the problem.

The quantum and classical methods are found to perform on par with each other. A detailed comparison of the results is presented in the paper. Given the combinatorial nature of the optimization problem, the classical approach may become intractable while solving the problem in an integrated manner (instead of solving individually for each side). Under such scenarios, there is a possibility to expect benefits by adopting the proposed strategies with the availability of large-sized quantum computers in the near future.

Index Terms—Variational quantum algorithms, ansatz, quantum annealing, maximum set-coverage, sensor-placement and at a given angle with respect to the surface, all of which determine its Field of View (FoV), which typically takes the shape of an elliptical cone [2]. A point in the RoI is said to be covered only if it lies within the FoV of at least one of the selected sensors.

The challenge lies in choosing the appropriate combination of sensor-configurations (type, position and orientation) that can maximally cover the entire RoI. Using too many sensors increases the overall cost and causing an overlap in their FoVs, resulting in a redundancy which ultimately does not improve the coverage [3]. The effective-FoV of a number of sensors places on the surface of a vehicle has been portrayed in Figure 1. As a result, the task is to make a trade-off to select the combination of sensor-configurations that maximize the coverage, while minimizing their cost, simultaneously.

The challenge dataset [4] provides, as RoI, the coordinates of about a 100,000 points spread over a volume of 40,000 cubic metres in the vicinity of the vehicle. The points are discretized at separations of 0.5 metres, and each point has a criticality index which determines how important covering that point is. The continuous-valued criticality index ranges from 0 to 1, with 0 having the lowest importance and 1 carrying the highest. The problem statement also inspires the use of four different types of sensors - Camera, LiDAR, Radar and Ultrasonic. The parameters in Table I were collated from various sources [5], [6] while avoiding situations where a sensor type can completely dominate another in terms of the cost and the coverage provided, and utilized during experimentation:

| SensorType | α_H(°) | α_V(°) | Range (m) | Cost ($) |
|------------|--------|--------|-----------|---------|
| LiDAR      | 80     | 40     | 120       | 200     |
| Radar      | 60     | 5      | 120       | 100     |
| Camera     | 90     | 60     | 20        | 120     |
| Ultrasonic | 90     | 5      | 10        | 20      |

TABLE I: Parameters considered for the various types of sensors.

From a top-view, 2D visualization of the RoI (see Figure 4 of the challenge document [1]), it is clear that the points in regions pertaining to the four sides - front, back, left and right - of the vehicle are mutually exclusive, with only a minor overlap between the front and the left sides. Exploiting this...
observation, the problem was decomposed into finding the optimal configurations of the sensors independently for each side, before coalescing the results from all sides to check for the coverage and cost for the overall vehicle.

II. METHODOLOGY AND RESULTS

The problem described in Section I, for each side of the vehicle, was expressed as an Integer Linear Problem (ILP) which aims to minimize an objective function composed of a linear combination of the coverage $V_{cov}$ provided by, and the cost $C$ of selected configurations of sensors which is given by:

$$
\min_{x_{t,p,o}} J = -\lambda_1 \cdot V_{cov} + \lambda_2 \cdot C
$$

where,

$$
= -\lambda_1 \sum_{r} c_r z_r + \lambda_2 \sum_{t,p,o} C_t x_{t,p,o}
$$

(1)

where, $T$, $P$ and $O$ are the sets of the possible sensor types, positions and orientations respectively, $t \in T$, $p \in P$, and $o \in O$ characterize the configuration of a sensor; the set $P$ is derived from the intersection points of a $G_1 \times G_2$ grid, mapped on each side of the vehicle. This discretization reduced the problem search space. $c_r$ is the criticality index of a point $r$ in the RoI, $z_r \in \{0, 1\}$ indicates whether $r$ is covered by at least one sensor or not. $C_t$ is the cost of a sensor of the type $t$, and $x_{t,p,o} \in \{0, 1\}$ is the binary decision variable to signify whether or not a sensor of configuration $(t, p, o)$ is selected. $\lambda_1$ and $\lambda_2$ are the weight factors for the coverage and cost terms, respectively. The above is subjected to the following constraints:

$$
\sum_{t,o} x_{t,p,o} \leq 1, \forall p
$$

i.e., at most one sensor can be placed at the position $p$, and,

$$
z_r \leq \sum_{t,p,o} x_{t,p,o} y_{r}^{t,p,o}, \forall r
$$

where, $y_{r}^{t,p,o}$ is an indicator which denotes whether or not the sensor with configuration $(t, p, o)$ covers the point $r$. This constraint forces the value of $z_r$ to 0 if $r$ is not covered by any of the selected sensors, while the presence of $z_r$ in the objective function makes $z_r = 1$ more favourable, thereby optimizing the value of $x_{t,p,o}$. The FoVs for all the possible sensor configurations were precomputed [7], and for the selected combinations of configurations at each step of the optimization process, the overlap amongst the FoVs and with the RoI was calculated to evaluate the objective function.

The paper presents a maximum coverage based quantum approach to solve the sensor-placement optimization problem. As a conventional Ising formulation of the set-coverage problem is qubit-intensive, an approximate version presented here, is expressed as a quadratic program with linear constraints, which is more resource friendly and conducive to the present Noisy, Intermediate-Scale Quantum era [8].

A version of the closely-related set cover problem where the “universe” set $U$ has $n$ elements, $N$ subsets are available to choose from, and $M \leq N$ sets are finally chosen, would lead to $N + n[\log_2 M]$ number of binary decision variables to formulate it as a Quadratic Unconstrained Binary Optimization (QUBO) problem [9]. Here, the universe set is analogous to the set of points in the RoI that need to be covered by the sensors, and the given subsets may be equated with the FoV of the sensors arranged in all possible configurations. The task is to find the subset of sensors that reasonably covers the RoI, while also minimizing the monetary cost of the selected set of sensors.

The number of qubits required to solve the set-cover version of the problem increases linearly with the cardinal number of the universe or RoI set, which typically ranges into tens or hundreds of thousands in the real-world. Solving the set-cover problem in its entirety may not be possible using quantum computing until there is a drastic increase in the count of qubits available. Hence, we resort to the following approximation, through the use of classical preprocessing, to reduce the qubit-requirements to the range of possible sensor configurations:

Let $R = \{r \mid r \text{ is a point in the RoI}\}$, $F_i$ be the set of points covered by the FoV of a sensor with configuration (type, position and orientation) $i \in S$, the set of all possible configurations, and $|S| = N$. Unlike the traditional maximum coverage problem where the number of subsets that can be
chosen is bounded by a predefined number $k$, we let it become a free parameter in our formulation, allowing it to be decided during the optimization process. Also, let us assume that all points in the RoI have equal, unit criticality, an assumption that will be generalized later.

The coverage obtained through a collection of sensors can then be expressed as:

$$V_{cov} = \left| R \cap \bigcup_{i=1}^{N} F_i \circ x_i \right|$$  \hspace{1cm} (4)

where $|S|$ represents the cardinality of any random set $S$, $x_i$ is a binary variable which denotes whether or not the sensor with configuration $i$ is selected,

$$F_i \circ x_i = \begin{cases} F_i, & \text{if } x_i = 1 \\ \emptyset, & \text{if } x_i = 0 \end{cases}$$ \hspace{1cm} (5)

and

$$|F_i \circ x_i| = |F_i| x_i$$ \hspace{1cm} (6)

Now, the cardinality of the union of $N$ sets can be approximated through the use of Bonferroni’s inequality [10] with only single and pairwise terms as:

$$\left| \bigcup_{i=1}^{N} F_i \right| \geq \sum_{i} |F_i| - \sum_{i<j} |F_i \cap F_j|$$ \hspace{1cm} (7)

which provides a lower bound to the actual coverage accorded by the selection of sensors. Since only pairwise terms are used, a large intersection of FoVs of three or more sensors is indirectly discouraged by the formulation. Further, the maximization of $V_{cov}$ occurs through the maximization of its lower bound. By exploiting the relation in Equation (7), and replacing the cardinality $|S|$ to the criticality-weighted cardinality $|S|_w$, Equation (4) is simplified to:

$$v_{cov} = \sum_i |R \cap F_i \circ x_i|_w - \sum_{i<j} |R \cap F_i \circ x_i \cap F_j \circ x_j|_w$$ \hspace{1cm} (8)

where $V_{cov} \geq v_{cov}$, and if $S = \{s_1, \ldots, s_m\}$ is a subset of $R$, then,

$$|S|_w = \sum_{j=1}^{m} c_{sj}$$ \hspace{1cm} (9)

The terms $\mu_i = |R \cap F_i|_w$ and $\sigma_{ij} = |R \cap F_i \cap F_j|_w$ are pre-computed for every single and pairwise configuration of sensors. As the possible configurations are typically limited by the spatial resolution, types of sensors and their orientations, such pre-calculations can be comfortably handled by classical computers even for large RoIs. The number of binary decision variables is correspondingly reduced from $N + n \lfloor \log_2 M \rfloor$ to $N$. As typically $n \gg N$, this makes the proposed approximate set-coverage formulation quite resource-friendly. The expression for coverage is thus approximated by the following quadratic form:

$$v_{cov} = \frac{3}{2} \mu^T x - \frac{1}{2} x^T \Sigma x$$ \hspace{1cm} (10)

The factor of $1/2$ before the quadratic term is to account for the symmetric nature of $\sigma$, and the coefficient of $3/2$ takes care of the $\mu_i = \sigma_{ii}$ elements in the diagonal of the $\sigma$ matrix. It is trivial to notice that the similarities between $\sigma$ and covariance matrices, considering that the (weighted) cardinality of intersection of sets can be expressed as $X^T X$, where the $n$-length columns of $X$ contain the square-roots of the criticality indices for the points covered by the FoVs, and zero for the points that are left uncovered. With the above expression of $\sigma$ as $X^T X$, the entries may even be calculated using quantum techniques which provide a benefit over classical inner product calculation [11], given an efficient way to load the data.

The sensor configuration $i$ can be resolved into the sensor’s type ($t$), position ($p$), and orientation ($o$), and all variables assume binary values, making the resultant Integer Quadratic Program (IQP) NP-hard [12]:

$$\begin{align*}
\min_{x_{t,p,o}} & \quad -\lambda_1 v_{cov} + \lambda_2 \sum_{t,p,o} c_{t} x_{t,p,o} \\
\text{s.t.} & \quad \sum_{t,o} x_{t,p,o} \leq 1, \forall p \\
& \quad x_{t,p,o} \in \{0,1\}
\end{align*}$$ \hspace{1cm} (11)

To find the minima of the above quadratic program, classical, quantum annealing, and gate-model based methods were employed. During experimentation, the values of $\lambda_1$ and $\lambda_2$ were fixed at 1 and $10^{-4}$ to emphasize coverage (which ranges between 0 and 1) over the cost (which is typically in the range of hundreds of dollars), each side was discretized into a $4 \times 4$ grid, and 4 possible angles of orientation were chosen that maximize the coverage of the RoI. IBM® ILOG® CPLEX® was used to classically solve the IQP. For experimentation on quantum annealing simulators, the objective function and the constraints in Equation (11) were converted to Binary Quadratic Model (BQM) and 1000 samples were drawn using the simulated annealing sampler on D-Wave Ocean SDK [13]. On gate-model, Variational Quantum Eigensolver (VQE) was employed with three layers of the ansatz shown in Figure 2, to solve a scaled-down version of the problem with a $2 \times 2$ grid, orientation fixed as perpendicular to the surface, and with only 2 types of sensors considered.

![Fig. 2: First layer of the ansatz acting on 4 qubits. The target qubit of each gate is determined using the relation $t = (c + l + 1) \mod n$, where $c$ and $t$ are the control and target qubit numbers, $l$ is the ansatz-layer being constructed and $n$ is the number of qubits the ansatz operates on. An $L$-layer ansatz avails $nL$ optimizable parameters.](image-url)
The inequality constraint in model 11, was ignored to reduce the qubits required in the realized QUBO implementation. The performance of the models with and without the inequality constraint was found to be comparable. Further, each of the $4 \times 4$ grids on the vehicle’s sides are large enough to accommodate multiple sensors, even if the constraint in Equation 2 was violated. Keeping these points in mind, the constraint was forgone for both the quantum and classical models. The results from the classical and annealer-based models are presented in Table II.

| Side    | Classical | Quantum Annealer |
|---------|-----------|------------------|
|         | Coverage % | Cost ($)         | Coverage % | Cost ($)         |
| Front   | 86.37      | 360              | 86.67      | 380              |
| Left    | 90.33      | 400              | 90.30      | 400              |
| Right   | 94.75      | 400              | 94.70      | 400              |
| Back    | 80.19      | 340              | 80.19      | 340              |
| Aggregate | 91.36    | 1500             | 91.10      | 1520             |

**TABLE II:** Results from the classical and annealer-based approximate max set-coverage models.

A comparison of the results for the downscaled problem from VQE and classical model is portrayed in Table III and Figure 3.

| Side    | Classical | VQE |
|---------|-----------|-----|
|         | Coverage % | Cost ($) | Coverage % | Cost ($) |
| Front   | 84.20      | 320    | 84.15      | 320    |
| Left    | 61.60      | 320    | 61.56      | 320    |
| Right   | 72.20      | 320    | 72.19      | 320    |
| Back    | 75.10      | 320    | 75.09      | 320    |
| Aggregate | 86.68   | 1280   | 87.77      | 1280   |

**TABLE III:** Results for the downsized problem from the classical and VQE-based approximate max set-coverage models with $2 \times 2$ grid with only 2 sensor-types and fixed angles considered.

More details on the approach, extensive results and reference are available at [14].

### III. CONCLUSION

An approach to solve the sensor-placement problem were presented and its classical and quantum implementations were discussed in detail. It is encouraging to note that the results from the semi-metaheuristic based quantum methods are within 1% of the optimal results obtained from classical exact optimization techniques. The suggested approximation to the max set-coverage algorithm, which uses a significantly lower qubit-count compared to the traditional Ising formulation, is generic enough to find applications in use cases across various domains. The results obtained from it may then be passed on to a classical evolutionary algorithm, as a warm-starting point, to overcome the suboptimality introduced by the approximation.

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