Model Comparison tests of modified gravity from the Eötvös experiment

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Perivolaropoulos et al [1] have argued that the residual torque data in the Eötvös experiment shows evidence for an oscillating potential, which could be a signature of non-local modified gravity theories. We independently assess the viability of this claim by analyzing the same data. We fit this data to three different parametrizations (an offset Newtonian, Yukawa model, oscillating model) and assess the significance of the oscillating model using four distinct model comparison techniques: frequentist, Bayesian, and information theoretic criterion such as AIC and BIC. We find that the frequentist test favors the Newtonian model over the oscillating one. The other techniques on the other hand favor the oscillating potential. However, only the BIC test decisively favors the oscillating parametrization as compared to a constant offset model. Our analysis codes have been made publicly available.

I. INTRODUCTION

Despite being more than a century old, General relativity (GR) currently agrees with all observational tests using solar system, binary pulsar, and gravitational waves [2–9]. In spite of all this, a large number of modified theories of gravity have been explored since the inception of GR. Most of the recent resurgence in these alternate theories of gravity has been driven by the need to address problems in Cosmology such as Dark matter, Dark Energy, Inflation, Baryogenesis, and data driven tensions such as discrepancy in Hubble constant and \( \sigma_8 \) measurements, low-\( l \) CMB anomalies, Lithium-7 problem in Big-Bang Nucleosynthesis, etc [10–24]. Apart from this, a number of alternatives have also been proposed to resolve conceptual issues, such as the Big-Bang singularity [25], arrow of time [26, 27], or the quantization of gravity [28–30].

For over three decades, the Eötvös group at the University of Washington has been conducting a series of high precision laboratory-based tests of gravity at sub-millimeter scales using torsion balances to look for departures from Newtonian gravity. Their own analysis has not revealed any signs of new physics. However, other authors [1] (P19 hereafter) have independently analyzed the data from these experiments and argued that the data show evidence for an oscillating Newtonian potential. They concluded that one possible explanation for the observed oscillations in the data, could be that this is a potential signature of non-local theories of gravity [35–38]. In this work, we independently assess the statistical significance of this result using multiple model selection methods motivated from frequentist, Bayesian and information theoretical considerations. We have previously used these same techniques to address a number of model selection problems in Astrophysics and Cosmology [39–44].

The outline of this manuscript is as follows. We recap the details of the Eötvös experiment and analysis of their data by P19 in Sect. II. We provided an abridged summary of different model comparison techniques used in Sect. III. We present the results of our analysis in Sect. IV. We conclude in Sect. V. Our analysis codes are publicly available at https://github.com/aditikrishak/EotWash_analysis.

II. SUMMARY OF EÖTVÖS RESULTS AND P19

P19 independently analyzed the data from the Eötvös experiment [31]. The Eötvös group used a torsion pendulum detector to look for departures from Newton’s law of gravity at sub-millimeter scales. The torsion pendulum was suspended by a thin tungsten fiber. The gravitational interactions were measured via a torque that developed between the holes in the torsion pendulum and similar holes machined on a molybdenum detector ring below the detector (See Fig. 1 of Ref. [31]). The Eötvös group conducted three different experiments named as Experiment I, II and III, consisting of the same detector ring and upper attractor disk, but using different sizes for the lower attractor disks. This feature was introduced to discriminate between potential new Physics and systematic errors. For each of these three setups, residuals between the measured torques and the expected Newtonian values were published. Based on these differences, 95% c.l. upper limits were set on possible Yukawa interactions.

P19 (and also Ref. [45]) fitted the 87 residual torque data points from these experiments (\( \delta\tau \)) to three different
functions:
\[
\begin{align*}
\delta \tau_1(a', m', r) &= a' \\
\delta \tau_2(a', m', r) &= a' e^{-m'r} \\
\delta \tau_3(a', m', r) &= a' \cos(m'r + \frac{3\pi}{4})
\end{align*}
\]

These correspond to an offset Newtonian potential (which can be considered as white noise for the purpose of fitting), a Yukawa potential, and an oscillating potential respectively. Here, \(a'\) and \(m'\) are two ad-hoc parameters, and the relations between these parameters and the corresponding parameters of any modified gravity theory depend on the details of the experimental setup [45]. For each of these functions, P19 carried out \(\chi^2\) minimization to obtain best-fit parameters. Among the three models, they found that the oscillating function Eq. 3 has the smallest \(\chi^2\) (cf. Table IV of P19), with \(\Delta \chi^2 \sim -15\) with respect to the other models. They assess the statistical significance based on this difference in \(\chi^2\) with respect to the other models to be 3\(\sigma\). P19 also carried out Monte-Carlo simulations of the null hypothesis and showed that the probability for the oscillation signal to be a statistical fluctuation is about 10\%. Therefore, P19 concluded that what they have found is one of three possibilities: either a statistical fluctuation in one of the experiments causing this anomaly, or a periodic distance-dependent systematic endemic to such laboratory based experiments; or a signature for a short-distance modification to GR. P19 also pointed out that such an oscillating potential at short distances is a characteristic of many non-local gravity theories [35–38], involving infinite derivatives of the Lagrangian. These theories have been constructed to solve the black hole and big-bang singularity problem [25].

Given the potential ramification for the third possibility, we independently do a fit to the same residual torque data and assess the statistical significance of the oscillating parametrization using multiple model selection techniques, which we have previously used to address multiple problems in Astrophysics and Cosmology [39–44].

III. MODEL COMPARISON TECHNIQUES

We briefly summarize the different selection techniques used to rank between multiple models. More details can be found in specialized reviews [46–48] or in some of our previous works [42–44].

• **Frequentist Test:** In this method (also known as the likelihood ratio test [48]), one calculates the goodness of fit for each model, given by the \(\chi^2\) probability distribution function [49]. The best-fit model between the two is the one with the larger value of \(\chi^2\) goodness of fit. If the two models are nested, then from Wilk’s theorem [50], the difference in \(\chi^2\) between the two models satisfies a \(\chi^2\) distribution with degrees of freedom equal to the difference in the number of free parameters for the two hypotheses [48]. Therefore, the \(\chi^2\) c.d.f. can be used to quantify the p-value of a given hypothesis. From the p-value, one usually calculates a statistical significance based on the Z-score, which is the number of standard deviations that a Gaussian variable would fluctuate in one direction to give the corresponding p-value [42, 51]. For the models considered in this work, Eq. 1 is nested within both Eq. 2 and Eq. 3.

• **AIC and BIC:** The Akaike Information Criterion (AIC) as well as the Bayesian Inference Criterion (BIC) are used to penalize for any free parameters to avoid overfitting. AIC is an approximate minimization of Kullback-Leibler information entropy, which estimates the distance between two probability distributions [46] and is given by:

\[
AIC = -2 \ln \mathcal{L}_{\text{max}} + 2p,
\]

where \(p\) is the number of free parameters, and \(\mathcal{L}\) is the likelihood.

BIC is an approximation to the Bayes factor and is given by [52]:

\[
BIC = -2 \ln \mathcal{L}_{\text{max}} + p \ln N,
\]

where all the parameters have the same interpretation as in Eq. 4. For both AIC and BIC, the model with the smaller value is the preferred model. To evaluate the significance, qualitative strength of evidence rules are used [53] based on the difference in AIC and BIC.

• **Bayesian Model Selection:** The Bayesian model selection technique used to compare two models \((M_1\) and \(M_2\)) is based on the calculation of the Bayesian odds ratio, given by:

\[
O_{21} = \frac{P(M_2 | D)}{P(M_1 | D)},
\]

where \(P(M_2 | D)\) is the posterior probability for \(M_2\) given data \(D\), and similarly for \(P(M_1 | D)\). The posterior probability for a general model \(M\) is given by

\[
P(M | D) = \frac{P(D | M) P(M)}{P(D)},
\]

where \(P(M)\) is the prior probability for the model \(M\), \(P(D)\) is the probability for the data \(D\) and \(P(D | M)\) is the marginal likelihood or Bayesian evidence for model \(M\) and is given by:

\[
P(D | M) = \int P(D | M, \theta) P(\theta | M) d\theta,
\]

where \(\theta\) is a vector of parameters, encapsulating the model \(M\). If the prior probabilities of the two models are equal, the odds ratio can be written as:

\[
B_{21} = \frac{\int P(D | M_2, \theta_2) P(\theta_2) d\theta_2}{\int P(D | M_1, \theta_1) P(\theta_1) d\theta_1},
\]
| parametrization | $a'_{\text{fN.m}}$ | $m'_{\text{mm}^{-1}}$ |
|-----------------|------------------|------------------|
| Newtonian       | 0.0001           | -                |
| Yukawa          | 0.0022           | 25               |
| Oscillating     | 0.0042           | 65.29            |

TABLE I: The best-fit parameters obtained for the three parametrizations (Eq. 1, Eq. 2, Eq. 3) by $\chi^2$ minimization

The quantity $B_{21}$ in Eq. 9 is known as the Bayes factor. We shall compute this quantity in order to obtain a Bayesian estimate of the statistical significance. Note that unlike the frequentist and AIC/BIC based tests, the Bayesian test does not use the best-fit values of the parameters.

To assess the significance, a qualitative criterion based on Jeffreys’s scale is used to interpret the odds ratio or Bayes factor [47, 49]. A value of $>10$ represents strong evidence in favor of $M_2$, and a value of $>100$ represents decisive evidence [47].

IV. ANALYSIS AND RESULTS

For each of the three models, we find the best-fit parameters by $\chi^2$ minimization, where $\chi^2$ is defined as follows:

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{\delta_{\tau_i} - \delta_{\tau_{\text{model}}}}{\sigma_{\delta_{\tau_i}}} \right)^2,$$

where $\delta_{\tau_i}$ denotes the data for the residual torques (provided to us by L. Kazantzidis) and $\sigma_{\delta_{\tau_i}}$ indicates its associated error; $\delta_{\tau_{\text{model}}}$ encapsulates the three model functions (defined in Eqs 1, 2, 3) used to fit the residual torque. We use the SLSQP algorithm as coded in the scipy Python module to carry out constrained optimization, with the parameter values lying between certain bounds. The $a'$ value is bounded between the lowest and highest data value of the residual torque for all the three models. For the Yukawa parametrization, $m'$ value is bound between 0 and 25, and for the oscillating model, between 0 and 70. The best-fit parameters for each of these models are shown in Table I and the corresponding plots with all three functions superposed on the data are shown in Fig. 1. We note that the Yukawa best-fit is indistinguishable by eye compared to the Newtonian model. We now present results from all the different model comparison tests.

A. Frequentist Model Comparison Test

We perform a frequentist model comparison by calculating the $\chi^2$ value for the three hypotheses with the best fit parameters in Eq. 1, 2 and 3, and finding the $\chi^2$ p.d.f. for each of them. The model with the higher value for $\chi^2$ p.d.f. is favored. We find that the $\chi^2$/dof values for Newtonian and Yukawa models are closer to one, as compared to that for the oscillating model, indicating that the former models fit the data more reasonably than the latter. For the oscillating model, the $\chi^2$/dof is very much less than one. This is a consequence of over-fitting due to the sinusoidal function, since for such a function increasing the frequency can fit any dataset [54]. We note that for calculating $\chi^2$ pdf, dof is given by the total number of data points minus the number of free parameters. For non-linear models, this relation however is not strictly valid. However, estimating the number of degrees of freedom in the general case is highly non-trivial [54]. Therefore for model comparison, we compute the dof in the same way as for a linear model.

According to the frequentist model comparison, the Newtonian parametrization has a marginally higher likelihood than the Yukawa model and a much higher likelihood than the oscillating model. To estimate the significance, we consider the Newtonian parametrization to be the null hypothesis. Making use of the fact that the null hypothesis is nested within the other two models, and thus applying Wilk’s theorem, the $\chi^2$ c.d.f. is calculated to compare Yukawa parametrization with the null hypothesis, as well as to compare the oscillating parametrization with the null hypothesis. The $p$-value and significance (or Z-score, computed using the prescription in Ref. [51]) for both these model comparisons are listed in Table II. For the oscillating model, we find that the $p$-value is 0.0001 (corresponding to 3.67σ), which implies that the probability of seeing the oscillating model from a fluctuation of the null hypothesis is $10^{-4}$.

B. AIC and BIC

We calculate the AIC and BIC values for each of the three models in Eqs. 1, 2, 3 using AIC and BIC. Note that AIC and BIC are computed from the maximum value of the Gaussian likelihood using the best-fit parameters obtained by $\chi^2$ minimization. These values are listed in Table II. We find that for both AIC and BIC, the values are the highest for Yukawa and lowest for the oscillating model, indicating that the null hypothesis is favored over the Yukawa model, whereas the oscillating model is favored over the null hypothesis. Using the empirical strength of evidence rules [53], we evaluate the level of empirical support for one model over the other. According to AIC, the support for the Newtonian model is “considerably less” compared to the oscillating model. Based on the difference in BIC, there is “very strong”
FIG. 1: Plots showing the data points for the residual torque (obtained from P19) from the Eötvös experiment, along with the best fits obtained for the three models using $\chi^2$ minimization. The best-fit values for each of the three models are shown in Table I. We note that for the best-fit Yukawa model, the exponential factor is very much suppressed, making it indistinguishable by eye compared to the Newtonian model.

evidence for the oscillating model compared to the Newtonian model. Thus, only the BIC-based test provides decisive evidence for the oscillating model.

C. Bayesian Model Comparison

Considering the Newtonian model to be the null hypothesis, we calculate the Bayes factor for the Yukawa model as well as the oscillating model in comparison with the constant Newtonian offset as the null hypothesis. We use a Gaussian likelihood function to calculate the Bayesian evidence for each of the models. The prior on $\alpha'$ is chosen to be uniform over $[d_{\text{min}}, d_{\text{max}}]$, where $d_{\text{min}} = -0.02fN.m$ and $d_{\text{max}} = 0.07fN.m$ are the minimum and maximum data values of residual torque respectively. The prior on $m'$ is chosen to be uniform over $[0, 25\text{mm}^{-1}]$ for the Yukawa model and over $[0, 100\text{mm}^{-1}]$ for the oscillating model.

We use the Nestle package in Python to calculate the Bayesian evidence for each of the hypotheses. The Bayes factor is calculated using Eq. 9 for both the Yukawa and the oscillating models in comparison with the null hypothesis, and the value of Bayes factor is found to be 7.5 for Yukawa model and 11.1 for oscillating model (cf. Table II). According to the Jeffreys' scale [47], Bayesian model comparison provides “substantial” evidence for the Yukawa model and “strong” evidence in favor of the oscillating model, when compared with the null hypothesis. However, none of the Bayes factors exceed the value of 100 needed for any one model to be decisively favored.

V. CONCLUSIONS

In a series of papers, Perivolaropoulos et al [1, 45] independently analyzed the residual torque data from the Eötvös-Wash experiment [31], obtained after subtracting the torques due to a Newtonian potential. They argued that an oscillating parametrization provides a better fit to this data as compared to a constant term (equivalent to an offset Newtonian potential). If the oscillating fit is the true description of the data, one possible implication is that this could be a signature of a non-local modified theory of gravity [35–38].

To further investigate the viability and statistical significance of this claim, we independently analyze this residual torque data, and carry out a fit to the same three functions as in Refs. [1, 45]. To discern the significance of the fit for the oscillating parametrization, we carry out model comparison techniques using four different methods: frequentist or likelihood ratio test, AIC and

\[\text{http://kylebarbary.com/nestle/}\]
|                  | Newtonian | Yukawa   | Oscillating |
|------------------|-----------|----------|-------------|
| **Frequentist**  |           |          |             |
| $\chi^2$/dof    | 85.52/86  | 85.50/85 | 70.73/85    |
| $\chi^2$ p.d.f. | 0.0305    | 0.0303   | 0.0187      |
| p-value         | -         | 0.9      | 0.0001      |
| Significance    | -         | 1.25σ    | 3.67σ       |
| **AIC values**  | -571.3    | -569.4   | -584.1      |
| $\Delta$AIC     | -         | -1.9     | 12.8        |
| **BIC values**  | -568.9    | -564.4   | -579.2      |
| $\Delta$BIC     | -         | -4.5     | 10.3        |
| **Bayes Factor**| -         | 7.5      | 11.1        |

**TABLE II:** Summary of results for the model comparison tests using frequentist, Bayesian and information theoretic techniques. The Bayes factor (in this table) is the ratio of Bayesian evidence for the Yukawa or oscillating model to the same for Newtonian model. $\Delta$AIC (and BIC) indicates the difference in AIC/BIC between the Newtonian model and the Yukawa/oscillating model. The frequentist test favors the Newtonian model over the oscillating model. The AIC, BIC, and Bayesian tests favor the oscillating model. However, only the BIC test shows decisive evidence in favor of the oscillating model.

BIC based information theory test, and finally a Bayesian model comparison technique based on calculation of the Bayes factor.

Our results from this model comparison analyses are summarized in Table II. When we compare the Yukawa and Newtonian parametrizations, we find that the frequentist and information theoretical tests prefer the Newtonian model. The Bayesian test on the other hand prefers the Yukawa parametrization, where according to Jeffreys’ criterion, the support for the Yukawa model is substantial, but not very strong. We find that the frequentist model comparison test favors the Newtonian parametrization, whereas the Bayesian comparison and information theoretic tests support the oscillating parametrization. However, only the BIC tests shows decisive evidence for the oscillating model.

To improve transparency in data analysis, we have made our analysis codes and datasets analyzed publicly accessible. These can be found at https://github.com/aditikrishak/EotWash_analysis.

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