Applied mathematical model for a heat transfer

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ABSTRACT

It is implicit for some time ago, that the radiative magneto hydrodynamic stands for an interacting between a radiation field and magneto hydrodynamic (MHD) field that is concerned with interacted electrical conducting fluids and electromagnetic field. An accurate solution to the heat transfer equation is obtained by calculating the radiant heat emission in the boundary layer MHD because of the eccentric rotations of the pore disk and the fluid for long time. It is found that the asymptotic solution is present in both the cases of suction and blowing states, whereas no such solution exists for the blowing state in the absence of radiative emission of heat. The heat transfer rate at the disk has been determined and the condition is gotten for the heat to flow from the liquid to the disk.

Keywords: Heat equation, Modeling, Ordinary and partial differential equations, Suction, Blowing

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1. Introduction

The investigation of a MHD layer with radiative heat transfer stands for a great importance in numerous astrophysical and aeronautical problems. Additionally, they also server to enhance our broad understanding of motions involving plasmas and conducting fluid generally. Recently Neild and Bejan [1], Chatti and et al [2], Khan and et al [3], Lee [4], Guenther and Lee [5], Khan and Elkamel [6] studied the boundary Layer because of no coaxial rotations of pores disc and fluid for long time, but they neglected the effects of radiative heat transfer which plays an important role in analyzing heat transfer characteristics under high temperature conditions. Numerical analysis of 3D flow among parallel rotating plates for a common axis had accomplished by Hud and et al [7], by Lai and Rajagopal [8]. Coirier [9] had been firstly studied a flow of Newtonian fluid caused by non-coaxial fluid owing to non-coaxial rotation of fluid and a disk for long time, he takes a rotation for both cases the dissimilar and identical angular velocity. Erdogan [10] studied the same flow in the case of pore disk when the rotation with both cases the same and different angular velocities, respectively. Numerical analysis of 3D flow among parallel rotating plates regarding a common axis has performed by Siddiqui and et al [11] and Maqbool and et al [12]. Impulsively started unsteady flow produced by eccentric rotation of a disk and fluid for long time has been contributed by Pop [13]. Kaloni and Siddiqui [14] investigated the time dependent Navier-stokes equations for a viscous fluid flow among eccentric rotating disks. Dupont and Crochet [15] and Guo and et al [16] extended the analysis done for the case of magnet-hydrodynamic effect. Wazwaz [17] obtained the analytical solution of the unsteady flow induced by non-coaxial rotations of a pore disk and fluid for long time. Bali’s and et al [18-22] give the Numerical studies of heat transfer between two eccentric rotating disks. Heat transfer phenomenon due to free convective have ample application in many engineering practices. The objective of this communication is to study the radiative heat emission influence on the flow of heat from the liquid to the disk or from the disk to the liquid[23-36]. This paper is organized as follow, the next
sections significance of the problem, section three problem description, section four the mathematical analysis and solution of the problem, exact solution has been established and description of parameters and discussion followed by conclusion in section 5.

2. Significance of the problem
This research we expect is important in applied mathematics and useful for fieldwork in engineering.

3. Problem Formulation
We consider a system of the Cartesian coordinate in which the z-axis has been taken normally to the plan z=0 of the pore disc. It is assumed that the disc and the fluid have been rotating with uniform angular velocity Ω regarding the axes with distance L in the plane x=0 between them, so that the boundary conditions are

\[
\begin{align*}
\{ & u = -\Omega y, v = x\Omega, w = -w_0 \text{ at } z = 0, \\
& u = -\Omega y + f(z), v = x\Omega, w = w_0 \text{ as } z \to \infty \quad \cdots (1)
\end{align*}
\]

Where the components of the fluid velocity are w,v,u . The fluid is assumed electrically conducting with a transverse applied magnetic field \(B_0\) in the direction of z-axis. Reynolds number magnetic \(R_m\) has been assumed so small that the induced magnetic field is negligible as compared to the applied magnetic field. Following the velocity profits in the magnetohydrodynamic (MHD) boundary layer caused by the rotation of disc are caused by the rotation of disc given by:

\[
\begin{align*}
u & = -\Omega y + f(z), v = \Omega x + g(z), w = -w_0 
\end{align*}
\]

where,

\[
\begin{align*}
 f & = \Omega L(1-e^{-\alpha z} \cos \beta z) \\
g & = \Omega L e^{-\alpha z} \sin \beta z \\
z & = \left(\frac{2s}{\rho v}\right)^{\frac{1}{2}} z, \alpha = \sqrt{2s + \left(\frac{s^4 + N + 1}{s^2} + s^2 + N\right)^{\frac{1}{2}}} \\
\beta & = \left(\frac{s^4 + N + 1}{s^2} - (s^2 + N)\right)^{\frac{1}{2}} \\
S & = w_0 (2\Omega v)^{-\frac{1}{2}}, N = \frac{\mu \beta_0^2}{(\rho v)}
\end{align*}
\]

Here,

S: is suction parameter
N: is magnetic parameter

The heat transfer equation to be solved is of the form.

\[
PC_Cw \frac{dT}{dz} = k \frac{d^2\rho}{dz^2} - q_R + pv\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] 
\]

where

\[C_P\] stands for the specific heat at a constant pressure
\[T\] denotes absolute temperature
\[K\] denotes a coefficient of thermal condition
\[q_R\] denotes the flux of radiative heat
\[V\] denotes the kinematic coefficient of viscosity.

4. Solution of the problem
Based on optically thin approximation, the mean free path of radiation is much larger than the characteristic length of flow field. The situation is sometimes expressed as that of radiation with negligible self-absorption so that the plank mean absorption coefficient \(k_p = \frac{1}{L_R}\) is very small, where \(L_R\) is the characteristic length of mean free path of radiation. Each element of the fluid exchanges radiation straightforwardly with the bounding surfaces and so there is no rad-dative interaction between various fluid elements. Sparrow, and Cess [24] have shown that the equation of radiative heat transfer for a non-grey gas near equilibrium under optically thin limit assumes a simple from:
\[
\frac{d\theta}{dz} = 4\pi k_p \sigma T^4 \quad \cdots (4)
\]

Where

\(\sigma:\text{is the Stefan – Boltzmann constant} \ k_p:\text{is the Planck mean absorption coefficient}\)

For the case of extremely optically thin limit \(k_p \to 0\) as \(L_R \to \infty\) and thus the flux of radiative heat will be a constant as has been shown.

Since the temperature \(T_0\) at the pore disc is the highest and the temperature \(T_\infty\) of the liquid for long time is the lowest, it can be assumed that the liquid at infinity is the lowest, it can be assumed that the liquid at infinity attains an isothermal state of equilibrium whereas the temperature difference in the whole flow filed is small. Under such condition it is possible to linearize the equation (4), and for the present case the best-fit linear approximation of (4) can be expressed in the form:

\[
\frac{d\theta}{dz} = c_1 T - c_2 \quad \cdots (5)
\]

where

\[c_1 = \frac{16}{5} \pi k_p \sigma T_0^3 \left(1 - \frac{T_\infty}{T_0}\right)^3\]
\[c_2 = \frac{4}{5} \pi k_p \sigma T_0^4 \left(1 - \frac{T_\infty}{T_0}\right)^4\]

The liquid at infinity is in an isothermal state of equilibrium. The boundary conditions to be satisfied by the temperature profile of (4) are:

\[T(0) = T_0, \quad T \to \frac{1}{5} T_0 \text{ as } z \to \infty \quad \cdots (6)\]

Which mean that the pore plate is maintained at a constant temperature \(T_0\) and the liquid temperature at infinity has stationary value \(\frac{1}{5} T_0\), we introduce the following dimension less parameters:

\[
\frac{\theta}{\frac{1}{2} k_p T_0}, \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad P_r = \frac{2 v' \rho p}{k}, \quad E_c = \frac{\alpha^2 r^2}{2 T_0 \sigma p},
\]

where \(P_r\) is called a Prandtl number and \(E_c\) is called Eckert number. Transforming (3) in to a dimension less form we get

\[
\frac{d^2 \theta}{dz^2} + P_r \frac{d \theta}{dz} - F \theta = -F + \frac{5}{4} P_r E_c (\alpha^2 + \beta^2) e^{-2 \alpha \delta} \quad \cdots (7)
\]

Where \(F = \frac{(4) (\frac{4}{5} \pi k_p \sigma P_r T_0^3)}{c_p \theta} > 0\)

The boundary conditions (6) assume the form,

\[\theta(0) = 0, \quad \theta(\infty) = 1 \quad \cdots (8)\]

Solution (7) satisfying (8) is given by.

\[\theta(\delta) = 1 - e^{-\lambda \delta} + \frac{5 E_c P_r (\alpha^2 + \beta^2)}{4(\alpha^2 - 2 \alpha P_r S - F)} e^{-2 \alpha \delta} \quad \cdots (9)\]

For \(4 \alpha^2 - 2 \alpha P_r S - F \neq 0\) and

\[\theta(\delta) = 1 - e^{-\lambda \delta} - \frac{5 E_c P_r (\alpha^2 + \beta^2) e^{-2 \alpha \delta}}{4(\alpha^2 - P_r S)} \quad \cdots (10)\]

For \(4 \alpha^2 - 2 \alpha P_r S - F = 0\), where \(\lambda = \frac{1}{2} \sqrt{P_r^2 S^2 + 4 F + P_r S} > 0\). Since \(F > 0\) and \(\lambda > 0\) for both cases of suction (s>0) & blowing (s<0), the asymptotic solution (9) and (10) are valid for the whole flow filed, where as in the absence of radiative heat transfer the asymptotic solution does not exist for the case of blowing [11].

In the limit of vanishing radiative effects (F=0), the solution (9) and (10) reduce to

\[\theta(\delta) = 1 - e^{-P_r S} + \frac{5}{4} \frac{E_c P_r (\alpha^2 + \beta^2)}{4 \alpha P_r S} e^{-2 \alpha \delta} \quad \cdots (11)\]

For \(2 \alpha - P_r S \neq 0\) and

\[\theta(\delta) = 1 - e^{-P_r S} - \frac{5}{4} \frac{E_c P_r (\alpha^2 + \beta^2)}{4 \alpha P_r S} e^{-2 \alpha \delta} \quad \cdots (12)\]

For \(2 \alpha = P_r S\).
The equation (11) and (12) show that the solution is valid only for the suction case \((s>0)\) and no asymptotic solution exists for the blowing case \((s<0)\). At the disc the rate of heat transfer is given by

\[
c_w = -k \frac{dT}{dz} z = 0
\]

\[
\frac{4}{5} T_0 \frac{\partial}{\partial \nu} \left[ \lambda \cdot \frac{5}{4} e_{sp} (a^2 + \beta^2) \right]
\]

The heat will flow from the disc to the liquid, if the following condition is satisfied:

\[
\lambda < \frac{5}{4} \left[ \frac{e_{sp} (a^2 + \beta^2)}{2\alpha + \frac{1}{2} \left( -p_s + \sqrt{p_s^2 + 4F} \right)} \right]
\]

or otherwise, there will be flow of heat from the liquid to the disc.

5. Conclusion and recommendation

1- This research is about model in applied mathematics for an interact with fluid flow, heat transfer and radiative optical thin limit, it includes a solution of heat transfer equation and radiative heat emission in magneto hydrodynamic (MHD) boundary layer with eccentric rotations of pores and disk and fluid for long time.

2- It is found that the asymptotic solution exists in both the cases of blowing and suction whereas no such solution exists for the case of blowing in the absence of radiative heat emission.

3- The heat transfer rate at the disc is determined, and the condition is obtained for the flowing of heat from the liquid to the disc.

4- The equations (11) and (12) show that the solution is valid only for the suction case of \((s>0)\), and the asymptotic solution does not exist for the blowing case, \((s<0)\).

5- This research is an important and wide subject especially for engineering students in undergraduate and postgraduate in most of the Universities. Also the important of the researches from the above material and its equations (ordinary differential and partial differential) and their solutions especially for the faculty interest in applied mathematics

References

[1] D. A. Nield, and A. Bejan, Convection in porous media (Vol. 3), New York: springer, 2006.

[2] S. Chatti, C. Ghabi, and A. Mhimid, Fluid flow and heat transfer in porous media and post heated obstacle: Lattice boltzmann simulation, International Journal of Heat and Technology, vol. 34, no.3, pp.377-385, 2016.

[3] I. Khan, T. N. A. Abdelhameed, and L. C. Dennis, Heat transfer in eccentric-concentric rotation of a disk and fluid at infinity. Journal of Computational and Theoretical Nanoscience, vol.13, no.10, pp.6482-6487, 2016.

[4] J. H. Lee, Mathematical Modeling to Calculate the Ratio of Instantaneous Stop and MTBE using Instantaneous Frequency, international Journal of Advanced Research in Science, Engineering and Technology, vol.5, no.8, pp.6503-6506, 2018.

[5] R. B. Guenther, and J. W. Lee, Partial differential equations of mathematical physics and integral equations. Courier Corporation, 1996.

[6] A. R. Khan and A. Elkamel, Mathematical model for heat transfer mechanism for particulate system. Applied mathematics and computation, vol.129, no. 2-3, pp. 295-316, 2002.

[7] V. Hud, O. Pinchuk, P. Martyniuk, I. Gerasimov, and P. Volk, Mathematical modelling of heat transfer in a greenhouse with surface soil heating system. Przegląd Naukowy. Inżynieria i Kształtowanie Środowiska, vol.28, no.4, p.86, 2019.

[8] C. Y. Lai, K. R. Rajagopal, and A. Z. Szeri, Asymmetric flow between parallel rotating disks. Journal of Fluid Mechanics, vol.146, pp.203-225, 1984.
J. Coirier, Rotations non coaxiales d’un disque et d’un fluide à l’infini. Journal de Mecanique, vol.11, pp.317-340, 1972.

M. E. Erdoğan, Unsteady flow between two eccentric rotating disks executing non-torsional oscillations. International journal of non-linear mechanics, vol.35, no.4, pp.691-699, 2000.

A. M. Siddiqui, T. Haroon, T. Hayat, and S. Asghar, Unsteady MHD flow of a non-Newtonian fluid due to eccentric rotations of a porous disk and a fluid at infinity, Acta mechanica, vol.147, no.1, pp.99-109, 2001.

K. Maqbool, A. Sohail, and A. B. Mann, Numerical study of heat transfer and Hall current effects on the flow of Johnson-Segalman fluid between two eccentric rotating disks. Journal of Physics Communications, vol.2, no.5, p.055017, 2018.

I. Pop, Unsteady flow due to noncoaxially rotating disk and a fluid at infinity, 1979.

P. N. Kaloni, and A. M. Siddiqui, A note on the flow of a viscoelastic fluid between eccentric disks. Journal of non-newtonian fluid mechanics, vol.26, no.1, pp.125-133, 1987.

S. Dupont, and M. J. Crochet, The vortex growth of a KBKZ fluid in an abrupt contraction. Journal of Non-Newtonian Fluid Mechanics, vol.29, pp.81-91, 1988.

B. Guo, J. Guo, J. Song, and G. Li, Mathematical modeling of heat transfer in counter-current multiphase flow found in gas-drilling systems with formation fluid influx. Petroleum Science, vol.14, no.4, pp.711-719, 2017.

A. M. Wazwaz, The variational iteration method for analytic treatment for linear and nonlinear ODEs. Applied Mathematics and Computation, vol.212, no.1, pp.120-134, 2009.

E. Balvis, A. Paredes, I. Area, R. Bendana, A. V. Carpentier, H. Michinel, and S. Zaragoza, A Fractional Derivative Modeling of Heating and Cooling of LED Luminaires. Mathematics, vol.8, no.3, p.362, 2020.

B. K. Mohammed, S. A. Alsaidi, R. F. Chisab, and H. T. S. ALRikabi, "Efficient RTS and CTS mechanism which save time," International Journal of Interactive Mobile Technologies, vol. 14, no. 4, pp. 204-211, 2020.

Y. S. Mezaal, H. H. Madhi, T. Abd, S. K. Khaleel, “Cloud computing investigation for cloud computer networks using cloudanalyst,” Journal of Theoretical and Applied Information Technology, vol. 96, no. 20, 2018

M. Al-dabag, H. Salim, and R. Al-Nima, "Anticipating Atrial Fibrillation Signal Using Efficient Algorithm," International Journal of Online and Biomedical Engineering (iJOE), vol. 17, no. 2, pp. 106-120, 2021.

A. MOHAMMED "HYBRID COARSE AND DENSE WDM OVER FSO LINK UNDER THE EFFECT OF MODERATE AND HEAVY RAIN WEATHER ATTENUATIONS," vol. 15, no. 5, pp. 3494-3501, 2020.

L. F. Jawad, B. H. Majeed, and H. TH. Salim, "The Impact of CATs on Mathematical Thinking and Logical Thinking Among Fourth-Class Scientific Students," International Journal of Emerging Technologies in Learning (iJET), vol. 16, no. 10, 2021.

A. Ghazi, S. Aljunid, A. Fareed, S. Z. S. Idrus, C. M. Rashidi, and A. M. Fakhrudeen, "Performance Analysis of ZCC-Optical-CDMA over SMF for Fiber-To-The-Home Access Network," in Journal of Physics: Conference Series, 2020, vol. 1529, no. 2, p. 022013: IOP Publishing.

A. H. M. Alaidi, O. H. Yahya, and H. T. Salim, "Using modern education technique in Wasit university," International Journal of Interactive Mobile Technologies, Article vol. 14, no. 6, pp. 82-94, 2020.

T. M. A. Amphawan, S. Alshwani, and A. Aldawoodi, "Modal Properties of a Varied High Indexed Large Core 4 Mode Photonic Crystal Fiber."

A. S. Hussein, R. S. Khairy, S. M. M. Najeeb, and H. ALRikabi, "Credit Card Fraud Detection Using Fuzzy Rough Nearest Neighbor and Sequential Minimal Optimization with Logistic Regression," vol. 15, no. 5, 2021.
[28] T. Masunda, A. Amphawan, S. Alshwani, and A. Aldawoodi, "Modal properties of a varied high indexed large core 4-mode photonic crystal fiber," in 2018 IEEE 7th International Conference on Photonics (ICP), 2018, pp. 1-3: IEEE.

[29] A. Salah, R. S. Khairy, and H. ALRikabi, "The Detection of Counterfeit Banknotes Using Ensemble Learning Techniques of AdaBoost and Voting," International Journal of Intelligent Engineering and Systems, vol. 14, no. 1, pp. 326-339, 2021.

[30] B. H. Majeed, L. F. Jawad, and H. Salim, "Tactical Thinking and its Relationship with Solving Mathematical Problems Among Mathematics Department Students," International Journal of Emerging Technologies in Learning (IJET), vol. 16, no. 9, 2021.

[31] N. A. Hussein, H. A. Naman, M. Al-dabag, H. Alrikabi, "Encryption System for Hiding Information Based on Internet of Things," International Journal of Interactive Mobile Technologies (iJIM), vol. 15, no. 2, 2021.

[32] A. Fareed, A. Ghazi, A. Al-Dawoodi, S. Aljunid, S. Z. S. Idrus, C. Rashidi, A. Amphawan, A. M. Fakhrudeen, and I. E. I. Fadhel, "Comparison of Laguerrre-Gaussian, Hermite-Gaussian and linearly polarized modes in SDM over FMF with electrical nonlinear equalizer," in AIP Conference Proceedings, 2020, vol. 2203, no. 1, p. 020045: AIP Publishing LLC.

[33] B. K. Mohammed, M. B. Mortatha, A. S. Abdalrada, and H. T. Salim, "A comprehensive system for detection of flammable and toxic gases using IoT," Periodicals of Engineering and Natural Sciences (PEN), vol. 9, no. 2, pp. 702-711, 2021.

[34] Y. S. Mezaal, and S. A. Hashim. Design and Simulation of Square Based Fractal Slot Antennas for Wireless Applications, Journal of Engineering and Applied Sciences, 13(17), 7266-7270, 2018.

[35] Y. S. Mezaal, L. N. Yousif, Z. J. Abdulkareem, H. A. Hussein, S. K. Khaleel, “Review about effects of IOT and Nano-technology techniques in the development of IONT in wireless systems,” International Journal of Engineering and Technology (UAE), vol. 7, no. 4, 2018.

[36] E. Sparrow, and R. Cess, Radiation Heat Transfer, McGraw Hill. New York, 1978.