Hydrodynamic surrogate models for bio-inspired micro-swimming robots

Ahmet Fatih Tabak, Serhat Yesilyurt*

Mechatronics Program, Faculty of Engineering and Natural Sciences,
Sabanci University, Tuzla, 34956 Istanbul, Turkey

Summary

Research on untethered micro-swimming robots is growing fast owing to their potential impact on minimally invasive medical procedures. Candidate propulsion mechanisms of robots are based on flagellar mechanisms of micro organisms such as rotating rigid helices and traveling plane-waves on flexible rods and parameterized by wavelength, amplitude and frequency. For design and control of swimming robots, accurate real-time models are necessary to compute trajectories, velocities and hydrodynamic forces acting on robots. Resistive force theory (RFT) provides an excellent framework for the development of real-time six degrees-of-freedom surrogate models for design optimization and control. However the accuracy of RFT-based models depends strongly on hydrodynamic interactions. Here, we introduce interaction coefficients that only multiply body resistance coefficients with no modification to local resistance coefficients on the tail. Interaction coefficients are obtained for a single specimen of Vibrio Algino reported in literature, and used in the RFT model for comparisons of forward velocities and body rotation rates against other specimens. Furthermore, CFD simulations are used to obtain forward and lateral velocities and body rotation rates of bio-inspired swimmers with helical tails and traveling-plane waves for a range of amplitudes and wavelengths. Interaction coefficients are obtained from the CFD simulation for the helical tail with the specified amplitude and wavelength, and used in the RFT model for comparisons of velocities and body rotation rates for other designs. Comparisons indicate that hydrodynamic models that employ interaction coefficients prove to be viable surrogates for computationally intensive three-dimensional time-dependent CFD models. Lastly, hydrodynamic models of

* Author to whom correspondence should be addressed. Tel: +902164839579, Fax: +902164839550, Email: syesilyurt@sabanciuniv.edu
bio-inspired swimmers are used to obtain optimal amplitudes and wavelengths of flagellar mechanisms, as a demonstration of the approach.

**Keywords:** micro swimming, micro flows, resistive force theory, hydrodynamic interaction, bio-inspired robots, surrogate models

1. **Introduction**

Potential advantages of micro swimming robots can revolutionize the modern medicine: procedures such as kidney stone destruction, cleaning of clogged arteries, reaching tumors deep inside vital organs or retina restoration can be performed with minimal side-effects [1, 2]. Conventional mechanisms such as propellers cannot achieve propulsion at low Reynolds numbers in simple fluids, such as in micro fluids, as stated by Purcell’s scallop theorem [3]. However, propulsion mechanisms of natural micro organisms are viable candidates for autonomous micro swimming robots [4]. Bio-inspired propulsion mechanisms have been demonstrated successfully in literature: a representative review is presented next.

Dreyfus et al. [5] demonstrated a novel artificial micro swimmer whose tail is chemically glued to a red blood cell and composed of magnetic particles attached to each other by DNA-protein strands. The tail was actuated by a combination of external dynamic magnetic fields to attain travelling planar waves and the swimmer moved in the opposite direction to the wave propagation. Yu et al. [6] carried out experiments with cm-scale flexible filaments, which mimic the whipping motion of spermatozoa, in viscous oils. The thrust force was measured by a strain gauge and the undulatory motion of the flexible tail was recorded by a camera; experimentally measured forces agreed well with theoretical results of Wiggins and Goldstein [7]. Kosa et al. [8] proposed an actuation mechanism composed of piezoelectric laminates that deform and induce traveling plane waves. The propulsive force is calculated from the pendulum-like motion of the power cable that holds the robot.
Zhang et al. [9,10] manufactured artificial helical flagella as small as a few tens of microns long. Metal and polymer layers are deposited in the shape of a narrow tape and formed into a helix due to the tensile stress exerted on the inner layers during the manufacturing process. A magnetic bead of 4.5 x 4.5 x 0.2 µm is attached to one end of the artificial flagellum. In the presence of a rotating external magnetic field, the torque on the magnetic head enabled the rotation of the helical flagellum and the forward motion of the artificial swimmer [9,10]. Ghosh and Fischer [11] demonstrated the use of glancing angle deposition on the silicon wafer in an electron beam evaporator to obtain about a micron long helical screw like structures with diameters of a few hundreds of nanometers. Helical structures are removed from the wafer, laid onto a surface and deposited by magnetic cobalt on one side. By means of a tri-axial Helmholtz coil, a rotating magnetic field is generated and modified by an open loop control scheme to navigate the micro robot on a preselected trajectory. Gao et al. [12] demonstrated swimming of a few microns long, flexible, Au/Ag/Ni nanowires, with a Au head and Ni tail linked by partially dissolved silver bridge under an external rotating magnetic field. With a similar process presented in [12], Pak et al [13] developed artificial swimmers with 1.5 µm-long Ni heads and 4 µm-long Ag tails, and reported that the structure swims at about 20 µm/s under an external magnetic field rotating at 35 Hz. Tottori et al. [14] used 3-D direct laser writing process and physical vapor deposition to design and fabricate helical devices of varying lengths between 4 and 65 µm. Authors demonstrated corkscrew motion of helical structures in water and fetal bovine serum with rotating magnetic fields. Steering of the helical structure with a micro holder is achieved by changing the direction of the axis of rotation, and utilized for transportation of micro particles by pick-and-place manipulation.

Hydrodynamic models of the propulsion of micro organisms date back to G.I. Taylor [15] who presented an analysis of the swimming of an infinite sheet, which deforms as traveling plane waves in an unbounded fluid; effects of the amplitude, wavelength and frequency on the
swimming speed of the sheet are formulated with the first-order perturbative approximation of the Stokes flow generated by the sheet. Gray and Hancock [16] presented the application of resistive force coefficients to calculate fluid forces due to undulatory motion of slender filaments; force coefficients are obtained from the approximate solution of the fluid motion due to doublets and stokeslets on the filament [17]. Sir James Lighthill [18] used slender body theory for swimmers with helical tails based on the representation of the velocity field by stokeslets and distribution of point forces on the tail. Higdon [19] used a numerical integration method for the integrals approximated by Lighthill [18] to calculate the velocity of the swimmer with the spherical head and the helical tail, and reported the variation of the swimming velocity with the tail length, wavelength and amplitude given in dimensionless forms with respect to the diameter of the body. Lauga and Powers [20] present a comprehensive review of hydrodynamic models of swimming in micro scales.

There are a number of studies reported in literature for artificial swimmers with flexible flagella, such as the structure developed by Dreyfus et al. [5]. Here, we present a representative review: Roper et al [21] modeled the artificial swimmer as a slender elastica driven by magnetic body torque the magnetostatic number, which is the ratio of magnetic and elastic forces. Gauger and Stark [22] presented a bead-spring model of the artificial swimmer to study the mean velocity and the efficiency of the swimmer as a function of the size of the load particle, dimensionless Sperm number, which is the ratio of viscous forces to strength of the flexible filament, magnitude of the applied magnetic field and angular amplitude of the oscillating component of the field. According to results, optimum size of the load is a compromise between the swimming velocity and the efficiency. Keaveny and Maxey [23] presented a particle-based numerical model of the artificial micro swimmer that consisted of paramagnetic beads as rigid spheres connected by inextensible flexible links, demonstrated that the model could be used to study corkscrew form of swimming driven by a rotating magnetic field, and
obtained the velocity of the swimmer as a function of the Sperm number and the magnetostatic number. In [23], authors also presented a resistive force model for three-dimensional deformations of the flexible tail, and an analytical result for the swimming velocity at the low frequency limit. Lastly, a slender-body model is presented for the swimmer developed by Pak et al. [13] by the authors, the model uses standard resistive force coefficients. In the model, bending stiffness of the tail is treated as a fitted parameter for a specified strength of the magnetic field and results agree well with experiments conducted for other magnetic field strengths.

Numerical solutions of the flow coupled with the equation of motion of the swimmer are carried out extensively in literature: the following is a representative review. Fauci and McDonald [24] presented a study of sperm motility near both rigid and elastic walls using the immersed boundary method to solve two-dimensional time-dependent Navier-Stokes equations; authors report that the method proves to be useful especially for handling interactions with elastic walls. Ramia et al. [25] obtained instantaneous velocities of swimming of microorganisms from the solution of Stokes equations with the boundary element method (BEM) in order to study hydrodynamic interactions between cells and solid boundaries as well as the interaction between the body and the tail of the cell. Goto et al. [26] used a BEM model to solve Stokes equations, calculated the velocity vector of a natural micro swimmer, compared their results with observations of actual swimmers, *Vibrio Alginolyticus*, and concluded that the BEM solutions agree reasonably well with observations. Qin et al. [27] studied the wall effects on a swimmer based on spermatozoa undergoing translations on a plane while fully submerged in a highly viscous fluid. Authors used immerse boundary method incorporating Navier-Stokes equations with Newton’s second law to include swimmer’s rigid-body accelerations, and computed the effective hydrodynamic interaction between the swimmer and parallel walls based on the ratio of the half-distance and the wavelength.
Swimming velocities of micro organisms and artificial structures depend on a number of parameters, such as shape and size of the body, length and radius of the tail, amplitude and wavelength of deformations, or propagation of plane or helical waves on the tail depending on the type of propulsion. Fast and accurate RFT models can be used to calculate full three-dimensional trajectories of swimmers in order to obtain optimal shapes, propulsion mechanisms, flagellar parameters, or to develop a model-based control for a specific artificial structure [28,29,30,31]. Hydrodynamic forces on the swimmer are obtained from the integration of local forces in tangent and perpendicular directions to the motion and expressed by resistive force coefficients over the tail. Typically, force coefficients can be calculated from analytical formulas available in the literature, such as from Lighthill’s slender body theory. However, RFT models do not often yield accurate predictions for even the forward velocity of a swimmer for all configurations of flagellar parameters such as the length of the tail, wavelength and amplitude especially in the presence of a sizable body of the swimmer compared to the tail [19,28,29,31].

Effects of hydrodynamic interactions have utmost importance in accuracy of RFT models; in principle, resistive force coefficients can be modified to include those effects. Hydrodynamic interactions are studied for organisms near planar walls, e.g. [6,30], and for two or more organisms swimming together, e.g. [32,33], however the influence of the flagellar motion on the body of swimmers has not been addressed thoroughly in literature to the best of our knowledge. Lighthill included the effect of the cell body on the distribution of stokeslets on the tail using slender body theory, and concluded that the correction to the velocity of the swimmer is very small compared to an isolated infinite flagellum [18]. Chattopadhyay and Wu demonstrated that Lighthill’s correction is very small for micro swimming species such as *Vibrio Algino* [31]. Hydrodynamic interaction between the body and the flagellum was studied numerically by Ramia et al. [25]. Authors concluded that the presence of the cell body does not
alter the flagellar propulsion force as significantly as the flagellar force alters the total drag force on the cell. However, authors did not provide detailed results for the effect of flagellar parameters on the drag force on the body, which is attached to an actuated flagellum, such as a helix, compared to an isolated body with the same shape and size.

Body resistance coefficients are usually available for well-known isolated objects in fluids; for example the resistance coefficient for a sphere of radius $a$ inside a fluid of viscosity $\mu$ is $6\pi \mu a$ regardless of the direction of the motion. In the presence of an actuated tail attached to the body, it is clear that the symmetrical drag relationship breaks, and the drag force on the body deviates from the case when the body is isolated in fluid.

The effect of the motion of the tail on the resistance of the body can be quantified by hydrodynamic interaction coefficients, $\gamma_i$, which scale the resistance of the body in the $i^{th}$ direction; for example for spherical objects $F_i / U_i = \gamma_i (6\pi \mu a)$ for the translation in the $i^{th}$ direction, and $\tau_j / \Omega_j = \gamma_j (8\pi \mu a^3)$ for the rotation about the $j^{th}$ direction. Hydrodynamic interaction coefficients can be different for each direction due to the motion of the tail, which breaks the symmetry of the flow over the spherical body. Calculation of hydrodynamic interactions is extremely difficult analytically; however a single experiment or a computational solution of Stokes equations can be used to obtain the coefficients for at least a fixed representative design. Then, the model with the fixed values of interaction coefficients is essentially a surrogate model for the computational model used to solve the Stokes equations, and can be validated with further simulations, which are carried out for swimmers with different dimensions.

The RFT-based hydrodynamic model, which is presented here, includes hydrodynamic interactions between the tail and the body to improve its accuracy, then, serves as a surrogate for accurate numerical solutions of Stokes equations, and can be used as a real-time model in model-based control and design optimization studies such as searching for the optimum design in the neighborhood of a base-case design. The approach can be generalized for arbitrary body
shapes and propulsion mechanisms. However, the extent of design variations, for which the RFT model with interaction coefficients retains its accuracy, needs to be identified in order to demonstrate the effectiveness of the approach. Lastly, the approach is valid only for design optimization problems that can be represented by a finite number of design variables, which are used to parameterize a given waveform of the tail and the geometry of the body, and is not an alternative for generalized shape optimization strategies, for example as recently presented by Keaveny et al. [34].

In this work, the RFT-based hydrodynamic model is used to model micro organisms with varying body and tail dimensions and studied by Goto et al. [26]. The forward velocity and the body rotation rate of a fixed specimen, are used to obtain interaction coefficients for body resistances in the model with a simple search algorithm; then, the model is validated against measurements for other specimens in order to demonstrate the feasibility of the approach. Moreover, for a base-case swimmer with the spherical body and helical tail, which consists of two full turns (waves) and radius (amplitude) of 1/5th of the body radius, hydrodynamic interaction coefficients are obtained from the CFD simulation and used in the RFT model, which is validated against other CFD simulations for different numbers of waves and amplitudes of the tail than the base-case values. Furthermore, resistive force coefficients for the tail are directly computed from the CFD simulation for the base-case swimmer and compared with force coefficients obtained from Lighthill [18]. Lastly, the validated hydrodynamic model is used to obtain optimal tail parameters for efficiency and speed as a demonstration of the approach.

2. Methodology

2.1 Hydrodynamic model

Time-dependent trajectory of a two-link, micro-swimmer is obtained from the equation of motion, which balances forces on the swimmer’s body and the tail:

\[ F_b + F_t = 0 \]  

(1)
where \( \mathbf{F} = [\mathbf{F}', \mathbf{T}'] \) is the generalized force vector, \( \mathbf{F} \) and \( \mathbf{T} \) are force and torque vectors, ‘’ is the transpose, and subscripts \( b \) and \( t \) refer to the body and the tail. For simplicity and demonstration of the approach, we assume that the body of the swimmer is a blunt object such as a sphere, and the flexible tail is subject to a motion that generates propulsion force in viscous flows, such as the rotation of a helix, or traveling-plane waves on a slender rod as commonly observed among micro swimming organisms.

For creeping flows at low Reynolds numbers, the equation of motion can be cast in a linear system of equations relating the generalized force and velocity vectors by means of the resistance matrix, \( \mathbf{B} \), as follows:

\[
\mathbf{F}_i = -\mathbf{B}_i \mathbf{V}_i
\]

Here, \( i = \{b, t\} \), \( \mathbf{B} \) is the resistance matrix, \( \mathbf{V} = [\mathbf{U}', \mathbf{\Omega}'] \) is the generalized velocity vector, \( \mathbf{U} \) and \( \mathbf{\Omega} \) are translational and rotational velocity vectors respectively.

The resistance matrix for the rigid body of the swimmer, \( \mathbf{B}_b \), is obtained from the linear and rotational resistances of the body, and can be considered as a combination of four subcomponents which relate linear and angular velocities to forces and torques:

\[
\mathbf{B}_b = \begin{bmatrix} \mathbf{D}_N & \mathbf{E} \\ \mathbf{E}' & \mathbf{D}_R \end{bmatrix}
\]

where matrices \( \mathbf{D}_N \) and \( \mathbf{D}_R \) are 3x3 diagonal matrices that correspond to translational and rotational resistances of the body, matrix \( \mathbf{E} \), contains nonzero elements only if the center of masses of the body and the swimmer are far apart. For a spherical isolated body in an unbounded fluid, each diagonal element of \( \mathbf{D}_N \) is \( 6\pi \mu \eta b \) and each diagonal element of \( \mathbf{D}_R \) is \( 8\pi \mu \eta b^3 \), where \( \mu \) is the dynamic viscosity and \( r_b \) is the radius of the spherical body. Drag coefficients for generalized ellipsoids and other body shapes are also known [35,36,37]. However, even for a simple body such as a sphere, resistance matrices \( \mathbf{D}_N \) and \( \mathbf{D}_R \) must be modified due to motion of the tail attached to the body as well as for flows inside channels and nearby boundaries [38,39]. For
example, interaction coefficients can be introduced to modify the translational and rotational resistance matrices for spherical bodies as follows:

\[
D_{N,k,k} = \gamma_{N,k} 6\pi \mu r_b \quad \text{and} \quad D_{R,k,k} = \gamma_{R,k} 8\pi \mu r_b^3, \quad k = \{s,q,r\} \tag{4}
\]

where \(\gamma_{N,k}\) and \(\gamma_{R,k}\) are interaction coefficients that modifies corresponding drag components in the \(k^{th}\) direction.

In effect, interaction coefficients are only applied to body resistances; due to the linearity of the equation of motion, Eq. (1), relative effect of hydrodynamic interactions can be applied to either the body of the swimmer or its tail. Interaction coefficients need to be estimated well in order to ensure accuracy of the hydrodynamic model.

Time-dependent resistance matrix of the tail, \(B_t\) in (2), is obtained from integration of local forces:

\[
B_t = \int_0^L \begin{bmatrix} RCR' & -RCR'S \\ S\hat{CR'} & -S\hat{RCR'}S \end{bmatrix} ds \tag{5}
\]

where \(L\) is the apparent length of the tail in the \(s\)-direction, \(S\) is the skew-symmetric matrix that corresponds to the cross product with the position vector on the tail, \(R\) is the rotation matrix between the local Frenet-Serret coordinates, \(t-b-n\), and the \(s-q-r\) coordinates of the swimmer (Fig. 1), and formed by local tangential, \(t\), bi-normal, \(b\), and normal, \(n\), vectors [40]:

\[
R = \begin{bmatrix} t(s,t) & b(s,t) & n(s,t) \end{bmatrix} \tag{6}
\]
The local resistance matrix at a given position on the tail, $C$ in Eq. (5), is a diagonal matrix that consists of the local resistance coefficients in the tangent, $c_t$, bi-normal and normal directions, $c_n$. Local resistance coefficients are the same in the bi-normal and normal directions as both are perpendicular to the tangential direction.

Accurate calculation of resistance coefficients is extremely difficult. Lighthill derived resistance coefficients from the distribution of stokeslets and point forces on infinite helices in unbounded fluids [18]. A number of simplifying assumptions are used in the derivation of resistance coefficients. The local normal and tangential components of resistive force coefficients are obtained as follows [18]:

\[
\frac{4\pi\mu}{-\ln \varepsilon + (2\alpha^2 - 1)A_1 + 2(1 - \alpha^2)A_2}
\]

(7)

and

\[
\frac{2\pi\mu}{-\ln \varepsilon - 0.5 + \alpha^2A_1 + (1 - \alpha^2)A_2}
\]

(8)

Here $\alpha$ is the ratio between apparent and actual lengths of the tail; $\varepsilon$ is given by a relationship based on tail’s radius, $a$, $\alpha$, and wavelength, $\lambda$: $\varepsilon = 5.2a\alpha/\lambda$. $A_1$ and $A_2$ are periodic integrals of functions of assumed local flow fields and specified in [18] as follows:
Local velocity on the tail is the summation of the swimmer’s net velocity and the motion of the tail with respect to body coordinates i.e.

\[ \mathbf{U}_t = \mathbf{U}_b + \mathbf{\hat{u}} \]  

(10)

where \( \mathbf{\hat{u}} \) is the local velocity on the tail, and can be obtained from the deformation or the rotation of the tail:

\[ \mathbf{\hat{u}} = \frac{d\mathbf{P}}{dt} \]  

(11)

For example for a left-handed helical tail as shown in Fig. 1, position of the centerline of the helical rod, \( \mathbf{P} = [s \ q \ r]' \), is specified in the body coordinate frame as follows:

\[
\mathbf{P} = \begin{bmatrix}
s \\ q(s,t) \\ r(s,t)
\end{bmatrix} = \begin{bmatrix}
s \\ b_s(s) \cos \left( ks - \omega t \right) \\ -b_s(s) \sin \left( ks - \omega t \right)
\end{bmatrix}
\]  

(12)

where \( k \) is the wave number, \( \omega \) is the angular frequency of rotations, and \( b(s) \) is the local radius of the helix, which is modified with a ramp function to ensure a fixed connection with the body (Fig. 1), e.g. \( b_s(s) = 10b(s/L) \) for \( s/L < 0.1 \) and \( b_s(s) = b \) for \( (s/L) \geq 0.1 \).

In the case of plane-wave deformations, local position is specified by the \( q \)-displacement and the \( r \)-displacement is set to zero. For an arbitrary actuation mechanism, the velocity of the tail can be calculated from Eq. (10) once position of the centerline is specified similarly to Eq. (12).

Forces on the tail can be decomposed into propulsion and drag forces; according to Eqs. (2) and (10) as follows:

\[
\begin{bmatrix}
\mathbf{F}_t \\ \mathbf{T}_t
\end{bmatrix} = -\mathbf{B}_t \begin{bmatrix}
\mathbf{U}_t \\ \mathbf{\Omega}_t
\end{bmatrix} = -\mathbf{B}_t \left( \begin{bmatrix} \mathbf{U}_b \\ \mathbf{\Omega}_b \end{bmatrix} + \mathbf{\hat{u}} \right) = -\mathbf{B}_t \begin{bmatrix} \mathbf{U}_b \\ \mathbf{\Omega}_b \\ \mathbf{F}_p \\ \mathbf{T}_p
\end{bmatrix}
\]  

(13)
The first term in the right-hand-side of the last equation in Eq. (13) is the total drag force on the tail due to its motion with the body, and the second term is the propulsion force and torque generated by the tail due to its motion relative to the body, i.e. \( \mathbf{B}_\tau \hat{u} \), and obtained from Eq. (5) as follows:

\[
\begin{bmatrix}
F_p \\
T_p
\end{bmatrix} = \int_0^L \left[ \mathbf{R} C \mathbf{R} \hat{u} \right] \left( \mathbf{P} - \mathbf{P}_{com} \right) \times \left( \mathbf{R} C \mathbf{R} \hat{u} \right) \, ds
\]

(14)

where \( \mathbf{R} \) is given by Eq. (6), \( \mathbf{C} \) is the local resistance matrix. \( \mathbf{P}_{com} \) is the position of the center of mass in \( \text{sqr} \) coordinates. In case of traveling plane-waves, the propulsion force and torque are only due to the \( q \)-component of \( \hat{u} \), whereas \( r \)-component of \( \mathbf{U}_b \), \( s \) and \( q \)-components of \( \mathbf{\Omega}_b \) are zero. Furthermore, the formulation can easily be extended to swimmers for which the body and the tail are attached, and rotate together, i.e. \( \mathbf{\Omega}_{b,s} \) is equal to \( \omega \), and computed from the specified external magnetic torque.

Substituting (2), (5) and (13) into (1), one obtains the instantaneous velocity vector of the swimmer in the \( \text{sqr} \) coordinates:

\[
\begin{bmatrix}
\mathbf{U}_b \\
\mathbf{\Omega}_b
\end{bmatrix} = (\mathbf{B}_b + \mathbf{B}_\tau)^{-1} \begin{bmatrix}
F_p \\
T_p
\end{bmatrix}
\]

(15)

In order to obtain the velocity of the swimmer in the lab frame, e.g. for control studies or to generate trajectories, the rotation matrix, \( \mathbf{R}_L \), between the \( \text{sqr} \) and XYZ frames (see Fig. 1) must be calculated either explicitly from Euler angles or from quaternion transformations [41]. In order to alleviate the representation problem, we implement the latter and obtain the velocity vector in the lab frame:

\[
\begin{bmatrix}
\mathbf{U}_b^{XYZ} \\
\mathbf{\Omega}_b
\end{bmatrix} = \begin{bmatrix}
\mathbf{R}_L & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
\mathbf{U}_b \\
\mathbf{\Omega}_b
\end{bmatrix}
\]

(16)

Once the velocity vector in the lab frame is obtained, the position of the swimmer is obtained kinematically, for example, with the Runge-Kutta scheme. The quaternion for the rota-
tion matrix $R_s$ is also part of the integration scheme to keep track of the orientation of the swimmer. Typical simulation time for a swimmer with a helical tail is less than a second for the full rotation of the tail on a high-end mobile workstation.

2.2 CFD model

Computational fluid dynamics (CFD), which provides numerical solution of three-dimensional time-dependent Navier-Stokes equations, can be used to compute reliably fluid forces especially at low Reynolds number flows. In order to model the motion of a swimmer in an unbounded fluid, here, we use a relatively large channel around the swimmer with the diameter as large as ten times the diameter of the body, and length five times the total length of the swimmer with negligible distortion to the flow field nearby the swimmer.

Fluid forces are calculated from the finite-element method solution of incompressible Navier-Stokes equations in the moving domain due to the motion of the tail and the overall swimmer. Arbitrary-Lagrangian-Eulerian (ALE) scheme [42] is used to handle the deforming mesh. Equations are nondimensionalized with the diameter of the body, $D_b$, as the length scale and $2\pi/\omega$ as the time-scale; hence the velocity scale is $\omega D_b/2\pi$, which varies linearly with the frequency, and the scaling Reynolds number is given by

$$Re = \frac{\rho \omega D_b^2}{2\pi \mu}$$

(17)

Complete list of variables used in the representative, base-case design is shown in Table 1.

Hydrodynamic forces on the swimmer are computed from the integration of fluid stresses over the surface of the swimmer, and set to zero as a set of constraint equations in order to obtain forward and lateral velocities and body rotation rates from no-slip moving boundary conditions on the swimmer:
\[
\begin{bmatrix}
F_r \\
T_r
\end{bmatrix} + \begin{bmatrix}
F_b \\
T_b
\end{bmatrix} = \begin{bmatrix}
\int_{\text{Tail + Body}} \tau \mathbf{n} \, dA \\
\int_{\text{Tail + Body}} (\mathbf{x} - \mathbf{x}_{\text{com}} \times \tau) \mathbf{n} \, dA
\end{bmatrix} = 0
\] (18)

Here, \( \tau \) is the fluid stress tensor, \( \mathbf{n} \) is the time-dependent three-dimensional local surface normal, \( \mathbf{x} \) is the position vector, and \( \mathbf{x}_{\text{com}} \) is the position of the center of mass, which is assumed to be the geometric center of the spherical body.

Full translations of the rigid body and the rotation of the body along the \( s \)-axis are obtained for swimmers with helical tails from (18). Specifically, the first row of (18) is the constraint equation for the forward velocity of the swimmer, the second row for the \( \mathbf{q} \)-velocity, third row for the \( \mathbf{r} \)-velocity, and the fourth row is for the angular velocity of the body around the \( s \)-axis.

CFD simulations are carried out for two types of flagellar propulsion mechanisms: first one is the left-handed helical tail rotating in the positive direction with respect to the \( s \)-coordinate as shown in Fig. 1a; and the second one is for the tail with traveling waves in the \( \mathbf{q} \)-\( s \) plane as shown in Fig. 1b. Independent rotation of helical tails is observed in micro organisms and demonstrated as an effective mechanism with large scale experiments in viscous fluids [43,44]. Traveling waves are used to simplify the deformation of flexible filaments; actual deformations can be modeled with elastic properties to replace the amplitude and wavelength, which are used as independent variables here. Spherical body is chosen for its simplicity and well-known drag coefficients. The approach, which is presented here, can easily be extended to study magnetized artificial structures with arbitrary body shapes and flexible filaments without loss of generality.

Commercial software, COMSOL Multiphysics [45], which is based on the finite-element method, is used to perform CFD simulations with the second order Lagrangian tetrahedral elements. For all simulations 300,000 degrees of freedom are used. Linear system of equations is solved with the PARDISO linear solver and a second order backward difference formula with variable time-stepping for the numerical integration in time (maximum time step is set to
Simulations require up to twenty hours on a high-end workstation in order to complete two full periods of the wave propagation (helical or planar) on the tail depending on its geometric parameters.

### Table 1
Base case parameters and their dimensionless values for swimmers in CFD simulations.

| Parameter Name                        | Dimensionless Value |
|---------------------------------------|---------------------|
| Radius of the spherical body, $r_b$   | 0.5                 |
| Chord radius of the tail, $r_t$       | 0.05                |
| Apparent length of the tail, $L$      | 2                   |
| Apparent wavelength, $\lambda$       | 2/3                 |
| Wave amplitude, $b$                   | 0.1                 |
| Actuation frequency of the tail, $\omega/2\pi$ | 1                   |
| Fluid density, $\rho$                | 1                   |
| Scaling Reynolds number, $Re = \rho \omega D_p^2 / 2\pi \mu$ | $10^{-2}$ |
| Cylindrical channel length, $L_{ch}$ | 10                  |
| Cylindrical channel diameter, $2r_{ch}$ | 10                  |
| Spherical body resistances, $\{6\pi \mu r_b, 8\pi \mu r_b^3\}$ | $\{942.5, 314.2\}$ |

3. **Results**

3.1 **Validation of the hydrodynamic model with measurements**

Goto et al. [26] measured forward velocity and body rotation rates for a number of specimens of *Vibrio Alginolyticus*, whose dimensions and tail rotation rates vary individually.

Authors could not measure the frequency of rotations of the helical tail, due to relatively high frequency of tail rotations compared to body rotations, and used the boundary element method (BEM) to calculate the frequency of tail rotations from the measured frequency of body rotations. Table 2 shows geometric parameters of individual organisms reported in [26]; for all cases radius of the tail is 16 nm, wavelength of the helical waves is 1.37 µm and the amplitude (helical radius) is 0.1487 µm [26]. In effect, the amplitude and wavelength are constant, and tail lengths and body dimensions vary.
Resistive force coefficients from Lighthill’s slender body theory [18], which are given by Eqs. (7) and (8), are used to calculate fluid forces on the helical tail. Translational and rotational resistance coefficients for the body’s motion are calculated from drag relationships for oblique spheroids given by [36]:

\[
D_{N,s} = \Gamma_{N,s} \frac{4\pi \mu r_s}{\left(\log \left(\frac{2r_s}{r_q}\right) - 0.5\right)}
\]

(19) and

\[
D_{R,s} = \Gamma_{R,s} \left(\frac{16}{3}\right) \frac{\pi \mu r_s r_q^2}{r_q^{3/2}}
\]

(20)

In (19) and (20), \(r_{(s,q)}\) are the radii of the body in the \(s\) and \(q\)-directions respectively, and \(\Gamma_{(N,R),s}\) are hydrodynamic interaction coefficients, which, in essence, quantify variations in body’s translational and rotational drags due to the flow by the rotating tail. If interaction coefficients in (19) and (20) are set to unity, translational and rotational drag factors, \(D_{N,s}\) and \(D_{R,s}\), would be those of isolated spheroids in infinite media.

Table 2
Geometric parameters of \(V.\ Alginolyticus\) specimens.

| Specimen | Frequency (Hz) | Tail Length (µm) | Body \(s\)-semi-axes, \(r_s\) (µm) | Body \(q\) and \(r\)-semi-axes, \(r_q\) (µm) |
|----------|----------------|------------------|----------------------------------|----------------------------------|
| A        | 187.70         | 4.89             | 1.885                            | 0.415                            |
| B        | 123.20         | 4.90             | 1.320                            | 0.380                            |
| C        | 73.95          | 5.24             | 1.380                            | 0.405                            |
| D        | 244.70         | 5.19             | 1.975                            | 0.400                            |
| E        | 126.20         | 5.03             | 1.785                            | 0.405                            |
| F        | 220.10         | 5.07             | 2.260                            | 0.380                            |
| G        | 477.10         | 4.87             | 2.280                            | 0.410                            |

Time-averaged forward velocity and the body-rotation rate of natural swimmers are calculated from (15) and compared with the measurements of Goto et al. [26] in Fig. 2. There is a significant discrepancy between the measurements and model results when interaction coeffi-
cients are set to unity, i.e. for $\Gamma^{N,R}_{s} = 1$: maximum error is 87% in the average forward velocity for specimen G, and 47.2% in the body-rotation rate for specimen B.

Values of interaction coefficients, $\Gamma_{N,s}$ and $\Gamma_{R,s}$, can be determined from the solution of the inverse problem for observed values of the forward velocity and the body rotation rate of a selected swimmer as the representative design. Here, specimen C is used as the representative design, and interaction coefficients in translational and rotational drag relationships given by Eqs. (19) and (20) are calculated as $\Gamma_{N,s} = 2.37$ and $\Gamma_{R,s} = 1.49$ respectively from the minimization of the squared error between the model results and measured values with respect to interaction coefficients, $\Gamma_{N,s}$ and $\Gamma_{R,s}$, for specimen C. In effect, the forward drag of the spheroid body increases by a factor of 2.37, and the rotational drag by a factor of 1.49 compared to an isolated spheroid without a rotating tail.

As shown in Fig. 2, the agreement between the hydrodynamic model and measurements is very good when the interaction coefficients for specimen C is used in the model: maximum error is 8.2% in the average forward velocity for specimen G, and 6.5% in the body rotation rate for specimen F. Despite that specimens have different body dimensions, tail lengths and tail rotation frequencies (Table 2), interaction coefficients obtained from the solution of the inverse problem for an arbitrary selected specimen work very well other specimens as well.

It is reasonable to expect that hydrodynamic interactions between the body and the tail would have an effect on the resistance force coefficients of the tail as well. In effect, the linearity of the equation of motion, which consists the force-free swimming condition given by Eq. (1) and the resistance relationship between the forces and velocities given by Eq. (2), allows that hydrodynamic interactions can be included in the resistance matrix of only one component, either the body or the tail. Furthermore, results of previous numerical studies show that the total drag force on the tail is not affected by the choice of body as much as the total drag force on the body of the swimmer [25].
3.2 CFD simulations

3.2.1 Estimation of hydrodynamic model parameters

Two sets of resistive force coefficients are used for tails in the hydrodynamic model: the first set is by Lighthill [18] and obtained from Eqs. (7) and (8); and the second set is obtained from the CFD simulation for the stationary swimmer with the rotating helical tail. The helical radius (amplitude) of the tail is set to 0.1 and the wavelength to 2/3 as the base case design. Complete list of base-case design parameters and their values are given in Table 1.

Stationary swimmer is not subject to force-free swimming constraints given by Eq. (18), thus the rotating left-handed helical tail of the swimmer generates the propulsion, which can be calculated from the integration of fluid stresses over the tail in the CFD model. Furthermore, integrations on the right-hand-side of Eq. (14) are carried out explicitly only in the swimming
direction (s-direction in Fig. 1) to obtain a closed-form expression for the force and the torque generated by the tail as a function of angular and translation velocities as follows:

$$\begin{bmatrix} \bar{F}_{t,s} \\ \bar{T}_{t,s} \end{bmatrix} = -\alpha L \begin{pmatrix} b^2 k^2 & 1 \\ b^2 k & -b^2 k \end{pmatrix} \bar{u}_s + \begin{pmatrix} k & -k \\ 1 & b^2 k^2 \end{pmatrix} b^2 \omega \begin{bmatrix} c_n \\ c_t \end{bmatrix}$$

(21)

where $\alpha = (1 + b^2 k^2)^{-1/2}$ is the ratio of the apparent length of the helix to the actual rod length of the tail, $b$ is the helical radius, which is 0.1 for the base case, $k$ is the wavenumber, which is $3\pi$ for the base case, $\bar{u}_s$ is the average swimming speed, which is zero for the stationary swimmer, and $\omega$ is the frequency of rotations of the tail. Once the left-hand-side of Eq. (21) is computed from the CFD model for the stationary swimmer, resistive force coefficients, $c_n$ and $c_t$ are, then, obtained as 995.5 and 775.2, respectively. Arguably, the constant pair of force coefficients incorporates realistic flow conditions such as the finite length and radius of the tail and the trailing-edge force due to the motion of the tip of the tail, which are not taken into account in the derivation of the resistance coefficients by Lighthill [18].

According to Eqs. (7) and (8), resistive force coefficients vary with the parameter $\alpha$, which is the ratio of the chord length of the tail to its apparent length and varies with the amplitude and wavelength. Fig. 3a shows the variation of the $c_n / c_t$ ratio with respect to number of waves, and Fig. 3b shows the variation of the ratio with respect to amplitude for helical tails and traveling plane waves. The constant $c_n / c_t$ ratio obtained from the CFD simulation for the base case is also shown in Fig. 3. For traveling-plane-wave tails, we used the wavelength-averaged value of $\alpha$, the ratio of the chord length to apparent length, as it varies locally unlike the ratio for the helical tail, which remains constant independent of the local position on the tail.
Resistance coefficients for the body are obtained from the well-known drag coefficients of spherical objects multiplied by translational and rotational hydrodynamic interaction coefficients in the $k^{\text{th}}$ direction ($k=\{s,q,r\}$), $\gamma_{N,k}$ and $\gamma_{R,k}$, respectively, and used as diagonal factors in the body resistance sub-matrices in Eq. (3) as follows:

$$D_{N,k,k} = \gamma_{N,k} \frac{6\pi\mu r_b}{\rho} \quad \text{and} \quad D_{R,k,k} = \gamma_{R,k} \frac{8\pi\mu r_b^3}{\rho}$$

Interaction coefficients account for the hydrodynamic effect of the tail’s motion on the body’s resistance coefficients, which are diagonal elements of the resistance matrix given in Eq. (3). Off-diagonal elements of resistance matrices can be used to account for more general interactions between the directions of motion, for example the well-known Magnus effect, which is recently observed for micro particles at very low Reynolds numbers [46], can be described as the force in the $q$-direction due to the translation of the body in the $s$-direction and the rotation in the $r$-direction (see Fig. 1 for directions). Moreover, strictly-diagonal form of the body resistance matrix, which is considered here, can be viewed as the result of the diagonalization of a general form that includes all hydrodynamic interactions and is currently under investigation [47].
For helical propulsion, one interaction coefficient in the swimming direction, one for lateral directions, and one for the rotation of the body in the swimming direction are necessary; body rotations in other directions are neglected in the model. Interaction coefficients for the spherical body of the swimmer that corresponds to the base-case representative design are calculated directly from the ratio of forces and velocities obtained from the CFD simulation.

Time-dependent forward velocity of the swimmer is within 0.6% of its average value, -0.038. The net hydrodynamic drag force in the swimming direction on the spherical body of swimmer is obtained as 81.7, which corresponds to 2.28 times the well-known drag force on spherical objects. Therefore the interaction coefficient in the swimming direction is obtained as \( \gamma_{N,s}^{\text{Helix, CFD}} = 2.28 \). Similarly, the angular velocity of the swimmer is within 0.2% of its time-averaged value, which is obtained as -0.4; the torque exerted on the spherical body is 1.09 times its well-known value for spherical objects, and sets the value of the interaction coefficient for rotations in the swimming direction as \( \gamma_{R,s}^{\text{Helix, CFD}} = 1.09 \).

Lateral (q- and r-directions in Fig. 1) velocities and forces are both sinusoidal in time with zero mean and amplitude of 0.015 and 7.235 respectively. The phase between the wave forms of lateral velocities and forces is equal to \( \pi/2 \). The ratio of amplitudes of the lateral forces and the lateral velocities is 0.51 times the spherical drag; however for lateral directions, we use the interaction coefficient from the minimization of the squared error, which is obtained as \( \gamma_{N,q}^{\text{Helix, CFD}} = \gamma_{N,r}^{\text{Helix, CFD}} = 1.24 \).

The effect of hydrodynamic interactions between the body and the tail is evaluated by interaction coefficients applied only to body resistances. Therefore, values of interaction coefficients vary with the choice of resistive force coefficients used for the tail. For resistive force coefficients obtained from Lighthill’s slender-body-theory [18], a new set of interaction coefficients are necessary. In this case, interaction coefficients are obtained from the minimization of the squared error between the RFT model that uses Lighthill’s coefficients and already
calculated velocities from the CFD simulation; interaction coefficients for the body resistance matrices are obtained as $\gamma_{N,s}^{Helix,SBT} = 3.35$, $\gamma_{N,q}^{Helix,SBT} = \gamma_{N,r}^{Helix,SBT} = 1.1$, and $\gamma_{R,s}^{Helix,SBT} = 0.85$.

Flagellar propulsion with traveling plane waves (TPW), in essence, can be considered as a special case of the helical propulsion since the deformation of the tail in the $r$-direction is set to zero in Eq. (12). Therefore, it is assumed that resistive force coefficients obtained from Eq. (21) for the stationary swimmer with the helical tail should perform reasonably well here. In this case, interaction coefficients for the forward motion of the swimmer in the $s$-direction, for the lateral motion of the swimmer in the $q$-direction, and for the rotation of the body in the $r$-direction are needed. The interaction coefficient in the swimming direction is calculated from the ratio of the time-averaged force and the time-averaged velocity in that direction, which are obtained from the CFD simulation of the swimmer with the traveling-plane-wave tail whose amplitude and wavelength are set to the base-case values, 0.1 and $2/3$ respectively; the calculated value of the interaction coefficient is obtained as, $\gamma_{N,s}^{TPW,CFD} = 2.21$. This value is very close to the one obtained for the swimmer with the helical tail, which is $\gamma_{N,s}^{Helix,CFD} = 2.28$.

The lateral interaction coefficient is obtained as: $\gamma_{N,q}^{TPW,CFD} = 3.14$, from the ratio of amplitudes of the lateral force and the lateral velocity, which are zero in average. Lastly, the interaction coefficient for the rotational resistance of the body in the $r$-direction perpendicular to the plane of propagating wave is obtained from the ratio of amplitudes of the torque and the angular velocity in that direction; as, $\gamma_{R,r}^{TPW,CFD} = 0.45$.

Interaction coefficients are also calculated from the minimization of the squared-error between the RFT model and the simulation for the base case for both sets of resistive force coefficients; results are presented in Table 3. It is somewhat surprising to see that the interaction coefficient for the $r$-rotation of the swimmer is negative. We suspect that the result is an artifact of using only the diagonal components of the body resistance matrix. In effect, the $r$-rotation of
the swimmer is strongly linked with the \( q \)-translation due to strong coupling between the \( r \)-torque and the \( q \)-force, which is generated by the tail. In essence, having a very large interaction coefficient in the \( q \)-translation and a negative one for the \( r \)-rotation could be the result of more complex interactions between the two modes of the motion. For example, the Magnus effect, although does not apply here but observed in micro flow conditions recently [46], can be represented by a negative lateral resistance due to the rotation and forward motion of a spherical body. Our efforts continue to investigate this matter further to elucidate the extent of hydrodynamic interactions between the body and the tail.

### Table 3
Interaction coefficients for body the resistance matrices of the spherical body given by Eqs. (22) and Error! Reference source not found..

| Propulsion type | Tail resistance coefficient | Body drag factor |
|----------------|---------------------------|-----------------|
| Helical        | From (7) and (8)          | \( \gamma_{Helix,SBT}^{(N,s;N,q;R,s)} = \{3.35; 1.1; 0.85\} \) |
| Helical        | CFD: \( c_{n,t} = \{995.5, 775.2\} \) | \( \gamma_{Helix,CFD}^{(N,s;N,q;R,s)} = \{2.24; 1.25; 1.09\} \) |
| TPW           | From (7) and (8)          | \( \gamma_{TPW,SBT}^{(N,s;N,q;R,r)} = \{1.95; 6.75; -2.5\} \) |
| TPW           | CFD: \( c_{n,t} = \{995.5, 775.2\} \) | \( \gamma_{TPW,CFD}^{(N,s;N,q;R,r)} = \{1.65; 8; -3\} \) |

#### 3.2.2 Validation of the hydrodynamic model

The hydrodynamic model is validated with additional CFD simulations for different values of amplitudes and wavelengths than the values used for the estimation of interaction coefficients. The study can be extended to other parameters such as the tail length and body type etc. Here, we considered only wavelength and amplitude scaled with the radius of the body, which affect the swimmer’s performance greatly. Moreover, frequency, diameter of the body and fluid properties are lumped into the scaling Reynolds number given by Eq. (17). Thus, for small Reynolds numbers, the velocity of the robot scales linearly with the frequency of tail rotations.
and its body size; therefore there is no need to conduct additional simulations for frequency
dependence and the radius of the body.

For swimmers with helical tails, hydrodynamic model results are compared with CFD
simulation results in Figs. 4a-f. Average forward velocity (Fig. 4a), amplitude of the lateral
velocity (Fig. 4b) and the body rotation rate (Fig. 4c) are plotted against the amplitude, which is
the radius of the helix. According to hydrodynamic model results with resistive force coeffi-
cients from Lighthill’s slender body theory (SBT) [18], magnitude of the time-averaged for-
ward velocity increases with the amplitude with a rate that slows down at higher values. The
model results with CFD-based force coefficients also show that the average velocity increases
with the amplitude; in this case, a slightly better agreement with actual simulation results is
observed than the case with SBT-based force coefficients. The agreement between the hy-
drodynamic surrogate model and simulation results is better at small values of the amplitude
than large ones (Fig. 4a), indicating that hydrodynamic interaction coefficients must be updated
for large amplitudes. Percentage errors from the plots are listed in Table 4 for all cases.

Time-dependent lateral motion of the swimmer is periodic with zero mean-value. However,
the amplitude of the lateral velocity increases with the amplitude of helical waves almost lin-
early; the agreement is slightly better for the force coefficients from the slender body theory
than the force coefficients obtained from the CFD simulation for the stationary swimmer (Fig.
4b).

For the body rotation rate in Fig. 4c, model results with analytically obtained force coeffi-
cients from the slender body theory agree with simulation results better than the results with
constant force coefficients (11.8% error vs. 42.7%) for large wave amplitudes; agreement is
somewhat poor for both sets of coefficients at small amplitudes.

Average forward velocity, amplitude of the lateral velocity and the average body rotation
rate are plotted against the number of waves in Figs. 4d-f, respectively. The forward velocity
predicted by the hydrodynamic model indicates that the wavelength does not have a significant effect, and agrees well with CFD simulation results for both sets of parameters (Fig. 4d) (6.9% for constant $c_n$ and $c_t$, and 9.3% for $c_n$ and $c_t$ from the slender body theory (SBT)).

The amplitude of the lateral velocity peaks at half integer values of the number of waves, i.e. for $N_\lambda = 1.5, 2.5, 3.5$, etc, and falls at full integer values. When the helical waves are in full-periods, forces in the lateral directions are minimal, and emerge only due to the bias introduced by the shape function $b_s(s)$ in Eq. (12). However when the helical waves do not have full turns, the symmetry is broken and hydrodynamic forces in lateral directions emerge and the effect is maximized when the incomplete wave is half. Moreover, the strength of the lateral motion diminishes as the number of waves increases indicating that the effect of the incomplete wave is diluted as the total number of waves increases. Overall, the hydrodynamic model predicts the lateral motion well especially with analytical resistive force coefficients compared

![Fig. 4. Time-averaged forward (a,d), amplitude of the lateral (b,e), and rotation of the body (c,f) against the amplitude (a-c) and number of waves (d-f) for helical tails: circles are CFD results, solid lines are for hydrodynamics model with resistive force coefficients obtained from the CFD simulation for a stationary swimmer, and dashed lines are for hydrodynamic model results with resistive force coefficients obtained from Lighthill’s slender body theory (SBT) [18].](image-url)
to resistive force coefficients computed from the CFD simulation for the stationary swimmer (see Fig. 4e). Lastly, the rotation rate of the body does not vary with the number of waves on the tail significantly, and predicted reasonably well with the hydrodynamic model as shown in Fig. 4f.

For swimmers with traveling-plane-wave tails, hydrodynamic model results are compared with CFD-simulation results in Figs. 5a-f for both sets of parameters: resistive force coefficients from the slender body theory (RFC from SBT) and corresponding hydrodynamic interaction coefficients of the body; and resistive force coefficients determined from the CFD-simulation for stationary swimmer with the helical tail (constant RFC) and corresponding interaction coefficients.

Time-averaged forward velocity, amplitude of the lateral velocity and the amplitude of the angular velocity of the body in the r-direction increase with the amplitude, and are predicted very well with the hydrodynamic model for both sets of parameters (Figs. 5a-c). Time-averaged forward velocity of the swimmer is plotted against the number of waves on the tail in Fig. 5d. For small number of waves, hydrodynamic model results agree well with CFD-simulation results for both sets of coefficients. However the time-averaged velocity calculated by the model with force coefficients from the slender body theory decreases with increasing number of waves for large values. Model results with constant force coefficients agree very well with CFD simulation results for large number of waves as well.

The lateral velocity of the swimmer in the q-direction is periodic in time with zero-average value. Amplitude of the lateral velocity varies with the number of waves on the tail similarly to helical tails with the exception that peaks are observed at number of waves equal to full integers and bottoms at half-integers as shown in Fig. 5e. In part this is because of the effect of the amplitude-shape function, which introduces a bias near the body and breaks the balance of forces towards the tip of the tail: for half integer waves on the tail q-direction forces are
symmetric and net force is small, on the other hand for full integer waves the motion of the tip of the tail is not balanced by the motion of the tail near the body. Hydrodynamic model results agree very well qualitatively with CFD results, despite a slide shift in the results for number of waves larger than 3.5. Moreover, the hydrodynamic model with resistive force coefficients from the slender body theory predicts that overall trend of the amplitude of the lateral velocity decreases slowly with respect to number of waves on the tail, although results of the model with constant force coefficients show a decreasing trend as the number of waves increases, and agrees well with the CFD simulation results (see Fig. 5e). Lastly, the amplitude of r-rotations of the body follows a trend with peaks near the half integer waves and falls at slightly larger values than the full integer number of waves on the tail (see Fig. 5f). Although the overall trend agrees well with the CFD simulation results, the range of the falls and peaks from the hydrodynamic model are not as large as the ones from CFD simulations.

Fig. 5. Time-averaged forward velocity (a), amplitude of the lateral velocity (b), and amplitude of body rotation around the r-axis (c) are plotted against the amplitude of waves for TPW tails (a-c) and number of waves (d-f). Circles are CFD results, solid lines are for hydrodynamics model with resistive force coefficients obtained from the CFD simulation for a stationary swimmer with a helical tail, and dashed lines are for hydrodynamic model results with resistive force coefficients obtained from Lighthill’s slender body theory (SBT) [18].
Summary of the performance of the hydrodynamic surrogate model is presented in Table 4. Overall, the surrogate model agrees very well with CFD simulation results for both sets of resistive force coefficients (RFC) and for both actuation types.

**Table 4**
Errors in predictions of the hydrodynamic model, (absolute error; range)

| Number of Waves | Amplitude |
|-----------------|-----------|
|                 | CFD-based constant | Analytical | CFD-based constant | Analytical |
|                 | $c_n$, $c_t$ | $c_n$, $c_t$ | $c_n$, $c_t$ | $c_n$, $c_t$ |
| Helical tail $\bar{u}_t$ | .0025, [-.0327,-.0393] | .0033, [-.0319,-.03939] | .0047, [-.0011,-.0596] | .0046, [-.0008,-.0597] |
| $v_{q,max}$ | .0085, [.0073,.0435] | .0102, [.0052,.0356] | .003, [.002,.0251] | .0074, [.002,.0212] |
| $\bar{\Omega}$ | .0011, [-.0574,-.0694] | .00843, [-.0708,-.4783] | .0516, [-.0009,-.1722] | .0143, [-.0009,-.1349] |
| Traveling plane waves $\bar{u}_t$ | .0011, [-.020,.024] | .0031; [.0023, .0022] | .0056; [.0012, .0012] | .0023; [.0017, .0017] |
| $v_{q,max}$ | .0013; [.0023, .0015] | .0022; [.0015, .0015] | .0012; [.0016, .0016] | .0017; [.0016, .0016] |
| $\Omega_{r,max}$ | .0014; [.0059,.0096] | .0010; [.0058,.0095] | $4.1 \times 10^{-4}$; $[8.2, 95.1] \times 10^{-4}$ | $5.5 \times 10^{-4}$; $[8.1, 93.8] \times 10^{-4}$ |

4. **Applications of the hydrodynamic model**

4.1 **Design with the hydrodynamic model**

Design of an artificial micro swimmer can be carried out with the validated hydrodynamic surrogate model that can replace the computationally exhaustive three-dimensional CFD model. For example energy consumption of the robot, for which the base case parameters are given in Table 1, can be minimized with the maximization of its efficiency, which is the ratio of the useful work done to move the body of the swimmer and the work done to rotate the tail as defined by Purcell [3], and given by:

$$\eta = \frac{\Pi_{body}}{\Pi_{tail}}$$  \hspace{1cm} (23)$$

where $\Pi_{body} = F_s u_s$ is the average rate of work done to move the body of the robot with the velocity of $u_s$ against the drag force on the body, $F_s$, and $\Pi_{tail}$ is the rate of work done to actuate...
the tail of the robot and calculated from $\Pi_{\text{tail}} = T_s \omega$ for helical tails, where $T_s$ is the torque needed to rotate the tail with angular velocity, $\omega$. For traveling plane waves, the rate of actuation work is calculated from the integration of the product of the local force and the local net velocity in the lateral direction, i.e. $F_q dq/dt$, over the entire tail length.

Average forward velocity (Fig 6a,b) and the hydrodynamic efficiency of swimmers (Fig. 6c,d) are calculated with the hydrodynamic model for amplitudes varying between .01 and .5 and for number of waves between .5 and 5. Admittedly, we use a broader range of amplitudes than the one used in the validation of the RFT model in order to demonstrate the effect of the amplitude on the swimming speed, and find out if there is a local maximum within the range.

According to Figs. 6a and 6b, there is a similarity between the forward velocity of swimmers with helical tails and traveling plane waves, former with the maximum velocity of .21 for $b = 0.5$ and $N_\lambda = 1$, and the latter with the maximum velocity of .12 for the same amplitude and $N_\lambda = 0.8$. Therefore, first, swimmers with helical tails are considerably faster than the ones with traveling plane waves; and second, the fastest swimmer with a tail twice as long as the diameter of the body consists of a single helical turn with the largest amplitude. Unfortunately, we do not have CFD results that can confirm the accuracy of RFT model results for the amplitude that maximizes the speed due to numerical difficulties.

From Figs. 6c and 6d, the efficiency of swimmers with helical tails is considerably larger than the efficiency of swimmers with traveling plane waves; the maximum efficiency for the helical tails is obtained as 2.5%, and as 0.29% for traveling plane waves for robots with geometric parameters given in Table 3.

In addition to geometric design, hydrodynamic model can also be used to estimate physical properties of natural swimmers. Geometric properties and wave propagation parameters of a natural swimmer can be determined from the hydrodynamic model. For example, given the swimming trajectory of a particular specimen with unknown geometric properties, such as a
parameterized waveform, amplitude, wavelength, etc., as in the study reported by Friedrich et al. [48] for bull sperm cells, the RFT model can be used to generate trajectories from the solution of (16) by varying the geometric variables of the swimmer in the model. The error between the target and model-generated trajectories in an appropriate norm can be minimized to obtain the unknown geometric properties of the specimen. Moreover, Gurarie et al [49] demonstrated that stochastic model can be used for the prediction of the full three-dimensional trajectory of the swimmer based on two-dimensional observations; hydrodynamic models can be used to improve the predictability of complex trajectories.

![Fig. 6](image-url) Time-averaged forward velocity, $u_s$ (a,b) and hydrodynamic efficiency, $\eta$ (c,d); obtained by the hydrodynamic model for robots with helical tails and traveling plane waves respectively.

5. Conclusions

Forward and lateral translational and rotation of bio-inspired micro swimmers that consist of a body and an actuated tail are computed with three-dimensional, time-dependent CFD
simulations and predicted with a hydrodynamic surrogate model, which is based on the resistive force theory (RFT) that uses a resistive relationship between the force and velocity vectors on the tail and the body. The hydrodynamic model runs essentially in real-time to predict the full trajectory of swimmers unlike computationally intensive three-dimensional numerical models of Stokes or Navier-Stokes equations. The accuracy of the RFT-based hydrodynamic model is improved by the consideration of hydrodynamic interaction coefficients for the resistance coefficients of the body. Interaction coefficients are calculated from the solution of the inverse problem for the representative design of the tail with the amplitude and wavelength are set to 0.1 and 2/3 respectively in non-dimensional units.

Two sets of resistive force coefficients are used for tails, which are rotating helices or traveling-plane-wave deformations: one set is from the slender body theory of Lighthill [18], and the second set is directly calculated from the CFD simulation for the stationary swimmer with the helical tail that has a fixed amplitude and wavelength. For each form of flagellar actuation and force coefficients, hydrodynamic interaction coefficients are estimated for the body of the swimmer from the minimization of the squared error between the velocities for the base case. Then the hydrodynamic model is validated directly against CFD results for swimmers with helical and traveling-plane-wave tails, for which the amplitude is varied between .01 and .15 and the wavelength is varied between 0.5 and 1. For all cases, the surrogate hydrodynamic model results agree reasonably well with CFD model results indicating that interaction coefficients obtained for a fixed pair of amplitude and wavelength can be used for a range of amplitudes and wavelengths. Accuracy of the surrogate model starts to deteriorate for amplitude values significantly larger than the base-case value.

Furthermore, experimentally measured time-averaged forward velocity and body rotation rates for micro organisms that are presented in literature are compared with the results of the hydrodynamic model with resistive force coefficients obtained from the slender body theory.
Once the hydrodynamic interaction coefficients of the body are determined from the inverse problem for a fixed specimen, predicted forward velocities and body rotation rates agree very well with the measurement results for other species with different body and tail dimensions.

Lastly, we demonstrated the application of validated hydrodynamic surrogate models in design of bio-mimetic robots to obtain optimal propulsion type, amplitude and number of wavelengths.

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