The Cauchy problem for $\mathcal{D}$-modules on Ran spaces.

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Abstract

We will adopt an elementary approach to $\mathcal{D}$-modules on Ran spaces in terms of two-limits; the aim here is to define the category of coherent $\mathcal{D}$-modules, characteristic varieties and non-characteristic maps. An application will be the proof of the Cauchy-Kowaleski-Kashiwara theorem in this setting.

1 Introduction

The pioneering work [BD04] on the geometric foundations of Quantum Field Theory and Conformal Field Theory stressed the importance played by so-called Ran spaces. For example chiral, Lie* and factorization algebras can be considered as geometric objects living on such spaces (see [BD04]). The most appropriate approach to the theory of $\mathcal{D}$-modules on Ran spaces (as presented in [FG11]) requires many sophisticated tools from Lurie's work on ($\infty$, 1)-categories. The main goal of this paper is to explore some definitions and properties of $\mathcal{D}$-modules on Ran spaces employing the classical theory of prestacks and two limits (SGA4). This simplified approach makes the theory accessible and sheds light on some definitions: the notions of characteristic varieties and of coherent $\mathcal{D}$-modules are really natural in this language. In paragraph §2 we recall some basics on the classical theory of $\mathcal{D}$-modules and microlocal geometry; in §3 we recall some definitions relative to prestacks, two-limits, etc.; in paragraph §4 we introduce the theory of modules on Ran manifolds by the use of §3 as an application we prove the Cauchy-Kowaleski-Kashiwara theorem for Ran manifolds.

Acknowledgments. I am deeply grateful to Pierre Schapira for having called my attention on the study of coherent $\mathcal{D}$-modules over Ran spaces. I have much benefited from the reading of the unpublished manuscript [GS06] and I wish to kindly thank the authors. Moreover I thank the Luxembourgian National Research Fund for support via AFR grant PhD 09-072.

2 Preliminaries

Sheaves

We shall mainly follow the notations of [KS90] for sheaves, that of [KS06] for categories and that of [Ka03] for $\mathcal{D}$-modules.
Let $X$ be a real manifold and let $k$ be a field. One denotes by $k_X$ the constant sheaf on $X$ with stalk $k$ and by $\text{Mod}(k_X)$ the abelian category of sheaves of $k$-modules on $X$. We denote by $D^b(k_X)$ the bounded derived category of $\text{Mod}(k_X)$. One simply calls an object of this category, “a sheaf”.

We shall use the classical six operations on sheaves, the internal hom $\mathcal{H}om$, the tensor product $\otimes$, the direct image $f_*$, the proper direct image $f_!$, the inverse image $f^{-1}$ and the derived functors $R\mathcal{H}om$, $L\otimes$, $Rf_*$, $Rf_!$, $f^{-1}$, as well as the extraordinary inverse image $f_!$, right adjoint to $Rf_!$.

We also use the notation $\boxtimes$ for the external product: if $X$ and $Y$ are two manifolds and where $q_1$ and $q_2$ denote the first and second projection on $X \times Y$, we set for $F \in \text{Mod}(k_X)$ and $G \in \text{Mod}(k_Y)$, one sets

$$F \boxtimes G := q_1^{-1}F \otimes q_2^{-1}G.$$  

**Microlocal geometry**

For a real or complex manifold $X$, we denote by $\tau : TX \to X$ its tangent vector bundle and by $\pi : T^*X \to X$ its cotangent vector bundle. If $E \to X$ is a vector bundle, we identify $X$ with the zero-section.

Let $f : X \to Y$ be a morphism of real or complex manifolds. To $f$ are associated the tangent morphisms

\begin{equation}
\begin{array}{ccc}
TX & \xrightarrow{f'} & X \times_Y T^*Y \\
\downarrow{\tau} & & \downarrow{\tau} \\
X & \xrightarrow{f} & Y.
\end{array}
\end{equation}

By duality, we deduce the diagram:

\begin{equation}
\begin{array}{ccc}
T^*X & \xleftarrow{f_!} & X \times_Y T^*Y \\
\downarrow{\pi} & & \downarrow{\pi} \\
X & \xrightarrow{f} & Y.
\end{array}
\end{equation}

One sets

$$T^*_X Y := \ker f_! = f_{d!}^{-1}(T^*_X X).$$

**Definition 2.1.** Let $\Lambda \subset T^*Y$ be a closed $\mathbb{R}^+$-conic subset. One says that $f$ is non-characteristic for $\Lambda$, or else, $\Lambda$ is non-characteristic for $f$, if

$$f^{-1}_!(\Lambda) \cap T^*_X Y \subset X \times_Y T^*_Y Y.$$  

**$\mathcal{O}$-modules**

Now assume that $(X, \mathcal{O}_X)$ is a complex manifold of complex dimension $d_X$. We denote by $\text{Mod}(\mathcal{O}_X)$ the abelian category of sheaves of $\mathcal{O}_X$-modules and by $\mathcal{H}om_{\mathcal{O}_X}$ and $\otimes_{\mathcal{O}_X}$ the internal hom and tensor product in this category. The sheaf of rings $\mathcal{O}_X$ is Noetherian and we denote by $\text{Mod}_{\text{coh}}(\mathcal{O}_X)$ the thick abelian subcategory of $\text{Mod}(\mathcal{O}_X)$ consisting of coherent sheaves.
We denote by $\text{D}^b(\mathcal{O}_X)$ the bounded derived category of $\text{Mod}(\mathcal{O}_X)$ and by $\text{D}^b_{\text{coh}}(\mathcal{O}_X)$ the full triangulated subcategory consisting of objects with coherent cohomology.

Let $f : X \to Y$ be a morphism of complex manifolds. We keep the notation $f_*$ and $f_!$ for the two direct image functors for $\mathcal{O}$-modules and we denote by $f^*$ the inverse image functor for $\mathcal{O}$-modules:

$$f^* \mathcal{G} := \mathcal{O}_X \otimes_{f^{-1}\mathcal{O}_Y} f^{-1}\mathcal{G}.$$  

Hence we have the pair of adjoint functors $(f^*, f_*)$

$$\text{Mod}(\mathcal{O}_X) \xrightarrow{f^*} \text{Mod}(\mathcal{O}_Y)\xleftarrow{f_*} \text{Mod}(\mathcal{O}_Y)$$

It follows that $f_*$ is left exact and $f^*$ is right exact. We denote by $L f^*$ the left derived functor of $f^*$.

Given two manifolds $X$ and $Y$, we denote by $\boxtimes$ the external product for $\mathcal{O}$-modules. Hence,

$$\mathcal{F} \boxtimes \mathcal{G} := \mathcal{O}_{X \times Y} \otimes_{\mathcal{O}_X \otimes \mathcal{O}_Y} (\mathcal{F} \boxtimes \mathcal{G}).$$

\section*{$\mathcal{D}$-modules}

We denote by $\mathcal{D}_X$ the sheaf of rings of holomorphic differential operators, the subalgebra of $\text{End}(\mathcal{O}_X)$ generated by $\mathcal{O}_X$ and $\Theta_X$, the sheaf of holomorphic vector fields.

Unless otherwise specified, a $\mathcal{D}_X$-module is a left-$\mathcal{D}_X$-module. Hence a right $\mathcal{D}_X$-module is a ($\mathcal{D}_X^\text{op}$)-module. We denote by $\text{Mod}(\mathcal{D}_X)$ the abelian category of $\mathcal{D}_X$-modules and by $\text{D}^b(\mathcal{D}_X)$ its bounded derived category.

Recall the operations

$$\mathcal{H}om_{\mathcal{D}_X} : \text{Mod}(\mathcal{D}_X)^\text{op} \times \text{Mod}(\mathcal{D}_X) \to \text{Mod}(\mathcal{D}_X),$$

$$\mathcal{H}om_{\mathcal{D}_X} : \text{Mod}(\mathcal{D}_X)^{\text{op}} \times \text{Mod}(\mathcal{D}_X) \to \text{Mod}(\mathcal{D}_X),$$

$$\otimes_{\mathcal{D}_X} : \text{Mod}(\mathcal{D}_X) \times \text{Mod}(\mathcal{D}_X) \to \text{Mod}(\mathcal{D}_X),$$

$$\otimes_{\mathcal{D}_X} : \text{Mod}(\mathcal{D}_X)^{\text{op}} \times \text{Mod}(\mathcal{D}_X) \to \text{Mod}(\mathcal{D}_X^{\text{op}}).$$

We denote by $\boxtimes$ the external product for $\mathcal{D}$-modules:

$$\mathcal{M} \boxtimes \mathcal{N} := \mathcal{D}_{X \times Y} \otimes_{\mathcal{D}_X \otimes \mathcal{D}_Y} (\mathcal{M} \boxtimes \mathcal{N}).$$

For a morphism $f : X \to Y$ of complex manifolds, we denote as usual by $\mathcal{D}_X \to Y$ the transfert bimodule, a ($\mathcal{D}_X, f^{-1}\mathcal{D}_Y$)-bimodule

$$\mathcal{D}_{X \to Y} := \mathcal{O}_X \otimes_{f^{-1}\mathcal{O}_Y} f^{-1}\mathcal{D}_Y.$$  

One should be aware that the left $\mathcal{D}_X$ structure of $\mathcal{D}_{X \to Y}$ is not the one induced by that of $\mathcal{O}_X$. We then have the inverse and direct image functors

$$f^D : \text{D}^b(\mathcal{D}_Y) \to \text{D}^b(\mathcal{D}_X), \quad f^D \mathcal{N} = \mathcal{D}_{X \to Y} \otimes_{f^{-1}\mathcal{D}_Y} f^{-1}\mathcal{N};$$

$$f_D : \text{D}^b(\mathcal{D}_X) \to \text{D}^b(\mathcal{D}_Y), \quad f_D(\mathcal{M}) = R f_* (\mathcal{L} \otimes_{\mathcal{D}_X} \mathcal{D}_{X \to Y}).$$
We denote by $\text{Mod}_{\text{coh}}(\mathcal{D}_X)$ the thick abelian subcategory of $\text{Mod}(\mathcal{D}_X)$ consisting of coherent $\mathcal{D}_X$-modules and by $\text{D}^b_{\text{coh}}(\mathcal{D}_X)$ the full triangulated subcategory consisting of objects with coherent cohomology.

For $\mathcal{M} \in \text{D}^b_{\text{coh}}(\mathcal{D}_X)$, we denote by $\text{char}(\mathcal{M})$ its characteristic variety, a closed $\mathbb{C}^\times$-conic complex analytic subvariety of $T^*X$.

### The Cauchy-Kowaleski-Kashiwara theorem

**Definition 2.2.** Let $f : X \to Y$ be a morphism of manifolds and $\mathcal{N}$ be a coherent $\mathcal{D}_Y$-module. One says that $f$ is non-characteristic for $\mathcal{N}$ if $f$ is non-characteristic for $\text{char}(\mathcal{N})$.

**Proposition 2.3.** ([Ka70]) Let $\mathcal{N}$ be a coherent $\mathcal{D}_Y$-module. Suppose that $f : X \to Y$ is non-characteristic for $\mathcal{N}$. Then

(a) $H^k(f^D \mathcal{N}) = 0$ for $k \neq 0$,

(b) $H^0(f^D \mathcal{N})$ is a coherent $\mathcal{D}_X$-module,

(c) $\text{char}(f^D \mathcal{N}) = f_d f^{-1} \pi \text{char} \mathcal{N}$.

For $\mathcal{N}_1$ and $\mathcal{N}_2$ in $\text{D}^b(\mathcal{D}_Y)$ we have a natural morphism

$$f^{-1} R\mathcal{H}om_{\mathcal{D}_Y}(\mathcal{N}_1, \mathcal{N}_2) \to R\mathcal{H}om_{\mathcal{D}_X}(f^D \mathcal{N}_1, f^D \mathcal{N}_2).$$

Note that $f^D \mathcal{O}_X \simeq \mathcal{O}_Y$.

The following theorem is known as the Cauchy-Kovalesky-Kashiwara theorem:

**Theorem 2.4.** ([Ka70]) Let $f : X \to Y$ be a morphism of complex manifolds and $\mathcal{N}$ be a coherent $\mathcal{D}_Y$-module. If $f$ is non-characteristic for $\mathcal{N}$, then

$$f^{-1} R\mathcal{H}om_{\mathcal{D}_Y}(\mathcal{N}, \mathcal{O}_Y) \to R\mathcal{H}om_{\mathcal{D}_X}(f^D \mathcal{N}, \mathcal{O}_X)(2.3)$$

is an isomorphism.

### 3 Grothendieck stacks

References for this section are made to [SGA4] (see also [KS06] for an exposition). The results of this section are well-known to the specialists. We will work in a given universe $\mathcal{U}$, a category means a $\mathcal{U}$-category and a set means a small set. Here $\text{Cat}$ denotes the big 2-category of all $\mathcal{U}$-categories.

**Prestack**

Let $\mathcal{S}$ be a 2-projective system of categories indexed by a small category $\mathcal{I}$, that is, a functor $\mathcal{S} : \mathcal{I}^{\text{op}} \to \text{Cat}$. For short, we consider $\mathcal{I}$ as a presite and call $\mathcal{S}$ a prestack on $\mathcal{I}$. Hence

(a) for each $i \in \mathcal{I}$, $\mathcal{S}(i)$ is a category;

(b) for each morphism $s : i_1 \to i_2$ in $\mathcal{I}$, $\mathcal{S}(s)$ is a functor $\mathcal{S}(i_2) \to \mathcal{S}(i_1)$, called the restriction functor (for convenience, we shall write in the sequel $\rho_s$ instead of $\mathcal{S}(s)$);
(c) for $s: i_1 \rightarrow i_2$ and $t: i_2 \rightarrow i_3$, we have an isomorphism of functor $c_{s,t}: \rho_s \circ \rho_t \sim \rho_{tos}$, making the diagram below commutative for each $u: i_3 \rightarrow i_4$:

\[
\begin{array}{ccc}
\rho_s \circ \rho_t \circ \rho_u & \xrightarrow{c_{s,t}} & \rho_s \circ \rho_{uot} \\
\rho_{tos} \circ \rho_u & \xrightarrow{c_{s,uot}} & \rho_{uotos} \\
\end{array}
\]

(d) finally $\rho_{id_i} = \text{id}_{\mathcal{S}(i)}$ and $c_{id_i, id_i} = \text{id}_{\mathcal{S}(i)}$ for any $i \in \mathcal{I}$.

**Functor of prestacks**

Let $\mathcal{S}_\nu (\nu = 1, 2)$ be prestacks on $\mathcal{I}$ with the restrictions $\rho_s^\nu$ and the composition isomorphisms $c_{s,t}^\nu$.

Recall that a functor of prestacks $\Phi: \mathcal{S}_1 \rightarrow \mathcal{S}_2$ on $\mathcal{I}$ is the data of:

(a) for any $i \in \mathcal{I}$, a functor $\Phi(i): \mathcal{S}_1(i) \rightarrow \mathcal{S}_2(i)$;

(b) for any morphism $s: i_1 \rightarrow i_2$ an isomorphism $\Phi_s$ of functors from $\mathcal{S}_1(i_2)$ to $\mathcal{S}_2(i_1)$

\[
\Phi_s: \Phi(i_1) \circ \rho_s^1 \sim \rho_s^2 \circ \Phi(i_2);
\]

these data satisfying: for any sequence of morphisms $i_1 \xrightarrow{s} i_2 \xrightarrow{t} i_3$ the following diagram commutes

\[
\begin{array}{ccc}
\Phi(i_1) \circ \rho_s^1 \circ \rho_t^1 & \xrightarrow{c_{s,t}^1} & \Phi(i_1) \circ \rho_s^2 \circ \Phi(i_2) \\
\Phi(i_1) \circ \rho_{tos}^1 & \xrightarrow{c_{s,t}^2} & \Phi(i_1) \circ \rho_{tos}^2 \circ \Phi(i_3) \\
\end{array}
\]

(3.1)

**Morphism of functors of prestacks**

We will need the following definition

**Definition 3.1.** Let $\Phi_\nu: \mathcal{S}_1 \rightarrow \mathcal{S}_2 (\nu = 1, 2)$ be two functors of prestacks on $\mathcal{I}$. A morphism of functors of prestacks $\theta: \Phi_1 \rightarrow \Phi_2$ is the data for any $i \in \mathcal{I}$ of a morphism of functors $\theta(i): \Phi_1(i) \rightarrow \Phi_2(i)$ such that for any morphism $s: i_1 \rightarrow i_2$ in $\mathcal{I}$, the following diagram commutes

\[
\begin{array}{ccc}
\Phi_1(i_1) \circ \rho_s^1 & \xrightarrow{\theta(i_1)} & \Phi_2(i_1) \circ \rho_s^1 \\
\Phi_1(i_1) \circ \rho_{tos}^1 & \xrightarrow{\theta(i_2)} & \Phi_2(i_1) \circ \rho_{tos}^1 \\
\end{array}
\]

(3.2)

**2-projective limits**

For a prestack $\mathcal{S}$ on $\mathcal{I}$, one defines the category $\mathcal{S}(\mathcal{I})$ as the 2-projective limit of the functor $\mathcal{S}: \mathcal{I}_{\text{op}} \rightarrow \text{Cat}$:

\[
\mathcal{S}(\mathcal{I}) = \lim_{\longrightarrow}^{\mathcal{I}, s} \mathcal{S}(i).
\]

More explicitly $\mathcal{S}(\mathcal{I})$ is given as follows.
Definition 3.2. (a) An object $F$ of $\mathcal{G}(\mathcal{J})$ is a family $\{(F_i, \varphi_s)\}_{i,s}$ ($i \in \mathcal{J}$, $s \in \text{Mor}(\mathcal{J})$) where

(i) for any $i \in \mathcal{J}$, $F_i$ is an object of $\mathcal{G}(i)$,

(ii) for any morphism $s: i_1 \to i_2$ in $\mathcal{J}$, $\varphi_s: \rho_{sF_i} \simeq F_{i_1}$ is an isomorphism such that

• for all $i \in \mathcal{J}$, $\varphi_{i\text{id}_i} = \text{id}_{F_i}$,

• for any sequence $i_1 \overset{s}{\to} i_2 \overset{t}{\to} i_3$ of morphisms in $\mathcal{J}$, the following diagram commutes

\[
\begin{array}{ccc}
\rho_{s\rho_{tF_i}} & \cong & \rho_{sF_{i_2}} \\
\varphi_{st} & \downarrow & \varphi_{s} \\
\rho_{\psi sF_i} & \cong & F_{i_1}.
\end{array}
\]

(3.3)

(b) A morphism $f: \{(F_i, \varphi_s)\}_{i,s} \to \{(F'_i, \varphi'_s)\}_{i,s}$ in $\mathcal{G}(\mathcal{J})$ is a family of morphisms $\{f_i: F_i \to F'_i\}_{i \in \mathcal{J}}$ such that for any $s: i_1 \to i_2$, the diagram below commutes:

\[
\begin{array}{ccc}
\rho_s(F_{i_2}) & \xrightarrow{\rho_s(f_{i_2})} & \rho_s(F'_{i_2}) \\
\varphi_s & \downarrow & \varphi_s' \\
F_{i_1} & \xrightarrow{f_{i_1}} & F'_{i_1}.
\end{array}
\]

(3.4)

Remark 3.3. Denote by $\text{Mor}(\mathcal{J})$ the category whose objects are the morphisms of $\mathcal{J}$, a morphism $(s: i \to j) \to (s': i' \to j')$ being visualized by the commutative diagram

\[
\begin{array}{ccc}
i & \xrightarrow{s} & j \\
i' & \downarrow & \downarrow \\
i' & \xrightarrow{s'} & j'.
\end{array}
\]

For two objects $F = \{(F_i, \varphi_s)\}_{i,s}$ and $F' = \{(F'_i, \varphi'_s)\}_{i,s}$ in $\mathcal{G}(\mathcal{J})$, consider the functor $\Psi: \text{Mor}(\mathcal{J})^{\text{op}} \to \text{Set}$ defined as follows. For $(s: i \to j) \in \text{Mor}(\mathcal{J})$, set

\[\Psi(s) = \text{Hom}_{\mathcal{G}(i)}(F_i, \rho_sF'_j).\]

For $(s': i' \to j') \in \text{Mor}(\mathcal{J})$ and for a morphism $(t, t'): s \to s'$ in $\text{Mor}(\mathcal{J})$, define $\Psi(s') \to \Psi(s)$ as the composition

\[
\text{Hom}_{\mathcal{G}(i')}((F'_{i'}, \rho_{s'}F'_{j'})) \to \text{Hom}_{\mathcal{G}(i')}((\rho_{tF_{i'}}, \rho_{s'}F'_{j'})) \\
\to \text{Hom}_{\mathcal{G}(i)}((F_i, \rho_{s}F'_{j}))
\]

where the last map is associated with the morphisms $F_i \to \rho_{tF_{i'}}$ and $\rho_{tF_{i'}}F'_{j'} \to \rho_{t\rho_{s'}F'_{j}} \simeq \rho_{s}F'_{j}$.

Then

\[
\text{Hom}_{\mathcal{G}(\mathcal{J})}(F, F') \simeq \lim_{s \in \text{Mor}(\mathcal{J})^{\text{op}}} \Psi(s).
\]
Proposition 3.4. Let $\mathcal{S}$ be a prestack on $\mathcal{I}$. Assume that, for each object $i \in \mathcal{I}$, the category $\mathcal{S}(i)$ admits inductive (resp. projective) limits indexed by a small category $A$ and that, for each morphism $s : i \to j$ in $\mathcal{I}$, the restriction functor $\rho_s$ commutes with such limits. Then the category $\mathcal{S}(\mathcal{I})$ admits inductive (resp. projective) limits indexed by $A$ and the restriction functors $\rho_i : \mathcal{S}(\mathcal{I}) \to \mathcal{S}(i)$ commute with such limits.

Some Lemmas

Lemma 3.5. Every functor of prestacks $\Psi : \mathcal{S}_1 \to \mathcal{S}_2$ on $\mathcal{I}$ induces a functor $\Psi_\infty : \mathcal{S}_1(\mathcal{I}) \to \mathcal{S}_2(\mathcal{I})$ on the corresponding two-limit categories.

Proof. To each object $\{(F_i, \varphi_s)\}_{i,s} \in \mathcal{S}_1(\mathcal{I})$, $\Psi$ associates $\{(\Psi(i)F_i, \varphi_s)\}_{i,s} \in \mathcal{S}_2(\mathcal{I})$, with $\varphi_s$ defined as the composition $\rho_2^2 \Psi(j)F_j \cong \Psi(i)\rho_1^1 F_j \cong \Psi(i)F_i$ where the first isomorphism is the inverse of $\Psi_s$ and the second is $\Psi(i)(\varphi_s)$. Notice that for any sequence $i_1 \xrightarrow{s} i_2 \xrightarrow{t} i_3$ of morphisms in $\mathcal{I}$, the following diagram commutes

\[
\begin{array}{ccc}
\rho_2^2 \Psi(i_3)F_{i_3} & \xrightarrow{\rho_2^1(\varphi_t)} & \rho_2^2 \Psi(i_2)F_{i_2} \\
\varphi_{i_2,i_3} \downarrow & & \downarrow \varphi_{i_1,i_2} \\
\rho_{i_2,i_3} \Psi(i_3)F_{i_3} & \xrightarrow{\varphi_{i_3,i_2}} & \Psi(i_1)F_{i_1};
\end{array}
\]

in fact it is enough to apply $\Psi(i_1)$ to the corresponding diagram in def (3.2) and then use property (3.1) of the morphism $\Psi$.

Finally $\Psi_\infty$ associates to each morphism $f : \{(F_i, \varphi_s)\}_{i,s} \to \{(F'_i, \varphi'_s)\}_{i,s}$, $\Psi_\infty f = \{(\Psi(f)i)\} : \mathcal{S}_1(\mathcal{I}) \to \mathcal{S}_2(\mathcal{I})$ with

\[
\begin{array}{ccc}
\rho_s(\Psi(i_2)F_{i_2}) & \xrightarrow{\rho_s(\Psi(i_2)(f_{i_2})} & \rho_s(\Psi(i_2)F'_{i_2}) \\
\varphi_s \downarrow & & \downarrow \varphi_s \\
\Psi(i_1)F_{i_1} & \xrightarrow{\Psi(i_1)(f_{i_1})} & \Psi(i_1)F'_{i_1};
\end{array}
\]

(it is enough to apply the functor $\Psi(i_1)$ to the diagram (3.1))

\[\Box\]

The next lemma is obvious

Lemma 3.6. Let $\Psi_1 : \mathcal{S}_1 \to \mathcal{S}_2$ and $\Psi_2 : \mathcal{S}_2 \to \mathcal{S}_3$ be two morphisms of prestacks on $\mathcal{I}$. There exists an isomorphism of functors $\Psi_1 \circ \Psi_2 \cong (\Psi_1 \circ \Psi_2)_\infty : \mathcal{S}_1(\mathcal{I}) \to \mathcal{S}_3(\mathcal{I})$.

Define

\[
\pi : \mathcal{S}(\mathcal{I}) \to \prod_{i \in \mathcal{I}} \mathcal{S}(i)
\]

as the functor that associates to each $\{(F_i, \varphi_s)\}_{i,s}$, the collection $\{F_i\}_i$ and acts on each morphism $\{f_i\}_{i \in \mathcal{I}}$ forgetting the property (3.4).
Lemma 3.7. The functor $\pi$ is conservative.

Proof. Consider $\{f_i\}_{i \in I}$ a morphism in $\mathcal{G}(J)$ between $\{F_i, \varphi_s\}_{i,s}$ and $\{F_i', \varphi_s'\}_{i,s}$. Assume $\{f_i\}_{i \in I}$ is an isomorphism in $\prod_{i \in J} \mathcal{G}(i)$. For each $f_i$ denote by $g_i$ the corresponding inverse morphism. Consider the diagram

\[
\begin{array}{ccc}
\rho_s(F_{i_2}') & \rho_s(g_{i_2}) & \rho_s(F_{i_2}) \\
\varphi_s' \downarrow & \downarrow \varphi_s & \downarrow \varphi_s' \\
F_{i_1} & F_{i_1} & F_{i_1}'.
\end{array}
\]

the right square is commutative by hypothesis and the large rectangle is trivially commutative. Then the equality $f_{i_1} \circ \varphi_s \circ \rho_s(g_{i_2}) = f_{i_1} \circ g_{i_1} \circ \varphi'_s$ implies $\varphi_s \circ \rho_s(g_{i_2}) = g_{i_1} \circ \varphi'_s$ since each $f_{i_1}$ is an isomorphism.

Lemma 3.8. Let $\Psi_\nu: \mathcal{G}_1 \to \mathcal{G}_2$ ($\nu = 1, 2$) be two functors of prestacks on $I$, and $\theta: \Psi_1 \to \Psi_2$ a morphisms of functors.

(a) $\theta$ defines a morphism of functors $\theta_\infty: \Psi_{1 \infty} \to \Psi_{2 \infty}$,
(b) if $\theta$ is an isomorphism, then $\theta_\infty$ is an isomorphism as well.

Proof. Obvious.

Grothendieck prestacks

Recall that a $k$-abelian prestack $\mathcal{G}$ on a presite $I$ is a prestack such that $\mathcal{G}(i)$ is a $k$-abelian category for each $i \in I$ and the restriction functors $\rho_s$ are exact. For a site $I$, a $k$-abelian stack is a $k$-abelian prestack which is a stack. If there is no risk of confusion, we shall not mention the field $k$.

Definition 3.9. (see [GS06])

(i) A Grothendieck prestack $\mathcal{G}$ over $k$ on a presite $I$ is a $k$-abelian prestack such that the abelian category $\mathcal{G}(i)$ is a Grothendieck category for each object $i \in I$ and the restriction functor $\rho_s$ commutes with small inductive limits for each morphism $s: i \to j$. (Recall that $\rho_s$ is exact.)

(ii) For a site $I$, a Grothendieck stack is a Grothendieck prestack which is a stack.

The next result is a deep results of [SGA4] (see Exposé I, Theorem 9.22).

Theorem 3.10. Let $\mathcal{G}$ be a Grothendieck prestack on $I$. Then $\mathcal{G}(J)$ is a Grothendieck category.

If $\mathcal{C}$ is an abelian category, we denote as usual by $D^b(\mathcal{C})$ (resp. $D^+(\mathcal{C})$) its bounded (resp. bounded from below) derived category. Hence, for each $i \in J$, the functor $\rho_i: \mathcal{G}(J) \to \mathcal{G}(i)$ extends as a functor $\tilde{\rho}_i: D^+(\mathcal{G}(J)) \to D^+(\mathcal{G}(i))$ and one checks easily that these functors define a functor

\[
\tilde{\rho}: D^+(\mathcal{G}(J)) \to 2 \lim_{\underset{i,s}{\leftarrow}} D^+(\mathcal{G}(i)).
\]
The 2\(\lim\) of triangulated categories is no more triangulated in general; the functor \(\tilde{\rho}\) is neither full nor faithful. However it would be interesting to check if this functor is conservative.

### 4 Ran manifolds

We shall specialize the general theory of 2-limits presented in the previous section. Let \(I\) be a small category.

**Definition 4.1.** (a) A real (resp. complex) Ran-manifold indexed by \(I\) is a projective system \(X = \{X_i\}_{i \in I}\) of real (resp. complex) manifolds indexed by \(I\) such that for every morphism \(s \in \text{Hom}_I(j, i)\) the corresponding morphism \(\Delta_s : X_i \to X_j\) is a closed embedding.

(b) Let \(X = \{X_i\}_{i \in I}\) and \(Y = \{Y_i\}_{i \in I}\) be two Ran-manifolds. A morphism \(f : X \to Y\) is defined as a collection of morphisms \(\{f_i\}_{i \in I}\) such that all the diagrams below commute

\[
\begin{array}{ccc}
X_i & \xrightarrow{f_i} & Y_i \\
\downarrow^{\Delta_s} & & \downarrow^{\Delta_s} \\
X_j & \xrightarrow{f_j} & Y_j
\end{array}
\]

For each morphism \(s : j \to i\), with the corresponding embedding \(\Delta_s : X_i \hookrightarrow X_j\), we consider the following diagram

\[
\begin{array}{ccc}
T^*X_i & \xleftarrow{\Delta_{sd}} & X_i \times X_j \\
\downarrow^{\pi} & & \downarrow^{\pi} \\
X_i & \xrightarrow{\Delta_s} & X_j
\end{array}
\]

We denote by \(T^*_sX_j\) the kernel of \(\Delta_{sd}\), that is, \(\Delta_{sd}^{-1}T^*_X X_i\).

**Definition 4.2.** (a) We define \(T^*X\) as the collection of all the diagrams (4.2).

(b) A family \(\{\Lambda_i\}_{i \in I}\) of closed \(\mathbb{R}^+\)-conic subsets \(\Lambda_i \subset T^*X_i\) satisfying

\[
\Lambda_i \subset \Delta_{sd}(\Delta_{sd}^{-1}(\Lambda_j))
\]

for all \(s : j \to i\) is said to be a closed \(\mathbb{R}^+\)-conic subset \(\Lambda \subset T^*X\).

(c) A closed \(\mathbb{R}^+\)-conic subset \(\Lambda\) of \(T^*X\) is said to be transversal to the identity, if for each \(s : j \to i\), the map \(\Delta_s\) is non characteristic with respect to \(\Lambda_j\), that is, \(\Delta_{sd}^{-1}T^*_X X_i \cap \Delta_{sd}^{-1}\Lambda_j \subset X_i \times X_j T^*_X X_j\).
Consider a morphism $f : X \to Y$ of Ran-manifolds. It gives rise to the commutative diagrams associated to $s : j \to i$:

\[
\begin{array}{ccc}
T^*X_i & \leftarrow & X_i \times Y_j \xrightarrow{T^*f} T^*X_j \\
\downarrow & & \downarrow \Delta_{f,s,d} & & \downarrow f_{jd} \\
X_i \times Y_i & \leftarrow & X_i \times Y_j \xrightarrow{T^*f} X_j \times Y_j \\
\downarrow & & \downarrow \Delta_{f,s,\pi} & & \downarrow f_{jd} \\
T^*Y_i & \leftarrow & Y_i \times Y_j \xrightarrow{T^*f} T^*Y_j \\
\end{array}
\]

Notice that in Diagram (4.4) we have defined the maps

\[\Delta_{f,s,d} : X_i \times Y_j \to T^*X_i, \quad \Delta_{f,s,\pi} : X_i \times Y_j \to T^*Y_j,\]

where $f_s : X_i \to Y_j$ is given by the sequence of morphisms of manifolds

\[\Delta_s f_s : X_i \to Y_j \text{ is non-characteristic with respect to } \Lambda_j \text{ in a neighborhood of } f_i(X_i).\]

Definition 4.3. Let us fix a closed $\mathbb{R}^+$-conic subset $\Lambda \subset T^*Y$.

We say that $\Lambda$ is transversal to $f$ if for each $s : j \to i$, the map $f_s : X_i \to Y_j$ is non-characteristic with respect to $\Lambda_j$, that is $\Delta_{f,s,d}(T^*X_i) \cap \Delta_{f,s,\pi}(\Lambda_j) \subset X_i \times Y_j \times T^*Y_j$.

Note that to be transversal is stronger than to be non characteristic. When $f : X \to Y = X$ is the identity, we recover the notion of Definition 4.2 (c).

Notation 4.4. We will denote by $\Delta_s^{(Y)}$, $\Delta_s^{(X)}$, $\forall s \in \text{Hom}(J)$, the closed embeddings of $Y$ and $X$ respectively. However when it is clear from the context we will omit the superscript.

Lemma 4.5. Let $\Lambda$ be a closed $\mathbb{R}^+$-conic subset of $T^*Y$; moreover assume $\Lambda$ is transversal to $f$. Then for every $s : j \to i$

(a) $\Delta_s^{(Y)}$ is non-characteristic for $\Lambda_j$ in a neighborhood of $f_i(X_i)$;
(b) $\Delta_s^{(X)}$ is non-characteristic for $f_{jd}f_{jd}^{-1}(\Lambda_j)$.

Proof. (a) By hypothesis the morphism $f_s$ is non-characteristic for $\Lambda_j$. The result follows from Lemma 4.10 pag.65 in [Ka03] applied to the following commutative diagram:

\[
\begin{array}{ccc}
T^*X_i & \xrightarrow{f_{id}} & X_i \times Y_j \xrightarrow{T^*f} X_i \times Y_j \xrightarrow{T^*f} T^*X_j \\
\downarrow f_{id} & & \downarrow f_{jd} & & \downarrow f_{jd} \\
T^*Y_i & \xrightarrow{f_{id}} & Y_i \times Y_j \xrightarrow{T^*f} T^*Y_j \\
\end{array}
\]

\[\Delta_{s,d} : X_i \times Y_j \to T^*X_i, \quad \Delta_{s,\pi} : X_i \times Y_j \to T^*Y_j.\]
(b) Same argument applied to $f_s = f_j \circ \Delta_s^{(X)}$. \hfill \qed

5 Sheaves on Ran-manifolds

Let $X$ be a Ran-manifold and $\mathcal{I}$ a small category. Consider $\mathbb{D}^b(\mathcal{K}_X)$ the bounded derived category of $\text{Mod}(\mathcal{K}_X)$. Define the functor $\mathcal{G}: \mathcal{I}^{op} \to \text{Cat}$ as follows:

(a) to any $i \in \mathcal{I}$ set $\mathbb{D}^b(\mathcal{K}_X)$;
(b) for any morphism $s: i_1 \to i_2$ set $\rho_s : \mathbb{D}^b(\mathcal{K}_{X_{i_1}}) \to \mathbb{D}^b(\mathcal{K}_{X_{i_2}})$, with $\rho_s := \Delta_s^{-1}$;
(c) for $s: i_1 \to i_2$, $t: i_2 \to i_3$ the isomorphism of functors $c_{s,t}$ is $\Delta_t^{-1} \circ \Delta_s^{-1} \simeq \Delta_{t,s}^{-1}$.

Definition 5.1. We set

$$\mathbb{D}^b(\mathcal{K}_X) := 2 \lim_{i,s} (\Delta_s^{-1}).$$

Hence, an object $\mathbb{D}^b(\mathcal{K}_X)$ is the data of $\{(F_i, \varphi_s)\}_{i,s}$ ($i \in \mathcal{I}, s \in \text{Mor}(\mathcal{I})$) with $F_i \in \mathbb{D}^b(\mathcal{K}_X)$ and $\varphi_s : \Delta_s^{-1} F_j \simeq F_i$ satisfying the compatibility conditions in def 5.2.

Inverse images

Let $f: X \to Y$ be a morphism of Ran manifolds.

Definition 5.2. The inverse image functor $f^{-1}: \mathbb{D}^b(\mathcal{K}_Y) \to \mathbb{D}^b(\mathcal{K}_X)$ is defined as follows. Given $G \in \mathbb{D}^b(\mathcal{K}_Y), G = \{(G_i, \psi_s)\}_{i,s}$ with $i \in \mathcal{I}, s \in \text{Hom}_\mathcal{I}(j, i)$, we set $f^{-1} G = \{(F_i, \varphi_s)\}_{i,s}$ where $F_i = f_i^{-1} G_i$ and $\varphi_s : \Delta_s^{-1} F_j \simeq F_i$ is given by the isomorphisms

$$\Delta_s^{-1} F_j = \Delta_s^{-1} f_j^{-1} G_j \simeq f_i^{-1} \Delta_s^{-1} G_j \simeq f_i^{-1} G_i = F_i.$$

6 $\mathcal{D}$-modules on complex Ran-manifolds

The $\infty$-category of $\mathcal{D}$-modules on Ran spaces has been introduced and studied in [FG11]. We propose here a more elementary approach.

First we shall adapt Definition 5.1 in the $\mathcal{D}$-modules setting. Let $X$ be a Ran-manifold and $\mathcal{I}$ a small category. Define the functor $\mathcal{G}: \mathcal{I}^{op} \to \text{Cat}$ by

(a) to $i \in \mathcal{I}$, $\mathcal{G}(i) = \mathbb{D}^b(\mathcal{D}_X)$;
(b) for any morphism $s: i_1 \to i_2$, $\rho_s : \mathbb{D}^b_{\text{coh}}(\mathcal{D}_{X_{i_1}}) \to \mathbb{D}^b_{\text{coh}}(\mathcal{D}_{X_{i_2}})$ is the functor $\Delta_s^D$;
(c) for $s: i_1 \to i_2$, $t: i_2 \to i_3$, the isomorphism of functors $c_{s,t}$ is the obvious one $\Delta^D_t \circ \Delta^D_s \simeq \Delta^D_{t,s}$.

Definition 6.1. We set

$$\mathbb{D}^b(\mathcal{D}_X) = 2 \lim_{i,s} (\Delta_s^D).$$
Definition 6.2. Denote by $D_{\text{coh nc}}^b(D_X)$ the full triangulated subcategory of $D_{\text{coh}}^b(D_X)$ whose objects $\mathcal{M}_j$ are such that for every $s \in \text{Hom}_j(j,i)$ the corresponding morphism $\Delta_s: X_i \to X_j$ is non-characteristic with respect to $\mathcal{M}_j$.

Definition 6.3. We set

$$D_{\text{coh nc}}^b(D_X) = 2 \lim_{\leftarrow} \text{lim}_{i,s}(D_{\text{coh nc}}^b(D_X), \Delta_s^D).$$

Let $X$ and $Y$ be two complex Ran-manifolds and $f: X \to Y$. We shall mimic the previous definitions.

Definition 6.4. Denote with $D_{\text{coh f-nc}}^b(D_Y)$ the full triangulated subcategory of $D_{\text{coh}}^b(D_Y)$ whose objects $N_j$ are such that for every $s \in \text{Hom}_j(j,i)$ the corresponding morphism $f_s = f_j \circ \Delta_s(X): X_i \to Y_j$ is non-characteristic with respect to $\mathcal{N}_j$.

Definition 6.5. We set $D_{\text{coh f-nc}}^b(D_Y)$ as the 2-limit category

$$D_{\text{coh f-nc}}^b(D_Y) = 2 \lim_{\leftarrow} \text{lim}_{i,s}(D_{\text{coh f-nc}}^b(D_Y), \Delta_s^D).$$

Of course the two definitions coincide in the case $f = \text{id}$.

Example 6.6. Let $X$ be a complex Ran-manifold. Since $\Delta_s^D \theta_X \simeq \theta_X$, the family $\{\theta_X\}_i$ defines an object in $D_{\text{coh}}^b(D_X)$.

Lemma 6.7. Any object $\mathcal{N} \in D_{\text{coh f-nc}}^b(D_Y)$ belongs to the subcategory of $D_{\text{coh}}^b(D_Y)$ consisting of objects s.t. for every $s: j \to i$, $\Delta_s$ is non-characteristic for $\mathcal{N}_j$ in a neighborhood of $f_i(X_i)$.

Proof. It is a consequence of Lemma 4.5.

The inverse image functor $f^D$

Let $f: X \to Y$ be a morphism of Ran-manifolds. We will need the following result (which holds in view of Lemma 4.5).

Lemma 6.8. For all $s: j \to i$ the corresponding map $\Delta_s: X_i \to X_j$ is non characterisitc for the corresponding $f^D_s$.

Proposition 6.9. The map $f$ induces a functor $f^D: D_{\text{coh f-nc}}^b(D_Y) \to D_{\text{coh nc}}^b(D_X)$

Proof. We set $f^D := \{f_i^D\}_{i \in I}$. This inverse image functor is well-defined due to Lemma 6.8. In view of Lemma 3.5 it is enough to prove that $\{f_i^D\}_{i \in I}$ is a functor of prestack. In particular this means that each functor $f_i^D$ commutes with the restriction morphisms. By Definition 4.1 of morphism of Ran manifolds we have $\Delta_s \circ f_i = f_j \circ \Delta_s$ and applying the inverse image functor we obtain the desired property $f_i^D \circ \Delta_s^D \simeq \Delta_s^D \circ f_j^D$. \qed
The functor $R \mathcal{H}om_{\mathcal{D}Y}(-, \mathcal{O}_Y)$

**Lemma 6.10.** The functor $R \mathcal{H}om_{\mathcal{D}Y}(-, \mathcal{O}_Y) := \{R \mathcal{H}om_{\mathcal{D}Y_i}(-, \mathcal{O}_{Y_i})\}_{i \in I},$

$$R \mathcal{H}om_{\mathcal{D}Y}(-, \mathcal{O}_Y) : D^b_{\text{coh}}(\mathcal{D}Y) \to D^b_{\text{coh}}(k_X)$$

is well defined.

**Proof.** Consider an object $\{\mathcal{N}_i, \varphi_s\}_{i,s} \in D^b_{\text{coh}}(\mathcal{D}Y);$ by the CKK theorem (Theorem 2.4), there is the isomorphism $\Delta_s^{-1}R \mathcal{H}om_{\mathcal{D}Y_i}(\mathcal{N}_i, \mathcal{O}_{Y_i}) \simeq R \mathcal{H}om_{\mathcal{D}Y_j}(\Delta_s^D \mathcal{N}_i, \mathcal{O}_{Y_j}).$

By the composition of the last isomorphism with $\varphi_s : \Delta_s^D \mathcal{N}_i \cong \mathcal{N}_j$ we obtain the isomorphism $\Delta_s^{-1}R \mathcal{H}om_{\mathcal{D}Y_i}(\mathcal{N}_i, \mathcal{O}_{Y_i}) \cong R \mathcal{H}om_{\mathcal{D}Y_j}(\mathcal{N}_j, \mathcal{O}_{Y_j}).$ Lemma 3.5 completes the proof. \qed

The Cauchy-Kowaleski-Kashiwara theorem for Ran spaces

**Theorem 6.11.** There is an isomorphism of functors of prestacks $\theta_\infty : f^{-1} \circ R \mathcal{H}om_{\mathcal{D}Y}(-, \mathcal{O}_Y) \cong R \mathcal{H}om_{\mathcal{D}Y}(-, \mathcal{O}_Y) \circ f^D.$

In other words for any $\mathcal{N} \in D^b_{\text{coh}} f_{-\text{nc}}(\mathcal{D}Y)$ we have the isomorphism

$$f^{-1}R \mathcal{H}om_{\mathcal{D}Y}(\mathcal{N}, \mathcal{O}_Y) \cong R \mathcal{H}om_{\mathcal{D}X}(f^D \mathcal{N}, \mathcal{O}_X)$$

functorial in $\mathcal{N} \in D^b_{\text{coh}} f_{-\text{nc}}(\mathcal{D}Y)$.

The theorem is visualized by the following quasi commutative diagram:

$$\begin{array}{ccc}
D^b_{\text{coh}} f_{-\text{nc}}(\mathcal{D}Y) & \xrightarrow{f^D} & D^b_{\text{coh}}(\mathcal{D}X) \\
R \mathcal{H}om_{\mathcal{D}Y}(-, \mathcal{O}_Y) \downarrow & & \downarrow R \mathcal{H}om_{\mathcal{D}X}(-, \mathcal{O}_X) \\
D^b(k_Y) & \xrightarrow{f^{-1}} & D^b(k_X).
\end{array}$$

**Proof.** In view of Lemmas 3.6, 3.6, 3.8 it is enough to observe that each morphism $\theta_i : f_i^{-1} \circ R \mathcal{H}om_{\mathcal{D}Y_i}(-, \mathcal{O}_{Y_i}) \cong R \mathcal{H}om_{\mathcal{D}Y_i}(-, \mathcal{O}_{Y_i}) \circ f_i^D$ is an isomorphism due to Theorem 2.4. \qed

**Example 6.12.** Let $I$ be the category of finite non-empty sets and surjective maps. Let $X$ be a complex manifold. The Ran space $Ran \ X$ in [BD04] is defined as follows: to $I \in I$ one associates the product manifold $X^I$ and to a surjection $s : J \to I$ is associated the diagonal embedding

$$\delta_s : X^I \hookrightarrow X^J$$

which maps $\{x_i\}_{i \in I} \in X^I$ to $\{x_j\}_{j \in J} \in X^J$ with $x_j = x_i$ if $s(j) = i.$
Appendix

There are two other natural categories, \( S^+(J) \) and \( S^-(J) \).

**Definition 6.13.** (a) An object \( F \) of \( S^+(J) \) (resp. \( S^-(J) \)) is a family \( \{(F_i, \phi_s)\}_{i,s} \) (\( i \in J, s \in \text{Mor}(J) \)) where

(i) for any \( i \in J \), \( F_i \) is an object of \( S(i) \),

(ii) for any morphism \( s : i_1 \rightarrow i_2 \in J \), \( \phi_s : F_{i_1} \rightarrow \rho_s(F_{i_2}) \) (resp. \( \phi_s : \rho_s(F_{i_2}) \rightarrow F_{i_1} \)) is a morphism such that

- for all \( i \in J \), \( \phi_{id_i} = \text{id}_{F_i} \),
- for any sequence \( i_1 \rightarrow i_2 \rightarrow i_3 \) of morphisms in \( J \), the following diagram commutes:

\[
\begin{array}{ccc}
\rho_s \rho_t(F_{i_3}) & \rightarrow & \rho_s(F_{i_2}) \\
\rho_t & \downarrow & \phi_s \\
\rho_{toss}(F_{i_3}) & \rightarrow & F_{i_1}
\end{array}
\]  

(6.3)

(resp. the following diagram commutes:

\[
\begin{array}{ccc}
\rho_s \rho_t(F_{i_3}) & \rightarrow & \rho_s(F_{i_2}) \\
\rho_t & \downarrow & \phi_s \\
\rho_{toss}(F_{i_3}) & \rightarrow & F_{i_1}
\end{array}
\]  

(6.4)

(b) A morphism \( f : \{(F_i, \phi_s)\}_{i,s} \rightarrow \{(F'_i, \phi'_s)\}_{i,s} \) in \( S^+(J) \) (resp. \( S^-(J) \)) is a family of morphisms \( f_i : F_i \rightarrow F'_i \) such that for any \( s : i_1 \rightarrow i_2 \), the diagram below commutes:

\[
\begin{array}{ccc}
\rho_s(F_{i_2}) & \rightarrow & \rho_s(F'_{i_2}) \\
\phi_s & \downarrow & \phi'_s \\
F_{i_1} & \rightarrow & F'_{i_1}
\end{array}
\]  

(6.5)

(resp. the diagram below commutes:

\[
\begin{array}{ccc}
\rho_s(F_{i_2}) & \rightarrow & \rho_s(F'_{i_2}) \\
\phi_s & \downarrow & \phi'_s \\
F_{i_1} & \rightarrow & F'_{i_1}
\end{array}
\]  

(6.6)

(c) We consider \( S(J) \) as the full subcategory of \( S^+(J) \) or \( S^-(J) \) consisting of objects \( \{(F_i, \phi_s)\}_{i,s} \in J, s \in \text{Mor}(J) \) such that for all \( s \in \text{Mor}(J) \), the morphisms \( \phi_s \) are isomorphisms and we denote by \( i^+_J : S(J) \rightarrow S^+(J) \) and \( i^-_J : S(J) \rightarrow S^-(J) \) the natural faithful functors.
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