Title:
Slice Interpolation for Medical Image based on Spatial Geometry Polynomial Fitting

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Introduction:
Medical imaging technologies are essential to disease diagnosis and surgery planning, such as computa-
tional tomography (CT), which expresses the acquired tomographic medical image data as a set of slice
sequences. In order to decrease the radiation amount received by the patients, it is a common practice
to reduce the sampling rate and improve the scanning speed, which resulting in the loss of some valuable
temporal information and remarkably large slice interval. Therefore, most medical imaging volumes are
taken anisotropically with a high intra-slice resolution and a low inter-slice resolution. This phenomenon
leads to problems such as rough or even broken tissue boundaries in 3D reconstructed models, which
will undoubtedly aect the accuracy of lesion analysis result. As such, an accurate and reliable method
to upsample the low inter-slice resolution, which we refer to as the medical image slice interpolation
techniques, is much needed in research. In addition to generating accurate 3D reconstructions, medical
slice interpolation can also be widely used in medical image segmentation, multi-frame super-resolution
(MFSR) reconstruction, and other fields. By adding new virtual slices between every two consecutive
images, as shown in Fig. 1, the number and information of experimental data sets are increased, in
order to boost MFSR and medical image segmentation accuracy. Especially, increasing the amount of
training samples is indispensable for the popular research method such as neural network. Therefore, it
is necessary to improve the inter-slice interpolation techniques to increase the axial spatial resolution of
the data acquired using medical imaging modalities.

Image interpolation technology has been wide-spread used in various area of image processing, es-
pecially in the field of medical image processing. The methods for this task can be categorized into
two groups. (1) Grayscale-based interpolation methods [12, 13] directly use the grayscale information of
two consecutive images, to interpolate the inter-layer images through a set of basis functions. Nearest
neighbor interpolation [2], linear interpolation [1] and cubic B spline interpolation [11] are the common
types of such interpolation methods. This method is widely used in image interpolation because of its
computational simplicity and less computationally expensive. However, the interpolated images obtained
by these methods are usually too smooth and contain the artifacts. (2) The shape-based interpolation
methods [3, 7] generate contours of the image to be interpolated directly based on the contour shapes of
two consecutive images. Compared with the grayscale-based interpolation methods, it can effectively
eliminate the artifacts and improve the quality of interpolated images. However, the shape-based methods still have significant limitations. On the one hand, it requires high quality of tissue contours for the case where the input images are very similar. On the other hand, the extraction and representation of contours have complicated the method and reduced the efficiency of the algorithms. (3) Methods based on registration [8] achieve inter-layer interpolation by finding correspondences between consecutive images to match local anatomical structures, and obtain the deformation information of the pixel points to be interpolated according to the found deformation fields. This kind of methods can eliminate the boundary artifacts while taking into account various non-rigid deformations of human organ structures, which can produce the satisfactory visual effects. The currently popular non-rigid registration methods can be divided into two categories: one is based on spatial transformation [10], the other is based on physical models, such as the alignment methods based on optical flow estimation [5, 6]. However, registration-based interpolation methods have approximation calculations in the alignment process, which may generating anomalies of missing pixels. Furthermore, the computational complexity is higher compared to the shape and grayscale based interpolation methods. (4) In recent years, the rapid development of deep learning has enabled neural networks to bring new breakthroughs on medical image slice interpolation. Thus, deep learning based medical image slice interpolation techniques [9, 4] are promising. However, their inherent unpredictability and uninterprettability in handling sophisticated tasks remain to be explored. In addition, deep learning based methods rely on large amounts of data, which is difficult to meet in most practical application scenarios.

Fig. 1: Slice interpolation in fault medical images.

Main Idea:
As we all known, image interpolation technology consumes a certain amount of time inevitably while enhancing the image data. In view of the high reliability and real-time requirements of medical image processing system, it is essential to study a slice interpolation method with low computing complexity, in order to really improve the accuracy and efficiency of 3D reconstruction. Therefore, in this paper we propose a method that generates inter-layer images based on spatial geometry polynomial fitting, which can reconstruct an arbitrary number of intermediate medical slices from two consecutive slices via a priori information from three consecutive slices. An overview of our method is illustrated in Fig. 2. At first, the reverse sampling on original slice sequence is adopted to locally construct a ternary quadratic polynomial space geometry, which is used to generate the unit space geometry by weighted average. Then, all unit space geometries are pieced together to fit the space geometry. Finally, the spatial geometry is resampled to reconstruct the inter-layer slice. Our method is used to increase the axial spatial resolution in the CT sequence image at any scale. Meanwhile, the proposed interpolation method can reduce the time consuming and obtain the interpolation results quickly.

In summary, the main contributions of this work are summarized as follows:
1) We propose a fast and flexible medical image volume interpolation method based on polynomial

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fitting of spatial geometry. Unlike the traditional model in which two consecutive frames are used as input, it uses three adjacent frames as input to predict the motion trajectory information of the intermediate frames.

2) Using the relevant theory of computer graphics, the spatial geometry structure is constructed in 3D space and resampled to generate new pixels to interpolate the inter-slice images. This method can be applied to fast interpolation of images in arbitrary directions and scales, and can greatly reduce the time complexity while generating good visual results.

Next, we will briefly present our interpolation method, including the constraints, key solution procedure, etc. The proposed method relies on two constraints:

Condition 1: The pixels between the inter-slice image to be interpolated and the original image sequence are continuously varying, and they contain similar structures.

Condition 2: The similarity between the inter-slice image to be reconstructed and the two adjacent images of the original is inversely related to its distance from these two images.

Suppose three consecutive slices $S_{k-1}(0 < k < t)$, $S_k$ and $S_{k+1}(0 < k < t)$, the interpolated image $S_{k+d}(0 < d < 1)$ consists of $m \times n$ pixels $V_{i,j,k+d}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$, and the pixels to be generated can be regarded as the sampled values on the spatial geometry $F(x, y, z)$. For the sake of discussion, let each pixel $V_{i,j,k+d}$ be sampled from the unit volume, that is,

$$V_{i,j,k+d} = \int_{k+d-0.5}^{k+d+0.5} \int_{j-0.5}^{j+0.5} \int_{w-0.5}^{w+0.5} w(x, y, z) F(x, y, z) dx dy dz. \quad (1.1)$$

In three dimensions, the space geometry, $f_{i,j,k}(x, y, z), i = 2,3,\ldots,m - 1, j = 2,3,\ldots,n - 1, k = 2,3,\ldots,t - 1$, is subject to the following condition: if the $F(x, y, z)$ in Eqn. (1.1) is ternary quadratic polynomial, then $f_{i,j,k}(x, y, z)$ can accurately reconstruct $F(x, y, z)$, that is, $f_{i,j,k}(x, y, z) = F(x, y, z)$. Let $u = x - i, v = y - j, w = z - k$. Then ternary quadratic polynomial $f_{i,j,k}(x, y, z)$ can be written on the $[-1.5, 1.5] \times [-1.5, 1.5] \times [-1.5, 1.5]$ space as,

$$f_{i,j,k}(x, y, z) = a_1 u^2 + a_2 v^2 + a_3 w^2 + a_4 uv + a_5uw + a_6 vw + a_7u + a_8v + a_9w + a_{10}, \quad (1.2)$$

where, $a_1, a_2, \ldots, a_{10}$ are all unknown coefficients. Next, these unknown coefficients are solved for. To reduce the computational effort, first discuss how to determine $a_7, a_8, a_9$, and then calculate the remaining seven coefficients.

Assuming that $F(x, y, z)$ can be defined by Eqn. (1.2), $P_{i,j,k}$ is defined by sampling Eqn. (1.1), brought into the solution can be obtained:
\[ a_7 = e_1 = \frac{(P_{i+1,j,k} - P_{i-1,j,k})}{2} \]
\[ a_8 = e_2 = \frac{(P_{i,j+1,k} - P_{i,j-1,k})}{2} \]
\[ \vdots \]
\[ a_7 + a_8 = e_4 = \frac{(P_{i+1,j+1,k} - P_{i-1,j-1,k})}{2} \]
\[ \vdots \]
\[ a_7 + a_8 + a_9 = e_{13} = \frac{(P_{i+1,j+1,k+1} - P_{i-1,j-1,k-1})}{2}. \]

To reflect the contour properties of the image as much as possible, the unknown coefficients \( a_7, a_8, a_9 \) are determined by the least squares method with constraints. The objective function is

\[(G(a_7, a_8, a_9) = w_1(a_7 - e_1)^2 + w_2(a_8 - e_2)^2 + \ldots + w_{13}(a_7 + a_8 + a_9 - e_{13})^2). \quad (1.4)\]

The unknown coefficients can be found by minimizing the objective function:

\[
\frac{\partial G(a_7, a_8, a_9)}{\partial a_7} = 0, \quad \frac{\partial G(a_7, a_8, a_9)}{\partial a_8} = 0, \quad \frac{\partial G(a_7, a_8, a_9)}{\partial a_9} = 0. \quad (1.5)
\]

The same least-squares method with constraints is used to determine the remaining parameters.

Conclusions:
A fast inter-slice interpolation method for medical image based on spatial geometry polynomial fitting is proposed in this paper. This method enables generating an arbitrary number of intermediate medical slices from every two consecutive slices, and takes into consideration the time costs. In order to comprehensively evaluate the performance of the proposed method, we present relevant simulation experiments based on three different sets of CT datasets. The performance of the proposed method is evaluated from three aspects: visual effect, quantitative analysis and time complexity. The interpolation results are shown in Fig. 3, which show that the intermediate image obtained by this method does not lose the relevant information. The internal details are clear, and the effect of smooth transition can be achieved, which can effectively improve the resolution between slices. It is worth noting that the interpolation on CT sequence images does not generate new lesion information, but the resolution of CT images is improved to make subsequent three-dimensional reconstruction processing more convenient.

It is well known that the existing quantitative evaluation results are not a good substitute for human visual perception, and temporal coherence cannot be measured by numerical standards. We find that compared to the state-of-the-art methods, the visual quality differences and competitiveness of this methods are much greater than the numerical differences. In summary, it can be stated that the proposed method effectively improves the axial resolution of medical image sequence by quickly generating an arbitrary number of intermediate images. In the future works, we will further investigate the new target functions to construct the spatial geometry corresponding to the image sequence, for improving the interpolation accuracy.

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References:
Fig. 3: Interpolation results of tomographic medical images: (a) Input image 1; (b) Interpolation result 1; (c) Input image 2; (d) Interpolation result 2; (e) Input image 3; (f) Interpolation result 3; (g) Input image 4.

[1] Blu T, Thévenaz P, Unser M. Linear interpolation revitalized. IEEE Transactions on Image Processing, 2004, 13(5): 710-719. https://doi.org/10.1109/TIP.2004.826093

[2] Fadnavis S. Image interpolation techniques in digital image processing: an overview. International Journal of Engineering Research and Applications, 2014, 4(10), https://doi.org/10.1109/TIP.2004.826093

[3] Grevera G. J, Udupa J. K. Shape-based interpolation of multidimensional grey-level images. IEEE Transactions on Medical Imaging, 1996, 15(6): 881-892. https://doi.org/10.1109/42.544506

[4] Guo Y, Bi L, Ahn E, et al. A Spatiotemporal Volumetric Interpolation Network for 4D Dynamic Medical Image. Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2020: 4726-4735.

[5] Hu, Yinlin, Yunsong Li, and Rui Song. "Robust interpolation of correspondences for large displacement optical flow." Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2017.

[6] Lyasheva S, Rakhmankulov R, Shleymovich M. Frame interpolation in video stream using optical flow methods. Journal of Physics: Conference Series, 2020, 1488: 012024.

[7] Oliveira W. A. A, Guliato D, Oliveira D. C. B, et al. New Technique for Binary Morphological Shape-Based Interpolation. International Journal of Image and Graphics, 2019, 19(02): 1950007. https://doi.org/10.1142/S0219467819500074

[8] Penney G. P, Schnabel J. A, Rueckert D, et al. Registration-based interpolation. IEEE Transactions on Medical Imaging, 2004, 23(7): 922-926. https://doi.org/10.1109/TMI.2004.828352

[9] Peng C, Lin W A, Liao H, et al. SAINT: Spatially Aware Interpolation NeTwork for Medical Slice Synthesis. Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2020: 7750-7759.

[10] Rueckert D, Aljabar P, Heckemann R. A, et al. Diffeomorphic registration using B-splines. International Conference on Medical Image Computing and Computer-Assisted Intervention, 2006: 702-709. https://doi.org/10.1007/11866763_86

[11] Smith P. R. Bilinear interpolation of digital images. Ultramicroscopy, 1981, 6(2): 201-204. https://doi.org/10.1016/0304-3991(81)90061-9

[12] Yuan S, Abe M, Taguchi A, et al. High accuracy bicubic interpolation using image local features. IEEE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, 2007, 90(8): 1611-1615. https://doi.org/10.1093/ietfec/e90-a.8.1611

[13] Zulkifli N. A. B, Karim S. A. A, Sarfraz M, et al. Image interpolation using a rational bi-cubic ball. Mathematics, 2019, 7(11): 1045. https://doi.org/10.3390/math7110145