Massless surface wave

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An interface between two media is topologically stable two-dimensional object where 3D-symmetry breaks which allows for existence of many exotic excitations. A direct way to explore surface excitations is to investigate their interaction with the surface waves, such as very well known capillary-gravity waves and crystallization waves. Helium remains liquid down to absolute zero where bulk excitations are frozen out and do not mask the interaction of the waves with the surface states. Here we show the possibility of the new, massless wave which can propagate along the surface between two different superfluids phases of $^3$He. The displacement of the surface in this wave occurs due to the transition of helium atoms from one phase to another, so that there is no flow of particles as densities of phases are equal. We calculate the dispersion of the wave in which the inertia is provided by spin supercurrents, and the restoring force is magnetic field gradient. We calculate the dissipation of the wave and show the preferable conditions to observe it.

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Watching the waves on a surface of ocean is probably one of the oldest physical observation. Similar waves can propagate along the surface between any two immiscible fluids, and particularly between liquid and its vapour, see Fig. 1a. The inertia of these waves is due to the motion of the liquid while the restoring force is due to gravity (long waves) and surface tension (short waves). Unlike the most of known waves, dispersion relation of surface wave is strongly nonlinear,

$$\rho \omega^2 = \rho g q + \alpha q^3, \quad (1)$$

so that there is no phonon-like spectra even at $q \to 0$. Here $\rho$ is the density of the liquid, $\omega$ is the angular frequency, $g$ is the gravitational acceleration, $\alpha$ is the surface tension, and $q$ is the wavevector. Capillary-gravity waves have been observed on surface of $^4$He and $^3$He and have been utilized for high-precesion measurements on the surface tension. In this way critical exponents near critical liquid-gas point have been measured [1].

For helium liquid-solid interface there opens a possibility for quite exotic wave, a crystallization wave. At low enough temperatures, where liquid phase is superfluid and thus provides extremely fast mass and heat transport, the interface between solid and liquid becomes mobile enough to support wave of crystallization and melting. The dispersion relation is similar to that of waves on free liquid surface, and the only difference is that for the same amplitude of the wave, liquid phase carries smaller mass flux (see Fig. 1b) which is proportional to density difference $\Delta \rho$ between the solid and the liquid,

$$\frac{\Delta \rho^2}{\rho} \omega^2 = \Delta \rho g q + \alpha q^3. \quad (2)$$

Crystallization waves have been predicted by Andreev and Parshin in 1978 [2] and discovered by Keshishev et al. two years later in $^4$He below 0.5 K [3]. By measuring crystallization waves at surfaces of different orientations the singularity of the surface tension at the basal c-facet orientation has been observed [4].

In this Letter we show the possibility of the new kind of surface wave which does not have any associated mass flux. Imagine an interface between two immiscible liquids which both consist of the same atoms or molecules. Such interface will support, in addition to usual capillary-gravity wave described above, another type of wave which is associated with the transition of particles near the surface from one phase to another one. Basically, such wave is an analog of the crystallization wave as one of the phases locally “grows” into another phase. The hydrodynamical motion in both liquids is thus only due to the difference in mass densities of the phases.

Generally, it is possible that both phases have the same or nearly the same mass densities. Then such a wave will not have any hydrodynamical inertia and its frequency will be infinitely high at any finite wavevector $q$ unless some other kind of inertia is involved. The simplest example of the discussed system is liquid in contact with its vapour close to the critical liquid-vapour point where densities of the both phases equalize. Moreover, at the critical point surface tension also vanishes, and thus there is no restoring force unless some other force stabilizing the surface is introduced.

Although it seems impossible to satisfy all the conditions for the discussed massless wave, nevertheless, there is an example of such system, a surface between two superfluid phases of helium-3, stabilized with the magnetic field [5, 6]. These two phases, being referred to as “A” and “B” phases, have the same mass densities, and the described phase wave on the A-B interface does not require mass transport in the bulk phases, see Fig. 1b:

However, the A-phase has larger magnetic susceptibil-
We therefore have a wave on the surface between two liquids in which there is no mechanical motion at all: $^3$He particles transit from one phase to another staying at rest, and both kinetic and potential energy of the wave are provided by the spin degree of freedom.

As it was shown by Leggett \cite{8}, the spin dynamics in the longitudinal direction can be described in the terms of the angle $\theta$ of rotation of the order parameter vector $\vec{d}$ around the field direction. The corresponding equations for the A-phase ($z > 0$) and for the B-phase ($z < 0$) are \cite{7}:

\begin{align}
\ddot{\theta}_a - c_a^2 \frac{\partial^2 \theta_a}{\partial z^2} + \Omega_a^2 \theta_a &= 0, \\
\ddot{\theta}_b - c_b^2 \frac{\partial^2 \theta_b}{\partial z^2} + \Omega_b^2 \theta_b &= 0, \tag{3}
\end{align}

Here $\Omega_a$, $\Omega_b$, $c_a$, and $c_b$ are longitudinal NMR frequencies and spin waves velocities of A- and B- phases, correspondingly. We consider a slow wave, $\omega \ll \Omega_a, \Omega_b$, for which the solution of (3) decaying at $z = \pm \infty$ is

$$\theta_{a,b}(z,t) = \theta_{0a,0b}(t) \exp \left( \mp \frac{\Omega_a,b}{c_a,b} z \right). \tag{4}$$

Following Andreev \cite{7}, we write the energy density of the spin waves as

$$\varepsilon_{a,b} = \frac{\chi_{a,b}}{2} \left[ c_{a,b} \frac{\partial \theta}{\partial z} \right]^2 + \Omega_{a,b}^2 \theta_{a,b}^2, \tag{5}$$

where $\chi_a$ and $\chi_b$ are magnetic susceptibilities of the phases, and $\gamma = 2\mu/\hbar$ is the gyromagnetic ratio. After integration over $z$ we find the total kinetic energy of the wave,

$$E_{\text{kin}} = \frac{1}{2} \chi_a c_a \Omega_a \theta_{0a}^2 + \frac{1}{2} \chi_b c_b \Omega_b \theta_{0b}^2. \tag{6}$$

The fluxes of the $z$ component of the spin at the boundary are given by

$$j_{a,b} = -\frac{\chi_{a,b} c_{a,b}}{\gamma^2} \frac{\partial \theta}{\partial z} = \pm \frac{\chi_{a,b} c_{a,b} \Omega_{a,b}}{\gamma^2} \theta_{0a,0b}. \tag{7}$$

The fluxes are connected by the boundary condition

$$j_a - j_b = \frac{H}{\gamma} (\chi_a - \chi_b) \zeta, \tag{8}$$

where $\zeta(x,t) = \zeta_0(t) \exp(iqx)$ is the displacement of the boundary from its equilibrium $z = 0$ position. Using (8) and minimizing kinetic energy (6) we obtain amplitudes of the spin currents,

$$\theta_{0a} = \theta_{0b} = \frac{\gamma H (\chi_a - \chi_b)}{\chi_a c_a \Omega_a + \chi_b c_b \Omega_b} \zeta. \tag{9}$$
Substituting amplitudes (9) to (6) we find the kinetic energy of the wave, $E_{kin} = (1/4)M\dot{\zeta}^2$ with the effective “mass” of spin currents

$$M = \frac{(\chi_a - \chi_b)^2 H^2}{\chi_a c a \Omega_a + \chi_b c b \Omega_b} \quad (10)$$

The potential energy of the wave is due to the field gradient and due to the small surface tension $\alpha$, $E_{pot} = (1/4)[(\chi_a - \chi_b)H \nabla H + \alpha q^2] \zeta^2$, and the dispersion relation of the wave is

$$\omega^2 = \frac{(\chi_a - \chi_b)H \nabla H + \alpha q^2}{M} \quad (11)$$

Note that in the limit of long waves frequency does not depend on the wave vector and is determined only by magnetic parameters of superfluid phases and by the geometry of the magnetic field. At low temperatures and low pressure the A-B interface is stabilized in the field of $H_{AB} \approx 0.34$ T [4], and the surface tension is $\alpha \approx 5$ nJ/m$^2$ [10]. If we assume the field gradient of $\nabla H \sim 10$ T/m, then the characteristic wavelength at which the two terms in Eq. (11) equalize is

$$\lambda_c \sim 2\pi \sqrt{\frac{\alpha}{(\chi_a - \chi_b)H \nabla H}} \sim 1 \text{ mm}. \quad (12)$$

Longer waves are inherently magnetic, and their frequency $\omega_0 \sim \sqrt{\alpha \nabla H / H}$ does not depend on wavevector but can be tuned by the magnetic gradient field.

The wave can be exited by applying an oscillating magnetic field $h = h_0 \exp(i\omega t)$ parallel to the constant stabilizing field $H$. This kind of experiment has been done in Lancaster by Bartkowiak et al., who have measured the heat produced by the oscillating A-B interface in the ultra low temperature limit at zero pressure [11]. The oscillating field $h$ causes the equilibrium vertical position of the interface to oscillate with the amplitude $\delta \zeta = h/\nabla H$. The equation of motion of the interface can be written as

$$M\ddot{\zeta} + \Gamma \dot{\zeta} = -H \nabla H (\chi_a - \chi_b) (\zeta - \delta \zeta \exp(i\omega t)) = -M\omega_0^2 (\zeta - h/\nabla H \exp(i\omega t)) \quad (13)$$

where $\Gamma$ is the friction coefficient. The frequency dependence of the power dissipated in the wave per unit area is given by

$$P = \Gamma |\dot{\zeta}|^2 = \Gamma \omega_0^2 |\dot{\zeta}|^2 = \frac{\Gamma}{\omega_0^2} \frac{h^2}{(\omega_0^2 - \omega^2)^2 + (\omega \Gamma / M)^2 \nabla H^2}. \quad (14)$$

The quality factor of the wave $Q = \omega_0 M / \Gamma$ is proportional to the square root of the effective mass of the wave $M$ which is very small because of the smallness of nuclear susceptibilities, $\chi_a = 3.8 \cdot 10^{-8}$, $\chi_b = 1.2 \cdot 10^{-8}$ at zero bar [12]. The longitudinal NMR frequencies in the low temperature limit at high pressure have been measured to be $\Omega_a |_{hp} = 2\pi \cdot 100$ kHz, $\Omega_b |_{hp} = 2\pi \cdot 250$ kHz [13] [16].

As it was shown by Leggett, the longitudinal resonance frequency is scaled as $\Omega \propto \sqrt{N(0) \alpha T_c}$ where $N(0)$ is the density of states at the Fermi surface, and $\alpha$ is the ratio of the relative specific heat jump to its BCS value (1.43) [17]. Using $N(0) |_{bar} = 1.26 \cdot 10^{51}$ J$^{-1}$m$^{-3}$, $N(0) |_{bar} = 0.54 \cdot 10^{51}$ J$^{-1}$m$^{-3}$ [12], $\Omega |_{hp} = 1.4$, $\Omega |_{bar} = 1$, $\Omega |_{hp} = 2.49$ mK, and $\Omega |_{bar} = 0.93$ mK [18] [20] we find $\Omega |_{bar} \approx 2\pi \cdot 25$ kHz, and $\Omega |_{bar} \approx 2\pi \cdot 65$ kHz. For the estimation of the velocity $c$ of spin waves one can use the relation $c = l(\Omega / 2\pi)$ where $l \approx 10 \mu$m is the dipole healing length. Finally, we find the mass of the wave at high pressures and at zero bar, $M |_{bar} \approx 8 \cdot 10^{-9}$ kg/m$^2$ and $M |_{hp} \approx 2 \cdot 10^{-9}$ kg/m$^2$.

Another kind of the inertial mass $M^*$ of the interface originates from the time variation of the order parameter near the moving interface. This mass has been first considered by Yip and Leggett who have given a rough estimation $M^* \approx 10^{-11}$ kg/m$^2$ [21] [22]. Roughly, this mass is smaller than the magnetic mass by the factor $\xi_0/l$, where $\xi_0 = 18 ... 88$ nm is the coherence length.

There also exists usual material mass of the wave because of the density difference $\Delta \rho_{AB} \approx 10^{-6}$ kg/m$^3$ between phases [22]. The applied magnetic field increases the density difference because the A-phase has large susceptibility. The magnetostriction effect can be estimated as the additional effective pressure $\delta P = (1/2) \chi H^2$ which causes compression of the liquid by an amount $\delta \rho / \rho = 3 \delta P / \beta \rho$, where $\beta = 5.7 \cdot 10^{-8}$ 1/Pa is the compressibility [23]. The additional density difference due to magnetostriction is thus $\Delta \rho_{AB}(H) = (1/2) (\chi_A - \chi_B) H^2 \beta \rho \approx 10^{-8}$ kg/m$^3$ which is negligible compared to zero field difference $\Delta \rho_{AB}$. The contribution to the mass of the wave from density difference of the phases is given by $m = \Delta \rho_{AB}^2 / (\rho q) \approx 10^{-17}$ kg/m$^2$ which is nine orders of magnitude smaller than the spin current mass $M$.

With the estimated above mass $M \sim 10^{-8}$ kg/m$^2$ and with the field gradient of few Tesla per meter we can estimate the resonant frequency of the wave $\omega$ to be of the order of $2\pi \cdot 1$ kHz. This value is indeed much smaller than the longitudinal NMR frequency, and thus the condition of slow wave used for calculations of the mass is valid. In the opposite case of fast wave (very short wavelength and/or very strong field gradient), the magnetizations of the bulk phases will not have time to reach their equilibrium values, and both the solution (14) and the boundary condition (8) fail. For this case more complicated analysis is needed.

The friction coefficient $\Gamma$ is determined by scattering of quasiparticles in the A-phase and decreases at ultra low temperatures as $T^4$ [21] [24]. It has been measured at temperatures close to the A-B transition at high pressures in Los Alamos [22], $\Gamma |_{0.75 T_c, hp} = 0.07$ kg/(m$^2$ s). This value is in very good agreement with theoretical es-
timation of Kopnin, $\Gamma \approx 7\pi^4 N(0) T^4/(30 v_F^2 \Delta^2)$ ($v_F$ is Fermi velocity, $\Delta$ is the BSC energy gap) [24]. The low temperature value of the friction coefficient can be found from the Lancaster experiment [11]. According to the Eq. (13), in the case of strong damping $\Gamma/M \gg \omega_0$, the dissipation has a plateau at high frequencies,

$$P_{pl} = \frac{1}{2} \frac{\omega_0^2 M^2 h^2}{\nabla H^2} = h^2 \frac{H^2(\chi_a - \chi_b)^2}{2M},$$

(15)

which depends only on the amplitude $h$ of the oscillating field and on the friction coefficient $\Gamma$. Indeed, such plateau has been observed in Lancaster at frequencies larger than 10 Hz with the level $P_{pl} = 2 \cdot 10^{-7}$ W/m$^2$ independent on the field gradient [11]. The amplitude of the oscillating field used in Lancaster experiment was $h = 0.64$ mT which gives $\Gamma_{0.17Tc, \text{obar}} = 3 \cdot 10^{-5}$ kg/(m$^2$s). This is about three times larger than the theoretical estimate.

Due to the smallness of the mass of the wave, the characteristic attenuation rate of the wave is very fast even at ultra low temperatures, $\Gamma_{0.17Tc, \text{obar}}/M \approx 4 \cdot 10^3$ s$^{-1}$ which should be compared to $\omega_0 \approx 1.4 \cdot 10^3$ s$^{-1}$ for the strongest field gradient $\nabla H = 2$ T/m used in Lancaster experiments. To observe resonance of the A-B wave one should increase the field gradient by at least order of magnitude to shift the resonant frequency $\omega_0$ above the dissipation rate.

The excess by factor of three of the dissipation measured in Lancaster over the theoretical value might be the indication of the contribution of the surface states which should dominate at zero temperature limit. The A-B interface between two fermionic superfluids is probably the richest surface in nature and promises to support variety of surface states such as Majorana fermions and anyons (particles which are neither bosons nor fermions) [26]. The proposed magnetic massless surface wave could open access to these exotic surface states which may cause additional dissipation and contribute to the mass of the wave.

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