Wormholes in Bulk Viscous Cosmology

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Abstract

We investigate the effects of the accretion of phantom energy with non-zero bulk viscosity onto a Morris-Thorne wormhole. We have found that if the bulk viscosity is large then the mass of wormhole increases rapidly as compared to small or zero bulk viscosity.

Keywords: Accretion; Phantom Energy; Wormhole

1 Introduction

Exotic tunnel like topology in spacetime commonly called wormhole arises as a solution to the Einstein field equations. A typical two mouth wormhole joins two arbitrary points either of the same spacetime or two different spacetimes. Morris and Thorne [1] suggested the existence of exotic matter for the stability of a wormhole. The stress energy tensor $T_{\mu \nu}$ of the exotic matter must violate the Null energy condition ($T_{\mu \nu}u^\mu u^\nu \geq 0$) where $u^\mu$ is the future directed null vector. They concluded that an advanced civilization can produce a wormhole for interstellar travel by injecting sufficient amount of exotic matter in it. Recent interest in wormhole has arose due to the discovery of exotic phantom energy driving the accelerated expansion of the universe [2, 3]. It has been proposed that wormhole can be stabilized by the accretion of phantom energy and it can result in increasing the size of the wormhole to engulf the observable universe [4]. We here consider a similar scenario where we incorporate the effects of bulk viscous stress in our calculations. The bulk viscosity is quite relevant in physical cosmology as it can cause expansion of the universe due to its negative pressure [5]. The presence of viscous fluid can also explain the observed high entropy per baryon ratio in the universe [6]. The formulation of the paper is adopted from [4, 7].

The plan of this paper is as follows. In the second section, we present the relativistic model of accretion of viscous phantom energy onto a stationary wormhole. In third section, we consider two special cases of bulk viscosity in our model. Finally we conclude our paper.
2 Accretion onto wormhole

We consider a stationary and spherically symmetric wormhole specified by the line element:

$$ds^2 = -e^{\Phi(r)} + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

(1)

The above metric has the following properties [8, 9]: (1) The redshift function $\Phi(r)$ must be finite for all values of $r$ thus no horizon exists outside the spacetime, (2) the shape function $b(r)$ must satisfy the following conditions at the throat $r = r_o$: $b(r_o) = r_o$ and $b'(r_o) < 1$, (3) $b(r) < r$ for $r > r_o$ and (4) the spacetime is asymptotically flat i.e. $\frac{b(r)}{r} \to 0$ as $|r| \to \infty$.

Outside the wormhole, we assume the spacetime to be Friedmann-Robertson-Walker containing only one fluid, namely the phantom energy with non-vanishing bulk viscosity. The fluid is assumed to fall onto the WH horizon in the radial direction only which is in conformity with the spherical symmetry of the WH. Thus the fluid four velocity is $u^\mu = (u^t(r), u^r(r), 0, 0)$ which satisfies the normalization condition $u^\mu u_\mu = 1$. The corresponding stress energy tensor for the exotic phantom energy is

$$T^{\mu\nu} = (\rho + p_{\text{eff}})u^\mu u^\nu + p_{\text{eff}}g^{\mu\nu}.$$  

(2)

Here $p_{\text{eff}} = p + p_{\text{visc}}$, where $p$ is the isotropic pressure and $p_{\text{visc}} = -3\xi H$ is the bulk viscous pressure with $\xi$ is bulk viscosity. Using the energy momentum conservation $T^{\mu\nu}_{;\nu} = 0$, we get

$$ur^2 M^{-2}(\rho + p_{\text{eff}}) \left(1 - \frac{b(r)}{r}\right)^{-1} \sqrt{u^2 + \frac{b(r)}{r}} - 1 = C_1,$$

(3)

where $u^r = u = dr/ds$ is the radial component of the velocity four vector and $C_1$ is a constant of integration. The second constant of motion is obtained by contracting the four velocity with the energy momentum conservation equation $u^\mu T^{\mu\nu}_{;\nu} = 0$, which gives

$$u^\mu \rho_{;\mu} + (\rho + p_{\text{eff}})u^\mu_{;\mu} = 0.$$

(4)

Integration of Eq. (4) gives the second constant of motion

$$ur^2 M^{-2} \left(1 - \frac{b(r)}{r}\right)^{-1} \exp \left[\int_{\rho_{\infty}}^{\rho_h} \frac{d\rho'}{\rho' + \rho_{\text{eff}}(\rho')}\right] = -A,$$

(5)

where $A$ is a constant of integration. Also $\rho_h$ and $\rho_{\infty}$ is the energy density of the phantom fluid at the horizon of the WH, and at infinity respectively. From Eqs. (3) and (5) we have

$$(\rho + p_{\text{eff}}) \left(1 - \frac{b(r)}{r}\right)^{-1/2} \sqrt{u^2 + \frac{b(r)}{r}} - 1 \exp \left[\int_{\rho_{\infty}}^{\rho_h} \frac{d\rho'}{\rho' + \rho_{\text{eff}}(\rho')}\right] = C_2,$$

(6)

here $C_2 = -C_1/A = \tilde{A}(\rho_{\infty} + p(\rho_{\infty}))$. In order to calculate the rate of change of mass of WH $\dot{M}$, we integrate the flux of the bulk viscous phantom fluid over the entire WH
horizon i.e.
\[ \dot{M} = \oint T_r^r dS. \] (7)
Here \( T_r^r \) determines the energy momentum flux in the radial direction only and \( dS = \sqrt{-g} d\theta d\varphi \) is the infinitesimal surface element of the WH horizon. Using Eqs. (3) - (7), we get
\[ \frac{dM}{dt} = -4\pi DM^2 \sqrt{1 - \frac{b(r)}{r}(\rho + p_{\text{eff}})}, \] (8)
where \( D = A\tilde{A} \) is a positive constant. In the asymptotic regime \( r \to \infty \), we have
\[ \dot{M} = -4\pi DM^2(\rho + p_{\text{eff}}), \] (9)
which clearly demonstrates the increase in mass of the wormhole if \( \rho + p_{\text{eff}} < 0 \) and vanishing in the opposite case.

3 Accretion of viscous phantom energy

We now study the evolution of mass of WH in two special cases: (a) the constant viscosity; and (b) the power law viscosity.

3.1 Constant bulk viscosity

For constant viscosity, the evolution of \( a(t) \) is given by [10]
\[ a(t) = a_o \xi_o^{-2} [\xi_o + \gamma H_o B(t)]^{\frac{1}{2}}, \] (10)
where
\[ B(t) \equiv \exp \left( \frac{3t\xi_o}{2} \right) - 1, \] (11)
and \( \gamma < 0 \). The density evolution is given by
\[ \rho(t) = \frac{\rho_o \xi_o^2 \exp (3\xi_o t)}{[\xi_o + \gamma H_o B(t)]^2}. \] (12)
Further, for \( \gamma < 0 \) the big rip singularity occurs in a finite time at
\[ \tau = \frac{2}{3\xi_o} \ln \left( 1 - \frac{\xi_o}{H_o \gamma} \right). \] (13)
Finally, the WH mass evolution is determined by using Eqs. (10)-(12) in (9) to get
\[ M = M_o \left[ 1 - 4\pi DM_o \left\{ \frac{2\xi_o}{\gamma} \ln \left( \frac{\xi_o}{\xi_o + \gamma H_o B(t)} \right) + \frac{2H_o B(t)(\xi_o - \gamma H_o)}{\xi_o + \gamma H_o B(t)} + 3\xi_o \ln \left( \frac{a}{a_o} \right) \right\} \right]^{-1}. \] (14)
We can analyze this expression in some asymptotic limits. Assume \( \xi_o \) is very large i.e. \( \xi_o \gg \gamma H_o B(t) \) and \( \xi_o \gg \gamma H_o \) while \( a \sim a_o \). Hence Eq. (14) reduces to
\[ M \approx M_o [1 - 8\pi DM_o H_o B(t)]^{-1}. \] (15)
Using the approximation \( t \sim \xi_o^{-1} \), Eq. (15) becomes
\[
M \approx M_o (1 - 13\pi DM_o H_o)^{-1}. \quad (16)
\]
which finally becomes
\[
M \approx M_o [1 + 13\pi DM_o H_o + O(H_o^2)]. \quad (17)
\]
Notice that the current value of Hubble parameter is \( H_o \approx 2.3 \times 10^{-18} s^{-1} \). Due to smallness of \( H_o \), its contribution to higher order terms in Eq. (17) is negligible. One can see from Eq. (17) that mass of the wormhole increases under the assumption of large viscosity. Hence the wormhole is perfectly supported with the viscous phantom energy.

Similarly, Eq. (14) can be analyzed when \( \xi_o \ll 1 \) and \( \xi_o \ll |\gamma H_o B(t)| \), hence we get
\[
M \approx M_o \left[ 1 - 4\pi DM_o \left\{ \frac{2\xi_o}{\gamma} \ln \left( \frac{\xi_o}{\gamma H_o B(t)} \right) + 3\xi o \ln \left( \frac{a}{a_o} \right) \right\} \right]^{-1}. \quad (18)
\]
In the big rip scenario when \( a(t) \to \infty \), the two quantities in curly brackets in Eq. (18) will be of the same order of magnitude having opposite sign and hence cancel each other. Thus Eq. (18) reduces to
\[
M \approx M_o. \quad (19)
\]

### 3.2 Power law bulk viscosity

Now we consider the bulk viscosity to possess power law dependence upon density i.e. \( \xi = \alpha \rho^s \), where \( \alpha \) and \( s \) are constant parameters. Let us take \( \xi = \alpha \rho^{1/2} \) as a special case. Then the scale factor evolves as
\[
a(t) = a_o \left( 1 - \frac{t}{\tau} \right)^{\frac{2}{3(\gamma - \sqrt{3}\alpha)}}. \quad (20)
\]
The density of phantom fluid evolves as
\[
\rho(t) = \frac{4}{3\tau^2(\gamma - \sqrt{3}\alpha)^2} \left( 1 - \frac{t}{\tau} \right)^{-2}, \quad (21)
\]
or in terms of critical density \( \rho_{cr} \) as
\[
\rho(t) = \rho_{cr} \left( 1 - \frac{t}{\tau} \right)^{-2}. \quad (22)
\]
The corresponding big rip time \( \tau \) is given by
\[
\tau = \frac{2}{3(\sqrt{3}\alpha - \gamma)} H_o^{-1}. \quad (23)
\]
Finally, the mass evolution of WH is determined by using Eqs. (20)-(22) in (9), we get
\[
M = M_o \left[ 1 - 4\pi DM_o \left\{ \gamma \tau \rho_{cr} \left[ 1 - \left( 1 - \frac{t}{\tau} \right)^{-1} \right] + 3\alpha \sqrt{\rho_{cr} \left[ \left( 1 - \frac{t}{\tau} \right)^{-1} \ln a - \ln a_o \right]} \right\} \right]^{-1}. \quad (24)
\]
In the big rip scenario when \( t \to \tau \) and \( a \to \infty \), the two quantities in the square brackets in Eq. (24) will be added. This expression gives the growth of the wormhole if the dominant energy condition \( (\rho + p > 0) \) is violated.
4 Conclusion

Wormholes are tunnel like topological structures supported by the exotic matter like phantom energy. In our model, we have incorporated the viscous pressure along the usual isotropic pressure in the accretion model. Our model predicts the growth of wormholes by the accretion of bulk viscous phantom energy. For large bulk viscosity, the increase in the mass of wormhole is large as compared to small viscosity.

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