The prediction method of supporting force distribution for the refitting vehicle

Hongfeng Ma1*, Youwei Guo1 and Gaojie Liu2

1 School of Mechanical and Aerospace Engineering, Jilin University, Changchun, Jilin Province, 130022, China
2 Taizhou First Technician College, Taizhou, Zhejiang Province, 318000, China
*Corresponding author’s e-mail: 290297738@qq.com

Abstract. The supporting force distribution is very important for the vehicle to maintain safety and stability, especially for the heavy-duty refitting vehicle with eccentric load. A simple and generic model to compute the supporting force distribution is essential in the design stage of refitting vehicle, whereas there is little research involving in solving this problem by now. This article presents an efficient and simple method to predict the supporting force of tire (supporting point). Firstly, the model for the vehicle with three or four axles is simplified. Then, the mathematical matrix is established by force equilibrium equation, moment equilibrium equation and deformation compatibility equation. With the known parameters of load, position and dimension, the force of every supporting point can be calculated, and so the supporting force distribution is acquired. Finally, the method of predicting force distribution is utilized to computing the supporting force of a vehicle, and a corresponding experiment is also implemented. The results indicate that the general prediction method to calculate supporting force distribution presented in this paper is accurate and validated in the pre-design stage of the refitting vehicle.

1. Introduction
In recent years, as the development of the economy, the demand for refitting vehicle has been increasing sharply, such as rubbish vehicle, lorry crane, bridge inspection vehicle, and so on. These vehicles are all redesigned based on the vehicle chassis, majority has four or three axles, and multi tires (supporting points) are supporting on the ground. As the heavy load and complexity of working device mounted on the chassis, the supporting force of every tire is different, especially under eccentric load. The supporting force distribution can influence the layout of the whole vehicle, which is very important for the vehicle to ensure the safety and stability. The FEA of the whole vehicle can acquire the force distribution accurately. However, in the initial stage of design, the frequent improvement of design needs the FE model changes frequently, resulting in wasting a lot of time. Obviously, a fast and accurate method to calculate the force distribution is needed urgently.

There is little research involving in proposing simple and generic model to compute the force distribution for the vehicle by now. Some researchers have studied the axle load and have solved some engineering problems in the recent years [1-4]. Timm et al. proposed a mixed distribution model of two or more theoretical distributions to accurately characterize axle load spectra [5]. Battini et al. used a plane finite element analysis to study the axle load distribution, and on the basis of this, a triangular load distribution was proposed [6]. Law et al. presented a new moving load identification method to identify a system of general moving loads or interaction forces between the vehicle and the bridge.
deck [7]. The force or load distribution is also studied or used to solve engineering problem. Tong et al. researched the energy saving design of overhead travelling crane camber based on probability load distribution [8]. Ricci analyzed the internal loading distribution in statically loaded ball bearings subjected to an eccentric thrust load [9].

2. Mathematical model

For the vehicle with four axles, a balanced suspension is typically designed between the rear two axles to homogenize the supporting force, as shown in Figure 1. According to the force analysis, the vehicle with four axles can be simplified to the force state of the vehicle with three axles. Each axle has two supporting points of tire, so there are 8 supporting points for the vehicle with four axles and 6 supporting points for the vehicle with three axles. In the following study, the calculation formula of the force distribution will be deduced based on the force model of the vehicle with three axles.

![Figure 1. Force analysis diagram in the front view](image)

The main factor affecting the supporting force is the vertical stiffness of the supporting point, so in this study, the vertical stiffness of leaf spring and tire is considered, but the stiffness in other directions is ignored. As the stiffness of the chassis is much higher than leaf spring and tire, the structure of the chassis is assumed to be non-deformed. The vehicle is in the static and stable state. The supporting force distribution is as follows.

\[ G + F = \sum_{i=1}^{6} N_i \quad (i=1, 2, \ldots, 6) \]  

(1)

where \( G \) is the gravity of the whole vehicle, \( F \) is the load, and \( N_i \) represents the force of the \( i \)-th supporting point. \( N_1 \) and \( N_2 \) are the supporting forces of the front axle, respectively. \( N_3 \) and \( N_4 \) are the supporting forces of the mid axle, respectively. \( N_5 \) and \( N_6 \) are the supporting force of the rear axle, respectively.

\[ N_f = N_1 + N_2 \]  

(2)

\[ N_m = N_3 + N_4 \]  

(3)

\[ N_r = N_5 + N_6 \]  

(4)

where \( N_f \) is the supporting force of the front axle, \( N_m \) is the supporting for of mid axle, and \( N_r \) is the supporting force of the rear axle. The calculation model is simplified, the deformation form of the vehicle is assumed to be the configuration shown in Figure 2, and the stiffness of the leaf spring and the tire is assumed linear elasticity. The point \( o \) represents the center of gravity, \( M \) represents the moment caused by \( F \), \( \delta_f \) is the deformation of front axle, \( \delta_m \) is the deformation of mid axle, and \( \delta_r \) is the deformation of rear axle. \( L \) means the distance from the center of gravity to the front axle, \( L_m \) means the distance from the mid axle to the front axle, and \( L_r \) means the distance from the rear axle to the front axle.
The static force equilibrium equation can be defined as

\[ N_f + N_m + N_r = G + F \]  

(5)

The moment equilibrium equation can be defined as

\[ N_r L_r + N_m L_m = (G + F) L - M \]  

(6)

According to the geometric relation, the deformation compatibility equation can be defined as

\[ \frac{\delta_m - \delta_r}{\delta_m - \delta_r} = \frac{L_m - L_r}{L_r} \]  

(7)

The stiffness of the leaf spring and the tire is assumed linear elasticity, the elastic equation can be defined as

\[ N_f = k_f \delta_f \]  

(8)

\[ N_m = k_m \delta_m \]  

(9)

\[ N_r = k_r \delta_r \]  

(10)

where \( k_f, k_m \) and \( k_r \) represent the stiffness of the front axle, the stiffness of the mid axle and the stiffness of the rear axle, respectively.

The mathematical matrix can be acquired based on the Eq. (5-10), as shown in Eq. 11. For the structure designed in the initial stage, the parameters of \( G, F, L, M, k_f, k_m, k_r \) and \( k_r \) can be acquired from geometric model and the stiffness of the leaf spring and the tire. So \( \delta_f, \delta_m \) and \( \delta_r \) are obtained by calculating the mathematical matrix, and the supporting force of each axle is acquired by the Eq. (8-10).

\[
\begin{bmatrix}
k_f & k_m & k_r \\
k_f L_r & k_m L_m & 0 \\
k_f L_m & -L_f & L_r - L_m
\end{bmatrix}
\begin{bmatrix}
\delta_f \\
\delta_m \\
\delta_r
\end{bmatrix} =
\begin{bmatrix}
G + F \\
(G + F) L - M \\
0
\end{bmatrix}
\]  

(11)

The force state in left view of the vehicle is presented in Figure 3. The point \( o \) is the gravity center of the chassis of the vehicle. The point \( o' \) is the equivalent loading position. When the load is on the symmetric plane, the forces of the two supporting points in every axle are the same. While the eccentric load occurs, the additional moment is formed, the forces of the two supporting points in every axle are different, and resulting in every force of supporting points of the vehicle is different.
As shown in Figure 3, $G$ is the gravity of the chassis, $F$ is the load, and $a$ is the distance from the symmetric plane to the supporting point. Based on the force translation theorem, the additional moment can be equivalent from point $o$ to point $o'$, so

$$l' = M'(G+F)^{-1}$$

(12)

where $l'$ is the equivalent distance from the point $o$ to the point $o'$; $M'$ is the additional moment caused by the eccentric load. ($G+F$) is the total load applied to the vehicle. As the structure of the chassis is assumed to be non-deformed, so the distance $l'$ is also equivalent and applicable to the force state of each axle, as shown in Figure 4.

where $F'$ is the load applied on the axle; $N'$ and $N''$ are the forces of the two supporting points of each axle, so the static force equilibrium equation can be defined as

$$N = N' + N''$$

(13)

$$N = F'$$

(14)

The moment equilibrium equation can be defined as

$$N'' \cdot 2a = F'(a - l')$$

(15)

$N'$ and $N''$ can be calculated by Eq. (12), Eq. (13) and Eq. (15)

$$N' = N + \frac{M'}{2(G+F)a} N$$

(16)

$$N'' = N - \frac{M'}{2(G+F)a} N$$

(17)

where $N$ is the supporting force of each axle, so the force distribution of the vehicle is as follows...
\[ \begin{align*}
N_1 &= \frac{N_f}{2} + \frac{M'}{2(G + F)a} N_f \\
N_2 &= \frac{N_f}{2} - \frac{M'}{2(G + F)a} N_f \\
N_3 &= \frac{N_m}{2} + \frac{M'}{2(G + F)a} N_m \\
N_4 &= \frac{N_m}{2} - \frac{M'}{2(G + F)a} N_m \\
N_5 &= \frac{N_r}{2} + \frac{M'}{2(G + F)a} N_r \\
N_6 &= \frac{N_r}{2} - \frac{M'}{2(G + F)a} N_r \\
N_7 &= \frac{N_r}{4} + \frac{M'}{4(G + F)a} N_r \\
N_8 &= \frac{N_r}{4} - \frac{M'}{4(G + F)a} N_r
\end{align*} \]

For the vehicle with four axles, the force distribution of the vehicle is as follows

\[ \begin{align*}
N_1 &= \frac{N_f}{2} + \frac{M'}{2(G + F)a} N_f \\
N_2 &= \frac{N_f}{2} - \frac{M'}{2(G + F)a} N_f \\
N_3 &= \frac{N_m}{2} + \frac{M'}{2(G + F)a} N_m \\
N_4 &= \frac{N_m}{2} - \frac{M'}{2(G + F)a} N_m \\
N_5 &= \frac{N_r}{2} + \frac{M'}{2(G + F)a} N_r \\
N_6 &= \frac{N_r}{2} - \frac{M'}{2(G + F)a} N_r \\
N_7 &= \frac{N_r}{4} + \frac{M'}{4(G + F)a} N_r \\
N_8 &= \frac{N_r}{4} - \frac{M'}{4(G + F)a} N_r
\end{align*} \]

3. Experiment
In order to further verify the validity of the computational model proposed in this paper, the vehicle with 4 axles is presented in the laboratory, as shown in Figure 5a. The supporting force is tested by the weighing sensor under the tire, as shown in Figure 5b. The counterweight of 1200 Kg is applied on the chassis, which is used to simulate the load, as shown in Figure 5c.
The counterweight is placed in 3 different locations, which are defined as load case A, load case B and load case C, respectively. The supporting points are shown in Figure 6. The calculation process based on the Eq. 19 is the same with the process in section 3.2. Due to the confidentiality of technical parameters, the relevant parameters of the vehicle and the weight of the structural parts can’t be shown here. The force distribution predicted by the mathematical model proposed in this paper is listed in Table 3. The experimental results are close to the calculated results, where the maximum error is 6.9% and the average error is only 5.5%. The data and the comparison result indicate that the prediction method is effective and feasible for product design.

Table 1. The comparison between calculation and experiment for the vehicle

| Supporting points | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------------------|---|---|---|---|---|---|---|---|
| **Load case A**    |   |   |   |   |   |   |   |   |
| Calculation / kN   | 15.20 | 13.98 | 13.65 | 12.56 | 13.02 | 11.98 | 13.02 | 11.98 |
| Experiment / kN    | 15.95 | 14.40 | 14.20 | 13.10 | 13.50 | 12.15 | 12.25 | 11.20 |
| Error              | 4.7% | 2.9% | 3.9% | 4.1% | 3.6% | 1.4% | 6.3% | 6.9% |
| **Load case B**    |   |   |   |   |   |   |   |   |
| Calculation / kN   | 16.15 | 14.54 | 14.32 | 12.89 | 12.50 | 11.25 | 12.50 | 11.25 |
| Experiment / kN    | 16.60 | 15.05 | 14.60 | 13.40 | 12.95 | 11.90 | 11.75 | 10.65 |
| Error              | 2.7% | 3.4% | 1.9% | 3.8% | 3.5% | 5.5% | 6.4% | 5.6% |
| **Load case C**    |   |   |   |   |   |   |   |   |
| Calculation / kN   | 14.95 | 14.20 | 13.59 | 12.91 | 12.75 | 12.11 | 12.75 | 12.11 |
| Experiment / kN    | 15.50 | 14.95 | 13.15 | 13.15 | 13.10 | 12.50 | 12.05 | 11.35 |
| Error              | 3.5% | 5.0% | 3.3% | 1.8% | 2.7% | 3.1% | 5.8% | 6.7% |
4. Summary and discussion
The mathematical model proposed in this paper can solve the problem of force distribution for the refitting vehicle. However, there are some assumptions in this prediction method and they are also the limiting factors for the application of this model. Firstly, the chassis or the frame should have enough stiffness that the structural deformation is very small. Secondly, the nonlinear characteristic of stiffness for the leaf spring and tire is not big. To further improve the calculation model proposed in this paper, nonlinear factors need to be further considered and studied.

References
[1] Wu S Q, Law S S. (2011) Vehicle axle load identification on bridge deck with irregular road surface profile. Engineering Structures, 33: 591-601.
[2] Mutton P J, Epp C J, Dudek J. (1991) Rolling contact fatigue in railway wheels under high axle loads. Wear, 144: 139-152.
[3] Pinkaew T. (2006) Identification of vehicle axle loads from bridge responses using updated static component technique. Engineering Structures, 28: 1599-1608.
[4] Mutton P J, Alvarez E F. (2004) Failure modes in aluminothermy rail welds under high axle load conditions. Engineering Failure Analysis, 11: 151-166.
[5] Timm D H, Tisdale S M, Turochy R E. (2005) Axle Load Spectra Characterization by Mixed Distribution Modelling. Journal of Transportation Engineering, 131: 83-88.
[6] Battini J M, Mahir Älker-Kaustell, Syk A, et al. (2014) Effect of Axle Load Spreading and Support Stiffness on the Dynamic Response of Short Span Railway Bridges. Structural Engineering International, 24: 1-10.
[7] Law S S, Bu J Q, Zhu X Q, et al. (2004) Vehicle axle loads identification using finite element method. Engineering Structures, 26: 1143-1153.
[8] Tong Y, Tang Z, Mei S, et al. (2014) Research on Energy-Saving Design of Overhead Travelling Crane Camber Based on Probability Load Distribution. Mathematical Problems in Engineering, 2014:1-9.
[9] Ricci Márió César. (2009) Internal Loading Distribution in Statically Loaded Ball Bearings Subjected to an Eccentric Thrust Load. Mathematical Problems in Engineering, 2009:1-36.