Gauged $B - 3L_\tau$ and Radiative Neutrino Masses

Ernest Ma

Department of Physics, University of California
Riverside, California 92521, USA

Abstract

If the minimal standard model of quarks and leptons is extended to include just a righthanded partner to $\nu_\tau$, then the quantum number $B - 3L_\tau$ can be added as a gauge symmetry without the appearance of anomalies. A suitable extension of the scalar sector allows one neutrino to have a seesaw mass, and the other two to have radiative masses, with acceptable phenomenological values for neutrino oscillations. The $B - 3L_\tau$ gauge boson may be light and be observable through its decay into $\tau^+\tau^-$. 
The minimal standard model of quarks and leptons possesses four global symmetries, \( i.e. \) baryon number (\( B \)) and the three lepton numbers (\( L_e, L_\mu, L_\tau \)). They are all conserved at the classical level, but are all violated at the quantum level because of the well-known axial-vector-current triangle anomaly.\(^1\) Hence they cannot be individually gauged, \( i.e. \) promoted from a global to a local symmetry. The linear combination \( B - L_e - L_\mu - L_\tau \) is a divergenceless current, but it cannot be gauged because the corresponding \( U(1) \) is anomalous itself without the addition of three right-handed neutrino singlets. In fact, only one of the three lepton number differences (\( L_e - L_\mu, L_e - L_\tau, L_\mu - L_\tau \)) can be gauged, leading to some possible interesting phenomenological consequences.\(^2\)

If three right-handed neutrino singlets are added, then in general there will be mixing among the three lepton families. Hence it is more appropriate to consider a single lepton number \( L \) and it is well-known that \( B - L \) can now be gauged. In fact, the standard electroweak gauge group \( SU(2)_L \times U(1)_Y \) is embedded naturally in the left-right symmetric gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \).

In this paper, I will consider instead the addition of just one right-handed neutrino singlet and pair it with \( \nu_\tau \). It will be shown that the quantum number \( B - 3L_\tau \) can now be added as a gauge symmetry without the appearance of anomalies. The possible existence of the associated gauge boson, call it \( X \), is very interesting phenomenologically because the \( B - 3L_\tau \) gauge symmetry may be broken at an energy scale below that of electroweak symmetry breaking and yet remains undiscovered. Although \( X \) shares many of the properties of a gauge boson coupled only to \( B \), as already discussed in the literature,\(^3\) it will be much easier for experiments to confirm or refute because it also couples to \( \tau \) and \( \nu_\tau \).

This model also has important implications regarding neutrinos. The \( \tau \) neutrino may acquire a seesaw mass\(^4\) of order 1 eV or 0.1 eV. The other two neutrinos are massless at tree level, but if they mix with \( \nu_\tau \) through a scalar doublet carrying \( L_\tau \) number, then they will
acquire radiative masses. A suitable implementation of the scalar sector allows realistic
values of the masses and mixings of the three neutrinos which are consistent with the present
experimental evidence for neutrino oscillations.

Cancellation of Anomalies

Consider the extension of the standard gauge group of particle interactions to $SU(3)_C \times
SU(2)_L \times U(1)_Y \times U(1)_X$. Let the quarks and leptons transform as follows.

$$
\begin{pmatrix}
    u_l \\
    d_l
\end{pmatrix}_L \sim (3, 2, 1/6; x), \quad u_{iR} \sim (3, 1, 2/3; x), \quad d_{iR} \sim (3, 1, -1/3; x);
\quad (1)
$$

$$
\begin{pmatrix}
    \nu_e \\
    e
\end{pmatrix}_L, \quad \begin{pmatrix}
    \nu_\mu \\
    \mu
\end{pmatrix}_L \sim (1, 2, -1/2; 0), \quad e_R, \mu_R \sim (1, 1, -1; 0);
\quad (2)
$$

$$
\begin{pmatrix}
    \nu_\tau \\
    \tau
\end{pmatrix}_L \sim (1, 2, -1/2; x'), \quad \tau_R \sim (1, 1, -1; x'), \quad \nu_{\tau R} \sim (1, 1, 0; x').
\quad (3)
$$

In the above, only one right-handed neutrino singlet, i.e. $\nu_{\tau R}$, has been added to the minimal
standard model. Since the number of $SU(2)_L$ doublets remains even (it is in fact unchanged),
the global $SU(2)$ chiral gauge anomaly is absent. Since the quarks and leptons are chosen
to transform vectorially under the new $U(1)_X$, the mixed gravitational-gauge anomaly
is also absent. To ensure the absence of the axial-vector anomaly, the following conditions
are considered.

The $[SU(3)]^2 U(1)_X$ and $[U(1)_X]^3$ anomalies are automatically zero because of the vectorial
nature of $SU(3)$ and $U(1)_X$. It is also easy to show that the $[U(1)_X]^2 U(1)_Y$ anomaly is
zero independent of $x$ and $x'$. The remaining two conditions are:

$$
[SU(2)]^2 U(1)_X : \quad (3)(3)x + x' = 0; \quad (4)
$$

and

$$
[U(1)_Y]^2 U(1)_X : \quad (3)(3)[2(1/6)^2 - (2/3)^2 - (-1/3)^2]x + [2(-1/2)^2 - (-1)^2]x' = 0. \quad (5)
$$
Both have the solution: $x' = -9x$. Let $x = 1/3$ and $x' = -3$, then the addition of $U(1)_X$ is recognized as the gauging of $B - 3L_\tau$.

**Properties of $X$**

Since $X$ does not couple to $e$ or $\mu$ or their corresponding neutrinos, there is no direct phenomenological constraint from the best known high-energy physics experiments, such as $e^+e^-$ annihilation, deep-inelastic scattering of $e$ or $\mu$ or $\nu_\mu$ on nuclei, or the observation of $e^+e^-$ or $\mu^+\mu^-$ pairs in hadronic collisions. Although $X$ does contribute to purely hadronic interactions, its presence is effectively masked by the enormous background due to quantum chromodynamics (QCD). However, unlike the case of a gauge boson coupled only to baryon number, $X$ also couples to $L_\tau$. Assuming that $\nu_{\tau R}$ and the $t$ quark are too heavy to be decay products of $X$, the branching fraction of $X \rightarrow \tau^+\tau^-$ is roughly given by

$$B(X \rightarrow \tau^+\tau^-) = \frac{(2)(-3)^2}{(3)(-3)^2 + (5)(3)(2)(1/3)^2} = \frac{54}{91}.$$  \hspace{1cm} (6)

Thus $X$ may be produced in hadronic collisions and be observed through its $\tau^+\tau^-$ signature.

**Scalar Sector**

The minimal scalar content of this model consists of just the usual doublet

$$\left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) \sim (1, 2, 1/2; 0) \hspace{1cm} (7)$$

and a neutral singlet

$$\chi^0 \sim (1, 1, 0; 6) \hspace{1cm} (8)$$

which couples to $\nu_{\tau R}\nu_{\tau R}$. As the former acquires a nonzero vacuum expectation value, the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ breaks down to $U(1)_{em}$, whereas $\langle \chi^0 \rangle \neq 0$ breaks $U(1)_X$. The resulting theory allows $\nu_{\tau L}$ to obtain a seesaw mass and retains $B$ as
an additively conserved quantum number and $L_\tau$ as a multiplicatively conserved quantum number. The two other neutrinos, i.e. $\nu_e$ and $\nu_\mu$, are massless in this minimal scenario and cannot mix with $\nu_\tau$.

To allow $\nu_e$ and $\nu_\mu$ to become massive without introducing two additional right-handed neutrino singlets, the scalar sector is now extended to include a doublet

$$\begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} \sim (1, 2, 1/2; -3)$$

and a charged singlet

$$\chi^- \sim (1, 1, -1; -3).$$

The doublet allows mixing among all three charged leptons and pairs $\nu_\tau R$ with one linear combination of the three left-handed neutrinos. It appears at first sight that there are then two massless neutrinos left. However, since the three lepton numbers are no longer individually conserved, these two neutrinos necessarily pick up radiative masses. This generally happens in two loops through double $W$ exchange, but the masses so obtained are extremely small.

To obtain phenomenologically interesting radiative neutrino masses, the singlet $\chi^-$ has been added so that there can be the following new interactions:

$$f_l(\nu_\tau \tau_L - l_L \nu_\tau)\chi^+, \quad (\phi^+ \eta^0 - \phi^0 \eta^+)\chi^0,$$

where $l = e, \mu$. The mass-generating radiative mechanism of Ref. [6] is now operative. See Figure 1.

One should note that the above scalar sector contains a pseudo-Goldstone boson which comes about because there are 3 global U(1) symmetries in the Higgs potential and only 2 local U(1) symmetries which get broken. However, if an extra neutral scalar transforming as $(1, 1, 0; -3)$ is added, then the Higgs potential will have two more terms and the extra unwanted U(1) symmetry is eliminated.
Neutrino Masses and Mixing

From the Yukawa couplings $\bar{L}_L L_R \phi^0$, $\tau_L L_R \phi^0$, and $\tau_L L_R \eta^0$, the charged-lepton mass matrix linking $\bar{e}_L, \mu_L, \tau_L$ to $e_R, \mu_R, \tau_R$ can be chosen to be of the form

$$M_l = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ a_e & a_\mu & m_\tau \end{bmatrix}.$$  \hspace{1cm} (12)

The corresponding neutrino mass matrix spanning $\nu_e, \nu_\mu, \nu_\tau$, and $\nu_R$ is then given at tree level by

$$M_\nu = \begin{bmatrix} 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & 0 & m_3 \\ m_1 & m_2 & m_3 & M \end{bmatrix},$$  \hspace{1cm} (13)

where $m_{1,2}$ come from $\bar{\nu}_L \nu_R \phi^0$, $m_3$ comes from $\bar{\nu}_\tau \nu_R \phi^0$, and $M$ from $\nu_\tau R \nu_\tau R \chi^0$. Assuming $m_{1,2} \ll M$, the reduced $3 \times 3$ mass matrix spanning the 3 light neutrinos becomes

$$M_\nu = \begin{bmatrix} m_1^2/M & m_1 m_2/M & m_1 m_3/M \\ m_1 m_2/M & m_2^2/M & m_2 m_3/M \\ m_1 m_3/M & m_2 m_3/M & m_3^2/M \end{bmatrix}.$$  \hspace{1cm} (14)

At this stage, there is exactly one neutrino with mass $(m_1^2 + m_2^2 + m_3^2)/M$, but there are still 2 massless neutrinos. Now consider the one-loop diagram of Fig. 1. Of the 3 charged scalars, the linear combination $\phi^\pm \cos \theta + \eta^\pm \sin \theta$ (where $\tan \theta = (\langle \eta^0 \rangle / \langle \phi^0 \rangle)$) becomes the longitudinal component of $W^\pm$. The orthogonal combination is physical and mixes with $\chi^\pm$. Let $M_{1,2}$ be the resulting mass eigenvalues and $\alpha$ be the mixing angle. The radiative mass is then easily calculated to be

$$m_{\mu\mu} = f_\mu a_\mu m_\mu \sin \theta \frac{\sin \alpha \cos \alpha [F(M_1, m_\tau) - F(M_2, m_\tau)]}{16 \pi^2 \langle \phi^0 \rangle},$$  \hspace{1cm} (15)

where the function $F$ is given by

$$F(a, b) = \frac{a^2 \ln(a^2/b^2)}{a^2 - b^2}.$$  \hspace{1cm} (16)
To illustrate, let $f_\mu = 0.6$, $a_\mu = 10$ MeV, $\sin \theta = 0.01$, and $\sin \alpha \cos \alpha [F(M_1, m_\tau) - F(M_2, m_\tau)] = 0.01$, then $m_{\mu\mu} = 2.3 \times 10^{-3}$ eV, which is suitable for solar neutrino oscillations. The other entries of the radiative $(\nu_e, \nu_\mu)$ mass matrix are

$$m_{ee} = \left( \frac{f_e a_e m_e}{f_\mu a_\mu m_\mu} \right) m_{\mu\mu}$$

(17)

and

$$m_{e\mu} = m_{\mu e} = \left( \frac{f_e a_\mu m_\mu + f_\mu a_e m_e}{2 f_\mu a_\mu m_\mu} \right) m_{\mu\mu}.$$  

(18)

It is clear that the hierarchy $m_{ee} << m_{e\mu} = m_{\mu e} << m_{\mu\mu}$ may be naturally established and phenomenologically realistic $\nu_e - \nu_\mu$ mixing can be obtained for solar neutrino oscillations. If the tree-level neutrino mass of Eq. (14) is assumed to be of order 1 eV, then the mixing of $\nu_\mu$ with $\nu_\tau$ can induce neutrino oscillations between $\nu_e$ and $\nu_\mu$ governed by $\Delta m^2 \sim 1$ eV$^2$ as a possible explanation of the LSND observations. If it is assumed to be of order 0.1 eV, then atmospheric neutrino oscillations may be explained. Note that $\nu_\tau$-quark interactions may affect neutrino oscillations inside the sun and the earth, and be a potential explanation of the zenith-angle dependence of the atmospheric neutrino deficit.

Because of Eq. (12), there is also some small mixing of $e, \mu$ into the $\tau$ sector. However, as long as $a_e, a_\mu << m_\tau$, this has negligible effects on the current phenomenology. Details will be presented elsewhere.

**Z – X Mixing**

Let $\langle \phi^0 \rangle = v \cos \theta$, $\langle \eta^0 \rangle = v \sin \theta$, and $\langle \chi^0 \rangle = u$, then the mass-squared matrix spanning $Z$ and $X$ is given by

$$M^2_{ZX} = \begin{bmatrix}
(1/2)g_Z^2v^2 & 3g_Zg_Xv^2 \sin^2 \theta \\
3g_Zg_Xv^2 \sin^2 \theta & 18g_X^2(4u^2 + v^2 \sin^2 \theta)
\end{bmatrix}.$$  

(19)

Precision measurements of the observed $Z$ require this mixing to be very small. Assuming the value $\sin \theta = 0.01$ which was used earlier in estimating radiative neutrino masses, this
mixing is at most of order \(10^{-3}\), and that is certainly allowed by the present data. Note that for small \(\sin \theta\), \(M_X < M_Z\) is indeed possible. As mentioned earlier, this model can be tested by looking for the hadronic production of \(X\) and observing the decay of \(X\) into a \(\tau^+\tau^-\) pair.

It should be noted that there is also mixing of the two \(U(1)\) gauge factors (i.e. \(U(1)_Y\) and \(U(1)_X\)) in general through the kinetic energy terms. This mixing is assumed to be small here as well.

**Decay of \(Z\) to \(X\)**

If \(M_X < M_Z\), then the decay of \(Z\) to \(X\) and \(\bar{q}q\) for example would be possible. If \(X\) couples only to \(B\), this would result in four hadronic jets, and as shown in Ref. [3], it would be difficult to discover experimentally against the very large QCD background. However, \(X\) also couples to \(L_\tau\), hence there are the following interesting possibilities:

\[
\begin{align*}
(1) & \quad Z \rightarrow \bar{q}qX \rightarrow \bar{q}q\tau^+\tau^- , \\
(2) & \quad Z \rightarrow \bar{q}qX \rightarrow \bar{q}q\bar{\nu}_\tau\nu_\tau , \\
(3) & \quad Z \rightarrow \tau^+\tau^- X \rightarrow \tau^+\tau^-\bar{q}q , \\
(4) & \quad Z \rightarrow \tau^+\tau^- X \rightarrow \tau^+\tau^-\tau^+\tau^- , \\
(5) & \quad Z \rightarrow \tau^+\tau^- X \rightarrow \tau^+\tau^-\bar{\nu}_\tau\nu_\tau , \\
(6) & \quad Z \rightarrow \bar{\nu}_\tau\nu_\tau X \rightarrow \bar{\nu}_\tau\nu_\tau\bar{q}q , \\
(7) & \quad Z \rightarrow \bar{\nu}_\tau\nu_\tau X \rightarrow \bar{\nu}_\tau\nu_\tau\tau^+\tau^- .
\end{align*}
\]

Thus many distinct signatures are available, such as 2 jets + 2 charged leptons + missing energy from (1) and (3); 2 jets + missing energy from (2) and (6); 2 charged leptons + missing energy from (5) and (7); and 4 charged leptons + missing energy from (4). Some of the above final states are already being searched for in connection with processes such as \(Z \rightarrow \bar{q}qH\), where \(H\) is a neutral Higgs boson which may decay into \(\tau^+\tau^-\), and \(Z \rightarrow hA\),
where \( h \) is a scalar and \( A \) a pseudoscalar boson, and both decay into \( \tau^+\tau^- \). However, since the decay of Higgs bosons into neutrinos is essentially zero, processes (2) and (5) are unique to this model. In (2), one looks for 2 hadronic jets recoiling against nothing, but the missing mass has a sharp peak at \( M_X \). Dedicated searches of the above 7 processes will put limits on \( M_X \) as well as \( g_X \) or discover \( X \). More details of the phenomenology will be presented elsewhere.

**Summary and Conclusion**

In this paper, it is shown for the first time that the addition of just one right-handed neutrino singlet to the minimal standard model allows the quantum number \( B - 3L_\tau \) to be gauged. This is a novel possibility which has hitherto been unrecognized. Analogously, the associated \( X \) gauge boson may actually be light and yet remains undiscovered. However, unlike a gauge boson which couples only to baryon number, \( X \) decays also to \( \tau^+\tau^- \), and that will allow it to be observed in the future if it is produced. Meanwhile, if \( M_X < M_Z \), one should look for the decay of \( Z \) to \( X \) in the 7 processes given in Eqs. (20) to (26). Unique signatures among them are \( Z \to \bar{q}qX \) and \( Z \to \tau^+\tau^-X \), with \( X \to \bar{\nu}_\tau \nu_\tau \), *i.e.* nothing.

The spontaneous breaking of \( B - 3L_\tau \) through a scalar singlet \( \chi^0 \) retains \( B \) as an additively conserved quantum number and \( L \) as a multiplicatively conserved quantum number. The addition of a scalar doublet and a charged scalar singlet carrying \( L_\tau \) number allows the 3 light neutrinos to become massive, one by the seesaw mechanism and the other two radiatively with possible mixing among all. Acceptable phenomenological values for neutrino oscillations can be obtained.

**Note added:** After the completion of this paper, it became known to the author that the \( K^+ \to \pi^+\nu\bar{\nu} \) branching fraction has just recently been measured:\[19\]

\[
B = 4.2^{+9.7}_{-3.5} \times 10^{-10}. \quad (27)
\]
This may be a manifestation of the contribution of $X$ to the one-loop $W$-induced process $s \to d\nu_\tau\bar{\nu}_\tau$. Details will be presented elsewhere.

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References

[1] S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969); W. A. Bardeen, Phys. Rev. 184, 1848 (1969).

[2] X.G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. D43, R22 (1991); ibid. D44, 2118 (1991).

[3] C. D. Carone and H. Murayama, Phys. Rev. Lett. 74, 3122 (1995); Phys. Rev. D52, 484 (1995).

[4] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, Proc. of the Workshop, Stony Brook, New York, 1979, eds. P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, 1979), p. 315; T. Yanagida, in Proc. of the Workshop on the Unified Theory and the Baryon Number in the Universe, Tsukuba, Japan, 1979, eds. O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979).

[5] For a review, see for example K. S. Babu and E. Ma, Mod. Phys. Lett. A4, 1975 (1989).

[6] A. Zee, Phys. Lett. 93B, 389 (1980).

[7] E. Witten, Phys. Lett. B117, 324 (1982).
[8] R. Delbourgo and A. Salam, Phys. Lett. 40B, 381 (1972); T. Eguchi and P. G. O. Freund, Phys. Rev. Lett. 37, 1251 (1976); L. Alvarez-Gaume and E. Witten, Nucl. Phys. B234, 269 (1984).

[9] See for example C. Q. Geng and R. E. Marshak, Phys. Rev. D39, 693 (1989); X. G. He, G. C. Joshi, and R. R. Volkas, ibid. D41, 278 (1990).

[10] K. S. Babu and E. Ma, Phys. Rev. Lett. 61, 674 (1988); Phys. Lett. B228, 508 (1989). See also S. T. Petcov and S. T. Toshev, Phys. Lett. B143, 175 (1984).

[11] See for example E. Ma, Phys. Rev. Lett. 62, 1228 (1989), ibid. 63, 1042 (1989); Phys. Rev. D51, R3145 (1995).

[12] R. Davis, Prog. Part. Nucl. Phys. 32, 13 (1994); Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996); P. Anselmann et al., Phys. Lett. B327, 377 (1994); 342, 440 (1995); J. N. Abdurashitov et al., ibid. B328, 234 (1994).

[13] S. M. Bilenky, A. Bottino, C. Giunti, and C. W. Kim, Phys. Lett. B356, 273 (1995); K. S. Babu, J. C. Pati, and F. Wilczek, ibid., B359, 351 (1995); C. Y. Cardall and G. M. Fuller, Phys. Rev. D53, 4421 (1996).

[14] C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995); 77, 3082 (1996); nucl-ex/9706006.

[15] Y. Fukuda et al., Phys. Lett. B335, 237 (1994) and earlier references therein; R. Clark et al., Phys. Rev. Lett. 79, 345 (1997). See also R. Lipari et al., Phys. Rev. Lett. 74, 4384 (1995).

[16] E. Ma and P. Roy, Univ. of Calif., Riverside Report No. UCRHEP-T186 (September 1997), hep-ph/9706303.
[17] B. Holdom, Phys. Lett. 166B, 196 (1986).

[18] Actually, a Higgs boson may also decay invisibly, into Majorons for example. However, there should be no confusion because the ratio of the $X \rightarrow \bar{\nu}_\tau \nu_\tau$ to $X \rightarrow \tau^+ \tau^-$ widths is predicted here to be exactly $1/2$.

[19] S. Adler et al., hep-ex/9708031, Phys. Rev. Lett. (in press).

FIGURE CAPTION

Fig. 1. Diagram for generating the one-loop diagonal radiative mass for $\nu_\mu$. Similar diagrams are operative for the rest of the $(\nu_e, \nu_\mu)$ mass matrix.