Cosmic structures and gravitational waves in ghost-free scalar-tensor theories of gravity

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Abstract. We study cosmic structures in the quadratic Degenerate Higher Order Scalar Tensor (qDHOST) model, which has been proposed as the most general scalar-tensor theory (up to quadratic dependence on the covariant derivatives of the scalar field), which is not plagued by the presence of ghost instabilities. We then study a static, spherically symmetric object embedded in de Sitter space-time for the qDHOST model. This model exhibits breaking of the Vainshtein mechanism inside the cosmic structure and Schwarzschild-de Sitter space-time outside, where General Relativity (GR) can be recovered within the Vainshtein radius. We then look for the conditions on the parameters on the considered qDHOST scenario which ensure the validity of the Vainshtein screening mechanism inside the object and the fulfilment of the recent GW170817/GRB170817A constraint on the speed of propagation of gravitational waves. We find that these two constraints rule out the same set of parameters, corresponding to the Lagrangians that are quadratic in second-order derivatives of the scalar field, for the shift symmetric qDHOST.
Introduction

Modelling the recent cosmic acceleration phase of the universe expansion [1–3] has become one of the greatest challenges of modern theoretical cosmology. A very useful general framework in this direction is provided by scalar-tensor gravity theories, where GR is extended by introducing one or more scalar degrees of freedom. A necessary requirement for any such extension is that it should not introduce higher-order time derivatives in the equation of motion, known as Ostrogradsky theorem [4–6]. The Horndeski or “generalized Galileon” theory [7] was originally proposed as the most general Ostrogradsky ghost-free scalar-tensor theory. The authors of Ref. [8, 9] claimed that their extension of the Horndeski theory to the so-called Gleyzes-Langlois-Piazza-Vernizzi (GLPV) theory, leads to a new class of models without the Ostrogradsky ghost instability. A later proposal showed that higher-order derivatives in the Lagrangian may not necessarily introduce Ostrogradsky ghosts, provided certain degeneracy conditions are met [10]. Indeed, “degenerate” Lagrangians with non-invertible kinetic matrix will ensure that the number of degrees of freedom is preserved, thus making the theory free from the Ostrogradsky ghost; this new class was named “degenerate higher order scalar-tensor” (DHOST) theory [11–13]. A particular extension of Horndeski was proposed earlier in Ref. [14], which appeared as a result of a disformal transformation on the Einstein-Hilbert action, later found a specific subclass of HOST theory. DHOST theory is categorised into several classes [13]. Class I DHOST theories are the only one which are healthy from the gradient instability, i.e., the square of the speed of the tensor modes (gravitational-wave speed) and that of the scalar mode (sound speed) do not have opposite sign, \( c^2_s \propto -c_T^2 \) [15].

Gravity is well tested and established on small scales (e.g. laboratory, solar system, ...). Therefore, there must be a screening mechanism able to suppress the fifth-force mediated by the new scalar degree of freedom, without destroying the modifications on large scales, while recovering GR on a small scale. In general, the so-called Vainshtein screening is widely used for higher-order scalar-tensor theories [16]. In the Vainshtein screening, the non-linear self-interactions of the scalar field suppress the propagation of the fifth-force near the matter source [17, 18]. The Vainshtein mechanism in the Horndeski framework has been studied intensively...
in Refs. [19–22]. Although standard gravity is recovered outside a non-relativistic static and spherically symmetric cosmic structure in the small scale limit, the Vainshtein mechanism breaks down inside the matter source for the specific case of GLPV-beyond Horndeski models [23–30]. The consequences of these predictions for astrophysical and cosmological observations have been discussed in [31, 32]. A similar breakdown of the Vainshtein mechanism in some classes of the qDHOST model has been studied in some recent articles [33, 34].

The recent multi-messenger gravitational wave (GW) event, GW170817 [35] by the LIGO/VIRGO collaboration and the associate gamma-ray burst (GRB) event, GRB170817A [36] put a very tight constraint on the speed of gravitational wave propagation with respect to the speed of light, $|c_T^2/c^2 - 1| \leq 5 \times 10^{-16}$. This constraint is so narrow that we may generically consider $c_T^2 = c^2$. Many of the aforementioned Horndeski and beyond Horndeski models predict significant deviations in the speed of GW from the speed of light [37]. Constraining the scalar-tensor theories in the light of the anomalous speed of GW propagation have been first studied in Ref. [38], and followed by many recent articles [28, 32, 39–43].

Here we focus on the specific subclass of class I of qDHOST models for simplicity, where the degeneracy conditions are on the scalar sector alone, which does not suffer from the gradient and ghost instability [15]. We shall study the ‘Vainshtein mechanism’ in Ia* class of models in the presence of spherically symmetric cosmic structures. In section 2, we briefly discuss the shift-symmetric qDHOST theory of gravity. The covariant field equations of qHOST theories, therefore including all qDHOST classes, are derived in section 3. In the following section, section 4, we discuss the field equations in a de Sitter background. In section 5 we derive the perturbed equations around the de Sitter background for a static spherically symmetric matter source, taking the sub-Horizon, non-relativistic weak-field limit. We thus obtain the explicit form of the modified Newton’s constant and the two gravitational potentials. In section 6 we compute the speed of GW propagation for that model and show that the speed of GWs and the screening mechanism are governed by the same functional parameters for this class of models. Finally, section 7 presents our main conclusions. We use the metric signature $(-, +, +, +)$ and we set the speed of light and reduced Planck mass to unity. Greek indices run from 0 to 3.

2 The qDHOST theory

Let us consider the action for the shift-symmetric quadratic higher-order scalar tensor theory, which is expressed as follows [12],

$$S = \int d^4 x \sqrt{-g} \mathcal{L},$$

(2.1)

where the total Lagrangian, $\mathcal{L}$ is defined as the sum of the following four parts,

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_\varphi + \mathcal{L}_{\text{oth}} + \mathcal{L}_m,$$

(2.2)

with

$$\mathcal{L}_g \equiv f R,$$

(2.3)

$$\mathcal{L}_\varphi \equiv \sum_{I=1}^{5} \zeta_I(X)\mathcal{L}_I,$$

(2.4)

$$\mathcal{L}_{\text{oth}} \equiv (A X - BA),$$

(2.5)
where \( g \) is the determinant of the metric \( g_{\mu\nu} \). The Lagrangian \( \mathcal{L}_g \) is built in terms of the metric tensor and Ricci scalar, \( \mathcal{L}_\varphi \) is the second-order contraction of the derivatives of a scalar field, \( \varphi \); \( \mathcal{L}_{\text{oth}} \) are the terms which do not affect the degeneracy condition, and \( \mathcal{L}_m \) is the matter Lagrangian coupled only with the metric \( g_{\mu\nu} \). For simplicity, we have considered the free coefficients \( f, A, B \) to be constant, \( \Lambda \) is a (positive) cosmological constant \(^1\). The above action is shift-symmetric with respect to the scalar field, \( \varphi \to \varphi + \text{const} \). Therefore all \( \varphi \) contributions and dependences appear only as contractions of first and second-order covariant derivatives.

In our notation the all \( \varphi \) above action is shift-symmetric with respect to the scalar field, the free coefficients \( f, A, B \) the matter Lagrangian coupled only with the metric \( \varphi \) scalar field, \( \varphi \) L metric tensor and Ricci scalar, \( \varphi \) is the determinant of the metric \( g \).

The condition on the coefficients for such a ghost-free subclass is given as follows, the subclass characterised by the degeneracy of the scalar sector alone, and we call this class Ia*. The five arbitrary functions, \( \zeta_I = \zeta_I(X) \) depend only on the kinetic term, \( X \equiv \nabla_\mu \varphi \nabla^\mu \varphi \).

In our notation the \( \nabla_\mu \) symbol indicates the covariant derivative.

The five possible quadratic dependences on the covariant derivatives of the scalar field, \( \varphi \), are, \( \mathcal{L}_I \) \([12]\),

\[
\begin{align*}
\mathcal{L}_1 &\equiv \nabla^\mu \nabla_\nu \varphi \nabla_\mu \nabla_\nu \varphi, \\
\mathcal{L}_2 &\equiv (\Box \varphi)^2, \\
\mathcal{L}_3 &\equiv (\Box \varphi)\nabla^\mu \varphi \nabla_\nu \varphi \nabla_\mu \nabla_\nu \varphi, \\
\mathcal{L}_4 &\equiv \nabla^\mu \varphi \nabla_\nu \varphi \nabla_\mu \nabla^\rho \varphi \nabla_\nu \nabla_\rho \varphi, \\
\mathcal{L}_5 &\equiv (\nabla^\mu \varphi \nabla_\mu \nabla_\nu \varphi \nabla^\nu \varphi)^2.
\end{align*}
\] (2.6)

The vanishing of the determinant of the kinetic matrix leads to the three different degeneracy conditions associated with second-class constraints, which lead to the seven classes of the ghost-free degenerate theories within the qHOST \([12]\). We will study the degenerate subclass characterised by the degeneracy of the scalar sector alone, and we call this class Ia*. The condition on the coefficients for such a ghost-free subclass is given as follows,

\[
\zeta_2(X) = -\zeta_1(X) \quad \zeta_3(X) = -\zeta_4(X) = 2X^{-1} \zeta_1(X) \quad \zeta_5(X) = 0. \quad (2.7)
\]

\( \zeta_5(X) = 0 \) is in general non-zero in the general zero for the class I qDHOST. It turns out to be zero for this restricted class of model.

### 3 Covariant field equations of qHOST theories

In this section, we will obtain the explicit expression of the covariant field equations for the qHOST theory. The scalar field equation of the qHOST action, Eq. (2.1), is \( \delta S/\delta \varphi = 0 \). Considering that our action is shift-symmetric, the scalar field equation leads to the current-conservation law

\[
\nabla_\mu J^\mu = 0, \quad (3.1)
\]

where the \( J \) is the current for qHOST theories, whose expression is

\[
\begin{align*}
J^\mu &= \left\{ -2A \nabla^\mu \varphi \right\}_{(\text{oth})} \\
&+ \left\{ \zeta_1X \left( 4 \nabla^\nu \varphi \nabla^\mu \nabla_\nu \varphi \nabla_\mu \varphi - 2 \nabla^\mu \varphi \nabla^\nu \nabla^\rho \varphi \nabla_\nu \nabla_\rho \varphi \right) + \zeta_1 \left( 2 \nabla^\nu \nabla_\nu \nabla^\mu \varphi \right) \right\}_{(1)} \\
&+ \left\{ \zeta_2X \left( 4 \nabla^\nu \varphi \nabla^\mu \nabla_\nu \varphi \nabla^\rho \varphi \nabla_\rho \varphi - 2 \nabla^\mu \varphi \nabla^\nu \nabla^\rho \varphi \nabla_\nu \nabla_\rho \varphi \right) + \zeta_2 \left( 2 \nabla^\nu \nabla^\mu \nabla_\nu \varphi \right) \right\}_{(2)}
\end{align*}
\]

\(^1\)The dimensions of the introduced parameters are as follows: \([A] = [B] = [M]^0, [f] = [M]^2\), and \([\zeta_1] = [\zeta_2] = [M]^{-2}\) and \([\zeta_3] = [\zeta_4] = [M]^{-6}, [\zeta_5] = [M]^{-10}, [A] = [M]^4\) and \([H] = [M]\).
with

\[
\mathcal{H}_{\mu\nu} \equiv \left\{ 2fG_{\mu\nu} \right\}_{(g)} - \left\{ T_{\mu\nu} \right\}_{(m)} + \left\{ A(2\nabla_{\mu}\varphi\nabla_{\nu}\varphi - g_{\mu\nu}X) + B\Lambda g_{\mu\nu} \right\}_{(oth)}
\]

\[
+ \left\{ \zeta_1 \left( 2\nabla_{\mu}\varphi\nabla_{\nu}\nabla^\rho\nabla_\rho\varphi\nabla_\sigma\varphi + 4\nabla^\rho\nabla_\rho\varphi\nabla_{\mu}\nabla_\nu\varphi\nabla_\rho\nabla_\sigma\varphi - 8\nabla_\rho\varphi\nabla^\rho\nabla_\nu\varphi\nabla_\rho\nabla_\sigma\varphi \right) \right\}
\]

\[
+ \left\{ \zeta_2 \left( 2\nabla_\nu\varphi\nabla_\rho\varphi\nabla^\sigma\nabla_\sigma\varphi + 4g_{\mu\nu}\nabla^\rho\nabla_\rho\varphi\nabla_\alpha\nabla_\rho\varphi\nabla_\sigma\varphi - 8\nabla_\rho\varphi\nabla^\rho\nabla_\nu\varphi\nabla_\rho\nabla_\sigma\varphi \right) \right\}
\]

\[
+ \left\{ \zeta_3 \left( 2g_{\mu\nu}\nabla^\alpha\varphi\nabla^\beta\varphi\nabla_\alpha\nabla_\beta\varphi\nabla_\rho\nabla_\sigma\varphi - 4\nabla_\rho\varphi\nabla^\rho\varphi\nabla_\alpha\nabla_\beta\varphi\nabla_\alpha\nabla_\beta\varphi\nabla_\rho\nabla_\sigma\varphi \right) \right\}
\]

\[
+ \left\{ \zeta_4 \left( -2\nabla_\nu\varphi\nabla_\rho\varphi\nabla^\alpha\varphi\nabla_\alpha\nabla_\beta\varphi\nabla_\rho\nabla_\sigma\varphi + 2\nabla_\nu\varphi\nabla_\rho\varphi\nabla^\alpha\varphi\nabla_\alpha\nabla_\beta\varphi\nabla_\rho\nabla_\sigma\varphi \right) \right\}
\]

\[
+ \left\{ \zeta_5 \left( -2\nabla_\nu\varphi\nabla_\rho\varphi\nabla^\alpha\varphi\nabla_\alpha\nabla_\beta\varphi\nabla_\rho\nabla_\sigma\varphi \right) \right\}
\]

The subscripts \((g), (I), (oth),\) and \((m)\) in parentheses indicate the correspondence with the term of the Lagrangian, \(L_g, L_I, L_{others}\) and \(L_m.\)

The equation of motion of qHOST with respect to the metric field, \(g_{\mu\nu},\) is \(\delta S/\delta g^{\mu\nu} = 0,\)

\[
\mathcal{H}_{\mu\nu} = 0,
\]
where $G_{\mu\nu}$ is the standard Einstein tensor and the energy-momentum tensor $T_{\mu\nu}$ is defined as,

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(-g\mathcal{L}_m)}{\delta g_{\mu\nu}}. \quad (3.5)$$

These field equations, (3.1) and (3.3) in general contain higher-order derivatives of the fields [13]. The proper degeneracy condition on $f$ and $\zeta_f$ leads to the field equations corresponding to all the different classes of the qHOST theories. As explained in the previous section, we have restricted our analysis to the Ia* qHOST subclass, defined by Eq. (2.7).

### 3.1 Covariant field equations of qHOST theories in degenerate subclass Ia*

One can derive the field equations of the Ia* qHOST subclass from Eqs. (3.2) and (3.4), by applying the degeneracy conditions given in Eq. (2.7).

$$\nabla J^\mu = 0, \quad H_{\mu\nu} = 0. \quad (3.6)$$

To formally distinguish the qHOST EOMs of class Ia* from qHOST EOMs, we change the notation $\mathcal{J}^\mu \rightarrow J^\mu$ and $\mathcal{H}_{\mu\nu} \rightarrow H_{\mu\nu}$, where

$$J^\mu = -2A\nabla^\mu \varphi + \zeta_1X \left( 4\nabla^\nu \varphi \nabla^\alpha \varphi \nabla^\beta \varphi \nabla^\gamma \varphi \nabla^\delta \varphi - 2\nabla^\mu \varphi \nabla^\nu \varphi \nabla^\rho \varphi - 2\nabla^\mu \varphi \nabla^\nu \varphi \nabla^\rho \varphi - 4\nabla^\mu \varphi \nabla^\nu \varphi \nabla^\rho \varphi \nabla^\sigma \varphi \right)$$

and where

$$H_{\mu\nu} = 2fG_{\mu\nu} + A\left( 2\nabla_{,\mu} \varphi \nabla_{,\nu} \varphi - g_{\mu\nu} X \right) + B\nabla g_{\mu\nu} - T_{\mu\nu} + \zeta_1X \left( 2\nabla_{,\mu} \varphi \nabla_{,\nu} \varphi \nabla_{,\rho} \varphi \nabla_{,\sigma} \varphi \right)$$
4 qDHOST in de Sitter background

Let us assume that the background is a spatially flat de Sitter universe, with expansion rate \( H \). The expansion is dominated by a positive cosmological constant in vacuum. The metric is written as follows, in Friedmann Lemaître Robertson Walker (FLRW) coordinates \((\tau, \rho, \theta, \phi)\),

\[
ds^2_{(0)} = -d\tau^2 + e^{2H\tau}(d\rho^2 + \rho^2d\Omega^2),
\]

and the linear background scalar field profile \([30, 31, 34]\) is,

\[
\varphi(0)(\tau) = v_0\tau,
\]

where, \(v_0\) is a free constant coefficient and \(d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2\).

In reality, the background scalar profile can be non-linear if one accounts for the deviation from pure de Sitter space-time arising from the presence of matter. We leave the analysis of this point for future investigation.

At the background level, the scalar field equation, Eq. (3.6), reduces to

\[
\partial_\tau(e^{3H\tau}J^\tau) = 0.
\]

The solution, \(J^\tau \sim e^{-3H\tau}\) approaches to zero very fast over time. Therefore, \(J^\tau = 0\) can be considered the acceptable particular solution.

\[
2v_0(6H^2\xi_{1,X}v_0^2 - 6H^2\xi_{1}) - A) = 0.
\]

The subscript \(, X\) denotes differentiation with respect to \(X\) and the sub and superscript \((0)\) denotes the quantity in the de Sitter background profile. The background value of the kinetic term is

\[
X(0) = -v_0^2,
\]

and we introduced the short-hand notation of the scalar function and its derivative on the background, as

\[
\xi_{1}(0) \equiv \xi_{1}|_{X(0)} , \quad \xi_{1,X} \equiv \xi_{1,X}|_{X(0)}.
\]

The only non-vanishing and independent component\(^2\) is that of the background metric equation \(H_{\tau\tau} = 0\), which implies

\[
12H^2\xi_{1,X}v_0^4 - 18H^2\xi_{1}v_0^2 - 6H^2f + BA - Av_0^2 = 0.
\]

One can find the relation of the free parameters from the background solution, Eq. (4.4) and Eq. (4.7),

\[
A = 6H^2 \left(v_0^2\xi_{1,X} - \xi_{1}(0)\right),
\]

and

\[
f = \frac{\xi_{1}(0) - 2\xi_{1}(0)}{1 - B\sigma^2}.
\]

\(^2\)The other non-vanishing components of the background metric equation are \(H_{\rho\rho} = 0\) and \(H_{\phi\phi} = 0\). However these component expressions are related together and to the scalar equation \(J^\tau = 0\) by the relation \(H_{\tau\tau} + v_0J^\tau = H_{\rho\rho} = H_{\phi\phi} = H_{\phi\phi}\), so the only independent non-vanishing component of the background metric equation is \(H_{\tau\tau} = 0\).
Let us introduce the dimensionless quantity $\sigma^2$, defined as

$$\sigma^2 = \frac{\Lambda}{6H^2f},$$

(4.10)

which depends on the model parameter $f$, which we assume to be positive.

Vainshtein mechanism is studied in the spherical coordinate. Therefore, we now incorporate the previously obtained de Sitter background solution into the Schwarzschild-like coordinates $(t, r, \theta, \phi)$, performing the transformation

$$\tau(t, r) = t + \frac{1}{2H} \ln (1 - H^2 r^2) , \quad \rho(t, r) = \frac{r e^{-Ht}}{\sqrt{1 - H^2 r^2}},$$

(4.11)

with $1 - H^2 r^2 > 0$.

Using the above transformations in Eq. (4.11), the cosmological background metric in (4.1) and scalar profile (4.2) can be rewritten in terms of the new Schwarzschild-like coordinates as

$$ds^2(0) = -(1 - H^2 r^2) dt^2 + \frac{d\rho^2}{(1 - H^2 r^2)} + \rho^2 d\Omega_2^2,$$

(4.12)

$$\varphi^{(0)}(t, r) = v_0 t + \frac{v_0}{2H} \ln (1 - H^2 r^2).$$

(4.13)

5 Static spherically symmetric matter distribution in qDHOST class Ia*

Introducing static and spherically symmetric energy density, $\varepsilon$, and pressure, $P$, one gets the energy-momentum tensor

$$T^\mu_\nu \equiv \text{diag}\{-\varepsilon(r), P(r), P(r), P(r)\},$$

(5.1)

which modifies the background space-time and the scalar profiles into

$$ds^2 = -\varepsilon(r) dt^2 + \lambda(r) dr^2 + r^2 d\Omega_2^2.$$

(5.2)

Expressions (5.2) include two radial-dependent metric potentials $\nu(r), \lambda(r)$. We may neglect the time dependency in the perturbation of the scalar field in the sub-Horizon scale. The scalar field equation, Eq. (3.6) leads to $J^r = 0$ in the above coordinate system, which is expressed as

$$-2 e^{-3\lambda - \nu} \varphi' \left[ r^2 (v_0^2 e^\lambda - e^\nu \varphi^2) \right]^{-1} \left( Ar^2 v_0^2 e^{3\lambda + \nu} - Ar^2 e^{2\lambda + 2\nu} \varphi^2 + 2 Ar^4 1 \lambda X \right)$$

$$- r v_0^2 e^{2\lambda + \nu} - 3 r v_0^2 e^{2\lambda + \nu} \nu' - 4 r v_0^2 e^{2\lambda + \nu} \varphi^2 + 2 r v_0^4 1 \lambda X \right) + 2 r v_0^2 e^{2\nu} \varphi' + 2 r v_0^4 1 \lambda X \right) = 0.$$
\[ + BAr^2e^{3\lambda+2\nu}\varphi^2 + 2frv_2^2e^{3\lambda+\nu}\lambda' - 2fre^{2\lambda+2\nu}\chi'\varphi^2 - 2f_0v_0^2e^{3\lambda+\nu} + 2f_0v_0^2e^{4\lambda+\nu} \\
+ 2fe^{2\lambda+2\nu}\varphi'^2 - 2fe^{3\lambda+2\nu}\varphi'^2 + 4rv_0^4\xi_1e^{2\lambda}\lambda'\varphi'^2 - 8rv_0^4\xi_1e^{2\lambda}\lambda'\varphi'' - 10rv_0^2\xi_1e^{2\lambda+2\nu}\lambda'\varphi'^2 - 12rv_0^2\xi_1e^{2\lambda+2\nu}\lambda'\varphi'' - 8rv_0^2\xi_1e^{2\lambda+\nu}\lambda'\varphi'^{\prime\prime} + 16rv_0^2\xi_1e^{2\lambda+\nu}\lambda'\varphi'^{\prime\prime} + 6r\xi_1e^{\lambda+2\nu}\lambda'\varphi'^2 - 4r\xi_1e^{2\lambda}\varphi'' \to 6r\xi_1e^{2\lambda}\varphi'' - 8r\xi_1e^{2\lambda}\varphi'' - 8r\xi_1e^{2\lambda}\varphi'' - 4\xi_1e^{2\lambda}\varphi^2 \\
+ 6\xi_1^2e^{2\lambda+\nu}\varphi'^2 + 4v_0^2\xi_1e^{\lambda+2\nu}\lambda'\varphi'^4 - 2\xi_1e^{\lambda+2\nu}\lambda'\varphi'^4, \tag{5.4} \]

The matter source perturbs the metric potentials and the scalar field profiles about their cosmological values as

\[ \nu(r) \sim \nu^{(0)}(r) + \delta\nu(r), \quad \lambda(r) \sim \lambda^{(0)}(r) + \delta\lambda(r), \quad \varphi(r, t) \sim \varphi^{(0)}(r, t) + \delta\varphi(r), \quad \tag{5.6} \]

where \( \delta\nu \ll \nu^{(0)}, \delta\lambda \ll \lambda^{(0)}, \delta\varphi \ll \varphi^{(0)} \). By definition, as distance approaches to the de Sitter horizon, these perturbations vanish and the background de Sitter solutions become important and approaches to the potentials and scalar field profile given in Eqs. (4.12) and (4.13). The kinetic energy and the scalar functions are perturbed as

\[ X \sim X^{(0)} + \delta X, \quad \delta\zeta_1 \sim \zeta_1^{(0)} + \delta\zeta_1, \tag{5.7} \]

where we have defined \( \delta\zeta_1 = \zeta_1^{(0)} \delta X \) and

\[ \delta X = \left( \frac{1}{1 - H^2 r^2} \right) v_0^2 \delta\nu - \left( \frac{H^2 r^2}{1 - H^2 r^2} \right) v_0^2 \delta\lambda - 2(Hr)v_0\delta\varphi'. \tag{5.8} \]

The background expression for \( X^{(0)} \) is given in (4.5).

### 5.1 Sub-Horizon non-relativistic Weak-Field Limit

The mass distribution of the matter source is

\[ M(r) = 4\pi \int_0^r x^2 \varepsilon(x) \, dx. \tag{5.9} \]

Obviously, \( M(r \to \infty) = M \), where \( M \) is the total mass of the structure, and \( M(r = 0) = 0 \).

There are three branches of solutions for \( \delta\varphi' \). The only solution where \( \delta\varphi \) decays at large scales, i.e., \( \delta\varphi' < 0 \), leads to the physical solution. In order to write the physical solution, one has to choose the weak-field limit, \( \delta\nu \sim \delta\lambda \sim G_N M/r \ll 1 \), and also consider the perturbation in the sub-horizon limit, \( Hr \ll 1 \). We keep all the non-linear terms of \( \varphi \) which dominate
over the power of $\delta \nu$ and $\delta \lambda$. The $\delta \varphi'^4/\nu^4$ term is neglected with respect to $\delta \varphi'^2/\nu^2$ and $r \delta \varphi'' \delta \varphi''/\nu^2$. We refer to Ref. [30], for a more detailed discussion of these approximations.

In the sub-horizon weak-field limit, $\delta X \sim \nu^2 \delta \nu$ and the field equations, (5.3), (5.4), and (5.5), become

\[
2r v_0^2 \zeta_1^{(0)} \delta \lambda' + 2v_0^2 r \left(3 \zeta_1^{(0)} - 2 v_0^2 \zeta_{1.X} \right) \delta \nu' - 4 \left( \zeta_1^{(0)} - v_0^2 \zeta_{1.X} \right) \delta \varphi'^2 = 0, \tag{5.10}
\]

\[
2f \left( \delta \lambda + r \delta \lambda' \right) + 2 \left(3 \zeta_1^{(0)} - 2 v_0^2 \zeta_{1.X} \right) \left( \delta \varphi'^2 + 2r \delta \varphi' \delta \varphi'' \right) - \frac{M'}{4\pi} = 0, \tag{5.11}
\]

\[
2f \left( \delta \lambda - r \delta \nu' \right) + 2\zeta_1^{(0)} \left( \delta \varphi'^2 + 2r \delta \varphi' \delta \varphi'' \right) + r^2 P = 0. \tag{5.12}
\]

Integrating (5.11) one can obtain

\[
2fr \delta \lambda + 2r \left(3 \zeta_1^{(0)} - 2 v_0^2 \zeta_{1.X} v_0^2 \right) \delta \varphi'^2 - \frac{M}{4\pi} + k = 0. \tag{5.13}
\]

The integrating constant, $k = 0$ under such physical system (because first three terms will be zero at the center), leaves

\[
\delta \lambda = -\frac{3\zeta_1^{(0)} f}{f} \delta \varphi'^2 + \frac{2\zeta_{1.X} v_0^2}{f} \delta \varphi^2 + \frac{M}{8\pi rv_0^2}. \tag{5.14}
\]

In the non-relativistic limit, $\epsilon \gg P$, the combination of (5.14) and (5.12) leaves

\[
\delta \nu' = \frac{2\zeta_1^{(0)} f}{f} \delta \varphi' \delta \varphi'' - \frac{2\zeta_1^{(0)} f}{rf} \delta \varphi'^2 + \frac{2\zeta_{1.X} v_0^2}{rf} \delta \varphi^2 + \frac{M}{8\pi rv_0^2}. \tag{5.15}
\]

By substituting (5.14) and (5.15) into (5.10), we may find three branches of solutions for $\delta \varphi'$. The only physical branch, which decays at the large scale and leads to the asymptotic de Sitter expansion is

\[
\delta \varphi'^2 = -\frac{v_0^2 \left(\zeta_1^{(0)} r M' + 2\zeta_1^{(0)} M - 2\zeta_{1.X} v_0^2 M \right)}{16\pi rv_0^2 \left(-\zeta_1^{(0)} + \zeta_{1.X} v_0^2 \right) \left(3\zeta_1^{(0)} v_0^2 - 2\zeta_{1.X} v_0^4 + f \right)}. \tag{5.16}
\]

In such a weak-field limit, the Schwarzschild potentials $\delta \lambda$ and $\delta \nu$ are related to the Newtonian potential and curvature perturbations by

\[
\frac{d\Phi(r)}{dr} = \frac{\delta \nu'(r)}{2}, \quad \frac{d\Psi(r)}{dr} = \frac{\delta \lambda(r)}{2r}. \tag{5.17}
\]

Inserting the relation (5.16) in (5.14) and (5.15) we find the potentials of the qDHOST model in terms of $G_N$.

\[
\frac{d\Phi(r)}{dr} = \frac{G_N M(r)}{r^2} + \frac{\Upsilon_1 G_N M''(r)}{4}, \tag{5.18}
\]

\[
\frac{d\Psi(r)}{dr} = \frac{G_N M(r)}{r^2} - \frac{5\Upsilon_2 G_N M'(r)}{4r^2}, \tag{5.19}
\]

where $G_N$, $\Upsilon_{1,2}$ parameters defined as

\[
G_N = \frac{1}{16\pi \left(3\zeta_1^{(0)} v_0^2 - 2\zeta_{1.X} v_0^4 + f \right)}. \tag{5.20}
\]
\[ \Upsilon_1 = -\frac{2\zeta_1^{(0)} v_0^2}{f \left( -\zeta_1^{(0)} + \zeta_1^{(0)} v_0^2 \right)^2}, \quad (5.21) \]

\[ \Upsilon_2 = \frac{2\zeta_1^{(0)} v_0^2 \left( -3\zeta_1^{(0)} + 2\zeta_1^{(0)} v_0^2 \right)}{5f \left( -\zeta_1^{(0)} + \zeta_1^{(0)} v_0^2 \right)^2} \cdot (5.22) \]

After using the background equations (4.9) and restoring the proper dimensions to the parameters, we find

\[ G_N = \frac{1}{2\bar{f}} \left[ \frac{3\zeta_1^{(0)} - 2v_0^2\zeta_1^{(0)} - v_0^2\zeta_1^{(0)}}{2\zeta_1^{(0)} - v_0^2\zeta_1^{(0)}} \right] \left( B\sigma^2 - 1 \right) + 1 \cdot (5.23) \]

\[ \Upsilon_1 = \frac{2\zeta_1^{(0)} v_0^2 \left( \zeta_1^{(0)} v_0^2 - \zeta_1^{(0)} \right) \left( \zeta_1^{(0)} v_0^2 - 2\zeta_1^{(0)} \right)}{(\zeta_1^{(0)} v_0^2 - \zeta_1^{(0)}) \left( \zeta_1^{(0)} v_0^2 - 2\zeta_1^{(0)} \right)} \left( B\sigma^2 - 1 \right), \quad (5.24) \]

\[ \Upsilon_2 = -\frac{2\zeta_1^{(0)} (2\zeta_1^{(0)} v_0^2 - 3\zeta_1^{(0)})}{5 \left( \zeta_1^{(0)} v_0^2 - \zeta_1^{(0)} \right) \left( \zeta_1^{(0)} v_0^2 - 2\zeta_1^{(0)} \right)} \left( B\sigma^2 - 1 \right). \quad (5.25) \]

where we introduced the normalised \( \bar{f} = f m_{pl}^{-2} \) where the Planck mass is defined as \( m_{pl}^{-2} = 8\pi G \). \( G \) is the gravitational constant.

Non-vanishing \( \Upsilon_{1,2} \) parameters in expressions (5.18) and (5.19) determine the breaking of Vainshtein screening. Inside the matter source, the radial dependency of mass suggests non-zero value of \( M'(r) \) and \( M''(r) \). However, the mass of the source is constant outside the source, hence \( M'(r) = M''(r) = 0 \) and \( \Upsilon_1 = \Upsilon_2 = 0 \), which confirm that one can recover GR outside extended sources within the Vainshtein radius, therefore, \( \gamma_{PPN} = 1 \).

We have found that Vainshtein screening can be recovered inside the matter if \( \Upsilon_1 = \Upsilon_2 = 0 \). The common factor in the numerator of the expressions of the \( \Upsilon_1 \) and \( \Upsilon_1 \) in Eq. (5.21) and (5.21) is \( \zeta_1^{(0)} \phi'(0) \). Therefore, \( \Upsilon_{1,2} \) can set to zero in principle by setting either \( \zeta_1^{(0)} = 0 \) or \( \phi'(0) = 0 \). The second condition will kill the scalar degrees of freedom, thus making the model not interesting. It is possible to find such a non-zero function, \( \zeta_1(X) \), whose background value is zero, \( \zeta_1^{(0)} = 0 \), while the derivative of the function with respect to the kinetic energy in the background is non-zero, \( \phi'(0) \neq 0 \). We have found that Vainshtein screening can be recovered fully within qDHOST, if we impose the condition, \( \zeta_1^{(0)} = 0 \), on the free functions of the qDHOST theories, and the modified Newton constant becomes

\[ G_N = \frac{1}{2\bar{f}(2B\sigma^2 - 1)} \mathcal{G}, \quad \Upsilon_1 = \Upsilon_2 = 0. \quad (5.26) \]

As a side note, we present a special case, where both, \( \Upsilon_{1,2} \) are equal but non-zero, i.e., both the Newtonian potentials and the curvature perturbations break down the standard Newtonian behaviour in the same way, for the condition, \( \zeta_1^{(0)} + v_0^2\zeta_1^{(0)} = 0 \) and we call it symmetric breaking of Vainshtein screening. The expression of the \( G_N \) is

\[ G_N = \frac{3}{2\bar{f}(5B\sigma^2 - 2)} \mathcal{G}, \quad \Upsilon_1 = \Upsilon_2 = -\frac{1}{3}(1 - B\sigma^2). \quad (5.27) \]
For the specific choices of the parameters of qDHOST \( \zeta_1 \to f_1 X, A \to -k_2, B \to 1 \), will lead to the restricted GLPV-beyond Horndeski class of theories studied in [30]. One can obtain the same condition, \( \zeta_1^{(0)} + v_1^{(0)} Y_1.X = 0 \), for the restricted GLPV-beyond Horndeski model and the full results and analysis can be recovered from the above expressions into the Eq. (5.27) [30].

### 6 Propagation of Gravitational Waves

We are interested in the propagation of GW in a cosmological background. The tensor perturbation to the metric around the cosmological background is defined as

\[
g_{\mu\nu} = \begin{pmatrix} -a^2(\eta) & 0 \\ 0 & a^2(\eta)\delta_{ij} + 2a^2(\eta)h_{ij} \end{pmatrix},
\]

where \( h_{ij} \) is the tensor perturbation. We are using conformal time, \( \eta \). The tensor perturbations are traceless and transverse, i.e., \( h_{ii} = 0 = \partial^i h_{ij} \), where the latin indices \( i, j \) refer to spatial coordinates.

The equation of motion for tensor modes in the DHOST Class Ia* reads

\[
2f\nabla^2 h_{ij} + \ddot{h}_{ij} \left(-2f + \frac{1}{a^4} \zeta_3 \dot{\varphi}^4\right) + h_{ij} \left(-4f\dot{H} + \frac{1}{a^4} (4\zeta_3 \dot{\varphi}^3 \dot{\varphi} - 2\zeta_3 \varphi^4) \right) + \frac{1}{a^4} \left(2\zeta_3 \varphi^6 - 2\zeta_3 \dot{\varphi} \cdot \dot{\varphi}^3\right) + h_{ij} \left(-2A\dot{\varphi}^2 - 2BAa^2 + 4f\dot{H} + 8f\ddot{H} \right)
\]

\[
+ \frac{1}{a^4} \left(14\zeta_3 \varphi^2 \cdot \dot{\varphi} + 4\zeta_3 \ddot{\varphi}^4 - 16\zeta_3 \varphi^4 \right) + \frac{1}{a^4} \left(8\zeta_3 \varphi^4 \dot{\varphi}^2 - 8\zeta_3 \varphi^2 \dot{\varphi} \cdot \varphi^3\right) + \frac{1}{a^4} \left(8\zeta_3 \dot{\varphi} \cdot \varphi^5 - 8\zeta_3 \varphi^4 \dot{\varphi} \cdot \varphi^2\right) \right) = 0.
\]

Dot denotes the derivative with respect to the conformal time, and \( \dot{\varphi} = \dot{\varphi}/a \). As we seek solutions whose spatial dependence is given by \( \exp(i\vec{k} \cdot \vec{x}) \), \( \nabla^2 h_{ij} \) implies \(-k^2 h_{ij}\). The squared speed of GW propagation is

\[
c_T^2 = \frac{f}{f - \frac{1}{2a} \zeta_3^{(0)} \dot{\varphi}^2}, \quad (6.3)
\]

\[
= 1 - \frac{\zeta_3^{(0)} \dot{\varphi}^2}{f + \zeta_3^{(0)} \varphi^2}, \quad (6.4)
\]

\[
= 1 + \alpha_T, \quad (6.5)
\]

as \( \zeta_3 = \frac{2}{3} \zeta_1, \zeta_3^{(0)} = -\frac{2a^2}{\varphi^2(0)} \zeta_1^{(0)} \) and newly defined [44], \( \alpha_T = -\frac{\zeta_3^{(0)} \varphi^2}{f + \zeta_3^{(0)} \varphi^2} \).

Therefore \( c_T^2 = c_s^2 = 1 \) when \( \alpha_T \) vanishes.

Note that the parameters in Eq. (6.4) depend on the background only [37]. It is interesting to note that the numerator of \( \alpha_T \) is the same as in the expression of \( \dot{Y}_{1,2} \) given in Eq. (5.21) and (5.22). Therefore, similarly to the discussion on recovering screening in subsection 5.1, \( c_T^2 = c_s^2 = 1 \) can be obtained in principle by setting \( \zeta_3^{(0)} = 0 \), without setting \( \zeta_1(X) = 0 \). Considering, however, the tight constraint arising from GW170817/GRB170817A, we need to ensure that even a small deviation of \( \zeta_1^{(0)} \) in the background from zero will not lead to huge contributions to the \( c_T^2 \) or \( \alpha_T \).
Eliminating $H^2$ from Eq. (4.4) and (4.7), and then solving the equation for $v_0^2$, and use the expression of $\varphi'(0)^2$ given in (4.5)

$$\varphi'(0)^2 = -v_0^2$$

$$= \frac{Af + \Lambda B\zeta_{1}^{(0)}}{2A\zeta_{1}^{(0)} - \Lambda B\zeta_{1,X}^{(0)}},$$

and

$$6H^2 = \frac{-2A\zeta_{1}^{(0)} + \Lambda B\zeta_{1,X}^{(0)}}{2\zeta_{1}^{(0)^2} + f\zeta_{1,X}^{(0)}}.$$ 

Now, the extra quantity in the denominator of Eq. (6.4), which contributes to the $c_T$ different than 1, will be same in the cosmic time (as it is dimensionless) and becomes

$$\zeta_{1}^{(0)} \varphi'(0)^2$$

$$f$$

$$= \frac{A + \Lambda B\zeta_{1}^{(0)}}{2A - \Lambda B\zeta_{1,X}^{(0)}}.$$ 

We can see from Eq. (6.10), that a small deviation of $\zeta_{1}^{(0)}$ from zero will contribute by a large amount to $\alpha_T$, thus requiring a huge fine-tuning of the constants, $f$, $A$ and $B$. This conclusion would have been more complicated to reach, if $f$, $A$, and $B$ were functions instead of constants. Therefore, at least for the restricted shift-symmetric Ia* class qDHOST theory analysed here, one has to require $\zeta_{1} = 0$ in order to satisfy $c_T^2 = 1$.

In summary, the two expressions for the gravitational potentials in Eqs. (5.21) and (5.22) lead to exactly the same condition, in order to recover GR on small scales in Eq. (6.10). $\zeta_{1} = 0$ implies that $\mathcal{L}_{\varphi} = 0$ in our action (2.1), corresponding to the Lagrangians that are quadratic in second-order derivatives of the scalar field. Thus these two independent constraints leave only GR plus $\mathcal{L}_{oth}$, which is k-essence field for the restricted qDHOST framework. On the other hand, some sector of the class Ia of qDHOST theories are surviving after the GW event; the Vainshtein screening for these classes is discussed in [33, 34].

7 Discussion and Conclusions

In this article we studied the gravitational dynamics generated by a non-relativistic static and spherically symmetric cosmic structure within the framework of qDHOST theories of gravity. We restricted our study to the shift-symmetric Ia* class qDHOST gravitational model. We have explicitly deduced the covariant scalar-tensor field equations of qHOST and qDSOT theories. We then studied a static, spherically symmetric cosmic structure embedded in de Sitter space-time for our qDHOST model. Similarly to the GLPV-beyond Horndeski theory, for the Ia* class of DHOST theory, the Vainshtein mechanism breaks down inside the cosmic structure, while GR can be recovered outside the matter source, within the Vainshtein radius. The expressions for the two gravitational potentials suggest us how to constrain the theory in order to recover GR within the Vainshtein radius. Then we explicitly derived the equation
which governs the propagation of GWs in a cosmological background and found the conditions on the parameters of the theory, which allow to satisfy the \( c_T^2 = c^2 \) constraint. We then showed that the condition obtained from the screening of the fifth-force and the one on the GW propagation speed lead to the same constraint in Ia* class qDHOST theories.

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While completing our manuscript, two papers appeared on the arXiv [33, 34], which consider related classes of models. As long as the analyses overlap we reach the same conclusions.

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