Pion-Nucleus Microscopic Optical Potential at Intermediate Energies and In-Medium Effect on the Elementary $\pi N$ Scattering Amplitude

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Abstract. Analysis is performed of calculations of the elastic scattering differential cross sections of pions on the $^{28}$Si, $^{40}$Ca, $^{58}$Ni and $^{208}$Pb nuclei at energies from 130 to 290 MeV basing on the microscopic optical potential (OP) constructed as an optical limit of a Glauber theory. Such an OP is defined by the corresponding target nucleus density distribution function and by the elementary $\pi N$ amplitude of scattering. The three (say, “in-medium”) parameters of the $\pi N$ scattering amplitude: total cross section, the ratio of real to imaginary part of the forward $\pi N$ amplitude, and the slope parameter, were obtained by fitting them to the data on the respective pion-nucleus cross sections calculated by means of the corresponding relativistic wave equation with the above OP. A difference is discussed between the best-fit “in-medium” parameters and the “free” parameters of the $\pi N$ scattering amplitudes known from the experimental data on scattering of pions on free nucleons.

1 Introduction

There is a great number of papers on pion-nucleus scattering at different energies. In theoretical study two approaches are usually employed. First, the microscopic Kisslinger potential is based on $s$-, $p$-, and $d$-$\pi N$ scattering amplitudes having six and more parameters obtained from phase analysis of $\pi N$ data [1].

Second approach is the Glauber high-energy approximation (HEA) that uses analytic form of the $\pi N$ amplitude inherent in high energy scattering [2]. Such approach was employed, for example, in [3].

Here we utilize our HEA-based microscopic optical potential (OP) [4] for calculation of $\pi$-nucleus elastic scattering. This potential is constructed as an optical limit of a Glauber theory. Such an OP is defined by the known density distribution of a target nucleus and by the elementary $\pi N$ amplitude of scattering.

The $\pi N$ amplitude itself depends on three parameters: total cross section $\sigma$, the ratio $\alpha$ of real to imaginary part of the forward scattering $\pi N$ amplitude, and the slope parameter $\beta$. For $\pi$-scattering on “free” nucleons they are known, in
principle, from the phase analysis of the pion-nucleon scattering data. However, if one studies the pion-nucleus data then respective “in-medium” pion-nucleon amplitudes can be extracted. Thus the established best-fit “in-medium” $\pi N$ parameters can be compared with the corresponding parameters of the “free” $\pi N$ scattering amplitudes.

The aim of our study is an explanation of the experimental pion-nucleus data in the region of $(3\,3)$-resonance energies and estimation of the “in-medium” effect on the elementary pion-nucleon amplitude.

## 2 Basic equations

The differential cross sections are calculated by solving the relativistic wave equation [5] with the help of the standard DWUCK4 computer code [6]:

$$ (\Delta + k^2) \psi(\vec{r}) = 2\bar{\mu} U(r) \psi(\vec{r}), \quad U(r) = U^H(r) + U_C(r). \quad (1) $$

Here $k$ is relativistic momentum of pion in c.m. system:

$$ k = \frac{M_A k_{\text{lab}}}{\sqrt{(M_A + m_\pi)^2 + 2M_A T_{\text{lab}}}}, \quad k_{\text{lab}} = \sqrt{T_{\text{lab}} (T_{\text{lab}} + 2m_\pi)}, \quad (2) $$

$\bar{\mu} = E M_A / |E + M_A|$ - relativistic reduced mass, $E = \sqrt{k^2 + m_\pi^2}$ - total energy, $m_\pi$ and $M_A$ - the pion and nucleus masses, $T_{\text{lab}}$ and $k_{\text{lab}}$ - kinetic energy and momentum of pion in the laboratory system.

The HEA-based microscopic optical potential $U$ consists of nuclear and Coulomb parts. The nuclear part is as that derived in [4]:

$$ U^H = -\sigma (\alpha + i) \frac{\hbar c \beta_c}{(2\pi)^2} \int_0^\infty dq q^2 j_0(qr) \rho(q) f_\pi(q), \quad f_\pi(q) = e^{-\beta q^2/2}, \quad (3) $$

where $\hbar c = 197.327\text{MeV-fm}$, $\beta_c = k/E$, $j_0$ is the spherical Bessel function, $f_\pi(q)$ - formfactor of $\pi N$-scattering amplitude, $\rho(q)$ - formfactor of the nuclear density distribution in the form of symmetrized Fermi-function:

$$ \rho_{SF}(r) = \rho_0 \frac{\sinh(R/a)}{\cosh(R/a) + \cosh(r/a)}, \quad \rho_0 = \frac{A}{1.25 \pi R^3} \left[ 1 + \left( \frac{\pi a}{R} \right)^2 \right]^{-1} \quad (4) $$

Parameters of radius $R$ and diffuseness $a$ are known from electron-nucleus scattering data.

Three parameters of the $\pi N$ scattering amplitude are obtained by fitting to the experimental $\pi A$ differential cross sections:

- $\sigma$, total cross section $\pi N$,
- $\alpha$, ratio of real to imaginary part of the forward $\pi N$ amplitude,
- $\beta$, the slope parameter.
Figure 1. Comparison of the calculated pion-nucleus elastic scattering differential cross sections at $T_{\text{lab}} = 291$ MeV with experimental data from [12]. The best-fit “in-medium” parameters $\sigma$, $\alpha$, and $\beta$ are given in the Table 1.

We minimize the function

$$\chi^2 = f(\sigma, \alpha, \beta) = \sum_i^{k} \frac{(y_i - \hat{y}_i(\sigma, \alpha, \beta))^2}{(s_{as}^i)^2},$$

(5)

where $y_i = \log \left[ \frac{d\sigma}{d\Omega} \right]_i$ and $\hat{y}_i = \log \left[ \frac{d\sigma}{d\Omega}(\sigma, \alpha, \beta) \right]_i$ are, respectively, experimental and theoretical differential cross sections of elastic scattering. Asymmetric experimental errors $s_{as}^i$ are calculated at each $i$-th experimental point as follows

$$s_{as}^i = \begin{cases} y_i^{(+)} - y_i & \text{if } \hat{y}_i > y_i \\ y_i - y_i^{(-)} & \text{if } \hat{y}_i < y_i \end{cases}$$

(6)

where $y_i^{(+)}$ and $y_i^{(-)}$ are, respectively, maximal and minimal estimations of the experimental value $y_i$.

The fitting technique is based on the asynchronous differential evolution algorithm [7,8].
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Figure 2. The same as in Figure 1 but for $T_{\text{lab}} = 162$ MeV. The experimental data are from [13].

Figure 3. The same as in Figure 1 but for $T_{\text{lab}} = 130$ MeV. The experimental data are from [14] and [15].
3 Results and discussion

Figures 1–5 show the differential cross sections of elastic pion-nucleus scattering at energies between 291 and 130 MeV calculated using the OP presented in Section 2. Parameters (radius \( R \) and diffuseness \( a \)) of the target nuclear density distribution are following: \( R = 3.134 \) fm and \( a = 0.477 \) fm for \(^{28}\text{Si}\) \([9]\); \( R = 4.2 \) fm and \( a = 0.475 \) fm for \(^{58}\text{Ni}\) \([10]\); \( R = 3.593 \) fm and \( a = 0.493 \) fm for \(^{40}\text{Ca}\) \([9]\); \( R = 6.654 \) fm and \( a = 0.475 \) fm for \(^{208}\text{Pb}\) \([11]\). Calculated best-fit parameters \( \sigma, \alpha, \) and \( \beta \) of the in-medium pion scattering amplitude and respective \( \chi^2 \) values are given in the Table 1.

It is seen that our results are in a reasonable agreement with experimental data. Some dissimilarity is observed only at large angles (discussed below).

Figure 6 shows the averaged values \( X = (X_{\pi^+} + X_{\pi^-})/2 \) where \( X = \sigma, \alpha, \beta \) for the “free” \( \pi^\pm N \)-scattering parameters from \([16]\) in comparison with the obtained, also averaged, best-fit “in-medium” parameters in dependence on \( T_{\text{lab}} \).

Note, the bell-like forms of \( \sigma^{\text{free}} \) and \( \sigma^{\text{eff}}(T_{\text{lab}}) \) have maximum at the same \( T_{\text{lab}} \). The dark gray (blue) domain \( \sigma^{\text{eff}} \) is located below the light gray (yellow) \( \sigma^{\text{free}} \) region. This means that the “in-medium” \( \pi^\pm N \)-interaction be-
Figure 5. The same as in Figure 1 but for $T_{\text{lab}} = 226$ and 230 MeV. Experimental data are, respectively, from [14] and [15].

comes weaker as compared with that for “free” $\pi^\pm N$-scattering.

“In-medium” $\alpha_{\text{eff}}(T_{\text{lab}})$ behavior indicates that refraction increases at energy $T_{\text{lab}} > T_{\text{res}} \simeq 170$ MeV. It can be seen also that dark gray (blue) and light gray (yellow) regions become closer at $T_{\text{lab}} > 250$ MeV.

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Figure 6. (Color online) Light gray (yellow): “free” $\pi^\pm N$-scattering parameters from the paper of Locher et al [16]. Dark gray (blue): the best fit values $X^{\text{eff}} = (X^+ + X^-)/2$; $X = \sigma, \alpha, \beta$. 

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Figure 7. (a) Differential cross sections of $\pi^+ + ^{28}$Si scattering at 180 MeV. Solid curve: calculation with full set of experimental points from [14]. Best-fit parameters are given in the Table 1. Dashed curve: calculation with reduced set of experimental points; the removed points are indicated by dark solid circles, and thus the obtained parameters are $\sigma = 7.54$, $\alpha = 0.771$, $\beta = 0.538$, $\chi^2/k = 5.1$. (b) Differential cross sections of $\pi^- + ^{28}$Si elastic scattering at 130 MeV. Solid curve: calculation with the best-fit parameters from the Table 1. Dashed curve: calculation with parameters corresponding to the second minimum of the $\chi^2$ function where one gets $\sigma = 10.95$, $\alpha = -0.328$, $\beta = 0.598$, $\chi^2/k = 4.2$.

4 Summary

- We show that the HEA-based three-parametric microscopic OP provides a reasonable agreement with experimental data of pion-nucleus elastic scattering at intermediate energies between 130 and 290 MeV.
- Comparison of $\sigma^\text{free}$ and $\sigma^\text{eff}$ shows that, at (3 3)-resonance energies, the $\pi N$-interaction in nuclear matter is weaker than in the case of free $\pi N$ collisions.
- Behavior of parameter $\alpha$ indicates that the refraction increases at energies more than $T_{\text{res}}^\text{lab} \approx 170$ MeV.
- The decrease of the inmedium slope parameter $\beta^\text{eff}$ in comparison to the free one $\beta^\text{free}$ means that effective rms radius of the $\pi N$-system in nuclear medium becomes less than in the pion collisions with free nucleons.
- Total cross section data are desirable to be involved to resolve the ambi-
Table 1. The best-fit parameters $\sigma$, $\alpha$, $\beta$ and corresponding $\chi^2/k$ quantities where $k$ is the number of experimental points.

| reaction | $T^{(ab)}$ | $\chi^2/k$ | $\sigma$ | $\alpha$  | $\beta$  |
|----------|-----------|------------|----------|-----------|----------|
| $\pi^+ +^{28}\text{Si}$ | 130 | 2.1 | 7.08±0.16 | 0.87±0.05 | 0.81±0.05 |
| $\pi^+ +^{28}\text{Si}$ | 5.5 | 9.61±0.14 | 0.04±0.02 | 0.85±0.04 |
| $\pi^+ +^{40}\text{Ca}$ | 3.9 | 6.97±0.11 | 0.89±0.01 | 0.87±0.03 |
| $\pi^+ +^{40}\text{Ca}$ | 13.3 | 8.58±0.08 | 0.11±0.01 | 0.76±0.02 |
| $\pi^+ +^{68}\text{Si}$ | 162 | 3.5 | 11.02±0.1 | 0.04±0.02 | 0.39±0.02 |
| $\pi^+ +^{68}\text{Si}$ | 6.7 | 8.48±0.06 | 0.71±0.01 | 0.71±0.01 |
| $\pi^+ +^{58}\text{Ni}$ | 10.7 | 10.95±0.1 | -0.146±0.01 | 1.08±0.02 |
| $\pi^+ +^{58}\text{Ni}$ | 7.5 | 9.28±0.04 | 0.444±0.01 | 0.77±0.01 |
| $\pi^+ +^{208}\text{Pb}$ | 3.7 | 9.62±0.09 | 0.36±0.01 | 1.02±0.01 |
| $\pi^+ +^{208}\text{Pb}$ | 10.3 | 6.60±0.03 | 0.61±0.01 | 0.01±0.01 |
| $\pi^+ +^{28}\text{Si}$ | 180 | 10.5 | 10.03±0.06 | 0.33±0.01 | 0.266±0.01 |
| $\pi^+ +^{28}\text{Si}$ | 12.1 | 10.24±0.07 | 0.31±0.01 | 0.323±0.01 |
| $\pi^+ +^{40}\text{Ca}$ | 3.3 | 9.44±0.11 | 0.25±0.02 | 0.29±0.01 |
| $\pi^+ +^{40}\text{Ca}$ | 4.2 | 5.78±0.07 | 1.08±0.02 | 0.70±0.02 |
| $\pi^- +^{28}\text{Si}$ | 226 | 13.8 | 7.36±0.06 | 0.596±0.01 | 0.175±0.01 |
| $\pi^- +^{28}\text{Si}$ | 23.8 | 9.79±0.014 | -0.142±0.02 | 0.162±0.013 |
| $\pi^- +^{40}\text{Ca}$ | 230 | 7.56 | 5.25±0.06 | 0.796±0.01 | 0.253±0.01 |
| $\pi^- +^{40}\text{Ca}$ | 7.70 | 8.95±0.02 | -0.122±0.02 | 0.277±0.01 |
| $\pi^- +^{68}\text{Si}$ | 291 | 6.2 | 5.03±0.08 | -0.82±0.02 | 0.173±0.012 |
| $\pi^- +^{28}\text{Si}$ | 4.9 | 5.35±0.13 | -0.79±0.02 | 0.38±0.013 |
| $\pi^- +^{58}\text{Ni}$ | 3.8 | 4.78±0.08 | -0.85±0.02 | 0.28±0.02 |
| $\pi^- +^{58}\text{Ni}$ | 2.6 | 5.63±0.15 | -0.66±0.02 | 0.36±0.01 |
| $\pi^- +^{208}\text{Pb}$ | 4.1 | 4.50±0.07 | -1.06±0.02 | 0.666±0.02 |
| $\pi^- +^{208}\text{Pb}$ | 3.0 | 5.56±0.15 | -0.45±0.02 | 0.588±0.02 |

We should note that the usage of isotopically averaged parameters of $\pi^\pm N$-scattering amplitudes in the microscopic OP [13] is available for nuclei with the same numbers of protons and neutrons $Z \simeq A - Z$ [18]. Hence the case of $\pi$-scattering on $^{208}\text{Pb}$ with significant difference between numbers of protons and neutrons requires a special consideration.

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