On the magnetisation of gamma-ray burst blast waves

Martin Lemoine1*, Zhuo Li2,3†, Xiang-Yu Wang4,5‡

1 Institut d’Astrophysique de Paris, CNRS, UPMC, 98 bis boulevard Arago, F-75014 Paris, France
2 Department of Astronomy / Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China
3 Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, Kunming 650011, China
4 School of Astronomy and Space Science, Nanjing University, Nanjing, 210093, China
5 Key laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, China

ABSTRACT
The origin of magnetic fields that permeate the blast waves of gamma-ray bursts is a long-standing problem. The present paper argues that in four GRBs revealing extended emission at > 100 MeV, with follow-up in the radio, optical and X-ray domains at later times, this magnetisation can be described as the partial decay of the micro-turbulence that is generated in the shock precursor. Assuming that the extended high energy emission can be interpreted as synchrotron emission of shock accelerated electrons, we model the multi-wavelength light curves of GRB 090902B, GRB 090323, GRB 090328 and GRB 110731A, using a simplified micro-turbulence in the shock precursor. Assuming that the extended high energy emission can be interpreted as synchrotron emission of shock accelerated electrons, we model the multi-wavelength light curves of GRB 090902B, GRB 090323, GRB 090328 and GRB 110731A, using a simplified micro-turbulence in the shock precursor. We find that these models point to a consistent value of the decay exponent −0.5 ≲ αt ≲ −0.4.

Key words: Acceleration of particles – Shock waves – Gamma-ray bursts

1 INTRODUCTION

In principle, the multi-wavelength light curves of gamma-ray bursts (GRB) in the afterglow phase open a remarkable window on the physics of relativistic, weakly magnetized collisionless shock waves: these light curves are indeed thought to result from the synchrotron process of electrons accelerated at the external shock. Additionally, the electromagnetic turbulence, ϵB, should decay rapidly behind the shock (e.g. Gruzinov & Waxman 1999), whereas early afterglow models of GRBs have pointed to finite, substantial values of ϵB on the (comoving) scale of the blast ∼ ctdown (with tdown ∼ τ/(γb)c) the dynamical timescale, γb the blast Lorentz factor], many orders of magnitude larger than the skin depth scale (e.g. Piran 2004 and references therein):

\[ t_{\text{down}} \pi \epsilon_B \sim 2 \times 10^7 E_5^{1/8} n_0^{-3/8} \mu G^{-5/8} \] .

Nevertheless, the decay of Weibel turbulence has been observed in dedicated numerical experiments (Chang et al. 2008, Keshet et al. 2009, Medvedev et al. 2011), although admittedly, such simulations can probe only a small fraction of a GRB dynamical timescale.

The detection of extended high energy emission > 100 MeV by the Fermi-LAT instrument in several GRBs has brought in new constraints in this picture. Most notably, the synchrotron model of this emission has pointed to values of ϵB much smaller than unity in an adiabatic scenario (Kumar & Barniol-Duran 2009, 2010, Barniol-Duran & Kumar 2011, He et al. 2011, Liu & Wang 2011). Kumar & Barniol-Duran (2009) have noted that the magnetic field in which the electrons radiate corresponds to a strength ∼ 10 μG in the upstream frame, before shock compression; they therefore interpret this magnetic field as the simple shock compression of the interstellar field. However, the fact that the inferred ϵB lies a few orders of magnitude above the interstellar magnetization level ∼ 10−9 rather suggests that the electrons radiate in a partially decayed micro-turbulence (Lemoine 2013); theoretically, such a picture could reconcile the results of PIC simulations with the observational determinations of ϵB.

In the present work, we push forward this idea and put it to the test by considering the multi-wavelength light curves of four GRBs observed in radio, optical, X-ray and GeV in the framework

---

* e-mail: lemoine@iap.fr
† e-mail: zhuo.li@pku.edu.cn
‡ e-mail: xywang@nju.edu.cn

1 We use the standard notation \( Q_x \equiv Q/10^x \) in cgs units, unless otherwise noted.
2 AFTERGLOW MODEL

2.1 General considerations

The calculation of the synchrotron spectrum of a relativistic blast wave with decaying micro-turbulence can be approximated (and much simplified) by noting that photons in different frequency bands have been emitted by electrons of different Lorentz factors, which cool at different times after their injection, hence in regions of different magnetic field strengths. In this approximate treatment, one can therefore use the standard homogeneous afterglow model for each frequency band, allowing for a possibly different $\epsilon_B$ in each band. When compared to the detailed calculations with decaying micro-turbulence, one finds that the above provides a reasonable approximation, provided the decay index $\alpha_t \gtrsim -1$. We make this approximation in the present work and justify it at a posteriori.

According to the above picture, one should take a similar $\epsilon_B$ for all frequencies $\nu < \nu_c$ that correspond to Lorentz factors $\gamma < \gamma_c$ such that the cooling timescale exceeds the dynamical timescale.

For GRB afterglows with extended $>100$ MeV emission, which we are interested here, this concerns the radio and optical range, and possibly the X-ray range at late times. For those frequencies, one can therefore use the standard homogeneous approximation of slowly cooling particles for the calculation of $F_{\nu}$. At the other extreme, GeV photons are likely produced in a region of strong $\epsilon_B$, due to the short cooling timescale of the emitting parent electrons. The large Lorentz factors also generally imply that inverse Compton losses are negligible in this frequency range due to Klein-Nishina suppression, although this should be verified on a case-to-case basis. Given these assumptions, the expected flux depends on the ejecta kinetic energy $E$ and $\epsilon_e$, but very little on the other parameters, $\epsilon_B$ in particular. Indeed, the energy radiated in the GeV range corresponds to $\sim \gamma_B(\gamma_{\text{min}})^{2-4}$ times the blast energy stored in the electron distribution $\sim \epsilon_e E$, where $\gamma_B$ denotes the minimum Lorentz factor of electrons radiating at $>100$ MeV.

It is easy to see that $\gamma_B(\gamma_{\text{min}}) \propto \epsilon_B^{-1/4}$, so that the residual dependence of $F_{\nu}(>100$ MeV) on $\epsilon_B$ is quite small. As inverse Compton losses can be neglected at those energies, the flux does not depend either on the external density $n$. As already noted in Kumar & Barniol-Duran (2009), the flux density $F_{\nu}(>100$ MeV) provides a unique constraint on the model parameters, all the more so in the present case of decaying micro-turbulence.

The application of the above simple algorithm allows to evaluate the parameters of the afterglow in the framework of the standard model. One outcome of this analysis is the measurement of $\epsilon_B$, which represents the value of $\epsilon_B$ at the back of the blast, through the modelling of the radio, optical and X-ray flux. Since the dynamical timescale is determined by the standard parameters of the blast, one can constrain directly the exponent of power law decay $\alpha_t$:

$$\alpha_t = \frac{\log \left[ \epsilon_B / \epsilon_B^+ \right]}{\log [\delta_B / \tau_B]} ,$$

up to logarithmic corrections dependent on $\tau_B \sim 100\omega_{\text{min}}^{-1}$, the time scale beyond which turbulence starts to decay and $\epsilon_B^+ \sim 0.01$, the value of the micro-turbulence close to the shock front, both of which are constrained by PIC simulations.

Care must be taken in the course of this exercise, because for low $\epsilon_B$, the Compton parameter at the cooling frequency $Y_c \gg 1$, and Klein-Nishina suppression of the inverse Compton process may be efficient in the X-ray range at late times. The magnitude of KN suppression at frequency $\nu$ can be quantified through the following

$$\Upsilon_{KN}(\nu) = \frac{h\nu \gamma_c(1+z)}{m_e c^2} \approx 50 \frac{E_\gamma^{1/4}}{t_5^{1/3}} A_{35}^{-1/3} \epsilon_{B-5}^{-1/2} \epsilon_{e-1}^{-0.80} \gamma_c^{-1/2}$$

where $\gamma_c(\nu)$ denotes the Lorentz factor of electrons whose (observer frame) synchrotron peak frequency equals $\nu$. For the numerical values, we have assumed a wind profile of external density $n = 10^{-15} A_{35}^{-1} r^{-2}$ cm$^{-3}$, an electron spectral index $p = 2.2$, $\nu > \nu_c$ with $Y_c$ given by Sari & Esin (2001) in the slow cooling regime, and $z = 1$. $Y_{KN} > 1$ at X-ray frequencies means that Klein-Nishina suppression of the inverse Compton cooling is efficient, and cannot be ignored.

The optical and radio data of the following light curves always lie below $\nu_c$, in which case the Compton parameter does not depend on the electron Lorentz factor, $Y(\gamma) = Y_c$, the Compton parameter at $\gamma_c$ (or equivalently, $\nu_c$). In contrast, at GeV energies KN suppression is so efficient that the Compton parameter $Y_{>100}$ MeV $\ll 1$ (e.g. Wang et al. 2010, Liu & Wang 2011). Therefore inverse Compton losses with substantial KN suppression, which modify the synchrotron spectrum (e.g. Nakar et al. 2009, Wang et al. 2010), concern only the X-ray domain at late times.

We therefore proceed as follows. We first search a solution assuming $Y_{KN} < 1$ in the X-ray range, with possibly large $Y_c$. We then compute $Y_{KN}$, and if $Y_{KN} > 1$, we look for another solution in which we take into account the effect of KN suppression in the X-ray domain, following Li & Waxman (2006), Nakar et al. (2009), Wang et al. (2010). In particular, we solve the following equations for the cooling Lorentz factor $\gamma_c$ and Compton parameter $Y_c$ at the cooling frequency:

$$(1 + Y_c) \gamma_c \equiv \gamma_{c,syn} ,$$

$$Y_c (1 + Y_c) = \frac{\epsilon_e}{\epsilon_B^{-1/4}} \left( \frac{\gamma_c}{\gamma_{\text{min}}} \right)^{2-p} \min \left( 1, \frac{\gamma_c}{\gamma_{\text{min}}} \right)^{(3-p)/2}$$

with (see Nakar et al. 2009, Wang et al. 2010)

$$\gamma_{c,syn} = \gamma_c |_{\nu_c \to 0} \ , \ \gamma_{c} \equiv \frac{\gamma_{c,syn}}{\epsilon_{\nu_c}(1+z)} .$$

We neglect more extreme cases in which the electron interacts with low frequency bands of the synchrotron spectrum, below $\nu_{\text{min}}$. We then consider a synchrotron spectrum in the slow cooling phase (generic in the cases that we study $F_{\nu} \propto t_{\text{dyn}}^{-\alpha} \nu^{-\beta}$ with $\beta = 3(p-1)/4$ above $\nu_c$ instead of $\beta = p/2$ when $Y_{KN} < 1$). We then verify a posteriori that the Compton parameter in the X-ray
range $Y_X > 1$, if the X-ray range is fitted with this modified spectrum. In the GeV range, we always find $Y_{>100\text{MeV}} \ll 1$ due to KN suppression, therefore we keep $\beta = p/2$ in that range, in Sec. 2.2 we incorporate the influence of decaying micro-turbulence, which modifies further the time and frequency dependencies of the synchrotron afterglow flux.

2.2 Application to four Fermi-LAT GRBs

We now consider the application of this exercise to four GRBs observed in the radio, optical, X-ray and GeV range: GRB 090902B, GRB 090323, GRB 090328, GRB 110731A. We select them because four observational constraints (corresponding to the four frequency bands) are required to determine unambiguously the four parameters $\epsilon_e, \epsilon_B, E, n$. These four bursts have been discussed in the literature, the first three by Cenko et al. (2011), the last one by Ackermann et al. (2013). We will compare our results to these studies in Sec. 3.

2.2.1 GRB 090902B

We assume in the following $p = 2.3$, as suggested by the previous analyses of Cenko et al. (2011), Kumar & Barniol-Duran (2010), Barniol-Duran & Kumar (2011), Liu & Wang (2011), and $k = 0$ (constant density profile). The flux density at 100 MeV reads

$$F_{\nu} \simeq 6 \times 10^{-9} \text{Jy} \ E_{54}^{0.5} B_{-5}^{0.5} \nu_{-2}^{1.2} t_{50}^{1.15},$$

so that its measured value $\simeq 0.22 \mu\text{Jy}$ at a time $t_{\text{obs}} = 50 \text{s}$ leads to

$$E_{54} \simeq 11.1 \epsilon_{e,-1}^{-21}.$$  

We have discarded the dependence on the Compton parameter $Y_{>100\text{MeV}} \ll 1$ and on $\epsilon_B$, since we assume that the value of $\epsilon_B$, that would enter this equation is close to 0.01, and its exponent is small.

For the optical range in the R-band at $\nu_{\text{opt}}$, we assume $\nu_{\text{min}} < \nu_{\text{opt}} < \nu_c$ at $t_{\text{obs}} = 65\,000 \text{s}$, with flux density $1.8 \times 10^{-5}$ Jy. Therefore the optical flux

$$F_{\nu} \simeq 0.088 \text{Jy} \ E_{54}^{1.3} n_{-2}^{0.5} \epsilon_{e,-1}^{0.8} t_{50}^{1.2} n_0^{-0.9},$$

leads to the constraint, once Eq. [6] has been taken into account:

$$\epsilon_{B,-2} \simeq 4.1 \times 10^{-5} \epsilon_{e,-1}^{0.37} n_0^{-0.61}.$$  

Quite interestingly, these two GeV and optical determinations lead by themselves to very low values of $\epsilon_B$, provided $\epsilon_e$ and $n_0$ do not differ strongly from unity. The radio flux at $\nu_{\text{rad}} = 8.5 \text{GHz}$ lies in the range $\nu_{\text{rad}} < \nu_{\text{min}} < \nu_c$ at $t_{\text{obs}} \sim 10^5 \text{s}$, so that

$$F_{\nu} \simeq 4.2 \times 10^{-5} \text{Jy} \ E_{54}^{0.5} B_{-5}^{1/3} \epsilon_{e,-1}^{1/2} n_0^{2/3},$$

leads to $F_{\nu} \sim 1.3 \times 10^{-4} \text{Jy}$ at $4.8 \times 10^5 \text{s}$; when combined with the above Eqs. [6] and [8], this implies

$$n_0 \simeq 2.5 \times 10^{-6} \epsilon_{e,-1}^{5.21}.$$  

The decay rate in the X-ray range at $t_{\text{obs}} > 10^5 \text{s}$ suggests that $\nu_c < \nu_c$ (see Liu & Wang 2011), which therefore brings in complementary constraints relatively to the optical and radio domains. In principle, one should allow for a different $\epsilon_B$ parameter in the region in which X-rays are produced; here, we make however the approximation that this $\epsilon_B \sim \epsilon_{B,-2}$. In Sec. 2.3 we compute the afterglow allowing for the dependence of $\epsilon_B$ on location, thus correcting this approximation.

If one first neglects KN suppression in the X-ray range, one is led to a solution with $t_{\text{rad}} \sim 2.7$, but with $Y_{\text{KN}} \sim 350$ at times $5 \times 10^5 \text{s}$, so that one needs to include the KN suppression. Following the above algorithm, and using the X-ray flux measurement between $0.3 \text{keV}$ and $10 \text{keV}$ of $2.2 \times 10^{-13}$ erg/cm$^2$/s at $5.2 \times 10^7 \text{s}$, with $\nu_{\infty} > \nu_c$, one derives $\epsilon_e$, hence the parameter set

$$\epsilon_e \simeq 0.46, \quad E \simeq 1.8 \times 10^{54} \text{erg}, \quad \epsilon_{B,-2} \simeq 1.5 \times 10^{-5}, \quad n \simeq 7.0 \times 10^{-3} \text{cm}^{-3}.$$  

We also note that $\nu_c \sim 8.2 \times 10^{16} \text{Hz}$ at $5.2 \times 10^5 \text{s}, Y_c \sim 27$, just as $\nu_{\text{rad}} < \nu_{\text{min}}$ and $\nu_{\text{min}} < \nu_{\text{opt}} < \nu_c$ at the respective times; the solution is therefore consistent.

This light curve therefore indicates a low value for $\epsilon_{B,-2}$, corresponding to a decay exponent

$$\alpha_t \simeq -0.44 \pm 0.10,$$

assuming $\epsilon_{B,+} = 0.01$ at $t = 100 \omega_{pi}^{-1}$. We used the value of $t_{\text{dyn}}$ at time $10^5 \text{s}$, at which the predicted spectrum has been normalized to the optical and radio data. We derive the uncertainty on $\alpha_t$ by propagating conservative estimates of the uncertainties in the value of $p$, of $k$ and the statistical errors of the data used for normalization. As $p$ goes from 2.1 to 2.5, $\alpha_t$ changes from $-0.36$ to $-0.48$. If $k = 2$ instead of 0, one finds $\alpha_t = -0.51$. For this burst, scintillation in the radio range provides the largest source of uncertainty, leading to a conservative factor $\sim 3$ uncertainty on the flux, which in turn leads to an error $\simeq 0.03$ on $\alpha_t$. In total, we estimate the uncertainty $\Delta \alpha_t \simeq 0.10$.

2.2.2 GRB 090323

We repeat the same exercise with GRB 090323, which has been observed at $> 100 \text{MeV}$ up to a few hundred seconds, and in the X, optical and radio domains, short of a day onwards. In what follows, we use $p = 2.5$, slightly smaller than the value found by Cenko et al. (2011) in their best fit, and $k = 2$. The $> 100 \text{MeV}$ flux is normalized to $\phi(> 100 \text{MeV}) = 1.5 \times 10^{-2} \text{ph/cm}^2$/s at $350 \text{s}$, leading to

$$E_{54} \simeq 27.7 \epsilon_{e,-1}^{-33},$$

while the optical flux is normalized to $1.3 \times 10^{-5} \text{Jy}$ at $1.6 \times 10^5 \text{s}$, assuming $\nu_{\text{min}} < \nu_{\text{opt}} < \nu_c$, leading to

$$\epsilon_{B,-2} \simeq 2.1 \times 10^{-3} A_{35}^{-1.14} \epsilon_{e,-38},$$

once Eq. [13] has been taken into account; then, normalization to the radio flux $2 \times 10^{-4} \text{Jy}$ at $4.3 \times 10^5 \text{s}$ with $\nu_{\text{rad}} < \nu_{\text{min}} < \nu_c$ leads to

$$A_{35} \simeq 0.98 \epsilon_{e,-1}^2.$$  

Here we note, that the radio, optical and GeV constraints lead to a very low value for $\epsilon_{B,+}$ if one assumes a parameter $\epsilon_e$ close to the value inferred in PIC simulations, $\epsilon_{e,-1} \sim 1$. To account for the X-ray flux, $\simeq 10^{-13}$ erg/cm$^2$/s at $2.5 \times 10^7 \text{s}$, it is here as

2 The multi-wavelength light curve with a wind profile $k = 2$ does not provide as good a fit to the data as that with $k = 0$; however, it leads to a relatively high external wind parameter at early times, $A \sim 10^{35} \text{cm}^{-1}$, which in turn implies a significant inverse Compton contribution at $> 100 \text{MeV}$. Such a contribution could potentially explain the origin of the highest energy photon at $\sim 30 \text{GeV}$, which is difficult to account for in a scenario with $k = 0$, see Wang et al. (2013).
well necessary to consider the influence of KN suppression, which eventually leads to
\[\epsilon_c \simeq 0.25, \quad E \simeq 8.1 \times 10^{54} \text{ erg},\]
\[\epsilon_{B,2} \simeq 1.8 \times 10^{-6}, \quad A \simeq 6.1 \times 10^{35} \text{ cm}^{-1}. \quad (16)\]
This corresponds to a decay index
\[\alpha_t \simeq -0.54 \pm 0.09, \quad (17)\]
where the error accounts for a factor 2 uncertainty on the GeV flux (leading to \pm 0.06 on \alpha_t), a factor uncertainty on the radio determination (leading to \pm 0.03) and an uncertainty \Delta \rho = \pm 0.2 (leading to \pm 0.04); finally, if \( k = 0 \) instead of \( k = 2 \), one finds \( \alpha_t = -0.50 \).

### 2.2.3 GRB 090328

The multi-wavelength light curve for this burst is rather similar to that of GRB 090323, and we proceed analogously. Using a \( > 100 \text{ MeV} \) flux of \( 2.9 \times 10^{-6} \text{ ph/cm}^2/\text{s} \) at \( 1.1 \times 10^3 \text{ s} \), we obtain
\[E_{54} \simeq 2.1 \epsilon_{-1,1}^{1.33}, \quad (18)\]
while the optical flux is normalized to \( 3 \times 10^{-5} \text{ Jy at } 0.6 \times 10^9 \text{ s} \) (with \( \nu_{\text{min}} < \nu_{\text{opt}} < \nu_c \)), leading to
\[\epsilon_{B,2} \simeq 1.5 \times 10^{-3} A_{35}^{0.14} \epsilon_{-1,1}^{-0.38}. \quad (19)\]
Normalization to the radio flux \( 6 \times 10^{-4} \text{ Jy at } 3 \times 10^5 \text{ s} \) (\( \nu_{\text{min}} < \nu_{\text{ad}} < \nu_{\text{c}} \)) leads to
\[A_{35} \simeq 0.4 \epsilon_{-1,1}^2. \quad (20)\]
The X-ray flux is normalized to \( 2.7 \times 10^{-12} \text{ erg/cm}^2/\text{s} \) at \( 0.63 \times 10^5 \text{ s} \), in the KN regime, which leads to \( \epsilon_c \), hence
\[\epsilon_c \simeq 0.19, \quad E \simeq 0.88 \times 10^{54} \text{ erg},\]
\[\epsilon_{B,2} \simeq 7.6 \times 10^{-6}, \quad A \simeq 1.5 \times 10^{35} \text{ cm}^{-1}. \quad (21)\]
This corresponds to a decay index
\[\alpha_t \simeq -0.46 \pm 0.11, \quad (22)\]
at time \( 10^5 \text{ s} \). The error accounts for a factor 2 uncertainty on the GeV flux (leading to \pm 0.06 on \alpha_t), a factor uncertainty on the radio determination (leading to \pm 0.02) and an uncertainty \Delta \rho = \pm 0.2 (leading to \pm 0.08); finally, if \( k = 0 \) instead of \( k = 2 \), one finds \( \alpha_t = -0.42 \).

### 2.2.4 GRB 110731A

This burst presents the most comprehensive multi-wavelength follow-up of a LAT burst with extended emission at \( > 100 \text{ MeV} \); X-ray and optical start short of 100 s, while \( > 100 \text{ MeV} \) emission is still ongoing. Unfortunately, there are no radio detections for this burst, only an upper limit of \( 5 \times 10^{-5} \text{ Jy at } 0.58 \times 10^5 \text{ s} \) (Zauderer et al., 2011). Nevertheless, one can obtain strong constraints on \( \epsilon_{B,2} \) by noting that the optical frequency \( \nu_{\text{opt}} = 5.5 \times 10^{14} \text{ Hz} \) must satisfy \( \nu_{\text{opt}} > \nu_{\text{min}} \text{ at } t_{\text{obs}} = 100 \text{ s} \), because the optical decays as a power law with index \( \alpha_c \simeq 1.37 \); if the opposite inequality were to hold at this time, one would rather observe \( \alpha = 0 \) for slow cooling, or \( \alpha = 1/4 \) for fast cooling. We thus write \( \nu_{\text{min}} = C_{\nu,2} \nu_{\text{opt}} \) with \( C_{\nu,2} > 1 \) at 100 s, which imposes
\[\epsilon_{B,2} \simeq 5.1 \times 10^{-4} C_{\nu,2}^{-2} E_{54}^{-1} \epsilon_{-4,1}. \quad (23)\]
Here and in the following, we assume \( p = 2.1, k = 2 \). Given that \( C_{\nu} > 1 \), this obviously restricts \( \epsilon_{B,2} \) to very low values, if \( E \) and \( \epsilon_c \) take close to standard values. We next normalize the predicted \( F_{\nu} \) to the observed optical flux density \( \simeq 3.5 \times 10^{-5} \text{ Jy at } 1100 \text{ s}, \) assuming \( \nu_{\text{min}} < \nu_{\text{opt}} < \nu_c \) (verified a posteriori), which leads to
\[A_{35} \simeq 1.5 C_{\nu}^{1.55} \epsilon_{-1,1}^2. \quad (24)\]

The above two conditions imply a radio flux which is a factor \( \geq 4 \) in excess of the observational upper bound; this remains reasonable given the amount of scintillation typically expected at this time, and seen in the other bursts. We then use the \( > 100 \text{ MeV} \) flux, \( \phi(> 100 \text{ MeV}) \simeq 8.4 \times 10^{-8} \text{ ph/cm}^2/\text{s} \) at 26 s, to derive
\[E_{54} \simeq 2.5 \epsilon_{-1,1}^{1.07}. \quad (25)\]
and finally the X-ray flux, \( 2 \times 10^{-9} \text{ erg/cm}^2/\text{s} \) at 100 s, assuming \( \nu_c < \nu_X \). For this burst, KN suppression is not effective at such an early time and it can be neglected in the normalization; however, \( \nu_c \) is eventually found to be close to 1 keV, which makes this solution only approximate. In Sec. 2.2 we derive a better fit by adjusting by hand the missing parameter \( \epsilon_{-1,1} \) under the above constraints. Modulo this small uncertainty, the X-ray flux leads to
\[\epsilon_c \simeq 0.021 C_{\nu}^{-0.50}, \quad E \simeq 13 \times 10^{54} C_{\nu}^{0.54} \text{ erg}, \]
\[\epsilon_{B,2} \simeq 1.9 \times 10^{-4} C_{\nu}^{-0.53}, \quad A \simeq 0.068 \times 10^{35} C_{\nu}^{0.54} \text{ cm}^{-1}. \quad (26)\]
This implies a decay index
\[\alpha_t \simeq -0.35^{1+0.51 \ln C_{\nu}} \frac{1+0.10 \ln C_{\nu}}{1+0.10 \ln C_{\nu}} \text{ at } t_{\text{obs}} = 1100 \text{ s}. \quad (27)\]
Assuming \( C_{\nu} = 1 \), we estimate a conservative uncertainty on \( \alpha_t \) to be \( \Delta \alpha_t \simeq \pm 0.2 \) given that a factor 2 uncertainty on the GeV flux leads to an error \pm 0.10, \( p = 2.01 \) leads to \( \alpha_t = -0.14 \) while \( p = 2.3 \) leads to \( \alpha_t = -0.53 \). Note that the light curves leave very little ambiguity on the density profile (Ackermann et al. 2013), therefore we do not consider \( k = 0 \).

### 2.3 Multi-wavelength light curves in a decaying turbulence

We now include the effect of decaying micro-turbulence. The changing magnetic field modifies the spectral shape of electrons with \( \gamma > \gamma_c \), as well as the characteristic frequencies and their evolution in time (Lemoine 2013). With respect to the previous two-zone slow-cooling model, most of the difference concerns the X-ray domain, which lies above \( \nu_c \). The spectrum is computed as follows.

At frequencies \( < \nu_c \), the standard synchrotron spectrum holds, although the magnetic field value should be taken as the partially decaying microturbulent value at the back of the blast, which evolves in time:
\[\delta B_{\ast} \simeq \delta B_{\ast} \left( \frac{\nu_{\text{obs}}}{\delta B_{\ast}} \right)^{\alpha_t/2} \simeq \frac{1}{2} \frac{\nu_{\text{obs}}}{\nu_{\text{obs}}^{\alpha_t} + \nu_{\ast}}. \quad (28)\]
Of course, one recovers the standard time evolution in the limit \( \alpha_t \rightarrow 0 \).

At frequencies \( \nu_c < \nu < \nu_X \), that is if \( \nu_c < \nu_X \) (\( \nu_c \) designing the synchrotron peak frequency associated to \( \gamma_c \)), KN suppression is ineffective, \( T_{\text{KN}}(\nu) < 1 \), therefore the electrons cool in a uniform radiation background, but radiate their synchrotron flux in a changing magnetic field, all along their cooling history. This leads to a synchrotron spectral index
\[\beta = \frac{p + \alpha_t/2}{2 - \alpha_t/2} \left[ \nu > \nu_c, \quad T_{\text{KN}}(\nu) < 1 \right], \quad (29)\]
see the Appendix of Lemoine (2013), Sec. A3.

To account for the influence of KN suppressed inverse Compton losses at frequencies \( \nu > \max (\nu_c, \nu_e) \), we proceed as follows. We first solve for \( \gamma_c \) and \( Y_c \) as in Eqs. (2) using however a value \( \delta B_\gamma \) for the magnetic field at the back of the blast. We then solve for the cooling history \( \gamma_c(t) \) of an electron with initial Lorentz factor (meaning at the shock front, \( t \) representing the comoving since acceleration at the shock) \( \gamma_{c,0} > \gamma_c \), considering that if \( T_{KN}(\nu) > 1 \), this electron interacts with a radiation field of energy density \( Y(\gamma_c) \delta B^2 / (8\pi) \), characterized by the Lorentz factor dependent Compton parameter \( Y(\gamma_c) \) (e.g. Li & Waxman 2006, Nakar et al. 2009, Wang et al. 2010):

\[
Y(\gamma_c) \simeq Y_c \left( \frac{\gamma_c}{\gamma_e} \right)^{(p-3)/2}, \tag{30}
\]

assuming \( \gamma_e < \gamma_c < \gamma_c \). Here as well, we can neglect extreme cases in which the electron interacts with the low frequency bands of the spectrum, below \( \nu_{\text{min}} \). Solving for the cooling history in this radiation field, one determines a cooling timescale \( t_{\text{cool}}(\gamma_c, \nu) \simeq t_{\text{dyn}}(\gamma_{e,0}/\gamma_c)^{-1}(p-1)/2 \), and \( \gamma_c(t) \simeq \gamma_c(t_{\text{dyn}}/t)^{-1}(p-1)/2 \) for \( t \gg t_{\text{cool}}(\gamma_c, \nu) \). Following Lemoine (2013), we then calculate the individual electron synchrotron contribution, by integrating the synchrotron power \( \gamma_c^2(t) \delta B^2(t) \) over this cooling history; then we evaluate the contribution of the electron population by folding the latter result over the injection distribution function of electron Lorentz factors. This leads to a synchrotron spectral index

\[
\beta = \frac{3(p-1)}{4} + \frac{1 + \alpha\epsilon/6}{1 - \alpha(p-1)/8} \left[ \nu > \nu_c, \ T_{KN}(\nu) > 1 \right], \tag{31}
\]

which tends to \( 3(p-1)/4 \) as it should when \( \alpha \epsilon \to 0 \) (non-decaying turbulence).

Finally, at \( \nu > 100 \text{ MeV} \), we assume that inverse Compton losses are negligible, hence we use the above \( \beta = (p + \alpha\epsilon/2)/(2 - \alpha\epsilon/2) \). This slight change of slope, as compared to the two-zone determinations, implies slightly different parameter values. The final estimates are given in the captions of Figs. 1-3, which present the models of these multi-wavelength light curves.

We have not attempted to obtain least squares fits to these multi-wavelength light curves, rather we have used the normalization of the flux at several data points, as discussed in the previous sections, derived the parameters, then plotted the predicted multi-wavelength light curves. We have also neglected the possibility of significant extinction in the optical domain, which could improve the quality of the fit for GRB 110731A in particular. Moreover, our numerical code computes the light curves for a decelerating blast wave; it does not account for the initial ballistic stage, and neither does it account for sideways expansion beyond jet break. We therefore start plotting the \( > 100 \text{ MeV} \) lightcurve at 10 s, which corresponds to initial Lorentz factors \( > 700 \) for GRB 110731A and GRB 090902B, for which \( > 100 \text{ MeV} \) data exist at 10 s. Evidence for jet break is lacking in the 4 bursts, except possibly for GRB 090902B (Cenko et al. 2011), in which case it would improve the fit at times \( \geq 10^6 \text{ s} \). Thus, there is room for improving the quality of these fits, but it should not modify the value of \( \alpha \epsilon \) derived in the previous sections beyond the quoted uncertainties.

Finally, using the solutions indicated in the captions of the figures, one can verify that synchrotron self-absorption effects are negligible in the radio domain at the time at which the flux was normalized to the data. One can also verify that for all bursts except GRB 090323, the inverse Compton component provides a negligible contribution at \( > 100 \text{ MeV} \) at early times; for GRB 090323, this contribution is a factor 0.6 of the observed flux at \( t_{\text{obs}} = 360 \text{ s} \), thus non negligible. However, this remains within the error bars on the flux normalization that we have adopted for this GRB, therefore we neglect its influence. Future work should consider more detailed multi-wavelength light curves including this inverse Compton component, and possibly as well the effect of the maximal energy in the \( > 100 \text{ MeV} \) domain, as in Wang et al. (2013).
There are however important differences in the interpretation of these low values: Kumar & Barniol-Duran (2009, 2010) argue that all particles cool in the background shock compressed magnetic field (including those producing \( > 100 \text{ MeV} \) photons), which is inferred of the order of \( \sim 10 \mu \text{G} \) (upstream rest frame). We rather argue that the particles cool in the post-shock decaying micro-turbulence, which is self-generated in the shock precursor through microinstabilities, and which actually builds up the collisionless shock. As discussed in the introduction, this latter interpretation is motivated by the large hierarchy between the inferred values of \( \epsilon_{B-} \sim 10^{-6} - 10^{-4} \) and the much smaller interstellar magnetization level \( \sim 10^{-9} \), indicating that the background shock compressed field plays no role in shaping the light curves. A power law decay of the micro-turbulence behind the shock front is also theoretically expected, e.g. Chang et al. (2008). Furthermore, we provide a complete self-consistent model of the synchrotron afterglow light curves in this scenario, based on and improving the results of Lemoine (2013). Within our interpretation, we are thus able to constrain the value of the exponent of the decaying micro-turbulence (assuming power law decay), and we find a consistent value among all bursts studied, \(-0.5 \lesssim \alpha_\epsilon \lesssim -0.4\). This value turns out to agree quite well with the results of the PIC simulations of Kesht et al. (2009), see the discussion in Lemoine (2013).

These low values of \( \epsilon_{B-} \) stand in stark contrast with other determinations by Cenko et al. (2011) for GRB 090902B, GRB 090323, GRB 090328, and by Ackermann et al. (2013) for GRB 110731A, who systematically find values \( \epsilon_{B-} \sim 0.01 \). The key difference turns out to come from the high energy component above 100 MeV. While in the present work, we assume that this GeV extended emission is synchrotron radiation from shock accelerated electrons, those studies do not incorporate the constraints from this high energy component. Using the best fit models of Cenko et al. (2011) and Ackermann et al. (2013), it is straightforward to calculate the ratio \( R_{>100 \text{ MeV}} \) of the predicted photon flux \( \phi(>100 \text{ MeV}) \) to the observed value\(^3\).

The result is rather striking: those models do not explain the high energy component, in spite of the excellent quality of the fits obtained in the other domains, e.g. Cenko et al. (2011). Ultimately, this results from degeneracy in the parameter space, when only 3 wavelength bands are used to determine the 4 parameters \( E, n, \epsilon_\epsilon \) and \( \epsilon_B \) (assuming some extra information is available to determine \( p \) and \( k \), e.g. the time behavior). Specifically, the models of Cenko et al. (2011) and Ackermann et al. (2013) present solutions that are degenerate up to the choice of one of the above parameters, say \( \epsilon_\epsilon \). To verify this, one can explicitly repeat the above exercises, neglecting the \( > 100 \text{ MeV} \) data. By tuning \( \epsilon_\epsilon \), one can then find similar light curves, with different values of the parameters. These different sets of solutions also correspond to different values of \( Y_C \); the solutions of Cenko et al. (2011), Ackermann et al. (2013) systematically have \( Y_C \lesssim 1 \), while ours rather corresponds to \( Y_C \gg 1 \). When \( Y_C \gg 1 \), the solution scales differently with \( \epsilon_\epsilon \), because of the influence of inverse Compton losses in the X-ray domain (notwithstanding possible KN suppression). As \( Y_C \gg 1 \), one recovers our solutions up to the ambiguity in the choice of \( \epsilon_\epsilon \). This ambiguity is eventually raised by the normalization to the \( > 100 \text{ MeV} \) flux, leading to the present low \( \epsilon_B \) values.

Going one step further, one should envisage the possibility that earlier (pre-Fermi) determinations of the microphysical param-

---

\( ^3 \) for GRB090902B, we rather compare the spectral flux density at \( 2.4 \times 10^{22} \text{ Hz} \) to the observed value.
The magnetisation of gamma-ray bursts afterglows

eters could be affected by a similar bias. The detailed analysis of Panaitescu & Kumar (2001, 2002) indicates indeed a broad range of values of $\epsilon_{B-}$ for any GRB, spanning values from $\sim 10^{-6}$ up to $10^{-1}$. Thus $\epsilon_{B-}$ is poorly known. In very few cases, such as the famous GRB 970508, a synchrotron self absorption break seems to appear in the radio band. In these cases, using the radio data in both optically thin and thick regimes, as well as the optical and X-ray data, one has 4 bands for 4 parameters, then all the parameters can be determined. A large value for the magnetic field, $\epsilon_{B-} \sim 0.01$, is obtained for GRB 970508 by Wijers & Galama (1999). However, the absorption break in radio may not be clear given the bad quality of radio data (due to strong scintillation). A recent re-analysis of GRB 970508 by Leventis et al. (2013) also finds a variety of solutions, including one with a low value of $\epsilon_{B-}$, when no ad-hoc extra constraint is imposed on the parameters. Future work should consider carefully the uncertainty in the determination of $\epsilon_{B-}$ in such bursts.

Taken at face value, the present results suggest that the magnetization of the blast can be described as the partial decay of the micro-turbulence that is self-generated at the shock; it also suggests that evidence for further amplification of this turbulence is lacking, at least in the bursts observed by the Fermi-LAT instrument.

Acknowledgments: This work has been supported in part by the PEPS/PTI program of the INP (CNRS), by the NSFC (11273005), the MOE Ph.D. Programs Foundation, China (20120001110064) and the CAS Open Research Program of Key Laboratory for the Structure and Evolution of Celestial Objects, as well as the 973 program under grant 2009CB284800, the NSFC under grants 11273016, 10973008, and 11033002, the Excellent Youth Foundation of Jiangsu Province (BK2012011). This work made use of data supplied by the UK Swift Science Data Centre at the University of Leicester.

REFERENCES

Abdo, A. A. et al. (Fermi Collaboration), 2010, ApJ, 736, L138
Ackermann et al. (Fermi Collaboration), 2013, ApJ, 763, 71
Barniol-Duran, R., Kumar, P., 2011, MNRAS, 417, 1584
Cenko, S. B. et al., 2011, ApJ, 732, 29
Chang, P., Spitkovsky, A., Arons, J., 2008, ApJ, 674, 378
Evans, P. A. et al. (Swift-XRT), 2007, A&A, 469, 379
Evans, P. A. et al. (Swift-XRT), 2009, MNRAS, 397, 1177
Gruzinov, A., Waxman, E., 1999, ApJ, 511, 852
He, H.-N., Wu, X.-F., Toma, K., Wang, X.-Y., Mészáros, P., 2011, ApJ, 733, 22
Keshet, U., Katz, B., Spitkovsky, A., Waxman E., 2009, ApJ, 693, L127
Kumar, P., Barniol-Duran, R., 2009, MNRAS, 400, L75
Kumar, P., Barniol-Duran, R., 2010, MNRAS, 409, 226
Lemoine, M., 2013, MNRAS, 428, 845
Leventis, K., van der Horst, A. J., van Eerten, H. J., Wijers, R. A. M. J., 2013, MNRAS, 431, 1026
Li, Z., Waxman E., 2006, ApJ, 651, L328
Liu, R., Wang, X.-Y., 2011, ApJ, 730, 1
Martins, S. F., Fonseca, R. A., Silva, L. O., Mori, W. B., 2009, ApJ, 695, L189
Medvedev, M. V., Loeb, A., 1999, ApJ, 526, 697
Medvedev, M. V., Trier Frederiksen, J., Haugboelle, T., Nordlund, A., 2011, ApJ, 737, 55
Nakar, E., Ando, S., Sari, R., 2009, ApJ, 703, 675
Panaitescu, A., Kumar, P., 2001, ApJ, 544, 667
Panaitescu, A., Kumar, P., 2002, ApJ, 571, 779
Piran, T., 2004, Rev. Mod. Phys., 76, 1143
Piron, F., 2010, Gamma-ray bursts 2010 Conference (Annapolis)