Large $\tan \beta$ from $SU(2)_R$ Gauge Symmetry

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Abstract

Sufficient conditions for the relation $\tan \beta \simeq m_t/m_b$ to hold in supersymmetric grand unified theories are formulated. Essential ingredients are the $SU(2)_R$ gauge symmetry and a discrete matter parity. The applicability of our conditions is illustrated by specific examples. Implications for neutrino masses are discussed.
The minimal supersymmetric standard model (MSSM), in spite of the large number of undetermined parameters that it contains, is widely believed to be the correct effective low energy theory of strong, weak and electromagnetic interactions. One hopes, of course, that at least some of the parameters of this model will be determined by embedding the standard gauge group in a larger (unifying) one. This hope seems to be closer to fulfillment as far as the well-known parameters $\sin^2\theta_w$ and $\alpha_s$ of the model are concerned. Assuming a supersymmetry breaking scale $M_s \sim 1\text{ TeV}$, the renormalization group (RG) equations of the MSSM are remarkably consistent\(^{(1)}\) with the observed values of $\sin^2\theta_w$ and $\alpha_s$ and the unification of the three gauge couplings at a superheavy scale $\sim 10^{16}\text{ GeV}$.

The supersymmetric version of the standard electroweak theory, however, although more successful than the non-supersymmetric one with respect to ”predicting” the ratio of the gauge couplings, introduces an important undetermined new parameter by necessitating the doubling of the number of electroweak higgs doublets. This new parameter, known as $\tan\beta$, is the ratio of the vacuum expectation values (VEVs) of the doublet $h^{(1)}$ giving mass to the up-type quarks and the doublet $h^{(2)}$ giving mass to the down-type quarks and charged leptons. The embedding of the MSSM in the simplest supersymmetric grand unified theory (SUSY GUT), the minimal SUSY SU(5) model, fails to determine $\tan\beta$. However, SUSY GUTs based on larger groups, like SO(10), may lead\(^{(2)}\) to the asymptotic relation $\tan\beta = m_t/m_b$. In deriving this relation in SO(10), the assumption is made that the third generation fermions (mostly) acquire mass from a single $16 \times 16 \times 10$ coupling with the electroweak higgs doublet pair contained primarily in the 10-plet.
This way the large mass of the t-quark is explained without invoking large ratios of yukawa couplings within the third generation. Besides, detailed investigations\(^{(3)}\) have shown that large tan$\beta$ scenarios combined with radiative electroweak breaking lead to very interesting sparticle spectroscopy.

Our purpose here is to formulate, in the context of SUSY GUTs, widely applicable conditions under which the relation $\tan\beta \approx m_t/m_b$ holds. A discrete matter parity (MP) and the $SU(2)_R$ gauge group are essential ingredients of these conditions which we apply to specific SUSY GUT examples. In particular, we are primarily interested in embeddings of the MSSM into SUSY GUTs with semi-simple gauge groups.

We consider SUSY GUT models which, below an energy scale $M$, have a symmetry group $P \times G$. $P$ is a global symmetry group and $G \supseteq G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is a gauge group. At a scale $M_R \ll M$, $G_{LR}$ breaks spontaneously down to the standard gauge group $G_S \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$. At energies $\sim M_s$, the unbroken symmetry group contains $C \times G_S$, where $C$ is a $Z_n$ symmetry. We will assume, without any essential loss of generality, that the generator of $C$ is obtained by multiplying an element of $P$ with an element of the Cartan subgroup of $G$ commuting with $G_S$. For the spectrum of the model, we assume that all $G$-non-singlet states are either heavier than $M$ or lighter than $M'$ ($M' \gg M'R$) and the only states which are massless in the supersymmetric limit are the states of the MSSM on which $C$ acts as a (generation blind) $Z_n$ matter parity\(^{(4)}\). The unique pair of electroweak higgs doublets $h^{(1)}, h^{(2)}$ is assumed to form a single $SU(2)_R$-doublet $h$. The massive spectrum below $M$ should not contain any other states with the $C \times G_S$ quantum numbers of the ordinary light quarks.
and leptons. We will show that under the above conditions the approximate relation \( \tan \beta \simeq m_t/m_b \) holds. If, in addition, (below \( M' \)) there are exactly three \( G_S \)-singlet states \( \nu^c_i (i = 1, 2, 3) \) which could possess Dirac mass terms with the ordinary left-handed neutrinos, then we also have \( \tan \beta \simeq m_{\nu^c_\tau}/m_\tau \) (with \( m_{\nu^c_\tau} \) being the Dirac mass of the \( \tau \)-neutrino).

The states with masses \( \lesssim M' \), on which we focus, fall (up to corrections of order \( M'/M \) from heavier states) into multiplets of the gauge group \( SU(2)_R \). Moreover, our assumption about the generator of \( C \) implies that, if one component of a \( SU(2)_R \)-multiplet happens to be an eigenstate of \( C \), then all its components are also eigenstates of \( C \) (not necessarily with the same eigenvalue). Let \( q_i (i = 1, 2, 3) \) denote the ordinary \( SU(2)_L \)-doublet quarks and \( u^c_i (d^c_i) \) the ordinary up (down) - type antiquarks, all of them being massless superfields in the supersymmetric limit. The \( q_i \)'s are assumed to be \( SU(2)_R \)-singlets with \( B - L = 1/3 \) and the \( u^c_i \)'s to belong to \( SU(2)_R \)-doublets with \( B - L = -1/3 \). Let \( d^c_i \) be the partner of \( u^c_i \) in its \( SU(2)_R \)-doublet. The above discussion shows that \( d^c_i \), like \( u^c_i \), is an eigenstate of \( C \). The existence of the terms \( h^{(1)} q_i u^c_j (i, j = 1, 2, 3) \) implies (using the \( SU(2)_R \) symmetry) the existence of the terms \( h^{(2)} q_i d^c_j \) which combined with the assumed existence of the terms \( h^{(2)} q_i d^c_j \) leads to the conclusion that \( d^c_i \)'s and \( d^c_i \)'s have the same MP.

Our assumption that the only states (below \( M \)) with the \( C \times G_S \) quantum numbers of ordinary quarks and leptons are the ordinary quarks and leptons themselves implies that (up to a simple renaming) the \( d^c_i \)'s coincide with the \( d^c_i \)'s. Therefore, the \( u^c_i \)'s and the \( d^c_i \)'s fall by themselves into three \( SU(2)_R \)-doublets, which we denote by \( q^c_i \). Analogous arguments lead to the conclusion that the three ordinary light charged antileptons \( e^c_i \) (assumed to belong
to $SU(2)_R$-doublets with $B - L = 1$) and the three right-handed neutrinos $\nu_i^c$ (provided there are exactly three such states) form three $SU(2)_R$-doublets $\ell_i^c$. Consequently, the tree-level light quark mass terms come from couplings of the type $hq_iq_j^c$ while the tree-level light charged lepton and neutrino Dirac mass terms from couplings of the type $h\ell_i\ell_j^c$ ($\ell_i$'s are the three ordinary light lepton $SU(2)_L$-doublets). We see that the tree-level up and down quark mass matrices are proportional with proportionality constant equal to $\tan\beta$. A similar proportionality holds between neutrino Dirac mass matrices and charged lepton mass matrices. Phenomenologically, this proportionality is unacceptable for the quarks of the first two families. We assume that for the first two families there should be considerable corrections to the tree-level mass matrices from other sources whereas there should be no such sizeable corrections for the third family\(^5\). Thus, we conclude that our scheme leads to the relations mentioned earlier.

In the special but very common case of a $Z_2$ MP, one could relax the assumption that $h^{(1)}, h^{(2)}$ are partners in a single $SU(2)_R$-doublet $h$. It is enough to assume that they just belong to $SU(2)_R$-doublets and that (below $M$) there are no other states which belong to $SU(2)_R$-doublets and carry the same $C \times G_S$ quantum numbers as $h^{(1)}, h^{(2)}$. We remind the reader that under the $Z_2$ MP the electroweak higgs doublet pair remains invariant while the ordinary quarks and leptons change sign. Let $h^{(2)'}$ be the partner of $h^{(1)}$ in its $SU(2)_R$-doublet. It is easily seen that, under $G_S, h^{(2)'}$ and $\ell_i$'s have the same quantum numbers. Then, if $h^{(2)'}$ has different MP from $h^{(2)}$, namely minus, it must coincide with one of the light leptons $\ell_i$. This is, of course, impossible because the $\ell_i$'s are not $SU(2)_R$-doublets. Therefore,
$h^{(2)\prime}$ belongs to a $SU(2)_R$ -doublet and is identical under $C \times G_S$ with $h^{(2)}$. Consequently, $h^{(2)\prime}$ coincides with $h^{(2)}$. Thus, $h^{(1)}$ and $h^{(2)}$ are partners in a single $SU(2)_R$ -doublet $h$.

Finally, even in the general case of a $Z_n$ MP, there are situations in which one could relax the assumption that $h^{(1)}, h^{(2)}$ belong to the same $SU(2)_R$ -doublet. In fact, no assumption whatsoever on the $SU(2)_R$ properties of $h^{(1)}, h^{(2)}$ is needed provided (below $M$) there are no other states identical under $C \times G_S$ with them and the light $u_i^c$’s, $d_i^c$’s form $SU(2)_R$ -doublets $q_i^c$. The last requirement is readily fulfilled if there are just three states with the $G_S$ quantum numbers of a $d^c$. The existence of the terms $h^{(1)} q_i u_j^c$ together with the uniqueness of $h^{(1)}$ under $C \times G_S$ imply that $h^{(1)}$ belongs to a $SU(2)_R$ -doublet $h$. Let $h^{(2)\prime}$ be the partner of $h^{(1)}$ in $h$. Then the terms $h^{(2)\prime} q_i d_j^c$ are certainly allowed (by $SU(2)_R$ symmetry) leading to the conclusion that $h^{(2)\prime}$ and $h^{(2)}$ are indistinguishable under $C \times G_S$ and, consequently, identical. Thus, $h^{(1)}$ and $h^{(2)}$ are partners in a single $SU(2)_R$ -doublet $h$.

We should point out here that global symmetries acting as matter parities are standard ingredients of supersymmetric models. They are introduced in order to deal with the well-known problem of rapid proton decay through $d = 4$ operators at the level of the supersymmetric standard model. Therefore, our assumption that the gauge group $G_S$ is supplemented with a MP $C$ should not at all be considered as an extra restriction.

As a first application, we consider a SUSY GUT model with a $P \times G$ symmetry, where $P$ is a $Z_2$ global symmetry and the gauge group $G \equiv SO(10)$ $\supset G_{LR}$. The scale $M$ here could be taken to be the Planck mass $M_P \sim 10^{19}$ GeV. $G$ breaks down at a scale $M' \equiv M_R \sim 10^{16}$ GeV directly to $G_S$ using
higgs fields with $P = +1$ in the $16$, $\bar{16}$, $126$, $\bar{126}$, 45, 54 and 210 representations of SO(10). The three ordinary light generations are contained in three 16-plets with $P = -1$. There are also 10, $\bar{126}$ and 120 representations of SO(10) with $P = +1$. We assume that the electroweak higgs doublets $h^{(1)}, h^{(2)}$ are partners in a single $SU(2)_R$ -doublet and belong to an otherwise arbitrary linear combination of the 10, $\bar{126}$, 120 and 210 representations. Higgs fields in the $16$, $\bar{16}$ and 210 representations could also contribute to $h^{(1)}, h^{(2)}$. These contributions, however, cannot belong to a single $SU(2)_R$ -doublet and, thus, should be suppressed. Notice that $P$ remains unbroken by all the VEVs and coincides with the MP $C$. Furthermore, there are no other states having the same $C \times G_S$ quantum numbers with the light quarks and leptons and there are just three $\nu^c_i$’s. Thus, we conclude that all the conditions for the relation $\tan \beta \simeq m_t/m_b \simeq m^D_{\nu\tau}/m_{\tau}$ to hold are satisfied. It is important to note that the validity of this relation is not automatic even in the minimal SUSY SO(10) model which contains higgs fields in the $16$, $\bar{16}$ representations.

To construct an example with automatically large $\tan \beta$ and the usual MSSM predictions of $\sin^2 \theta_w$ and $\alpha_s$, we consider a model with symmetry $P_1 \times P_2 \times P_3 \times G$, where $P_1, P_2$ and $P_3$ are $Z_2, Z_2$ and $Z_7$ global symmetries respectively and $G$ the gauge group $SU(4)_c \times SU(2)_L \times SU(2)_R$. The scale $M$ is a superheavy scale close to $M_P$, where we assume a common value for the gauge couplings of the three factors of $G$. Let $A_L, A_R, D, B, H$ and $T$ denote left-handed fields transforming under $G$ as $(4,2,1)$, $(\bar{4},1,2)$, $(6,1,1)$, $(15,1,1)$, $(1,2,2)$ and $(1,1,1)$ respectively. The field content of the model consists of three $A_L(-1,+1,\alpha)$, three $A_R(-1,+1,\alpha)$, three $D(+1,+1,1)$, one $B(+1,+1,1)$, one $H(+1,+1,\alpha^5)$, one $T(+1,+1,\alpha^3)$, one $A_L(+1,-1,\alpha)$,
one $A_L(+1, -1, \alpha^6)$, one $A_R(+1, +1, \alpha)$ and one $\bar{A}_R(+1, +1, \alpha^6)$, where the transformations of the various fields under the generators of $P_1, P_2, P_3$ are given in parenthesis and $\alpha = e^{2\pi i/7}$. The large VEVs, all of the order of $M_R \simeq 10^{16}$ GeV, are acquired by the fields $B(+1, +1, 1)$, $A_R(+1, +1, \alpha)$, $\bar{A}_R(+1, +1, \alpha^6)$ and $T(+1, +1, \alpha^3)$. As a result, the gauge group $G$ breaks down to $G_S$ and the symmetry $P_3$ breaks down completely. The symmetry $P_1 \times P_2$ remains unbroken and is identified with the MP $C$. All the allowed tree-level mass terms are assumed $\sim M_R \equiv M'$. Below the scale $M_R$, we recover the MSSM supplemented with exactly three right-handed neutrinos (mass $\sim M_R^2/M$). Ordinary light quarks and leptons are contained in the $A_L$ 's and $A_R$ 's with $C$-charge $(-1, +1)$ and are unique under $C \times G_S$. Besides, $h^{(1)}$ and $h^{(2)}$ are the $SU(2)_R$ partners in $H$. Their mass is $\sim< T^6/M^5 \sim M_s$. Therefore, $\tan \beta$ is automatically large. The role of the symmetry $P_3$ is to protect $H$ from acquiring a mass $\sim M_R$. It also suppresses the rate of proton decay mediated by the $D$'s. Due to the appropriately chosen spectrum, the one-loop RG equations above $M_R$ predict identical running for the gauge couplings of the groups $SU(4)_c, SU(2)_L$ and $SU(2)_R$. Combining this fact with their assumed equality at $M$, we conclude that equality of the gauge couplings of $G_S$ at the scale $M_R$ is a justified boundary condition. The successful MSSM predictions for $\sin^2 \theta_w$ and $\alpha_s$ follow immediately.

As another example, we consider a SUSY model with symmetry group $P_1 \times P_2 \times G$. $P_1$ is a $Z_2$ global symmetry which remains unbroken and coincides with the $Z_2$ MP $C$. $P_2$ is a $Z_6$ global symmetry and $G$ coincides with the gauge group $G_{LR}$. The coupling constants of the four factors in $G_{LR}$ are assumed to become equal at a scale $M_c = 2.4 \times 10^{18}$ GeV probably related
to a more fundamental theory. The spectrum below $M_c$ contains the three quark, $q_i$, antiquark, $q^c_i$, lepton, $\ell_i$, and antilepton, $\ell^c_i$, fields, one pair of colorless $SU(2)_L$ -doublet fields, $H^{(1)}, H^{(2)}$, which form a $SU(2)_R$ -doublet with zero $(B - L)$-charge, one $\ell_o, \bar{\ell}_o$ pair, one $\ell^c_o, \bar{\ell}^c_o$ pair, eight $q_m, \bar{q}_m$ and $q^c_m, \bar{q}^c_m$ ($m = 1, \ldots, 8$) pairs and one singlet $T$. All fields, except the ordinary quark and lepton generations, have $P_1 = +1$. Under the $P_2$ symmetry generator all fields get multiplied by $\alpha = e^{2\pi i/6}$ except $H^{(1)}, H^{(2)}$ which get multiplied by $\alpha^4$ and $\bar{\ell}_o, \bar{\ell}^c_o, q_m, \bar{q}^c_m$ by $\alpha^5$. The $\ell^c_o, \bar{\ell}^c_o$ pair is assumed to have appropriate superpotential couplings in order to break the $SU(2)_R \times U(1)_{B - L}$ gauge symmetry down to $U(1)_Y$ at a scale $M_R = 10^{15}$ GeV. All other allowed superpotential mass terms are assumed to be equal to $M = 2.4 \times 10^{17}$ GeV.

In the absence of $P_2$, there are no light states which could play the role of the electroweak higgs doublets. This is due to the fact that the only states able to play such a role, namely $H^{(1)}, H^{(2)}, \ell_o$ and $\bar{\ell}_o$, acquire tree-level masses $\sim M$. $P_2$, however, allows the mass terms $< \nu^c > H^{(1)}\ell_o$ and $M\bar{\ell}_o\ell_o$ but forbids the mass terms $MH^{(1)}H^{(2)}$ and $< \bar{\nu}^c > \bar{\ell}_oH^{(2)}$. As a result, there is an electroweak higgs doublet pair $h^{(1)} \simeq \rho[H^{(1)} - (M_R/M)\bar{\ell}_o], h^{(2)} = H^{(2)}$ (where $\rho = [1 + (M_R/M)^2]^{-1/2}$) left massless with the orthogonal states $\rho[\bar{\ell}_o + (M_R/M)H^{(1)}]$ and $\ell_o$ acquiring superheavy masses $\sim M$. $P_2$ must, of course, be broken in order for a higgsino mass term $\sim M_s$ to be generated. A VEV $< T > \simeq \frac{M_R}{2}$ achieves this breaking. The superpotential coupling $H^{(1)}H^{(2)} < T^4 > /M_c^3$ generates, then, a mass term with the desired order of magnitude without significantly altering the above discussion.

It is easily seen that the only states below the scale $M \gg M_R \equiv M'$ are the states of the MSSM supplemented with three right-handed neutrinos.
with masses $\sim M_R^2/M_c$, the massive gauge supermultiplet associated with the generators of $SU(2)_R \times U(1)_{B-L}/U(1)_Y$ and the higgs associated with the breaking of $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$ with mass $\sim M_R$. Notice that (below $M$) the states of the MSSM are unique under $C \times G_S$ and, moreover, there are only three states with the $G_S$ quantum numbers of a $d^c$.

Our relation for $\tan \beta$ follows immediately.

We can easily estimate the corrections to the relation for $\tan \beta$ due to the spectrum above $M$. It should be clear that the only sizable correction could come from the fact that, due to the small ($\sim M_R/M$) contribution of $\bar{\ell}_o$ in $h^{(1)}$, $h^{(1)}$ is not exactly the $SU(2)_R$-partner of $h^{(2)}$. The exact relation is $m_t/m_b = <H^{(1)}> / <H^{(2)}> \text{ instead of } m_t/m_b = <h^{(1)}> / <h^{(2)}> \text{ because } H^{(1)}$, and not $\bar{\ell}_o$, couples to quarks and leptons. Using the readily obtainable relation $<H^{(1)}>=\rho <h^{(1)}>$, we get $m_t/m_b = \rho \tan \beta$. It seems that in this case the departure of the coefficient $\rho$ from unity is $\simeq \frac{1}{2}(M_R/M)^2$ and not $\sim M_R/M$ as one would naively have thought.

For the sake of completeness we mention that our simple model with the chosen field content is consistent with the presently favored values for $\sin^2 \theta_w$ and $\alpha_s$. An one-loop calculation gives the perfectly acceptable values $\sin^2 \theta_w = 0.234$ and $\alpha_s(M_Z) = 0.120$ with the perturbative value $\alpha_G = 0.06$ for the unified gauge structure constant at $M_c$.

Our last example concerns a natural and very attractive model based on the gauge group $\bar{G} \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$. An advantage of SUSY $SU(3)^3$ models over the usual GUTs is that the proton decay amplitude through $d = 5$ operators is not related to the light quark masses and, thus, can be suppressed through suitable discrete symmetries. The present model,
a variation of the model in ref.(6), possesses a symmetry group $P \times \tilde{G}$ with $P \equiv P_1 \times P_2 \times P_3$ a $Z_2 \times Z_2 \times Z_2$ global symmetry. We assume a common unified gauge coupling at a scale $M_c = 2.4 \times 10^{18}$ GeV as before. The model makes use only of gauge singlets, $\lambda = (1, \bar{3}, 3), Q = (3, 3, 1)$ and $Q_c = (\bar{3}, 1, \bar{3})$ fields and their mirrors. The only difference from the model of ref.(6) is that we omit one $P$-invariant $\lambda, \bar{\lambda}$ pair in order to obtain the correct values of $\sin^2 \theta_w$ and $\alpha_s$ since we now assume a two step symmetry breaking chain. Therefore, there are seven $\lambda$’s, eight $Q$’s and $Q_c$’s, four $\bar{\lambda}$’s and five $\bar{Q}$’s and $\bar{Q}_c$’s. Under $P_1$, all color singlets remain invariant while all color triplets and antitriplets change sign. Under $P_2$, all fields remain invariant except $\lambda_4, \lambda_7, \bar{\lambda}_2$ and $\bar{\lambda}_4$ which change sign. Finally, under $P_3$, all fields remain invariant except $\lambda_4, \lambda_5, \lambda_6, \bar{\lambda}_2, \bar{\lambda}_3, Q_1, Q_2$ and $Q_3^c$ which change sign. At the scale $M_X$, the symmetry $\tilde{G}$ breaks down to $G \equiv G_{LR}$(by the VEVs $|<\lambda_6>|=|<\bar{\lambda}_3>|$) which, in turn, breaks down to $G_S$ (by the VEVs $|<\lambda_7>|=|<\bar{\lambda}_4>|$) at a scale $M_R$. Moreover, a $Z_2$ MP $C$ (the combination of $P_2$ with the center of $SU(2)_R$) commuting with $G_S$ survives as an exact symmetry. All $\tilde{G}$-non-singlet states acquire masses $\sim M_X$ except the states of the MSSM together with three right-handed neutrinos (with mass $\sim M_R^2/M_c$), the massive gauge supermultiplet associated with the generators of $SU(2)_R \times U(1)_{B-L}/U(1)_Y$ together with the higgs fields necessary for the breaking of $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$ and one $\ell, \bar{\ell}$ pair with a mass $\sim M_X^2/M_c$. This pair is identical under $C \times G_S$ with the $h^{(1)}, h^{(2)}$ pair. The situation coincides with the one encountered in the previous example after identification of $M$ with $M_X^2/M_c$. The only question that remains to be answered is whether there are values of $M_X$ and $M_R$ consistent with the measured values of $\sin^2 \theta_w$ and $\alpha_s$ and
satisfying the inequality $M' \equiv M_R \ll M \equiv M_X^2/M_c$. It turns out that, for $M_X = 10^{17.1}$ GeV and $M_R = 10^{15}$ GeV, the one-loop RG equations give the perfectly acceptable values $\sin^2 \theta_w(M_Z) = 0.232$ and $\alpha_s(M_Z) = 0.121$.

For the above values, the expected departure from unity of the coefficient $\rho$, in the relation $m_t/m_b = \rho \tan \beta$, is only $\approx \frac{1}{2}(M_R/M)^2 = 0.0126$ which is sufficiently small.

It is, perhaps, appropriate at this point to make some remarks concerning the implications of the relation $\tan \beta \approx m_t/m_b \approx m^{D}_{\nu_\tau}/m_\tau$ for neutrino masses. The large value of $\tan \beta$ implied by this relation forces $m^{D}_{\nu_\tau} \sim 100$ GeV. Consequently, in order for $m_{\nu_\tau}$ to satisfy the cosmological bounds, the Majorana mass of $\nu^c_\tau \gtrsim 10^{12}$ GeV. This fact certainly places a lower bound on the $SU(2)_R$ symmetry breaking scale $M_R$. In the very common case in which the Majorana $\nu^c$ mass is due to non-renormalizable terms, this bound on $M_R$ is $\sim 10^{15}$ GeV. $\nu^c$ Majorana masses typically $\sim 10^{12}$ GeV and $m^{D}_{\nu_\tau} \sim 100$ GeV offer the exciting possibility that $\nu^c_\tau$’s contribute significantly to the ”hot” component of the dark matter of the universe. At the same time the two lighter neutrinos could have masses small enough to be consistent with the MSW solution of the solar neutrino problem.

In summary, we investigated the possibility that the well-known free parameter $\tan \beta$ of the MSSM can be determined by embedding MSSM in a SUSY GUT. Sufficient conditions for the relation $\tan \beta \approx m_t/m_b \approx m^{D}_{\nu_\tau}/m_\tau$ to hold were given and the importance of the $SU(2)_R$ gauge symmetry in this connection was emphasized. We discussed examples of such SUSY GUTs in some detail with particular emphasis on semi-simple gauge groups.
References

1. J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B249 (1990) 441; V. Amaldi, W. de Boer and H. Furstenan, Phys. Lett. B260 (1991) 447; P. Langacker and M.X. Luo, Phys. Rev. D44 (1991) 817.

2. B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. D44 (1991) 1613; H. Arason, D.J. Castaño, B.E. Keszhelyi, S. Mikaelian, E. J. Piard, P. Ramond and B.D. Wright, Phys. Rev.Lett. 67 (1991) 2933; S.Kelley, J.L.Lopez and D.V. Nanopoulos, Phys. Lett B274(1992) 387.

3. B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Lett. B300 (1993)245; V.Barger, M.S. Berger and P. Ohmann, Wisconsin Univ. preprint MAD/PH/801 (1993).

4. L.E. Ibáñez and G.G.Ross, Nucl.Phys. B368 (1992)3.

5. L.J.Hall, R. Rattazzi and U.Sarid, LBL preprint 33997(1993).

6. G. Lazarides and C. Panagiotakopoulos, Thessaloniki Univ. preprint UT-STPD-2-93(1993).