THEORETICAL ASPECTS OF AZIMUTHAL AND TRANSVERSE SPIN ASYMMETRIES

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We use Lorentz invariance and the QCD equations of motion to study the evolution of functions that appear at leading (zeroth) order in a 1/Q expansion in azimuthal asymmetries. This includes the evolution equation of the Collins fragmentation function. The moments of these functions are matrix elements of known twist two and twist three operators. We present the evolution in the large \( N_c \) limit, restricted to the non-singlet case for the chiral-even functions.

In this contribution I want to present one possible way to investigate the QCD evolution of azimuthal asymmetries. These asymmetries appear in hard scattering processes with at least two relevant hadrons and constitute a rich phenomenology, suitable for studying quark and gluon correlations in hadrons. By relevant hadrons we mean hadrons used as target or detected in the final state. A well-known azimuthal asymmetry appears in the semi-inclusive deep inelastic polarized lepton production of pions (\( ep^+ \rightarrow e'\pi X \)) generated by the so-called Collins effect. This asymmetry is one of the possibilities to gain access to the so-called transversity or transverse spin distribution function, which is the third distribution function needed for the complete characterization of the (collinear) spin state of a proton as probed in hard scattering processes. In contrast to the transversity function, the evolution of the Collins fragmentation function had not been investigated so far. Knowledge of this evolution is indispensable for relating measurements at different energies.

For azimuthal asymmetries in processes like semi-inclusive lepton production, often appearing coupled to the spin of the partons and/or hadrons, it is important to take transverse momentum of partons into account, first studied by Ralston and Soper for the Drell-Yan process at tree level. Also the Collins function involves transverse momenta. Furthermore, it is a so-called T-odd function allowed because time-reversal symmetry does not pose constraints for fragmentation functions. Its evolution will be one of the new results presented here, although we limit ourselves to the large \( N_c \) limit, in which case

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the evolution for T-odd $p_T$-dependent functions is autonomous.

Factorization crucially depends on the presence of a large energy scale in the process, such as the space-like momentum transfer squared $q^2 = -Q^2$ in lepton production. In this paper we will be concerned with functions that appear in processes which have, apart from such a hard scale, an additional soft momentum scale, related to the transverse momentum of the partons. In one-hadron inclusive lepton production this scale appears because one deals with three momenta: the large momentum transfer $q$, the target momentum $P$ and the momentum of the produced hadron $P_h$. The noncollinearity at the quark level appears via $q_T = q + x_B P - P_h / z_h$, where $x_B = Q^2 / 2 P \cdot q$ and $z_h = P \cdot P_h / P \cdot q$ are the usual semiinclusive scaling variables, at large $Q^2$ identified with lightcone momentum fractions. The hadron momenta $P$ and $P_h$ define in essence two lightlike directions $n_+$ and $n_-$, respectively. The soft scale is $Q^2_T = -q^2_T$.

To study the scale dependence of the various distribution and fragmentation functions appearing in these (polarized) processes we construct specific moments in both $p_T$ and $x$, employ Lorentz invariance and use the QCD equations of motion. The moments in $x$ for leading (collinear) distribution functions (appearing for instance in inclusive lepton production) are related to matrix elements of twist two operators. On the other hand, for the transverse moments entering the azimuthal asymmetry expressions of interest, one finds relations to matrix elements of twist two and twist three operators, for which the evolution, however, is known. In the large $N_c$ limit this evolution becomes particularly simple.

In hard processes the effects of hadrons can be studied via quark and gluon correlators. In inclusive deep inelastic scattering (DIS), these are lightcone correlators depending on $x \equiv p^+ / P^+$ of the type

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi} e^{i P^- \xi^-} \langle P, S | \psi_j(0) U(0, \xi^-) \psi_i(\xi^-) | P, S \rangle \bigg|_{LC}.$$  \hspace{1cm} (1)

where the subscript ‘LC’ indicates $\xi^+ = \xi^- = 0$ and $U(0, \xi)$ is a gauge link with the path running along the minus direction. The parametrization relevant for DIS at leading (zeroth) order in a $1/Q$ expansion is

$$\Phi^{\text{twist}^{-2}}(x) = \frac{1}{2} \left\{ f_1(x) \not{p}_+ + S_L g_1(x) \gamma_5 \not{p}_+ + h_1(x) \gamma_5 S_T \not{s}_\tau \not{p}_+ \right\},$$  \hspace{1cm} (2)

where longitudinal spin $S_L$ refers to the component along the same lightlike direction as defined by the hadron. Specifying also the flavor one also encounters the notations $q(x) = f_1^f(x)$, $\Delta q(x) = g_1^f(x)$ and $\delta q(x) = \Delta T q(x) = h_1^f(x)$. The evolution equations for these functions are known to next-to-leading order and
for the singlet $f_1$ and $g_1$ there is mixing with the unpolarized and polarized gluon distribution functions $g(x)$ and $\Delta g(x)$, respectively.

For DIS up to order $1/Q$ one needs also the $M/P^+$ parts in the parameterization of $\Phi(x)$,

$$\Phi^{\text{twist}^-}(x) = \frac{M}{2P^+} \left\{ e(x) + g_T(x) \gamma_5 P_T + S_L h_L(x) \gamma_5 \frac{[h_+^+, h_{-}^-]}{2} \right\} + \frac{M}{2P^+} \left\{ -i S_L e_L(x) \gamma_5 - f_T(x) e_T^\gamma \gamma_{\rho \sigma} S_{\rho \sigma} + i h(x) \frac{[h_+^+, h_{-}^-]}{2} \right\}$$

We have not imposed time-reversal invariance in order to study also the T-odd functions, which are particularly important in the study of fragmentation. The functions $e$, $g_T$ and $h_L$ are T-even, the functions $e_L$, $f_T$ and $h$ are T-odd. The leading order evolution of $e$, $g_T$ and $h_L$ is known \[\text{and for the non-singlet case this also provides the evolution of the T-odd functions $e_L$, $f_T$ and $h$ respectively, for which the operators involved differ only from those of the T-even functions by a $\gamma_5$ matrix. The twist assignment is more evident by connecting these functions to the Fourier transforms of matrix elements of the form $\langle P, S|U(0,\eta) i D^\gamma_\nu(\eta) U(\eta, \xi) \psi_i(\xi)|P, S\rangle$ via the QCD equations of motion.}

For a semi-inclusive hard scattering process in which two hadrons are identified (in either initial or final state) the treatment of transverse momentum is important. Instead of light-cone correlations one needs light-cone correlations (where only $x^n = 0$). The parametrization of the $x$ and $p_T$ dependent correlator becomes

$$\Phi(x, p_T) = \frac{1}{2} \left\{ f_1(x, p_T^2) \gamma_5 + f_{1T}(x, p_T^2) \frac{\gamma_5^\nu n_+^\mu p_+^\rho S_+^\gamma}{M} \right\} - g_{1L}(x, p_T^2) \gamma_5 - h_{1T}(x, p_T^2) i \sigma_{\mu \nu} \gamma_5 S_+^\nu n_+^\mu$$

We used the shorthand notation $g_{1L}(x, p_T^2) \equiv S_L g_{1L}(x, p_T^2) + \frac{\langle p_T^2, S_T \rangle}{M} g_{1T}(x, p_T^2)$, and similarly for $h_{1L}$. The parameterization contains two T-odd functions, the Sivers function $f_{1T}$ and the function $h_{1T}$, the distribution function analogue of the Collins fragmentation function $H_{1T}$. The whole treatment of the fragmentation functions is analogous with dependence on the quark momentum fraction $z = P^-_h/k^-$ and $k_T$. We use capital letters for the fragmentation functions. At measured $q_T$ one deals with a convolution of two transverse momentum dependent functions, where the transverse momenta of the partons

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from different hadrons combine to \( q_T \). A decoupling is achieved by studying cross sections weighted with the momentum \( q_T^\alpha \), leaving only the directional (azimuthal) dependence. The functions that appear in that case are contained in \( \Phi^\alpha_B(x) = \int d^2p_T \frac{p_T^\alpha}{M} \Phi(x, p_T) \) which projects out the functions in \( \Phi(x, p_T) \) where \( p_T \) appears linearly,

\[
\Phi^\alpha_B(x) = \frac{1}{2} \left\{ -g_{1T}^{(1)}(x) S_T^\alpha \not{n} \gamma_5 - S_L \not{h}^{\perp 1}(x) \frac{[\gamma^\alpha, \not{n} \gamma_5]}{2} \\
- f_{1T}^{\perp 1}(x) \epsilon_{\mu \nu \rho \gamma} \gamma^{\mu} n^{\nu} S_T^\rho - i h_1^{\perp 1}(x) \frac{[\gamma^\alpha, \not{n} \gamma_5]}{2} \right\},
\]

and transverse moments are defined as \( f^{(n)}(x) = \int d^2p_T \left( \frac{p_T^\alpha}{M^2} \right)^n f(x, p_T) \).

At this point one can invoke Lorentz invariance as a possibility to rewrite some functions. All functions in \( \Phi(x) \) and \( \Phi^\alpha_B(x) \) involve nonlocal matrix elements of two quark fields. Before constraining the matrix elements to the light-cone or lightfront only a limited number of amplitudes can be written down. This leads to the following Lorentz-invariance relations

\[
g_{1T} = g_1 + \frac{d}{dx} g_{1T}^{(1)}, \quad h_1^{\perp 1} = h_1 - \frac{d}{dx} h_{1L}^{\perp 1}, \quad f_{1T}^{\perp 1} = f_{1T} + i \frac{d}{dx} f_{1T}^{(1)}, \quad h = - \frac{d}{dx} h_1^{\perp 1}.
\]

From these relations, it is clear that the \( p_T^2/2M^2 \) moments of the \( p_T \)-dependent functions, appearing in \( \Phi^\alpha_B(x) \), involve both twist-2 and twist-3 operators.

Another useful set of functions is obtained as the difference between the correlator \( \Phi_D(x) \) which via equations of motion is connected to \( \Phi^{\text{twist-3}} \) and \( \Phi_\partial \). This difference corresponds in \( A^+ = 0 \) gauge to correlators \( \Phi_A \), involving \( \langle P, S | \overline{\psi}_j(0) \mathcal{U}(0, \eta) A^\alpha_v(\eta) \mathcal{U}(\eta, \xi) \psi_i(\xi) | P, S \rangle \). The difference defines interaction-dependent (tilde) functions,

\[
x g_T(x) - \frac{m}{M} h_1(x) - g_1^{(1)}(x) + i \left[ x f_T(x) + f_{1T}^{(1)}(x) \right] = x \tilde{g}_T(x) + i x \tilde{f}_T(x), \quad x h_L(x) = x h_L + 2 h_{1L}^{\perp 1}(x) - i x e_L(x) = \tilde{h}_L(x) - i \tilde{e}_L(x),
\]

\[
x e(x) - \frac{m}{M} f_1(x) + i \left[ x h(x) + 2 h_1^{\perp 1}(x) \right] = \tilde{e}(x) + i \tilde{h}(x).
\]

Using the equations of motion relations in Eqs. (8) - (11) and the relations based on Lorentz invariance in Eqs. (6) - (7), it is straightforward to relate
the various twist-3 functions and the $p_T^2/2M^2$ (transverse) moments of $p_T$-dependent functions. The results e.g. for the $h$-functions are (omitting quark mass terms) are

$$h_L(x) = 2x \int_x^1 dy \frac{h_1(y)}{y^2} + \left[ \tilde{h}_L(x) - 2x \int_x^1 dy \frac{\tilde{h}_L(y)}{y^2} \right], \quad (11)$$

$$\frac{h_L^{(1)}(x)}{x^2} = -\int_x^1 dy \frac{h_1(y)}{y^2} + \int_x^1 dy \frac{\tilde{h}_L(y)}{y^2}, \quad (12)$$

$$h(x) = \left[ \tilde{h}(x) - 2x \int_x^1 dy \frac{\tilde{h}(y)}{y^2} \right], \quad (13)$$

$$\frac{h_L^{(1)}(x)}{x^2} = \int_x^1 dy \frac{\tilde{h}(y)}{y^2}. \quad (14)$$

Actually, we need not consider the T-odd functions separately. They can be simply considered as imaginary parts of other functions, when we allow complex functions. In particular one can expand the correlation functions into matrices in Dirac space $\gamma_5$ to show that the relevant combinations are $(g_{1T} - i f_{1T})$ which we can treat together as one complex function $g_{1T}$. Similarly, we can use complex functions $(h_{1L}^{+} + i h_{1L}^{-}) \to h_{1L}^{\pm}$, $(g_T + i f_T) \to g_T$, $(h_L + i h) \to h_L$, $(e + i e_L) \to e$. The functions $f_1$, $g_1$ and $h_1$ remain real, they don’t have T-odd partners.

As mentioned the evolution of the twist-2 functions and the tilde functions in known. The twist-2 functions have an autonomous evolution of the form

$$\frac{d}{d\tau} f(x, \tau) = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 dy \frac{P[f]}{y} f(y, \tau), \quad (15)$$

where $\tau = \ln Q^2$ and $P[f]$ are the splitting functions. In the large $N_c$ limit, also the tilde functions have an autonomous evolution. Using the relations given above, we then find the evolution of the transverse moments,

$$\frac{d}{d\tau} g_{1T}^{(1)}(x, \tau) = \frac{\alpha_s(\tau)}{4\pi} N_c \int_x^1 dy \left\{ \left[ \frac{1}{2} \delta(y - x) + \frac{x^2 + xy}{y^2(y - x)_+} \right] g_{1T}^{(1)}(y, \tau) + \frac{x^2}{y^2} g_1(y, \tau) \right\}, \quad (16)$$

$$\frac{d}{d\tau} h_{1L}^{\perp(1)}(x, \tau) = \frac{\alpha_s(\tau)}{4\pi} N_c \int_x^1 dy \left\{ \left[ \frac{1}{2} \delta(y - x) + \frac{3x^2 - xy}{y^2(y - x)_+} \right] h_{1L}^{\perp(1)}(y, \tau) \right\}.$$
Next we note that apart from a $\gamma_5$ matrix the operator structures of the T-odd functions $f_{1T}^{(1)}$ and $h_{1T}^{(1)}$ are in fact the same as those of $g_{1T}^{(1)}$ and $h_{1L}^{(1)}$ (or as mentioned before, they can be considered as the imaginary part of these functions). This implies that for the non-singlet functions, one immediately obtains the (autonomous) evolution of these T-odd functions. In particular we obtain for the Collins fragmentation function (at large $N_c$),

$$\frac{d}{d\tau} z H_1^{(1)}(z, \tau) = \frac{\alpha_s}{4\pi} N_c \int_z^1 dy \left[ \frac{1}{2} \delta(y - z) + \frac{3y - z}{y(y - z)\tau} \right] y H_1^{(1)}(y, \tau),$$

(18)

which should prove useful for the comparison of data on Collins function asymmetries from different experiments, performed at different energies.

Summarizing, we have obtained evolution equations of the $p_T$-dependent functions that appear in asymmetries and that are not suppressed by explicit powers of the hard momentum. But as functions of transverse momentum they are not of definite twist

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