Data mining and data driving of harmonic in AC arc furnaces based on functional analysis

P Deng¹, X Xu²,³, F Y Wang¹ and Z Chen²
¹Power Grid Planning and Research Centre, Guizhou Power Grid Corporation, Guiyang Guizhou, 550002, China
²College of Electrical Engineering, Guizhou University, Guiyang Guizhou, 550025, China
E-mail: 1055376461@qq.com

Abstract. This paper use big data technology, mine synchronous harmonic measuring data of AC arc-furnaces whole production process with functional analysis, verifies that harmonic current released from AC arc-furnaces is in independence and orthogonality approximately between each other. The discovered knowledge shows that multi-arc-furnaces harmonic current can be largely counteracted each other. If these AC arc-furnaces were connected to a PCC in which short-circuit capacity is enough for that the superposed harmonic current of AC arc-furnaces can be electromagnetic compatible, then power quality of the PCC can reach the qualified level discovered knowledge further reveals the model of AC arc-furnace, disorder data stream generated from harmonic measuring data is derived to calculation composition harmonic current at PCC(Point of Common Connection) in which AC arc-furnaces were connected.

1. Introduction
AC electric arc furnace (EAF) [1] is a load with a large amount of harmonic emission. When the public access point of power grid is connected to the load of EAF, serious harmonic problems often occur, which requires a great deal of cost for harmonic control.

In order to realize the idea of pre-evaluation, it is necessary to solve the problem of synthesizing harmonic currents released by several AC arc furnaces. It is well known that harmonic current synthesis is not arithmetic and superposition. A general formula for harmonic current synthesis is recommended in the annex of harmonic of public power grid (GB/T 14549-93):

\[ I_h = \sqrt{I_{h1}^2 + I_{h2}^2 + K_h \cdot I_{h1} \cdot I_{h2}} \]  

(1)

The relational coefficient KH is taken from table 1.

| Harmonic number | 3    | 5    | 7    | 9    | 11   | 13   | 9, >13, even order |
|-----------------|------|------|------|------|------|------|-------------------|
| \( K_h \)       | 1.62 | 1.28 | 0.72 | 0.18 | 0.08 | 0    |                   |

References [2-5] point out that formula (1) is too large for AC arc furnace. If the filter design is based on the calculation conclusion, it will cause a lot of waste.
Based on the arc column model, the AC arc furnace model is established in reference [6], which can reflect the fast change process of single arc furnace current. However, this deterministic physical model does not possess the random statistical characteristics of arc furnace harmonics and cannot be applied to the harmonic synthesis of multiple arc furnaces.

2. Synchronized measurement of harmonic current in AC arc furnace

On the high voltage side of the electric furnace, synchronous measurements of the harmonic currents in the same phase of two AC arc furnaces A and B, and the combined harmonic currents in the same phase of the power supply line [7] and the bus voltage are carried out (see figure 1).

![Figure 1. The wiring diagram of synchronous measuring harmonic current in AC arc furnaces.](image)

The $h$ sub harmonic current [8] of AC arc furnace A is at $t=1,2,...$. The complex time series [9] of n-time measurements is defined as the generalized function

$$f_{hA}(t) = I_{hA}(t)\cos\varphi_{hA}(t) + j \cdot I_{hA}(t)\sin\varphi_{hA}(t)$$

$t=1, 2... n$ measuring moments; 2 to 50 harmonics are measured at each time, and $h$ is the number of harmonics.

$f_{hA}(t)$ is a complex sequence [10], arranged in the order of measurement, in which $\varphi_{hA}(t)$ is the lag angle [11] between $h$-second harmonic current and $h$-second harmonic voltage current of bus bar, and $I_{hA}(t)$ is the $h$-second harmonic current of furnace A at $t$ time.

3. Data mining and knowledge discovery of synchronized harmonic current measurement sequences

When observing the alternating current arc furnace, the author finds that the brightness and path of the arc passing through the furnace charge [12] are completely irregular, and there is no similarity between the sound produced by harmonics and the change of the sound produced by harmonics.

In order to confirm these conjectures [13], the harmonic synchronization test of several AC arc furnaces is used for analysis.

correlation coefficients and regularized correlation coefficients (Pearson correlation coefficients) are used to process scalar sequences.

$$\rho(x, y) = \frac{E[(x - E(x))(y - E(y))]}{\sqrt{E[(x - E(x))^2]} \cdot \sqrt{E[(y - E(y))^2]}}$$

$x = f_{hA}(t), y = f_{hB}(t)$

$\rho(x, y)$ is also a complex number. It is conjugated with $\rho(y, x)$, the modulus of the two is the same. $x$ and $y$ are functional $f_{hA}(t)$ and $f_{hB}(t)$, respectively.

The modulus of the regularized co-correlation coefficient of complex values is between 0 and 1. The
correlation coefficient is 0, which indicates that the harmonics of the two electric arc furnaces are independent of each other.

Functional $f_{ia}(t)$ and $f_{ib}(t)$ are complex numbers belonging to n-dimensional unitary space.

The inner product of functional $f_{ia}(t)$ and $f_{ib}(t)$ is:

$$\langle f_{ia}(t) \cdot f_{ib}(t) \rangle = \frac{1}{\Delta} \int_{t} f_{ia}(t) \times \overline{f_{ib}(t)} \, dt$$

$$= \Delta \times \sum_{i=1}^{n} (f_{ia}(t) \times \overline{f_{ib}(t)})$$  \hspace{1cm} (4)

Among them, $\int f_{ia}(t) \overline{f_{ib}(t)} \, dt$ is the Lebesgue integral [14], $\overline{f_{ib}(t)}$ is the conjugate complex of $f_{ib}(t)$, and $t$ is the interval of harmonic measurement. Notice that the inner product space of $(f_{ia}(t) \overline{f_{ib}(t)})$ belongs to Hilbert space.

Norms of functional $f_{ia}(t)$ and $f_{ib}(t)$ in Hilbert space:

$$\left\{ f_{ia}(t) \right\} = \sqrt{\int f_{ia}(t) \times \overline{f_{ia}(t)} \, dt} = \sqrt{\Delta \times \sum_{i=1}^{n} (i_{ia}(t))^{2}}$$  \hspace{1cm} (5)

In Hilbert space, the mutual angle $\Phi$ of functional $f_{ia}(t)$ and $f_{ib}(t)$ is:

$$\cos \Phi = \frac{\langle f_{ia}(t) \cdot f_{ib}(t) \rangle}{\left| f_{ia}(t) \right| \times \left| \overline{f_{ib}(t)} \right|} = \frac{\sum_{i=1}^{n} (f_{ia}(t) \times \overline{f_{ib}(t)})}{\sqrt{\sum_{i=1}^{n} (i_{ia}(t))^{2}} \times \sqrt{\sum_{i=1}^{n} (i_{ia}(t))^{2}}}$$  \hspace{1cm} (6)

If $\Phi = 90$ degrees, $f_{ia}(t)$ and $f_{ib}(t)$ are orthogonal to each other.

According to formula (6), the angle between the complex measurement sequence of the same harmonic current of two AC arc furnaces is calculated, as shown in table 2.

**Table 2.** Interaction angle $\Phi$ between measuring serials of same order harmonic current in two AC arc furnaces.

| Harmonic number | Two Similar Silicon-Manganese Furnaces | Two identical yellow phosphorus furnaces | A Silicon-manganese furnace and a ferrosilicon furnace |
|-----------------|---------------------------------------|------------------------------------------|------------------------------------------------------|
| 2               | 99.7°                                 | 100.2°                                   | 98.1°                                                |
| 3               | 93.0°                                 | 79.7°                                    | 82.2°                                                |
| 5               | 94.3°                                 | 82.8°                                    | 93.2°                                                |
| 7               | 93.3°                                 | 81.4°                                    | 92.0°                                                |
| 9               | 94.8°                                 | 86.6°                                    | 94.7°                                                |
| 11              | 93.7°                                 | 87.0°                                    | 89.7°                                                |
| 13              | 92.9°                                 | 85.6°                                    | 91.0°                                                |
| 15              | 90.8°                                 | 94.9°                                    | 88.3°                                                |
| 17              | 85.5°                                 | 85.1°                                    | 96.3°                                                |
| 19              | 88.7°                                 | 84.1°                                    | 91.5°                                                |

Table 2 shows that the angle of the complex sequence of harmonic currents of two AC arc furnaces with the same number of times is approximate to 90 degrees and is approximate to two orthogonal ones.

For example, $f_{ia}(t)$ and $f_{ib}(t)$ are mutually orthogonal functions [15]. According to the quotient height theorem, there are:
\[ |f_h(t)|^2 = |f_{ha}(t)|^2 + |f_{hb}(t)|^2 \]  

(7)

Simplify and get:

\[ \sum_{i=1}^{n} i_h^2(t) = \sum_{i=1}^{n} i_{ha}^2(t) + \sum_{i=1}^{n} i_{hb}^2(t) (h = 2, 3 \cdots 50) \]  

(8)

\( t \) is the time of harmonic measurement. There are \( n \) times in total.

The harmonic currents synthesized by more than two orthogonal AC arc furnaces (A, B... M) are as follows:

\[ \sum_{i=1}^{n} i_h^2(t) = \sum_{i=1}^{n} i_{ha}^2(t) + \sum_{i=1}^{n} i_{hb}^2(t) + \cdots + \sum_{i=1}^{n} i_{hM}^2(t) \]  

(9)

The formula of harmonic synthesis (9) shows that the harmonic current of AC arc furnace is synthesized according to the root of square sum.

4. Harmonic data drive

Set up 1~\( m \) AC electric arc furnaces to connect to the common access points of the power grid, and the harmonic impedance [16] of the common access points of the power grid is inductive. Its h-harmonic impedance is:

\[ X_h = 2\pi hL = hX_i \]  

(10)

The total harmonic distortion rate of voltage of public access point \( THD_u \) is:

\[ THD_u = \sqrt{\sum_{i=1}^{n} U_s^2} \sqrt{\sum_{i=1}^{n} (X_s - i_s)^2} \]

\[ = \frac{X_i \sum_{i=1}^{n} (h \cdot i_s)^2}{U_s} = \sqrt{\sum_{i=1}^{n} (h \cdot i_s)^2} \]  

(11)

In this paper, \( \sum_{i=1}^{n} (h \cdot i_s)^2 \) is called equivalent harmonic current \( I_H \).

The relationship between \( I_H \) and total harmonic distortion rate of voltage is as follows:

\[ THD_u = \frac{I_H}{I_s} \]  

(12)

Formula (13) can be used to evaluate the voltage harmonic level of PCC.

\[ \sum_{i=1}^{n} I_H^2(t) = \sum_{i=1}^{n} I_{HA}^2(t) + \sum_{i=1}^{n} I_{HB}^2(t) + \cdots + \sum_{i=1}^{n} I_{HM}^2(t) \]  

(13)

In statistical sense, the equivalent harmonic current is also synthesized according to the quotient high theorem. But only the mean value of synthesized equivalent harmonic current can be obtained by formula (13).

By changing the order of harmonic data set by random generator, the test sequence of equivalent harmonic current data of AC arc furnace A is transformed into equivalent harmonic disordered data stream \( I_{Hs} \).

\[ \overline{I_{Hs}} = \sqrt{\sum_{k=1}^{n} h^2 \cdot \overline{i_{h}}^2(k)} \quad k \in [1, n] \]  

(14)
The equivalent harmonic current $I_H$ injected into PCC is synthesized by using the equivalent harmonic disordered data streams $I_{HA}$, $I_{HB}$ and $I_{HM}$ of each AC arc furnace according to the quotient high law.

$$I_H = \sqrt{I_{HA}^2 + I_{HB}^2 + ... + I_{HM}^2}$$  \hspace{1cm} (15)

Synthesis algorithm is shown in figure 2.

**Figure 2.** Synthesis of disorder data streams of Multiple AC Arc furnaces.

**Figure 3.** Algorithm of harmonic current synthesis error.

5. **Errors in harmonic current synthesis and harmonic voltage evaluation**

According to the measured total harmonic current, the error of $I_{H95\%}$ synthesis algorithm is evaluated and analyzed. The analysis algorithm is shown in figure 3.

In this algorithm, 1000 $I_{H95\%}$ synthetic data are plotted as interval number distributions, as shown in figures 4-6.

From figures 4-6, the error between calculated and measured values of synthetic $I_{H95\%}$ is obtained, as shown in table 3.

**Figure 4.** The histogram of synthesis harmonic current of two silicon manganese arc-furnace harmonic test data of different types of ac arc-furnace.

**Figure 5.** The histogram of synthesis harmonic current of two yellow phosphorus arc-furnace.
Figure 6. The histogram of synthesis harmonic current of a silicon manganese arc furnace and a silicon iron arc furnace.

Table 3. The calculated value, measured value and error of \( I_{95\%} \).

|                      | Two Silicon Manganese Furnaces | Two yellow phosphorus furnaces | A silicon manganese, A ferrosilicon |
|----------------------|--------------------------------|-------------------------------|-------------------------------------|
| Synthesis of \( I_{95\%} \) | 90.83~96.41                    | 18.08~20.54                   | 72.29~79.22                         |
| Measured \( I_{95\%} \)    | 92.02                           | 19.83                         | 72.53                               |
| error                 | -1.3%~4.7%                      | -8.8%~3.6%                    | 0.3%~9.6%                           |

The error in table 3 corresponds to the relative error of confidence interval (2.5%~97.5%) [17]. The maximum relative error is -8.8%~9.6%.

According to the algorithm of figure 2, the equivalent harmonic current \( I_{95\%} \) synthesized by 34 AC arc furnaces is 106.05 A. The total harmonic distortion rate [18] of 35 kV bus voltage is evaluated as follows:

\[
THD_u = \frac{I_u}{I_s} = \frac{106.05}{2813} = 3.77\%
\]

It is concluded that although the number of AC electric arc furnaces in the plant is large, the power quality is qualified and can be directly connected to the power grid without taking control measures.

The plant has 25 AC arc furnaces in operation. The measured value of \( T_{h\%} \) of 35 kV bus voltage is 3.2%. The injected 35 kV equivalent harmonic current [19] \( T_{h\%} \) of 25 AC arc furnaces is 94.24 A, and the voltage harmonic level \( T_{h\%} \) is evaluated as 94.24 A.

\[
THD_u = \frac{I_u}{I_s} = \frac{94.24}{2813} = 3.35\%
\]

The evaluation is very close to the measured data, which confirms the correctness of the data driving method for harmonics of AC arc furnace.

6. Conclusions

There is orthogonality and independence between the harmonic currents in the same phase of several AC arc furnaces. The knowledge discovery shows that the harmonic currents of several AC arc furnaces are synthesized according to the theorem of equal height. With the increase of the number of AC arc furnaces, the synthesized harmonic currents increase little. Therefore, concentrating multiple AC [20] arc furnaces in an independent power supply area can largely offset the harmonics released by each of the multiple electric arc furnaces, and then connect the area planning of the electric arc furnaces to a public access point with relatively large short-circuit current [21]. Through the electromagnetic compatibility of the electric network, the harmonic power quality of the area of the AC arc furnaces [22]...
can be combined without harmonic control measures.

**Acknowledgments**

The authors gratefully acknowledge the support of the National Natural Science Fund of China (51567005) and Guizhou Province Joint Fund Project (LH [2017]7230, [2017]5788). Guizhou Science and Technology Innovation Talents Team Project [2018]5615.

**References**

[1] GB/T14549-1993 Quality of Electric Supply 1993 Harmonics in public supply network Chinese Standard SN for Books (Beijing, China: China Standard Press).

[2] Bai H Z, Xu R and Xu R L 2007 Approach of forecasting evaluation methodology to harmonic wave emitting level of current electric arc furnace Inner Mongolia Electric Power 25 26-9

[3] Hu W, Zha X M and Sun J J 2006 Harmonic sources modeling and harmonic superposition simulation of power system with multi-harmonic sources Electric Power 39 61-5

[4] Zhang W and Yang H G 2004 A method for assessing harmonic emission level based on binary linear regression Pro. CSEE 24 50-3

[5] Hui J, Yang H G, Lin S F et al 2009 Assessment method of harmonic emission level based on covariance characteristic of random vectors Autom. Electr. Power Syst. 33 27-31

[6] Zhang X 2016 Real-time active and reactive power regulation in power systems with tap-changing transformersand controllable loads Sustainable Energy, Grids and Networks 5 27-38

[7] Liu X H and Yang X Y 2006 Research on harmonic analysis of arc furnace electric system based on frequency domain method. Pro. CSEE 26 30-5

[8] Sherman W G 1972 Summation of harmonics with random phase angles IEE Proc 119 1643-8

[9] Zhang J 1995 Studies of harmonics superposition method in multi-harmonic sources system Power Syst. Technol. 19 23-7

[10] Zhang D X, Miao X, Liu L P et al 2015 Research on development strategy for smart grid big data Pro. CSEE 35 2-11

[11] Hey T, Tansley S and Tolle K (edi) 2009 The Fourth Paradigm: Data-Intensive Scientific Discovery. (USA: Microsoft)

[12] Li G J 2015 Further understanding of big data Big Data Research 01 65-7

[13] Weng L M, Chen Y P and Shu L P 2004 Influence of electric arc steel furnace on the power system and its suppression Power Syst. Technol. 28 64-7

[14] Cheng W H and Lai X R 2001 Covariance and correlation coefficient of complex random variables. Journal of the CUN (Natural Sciences Edition) 10 19-21

[15] Luo C, Wang X D and Zhang Y S 2010 Orthogonality and independence-new thinking of dealing with complex systems series three. J. Shanghai Institute Technol. (Natural Science) 10 271-7

[16] Zhao H G 2005 Introduction to Functional Analysis (Sichuan, China: Sichuan University Press) pp 95-6

[17] Cheng Q X, Zhang D Z, Wei G Q 2004 Real Variable Function Theoyt and Functional Analysis Foundation (2nd). (Beijing, China: Higher Education Press) 2 242-3.

[18] GB/T 1024.1-2000 2000 Electromagnetic compatibility-limits-assessment of emission limits for distorting loads in MV and HV power systems Chinese Standard SN for Books (Beijing, China: China Standard Press)

[19] Dong G Z and HE J H 2007 Causal analysis and countermeasure on harmonic resonance in local circuit of electric power systems. Power Syst. Prot. Contr. 35 77-80

[20] Baggini A and Hanzelka Z 2008 Handbook of Power Quality (Italy: John Wiley & Sons)

[21] Wakileh G J 2005 Power Systems Harmonics Fundamentals, Analysis and Filter Design (Beijing, China: China Machine Press)

[22] Yu X L, Li H M and Li T Y 2014 An harmonic and inter-harmonic detection method based on EEMD and TLS-ESPRIT. Power Syst. Prot. Contr. 42 67-72