Static calculation of the rotor unloading automatic machine for a high-pressure centrifugal pump

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Abstract. The article presents static calculation of an automatic machine for unloading of rotor axial forces in a high-pressure and high-speed pump. Analysis of the rotor displacement influence to values of mass flow rate and pressure in the unloading chamber of the unloading system is given. The main geometric parameters of the system are obtained.

1. Introduction
Most turbo pump units (TPU) are equipped with an automatic unloading machine (AUM) for the axial forces discharge, which consists of two tandem throttles (Figure 1) Throttle 5 is an annular gap which resistance is not influenced by the rotor axial displacement. The second throttle is a face gap, the resistance of which in turbulent flow regimes is inversely proportional to the axial gap value to the power of 1.5. In the first approximation, one can assume that radial vibrations of the impeller have no effect on the throttle’s resistance. Rotor displacement to the left side reduces the face gap 4 and increases pressure in the chamber 3. It increases axial force pressure acting on the impeller, and rotor shifts to the right, approaching to its initial state. Thus a face throttle 4 provides a negative feedback between rotor position and the balancing axial force [1]. Gaps 4 and 5 located between fixed and rotating surfaces pose a threat of scoring, since small force and temperature deformations, as well as deformations from centrifugal loads, can cause local contact of rubbing surfaces [2].

Figure 1. Traditional TPU scheme with automatic unloading machine.
Figure 2 shows a calculation scheme which differs from the traditional one, primarily by the feedback mechanism: unloading chamber is limited by two changing reciprocal slotted throttles. When rotor is moved to the right, the conductance $g_1$ of the upper throttle increases (hydraulic resistance decreases), and conductance of the lower one - $g_2$ decreases. Pressure $p_2$ in the unloading chamber increases, i.e. balancing force, returning rotor to its initial position increases. Scoring risk is much less due to reducing surface area forming the throttling groove.

High pressure values developed by the TPU cause turbulent flow regimes in the main hydraulic channel, so the relationship between mass flow rate and throttled pressure drop is characterized by a quadratic dependence

$$Q = g \sqrt{\Delta p}$$

where

- $\Delta p$ - throttled pressure drop in this section;
- $g$ - hydraulic conductance of the section.

Hydraulic system and conductance dependence of the rotor axial displacements significantly affect the AUM characteristics. Analytical representation of such dependences or consumption characteristics of the throttles are possible only for the channels having the simplest geometric form [3]. As a rule, it is necessary to use the results of experiments or perform numerical calculations with the help of appropriate computer programs.

Figure 2. Calculation scheme of a centrifugal stage with an automatic unloading machine (a) and a hydraulic channel scheme (b).
Basic geometric parameters of the AUM are selected based on the static calculation results. It comes down to the fact that using the flow balance equation of the series-connected throttles and rotor axial balance equation one can obtain a dependence of steady axial displacement on the discharge pressure, unbalanced force $T_0$ and rotational speed, i.e. static characteristics are built. Static calculation also makes it possible to determine hydrostatic stiffness of the system - dependence of the regulatory impact (pressure $p_2$ in the unloading chamber) on the adjustable gap in the throttles. Negative value of the hydrostatic stiffness coefficient is a sign of the system equilibrium stability [4,5].

Unit rotor together with the automatic unloading machine is a complex dynamic system with distributed parameters that is a subject to periodic external perturbations. Each cross section of the rotor performs interrelated radial, angular and axial oscillations [6]. In regards to reliability, primarily axial vibrations can be dangerous, since their relatively large amplitudes limit the operating life of ball bearings wherein rotor is located. In this paper, as a first approximation, a simplified problem is considered: a rotor is studied as a solid body with an auto-loading system which performs one-dimensional axial oscillations along the support axis. Such a simplified model enables to obtain static and dynamic characteristics in an analytical way and accurately describe the main regularities of the system oscillations.

2. Calculation of the axial forces acting on the impeller
Purpose of the static calculation is to select such basic geometric parameters so that in the given variation range of the changing balancing force $T_0$ face gaps $z_{1,2}$ and flow rate $Q$ values do not exceed the permissible limits. Analytical model of the automatic balancing system should take into account all the factors that have a significant effect on the system operation and at the same time it should not be cluttered with unnecessary details that complicate the analysis and give only minor quantitative corrections to the calculation results. When assessing the significance of certain factors, it must be borne in mind that an error in calculating equilibrated axial force, even in the nominal mode, can reach 50%, not to mention changing of this force during pump operation [7,8].

Balancing system will be considered as a hydro-mechanical automatic control system, for which the adjustable value is a size of the face gap. Regulatory impact $F_z$ is a resultant axial force of pressure acting on the outer side of the main disk (balancing force). External influences are the discharge pressure $p_1$ and rotor speed $\omega$. If a squeezing device is available the force of pre-compression of the springs $kA$ acts as a control input. Block diagram of the balancing device as an automatic control system is shown in Figure 3.

![Block diagram of the balancing system of axial forces.](image)

**Figure 3.** Structural scheme of the balancing system of axial forces.
Developing of static characteristics is based on the condition of equilibrium of the pressure axial forces acting on the impeller and the residual axial force $T_0$ on the side of the drive turbine [9]. The last force is considered to be given.

At present, an approximate estimate of the axial forces is obtained, as a rule, by taking the average liquid angular velocity across the chamber width to be stable: $\omega_c = 0.5\omega$ [10, 11]. A precise calculation of the angular velocity and pressure in the pockets of the impeller comes down to solving the problem of hydromechanics of a nonstationary three-dimensional turbulent flow with complex boundary conditions.

For preliminary estimation of axial forces we will use the recommendations of prof. V. Endrala [12], based on the numerical solution of the Reynolds equations of turbulent flow, both in the circumferential and radial directions. The recommendations’ main message is that the average liquid angular velocity across the gap width $\omega_c$ is expressed with the swirl coefficient $\kappa_0$:

$$\omega_c = \kappa_0 \omega$$  \hspace{1cm} (1)

Swirl coefficient of the flow depends primarily on the radial flow direction and for the relative widths of the chambers $h/r_2 = 0.02...0.04$ it has the following radial averaged values:

- flow from the periphery to the center $\kappa_1 = 0.7...0.9$,
- flow from the center to the periphery $\kappa_2 = 0.3...0.4$,
- In the absence of radial flow $\kappa_0 = 0.45...0.5$.

In our case, in both pockets, from the covering disk and from the main disk, the radial (feed) flow is directed from the periphery to the center, so both pocket flows will be assumed: $\kappa = \kappa_1$.

To calculate the axial forces acting on the impeller one should consider a section of the main disk $r_1 - r_2$ (chamber $B$, Figre 2a). Sections of individual disk parts are indicated by capital letters $A, A_0, B, C, D$.

To find pressure distribution along the radii of the disks it is necessary to consider equilibrium of the elementary liquid volume $dV$ (Figure 4), bounded by two cylindrical surfaces with radii $r$ and $r + dr$ and meridional planes placed at an angle $d\phi$ to each other: $dV = bdrd\phi$. The average angular velocity is assumed to be constant for all chambers with a swirl coefficient of 0.7: $\omega_c = \kappa_1 \omega = 0.7\omega$.

In the radial direction centrifugal force $dF_g = \omega^2 r^2 \rho dV$ is balanced by the pressure force $dP = b\rho \omega^2 r dr d\phi$. Comparing these forces, an equation $dP = \rho \omega_c^2 r dr$ is derived. Having integrated it considering the boundary condition $r = r_0$, $p = p_1$, it is obtained a parabolic law of pressure variation along the radius:

$$p_b = p_1 - 0.5 \rho \omega_c^2 (r_1^2 - r^2) = p_1 - \frac{1}{2} \rho \omega^2 \kappa_1^2 (r_1^2 - r^2)$$  \hspace{1cm} (2)

By analogy for the section $r_2 - r_3$ (chamber $C$) and for the left chamber $A$ the equations are the following:

$$p_c = p_2 - 0.5 \rho \omega_c^2 (r_2^2 - r^2) = p_2 - \frac{1}{2} \rho \omega^2 \kappa_1^2 (r_2^2 - r^2)$$

$$p_A = p_1 - 0.5 \rho \omega_c^2 (r_1^2 - r^2) = p_1 - \frac{1}{2} \rho \omega^2 \kappa_1^2 (r_1^2 - r^2)$$  \hspace{1cm} (3)
Pressure in the chamber $D$ and in the inlet orifice $A_0$ can be considered constant along the radius: $p_D = p_{i3} = const$. It is also assumed that pressure $p_i$ on the radius $r_1$ in both chambers is approximately the same.

Pressure values $p'_1, p'_2$ before the throttles differ from the pressure values $p_1, p_2$ at the inlet to the corresponding chamber by the amount of the inertial pressure:

$$p'_1 = p_1 - \frac{1}{2} \rho \omega^2 \kappa^2 (r_1^2 - r_4^2) = \frac{1}{2\pi} \rho \omega^2 \kappa^2 A$$

$$p'_2 = p_2 - \frac{1}{2} \rho \omega^2 (r_2^2 - r_3^2) = \frac{1}{2\pi} \rho \omega^2 \kappa^2 B$$

Pressure values $p_2, p_3$ after the throttles differ from pressure values $p'_1, p'_2$ before throttles by the amount of pressure being lost to overcome resistance of the corresponding slotted throttles. The approximate pressure diagrams for the main disk are shown in Figure 2.

From the obtained equations, it is evident that the average pressure and, correspondingly, the pressure force on the wheel disks decreases if liquid rotation frequency in the pocket increases. As the $\omega$ decreases, the pressure diagram becomes more complete. This peculiarity is widely used in various designs: to reduce unbalanced axial force, all possible measures are taken to increase the average frequency of liquid rotation in the back pocket and to reduce it in the front one [6].

Pressure in pockets is axisymmetric, so it is possible to summarize axial forces acting on the elementary annular sections $2\pi \rho dr$. As a result, pressure forces on the cover and main disks can be determined:

$$T_A = 2\pi \int_{r_6}^{r_1} p_A dr = \pi (r_1^2 - r_6^2) \left[p_1 - 0.5 \rho \omega^2 \kappa^2 (r_1^2 - r_4^2)\right] = A \left[p_1 - \frac{\rho \omega^2}{2\pi} \kappa^2 A\right]$$

$$T_{A_0} = \pi (r_6^2 - r_3^2) p_3 = A_0 p_3$$

$$T_B = 2\pi \int_{r_3}^{r_1} p_B dr = \pi (r_1^2 - r_3^2) \left[p_1 - 0.5 \rho \omega^2 \kappa^2 (r_1^2 - r_4^2)\right] = B \left[p_1 - \frac{\rho \omega^2}{2\pi} \kappa^2 B\right]$$

$$T_C = 2\pi \int_{r_3}^{r_1} p_C dr = \pi (r_1^2 - r_3^2) \left[p_2 - 0.5 \rho \omega^2 \kappa^2 (r_2^2 - r_3^2)\right] = C \left[p_2 - \frac{\rho \omega^2}{2\pi} \kappa^2 C\right]$$

$$T_D = \pi (r_1^2 - r_4^2) p_3 = D p_3.$$
where
\[ A = \pi \left( r_1^2 - r_6^2 \right), B = \pi \left( r_2^2 - r_3^2 \right), C = \pi \left( r_4^2 - r_5^2 \right), D = \pi \left( r_7^2 - r_8^2 \right) \]
\[ (7) \]

In the static equilibrium position the sum of all forces (5,6) is equal to the residual force \( T_0 \). This equation comes down to the form:
\[ Cp_2 = T_0 + (A - B)p_1 + (A_0 - D)p_3 - \frac{D0^2r^2}{2\pi} \left( A^2 - B^2 - C^2 \right) \]
\[ (8) \]

Pressure \( p_2 \) in equation (8) depends on the gaps in the slotted throttles, i.e. from the axial rotor position. Having derived \( p_2 \) from the flow balance equation one substitutes it in (8) and obtains an equation to find a static characteristic – a dependence \( z \) on the discharge and suction pressure, on the axial force \( T_0 \), and on the rotor speed.

3. Determination of the turbulent conductance of slotted throttles
The equation of flow rates through the upper and lower slotted throttles for the auto-modeling section of the turbulent flow has a form:
\[ \sqrt{p_1 - p_2 - p'_1} = g_2 \sqrt{p_2 - p_3 - p'_2} \]
\[ (9) \]

\( g_1, g_2 \) - turbulent conductance values that represent a separate problem to be solved. To solve this problem, a liquid oxygen flow in a system of two serial slotted throttles at a different size of the axial gap are simulated. Turbulence model \( k-\varepsilon \) was used in the calculation. As a result of each calculation, values of mass flow rate and averaged pressure \( p_2 \) in the chamber between the throttles were obtained, by means of which throttle turbulent conductance was calculated (Table 1). Figures 5 - 7 show the distribution of pressure \( p_2 \) for different axial gap values of the upper throttle.

Axial displacement of a rotor \( z \) and face gaps of the upper and lower throttles are interrelated by the following dependences: \( z_1 = z_n - \delta z, z_2 = z_\delta, z_\| = 0.65 \text{mm} \) is a maximum possible value of the face gap protecting another throttle from being overlapped.

![Figure 5. Pressure distribution \( p_2 \): a-\( z = 0.2 \text{mm} \); b-\( z = 0.25 \text{ mm} \).](image-url)
Based on the calculation results, summarized in Table 1, dependences of different throttles conductance on the rotor \( z \) displacement were obtained. Figure 8 shows that these dependences are close to linear, therefore it is adequate to take: 

\[
g_1 = a_1 u + b_1, \quad g_2 = a_2 u + b_2.
\]

\[
a_1 = -10.8 \times 10^{-6}, \quad b_1 = 14.26 \times 10^{-6}, \quad a_2 = 8.14 \times 10^{-6}, \quad b_2 = 2.49 \times 10^{-6}
\]

### Table 1. Numerical calculation of conductance

| Axial gap, \( z \) \( \times 10^{-6} \) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 |
|------------------------------------------|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| Mass flow rate, \( \text{kg/s} \)         | 18.6 | 19.9 | 21.7 | 23.8 | 26.0 | 28.0 | 29.6 | 30.6 | 30.8 | 30.0 | 28.4 | 26.4 | 25.5 | 23.9 |
| Average pressure, MPa                   | 30.4 | 29.9 | 29.3 | 28.3 | 27.1 | 25.2 | 23.0 | 20.2 | 16.8 | 13.7 | 10.5 | 8.1  | 7.3  | 5.8  |
| Conductance \( g_1 \times 10^6, (m^2/kg)^{0.5} \) | 14.8 | 13.8 | 12.7 | 11.7 | 11.0 | 10.0 | 9.1  | 8.2  | 7.2  | 6.4  | 4.9  | 4.6  | 4.2  | 4.0  |
| Conductance \( g_2 \times 10^6, (m^2/kg)^{0.5} \) | 3.2 | 3.4 | 3.8 | 4.2 | 4.7 | 5.3 | 5.9 | 6.5 | 7.3 | 8.0 | 8.8 | 9.7 | 10.1 | 11.1 |
4. Static characteristics and hydrostatic stiffness of the system

First, it is necessary to square both parts (9) \( g_1^2(p_1 - p_2 - p'_1) = g_2^2(p_2 - p_3 - p'_2) \) and find pressure in the chamber \( C \):

\[
p_2 = \frac{g_1^2 p_1 + g_2^2 p_3}{g_1^2 + g_2^2} = \frac{(a_u + b_1)^2(p_1 - p'_1) + (a_u + b_2)^2(p_3 + p'_2)}{(a_u + b_1)^2 + (a_u + b_2)^2}
\]

(10)

Further, we substitute this expression in (8) and an equation to determine the relative dimensionless gap will look as following:

\[
\frac{C}{(a_u + b_1)^2 + (a_u + b_2)^2} \psi_1 + \frac{(a_u + b_2)^2}{(a_u + b_1)^2 + (a_u + b_2)^2} \psi_3 = T_0 + (\overline{A} - \overline{B}) \psi_1 + (\overline{A}_0 - \overline{D}) \psi_3 - \overline{K} (A^2 - B^2 - C^2) \Omega^2
\]

(11)

where

\[
\overline{A} = \frac{A}{A}, \overline{B} = \frac{B}{A}, \overline{C} = \frac{C}{A}, \overline{D} = \frac{D}{A}, \psi_1 = \frac{p_1}{p_n}, \psi_3 = \frac{p_3}{p_n}, \psi'_1 = \frac{p'_1}{p_n}, \psi'_3 = \frac{p'_3}{p_n}
\]

\[
T_0 = \frac{T_0}{Ap_n}, \overline{K} = \frac{\rho \alpha^2 \kappa^2}{2 \pi \overline{A} p_n}, \overline{\Omega} = \frac{\omega}{\alpha \overline{\nu}}.
\]

From the equation (10) a dependence of the relative numerical gap on the dimensionless pressure can be derived \( \psi_1 \):

\[
u = \frac{b_2}{a_1 - a_2} \frac{f(\psi_1) - \psi'_1 - \psi'_3}{\psi_1 - \psi'_1 - f(\psi_1)} - b_1
\]

(12)

where

\[
f(\psi_1) = T_0 + (\overline{A} - \overline{B}) \psi_1 + (\overline{A}_0 - \overline{D}) \psi_3 - \overline{K} (A^2 - B^2 - C^2) \Omega^2
\]
The resulting equation makes it possible to construct a dependence of rotor axial displacement on any of the external influences, and also to evaluate effect on the static characteristics of independent parameters.

Calculation example. Initial data:

\[ p_u = p_n = 31.7 \text{MPa}, \quad p_1 = 2.0 \text{MPa}, \quad \omega = \omega_n = 1963 \text{s}^{-1}, \quad \rho = 1102 \text{kg/m}^3, \]

\[ r_1 = 13 \text{mm}, \quad r_2 = 119 \text{mm}, \quad r_3 = 77 \text{mm}, \quad r_4 = 46 \text{mm}, \quad r_5 = 50 \text{mm} \]

It can be seen from Figure 9 that a working area of the discharge pressure when a pump operates at the nominal frequency starts at a value \(0.5\psi_1\), and at a nominal pressure \(p_n = 31.7\text{MPa}\) it is equal to 16.8 MPa. At \(p_1 = p_u = 31.7\text{MPa}\) \(u = 0.526\).

Derivative

\[ \frac{\partial \psi_2}{\partial u} = \frac{\partial}{\partial u} \left[ \frac{(a_u + b_1)^2(\psi_1 - \psi_1') + (a_u + b_2)^2(\psi_1 + \psi_1')}{(a_u + b_1)^2 + (a_u + b_2)^2} \right] \]

represents hydrostatic stiffness of the support-seal assembly in question. Negative value of stiffness is a sign of stability of the rotor equilibrium position.

\[ \frac{\partial \psi_2}{\partial u} = \left[ 2(a_u + b_1)(\psi_1 - \psi_1') + 2(a_u + b_2)(\psi_3 + \psi_3') \right] \left[ (a_u + b_1)^2 + (a_u + b_2)^2 \right]^{-1} \]

\[ \times \left[ (a_u + b_1)^2(\psi_1 - \psi_1') + (a_u + b_2)^2(\psi_3 + \psi_3') \right] \left[ 2(a_u + b_1) + 2a_2(a_u + b_2) \right] \]

or

\[ \kappa_2 = \frac{\partial \psi_2}{\partial u} = \frac{2a_1a_2(\psi_1 - \psi_1' - \psi_3 - \psi_3')(a_2b_2 - a_2b_1)(u + b_1)\left( u + \frac{b_1}{a_1} \right) + (u + b_2)\left( u + \frac{b_2}{a_2} \right)}{(a_u + b_1)^2 + (a_u + b_2)^2} \] (12)
Figure 10 shows that in the working section where \( \mu < 1.32 \) hydrostatic stiffness has a negative value and, consequently, equilibrium position of the rotor is stable.

5. Conclusion

The article presents a calculation of rotor axial vibrations for the pump with the automatic machine for unloading axial forces. To calculate static characteristics of a system there a numerical experiment was performed and there mass flow rates and pressure values in the discharge chamber depending on the rotor axial displacement (axial gap of the lower throttle) were obtained. Based on the results of numerical calculations, turbulent conductance of the upper and lower slotted throttles were calculated. This made it possible to obtain a dependence of the axial displacement on the discharge pressure and, as a result, determine axial gaps sizes of the slotted throttles, as well as the mass flow rate at the nominal operating mode of the pump. Therefore, at a nominal frequency of 18750 rpm mass flow rate is 25.4 kg/s, axial gap of the lower throttle is 0.34 mm, the upper one - 0.31 mm. At a minimum frequency of 10500 rpm mass flow rate is 12.1 kg/s, axial gap of the lower throttle is 0.34 mm, the upper one - 0.31 mm. At a maximum frequency of 21150 rpm, mass flow rate is 27.55 kg/s, axial gap of the lower throttle is 0.33 mm, the upper one - 0.32 mm. Thus, it can be stated that a face gap has almost constant value in the entire range of operating frequencies of the rotor. As it can be seen from the static characteristics (Figure 9), if discharge pressure increases, the face gap value increases, and when rotation speed goes up then the face gap value goes down. Analysis of hydrostatic stiffness (Figure 10) makes it possible to conclude that equilibrium position of the rotor is stable.

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