Constraints on Dark Matter in the Solar System

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Abstract – We have searched for and estimated the possible gravitational influence of dark matter in the Solar system based on the EPM2011 planetary ephemerides using about 677 thousand positional observations of planets and spacecraft. Most of the observations belong to present-day ranging measurements. Our estimates of the dark matter density and mass at various distances from the Sun are generally overridden by their errors ($\sigma$). This suggests that the density of dark matter $\rho_{dm}$, if present, is very low and is much less than the currently achieved error of these parameters. We have found that $\rho_{dm}$ is less than $1.1 \cdot 10^{-20} \text{ g cm}^{-3}$ at the orbital distance of Saturn, $\rho_{dm} < 1.4 \cdot 10^{-20} \text{ g cm}^{-3}$ at the orbital distance of Mars, and $\rho_{dm} < 1.4 \cdot 10^{-19} \text{ g cm}^{-3}$ at the orbital distance of the Earth. We also have considered the case of a possible concentration of dark matter to the Solar system center. The dark matter mass in the sphere within Saturn’s orbit should be less than $1.7 \cdot 10^{-10} M_\odot$ even if its possible concentration is taken into account.

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INTRODUCTION

At present, the subject of dark matter attracts rapt attention of physicists and astronomers. It is one of the most popular in theoretical and observational works concerning cosmology and studies of galactic structures. Dark matter in galaxies has long been discussed in stellar dynamics. Its existence was suggested by the virial paradox concerning galaxy clusters (Zwicky 1933; Karachentsev 1966) and a flat rotation curve for many spiral galaxies. For an explanation, it was hypothesized that an additional invisible mass was in the halos of galaxies and its value could exceed the visible one by several times. Massive halos (Flynn et al. 1996; Karachentsev 2001; Fridman and Khoperskov 2011) are generally included when describing galactic structures and in galactic models.

In present-day cosmological theories, it is hypothesized that the bulk ($\sim 73\%$) of the mean density of the Universe is accounted for by dark energy, while about 4% and 23% of the remaining part are accounted for by baryonic and dark matter, respectively (Kowalski et al. 2008; Komatsu et al. 2011; Keisler et al. 2011). The dark matter is deemed to be nonbaryonic in nature and its properties are speculative. It is believed that this matter is formed not from atoms, does not interact with ordinary matter through electromagnetic forces, and its particles carry no electric charge. Various hypothetical and exotic particles are proposed as candidates for dark matter (see the review by Bertone et al. (2005) and Peter (2012)). If the hypothesis about particles is correct, then our Galaxy, along with other galaxies, is immersed in a halo of such dark matter particles, and these particles with exceptional penetrability must be everywhere, including the Solar system and the Earth. Although the particles interact very weakly with matter, attempts are made to find them by rare interactions with atoms of ordinary matter. Experiments using special detectors and telescopes (CRESST, CoGeNT, DAMA, XENON100, PAMELA, FERMI, HESS, CDMS, ANTARES, WMAP, SPT, etc.) are carried out to find and investigate dark matter particles or traces of their interaction, and their data are carefully analyzed. New experiments are planned. The hypothetical particles can interact with ordinary matter through their elastic scattering by atomic nuclei (Goodman and Witten 1985), and various experiments are conducted to find this effect: CDMS II, Xenon100, Zeplin III, etc. The goal of these studies is an attempt to detect and measure the number of outlier events per unit energy, their time and angular dependences. These quantities are assumed to depend on the local density and
velocity distribution of dark matter particles.

Despite the possible absence or very weak interaction of dark matter with ordinary one, the dark matter must have gravitational properties. Since it can be in the Solar system, the dark matter can manifest itself through its gravitational influence on Solar system bodies.

Attempts to detect the influence of possible dark matter on the motion of bodies in the Solar system have already been made. Nordtwedt (1994) and Nordtwedt et al. (1995) used laser observations of the Moon and found an upper limit for the possible acceleration in the presence of dark matter: $3 \times 10^{-14}$ cm s$^{-2}$. There are several works where the effects are searched for in the motions of planets and other bodies in the Solar system (Anderson et al. 1989, 1995; Khriplovich and Pitjeva 2006; Iorio 2006; Khriplovich 2007; Frère et al. 2008). Table 1 lists previous estimates of the dark matter density $\rho_{dm}$ and $M_{dm}$ in the Solar system. The third column gives the distance $r$ in astronomical units (AU) from the Sun corresponding either to the distance at which the density $\rho_{dm}$ was estimated or the radius of the sphere within which the mass $M_{dm}$ was estimated. The goal of this paper is an attempt to detect the gravitational manifestation of dark matter or to give a constraining upper limit for the dark matter density and mass in the Solar system using a new version of the planetary ephemerides, EPM2011, and new observations of planets and spacecraft.

Table 1. Estimates of the dark matter density and mass in the Solar system

| Year | Authors | Distance $r$, AU | Density $\rho_{dm}$, g cm$^{-3}$ | Mass $M(r)_{dm} (M_{\odot})$ |
|------|---------|-----------------|-----------------|-------------------|
| 1989 | Anderson et al. | 19.2 | | $< 3 \times 10^{-6}$ |
| 1995 | Anderson et al. | 19.2 | | $< 5 \times 10^{-7}$ |
| | | 30.1 | | $< 3 \times 10^{-6}$ |
| 1996 | Gron and Soleng | 1.08 | $< 1.8 \times 10^{-16}$ | $< 2 \times 10^{-6}$ |
| | | 19.2 | | |
| 2006 | Khriplovich and Pitjeva | 1.52 | $< 3 \times 10^{-19}$ | |
| 2006 | Iorio | 1.52 | $< 4 \times 10^{-19}$ | |
| 2006 | Sereno and Jetzer | 1.52 | $< 3 \times 10^{-19}$ | |
| 2008 | Frère et al. | 1.52 | $< 3 \times 10^{-19}$ | |

POSSIBLE OBSERVATIONAL EFFECTS

If dark matter is present in the Solar system, then it should lead to some additional
gravitational influence on all bodies. The effect will depend on the density of dark matter, on its distribution in space, etc. Let us assume, as is usually done (Anderson et al. 1989, 1995; Gron and Soleng 1996; Khriplovich and Pitjeva 2006; Sereno and Jetzer 2006; Frère et al. 2008) that dark matter of an unknown nature is distributed in the Solar system spherically symmetrically relative to the Sun. Apart from the already accountable accelerations from the Sun, planets, asteroids, and trans-Neptunian objects (TNOs), any planet at distance $r$ from the Sun can then be assumed to undergo an additional acceleration from dark matter:

$$\ddot{r}_{dm} = -\frac{GM(r)_{dm}}{r^3}r,$$

(1)

where $M(r)_{dm}$ is the mass of the additional matter in a sphere of radius $r$ around the Sun.

Thus, if we assume that there is an extended gravitating medium in the Solar system, then when finding the central attractive mass (or the correction to the heliocentric gravitational constant $GM_{\odot}$) from observational data separately for each planet, we would obtain an increasing value of this mass in accordance with the additional mass within the sphere with the mean radius of the planetary orbit. With sufficiently accurate observational data and a sufficiently large amount of interplanetary matter, this dependence of the central attractive mass on the semimajor axis of the planetary orbit not only could be an indicator for the presence of dark matter but also could characterize the increasing amount of additional mass with distance from the Sun, i.e., the density distribution. In particular, the processing high-accuracy observations for Mars and Saturn located at different distances from the Sun (1.52 and 9.58 AU, respectively) could provide data on the presence or absence of an appreciable amount of dark matter between the orbits of these planets.

Another consequence of the presence of a continuous gravitating medium in interplanetary space is its influence on the motion of the perihelion of a planetary orbit. At a uniform density $\rho_{dm}$ of the gravitating medium filling the Solar system, the additional acceleration on a body will be proportional to $r$:

$$\ddot{r}_{dm} = -kr,$$

(2)

where $k$ is a constant coefficient related to the density of the medium $\rho_{dm}$: $k = 4/3\pi G\rho_{dm}$. $G$ is the gravitational constant. In other cases, the dependence on $r$ is more complex. If we
denote the energy and area integrals per unit mass by \( E \) and \( J \) and a spherically symmetric potential by \( U(r) \), then (Landau and Lifshitz 1988) the equation of motion along the radius \( r \) can be written as

\[
\dot{r} = \left(2[E + U(r)] - (J/r)^2\right)^{1/2},
\]

and the equation along the azimuth \( \theta \) is

\[
\frac{d\theta}{dr} = \frac{J/r^2}{(2[E + U(r)] - (J/r)^2)^{1/2}}.
\]

In the Newtonian two-body problem, the oscillation periods along the radius \( r \) (from the pericenter to the apocenter and back) and azimuth \( \theta \) around the center coincide, and the positions of the pericenters and apocenters are not shifted from revolution to revolution. In the general case of a spherically symmetric potential different from the central field of a point mass or a homogeneous sphere, the bounded trajectory is not closed and fills everywhere densely the flat ring between the pericenter and apocenter distances. Since the trajectory is not closed, the pericenter and apocenter positions are shifted from revolution to revolution:

\[
\theta_1 - \theta_0 = 2\int_a^b \frac{J/r^2}{(2[E + U(r)] - (J/r)^2)^{1/2}} dr.
\]

where \( a, b \) are the minimum and maximum radial distances, \( \theta_0, \theta_1 \) are the initial and next positions of the pericenter. The presence of an additional gravitating medium leads to a shorter radial period and a negative drift of the pericenter and apocenter positions (in a direction opposite to the planetary motion). The perihelion drift for uniformly distributed matter (\( \rho_{dm} = \text{const} \)) depends on the orbital semimajor axis \( a \) and eccentricity \( e \) (Khriplovich and Pitjeva 2006):

\[
\Delta \theta_0 = -4\pi^2 \rho_{dm} a^3 (1 - e^2)^{1/2} / M_\odot,
\]

where \( \Delta \theta_0 \) is the perihelion drift in one complete radial oscillation. Since the eccentricity \( e \) for the planets in the Solar system is small in most cases, the dependence on \( e \) is occasionally neglected in Eq. (6) for the perihelion drift, as was done in Fr`ere et al. (2008).

It should be taken into account that the Solar system has its own extended medium associated with the solar wind and plasma. The solar wind produces an almost spherically
symmetric distribution of the particle flux (Parker 1963) whose space density decreases rapidly with increasing distance from the Sun, becoming vanishingly small on the periphery. Data from interplanetary spacecraft revealed that the solar wind particle flux density changes approximately as $r^{-2}$, where $r$ is the distance from the Sun, up to the orbit of Jupiter (Parker 1963, 1968; Hundhausen 1972). The solar wind density is $10^{-23}$ g cm$^{-3}$ near the Earth’s orbit and decreases to $10^{-25}$ g cm$^{-3}$ at the distance of Saturn. The total mass of the solar wind plasma up to Saturn’s orbit is approximately $10^{-15}$. These values are too small to be detected at present. Provided that the dark matter exceeds appreciably these estimates, it becomes possible to find its manifestations and to separate its effects from the medium with ordinary properties associated with the Solar system.

The density and mass of the dark matter are more commonly estimated by assuming that it changes very slowly or is constant within the Solar system, i.e., by assuming its distribution to be uniform. The concentration of dark matter to the center and even its capture and direct fall to the Sun are assumed in a number of papers (Lundberg and Edsjö 2004; Peter 2009; Iorio 2010). The latter assumptions should be made with caution. Pitjeva and Pitjev (2012) found a secular decrease in the heliocentric gravitational constant $GM_\odot$: 

\[
\dot{GM}_\odot/GM_\odot \simeq (-5.0) \cdot 10^{-14} \quad \text{per year.}
\]

This is primarily due to the decrease in solar mass through radiation and the solar wind. Therefore, there is a stringent constraint on the amount of possible dark matter falling to the Sun. In any case, it is less than that assumed by Iorio (2010) by several orders of magnitude. A serious constraint on the possible presence of dark matter inside the Sun (no more than 2-5% of the solar mass) was also obtained by Kardashev et al. (2005), who carefully analyzed the physical characteristics of the Sun.

Whereas the integral effect of the entire additional mass in the volume up to the planetary orbit is important in estimating the change of the central attractive mass, the local effect of the gravitational field difference near the planetary orbit due to the presence of an additional gravitating medium is important in searching for an additional change in the perihelion position. The additional planetary perihelion precession is investigated by taking into account all other known effects affecting the perihelion drift. Note that in the case of a small change in the central mass with time, there is no precession of the pericenter and apocenter positions (Pitjeva and Pitjev 2012), but if there is also an additional gravitating medium, then a negative drift of the perihelion and aphelion occurs from revolution to
revolution in accordance with Eqs. (5) and (6). Since the growth of the perihelion shift is accumulated, this criterion (effect) can be fairly sensitive for testing the presence of additional matter.

**OBSERVATIONAL DATA AND THEIR PROCESSING**

Finding the effects related to the possible presence of dark matter in the Solar system requires using highly accurate observations and a careful allowance for other small effects that may turn out to be comparable to the sought-for ones. For example, the weak effect from solar oblateness on the motion of Mercury and on the drift of its perihelion may turn out to be of the same order of magnitude with the action of dark matter. Different parameters of the planetary ephemerides are estimated from processing observations of different types, from classical meridian ones to present-day radio and spacecraft observations (Pitjeva 2008). Here, we use the optical observations since 1913, when an improved micrometer was installed at the US Naval Observatory and the observations became more accurate (about 0″5), up to all the available present-day observations in 2011. It should be noted that the accuracies of present-day optical CCD observations and spacecraft trajectory observations reach, respectively, a few hundredths of an arcsecond and a few meters (at the distance of Saturn). Most of the observations were retrieved from the database of the US Jet Propulsion Laboratory (JPL) created by E.M. Standish and updated and maintained at present by W.M. Folkner: [http://iau-comm4.jpl.nasa.gov/plan-eph-data/index.html](http://iau-comm4.jpl.nasa.gov/plan-eph-data/index.html). These data were supplemented with the Russian radar observations of planets (1961-1995), [http://www.ipa.nw.ru/PAGE/DEPENDFUND/LEA/ENG/rrr.html](http://www.ipa.nw.ru/PAGE/DEPENDFUND/LEA/ENG/rrr.html), and with the Venus Express and Mars Express data obtained due courtesy of A. Fienga. The volume of highly accurate observations on which the next EPM (Ephemerides of Planets and the Moon) versions are based increases continuously, and the total number of observations used in the current version of the EPM2011 planetary theory is 676 804 (Table 2).

**Table 2.** Observational material
| Planet                  | Radio interval | Radio number | Optical interval | Optical number |
|------------------------|----------------|--------------|------------------|----------------|
| Mercury                | 1964-2009      | 948          | –                | –              |
| Venus                  | 1961-2010      | 40281        | –                | –              |
| Mars                   | 1965-2010      | 578918       | –                | –              |
| Jupiter + 4 satellites | 1973-1997      | 51           | 1914-2010        | 13023          |
| Saturn + 9 satellites  | 1979-2009      | 126          | 1913-2010        | 14744          |
| Uranus + 4 satellites  | 1986           | 3            | 1914-2010        | 11681          |
| Neptune + 1 satellite  | 1989           | 3            | 1913-2010        | 11474          |
| Pluto                  | –              | –            | 1914-2010        | 5552           |
| **Total**              | **620330**     | **56474**    |                  |                |

The observations were processed using proved and tested techniques by taking into account all of the necessary reductions (Pitjeva 2005).

The reductions of the radar observations:

- the reduction of the instants of time to a uniform scale;
- the relativistic corrections - the time delay in the gravitational fields of the Sun, Jupiter, and Saturn (Shapiro effect) and the transition from the coordinate time (ephemeris argument) to the observer’s proper time;
- the time delay in the Earth’s troposphere;
- the time delay in the solar coronal plasma;
- the correction for the planetary surface topography (Mercury, Venus, Mars).

The reductions of the optical observations:

- the reduction of the observations to the ICRF: the reference catalogs => FK4 => FK5 => ICRF;
- the correction for the additional phase effect;
- the correction for the gravitational deflection of light by the Sun.

Present-day radio observations of planets and spacecraft with a 1-m accuracy (a relative error of $10^{-12} \div 10^{-11}$) make it possible to estimate very subtle and small (in magnitude) effects in the Solar system (see, e.g., Konopliv et al. 2011; Pitjeva 2010; Fienga et al. 2011).
Substantial progress is related to several factors: an increase in the accuracy of observational data reduction procedures and dynamical models of motion and an improvement in the quality of observational data, an increase in their accuracy and the extent of the time interval on which these observations were obtained.

**THE EPM2011 PLANETARY EPHEMERIDES**

The EPM2011 (Ephemerides of Planets and the Moon) numerical ephemerides were constructed using about 677 thousand observations (1913-2010) of various types. The equations of motion for bodies were taken for a parameterized post-Newtonian n-body metric. The integration in the barycentric coordinate system in the TDB scale at epoch J2000.0 was performed by Everhart’s method in an interval of 400 years (1800-2200) by the lunar-planetary integrator of the ERA-7 software package (Krasinsky and Vasilyev 1997). The EPM ephemerides, along with the corresponding TT-TDB time differences and the coordinates of seven additional objects (Ceres, Pallas, Vesta, Eris, Haumea, Makemake, Sedna), are accessible via FTP: [ftp://quasar.ipa.nw.ru/incoming/EPM/](ftp://quasar.ipa.nw.ru/incoming/EPM/).

Apart from the mutual perturbations of the major planets and the Moon, the EPM2011 dynamical model includes:

- the perturbations from 301 most massive asteroids;
- the perturbations from the remaining minor planets of the main asteroid belt modeled by a homogeneous ring;
- the perturbations from 21 largest trans-Neptunian objects (TNOs);
- the perturbations from the remaining trans-Neptunian planets modeled by a homogeneous ring at a mean distance of 43 AU;
- the perturbations from solar oblateness ($2 \times 10^{-7}$);
- the perturbations caused by the nonsphericity of the Earth’s and Moon’s figures;
- the relativistic perturbations from the Sun, the Moon, planets and five largest asteroids.

Since the radio measurements where the distances are predominantly measured were the main observational material when creating the next version of planetary ephemerides, EPM2011, controlling the orientation of the coordinate system for the ephemerides with respect to the ICRF requires particular attention and carefulness. The orientation was performed using VLBI observations of spacecraft near planets against the background of
quasars whose coordinates are given in the ICRF (Table 3). An example of VLBI observations for Cassini near Saturn are given in Jones et al. (2011).

**Table 3.** VLBI observations of spacecraft near planets against the background of quasars

| Spacecraft | Planet | Interval   | Number of measurements |
|------------|--------|------------|------------------------|
| Magellan   | Венера | 1990-1994  | 18(α + δ)               |
| Venus Express | Венера | 2007-2010  | 29(α + δ)               |
| Phobos     | Марс   | 1989       | 2(α + δ)                |
| MGC        | Марс   | 2001-2003  | 15(α + δ)               |
| Odyssey    | Марс   | 2002-2010  | 86(α + δ)               |
| MRO        | Марс   | 2006-2010  | 41(α + δ)               |
| Cassini    | Сатурн | 2004-2009  | 22(α, δ)                |

*Note:* (α + δ) denotes one-dimensional measurements of the combination of α and δ, (α, δ) denotes two-dimensional measurements.

The accuracy of such observations improved to a few tenths of mas (1 mas = 0′′001) for Mars and Saturn in 2001-2010, which allowed the orientation of the coordinate system for the EPM ephemerides to be refined (Table 4).

**Table 4.** Rotation angles of the coordinate system for the EPM ephemerides in the ICRF (1 mas = 0′′001)

| Interval of observ. | Number of observ. | εₓ, mas    | εᵧ, mas    | εz, mas    |
|---------------------|--------------------|------------|------------|------------|
| 1989-1994           | 20                 | 4.5 ± 0.8  | −0.8 ± 0.6 | −0.6 ± 0.4 |
| 1989-2003           | 62                 | 1.9 ± 0.1  | −0.5 ± 0.2 | −1.5 ± 0.1 |
| 1989-2007           | 118                | −1.528 ± 0.062 | 1.025 ± 0.060 | 1.271 ± 0.046 |
| 1989-2010           | 213                | −0.000 ± 0.042 | −0.025 ± 0.048 | 0.004 ± 0.028 |

More than 260 parameters were determined and refined in the main version of the improvement of the planetary part of the EPM2011 ephemerides:

- the orbital elements of the planets and satellites of the outer planets;
- the astronomical unit or \( GM_\odot \);
- the orientation angles of the ephemerides relative to the ICRF;
- the rotation parameters of Mars and the coordinates of three landers on Mars;
- the masses of 21 asteroids, the mean densities of the taxonomic classes of asteroids (C, S, M);
• the mass and radius of the asteroid ring, the mass of the TNO ring;
• the Earth-to-Moon mass ratio;
• the quadrupole moment of the Sun and solar corona parameters for different conjunctions of the planets with the Sun;
• the coefficients of Mercury’s topography and the corrections to the level surfaces of Venus and Mars;
• the coefficients for the additional phase effects of the outer planets.

The parameters determined while fitting the DE and EPM ephemerides (Pitjeva and Standish 2009) and approved by the 27th General Assembly of the International Astronomical Union in 2009 as the current best values for ephemeris astronomy (Luzum et al. 2011) were used in EPM2011 as the initial ones; they were subsequently improved based on all observations. Among them, there are the masses of the largest asteroids: 

\[ M_{\text{Ceres}}/M_\odot = 4.726(8) \cdot 10^{-10}, \quad M_{\text{Pallas}}/M_\odot = 1.048(9) \cdot 10^{-10}, \quad M_{\text{Vesta}}/M_\odot = 1.297(5) \cdot 10^{-10}; \]

the Earth-to-Moon mass ratio: \[ M_{\text{Earth}}/M_{\text{Moon}} = 81.3005676 \pm 0.0000006; \] and the astronomical unit in meters: \[ \text{AU} = (149597870695.88 \pm 0.14) \text{ m}. \]

Special efforts were made for a more accurate allowance for the overall influence of asteroids on the motion of planets, most of which are located in the main asteroid belt. In EPM the main asteroid ring is modeled by the motion of 301 large asteroids and a homogeneous material ring representing the influence of the remaining numerous small asteroids. The parameters characterizing the asteroid ring of small asteroids were determined from processing observations:

\[ M_{\text{ring}} = (1.06 \pm 1.12) \cdot 10^{-10} M_\odot, \quad R_{\text{ring}} = (3.57 \pm 0.26) \text{ a.e.} \]

The total mass of the main-belt asteroids represented by the sum of the masses of 301 largest asteroids and the asteroid ring is \[ M_{\text{belt}} = (12.3 \pm 2.1) \cdot 10^{-10} M_\odot \approx 3 \text{ masses of Ceres}. \] The gravitational perturbations from TNOs is similarly modeled by 21 known TNOs and an additional homogeneous ring with a radius of 43 AU representing the influence of the remaining smaller objects. The mass of the TNO ring found from processing observations is

\[ M_{TNO_{\text{ring}}} = (501 \pm 249) \cdot 10^{-10} M_\odot \]

The total mass of all TNOs, including the mass of Pluto, 21 largest TNOs, and the TNO ring, is \[ M_{\text{TNO}} = 790 \cdot 10^{-10} M_\odot, \approx 164 \text{ masses of Ceres or two masses of the Moon}. \]
The uncertainties of parameters of this section correspond to a $3\sigma$ formal standard error of the least-squares method.

## RESULTS

We used the following approach to find and test the possible effects in the motions of planets.

At the first step, we improved the heliocentric gravitational constant $GM_\odot$ from processing the observations of all planets. If there is actually an additional gravitating medium, then the value obtained will be some mean value with allowance made for the extended matter. At the next step, having fixed most of the derived parameters, we processed the observations for one of the chosen planets and searched for $GM_\odot$ and an additional perihelion shift based on the observational data only for this planet. Since the expected dark matter density is low (Table 1), it is desirable to use the data for more distant planets to include a larger volume of the influencing invisible medium in our analysis. Since the expected pattern of change in the central attractive mass and the perihelion drift with increasing distance in the presence of an additional gravitating medium are known, comparison with the results obtained can be made. If the values found turned out to be larger than their errors, then the correspondence of the derived pattern of change in the central attractive mass and the sign of the perihelion drift to the expected ones should be tested. If the errors exceed the values themselves, then only the upper limit for the possible additional mass and the density of the distributed matter can be judged.

Tables 5 and 6 list the corrections to the perihelion precessions and the central mass found from the observations of planets and spacecraft.

**Table 5.** Additional perihelion precessions from the observations of planets and spacecraft
Planets | $\dot{\pi}$ | $|\sigma_{\dot{\pi}}/\dot{\pi}|$  
Mercury | $-0.020 \pm 0.030$ | 1.5  
Venus | $0.026 \pm 0.016$ | 0.62  
Earth | $0.0019 \pm 0.0019$ | 1.0  
Mars | $-0.00020 \pm 0.00037$ | 1.9  
Jupiter | $0.587 \pm 0.283$ | 0.48  
Saturn | $-0.0032 \pm 0.0047$ | 1.5  

To control the derived quantities and their errors, to check the stability of their values, and to reduce the influence of possible correlations, we considered various cases with different numbers of simultaneously determined parameters. This makes it possible to obtain more reliable errors ($\sigma$) for the corrections found that better reflect and correspond to the actual accuracy of the results obtained. Therefore, the errors in Tables 5 and 6 generally exceed the formal ones by several times.

**Table 6.** Corrections to the central attractive mass

| Planets   | $\Delta M_0$ | $|\sigma_{\Delta M_0}/\Delta M_0|$ |
|-----------|---------------|----------------------------------|
| Mercury   | $-0.5 \pm 117.7$ | 235.4 |
| Venus     | $-0.67 \pm 5.86$ | 8.7 |
| Mars      | $0.20 \pm 2.65$ | 13.3 |
| Jupiter   | $0.4 \pm 1671.4$ | 4178.5 |
| Saturn    | $-0.27 \pm 15.16$ | 56.1 |

The values found in both Tables 5 and 6 are generally overridden by their errors ($\sigma$), indicating that the density of the dark matter $\rho_{dm}$, if present, is very low and is much less than the currently achieved error in these parameters. The derived opposite signs and the absence of general trends in the change of the corrections themselves to the attractive central mass and the perihelion precession depending on the distance from planet to planet also suggest that the sought-for effects are small.

The relative error in the correction to the central mass from the observational data separately for each planet (Table 6) turned out to be considerably larger than that for the
additional perihelion precession (Table 5) and exceeds the corrections to the central mass themselves by several times or even by orders of magnitude. It should be kept in mind that the accuracy of allowance for and knowledge of all masses of the bodies that fell into a spherically symmetric volume relative to the Sun plays a major role in the integral estimate of the dark matter mass in this volume. The total amount of dust, meteoric matter, and solar wind plasma is comparatively small (less than $10^{-15} \div 10^{-13} M_\odot$ in the volume of Saturn’s orbit). Incomplete and inaccurate knowledge of the asteroid masses play a major role in the uncertainty; in particular, the error in the mass of the main asteroid belt is $2 \cdot 10^{-10} M_\odot$.

The problem of improving the asteroid masses, their number, and distribution in the main asteroid belt is topical and important for increasing the accuracy of planetary theories. More accurate results were obtained for the perihelion precession estimates, which allow the local dark matter density at the mean orbital distance of a planet to be estimated. Here, the error is comparable to the values themselves (Table 5) and, therefore, the data from Table 5 were used for constraining estimates.

To a first approximation, a uniform distribution can be assumed for the distributed medium, as is done most often for such estimates, and its density is then determined from the planets with the most accurately estimated perihelion precessions that are farthest from the Sun. Although there is a negative secular perihelion drift for some of the planets, the error for all planets is comparable to or appreciably larger than the absolute values of the derived perihelion precessions (Table 5). Therefore, attention should be focused on the errors themselves. The latter may be considered as an upper limit for the possible perihelion precession in absolute value $|\delta \pi|$ (arcsec yr$^{-1}$) and, thus, using Eq. (6) it can give an upper limit for the density of the distributed matter. Our $\rho_{dm}$ estimates are given in Table 7.

**Table 7.** Estimates of the density $\rho_{dm}$ from the perihelion precessions
| Planets | $\sigma_\varepsilon$, arcsec yr$^{-1}$ | $\rho_{dm}$, g cm$^{-3}$ |
|---------|-----------------|----------------|
| Mercury | 0.000030        | $< 9.3 \cdot 10^{-18}$ |
| Venus   | 0.000016        | $< 1.9 \cdot 10^{-18}$ |
| Earth   | 0.0000019       | $< 1.4 \cdot 10^{-19}$ |
| Mars    | 0.0000037       | $< 1.4 \cdot 10^{-20}$ |
| Jupiter | 0.000283        | $< 1.7 \cdot 10^{-18}$ |
| Saturn  | 0.0000047       | $< 1.1 \cdot 10^{-20}$ |

The estimates from the data for the Earth, Mars, and Saturn give the most stringent constraints on the density. If we proceed from the assumption of a uniform $\rho_{dm}$ distribution in the Solar system, then the most stringent constraint $\rho_{dm} < 1.1 \cdot 10^{-20}$ g cm$^{-3}$ is obtained from the data for Saturn. The mass within the spherical volume with the size of Saturn’s orbit is then $M_{dm} < 7.1 \cdot 10^{-11}M_\odot$, which is within the error of the total mass of the main asteroid belt.

We can also consider the case where a continuous gravitating medium has some concentration to the Solar system center. Studies under the assumption of density concentration to the center have already been carried out, for example, in Frère et al. (2008). As a model of the $\rho_{dm}$ distribution, we took the expression

$$\rho_{dm} = \rho_0 \cdot e^{-cr},$$

where $\rho_0$ is the central density and $c$ is a positive parameter characterizing an exponential decrease in density to the periphery. The value of $c = 0$ corresponds to a uniform density. Function (7) is everywhere finite, has no singularities at the center and on the periphery, and is integrable. The expressions for the gravitational potential for an inner point at distance $r$ and the mass inside a sphere of radius $r$ for distribution (7) are, respectively,

$$U(r) = 4\pi G\rho_0/r \cdot [2 - e^{-cr}(cr + 2)]/c^3,$$

$$M_{dm} = 4\pi\rho_0[2/c^3 - e^{-cr}(r^2/c + 2r/c^2 + 2/c^3)].$$

In contrast to the potential $U(r)$ (8), Eq. (9) for the mass $M_{dm}$ has no singularities for $c \to 0$, despite the presence of $c^3$ in the denominator, and transforms into the expression for a homogeneous sphere

$$M(r)_{dm} = \frac{4}{3}\pi r^3 \rho_0.$$
The values in Table 7 may be considered as estimates of the dark matter density at various distances. Indeed, if we take into account the fact that the dark matter density is almost constant in a comparatively narrow range of radial distances (the value of $c$ in Eq. (7) is not too large), then the density of the extended medium can be roughly assumed to be constant in the range of $r$ due to the eccentricity of the planetary orbit. Thus, when a changing density is considered, the estimates from each planet may be considered as a local estimate of $\rho_{dm}$ for the distance $r = a_{orb}$, where $a_{orb}$ is the semimajor axis of the planetary orbit. Allowance for the distributed dark matter $M_{dm}$ between the Sun and the planetary orbit gives very small corrections and contribution (in the tenths or elevenths decimal digit) to the total attractive central mass determined by the solar mass. Therefore, we can use Eq. (6) with a sufficient accuracy and estimate the density $\rho_{dm}$ near the planetary orbit from the perihelion precession produced by the dark matter.

When constructing Table 7 to estimate the dark matter density, we took overestimated perihelion precessions of planets corresponding to the errors of their determination, i.e., the table contains constraining upper limits. Using the data from it with similar properties, we will obtain the density distribution (7). To find the parameters $\rho_0$ and $c$ in (7), we took the most reliable data in Table 7 for Saturn ($\rho_{dm} < 1.1 \cdot 10^{-20}$ g cm$^{-3}$), Mars ($\rho_{dm} < 1.4 \cdot 10^{-20}$ g cm$^{-3}$), and the Earth ($\rho_{dm} < 1.4 \cdot 10^{-19}$ g cm$^{-3}$). After the arrival of the Cassini spacecraft to Saturn, a highly accurate series of observations since 2004 has appeared. For Mars, there is a large and long set of observations related to spacecraft on its surface and near it. Since the observations are performed from the Earth, the improvement of the Earth’s orbit is based on all observations and includes measurements of different accuracies.

For the Saturn-Mars pair, we obtained $\rho_0 = 1.50 \cdot 10^{-20}$ g cm$^{-3}$ and $c = 0.0279$ AU$^{-1}$. This corresponds to a very flat density curve (7). The dark matter mass within the spherical volume corresponding to Saturn’s orbit turned out to be $M_{dm} < 7.6 \cdot 10^{-11} M_\odot$.

The Saturn-Earth pair gives $\rho_0 = 1.86 \cdot 10^{-19}$ g cm$^{-3}$ and $c = 0.290$ AU$^{-1}$ and a steeper rise in $\rho_{dm}$ to the center, majorizing the density estimate for Mars. For these parameters, the mass $M_{dm}$ within Saturn’s orbit is $M_{dm} < 1.7 \cdot 10^{-10} M_\odot$, which is also within the error in the total mass of the main asteroid belt ($\pm 2.1 \cdot 10^{-10} M_\odot$).

The situation and the results did not change greatly compared to the hypothesis of a uniform density $\rho_{dm}$ – the estimated total dark matter mass within Saturn’s orbit increased
by a factor of $\sim 2.5$, although the density distribution in the latter case gives a significant increase to the center. Note that a change in the parameter $\rho_0$ in the exponential distribution (7), just as in the density $\rho$ for a uniform distribution, leads to the corresponding almost linear change in the secular perihelion drift in the entire range of distances from Mercury to Saturn. An increase in the parameter $c$ causes the perihelion precession to decrease in accordance with the decrease in density $\rho_{dm}$ with distance $r$.

CONCLUSIONS

We investigated and estimated the possible gravitational influence of dark matter in the Solar system on the motion of planets based on the EPM2011 planetary ephemerides using about 677 thousand positional observations of planets and spacecraft, most of which belong to present-day ranging. Our results show that the mass of the dark matter, if present, and its density $\rho_{dm}$ are much lower than the present-day errors in these parameters. We found that the density $\rho_{dm}$ is less than $1.1 \cdot 10^{-20}$, $1.4 \cdot 10^{-20}$, and $1.4 \cdot 10^{-19}$ g cm$^{-3}$ at the orbital distances of Saturn, Mars, and the Earth, respectively. Taking into account our constraining estimates, we considered the case of a possible concentration of dark matter to the Solar system center. The dark matter mass in the sphere within Saturn’s orbit should be less than $1.7 \cdot 10^{-10} M_{\odot}$ even if its possible concentration is taken into account.

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