Conditional analysis near strong shear layers in DNS of isotropic turbulence at high Reynolds number

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Abstract. Data analysis of high resolution DNS of isotropic turbulence with the Taylor scale Reynolds number \(R_\lambda = 1131\) shows that there are thin shear layers consisting of a cluster of strong vortex tubes with typical diameter of order \(10\eta\), where \(\eta\) is the Kolmogorov length scale. The widths of the layers are of the order of the Taylor micro length scale. According to the analysis of one of the layers, coarse grained vorticity in the layer are aligned approximately in the plane of the layer so that there is a net mean shear across the layer with a mean velocity jump of the order of the root-mean-square of the fluctuating velocity, and energy dissipation averaged over the layer is larger than ten times the average over the whole flow. The mean and the standard deviation of the energy transfer \(T(x,k)\) from scales larger than \(1/k\) to scales smaller than \(1/k\) at position \(x\) are largest within the layers (where the most intense vortices and dissipation occur), but are also large just outside the layers (where viscous stresses are weak), by comparison with the average values of \(T\) over the whole region. The DNS data are consistent with exterior fluctuation being damped/filtered at the interface of the layer and then selectively amplified within the layer.

1. Introduction

Visualization of intense vorticity iso-surfaces in direct numerical simulation (DNS) of homogeneous isotropic turbulence with \(4096^3\) grid points and the Taylor scale Reynolds number \(R_\lambda = 1131\) (Ishihara \textit{et al.}, 2009) shows that there are very sharp interfaces between high vorticity and low vorticity regions. Comparison of the figure with those in DNS at lower \(R_\lambda\) values shows that the interface gets sharper as the Reynolds number (Re) increases. Another visualization of a snapshot of intensity distribution of energy dissipation and enstrophy on a cross section in DNS of turbulence with \(R_\lambda = 675\) (Ishihara \textit{et al.}, 2009) suggests that intense energy dissipation as well as intense enstrophy in high \(Re\) turbulence occurs in layer-like regions, which have sharp interfaces. The interfaces across which vorticity distribution changes sharply are often observed in various high \(Re\) flows, including turbulent jets, mixing layers, boundary layers, and can be regarded as the key to high \(Re\) turbulence structure, (see Hunt \textit{et al.}, 2010, 2011).

Conditional statistics have been analysed to characterise the turbulent/non-turbulent interfaces of far wake (Bisset \textit{et al.}, 2002) and of jet (da Silva \& Pereira, 2008; Westerweel \textit{et al.}, 2009). To study the properties of strong thin shear layers observed in high \(Re\) turbulence, we use the DNS data of homogeneous isotropic turbulence with \(4096^3\) grid points and \(R_\lambda = 1131\).
(Kaneda et al., 2003) and analyse the conditional statistics near the strong thin shear layers in the homogeneous isotropic high Re turbulence. Particular attention is paid to the conditional statistics to explore the dynamics near the layer. For this purpose, we analyze the conditional statistics of the energy transfer and the conditional correlation of velocity fluctuations.

2. Conditional data analysis

Characteristic parameters of the DNS data we use in this study are listed in Table 1. For the detail of the DNS data, see Kaneda et al. (2003); Ishihara et al. (2007). To analyze the 4096³ DNS data we divide the total DNS domain into 512(=8x8x8) sub-domains, each of which has 512³ grid points. The size of each sub-domain is 512Δx, which corresponds to 0.72L ≈ 23.3λ ≈ 1542η, where Δx is the mesh size used in the DNS, L the integral length scale, λ the Taylor micro scale, and η the Kolmogorov length scale.

A typical shear layer was observed in a sub-domain whose averaged enstrophy is about 2.42Ω, where $\Omega = \langle \omega^2 \rangle / 2$ is the enstrophy averaged over the whole region. The visualization of intensity distribution of enstrophy on a cross section (a x-z plane) at a fixed y in the sub-domain is shown in Figure 1, in which we can observe a shear layer with sharp interfaces. We call it “shear layer” because coarse grained vorticity in the layer aligns roughly in the plane of the layer so that there is a net mean shear across the layer with a mean velocity jump of the order of the root-mean-square of the fluctuating velocity. Detailed conditional studies were done using the planar data near the shear layer observed in Figure 1.

The interfaces on either side of the layer, i.e., the left interface and the right interface, were defined as the set of the outermost points of the region with the local enstrophy $\Omega_{\text{local}} = \omega^2 / 2$ greater than a certain threshold, say $\Omega_T$ (following Westerweel et al., 2009). In practice we set $\Omega_T = 7.8\Omega$ and determined the locations (x coordinates), $x^L$ and $x^R$, of the left and right interfaces, respectively, as functions of z on the x-z plane shown in Figure 1. Note that the locations of the interfaces thus determined are quite insensitive to the choice of $\Omega_T$, as far as $\Omega_T$ is in a certain appropriate range. Conditional averages $\langle A \rangle_L$ and $\langle A \rangle_R$ near the left and right interfaces are determined by averaging data of A over a certain z range (for which the interfaces can be clearly defined) at fixed distances from $x^L$ and $x^R$, respectively, and are functions of $\xi = x - x^L$ and $\eta = x - x^R$, respectively.

| Table 1. DNS parameters and turbulence characteristics |
|-------------------------------------------------------|
| $N$ | $Re$ | $R_\lambda$ | $k_{\text{max}}$ | $10^3f_{\nu}$ | $\langle \epsilon \rangle$ | $\lambda$ | $\eta$ | $T$ | $\tau_\eta$ |
| 4096 | 36500 | 1131 | 1930 | 0.173 | 0.0752 | 1.09 | 0.0339 | 0.0051 | 1.89 | 0.015 |

3. Results

As a typical example among many conditional statistics, we show the conditional average of normalized dissipation as functions of $x - x^L$ and $x - x^R$ in Figure 2. It can be observed in Figure 2 that (i) the thickness of the layer is 4 ~ 5λ, (ii) energy dissipation averaged over the layer is larger than ten times the average over the whole flow, and (iii) the right interface (upstream face of the layer) is sharper than the left one (downstream face).

Energy transfer $T(x, k)$ from scales larger than $1/k$ to scales smaller than $1/k$ at position $x$ is defined by

$$T(x, k) \equiv \sum_{ij} (u_i u_j - \bar{u}_i \bar{u}_j) S_{ij},$$
where $\tilde{S}_{ij} \equiv (1/2)(\partial \tilde{u}_i/\partial x_j + \partial \tilde{u}_j/\partial x_i)$ and $\mathcal{F}$ represents the filtering operation to remove all the Fourier modes with wave numbers higher than a threshold $k$, i.e., the so-called spectral cut-off filter. Computations of $T(x,k)$ using the DNS data of high $Re$ turbulence (Aoyama et al., 2005; Ishihara et al., 2009) show that upscale transfer (from small to large) is almost as large as the downscale transfer and also that $T(x,k)$ is very intermittent; its fluctuations being much larger than its mean value and increasing functions of $k$. Conditional analysis of $T(x,k)$ near the interfaces shows that the mean and the standard deviation of the energy transfer $T(x,k)$ are largest within the layers (where the most intense vortices and dissipation occur), but are also large just outside the layers (where viscous stresses are weak), by comparison with the average values of $T$ over the whole region. Note however that the shape of the probability density function of $T$ in the layer under an appropriate normalization is similar to that outside the layer.

To test whether this thin shear layer acts as a barrier to the fluctuations outside the layer, we consider the conditional cross correlations of the velocity fluctuations near the interfaces $x^L$ and $x^R$. Let the fluctuation $\tilde{f}_L(\xi,z)$ and its two-point correlation $C^L(\xi,\xi')$ be defined by

$$\tilde{f}_L(\xi,z) \equiv f(x^L + \xi,z) - \langle f \rangle_L(\xi),$$

where $f$ represents the velocity fluctuations.
and

\[ C_L^L(\xi, \xi') \equiv \frac{\langle f_L(\xi, z) \hat{f}_L(\xi', z) \rangle_C}{\langle f_L(\xi, z) \rangle_C^{1/2} \langle \hat{f}_L(\xi', z) \rangle_C^{1/2}}, \]

respectively, where \( \langle f \rangle_C \equiv \langle f(x^L + \xi, z) \rangle_C \) is the average of \( f \) over the \( z \) range conditioned on the distance \( \xi \) from the left interface \( x = x^L \) and \( \langle \rangle_C \) is the average over the same range of \( z \) conditioned on \( \xi \). The cross correlation \( C_R(\xi, \xi') \) is defined similarly by using \( x^R \). The results of the cross correlations \( C_L^L(\xi, \xi') \) and \( C_R(\xi, \xi') \) of velocity fluctuation show high correlations between two points on the same side and low correlations between points at different sides of the boundaries of the layer. The two-point correlation \( C_R(\xi, \xi') \) of velocity fluctuation falls sharply near \( \xi = 0 \) when \( \xi' \) is in the right side (upstream) of the layer. The pdfs of the velocity components in the layer are strongly non-Gaussian and the shapes of the pdfs correlate directly with the shapes of the pdfs of those outside the layer. These data are consistent with exterior fluctuation being damped/filtered at interface and then selectively amplified within the layer.

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