Air shower universality from $10^{14}$ to $10^{22}$ eV

A A Lagutin, R I Raikin, T L Serebryakova
Altai State University, Barnaul, Russia
E-mail: raikin@theory.asu.ru

Abstract. Scaling properties of nuclei- and photon-initiated air showers are examined in wide primary energy range ($10^{14} ÷ 10^{22}$ eV) taking into account Landau-Pomeranchuk-Migdal and geomagnetic field effects. It is shown that the invariance in lateral distribution and also the universal dependence between shower age and root mean square radius of electron component exist up to the highest energies. The implications of shower universality for reanalysis, cross-checking and validation of the results of different experiments together with decreasing of the influence of hadronic model uncertainties are discussed in detail.

1. Introduction
The concept of air shower universality dating back to pioneering works [1–3] is subjected to the remarkable rise of interest during the last decade (see e.g. [4–12]). In a broad sense, the so-called universality is expressed in the similarity of the spectra of low-energy particles in air showers at the same development stage. Phenomenology, possible generalisations, deeper understanding of the origin and limitations of such properties are obviously important to unveil their potential in the achievement of the unambiguous physical interpretation of the results of comparisons between air shower observables and the appropriate theoretical calculations. Various parameterizations of the correlations between different air showers characteristics have been obtained on the basis of universality; attempts have been made to work out new experimental data processing techniques that could, for example, suppress or separate the influence of the hadronic interaction model and improve the reliability of the results.

In this paper we investigate perspectives of model-independent scaling approach for lateral distribution of electrons in nuclei- and photon-initiated air showers and also the universal dependence between shower age and root mean square radius of electron component for the improved analysis of the experimental data, particulary to extract information about the variations in elemental composition of cosmic rays in almost a model-independent way.

2. Scale-invariance of electron lateral distribution in air showers
According to the theoretically motivated scaling approach [13, 14], the electron lateral distribution function in both electromagnetic cascade showers (EMC) initiated by high-energy photons and hadron-induced extensive air showers (EAS) can be described with high accuracy by the scale-invariant form

$$\rho(r; E, t) = \frac{N(E, t)}{R_0^2(E, t)} F\left(\frac{r}{R_0(E, t)}\right).$$

(1)
Here $\rho(r; E, t)$ – local particle density at radial distance $r$ from the core in shower of primary energy $E$ observing at depth $t$ in the atmosphere, $N$ – total number of particles at observation level (shower size), $R_0$ – radial scale factor, which, in contrast to commonly used LDF representations, depends on primary particle type and mass, shower age and (in case of extensive air showers) properties of hadronic interactions. Using the results of extensive series of calculations we found [14] the following overall fit for scale-invariant part of lateral distribution:

$$F(x) = Cx^{-\alpha}(1 + x)^{-(\beta-\alpha)}(1 + (x/10)^\gamma)^{-\delta}. \quad (2)$$

For electron LDF in EMC and EAS parameters of function (2) are determined as $C = 0.28$, $\alpha = 1.2$, $\beta = 4.53$, $\gamma = 2.0$, $\delta = 0.6$, and scale factor $R_0$ in both cases is equal to the root mean square radius of electron component $R_{\text{ms}}$, defined in a standard way as:

$$R_{\text{ms}}^2(E, t) = \frac{2\pi}{N(E, t)} \int_0^\infty r^3 \rho(r; E, t) dr. \quad (3)$$

The validity of scaling function was confirmed by simulations for $E = (10^{14} \div 10^{18})$ eV, $t = (600 \div 1030)$ g/cm$^2$, $x = (0.05 \div 25)$. The last condition corresponds to the radial distance range from $r \sim (5 \div 10)$ m to $r \sim (2.5 \div 4)$ km depending on the shower age.

3. Scaling LDF and universality property

It was also found [15,16] that for average extensive air shower of particular energy, developing in real atmosphere, there is a one-to-one mapping between $R_{\text{ms}}(E, t) \times \rho(t)$, where $\rho(t)$ is air density at the observation depth, and $s' = t/(t_{\text{max}} + 100 \text{ g/cm}^2)$, where $t_{\text{max}}$ is the depth of maximum of an average cascade curve. This allows us to propose the following parameterisation, which does not depend on energy, observation depth and also primary nuclear mass:

$$R_{\text{ms}}(E, t) = \frac{\rho_0}{\rho(t)} A \left[ B + \frac{2}{\pi} \arctg (s' - 1) \right] \text{ m,} \quad (4)$$

where $\rho_0 = 1.225 \times 10^{-3} \text{ g/cm}^2$, $A = 173.0$ and $B = 0.546$.

Equation (4) obviously represents the universal dependence between average shower width and age. The most remarkable feature of the described approach is that scaling property (1), scaling function (2) and parameterisation (4) are almost insensitive to hadronic interactions model implemented in calculations [14–16].

In figure 1 energy dependences of root mean square radius of electron component are shown for vertical EAS initiated by protons and iron nuclei at observation depth $t = 890 \text{ g/cm}^2$. The data were obtained from relation (4) using depths of maximum calculated with hadronic interactions models retuned on the basis of recent accelerator data as reported in [17]. The uncertainty in $t_{\text{max}}$ predictions is currently reduced to $\sim 20 \text{ g/cm}^2$ obviously improving the accuracy of conclusions based on comparisons between simulated and measured depths of maximum. As it is seen from figure 1, considering radial scale factors of electron LDF estimated from the experimental data gives us another important source of information on primary composition.

We also show in figure 1 the mean square radii of pure electromagnetic cascades initiated by primary gammas examining the influence of ultra-high energy effects – Landau-Pomeranchuk-Migdal (LPM) effect and interaction of UHE photons and electrons with the geomagnetic field (GMF). These data were obtained [18, 19] on the basis of the numerical solution of adjoint cascade equations. The GMF intensity profile corresponds to conditions of Auger Observatory in Argentina. One can see that for UHE primary gammas $R_{\text{ms}}$, keeping much smaller values comparing to EAS, increases considerably with energy from $E \sim 3 \times 10^{19}$ eV to $E \sim 3 \times 10^{20}$ eV.
Figure 1. Energy dependences of root mean square radius of electron component in vertical air showers initiated by protons, iron nuclei (corresponding to recently retuned hadronic models) and gammas (with and without UHE effects – LPM and geomagnetic) at 890 g/cm² (see text for details).

Figure 2. $R_{\text{ms}} - t_{\text{max}}$ scatter plot for 500 simulated vertical extensive air showers initiated by protons and iron nuclei with $E = 10^{18}$ eV at three observation depths (614, 830, 1028 g/cm²).

after interaction with geomagnetic field begins to affect shower development. This should be taken into account when $R_{\text{ms}}$ is used for testing gamma-ray fraction in UHE cosmic rays flux.

On the other hand, our calculations show that scaling formalism (1)-(2) remains valid when LPM- and GMF-effects in EMC are taken into account [18, 19]. Since for electron LDF in UHE extensive air showers only LPM-effect in partial electromagnetic cascades contributes to $R_{\text{ms}}$ we expect that, barring dramatic changes in nuclear interactions, both scaling formalism (1)-(2) and universality in form (4) will be kept up to $10^{22}$ eV.

4. Implications for experimental data analysis at different energies

In our recent works [20–23] the experimental data of KASCADE, KASCADE-Grande and MSU EAS arrays were analyzed with respect to the radial scale factors of electron lateral distribution in the framework of generalized scaling formalism taking into account the appropriate shower classification procedure.

It was shown that uncertainty in the explicit form of lateral distribution function chosen for data fitting and bias of different nature in estimating the mean square radius of the EAS electron component do not impede the extraction of information about mean primary mass change with energy. The variations of $R_0$ with energy obtained using different model LDFs from the experimental data of the above mentioned arrays give a consistent evidence of weighting of primary composition above the knee. According to the KASCADE-Grande data it is followed by a rather sharp decrease of mean primary mass at $\lg N_e \gtrsim 7.5$ [23], indicative in favor of a “heavy knee” in primary spectrum.

Considering the applicability of such an analysis at very high energies for the experimental lateral distributions measured by largest air shower arrays it is important in what extent the universality property is affected by fluctuations in individual showers. In figure 2 the $R_{\text{ms}} - t_{\text{max}}$ scatter plot is presented for 500 simulated vertical extensive air showers initiated by protons and iron nuclei with $E = 10^{18}$ eV at three observation depths (614, 830, 1030 g/cm²). Observation
depths are chosen in such a way as to analyze $R_{\text{rms}} - t_{\text{max}}$ correlations in individual showers of different age. It is clear that for the majority of individual EAS of particular energy 614 g/cm$^2$ level is located higher than depth of maximum, while sea level is considerably deeper than $t_{\text{max}}$. At the same time 830 g/cm$^2$ roughly corresponds to the depth where fluctuations in shower size are minimal. Dependence on the observation level could also be used to infer the dependence on the shower zenith angle.

It is seen from figure 2 that strong correlation between root mean square radius and depth of maximum in individual extensive air showers is observed for showers of different ages and for both light and heavy primary nuclei, which, in addition, are separated quite well from each other on scatter plot. This fact suggests that implementation of radial scale factors into multicomponent analysis procedures, e.g., the principal component analysis, could be more efficient comparing with utilizing of other observables sensitive to primary particle type such as electron and muon shower sizes and local densities far from shower core position. Of course, additional studies devoted to the reliable estimation of radial scale factors of electron LDF from the experimental data of largest ground-based air shower arrays taking into account realistic measurement uncertainties, detectors response and other experimental conditions are needed. This is essential especially for hybrid arrays measuring longitudinal and lateral profiles of the shower independently.

5. Conclusion
The universality in extensive air shower development expressed by means of scale-invariant lateral distribution of electrons and the relation between root mean square radius and depth of shower maximum is evidently retaining in a common form in extremely wide primary energy range (from $10^{14}$ to $10^{22}$ eV) neglecting possible exotic hadronic phenomena. This formalism could be useful in development of new effective techniques for reanalysis, cross-checking and validation of the results of different experiments together with decreasing of the influence of hadronic model uncertainties allowing to raise confidence in the results coming from the comparisons between air showers observables and theoretical calculations.

References
[1] Rossi B, Greisen K 1941 Rev. Mod. Phys. 13 240
[2] Nishimura J, Kamata K 1958 Progr. Theor. Phys. 6 93
[3] Greisen K 1960 Ann. Rev. Nucl. Part. Sci. 10 63
[4] Capdevielle J N. and Cohen F 2005 J. Phys. G 31 507
[5] Nerling F et al 2006 Astropart. Phys. 24 421
[6] Giller M et al 2005 J. Phys. G 31 947
[7] Gora D et al 2006 Astropart. Phys. 24 484
[8] Apel W D et al 2008 Astropart. Phys. 29 412
[9] Lipari P 2009 Phys. Rev. D 79 063001
[10] Lafèbre S et al 2009 Astropart. Phys. 31 243
[11] Matthews J A J et al 2010 J. Phys. G: Nucl. Part. Phys. 37 025202
[12] Yushkov A et al 2010 Phys. Rev. D 81 123004
[13] Lagutin A A et al 1997 Proc. 25 ICRC (Darban) V. 6 285-288
[14] Lagutin A A, Raikin R I 2001 Nucl. Phys. B (Proc. Suppl.) 97B 274-277
[15] Raikin R I et al 2001 Proc. 27 ICRC (Hamburg) V. 1 290-293, 294-297
[16] Lagutin A A et al 2002 J. Phys. G: Nucl. Part. Phys. 28 1259-1274
[17] Pierog T 2012 Proc. Int. Symp. on Future Directions in UHECR Physics (CERN)
[18] Goncharov A I et al 2003 Proc. 28 ICRC (Tsukuba) V. 1 575-578
[19] Goncharov A I et al. 2004 Russian Physics Journal 47 N4 444-452
[20] Raikin R I, Lagutin A A 2008 Izv. Altai Gos. Univ. 1 66-71
[21] Raikin R I et al 2008 Nucl. Phys. B (Proc. Suppl.) 175-176 559-563
[22] Raikin R I et al 2009 Nucl. Phys. B (Proc. Suppl.) 196 383-386
[23] Raikin R I, Lagutin A A 2011 Proc. 32 ICRC (Beijing) V. 1 299-302