Systematic study for particle transverse momentum asymmetry in minimum bias pp collisions at LHC energies

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A method of randomly rearranging the $p_x$ and $p_y$ components of the produced particle on the circumference of an ellipse with the half major and minor axes being $p_T(1 + \delta_p)$ and $p_T(1 - \delta_p)$ is introduced. The ALICE data on the transverse sphericity as a function of charged multiplicity in the minimum bias pp collisions at $\sqrt{s}=0.9$ and 7 TeV are well reproduced by this method based on the particles generated in the PYTHIA6.4 simulations. The correspondingly predicted charged particle $v_2$ upper limit is a measurable value of ~0.2-0.3 . We suggest a systematic measurements for the particle transverse momentum sphericity and the elliptic flow parameter.

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I. INTRODUCTION

The event shapes are relevant to the properties of hadronic final state. One has employed the hadronic event shape measurements to test asymptotic freedom and to extract the strong coupling constant, etc. in the $e^+e^-$ annihilation and lepton deep inelastic scattering, respectively, for a long time $^1,^2$. Recently, the final hadronic event shapes in pp collisions at the LHC energies have been measured by CMS $^3$, ALICE $^4,^5$, and ATLAS $^6,^7$.

In order to avoid bias from the boost along beam axis $^8$, this study is restricted to the transverse momentum plane. We start from the transverse momentum matrix of the produced charged particles $^9$

$$S_{XY} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix},$$

$$a_{11} = \frac{1}{\sum_i p_T^2} \sum_i p_x^2 p_T, \quad a_{22} = \frac{1}{\sum_i p_T^2} \sum_i p_y^2 p_T, \quad a_{12} = a_{21} = \frac{1}{\sum_i p_T^2} \sum_i p_x p_y p_T,$$

where $p_T$ is the transverse momentum of particle $i$, $p_x$, and $p_y$ are the corresponding transverse momentum components, and the sum runs over the charged particles in a single event. The two eigenvalues of this transverse momentum matrix satisfy

$$\lambda_1 + \lambda_2 = a_{11} + a_{22} = 1.$$ (2)

If they are ordered in $\lambda_1 > \lambda_2$, the transverse momentum sphericity is then defined as $^9$

$$S_T = 2\lambda_2.$$ (3)

By intuitive construction one knows that $S_T$ possesses limits of

$$S_T = \begin{cases} 0, \text{ pencil – like limit}, \\
1, \text{ isotropic limit}, \end{cases}$$ (4)

and hence $S_T$ is a measure of the degree of transverse momentum azimuthal symmetry. The event averaged transverse momentum sphericity is then denoted as $\langle S_T \rangle$, where $\langle \ldots \rangle$ indicates an average over events.

As mentioned in $^9$ that the transverse momentum azimuthal asymmetry is measured by the dimensionless observable

$$A_T = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{(a_{11} - a_{22})^2 + 4a_{12}^2}{(a_{11} + a_{22})^2}.$$ (5)

It is easy to prove

$$S_T + A_T = 1.$$ (6)

Corresponding to the $S_T$ limits in Eq. (4) $A_T$ possesses the limits of

$$A_T = \begin{cases} 1, \text{ pencil – like limit}, \\
0, \text{ isotropic limit}, \end{cases}$$ (7)

and therefore $A_T$ is also a measure of the degree of transverse momentum azimuthal asymmetry. The event averaged transverse momentum asymmetry is denoted as $\langle A_T \rangle$. 

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II. MODELS AND SIMULATIONS

A. Sphericity from PYTHIA6.4

PYTHIA6.4 \cite{10} is used to calculate the charged particle transverse momentum sphericity as a function of charge multiplicity, \(\langle S_T\rangle(N_{ch})\), in \(p_T \geq 0.5\) GeV/c and \(|\eta| \leq 0.8\), for the minimum bias pp collisions at \(\sqrt{s} = 0.9\) and 7 TeV. These results (red solid diamonds, indicated as PYTHIA6) are compared with the corresponding ALICE data of “all” events (black open circles, indicated as ALICE) \cite{5} in Fig. 1 (a) and (b) for 0.9 and 7 TeV, respectively. The black solid circles indicated as PYTHIA8 in this figure are the results of PYTHIA8 copied from \cite{5}. One sees here that the results of PYTHIA6, like PYTHIA8, are not consistent with the ALICE data.

One way out is to invoke the various PYTHIA6 tunes. They are based on PYTHIA6.4 but with a couple of extra model parameters, relevant to the soft- and/or hard-processes as well as the parton distribution function etc., fitting to the experimental data. For example, in the PERUGIA-2011 tune \cite{11} “the data sets used to constrain the models include hadronic Z^0 decay at LEP, Tevatron min-bias data at 630, 1800, and 1960 GeV, Tevatron Drell-Yan data at 1800 and 1960 GeV, and SPS min-bias data at 200, 546, and 900 GeV”. As mentioned in \cite{5}, the above ALICE data were best reproduced by the PERUGIA-2011 tune (cf. the blue open triangles indicated as PERUGIA in Fig. 1) among the PYTHIA6 tunes.\[p_T^2 = \frac{p_x^2}{a^2} + \frac{p_y^2}{b^2} = 1]\[a = p_T(1 + \delta_p) \quad b = p_T(1 - \delta_p), \quad (8)\]

where \(p_T\) is the transverse momentum of the produced particle and \(0 < \delta_p < 1\) is an extra introduced deformation parameter. This is equivalent to randomly resample \(p_x\) and \(p_y\) of the produced particle according to \(p_x = p_T(1 + \delta_p) \cos \phi, \quad p_y = p_T(1 - \delta_p) \sin \phi, \quad (9)\)
as the equation of ellipse \[\frac{p_x^2}{a^2} + \frac{p_y^2}{b^2} = 1 \quad (10)\]
is held, where the half major and minor axes are given by Eq. (8). In the Eq. (9) \(\phi\) is randomly distributed in \([0, 2\pi]\). Then we may fit the deformation parameter \(\delta_p\) to the (ALICE) sphericity data and make prediction for other observables such as the elliptic flow parameter \(v_2\), or vice versa.

As the particle transverse momentum may be change in the rearrangement, one has to re-calculate the transverse momentum by \(p_T^2 = p_x^2 + p_y^2\) after the rearrangement. However we will see later that provided \(\delta_p\) is far less than unity (small perturbation) the change in particle transverse momentum distribution caused by the rearrangement is not visual.
C. Fitting process

The fitting process could be started from the small $\delta_p$ side, $\delta_p = 0.01$ for instance. If the calculated $\langle S_T \rangle_{N_{ch}}$ is larger than the ALICE data we increase the $\delta_p$ value and repeat the calculation, vice versa otherwise. At last, we obtain the results (red curves indicated PYTHIA6_1 in Fig. 1) calculated with $\delta_p = 0.092$ and 0.091, consistent well with the ALICE data of $\langle S_T \rangle_{N_{ch}}$ in the minimum bias pp collisions at $\sqrt{s} = 0.9$ and 7 TeV, respectively.

III. PREDICTIONS FOR THE ELLIPTIC FLOW PARAMETER

The Fourier expansion is another method investigating the particle transverse momentum azimuthal asymmetry 13,14. In 14 the Fourier expansion of the particle number (multiplicity) distribution is expressed as

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T} [1 + \sum_{n=1,2,...} 2\nu_n \cos(n\phi - \Psi_r)],$$

(11)

where $\phi$ refers to the azimuthal angle of particle transverse momentum, $\Psi_r$ stands for the azimuthal angle of reaction plane. In the theoretical study, if the beam direction and impact parameter vector are fixed, respectively, on the $p_z$ and $p_x$ axes in the laboratory (Lab.) frame, then the reaction plane is just the $p_x - p_z$ plane 13. Therefore the reaction plane angle, $\Psi_r$, between the reaction plane and $p_x$ axis 13 introduced for the extracting elliptic flow experimentally 14 is zero. The equation (11) and the harmonic coefficients there reduce to

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T} [1 + \sum_{n=1,2,...} 2\nu_n \cos(n\phi)],$$

$$\langle \nu_n \rangle_p = \langle \cos(n\phi) \rangle_p,$$

$$\langle \nu_1 \rangle_p = \langle \frac{p_x}{p_T} \rangle_p,$$

$$\langle \nu_2 \rangle_p = \langle \frac{p_x^2 - p_y^2}{p_T^2} \rangle_p,$$

... (12)

where $\langle ... \rangle_p$ denotes the particle-wise average, i.e. averaged over all particles in all events 14.

The second harmonic coefficient (elliptic flow parameter $v_2$) is specially important because the large $v_2$ of emitted particles is a characteristic feature of the hot and dense medium created in the ultra-relativistic nuclear collisions. It has contributed to the suggestion of a strongly coupled quark-gluon plasma (sQGP) observed in the nucleus-nucleus collisions at RHIC energies 15-18.

There are a number of predictions for the elliptic flow parameter $v_2$ in high energy (multiplicity) pp collisions 19-22. We pursue having a prediction based on the experiments, the sphericity measurements 3-6.

The predictions of the above PYTHIA6_1 simulations are given in Fig. 2 (a) and (b) for the charged particle $v_2(\eta)$ and $v_2(p_T)$ in the minimum bias pp collisions at $\sqrt{s} = 0.9$, 7 and 14 TeV (which is calculated with $\delta_p = 0.091$ in addition), respectively. Table I gives the upper limit of the charged particle elliptic flow parameter $v_2$ obtained by averaging $v_2(p_T)$ over $p_T$. It is found in the table that the $v_2$ upper limits are estimated as
0.2-0.3 for minimum bias pp collisions at the LHC energies, which is consistent with the existing results of 0.04 - 0.2 in [18,22]. We also see in the table that the $v_2$ upper limit decreases with increasing the reaction energy. This is consistent with the ALICE results that the charged particle event averaged transverse sphericity increases from 0.613 at 0.9 TeV to 0.700 at 7 TeV.

To check how the momentum rearrangement defined in Eq. (8) change the transverse momentum distribution, we calculate the charged particle transverse momentum distributions. Shown in Fig. 3 are the results of PYTHIA6 (black solid circles) and PYTHIA6.1 (red open circles) simulations for the minimum bias pp collisions at $\sqrt{s}=0.9$ (a) and 7 TeV (b). It is found that the results of PYTHIA6 and PYTHIA6.1 are so close that one may conclude that provided the deformation parameter $\delta_p$ is far less than unity (small perturbation) the change in transverse momentum distribution caused by the rearrangement is trivial.

![FIG. 3](Color online) Charged particle transverse momentum distributions in the minimum bias pp collisions at $\sqrt{s}=0.9$ (a) and 7 TeV (b).

### TABLE I: $v_2$ upper limit obtained by averaging the $v_2(p_T)$ over $p_T$ in the minimum bias pp collisions at $\sqrt{s}=0.9$, 7 and 14 TeV.

| $\sqrt{s}$ (TeV) | PYTHIA6 | PYTHIA6.1 | PYTHIA6 | PYTHIA6.1 | PYTHIA6.1 |
|------------------|---------|-----------|---------|-----------|-----------|
| $\sqrt{s}=0.9$   | partial† | $\sim0$   | 0.297   | $\sim0$   | 0.275     | 0.267     |
|                  | full‡    | $\sim0$   | 0.291   | $\sim0$   | 0.243     | 0.237     |

† calculated in partial phase space of $p_T \geq 0.5$ GeV/c and $|\eta| \leq 0.8$.
‡ calculated in full $p_T$ and $\eta$ phase space.

As mentioned in [6] that the charged particle transverse momentum sphericity is measured (defined) "using jets to represent the final state four-momentum." This measurement is only influenced by the jet reconstruction. However, the measurement of $v_2$, whatever the event plane method [14] or the Lee-Yang zero point method [23] or the cumulant method [24], is much more model dependence, such as the dependence on the model selected for the nonflow decomposition [25]. The cumulant method is even distinguished by two-, four-, and six-particle cumulants. So the discrepancy among the measured $v_2$ values with the different methods may reach 10-

### IV. DISCUSSION AND CONCLUSION

In summary, we have proposed a method of randomly rearranging the $p_x$ and $p_y$ components of produced particle from the PYTHIA6.4 simulation on the circumference of an ellipse with the half major and minor axes being $p_T(1+\delta_p)$ and $p_T(1-\delta_p)$. The charged particle transverse momentum sphericity and the elliptic flow parameter in the minimum bias pp collisions at $\sqrt{s}=0.9$ and 7 TeV are then calculated systematically. The ALICE data of event averaged charged particle transverse sphericity as a function of charged multiplicity in the above pp collisions are well reproduced. The elliptic flow parameter as a function of $\eta$ ($v_2(\eta)$) and $p_T$ ($v_2(p_T)$) as well as the $v_2$ upper limits of $\sim0.2-0.3$ obtained by averaging $v_2(p_T)$ over $p_T$ are predicted for the minimum bias pp collisions at the LHC energies.
100%. Recently, one even argued that the event plane method is obsolete. Therefore, we strongly suggest that the transverse momentum sphericity and the elliptic flow parameter should be measured simultaneously for the benefit of cross checking and the reliable measurements.

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