Constraining the range of Yukawa gravity interaction from S2 star orbits III: improvement expectations for graviton mass bounds

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Abstract. Recently, the LIGO-Virgo collaboration discovered gravitational waves and in their first publication on the subject the authors also presented a graviton mass constraint as \( m_g < 1.2 \times 10^{-22} \text{ eV} \) [1] (see also more details in a complimentary paper [2]). In our previous papers we considered constraints on Yukawa gravity parameters [3] and on graviton mass from analysis of the trajectory of S2 star near the Galactic Center [4]. In the paper we analyze a potential to reduce upper bounds for graviton mass with future observational data on trajectories of bright stars near the Galactic Center. Since gravitational potentials are different for these two cases, expressions for relativistic advance for general relativity and Yukawa potential are different functions on eccentricity and semimajor axis, it gives an opportunity to improve current estimates of graviton mass with future observational facilities. In our considerations of an improvement potential for a graviton mass estimate we adopt a conservative strategy and assume that trajectories of bright stars and their apocenter advance will be described with general relativity expressions and it gives opportunities to improve graviton mass constraints. In contrast with our previous studies, where we present current constraints on parameters of Yukawa gravity [3] and graviton mass [4] from observations of S2 star, in the paper we express expectations to improve current constraints for graviton mass, assuming the GR predictions about apocenter shifts will be confirmed with future observations. We concluded that if future observations of bright star orbits during around fifty years will confirm GR predictions about apocenter shifts of bright star orbits it give an opportunity to constrain a graviton mass at a level around \( 5 \times 10^{-23} \text{ eV} \) or slightly better than current estimates obtained with LIGO observations.

Keywords: black hole physics; gravity; modified gravity; massive graviton theories; graviton; gravitational waves
1 Introduction

1.1 Theories of massive gravity

A development of general relativity for more than 100 years was extremely successful. Currently, predictions of GR have been confirmed with many different experiments and observations. Recently, gravitational waves and binary black holes have been discovered with LIGO detectors. Now, one can say that "Einstein was 100% right" [5] but the correctness of the claim may be changed with new observational and experimental data and GR phenomena must be checked again and again with better precision. There are many proposals to check GR predictions with future observations and experiments. However, if we adopt GR as the universal theory of gravity, then we face the dark matter (DM) and dark energy (DE) problems. There is a slow progress to find solutions of the DM and DE puzzles. There exists a natural way to avoid DM and DE problems with changing the fundamental gravity law. Now there are many alternative theories of gravity and some of these theories have no Newtonian limit in a weak gravitational field approximation. A theory of massive gravity where graviton has non-vanishing mass is among popular alternative theories of gravity. A direct graviton detection is an extremely difficult problem [6], however, if a graviton has a mass one can discuss opportunities to constrain its mass with current and future observations. In the paper we discuss the issue in more details. A theory of massive gravity was introduced by M. Fierz and W. Pauli in 1939 [7]. At the beginning a development of the theory was very slow. In seventies of the last century a couple of pathologies of such a gravity theory have been found, such as van Dam – Veltman – Zakharov – Iwasaki discontinuity [8–10] for \( m_g \rightarrow 0 \) (where \( m_g \) is a graviton mass) and if one considers a deflection of light, for instance, in the framework of Fierz – Pauli theory with vanishing graviton mass, one obtains a result which is different from the GR expression. However, the analysis presented in these papers based on linear approximation of weak gravitational field and as it was shown in [11] the approximation is
valid only for \( r \gg r_V \) (where \( r_V \) is the Vainshtein radius and \( r_V = \left( \frac{M}{m_g} \right)^{1/5} \) in Planck units, therefore, for typical stellar masses and current graviton mass estimates the length is much greater than a size of Solar system). The phenomenon is called now Vainshtein screening (see, also [12–16] for a more detailed discussion).

Boulware and Deser found a presence of ghosts and instabilities in theory of massive gravity [17, 18]. Nowadays, a number of different techniques have been proposed to construct theories of massive gravity without Boulware – Deser ghosts [13, 19]. A class of ghost-free massive gravity has been proposed in papers [20, 21], such a theory are called now de Rham – Gabadadze – Tolley (dRGT) gravity model (see also more comprehensive reviews [22, 23]).

A. A. Logunov and his group proposed a bi-metric theory of gravity with a massive graviton and investigated its cosmological consequences in papers [24–29] at the period of low popularity of such theories of massive gravity due to a presence of pathologies such as discontinuities and ghosts, see also recent publications of this group and references therein [30–32].

In the last years it has been a great progress in theoretical development of massive gravity theories and they may be treated now as a reasonable alternative for the conventional GR and people discuss observational signatures of such theories.

1.2 Constraints on parameters of Yukawa gravity

Massive gravity is a specific case of Yukawa gravity. Any universal theoretical model of gravity has to be checked at different scales: in laboratory, Solar system [33], our Galaxy (including both its innermost region around the central supermassive black hole, as well as its outer parts), other galaxies, clusters of galaxies and at cosmological scales. Geophysical, astronomical and laboratory constraints on Yukawa gravity are summarized in Fig. 9 (for 1 \( \mu \)m \( \lesssim \Lambda \lesssim 1 \) cm), and Fig. 10 (for 1 cm \( \lesssim \Lambda \lesssim 10^{15} \) m) in [34]. From these data one can see that there is a growing trend for strength of Yukawa interaction with increase of its range, especially for \( \Lambda > 10^{11} \) m. Also, one can see that gravity is relatively well constrained at short ranges (see Figs. 9 and 10 in [34]). Solar System, Lunar Laser Ranging (LLR) and LAGEOS constraints give \( \Lambda \gg 1.5 \times 10^{11} \) m and \( \Lambda \gg 4 \times 10^8 \) m [34, 38]. Clearly, that forthcoming data analysis of LARES data will improve LAGEOS constraints on Yukawa theory parameters [35–37]. Toy models with the Yukawa potential derived in \( f(R) \) gravity have been considered for Solar System and the Galactic Center and it was shown how to evaluate Yukawa potential parameters from observations for these systems [39, 40]. Yukawa potential have been successfully used to fit observational data for galactic rotation curves and clusters of galaxies [41–43]. It is important to note that there are not too many constraints for parsec (and sub-parsec) scales and such constraints could be obtained from an analysis of star trajectories in the Galactic Central Parsec and we try to evaluate expectations to improve previous results on graviton mass constraints with S2 star trajectory. The obtained constraints from S-star orbits (\( 10^{15} \lesssim \Lambda \lesssim 10^{18} \) m) are beyond the parameter region given in Figs. 9 and 10 from Ref. [34]. Besides, it is very important to investigate gravity in the vicinity of black holes to probe a gravity law in a strong gravitational field limit.

1.3 Observational constraints on graviton mass

Probably first estimates of graviton mass have been given by F. Zwicky as \( m_g < 5 \times 10^{-64} \)g [44] (to obtain this estimate he assumed that the Newtonian law has to be valid until the
typical scales for galactic clusters\(^1\) (some time before Zwicky discussed an opportunity that the fundamental gravity law has to be valid for scales of galactic clusters because in this case it is checked in a reliable way while for longer length scales it may be changed\(^46\) and for such scales an impact of the cosmological \(\Lambda\)-term may be significant). M. Hare used Galactic scale to estimate Compton length for graviton\(^47\) \(m_g < 1.2 \times 10^{-66}\) g. If one uses the cosmological length scale\(^26, \ 27\), then \(m_g < m_0^H\), where \(m_0^H = \frac{\hbar H_0}{c^2} = 3.8 h \times 10^{-66}\) g is "Hubble mass" and \(H_0 = h_{100} \times 100\) km/(s Mpc) is the current Hubble constant (\(h_{100}\) is a useful dimensionless parameter).

Observations of pulsars give an opportunity to evaluate a graviton mass. As it is well-known binary pulsars provide a remarkable test of GR predictions that their orbits have to be shrinking due to gravitational radiation and it firstly was observed for Hulse – Taylor pulsar PSR B1913+16\(^48\) (see also\(^49\) for a more modern review). In paper\(^50\) Finn and Sutton showed that evolutions of orbits for the binary pulsars PSR B1913+16 and PSR B1534+12 constrained a graviton mass at a level \(7.6 \times 10^{-20}\) eV with 90% C.L. A review of opportunities to evaluate parameters of alternative theories of gravity with binary pulsars is given\(^51\). In paper\(^52\) it was evaluated an opportunity to estimate a graviton mass from future LISA observations of evolution of eccentric binary systems.

As it was noted many years ago predicted and observed time of arrivals for pulsar residuals may be used for detection of gravitational waves with long wavelengths\(^53\). This idea is widely used to detect (or constrain) long gravitational waves with pulsar timing observations\(^54, \ 55\). As it was shown in\(^56\), current and future pulsar timing arrays could be used not only for detection of gravitational waves but also to obtain graviton mass constraints for different programs for observations at a level around \([3 \times 10^{-23}, 3 \times 10^{-22}]\) eV as and later Lee showed that these constraints can be improved with a more sophisticated data analysis\(^57\).

In paper\(^58\) the authors considered study of massive spin-2 fluctuations (including massive gravitons) around Schwarzschild and slowly rotating Kerr black holes and concluded that such black holes are generically unstable and solid observations of such objects will lead to conclusion that graviton mass is less than \(5 \times 10^{-23}\) eV.

Considering weak lensing for galactic clusters, one could obtain a more stringent constraint on a graviton mass \(6 \times 10^{-32}\) eV\(^59\), in the paper the authors argued that the corresponding Compton wavelength is more than 100 Mpc, while an analysis of CMB anisotropy leads to conclusion that Compton wavelength should be more than 4 Mpc\(^60\).

In papers\(^61, \ 62\) a minimal speed of gravity \(v_g = c(1 - 0.15)\) has been evaluated from the relativistic light deflection of the quasar J0842+1835 by gravitational field of Jupiter on September 8, 2002. In principle one could evaluate a graviton mass from the constraint on speed of gravity and the dispersion relation.

A number of other ways to constrain a graviton mass from observations are given in comprehensive reviews\(^23, \ 63\). One should note that very often when people discussed observational constraints on graviton mass they presented their expectations or forecasts from future observations since uncertainties and systematics were not carefully analyzed. Therefore, such estimates are model dependent.

\(^1\)H"ida and Yamaguchi used a similar assumption in paper\(^45\) where they obtained the estimate \(m_g < 5 \times 10^{-62}\) g. As V. L. Ginzburg noted a couple of times, in astronomy ten is equal to one, moreover, in astronomy estimates for lengths and masses (and related quantities) are significantly changing with time because an evaluation of lengths in astronomy is often model dependent.
1.4 Observational constraints on graviton mass from observations of gravitational waves

In times when theories of massive gravity were not very popular due a presence of pathologies C. Will considered an opportunity to evaluate a graviton mass from observations of gravitational waves [64] (see also [65] for a more detailed discussion).

Assuming that a graviton mass is small in comparison with energy of gravitational waves $h f \gg m_g c^2$, then

$$v_g/c \approx 1 - \frac{1}{2} \left( \frac{c}{\lambda_g f} \right)^2,$$

where $\lambda_g = h/(m_g c)$ is the graviton Compton wavelength and one could obtain [64]

$$\lambda_g > 3 \times 10^{12} \text{km} \left( \frac{D}{200 \text{ Mpc}} \right)^{1/2} \left( \frac{100 \text{ Hz}}{f} \right)^{1/2} (1.1)$$

$$\Delta t = \Delta t_a - (1+z)\Delta t_e,$$  (1.3)

where $\Delta t_a = t_a^{EM} - t_a^{GW}$, $\Delta t_e = t_e^{EM} - t_e^{GW}$, $t_a^{EM}$ and $t_a^{GW}$ are arrival (emission) instants of electromagnetic radiation and arrival (emission) instant for gravitational waves. As it was pointed out [64], one can use Eq. (1.2) if observers detected gravitational waves and electromagnetic radiations from one source and $\Delta t_e$ is known or can be evaluated with a sufficient accuracy. Moreover, there is an opportunity to constrain a graviton mass in the case if there is only a gravitational wave signal. For numerical estimate, one can estimate $f \Delta t \sim \rho^{-1} \approx 10$ (where $\rho$ is a signal-to-noise ratio) for LIGO-Virgo ground based interferometers [64], therefore, a graviton mass constraint can be at a level $2.5 \times 10^{-22}$ eV for ground based LIGO-Virgo detectors.

The LIGO-Virgo collaboration reported about the first detection of gravitational waves from a merger of two black holes (it was detected on September 14, 2015 and it is called GW150914) [1]. According to estimates from the shape of gravitational wave signal the source is located at a luminosity distance of around 410 Mpc (which corresponds to a redshift $z \approx 0.09$), the initial black hole masses were $36 M_\odot$ and $29 M_\odot$ and the final black hole mass is $62 M_\odot$, therefore, around $3 M_\odot$ was emitted in gravitational waves in 0.1 s. The collaboration not only discovered gravitational waves but also detected the first binary black hole system and one of the most powerful source of radiation in the Universe and the energy was released in gravitational waves. Moreover, the team constrained the graviton Compton wavelength $\lambda_g > 10^{13}$ km which could be interpreted as a constraint for a graviton mass $m_g < 1.2 \times 10^{-22}$ eV [1] (the estimate roughly coincides with theoretical predictions [64]).

In June 2017 the LIGO-Virgo collaboration published a paper where the authors described a detection of gravitational wave signal from a merger of binary black hole system with masses of components $31.2 M_\odot$ and $19.4 M_\odot$ at distance around 880 Mpc which corresponds to $z \approx 0.18$ [66]. In this case, around $2 M_\odot$ was emitted in gravitational waves in around 0.4 s. The event is named GW170104. In this paper the authors significantly improved their previous graviton mass constraint, $m_g < 7.7 \times 10^{-23}$ eV [66].

On August 17, 2017 the LIGO-Virgo collaboration detected a merger of binary neutron stars with masses around 0.86$M_\odot$ and 2.26$M_\odot$ at a distance around 40 Mpc (GW170817) and after 1.7 s the Fermi-GBM detected $\gamma$-ray burst GRB 170817A associated with the GW170817 [67, 68]. Since gravitational wave signal was observed before GRB 170817A one could conclude that the observational data are consistent with massless or very light graviton,
otherwise, electromagnetic signal could be detected before gravitational one because in the case of relatively heavy gravitons gravitational waves could propagate slower than light.

In the consideration one assumes that photon is massless (but graviton may be massive). In the case of massive photon \( m_\gamma \geq 0 \) (see, for instance [69] for introduction of Proca theory which describes a massive photon case) to use the same logic at least we have to have \( (c - v_\gamma) \ll (c - v_g) \) \( (c \) is a limiting speed of ultra high energy quanta, \( v_\gamma \) and \( v_g \) are velocities of quanta and gravitons respectively) or

\[
m_g/f \gg m_\gamma/\nu,
\]

as we see from Eq. (1.1), where \( m_g \) and \( m_\gamma \) are masses of graviton and photon, respectively; \( f \) and \( \nu \) are their typical frequencies) and photon mass is constrained with another experimental (or observational) data. Different ways to evaluate photon mass are discussed in couple of reviews [63, 70, 71] and original papers [72–77]. Laboratory experiments gave the upper limit as \( m_\gamma < 7 \times 10^{-19} \) eV [72] or \( m_\gamma < 5 \times 10^{-20} \) eV [73], while astrophysical constraint from analysis of plasma in Solar wind gave \( m_\gamma < 10^{-18} \) eV [74], analysis of Fast Radio Bursts gave weaker constraints on photon mass \( m_\gamma < 10^{-14} \) eV [75–77].

One could roughly estimate frequency band for quanta where inequality (1.4) is hold. If we adopt the upper limit of graviton mass (around \( 10^{-22} \) eV) obtained by the LIGO collaboration from the first GW events without electromagnetic counterpart and we assume \( f \approx 100 \), then the inequality (1.4) is hold for spectral band of quanta from radio up to higher frequencies if we use upper limit estimates from papers [72–74] and the inequality (1.4) is hold for spectral band of quanta from optical band up to higher frequencies if we use upper limit estimates from papers [75–77].

Constraints on speed of gravitational waves have been found \(-3 \times 10^{-15} < (v_g - c)/c < 7 \times 10^{-16}\) [68]. Graviton energy is \( E = hf \), therefore, assuming a typical LIGO frequency range \( f \in (10, 100) \), from the dispersion relation one could obtain a graviton mass estimate \( m_g \approx 3 \times (10^{-21} - 10^{-20}) \) eV which a slightly weaker estimate than previous ones obtained from binary black hole signals detected by the LIGO team [78].

### 1.5 Observations of bright stars near the Galactic Center

Two groups of observers are monitoring bright stars (including S2 one) to evaluate gravitational potential at the Galactic Center [79–93]. The precision observations of S2 star [94] were used to evaluate parameters of black hole and to test and constrain several models of modified gravity at mpc scales [4, 95–99]. The simulations of the S2 star orbit around the supermassive black hole at the Galactic Centre (GC) in Yukawa gravity [3] and their comparisons with the NTT/VLT astrometric observations of S2 star [94] resulted with the constraints on the range of Yukawa interaction \( \Lambda \). These constrains showed that \( \Lambda \) is most likely on the order of several thousand astronomical units. Assuming Yukawa gravitational potential of a form \( \propto r^{-1} \exp(-r/\lambda_g) \) [see e.g. 64] this result indicates that it can be used to constrain the lower bound for Compton wavelength \( \lambda_g \) of the graviton, i.e. the upper bound for its mass

\[
m_{g(upper)} = h c/\lambda_g.
\]

### 2 Estimates

The goal of this paper is to discuss perspectives to reduce an upper limit for graviton mass \( m_g \) with the future observations of bright stars assuming the bright star apocenter shifts calculated in the framework of classical GR will be confirmed with future observations, therefore,
analyzing possible range for $\Lambda_{\text{crit}}$ parameters, we conclude that $\Lambda$ parameters corresponding to greater shifts $\Lambda < \Lambda_{\text{crit}}$ should be ruled out with these observations. We think that our assumption is rather conservative because earlier GR predictions were always confirmed (if the GR predictions will not be confirmed with future observations we have to introduce new object such as a bulk distribution of stellar cluster or dark matter or to change a fundamental gravity law according to Le Verrier’s suggestions [100]). Therefore, we have the condition for $\Lambda_{\text{crit}}$, that Yukawa gravity induces the same orbital precession as GR. Assuming that $\Lambda_{\text{crit}}$ gives a lower limit for Compton length for graviton one obtains an upper graviton mass constraint. Thus, we expect to improve a current estimate of graviton mass with the future observations analyzing apocenter advances in Yukawa potential and GR.

In other words, we consider $\Lambda$ parameter of Yukawa gravity which induces the same orbital precession as predicted by GR. We also show that, if such precessions will be detected in the observed orbits of S-stars around the Galactic Center, they could be used to put more stringent constraints on the range of Yukawa interaction, as well as on the mass of graviton. For that purpose, we first find the conditions which must be satisfied by the parameters of Yukawa gravity in order to induce the same stellar orbits as GR, then we use these forecasts to constrain the range of Yukawa interaction and graviton mass, taking into account the latest results about orbital parameters of stellar orbits at the Galactic Center (GC), given in [90, 91]. We also perform graphical comparisons of the simulated stellar orbits with the same apocenter shifts in these two theories of gravity.

2.1 Orbital precession due to general central-force perturbations

A general expression for apocenter shifts for Newtonian potential and small perturbing potential is given as a solution of problem 3 in Section 15 in the classical Landau & Lifshitz (L & L) textbook [101]. In paper [104], the authors derived the expression which is equivalent to the (L & L) relation in an alternative way and showed that the expressions are equivalent and after that they calculated apocenter shifts for several examples of perturbing functions.

According to [104], orbital precession $\Delta \varphi$ per orbital period, induced by small perturbations to the Newtonian gravitational potential $\Phi_N(r) = -\frac{GM}{r}$ could be evaluated as:

$$\Delta \varphi^{\text{rad}} = -\frac{2L}{GMe^2} \int_{-1}^{1} \frac{z \cdot dz}{\sqrt{1-z^2}} \frac{dV(z)}{dz}, \quad (2.1)$$

while in the textbook [101] it was given in the form

$$\Delta \varphi^{\text{rad}} = -\frac{2L}{GMe^2} \int_{0}^{\pi} \cos \varphi r^2 \frac{dV(r)}{dr} d\varphi, \quad (2.2)$$

where $V(z)$ is the perturbing potential, $r$ is related to $z$ via: $r = \frac{L}{1 + ez}$ in Eq. (2.1) (and $r = \frac{L}{1 + e \cos \varphi}$ in Eq. (2.2)) , and $L$ being the semilatus rectum of the orbital ellipse with semi-major axis $a$ and eccentricity $e$.

\[\text{Note:} \text{See applications of the expressions for calculations of stellar orbit precessions in presence of supermassive black hole and dark matter [102, 103].}\]
\[
L = a \left(1 - e^2\right),
\]
while \(\Delta \varphi\) represents true precession in the orbital plane, and the corresponding apparent value \(\Delta s\), as seen from Earth at distance \(R_0\), is \(\text{[105]}\) (assuming that stellar orbit is perpendicular to line of sight and taking into account an inclination of orbit one has to write an additional factor which is slightly less than 1 in the following expression):
\[
\Delta s \approx \frac{a(1+e)}{R_0} \Delta \varphi.
\]

In order to compare the orbital precession of S-stars in both GR and Yukawa gravity, we applied the same procedure as described in \(\text{[3]}\) to perform the two-body simulations of the stellar orbits in the framework of these two theories.

### 2.2 Orbital precession in General Relativity

In order to simulate the orbits of S-stars in GR we used parameterized post-Newtonian (PPN) equation for acceleration of a test particle orbiting the central mass, given in \(\text{[106]}\). From their Eqs. (11) and (12), one can easily obtain the following PPN equation of motion for two-body problem \(\text{[106]}\):
\[
\ddot{\mathbf{r}} = -G(M + m) \frac{\mathbf{r}}{r^3} + \frac{GM}{c^2r^3} \left\{ 2 \left( \beta + \gamma \right) \frac{GM}{r} - \gamma \left( \mathbf{\dot{r}} \cdot \mathbf{\dot{r}} \right) \mathbf{\dot{r}} + 2 \left( 1 + \gamma \right) \left( \mathbf{\dot{r}} \cdot \mathbf{\dot{r}} \right) \mathbf{\ddot{r}} \right\}.
\]
(2.5)

The second term in r.h.s. of the above expression (2.5) represents the PPN correction to the acceleration due to contribution of central mass, and it depends on two PPN parameters: \(\beta\) and \(\gamma\). In the case of GR: \(\beta = 1\) and \(\gamma = 1\), and then this PPN correction induces the well known expression for Schwarzschild precession [see e.g. \(\text{65, 107}\)]:
\[
\Delta \varphi^{\text{rad}} \approx \frac{2\pi GM}{c^2a(1-e^2)} (2 + 2\gamma - \beta) \Rightarrow \Delta \varphi^{\text{rad}}_{GR} \approx \frac{6\pi GM}{c^2a(1-e^2)}.
\]
(2.6)

The corresponding formula for apparent precession \(\Delta s_{GR}\), as seen from Earth at distance \(R_0\), could be calculated according to \(\text{[105]}\):
\[
\Delta s_{GR} \approx \frac{6\pi GM}{c^2(1-e)R_0},
\]
(2.7)
which in the case of GR does not depend on \(a\).

### 2.3 Orbital precession in Yukawa gravity

In order to simulate orbits of S-stars in Yukawa gravity we assumed the following gravitational potential [see e.g. 3, and references therein]:
\[
\Phi_Y(r) = -\frac{GM}{(1+\delta)r} \left[ 1 + \delta e^{-\frac{r}{\Lambda}} \right],
\]
(2.8)
where \(\Lambda\) is the range of Yukawa interaction and \(\delta\) is a universal constant. Here we will assume that \(\delta > 0\), as indicated by data analysis of astronomical observations [see e.g. 42, 43]. Yukawa gravity induces a perturbation to the Newtonian gravitational potential described by the following perturbing potential:
\[ V_Y(r) = \Phi_Y(r) + \frac{GM}{r} = \frac{\delta GM}{1 + \delta} \left( 1 - e \frac{r}{\Lambda} \right) \]  

(2.9)

The exact analytical expression for orbital precession in the case of the above perturbing potential could be presented in the integral form Eqs. (2.1) and (2.2), but we will calculate the approximate expression for \( \Delta \phi \) using power series expansion of \( V_Y(r) \), assuming that \( r \ll \Lambda \):

\[ V_Y(r) \approx -\frac{\delta GMr}{2(1 + \delta)\Lambda^2} \left[ 1 - \frac{r}{3\Lambda} + \frac{r^2}{12\Lambda^2} - \ldots \right], \quad r \ll \Lambda, \]  

(2.10)

where we neglected the constant term since it does not affect \( \Delta \phi \). By substituting the above expression into (2.1) we obtain the following approximation for the angle of orbital precession in Yukawa gravity:

\[ \Delta \varphi_Y^\text{rad} \approx \frac{\pi \delta \sqrt{1 - e^2}}{1 + \delta} \left( \frac{a^2}{\Lambda^2} - \frac{a^3}{\Lambda^3} + \frac{4 + e^2}{8} \frac{a^4}{\Lambda^4} - \ldots \right). \]  

(2.11)

As it was shown in [104] the right-hand side in Eq. (2.11) could be presented as series of Gauss’s hypergeometric function \( _2F_1 \) with different arguments.

Since \( r \ll \Lambda \) also implies that \( a \ll \Lambda \), we can neglect higher order terms in the above expansion. The first order term then yields the following approximate formula for orbital precession:

\[ \Delta \varphi_Y^\text{rad} \approx \frac{\pi \delta \sqrt{1 - e^2}}{1 + \delta} \frac{a^2}{\Lambda^2}, \quad a \ll \Lambda. \]  

(2.12)

Both, \( \Delta \varphi_{GR} \) and \( \Delta \varphi_Y \) represent the angles of orbital precession per orbital period in the orbital plane (i.e. true precession). The corresponding apparent values in Yukawa case \( \Delta s_Y \), as seen from Earth at distance \( R_0 \) is (for \( \delta = 1 \), according to paper [105]):

\[ \Delta s_Y \approx \frac{a(1 + e)}{R_0} \Delta \varphi_Y^\text{rad} \approx 0.5\pi \frac{a^3}{R_0\Lambda^2} (1 + e) \sqrt{1 - e^2}. \]  

(2.13)

If one believes that a gravitational field at the Galactic Center is described with a Yukawa potential, then the maximal \( \Delta s_Y \) value corresponds to \( e = 1/2 \) when function \( (1 + e)\sqrt{1 - e^2} \) has its maximal value (assuming that all other parameters are fixed).

### 2.4 Expectations to constrain the range of Yukawa gravity with future observations

We assume that in future GR predictions about precession angles for bright star orbits around the Galactic Center will be successfully confirmed, therefore, for each star we have a constraint on \( \Lambda \) which can be obtained from the condition for \( \Lambda \), so that Yukawa gravity induces the same orbital precession as GR. This constraint can be obtained directly from (2.6) and (2.12), assuming that \( \Delta \varphi_Y = \Delta \varphi_{GR} \). In this way we obtain that:

\[ \Lambda \approx \frac{\delta c^3 (a \sqrt{1 - e^2})^3}{6(1 + \delta)GM}. \]  

(2.14)

As it can be seen from the above expression, taking into account that \( \delta \) is universal constant, the corresponding values of \( \Lambda \) in the case of all S-stars depend only on the semi-major axes.
and eccentricities of their orbits. In order to stay in accordance with [4], here we will also assume that $\delta = 1$, in which case formula (2.14) reduces to:

$$\Lambda \approx \frac{c}{2} \sqrt{\frac{(a\sqrt{1-e^2})^3}{3GM}} \approx \sqrt{\frac{(a\sqrt{1-e^2})^3}{6R_S}},$$  

(2.15)

Using Kepler law we could write the previous equation in the following form

$$\Lambda \approx \frac{T}{T_0} \sqrt{\frac{(a_0\sqrt{1-e^2})^3}{6R_S}}.$$  

(2.16)

where $T_0$ and $a_0$ are periods and semi-major axis for a selected orbit of S2 star, for instance. Inspecting Eq. (2.16) one can see that greater constraints on $\Lambda$ parameter (smaller graviton mass bounds) correspond to orbits with longer periods and smaller eccentricities (usually orbits with small eccentricities are not interesting to test GR precession phenomena because shifts for more eccentric orbits are greater).

3 Numerical results

We first applied the formula (2.15) to estimate the value of $\Lambda$ in the case of S2-star, so that its orbits in GR and Yukawa gravity almost coincide with each other. For that purpose we used the results presented in [90], according to which mass of the central black hole of the Milky Way is $M = 4.28 \times 10^6 M_\odot$, distance to the GC is $R_0 = 8.32$ kpc, semi-major axis of the S2-star orbit is $a = 0.^\prime\prime 1255$, and its eccentricity $e = 0.8839$. The corresponding range of Yukawa gravity according to Eq. (2.15) is $\Lambda = 15125.5$ AU. We then compared the orbital precession of S2-star in both GR and Yukawa gravity by performing the two-body simulations of its orbit in the same way as described in [3]. Fig. 1 presents a graphical comparison between the obtained simulated orbits during five orbital periods. As it can be seen from this figure, the values of parameter $\Lambda$ calculated from (2.14) produce the simulated orbits in Yukawa gravity which are almost identical to those in GR, therefore, if the geodesics for these two different theories of gravity produce the same apocenter advances, they have tiny differences in their shapes.

Besides, we also used this approach to calculate the values for $\Lambda$ in the case of all S-stars from the Table 3 of [90] except S111. The observed orbital elements, together with their uncertainties, of these stars are given in Table 1, while the obtained numerical results are presented in Table 2, where $\Delta \varphi$ and $\Delta s$ represent orbital precession in both GR and Yukawa gravity, since their numerical values are identical up to three decimals, at least.

The obtained estimates of $\Lambda$ are then used to find the corresponding constraints for the graviton mass $m_g$, according to Eq. (1.5), where we assumed that the Compton wavelength $\lambda_g$ of graviton is equal to the obtained values of $\Lambda$: $\lambda_g = \Lambda$. The resulting graviton masses are also given in Table 1. The first column in Table 1 contains the observed semi-major axes of all S-stars which are converted from arcsec to AU using the following distance to Sgr A*: $R_0 = 8.32$ kpc [90].

The relative errors of the range of Yukawa interaction and graviton mass can be obtained by differentiating the logarithmic versions of Eqs. (2.15,1.5), which gives the following expression:

$$\frac{\Delta \Lambda}{\Lambda} = \frac{\Delta m_g}{m_g} \approx \pm 3 \left( \frac{|\Delta a|}{a} + \frac{e|\Delta e|}{1-e^2} + \frac{1}{3} \frac{|\Delta M|}{M} \right),$$  

(3.1)
Figure 1. Comparison between the simulated orbits of S2-star in GR (blue solid line) and in Yukawa gravity (red dashed line) during five orbital periods. Region around apocenter is zoomed in the right panel in order that small orbital precession of $\Delta \varphi = 722.71 = 0.82$ is visible.

Currently, we know that $\Delta M/M$ is around 7.2% [90] is a relatively small quantity and it is the same for all orbits, so that the uncertainty contributes the relative error (around 3.6%) to relative errors which are presented in column 6 of Table 2. There is a hope that in future the relative error for the black hole mass estimate will be improved. We used the above analytic expression to estimate the numerical values of relative and absolute errors for $\Lambda$ and $m_g$ which are also given in Table 2. As it can be seen from this table, the obtained relative errors range from a few percent (as e.g. for S6 and S38) up to more than a hundred of percent (as e.g. for S54 and S85), depending on the current observational uncertainties for the semi-major axis and eccentricity of the S-star orbits (see Table 1). On the other hand, here we showed that the relative error of the graviton mass estimate in the case of the S2 star is $\approx 5.8\%$ (see Table 2). Therefore, one could expect that with future observations apocenter shift for S2 star one should obtain graviton mass estimate around $5.48 \times 10^{-22}$ eV with relative error $\approx 5.8\%$, while, for instance, for S4 star with orbital period $T = 77$ yr we have $4.1 \times 10^{-23}$ eV with relative error $\approx 5.6\%$, assuming current uncertainties for mass, eccentricity and semi-major axis. The expectation for graviton mass estimate with S4 star trajectory is better than the graviton mass LIGO estimate obtained from GW170104 event. One should note that a) current estimates for errors of orbital parameters and mass are changing with time since subsequent observations will be performed; b) GR predictions about pericenter shifts will be confirmed with additional uncertainties; c) when subsequent observations will be done, perhaps, a theoretical model for motion of bright stars near the Galactic Center should be clarified, namely one should add into account a bulk mass distribution of stellar cluster and dark matter. However, one should keep in mind that bulk distribution of matter introduces apocenter shifts in the opposite direction in respect to relativistic advance [80, 95, 96] but Yukawa potential and PN potential in GR cause apocenter advances. It gives an opportunity to compare values for these two shifts in positive directions.

As can be seen from Table 1 for all studied S-stars obtained results are $\gtrsim 10^{-24}$ eV, and it indicates that this value represents a limit of graviton mass which could be reached with the discussed procedure. Besides, one can see that the stars with the largest semi-major axes and periods (e.g. R44) give the smallest upper bounds on graviton mass (but orbital periods are too long for these cases), while the stars with the shortest orbits (e.g. S2) give
the highest upper limits on graviton mass. Therefore, more stringent constraints on graviton mass could be achieved in future from observed orbits of S-stars with longer periods and, preferably, lower eccentricities.
except S111. The semi-major axes are converted from arcsec to AU assuming the following distance to Sgr A*: $R_0 = 8.32$ kpc [90].

| Star | $a \pm \Delta a$ (AU) | $e \pm \Delta e$ | $T \pm \Delta T$ (yr) |
|------|----------------------|-----------------|---------------------|
| S1   | 4950.4 ± 199.7       | 0.556 ± 0.018   | 166.0 ± 5.8         |
| S2   | 1044.2 ± 7.5         | 0.8393 ± 0.0019 | 16.00 ± 0.02        |
| S4   | 2970.2 ± 30.8        | 0.3905 ± 0.0059 | 77.0 ± 1.0          |
| S6   | 5469.6 ± 5.0         | 0.8400 ± 0.0003 | 192.0 ± 0.17        |
| S8   | 3367.1 ± 11.6        | 0.8031 ± 0.0075 | 92.9 ± 0.41         |
| S9   | 2266.4 ± 34.1        | 0.644 ± 0.020   | 51.3 ± 0.70         |
| S12  | 2485.2 ± 15.0        | 0.8883 ± 0.0017 | 58.9 ± 0.22         |
| S13  | 2197.3 ± 13.3        | 0.4250 ± 0.0023 | 49.00 ± 0.14        |
| S14  | 2382.0 ± 30.0        | 0.9761 ± 0.0037 | 55.3 ± 0.48         |
| S17  | 2961.1 ± 79.9        | 0.397 ± 0.011   | 76.6 ± 1.0          |
| S18  | 1979.3 ± 12.5        | 0.471 ± 0.012   | 41.9 ± 0.18         |
| S19  | 4326.4 ± 782.1       | 0.750 ± 0.043   | 135 ± 14            |
| S21  | 1822.1 ± 14.1        | 0.764 ± 0.014   | 37.00 ± 0.28        |
| S22  | 10899.2 ± 2329.6     | 0.449 ± 0.088   | 540 ± 63            |
| S23  | 2105.0 ± 99.8        | 0.56 ± 0.14     | 45.8 ± 1.6          |
| S24  | 7854.1 ± 399.4       | 0.8970 ± 0.0049 | 331 ± 16            |
| S29  | 3561.0 ± 158.1       | 0.728 ± 0.052   | 101.0 ± 2.0         |
| S31  | 3735.7 ± 83.2        | 0.5497 ± 0.0025 | 108 ± 1.2           |
| S33  | 5466.2 ± 216.3       | 0.608 ± 0.064   | 192.0 ± 5.2         |
| S38  | 1178.1 ± 1.7         | 0.8201 ± 0.0007 | 19.2 ± 0.02         |
| S39  | 3078.4 ± 124.8       | 0.9236 ± 0.0021 | 81.1 ± 1.5          |
| S42  | 7904.0 ± 1497.6      | 0.567 ± 0.083   | 335 ± 58            |
| S54  | 9984.0 ± 7238.4      | 0.893 ± 0.078   | 477 ± 199           |
| S55  | 896.9 ± 8.3          | 0.7209 ± 0.0077 | 12.80 ± 0.11        |
| S60  | 3225.7 ± 58.2        | 0.7179 ± 0.0051 | 87.1 ± 1.4          |
| S66  | 12496.6 ± 790.4      | 0.128 ± 0.043   | 664 ± 37            |
| S67  | 9368.3 ± 216.3       | 0.293 ± 0.057   | 431 ± 10            |
| S71  | 8095.4 ± 332.8       | 0.899 ± 0.013   | 346 ± 11            |
| S83  | 12396.8 ± 1580.8     | 0.365 ± 0.075   | 656 ± 69            |
| S85  | 38272.0 ± 27456.0    | 0.78 ± 0.15     | 3580 ± 2550         |
| S87  | 22796.8 ± 13312.2    | 0.224 ± 0.027   | 1640 ± 105          |
| S89  | 8993.9 ± 457.6       | 0.639 ± 0.038   | 406 ± 27            |
| S91  | 15949.4 ± 740.5      | 0.303 ± 0.034   | 958 ± 50            |
| S96  | 12471.7 ± 474.2      | 0.174 ± 0.022   | 662 ± 29            |
| S97  | 19302.4 ± 3827.2     | 0.35 ± 0.11     | 1270 ± 309          |
| S145 | 9318.4 ± 1497.6      | 0.50 ± 0.25     | 426 ± 71            |
| S175 | 3444.5 ± 324.5       | 0.9867 ± 0.0018 | 96.2 ± 5.0          |
| R34  | 15059.2 ± 1248.0     | 0.641 ± 0.098   | 877 ± 83            |
| R44  | 32448.0 ± 11648.0    | 0.27 ± 0.27     | 2730 ± 1350         |
4 Conclusions

A precession of orbit in Yukawa potential is in the same direction as in GR, but dependences of precession angles on semi-major axis and eccentricity are different in these two models, therefore, after observations of bright star orbits for one or a few periods one could select the best fit from two considered cases.

In paper [4] we presented an upper bound for graviton mass $2.9 \times 10^{-21}$ eV using previous observations of S2 star (see also [108–112] for a more detailed discussion) and now we also demonstrate our forecasts to reduce this upper limit. As it was noted earlier, our current estimates for graviton mass is slightly weaker than the LIGO ones, but it is independent and consistent with LIGO results and we expect that the graviton mass estimate will be significantly improved with new observations.

The obtained results show that (see Fig. 1 and Table 2):

1. Range of Yukawa gravity $\Lambda$ can be constrained in such a way to induce the same orbital precession of stellar orbits as in GR;

2. Orbits with small eccentricities provide better constraints on graviton mass (see Eq. (2.16));

3. There is a linear dependence of $\Lambda$ constraint on orbital periods of bright stars (perhaps, monitoring bright stars for more than 50 – 100 years looks rather problematic, but people monitor comets for centuries);

4. Such a precession of stellar orbits around GC, if observed, could provide strong constraints on the mass of graviton, indicating that it is most likely $\approx 8 \times 10^{-23}$ eV for orbital periods around a several decades, (see, parameters for S9 star, for instance).

5. If we assume that the future telescopes give an opportunity to a bright star with a period around 50 years and small eccentricity then the similar procedure give a graviton mass constraint around $\approx 5 - 6 \times 10^{-23}$ eV, while pericenter advance for such a star will be $\Delta s_{GR} \approx 0.1$ mas which is in principle should be detectable in the future.

We expect that future observations of trajectories of bright stars near the Galactic Center like GRAVITY [113], E-ELT [114] and TMT [115] will realize our forecasts and improve the current graviton mass constraints.

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References

[1] B. P. Abbott et al., (LIGO Scientific Collaboration and Virgo Collaboration), Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116 (2016) 061102.

[2] B. P. Abbott et al., (LIGO Scientific Collaboration and Virgo Collaboration), Tests of general relativity with GW150914, Phys. Rev. Lett. 116 (2016) 221101; LIGO Document P1500213-v27, arXiv:1602.03841 (2016).

[3] D. Borka, P. Jovanović, V. Borka Jovanović, A. F. Zakharov, Constraining the range of Yukawa gravity interaction from S2 star orbits, JCAP 11 (2013) 050.

[4] A. F. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, Constraining the range of Yukawa gravity interaction from S2 star orbits II: bounds on graviton mass, JCAP 5 (2016) 045.

[5] T. Damour, Was Einstein 100% Right?, in Proceedings of the Albert Einstein Centenary International Conference, Eds. J.-M. Alimi and André Füzfa, AIP Conference Proceedings 861 (2006) 135.

[6] F. Dyson, Is a Graviton Detectable? Intern. J. Mod. Phys. A 28 (2013) 1330041 (14 pages).

[7] M. Fierz and W. Pauli, On Relativistic Wave Equations for Particles of Arbitrary Spin in an Electromagnetic Field, Proc. of the Royal Society of London A173 (1939) 211.

[8] H. van Dam and M. Veltman, Massive and mass-less Yang-Mills and gravitational fields, Nucl. Phys. B 22 (1970) 397.

[9] V. I. Zakharov, Linearized Gravitation Theory and the Graviton Mass, JETP Lett. 12 (1970) 447.

[10] Y. Iwasaki, Consistency Condition for Propagators, Phys. Rev. D 2 (1970) 2255.

[11] A.I. Vainshtein, To the problem of non-vanishing gravitation mass, Phys. Lett. B 39 (1972) 393.

[12] C. Deffayet, G. Dvali, G. Gabadadze and A. Vainshtein, Nonperturbative continuity in graviton mass versus perturbative discontinuity, Phys. Rev. D 65 (2002) 044026.

[13] V. A. Rubakov and P. G. Tinyakov, Infrared-modified gravities and massive gravitons, Physics – Uspekhi 51 (2008) 759.

[14] E. Babichev, C. Deffayet and R. Ziour, Recovering General Relativity from massive gravity, Phys. Rev. Lett. 103 (2009) 201102.

[15] E. Babichev and C. Deffayet, An introduction to the Vainshtein mechanism, Class. Quantum Grav. 30 (2013) 184001 (25pp).

[16] A. Zee, Quantum field theory in a nutshell, Princeton University Press, Princeton (2010).

[17] D. G. Boulware and S. Deser, Can Gravitation Have a Finite Range? Phys. Rev. 6 (1972) 3368 (1972).

[18] D. G. Boulware and S. Deser, Inconsistency of finite range gravitation Phys. Lett. B 40 (1972) 227.

[19] K. Hinterbichler, Theoretical aspects of massive gravity, Rev. Mod. Phys. 84 (2012) 671.

[20] C. de Rham and G. Gabadadze, Generalization of the Fierz-Pauli Action, Phys.Rev. D 82, 044020 (2010).

[21] C. de Rham, G. Gabadadze and A. J. Tolley, Resummation of Massive Gravity, Phys. Rev. Lett. 106, 231101 (2011).

[22] C. de Rham, Massive Gravity, Living Rev. Rel. 17, 7 (2014).

[23] C. de Rham, J. T. Deskins, A. J. Tolley et al., Graviton mass bounds, Rev. Mod. Phys. 89, 025004 (2017).
[24] A. A. Logunov, M. A. Mestvirishvili and Yu. V. Chugreev, Graviton mass and evolution of a Friedmann universe, Theor. Math. Phys. 74 (1988) 1.
[25] Yu. V. Chugreev, Cosmological consequences of the relativistic theory of gravitation with massive gravitons, Theor. Math. Phys. 79 (1989) 554.
[26] S. S. Gershtein, A. A. Logunov and M. A. Mestvirishvili, Graviton Mass and the Total Relative Mass Density $\Omega_{tot}$ in the Universe, Dokl. Phys. 48 (2003) 282.
[27] S. S. Gershtein, A. A. Logunov, M. A. Mestvirishvili and N. P. Tkachenko, Graviton Mass, Quintessence, and Oscillatory Character of Universe Evolution, Phys. Atom. Nucl. 67 (2004) 1596.
[28] A. A. Logunov, Relativistic Theory of Gravitation, (Nauka, Moscow, 2006, in Russian).
[29] S. S. Gershtein, A. A. Logunov and M. A. Mestvirishvili, Graviton Mass and the Total Relative Mass Density $\Omega_{tot}$ in the Universe, Dokl. Phys. 48 (2003) 282.
[30] V. O. Soloviev, Hamiltonian Cosmology of Bigravity, Phys. Part. Nucl. 48 (2017) 287.
[31] V. O. Soloviev, Evolution of the equations of dynamics of the Universe: From Friedmann to the present day, Theor. Math. Phys. 191 (2017) 674.
[32] Yu. V. Chugreev, Cosmological Constraints on the Graviton Mass in RTG, Phys. Part. Nucl. Lett. 14 (2017) 539.
[33] C. Talmadge, J.-P. Berthias, R. W. Hellings and E. M. Standish, Model-Independent Constraints on Possible Modifications of Newtonian Gravity, Phys. Rev. D 61 (2007) 1159.
[34] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl and S. Schlamminger, Torsion balance experiments: A low-energy frontier of particle physics, Prog. Part. Nucl. Phys. 62 (2009) 102.
[35] I. Ciufolini, B. Moreno Monge, A. Paolozzi, R. Koenig, G. Sindoni, G. Michalak and E. C. Pavlis, Monte Carlo simulations of the LARES space experiment to test General Relativity and fundamental physics, Class. Quantum Grav. 30 (2013) 235009 (11pp).
[36] I. Ciufolini, A. Paolozzi, R. Koenig, E. C. Pavlis, J. Ries, R. Matzner, V. Gurzadyan, R. Penrose, G. Sindoni and C. Paris, Fundamental Physics and General Relativity with the LARES and LAGEOS satellites, Nucl. Phys. B - Proc. Suppl. 243-244 (2013) 180.
[37] I. Ciufolini, A. Paolozzi, E. C. Pavlis, R. Koenig, J. Ries, V. Gurzadyan, R. Matzner, R. Penrose, G. Sindoni, C. Paris, H. Khachatryan and S. Mirzoyan, A test of general relativity using the LARES and LAGEOS satellites and a GRACE Earth gravity model. Measurement of Earth’s dragging of inertial frames, Eur. Phys. J. C 76(2016) 120.
[38] J. W. Moffat, A Modified Gravity and its Consequences for the Solar System, Astrophysics and Cosmology, Int. J. Mod. Phys. D 16 (2008) 2075.
[39] I. De Martino, R. Lazkoz and M. De Laurentis, Analysis of the Yukawa gravitational potential in f(R) gravity I: semiclassical periastron advance [arXiv: 1801.08135].
[40] M. De Laurentis, I. De Martino, and R. Lazkoz, Analysis of the Yukawa gravitational potential in f (R) gravity II: relativistic periastron advance arXiv: [1801.08136].
[41] S. Capozziello, A. Stabile and A. Troisi, The Newtonian Limit of f(R) gravity, Phys. Rev. D 76 (2007) 104019.
[42] S. Capozziello, E. De Filippis and V. Salzano, Modelling clusters of galaxies by f(R)-gravity, Mon. Not. Roy. Astron. Soc. 394 (2009) 947.
[43] V. F. Cardone and S. Capozziello, Systematic biases on galaxy haloes parameters from Yukawa-like gravitational potentials, Mon. Not. R. Astron. Soc. 414 (2011) 1301.
[44] F. Zwicky, Cosmic and Terrestrial Tests for the Rest Mass of Gravitons, Publ. Astron. Soc. Pac. 73 (1961) 314.
[45] K. Hiida and Y. Yamaguchi, Gravitation Physics, Prog. Theor. Phys. Suppl. E 65, (1965) 261.
[46] F. Zwicky, Non-Uniformities in the Apparent Distribution of Clusters of Galaxies, Publ. Astron. Soc. Pac. 69 (1957) 518.
[47] M. G. Hare, Mass of the Graviton, Can. J. Phys. 51 (1973) 431.
[48] R. A. Hulse and J. H. Taylor, Discovery of a pulsar in a binary system, Astrophys. J. 195 (1975) L51.
[49] J. M. Weisberg and Y. Huang, Relativistic measurements from timing the binary pulsar PSR B1913+16, Astrophys. J. 829 (2016) 55 (10pp).
[50] L.S. Finn and P.J. Sutton, Bounding the mass of the graviton using binary pulsar observations, Phys. Rev. D 65 (2002) 044022.
[51] G. Esposito-Farese, Motion in Alternative Theories of Gravity, in Mass and Motion in General Relativity, Fundamental Theories of Physics 162, ed. L. Blanchet, A. Spallicci, B. Whiting, Springer, Dordrecht – Heidelberg – London – New York (2011), pg. 461.
[52] D. I. Jones, Bounding the Mass of the Graviton Using Eccentric Binaries, Astrophys. J. 618 (2005) L115.
[53] M. V. Sazhin, Opportunities for detecting ultralong gravitational waves, Sov. Astron. 22 (1978) 36.
[54] G. Desvignes, R. N. Caballero, L. Lentati et al., High-precision timing of 42 millisecond pulsars with the European Pulsar Timing Array, Month. Not. R. Astron. Soc. 458 (2016) 3341.
[55] M. T. Lam, J. M. Cordes, S. Chatterjee et al., The NANOGrav nine-year data set: excess noise in millisecond pulsar arrival times, Astrophys. J. 834 (2017) 35 (19pp).
[56] K. Lee, F. A. Jenet, R. H. Price et al., Detecting massive gravitons using pulsar timing arrays, Astrophys. J. 722 (2010) 1589.
[57] K. Lee, Pulsar timing arrays and gravity tests in the radiative regime, Class. Quant. Grav. 30 (2013) 224016 (12pp).
[58] R. Brito, V. Cardoso and P. Pani, Massive spin-2 fields on black hole spacetimes: Instability of the Schwarzschild and Kerr solutions and bounds on the graviton mass, Phys. Rev. D 88 (2013) 023514.
[59] S. R. Choudhury, G. C. Joshi, S. Mahajan, B.H.J. McKellar, Probing large distance higher dimensional gravity from lensing data, Astropart. Phys. 21 (2004) 559.
[60] Binetruy and J. Silk, Probing Large Distance Higher-Dimensional Gravity with Cosmic Microwave Background Measurements, Phys. Rev. Lett. 87 (2001) 031102.
[61] E. B. Fomalont and S. M. Kopeikin, The measurement of the light deflection from Jupiter: experimental results, Astrophys. J. 598 (2003) 704; [astro-ph/0302294].
[62] S. M. Kopeikin, The speed of gravity in general relativity and theoretical interpretation of the Jovian deflection experiment, Class. Quant. Grav. 21 (2004) 3251 (2004), [gr-qc/0310059].
[63] A. S. Goldhaber and M. M. Nieto, Photon and graviton mass limits, Rev. Mod. Phys. 82 (2010) 939.
[64] C. Will, Bounding the mass of the graviton using gravitational-wave observations of inspiralling compact binaries, Phys. Rev. D 57 (1998) 2061; [gr-qc/9709011].
[65] C. Will, The Confrontation between General Relativity and Experiment, Living Reviews in Relativity 17 (2014) 4 ; arXiv:1403.7377v1[gr-qc].
[66] B. P. Abbott et al., GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2, Phys. Rev. Lett. 118 (2017) 221101.
Q. Yu, F. Zhang and Y. Lu, Prospects for Constraining the Spin of the Massive Black Hole at the Galactic Center via the Relativistic Motion of a Surrounding Star, Astrophys. J. 827 (2016) 114.

S. Gillessen, P. M. Plewa, F. Eisenhauer et al., An update on monitoring stellar orbits in the Galactic Center, Astrophys. J. 837 (2017) 30.

A. Hees, T. Do, A. M. Ghez et al., Testing General Relativity with stellar orbits around the supermassive black hole in our Galactic center, Phys. Rev. Lett. 118 (2017) 211101.

A. Hees, A. M. Ghez, T. Do et al., Testing the gravitational theory with short-period stars around our Galactic Center, arXiv:1705.10792 [astro-ph.GA].

D. S. Chu, T. Do, A. Hees et al., Investigating the Binarity of S0-2: Implications for its Origins and Robustness as a Probe of the Laws of Gravity around a Supermassive Black Hole, Astrophys. J. 854 (2018) 12 . arXiv:1709.04890 [astro-ph.SR].

S. Gillessen, F. Eisenhauer, T. K. Fritz, et al., The Orbit of the Star S2 Around SGR A* from Very Large Telescope and Keck Data, Astrophys. J. 707 (2009) L114.

A. A. Nucita, F. De Paolis, G. Ingrosso, A. Qadir and A. F. Zakharov, Sgr A*: A Laboratory to Measure the Central Black Hole and Stellar Cluster Parameters, Publ. Astron. Soc. Pac. 119 (2007) 349.

A. F. Zakharov, A. A. Nucita, F. De Paolis and G. Ingrosso, Apoastron Shift Constraints on Dark Matter Distribution at the Galactic Center, Phys. Rev. D 76 (2007) 062001.

D. Borka, P. Jovanović, V. Borka Jovanović and A. F. Zakharov, Constraints on $R^n$ gravity from precession of orbits of S2-like stars, Phys. Rev. D 85 (2012) 124004.

A. F. Zakharov, D. Borka, V. Borka Jovanović and P. Jovanović, Constraints on $R^n$ gravity from precession of orbits of S2-like stars: A case of a bulk distribution of mass, Advances in Space Research 54 (2014) 1108.

A. F. Zakharov, Possible Alternatives to the Supermassive Black Hole at the Galactic Center, J. Astrophys. Astron. 36 (2015) 539.

A. F. Zakharov, S. Capozziello, F. De Paolis, G. Ingrosso and A. A. Nucita, The Role of Dark Matter and Dark Energy in Cosmological Models: Theoretical Overview, Space Sci. Rev. 48 (2009) 301.

L. D. Landau and E. M. Lifshitz, Mechanics, Butterworth-Heinemann, Oxford (1976).

V. I. Dokuchaev and Yu. N. Eroshenko, Weighing of the Dark Matter at the Center of the Galaxy, JETP Letters 101 (2015) 777.

V. I. Dokuchaev and Yu. N. Eroshenko, Physical laboratory at the center of the Galaxy, Physics - Uspekhi 58 (2015) 772.

G. S. Adkins and J. McDonnell, Orbital precession due to central-force perturbations, Phys. Rev. D 75 (2007) 082001.

N. N. Weinberg, Milosavljević and A. M. Ghez, Stellar dynamics at the Galactic Center with an extremely large telescope, Astrophys. J. 622 (2005) 878.

J. D. Anderson, P. B. Esposito, W., Martin, C. L. Thornton and D. O. Muhleman, Experimental test of general relativity using time-delay data from Mariner 6 and Mariner 7, Astrophys. J. 200 (1975) 221.

S. Weinberg, Gravitation and cosmology: principles and applications of general relativity, John Wiley & Sons, New York (1972).

A. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, Trajectories of bright stars at the Galactic Center as a tool to evaluate a graviton mass, EPJ Web of Conferences 125 (2016) 01011.
[109] A. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, Graviton mass bounds from an analysis of bright star trajectories at the Galactic Center, *EPJ Web of Conferences* **138** (2017) 01010.

[110] A. Zakharov, P. Jovanović, D. Borka and V. Borka Jovanović, Graviton mass evaluation with trajectories of bright stars at the Galactic Center, *Journal of Physics: Conf. Ser.* **798** (2017) 012081.

[111] A. Zakharov, The black hole at the Galactic Center: observations and models in a nutshell, *Journal of Physics: Conf. Ser.* **934** (2017) 012037.

[112] A.F. Zakharov, The black hole at the Galactic Center: observations and models, *Int. J. Mod. Phys. D* **27** (2018) 1841009 (15 pp.). arXiv:1801.09920[qr-qc].

[113] N. Blind, F. Eisenhauer, S. Gillessen et al., GRAVITY: the VLTI 4-beam combiner for narrow-angle astrometry and interferometric imaging, arXiv:1503.07303 [astro-ph.IM].

[114] A. Ardeberg, J. Bergeron, A. Cimatti et al., An Expanded View of the Universe Science with the European Extremely Large Telescope, ed. by M. Lyubenova and M. Kissler-Patig, ESO, Munich (2014).

[115] W. Skidmore and TMT International Science Development Teams and TMT Science Advisory Committee, Thirty Meter Telescope Detailed Science Case: 2015, *Res. Astron. Astrophys.* **15** (2015) 1945.
Table 2. The estimated true and apparent orbital precession (columns 3 and 4), range of Yukawa interaction (column 5) and graviton mass for $\lambda = \Lambda$ (column 6), as well as the corresponding absolute errors, for all S-stars in Table 3 from [90], except S111. The last column contains the corresponding relative errors calculated according to expression (3.1), assuming the following mass of the central black hole: $M = 4.28 \pm 0.31 \times 10^6 M_{\odot}$ [90]. The orbital period obtained from the third Kepler’s law is also shown in column 2.

| Star name | $T_{Kep}$ (yr) | $\Delta \varphi$ (°) | $\Delta s$ (mas) | $\Lambda \pm \Delta \Lambda$ (AU) | $m_g \pm \Delta m_g$ (10^{-24} eV) | R.E. (%) |
|-----------|----------------|---------------------|-----------------|---------------------------------|----------------------------------|---------|
| S1        | 168.4         | 48.2                | 0.22            | 369952.9 ± 43820.4               | 22.4 ± 2.7                        | 11.8    |
| S2        | 16.3          | 722.1               | 0.83            | 15125.5 ± 884.7                 | 547.9 ± 32.0                       | 5.8     |
| S4        | 78.2          | 65.5                | 0.16            | 200418.3 ± 11191.1               | 41.4 ± 2.3                        | 5.6     |
| S6        | 195.5         | 102.4               | 0.60            | 226607.6 ± 8807.8                | 36.6 ± 1.4                        | 3.9     |
| S8        | 94.4          | 138.0               | 0.49            | 125957.9 ± 8420.6                | 65.8 ± 4.4                        | 6.7     |
| S9        | 52.2          | 124.3               | 0.27            | 101193.1 ± 9289.8                | 81.9 ± 7.5                        | 9.2     |
| S12       | 59.9          | 314.6               | 0.86            | 54047.1 ± 3026.3                 | 153.3 ± 8.6                       | 5.6     |
| S13       | 49.8          | 91.6                | 0.17            | 124334.4 ± 5855.1                | 66.7 ± 3.1                        | 4.7     |
| S14       | 56.2          | 1465.9              | 4.02            | 16508.7 ± 2802.9                 | 502.0 ± 85.2                       | 17.0    |
| S17       | 77.9          | 66.1                | 0.16            | 198588.4 ± 16771.2               | 41.7 ± 3.5                        | 8.4     |
| S18       | 42.6          | 107.1               | 0.18            | 102236.2 ± 5784.8                | 81.0 ± 4.6                        | 5.7     |
| S19       | 137.6         | 87.1                | 0.38            | 214568.1 ± 89676.6               | 38.6 ± 16.1                       | 41.8    |
| S21       | 37.6          | 217.4               | 0.41            | 56500.3 ± 4881.5                 | 146.7 ± 12.7                       | 8.6     |
| S22       | 550.0         | 19.0                | 0.17            | 1347095.9 ± 580678.1             | 6.2 ± 2.7                         | 43.1    |
| S23       | 46.7          | 114.1               | 0.22            | 102079.9 ± 28448.6               | 81.2 ± 22.6                       | 27.9    |
| S24       | 336.5         | 107.5               | 0.93            | 286723.3 ± 41927.1               | 28.9 ± 4.2                        | 14.6    |
| S29       | 102.7         | 98.5                | 0.35            | 169074.0 ± 37087.0               | 49.0 ± 11.0                       | 22.4    |
| S31       | 110.4         | 63.3                | 0.21            | 244347.7 ± 17733.9               | 33.9 ± 2.5                        | 7.3     |
| S33       | 195.3         | 47.9                | 0.25            | 400731.7 ± 75407.3               | 20.7 ± 3.9                        | 18.8    |
| S38       | 19.5          | 427.5               | 0.53            | 24533.9 ± 1005.0                 | 337.8 ± 13.8                       | 4.1     |
| S39       | 82.6          | 364.5               | 1.26            | 56824.5 ± 6638.4                 | 145.8 ± 17.0                       | 11.7    |
| S42       | 339.7         | 30.8                | 0.22            | 736342.1 ± 312551.0              | 11.3 ± 4.8                        | 42.4    |
| S54       | 482.2         | 81.5                | 0.90            | 422181.5 ± 692183.8              | 19.6 ± 32.2                       | 164.0   |
| S55       | 13.0          | 382.8               | 0.34            | 21721.6 ± 1465.5                 | 381.5 ± 25.7                       | 6.7     |
| S60       | 88.6          | 105.5               | 0.34            | 149149.2 ± 11131.0               | 55.6 ± 4.1                        | 7.5     |
| S66       | 675.3         | 13.4                | 0.11            | 1933974.1 ± 269754.5             | 4.3 ± 0.6                         | 13.9    |
| S67       | 438.3         | 19.3                | 0.14            | 118822.1 ± 116748.7              | 7.0 ± 0.7                         | 9.8     |
| S71       | 352.1         | 106.2               | 0.95            | 295890.2 ± 56006.2               | 28.0 ± 5.3                        | 18.9    |
| S83       | 667.2         | 15.3                | 0.15            | 1737943.9 ± 477698.2             | 4.8 ± 1.3                         | 27.5    |
| S85       | 3619.1        | 11.0                | 0.44            | 519511.7 ± 8106789.5             | 1.6 ± 2.5                         | 156.0   |
| S87       | 1663.8        | 7.6                 | 0.12            | 4641782.4 ± 619016.2             | 1.8 ± 0.2                         | 13.3    |
| S89       | 412.3         | 31.0                | 0.27            | 806547.1 ± 140413.3              | 10.3 ± 1.8                        | 17.4    |
| S91       | 973.6         | 11.4                | 0.14            | 2626592.1 ± 322729.8             | 3.2 ± 0.4                         | 12.3    |
| S96       | 673.2         | 13.6                | 0.12            | 1907722.2 ± 189196.9             | 4.3 ± 0.4                         | 9.9     |
| S97       | 1296.3        | 9.7                 | 0.15            | 3407955.4 ± 1361276.1            | 2.4 ± 1.0                         | 39.9    |
| S145      | 434.8         | 23.6                | 0.19            | 1016130.7 ± 535791.9             | 8.2 ± 4.3                         | 52.7    |
| S175      | 97.7          | 1812.0              | 7.23            | 18570.1 ± 5168.9                 | 446.3 ± 124.2                      | 27.8    |
| R34       | 893.3         | 18.6                | 0.27            | 1741793.7 ± 558192.6             | 4.8 ± 1.5                         | 32.0    |
| R44       | 2825.3        | 5.5                 | 0.13            | 7740489.4 ± 5361256.0            | 1.1 ± 0.7                         | 69.3    |