Topological effects on magnitude of persistent current in a ring

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Abstract

We show that defects in 1D rings decrease the conductance (when the ring is opened up) many more times than it decreases the persistent currents. This means that the states in such a 1D ring are very sensitive to twisting of boundary condition but conductance of the system is small. In 1D the electron effectively sees a periodic potential and escapes localization. This does not happen in higher dimensions. This helps us to understand the simulations of G. Kirczenow[13] and also suggests that a rough boundary in a 3D ring that provide effectively a strong 1D potential to the electron will have this type of different effects on conductance and persistent currents.

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Much before the experimental observations Böttiker et al [1] had predicted that equilibrium persistent currents may be observed in a normal metal or semiconductor ring. But the experimental observations[2-4] of these currents have posed many serious problems. Prominent among these problems is that persistent currents observed in a single gold ring[3] is orders of magnitude larger than that predicted theoretically.

There has been a large number of works[5-7] trying to give a many body explanation for the large observed value of persistent current. But recent detailed calculations[8-11] show no significant enhancement of persistent currents. Now it is considered that search for a many body explanation of the experiments of Chandrasekhar et. al. is far from over[12].

In the experiment[3] the conductance of the ring was measured by opening up the ring and forming it into a wire. The conductance across it was measured and the elastic mean free path was estimated from that. Then it was estimated that the small elastic mean free path of electron thus obtained cannot sustain the large value of observed persistent current. Ref [13] has shown that a special kind of defects known as grain boundaries can decrease the conductance much more drastically than the persistent currents in a 3D multichannel ring and this does not happen with point defects. Such defects as grain boundaries are possible in a gold ring and in case such defects are present (can be observed with a microscope) then it is incorrect to estimate the persistent current from the conductance. In this paper we show that it is a basic feature of 1D rings to exhibit this effect where an impurity can decreases conductance many more times than it decreases persistent currents. This helps us to bring out deeper physics guiding the behavior of an electron in a ring. Understanding the basic physics help us to argue that another type of defect that are invariably present in the experimental rings, can produce similar effects.

To this end we consider a single delta potential impurity in a ring of length L and consider the electrons in the ring to be free. To measure the conductance of the ring in the sense of Chandrasekhar et. al. we have to sever the ring at a point say P shown in fig 1 and form it into a wire shown in fig 2. The two ends of the wire i.e. A and B are then connected to two semi-infinite leads to calculate the conductance using Landauer’s conductance formula.
The persistent current is calculated using the procedure of ref [14] based on first principles that involves no approximations. After some simplification we find that for our system the allowed modes $k$ are given by the solution of the following equation.

$$\cos(\alpha) = \frac{V \sin(kL)}{2k} + \cos(kL)$$  \hspace{1cm} (1)

where $\alpha$ is the Ahoronov Bohm phase acquired by the electron in moving round the ring once and $V$ is the strength of the delta potential in the ring. The persistent current at a certain allowed value of $k$ is given by

$$I_k = \left( -\frac{e}{h} \right) \frac{2k \sin(\alpha)}{V \cos(kL) - \sin(kL)(\frac{V}{2k^2} + L)}$$ \hspace{1cm} (2)

Table 1. and table 2. lists a few values showing by how much the impurity decreases the conductance and by how much it decreases the total persistent current. The value of parameters chosen is given in the table captions.

As we are studying the relative change in conductance and persistent current we can calculate the persistent current for any value of $\alpha$ but we have calculated it at a value of $\alpha = \pi/2$ because the magnitude of the persistent current at this value is approximately maximum. $I(n)$ means the total persistent current when $n$ electrons are present in the ring with a delta potential impurity of strength $V$. $I_0(n)$ is the total persistent current when $n$ electrons are present in a clean ring. Fermi energy of a ring with a delta potential and having $n$ electrons is slightly higher than that of a clean ring with $n$ electrons. $g_E$ is the conductance evaluated at a Fermi energy (chemical potential of the reservoir determines this Fermi energy) which corresponds to the Fermi energy of the ring with the delta potential (the one that has a higher Fermi energy; although we could have well evaluated at a Fermi energy which is average of the Fermi energies of the two rings). $g_E^0$ is the conductance of a clean wire which at all Fermi energy is $2e^2/h$. For a few electrons in the ring it is found that the impurity decreases the conductance many more times than it decreases the persistent currents(see table 1). This effect becomes smaller as the number of electrons in the ring increases but the conductance always decreases more than the persistent currents.
As more electrons are put in the ring the higher energy states get filled up. It is well known that higher energy electrons do not feel the delta potential and if the potential is not felt at all it cannot be expected to produce different types of effect on persistent currents and conductance. We can increase the length of the ring and thus decrease the level spacings to create more levels in the region where the potential is strongly felt. Note that this does not affect the conductance of the ring (when opened up) because in the scattering problem this length scale is not involved in any way as will be explained soon. Table 2 compares the decrease in persistent current and conductance in this case. We find that to a larger filling the difference between decrease in persistent current and decrease in conductance is formidable. It also shows that for the first 2 levels this difference has increased by another order of magnitude. This formidable difference (orders of magnitude difference) when the delta potential is strongly felt is very counterintuitive. It is well known that more is the sensitivity of the states to twisting of boundary condition more will be the persistent current carried by the states. It is also well known that more is the sensitivity of the states to twisting of boundary condition more will be the conductance of the system. But here we have a simplest situation where the states are very sensitive to twisting of boundary condition and yet conductance is very small.

The mechanism by which the impurity can decrease persistent current is different from that by which it can decrease the conductance and this can make a lot of difference in 1D. When we are finding the conductance we are just solving the scattering problem of a geometry shown in fig 2. There is absolutely no length scale in the problem as is evident from fig. 2. But as soon as we join the points A and B to form a ring (for the time being there is no magnetic field) there is a length scale which is the length L of the ring. Ref [15] has shown that even in absence of magnetic field the electron in a 1D ring is moving with a momentum K given by KL=re(1/t) which is exactly the Block momentum of a periodic system of delta potentials of strength V and L is the length separating the delta potentials. This means the electron is effectively moving in such a periodic potential system. This does not happen in higher dimensions. Now if we put a magnetic field the effect of the magnetic field is to twist
the boundary condition and produce a dispersion with $\phi$. This $\phi$ is called a pseudo Block momentum[1]. So as soon as we connect points A and B we change the topology of the system in which the electron is moving. As a result in its motion the electron encounters an identical delta potential after traversing identical path lengths whereas earlier in case of the scattering problem it could encounter the delta potential only once and then get dissipated into the reservoir. It is well known that the probability of scattering by an infinite periodic array of potentials in certain range of Fermi energies (band energies) is much too smaller than the probability of scattering by any one of the delta potentials making the periodic array and this difference increases as the strength of the delta potential increases. But as the strength increases this range of Fermi energy or the band becomes narrow. At other range of Fermi energies (band gap energies) where the probability of scattering by an infinite periodic potential is much smaller than the probability of scattering by single delta potential, most probably we do not get an allowed mode in the ring. This is so because these modes are evanescent modes and cannot carry current. Only under some special situations one can excite evanescent modes that contribute to persistent currents[15,16]. So an electron in any one of the allowed modes in the ring experience very little resistance from the scatterer than it experiences when the ring is cut open and the electron is incident on the scatterer from one side.

When we are studying the conductance the impurity breaks translational symmetry completely and hence scatters vigorously. But as soon as we form it into a ring (we have not put any magnetic field yet) a discrete symmetry is restored. As now it is a ring it has a rotational symmetry instead of a translational symmetry and it corresponds to rotations like $2\pi$, $4\pi$, $6\pi$ etc. Hence we have symmetry dictated good quantum numbers in the ring (1D ring) called topological quantum numbers and they never mix. An electron left to a state characterized by a topological quantum number will stay in this state for ever unscattered and it will have a definite group velocity constant in direction and magnitude. It is just like the case of an electron that can pass through a discrete but regular lattice without being scattered. However increasing the potential strength narrows the band and decrease
the group velocity of the electron. It is this decrease in group velocity that decreases the persistent current and not the scattering. In higher dimensions also we have the same discrete rotational symmetry and topological quantum numbers along with a transverse or subband quantum number. There are actually a few propagating subbands that are degenerate and impurities can mix these different subbands and hence the different topological states having the same total energy. This is in spite of the discrete rotational symmetry. However due to the transverse degree of freedom an additional effect comes into play which does not happen in 1D rings. The second time the electron goes round the ring it may be encountering a different disorder configuration than it did the first time because of the transverse degree of freedom. So in higher dimension the electron in a particular state characterized by a topological quantum number will not see an effective periodic potential but rather an effective random potential in the absence of magnetic field. Hence it has an extremely small group velocity. Smaller is the group velocity lesser will be the sensitivity of the state to twisting of boundary condition by the magnetic field and smaller will be the persistent current carried by it. Hence the amount by which the impurities decrease the conductance is comparable to the amount by which it decreases the sensitivity of the states to twisting of boundary conditions. This is what is found in the simulations of ref[13] that for point defects in a ring the persistent current decreases as many times as the conductance. But the physics is completely different in 1D. When the ring is closed the electrons feel a periodic potential and are very sensitive to twisting of boundary condition. Ref[15] has shown that for a ring with defects the equation that determines the twisting of boundary condition with magnetic field i.e. \( \alpha = e^{i(KL+\alpha)} \) where K is the momentum in the periodic system and hence the states can be very sensitive to the twisting of boundary condition. As soon as the ring is opened up the electrons do not feel the periodic potential and hence contribute very little to conduction. The numbers given in the table and their orders of magnitude establish this.

Grain boundaries are extended defects that run across the whole cross section of the ring. If the grain boundaries are radial then they are more effective in decreasing the conductance
much more than the persistent current. For the time being consider a single radial grain boundary. If the defect be such as the grain boundary then again in higher dimensional ring also the electron will encounter the grain boundary at regular intervals in going round the ring again and again. This also explains why non radial grain boundaries are worse than radial ones but obviously they will be better than point defects.

It is also well known that boundary roughness can be mapped into an effective 1D potential[17] and it has also been shown that smaller is the boundary roughness greater will be the strength of the effective 1D potential[15]. Because of the later fact it is quiet possible that there are very strong effectively 1D potentials in the experimental ring and hence they will exhibit the same effect of decreasing the conductance much more than decreasing the persistent currents. A simulation of boundary roughness in a multichannel ring will be reported very soon.
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