Fluctuations in the Quantum Vacuum

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Abstract
We argue that it is a fluctuational underpinning of the Quantum vacuum which on the one hand gives a stochastic character to the conservation laws, and on the other is required for explaining the recently observed acceleration of the universe. This also provides us with the arrow of time, in a consistent cosmology.

1 Introduction
Classical Physics is based on a deterministic spacetime. It is also true that the equations of Classical Physics are time reversible. The equations of Quantum Mechanics too are time reversible, though here it is the probabilities which are determinable. Irreversible processes are phenomenological, according to this thinking. That is irreversibility arises due to the role of the observer. One important difference which is brought about by Quantum Theory is that the smaller the spacetime intervals we consider, the greater the Uncertainty in the energies and momenta wherein the Uncertainty in energy results in the creation and destruction of particles. As Feynman put it, "created and annihilated, created and annihilated - what a waste of time". The Quantum vacuum is thus an arena of frenzied activity at the micro scale. Nevertheless, conservation of energy is respected. Particles may appear to be created, but this energy is returned to the Quantum vacuum within the Uncertainty time interval. There are wild incessant fluctuations in energy, but these miraculously cancel out on the average. Energy is conserved on the average. However all of this still respects a smooth spacetime manifold, what Witten has termed "Bosonic spacetime"
2 Non Commutative Spacetime

Latest studies for example Quantum SuperString Theory or the author’s stochastic non commutative spacetime, take us beyond the above approximate description to what Witten calls Fermionic spacetime\[2, 3, 4, 5\]. In this case two spacetime coordinates like $x$ and $y$ do not commute but rather we have the equation

$$[x, y] \approx 0(l^2)$$  \hspace{1cm} (1)

and similar equations, where $l$ defines the Compton scale.

Another way of looking at (1) is that the usual Uncertainity Principle is modified:

$$\Delta x \cdot \Delta p \sim \hbar + \hbar'$$,  \hspace{1cm} (2)

$$\hbar' \sim l^2 \cdot (\Delta p)^2$$

This is an expression of the well known duality\[2, 6\], which provides an interface between the micro world and the macro universe. Infact $\hbar'$ defines this interface. We can now show that $\hbar'$ results in a non conservation of energy. Infact from (2) we have

$$\Delta p \sim \frac{hR}{l^2}$$ \hspace{1cm} (3)

where $R = \Delta x$ is the radius of the universe.

We have an equation for energy similar to (3) also. Using the well known Eddington formula, $R \sim \sqrt{N}l$, where $N \sim 10^{80}$ is the number of particles in the universe, (3) becomes

$$\Delta p \sim \sqrt{N}mc$$

and the similar equation for energy is

$$\Delta E \sim \sqrt{N}mc^2$$  \hspace{1cm} (4)

An interpretation of (4) follows naturally from the model of fluctuational cosmology\[4, 8\]. Here the $\sqrt{N}$ particles are created fluctuationally out of the Quantum vacuum, and equation (4) gives the energy of these particles. It must be emphasized that the above cosmological model provides an explanation for the otherwise miraculous large number coincidences, apart from
predicting an ever expanding, accelerating universe, as indeed the latest observations confirm.

It must also be emphasized that in the above model the particles are created unidirectionally, that is there is no destruction of particles. It is in this sense that the above model goes beyond the conventional conservation laws, as symbolised by the extra Uncertainty term $h'$ in equation (3). In fact as argued elsewhere[9], the conservation laws themselves are not iron clad, but are stochastic in nature. There is a certain resemblance of this model to Dirac’s large number cosmology[10], wherein also particles are created. However this latter model has some inconsistencies. What is interesting is that Dirac, on the one hand brilliantly hypothesized that the large number relations like the Eddington formula referred to earlier were not mere accidents, but rather were symptomatic of a profound underlying principle. On the other hand, Dirac himself vacillated inconclusively between two versions of his cosmology, one in which energy was not conserved and another in which energy was conserved[11].

In any case we have here a resolution to the so called time paradox, that is manifest irreversibility within the framework of reversible equations as alluded to at the beginning[12]. In the words of Prigogine (loc.cit)"As is well known, Albert Einstein often asserted, "Time is an illusion." Indeed time, as described by the basic laws of physics, from classical Newtonian dynamics to relativity and quantum physics, does not include any distinction between past and future. Even today, for many physicists it is a matter of faith that as far as the fundamental description of nature is concerned, there is no arrow of time.

"Yet everywhere-in chemistry, geology, cosmology, biology, and the human sciences-past and future play different roles. How can the arrow of time emerge from what physics describes as a time-symmetrical world? This is the time paradox..."

In fact the time of conventional physics, as argued elsewhere[8, 13] is an approximate time, an approximation that fudges the above fluctuations. Such a time is what may be called a stationary time, or time without time in phraseology a la Wheeler. This is also the stationary time of Quantum Mechanics in which the three space coordinates and time are on the same footing as in Special Relativity, and the displacement operators represent the energy-momenta[14]. It is the fluctuational creation of particles which gives rise to irreversibility, and infact time itself (Cf.[15]. It is the age old divide between
being and becoming.
Another way of looking at this is, that in the above fluctuational cosmological scheme, we have the analogue of the Eddington formula for time, viz.,

\[ T \approx \sqrt{N\tau}, \]

where \( T \) is the age of the universe and \( \tau \) the Compton time. Not only is this relation correct, but it also provides the arrow of time.
We would also like to remark that the vacuum energy, as given in (4) is, according to latest observations, required to explain the cosmological constant and that is, to explain the acceleration of the universe.

3 Fluctuational Cosmology

We now briefly describe the cosmological model referred to in Section 2. Dirac’s Large Number Hypothesis (LNH) has been much written about, ever since he spelt it out. As is well known this is based on apparently mysterious ratios of certain physical constants which coincide or show a relationship. Let us start with,

\[ N_1 = \frac{e^2}{Gm^2} \approx 10^{40} \]  

(5)

where \( m \) is the pion mass, the pion being a typical elementary particle, this being the ratio of the electromagnetic and gravitational forces, and

\[ N_2 = \frac{cT}{l} \approx 10^{40} \]  

(6)

where \( T \) is the age of the universe and \( l \) the pion Compton wavelength.
In this light, the LNH can be stated as (cf.22).
"Any two of the very large dimensionless numbers occurring in nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity”
An application of this to (5) and (6) means that their equality is not accidental but rather leads immediately to, Dirac’s well known relation,

\[ G \ \alpha \ \frac{1}{T} \]

(7)
Dirac’s approach further leads to

\[ R \propto T^{1/3} \]  
\[ \text{(8)} \]

which appears to be inconsistent\,[18, 25].

Another "accidental" relation is,

\[ m \approx (\frac{\hbar^2 H}{Gc})^{1/3} \]  
\[ \text{(9)} \]

As observed by Weinberg \,(7), this is in a different category and is unexplained: it relates a single cosmological parameter \( H \) to constants from microphysics.

In the spirit of LNH, one could also deduce that\,[23],

\[ \rho \propto T^{-1} \]  
\[ \text{(10)} \]

and

\[ \wedge \propto T^{-2} \]  
\[ \text{(11)} \]

where \( \rho \) is the average density of the universe and \( \wedge \) is the cosmological constant.

It may be mentioned that attempts to generalise or modify the LNH have been made (cf.e.g.\,[25, 26]) but without gaining much further insight.

We now deduce equations \,(7), (9), (10) and (11) from an alternative standpoint. Moreover in place of the troublesome equation \,(8), we will get a consistent equation. Our starting point is the Zero Point Field (ZPF). According to QFT, this field is secondary, while according to Stochastic Electrodynamics (SED), this field is primary.

We observe that the ZPF leads to divergences in QFT\,[27] if no large frequency cut off is arbitrarily prescribed, e.g. the Compton wavelength. On the contrary, we argue that it is these fluctuations within the Compton wavelength and in time intervals \( \sim \hbar/mc^2 \), which create the particles. Thus choosing the pion again as a typical particle, we get\,[27, 28]

\[ (\text{Energy density of ZPF})X^3 = mc^2 \]  
\[ \text{(12)} \]

Further as there are \( N \sim 10^{80} \) such particles in the Universe, we get,

\[ Nm = M \]  
\[ \text{(13)} \]
where $M$ is the mass of the universe.

In the following we will use $N$ as the sole cosmological parameter.

Equating the gravitational potential energy of the pion in a three dimensional isotropic sphere of pions of radius $R$, the radius of the universe, with the rest energy of the pion, we can deduce the well known relation

$$R = \frac{GM}{c^2} \quad (14)$$

where $M$ can be obtained from (13).

We now use the fact that the fluctuation in the particle number is of the order $\sqrt{N}$ [29, 30, 8], while a typical time interval for the fluctuations is $\sim \frac{\hbar}{mc^2}$ as seen above. (That is particles induce more particles by fluctuations). This leads to the relation [28]

$$T = \frac{\hbar}{mc^2} \sqrt{N} \quad (15)$$

where $T$ is the age of the universe, and

$$\frac{dR}{dt} \approx HR \quad (16)$$

while from (16), we get the cosmological constant as,

$$\Lambda \leq H^2 \quad (17)$$

where $H$ in (16) can be identified with the Hubble Constant, and is given by,

$$H = \frac{Gm^3c}{\hbar^2} \quad (18)$$

Equation (14) and (15) show that in this formulation, the correct radius and age of the universe can be deduced given $N$ as the sole cosmological or large scale parameter. Equation (17) for $\Lambda$ is consistent and exactly agrees with an upper limit deduced for it [31]. Equation (18) is identical to equation (9).

In other words, equation (9) is no longer a mysterious coincidence but rather a consequence.

To proceed we observe that the fluctuation of $\sim \sqrt{N}$ (due to the ZPF) leads to the empirically well known and apparently mysterious relation [28], with $N_1 = \sqrt{N}$, whence we get,

$$R = \sqrt{Nl} \quad (19)$$
If we combine (19) and (14), we get,
\[
\frac{Gm}{lc^2} = \frac{1}{\sqrt{N}}
\]  
(20)

If we combine (20) and (15), we get Dirac’s original equation (4). It must
be mentioned that, as argued by Dirac (cf.also ref.[32]) we treat \( G \) as the
variable, rather than the quantities \( m, l, c \) and \( \hbar \) (which we will call micro
physical constants) because of their central role in atomic (and sub atomic)
physics.

Further, using (20) in (5), with \( N_1 = \sqrt{N} \), as pointed out before (19), we
can see that the charge \( e \) also is independant of time or \( N \). So \( e \) also must
be added to the list of microphysical constants.

Next if we use \( G \) from (20) in (18), we can see that
\[
H = \frac{c}{l} \frac{1}{\sqrt{N}} \approx \frac{Gm^3c}{h}
\]  
(21)

Thus apart from the fact that \( H \) has the same inverse time dependance on
\( T \) as \( G \), (21) shows that given the microphysical constants, and \( N \), we can
deduce the Hubble Constant also, as from (18).

Use of (17) in (21) now gives equation (11).

Using (13) and (19), we can now deduce that
\[
\rho \approx \frac{m}{l^3} \frac{1}{\sqrt{N}}
\]  
(22)

Equation (22) gives the equation (10).

Next (19) and (15) give,
\[
R = cT
\]  
(23)

The equation (23) differs from the troublesome Dirac dependence (8).

Finally, we observe that using \( M, G \) and \( H \) from the above, we get
\[
M = \frac{c^3}{GH}
\]  
(24)

a relation which is required in the Friedman model of the expanding universe
(and the Steady State model also (cf.refs.[24] and [25])).

We finally make four comments:
Firstly, in our model of particle production through fluctuations of the ZPF, the equation (15) actually provides an arrow of time, at least at the cosmological scale, in terms of the particle number \( N \).

Secondly, in the spirit of the uniform cosmic dust approximation, the newly created particles are uniformly spread out. In practice, as the number of the fluctuationally created particles is proportional to the square root of the particles already present, more of the new particles are created, for example near Galactic centres, than in empty voids, reminiscent of the jets which are observed.

Thirdly, the reason why the Compton wavelength emerges as a fundamental length has been seen in previous communications\(^\text{[28, 8, 33]}\).

Finally, in this model, while the mass of the universe increases as \( N \) or \( T^2 \), the volume increases as \( T^3 \) so that the mean density decreases as \( T^{-1} \) (equation (22)), unlike in the steady state cosmology.

References

[1] T. Ferris, ”The Whole Shebang”, Simont Schuster, New York, 1997, p.97.

[2] W. Witten, Physics Today, April 1996, pp.24-30.

[3] Y. Ne’eman, in Proceedings of the First International Symposium, ”Frontiers of Fundamental Physics”, Eds. B.G. Sidharth and A. Burinskii, Universities Press, Hyderabad, 1999, pp.83ff.

[4] B.G. Sidharth, Chaos, Solitons & Fractals, 11(2000), p.1269-1278.

[5] B.G. Sidharth, ”Quantum Super Strings and Quantized Fractal Spacetime”, to appear in Chaos, Solitons & Fractals.

[6] B.G. Sidharth, Proceedings of Fourth International Symposium on ”Frontiers of Fundamental Physics” Kluwer Academy, New York (in press).

[7] B.G. Sidharth, Int.J.Th.Phys., 37 (4), 1998, p.1307ff.

[8] B.G. Sidharth, Int.J.Mod.Phys.A, 13 (15), 1998, p.2599ff.
[9] B.G. Sidharth, Chaos, Solitons & Fractals, 11(2000), p.1037-1039.

[10] J.V. Narlikar, "Introduction to Cosmology", Cambridge University Press, Cambridge, 1993, p.57.

[11] H. Kragh, ”Dirac A Scientific Biography”, Cambridge University Press, New York, 1990, pp.223ff. Also P.A.M. Dirac, ”Directions in Physics”, Eds. H. Hora and J.R. Shepanski, John Wiley & Sons, New York, 1978, pp.76-77.

[12] I. Prigogine, ”End of Certainty”, Free Press, New York, 1997.

[13] B.G. Sidharth, "The Chaotic Universe”, Nova Science Publishers, New York, 2001 (in press).

[14] A.S. Davydov, "Quantum Mechanics, Pergamon Press, Oxford, 1965, p.655.

[15] B.G. Sidharth, "The Substructure of Space Time and Some Related Issues", to appear in Chaos, Solitons and Fractals.

[16] P.A.M. Dirac, Proc. Royal Soc. of London A 165 1938, 199.

[17] J.D. Barrow, and F.J. Tipler, ”The Anthropic Cosmological Principle”, Oxford University Press, Oxford, 1986.

[18] S. Weinberg, ”Gravitation and Cosmology”, Wiley, New York, 1972.

[19] M. Rees, R. Ruffini, and J.A. Wheeler, ”Black Holes, Gravitational Waves and Cosmology: An Introduction to Current Research”, Gordon and Breach, New York, 1974.

[20] M.S. Berman, International Journal of Theoretical Physics, 31 (7), 1992.

[21] M.S. Berman, and F. de M., Gomide, International Journal of Theoretical Physics, 33 (9), 1994.

[22] A.K. Beesham, International Journal of Theoretical Physics, 33 (6), 1994.

[23] M.S. Berman, International Journal of Theoretical Physics, 35 (5), 1996.
[24] A.K. Beesham, International Journal of Theoretical Physics, 33 (9), 1994.

[25] G.W. Ma, International Journal of Theoretical Physics, 34 (12), 1995.

[26] J.C. Carvalho, International Journal of Theoretical Physics, 34 (12), 1995.

[27] R.P. Feynman, and A.R. Hubbs, "Quantum Mechanics and Path Integrals", McGraw Hill, New York, 1965.

[28] B.G. Sidharth, in "Frontiers of Quantum Physics", Eds., Lim, S.C., et al, Springer Verlag, Singapore, 1998.

[29] S. Hayakawa, Suppl of PTP Commemorative Issue, 1965.

[30] K. Huang, "Statistical Mechanics", Wiley Eastern, New Delhi, 1975.

[31] C.W. Misner, K.S. Thorne, and J.A. Wheeler, "Gravitation", Freeman (San Francisco), 1973.

[32] V.N. Melnikov, International Journal of Theoretical Physics, 33 (7), 1994.

[33] B.G. Sidharth, Ind.J.Pure and Appl.Phys., Vol.35, July 1997, pp.456-471.