Optical bistability and absorption characteristic of an optomechanical system embedded with double quantum dot and nonlinear medium

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Abstract
This paper theoretically studies a hybrid optomechanical system embedded with two coupled quantum dots and a third-order Kerr nonlinear medium inside the cavity. The optical bistability and absorption spectrum are analyzed for the proposed system. From the Hamiltonian which describes the proposed system, a set of quantum Langevin equations are derived. Using these equations of motion, the steady-state mean field analysis is done which gives the phenomena of optical bistability. The performance of the optical switch is also analyzed in terms of gain and switching ratio. Further, the absorption spectrum of the system is derived and analyzed from the fluctuation dynamics. The optical bistability has the potential to design tunable all-optical switches. The absorption spectra display peculiar characteristics of negative absorption (transparency dip). The transparency dips are found to be strongly dependent on the frequency of the mechanical resonator. The results of our investigation reveal that the proposed system can be used as an optical switch and has numerous other applications in quantum communication systems.

Keywords Optomechanics · Optical Bistability · Negative Absorption · DBRs
1 Introduction

With recent theoretical studies and tremendous technological developments, quantum optomechanics has emerged as a well-developed and effective tool for many quantum manipulation. In an optomechanical system, an optical field coupled to the mechanical oscillator via radiation pressure has emerged as a rapidly developing field of research (Kippenberg and Vahala 2008; Aspelmeyer et al. 2012). This optomechanical interaction has led to various applications such as atomic force microscopes Mertz et al. (1993), quantum entanglement (Hofer et al. 2011; Aggarwal et al. 2014), gravitational wave interferometers (Loudon 1981), quantum information processing Wang and Clerk (2012), optomechanically induced transparency (Agarwal and Huang 2010; Tassin et al. 2012), and ultra-high precision measurement Teufel et al. (2009). In addition placing a $\chi^{(3)}$ medium inside a cavity produces massive optical Kerr nonlinearities Imamoglu et al. (1997). There is a strong nonlinear interaction between photons as a result of the $\chi^{(3)}$ medium. In a related context, the progress in the development of micro-scale optical frequency combs using parametric frequency conversion has led to the development of dissipative Kerr solitons in microresonators with applications such as soliton combs (Kippenberg et al. 2018). The Kerr medium is a new handle to efficiently control the micro-mirror dynamics, which suggests possibilities of designing new quantum devices.

The Quantum dots are optically active semiconductor nanocrystals that constrain the movement of both holes and electrons in regions of space comparable to or smaller than the exciton Bohr radius. They are also called "artificial atoms" because of the confinement of charge carriers in three-dimensions. In one-dimensional nanostructures, a novel technique of synthesis was developed using selenium and gold and was shown to be a catalyst for clock reaction (Ray et al. 2013). Semiconductor quantum dots (QDs) are promising candidates for developing hybrid quantum devices due to their large density of states, narrow line-widths, and capability to implement optoelectronic devices with optical tunability (Vahala 2003; Hill et al. 2012; Yadav and Bhattacherjee 2022). Quantum dots confined in micro-cavities provides an ideal system to study cavity quantum electrodynamics which is the most straightforward way of obtaining single-photon nonlinear behavior. Quantum dot molecules (QDMs) are systems formed by two or more closely packed and interacting QDs. Quantum dot molecules are experimentally fabricated using either epitaxial growth or chemical synthesis by self-assembly technique Wang et al. (2009). Compared to a single QD, the states in a double QD are significantly changed, leading to applications such as conditional quantum control Unold et al. (2005), entangled photon source Gywat et al. (2002), and spin flip-flop Benny et al. (2014), providing a new way to implement a two-qubit quantum gate. Various theoretical methods have studied coupled double quantum dots (CDQDs). Gómez et al. (2017) studied a dissipative quantum dot microcavity system interacting with a nonlinear optical interaction. Dipole-dipole interaction in which one QD absorbs a photon released by the other excited QD is also studied to characterize CDQDs. This interaction has been used to study remote entanglement states Chen et al. (2011). A work of related observation is the theoretical demonstration of quantum state transfer through coherent atom-molecule conversion is ultracold atoms (Modak et al. 2017). Quantum interference and coherence can be induced by the tunneling of electrons between the dots, controlled by an external electric field (de la Giroday et al. 2011; Kim et al. 2011). As a result, fundamental double quantum dot studies such as optical bistability Wang et al. (2013), entanglement Cheng et al. (2011), enhanced Kerr nonlinearity Peng et al. (2014),
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Optical bistability (OB) is an interesting nonlinear quantum optical phenomenon that has attracted a great deal of attention because of its potential in wide applications such as in optical memory, logic circuits, and all-optical switches which are critical in telecommunication applications (Gibbs et al. 1976; Lugiato 1984). Over the past few years, studies have shown various ways Wang et al. (2013) to control OB, such as electromagnetically induced transparency (EIT), spontaneously generated coherence, and phase fluctuation. Optical switching (OS) in the semiconductor QDs and quantum wells (QWs) have been followed with a great deal of interest for many properties such as strong optical nonlinearity and excellent flexibility in devices Joshi and Xiao (2004). In recent years, there has been much interest in exploring OB in nonlinear nanophotonic systems Zhou et al. (2010). Optical bistability has been investigated experimentally and theoretically predicted to exist in photonic crystal cavities Yanik and Fan (2003), metal gap waveguide nanocavities Shen and Wang (2008), and waveguide-ring resonators Priem et al. (2005). It has also shown that the optical bistable switching is also possible in a hybrid magnetic semiconductor (Tripathy and Hota 2009). The loss of power at frequencies in metal interconnects is one of the most significant problems associated with high-speed electronic devices. All-optical switches with low laser power consumption are new for developing novel quantum technologies.

Electromagnetically induced transparency has played a crucial role in many sub-fields of quantum optics. EIT is a quantum interference effect caused by different optical field transition pathways Liu et al. (2017). Both absorption and dispersion properties change dramatically within the transparency window, leading to many applications, including slow light and optical storage Kasapi et al. (1995).

This paper investigates the OB and absorption characteristics of a double quantum dot molecule embedded in an optomechanical cavity interacting with a Kerr nonlinear medium.

2 Theoretical model

In our proposed model as shown in Fig. 1, we consider a hybrid optomechanical system comprising of two coupled quantum dots embedded in a $\chi^{(3)}$ nonlinear photonic crystal optomechanical cavity. InAs QDs and GaAs photonic crystal cavity (PhC) have been easily integrated using well-known techniques (Hennessy et al. 2007; Faraon et al. 2010).

![Fig. 1 Schematic diagram of an optomechanical system comprising two CDQDs embedded in a photonic crystal cavity. DBR mirrors form the cavity mode. The blue stripe is the AlGaAs layer, whereas the GaAs layer is the white stripe. A Kerr nonlinear medium is attached to the cavity, and the resulting photons from the nonlinear process are directly injected into the cavity. The cavity is driven by the strong pump field and weak probe field](image-url)
The photonic crystal cavity mode can interact with either one or two nearby QDs located precisely at the cavity electric field maximum Hennessy et al. (2007). The density of the embedded QDs can range between zero to 3 per $\mu m^2$ (Englund et al. 2009; Thon et al. 2009; Faraon et al. 2011). Numerous experimental techniques are available to fabricate a photonic crystal cavity integrated optomechanical system (Ji et al. 2020; Yamaguchi 2017). In addition, a $\chi^{(3)}$ nonlinear 2D layered medium is grown in the photonic crystal cavity using known techniques (Majumdar et al. 2015; Fryett et al. 2018; Janisch et al. 2014). Two-dimensional materials which are atomically thin, exhibit unique physical properties such as large second and third order optical nonlinearities (Majumdar et al. 2015; Fryett et al. 2018). Numerous experimental techniques are available to fabricate a photonic crystal cavity integrated optomechanical system (Ji et al. 2020; Yamaguchi 2017). In addition, a $\chi^{(3)}$ nonlinear 2D layered medium is grown in the photonic crystal cavity using known techniques (Majumdar et al. 2015; Fryett et al. 2018; Janisch et al. 2014). Two-dimensional materials which are atomically thin, exhibit unique physical properties such as large second and third order optical nonlinearities (Majumdar et al. 2015; Fryett et al. 2018). These materials can be grown by the simple technique of chemical vapor deposition on a medium like silicon dioxide and then can be easily transferred onto a pre-existing device like a photonic crystal cavity made out of distributed Bragg reflectors Fryett et al. (2018). These 2D materials after fabrication can be transferred easily on top of a pre-fabricated cavity, thus the growth process of the 2D material is not dependent on the etching treatment Majumdar et al. (2015). Unlike the molecular beam epitaxy process, this process does not require lattice matching between the device and the 2D material Majumdar et al. (2015). Some of the experimentally reported third-order nonlinear 2D layered materials are graphene, MoS$_2$, GaSe Fryett et al. (2018). The first demonstration of optical bistability was with third-order nonlinearity of graphene and a silicon photonic crystal Gu et al. (2012).

In general, PhC cavities are fabricated by introducing defects into the cavities which could be point or line defect in the periodic structure to create a microcavity. The periodic structure is made up of periodic dielectric structures which has a periodic variation in the dielectric constant Ji et al. (2020). Various examples of 2D PhC based mechanical resonators include tunable air slot 2D PhC slab with in-plane motion driven by an micro-electro-mechanical system (MEMS) actuator which was already integrated into the system (Gao et al. 2010; Pitanti et al. 2015). The dielectric discontinuity in the air-slot leads to localization of the optical mode inside the air-slot gap. A 2D planar PhC cavity can support both phononic as well as photonic band gaps simultaneously. In a radiation pressure driven PhC structure, the optical modes can be strongly coupled to localized mechanical modes Gavartin et al. (2011). The optomechanical system was established by confining photons into the air-slot between two flexible and suspended membranes Safavi-Naeini et al. (2010). On the other hand, mechanical resonator based on GaAs/AlGaAs are fabricated by utilizing micromachining and selective etching methods. Thin-film crystal growth techniques are used to prepare a larger structure where a sacrificial layer is grown under the resonator. The sacrificial layer is later selectively etched to release the structure from the substrate Yamaguchi (2017).

The proposed model in the rotating wave (frame rotating at the pump frequency $\omega_p$) and dipole approximation can be described by the following optomechanical Hamiltonian, taking $\hbar = 1$ as,

$$H = \Delta_a \sigma_-^a + \Delta_d \sigma_-^{(1)} + \Delta_d \sigma_-^{(2)} + \frac{\omega_m}{2} (p^2 + q^2) + \beta \sigma_-^a \sigma_-^a + \Omega_1 (\sigma_-^{(1)} + \sigma_-^{(1)})$$

$$+ \Omega_2 (\sigma_-^{(2)} + \sigma_-^{(2)}) + 2M \sigma_z^{(1)} \sigma_z^{(2)} + \omega_p \sigma_+^{(1)} \sigma_-^{(2)} + \omega_p \sigma_-^{(1)} \sigma_+^{(2)}$$

$$- G \sigma_-^{(1)} a q + E_p (a + a^+) + E_i (a e^{i \delta t} + a^+ e^{-i \delta t}),$$

where $\Delta_a = \omega_a - \omega_p$, $\Delta_d = \frac{\omega_m}{2} - \omega_p$, $\Delta_d = \frac{\omega_m}{2} = \omega_p$. 

\[\text{Springer}\]
Here, $a$ ($a^\dagger$) are the annihilation (creation) operators of the photonic crystal cavity mode. The first four terms are the free energies of the cavity mode, the two quantum dots and the mechanical mode respectively. The cavity frequency is $\omega_c$, the QD frequencies are $\omega_{d1}$ and $\omega_{d2}$ while $\omega_m$ is the natural frequency of the mechanical oscillator. Here $\sigma_z^{(1)}$ and $\sigma_z^{(2)}$ are the electronic transition operators of the two levels in first and second QD respectively. Further $\sigma_z^{(1)}$ and $\sigma_z^{(2)}$ denotes the population difference between the two energy levels of the first and second QD respectively. Here $p$ and $q$ are the normalized momentum and the position operators of the mechanical oscillator respectively. The fifth term is the Kerr nonlinearity with $\beta$ as the strength of the nonlinearity. $\beta$ depends on the nonlinear susceptibility $\chi^{(3)}$ and the cavity mode volume as $\beta = 3\omega_c^2 \Re[\frac{\chi^{(3)}}{2\omega_0 V_c}]$ with $V_c$ as the cavity volume (Ferretti and Gerace 2012; Drummond and Walls 1980; Sipe et al. 2004). The sixth and seventh terms represent the QD-optical cavity mode coupling with interaction strengths $\Omega_1$ and $\Omega_2$ for the first and second QD respectively. The eighth term corresponds to the exciton-exciton interaction between the two QDs with $\omega_{D}$ as the rate at which energy is transferred between the QDs. The eleventh term is as the rate at which energy is pumped into the cavity and $\delta = \omega_z - \omega_p$, where $\omega_z$ is the signal field frequency. It is important to note that the nonlinear $\chi^{(3)}$ material is not directly pumped. Using the Hamiltonian of Eq. (1), the dynamics of the proposed system can be described by the quantum Langevin equations which are derived as,

$$\dot{a} = -i\Delta_ao + \kappa_1a - i\Omega_1\sigma_{z}^{(1)} - i\Omega_2\sigma_{z}^{(2)} - 2i\beta|a|^2a + iGa + \Delta \sigma_{z}(t)e^{-i\delta t},$$

$$\dot{\sigma}_{z}^{(1)} = -i\Delta_{d1}\sigma_{z}^{(1)} - \kappa_{d1}\sigma_{z}^{(1)} + 2i\Omega_1a\sigma_{z}^{(1)} - 2i\omega_{D}\sigma_{z}^{(2)} + 2i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)} + 2i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)},$$

$$\dot{\sigma}_{z}^{(2)} = -i\Delta_{d2}\sigma_{z}^{(2)} - \kappa_{d2}\sigma_{z}^{(2)} + 2i\Omega_2a\sigma_{z}^{(2)} - 2i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)} + 2i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)},$$

$$\dot{\sigma}_{z} = -\Gamma_1(\sigma_{z}^{(1)} + 1) + i\Gamma_1(a^\dagger\sigma_{z}^{(1)} - \sigma_{z}^{(1)}a) - i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(2)} + i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)} + i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)},$$

$$\dot{\sigma}_{z} = -\Gamma_2(\sigma_{z}^{(2)} + 1) + i\Gamma_2(a^\dagger\sigma_{z}^{(2)} - \sigma_{z}^{(2)}a) - i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(2)} + i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)} + i\omega_{D}\sigma_{z}^{(2)}\sigma_{z}^{(1)},$$

$$\dot{q} = \omega_m p,$$

$$\dot{p} = -\omega_m q + Ga^\dagger - \gamma_m p.$$

Here $\kappa_{d1}$ and $\kappa_{d2}$ respectively. The decay rate of the mechanical mode is $\gamma_m$ and $\Gamma_1$ and $\Gamma_2$ are the relaxation rate of the first and second QD respectively. In order to solve Eqs. (2)–(8), we make the following ansatz (Xu et al. 2016; Boyd 2008),
\[ a(t) = a_0 + a_{(+)} e^{-i\delta t} + a_{(-)} e^{i\delta t}, \]
\[ \sigma_-^{(1)}(t) = \sigma_0^{(1)} + \sigma_{(+)}^{(1)} e^{-i\delta t} + \sigma_{(-)}^{(1)} e^{i\delta t}, \]
\[ \sigma_-^{(2)}(t) = \sigma_0^{(2)} + \sigma_{(+)}^{(2)} e^{-i\delta t} + \sigma_{(-)}^{(2)} e^{i\delta t}, \]
\[ \sigma_+^{(1)}(t) = \sigma_0^{(1)} + \sigma_{(+)}^{(1)} e^{-i\delta t} + \sigma_{(-)}^{(1)} e^{i\delta t}, \]
\[ \sigma_+^{(2)}(t) = \sigma_0^{(2)} + \sigma_{(+)}^{(2)} e^{-i\delta t} + \sigma_{(-)}^{(2)} e^{i\delta t}, \]
\[ q(t) = q_0 + q_{(+)} e^{-i\delta t} + q_{(-)} e^{i\delta t}, \]
\[ p(t) = p_0 + p_{(+)} e^{-i\delta t} + p_{(-)} e^{i\delta t}. \]

(9)

We will be working to the lowest order in \( E_s \), but to all orders in \( E_p \). The linear susceptibility for first exciton is given as,

\[ \chi_{\text{eff}}^{(1)} = \frac{\sigma_+^{(1)}}{E_s} \]  

(10)

where, \( \chi_{\text{eff}}^{(1)} \) is the linear optical susceptibility. Here all system parameters are dimensionless w.r.t. \( \Gamma_i \) and written as,

\[
\delta_0 = \frac{\delta}{\Gamma_1}, \quad \Omega_{10} = \frac{\Omega_{10}}{\Gamma_1}, \quad \Omega_{20} = \frac{\Omega_{20}}{\Gamma_1}, \quad \epsilon_{0c}^{(1)} = \frac{\epsilon_{0c}^{(1)}}{\Gamma_1}, \quad \epsilon_{0c}^{(2)} = \frac{\epsilon_{0c}^{(2)}}{\Gamma_1}, \quad \beta_0 = \frac{\beta}{\Gamma_1}, \quad \Delta = \frac{\Delta}{\Gamma_1}, \quad \omega_D = \frac{\omega_D}{\Gamma_1}, \quad \Delta_{d10} = \frac{\Delta_{d10}}{\Gamma_1}, \quad \Delta_{d20} = \frac{\Delta_{d20}}{\Gamma_1}, \quad \kappa_{d10} = \frac{\kappa_{d10}}{\Gamma_1}, \quad \kappa_{d20} = \frac{\kappa_{d20}}{\Gamma_1}, \quad M = \frac{M}{\Gamma_1}, \quad E_p = \frac{E_p}{\Gamma_1}, \quad G_0 = \frac{G_0}{\Gamma_1}, \quad \omega_{m0} = \frac{\omega_{m0}}{\Gamma_1}.
\]

### 3 Optical bistability

Optical bistability is a quantum phenomenon that displays two output states for the same input state. It has been observed in different optical and optomechanical systems (Vengalattore et al. 2008; Jiang et al. 2013; Sete and Eleuch 2012) and investigated experimentally in semiconductor microcavities Baas et al. (2004). It has received considerable attention due to its promising application in all-optical switching devices Sarma and Sarma (2016). The optomechanical coupling and the third-order Kerr nonlinear medium introduce nonlinearities in our proposed model. Here, we studied the optical response of \( N_0 = |a_0|^2 \) as a function of dimensionless external pump power \( E_p0 \). We will discuss how the different system parameters affect optical bistability. All-optical switching strongly relies on the demonstration of optical bistability by the system. On gradually increasing the amplitude of the input control (pump) laser, the intracavity steady state photon number \( N_0 \) makes a sudden transition from lower stable state (OFF-STATE) to the upper stable state (ON-STATE). On decreasing the control laser power, the system jumps from the ON-STATE to OFF-STATE.

In Fig. 2a, such an optical bistability is demonstrated for three different values of the Kerr nonlinearity strength \( \beta_{0} \). A higher value of Kerr nonlinearity demonstrates optical bistability at a higher pump power with a corresponding higher value of \( N_0 \). For efficient optical switching, low power consumption and a higher value of \( N_0 \) is desirable. Given the results of Fig. 2a, we need to choose an optimal value of \( \beta_0 \) such that the system displays optical bistability at a low value of pump power with a moderate value of \( N_0 \) which the external detector can detect. Figure 2b illustrates optical bistability for different values of exciton-exciton interaction between the two quantum dots \( (M_{z0}) \). A higher value of \( M_{z0} \) seems desirable since the switching occurs at a relatively low value of the input power with no substantial change in \( N_0 \) for different values
Mean number of photons $|a_0|^2$ as a function of pump field $E_{p0}$ for different values of (a) nonlinear strength ($\beta_0$), b exciton-exciton interaction ($M_{z0}$). c cavity detuning ($\Delta_{a0}$). d first exciton detuning ($\Delta_{d10}$). e optomechanical coupling ($G_0$). f Photon intensity $|a_0|^2$ versus cavity detuning $\Delta_{a0}$ for different values of optomechanical coupling $G_0$. Other parameters used are $\Omega_{10} = 0.2$, $\Omega_{20} = 0.2$, $\Delta_{d10} = 1$, $\Delta_{d20} = 1$, $\omega_{r0} = 0.01$, $\kappa_{a0} = 0.1$, $\kappa_{d10} = 0.2$, $\kappa_{d20} = 0.2$, $\omega_{\Delta0} = 0.01$.

Optical bistability for different values of cavity detuning $\Delta_{a0}$ is shown in Fig. 2c. It is noted that as $\Delta_{a0}$ increases, optical bistability occurs at a higher value of $E_{p0}$ accompanied by a higher intracavity photon number. Thus a moderate value of $\Delta_{a0}$ is required for an efficient optical switching as discussed for Fig. 2a. The influence of quantum dot detuning $\Delta_{d10}$ with a fixed $\Delta_{d20}$ is shown in Fig. 2d. Optical bistability for a lower value of input pump power is seen to occur for a higher value of $\Delta_{d10}$. No substantial change in $N_0$ is noticed by changing the $\Delta_{d10}$. Finally Fig. 2e illustrates the influence of optomechanical coupling $G_0$ on the optical switching response. As noted, a higher value of $G_0$ leads to optical switching at low values of $E_{p0}$ with a corresponding low value of $N_0$. Depending upon the sensitivity of the external detector, a moderate value of $G_0$ is required such that detectable value of $N_0$ and low power consumption is satisfied. In Fig. 2f, a plot of $|a_0|^2$ as a function of cavity detuning $\Delta_{a0}$ is shown.
for two different values of optomechanical coupling strength $G_0$. We see that for a low value of $G_0 = 0.005$, there is a complete absence of bistability (dashed line) and for a higher value of $G_0 = 0.01$, the plot (solid line) displays optical bistability. As $G_0$ increases further, the bistable behavior becomes more prominent. Thus, we observe the influence of various system parameters on the optical switching characteristics of the system and conclude that a careful choice of these parameters are required to operate the system as an efficient "all-optical switch".

4 Performance of the optical switch

With the growing trend of building optical interconnects for short-distance data transfer, optical systems could become a new means of computation rather than just a channel for signal transportation Caulfield and Dolev (2010). The nonlinear Kerr effect, caused by a control light changing the refractive index of nonlinear materials and the nonlinear optomechanical effect is the basis for all-optical switching. As a result, all-optical switching can be considered a key component in developing on-chip ultrafast all-optical switch networks. We now analyze the performance of an optical switch based on the proposed bistable device. The switching ratio is defined as the ratio of maximum to minimum cavity output and is given by,

$$\text{SR} = \frac{(P_{\text{out}})_{\text{max}}}{(P_{\text{out}})_{\text{min}}}.$$  

Here, $P_{\text{out}} = <a^\dagger a>$. The gain, is defined as,

$$\text{gain} = \frac{(P_{\text{out}})_{\text{max}} - (P_{\text{out}})_{\text{min}}}{P_{\text{in}}},$$  

and $P_{\text{in}} = E_{\text{p0}}$.

In Fig. 3a and b, we plot the switching ratio (SR) and the gain versus the optomechanical strength $G_0$, respectively. It is noted that, as the value of $G_0$ increases, both the switching ratio and the gain decreases. For an efficient optical switching device, both the gain and switching ratio should be large but at the same time power consumption should be less. Keeping in view the result of Fig. 2e, we need to choose $G_0$ in such a way that switching ratio, gain and power consumption is optimized.

\[\text{Fig. 3} \quad \text{a} \quad \text{The graph of switching ratio as a function of optomechanical interaction strength ($G_0$).} \quad \text{b} \quad \text{The graph of gain as a function of optomechanical coupling strength ($G_0$). All other system parameters are same as used in Fig. 2}\]
Absorption spectrum

In any optical device, tuning and controlling the optical absorption or transmission is important for practical applications. Consequently, we now calculate and plot the linear optical absorption \( \text{Im}[\chi^{(1)}_{\text{opt}}] \) as a function of signal-pump detuning \( \delta_0 \) in realistic experimental parameter domain. Figure 4a illustrates the spectral features of \( \text{Im}[\chi^{(1)}_{\text{opt}}] \) for two different values of (a) Kerr nonlinear coupling strength \( (\beta_0) \), (b) exciton-exciton interaction \( (M_{\text{exc}}) \), (c) optomechanical coupling \( (G_0) \), (d) mechanical frequency \( (\omega_m) \). Other parameters used are \( \Omega_{\text{10}} = 0.5, \Omega_{\text{20}} = 0.5, \Delta_{\text{20}} = 1, \omega_{\text{D0}} = 0.01, \kappa_{\text{d0}} = 0.1, \kappa_{\text{d10}} = 0.2, \kappa_{\text{d20}} = 0.2, E_{\text{p0}} = 0.1 \)

Fig. 5 Depiction of dressed state picture of the exciton

### 5 Absorption spectrum

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electron can make a transition from $|g, n + 1\rangle$ to $|e, n\rangle$ by the absorption of two pump photons and emission of a photon at $\omega_p + \omega_m$ as shown in Fig. 5b. This leads to amplification at $\delta_0 = \omega_m = 2$. As seen from Fig. 4a, a higher Kerr nonlinearity ($\beta_0 = 0.2$) leads to a transparency dip at $\delta_0 = \omega_m = 0$ with larger amplitude. This is expected since a large $\beta_0$ implies more number of two photon process and hence large amplitude signal field. The second broad transparency dip is due to a similar three photon process as shown in Fig. 5b but modified by the exciton-exciton interaction, Kerr nonlinearity and the optomechanical coupling. Here the electron makes a transition from $|g, n + 1\rangle$ to $|e, n\rangle$ by the absorption of two pump photons and emission of a photon at $\omega_p + \Omega$, where $\Omega$ is the Rabi frequency modified due to $M_{z0}$, $\beta_0$ and $G_0$. In this case, the dressed states of the excitons shown in Fig. 5b would be changed and much complicated due to $M_{z0}$, $\beta_0$ and $G_0$. The broad absorption peak and small dip centered near $\delta_0 = 0$ is due to optomechanically induced stimulated Rayleigh resonance which corresponds to the transition from $|g, n\rangle$ to $|e, n\rangle$ as shown in Fig. 5c.

Figure 4b illustrates spectral features for two different values of exciton-exciton interaction $M_{z0}$. For a higher value of $M_{z0}$, the transparency dips at $\delta_0 = \omega_m = 0$ and the broad dip has less amplitude compared to that for smaller value of $M_{z0}$. The absorption peak also has a less amplitude for higher value of $M_{z0}$. These features are due to the fact that for a relatively stronger exciton-exciton coupling, a significant amount of energy is transferred to the second QD which is not probed. Figure 4c shows $\text{Im} \chi^{(1)}$ versus $\delta_0$ for two values of optomechanical coupling $G_0$. The transparency dip for $\delta_0 = \omega_m = 0$ is extremely small for $G_0 = 0.3$ compared to that for $G_0 = 0.6$. A stronger optomechanical coupling leads to multiple photons being emitted at $\omega_p + \omega_m$ and thus leading to a relatively stronger signal wave. On making $G_0 << M_{z0}$, $\beta_0$, the transparency dip at $\delta_0 = \omega_m = 0$ vanished (figure not shown). Figure 4d illustrates the influence of changing the mechanical frequency $\omega_m = 0$ on the optical absorption spectrum. As evident, the sharp transparency dip at $\delta_0 = \omega_m = 0$ shifts from $\delta_0 = 2$ to $\delta_0 = 1.5$ as $\omega_m = 0$ changes from $\omega_m = 2$ to $\omega_m = 1.5$ respectively. It is also observed that keeping $M_{z0}$, $\beta_0 << G_0$, the broad transparency dip shifts towards $\delta_0 = \Omega'$, where $\Omega' = \sqrt{\Omega_{10}^2 + \Delta_{d10}^2}$ is the generalized Rabi frequency unmodified by $M_{z0}$ and $\beta_0$.

6 Conclusion

In summary, we have investigated the nonlinear optical response properties in a double quantum dot optomechanical microcavity, interacting with the photons generated through a third-order nonlinear medium. The steady-state mean-field analysis shows the existence of tunable optical bistability. This property can be used in all-optical switching by tuning the different system parameters. We have also calculated the performance of the optical switch. Based on our mean-field analysis, we have concluded that to develop an efficient optical switching device, the gain and switching ratio should be significant, and power consumption should be less. We have demonstrated that the fluctuation dynamics give rise to the absorption spectrum, which shows distinct characteristics of negative absorption that can be controlled and tuned by varying various system parameters, thus making our proposed model suitable for the next-generation optoelectronic device.
Author contributions The theoretical model was proposed by A.B.B and P.K.J. Analytical calculations and plots were done by S.Y and V.B in consultation with A.B.B, P.K.J and S.R. All authors contributed to writing the manuscript and discussing the results. This work forms a part of the PhD thesis of S.Y.

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Declarations

Conflict of interest The authors declare no conflicts of interest.

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