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Mutual information as a measure of reconstruction quality in 3D dynamic lung EIT

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Abstract. We report on a pilot study with healthy subjects who had an MR scan in addition to EIT data acquired with the Manchester fEITER system. The MR images are used to inform the external shape of a 3D EIT reconstruction model of the thorax, and small changes in the boundary that occur during respiration are addressed by incorporating the sensitivity with respect to boundary shape into a robust reconstruction algorithm. A quantitative comparison of the image quality for different EIT reconstructions is achieved through calculation of their mutual information with a segmented MR image. A shape corrected reconstruction algorithm reduces boundary artefacts relative to a standard reconstruction, and has a greater mutual information of approximately 10% with the segmented MR image.

1. Introduction
The majority of EIT reconstruction algorithms do not exhibit an acceptable level of reproducibility across different patients, or for the same patient at different data acquisition times. The reason being a combination of poor modelling approximations as well as an insufficient assessment of EIT reconstruction quality. An inaccurately known boundary shape is known to result in major artefacts in EIT image reconstruction [1, 2], and so an MR image of the subject is used to inform the boundary shape of a 3D tetrahedral reconstruction model of the thorax. Reconstruction artefacts associated with small changes in the boundary shape that occur during respiration are addressed by calculating a derivative of the EIT forward model w.r.t. the boundary shape. An assessment of the spatial resolution and reconstruction quality of various EIT reconstructions is carried out via image co-registration and calculation of the mutual information with a segmented MR image.

2. Methods
2.1. Data acquisition and model generation
Two planes of 16 electrodes were placed equidistantly around the chest at the 4th and 6th intercostal spaces and EIT measurements were acquired with the fEITER instrument [3] with sinusoidal current injection of 0.5 mA amplitude, 10 kHz frequency operating at 100 fps. A nearest-neighbour current injection and voltage measurement protocol was deployed with 20
problem can be linearised using a Taylor series expansion as
and small corrections to the electrode positions are accounted for due to breathing. The forward
can be represented by a constant and piecewise linear representations of the conductivity and potential respectively.

\[ \mathbf{V}_{\sigma+\delta\sigma,\mathbf{v}+\delta\mathbf{v}} = \mathbf{V}_{\sigma,\mathbf{v}} + J_{\nabla,\sigma}^c(\delta\sigma) + J_{\nabla,\sigma}^m(\delta\mathbf{v}) + O(||(\delta\sigma, \delta\mathbf{v})||^2), \]

where \( \| (x, y) \| := \| x \| + \| y \| \) and \( J^c : \mathbb{R}^{3L} \rightarrow \mathbb{R}^m \) and \( J^m : \mathbb{R}^{3L} \rightarrow \mathbb{R}^m \) are the movement and conductivity Jacobians respectively (see [5, 6] for details).

The estimation of conductivity and electrode positions \( \mathbf{x} := (\sigma, \mathbf{v}) \) from the data is approached from a semi-Bayesian viewpoint. The voltages are assumed to be related by \( \mathbf{V} = Z_I F(\sigma, \mathbf{v}) I + n \), where \( n \in \mathbb{R}^m \) represents the measurement channel noise. We assume mean zero Gaussian noise with covariance matrix \( \Gamma_e \in \mathbb{R}^{m \times m} \) and a Gaussian smoothness prior on the conductivity and electrode positions, with covariance matrix \( \Gamma_{\sigma,v} \in \mathbb{R}^{3L+N \times 3L+N} \) and mean \( \mathbf{x}_r := (\sigma_r, \mathbf{v}_r) \). We assume further that measurement noise is independent and identically distributed (i.i.d) and that conductivity and electrode position changes are independent from one another and are separately i.i.d. These assumptions imply that \( \Gamma_e^{-1} = I \in \mathbb{R}^{m \times m} \) and \( \Gamma_{\sigma,v}^{-1} \) is a diagonal matrix with entries \( \Gamma_{\sigma,v}^{-1}(i, i) = \alpha^2 \) if \( i \leq N \) and \( \Gamma_{\sigma,v}^{-1}(i, i) = \beta^2 \) if \( N < i \leq N + 3L \). \( \alpha \) and \( \beta \) represent the ratios of expected changes of conductivity and electrode position to the measurement noise standard deviation. The maximum \( a-posteriori \) (MAP) estimate of \( \mathbf{x} \), at the \( i^{th} \) temporal data frame, \( \mathbf{V}_i \), is equivalent to the following minimisation problem

\[ \mathbf{x}_i = \arg \min_{\mathbf{y}} \left\{ (Z_I F(\mathbf{y}) I - \mathbf{V}_i)^T \Gamma_e^{-1} (Z_I F(\mathbf{y}) I - \mathbf{V}_i) + (\mathbf{y} - \mathbf{x}_r)^T \Gamma_{\sigma,v}^{-1} (\mathbf{y} - \mathbf{x}_r) \right\}. \]

An update \( \mathbf{x}_i^{(1)} \) is computed from an initial guess, chosen to be the prior, \( \mathbf{x}_r \), computed using a one step Gauss-Newton method. Denoting \( J = [J_c | J_m] \in \mathbb{R}^{m \times (N+3L)} \) and neglecting second order partial derivatives of the forward problem w.r.t. \( \sigma \) and \( \mathbf{v} \), the linearised conductivity difference, \( \delta\mathbf{x}_{ij} \), between \( \mathbf{V}_i \) and \( \mathbf{V}_j \) is then

\[ \delta\mathbf{x}_{ij} := \mathbf{x}_i^{(1)} - \mathbf{x}_j^{(1)} = (J^T J + \Gamma_{\sigma,v}^{-1})^{-1} J^T (\mathbf{V}_i - \mathbf{V}_j). \]

2.3. Co-registration and mutual information
The reconstructed EIT image is mapped from the tetrahedral mesh into a voxel based image with 2 mm cubic voxels in the same space as the MRI data. The EIT images were thresholded so approximately 5\% of extreme points were discarded from the visualisation and the dataset imported into the 3D Slicer open source visualisation software [7].
Mutual information has been used in previous multi modal imaging studies [8]. We denote two images $A$ and $B$, each with $N$ cubic voxels, and each voxel having a positive grayscale value. The probability distribution of $A$, $p_A(a)$, is defined as the number of voxels in image $A$ that have pixel value $a$, normalised by $N$. The joint probability distribution of $A$ and $B$, denoted $p_{A,B}(a,b)$, is then calculated as the number of times out of $N$ that pixel in $A$ contains $a$ and the same pixel in $B$ contains $b$ normalised by $N$.

The Shannon entropy, in the imaging context, is the ability for an image to convey information and is measured in \textit{bits}. The information content of a single event is proportional to the log of the inverse of the probability of an event. The information content of a single event is weighted by the probability that the event occurs and summed over all events to give the total information content, or entropy. The total entropy of $A$, and the joint entropy of $A$ and $B$, are thus

$$H_A = -\sum_a p_A(a) \log(p_A(a)), \quad H_{A,B} = -\sum_a \sum_b p_{A,B}(a,b) \log(p_{A,B}(a,b)). \quad (3)$$

It can be shown that $0 \leq H_A \leq \log(N)$, where $H_A = 0$ when the image conveys no information i.e. it is featureless or homogeneous, and $H_A = \log(N)$ for white noise. The mutual information of $A$ and $B$, $I_{A,B}$, is defined as the total entropy of $A$ and $B$ minus the joint entropy, $I_{A,B} = H_A + H_B - H_{A,B}$. It is also true that $0 \leq I_{A,B} \leq H_A$, where $I_{A,B} = 0$ when $A$ and $B$ have no features in common and $I_{A,B} = H_A = H_B$ when $A$ and $B$ are the same.

To calculate the mutual information, the MR image is first thresholded to create a binary MR image representing just lungs and chest. The voxel based EIT image is then scaled linearly to lie within the same range as the binary MR image. The images have a finite number of voxels, and so the probabilities above are interpreted as histograms with 256 equispaced bins, and if a bin happens to be empty, then $0 \times \log(0)$ is interpreted as 0. The MR and EIT images are both sampled at their voxel centres to estimate $H_A$, $H_B$ and $H_{A,B}$ via (3), and hence $I_{A,B}$. This calculation is thus measuring how well a linearised EIT reconstruction is able to find the lungs as a whole. We perform the mutual information calculation over a range of regularisation parameters, $(\alpha,\beta)$, for both a standard $(\beta = 0)$ and shape corrected reconstruction algorithm. In doing so we effectively perform a numerical parameter study to maximise the mutual information between MRI and EIT as a function of $\alpha$ and $\beta$.

3. Results

Figure 1 illustrates an image co-registration example of MRI and EIT for the subject in the sitting position at three transverse planes for both a standard and movement reconstruction. The images are at max inhalation (relative to max exhalation) and blue indicates negative conductivity changes. The EIT reconstruction typically underestimates the physiological size of the cross-sectional areas of the lung. Figure 2 illustrates the mutual information with segmented MRI as a function of the parameter $\alpha$, for a movement $(\beta = 4\times10^{-2})$ and standard reconstruction $(\beta = 0)$. The maximum possible mutual information (the lung segmented MR image with itself) was 1.51 bits. There is a peak of mutual information of 1.11 bits at $\alpha \approx 3 \times 10^{-2}$, for the standard reconstruction and 1.21 bits at $\alpha \approx 10^{-2}$, for the shape corrected reconstruction, corresponding to a 10% increase in mutual information with shape correction. In general, the shape corrected reconstruction had larger mutual information then the standard reconstruction when taken over a range of $\alpha$ and $\beta$.

4. Conclusions and future work

The shape corrected reconstruction algorithm deployed results in a visual reduction of boundary artefacts relative to a standard reconstruction, and an increase in mutual information when compared with a segmented MR image. If EIT were to be routinely used as a bedside monitoring
Figure 1. MR and EIT image co-registration. Left to right: Standard and movement reconstruction. Top to bottom: Transverse planes from superior to inferior.

Figure 2. Mutual information parameter study as a function of $\alpha$ for a shape corrected reconstruction ($\beta = 4 \times 10^{-2}$) and a standard reconstruction ($\beta = 0$).

tool, optical data capture techniques could be used to inform the boundary shape in real time. The strength of MRI to inform the shape, however, is that we can also measure reconstruction quality through co-registration and mutual information as proposed here, and begin to assess the quality of EIT reconstructions over a range of available algorithms.

We would like to continue with this work by developing fully 3D models, that have variation in the cross section along the caudal-distal axis, and also incorporating more prior information such as the spine, liver and heart into a reconstruction model. An assessment of the best prior smoothness constraints on the conductivity, to offset the ill-posedness of EIT reconstruction, is another topic of importance and we believe significant improvements to reconstruction could be made from using both generalised Tikhonov regularisation and non-smooth penalisation norms.

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