MORPHOLOGY OF GALAXY CLUSTERS AND SUNYAЕV-ZЕL’DOVICH EFFECT

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ABSTRACT

Observations of the X-ray band wavelength reveal an evident ellipticity of many galaxy clusters atmospheres. The modeling of the intracluster gas with an ellipsoidal β-model leads to different estimates for the total gravitational mass and the gas mass fraction of the cluster than those one finds for a spherical β-model. An analysis of a recent Chandra image of the galaxy cluster RBS797 indicates a strong ellipticity and thus a pronounced aspherical geometry. A preliminary investigation which takes into account an ellipsoidal shape for this cluster gives different mass estimates than by assuming spherical symmetry.

We have also investigated the influence of aspherical geometries of galaxy clusters, and of polytropic profiles of the temperature on the estimate of the Hubble constant through the Sunyaev-Zel’dovich (SZ) effect. We find that the non-inclusion of such effects can induce errors up to 40 % on the Hubble constant value.

Key words: Missions: Chandra and XMM-Newton, Galaxy clusters, Sunyaev-Zel’dovich effect

1. Introduction

Clusters of galaxies can be used to study how structures form on large scales. The formation and evolution of clusters depend very sensitively on cosmological parameters like the mean matter density $\Omega_m$ in the Universe. Thus it is of great importance to determine the dynamical state of clusters at different redshifts, see e.g. Schindler (2001). Geometry can give important insight in the dynamics of galaxy clusters. For example, the fitting of the cluster X-ray surface brightness with β-models usually provides different best fit parameters depending on the shape one assumes for the intracluster gas, but generally the classical calculations of the mass of galaxy clusters suppose a spherical distribution of the density.

The SZ effect, Sunyaev & Zel’dovich (1972) offers the possibility to put important constraints on the cosmological models. Combining the temperature change in the cosmic microwave background due to the SZ effect and the X-ray emission observations, the angular distance to galaxy clusters and thus the Hubble constant $H_0$ can be derived. Nevertheless, geometrical shape of galaxy clusters can also introduce some errors on the analysis of the SZ effect.

In Section 2 we will analyze some consequences of the geometry of the clusters of galaxies and particularly on the mass, by taking as an example a recent observation of the galaxy cluster RBS797. Then we will describe, in Section 3, the influence of the cluster shape on the estimate of the SZ effect. In section 4 we will give an outlook on the role played by the geometry of galaxy clusters.

2. Morphology of Galaxy Clusters

The β-model, Cavaliere & Fusco-Femiano (1976), is widely used in X-ray astronomy to parametrise the gas density profile in clusters of galaxies by fitting their surface brightness profile. In this fitting procedure spherical symmetry is usually assumed, also in cases where the ellipticity of the surface brightness isophotes is manifest. For example Fabricant, Rybicki and Gorenstein (1984) showed a pronounced ellipticity of the surface brightness for the cluster Abell 2256, Allen et al. (1993) obtained the same result for the profile of Abell 478 and Neumann & Böhringer (1997) for CL0016+16. The asphericity of the observed surface brightness lets us also ponder on the possible asphericity of the intracluster medium, which can be modelled with an ellipsoidal β-model rather than with the less accurate spherical one. Hughes & Birkinshaw (1998) fitted the surface brightness of CL0016+16, which shows an axis ratio of major to minor axis of 1.176, with both circular and elliptical isothermal β-models obtaining for the best fit parameters $\beta_{\text{circ}} = 0.728^{+0.025}_{-0.022}$, $\sigma_{\text{circ}} = 0.679^{+0.045}_{-0.039}$ arcmin and $\beta_{\text{ell}} = 0.737^{+0.027}_{-0.022}$, $\sigma_{\text{ell}} = 0.746^{+0.044}_{-0.039}$ arcmin ($\sigma_{\text{ell}}$ is the core radius along the major axis), respectively, with the latter model providing a considerably better fit.

More recently, Schindler et al. (2001) and De Filippis, Schindler and Castillo-Morales (2002) with a Chandra observation of the galaxy cluster RBS797 revealed a pronounced aspherical geometry. The analysis of the image (see Figure 1) gives a strong ellipticity, where the axis ratio of major to minor axis varies slightly from 1.3 at a radius of 0.26 arcmin to 1.4 at a radius of 1.7 arcmin. A preliminary analysis of the surface brightness profile for RBS797 gives best fit parameters: $\beta_{\text{circ}} = 0.62^{+0.03}_{-0.01}$, $\sigma_{\text{circ}} = 7.32^{+0.07}_{-0.07}$ arcsec, and $\beta_{\text{ell}} = 0.59^{+0.02}_{-0.02}$, $\sigma_{\text{ell}} = 7.89^{+0.09}_{-0.09}$ arcsec (along the major axis), for the circular and elliptical models re-
and for ellipsoidal shapes, which are, compared at the same values of $R$, lower than those for the spherical symmetry by $\sim 10\%$ and $\sim 17\%$ for oblate or prolate shapes, respectively. For a detailed analysis of the influence of the ellipsoidal shape of the intracluster medium distribution on the total mass and gas mass fraction estimations see [Piffaretti, Jetzer and Schindler (2002)]

3. SZ EFFECT AND HUBBLE CONSTANT

The SZ effect is difficult to measure, since systematic errors can be important. For example, [Inagaki, Suginohara and Suto (1995)] analysed the reliability of the Hubble constant measurement based on the SZ effect. [Cooray (1998)] showed that projection effects of clusters can lead to dependence on the calculations of the Hubble constant and the gas mass fraction, and [Hughes & Birkinshaw (1998)] as well as [Sulkanen (1999)] pointed out that galaxy cluster shapes can produce systematic errors on the measured value of $H_0$.

We assume an ellipsoidal $\beta$-model [1]:

$$n_e(r_x, r_y, r_z) = n_{eo} \left[ 1 + \frac{r_x^2}{\zeta_1^2} + \frac{r_y^2}{\zeta_2^2} + \frac{r_z^2}{\zeta_3^2} \right]^{-3\beta/2},$$

where $n_{eo}$ is the electron number density at the center of the cluster and $\beta$ is a free fitting parameter which lies in the range $1/2 \leq \beta \leq 1$.

The Compton parameter $y$ and the X-ray surface brightness $S_X$ depend on the temperature of the hot gas $T_e$ and the electron number density $n_e$ as follows:

$$y \propto 2 \int_0^l n_e T_e dr_y ,$$

$$S_X \propto 2 \int_0^l n_e^2 \sqrt{T_e} dy ,$$

where $l$ is the maximal extension of the hot gas along the line of sight in units of the core radius $r_c$. We have chosen the line of sight along the $r_y$ axis.

For a detailed calculation of the Compton parameter and the X-ray surface brightness with an isothermal profile $T = T_{eo}$, we refer to the paper by [Puy et al. (2000)].

$$y(r_x, r_z) = T_{eo} n_{eo} \zeta_2 r_c \times \left( 1 + \frac{r_x^2}{\zeta_1^2} + \frac{r_z^2}{\zeta_3^2} \right)^{-3\beta/2} \times \Gamma_y(\beta, m),$$

and

$$\Gamma_y(\beta, m) = \left[ B \left( \frac{3}{2} \beta - \frac{1}{2}, \frac{1}{2} \right) - B_m \left( \frac{3}{2} \beta - \frac{1}{2}, \frac{1}{2} \right) \right],$$

$$S_X(r_x, r_z) = \frac{n_{eo}^2 \sqrt{T_{eo} \zeta_2 r_c}}{(1 + z)^3} \times \left( 1 + \frac{r_x^2}{\zeta_1^2} + \frac{r_z^2}{\zeta_3^2} \right)^{-3\beta/2} \times \Gamma_{S_X}(\beta, m).$$

[1] The set of coordinates $r_x$, $r_y$ and $r_z$, as well as the characteristic lengths of the half axes of the ellipsoid $\zeta_1$, $\zeta_2$ and $\zeta_3$ are defined in units of the core radius $r_c$. 

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Figure 1. Chandra image of the cluster RBS797. The cluster is rather regular with, however, an ellipticity of 1.3-1.4 in E-W direction. The center and the position angle (≈ −70°, N over E) of the various isophotes are almost the same over the entire radius range, from Schindler et al. (2001).
at the redshift $z$, with

$$
\Gamma_{S_X}(\beta, m) = \left[ B \left( 3\beta - \frac{1}{2}, \frac{1}{2} \right) - B_m \left( 3\beta - \frac{1}{2}, \frac{1}{2} \right) \right] \tag{7}
$$

where we introduced the Beta $B$ and the incomplete Beta-functions $B_m$ with the cut-off parameter $m$ given by:

$$
m = \frac{1 + (r_x/\zeta_1)^2 + (r_z/\zeta_3)^2 + (l/\zeta_3)^2}{1 + (r_x/\zeta_1)^2 + (r_z/\zeta_3)^2 + (l/\zeta_3)^2}. \tag{8}
$$

Introducing the angular core radius $\theta_c = r_c/D_A$, where $D_A$ is the angular diameter distance of the cluster, we can estimate the Hubble constant from the ratio between $y^2(r_x,r_z)$ and $S_X(r_x,r_z)$. If we choose the line of sight through the cluster center we get, see Puy et al. (2000):

$$
H_0(l) = T_{co}^{3/2} \alpha(l) \theta_c \frac{\Gamma_y(\beta, m)}{\Gamma_{S_X}(\beta, m)} \tag{9}
$$

with $\alpha(l) = S_X(l)/y^2(l)$ for a finite extension $l$ and, for an infinitely extended cluster, we get instead

$$
H_0(\infty) = T_{co}^{3/2} \alpha(\infty) \theta_c \frac{\Gamma_y(\beta, 0)}{\Gamma_{S_X}(\beta, 0)} \tag{10}
$$

with $\alpha(\infty) = S_X(\infty)/y^2(\infty)$. Since $S_X$ and $y^2$ are observed quantities, the ratios $\alpha(\infty)$ and $\alpha(l)$ are in the following both set equal to the measured value $\alpha_{obs}$.

Mauskopf et al. (2000) determined $H_o$ from X-ray measurements of A1835 obtained with ROSAT and from the corresponding millimetre observations of the SZ effect with the Suisie experiment. Assuming an infinitely extended, spherical gas distribution with an isothermal profile $\beta = 0.58 \pm 0.02$, $T_{co} = 9.8^{+2.3}_{-1.3}$ keV, $n_{co} = 5.64^{+1.01}_{-1.12} \times 10^{-2}$ cm$^{-3}$, they found $H_o = 59^{+36}_{-28}$ km s$^{-1}$ Mpc$^{-1}$. The figures on the right show the influence of geometry and of the assumption of finite extension on the above result using the same input parameters. The left figure 2 shows that for a spherical geometry $H_o$ displays a strong dependence on the cluster extension. The right figure 2 gives the value of $H_o$ assuming an infinite extended ellipsoid shaped cluster as a function of its axis ratio $\zeta_1/\zeta_3$. We find $H_o \sim 51.6$ km Mpc$^{-1}$ s$^{-1}$ for $\zeta_1/\zeta_3 = 1.5$.

Although the isothermal distribution is often a reasonable approximation of the actual observed clusters, some clusters do show non-isothermal features. A polytropic profile has the following form:

$$
T_e = T_{co} \left( \frac{n_e}{n_{co}} \right)^{\gamma^{-1}}. \tag{11}
$$

The isothermal profile is obtained by setting $\gamma = 1$. The polytropic profile can play a role in the estimates of the Hubble constant. We refer to the paper of Puy et al. (2000). The figure 3 shows that a small deviation from the isothermal case ($\gamma = 1$) leads to a significant change in the values of the Hubble constant. For instance, assuming an isothermal instead of a polytropic profile with index $\gamma = 1.2$ leads to an overestimation of 34 % for $H_o$. Indeed for A3562 a polytropic profile with index $\gamma = 1.16 \pm 0.03$ has been determined by Battocchi et al. (2000).

**Outlook**

We see thus that the geometry of galaxy clusters can affect the estimates of the masses and can substantially change the analysis of the SZ effect. The temperature profile is another important factor of change, indeed the hypothesis of isothermal profile might be too simple for some clusters. The recent XMM observation of the Coma cluster by Arnaud et al. (2001) points out that the temperature distribution is remarkably homogeneous suggesting that the core is actually in a relaxed state, but also that the NGC 4839 group is falling into the Coma cluster, see Neumann et al. (2001). This last result suggests the presence of more

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**Figure 2.** The dotted line corresponds to the value of the Hubble constant $H_0 = 59$ km s$^{-1}$ Mpc$^{-1}$ derived from the data of Mauskopf et al. (2000). The left figure shows the influence of finite extension, while the right figure gives the value of $H_o$ assuming an axisymmetric ellipsoidal geometry. In the latter case, oblate or prolate ellipsoidal geometry give the same value of $H_o$ when taking a line of sight through the

**Figure 3.** The Hubble constant (in km s$^{-1}$ Mpc$^{-1}$) for different values of the polytropic index. The line of sight is taken to go through the center of the cluster, which is assumed to have a spherical profile with infinite extension ($\beta = 2/3$-model). $H_o = 59$ km s$^{-1}$ Mpc$^{-1}$ is the dotted line according to Mauskopf et al. (2001).
complex substructures which can be explained by a first infall onto the Coma cluster.
In some clusters there is evidence of cooling flows in the central regions. Cooling flows in galaxy clusters can substantially change the temperature profiles, especially in the inner regions. Schlickeiser (1991) and Majumdar & Nath (2000) investigated the changes induced by a cooling flow on the temperature and density profiles, and their implication on the SZ effect. An analogous effect might be observed if the fluid speed and density are higher, as it is expected for merging processes in clusters of galaxies. For example very recently De Filippis, Schindler and Castillo-Morales (2002) revealed that the CL 0939+4713 cluster has an irregular morphology with evident substructures which seem to be in the process of merging.

The existence of a magnetic field in the intra-cluster medium might also be important. A first analysis by Koch, Jetzer and Puy (2002) suggests a possible influence on the SZ effect.

In these contexts an aspherical distribution of the density could play an additional important role. In summary, we see that it is crucial to know the shape of a cluster and its temperature profile. To address these problems, the Chandra and XMM-Newton satellites have the necessary spatial resolution for a complete analysis of the geometrical parameters of galaxy clusters.

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