Non-stationary oscillations of a box-like structure of a building

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Abstract. The problem of forced oscillations of a spatial box of a building is considered in the paper; it consists of interacting beams and rectangular panels under dynamic effect set by base displacement according to a sinusoidal law. The problem is solved by the finite difference method. Numerical results of displacements, stresses in the risk areas of the building box are obtained.

1. Introduction

In scientific studies [1-4], various dynamic problems in a plane and spatial statements are devoted to estimating and predicting the dynamic behaviour of various structures taking into account physical and geometric nonlinearity, inelastic properties of the material and inhomogeneous structural features under multicomponent kinematic effects.

In [5,6], oscillations of structural elements from an isotropic viscoelastic plate of variable thickness under uniformly distributed vibrational load applied on one of the parallel sides, leading (at certain combinations of natural vibration frequencies and disturbing force) to parametric resonance, are considered.

The papers [7, 8] are devoted to the development of the methods for dynamic spatial calculation of a structure based on the finite difference method in the framework of the bimoment theory, which takes into account the spatial stress-strain state. Solutions were obtained to the problem of transverse and longitudinal vibrations of buildings and structures using a plate model developed in the framework of the bimoment theory of plates [9-11].

This article poses the problem of forced oscillations of a spatial box of a building, consisting of interacting beams and rectangular panels under dynamic influences.

As a design scheme of the building, consider a spatial box with a fixed lower end, consisting of beams and rectangular panels, Fig. 1. All elements of the spatial frame are the beams of square section of the size $\delta$ made of the same material, with the same elastic and shear moduli $E$ and $G$, Poisson's ratio $\nu$ and density $\rho$. $J$ and $I_{tp}$ are the moments of inertia of the beam section under bending and torsion.
To study the motion of the box elements, introduce a Cartesian coordinate system, figure 1. The following notation for the panels of the spatial box-like structure of the building are introduced:

- $E_k$, $v_k$, $\rho_k$ and $h_k$ - the elastic modulus, Poisson's ratio, density and thickness of the $k$-th panel, respectively.

2. Statement of the Problem

The strain scheme of the box elements in the form of panels and beams is very complex. For simplicity, assume that the supporting walls of the building (panels 1,3) are perpendicular to the direction of seismic effect and work only on transverse dynamic bending. Parallel panels 2 and 4 in the direction of external effect work on shear in the OXZ plane.

The floor (panel 5) is also considered deformable. The law of motion of its points is determined in accordance with the forms of deformation of the upper edges of the vertical contacted panels. Box panels are supported by elastic beams. Therefore, the displacements of the panel edges are equal to the beam deflection, and the angle of end rotation of the panel is equal to the angle of twist of the beam. Bending of a beam is caused by the cutting force of the edge of the bending panel attached to it (panel 1 or 3) and the longitudinal force of the edge of the other attached beam (see panel 2 or 4).

The general kinematic law of box motion is taken in the form

$$ U_0 = A_0 \sin \omega_0 t, $$

where $A_0$ and $\omega_0$ are the amplitude and frequency of forced oscillations.

Based on the representation (1), rewrite the kinematic laws of motion of the panel points. Normal displacements of bending panels points are

$$ u_3 = A_0 \sin \omega_0 t + W(x, y, t), $$

where $W(x, y, t)$ is the deflection of bending panels, $A_0$ and $\omega_0$ are the amplitude and frequency of forced vibrations.

The displacement field of the panels working on shear is described by functions

$$ u_1 = A_0 \sin \omega_0 t + u(x, y, t), $$

$$ u_2 = u(x, y, t). $$
Here \( u, \nu \) are the displacements of panels working on shear.

As an equation of motion of the bending panel, we take \([9,10]\) with (2, a) in the form

\[
D\left(\frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^2}{\partial y^2} \left(\frac{\partial^2 W}{\partial y^2}\right) + \frac{\partial^4 W}{\partial y^4}\right) + \rho \ddot{h} W = \rho \dot{h} A_0 \dot{\omega}_0^2 \sin \omega_0 t
\]

where \( D \) is the cylindrical rigidity of panels under lateral bending, \( D = \frac{E h^3}{12(1 - \nu^2)} \), \( h_b \) is the thickness of the beam, \( W \) is the deflection of the panel (working on bending).

Two-dimensional equations of motion of the panels working on shear, are taken \([9,10]\) in the form:

\[
B\left(\frac{\partial^2 u}{\partial z^2} + \frac{1 + \nu}{2} \frac{\partial^2 \nu}{\partial x \partial z} + \frac{1 - \nu}{2} \frac{\partial^2 u}{\partial x^2}\right) = \rho h \ddot{u} - \rho h A_0 \dot{\omega}_0^2 \sin \omega_0 t,
\]

\[
B\left(\frac{\partial^2 \nu}{\partial x^2} + \frac{1 + \nu}{2} \frac{\partial^2 u}{\partial x \partial z} + \frac{1 - \nu}{2} \frac{\partial^2 \nu}{\partial z^2}\right) = \rho h \ddot{\nu},
\]

where \( B \) is the cylindrical rigidity of panels under extension and compression. \( B = \frac{E h}{1 - \nu^2} \), \( u, \nu \) are the displacements along the axes \( OX \) and \( OY \).

The equation of bending oscillations of the beam located between the panels is written in the form:

\[
E J \frac{\partial^4 W_0}{\partial x^4} + \rho_0 F \ddot{W}_0 = R_0' - P_1,
\]

The equation of torsional oscillations of the beam has the form:

\[
\frac{\partial}{\partial x} M_{sp} = \rho_0 I_{sp} \ddot{\omega} + M_{\gamma}' + \frac{\partial}{\partial x} R_0'.
\]

where: \( R_0' \) and \( P \) are the values of the reactive cutting force, bending panel and longitudinal force of the panel working on shear in the area of the butt connection of panels and beam; \( W_0(x, t) \) is the deflection of the beam.

The equations of bending and torsional oscillations for the remaining beams and panels are compiled similarly.

The boundary conditions on the base of the building \( (x = 0) \) are written for the rigid fixing. The lower part of the building moves with the base and there is no turning

\[
u_1, \nu_2 = 0, \quad \frac{\partial W}{\partial x} = 0.
\]

The boundary conditions (7) and (8) at \( x = 0 \) with (2) are rewritten in the form:

\[
W = 0, \quad \frac{\partial W}{\partial x} = 0, \quad u = 0, \quad \nu = 0.
\]

The boundary conditions at the upper end \( x = H \) are:

The contact conditions at the joints of the floor and the bending wall have the form

\[
-R_0^b + \eta_0 \rho_n h_n \dot{W}_n, \dot{r}_n = h_n \ddot{r}_n h_n - \eta_0 \rho_n h_n \dot{U}_0
\]
The contact conditions at the joints of the floor and the shear wall relative to the contacts hear stress are written as

\[ -ch_{t}v_{,z} + m_{w}u_{,z} = ch_{t} h_{\mu} \frac{\partial \sigma_{\mu}^z}{\partial z} - m_{w} \dot{U}_{0}. \]  

(10)

The contact conditions at the joint of the floor and the shear wall relative to the contact normal stress are written as

\[ -ch_{t} \sigma_{\mu} + m_{w} \dot{v}_{,z} = ch_{t} h_{\mu} \frac{\partial \sigma_{\mu}^z}{\partial z}. \]  

(11)

3. Method of the solution

The general solution to the problem of forced oscillations of a bending box panel is described by a function represented as the sum of the solution to the problem of forced and natural oscillations:

\[ W(x, y, t) = A_{n}W_{n}(x, y)\sin \omega_{n}t + C_{1}W_{1}(x, y)\sin p_{1}t, \]  

(12)

where \( p_{1} \) is the first natural frequency, \( C_{1} \) is a constant, to be determined.

The solution to the problem of forced oscillations is expressed through the principal mode of oscillations:

\[ W_{n}(x, y) = A_{n}W_{n}(x, y), \]  

(13)

where \( A_{n} \) is the expansion coefficient, the value of which is obtained by the formula

\[ A_{n} = \frac{\int \int f_{0}(x, y)f_{1}(x, y)dxdy}{\int \int f_{1}^{2}(x, y)dxdy} = 0,1739. \]

Here \( f_{0}(x, y) \) and \( f_{1}(x, y) \) are the forced and principal natural modes of vibrations.

Substituting (12) into (9) and subjecting to zero initial conditions, \( C_{1} = -A_{n}A_{t} \frac{\partial \omega_{k}}{p_{1}} \) is obtained.

With this expression and taking into account (13), we obtain the general solution of the problem for a flexible panel in the form:

\[ W(x, y, t) = A_{0}(\sin \omega_{0}t - A_{t} \frac{\partial \omega_{0}}{p_{1}}\sin p_{1}t)W_{n}(x, y). \]  

(14)

The expressions for displacement of the panel working on shear have the form

\[ u(x, z, t) = A_{0}(\sin \omega_{0}t - A_{t} \frac{\partial \omega_{0}}{p_{1}}\sin p_{1}t)u_{z}(x, z, t), \]  

(15)

\[ v(x, z, t) = A_{0}(\sin \omega_{0}t - A_{t} \frac{\partial \omega_{0}}{p_{1}}\sin p_{1}t)v_{z}(x, z, t). \]

The kinematic functions of the beams can be written as:

\[ W^{(z)}(x, t) = A_{0}(\sin \omega_{0}t - A_{t} \frac{\partial \omega_{0}}{p_{1}}\sin p_{1}t)W^{(z)}(x), \]  

(16)

\[ \alpha^{(z)}(x, t) = A_{0}(\sin \omega_{0}t - A_{t} \frac{\partial \omega_{0}}{p_{1}}\sin p_{1}t)\alpha^{(z)}(x). \]
Thus, a general solution of the problem is constructed in the form of (14), (15) and (16), satisfying the boundary conditions (7), (8) and the contact conditions between the floors (9) - (11), panels and beams (5), (6), and zero initial conditions.

The problem of determining the unknown coordinate functions in expressions (14), (15) and (16) is solved by the finite difference method.

4. Analysis of numerical results

Figure 2 shows the graphs of changes in dimensionless deflections of the beam $\xi(t) = \frac{u(t)}{A_0}$ in the middle (see figure 2, a) and the upper points (see figure 2, b) depending on time.

![Graphs of changes in beam deflections over time:](image)

(a) in the middle of the beam; (b) in the upper point

The dash lines show the graphs of solution $u_0(t)$ for stationary problems. The solid lines correspond to the deflections $w(t)$ obtained taking into account the initial conditions of the problem.

Calculations show that an account for the initial conditions of the problem at points located in the middle of the beam leads to insignificant increase in displacement (see figure 2, a), and at the upper point of the beam the displacement increase by 20% (see figure 2, b).

Figure 3 shows the graphs of changes over time of deflection at the top point of the panel, working on bending; without $u_0(t)$ (dashed line) and with account $w(t)$ (solid line) of the initial conditions. As can be seen, the deflections are 20% greater (see figure 3) when the initial conditions of the problem are taken into account. In the figures, the graphs of deflections located closer to the middle and below in height of the panel, differ by 10-12%.
Figure 3. The change in deflection over time at the upper point of the panel working on bending.

Figure 4 shows the graphs of changes in horizontal displacement at the top point of the panel working on shear.

Figure 4. Graphs of dependence of horizontal displacements over time at the upper point of the panel working on shear.

Figure 5 shows the graphs of normal stresses $\sigma_{xx}$ over time at the upper and middle points in the extreme section of the panel working on shear. Calculations show that the maximum values of normal stresses $\sigma_{xx}$ with account for the initial conditions of the problem are 25-30% greater than the values of the normal stresses $\sigma_{xx}^0$ obtained when solving the stationary problem. The maximum value of the
normal stress $\sigma_{xx}$ at the upper point is 1.5-2 times greater than the corresponding value at the midpoint of the cross section.

Figure 5. The change in stresses $\sigma_{xx}$ over time at the upper point of the panels working on shear

Figure 6 shows the graphs of changes in the normal stress $\sigma_{zz}$ over time at the upper point in the extreme section of the panel working on shear.

Figure 6. The change in stress $\sigma_{zz}$ over time at the upper point of the panel working on shear
Calculations show that the maximum values of the normal stress $\sigma_{zz}$, when taking into account the initial conditions of the problem, are 30-35% greater than the values of the normal stress $\sigma_{zz0}$ obtained when solving the stationary problem.

The maximum value of the normal stress $\sigma_{xx}$ at the upper point is 10-12 times greater than the same value at the midpoint of the cross section. Note that the values of $\sigma_{zz}$ obtained in vertical section are the contact stresses between the panels and the beams.

Figure 7 shows the graphs of changes in shear stress $\tau_{zx}$ over time at a midpoint in the extreme section of the panel working on shear.

![Graph of changes in shear stress $\tau_{zx}$ over time at a midpoint in the extreme section of the panel working on shear](image)

**Figure 7.** Change in stresses $\tau_{zx}$ over time at the midpoint of the panel working on shear

Note that the values $\tau_{zx}$ obtained in this vertical section are the contact stresses between the panels and the beams.

Figure 8 shows the graphs of changes in normal bending stress $\sigma_{xx}$ over time at the lower point of the extreme section of the bending panels.

Calculations show that the maximum values of normal stress $\sigma_{xx}$ when taking into account the initial conditions of the problem are 15-20% greater than the ones obtained when solving $\sigma_{0xx}$ the stationary problem.
**Figure 8.** The change in stresses $\sigma_{xx}$ over time at the lower point of the panel working on bending

5. Conclusion

The equations of motion of the panels and beams points of the box of a building, the boundary, contact and initial conditions of the problem of forced oscillations are given in the paper. Within the framework of the finite difference method, a method for dynamic calculation of displacements and stresses in beam and panel elements of box-like structures of buildings has been developed.

The stress-strain state of panels and butt joints is determined. The laws of changes in the maximum values of deflections and stresses at characteristic points of panels and beam elements depending on time are graphically presented.

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