Enhancing Spectrum Efficiency for Multiple Users in Hybrid Satellite-Terrestrial Networks

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\textbf{ABSTRACT} In the paper, we present a study on the performance analysis of a non-orthogonal multiple access (NOMA) underlay cognitive hybrid satellite-terrestrial relaying network (CSTRN) and highlight the performance gaps between multiple users. The satellite source communicates with users by enabling cognitive radio scheme to forward signals to secondary destinations on the ground which belong to dedicated groups following the principle of NOMA. In this scenario, the secondary source acts as a relay and employs Amplify and Forward (AF) mode to serve distant NOMA users under a given interference constraint. To characterize the transmission environment, the shadowed-Rician fading and Nakagami-$m$ fading models are widely adopted to the relevant hybrid channels. To provide detailed examination of the system performance metrics, we aim to derive closed-form formulas for the outage probability of the secondary destinations in the presence of the primary interference power constraint imposed by the adjacent primary satellite network. Finally, our simulation results showed that a greater number of antennas, better quality of wireless channels and power allocation factors exhibit the main effects on system performance.

\textbf{INDEX TERMS} Non-orthogonal multiple access, cognitive hybrid satellite-terrestrial relaying network, cognitive radio.

\textbf{I. INTRODUCTION}

Regarded as an attractive method for achieving high throughput with a broad coverage area, hybrid satellite systems and terrestrial networks can be integrated to form hybrid satellite terrestrial networks (HSTNs) [1]. In a HSTN, a wide range of applications can be offered for the purposes of navigation, broadcasting and disaster relief [2]. To improve coverage, system performance can be achieved in the system models by employing cooperative relaying techniques which are reported in [3], [4]. In addition, both the relaying network and cognitive radio (CR) technology can benefit to major applications of HSTNs by enhancing the efficiency of spectrum utilization. The promising architecture is studied as cognitive HSTN (CHSTN) [5]–[10]. In such a CHSTN, a secondary terrestrial network is permitted to operate in the same spectrum resource as the primary satellite network.

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The two main factors in CHSTNs, i.e. secondary user (SU) transmit power constraints and inefficient use of available spectrum resources, may degrade the performance of the secondary network.

Because data traffic is ever rapidly growing, it is crucial to study new multiple access methods. There are other challenges since current communication systems possess limited power and spectrum resources. Fortunately, existing systems can potentially overcome these challenges by enabling the recently proposed non-orthogonal multiple access (NOMA). Unlike conventional orthogonal multiple access (OMA), NOMA-aided base stations employ the power domain to assist multiplexing of multiple users before sending to mobile users. The NOMA technique, superposition coding (SC) and successive interference cancellation (SIC) performed at the corresponding transmitters and receivers allow users to achieve higher spectrum efficiency [11]. In the principle of NOMA, more power levels are allocated to users with poorer channels. As such, weak users can decode signals...
directly by considering other signals, such as noise. Before detecting the main signal, the user which experiences a better channel decodes the strong signals and eliminates them. The NOMA technique has three main benefits: low latency, massive connectivity, and high spectral efficiency [12]. The authors in [13] investigated a downlink NOMA scenario with randomly deployed users to demonstrate its superior performance, i.e., ergodic capacity. The authors in [14] studied a real scenario with a transmitter that only achieved statistical channel state information (CSI) associated with each user. They proposed this model to implement a NOMA system by employing the Nakagami-\(m\) fading channels as a practical downlink. The authors in [15] presented the capability of unmanned aerial vehicle (UAV) communications employing full-duplex NOMA (FD-NOMA) to enhance spectrum utilization. They described closed-form outage probability formulas for several scenarios such as FD-NOMA, half-duplex NOMA (HD-NOMA), and half-duplex OMA (HD-OMA) schemes over Rician shadowed fading channels. The authors in [16] studied an interesting alternative to conventional power domain-NOMA, known as NOMA-2000, in which multiple access was implemented by applying two sets of orthogonal waveforms. The simulation results in [17] for a fair-NOMA approach reported that each user’s capacity was greater than or equal to the capacity of a system relying on OMA. Recent studies such as [18], [19] have also developed the application of device-to-device (D2D) communication in the context of NOMA. A cooperative NOMA scheme was studied in [20]–[23] to increase reliability and coverage. Users experiencing better channels were treated as relays to forward the messages of distant users experiencing poorer channels. The authors in [24] and [25] proposed a UAV-NOMA network architecture. The authors in [25] indicated that D2D can be used to increase file dispatching efficiency. In a D2D-enhanced UAV-NOMA network, ground users (GUEs) may reuse the time-frequency resources assigned to NOMA links to share with other GUEs. The study in [26] presented two practical schemes of downlink UAV-NOMA. The first method minimized the transmit power of the UAV, satisfying the minimum achievable rate requirements. The second proposed method maximized the achievable rate of a specific user while maintaining the minimum achievable rate requirements for other users.

To further improve the efficiency of spectrum utilization, NOMA can gain more benefit from cognitive radio (CR) and forming CR-inspired NOMA scenarios [27]–[30]. The work in [27] investigated the integration of NOMA with CR to introduce more intelligent spectrum sharing paradigms. To examine the performance of secondary NOMA users, end-to-end outage probability is considered as the main performance metric. The recent work in [28] studied error rate performance in a scenario of relay-assisted NOMA with partial relay selection applied in an underlay CR network. Based on this system model, \(K\) relays were used to support transmission between secondary NOMA users and a secondary base station (SBS). In such a system, the relay which experiences the strongest link with the SBS is selected to forward the received signals to secondary receivers.

A. RELATED WORK
The authors in [31] considered the performance of an overlay cognitive hybrid satellite-terrestrial relaying network (CSTRN). This system contains a primary satellite transmitter set up to send signals to multiple terrestrial receivers and a secondary transmitter-receiver pair located on the ground. The primary satellite transmitter employing NOMA architecture to serve all users simultaneously. To deal with spectrum access, the secondary transmitter assisted primary communication through a cooperative relaying method. The authors in [32] studied the widely adopted shadowed-Rician fading and Nakagami-\(m\) fading models for relevant hybrid channels. Their main results were the derivation of a closed-form formula for the outage probability of a secondary network by considering the conditions on primary interference power constraints associated with the adjacent primary satellite network. The system reported in [33] introduced two main performance metrics: closed-form outage probability and approximate ergodic capacity expressions for the primary user (PU) and secondary user (SU). In this system, generalized Shadowed-Rician fading and a Nakagami-\(m\) fading were applied to satellite links and terrestrial links, respectively. The simulation results verified the superiority of NOMA compared to conventional OMA schemes by varying certain key parameters. The authors in [34] presented the critical effect of hardware impairments (HIs) at user devices in an overlay cognitive hybrid terrestrial network which included a primary satellite source-receiver pair and a secondary transmitter-receiver pair on the ground. To mitigate the effect of HIs, the authors studied an adaptive relaying (AR) protocol for both amplify-and-forward (AF) and decode-and-forward (DF) modes and compared its performance to competitor fixed relaying schemes.

However, the most benefit of NOMA in the CSTRN system is enabler of multiple served users in dedicated group of group. This paper motivated by recent studies [31]–[34] to highlight how we benefit from enabling CR and NOMA in the CSTRN system.

B. OUR CONTRIBUTION AND ORGANIZATION
The key findings of this paper can be summarized as follows:

- This study aims to propose a framework of multiple NOMA user in the CSTRN system by exploiting set of analytical expressions to indicate the system performance metrics. In particular, we consider the outage probability (OP) and throughput of a complicated system which is designed with enablers of NOMA, CR, multiple secondary users and multiple-antenna satellite.
- Although reference [31] quantified the performance of a NOMA-assisted the CSTRN of the primary and secondary networks in terms of outage probability by considering the pertinent heterogeneous fading models, our
study mentions on complicated scenario of multiple antennas at the satellite and multiple users in a considered group. It is beneficial to improvement performance of NOMA-assisted CSTRN. Due to complicated results computed in studies related to CSTRN system [31]–[34], we also investigate the approximation performance of such outage behavior to look insights of such system. This important finding should be guidelines to design NOMA-CSTRN in practice. Table I should emphasize our key findings compared with recent studies.

- Our numerical simulations are expected to provided main parameters which are used to improve specific performance metrics for the NOMA-CSTRN. These parameters are power allocation factors, the number of transmit antennas, channel fading gains, and transmit signal-to-noise ratio at the satellite.

The other parts of the paper can be emphasized as follows. Section II presents the details of the system model along with computation of received signals and channel distributions. Section III considers main metric, namely outage probability and asymptotic computation of such performance metric. Section IV presents the numerical results and discussion. Finally, Section V intends to provide concluding remarks and outlines of future research directions.

**II. SYSTEM MODEL**

In the present study, we examine a satellite (S) equipped $N_S$ antennae, a single antenna secondary relay (R), $K$ secondary terrestrial destinations $D_k$ ($1 \leq k \leq K$) and a single primary terrestrial destination (PD). The key parameters and denotations can be seen in Table 2.

Two phases are required to process overall communication of a secondary network in such CSTRN. In the first phase, the superposition signal $s = \sum_{i=1}^{K} \sqrt{P_S} \phi_i x_i$ is sent from the S to the R using a weight vector $w_{SR}$. The received signal at R is given by

$$y_R = h_{SR}^T w_{SR} \sum_{i=1}^{K} \sqrt{P_S} \phi_i x_i + n_R,$$

where $h_{SR} = \left[ h_{SR}^1 \cdots h_{SR}^N \right]^T$ and $n_R$ is the additive white Gaussian noise (AWGN) with $n_R \sim CN(0, N_0)$. By employing a maximum ratio transmission (MRT) scheme [35], the transmit beamforming vector is formulated as $w_{SR} = h_{SR} / \|h_{SR}\|_F$.

After signal processing in the first phase, the R first amplifies the received signal $y_R$ with a gain factor $G$. This variable gain is defined as $G = \sqrt{\frac{1}{P_S z_R + N_0}}$. Then, R forwards the processed signal to all distant users. The received signal at the 4th user is given by

$$y_{D_k} = \sqrt{P_R} h_{RD_k} y_R + n_{D_k}$$

$$= \sqrt{P_R} h_{RD_k}^T h_{SR} w_{SR} \sum_{i=1}^{K} \sqrt{P_S} P_R \phi_i x_i$$

$$+ G \sqrt{P_R} h_{RD_k} n_R + n_{D_k}, \quad (2)$$

Without loss of generality, the channel gains from R to $D_k$ are ordered $h_{RD_1} \leq h_{RD_2} \leq \cdots \leq h_{RD_K}$. This ordering procedure is associated with NOMA requirements. $n_{D_k} \sim CN(0, N_0)$ is the AWGN.

To prevent interference at the primary user beyond an acceptable level $Q$, the transmit power at S and R are given by $[36] P_S = \frac{Q}{P}$. By $P_R = \frac{Q}{|h_{RP}|^2}$, respectively, where $Z_v = \|h_{SV}\|_F^2, v \in \{R, P\}$.

Considering how the system precisely detects the required signal at each user, we mention SIC in the context of NOMA. Since the desired signal encouters interference from the other users’ signals, SIC is required at each user to eliminate
the adverse effect of inter-user interference. Therefore, at the k-th user, the m-th user’s signal, \( k < m \), must be detected. The system then deletes it from the received signal of the k-th user in a successive manner. In this situation, the m-th user’s signal is treated as noise at the k-th user. We can therefore compute the signal-to-interference-and-noise ratio (SINR) for the k-th user to decode the m-th user’s signal, \( k \leq m \), as follows:

\[
\Gamma_{m \rightarrow k} = \frac{G^2 P_R P_S |h_{RD_k}|^2 \phi_m}{G^2 P_R P_S |h_{RD_k}|^2 \sum_{i=m+1}^{K} \phi_i + G^2 \sqrt{P_R} |h_{RD_k}|^2 N_0 + N_0}
\]

\[
= \frac{\gamma_R \gamma_k \phi_m}{\gamma_R \gamma_k \sum_{i=p+1}^{K} \phi_i + \gamma_k + \gamma_R + 1}, \quad (3)
\]

where \( \rho = \phi / N_0 \), \( \gamma_R = \phi |Z_R| / Z_P \) and \( \gamma_k = \phi |h_{RD_k}|^2 / |h_{RP}|^2 \). If \( x_m \) can be detected successfully, signal \( x_m \) can be deleted before the signal for user \( D_k \) is detected. The SINR of \( D_k \) to decode its own signal is given by

\[
\Gamma_k = \frac{\gamma_R \gamma_k \phi_k}{\gamma_K + \gamma_R + 1}. \quad (4)
\]

For successive processing of many users, the standard SIC component is operated until all other users’ signals are detected. For the final user, the SINR of \( D_K \) to decode its own signal can be computed according to

\[
\Gamma_K = \frac{\gamma_K \phi_K}{\gamma_K + \gamma_R + 1}. \quad (5)
\]

In terms of the characteristics of wireless channels, the channel vector \( h_{SV} \) is associated with i.i.d. Shadowed-Rician fading entries. By denoting \( \delta_v = \Omega_{SV}/(2b_{SV}) (2b_{SV}m_{SV} + \Omega_{SV}) \) in which \( m_{SV} \) represents the fading severity parameter, \( \Omega_{SV} \) and \( b_{SV} \) are the average power of Line-of-Sight (LOS) and multipath components, respectively, and \( F_1(\cdot;\cdot) \) is the confluent hypergeometric function of the first kind [37, Eq. 9.210.1], the probability density function (PDF) of the squared amplitude of the channel coefficient \( |h_{SV}^{(i)}|^2 \) is given by [4], [35]

\[
f_{|h_{SV}^{(i)}|^2}(x) = \alpha_v e^{-\beta_v x} F_1(m_{SV}; 1; \delta_v x), \quad x > 0,
\]

where \( \alpha_v = (2b_{SV}m_{SV}/(2b_{SV}m_{SV} + \Omega_{SV}))^{m_{SV}} / 2b_{SV}, \delta_v = 0.5b_{SV} \). For Shadowed-Rician fading with integer-valued severity parameters, we can therefore rewrite (6) as

\[
f_{|h_{SV}^{(i)}|^2}(x) = \alpha_v \sum_{k=0}^{m_{SV}-1} \zeta_v(k)x^k e^{-\Psi_v x}, \quad (7)
\]

where \( \zeta_v(k) = (-1)^k (1 - m_{SV})_k \delta_v^k / (k!)^2, \Psi_v = \beta_v - \delta_v \) and \((,)_k\) is the Pochhammer symbol [37, p. xliii], the PDF of \( Z_v \) under i.i.d. Shadowed Rician fading is given as

\[
f_{Z_v}(x) = \sum_{i_1=0}^{m_{SV}-1} \cdots \sum_{i_{NS}=0}^{m_{SV}-1} \Theta(v, N_S)x^{\Delta_v-1} e^{-\Psi_v x}, \quad (8)
\]

where

\[
\Theta(v, N_S) = \alpha_v^{N_S} \prod_{\ell=1}^{N_S} \zeta_v(i_\ell) \prod_{j=1}^{N_S-1} \beta \left( \sum_{i=j}^{j} i, j+1 \right), \quad (9a)
\]
\[
\Delta_v = \sum_{q=1}^{N_S} \ell_q + N_S. \tag{9b}
\]

and \(\theta(\cdot, \cdot)\) is the Beta function [37, Eq. 8.384.1].

The cumulative distribution functions (CDF) of \(Z\) can be obtained as [37, Eq. 3.351.2]

\[
F_{Z_v} (x) = 1 - \sum_{i_1=0}^{m_{v,1}-1} \cdots \sum_{i_{S_v}=0}^{m_{v,S_v}-1} \Theta (v, N_S) \times \sum_{p=0}^{\Delta_v-1} \frac{\Gamma \left( \Delta_v \right) \Psi_v^{\Delta_v-p}}{p!} x^{p} e^{-\Psi_v x}. \tag{10}
\]

Considering the characterization of Nakagami-\(m\) fading, the PDF and CDF of channel gain \(|h_{RD_k}|^2\) are given, respectively, by [38]

\[
f_{|h_{RD_k}|^2} (x) = \frac{x^{m_{RD}-1}}{\Gamma (m_{RD}) \lambda_{RD}} e^{-\frac{x}{\lambda_{RD}}}, \tag{11}
\]

and

\[
F_{|h_{RD_k}|^2} (x) = \frac{\gamma (m_{RD}, x/\lambda_{RD})}{\Gamma (m_{RD})} \times 1 - e^{-\frac{x}{\lambda_{RD}}} \sum_{n=0}^{m_{RD}-1} \frac{x^{n}}{n!}. \tag{12}
\]

where \(\lambda_{RD} = \frac{2R_{m}}{m_{RD}}\) and \(\Omega_{RD} = \Lambda = \Omega_2 \cdots = \Omega_K\) in this case are the fading severity and average power, respectively, and \(\gamma (\cdot, \cdot)\) is the lower incomplete gamma function [37, Eq. 8.350.1]. Using order statistics, the PDF of \(|h_{RD_k}|^2\) can be expressed as

\[
f_{|h_{RD_k}|^2} (x) = \Upsilon \sum_{m=0}^{K-k} \frac{(-1)^m (K-k)}{m} \times f_{|h_{RD_k}|^2} (x)^{K-k} \times F_{|h_{RD_k}|^2} (x)^{m-1}. \tag{13}
\]

where \(\Upsilon = \frac{1}{(K-k)(K-k-1)}\). Furthermore, the CDF of \(|h_{RD_k}|^2\) is expressed as

\[
F_{|h_{RD_k}|^2} (x) = \Upsilon \sum_{m=0}^{K-k} \frac{(K-k)}{m} \times F_{|h_{RD_k}|^2} (x)^{m}. \tag{14}
\]

The PDF and CDF of \(|h_{RP}|^2\) are computed, respectively, according to

\[
f_{|h_{RP}|^2} (x) = \frac{x^{m_{RP}-1}}{\Gamma (m_{RP}) \gamma_{RP}} e^{-\frac{x}{\gamma_{RP}}}, \tag{15}
\]

\[
F_{|h_{RP}|^2} (x) = \frac{\gamma (m_{RP}, x/\lambda_{D})}{\Gamma (m_{RP})} \times 1 - e^{-\frac{x}{\gamma_{RP}}} \sum_{n=0}^{m_{RP}-1} \frac{x^{n}}{n! \gamma_{RP}^{n+1}}. \tag{16}
\]

where \(\lambda_{RP} = \frac{2R_{RP}}{m_{RP}}\) represents the fading severity, and \(\Omega_{RP}\) represents the average power.

III. SYSTEM PERFORMANCE ANALYSIS

A. OUTAGE PROBABILITY

The main performance metric, i.e. outage probability, requires study. In this case, the SIC is applied to many users. In particular, SIC is performed at the \(k\)-th user in two steps. The first and the second steps correspond to detecting and canceling the \(m\)-th user’s signal (\(m \leq k\)). The intended users are then able to decode its own signal. In an unwanted scenario, the \(k\)-th user cannot detect the \(k\)-th user’s signal and outage occurs. It is denoted by \(E_{k,m}\). In particular, the outage probability of the \(k\)-th user is given by

\[
OP_{k}^{out} = 1 - Pr \left( E_{k,1}^{c} \cap \ldots \cap E_{k,k}^{c} \right), \tag{17}
\]

where \(E_{k,m}^{c}\) is the complement event of \(E_{k,m}\). It can be written as

\[
E_{k,m}^{c} = \gamma_{m} > \gamma_{K}, \gamma_{K} > \gamma_{m} + \gamma_{K} + 1, \tag{18}
\]

where \(\gamma_{m} = 2R_{m} - 1\), \(R_{m}\) is the target rate at the \(m\)-th user, \(\delta_{m} = \frac{\gamma_{m}}{\phi_{m} - \gamma_{m}}\), and step (a) follows as \(\phi_{m} > \gamma_{m} \sum_{i=p+1}^{K} \phi_{i}\). We can then rewrite:

\[
OP_{k}^{out} = 1 - Pr \left( \gamma_{k} > \delta_{k}^{a}, \gamma_{K} > \gamma_{k} + 1 \right), \tag{19}
\]

where \(\delta_{k}^{a} = \max_{1 \leq m \leq k} \delta_{m}\).

Proposition 1: The CDF of \(\gamma_{K}\) is obtained as

\[
F_{\gamma_{K}} (x) = 1 - \sum_{(P, R)} \Gamma (\Delta_{P}^{k}) \times \sum_{p=0}^{\Delta_{K}-1} \frac{\Gamma (\Delta_{K}) \Gamma (\Delta_{P}+p) \Psi_{k}^{\Delta_{K}+p} \rho^{\Delta_{P}+p}}{p! \Psi_{k}^{\Delta_{P}+p}}. \tag{20}
\]

1 Although derivations of outage probability have studied in most of paper, such metric is still necessary to evaluate quality of links in term of satellite and devices in ground. Further, since the characteristic of SINR of multiple users limit us to consider other metric, we still aim to find insights in next sections.
where
\[
\sum (R, P) = \sum_{i_1=0}^{m_{sg}-1} \cdots \sum_{i_{sg}=0}^{m_{sg}-1} \varTheta (R, N_{S}) \times \sum_{i_1=0}^{m_{sp}-1} \cdots \sum_{i_{sp}=0}^{m_{sp}-1} \varTheta (P, N_{S})
\]  
(21)

**Proof:** See Appendix A.

Since \( \gamma_k = \frac{\rho |h_{RD_k}|^2}{|h_{RP}|^2} \) as the ratio of two Gamma random variables, we can derive the PDF of \( \gamma_k \) with \( x > 0 \) as
\[
F_{\gamma_k} (x) = 1 - \sum_{m=0}^{K-k-k+m-a(m_{rd})} \left( \begin{array}{c} K-k \\ m \end{array} \right) \left( \begin{array}{c} k + m \\ a \end{array} \right) \times \frac{\Gamma (m_{sp} + b) (-1)^{m+b} \vartheta_b (a)}{(k + m)(m_{sp} + b)}
\]
\[
\times \frac{x^b \rho^{m_{sp}b} (\lambda_{D}^{m_{sp}+b})}{(\lambda_{D}^{m_{sp}+b} + \lambda_{sp} (a + 1)) x^{m_{sp}+m_{sp}+b}}.
\]  
(22)

By taking the first derivative of (22), the corresponding PDF can be obtained as
\[
f_{\gamma_k} (x) = \frac{K-k-k+m-1(a(m_{rd})-1)x}{\Gamma (m_{sp} + b) (-1)^{m+b} \vartheta_b (a)} \times \frac{\Gamma (m_{sp} + b)}{(m_{sp} + b)}
\]
\[
\times \frac{x^{b-1} \rho^{m_{sp}b-1} (\lambda_{D}^{m_{sp}+b-1})}{(\lambda_{D}^{m_{sp}+b-1} + \lambda_{sp} (a + 1)) x^{m_{sp}+m_{sp}+b}}.
\]  
(23)

**Proposition 2:** The closed-form formula of \( \text{OP}^\text{out}_{out} \) of the \( k \)-th user is given by (24), shown at the bottom of the next page.

**Proof:** See Appendix B.

**Remark 1:** Since (24) contains lots of main parameters, the considered system performance relies on quality of channels and transmit SNR at the satellite, interference power constraint. We treat system performance of small group of users as basic requirement to retain normal operation. However, it is still challenging task to know how large the number of users in dedicated group the system can serve. We expect to further give suggestions in the section of numerical simulation.

**B. ASYMPTOTIC OUTAGE PROBABILITY AND DIVERSITY ORDER ANALYSIS**

To gain more insight into CSTRN performance, the asymptotic outage probability should be considered in the high SNR region \( \rho \rightarrow \infty \). Interestingly, when \( \rho \rightarrow \infty \) we can apply the Maclaurin series expansion of the exponential function in (7) to approximate. The PDF of \( \rho_{Z_k} \) is then given as
\[
f_{\rho_{Z_k}} (x) \simeq \frac{\alpha_{N_{S}}}{(N_{S} - 1)! \rho_{N_{S}}^{N_{S} - 1} \chi_{N_{S}}^{N_{S} - 1}},
\]  
(25)

and the corresponding CDF follows asymptotic behavior according to
\[
F_{\rho_{Z_k}} (x) \simeq \frac{\alpha_{N_{S}}}{(N_{S}! \rho_{N_{S}}^{N_{S}} \chi_{N_{S}}^{N_{S}}).}
\]  
(26)

Hence, substituting (26) and (8) into (32) and together with [37, Eq. 3.351.3], the asymptotic CDF of \( \gamma_k \) is obtained as follows
\[
F_{\gamma_k} (x) \simeq \sum_{i_1=0}^{m_{sp}-1} \cdots \sum_{i_{sp}=0}^{m_{sp}-1} \theta (P, N_{S})
\]
\[
\times \frac{\alpha_{N_{S}}^{N_{S}} (N_{S} + \Delta_{P} - 1)!}{(N_{S}! \rho_{N_{S}}^{N_{S}} \chi_{N_{S}}^{N_{S}+\Delta_{P}}).}
\]  
(27)

Similarly, by applying the Maclaurin series representation of the exponential function, the CDF of \( \rho_{Y} \) can be obtained as
\[
F_{\rho_{\lbrack h_{RD_k} \rbrack}^2} (x) \simeq \frac{1}{[\Gamma (m_{d} + 1)]^{K}} \frac{x^{m_{d}K}}{\Gamma (m_{sp} + b)}
\]  
(28)

Assisted by (30) and (15), the asymptotic behavior of \( \gamma_{Y} \) is written as
\[
F_{\gamma_{k}} (x) \simeq \frac{\Gamma (m_{d}K + m_{sp})}{\Gamma (m_{sp} + b)} \frac{x^{m_{d}K}}{\Gamma (m_{sp} + b)}
\]  
(29)

With a large SNR, we can rewrite \( \Gamma_{RD} \simeq \frac{\gamma_{Y_{SD}}}{\gamma_{Y_{RD}} + \gamma_{Y_{DF}}} \), and hence, system performance is dominated by the weakest link. We can thus represent the asymptotic OP as
\[
\text{OP}^\text{out}_{out}, \rho \rightarrow \infty
\]
\[
\simeq \sum_{i_1=0}^{m_{sp}-1} \cdots \sum_{i_{sp}=0}^{m_{sp}-1} \theta (P, N_{S})
\]
\[
\times \frac{\alpha_{N_{S}}^{N_{S}} (\delta_{k}^{N_{S}} \Gamma (N_{S} + \Delta_{P})}{(N_{S}! \rho_{N_{S}}^{N_{S}} \chi_{N_{S}}^{N_{S}+\Delta_{P}})
\]
\[
+ \frac{\Gamma (N_{S}m_{d}K + m_{sp})}{\Gamma (m_{sp} + b)} \frac{x^{m_{d}K}}{\Gamma (m_{sp} + b)} \frac{\delta_{k}^{N_{S}} \chi_{N_{S}}^{N_{S}+\Delta_{P}}}{\Gamma (m_{sp} + b)}.
\]  
(30)

It is straightforward to indicate that when \( \rho \rightarrow \infty \), the diversity order is \( \min(N_{S}, m_{d}K) \).

**Remark 2:** To evaluate the outage behavior, we conduct such asymptotic computations as helpful guideline of system design in practice. We aim to enhance the transmit SNR at the satellite and then to improve system performance of each user. Since SNR mainly depends on \( N_{S} \), design of multiple antenna for satellite is highly demand to improve performance for users in ground compared with adjusting other system parameters. We expect to further verify such explanation in the section of numerical simulation.

**C. THROUGHPUT**

Based on outage probability, we can further evaluate throughput in delay-limited transmission mode. For a fixed target rate \( R_{k} \), we can compute the overall throughput as follows [39]
\[
T = \frac{1}{2} \sum_{k=1}^{K} R_{k} (1 - \text{OP}^\text{out}_{out}).
\]  
(31)
The performance of three users by changing the number of satellite links are described in the Table 3.

**TABLE 3. Satellite channel parameters.**

| Parameter          | $m_{SV}$ | $b_{SV}$ | $\Omega_{SV}$ |
|--------------------|----------|----------|---------------|
| The heavy shadowing | 1        | 0.083    | 0.007         |
| The average shadowing | 5        | 0.251    | 279           |

**FIGURE 2. Outage probability versus transmit $\rho$ varying $N_S$ with $m = 1$ and satellite link in the HS scenario.**

**IV. NUMERICAL RESULTS**

This section provides and discusses the numerical results. To verify the accuracy of the expressions, we compared the analytical results with Monte Carlo simulation results. Unless mentioned otherwise, we set $K = 3$, $\phi_1 = 0.5$, $\phi_2 = 0.4$, $\phi_3 = 0.1$, the target rate $R_1 = 0.1$, $R_2 = 0.5$ and $R_3 = 1$, and the main parameters before simulation as $m_D = m_{RP} = m$ and $\Omega_D = \Omega_{RP} = 1$. The Shadowed-Rician fading parameters for the satellite links are described in the Table 3.

Fig. 2 demonstrates the comparison of outage probability performance of three users by changing the number of satellite’s antenna $N_S$. It can be intuitively seen that, by increasing $N_S$ the considered system obviously improves the outage performance of three users. The first user $D_1$ shows its superiority in term of outage performance in case of single antenna at the satellite $N_S = 1$, then the worst performance occurs in user $D_3$ which is allocated less percentage of transmit power. The system based on OMA scheme shows its performance, but such outage behavior just better than user $D_3$. The main reason is that OMA-based system need more time slots to process the same number of users compared with NOMA-based system. It is valuable finding since Monte-Carlo based simulation and analytical result are matched very well. If we increase the number of antennas at the satellite, such performance metrics show its improvement which demonstrating the benefits of introducing multiple antennas in the NOMA-CSTRN. Furthermore, asymptotic lines of outage probability match exact curves at high SNR regime which indicates the exactness of our derived expressions in term of outage probability.

Fig. 3 depicts the outage performance of the secondary users against two crucial parameters i.e., Nakagami channel parameter $m$ with the number of transmit antennas set as $N_S = 2$. Similarly with previous figure, we can see significant improvement in term of outage behavior once SNR is greater than 25 dB. It can be explained that since the outage probability is a function of $\rho$, we can see that the outage probability decreases since $\rho$ increases, shown in (4), (5), and (6). However, outage performance improves significantly for case of $m = 2$. Consequently, by improving quality of channels at ground, an increasing $m$ will improve the outage performance of all users.

Fig. 4 illustrates the effect of different channel conditions about satellite link from S to R, the outage performance of these users can be improved when either the terrestrial link quality gets better, i.e. AS case is better case among two considered cases. In particular, comparing those analytical and asymptotic OP curves in Figs. 2 and Fig.3, we can see that asymptotic results agree with analytical results across the entire average SNR range.

As can be seen in Fig. 5 and Fig. 6 the throughput performance of such NOMA-CSTRN system can be improved by changing configurations of transmit antennas at the satellite and links from the satellite to base station at ground. From (31), it can be explained that further metric, throughput $OP_{out}$ improves with the number of users compared with NOMA-based systems and the exactness of our derived expressions in term of outage probability.
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V. CONCLUSION

This study has investigated the performance of an NOMA-CSTRN system wherein multiple secondary terrestrial users enjoys the access to spectrum with a primary satellite network to further achieve higher spectrum efficiency. Different from the current studies, we have considered the multiple antennas designed at the satellite and NOMA to indicate performance of several users which are groups in the context of NOMA scheme. We also consider throughput performance with different configurations of channels associated with transmission links and key parameters are necessary to improve performance metrics, i.e. outage probability and throughput. Above all, a comparison with many scenarios revealed that proposed NOMA-CSTRN system provides different performance for many users while utilizing the spectrum resource is conducted efficiently. Further metrics are expected to study in future work.

APPENDIX A

The CDF of $\gamma_R$ can be rewritten as

$$F_{\gamma_R}(x) = \Pr \left( Z_R < \frac{xZ_p}{\rho} \right)$$

$$= \int_0^{\infty} f_{Z_p}(z) F_{Z_R} \left( \frac{xz}{\rho} \right) dz$$

$$= 1 - \int_0^{\infty} f_{Z_p}(z) \left( 1 - F_{Z_R} \left( \frac{xz}{\rho} \right) \right) dz. \quad (32)$$

With the help of (8) and (10), we can express (32) as

$$F_{\gamma_R}(x) = 1 - \sum_{(R,P)} (R,P) \times \sum_{p=0}^{\Delta R-1} \Gamma \left( \Delta R \right) \Psi_R^{-\Delta R+p} \left( \frac{x}{\rho} \right)^p \times \int_0^{\infty} z^{\Delta p+p-1} e^{-\left( \frac{xz}{\rho} + \Psi_R \right) z} dz. \quad (33)$$

Based on [37, Eq. 3.351], we obtain

$$F_{\gamma_R}(x) = 1 - \sum_{(R,P)} (R,P) \times \sum_{p=0}^{\Delta R-1} \Gamma \left( \Delta R \right) \Gamma \left( \Delta R + p \right) \Psi_R^{-\Delta R+p} \rho^{\Delta R+p} \left( \Psi_R x + \rho \Psi_R \right)^{\Delta R + p} \rho^{\Delta R+p}. \quad (34)$$

This is the end of the proof.

APPENDIX B

Then, we can calculate (19) as

$$OP_{out}^k = 1 - \sum_{(R,P)} (R,P) \sum_{m=0}^{\Delta R-1} \sum_{k=m-1}^{a(m_p-1)} \sum_{b=0}^{K-k} a(m_p-1)$$

depends on the outage probability achieved in previous steps. Therefore, quality of channels results in the curves of the throughput of system. The higher value of $N_S$ contributes to improve SINRs, then the higher throughput performance can be benefited. These observations become important guidelines to design such NOMA-CSTRN system.
where  

\[
\begin{align*}
\Gamma = & \frac{\Gamma(\alpha, \beta)}{\Gamma(\alpha) \Gamma(\beta)} \quad \text{Here, we apply the identity [40, Eq. 10]}
\end{align*}
\]

\[
(1 + x)^{-b} = \frac{1}{\Gamma(b)} G^{(1, 1)}_{1, 1} \left[ a \left| \begin{array}{c}
1 - b \\
0
\end{array} \right. \right].
\]

where \( G^{(1, 1)}_{1, 1} \) denotes Meijer’s G-function [37, Eq. 8.2.1.1]. Then, we represent (35) as

\[
\begin{align*}
\mathcal{O}^\text{out}_{p,k} & = 1 - \sum_{(R, P)} \sum_{m=0}^{K-k-1} \sum_{a=0}^{K-m} \sum_{b=0}^{m+b+1} \sum_{l_0=0}^{l_1=1} \sum_{l_2=0}^{l_3=1} \\
& \times \left( \begin{array}{c}
k + m - 1 \\
a
\end{array} \right) \left( \begin{array}{c}
k - k \\
m
\end{array} \right) \left( \begin{array}{c}
p \\
l_1
\end{array} \right) \left( \begin{array}{c}
M_D + b + l_3 - 1 \\
l_2
\end{array} \right) \\
& \times \frac{\Gamma \Gamma(\mu_R + \lambda b + 1 - \mu_R) (\delta_k^a + 1)}{\delta_k^a (\delta_k^a + 1) \delta_k^a \frac{1}{\delta_k^a} - 1} \\
& \times \psi_{\gamma R}^\delta_{k} \left( \delta_k^a + 1 \right) \frac{1}{\delta_k^a} - 1
\end{align*}
\]

This is the end of the proof.

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