EFloat: Entropy-coded Floating Point Format for Compressing Vector Embedding Models

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ABSTRACT
In a large class of deep learning models, including vector embedding models such as word and database embeddings, we observe that floating point exponent values cluster around a few unique values, permitting entropy based data compression. Entropy coding compresses fixed-length values with variable-length codes, encoding most probable values with fewer bits. We propose the EFloat compressed floating point number format that uses a variable field boundary between the exponent and significand fields. EFloat uses entropy coding on exponent values and signs to minimize the average width of the exponent and sign fields, while preserving the original FP32 exponent range unchanged. Saved bits become part of the significand field increasing the EFloat numeric precision by 4.3 bits on average compared to other reduced-precision floating point formats. EFloat makes 8-bit and even smaller floats practical without sacrificing the exponent range of a 32-bit floating point representation. We currently use the EFloat format for saving memory capacity and bandwidth consumption of large vector embedding models such as those used for database embeddings. Using the RMS error as metric, we demonstrate that EFloat provides higher accuracy than other floating point formats with equal bit budget. The EF12 format with 12-bit budget has less end-to-end application error than the 16-bit BFloat16. EF16 with 16-bit budget has an RMS-error 17 to 35 times less than BF16 RMS-error for a diverse set of embedding models.

1 INTRODUCTION
As natural language processing (NLP) models expand their capabilities and complexity, their sizes have been increasing dramatically demanding higher memory capacity and bandwidth. For example, state-of-the-art transformer-based NLP models such as BERT (Vaswani et al. [63]), Megatron-LM (Shoeybi et al. [59]), Open AI GPT-3 (Brown et al. [13]), or Google Switch-C Transformers (Fedus et al. [20]), contain from hundreds of millions, to even trillion parameters (Fedus et al. [20], Hoefler [32]). Although NLP model transformation is a very active area of research (Section 6), its current focus is on model inference scenarios in which reduced precision and integer quantization are commonly used, under the assumption that the original model need not be restored (i.e., it is destructive).

The primary goal of this work is to explore compression strategies for large vector embedding models used in NLP and related use cases, such that one can decompress and recover the original model with minimum loss of accuracy. The database embedding (db2Vec), a vector embedding technique designed to develop semantic models from multimodal relational database tables (Bordawekar and Shmueli [11, 12]), forms the impetus behind this exploration. Although related, db2Vec differs from its NLP counterparts such as Word2Vec (Mikolov et al. [49]) and GloVe ([52]), in that its source data follows the relational data model (Date [18]) (the source data is not a natural language document but a relational database table). Considering the relational database tables can be very large (e.g., billions of rows in a table) with a large number of unique tokens, a much larger vocabulary is used than a traditional natural language document. As a result, trained db2Vec models have high demand on memory capacity and bandwidth (i.e., both memory and AI accelerator I/O bandwidth).

A trained vector embedding model is a snapshot of its weight matrices and consists of weight values represented in IEEE 32-bit single-precision floating point (FP32) format. Therefore, we focus on compression approaches that compress FP32 to low-precision floating point formats. In our approach, active rows of the compressed db2Vec model are uncompressed on-demand during inference computations and the original FP32 values are restored with smaller loss of accuracy than other low-precision methods, as we quantify in later sections.

A floating-point(FP) number is of the form (Goldberg [23]):

$$-1^{signbit} \times 2^{exponent{-bias}} \times significand$$

The exponent largely determines the range of minimum and maximum values representable by the format. The number of significand bits determine the precision (a constant bias is
typically present to represent exponents as all positive integers which simplifies FP magnitude comparisons.) The two most widely used low-precision 16-bit FP formats, BFloat16 (BF16) and IEEE 754-2019 Half-precision (FP16), make a tradeoff between the number of exponent and significand bits (Figures 1(a,b,c)). BF16 with an 8-bit exponent and 7-bit significand has a wide exponent range but low precision as compared to FP32 and FP16 (Wang and Kumar [65]). In contrast, FP16 with a 5-bit exponent and 10-bit significand has a greater precision but in a much narrower exponent range than BF16 due to smaller number of exponent bits (IEEE [35]). One could convert a FP32 model to either of these two 16-bit representations, but with a big loss of either accuracy or range, and the data may not be recovered when converted back to FP32.

Therefore, we have developed a flexible and portable low-precision compressed FP format, EFloat (EFn), that can uncompres and recover the original data with minimum loss for runtime inference computations. Figure 1(e) presents the new low-precision FP format, EFloat (EFn), with a fixed total bit budget of n bits, e.g., n = 16. EF16 uses an entropy-coded variable-width N bit exponent and a variable-width 15 – N bit significand (mantissa) with a total number of 16 bits, including the sign bit. The EF width is adjustable, e.g., EF12, permitting a tradeoff between compression ratio and accuracy, as we demonstrate in later sections. We have used the EF format to compress db2Vec models such that relevant rows of the compressed db2Vec model can then be uncompressed on-demand during inference computations into any target representation (e.g., FP32 or a 16-bit format) with smaller loss of accuracy.

Our design is motivated by a key pattern that we observed across a wide range of vector embedding models: the trained models contain only few of the \(2^8 = 256\) unique exponents available in FP32 and with a bell-shaped histogram distribution caused by a certain class of non-linear activation functions used in training. The EFloat design exploits this behavior and assigns the least number of bits to most common exponent values, however preserving the original exponent range of the original FP32 format.

Our work makes the following contributions:

- EFloat provides flexible variable-length reduced-bit representation of any floating point format (e.g., FP32, FP16) by using fewer exponent bits to map the same exponent range as the original value.
- For a given bit budget (e.g., 16), EFloat provides more accurate representation of the FP32 values than BF16 and FP16 by using fewer exponent bits to capture the same range as before, and then using the remaining bits to increase significand precision. Thus, EFloat provides portable low-precision representation of FP values that can be converted to other representations with minimum loss (e.g., 16-bit EFloat (EF16) to FP32, BF16, or FP16).
- We employ the length-limited variant of the Huffman encoding approach to map the exponent values to bit codes that satisfy the maximum bit length constraint.
- Although designed specifically for improving both memory and bandwidth via compression, the EFloat format can also support native reduced-bit computations over pre-trained vector embedding models. EFn support in hardware, if required, is basically via 256-entry table lookup converting from EFn to conventional FP and vice versa.
- For a given dataset, multiple floating point to EFloat conversion tables are possible. Tables may be optimized for maximum significand width (highest exponent compression) at the expense of worse significand precision for few floats with infrequent exponents (i.e., using less significand bits for outliers) and vice versa.
- Since vector embedding models are used in a wide array of NLP architectures including transformers, in addition to db2Vec, the EFloat format can be used for a much wider (and more space consuming) class of NLP models.
- A hardware implementation of EFloat supporting both inference and training is sketched. EFloat to/from FP conversion tables may be used in the memory and I/O interfaces of AI accelerators to save memory capacity and I/O bandwidth resulting in higher AI accelerator performance.

In Section 2, we first present the analysis of various vector embedding models. The EFloat format is presented in Section 3. Section 4 describes key steps in conversion between EFloat and other floating point formats. Section 5 presents an error analysis of various EFloat widths (EFn) against BF16 and FP16. In Section 6, a review of related work on model compression and floating-point formats for deep learning is presented. Finally, Section 7 presents conclusions and outlines future directions.

### 2 ANALYZING VECTOR EMBEDDING MODELS

Vector embedding models are extensively used in natural language processing (NLP) to capture and exploit semantic relationships of word entities (e.g., words, sentences, phrases, paragraphs, or documents). A trained vector embedding model consists of a set of vectors, each vector encoding a distributed representation of inferred semantics of a word entity, i.e., a single vector captures different attributes of the inferred semantics ([31]), created in part by contributions
Figure 1: Floating point formats are compared. EFloat has a fixed total width, but the boundary between the exponent and the significand is variable (e). The exponent is entropy coded, providing an average of 4.3 extra bits of precision to the significand (e.g., (h)), while keeping the logical exponent range at 8 bits, same as that of FP32. EFloat has greater precision and range than the existing FP formats having the same bit budget.

Figure 2: Histogram of the exponent fields of 32-bit floating-point (FP32) values found in vector-embedding and related NLP models. Only the db2Vec, word2vec, doc2vec, and sentence-encoder models were generated. Others were downloaded as publically available pretrained models.
by other word entities. Every vector embedding model implements some variant of the log-bilinear language (LBL) model that predicts the probability of the next word \(w_t\) given the previous words (context) \((5, 7, 30)\). The LBL model first predicts a real-valued vector representation of a word by linearly combining the real-valued vector representations of its context words. Then, the distributed representation of the word is computed based on the similarity between the predicted representation and the representations of all words in the vocabulary. This step is accomplished using the normalized exponential or Softmax function over the associated feature vectors. The output of the Softmax function is the probability distribution over \(V\) different possible outcomes, where \(V\) is the vocabulary size.

![Distribution of Most Significant Significand Decimal Digit](image1)

**Figure 3:** The most significant decimal digit in the significand of FP32 values in various vector embedding models follows the Benford distribution.

![Probability of bit value being 1](image2)

**Figure 4:** Distribution of a bit value being 1 at different bit locations in the significand of FP32 values in various vector embedding models.

Characterizing exponent behavior: Figure 2 presents histograms of exponent values in multiple pre-trained vector embedding models, where the X-axis represents exponent values (from the 8-bit exponent portion of a 32-bit IEEE 754 floating point value). The Y-axis represents normalized number of occurrences of the exponent values, i.e., a histogram. For all models, exponents mostly vary between -17 to 3 (i.e., \(2^{-17}\) to \(2^3\)). The most frequent two exponents are -2 and -3 as apparent on Fig.2 except for few models.

Vector embedding and related NLP models presented in Fig. 2 include word embedding (word2Vec) (Mikolov et al. [49], Zhang et al. [70]), sentence (sent2Vec) and document embedding (doc2Vec) (Chen et al. [17], Le and Mikolov [43]), GloVe (Pennington et al. [51]), subword embedding (FastText) (Bojanowski et al. [9, 10]), database embedding (db2Vec) (Bordawekar and Shmueli [11, 12]), graph embedding (PyTorch BigGraph) (Lerer et al. [44]), and Google’s transformer-based universal sentence encoder (Cer et al. [16], Google [24]) using the Brown corpus [14]. All these models implement different variations of the LBL model. The word2Vec based models, e.g., word2Vec, sent2Vec, doc2Vec, db2Vec, and FastText, use a neural network with different versions of Softmax as the activation function. GloVe, on the other hand, is a count-based optimization approach that uses a word co-occurrence matrix and weighted least-square as the optimization function. The FastText subword model ([9, 38]) assigns a vector for every character n-gram, using an extended skip-gram model ([49]) and then, words are represented as the sum of these representations. The universal sentence encoder generates embedding vectors for sentences using a standard Transformer architecture that takes word embedding vectors as input and uses a Softmax function to compute attention ([63]).

Irrespective of the model type, we observe in Fig. 2 that exponent values cluster in a narrow range of values, and display a distinct histogram peak. The only exception is the doc2Vec model that exhibits two peaks as the doc2Vec first builds fine-grained embeddings for words and then uses them to build embeddings for coarser-grained entities such as paragraphs via concatenating and averaging individual word vectors which results in a smaller second peak. Finally, multiple models of a given type (e.g., churn and credit-fraud db2Vec models), exhibit the same exponent behavior.

Characterizing significand behavior: Figures 3 and 4 present patterns observed in the significands of FP32 values of various pre-trained vector embedding models used in the evaluation of the EFoat format. Figure 3 plots the frequency distribution of the most significant significand digit in the decimal representation of the FP32 values. Surprisingly, occurrences of the digits in these trained models follow a logarithmically decreasing distribution consistent with the Benford’s Law ([8], Newcomb [50]). Benford’s Law predicts the frequency of numbers 1–9 in the leading non-zero digit of most datasets found in nature. According to the law, the number 1 occurs in the most significant digit 30.1% of time, the number 2 occurs 17.6% of time, and so on. The logarithmic reduction leads to the number 9 being at the most significant location around 4.6% of time.
Figure 4 presents the binary (0/1 bit) view of the significand in few trained models. These models also approximately follow the Benford’s Law for binary numbers. We extended Benford’s Law to bits 1 through 23 of the significand. (Note that the leading non-zero bit in a binary number is always 1 obviously; but that is stored in the hidden leading bit called the implied-bit of normalized FP values, not visible in Figure 4.) In this figure, we report the aggregate distribution of a particular FP32 significand bit location having the value 1, for the 23 significand bits in selected vector embedding models. As we observe, the distribution exhibits a knee at location 7; the first 7 bits show an increase in the probability of the bit value being 1 from 41.51% to 49.88%; the remaining bits exhibit probabilities around 50%. In other words, it is more likely to see the bit value being 0 in the most significant bit of an FP32 significand in trained vector embedding models. That the bit values in trained models are skewed towards 0 in the leading bits presents additional compression and FP rounding-mode opportunities to improve precision, although the focus of this paper is mainly on the FP exponent.

Figure 5: The Sigmoid $\sigma(x)$ curve and its gradient. The floating-point (FP32) exponent of few neural weights are overlaid on $\sigma(x)$.

**Impact of activation function:** The Softmax family of activation functions used in vector embedding models is responsible for the clustering behavior of exponents (Figure 2). To understand the reasons, let us delve deeper into the training of an embedding model. For illustration purposes, we use database embedding (db2Vec) of the Telecom Churn data (IBM [34]) as an example. db2Vec is an adaptation of the word2Vec approach, and has been designed to build an embedding model from structured database tables that adhere to the relational data model. Like word2Vec, db2Vec also uses Skipgram with Negative Sampling (SGNS) as the training approach. The SGNS approach uses a binary classifier based on the logistic (Sigmoid) function instead of using the Softmax-based predictor. The Sigmoid function $\sigma(x)$ is defined as:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

The Sigmoid curve and its gradient $\sigma'(x) = \sigma(x)(1 - \sigma(x))$ is shown in Figure 5. The gradient of the Sigmoid activation function and the back-propagation process can be specified as:

$$\text{weights}_{\text{iter}+1} = \text{weights}_{\text{iter}} + \text{learning\_rate} \times \sigma'(x)$$

The overall training process involves multiple back-propagation iterations to update model weights using the gradient of the Sigmoid function. Weights get updated iteratively during the back-propagation process by the error computed for that iteration. Practically, the error is computed using the gradient of the activation function. During model training, we observe that the weights rapidly converge (Fig.6) to their final values. Their exponents are substantially clustered at the slope of the Sigmoid curve, the $2^{-8}$ to $2^0$ output range of Sigmoid, as evidenced by Figures 5 and 6. Training eliminates smaller exponents from the model because the activation function output is practically zero for any input value when weights are small. Large exponents are non-existent due to normalization of weights. Accordingly, most exponents cluster at the slope of the Sigmoid curve. Further, we observe that early in the training process, the significand precision is not as important as the magnitude of model weight updates are dominated by exponent updates from one iteration to the next (Figures 7 and 8). Once an exponent settled to its final value, the significand precision becomes more important since weight starts converging to its final value in small increments.

This observation is useful for amortizing the overhead of Huffman code-table generation over many training iterations and inferencing. One could produce a static code-table covering multiple NLP models (except a few models such as Wiki-doc2vec) shown in Figure 2. Then, one could use the same code-table any number times because the exponent
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The key idea behind the EFloat format is the variable-width encoding of exponents using the well-known Huffman algorithm. Frequency of unique exponent values in the dataset determine the coded-exponent widths which may vary between as small as 1-bit and some software configurable maximum, e.g., 8-bit (Figure 1(e,f,g)). Thanks to the entropy coding of the Huffman algorithm, frequent exponent values are coded with fewer bits and infrequent exponents are coded with more bits as observed in Fig.9. For example, the most frequent exponent -2 is encoded with 2 bits. That shaves 6-bits out of the 8-bit exponent field used in the conventional FP32 format.

Bits saved from the exponent become part of the significand, therefore increasing the floating point precision compared to other float formats with the same bit budget. An N-bit coded-exponent in an EF16 float results in a (15-N) bit significand as shown in Fig.1(e). Since EFloats with frequent exponents have wide significands, the entire dataset has a greater precision on average. EFloats with infrequent exponents have narrow significands. But, their contribution are relatively small in the common calculations used in model training and inferencing, such as dot-products, vector-sums, and cosine-similarity (EFloat precision is quantified and compared to prior formats in Section 5.).

EFloat on average have greater precision and range than any other fixed-field FP format with the same bit budget. For example, EF16 with a 3-bit coded-exponent has 12-bits of significand compared to the 7-bit significand found in a BF16 (Figures 1(h,b)). However, EFloat exponent’s logical width is always 8-bit, the same as for FP32 and BF16, irrespective of EFloat width. Even for extremely narrow floats such as EF8, the logical exponent width can be 8-bit since encoding compresses the exponent field.

The EFloat format compresses special values of IEEE 754, such as signed zeros and infinities losslessly. NaNs are semantically compressed losslessly: converting a NaN to and from FP32 to EFn and vice-versa still results in a NaN. Denormal floats may round to zero since least significant bits of significands are truncated during encoding.

4 EFLOAT ENCODING AND DECODING

4.1 Entropy Compression

The Huffman algorithm: is a popular lossless compression algorithm used in many compression tools and compressed data formats (Salomon [57]). Data symbols are encoded with variable-length binary codes whose length are determined by the symbol probabilities in the data stream. The algorithm builds a binary tree with each leaf assigned a symbol. Higher probability symbols are closer to the tree root than others. The path from the tree root to the leaf is the binary coding of the symbol. To demonstrate with a trivial example, the letters A, B, C occurring with probabilities of 0.5, 0.25, and 0.25 may be encoded with the bit patterns 0, 10, and 11, respectively. The algorithm yields 1.5-bit/symbol compression efficiency, better than 8-bits/symbol using an ASCII representation or 2-bits/symbol using a simplistic mapping of the 3 letters to 2-bit integers. Huffman coding is optimal when symbol probabilities are negative powers of 2. However, it is an effective compression method even for non powers of 2 distributions. Fig.9 shows the Huffman coded exponent widths as a function of exponent frequencies of a word2vec trained model.
Huffman codes have the \textit{prefix} property that states that no code is a prefix of a longer code (due to the binary tree construction.) As a result, the Huffman code not only encodes the original symbol but the code-length as well. Thanks to the prefix property, the movable boundary between the exponent and the significand is easily identified while decoding EFloats (Fig.1(e)).

\textbf{Length-limited Huffman Encoding}: The Huffman algorithm builds probabilities based on frequency histogram of exponents and outputs a code-table mapping 8-bit exponents to variable-width coded-exponents.

If the dataset contains \( N \) unique exponents, for some distributions the algorithm may produce codes with \( N - 1 \) bit-widths in the worst case exceeding EFloat’s entire width. For example, a word2vec dataset in Fig.9 contains 23 unique exponent values. With a worst case distribution, some coded exponent widths may not fit in an EF16 number (e.g., a 22 bit exponent). Therefore, we use the \textit{Length-Limiting} variant of the Huffman algorithm to set a maximum coded-exponent width (Abali et al. [1]). In Fig.9, the maximum code width is set to 8-bits resulting in the infrequent exponents coded with that maximum. Note that length-limited Huffman coding is a well know compression technique and has been in use in many popular compression tools. Our contribution here is application of it in a floating-point number format.

This software-defined maximum exponent width presents an opportunity to tune the EFloat precision to the particular NLP application requirements. Figure 10 shows the inverse relationship between the maximum code-width and the minimum code-width. As the limit is increased from 5 to 8 and 10-bits we observe that the least-frequent exponents are coded with the maximum-width codes as expected, therefore their respective significands lose precision. At the same time, with an increased limit the most-frequent exponents are now coded with fewer bits. This reduces the average coded-exponent width as Figure 10 shows. Therefore, on average the floating point values gain precision while few outliers lose precision. Therefore, EFloat not only compresses the regular floats but for a given EFn budget of \( n \)-bits the software application can optimize the end-to-end precision and compression ratio by adjusting both the floating point bit-budget \( n \) and the maximum coded-exponent width.

\subsection*{4.2 EFloat Encoding and Decoding}

\textbf{EFloat Encoding and the Code-Table}: During the conversion from FP32 to an EFn (e.g., EF16), exponents in the original dataset are histogrammed first, e.g., Fig.2. The histogram representing probabilities of the exponents is an input to Length-Limiting Huffman algorithm that produces a code table for translating the FP32 exponents to EFn coded-exponents. The output is a 256-entry (\( 2^8 \)) code-table indexed by the original 8-bit exponent. Each table entry contains a pair, the variable-width coded-exponent and its bit-width. Note that the code-table is quite small, tens of bytes in practice, since few unique exponents are present in most NLP datasets as Fig.2 shows.

When the sign bits have a skewed distribution, e.g., if they are substantially positive, then the sign bit and the 8-bit exponent may be treated as a single 9-bit integer when histogramming. Thanks to the entropy coding, a skewed sign bit distribution may save up to one additional bit in the exponent, becoming available for further increasing precision of the significand.

Using the code-table, the entire dataset is converted from FP32 to the chosen EFn width (e.g., EF16) replacing original exponents with coded-exponents. Least significant bits of the FP32 significand are truncated to match the EFn width. For example, in Fig.9, the algorithm encodes the most frequent exponent with 2-bits. Accounting for the sign bit, this yields a 13-bit EF16 significand by truncating the bottom 10-bit of the 23-bit significand of FP32. We use the \textit{round-to-nearest} method to provide on average 0.5 bits of additional precision during this truncation step (many different rounding modes may be used, although it’s not a subject of this paper.)
For large datasets, a statistically representative subset may also be used to reduce histogram collection time. Furthermore, when the histogram is known in advance, a pre-built code-table may be used, skipping the histogram collection and Huffman algorithm steps. During training exponents rapidly converge to their final values as observed in Fig. 6. Past iteration 10, the exponent distribution is practically identical for all iterations 11 through 2481, which suggests that a single pre-built code-table optimized for final iterations may serve for all iterations to start. The same pre-built table, although suboptimal for early iterations, may be used because significant precision is not as important at that point in time; model weight updates are dominated by exponent updates. Once exponents settled to their final values the significant precision becomes important since model weights updates progressively get smaller.

**EFloat Decoding:** For EFloat to FP32 conversion we use a inverse mapping of the code-table described earlier. Note that Huffman codes are prefix codes which encode both the original value and the code-width. Therefore, the movable boundary between the exponent and the significand is not ambiguous; a "boundary marker" is not necessary. A decoder-table indexed by the coded-exponent may be used to decode the original exponent value and the significand’s leading bit position in constant time. The decoder-table has as many entries as \(2^{\text{max code width}}\) e.g., \(2^8\). Each table entry contains the original exponent and bit-width of the coded-exponent. To index the decoder-table with variable-width codes many entries are filled with duplicates. For example, a 2-bit coded-exponent 00 is duplicated 64 times in the table at locations 00000000 through 00111111 with each location containing the original exponent and code-width= 2. Duplicating entries is equivalent to having logical don’t care bits in the index which is a common technique used in hardware lookup tables.

The second element of each table entry contains the EFloat significand width. Since the significand was truncated earlier during the FP32 to EFloat conversion, the missing least significant bits must be padded with zeros to match the original FP32 width.

### 4.3 Implementation Alternatives

Currently, both EFloat encoding and decoding are completely implemented in software that allows us to compress an FP32 value to an EFn format of variable size \(n\), and conversely, given a value in the EFn format, generate its FP32, BF16, or FP16 representations.

As hardware arithmetic functional units natively supporting EFloat operations currently do not exist, it is necessary to recover data from the compressed EFloat representation into the target floating point representation used in the computation. EFloat encoding is not time critical as it’s a trivial fraction of training time. However, hardware support for EFloat decoding may be necessary so that the numerical values in the correct format are fed to the functional units with minimum delay. The EFloat decoding hardware may be implemented with a Static Random Access Memory (SRAM) based lookup table. With a maximum code width of \(K\) bits, a \(2^K\) entry SRAM based table may be used. \(K = 8\) is desirable to cover the worst case condition of all 256 exponents utilized by an FP32 dataset.

If the computer processor has many input ports, for example a systolic array such as the Google TPU [39], or a wide SIMD architecture [28, 29], many decoder tables will be necessary for parallel access. To save area, the maximum code width may be set to \(K < 8\) at the expense of giving up some compression quality. An alternative approach for saving area might be using a two level table which will substantially reduce SRAM capacity requirements. For example, the Zlib software [3] uses a maximum code width of 15-bits. But for encoding the Zlib symbols, a 2-level table requires only 852 table entries compared to \(2^{15} = 32768\) entries required in a 1-level table.

### 5 Evaluating the EFloat Representation

In this section, we evaluate the efficacy of the EFloat format using two sets of experiments. The first set measures the loss of precision in representing FP32 data in BF16, FP16, and EFloat formats with bit budgets from 16 down to 8 bits. The second set of experiments compares the quality of ranked results for similarity and dissimilarity queries using the Normalized Discounted Cumulative Gain (NDCG) score for BF16, FP16, and various EFloat formulations. Table 1 presents the list of the pre-trained vector embedding models used in these experiments, along with their characteristics: mode types, number of tokens in the model, the vector dimension and the model size (vector values are stored using FP32).

| Name          | Type      | #Tokens | Dimension | Size    |
|---------------|-----------|---------|-----------|---------|
| churn         | db2vec    | 7104    | 300       | 20 MB   |
| crawl         | fast-text | 1999995 | 300       | 4.3 GB  |
| enwiki        | word2vec  | 5427849 | 200       | 9.6 GB  |
| MDM           | db2vec    | 5050264 | 300       | 14 GB   |
| 840B          | GloVe     | 2196017 | 300       | 5.3 GB  |
| wiki-sw       | fast-text | 999994  | 300       | 2.2 GB  |
| virginia      | db2Vec    | 80772   | 300       | 222 MB  |
Table 2 presents characteristics for the models used in these experiments: code table sizes, number of unique exponents, range of exponent bits generated by the Huffman algorithm, the average count of exponent bits, and minimum and maximum average count of significant bits. For EF16, the average significant length is 4.3 bits higher than BF16 (with a 7-bit significand) and 1.2 bits higher than FP16 (with a 10-bit significand). EF8 is another demonstration of the EFloat benefit: the full 8-bit exponent range, a sign bit, and a 3.4 bit avg. width significant (a total of 12.4 bits) fit in a budget of 8 bits. As the Table 2 illustrates the code tables used for encoding and decoding these models are very small in size (average size is 86 bytes and the maximum is 116 bytes).

5.1 Evaluation of Numerical Precision

The first set of experiments compares the loss of precision due to the least significant significant bits being truncated during conversion from FP32 to various lower-precision formats. Given a low-precision format (e.g., EF16 or BF16), the values are converted back to FP32, and the arithmetic difference, \( f^o - f^c \), of the original FP32 value, \( f^o \), and the regenerated FP32 value, \( f^c \), is computed. This difference represents the precision loss due to conversion. Root Mean Square Error (RMSE) metric is then used to summarize the loss of precision across a dataset of \( N \) floats as:

\[
RMSE = \sqrt{\frac{1}{N} \sum_{k} (f^e_k - f^c_k)^2}
\]

We then compare the errors of BF16/FP16 and EFn by dividing \( RMSE_{BF16/FP16} \) by \( RMSE_{EFn} \) in Table 3. Ratios greater than 1.0 indicate that the EFloat error is less than BF16 or FP16 errors. For EF16, across all models, we observe an average RMSE error ratio of 24.1 for BF16, and 1.5 for FP16. Note that for these experimental results, the datasets were encoded with a minimum of 3-bit and a maximum of 6-bit coded-exponents resulting in an average width in the range of 3.4 to 4.2-bits (Table 2). Accordingly, for EF16, the minimum significant width is 10-bit which is 3-bit wider than BF16, and of the same length as FP16. Therefore, EF16 has significantly higher precision against BF16 than FP16. Also, Table 3 shows that EF12 has the same to slightly better RMSE than BF16 since the RMSE ratios are in the 1.0 to 2.2 range. Thus, EF12 uses 25% less bandwidth and memory capacity than BF16 for similar floating-point precision.

Table 4 presents the relative RMS error ratios of BF16, FP16, and EF16 to evaluate the impact of two different rounding strategies: deterministic (DETR) and stochastic (STOC) roundings. In the DETR approach, if the leading bit of the truncated part is 1 the significant is incremented by 1 (using integer arithmetic) provided it doesn’t overflow in to the exponent field. If the leading bit is 0, the significand is not incremented. In STOC, we adopt the stochastic rounding approach [27, 33, 48, 69] as follows: given \( n \) bits to be chopped from the significand, we increment the significand by 1 with probability of \( \text{value}(n)/(2^{n+1}) \) and do not increment with probability \( 1-\text{value}(n)/(2^{n+1}) \). As Table 4 shows, for all datasets, the we observed higher RMS error ratios (i.e., better accuracy) when deterministic rounding was used. Therefore, we use deterministic rounding as the default rounding approach.

5.2 Evaluation of Result Quality

Note that the RMSE method amplifies larger errors due to the squaring of differences. EFloat coded floating point values with short significands (i.e., those with infrequent exponents) are disproportionately represented in the RMSE summation. However, the true measure of error for vector embedding models will be the evaluation of ranked results for similarity queries for different floating point formats. Unlike the binning in traditional classification inference tasks, ranked results from similarity queries are far more sensitive to numerical precision. We use the Normalized Discounted Cumulative Gain (NDCG) metric [Järvelin and Kekäläinen [37], Wang et al. [66]), to evaluate the quality of ranked results for different floating point formats. NDCG is widely used in information retrieval and web search to evaluate the relevance of retrieved documents. NDCG is a normalization of the Discounted Cumulative Gain (DCG) measure. DCG is calculated as a weighted sum of the degree of relevancy of the ranked items, where the weight is a decreasing function of the position of an item. NDCG is computed by normalizing DCG by IDCG, which is the DCG measure for a perceived ideal ranking result. Thus, the NDCG measure always lies within \([0.0,1.0]\). The NDCG metric provides us a common evaluation format across multiple vector embedding models, irrespective of their target use cases.

For a given vector embedding model, we choose \( q = 20 \) randomly selected distinct query points. For each query point, we compute similar and dissimilar points by computing cosine similarities over the corresponding vectors. For similarity queries, the result contains a list of points sorted in increasing order of their similarity scores (most similar pair of items will have score closer to 1.0), and for dissimilarity queries, the result list is sorted in increasing order of their similarity scores (most dissimilar pair of items will have score closer to -1.0). For each query point, we run similarity and dissimilarity queries for different floating point formats, and use the top \( k = 10 \) results for each test to compute the NDCG score, \((\text{NDCG}@10)\). In our evaluation, we use the ranked results for FP32 as the baseline for calculating the IDCG. For each model, we report the average NDCG@10
Table 2: EFloat characteristics from EF16 to EF8 for different datasets

| Model     | Code Bytes | Table Size | Unique exponents | EFn exponent bits | EFn significand bits (Avg.) | EFn exponent bits (Max) | EFn significand bits (Avg) | (Avg.) | (Max) |
|-----------|------------|------------|-------------------|-------------------|----------------------------|-------------------------|---------------------------|--------|-------|
| churn     | 80         | 23         | 3                 | 5                 | 3.6                       | 11.4                    | 3.4                       |        |       |
| crawl     | 71         | 30         | 3                 | 5                 | 3.4                       | 11.6                    | 3.6                       |        |       |
| enwiki    | 92         | 27         | 4                 | 5                 | 4.2                       | 10.8                    | 4.8                       |        |       |
| MDM       | 83         | 24         | 3                 | 6                 | 3.6                       | 11.4                    | 3.4                       |        |       |
| 840B      | 116        | 35         | 3                 | 6                 | 3.5                       | 11.5                    | 3.5                       |        |       |
| wiki-sw   | 77         | 22         | 3                 | 5                 | 3.6                       | 10.5                    | 3.4                       |        |       |
| virginia  | 83         | 24         | 3                 | 5                 | 3.7                       | 11.3                    | 3.4                       |        |       |

Table 3: BFloat16 (BF16), IEEE Half (FP16), and EF16–8 precision comparisons using FP32 RMSE ratio. Higher is better.

| Base | DETR | STOC | Base | DETR | STOC | Base | DETR | STOC | Base | DETR | STOC | Base | DETR | STOC |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| churn | 22.5 | 14.1 | churn | 11.3 | 7.6 | churn | 5.6 | 0.4 | churn | 2.8 | 0.2 | churn | 1.4 | 0.009 | 0.7 | 0.04 | 0.3 | 0.02 | 0.2 | 0.01 | 0.08 | 0.005 |
| crawl | 34.6 | 22.2 | crawl | 17.3 | 11.1 | crawl | 8.6 | 0.5 | crawl | 4.3 | 0.3 | crawl | 2.2 | 0.1 | 1.1 | 0.07 | 0.5 | 0.02 | 0.3 | 0.02 | 0.1 | 0.008 | 0.06 | 0.004 |
| enwiki | 16.9 | 10.0 | enwiki | 8.5 | 0.5 | enwiki | 4.2 | 0.3 | enwiki | 2.1 | 0.1 | enwiki | 1.0 | 0.07 | 0.5 | 0.03 | 0.3 | 0.02 | 0.1 | 0.008 | 0.06 | 0.004 |
| MDM | 27.9 | 18.9 | MDM | 13.9 | 9.0 | MDM | 6.9 | 0.4 | MDM | 3.5 | 0.2 | MDM | 1.8 | 0.1 | 0.9 | 0.05 | 0.4 | 0.03 | 0.3 | 0.01 | 0.1 | 0.007 |
| 840B | 25.0 | 16.1 | 840B | 12.5 | 8.6 | 840B | 6.3 | 0.4 | 840B | 3.1 | 0.2 | 840B | 1.6 | 0.09 | 0.8 | 0.05 | 0.4 | 0.1 | 0.02 | 0.01 | 0.09 | 0.006 |
| wiki | 22.0 | 14.1 | wiki | 11.0 | 7.6 | wiki | 5.5 | 0.3 | wiki | 2.7 | 0.2 | wiki | 1.2 | 0.08 | 0.6 | 0.04 | 0.3 | 0.02 | 0.2 | 0.01 | 0.08 | 0.005 |
| virginia | 19.6 | 12.1 | virginia | 7.8 | 0.6 | virginia | 4.9 | 0.3 | virginia | 2.4 | 0.2 | virginia | 1.2 | 0.08 | 0.6 | 0.04 | 0.3 | 0.02 | 0.2 | 0.09 | 0.07 | 0.004 |

Table 4: Impact of deterministic(DETR) and stochastic(STOC) rounding on BF16, FP16, and EF16, using the RMSE-with-FP32 ratio. Higher is better.

score computed over 20 query points using BF16, FP16, and various EFn from EF16 to EF8.

Figure 11 presents NDCG10 results for similarity queries, and Figure 12 presents NDCG10 results for dissimilarity queries. For both similarity and dissimilarity queries, EF16 matches or exceeds the quality of BF16 or FP16 (in particular, among the three formats, BF16 provides the worst quality results). Furthermore, EF14 and EF12 provide similar quality results as EF16 in many instances. The two lower-precision EFn, EF10 and EF8, consistently generate the least quality results.

In summary, results from the two sets of experiments (Table 3, and Figures 11 and 12), conclusively demonstrate that: (1) Given a bit budget, EFloat has higher accuracy than other formats, (2) In many scenarios, EFn with reduced bit budget (e.g., EF14 or EF12) provides results of quality comparable to higher precision formats, e.g., BF16, and FP16. These results validate the design of the EFloat format, and demonstrate that EFloats can be used for compressing and computing using vector embedding models.

6 RELATED WORK

In this section, we overview relevant work in lossless compression techniques, deep learning model compression, and floating point representations for deep learning.

Entropy coding is a statistical method for lossless compression [57]. Fixed-size items are replaced with variable-size codes with the shorter codes assigned to the frequently occurring items in the data. Huffman coding, Arithmetic coding and Range coding are commonly used entropy methods. Variable-size codes, for example the Huffman codes, generally have the prefix-property permitting their concatenation without any separating markers in between.
Figure 11: Evaluation of similarity query accuracy using NDCG score across different floating point formats. Higher score (closer to 1.0) is better.

Figure 12: Evaluation of dissimilarity query accuracy using NDCG score across different floating point formats. Higher score (closer to 1.0) is better.

Dictionary methods for lossless compression, most popularly the Lempel-Ziv (LZ) algorithms, use a dictionary of strings [57]. Strings in the data stream, when found in the dictionary, are replaced with distance and length pairs pointing to their dictionary location therefore achieving compression. A dynamic dictionary is typically the most recent set of input strings, e.g., the recent 32KB of input in the popular Deflate method [3].

Over the years, the size and complexity of deep learning models has increased substantially. In particular, advent of new transformer-based NLP models (e.g., BERT and friends, T5, Megatron-LM, Open AI GPT-2/3) has highlighted the very high space and computational costs associated with these models [6, 22, 25, 26, 45, 53, 54, 59]. Given potential uses of NLP models in enterprise and consumer domains, a lot of attention is being devoted to compressing such models. The primary goal of these compression efforts is to reduce the size of a pre-trained model to enable its deployment in real world industrial applications that demand low memory footprint, low response times, and smaller computational and power budget during the inference phase. Gupta and Agarwal [26] have identified six different types of compression techniques currently being used for the NLP models: pruning, quantization, parameter sharing, knowledge distillation, tensor decomposition, and Linear Transformer based methods. The pruning approach is the most obvious way to
reduce model size by sparsifying weight matrices. Pruning is related to quantization which aims to reduce the number of bits to represent each weight. Quantization covers two broad approaches: the first represents a full-precision (e.g., 32-bit) floating point value using reduced or mixed precision representations, and the second converts full-precision floating point values into integer values with fewer bits (e.g., INT8, INT4, and INT1 [36, 47, 62, 67]). Another way to reduce model size is parameter sharing that uses fewer shared values to represent similar weights. Knowledge distillation aims to build a student-trainer model where the student is trained to mimic a pre-trained larger teacher model. The deeper teacher model is trained first, and then the student model is trained via knowledge transfer. Tensor factorization covers a set of techniques that can be used to approximate a larger matrix using a combination of smaller matrices computed via tensor decomposition methods. The final technique aims to develop transformer-based models that are linear in terms of input sequence size, rather than the current quadratic complexity. In general, these techniques follow a destructive approach that throws out the original large model and can not recreate it from the compressed smaller model.

Compression techniques have been explored to reduce data communication costs during deep learning training. Floating-point quantization approaches are often used to reduce the communication volume during the deep learning training process [68]. Gajjala et al [21] use Huffman encoding based techniques to encode quantized gradients for optimizing communication volume in distributed deep learning training. Recently announced Nvidia nvcomp [56] uses LZ4 and run-length encoding (RLE) based approaches to compress data being communicated between GPUs during training of deep learning models. While the approaches used in nvcomp work well for string and integer datatypes, these techniques do not support floating point values very well. Also, LZ4 and RLE approaches work well only for repeated values or sequences. The EFloat approach is compatible and orthogonal to these existing compression techniques employed in NLP models.

In conjunction with the model compression work, there has been significant activity in devising reduced-precision floating point formats tuned for broader machine learning and HPC applications [2, 58]. Unlike the inference-focused model compression work, reduced-precision floating point representations are designed to work for both model training and inference phases. The most common reduced-precision floating point representation uses 16 bits. Current 16-bit implementations include IEEE 754 half-precision (FP16), with 1 sign bit, 5 exponent bits, and 10 fraction bits; Brain Floating Point (BFLOAT16) [40, 65], with 1 sign bit, 8 exponent bits, and 7 fraction bits; and Deep Learning Float (DLFloat) [4], with 1 sign bit, 6 exponent bits, and 7 fraction bits. TensorFloat-32 (TF32) from Nvidia is a 19-bit format that combines 8 exponent bits from BFLOAT16 and 10 exponent bits from IEEE FP16. Hybrid Block Floating Point (HBFP) [19], Intel Nervana’s Flexpoint [41], and Microsoft MSFP [55] formats combine the advantages of fixed point and floating point representations by splitting up the mantissa and the exponent part which is shared across multiple numeric values. Recent research proposals have described training of key deep learning models using even reduced precision floating point values (8- and 4-bit representations) [15, 46, 60, 64]. Recently proposed AdaptiveFloat [61] and AdaptivePosit [42] are inference-targeted floating-point formats that maximize their dynamic range per network layer by dynamically shifting its exponent range via modifications to the exponent bias and by optimally clipping (quantizing) its representable datapoints. Our proposed EFloat design practically achieves the same result without altering the exponent range and quantizing full-precision values, and it does not need to change per neural network layer.

7 CONCLUSION

We introduced EFloat, a novel entropy-coded variable length floating point format for deep learning applications. This format can be used for compressing a trained deep learning model, as well as for enabling more accurate model representations using reduced-precision floating point formats. While our intended use cases were initially for the database embedding (db2Vec) workloads, we demonstrate that the proposed format works effectively for other vector embedding models, and can be used for a much broader class of NLP models including transformer-based models. Broadly, EFloat may be used in deep learning applications where tradeoffs need to be made between range, precision, memory capacity and bandwidth savings. As a future work, we plan to explore the Benford distribution pattern (Benford [8], Newcomb [50]) exhibited by significands of vector embedding models (Section 4) and investigate its application in rounding EFloat values. A follow-up study on 8-bit floats and integers is being considered as well.

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