ADDRESSING TRANSVERSITY WITH INTERFERENCE FRAGMENTATION FUNCTIONS

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The class of interference fragmentation functions, arising from interference among different hadron production channels, is reconsidered. Their symmetry properties with respect to chiral transformations allow building spin asymmetries where the quark transversity distribution can be factorized out at leading twist. For the case of two leading spinless hadrons inside the same current jet, the pair system is expanded in relative partial waves. The cross section is represented on the helicity basis of the target and the fragmenting quark, as well as on the relative orbital angular momentum of the pair. From the decay matrix being positive semi-definite, new bounds on the interference fragmentation functions can be derived. The expansion in partial waves allows to naturally incorporate in a unified formalism specific cases already studied in the literature, such as the fragmentation functions arising from the interference of two mesons in relative $s$ and $p$ waves, as well as the fragmentation of a spin-1 hadron.

1. Introduction

The interest in two-hadron fragmentation functions is justified by the search for a leading-twist mechanism capable to single out the chiral-odd transversity distribution in an alternative and technically simpler way than the Collins effect $^1$. In fact, for semi-inclusive process where two leading spinless hadrons are produced inside the same current jet, the analysis of the leading-twist quark-quark correlator reveals a rich structure $^2$. Four fragmentation functions appear, related to various polarization states of the fragmenting quark. After integrating over the transverse momentum of the fragmenting quark, one of them survives, which is naive time-reversal odd (being related to the interference of different channels leading to the same final hadronic state; hence the naming convention of interference fragmentation function, IFF) and chiral odd; a single-spin asymmetry (SSA) can then be built to extract transversity at leading twist $^3$. In the SSA the criti-
cal parameter is just the azimuthal angle between the two-hadron plane and the laboratory plane; the insensitivity to the quark transverse momentum preserves collinear factorization, thus avoiding the introduction of Sudakov form factors that take into account the resummation of the soft-gluon radiation and possibly dilute the SSA as in the case of the Collins effect.

From numerical simulations, it turns out that the SSA are measurable, at least for the semi-inclusive deep-inelastic scattering (SIDIS).

In this work we reanalyze the general quark-quark correlator for the fragmentation in two spinless hadrons in the helicity basis of the fragmenting quark and in the basis of the relative orbital angular momentum of the hadron pair. The advantage is twofold. First, a certain number of useful bounds on IFF can be deduced from the fact that the helicity matrices are positive semi-definite. Second, a general unifying formalism is deduced that naturally incorporates the specific case describing the interference between relative s and p waves of the hadron pair, as well as the case of spin-1 hadron fragmentation in the subsector of the two-hadron relative p wave.

![Figure 1. The usual quark handbag diagram contributing at leading twist to the semi-inclusive DIS in two leading hadrons, with total momentum $P_h = P_1 + P_2$ and relative momentum $R = (P_1 - P_2)/2$. There is a similar diagram for anti-quarks.](image)

2. The quark-quark correlator in the helicity basis

The semi-inclusive production of two spinless hadrons is depicted in Fig. 1. Inside a target nucleon with mass $M$, polarization $S$ and momentum $P$ ($P^2 = M^2, S^2 = -1, P \cdot S = 0$), a virtual hard photon with momentum $q$ hits a quark with momentum $p$ promoting it to the state with momentum $k = p + q$. The quark then fragments into a residual jet and two leading
unpolarized hadrons with masses $M_1, M_2$, and momenta $P_1$ and $P_2$ ($P_1^2 = M_1^2, P_2^2 = M_2^2$), with $P_h = P_1 + P_2$ ($P_h^2 = M_h^2$) and $R = (P_1 - P_2)/2$.

We further define the light-cone fractions $x = p^+/P^+, \ z = P_h^-/k^-$, and $\zeta = 2R^-/P_h^-$, which describes how the total momentum of the hadron pair is split into the two single hadrons ($-1 \leq \zeta \leq 1$ and $\zeta = 2\xi - 1$, with $\xi$ defined in Ref. 2). On-shell conditions and the positivity requirement $R_T^2 \geq 0$ constrain the light-cone components of all the relevant 4-momenta.

Following the factorization hypothesis, the leading-twist cross section in the helicity basis (including polarizations of beam and target) can be written as

$$
\frac{d^7 \sigma}{d\zeta \, dM_h^2 \, d\phi_R \, dz \, dy \, d\phi_S} = \sum_a \rho_{\Lambda\Lambda'}(S) \frac{[\Phi_a(x)]_{\Lambda\Lambda'}}{[\Lambda\Lambda]} \left( \frac{d\sigma_{eq}}{dy} \right)_{\chi_1\chi_2'} \, [\Delta_a(z, \zeta, M_h^2, \phi_R)]_{\chi_2'\chi_2},
$$

where $\phi_R, \phi_S$ are the azimuthal angles of the transverse components of $R$ ($R_T$) and of $S$ ($S_T$) with respect to the lepton scattering plane, respectively.

In Eq. (1), $\rho_{\Lambda\Lambda'}(S)$ is the target helicity density matrix (with $\Lambda, \Lambda'$ the helicities of the nucleon legs in Fig. 1). The q-q correlator $\Phi_a(x)$ describes the lower blob in Fig. 1 and contains the well known momentum $f_1^a$, helicity $g_1^a$ and transversity $h_1^a$ distributions for the flavor $a$. The indices $(\chi_1, \chi_2')$ identify the parton helicities for the emerging quark legs in Fig. 1. The matrix satisfies general requirements, in particular it is positive semidefinite, from which the well known Soffer bound is obtained.

Then, $(d\sigma_{eq}/dy)_{\chi_1\chi_2', \chi_2'\chi_2}$ represents the standard elementary electron-quark cross section for a flavor $a$ with $y = E - E'/E$ the beam energy fraction delivered to the target. Finally, the q-q correlator $\Delta_a(z, \zeta, M_h^2, \phi_R)$ refers to the upper blob in Fig. 1 and is described as

$$
[\Delta_a(z, \zeta, M_h^2, \phi_R)]_{\chi_2'\chi_2} = \frac{1}{2} \left( \begin{array}{c} D_1^a \ D_1^a \ i e^{i\phi_R} \frac{[R_T]}{M_h} H_1^{q_1a} \\ -i e^{-i\phi_R} \frac{[R_T]}{M_h} H_1^{q_2a} \ D_1^a \end{array} \right). \quad (2)
$$

The function $D_1^a(z, \zeta, M_h^2)$ represents at leading twist the probability that two spineless hadrons are produced with $M_h$ invariant mass and carrying a $z$ fraction of the parent fragmenting quark with flavor $a$, sharing it in $(1 + \zeta)/2$ and $(1 - \zeta)/2$ parts inside the pair. The same interpretation can be ascribed to $H_1^{q_1a}$ but for transversely polarized quarks. The following bounds derive:

$$
D_1^a(z, \zeta, M_h^2) \geq 0 \quad D_1^a(z, \zeta, M_h^2) \geq \frac{[R_T]}{M_h} |H_1^{q_1a}(z, \zeta, M_h^2)|. \quad (3)
$$
The function $H_{1}^{4a}$ is the only naive $T$-odd IFF surviving the integration upon $\vec{k}_{T}$; it is also chiral odd and can be used to isolate the transversity $h_{1}^{T}$ by considering the SSA for unpolarized lepton beam and transversely polarized target $^{3}$.

![Figure 2. The hadron pair in the cm frame. The light-cone direction $P_{h}^{-}$ identifies the current jet, while $\theta$ is the cm polar angle of the pair.]

**3. Partial-wave expansion**

If the invariant mass $M_{h}$ of the two hadrons is not very large, the pair can be assumed to be produced mainly in the relative $s$- or $p$-wave channels in the cm frame, which is defined by the back-to-back emission of the two hadrons. The direction identified by this emission forms an angle $\theta$ with the dominating light-cone direction $P_{h}^{-}$ (see Fig. 2). The crucial remark is that in this frame $\zeta = a + b \cos \theta$, with $a, b$, functions only of the invariant mass. This suggests to expand the IFF in Eq. (2) onto the basis of Legendre polynomials in $\cos \theta$, namely

$$D_{1}(z, \zeta(\cos \theta), M_{h}^{2}) = \sum_{n} D_{n}(z, M_{h}^{2}) P_{n}(\cos \theta)$$

$$H_{1}^{4}(z, \zeta(\cos \theta), M_{h}^{2}) = \sum_{n} H_{n}^{4}(z, M_{h}^{2}) P_{n}(\cos \theta). \quad (4)$$

Here, the expansion includes the first three terms only ($n \leq 2$), which are the minimal set required to describe all the “polarization” states of the system in the cm frame for relative partial waves $L = 0, 1$; they will be conveniently indicated by the pair of indices $(i, j)$, with $i, j = O, L, T$ for unpolarized $s$-wave, longitudinal and transversely polarized $p$-wave amplitudes.

After inserting the expansion (4) in the correlator (2), it is also useful to project out of the obtained matrix the information about the orbital angular momentum of the system encoded in the angular distribution. In fact, for $L \leq 1$ the decay matrix for the spinless hadron pair is given by specific bilinear combinations of spherical harmonics as $D_{LM}^{L'}(\theta, \phi_{R}) = 4\pi Y_{LM} Y_{L'M'}^{*}$, from which the q-q correlator (2) can be decomposed as

$$[\Delta(z, \zeta, M_{h}^{2}, \phi_{R})]_{\chi_{2}\chi_{2}} = [\Delta(z, M_{h}^{2})]_{LM}^{L'} D_{LM}^{L'}(\theta, \phi_{R}). \quad (5)$$
where \( \Delta(z, M_2^2)_{MM', \chi_2^2}^{LL'} \) contains the various \( D_{ij} \) and \( H_{ij}^{<} \) components and, being positive semi-definite, gives their corresponding bounds.

Using Eq. (5) inside Eq. (1), we can take advantage of the full power of the analysis in the helicity formalism. It is particularly interesting to consider the case for an unpolarized beam and a transversely polarized target, i.e.

\[
d^7\sigma_{OT} \propto \sin(\phi_R + \phi_S) h_1^a(x) \sin \theta \left[ H_{OT}^3 + \cos \theta H_{LT}^3 \right],
\]

because we can see that the transversity \( h_1 \) can be matched by two different chiral-odd, naive \( T \)-odd IFF: one pertaining the interference between \( s \)- and \( p \)-wave states of the hadron pair, \( H_{OT}^3 \) (corresponding to the hypothesis first formulated in Ref. \(^5\)), the other pertaining the interference of the system in the \( p \) wave only, \( H_{LT}^3 \) (naturally linked to the analysis developed in the case of a spin-1 hadron fragmentation \(^6\)). As for the former, it is enough to take \( \phi_S = 0 \) in Eq. (6), to integrate the \( \theta \) dependence away in one emisphere and to realize that the azimuthal angle defined in Ref. \(^5\) is \( \phi = \frac{1}{2} \pi - \phi_R \).

However, as it is evident from Eq. (6) itself, the most general approach leads to an unfactorized \( (z, M_2^2) \) dependence of the fragmentation function and to a completely different behaviour of the single-spin asymmetry with respect to Ref. \(^5\) (for a quantitative comparison, see Ref. \(^3\)).

Finally, it is also possible to generalize the previous analysis to the case of unintegrated transverse momenta. The formulae, however, are more involved and will be shown elsewhere \(^7\).

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