Prediction of presence and severity of damages using experimental Mode Shape

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Abstract. Global dynamic technique based on frequencies change is used to detect the damages/ cracks in structures. This technique is based on changes in natural frequencies. However, it is found that changes in frequencies are negligible even in major damage cases. Several researchers have used modes shape instead of frequencies to detect the damage/ crack in structure. It is true that mode shapes approach is better compared to frequencies based analysis for damage detection; however, it fetches excellent result, if real mode shape is used. Researchers are using theoretical/ numerical/ interpolated modes shape, which produce erroneous results. In this paper, a method is proposed to extract the real mode shape of structure experimentally using only one accelerometer and damage was detected successfully in the steel beam using this mode shape. An artificial damages were created in the steel beam in four stages and the mode shapes were determined after damage for each stage. The damage was also located and severity was predicted with the help of the extracted modes shape. It was found that location predicted by proposed method was almost correct. This technique is useful for sensitive structures.

1. Introduction
Civil structures are backbone of the economy of any nation. It has been emphasized that failure of civil-infrastructure to perform well could have an adverse effect on the gross domestic product [1]. It is most important to maintain these structures after completing construction because the overall performance depend upon the initial strength as well as post construction deterioration. It is not possible to ensure satisfactory performance the structure by only providing greater initial strength. Even lower strength structure performs satisfactorily if proper maintenance/ monitoring are insured. It concludes that monitoring of any structure is a must. During the last two decades, innumerable research attempts have been made to develop techniques and methodologies to automatically monitor the health of structures. Global dynamic techniques have gained significant popularity during the last one and half decade. These are based on two parameters, the natural frequencies and the mode shape of the structure. Lower frequencies of structure can be determined easily but it is generally observed that at around 60 to 75 % of their ultimate failure load, the measured first frequency hardly shift by 1-2% only [2]. Pandey and Biswas [3] also reported that a 50% reduction in the Young’s modulus of elasticity, over the central 3% length of a 2.44m long beam, only resulted in about 3% reduction in the observed first natural frequency. In addition, most low frequency vibration based structural health monitoring (SHM)/ non destructive evaluation (NDE) methods on real-life structures are likely to encounter the presence of noise. The noise could be (a) mechanical noise, caused by sources such as...
vehicle movement or wind; (b) electrical noise, generated by variations in the power supply; or (c) electromagnetic noise, caused by communication waves, which affect the signal acquisition and transmission through cables and other susceptible circuitry \[4\]. The contamination by electrical interference and mechanical ambient noise degrades the quality of the emission signals \[5, 6\].

This paper describes the new approach of health monitoring using single accelerometer on the steel beam.

2. Sensor system: Accelerometer
An accelerometer essentially consists of a seismic mass connected to a base (which is attached to the host structure) by means of a spring and a viscous damper. If \(x\) denote the acceleration of the base, \(z\) the relative displacement between the host structure and the seismic mass, the acceleration of the host structure is given by

\[
\ddot{x} \approx -z \omega_n^2
\]

Thus, the acceleration of the host system can be determined by measuring the relative displacement, \(z\), between the seismic mass and the base. Equation (1) is valid for small frequencies, typically lower than the natural frequency \(\omega_n\) of the spring-damper system. In order to ensure the necessary condition \(\omega << \omega_n\), the system should have very stiff spring and a small seismic mass so that \(\omega_n\) is maximum possible. Most commercial accelerometers employ piezoelectric transducers as displacement sensors, due to their small mass, high stiffness and low damping. Generally, the accelerometers produce a voltage across their terminals proportional to the measured acceleration.

3. Global techniques
In these techniques, the test-structure is subjected to low-frequency excitations, either harmonic or impulse, and the resulting vibration responses such as displacements, velocities or accelerations are picked up at specified locations along the structure. The vibration picked-up data is processed to extract the first few mode shapes and the corresponding natural frequencies of the structure, which, when compared with the corresponding data of healthy state, yields information pertaining to the locations and severity of the damages. Damage in a structure alters its modal parameters such as modal frequencies, modal damping, and mode shape associated with each model frequency. Changes also occur in structural parameters, namely stiffness and damping matrices.

4. Experimental procedure and results
In this paper, an experimental approach is outlined to obtain the mode shapes directly using the accelerometer. For a linear structure, by Betti’s theorem, the response at point ‘a’ due to a unit load acting at point ‘b’ equals the response at point ‘b’ due to a unit load acting at point ‘a’[7]. This theorem facilitates measuring responses along the entire structure using single accelerometer. By varying the point of application of a fixed load, the response of the sensor (located at a fixed point) is equivalent to the response at the point of application of the force. Plotting the ordinates of the FFT curve corresponding to a particular frequency for all measurement points represents the mode shape of the structure corresponding to that frequency.

Figure 1 shows the experimental setup used for demonstration of the proposed approach. The test structure was a 4m long simply supported steel beam (ISBM 150 as per Indian Standards), instrumented with the accelerometer at a distance of 130.5 cm from the right support. The beam was divided into twelve equal parts (i.e. thirteen equally spaced nodes), at each of which a standard excitation was made by dropping a steel ball of 0.2kg from a fixed height of 1m as shown in the figure.
1. For each excitation, the voltage response of the accelerometer was recorded for a period of two seconds using the data acquisition systems at a sampling interval of 200 µs, automatically through programs running in LAB View [8].

The recorded time-domain data was transformed into frequency domain. Figure 2 shows the typical FFT response of the accelerometer. From this plot, the first three experimental bending frequencies of the steel beam can be identified as 30 Hz, 128Hz and 228Hz. Applying this procedure, frequency plots were determined by making impact at each of the 13 nodes.

![Figure 1. Experimental Set up.](image)

![Figure 2. Frequency responses of 4m beam.](image)
(a) Mode 1.

(b) Mode 2.
The ordinates of the FFT curves were noted corresponding to the first three bending frequencies for all the nodes, through which the first three bending mode shapes were plotted, as shown in figure 3. It should be noted that this measurement approach, the number of mode shapes extracted will depend upon the minimum sampling interval of measurement. Also, the accuracy of a particular mode shape depends into how many nodes the structure is divided into, the higher the number of measurement points, higher the accuracy.

5. Localization of damages

However, in several structures, incipient level damage does not pose a serious risk. In such scenarios, it may be more meaningful to closely monitor damage growth and wait till the damage grows to moderate or severe nature. This section explains how moderate to severe damage types can be quantitatively monitored using the global dynamic techniques. In addition, the global dynamic techniques also provide a back up to capture any damages that miss out detection at the incipient stage due to any unavoidable reasons.

The experimental mode shapes of the structure obtained using the accelerometer as explained in previous sections can be directly utilized. If an element undergoes damage, its flexural stiffness, namely $EI$ (the product of Young’s modulus and the moment of inertia) reduces, thereby increasing its curvature. In case of beam or one dimensional structures, the damage index for each element can be defined as [9]

$$D.I. = \sum_{n=1}^{N} (C_{\text{damaged}} - C_{\text{undamaged}})$$

\[ (2) \]
where $C_{undamaged}$ is the curvature before damage, $C_{damaged}$ is the curvature after damage and $N$ is the number of the modes considered. In this study, the curvature of an element was determined as the average of the curvature at the two nodes of the element.

After determination of experimental mode shapes of the steel beam at healthy state, the artificial damages were induced in four stages. In the first stage damage, a hole of 0.6 cm diameter was drilled in the flange at a distance of 1.5 cm from the left corner. In the second stage, a cut of 6.5 cm length was made through the entire thickness of the flange starting from the left corner. In third damage stage, the severity of the previous damage was increased by increasing the depth of the cut to one forth depth of the web. In the fourth stage, the depth of the cut was increased till the middle of the web. The experimental mode shapes were determined after each stage of damage. Compared to the undamaged structure, the mode shapes appear to change significantly. In fact, a closer inspection of the first mode clearly points maximum change occurring in the vicinity of the damage location. It may be noted here that the modes shapes were determined directly using a single accelerometer. Next, the damage index was determined using equation (2) for all the damage stages. A plot of the damage index versus the element number is shown in figure 4.

It can be observed from figure that at the all stages of damage, the damage index was highest at the damaged element as compared to other elements. The index increased numerically as the damage severity increased. Thus, the damage location was correctly identified. It should also be noted that the damage index of the adjacent elements is comparable but smaller than the damaged element. Hence, at the location of the damage, it appears like an umbrella shaped structure.

![Figure 4. Damage index of steel Beam (damage stage 1-4).](image)

For a single damage scenario, greatest value of the damage index will indicate the location of damage. Since it represents value for the whole element, if greater accuracy is desired, the distance between the nodes can be further subdivided into suitable number of elements, which can be treated as new nodes and the process be continued till damage is located with desired accuracy. There will be no requirement to determine the damage index for other elements which had lesser damage index in the
previous cycle of measurements. With the use of Betti’s theorem, this can be performed without needing extra sensors.

Typically, the damage location can only be predicted with a resolution equal to the distance between the measurement points [10]. Hence, the greater the measurement points in the structure, the higher the accuracy of damage localization.

6. Severity of damage

Denoting the amplitude of the motion by the generalized coordinate \( Z(t) \), the displacement at any point in a structure can be expressed as [11]

\[
y(x, t) = \phi(x)Z(t)
\]

(3)

where \( \phi(x) \) is the mode shape function. Thus, the harmonic variation of the generalized coordinate at a given time in free vibration can be expressed as

\[
y(x, t) = \phi(x)Z_0 \sin \omega t
\]

(4)

This equation expresses the assumption that the shape of the vibrating beam does not change with time, only the amplitude of motion varies, as, it varies harmonically in a free-vibration condition. At the point of maximum displacement,

\[
y(x, t) = \phi(x)Z_0
\]

(5)

The strain energy of this flexural system is given by

\[
U = \frac{1}{2} \int_0^L EI(x) \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx
\]

(6)

where \( L \) is the length of the beam, \( E \) the Young’s modulus of the material of the structure and \( I \) the moment of inertia. Thus, substituting the mode shape function and letting the displacement amplitude take its maximum value leads to

\[
U_{\text{max}} = \frac{1}{2} Z_0^2 \int_0^L EI(x)(\phi''(x))^2 dx
\]

(7)

where \( \phi''(x) \) denotes the second derivative of a particular mode shape that is the curvature mode shape.

In general, the total energy of the system is distributed among various mode shapes. If the structure is damaged, it is assumed that there is no significant change in the distribution of the energy in various modes if the excitation force and the boundary conditions remain unchanged. This is true in the context of the experiments reported in this paper, due to the fact that the height of the free fall of the mass used for excitation during the test was constant. If there occurs any damage/crack in structure in any element, the value of \( EI \) will reduce more at that element compared to other parts. This change
reflects the severity of damage. In the present approach, only average value of ‘EI’ of whole structure has been determined. If any element of structure is damaged, the tendency of that element is to undergo greater displacement compared to the undamaged case. However, due to the inertia and the adhesive forces, the neighbouring elements, which are less damaged, oppose it and undergo somewhat greater displacement to balance the equilibrium. Hence, damage affects the entire mode shape of the structure. As the severity of the damage increases, the strain at the damaged section increases further. By conversation of energy for any mode,

\[
\left( U_{\text{max}} \right)_{\text{undamaged}} = \left( U_{\text{max}} \right)_{\text{damaged}}
\]  

(8)

Using (8) and solving, we can derive

\[
\frac{(EI(x))_{\text{damaged}}}{(EI(x))_{\text{undamaged}}} = \eta = \frac{\left( Z_0^2 \right)_{\text{undamaged}} \left( \int_0^L \phi''(x)dx \right)_{\text{undamaged}}}{\left( Z_0^2 \right)_{\text{damaged}} \left( \int_0^L \phi''(x)dx \right)_{\text{damaged}}}
\]  

(9)

where \( \eta \) represent the ratio of the current stiffness of structure to the undamaged stiffness. Using the experimental damaged and undamaged mode shapes of steel beam at different damage states, the percentage change of stiffness was determined using the equation (9). Integration of \( [\phi''(x)]^2 \) with respect to \( x \) gives the area of square of the mode shape. Hence, the damage severity can be determined in terms of the original stiffness after drawing the experimental mode shape of structure and the computing area of the square of the mode shape and the maximum relative amplitude \( Z_0 \) of the mode shapes.

The above approach was applied to steel beam described earlier. Severity of the damage was increased in four stages in steel beam, as explained earlier. Severity of damage of steel beam for various states is listed in Table 1.

| S.No. | Damage State | \( \eta \) = ratio of current stiffness with undamaged stiffness |
|-------|--------------|---------------------------------------------------------------|
| 1     | State-1      | 0.93                                                          |
| 2     | State-2      | 0.90                                                          |
| 3     | State-3      | 0.86                                                          |
| 4     | State-4      | 0.80                                                          |

It is observed that as the damage severity increases, \( \eta \) decreases which is clearly visible from the tables. The main advantage is that the severity is determined in term of the ratio of the residual stiffness with the original stiffness, which provides the most direct estimate of the overall damage severity of the structure.
8. Conclusions
This paper has presented the theoretical background and the experimental details of a new approach to monitor structural health. The experimental modes shape of steel beam has been extracted using only one accelerometer. The damage was located and severity was predicted using extracted experimental mode shape. Damage severity is defined in term of the original stiffness of the structure. The approach does not require any numerical model of the structure \textit{a-priori}, approach has considerable potential on the real-life structures.

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