The Spacetime Superalgebras
from M-branes
in M-brane Backgrounds

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Abstract

We derive the spacetime superalgebras explicitly from “test” M-brane actions in M-brane backgrounds to the lowest order in $\theta$ via canonical formalism, and discuss various BPS saturated configurations on the basis of their central charges which depend on the harmonic functions determined by the backgrounds. All the 1/4 supersymmetric intersections of two M-branes obtained previously are deduced from the requirement of the test branes to be so “gauge fixed” in the brane backgrounds as to preserve 1/4 supersymmetry. Furthermore, some of 1/2-supersymmetric bound states of two M-branes are deduced from the behavior of the harmonic functions in the limits of vanishing distances of the two branes. The possibilities of some triple intersections preserving 1/4 supersymmetry are also discussed.

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1 Introduction

The M-theory is currently a most hopeful candidate for a unified theory of particle interactions\cite{1,2} and is extensively studied from various points of view\cite{3,1}. Among them, the analysis via superalgebra is one of the most powerful approaches to investigate its various properties\cite{3,3}. Since there are, of course, two kinds of supersymmetries, two kinds of algebras have been discussed so far: the spacetime superalgebra and the worldvolume ones. The former is initially constructed as the most general modification of the standard D=11 supersymmetry algebra\cite{7,8}, and then deduced explicitly from the M-5-brane action in the flat background via (anti-)canonical commutation relations of the worldvolume fields\cite{9}(see also ref.\cite{10}). It is called M-theory superalgebra, from which it is shown that only five basic 1/2 supersymmetric constituents of M-theory can be permitted. And the latters, defined on the flat (p+1)-dimensional worldvolumes of p-branes, are the maximal extensions of the (p+1)-dimensional supertranslation algebras, from which all the 1/4 supersymmetric M-brane intersections as worldvolume solitons can be deduced\cite{11,12,13}. Both of these analyses are also applied to D-branes\cite{12,14}, although there are some subtleties in the worldvolume cases. In this way these discussions so far have been based only on the flat spacetime.

Now, we will discuss the extension, in particular, of the former to those from branes in \textit{nontrivial} backgrounds.\footnote{The possibility of this extension has been already pointed out in the earlier paper\cite{10} for a different purpose(related to nontrivial topologies), although it is not shown explicitly there.} One of our most important motivation for it is to get the superalgebra of the 10-dimensional massive IIA theory\cite{15,16,17}. It has not been obtained yet since this theory does not admit the flat background because of the existence of cosmological constant\cite{18,19}. This superalgebra is significant not only to investigate the properties of the theory like the above cases, but also to understand the 11-dimensional origin of the massive IIA theory, which is not yet known satisfactorily because of the no-go theorem presented in ref.\cite{20}, although several trials were made\cite{21,22,23}(and recently\cite{42}). Therefore, the extension is urgently necessary.

Let's reconsider the computation in ref.\cite{9}. We might be able to interpret it as follows: the M-5-brane action is originally invariant under \textit{local} super-transformation before taking its background to be any specific geometry. Suppose we float the M-5-brane as a “test” brane in the flat background. Then, the system and hence the action have the supertranslation symmetry in all the directions for the following two reasons: first, the background has the supertranslation symmetry and the test brane is assumed to be so light that it has no effect on the background. Second, the configuration of the test brane including its orientation has not yet been fixed at this time. Therefore, we can define the Noether supercharge, compute its anti-commutator as the superalgebra and discuss the supersymmetric configurations permitted in the flat background on the basis of its central
This interpretation will suggest us to think of the following extension: if we float a test M-brane in a certain non-trivial background which has some portions of supersymmetry, then the system and the action should have the same supersymmetry because of the same reasons stated above. Therefore, in the same way, we can define the corresponding Noether supercharge, compute its commutator as the superalgebra, and finally, we are able to deduce from it the supersymmetric configurations of the test brane (including intersections with the background) permitted in the non-trivial background.

The aim of this paper is to examine this idea explicitly in the cases of a test M-2-brane and a test M-5-brane in an M-2-brane and an M-5-brane background, i.e., four cases in all. The concrete procedures are as follows: we will take the background to be a M-brane solution which actually consists of such a large number of coincident M-5-branes that our “test brane” approximation is justified, and substitute the solution for the test brane action as was done in ref. [25]. Then, we check the invariance of the action under the unbroken supersymmetry transformation, derive the representation of the supercharge in terms of the worldvolume fields of the test brane and their conjugate momenta, compute its algebra and discuss various supersymmetric configurations permitted in the background on the basis of the algebra.

We note that our computations are performed only up to the low orders in $\theta$ which might contribute to the central charges at zeroth order in $\theta$ because they suffice to discuss the supersymmetric configurations which we want to know. The most important fact throughout our computations is that we can reduce the superspace with the supercoordinates $(x, \theta)$ to that with coordinates $(x, \theta^+)$, where the sign $+$ of $\theta^+$ implies that $\theta^+$ has a definite worldvolume chirality of the background. The reason is the following: since half of supersymmetry is already not the symmetry of the system owing to the existence of the background brane, the corresponding parameter $\theta^-$ must not be transformed. Thus, the conjugate momentum of $\theta^-$ does not appear in the supercharge $Q^+$, which means that the terms including $\theta^-$ do not contribute to the central charges at zeroth order in $\theta$. Therefore, we set $\theta^- = 0$ from the beginning.

The consequence is that all the 1/4-supersymmetric intersections of two M-branes obtained previously both in 11D supergravity [25] [26] and via worldvolume superalgebras [27] are deduced from the requirement of the test branes to be so “gauge fixed” in the backgrounds as to preserve 1/4-supersymmetry. In addition, one outstanding characteristic of the results is the dependence of (the r.h.s. of) the superalgebras on the harmonic

\[\text{In the middle of completing this work, this idea is pointed out and proved generically by ref. [24] in some other context (about the new actions presented in it). Our work will be worth doing in order to examine this idea explicitly to low orders in } \theta.\]

\[\text{We apply the method to the case in 10D massive IIA background in the next paper [40], which will be completed soon.}\]
functions determined by the backgrounds. By using this property we derive the following two kinds of bound states composed of two M-branes preserving 1/2 supersymmetry: the first ones are deduced from the configurations of the test M-p-branes parallel to the background M-p-brane with the converse orientation. The second is from the configuration of the test M-2-branes parallel to a two-dimensional subspace of the worldvolume of the background M-5-brane. In both cases all the supersymmetry is broken because there are only attractive forces between the two branes. Then, the “absorption” limits, namely the limits of zero distances are expected to lead to the restorations of 1/2 spacetime supersymmetry because the potential energies are minimized in the limit. It is shown that these are really the cases, which correspond to M-p-brane/M-p-brane bound states and a M-2-brane/M-5-brane bound state[28][9] (preserving 1/2 supersymmetry), respectively. Another merit of the results is that the possibilities of some supersymmetric triple intersections can be discussed directly on the basis of the spacetime superalgebras, although not systematically.

This paper is organized as follows: in section 2 we discuss the superalgebras from the test M-2-brane in an M-2-brane and an M-5-brane background. In section 3 we discuss the superalgebras from the test M-5-brane in an M-2-brane and an M-5-brane background. In section 4 we give short summary and discussions.

Before presenting our results we give some preliminaries. We use “mostly plus” metrics for both worldvolume and spacetime. And we use Majorana (32 × 32) representation for Gamma matrices $\Gamma_{\hat{m}}$ which are all real and satisfy \[ \{ \Gamma_{\hat{m}}, \Gamma_{\hat{n}} \} = 2\eta_{\hat{m}\hat{n}}. \] \( \Gamma_0 \) is antisymmetric and others symmetric. Charge Conjugation is $\mathcal{C} = \Gamma_0$. We use the symbol $\sharp$ to denote the number 10 as used in [9]. We use capital latin letters($M, N, ..$) for superspace indices, small latin letters($m, n, ..$) for spacetime vectors and early small greek letters ($\alpha, \beta, ..$) for spinors. Furthermore, we use late greek letters ($\mu, \nu, ..$) for spacetime vectors parallel to the background branes and early latin letters($a, b, ..$) for spacetime vectors transverse to them. We use hatted letters ($\hat{M}, \hat{m}, \hat{a}, \hat{\alpha} ..$) for all the inertial frame indices and finally middle latin letters($i, j, ..$) for worldvolume vectors.

2 The Superalgebras from the M-2-brane in M-brane Backgrounds

In this section we will deal with the M-2-brane in M-brane backgrounds. (2a) the M-2-brane in an M-2-brane background

At first we will begin with the case of the test M-2-brane floating in an M-2-brane background. The M-2-brane action in a D=11 supergravity background is[29]

\[
S_{M2} = -T \int d\xi^3 \sqrt{-\det g_{ij}} + T \int d\xi^3 \frac{1}{3!} \epsilon^{ijk} C_{ijk}^{(3)}
\] (2.1)
where \( g_{ij} = E_i^m E_j^n \eta_{mn} \) is the induced worldvolume metric and \( C_{ijk}^{(3)} \) is the worldvolume 3-form induced by the superspace 3-form gauge potential. \( E_i^A = \partial_i Z^M \bar{E}_M \hat{A} \) where \( E_M \hat{A} \) is the supervielbein. Note that at this time the action is invariant under local super-transformation.

Let’s fix the background to an M-2-brane solution given by

\[
\begin{align*}
    ds^2 &= H^{-2/3} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/3} dy^a dy^b \delta_{ab} \\
    C_{\mu_1 \mu_2 \mu_3} &= \frac{\varepsilon_{\mu_1 \mu_2 \mu_3}}{\det g_{\mu\nu}} H^{-1}, \quad (\text{the others}) = 0
\end{align*}
\]

(2.2)

where \( \eta_{\mu\nu} \) is the 3-dimensional Minkovski metric with coordinates \( x^\mu \) and \( H \) is a harmonic function on the transverse 8-space with coordinates \( y^a \), that is, \( H = 1 + \frac{q_2}{y^a} \) where \( y = \sqrt{y^a y^b \delta_{ab}} \) and \( q_2 \) is a constant. \( \varepsilon_{\mu_1 \mu_2 \mu_3} = g_{\mu_1 \nu_1} g_{\mu_2 \nu_2} g_{\mu_3 \nu_3} \varepsilon^{\nu_1 \nu_2 \nu_3} \) and \( \varepsilon^{012} = 1 \). This background admits a Killing spinor \( \varepsilon \) which satisfies

\[
    \delta \psi_m = (\partial_m + \frac{1}{4} \omega_m \bar{\Gamma} \Gamma + T_m^{n_1 n_2 n_3 n_4} F_{n_1 n_2 n_3 n_4}) \varepsilon = 0
\]

(2.3)

where \( T_m^{n_1 n_2 n_3 n_4} = -\frac{1}{288} (\Gamma_m^{n_1 n_2 n_3 n_4} + 8 \Gamma^{[n_1 n_2 n_3} \delta^{n_4]}_m) \). Then the Killing spinor has the form \( \varepsilon = H^{-1/6} \varepsilon_0 \) where \( \varepsilon_0 \) has the positive worldvolume chirality, i.e. \( \bar{\Gamma}_0 \varepsilon_0 \equiv \Gamma_0 \varepsilon_0 = +\varepsilon_0 \).

Since \( \bar{\Gamma} \) satisfies \( \bar{\Gamma}^T = \bar{\Gamma} \) and \( \bar{\Gamma}^2 = 1 \), both \( \frac{1 + \bar{\Gamma}}{2} \) and \( \frac{1 - \bar{\Gamma}}{2} \) are projection operators. So, if we denote \( \frac{1 + \bar{\Gamma}}{2} \) as \( \zeta^+ \) for a spinor \( \zeta \), the background is invariant under the transformation generated by the supercharge \( Q^+ \) and so is the system because the negligibly light test brane is assumed not to affect the background geometry and its configuration is not fixed yet at this moment. On the other hand, the background and the brane action are not invariant under the transformation by \( Q^- \), which means that we should set the corresponding transformation parameter \( \varepsilon^- \) to be zero. Then, the conjugate momentum \( \Pi^- \) of \( \theta^- \) does not appear in the Noether charge \( Q^+ \) only whose algebra we are interested in. Therefore, the terms including \( \theta^- \) never contribute to the central charges at zeroth order in \( \theta \). Thus, we can set from the beginning

\[
    \theta^- = 0.
\]

(2.4)

From now on, we will use these freely in all the cases we treat in this paper. Related with this, we exhibit the properties of \( \bar{\Gamma} \):

\[
    [\bar{\Gamma}, \Gamma_\dot{\mu}] = [\bar{\Gamma}, \mathcal{C}] = \{\bar{\Gamma}, \Gamma_\dot{a}\} = 0.
\]

(2.5)

Now, we are prepared to get the explicit representations of the superfields and the super-coordinate transformation in terms of component fields to low orders in \( \theta \). By substituting (2.2) to the usual expressions (31) and using (2.4) and (2.5) we see that only the
$E_\alpha^a$ has the nontrivial contribution from the background. From the results the superspace 1-form on the inertial frame $E_\mathcal{A} = dZ^M E_M^\mathcal{A}$ is given by

\[
E_\mathcal{\bar{\nu}} = dx^\nu H^{-1/3} \delta_\mathcal{\bar{\nu}}^\mathcal{\bar{\mu}} - i \bar{\theta}^+ \Gamma_\mathcal{\bar{\nu}} d\theta^+ + \mathcal{O}(\theta^4)
\]
\[
E_\mathcal{\alpha} = dy^b H^{1/6} \delta_\mathcal{\alpha}^b + \mathcal{O}(\theta^4)
\]
\[
E_{\tilde{\alpha}} = d\theta^{\alpha^+} + \frac{1}{6} H^{-1} dH \theta^{\alpha^+} + \mathcal{O}(\theta^3).
\]

(2.6)

Since the 1-form $E_\mathcal{A}$ has no superspace (curved) indices, $E_\mathcal{\bar{\nu}}$, and hence the Nambu-Goto action, are invariant under the super-coordinate transformation\[31\] $\delta Z^M = \Xi^M$ in this background given by

\[
\Xi^\mu = \bar{\xi}^+ \Gamma_\mu \theta^+ + \mathcal{O}(\theta^3)
\]
\[
\Xi^a = 0 + \mathcal{O}(\theta^3)
\]
\[
\Xi^{\alpha} = \bar{\epsilon}^{\alpha^+} + \mathcal{O}(\theta^2).
\]

(2.7)

We can easily check the invariance of $E_\mathcal{\bar{\nu}}$ explicitly up to second order in $\theta$. Note that the coordinates $y^a$ transverse to the background brane are not transformed (at least up to the second order in $\theta$), i.e. this is the supertranslation parallel to the background brane. (Thus, we can define the corresponding Noether supercharge.) And it is also to be noted that $\Gamma_\mu = H^{1/3} \Gamma^a \delta_\mu^a$, i.e. the gamma matrices with the spacetime indices depend on the harmonic function.

The remaining field is the super-3-form gauge potential. It is introduced by the gauge invariant 4-form field strength\[31\] \[33\]

\[
R^{(4)} = dC^{(3)} = \frac{i}{2} E_\mathcal{\bar{m}} E_\mathcal{\bar{n}} \bar{E}_\mathcal{\bar{\nu}} \Gamma_{\mathcal{\bar{m}}\mathcal{\bar{n}}} E^\mathcal{\bar{\nu}} \mathcal{\bar{\psi}} + \frac{1}{4!} E^{\mathcal{\bar{m}} \mathcal{\bar{n}}} E^{\mathcal{\bar{m}}_2 \mathcal{\bar{n}}_2} E^{\mathcal{\bar{m}}_3 \mathcal{\bar{n}}_3} F_{\mathcal{\bar{m}}_4 \mathcal{\bar{n}}_4 \mathcal{\bar{m}}_2 \mathcal{\bar{n}}_2},
\]

(2.8)

where $F_{\mathcal{\bar{m}}_4 \mathcal{\bar{m}}_3 \mathcal{\bar{m}}_2 \mathcal{\bar{n}}_1}$ is the bosonic field strength which is in this case associated with the electric M-2-brane background. From (2.8) we get\[3\]

\[
C^{(3)} = \frac{1}{3!} H^{-1}( -\epsilon_{\mu \nu \rho} ) dx^\mu dx^\nu dx^\rho - \frac{i}{2} H^{-2/3} dx^\rho \delta_\mathcal{\bar{\nu}}^\mathcal{\bar{\mu}} dx^\sigma \delta_\mathcal{\nu}^\mathcal{\alpha} \bar{\theta}^+ \Gamma_{\mathcal{\bar{\mu}} \mathcal{\bar{\nu}}} d\theta^+
\]
\[
- \frac{i}{2} H^{1/3} dx^\mathcal{\bar{\nu}} d\theta^+ \delta_\mathcal{\nu}^\mathcal{\alpha} \bar{\theta}^+ \Gamma_{\mathcal{\bar{\nu}} \mathcal{\bar{\mu}}} + \mathcal{O}(\theta^4) \quad (\epsilon_{012} = -1),
\]

and hence the supertransformation of $C^{(3)}$\[4\]

\[
\delta C^{(3)} = d( -\frac{i}{2} H^{1/3} dy^b \delta_\mathcal{\alpha}^b dy^d \delta_\mathcal{\alpha}^d \bar{\delta} \bar{\xi}^+ \Gamma_{\mathcal{\bar{\alpha}} \mathcal{\bar{\nu}}} \theta^+ + \mathcal{O}(\theta^3) ) = d( \bar{\xi}^+ \Delta_2 ).
\]

(2.9)

In fact we need to know the (vanishing of the) contribution from $E_\mathcal{\bar{m}}^\mathcal{\bar{n}}$ at order $\theta^2$. We can infer its vanishing in this specific simple background, but the expression of $E_\mathcal{\bar{m}}^\mathcal{\bar{n}}$ at order $\theta^2$ in general background was obtained recently\[32\], by which our inference is confirmed.

\[4\] Although the $\mathcal{\bar{\alpha}}$ of $\theta^{\alpha^+}$ is the index of the inertial frame, $\theta^{\alpha^+} = \theta^2 \delta_\alpha^\beta + \mathcal{O}(\theta^4)$. So, we need not distinguish the two indices in this paper.

\[**\] Throughout this paper we make an assumption that all the fermionic (but not bosonic) cocycles in the superspaces are trivial. Then, the invariance of $R^{(4)}$ under the super-transformation means that $\delta C^{(3)}$ in any of the supersymmetric backgrounds under the assumption can be written as certain $d$-exact forms to full order in $\theta$.\[6\]
Thus, the M-2-brane action (2.1) is invariant under (2.7) up to total derivative, and we define the Noether supercharge $Q^+_\alpha$ in the Hamiltonian formulation as an integral over the test brane at fixed time $M_2$, given by

$$Q^+_\alpha \equiv Q^{+0}_\alpha - i \int_{M_2} (\mathcal{C} \Delta_2) \alpha$$

$$= \int_{M_2} d^2 \xi (i \Pi^+_{\alpha} - \Pi_{\mu} (C^{\mu} \theta^+_\alpha)) - \frac{1}{2} T \int_{M_2} dy^a dy^b (C \Gamma_{ab} \theta^+_\alpha) + \mathcal{O}(\theta^3)$$

(2.11)

where $\Pi_{\mu}$ and $\Pi^+_\alpha$ are the conjugate momenta of $x^\mu$ and $\theta^+_\alpha$, respectively, and $Q^{+0}_\alpha$ is the contribution from the Nambu-Goto action whose form is almost common to all the p-brane. In this way we get the superalgebra of $Q^+_\alpha$:

$$\{Q^+_\alpha, Q^+_\beta\} = 2 \int_{M_2} d^2 \xi \Pi_{\mu} (C^{\mu})_{\alpha\beta} + 2 \cdot \frac{1}{2} T \int_{M_2} dy^a dy^b (C \Gamma_{ab})_{\alpha\beta} + \mathcal{O}(\theta^2).$$

(2.12)

Before discussing this result, we give the explicit expression of $\Pi_{\mu}$:

$$\Pi_{\mu} = \frac{\delta \mathcal{L}^{(0)}}{\delta \dot{\epsilon}_{\mu}} + \frac{T}{2} \varepsilon^{0ij} \partial_i x^0 \partial_j x^\rho C^{(3)}_{\mu\nu\rho} + \mathcal{O}(\theta^2) \equiv \Pi^{(0)}_{\mu} + \Pi^{WZ}_{\mu}$$

(2.13)

where $\mathcal{L}^{(0)}$ is the Nambu-Goto Lagrangian.

We now discuss the implications of this algebra. First, if the test brane is oriented parallel to the background brane, the term like a central charge arises from the $\Pi^{WZ}_{\mu}$, although the original central charge vanishes. If we choose the static gauge $\partial_0 x^\mu = \delta^\mu_0$ and $\partial_i x^0 = \delta_i^0$, $\Pi^{(0)}_{\mu}$ and $\Pi^{WZ}_{\mu}$ are obtained respectively as

$$\begin{align*}
\int_{M_2} d^2 \xi \Pi^{(0)}_{\mu} &= T | \int_{M_2} dx^1 dx^2 | H^{-1} \delta^0_{\mu} + \mathcal{O}(\theta^2) \\
\int_{M_2} d^2 \xi \Pi^{WZ}_{\mu} &= T | \int_{M_2} dx^1 dx^2 H^{-1} \delta^0_{\mu} + \mathcal{O}(\theta^2).
\end{align*}$$

(2.14) (2.15)

The symbol of the absolute value in $\Pi^{(0)}_{\mu}$ is due to the Jacobian originated from the determinant in $\mathcal{L}^{(0)}$. Thus, we conclude that the parallel configuration with a certain orientation of the test brane has the 1/2 spacetime supersymmetry and the one with the other orientation breaks all the supersymmetry. Note that (2.14) and (2.15) are invariant under the 12-plane rotation and hence the discussion above holds, as it should be. Furthermore, even in the case that all the supersymmetry is broken, 1/2 supersymmetry is restored in the limit $y \to 0$ (i.e. $H \to \infty$). The reason is as follows: since $\Pi_0 \propto H^{-1}$ and $\Gamma^{\mu} \propto H^{1/3}$, the r.h.s. of the superalgebra (2.12) is proportional to $H^{-2/3}$, hence vanishes in the limit. In fact this restoration is reasonable because both of the forces via graviton and anti-symmetric tensor are attractive in this case and the potential energy is formally minimized in this “absorption” limit. This is the M-2-brane/M-2-brane bound state preserving 1/2 supersymmetry.
On the other hand, if the test brane is oriented orthogonally to the background brane, the central charge *does* have the nonzero value. In the static gauge with the test brane to be fixed, for example, to 34-plane, the algebra becomes

\[ \{Q^+_\alpha, Q^+_\beta\} = 2T \int_{M_2} d^2 \xi H^{1/3}(1 - \hat{ \Gamma}_{\bar{3}4})_{\alpha\beta}, \]  

which means that 1/4 spacetime supersymmetry is preserved in this configuration.

We note that the r.h.s. of the superalgebra does not vanish completely in any limit if the test brane has at least one coordinate transverse to the background brane. This fact is common to all the other cases. The reason is the following: suppose that one of the worldvolume coordinates of the test brane \( \xi^1 \) equals to the transverse coordinate \( y^{a'} \). Then, \( H \) is expressed as

\[ H = 1 + \frac{q}{((\xi^1)^2 + (y^{a'})^2)^3}, \]

where \( q = 3, ..., 9 \). Thus, \( \int d^2 \xi H^K \) does not vanish for any constant \( K \), even in the limit of \( (y^{a'})^2 \to 0 \). In other words, the r.h.s. of the superalgebras do not vanish completely for the cases of the string intersection and the orthogonal orientation.

Thus, we have derived the superalgebra from the M-2-brane action in an M-2-brane background. All the 1/4-supersymmetric intersections and a 1/2-supersymmetric bound state of two M-2-branes known before[27][34] have been deduced from the algebra.

Next, we will consider the M-2-brane in an M-5-brane background. The M-5-brane background solution is given by[35]

\[ ds_{11}^2 = H^{-1/3} \eta_{\mu\nu} dx^\mu dx^\nu + H^{2/3} dy^a dy^b \delta_{ab}, \]

\[ F_{abcd} = -\varepsilon_{abcd} \partial_c H \]  

(2.17)

where \( \mu = 0, 1, ..., 5 \) and \( a = 6, ..., 9, \bar{z} \). The Killing spinor \( \varepsilon \) has the form \( \varepsilon = H^{-1/12} \varepsilon_0 \) where \( \varepsilon_0 \) has again the positive chirality of the worldvolume of the background: \( \hat{\Gamma}'\varepsilon_0 \equiv \Gamma_{012345}\varepsilon_0 = +\varepsilon_0 \). Since \( \frac{1+\Gamma'_{(C)}}{2} \) are again projection operators, only the supersymmetry corresponding to \( Q^+ \) is the symmetry of the background. Thus, for the same reason stated in the case of the M-2-brane background, only \( Q^+ \) is the symmetry of the system and we set \( \varepsilon^- = 0 \) and hence \( \theta^- = 0 \). We note that \( \Gamma' \) satisfies the (anti-)commutators \( \{\hat{\Gamma}', C\} = \{\hat{\Gamma}', \Gamma_{\beta}\} = [\hat{\Gamma}', \Gamma_{\dot{a}}] = 0 \). By using this relations and the formula presented in ref.[31] the superspace 1-form on the inertial frame is given by

\[ E^{\dot{\mu}} = dx^\mu H^{-1/6} \delta_{\dot{\mu}}^\mu - i\bar{\theta}^+ \Gamma^\mu \dot{\theta}^+ + O(\theta^4) \]

\[ E^\mu = dy^b H^{1/3} \delta_b^\mu + O(\theta^4) \]

\[ E^\dot{a} = d\theta^\dot{a} + \frac{1}{12} H^{-1} dH \theta^\dot{a} + O(\theta^3). \]  

(2.18)

The super-coordinate transformation is formally the same form as that in the M-2-brane background [2.7] except for the ranges of \( \mu \) and \( a \). The superspace 3-form \( C^{(3)} \) is introduced in the same way as (2.8). Note that \( C^{(3)} \) cannot be expressed globally in this case.
because it is associated with the magnetic M-5-brane solution. However, neither does it contribute to $\delta \mathcal{L}^{WZ}$ nor the $\Pi_\mu$ up to first order in $\theta$, owing to the inertness of the transverse coordinates $y^a$ under the super-transformation. As a result,

$$
\delta C^{(3)} = d(-iH^{1/6} dx^\nu \delta_\nu^a dy^b \delta_\beta^b \varepsilon^{+\mu} \Pi_{\mu\alpha} \theta^+ + \mathcal{O}(\theta^3)) \equiv d(\varepsilon^{+}(\Delta^{(3)}_2)). \quad (2.19)
$$

Thus, the action is invariant under the super-transformation up to total derivative. After the same procedures as in the M-2-brane background, we get the expression of the supercharge $Q^{+}_\alpha \equiv Q^{+(0)}_\alpha - i \int_{M_2} (C \Delta^{(3)}_2)_\alpha$. Then, the superalgebra is obtained as

$$
\{Q^{+}_\alpha, Q^{+}_\beta\} = 2 \int_{M_2} d^2 \xi (\Pi_\mu (C \Gamma^\mu))_{\alpha\beta} + 2 \cdot T \int_{M_2} dx^\mu dy^a (C \Gamma^{\mu a})_{\alpha\beta} + \mathcal{O}(\theta^2). \quad (2.20)
$$

We conclude from the form of its central charge that the string intersection of the test brane with the background is the only 1/4-supersymmetric configuration permitted in this background. This is consistent with ref.[27][34], too. In this case the interpretation of the “boundary” of the test M-2-brane is as follows, as given in ref.[36]: if we choose the gauge $\xi^1 = x^1$ and $\xi^2 = y^2$, the intersection on the hypersurface $y \equiv \sqrt{y^a y^b \delta_{ab}} = 0$, (i.e., $\xi^2 = 0$) does not correspond to any points of the M-2-brane because the proper distance on it is infinite. So, the “edge” of the M-2-brane is interpreted to disappear down the infinite M-5-brane throat. In other words the test M-brane has no boundary in this method.††

On the other hand, if the test M-2-brane is parallel to any two-dimensional subspaces of the worldvolume of the background M-5-brane, all the supersymmetry is broken. However, 1/2 spacetime supersymmetry is restored in the limit $y \to 0$ (i.e. $H \to \infty$) since $\Pi_0 C \Gamma^0 \propto H^{-1/3}$. Thus, we can deduce the M-2-brane/M-5-brane bound state with 1/2 spacetime supersymmetry given in ref.[28][9].

3 The Superalgebras from the M-5-brane in M-brane Backgrounds

In this section we discuss the test M-5-brane in M-brane backgrounds. There are two new features which do not emerge in the previous cases of the test M-2-brane: one is the fact that the M-5-brane action contains worldvolume self-dual 2-form gauge potential $A_2$ in addition to the usual scalar fields[37][38]. The super-transformation of $A_2$ is determined by the requirement of the invariance of the “modified” field strength [4] given by

$$
\mathcal{H} = dA_2 - C^{(3)}. \quad (3.1)
$$

††If we set the worldvolume coordinate $\xi^2$ to take values in the open interval $(0, \infty)$, it is interpreted as the M-2-brane ending on the M-5-brane[34], on so large scales compared to that determined by M-5-brane tension that the background solution can be replaced with the M-brane source.
The other is the introduction of the superspace 6-form field strength $C^{(6)}$ [39] whose field strength takes the form

$$R_{(7)}^{(7)} \equiv dC^{(6)} - \frac{1}{2} C^{(3)} R_{(4)}^{(4)}$$

$$= \frac{i}{3!} E_{\hat{m}_{1}} ... E_{\hat{m}_{5}} \bar{E}^{\hat{a}} (\Gamma_{\hat{m}_{5} ... \hat{m}_{1}}) \delta^{\hat{a}} \delta^{\hat{b}} E^{\hat{b}} + \frac{1}{7!} E_{\hat{m}_{1}} ... E_{\hat{m}_{7}} F_{\hat{m}_{7} ... \hat{m}_{1}}^{(7)}$$

(3.2)

where the 7-form $F^{(7)}$ is the Hodge dual of the bosonic 4-form field strength.

(3a) the M-5-brane in the M-2-brane background

First, we will consider the test M-5-brane in the M-2-brane background (2.2). This set-up might seem to be unreasonable because M-5-branes cannot have the boundary [38]. However, test branes have no boundary in this method as stated above in (2b) case. Thus, we can compute the superalgebra and discuss supersymmetric configurations in this case in the same way. The M-5-brane action is [37]

$$S_{M5} = -T \int d^8 \xi \left[ \sqrt{-\det(g_{ij} + \hat{H}_{ij})} + \frac{\sqrt{-g}}{4(\partial a)^2} (\partial_i a)(\mathcal{H}^{*i j k} \mathcal{H}_{j k l}(\partial^a a)) \right]$$

$$+ \int (C^{(6)} + \frac{1}{2} \mathcal{H} C^{(3)})$$

(3.3)

where $(\mathcal{H}^{*i j k} = \frac{1}{3! \sqrt{-g}} \epsilon^{i j k l r s} \mathcal{H}_{l r s}, \mathcal{H}^{*i j k} \partial_k a$ and $a$ is an auxiliary world-volume scalar field. Since the superspace 1-form and the (transformation of ) $C^{(3)}$ in the M-2-brane background are already given by (2.9) and (2.3), the transformation of $A_2$ is

$$\delta A_2 = -\frac{i}{2} H^{1/3} \delta^b d\xi^a \delta^b d\psi^a + \Gamma_{a b} \theta^+ (\equiv \xi^+ \Delta_2)$$

(3.4)

and the transformation of $C^{(6)}$ is deduced in the same way as the 3-form $C^{(3)}$. As a result, we get $\delta L^{WZ} \equiv d(\xi^+ \Delta)$ where.

$$\Delta^\alpha = -\frac{i}{4!} H^{1/3} d\xi^a \delta^b d\psi^a \delta^c d\psi^c (\Gamma_{a b c d} \theta^+)^\alpha - \frac{i}{4} H^{1/3} d\xi^a \delta^b d\psi^b \delta^c d\psi^c dA_2 (\Gamma_{a b c d} \theta^+)^\alpha$$

(3.5)

The supercharge is given as before by $Q^+_\alpha \equiv Q^+_\alpha - i \int_{M5} (C \Delta)_\alpha$ where $Q^+_\alpha$ takes the form [4]

$$Q^+_\alpha = \int_{M5} d^5 \xi [i \Pi^+ - \Pi_{\mu} (C \Gamma^\mu \theta^+) + \frac{i}{2} P^{i j} (C \Delta_2)_{i j}]$$

(3.6)

where $i$ is the space index of the test M-5-brane worldvolume and $\Pi_{\mu}, \Pi^+ \alpha$ and $P^{i j}$ are the conjugate momenta of $x^\mu$, $\theta^+$ and $A_{i j}$, respectively.

Because of the invariance of $R^{(7)}$ under the super-transformation, $\delta C^{(6)} - \frac{1}{2} C^{(3)} \delta C^{(3)}$ in the background can be written as a d-exact form $d(\bar{\xi}^+ \Delta_5)$. Then, it is shown that it holds $dL^{WZ} = d(\bar{\xi}^+ \Delta_5 - \frac{1}{2} dA_2 \delta A_2)$ to full order in $\theta$.
Thus, the superalgebra is obtained as
\[
\{Q^+, Q^+_\beta\} = 2 \int_{M_5} d^5 \xi \Pi_\mu (C \Gamma^\mu)_{\alpha \beta} + 2 \int_{M_5} d^5 \xi \frac{1}{2} P^i_\perp H^{1/3} \partial_\nu \delta^i_\nu \delta_\mu \delta_\nu (C \Gamma^\mu)_{\alpha \beta} \\
-2 \cdot \frac{1}{4!} T \int_{M_5} dx^\mu dy^a ... dy^a (C \Gamma^a_{\mu 1 ... a_4})_{\alpha \beta} - 2 \cdot \frac{1}{4} T \int_{M_5} dy^a dy^b dA_2 (C \Gamma^a b)_{\alpha \beta} + O(\theta^3) \quad (3.7)
\]
The third term in (3.7) means the string intersection with the M-2-brane background leads to the preservation of 1/4 supersymmetry, which is again consistent with ref.[27], ref.[34] and the result of (2b) case. And if we choose the temporal gauge \(a(\xi) = t\), the second term including \(P_\perp\) turns to be the same as the last term as in ref.[9]. Thus, these terms imply the possibility of the triple intersection with the configuration

| background M-2 | 1 | 2 | - | - | - | - | - | - | - | - |
| test M-5 | - | 2 | 3 | 4 | 5 | 6 | - | - | - | - |
| M-2 | - | - | 3 | 4 | - | - | - | - | - | - |

where the third M-2 brane is within the M-5-brane (to form the bound state)[9]. This configuration would preserve 1/4 spacetime supersymmetry since \(C \Gamma_{23456}\) anticommutes with \(C \Gamma_{34}\). We note that the possibility of this triple intersection is discussed directly on the basis of the spacetime superalgebra, by making use of the fact that there exist an M-brane as a background from the beginning.

Next, we will consider the M-2-brane/M-5-brane bound state. Suppose \(\xi^1 = x^1, \xi^2 = x^2\) (that is, a two-dimensional worldvolume subspace of the test M-5-brane is parallel to the background M-2-brane). Then, if \(A_2 = 0\), all the terms except \(\Pi_\theta C \Gamma^\theta (x \ H^{1/6})\) in the r.h.s. of (3.7) vanish in the static gauge. However, since the harmonic function depends on the “relatively transverse” coordinates \(\xi^3, \xi^4\) and \(\xi^5\), \(\int d^5 \xi H^K\) do not go to zero even in the limit of vanishing distance. This means that we cannot deduce the M-2-brane/M-5-brane bound state from the algebra at least straightforward. But if we consider the background whose harmonic function does not depend on the “relatively transverse” coordinates as in ref.[27], the r.h.s. of the algebra vanishes in the limit, and we can obtain the 1/2-supersymmetric M-2-brane/M-5-brane bound state.

(3b) the M-5 brane in the M-5-brane background

Finally we will deal with the test M-5 brane in the M-5-brane background(2.17). Since almost all the preparations have been already given above and the procedures are similar to before, we present only the transformation of \(A_2\), the result of the superalgebra as well as its interpretations in this case. The transformation of \(A_2\) in the M-5 brane background is
\[
\delta A_2 = -i H^{1/6} dx^\nu \delta^\nu_\nu dy^b \delta^b_\mu \bar{\varepsilon}^+ \Gamma_{\bar{a} a} \theta^+ + O(\theta^3).
\]
The superalgebra is given by
\[
\{Q^+_\alpha, Q^+_\beta\} = 2 \int_{M_5} d^5 \xi \Pi_\mu (C \Gamma^\mu)_{\alpha \beta} + 2 \int_{M_5} d^5 \xi i P^\perp H^{1/6} \partial_\nu x^\nu \delta^\nu_\nu \bar{\varepsilon}^+ \delta_\mu \delta_\nu (C \Gamma^\mu)_{\alpha \beta}
\]
\[-\frac{2}{12}T \int_{M_5} dx^{\mu_1} \cdots dx^{\mu_3} dy^{a_1} dy^{a_2} (\mathcal{C} \Gamma_{\mu_1 \cdots \mu_3 a_1 a_2})_{\alpha \beta} - \frac{2}{4!} T \int_{M_5} dx^{\mu} dy^{a_1} \cdots dy^{a_4} (\mathcal{C} \Gamma_{\mu a_1 \cdots a_4})_{\alpha \beta} \]
\[-\frac{2}{2} T \int_{M_5} dy^a dx^{\mu} dA_2 (\mathcal{C} \Gamma_{\mu a})_{\alpha \beta} + O(\theta^2). (3.9)\]

In fact only the fourth term is difficult to derive straightforward in this case because the magnetic 3-form potential which cannot be globally expressed contributes to \(\delta \mathcal{L}^{WZ}\). However, since they can be expressed locally in a certain gauge, we can confirm in a gauge that some of these terms do not vanish, although we do not exhibit the computation because is very primitive and awkward to present. Then, we reproduce the term on the ground that it should be gauge invariant and Lorentz \((SO(1,5) \times SO(5))\) covariant.

Now, we will interpret the result. If the test M-5-brane is oriented parallel to the background M-5-brane, perfectly the same circumstances occur as in the case of the M-2-brane parallel to the background M-2-brane. So, we present only the result, without repeating the explanations. The configuration of two parallel M-5-branes and an M-5-brane/M-5-brane bound state, both of which preserve 1/2 spacetime supersymmetry, are deduced. If the test 5-brane intersects on 3-brane with the background M-5-brane, 1/4 spacetime supersymmetry is preserved because of the third term (as in ref.[27]). The fourth term shows that the string intersection of the two M-5-brane leads to the preservation of 1/4 supersymmetry[28]. The last term, together with the second term with the temporal gauge condition \(a(\xi) = t\), implies the possibility of the existence of the triple intersections preserving 1/4 supersymmetry with the following configurations:

| Configuration         | 1 | 2 | 3 | 4 | 5 | - | - | - | - |
|-----------------------|---|---|---|---|---|---|---|---|---|
| Background M-5        | 1 | 2 | 3 | 4 | 5 | - | - | - | - |
| Test M-5              | 1 | 2 | 3 | - | - | 6 | 7 | - | - |
| M-2                   | - | - | 3 | - | - | 6 | - | - | - |

and

| Configuration         | 1 | 2 | 3 | 4 | 5 | - | - | - | - |
|-----------------------|---|---|---|---|---|---|---|---|---|
| Background M-5        | 1 | 2 | 3 | 4 | 5 | - | - | - | - |
| Test M-5              | - | - | - | - | 5 | 6 | 7 | 8 | 9 |
| M-2                   | - | - | - | - | 5 | 6 | - | - | - |

where each of the third M-2 branes is within the test M-5-brane.

4 Discussion

In summary we have discussed the method of computing explicitly the spacetime superalgebras from the test M-brane actions in M-brane backgrounds to the lowest order in \(\theta\). As the consequences we have derived all the 1/4-supersymmetric intersections of the two M-branes known before from the central charges of the spacetime superalgebras, as the supersymmetric “gauge fixing” of the test brane permitted in the background. We
have also deduced some of 1/2 supersymmetric bound states of two M-branes from examining the behavior of the harmonic functions in the limit of vanishing distances. In addition, the possibilities of some triple intersections preserving 1/4 supersymmetry have been discussed.

In order to obtain the 1/2 supersymmetric bound state (M-2-brane within M-5-brane), we need to assume only in the case of the M-5-brane in the M-2-brane background that the harmonic function is independent of the “relatively transverse coordinates. Thus, when we deal with the system composed of a p-brane and a q-brane with the inequality $q \geq p$, it is more suitable for this method to discuss the system as the p-brane in the q-brane background. Except for this subtlety, the method discussed in this paper is confirmed to be consistent with those presented before, and hence reliable.

Having confirmed its reliability, we will apply this method to the p-branes in 10-dimensional massive IIA backgrounds[40] as stated in the introduction, in which case the background have to be nontrivial[18][19]. It will also be interesting to apply this to the cases of other backgrounds[11], or use it to the new supersymmetic invariant p-brane action presented recently[24].

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References

[1] P. K. Townsend, Phys. Lett. B350 (1995) 184.

[2] E. Witten, Nucl. Phys. B443 (1995) 85.

[3] For a review, P. K. Townsend, Four Lectures on M-theory, hep-th/9612121 and references therein.

[4] For a review, D. Bigatti, L. Susskind, Review of Matrix Theory, hep-th/9712072 and references therein.

[5] C. M. Hull, Nucl. Phys. B509 (1998) 216.

[6] For a review, P. K. Townsend, M-theory from its superalgebra, hep-th/9712004 and references therein.
[7] J. W. van Holten and A. Van Proeyen, J. Phys. A:Math Gen. 15 (1982) 3763.

[8] P. K. Townsend, p-brane democracy, hep-th/9507048.

[9] D. Sorokin and P. K. Townsend, M-theory superalgebra from the M-5-brane, Phys. Lett. 412 (1997) 265.

[10] J. A. de Azcarraga, J. P. Gauntlett, J. M. Izquierdo and P. K. Townsend, Phys. Rev. Lett. 63 (1989) 2443.

[11] P. S. Howe, N. D. Lambert and P. C. West, Phys. Lett. B419 (1998) 79.

[12] E. Bergshoeff, J. Gomis and P.K. Townsend, Phys. Lett. B421 (1998) 109.

[13] J. P. Gauntlett, J. Gomis and P.K. Townsend, BPS bounds for worldvolume branes, JHEP 001, (1998), 033, hep-th/9711205.

[14] H. Hammer, Nucl. Phys. B521 (1998) 503; M. Hatsuda and K. Kamimura, Wess-Zumino actions for IIA D-branes and their supersymmetry, hep-th/9804087.

[15] E. Bergshoeff and M. de Roo, Phys. Lett. B380 (1996) 265.

[16] M. B. Green, C. M. Hull and P.K. Townsend, Phys. Lett. B382 (1996) 65.

[17] E. Bergshoeff and P.K. Townsend, Nucl. Phys. B490 (1997) 145.

[18] L. Romans, Phys. Lett. 169B (1986) 374.

[19] E. Bergshoeff, M. de Roo, M. B. Green, G. Papadopoulos and P. K. Townsend, Nucl. Phys. B470 (1996) 113.

[20] K. Bautier, S. Deser, M. Henneaux and D. Seminara, Phys. Lett. B406 (1997) 49; S. Deser, hep-th/9712064.

[21] P. S. Howe, N. D. Lambert and P. C. West, Phys. Lett. B416 (1998) 303.

[22] E. Bergshoeff, Y. Lozano and T. Ortin, Nucl. Phys. B518 (1998) 363.

[23] E. Bergshoeff, E. Eyras and Y. Lozano, The massive Kaluza-Klein monopole, hep-th/9802199.

[24] E. Bergshoeff and P.K. Townsend, Super D-brane revisited, hep-th/9804011.

[25] J. Gomis, D. Mateos, J. Simon and P. K. Townsend, Brane intersection dynamics from branes in brane backgrounds, hep-th/9803040.
[26] J. P. Gauntlett, D. A. Kastor and J. Traschen, Nucl. Phys. 478 (1996) 544.

[27] G. Papadopoulos and P. K. Townsend, Phys. Lett. B380 (1996) 273.

[28] J. M. Izquierdo, N. D. Lambert, G. Papadopoulos and P. K. Townsend, Nucl. Phys. 460 (1996) 560.

[29] E. Bergshoeff, E. Sezgin and P. K. Townsend, Phys. Lett. B189 (1987) 75; Ann. Phys. (NY) 185 (1988) 330.

[30] M. J. Duff and K. S. Stelle, Phys. Lett. 350B (1991) 113.

[31] E. Cremmer and S. Ferrara, Phys. Lett. 91B (1980) 61.

[32] B. de Wit, K. Peeters and J. Plefka, Superspace Geometry for Supermembrane backgrounds, hep-th/9803209.

[33] L. Brink and P. S. Howe, Phys. Lett. 91B (1980) 384.

[34] A. A. Tseytlin, Nucl. Phys. 487 (1997) 141.

[35] R. Güven, Phys. Lett. 276B (1992) 49.

[36] P. K. Townsend, Brane surgery, Nucl. Phys. Proc. Suppl. 58 (1997) 163.

[37] P. Pasti, D. Sorokin and M. Tonin, Phys. Lett. 398B (1997) 41; I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, Phys. Rev. Lett. 78 (1997) 4332.

[38] M. Aganagic, J. Park, C. Popescu and J. H. Schwarz, Nucl. Phys. B496 (1997) 191.

[39] A. Candiello and K. Lechner, Nucl. Phys. B412 (1994) 479.

[40] Takeshi Sato, The spacetime superalgebras in a massive IIA background via brane probes, hep-th/9805209.

[41] Takeshi Sato, in preparation.

[42] E. Bergshoeff and J. P. van der Schaar, On M-9-branes, hep-th/9806069.