Implemented preconditioning conjugate gradient algorithm on electrical resistance tomography to modelling structural geology

A Kusmiaran1*, T A Sanny2, S Nurrahmi1, F Yanti1 and Minarti1

1 Physics Department, Universitas Islam Negeri Alauddin Makassar, Makassar, Indonesia
2 Geophysics Engineering Department, Institut Teknologi Bandung, Bandung, Indonesia

*Email: amirin19@gmail.com

Abstract. The development of a method to enhance resolution in imaging subsurface structural geology has continuously developed. The method of tomography field is electrical resistance tomography that is a powerful method for imaging subsurface structural geology. The electrical resistance tomography method is divided into two parts that are forward modelling and inverse modelling. The principle of forward modelling is to solve elliptic differential equation second order by finite volume method. The result of this part is the subsurface resistivity magnitude where it is used to model structural geology. For modelling inverse, the algorithm that has used to the model of subsurface structural geology is the Gauss-Newton method with an updated model that is preconditioning conjugate gradient. The ability of the electrical resistance tomography has tested to models of subsurface structural geology such us layering, and fault model. The results have revealed that this method able to model the subsurface of the structural geology, delineate the edge of the layer, and enhance the resolution of the model.

1. Introduction

Methods for imaging subsurface in geophysics have developed for many applications such as coal mine detection, subsurface void, fluid of underground, and upper-crust [1, 2]. One of some technique that used to map subsurface is electrical resistance where this method base on the electrical principle in imaging geology subsurface, such as disturbance density of materials. The density of any materials is different from others, and density is reciprocal of resistivity.

The resistivity distribution of the subsurface is successfully in mapping the subsurface structure where is used conventional and tomography resistance. The electrical resistivity conventional is only mapping subsurface in either lateral or vertical direction while electrical resistance tomography is mapping both lateral and vertical magnitude [3]. The number of data which is electrical resistance tomography is more data than electrical resistivity conventional. Therefore, the electrical resistivity tomography method is higher resolution than the electrical resistivity method [4, 5]. The problem in electrical resistance tomography modelling, besides high resolution, is ill-conditioned [6, 7, 8]. In other to meet the requirement, some resistivity tomography algorithm has developed for data tomography [9,10].

Data tomography of the subsurface has generated by current injection in bore-hole which current will be propagating in the subsurface. So the disturbance current is measured by potential electrodes. Hence from difference potential, the electrical resistivity tomography data can be modelled by the inversion method. The performance of inversion data based on the algorithm that has used. In this research,
geology subsurface modelling used algorithm preconditioning conjugate gradient and Gauss-Newton. Algorithm Gauss-Newton uses to reach out an initial model, whereas update models use the preconditioning conjugate gradient to efficiency convergent model [11]. Hessian matrix or gradient vector for the variable update model has computed by a finite volume method [7, 8].

2. Materials and Methods

The resistivity method is one of the techniques in geophysics that is used to geologies subsurface imaging such as mine detection, and subsurface void, and environmental application [12, 13]. In the subsurface imaging, this method is used electrical current injection in bore-hole [14], so that electrical current is propagated in underground. The disturbance current has measured by the potential electrodes where those electrodes have placed on either bore-hole [15, 16] or ground surface. Relation both current density, J and conductivity, are described by Ohm Law

\[ \nabla J = -\sigma \nabla \phi \]  

(1)

where \( \phi \) is potential, and conductivity \( \sigma \) is reciprocal of resistivity, where each material have difference resistivity as revealed by Table 1. Equation 1 is basic formula that use in electrical tomography, and can be modified using current principle in volume. Construction by this formula is showed by:

\[ \nabla J = \frac{\partial m}{\partial t} \delta(x)\delta(y)\delta(z) \]  

(2)

where \( m \) is current density in x-y-z dimension of delta Dirac function.

| Rock Type          | Resistivity (\( \Omega \)m)          |
|--------------------|-------------------------------------|
| Granite porphyry   | 4.5 x 10^3 (wet) - 1.3 x 10^6 (dry) |
| Feldspar porphyry  | 4 x 10^3 (wet)                      |
| Diorite porphyry   | 1.9 x 10^3 (wet) - 6 x 10^4 (dry)   |
| Dacite             | 2 x 10^4 (wet)                      |
| Andesite           | 4.5 x 10^4 (wet) - 1.7 x 102 (dry)  |
| Basalt             | 10 - 1.3 x 10^7 (dry)               |
| Gabbro             | 10^3 - 10^6                         |
| Olivine            | 10^3 - 6 x 10^4 (dry)               |
| Peridotite         | 3 x 10^3(wet) - 6.5 x 10^3(dry)     |
| Limestone          | 50 - 10^7                           |
| Sandstone          | 1 - 6.4 x 10^7                      |
| Clay               | 1 – 100                             |

The physical properties distribution in Table 1 has modelled by both traditional electrical resistance and electrical resistance tomography [17, 18]. The basic principle of electrical resistance tomography method is dived subsurface into the part by part or slice. This method has been widely mapped subsurface structure such as fracture zone, fluid current, and mine detection.

Mathematically, resistivity distributions in the subsurface are calculated based on combination both equation 1 and equation 2 such as:

\[ -\nabla [\sigma \nabla \phi] = \frac{\partial m}{\partial t} \delta(x)\delta(y)\delta(z) \]  

(3)
where $\nabla$ is linear divergence operator. The boundary condition, applied in this model, is Neumann boundary as constrain of the measurement. Dipole technique is applied to acquisition data of electrical resistance tomography that is modified equation 3 [7, 8]:

$$\nabla. [\sigma(x, y, z) \nabla \phi(x, y, z)] = I (\delta(r - r_+) - \delta(r - r_-))$$

(4)

where $I$ is an electrical current, $r_+$ and $r_-$ are positive and negative of source electrical current, and $\delta(r - r_+)$ and $\delta(r - r_-)$ are delta Dirac function. According to Dey and Morrison, 1979 [19], Equation 4 is effectively solved by Fourier domain that is formulated to be:

$$\nabla. [\sigma(x, y, z) \nabla \tilde{\phi}(x, K_y, z)] + K_y^2 \sigma(x, z) \tilde{\phi}(x, K_y, z) = \frac{1}{\pi} I \delta(r - r_+) \delta(r - r_-)$$

(5)

where $\tilde{\phi}$ is potential transformation. Simplification of equation 5 is wrote by equation 6 [7]:

$$(D.S(\sigma)G) + K_y^2.S(\sigma)\tilde{U} = A(\sigma, K_y^2)\tilde{U} = q$$

(6)

where both $B$ and $G$ are divergence and gradient operator, $S(\sigma)$ is diagonal matrix that conductivity value is contained. $\tilde{U}$ is transformation potential vector, $A(\sigma, K_y^2)$ is matrix forward operator, and $q$ is source of electrical current. Forward transformation of vector potential, $\tilde{U}$ forward transform to vector potential, $U$, so Equation 6 can be written to [7]:

$$U = A^{-1}(\sigma)q$$

(7)

that is linear algebra equation that is used to solve distribution resistivity as observation data.

Electrical resistivity tomography data in inversion geophysics is highly ill-conditioned, undetermined, and nonlinear. Thus, the problem data can be solved by least square function [6, 20], so that equation 6 is modified to be:

$$\phi(m) = \frac{1}{2} \| d(m) - d_{obs} \|^2 + \frac{\beta}{2} \| W(m - m_{ref}) \|^2$$

(8)

where $d(m)$ is the vector of calculated data which use to conductivity models, $d_{obs}$ is observation data, and $\beta$ is regularisation parameter. This parameter is used as solution stabilization so that the model is not ill-conditioned. $m$ is a parameter model that is generated by $\ln \sigma$, and $W$ is the weight parameter. For minimum condition is generated, Equation 7 is modified by Taylor series approximation, and we get [11, 7]

$$\left( J^T J + \beta W^T W \right) \delta m = -J(QA^{-1}q - d_{obs}) + \frac{\beta}{2} \left( W(m - m_{ref}) \right)$$

(9)

where $J$ is matrix Jacobian or matrix sensitivity, $g$ is gradient function, and $H$ is matrix Hessian. The matrix Jacobian is calculated by [11, 7]:

$$J = -QA^{-1}B$$

(10)

$B$ is the diagonal negative exponential of model $m$. update model is generated by equation 11.

$$\delta m = -H^{-1} g$$

(11)
Update model from equation 11 is generated by the preconditioned conjugate gradient algorithm [9], so the equation 11 can be written to:

\[
\mathbf{m}_{i+1} = \mathbf{m}_i + \delta \mathbf{m}
\]

where \(\mathbf{m}_{i+1}\) is the new model and \(\mathbf{m}_i\) is the initial model. It suggests that the algorithm is applied to the numerical equation such as finite volume method that is based on Haber [11] formulation. Grid model of finite volume method is presented [11, 7]. Update model can use some algorithm such as the conjugate gradient and preconditioning conjugate gradient [21]. A flowchart of this research is revealed in Figure 2.

![Flowchart](image)

**Figure 1.** Implemented preconditioning conjugate gradient (*PCG) algorithm procedure.

Figure 1 is exhibited how to precondition the conjugate gradient algorithm is applied to structural geology modelling such as layering and fault modelling. In this research, the inversion process is conducted by two processes which are misfit error between observation and calculation data and find out update models. If misfit errorless and equal than 0.0005, preconditioning conjugate gradient (PCG) is applied to the modelling process to reach out update model with solve equation 8 (Hessian and Jacobi matrix). The last process is solved equation 12 with a numerical method such as the Gauss-Newton Method.

### 3. Results and discussion

This research, preconditioning conjugate gradient is applied to compute electrical resistance tomography in the updated model. It is because of preconditioning conjugate gradient is more effective than others algorithm such as conjugate gradient to generated model. This statement is validated by Figure 2.

The algorithm is applied to modelling geology structure data syntactic such as layering model in Figure 3 and the fault model in Figure 4, where the model construction has used a variation of resistivity data.
Figure 2. Preconditioning conjugate gradient and conjugate gradient comparison.

Figure 3. a) Preconditioning conjugate gradient convergent b) Gauss-Newton Convergent c) layering synthetic model d) inversion of layering synthetic model.
The synthetic model is generated by the Gauss-Newton method for electrical resistance tomography data inversion, and algorithm preconditioned conjugate gradient is applied to this model. Figure 3 and 4 are revealed of inversion models of the structural geology such as layering and fault structure. Based on that results, this algorithm is exhibited better imaging structural geology [5] and more efficient in convergent [20, 21]. The figure also reveals there are some parts after the inversion which does not agree with the initial model. Appropriate this model caused by there is any data that is not interpolated as well as, and this limitation of the algorithm. But electrical resistance tomography able to delineate the edge of structure models [5, 18, 21]. The misfit error of the models is less than 0.7 percent for layering structural geology model (Fig. 3b) and less than 0.95 percent for fault structural geology model (Fig. 4b).

Figure 4. a) Preconditioning conjugate gradient convergent b) Gauss-Newton Convergent c) fault synthetic model d) inversion of fault synthetic.

4. Conclusion
Electrical resistance tomography method is one of the methods to enhance the resolution of models in both lateral and vertical direction. Algorithm Gauss-Newton is applied to the inversion of electrical resistance tomography data, and the algorithm preconditioning conjugate gradient is applied to calculate the updated model. The combination thus method have given time efficiency of time because of fast converge. Based on these results, the misfit error structural geology model is less than 0.7 percent for layering model and less than 0.95 percent for fault model. So, incorporated the algorithm both Gauss-Newton and preconditioning conjugate gradient are applied to generate layering and fault model and enhance imaging resolution.
Acknowledgement
Thanks to Head of Physics Department and Head of Computation Physics Laboratory Universitas Islam Negeri Alauddin Makassar that has given authority to laboratory access.

References
[1] Cardarelli E, Cercato M, Cerreto A, Filippo G D 2010 Geophysical Prospecting 58 685–695
[2] Storz H, Storz W and Jacobs F 2000 Geophysical Prospecting 48 455–471
[3] Daily W and Owen E 1991 Geophysics 56 1228–1235
[4] Christensen N B, Sherlock D and Dodds K 2006, Exploration Geophysics 37 44–49
[5] Hani Al-Amoush, Jafar Abu Rajab, Eid Al-Tarazi, Abdel Rahman Al-Shabeeb, Rida Al-Adamat and A’kif Al-Fugara 2017 Jordan Journal of Earth and Environmental Sciences 8 27-34
[6] Aster R C, Brochers B and Thurber C H 2019 Parameter Estimation and Invers Problem Third Edition (Kidlington:Elsevier)
[7] Pidlisecky A, Haber E and Knight R 2007 Geophysics 72 P.H1-H10
[8] Pidlisecky A and Knight R 2008 Computer and Geosciences 34 1645–1654
[9] Lowry T, Allen MB and Shive PN 1989 Geophysics 54 766–774
[10] Sasaki Y 1994 Geophysics 59 1839 1848
[11] Haber E, U M Ascher, D A Aruliah and D W Oldenburg 2000 Journal of Computational Physics 163 150–171
[12] Manuel Herzog, Maximilian Kanig and Olaf Bubenzer 2019 Zeitschrift für Geomorphologie 62 119-135
[13] Nobes D C 1996 Surveys in Geophysics 17 393–454
[14] Zhou B and Greenhalgh S A 2000 Geophysical Prospecting 48 887–912
[15] Hagrey S T A 2012 Pure and Applied Geophysics 169 1283–1292
[16] Pain C C, Herwanger J V, Worthington M H and Oliveira C R E 2002 Geophysics 151 710-728
[17] Dahlin T and Zhao B 2004 Geophysical Prospecting 52 379–398
[18] Kemna A, Binley A, Ramires A and Daily 2000 Chemical Engineering Journal 77 11–18
[19] Dey A and Morrison H F 1979 Geophysical Prospecting 27 106–136
[20] Saad Y 2003 Iterative Methods for Sparse Linear Systems Second Edition (Philadelphia:Siam)
[21] Wang Tai-Han, Huang Da-Nian, Ma Guo-Qing, Meng Zhao-Hai and Li Ye 2017 Applied Geophysics 14 P.301-313