A Dimensional Analysis of Local Sandy Soil Erosion Induced By Leaky Sewer Pipes

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Abstract

Soil erosion is the most important reason that leads directly to the formation of sinkholes in the roads of urban areas. Soil erosion occurs due to the movement of water from and into the surrounding soil through defects in sewer pipes. In the present study, dimensional analysis of the erosion process was carried out and a dimensionless model was proposed to estimate the rate of erosion for local sandy soil. Model tests were conducted under a varying matrix of influencing parameters including: leak size, soil particle size, initial water content, water flow rate, height of water level above the defect in soil, and number of water flow cycle. The experimental model involved soil exposed to cyclic water flow through leaks located at the bottom of the model. Eroded soil is collected, dried, weighted and sieved for each cycle. From the results, it was found that the proposed dimensional analysis prediction model can reasonably estimate the rate of erosion for the local sandy soil.

Key words: sewer pipes, leakage, model test, soil erosion, dimensionless

1. Introduction

Buried pipelines networks are widely distributed around the world especially in urban areas where they form an important part of investments [1–2]. With the aging of the pipes, and under the different surrounding circumstances and loading conditions, defects begin to appear in these pipes in the form of openings, cracks or breakage [3–4]. These defects, allow the water to move from the soil to the inside of the sewer pipe, carrying with it the detached
soil particles [5–6]. This process loosens the soil and forms cavities [7]. The Repetition of the process of soil erosion over time leads to the expansion of the cavities and eventually to the failure of the soil and the formation of the sinkhole [3–8–9]. Sinkhole accidents are becoming more frequent and new cases are constantly being reported [7–10]. These accidents have caused great economic losses including damage to buried service lines, interruption to traffic, damage to roads, structures and sometimes even loss of life [9–11–12]. Previous studies have addressed the process of soil erosion and the factors affecting it. Several parameters are found to be potentially important. Soil particle size distribution, initial water content, water flow rate, height of water level above the defect in soil, and number of water flow cycle [4–13–16]. Leak width (defect size) is a key factor which determines the amount of eroded soil, soil lose is related to the ratio between leak width or width of the opening and the soil particle size [4–13–17]. Rogers, 1986, studied the soil erosion through pipe defects and found a relationship between the ratio of leak width to the size of the soil particles (B/D85) and the amount of soil loss, where: (B) is the leak width and (D85) is the size of the sieve through which 85% of the soil sample will pass [13]. He found that the continuous migration of soil through the pipe defect occur when the leak width have a value of 2.5D85 to 4.5D85 or more. For freely flow sand under gravity, Kamel, 2008, observed a reduction in volume of soil loss as the particle to opening size ratio (D60/B) increased [18]. In addition, he observed that the location of the hole or leak has a significant effect on the rate of sand movement into the pipe. A very small ratio or no sand erosion was observed for the case where the hole was located at the springline of the pipe. As the induced hole is moved towards the crown, more sand eroded into the pipe the maximum erosion was measured at the pipe crown. Mukunoki, 2012, studied the failure of soil (for fine sand and gravels) due to defective underground pipe and demonstrated that the critical value of leak width is 5.9Dmax [4]. Several studies have shown that initial water content affects the soil erosion process as the erosion becomes faster and larger when the initial water content is high[19–21]. Many studies have also been addressed the effect of cyclic flow behavior of water through pipe defects (exfiltration/infiltration) and it was found that it lead to fatal failure in the soil where, the ground loosening caused by a succession of water supply and drainage cycles leads to a faster collapse than a continuous leakage system[4–21–22].

The aim of this study is to develop a dimensionless model to estimate the rate of soil erosion of local soil due to water exfiltration/infiltration cycle through a defect at the crown of sewer pipe. The local soil in (Karbala - Iraq) is mostly sandy soil, therefore it is more affected by erosion due to the movement of water through the defects of pipes [23–24]. Stormwater networks in Karbala are subjected to flooding, especially near the downstream areas, due to annual rains and groundwater. Furthermore, the city receives annually millions of pilgrims at short time, which puts enormous pressure on sewage network and leads to flooding and increases the risk of the emergence of sinkholes [6–25–26]. An economic apparatus has been used in this study which has flexibility to change the leak size and soil conditions to facilitate the measurement and observation process of soil erosion.

2. Dimensional analysis of soil erosion

Dimensional analysis was utilized as a tool in the present study, for suggesting a dimensionless model to predict the soil erosion of local experimental soil, due to water exfiltration/infiltration cycle through defective sewer pipe. Cyclic water flow and soil drainage through sewer pipe defect could lead to fatal failure in the soil and roads near the
defected sewer pipe and this could endanger lives and property. In the present study, cyclic water exfiltration/infiltration case was adopted, where a typical way of soil erosion to happen is when the stormwater and the sewage water are filling the sewers, and may exfiltrate from the sewer pipe through the defects, which cause disturbing the surrounding soil and leads to the fluidization of it Fig (1a). When the rain ends, the level of groundwater decreases, accompanied by the migration of the irritated soil granules into the sewer pipe through the pipe defects, the repetition of this process leads to the creation of the cavity, the ground cavity gradually expands and eventually a sinkhole Fig (1b).

(A) Exfiltration

(B) Infiltration

*Figure 1 typical way of soil erosion due to cyclic flow through pipe defect*
The application of dimensional analysis accounts for most of the factors influencing the erosion process. In the present study, leakage width, particle size distribution, dry density of the soil, initial water content, the height of water level in soil, the flow rate through the leakage and the number of cycle, assumed to be primary factors Table (1).

| factors                      | Abbreviation | units   | Basic dimension |
|------------------------------|--------------|---------|----------------|
| Leak width                   | B            | mm      | [L]            |
| Particle size distribution   | D70          | mm      | [L]            |
| Dry density of the soil      | ρ            | Kg/m³   | [M].[L]⁻³      |
| water flow rate              | Q            | ml/sec  | [L]².[T]⁻¹     |
| Initial water content        | W            | -       | -              |
| Number of cycle              | C            | -       | -              |
| Height of water level above  | Hw           | cm      | [L]            |
| the defect in soil           |              |         |                 |
| Acceleration                 | g            | m/sec²  | [L].[T]⁻²      |

The total amount of eroded soil ($E_{s(gm)}$, the total amount of soil that discharges with water into sewer pipe through defects) is the depended variable, which depend on the previously mentioned factors and can be represented using the list of these parameters, as shown in Equation (1)

$$E_s = f(\rho, B, Q, W, H_w, C, D70, g) \quad \ldots \ldots (1)$$

Equation (1) involves nine variables and three dimensions (mass, length and time). According to Buckingham’s Pi theorem [27]. Number of dimensionless variables required to describe the problem equals the number of dimensional variables, nine as indicated by Equation (1) minus the number of primary dimensions required to describe the problem. Depending on the way that the presented variables are merged, equation (1) can be reduced to a simple equation including six dimensionless parameters. “For a group of variables appearing in each dimensionless parameter. They have to be combined in such a way that the powers of each dimensions appearing in the group are separately equal to zero. A number of groupings is possible to form dimensionless parameters. Yet, the correct groups of
Dimensionless parameters are required to be selected and have to be proved by the experimental data.

The six dimensionless groups are generated by choosing three repeating variables and grouping them with one of the remaining parameters, forcing the product to be dimensionless. In this way, all the dimensionless groups can be constructed.

For \( \pi_1 \) (which is the first dimensionless group)

\[
1 = (\rho^a D_{70}^b Q^c Es) \quad \ldots \ldots \ldots (2)
\]

Where \( \rho, D_{70}, \) and \( Q \) are repeating variables

\( Es \) is the depended variable

By substituting the basic dimensions in the previous equation, we have:

\[
1^0 = M^a L^{-3a} \times L^b \times L^{3c} T^c \times M^1
\]

\[
\begin{array}{|c|c|}
\hline
\text{M} & a + 1 = 0 \\
\text{L} & -3a + b + 3c = 0 \\
\text{T} & c = 0 \\
\hline
\end{array}
\]

And therefore \( \pi_1 = \frac{Es}{\rho \ D_{70}^3} \)

Following the same method for the five remaining independed variables, the dimensionless equation takes the following form:

\[
\frac{Es}{\rho \ D_{70}^3} = f(C, W, \frac{B}{D_{70}}, \frac{Hw}{D_{70}}, \frac{g \ D_{70}^5}{Q^2}) \quad \ldots \ldots \ldots \text{Equation (3)}
\]

Where:

\( \frac{Es}{(D_{70})^3} \): (The Rate of Erosion) : It is a parameter that represents the total accumulated eroded soil relative to the physical quality of sandy soils (particle size and dry density).

\( C \): (Number of Cycle): Represents the sequence of the cycle in the periodic flow.

\( W \): (Initial Water Content): It is the amount of primary water content of the soil at the beginning of the periodic flow.

\( \frac{B}{D_{70}} \): (Leakage Width Ratio): This term represents the ratio between the size of leakage and the size of soil granules, which is one of the most important factors affecting the process of
erosion. The larger the leakage and the smaller the size of the soil granules, the greater the possibility and ease of movement of the soil into the pipe through the leakage.

**Hw/D70**: (Water Height Ratio): This parameter represents the ratio of water height in the soil to the size of soil granules. Where it gives an indication of the hydraulic state of the soil. The increase and ease of the spread of water in the soil leads to an increase in the amounts of soil transferred to the inside of the sewer pipe.

\((\frac{g \ (D70)^5}{Q^2})\): (waterflow discharge factor): This coefficient represents the ratio between the size of soil granules and the amount of waterflow discharge and its effect on soil erosion. The amount and speed of waterflow play an important role in soil erosion by erosion due to the continuous flow of water.

Having defined the dimensionless groups, it becomes necessary to determine the correlation between them in order to establish which dimensionless charts correlated to the best in terms of behavior, to facilitate the interpretation of the experimental work data.

A testing apparatus is built to simulate the process of soil erosion due to cyclic water flow through sewer pipe defect; it is designed to facilitate the change of the influencing parameters for both dimensional analysis and soil erosion investigation Fig (2). The proposed apparatus, its parts, details and way of operation are described in the (Experimental work).

![Figure 2](image_url)  
*Figure 2 I Schematic diagram of experimental setup and key features that facilitate the dimensional analysis*
3. Experimental work

3.1. Test apparatus

The test apparatus used in this study is shown in Fig. 3. This consists of: soil chamber, eroded soil collection unit, water flow unit and loaded weights. The soil chamber has dimensions of 800 mm long, 100 mm wide and 500 mm high. The front and back walls of the soil chamber are made from 10 mm tempered glass and the frame is made of steel. The transparent walls are used to allow monitoring of the cavity formation process. The eroded soil collection unit is placed at the base of the soil chamber. It has 100 mm diameter and 100 mm height with conical shaped bottom. The surface of this unit then has the same level with the base of the soil chamber. This represents a defect at the crown of the pipe. More leak sizes can be used by changing the eroded soil collection unit with those which have a different leak size. An O-ring is placed between the soil chamber base and the eroded soil collection unit to avoid the leakage of water or soil through this connection. The eroded soil collection unit has a water inflow valve located on the side of the unit and a drainage plug located at the bottom of the unit, where the drainage plug remains closed during the water inflow period and is opened at drainage.

Steel weights are placed on timber beams that are to be placed on the soil surface to simulate the weight of backfill soil above the sewer pipe. Different sewer depths can be simulated by changing the amount of load. A constant head tank is used with a 4 mm diameter high stiff pipe from the tank to the water inflow valve. The water flow rate is calibrated to required value by measuring water volume with time.
Figure 3 Schematic diagrams of experimental setup
3.2. Testing Materials

Local sandy soil was used in this study. Soil is provided from local materials in Karbala governorate, more specifically from Al-Hur area. Soil samples were sieved, according to ((ASTM, 2007), Astm D 422 standard test method for particle size analysis of soils). The gradation is shown in Fig. (5). Other specifications is shown in Table 2. It is classified as a poorly graded sandy soil according to the unified soil classification system (ASTM D 2487 - 17).
### Table 2 Experimental sandy soil properties

| Property                        | ASTM Designation | Value  |
|---------------------------------|------------------|--------|
| Specific gravity                | ASTM D854-14     | 2.65   |
| Coefficient of Gradation        | ASTM D2487-11    | 1      |
| $C_c = D_{30}/D_{60} D_{10}$    |                  |        |
| Coefficient of Uniformity       | ASTM D2487-11    | 2.28   |
| $C_u = D_{60}/D_{10}$           |                  |        |
| $D_{70}$                        | -                | 0.85 mm|
| Optimum water content           | -                | 9%     |

### 3.3. Testing procedure

The eroded soil collection unit with the desired leak width was placed at the bottom of the soil chamber and connected with screws, where the surface of the unit is then at same level with the base plate of soil chamber and the leak length is perpendicular to the glass walls. To prevent the soil from leaking out through the defect while filling the soil chamber, icing sugar was placed in the eroded soil collection unit. This substance dissolves when water flows into the soil chamber. Soil was then added to the soil chamber in the form of layers. Each layer is 50 mm deep and compacted to 80% of relative density.

Steel weights were then placed on the timber beam that was placed on the soil surface to simulate 1m of soil depth above the sewer pipe. It was then left for 12 hours to reduce the potential creep effect. The desired volume of water was applied to the model ground through the leak. After 2 minutes, the drainage plug was opened to let water and eroded soil flow out. This process of water supply/drainage is called a cycle and was repeated 10 times for each run. For each cycle, the dry weight of the eroded soil was measured and then sieved.

### Table 3 Experimental tests matrix

| $B_{(mm)}$ | $H_{W_{(mm)}}$ | $W_{(\%)}$ | $Q_{ml/sec}$ | $C$                                      |
|------------|---------------|------------|--------------|------------------------------------------|
| 3          | 11.5          | 0          | 10           | 1 ... 10 cycle for each Hw value          |
|            | 12.5          |            |              |                                          |
|            | 13            |            |              |                                          |
|            | 11.5          | 10         |              |                                          |
|            | 12.5          |            |              |                                          |
|            | 13            |            |              |                                          |
| 4          | 11.5          | 0          |              |                                          |
|            | 12.5          |            |              |                                          |
|            | 13            |            |              |                                          |
| 5          | 11.5          | 0          |              |                                          |
|            | 11.5          |            |              |                                          |
4. Results and Discussion

Experimental tests performed on local soil using varying matrix of influencing factors as shown in Table (3), and the dry eroded soil mass was measured for each cycle. The collected results is 256 dataset of cycles, divided randomly into 189 cycle to generate the model and the other 67 dataset was used to validate the model. Since the dependent variable is the total (accumulated) eroded soil mass, then the other dimensionless groups was multiplied by the number of cycle (C) to show their effect in the cyclic flow. The data was analyzed using SPSS and correlated using Pearson correlation. The results show that the leakage width has the largest impact on the rate of erosion.

Fig. (6) shows the strong positive relationship between the ratio of leakage width to soil particle size \((B/D_{70})\), and the rate of erosion \((E_s/\rho D_{70}^{3})\). The correlation value is 0.87. The erosion rate increases with increasing of the ratio of \((B/D_{70})\), where larger leak width means larger amounts of soil that can be carried by the water into the sewer pipe, while the converse is true for the soil particle size.
Fig. (7) shows the effect of the ratio between the height of water in the soil and the soil particle size \( \frac{H_w}{D_{70}} \). The relation is positive, the correlation value is 0.62. Higher water level means more water spread in the soil and thus more soil becomes disturbed and able to leave with water. Furthermore, higher water level applies more pressure on the leakage and pushes more soil into the pipe.
Fig (8) shows the effect of initial water content on the rate of erosion. The effect is limited compared to the leakage width. The relation is positive, the correlation value is 0.6.

![Figure 8 The effect of initial water content on the rate of erosion](image)

Fig (9) shows the effect of different water flow rates on the rate of soil erosion. Based on the limitations of the experimental tests, it can be found that water flow rate has no noticeable effect on the rate of soil erosion. Previous studies were shown that the value of water flow rate through the leakage is important in the case of continuous sewer or water pipe exfiltration, while the present study focused on cyclic water flow.

![Figure 9 The effect of water flow rate value on the erosion](image)
SPSS software was carried out to achieve the analysis and build the required model. The prediction model was linear; some of the dimensionless groups have been modified by multiplying it with another or by using the natural logarithm. The analysis results of the model is shown in Tables (4, 5, 6 and 7). The other 67 data sets was used to validate the proposed dimensionless model. Fig (10) presents the comparison between the experimental data and the estimated values of the rate of soil erosion. Value of coefficient of determination was of \( R^2 = 0.864 \). It can be concluded that the rate of erosion for the local sandy soil under similar conditions can be reasonably estimated using the dimensional analysis.

### Table 4 Prediction Model

| Developed model | \( \frac{E_m}{\rho D^{7/3}} = -0.524 - 0.905 \ (C + CW) + 0.063 \ (C \ast \frac{H_w}{D^{7/3}} \ast \ln\left(\frac{B}{D^{7/3}}\right)) \) |

### Table 5 Model Summary

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate |
|-------|---|----------|-------------------|---------------------------|
| 1     | .935\(^a\) | .874     | .873              | .8620900                  |

\(^a\) Predictors: (Constant), CHLN(B), CW

### Table 6 The Coefficients Estimation

| Model | Unstandardized Coefficients | Standardized Coefficients | t | Sig. |
|-------|-----------------------------|---------------------------|---|------|
|       | B                           | Std. Error                | Beta |   |     |
| 1     | (Constant)                  | -.524                     | .135 | -3.884 | .000 |
|       | CW                          | -.905                     | .057 | -1.065 | .15956 |
|       | CHLN(B)                     | .063                      | .002 | 1.819 | 27.239 .000 |

\(^a\) Dependent Variable: Es

### Table 7 ANOVA Index
### ANOVA

| Model      | Sum of Squares | df | Mean Square | F      | Sig. |
|------------|----------------|----|-------------|--------|------|
| Regression | 958.777        | 2  | 479.388     | 645.034| .000 |
| Residual   | 138.235        | 186| .743        |        |      |
| Total      | 1097.012       | 188|             |        |      |

a. Dependent Variable: Es  
b. Predictors: (Constant), CHLN(B), CW

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**Figure 10** comparison between the experimental and predicted values of the rate of erosion

95% confidence interval
5. Conclusions

Based on the analysis of the present study, the following conclusions can be drawn:

- It was found that the proposed dimensional analysis prediction model can reasonably estimate the rate of erosion for the local sandy soil. The value of coefficient of determination was of \( R^2 = 0.873 \).
- It was found that both leakage width as well as soil particle size have significant effects on the amount of soil draining into the sewer pipe. The results of the experiments and data analysis showed that the ratio of B/D70 is the most influential factor on soil erosion among the other factors. Where: 7mm of leakage width has 14.5 times, 6mm has 13.4, 5mm has 13 and 4mm has 6.4 times the amount of eroded soil mass compared to the amount of eroded soil mass collected through the leakage width of 3mm.
- When the ratio of D70/B is less than 0.17, the eroded soil drains through the pipe leakage easily and continuously.

References

1. X. Cui, J. Li, A. Chan, & D. Chapman, Coupled DEM–LBM simulation of internal fluidisation induced by a leaking pipe. Powder Technology, 254 (2014) 299–306.
2. S. Guo & D. Z. Zhu, Soil and Groundwater Erosion Rates into a Sewer Pipe Crack. Journal of Hydraulic Engineering, 143 (2017) 06017008. https://doi.org/10.1061/(ASCE)HY.1943-7900.0001306.
3. J. P. Davies, B. A. Clarke, J. T. Whiter, & R. J. Cunningham, Factors influencing the structural deterioration and collapse of rigid sewer pipes. Urban Water, 3 (2001) 73–89. https://doi.org/10.1016/S1462-0758(01)00017-6.
4. T. Mukunoki, N. Kumano, & J. Otani, Image analysis of soil failure on defective underground pipe due to cyclic water supply and drainage using X-ray CT. Frontiers of Structural and Civil Engineering, 6 (2012) 85–100. https://doi.org/10.1007/s11709-012-0159-5.
5. K. A. Mohsen, B. K. Nile, & W. H. Hassan, Experimental work on improving the efficiency of storm networks using a new galley design filter bucket. IOP Conf. Ser. Mater. Sci. Eng. (IOP Publishing, 2020), p. 12094.
6. H. A. Obaid, S. Shamsuddin, K. N. Basim, & C. Shreeshivadasan, Modeling sewer overflow of a city with a large floating population. Hydrology: Current Research, 5 (2014) 1.
7. S. Guo, T. Zhang, Y. Zhang, & D. Z. Zhu, An approximate solution for two-dimensional groundwater infiltration in sewer systems. Water Science and Technology, 67 (2013) 347–352. https://doi.org/10.2166/wst.2012.568.
8. J. W. Delleur, Sewerage failure, diagnosis and rehabilitation. Urban Drain. Rehabil. Programs Tech. Sel. Pap. Urban Drain. Rehabil. from 1988-1993 Water Resour. Plan. Manag. Div. Conf. Sess. (ASCE, 1994), pp. 11–28.
9. K. Than, Guatemala Sinkhole Created by Humans, Not Nature. (2010). https://news.nationalgeographic.com/news/2010/06/100603-science-guatemala-sinkhole-2010-humans-caused/.
10. S. Bonelli, Erosion of Geomaterials (2013). https://doi.org/10.1002/9781118561737.
11. G. J. Weil, C. P. Court, & S. Louis, Remote Infrared Thermal Sensing of Sewer Voids. 2454 (1846).
12. J. P. Galve, F. Gutiérrez, J. Guerrero, J. Alonso, & I. Diego, Optimizing the
application of geosynthetics to roads in sinkhole-prone areas on the basis of hazard models and cost-benefit analyses. Geotextiles and Geomembranes, 34 (2012) 80–92. https://doi.org/10.1016/j.geotexmem.2012.02.010.

13. C. J. Rogers, Sewer deterioration studies the background to the structural assessment. (1986).

14. R. A. Fenner, Influence of sewer bedding arrangements on infiltration rates on soil migration. Proceedings of ICE, Municipal Engineer (Institution of Civil Engineers), 8 (1991) 105–117.

15. S. Guo, Y. Shao, T. Zhang, D. Z. Zhu, & Y. Zhang, Physical Modeling on Sand Erosion around Defective Sewer Pipes under the Influence of Groundwater. Journal of Hydraulic Engineering, 139 (2013) 1247–1257. https://doi.org/10.1061/(ASCE)HY.1943-7900.0000785.

16. T. Mukunoki, J. Otani, & R. Kuwano, Visualization of cavity generation in soils on sewerage defects using X-ray CT. Proc. 13th Asian Reg. Conf. SMGE (2007), pp. 485–488.

17. A. Ghulam, K. Basim, & J. Al-Baidhani, Evaluation of the Effect of Leak Size of Defective Sewer Pipes on Soil Erosion. Journal of Engineering and Applied Sciences, 13 (2018) 10708–10712. https://doi.org/10.36478/jesaci.2018.10708.10712.

18. M. Kamel, Sheriff.; Meguid, an Experimental Study of Soil Erosion Around Leaking Pipes. (2008).

19. M. Sato, Model tests for the evaluation of formation and expansion of a cavity in the ground. (2010) 581–586.

20. D. Zhang, W. Du, & C. Gao, Proceedings of GeoShanghai 2018 International Conference: Multi-physics Processes in Soil Mechanics and Advances in Geotechnical Testing (Springer Singapore, 2018). https://doi.org/10.1007/978-981-13-0095-0.

21. T. Karoui, S.-Y. Jeong, Y.-H. Jeong, & D.-S. Kim, Experimental study of ground subsidence mechanism caused by sewer pipe cracks. Applied Sciences (Switzerland), 8 (2018). https://doi.org/10.3390/app8050679.

22. T. MUKUNOKI, N. KUMANO, J. OTANI, & R. KUWANO, Visualization of Three Dimensional Failure in Sand Due To Water Inflow and Soil Drainage From Defective Underground Pipe Using X-Ray Ct. Soils and Foundations, 49 (2009) 959–968. https://doi.org/10.3208/sandf.49.959.

23. K. Basim, Estimation Of The Floods That Occur In The Drainage Network During The Rainy Season. (2018).

24. B. K. Nile, Effectiveness of hydraulic and hydrologic parameters in assessing storm system flooding. Advances in Civil Engineering, 2018 (2018).

25. B. K. Nile, W. H. Hassan, & B. A. Esmaeel, An evaluation of flood mitigation using a storm water management model [SWMM] in a residential area in Kerbala, Iraq. MS&E, 433 (2018) 12001.

26. K. Basim, W. Al-Mussawi, & G. Alshammaa, Analysis of the effect of climate change on rainfall intensity and expected flooding by using ANN and SWMM programs. ARPN Journal of Engineering and Applied Sciences, 14 (2019) 974–984.

27. E. Buckingham, On physically similar systems; illustrations of the use of dimensional equations. Physical review, 4 (1914) 345.