Electric field in stationary superconductors

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It is generally accepted that vortex core is charged, what illustrates that even in stationary superconductors electric field may be present. Vortex structure and other properties of superconductors are usually calculated in the framework of the Ginzburg-Landau theory, which does not cover electric field. We show that the generalization of the GL theory due to Bardeen allows one to derive a third GL equation for the electrostatic potential. Since the Bardeen theory applies to all temperatures, the presented theory enables one to calculate charge profiles of the vortex under quite general conditions. The theory is consistent with the BCS-Gorkov results.

I. INTRODUCTION

As was pointed out by Adkins and Waldram \cite{1} and later followed by others, see e.g. \cite{2}, a difference of the chemical potential in a superconducting versus normal state, is compensated by the electrostatic potential. If the gap is modulated in space, say due to supercurrents, this potential results in an internal electric field. This idea has been used later to estimate the charge of vortices \cite{3,4}. For superconductor of the first kind, the presence of nonzero electric field in the superconductor carrying transport current was first experimentally proved by Bok and Klein \cite{5} who measured the surface charge. For recent measurement with this method see e.g. \cite{6}. A new experiment, which allows one to measure the electric field in the bulk of the superconductor of the second kind in the mixed state, has been performed by Kumagai, Nozaki and Matsuda \cite{7}.

A vortex structure is usually calculated in the framework of the Ginzburg-Landau (GL) theory, which is applicable only for temperatures close to the critical temperature $T_c$. In its standard formulation does not cover the electric field. The electric field can be evaluated, however, from the Poisson equation, called by Jakeman and Pike \cite{8} the third GL equation, to reflect that the source term (density of unscreened charge) is a functional of the GL function.

Various expressions for the source term of the third GL equation can be found in the literature \cite{1,2,3,4}, most of contributions are of similar amplitude and seem to be merely alternative approximations of the same mechanism. This is not, however, the case. There are at least three distinguishable mechanisms which create the electric field in superconductors. To clarify this point, we derive the third GL equation with all these mechanisms from a simple phenomenologic theory of the GL type. Not to be restricted to temperatures close to $T_c$, we adopt Bardeen’s extension of the GL theory \cite{11} based on the Gorter-Casimir model \cite{12}.

II. GORTER-CASIMIR TWO FLUID MODEL

The free energy of the normal state metal without electric and magnetic fields reads

$$F_n = U - \frac{1}{2}\gamma T^2,$$

(1)

where $U$ is internal energy including lattice vibrations and $\gamma T$ is the electronic specific heat. According to the Gorter-Casimir two fluid model, the free energy of a superconducting state without fields can be written as

$$F_s = U - \frac{1}{4}\gamma T_c^2 \omega - \frac{1}{2}\gamma T^2 \sqrt{1-\omega}.$$

(2)

Equilibrium value of the order parameter $\omega$ makes $F_s$ minimum. From the condition $\frac{\partial F_s}{\partial \omega} = 0$ it follows that

$$\omega_{eq} = 1 - t^4,$$

(3)

where $t = T/T_c$ is the reduced temperature. At $T = 0$ the equilibrium value $\omega_{eq} = 1$ and $F_n - F_s = \frac{1}{2}\gamma T_c^2 = \frac{1}{2}\mu H_c^2$, where $H_c$ is the thermodynamic critical field. At $T = T_c$ the equilibrium value of $\omega$ is zero and $F_s = F_n = U - \frac{1}{4}\gamma T_c^2$ as it should. According to its temperature dependence, the order parameter $\omega$ can be identified with the square of the GL effective wave function $\psi$, normalized to the superfluid fraction, $|\psi|^2 = n_s$, where $n_s$ is the superconducting charge carriers density and $n$ is the total density of charge carriers.

Near the critical temperature the order parameter $\omega$ is small. Using the expansion $\sqrt{1-\omega} \approx 1 - \frac{1}{2}\omega - \frac{1}{4}\omega^2$ the Ginzburg-Landau free energy, $\alpha |\psi|^2 + \frac{1}{2}\beta |\psi|^4$ is recovered, where the parameters $\alpha$ and $\beta$ in the used normalization are $\alpha = -2\mu H_c^2(1-t)/n$ and $\beta = \mu H_c^2/n^2$. Accordingly, one can see the free energy of Ginzburg and Landau as an asymptotic form of the more general Gorter-Casimir model.

III. BARDEEN’S EXTENSION OF THE GL THEORY

To extend the region of applicability of the Ginzburg-Landau theory, Bardeen \cite{13} replaced the Ginzburg-
Landau polynomial free energy by the free energy due to Gorter and Casimir. The free energy then reads

\[ \mathcal{F} = U - \frac{1}{4} \frac{T^2}{\gamma} |\psi|^2 - \frac{1}{2} \gamma T^2 \sqrt{1 - |\psi|^2} + \frac{\mathbf{B}^2}{2 \mu} + \frac{n}{2} \left( \frac{(i\hbar \nabla + e^* \mathbf{A})|\psi|^2}{2m^*} - \frac{\mathbf{E}^2}{2} + \varphi \rho \right). \]  

(4)

We note that the last two terms, which describe the electrostatic field, have not been assumed by Bardeen \cite{11}. The magnetic and electric fields are given by standard definitions, \( \mathbf{B} = \nabla \times \mathbf{A} \) and \( \mathbf{E} = -\nabla \varphi \).

The state of the system is given by the minimum of the total free energy. Accordingly, variations of \( \mathcal{F} \) with respect to the vector potential \( \mathbf{A} \), the scalar potential \( \varphi \), the GL function \( \psi \) and the electron density \( n \) have to vanish.

Variation \( \frac{\delta \mathcal{F}}{\delta \mathbf{A}} = 0 \) leads to extended Ginzburg-Landau equation, see \cite{12}

\[ \frac{(i\hbar \nabla + e^* \mathbf{A})^2}{2m^*} \psi = \frac{\mu H^2_c}{n} \left( 1 + \frac{t^2}{\sqrt{1 - |\psi|^2}} \right) \psi. \]  

(5)

Variation \( \frac{\delta \mathcal{F}}{\delta \varphi} = 0 \) leads to the Maxwell equation,

\[ \nabla \times \mathbf{B} = -\mu n \frac{1}{m^*} \text{Re} \psi (i\hbar \nabla + e^* \mathbf{A}) \psi. \]  

(6)

In principle, density \( n \) of the total charge is modified by internal electric fields. Accordingly, these equations should be treated together with the Poisson equation presented below. Since both equations depend on the total density \( n \), not on the deviation from the homogeneous value, it is clear that for calculating the magnetic properties one can safely neglect the effect of the small electric field on the density. Within this approximation, the set \( (5) \) and \( (6) \) is closed.

IV. THIRD GL EQUATION

Variation \( \frac{\delta \mathcal{F}}{\delta n} = 0 \) leads to the Poisson equation, \(-\epsilon \nabla^2 \varphi = \rho \). Finally, variation \( \frac{\delta \mathcal{F}}{\delta \psi} = 0 \) yields the third GL equation,

\[ \epsilon \varphi = -\Delta_F^2 \nabla^2 \varphi + \frac{1}{4} \frac{\partial (\gamma T^2_c)}{\partial n} |\psi|^2 + \frac{1}{2} \frac{T^2}{\gamma} \frac{\partial^2}{\partial n^2} \sqrt{1 - |\psi|^2} - \frac{1}{2} \left( 1 - \frac{\partial \ln m^*}{\partial \ln n} \right) \frac{(i\hbar \nabla + e^* \mathbf{A})|\psi|^2}{2m^*}. \]  

(7)

The first term of \( (7) \) represents the screening on the Thomas-Fermi length \( \lambda^2_T = \frac{\epsilon}{2N \gamma^2} \), where \( N \) is the single-spin density of states. In deriving this term we have used the linear response approximation, \( \frac{\partial \ln}{\partial \ln n} = 2 \). The magnetic and electric fields are given by standard definitions, \( \mathbf{B} = \nabla \times \mathbf{A} \) and \( \mathbf{E} = -\nabla \varphi \).

\[ \mathcal{F} \approx E_F^0 n + \frac{\partial E_F^0}{\partial n} \delta n = \frac{\partial E_F^0}{\partial n} \delta n, \]  

with \( E_F^0 = 0 \), and employed the Poisson equation. Accordingly, equation \( (7) \) can be viewed as the Poisson equation with screening.

The rest of terms in the right hand side of \( (7) \) are source terms of the third GL equation corresponding to various mechanisms. The second term is the internal pressure due to the dependence of the condensation energy on the total density \( n \). The third term is thermoelectric field of the normal metal reduced by factor \( \sqrt{1 - |\psi|^2} \). The fourth term of is the non-local Bernoulli potential \( \int_0^\delta \frac{d\tilde{\psi}}{d\tilde{\psi}} \) but reduced by \( |\psi|^2 \) in the spirit of the so called quasiparticle screening \( \mathcal{F} \). The correction due to the density dependency of the Cooperon mass, \( \propto \frac{\partial \ln m^*}{\partial \ln n} \), has been derived by Rickayzen \cite{1} but it is usually omitted.

V. COMPARISON WITH THE BCS RESULTS

Using the BCS formula for the condensation energy \cite{13},

\[ \varepsilon_{\text{con}} = \frac{1}{4} \gamma T_c^2 = \frac{1}{2} \frac{\mu H^2_c}{n} = \frac{1}{2} \lambda^2 \Delta_0^2, \]  

(8)

with \( \Delta_0 = 2h \omega_D \exp(-1/\mathcal{N}V) \), and the relation of the gap to the GL function, \( \Delta = \Delta_0 \psi \), one recovers from the second term of \( (7) \) the BCS expression,

\[ \frac{1}{4} \frac{\partial \gamma T^2_c}{\partial \ln n} |\psi|^2 \approx \frac{\Delta^2}{2} \frac{\partial \ln \mathcal{N}}{\partial E_F} \ln \left( \frac{2\omega_D \hbar}{\Delta_0} \right) \approx \frac{\Delta^2}{2} \frac{1}{2E_F} \frac{1}{\mathcal{N}V}. \]  

(9)

The first expression has been derived e.g. in \cite{14}. The second form uses the parabolic band approximation, \( \frac{\partial \mathcal{N}}{\partial \ln n} = \frac{1}{2E_F} \). Together with the estimate, \( \mathcal{N}V \sim 1 \), it gives the formula used e.g. in \cite{1}.  

VI. CONCLUSION

Starting from the combination of the Gorter-Casimir and Ginzburg-Landau free energies further extended by the energy of the electrostatic field, we have derived a set of three GL equations. This set includes the extended GL equation \( (5) \), the Maxwell equation \( (6) \) for the magnetic field and the Poisson equation \( (7) \) for the electrostatic potential. In particular limits, the theory reproduces previous results.

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[1] C. J. Adkins and J. R. Waldram, Phys. Rev. Lett. 21 (1968) 76.
[2] K. M. Hong, Phys. Rev. B 12 (1975) 1766.
[3] G. Blatter, M. Fiege’l’m, V. Geshkenbein, A. Larkin and A. van Otterlo, Phys. Rev. Lett. 77 (1996) 566.
[4] D. I. Khomskii and A. Freimuth, Phys. Rev. Lett. 75 (1995) 1384.
[5] J. Bok and J. Klein, Phys. Rev. Lett 20 (1968) 660.
[6] Y. N. Chiang and O. G. Shevchenko, Fizika nizikh temperatur 22 (1996) 669.
[7] K. Kumagai, K. Nozakii and Y. Matsuda, cond-mat/0012492.
[8] E. Jakeman and E. R. Pike, Proc. Phys. Soc. 91 (1967) 422.
[9] G. Rickayzen, J. Phys. C 2 (1969) 1334.
[10] J. Koláˇcek, P. Lipavský and E. H. Brandt, Phys. Rev. Lett. 86 (2001) 312.
[11] J. Bardeen, Theory of Superconductivity in Handbuch der Physik, Bd. XV. (1955).
[12] C. J. Gorter and H. B. G. Casimir, Phys. Z. 35 (1934) 963.
[13] A. G. van Vijfeijken and F. S. Staas, Phys. Lett. 12 (1964) 175.
[14] M. Tinkham, Introduction to superconductivity (McGraw-Hill, New York,1996).