Effect of bulk viscosity in cosmic acceleration

Sankarsan Tarai, Pratik P Ray, B. Mishra, S.K. Tripathy

In the framework of extended theory of gravity, an accelerating cosmological model has been presented in this paper in an anisotropic space-time. The extended gravity considered as \( f(R,T) \) gravity with the matter field as viscous fluid. The role of viscous coefficient has been discussed on the cosmic expansion issue. In the developed formalism a variable Hubble parameter has been incorporated to express the dynamical parameter with respect to redshift. The model is observed to be compatible with recent observational data.

**PACS number:** 04.50kd.
**Keywords:** Extended gravity, Anisotropic metric, Viscous fluid, Deceleration parameter.

## I. INTRODUCTION

The need of an alternative theory to Einstein’s General Relativity (GR) or any other alternative theory had become inevitable, when the existing theories unable to answer some key issues on the gravitational phenomena. Some theories are more phenomenological and others motivated by theoretical and observational results. In a similar note, an alternative to GR has become inevitable when several cosmological experiments claim the accelerated expansion of the Universe [1–5]. In addition, several experiments have indicated the presence of some kind of aether or dark energy in the Universe. GR and other alternative theories of gravity could not make a satisfactory answer to this claim. Therefore, researchers motivated to find an alternative gravity theory to get an explanation pertaining to the present astrophysical observations. The search for a new gravity has not been confined to the reason of presence of dark energy, which is the main part of the energy budget of the Universe, but also for the huge amounts of unseen matter. It is a belief that this unseen matter may give some explanation to the formation of structure, gravitational lensing and the rotation curves of galaxies. Of course the study of accelerated expansion of the Universe can be performed with the unknown form of matter and energy driven through negative pressure, but the change in the geometric part of gravitational sector of GR received a lot of attention.

Among the different suggested modifications to GR, the \( f(R) \) gravity theory requires a geometry modification and generalizes GR. Actually \( f(R) \) gravity is a family of theories defined by different functions of the Ricci scalar \( R \) else than the linear one used in GR. Again, this gravity can be studied in two approaches such as the Palatini \( f(R) \) gravity and metric-affine \( f(R) \) gravity. So, the action of \( f(R) \) gravity becomes, \( \mathcal{S} = \frac{1}{16\pi} \int f(R, T) \sqrt{-\gamma} d^4 x + \mathcal{L}_m \sqrt{-\gamma} d^4 x, \) \( (1) \) where \( f(R, T) \) is an arbitrary function of Ricci scalar \( R \) and \( T = T_{ij} g^{ij} \), where \( T_{ij} \) is the energy momentum tensor, \( (T_{ij}) \). The matter Lagrangian, \( \mathcal{L}_m = -p \). The Lagrangian density of matter field depends only on the metric tensor component \( g_{ij} \) and not on its derivatives. So, the stress energy tensor of matter can be,

\[
T_{ij} = g_{ij} \mathcal{L}_m - 2 \frac{\partial \mathcal{L}_m}{\partial g^{ij}}.
\]  

\( \text{[a] Centre of High Energy and Condensed Matter Physics, Department of Physics, Utkal University, Vani Vihar, Bhubaneswar, India-751004 E-mail: tsankarsan87@gmail.com} \)
\( \text{[b] Department of Mathematics (SSL), Vellore Institute of Technology-Andhra Pradesh University, Andhra Pradesh - 522237, India, E-mail: pratik.chika9876@gmail.com} \)
\( \text{[c] Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad-500078, India, E-mail:bivudutta@yahoo.com} \)
\( \text{[d] Department of Physics, Indira Gandhi Institute of Technology, Sarang, Dhenkanal, Odisha 759146, India, E-mail:tripathy_sunil@rediffmail.com} \)
By varying the modified four-dimensional Einstein-Hilbert action [1] with respect to the metric tensor components $g^{ij}$, the algebraic function $f(R, T)$ has been chosen as a sum of two independent functions $f(R, T) = f_1(R) + f_2(T)$, where $f_1(R)$ and $f_2(T)$ are respectively some functions of curvature $R$ and trace $T$. The field equations of $f(R, T)$ gravity can be obtained as [21],

$$f_R R_{ij} - \frac{1}{2} f(R) g_{ij} + (g_{ij} \Box - \nabla_i \nabla_j) f_R = 8\pi T_{ij} + f_T T_{ij} + \left[ \bar{p} f_T + \frac{1}{2} f(T) \right] g_{ij}. \quad (3)$$

Here, $f_R = \frac{\partial f(R)}{\partial R}$, $f_T = \frac{\partial f(T)}{\partial T}$ and $\bar{p}$ be the effective viscous pressure. Three functional forms are suggested as (i) $f(R, T) = R + 2f(T)$, (ii) $f(R, T) = f_1(R) + f_2(T)$ and (iii) $f(R, T) = f_1(R) + f_2(R)f_3(T)$. In order to formulate the cosmological model, in this problem, we have assumed $f(R, T) = f_1(R) + f_2(T)$, where $f_1(R) = \mu R$ and $f_2(T) = \mu T$, $\mu$ be the scaling constant. Now, the $f(R, T)$ field equations (3) reduced to,

$$R_{ij} - \frac{1}{2} R g_{ij} = \left( \frac{8\pi + \mu}{\mu} \right) T_{ij} + \Lambda(T) g_{ij}. \quad (4)$$

where, $\Lambda(T) = \bar{p} + \frac{1}{2}T$ can be treated as the effective cosmological constant that evolves with cosmic time. Many investigations are surfaced to find the cosmic nature of the Universe particularly on the expanding Universe in $f(R, T)$ gravity. Under this gravity, Das et al. [25] have obtained the solutions that describes the interior of a compact star whereas Deb et al. [26] have presented the spherically symmetric strange star solution. Fisher and Carlson [27] have reexamined $f(R, T)$ gravity and argued that the separable trace term should be included in the matter Lagrangian. Shabani and Zaia [28] have studied the classical bouncing solutions in FRW background and Tripathy et al. [29] have presented the bouncing solution with exponential and power law cosmology. Barbar et al. [30] have analyzed the viability of bouncing cosmology by incorporating the square of energy momentum tensor. Wu et al. [31] have obtained the generalized Friedmann equations and studied its cosmological implications. Elizalde and Khurshudyan [32] have investigated static wormhole cosmological model in $f(R, T)$ gravity. In this gravity, Khan et al. have studied the gravitational collapse of perfect fluid in a spherically symmetric space-time [33], and effects of electromagnetic field on gravitational collapse [34]. Triapthy and Mishra [35] have studied the phantom cosmology whereas Mishra and Triapthy [36] investigated the little rip and hyperbolic form of scale factor in $f(R, T)$ gravity. Triapthy et al. have constructed some cosmic transit model in $f(R, T)$ gravity theory and studied their cosmographic aspects [37]. Saridakis et al. [38] have investigated the cosmological implications of Myrzakulov $f(R, T)$ gravity. Recently Triapthy [39] has modelled traversable wormholes in this modified gravity considering the Casimir effect along with corrections arising out of the generalized uncertainty principle. Several cosmological models have been studied in $f(R, T)$ gravity to study different different aspects of the cosmic phenomena issue.

We have organised the paper as follow: In section 2, the basic equations are formed in an anisotropic space-time along with the dynamical parameters. In section 3, the dynamics of the model derived with a variable deceleration parameter and the behaviours are analysed. Section 4 contains the results and discussions on the model.

II. FORMATION OF BASIC EQUATIONS

In this section, we have presented the mathematical formalism of the cosmological model to study the dynamical behaviour with an anisotropic space-time. The standard FLRW model is homogeneous and isotropic. The CMB observation has suggested a small amount of anisotropy at the late time of cosmic evolution. So, in order to get into the study of anisotropy, we consider an anisotropic but homogeneous space-time dubbed as Bianchi $VI_h$ space-time. The subscript can take integral values namely $h = -1, 0, 1$. In some of our earlier studies [40], we have observed that $h = -1$ provides significant results on the cosmological problems as compared to $h = 0, 1$, therefore we shall consider Bianchi type $VI_{-1}$ space-time as,

$$ds^2 = dt^2 - C_1^2 dx^2 - C_2^2 e^{2x} dy^2 - C_3^2 e^{-2x} dz^2, \quad (5)$$

where, $C_i = C_i(t), i = 1, 2, 3$. It has been seen that several cosmological models are framed with isotropic space-time and the matter in the form of perfect fluid. However, there is a scope to have a better cosmological model as compared to the standard perfect fluid model. So, here we are intending to introduce the bulk viscous fluid which resembles an important part in the the study of cosmic expansion history of the Universe. It is worth to mention here that the viscous fluid models are instrumental in order to explain the observed highly isotropic matter distribution on the high entropy per baryon. Also, the the strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of black- body radiation. There have been considerable interests in cosmological models with bulk viscosity, since bulk viscosity leads to the accelerated expansion phase of the early Universe, popularly known as the inflationary phase [41][43]. Mishra et al. [44] have studied the dynamical properties of the cosmological model with...
viscous cosmology in $f(R, T)$ gravity. Applying the Misner-Sharp approach, Ahmed and Abbas \cite{45} have studied the dissipative gravitational collapse in $f(R, T)$ gravity. Also, Singh and Kumar \cite{46} have studied the holographic dark energy model in extended modified gravity with the presence of bulk viscosity. The energy momentum tensor $T_{ij}$ for the viscous fluid can be expressed as

$$T_{ij} = (\rho + \bar{p})u_iu_j - \bar{p}g_{ij},$$

(6)

where $\rho$ is the proper energy density and $\bar{p} = p - \xi \theta$ is the viscous pressure and $\xi$ is the bulk viscous coefficient. In the co-moving coordinate system, the field equations (4) for the metric (5) and energy momentum tensor (6) can be obtained as,

$$\frac{\dot{C}_2}{C_2} + \frac{\dot{C}_3}{C_3} + \frac{\dot{C}_2 C_3}{C_2 C_3} + \frac{1}{C_1^2} = -\beta \bar{p} + \frac{\rho}{2}$$

(7)

$$\frac{\dot{C}_1}{C_1} + \frac{\dot{C}_3}{C_3} + \frac{\dot{C}_1 C_3}{C_1 C_3} - \frac{1}{C_1^2} = -\beta \bar{p} + \frac{\rho}{2}$$

(8)

$$\frac{\dot{C}_1 \dot{C}_2}{C_1 C_2} + \frac{\dot{C}_2 C_3}{C_2 C_3} + \frac{\dot{C}_3 C_1}{C_3 C_1} - \frac{1}{C_1^2} = \beta \rho - \frac{\bar{p}}{2}$$

(9)

$$\frac{\dot{C}_2}{C_2} - \frac{\dot{C}_3}{C_3} = 0$$

(10)

$$\frac{\dot{C}_1}{C_1} - \frac{\dot{C}_2}{C_2} + \frac{\dot{C}_3}{C_3} - \frac{1}{C_1^2} = -\beta \bar{p} + \frac{\rho}{2}$$

(11)

An over dot on the field variable denotes the differentiation with respect to cosmic time $t$. Also, $\beta$ is chosen to be a value such as, $\beta = \left( \frac{8\pi}{7} + \frac{3}{2} \right)$. The reason behind this particular choice is to look at the present problem from the backdrop of GR where the modified gravity field equation appears as Einstein field equation with a time varying effective cosmological constant $\Lambda$. Due to this non-linearity nature of the field equations (7)-(11), it would be difficult to determine the physical and geometrical parameters. Therefore, we shall relate the metric potentials in terms of the directional Hubble parameters $H_j$ along the orthogonal coordinate axes as, $H_j = \frac{\dot{C}_j}{C_j}$, $j = 1, 2, 3$. As a consequence, the mean Hubble parameter, $H = \frac{1}{3} (H_j)$, $j = 1, 2, 3$. We can infer from the field equation (11), $H_2 = H_3$ by suitably absorbing the integration constant. Also, the Hubble parameter and scale factor can be related as $H = \frac{\dot{R}}{R}$. Still then no direct relation is possible between $H_1$ and $H_3$, unless we make a prior assumption. Since our study is basically on an anisotropic cosmological model, apart from the anisotropic incorporated in the space-time, here we shall incorporate some amount of anisotropy among the field variable in the form $H_1 = kH_3$, $k \neq 1$ such that the dynamical behaviour can be studied with additional anisotropy at the background. The Hubble parameter defined field equations can be derived from (7)-(11) as,

$$\left( \frac{6}{k+2} \right) \dot{H} + \left( \frac{27}{k^2 + 4k + 4} \right) H^2 - R^{-\frac{4\bar{p}}{2}} = -\beta \bar{p} + \frac{\rho}{2}$$

(12)

$$3 \left( \frac{k+1}{k+2} \right) \dot{H} + 9 \left( \frac{k^2 + k + 1}{k^2 + 4k + 4} \right) H^2 - R^{-\frac{4\bar{p}}{2}} = -\beta \bar{p} + \frac{\rho}{2}$$

(13)

$$9 \left( \frac{2k+1}{k^2 + 4k + 4} \right) H^2 - R^{-\frac{4\bar{p}}{2}} = \beta \rho - \frac{\bar{p}}{2}$$

(14)

The above set of field equations still poses some degree of difficulty to provide an explicit solution for the dynamical parameters. In view of this, we wish to adopt some algebraic approaches to express the effective pressure and energy density of the matter as a function of the Hubble parameter. We express the Einstein tensor of (12)-(14) with the respective indices as, $S_1(H, k) = \left( \frac{6}{k+2} \right) \dot{H} + \left( \frac{27}{k^2 + 4k + 4} \right) H^2 - R^{-\frac{4\bar{p}}{2}}$, $S_2(H, k) = 3 \left( \frac{k+1}{k+2} \right) \dot{H} + 9 \left( \frac{k^2 + k + 1}{k^2 + 4k + 4} \right) H^2 - R^{-\frac{4\bar{p}}{2}}$, $S_3(H, k) = 9 \left( \frac{2k+1}{k^2 + 4k + 4} \right) H^2 - R^{-\frac{4\bar{p}}{2}}$. Now, we can express the effective pressure $\bar{p}$ and energy density $\rho$ in the following. It is to note here that the effective pressure that consists of both the proper pressure $p$ and barotropic bulk viscous pressure can be written as, $\bar{p} = p - \xi \theta = p - 3\xi H$, where $\xi$ be the coefficient of bulk viscosity. From (12)-(14), a general expression based on directional Hubble parameter for the effective pressure $\bar{p}$ and rest energy density $\rho$ may be established as,
\[ \bar{p} = p - 3\xi H = - [S_1(H,k) + S_3(H,k)] \left( \frac{2}{1 - 4\beta^2} \right) + [S_2(H,k)] \left( \frac{2}{1 - 2\beta} \right) \] (15)

\[ \rho = [S_1(H,k)] \left( \frac{2}{1 - 4\beta^2} \right) - [S_3(H,k)] \left( \frac{4\beta}{1 - 4\beta^2} \right) \] (16)

Also, we can obtain \( \omega_{eff} = \bar{p} / \rho \) and \( \Lambda \) as,

\[ \omega_{eff} = -1 + \left[ \frac{S_2(H,k) - S_3(H,k)}{S_1(H,k) - 2\beta S_3(H,k)} \right] (1 + 2\beta), \] (17)

\[ \Lambda = [S_1(H,k) + S_3(H,k)] \left( \frac{1}{1 + 2\beta} \right). \] (18)

III. SIMPLIFIED MODEL DYNAMICS

In eqns. (15)-(18), the dynamical parameters of the cosmological models are expressed which will enable us to study the dynamical behaviour of the model. Since the parameters are expressed in terms of Hubble rate, we shall consider a form of the Hubble parameter as \( H = \frac{\dot{R}}{R} = \frac{1}{3} (a + \frac{b}{\xi}) \) such that the scale factor can be obtained as, \( R = e^{\alpha t} b \). This scale factor is called hybrid scale factor and the constants \( a, b \) are positive arbitrary constant calculated from the background cosmology. The values of the parameters \( a \) and \( b \) have been found in the range \( 0 < [a, b] < 1 \). These values have been further refined in some earlier research works \( [17] \) where \( b \) remains in \( [0, \frac{1}{3}] \) and \( a \) is left open. In this proposed scale factor, the deceleration parameter approaches to \(-1\) at late time and at the initial stage remain as constant \( \left( \frac{1 + \beta}{3} \right) \). In this range of the model parameters value, we are intending to examine the accelerating behaviour of the model in viscous fluid scenario. So, we shall express all the dynamical parameters with respect to cosmic time for elaborating its behaviour during the cosmological evolution. Here, the scale has been fixed such that 1 unit of cosmic time = 10 billion years. However, for comparing the result with recent observational data, the physical parameters of the model can be illustrated graphically with respect to the redshift \( z \). This is possible with the defined relation between the scale factor and the redshift as, \( z = \frac{1}{R} - 1 \). The recent observational results predicts a transition redshift in the range \( 0.4 \leq z_t \leq 0.8 \) \( [18] \); further the Planck collaboration suggested the range as \( 0.19 \leq z_t \leq 0.76 \) \( [19] \). So, in order to keep the redshift in the observed range, we have analysed and chosen representative values for the parameters \( a = 0.14, b = 0.32, k = 1.0001633 \). In the same note, \( \beta = \left( \frac{2\pi}{c^2} + \frac{3}{2} \right) \) and \( \xi \) have been analysed within preferred ranges. The value of anisotropic parameter \( k \) have been constrained keeping the amount of anisotropy added and the recent anisotropic behavior of Universe at a small scale \( [50] \). Throughout the paper, we express the physical quantities in Planckian unit system \( (c = G = \hbar = 1) \), where \( c, G \) and \( \hbar \) are the generic constants in the Einstein field equation of GR. Substituting the presumed form of Hubble parameter, eqns. (15)-(16) reduce to,

\[ \bar{p} = -\frac{2}{1 - 4\beta^2} \left[ \frac{(k - 1) + 2\beta(k + 1)}{(k + 2)} \right] \left( \frac{b}{t^2} \right) \]

\[ + \frac{2}{1 - 4\beta^2} \left[ \frac{(k^2 - k - 3) + 2\beta(k^2 + k + 1)}{(k + 2)^2} \right] \left( a + \frac{b}{\xi} \right)^2 \] (19)

\[ \rho = \frac{2}{1 - 4\beta^2} \left[ -\frac{2}{(k + 2)} \right] \left( \frac{b}{t^2} \right) \]

\[ + \frac{2}{1 - 4\beta^2} \left[ 3 - 2\beta(2k + 1) \right] \left( a + \frac{b}{\xi} \right)^2 \]

\[ + \frac{2}{1 - 2\beta} \left( e^{\alpha t} b \right) - \frac{\alpha}{e^{\alpha t}} \] (20)

Fig. 1 shows that the bulk viscous pressure remains negative for all suitably chosen scaling constant \( (\mu) \) values throughout the evolution which in turn indicates the accelerated expansion of the Universe. The rate of negative pressure is suddenly high in the figure at the initial phase of evolution which may explain the inflationary epoch (exponential expansion of space in the early Universe) that lasted between \( 10^{-36} \) seconds to \( 10^{-32} \) seconds after the big bang. Following the inflation period, the Universe continued to expand but at a slower rate and hence the slower
increasing rate of the figure. Eventually, the rate of negative pressure increases exponentially towards late phase of evolution that mimics the accelerated expansion. This sudden increase in negative pressure may sufficiently explain the theory of dark energy that began over 4.82 billion years ago. The nature of energy density parameter (Fig. 2), as expected, is positive and decreasing. It decreases from a high value at an early time to small values at late time. This observation confirms the fact that the density of matter decreases as the Universe expands because the volume of the space increases. One can also note that with decrease in coupling constant value, the energy density increases in early epoch showing a clear effect. However, the effect of \( \mu \) vanishes towards the late epoch which can be observed as the different curves in the figure merge together towards \( z = -1 \).

Since the dynamical properties of the Universe are computed through some physical natures with hybrid scale factor, effective cosmological constant and bulk viscous coefficient can be obtained as follows,

\[
\Lambda = \frac{2}{(k+2)(1+2\beta)} \left[ -\frac{b}{t^2} + \left( a + \frac{b}{t} \right)^2 \right]
\]

\[
\xi = \frac{2}{3(1-4\beta^2)} \left[ \frac{(2k+1)(\beta - 1) + (k^2 + k + 1)(2\beta - 0.5)}{(k+2)^2 (bt^{-1} + a)^{-1}} + \frac{b(k^2 + 3k + 2)(0.5 - 2\beta)}{t^2 (bt^{-1} + a)(k+2)^2} \right]
\]

\[
+ \frac{2}{3(1-4\beta^2)} \left[ \frac{3(1 - 2\beta)}{2\pi^2 t^{2\beta + 2} (bt^{-1} + a)} \right] \quad (22)
\]

Since the model parameter \( \mu \) is chosen to compare the present model in the frame of general relativity where the field equations appear with a time varying cosmological constant (\( \Lambda \)), therefore we have shown the dynamical variation of the effective cosmological constant for some representative values of \( \mu \) in Fig. 3. It is observed that the evolutionary behavior of \( \Lambda \) is affected by the choice of the values of \( \mu \). \( \Lambda \) curves with high values of \( \mu \) remain
below the curves with lower values of $\mu$. However, at remote past, the $\lambda$ values merge for negative values of $\mu$ and going to vanish gradually. In fact, as it appears from the figure in general, $\Lambda$ varies from large positive values in early epoch to almost vanishingly small values at late time. This behaviour may describe the phenomenon of late time cosmic acceleration where cosmological constant vanishes. One may note that, the repulsive nature of the cosmological constant is due to a negative pressure which can be considered as bulk viscous pressure for the present model.

Fig. 4 conveys the evolution of bulk viscous coefficient ($\xi$) with representative values of model parameter $\mu$. For negative values of $\mu$, the viscous coefficient shows an oscillating nature and gradually attains a vanishingly small value for higher positive values of $\mu$. The reason behind this nature may be due to some deviation in bulk viscous coefficient. So, one should choose suitable negative values of $\xi$ in order to know its behavior through the evolution. An important observation in this figure shows that the curves are more dominant in the early deceleration phase of evolution but the effect reduces gradually and vanishes at the late epoch. This behavior indicates that, the bulk viscous coefficient plays a vital role at early epoch, possibly providing a strong source for anisotropy.

Consequently, to have an insight in the dynamics of state of matter along with the cosmic evolution, we have derived the EoS parameter for the model as:

$$\omega_{\text{eff}} = -1 + (1 + 2\beta) \times \left[ \frac{-k^2 + 3k + 2}{(k^2 + k)^2} \left( \frac{a}{b} \right) + (k^2 - k) \left( a + \frac{1}{k} \right)^2 \right]$$

Fig. 5 shows a concrete dynamical behavior through the effective EoS parameter ($\omega_{\text{eff}}$) against the redshift of evolution. While plotting the figure, we considered the free parameters ensuring a positive energy density and negative pressure throughout the cosmic evolution in the model. To keep $\omega_{\text{eff}}$ in preferred observational data range, we have incorporated a set of values for the free parameters as discussed earlier in this section. More precisely, the
EoS parameter stays in the quintessence region till late phase. On the same note, we have taken the negative value of the coupling parameter $\mu$ in Fig. 6. It is clear from the figure that for negative $\mu$ values, $\omega_{\text{eff}}$ starts evolving almost from a quintessence region ($\omega_{\text{eff}} \leq -1$) during the early phase of evolution. Though the $\omega_{\text{eff}}$ curves have started evolving from different regions for different values of $\mu$ but eventually towards late phase of evolution they show same nature falling in the quintessence region as indicated in previous figure. It can be noted that when the value of $\mu$ increases, $\omega_{\text{eff}}$ increases most rapidly in the initial phase of evolution. Fig. 6 represents the behaviour of the EoS parameter for the representative value of the anisotropic parameter. It has been observed that for a smaller $k$ value the evolution starts from small negative value and increases with the increase in the value of $k$. But in spite of different values of the anisotropic parameter at late phase all merged and stay in the range $[-0.88, -0.86]$.

The evolution of effective EoS parameter is studied for deviations in viscous coefficient ($\xi$) in Fig. 7. The effective EoS parameter is not sensitive to the choice of parameter $a$, $b$ and $k$, at least at cosmic times spanning from recent past to late epoch. However, the presence of viscous co-efficient strongly affect at early time of evolution. In order to investigate this effect of viscous coefficient on the prescribed model, we have considered representative values of $\xi$ in the range $[-1, 1]$. The viscous coefficient behaves same outside the range in a larger scale approach. It is evident from the figure that the presence of viscous fluid incorporates a substantial amount of anisotropy and affect the early phase of cosmic evolution. This can be observed by the increasing trend of $\omega_{\text{eff}}$ for different $\xi$ values as we move back to past. In addition, the EoS parameters for $\xi = -1$ (red line) and $\xi = -0.5$ (blue line) changes sign from negative to positive at redshift values $z_t = 0.38$ and $z_t = 0.62$ respectively which is in agreement with planck observational data [49]. Where as, EoS parameter for $\xi = 1$ (green line) and $\xi = 0.5$ (black line) show a decreasing behaviours as we go back to early phase of evolution.

With increase in the positive values and decrease in the negative values of $\xi$, the effective EoS parameter decreases to lower values showing a less effect at present time ($z=0$). However, the effective EoS is affected by deviation in viscous coefficient at late phase of evolution which can be observed by smoothly merging of all four lines for different values of $\xi$ (Fig. 7). This nature is also obvious in Fig. 1, Fig. 2, Fig. 3, Fig. 4 and Fig. 6 which concretes the facts that at early phase, the viscous coefficient has a substantial contribution to energy density as well as viscous pressure.
FIG. 7. Evolution of EoS parameter for representative values of viscous coefficient $\xi$ for which the dynamics of effective EoS is greatly affected. The reason behind nature of effective EoS parameter in the late phase of expansion may be due to the presence of strongly dominant dark energy over the presence of viscous coefficient $[48, 50]$. One can also note that, for different values of viscous coefficient in the preferred range $[-1, 1]$, $\omega_{eff}$ decreases smoothly and lies in the quintessence region as mentioned by recent Planck collaboration data. Hence, the present model behaves like quintessence field. The brown dots on green and black lines along the $\Lambda CDM$ line ($\omega_{eff} = -1$) indicate the transition of effective EoS parameters from phantom region to quintessence region. Similarly, the dots on blue and red lines show that the Universe travels through stiff dominated region to quintessence region in the cosmic evolution process for different $\xi$ values. Moreover, the EoS parameter plot depicts the brief evolutionary history of Universe. Staring from inflationary phase, it travels through radiation dominated phase followed by matter dominated phase and finally lies in the accelerated expansion phase.

Energy Conditions are experimentally shown that it always seem to be positive and Null or Time like geodesics in nature and the common characteristics shared by almost every matter field. As standard matter is assumed to satisfy the necessary energy conditions, so for a viscous fluid distribution, the general inequalities of energy conditions are

Null Energy Condition (NEC): $\rho + p \geq 0$
Weak Energy Condition (WEC): $\rho + p \geq 0$, $\rho \geq 0$
Strong Energy Condition (SEC): $\rho + 3p \geq 0$
Dominant Energy Condition (DEC): $\rho - p \geq 0$, $\rho \geq 0$.

In $f(R, T)$ gravity with hybrid scale factor, the energy conditions can be obtained as

$$\rho + p = \frac{2}{1 - 4\beta^2} \left[ \left( \frac{(k + 1)(2\beta - 1)}{k + 2} \right) \frac{b}{t^2} + \frac{(k^2 - k)(2\beta + 1)}{(k + 2)^2} \left( a + \frac{b}{t} \right)^2 \right]$$

$$\rho - p = \frac{2}{1 - 4\beta^2} \left[ \left( \frac{k - 3 + 2\beta(k + 1)}{k + 2} \right) \frac{b}{t^2} + \left( -k^2 + k - 6 - 2\beta(k^2 + 3k + 2) \right) \left( a + \frac{b}{t} \right)^2 \right]$$

$$+ \frac{4}{1 - 2\beta} \left( e^{at} t^b \right)^{\frac{-6\beta}{4\beta^2}}$$

$$\rho + 3p = \frac{2}{1 - 4\beta^2} \left[ \left( -3(k - 1) - 6\beta(k + 1) \right) \frac{b}{t^2} + \left( 3(k^2 - k - 2) + 2\beta(3k^2 + k - 2) \right) \left( a + \frac{b}{t} \right)^2 \right]$$

$$- \frac{4}{1 - 2\beta} \left( e^{at} t^b \right)^{\frac{-6\beta}{4\beta^2}}$$

The energy conditions, in Fig. 8, are observed to change dynamically with the cosmic evolution. The SEC is satisfied in early phase of evolution upto $t = 0.55$ Gyr and afterwards, there is a clear violation of this condition (green line). The blue curve shows that DEC is satisfied throughout the evolution. Moreover, the WEC and NEC are satisfied in the model. The characteristics of the energy conditions might be due to the presence of viscous pressure in the matter field. Also, it can be inferred that the present model in the framework of extended gravity favours an accelerated expansion of the universe.
IV. RESULTS AND DISCUSSIONS

The cosmological model of the Universe in an extended theory of gravity has been presented with an anisotropic space-time. A dissipative cosmic fluid in the form of bulk viscosity is chosen to study its effect on the cosmic dynamics particularly the equation of state parameter. A mathematical formalism is discussed for the construction of the cosmological model, where we obtain a time varying cosmological constant as a consequence of the geometry modification. The viscous pressure, energy density, EoS parameter, viscous coefficient and effective cosmological constant are calculated with the developed mathematical formalism and its graphical representations are given. Throughout this work the model parameter $\mu$ plays a key role to describe an anisotropic acceleration Universe. It is observed that, the choice of the extended gravity parameter $\beta$ affects the behaviour of the effective cosmological constant ($\Lambda$) (Fig.3). A varies from large positive values with high value of $\mu$ at early epoch and remains below with lower value of $\mu$ at late epoch. The bulk viscous coefficient initially increases from large negative values and after attending a peak decreases to a vanishingly small value(Fig.4). The extended gravity parameter $\mu$ decreases the requirement of bulk viscous fluid. The dynamics of the Universe in the form of the evolution of the EoS parameter is discussed in Fig.5 and Fig.6. We have shown the behaviour both with the representative value sof the model parameter and anisotropic parameter. We have the EoS parameter as predicted from the negative choice of $\beta$, lying in the range of $-0.9$ to $-1$ at the present epoch. Similarly, the effect of viscous coefficient has shown through the evolution of EoS parameter(Fig.7). We have obtained that the presence of viscous fluid incorporates a substantial amount of anisotropy and affect at early stage of cosmic evolution. With different values of viscous coefficient($\xi$), $\omega_{eff}$ decreases slowly and lies in the quintessence region in the cosmic evolution process for different $\xi$ values.

The present investigation provides some useful insight into the cosmic dynamics and evolution of the EoS parameter in presence of a dissipative fluid. Our model is quite compatible with recent observations. However, we have used a very specific form of the functional $f(R,T)$. A more general form of the functional $f(R,T)$ in place of the one used in the present work, may provide some more information about the viscous cosmic dynamics.

ACKNOWLEDGEMENTS

ST acknowledges Rashtriya Uchchatar Shiksha Abhiyan (RUSA), Ministry of HRD, Govt. of India for the financial support. BM and SKT acknowledges Inter-University Center for Astronomy and Astrophysics (IUCAA), Pune, India for hospitality and support during an academic visit where a part of this work is accomplished.

[1] A.G. Riess et al., *Astron. J.*, **116**, 1009 (1998).
[2] A.G. Riess et al., *Astron. J.*, **117**, 707 (1999).
[3] D.N. Spergel et al., *Astrophys. J. Suppl.*, **148**, 175 (2003).
[4] K. Abazajian, *Astron. J.*, **128**, 502 (2004).
[5] A. C. Pope et al., *Astrophys. J.*, **607**, 655 (2004).
[6] S. Capozziello, *Int. J. Mod. Phys. D.*, **11**, 483 (2002).
[7] S. Nojiri, S.D. Odintsov, *Phys. Rev. D.*, **68**, 123512 (2003).
[8] A.A. Starobinsky, *JETP Lett.*, **86**, 157, (2007).
[9] C.Y. Zhang, Z.Y. Tang, B. Wang, *Phys. Rev. D*, **94**, 104013 (2016).
[10] A. Paliathanasis, J. D. Barrow, P.G.L. Leach, *Phys. Rev. D*, 94, 023525 (2016).
[11] G. Otalora, M. J. Reboucas, *Eur. Phys. J. C*, **77**, 799 (2017).
[12] K. Rezaazadeh, A. Abdolmaleki, K. Karami, *Astrphys. J.*, **836**, 228 (2017).
[13] P. Chanmuie, D. Momeni, *Nucl. Phys B*, **935**, 256 (2018).
[14] S. Nojiri, S. D. Odintsov, *Phys. Lett. B*, **631**, 1 (2005).
[15] B. Li, J. D. Barrow, D. F. Mota, *Phys. Rev. D*, **76**, 044027, (2007).
[16] G. Kofinas, E. N. Saridakis, *Phys. Rev. D*, **90**, 084044 (2014).
[17] F. M. Shamir, *Astrophys. Space Sci.*, **361**, 147 (2016).
[18] M. Sharif, S. Saba, *Chin. J. Phys.*, **58**, 2020 (2019).
[19] R. Lazkoz, F. S. N. Lobo, M. O. Banos, V., *Phys. Rev. D*, **100**, 104027 (2019).
[20] Y. Xu, G. Li, T. Harko, S. D. Liang, *Eur. Phys. J. C*, **79**, 708 (2019).
[21] Y. Xu, T. Harko, S. Shahidi, S. D. Liang, *Eur. Phys. J. C*, **80**, 449 (2020).
[22] S. Bahamonde, S. Capozziello, *Eur. Phys. J. C*, **77**, 107 (2017).
[23] M. Caruana, G. Farrugia, J. L. Said, *Eur. Phys. J. C*, **80**, 640 (2020).
[24] T. Harko, F.S.N. Lobo, S. Nojiri, S. D. Odintsov, *Phys. Rev.D* **84**, 024020 (2011).
[25] A. Das, F. Rahaman, B. K. Guha, S. Ray, *Eur. Phys. J. C*, **76**, 654 (2016).
[26] D. Deb, F. Rahaman, S. Ray, B. K. Guha, *JCAP*, **03**, 44 (2018).
[27] S. B. Fisher, E. D. Carlson, *Phys. Rev. D*, **100**, 064059 (2019).
[28] H. Shabani, A. H. Ziaie, *Eur. Phys. J. C*, **78**, 397 (2018).
[29] S. K. Tripathy, R. K. Khuntia, P. Parida, *Eur. Phys. J. Plus*, **134**, 504 (2019).
[30] A. H. Barbar, A. M. Awad, M. T. AlFiky, *Phys. Rev. D*, **101**, 044058 (2020).
[31] J. Wu, G. Li, T. Harko, S. D. Liang, *Eur. Phys. J. C*, **78**, 430 (2018).
[32] E. Elizalde, M. Khurshudyan, *Int. J. Mod. Phys. D*, **28**, 1950172 (2019).
[33] S. M. Khan, A. Ali, *Mod. Phys. Lett. A*, **33**, 1850065.
[34] M. S. Khan, S. Khan, *Gem Relativ. Grav.*, **51**, 148 (2019).
[35] S. K. Tripathy, B. Mishra, *Chin. J. Phys.*, **63**, 448 (2020).
[36] B. Mishra, S. K. Tripathy, *Phys. Scr.*, **95**, 095004 (2020).
[37] S. K. Tripathy et al., *Phys. Scr.*, **95**, 115001 (2020).
[38] E. N. Saridakis, K. Myrzakul, K. Myrzakulov, K. Yerzhanov, *Phys. Rev. D*, **102**, 023525 (2020).
[39] S. K. Tripathy, *Phys. of Dark Univ.*, **31**, 100757 (2021).
[40] B. Mishra, S. Tarai, S. K. Tripathy, *Mod. Phys. Lett. A*, **33**, 1850052 (2018).
[41] T. Padmanabhan, S. M. Chitre, *Phys. Lett. A*, **120**, 433 (1987).
[42] I. Brevik, A. V. Timoshkin, *Int. J. Geom. Meth. Mod. Phys.*, **14**, 1750061, (2017).
[43] I. Brevik et al., *Int. J. Mod. Phys. D*, **26**, 1730024 (2017).
[44] B. Mishra, S. Tarai, S. K.J. Pacif, *Int. J. Geom. Meth. Mod. Phys.*, **15**, 1850036 (2018).
[45] R. Ahmed, G. Abbas, *Can. J. Phys.*, **97**, 994 (2019).
[46] C.P. Singh, V. Kumar, *Gravit. Cosmol.*, **25**, 58 (2019).
[47] B. Mishra, S. K. Tripathy, *Mod. Phys. Lett. A*, **30**, 1550175 (2015).
[48] B. Mishra, Pratik P. Ray, S. K.J. Pacif, *Eur. Phys. J. Plus*, **132**, 429 (2017).
[49] Planck collaboration, *Astron. and Astrophys.*, **641**, A6, (2020).
[50] B. Mishra, S. K. Tripathy, Pratik P. Ray, *Astrophys. Space Sci.*, **363**, 86 (2018).