Extended gravitational action and novel consequences

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Abstract. It is found that the inclusion of higher derivative terms in the gravitational action along with concepts of phase transition and spontaneous symmetry breaking leads to some novel consequence. The Ricci scalar plays the dual role, like a physical field as well as a geometrical field. One gets Klein-Gordon equation for the emerging field and the corresponding quanta of geometry are called Riccions. For the early universe the model removes singularity along with inflation. In higher dimensional gravity the Riccions can break into spin half particle and antiparticle along with breaking of left-right symmetry. Most tantalizing consequences is the emergence of the physical universe from the geometry in the extreme past. Riccions can Bose condense and may account for the dark matter.

1. Introduction

The concept of the superfluid vacuum state connected with the cosmological constant was suggested several decades ago in collaboration with Sudarshan [1,2,3]. These researches involved the study of non-singular cosmological models, symmetry breaking and phase transitions at various stages of the evolution of the universe [4-7]. In the context of cosmology the geometry of space-time (whether flat or curved) of four or higher dimension is intimately connected with the interplay of matter and energy.

2. Formalism and results

In what follows, we present our recent results arising out of our study on extended gravitational action which contains higher derivative terms. The Einstein-Hilbert action.

\[ S_g = \int d^4x \sqrt{g} \left( R - \frac{1}{16\pi G_N} + \Lambda \right) \] (1)

contains only the first power of Ricci scalar \( R \), \( 1/16\pi G_N = M_p^2 \), \( M_p \) = Planck mass, \( G_N \) is the Newtonian constant. In natural units (\( \hbar = c = 1 \)) \( 16\pi G_N \) has the dimension of \( (\text{length})^2 \). \( \Lambda \) is the cosmological constant and \( g \) is the determinant of the metric tensor \( g_{\mu\nu} \). Varying the action yields the Einstein field equation

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G_N T_{\mu\nu} \] (2)

where \( R_{\mu\nu} \) is the Ricci tensor and \( T_{\mu\nu} \) the energy-momentum tensor. Here we have taken into account other physical fields also \( S_m = \int d^4x L_m \), \( L_m \) being the Lagrangian density for fields other than gravity.

The above Einstein field equation had great success in explaining low energy phenomena involving gravity. However, in the high energy regime the higher derivative terms such as \( R^2 \), \( R_{\mu\nu} R^{\mu\nu} \),
and $R_{\mu\nu\rho\sigma}$ become important. They play an important role in the context of renormalizability of the gravitational theory and stringy corrections in the string theory.

A generalized version of the extended gravitational action has the form \[S_g = \int d^4x \sqrt{-g} \left[ R - \frac{1}{16\pi G} \Lambda + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 \right], \tag{3}\]

where $\alpha, \beta$ are dimensionless coupling constants and $G$ may have value other $G_N$ to account for strong gravitational interaction in the early Universe [1]. We confine to lengths larger than $L_P$ (the Planck length) to avoid unitarity problem. For the extended action the field equation is obtained by demanding the condition

\[(1/\sqrt{-g})\delta S_g/\delta g = 0 \tag{4}\]

One finally gets

\[R + [32\pi G(\alpha + 3\beta)^{1/2}]R = - \Lambda/(2\alpha + 6\beta), \tag{5}\]

which is the dynamical equation for $R$ in curved space with a potential

\[\Lambda R / (2\alpha + 6\beta) \text{ as a source term.} \]

\[\eta = \nabla_{\mu} \nabla^\mu = 1/\sqrt{-g} \delta/\delta x^\mu (\sqrt{-g} g^{\mu\nu} \delta/\delta x^\nu) \]

In the absence of $\Lambda$, (5) can be written as

\[\alpha R + m^2 R = 0 \tag{6}\]

with $m_R = [32\pi G(\alpha + 3\beta)]^{1/2} \tag{7}$

This shows that in the high energy regime $R$ behaves like a scalar material field with its mass given by (7). The structure of (6) is exactly like Klein-Gordon (K – G) equation and from geometry we have made transition to physical field whose quanta are massive. We have named the quanta “Riccion”. Note the mass depends on the coupling constants. To avoid ghost problem it is necessary to take $(\alpha + 3\beta) > 0$ [ghost fields are not associated with physical particles].

Compared to the usual K – G equation (where the mass dimension of the field is 1) the field dimension of $R$ is 2. This can be circumvented by multiplying (6) by $\eta$ (where $|\eta| = \text{some number and dimension of } \eta = \text{(mass)}^{-1}$). Thus with $R^\mu = \eta R$

\[\nabla_{\mu} R_{\mu} = 0 \tag{8}\]

An important point to note is that the above formulation involves the modification of Einstein’s theory of gravity in the high energy regime. The Hawking-Penrose theorem is violated and the model should be singularly free and there will be bounce at $t = 0$. It is found that [7]

\[a^2 = a^2 \sinh [(t - t_c)/\sqrt{T_c/6} + 0.89] \tag{9}\]

t = time, $a$ the radius of the universe, $T_c$, $t_c$ critical temperature and time. The model universe expands initially according to (9) and for $t$ sufficiently larger than $t_c$.
\[ a = a_c \exp \left[ \left( t - t_c \right) \sqrt{T_c/24} \right] \]  
(10)

giving an exponential expansion in conformity with the inflationary model.

With the expansion the temperature falls and when \( T \) is well below \( M_p \) the Einstein's theory of gravity takes over with \( R \) behaving like a geometrical field.

So far we have considered four-dimensional space-time only. We extend the model to \((4 + D)\) dimensional action for \( R^2 \) gravity. The action is now [11]

\[ S_g = \int d^4x d^Dy \sqrt{|g_{4+D}|} \left[ (R_4 + D)/(16\pi G_{4+D}) + \alpha R^2_{4+D} + \beta R_{4+D} \right] \]

\[ \mathcal{L}_{4+D} = \left( \sqrt{|g_{4+D}|} \right)^{-1} \frac{\partial}{\partial x^M} \left( \sqrt{|g_{4+D}|} g^{MN} \frac{\partial}{\partial x^N} \right) \]

(11)

\( R^2 \) terms dominate over \( R_{4+D} / (16\pi G_{4+D}) \)

when the energy mass scale \( M \geq \left[ 16\pi (\alpha + \beta) \right]^{-1} M_p \)

The equation obtained is

\[ \alpha R + \lambda R^2 + m^2 R = 0 \]  
(12)

\( (R_{4+D} = R_4 = R, \mathcal{L}_{4+D} = 1) \)

\[ m^2_R = (D + 2) / (16\pi G \left[ 4(D+3) \alpha + D\beta \right] ) \]  
(13)

and

\[ \lambda = D \alpha / \left( 4(D+3)\alpha + D\beta \right) \]  
(14)

subject to the condition \( [4(D+3)\alpha + D\beta] > 0 \), to avoid ghost problem. The operator \( \mathcal{L}_{D+4} \) reduces to \( \mathcal{L}_4 \) as \( R \) is independent of \( y \) - coordinates.

Thus one obtains the four dimensional K-G equation for \( R^- \) from higher-dimensional space-time.

\[ \mathcal{L} R^- + \lambda R R^- + m^2_R R^- = 0. \]  
(15a)

as a result of spontaneous compactification. (For \( \lambda = 0 \) for \( D = 0 \), as discussed earlier, the above reduces to the usual K-G eqn.). In fact \( \lambda R R^- \) is a manifestation of higher-dimensional geometry in four-dimensional space-time.

For special reasons we choose \( \lambda = \frac{1}{4} \); we have

\[ (1 + \frac{1}{4} R + m^2_R) R^- = 0 \]  
(15b)

Now a scalar field can be expressed as \( R^- = \Psi^- \bar{\Psi} \) (where \( \Psi \) is a Dirac spinor, \( \bar{\Psi} = \Psi^* \gamma^0 \), \( \Psi^* \) being the Hermitian conjugate of \( \Psi \), \( \gamma^\mu \) is a Dirac matrix. This amounts to taking Riccion as a composite of a spin-1/2 Dirac fermion and anti-fermion. Using the algebra of Dirac matrices and vanishing covariant derivative of tetrad components we can write (15b) as

\[ \Psi (\gamma^\mu \nabla_\mu - im_k) (\gamma^\nu \nabla_\nu + im_k) \Psi = 0 \]  
(16)

provided \( \nabla_\mu \Psi \gamma^\mu \gamma^\nu \Psi = 0 \)  
(17)

This condition can also reexpressed as
\[ \nabla_\mu \nabla_\nu \Psi_L \gamma^\mu \gamma^\nu \Psi_R = -\nabla_\mu \nabla_\nu \Psi_R \gamma^\mu \gamma^\nu \Psi_L \]  \hspace{1cm} (18)

where

\[ \Psi_{LR} = [ (1 \pm \gamma^5) / 2 ] \Psi, \Psi_{LR} = \Psi \left( (1 + \gamma^5) / 2 \right) \]  \hspace{1cm} (19)

with

\[ \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \]  \hspace{1cm} (20)

Equation (18) implies breaking of left-right (L-R) symmetry. Equation (16) involves the Dirac equation for \( \Psi \)

\[ (\gamma^\mu \nabla_\mu + \text{im}) \Psi = 0 \]  \hspace{1cm} (21)

and

\[ \Psi (\gamma^\mu \nabla_\mu - \text{im}) = 0 \]  \hspace{1cm} (22)

The important point to note is that a Riccion \( R \) (with \( \lambda = \gamma^5 \)) can break into fermion and antifermion pair only when L-R symmetry is broken. Thus

\[ R \to \Psi + \bar{\Psi} \]  \hspace{1cm} (23a)

provided that parity is violated. However, the reverse reaction

\[ \Psi + \bar{\Psi} \to R \]  \hspace{1cm} (23b)

is not allowed and hence no back reaction. We have named these spin \( \frac{1}{2} \) particles as Riccinos.

The Riccinos are also very heavy particles and can decay further according to

\[ \Psi \to \text{baryon} + \text{Photons} \]  \hspace{1cm} (24a)

\[ \bar{\Psi} \to \text{antibaryons} + \text{Photons} \]  \hspace{1cm} (24b)

In the reaction (23a) and (23b) CP invariance is broken. As a consequence there will be excess of baryons over antibaryons. This is in agreement with the observed matter distribution in the early universe. This would suggest that the observed baryons and antibaryons were produced through the decay of riccinos and anti-riccinos with the condition that the process (24b) is slower than (24a).

Note that the above situation of massive Riccions and Riccinos and Antiriccinos appearing out of the geometry of space-time is possible only for curved space-time. For flat space time \( R = 0 \).

The above scenario prompts us to speculate that our physical universe emerged from higher-dimensional geometry in the extreme past through decay of Riccions and Riccinos.

Besides the spin-1/2 particles that will be created, other scalar particles can also be created [9, 10]. This arises from the non-minimal coupling of the Riccion field with a scalar field \( \phi \) of the form \( L_i = \frac{1}{2} \chi R \phi^2 \). We have noted earlier the inflation. When \( R \) undergoes spontaneous symmetry breaking resulting in a phase transition from the state \( <R>_{\text{VEV}} = 0 \) to \( <R>_{\text{VEV}} = \pm \sigma \) (\( \sigma \) is the spontaneous symmetry breaking mass scale). As long as \( T > T_c \), \( <R>_{\text{VEV}} = 0 \). But as \( T \) falls, \( <R> \) tunnels through temperature barrier (\( T = T_c \)) and acquires \( <R>_{\text{VEV}} = \pm \sigma \). The scalar field \( \phi \) is mass less for \( <R> = 0 \) and acquires mass in the state \( <R> = \pm \sigma \). Thus a large number of particle-antiparticle pairs are created during the exponentially expanding stage in the state \( <R> = \pm \sigma \).

The scalar particles, in the cooling stage of the inflating universe, will undergo Bose-Einstein condensation below a temperature given by \( T_{\text{BEC}} = (3.31) / (g_{\text{deg}}^{2/3})(h^3/m_R (N/V)^{2/3}) \), \( g_{\text{deg}} = \text{degeneracy} \).
factor. These massive scalars in the universe play the role of dark matter and those massless as dark energy [13]. With \( m_R \approx 100 \text{ GeV} \), \((N/V) = 10^{48}, T_{\text{BEC}} \approx \text{a few degree K.}\)

3. Concluding remarks

The model presented involves many novel features:

The TRINITY (Geometry, Matter and Energy) describing the universe are inter-convertible.

\[
\begin{array}{ccc}
G & E & M \\
\end{array}
\]

\((G\ E\ M\ \text{Model})\)

Einstein showed that \( E = mc^2 \), the interconvertibility of energy and matter. The GEM model shows that geometry (of the spacetime), energy and matter are interconvertible to each other. In keeping with the superstring theory, the geometry of the Universe may comprise higher than \((3 \text{ space} + 1 \text{ time})\) dimensions. The extra dimensions are curled up and do not manifest to direct observation. In the context of the above model it is appropriate to note Einstein’s remark. “The geometry of space-time is physical, much like matter”. The geometry may indeed have lattice structure and other attributes of matter e.g. charge, mass, spin, isospin, hypercharges, charm arise from the topological defects of the lattice.

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