Science with the TianQin observatory: Preliminary result on extreme-mass-ratio inspirals

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Systems consisting of a massive black hole (MBH) and a stellar-origin compact object (CO), known as extreme mass-ratio inspirals (EMRIs), are of great significance for space-based gravitational-wave detectors, as they will allow for testing gravitational theories in the strong field regime, and for checking the validity of the black hole no-hair theorem. In this work, we present a calculation of the EMRI rate and parameter estimation capabilities of the TianQin observatory, for various astrophysical models for these sources. We find that TianQin can observe EMRIs involving COs with mass of $10M_\odot$ up to redshift $\sim 2$. We also find that detections could reach tens or hundreds per year in the most optimistic astrophysical scenarios. Intrinsic parameters are expected to be recovered to within fractional errors of $10^{-6}$, while typical errors on the luminosity distance and sky localization are 10% and 10 deg$^2$, respectively. TianQin observation of EMRIs can also constrain possible deviations from the Kerr quadrupole moment to within fractional errors of $10^{-4}$. We also find that a network of multiple detectors would allow for improvements in both detection rates (by a factor $\sim 1.5$–3) and in parameter estimation precision (20-fold improvement for the sky localization and 5-fold improvement for the other parameters.)

I. INTRODUCTION

Gravitational-wave (GW) observations provide information on the minute vibrations of the spacetime, and promise to revolutionize astronomy and astrophysics by opening a new window on the Universe. To date, the ground-based GW observatories, LIGO and Virgo, have detected several GW events [1]. Limited by seismic noise and their relatively short armlengths, however, ground-based detectors are only sensitive to high frequency GWs (above a few Hz) generated by low mass sources (e.g. mergers of stellar-origin COs). In order to detect heavier sources, such as ones involving MBHs, or even the low frequency (sub Hz) inspiral phase of stellar-origin compact binaries [2], a significant increase in the size of GW detectors is necessary, which can only be achieved in space. LISA, for example, will present armlengths of about 2.5 million km, and will be sensitive to GWs in the frequency band $10^{-5}$–0.1 Hz [3, 4].

TianQin is a proposed space-based, geocentric GW observatory with armlengths of about $1.7 \times 10^7$ km, aiming to detect GW signals in the frequency band $10^{-4}$–1 Hz [5, 6]. In the last few years, a systematic effort has been undertaken to study the science prospects of TianQin [7]. On the astrophysics side, this included the study of the detection prospects for Galactic ultra-compact binaries [8], coalescing MBHs [9, 10], the low-frequency inspiral of stellar-mass black holes [11], and stochastic GW backgrounds [12]. On the fundamental physics side, TianQin’s ability to test the black hole no-hair theorem with the ringdown of MBHs resulting from a merger has been analyzed, both in a theory-agnostic framework [13] and within specific gravitational theories extending general relativity [14], and more work is in preparation in this direction.

In this paper, we focus on EMRIs, i.e. binaries consisting of a stellar origin CO (a stellar mass black hole or a neutron star) orbiting around a MBH in a long inspiral. These sources are expected to be relatively numerous in the mHz GW sky probed by LISA and TianQin. Indeed, strong observational evidence suggests the presence of MBHs at the center of most local galaxies [15–18], typically surrounded by stellar clusters or cusps [19, 20] of a few pc scale. Relaxation processes in such a high density environment occasionally force stars and compact objects onto extremely eccentric, low angular momentum orbits, resulting in a close encounter with the central MBH. While main sequence stars are torn apart

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and tidally disrupted, potentially resulting in luminous flares and prompting gas accretion onto the central MBH [21–24], COs typically survive intact until merger [25–27]. Depending on their orbital angular momentum, COs can directly plunge onto the MBH or are captured in eccentric bound orbits, whose secular evolution decouples from the cluster’s dynamics, and is dominated by GW emission [28]. These latter systems are usually referred to as EMRIs.

Detecting GWs from EMRIs will be very significant for our understanding of the astrophysics of these sources [28]. For instance, it will allow for gaining information on the mass distribution of MBHs [29] and their host stellar environments [28]. It may also provide information on the expansion of the Universe [30], as well as allow for mapping the spacetime geometry of the MBH in great detail, testing general relativity and the no-hair theorem [31–37] and revealing the possible presence of matter surrounding MBHs [38–43].

In this paper, we use previously published astrophysical models [44] for the formation and evolution of EMRIs across cosmic time to assess TianQin’s capability to detect these sources and estimate their parameters. In more detail, by adopting, as was done in [44] for LISA, analytic kludge waveforms and using a simple Fisher information matrix (FIM) method to analyze the parameter estimation prospects, we find that EMRIs can be observed up to redshift \( \sim 10^{5–3} \). Intrinsic parameters are projected to be estimated to within fractional errors of \( \sim 10^{-6} \), while typical errors on the luminosity distance and sky localization are 10% and 10 deg\(^2\), respectively. We also estimate the potential scientific gain of operating TianQin within a network of detectors, e.g. together with LISA and/or a twin TianQin detector (TQ II). We find a twin TianQin detector would increase detection rates by a factor \( \sim 1.5–3 \).

The paper is organized as following. In section II, we describe our model for the extreme mass ratio inspiral (EMRI) astrophysical population, the gravitational waveforms, the response of TianQin to EMRI signals and the TianQin noise model used in this study. In section III, we describe the method for calculating the signal-to-noise ratio (SNR) and the precision of the parameter estimation. The main results of our study are presented in section IV. In section V, we present our conclusions.

II. THE MODEL

A. EMRI rate

Extensive evidence exists for the ubiquitous presence of MBHs at the center of virtually every galaxy at low redshifts [45–48], including our own Milky Way [49–52] and, as recently confirmed by the Event Horizon Telescope, M87 [53]. Moreover, nuclear stellar clusters with sizes of a few parsec (pc) and masses up to \( 10^5–10^8 M_\odot \) are also known to coexist with MBHs in the local universe (except at the high mass end of the MBH mass function) [20]. The high densities of these clusters make them the perfect cradles for the formation of EMRIs, as two-body relaxation will make the system tend toward energy equipartition and thus mass segregation, with the heavier objects (i.e. stellar-mass black holes) sinking deeper in the MBH gravitational potential well. This process can eventually lead to COs plunging or inspiralling into the MBH, depending on their angular momenta.

The rate of EMRIs and their properties depend on a variety of (astro)physical processes, which determine the evolution of the population of MBHs along cosmic history and the accumulation of COs in their vicinity. In this paper, we make use of the EMRI population models developed by Babak et al. [44]. For the convenience of the readers, we summarize the main features of the model here, but refer to [44] for more details. The intrinsic EMRI rate is given by the following function:

\[
R(M, z, a) = \frac{d^3N}{dM dz da} \times p_0(M, z) \times \kappa \Gamma R_0(M),
\]

where \( M, z \) and \( a \) are the mass, redshift and spin of the MBH, respectively. Terms on the right hand side of this equation are explained below:

- \( \frac{d^3N}{dM dz da} \) is the redshift dependent MBH mass function.
- The two scenarios have been adopted that bracket the uncertainties in the low mass end of the MBH mass function at \( z = 0 \). One scenario is based on the semianalytic model (SAM) developed by Barausse and collaborators in a series of papers [54–56]. The SAM follows the formation and evolution of MBH masses and spins along cosmic history, and produces a mass function \( \frac{dN}{d\log M} \propto M^{-0.3} \) in the range \( 10^4 M_\odot < M < 10^7 M_\odot \), consistent with the upper bound of current observations (see Figure 1 in [44]). A second scenario employs an empirical mass function where \( \frac{dN}{d\log M} \propto M^0.3 \) in the same mass range \( 10^4 M_\odot < M < 10^7 M_\odot \) [29], consistent with the lower bound of current observations.
- \( p_0(M, z) \) describes the probability that a MBH with mass \( M \) and redshift \( z \) is surrounded by a cusp of stars and COs, thus potentially giving rise to an active EMRI source. When two galaxies merge, the cusps of stars and COs around the central MBHs of the parent galaxies are eroded by the action of the inspiralling MBH binary. The cusp in the merger remnant is therefore destroyed, and is rebuilt only after a fraction of the relaxation time [57]. The SAM model allows one to follow the galaxy and MBH merger rate and to estimate the time needed to rebuild the cusp, from which the probability function for each MBH to be a potential EMRI source is constructed.
- \( R_0(M) \) is the rate at which a galaxy hosting a MBH with mass \( M \) surrounded by a stellar (and CO)
cusp actually generates an EMRI. Note that this probability depends on the density profile of the CO population, which might depend on the redshift and other parameters unrelated to the MBH mass. For simplicity, however, any such possible dependence is dropped and $R_0(M)$ is assumed to be function of the MBH mass only, following [57].

- Finally $\kappa$ and $\Gamma$ are two “ad hoc” correction factors to $R_0(M)$ that ensure that the overall EMRI rate is consistent with the observed MBH mass function, i.e. that the MBHs do not “overgrow” their present masses by capturing too many EMRIs and plunges.

Besides the choice of two different MBH mass functions, Eq. (1) – and the observed EMRI rate – also depend on a number of additional astrophysical factors, including:

- The relative occurrence rate of plunges vs. EMRIs, which enters in the $\Gamma$ factor. The plunge cross section of the MBH itself ($4GM/c^2$ for a non rotating MBH) is not negligible compared to the EMRI capture cross section, which is generally $<10GM/c^2$. Recent simulations have actually found that plunges are typically more frequent than EMRIs [25], and this has an impact on the intrinsic EMRI rate for a given MBH.

- The choice of parameters of the MBH-galaxy scaling relations, which is important to compute the $p_0(M, z)$ function. In fact, the time needed to rebuild the cusp depends on the properties of the galactic nucleus, whose mass can be computed from the MBH mass via the MBH-galaxy scaling relations.

- The MBH spin distribution, which has an impact both on the EMRI capture rate and the EMRI waveforms, and hence on their detectability with GWs (as shown in the following section).

- The mass of the CO, which affects the rate normalization and which enters the EMRI waveforms. Most models in [44] assume COs with $10M_\odot$, but some consider COs with $30M_\odot$.

The above ingredients have been suitably combined in [44] to build a suite of 12 models encompassing three orders of magnitudes in the expected cosmic EMRI rate, from $10 \text{ yr}^{-1}$ to about $2 \times 10^4 \text{ yr}^{-1}$. These are also the models that we employ in this investigation. We label the models as M-$i$ with $i = 1, \ldots , 12$, following the original nomenclature. Detailed prescibrions for each model can be found in Table I of [44].

### B. Waveform

The calculation of the waveforms for EMRIs is a challenging task. Although much progress has been attained with the goal of producing accurate and efficient EMRI waveforms including the effect of the self-force [58–64], the problem has not been fully solved yet. Here, we adopt simple waveforms suitable for predicting the detection and parameter estimation capabilities of TianQin, but one should keep in mind that full waveforms including the effect of the self-force will be needed to analyze the real data.

In more detail, in this paper we follow [44] and utilize a class of simplified and approximate but computationally inexpensive EMRI waveforms, the analytic kludge (AK) model of [65]. (See also [66–69] for other EMRI kludge waveforms.)

The AK waveform is calculated simply from the quadrupole formula, while the orbital evolution of the CO includes post-Newtonian (PN) corrections accounting for pericenter precession, Lense-Thirring precession and (leading-order) radiation reaction.

The waveform far away from the source is given in the transverse traceless gauge by

$$h_{ij} = \frac{2}{D}(P_{ik}P_{jl} - \frac{1}{2}P_{ij}P_{kl})\tilde{T}^{kl},$$

where $D$ is the source distance, $P_{ij} = \eta_{ij} - \hat{n}_i\hat{n}_j$ is the projection operator on the space orthogonal to the unit vector of the source position $\hat{n}$, and $\tilde{T}^{ij}$ is the second time derivative of the quadrupole. For an EMRI system with CO mass $m$, central MBH mass $M$ and mass ratio $m/M \ll 1$, we have $\tilde{T}^{ij}(t) = m\tilde{r}^i(t)\tilde{r}^j(t)$, where $\tilde{r}$ is the displacement vector of the CO from the MBH.

The orbit evolution of the CO is described by the first order derivative of the following five quantities:

- $\Phi$: the mean anomaly of the CO’s orbit;
- $\nu$: the orbital frequency;
- $e$: the orbital eccentricity;
- $\alpha$: the azimuthal angle of the CO orbital angular momentum $\hat{L}$ with respect to the MBH’s spin angular momentum $\hat{S}$;
- $\tilde{\gamma}$: the direction of the pericenter relative to $\hat{L} \times \hat{S}$.

The evolution equations for $\Phi$, $\nu$ and $e$ include terms up to 3.5PN order. The evolution of $\alpha$ caused by Lense-Thirring precession and that of $\tilde{\gamma}$ caused by pericenter precession are instead followed up to 2PN order. The equations depend on the two masses $m$ and $M$, the dimensionless spin $\alpha = S/M^2$, and the angle $\lambda$ between $\hat{L}$ and $\hat{S}$. For a distant source, the masses should be replaced by “redshifted” mass $m_z = m(1+z)$ and $M_z = M(1+z)$. To test the no-hair theorem, one can also introduce an arbitrary quadrupole moment $Q$ for the central MBH in the evolution equations. The explicit form of the evolution equations is given in equations (27-31) of [65] (without $Q$) and in equations (4-8) of [35] (with an arbitrary $Q$). Obviously, the orbital evolution depends on the initial conditions at some initial time $t_0$. To obtain the waveform,
we also need five other extrinsic parameters, namely the source’s sky position ($\theta_s, \phi_s$), the luminosity distance $D$ and direction of the MBH’s spin $\hat{S}$ relative to the line of sight ($\theta_K$ and $\phi_K$). In summary, there are 14 parameters, $(t_0, m, M, a, e, \theta_0, \phi_0, \theta_K, \phi_K, \lambda, \alpha_0, \theta_K, \phi_K, D_L)$, with the additional parameter $Q$ introduced when testing the no-hair theorem.

In [65], the waveform was conventionally cut off at the last stable orbit (LSO), $r_{\text{LSO}}$, of the Schwarzschild spacetime. We refer to this as the AK Schwarzschild (AKS) case. However, since the exact value of the cutoff frequency can have significant impact on the parameter estimation, we also consider, as in [44], an AK Kerr (AKK) waveform model where we cut the waveform off at the Kerr LSO. As argued in [44], more realistic EMRI waveform models including dissipative self-effects should yield results between the (more conservative) AKS results and the (more optimistic) AKK ones.

C. Detector response

TianQin will be consist of three satellites orbiting the Earth, forming a regular triangular constellation, with each side measuring about $L = 1.7 \times 10^6$ m. The detector orientation, i.e. the direction normal to the plane of the constellation, will point to a reference source, the white dwarf binary system RX J0806.3+1527 (J0806 for short). The nominal operation time of TianQin will be of five years, which is assumed throughout this paper. There is expected to be only a very small drift of the detector orientation over a five-year period, and since that should have negligible effect on the present study, we will not consider it. The location of J0806 is close to the ecliptic plane, so the constellation plane of TianQin will be nearly perpendicular to the ecliptic plane. The TianQin satellites will have nearly identical orbits and nearly identical periods, which will be about $T = 3.6$ days [5–7].

In the Solar Ecliptic Coordinate System where the x-axis points towards the direction of the vernal equinox and the z-axis is normal to and northward from the ecliptic plane, the location of the satellites at a given time can be formulated as [10, 70]:

$$
x(t) = R \cos \alpha_e + \frac{1}{2} R \cdot e_c \cdot \cos(2\alpha_e - 3) + \frac{1}{\sqrt{3}} L \cdot \cos \theta \cos \phi \cos \gamma_e - \sin \phi \sin \gamma_e,
$$

$$
y(t) = R \sin \alpha_e + \frac{1}{2} R \cdot e_c \cdot \sin(2\alpha_e) + \frac{1}{\sqrt{3}} L \cdot \cos \theta \sin \phi \cos \gamma_e + \cos \phi \sin \gamma_e,
$$

$$
z(t) = -\frac{1}{\sqrt{3}} L \cdot \sin \theta \cos \gamma_e,
$$

(3)

where $R = 1 \text{AU}$ is the semi-major axis, $e_c = 0.0167$ is the eccentricity, and $\alpha_e = 2\pi f_c t + \Phi_e$ is the phase, with $f_c = 1/yr$ and $\Phi_e$ being respectively the frequency and initial phase of the orbit of the Earth. The angular parameters ($\theta, \phi$) specify the space direction to J0806, $\gamma_e = 2\pi t/T + 2\pi n/3 + \delta, (n = 1, 2, 3)$ are the phases of the satellites in their geocentric orbits, where $\delta$ is some reference phase that can be set to zero.

The effect of a propagating GW, $h(\xi)$, on the optical path length $L_{ij}$ starting from the satellite $i$ at time $t - L_{ij}$ and arriving at the satellite $j$ at time $t$ can be expressed as [71, 72],

$$
\delta L_{ij}(t) = \frac{1}{2} \frac{\hat{r}_{ij}(t) \otimes \hat{r}_{ij}(t)}{1 - k \cdot \hat{r}_{ij}(t)} \cdot \int_{\xi_i}^{\xi_j} h(\xi) d\xi,
$$

(4)

where $\xi$ is the phase, $\hat{r}_{ij}(t)$ is the unit vector from the satellite $i$ to the satellite $j$ at time $t$. Two independent Michelson interferometer signals can be constructed [73],

$$
h_1(t) = [\delta L_{12}(t) - \delta L_{13}(t)]/L,
$$

$$
h_2(t) = \frac{1}{\sqrt{3}} [\delta L_{12}(t) + \delta L_{13}(t) - \delta L_{23}(t)]/L.
$$

(5)

Note that the orbital motion of the TianQin satellites also contributes a phase modulation (due to the Doppler shift) to the observed signal,

$$
\Phi^D(t) = 2\pi v(t) R \sin \theta \cos[\phi(t) - \phi_S],
$$

(6)

where $2\nu$ is the frequency of the GW signal, $(\theta_S, \phi_S)$ are the angular coordinates of the source, and $\phi(t) = \alpha_e$ is the orbit phase.

D. Detector noise

The noise model of TianQin is encoded in the following sensitivity curve [8, 9, 13, 74],

$$
S_n(f) = \frac{1}{L^2} \left[ \frac{4S_n}{(2\pi f)^2} \left( 1 + \frac{10^{-4}\text{Hz}}{f} \right) + S_z \right] \times \left[ 1 + 0.6 \left( \frac{f}{f_s} \right)^2 \right],
$$

(7)

where $S^{1/2}_n = 1 \times 10^{-17} \text{m s}^{-2}/\text{Hz}^{1/2}$ characterizes the residual acceleration on a test mass playing the role of an inertial reference, $S_z^{1/2} = 1 \times 10^{-12} \text{Hz}^{1/2}$ characterizes the one-way noise of the displacement measurement with inter-satellite laser interferometry, and $f_s = 1/(2\pi L)$ is the transfer frequency [5]. An illustration of the sensitivity curve of TianQin is given in Fig. 1.

At the low-frequency end of the TianQin observation range, there exist numerous compact binaries in the Galaxy, whose GW signals overlap to give rise to a stochastic unresolved signal, sometimes referred to as the foreground. Preliminary analyses suggest that such a foreground will be consistently below the sensitivity curve for resolved sources, given that the operation time of TianQin is limited to five years [12]. We therefore do not consider the effect of Galactic compact binaries throughout this work.
### III. METHOD

#### A. Signal-to-noise ratio

In order to study the prospects of detecting EMRIs with TianQin, one can calculate the SNR. Previous studies have shown that EMRIs with SNRs as low as 15 can be detected by LISA under favorable circumstances [75]. In this paper, we adopt the more conservative SNR threshold of 20 for detection, following [44].

Using the noise-weighted inner product between two signals $s_1(t)$ and $s_2(t)$,

$$ (s_1 | s_2) = 2 \int_{0}^{\infty} \tilde{s}_1(f) \tilde{s}_2(f) \, df, $$

where $\tilde{s}_i(f)$, $i = 1, 2$, are the Fourier transforms of $s_i(t)$, the SNR can be defined as:

$$ \rho = (h|h)^{1/2} = 2 \left[ \int_{0}^{\infty} \tilde{h}(f) \tilde{h}^*(f) \, df \right]^{1/2}, $$

where $h(t)$ is the GW induced signal in the detector. The Fourier transform $\tilde{h}(f)$ is obtained from $h(t)$ by applying a discrete Fourier transform,

$$ \tilde{h} \left( \frac{k}{N \Delta t} \right) = \Delta t \sum_{n=1}^{N} h(n \Delta t) e^{-i 2\pi kn/N}, $$

where $\Delta t$ is the sampling interval.

The basic TianQin data stream consists of data segments each lasting for three months, and the total accrued SNR is obtained as the root sum square of the individual SNR from each data segment. The same root-sum-square rule is also used when combining the contributions from different interferometer signals (when we consider TianQin operating within a detector network).

#### B. Fisher Information Matrix

The existence of noise leads to uncertainties in the inference on source parameters. To quantify these uncertainties, we use the FIM method to obtain the lowest order expansion of the posteriors (valid in the high SNR limit), which can be more accurately estimated through a full Bayesian parameter estimation analysis. Indeed, we note that the FIM method can be used as a fast assessment of the expected parameter estimation capabilities of an experiment, but the obtained $\Sigma$ only represents the Cramer-Rao bound of the covariance matrix. More advanced techniques are required to obtain more realistic results [76, 77].

The FIM is defined as

$$ \Gamma_{ij} = \left( \frac{\partial \tilde{h}(f)}{\partial \theta^i} \frac{\partial \tilde{h}(f)}{\partial \theta^j} \right), $$

where $\theta^i, i = 1, 2, \ldots$, are the parameters appearing in the template $\tilde{h}(f)$. When multiple interferometers are present, the network’s FIM can be obtained as the sum of the individual FIM from each interferometer.

The EMRI waveform, even assuming the relatively simple AK model, is rather complicated, and it is difficult to obtain general analytical expressions for the partial derivatives, $\partial \tilde{h}(f)/\partial \theta^i$. We therefore approximate derivatives with respect to the parameters by numerical finite differences. In the lowest order expansion (i.e. in the high SNR limit), the variance-covariance matrix can be obtained as the inverse of the FIM,

$$ \Sigma_{ij} \equiv \langle \delta \theta_i \delta \theta_j \rangle = (\Gamma^{-1})_{ij}. $$

From the variance-covariance matrix, the uncertainty $\sigma_i$ of the $i$th parameter $\theta_i$ can be obtained as

$$ \sigma_i = \Sigma_{ii}^{1/2}. $$

We also note that it is often meaningful to discuss the sky localization in terms of the solid angle $\Delta \Omega$ corresponding to the error ellipse for which there is a probability $\exp(-1)$ for the source to be outside of it [44], which can be expressed as a combination of the uncertainties on the ecliptic longitude angle $\phi_S$ and the ecliptic latitude angle $\theta_S$,

$$ \Delta \Omega = 2 \pi \sin \theta_S \sqrt{\Sigma_{\phi_S} \Sigma_{\phi_S} - \sigma_{\phi_S}^2}. $$

### IV. RESULTS

#### A. Horizon distance

As a first assessment of TianQin’s capability of detecting EMRIs, we compute the horizon distance, i.e. the farthest distance at which an EMRI source can be detected, or equivalently the farthest distance at which the

![The sensitivity curve for TianQin.](image)
SNR exceeds our detection threshold of 20, under the most favourable conditions possible [78].

For the 7 intrinsic parameters, \( m, M, S/M^2, e_0, \gamma_0, \phi_0, \) and \( \lambda \), we fix the mass of CO to \( m = 10M_\odot \), the orbital eccentricity to \( e_0 = 0.1 \), the MBH’s spin to \( a = 0.98 \) and the inclination angle to \( \lambda = \pi/3 \), while the initial condition for \( \gamma_0 \) is set to 0, although that choice has marginal effect on the SNR. One can see from Eq. (4) that TianQin has the strongest response to sources sitting on the line passing through the detector and J0806, so the source is placed in the direction of J0806. We also fix \( \Phi_0, \alpha_0 \) to 0, \( \theta_K, \phi_K \) to \( \pi/4 \), while the plunge time is taken to be 5 yr, which is the mission time of TianQin. These values of the parameters are all assumed to hold at the moment when the CO plunges into the MBH, which happens at different \( r_{LSO} \) depending on whether AKS or AKK waveforms are used.

The maximum redshift at which EMRIs can be detected by TianQin with a threshold SNR of 20 is illustrated in Fig. 2 as a function of MBH mass. The black curve (corresponding to AKS waveforms) would allow for a larger detection range, and features better sensitivity to systems with heavier MBHs than the red curve (which more conservatively uses AKK waveforms). This is mainly due to the fact that by adopting a cutoff at later times (i.e. higher frequencies), the AKK waveforms include the larger GW amplitudes emitted when the CO is very close to the MBH. Thus, farther events are expected to be detectable under the fixed SNR threshold. The maximum horizon distance for AKK waveforms corresponds to \( z \approx 2.6 \), and to MBH mass around \( 4 \times 10^5 M_\odot \). For the AKS waveform, on the other hand, the maximum horizon distance is smaller, with a corresponding redshift of about 1.6. The MBH mass for which the AKS horizon distance peaks is also smaller, and around \( 2 \times 10^5 M_\odot \). This feature can also be explained in a simple manner: for the same EMRI system, AKK waveforms extend to higher frequencies, but that high frequency component is most important for high mass systems, whose low frequency inspiral produces little SNR as it lies at lower frequencies than TianQin’s sensitivity sweet spot. Therefore, when using AKS waveforms (for which that high frequency part is absent), the SNR of high mass EMRIs is suppressed.

B. Detection rate

We compute the expected detection rates for EMRI systems with TianQin by using the 12 astrophysical models developed in [44] and reviewed in section II A. For each of the 12 models, we construct catalogs of simulated events with both the number of events and their physical parameters randomly generated according to the underlying distribution. Five parameters, including \( M, m, a, \lambda, z \), are inherited from the catalog realizations used by [44], while all other parameters are randomly extracted again. In more detail, the plunge time is distributed uniformly within the mission lifetime of TianQin (5 yr). The sky positions of the sources \( (\theta_S, \phi_s) \) and their spin orientations \( (\theta_K, \phi_K) \) are drawn from an isotropic distribution on the sphere. The phase parameters \( \Phi, \gamma, \alpha \) at plunge are drawn from a uniform distribution in \([0, 2\pi]\). The orbital eccentricity at plunge is drawn from a uniform distribution in \([0, 0.2]\).

The SNR for all events is calculated, and events with
TABLE I. The expected detection rate of EMRIs with TianQin for different astrophysical models. The physical assumptions of the 12 models are described in Table I of [44]. The numbers are broken into different MBH mass ranges in the middle three columns, while the rightmost column summarizes the total detection rate. Numbers in brackets correspond to detection rates with AKS waveforms, while numbers outside brackets assume AKK waveform.

| Model | event rate ($\text{yr}^{-1}$) | Detection rate of TianQin in mass range ($\text{yr}^{-1}$) | Total ($\text{yr}^{-1}$) |
|-------|-------------------------------|-------------------------------------------------|-------------------|
|       |                               | $M_{10} < 5$ | $5 < M_{10} < 6$ | $6 < M_{10}$ | |
| M1    | 1600                          | 1(1)       | 25(11)           | 8 (1)     | 34 (13) |
| M2    | 1400                          | 0(0)       | 18(12)           | 2(0)      | 20 (12) |
| M3    | 2770                          | 0(0)       | 83(28)           | 27 (2)    | 110(30) |
| M4    | 520(620)                      | 1 (0)      | 42(28)           | 7(3)      | 50 (31) |
| M5    | 140                           | 0(0)       | 4(2)             | 4(0)      | 8(2)    |
| M6    | 2080                          | 1(1)       | 40(22)           | 23(0)     | 64(23)  |
| M7    | 15800                         | 18(18)     | 187(121)         | 55(4)     | 260(143) |
| M8    | 180                           | 0(0)       | 5(0)             | 1(0)      | 6(0)    |
| M9    | 1530                          | 2(1)       | 16(14)           | 2(1)      | 20(16)  |
| M10   | 1520                          | 0(0)       | 18(14)           | 0(0)      | 18(14)  |
| M11   | 13                            | 0(0)       | 0(0)             | 0(0)      | 0(0)    |
| M12   | 20000                         | 13(11)     | 273(113)         | 150(2)    | 436(126)|

SNR larger than 20 are considered as detected. Again, we perform calculations with AKS and AKK waveforms separately (c.f. section II B). For both AKK and AKS waveforms, the physical parameters are assumed to be measured at the Schwarzschild LSO. In both cases, each system is then evolved backwards to a sufficiently long time before merger.

We present the expected detection rates for different models in Table I, where the results for AKS (AKK) waveforms are inside (outside) the brackets. The overall detection rates are summarised in the rightmost column, and a breakdown of the rates for three different MBH mass ranges, $M_{10} < 5$, $5 < M_{10} < 6$ and $M_{10} > 6$ with $M_{10} \equiv \log(M/\text{M}_\odot)$, is also given.

For most of the 12 models, the expected detection rates vary from dozens to hundreds of events per year, regardless of whether AKS or AKK waveforms are used. Models M5, M8 and M11 predict however significantly smaller detection rates, mainly because of the significantly lower intrinsic EMRI rate in the models themselves. Note that using the AKK waveforms generally predicts a significantly higher detection rate than using AKS waveforms. This happens because sources with prograde orbits are about 40% more numerous than those with retrograde orbits. Sources on prograde orbits can have Kerr LSO closer to the MBH, and thus AKK waveforms accrue much higher SNRs.

One can also see from Table I and Fig. 2 that the majority of EMRIs exceeding the SNR detection threshold are those with masses $10^5\text{M}_\odot \sim 10^6\text{M}_\odot$. A similar feature was also found to hold for the detectability of massive binary black hole mergers using TianQin [9]. This feature is mostly related to the frequency dependence of the sensitivity curve and to the relation between the MBH mass and the peak frequency of a GW signal.

We remark that the calculations performed in this section account for EMRIs and not for direct plunges into the MBH. In fact, as mentioned in section II A, for each EMRI there is expected to be a potentially sizeable number $N_p$ of COs plunging directly into the MBH along low angular momentum orbits. We have however verified that these plunges typically have SNR of a few [79, 80], and are thus not easily detectable by TianQin.

C. Parameter estimation

As already mentioned, observation of the GW signal from EMRIs by space based detectors may allow for testing the no-hair theorem [31–37] and for revealing the possible presence with matter surrounding MBHs [38–43]. Moreover, EMRIs may permit gaining information on the mass distribution of MBHs [29], on their host stellar environments [28], as well as on the expansion of the Universe [30]. All these goals, however, rely on high precision measurements of the source parameters. In this section, we therefore investigate the parameter estimation of EMRIs with TianQin, using a FIM approach.

Among the 14 parameters introduced in section II B, one is generally mostly interested in the redshifted mass $m_z$ and the orbital eccentricity $e_0$ of the CO, the redshifted mass $M_z$ and the spin $a$ of the MBH, the luminosity
distance to the source $D_L$, and the sky localization (the solid angle within which the source is located) $\Omega$. The FIM-predicted uncertainties in the estimation of these parameters are given in Figs. 3 and 4, respectively for AKS and AKK waveforms. A kernel density estimation has been used to smooth out the random fluctuations.

We first notice that although the 12 models cover a wide range of different astrophysical setups, the probability distributions of the predicted 1σ errors are quite similar, as expected from earlier studies conducted for the LISA mission [44]. There is also an intriguing similarity between the results in Figs. 3 and 4, as one may have expected much better results for AKK waveforms. Indeed, for a source that can be detected with both AKS and AKK waveforms, the precision of parameter estimation is certainly better with AKK waveforms, because the SNR is greater. However, there is a large portion of events that can be detected with AKK waveforms, but
FIG. 4. The distributions of parameter estimation 1σ errors for various astrophysical models, calculated using AKK waveforms.

which do not have enough SNR when using AKS waveforms. In Fig. 4, the high precision achieved for the high SNR events is largely diluted by these relatively low SNR events. In this sense, the similarity between Figs. 3 and 4 is a demonstration of the strong link between the SNR and the precision of parameter estimation.

Figs. 3 and 4 also show a stark difference between intrinsic and extrinsic parameters. The intrinsic parameters are those that contribute to the phase of the GWs, such as the redshifted mass $M_z$ and the spin $a$ of the MBH, and the redshifted mass $m_z$ and the orbit eccentricity $e_0$ of the CO. Parameters such as the luminosity distance $D_L$ and the sky localization $\Omega$ only affect the amplitude of the GWs, and are referred to as extrinsic. Assuming a typical observation time of $10^8$ second and a typical frequency of $10^{-2}$Hz, the number of wave cycles in an EMRI signal is of the order of $10^6$. Due to the huge number of cycles, a very slight change in the in-
trintrinsic parameters (and hence in the phase) could change the cycle number by one, which is in principle detectable. Therefore, a (relative) precision of the order of $10^{-6}$ is expected for the intrinsic parameters. We indeed observe peaks roughly at this precision in both Fig. 3 and Fig. 4, for all models. On the other hand, the estimation of the extrinsic parameters cannot benefit from the accumulation of a large number of wave cycles, and therefore the expected precision is much worse than for the intrinsic parameters.

We have also considered the possibility of testing the no-hair theorem by measuring the multipole moments of the MBH [35, 71, 81]. A Kerr black hole satisfies the no-hair theorem and has a quadrupole moment $Q_K$ determined completely by its mass and spin, $Q_K = -a^2 M^3$ [82]. Here, we relax the Kerr hypothesis and allow for the quadrupole moment $Q$ to deviate from the Kerr value. In Fig. 5, we present the predicted errors on the dimensionless quantity $Q \equiv (Q - Q_K)/M^3$, with the two distributions corresponding to AKS and AKK waveform for the 12 models, respectively.

D. TianQin in a network of detectors

We briefly discuss the intriguing possibility that TianQin could be observing within a network of detectors, such as TQ I+II, TQ + LISA and TQ I+II + LISA (see [8] for a detailed explanation of each of the detector networks).

In Fig. 6, we plot the expected detection rate using TQ and TQ I+II, adopting AKK waveforms. Note that for burst signals, extending the observation time to full coverage would effectively double the detection rate. However, since EMRIs are long-lived sources, the increase in the detection rate will not scale with the duty cycle, and is shown in Fig. 6. As can be seen, detection rates could increase by a factor ranging from $\sim 2$ in model 8 up to even $\sim 3$.

A comparison of the precision of parameter estimation with TQ, TQ I+II, TQ + LISA and TQ I+II + LISA is given in Table II. In the calculation, we employed 12 astrophysical models encapsulating a wide range of different scenarios for the underlying EMRI populations, which result in significantly different intrinsic EMRI rates (ranging from $\sim 10$ to $\sim 20000$ per year). Waveforms are described by simple analytic kludge templates.

Adopting a detection threshold of SNR=20, we find that most of the 12 astrophysical models predict that TianQin will detect dozens to thousands of EMRIs. The only model in which this is not the case (model 11) predicts that the vast majority of events should involve a CO plunging directly into the MBHs, which results in very low rates irrespective of the GW detector configuration.

As for the horizon distance, we find that EMRIs can be detected up to maximum redshifts varying from 1.6 to $\sim 2.6$ according to what waveform model is adopted (AKS vs AKK). The MBH mass yielding the maximum horizon distance also changes from around $2 \times 10^5 M_\odot$ if AKS waveforms are used, to around $4 \times 10^5 M_\odot$ for AKK waveforms. As a result, AKK waveforms also predict larger detection rates. Overall, this dependence on the waveform model highlights the need to develop fast and accurate EMRI waveforms beyond the kludge approximation.

The expected precision of the parameter estimation is calculated using the FIM method. We find that the majority of detected events can determine the intrinsic parameters to within fractional errors of $\sim 10^{-6}$, while the errors on the extrinsic parameters are much less stringent. However, the majority of detected events can still determine the relative uncertainty in the luminosity distance with 10% and the uncertainty in the sky localization to the level of about 10 deg$^2$. The precise determination of the three dimensional location might make it possible for EMRIs to be used as standard sirens for cosmology [83, 84], although further detailed studies are needed in this direction.

We briefly consider using EMRI to put constraints on possible deviations from the Kerr quadrupole moment, and we find the uncertainty in the dimensionless parameter $Q$ peaks at about $\Delta Q \sim 10^{-4}$.

We also briefly consider the possible cases when TianQin is observing within a network of detectors. We find that such networks of detectors can improve the precision on sky localization by more than 20 times and the precision on other parameters as large as 5 times.

V. SUMMARY AND FUTURE WORK

In this work, we have performed a preliminary study of the horizon distance, detection rate and precision of parameter estimation for EMRIs with TianQin. We have employed 12 astrophysical models encapsulating a wide range of different scenarios for the underlying EMRI populations, which result in significantly different intrinsic EMRI rates (ranging from $\sim 10$ to $\sim 20000$ per year). Waveforms are described by simple analytic kludge templates.

Adopting a detection threshold of SNR=20, we find that most of the 12 astrophysical models predict that TianQin will detect dozens to thousands of EMRIs. The only model in which this is not the case (model 11) predicts that the vast majority of events should involve a CO plunging directly into the MBHs, which results in very low rates irrespective of the GW detector configuration.

As for the horizon distance, we find that EMRIs can be detected up to maximum redshifts varying from 1.6 to $\sim 2.6$ according to what waveform model is adopted (AKS vs AKK). The MBH mass yielding the maximum horizon distance also changes from around $2 \times 10^5 M_\odot$ if AKS waveforms are used, to around $4 \times 10^5 M_\odot$ for AKK waveforms. As a result, AKK waveforms also predict larger detection rates. Overall, this dependence on the waveform model highlights the need to develop fast and accurate EMRI waveforms beyond the kludge approximation.

The expected precision of the parameter estimation is calculated using the FIM method. We find that the majority of detected events can determine the intrinsic parameters to within fractional errors of $\sim 10^{-6}$, while the errors on the extrinsic parameters are much less stringent. However, the majority of detected events can still determine the relative uncertainty in the luminosity distance with 10% and the uncertainty in the sky localization to the level of about 10 deg$^2$. The precise determination of the three dimensional location might make it possible for EMRIs to be used as standard sirens for cosmology [83, 84], although further detailed studies are needed in this direction.

We briefly consider using EMRI to put constraints on possible deviations from the Kerr quadrupole moment, and we find the uncertainty in the dimensionless parameter $Q$ peaks at about $\Delta Q \sim 10^{-4}$.

We also briefly consider the possible cases when TianQin is observing within a network of detectors. We find that such networks of detectors can improve the precision on sky localization by more than 20 times and the precision on other parameters as large as 5 times.

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FIG. 5. Probability distribution of the 1σ errors for the “anomalous” quadrupole moment $Q$ defined in the text, for AKS (left) and AKK (right) waveforms.

| MBH mass | configuration | $\Delta M_1/M_1$ | $\Delta M_2/M_2$ | $\Delta a_0$ | $\Delta c_0$ | $\Delta D_L/D_L$ | $\Delta (\text{deg}^2)$ |
|----------|----------------|--------------------|--------------------|--------------|--------------|-------------------|---------------------|
| log($\frac{M}{L}$) = 5.0 | TQ | $2.25 \times 10^{-7}$ | $9.15 \times 10^{-7}$ | $1.46 \times 10^{-7}$ | $1.71 \times 10^{-8}$ | $2.23 \times 10^{-2}$ | 0.31 |
| | TQ I+II | $1.54 \times 10^{-7}$ | $7.3 \times 10^{-7}$ | $1.15 \times 10^{-7}$ | $1.25 \times 10^{-8}$ | $2.19 \times 10^{-2}$ | 0.19 |
| | TQ+LISA | $1.01 \times 10^{-7}$ | $4.86 \times 10^{-7}$ | $8.0 \times 10^{-7}$ | $8.4 \times 10^{-8}$ | $1.70 \times 10^{-2}$ | 0.09 |
| | TQ I+II+LISA | $0.91 \times 10^{-7}$ | $4.67 \times 10^{-7}$ | $0.75 \times 10^{-7}$ | $0.76 \times 10^{-7}$ | $1.69 \times 10^{-2}$ | 0.08 |
| log($\frac{M}{L}$) = 5.5 | TQ | $1.87 \times 10^{-6}$ | $1.41 \times 10^{-6}$ | $6.3 \times 10^{-7}$ | $4.09 \times 10^{-7}$ | $1.27 \times 10^{-2}$ | 1.51 |
| | TQ I+II | $1.23 \times 10^{-6}$ | $0.82 \times 10^{-6}$ | $5.47 \times 10^{-7}$ | $3.22 \times 10^{-7}$ | $1.24 \times 10^{-2}$ | 0.46 |
| | TQ+LISA | $0.72 \times 10^{-6}$ | $0.51 \times 10^{-6}$ | $3.59 \times 10^{-7}$ | $1.51 \times 10^{-7}$ | $8.5 \times 10^{-2}$ | 0.15 |
| | TQ I+II+LISA | $0.69 \times 10^{-6}$ | $0.49 \times 10^{-6}$ | $3.51 \times 10^{-7}$ | $1.48 \times 10^{-7}$ | $8.4 \times 10^{-2}$ | 0.14 |
| log($\frac{M}{L}$) = 6.0 | TQ | $6.63 \times 10^{-6}$ | $3.53 \times 10^{-6}$ | $9.66 \times 10^{-7}$ | $5.53 \times 10^{-6}$ | $9.7 \times 10^{-3}$ | 4.88 |
| | TQ I+II | $3.08 \times 10^{-6}$ | $1.87 \times 10^{-6}$ | $6.3 \times 10^{-7}$ | $2.55 \times 10^{-6}$ | $9.11 \times 10^{-2}$ | 1.61 |
| | TQ+LISA | $0.57 \times 10^{-6}$ | $0.40 \times 10^{-6}$ | $2.74 \times 10^{-7}$ | $0.48 \times 10^{-6}$ | $5.48 \times 10^{-3}$ | 0.16 |
| | TQ I+II+LISA | $0.57 \times 10^{-6}$ | $0.40 \times 10^{-6}$ | $2.72 \times 10^{-7}$ | $0.48 \times 10^{-6}$ | $5.45 \times 10^{-3}$ | 0.15 |
| log($\frac{M}{L}$) = 6.5 | TQ | $3.4 \times 10^{-6}$ | $5.01 \times 10^{-6}$ | $9.07 \times 10^{-7}$ | $3.38 \times 10^{-6}$ | $1.5 \times 10^{-2}$ | 11.8 |
| | TQ I+II | $3.04 \times 10^{-6}$ | $2.68 \times 10^{-6}$ | $8.19 \times 10^{-7}$ | $2.94 \times 10^{-6}$ | $1.46 \times 10^{-2}$ | 4.51 |
| | TQ+LISA | $0.51 \times 10^{-6}$ | $0.83 \times 10^{-6}$ | $2.59 \times 10^{-7}$ | $0.64 \times 10^{-6}$ | $0.57 \times 10^{-2}$ | 0.44 |
| | TQ I+II+LISA | $0.51 \times 10^{-6}$ | $0.82 \times 10^{-6}$ | $2.57 \times 10^{-7}$ | $0.64 \times 10^{-6}$ | $0.57 \times 10^{-2}$ | 0.42 |

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