Symmetry Breaking Using Fluids II: Velocity Potential Method.

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February 7, 2008

Published: Hadronic Journal 20(1997)73-83.

Eprint: hep-th/9904079
Comments: 13 pages, no diagrams, one table, Latex2e.

3 KEYWORDS:
Covariantly Charged Fluid: Symmetry Breaking: Higg’s Model.

1999 PACS Classification Scheme:
http://publish.aps.org/eprint/gateway/pacstable
11.15Ex, 14.80Gt,05.70Jk,04.40+c

1991 Mathematics Subject Classification:
http://www.ams.org/msc
831T13, 83C55.
Abstract

A generalization of scalar electrodynamics called fluid electrodynamics is presented. In this theory a fluid replaces the Higgs scalar field. Fluid electrodynamics might have application to the theory of low temperature Helium superfluids, but here it is argued that it provides an alternative method of approaching symmetry breaking in particle physics. The method of constructing fluid electrodynamics is to start with the velocity decomposition of a perfect fluid as in general relativity. A unit vector tangent to the flow lines of an isentropic fluid can be written in terms of scalar potentials:

\[ V_a = h^{-1}(\phi_a + \alpha \beta_a - \theta S). \]

A novel interacting charged fluid can be obtained by applying the covariant derivative: \( D_a = \partial_a + ieA_a \) to these scalar potentials. This fluid is no longer isentropic and there are choices for which it either obeys the second law of thermodynamics or not. A mass term of the correct sign occurs for the \( A \) term in the stress, and this mass term depends on the potentials in the above vector. The charged fluid can be reduced to scalar electrodynamics and the standard approach to symmetry breaking applied; alternatively a mass can be induced by the fluid by using just the thermodynamic potentials and then fixing at a critical point, if this is taken to be the Bose condensation point then the induced mass is negligible.

1 Introduction.

In particle physics fields are given mass by assuming the existence of Higgs scalar fields. There are a number of unsatisfactory aspects to this procedure, for example: the Higgs scalar has not been experimentally found, also it is required to have a mass term of the wrong sign. An alternative procedure has been suggested, Roberts (1989) [1] in which fluids rather than scalar fields are responsible for vector fields acquiring mass. The motivation for using fluids is a principle [1] which states that there is only one concept of mass in physics; furthermore this concept of mass must be ultimately of gravitational origin. The picture envisaged can be thought of as occurring in four steps or stages, these words have unwanted temporal connotations so that we refer instead to four basic ingredients. The first ingredient is that the vector field is taken to exist, and it is assumed to have a primitive stress. The second ingredient is that the vector field has statistical properties which produce an effective fluid that couples to the primitive stress. The third ingredient is that the coupling between the fluid and the primitive stress produces a mass term. The fourth ingredient is that the gravitational origin of this
is in some as yet undiscovered relationship between statistical mechanics and gravitational theory. Those of a more prosaic disposition can simply regard fluids as an alternative technical means for inducing mass, perhaps with application in the theory of superfluids, Israel (1981) [22]. Scalar fields and fluids can be equated in several ways as shown for example in [1]. Usually fluids exhibit more freedom as they are parameterised by \( p, \mu, V_a \) however static scalar fields are equivalent to fluids with imaginary vector \( V_a \) so that there are sometimes different qualitative properties, an example of this is the asymptotic properties, Roberts (1998) [3] of the space-time. The technical methods in [4], uses what in section 2 is called the "already interacting" fluid. Here the technical procedure is to replace derivatives in the velocity decomposition of the fluid tangent vector \( V_a \) by vector covariant derivatives. The resulting "covariantly interacting" fluid can be reduced to scalar electrodynamics, and then the standard symmetry breaking procedure applied. The covariantly interacting fluid generalizes scalar electrodynamics and the extra degrees of freedom provide the scope for other symmetry breaking mechanisms to be used. One mentioned here requires that only quantities of thermodynamic importance be retained in the mass breaking term, these thermodynamic quantities can then be fixed at a critical value such as the Bose condensation value. The drawback of this approach in its present form is that it necessitates the introduction of an ad hoc time interval; here this is taken to be the Planck time. It is found that the induced mass is negligible, i.e. well beneath the experimental limit of \( 10^{-50} \) Kg. [3]. It is of interest to know if it could be demonstrated that the photon mass is exactly zero. The Proca equation [4] p.135 necessitates \( m^2 \nabla_a A^a = 0 \), so that the existence of photon mass fixes the gauge. The Bohm-Aharonov effect [5] shows that the gauge must be chosen such that \( A_a \) is continuous; it might be possible to devise a geometric configuration in which the continuity of \( A_a \) requires a different gauge choice from \( \nabla_a A^a = 0 \), thus giving \( m = 0 \) from \( m^2 \nabla_a A^a = 0 \). In this paper section 2 briefly introduces the standard scalar electrodynamics symmetry breaking, section 3 introduces the velocity decomposition for the velocity tangent to a fluid, section 4 applies the vector covariant derivative to this, section 5 mentions some possible ways in which the resulting fluid could break symmetry. The conventions used are: signature \(-+++
\)”, \(^\n\) signifies Christoffel covariant derivative, symmetrization is denoted by round brackets, e.g. \( T_{(ab)} = (T_{ab} + T_{ba})/2 \), anti-symmetrization is denoted by square brackets, e.g. \( T_{[ab]} = (T_{ab} - T_{ba})/2 \).
2 Scalar Electrodynamics.

The scalar electrodynamic Lagrangian [6], [4]p.68, [7]p.699 is

\[ L_{sel} = -D_a \psi D^a \bar{\psi} - V(\psi \bar{\psi}) - \frac{1}{4} F^2, \quad (1) \]

where the covariant derivative is

\[ D_a \psi = \partial_a \psi + i e A_a, \quad (2) \]

and \( D_a \bar{\psi} = \bar{D}_a \psi \). The variation of the corresponding action with respect to \( A_a, \psi, \) and \( \bar{\psi} \) are given by

\[ \delta I \delta A_a = F_{ab} \ldots \delta I \delta \psi = (D_a D^a - V') \bar{\psi}, \quad V' = \frac{dV}{d(\psi \bar{\psi})}, \quad (3) \]

and its complex conjugate. Variations of the metric give the stress

\[ T_{ab} = 2D_a \psi D_b \bar{\psi} + F_{ac} F_{b}^{\cdot \cdot c} + g_{ab} L. \quad (4) \]

The complex scalar field can be put in ’polar’ form by defining

\[ \psi = \exp(i\nu), \quad (5) \]

giving the Lagrangian

\[ L = \rho^2 + (D_a \nu)^2 - V(\rho^2) - \frac{1}{4} F^2, \quad (6) \]

where

\[ D_a \nu = \rho (\nu_a + eA_a). \quad (7) \]

The variations of the corresponding action with respect to \( A_a, \rho, \) and \( \nu \) are given by

\[ \delta I \delta A_a = F_{ab} + 2 \rho D^a \nu, \]

\[ \delta I \delta \rho = 2 \left( \Box + (\nu_a + eA_a)^2 + V' \right), \]

\[ \delta I \delta \nu = 2 \left( \Box \nu + eA^a_{a; a} \right). \quad (8) \]

\[ 4 \]
Variation of the metric gives the stress

\[ T_{ab} = 2\rho_a \rho_b + 2\nabla_a \nu \nabla_b \nu + F_{ac} F_{b.}^c + g_{ab} L. \]  

(9)

Defining

\[ B_a = A_a + \nu_a / e, \]  

(10)

\( \nu \) is absorbed to give Lagrangian

\[ L = \rho_a^2 + \rho^2 e^2 B_a^2 - V(\rho^2) - \frac{1}{4} F^2; \]  

(11)

which does not contain \( \nu \); equation (10) is a gauge transformation when there are no discontinuities in \( \nu \) i.e. \( \nu_{[ab]} = 0 \).

The requirement that the corresponding quantum theory is renormalizable restricts the potential to the form

\[ V(\rho^2) = m^2 \rho^2 + \lambda \rho^4. \]  

(12)

The ground state is when there is a minimum, for \( m^2, \lambda > 0 \) this is \( \rho = 0 \), but for \( m^2 < 0, \lambda > 0 \) this is

\[ \rho^2 = \frac{-m^2}{2\lambda} = a^2, \]  

(13)

thus the vacuum energy is

\[ <0|\rho|0> = a. \]  

(14)

To transform the Lagrangian to take this into account substitute

\[ \rho \to \rho' = \rho + a, \]  

(15)

to give

\[ L = \rho_a^2 + (\rho + a)^2 e^2 B_a^2 - V((\rho + a)^2) - \frac{1}{4} F^2. \]  

(16)

Now apparently the vector field has a mass \( m \) from the \( a^2 e^2 B_a^2 \) term it is given by \( m = ae \). The cross term \( 2a\rho e^2 B_a^2 \) is ignored.
3 Velocity Potentials.

A Newtonian 3-vector, can be decomposed: \( \mathbf{v} = \nabla \phi + \alpha \nabla \beta \), where \( \phi, \alpha \), and \( \beta \) are the Clebsch potentials. A particular case of Paff’s theorem shows that a 4-vector can be decomposed: \( \mathbf{V} = A \mathbf{B} + C \mathbf{D} \) where \( A, B, C, \) and \( D \) are the potentials. The work of Refs. [1]–[13] shows that a non-minimal decomposition

\[
\mathbf{V} = h^{-1} (\phi_a + \alpha \beta_a - \theta S_a), \quad V_a V^a = -1
\]

is more useful, because for an isentropic fluid all the potentials have evolution equations. \( \theta \) is the thermasy of van Dantzig \[14\] eq.4.9 it is usually defined by

\[
d\theta = -kT \, dt,
\]

where \( T \) is the temperature; also \( h \) is the enthalpy and \( S \) is the entropy. The three other potentials do not have a thermodynamic interpretation. A current vector can be defined by

\[
\mathbf{W} = h \mathbf{V},
\]

c.f.\[7\] p.69, because \( W_{a:b} = 0 \) it is not conserved. The Christoffel derivative of \( W_a \) can be in the usual manner \[7\] p.82-3

\[
W_{a:b} = \omega_{ab} + \sigma_{ab} + \frac{1}{3} \Theta h_{ab} - \dot{W}_a W_b,
\]

and then defining the vorticity tensor, vorticity vector, expansion tensor, expansion scalar, shear tensor, and acceleration vector in the usual manner, it is found that only the vorticity tensor, expansion scalar, and acceleration vector show any simplification, they are

\[
\omega_{ab} \equiv h_a^{\,c} h_b^{\,a} W_{c:d} = h_a^{\,c} h_b^{\,a} (\alpha_{[d} \beta_{c]} - \theta_{[d} S_{c]}),
\]

\[
\Theta \equiv W_{a:a} = \nabla \phi + \alpha \nabla \beta - \theta \nabla S = \alpha^a \beta_a - \theta^a S_a,
\]

\[
\dot{W}_a \equiv V^b W_{a:b} = -T S_a + V^a (\phi_{ab} + \alpha \beta_{ab} - \theta S_{ab}),
\]

respectively, where the projection tensor is given by

\[
h_{ab} = g_{ab} - V_a V_b,
\]

and the evolution equations \[21\] have been used in deriving the equation for the acceleration \( \dot{W}_a \).
The evolution equations can be derived from a Lagrangian; to do this it is necessary to assume both the equation
\[ nh = p + \mu, \]  
where \( n \) is the particle number, \( p \) is the pressure, and \( \mu \) is the density; and also the first law of thermodynamics in the form
\[ dp = n\, dh - nT\, dS. \]

Usually \cite{15} the the Lagrangian of a fluid is taken to be the pressure, this is done here; but occasionally, e.g. \cite{17} p.69, other quantities are used. The action is
\[ I = \int \sqrt{-g}\, p\, dx. \]

Using \cite{24} and then \cite{17} shows that variations in the pressure depend on the velocity potentials. The variations are
\[
\frac{\delta I}{\delta g^{ab}} = -n\, V_a V_b,
\]
\[
\frac{\delta I}{\delta \phi} = -\sqrt{-g}\, (nV^a)_a,
\]
\[
\frac{\delta I}{\delta \alpha} = -\sqrt{-g}\, n\beta^a V_a,
\]
\[
\frac{\delta I}{\delta \beta} = -\sqrt{-g}\, (n\alpha V^a)_a,
\]
\[
\frac{\delta I}{\delta \theta} = +\sqrt{-g}\, S^a V_a,
\]
\[
\frac{\delta I}{\delta S} = -\sqrt{-g}\, [(n\theta V^a) + nT].
\]

The first of these variations give the stress
\[ T_{ab} = (p + \mu) V_a V_b + pg_{ab}. \]

The absolute derivative is given by
\[ \dot{X}_{ab...} = \frac{D}{d\tau} X_{ab...} = V^c X_{ab...;c}, \]
then the requirement that the second of the variations \[26\] vanishes is

\[\dot{n} + nV^a_{;a} = 0, \tag{29}\]

and this is just the conservation of particle number \[16\]eq.2.3. Using this the vanishing of the remaining variations gives

\[\dot{\alpha} = \dot{\beta} = \dot{S} = 0, \quad \dot{\theta} = -T. \tag{30}\]

The third of these shows that the fluid is isentropic, the fourth is just \[18\] in another forms. Using \[17\] and \[31\] shows that

\[\dot{\phi} = -h. \tag{31}\]

The Bianchi identities give the fluid conservation equations

\[-V_a T^{ab}_{;b} = \dot{\mu} + (p + \mu)V^a_{;a},\]

\[h_{ab} T^{bc}_{;c} = (p + \mu)V^b_{;a} + h_{ab}p_{;b}, \tag{32}\]

where the projection tensor is given by \[21\]; these equations do not immediately occur as a result of varying the Lagrangian, but only by applying the Bianchi identities to \[27\]. The first law of thermodynamics has been assumed \[21\]. In the present case the second law is obeyed as an equality as in \[30\] $\dot{S} = 0$. The situation is more complex in the next section. Anticipating some of the problems and how to approach them - In particle physics there is not always invariance under time and space reflections. The above involves no inequalities: there are no assumed energy inequalities, and the fluid is isentropic (in equilibrium with $\dot{S} = 0$) rather than $\dot{S} > 0$. Recall that a vector is future pointing iff $V_t > 0$; for \[17\] $hV_t = \phi_t + \alpha\beta_t - \theta S_t$; ignoring $\phi, \alpha, \text{ and } \beta$, $V_a$ is past pointing when $\theta S_t > 0$. This can be reversed by either defining a new vector $V'_a = -V_a$ or a new time coordinate $t' = -t$.

4 The ”Covariantly Interacting” Fluid.

There are several ways, four of which are mentioned here, of introducing an interacting fluid and vector field, with the intention of breaking the vector fields symmetries. In the first method a ”plasma interacting” fluid is produced by generalizing the treatment of \[14\]#7. In the second method, the fluid is ”already interacting”, the stress \[20\] is directly equated with the stress calculated from \[11\]; this gives $-2(\rho_a \rho_b + \rho^2 \epsilon^2 B_a B_b) = (\mu + p)V_a V_b$ and it
is impossible to proceed with one real fluid; this method is similar to the method discussed in [1]. The third method is the "traditionally interacting" fluid [7] p.70, this is produced by adding a term $L_I = -\frac{1}{2} V_a A^a$ to the Lagrangian; this method is of no use for present purposes as there is only a single term $A_a$ in the Lagrangian and stress, and for symmetry breaking products $A_a A_B$ are required. The fourth method is the "covariantly interacting" fluid; this is produced by simply applying the covariant derivative $\nabla$ to the vector $\vec{V}$ to produce

$$V_a = h^{-1} (\phi_a + \alpha \beta_a - \theta S_a + ie(\phi + \alpha \beta - \theta S) A_a).$$  \hfill (33)

It is required that

$$-1 = g^{ab} V_a \vec{V}_b, \quad \hfill (34)$$

and also that the projection tensor is

$$h_{ab} = g_{ab} - V_a \vec{V}_b, \quad \hfill (35)$$

this ensures that most quantities are real. After using $W_a = h V_a$, [34] becomes

$$h = -g^{ab} W_a \vec{V}_b, \quad \hfill (36)$$

then via the first law of thermodynamics [23] the pressure and the Lagrangian are real; however the expansion, shear, ...etc. are complex, this can be verified by direct computation or by noting from [20] that as $W_a$ is complex the quantities involved in this decomposition should also be complex.

The evolution equations are derived in a similar manner to the derivation in the last section. The action is taken to be [23] and the first law of thermodynamics is assumed with the enthalpy $h$ now given by [36]. The variations of the action are

$$\frac{\delta I}{\delta g^{ab}} = -nh V_a \vec{V}_b,$$

$$\frac{\delta I}{\delta A^a} = -n\sqrt{-g} \frac{e^2}{h^2} A_a, \quad e^2 = e^2(\phi + \alpha \beta - \theta S),$$

$$\frac{\delta I}{\delta \phi} = -\sqrt{-g} [(n \Re(V^a)) + e^2 A_a^2 n],$$

$$\frac{\delta I}{\delta \alpha} = -n\sqrt{-g} [\beta_a \Re(V^a) + e^2 A_a^2 \beta],$$
\[ \frac{\delta I}{\delta \beta} = -\sqrt{-g}[(n\alpha)\mathbb{R}(V^a)_{;a} + e'^2A^2_\alpha n\alpha], \]
\[ \frac{\delta I}{\delta \theta} = +n\sqrt{-g}[S_a\mathbb{R}(V^a) + e'^2A^2S], \]
\[ \frac{\delta I}{\delta S} = +\sqrt{-g}[(n\theta)\mathbb{R}(V^a)_{;a} + e'^2A^2\theta n - nT], \] (37)

and the vanishing of these give
\[-h = \mathbb{R}(V^a) + e'^2A^2\phi]. \] (38)

The stress and Maxwell equation are
\[ T_{ab} = (p + \mu)V_{(a}\dot{V}^b) + p g_{ab}, \]
\[ F_{\alpha\beta} + \frac{e}{\hbar^2}(\phi + \alpha\beta - \theta S)A^a = 0. \] (39)

Introducing the notation
\[ \dot{X}_{ab...} = \frac{DX_{ab...}}{d\tau} = \mathbb{R}(V^c)X_{ab...;c}, \]
\[ \dot{\hat{X}}_{ab...} = \left( \frac{D}{d\tau} + e'^2A^2_a \right)X_{ab...}, \] (40)

and the vanishing of the variations can be written in the simple form
\[ \dot{\alpha} = \dot{\beta} = \dot{S} = 0, \quad \dot{\phi} = -h, \quad \dot{\theta} = T, \quad \dot{n} = n\Theta \neq 0. \] (41)

Changing the sign and index of the last term produces the changes in the table. The fluid has \( n \neq 0 \) implying that particle number is not conserved, this appears unavoidable. Our choice has \( \theta = T \) so that the standard expression for thermasy is recovered; also \( \dot{S} = 0 \) implies that the second law of thermodynamics is obeyed when \( \theta S > \phi + \alpha\beta \).
Table: The Choice for the Thermodynamic Term in the Velocity Vector.

| $-\theta S_a$ | $+\theta S_a$ | $+\theta S_a$ | $-\theta S_a$ |
|----------------|----------------|----------------|----------------|
| $\phi = ST - h$ | $\dot{\phi} = -\theta$ | $\phi = ST - h$ | $\phi = -h$ |
| $\dot{S} = 0$ | $\dot{S} = 0$ | $\dot{S} = 0$ | $\dot{S} = 0$ |
| $\theta = -T$ | $\dot{\theta} = -T$ | $\dot{\theta} = -T$ | $\dot{\theta} = -T$ |

5 Symmetry Breaking.

The straightforward way of proceeding to break symmetry is to note that the covariantly interacting fluid of the previous section contains scalar electrodynamics as a special case. There are several choices of the parameters by which scalar electrodynamics can be recovered, an example is

$$\alpha = \beta = \theta = S = \partial_a h = 0, \quad \phi = \sqrt{2h}\psi, \quad p = l_{sel}, \quad \mu = 1 - p, \quad (42)$$

in the gauge refer:2.10. Thus the standard way of breaking symmetry can then be applied, with the aesthetic difference the fundamental cause of breaking is a fluid not a field. There is the possibility of other velocity potentials such as $\alpha$ being non-vanishing operators leading to modifications of the standard treatment; the velocity potentials themselves have been subject to quantization for a fluid coupled to gravity in the ADM formalism, see for example [17] [18] [19].

The Proca equation [4]p.135 is

$$F^{ab}_{\cdots} + m^2 eA^a = 0, \quad (43)$$

comparing with (40), the covariantly interacting fluid has a mass

$$m^2 = h^{-2}(\phi + \alpha \beta - \theta S)^2. \quad (44)$$

Choosing only to retain thermodynamic quantities $m^2 = h^{-2}\theta^2 S^2$, and the thermasy $\theta$ needs to be evaluated. The tempreature can be taken to be independent of the proper time $\tau$, but the mass in (44) still depends on proper time; to proceed it is necessary to introduce an artificial time interval and the Plank time is chosen, thus

$$-\theta = k \int_0^{t_{aPL}} T \, d\tau = kT \int_0^{\tau_{PL}} d\tau = kT |_{\tau_{PL}} = \sqrt{\frac{Gh}{c^5}} kT. \quad (45)$$
Restoring constants and substituting into $44$ gives

$$m = \sqrt{\frac{ch}{Gk}} \frac{TS}{h} \simeq 3.10^{-31} \frac{TS}{h} \text{ Kg.} \quad (46)$$

For $m$ to be constant it is necessary to evaluate $TS/h$ at the critical point and the Bose condensation point is a choice. This choice turns out to give zero mass because Bose condensation requires that the lowest state has zero kinetic energy, as there is no extra energy there is no extra heat content and hence no entropy, there is no disorder because all the particles are in the same state, c.f. [20] p.78, no entropy implies that there is no induced mass. The Bose condensation point is also unsatisfactory choice because strictly speaking $N/V$ is not required to have a given value for radiation in a cavity for a vector fields, c.f. [21] prob.16.2.

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