Mass dependence of the heavy quark potential and its effects on quarkonium states

Alexander Laschka
Norbert Kaiser   Wolfram Weise

Physik Department
Technische Universität München

XIV International Conference on Hadron Spectroscopy
June 14, 2011
**History:** phenomenological potential models
- Fitted to low lying charmonium and bottomonium states
- Typical shape: “Coulomb-plus-linear”

**Today:** heavy quark-antiquark potential from QCD
- Characteristic scales of non-relativistic bound states
  - $m$: heavy quark mass, hard scale
  - $mv$: heavy quark momentum, soft scale
  - $mv^2$: heavy quark energy, ultrasoft scale
- Effective field theory (EFT) methods
  - QCD $\Rightarrow$ non-relativistic QCD (NRQCD, pNRQCD, vNRQCD)

**Topics:**
- Extended range of validity of perturbative potential
- Spectroscopy at order $1/m$
- Detailed analysis of the role of quark masses
1. Static quark-antiquark potential

2. Heavy quark potential at order $1/m$
Non-perturbative sector: lattice studies of quenched and full QCD

Static QCD potential (from static Wilson loop)

G.S. Bali et al., Phys.Rev.D62 (2000)

Y. & M. Koma, Nucl.Phys.B769 (2007)

Sea quark effects important at small distances
Perturbative sector: static potential is known at three-loop order

M. Peter, Phys.Rev.Lett.78 (1997), Y. Schröder, Phys.Lett.B447 (1999)

Three-loop: C. Anzai, Y. Kiyo, Y. Sumino, Phys.Rev.Lett.104 (2010),
A. & V. Smirnov, M. Steinhauser, Phys.Rev.Lett.104 (2010)

Momentum space

\[ \tilde{V}^{(0)}(|q|) = -\frac{4\pi C_F \alpha_s(|q|)}{q^2} \left[ 1 + \frac{\alpha_s(|q|)}{4\pi} a_1 + \left( \frac{\alpha_s(|q|)}{4\pi} \right)^2 a_2 \\
+ \left( \frac{\alpha_s(|q|)}{4\pi} \right)^3 \left( a_3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{q^2} \right) + \ldots \right] \]

where \( C_F = 4/3, C_A = 3, \)
\[ a_1 = 7, a_2 \approx 268.8, a_3 \approx 5199.8 \ (n_f = 3) \]

At N^3LO (three-loop order):

infrared divergences \( \mu_{IR}^2 \) from ultrasoft gluons

Avoid expansion of \( \alpha_s(|q|) \) about a fixed scale \( \mu \)

Reliable potential from extremely small distances up to \( r \approx 0.15 \text{ fm} \) needed
Potential subtracted (PS) scheme

**PS scheme with numerical Fourier transform**

- Evaluate numerically (with a low-momentum cutoff $\mu_f$)

\[
V^{(0)}(r, \mu_f) = -4\pi C_F \int_{|\bar{q}| > \mu_f} \frac{d^3 \bar{q}}{(2\pi)^3} e^{i \bar{q} \cdot \bar{r}} \frac{\alpha_s(|\bar{q}|)}{\bar{q}^2} \left[ 1 + \frac{\alpha_s(|\bar{q}|)}{4\pi} a_1 + \left( \frac{\alpha_s(|\bar{q}|)}{4\pi} \right)^2 a_2 + \ldots \right]
\]

- No free scale parameter $\mu$

- Unknown constant is moved into the definition of $m_{PS}$:

\[
2m_{pole} + V^{(0)}(r) = 2m_{PS}(\mu_f) + V^{(0)}(r, \mu_f)
\]
Matching and uncertainty estimate

- Perturbative potential (here NNLO) and lattice potential matched

![Graph showing matching and uncertainty estimate](image)

- Differentiable quark-antiquark potential for distances up to \( \sim 1 \) fm
- Matching at 0.14 fm gives \( \mu_f = 0.9^{+0.3}_{-0.2} \) GeV
  (for charmonium and bottomonium)
- Grey band: uncertainty of lattice calculation and uncertainty of \( \alpha_s \)
- Dot-dashed curve: continuation of the “Coulomb-plus-linear” fit
Bottomonium spectrum

Solve the Schrödinger equation with this matched potential

\begin{align*}
\text{Mass [GeV]} & \quad & \text{BB-threshold} & & \Upsilon(4S) & & \chi_{bJ}(2P) \\
10.6 & & & & & & \\
10.4 & & & & & & \\
10.2 & & & & & & \\
10.0 & & & & & & \\
9.8 & & & & & & \\
9.6 & & & & & & \\
9.4 & & & & & & \\
9.2 & & \eta_b(1S) & & \Upsilon(1S) & & \chi_{bJ}(1P) \\
1S_0 & & & & & & \\
3S_1 & & & & & & \\
1P_1 & & & & & & \\
3P_j & & & & & & \\
\end{align*}

- Single parameter $m_{PS}(0.908 \text{ GeV}) = 4.78 \text{ GeV}$
- Can be converted to the $\overline{\text{MS}}$ scheme

\begin{tabular}{|c|c|c|}
\hline
$\overline{\text{MS}}$ masses [GeV] & $m_{\overline{\text{MS}}}$ & PDG 2010 \\
\hline
bottom quark & 4.20$\pm0.04$ & 4.19$^{+0.18}_{-0.06}$ \\
charm quark & 1.23$\pm0.04$ & 1.27$^{+0.07}_{-0.09}$ \\
\hline
\end{tabular}
1. Static quark-antiquark potential

2. Heavy quark potential at order $1/m$
Expansion in inverse powers of the heavy quark mass \( m \)

\[
V(r) = V^{(0)}(r) + \frac{V^{(1)}(r)}{m/2} + \frac{V^{(2)}(r)}{(m/2)^2} + \ldots
\]

Non-perturbative expression for \( 1/m \) potential is known

N. Brambilla et al., Phys.Rev.D63 (2001) 014023

Lattice simulations

Efficient method from M. & Y. Koma and H. Wittig

Quenched simulation, renormalization issues (\( \approx 15\% \) error estimated)

Contains a non-perturbative contribution

Fit function

\[
V_{\ln}^{(1)}(r) = -\frac{A_2}{r^2} + B_2 \ln r + C_2
\]

Effective string theory suggests logarithmic shape: \( V^{(1)} \propto \ln r + C \)

G. Perez-Nadal, J. Soto, Phys.Rev.D79 (2009)
Quark-antiquark potential at order $1/m$

- **Perturbative potential at order $1/m$** ($C_F = \frac{4}{3}$, $C_A = 3$)

\[
\tilde{V}^{(1)}(|\vec{q}|) = \frac{C_F \pi^2 \alpha_s^2(|\vec{q}|)}{2|\vec{q}|} \left[-C_A + \mathcal{O}(\alpha_s)\right]
\]

- **Restricted numerical Fourier transform**

- **Differentiable quark-antiquark potential** for distances up to $\sim 1$ fm

- Matching at 0.14 fm gives $\mu_f' = 1.6^{+0.5}_{-0.8}$ GeV (for charmonium)

- $\mu_f' = 1.9^{+0.4}_{-0.6}$ GeV (for bottomonium)

- Grey band: uncertainty of lattice calculation and uncertainty of $\alpha_s$
Heavy quark masses at order $1/m$

PS mass needs redefinition $m_{PS}(\mu_f) \rightarrow m_{\tilde{PS}}(\mu_f, \mu'_f)$

$m_{\tilde{PS}}(\mu_f, \mu'_f) \equiv m_{PS}(\mu_f) - \frac{1}{8m} C_F C_A \alpha_s^2 \mu'_f$

Quark masses from comparison with empirical quarkonium states

| $\overline{\text{MS}}$ masses [GeV] | static        | static + $1/m$ | PDG 2010   |
|-------------------------------------|---------------|----------------|-------------|
| bottom quark                        | 4.20±0.04     | 4.18$^{+0.05}_{-0.04}$ | 4.19$^{+0.18}_{-0.06}$ |
| charm quark                         | 1.23±0.04     | 1.28$^{+0.07}_{-0.06}$ | 1.27$^{+0.07}_{-0.09}$ |

Error estimates include:
- uncertainties in the potentials (static and order $1/m$)
- uncertainties from matching to experimental spectra
**Bottomonium spectrum**

- Tightly bound $\eta_b(1S)$ and $\Upsilon(1S)$ states are most sensitive to $1/m$-effects.
- Hyperfine effects (h.f.) added phenomenologically (one-gluon exchange) with $\alpha_s^{\text{eff}} = 0.3$.
  - ... (work in progress)
  - to be substituted by the full $1/m^2$ potential.
- String tension $\sigma = 1.01$ GeV/fm.
- Different strategies needed above $B\bar{B}$ threshold.

| Mass [GeV] | $\eta_b(1S)$ | $\Upsilon(1S)$ |
|------------|---------------|----------------|
| $1S$       | $^1S_0$       | $^3S_1$        |
| $2S$       | $^3S_1$       |                |
| $3S$       |                | $\Upsilon(3S)$ |

| Mass [GeV] | $\chi_{bj}(1P)$ | $\chi_{bj}(2P)$ |
|------------|------------------|------------------|
| $1P$       | $^3P_j$          |                  |
| $1D$       | $^3D_j$          |                  |
| $2P$       | $^3P_j$          | $\Upsilon(1D)$  |

String tension $\sigma = 1.01$ GeV/fm.
Charmonium spectrum

- Downward shift from $V^{(1)}$ in the $1S$ states ($\eta_c$ and $J/\psi$) to large $1/m^2$ effects significant
- Hyperfine effects (h.f.) added phenomenologically (one-gluon exchange) with $\alpha_s^{\text{eff}} = 0.3$
  
  ... (work in progress)
  to be substituted by the full $1/m^2$ potential
- String tension $\sigma = 1.01$ GeV/fm
- Different strategies needed above DD threshold

\[\begin{array}{cccc}
\text{Mass [GeV]} & \text{D ¯D-threshold} & \text{D ¯D-threshold} \\
\hline
\text{ηc (1S)} & \text{ηc (2S)} & \text{ψ (2S)} \\
\text{1S0} & \text{3S1} & \text{J/ψ (1S)} \\
\text{ηc (2S)} & \text{ψ (2S)} & \\
\text{1S0} & \text{3S1} & \\
\end{array}\]
Heavy quark-antiquark potential from QCD
(perturbative QCD ↔ lattice QCD)

Excellent matching in $r$-space up to order $1/m$

Spectroscopy at order $1/m$

- Works well for bottomonium
- Less successful for charmonium
  ($1/m^2$ effects sizeable: work in progress)

Quark masses can be extracted

| $\overline{\text{MS}}$ masses [GeV] | static | static + $1/m$ |
|-------------------------------------|--------|----------------|
| charm quark                         | 1.23±0.04 | 1.28±0.07      |
|                                    |         | −0.06          |
| bottom quark                        | 4.20±0.04 | 4.18±0.05      |
|                                    |         | −0.04          |

See for details: A. Laschka, N. Kaiser, W. Weise, Phys.Rev.D83 (2011) 094002
Thank you for your attention!