Reversal of Circular Bragg Phenomenon in Ferrocholesteric Materials with Negative Real Permittivities and Permeabilities

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A structurally right/left–handed ferrocholesteric slab with negative real permittivities and permeabilities is theoretically shown to display the Bragg phenomenon on axial excitation as if it were a structurally left/right–handed ferrocholesteric slab with positive real permittivities and positive real permeabilities. In addition to the promise of isotropic homogeneous materials with negative real permittivities and permeabilities for unexpected applications, the presented results underscore the similar potential of anisotropic nonhomogeneous materials with analogous characteristics.

Dielectric and magnetic materials are ubiquitous. Their linear electromagnetic response properties are characterized by permittivity and permeability dyadics which depend on the frequency of excitation and comprise complex–valued scalar components. The permittivity and the permeability dyadics (i.e., second–order tensors) of an isotropic material reduce to complex–valued scalars. The real parts of these scalars can be negative or positive, but the latter possibility is the normative one of the two, as the perusal of almost any undergraduate electromagnetics textbook will show. However, the former possibility does exist for natural materi-
als such as metals, plasmas and ferrites [1, 2]. Most recently, composite materials which effectively have both negative real permittivity and negative real permeability in a certain frequency range have been fabricated and satisfactorily tested [3, 4], notwithstanding the discounting of anisotropy, nonhomogeneity and dissipation in the sample materials [5].

Technological bonanzas have been proffered, provided homogeneous, isotropic and virtually non–dissipative materials with negative permittivity and negative permeability can be economically manufactured [5, 6, 7]. These potential benefits are based on the opposite directions of the phase velocity and the velocity of energy transport in these materials. Available results indicate that these materials would be realized in the form of multilaminar slabs, each lamina itself being anisotropic due to the imprinting of various features thereon [4, 7]. Feature geometries other than the only one in current use will also arise, sooner or later. These open up the possibility of an entirely new class of prospective materials: ferrocholesterics with negative permittivity and negative permeability. This Communication is devoted to these unidirectionally nonhomogeneous materials.

About twenty years before the discovery of cholesteric liquid crystals by Reinitzer in 1888 [8], Reusch [9] presented structurally similar materials made from uniaxial dielectric laminas, which can possibly be fabricated as unidirectional fibrous composites. The laminas are identical, with the sole optic axis lying in the laminar plane. The laminas are stacked sequentially, the optic axis in any particular lamina offset by a small angle in the laminar plane from the optic axis of the lamina lying immediately below it. The successive optic axes rotate helicoidally, and the optical response properties of the entire structure resemble those of a cholesteric liquid crystal at frequencies below a certain maximum [10]. The optical response of
cholesteric materials in the liquid crystalline form has been intensively studied and technologically exploited \[11, 12\].

Just about a century later, Brochard and de Gennes \[13\] incorporated parallel magnetic needles in the laminas, giving rise to the so–called ferrocholesteric materials. Let the thickness direction of a ferrocholesteric material be parallel to the \(x_3\) axis of a cartesian coordinate system \((x_1, x_2, x_3)\). The simplest effective constitutive equations of this material are as follows:

\[
D(x) = \varepsilon_0 \left[ \varepsilon_a \mathbb{I} + (\varepsilon_b - \varepsilon_a) cc \right] \cdot E(x),
\]

\[
B(x) = \mu_0 \left[ \mu_a \mathbb{I} + (\mu_b - \mu_a) cc \right] \cdot H(x).
\]

Here, \(\varepsilon_0\) and \(\mu_0\) are the permittivity and the permeability of free space (i.e., vacuum); \(\mathbb{I}\) is the identity dyadic; the unit vector

\[
c = \hat{x}_1 \cos(\pi x_3 / \Omega) + h \hat{x}_2 \sin(\pi x_3 / \Omega)
\]

involves the helicoidal pitch \(2\Omega\); the parameter \(h = +1\) for structural right–handedness, and \(h = -1\) for structural left–handedness, while the scalars \(\varepsilon_{a,b}\) and \(\mu_{a,b}\) are complex–valued functions of the angular frequency \(\omega\).

As a ferrocholesteric material is periodically nonhomogeneous, it must exhibit the circular Bragg phenomenon \[14\]. Most significantly, if a circularly polarized plane wave with angular frequency in the so–called Bragg regime were normally incident on a ferrocholesteric slab \(0 \leq x_3 \leq L\) (which is thus excited axially), and the ratio \(L/\Omega\) were sufficiently high, it will be almost completely reflected if its handedness matches the structural handedness of the slab. Virtually no reflection would occur if the two handednesses do not coincide. The importance of this polarization–discriminatory characteristic in optics cannot be exaggerated, as it is exploited for a variety of polarization–sensitive filters and laser mirrors \[12\].
The reflectances and the transmittances of axially excited ferrocholesteric slabs of infinite lateral extent were calculated by following an established procedure \[15\]. These quantities were organized as the $2 \times 2$ matrixes
\[
\begin{bmatrix}
R_{RR} & R_{RL} \\
R_{LR} & R_{LL}
\end{bmatrix}, \quad
\begin{bmatrix}
T_{RR} & T_{RL} \\
T_{LR} & T_{LL}
\end{bmatrix}.
\]
Here, the subscript RL indicates the intensity of either a reflected or a transmitted right circularly polarized plane wave in relation to the intensity of an incident plane wave that is left circularly polarized; and so on.

In Figure 1 are shown the reflectances and the transmittances as functions of the free–space wavelength $\lambda_0 = 2\pi/\sqrt{\varepsilon_0\mu_0}$ for a ferrocholesteric slab with the following properties: $\varepsilon_a = 3(1 + 0.001i)$, $\varepsilon_b = 3.3(1 + 0.001i)$, $\mu_a = 1.2(1 + 0.002i)$, $\mu_b = 1.5(1 + 0.002i)$, $h = 1$, $\Omega = 14$ mm and $L = 40\Omega$. Dispersion was ignored in this illustrative study, and an $\exp(-i\omega t)$ time–dependence was assumed. Differential reflection of incident left/right circularly polarized plane waves is clearly in evidence in the Bragg regime, which spans the wavelength range $\lambda_0 \in [53.2, 62.3]$ mm. As the chosen material is structurally right–handed, $R_{RR}$ and $T_{LL}$ are enormously larger than the negligibly small $R_{LL}$ and $T_{RR}$ in the Bragg regime. The cross–polarized reflectances and transmittances — $R_{RL}$, etc., — are also very small, and can be further reduced by the use of impedance–matching layers.

The calculations for Figure 1 were repeated, except with the following changes in the input parameters: $\varepsilon_a = -3(1 - 0.001i)$, $\varepsilon_b = -3.3(1 - 0.001i)$, $\mu_a = -1.2(1 - 0.002i)$, $\mu_b = -1.5(1 - 0.002i)$. These values are in accord with the Lorentz oscillator model \[4, 11\], and therefore do not violate the principles of causality and energy conservation. The calculated reflectance and transmittance spectrums are shown in Figure 2. The circular Bragg phenomenon is preserved, but with a difference.
Despite the fact that the chosen material is still structurally right–handed, now $R_{LL}$ and $T_{RR}$ are enormously larger than the negligibly small $R_{RR}$ and $T_{LL}$. The cross–polarized quantities remain unaffected.

In other words, a structurally right/left–handed ferrocholesteric slab with negative real permittivities and negative real permeabilities will display the circular Bragg phenomenon on axial excitation as if it were a structurally left/right–handed ferrocholesteric slab with positive real permittivities and positive real permeabilities. Due to mathematical isomorphism, this conclusion would also hold for ferrosmectic materials [16, 17].

The reason underlying the foregoing symmetry can be understood by examining the characteristics of the axial propagation modes in ferrocholesteric and ferrosmectic materials; detailed expressions are available elsewhere [16]. The propagation modes are either left or right elliptically polarized, with their respective vibration ellipses rotating along the $x_3$ axis in accordance with the structural handedness of the material [18]. Let dissipation be weak. When the real parts $\text{Re}[\varepsilon_{a,b}] > 0$ and $\text{Re}[\mu_{a,b}] > 0$, the direction of the phase velocity of a particular mode is the same as the (common) direction of energy transport and attenuation. However, when $\text{Re}[\varepsilon_{a,b}] < 0$ and $\text{Re}[\mu_{a,b}] < 0$, not only does the phase velocity reverse in direction, but the handedness of the vibration ellipse also reverses, while the direction of energy flow and attenuation as well as the sense of rotation of the vibration ellipse remain unchanged. The reversal of the modal handedness is thus responsible for the left/right switching between the spectrums of Figures 1 and 2.

The Lorentz and the Drude models for oscillators are well–known [1]. These models yield negative as well as positive values for $\text{Re}[\varepsilon_{a,b}, \mu_{a,b}]$ in different parts of the electromagnetic spectrum, while the imaginary parts $\text{Im}[\varepsilon_{a,b}, \mu_{a,b}]$ must be
always positive. If the electric and the magnetic resonances are close to one another, the spectral regime of negative permittivities and permeabilities may be accessible simply by increasing the frequency across all the resonances.

To conclude, many interesting phenomena (such as anomalous refraction and reversed Doppler shifts) and applications (such as distortion–free lenses) have been forecasted for isotropic homogeneous materials with negative real permittivity and permeability \[ \varepsilon < 0, \mu < 0 \]. The technology for fabricating these materials can also be used for structurally nonhomogeneous materials with analogous constitutive properties. Due to their anisotropic and nonhomogeneous constitution, many unexpected phenomena and applications are likely to emerge.

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Figure Captions

Figure 1. Computed spectrums of the reflectances and transmittances of a ferrocholesteric slab with Re $[\epsilon_{a,b}, \mu_{a,b}] > 0$; $\epsilon_a = 3(1 + 0.001i)$, $\epsilon_b = 3.3(1 + 0.001i)$, $\mu_a = 1.2(1 + 0.002i)$, $\mu_b = 1.5(1 + 0.002i)$, $h = 1$, $\Omega = 14$ mm and $L = 40\Omega$. Note that $R_{LR} = R_{RL}$ and $T_{LR} = T_{RL}$, correct to graphical accuracy.

Figure 2. Same as Figure 1, but for $\epsilon_a = -3(1 - 0.001i)$, $\epsilon_b = -3.3(1 - 0.001i)$, $\mu_a = -1.2(1 - 0.002i)$, and $\mu_b = -1.5(1 - 0.002i)$. 