On Mass Gap in Type IIB Quantum Hall Solitons

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Abstract

We discuss the mass gap in quantum Hall solitons (QHS) embedded in superstring theory. In particular, we give two holographic models which are obtained from D-brane configurations in type IIB superstring compactifications. The first one deals with the monolayered system in the D3/D7 brane set up. The second model corresponds to a multilayered system which is described by intersecting D5-branes wrapping a particular set of 3-cycles. In both models, we have shown that the mass gap is related to the filling factor.

KeyWords: Quantum Hall Solitons, Quiver gauge theory, Type IIB superstring, ALE space fibrations.
1 Introduction

Quantum Hall effect is one of the most interesting strongly correlated electron system in condensed matter physics [1]. At low energies, this effect which is described by a topological field theory has been subject to great interest not only because of its experimental results but also from its connection with recent developments in string theory [2, 3, 4, 5].

The quantum Hall states are characterized by the filling factor $\nu$ describing the ratio between the electronic density and the magnetic flux density. When $\nu$ is a fractional value, it is called fractional quantum Hall effect (FQHE) describing interacting electron systems. The first proposed series of the fractional quantum states was given by Laughlin and they are characterized by the filling factor $\nu_L = \frac{1}{k}$ where $k$ is an even integer for bosons and an odd integer for fermions [6]. According to Susskind [7], the non-commutative three dimensional Chern-Simons gauge theory can provide a nice description for such Laughlin states. This framework has opened a modeling of these sort of phenomena in terms of D-brane solitons of type II superstring theory. In particular, it has been shown that the Susskind model can be constructed by a ten dimensional system of D0, D2 and D6-branes and F1 strings stretched between the D2 and D6-branes[2]. Using D-brane configurations on the K3 surface, a six dimensional type IIA stringy realization of these type of models, has been given in terms of D2 and D6-branes wrapping the K3 surface [8].

On the other hand, quantum Hall soliton (QHS) have been studied using Anti de Sitter/conformal field theory (AdS/CFT) correspondence [4, 5]. In particular, based on the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory in (2+1)-dimensions [9], Chern-Simons (CS) descriptions of QHS have been discussed in [10, 11]. These models are embedded in the 3-dimensional supersymmetric $N = 6$ CS quiver theory with $U(N)_k \times U(N)_{-k}$ gauge symmetry. Recall that the ABJM proposal is dual to M-theory propagating on $AdS_4 \times S^7 / \mathbb{Z}_k$, with an appropriate amount of fluxes, or equivalently to type IIA superstring on $AdS_4 \times \mathbb{C}P^3$. Indeed, it has been shown that the FQHE can be obtained from the world-volume action of the M5-brane filling $AdS_3$ inside $AdS_4$ space [10]. The corresponding model has been derived from $d = 3$ flavored ABJM theory with the CS levels $(1,1)$. Alternatively, a FQHE system in $AdS_4$/CFT$_3$ has been realized by adding fractional D4-branes, wrapping $\mathbb{C}P^1$, to the ABJM theory [11]. Moreover, an extended model based on D6-branes wrapping del Pezzo surfaces has been discussed in [12, 13].

A more generic class of FQHE models which can appear in real materials, including graphene [14, 15] is characterized by a vector $q_i$ and a real symmetric invertible matrix $K_{ij}$ which are related by the filling factor [1]. They have a nice interpretation using solitonic D-branes wrapping non trivial cycles in Calabi-Yau manifolds [8]. In particular, the matrix $K_{ij}$ and the vector charge $q_i$ play an important role in the quiver approach of FQHS, embedded
in type II superstrings and M-theory, compactified on deformed singular geometries that are classified by Dynkin diagrams of Lie algebras [8, 16, 17].

The aim of this work is to contribute to these activities by studying the mass gap in QHS models, using type IIB D-brane configurations. First, we give a simple model involving a monolayer system based on the D3/D7 brane setup. Then we discuss the case of the multilayered systems with several abelian gauge factors, using the quiver method that is based on intersecting D5-branes wrapping 3-cycles of the internal space. These 3-cycles are given by a line segment fibered by intersecting spheres according to extended Dynkin diagrams. In both examples, the mass gap can be related to the filling factor of the corresponding QHS model. The last section is devoted to discussions.

The organization of the paper is as follows. In section 2 we review briefly the three-dimensional gauge theory obtained from the D3/D7 setup. In section 3, we give a stringy realization of the QHE in 1+2 dimensions from such a brane configuration, and then we discuss the corresponding mass gap. In section 4, we study the mass gap in a hierarchical stringy description. This is obtained from quiver gauge theories living on the world-volume of D5-branes that wrap a particular set of 3-cycles.

2 The D3/D7 brane system

The starting point is the (D3, D7) brane system that is embedded in type IIB superstring[18]. We consider D3-branes which are extended in (0123) directions. This leads to four-dimensional gauge interactions living on its world-volume. As we will see, the three-dimensional fermions can be modeled by a stack of $k$ D7-branes oriented along (01245678) directions. The system is represented in the Table 1.

|        | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|---|---|---|---|---|---|---|---|---|---|
| D3-brane | ✓ | ✓ | ✓ | ✓ |   |   |   |   |   |   |
| D7-brane | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

Table 1: The D3/D7 brane configuration of the studied model

Open strings have two-end points satisfying either the Neumann (N) or Dirichlet (D) boundary conditions. One has three possibilities: (NN), (DD) and (ND). These are completely fixed once the directions in which the D-branes live are determined. In the above configuration, the D3-D7 open strings have six ND boundary conditions. We know that the NS sector is massive and only the R sector contains massless states. In this case, each D3-D7 pair gives a complex massless two-component spinor living in the $2 + 1$ common directions. This configuration is non-supersymmetric and unstable because the branes are repelled one
from another in the direction $x_9$. The excitations of RR open strings stretched between D3 and D7-branes give massless fermions.

The dual bulk description is obtained by taking a large number of D3-branes and a finite number of D7-branes (i.e., $N_f << N_c$). In this way, the D7-branes can be treated as probes in the near-horizon geometry, $AdS_5 \times S^5$, of the D3-branes. The D3-branes describe the four-dimensional gauge field dynamics, and the D7-brane embedded in that background captures the three-dimensional physics of the fermions. The corresponding effective theory, which has been used in [18] to engineer graphene multilayered, reads as

$$S_4 = \int_{\mathbb{R}^{1,3}} \left[ \frac{1}{4g^2} F_{MN}^2 + \frac{1}{2g^2} \left( \partial^M A_M \right)^2 + L_f \right], \quad (2.1)$$

where $A_M$ is the gauge field living on the D3-brane ($M = 0, 1, 2, 3$). The second term on this equation is a covariant gauge fixing term, while the last one reflects the behavior of the fermions localized in the shared three dimensions of the D3/D7 system. This term is given by the following Lagrangian

$$L_f = \sum_a \left( \bar{\psi}^a \gamma^M \left( i \partial_M - A_M \right) \psi^a \delta \left( x^3 \right) \right), \quad (2.2)$$

where the $\gamma^M$ gamma matrices are given in terms of the $2 \times 2$ Pauli matrices. $\psi^a$ are the complex two-dimensional spinors and $a = 1, \ldots, k$.

The physics at the intersection of the D7-branes and D3-branes can be described by an effective $2 + 1$-dimensional field theory, with SO(3,2) conformal invariance which is a subgroup of the SO(4,2) symmetry of the D3 subsector. Following [18], the action (2.1) can be integrated out and reduced to

$$S_3 = \int_{\mathbb{R}^{1,2}} \left[ \frac{1}{4g^2} F_{\mu \nu} \frac{1}{\sqrt{-\partial^2}} F_{\mu \nu} + \frac{1}{1 + \xi} \left( \partial^\mu A_\mu \right) \frac{1}{\sqrt{-\partial^2}} \left( \partial^\mu A_\mu \right) + \left( \bar{\psi}^a \gamma^\mu \left( i \partial_\mu - A_\mu \right) \psi^a \right) \right], \quad (2.3)$$

where now $\mu = 0, 1, 2$.

In the following sections, we will discuss the generation of a mass gap of the gauge field of the QHS living on the three-dimensional intersecting IIB D-branes. In particular, we will propose a relationship between the Kaluza-Klein mass gap and the filling factor $\nu$. Then we extend this result by constructing a holographic model based on D5-branes wrapping non trivial 3-cycles.
3 Mass gap for D3/D7 brane quantum Hall solitons

In this section, we study the mass gap for QHS constructed in the D3/D7 brane set up. First we recall that the simplest series of the fractional quantum states with the filling factor $\nu = \frac{1}{k}$ can be described by a 3-dimensional U(1) Chern-Simons theory coupled to an external electromagnetic field $\tilde{A}$. The effective action of the system is

$$S_{CS} = -\frac{k}{4\pi} \int_{\mathbb{R}^{1,2}} A \wedge dA + \frac{q}{2\pi} \int_{\mathbb{R}^{1,2}} \tilde{A} \wedge dA, \quad (3.1)$$

where $A$ is the dynamical gauge field and $q$ is the charge of the electron[1, 19].

Susskind has conjectured that a two-dimensional quantum Hall fluid of charged particles with the filling factor $\nu = \frac{1}{k}$ can be modeled by a non-commutative Chern-Simons gauge theory at level $k$ [7]. This conjecture has opened a new way to apply string theory for studying low-energy systems in condensed matter physics. This has been based on the recent developments in string dualities, mainly AdS/CFT. The first connection with string theory was given by Bernevig, Brodie, Susskind and Toumbas reproducing the QHE on the 2-sphere, $S^2$ [2]. The corresponding solitonic D-brane configuration involves a spherical D2-brane and dissolved D0-branes on it. The system is placed in a background of coincident D6-branes extended in the directions perpendicular to the world-volume of the D2-brane on which the QHE resides. After a compactification on the K3 surface, this ten dimensional type IIA superstring construction can be reduced to a six dimensional one [8]. Other connections between string theory and QHS in $1+2$-dimensions were found recently using the AdS/CFT correspondence [4, 5].

The specific model that we deal with here is based on the D3/D7 brane system given in the previous section. In particular, we will see that the action (3.1) can be derived from the D3/D7 brane system in type IIB superstring. In type II superstring theory, the Chern-Simons terms can be obtained from the Wess-Zumino (WZ) part of the action of a D-brane wrapping non trivial cycles and interacting with the R-R flux. Here, it will be shown that it is possible to get the first term of (3.1) from the WZ action of the D3-brane. Similarly, the second part of (3.1) can be obtained from the interaction between the D3-branes and the D7-branes in the three dimensional shared space. This indicates how the D3-brane couples to the various backgrounds appearing in type IIB superstring. Indeed, on the 4-dimensional world-volume of each D3-brane one has a U(1) gauge symmetry. The corresponding WZ action reads as

$$S_{WZ} = \int_{\mathbb{R}^{1,3}} C_0 \wedge F \wedge F + C_2 \wedge F + C_4, \quad (3.2)$$

where $C_n$ are $n$-forms belonging to the R-R sector.
To get the first part of the action (3.1), we consider a particular background given by the vanishing condition $C_2 = C_4 = 0$ on the D3-brane world-volume. In this case, the action (3.2) reduces to

$$S_{WZ} = \int_{\mathbb{R}^{1,3}} C_0 \wedge F \wedge F,$$

(3.3)

where $C_0$ is the axion scalar field. After a simple integration by part, we get

$$S_{WZ} = -\int_{\mathbb{R}^{1,3}} dC_0 \wedge A \wedge F.$$

(3.4)

Now we take a particular brane configuration, in which the $x^3$-direction is compact and the axion field $C_0$ behaves like $k x^3$. $L$ is the size of the compact dimension $x^3 \sim x^3 + L$. This brane configuration can be supported by a stack of $k$ D7-branes coupled to the gauge field living on the D3-brane. Integrating over the $x^3$-direction, we obtain the first Chern-Simons term of (3.1)

$$S_{WZ} = -\frac{k}{4\pi} \int_{\mathbb{R}^{1,2}} A \wedge dA,$$

(3.5)

where $\frac{k}{4\pi} = \int dC_0$.

To obtain the full action of quantum Hall effect (3.1), one needs to couple the above D3-brane system to an external magnetic source. A new term containing the coupling between the external gauge field $\tilde{A}_\mu$ and the gauge potentials $A_\mu$ should be added. Then, the last term of the effective Lagrangian describing the $2+1$-dimensional interaction between the D3-branes and D7-branes that appears in (2.3) becomes

$$\bar{\psi}^a \left( \gamma^\mu \left( i \partial_\mu - A_\mu - \tilde{A}_\mu \right) \right) \psi^a.$$

(3.6)

Like the dynamical gauge field, that was obtained from the D3-brane world-volume, the external gauge field $\tilde{A}_\mu$ can be derived using several brane configurations interacting with the R-R fields. However, for the external field we make use of the gauge field living on the D7-brane worldvolume. This field will play the role of the external source used in the quantum Hall description. On a D7-brane lives an eight dimensional U(1) gauge field obtained from the quantization of the open string with the two ends on it. Wrapping this D7-brane on $S^5$ we generate a three dimensional gauge field $\tilde{A}_\mu$. In this way, this gauge field $\tilde{A}_\mu$ can be obtained by reducing dimensionally the gauge field living in eight dimensions down to three dimensions. This produces a direct coupling with the D3-brane gauge field because now the two fields live in the same space-time. The three dimensional coupling is given by
the following action

\[ S \sim \int_{\mathbb{R}^{1,2}} \tilde{A} \wedge F. \] (3.7)

For \( k \) D7-branes, the system is coupled to a U(\( k \)) gauge fields \( \tilde{A} \). Taking the CS actions (3.7) and (3.5), integrating out the gauge field \( \tilde{A} \) and using the equations of motion, one gets the following filling factor

\[ \nu = \frac{1}{k}. \] (3.8)

To obtain the relationship between the mass gap and the filling factor, we make use of the following steps. First, we add the Chern-Simons action (3.1) to the three dimensional interacting action given in (2.3)

\[ S_3 + S_{CS}. \] (3.9)

Second, we take the \( \xi \) large limit. Then, by computing the powers of the derivatives of the dynamical gauge field \( A \), we find the form of the inverse of the propagator for the gauge boson in momentum space

\[ kp - \frac{1}{\xi^2} p^2 - p. \] (3.10)

Ignoring the Kaluza-Klein contribution, the spectrum has a mass gap given by

\[ m \sim (k - 1)g^2. \] (3.11)

Besides the three-dimensional gauge coupling constant, the mass gap depends also on the order parameter \( k \) responsible for classification of the various quantum Hall states. Using the relation \( \nu = \frac{1}{k} \), the mass gap takes the form

\[ m \sim \frac{1 - \nu}{\nu} g^2. \] (3.12)

We see that the massless case is given by the line \( \nu = 1 \) in the \( (\nu, g) \) parameter space. This happens if the condition \( k = 1 \) is satisfied, that is, the number of the D7-branes should be one.

4 Multi-layered systems

So far, we have discussed the mass gap in a QHS model with a single gauge group. In this section, we give a holographic model based on several gauge fields. In such a multi-layered system, we can associate a separate gauge potential \( A_\mu \) with each layer. This is due to the fact that there is not tunneling and the current in each layer is separately conserved.

The most general fractional quantum Hall system is described by the following Chern-
Simons action

\[ S \sim \frac{1}{4\pi} \int_{\mathbb{R}^{1,2}} \sum_{i,j} K_{ij} A^i \wedge dA^j + 2 \sum_i q_i \tilde{A} \wedge dA^i, \]  

(4.1)

where \( K_{ij} \) is a real, symmetric and invertible matrix (\( \det K \neq 0 \)) and where \( q_i \) is a vector of integer charges. The external gauge field \( \tilde{A} \) couples now to each current \( *dA^i \) with charge strengths \( e q_i, e \) being the fundamental charge. The \( K_{ij} \) matrix and the \( q_i \) charge vector in this effective field action are very suggestive in many physical senses. In the Wen-Zee model [19], \( K_{ij} \) and \( q_i \) are interpreted as order parameters of the model, classifying the various QHS states. These parameters can be also explored to give a quiver gauge theory description of QHS embedded in type II superstrings and M-theory compactifications [8, 16, 17].

Using eqs(4.1) and integrating over the gauge fields \( A_{i\mu} \) in the same way as in the Susskind model [7], we obtain the following filling factor

\[ \nu = q_i K_{ij}^{-1} q_j, \]  

(4.2)

generalizing eqs(3.8). This quadratic form can be split as

\[ \nu = \sum_i K_{ii}^{-1} q_i^2 + 2 \sum_{i<j} K_{ij}^{-1} q_i q_j, \]  

(4.3)

From this equation we can observe that different values of \( \nu \) correspond to different forms for the matrix \( K_{ij} \) and the vector charge \( q_i \). However, it is possible to have different models \((K_{ij} \text{ and } q_i)\) with the same filling factor.

From the string theoretic point of view, the matrix \( K_{ij} \) can be related with the intersection matrices of the compactified manifolds, considered as orbifold geometries [4]. The resolution of the singularities of such geometries is nicely interpreted in terms of the Dynking diagrams. In particular, the intersection matrix used for the singularity resolutions is, up to some details, the opposite of the Cartan matrices of the corresponding Lie algebras.

In this section, we consider models which can be obtained from the IIB superstring living on orbifolds like \( AdS_5 \times S^5 / \Gamma \), where \( \Gamma \) is a discrete subgroup of the holonomy group of the dual geometry. The corresponding four dimensional gauge theory version, can be obtained by considering D5-branes wrapping the blown up two-cycles of ALE spaces [20, 21, 22, 23]. In this way, \( N \) D5-branes are partitioned into \( \ell \) subsets of \( N_i \) D5-branes \((N = \sum_{i=1}^{\ell} N_i)\), where \( \ell \) is related to the rank of the corresponding Lie algebras. The four-dimensional gauge symmetry we will use in this section can be obtained by breaking the original symmetry to its abelian part involving only U(1) factor that describes the Coulomb phase of the theory. Using the quiver gauge theory results of [8], the action (4.1) can be obtained by wrapping D5-branes on particular 3-cycles. We will see that these cycles can be viewed as a collection of 2-cycles.
fibered over a one dimensional compact base.

To see how to get (4.1) using D5-branes, first we examine the model that gives a U(1) gauge theory. This will be subsequently extended to more general cases, with many gauge fields. A single D5-brane supports a U(1) gauge field. The corresponding DBI action gives (2.1) and must be supplemented with the WZ term that leads to the Chern-Simom action. The first term of (3.1) can be obtained from the WZ action on a single D5-brane wrapping a particular 3-cycle. This three-cycle is embedded in a Calabi-Yau threefolds given by ALE spaces fibered over the complex plane. To be explicit we consider the ALE space with $A_1$ geometry which is identified with the cotangent bundle over the two dimensional sphere $S^2$. This complex geometry is

$$z_1^2 + z_2^2 + z_3^2 = \mu,$$  \hspace{1cm} (4.4)

where $z_i$ are complex variables and where $\mu$ is a complex parameter. The real part of this parameter is the radius squared of the sphere $S^2$, while the imaginary part can be identified with the $B$-field in superstring compactifications.

The 3-cycle that we consider here will be obtained from an $A_1$ threefold, by varying the parameter $\mu$ over the complex plane parametrized by $w$. Consequently, the equation (4.4) is given by

$$z_1^2 + z_2^2 + z_3^2 = \mu(w).$$  \hspace{1cm} (4.5)

The 3-cycle can be given explicitly by a finite line segment with a $S^2$ fibration, where the radius $r$ of $S^2$ vanishes at the two interval end points and only there. This ensures that no more singular points appear. One way to realize this geometry is to use the parametrization $r \sim \sin x$, where $x$ is a real variable parameterizing the interval $[0, \pi]$ in the $w$-plane. This construction may be extended to more general geometries where we have intersecting $S^2$s according to extended Dynkin diagrams.

Having specified the 3-cycle, we will discuss the corresponding CS quivers of the compactified type IIB superstring. The analysis here will be based on the WZ action on a single D5-brane wrapping the 3-cycle $S^2 \times I_{[0,\pi]}$ previously constructed. In fact, on the six-dimensional world-volume of each D5-brane we can have a U(1) symmetry with the following WZ action

$$S_{WZ} = \int_{R^{1,5}} F \wedge F \wedge C_2,$$  \hspace{1cm} (4.6)

where $C_2$ is the R-R 2-form. Integrating by part, this WZ action becomes

$$S_{WZ} = - \int_{R^{1,5}} A \wedge F \wedge dC_2.$$  \hspace{1cm} (4.7)
Now we integrate the $dC_2$ three-form over $S^2 \times I_{[0,\pi]}$ and we find exactly the first part of the action (3.1), where $\frac{k}{4\pi} = \int_{S^2 \times I_{[0,\pi]}} dC_2$. Similarly, the second part of (3.1) is obtained from the following WZ term

$$S_{WZ} = -\int_{\mathbb{R}^{1,5}} F \wedge C_4,$$

(4.8)

where $C_4$ is now the R-R 4-from which couples to the D3-brane. To derive the external source $\tilde{A}$, the $C_4$ gauge field must be decomposed as follows

$$C_4 \rightarrow \tilde{A} \wedge \Omega,$$

(4.9)

where $\Omega$ is a harmonic 3-form on the compact space. After integrating $\Omega$ over $S^2 \times I_{[0,\pi]}$, the equation (4.8) takes the form

$$q \int_{\mathbb{R}^{1,2}} \tilde{A} \wedge F.$$

(4.10)

Now, the gauge field $\tilde{A}$ is interpreted as a magnetic external source interacting with the D5-brane system.

We can follow the same analysis to get a general solution with an arbitrary number of 3-cycles. However, the general study is beyond the scope of the present work, and we will restrict ourselves to only two isolated three-cycles ($K_{i<j} = 0$, $(i,j = 1,2)$). In this case, (4.3) reduces to

$$\nu = \sum_i K_{ii} q_i^2.$$  

(4.11)

The explicit model we deal with here represents a two layers QHS, and it is associated with a $U(1) \times U(1)$ quiver gauge theory with the following $K_{ij}$ matrix

$$
\begin{pmatrix}
  k_1 & 0 \\
  0 & k_2
\end{pmatrix}.
$$

(4.12)

This matrix can be considered as an extended Cartan matrix involving two simple roots. Geometrically, this may correspond to two isolated Riemann surfaces with a positive genus (instead of $S^2$’s). If we take the charges values $q_i = (1,1)$, then the filling factor of (4.11) is

$$\nu = \frac{1}{k_1} + \frac{1}{k_2}.$$  

(4.13)

As we have done in the previous section, and using the fact that $\nu = \sum_i v_i = v_1 + v_2$, we get
the following mass gap formula for the bi-layer system

\[ m \sim \sum_i \frac{1 - \nu_i}{\nu_i} g_i^2. \]  

(4.14)

In string theory compactification, the gauge coupling \( g_i \) is related to the volume \( V_i \) of the cycles on which the D5-branes are wrapped, \( g_i \sim \frac{1}{V_i} \). We see that the mass gap can also be related to the closed string moduli. We believe that this connection deserves to be studied further.

5 Discussions

In this work, we have discussed the mass gap for quantum Hall solitons which have been constructed from string theory compactification. We have shown that the mass gap can be related to the filling factor of the quantum Hall systems. In particular, we have obtained two simple holographic models for quantum Hall solitons that are based on D-brane configurations in type IIB superstring. The first one has been constructed from the monolayer system based on the D3/D7 brane configuration by adding Chern-Simons terms. The second model is associated with a quiver gauge theory describing bi-layered system constructed with intersecting D5-branes wrapping on a particular set of 3-cycles. This analysis can be generalized to multilayered systems by considering an arbitrary number of three-cycles with arbitrary charges. A simple generalization can be obtained from geometries whose intersection forms may be represented by \( K_{ij} = k_i \delta_{ij} \).

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