The Light Higgsino-dominated NLSPs in the Semi-constrained NMSSM

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Abstract: In the semi-constrained NMSSM (scNMSSM, or NMSSM with non-universal Higgs mass) under current constraints, we consider a scenario where \( h_2 \) is the SM-like Higgs, \( \tilde{\chi}^0_1 \) is singlino-dominated LSP, \( \tilde{\chi}^+_1 \) and \( \tilde{\chi}^0_{2,3} \) are mass-degenerated, light and higgsino-dominated NLSPs (next-to-lightest supersymmetric particles). We investigate the constraints to these NLSPs from searching for SUSY particles at the LHC Run-I and Run-II, discuss the possibility of discovering these NLSPs in the future, and come to the following conclusions regarding the higgsino-dominated 100 \( \sim \) 200 GeV NLSPs in scNMSSM: (i) Among the search results for electroweakinos, the multilepton final state constrain our scenario most, and can exclude some of our samples. While up to now, the search results by Atlas and CMS with Run I and Run II data at the LHC can still not exclude the higgsino-dominated NLSPs of 100 \( \sim \) 200 GeV. (ii) When the mass difference with \( \tilde{\chi}^0_1 \) is smaller than \( m_{h_2} \), \( \tilde{\chi}^0_2 \) and \( \tilde{\chi}^0_3 \) have opposite preference on decaying to \( Z/Z^* \) or \( h_1 \). (iii) The best channels to detect the NLSPs are though the real two-body decay \( \tilde{\chi}^+_1 \rightarrow \tilde{\chi}^0_1 W^\pm \) and \( \tilde{\chi}^0_{2,3} \rightarrow \tilde{\chi}^0_1 Z/h_2 \). When the mass difference is sufficient, most of the samples can be checked at 5\( \sigma \) level with future 300 fb\(^{-1}\) data at the LHC. While with 3000 fb\(^{-1}\) data at the High Luminosity LHC (HL-LHC), nearly all of the samples can be checked at 5\( \sigma \) level even if the mass difference is insufficient. (iv) The \( a_1 \) funnel and the \( h_2/Z \) funnel are the two main mechanisms for the singlino-dominated LSP annihilation, which can not be distinguished by searching for NLSPs.
1 Introduction

As an internal symmetry between fermions and bosons, Supersymmetry (SUSY) is an attractive idea. In the framework of SUSY, the strong, weak and hypercharge gauge couplings \((g_3, g_2, g_1)\) can be unified at the GUT scale \(\sim 10^{16} \text{ GeV}\), and the large hierarchy problem between the electroweak scale and the Planck scale can be solved. Besides, with the R-parity conserved, the lightest SUSY particle (LSP) is stable and can be a good candidate for weakly-interaction-massive-particle (WIMP) dark matter (DM).

SUSY at TeV scale is motivated by the possible cancellation of quadratic divergences of the Higgs boson mass. And the simplest implementation of SUSY is the Minimal Supersymmetric extension to the Standard Model (MSSM). Since the soft SUSY breaking parameters is totally free in the MSSM, a dynamic way to get these parameters is more favoured. In the minimal supergravity (mSUGRA) framework, the dynamics is used to derive soft SUSY breaking parameters, which is a top-down approach and all soft SUSY breaking parameters unified at the GUT scale. The fully constrained MSSM (CMSSM) is the MSSM with the boundary conditions same as the mSUGRA. But in order to get the 125 GeV SM-like Higgs, the MSSM need very large one-loop radiative corrections to Higgs mass, which makes the MSSM not natural. And there is a so-called \(\mu\)-problem [1] in the MSSM, where the superpotential contained a term \(\mu \hat{H}_u \hat{H}_d\), and \(\mu\) is a dimensionful parameter and can be chosen at any value artificially.

The Next-to Minimal Supersymmetric Standard Model (NMSSM) can solve the \(\mu\)-problem by introducing a complex singlet superfield \(\hat{S}\), which can generate an effective \(\mu\)-term dynamically. And it can easily predict a SM-like 125 GeV Higgs, under all the constraints and with low fine-tuning [2]. The fully constrained NMSSM (cNMSSM) contains none or only one more parameter than the CMSSM/mSUGRA, thus both of them are in tension with current experimental constraints including 125 GeV Higgs mass, high mass
bound of gluino, muon g-2, and dark matter [3, 4]. So, we consider the semi-constrained
NMSSM (scNMSSM), which relaxing the universality of scalar masses by decoupling
the squared-masses of the Higgs bosons and the squarks/sleptons, which is also called NMSSM
with non-universal Higgs mass (NUHM) [5–7]. In the scNMSSM, the bino and wino are
heavy because the high mass bound of gluino and the unification of gaugino masses at
GUT scale, thus the light neutralinos and charginos can only be singlino-dominated or
higgsino-dominated.

In recent years, the Atlas and CMS collaborations have carried out many searches for
SUSY particles, which pushed the gluino and squarks masses bounds in simple models up
to several hundreds GeV and even TeV scale. While it is still possible for the electroweakino
sector to be very light. The electroweakino sector of NMSSM was studied in [8–14], among
which different search channels were provided, such as trileptons [11], multi-lepton [12], and
jets with missing transverse energy (E_{T}) [13]. These motivated us to check the current status
of higgsino, in special SUSY models such as the scNMSSM, under direct-search constraints
and their possibility of discovery by detailed simulation.

In this work, we discuss the light higgsino-dominated NLSPs (next-to-lightest super-
symmetric particles) in the scNMSSM. We use the scenario of singlino-dominated \tilde{\chi}_1^0
and SM-like \tilde{h}_2 in the scan result in our former work on scNMSSM [7], where we considered
the constraints including theoretical constraints of vacuum stability and Landau pole, ex-
perimental constraints of Higgs data, muon g-2, B physics, dark matter relic density and
direct searches, etc. Thus in this scenario the \tilde{\chi}_1^\pm and \tilde{\chi}_2^0,3 are higgsino-dominated, light and
mass-degenerated NLSPs. We first investigate the constraints to these NLSPs, including
searching for SUSY particle at the LHC Run-I and Run-II. We use Monte Carlo to do the
detailed simulations to add these constraints. Then we discuss the possibility of discovering
the higgsino-dominated NLSPs in the future at the High Luminosity LHC (HL-LHC).

This paper is organized as follows. First, in Section 2, we briefly introduce the model of
NMSSM and scNMSSM, especially the Higgs and electroweakino sector. Latter in Section
3, we discuss the constraints to the light higgsino-dominated NLSPs, and the possibility of
discovering them at the HL-LHC. Finally, we draw our conclusions in Section 4.

2 Introduction to NMSSM and scNMSSM

The superpotential of the NMSSM with Z_3 symmetry:

\begin{equation}
W_{NMSSM} = W_{MSSM}|_{\mu=0} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{\kappa}{3} \hat{S}\bar{\beta},
\end{equation}

where the superfields \hat{H}_u and \hat{H}_d are two complex doublet superfields, the superfield \hat{S}
is the singlet superfield, the coupling constrats \lambda and \kappa are dimensionless, and the \hat{W}_{MSSM}|_{\mu=0}
is actually the Yukawa couplings of the \hat{H}_u and \hat{H}_d to the quark and lepton superfields.
When electroweak symmetry breaking, the scalar component of superfields \hat{H}_u, \hat{H}_d and
\hat{S} get their vacuum expectation values(VEVs) v_u, v_d and v_s respectively. The relations
between the VEVs are

\begin{equation}
tan\beta = v_u/v_d
\end{equation}

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\[ v = \sqrt{v_u^2 + v_d^2} = 174 \text{ GeV} \]  

\[ \mu_{eff} = \lambda v_s, \]  

where the \( \mu_{eff} \) is the mass of Higgsino, like in the MSSM. In the following, for the sake of convenience, we refer to \( \mu \) as \( \mu_{eff} \).

The soft SUSY breaking terms in the NMSSM is only different from the MSSM in several terms:

\[ -\mathcal{L}_{\text{soft}}^{\text{NMSSM}} = -\mathcal{L}_{\text{soft}}^{\text{MSSM}}|_{\mu=0} + m_S^2 |S|^2 + \lambda A_\lambda S H_u \cdot H_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}, \]  

where the \( S, H_u \) and \( H_d \) is the scalar component of the superfields, the \( m_S^2 \) is the soft SUSY breaking mass for single field \( S \), and the trilinear coupling constants \( A_\lambda, A_\kappa \) have mass dimension.

In the semi-constrained NMSSM (scNMSSM), the Higgs sector are considered non-universal, that is, the Higgs soft mass \( m_{H_u}, m_{H_d} \) and \( m_S^2 \) are allowed to be different from \( M^2_0 \), and the trilinear couplings \( A_\lambda, A_\kappa \) can be different from \( A_0 \). Hence, in the scNMSSM, the complete parameter sector is usually chosen as:

\[ \lambda, \kappa, \tan\beta, \mu, A_\lambda, A_\kappa, A_0, M_{1/2}, M_0. \]  

### 2.1 The Higgs sector of NMSSM and scNMSSM

When the electroweak symmetry breaking, the scalar component of superfields \( \hat{H}_u, \hat{H}_d \) and \( \hat{S} \) can be written as

\[ H_u = \left( \frac{H_u^R + H_u^L}{\sqrt{2}} \right), \quad H_d = \left( v_d + \frac{H_d^R + iH_d^I}{H_d^R} \right), \quad S = v_s + \frac{S^R + iS^I}{\sqrt{2}}, \]  

where \( H_u^R, H_d^R \), and \( S^R \) are CP-even component fields, \( H_u^L, H_d^L \), and \( S^I \) are the CP-odd component fields, and the \( H_u^+ \) and \( H_d^- \) are charged component fields.

In the basis \( (H_d^R, H_u^R, S^R) \), the CP-even scalar mass matrix is [15]

\[ \mathcal{L} \ni \frac{1}{2} \left( H_d^R, H_u^R, S^R \right) M_S^2 \left( \begin{array}{c} H_d^R \\ H_u^R \\ S^R \end{array} \right) \]  

with

\[ M_S^2 = \begin{pmatrix} M_{A\beta}^2 s_\beta^2 + M_{\beta\beta}^2 c_\beta^2 & (2\lambda v^2 - M_A^2 - M_S^2) s_\beta c_\beta & C c_\beta + C' s_\beta \\ (2\lambda v^2 - M_A^2 - M_S^2) s_\beta c_\beta & M_A^2 c_\beta^2 + M_S^2 s_\beta^2 & C s_\beta + C' c_\beta \\ C c_\beta + C' s_\beta & C s_\beta + C' c_\beta & M_{S,S,R}^2 \end{pmatrix} \]  

where

\[ M_A^2 = \frac{2\mu(A_\lambda + \kappa v_s)}{\sin 2\beta} \]  

\[ C = 2\lambda^2 v v_s \]  

\[ C' = \lambda v(A_\lambda - 2\kappa v_s) \]  

\[ M_{S,S,R}^2 = \lambda A_\lambda \frac{v_u v_d}{v_s} + \kappa v_s (A_\kappa + 4\kappa v_s) \]
and $s_\beta = \sin \beta, c_\beta = \cos \beta$. Actually, there is a more common basis $H_1, H_2, S^R$, where

$$H_1 = H_u^c s_\beta - H_d^c s_\beta$$  \hspace{1cm} (2.14)

$$H_2 = H_u^c s_\beta + H_d^c c_\beta$$  \hspace{1cm} (2.15)

and the $H_2$ is the SM-like Higgs field. In the basis $(H_1, H_2, S^R)$, the scalar mass matrix is different from eq.(2.9). But, since the rotation of the basis do not touch the third component $S^R$, the $M_{S^R}^2$ will keep the same as in eq.(2.13). The Higgs boson mass matrix $M_{S^R}^2$ in basis $(H_1, H_2, S^R)$ is given by [16]

$$M_{S^R,H_1,H_1}^2 = M_A^2 + (m_Z^2 - \lambda^2 v^2) \sin^2 2\beta,$$

$$M_{S^R,H_1,H_2}^2 = -\frac{1}{2} \left( m_Z^2 - \lambda^2 v^2 \right) \sin 4\beta,$$

$$M_{S^R,H_2,S^R}^2 = 2\lambda v \left[ 1 - \left( \frac{M_A}{\mu} \sin 2\beta \right)^2 + \frac{\kappa v}{\lambda} \sin 2\beta, \right],$$

$$M_{S^R,S^R}^2 = \frac{1}{4} \lambda^2 v^2 \left( \frac{M_A}{\mu} \sin 2\beta \right)^2 + \kappa v A_\kappa + 4(\kappa v)^2 - \frac{1}{2} \lambda v^2 \sin 2\beta, \right. \hspace{1cm} (2.21)$$

And comparing eq.(2.21) with eq.(2.13), it’s not hard to get $M_{S^R,S^R}^2 = M_{S^R}^2$.

In order to get the physical CP-odd scalar Higgs bosons, one can rotate the Higgs fields,

$$A = H_u^c s_\beta + H_d^c c_\beta.$$

(2.22)

Then the Goldstone mode can be dropped off, and the CP-odd scalar mass matrix in the basis $(A, S^I)$ become [15]

$$\mathcal{L} \equiv \frac{1}{2} (A, S^I) M_P^2 \begin{pmatrix} A \\ S^I \end{pmatrix},$$

with

$$M_P^2 = \begin{pmatrix} M_A^2 & \lambda v (A_\lambda - 2\kappa v) \\ \lambda v (A_\lambda - 2\kappa v) & M_{P,S^I}^2 \end{pmatrix},$$

(2.23)

(2.24)

where

$$M_{P,S^I}^2 = \lambda (A_\lambda + 4\kappa v) \frac{v_u v_d}{v_s} - 3\kappa v A_\kappa.$$

(2.25)

The mass eigenstates of the CP-even Higgs $h_i$ ($i = 1, 2, 3$) and the CP-odd Higgs $A_i(i = 1, 2)$ can be obtained by

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = S_{ij} \begin{pmatrix} H_1 \\ H_2 \\ S^R \end{pmatrix}, \hspace{1cm} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = P_{ij} \begin{pmatrix} A \\ S^I \end{pmatrix}, \hspace{1cm} (2.26)$$

where the matrix $S_{ij}$ can diagonalize the mass matrix $M_S^2$, and the matrix $P_{ij}$ can diagonalize the mass matrix $M_P^2$. 

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2.2 The electroweakino sector of NMSSM and scNMSSM

In the NMSSM, there are five neutralinos $\tilde{\chi}_i^0$ ($i = 1, 2, 3, 4, 5$), which are the mixture of $\tilde{B}$ (bino), $\tilde{W}^0$ (wino), $\tilde{H}_d^0$, $\tilde{H}_u^0$ (higgsino) and $\tilde{S}$ (singlino). In the gauge-eigenstate basis $\psi^0 = (B, W^0, H^0_d, H^0_u, S)$, the neutralino mass matrix takes the form [15]

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z & 0 \\
0 & M_2 & c_\beta c_w m_z & -s_\beta c_W m_Z & 0 \\
-c_\beta s_W m_Z & c_\beta c_w m_z & 0 & -\mu & -\lambda v_d \\
s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 & -\lambda v_u \\
0 & 0 & -\lambda v_d & -\lambda v_u & 2\kappa v_s
\end{pmatrix}
\] (2.27)

where $s_\beta = \sin \beta, c_\beta = \cos \beta, s_W = \sin \theta_W, c_W = \cos \theta_W$. To get the mass eigenstates, one can diagonalize the neutralino mass matrix $M_{\tilde{\chi}^0}$

\[N^* M_{\tilde{\chi}^0} N^{-1} = M_{\tilde{\chi}^0}^D = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0})\] (2.28)

where $M_{\tilde{\chi}^0}^D$ means the diagonal mass matrix, and the order of eigenvalues is $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0} < m_{\tilde{\chi}_5^0}$. Meanwhile, one can get the mass eigenstates

\[
\begin{pmatrix}
\tilde{\chi}_1^0 \\
\tilde{\chi}_2^0 \\
\tilde{\chi}_3^0 \\
\tilde{\chi}_4^0 \\
\tilde{\chi}_5^0
\end{pmatrix} = N_{ij} \begin{pmatrix}
\tilde{B} \\
W^0 \\
\tilde{H}_d \\
\tilde{H}_u \\
\tilde{S}
\end{pmatrix}
\] (2.29)

In the scNMSSM, bino and wino were constrained to be very heavy, because of the high mass bounds of gluino and the universal gaugino mass at GUT scale, thus they can be decoupled from the light sector. Then the following relations for the $N_{ij}$ can be can be found [17]:

\[
N_{i3} : N_{i4} : N_{i5} = \frac{m_{\tilde{\chi}_1^0}}{\mu} s_\beta - c_\beta : \frac{m_{\tilde{\chi}_1^0}}{\mu} c_\beta - s_\beta : \frac{\mu - m_{\tilde{\chi}_1^0}}{\lambda v_u} (2.30)
\]

We assume the lightest neutralino $\tilde{\chi}_1^0$ is the lightest supersymmetric particle (LSP) and makes up of the cosmic dark matter. If the LSP $\tilde{\chi}_1^0$ satisfies $N_{15}^2 > 0.5$, we call it singlet-dominated. And the coupling of such an LSP with the CP-even Higgs bosons is given by [17]

\[
C_{h_1\tilde{\chi}_1^0} = \sqrt{2} \lambda [S_{11} N_{15} (N_{13} c_\beta - N_{14} s_\beta) + S_{12} N_{15} (N_{14} c_\beta + N_{13} s_\beta) + S_{13} N_{15} (N_{13} N_{14} - \frac{\kappa}{\lambda} N_{15}^2)]
\] (2.31)

In the singlet-dominated-LSP scenario, taking $N_{11} = N_{12} = 0$, the mass of LSP can be written as $m_{\chi_1^0} \approx M_{\tilde{\chi}_1^0, \tilde{S}\tilde{S}} = 2\kappa v_s$. And from eq.(2.27), eq.(2.13) and eq.(2.25), one can find the sum rule [18]:

\[
M_{\tilde{\chi}_1^0, \tilde{S}\tilde{S}}^2 = 4\kappa^2 v_s^2 = M_{S, S}^2 + \frac{1}{3} M_{P, S}^2 + \frac{4}{3} v_u v_d \left( \frac{\lambda^2 A_\lambda}{\mu} + \kappa \right)
\] (2.32)
In the case that $h_1$ singlet-like, $\tan\beta$ sizable, $\lambda, \kappa$ and $A_\lambda$ not too large, this equation can become
\[ m_{\tilde{\chi}_0^1}^2 \approx m_{h_1}^2 + \frac{1}{3} m_{a_1}^2 \]  
\[ (2.33) \]

The Chargino sector in the NMSSM is very similar to neutralino sector. The charged Higgsino $\tilde{H}_u^+, \tilde{H}_d^-$ (with mass scale around $\mu$) and the charged gaugino $\tilde{W}^\pm$ (with mass scale $M_2$) can also mixed respectively, forming two couples of physical chargino $\chi^+_1, \chi^+_2$. In the gauge-eigenstate basis $(\tilde{W}^\pm, \tilde{H}_{u,d}^\pm)$, the chargino mass matrix is given by [15]
\[ M_{\tilde{\chi}^\pm} = \left( \begin{array}{cc} M_2 & \sqrt{2} c_\beta m_W \\ \sqrt{2} s_\beta m_W & \mu \end{array} \right) \]  
\[ (2.34) \]

To obtain the chargino mass eigenstates, one can use two unitary matrix to diagonalize the chargino mass matrix
\[ U^* M_{\tilde{\chi}^\pm} V^{-1} = M_{\tilde{\chi}^\pm} = diag(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}) \]  
\[ (2.35) \]

where $M_{\tilde{\chi}^\pm}^D$ means the diagonal mass matrix, and the order of eigenvalues is $m_{\tilde{\chi}^+_1} < m_{\tilde{\chi}^+_2}$. Meanwhile, we can get the mass eigenstates
\[ \begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = V_{ij} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_{u}^+ \end{pmatrix}, \quad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = U_{ij} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_{d}^- \end{pmatrix} \]  
\[ (2.36) \]

In the scNMSSM, since $M_2 \gg \mu$, $\chi^+_1$ can be higgsino-dominated, with mass around $\mu$. With $\chi^0_1$ singlino-dominated, $\chi^0_{2,3}$ can be higgsino-dominated, with masses nearly degenerate also around $\mu$, and with $N_{23}^2 + N_{24}^2 > 0.5$. Then with $\mu$ not large, smaller than other sparticle mass, the nearly-degenerate $\chi^+_1$ and $\chi^0_{2,3}$ can be called the next-to-lightest SUSY particles (NLSPs). In this work, we will focus on the detection of the higgsino-dominated NLSPs ($\chi^+_1$ and $\chi^0_{2,3}$) in the scNMSSM.

3 The Light Higgs-dominated NLSPs in scNMSSM

In this work, we use the scan result in our former work about scNMSSM [7], but only consider the surviving samples with singlino-dominated $\chi^0_1$ ($|N_{15}|^2 > 0.5$) as the LSP, and impose the SUSY search constraints with CheckMATE [19]. We did the scan with the program NMSSMTools-5.4.1 [20], and considered the constraints there, including theoretical constraints of vacuum stability and Landau pole, experimental constraints of Higgs data, muon $g-2$, B physics, dark matter relic density and direct searches, etc. We also use HiggsBounds-5.1.1beta [21] to constrain the Higgs sector (with $h_2$ as the SM-like Higgs and $123 < m_{h_2} < 127$ GeV), and SModelS-v1.1.1 [22] to to constrain the SUSY particles. The scanned spaces of the parameters are:
\[ 0 < M_0 < 500 \text{ GeV}, \quad 0 < M_{1/2} < 2 \text{ TeV}, \quad |A_0| < 10 \text{ TeV}, \]  
\[ 100 < \mu < 200 \text{ GeV}, \quad 1 < \tan\beta < 30, \quad 0.3 < \lambda < 0.7, \]  
\[ 0 < \kappa < 0.7, \quad |A_\lambda| < 10 \text{ TeV}, \quad |A_\kappa| < 10 \text{ TeV}. \]  
\[ (3.1) \]
Since in scNMSSM the bino and wino are heavy because of the high bounds of gluino mass, and the $\chi^0_1$ LSP is singlino-dominated in our samples, the neutralino and chargino NLSPs ($\chi^\pm_1$ and $\chi^0_{2,3}$) are Higgsino dominated in this work. In the following, we focus on the Higgsino-dominated NLSPs in the scNMSSM, considering the constraints from the recent search results at the LHC Run I and Run II, and possibility of discovery at the HL-LHC in the future.

3.1 Constraints from the recent search results at the LHC

Unlike colored particles, the production rates of electroweakinos are very low at the LHC. But they can mainly decay to leptons plus $E_T$, and the SM backgrounds in these leptons channels are relatively cleaner than jet channels at the LHC. In recent years, Atlas and CMS collaborations have released several search results with the LHC Run-I & Run-II data in such channels as $2\ell + E_T$ [23], $3\ell + E_T$ [24, 25], $2\gamma + E_T$ [26] and Higgs + $E_T$ [27]. In their analyses, they considered simple models, where purely higgsino or wino NLSP produced in pair, each decaying to $\chi^0_1$ plus $h$, $Z$, or $W^\pm$ in 100%.

In this work, we use these result to constrain our specific surviving samples in scNMSSM. We consider the production and decay of $\tilde{\chi}^+_1 \tilde{\chi}^-_1$, $\tilde{\chi}^+_1 \tilde{\chi}^0_{2,3}$ and $\tilde{\chi}^0_2 \tilde{\chi}^0_{2,3}$ at the LHC, using CheckMATE 2.0.26 [19] to impose these constraints.

Firstly, We use MadGraph5_aMC@NLO 2.6.6 [28] to generate three types tree level process at 8 TeV and 13 TeV:

$$pp \to \tilde{\chi}^+_1 \tilde{\chi}^-_1, \quad pp \to \tilde{\chi}^+_1 \tilde{\chi}^0_{2,3}, \quad pp \to \tilde{\chi}^0_{2,3} \tilde{\chi}^0_{2,3}.$$ (3.2)

Since the cross sections by the MadGraph are at tree level, we multiply them by a NLO K-factor calculated with the Prospino2 [29]. Then, we use the PYTHIA 8.2 [30] to deal with particle decay, parton showering, and hardronization, use Delphes 3.4.1 [31] to simulate the detector response, and use the anti-$k_T$ algorithm [32] for jet clustering.

After the simulation, we can get a `.root` file. We use the CheckMATE2 to read this `.root` file. Then, we apply the same cuts in signal regions of the CMS and ATLAS experiments at 8 TeV and 13 TeV, by using analysis cards which have been implement in CheckMATE2. At the last step, with the CheckMATE2 we get a r-value for each samples, which is defined as

$$r \equiv \frac{S - 1.96\Delta S}{S_{95}^{Exp}}.$$ (3.3)

where $S$ is the total number of expected signal events, $\Delta S$ is the uncertainty of $S$, and $S_{95}^{Exp}$ is the experimentally measured 95% confidence limit of signal events number. So, a model can be considered excluded at 95% confidence level, if $r \geq 1$. If the $r \geq 1$ in only one signal region, the model can also be excluded. We can get $r_{max}$, the maximal value of $r$ in different signal regions. The model is excluded if $r_{max} \geq 1$.

In Fig.1, we show the production cross sections of $\tilde{\chi}^+_1 \tilde{\chi}^0_2$, $\tilde{\chi}^+_1 \tilde{\chi}^0_3$, $\tilde{\chi}^-_1 \tilde{\chi}^0_2$, $\tilde{\chi}^-_1 \tilde{\chi}^0_3$, $\tilde{\chi}^+_1 \tilde{\chi}^0_1$, $\tilde{\chi}^0_1 \tilde{\chi}^0_2$, $\tilde{\chi}^0_2 \tilde{\chi}^0_2$, and $\tilde{\chi}^0_3 \tilde{\chi}^0_3$ at the 14 TeV LHC.

- The relation between the cross sections and the masses of final states is clear shown in all of these plots, that is, the masses of final states particles are heavier, its production...
cross section is lower, because that the phases space is suppressed by the mass of final state.

- From the second and fourth plots in the upper panel, the production cross section of $\tilde{\chi}_1^+\tilde{\chi}_1^0$ is about 2 times of $\tilde{\chi}_1^-\tilde{\chi}_1^0$, for both $i = 2$ and 3. The Feynman Amplitudes of the productions of $\tilde{\chi}_1^+\tilde{\chi}_1^0$ and $\tilde{\chi}_1^-\tilde{\chi}_1^0$ are the same, and the production of $\tilde{\chi}_1^+\tilde{\chi}_1^0$ or $\tilde{\chi}_1^-\tilde{\chi}_1^0$ need the initial state to be $ud$ or $du$, respectively. So, the reason is that the LHC is proton-proton collider, and the proton is a bound state of $uud$, so the parton distribution functions (PDF) for up quark is more than down quark, that leads to a linear relation of 2 times.

- From the third and fourth plots in lower panel, we can see that the production cross section of $\tilde{\chi}_2^0\tilde{\chi}_2^0$ ($i = 2$ or 3) is very small, only a few fb. The reason is that the squarks are very heavy, so $\sigma(pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0)$ mainly contribute from s channel through $Z$ boson
resonance. The coupling of $Z - \tilde{\chi}_i^0 - \tilde{\chi}_j^0$ is given by

$$C_{Z\tilde{\chi}_i^0\tilde{\chi}_j^0} = -\frac{i}{2}(g_1 s_W + g_2 c_W)(N_{j3}^* N_{i3} - N_{j4}^* N_{i4})(\gamma_\mu P_L) + \frac{i}{2}(g_1 s_W + g_2 c_W)(N_{i3}^* N_{j3} - N_{i4}^* N_{j4})(\gamma_\mu P_R)$$

(3.4)

where the matrix $N$ is neutralino mixing matrix. And we can see that if $|N_{i3}|^2 - |N_{i4}|^2 \approx 0$, then $\sigma(pp \to \tilde{\chi}_i^0\tilde{\chi}_j^0) \approx 0$.

After using CheckMATE to checking our surviving samples, we notice that most of the samples excluded are by the CMS analysis in multilepton final states at 13 TeV LHC with 35.9 fb$^{-1}$ data [24]. We checked that the relevant mechanism is $\tilde{\chi}_1^+\tilde{\chi}_2^0$ produced and each decaying to 2 body. Since the sleptons are heavier, the LSP plus a $pp$ in the final state. The CMS searches related to our process included the following signal particles, since the leptons coming form a virtual $W$ boson are very soft and hard to detect.

The main decay modes of neutralino $\tilde{\chi}_i^0$ ($i = 2, 3$) are to a $\tilde{\chi}_1^0$ plus a $Z$ boson or a Higgs boson. The most effective process excluding the samples are $pp \to \tilde{\chi}_1^+ (W^+\tilde{\chi}_1^0)\tilde{\chi}_2^0 (Z\tilde{\chi}_1^0)$ and $pp \to \tilde{\chi}_1^+ (W^+\tilde{\chi}_1^0)\tilde{\chi}_2^0 (h\tilde{\chi}_1^0)$.

The searching strategy for these two process is three or more leptons plus large $E_T$ in the final state. The CMS searches related to our process included the following signal regions (SR) SR-A, SR-C and SR-F

- **SR-A**: events with three light leptons ($e$ or $\mu$), two of which forming an opposite sign same-flavor (OSSF) pair. The SR-A is divided into 44 bins, according to the invariant mass of OSSF pair $M_{\ell\ell}$, the third lepton’s transverse mass $M_T$ and the missing energy $E_T$.

- **SR-C**: events with two light leptons ($e$ or $\mu$) forming an OSSF pair, and one $\tau_h$ candidate. The SR-C is divided into 18 bins, according to the invariant mass $M_{\ell\ell}$, the two-lepton ‘transverse mass’ $M_{T2}(\ell_1, \ell_2)$ [33] instead of $M_T$ on the off-Z regions, and the $E_T$. The $M_{T2}$ is defined as:

$$M_{T2} = \min_{E_{T1} + E_{T2} = E_T} \left\{ \max \left\{ M_T(p_{T1}, E_{T1}), M_T(p_{T2}, E_{T2}) \right\} \right\} , \quad (3.5)$$

where the $E_{T1}$ and $E_{T2}$ stand for the missing transverse energy for the two leptons respectively. And it’s used to suppressed the SM background, since the large $t\bar{t}$ background is at low $M_{T2}$.

- **SR-F**: events with one electron or muon plus two $\tau_h$ candidates. SR-F is divided into 12 bins, according to $M_{\ell\ell}$, $M_{T2}(\ell, \tau_1)$ and the $E_T$.

In Fig.2, we show the branching ratios of $\tilde{\chi}_1^+$ on the plane of $m_{\tilde{\chi}_1^0}$ vs $m_{\tilde{\chi}_1^+}$. We can see that, the chargino $\tilde{\chi}_1^+$ decay to $\tilde{\chi}_1^0$ plus a $W$ boson in 100%: when the mass difference between $\tilde{\chi}_1^+$ and $\tilde{\chi}_1^0$ is greater than $m_W$, the $W$ boson is a real one; while when the mass difference is insufficient, the $W$ boson is a virtual one, that is, the decay is a three-body decay $\tilde{\chi}_1^+ \to \ell \nu \tilde{\chi}_1^0$. The low mass difference is negative for us to search for the SUSY particles, since the leptons coming form a virtual $W$ boson are very soft and hard to detect.

The main decay modes of neutralino $\tilde{\chi}_i^0$ ($i = 2, 3$) are to a $\tilde{\chi}_1^0$ plus a $Z$ boson or a Higgs boson. In Fig.3, we show the branching ratios of the neutralinos $\tilde{\chi}_i^0$ on the plane of $m_{\tilde{\chi}_i^0}$ vs
Figure 2. The samples in the $m_{\tilde{\chi}_i^0}$ versus $m_{\tilde{\chi}_i^+}$ plane, where colors indicate the branching ratios of the chargino $\tilde{\chi}_i^+$ to $\tilde{\chi}_i^0$ plus $W$ boson. In the left panel, the $W$ boson is a real one and the decay is real two-body decay; while in the right panel the $W$ boson is a virtual one and the decay is virtual three-body decay.

Figure 3. The samples in the $m_{\tilde{\chi}_i^0}$ versus $m_{\tilde{\chi}_j^0}$ planes (upper $i = 2$, lower $i = 3$). From left to the right, colors indicate the branching ratios $Br(\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 Z)$, $Br(\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 Z^*)$ ($Z^*$ means a virtual $Z$ boson), $Br(\tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 h_1)$ and $Br(\tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 h_2)$, respectively. The dash line and the dotted line means that the mass difference, $m_{\tilde{\chi}_i^+} - m_{\tilde{\chi}_i^0}$, equal to $m_Z$ and $m_{h_2}$ respectively.

$m_{\tilde{\chi}_i^0}$, where $i = 2, 3$. In these plots, we use the dash line, $m_{\tilde{\chi}_i^+} - m_{\tilde{\chi}_i^0} = m_Z$, and the dotted line, $m_{\tilde{\chi}_i^+} - m_{\tilde{\chi}_i^0} = m_{h_2}$, divided the plane into 3 parts.

- **Case 1:** In the region $m_{\tilde{\chi}_i^+} - m_{\tilde{\chi}_i^0} < m_Z$, the neutralino $\tilde{\chi}_i^0$ can only decay to $\tilde{\chi}_i^0$ plus a virtual $Z$ boson or a light Higgs boson $h_1$. And we can see from the second and third plots on the upper panel, the $\tilde{\chi}_2^0$ mainly decay to a virtual $Z$ boson plus a $\tilde{\chi}_1^0$, with only a small fraction to the light Higgs boson $h_1$ plus $\tilde{\chi}_1^0$. On the contrary, the second and third plots on the lower panel show that the $\tilde{\chi}_3^0$ mainly decay to the light Higgs boson $h_1$. 
• **Case II:** In the region \(m_Z \leq m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} < m_{h_2}\), the neutralino \(\tilde{\chi}_1^0\) can decay to \(\tilde{\chi}_1^0\) plus a real Z boson or a light Higgs boson \(h_1\). As showed in the first and third plots on the upper panel, the \(\chi_0^0\) mainly decay to a real Z boson with \(\tilde{\chi}_1^0\). While according to the first and third plots on the lower panel, the \(\chi_0^0\) mainly decay to a light Higgs boson \(h_1\) plus a LSP.

• **Case III:** In the region \(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} \geq m_{h_2}\), the neutralino \(\tilde{\chi}_1^0\) can decay to \(\tilde{\chi}_1^0\) plus a 125 GeV SM-like Higgs boson \(h_2\), which is showed in the fourth plot (upper and lower panels).

In the channel \(\tilde{\chi}_1^0 \to \tilde{\chi}_1^0 Z \) (\(i = 2, 3\)), like the \(\tilde{\chi}_1^+ \to W^+ \tilde{\chi}_1^0\), when the mass difference is insufficient the Z boson becomes a virtual one, which make it hard to detect. In the channel \(\tilde{\chi}_1^0 \to \chi_1^0 H \) (\(i = 2, 3\)), where the Higgs boson can be \(h_1\) or \(h_2\) and \(h_2\) is the SM-like one. Both \(h_1\) and \(h_2\) mainly decay to \(b\bar{b}\), thus the \(t\bar{t}\) background is sizable at the LHC. In the case that Higgs decay to \(WW\), \(ZZ\), or \(\tau\tau\), and \(W\) or \(Z\) decays leptonically, it might contribution to the multilepton final state. Since the light Higgs \(h_1\) is highly singlet-dominated, the \(\tilde{\chi}_1^0 \to \chi_1^0 h_1\) is very hard to contribute to the multilepton signal regions. Thus only the \(\tilde{\chi}_1^0 \to \tilde{\chi}_1^0 h_2\) can contribute to the multilepton signal regions visibly.

It is worth to mention that, when the heavier neutralinos decay to the \(\tilde{\chi}_1^0\) LSP, the \(\tilde{\chi}_2^0\) and \(\tilde{\chi}_3^0\) behave differently. Especially in the case II, \(\tilde{\chi}_2^0\) prefers to decay to a Z boson plus \(\tilde{\chi}_1^0\), \(Br(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z) > Br(\tilde{\chi}_2^0 \to \chi_1^0 h_1)\); while \(\tilde{\chi}_3^0\) tends to decay to a light Higgs boson \(h_1\) plus \(\tilde{\chi}_1^0\), \(Br(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z) < Br(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 h_1)\). The couplings \(C_{h_1 \tilde{\chi}_2^0 \tilde{\chi}_1^0}\) and \(C_{h_1 \tilde{\chi}_3^0 \tilde{\chi}_1^0}\) can be written down as

\[
C_{h_1 \tilde{\chi}_2^0 \tilde{\chi}_1^0} \sim \frac{\lambda(N_{14}N_{23} + N_{12}N_{24})S_{13}}{\sqrt{2}} - \sqrt{2} \kappa N_{15} N_{25} S_{13}
\]

\[
C_{h_1 \tilde{\chi}_3^0 \tilde{\chi}_1^0} \sim \frac{\lambda(N_{14}N_{33} + N_{13}N_{34})S_{13}}{\sqrt{2}} - \sqrt{2} \kappa N_{15} N_{35} S_{13}
\]

where the \(N_{11}, N_{12}, N_{21}, N_{22}, N_{31}\) and \(N_{32}\) was set to 0 since the wino and bino are very heavy and decoupled in the sCNMSSM, and the \(S_{11}\) and \(S_{12}\) was set to 0 since \(|S_{13}| > |S_{11}|, |S_{12}|\). \(\lambda/\sqrt{2} \ll 1\) and \(\sqrt{2} \kappa \ll 1\), so the couplings \(C_{h_1 \tilde{\chi}_2^0 \tilde{\chi}_1^0}\) and \(C_{h_1 \tilde{\chi}_3^0 \tilde{\chi}_1^0}\) are both very small and roughly the same. While the couplings \(C_{Z\tilde{\chi}_2^0 \tilde{\chi}_1^0}\) and \(C_{Z\tilde{\chi}_3^0 \tilde{\chi}_1^0}\) can be different from each other according to eq.(3.4), which can be approximated to

\[
C_{Z\tilde{\chi}_2^0 \tilde{\chi}_1^0} \sim \frac{g_2}{c_W} (N_{13}N_{23} - N_{14}N_{24})
\]

\[
C_{Z\tilde{\chi}_3^0 \tilde{\chi}_1^0} \sim \frac{g_2}{c_W} (N_{13}N_{33} - N_{14}N_{34})
\]

where the \(g_2/c_W \sim 1\). When the two terms in eq.(3.8) or eq.(3.9) have different sign, and do not cancel with each other, the couplings \(C_{Z\tilde{\chi}_2^0 \tilde{\chi}_1^0}\) can be much larger than \(C_{h_1 \tilde{\chi}_2^0 \tilde{\chi}_1^0}\); otherwise the cancel between the two terms can make \(C_{Z\tilde{\chi}_2^0 \tilde{\chi}_1^0}\) smaller than \(C_{h_1 \tilde{\chi}_2^0 \tilde{\chi}_1^0}\). For some surviving samples, \(C_{Z\tilde{\chi}_2^0 \tilde{\chi}_1^0}\) have the cancellation between the two terms, and that leads to small \(Br(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 Z)\) and large \(Br(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 h_1)\). Six benchmark points are listed in the Table 1.
Table 1. Masses and branching ratios for 6 benchmark points in the scNMSSM. The signal significances in the last line are calculated with the luminosity of 300 fb$^{-1}$, and with similar analysis of multi-lepton final state as in Ref. [24].

|                  | P1   | P2   | P3   | P4   | P5   | P6   |
|------------------|------|------|------|------|------|------|
| $m_{\tilde{\chi}_1^\pm}$ (GeV) | 183  | 173  | 175  | 189  | 175  | 187  |
| $m_{\tilde{\chi}_1^0}$ (GeV)    | 120  | 119  | 103  | 108  | 82   | 92   |
| $m_{\tilde{\chi}_2^0}$ (GeV)    | 200  | 187  | 200  | 209  | 202  | 206  |
| $m_{\tilde{\chi}_3^0}$ (GeV)    | 216  | 200  | 214  | 216  | 212  | 209  |
| $Br(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 W)$ | 0%   | 0%   | 0%   | 100% | 100% | 100% |
| $Br(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 W^*)$ | 100% | 100% | 100% | 0%   | 0%   | 0%   |
| $Br(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z)$ | 0%   | 0%   | 90%  | 93%  | 94%  | 95%  |
| $Br(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^*)$ | 80%  | 65%  | 0%   | 0%   | 0%   | 0%   |
| $Br(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 h_1)$ | 20%  | 35%  | 10%  | 7%   | 6%   | 5%   |
| $Br(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 h_2)$ | 0%   | 0%   | 0%   | 0%   | 0%   | 0%   |
| $Br(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 Z)$ | 1%   | 0%   | 1%   | 13%  | 13%  | 38%  |
| $Br(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 Z^*)$ | 0%   | 0%   | 0%   | 0%   | 0%   | 0%   |
| $Br(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 h_1)$ | 99%  | 100% | 99%  | 87%  | 33%  | 62%  |
| $Br(\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 h_2)$ | 0%   | 0%   | 0%   | 0%   | 54%  | 0%   |

$s_{ss} = S/\sqrt{B}$ @300 fb$^{-1}$ ($\sigma$) | 3.1  | 2.9  | 2.1  | 3.8  | 10.8 | 8.8  |

3.2 Possibility of discovery at the HL-LHC in the future

In this part, we investigate the possibility of detect electrowekinos in the future High Luminosity LHC (HL-LHC). We adopt the same analysis of multilepton final state by CMS [24], only increasing the integrate luminosity from 35.9 fb$^{-1}$ to 300 fb$^{-1}$, to see the possibility of discovery in the future. And we evaluate the signal significance by

$$ss = S/\sqrt{B}$$

where $S$ and $B$ are the number of events from signal and background process respectively.

In the Fig.4, we show $ss$ on the planes of $m_{\tilde{\chi}_1^0}$ versus $m_{\tilde{\chi}_1^\pm}$, $m_{\tilde{\chi}_2^0}$ and $m_{\tilde{\chi}_3^0}$ respectively. We can see that most of the samples can be checked at 5$\sigma$ level when the mass difference between LSP $\tilde{\chi}_1^0$ and NLSPs $\tilde{\chi}_i^\pm (i = 2,3)$ is sufficient. However, there are still some samples can not be checked at level above 3 or 5 sigma. The main reasons is that the mass spectra is compacted, so that the leptons from the decay of NLSPs are very soft. Because $P_T$ cut has to be very large at the LHC due to the large background, detecting soft particles is not easy. Combining with Fig.2 and 3, we can learn the following facts:

- If $\tilde{\chi}_i^\pm (i = 2,3)$ decays to a virtual vector boson, the area over the dash line in all the planes, the signal significance is less than 5$\sigma$, and it is hard to check at the LHC with 300 fb$^{-1}$ data.
- If $\tilde{\chi}_i^\pm$ or $\tilde{\chi}_i^0 (i = 2,3)$ decays to a real vector boson, the area between the dash and dotted line in left and middle the planes, the signal significance can be larger than 5$\sigma$, and it is easy to check at the LHC with 300 fb$^{-1}$ data.
The dash, dotted and dash dotted lines indicate \( m_{\chi_{i}^0} \) versus \( m_{\chi_{i}^\pm} \) (left), \( m_{\chi_{i}^0} \) versus \( m_{\chi_{2}^0} \) (middle), \( m_{\chi_{i}^0} \) versus \( m_{\chi_{3}^0} \) (right) planes. The colors indicates the signal significance, where red represents \( ss < 3\sigma \), green represents \( 3\sigma < ss < 5\sigma \), and gray represents \( 5\sigma < ss \). In the left plane, the dash line indicates the mass difference equal to \( m_W \), \( m_{\chi_2^0} - m_{\chi_1^0} = m_W \). In the middle and right planes, the dash line and dotted line indicate the mass difference equal to \( m_Z \) and \( m_{h_2} \) respectively, that is, \( m_{\chi_{i}^0} - m_{\chi_{1}^0} = m_Z \) and \( m_{\chi_{i}^0} - m_{\chi_{3}^0} = m_{h_2} \), where \( i = 2, 3 \) for the middle and right planes respectively.

The \( \tilde{\chi}_i^0 \) decay to a light Higgs \( h_1 \), the area between the dash and dotted line in the right plane, the signal significance is less than \( 5\sigma \) for some samples. The reason is that the decay width of light Higgs \( h_1 \) has been constrained by many experiments, so it's hard to detect it.

If \( \tilde{\chi}_i^0 \) (\( i = 2, 3 \)) decays to a SM-Like Higgs, the area under the dotted line in middle and right planes, it is also have \( ss > 5\sigma \) for most samples.

For the samples with insufficient mass difference between the NLSPs and the LSP, the integrate luminosity at 300 fb\(^{-1}\) is not enough. So we also tried to increase the luminosity to 3000 fb\(^{-1}\), the result is that almost all samples can be checked with \( ss > 5 \) at 3000 fb\(^{-1}\).
In Fig.5, we show ss on the planes of \(m_{\tilde{\chi}_0^0}\) versus \(m_{a_1}\). We can see that there are mainly two mechanisms for dark matter annihilation: the \(a_1\) funnel where \(2m_{\tilde{\chi}_0^0} \simeq m_{a_1}\), and the \(h_2/Z\) funnel where \(2m_{\tilde{\chi}_0^0} \simeq m_{h_2}\) or \(2m_{\tilde{\chi}_0^0} \simeq m_Z\). Unfortunately, searching for the NLSPs is helpless to distinguish these two mechanisms.

4 Conclusions

In this work, we have discussed the light higgsino-dominated NLSPs in the scNMSSM, which is also called the non-universal Higgs mass (NUHM) version of the NMSSM. We use the scenario of singlino-dominated \(\tilde{\chi}_1^0\) and SM-like \(h_2\) in the scan result in our former work on scNMSSM, where we considered the constraints including theoretical constraints of vacuum stability and Landau pole, experimental constraints of Higgs data, muon g-2, B physics, dark matter relic density and direct searches, etc. In our scenario, the bino and wino are heavy because the high mass bound of gluino and the unification of gaugino masses at GUT scale. Thus the \(\tilde{\chi}_1^\pm\) and \(\tilde{\chi}_{2,3}^0\) are higgsino-dominated and mass-degenerated NLSPs.

We first investigate the constraints to these light higgsino-dominated NLSPs, including searching for SUSY particle at the LHC Run-I and Run-II. We use Monte Carlo to do the detailed simulations to add these constraints from search SUSY particles at LHC. Then we discuss the possibility of discovering the higgsino-dominated NLSPs at the HL-LHC in the future. We use the same analysis by increasing the integrate luminosity to 300 \(\text{fb}^{-1}\) and 3000 \(\text{fb}^{-1}\).

Finally, we come to the following conclusions regarding the higgsino-dominated 100 \(\sim\) 200 GeV NLSPs in scNMSSM or NUHM-NMSSM:

- Among the search results for electrowekinos, the ‘multi-lepton final state’ constrain our scenario most, and can exclude some of our samples. While up to now, the search results by Atlas and CMS with Run I and Run II data at the LHC can still not exclude the light higgsino-dominated NLSPs of 100 \(\sim\) 200 GeV.

- When the mass difference with \(\tilde{\chi}_1^0\) is smaller than \(m_{h_2}\), \(\tilde{\chi}_2^0\) and \(\tilde{\chi}_3^0\) have different preference on decaying to \(Z/Z^*\) or \(h_1\).

- The best channels to detect the NLSPs are though the real two-body decay \(\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0W\) and \(\tilde{\chi}_{2,3}^0 \rightarrow \tilde{\chi}_1^0Z/h_2\). When the mass difference is sufficient, most of the samples can be checked at 5 \(\sigma\) level with future 300 \(\text{fb}^{-1}\) data at the LHC. While with 3000 \(\text{fb}^{-1}\) data at the LHC, nearly all of the samples can be checked at 5\(\sigma\) level even if the mass difference is insufficient.

- The \(a_1\) funnel and the \(h_2/Z\) funnel are the mainly two mechanisms for the singlino-dominated LSP annihilation, which can not be distinguished by searching for NLSPs.
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