Does theory of quantum correction to conductivity agree with experimental data in 2D systems?

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The quantum corrections to the conductivity have been studied in the two types of 2D heterostructures: with doped quantum well and doped barriers. The consistent analysis shows that in the structures where electrons occupy the states in quantum well only, all the temperature and magnetic field dependences of the components of resistivity tensor are well described by the theories of quantum corrections. Contribution of the electron-electron interaction to the conductivity has been determined reliably for the structures with different electron density. A possible reason of large scatter in experimental data relating to the contribution of electron-electron interaction, obtained in previous papers, and the role of the carriers, occupied the states of the doped layers, is discussed.

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I. INTRODUCTION

The quantum corrections to the Drude conductivity in disordered metals and doped semiconductors are intensively studied since 1980.1,2 Two mechanisms lead to these corrections: (i) the interference of the electron waves propagating in opposite directions along closed paths; (ii) the electron-electron interaction (EEI). These corrections increase with decreasing temperature and/or increasing disorder, and the low temperature transport in 2D systems is largely determined by those. Recently, two different behaviors of the conductivity of 2D system with decrease of temperature $T$ have come to light: (i) the conductivity decreases monotonically for one type of systems; (ii) the conductivity decreases at sufficiently high temperature, but reveals surprising growth at low enough temperature for other ones.3

It is commonly accepted that decrease of the conductivity for the first type of systems results from temperature dependence of the quantum corrections that are negative and logarithmically diverge at $T \to 0$. In such systems the crossover from the weak localization (WL) regime, when the corrections are small compared with the Drude conductivity, to the strong localization (SL) regime is observed. The role of interference and electron-electron interaction in this crossover is of special interest and attracts much attention in recent years.

As for the second type of the 2D systems, no general consensus on the origin of metallic behavior has been reached as yet. The study of the role of the EEI and interference can be useful for understanding the origin of the metallic-like behavior of conductivity in such systems.

The WL-SL crossover was intensively studied in thin metal films. It is generally assumed that the EEI has a crucial role because the interference is suppressed by the strong spin-orbit interaction in metals. Different aspects of the WL-SL crossover were studied in semiconductor 2D structures but there is no conventional view on the role of EEI and interference in this crossover up to now. Moreover, the magnitudes of the EEI and interference corrections to the conductivity are not well established experimentally in the WL regime, when theories of the quantum corrections are applicable. It is especially concerns the EEI contribution. It was shown in Ref. 1,4 that the EEI contribute to $\sigma_{xx}$ only. For $g\mu_B B/kT \lesssim 1$ the correction has the form

$$\Delta \sigma_{xx}^{ee} = G_0 \left( 1 + \frac{3}{4} \lambda \right) \ln \left( \frac{kT \tau}{\hbar} \right),$$

whereas for $g\mu_B B/kT \gg 1$ it is given by

$$\Delta \sigma_{xx}^{ee} = G_0 \left( 1 + \frac{1}{4} \lambda \right) \ln \left( \frac{kT \tau}{\hbar} \right).$$

Here, $G_0 = e^2/(2\pi^2\hbar)$, $\tau$ is the momentum relaxation time, $\lambda$ is a function of $k_F/K$ with $k_F$ as the Fermi quasimomentum and $K$ as the screening parameter, which for 2D case is equal to $2/a_B$, where $a_B$ is the effective Bohr radius. For the most intensively studied Si MOS-structures, the transition to the case $g\mu_B B/kT \gg 1$ occurs within the actual range of temperature and magnetic field. It results in complicated magnetic field dependence of the resistance:
decreasing at low magnetic field it increases at high ones. This makes the quantitative interpretation of experimental data difficult.

For electron 2D systems based on GaAs, in which the electron $g$-factor is much smaller than that in Si, the condition $g\mu_B B/kT \lesssim 1$ is fulfilled in wide range of temperature and magnetic field except extremely low temperature or high magnetic field. Therefore, the experimental results can be interpreted in the most simple way for these systems. The multiplier before logarithm in Eq. (1) is determined experimentally and just its value is shown in Fig. 1 as function of $k_F/K$. Theoretical curve from Ref. 5 is also shown in the figure. The large scatter of the experimental data shows that there are no reliable data on the contribution of the EEI and it is impossible to conclude whether the theory describes the experiment.

From our point of view, before the discussion of the very interesting problem concerning the role of the EEI in WL-SL crossover and in the new low temperature metallic phase, it would be essential to acquire reliable data in the WL regime far from the WL-SL crossover. Just this problem our paper is devoted to.

Let us discuss what type of semiconductor heterostructures would be the most suitable for quantitative study of the quantum corrections to the conductivity in the 2D systems. First of all, the Drude conductivity $\sigma_0$ should be high: $\sigma_0/(\pi G_0) = k_F l \gg 1$, where $l$ is mean-free path. In this case the WL-theory can be applied. On the other hand, the quantum corrections must not be very small, lest the measurement accuracy restricts the quantitative analysis. This means that the electron scattering must be strong enough, i.e., the mobility must not be high. It should be the single-quantum-well heterostructure with a single subband occupied, as the theories of quantum corrections have been developed mainly for such case. Quantum well should be symmetric in shape. It allows to eliminate the peculiarities caused by the spin-orbit interaction\textsuperscript{6,7} and to neglect the spin effects under analysis of experimental data. It should be the structure with electrons in quantum well only, i.e. with empty doped (modulation- or $\delta$-) layer. It enables one to avoid the shunting of the quantum well. The role of the carriers in doped layers is not restricted by shunting. Their redistribution within the layers with temperature can lead to the temperature-dependent disorder, and, hence, to additional temperature dependence of the mobility.\textsuperscript{12}

Thus, two types of structures meet these requirements: (i) the structures with doped quantum well; (ii) the structures with symmetrically doped barriers and low carrier density, when the Fermi level lies significantly lower than any states in doped layers. Exactly these types of structures was studied in this work.

FIG. 1: The value of multiplier $(1 + 3/4\lambda)$ in Eq. (2) as function of $k_F/K$. Symbols are the experimental results from Refs. 8 (□), 9 (○), 10(□), 11(○) , and our data (●, ⊗). The solid curve represents the theoretical result from Ref. 5, dotted line is the guide for an eye. Arrow indicates the shift of experimental point for structure 4 after extraction of temperature dependence of electron mobility (see Section VII).
II. THEORETICAL BASIS

In this section we present the main theoretical results which will be used in the analysis of the experimental data. These theories are valid when $k_F l \gg 1$ and the quantum corrections to the conductivity are small compared with the Drude conductivity.

Without an external magnetic field the total quantum correction to conductivity is

$$\delta \sigma(T) = G_0 \left[ \ln \left( \frac{\tau}{\tau_\varphi(T)} \right) + \left( 1 + \frac{3}{4} \lambda \right) \ln \left( \frac{kT \tau}{\hbar} \right) \right], \quad (3)$$

where $\tau_\varphi$ is the phase-breaking time. The first term in Eq. (3) is the interference correction, the second one is the EEI contribution. At low temperatures the phase-breaking time is determined by inelasticity of the electron-electron interaction and is

$$\tau_\varphi^{-1} = \frac{kT 2\pi G_0}{\hbar} \frac{\sigma_0}{\sigma_0} \ln \left( \frac{\sigma_0}{2\pi G_0} \right). \quad (4)$$

The value of $\lambda$ was obtained in Ref. 5

$$\lambda = 4 \left[ 1 - 2 \left( 1 + \frac{F}{2} \right) \ln \left( 1 + \frac{F}{2} \right) \right], \quad (5)$$

where

$$F = \int \frac{d\theta}{2\pi} \left[ 1 + \frac{2k_F}{\mathcal{K}} \sin \frac{\theta}{2} \right]^{-1}. \quad (6)$$

In a magnetic field the classical conductivity tensor has the following form:

$$\sigma_{xx}^0 = \frac{e n \mu}{1 + \mu^2 B^2}, \quad (7)$$

$$\sigma_{xy}^0 = \frac{e n \mu^2 B}{1 + \mu^2 B^2}. \quad (8)$$

The electron-electron interaction contributes to $\sigma_{xx}$ only [see Eqs. (1) and (2) for $\Delta \sigma_{xx}^{ee}$], whereas $\Delta \sigma_{xy}^{ee} = 0$. It is easy to show that the magnetoresistance

$$\rho_{xx}(B, T) = \frac{\sigma_{xx}^0(B) + \Delta \sigma_{xx}^{ee}(T)}{(\sigma_{xy}^0(B))^2 + (\sigma_{xx}^0(B) + \Delta \sigma_{xx}^{ee}(T))^2} \quad (9)$$

is parabolic in the form when $\Delta \sigma_{xx}^{ee} \ll \sigma_{xx}^0$:

$$\rho_{xx}(B, T) \approx \frac{1}{\sigma_0^0} - \frac{1}{\sigma_0^0} \left( 1 - \mu^2 B^2 \right) \Delta \sigma_{xx}^{ee}(T). \quad (10)$$

So, $\rho_{xx}$-versus-$B$ curves for different temperatures should cross one another at fixed point $B_{cr} = 1/\mu$ and the value of $\rho_{xx}^{-1}(B_{cr})$ should be equal to the Drude conductivity.

The interference correction to the conductivity gives the contributions both to $\sigma_{xx}$ and $\sigma_{xy}$ but their ratio is such that $\rho_{xy}$ remains unchanged. Within the framework of the diffusion approximation, which is valid when $\tau_\varphi/\tau \gg 1$ and $B < B_{cr} = \hbar c/(2e l^2)$, the magnetic field dependence of $\Delta(1/\rho_{xx}^{int}) = 1/\rho_{xx}(B) - 1/\rho(0)$ is described by the well-known expression

$$\Delta(1/\rho_{xx}^{int}(B)) = \alpha G_0 \left\{ \psi \left( \frac{1}{2} + \frac{\tau}{\tau_\varphi} \frac{B_{tr}}{B} \right) - \psi \left( \frac{1}{2} + \frac{B_{tr}}{B} \right) - \ln \left( \frac{\tau}{\tau_\varphi} \right) \right\}, \quad (11)$$

where $\psi(x)$ is a digamma function, the value of $\alpha$ is equal to unity. The magnetic field dependence of the interference correction beyond the diffusion approximation was studied in Refs. 14,15,16,17,18,19. The analytical expression suitable for fitting of the experimental data was not obtained for this case, however, as is shown in Ref. 19, Eq. (11) also describes the $\Delta(1/\rho_{xx}^{int})$-versus-$B$ curve well but with prefactor $\alpha < 1$.

It follows from Eqs. (10) and (11) that the interference correction gives the strong magnetic field dependence of the resistivity at $B \leq B_{tr}$, whereas the EEI does it at magnetic field $B \geq B_{cr} = 1/\mu$. Since the ratio $B_{cr}/B_{tr}$ is equal to $2k_FL$, these magnetic field ranges are well separated. Thus, application of magnetic field allows to obtain the interference and interaction contributions to the conductivity separately.

To answer the question “Does the theory of quantum corrections agree with experiment in 2D systems?”, all the theoretical predictions given above should be checked step by step.
FIG. 2: Magnetic field dependences of $\rho_{xx}$ (a), $\tilde{\rho}_{xx}$ (b) and $\rho_{xy}$ (lines), $\tilde{\rho}_{xy}$ (symbols) (c) for different temperatures for structure 2.

FIG. 3: Temperature dependence of $\sigma_{xx}$ (a) and $\sigma_{xy}$ (b) for two magnetic fields for structure 2.

### III. SAMPLES

The heterostructures with 50Å In$_{0.15}$Ga$_{0.85}$As single quantum well in GaAs with Si $\delta$-doping layers were investigated. Two types of heterostructures were studied: with doped quantum well (structures 1 and 2), and with doped barrier (structures 3 and 4). In the first case $\delta$-layer was arranged in the center of quantum well. In the second one, two $\delta$-layers, separated by the 60Å GaAs spacer, were disposed on each side of the quantum well. The thickness of undoped GaAs cap layer was 3000 Å for all structures. The samples were mesa etched into standard Hall bridges. The parameters of the structures are presented in Table I.

| Str. | $\sigma_0$ (10$^{-4}$ Ohm$^{-1}$) | $n$ (10$^{12}$ cm$^{-2}$) | $\tau$ (10$^{-11}$ sec) | $B_{tr}$(T) | $B_{cr}$(T) | $\rho_{xx}^{-1}$($B_{cr}$)(Ohm$^{-1}$) | $\mu^{-1}$a(T) |
|------|---------------------------------|-------------------------|--------------------------|-------------|-------------|---------------------------------|---------------|
| 1    | (4.13 ± 0.02)                   | 1.35 ± 0.05             | 6.5                      | 0.25        | 10.7        | 3.95 × 10$^{-4}$                | 5.26          |
| 2    | (3.55 ± 0.03)                   | 0.87 ± 0.02             | 8.8                      | 0.21        | 9.2         | 4.15 × 10$^{-4}$                | 4.17          |
| 3    | (1.90 ± 0.05)                   | 0.19 ± 0.02             | 21.0                     | 0.16        | 4.9         | 1.67 × 10$^{-4}$                | 1.64          |
| 4    | (6.50 ± 0.05)                   | 1.0 ± 0.05              | 13.7                     | 0.076       | 16.6        | 6.45 × 10$^{-4}$                | 2.50          |

$^a$The value of mobility has been determined as $\mu = \rho_{xy}/(\rho_{xx}B)$ at $B = B_{cr}$. 
FIG. 4: The magnetic field dependence of $\Delta(1/\rho_{xx}(B))$ for structure 2, $T = 0.45$ K. Symbols are the experimental data. Lines are best fit to Eq. (11) made over different magnetic field ranges: $\Delta B = (0 - 0.1)B_{tr}$ (upper line), and $\Delta B = (0 - 0.3)B_{tr}$ (lower line).

IV. TEMPERATURE DEPENDENCE OF CONDUCTIVITY AT HIGH MAGNETIC FIELD. CONTRIBUTION OF ELECTRON-ELECTRON INTERACTION.

The experimental magnetic field dependences of $\rho_{xx}$ and $\rho_{xy}$ are presented in Fig. 2(a), (c) for one of the structures at different temperatures. The two different magnetic field ranges are evident: the range of sharp dependence of $\rho_{xx}$ at low field $B \leq 0.5 - 1$ T, and the range of moderate dependence, which is close to parabolic, at higher field. All $\rho_{xx}$-vs-$B$ curves cross each other at fixed magnetic field $B_{cr} = 4.15$ T. This value is close to $1/\mu$ (see Table I).

The Hall resistance is practically linear with magnetic field. However, despite the strong degeneracy of electron gas ($E_F/(kT) > 100$, where $E_F$ is the Fermi energy) the Hall resistance decreases with increasing temperature. Low-magnetic-field behavior of $\rho_{xx}$ is a consequence of suppression of the interference correction by magnetic field. This effect will be discussed below.

At high magnetic field ($B > 1 - 2$ T), $\rho_{xx}(B,T)$ and $\rho_{xy}(B,T)$ differ from the classical behavior following from Eqs. (7) and (8), by contribution of electron-electron interaction only. To assure in this fact we plot the temperature dependences of the conductivity tensor components in Fig. 3. It is clearly seen that the change of $\sigma_{xx}$ with temperature does not depend on $B$ and significantly larger than that of $\sigma_{xy}$. Namely such a behavior is in full agreement with the predictions of the EEI theory. Thus, the absence of $\sigma_{xy}$ temperature dependence allows us to attribute the $\sigma_{xx}$ temperature dependence to contribution of the EEI and determine the multiplier before logarithm in Eq. (1) from the slope of $\sigma_{xx}$-vs-$\ln T$ dependence: $(1 + 3/4\lambda) = 0.5 \pm 0.1$ (see Fig. 3(a)). Notice that practically in all papers where the EEI was studied, $\sigma_{xy}$ temperature independence was not demonstrated over the magnetic field range where the EEI contribution was determined. Below we show that the existence of $\sigma_{xy}$ temperature dependence introduces a large error into the determination of $(1 + 3/4\lambda)$.

Now when we have determined the EEI contribution let us extract it from $\sigma_{xx}$, invert the conductivity tensor and plot the components $\tilde{\rho}_{xx}$ and $\tilde{\rho}_{xy}$ without this correction (Fig. 2 (b), (c)). Disappearance of the temperature dependence of $\tilde{\rho}_{xy}$ and $\tilde{\rho}_{xx}$ confirms the correctness of determination of the EEI contribution to the conductivity, and absence of any mechanisms that can lead to additional temperature dependence of the conductivity.

Note, that after extraction of the EEI contribution the electron density determined by the different ways: (i) $B/(e\tilde{\rho}_{xy})$; (ii) $B_{cr}/(e\rho_{xx}(B_{cr}))$; and (iii) from the Shubnikov - de Haas oscillations, are very close to each other and lie within the error interval given in Table I.

V. THE LOW FIELD MAGNETORESISTANCE. INTERFERENCE CORRECTION TO THE CONDUCTIVITY

Let us consider the low magnetic field range. The temperature dependence of $\rho_{xx}$ in this range is determined by both the EEI and interference contributions whereas the magnetic field dependence is determined by interference
FIG. 5: Temperature dependence of $\tau_\varphi$. Symbols are the results of fitting of the experimental data to Eq. (11) for different fitting range $\Delta B$: $\Delta B = (0 - 0.1)B_{tr}$ (full circles), and $\Delta B = (0 - 0.3)B_{tr}$ (open circles).

FIG. 6: (a) The dependence $\Delta \sigma(T) = \sigma(T) - \sigma(T_0)$ with $T_0 = 0.49$ K. Symbols are the experimental data, lines are given by Eq. (12) with $(1 + 3/4\lambda) = 0.5$ and $p = 1$ (dashed line), $p = 0.85$ (dotted line). (b) The temperature dependence of absolute value of total quantum correction to the conductivity determined by different ways (see text).

contribution only, because $\rho_{xx}(B)$ is unaffected by the EEI at $B \ll 1/\mu$. Thus, the dependence of $\Delta(1/\rho_{xx}(B))$ must be described by Eq. (11) over this magnetic field range and one can determine the value of phase-breaking time using $\alpha$ and $\tau_\varphi$ as fitting parameters. The low-field magnetoresistance for structure 2 is presented in Fig. 4. Detailed analysis of the dependences $\Delta(1/\rho_{xx}(B))$ shows that the fitting values of $\alpha$ do not depend on the temperature but to some extent depend on the fitting range $\Delta B$: $\alpha = 1.2$ at $\Delta B = (0 - 0.1)B_{tr}$, and $\alpha = 0.9$ at $\Delta B = (0 - 0.3)B_{tr}$ (Fig. 5). The temperature dependences of $\tau_\varphi$ are close to $T^{-p}$ with $p \simeq 0.85^{20}$ for both fitting ranges but absolute values of $\tau_\varphi$ are somewhat different. Theoretical dependences $\tau_\varphi(T)$ calculated in accordance with Eq. (4) are also shown in Fig. 5. It is seen that magnitudes of $\tau_\varphi$ are close to theoretical values. Notice that close to linear $\tau_\varphi$-vs-$1/T$ dependence was observed in most papers but the magnitude often happened less as much as $3 - 5$ times. The reasons for that are unclear and one ought to suppose that additional phase-breaking mechanisms with close temperature dependence are essential in such structures. Therefore, the quantitative results for quantum correction, obtained for such structures seems to be inconclusive.
FIG. 7: The magnetic field dependence of the interference quantum correction \( \delta \sigma_{\text{int}} \) over the entire magnetic field range for structure 2, \( T = 1.5 \) K. The shadowed area is the experimental result, spread is caused by error in determination of \( \sigma_0 \) (see Section VI). The dashed line is the result of the diffusion approximation given by Eq. (11), the dot-dashed line takes into account both the back-scattering and non-back-scattering processes, the dotted line represents only the back-scattering contribution.\(^{17}\)

VI. TEMPERATURE DEPENDENCE OF THE CONDUCTIVITY AT B=0. ABSOLUTE VALUE OF THE QUANTUM CORRECTIONS

We turn now to temperature dependence of the conductivity at \( B = 0 \) (Fig. 6). It is determined by the temperature dependence of both the interference correction and correction due to the EEI. As seen from Eqs. (3) and (4), the relative value of \( \sigma(T) \) has to decrease logarithmically with decreasing temperature

\[
\Delta \sigma(T) = \sigma(T) - \sigma(T_0) = G_0 \left[ p \ln \left( \frac{T}{T_0} \right) + \left( 1 + \frac{3}{4} \lambda \right) \ln \left( \frac{T}{T_0} \right) \right],
\]

where \( T_0 \) is some arbitrary temperature. Thus, the slope in \( \Delta \sigma \text{-vs-} \ln T \) dependence has to be equal to \( G_0 \left[ p + (1 + 3/4\lambda) \right] \). Lines in Fig. 6(a) show the dependences (12) calculated with \( (1 + 3/4\lambda) = 0.5 \) determined above (see Section IV) and with two values of \( p \): with theoretical value \( p = 1 \) [see Eq. (4)], and \( p = 0.85 \) describing the experimental \( \tau_\sigma(T) \) dependence (see Fig. 5). It is evident that the experimental results coincide with these dependences within experimental error.

Now let us consider the absolute value of the total quantum correction \( \delta \sigma \). On the one hand, we can find it from Eq. (3), using the parameters \( \tau_\sigma \), \( (1 + 3/4\lambda) \) determined above, and \( \tau = \mu \rho / e \), where \( m = 0.06 m_0 \) is the electron effective mass in \( In_{0.15}Ga_{0.85} \). As quantum well. This values are plotted in Fig. 6(b) with open circles. On the other hand, the absolute value of the quantum corrections at given \( T \) is equal to the difference between \( \sigma(T) \) and the Drude conductivity: \( \delta \sigma(T) = \sigma(T) - \sigma_0 \). This value obtained with \( 1/\rho_{xx}(B_{cr}) \) as \( \sigma_0 \) is represented by solid circles. As is seen these plots are parallel to each other, but differ by the value about \( (1.3 \pm 0.3)G_0 \).

What is the reason for noticeable difference between the absolute values of quantum correction, obtained by different ways? When evaluating the Drude conductivity we supposed that at \( B_{cr} \) the interference contribution was fully suppressed by magnetic field. In fact this correction does not equal to zero even at \( B_{cr} \gg B_{tr} \), because at \( B \gg B_{tr} \) it decreases with increasing magnetic field very slowly.\(^{14,15,17,19}\) Therefore, it is naturally to associate the difference in Fig. 6(b) with residual interference contribution to the conductivity at \( B = B_{cr} \). Thus, the proper values of the total quantum correction are represented in Fig. 6(b) by open circles, and the Drude conductivity should be more correctly estimated as \( \sigma_0 \approx \rho_{xx}^{-1}(B_{cr}) + (1.3 \pm 0.3)G_0 \) (see Table I). Note, that the presence of some interference correction at large magnetic field does not affect the determination of \( (1 + 3/4\lambda) \) in Section IV because at \( B \gg B_{tr} \) the interference correction is practically temperature independent.

After we have found the Drude conductivity and the EEI contribution, we can obtain the interference correction to the conductivity over entire magnetic field range as

\[
\delta \sigma_{\text{int}}(B) = \frac{(\sigma_{xx} - \Delta \sigma_{xx}^{ee})^2 + \sigma_{xy}^2}{\sigma_{xx} - \Delta \sigma_{xx}^{ee}} - \sigma_0.
\]

In Figure 7 the magnetic field dependence of \( \delta \sigma_{\text{int}} \) is presented together with theoretical dependences. One can see
FIG. 8: Magnetic field dependences of $\rho_{xx}$ at different temperatures as they have been measured (a), and those after extraction of the temperature dependence of mobility (b), structure 4.

that $\delta \sigma_{\text{int}}(B)$ calculated from Eq. (11) as

$$\delta \sigma_{\text{int}}(B) = \Delta(1/\rho_{xx}^\text{int}(B)) - \Delta(1/\rho_{xx}^\text{int}(\infty))$$

well describes the magnetoresistance at low magnetic field (see also Fig. 4), but significantly deviates at $B > 0.1$ T. It is not surprising because Eq. (11) was obtained within the diffusion approximation which is valid at $B < B_{tr}$ ($B_{tr} = 0.21$ T for this structure). The dependences $\delta \sigma_{\text{int}}(B)$ obtained beyond the diffusion approximation for back-scattering processes and those taking into account non-back-scattering processes are presented also. One can see that the experimental data lie closely to the curves obtained beyond the diffusion approximation. However, our results do not allow to judge the role of non-back-scattering processes.

VII. DISCUSSION

As shown in previous sections, all the temperature and magnetic field dependences for structure 2 are consistently described by the theory of quantum corrections. Namely: (i) for $B \gg B_{tr}$, the temperature dependence of $\sigma_{xx}$ is logarithmic, whereas the temperature dependence of $\sigma_{xy}$ is negligible; (ii) the low-field magnetoresistance is well described by the weak-localization theory with the value and temperature dependence of $\tau_{\varphi}$ close to the theoretical ones; (iii) the temperature dependence of the conductivity at $B = 0$ is logarithmic and quantitatively described by the interference and EEI contributions determined experimentally from the analysis of low and high magnetic field magnetoresistance, respectively.

Above we have analyzed in detail the results obtained for structure 2. Similar accordance with the theoretical predictions has been observed for structures 1 and 3. It allows to determine the EEI contribution to the conductivity and the values of $(1 + 3/4\lambda)$ for different $2k_F/K$ values (see Fig. 1). The results for these three structures are seen to fall on the curve, which is close in shape to the theoretical one, but lies somewhat lower.

What is the possible reason for large scatter in the results obtained in other papers (see Fig. 1)? To understand this let us analyze the results for structure 4. It is the structure with $\delta$-doped barriers, like structure 3, but with higher electron density (see Table I). At the first sight the magnetic field dependences of $\rho_{xx}$ and $\rho_{xy}$ at different temperatures (see Fig. 8(a)) are similar to those for structure 2 (Fig. 2), but unlike structure 2, the value of $B_{tr}$ is much greater than $1/\mu$. Moreover, the significant temperature dependence of $\sigma_{xy}$ is evident at high magnetic field (see Fig. 9). Such behavior of $\sigma_{xy}$ is in conflict with the theoretical prediction for the EEI correction, but yet if one uses the slope of $\sigma_{xx}$-vs-ln $T$ dependence at high magnetic field for evaluation of the EEI contribution, we obtain the value of $(1 + 3/4\lambda)$ about 1.1 that is much greater than that for other structures. Notice, the temperature dependence of $\sigma$ at $B = 0$ for this structure remains logarithmic, but with the slope about 2.4 that is essentially greater than the slope for structures 1−3: $p + (1 + 3/4\lambda) = 1.35 - 1.5$ (see Fig. 6).

The temperature dependence of $\sigma_{xy}$ at high magnetic field in structure 4 seems to be incomprehensible. The interference correction does not depend on temperature at $B \simeq 10B_{tr}$. The EEI does not affect $\sigma_{xy}$. Finally, the classical part $\sigma_{xy}^0$ is temperature independent at such strong degeneracy of electron gas ($E_F/kT > 100$).
It should be noted that in this structure some fraction of the electrons occupies the states in $\delta$-doped layers in contrast to other structures investigated. Analysis of the magnetic field dependences of $\rho_{xx}$ and $\rho_{xy}$ from the point of view of two-types of carriers shows that these electrons do not contribute to the conductivity of the structure. However, at temperature change the redistribution of the electrons within the $\delta$-layers can lead to the temperature dependent disorder and, hence, to the temperature dependence of the mobility of electrons in the quantum well. Estimations show that as low as 1% increase of the mobility with increasing temperature is enough to cause the temperature dependence of $\sigma_{xy}$ observed experimentally. If we extract this 1%-changing from $\sigma_{xx}$ and $\sigma_{xy}$, all the results for this structure will be in accordance with the theoretical predictions, as for structures 1–3: the value of $B_{cr}$ will coincide with $1/\mu$ (Fig. 8(b)); the value of $(1 + 3/4\lambda)$ will be equal to $(0.40 \pm 0.05)$; and the slope in temperature dependence of $\sigma$ at $B = 0$ will be equal to $(1.5 \pm 0.2)$.

It should be noted that only in Ref. 11 the temperature independence of $\sigma_{xy}$ in high magnetic field was demonstrated, and, as seen in Figure 1 these results accord well with our data. Thus, the results for structure 4 show that existence of carriers in doped layers can bring about additional temperature dependence of the mobility, and this dependence should be taken into account when the parameters of the electron-electron interaction are determined in such type of structures. From our point of view, disregard of this fact can be one of the reasons for large scatter of results obtained by other authors (Fig. 1).

**VIII. CONCLUSION**

We have studied the quantum corrections to the conductivity in the two types of 2D structures: with doped quantum well and with doped barriers. Successive analysis of experimental data for the structures where electrons occupy the states in the quantum well only, shows that all the results are self-consistently described by the theory of quantum corrections. This allows us to determine the reliable value of the EEI contribution to the conductivity and its $k_F$-dependence. It has been shown that the existence of carriers in doped layers can lead to the temperature dependent mobility in the structures even at low temperature, when the electron gas in quantum well is strongly degenerated.

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13. Recently, the paper on classical mechanism of negative magnetoresistance in two dimensions has appeared [Alexander Dmitriev, Michel Dyakonov and Remi Jullien, cond-mat/0103490]. Estimations for our structures show that the negative magnetoresistance due to this mechanism is no more than $0.3 - 0.4 \mu_0$ at magnetic field $B = \mu^{-1}$. As result $\rho_{xx}(B_{cr})$ differs from the Drude conductivity on the same value. It is within the limits of our experimental error. Moreover, this mechanism is temperature independent, therefore taking it into account does not change the values of parameters which are obtained from the temperature dependences.

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