Dynamics of large-scale instabilities in conductors electrically exploded in strong magnetic fields

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Abstract. The growth of large-scale instabilities during the propagation of a nonlinear magnetic diffusion wave through a conductor was studied experimentally. The experiment was carried out using the MIG terawatt pulsed power generator at a peak current up to 2.5 MA with 100 ns rise time. It was observed that instabilities with a wavelength of 150 µm developed on the surface of the conductor hollow part within 160 ns after the onset of current flow, whereas the surface of the solid rod remained almost unperturbed. A system of equations describing the propagation of a nonlinear diffusion wave through a conductor and the growth of thermal instabilities has been solved numerically. It has been revealed that the development of large-scale instabilities is obviously related to the propagation of a nonlinear magnetic diffusion wave.

1. Introduction

Interest in the electrical explosion of conductors (EEC) is related to various applications, such as generation of superstrong magnetic fields, implosion of heavy metal liners, energy transmission using vacuum transmission lines, etc. The current flow in an electrically exploded conductor can occur in three modes: a mode of uniform current distribution in the load, a skin effect mode, and a transient mode. The basic phenomena characteristic of EEC occurring in the skin-effect mode are a shock wave (SW) and a nonlinear magnetic diffusion wave (NMDW) propagating in the conductor, formation of low-temperature plasma at the conductor surface, and thermal instabilities [1, 2, 3]. The nonlinear diffusion of an electromagnetic field into a conductor features anomalously high speed compared to conventional diffusion. The higher diffusion speed is related to a decrease in conductivity of the metal due to its heating by electric current. Nonlinear diffusion takes place in rather high magnetic fields with the induction at the conductor surface reaching several tens of Teslas (for aluminum, the characteristic field $B_0 \approx 27$ T). The upper limit of a magnetic field for the propagation of an NMDW is its value at which plasma generation begins at the conductor surface and, hence, the temperature dependence of conductivity changes in nature. This limiting field is approximately equal to 10 $B_0$. The critical field for a conductor can be estimated by the formula:

$$B_0 = \left( \frac{6 \mu_0 \rho_0 k}{a m_u} \right)^{1/2}$$

(1)
where $\alpha$ is the temperature coefficient of resistance, $k$ is Boltzmann’s constant, $m_a$ is the atomic mass, $\rho_0$ is the density of the conductor, and $\mu_0$ is the permeability of vacuum.

The development of large-scale instabilities that arise in a conductor during the propagation of a nonlinear magnetic diffusion wave was studied experimentally. The experiment was carried out with the use of the MIG terawatt pulsed power generator at a peak current up to 2.5 MA with 100 ns rise time.

2. Experiment
The growth of large-scale instabilities during the propagation of a nonlinear magnetic diffusion wave through a conductor was studied experimentally. The experiment was carried out with the MIG terawatt pulsed power generator at a peak current of up to 2.5 MA with 100 ns rise time. The diagnostic complex of the setup consisted of Rogowski coils, magnetic probes, voltage dividers, vacuum x-ray diodes (XRD) and an HSFC-Pro four-frame optical camera capable of shooting pictures with a minimum exposure time of 3 ns. The generator load was a cylindrical aluminum conductor 3 mm in diameter. With this diameter, the field strength at the conductor surface was not above 10 $B_0$, that is, plasma formation processes were ineffective. The load is sketched in figure 1. It consisted of two parts: a solid rod of external diameter $D = 3$ mm with a cavity of diameter 2.5 mm drilled from the rod end face to a certain depth, so that the solid rod of length 7.5 mm served as a load, and a hollow tube of the same diameter and length with 250 $\mu$m wall thickness.

![Figure 1. Sketch of the load of the MIG generator.](image)

The choice of this design was dictated by the different times it takes for a nonlinear diffusion wave to propagate through a rod and through a hollow tube. We expected that this difference would allow us to elucidate whether an NMDW is related to the growth of large-scale instabilities.

Figure 2 presents pictures of an aluminum load taken in the optical range at different times from the onset of current flow. It can be seen that after the 160th nanosecond, instabilities of wavelength 150 $\mu$m developed on the surface of the hollow part of the conductor, whereas the surface of the solid rod remained in fact unperturbed.
It should be noted that for a 2 mm diameter aluminum conductor of the same design, which was exploded when the magnetic field induction at the conductor surface was above 10$B_0$, surface instabilities occurred after the onset of plasma generation and developed over the hollow and the continuous part of the conductor simultaneously. This can clearly be seen in figure 3.

3. Results and discussion

To interpret the experimental results, a numerical model was used which describes the propagation of a nonlinear magnetic diffusion wave [1, 5]. Nonlinear diffusion of a magnetic field is described by the Maxwell equations written in a quasi-stationary approximation (not taking into account displacement currents) and complemented with Ohm's law and the Joule–Lentz law [1, 4, 5]:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

(2)

$$\mathbf{E} = \mathbf{j}\delta$$

(3)

$$\frac{\partial Q}{\partial t} = \mathbf{j}^2 \delta$$

(4)

where $\mathbf{E}$ is the electric field strength, $\mathbf{B}$ is the magnetic field induction, $\mathbf{j}$ is the current density, $Q$ is the thermal energy density, $\delta$ is the resistivity, and $\mu_0$ is the permeability of vacuum. The temperature dependence of resistivity is given by:
\[ \delta(T) = \delta_0 \left( 1 + \frac{\partial \delta}{\partial T} T \right) \]  

(5)

where \( \delta_0 \) is the resistivity under normal conditions and \( \frac{\partial \delta}{\partial T} \) is the derivative of resistivity with respect to temperature. As \( Q = \rho c_v T \), where \( \rho \) is the density of the conductor and \( c_v \) is its heat capacity at constant volume, the resistivity can be expressed in terms of thermal energy density as:

\[ \delta(Q) = \delta_0 \left( 1 + \beta Q \right) \]  

(6)

where \( \beta = \frac{1}{\rho c_v \delta_0} \frac{\partial \delta}{\partial T} \).

The system of equations (2)–(6) allows us to describe not only the propagation of an NMDW through a conductor, but also the growth of thermal instabilities whose structure is determined by the functional form of the temperature dependence of resistivity. In cases where resistivity increases with temperature, as this takes place for most of the metals in liquid and solid states, thermal instabilities are responsible for the formation of layered structures (strata) with the layers arranged normal to the direction of current flow. In a skin effect mode, the increments of thermal instabilities peak when the NMDW arrives at the boundary of the conductor.

The system of equations (2)–(6) was solved numerically, and the results are presented in figure 4. Curve 1 in this figure depicts the evolution of the magnetic field induction at the conductor surface observed in the experiment; curves 2 and 3 show the time-varied location of the NMDW front in the solid rod and in the hollow conductor, respectively. The location of the wave front was determined by the position of the current density maximum. In the hollow conductor, once the NMDW front had arrived at the internal boundary of the conductor, a reflected NMDW was generated which propagated outward.

As can be seen from figure 4, the reflected nonlinear diffusion wave arrived at the external boundary of the conductor at about the 160th nanosecond.

![Figure 4](image_url)

**Figure 4.** Evolution of the magnetic field inductance at the surface (curve 1) and of the location of the NMDW front in the solid rod (curve 2) and in the hollow conductor (curve 3).
The experiment has shown (see figure 2) that large-scale instabilities (strata) started developing on the conductor surface after the 160th nanosecond; that is, the relation between the growth of instabilities and the propagation of an NMDW is evident.

As thermal instabilities occur preferentially during the propagation of an NMDW through a conductor [1], it can be concluded that the growth of thermal instabilities is the most probable mechanism of strata formation.

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References
[1] Oreshkin V I and Chaikovsky S A 2012 Phys. Plasmas 19 022706
[2] Peterson K J, Sinars D B, Yu E P, Herrmann M C, Cuneo M E, Slutz S A, Smith I C, Atherton B W, Knudson M D and Nakhleh C 2012 Phys. Plasmas 19, 092701
[3] Chaikovsky S A, Oreshkin V I, Mesyats G A, Ratakhin N A, Datsko I M and Kablambaev B A 2009 Phys. Plasmas 16 042701
[4] Shneerson G A 1992 Fields and Transient Processes in High-Current Devices [in Russian], Energoatomizdat, Moscow
[5] Chaikovsky S A, Oreshkin V I, Datsko I M, Labetskaya N A and Ratakhin N A 2014 Phys. Plasmas 21 042706