A dynamic weapon target assignment based on receding horizon strategy by heuristic algorithm

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Abstract. Weapon-target assignment problem is the crucial decision support in Command & Control system. As a typical operational scenario, asset defense has less research on the issue of asset-based dynamic weapon target assignment, and the major models of A-DWTA are challenging in practical application. In this paper, an A-DWTA model is first established by receding horizon decomposition strategy, which closely reflects the operational requirement. Then a heuristic algorithm based on statistical marginal return is proposed to solve the A-DWTA model. Experimental results show that the proposed algorithm for solving the A-DWTA model has advantages of real-time and robustness. The obtained decision plan can complete the operational mission in fewer stages and be adjusted adaptively by model and algorithm parameters.

1. Introduction

Weapon target assignment (WTA), which is also known as Weapon Allocation or Weapon Assignment (WA), refers to the reactive assignment of defensive weapons to counter identified threats [1]. With the development of advanced weapons and combat theory, it is difficult for human decision-makers to counter the fire allocation problem effectively in the complex operational environment [2]. WTA is studied as a critical problem in an intelligent decision support system in order to reduce the decision pressure of human decision-makers or replace them.

WTA problem first was introduced by Manne in 1959 [3]. In the following decades, various types of WTA problems have evolved. From the decision process, WTA problem can be divided into static WTA (SWTA) [4] and dynamic WTA (DWTA) [5][6]. The differences between SWTA and DWTA is whether the time is considered as a dimension. SWTA launches weapons in a salvo to maximize operational effectiveness. DWTA assigns the sequence of weapons for the equilibrium plan during multi-stage [7]. According the operational mission, there are mainly target-based WTA (T-WTA) [8], asset-based WTA (A-WTA) [9] and sensor-WTA (S-WTA) [10][11] model. T-WTA model adopts the kill effectiveness of weapons against targets as the optimization objective. The purpose of A-WTA problem is to maximize the survival values of own assets. S-WTA considers the collaborations between sensors and weapons. This paper focuses on the A-DWTA problem. The operational scenario of A-DWTA is: enemy units launch an attack to destroy the assets with military value but no defensive ability, such as military installations, personnel gathering placing, command and control nodes, weapons warehouses and ports. At this time, the defense unit with combat ability is the decision-maker and is to maximize the assets’ survival value in multiple attack and defense stages by allocating weapon resources to kill the enemy targets.
A-DWTA problem is a more complex WTA problem combining the characteristics of DWTA and A-WTA problems, and the most WTA models can be seen as a special case of A-DWTA model. For example, DWTA model can be considered as the zero asset value of A-DWTA model, and A-WTA model can be considered as one stage model of A-DWTA. Compared with the other WTA problems, A-DWTA problem is less studied and the research frameworks are different. WTA problem is proved to be NP-complete, and the objective function is convex [12]. The exact algorithms solving the WTA model have a "dimension explosion" dilemma, which does not satisfy the real-time requirements in practical applications [13]. Hence the alternate approaches, such as model transformation, heuristics algorithm, are studied to approximate the optimal solution [14]. The swarm intelligence algorithm, which is widely used to solve WTA models, has fewer limitations on constraints, and the computational complexity is less sensitive to the problem scale [15][16].

Motivated by the above research, we described the OODA/A-DWTA loop system, and decomposed the A-DWTA model by receding horizon strategy (A-DWTA/RH). Then a heuristic algorithm based on the statistical marginal return (HA-SMR) is proposed to obtain the optimal plan for decision-maker. The rest of this paper is organized as follows. Section 2 formulates the objective and constraints of A-DWTA decision model. Section 3 presents the proposed HA-SMR for A-DWTA model. Section 4 verifies the proposed HA-SMR solving A-DWTA problem by experimental studies. The conclusion is finally summarized in Section 5.

2. Problem formulation

The notation employed in the context is listed in Table 1.

| Notation | Description |
|----------|-------------|
| \( m \)  | the number of available weapons at the initial stage; |
| \( n \)  | the number of hostile targets at the initial stage; |
| \( l \)  | the number of defense asset at the initial stage; |
| \( s \)  | the index of decision stage, \( s = 1, 2, \ldots, s_m \); |
| \( W(s) = [w_i(s)]_{1 \times m} \) | the weapon state of stage \( s \); |
| \( T(s) = [t_j(s)]_{1 \times n} \) | the target state of stage \( s \); |
| \( A(s) = [a_k(s)]_{1 \times l} \) | the asset state of stage \( s \); |
| \( P(s) = [p_{ij}(s)]_{m \times n} \) | the weapon-target kill probability of stage \( s \), |
| \( Q(s) = [q_{jk}(s)]_{n \times l} \) | the target-asset intention matrix of stage \( s \); |
| \( D(s) = [d_{ij}(s)]_{m \times n} \) | the weapon-target decision variable of stage \( s \); |
| \( C(s) = [c_{ij}(s)]_{m \times n} \) | the weapon-target attack condition of stage \( s \); |
| \( H(s) = [h_j(s)]_{1 \times n} \) | the target survival probability after the act phase of stage \( s \); |
| \( U^u(s) = [u^u_{s}(s)]_{1 \times l} \) | the asset survival probability after the act phase of stage \( s \). |

In the A-DWTA decision model, it is assumed that there are \( s_m \) stages of attack and defense decision-making before the enemy targets are annihilated, or assets are destroyed. In stage \( s \), the problem variables are the destroy effectiveness \( Q \) of enemy targets against assets obtained by threat assessment; the survival value \( V \) of assets; the defensive effectiveness \( P \) of weapons against targets and the attack constraints \( C \) of weapon-target. The output variable is the defense decision matrix \( D \). The research goal of the A-DWTA problem is to solve the decision model through algorithms under the current
situation \( \{V, Q, P, A, T, W, C, D, s\} \), obtain an optimal firepower decision plan \( D^* \), and maximize the effectiveness of reducing enemy targets' threat to assets.

\[
D = \arg \max_{D} F(V, Q, P, A, T, W, C, D, s) \tag{1}
\]

In formula (1), the design of the objective function \( F \) directly reflects the operational purpose of the command and control system, and the constraints \( C \) reflect the battlefield environment. The solving algorithm directly reflects the reasonable degree of firepower decision.

To solve the problem that the standard A-DWTA model pursues the theoretical optimal solution in mathematics and is not tightly integrated with the operational requirements, this section establishes a two-stage A-DWTA model based on Receding Horizon Strategy (A-DWTA/RH).

2.1. Absolute return expectation at the current stage

The absolute return expectation at the current stage can be expressed by the combat effectiveness of the used weapons at the current stage: the survival value expectation of assets under the weapon-target assignment scheme. Let the current stage be the stage \( s \) and the decision variable is \( D(s) \), then the normalized combat effectiveness of the current stage is

\[
f_1 = \frac{1}{\| V \cdot A(s) \|} \sum_{i=1}^{l} v_i a_i(s) \prod_{j=1}^{n} \left( 1 - q_{jk}(s) \right) \prod_{j=1}^{n} \left( 1 - p_y(s) \right)^{d_j(s)} \tag{2}
\]

2.2. Return assessment of remaining weapons

Since the current stage cannot obtain the situation information such as the weapon/target/asset status and target attack intention of the next stage, it is necessary to design the return evaluation method of combat mission under the prediction situation. The designed evaluation method is: due to the event "weapons attack the target," the threat of the target to the asset will be attenuated, and the value expectation of the asset will rise. Therefore, according to the state distribution of the situation information in the next stage, the value expectation of assets under the situation of "threatening target-surviving asset" is calculated first. Then the value expectation under the situation of "surplus weapon-threatening target-surviving asset" is calculated. The difference between the two value expectation can be used as the return expectation of remaining weapons in the prediction situation.

Based on the above ideas, the normalized prediction return of remaining weapons in the next stage is presented as

\[
f_2 = \frac{1}{\| V(s) \|} \sum_{i=1}^{l} \sum_{j=1}^{n} u_i^e(s)v_i(q_{jk})s h_j(s)q_{jk}(s) \left( 1 - \frac{1}{\| W(s) + 1 \|} \sum_{j=1}^{n} p_y(s)w_i(s + 1) \right)^{\| W(s+1) \|} \| T(s) \|} \tag{3}
\]

In summary, the objective function of the A-DWTA/RH model is shown as

\[
\max F(V, Q, P, A, T, W, C, D, s) = \lambda f_1(V, Q, P, A, D, s) + (1 - \lambda) f_2(V, Q, P, W, s) \tag{4}
\]

Where \( \lambda \) is a weight parameter reflecting the decision maker’s preference for \( f_1 \) and \( f_2 \) in the objective function.

2.3. Constraints

In the A-DWTA model, the necessary mathematical constraints include 0-1 integer constraints of decision variables, and total constraints of weapons, as shown below:
\[ d_{ij}(s) \in \{0,1\}, \text{ for } i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; s = 1, 2, \ldots, s_m \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} \leq m, \text{ for } i = 1, 2, \ldots, m \]  

(5)

Besides, model constraints can be designed according to specific DWTA models. Commonly used model constraints include multi-target attack constraints and weapon consumption constraints on single-target.

\[ \sum_{i=1}^{m} d_{ij}(s) \leq m_j, \text{ for } i = 1, 2, \ldots, m; s = 1, 2, \ldots, s_m \]  

\[ \sum_{i=1}^{m} d_{ij}(s) \leq n_j, \text{ for } j = 1, 2, \ldots, n; s = 1, 2, \ldots, s_m \]  

(6)

3. Solving algorithm

Under the current decision \( D \) and situation \( \{V, P, Q, C, A, T, W\} \), the weapon-target marginal return expectation matrix \( E \) of the A-DWTA model is

\[
\begin{align*}
    e_{ij} &= v_{kj} \prod_{j=1}^{n} (1-q_{jk}^{'}h_{i}) \frac{p_{d_{jk}}}{1-q_{jk}} \\
    k_{a} &= \{k | q_{jk} > 0\}, \text{ for } i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \\
    q_{jk}^{'} &= q_{jk} \prod_{j=1}^{n} (1-p_{d_{jk}})q_{jk} \\
\end{align*}
\]

(7)

Where \( e_{ij} \) represents the marginal return expectation of the value of asset \( k_{ja} \) generated by weapon \( i \) attacking target \( j \), and asset \( k_{ja} \) is the attack intention of target \( j \); \( q_{jk}^{'} \) denotes the equivalent kill condition probability of target \( h \) against asset \( k \).

From the perspective of weapon \( w_{i} \), we define the target set whose killing efficiency is higher than \( w_{i} - t_{j} \) as the high-return target set \( T_{p} = \{j | e_{ij} > e_{itp}\} \), and the target set lower than \( w_{i} - t_{j} \) as the low-return target set \( T_{s} = \{j | 0 < e_{ij} < e_{itp}\} \), where the available weapon condition is not satisfied, that is, the weapon of \( e_{ij} = 0 \) Do not enter this set division. So the relative return is designed as

\[ e_{z}(i,t_{p}) = \frac{1}{|T_{p}|} \sqrt{\frac{\sum_{j \in T_{p}} (e_{ij} - e_{itp})^2}{|T_{p}|}}, i = 1, 2, \ldots, m \]  

(8)

The heuristic information pair constructed in this paper is
\[
(g_1, g_2) = \left( e_t \left| T_p \right| + \frac{1}{|T_p|} \sum_{j \in T_s} (e_{ij} - e_{ij})^2 \right)
\]

\[
T_p = \{ j \mid e_{ij} > e_{ij} \}
\]

\[
T_s = \{ j \mid 0 < e_{ij} < e_{ij} \}
\]

(9)

4. Experiment studies

From the application background of A-DWTA, the normalized asset value (NAV) is proposed as the dynamic metric. The weapon consumption and the number of stages are taken as the mission completion metric. Algorithm complexity and compute time are used as the real-time metric. The above metrics cover the operational requirements of decision-makers.

In the A-DWTA/RH model, parameters are used to weigh the weapon consumption's killing efficiency at the current stage against the counter-effectiveness of the remaining weapon. To analyze the influence on operational mission, \( \lambda = [0.1:0.1:0.9] \) is set up for simulation experiments. The problem scale of the example was W100T50A50, and the other variables are generated as the previous experiment. Under the different values of \( \lambda \), the following three operational mission termination conditions are set:

- All assets are destroyed \( (l(s_m) = 0) \), defense mission fails, let \( \Gamma = 0 \).
- Assets are not all damaged and targets are all killed \( (l(s_m) > 0 \text{ and } n(s_m) = 0) \), the defense task is completed, let \( \Gamma = 1 \).
- Assets have not been destroyed and targets have not been killed, but weapons have been consumed \( (l(s_m) > 0, n(s_m) > 0 \text{ and } m(s_m) = 0) \). It is predicted that the defense mission will fail, and let \( \Gamma = -1 \).

The experimental results of each case under the above parameter settings are shown in Table 2.

| Parameter \( \lambda \) | Stage \( s_m \) | Compute time | NVSA | Plan fitness | Termination state |
|------------------------|--------------|--------------|-----|-------------|-------------------|
| 0.1                    | 4            | 3.3302       | 0.3461 | 0.5491 | \( \Gamma : W54T0A24 \) |
| 0.2                    | 3            | 1.9247       | 0.5417 | 0.6369 | \( \Gamma : W47T0A34 \) |
| 0.3                    | 2            | 1.7992       | 0.8096 | 0.6713 | \( \Gamma : W47T0A43 \) |
| 0.4                    | 3            | 1.8254       | 0.7599 | 0.7235 | \( \Gamma : W36T0A41 \) |
| 0.5                    | 2            | 1.4043       | 0.8882 | 0.7640 | \( \Gamma : W31T0A46 \) |
| 0.6                    | 2            | 1.6805       | 0.9328 | 0.8012 | \( \Gamma : W21T0A47 \) |
| 0.7                    | 2            | 1.2787       | 0.9832 | 0.8604 | \( \Gamma : W15T0A49 \) |
| 0.8                    | 2            | 1.3525       | 0.9721 | 0.9057 | \( \Gamma : W7T0A49 \) |
| 0.9                    | 2            | 1.2706       | 0.9615 | 0.9448 | \( \Gamma : W4T0A49 \) |

To further analyze the dynamic performance of HA-SMR solving the A-DWTA/RH under different \( \lambda \) value, Figure 1 gives the plot of the number of surviving assets, the number of surviving assets targets, the number of remaining weapons, NAV, and plan fitness observed at each stage.
In Table 2 and Figure 1, with the parameter increases from 0.1 to 0.9, the number of decision stages decreases from 4 to 2, and the compute time decreases from 3.3302s to 1.2706s. The reason is that A-DWTA/RH's objective $F$ is composed of the current decision's kill effect $f_1$ and the remaining weapons' counter effect $f_2$ after execution. $\lambda$ represents the weight of the former. When $\lambda$ is small, the decision plan will be guided to limit the weapon consumption of the current stage and retain the countermeasures in subsequent stages, which leads to an increased number of stages and compute time. The maximum value of NAV is 0.9832 at $\lambda = 0.7$, and the fitness metric increases with the increase of $\lambda$. It can be seen that the objective $F$ of A-DWTA/RH is more sensitive to $f_1$ than $f_2$. Even when $\lambda = 0.9$, there is no situation of excessive weapon consumption at the current stage and no counter weapons at the later stage ($\Gamma = -1$). In the mission termination state, the defender can complete the mission ($\Gamma = 1$) under the different $\lambda$ value. However, as the $\lambda$ value increases, the proportion of the current stage's decision return increases, and the weapon consumption of the current stage also increases. So the number of surviving assets increases when the operational mission is completed. It is worth noting that when $\lambda$ is 0.7, 0.8, and 0.9, the number of surplus weapons keeps decreasing, 15, 7 and 4 respectively. However, the number of decision stages and surviving targets remains unchanged, 2 and 49 respectively. It can be seen that if $\lambda$ is higher than 0.7, weapon resource is easily wasted. In conclusion, model parameter $\lambda$ has the effect of balancing the "radical - conservative" degree of the decision plan, and the reasonable value is between 0.6 and 0.8.

The relationship between the scale of weapon-target-asset directly affects the completion of the combat mission. In order to provide reference and basis for the configuration of completing the A-DWTA/RH mission, the dynamic relationship among the scale of weapon-target-asset is experimentally
analyzed. In this experiment, the target number is fixed to 50. To ensure the possibility of completing mission, the weapons should be more than targets. The weapon number is set to \( m = [60:10:100] \) in turn, and the asset number is set to \( l = [20:10:80] \). The model parameter is set to 0.8. The remaining variables are consistent with the previous experiment. As the scales and parameters of each case in this simulation are different, they can only be analyzed based on the win-loss indicator's statistical data. The distribution of defense mission completion indicator is shown in Figure 2.

Illustrated by Figure 2, all assets are destroyed (\( \Gamma = 0 \)) only when the asset number is 5, and the weapon number is 55, 60, and 70. The reason is that the asset number is too small, and the weapon number is only 10% to 20% higher than the incoming target number, which cannot guarantee the survival probability of assets in a few stages. When the weapon number is less than 70, and the assets number rises to the target number (50), the situation of \( \Gamma = -1 \) is basically obtained, that is, the weapons are used out. The reason is that the weapon number is close to the target number, making it impossible to complete the defense mission. With the increase of asset number (\( l > 50 \)), the target cannot destroy all assets in a few stages under the attack of weapons, so the weapons are consumed first. Finally, when the number of weapons is no less than 75, the probability of successful defense (\( \Gamma = 1 \)) increases significantly. When the number of weapons is no less than 95, the proportion of \( \Gamma = 1 \) is 100%. In summary, regardless of the state of assets, the weapon number of 95% higher than the target number can ensure the completion of the defense mission.

![Figure 2](image-url)

**Figure 2.** The distribution of mission completion indicator under the scale of \( n = 50, 55 \leq m \leq 100, 5 \leq l \leq 100 \)

5. **Conclusions**

At present, the major A-DWTA researches adopt the global return model based on multi-stage prediction, which can not fully reflect the operational requirement at the current stage and burden computing resource. This paper established the A-DWTA/RH model and designed the solving algorithm HA-SMR. Experimental results show HA-SMR has certain robustness for A-DWTA scale.
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References
[1] Roux, J.N.; Van Vuuren, J.H. Threat evaluation and weapon assignment decision support: A review of the 569 state of the art. ORiON 2007, 23, 151-187.
[2] Yang, Z.; Zhou, D.; Kong, W.; Piao, H.; Zhang, K.; Zhao, Y. Nondominated Maneuver Strategy Set With Tactical Requirements for a Fighter Against Missiles in a Dogfight. IEEE Access 2020, 8, 117298-117312.
[3] Manne, A.S. A target-assignment problem. Operations Research 1958, 6, 346-351.
[4] Hosein, P.A.; Athans, M. Preferential defense strategies. Part I: The static case. this issue 1990.
[5] Hosein, P.; Athans, M. Preferential Defense Strategies. Part 2: The Dynamic Case. LIDS-P-2003. MIT 1990.
[6] Cai, H.; Liu, J.; Chen, Y.; Wang, H. Survey of the research on dynamic weapon-target assignment problem. 576 Journal of Systems Engineering and Electronics 2006, 17, 559-565.
[7] Zhang, K.; Zhou, D.; Yang, Z.; Pan, Q.; Kong, W. Constrained Multi-Objective Weapon Target Assignment 578 for Area Targets by Efficient Evolutionary Algorithm. IEEE Access 2019, 7, 176339-176360.
[8] Murphey, R.A. Target-based weapon target assignment problems. In Nonlinear assignment problems; Springer, 2000; pp. 39-53.
[9] Cho, D.H.; Choi, H.L. Greedy maximization for asset-based weapon-target assignment with time-dependent rewards. Cooperative Control of Multi-Agent Systems: Theory and Applications 2017, pp. 115-139.
[10] Xin, B.; Wang, Y.; Chen, J. An efficient marginal-return-based constructive heuristic to solve the sensor-weapon-target assignment problem. IEEE Transaction on System, Man, and Cybernetics: Systems 2018, 49, 2536-2547.
[11] Li, X; Zhou D.; Yang, Z; Pan Q.; Huang, J. A novel genetic algorithm for the synthetical Sensor-Weapon-Target assignment problem. Applied Sciences 2019, 9, 3803.
[12] Lloyd, S.P.; Witsenhausen, H.S. Weapons allocation is NP-complete. 1986 Summer Computer Simulation 609 Conference, 1986, pp. 1054-1058.
[13] Hocaoglu, M.F. Weapon target assignment optimization for land based multi-air defense systems: A goal programming approach. Computers & Industrial Engineering 2019, 128, 681-689.
[14] Kline, A.G.; Ahner, D.K.; Lunday, B.J. Real-time heuristic algorithms for the static weapon target assignment problem. Journal of Heuristics 2019, 25, 377-397.
[15] Fu, G.; Wang, C.; Zhang, D.; Zhao, J.; Wang, H. A Multiobjective Particle Swarm Optimization 621 Algorithm Based on Multipopulation Coevolution for Weapon-Target Assignment. Mathematical Problems 622 in Engineering 2019, 2019
[16] Guo, D.; Lian, C.; Grand, P.; Dong, X.; Li, Q.; Ren, Z. Weapon-target assignment for multi-to- multi interception with grouping constraint. IEEE Access 2019, 7, 34838-34849.