Decay Modes of Intersecting Fluxbranes

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Abstract

Just as the single fluxbrane is quantum mechanically unstable to the nucleation of a locally charged spherical brane, so intersecting fluxbranes are unstable to various decay modes. Each individual element of the intersection can decay via the nucleation of a spherical brane, but uncharged spheres can also be nucleated in the region of intersection. For special values of the fluxes, however, intersecting fluxbranes are supersymmetric, and so are expected to be stable. We explicitly consider the instanton describing the decay modes of the two–element intersection (an F5-brane in the string theory context), and show that in dimensions greater than four the action for the decay mode of the supersymmetric intersection diverges. This observation allows us to show that stable intersecting fluxbranes should also exist in type 0A string theory.
1 Introduction

The four–dimensional Melvin universe [1] describes a gravitating magnetic fluxtube which, in the context of Kaluza–Klein theory [2, 3], can be obtained from five–dimensional Minkowski space by identifying coordinates in a non–standard way [4, 5, 6, 7]. By adding six flat spectator dimensions, one obtains [7, 8] a seven–dimensional fluxbrane (the F7-brane) which can be generated via a similar identification of points in eleven–dimensional Minkowski space. The F7-brane, and its intersecting cousins to be considered below, are thus expected to be exact solutions of M-theory, including all curvature corrections. For the same reason, string theory on the similar solution with Neveu–Schwarz–Neveu–Schwarz (NS–NS) flux obtained from ten–dimensional Minkowski space is exactly solvable [9, 10, 11, 12].

One can further generate spherical branes immersed within the background of a single fluxbrane [6, 7], spherical D6-branes within the context of string theory, and supergravity solutions [13, 14, 15] describing the dielectric effect of Myers [16]. Of course, a spherical brane is held only in metastable equilibrium by the ambient flux; it is easy to see that the fluxbrane itself will be unstable to the semi–classical nucleation of just such a spherical brane. In fact, there is a second decay mode [6, 7], corresponding to Witten’s bubble of nothing [17].

More explicitly, to obtain a flux \((D−4)\)-brane from \(D\) dimensional flat space in terms of cylindrical polar coordinates, one dimensionally reduces by identifying points separated by \(2\pi R\) along the closed orbits of the Killing vector

\[
K = \frac{\partial}{\partial z} + B \frac{\partial}{\partial \phi},
\]

where the parameter \(B\) becomes the magnetic field in the reduced theory. In other words, the identification of points is

\[
z \equiv z + 2\pi n R, \quad \phi \equiv \phi + 2\pi n B R + 2\pi m \quad n, m \in \mathbb{Z}.
\]

These identifications are unchanged under \(B \rightarrow B + n/R\), for some integer \(n\), so physically inequivalent spacetimes have [6, 7]

\[
-\frac{1}{2R} \leq B < \frac{1}{2R}.
\]

In a theory with fermions, however, one must be more careful [8]. Fermionic boundary conditions are unchanged only under \(B \rightarrow B + 2n/R\), so we should consider the magnetic field to lie in the range

\[
-\frac{1}{R} \leq B < \frac{1}{R}.
\]

This becomes clearer when one thinks about spin structures [8]. The original \(D\)–dimensional space is topologically \(M^{D−1} \times S^1\), so there are two distinct spin structures; under parallel transport
around the orbits of $K$, spinors pick up a phase

$$ +e^{\pi R B \Gamma} \quad \text{or} \quad -e^{\pi R B \Gamma}, \tag{1.5} $$

where $\Gamma$ is an element of the Lie algebra of Spin$(1,D - 1)$ which generates rotations along $\phi$ and has $\Gamma^2 = -1$. As in the usual Kaluza–Klein scenario [17], we are free to choose the overall sign. For $B = 0$, and in the string theory context, the first choice corresponds to the type IIA theory with supersymmetric boundary conditions and the second corresponds to the non–supersymmetric type 0A theory [18]. By continuity we make the same assignments for non–zero $B$.

Since the twisted identifications (1.2) break all supersymmetry, there is no immediate obstruction to decay processes. The two semi–classical decay modes of the fluxbrane [6, 7] are determined by choosing a spin structure [8]. In both cases [6, 7], the relevant instanton is the Euclidean version of the $D$–dimensional Kerr black hole found by Myers & Perry [19]. We denote this as Kerr$(\Omega)$, where $\Omega$ is the Euclidean “rotation” parameter.

To avoid a conical singularity in Kerr$(\Omega)$, the Euclidean time coordinate must be periodic in the usual way. But one must also identify coordinates as in (1.2), with $B = \Omega$, where $\Omega$ lies in the range [6, 7]

$$ -\frac{1}{R} < \Omega < \frac{1}{R}. \tag{1.6} $$

Reducing Kerr$(\Omega = B)$ along

$$ K = \frac{\partial}{\partial z} + B \frac{\partial}{\partial \phi}, \tag{1.7} $$

thus gives a $(D - 1)$–dimensional solution with the same asymptotic magnetic field, $B$, as reducing Kerr$(\Omega = B \mp 1/R)$ along

$$ K' = K \pm \frac{1}{R} \frac{\partial}{\partial \phi}. \tag{1.8} $$

The latter is the “shifted instanton” of [6, 7], and we use the upper (lower) sign if $B$ is positive (negative). As far as the bosonic fields are concerned, one would thus conclude that a fluxbrane with magnetic field $B$ has two decay modes, corresponding to the shifted and unshifted instanton. However, due to the shift in $B$, the two $(D - 1)$–dimensional theories will have opposite spin structures. To determine which is which [6], note that under parallel transport around integral curves at infinity of (1.7), spinors in Kerr$(\Omega = B)$ pick up a phase $-\exp(\pi R B \Gamma)$ so, in the string theory context, this corresponds to the 0A theory. On the other hand, under parallel transport around integral curves at infinity of (1.8), spinors in Kerr$(\Omega = B \mp 1/R)$ pick up a phase $-\exp(\pi R(B \pm 1/R) \Gamma) = +\exp(\pi R B \Gamma)$, which corresponds to the IIA theory.

We thus have the following picture [6, 7, 8]. Reducing Kerr$(\Omega = B)$ along $K$ gives the decay mode relevant to the F7-brane in the 0A theory; the fixed point set of $K$ is a nine–sphere so, upon analytically continuing one of the ignorable angles, this describes a deformed version of Witten’s bubble of nothing [17]. Reducing instead Kerr$(\Omega = B \mp 1/R)$ along $K'$ gives the decay mode
relevant to the F7-brane in the IIA theory; the fixed point set of $K'$ gives rise to an expanding six-sphere upon analytic continuation.

As we will review below, intersecting fluxbranes can be constructed \textit{via} a similar twisted reduction of flat Minkowski space; one simply adds extra rotations to the Killing vector along which one dimensionally reduces \cite{7}. But here there is a surprise, in that supersymmetry can be preserved for specific values of the magnetic fields associated with each of the individual fluxbranes. In the ten-dimensional context of two intersecting F7-branes, this was first pointed out in \cite{20} and a complete classification of the ten-dimensional supersymmetric fluxbrane configurations was made in \cite{21}. Yet instantons describing the semi-classical decay modes of intersecting fluxbranes certainly exist, having already been discussed in \cite{7}; they are constructed from the Euclideanized Myers–Perry black holes \cite{19} with more than one plane of rotation\footnote{Upon a further analytic continuation of one of the ignorable angles, and after dimensional reduction, these spaces describe the evolution of intersecting fluxbranes after they decay but, prior to dimensional reduction, they are interesting potential string theory backgrounds in their own right \cite{22}.}.

The question we want to address here is what happens to the instantons describing these decay modes for the cases in which supersymmetry is preserved. Some mention of these issues has already appeared in \cite{24}, where string theory on intersecting NS–NS fluxbranes is shown to be solvable, just as for the single fluxbrane. As in that paper, one can argue that the semi-classical amplitude for the decay of a supersymmetric solution must vanish, due to the presence of fermion zero-modes. For the dual solution in type 0A however, there is no supersymmetry, so stability does not arise from fermion zero modes. For this reason we calculate the action of the instantons, showing that in the supersymmetric case the action diverges, giving a vanishing semi-classical decay amplitude.

This note is organized as follows. In the next section, we briefly review the construction of the flux $(D-6)$-brane (two intersecting flux $(D-4)$-branes), before turning to consider the instantons describing the various decay modes in section 3. We discuss spin structures and how they determine the possible decay modes in section 4 and compute the action for the instanton in section 5. We conclude in section 6. We try to be dimension-independent although, since we will ultimately be interested in string theory, we will sometimes specialize to the ten-dimensional case.

## 2 Intersecting fluxbranes

Although most of what we discuss here can be made more general, we will concentrate on the case of two flux $(D-4)$-branes intersecting over a flux $(D-6)$-brane. To generate such a solution \cite{6, 7}, we start with $D$-dimensional Minkowski space written as

$$
\text{d}s_D^2 = \text{d}s^2(M^{D-5}) + \sum_{i=1,2} (d\rho_i^2 + \rho_i^2 d\phi_i^2) + dz^2, \quad (2.1)
$$

...
and, with \( i = 1, 2 \), make the identifications

\[
z \equiv z + 2\pi nR, \quad \phi_i \equiv \phi_i + 2\pi nB_i R + 2\pi m_i, \quad n, m_i \in \mathbb{Z}.
\] (2.2)

At least as far as the bosons are concerned, inequivalent spacetimes are thus obtained for both

\[
-\frac{1}{2R} \leq B_i < \frac{1}{2R}.
\] (2.3)

Geometrically, we dimensionally reduce along the closed orbits of the Killing vector

\[
K = \frac{\partial}{\partial z} + B_1 \frac{\partial}{\partial \phi_1} + B_2 \frac{\partial}{\partial \phi_2},
\] (2.4)

which in practice involves introducing coordinates

\[
\tilde{\phi}_i = \phi_i - B_i z, \quad \tilde{\phi}_i \equiv \tilde{\phi}_i + 2\pi m_i,
\] (2.5)

with standard periodicity and which are constant along orbits of \( K \), \( K(\tilde{\phi}) = 0 \). In these coordinates, the Killing vector is simply \( K = \partial / \partial z \).

To show that this identification need not break supersymmetry [20, 21], we work with the shifted coordinates \( \tilde{\phi} \) in the obvious orthonormal basis. Then it is easy to see that Killing spinors must have the form

\[
\epsilon(\tilde{\phi}_i, z) = \exp \left( \frac{1}{2}(\tilde{\phi}_1 \Gamma_1 + \tilde{\phi}_2 \Gamma_2) \right) \exp \left( -\frac{1}{2}(B_1 \Gamma_1 + B_2 \Gamma_2)z \right) \xi_0,
\] (2.6)

\( \Gamma_i \) being the element of the Lie algebra of \( \text{Spin}(1,10) \) which generates rotations along \( \tilde{\phi}_i \) and has \( \Gamma_i^2 = -1 \) and where \( \xi_0 \) is an arbitrary constant spinor. We can thus preserve one-half of the \( D \)-dimensional supersymmetries if and only if \( B_2 = \pm B_1 \), the identifications (2.2) preserving those spinors which satisfy the projection condition

\[
(1 \mp \Gamma_1 \Gamma_2) \xi_0 = 0.
\] (2.7)

Reducing along orbits of \( K \), the \( (D-1) \)-dimensional solution describing the two–element intersecting fluxbrane is, in the Einstein frame [7],

\[
\begin{align*}
ds_{D-1}^2 &= \Lambda^{\frac{D-3}{2}} (ds^2(M^{D-5}) + d\rho_1^2 + d\rho_2^2) + \Lambda^{-\frac{1}{D-4}} \left( \rho_1^2 d\phi_1^2 + \rho_2^2 d\phi_2^2 + \rho_1^2 \rho_2^2 (B_2 d\phi_1 - B_1 d\phi_2)^2 \right), \\
A &= \Lambda^{-1} \left( \rho_1^2 B_1 d\phi_1 + \rho_2^2 B_2 d\phi_2 \right), \\
\epsilon^{\sqrt{D-2}} &= \Lambda \equiv 1 + B_1^2 \rho_1^2 + B_2^2 \rho_2^2.
\end{align*}
\] (2.8)

For \( D = 11 \), this has been referred to as an F5-brane in the literature [20, 21], due to the Poincaré invariance in a six–dimensional “worldvolume”, although strictly speaking this terminology has
been applied only to the supersymmetric case; the polarization of D-branes due to an F5-brane was studied in [23]. The field strength has non–vanishing second Chern class [20]
\[ \int_{\mathbb{R}^2 \times \mathbb{R}^2} F \wedge F = \int_{S^3_\infty} A \wedge dA = \frac{(2\pi)^2}{B_1 B_2}. \tag{2.9} \]

As for the single fluxbrane [6, 7], the intersecting solution should be thought of as a good description of the configuration of magnetic fields only when they are, in some sense, small. For the \((D - 1)\)-dimensional solution to be weakly curved, and, when \(D = 11\), for string theory to be weakly coupled, we need both the \(\rho_i \ll B_i\). But for the Kaluza–Klein ansatz to be valid, we need all length scales to be much larger than the radius of compactification; that is, \(\rho_i \gg R\). Thus, the ten–dimensional solution is valid only for both \(B_i \ll 1/R\). For either \(B_i \sim 1/R\), and to keep weak curvature and string coupling, we need \(\rho_i \gg R\), so the ten–dimensional description is no longer a good one.

There is an interesting way to view the supersymmetric intersecting fluxbrane [25, 26], which is worth making explicit, and shows why it preserves supersymmetry in the first place. To see this, take flat Minkowski space as in (2.1). Think of the \(\mathbb{R}^4\) spanned by \(\{\rho_i, \phi_i\}\) as \(\mathbb{R} \times S^3\) and the \(S^3\) as an \(U(1)\) bundle over \(S^2\). Then one can show that the Killing vector
\[ K = B \left( \frac{\partial}{\partial \phi_1} + \frac{\partial}{\partial \phi_2} \right), \tag{2.10} \]
generates translations along the \(U(1)\) Hopf fibre [6, 25, 26]. In fact, identifying points along the closed orbits of \(K\) gives the near–core geometry of Taub–NUT space [6]. Introduce the coordinates
\[ \psi = \frac{\phi_1}{B}, \quad \phi = \phi_2 - B\psi, \tag{2.11} \]
so that \(K = \partial/\partial \psi\). Then, upon setting
\[ \rho_1 = \frac{1}{B} \sqrt{\frac{r}{\mu}} \cos(\theta/2), \quad \rho_2 = \frac{1}{B} \sqrt{\frac{r}{\mu}} \sin(\theta/2), \tag{2.12} \]
and taking \(B = 1/(2\mu)\), the metric (2.1) becomes
\[ ds^2_D = ds^2(M^{D-5}) + dz^2 + V^{-1} (d\psi + \mu(1 - \cos \theta)d\phi)^2 + V(dr^2 + r^2d\Omega_2^2), \tag{2.13} \]
where \(V = \mu/r\). This is easily recognized as the near–core geometry of the \(D\)-dimensional Kaluza–Klein monopole [27, 28], with points identified along orbits of \(K = \partial/\partial \psi\) i.e. along the Hopf fibre as promised.

\footnote{In the same way that the single fluxbrane describes a “constant” two–form magnetic field in general relativity, one might think that the above intersecting fluxbrane in some sense describes a “constant” four–form magnetic field and this, of course, is consistent with the terminology. However, there is only ever a two–form in the game, so it is unclear to what extent the intersecting fluxbrane is an independent entity.}
To generate the two intersecting fluxbranes, we simply add a translation along $z$ to the Killing vector $K$, identifying points along orbits of

$$K' = \frac{\partial}{\partial z} + K = \frac{\partial}{\partial z} + \frac{\partial}{\partial \psi}. \quad (2.14)$$

Since the double twist involved in generating the above fluxbrane intersection is in the Hopf fibre direction $\psi$, supersymmetry is guaranteed; none of the Killing spinors of (the near–core region of) Taub–NUT space are broken by the twist. If we now set $\tau = \psi - z$ such that $K'(\tau) = 0$, and reduce along orbits of $K' = \partial/\partial z$ as above, we find, in the Einstein frame

$$\begin{align*}
\text{d}s^2 &= H^{1\over D-5}V^{D-3\over D-4} \left( V^{-1}\text{d}s^2(\mathbb{M}^{D-5}) + (HV)^{-1}(d\tau + \omega)^2 + \text{d}s^2(\mathbb{E}^3) \right), \\
A &= H^{-1}(d\tau + \omega), \\
e^{\text{d}\phi} &= HV^{-1},
\end{align*} \quad (2.15)$$

where $H = 1 + V$, $V = \mu/r$, $\omega = \mu(1 - \cos \theta)d\phi$ and we have again taken $B = 1/(2\mu)$. As discussed in [25, 26], for $D = 11$, this form of the F5-brane is reminiscent of that describing bound states of D6-branes and Kaluza–Klein monopoles found in [29] where the charges, $\mu$, of both elements are equal. The function $V$ would be associated with the D6-brane, and the function $H$ with the monopole, so the D6-brane is actually wrapped on the monopole circle. However, the match cannot be made exact and we have also gone to the near–core of the D6-brane, but not of the monopole, despite the fact that the charges are the same. In fact, the above form of the F5-brane is what one would find by taking this “near–core” limit of the fluxbranes considered in [30, 26]. In these papers, supersymmetric fluxbranes on curved space were found via twisted reductions of the Taub–NUT geometry, whereas to generate the F5-brane itself, one simply starts instead with the near–core limit of the Taub–NUT space.

3 Instantons for intersecting fluxbranes

In general, one should expect similar instabilities of the intersecting fluxbrane solution as for the single fluxbrane. Indeed, the relevant decay modes and corresponding instantons have already been briefly described [7]: decay via Witten’s bubble of nothing; nucleation of a locally charged $(D-5)$–sphere in either one of the fluxbrane elements; or nucleation of an uncharged $(D-7)$–sphere in the intersecting region $^3$. However, the authors of [7] did not appreciate that, for $B_1 = \pm B_2$, the

$^3$With $D = 11$, one should have a nice interpretation of this four–sphere in terms of branes, but it is unclear to us as to what this should be. One clue is the reduction of the ten–dimensional Euclidean Schwarzschild solution with a trivial time direction. In the case of a single twist, this gives a six–sphere held in metastable equilibrium by the background flux [6, 7]. By going near the core, one can see explicitly that this is a spherical D6-brane. We have looked at the case with two twists instead, in which one can indeed identify a four–sphere in the region of intersection, but have been unable to give this a satisfactory interpretation in terms of branes. This is related to our discussion of the F5-brane in section 2 which is similar to a bound state of D6-branes and monopoles.
intersecting solution is supersymmetric. In this case, we would expect stability, and so would not expect to find such instantons.

Our starting point is the Myers–Perry [19] black hole in arbitrary odd dimension \( D = 1 + N \), with angular momenta, \( a_i \), in two orthogonal planes. Ultimately, we will be interested in taking \( D = 11 \). Analytically continuing \( t \to iz \) and \( a_i \to i\alpha_i \), the Euclidean metric is

\[
\text{d}s^2_D = \text{d}z^2 + (r^2 - \alpha_1^2)(\text{d}r_1^2 + \mu_1^2 \text{d}\phi_1^2) + (r^2 - \alpha_2^2)(\text{d}r_2^2 + \mu_2^2 \text{d}\phi_2^2) + r^2 \sum_{i=3}^{N/2} (\text{d}\mu_i^2 + \mu_i^2 \text{d}\phi_i^2) - \frac{\mu r^2}{\Pi F} (\text{d}z + \alpha_1 \mu_1^2 \text{d}\phi_1 + \alpha_2 \mu_2^2 \text{d}\phi_2)^2 + \frac{\Pi F}{\Pi - \mu r^2} \text{d}r^2,
\]

where the direction cosines are constrained as

\[
\mu_1^2 + \mu_2^2 + \sum_{i=3}^{N/2} \mu_i^2 = 1,
\]

and

\[
\Pi = r^{N-4}(r^2 - \alpha_1^2)(r^2 - \alpha_2^2), \quad F = 1 + \frac{\alpha_1^2 \mu_1^2}{(r^2 - \alpha_1^2)} + \frac{\alpha_2^2 \mu_2^2}{(r^2 - \alpha_2^2)}.
\]

There is a “bolt” at the origin, \( r = r_H \), of polar coordinates, given by the largest root of \( \Pi(r_H) = \mu r_H^2 \), so that

\[
\mu = r_H^{N-6}(r_H^2 - \alpha_1^2)(r_H^2 - \alpha_2^2),
\]

and where \( \mu \geq 0 \). To avoid a conical singularity at \( r = r_H \), we have to identify the coordinates as

\[
z \equiv z + 2\pi R, \quad \phi_1 \equiv \phi_1 + 2\pi n\Omega_1 R + 2\pi m_1, \quad \phi_2 \equiv \phi_2 + 2\pi n\Omega_2 R + 2\pi m_2,
\]

where the radius of the \( z \) circle is given by

\[
R = \frac{2\mu r_H^2}{\Pi(r_H) - 2\mu r_H} = 2\mu \frac{1}{r_H^{N-5}} \left( (N-2)r_H^2 - (N-4)(\alpha_1^2 + \alpha_2^2) + (N-6)\frac{\alpha_1^2 \alpha_2^2}{r_H^2} \right)^{-1},
\]

and the “Euclidean angular velocities” are

\[
\Omega_1 = \frac{\alpha_1}{r_H^2 - \alpha_1^2} = \frac{\alpha_1^2 (r_H^2 - \alpha_2^2)}{\mu (r_H^2 - \alpha_1^2)} r_H^{N-6},
\]

\[
\Omega_2 = \frac{\alpha_2}{r_H^2 - \alpha_2^2} = \frac{\alpha_2^2 (r_H^2 - \alpha_1^2)}{\mu (r_H^2 - \alpha_2^2)} r_H^{N-6}.
\]

Asymptotically, the solution tends to flat space, but with the non–standard identifications (3.5). In other words, asymptotically the solution looks like the Euclidean version of the \( D \)-dimensional intersecting fluxbrane solution considered above. We simply identify the magnetic fields as

\[
B_1 = \Omega_1, \quad B_2 = \Omega_2,
\]

\footnote{Of course, we are concerned here with non–perturbative stability. One should also expect perturbative stability of the supersymmetric intersecting solutions. However, they are not asymptotically flat so, as for supersymmetric plane waves [31, 32], it is unclear as to what extent the presence of Killing spinors guarantees stability.}
and we have the correct asymptotics for this to be a valid instanton describing various decay modes of the intersecting fluxbrane.

An important question concerns the parameter space of this instanton. That is, what are the possible ranges of the $\Omega_i$? To analyse this, we first consider the two obvious limits:

(i) $\alpha_1 \to \pm \infty$, $\alpha_2 = \text{constant} << \alpha_1$, \hspace{1cm} (3.10)

(ii) $\alpha_2 \to \pm \infty$, $\alpha_1 = \text{constant} << \alpha_2$. \hspace{1cm} (3.11)

From (3.4), we must have $r_H \to |\alpha_1|$ ($r_H \to |\alpha_2|$) in case (i) ((ii)). Then, from (3.6), we can keep the radius fixed if we take $\mu \to R|\alpha_2|^{N-3}$ ($\mu \to R|\alpha_1|^{N-3}$) in case (i) ((ii)). This gives

(i) $\Omega_1 R \to \pm 1$, $\Omega_2 \to 0$, \hspace{1cm} (3.12)

(ii) $\Omega_2 R \to \pm 1$, $\Omega_1 \to 0$. \hspace{1cm} (3.13)

These are the two limits in which the parameters effectively reduce to those of the case with a single rotation, with one of the $\Omega_i = 0$.

To understand what happens in the intermediate cases, the obvious limit to take is that in which both $\alpha_1$ and $\alpha_2$ go to infinity at the same rate, namely

$\alpha_1 = \alpha_2 \equiv \alpha \to \infty$. \hspace{1cm} (3.14)

Then (3.4) gives $r_H \to \alpha$ as before, but the radius (3.6) would seem to be ill-defined. To analyse this case further, and with an eye on the evaluation of the action in the following section, we introduce the dimensionless parameters

$\hat{\alpha}_i = \frac{\alpha_i}{r_H}$, $\hat{\Omega}_i = \Omega_i r_H$, $\hat{\mu} = \frac{\mu}{r_H^{N-2}}$, $\hat{R} = \frac{R}{r_H}$, \hspace{1cm} (3.15)

in terms of which

$\hat{\mu} = (1 - \hat{\alpha}_1^2)(1 - \hat{\alpha}_2^2)$, $\hat{\Omega}_i = \frac{\hat{\alpha}_i}{1 - \hat{\alpha}_i^2}$, \hspace{1cm} (3.16)

$\hat{R} = \frac{2\hat{\mu}}{N - 2 - (N - 4)(\hat{\alpha}_1^2 + \hat{\alpha}_2^2) + (N - 6)\hat{\alpha}_1^2 \hat{\alpha}_2^2}$, \hspace{1cm} (3.17)

so that

$\Omega_1 R = \hat{\Omega}_1 \hat{R} = 2\hat{\alpha}_1(1 - \hat{\alpha}_2^2) \left( N - 2 - (N - 4)(\hat{\alpha}_1^2 + \hat{\alpha}_2^2) + (N - 6)\hat{\alpha}_1^2 \hat{\alpha}_2^2 \right)^{-1}$, \hspace{1cm} (3.18)

$\Omega_2 R = \hat{\Omega}_2 \hat{R} = 2\hat{\alpha}_2(1 - \hat{\alpha}_1^2) \left( N - 2 - (N - 4)(\hat{\alpha}_1^2 + \hat{\alpha}_2^2) + (N - 6)\hat{\alpha}_1^2 \hat{\alpha}_2^2 \right)^{-1}$. \hspace{1cm} (3.19)

Note that for $\mu \geq 0$, we need either both $|\hat{\alpha}_i| \leq 1$ or both $|\hat{\alpha}_i| \geq 1$, but for $\hat{R}$ positive we are restricted to $|\hat{\alpha}_i| \leq 1$. Of the four possible combinations of limiting cases ($\hat{\alpha}_1 \to \pm 1$, $\hat{\alpha}_2 \to \pm 1$) we shall consider the limit $\hat{\alpha}_1 \sim \hat{\alpha}_2 \sim 1$, so take

$\hat{\alpha}_1 = 1 - \epsilon_1$, $\hat{\alpha}_2 = 1 - \epsilon_2$. \hspace{1cm} (3.20)
where $\epsilon_1, \epsilon_2$ are small and positive. We then find that
\[
\Omega_1 R = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2}, \quad \Omega_2 R = \frac{\epsilon_1}{\epsilon_1 + \epsilon_2},
\] (3.21)
This shows that the limit $\hat{\alpha}_1 \sim \hat{\alpha}_2 \sim 1$ corresponds to $\Omega_1 R + \Omega_2 R = 1$. Taking the other three possibilities for the $\hat{\alpha}_i$ limits we find that the $\Omega_i R$ are bounded to lie in the diamond region defined by the vertices $(\Omega_1 R, \Omega_2 R) = (0, \pm 1), (\pm 1, 0)$.

To higher order one finds that
\[
\Omega_1 R + \Omega_2 R = 1 - \epsilon_1 \epsilon_2 \quad (N = 4),
\] (3.22)
\[
\Omega_1 R + \Omega_2 R = 1 - (N - 4) \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \quad (N > 4).
\] (3.23)
If we wish to reach the boundary $(\Omega_1 R + \Omega_2 R = 1)$ with both $\Omega_1 R$ and $\Omega_2 R$ non-zero then we see from (3.21) that we can take $\epsilon_1 = \kappa \epsilon_2$ and then take $\epsilon_2 \to 0$. So, in the sense of a limiting procedure, we may find the instanton which has parameters on the boundary of the fundamental region defined by the diamond with vertices $(\Omega_1 R, \Omega_2 R) = (0, \pm 1), (\pm 1, 0)$; this will be relevant when we consider the supersymmetric intersections.

### 4 Spin structures and decay modes

At any rate, just as in the case of a single rotation, there are various possibilities when we come to dimensionally reduce the instanton [7]. In each case, demanding that the $(D - 1)$-dimensional solution be free from conical singularities will restrict the magnetic field. Since we can shift either of the $\hat{\Omega}_i$ by at most $\pm 1$, there are four cases to consider. Each describes a different decay mode, one relevant to either the IIA or 0A theory. To describe the subsequent evolution of the intersecting fluxbranes, we should analytically continue one of the ignorable angles into time, $t$. The fixed point set of the Killing vector along which we reduce in each case, restricted to the initial $t = 0$ surface, will give rise to a naked singularity, which can be interpreted as the “worldvolume” of a brane in the usual way.

1 Reduction along
\[
K = \frac{\partial}{\partial z} + \Omega_1 \frac{\partial}{\partial \phi_1} + \Omega_2 \frac{\partial}{\partial \phi_2},
\]
will give a magnetic field $B_1 = \Omega_1, B_2 = \Omega_2$ and a fixed point set corresponding to the entire origin, a $(D - 2)$-sphere on the $t = 0$ surface. This is Witten’s bubble of nothing [17].

2 Reduction along
\[
K' = K \mp \frac{1}{R} \frac{\partial}{\partial \phi_1},
\]
will give a magnetic field of \( B_1 = \Omega_1 \mp 1/R \), \( B_2 = \Omega_2 \mp 1/R \) and the fixed point set will be a \((D - 5)\)-sphere expanding in one of the fluxbranes (the one associated with \( B_1 \)).

3 Reduction along

\[ K' = K \mp \frac{1}{R} \frac{\partial}{\partial \phi_2}, \]

will give a magnetic field of \( B_1 = \Omega_1 \mp 1/R \), \( B_2 = \Omega_2 \mp 1/R \) and the fixed point set will again be a \((D - 5)\)-sphere, but now expanding in the fluxbrane associated with \( B_2 \).

4 Reduction along

\[ K' = K \mp \frac{1}{R} \frac{\partial}{\partial \phi_1} \mp \frac{1}{R} \frac{\partial}{\partial \phi_2}, \]

will give a magnetic field of \( B_1 = \Omega_1 \mp 1/R \), \( B_2 = \Omega_2 \mp 1/R \) and the fixed point set will now be a \((D - 7)\)-sphere expanding in the region of intersection.

Again, as for the case of a single rotation [8], the spin structures determine which of these particular decay modes is allowed for any given solution. To see this, we compute the phase which a spinor picks up under parallel transport at infinity around the orbits of the Killing vector \( K \). We first define the shifted angular coordinates analogous to (2.5)

\[ \phi_1 = \tilde{\phi}_1 + \Omega_1 z, \quad \phi_2 = \tilde{\phi}_2 + \Omega_2 z, \quad (4.1) \]

so that \( K = \partial/\partial z \) and then send \( r \to \infty \) in the metric (3.1). The covariant derivative is then

\[ D_z = \partial_z + \frac{1}{2} \Omega_1 \Gamma_1 + \frac{1}{2} \Omega_2 \Gamma_2. \quad (4.2) \]

Parallel transport around a closed loop thus gives

\[ \psi(2\pi R) = -\exp(-\pi R(\Omega_1 \Gamma_1 + \Omega_2 \Gamma_2)) \psi_0, \quad (4.3) \]

where, since the instanton is simply connected (it is topologically \( \mathbb{R}^2 \times S^{D-2} \)), there is a unique spin structure; the overall minus sign can be determined by continuity with the \( D \)-dimensional Euclidean Schwarzschild solution.

Now we see how the spin structure determines the allowed decay modes 1–4 above, since under each independent shift of the Killing vector, the phase picks up an extra overall minus sign. Thus, decay mode 1 will just see the phase (4.3). For \( D = 11 \), it corresponds to the non–supersymmetric spin structure of the \( 0A \) theory. Decay modes 2 and 3, however, have an extra overall minus sign, so correspond to the supersymmetric spin structure of the IIA theory. Finally, the decay mode 4 picks up a further minus sign, and so is again only allowed in the \( 0A \) theory. We thus arrive at the following conclusions; the F5-brane in the IIA theory can decay via nucleation of a spherical D6-brane, which expands into one of the two elements of the intersection (that with the larger of the
magnetic fields); and the F5-brane in the 0A theory can decay via Witten’s bubble of nothing [17] or via the nucleation of a four–sphere in the intersection region. Note that the final, perhaps most interesting, decay mode is pertinent to the non–supersymmetric 0A theory only.

We still have not answered the question as to how the decay of the supersymmetric F5-brane, with $B_1 = \pm B_2$, is eliminated. To show that it is not allowed, we consider the action for the above instanton. We want to compute the action for the range of possible parameters, that is for each of the above four cases. We will be especially interested in what happens when $B_1 = \pm B_2$ for, in that case, we do not expect any decay modes to be possible at all.

5 The action

Since the metric (3.1) is Ricci–flat, the only contribution to the action is the boundary term

$$I_D = -\frac{1}{8\pi G_D} \int_{r\to\infty} d^N x \sqrt{h} (K - K_0),$$

where $h$ is the determinant of the metric induced on surfaces of constant $r$ and $K$ is the extrinsic curvature of this surface. $K_0$ is the curvature of a reference background, which in this case is just flat space, corresponding to $\mu = 0$.

To compute the action, we have to take account of the constraint (3.2) on the $\mu_i$, so we substitute for

$$\mu_{N/2}^2 = 1 - \left( \mu_1^2 + \mu_2^2 + \sum_{i=3}^{N/2-1} \mu_i^2 \right),$$

and write the metric as

$$ds_D^2|_{r=\text{const}} = \left( 1 - \frac{\mu r^2}{\Pi F} \right) dz^2 + \sum_{i,j=1,2} \left( (r^2 - \alpha_i^2) \delta_{ij} - \frac{\mu r^2}{\Pi F} \alpha_i \alpha_j \mu_i^2 \mu_j^2 \right) d\phi_i d\phi_j$$

$$-2\frac{\mu r^2}{\Pi F} \sum_{i,j=1,2} \alpha_i \mu_i^2 d\phi_i dz + \sum_{i,j=1}^{N/2-1} \left( (r^2 - \alpha_i^2) \delta_{ij} + r^2 \frac{\mu_i \mu_j}{\mu_{N/2}^2} \right) d\mu_i d\mu_j + r^2 \sum_{i=3}^{N/2} \mu_i^2 d\phi_i^2,$$

where in the second to last term, we have to remember only $\alpha_1$ and $\alpha_2$ are non–zero. It is easy to compute the determinant of the metric in the \{z, $\phi_1$, $\phi_2$\} directions explicitly, and one can use the identity involving alternating tensors to compute the determinant in the $\mu_i$ directions. The result is

$$h = (\mu_1 \cdots \mu_{N/2-1})^2 \frac{\Pi F}{r^2} \left( 1 - \frac{\mu r^2}{\Pi} \right).$$

We now proceed as in [6], using the fact that $\sqrt{hK} = n(\sqrt{h})$, where $n$ is the unit normal

$$n = \sqrt{\frac{\Pi - \mu r^2}{\Pi F} \frac{\partial}{\partial r}}.$$
which, with a prime denoting $\partial/\partial r$, gives
\[ \sqrt{\hbar}K = \frac{1}{2} \frac{\mu_1 \cdots \mu_{N/2-1}}{r^2} \left( -2\Pi + r \frac{F'}{F} (\Pi - \mu r^2) + r \frac{\Pi'}{\Pi} (2\Pi - \mu r^2) \right). \]

(5.9)

Since
\[ \lim_{r \to \infty} \frac{\sqrt{\hbar}(K - K_0)}{\sqrt{\hbar}} = 1 - \frac{1}{2} \frac{\mu}{r^{N-2}}, \]

(5.10)

we have
\[ \lim_{r \to \infty} \sqrt{\hbar}(K - K_0) = \lim_{r \to \infty} \left( n(\sqrt{\hbar}) - n(\sqrt{\hbar}) \right)_{\mu=0} + \frac{1}{2} \frac{\mu}{r^{N-2}} n(\sqrt{\hbar})_{\mu=0} \]
\[ = \lim_{r \to \infty} \left( \frac{\partial}{\partial \mu} n(\sqrt{\hbar}) + \frac{1}{2} \frac{1}{r^{N-2}} n(\sqrt{\hbar})_{\mu=0} \right) \mu. \]

(5.11)

This gives
\[ \lim_{r \to \infty} \sqrt{\hbar}(K - K_0) = -\frac{1}{2} \mu_1 \cdots \mu_{N/2-1} \lim_{r \to \infty} \left( r \frac{F'}{F} (1 - \frac{1}{2} \frac{\Pi}{r^N}) + r \frac{\Pi'}{\Pi} (1 - \frac{\Pi}{r^N}) + \frac{\Pi}{r^N} \right) \]
\[ = -\frac{1}{2} \left( \mu_1 \cdots \mu_{N/2-1} \right) \mu. \]

(5.12)

We are ultimately interested in the $(D-1)$-dimensional interpretation as a decay of magnetic fields so we should consider $G_{D-1} = 2\pi RG_D$ as a constant, giving
\[ I_D = \frac{1}{16\pi} \frac{\Omega_{N-1}}{G_N} \mu, \]

(5.13)

where the volume, $\Omega_{N-1}$, of the unit $(N-1)$-sphere comes from the factor
\[ \Omega_{N-1} = \int_0^{2\pi} d\phi_1 \cdots d\phi_{N/2} \int d\mu_1 \cdots d\mu_{N/2-1} \mu_1 \cdots \mu_{N/2-1}. \]

(5.14)

We also want to hold the string coupling constant fixed so the dimensionless parameter of interest is actually
\[ \hat{I} = \frac{\kappa_N^2}{R^{N-2} \Omega_{N-1}} I = \frac{1}{2} \frac{\mu}{R^{N-2}} = \frac{1}{2} \frac{\mu}{\hat{R}^{N-2}}, \]

(5.15)

which is the quantity we will plot.

This is a numerical problem; we pick values of $(\hat{\Omega}_1 \hat{R}, \hat{\Omega}_2 \hat{R})$ within the region of inequivalent spacetimes, $-\frac{1}{\hat{R}} \leq B_i < \frac{1}{\hat{R}}$, then solve the equations (3.18) and (3.19) for $(\hat{\alpha}_1, \hat{\alpha}_2)$, and finally evaluate the action for these quantities. This numerical approach confirms the earlier argument that solutions are restricted to lie in a diamond region. Consider, first, the cases 2 and 3 of section 4, those relevant to the type IIA theory. The action as a function of $RB_i$ is in these cases is plotted in Fig. 2 and 3. Taken together they show that an instanton exists for every value of $-1 < RB_i < 1$, i.e. all inequivalent fluxbranes have an instanton solution. Also note that Fig. 2 governs instanton solutions with $|B_1| > |B_2|$. As such, the relevant instanton for $|B_1| > |B_2|$ is case 2, i.e. the decay
of the fluxbrane with magnetic field $B_1$. Fig. 5 shows the complete case for IIA, combining cases 2 and 3: the action for $B_1 = \pm B_2$ diverges.

Similarly, cases 1 and 4, plotted in Fig. 1 and 4, together give the complete picture for decay in type $0A$, shown in Fig. 6. Each case covers a different part of $RB_i$ parameter space and together they give the complete space, showing that an instanton exists for every inequivalent value of $RB_i$. The action now diverges along $RB_1 = \pm 1 \pm RB_2$, these values being the type $0A$ dual of the supersymmetric IIA solutions. We can thus predict that there are stable intersecting fluxbranes in the type $0A$ theory for these values of the magnetic fields.

To see how the action behaves near the supersymmetric point, consider the limit (3.20). In that case, the action is found to be

$$\hat{\mu} = 4\epsilon_1\epsilon_2 \left( \frac{\epsilon_1 + \epsilon_2}{\epsilon_1\epsilon_2} \right)^{N-2}, \quad (5.16)$$

and if we look near a point on the boundary by taking $\epsilon_1 = \kappa \epsilon_2$ we see that

$$\hat{\mu} = 4\kappa \left( \frac{1 + \kappa}{\kappa} \right)^{N-2} \frac{1}{\epsilon_2^{N-4}}. \quad (5.17)$$

So, as the boundary is approached ($\epsilon_2 = 0$) the action diverges (if $N > 4$) and the semi-classical decay probability goes to zero. In the special case of $N = 4$ we note that the action has a finite limit as we approach the boundary, so how do we square this with the fact that this situation should be supersymmetric? As discussed earlier we expect that the fermion zero modes, which are present due to supersymmetry, will cause the decay amplitude to vanish. Exactly what happens in the dual type $0A$ case is unclear and is under investigation. It is not unexpected that the case $N = 4$ is singled out, because the two independent rotation planes use up the whole space. It is possible therefore that similar effects would occur in higher dimensions when there are more intersecting fluxbranes.

6 Conclusions

We have discussed the instantons relevant to the semi-classical decay of intersecting fluxbranes. Although we have concentrated on the two–element intersection, we suspect that similar results hold in general. The action, as a function of more than two variables, however, would soon become difficult to picture. We have argued that the parameter space of the instanton includes the supersymmetric solutions, with $B_1 = \pm B_2$, but that the action diverges for such solutions.

Of course, due to the presence of fermion zero–modes in the supersymmetric cases, one can argue that the amplitude for their decay must vanish identically. However, it is difficult to verify this without an explicit computation. Moreover, by analysing the instanton, we have been able to argue that stable intersecting fluxbranes in type $0A$ string theory (with $B_1 = \pm 1/R \pm B_2$) should
Figure 1: This shows the value of the action as a function of $RB_i$ for the unshifted instanton (case 1).

Figure 2: This shows the value of the action as a function of $RB_i$ for the first shifted instanton (case 2). Note that this has $|B_1| > |B_2|$. 
Figure 3: This shows the value of the action as a function of $RB_i$ for the second shifted instanton (case 3). Note that this has $|B_2| > |B_1|$.

Figure 4: This shows the value of the action as a function of $RB_i$ for the third shifted instanton (case 4).
Figure 5: This shows the value of the action as a function of $RB_i$ for the type IIA theory.

Figure 6: This shows the value of the action as a function of $RB_i$ for the type 0A theory.
exist, and arguments about fermion zero–modes due to supersymmetry would have missed this. It would be of interest to think further about the implications of these stable configurations of the 0A string theory.

It would also be of interest to see if the interpretation of the F5-brane in terms of D6-branes and monopoles can be made clearer. This would give a better understanding of what it is that is actually nucleated in region of intersection (decay mode 4 of section 4), and might provide a hints as to a possible “worldvolume” theory of the F5-brane. We have also tried to analyse the late–time limit of the post–decay evolution of the F5-brane, along the lines of [33], but the complicated metric makes it difficult to extract any interesting results. Finally, it would be nice to understand the parameter space of the Euclidean Myers–Perry black holes for more than two rotation parameters, in relation to intersecting flux branes with more than two elements.

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