Random 3D Spin System Under the External Field and Dielectric Permittivity Superlattice Formation

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A dielectric medium consisting of roughly polarized molecules is treated as a 3D disordered spin system (spin glass). A microscopic approach for the study of statistical properties of this system on micrometer space scale and nanosecond time scale of standing electromagnetic wave is developed. Using ergodic hypothesis the initial 3D spin problem is reduced to two separate 1D problems along external field propagation. The first problem describes the disordered spin chain system while the second one describes a disordered \textit{N}-particle quantum system with relaxation in the framework of Langevin-Schrödinger (L-Sch) type equation. Statistical properties of both systems are investigated in detail. Basing on these constructions, the coefficient of polarizability, related to collective orientational effects, is calculated. Clausius-Mossotti formula for dielectric constant is generalized. For dielectric permittivity function generalized equation is found taking into account Clausius-Mossotti generalized formula.

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I. INTRODUCTION

The formation and control of periodic nanostructures in various type media (Media with Periodically Modulated Refractive Index (MPMRI)) are one of most important applied problems. First of all it is related to the possibility of the creation of compact UV or X-ray Free-Electron Lasers (FEL) based on the principle of transition radiation (TR) (see for example \textsuperscript{[1]}). Currently the following two problems are discussed intensively:
1. A gas-plasma medium with periodically varied ionization density

2. A specially periodical solid-state superlattice-like (SSL) structures, which are composed of layers with different refracted indices

TR is generated due to the difference in frequency-dependent dielectric constants (dielectric permittivity functions) of adjacent layers (recall that radiation power is proportional to $[\epsilon_1(w) - \epsilon_2(w)]^2$). Therefore, an important problem is the possibility of the control of this difference by means of some external field. In other words, a problem is to construct a superlattice with dielectric constant difference between neighboring domains having a form $[\epsilon_1(w, g) - \epsilon_2(w, g)]^2$, where $g$ describes controlling parameters, $\epsilon_1(w, g)$ and $\epsilon_2(w, g)$ are dielectric permittivity functions in the neighboring regions. According to theoretical and experimental studies the periodical structures can be created in condensed matter by means of external electromagnetic or acoustic fields (see in particular [20, 21, 22, 23]).

This idea have been applied recently for experimental TR generation [24]. In particular, it was shown that the electron beam of 20 Mev, passing through amorphous silicon dioxide $a-SiO_2$ with the standing electromagnetic wave (of 10 GHz frequency and 3 $\mu m$ wavelength) inside, produces anomaly high radiation. Preliminary investigations explain its high intensity as a result of the multiple passing of the electron beam through interfaces between regions with different dielectric constants. The theoretical study explained the appearance of one-dimensional superlattice order in random media by media polarization, caused by orientation relaxation of elastic molecular dipoles in external field [25].

The main goal of this paper is systematic investigation of the relaxation processes and critical effects in the disordered 3D spin system type of $a-SiO_2$ under the influence of external electromagnetic field, which forms a standing wave in the medium. In particular, we develop a mathematical approach for description of statistical behavior of disordered 3D spin system along the direction of standing wave propagation on space-time scale of its period.

In the framework of this approach we generalize Clausius-Mossotti equation for dielectric constant and the equation for frequency-dependent dielectric permittivity on the scale of space-time period of standing electromagnetic wave.
II. FORMULATION OF THE PROBLEM. NEW IDEAS AND BASIC FORMULAS

Let us begin by discussing the fundamental problem of the dielectric constant space-time modulation (generation of dielectric constant superlattice with controlling parameters) in some types of amorphous materials.

In this subsection, we shall give mathematical formulations for dielectric constant in the presence of an standing low electrical field. A particular attention will be pointed to investigation of dispersion properties of dielectric permittivity function where generation of collective orientation effects is possible under the standing low electromagnetic field.

The starting point in our discussion will be Clausius-Mossotti relation for dielectric constant. It is known that in isotropic mediums (as well as in the crystals with cubic symmetry) the dielectric constant is well described by the Clausius-Mossotti formula (see for example [26, 27, 28])

\[
\frac{\varepsilon_s - 1}{\varepsilon_s + 2} = \frac{4\pi}{3} \sum_m N_m^0 \alpha_m^0,
\]

where \(N_m^0\) is the concentration of particles (electrons, atoms, ions, molecules (or dipoles)) with the given \(m\) types of polarizabilities and \(\alpha_m^0\) correspondingly are coefficients of polarizabilities. From this formula it follows that the static dielectric constant \(\varepsilon_s\) depends on the polarizabilities properties of the particles as well as on their topological order. In the external alternating field the homogeneity and the isotropy of the medium is often lost. Then, it is expected that the formula (1) will be applicable after the minor generalization.

The object of our investigation will be solid state dielectrics type of amorphous silicon dioxide \(a - SiO_2\). According to numerical ab initio simulations [29], the structure of this type compound is well described by 3D random network (see Fig. 1.)

The white and black nodes in this figure correspond to different atoms while links between them correspond to covalent bonds. The redistribution of charges in outer electronic shells takes place because of the asymmetry of the bounded atoms. As a result some atoms acquire positive charge while others acquire negative ones. Thus this type of compound can be treated as a disordered 3D rigid dipoles system (see Fig. 2). Below this system will be called a 3D disordered spin system. For the description of amorphous media we use 3D lattice with the lattice constant \(d_0(T) = \{m_0/\rho_0(T)\}^{1/3}\), where \(m_0\) is the molecule mass, \(\rho_0\) is the...
density and $T$ is the temperature. The lattice contains one random spin per elementary cell. Note that it has random direction as well as random location inside the cell. Suppose now that with the help of external electromagnetic filed a standing wave is formed in the medium:

$$E(x; E_0, \Omega, \varphi_0) = 2E_0 \sin(\varphi_0) \cos(kx), \quad \varphi_0 = \Omega t_0,$$  \hspace{1cm} (2)
where \( \varphi_0 \) and \( t_0 \) are correspondingly the initial phase and time, \( \Omega \) is the wave frequency, \( k = 2\pi/\lambda_s \) and \( \lambda_s \) is the wavelength (see Fig. 3). The following natural question arises. How
does dielectric constant change on scale of wavelength period and on time interval \( \Delta t \ll \Omega^{-1} \sim 10^{-9} \text{sec} \) when the relaxation time of molecular dipoles is \( \tau \sim 10^{-11} \div 10^{-12} \text{sec} \ll \Omega^{-1} \). This question is important because faster processes, like the transition or Cherenkov radiation, can go in the media. Note, that the time of relativistic electron is passing the wavelength \( (\lambda_s \sim 10^{-4} \text{cm}) \) of standing wave and the time of formation of transition or Cherenkov photons in this layer is smaller than \( 10^{-15} \text{sec} \). This time interval is essentially lower than the time during which the standing wave is stationary. Since the wavelength is supposed to be much bigger than the inter-dipole distance \( \lambda_s \gg d_0 \), the Clausius-Mossotti relation still remains true. In this case the main problem is to calculate the polarizability coefficient related to orientational effects.

Taking into account the external field, one can express the polarization of the matter at arbitrary point as the macroscopic self-consistent relation:

\[
P(\vec{r}, d_0(T)) = \sum_i \vec{p} (\vec{l} - \vec{r}) = \sum_i \left[ \sum_m n_m \alpha_m (\vec{l} - \vec{r}) \vec{E}_{\text{loc}} (\vec{l} - \vec{r}) \right], \quad \vec{l} \equiv (l_x, l_y, l_z), \quad (3)
\]

where \( \vec{l} \) is the vector of 3D lattice, \( \vec{p} \) correspondingly the dipole moment of molecule. The second equation in (3) contributes in the value of the dipole moment (spin). Note, that the number of the carriers of given polarization type in elementary cell is \( n_m \sim (d_0(T))^{-3}, \alpha_m \)
coefficients of corresponding types polarizabilities taking into account the external field and \( \vec{E}_{\text{loc}} \) is local field, the effective field, at the site of an individual molecule that causes the induced polarization. Each effect adds linearly to the net dipole moment per molecule which is a fact verified by experiments. Under the action of external field different polarization types arise in media. However as the simple analysis shows the value of polarizabilities coefficient determined by the orientational effects is essentially higher than others.

Note that the elastic orientational polarizability coefficient in amorphous media \( \alpha_{\text{dip}}(\vec{l} - \vec{r}) \) randomly depends on the cell location. This fact is the consequence of random orientation of the local field strengths \( \vec{E}_{\text{loc}}(\vec{l} - \vec{r}) \) with respect to the external field \( \vec{E}(x; E_0, \Omega, \varphi_0) \). Therefore all the terms in the right side of (1) are basically known and well studied in literature (see for example [26, 27, 28]) except from those which are connected with orientational effects.

The orientational effects have a collective nature and are characterized by average value of random sum \( \sum_{\vec{l}} \alpha_{\text{dip}}(\vec{l} - \vec{r}) \) (sum of random coefficients of orientational polarizabilities).

Multiplying both sides of the relation (3) on the external field, we obtain:

\[
P(\vec{r}, g) \vec{E}(x, g) = -\delta U(\vec{r}, g) = \sum_{m} \sum_{\vec{l}} \alpha_{\text{dip}}(\vec{l} - \vec{r}) \vec{E}_{\text{loc}}(\vec{l} - \vec{r}) \vec{E}(x; E_0, \Omega, \varphi_0),
\]

where \( -\delta U(\vec{r}, g) \) describes the potential energy of amorphous matter in the external field, symbol \( g \) shows parameters of standing wave \( (E_0, \Omega, \varphi_0) \) (controlling parameters). Later we will consider the statistical properties of the medium in the direction of wave propagation.

Taking into account formula (4), one can obtain the following expression for the part of potential energy of the 3D spin system, which is related with orientation effects in external field:

\[
-\delta U_{\text{dip}}(\vec{r}, g) = \sum_{\vec{l}} \alpha_{\text{dip}}(\vec{l} - \vec{r}) \vec{E}_{\text{loc}}(\vec{l} - \vec{r}) \vec{E}(x; E_0, \Omega, \varphi_0).
\]
FIG. 4: The steric 1D random spin chain system.

$$-\delta U_{L_x}(\vec{l}_\perp | \vec{r}, g) = \sum_{l_x} \alpha_{dip}(\vec{l} - \vec{r}) \vec{E}_{loc}(\vec{l} - \vec{r}) \vec{E}(x; E_0, \Omega, \varphi_0), \quad \vec{l}_\perp \equiv \vec{l}_\perp(l_x, l_y),$$

(6)

where $-\delta U_{L_x}(\vec{l}_\perp | \vec{r}, g)$ is the interaction potential between the steric spin chain and external field. First, we take the mean value of the potential $-\delta U_V(\vec{r}, g)$ on $(y, z)$ plane:

$$\lim_{S_\perp \to \infty} \frac{1}{S_\perp} \int \delta U_V(x, g) dS_\perp = \lim_{S_\perp \to \infty} \frac{1}{S_\perp} \sum_{\vec{l}_\perp} \delta U_{L_x}(\vec{l}_\perp | x, g) dS_\perp = \langle \delta U_{L_x}(x; g) \rangle_\perp,$$

where $S_\perp = L_y \times L_z$ and $\langle ... \rangle_\perp$ is averaging over all possible stable steric 1D spin configurations. The integral corresponds to the potential energy density $-\langle \delta U_{L_x}(x; g) \rangle_\perp$ on $(y, z)$ plane depends on the distance $x$. Taking into account the fact that the distribution of spins in the plan $(y, z)$ is random but isotropic (see Fig. 5) it is simple to prove that in the limit of $S_\perp \to \infty$ it becomes the full self-averaging of spin system.

FIG. 5: The projection of 3D disordered spin system on the plan $(y, z)$. 
This implies that one may use Birgoff ergodic hypothesis \[30\] and averages by plan \((y, z)\) interaction potential \(-\langle \delta U_{L_x}(x; g) \rangle_{(\hat{\mathbf{z})}}\) obtains also by the integration of chain’s intrinsic energy distribution:

\[
\langle \delta U_{L_x}(x, g) \rangle_{(\hat{\mathbf{z})}} = \frac{\int_{-\infty}^{0} \delta U_{L_x}(E|x, g) Z(E; g) dE}{\int_{-\infty}^{0} Z(E; g) dE},
\]

where \(-\delta U_{L_x}(E|x, g)\) shows the interaction potential energy between some steric spin chain with energy of \(E\) and external field \(\mathbf{g}\), and \(Z(E; g)\) denotes the energy distribution function (partition function) of 1D steric spin chain configurations. The definition of distribution function will be given in section 4. Recall, that in \((7)\) we take into account only negative values of \(E\), because only for these values the spin chains maybe stable.

Taking the average value of expression \((7)\), we obtain the following relation for the mean value of the potential increment:

\[
\langle \delta U_{V}(\vec{r}; g) \rangle_{V} = -\langle \delta U_{L_x}(E|x, g) \rangle_{(x, \hat{\mathbf{z})}} = \overline{\alpha}_{\text{ch}} \langle \vec{E}^2(x; \mathbf{g}) \rangle_x,
\]

\[
\langle \delta U_{L_x}(E|x, g) \rangle_{(x, \hat{\mathbf{z})}} = \frac{1}{L_x} \int_{0}^{L_x} \langle \delta U_{L_x}(E|x, g) \rangle_{(\hat{\mathbf{z})}} d x,
\]

where the bracket \(\langle ... \rangle_x\) means the integration over \(x\) on a scale \(L_x\), the mean value of a full averaging potential energy of interaction between the spin chains and the external field is defined by term \(-\langle \delta U_{L_x}(E|x, g) \rangle_{(x, \hat{\mathbf{z})}}\).

**Definition 1.** In the relation \((8)\) parameter \(\overline{\alpha}_{\text{ch}}\) will be named a collective polarization coefficient of steric 1D spin chain. It is given by formula:

\[
\overline{\alpha}_{\text{ch}} = -\frac{\langle \delta U_{L_x}(E|x, g) \rangle_{(x, \hat{\mathbf{z})}}}{\langle \vec{E}^2(x; E_0, \Omega, \phi_0) \rangle_x}.
\]

Note, that \(\overline{\alpha}_{\text{ch}}\) is complex amount and describes the average value of steric spin chain polarizability with taking into account the lattice relaxation. When \(E_0 \to 0\) it is simple to show that \(\overline{\alpha}_{\text{ch}} \to 0\).

Now one can write down the expression for the sum in the right part of the relation \((1)\), which takes into account the spin chains orientation effects in the external field:

\[
\sum_{m} N_m \alpha_m = \sum_{m} N^0_m \alpha^0_m + N_{\text{ch}} \overline{\alpha}_{\text{ch}} = \sum_{m} N^0_m \alpha^0_m - n_{\text{dip}} N_x^{-1} \langle \delta U_{L_x}(E|x, g) \rangle_{(x, \hat{\mathbf{z})}} \langle \vec{E}^2(x; E_0, \Omega, \phi_0) \rangle_x,
\]
where \( N_{ch} = n_{dip}N^{-1} = (d_0(T))^{-3}N^{-1} \) -concentration of steric spin chains. From (10) it follows that in areas where the field strength is small the orientation correction vanishes. In other words the external field on wavelength scale (2) creates the alternating inhomogeneities with different dielectric constants. These layers are stable in nanosecond scale \( \Delta t \sim 10^{-10} \text{sec} = 0.1 \text{ns} \).

Using (10) we can generalize Clausius-Mossotti equation on the space-time scale of standing wave taking into account orientation effects:

\[
\frac{\varepsilon_{st}(g) - 1}{\varepsilon_{st}(g) + 2} = \Lambda(g), \quad \text{where} \quad \Lambda(g) \approx \frac{4\pi}{3} \sum_m N^0_m \alpha^0_m - \frac{1}{d^3_0N_x} \cdot \frac{\langle \delta U_{L_z}(E|x, g) \rangle(x, \uparrow)}{\langle E^2(x; E_0, \Omega, \varphi_0) \rangle_x}.
\]

(11)

Note that \( \varepsilon_{st} \) is the label of stationary dielectric constant.

Now we turn to the study of the dispersion property of frequency-depending dielectric constant (dielectric permittivity function).

In the theory of dielectric relaxation, one writes the frequency-dependent dielectric constant \( \varepsilon(\omega) \) with Williams-Watts [31] function of dielectric relaxation by relation [32]:

\[
\frac{\varepsilon(\omega) - \varepsilon_\infty}{\varepsilon_s - \varepsilon_\infty} = \varrho(\sigma, \omega), \quad \varrho(\sigma, \omega) = -\int_0^\infty e^{-\lambda s - s^\sigma} ds, \quad \lambda = \omega\tau, \quad s = t/\tau.
\]

(12)

\( \varepsilon_\infty = \varepsilon(\omega \to \infty) \) is the high-frequency limit of the dielectric constant and \( \varepsilon_s = \varepsilon(\omega \to 0) \), the static dielectric constant which can be defined from generalized Clausius-Mossotti equation. In (12) the function \( F(t) \) describes the decay of polarization of a dielectric sample with time after sudden removal of steady polarizing electric field. The frequency-dependent dielectric constants of broad class of materials including polymeric systems and glasses may be interpreted in terms of the Williams-Watts [31] polarization decay function:

\[
F_\sigma(t) = \exp[-(t/\tau)^\sigma], \quad 0 < \sigma < 1,
\]

(13)

where exponent \( \sigma \) and the time constant \( \tau \) depending on the material and fixed external conditions such as temperature \( T \) and pressure.

The relation [12] can be generalized for case of external field as we are interested in the time scale lesser than the time interval during which an external standing electrical wave may be considered stationary. After substitution \( \varepsilon(\omega) \to \varepsilon(\omega, g) \) and \( \varepsilon_s \to \varepsilon_{st}(g) \) the Eq. [12] we can transform and write in the form:

\[
\frac{\varepsilon(\omega, g) - \varepsilon_\infty}{\varepsilon_{st}(g) - \varepsilon_\infty} = \varrho(\sigma, \omega), \quad \varrho(\sigma, \omega) = -\sigma \int_0^\infty e^{-\lambda s - s^\sigma} s^\sigma ds, \quad \lambda = \omega\tau, \quad s = t/\tau.
\]

(14)
For example in Debye’s classical theory of dielectric relaxation \( \sigma = 1 \), the integral in the right side of (12) has the form \((1 + i\omega \tau)^{-1}\).

### III. AVERA GE INTERACTION POTENTIAL BETWEEN STERIC SPIN CHAIN AND EXTERNAL FIELD

As we have already shown in order to take into account the contribution of orientation effects into the polarization one has to calculate the total average value of the interaction potential of steric spin chain with the external field \(-\langle \delta U_{L_x}(E_0, \Omega, \varphi_0) \rangle_{(L_x, \mathbf{g})}\).

Taking into account the fact that the external field is low i.e. \(|\bar{E}(l_x - x)| \ll |\bar{E}_{loc}(l_x - x)| \approx |\bar{E}_{int}(l_x - x)|\), we can apply the Taylor decomposition of dipole angular momentum:

\[
\bar{p}(l_x - x) \simeq \bar{p}^0(l_x - x) + \delta \bar{p}(l_x - x), \quad \delta \bar{p}(x) \sim \bar{E}(x; E_0, \Omega, \varphi_0),
\]

where \(|\delta \bar{p}(x)| \ll |\bar{p}^0(x)|\), as well as \(\bar{E}_{int}^0(x - l_x)\) and \(\bar{p}^0(x - l_x)\) are correspondingly the field strength and the dipole angular momentum of the molecule, located in the \(l_x\)-th cell in the absence of the external field. In case when the coordinate \(x\) is outside of \(l_x\)-th cell, the field vanishes. Inside the cell they have constant values. From the discussion above and taking into account (7), the relation (15) can be represented in the form:

\[
-\delta U_{L_x}(E|x, \mathbf{g}) = \sum_{l_x=0}^{L_x} \bar{p}^0(l_x - x)\bar{E}(x; E_0, \Omega, \varphi_0) + \sum_{l_x=0}^{L_x} \delta \bar{p}(l_x - x)\bar{E}(x; E_0, \Omega, \varphi_0).
\]

Using the fact that without an external field the spin system has no polarization we conclude that first sum in (16) vanishes. In other words, the interaction potential has the following form:

\[
-\delta U_{L_x}(E|x, \mathbf{g}) = \sum_{l_x=0}^{L_x} \delta \bar{p}(l_x - x)\bar{E}(x; E_0, \Omega, \varphi_0).
\]

Now we turn to the equation of motion for the steric spin-chain with relaxation in 3D spin lattice in external field. Recall that the interaction potential \(-\delta U_{L_x}(E|x, \mathbf{g})\) (see (11) and (17)) between 1D disordered spin chain and external field does not take into account relaxation with environmental spin chains. It is possible only after solution of the dynamical problem. The resulting interaction potential in this case will be complex, where the imaginary part characterizes the relaxation processes in 3D lattice.
In more general case those motions can have quantum characters and can be represented by complex Langevin-Schrödinger type stochastic differential equation:

$$\lambda \delta U_L(x; g) = \lambda E + \Psi^{-1}(dt)^2 \Psi,$$  \hspace{1cm} (18)

where

$$t = x/d_0, \quad \lambda = 2\mu/(\hbar^2d_0^2), \quad \mu = m_0/N^{1/(N-1)}, \quad (dt)^2 = d^2/dt^2,$$

$m_0$ and $\mu$ are the molecule (spin) mass and the spin chain's effective mass correspondingly, $t$ denote of natural parameter of evolution along the spin chain. In the equation (18) interaction potential $U_L(x; g)$ is a random complex function. Below we present its detailed description.

Substituting

$$\Psi(t) = \exp\left(\int_0^t \Xi(t')dt'\right),$$ \hspace{1cm} (19)

into (18) and using the relations (19), we obtain the following nonlinear complex SDE [34]:

$$\Xi_t + \Xi^2 + \lambda(E - \bar{\nabla}) + \lambda \bar{f}(t) = 0, \quad \Xi(t) = \theta(t) + i\vartheta(t), \quad \Xi_t = d\Xi/dt,$$ \hspace{1cm} (20)

where

$$\sum_{l_x=0}^{L_x} \vec{p}(l_x - x) \bar{E}(x; E_0, \Omega, \varphi_0) = \bar{\nabla} + \bar{f}(t), \quad \bar{\nabla} = \left< \sum_{l_x=0}^{L_x} \delta \vec{p}(l_x - x) \bar{E}(x; E_0, \Omega, \varphi_0) \right>_x.$$ \hspace{1cm} (21)

In formulas (20) and (21) we have denoted by $\bar{\nabla}$ the mean value of the sum, and by $\bar{f}(t)$ its complex random part. Analyzing the contribution from different mechanisms of molecule polarization in glass medium, we conclude that under the influence of the external field $\vec{E}$ with frequency $\Omega \sim 10^9 Hz$ the main part comes from elastic dipole (dipole thermal polarization is not essential in this case due to the large relaxation time $\tau \sim 10^{-4} \div 10^{-5} sec$ [26], [35]). Let us note that the coefficient of elastic dipole polarization at low external fields is determined by

$$\alpha_{dip}(l_x - x) = \Lambda^{-1}\left(p^0 \sin[\beta(l_x - x)]\right)^2 = \frac{p^0}{E^0_{int}} \sin^2[\beta(l_x - x)],$$

$$\delta \vec{p}(l_x - x) = \alpha_{dip}(l_x - x) \bar{E}(x; E_0, \Omega, \varphi_0),$$ \hspace{1cm} (22)

where $\beta(l_x - x)$ is the angle between the external $\bar{E}(x; E_0, \Omega, \varphi_0)$ and the internal $\bar{E}^0_{int}(l_x - x)$ fields, $\Lambda(l_x) = \bar{p}^0(l_x) \bar{E}^0_{int}(l_x) \cong p^0E^0_{int}$ is the dipole energy in the field $\bar{E}^0_{int}$. Following
heuristic argumentation of Debby [20], [31, 32] one can write down the expression for the elastic dipole polarization, which takes into account the spin (polar molecule) relaxation process in the glass:

\[
\Upsilon_{ch}(l_x - x) = \alpha_{dip}(l_x - x)/(1 - i\Omega\tau) \approx \frac{p^0 \sin^2[\beta(l_x - x)]}{E^0_{int}(1 - i\Omega\tau)}, \tag{23}
\]

where \(\tau\) is the spin relaxation time in the glass. It is very small in the aforementioned media \(\tau \sim 10^{-11} \div 10^{-15}\) sec (see, for example [20]).

So, the equation (18) with generalized coefficient of elastic dipole polarizability (23) will describe the motion of 1D random steric spin chain in the external field with relaxation.

After substitution of (23) in (21) and simple calculations we obtain:

\[
\mathbf{V} = \left\langle \sum_{l_x=0}^{L_x} \Upsilon_{ch}(l_x - x) \mathbf{E}^2(x; E_0, \Omega, \phi_0) \right\rangle_x = -\frac{1 + i\Omega\tau}{2} \frac{N_x p^0 E^2_0}{4 d_3^0 E^0_{int}} \left(1 - \frac{\sin(2kL_x)}{2kL_x}\right), \tag{24}
\]

where \(E_0(\phi_0) = 2E_0 \sin \phi_0\).

Let us now investigate the properties of the random function \(\mathbf{f}(t)\). From the relations (21) and (23) it is easy to find the random strength:

\[
\mathbf{f}(t) = -\frac{1 + i\Omega\tau}{1 + (\Omega\tau)^2} \cdot \frac{p^0 E^2_0}{4 d_3^0 E^0_{int}} \xi(t), \quad \xi(t) = \left(1 + \cos(2k_t t)\right) \sum_{l_x=0}^{L_x} \cos 2\beta(l_t - t)
\]

\[
\beta(l_t - t) = \beta(l_x - x), \quad k_t = h^{-1} \sqrt{2}\mu k, \quad l_t = hl/\sqrt{2}\mu. \tag{25}
\]

If the phase \(\beta\) is homogeneously distributed along the interval \([0, \pi]\), then for the mean value we obtain:

\[
M\mathbf{f}(t) = \langle \mathbf{f}(t) \rangle = 0, \quad M\xi(t) = \langle \xi(t) \rangle = 0. \tag{26}
\]

For the autocorrelation function one can write the following expression (see, for example [37]):

\[
R_{ff}(\tau) = \langle \mathbf{f}(t)\mathbf{f}(t') \rangle = \frac{1}{2} \left(\frac{1 + i\Omega\tau}{1 + (\Omega\tau)^2}\right)^2 \left(\frac{N_x p^0 E^2_0}{4 d_3^0 E^0_{int}}\right)^2 (M\xi^2(\tau)) \tag{27}
\]

where \(\tau = t - t'\). Substituting (23) in (27) and carrying out straightforward calculations, we may obtain:

\[
R_{ff}(\tau) = \langle \mathbf{f}(t)\mathbf{f}(t') \rangle = 2D\delta(t - t'), \quad D = \frac{1}{4} \left(\frac{1 + i\Omega\tau}{1 + (\Omega\tau)^2}\right)^2 \left(\frac{N_x p^0 E^2_0}{4 d_3^0 E^0_{int}}\right)^2. \tag{28}
\]
So, we have shown that the dispersion equation (16), which describes the relation between the external field and the local polarization in disordered system, is reduced to the investigation of the Langevin type nonlinear complex stochastic SDE with the auto correlation function (26)-(28) of white noise. For further investigation, it is convenient to represent the complex equation (20) as a system of two real equations:

\[
\dot{\theta} + \theta^2 - \vartheta^2 + \lambda (E - V_r + f_r(t)) = 0, \quad (29)
\]

\[
\dot{\vartheta} + 2\theta \vartheta + \lambda (V_i + f_i(t)) = 0, \quad (30)
\]

where \(\dot{\theta} = d_\theta \theta, \dot{\vartheta} = d_\theta \vartheta, V_r = Re\vec{V}, V_i = Im\vec{V}, f_r(t) = Re\vec{f}(t), \) and \(f_i(t) = Im\vec{f}(t)\).

Now the problem is to find of evolution equation for the conditional probability:

\[
Q(\theta, \vartheta, t|\theta_0, \vartheta_0, t_0) = \left\{ \delta(\theta(t) - \theta(0))\delta(\vartheta(t) - \vartheta(0)) \right\}_{\{\theta_0 = \theta(t_0); \vartheta_0 = \vartheta(t_0)\}}, \quad (31)
\]

describing the probability of fact that, the trajectory \((\theta_0 \equiv \theta(t_0), \vartheta_0 \equiv \vartheta(t_0))\) at the initial moment leaving of natural parameter \(t_0\) from the point \((\theta_0, \vartheta_0)\), at the arbitrary moment of \(t\) it will be turned out to the suburb of point \((\theta, \vartheta)\). Subject to SDE system (29), (30) the Fokker-Plank equation can be easily found (33) (see also (34)):

\[
\frac{\partial Q}{\partial t} = D_r \frac{\partial^2 Q}{\partial \theta^2} + D_i \frac{\partial^2 Q}{\partial \vartheta^2} + (\theta^2 - \vartheta^2 + \lambda (E - V_r)) \frac{\partial Q}{\partial \theta} + (2\theta \vartheta + \lambda V_i) \frac{\partial Q}{\partial \vartheta} + 4\theta Q, \quad (32)
\]

where \(Q \equiv Q(E|\theta, \vartheta; t), D_r = \lambda^2 Re\vec{D} \) and \(D_i = \lambda^2 Im\vec{D}\). Note, that the solution of equation (32) must satisfy the initial condition:

\[
Q(E|\theta, \vartheta; t)|_{t=t_0} = \delta(\theta - \theta_0)\delta(\vartheta - \vartheta_0), \quad (33)
\]

where the initial phases \(\theta_0\) and \(\vartheta_0\) are equal to zero. Later we will be more interested in the fixed limit of the solution (32), which is obviously received under values \(t \gg \Delta t = O(1)\) which is equivalent to the condition \(t \to \infty\). In that limit the equation (32) is simplified and accurate to the value \(O(\Delta t/t_{N_x}) \ll O(1)\) may take the following form:

\[
D_r \frac{\partial^2 Q_s}{\partial \theta^2} + D_i \frac{\partial^2 Q_s}{\partial \vartheta^2} + (\theta^2 - \vartheta^2 + \lambda (E - V_r)) \frac{\partial Q_s}{\partial \theta} + (2\theta \vartheta + \lambda V_i) \frac{\partial Q_s}{\partial \vartheta} + 4\theta Q_s = 0, \quad (34)
\]

where \(Q_s \equiv Q_s(E|\theta, \vartheta) \equiv \lim_{t \to \infty} Q(\theta, \vartheta, t)\) and \(t_{N_x} = N_x\).

The (34) is an elliptic type differential equation for which there are not real characteristics. This implies that the type of quasilinear equation depends on which solution is considered.
and can be different for different solutions. The solution of the equation \(34\) must satisfy the conditions:

\[ Q_s \Big|_S = \frac{\partial Q_s}{\partial n} \Big|_S = 0, \quad |n| = (\theta^2 + \vartheta^2)^{1/2}, \quad (35) \]

where the border condition is set on a curve \(S\) and \(n\) is correspondingly the normal of curve.

Now one can write the value of the average interaction potential:

\[
\langle \delta U_L (E|x, g) \rangle_x = E + \frac{1}{\lambda} \lim_{t \to \infty} \frac{1}{t} \int_0^t \left( \dot{\theta} + i \dot{\vartheta} + \theta^2 - \vartheta^2 - i 2 \theta \vartheta \right) dt
\]

\[
= E + \frac{1}{\lambda} \lim_{t \to +\infty} \frac{1}{t} \int_0^{+\infty} \left( \dot{\theta} + i \dot{\vartheta} + \theta^2 - \vartheta^2 - i 2 \theta \vartheta \right) dt + O \left( \frac{\Delta t}{t N_x} \right). \quad (36)
\]

Using the Birgoff ergotic hypothesis \([30]\) we can change integration by natural parameter \(t\) to integration by stationary distribution \(Q_s(E|\theta, \vartheta)\) and finally find the following expression for mean value of the potential energy increment:

\[
\langle \delta U_L (E|x, g) \rangle_x = E + \frac{1}{\lambda R} \int_{-\infty}^{+\infty} \left[ \theta^2 - \vartheta^2 - i 2 \theta \vartheta \right] Q_s(E|\theta, \vartheta) \, d\theta \, d\vartheta + O(N_x^{-1}), \quad (37)
\]

where \(R = \int_{-\infty}^{+\infty} Q_s(E|\theta, \vartheta) \, d\theta \, d\vartheta\) denoted normalization constant.

### IV. STATISTICAL MECHANICS OF THE STERIC 1D RANDOM SPIN CHAIN

Before turning to the effective statical dielectric constant calculation the two important problems have to be solved. The first one is the calculation of the partition function \(Z(E; g)\) for the steric 1D random spin chain with energy \(E\), whereas the second one is the determination of the thickness of layers on a scale of this chain (i.e. the fragmentation of the media on layers with different dielectric constant).

First, let us consider the statistical properties of the 1D steric spin chain. As it is shown, the type of a compound \((a - SiO_2)\) can be used as a canonical model for such network glasses describing continuous random networks of atom and bonds \([38], [29]\) (see Fig 1). It means that Heisenberg spin-glass Hamiltonian \([39]\) can be suitable for describing of 3D disordered spin system. Note that this allows to use the ergodic hypothesis (see \([7]\)) and make reduction of 3D spin lattice dynamical problem to the problem of 1D complex Langevin-Schrödinger type stochastic differential equation (see \([18], [20]\)) and the problem of construction statistical mechanics of steric disordered spin chain system. As it is easy to see, this type of reduction in a certain sense is conditional because both problems are interrelated.
For further investigations it is favorable to consider the spin-glass Hamiltonian of type:

\[ H(N_x; g) = - \sum_{ij} J_{ij}(r_{ij}) \vec{S}_i \cdot \vec{S}_j + p^0 \sum_i \vec{E}_i \vec{S}_i, \quad \vec{S}_i \equiv \vec{p}_i^0, \]  

(38)

where \( r_{ij} = |i - j|d_0(T) + \eta_{ij} \) is a distance between 1D spins \( \vec{S}_i \) and \( \vec{S}_j \) (classical vectors of unit length), \( \eta_{ij} \) is its random part, subject to Gaussian distribution with zero mean value and unit variance, \( J_{ij} \) is nearest-neighbor interaction constant, which depends on the distance between spins. It may be positive or negative, the external field \( \vec{E}_i \) is defined by means of formulas (2).

Now the problem is the construction of the energy distribution function of spin-chains \( Z(E; g) \). However, the problem is that the time scale, on which we make statistical study of the system, is very short (\( \lesssim 10^{-10} \text{sec} = 0.1 \text{nsec} \)), while the characteristic thermal relaxation time in amorphous media \[26\] is of order \( \Omega_T^{-1} \sim 10^{-4} \div 10^{-5} \text{sec} \), where \( \Omega_T \) is the frequency of thermal fluctuations. The last fact means that the temperature and related thermodynamical constructions become meaningless in our problem. Nevertheless, some structural similarity between gas and amorphous media is evident. At that a steric spin chain will correspond to an atom in gas. Since in the equilibrium state the average energy value per atom in the gas is \( \frac{3}{2} kT \), a corresponding value in this case will be the chain’s energy in the equilibrium state (chain’s energy without external field). However, the considered system has a specific peculiarity. The point is that the equilibrium state in gas is characterized by one temperature whereas the spin system can be in the equilibrium state in any negative energy. These energies coincide with the local minimums of non-perturbed Hamiltonian and as it is well-known their numbers can be high \[39, 40\]. In other words, in this case the phase space can be decomposed uniquely into micro-canonical states associated with different thermodynamic equilibrium states \[41\].

The Hamiltonian (38) in case of the external field absence can be rewritten in spherical coordinate system as follows:

\[ H_0(d_0(T), N_x) = \sum_{ij=1, i \neq j}^{N_x} J_{ij}(r_{ij}) \left( \cos \psi_i \cos \psi_j \cos(\phi_i - \phi_j) + \sin \psi_i \sin \psi_j \right), \]  

(39)

For the determination of local minimums one has to solve the following algebraic equations:

\[ \Phi_{\psi_i}(\Theta) + \frac{\partial H}{\partial \psi_i} = \sum_{j=1}^{N_x} J_{ij}(r_{ij}) \left( - \sin \psi_i \cos \psi_j \cos(\phi_i - \phi_j) + \cos \psi_i \sin \psi_j \right) = 0, \]
\[ \Phi_{\phi_i}(\Theta) = \frac{\partial H}{\partial \phi_i} = -\sum_{j=1}^{N_x} J_{ij}(r_{ij}) \cos \psi_i \cos \psi_j \sin(\phi_i - \phi_j) = 0, \]

\[ \Phi_{r_{ij}}(\Theta) = \frac{\partial H}{\partial r_{ij}} = \sum_{j=1}^{N_x} \frac{\partial J_{ij}}{\partial r_{ij}} \left( \cos \psi_i \cos \psi_j \cos(\phi_i - \phi_j) + \sin \psi_i \sin \psi_j \right) = 0, \] (40)

where \( \Theta_i = (\psi_i, \phi_i) \) are angles of \( i \)-th spin (\( \psi_i \) is the polar and \( \phi_i \) azimuthal angles), \( \Theta = (\Theta_1, \Theta_2, ..., \Theta_{N_x}) \) correspondingly describe the angular part of spin configuration. Now suppose that non-perturbed Hamiltonian for fixed averaged distance between spins \( d_0(T) \) has \( n \) local minimums of function \( E(d_0(T)) \), each corresponds to \( M_j \) spin configurations \( \{ \Theta^{(\nu)}_i \} \), where \( i = 0, 1, ..., M_\nu \). The number of all configurations, which correspond to local equilibrium states \( M_{\text{full}} = \sum_{j=1}^{N_x} M_\nu \). Correspondingly, the weight of every equilibrium state may be defined by formulas:

\[ P_\nu(E_\nu; d_0(T)) = M_\nu / M_{\text{full}}, \quad \sum_{\nu=1}^{N_x} P_\nu(E_\nu; d_0(T)) = 1. \] (41)

Thus we can propose to use the following statistical weight instead of the canonical distribution at multi-equilibrium state:

\[ W_\nu(H; g, \eta) = P_\nu(E_\nu; d_0(T)) \exp \left\{ -\frac{H(N_x; g)}{E_\nu(d_0(T))} \right\}, \] (42)

where \( E_\nu(d_0(T)) \) is the energy of spin chain in the absence of an external field. Recall, that the multicanonical ensemble was introduced in the \[42\] as an approach to simulate a strong first-order phase transitions. It has been very successful in sampling a large barrier.

Now, taking into account (39)-(42), one can present the expression of the energy distribution function for \( N_x \) spin system (this expression can be explained as a local partition function in above mentioned multicanonical thermodynamics):

\[ Z_\nu(E_\nu; g, \eta) = P(E_\nu; d_0(T)) \int \frac{d\Omega_1}{4\pi} \cdots \frac{d\Omega_{N_x}}{4\pi} \exp \left\{ -\frac{H(N_x; g)}{E_\nu(d_0(T))} \right\}, \] (43)

where \( \eta \) describes the set of random distances \( \eta_{ij} \) and random angles \( \beta_i \), correspondingly \( d\Omega_i \) is an element of the solid angle \( \Omega_i \) containing the unit vector \( \vec{S}_i \).

Energy distribution function of spin chain can be essentially simplified after the suggestion that in Hamiltonian (38) only nearest-neighboring spins interact, i.e. \( J_{ij} = 0 \) if \( |j - i| > 2 \). In this case the multidimensional integral can be taken exactly as follows. The integration starts from the end of the chain. When integrating over \( d\Omega_i \) we take as a polar axis the
direction of the vector \((J_{i(i-1)}\vec{S}_i + p^0\vec{E}_i)\). Then it is easy to obtain the following expression:

\[
Z_\nu(E_\nu; g, q) = P_\nu \prod_{i=1}^{N_x} \left[ \frac{1}{2} \int_0^\pi \exp\{K_{i(\nu)} \cos \psi \} \sin \psi \, d\psi \right] = P_\nu(E_\nu; d_0(T)) \prod_{i=1}^{N_x} \frac{\sinh K_{i(\nu)}}{K_{i(\nu)}},
\]

where \(\beta_i\) is the random angle between the vectors \(\vec{S}_{i-1}\) and \(\vec{E}_i\). Supposing that the distribution of spin \(\vec{S}_i\) around field \(\vec{E}_i\) direction is isotropic, one can perform an integration by angle \(\beta_i\). After simple integration with the help of formula (43):

\[
E_i(ax) = \int \frac{e^{ax}}{x} \, dx = \ln |x| + \sum_{k=1}^{\infty} \frac{(ax)^k}{k!k}, \quad a \neq 0,
\]

we can find:

\[
Z(E; g, \eta) = P(E; d_0(T)) \prod_{i=1}^{N_x} \left[ \frac{1}{2} \int_0^\pi \frac{\sinh K_i}{K_i} \sin \beta_i \, d\beta_i \right]
= P(E; d_0(T)) \prod_{i=1}^{N_x} \frac{1}{4a_{i(\nu)}} \left\{ \left[ E_i(b_i + a_i) - E_i(-b_i - a_i) \right] - \left[ E_i(b_i - a_i) - E_i(-b_i + a_i) \right] \right\},
\]

\[
a_i = \frac{1}{E} 2p^0E_iJ_{(i-1)i}, \quad b_i = \frac{1}{E} \left[ J_{(i-1)i}^2 + (p^0)^2E_i^2 \right].
\]  

(45)

For simplification of expression (45) and below will be admit index \(\nu\). In the \(\eta\) symbol \(\eta\) describes the set of random distances, \(E_i(x)\) is the exponential integral function. It is simple that in spite of the fact that every member in parentheses near the points \(E^+_c = J_0/p^0\) or \(E^-_c = -J_0/p^0\) has a singularity nevertheless their sum is an analytical function.

In the paper [44] at the first time it was suggested that one may describe spin glass by a Hamiltonian of the type [8], where spins are put onto the sites of a regular lattice, and disorder is introduced by a suitable distribution \(W(J_{(i-1)i})\) of exchange bonds. A standard choice is the Gaussian Edwards-Anderson model [44] (see also [45]):

\[
W(J_{(i-1)i}) = \frac{1}{\sqrt{2\pi(\Delta J_{(i-1)i})^2}} \exp\left\{ -\frac{(J_{(i-1)i} - J_0)^2}{2(\Delta J_{(i-1)i})^2} \right\}
\]

\[
J_0 = \langle J_{(i-1)i} \rangle_{av}, \quad (\Delta J_{(i-1)i})^2 = \langle J_{(i-1)i}^2 \rangle_{av} - \langle J_{(i-1)i} \rangle_{av}^2.
\]  

(46)

Recall that for this model \(J_0\) and \(\Delta J_{(i-1)i}\) are independent on distance and scaled with spin number \(N_x\) as

\[
\langle J_{(i-1)i} \rangle_{av} = J_0 \propto N_x^{-1}, \quad \Delta J_{(i-1)i} \propto N_x^{-1/2}.
\]  

(47)
to ensure a sensible thermodynamic limit. Eqs (46) and (47) \( \langle ... \rangle_{av} \) describe the averaging procedure. Now we can conduct averaging of function (45) by distribution (46) and find the local partition function near some equilibrium energy \( E_\nu \) of disordered spin chain:

\[
Z(E; g) = \frac{1}{2a_i} \left\{ \left[ E_i(b_i + a_i) - E_i(-b_i - a_i) \right] - \left[ E_i(b_i - a_i) - E_i(-b_i + a_i) \right] \right\}. \tag{48}
\]

Taking into account the fact that on the scale of half-wavelength \( N_x \gg 1 \) as well as that \( G(E, J_i; g) \) is an analytical function for computation of integrals in partial partition function (48), the Laplace asymptotic method [46] may be used:

\[
Z(E; g) \approx P(E; d_0(T)) \prod_{i=1}^{N_x} \frac{1}{4a_i^0} \left\{ \left[ E_i(b_i^0 + a_i^0) - E_i(-b_i^0 - a_i^0) \right] - \left[ E_i(b_i^0 - a_i^0) - E_i(-b_i^0 + a_i^0) \right] \right\}
\]

\[a_i^0 = \frac{1}{E} 2p^0 E_i J_0, \quad b_i^0 = \frac{1}{E} \left[ J_0^2 + (p^0)^2 E_i^2 \right]. \tag{49}\]

It is simple to show that in the limit of \( E_i \to 0 \) the correspondence \( Z(E, g) \to P(E; d_0(T)) \) takes place.

Note, that averaging procedure in the framework of multicanonical ensemble is carried by the following formula:

\[
\langle Y(x, g) \rangle_{\uparrow \downarrow} = \lim_{N \to \infty} \frac{\sum_{\nu=1}^{N} Y(E_\nu|x, g) Z(E_\nu; g)}{\sum_{\nu=1}^{N} Z(E_\nu; g)} = \frac{\int_{-\infty}^{0} Y(E|x, g) Z(E; g) dE}{\int_{-\infty}^{0} Z(E; g) dE}, \tag{50}\]

where \( Y(x, g) \) is an averaged physical parameter.

For 1D random steric spin chain system in analogy to usual thermodynamics Helmholtz type free energy may be specified near the local equilibrium state with energy \( E_\nu \). In this case the free energy amount with the expectation of one spin and the per unit of equilibrium state energy \( E_\nu \) is defined:

\[
F(E; g) = -\frac{1}{N_x} \ln Z(E; g). \tag{51}\]

All macroscopic thermodynamic properties of 1D random steric spin chain can be obtained by free energy derivatives. The simple investigation of expression for free energy shows that in the steric 1D random spin chain model with the nearest-neighbor interaction depending
on amplitude of external field $E_0$ there are not phase transition phenomena. However in this case under the even low fields $E_i \sim 1/N_x$ the free energy of spin system essentially changes (see Fig 6).

![Graphs showing the Helmholtz free energy](image)

**FIG. 6:** The Helmholtz free energy for any average energy of spin chain $E_\nu$ (see figures (a),(b) and (c)) and amplitude of external field $E_0$. On the figures axis $k$ shows the parameter which is proportional to external fields amplitude $E_0$.

Particularly, the order parameter for disordered spin-glass medium is described by $\sum_i p_i^x$ and without the external field on the scale of the period of standing wave it is equal to zero, where $p_i^x$ is a projection of spin on the direction of external field propagation. It becomes non-zero at the weak external field $E_i \sim |E^\pm_0|$ due to a symmetry breaking. In this case nanoparticles (spin chains) with the super spins (macropolarizations) are generated on the microscale space and nanoscale time.

**V. DIELECTRIC PERMITTIVITY OF THE NEIGHBORING LAYERS**

Now we can examine the question of dividing the dielectric medium on the regions with extremely different polarizations. Recall that the polarized medium without an external field has a zero macroscopic polarization on the space scale $10^{-4} \div 10^{-5}cm$. After turning on the external field in the medium (on the scale of wavelength of standing electrical field) the amplification of the orientational effects and, as a result, the initiation of macroscopic
polarization is possible. Assuming that the collective orientation effects, generated in those regions of media, where voltage of external field \( E \) is of order to the critical value \( E_c \), we can divide the scale of wavelength of external field on the forth region. In the first and third regions the macroscopic polarization of medium is zero whereas in the second and forth regions it is different from zero (see Fig. 7). Note that the height of these layers may be computed with the help of numerical experiments.

\[ \text{FIG. 7: } \text{The regions with different polarizations and correspondingly different dielectric constants on the one wavelength } \lambda_s \text{ scale.} \]

Now, with calculation of effective dielectric constant at the half-wavelength scale of external electromagnetic field at the time intervals \( \Delta t \ll 2\pi/\Omega \). Using (11), we can obtain:

\[ \epsilon_{st}(g) = 1 + 2 \Lambda(g). \]  \hspace{1cm} (52)

Remember, that here we have taken into account the contribution from the elastic dipole polarization only. It is easy to see that the nominator of the expression (52) has no singularities and is bounded by the imaginary part of the potential increment (37). The important characteristic is the dielectric constant’s difference in neighboring layers. Taking into account the relations (11) and (52) one can get:

\[ \delta \epsilon_{st}(g) = \epsilon_{st}(g) - \epsilon_{st}(0) = (1 - \epsilon_{st}(0)) + \frac{3 \Lambda(g)}{1 - \Lambda(g)}. \]  \hspace{1cm} (53)

Dielectric permittivity of layer in the external field can be calculated simply:

\[ \epsilon_{st}(g, \omega) = \epsilon_{\infty} + (\epsilon_{st}(g) - \epsilon_{\infty}) \varrho(\sigma, \omega). \]  \hspace{1cm} (54)
From equation (54) taking into account Debye relaxation model in the $x$-rays region we find:

$$
\epsilon_{st}(g, \omega) \simeq 1 - (\epsilon_{st}(g) - 1) \left\{ \frac{i \omega_p}{\omega} + \left( \frac{\omega_p}{\omega} \right)^2 \right\},
$$

(55)

where $\omega_p$ is the plasma frequency of media.

Now we can write the expression for difference of frequency-dependent dielectric constants (dielectric permittivity function) in neighboring layers:

$$
\delta\epsilon(g, \omega) = \epsilon_{st}(g, \omega) - \epsilon_{st}(\omega) = \delta\epsilon_{st}(g) \varrho(\sigma, \omega),
$$

(56)

which in the $x$-rays region has a form:

$$
\delta\epsilon(g, \omega) \simeq -\delta\epsilon_{st}(g) \left\{ \frac{i \omega_p}{\omega} + \left( \frac{\omega_p}{\omega} \right)^2 \right\}.
$$

(57)

So, we constructed the dielectric permittivity function (54)-(55) for spin-glass medium under the external standing electromagnetic wave (2), which is correct on the micrometer space scale and nanosecond time scale.

Note, that the main peculiarity of expression (54) concludes in the fact that $\epsilon_{st}(g)$ is a complex function. Moreover depending on parameters of weak external field $\epsilon_{st}(g)$ can have big imaginary value. This fact in contrast to usual case builds the media with radically new properties. Recall when the external field is absent the dielectric permittivity function depends on $\epsilon_s$, which is order of unit real positive constant.

VI. CONCLUSION

In this article a new microscopic approach is developed for the study of the properties of stationary dielectric constant and dielectric permittivity function in the dielectric medium under the external standing electromagnetic field. The approach consists of the following two general steps:

1. Generalization of Clausius-Mossotti equation for dielectric constant in the external standing electromagnetic wave;

2. generalization of the equation for dielectric permittivity function taking into account the previous results.
Mathematically the problem is solved as follows. The dielectric medium in the external electromagnetic field is modelled as a 3D spin-glass system under the external field. Note that all general changes of properties of media take place in the scale of wavelength of external field. For that the layer of medium which consist from disordered 1D spin chains with length of order of wavelength in detail has been investigated. Taking into account the fact that the distribution of spin chains in the infinite \((x, y)\) plan is isotropic we can use the Birgoff ergodic hypothesis (see (6)-(7)) and conditionally reduce the initial 3D spin-glass problem to two 1D problems. It means that we can investigate two 1D problems separately only in this case the parameters of the first 1D problem must be taken into account during the solution of the second 1D problem.

In the work all formal definitions for accounting the investment of orientational effects in the stationary and frequency-depended dielectric constants computation are adduced.

The first 1D problem is related to one-dimensional disordered quantum \(N\)-particle system with relaxation. The investigation of the motion in this system takes place in the framework of complex Langevin-Schrödinger type SDE \((18)\), which can be transformed to 2D system of nonlinear Langevin type SDE \((29)-(30)\). For probability distribution of interaction potential in 1D spin chain with certain energies \(E\) and external standing electromagnetic wave Fokker-Plank equation \((32)\) is obtained, which is found after using white noise model for stochastic forces and system of SDE \((29)-(30)\). In the limit of long-range distance \(t \to \infty\), the probability distribution \(Q(E|\theta, \vartheta; t)\) tends to the limit of its stationary limit \(Q_s(E|\theta, \vartheta)\) which satisfies the elliptic type differential equation \((34)\). Note, that this type of differential equation \((34)\) depending on parameters can have extremely various solutions between which there aren’t smooth transitions. In another words this type of behavior in the solutions we can interpret as a critical phenomenon in the 3D spin-glass system.

The second 1D problem includes the computation of 1D random spin chain energy distribution in the external field. In other words, it means the development of multicanonical statistical mechanics for disordered 1D spin chain system in the external field. In order to do this the classical spin-glass Heisenberg type Hamiltonian is investigated \((38)\). In the first step, the nonperturbed Hamiltonian \((38)\) is used and the system of algebraic equations \((40)\) is found for computation of all possible stable spin configurations on the spin chain scale. After this, the statistic weight \(P_\nu(E_\nu; d_0(T))\) for certain energies \(E_\nu\) of stable non-perturbed spin chain is found simply by formula \((41)\). With the help of formulas \((41)-(46)\) the partition
function $Z(E; g)$, which gives an energy distribution in the spin chain system after inserting of external field, is defined \( [48] \). The Helmholtz free energy \( [51] \) is constructed and it is shown that the 1\( D \) random spin chain system subjected to weak external field \( E_i \) and the order \( E_c^\pm \propto 1/N_x \) can be severely changed (see Fig. 6).

The mean value of complex interaction potential \( \langle \delta U_{L_x}(x, g) \rangle \) between spin chain and external field is found from formula \( [7] \) taking into account expressions \( [41] \) and \( [48] \). An averaging potential is used for computation of polarizability coefficient related with the orientation effects \( [9] \) and correspondingly for generalization of Clausius-Mossotti equation \( [11] \). Remind that Eq. \( [11] \) has a content only on the micro-scale space and the nano-scale time. Note, that the analysis of Eq. \( [11] \) together with \( [23] \) show that in the spin-glass medium with the help of weak external field it is possible to form regions (layers) with different stationary dielectric constants \( \epsilon_{st}(g) \) (see Fig 8). As it is easy to understand the value of stationary dielectric constant strongly depends on initial electrostatic dielectric constant \( \epsilon_s \) of media. In particular, it is shown that in the spin-glass mediums with static dielectric constants \( \epsilon_s < 4 \) under the external field the value of the stationary dielectric constant can be changed maximum 3-4 times. In the case \( \epsilon_s > 4 \) analysis shows that the value of \( Re(\Lambda(g)) \to 1 \) and correspondingly stationary dielectric constant can be complex function \( \epsilon_{st}(g) \) depending on the parameters of external field \( g \). Moreover for a set of some parameters of weak external field it is safe to say that the \( \epsilon_{st}(g) \) can have a large imaginary...
part (see 52).

Obviously, near to this value critical effecters are hold. In this case taking into account the relaxation processes, which are going in the lattice, the system becomes ordered and is characterized by macroscopic classical polarization.

The second main step of investigation is the frequency-dependant dielectric constant or dielectric permittivity. Generalized Clausius-Mossotti Eq. (11) may be used for derivation of generalized equation for dielectric permittivity (14) for the frequencies range $\omega \gg \Omega$. It is important to say that the analysis of stationary dielectric constant value $\epsilon_{st}(g)$ shows that dielectric permittivity function $\epsilon(\omega, g)$ (see 51 and 55) can have positive, as well as negative dispersions. The last circumstance is very important especially for the $x$-range frequency $\omega$ because on the scale of wavelength of external field $\lambda_s$ (see Fig. 7) the dielectric permittivity in the region of positive dispersion "I" and "III" $Re \epsilon_1(\omega, g) < 1$ whereas in the neighbor region of negative dispersion "II" and "IV" correspondingly $Re \epsilon_2(\omega, g) > 1$ (see for example [49, 50, 51]). Recall that this occurs when initial electrostatic dielectric constant $\epsilon_s > 4$. Evidently that the difference between dielectric permittivity of neighboring layers [57] in the $x$-ray region as compared with usual cases when the external field is absent can differ drastically.

In this case collective orientation effects of spins make main investment in the permittivity function (macroscopic polarization) which radically changes behavior of media on the space-time scale of external standing electromagnetic wave. Particularly only a presence of large imaginary part in the coefficient of $\delta \epsilon_{st}(g)$ brings to an essential growth of the real part of difference $\{i(\omega_p/\omega) + (\omega_p/\omega)^2\}$.

Thus, obviously it is possible to make and control the periodic structure from dielectric permittivities in the spin-glass type dielectric medium even with the help of weak external standing electromagnetic field. In other words a new type of controlled radiator for generation of coherent $x$-radiation is suggested.

On the other hand the periodic dielectric permittivity function may be illustrated as a 1D lattice of quantum dots (quantum strings) which can be interesting in the view of quantum computer making.

Finally it is important to note that the first time a new mathematical method for reducing the problem of dynamical $3D$ disordered spin system in the external field to the two conditionally separated 1D problems is developed. The last circumstance can be used
for elaboration of new extremely effective parallel algorithms which is very important for systematic investigations of mentioned problem in the framework of numerical simulations method.

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