CALCULATING CANONICAL DISTINGUISHED INVOLUTIONS IN THE AFFINE WEA TL GROUPS

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Abstract. Distinguished involutions in the affine Weyl groups, defined by G. Lusztig, play an essential role in the Kazhdan-Lusztig combinatorics of these groups. A distinguished involution is called canonical if it is the shortest element in its double coset with respect to the finite Weyl group. Each two-sided cell in the affine Weyl group contains precisely one canonical distinguished involution. In this note we calculate the canonical distinguished involutions in the affine Weyl groups of rank \( \leq 7 \). We also prove some partial results relating canonical distinguished involutions and Dynkin’s diagrams of the nilpotent orbits in the Langlands dual group.

1. Introduction

It is well known that the Kazhdan-Lusztig combinatorics of the affine Hecke algebra is deeply related with the geometry of the corresponding algebraic group (over the complex numbers) \( G \). The central result here is the deep Theorem of George Lusztig establishing a bijection between the set \( \mathcal{U} \) of unipotent classes in \( G \) and the set of two-sided cells in the corresponding affine Weyl group \( W_a \), see \[7\]. Using this one defines a map from \( \mathcal{U} \) to the set \( X_+ \) of the dominant weights in the following way: let \( O \) be an unipotent orbit and let \( c_O \) be the corresponding two-sided cell. George Lusztig and Nanhua Xi attached to \( c_O \) a canonical left cell \( C_O \), see \[8\]. In turn the left cell \( C_O \) contains a unique distinguished involution \( d_O \in W_a \), see \[8\], which is the shortest element in its double coset \( W d_O W \) with respect to the finite Weyl group \( W \subset W_a \). It is well known that the set of double cosets \( W \backslash W_a / W \) is bijective to the set \( X_+ \), since any such coset contains unique translation by dominant weight. Combining maps above we get a canonical map \( \mathcal{L}: \mathcal{U} \to X_+ \). The explicit calculation of this map is equivalent to the determination of the distinguished involutions lying in the canonical cells (or, equivalently, which are shortest in their left coset with respect to the finite Weyl group). We call such involutions canonical distinguished involutions. We believe that understanding of these involutions is an important step towards understanding of all distinguished involutions and cells in the affine Weyl group.

In \[10\] one of us suggested a conjectural algorithm for the calculation of \( \mathcal{L} \) and now this algorithm is known to be correct thanks to the (unfortunately still undocumented) work of R. Bezrukavnikov. The aim of this note is to present results of calculations using this algorithm.

The paper is organized as follows. In the section 2 we recollect necessary facts. In section 3 we present our main results: calculation of the map \( \mathcal{L} \) for group \( GL_n \) and partial results for other groups. These results seems to be known to the experts but to the best of our knowledge were never published. In section 4 we present tables with the results of explicit calculation of the map \( \mathcal{L} \) for groups of small rank. These tables should be considered as a main result of this work.

We would like to thank David Vogan and Pramod Achar for useful conversations.

2. Recollections

2.1. Notations. Let \( G \) be a semisimple algebraic group over the complex numbers. Let \( \mathfrak{g} \) denote the Lie algebra of \( G \) and let \( N \subset \mathfrak{g} \) denote the nilpotent cone. As a \( G \)–variety \( N \) is isomorphic to the subvariety of unipotent elements of \( G \) via the exponential map but it has one additional virtue — an obvious action of \( \mathbb{C}^\ast \) by dilations commuting with
the $G$–action. We will consider $\mathcal{N}$ as $G \times \mathbb{C}^\ast$–variety via the following action $(g,z)n = z^{-2}Ad(g)n$ for $(g,z) \in G \times \mathbb{C}^\ast$ and $n \in \mathcal{N}$.

The variety $\mathcal{N}$ consists of finitely many $G$–orbits, see e.g. [8]. These orbits called nilpotent orbits are the main subject of our study. Any nilpotent orbit $O$ is identified via its Dynkin diagram defined as follows: let $e \in O$ be a representative, by the Jacobson—Morozov Theorem it can be included in $sl_2$–triple $(e,f,h)$ (i.e. $[h,e] = 2e, [h,f] = -2f, [e,f] = h$). The semisimple element $h$ is uniquely defined up to $G$–conjugacy by $O$ and vice versa, see e.g. [3]. Let $\mathfrak{h} \subset \mathfrak{g}$ be a Cartan subalgebra, let $R \subset \mathfrak{h}^\ast$ be the root system, choose a subset $R_+ \subset R$ of positive roots and let $\{\alpha_i, i \in I\}$ be the set of simple roots ($I$ is the set of vertices of Dynkin diagram of $\mathfrak{g}$). The element $h$ is conjugate to a unique $h_0 \in \mathfrak{h}$ such that $\alpha_i(h_0)$ is positive for any $i \in I$ and moreover $\alpha_i(h_0) \in \{0,1,2\}$, see [6]. Thus any nilpotent orbit can be specialized by labeling the Dynkin diagram of $\mathfrak{g}$ by numbers 0, 1, 2 and this is called the (labeled) Dynkin diagram of $\mathfrak{g}$. We note that the Dynkin diagram $h_0$ is naturally integral dominant coweight for group $G$.

Let $X$ denote the weight lattice of $G$ and let $X_+$ denote the set of dominant weights.

2.2. Review of [11]. Let $K_{G \times \mathbb{C}^\ast}(\mathcal{N})$ denote the Grothendieck group of the category of $G \times \mathbb{C}^\ast$–equivariant coherent sheaves on $\mathcal{N}$. It has an obvious structure of a module over representation ring $Rep(\mathbb{C}^\ast)$ of $\mathbb{C}^\ast$. Let $v$ denote tautological representation of $\mathbb{C}^\ast$. Then $Rep(\mathbb{C}^\ast) = \mathbb{Z}[v,v^{-1}]$ and the rule $v \mapsto v^{-1}$ defines an involution $\overline{\cdot} : Rep(\mathbb{C}^\ast) \rightarrow Rep(\mathbb{C}^\ast)$.

In [11] it was constructed the following:

1) the basis $\{AJ(\lambda)\}$ of $K_{G \times \mathbb{C}^\ast}(\mathcal{N})$ over $Rep(\mathbb{C}^\ast)$ was labeled by dominant weights $\lambda \in X_+$.

2) $Rep(\mathbb{C}^\ast)$–antilinear involution $K_{G \times \mathbb{C}^\ast}(\mathcal{N}) \rightarrow K_{G \times \mathbb{C}^\ast}(\mathcal{N}), x \mapsto \overline{x}$.

Then using usual Kazhdan-Lusztig machinery the basis $\{C(\lambda)\}$ was defined. Thus $C(\lambda)$ is a unique selfdual element of the form $AJ(\lambda) + \sum_{\mu < \lambda} b_{\mu,\lambda}AJ(\mu)$ where $b_{\mu,\lambda} \in \mathbb{Z}[v,v^{-1}] \cap \mathbb{C}^\ast$. The main Conjecture of [11] is that for any $\lambda \in X_+$ the support of $C(\lambda)$ is the closure of nilpotent orbit $O_\lambda$ and $C(\lambda)|_{O_\lambda}$ represents up to sign class of irreducible $G$–equivariant bundle on $O_\lambda$. In this way one can recover Lusztig’s bijection between dominant weights $X_+$ and pairs consisting of nilpotent orbit and $G$–equivariant irreducible bundle on it (see [7], [11], [3], [13] for various approaches to Lusztig’s bijection). All these Conjectures are now known to be true thanks to the work of R. Bezrukavnikov.

Now let $e^\lambda \in K_{G}(\mathcal{N})$ be the image of $AJ(\lambda)$ under forgetting map $K_{G \times \mathbb{C}^\ast}(\mathcal{N}) \rightarrow K_{G}(\mathcal{N})$. By definition $e^\lambda$ can be constructed as follows: let $\mathcal{L}(\lambda)$ be the line bundle on $G/B$ corresponding to the weight $\lambda$ (we choose notations in such a way that $H^0(G/B, \mathcal{L}(\lambda)) \neq 0$ for dominant $\lambda$); then $e^\lambda = |sp_*\pi^*\mathcal{L}(\lambda)|$ where $\pi : T^*G/B \rightarrow G/B$ is natural projection and $sp : T^*G/B \rightarrow \mathcal{N}$ is the Springer resolution. This definition makes sense for any (not necessarily dominant) weight $\lambda$ and one knows that $e^{\omega_\lambda} = e^\lambda$ for any $w \in W, \lambda \in X$.

2.3. McGovern’s formula. In this note we are especially interested in weights corresponding to the trivial bundles on nilpotent orbits under Lusztig’s bijection. One of the Conjectures in [11] states that these weights are exactly $\mathcal{L}(O)$ where $\mathcal{L}$ is defined in the Introduction. Moreover, $C(\mathcal{L}(O))$ should represent a class $j_*(\mathcal{C}_O)$ where $j : O \rightarrow \mathcal{N}$ is natural inclusion and $\mathcal{C}_O$ is trivial $G$–equivariant bundle on $O$. Again this is known to be true thanks to the work of R. Bezrukavnikov.

Now there is a simple formula for $j_*(\mathcal{C}_O)$ due to W. McGovern. Namely, let $h$ be the Dynkin diagram of the orbit $O$. Then it defines a grading of Lie algebra $\mathfrak{g} = \bigoplus_{i \in \mathbb{Z}} \mathfrak{g}_i$, where $\mathfrak{g}_i = \{x \in \mathfrak{g} | [h,x] = ix\}$. Furthermore $\mathfrak{g}_{\geq 0} = \bigoplus_{i \geq 0} \mathfrak{g}_i$ is a parabolic subalgebra of $\mathfrak{g}$ and $\mathfrak{g}_{\geq 2} = \bigoplus_{i \geq 2} \mathfrak{g}_i$ is a module over $\mathfrak{g}_{\geq 0}$. Let $G_{\geq 0}$ be a parabolic subgroup with Lie algebra $\mathfrak{g}_{\geq 0}$ and let $M = G \times G_{\geq 0}$ be the homogeneous bundle on $G/G_{\geq 0}$ corresponding to the $G_0$–module $\mathfrak{g}_{\geq 2}$. There is a natural map $r : G \times G_{\geq 0} \rightarrow \mathfrak{g}$. An image of $r$ is exactly $\mathcal{O}$ and moreover this map is proper and generically one to one and therefore is a resolution of singularities of $\mathcal{O}$, see e.g. [3]. In loc. cit. McGovern proved that $[j_*\mathcal{C}_O] = [r_*\mathcal{C}_M]$.

Now let $R_{+,0}$ (resp. $R_{+,1}$) be the subset of all positive roots such that $\alpha(h) = 0$ (resp. $\alpha(h) = 1$). Then the Koszul complex gives the following formula:

$$[j_*\mathcal{C}_O] = \prod_{\alpha \in R_{+,0} \cup R_{+,1}} (e^\alpha - e^\alpha)$$

(see loc. cit. for details). This implies the following algorithm for computing $L(O)$:

1) Multiply the brackets in the right hand side of (*) using usual rule $e^\lambda e^\mu = e^{\lambda + \mu}$. 

2) In expression from 1) replace each $e^\lambda$ by $e^{w\lambda}$ where $w \in W$ and $w\lambda$ is dominant. Then make all possible cancelations. (Warning: Step 1) and Step 2) don’t commute!)

3) In expression from 2) find the leading term $\pm e^\lambda$ that is such term that for any other term $e^\mu$ the inequality $\lambda > \mu$ holds (existence of such $\lambda$ is a consequence of the results in [3]). This $\lambda$ is exactly $L(\mathcal{O})$.

Unfortunately this algorithm is completely impractical for groups of large rank.

3. Theorems

The results of this section are probably well known to experts. Moreover, Theorem 3.3 was stated in [10] without proof.

3.1. Richardson resolutions. It follows from the Theorem of Hinich and Paniushev [4], [11] on rationality of singularities of normalizations of closures of nilpotent orbits that $j_*\mathcal{C}_\mathcal{O} = r_*\mathcal{C}_M$ where $r : M \to \mathcal{O}$ is a resolution of singularities of $\mathcal{O}$. One obtains from this McGovern’s formula using canonical resolution of a nilpotent orbit. Now let $P$ be a parabolic subgroup of $G$. The image of the moment map $m_P : T^*G/P \to g^* = g$ is the closure of the nilpotent orbit $O(P)$ and by the well known Theorem due to R. W. Richardson the map $m_P : T^*G/P \to \overline{O(P)}$ is generically finite to one.

Let $G'$ be a Langlands dual group of $G$. By definition there is a bijection between simple roots for $G$ and $G'$. In particular we can attach to any parabolic subgroup $P \subset G$ a Levi subgroup $L'_P \subset G'$. Recall that any coweight for $G'$ is by definition a weight for $G$.

**Theorem.** Let $P \subset G$ be a parabolic subgroup such that the map $m_P : T^*G/P \to \overline{O(P)}$ is birational. Then $L(\mathcal{O}_P)$ equals to the Dynkin diagram of the principal nilpotent element in $L'_P$.

**Proof.** By remarks above we know that $j_*\mathcal{C}_\mathcal{O}_P = m_*\mathcal{C}_{T^*G/P}$. Let $R_+(L) \subset R_+$ denote the subset of positive roots of subgroup $L$. Again the Koszul complex gives a formula

$$m_*\mathcal{C}_{T^*G/P} = \prod_{\alpha \in R_+(L)}(1 - e^{\alpha})$$

interpreted in the same way as McGovern’s formula [2,3]. Let $\rho_L = \frac{1}{2} \sum_{\alpha \in R_+(L)}(1 - e^{\alpha})$ and let $W_L \subset W$ denote Weyl group of $L$. The Weyl denominator formula gives

$$\prod_{\alpha \in R_+(L)}(1 - e^{\alpha}) = \sum_{w \in W_L} \det(w)e^{\rho_L - w\rho_L}.$$ 

In particular we have a leading term $e^{2\rho_L}$ corresponding to summand with $w = w_0(L)$ — the element of largest length in $W_L$. It cannot cancel with anything else since the scalar product $(2\rho_L, 2\rho_L)$ is clearly bigger than any scalar product $(\rho_L - w\rho_L, \rho_L - w\rho_L)$ for $w \neq w_0(L)$. By the same reason this term is the unique candidate for the leading term since $\lambda > \mu$ for dominant $\lambda$ and $\mu$ implies $(\lambda, \lambda) > (\mu, \mu)$. Since we know that the leading term exists (this is Bezrukavnikov’s result as we mentioned in [2,3] it should be equal to $e^{2\rho_L}$ where $x \in W$ is such that $x\rho_L$ is dominant.

Finally weight $2\rho_L$ considered as a coweight for $G'$ is clearly the Dynkin diagram of the regular nilpotent element in $L'$ since the Dynkin diagram of regular nilpotent element is always the sum of positive coweights, see [3]. □

3.2. We will say that nilpotent orbit is a *strongly Richardson orbit* if it admits a desingularization via momentum map $m_P : T^*G/P \to \overline{O}$. We have the following

**Theorem.** Let $\mathcal{O} = O_P$ be a strongly Richardson orbit. Suppose that the $G' -$orbit $\mathcal{O}'$ of regular nilpotent element of $L'_P$ is also strongly Richardson in $G'$. Then $L(\mathcal{O}') = $ Dynkin diagram of $\mathcal{O}$ and $L(\mathcal{O}) = $ Dynkin diagram of $\mathcal{O}'$.

**Proof.** The equality $L(\mathcal{O}) = $ Dynkin diagram of $\mathcal{O}$ is immediate from the Theorem 3.1.

Both nilpotent orbits $\mathcal{O}$ and $\mathcal{O}'$ being Richardson orbits are *special*, see [12,3]. N. Spaltenstein defined an order reversing involutive bijection $d$ between the sets of special nilpotent orbits for $G$ and $G'$, see [12]. It follows from the definition [12] that $d(\mathcal{O}') = \mathcal{O}$. Consequently $d(\mathcal{O}) = \mathcal{O}'$ and equality $L(\mathcal{O}') = $ Dynkin diagram of $\mathcal{O}$ follows by symmetry. □

We don’t know the answer to the following


**Question.** Is it true that any Richardson orbit is strongly Richardson?

The nilpotent orbit is called even if its Dynkin diagram is divisible by 2 as element of coweight lattice, see e.g. [3].

**Corollary.** Suppose the orbit $O$ is even and that $\mathcal{L}(O)$ is divisible by 2 as element of weight lattice. Then $\mathcal{L}(O)$ is a Dynkin diagram of some nilpotent orbit $O'$ for $G^\vee$ and $\mathcal{L}(O') = $ Dynkin diagram of $O$.

**Proof.** This is clear since even nilpotent orbits are strongly Richardson, see e.g. [3]. \hfill $\Box$

A lot of examples for this Corollary can be found in the Tables in the end of this note. In fact this is the only general statement we can prove (or even just formulate) about these Tables.

3.3. It is well known that nilpotent orbits in $GL_n$ are numbered by partitions of $n$: to the partition $p_1 \geq p_2 \geq \ldots$ one associates an orbit of nilpotent matrices with Jordan blocks of size $p_1, p_2, \ldots$. One finds Dynkin diagram of the orbit $O = O(p_1, p_2, \ldots)$ as follows, see e.g. [3]: order $n$ numbers $p_1 - 1, p_1 - 3, \ldots, -p_1 + 1, p_2 - 1, \ldots, -p_2 + 1, \ldots$ in decreasing order, then label of $i$-th vertice of Dynkin diagram will be difference of $i$-th and $i + 1$-th numbers.

3.4. The following Lemma is well known.

**Lemma.** For $G = GL_n$ and a parabolic subgroup $P \subset G$ the map $m_P : T^*G/P \to \overline{O(P)}$ is birational.

**Proof.** The isotropy group of any nilpotent element in $GL_n$ is connected. \hfill $\Box$

3.5. Let $O = O(p_1, p_2, \ldots)$ be a nilpotent orbit. Let $p_1', p_2', \ldots$ be a partition dual to $p_1, p_2, \ldots$ and let $O' = O(p_1', p_2', \ldots)$ be the orbit dual to $O$.

**Theorem.** The weight $\mathcal{L}(O)$ equals to the Dynkin diagram of the dual orbit $O'$ considered as a weight for $GL_n$.

**Proof.** It is well known that any nilpotent orbit $O$ in $G = GL_n$ is a Richardson orbit and it follows from the Lemma 3.4 that it is strongly Richardson. It follows from the proof of the Theorem 3.2 that $\mathcal{L}(O)$ is Dynkin diagram of $d(O)$ (here $d$ is Spaltenstein duality). Now the result follows from description of $d$ for the group $GL_n$ in [12]. \hfill $\Box$

3.6. **Remarks.**

(i) As we mentioned in the Introduction the calculation of the map $\mathcal{L}$ is equivalent to the calculation of canonical distinguished involutions in affine Weyl group. For type $A_n$ canonical distinguished involutions were recently calculated by Nanhua Xi [14] by a completely different method. Surely his answer coincides with ours, but he did not mention a relation with Dynkin diagrams of nilpotent orbits.

(ii) For group $G$ of type different from $A_n$ it is not true in general that weight attached to a nilpotent orbit and considered as coweight for Langlands dual group $G^\vee$ is a Dynkin diagram for some nilpotent orbit of $G^\vee$, see tables in the end of this note. But calculations made by Pramod Achar (private communication) suggest some evidence for the positive answer to the following

**Question.** Is it true that any Dynkin diagram considered as a weight for the dual group corresponds under Lusztig’s bijection to the local system on some nilpotent orbit? (Here local system is a coherent equivariant sheaf on nilpotent orbit such that the corresponding representation of isotropy group factors through finite quotient.)

4. **Tables**

The Tables below contain results of our calculations of map $\mathcal{L}$ for groups of small rank. We have almost complete results for groups of rank $\leq 7$ (there are some gaps for groups of types $B_6$, $C_6$, $B_7$, $C_7$ and $E_7$) and partial results in rank 8 (for groups $D_8$ and $E_8$). The Tables are organized as follows: the tables for classical groups consist of 4 columns, in the first column we give the partition identifying the nilpotent orbit $O$ (see e.g. [3]), in the second column we give the Dynkin diagram of $O$, in the third column the weight $\mathcal{L}(O)$ is contained and last column contains the square of the length of $\mathcal{L}(O)$ (we normalize scalar product by $\langle \alpha, \alpha \rangle = 2$ for a short root $\alpha$); for exceptional groups the tables consist of 3 columns, the first column contains Dynkin diagram of the nilpotent orbit $O$, the second column contains $\mathcal{L}(O)$ and the third column contains the square of the length of $\mathcal{L}(O)$. In a few cases we were unable to calculate $\mathcal{L}(O)$ but we could predict the value $\mathcal{L}(O)$ using duality reasoning, such cases are marked by a question mark.
### Calculating Canonical Distinguished Involution in the Affine Weyl Groups

#### SO

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (5)       |         | 0      |
| (3, 1²)   |         | 2      |
| (2², 1)   |         | 10     |
| (1⁵)      |         | 20     |

#### SP

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (4)       |         | 0      |
| (2²)      |         | 2      |
| (2, 1²)   |         | 10     |
| (1⁴)      |         | 20     |

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (7)       |         | 0      |
| (5, 1²)   |         | 2      |
| (3², 1)   |         | 6      |
| (3, 2²)   |         | 16     |
| (3, 1⁴)   |         | 20     |
| (2², 1³)  |         | 28     |
| (1⁷)      |         | 70     |

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (9)       |         | 0      |
| (7, 1²)   |         | 2      |
| (5, 3, 1) |         | 6      |
| (4², 1)   |         | 14     |
| (5, 2²)   |         | 16     |
| (3¹)      |         | 18     |
| (5, 1⁴)   |         | 20     |
| (3², 1³)  |         | 24     |
| (3, 2², 1²)|        | 40     |
| (2⁴, 1)   |         | 60     |
| (3, 1⁶)   |         | 70     |
| (2², 1⁵)  |         | 78     |
| (1⁵)      |         | 168    |

### SO

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (6)       |         | 0      |
| (4, 2)    |         | 2      |
| (3²)      |         | 6      |
| (2³)      |         | 8      |
| (4, 1²)   |         | 10     |
| (2², 1²)  |         | 20     |
| (2, 1⁴)   |         | 34     |
| (1⁶)      |         | 56     |

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (8)       |         | 0      |
| (6, 2)    |         | 2      |
| (4²)      |         | 4      |
| (4, 2²)   |         | 8      |
| (6, 1²)   |         | 10     |
| (3², 2)   |         | 12     |
| (4, 2, 1²)|         | 20     |
| (2⁴)      |         | 20     |
| (3², 1²)  |         | 22     |
| (4, 1⁴)   |         | 34     |
| (2³, 1²)  |         | 36     |
| (2⁴, 1⁴)  |         | 54     |
| Partition | Diagram | Weight |
|-----------|---------|--------|
| (11)      | ![Diagram] | 0      |
| (9, 1²)   | ![Diagram] | 2      |
| (7, 3, 1) | ![Diagram] | 6      |
| (5², 1)   | ![Diagram] | 10     |
| (7, 2²)   | ![Diagram] | 16     |
| (5, 3²)   | ![Diagram] | 18     |
| (7, 1⁴)   | ![Diagram] | 20     |
| (5, 3, 1³) | ![Diagram] | 24     |
| (4², 3)   | ![Diagram] | 26     |
| (4², 1³)  | ![Diagram] | 32     |
| (3², 1²)  | ![Diagram] | 36     |
| (5, 2², 1²) | ![Diagram] | 40     |
| (3², 2², 1) | ![Diagram] | 60     |
| (5, 1⁵)   | ![Diagram] | 70     |
| (3², 1⁵)  | ![Diagram] | 74     |
| (3, 4)    | ![Diagram] | 80     |
| (3, 2², 1⁴) | ![Diagram] | 90     |
| (2⁴, 1³)  | ![Diagram] | 110    |
| (3, 1⁸)   | ![Diagram] | 168    |
| (2², 1⁷)  | ![Diagram] | 176    |
| (1¹¹)     | ![Diagram] | 330    |

**so_{11}**

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (13)      | ![Diagram] | 0      |
| (11, 1²)  | ![Diagram] | 2      |
| (9, 3, 1) | ![Diagram] | 6      |
| (7, 5, 1) | ![Diagram] | 10     |
| (5, 7, 1) | ![Diagram] | 10     |
| (9, 2²)   | ![Diagram] | 16     |

**sp_{10}**

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (10)      | ![Diagram] | 0      |
| (8, 2)    | ![Diagram] | 2      |
| (6, 4)    | ![Diagram] | 4      |
| (6, 2²)   | ![Diagram] | 8      |
| (5²)      | ![Diagram] | 8      |
| (4², 2)   | ![Diagram] | 10     |
| (4², 3)   | ![Diagram] | 18     |
| (6, 2, 1²) | ![Diagram] | 20     |
| (4², 1²)  | ![Diagram] | 22     |
| (3², 2¹²) | ![Diagram] | 24     |
| (6, 1⁴)   | ![Diagram] | 34     |
| (4², 2²)  | ![Diagram] | 36     |
| (3², 2, 1²) | ![Diagram] | 40     |
| (2⁵)      | ![Diagram] | 40     |
| (4², 1⁴)  | ![Diagram] | 54     |
| (3², 1⁴)  | ![Diagram] | 58     |
| (2⁴, 1²)  | ![Diagram] | 60     |
| (2³, 1⁴)  | ![Diagram] | 82     |
| (4, 1⁶)   | ![Diagram] | 84     |
| (2², 1⁶)  | ![Diagram] | 120    |
| (2, 1⁸)   | ![Diagram] | 164    |
| (1¹⁰)     | ![Diagram] | 220    |
| Partition   | Diagram  | Weight |
|-------------|----------|--------|
| (7, 3)      |          | 18     |
| (6, 1)      |          | 18     |
| (9, 1)      |          | 20     |
| (5, 3)      |          | 22     |
| (7, 3, 1)   |          | 24     |
| (5, 1^3)    |          | 24     |
| (5, 4)      |          | 32     |
| (5, 3, 2)   |          | 36     |
| (7, 2, 1)   |          | 40     |
| (4, 3, 2)   |          | 44     |
| (5, 3, 2, 1)|          | 60     |
| (3, 1, 1)   |          | 60     |
| (4, 2, 1)   |          | 64     |
| (7, 1)      |          | 70     |
| (5, 3, 1)   |          | 74     |
| (5, 2)      |          | 80     |
| (4, 2, 1)   |          | 82     |
| (3, 2^2)    |          | 82     |
| (3, 1^3)    |          | 84     |
| (5, 2, 1)   |          | 90     |
| (3, 2, 1^3) |          | 110    |
| (3, 2^4, 1) |          | 138    |
| (5, 1^2)    |          | 168    |
| (3, 2, 1^2) |          | 172    |
| (2, 1^4)    |          | 182    |
| (3, 1^9)    |          | 330    |
| (3, 2, 1^6) |          |        |
| (2, 1^6)    |          |        |
| (3, 2^2, 1^6) |        |        |

| Partition   | Diagram  | Weight |
|-------------|----------|--------|
| (12)        |          | 18     |
| (10, 2)     |          | 18     |
| (8, 4)      |          | 20     |
| (6^2)       |          | 22     |
| (8, 2^2)    |          | 24     |
| (10, 1^2)   |          | 24     |
| (6, 4, 2)   |          | 32     |
| (5^2, 2)    |          | 36     |
| (7, 2, 1)   |          | 40     |
| (4, 3^2)    |          | 44     |
| (6, 2^3)    |          | 60     |
| (8, 2^2)    |          | 60     |
| (6, 4, 2)   |          | 64     |
| (4, 2^2)    |          | 70     |
| (5^2, 1^2)  |          | 74     |
| (8, 1^4)    |          | 80     |
| (6, 2^2, 1^2)|         | 82     |
| (4, 2^2)    |          | 82     |
| (3, 2^3)    |          | 84     |
| (4, 3^2, 1^2)|         | 90     |
| (3, 1^3, 1^3)|       |        |
| (6, 2^1^4)|          | 100    |
| (4, 2^1^4)|          | 168    |
| (4, 3^2, 2)|          | 172    |
| (4, 2^3, 1^2)|        | 182    |
| (3, 2^2, 1^3)|       |        |
| (6^6)|          | 330    |
| (4, 2^2, 1^4)|        |        |
| Partition | Diagram | Weight |
|-----------|---------|--------|
| $(2^2, 1^9)$ | ![Diag](image1.png) | 572 |
| $(1^{13})$ | ![Diag](image2.png) | |

| $\mathfrak{so}_{13}$ | $\mathfrak{so}_{15}$ | $\mathfrak{sp}_{12}$ | $\mathfrak{sp}_{14}$ |
|-------------------------|----------------------|-----------------------|----------------------|
| Partition | Diagram | Weight | Partition | Diagram | Weight | Partition | Diagram | Weight | Partition | Diagram | Weight |
| $(6, 1^6)$ | | | $(14)$ | | | |
| $(3^2, 2, 1^4)$ | | | $(12, 2)$ | | | |
| $(2^5, 1^2)$ | | | $(10, 4)$ | | | |
| $(4, 2, 1^6)$ | | | $(8, 6)$ | | | |
| $(3^2, 1^6)$ | | | $(10, 2^2)$ | | | |
| $(4, 1^8)$ | | | $(12, 1^2)$ | | | |
| $(2^4, 1^4)$ | | | $(8, 4, 2)$ | | | |
| $(2^3, 1^6)$ | | | $(7^2)$ | | | |
| $(2^2, 1^8)$ | | | $(6^2, 2)$ | | | |
| $(2, 1^{10})$ | | | $(6, 4^2)$ | | | |
| $(1^{12})$ | | | $\mathfrak{sp}_{14}$ | | | |
| Partition       | Diagram     | Weight |
|-----------------|-------------|--------|
| (7, 2^4)        |             | 80     |
| (5, 3^2, 2^2)   |             | 82     |
| (5, 3^2, 1^4)   |             | 86     |
| (4^3, 3, 2^2)   |             | 90     |
| (7, 2^2, 1^4)   |             | 90     |
| (4^2, 3, 1^4)   |             | 94     |
| (3^8)           |             | 100    |
| (3^4, 1^3)      |             | 110    |
| (5, 3^2, 1^3)   |             | 110    |
| (4^2, 2^2, 1^3) |             | 114    |
| (5, 2^4, 1^2)   |             | 138    |
| (7, 1^6)        |             | 168    |
| (5, 3, 1^7)     |             | 172    |
| (4^2, 1^7)      |             | 180    |
| (3^3, 1^6)      |             | 184    |
| (5, 1^10)       |             | 330    |
| (3^2, 1^9)      |             | 334    |
| (3, 1^12)       |             | 572    |
| (5, 2^2, 1^6)   |             |        |
| (3^3, 2^2, 1^2) |             |        |
| (3^2, 2^4, 1)   |             |        |
| (3^2, 2^2, 1^5) |             |        |
| (3, 2^6)        |             |        |
| (3, 2^4, 1^4)   |             |        |
| (3, 2^2, 1^8)   |             |        |
| (2^6, 1^3)      |             |        |
| (2^4, 1^7)      |             |        |
| (2^2, 1^11)     |             |        |
| (1^15)          |             |        |

| Partition       | Diagram     | Weight |
|-----------------|-------------|--------|
| (6, 2, 1^2)     |             | 24     |
| (5^2, 2^2)      |             | 26     |
| (4^3, 2)        |             | 28     |
| (6, 3^2, 2^2)   |             | 30     |
| (10, 1^4)       |             | 34     |
| (8, 2^2, 1^2)   |             | 36     |
| (6, 4, 2, 1^2)  |             | 38     |
| (6, 2^4)        |             | 40     |
| (4^2, 3^2)      |             | 40     |
| (4^2, 2^2)      |             | 42     |
| (6, 3^2, 1^2)   |             | 42     |
| (5^2, 2^2, 1^2) |             | 44     |
| (8, 2, 1^4)     |             | 50     |
| (6, 4, 1^4)     |             | 54     |
| (6, 2^3, 1^2)   |             | 60     |
| (5^2, 1^4)      |             | 60     |
| (3^4, 2)        |             |        |
| (4^2, 2^2, 1^2) |             |        |
| (4^3, 2, 1^2)   |             |        |
| (4^2, 5)        |             |        |
| (3^4, 1^2)      |             |        |
| (6, 2^2, 1^4)   |             |        |
| (4^2, 2, 1^4)   |             |        |
| (8, 1^6)        |             |        |
| (4, 3^2, 1^4)   |             |        |
| (4, 2^4, 1^2)   |             |        |
| (6, 2, 1^6)     |             | 120    |
| Partition | Diagram | Weight |
|-----------|---------|--------|
| (2^7)     | ![Diagram](#) | 112    |
| (4^2,1^6) | ![Diagram](#) | 122    |
| (6,1^8)   | ![Diagram](#) | 164    |
| (3^2,1^8) | ![Diagram](#) | 222    |
| (4,2^3,1^4)| ![Diagram](#) |       |
| (4,2^2,1^6)| ![Diagram](#) |       |
| (4,2,1^8) | ![Diagram](#) |       |
| (4,1^10)  | ![Diagram](#) |       |
| (3^2,2^4) | ![Diagram](#) |       |
| (3^2,2^3,1^2)| ![Diagram](#) |       |
| (3^2,2^2,1^4)| ![Diagram](#) |       |
| (3^2,2,1^6) | ![Diagram](#) |       |
| (2^6,1^2)  | ![Diagram](#) |       |
| (2^5,1^4)  | ![Diagram](#) |       |
| (2^4,1^6)  | ![Diagram](#) |       |
| (2^3,1^8)  | ![Diagram](#) |       |
| (2^2,1^10) | ![Diagram](#) |       |
| (2,1^12)   | ![Diagram](#) |       |
| (1^14)     | ![Diagram](#) | 560    |
### Calculating Canonical Distinguished Involution in the Affine Weyl Groups

#### SO\(_6\)

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (5, 1)   | ![Diagram] | 0 |
| (3\(^2\)) | ![Diagram] | 2 |
| (3, 1\(^3\)) | ![Diagram] | 4 |
| (2\(^2\), 1\(^2\)) | ![Diagram] | 8 |
| (1\(^6\)) | ![Diagram] | 20 |

#### SO\(_8\)

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (7, 1)   | ![Diagram] | 0 |
| (5, 3)   | ![Diagram] | 2 |
| (5, 1\(^3\)) | ![Diagram] | 4 |
| (4\(^2\)) \(_1\) | ![Diagram] | 4 |
| (4\(^2\)) \(_2\) | ![Diagram] | 4 |
| (3\(^2\), 1\(^2\)) | ![Diagram] | 6 |
| (3, 2\(^2\), 1) | ![Diagram] | 14 |
| (3, 1\(^5\)) | ![Diagram] | 20 |
| (2\(^4\)) \(_1\) | ![Diagram] | 20 |
| (2\(^4\)) \(_2\) | ![Diagram] | 20 |
| (2\(^2\), 1\(^4\)) | ![Diagram] | 24 |
| (1\(^8\)) | ![Diagram] | 56 |

#### SO\(_{10}\)

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (5\(^2\)) | ![Diagram] | 0 |
| (5, 3, 1\(^2\)) | ![Diagram] | 6 |
| (4\(^2\), 1\(^2\)) | ![Diagram] | 10 |
| (3\(^3\), 1) | ![Diagram] | 12 |
| (5, 2\(^2\), 1) | ![Diagram] | 14 |
| (5, 1\(^5\)) | ![Diagram] | 20 |
| (3\(^2\), 2\(^2\)) | ![Diagram] | 20 |
| (3, 2\(^2\), 1\(^3\)) | ![Diagram] | 30 |
| (2\(^4\), 1\(^2\)) | ![Diagram] | 40 |
| (3, 1\(^7\)) | ![Diagram] | 56 |
| (2\(^2\), 1\(^6\)) | ![Diagram] | 60 |
| (1\(^{10}\)) | ![Diagram] | 120 |

#### SO\(_{12}\)

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (11, 1) | ![Diagram] | 0 |
| (9, 3) | ![Diagram] | 2 |
| (9, 1\(^3\)) | ![Diagram] | 4 |
| (7, 5) | ![Diagram] | 4 |
| (7, 3, 1\(^2\)) | ![Diagram] | 6 |
| (6\(^2\)) \(_1\) | ![Diagram] | 6 |
| (6\(^2\)) \(_2\) | ![Diagram] | 6 |
| (5\(^2\), 1\(^2\)) | ![Diagram] | 8 |
| (5, 3\(^2\), 1) | ![Diagram] | 12 |
| Partition | Diagram | Weight |
|-----------|---------|--------|
| (7, 2^2, 1) | 2 2 1 0 | 1 1 1 0 |
| (4^2, 3, 1) | 0 1 1 0 | 0 0 0 2 |
| (7, 1^5) | 2 2 0 0 | 2 2 0 0 |
| (5, 3, 2^2) | 0 1 0 1 | 0 2 0 1 |
| (4^2, 2^2)_1 | 0 2 0 0 | 0 2 0 0 |
| (4^2, 2^2)_2 | 0 2 0 0 | 0 2 0 0 |
| (5, 3, 1^4) | 2 0 2 6 | 2 1 0 1 |
| (3^4) | 0 0 0 2 | 0 1 1 0 |
| (4^2, 1^4) | 0 2 0 1 | 2 0 1 0 |
| (3^3, 1^3) | 0 0 2 0 | 2 0 2 0 |
| (5, 2^2, 1^3) | 2 1 0 1 | 1 1 1 0 |
| (3^2, 2^2, 1^2) | 0 1 0 1 | 0 2 0 2 |
| (3, 2^4, 1) | 1 0 0 0 | 1 1 1 2 |
| (5, 1^5) | 2 2 0 0 | 2 2 0 0 |
| (3^2, 1^6) | 0 2 0 0 | 2 2 1 0 |
| (3, 2^2, 1^5) | 1 0 1 0 | 1 1 1 1 |
| (2^6)_1 | 0 0 0 0 | 0 2 0 2 |
| (2^6)_2 | 0 0 0 0 | 0 2 0 2 |
| (2^4, 1^4) | 0 0 0 1 | 0 2 0 2 |
| (3, 1^9) | 2 0 0 0 | 2 2 2 2 |
| (2^2, 1^8) | 0 1 0 0 | 2 2 2 0 |
| (1^12) | 0 0 0 0 | 2 2 2 2 |

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (13, 1) | 2 2 2 2 | 2 2 2 2 |
| (11, 3) | 2 2 2 2 | 2 2 2 2 |
| (11, 1^3) | 2 2 2 2 | 2 2 2 2 |
| (9, 5) | 2 2 2 2 | 2 2 2 2 |
| (9, 3, 1^2) | 2 2 2 2 | 2 2 2 2 |
| (7^2) | 2 2 2 2 | 2 2 2 2 |
| (7, 1^2) | 2 2 2 2 | 2 2 2 2 |
| (7, 3^2, 1) | 2 2 2 2 | 2 2 2 2 |
| (6^2, 1^2) | 2 2 2 2 | 2 2 2 2 |
| (9, 2^2, 1) | 2 2 2 2 | 2 2 2 2 |
| (5^2, 3, 1) | 2 2 2 2 | 2 2 2 2 |
| (9, 1^5) | 2 2 2 2 | 2 2 2 2 |
| (7, 3^2, 2) | 2 2 2 2 | 2 2 2 2 |
| (5^2, 2^2) | 2 2 2 2 | 2 2 2 2 |
| (7, 3^2, 1^3) | 2 2 2 2 | 2 2 2 2 |
| (5^4, 1^2) | 2 2 2 2 | 2 2 2 2 |
| (5^2, 1^4) | 2 2 2 2 | 2 2 2 2 |
| (5, 3^3) | 2 2 2 2 | 2 2 2 2 |
| (5, 3^2, 1^3) | 2 2 2 2 | 2 2 2 2 |
| (4^2, 3^2) | 2 2 2 2 | 2 2 2 2 |
| (7, 2^2, 1^3) | 2 2 2 2 | 2 2 2 2 |
| (4^2, 3, 1^3) | 2 2 2 2 | 2 2 2 2 |
| (5, 3^2, 1^2) | 2 2 2 2 | 2 2 2 2 |
### Calculating Canonical Distinguished Involutions in the Affine Weyl Groups

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (3, 1) | ![Diagram](3, 1) | ![Weight](3, 1) |
| (4, 1) | ![Diagram](4, 1) | ![Weight](4, 1) |
| (5, 1) | ![Diagram](5, 1) | ![Weight](5, 1) |
| (3, 2) | ![Diagram](3, 2) | ![Weight](3, 2) |
| (6, 1) | ![Diagram](6, 1) | ![Weight](6, 1) |
| (3, 3) | ![Diagram](3, 3) | ![Weight](3, 3) |
| (6, 2) | ![Diagram](6, 2) | ![Weight](6, 2) |
| (3, 4) | ![Diagram](3, 4) | ![Weight](3, 4) |
| (7, 1) | ![Diagram](7, 1) | ![Weight](7, 1) |
| (3, 5) | ![Diagram](3, 5) | ![Weight](3, 5) |
| (7, 2) | ![Diagram](7, 2) | ![Weight](7, 2) |
| (3, 6) | ![Diagram](3, 6) | ![Weight](3, 6) |
| (7, 3) | ![Diagram](7, 3) | ![Weight](7, 3) |

| Partition | Diagram | Weight |
|-----------|---------|--------|
| (15, 1) | ![Diagram](15, 1) | ![Weight](15, 1) |
| Partition | Diagram | Weight |
|-----------|---------|--------|
| $(3, 2^2, 1^9)$ | ![Diagram](image) | $q^0$ |
| $(2^6, 1^4)$ | ![Diagram](image) | $q^0$ |
| $(2^4, 1^8)$ | ![Diagram](image) | $q^0$ |
| $(2^2, 1^{12})$ | ![Diagram](image) | $q^0$ |
| $(1^{16})$ | ![Diagram](image) | $q^{2}$ |

$SO_{16}$
### The Exceptional groups

| $G_2$ | Diagram | Weight |
|-------|---------|--------|
|       | 0       | 0      |
|       | 2       | 2      |
|       | 14      | 10     |
|       | 18      | 18     |
|       | 56      | 56     |

| $F_4$ | Diagram | Weight |
|-------|---------|--------|
|       | 0       | 0      |
|       | 2       | 2      |
|       | 6       | 6      |
|       | 8       | 8      |
|       | 12      | 12     |
|       | 16      | 16     |
|       | 28      | 28     |
|       | 34      | 34     |
|       | 38      | 38     |
|       | 56      | 56     |
|       | 70      | 70     |
|       | 72      | 72     |
|       | 118     | 118    |
|       | 156     | 156    |
|       | 312     | 312    |

| $E_6$ | Diagram | Weight |
|-------|---------|--------|
|       | 0       | 0      |
|       | 2       | 2      |
|       | 4       | 4      |
|       | 6       | 6      |
|       | 10      | 10     |
|       | 12      | 12     |
|       | 16      | 16     |
|       | 18      | 18     |
|       | 20      | 20     |
|       | 30      | 30     |
|       | 40      | 40     |

| $E_7$ | Diagram | Weight |
|-------|---------|--------|
|       | 0       | 0      |
|       | 2       | 2      |
|       | 4       | 4      |
|       | 6       | 6      |

| $E_8$ | Diagram | Weight |
|-------|---------|--------|
|       | 0       | 0      |
|       | 2       | 2      |
|       | 4       | 4      |
|       | 6       | 6      |
| Diagram | Weight |  | Diagram | Weight |
|--------|--------|--|--------|--------|
| 0      | 2 2 2 0 2 0 | 0 0 0 0 0 0 | 0      | 1 0 1 0 1 2 | 0 1 1 0 0 0 |
| 8      | 2 0 2 0 2 0 | 0 0 0 0 0 1 | 40     | 2 0 1 0 1 0 | 2 0 0 0 2 0 |
| 12     | 2 0 2 0 0 2 | 0 0 1 0 0 0 | 42     | 1 0 1 0 1 0 | 1 0 1 0 1 0 |
| 14     | 2 1 0 1 1 2 | 1 1 0 0 0 0 | 48     | 0 0 0 2 0 0 | 0 0 2 0 0 0 |
| 14     | 0 0 2 0 2 0 | 0 0 0 0 0 0 | 54     | 2 1 0 0 0 1 | 1 1 0 0 2 0 |
| 16     | 2 1 0 1 1 0 | 0 0 0 0 2 0 | 56     | 2 0 0 0 2 2 | 2 2 0 0 0 0 |
| 18     | 0 0 2 0 0 2 | 0 1 0 0 1 0 | 58     | 2 0 0 0 2 0 | 2 1 0 0 0 1 |
| 20     | 2 1 0 1 0 2 | 2 0 0 0 1 0 | 62     | 0 0 1 0 1 0 | 2 0 0 2 0 0 |
| 22     | 2 2 0 0 2 0 | 2 0 0 0 0 2 | 70     | 1 0 1 0 0 1 | 1 0 1 0 2 0 |
| 24     | 0 2 0 0 2 0 | 1 0 0 1 0 1 | 70     | 2 2 0 0 0 0 | 2 0 0 0 2 2 |
| 28     | 2 0 2 0 0 0 | 0 0 1 0 1 0 | 72     | 0 2 0 0 0 0 | 1 0 1 0 1 2 |
| 30     | 0 1 0 1 0 2 | 1 1 0 0 1 0 | 78     | 1 0 0 1 0 1 | 1 1 1 0 1 0 |
| 30     | 0 0 2 0 0 0 | 0 0 0 2 0 0 | 84     | 1 0 1 0 0 0 | 1 0 1 0 1 1 |
| 32     | 1 0 1 0 2 0 | 1 0 0 1 0 0 | 88     | 0 1 0 0 1 0 | 0 1 1 1 0 1 |
| Diagram | Weight  | Diagram | Weight  |
|---------|---------|---------|---------|
| 0       | 0 0 0 2 0 0 2 | 0       | 0 0 0 2 0 0 0 |
| 0       | 0 0 2 0 0 2 2 | 0 1 0 1 0 0 0 |
| 1       | 2 1 0 0 0 1 2 | 2 0 0 0 0 1 1 |
| 0       | 1 0 1 0 1 1 0 |  |
| 0       | 1 0 1 0 1 0 1 |  |
| 0       | 2 0 0 0 2 2 2 | 0 0 0 0 2 2 2 |
| 0       | 2 0 0 0 2 0 2 | 0 0 0 0 1 2 |
| 0       | 0 0 0 2 0 0 0 | 0 0 1 0 0 1 0 |
| 0       | 2 0 0 0 2 0 0 | 0 0 0 0 0 2 2 |
| 0       | 1 0 0 1 0 1 2 | 1 0 0 0 1 1 1 |
| 0       | 0 0 1 0 1 0 0 |  |
| 0       | 1 0 0 1 0 1 0 |  |
| 1       | 0 1 0 0 0 1 0 |  |

| Diagram | Weight  | Diagram | Weight  |
|---------|---------|---------|---------|
| 0       | 48     | 0       | 1 0 1 0 0 0 1 |
| 0       | 50     | 0       | 0 0 1 0 0 1 0 |
| 0       | 54     | 0       | 0 0 1 0 1 0 2 |
| 0       | 0       | 0 0 2 0 0 2 0 |
| 0       | 0       | 0 1 0 0 0 1 1 |
| 0       | 56     | 1       | 0 1 0 0 1 2 |
| 0       | 58     | 0       | 0 2 0 0 0 2 |
| 0       | 60     | 0       | 0 0 0 0 0 2 |
| 0       | 0       | 2 0 0 0 2 0 0 |
| 0       | 0       | 0 0 0 0 0 2 |
| 0       | 78     | 0       | 2 0 0 0 0 2 0 |
| 0       | 0       | 0 1 0 0 1 0 0 |
| 0       | 0       | 0 0 1 0 1 0 2 |
| 0       | 0       | 2 0 0 0 1 0 1 |

| Diagram | Weight  | Diagram | Weight  |
|---------|---------|---------|---------|
| 0       | 80     | 0       | 0 0 2 0 0 0 |
| 0       | 82     | 1       | 1 0 0 1 0 0 0 |
| 0       | 98     | 0       | 0 1 0 0 0 1 1 |
| 0       | 116    | 1       | 1 0 1 0 0 1 1 |
| 0       | 174    | 0       | 0 2 0 0 1 0 |
| 0       | 120    | 2       | 2 0 0 0 0 2 2 |
| 0       | 122    | 0       | 1 0 0 1 0 1 2 |
| 0       | 126    | 0       | 0 1 0 0 1 0 2 |
| 0       | 128    | 0       | 0 0 0 2 0 0 2 |
| Diagram | Weight |
|---------|--------|
| ![Diagram 1](image1) | 1 0 0 0 0 0 1 |
| ![Diagram 2](image2) | 1 0 0 1 0 0 0 |
| ![Diagram 3](image3) | 1 0 0 0 1 0 1 |
| ![Diagram 4](image4) | 0 0 0 0 0 0 0 |
| ![Diagram 5](image5) | 2 0 0 0 2 0 2 |
| ![Diagram 6](image6) | 2 1 0 0 1 0 2 |
| ![Diagram 7](image7) | 1 0 0 0 0 0 2 |
| ![Diagram 8](image8) | 1 0 0 0 0 0 2 |
| ![Diagram 9](image9) | 0 0 0 0 0 0 1 |
| ![Diagram 10](image10) | 0 0 0 1 1 0 0 |
| ![Diagram 11](image11) | 0 1 0 0 0 0 1 |
| ![Diagram 12](image12) | 1 0 0 1 0 1 2 |
| ![Diagram 13](image13) | 1 0 1 0 0 0 2 |
| ![Diagram 14](image14) | 0 0 0 0 0 0 1 |
| ![Diagram 15](image15) | 0 0 0 1 0 0 0 |
| ![Diagram 16](image16) | 0 0 0 0 0 2 2 |
| ![Diagram 17](image17) | 0 0 0 0 0 0 2 |
| ![Diagram 18](image18) | 1 0 1 0 1 2 2 |
| ![Diagram 19](image19) | 0 0 0 0 0 2 2 |

| Diagram | Weight |
|---------|--------|
| ![Diagram 1](image1) | 1 0 0 0 1 0 2 |
| ![Diagram 2](image2) | 2 0 0 0 0 0 0 |
| ![Diagram 3](image3) | 1 0 0 0 0 1 0 |
| ![Diagram 4](image4) | 1 0 0 0 0 0 0 |
| ![Diagram 5](image5) | 0 0 0 0 0 0 0 |
| ![Diagram 6](image6) | 2 2 2 2 2 2 2 |
| ![Diagram 7](image7) | 2 2 2 2 2 2 2 |
| ![Diagram 8](image8) | 2 2 2 2 2 2 2 |
| ![Diagram 9](image9) | 2 2 2 2 2 2 2 |
| ![Diagram 10](image10) | 2 2 2 2 2 2 2 |
| ![Diagram 11](image11) | 2 2 2 2 2 2 2 |
| ![Diagram 12](image12) | 2 2 2 2 2 2 2 |
| ![Diagram 13](image13) | 2 2 2 2 2 2 2 |
| ![Diagram 14](image14) | 2 2 2 2 2 2 2 |
| ![Diagram 15](image15) | 2 2 2 2 2 2 2 |
| ![Diagram 16](image16) | 2 2 2 2 2 2 2 |
| ![Diagram 17](image17) | 2 2 2 2 2 2 2 |
| ![Diagram 18](image18) | 2 2 2 2 2 2 2 |
| ![Diagram 19](image19) | 2 2 2 2 2 2 2 |

**Weights:**
- 1040
- 368
- 462
- 168
- 336
- 368
- 462
- 464
- 560
- 798
- 800
- 1040
- 1520
- 2480
REFERENCES

[1] R. Bezrukavnikov, On tensor categories attached to cells in affine Weyl group, preprint.
[2] R. Bezrukavnikov, Quasi-exceptional sets and equivariant coherent sheaves on the nilpotent cone, preprint RT/0102039.
[3] D. Collingwood, W. McGovern, Nilpotent orbits in semisimple Lie algebras, Van Nostrand Reinhold Co., New York, 1993.
[4] V. Hinich, On the singularities of nilpotent orbits, Israel J. Math. 73 (1991), no. 3, p. 297-308.
[5] B. Kostant, Lie group representations on polynomial rings, Amer. J. Math. 85 (1963), p. 327-404.
[6] G. Lusztig, Cells in affine Weyl groups II, J. Algebra 109 (1987), no. 2, p. 223-243.
[7] G. Lusztig, Cells in affine Weyl groups IV, J. Fac. Sci. Univ. Tokyo Sect. IA Math. 36 (1989), no. 2, p. 297-328.
[8] G. Lusztig, N. Xi, Canonical left cells in affine Weyl groups, Adv. in Math. 72 (1988), no. 2, p. 284-288.
[9] W. McGovern, Rings of regular functions on nilpotent orbits and their covers, Invent. Math. 97 (1989), no. 1, p. 209-217.
[10] V. Ostrik, On the equivariant K−theory of the nilpotent cone, Representation Theory 4 (2000), p. 296-305.
[11] Panyushev, Rationality of singularities and the Gorenstein property of nilpotent orbits, Funct. Anal. Appl. 25 (1991), no. 3, p. 225-226.
[12] N. Spaltenstein, Classes unipotentes et sous-groupes de Borel, LNM 946.
[13] D. Vogan, The method of coadjoint orbits for real reductive groups, Representation theory of Lie groups (Park City, UT, 1998), p. 179-238, IAS/Park City Math. Ser., 8, Amer. Math. Soc., Providence, RI, 2000.
[14] N. Xi, The based ring of two-sided cells of affine Weyl groups of type \( \tilde{A}_{n-1} \), preprint QA/0010159.

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