SU(4) pure-gauge phase structure and string tensions*

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We present numerical evidence that the SU(4) pure-gauge dynamics has a finite-temperature first-order phase transition. For a $6 \times 20^3$ lattice, this transition occurs at the inverse-square coupling of $8/g^2 \sim 10.79$. Below this and above the known bulk phase transition at $8/g^2 \sim 10.2$ is a confined phase in which we find two different string tensions, one between the fundamental $4$ and $4^*$ representations and the other between the self-dual diquark $6$ representations. The ratio of these two is about 1.5. The correlation in the adjoint representation suggests no string forms between adjoint charges.

There are renewed interests in SU($N_c$) pure Yang-Mills theory with large $N_c$:

1) Finite-temperature phase structure of quantum chromodynamics (QCD) would be easier to understand if the SU($N_c$) pure Yang-Mills system has a second order phase transition for $N_c \geq 4$ \cite{1}. With standard large-$N_c$ analysis where $N_c g^2$ is held fixed, the Z($N_c$) deconfinement transition occurs at $T_d \sim O(1)$, separating confining phase with free energy $F \sim O(1)$ and deconfining phase with $F \sim O(N_c^2)$. The deconfining temperature $T_d \sim O(1)$ is not effected if $N_f$ and $g^2 N_c$ are held fixed and $N_c \to \infty$. If the transition is first order, it is not effected either. So large $N_c$ is not a reasonable guide for $T \neq 0$ QCD phase structure with Columbia phase diagram \cite{2}, unless SU($N_c$) pure Yang-Mills dynamics has second order deconfining phase transition for all $N_c \geq 4$.

2) New developments in M/string theory \cite{3} predict such things as glueball spectrum at large $N_c$ and large $g^2$ or ratio between different string tensions for $N_c \geq 4$ \cite{4}.

3) The dimensionless ratio $T_d/\sqrt{\sigma}$ of the deconfining temperature $T_d$ and string tension $\sigma$ is expected to be independent of $N_c$ with a value $\sqrt{3/\pi(D-2)}$ with $D$ being the space-time dimensions \cite{5}.

Here we report the results of our numerical investigation on the order of deconfining phase transition and the ratio of string tensions for $N_c = 4$ \cite{6}. We use single-plaquette action defined in the fundamental $4$-representation of the SU(4) gauge group. Combinations of pseudo-heatbath or Metropolis and over-relaxation algorithms are used in updating $4, 6$ and $8 \times 8^3, 12^3, 16^3$ or $20^3$ lattices. Various workstations are used for the numerical calculations, while migration to the RIKEN BNL QCDSF mother boards is planned. We look at the following observables: plaquette, Polyakov loops, $L(\vec{x}) = \langle (1/N_c) \text{tr} \prod_{t=1}^{L_t} U(\vec{x},t,t) \rangle$, in $4$ (fundamental), $6$ (antisymmetric diquark), $10$ (symmetric diquark) and $15$ (adjoint) representations, deconfinement fraction, and Polyakov loop correlation $\langle L(0) L(\vec{r})^* \rangle \sim r^{-1} \exp(-F(r)/T) \sim \exp(-L_d \sigma r - \ln r)$ in $4, 6, 10$ and $15$ representations.

This SU(4) pure Yang-Mills system is known to have a bulk phase transition near $\beta = 8/g^2 \sim 10.2$, separating two confining phases \cite{7}: across this transition the plaquette jumps discontinuously but the average Polyakov line in the fundamental $4$ representation remains zero on both sides. However if the lattice extent in temperature direction $L_t$ is too small this bulk transition drives a first-order finite-temperature deconfining

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Figure 1. Time histories of the fundamental Polyakov loop magnitude and argument (in units of $\pi$) from a $6 \times 20^3$ lattice at $\beta = 10.79$ (above) and magnitude histogram (below). Confined and deconfined phases coexist at this temperature suggesting a first-order deconfining phase transition.

As is shown in Figure 1 on a $6 \times 20^3$ lattice we confirmed coexistence of confined and deconfined phases at temperature $\beta=10.79$: This strongly suggests a first-order deconfining phase transition. Work in progress confirms this phase coexistence as we extend the simulation from the present 3500 evolution steps (1 evolution = 5 heat bath + 1 over relaxation steps) to 20000 steps. We plan further study with finite-size scaling.

Figure 2. Polyakov line correlation in $4$ (+) and $6$ ($\times$) representations on a $6 \times 16^3$ lattice at $\beta = 10.70$. From the slopes we find different string tensions for these representations. No signal was obtained for $10$ and $15$ representations, probably because the coupling is too strong.

String tensions in SU($N_c$) pure Yang-Mills system is classified by its center $Z(N_c)$ $N_c$-ality. With $N_c = 4$, the fundamental ($4$) charge has 4-ality $k = 1$, the two diquark ($6$ and $10$) charges $k = 2$, and adjoint ($15$) $k = 0$. The string tensions between these charges and their anticharges are predicted to behave as $\sigma_k \propto \min\{k,N_c-k\}$ by a standard strong-coupling analysis, $k(N_c-k)$ by another strong-coupling analysis [4], and $\sin\left(k\pi/N_c\right)$ by a SUSY strong coupling analysis [4]. Generally the ratio $\sigma_k/\sigma_1$ should fall in the interval $1 \leq \sigma_k/\sigma_1 \leq 2$ [3]. Note also that $N_c = 4$ is the first example with different string tensions: in SU(3) pure Yang-Mills system the fundamental ($3$) and the symmetric diquark ($6$) tensions are the same [10].

In our numerical calculation on a $6 \times 16^3$ lattice at $\beta = 10.70$ (see Figure 2): we find a clear difference between $4$- and $6$-Polyakov loop correlations. From fitting these data we have $\sigma_4 = 0.068(4)$ and $\sigma_6 = 0.108(17)$. At a stronger coupling of $\beta = 10.65$...
Figure 3. Polyakov line correlation in $4$ (+), $6$ ($\times$) and $15$ ($*$) representations on a $8 \times 12^3$ lattice at $\beta=10.85$. The adjoint ($15$) signals now suggest there is no string for this representation. From the slopes of the former two lower-dimensional representations we confirm different string tensions for them, and by comparison there is no tension seen in the adjoint representation.

their values are $0.086(3)$ and $0.142(57)$ respectively. Thus their ratio $\sigma_6/\sigma_4$ does not show much temperature dependence and falls in the interval (1.2) as it should. We are yet to see any signal for $10$ and $15$ representations from this lattice, probably because of too strong couplings. On the other hand at a weaker coupling of $\beta=10.85$ using a $L_t=8$ lattice we now have rather good evidence that there exists no string in the adjoint ($15$) representation. We plan further investigation on larger and finer lattices, probably using smaller partitions of the QCDSF parallel supercomputer at RIKEN BNL Research Center.

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