Sufficient conditions for H-infinity control on the finite time interval

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Abstract. The article considers a problem of control of linear non-stationary continuous
dynamical systems on a finite time interval. It is assumed that information about the linear
output of the controlled system is available instead of the full state vector. The problem of the
synthesis of H-infinity regulators is formulated. Based on the ideas of the extension principle,
sufficient conditions for H-infinity control are formulated and proved, the best control law for
an object and the worst control law for external disturbances with full state feedback are found.
Since the exact implementation of the obtained control laws in the absence of complete
information about the state is impossible, two approaches to the approximate solution of the
problem are proposed. Both approaches are associated with finding an estimate of the state
vector and using it in control. The first approach is based on the use of pseudoinverse matrices,
which allow finding the estimate based on incoming measurements. The second approach
utilizes the idea of using asymptotic observers that ensure that the estimation error tends to
zero as well as the asymptotic stability of the closed-loop system. The solution of a model
example is given, illustrating the application of the proposed approaches.

1. Introduction

The development of approximate methods for the synthesis of optimal controllers could be applied to
autopilot design problems for aircrafts and rotorcrafts, which are considered to be very important and
pressing problems, in which the entire state vector is not measurable, and only information about the
external disturbances limitations is available. Problems and methods for finding control laws have
already become classic [1-4]. However, it is often hard to find a solution to the problem of regulator
synthesis when the state information is incomplete. The situation is complicated by the imposed
constraints on the time of the system operation. The first purpose of the study is to prove sufficient
conditions for $H^\infty$ – control and find corresponding control laws. The second one is to reveal the
dependence between two different approaches for approximate solution of output control problem.
Using methods from [1,3,4], new sufficient conditions for $H^\infty$ – control on the finite time interval are
formulated and proved. At first, the control laws which depend on the full state vector are obtained.
Then, because the information about the state vector is generally incomplete, two approximate
methods for finding the $H^\infty$ – controllers are proposed. The main difference of the paper in
comparison to the papers of authors which have been published earlier is the study of linear non-
stationary dynamical system on a given finite time interval, previously considered linear stationary
dynamical system on semi-infinite time interval [5].
A model example of searching matrix of regulator’s coefficients is given, illustrating the applicability of the proposed approaches to solving various control problems [6,7].

Currently mathematical models based on classic models [8] and models which implement neural networks [9] for calculations are used for the synthesis of linear regulator. The main distinction between the similar themed papers and this paper is that they consider stationary systems with full information about the state, while the purpose of our study is to find a linear regulator with incomplete information by considering external disturbances with limited energy. The problem of analyzing the synthesis of H-inf control of linear systems with incomplete information about the state over a finite time interval is not fully investigated.

It is worth noting that software which can be used in the future for solving problems of synthesis of linear regulators are being developed [10]. The other aim of this article was to create a complex of programs in MATLAB implementing two of the proposed methods.

2. Methodology

2.1. Statement of the problem

The mathematical model of the plant is

$$\dot{x}(t) = A(t)x(t) + B_1(t)w(t) + B_2(t)u(t), \quad x(0) = 0,$$

and model of a measuring system is

$$y(t) = C(t)x(t),$$

where \(x \in \mathbb{R}^n\) -- state vector, \(u \in \mathbb{R}^q\) -- control input, \(w \in \mathbb{R}^p\) -- disturbance, \(y \in \mathbb{R}^m\) -- measured output, \(t \in [0, t_f]\) -- time. Matrices \(A(t) \in \mathbb{R}^{n \times n}\), \(B_1(t) \in \mathbb{R}^{n \times p}\), \(B_2(t) \in \mathbb{R}^{n \times q}\), \(C(t) \in \mathbb{R}^{m \times n}\) and a positive number \(t_f\) are given. It is assumed that \(w(.) \in L_2[0, \infty)\), \(u(.) \in L_2[0, \infty); m \leq n, \rg C(t) = m\;\text{and}\;\rg(C^T A^T B_2 \ldots A^{n-1} B_2) = n\;\text{and}\;\rg(C^T A^T C^T \ldots (A^T)^{n-1} C^T) = n\;\text{.}

Let us denote \(\|z(t)\|^2 = y(t)^T S y(t) + u^T(t)Q(t)u(t)\) as performance output, where \(Q(t)\) -- positive definite symmetric matrix, \(S(t)\) -- non-negative definite symmetric matrix; \(\|F(t_i)\|_1 = x(t_i)^T \Lambda x(t_i)\) as performance output at the end of the process, where \(\Lambda \in \mathbb{R}^{n \times n}\) -- non-negative definite symmetric matrix.

It is required to ensure (if possible) the fulfillment of the inequality

$$\gamma^2 \leq \frac{\int_0^t \|z(t)\|^2 dt + \int_0^t \|F(t_i)\|_1 dt}{\int_0^t \|w(t)\|^2 dt + \int_0^t \|w(t)\|^2 dt} \leq \frac{\int_0^t [(x(t)^T S C x(t) + u^T(t)Q u(t)) dt + x(t_i)^T \Lambda x(t_i)]}{\int_0^t w^T(t)w(t) dt},$$

where \(\gamma > 0\) -- a given non-negative number, as well as the asymptotic stability of a closed loop system. It is preferred to find the minimum value of \(\gamma\) at which these properties are still valid, which can be achieved by minimizing the value of the fraction numerator while maximizing the denominator. In other words, the quality functional must satisfy the following condition
which will be fulfilled when the output vector tends to zero, minimizing control costs under the worst influence of disturbances possible.

2.2. Synthesis of full state vector $H^\infty$ – control

Let there be a function $V(t, x) \in C^{1,1}$ and the construction

$$R(t, x, u, w) = \frac{\partial V(t, x)}{\partial t} + \left( \frac{\partial V(t, x)}{\partial x} \right)^T \left[ A(t)x + B_1(t)w + B_2(t)u \right] + x^T C(t)S(t)C(t)x + u^T Q(t)u - \gamma^2 w^T w,$$

where

$$\frac{\partial V(t, x)}{\partial x} = \left( \frac{\partial V(t, x)}{\partial x_1}, \ldots, \frac{\partial V(t, x)}{\partial x_n} \right)^T.$$

The following sufficient condition of $H^\infty$ – control was proved.

Theorem. If a function $V(t, x) \in C^{1,1}$ exists, satisfying the condition $V(t, 0) = 0$ and

$$R(t, x, u^*(x), w^*(x)) = \max_u \min_w R(t, x, u, w) = 0 \quad \forall x \in R^n, \forall t \in T,$$

where

$$u^*(t, x) = -\frac{1}{2} Q^T(t)B_2(t) \frac{\partial V(t, x)}{\partial x} = -Q^T(t)B_2^T(t)K_2(t)x,$$

$$w^*(t, x) = \frac{1}{2\gamma^2} B_1^T(t) \frac{\partial V(t, x)}{\partial x} = \frac{1}{\gamma^2} B_1^T(t)K_2(t)x,$$

matrix $K_2 \geq 0$ satisfies the Riccati differential equation

$$\dot{K}_2 + K_2A + A^T K_2 = K_2[B_2Q^{-1}B_2^T - \frac{1}{\gamma^2}B_1B_1^T]K_2 + C^T S C = 0, \quad K_2(t) = \Lambda,$$

and a condition $\sigma(A + [\frac{1}{\gamma^2}B_1B_1^T - B_2Q^{-1}B_2^T]K_2) \subset C^-$, where $\sigma$ – matrix spectrum, $C^-$ – open left half-plane of the complex plane, then inequality (3) is true.

Proof. Let the assertion conditions be satisfied. Let’s find $R(t, x, u, w)$ using the necessary conditions for an extremum:

$$\frac{\partial R(t, x, u, w)}{\partial u} = B_1^T(t) \frac{\partial V(t, x)}{\partial x} - 2Q(t)u = 0,$$

$$\frac{\partial R(t, x, u, w)}{\partial w} = B_2^T(t) \frac{\partial V(t, x)}{\partial x} - 2\gamma^2 w = 0.$$

Here $u^*(t, x), w^*(t, x)$ are structures for controlling the plant and disturbance.

Sufficient conditions for an unconstrained minimum with respect to $u$ are fulfilled because

$$\frac{\partial^2 R(t, x, u, w)}{\partial u^T \partial u} = 2Q(t) > 0,$$

as well as sufficient conditions for maximum with respect to $w$ are fulfilled because

$$\frac{\partial^2 R(t, x, u, w)}{\partial w^T \partial w} = -2\gamma^2 < 0.$$
Then
\[ R(t, x, u, w) = R(t, x, u^*(t, x), w^*(t, x)) - \gamma^2 [w - w^*(t, x)]^T [w - w^*(t, x)] + \] 
\[ + [u - u^*(t, x)]^T Q[u - u^*(t, x)]. \]

Therefore
\[ R(t, x, u^*(x), w(t, x)) \leq R(t, x, u^*(t, x), w^*(t, x)) \leq R(t, x, u(t, x), w^*(t, x)) \]  
(9)
i.e. there is a saddle point.

Let the available function \( V(t, x) \in C^2 \) satisfy the condition \( V(t, 0) = 0 \) and \( R(t, x, u^*(x), w^*(x)) = 0 \).

Since the system is completely controllable and observable, for any \( u(.) \in L_2[0, \infty) \), \( w(.) \in L_2[0, \infty) \) we have \( y(.) \in L_2[0, \infty) \).

Along the trajectories of the dynamical system it is true that
\[
\frac{\partial V(t, x(t))}{\partial t} + \left( \frac{\partial V(t, x(t))}{\partial x} \right)^T \left[ A(t)x(t) + B_1(t)w(t) + B_2(t)u(t) \right] + \|z(t)\|^2 - \gamma^2 \|w(t)\|^2 =
\]
\[ = \frac{dV(t, x(t))}{dt} + \|z(t)\|^2 - \gamma^2 \|w(t)\|^2 = \frac{R(t, x, u^*(t, x(t)), w^*(t, x(t))))}{0} - \] 
\[ -\gamma^2 [w(t) - w^*(t, x(t))]^T [w(t) - w^*(t, x(t))] + [u(t) - u^*(t, x(t))]^T Q(t)[u - u^*(t, x(t))]. \]

Let's consider the left hand side of inequality \( (9) \), namely
\[ R(t, x, u^*(t, x), w(t, x)) \leq R(t, x, u^*(t, x), w^*(t, x)) \], i.e. when \( u^*(t, x) \):
\[ \frac{dV(t, x(t))}{dt} + \|z(t)\|^2 - \gamma^2 \|w(t)\|^2 \leq 0. \]

We integrate the left and right hand parts in time from \( 0 \) to \( t_1 \):
\[ V(t_1, x(t_1)) - V(0, x(0)) + \int_0^{t_1} \|z(t)\|^2 \, dt - \gamma^2 \int_0^{t_1} \|w(t)\|^2 \leq 0, \]

Since \( G(t_1, x) = V(t_1, x) - x^T A x = 0 \) then \( V(t_1, x(t_1)) = x(t_1)^T A x(t_1) \). Since \( x(0) = 0 \), then \( V(0, x(0)) = V(0, 0) = 0 \). Hence
\[ \int_0^{t_1} \|z(t)\|^2 \, dt + x(t_1)^T A x(t_1) \leq \gamma^2 \int_0^{t_1} \|w(t)\|^2 \, dt, \]

Therefore, condition (3) is fulfilled.

Let \( V(t, x) = x^T K_2(t)x \), where \( K_2(t) \) - symmetric positive definite matrix. Then \( \frac{\partial V(t, x)}{\partial x} = 2K_2(t)x \) and
\[ u^*(t, x) = -\frac{1}{2} Q^{-1}(t)B_1^T(t) \frac{\partial V(t, x)}{\partial x} = -Q^{-1}(t)B_1^T(t)K_2(t)x, \]
\[ w^*(t, x) = \frac{1}{2\gamma^2} B_1^T(t) \frac{\partial V(t, x)}{\partial x} = \frac{1}{2\gamma^2} B_1^T(t)K_2(t)x. \]

Let's write down the condition \( R(t, x, u^*(t, x), w^*(t, x)) = 0 \) (argument \( t \) is omitted):
\[ \dot{K}_2 + 2x^T K_2 A x + 2x^T K_2 B_1 \frac{1}{2\gamma^2} B_1^T 2K_2 x - 2x^T K_2 B_2 \frac{1}{2} Q^{-1}(t)B_1^T2K_2 x + x^T C^T S C x + \]
\[ + x^T K_2 B_2 Q^{-1} B_1^T K_2 x - \frac{1}{\gamma^2} x^T K_2 B_1 B_1^T K_2 x = 0, \]
or
\[ x^T [\dot{K}_2 + 2K_2 A + \frac{1}{\gamma^2} K_2 B_1 B_1^T K_2 - K_2 B_2 Q^{-1} B_2^T K_2 + C^T S C] x = 0. \]  

(10)

Condition \( G(t_1, x) = V(t_1, x) - x^T \Lambda x = 0 \) has the form

\[ x^T [K_2 (t_1) - \Lambda] x = 0 \]  

(11)

Applying in (10), (11) the condition \([\cdot] + [\cdot]^T = 0\), one can obtain

\[ \dot{K}_2 + K_2 A + A^T K_2 - K_2 [B_2 Q^{-1} B_2^T - \frac{1}{\gamma^2} B_1 B_1^T] K_2 + C^T S C = 0, \quad K_2(t_1) = \Lambda. \]

The proof is over.

**Remark.** If the energy of disturbances acting on the system is limited, i.e. \( \int_0^\infty \| w(t) \|_2^2 \, dt \leq 1 \), then

\[ \int_0^\infty \| x(t) \|_2^2 \, dt + x(t_1)^T \Lambda x(t_1) \leq \gamma^2. \]

2.3. Synthesis of Output H∞ control

If \( m = n \) and the matrix \( C(t) \) is non-degenerate, then the state vector could be found directly from the output vector: \( x = C^{-1}(t) y \). Then output control \( u^*(t, y) = -Q^{-1}(t) B_2^T(t) K_2(t) C^{-1}(t) y \).

If \( m \leq n \), \( \text{rg} \, C(t) = m \), we can apply two approaches. The first one is related to finding a pseudo-solution \( \hat{x} \) of system \( y = C(t)x \) and its application in control law, the second - to the synthesis of the state observer that develops the estimate \( \hat{x} \) of the state vector, and using estimates in control.

First approach. Find system pseudo-solution \( y = C(t)x \) using a pseudo-inverse matrix \( C^{-1}(t) = C^T(t) [C(t) C^T(t)]^{-1} : \hat{x} = C^{-1}(t) y \), i.e. column with the smallest modulus among all columns minimizing \( |C(t)x - y| \), where the column modulus is \( |x| = \sqrt{x_1^2 + \ldots + x_n^2} \). Then the output control has the form

\[ u^*(t, y) = u^*(t, \hat{x}) = -Q^{-1}(t) B_2^T(t) K_2(t) C^T(t) [C(t) C^T(t)]^{-1} y. \]  

(12)

Second approach. Synthesize an asymptotic full order observer

\[ \frac{d\hat{x}}{dt} = A(t) \hat{x}(t) + B_1(t) w(t) + B_2(t) u(t) + K(t) [y(t) - C(t) \hat{x}(t)], \quad \hat{x}(0) = x_o^*, \]  

(13)

where \( K(t) \) is the gain observer matrix with a size of \( (n \times m) \), \( x_o^* \) - column containing the a priori information about the initial state. Matrix \( K(t) \) is selected with respect to condition \( \sigma(A - KC) \subset C^- \), providing that the error \( e(t) = x(t) - \hat{x}(t) \) asymptotically tends to zero. Then the control law has the form

\[ u^*(t, y_o^*) = u^*(t, \hat{x}) = -Q^{-1}(t) B_2^T(t) K_2(t) \hat{x}, \]  

(14)

where \( y_o^* = \{y(\tau), 0 \leq \tau \leq t\} \) is an accumulated information about the output measurement results.

2.4. Algorithm of Output H∞ control

**Step 1.** Set parameter \( \gamma > 0 \). Find a solution to the Riccati differential equation
\[
\dot{K}_2 + K_2A + A^TK_2 - K_2[B_2Q^{-1}B_2^T - \frac{1}{\gamma}B_1B_1^T]K_2 + C^TS^C = 0, \quad K_2(t_i) = \Lambda,
\]

at fixed \( \gamma \), satisfying conditions \( K_2 \geq 0 \) and \( \sigma(A + [\frac{1}{\gamma}B_1B_1^T - B_2Q^{-1}B_2^T]K_2) \subset C^- \).

Sequentially reducing \( \gamma \), find the minimum value \( \gamma^* \) under which all conditions remain fulfilled.

**Step 2.** Find the control of a plant of the form (12) or (14) and the law of change of the disturbing effect \( w^*(t, x) = \frac{1}{\gamma^2}B_1^T(t)K_2(t)x \).

**Step 3.** Find output control law and closed-loop system trajectories.

**Case 1.** Control system with arbitrary disturbances satisfying the condition \( \int_0^t \|w(t)\|^2 dt \leq 1 \):

\[
\dot{x} = A(t)x(t) + B_1(t)x(t) + B_1(t)u(t), \quad x(0) = 0, \quad y(t) = C(t)x(t),
\]

first approach: \( u(t) = u^*(t, y(t)) = -Q^{-1}(t)B_2(t)K_2(t)C^T(t)[C(t)C^T(t)]^{-1}y(t) \);

second approach: \( u(t) = u^*(t, \tilde{x}(t)) = -Q^{-1}(t)B_2^T(t)K_2(t)\tilde{x}(t), \)

\[
\frac{d\tilde{x}}{dt} = A(t)\tilde{x}(t) + B_1(t)x(t) + B_2(t)u(t) + K(t)[y(t) - C(t)\tilde{x}(t)], \quad \tilde{x}(0) = x_0^*.
\]

**Case 2.** Control system with the worst perturbations \( w(t) = w^*(t, x(t)) = \frac{1}{\gamma^2}B_1^T(t)K_2(t)x(t) \):

\[
\dot{x} = A(t)x(t) + B_1(t)x(t) + B_1(t)u(t), \quad x(0) = 0, \quad y(t) = C(t)x(t),
\]

first approach: \( u(t) = u^*(t, y(t)) = -Q^{-1}(t)B_2(t)K_2(t)C^T(t)[C(t)C^T(t)]^{-1}y(t) \);

second approach: \( u(t) = u^*(t, \tilde{x}(t)) = -Q^{-1}(t)B_2^T(t)K_2(t)\tilde{x}(t), \)

\[
\frac{d\tilde{x}}{dt} = A(t)\tilde{x}(t) + B_1(t)x(t) + B_2(t)u(t) + K(t)[y(t) - C(t)\tilde{x}(t)], \quad \tilde{x}(0) = x_0^*.
\]

Verify the asymptotic stability of the closed loop system for each case.

**3. Example**

Following is a theoretical example, for which an analytical solution is obtained and matrix of regulator’s coefficients \( K_2(t) \) is found.

**The mathematical model of the plant is**

\[
\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t), \quad x(0) = 0,
\]

and model of a measuring system is

\[
y(t) = Cx(t),
\]

where \( x \in R, y \in R, u \in R, w \in R \).

Then condition (3) has the form

\[
\int_0^t \|x(t)\|^2 dt + \|F(t)\|^2 \leq \int_0^t [SC^2 \dot{x}(t) + Qu^2(t)] dt + \Lambda \dot{x}^2(t_i)
\]

\[
\int_0^t \|w(t)\|^2 dt + \int_0^t \|w^2(t)\| dt \leq \gamma^2.
\]

**Step 1.** Compose the Riccati differential equation

\[
\dot{K}_2 = -\frac{2}{\gamma^2}A_{K_2} - K_2\left[\frac{B_1^2}{\gamma^2} - \frac{B_2^2}{Q}\right] - SC^2, \quad K_2(t_i) = \Lambda.
\]
Then, separating the variables, we get integral like \[ \frac{dK_2}{aK_2^2 + bK_2 + c} = -dt. \] Denote \( \Delta = 4ac - b^2 \).

If \( \Delta < 0 \), then
\[ \frac{1}{\sqrt{-\Delta}} \ln \frac{2aK_2 + b - \sqrt{-\Delta}}{2aK_2 + b + \sqrt{-\Delta}} = -t + \ln C. \] By potentiating one can find
\[ \frac{2aK_2 + b - \sqrt{-\Delta}}{2aK_2 + b + \sqrt{-\Delta}} = Ce^{-\Delta t_i}. \] When \( t = t_i \) from condition \( K_2(t_i) = \Lambda \) we have
\[ \frac{2a\Lambda + b - \sqrt{-\Delta}}{2a\Lambda + b + \sqrt{-\Delta}} e^{\sqrt{-\Delta}t_i} = C. \] Therefore
\[ \frac{2aK_2 + b - \sqrt{-\Delta}}{2aK_2 + b + \sqrt{-\Delta}} = \frac{2a\Lambda + b - \sqrt{-\Delta}}{2a\Lambda + b + \sqrt{-\Delta}} e^{\sqrt{-\Delta}(t_i - t)}. \] Hence
\[ K_2(t) = \frac{-b[1 - d(t)] + \sqrt{-\Delta}[1 + d(t)]}{2a[1 - d(t)]} = \frac{-b + \sqrt{-\Delta}[1 + d(t)]}{2a[1 - d(t)]}. \]

If \( \Delta > 0 \), then
\[ 2\arctg \frac{2aK_2 + b}{\sqrt{\Delta}} = t + C. \] When \( t = t_i \) from condition \( K_2(t_i) = \Lambda \) have
\[ 2\arctg \frac{2a\Lambda + b}{\sqrt{\Delta}} - t_i = C. \] Then
\[ \arctg \frac{2aK_2 + b}{\sqrt{\Delta}} = t - t_i + \frac{2}{\sqrt{\Delta}} \arctg \frac{2a\Lambda + b}{\sqrt{\Delta}}. \] Therefore
\[ \arctg \frac{2aK_2 + b}{\sqrt{\Delta}} = \frac{\sqrt{\Delta}}{2}(t - t_i) + \arctg \frac{2a\Lambda + b}{\sqrt{\Delta}}, \]
or
\[ \frac{2aK_2 + b}{\sqrt{\Delta}} = \operatorname{tg} \left[ \frac{\sqrt{\Delta}}{2}(t - t_i) \right] + \frac{2a\Lambda + b}{\sqrt{\Delta}}, \]
then
\[ K_2(t) = -\frac{b}{2a} + \frac{\sqrt{\Delta} \operatorname{tg} \left[ \frac{\sqrt{\Delta}}{2}(t - t_i) \right]}{2a \left[ 1 - \frac{2a\Lambda + b}{\sqrt{\Delta}} \operatorname{tg} \left[ \frac{\sqrt{\Delta}}{2}(t - t_i) \right] \right]} + \frac{2a\Lambda + b}{\sqrt{\Delta}}. \]

**Step 2.** The feedback control looks like

for the first approach: \( u(t) = u^*(t, y(t)) = -\frac{B_2}{QC} K_2(t)y(t) \);

for the second approach: \( u(t) = u^*(t, \hat{x}(t)) = -\frac{B_2}{Q} K_2(t)\hat{x}(t) \), \[ \frac{d\hat{x}}{dt} = A\hat{x}(t) + B_1w(t) + B_2u(t) + K[y(t) - C\hat{x}(t)], \] \( \hat{x}(0) = x_0^* \);

with the worst perturbations
\[ w(t) = w^*(t, x(t)) = \frac{B_1}{\gamma} K_2(t)x(t). \]

For the second approach we set the gain observer matrix \( K \) (value for this example), which is selected with respect to condition \( \sigma(A - KC) \subset C^- \), i.e. \( \lambda = A - KC < 0 \).
4. Conclusion
The aim of this paper was to prove sufficient conditions for control and to design output feedback controllers for linear non-stationary continuous dynamical systems on a finite time interval using $H^\infty$ – approach. New sufficient conditions for closed loop control on the finite time interval are formulated and proved. Two approximate methods for finding the $H^\infty$ – controllers are proposed. Application of $H^\infty$ – control allows to significantly decrease the negative influence of external disturbances on the system. The model example of finding the regulator’s coefficients is solved and it demonstrates the applicability of two approaches to solving the problem of synthesis of linear controllers. In a particular case, if we consider an infinite period of time, and make the system stationary, then the results obtained will coincide with the results of previous studies. In this case, instead of solving the differential Ricatti equation, it will be necessary to solve the algebraic Ricatti equation.
Possible applications of the used methods include autopilot design problems for aircrafts and rotorcrafts. For example, approaches can be used to design regulator for the F-16 as has been done in [7]. This is the one of the ways of future development of this paper results.

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