Generation of Z bosons in emission processes by neutrinos in 
early universe

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Abstract

Production of Z bosons in emission processes by neutrinos in the expanding de Sitter universe is studied. We use perturbative methods to investigate emission processes that are forbidden in flat space-time electro-weak theory by the energy and momentum conservation. The amplitude and probability for the spontaneous emission of a Z boson by a neutrino or an antineutrino are computed analytically, then we perform a graphical analysis in terms of the expansion parameter. Our results prove that this process is possible only for large expansion conditions of the early Universe. The total probability of the process is analysed and we explore the physical consequences of our results proving that in the Minkowski limit there is no emission of Z bosons by neutrinos. The limit of large space expansion when the expansion parameter is much more larger than the mass of the Z boson is also obtained and the results prove that in this limit the emission probability increase.

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I. INTRODUCTION

The problem of electro-weak interactions in a de Sitter space-time by using perturbative methods was studied only recently in [1]. In [1] the general formalism for study the neutral current interactions intermediated by the Z boson was constructed in a curved geometry. This allows us to explore processes of interaction that generate production of massive bosons and fermions in an expanding de Sitter universe by adapting the electro-weak theory [5–17] to a curved space-time. Since it is well known that the massive bosons were produced in early universe [5, 6] it is important to explore all the possible processes that could produce them including the first order perturbative processes that are forbidden in Minkowski theory [17, 23] by energy and momentum conservation. In a non-stationary space-time the translational invariance with respect to time is lost and the amplitudes and probabilities corresponding to such processes become possible.

The idea of exploring the problem of particle generation at fields interactions was first studied in [21], and the main results of this paper are related to the fact that the perturbative calculations can be translated in the number of particles. Another result established in [21] is related to the conditions in which the perturbative particle production becomes dominant in rapport with the cosmological particle production. Other important results related to the problem of particle generation at fields interactions in de Sitter space-time were obtained in [25–35], where the production of photons and electron-positron pairs in first order perturbative processes of the de Sitter QED were studied. The nonperturbative approach to the problem of particle generation prove that the phenomenon of particle production could be studied by using the fact that the in and out vacua are different such that one can write two sets of solutions that can be related by using Bogoliubov coefficients and some of the important results can be found in [18–20, 36–39]. The nonperturbative approach to the problem of massive boson generation was less studied since this will imply nontrivial computations with the solutions of the Proca equation in curved geometries. One of the interesting phenomena that could be studied is related to the massive bosons decays from rest in expanding backgrounds.

In the present paper we use the solutions of the Dirac equation and Proca equation in a de Sitter geometry which have a defined momentum and helicity for computing the amplitude of Z boson emission by a neutrino or an antineutrino. The first order transition amplitudes
for the leptons interactions with the massive Z boson will be computed and we will explore the physical consequences of our results. Our approach allows us to explore the interesting limit cases when the expansion parameter is vanishing and the case when the expansion is large comparatively with the Z boson mass.

The paper begin in the second section with the transition amplitude and probability computation for the process of Z boson emission by a neutrino. In section three we discuss the physical consequences of our analytical results and we perform a graphical analysis. Section four is dedicated to the study of the total probability and limit cases. We use natural units with $\hbar = 1, c = 1$.

II. Amplitude of Z Boson Emission by Neutrino

For study the emission of Z bosons by neutrinos in early universe we start with the de Sitter metric [2]:

$$ds^2 = dt^2 - e^{2\omega t}d\vec{x}^2 = \frac{1}{(\omega t_c)^2}(dt_c^2 - d\vec{x}^2),$$

where the conformal time is given in terms of proper time by $t_c = \frac{-e^{-\omega t}}{\omega}$, and $\omega$ is the expansion factor ($\omega > 0$). The first order transition amplitude in electro-weak theory on curved space-time for the interaction between Z bosons and neutrino-antineutrino field was obtained in [1], and read:

$$A_{\nu \rightarrow Z\nu} = \int d^4x\sqrt{-g}\left(\frac{e_0}{\sin(2\theta_W)}\right)\bar{\psi}_{\nu e}\gamma^{\mu}e_{\mu}^{\alpha}\left(1 - \gamma^5\right)\psi_{\nu e}A_{\alpha}(Z),$$

where $e_0$ is the electric charge, $\theta_W$ is the Weinberg angle and, $\psi_{\nu e}$ designates the neutrino-antineutrino field, while $A_{\alpha}(Z)$ designates the Z boson field. We also mention that we use point independent Dirac matrices $\gamma^{\mu}$ and the tetrad fields $e_{\mu}^{\alpha}$. For the line element (1), in the Cartesian gauge tetrad components are,

$$e_0^0 = -\omega t_c; \quad e_j^i = -\delta_j^i \omega t_c.$$  

Our computations are done in the chart with conformal time $t_c \in (-\infty, 0)$, which covers the expanding portion of de Sitter space.

The above amplitude can be adapted for the process of Z emission by a neutrino ($\nu$) or an antineutrino ($\bar{\nu}$) i.e. $\nu \rightarrow \nu + Z, \bar{\nu} \rightarrow \bar{\nu} + Z$. In the present paper we compute the amplitude corresponding to Z boson generation by a neutrino, the result for the process in
which the emission is done by antineutrino have just one different sign. Then the amplitude for the first order process $\nu \rightarrow \nu + Z$, is:

$$A_{\nu \rightarrow Z\nu} = -\int d^4x \sqrt{-g} \left( \frac{\epsilon_0}{\sin(2\theta_W)} \right) (U_{\nu',\sigma'})_{\nu}(x) \gamma^\mu \tilde{e}_\mu \left( \frac{1 - \gamma^5}{2} \right) (U_{\rho\sigma})_{\nu}(x)f^*_\alpha\rho\lambda,Z(x)$$

$$= -\int d^4x \sqrt{-g} \left( \frac{\epsilon_0}{\sin(2\theta_W)} \right) (U_{\nu',\sigma'})_{\nu}(x) \gamma^\mu \tilde{e}_\mu \left( \frac{1 - \gamma^5}{2} \right) (U_{\rho\sigma})_{\nu}(x)f^*_0\rho\lambda,Z(x)$$

$$- \int d^4x \sqrt{-g} \left( \frac{\epsilon_0}{\sin(2\theta_W)} \right) (U_{\nu',\sigma'})_{\nu}(x) \gamma^\mu \tilde{e}_\mu \left( \frac{1 - \gamma^5}{2} \right) (U_{\rho\sigma})_{\nu}(x)f^*_\pi\rho\lambda,Z(x).$$

In equation (4) the solutions of the Dirac equations for zero mass field, $(U_{\rho\sigma})_{\nu}(x)$ are used. These solutions describe the neutrino field in de Sitter geometry and their explicit form was obtained in [3]

$$(U_{\bar{\nu},\sigma}(x))_{\nu} = -\frac{\omega t_c}{2\pi} \left( \frac{1}{2} - \sigma \right) \xi_\sigma(\hat{p}) \left( \frac{1}{0} \right) e^{i\hat{p}\cdot\hat{x} - i\omega t_c}. \quad (5)$$

In the case of Proca field the temporal and spatial solutions in de Sitter geometry are obtained in [4]. These solutions will describe the massive Z free field. The spatial part of the solution is given by:

$$f^*_{\bar{\rho}\rho,\lambda}(x) = \left\{ \begin{array}{ll} \frac{i\sqrt{\pi} \omega \rho e^{-\pi k/2}}{2M_Z (2\pi)^{3/2}} \left[ \frac{1}{2} + ik \right] \sqrt{t_c} H^{(1)}_{ik} (-\mathcal{P}t_c) - \left( -t_c \right)^{3/2} H^{(1)}_{1+ik} (-\mathcal{P}t_c) \right. & \text{for } \lambda = 0 \\
\frac{\sqrt{\pi} \omega \rho e^{-\pi k/2}}{2(2\pi)^{3/2}} \sqrt{-t_c} H^{(1)}_{ik} (-\mathcal{P}t_c) e^{i\hat{\rho}\cdot\hat{x}} \epsilon(\bar{n}_\rho, \lambda) & \text{for } \lambda = \pm 1 \end{array} \right. \quad (6)$$

while the temporal part of the solution of Proca equation [4] is given by:

$$f_0_{\bar{\rho}\rho,\lambda}(x) = \left\{ \begin{array}{ll} \frac{\sqrt{\pi} \omega \rho e^{-\pi k/2}}{2M_Z (2\pi)^{3/2}} \left( -t_c \right)^{3/2} H^{(1)}_{ik} (-\mathcal{P}t_c) e^{i\hat{\rho}\cdot\hat{x}} & \text{for } \lambda = 0 \\
0 & \text{for } \lambda = \pm 1 \end{array} \right. \quad (7)$$

In the above equations which describe the plane wave solutions for the Proca field, $\bar{n}_\rho = \hat{\rho}/\mathcal{P}$ and $\epsilon(\bar{n}_\rho, \lambda)$ are the polarization vectors. For $\lambda = \pm 1$ the polarization vectors are transversal on the momentum i.e. $\hat{\rho} \cdot \epsilon(\bar{n}_\rho, \lambda = \pm 1) = 0$. In the case $\lambda = 0$ the polarization vectors are longitudinal on the momentum $\hat{\rho} \cdot \epsilon(\bar{n}_\rho, \lambda = 0) = \mathcal{P}$, since $\epsilon(\bar{n}_\rho, \lambda = 0) = \bar{n}_\rho$. The notation for the mass of the Z boson is $M_Z$, and the parameter $k = \sqrt{(M_Z \omega)^2 - \frac{1}{4}}$ is dependent on the ratio $\frac{M_Z}{\omega}$, and the condition $\frac{M_Z}{\omega} > \frac{1}{2}$ assures that the index of the Hankel functions are imaginary.
A. The calculation

Using the solutions given in equations (5), (6) and (7), the amplitude for longitudinal modes with \( \lambda = 0 \), can be brought in the form:

\[
\mathcal{A}_{\nu \rightarrow Z\nu}(\lambda = 0) = -i \int d^4x \sqrt{-g} \left( \frac{e_0}{\sin(2\theta_W)} \right) (U_{\nu',\sigma'})_\nu(x) \gamma^\delta e^\mu_0 \left( \frac{1 - \gamma^5}{2} \right) (U_{\rho\sigma})_\nu(x) f^*_{0\rho,\lambda=0,Z}(x)
- \int d^4x \sqrt{-g} \left( \frac{e_0}{\sin(2\theta_W)} \right) (U_{\nu',\sigma'})_\nu(x) \gamma^\delta e^\mu_0 \left( \frac{1 - \gamma^5}{2} \right) (U_{\rho\sigma})_\nu(x) f^*_{j\rho,\lambda=0,Z}(x).
\]

In the case of transversal modes with \( \lambda = \pm 1 \), only the spatial part of the solution give contributions since we do not have the temporal part of the solution i.e \( f_{0\rho,\lambda=\pm 1}(x) = 0 \), and we obtain:

\[
\mathcal{A}_{\nu \rightarrow Z\nu}(\lambda = \pm 1) = -i \int d^4x \sqrt{-g} \left( \frac{e_0}{\sin(2\theta_W)} \right) (U_{\nu',\sigma'})_\nu(x) \gamma^\delta e^\mu_0 \left( \frac{1 - \gamma^5}{2} \right) (U_{\rho\sigma})_\nu(x) f^*_{j\rho,\lambda=\pm 1,Z}(x).
\]

The spatial integrals give the delta Dirac function expressing the momentum conservation in the emission process. For the temporal integral the new integration variable is \( z = -t_c \) [24–26], and we use Bessel K functions by transforming the Hankel functions according to the equation:

\[
H^{(1,2)}_\nu(z) = \mp \left( \frac{2i}{\pi} \right) e^{\mp i\pi\nu/2} K_\nu(\mp iz).
\]

Then the amplitudes equations for \( \lambda = 0 \) and \( \lambda = \pm 1 \) are:

\[
\mathcal{A}_{\nu \rightarrow Z\nu}(\lambda = 0) = \frac{e_0}{\sin(2\theta_W)} \delta^3(\vec{P} + \vec{p}' - \vec{p}) \frac{1}{\sqrt{\pi}(2\pi)^{3/2}} \left( \frac{1}{2} - \sigma \right) \left( \frac{1}{2} - \sigma' \right) \times \left\{ \frac{\mathcal{P}}{M_Z} A(t_c) \xi_\sigma^+(\vec{p}') \vec{\sigma} \cdot \vec{e}^*(\vec{n}_\rho, \lambda = 0) \xi_\sigma(\vec{p}) + \frac{\mathcal{P}}{M_Z} C(t_c) \xi_\sigma^+(\vec{p}') \xi_\sigma(\vec{p}) \right\},
\]

\[
\mathcal{A}_{\nu \rightarrow Z\nu}(\lambda = \pm 1) = \frac{e_0}{\sin(2\theta_W)} \delta^3(\vec{P} + \vec{p}' - \vec{p}) \frac{1}{\sqrt{\pi}(2\pi)^{3/2}} \left( \frac{1}{2} - \sigma \right) \left( \frac{1}{2} - \sigma' \right) \times \left\{ B(t_c) \xi_\sigma^+(\vec{p}') \vec{\sigma} \cdot \vec{e}^*(\vec{n}_\rho, \lambda = \pm 1) \xi_\sigma(\vec{p}) \right\},
\]

The notations \( A(t_c), B(t_c), C(t_c) \) stands for the following temporal integrals:

\[
A(t_c) = \int_0^\infty d\tilde{z} \sqrt{\tilde{z}} e^{-i(\vec{p}' - \vec{p})\tilde{z}} K_{-\vec{p}}(i\mathcal{P} \tilde{z}) \frac{1}{\vec{p}} \left( \frac{1}{2} - ik \right) - i \int_0^\infty d\tilde{z} z^{3/2} e^{-i(\vec{p}' - \vec{p})\tilde{z}} K_{-\vec{p}}(i\mathcal{P} \tilde{z}),
\]

\[
C(t_c) = i \int_0^\infty d\tilde{z} z^{3/2} e^{-i(\vec{p}' - \vec{p})\tilde{z}} K_{-\vec{p}}(i\mathcal{P} \tilde{z}),
\]

\[
B(t_c) = i \int_0^\infty d\tilde{z} \sqrt{\tilde{z}} e^{-i(\vec{p}' - \vec{p})\tilde{z}} K_{-\vec{p}}(i\mathcal{P} \tilde{z}).
\]
By using equation (46) from Appendix the final result for the amplitudes is:

$$\mathcal{A}_{\nu \to Z\nu}(\lambda = 0) = \frac{e_0}{\sin(2\theta_W)} \delta^3(\vec{P} + \vec{p}' - \vec{p}) \frac{1}{(2\pi)^{3/2}} \left( \frac{1}{2} - \sigma \right) \left( \frac{1}{2} - \sigma' \right)$$

$$\times \left\{ A_k(\mathcal{P}, p, p') \xi_{\sigma \sigma'}^+(\vec{p}') \sigma \cdot \epsilon^*(\vec{n}_p, \lambda = 0) \xi_{\sigma}^+(\vec{p}) + C_k(\mathcal{P}, p, p') \xi_{\sigma \sigma'}^+(\vec{p}') \xi_{\sigma}^+(\vec{p}) \right\},$$

$$\mathcal{A}_{\nu \to Z\nu}(\lambda = \pm 1) = \frac{e_0}{\sin(2\theta_W)} \delta^3(\vec{P} + \vec{p}' + \vec{p}) \frac{1}{(2\pi)^{3/2}} \left( \frac{1}{2} - \sigma \right) \left( \frac{1}{2} - \sigma' \right)$$

$$\times \left\{ B_k(\mathcal{P}, p, p') \xi_{\sigma \sigma'}^+(\vec{p}') \sigma \cdot \epsilon^*(\vec{n}_p, \lambda = \pm 1) \xi_{\sigma}^+(\vec{p}) \right\}$$

(14)

The functions $A_k(\mathcal{P}, p, p')$, $B_k(\mathcal{P}, p, p')$, $C_k(\mathcal{P}, p, p')$ that define the amplitudes are:

$$A_k(\mathcal{P}, p, p') = \frac{i^{-3/2}(2\mathcal{P})^{-ik}}{(\mathcal{P} + p' - p)^{3/2-ik} M_Z} \Gamma \left( \frac{3}{2} - ik \right) \Gamma \left( \frac{3}{2} + ik \right) \left( \frac{1}{2} - ik \right)$$

$$\times \text{F$_2$F$_1$} \left( \frac{3}{2} - i k, \frac{1}{2} - i k; 2, \frac{p' - \mathcal{P} - p}{\mathcal{P} + p' - p} \right) - \frac{i^{-3/2}(2\mathcal{P})^{1-ik}}{2(\mathcal{P} + p' - p)^{7/2-ik} M_Z} \Gamma \left( \frac{7}{2} - ik \right)$$

$$\times \Gamma \left( \frac{3}{2} + ik \right) \text{F$_1$F$_2$} \left( \frac{7}{2} - i k, \frac{3}{2} - i k; 3, \frac{p' - \mathcal{P} - p}{\mathcal{P} + p' - p} \right),$$

(15)

$$C_k(\mathcal{P}, p, p') = \frac{i^{-3/2}(2\mathcal{P})^{-ik}}{2(\mathcal{P} + p' - p)^{5/2-ik} M_Z} \Gamma \left( \frac{5}{2} - ik \right) \Gamma \left( \frac{5}{2} + ik \right)$$

$$\times \text{F$_2$F$_1$} \left( \frac{5}{2} - i k, \frac{1}{2} - i k; 3, \frac{p' - \mathcal{P} - p}{\mathcal{P} + p' - p} \right),$$

$$B_k(\mathcal{P}, p, p') = \frac{i^{-1/2}(2\mathcal{P})^{-ik}}{(\mathcal{P} + p' - p)^{3/2-ik} M_Z} \Gamma \left( \frac{3}{2} - ik \right) \Gamma \left( \frac{3}{2} + ik \right)$$

$$\times \text{F$_2$F$_1$} \left( \frac{3}{2} - i k, \frac{1}{2} - i k; 2, \frac{p' - \mathcal{P} - p}{\mathcal{P} + p' - p} \right),$$

(16)

The final result is dependent on Gauss hypergeometric functions $\text{F$_2$F$_1$}$ and gamma Euler functions $\Gamma$. The amplitudes depend on gravity via the parameter $k = \sqrt{(\frac{M_Z}{\omega})^2 - \frac{1}{4}}$. We also observe that the ratio between $Z$ boson mass and the expansion parameter $\frac{M_Z}{\omega}$ and the momenta $p, p', \mathcal{P}$ determine the analytical structure of the amplitudes. The delta Dirac function $\delta^3(\vec{P} + \vec{p} - \vec{p})$ assures the momentum conservation in the process of $Z$ boson emission by neutrino, and this factor will play a key role in the computations for obtaining the total probability.
III. PROBABILITY OF Z BOSON EMISSION BY NEUTRINO

The probability is obtained by summing after the final helicities the square modulus of the amplitudes given in Eq. (14). First we observe that the amplitude is nonvanishing only for specific values of fermion helicities, \( \sigma = -\frac{1}{2}, \sigma' = -\frac{1}{2} \) and the summation after them is no longer necessary. If the helicity of the neutrino before emitting a Z boson is \( \sigma \), then after emission the helicity of the neutrino will remain the same and do not change sign, i.e. \( \sigma' = \sigma \). Because the amplitude is proportional with the delta Dirac function \( \delta^3(\vec{P} + \vec{p} + \vec{p}') \), one can define the transition probability per volume unit, i.e. \( |\delta^3(\vec{p})|^2 = V \delta^3(\vec{p}) \). For production of Z bosons with \( \lambda = 0 \) the probability is then:

\[
P_{\nu \to Z\nu}(\lambda = 0) = |A_{\nu \to Z\nu}(\lambda = 0)|^2 = \frac{e_0^2}{\sin^2(2\theta_W)} \delta^3(\vec{P} + \vec{p}' - \vec{p}) \frac{1}{(2\pi)^3} \left( \frac{1}{2} - \sigma \right)^2 \left( \frac{1}{2} - \sigma' \right)^2 \left\{ |A_k(\mathcal{P}, p, p')|^2 |\xi_0^+(\vec{p}') \vec{\sigma} \cdot \vec{e}^* (\vec{n}_\mathcal{P}, \lambda = 0) \xi_0(\vec{p})|^2 + |C_k(\mathcal{P}, p, p')|^2 |\xi_0^+(\vec{p}') \xi_0(\vec{p})|^2 \\
+ |A_k^*(\mathcal{P}, p, p') C_k(\mathcal{P}, p, p') (\xi_0^+(\vec{p}') \vec{\sigma} \cdot \vec{e}^* (\vec{n}_\mathcal{P}, \lambda = 0) \xi_0(\vec{p})) (\xi_0^+(\vec{p}') \xi_0(\vec{p}))|^2 \right\}.
\]  \tag{17}

The probability for generation of transversal modes with \( \lambda = \pm 1 \) is:

\[
P_{\nu \to Z\nu}(\lambda = \pm 1) = \frac{1}{2} \sum_{\lambda} |A_{\nu \to Z\nu}(\lambda = \pm 1)|^2 = \frac{e_0^2}{\sin^2(2\theta_W)} \delta^3(\vec{P} + \vec{p}' - \vec{p}) \frac{1}{(2\pi)^3} \left( \frac{1}{2} - \sigma \right)^2 \left( \frac{1}{2} - \sigma' \right)^2 \left\{ \frac{1}{2} \sum_{\lambda} |B_k(\mathcal{P}, p, p')|^2 |\xi_\sigma^+(\vec{p}') \vec{\sigma} \cdot \vec{e}^* (\vec{n}_\mathcal{P}, \lambda = \pm 1) \xi_\sigma(\vec{p})|^2 \right\}.
\]  \tag{18}

Further we will analyse graphically the probability computed with longitudinal modes (\( \lambda = 0 \)), and the probability computed with transversal modes (\( \lambda = \pm 1 \)). The functions that define the probability depend on the parameter \( k = \sqrt{\left( \frac{M_Z}{\omega} \right)^2 - \frac{1}{4}} \), and we must take into account that the condition in which our computations are done is \( \frac{M_Z}{\omega} > \frac{1}{2} \). In this case the Hankel functions have imaginary index, such that our graphs are done for the interval \( \frac{M_Z}{\omega} \in (\frac{1}{2}, \infty] \). The graphs that follows shows the behaviour of the functions that define the probability in terms of ratio \( \frac{M_Z}{\omega} \).

First we plot the functions that define the probability obtained with transversal modes.
FIG. 1: $|B_k|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.3, p' = 0.6, \mathcal{P} = 0.1$, while the point line is for $p = 0.2, p' = 0.6, \mathcal{P} = 0.3$.

FIG. 2: $|B_k|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.3, p' = 0.6, \mathcal{P} = 0.0001$, while the point line is for $p = 0.2, p' = 0.6, \mathcal{P} = 0.0003$.

The graphs (1)-(4) for the square modulus of $|A(0)|^2, |B_k|^2$, shows that the probability density for the Z boson emission process is non-vanishing only for small values of parameter $M_Z/\omega$. The factor $|A(0)|^2 = |A_k|^2 + |C_k|^2 + A_k^*C_k + A_kC_k^*$ contain all the contributions in
FIG. 3: $|A(0)|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.3, p' = 0.6, \mathcal{P} = 0.1$, while the point line is for $p = 0.2, p' = 0.6, \mathcal{P} = 0.3$.

FIG. 4: $|A(0)|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.3, p' = 0.6, \mathcal{P} = 0.0001$, while the point line is for $p = 0.2, p' = 0.6, \mathcal{P} = 0.0003$.

probability for the case when the computations were done with longitudinal modes, $\lambda = 0$. The process of Z boson emission by neutrino is possible only in large expansion conditions of the early universe. As the ratio $M_Z/\omega \sim 1$ the probability drops quickly to zero. For $M_Z/\omega \to \infty$ we obtain the Minkowski limit and the graphs show that probabilities are zero.
in this limit.

Another result obtained from our graphs is related to the probability density variation with the values taken for the momenta modulus \(|P|\) of the Z boson. Regarded to this we observe from Figs. (2)-(4) that the probability densities increase as the Z boson momenta \(|P|\) take small values, and the conclusion is that there are favoured the processes where emission of soft Z bosons occurs. Another observation is that for small \(|P|\) the probability begin to have an oscillatory behaviour.

IV. TOTAL PROBABILITY

The total probability of the process is obtained by integrating after the final momenta \(p', P\) the probability. One of the integrals is immediate because the probability depend on delta Dirac function \(\delta^3(\vec{P} + \vec{p}' - \vec{p})\). The square modulus of transition amplitude contain products of two hypergeometric functions and a polynomial factor and there are no such integrals known in literature. The computations will be done by following the method applied in [1], considering emission of ”soft” Z bosons with small momenta.

A. Total probability for Z boson emission by neutrino \(\lambda = \pm 1\)

The total probability for the transversal modes generation \((\lambda = \pm 1)\), is obtained by using equation (18), and we obtain:

\[
P_{\text{tot}}(\lambda = \pm 1) = \int d^3P \ d^3p' \ P_{\nu \rightarrow Z \nu}(\lambda = \pm 1) = \int d^3P \ d^3p' \frac{1}{2} \sum_{\lambda} |A_{\nu \rightarrow Z \nu}(\lambda = \pm 1)|^2
\]

\[
= \int d^3P \ d^3p' \ \frac{e^2_0 \delta^3(\vec{P} + \vec{p}' - \vec{p})}{(2\pi)^3 \sin^2(2\theta_W)} \left( \frac{1}{2} - \sigma \right)^2 \left( \frac{1}{2} - \sigma' \right)^2 \\
\times \frac{1}{2} \sum_{\lambda} |B_k(P, p, p')|^2 |\xi^+_\sigma'(\vec{p}')\vec{\sigma} \cdot \vec{\epsilon}^*(\vec{n}_P, \lambda = \pm 1)|^2
\]

(19)

Consider now the case when the momenta of Z boson is fixed along the z axis, \(\vec{P} = P\hat{e}_3\). Further using the momentum conservation, the bispinor \(\xi^k_2(\vec{p})\) can be rewritten in terms of the momenta \(p', P\), following the integration after \(d^3p'\). The next step is to set according to the above formulas the possible values of polarizations such that \(\sigma = \sigma' = -\frac{1}{2}\). The
helicity spinors and polarization vectors can be expressed by taking an orthogonal local frame defined by the basis vectors $\vec{e}_i$. In this frame we define the momenta $\vec{p} = p_i \vec{e}_i$ and fix the Z boson momenta on third axis $\vec{P} = P \vec{e}_3$. For the momenta of the neutrinos we will take the spherical coordinates, corresponding to the situation when the momenta are in the plane $(1, 3)$, by taking $\vec{p}(p, \alpha, \beta = 0)$ and $\vec{p}'(p', \gamma, \theta = \pi)$ [26]. In this case the angle between $\vec{p}$ and $\vec{p}'$ is just $\alpha + \gamma$. The momenta conservation in the plane $(1, 3)$ give:

$$\mathcal{P} = p \cos \alpha - p' \cos \gamma;$$

$$p \sin \alpha + p' \sin \gamma = 0. \quad (20)$$

For performing the computations with the helicity bispinors in the case with $\lambda = \pm 1$, we consider the circular polarizations

$$\vec{e}_{\pm 1} = \frac{1}{\sqrt{2}}(\pm \vec{e}_1 + i\vec{e}_2), \quad (21)$$

in the local orthogonal frame where $\vec{P} = \mathcal{P} \vec{e}_3$. Then perform the integration with the delta Dirac function after the momenta of the anti-neutrinos $d^3p'$:

$$\int d^3p' \delta^3(\vec{P} + \vec{p}' - \vec{p})|B_k(\mathcal{P}, p, p')|^2|\xi_{+}^+(\vec{p}')\vec{\sigma} \cdot \vec{e}^*(\vec{n}_P, \lambda = \pm 1)\xi_{\sigma}(\vec{p})|^2$$

$$= |B_k(\mathcal{P}, p, |\vec{p} - \vec{P}|)|^2|\xi_{+}^+\frac{1}{2}(\vec{p} - \vec{P})\vec{\sigma} \cdot \vec{e}^*(\vec{n}_P, \lambda = \pm 1)\xi_{\pm}^+(\vec{p})|^2, \quad (22)$$

where

$$|B_k(\mathcal{P}, p, |\vec{p} - \vec{P}|)|^2 = \frac{1}{(\mathcal{P} - p + |\vec{p} - \vec{P}|)^3} \left| \Gamma \left( \frac{3}{2} - ik \right) \right|^2 \left| \Gamma \left( \frac{3}{2} + ik \right) \right|^2$$

$$\times _2F_1 \left( \frac{3}{2} - ik, \frac{1}{2} - ik; 2; \frac{-\mathcal{P} - p + |\vec{p} - \vec{P}|}{\mathcal{P} - p + |\vec{p} - \vec{P}|} \right) _2F_1 \left( \frac{3}{2} + ik, \frac{1}{2} + ik; 2; \frac{-\mathcal{P} - p + |\vec{p} - \vec{P}|}{\mathcal{P} - p + |\vec{p} - \vec{P}|} \right), \quad (23)$$

and

$$\xi_{+}^+\frac{1}{2}(\vec{p} - \vec{P}) = \sqrt{\frac{-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|}{2|\vec{p} - \vec{P}|}} \left( \frac{-p \sin(\alpha) e^{-i\beta}}{-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|} \right), \quad (24)$$

$$\xi_{-}^+\frac{1}{2}(\vec{p}) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \left( \frac{-\sin(\alpha) e^{-i\beta}}{1 + \cos(\alpha)} \right). \quad (25)$$
Equation (24) is justified knowing that $\vec{P} = \mathcal{P}\hat{e}_3$ and the conservation law of momentum $\vec{p}' = \vec{p} - \vec{P}$, and one obtain that the components on the axes 1, 2 are $p_1 = p'_1$; $p_2 = p'_2$; $P_1 = P_2 = 0, P_3 = \mathcal{P}$. Now considering the spherical coordinates for the momenta $p(p, \alpha, \beta)$ and $p'(p', \gamma, \theta)$, together with the momenta conservation relations projected on axes we obtain:

- $p'_3 = p' \cos(\gamma) = -\mathcal{P} + p \cos(\alpha)$, and $-p'_1 + ip'_2 = -p_1 + ip_2 = -p \sin(\alpha) e^{-i\beta}$. In this way the helicity bispinor $\xi_{\pm \frac{1}{2}}(p')$, was expressed in terms of momenta $p, \mathcal{P}$, polar angle $\alpha$ and azimuthal angle $\beta$, and we can compute the integration after $d^3\mathcal{P}$. After a little calculation we obtain:

$$
\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \mathbf{s} \cdot \mathbf{e}^* (\vec{n}_p, \lambda = 1) \xi^-_{-\frac{1}{2}}(\vec{p}) = \left( \frac{-\sin(\alpha) e^{-i\beta} \sqrt{(-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|)}}{\sqrt{2|\vec{p} - \vec{P}|(1 + \cos(\alpha))}} \right),
$$

$$
\xi^+_{+\frac{1}{2}}(\vec{p} - \vec{P}) \mathbf{s} \cdot \mathbf{e}^* (\vec{n}_p, \lambda = -1) \xi^-_{+\frac{1}{2}}(\vec{p}) = \left( \frac{p \sin(\alpha) e^{i\beta} \sqrt{1 + \cos(\alpha)}}{\sqrt{2(-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|)|\vec{p} - \vec{P}|}} \right).
$$

The dependence of the momenta modulus $|\vec{p} - \vec{P}|$, allows to consider fixed directions by fixing the angle between momenta vectors $\vec{P}$ and $\vec{p}$, which is $\alpha$, since we know that the $Z$ boson momenta is on the third axis. The result is expressed in terms of integrals with products of two hypergeometric functions and a polynomial factor in momenta powers, and the integrals that define the total probability are of the form:

$$
\int d\Omega_P \int_0^\infty dP dP^2 \int \frac{1}{(P - p + |\vec{p} - \vec{P}|)^3} \times {}_2F_1 \left( \frac{3}{2} - ik; \frac{1}{2} - ik; \frac{P - p + |\vec{p} - \vec{P}|}{P - p + |\vec{p} - \vec{P}|} \right) \times \left\{ \begin{array}{ll}
\frac{\sin^2(\alpha)(-P + p \cos(\alpha) + |\vec{p} - \vec{P}|)}{2|\vec{p} - \vec{P}|(1 + \cos(\alpha))} & \text{for } \lambda = 1 \\
\frac{p^2 \sin^2(\alpha)(1 + \cos(\alpha))}{2(-P + p \cos(\alpha) + |\vec{p} - \vec{P}|)|\vec{p} - \vec{P}|} & \text{for } \lambda = -1
\end{array} \right.
$$

(27)

For computing the above integrals we will approximate the hypergeometric functions considering the situation when the momenta of the $Z$ boson $\mathcal{P}$ is small in rapport with the momenta of the neutrinos $p$. In addition we fix the polar angle $\alpha = \frac{\pi}{2}$ and compute the total probability for given directions. The integral after the azimuthal angle $\beta$, contribute with a factor of $2\pi$ since in equation (27) we do not have dependence on $\beta$. The modulus $|\vec{p} - \vec{P}|$
become for \( \alpha = \frac{\pi}{2} \):

\[
|\vec{p} - \vec{P}| \simeq p \sqrt{1 + \frac{p^2}{P^2}} \sim p
\]  

(28)

because for \( p \gg P \) we use \( \frac{p^2}{P^2} \to 0 \). With this simplification the argument of the hypergeometric function become:

\[
\frac{-P - p + |\vec{p} - \vec{P}|}{P - p + |\vec{p} - \vec{P}|} \sim -1,
\]  

(29)

and we can use the formula \( _2F_1(a, b; a - b + 1; -1) = \frac{2^{-a} \sqrt{\pi} \Gamma(a - b + 1)}{\Gamma((a+1)/2)\Gamma(a/2 - b + 1)} \) \([24]\). Then for small \( P \) the hypergeometric functions in our equation reduce to:

\[
_2F_1 \left( \frac{3}{2} - ik, \frac{1}{2} - ik; 2; -1 \right) = \frac{2^{-3/2 + ik} \sqrt{\pi} \Gamma(2)}{\Gamma \left( \frac{5}{4} - ik \right) \Gamma \left( \frac{5}{4} + ik \right)},
\]  

(30)

In addition when \( p \gg P \) and for fixed \( \alpha = \pi/2 \), the square modulus of bispinor products give if we use that \( \frac{P}{p} \to 0 \):

\[
|\xi_{\frac{1}{2}}^+(\vec{p} - \vec{P}) \vec{\sigma} \cdot \vec{\epsilon}^{*} (\vec{n}_P, \lambda = 1) \xi_{-\frac{1}{2}}(\vec{p})|^2 = \frac{p - P}{2p} \simeq \frac{1}{2},
\]

\[
|\xi_{\frac{1}{2}}^+(\vec{p} - \vec{P}) \vec{\sigma} \cdot \vec{\epsilon}^{*} (\vec{n}_P, \lambda = -1) \xi_{-\frac{1}{2}}(\vec{p})|^2 = \frac{p}{2(p - P)} \simeq \frac{1}{2}.
\]  

(31)

The momenta integrals that results by applying the above approximations for both \( \lambda = \pm 1 \) are of the type:

\[
\int_{P_{\min}}^{P_{\max}} \frac{dP}{P} \frac{1}{P} = \ln \left( \frac{P_{\max}}{P_{\min}} \right),
\]  

(32)

where we take a cutoff of the lower limits since we have a logarithmic divergence. The upper limit is established by considering the momenta corresponding to the Compton wavelength, which is \( \frac{P}{\omega} \). Collecting all results we arrive at the following equation for total probability:

\[
P_{\text{tot}}(\lambda = \pm 1) = \frac{\alpha}{8 \sin^2(2\theta_W)} \frac{|\Gamma \left( \frac{3}{2} - ik \right)|^2 |\Gamma \left( \frac{3}{4} + ik \right)|^2 \ln \left( \frac{P_{\max}}{P_{\min}} \right)}{|\Gamma \left( \frac{5}{4} - ik \right)|^2 |\Gamma \left( \frac{5}{4} + ik \right)|^2} \]  

(33)

where the result of the solid angle integral \( \int d\Omega_P = 4\pi \) was included. A graphical analysis in terms of parameter \( \frac{M_Z}{\omega} \) for given values of the logarithm argument \( \frac{P_{\max}}{P_{\min}} \) can be done and the results are given in Fig. (5). The graph Fig. (5) show that the probability of Z boson emission is large only when \( M_Z/\omega \sim 1/2 \) and the probability is zero for large values of the same parameter. In the limit \( \frac{M_Z}{\omega} \to \infty \) we recover the results from the flat space theory \([17, 22, 23]\), where the probability is zero due to the energy-momentum conservation.
FIG. 5: Total probability $P_{\text{tot}}(\lambda = \pm 1)$ as a function of parameter $M_Z/\omega$. Solid line is for $\frac{P_{\text{C}}}{P_{\text{min}}} = 5$, while the point line is for $\frac{P_{\text{C}}}{P_{\text{min}}} = 10$.

A closer look to the probability equation (33) prove that we can study with this formula all the interesting cases including the case when the argument of the Bessel K functions from solutions of the Proca equation become real. A simple calculation also proves that in the case $\frac{M_Z}{\omega} = \frac{1}{2}$ or $k = 0$ the probability equation give a finite value

$$P_{\text{tot}}(\lambda = \pm 1) = \frac{\alpha}{2\pi \sin^2(2\theta_W)} \frac{\Gamma^4\left(\frac{3}{2}\right)}{\Gamma^4\left(\frac{7}{4}\right)} \ln \left(\frac{P_{\text{C}}}{P_{\text{min}}}\right).$$  \hspace{1cm} (34)

In the limit $M_Z/\omega \to 0$ we obtain the case of large expansion $\omega >> M_Z$ and our equation for total probability reduce in this case to:

$$P_{\text{tot}}(\lambda = \pm 1) = \frac{\alpha}{2\pi \sin^2(2\theta_W)} \ln \left(\frac{P_{\text{C}}}{P_{\text{min}}}\right).$$  \hspace{1cm} (35)

Equation (35) was obtained by neglecting the ratio $M_Z/\omega << \frac{1}{4}$ in which case the parameter $i\sqrt{(\frac{M_Z}{\omega})^2 - \frac{1}{4}} = -\frac{1}{2}$. The general formula for probability (33) contain all the interest cases including the interesting case when $M_Z/\omega \to 0$. These results allows us to graphically study the total probability for all possible values of the parameter $M_Z/\omega$, and this result include the case when $\omega >> M_Z$, see Fig. (7). In the limit $\omega >> M_Z$ the total probability for a emission process have a significative increasing and this represents the case of large expansion from early universe.
FIG. 6: Total probability $P_{\text{tot}}(\lambda = \pm 1)$ as a function of parameter $M_Z/\omega$. Solid line is for $P_{\text{C}}/P_{\text{min}} = 5$, while the point line is for $P_{\text{C}}/P_{\text{min}} = 10$.

In the case when $\lambda = 0$, the total probability can be obtained integrating after the final momenta $p'$, $\mathcal{P}$ the probability equation (17):

$$P_{\text{tot}}(\lambda = 0) = \int d^3\mathcal{P} d^3p' P_{\nu \to Z\nu}(\lambda = 0) = \int d^3\mathcal{P} d^3p' \frac{e_0^2}{\sin^2(2\theta_W)} \delta^3(\vec{\mathcal{P}} + \vec{p}' - \vec{p})$$

$$\frac{1}{(2\pi)^3} \left( \frac{1}{2} - \sigma \right)^2 \left( \frac{1}{2} - \sigma' \right)^2 \left\{ |A_k(\mathcal{P}, p, p')|^2 \xi_{\sigma'}(\vec{p'}) \vec{\sigma} \cdot \vec{\epsilon}^*(\vec{n}_\mathcal{P}, \lambda = 0) \xi_\sigma(\vec{p})|^2 + |C_k(\mathcal{P}, p, p')|^2 |\xi_{\sigma'}(\vec{p'})|^2 |\xi_\sigma(\vec{p})|^2 + A_k(\mathcal{P}, p, p') C_k(\mathcal{P}, p, p')(\xi_{\sigma'}(\vec{p'}) \vec{\sigma} \cdot \vec{\epsilon}^*(\vec{n}_\mathcal{P}, \lambda = 0) \xi_\sigma(\vec{p})|^2 \xi_{\sigma'}(\vec{p'}) \xi_\sigma(\vec{p}) \right\} \right\}$$

Solving the integral after $p'$ we obtain that the functions $|A_k|^2$, $|C_k|^2$, $A_k^* C_k$, $A_k C_k^*$, defining the probability depend on the modulus $|\vec{p} - \vec{\mathcal{P}}|$. This is done by integrating after $d^3p'$, and
use the delta Dirac function properties, then for $\sigma = \sigma' = -\frac{1}{2}$ one obtain:

$$
\int d^3p' \delta^3(\vec{P} + \vec{p}' - \vec{p}) \left\{ |A_k(\mathcal{P}, p, p')|^2 |\xi^+_{-\frac{1}{2}}(\vec{p}') \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p})|^2 \\
+ |C_k(\mathcal{P}, p, p')|^2 |\xi^+_{-\frac{1}{2}}(\vec{p}') \xi_{-\frac{1}{2}}(\vec{p})|^2 \\
+ A_k(\mathcal{P}, p, p') C_k(\mathcal{P}, p, p') (\xi^+_{-\frac{1}{2}}(\vec{p}') \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p}))^* (\xi^+_{-\frac{1}{2}}(\vec{p}') \xi_{-\frac{1}{2}}(\vec{p})) \\
+ C_k^*(\mathcal{P}, p, p') A_k(\mathcal{P}, p, p') (\xi^+_{-\frac{1}{2}}(\vec{p}') \xi_{-\frac{1}{2}}(\vec{p}))^* (\xi^+_{-\frac{1}{2}}(\vec{p}') \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p})) \right\}
$$

$$
= |A_k(\mathcal{P}, p, |\vec{p} - \vec{P}|)|^2 |\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p})|^2 \\
+ |C_k(\mathcal{P}, p, |\vec{p} - \vec{P}|)|^2 |\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \xi_{-\frac{1}{2}}(\vec{p})|^2 \\
+ A_k(\mathcal{P}, p, |\vec{p} - \vec{P}|) C_k(\mathcal{P}, p, |\vec{p} - \vec{P}|) (\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p}))^* (\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \xi_{-\frac{1}{2}}(\vec{p})) \\
+ C_k^*(\mathcal{P}, p, |\vec{p} - \vec{P}|) A_k(\mathcal{P}, p, |\vec{p} - \vec{P}|) (\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \xi_{-\frac{1}{2}}(\vec{p}))^* (\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p})).
$$

(37)

The helicity bispinors products from equation (37) are computed by knowing that the polarization vector is on the momentum direction $\bar{e}(\vec{n}_P, \lambda = 0) = \bar{e}_3$, and by using equations (24), (25):

$$
\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \bar{\sigma} \cdot \bar{e}^*(\vec{n}_P, \lambda = 0)\xi_{-\frac{1}{2}}(\vec{p}) = \sqrt{-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|} \sqrt{\frac{1 + \cos(\alpha)}{2}} \\
\times \left( \frac{p \sin^2(\alpha)}{(1 + \cos(\alpha))(-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|)} - 1 \right); \\
\xi^+_{-\frac{1}{2}}(\vec{p} - \vec{P}) \xi_{-\frac{1}{2}}(\vec{p}) = \sqrt{-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|} \sqrt{\frac{1 + \cos(\alpha)}{2}} \times \left( \frac{p \sin^2(\alpha)}{(1 + \cos(\alpha))(-\mathcal{P} + p \cos(\alpha) + |\vec{p} - \vec{P}|)} + 1 \right).
$$

(38)

The other relations with helicity bispinors from probability (36), can be obtained using the above equation. The dependence of polar angle $\alpha$, in equation (38) is complicated and we fix the angle between $\vec{p}, \vec{P}$ to $\alpha = 0$ such that $|\vec{p} - \vec{P}| = p - \mathcal{P}$, the Z boson momenta being oriented along the z axis. Then the bispinors square modulus from probability give if we
use Eq.(38) for $\alpha = 0$:

$$|\xi_+^+ (\tilde{p} - \tilde{P}) \tilde{\sigma} \cdot \tilde{\varepsilon}^* (\tilde{n}_P, \lambda = 0)\xi_-^- (\tilde{p})|^2 = |\xi_-^- (\tilde{p} - \tilde{P}) \xi_+^+ (\tilde{n}_P, \lambda = 0)\xi_-^- (\tilde{p})|^2 = 1;$$

$$(\xi_+^+ (\tilde{p} - \tilde{P}) \xi_-^- (\tilde{p}))^* (\xi_-^- (\tilde{p} - \tilde{P}) \tilde{\sigma} \cdot \tilde{\varepsilon}^* (\tilde{n}_P, \lambda = 0)\xi_-^- (\tilde{p}))$$

$$= (\xi_-^- (\tilde{p} - \tilde{P}) \xi_+^+ (\tilde{n}_P, \lambda = 0)\xi_-^- (\tilde{p}))^* = -1. \quad (39)$$

The argument of the hypergeometric functions is approximated for $p >> P$ by taking into account that for $\alpha = 0$ we have:

$$|\tilde{p} - \tilde{P}| \simeq p \sqrt{1 + \frac{P^2}{p^2} - 2 \frac{P}{p}} \sim p, \quad (40)$$

and by using this approximation the hypergeometric functions can be rewritten by using equations (29), and equation of the type (30). Now taking into account equation (36), the integrals that define the total probability in the case $\lambda = 0$ are:

$$\int d\Omega_P \int_{\frac{p_{min}}{\omega}}^{\frac{p_{C/\omega}}{\omega}} dP \frac{1}{P} \left\{ \begin{array}{c}
\frac{A_1}{2^3} _2F_1 \left( \begin{array}{c} 3 \over 2 - ik, 1 \over 2 - ik; 2; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 3 \over 2 + ik, 1 \over 2 + ik; 2; -1 \end{array} \right) \\
\frac{A_2}{2^5} _2F_1 \left( \begin{array}{c} 7 \over 2 - ik, 3 \over 2 - ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 7 \over 2 + ik, 3 \over 2 + ik; 3; -1 \end{array} \right) \\
\frac{A_3}{2^4} _2F_1 \left( \begin{array}{c} 7 \over 2 + ik, 3 \over 2 + ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 7 \over 2 - ik, 3 \over 2 - ik; 2; -1 \end{array} \right) \\
\frac{A_4}{2^4} _2F_1 \left( \begin{array}{c} 7 \over 2 - ik, 3 \over 2 - ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 7 \over 2 + ik, 3 \over 2 + ik; 2; -1 \end{array} \right) \\
\frac{A_5}{2^3} _2F_1 \left( \begin{array}{c} 5 \over 2 + ik, 1 \over 2 + ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 5 \over 2 - ik, 1 \over 2 - ik; 3; -1 \end{array} \right) \\
\frac{A_6}{2^3} _2F_1 \left( \begin{array}{c} 5 \over 2 + ik, 1 \over 2 + ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 5 \over 2 - ik, 1 \over 2 - ik; 2; -1 \end{array} \right) \\
\frac{A_7}{2^4} _2F_1 \left( \begin{array}{c} 5 \over 2 + ik, 1 \over 2 + ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 5 \over 2 - ik, 1 \over 2 - ik; 3; -1 \end{array} \right) \\
\frac{A_8}{2^3} _2F_1 \left( \begin{array}{c} 3 \over 2 + ik, 1 \over 2 + ik; 2; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 3 \over 2 - ik, 1 \over 2 - ik; 3; -1 \end{array} \right) \\
\frac{A_9}{2^4} _2F_1 \left( \begin{array}{c} 7 \over 2 + ik, 3 \over 2 + ik; 3; -1 \end{array} \right) _2F_1 \left( \begin{array}{c} 7 \over 2 - ik, 1 \over 2 - ik; 3; -1 \end{array} \right) \end{array} \right\}, \quad (41)$$
where the coefficients of each integral $A_1...A_9$ are given by:

$$A_1 = \left| \Gamma\left(\frac{3}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{3}{2} + ik\right)\right|^2 \left(\frac{1}{4} + k^2\right), \quad A_2 = \left| \Gamma\left(\frac{7}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{3}{2} + ik\right)\right|^2$$

$$A_3 = \Gamma^2\left(\frac{3}{2} - ik\right) \Gamma\left(\frac{3}{2} + ik\right) \Gamma\left(\frac{7}{2} + ik\right) \left(\frac{1}{2} - ik\right),$$

$$A_4 = \Gamma^2\left(\frac{3}{2} + ik\right) \Gamma\left(\frac{3}{2} - ik\right) \Gamma\left(\frac{7}{2} - ik\right) \left(\frac{1}{2} + ik\right),$$

$$A_5 = \left| \Gamma\left(\frac{5}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{5}{2} + ik\right)\right|^2, \quad A_6 = \left| \Gamma\left(\frac{5}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{3}{2} - ik\right)\right|^2 \left(\frac{1}{2} - ik\right),$$

$$A_7 = \left| \Gamma\left(\frac{5}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{3}{2} + ik\right)\right| \Gamma\left(\frac{7}{2} - ik\right), \quad A_8 = \left| \Gamma\left(\frac{5}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{3}{2} - ik\right)\right|^2 \left(\frac{1}{2} + ik\right),$$

$$A_9 = \left| \Gamma\left(\frac{5}{2} - ik\right)\right|^2 \left| \Gamma\left(\frac{3}{2} - ik\right)\right| \Gamma\left(\frac{7}{2} + ik\right). \quad (42)$$

The final equation for total probability in the case $\lambda = 0$ is obtained by approximating the hypergeometric functions from Eq.(41) with the help of equations (29), (30):

$$P_{tot}(\lambda = 0) = \frac{2\alpha}{\pi \sin^2(2\theta_W)} \left(\frac{M_Z}{\omega}\right)^{-2} \ln \left(\frac{\mathcal{P}_C}{\mathcal{P}_{min}}\right) \left\{ \frac{A_1 B_1}{2^3} + \frac{A_2 B_2}{2^5} - \frac{A_3 B_3}{2^4} - \frac{A_4 B_4}{2^4} + \frac{A_5 B_5}{2^3} - \frac{A_6 B_6}{2^3} + \frac{A_7 B_7}{2^4} - \frac{A_8 B_8}{2^3} + \frac{A_9 B_9}{2^4} \right\}. \quad (43)$$

The new coefficients $B_1...B_9$ resulted from the hypergeometric functions have the following expression:

$$B_1 = \left(\frac{\Gamma\left(\frac{5}{4} + \frac{ik}{2}\right)}{\Gamma\left(\frac{5}{4} - \frac{ik}{2}\right)}\right)^2, \quad B_2 = \left(\frac{\Gamma\left(\frac{9}{4} - \frac{ik}{2}\right)}{\Gamma\left(\frac{5}{4} + \frac{ik}{2}\right)}\right)^2$$

$$B_3 = \frac{1}{\Gamma\left(\frac{5}{4} - \frac{ik}{2}\right)^2 \Gamma\left(\frac{9}{4} - \frac{ik}{2}\right) \Gamma\left(\frac{5}{4} + \frac{ik}{2}\right)}, \quad B_4 = 1$$

$$B_5 = \frac{1}{\Gamma\left(\frac{7}{4} + \frac{ik}{2}\right)^2 \Gamma\left(\frac{7}{4} - \frac{ik}{2}\right)^2}, \quad B_6 = B_8 = \frac{1}{\Gamma\left(\frac{7}{4} - \frac{ik}{2}\right)^2 \Gamma\left(\frac{7}{4} - \frac{ik}{2}\right)^2}$$

$$B_7 = \frac{1}{\Gamma\left(\frac{7}{4} - \frac{ik}{2}\right)^2 \Gamma\left(\frac{7}{4} + \frac{ik}{2}\right) \Gamma\left(\frac{7}{4} - \frac{ik}{2}\right)}, \quad B_9 = \frac{1}{\Gamma\left(\frac{7}{4} - \frac{ik}{2}\right)^2 \Gamma\left(\frac{7}{4} + \frac{ik}{2}\right) \Gamma\left(\frac{7}{4} + \frac{ik}{2}\right)}. \quad (44)$$

Equation (43) in the limit $\frac{M_Z}{\omega} = \frac{1}{2}$ or $k = 0$ is reduced to:

$$P_{tot}(\lambda = 0) = \frac{\alpha}{\pi \sin^2(2\theta_W)} \ln \left(\frac{\mathcal{P}_C}{\mathcal{P}_{min}}\right) \left\{ \frac{\Gamma^4\left(\frac{3}{2}\right)}{4 \Gamma^4\left(\frac{5}{4}\right)} + \frac{\Gamma^4\left(\frac{5}{4}\right)}{4 \Gamma^4\left(\frac{5}{4}\right)} + \frac{\Gamma^2\left(\frac{7}{4}\right) \Gamma^2\left(\frac{3}{4}\right)}{4 \Gamma^2\left(\frac{7}{4}\right) \Gamma^2\left(\frac{5}{4}\right)} \right\}$$

$$- \frac{\Gamma\left(\frac{7}{2}\right) \Gamma^3\left(\frac{3}{2}\right)}{2 \Gamma^3\left(\frac{7}{4}\right) \Gamma^3\left(\frac{5}{4}\right)} - \frac{\Gamma^2\left(\frac{5}{4}\right) \Gamma^2\left(\frac{3}{4}\right)}{\Gamma^2\left(\frac{5}{4}\right) \Gamma^2\left(\frac{3}{4}\right)} + \frac{\Gamma^2\left(\frac{5}{4}\right) \Gamma\left(\frac{3}{4}\right)}{\Gamma^2\left(\frac{5}{4}\right) \Gamma\left(\frac{3}{4}\right)} \right\}. \quad (45)$$

Plotting the total probability equation for $\lambda = 0$ in terms of $\frac{M_Z}{\omega}$ we observe that the nonvanishing values of this quantity are around $\frac{M_Z}{\omega} \sim \frac{1}{2}$ and vanish in the Minkowski limit $\frac{M_Z}{\omega} \to \infty$. 

18
FIG. 7: Total probability $P_{\text{tot}}(\lambda = 0)$ as a function of parameter $M_Z/\omega$. Solid line is for $\frac{P_C}{P_{\text{min}}} = 5$, while the point line is for $\frac{P_C}{P_{\text{min}}} = 10$.

We study in this paper the problem of Z bosons production in emission processes by neutrinos and we conclude that these phenomena are possible only in large expansion conditions of early universe. Our graphical analysis prove that this kind of reactions are allowed until the ratio between mass of the boson and expansion parameter is in the interval, $\frac{M_Z}{\omega} \in (0, 1.5]$. As this ratio increase up to values bigger than 1.5 the total probability drops to zero. Our result prove that the Z bosons could be produced in emission processes by fermions and these reactions need to be considered along side with the phenomenon of Z bosons production from vacuum [1]. This mechanism for generation of massive bosons in emission processes is one of the mechanisms that could explain the abundance of massive bosons in the early universe [5, 6].

V. APPENDIX

The temporal integrals from amplitude are of the type [24]:

$$
\int_0^\infty dz z^{\mu-1} e^{-\alpha z} K_\nu(\beta z) = \frac{\sqrt{\pi} (2\beta)^\nu}{(\alpha + \beta)^{\mu + \nu}} \Gamma (\mu + \nu) \Gamma (\mu - \nu) \Gamma (\mu + \frac{1}{2}) \times _2 F_1 \left( \mu + \nu, \nu + \frac{1}{2}; \mu + \frac{1}{2}; \frac{\alpha - \beta}{\alpha + \beta} \right),
\text{Re}(\alpha + \beta) > 0, |\text{Re}(\mu)| > |\text{Re}(\nu)|.
$$

(46)
The form of the helicity bispinors can be expressed as follows [17, 23]:

\[
\xi_{\frac{1}{2}}(\vec{p}) = \sqrt{\frac{p_3 + p}{2p}} \begin{pmatrix} 1 \\ \frac{p_1 + ip_2}{p_3 + p} \end{pmatrix}, \quad \xi_{-\frac{1}{2}}(\vec{p}) = \sqrt{\frac{p_3 + p}{2p}} \begin{pmatrix} -\frac{p_1 + ip_2}{p_3 + p} \\ 1 \end{pmatrix},
\]

(47)

while \( \eta_\sigma(\vec{p}) = i\sigma_2[\xi_\sigma(\vec{p})]^* \). These spinors satisfy the relation:

\[
\bar{\sigma} \vec{p} \xi_\sigma(\vec{p}) = 2p_\sigma \xi_\sigma(\vec{p})
\]

(48)

with \( \sigma = \pm \frac{1}{2} \), where \( \bar{\sigma} \) are the Pauli matrices and \( p = |\vec{p}| \) is the modulus of the momentum vector.

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