Liquefaction of a horizontally vibrated granular bed: Friction, Dilation and Segregation

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(November 15, 2018)

Abstract

We present experimental observations of the onset of flow (liquefaction) for horizontally vibrated granular materials. As the acceleration $\Gamma$ increases above $\Gamma^*$, the top layer of granular material liquefies, while the remainder of the layer moves with the shaker in solid body motion. With increasing $\Gamma$, more of the layer becomes fluidized. The initial bifurcation is backward, and the amount of hysteresis depends mainly on frictional properties of the granular media. A small amount of fluidization by gas flow lifts the hysteresis. Modest differences in the frictional properties of otherwise identical particles leads to rapid segregation.

PACS numbers: 46.10.+z, 47.20.-k
Although granular materials are common in nature and industrial applications, the complete understanding of their dynamical behavior is still an open problem. Consequently, the dynamics of granular materials has attracted considerable interest in recent years (for comprehensive reviews see reference [1] and citations therein). Granular materials can exhibit both fluid-like and solid-like properties depending on the circumstances: they resist shearing up to a point, but flow freely under strong enough shear or at low enough density. These materials also display a number of different dynamical states including liquefaction, heap formation and convection under vibration, the spontaneous formation of stable arches, segregation, a variety of density waves, stick-slip motion during avalanches, etc. [1]

Much recent attention has been focused on the dynamics of vertically vibrated granular materials [1]. Although there have been some studies [2–6], much less is known about the corresponding dynamics of granular materials subject to horizontal vibration, and of the existing work much is very recent. The aim of the present study is to explore the dynamics of a horizontally vibrated systems, with a particular focus on the transition to flow—sometimes referred to as liquefaction. A better understanding of this second case is of interest scientifically because it gives insight into segregation phenomena and into the competition between dilation and friction. It is also of interest because both horizontal and vertical vibration are commonly used in industries as an aid to mixing, segregating and transporting granular materials. Finally, soil liquefaction during earthquakes is a common and destructive phenomena associated with horizontal shaking.

Our experimental setup is shown in Fig 1. The heart of the experiment is a rectangular Plexiglas cell with cross-sectional dimensions of 1.93 cm by 12.1 cm. The base of the cell is made of a porous medium (average pore size 50μm) through which gas can flow in order to fluidize the granular medium. This provides an independent control over the dilation of the material. The cell is mounted on a Plexiglas base of the same cross-sectional dimensions as the cell, which is in turn mounted on a movable table; the base acts as the gas distributor to the system. The table is mounted on four linear bearings sliding on horizontal shafts rigidly attached to the fixed bottom frame. An electro-mechanical actuator provides a sinusoidal
drive of the form \( x = A \sin \omega t \) at frequencies, \( \omega \), spanning of 3–15 Hz and at amplitudes \( A \), spanning 0–15mm. A calibrated PCB accelerometer mounted on the moving table yields the acceleration of the system. This device indicates very little extraneous noise and very nearly sinusoidal motion.

In a typical run, we observed the evolution of the system as \( A \) was increased from zero while keeping \( \omega \) fixed. We used several types of approximately monodisperse granular materials, including spherical glass beads, smooth Ottawa sand, and sieved rough sand. For these experiments, there are several useful dimensionless dynamical measures, including a dimensionless measure of the acceleration \( \Gamma = A\omega^2/g \) where \( g \) is the acceleration due to gravity. We note that other parameters, such as \( E = (A\omega)^2/gd \), which are important in describing higher order phenomena in vertically shaken materials (e.g. traveling waves \( [7] \), coarsening \( [8] \)) are not necessary to describe the onset of flow in these experiments. However, frictional properties are important, and recent experiments \( [4] \) also show that the ratio \( A/d \) may be important.

Fig 2 shows a sketch of convective flow lines observed from the top and side for \( \Gamma \) somewhat above onset. We have obtained these images by coloring some of the particles and by then following them over time. (In this regard, considerable care must be taken, since coloring the grains can change the surface friction and lead to strong segregation, as discussed below.) Both lab-mounted and shaker-mounted cameras were used to observe the system. Grains rise up in the middle of the cell and flow along the surface towards the side walls and then sink at the wall boundaries. The top surface of the liquefied layer has a shape which is concave down, and the bottom surface of this layer has a shape which is concave upwards. Thus, the thickness of the fluidized layer is largest in the middle of the cell and smallest at the end walls, as seen previously by Evesque \( [2] \).

A useful measure of the strength of the flow is then the thickness, \( H \), of the liquefied material in the middle of the cell. We show typical behavior for \( H \) as a function of \( \Gamma \) in Fig. is hysteretic. With increasing \( \Gamma \), there is a well defined transition to finite amplitude (i.e. finite \( H \)) flow at \( \Gamma^* \). If we then decrease \( \Gamma \) below \( \Gamma^* \) once flow has begun, the thickness of the
layer also decreases until \( \Gamma \) reaches a critical value \( \Gamma_c \) where the relative motion completely stops. As \( \Gamma \) is decreased from above towards \( \Gamma_c \), grains near the walls stop moving first, while grains in the middle keep moving. It is perhaps not surprising that the initial transition to flow is hysteretic, since the onset of flow must occur by the breaking of static friction, whereas once flow has begun, dynamical friction is involved, assuming that grains remain in motion throughout much of the shaking cycle. In addition, the grains must dilate in order to flow \[9\], but once dilated, less energy is likely to be required to sustain the flow.

Our observations show that the location of \( \Gamma^* \) and \( \Gamma_c \) are reproducible for a given height of a particular material. That is, different \( \omega \)'s or \( A \)'s yield the same critical \( \Gamma \)'s for a given material and fill height. Thus, \( \Gamma \) is the relevant control parameter, as opposed to say \( E \) or some other dynamical measure.

These \( \Gamma \)'s do depend on the physical properties of the material, as shown in Fig. 3. For instance, \( \Gamma_c \) and \( \Gamma^* \) increase as the roughness of the granular materials increases. The same is true for the difference \( (\Gamma^* - \Gamma_c) \), which is higher for rougher granular beds than for smooth ones. This can be attributed to two factors: first, the ability of rough grains to roll is reduced because of the interlocking of grains; and second, the effective macroscopic frictional forces between grains and between grains and walls may also be higher for rough grains. These effects are in principle distinct, although in an experiment, it may be difficult to distinguish between them.

To obtain additional insight into the relative importance of dilatancy and friction, we have fluidized the granular bed by passing air through it, using the flow-controlled air supply (see Fig. 1), where the porous base of the cell acts as the gas distributor. The pressure gradient across the distributor is more than 50% of the total pressure gradient; hence, there is reasonably even air distribution across the granular bed. Fig 4, which presents \( \Gamma^* \) and \( \Gamma_c \) as a function of air flow through the bed, indicates a very strong dependence of these quantities on the air flow. In particular, a modest air flow reduces the critical \( \Gamma \)'s and ultimately effectively removes the hysteresis in the initial transition. A key point is that the measured dilation of the bed due to the air flow is very small; the maximum
dilation for these experiments corresponds to less than one granular layer for a bed of 45 layers, i.e. less than 2% dilation. This small value of the dilation suggests that it is the reduction in both static and dynamical friction which is most important here rather than the unlocking effect which occurs for larger dilations. However, this conclusion must be made with some caution, because it is possible to have motion in a granular material along a localized shear band where the overall dilation of the whole sample is also only a few percent. In the experiments described here, the dilation occurs over the whole sample (although not necessarily completely uniformly).

Segregation by size and shape is a very common phenomenon observed in granular systems and these experiments seem particularly sensitive to this phenomenon. We observed the usual segregation by size in flowing layers of polydisperse grains. The larger particles rose to the top surface and accumulated near the surface and wall boundaries and the small particles sank, mostly accumulating near the bottom of the liquefied layer during the shaking. However, even a modest difference in the surface preparation of these materials was sufficient to cause segregation—i.e. size differences are not necessary for segregation. In particular, colored particles, which had a somewhat higher friction coefficient than uncolored ones, tended to migrate to the upper surface, where they accumulated next to the horizontal walls. In the case of size segregation for vertically shaken granular materials, segregation is understood in terms of preferred downwards motion of smaller particles throughout the medium [10], or in terms of a dilated boundary layer which smaller but not larger particles can penetrate [11]. In the present case, the colored and uncolored particles are virtually identical in size, so that an explanation involving only friction-related mobility is necessary. Mobility-related segregation occurs in conventional fluids (i.e. separation of different species due to a temperature gradient), but we are unaware of descriptions of segregation by mobility for granular materials. The segregation takes much more complicated formations at high Ω values where the granular convection dominates in the liquefied layers.

To conclude, we have characterized the transition to flow, or liquefaction, for horizontal shaking of granular materials. In the absence of additional fluidization by vertical gas flow,
this transition is hysteretic. However, by applying a very modest vertical gas flow which creates a dilation in the vertical of $\sim 2\%$, the hysteresis is lifted, and the onset to flow occurs at lower $\Gamma$. This suggests, at least tentatively, that it is the solid friction and not the dilatancy which is responsible for the hysteresis. In addition, there is strong segregation of equal size particles if their surface properties differ. This phenomena is then qualitatively different from the size-related segregation which has been reported for vertical shaking.

Acknowledgments: This work was supported by the National Science Foundation under Grant DMS95-04577, and by NASA under Grant NAG3-1917. RPB would like to thank the P.M.M.H-E.S.P.C.I for its hospitality during the completion of this paper.
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FIG. 1. Schematic of the apparatus. The rectangular cell is made of Plexiglas, and is mounted on another Plexiglas cell of the same cross-sectional dimensions which is attached to a small table. The table is mounted on four linear bearings running on horizontal cylindrical guidance rods rigidly attached to a fixed bottom frame. The bottom section of the cell acts as a gas distributor. An electro-mechanical vibrator, driven by a sinusoidal AC signal provides the horizontal driving.
FIG. 2. Sketch of convection flow lines in the liquified layer induced by horizontal shaking (a) as seen from the top, and (b) as seen from the side. Grains rise in the middle of the cell, flow along the surface towards the side walls, and then sink at the wall boundaries.
FIG. 3. Thickness of the liquefied layer versus acceleration amplitude $\Gamma$ for (a) glass beads ($d = 0.6mm$), (b) smooth Ottawa sand ($d_{ave} = 0.6mm$), and (c) rough sand ($d_{ave} = 0.6mm$). In each of the figures, $\Gamma_*$ is the bifurcation point when $\Gamma$ is increased quasi-statically, and $\Gamma_c$ is the bifurcation point when $\Gamma$ is decreased quasi-statically.
FIG. 4. The bifurcation points $\Gamma^*$ and $\Gamma_c$ versus air flow rate for the rough sand.