\[ \Delta I = 1 \text{ axial-vector mixing and charge symmetry breaking} \]

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Abstract

Phenomenological Lagrangians that exhibit (broken) chiral symmetry as well
as isospin violation suggest short-range charge symmetry breaking (CSB)
nucleon-nucleon potentials with a \[ \sigma_1 \cdot \sigma_2 \] structure. This structure could be
realized by the mixing of axial-vector \((1^+)\) mesons in a single-meson exchange
picture. The Coleman-Glashow scheme for \(\Delta I = 1\) charge symmetry break-
ing applied to meson and baryon \(SU(2)\) mass splittings suggests a universal
scale. This scale can be extended to \(\Delta I = 1\) nonstrange CSB transitions
\[ \langle a_1^0 | H_{em} | f_1 \rangle \] of size \(-0.005 \text{ GeV}^2\). The resulting nucleon-nucleon axial-vector
meson exchange CSB potential then predicts \(\Delta I = 1\) effects which are small.

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I. INTRODUCTION

A recent assessment of isospin violation in the nucleon-nucleon force was made with the aid of phenomenological Lagrangians that exhibit (broken) chiral symmetry as well as isospin breaking [1]. The charge symmetry breaking (CSB) nucleon-nucleon (NN) forces arise from the Lagrangian with the aid of dimensional power counting [2], so that the Lagrangian suggests specific pion-exchange NN force mechanisms. In addition, four-nucleon terms in the Lagrangian lead to NN contact forces with a given spin and isospin structure. The undetermined (dimensionless) coefficients of these forces are expected to be “natural,” i.e. of order $\mathcal{O}(1)$ when written in the appropriate scale [3,4]. Therefore, a part of this assessment [1] included a comparison of these generic CSB forces with specific models such as the standard $\pi-\eta$ and $\rho-\omega$ mixing models [5–8] which successfully describe nuclear data [9].

The strength of the meson mixing forces and of their pseudoscalar and vector-meson mixing matrix elements was found to be “natural” by this criterion [1]. The latter meson-mixing matrix elements have been found to have a universal scale [10], obtained with the aid of the Coleman-Glashow picture that charge symmetry-violating processes are dominated by CSB tadpole diagrams [11,12]. Then the generic analysis of charge symmetry breaking in the NN force appears consistent with these earlier specific mechanisms. However, the generic Lagrangian of Refs. [1,2] includes a term of the form

$$\gamma_\sigma \left( \bar{N} \tau_3 N \right) \cdot \left( \bar{N} \sigma N \right),$$

which leads to a class III (i.e., proportional to $(\tau_3(1) + \tau_3(2))$ [13]) CSB NN force, apparently not envisaged before. In this paper, we study the suggestion that the simple $t$-channel exchange of axial-vector mesons ($1^+$) might be the dominant contribution to this proposed short-range CSB NN force.

The NN contact force of the chiral Lagrangian represents the sum of all $t$-channel exchanges (multi-pion resonances, uncorrelated pion loop diagrams, simultaneous pion-photon loop diagrams, etc.) with a given spin and isospin structure. However, the effective chiral Lagrangian analysis can say nothing about the relative size of the individual contributions to the parameter $\gamma_\sigma$ (nor to the other short-range term $\gamma_s (\bar{N} T_3/2 N) \cdot (\bar{N} N)$ ) and one must turn to modeling. (In any case, models are essential if one is to use the resulting CSB potentials in nuclear calculations.) In the well-studied mesonic sector it has been found that the ten phenomenological parameters $L_1 \ldots L_{10}$ of the chiral Lagrangian (to one loop) are saturated by the exchange of the low-lying multi-pion resonances ($\rho, \omega, a_1, \eta_8$) to the extent that “there is no indication for the presence of any other contribution in addition to the meson resonances” [13]. In the nucleonic sector, the analogous parameters of the Lagrangian which generates one-pion-range potentials have now been established, but the short-range
CSB NN parameters $\gamma_s$ and $\gamma_\sigma$ have not been determined phenomenologically \[1\]. All that one can say, at this stage, is that models based on isospin mixing of meson resonances are consistent with the naturalness criterion \[1\] and with the nuclear data \[3,10\] when the on-mass-shell mixing matrix element of the Coleman-Glashow picture is employed. Still, these results in the mesonic and nucleonic sectors suggest dominance of $\gamma_\sigma$ by a simple $t$-channel (axial-vector) meson resonance mixing mechanism. To study this hypothesis we employ the Coleman-Glashow mixing model which (unlike other mixing models) is immediately extended to the axial-vector mesonic resonances $a_1$ and $f_1$, where there is no experimental data on isospin mixing.

In this Coleman-Glashow picture, the meson-mixing matrix element $\langle a_1|H_{em}|f_1 \rangle$ is determined by the dominant single-quark operator $H^{(3)} = \frac{1}{2}(m_u - m_d)\bar{q}\lambda_3 q$, established in Ref. \[10\] from the electromagnetic mass differences in the pseudoscalar mesons, vector mesons, baryon octet, and baryon decuplet. When extended to the off-diagonal $\Delta I = 1$ transitions $\langle \rho|H_{em}|\omega \rangle$ and $\langle \pi^0|H_{em}|\eta_{NS} \rangle$, this gives the value $-0.005$ GeV$^2$ for the mixing matrix elements, independent of the particular mesons concerned. Indeed, for this one-body operator, it is to be expected that one obtains the same numerical value connecting any $I = 1$ state with an $I = 0$ nonstrange state of the same spin and parity. Thus, for this first estimate of this novel source of charge symmetry breaking we assume $\langle a_1|H_{em}|f_{1NS} \rangle \approx -0.005$ GeV$^2$, and refer to a discussion of delta meson tadpole graphs in Ref. \[10\] for more details.

From this single-quark operator picture, or the equivalent tree-level tadpole picture, the electromagnetic transition $\langle a_1|H_{em}|f_{1NS} \rangle$ could have no dependence on the four-momentum squared of the hadrons. Therefore, the analogous theoretical predictions for the pseudoscalar and vector mesons are compared with measured electromagnetic mass splittings and with (on-mass-shell) CSB violating transitions $\eta \rightarrow 3\pi$ and $\omega \rightarrow 2\pi$. The comparison is excellent \[10\] and encourages us to extend this scheme to the axial-vector mesons. We emphasize that single-quark operator or tadpole-generated mixings of the Coleman-Glashow scheme are not of the two-body current $\times$ current type with conserved currents for which the vector-meson mixing amplitude must identically vanish at four-momentum squared equal to zero \[14\]. Furthermore, the claims based on various grounds \[14,15\] that the on-mass-shell $\rho-\omega$ mixing matrix element is greatly suppressed would force an unnaturally small coefficient $\gamma_s$ for the resulting CSB term of the NN force if the $\rho-\omega$ mixing mechanism saturates that term \[1\]. By the criteria of Refs. \[1-4\], such an unnaturally small coefficient would presuppose a symmetry to preserve its small value. Such a symmetry has not been identified, nor has an alternate mechanism been found which could both restore naturalness to $\gamma_s$ and describe all the nuclear data. For these reasons, we will use the on-mass-shell $a_1-f_{1NS}$ mixing matrix element obtained from the Coleman-Glashow scheme, rather than follow the
suggestions of a suppression of particle mixing at the off-mass-shell kinematics of an $NN$ force diagram. We return to a discussion of the saturation argument after presenting our results.

In the following, we will derive the CSB potential due to $a_1-f_1$ mixing, give two estimates for the coupling constants involved, and present the results for the $NN$ singlet scattering lengths and for the $^3$H–$^3$He binding-energy difference. As we will show, the $f_1$ meson is primarily nonstrange ($NS$) so we drop the distinction between $f_{1NS}$ and $f_1$ at this point.

II. CSB POTENTIAL DUE TO $a_1-f_1$ MIXING

We begin with the total interaction Hamiltonian

$$H_I(a_1NN) = \frac{1}{2}g_{a_1NN}\bar{N}\tau\cdot A^\mu\gamma_\mu\gamma_5N + \frac{1}{2}g_{f_1NN}\bar{N}f^\mu\gamma_\mu\gamma_5N + XA^\mu f_\mu ,$$

which defines the $ANN$ coupling constants, and where $X$ simulates the electromagnetic $a_1-f_1$ transition. We have neglected the tensor couplings from the beginning. Experience with the vector mesons, where the momentum-dependent tensor coupling of the $\rho$ is especially large, shows that the class III CSB results are merely reduced by 10-20% by this neglect.

In the actual $a_1^2-f_1$ $NN$ force diagram, the Feynman rules give

$$T_{NN}^{a_1f_1} = \frac{1}{4}g_{a_1NN}g_{f_1NN}X \frac{m_{a_1}^2}{(m_{a_1}^2 - t)(m_{f_1}^2 - t)} \times \bar{u}(p_{1f})\gamma_\sigma\gamma_5u(p_{1i}) \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_{a_1}^2}\right) g_{\nu\sigma} \left(g^{\sigma\tau} - \frac{q^\sigma q^\tau}{m_{f_1}^2}\right) \bar{u}(p_{2f})\gamma_\tau\gamma_5u(p_{2i}) + (1 \longleftrightarrow 2) ,$$

with $q = (p_{1i} - p_{1f}) = (p_{2f} - p_{2i}), t \equiv q^2$, and we have taken a narrow $a_1$ width to simplify the calculation.

The term $X$ in the interaction Hamiltonian is related to the electromagnetic $a_1-f_1$ transition as explained in Ref. [3]. That is, the scalar $\langle a_1^2|H_{em}|f_1\rangle$ will not change the helicity of the massive spin-1 meson. So we need consider only the specific helicity states in Eqs. (1) and (2),

$$\langle a_1^{(\lambda)}|H_I|f_1^{(\lambda)}\rangle = X \langle a_1^{(\lambda)}|A^\mu f_\mu|f_1^{(\lambda)}\rangle = X\varepsilon_\mu^{(\lambda)}\varepsilon_\mu^{(\lambda)} = -X ,$$

where we have used the identity $\varepsilon_\mu^{(\lambda)}\varepsilon_\mu^{(\lambda)} = -\delta^{\lambda\lambda}$. Since the left-hand side of Eq. (3) is the electromagnetic $a_1-f_1$ transition, it is clear that the contact term replacing $X$ in the Feynman graph for the $NN$ force must be $-\langle a_1^2|H_{em}|f_1\rangle$. 

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Because the mass of the $a_1$ is poorly known \cite{19} and, in any event, nearly degenerate with that of the $f_1(1285)$, we now take the limit $m_{a_1} = m_{f_1}$ arguing that the corrections $O(m_{a_1} - m_{f_1})$ are surely smaller than any other uncertainty in this problem. With this approximation, we specialize to the $S$-wave, make a nonrelativistic reduction of Eq. (2) to $O(q^2)$, where $q$ is the momentum transferred to the axial-vector meson, $q = p_1 - p_{1f} = p_{2f} - p_{2i}$, and define

$$
\Delta T \equiv T_{nn}^{a_1 f_1}(S) - T_{pp}^{a_1 f_1}(S).
$$

(4a)

Then

$$
\Delta T = -g_{a_1 NN} g_{f_1 NN} \frac{\langle a_1^0|H_{em}|f_1 \rangle}{(q^2 + m_A^2)^2} \left\{ \sigma_1 \cdot \sigma_2 + \frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q)}{m_A^2} \left( 2 + \frac{q^2}{m_A^2} \right) \right\},
$$

(4b)

where we have dropped nonlocal terms and used the isospin convention $\tau_3 | p \rangle = +| p \rangle$. Finally, we take the Fourier transform to arrive at

$$
\Delta V = -\frac{2}{3} \frac{g_{a_1 NN} g_{f_1 NN}}{4\pi} \sigma_1 \cdot \sigma_2 \frac{\langle a_1^0|H_{em}|f_1 \rangle}{2m_A} e^{-m_A r},
$$

(4c)

where $m_A = (m_{a_1} + m_{f_1})/2$. This is of the familiar form of the dominant term of the class III CSB potential due to $\rho-\omega$ mixing \cite{17}, with the exception of the coefficient $-\frac{2}{3} \sigma_1 \cdot \sigma_2$ which takes the value $+2$ in the $^1S_0$ partial wave of interest in the few-nucleon systems. Therefore, in these systems axial-vector meson mixing will have the same CSB effect as vector meson mixing. Notice that the coordinate space potential in Eq. (4c) is exponential with a longer range than a Yukawa of the same mass.

III. ESTIMATES FOR COUPLING CONSTANTS

In numerical calculations with Eq. (4c) we have used two estimates of the needed coupling constants $g_{a_1 NN}$ and $g_{f_1 NN}$. The first estimate comes from a model by Sudarshan for strong, weak, and electromagnetic interactions \cite{20}. The strong-interaction Lagrangian of this model has been used to study violation of time-reversal invariance in low-energy $NN$ scattering due to interference of vector and tensor $a_1 NN$ couplings \cite{21}. The axial-vector coupling constants of this model were found to be barely consistent with these experimental tests of time-reversal asymmetry. Sudarshan’s model assumes that the $\rho$, $a_1$, and $f_1$ are coupled to the nucleon and $\Delta(1232)$ in an $SU(4)$ symmetry scheme. The predicted coupling constants are $g_{a_1 NN} = 5g_{\rho NN}/(3\sqrt{2})$ and $g_{f_1 NN} = g_{\rho NN}/\sqrt{2}$. The ratio $g_{a_1 NN}/g_{f_1 NN} = 5/3$ is the same as that obtained from the textbook derivation of $g_A/g_V = 5/3$ using expectation values of $I_3\sigma_z$ versus $\sigma_z$ between proton states with $SU(6)$ wave functions. The extra factor of $1/\sqrt{2}$
is peculiar to Sudarshan’s model and arises from setting the (time-reversal asymmetrical)
tensor coupling of the $a_1$ equal to the direct coupling so that $g_{a_1NN} \approx 1.18g_{\rho NN}$, a number desired at that time when the empirical $g_A/g_V$ was thought to be $\approx 1.18$. Taking the Sudarshan model literally, we find that $g_{a_1NN}^2/4\pi \approx 2.8$ and

$$\frac{g_{a_1NN}g_{f_1NN}}{4\pi} = \frac{5}{6} \frac{g_{NN}^2}{4\pi} \approx 1.7 \ ,$$

where the numerical values are derived from the vector dominance hypothesis, universality ($g_{\rho NN} \approx g_{\rho \pi\pi} \approx g_{\rho}$), and the data on $\Gamma(\rho \rightarrow e^+e^-)$ [13], which gives $g_{\rho}^2/4\pi \approx 2.02$.

The second estimate of the needed coupling constants uses the idea of axial-vector dominance to relate $g_{a_1NN}$ to the observed axial-vector coupling constant $g_A(0)$. The assumption that the nucleon matrix elements of the isovector axial-vector hadronic current,

$$\langle N_p'|A_{\mu}^i(0)|N_p \rangle = \bar{N}_p' \gamma^i \frac{1}{2} \tau^i \left[ g_A(q^2)\gamma_\mu \gamma_5 + h_A(q^2)q_\mu \gamma_5 \right] N_p e^{iq \cdot x} \ ,$$

are dominated by the contribution of the lowest-lying axial-vector mesons yields immediately the relation

$$g_A(0) = \frac{f_{a_1} g_{a_1pp}}{m_{a_1}^2} \ ,$$

where $g_A(0) = 1.2573 \pm 0.0028$ [13], and we define $g_{a_1pp}$ as in Eq. (1). The decay constant $f_{a_1}$ is defined by the isovector $a_1$-to-vacuum matrix element of the hadronic axial-vector current

$$\langle 0|A_{\mu}^i(0)|a_1^j \rangle = \delta^{ij} f_{a_1} \epsilon_\mu \ ,$$

where $\epsilon_\mu$ is the polarization vector. The definition (7b) is analogous to that of the pion decay constant $f_\pi \approx 93$ MeV and the “vector decay constant” $f_\rho \equiv m_{\rho}^2/g_\rho$, and so it is a factor of $\sqrt{2}$ smaller than the convention of Ref. [22]. The $a_1$ mass and decay constant have recently been calculated from lattice QCD [23]. The simulation finds $m_{a_1} = 1250 \pm 80$ MeV and

$$f_{a_1} = 0.21 \pm 0.02 \text{ GeV}^2 \ ,$$

in our convention (7b). The decay constant can be obtained from the measured partial width of the decay $\tau^- \rightarrow a_1^+ + \nu_\tau$ [22,24] but the calculation is complicated by the large width of the $a_1$ and of the intermediate $\rho$ mesons in the decay to three pions. An empirical value of $f_{a_1}$ has been obtained in which Breit-Wigner forms were taken for the $\rho$ and $a_1$ single-particle contributions to vector and axial-vector spectral functions of $\tau$ decay [25]. Redoing this calculation with the “new world average” branching fractions [26] $B(\tau^- \rightarrow 3\pi \nu_\tau) = (17.73 \pm 0.28)\%$ and $B(\tau^- \rightarrow \rho^- \nu_\tau) = (24.91 \pm 0.21)\%$, we obtain $f_{a_1} = 0.23 \text{ GeV}^2$, in good agreement with the lattice result. With the lattice value of $f_{a_1}$, Eq. (7a) gives
$g_{a_1 pp} \approx 9.4 \pm 1.2$ , \hspace{1cm} (7d)

for the $a_1$ mass in the range of $1250 \pm 80$ MeV.

This second estimate of $g_{a_1 pp}/4\pi \approx 7$ is larger than the one from Sudarshan’s model but still rather conservative compared to other estimates in the literature. It is within two standard deviations of the coupling constant $g_{a_1 pp}/4\pi \approx 29 \pm 12$ in our convention of Eq. (1) obtained from a best fit to the discrepancy functions of a forward dispersion relation analysis of $NN$ data \[27\]. In addition, Eq. (7d) is about a factor of two smaller than the $g_{a_1 pp}$ predicted by chiral-invariant effective Lagrangians \[28\] where $g_{a_1 pp}/(2m_{a_1}) = \sqrt{m_{a_1}^2 - m_\rho^2 f/(m_\rho m_\pi)}$ and $f$ is the $\pi NN$ pseudovector coupling constant ($f^2/4\pi = 0.075$). The disagreement worsens if one uses empirical masses instead of assuming $m_{a_1} \approx \sqrt{2m_\rho}$ which is suggested by the vector dominance argument of \[28\] but is not supported by experiment. The latter chiral relationship of \[28\] has been used to discuss the role of the $a_1$ meson in the $NN$ interaction \[29\], and to construct two-meson-exchange contributions to the $NN$ interaction when one or both nucleons contains a meson pair vertex \[30\]. On the other hand, Eq. (7d) is near what one would expect ($g_{a_1 NN} \approx \sqrt{2g_{\rho NN}}$) from dynamical generation of the vector and axial-vector gauge fields ($\rho$ and $a_1$) from a meson-quark Lagrangian \[31\]. The latter theory predicts the relation $m_{a_1} \approx \sqrt{3m_\rho} \approx 1330$ MeV, close to experiment and a motivation for considering a dependence of our results upon the $a_1$ mass.

In addition, we need the strength of the coupling of the $f_1(1285)$ meson to the nucleons. It is estimated in the same way as used previously for the pseudoscalar meson couplings \[4\]. The axial-vector mesons $f_1(1285)$ and $f_1(1420)$ are mixtures of nonstrange ($NS$) and strange ($S$) quark states $|NS\rangle = |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ and $|S\rangle = |\bar{s}s\rangle$ such that the standard mixing equations hold:

$$
|f_1(1285)\rangle = \cos \phi_A |A_{NS}\rangle - \sin \phi_A |A_S\rangle , \hspace{1cm} (8a)
$$

$$
|f_1(1420)\rangle = \sin \phi_A |A_{NS}\rangle + \cos \phi_A |A_S\rangle . \hspace{1cm} (8b)
$$

The small axial-vector mixing angle $\phi_A$ in this basis is directly estimated from the observed \[19\] widths of the axial-vector mesons $f_1(1285)$ and $f_1(1420)$. An update of the estimate of Ref. \[12\] finds $\phi_A \approx 12^\circ$. These mixing equations in the $SU(3)$ octet-singlet basis, $|8\rangle = |\bar{u}u + \bar{d}d - 2\bar{s}s\rangle/\sqrt{6}$ and $|0\rangle = |\bar{u}u + \bar{d}d + \bar{s}s\rangle/\sqrt{3}$, take the form

$$
|f_1(1285)\rangle = \cos \theta_A |A_8\rangle - \sin \theta_A |A_0\rangle , \hspace{1cm} (9a)
$$

$$
|f_1(1420)\rangle = \sin \theta_A |A_8\rangle + \cos \theta_A |A_0\rangle , \hspace{1cm} (9b)
$$

where $\theta_A = \phi_A - \arctan \sqrt{2}$. Now the needed coupling $g_{f_1(1285) pp}$ is estimated with the $SU(3)$ structure constants \[24\] and the quark model assumption that the strange quark state $|S\rangle =
\[ (-\sqrt{2}|8\rangle + |0\rangle)/\sqrt{3} \] does not couple to the nucleon. The latter Zweig rule assumption fixes the coupling of the singlet in broken \( SU(3) \) to be \( g_{A_{0}pp} = \sqrt{2}g_{A_{8}pp} \). Within the axial-vector octet the \( SU(3) \) invariant axial-vector–baryon–baryon (ABB) couplings include \( g_{A_{8}pp} = +g(3f - d)/\sqrt{3} \), where the \( f \) and \( d \) are the strengths of the antisymmetric and symmetric ABB couplings. They are normalized, \( f + d = 1 \), so that \( g \) is \( g_{a_{1}pp} \). Inserting the assumed singlet coupling into the octet-singlet mixing relations (9) and re-expressing the result in the \( NS - S \) basis of (8), one finds the simple result

\[ g_{f_{1}(1285)pp} = \cos \phi_A (3f - d)g_{a_{1}pp} , \]  
\[ g_{f_{1}(1420)pp} = \sin \phi_A (3f - d)g_{a_{1}pp} . \]  

The derived ratio

\[ g_{f_{1}(1420)pp}/g_{f_{1}(1285)pp} = \tan \phi_A \approx 0.21 , \]

is clearly seen in the \( NS - S \) basis of (8) if \( g_{A_{3}pp} = 0 \) (Zweig rule). The axial-vector current \( f/d \) ratio, obtained from the Cabibbo theory of semileptonic decays of baryons, is now known to be \( 0.575 \pm 0.0165 \) \cite{33}. Continuing in the spirit of axial-vector dominance, we assume the structure constants for the coupling of the axial-vector mesons to the nucleons to have the same ratio and find, with the small mixing angle \( \phi_A \approx 12^\circ \),

\[ g_{f_{1}pp} \approx (0.45 \pm 0.025)g_{a_{1}pp} . \]

We neglect mixing of the \( a_{1} \) with the \( f_{1}(1420) \) as it is a factor of five smaller than (12) from (11).

Next we examine the assumptions made to arrive at (12). The first was that the \( f_{1}(1420) \) does belong to the axial \( \bar{q}q \) nonet of Eqs. (8) and (9). This assumption has been supported \cite{34} and attacked \cite{35} on experimental grounds and remains uncertain \cite{19}. On the theory side, a more elaborate mixing scheme with nonstrange and strange quark states and gluon states has suggested specific admixtures of the three isoscalar axial-vector mesons \( f_{1}(1285) \), \( f_{1}(1420) \), and \( f_{1}(1510) \) \cite{36}. In this scheme the \( f_{1}(1285) \) is primarily a \( NS \) \( \bar{q}q \) state with a coefficient of about 0.9 and the other two have small \( NS \) components, so that the estimate (12) from the nonet picture of Eq. (8) would not be altered by more than 10%. The second (Zweig rule) assumption about the singlet coupling strength (\( g_{0}pp = \sqrt{2}g_{8}pp \)) can be compared to the pseudoscalar coupling constants of the most recent Nijmegen potential model of the nucleon-nucleon interaction \cite{37}. The fitting procedure of the one-boson-exchange potential Nijm93 fixes the \( \pi^0NN \) coupling constant, \( f/d \), and the mixing angle \( \theta_P \). A search on the \( NN \) data finds \( g_{0}pp = 1.9g_{8}pp \) at the meson poles, somewhat larger than the Zweig rule assumption.

Finally, taking central values from Eqs. (7a), (7c), and (12) we find
\[
\frac{g_{a1pp} g_{f1pp}}{4\pi} \approx 0.45 \frac{m_{a1}^4 (g_A(0))^2}{4\pi f_{a1}^2} \approx 3.1 ,
\]
(13)

for an assumed \(a_1\) mass of 1250 MeV. Note that the CSB potential Eq. (1c) depends on three powers of the poorly known \(a_1\) mass if the coupling constants of Eqs. (7) and (12) are used. We do not assume charge symmetry breaking in the meson-nucleon-nucleon vertices, as has been suggested for vector meson exchange, first with explicit vertex models \[38\], and then discussed in more general terms \[39\]. Charge symmetry breaking in our preferred model (7) for the \(ANN\) vertices could arise from a difference in \(g_A(0)_p\) and \(g_A(0)_n\). This difference has recently been calculated within the external field QCD sum rule approach and found to be at the 1% level or less \[40\], and we disregard it.

### IV. NUMERICAL RESULTS

Now we turn to the charge symmetry breaking in the \(NN\) interaction from the model potential (1c) with coupling constants given in Eqs. (5) or (13). Traditional measures of nuclear charge asymmetry have been obtained from the positive value for the difference \(\Delta a = |a_{nn}| - |a_{pp}| \approx \mathcal{O}(1 \text{ fm})\) of the \(NN\) singlet scattering lengths and the positive value for the \(^3\text{H} - ^3\text{He}\) binding-energy difference \(\Delta E \approx \mathcal{O}(100 \text{ keV})\). The measures \[41\]

\[
\begin{align*}
\Delta a_{\text{exp}} & \equiv (|a_{nn}| - |a_{pp}|) \approx +1.1 \pm 0.6 \text{ fm} , \\
\Delta r_{0\text{exp}} & \equiv (r_{nn} - r_{pp}) \approx -0.02 \pm 0.11 \text{ fm} , \\
\Delta E_{\text{exp}} & \equiv (^3\text{H} - ^3\text{He}) \approx 76 \pm 24 \text{ keV} ,
\end{align*}
\]

are quoted after correction of experiment for direct electromagnetic effects and are quite consistent in sign and magnitude. A positive \(\Delta a\) reflects an interaction between two neutrons which is more attractive than between two protons and more binding energy is provided for \(^3\text{H}\) as compared to \(^3\text{He}\). The binding-energy difference \(\Delta E\) is quantitatively tied to the \(^1\text{S}_0\) effective range parameters \(\Delta a\) and \(\Delta r_0\), as has been known for a long time, and demonstrated in recent three-body calculations (see Ref. \[41\] for a discussion).

The theoretical shifts in \(a\) and \(r\) are obtained by adding the model for \(\Delta V = V_{nn} - V_{pp}\) to a model for the charge symmetric reaction. For the latter we choose two new Nijmegen potential models \[37\] which give an excellent description of the data and an older potential \[42\] which has a “super-soft core” and should therefore allow the largest effects of the short-range model (4). Because all three charge symmetric potentials are repulsive at short range, we did not include form factors in the CSB model (4). The results are displayed in Table I which demonstrates that the dependence of the results upon the charge symmetric potential are noticeable but the CSB effect from axial-vector meson mixing is rather small. The mass of
the $a_1$ was taken to be 1250 MeV in Table I; an uncertainty of 80 MeV in the $a_1$ mass in the couplings of Eq. (13) gives an uncertainty of 0.02 fm in the calculated $\Delta a$. We also show two estimates of $\Delta E$, the first $\Delta E_{GS}$ obtained from a relationship shown in Ref. [11] from the triton calculations of Ref. [13], and the second $\Delta E_{FF}$ obtained from empirical charge form factors of $^3$H and $^3$He with the aid of the hyperspherical formula [8,44]. Both calculations are expected to overestimate $\Delta E$ somewhat, see Ref. [41] for details.

The results of Table I can be compared with the empirical measures above and with the effect of $\pi-\eta-\eta'$ mixing ($\Delta a \approx +0.026$ fm from Ref. [1]) and of $\rho-\omega$ mixing ($\Delta a \approx +1.5$ fm from Ref. [8]). Another source of CSB suggested is two-pion exchange ($\Delta a \approx +1$ fm from Ref. [41]). Simultaneous $\gamma-\pi$ exchange does not contribute to CSB to the lowest order [45].

By all these measures, the contribution of axial-vector mixing to nuclear charge asymmetry is small but in the direction of experiment.

We conclude by returning to the phenomenological Lagrangian analysis of isospin breaking [14] which gives a more general measure of charge symmetry breaking. First we compare the strength of our meson-mixing potential with the expected strength ($\gamma_{a_1} \sim \epsilon m^2_\pi/f^2_{\pi}\Lambda^2$) of the $NN$ contact force $\gamma_{a_1}(\vec{N}\cdot\vec{\pi}_N)(\vec{\eta}\cdot\vec{\pi}_N)$. The momentum space form of Eq. (4b) becomes in the small $q^2$ limit ($q^2 \ll m^2_N$)

$$V(q) = +g_{a_1NN}g_{f_1NN}\frac{\langle a_1^0|H_{em}|f_1 \rangle}{2m_A^4} (\sigma_1\cdot\sigma_2) \left(\frac{\tau_3(1)}{2} + \frac{\tau_3(2)}{2}\right),$$

(14)

where we have explicitly reinstated the isospin operators of Eq. (2) to facilitate the comparison. Then the axial-vector mixing mechanism of this paper gives a contribution to $\gamma_{a_1}$ of

$$\gamma_{a_1f_1} = g_{a_1NN}g_{f_1NN}\frac{\langle a_1^0|H_{em}|f_1 \rangle}{2m_A^4} \equiv c_{a_1f_1}(\epsilon m^2_\pi/f^2_{\pi}\Lambda^2).$$

(15)

Inserting Eq. (7d), Eq. (12), and the assumed universal value of the mixing matrix element $\langle a_1^0|H_{em}|f_1 \rangle \approx -0.005$ GeV$^2 = -0.85(\epsilon m^2_\pi)$ into Eq. (15) and choosing the large-mass scale $\Lambda = m_\rho$, determines $c_{a_1f_1} \simeq -0.03$, a number very far from $O(1)$. There is a reduction of the dimensionless coefficient from that of vector-meson mixing by about a factor of three since $m^2_{a_1} \sim 3m^2_\rho$. Such a reduction will always occur when the “object” exchanged and mixed in the $t$-channel is more massive than $\Lambda$ but it alone cannot account for an unnatural coefficient in such stark contrast with the vector and pseudoscalar mixing cases.

It is instructive to breakdown this unnatural result into a comparison of coupling constants with the $\rho-\omega$ mixing mechanism. This is because the (on-mass-shell) mixing matrix element is the same as that of the latter which has been found natural $O(\epsilon m^2_\pi)$ in Ref. [1] (which we follow by parameterizing isospin violation by $\epsilon \sim 0.3$). The coupling constant $g_x$
of this zero-range Lagrangian is natural if \( g_x/m_x \sim 1/f_\pi \) \[4,12\]. Thus \( g_{\rho \pi}/m_\rho \sim 0.60/f_\pi \) is nearly the same as \( g_{a_1\pi}/m_A \sim 0.70/f_\pi \) from Eq. (74) and they are both natural. Even the twice as large axial-vector coupling constants of Refs. \[27,28\] are natural in that the dimensionless number is near unity. The difference between the natural potential of \( \rho - \omega \) mixing and the unnatural one of \( a_1 - f_1 \) mixing lies in the contrast between the natural coupling of the \( I = 0 \) vector meson \( g_{\omega NN}/m_\omega \sim 3g_{\rho NN}/m_\rho \sim 1.8/f_\pi \) and the unnatural coupling of the \( I = 0 \) axial-vector \( g_{f_1NN}/m_A \sim 0.5g_{a_1NN}/m_A \sim 0.3/f_\pi \) from Eq. (12). This factor of six reduction can in turn be traced to the broken \( SU(3) \) analysis of vector and axial-vector couplings to nucleons. A treatment analogous to that of Eqs. (8)–(12) can be made for the vector meson coupling constants. With the aid of the vector dominance hypothesis, one finds i) a very small mixing angle \( \phi_V \) (“ideal mixing”) and ii) pure F vector-baryon-baryon coupling so that \( f = 1 \) and \( d = 0 \) \[14\]. Thus from Eq. (10a), the small coupling \( g_{f_1NN} \) is due to the ratio \( d/f = 1.74 \pm 0.05 \) (chosen from the axial-vector current \( d/f \)) rather than the \( d/f = 0 \) of the vector mesons. This guess for the coupling constant \( g_{f_1NN} \) seems reasonable and we must leave our result at that. In any event, the reduction by a factor of about 18 from the \( \rho - \omega \) mixing case has been exposed.

In conclusion, the contribution of axial-vector mixing to nuclear charge asymmetry in two few-body mirror systems is small but in the direction of experiment. The dimensionless coefficient \( c_{a_1f_1} \) which characterizes the charge asymmetric potential is much smaller than unity, indicating that axial-vector mixing (with the values for coupling constants and matrix elements used here) does not saturate the corresponding term of the phenomenological Lagrangian \[12\]. If this conclusion remains true as more is learned about the axial-vector mesons, one might suggest that nonresonant \( \rho - \pi \) exchange between two nucleons should be examined. This suggestion is based upon a calculation in the same nuclear systems \[41\] which indicates that nonresonant \( \pi - \pi \) exchange might dominate the other short range coefficient \( \gamma_8 \), if a convincing case for a suppression from its on-shell value of the \( \rho - \omega \) mixing element can be made.

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TABLE I. The axial-vector \((a_1 - f_1)\) mixing contribution to \(\Delta a\) and \(\Delta r_0\), and to \(\Delta E\), the binding-energy difference between \(^3\)He and \(^3\)H. The charge-asymmetric potentials are distinguished by the two choices [Eqs. (5) or (13)] of axial-vector couplings to the nucleon. The Nijmegen Reid-like (Reid93), one-boson exchange (Nijm93), and de Tourreil-Rouben-Sprung (dTRS) potentials are the charge-symmetric potentials \(V(CS)\) used in the calculation of \(\Delta a\), \(\Delta r_0\), and the estimate of \(\Delta E_{GS}\) based on these effective-range parameters. Another estimate, \(\Delta E_{FF}\), is obtained from the “model independent” hyperspherical formula.

| \(V(CS)\) | \(g_{a_1 g f_1 NN} / 4\pi\) | \(\Delta a \text{ (fm)}\) | \(\Delta r_0 \text{ (fm)}\) | \(\Delta E_{GS} \text{ (keV)}\) | \(\Delta E_{FF} \text{ (keV)}\) |
|------------|----------------|----------------|----------------|----------------|----------------|
| Reid93     | 1.7            | +0.06          | −0.001         | +4             | +8             |
| Nijm93     | 1.7            | +0.07          | −0.001         | +4             | +8             |
| dTRS       | 1.7            | +0.09          | −0.002         | +7             | +8             |
| Reid93     | 3.1            | +0.11          | −0.002         | +8             | +15            |
| Nijm93     | 3.1            | +0.13          | −0.003         | +10            | +15            |
| dTRS       | 3.1            | +0.16          | −0.004         | +13            | +15            |