BDoS: Blockchain Denial of Service

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Abstract—Proof-of-work (PoW) cryptocurrency blockchains like Bitcoin secure vast amounts of money. Participants expend resources to participate and receive monetary rewards for their efforts. Despite rivalry among cryptocurrencies and financial incentive to disrupt blockchain availability, Denial of Service (DoS) attacks against blockchains are rare. Arguably, this is due to their cost: Known attacks either target individual participants or require the control of the majority of the system resources.

In this work, we present an incentive-based attack on blockchain availability, Blockchain Denial of Service (BDoS), with a significantly lower cost. Despite a plethora of work on revenue-driven attacks, to the best of our knowledge, this is the first incentive-based sabotage DoS attack.

We consider an attacker with an exogenous motivation, who is willing to spend resources in order to stop blockchain progress. The attacker commits to a behavior that incentivizes the other participants to stop mining, bringing the blockchain to a halt.

We analyze the miner behavior as a game with iterated elimination of strictly dominated strategies (IESDS). We observe that the success of the attack depends on a variety of factors: the mining power of the attacker, the mining power of the largest non-attacking miner, and the profitability of the mining process. We find that under realistic conditions, based on a new analysis of public data, an attack on Bitcoin-like cryptocurrencies requires as little as 20% of the mining power. The situation is even worse if miners can use their equipment in another blockchain rather than turn it off. We propose countermeasures to deter BDoS.

I. INTRODUCTION

Cryptocurrencies such Bitcoin, implemented with Nakamoto-like blockchain protocols, have a market cap of about $180B [1]. Like classical state machine replication protocols, blockchains allow participants to agree on a state, namely the client balances of a cryptocurrency. But unlike those classical protocols, blockchains are decentralized and allow anyone to join the system at will.

To deter Sybil attacks [2], where an attacker masquerades as multiple entities, Nakamoto relies on incentives. Participants, called miners, are required to expend resources and present Proof of Work (PoW) [3], [4], and they are rewarded with cryptocurrency for their efforts. Miners aggregate cryptocurrency transactions into so-called blocks, each containing PoW, and form a tree data structure. A path in the tree is called a blockchain, and the path with most work is called the main chain; its contents define the system’s state.

An extensive line of work (§II) is studying revenue-driven incentive-based attacks against blockchains [5]–[9], but DoS attacks that aim to stop a cryptocurrency blockchain have received less attention. These are attacks where an attacker is driven by exogenous incentives to spend resources in order to sabotage a blockchain. Indeed, blockchain protocols have been demonstrating unprecedented availability [10]. This might be because known approaches [11]–[13] are costly. An attacker that wishes to stop a blockchain must generate empty blocks such that its chain of empty blocks would contain more PoW than the chain of well-behaved miners. This implies she needs to spend as many resources as all other miners combined. This could be done either by a huge miner [13] or by any entity that pays more than half of the miners to mine empty blocks [11], [14]. For example, the cost of such an attack on Bitcoin is over $2B for equipment and $7M per day in electricity cost (see Appendix B).

In this work, we present a new type of sabotage attack called Blockchain Denial of Service (BDoS). In BDoS, an attacker spends resources in order to incentivize rational miners to stop mining.

We model the system (§III) with an external attacker that controls some mining power. The attacker commits to an attack strategy, without deviation, and a set of rational miners aim to maximize their revenue. BDoS is possible due to the following property of PoW blockchain protocols. A miner that generated a block can publish its header, proving that she has spent the effort to generate a block, but without exposing the block’s content. Now, deviating from the protocol, miners can mine on this block header, that is, extend the chain without knowing its content. We make an additional assumption that weakens the attacker – we assume that the miners are compliant (or petty-compliant [15]), that is, they only take steps that almost follow the protocol, namely extending the longest chain, extending only the block header or cease mining activity.

The crux of the attack (§IV) is as follows. The attacker generates a block $B_A$ and publishes only its header. If a miner extends the chain after this header, the attacker never publishes the header, and continues mining on the chain before it. The block $B_A$ and all blocks following it are ignored, and the miner’s efforts spent in extending $B_A$ would have been wasted. Alternatively, a miner can ignore the header of $B_A$ and generate a block following its parent, resulting in a fork. In this case, the attacker publishes the contents of $B_A$, resulting in a race, where both branches have the full blocks published. The miner’s block might or might not end up in the main chain, depending on the properties of the system. The implication is that either way the profitability of the

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rational miners decreases, if she decides to actively participate in mining. The miner’s other alternative is to pause her mining efforts until she has received the content of the last block in the chain. If the profitability decrease is significant enough so that all miners stop mining, the attacker can cease mining as well, while she has an advantage of one block $B_A$. The blockchain thus grinds to a complete halt.

We formulate the behavior of the miners as a game, and look for an equilibrium after iterated elimination of strictly dominated strategies (IESDS) ($\S$ V). The analysis shows that extending a header-only block is dominated by either not mining or extending the full chain. The attack is successful when not mining is the best response of the miners, and it depends on several factors. For a larger attacker, her ability to generate a block and start the attack increases, and it is also more likely she would win a race. For a larger individual rational miner, the incentive to stop mining decreases. And finally, if the profitability in the system is large, an individual miner continues to mine despite the profitability decrease due to the attack. In general, the cost of the attack can be significantly lower than the 50% required by previous approaches.

Naturally, non-myopic miners invested in the success of the system might be willing to suffer a temporary profitability decline, and even a loss, to overcome an attack and keep the blockchain running. However, we find that their dilemma is even more difficult if this is the case. We consider a case where miners extend the longest chain, ignoring the attacker’s headers and compete on races with the attacker. In such cases, when others compete with the attacker, there is an even stronger incentive for each individual miner to pause mining until the race is complete. We conclude that in order to overcome such attacks, the system relies on a significant portion of altruistic miners blindly following the protocol.

The situation becomes significantly worse if miners have the option to use their resources in another blockchain rather than turn it off ($\S$ VI). We show that if two cryptocurrencies have similar initial profitability, BDoS always decreases the attacked coin’s profitability, achieving defection of rational miners from the attacked coin to a now more profitable coin.

To check the practicality of BDoS we calculate profitability in the longest running cryptocurrency, Bitcoin ($\S$ VII). We combined mining difficulty data with mining hardware consumption and power, historical Bitcoin price fluctuation and electricity costs. Our conservative estimate shows that during seasonal electricity price changes and value drops Bitcoin is vulnerable to a BDoS attack. We believe that the instantaneous drop in block reward (and thus profitability) that is expected to take place in 2020 will put Bitcoin’s security at risk. We estimate the price that would cause Bitcoin to be in immediate danger is roughly $8100, as of the end of October 2019. We also show that migration between coins is common, and the profitability of Bitcoin and the profitability of Bitcoin Cash are almost identical over time. Thus the two-coin model implies that BDoS poses an imminent threat for both coins.

Constructively, we propose some possible mitigations to BDoS ($\S$ VIII). First, honest miners can prefer non-attacker blocks on a fork with a heuristic time-based detector. While not perfect, it can sufficiently reduce the attack’s effectiveness in scenarios that are close to the vulnerability threshold. Secondly, Ethereum’s reward mechanism [16], [17] compensates miners on lost races. This will makes BDoS ineffective, but it exposes the system to other attacks [18].

The discovery of BDoS raises questions for future work ($\S$ IX). First, the header-only publication could be used for alternative attacks, specifically ones that aim to increase the attacker’s revenue rather than sabotage the system. Secondly, BDoS applies to heaviest-chain PoW blockchains such as Bitcoin, Litecoin, Bitcoin-Cash, Zcash, etc., and it is important to understand whether there are similar attacks against other protocols like Ethereum and whether our heuristic mitigation applies there as well.

**Responsible disclosure** We are performing a disclosure process with prominent blockchain development groups.

II. Related Work

To the best of our knowledge, this work is the first to study incentive-based denial of service attacks against blockchains. We present an overview here of previous work on denial-of-service attacks in the context of blockchains, incentive-related behavior, and other related work.

**DoS** Denial-of-Service (DoS) attacks [19] aim to prevent a system from serving its clients, often performed from multiple machines in Distributed DoS (DDoS) attacks. However, in the context of blockchain networks such techniques only apply to individual elements in the system [20]–[22] like cryptocurrency exchanges or mining coordinators called pools. In eclipse attacks [23]–[25] an adversary monopolizes all connections of a target node and isolates it from the network. When applied to blockchain systems [26], [27], the victim’s local view is no longer in sync with the network, disrupting the victim and amplifies other blockchain attacks [6].

Similar effects can be achieved with routing attacks, chiefly BGP hijacking [28]–[30]. However, due to the decentralized structure of the system, nodes outside the effect of the attack can continue to interact with the blockchain as usual, apart from possible reduction of attacked mining power. In contrast, BDoS stops all blockchain progress.

Other attacks [31]–[33] saturate the blockchain to prevent transactions from being placed. However, such attacks result in graceful deterioration as the attacker simply raises the cost of transaction writes. Clients can still place transactions, albeit with a higher fee, thus also increasing the attack’s cost. Additionally, and in contrast to BDoS, such attacks require continuous expenditure for the duration of the attack.

**Majority (51%) attacks** An attacker controlling a majority of the mining power violates the assumptions of PoW protocols, and can perform a full fledged DoS attack by simply generating empty blocks and ignoring other blocks. Since this is a majority attacker, her chain will be longer
(w.h.p.) than any other chain, making it the main chain, despite its empty contents. An attacker with such power can also perform other attacks violating the system’s safety properties. Majority attacks have been observed happening on smaller cryptocurrencies [34]–[36], but not on major ones, possibly due to their significant continuous cost. Goldfinger and bribery attacks [11]–[13], [37]–[39] utilize miner bribery to achieve similar effects, only without requiring the attacker to directly acquire mining power. In contrast to this family of attacks, BDoS requires significantly lower than 50% mining-power budget, and no continuous expenditure.

**Revenue-seeking deviations** one line of study [6], [7], [18], [40]–[42] considers the incentive compatibility of blockchain protocols. They analyze mining as a game, showing when correct behavior is an equilibrium, and when deviations allow the miners to increase their revenue, and correct behavior is not an equilibrium. Such attacks may bias the mining power structure, leading to centralization, or affect other desired blockchain properties like censorship resistance. However, their goal and analysis considers only the internal system revenue, they do not consider exogenous malicious motivations, and they cannot be directly applied to achieve complete denial of service.

Goren and Spiegelman [43] present an attack that manipulates mining difficulty to increase revenue. They analyze the attack when all other miners follow the protocol, and show that the attack is profitable when the profit margin is small. In contrast, the BDoS attacker’s goal is not revenue, we do not assume a small profit margin – matching our analysis of real-world data – and we assume all miners are rational and might deviate from the protocol to increase their revenue.

Miners can do *coin-hopping*, switching their mining efforts from one blockchain to another [44]. If too many miners make the switch due to difficulty and value changes, it can lead to a so-called “blockchain death spiral” [45], possibly leading a deserted blockchain to a halt. However, such events follow naturally from varying cryptocurrency valuation and difficulty adjustment algorithms, both of which are outside the scope of this work.

Incentive attacks against mining pools [8], [9], [46]–[48] affect individual pools, but do not lead directly to blockchain DoS, and usually do not significantly affect the total mining power in the system.

**Incentive-based attacks** Another line of work explores attacks that use incentives to affect blockchain properties, using a form of bribery. Judmayer et al. [49] categorizes incentives attacks by their goals into three groups: transaction revision, transaction ordering, and transaction exclusion. These attacks may not violate protocol safety directly, but can be used to force a particular order of transactions [50]–[52], or transaction omission [38], [53]–[55].

**Non-Nakamoto blockchains** The BDoS attack is designed specifically for a Nakamoto-like blockchain. For example, it does not directly apply to the Ethereum blockchain (that is more vulnerable to other attacks [18], [40], though), where blocks receive partial reward even if they are off the main chain, and so in case of a BDoS header publication a participant is indeed better off mining, getting at least a partial reward. Blockchain operators should be aware of this new type of attack, and evaluate the resilience of their individual designs.

Alternatives such as Proof of Stake (PoS) [56]–[61] typically do not have complete treatment of incentives, therefore it remains an open question whether a similar attack is applicable. Buterin [14] introduced so-called Discouragement Attack on PoS, where an attacker reduces the profit of other participants by censoring victims’ messages, leading to a temporary DoS.

Nakamoto-like protocols with alternatives to PoW [62]–[66] are as vulnerable to BDoS as standard PoW.

### III. Model

We describe the system model (§ III-A), namely the participants, their interaction, and network assumptions, and the *game model* (§ III-B), namely the players’ action space and utility.

#### A. Mining Model

We model the system in a similar way to that of previous works [67]–[69] and make the same network assumptions as in [5]–[7], but we define a different capability of the attacker. The attacker can release a block header while withholding the data and the system allows other participants to react to such behavior.

**Blockchain data structures** The basic component of a cryptocurrency system is *block*. A block $B$ contains *block data*, i.e., transaction details, denoted by $D$, and metadata called *block header*, denoted by $H$. We say that $B = (H,D)$. Each block contains inside its header ($H$) a pointer to another block it is linked to in its header, except $B_0$, which is called the *genesis block*.

Linked blocks form a *blockchain*, the central data structure in a cryptocurrency system. We denote the data structure of a blockchain with $L$ and it is a linked list of blocks. Each block $B$ in $L$ is either a *full block* containing the entire block information $(H,D)$, or a block header without the block data $(H,\bot)$. The fact that the blockchain can consist of partial block information is a refinement of our model compared to previous works [5]–[7], [18], [40], [41], [67]–[69], where a blockchain consists only of full blocks.

**Participants** We consider a system that comprises $n$ participants called miners, denoted by $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n$, and an adversary $\mathcal{A}$. Each miner $\mathcal{P}_i$ has an associated value $\alpha_i$ called its *hash rate*, and the adversary $\mathcal{A}$ has hash rate $\alpha_A$. The total hash rate is one, $\alpha_A + \sum_{i=1}^{n} \alpha_i = 1$.

Each rational miner $\mathcal{P}_i$ possesses a view of the blockchain $L_i$ locally. As mentioned before, each block $B$ in $L_i$ can either be a block header if $\mathcal{P}_i$ doesn’t receive the block data or a full block otherwise. $\mathcal{P}_i$ also has a local order function $O_i : L_i^{\text{Full}} \rightarrow \{0, 1, \ldots, |L_i^{\text{full}}|\}$, where $L_i^{\text{Full}}$
is the subset of \( L_i \) that consists only of the full blocks in \( L_i \). This function indicates the order of full blocks in \( L_i \) observed by miner \( P_i \). Note that \( O_i \) is not defined over blocks whose full information is not public, i.e., block headers. For all \( P_i \in \{ P_1, P_2, \ldots, P_n \} \) it holds that \( O_i(0) = 0 \) (all miners agree that the genesis block was the first block). Different miners may have different order functions on their full blocks depending on the order they receive blocks locally.

We call a set of full blocks that are all sequentially linked a chain. The longest chain of full blocks in \( L_i \) represents the state of the system for a miner \( P_i \) and is called the main chain. When multiple chains are the longest, \( P_i \) prefers the chain she observes first to be the main chain, i.e., the chain whose \( O_i(B) \) value of the last block \( B \) in the chain is the minimal among that of other chains.

**Rushing** As in selfish mining [5]–[7], we denote by \( \gamma \) the strength of \( A \)’s rushing ability. Specifically, \( \gamma \) represents the expected proportion of rational miners that adopt \( A \)’s block when \( A \) publishes it to compete with a newly published block by some other miner \( P_i \) at the same height, i.e., having the same sequential index in chains that contain them. Similarly, \((1 - \gamma)\) is the expected portion of rational miners that adopts \( P_i \)’s new block.

**Mining on a block header** Usually a miner mines on the last block in the main chain, but we assume that a miner can also mine on a block header which references the last block in the main chain. This type of mining is called SPV mining. Notice that according to our definition, no miner considers a block that references a header, as part of her main chain. Therefore, by engaging in SPV mining, a miner has to assume that the full block corresponding to the header would be published in the future. Throughout this paper, we don’t consider SPV mining as malicious behavior, which enlarges the possible action space for a rational miner. We give more details in §III-B.

**Scheduler** The system is orchestrated by an organizing entity called the scheduler. The scheduler progresses in rounds. During each round, the scheduler selects the miner to generate the new block in this round and propagates blocks to the miners. All the messages are delivered immediately and the system is synchronous.

We denote with \( \lambda \) a system constant called the block generation rate constant. It corresponds to the desired block creation rate (average number of blocks created per second) in the blockchain. For instance, in Bitcoin \( \lambda = \frac{1}{10 \text{min}} \text{ s}^{-1} \), thus a block is created on average every 10 minutes.

In the beginning of each round \( r \), the scheduler asks each miner whether she participates as a candidate to find a new block during this round. We say that a participating miner is active in this round. The scheduler also records the block template of each active miner, which consists of the miner’s identity and the preceding block it links to. Then the scheduler chooses a miner, to mine the next block, from the set of active miners by a weighted random distribution depending on the hash rate of each miner. The selected miner generates the block in round \( r \) and is called the winner of the round, denoted by \( w_r \). We index the blocks \( B = \{ B_0, B_1, B_2, \ldots \} \) by the order of their issuance, i.e., \( w_r \) generates the block \( B_r \).

The scheduler then simulates the block generation time which is determined using an exponential distribution with the rate \( \lambda \cdot \alpha_{\text{active}}^r \) where \( \alpha_{\text{active}}^r \) is the total hash rate of active miners in round \( r \). The only purpose of block generation time is to determine the cost of active miners (further details are given in §III-B). If all the miners are mining during a round (i.e., \( \alpha_{\text{active}}^r = 1 \)), which we call the honest setting, it holds that the exponential distribution has a rate of \( \lambda \). Note that we do not consider difficulty adjustment unless otherwise stated, thus, the expected block generation time in a round is always \( \frac{1}{\lambda} \).

In the second phase, the scheduler announces block data to miners. It treats the cases of an adversarial winner and a rational winner separately. If the adversary \( A \) was chosen by the scheduler to mine a block, she is notified by the scheduler to make a decision. She decides whether to publish the full block of \( B_r \) or only the block header to other miners, and sends a list of partially withheld blocks (i.e., only headers released in previous rounds) to be published in the current round. Receiving the adversary’s decisions, the scheduler first sends blocks in the list one by one sequentially to all miners, then either the full block or the block header of \( B_r \) depending on \( A \)’s decision.

If a rational miner \( P_i \) was chosen by the scheduler to mine a block, the scheduler notifies the adversary \( A \) of \( B_r \) before sending it to any rational miner. The adversary decides whether to race against \( B_r \) and sends a list of her partially withheld blocks for competing to the scheduler. If the list is empty, the scheduler simply broadcasts \( B_r \) to all miners. Otherwise, the scheduler sends \( B_r \) and \( A \)’s competing blocks in different orders to different miners: Because the miner of a block sees her own block locally before propagating it to others and getting a response in the real world, the winner \( w_r \) in this round is sent by the scheduler \( B_r \) first and then \( A \)’s competing blocks. In reverse, the adversary is sent her own competing blocks first and then \( B_r \). For each miner \( p \in \{ P_1, \ldots, P_n \} \setminus \{ w_r \} \), to simulate the connectivity factor \( \gamma \) with probability \( \frac{\gamma(1 - \alpha_A)}{1 - \alpha_A - \alpha_{\text{active}}} \) the scheduler sends \( A \)’s competing blocks first and then \( B_r \) to \( p \), and with probability \( 1 - \frac{\gamma(1 - \alpha_A)}{1 - \alpha_A - \alpha_{\text{active}}} \) sends \( B_r \) first and then \( A \)’s blocks.

Note that assuming the propagation phase in each round is atomic, all rational miners have the same view of the blockchain, i.e., the local blockchains \( L_i \) rational miners possess are the same. Miners of blocks on the main chain of the common blockchain receive block rewards. The pseudocode of the scheduler is in Appendix C.

**B. Game-Theoretic Model**

The model from §III-A gives rise to a game played by rational miners and adversary:

**Miners** A rational miner \( P_i \) can either be a mining pool or an individual miner with hash rate \( \alpha_i \). Each miner knows the adversary’s strategy and participates in a game with a finite
number of actions: \{mineMain, stop, mineSPV\}, which is defined later. The sole purpose of the rational miners in this game is to maximize their utility.

**Utility** For each rational miner \( \mathcal{P}_i \), we denote by \( \Pi_i(t), R_i(t), \) and \( C_i(t) \) her expected profit, revenue, and cost until time \( t \), respectively. It holds that: \( \Pi_i(t) = R_i(t) - C_i(t) \). We denote the average profit per time unit, for \( \mathcal{P}_i \), along the entire game by: \( \hat{\Pi}_i = \lim_{t \to \infty} \frac{\Pi_i(t)}{t} = \lim_{t \to \infty} \frac{R_i(t) - C_i(t)}{t} \).

We also define the average revenue and cost per second denoted by \( \hat{R}_i \) and \( \hat{C}_i \) similarly.

Every block has a constant \( K \) reward (in dollars). For simplicity, we assume that coin price is constant during the entire game and thus the block reward \( K \). Different miners may mine at different costs per hash. The cost of miner \( \mathcal{P}_i \) per one second of hashing is \( c_i \), where \( c_i \) is the normalized mining cost per second for \( \mathcal{P}_i \). When there is no attack and all miners mine on a single chain, the expected profit per time unit, denoted with \( \hat{\Pi}_i^G \), is: \( \hat{\Pi}_i^G = \alpha_i (K - c_i) \).

In order to define the utility function, we normalize the expected profit by the miner’s hash rate. Therefore we say that the utility function \( U \) of miner \( i \) is:

\[
U_i = \frac{\hat{\Pi}_i}{\alpha_i}.
\]

We conclude that the utility of rational miner \( \mathcal{P}_i \) during an honest game is:

\[
U_i^h = \lambda K - c_i.
\]

The mining cost factor \( c_i \) for \( \mathcal{P}_i \) is assumed to be constant throughout the game. We also define the profitability factor \( \omega_i^b \) for miner \( \mathcal{P}_i \) participating in an honest game. Intuitively, the profitability factor is the return per dollar investment for a miner in an honest game. Formally it is defined as:

\[
\omega_i^b = \lim_{t \to \infty} \frac{R_i(t)}{C_i(t)} = \frac{\lambda K}{c_i}.
\]

We note that when \( U_i^b > 0 \) it implies \( \omega_i^b > 1 \) and \( U_i^b < 0 \) implies \( \omega_i^b < 1 \).

We show that in some cases, an attacker can change rational miners’ utility to be below zero (negative profit over time) when \( \omega_i^b > 1 \), i.e., even when it is profitable for \( \mathcal{P}_i \) to mine in an honest game.

**Actions** Similarly to [15], we consider rational miners that are petty compliant, meaning that they do not engage in malicious behavior, but are trying to maximize their profit with slight protocol deviations, namely engage in SPV mining. We include SPV mining in our model, as it is a common behavior of miners while selfish mining is rarely observed. Each rational miner therefore has three possible actions:

1) mineMain - Mine on the latest full block in the main chain.
2) stop - Stop mining.
3) mineSPV - Mine on the block header that the adversary released.

A miner decides on the action to take at the beginning of a round and commits to it until the end of the round. Changing the action within a round does not increase \( \mathcal{P}_i \)'s utility, this is formally justified in Appendix A.

Notice that if \( \omega_i^b > 1 \), the rational miner always chooses mineMain in the honest game and if \( \omega_i^b < 1 \) she chooses stop. In case the adversary initiated an attack by releasing a block header, the rational miner can choose one of the three actions. The rational miner can change her action throughout the game when a new round begins, but never during the round. Therefore, at the beginning of each round, the rational miner chooses the best action from the action space (the method will be described in §IV). On the other hand, during an honest game, the rational miner can choose between stop and mineMain. The pseudocode of a rational miner’s actions is in Appendix C.

**IV. The BDoS Attack**

In general, BDoS-Mine attack aims to incentivize rational miners to stop mining, i.e., to choose the action stop. The key observation is that whether a rational miner \( \mathcal{P}_i \) chooses mineMain or mineSPV, \( A \) can invalidate a rational miner’s block \( B_P \)—in the first case with some probability and always in the second. Thus, \( A \) negates \( \mathcal{P}_i \)'s work, while \( \mathcal{P}_i \) incurs cost for performing mining (e.g., the cost of electricity). We show that in the right setting, \( \mathcal{P}_i \) incurs negative utility should she choose either mineMain or mineSPV. Her best choice as a rational agent is thus the action stop.

Let \( B^* \) denote the latest block on the main chain. \( A \)'s strategy is to mine on \( B^* \) (fig. 1a). If she successfully appends a new block \( B_A = (H_A, D_A) \) to \( B^* \) before a new rational one is mined, she does not publish \( B_A \) in full. She publishes only the block header \( (H_A, \perp) \) for \( B_A \). She withholds the rest of the block, namely its associated transactions. At this point, we refer to the state of the attack as in progress. We refer to \( B_A \) as the leading block in the attack. \( B_A \) is not part of the main chain, as it has not been published in full (fig. 1b).

Because \( A \) has announced her attack BDoS-Mine to rational miners, the header of \( B_A \) serves as a proof that \( A \) has successfully mined \( B_A \) and is currently withholding the full block. Until a rational miner produces a fresh block, \( A \) stops mining completely. Next, three things can happen:

**Block generated via mineMain:** If a rational miner performs mineMain and successfully generates a block \( B_P \) appended to \( B^* \) (fig. 1c), \( A \) immediately publishes \( B_A \) in full, i.e., attempts to add it to the main chain. A race then ensues: Mining power is now divided between \( B_P \) and \( B_A \). The first block to be extended “wins” the race in the sense of being appended to the main chain. (We discuss later the parameter \( \gamma \) that determines \( A \)'s success probability in this race.)

**Rational miners stop mining:** \( A \) stops mining as longs as there no new block generated by rational miners.

**Block generated via mineSPV:** If a rational miner performs mineSPV and successfully generates a block \( B_P \) appended
to $B_A$ (fig. 1d), then $A$ "abandons" $B_A$. $A$ starts mining on $B^*$, i.e., on the main chain, thereby rendering $B_P$ permanently invalid, since $B_A$ will never be part of the main chain.

The pseudocode for BDoS-Mine is in Appendix C.

V. ANALYSIS

We proceed to analyze the conditions for successful attack. We find what is the profitability factor needed to make "stop mining" the best response during an attack.

A. Strategies

We call the rational miner we use as case study $P_i$. We define with $\Lambda_{B^*}$ and $\Lambda_{B_A}$ the sets of miners actively mining on $B^*$ and $B_A$ while the attack is in progress. Next, we define:

$$\alpha_{B_A} = \sum_{j \in \Lambda_{B_A}} \alpha_j, \quad \alpha_{B^*} = \sum_{j \in \Lambda_{B^*}} \alpha_j.$$ 

Given BDoS-Mine and $\omega_i^B$, our goal is to find an optimal strategy for $P_i$, which she chooses at the beginning of the game, i.e., a map from $L_i$ and $O_i$ to an optimal action. We say that strategy $S_1$ is more beneficial than strategy $S_2$, for a rational miner $i$, if the utility by playing $S_1$ is larger than the utility by playing $S_2$. Note that we assume that $\omega_i^B > 1$ (as for $\omega_i^B \leq 1$ no rational miner mines), therefore while the attack is not in progress, a rational miner always chooses action mineMain. Consequently, we consider only three strategies: $S_{main}$, $S_{stop}$ and $S_{SPV}$ that differ only by the actions of $P_i$ during the attack: mineMain, stop and mineSPV respectively. We describe the game for each strategy with three state Markov chains. Strategy $S_{main}$ appears in fig. 2a, $S_{stop}$ in fig. 2b and $S_{SPV}$ in fig. 2c. State 0 represents the initial state where everyone mines on $B^*$. State 1 represents the state where the adversary managed to find a block. State 2 represents the race condition, where the players are divided between $A$’s block and the block generated by a rational miner. The states are graphically illustrated in fig. 1. For all the mentioned above strategies, $P_i$ chooses the action mineMain when not in state 1 (when the attack is not in progress). For a given strategy, the same action would be selected every time the attack starts. Therefore, each Markov chain would match to a strategy that differs only by the actions of the player in state 1.

B. State Probabilities

We denote with $S$, $P_i$’s strategy and with $\alpha_{B^*}(S)$ the total hash rate of miners that mine on $B^*$ in state 1, i.e., the portion of miners who kept mining on $B^*$ during the attack:

$$\alpha_{B^*}(S) = \begin{cases} \alpha_{B^*} + \alpha_i, & \text{if } S = S_{main} \\ \alpha_{B^*}, & \text{otherwise}. \end{cases}$$

Similarly we denote with $\alpha_{B_A}(S)$ the hash rate of miners that mine on $B_A$ in state 1:

$$\alpha_{B_A}(S) = \begin{cases} \alpha_{B_A} + \alpha_i, & \text{if } S = S_{SPV} \\ \alpha_{B_A}, & \text{otherwise}. \end{cases}$$

We proceed to calculating the state probabilities of all three markov chains in fig. 2. We get the following identities:

$$P_0^S = \frac{\alpha_{B^*}(S) + \alpha_{B_A}(S)}{\alpha_{A} \cdot \alpha_{B^*}(S) + \alpha_A + \alpha_{B^*}(S) + \alpha_{B^*}(S)}.$$

$$P_1^S = \frac{\alpha_A}{\alpha_{A} \cdot \alpha_{B^*}(S) + \alpha_A + \alpha_{B_A}(S) + \alpha_{B^*}(S)}.$$

$$P_2^S = \frac{\alpha_A \cdot \alpha_{B^*}(S) + \alpha_A + \alpha_{B_A}(S) + \alpha_{B^*}(S)}{\alpha_{A} \cdot \alpha_{B^*}(S) + \alpha_A + \alpha_{B_A}(S) + \alpha_{B^*}(S)}.$$
C. Utility For Each Strategy

As the first step in calculating the utility, we calculate the cost and the revenue of $P_i$. While a rational miner is mining, her cost per second is constant, but when she stops mining, her cost is zero. Therefore for $S_{stop}$ it holds that the average cost per time unit $\bar{C}_{i}^{S_{stop}}$ for $P_i$ is:

$$\bar{C}_{i}^{S_{stop}} = \lim_{t \to \infty} \frac{C_{i}^{S_{stop}}(t)}{t} = \alpha_i (1 - p_i^{S_{stop}}) \cdot c_i.$$  

On the other hand when $P_i$ chooses strategy $S_{main}$ and therefore keeps mining all the time, her cost $\bar{C}_{i}^{S_{main}}$ is constant:

$$\bar{C}_{i}^{S_{main}} = \lim_{t \to \infty} \frac{C_{i}^{S_{main}}(t)}{t} = \alpha_i \cdot c_i.$$  

Therefore it is left find the average revenues $\hat{R}_{i}^{S_{main}}$ and $\hat{R}_{i}^{S_{stop}}$ for $S_{stop}$ and $S_{main}$ respectively, in order to find the more beneficial strategy.

We now analyse the Markov chain: For both strategies the rational miner $P_i$ receives profit $K$ every time she passes from state 0 back to state 0 with rate $\alpha_i \lambda$ and from state 2 to state 2 with rate $\alpha_i \lambda$. For strategy $S_{main}$, $P_i$ receives profit $(1 - \gamma) (1 - \alpha_A) \cdot K$ when she passes from state 1 to state 2 with rate $\alpha_i \lambda$. Therefore expected utility for strategy $S_{stop}$ is (using the definition in eq. (1)):

$$U_i^{S_{stop}} = \frac{1}{\alpha_i} (\hat{R}_{i}^{S_{stop}} - \bar{C}_{i}^{S_{stop}}) = \frac{1}{\alpha_i} ([p_0^{S_{stop}} + p_2^{S_{stop}}] \cdot \alpha_i \lambda K - (1 - p_1^{S_{stop}}) \alpha_i c_i)$$  

$$= [p_0^{S_{stop}} + p_2^{S_{stop}}] \cdot \lambda K - (1 - p_1^{S_{stop}}) \cdot c_i.$$  

Similarly the expected utility for strategy $S_{main}$ is:

$$U_i^{S_{main}} = \frac{1}{\alpha_i} (\hat{R}_{i}^{S_{main}} - \bar{C}_{i}^{S_{main}}) = (p_0^{S_{main}} + p_2^{S_{main}} + (1 - \gamma) (1 - \alpha_A) \cdot p_1^{S_{main}}) \lambda K - c_i.$$  

Finally, the utility for $S_{SPV}$ is:

$$U_i^{S_{SPV}} = \frac{1}{\alpha_i} (\hat{R}_{i}^{S_{SPV}} - \bar{C}_{i}^{S_{SPV}}) = (p_0^{S_{SPV}} + p_2^{S_{SPV}}) \cdot \lambda K - c_i.$$  

D. Narrowing down the possible number of strategies

In order to simplify the analysis we spot dominated strategy, i.e., a strategy that is always less beneficial compared to another strategy.

Claim V.1. $S_{SPV}$ is strictly dominated by $S_{main}$.

Proof. We calculate the difference $D$ between the utility of playing $S_{main}$ (defined in eq. (6)) and the utility of playing $S_{SPV}$ (defined in eq. (7)):

$$D = U_i^{S_{main}} - U_i^{S_{SPV}}$$  

$$= p_0^{S_{main}} + p_2^{S_{main}} + (1 - \gamma) (1 - \alpha_A) \cdot p_1^{S_{main}} - (p_0^{S_{SPV}} + p_2^{S_{SPV}}) \cdot \lambda K.$$  

We notice that the probability $p_i^{S_{SPV}}$ (eq. (4)) decreases when $P_i$ chooses $S_{main}$ instead of $S_{SPV}$, the numerator stays the same while the denominator increases. We conclude that $p_i^{S_{main}} < p_i^{S_{SPV}}$ and therefore:

$$D = (p_0^{S_{main}} + p_2^{S_{main}} - (p_0^{S_{SPV}} + p_2^{S_{SPV}})) \cdot (1 - p_1^{S_{main}}) - (1 - p_1^{S_{SPV}}) > 0.$$  

From eq. (8) and eq. (9) we conclude that $D > 0$. Therefore by playing $S_{main}$, $P_i$ always has strictly larger profit than she would have if she would play $S_{SPV}$.

From now on we consider only two strategies for $H_1$ in our analysis: $S_{main}$ and $S_{SPV}$. As we proved that $H_1$ never chooses strategy $S_{SPV}$. Notice that we still have to consider $S_{SPV}$ for other miners in order to find conditions for $S_{SPV}$ to be dominant strategy (§V-E). In §V-F we relax this in order to argue about the more practical setting where no rational miner chooses dominated strategy.

E. Conditions for Successful Attack

We intend to calculate for what values of $w$ (defined in eq. (3)) the attack would be successful given $\alpha_A$ and $\alpha_i$, i.e., the hash rate of the attacker and a certain rational miner $P_i$. Note that in order for this attack to enforce complete shutdown, we have to examine the miner with the largest hash rate. Using eq. (5) and eq. (6) we define $D(\alpha_B^*, \alpha_{BA})$ to be the normalized difference between $U_i^{S_{SPV}}$ and $U_i^{S_{main}}$ to be

$$D(\alpha_B^*, \alpha_{BA}) = \frac{U_i^{S_{main}} - U_i^{S_{SPV}}}{c_i}$$  

$$= (p_0^{S_{main}} + p_2^{S_{main}} - p_0^{S_{SPV}} - p_2^{S_{SPV}}) - (1 - \gamma) (1 - \alpha_A) \cdot p_1^{S_{main}} \cdot \omega_i^b + p_1^{S_{SPV}}.$$  

Our goal is to find when the attack is successful and all miners stop, that is, what are the $\omega_i^b$ values for which for all possible $\alpha_B^*$ and $\alpha_{BA}$ values it holds that $D(\alpha_B^*, \alpha_{BA}) < 0$. We therefore calculate the condition on $\omega_i^b$ so that $D(\alpha_B^*, \alpha_{BA}) < 0$ using eq. (10):

$$\omega_i^b < \frac{p_0^{S_{SPV}} + p_2^{S_{SPV}} + (1 - \gamma) (1 - \alpha_A) \cdot p_1^{S_{main}} - (p_0^{S_{SPV}} + p_2^{S_{SPV}})}{Q(\alpha_B^*, \alpha_{BA})}.$$  

This is the general bound on $\omega_i^b$ that makes $S_{stop}$ the dominant strategy for $P_i$. This can be solved for specific values of $\gamma$, $\alpha_A$ and $\alpha_i$ and otherwise it’s not analytically solvable for the parametric case.

F. Iterated Elimination of Weakly Dominated Strategies

The result in eq. (11) is the condition for $S_{stop}$ to be dominating strategy among the three strategies: $\{S_{stop}, S_{main}, S_{SPV}\}$. We use a technique called iterated elimination of strictly dominated strategies (IESDS) [70]. In realistic setting, we assume that no rational miner chooses to mine on $B_A$ (and
that this is a common knowledge [71], as this is strictly dominated strategy as we showed in § V-D. Therefore, we analyze the case where $\alpha_{B_A} = 0$. We are looking for the condition on $\omega^b_i$ so that $S_B$ would be an optimal strategy (in the reduced strategy space $\{S_{stop}, S_{main}\}$). Therefore, we are looking for the minimal $Q(\alpha_{B^*}, \alpha_{B_A} = 0)$. We use calculus to find the minimum and we get that $Q(\alpha_{B^*}, \alpha_{B_A} = 0)$ receives minimal value when $\alpha_{B^*} = 0$ regardless of the parameters’ values.

This result implies that the motivation for a miner to keep mining during the attack decreases when other miners keep mining, as the minimum achieved when all other miners are following $S_{stop}$. By assigning $\alpha_{B^*} = 0$ and $\alpha_{B_A} = 0$ to eq. (11) and using the probabilities calculated in eq. (4), the tight condition on $\omega^b_i$ is:

$$\omega^b_i < \frac{\alpha_A + \alpha_i + \alpha_A \alpha_i}{\alpha_i + \alpha_A \alpha_i + (1 - \gamma) \alpha_A (1 - \alpha_A)}.$$  

(12)

G. Examples of Bound on $\omega^b_i$ for Different Adversary and Rational Miners

First we use the condition on $\omega^b_i$ that was obtained in eq. (12).

We draw the graph of $\omega^b_i$ for different values of $\alpha_A$, $\alpha_i$ and $\gamma$ in fig. 3. In addition we simulate the game, for each setting of $(\alpha_A, \alpha_i, \gamma)$ averaging the upper bound on $\omega^b_i$ over 100 epochs, each generating 1,000,000 blocks, and plot the results in the graph. All simulated results were within 0.2% error range from the analytical results. We can see from the graph, unlike the case in previous attacks, even an attacker with a relatively small computational power (e.g., $\alpha_A < 0.1$) can successfully mount an attack to stop all other miners from mining. The hash rate of the rational miner $\alpha_i$ is also important to the success of the attack. So that if $\alpha_A = 0.2$ and $\alpha_i = 0.05$, the borderline $\omega^b_i$ is almost 1.9. Therefore, even if all the rational miners have similar profitability, the attacker would be able to stop smaller miners easier. This shows that large mining pools have stronger protection against BDos-Mine. Moreover, from fig. 3 we can see that when $\gamma = 0$ and $\alpha_A = 0.2$, the attacker needs $\omega^b_i$ to be smaller than 1.15 in order to attack rational miner with $\alpha_i = 0.1$, compared to $\omega^b_i < 1.6$ when $\gamma = \frac{1}{2}$ and $\omega^b_i < 2.7$ when $\gamma = 1$. This highlights the importance of rushing ability for the attacker. Note that $\gamma = \frac{1}{2}$ is conservative assumption especially due to the fact that an adversary can control a relay network [72] and therefore potentially achieve $\gamma$ even closer to 1. In § VII we further show that even if the rational miners are deviating from Nakamoto’s protocol by boycotting $\mathcal{A}$’s blocks (and therefore decreasing $\gamma$), she can use smart contracts (on external cryptocurrency) to make her blocks indistinguishable from rational miners’ blocks.

H. Fixing $\alpha_{B^*}$

Previously we found the borderline $\omega^b_i$ for the worst case, i.e., for all possible chosen strategies of other players. We saw that if the portion of rational miners that keep mining $\alpha_{B^*}$ increases, the motivation for $\mathcal{P}_i$ to stop mining also increases. We will now consider a scenario where $\mathcal{P}_i$ can estimate accurately $\alpha_{B^*}$. In real setting this can be done by spying on other pools [8], [73] or by monitoring the recent inter block time. As before we assume that $\alpha_A = 0$. Using eq. (11), we conclude that the bound on $\omega^b_i$ is $Q(\alpha_{B^*}, \alpha_{B_A} = 0)$. We define: $\alpha_r = \frac{\alpha_{B^*}}{\alpha_{B_A}}$, which is the absolute portion of rational miners other than $\mathcal{P}_i$ that continue mining. We plot the borderline $\omega^b_i$, $\alpha_A$ and $\alpha_i$ for different $\alpha_r$ values in fig. 4.

We can see that if all other rational chose $S_{main}$ ($\alpha_i = 1$), then for $\alpha_A = 0.2$ and $\alpha_i = 0.16$, the attack is successful for $\omega^b_i < 2$ which is significantly higher than $\omega^b_i < 1.45$ for the case with $\alpha_r = 0$. We conclude that the threshold for a partial attack is significantly higher than the threshold for a complete shutdown.

Figure 3: $\omega^b_i$ that will allow an attack for different $\alpha_A$, $\gamma$ and $\alpha_i$ (Notice that $\gamma$ can’t reach 1 in real setting).

Figure 4: $\omega^b_i$ that will allow an attack for different $\alpha_r$, $\alpha_A$ and $\alpha_i$ while $\gamma = \frac{1}{2}$.
VI. TWO-COIN MODEL

So far, we used a model where the attacker initiates an attack on coin $C$ and the rational miners can either mine on this coin or not mine at all.

We now consider a two-coin model where miners can choose to mine between two coins with the same hashing function, which is similar to coin-hopping [44]. In real life, there are pools allowing miners to switch to mine on the most profitable coin [74], [75] and automatic switching tools [76], [77]. There is also an evidence that miners switch between coins during the “Hash Wars” between Bitcoin ABC and Bitcoin SV [78]. In this incident, Craig Wright who is affiliated with Bitcoin SV, was recording saying that Bitcoin SV miners would try to “reorganize” (invalidating block by building a longer fork) blocks in Bitcoin ABC. In response, miners in favor of Bitcoin ABC switched some of their mining power from Bitcoin to Bitcoin ABC to win this Hash War.

In our two-coin model, we assume a rational miner can choose between mining on $C$ or on a competing coin $C'$ with the same mining mechanism. The profitability and utility of $P_i$ for coin $C$ denoted with $\omega_i^b$ and $U_i^b$ respectively. Similarly the profitability and utility for player $i$ for coin $C'$ denoted with $\omega_i^{b'}$ and $U_i^{b'}$ respectively. If we assume that the initial mining profitability for the player with largest hash rate $P_i$ on both coins is equal, thus $\omega_i^b = \omega_i^{b'}$, the attacker has no longer an upper bound on $\omega_i^b$ that would be a threshold for an attack on coin $C$. This is because any attack always decreases the mining utility $U_i$ for $P_i$ (mining on coin $C$) and therefore every miner would choose to mine on $C'$ instead (as $U_i^{b'} > U_i^b$).

Due to the large number of coins in the blockchain world and the fact that some of them use the same or similar mining scheme as Bitcoin, the mentioned above alternative model is more realistic and can cause the attack to be much stronger. This assumption is supported by a previous works [79], [80].

In [79], the authors describe an incident where due to a significant increase of Bitcoin’s cash exchange rate compared to Bitcoin’s, resulted in a large increase of the total hash rate in Bitcoin Cash and a parallel drop in the hash rate of Bitcoin. In [80], the authors analyse mining pools from inside and showed that miners tend to switch between coin due to profitability differences.

We can think of the coin profitability difference as a profitability arbitrage. When there is a profitability difference miners are expected to switch coins to the more profitable coin. By doing that they cause the profitability to decrease in the long term (due to difficulty adjustment) and bring the coins’ profitability to equilibrium. We describe a way to create artificial profitability difference between the coins and consequently causing all rational miners to abandon one of the coins for the other(s).

A. Analysis

The model is almost the same as the one described in §III-B and the analysis would be similar to the analysis in §V. The main difference is that we no longer consider a choice between mining on $B^*$ vs. stop mining. We analyse the difference between mining on $B^*$ in the attacked coin $C$ vs. mining regularly on coin $C'$. The utility $U_i^{S_1}$ for $P_i$ for the first strategy $S_1$ (mining on $B^*$ in coin $C$) is the same as $U_i^{S_{main}}$ in eq. (6):

$$U_i^{S_1} = U_i^{S_{main}} = (p_0^{S_{main}} + p_2^{S_{main}} + (1 - \gamma)(1 - \alpha_A) \cdot p_1^{S_{main}}) \lambda K - c_i.$$  

(13)

While the utility $U_i^{S_2}$ for miner 1 for the second strategy $S_2$ (mining in the honest setting in coin $C'$) is the same as $U_i^b$ in eq. (2):

$$U_i^{S_2} = U_i^{b'} = \lambda K' - c_i.$$  

(14)

Notice that we used different $\lambda$, $c_i$ and $K$ parameters for the second coin ($\lambda'$, $c_i'$ and $K'$ respectively), as they are not necessary the same for both coins. To compare the two utilities in two different coin we can no longer use the normalized utility define in eq. (1), as the hashrates $\alpha_i$ and $\alpha_i'$ of coin $C$ and $C'$ respectively, are not necessarily the same. Notice that the mining cost per second of $P_i$ is equal for both coins, so that $\alpha_i c_i = \alpha_i' c_i'$. We define $D$ as the difference between the two utilities in eq. (13) and eq. (14), when each utility is multiplied by the respective hashrate:

$$D(\alpha_{B^*}, \alpha_{B_A}) = \frac{\alpha_i U_i^{S_1} - \alpha_i' U_i^{S_2}}{\alpha_i c_i} = \frac{\alpha_i U_i^{S_1} - \alpha_i' U_i^{S_2}}{\alpha_i c_i} = (p_0^{S_{main}} + p_2^{S_{main}} + (1 - \gamma)(1 - \alpha_A) \cdot p_1^{S_{main}}) \cdot \omega_i^b - \omega_i^{b'}.$$  

As before we ask when it holds that $D(\alpha_{B^*}, \alpha_{B_A}) < 0$. Therefore we are looking for the ratio $r = \frac{\omega_i^{b'}}{\omega_i^b}$ s.t:

$$r = \frac{\omega_i^{b'}}{\omega_i^b} > \frac{(p_0^{S_{main}} + p_2^{S_{main}} + (1 - \gamma)(1 - \alpha_A) \cdot p_1^{S_{main}})}{W(\alpha_{B^*}, \alpha_{B_A})}.$$  

As before, we look at the reduced strategy space, for other miners, where $\alpha_{B_A} = 0$. We ask what is the maximal value of $W(\alpha_{B^*}, \alpha_{B_A} = 0)$. Using calculus we derive that it receives maximum for $\alpha_{B^*} = 1 - \alpha_A - \alpha_i$ which means that all other miners do not switch coins (as this the maximum value it can get):

$$r > W(\alpha_{B^*} = 1 - \alpha_A - \alpha_i, \alpha_{B_A} = 0) = \frac{(1 - \alpha_A)(\alpha_A(\gamma - 2) - 1)}{\alpha_A^2 - \alpha_A - 1} = r^*.$$  

An interesting fact is that the minimal $r$ that allows the attack, which we denote with $r^*$, does not depend on the hash rate of $P_i$.

We plot $r^*$ that allows the attack for different $\gamma$ and $\alpha_A$ in fig. 5. When $\gamma = \frac{1}{2}$ and $\alpha_A = 0.2$, it holds that $r^* = 0.9$. This means such an attacker can attack as long as $C$ profitability is less than 11% more profitable than $C'$. Notice that the attack is always possible when $r = 1$ (when the profitability
This is hash power equivalent to that of the entire Bitcoin network. In Appendix B we calculate the cost per day of achieving as the attacker mines at a constant rate in all states but 1 as in such case the attacker’s cost is:

\[ \alpha \gamma \] recall that portion of time spent in state 1 aims to disable Bitcoin, ignoring the hardware cost. We first sizes. Note that for in section we assume an existing conditions exist currently between Bitcoin ABC and Bitcoin.

**A. Attack Cost**

We investigate the cost of attack for different attacker sizes. Note that for in section we assume an existing miner aims to disable Bitcoin, ignoring the hardware cost. We first recall that portion of time spent in state 1 of fig. 2 is \( p_1 \). It immediately follows that the attacker’s cost is: \( \alpha_A C_d (1 - p_1) \) as the attacker mines at a constant rate in all states but 1. In Appendix B we calculate the cost per day of achieving hash power equivalent to that of the entire Bitcoin network. This is \( 7,104,000 \) given use of Antminer S9 SE rigs and \( 3,508,438 \) for Antminer S17 Pro. For majority hashpower, an attacker needs only \( \alpha_A (1 - p_1) \) of these costs.

We plot the graph of \( p_1 \) vs. \( \alpha_A \) in fig. 6. We denote by \( \alpha' \) the total fraction of rational miners who keep mining. The results of the graph are not surprising for \( \alpha' = 1 \), as in such case \( A \) takes complete control of the system and the Markov chain stays in state 1 forever at cost 0. Notice that for a complete failed attack (\( \alpha' = 0 \)), the system still spends 0.17 of the time in state 1 and pays a total 0.165 of the mining cost ($580k a day for the Antminer S17 Pro).

For \( \alpha_A = 0.2 \) and given a small fraction of altruistic miners, for example, \( \alpha_A' = 0.1 \), we will spend 6.5% of the entire mining cost ($65k a day for the Antminer S17 Pro). We showed that the attack would be less costly than regular mining with \( \alpha_i \), and significantly cheaper if only a small portion of the miners are altruistic.

**B. Spreading Block Header**

We assume that the adversary can announce a block header without revealing a full block. There are there main practical approaches:

- **Bitcoin (Compact Blocks):** A well-connected adversary can announce block headers without sending any extra message since the adoption of BIP152. BIP152 states that a sending peer, before sending a full block to a receiving peer, can send a short version of the block called a Compact Block. A Compact Block message consists of a block header and condensed transaction hashes. If the receiving peer can recreate the entire block, i.e., she possesses the full transactions, no further communication is required. On the other hand, if her mempool lacks at least one of the transactions from the Compact Block, she sends to the sending peer a request for the missing transactions. The protocol then stipulates that the sending peer reply with the missing transactions. For our attack, the attacker can mine a block with at least one private transaction known only to her. She can then use BIP152 to propagate her block header to other miners, but withhold the private transaction to prevent receiving peers from reconstructing the block. We note, though, that a BIP152-compliant peer will only propagate a block header upon fully validating the entire block. Thus, to be successful, the attacker must be well connected enough to send her block header to sufficiently large fraction of miners—something easily achievable in practice.

- **Website:** The attacker can announce that she is committing to an attack, and attach a web link to her private page where she publishes her block headers. Rational miners would be incentivized to poll this external website, as more information means more long-term revenue. Miners would still need to validate the correctness of headers to prevent false claims by putative attackers.

- **Smart contract:** Instead of sending a block header, the adversary can use an Ethereum smart contract to demonstrate discovery of a block without revealing it. The idea is to use an economic mechanism to demonstrate knowledge of a valid block header \( H \). Briefly, the attacker escrows a large sum of money \( \$C \) in the contract, along with a cryptographic commitment \( \text{Comm} \) to \( H \), along with the previous block and the the current difficulty parameter. If at some predetermined (distant) future time, she decommits a valid \( H \) for the contract, i.e., one that points to the previous block and is valid for the difficulty parameter, she recovers \( \$C \). Otherwise she forfeits \( \$C \) to miners. Thus, the attacker
is incentivized to claim and commit only to a valid header, but need not reveal any information about $H$ (until $H$ is no longer useful to miners).

In more detail, to ensure that the attacker is indeed incentivized to commit to a valid $H$, the collateral $\$C$ should be significantly larger than the cost of mining blocks during the commitment period. The collateral $\$C$, if forfeited, can be split among a predefined list of mining pools (weighted by their hash rate). For example, this list might include miners of the last, e.g., 1000 Bitcoin blocks.

This approach to demonstrating block discovery has one key advantage over our other two approaches: until $H$ is decommitted (again, in the far future), no rational miner can distinguish the attacker’s block from an honest block. This prevents rational miners from forming a coalition that would ignore the attacker’s block.

This smart-contract-based approach is such that rational miners can no longer engage in SPV mining. The same results hold here, though, as we showed in $\S$ V, namely that no rational miner chooses to do SPV mining when BDoS-Mine is in progress. Further details are beyond the scope of this paper.

C. Estimating Practical $\omega^b$

We intend to attain some intuition about practical values of $\omega^b$ in Bitcoin. First, we can separate the miner’s cost into two categories: Operating expense (OPEX) and capital expense (CAPEX). The OPEX would include costs like Electricity cost of mining equipment, Electricity cost for hardware cooling and risk factor. The CAPEX would include costs like Buying/renting facilities, Buying mining equipment and employees’ salaries. In our theoretical analysis, we used the constant $c_1c_2 \alpha \beta$ to describe the cost of doing one hash per second, this value represents only the OPEX. Therefore, $\omega^b$ depends only on the OPEX as we assume that a miner has a constant hash rate in the system throughout the game. We believe that high CAPEX is a positive property of the system, as it keeps omega higher because miners have to enough profit to return the investment. This implies that if mining equipment would become cheaper and the main cost of the miner would be the electricity, the system would be at greater risk of BDoS attack. In [43], [82] it is assumed that $\omega^b$ is close to 1, as this is an equilibrium. This property can not hold in a world where the CAPEX is significant, as otherwise, the attacker would necessarily have a negative profit. Moreover, mining has to bring some revenue to the miners as it involves large investment and risk even if CAPEX is low, therefore the return on investment has to be worthwhile for miners compared to other secure investment tracks.

More than 50% of the hash power in Bitcoin is originated from Sichuan, China [83]. The reason for that is that the price of electricity in this region is extremely low during the wet season (as low as $0.04$ per kWh, which may vary by hydropower plants). During the rest of the time, dry season, the price is higher, approximately $0.06$ per kWh. We stress the fact that our attacker can choose the timing of the attack according to the seasonal electricity prices and therefore pick a time when the prices are the highest. Moreover, in the end of the wet season or in unexpected dry periods, the difficulty would still remain high, but with the raise of electricity prices the profitability of most miners is expected to be at its lowest point. This would be ideal moment to attack. Another important factor than can make the system vulnerable to the attack is the block reward adjustment that is estimated to happen in the year 2020 [84]. The block reward will then drop from 12.5BTC to 6.25BTC. The transition would be immediate, therefore, this will cause a significant drop in $\omega^b$.

A sophisticated attacker can prepare for such event and launch her attack at the exact moment of the drop.

In [85], [86] the authors define the term Cashflow Breakeven Level, which is the price of Bitcoin below which the miner has to turn off the mining equipment to stop loses. The most recent estimation (June 2019) of the average Cashflow Breakeven Level is $3,300$ this cost includes the additional running costs (such as cooling). We can conclude that if this estimation is accurate, and the current bitcoin price is $BP$, then it currently holds that: $\omega^b = \frac{BP}{3,300}$. For the single coin setting, if the adversary and the largest rational miner own 20% of the hash rate each and $\gamma = \frac{1}{2}$, the attack can be dangerous, for the largest miner, when Bitcoin’s price is roughly $4537$.

Which is realistic given the constant bitcoin fluctuations. Since the conduction of estimation in [86] (June 2019) and the moment of publishing this work (Nov 2019), the difficulty in Bitcoin increased roughly by 185%, which can suggest that $\omega^b$ decreased with the same rate. The new price that can cause the attack to be dangerous can be as high as $8100$.

Estimating $\omega^b$ in practice can be a challenging task. We try to give an upper bound based only electricity cost. We found hash rate and electricity consumption of different mining hardware on [87]. We analyzed blocks 471744 to 602784 (June 17, 2017 - Nov 7, 2019) using the Google BigQuery [88] Bitcoin dataset to collected mining difficulty data and compute expected number of hashes needed to find a block [89].

Assuming the electricity price is 6 cents per kWh, the profitability of mining Bitcoin with different hardware model is plotted in fig. 7. Note that Bitmain Antminer S9 was widely used even after more profitable models were released because of its low price and breakeven period. The decrease in Bitcoin hashrate in Nov 2018 [90] matches with the moment when the profitability of Antminer S9 went below 1. It is reasonable to infer that when mining with Antminer S9 is no longer profitable, miners shut down their Antminer S9 machinesrationally. From fig. 7, there is a trend in the profitability going below the upper bound of $\omega^b$ for a successful complete shutdown attack.

Even if the mining profitability is not low enough for a complete shutdown, a BDoS attacker can still discourage some small miners from mining. This is because, the upper bound on $\omega^b$ that drives a rational miner out of mining increases with the hashrate of $P_i$, as shown in fig. 3. Assuming all miners have the same profitability factor $\omega^b$, consider an adapting process in which rational miners are always aware of the total hashrate.
of active miners in the network, and choose between \( S_{\text{main}} \) and \( S_{\text{stop}} \) adaptively. A rational miner \( P_i \) chooses to mine if the real-world \( \omega^b \) is greater than the upper bound for \( P_i \) and stops mining otherwise. The process starts from the state in which everyone mines and ends when no rational miner changes her strategy any more. Thus at the end of the process rational miners reach a Nash equilibrium. We simulate the process to find an equilibrium where all rational miners may fall in. In the simulation, we assume \( A \)'s rushing ability factor \( \gamma \) is 0.5. We use the mining power of pools collected from [91] on Oct 25, 2019, which is estimated as the block generation rate of each mining pool during the past week, as the hash rate of rational miners. The pools’ mining power distribution is shown in fig. 8. We view each mining pool as an individual player. To be conservative, we assume all the hash power from unknown sources form a single entity. Note that we introduce the adversary as a new miner entering the game with existing mining pools, so the actual hash rates of rational miners in the simulation are scaled with a factor \( 1 - \alpha_A \). We plot the proportion of rational miners that stop mining in the Nash Equilibrium with different mining profitability \( \omega^b \) in separate curves in fig. 9. The simulation indicates that for an adversary with only 20% of mining power, she may stop more than half of rational miners even when the real life profitability is 2.

D. Estimating Practical \( r \)

To justify the analysis in § VI, we are interested to find whether miners tend to switch between coins, with the identical mining algorithms, according to their profitability. In addition, we are interested to see if this causes profitability of coins to be close to each other over time.

In [80], the authors collect data from inside mining pools and show that such migrations between coins happen. Moreover, we collected data of the real life values of \( r \) between Bitcoin and Bitcoin Cash [92]. Our results are shown in fig. 10. We can see that until Bitcoin Cash introduced their new difficulty adjustment algorithm [93] in Nov 2017, there were major fluctuations due to the migration of miners between coins. The new algorithm uses a moving window that changes difficulty of each block according to the time difference between recent blocks. Because the difficulty readjusted every block, we can see that after a short acclimation period, Bitcoin and Bitcoin cash started to have an almost identical profitability over time. This shows that in case there is a deviation of \( r \) from 1, caused by price or difficulty changes, the next difficulty adjustment would act as negative feedback to relink both factors.

VIII. Mitigation

We describe possible mitigations that can weaken the effect of BDoS.

Uncle blocks The attack describes in this work was designed to attack Bitcoin and similar coins. However, it is not effective in Ethereum [16], [17]. The main property of Ethereum that might prevent the BDoS attacks is the uncle block mechanism [17]. This mechanism rewards miners that mined block on the main chain but their block was excluded as result of a longer sequence of blocks. Currently the reward for such block

![Figure 7: Profitability of mining Bitcoin using different mining hardware.](image_url)

![Figure 8: Hashrate of pools.](image_url)

![Figure 9: Proportion of rational miners that stop mining in the Nash equilibrium.](image_url)
is 7/8 of the regular block reward. This imposes a significant challenge on our attack, as now, in case a rational miner loses the “race” in state 2 in fig. 2, her reward is almost as the original block reward. Therefore, by publishing a block header the attacker no longer reduces the expected profit of rational miners significantly in case she keeps mining. Notice that a miner receives a block reward only for blocks that are directly connected to the main chain. Due to this fact, we believe that there are similar attacks to the one described in this paper that still allow the attacker to decrease the expected reward, e.g., by publishing two block headers that fork the most recent block in the chain. Although, such attack design is beyond the scope of this work. In addition, few works suggest that uncle block mechanism also reduces the security of Ethereum against selfish mining attacks [18], [40], [94].

**Ignoring attacker’s block during race** Another possible way to weaken the attack is to add to change miner behavior such that if there is a fork, a miner should prefer blocks not generated due to an attack. The challenge is to identify attack blocks. A third party service for this goal is out of the question as it violates the decentralized nature of the system and allows false incrimination. Instead, we propose to classify according to the time interval between the reception of the header and the reception of the block. We can safely assume that for a non-attack block, this interval is bounded by, e.g., one minute, and blocks with a longer interval are suspect.

Notice that this mitigation is possible only when the adversary chooses to prove that she mined a block using a block header. When the proof is done using a smart contract (§VII-B), the rational miners cannot distinguish the attacker’s block due to the lack of identifying details in the contract.

**Changing the Limited Supply Mechanism** As we mentioned in §VII-C, the Bitcoin has a limited supply. Which means that the block reward is declining, until it reaches 0 sometime in the future. Nakamoto’s design states that the block reward should halve every 210,000 blocks. This means that the block reward behaves as a step function, with sudden changes. We saw in eq. (3) that by reducing the block reward by two, we cause the $\omega^b_i$ also to drop by two immediately. In case that the new profitability for $P_i$ holds $\omega^b_i < 1$, this causes her to stop mining immediately upon the drop. The situation worsens in case there is an attacker who takes advantage of the drop and initiates an attack. The new block reward can cause the $\omega^b_i$ to cross the threshold for a successful attack, for most if not all the miners in the system. Although this issue can be mitigated by changing the limited supply mechanism. One solution would be to decrease the block reward with smaller steps and more frequently, which would allow to keep a limited supply with smaller drops of $\omega^b_i$. Another solution would be to change the difficulty more frequently, similarly to Bitcoin’s ABC difficulty adjustment mechanism [93]. We believe that such solutions are critical for the security of the system.

**IX. Conclusion**

We present BDoS, the first Blockchain denial-of-service attack that uses incentive manipulation. BDoS sabotages the incentive mechanism behind Nakamoto’s consensus by proving the attacker has achieved an advantage in mining without releasing her complete block. Such proof reduces miners’ incentive to mine down to a negative mining profit. Thus, rational profit-driven miners would necessarily cease mining. We show that cryptocurrencies based on Nakamoto’s blockchain are valuable to BDoS under realistic settings, and propose a mitigation.

The header-only publication capability we present is a realistic extension of the standard model under which blockchain protocols are typically analyzed. This could open the door to study new equilibria and strategies where a miner manipulates the system to increase her revenue rather than sabotage the system.

Additionally, alternative incentive-based DoS attacks may be exist, possibly more efficient than BDoS. General bounds and mitigations are necessary to assure the security of blockchain protocols.
how-the-winner-got-fomo3d-prize-a-detailed-explanation-b30a697813f, Aug 2018.

[53] A. Judmayer, N. Stifter, P. Schindler, and E. Weippl, “Pitchforks in cryptocurrencies: Enforcing rule changes through offensive forking and,”

[54] A. Miller, “Feather-forces: enforcing a blacklist with sub-50% hash power,” https://bitcointalk.org/index.php?topic=312668.0, Oct 2013.

[55] F. Winzer, B. Herder, and S. Faust, “Temporary censorship attacks in the presence of rational miners,” in 2019 IEEE European Symposium on Security and Privacy Workshops (EuroS&PW), pp. 357–366, IEEE, 2019.

[56] Y. Gilad, R. Hemo, S. Micali, G. Vlachos, and N. Zeldovich, “Algorand: Scaling byzantine agreements for cryptocurrencies,” in Proceedings of the 26th Symposium on Operating Systems Principles, pp. 51–68, ACM, 2017.

[57] A. Kiayias, A. Russell, B. David, and R. Oliynykov, “Ouroboros: A provably secure proof-of-stake blockchain protocol,” in Annual International Cryptology Conference, pp. 357–388, Springer, 2017.

[58] I. Bentov, R. Pass, and E. Shi, “Snow white: Provably secure proofs of stake,” IACR Cryptology ePrint Archive, vol. 2016, p. 919, 2016.

[59] P. Daian, R. Pass, and E. Shi, “Snow white: Robustly reconfigurable consensus and applications to provably secure proof of stake,” in International Conference on Financial Cryptography and Data Security, pp. 23–41, Springer, 2019.

[60] B. David, P. Gázi, A. Kiayias, and A. Russell, “Ouroboros proasis: An adaptively-secure, semi-synchronous proof-of-stake blockchain,” in Annual International Conference on the Theory and Applications of Cryptographic Techniques, pp. 66–98, Springer, 2019.

[61] C. Badertscher, P. Gázi, A. Kiayias, A. Russell, and V. Zikas, “Ouroboros genesis: Composite proof-of-stake blockchains with dynamic availability,” in Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, pp. 913–930, ACM, 2018.

[62] F. Zhang, I. Eyal, R. Escriva, A. Juels, and R. Van Renesse, “{REM}: Resource-efficient mining for blockchains,” in 26th USENIX Security Symposium (USENIX Security 17), pp. 1427–1444, 2017.

[63] L. Chen, L. Xu, N. Shah, Z. Gao, Y. Lu, and W. Shi, “On security analysis of proof-of-elapsed-time (poet),” in International Symposium on Stabilization, Safety, and Security of Distributed Systems, pp. 282–297, Springer, 2017.

[64] S. Wilkinson, T. Boshevski, J. Brandoff, and V. Buterin, “Storj a peer-to-peer cloud storage network,” 2014.

[65] S. Wilkinson, J. Lowry, and T. Boshevski, “Metadisk a blockchain-based decentralized file storage application,” Tech. Rep., 2014.

[66] E. Cechetti, I. Miers, and A. Juels, “Plex: Public incompressible encodings for decentralized storage,” IACR Cryptology ePrint Archive, vol. 2018, p. 684, 2018.

[67] A. Miller and J. J. LaViola Jr, “Anonymous byzantine consensus from moderately-hard puzzles: A model for bitcoin,” Available on line: http://nakamotoinstitute.org/research/anonymous-byzantine-consensus, 2014.

[68] J. Garay, A. Kiayias, and N. Leonardos, “The bitcoin backplane protocol: Analysis and applications,” in Annual International Conference on the Theory and Applications of Cryptographic Techniques, pp. 281–310, Springer, 2015.

[69] R. Pass, L. Seeman, and A. Shelat, “Analysis of the blockchain protocol in asynchronous networks,” in Annual International Conference on the Theory and Applications of Cryptographic Techniques, pp. 643–673, Springer, 2017.

[70] D. Fudenberg and J. Tirole, “Game theory, 1991,” Cambridge, Massachusetts, vol. 393, no. 12, p. 80, 1991.

[71] R. J. Aumann, “Agreeing to disagree,” The annals of statistics, pp. 1236–1239, 1976.

[72] A. Gervais, G. O. Karam, K. Wüst, V. Glykantzis, H. Ritzdorf, and S. Capkun, “On the security and performance of proof of work blockchains,” in Proceedings of the 2016 ACM SIGSAC conference on computer and communications security, pp. 3–16, ACM, 2016.

[73] Y. Sompolinsky and A. Zohar, “Bitcoin’s underlying incentives,” Communications of the ACM, vol. 61, no. 3, pp. 46–53, 2018.

[74] “A Hash Pool.” https://www.ahashpool.com/, 2017.

[75] “Mining pool hub.” https://miningpoolhub.com/.

[76] “MultiPoolMiner.” https://github.com/MultiPoolMiner/MultiPoolMiner, 2019.

[77] “SmartMine – an intelligent way to mine cryptocurrency.” https://www.smartmine.org/.

[78] SFOX, “Bitcoin Cash vs. Bitcoin SV: Six Months after the Hash War.” https : / / blog . sfox . com / bitcoin-cash-vs-bitcoin-sv-six-months-after-the-hash-war-ef6d92af3b891, Jun 2019.

[79] A. Spiegelman, I. Keidar, and M. Tennenholz, “Game of coins,” arXiv preprint arXiv:1805.08979, 2018.

[80] Y. Kwon, H. Kim, J. Shin, and Y. Kim, “Bitcoin vs. Bitcoin Cash: Coexistence or downfall of Bitcoin Cash?,” arXiv preprint arXiv:1902.11064, 2019.

[81] M. Corallo, “Bip 152: compact block relay.” See https://github.com/bitcoin/bips/blob/master/bip-0152, mediawiki, 2016.

[82] G. Huberman, J. Leshno, and C. C. Moallemi, “An economic analysis of the bitcoin payment system,” Columbia Business School Research Paper, no. 17-92, 2019.

[83] W. Zhao, "Bitcoin miners halt operations as rainstorm triggers mudslides in china," Aug 2019.

[84] M. Adham, “What will the next ‘halving’ mean for the price of bitcoin?” https : / / www . forbes . com / sites / forbesfinancecouncil / 2019/05/10/what-will-the-next-halving-mean-for-the-price-of-bitcoin/ #1e695e225f34, May 2019.

[85] C. Bendiksen, S. Gibbons, and E. Lim, “The bitcoin mining network-trends, marginal creation cost, electricity consumption & sources,” CoinShares Research, vol. 21, 2018.

[86] C. Bendiksen, S. Gibbons, and E. Lim, “The bitcoin mining network-trends, marginal creation cost, electricity consumption & sources,” CoinShares Research, 2019.

[87] “Miners profitability.” https://www.asicminervalue.com/.

[88] A. Day and C. Bookman, “Bitcoin in bigquery: blockchain analytics on public data.” https://cloud.google.com/blog/products/gcp/bitcoin-in-bigquery-blockchain-analytics-on-public-data, 2018.

[89] “Difficulty.” https://en.bitcoin.it/wiki/Difficulty, 2017.

[90] B. Research, “The price crash & the impact on miners.” https://blog.bitmex.com/the-price-crash-the-impact-on-miners/, Dec 2018.

[91] BTC.com, “Pool distribution.” https://btc.com/stats/pool?pool_mode=week, Oct 2019.

[92] “Bitcoin Cash ABC vs. Bitcoin Cash SV – examining the Bitcoin Cash hash war.” https : / / bitcoinist . com / bitcoin-cash-abc-vs-bitcoin-cash-sv-examining-the-bitcoin-cash-hash-war/, Dec 2018.

[93] Bitcoin ABC, “Difficulty Adjustment Algorithm Update.” http://www.bitcoinnabc.org/2017-11-01-DAU/.

[94] S. Lerner, “Uncle mining, an ethereum consensus protocol flaw,” Bitslog blog, Apr. 2016.

[95] Y. Sompolinsky and A. Zohar, “Secure high-rate transaction processing in bitcoin,” in International Conference on Financial Cryptography and Data Security, pp. 507–527, Springer, 2015.

[96] “Bitmain. https : / / shop . bitmain . com / product / detail ? pid = 00020191023161554895eHhxL0TO6C2.

[97] “Bitmain Launches Low-Cost Special Edition Antminer S9.” https : / / news . bitcoin . com / bitmain-launches-low-cost-special-edition-antminer-s9/, June 2019.
APPENDIX A
CHANGING ACTION IN THE MIDDLE OF THE ROUND

In the model, we assumed that no rational miner changes her action in the middle of the round. We now justify this assumption. As mentioned earlier, the coin price is assumed to be constant during the entire game. Therefore, the honest game profitability factor \( \omega^i_b \) of \( P_i \) keeps its value constant during the round. In addition we assume that no miner withhold blocks. We define as \( \text{Time}_j \) the time when round \( j \) ends and round \( j + 1 \) starts.

Claim A.1. If \( P_i \) chooses an action \( a \) in the beginning of round \( j \) (\( \text{Time}_{j-1} \)), she does not gain anything from changing her action for all \( t \) that hold \( \text{Time}_{j-1} < t < \text{Time}_j \).

Proof. We know that the rational miner \( P_i \) chose the most beneficial action \( a \) in the beginning of round \( r \), assume by contradiction that it is beneficial for \( P_i \) to change her action in time \( t_1 \) that holds \( \text{Time}_{j-1} < t_1 < \text{Time}_j \) to a different action \( a' \) s.t. \( a \neq a' \). Previous works showed that new block appearance in the system is modeled with Poisson distribution, with the time between blocks correspond to exponential distribution [95]. One of the properties of this distribution is that it is memoryless. Since \( P_i \) has the same probability of finding a new block as she had in the beginning of the round (and so do other players), she has the same expected revenue from each action. If changing a action in the middle of a round is profitable, this implies that changing a action was also beneficial in the beginning of the round. This is contradiction to the fact that \( P_i \) is rational and chose the best action in the beginning of the round. \( \square \)

Note that for memorylessness we had to assume that there is no block withholding in the system, i.e., in every point during the round, it is known by everyone that there was no new block mined, by any miner, since the beginning of the round. For example, this assumption does not hold when there is an active selfish mining attack [5]. Although, it is reasonable to assume that no miner is withholding blocks during the attack as there is no evidence of cases selfish mining attacks on Bitcoin.

APPENDIX B
COST OF 51% ATTACK

We show our calculation for the cost of 51% attack. At the moment of writing this paper the total hash rate of Bitcoin is roughly 100,000,000 TH/s. The most advance mining equipment is believed to be Bitmain S17 Pro which has hashrate of 53 TH/s and power consumption of 2.094 kWh [96]. The official cost of a unit is $2128. Another widely used ASIC machine, which is significantly cheaper to acquire is Bitmain S9 SE [97]. The hash rate of this machine is 16 TH/s, its power consumption is 1.280 kWh and unit price $350. The number of S17 Pro rigs required to have the majority of hash rate in the network is: \( \lceil \frac{100,000,000}{53} \rceil = 1,886,793 \). With total cost of 1,886,793 · 2,128 = $4,015,095,504 and power consumption of 1,886,793 · 2.094 = 94,822,669 kWh which with electricity price of 0.037 $/kWh would cost $3,508,438 a day. Similarly for S9 SE, the equipment cost would be $2,187,500,000 and the daily electricity cost of $7,104,000.

APPENDIX C
PSEUDO-CODE FOR MODEL

In this section we describe the pseudo code for the attack described in § IV and additionally the allowed interaction between players and scheduler that are described in § III-A. Note that for simplicity of the pseudocode we denote the hash rate of rational miner \( P_i \) as \( \alpha P_i \), as well, so \( \alpha P_i := \alpha_i \).

Algorithm 1 Scheduler

1: \( r \leftarrow 0 \)
2: 
3: loop // The scheduler runs in an infinite loop.
4: \( r \leftarrow r + 1 \)
5: \( \text{active} \leftarrow \emptyset \)
6: for \( p \in \{A, P_1, \ldots, P_n\} \) do
7: if \( p.\text{Mine}_\text{This}_\text{Round} = \text{true} \) then
8: template \( \leftarrow p.\text{Get}_\text{Block}_\text{Template} \)
9: \( \text{active} \leftarrow \text{active} \cup \{p\} \)
10: end if
11: end for
12: \( T \leftarrow \text{Exp}_\text{Distribution}(\lambda \cdot \sum_{p \in \text{active}} \alpha_p) \)
13: \( \text{sleep}(T) \)/ Simulate block time.
14: \( w \leftarrow \text{Sample by weight of hashrate from active} \)
15: \( \text{B}_w \leftarrow \text{Generate}_\text{Valid}_\text{Block}(r, \text{template}_w) \)
16: if \( w = A \) then
17: publish, withheld \( \leftarrow A.\text{Find}_\text{New}_\text{Block}(\text{B}_w) \)
18: for \( p \in \{A, P_1, \ldots, P_n\} \) do
19: \( \text{Send}_\text{ Blocks}(p, \text{published}) \)
20: end for
21: end if
22: if publish = “header” then
23: \( H = \text{Get}_\text{Header}(\text{B}_w) \)
24: for \( p \in \{P_1, \ldots, P_n\} \) do \( p.\text{Add}_\text{Header}(H) \)
25: end if
26: else if publish = “full block” then
27: for \( p \in \{A, P_1, \ldots, P_n\} \) do \( p.\text{Add}_\text{Block}(\text{B}_w) \)
28: end if
29: else
30: competing \( \leftarrow A.\text{Get}_\text{Competing}_\text{Blocks}(B_w) \)
31: if competing empty = true then
32: for \( p \in \{A, P_1, \ldots, P_n\} \) do \( p.\text{Add}_\text{Block}(B_w) \)
33: else
34: \( \text{Send}_\text{ Blocks}(w, \{B_w\} + \text{competing}) \)
35: \( \text{Send}_\text{ Blocks}(A, \text{competing} + \{B_w\}) \)
36: for \( p \in \{P_1, \ldots, P_n\} \backslash \{w\} \) do
37: with probability \( \frac{1}{1 - \alpha \cdot \alpha P_i} \):
38: \( \text{Send}_\text{ Blocks}(p, \text{competing} + \{B_w\}) \)
39: with probability \( \frac{\alpha}{1 - \alpha \cdot \alpha P_i} \):
40: \( \text{Send}_\text{ Blocks}(p, \{B_w\} + \text{competing}) \)
41: end for
42: end if
43: end loop
44: 
45: function \( \text{Send}_\text{ Blocks}(p, \text{blocks}) \)
46: for \( B \in \text{blocks} \) do \( p.\text{Add}_\text{Block}(B) \)
47: end function

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Algorithm 2 Adversary $\mathcal{A}$

1: $L_A \leftarrow \{B_0\}, O_A[B_0] \leftarrow 0, r \leftarrow 0$
2: $B_{\text{withheld}} \leftarrow \bot, B_{\text{extend}} \leftarrow B_0$
3:
4: function Mine_This_Round
5: $r \leftarrow r + 1$
6: if $B_{\text{withheld}} = \bot$ then
7: return true
8: else
9: return false
10: end if
11: end function
12:
13: function Get_Block_Template
14: return Generate_Template($\mathcal{A}$, Get_Header($B_{\text{extend}}$))
15: end function
16:
17: function Find_New_Block($B$)
18: if Get_Miner($B_{\text{extend}}$) = $\mathcal{A}$ and $B_{\text{withheld}} = \bot$ then
19: return “full block”, []
20: else
21: $B_{\text{tmp}} \leftarrow B_{\text{withheld}}, B_{\text{withheld}} \leftarrow B$
22: return “header”, [$B_{\text{tmp}}$]
23: end if
24: end function
25:
26: function Get_Competing_Blocks($B$)
27: if Get_Height($B$) = Get_Height($B_{\text{withheld}}$) then
28: return [$B_{\text{withheld}}$]
29: else
30: return []
31: end if
32: end function
33:
34: function Add_Block($B$)
35: $L_A \leftarrow L_A \cup \{B\}, O_A[B] \leftarrow |L_A|$
36: if $B = B_{\text{withheld}}$ then
37: $B_{\text{withheld}} \leftarrow \bot$
38: end if
39: if Get_Height($B$) > Get_Height($B_{\text{extend}}$) then
40: $B_{\text{extend}} \leftarrow B$
41: end if
42: end function

Algorithm 3 Rational Player $\mathcal{P}_i$

1: $L_i \leftarrow \{B_0\}, O_i[B_0] \leftarrow 0, r \leftarrow 0$
2: $B_{\text{header}} \leftarrow \bot, B_{\text{extend}} \leftarrow B_0$
3: $M \leftarrow \text{Get_Best_Strategy}(\text{BDDoS-Mine}, \alpha_i, \omega_i)$
4:
5: function Mine_This_Round
6: $r \leftarrow r + 1$
7: if $M[L_i][O_i] = \text{stop}$ then
8: return false
9: else
10: return true
11: end if
12: end function
13:
14: function Get_Block_Template
15: if $M[L_i][O_i] = \text{mineSPV}$ then
16: return Generate_Template($\mathcal{P}_i$, Get_Header($B_{\text{header}}$))
17: else if $M[L_i][O_i] = \text{mineMain}$ then
18: return Generate_Template($\mathcal{P}_i$, Get_Header($B_{\text{extend}}$))
19: end if
20: end function
21:
22: function Add_Block($B$)
23: $L_A \leftarrow L_A \cup \{B\}, O_A[B] \leftarrow |L_A|$
24: if Get_Header($B$) = Get_Header($B_{\text{header}}$) then
25: $B_{\text{header}} \leftarrow \bot$
26: end if
27: if Get_Height($B$) > Get_Height($B_{\text{extend}}$) then
28: $B_{\text{extend}} \leftarrow B$
29: end if
30: end function
31:
32: function Add_Header($H$)
33: $B_{\text{header}} \leftarrow (H, \bot)$
34: end function