ERRATA

Erratum to: BOGOLIUBOV’S CAUSAL PERTURBATIVE QED AND WHITE NOISE. INTERACTING FIELDS

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Lemmas 1 and 2 (Theoretical and Mathematical Physics, Vol. 211, No. 3, pp. 792–793 and 795–796) must be stated as follows.

**Lemma 1.** Let \( d \in S(\mathbb{R}^4; \mathbb{C})^* \), and let

\[
\kappa_{l,m} \in \mathcal{L}(\mathcal{E}, (E_{i_1} \otimes \cdots \otimes E_{i_l+m})^*) \cong \mathcal{L}(E_{i_1} \otimes \cdots \otimes E_{i_l+m}, \mathcal{E}^*),
\]

with the kernel

\[
\kappa_{l,m} = (\kappa_{l_1,m_1}^{n_1}) \otimes \cdots \otimes (\kappa_{M,M}^{n_M})
\]

corresponding to the Wick product (at the same space–time point \( x \))

\[
\Xi_{l,m}(\kappa_{lm}(x)) = \Xi_{l_1,m_1}(\kappa_{l_1,m_1}(x)) \cdots \Xi_{M,M}(\kappa_{M,M}(x)):
\]

of the integral kernel operators \( \Xi_{l_1,m_1}(\kappa_{l_1,m_1}(x)) \).

Let the integral kernel \( d \ast \kappa_{l,m} \) be equal to

\[
(d \ast \kappa_{l,m}(\xi_{i_1} \otimes \cdots \otimes \xi_{i_l+m}), \phi) = \int_{\mathbb{R}^4} d \ast \kappa_{lm}(\xi_1, \ldots, \xi_{l+m})(x)\phi(x) d^4x \times
\]

\[
\times \int_{\mathbb{R}^4 \times \mathbb{R}^4} d(x - y)\kappa_{l,m}(w_1, \ldots, w_{i_l+m}; y)\xi_1(w_1), \ldots, \xi_{i_l+m}(w_{i_l+m})\phi(x) dw_1 \ldots dw_{i_l+m} d^4y d^4x,
\]

where \( \xi_{ik} \in E_{ik}, \phi \in \mathcal{E} \) and \( \mathcal{E} = S(\mathbb{R}^4; \mathbb{C}) \) or \( \mathcal{E} = S_0(\mathbb{R}^4; \mathbb{C}) \).

Then

1. If the convolution \( d_n \ast d_{n-1} \ast \cdots \ast d_1 \ast \kappa_{l,m} \) exists, then it is continuous, i.e.,

\[
d_n \ast d_{n-1} \ast \cdots \ast d_1 \ast \kappa_{l,m} \in \mathcal{L}(E_{i_1} \otimes \cdots \otimes E_{i_l+m}, \mathcal{E}^*),
\]

provided

\[
\kappa_{l,m} = (\kappa_{l_1,m_1}^{n_1}) \otimes \cdots \otimes (\kappa_{M,M}^{n_M}), \quad l + m = M,
\]

and each \( d_i \) is equal to the product of pairings or to the retarded or advanced part of the causal combinations of products of pairings and \( M > 1 \), which we encounter as higher-order contributions to interacting fields in spinor QED.

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2. Let, moreover, in the case \( M = 1 \), \( \kappa_{1,m}^{n_1} = \kappa_{0,1}, \kappa_{1,0} \) be equal to the kernel of a free field with a mass \( m_{i_1} \). If further among the distributions \( d_n, d_{n-1}, \ldots, d_1 \) there are no (retarded or advanced parts of the) commutation functions of a free field of the mass \( m_2 = m_{i_1} \), then the convolutions

\[
d_n \ast \cdots \ast d_1 \ast \kappa_{0,1}(\xi), \quad d_n \ast \cdots \ast d_1 \ast \kappa_{1,0}(\xi), \quad \xi \in E,
\]

are well-defined and

\[
d_n \ast \cdots \ast d_1 \ast \kappa_{0,1}, \quad d_n \ast \cdots \ast d_1 \ast \kappa_{1,0} \in \mathcal{L}(E_{i_1}, \mathcal{E}^*) .
\]

3. If \( \kappa_{1,m}^{n_1} = \kappa_{0,1}, \kappa_{1,0} \) is the kernel of a free field with the mass not equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to \( d \), then the convolutions

\[
d \ast \kappa_{0,1}, \quad d \ast \kappa_{1,0} \in \mathcal{L}(E_{i_1}^*, \mathcal{E}^*) = \mathcal{L}(\mathcal{E}, E_{i_1}) \subset \mathcal{L}(E_{i_1}, E^*)
\]

are well-defined.

4. If \( \kappa_{1,m}^{n_1} = \kappa_{0,1}, \kappa_{1,0} \) is the kernel of a free field with the mass equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to \( d \), then the convolutions \( d \ast \kappa_{0,1} \) and \( d \ast \kappa_{1,0} \) are not well-defined.

**Lemma 2.** The following statements hold.

1. Let \( d_i \) be equal to the product of pairings or to the retarded or advanced part of the causal combinations of products of pairings and \( M > 1 \), which we encounter as the kernels of higher-order contributions to interacting fields in spinor QED and with the “natural” splitting of the causal distributions in the computation of the scattering operator. Assume that the convolution \( d_n \ast d_{n-1} \ast \cdots \ast d_1 \ast \kappa_{1,m} \) exists. Then the operator

\[
d_n \ast \cdots \ast d_1 \ast \Xi_{l,m}(\kappa_{l,m})(x) = \int_{[\mathbb{R}^4]^n} d_n(x - y_n)d_{n-1}(y_n - y_{n-1}) \cdots d_1(y_2 - y_1)\Xi_{l,m}(\kappa_{l,m}(y_1)) d^4y_1 \cdots d^4y_n = \Xi_{l,m}\left( \int_{[\mathbb{R}^4]^n} d_n(x - y_n)d_{n-1}(y_{n-1} - y_{n-2}) \cdots d_1(y_2 - y_1)\kappa_{l,m}(y_1) d^4y_1 \cdots d^4y_n \right) = \Xi_{l,m}(d_n \ast \cdots \ast d_1 \ast \kappa_{l,m}(x))
\]

defines an integral kernel operator

\[
\Xi_{l,m}(d_n \ast \cdots \ast d_1 \ast \kappa_{l,m}) \in \mathcal{L}((\mathcal{E}) \otimes \mathcal{E}, (\mathcal{E})^*) \cong \mathcal{L}(\mathcal{E}, \mathcal{L}((\mathcal{E}), (\mathcal{E})^*))
\]

with the vector-valued kernel

\[
d_n \ast \cdots \ast d_1 \ast \kappa_{l,m} \in \mathcal{L}(\mathcal{E}, (E_{i_1} \otimes \cdots \otimes E_{i_{l+m}})^*) \cong \mathcal{L}(E_{i_1} \otimes \cdots \otimes E_{i_{l+m}}, \mathcal{E}^*) .
\]
2. Let, moreover, for the higher-order contributions, in the case $M = 1$, $\kappa_{\ell_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ be equal to the kernel of a free field with a mass $m_{i_1}$. If further among the distributions $d_n, d_{n-1}, \ldots, d_1$ there are no (retarded or advanced parts of the) commutation functions of a free field of the mass $m_2 = m_{i_1}$, then

\[
d_n \ast \ldots \ast d_1 \ast \Xi_{0,1}(\kappa_{0,1})(x) = \\
= \int_{[\mathbb{R}^4]^n} d_n(x - y_n)d_{n-1}(y_n - y_{n-1}) \ldots d_1(y_2 - y_1)\Xi_{0,1}(\kappa_{0,1}(y))d^4y_1 \ldots d^4y_n = \\
= \Xi_{0,1}\left(\int_{[\mathbb{R}^4]^n} d_n(x - y_n)d_{n-1}(y_n - y_{n-1}) \ldots d_1(y_2 - y_1)\kappa_{0,1}(y)d^4y_1 \ldots d^4y_n\right) = \\
= \Xi_{0,1}(d_n \ast \ldots \ast d_1 \ast \kappa_{0,1}(x))
\]

defines an integral kernel operator

\[
\Xi_{0,1}(d_n \ast \ldots \ast d_1 \ast \kappa_{lm}) \in \mathcal{L}(E \otimes \mathcal{E}, (E)^*) \cong \mathcal{L}(\mathcal{E}, \mathcal{L}((E), (E)^*))
\]

with the vector-valued kernel

\[
d_n \ast \ldots \ast d_1 \ast \kappa_{0,1} \in \mathcal{L}(E_{i_1}, \mathcal{E}^*)
\]

and similarly for the kernel $\kappa_{1,0}$.

3. If, moreover, in the case $M = 1$, $\kappa_{\ell_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ is the kernel of a free field with the mass not equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to $d$, then the integral kernel operators

\[
d \ast \Xi_{0,1}(\kappa_{0,1}) = \Xi_{0,1}(d \ast \kappa_{0,1}), \\
d \ast \Xi_{1,0}(\kappa_{1,0}) = \Xi_{0,1}(d \ast \kappa_{1,0})
\]

are well-defined and belong to

\[
\mathcal{L}(\mathcal{E}, E_{i_1}) = \mathcal{L}(E_{i_1}^*, \mathcal{E}^*) \subset \mathcal{L}(E_{i_1}, \mathcal{E}^*)
\]

4. If $\kappa_{\ell_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ is the kernel of a free field with the mass equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to $d$, then the integral kernel operators

\[
d \ast \Xi_{0,1}(\kappa_{0,1}) = \Xi_{0,1}(d \ast \kappa_{0,1}), \\
d \ast \Xi_{1,0}(\kappa_{1,0}) = \Xi_{0,1}(d \ast \kappa_{1,0})
\]

are not well-defined.

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