Cross-Channel Intragroup Sparsity Neural Network

Zhilin Yu1*, Chao Wang2*, Qing Wu1, Yong Zhao2, Xundong Wu1
1 Hangzou Dianzi University
2 Peking University
wuxundong@gmail.com

Abstract

Modern deep neural network models generally build upon heavy over-parameterization for their exceptional performance. Network pruning is one often employed approach to obtain less demanding models for their deployment. Fine-grained pruning, while can achieve good model compression ratio, introduces irregularity in the computing data flow, often does not give improved model inference efficiency. Coarse-grained model pruning, while allows good inference speed through removing network weights in whole groups, for example, a whole filter, can lead to significant model performance deterioration. In this study, we introduce the cross-channel intragroup (CCI) sparsity structure that can avoid the inference inefficiency of fine-grained pruning while maintaining outstanding model performance.

1 Introduction

The exceptional performance of deep neural networks in many computer vision tasks has set off the trend of deploying them for challenging real-world tasks. While it is desirable to employ networks of the best performance, superior performing networks are often associated with high computing complexity as the result of adopting very wide/deep configurations. Excessively high computing complexity is at least undesirable as it translates into high cost and latency. For some platforms, such as mobile devices, such a high complexity is simply beyond their reach.

Various approaches have been proposed to obtain lightweight networks while maintaining desirable model performance (Han et al. 2015; Howard et al. 2017). Network pruning removes redundant network components to reduce model complexity. Pruning can be fine-grained, in the sense individual weights are selected for removal (Han et al. 2015). Or it can be structured, that is weights are removed in larger units, such as a whole channel or blocks of weights inside a filter (Mao et al. 2017). Fine-grained pruning usually gives a good performance, though often does not improve inference efficiency due to the data accessing irregularity associated with those structures. Structured pruning, or coarse-grained pruning, can be constructed to give improved inference efficiency, however, often lead to worse performance (Mao et al. 2017).

In this study, we introduce a novel structure that is designed to avoid the irregularity in the inbound dataflow of the sparse network layer inference process. We entitle it as cross-channel intragroup sparsity or CCI-Sparsity as shown in Fig. 1. Differ from the typical fine-grained sparse network, in a network of CCI-Sparsity, we divide network weights in each network layer into small groups and perform active weight pruning to reach a fixed number of non-zero weights for each group. The weight groups are arranged to contain weights from contiguous network output channels of the same layer. Our experiment shows that the weight group size can be kept rather small (typical group size of 8, 16 weights) without significantly affecting network performance. Our analysis shows that the proposed CCI-Sparsity structure can remove data inflow irregularity that is associated with the ordinary fine-grained sparse networks yet does not suffer from the extent of accuracy loss of structural pruning. With proper hardware construction, CCI-Sparsity should enable network inference to speed up linearly with the model sparsity level.

![Figure 1](image_url)

(a) Weight matrix before pruning. Each row of the weight matrix contains weights for one output channel. (b) Weight matrix pruned with CCI-Sparsity structure. In this case four weights (illustrated as a block of same color) from four neighboring channels form one weight group. That is, we have a group size of 4, and there are exactly two nonzero weights per group, that is sparsity level of 50 percent.

Contributions: Overall this work makes the following contributions.

*Equal contribution
(1) We propose the CCI-Sparsity structure, which can enable efficient sparse neural network inference while maintaining the performance advantage of the fine-grained sparsity over structured pruning.

(2) We perform the theoretical analysis on how the weight group size and sparsity level would affect post-pruning network performance with CCI-Sparsity.

(3) We give a solution that can overcome the difficulty for training networks with CCI-Sparsity structure of small weight group size. Our approach outperforms the standard iterative model pruning approach on CCI-sparsity networks.

2 Related Work

Many different approaches have been explored to reduce the inference cost of neural networks. In this part, we discuss topics that are very related to this study.

2.1 Lightweight CNN architectures

Convolutional neural networks (CNNs) are inherently sparse in that each convolutional filter only receive inputs from a small neighborhood, which gives CNNs a great advantage over fully connected networks, for image processing, in both efficiency and performance. Still, in some widely used CNN models, such as the AlexNet, VGG network, convolutional filters receive inputs of a rather high dimensionality and embody significant redundancy.

Tensor decomposition curtails the redundancy in network models through approximating network filters with low-rank tensors, thereby reducing model complexity. Recent works such as MobileNet (Howard et al. 2017), MobileNetV2 (Sandler et al. 2018), decompose convolutional layers into filters on different spatial dimensions. Specifically, they use the composition of depthwise filters and pointwise filters to replace the regular convolutional filters, thereby reducing the computation complexity. The group convolution (Ma et al. 2018; Gao, Wang, and Ji 2018; Huang et al. 2018; Zhang et al. 2017) goes one step further by separating convolution operations into groups, in effect remove the weight connections between filters that are not in the same group.

A CCI-Sparsity network, when we employ it to sparsify a network layer, can be considered as an upgraded version of group convolution that with the advantage of reduced input dimensionality while not suffering from over-restrain caused by separating input into groups.

2.2 Sparse neural networks

Reducing network redundancy through pruning nonessential model parts is routinely used to construct sparse or compact networks. Unstructured pruning removes individual network weights that meet certain criteria (Han et al. 2015) and generates networks of fine-grained sparsity. Unstructured pruning approaches can be rather efficient in removing redundant network weights; thus can greatly reduce the model sizes. However, due to their unstructured nature, those models can often be rather inefficient when used for inference.

Structured pruning removes network connections while conforming to specific schematics that would allow efficient model inference. Those approaches, instead of removing individual weights, removes contiguous connections in blocks. The unit of pruning block can be a filter, channel or blocks of weights inside filters (Li et al. 2016; Wen et al. 2016; Mao et al. 2017; Narang, Undersander, and Diamos 2017). However, those structured pruning procedures can often lead to reduced model accuracies (Mao et al. 2017; Yao et al. 2019).

2.3 Intragroup sparsity

Intragroup sparsity (Zhou, Jin, and Hoi 2010) is originally proposed as a form of model regularization, where the $\ell_{1,2}$-norm is used to induce sparsity at the intragroup level under the context of model feature selection. In Wu et al. (Wu et al. 2018), the authors propose the concept of adopting intragroup sparsity structure to overcome the irregularity associated with their dendritic neural networks. In their study, the weight group is constructed across their dendritic sub-kernels with fixed connection maps. Their proposed intragroup sparsity structure mostly remains as a concept and no detailed analysis is given. In this study, weight groups are constructed across network channels with learned connection maps. Another related study is Balanced-Sparsity (Yao et al. 2019), where the authors also propose to use weight groups, to overcome the computing inefficiency associated with unstructured sparsity. Differ from CCI-Sparsity, the weight group in (Yao et al. 2019) is constructed through partitioning weights inside a kernel instead of across channels, and the group size used there are much larger than this study. We give more discussion on this in Sec. 3.3.

3 CCI-Sparsity

For a typical fine-grained sparse neural network, the pruning process clamps portion of network weights to zero based on certain weight-wise local criteria. In this way, we obtain a sparse neural network with reduced memory and computing complexity. However, the post-pruning networks attained through such a process are unstructured which are known to lead to inefficient model inference (Han et al. 2016; Parashar et al. 2017; Zhang et al. 2016; Zhou et al. 2018; Cao et al. 2019; Mao et al. 2017).

3.1 CCI-Sparsity structure

To address the computing inefficiency associated with fine-grained sparse neural network models, in this study, we propose CCI-Sparsity. CCI-Sparsity is compatible with either convolutional or fully connected networks. For clarity, we use a fully connected neural network layer as an example to explain the concept. For a typical fully connected layer, we denote the layer input as a column vector $X \in \mathbb{R}^N$. With network weights as a matrix $W \in \mathbb{R}^{M \times N}$, layer output $H \in \mathbb{R}^M$, we have $H = W \times X$. Each row of $W$ illustrated in Fig. 1a contains weights for one output channel in $H$. For the corresponding network layer with CCI-Sparsity, with a weight group size of $G$ and $s$ nonzero weights available per group as illustrated in Fig. 1b. For our example in Fig. 1, we have $M = 8, N = 5, G = 4, s = 2$. We denote the sparse matrix in Fig. 1b as $W$. We can compress every $G = 4$ rows
in \( \hat{W} \) with \( s = 2 \) rows of new format of weights. Each new weight row will have the same number of columns as the original weight matrix.

To identify which row a weight of \( \hat{W} \) is associated with in \( W \) we also need a row index for each weight in the compressed matrix (This index also tells us which row in \( H \) should we add the multiplication result to). For this example of \( G = 4 \), we need 2 bits index for each weight. With \( G = 4, s = 2 \), we need \( 2 \times 2 = 4 \) index bits for each weight group (Even though the theoretically index bits is slightly less, it is probably better to stay with the plain index to avoid extra decoding overhead.). For an arbitrary group size of \( G \), a plain weight index will be \( \log_2 G \) bits. Apparently, we would prefer to have a small group size, to reduce the overhead comes with the extra storage and I/O requirement associated with weight indexes. But we also show in Sec. 5, smaller group size can lead to a lowered model accuracy.

### 3.2 Efficient inference for CCI-Sparsity

To understand what advantage CCI-Sparsity can afford us for efficient neural network inference. We use a standard matrix-matrix multiplication case \( H = W \times X \) for illustration, with weight matrix \( W \), input \( X \), multiplication result \( H \). Matrix multiplication of this form is often called General Matrix Multiply (GEMM), which is the core computation routine used in neural network inference, for both convolutional neural networks and fully connected neural networks (Chetlur et al. 2014). Without losing generality, we have \( W, X, H \in \mathbb{R}^{N \times N} \) in their dense form as shown in Fig. 2a. We give a naive form of the GEMM as in the following algorithm.

**Algorithm 1 Dense matrix multiplication**

**Require:** \( W, X \in \mathbb{R}^{N \times N}, H = 0 \).

**Ensure:** \( H = W \times X \)

1. \( i \leftarrow 0 \) to \( N \) by 1
2. \( j \leftarrow 0 \) to \( N \) by 1
3. \( k \leftarrow 0 \) to \( N \) by 1
   
   \[ H(i, j) = H(i, j) + W(i, k) \times X(k, j) \]

The number of computation steps of such naive procedure for this case is \( N^3 \) steps. When we compress weight matrix \( W \) into CCI-Sparsity format \( \hat{W} \), in this case, assume group size \( G = 4 \) and \( s = 1 \), we reduce the size of weight matrix by \( G/s = 4 \) times. Each element in \( \hat{W} \) now contains two components, a weight value, and an index. We denote them as \( \hat{W}(i, k)[V] \) and \( \hat{W}(i, k)[I] \) respectively. In this case, we have computation as in Alg. 2 (for clarity we assume \( s = 1 \) from here on). As we observe, with CCI-Sparsity, we can reduce the computation steps by \( G \) times to \( N^3/G \). Besides, with CCI-Sparsity the reading dataflow of \( \hat{W} \) and \( X \) remain continuous as in the dense case. The dataflow of \( H \) matrix is also continuous if we look at it as the group level block. We still have irregularity inside each step when the \( Idx \) associated with each weight is used to access the specific element inside the output matrix group. This can be overcome through loading a whole group of \( H \) elements to on-chip registers, thus circumventing the latency and inefficiency caused by irregular off-chip memory access (Cao et al. 2019).

**Algorithm 2 CCI matrix matrix multiplication**

**Require:** \( \hat{W}, X \in \mathbb{R}^{N \times N}, H = 0 \).

**Ensure:** \( H = \hat{W} \times X \)

1. For \( i \leftarrow 0 \) to \( N/G \) by 1
2. For \( j \leftarrow 0 \) to \( N \) by 1
3. For \( k \leftarrow 0 \) to \( N \) by 1
   
   \[ Idx = \hat{W}(i, k)[I] \]
   
   \[ H(i + Idx, j) = H(i + Idx, j) + \hat{W}(i, k)[V] \times X(k, j) \]

As we show above that CCI-Sparsity can reduce computing complexity as regular fine-grained sparse networks. More importantly, due to the introduction of the weight group structure, CCI-Sparsity allows us to split network inference along the boundary of the weight groups, which is essential for efficient model inference as it enables us to take advantage of parallel processing and matrix tiling based data reuse (Zhou et al. 2018; Cao et al. 2019; Yao et al. 2019).

### 3.3 CCI-Sparsity vs Bank-balanced Sparsity

In (Cao et al. 2019; Yao et al. 2019), the authors propose and verify the effectiveness of the Balanced-Sparsity structure. CCI-Sparsity is rather similar to the Balanced-Sparsity in that both approaches introduce uniform size weight group in neural network models. Both approaches associate an index with each compressed weight. The central difference between the two structures is shown in Fig. 2. In CCI-Sparsity weight groups are formed between weights corresponding to different outputs \((G = 4 \) contiguous weights of the same color as in Fig. 2a). For example, \( W_{0,0}, W_{1,0}, W_{2,0}, W_{3,0} \) form one weight group, they are compressed into a single weight in \( \hat{W} \) as \( W_{0,0} \). Assume \( W_{2,0} \) is the weight that survives the pruning process, that is we have a weight index of 2 associated with \( W_{0,0} \). The weight value of \( W_{0,0} \) is then used to multiply with the \( X_{0,0} \), then we add the multiplication result to the corresponding \( H_{2,0} \). This computation procedure is illustrated in Fig. 2c.

As for Balanced-Sparsity (Yao et al. 2019), weight groups are formed between weights corresponding to different inputs \((G = 4 \) contiguous weights of the same color as in Fig. 2b). Thus the index associated with the compressed weights is used to allocate the corresponding input elements inside a group as illustrated in Fig. 2d.

Many recent studies on specialized neural network accelerators show that reading/writing of off-chip memory data consumes the greater part of total energy budget (Zhou et al. 2018). Rather drastically different data-flow settings are adopted in different accelerator designs to maximise system efficiency. While our empirical experiments have found that the CCI-Sparsity structure gives a rather similar model accuracy as the Balanced-Sparsity structure when applied on the model of same sparsity level and group setting. We believe
that either structure can show superiority over another one under different accelerator architectures and data-flow design. While a full-scale analysis on the data-flow optimization for both approaches is beyond the scope of this study, we give an analysis on a representative case that shows a clear advantage of CCI-Sparsity over Balanced-Sparsity.

In this analysis, we assume each processing element (PE) contains just enough registers that are necessary for either CCI-Sparsity or Balanced-Sparsity to work. No matrix tiling or other data parallelism is considered in this scenario. For the CCI-Sparsity case, we use the output stationary dataflow, that is for each stride we first use G registers for the output summation results of a group (contiguous blocks of the same color as in H matrix of Fig. 2a). We then keep those G registers stationary for their maximum reuse. Inside a stride, for each calculation step, we read one element from W and one element from X. There are N steps inside each stride. At the end of a stride we need to write G elements to H in memory. The total number of groups inside H is \( N^2/G \). So we have \((2N + G) \times N^2/G = 2N^3/G + N^2\) memory I/O requirement for the multiplication. In the case of Balanced-Sparsity, as shown in Fig. 2d, for each stride we first read G elements from X and store them in registers for maximum reuse. Then for each computation step we also need read one element from W, but for the element form H, we not only need to read but also need to write out the element as it contains a partial summation result except the first step when every element in H is zero. We have same number of stride as in CCI case. Therefore in Balanced-Sparsity case, we end up with \((3N + G) \times N^2/G - N^2 = 3N^3/G\) I/O counts. The difference between theses two approaches is \(N^3/G - N^2\). Given \(N\) is generally much larger than \(G\), CCI-Sparsity can significantly reduce the I/O complexity for this case.

4 Constraint imposed by CCI-Sparsity

We explained in the last section the advantage that CCI-Sparsity can provide in the aspect of inference efficiency. Since CCI-sparsity imposes special sparsity configuration on our network models, obviously this can impact our model performance. We give both theoretical and empirical analysis below.

To understand how CCI-Sparsity might impact model performance, we begin with a scheme of applying CCI-Sparsity constraint on a pre-trained network layer of fine-grained sparsity. Again we use a fully connected network layer as the example. We have a weight matrix \(W\) in dense form, where each row of \(W\) corresponding to one output channel. We have weight matrix \(W \in \mathbb{R}^{M \times N}\), and each weight in \(W\) has the same probability \(P_{s}\) (sparsity ratio) of being zero. Again we partition \(N\) weights of each column into \(g = N/G\) groups, with a group size of \(G\).

Denote \(s = G \times (1 - P_s)\), is the number of slots that are available for storing the nonzero weight values in each group. For simplification, we assume \(g, G, s \in \mathbb{N}^+\). Since each weight in \(W\) has the same probability of being nonzero, the number of nonzero weights \(i\) fall in each weight group follows the Binomial distribution \(B(i, G, 1 - P_s)\). If for a certain case, we have \(i > s\), then \(i - s\) nonzero weights have to be discarded. Denote the probability of each individual weight being discarded as \(P_d\). We have

\[
P_d = \sum_{i=1}^{G} \frac{1(i > s)(i - s)B(i, G, 1 - P_s)}{s} \tag{1}
\]

the probability of a certain weight fail to allocate an encoding slot. \(1(\_\_)\) here is the indicator function. With Equ. 1, We plot the relationship between \(P_d\) and group size \(G\) and sparsity ratio \(P_s\) in Fig. 3a. We can clearly tell that, with same sparsity ratio, \(P_d\) steadily increase as group size \(G\) decreases. On the other hand, we also observe that \(P_d\) is significantly higher if we adopt a higher sparsity level under same group size.

To gain insight on how CCI-Sparsity group size might affect model performance, we experiment on applying CCI-Sparsity structure on a pre-trained MobileNetV2 networks of 75% sparsity ratio (sparse on point-wise layers only, since point-wise layers hold the majority of the computing). As shown in Fig. 3b, while we aim to enforce same sparsity ratio of 75% on models, a smaller group size causes greater performance impairment, corresponding well with the rising allocation error rate when we decrease group size. Obviously, the result we show here is just to demonstrate the constraint associated with the CCI-Sparsity structure. We will discuss a proper way for training those networks in Sec. 5.

5 CCI-Sparsity model training

The result we show in Fig. 3b is performed on pre-trained fine-grained sparse models, that is, those models are not trained with CCI-Sparsity constraint. The constraint is imposed on them directly. Clearly, such an approach would
not take full advantage of a neural network, therefore we can observe rather significant performance loss. In this part, we will train sparse network models with CCI-Sparsity from scratch.

In order to train a sparse neural network, typically there are two approaches for attaining network sparsity. The first approach is to prune a pre-trained dense network followed by post-training (PT) fine-tune; that is used in (Han et al. 2015; Zhu and Gupta 2017). Another approach is to learn the sparse structure directly, targeted dropout (TD) (Gomez et al. 2018) and dynamic sparse reparameterization (Mocanu et al. 2018; Mostafa and Wang 2019) go into this category. Both approaches are known to give regular sparse neural network models of rather decent performance. In order to induce fine-grained network sparsity, typically, we select a limited amount of weights out of a rather large group of weights as in typical network pruning approaches. The weight group is generally rather big, for example, the group can be composed of all weights from a network layer, or even all weights from a network. For CCI-Sparsity models, the sparsity is constructed with the group constraint, that is only limited slots are available for nonzero weights encoding inside a group. The desirable group size is rather small, for example, a group size of 8. This put a rather strong constraint on our models as shown in Eq. 1 and Fig. 3a. Given that the probability of any important weights being pruned away is rather high if small group sizes are being used, we hypothesize we would get models of better performance if we allow model more opportunity to recover pruned weights, therefore, better adjusting to the CCI-structure instead of using PT approach.

Therefore we use an improved version of TD for our model training. We believe TD-training would allow network models more chance to adapt to CCI structure as it enforces the sparsity throughout the model training. TD begins the network training with a dense network structure. During the model training, a set of candidate network connections are selected based on a certain policy (In our case, we use the absolute magnitude of weights). Then the candidate connections are dropped by a certain probability in the same manner as in the original dropout (Srivastava et al. 2014) for model training. Differing from the PT pruning approach (Zhu and Gupta 2017; Cao et al. 2019), where a weight cannot recover once pruned, TD allows weights in the pruning pool to recover. This we believe is important for the proper CCI-Sparsity model training.

In (Gomez et al. 2018), the authors ramp up the size of the dropout candidate pool during the training and use a fixed 50% dropout rate. While their approach is successful in the small datasets they tried, we find such strategy often does not give us models of ideal accuracy when we train models on the ImageNet dataset (Russakovsky et al. 2015). In (Gomez et al. 2018), the authors suggest that TD cause the unimportant connections in the pruning pool shrinking toward zero because the $l^2$ regularization, also called weight decay, causes weights moving toward 0. Such property is important, as weights in the pruning pool participate in the model training by a fixed probability, if those weights hold significant magnitude, they will affect the running mean and variance statistics of the batch normalization layers used in the neural network models, which can lead to inferior model performance (Li et al. 2018).

As shown in Fig. 4a, we can observe that the pruned weights still holds a significant magnitude in our model. Ideally, we would like those pruned weights to stay at a value of exactly zero, in this way they cause no issue for batch normalization. A possible solution is to use a higher dropout rate for the TD-training, which allows weights in the pruning pool more chance to move toward zero. However, this also lowers the chance of a pruning candidate weight to recover. Hence for this study, we choose to ramp candidate dropout rate from the initial 50% toward 100% to encourage pruning candidate weights to shrink towards zero while allowing weights more chance to recover at the early stage of the model training. As shown in Fig. 4a, dropout rate ramping indeed cause more pruned weights to shrink toward zero. Such an approach leads to significantly improved model test accuracy. In a typical case with $G = 8, s = 2$, that is 75% sparsity ratio, a MobileNetV2 model trained with ImageNet achieve Top-1 accuracy of 69.8% (ramping dropout) vs 64.8% (fixed 50% dropout rate).

### 5.1 Group size effect under different pruning approaches

Ideally, we want to use small sparse group sizes, as in addition to the lower weight index storage requirement, a small group size would translate into smaller register file size which gives better data locality or small area/ less wiring for hardware design and lower register file accessing energy cost (Yang et al. 2018). However, as explained in Sec. 3, small group sizes can pose a strong constraint on models, hence may lead to inferior performances. We train MobileNetV2 models with all pointwise layer sparsity ratio set...
to 75% under different group sizes and with TD or PT. As longer training generally lead to better model accuracy, we chose the number of training epochs to allow model of no-group setting to have very close accuracy under TD and PT to simplify comparison. For TD, all models are trained for 120 epochs. For PT, models are first trained for 120 epochs then for 60 epochs of iterative pruning. As we show in Fig. 4b, smaller weight group sizes can indeed cause more accuracy loss than big group size. But more importantly, when we compare TD training with the PT, while PT gives similar accuracy as TD when no-group setting is used, TD clearly outperform PT under group setting (TD still outperform PT on group cases even if we do iterative pruning for 120 epochs for PT).

6 Compare with other lightweight structures

In this section, we investigate the characteristics of CCI-Sparsity architecture empirically. Experiments are performed on widely used ImageNet dataset (Russakovsky et al. 2015) and CIFAR-10 dataset (Krizhevsky and Hinton 2009). To gain a good understanding of how CCI-Sparsity interacts with different network architectures, We prune both heavyweight models: VGG-16 and ResNet and lightweight models: MobileNetV2.

For ImageNet experiments, we use the ILSVRC-2012 subset. Models are trained with the augmented standard training set and evaluated with the center crop of images from the validation set as in (Sandler et al. 2018). Unless explicitly specified, models are trained with SGD optimizer with the momentum of 0.9 and initial learning rate of 0.18 on 4 GPUs with a batch size of 64 per GPU for a total of 120 epochs with cosine decay schedule (Loshchilov and Hutter 2016). For standard models, weight decay is set to 0.00004. For a proper comparison, we don’t tune any of the hyperparameters described above throughout our experiments. For TD training, we use the first half of the training epochs to ramp up targeted rate while keeping the candidate dropout rate at 50%, and the second half to ramp the candidate dropout rate from 50% to 100%. We compress all the convolutional layers in a network with the same group size $G$ and $s$ setting. For MobileNetV2, we only compress the pointwise layer. For VGG-16 network we also compress fully connected layers.

For experiments on CIFAR dataset, we use 50,000 samples for model training and 10,000 for testing. Images are augmented as in (He et al. 2016) before used for model training. We train all models for a total of 400 epochs at a batch size of 64, with an initial learning rate of 0.1 and cosine decay schedule (Loshchilov and Hutter 2016). Weight decay is set to 0.0001. For each experiment, we run 5 times and report the median test accuracy. Models are always first trained for 80 epochs while ramping up the targeted rate at 50% candidate dropout rate, followed by 200 epoch ramping up candidate dropout rate to 100%, then trained for 80 epochs at 100% candidate dropout rate.

6.1 CIFAR-10 classification results

Results from ResNet-32 and VGG-16 models on CIFAR-10 dataset are shown in Table 1 and 2. Our improved TD training gives better performance than the original TD approach as proposed in (Gomez et al. 2018). When comparing models with or without CCI-Sparsity constraint under the same sparsity level, We observe a much bigger performance drop than in lightweight MobileNetV2 models shown in 4b. This further confirms our analysis in Sec. 3 that CCI-Sparsity works better when the sparsity level is not too high. We also compare our result on the VGG-16 model with a structured sparsity model result as in (Li et al. 2016), where filters are targeted for pruning. CCI-Sparsity also shows a clear advantage in this case.

| Model                     | Param Pruned (%) | Flops Pruned (%) | Top1 Accuracy (%) |
|---------------------------|------------------|------------------|-------------------|
| ResNet-32 (baseline)      | 0.00             | 0.00             | 92.98             |
| ResNet-32 (1/16)          | 93.51            | 93.15            | 90.56             |
| ResNet-32 (Without Group) | 93.51            | 93.15            | 91.21             |
| ResNet-32 (1/64)          | 97.40            | 95.50            | 88.39             |
| ResNet-32 (Without Group) | 97.40            | 95.50            | 89.42             |
| ResNet-32 (TD)(Gomez et al. 2018)) | 94.00 | 94.00 | 88.80 |
| ResNet-32 (TD)            | 97.00            | 97.00            | 88.67             |
| ResNet-32 (TD)            | 98.00            | 98.00            | 88.70             |

Table 1: Testing accuracy on CIFAR-10 dataset

| Model                     | Pruned (%) | Flops Pruned (%) | Top1 Accuracy (%) |
|---------------------------|------------|------------------|-------------------|
| VGG-16 (baseline)         | 0.00       | 0.00             | 93.55             |
| VGG-16 (1/16)             | 93.68      | 93.21            | 92.55             |
| VGG-16 (Without Group)    | 93.68      | 93.21            | 92.90             |
| VGG-16 (1/64)             | 98.36      | 97.87            | 90.48             |
| VGG-16 (Without Group)    | 98.36      | 97.87            | 91.46             |
| VGG-16 (1/16)             | 74.93      | 74.58            | 93.43             |
| VGG-16 (Li et al. 2016)   | 64.00      | 34.20            | 93.40             |

Table 2: Testing accuracy on CIFAR-10 dataset

6.2 ImageNet classification results

In this part of the study, we first compare the test accuracy of CCI-Sparsity models (pruned from MobileNetV2, pointwise layer only) with that of lightweight architectures from the latest literature (Huang et al. 2018; Sun et al. 2018;
Table 3: Performance comparison with structured sparsity on ResNet-18 model trained on ImageNet as shown in Tab. 3. Again, CCI-Sparsity outperforms structured sparsity by a significant gap.

| Method          | Baseline Top1 Accuracy | Flips Pruned (%) | Top1 Accuracy (%) | Accuracy Drop (%) |
|-----------------|------------------------|------------------|-------------------|-------------------|
| Ours            | 71.12                  | 70.55            | 70.31             | 0.81              |
| (Dong et al. 2017) | 69.98                  | 34.6             | 66.33             | 3.65              |
| (He et al. 2018) | 70.28                  | 41.8             | 67.10             | 3.18              |

Table 3: Performance comparison with structured sparsity on ResNet-18 model. CCI-Sparsity (Group size $G = 16, S = 2$) outperforms structured sparsity by a big gap.

**7 Conclusion**

This work presents the cross-filter intragroup (CCI) sparsity structure. CCI-Sparsity structure retains the performance (accuracy) advantage of fine-grained sparsity over coarse-grained structural pruning approach. At same time CCI-Sparsity avoids the inference inefficiency of fine-grained sparsity caused by irregularity in computing data-flow. Also, CCI-Sparsity provides good compatibility with tiling or blocking that often used in network accelerator designs, therefore, would enable better inference efficiency than fine-grained sparsity through data reuse and parallelism.

Through theoretical and empirical analysis, we reveal that a large group size $G$ generally impose less constraint on neural network models, while a small group size is desirable as it is associated with less computing overhead. Empirical experiments reveal that a weight group size of 16 generally does not cause significant performance loss when compared with regular sparsity case.

We also compare CCI-Sparsity with Balanced-Sparsity and give a typical setting where CCI-Sparsity requires significantly lower I/O complexity than Balanced-Sparsity for model inference.

Our proposed TD training strategy outperforms iterative pruning in training models with CCI-Sparsity structure. And finally when compared with several lightweight network architectures and structural pruned models. CCI-Sparsity show clear performance (classification accuracy) advantage over those models under a similar level of computing complexity.

**8 Future works**

**Exclusive Lasso regularization:** Proper model regularization can often lead to improved generalization performance (Li et al. 2016; Wen et al. 2016). For the main part of this study, we use a simple $l_2$ regularizer and rely on the TD to induce the intragroup sparsity structure. Our preliminary investigation in Appendix. A.1 shows exclusive Lasso regularization can improve model training and gives better model test accuracy. More extensive investigation on this would be necessary to better understand the interaction between exclusive Lasso regularization and TD training.

**Group size:** As shown in Fig. 4b, a group size of 16 can generally give a accuracy quite close to that of the regular fine-grained sparse model on the lightweight networks. A group size of 16 is probably a good choice from a hardware/software design perspective. However, with a group size of 16, the lowest sparsity ratio would be 1/16, which is not sufficient for compressing some of the huge networks. Besides, analysis in Section 3 tells us that CCI-Sparsity does not work well with a very high sparsity ratio. Therefore we believe that it is best to apply CCI-Sparsity on network models that do not contain too much redundancy in the first place. Alternatively, we might want to combine CCI-Sparsity with structured pruning for the best outcome.

**Multiple sparsity ratios:** As this study is more about proof of concept, all network layers of the neural network we studied use same weight group size $G$ and $s$, thus same sparse ratio. Such a setting is known to perform not as good as designating different sparse ratios for different network layers (Liu et al. 2018). With proper data-flow design, such a mixture of different $G$ and $s$ combination should not be an big issue.

**References**

Cao, S.; Zhang, C.; Yao, Z.; Xiao, W.; Nie, L.; Zhan, D.; Liu, Y.; Wu, M.; and Zhang, L. 2019. Efficient and effective sparse lstm on fpga with bank-balanced sparsity. In *Proceedings of the 2019
Chetlur, S.; Woolley, C.; Vandermerw, P.; Cohen, J.; Tran, J.; Catanzaro, B.; and Shelhamer, E. 2014. cudnn: Efficient primitives for deep learning. arXiv preprint arXiv:1410.0759.

Dong, X.; Huang, J.; Yang, Y.; and Yan, S. 2017. More is less: A more complicated network with less inference complexity. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR).

Gao, H.; Wang, Z.; and Ji, S. 2018. Channelnets: Compact and efficient convolutional neural networks via channel-wise convolutions. In Advances in Neural Information Processing Systems, 5203–5211.

Han, S.; Liu, X.; Mao, H.; Pu, J.; Pedram, A.; Horowitz, M. A.; and Dally, W. J. 2015. Learning both weights and connections for efficient neural network. In Advances in Neural Information Processing Systems, 2752–2761.

He, K.; Zhang, X.; Ren, S.; and Sun, J. 2016. Deep residual learning for image recognition. In Proceedings of the IEEE conference on computer vision and pattern recognition, 770–778.

He, Y.; Kang, G.; Dong, X.; Fu, Y.; and Yang, Y. 2018. Soft filter pruning for accelerating deep convolutional neural networks. CoRR abs/1808.06866.

Howard, A. G.; Zhu, M.; Chen, B.; Kalenichenko, D.; Wang, W.; Weyand, T.; Andreetto, M.; and Adam, H. 2017. Mobilenets: Efficient convolutional neural networks for mobile vision applications. arXiv preprint arXiv:1704.04861.

Huang, G.; Liu, S.; Van der Maaten, L.; and Weinberger, K. Q. 2018. Condensenet: An efficient densestnet used learning grouped convolutions. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2752–2761.

Krizhevsky, A., and Hinton, G. 2009. Learning multiple layers of features from tiny images. Technical report, Citeseer.

Li, H.; Kadav, A.; Durandovic, I.; Samet, H.; and Graf, H. P. 2016. Pruning filters for efficient convnets. arXiv preprint arXiv:1608.08710.

Li, X.; Chen, S.; Hu, X.; and Yang, J. 2018. Understanding the disharmony between dropout and batch normalization by variance shift. arXiv preprint arXiv:1801.05134.

Liu, Z.; Sun, M.; Zhou, T.; Huang, G.; and Darrell, T. 2018. Rethinking the value of network pruning. arXiv preprint arXiv:1810.05270.

Loshchilov, I., and Hutter, F. 2016. Sgdr: Stochastic gradient descent with warm restarts. arXiv preprint arXiv:1608.03983.

Ma, N.; Zhang, X.; Zheng, H.-T.; and Sun, J. 2018. Shufflenet v2: Practical guidelines for efficient cnn architecture design. In Proceedings of the European Conference on Computer Vision (ECCV), 116–131.

Mao, H.; Han, S.; Pool, J.; Li, W.; Liu, X.; Wang, Y.; and Dally, W. J. 2017. Exploring the regularity of sparse structure in convolutional neural networks. arXiv preprint arXiv:1705.08922.

Mocanu, D. C.; Mocanu, E.; Stone, P.; Nguyen, P. H.; Gibescu, M.; and Liotta, A. 2018. Scalable training of artificial neural networks with adaptive sparse connectivity inspired by network science. Nature communications 9(1):2383.

Mostafa, H., and Wang, X. 2019. Parameter efficient training of deep convolutional neural networks by dynamic sparse reparameterization. arXiv preprint arXiv:1902.05967.

Narang, S.; Undersander, E.; and Diamos, G. 2017. Block-sparse recurrent neural networks. arXiv preprint arXiv:1711.02782.

Parashar, A.; Rhu, M.; Mukkara, A.; Puglisielli, L.; Venkatesan, R.; Khailany, B.; Emer, J.; Keckler, S. W.; and Dally, W. J. 2017. Scnn: An accelerator for compressed-sparse convolutional neural networks. In 2017 ACM/IEEE 44th Annual International Symposium on Computer Architecture (ISCA), 27–40. IEEE.

Russakovsky, O.; Deng, J.; Su, H.; Krause, J.; Satheesh, S.; Ma, S.; Huang, Z.; Karpathy, A.; Khosla, A.; Bernstein, M.; Berg, A. C.; and Fei-Fei, L. 2015. ImageNet Large Scale Visual Recognition Challenge. International Journal of Computer Vision (IJCV) 115(3):211–252.

Sandler, M.; Howard, A.; Zhu, M.; Zhmoginov, A.; and Chen, L.-C. 2018. Mobilenetv2: Inverted residuals and linear bottlenecks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 4510–4520.

Srivastava, N.; Hinton, G.; Krizhevsky, A.; Sutskever, I.; and Salakhutdinov, R. 2014. Dropout: a simple way to prevent neural networks from overfitting. The Journal of Machine Learning Research 15(1):1929–1958.

Sun, K.; Li, M.; Liu, D.; and Wang, J. 2018. Igcv3: Interleaved low-rank group convolutions for efficient deep neural networks. arXiv preprint arXiv:1806.00178.

Wen, W.; Wu, C.; Wang, Y.; Chen, Y.; and Li, H. 2016. Learning structured sparsity in deep neural networks. In Advances in neural information processing systems, 2074–2082.

Wu, X.; Liu, X.; Li, W.; and Wu, Q. 2018. Improved expressivity through dendritic neural networks. In Advances in neural information processing systems, 8057–8068.

Yang, X.; Gao, M.; Pu, J.; Nayak, A.; Liu, Q.; Bell, S. E.; Setter, J. O.; Cao, K.; Ha, H.; Kozyrakis, C.; et al. 2018. Dnn dataflow choice is overrated. arXiv preprint arXiv:1809.04070.

Yao, Z.; Cao, S.; Xiao, W.; Zhang, C.; and Nie, L. 2019. Balanced sparsity for efficient dnn inference on gpus. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 33, 5676–5683.

Zhang, S.; Du, Z.; Zhang, L.; Han, H.; Liu, S.; Li, L.; Guo, Q.; Chen, T.; and Chen, Y. 2016. Cambricon-x: An accelerator for sparse neural networks. In The 49th Annual IEEE/ACM International Symposium on Microarchitecture, 20. IEEE Press.

Zhang, T.; Qi, G.-J.; Xiao, B.; and Wang, J. 2017. Interleaved group convolutions. In Proceedings of the IEEE International Conference on Computer Vision, 4373–4382.

Zhou, X.; Du, Z.; Guo, Q.; Liu, S.; Liu, C.; Wang, Z.; Zhou, X.; Li, L.; Chen, T.; and Chen, Y. 2018. Cambricon-s: Addressing irregularity in sparse neural networks through a cooperative software/hardware approach. In 2018 51st Annual IEEE/ACM International Symposium on Microarchitecture (MICRO), 15–28. IEEE.

Zhou, Y.; Jin, R.; and Hou, S. C.-H. 2010. Exclusive lasso for multi-task feature selection. In Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics, 988–995.

Zhu, M., and Gupta, S. 2017. To prune, or not to prune: exploring the efficacy of pruning for model compression. arXiv preprint arXiv:1710.01878.
A Appendix

A.1 Exclusive Lasso regularization

Exclusive Lasso regularization models the competition between components inside a group (Zhou, Jin, and Hoi 2010) through applying the following regularizer:

\[
\ell(W) = \sum_{j=1}^{d} \left( \sum_{k=1}^{G} |W_{jk}| \right)^2
\]

(A.1)

By combining the \( \ell_1 \) norm to combine the weights from the same group, which tends to give sparse solution, and \( \ell_2 \) norm to combine different groups together, which tends to minimize the regularizer loss, the exclusive Lasso regularizer essentially encourages the weights inside a group to compete for non-zero weights positions (Zhou, Jin, and Hoi 2010). Such a property is desirable for models with CCI-Sparsity. We performed experiments on combining the exclusive Lasso regularization with CCI-Sparsity. When we use the exclusive Lasso regularizer alone for CCI-Sparsity models training, we find that models generally perform rather poor. However, when we combine exclusive Lasso regularizer with the TD training, we observe a much smoother transition in the validation accuracy curve when training ramp reaches the last step, as shown in Fig. A.1, indicating better model convergence. We also observe that a better final model validation accuracy than a model trained with TD (with \( \ell_2 \) norm regularizer) can be achieved through tuning exclusive Lasso regularization strength. As this paper is more about the testing of concept instead of achieving state-of-the-art results, the exclusive Lasso regularizer is not used for the main-body of this paper.

![Figure A.1: The exclusive Lasso regularizer improves the TD model training.](image)

| Model | Params | Flops | Top1Accuracy |
|-------|--------|-------|--------------|
| MobileNet V1 | 4.2M | 1235M | 70.60 |
| MobileNet V1 (0.75) | 2.6M | 515M | 68.30 |
| MobileNet V1 (0.5) | 1.3M | 253M | 63.70 |
| MobileNet V2 | 1.9M | 500M | 71.00 |
| MobileNet V2 (0.75) | 1.3M | 300M | 69.80 |
| MobileNet V2 (0.5) | 0.95M | 177M | 65.40 |
| SqueezeNet 0.5 | 1.6M | 338M | 72.20 |
| SqueezeNet 0.5 (0.75) | 1.2M | 220M | 68.49 |
| SqueezeNet 0.25 | 0.9M | 210M | 71.00 |
| ShuffleNet V1 (g = 1) | 4.8M | 252M | 70.00 |
| ShuffleNet V1 (g = 2) | 2.2M | 140M | 65.86 |
| ShuffleNet 0.25 (shallow, g = 2) | 1.2M | 80M | 57.20 |
| C-Sparsity on MobileNet V2 (width=1, epoch=300, G=8, S=2) | 5.7M | 252M | 65.40 |
| C-Sparsity on MobileNet V2 (width=4, epoch=120, G=8, S=2) | 2.5M | 120M | 72.25 |
| C-Sparsity on MobileNet V2 (width=2.0, epoch=120, G=8, S=2) | 1.2M | 60M | 72.25 |
| C-Sparsity on MobileNet V2 (width=2.0, epoch=120, G=8, S=2) | 0.97M | 30M | 71.25 |
| C-Sparsity on MobileNet V2 (width=2.0, epoch=120, G=8, S=2) | 1.3M | 60M | 70.15 |

Table A.1: Benchmark results on lightweight architectures