PRIMORDIAL NON-GAUSSIANITY: BARYON BIAS AND GRAVITATIONAL COLLAPSE OF COSMIC STRING WAKES

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ABSTRACT

I compute the three-dimensional nonlinear evolution of gas and dark matter fluids in the neighborhood of cosmic string wakes that are formed at high redshift (z ≈ 2240) for a "realistic" scenario of wake formation. These wakes are the ones that stand out most prominently as cosmological sheets and are expected to play a dominant role in the cosmic string model of structure formation. Employing a high-resolution three-dimensional hydrodynamic code to evolve these wakes until the present day yields results for the baryon bias generated in the inner wake region. I find that today, wakes would be 1.5 h⁻¹ Mpc thick and contain a 70% excess in the density of baryons over the dark matter density in their centers. However, high-density peaks in the wake region do not inherit a baryon enhancement. I propose a mechanism for this erasure of the baryon excess in spherically collapsed objects based on the geometry change around the collapsing region. Furthermore, I present heuristic arguments for the consequences of this work for large-scale structure in the cosmic string model and conclude that the peculiarities of wake formation are unlikely to have significant import for the discrepancy between power spectrum predictions and observations in this model. If one invokes the nucleosynthesis bound on Ω_b, this could be seen as strengthening the case against Ω_b = 1 or for low Hubble constants.

Subject headings: cosmic strings — cosmology: theory — galaxies: clusters: general — hydrodynamics

1. INTRODUCTION

Two widely studied paradigms for explaining the origin of structure are inflation (see Kolb & Turner 1990 for an introduction and references) and topological defects (Kibble 1976; Vilenkin & Shellard 1994), such as cosmic strings or textures. Finding theoretical predictions that lead to observational strategies for distinguishing between them or ruling any one of them out has been a major field of activity in modern cosmology.

The family of inflationary cosmogonies (with various inflation potentials, dark matter contents, open, flat, and closed topologies) give rise to a wide range of physical predictions. However, the nonlinear evolution of defect networks have made it difficult to pin down theoretical predictions for a long time. In addition, the inherent presence of non-Gaussianity in such models makes calculating the full predictions for such theories an enormous task, as two-point functions contain only a limited amount of information about the resulting density fields.

Recently it was shown that the cosmic string scenario of structure formation suffers from difficulties in fitting the galaxy power spectrum when it is normalized to COBE (Pen, Seljak, & Turok 1997; Turok, Pen, & Seljak 1997; Albrecth et al. 1997a, 1997b). However, these calculations describe fluctuations in the dark matter component, which are then compared to the galaxy power spectrum. To make contact between theory and observation, and to respond to what Albrecth et al. (1997a) termed the b_{100} problem, the differences between baryon and dark matter dynamics must be studied for active perturbation models.

This is particularly interesting in the case of cosmic strings, as non-Gaussianity leads to rich physics in the wake of a cosmic string (Vachaspati 1992; Stebbins et al. 1987; Perivolaropoulos, Brandenberger, & Stebbins 1998). The dark matter dynamics of cosmic string wakes and their role in the generation of large-scale structure has been actively studied (Vachaspati 1986; Vachaspati & Vilenkin 1991; Albrecht & Stebbins 1992, 1992b; Robinson & Albrecht 1996). The nonlinear evolution of gas/CDM wakes in three dimensions was first studied by Sornborger et al. (1997). These authors consider two extreme cases: a fast-moving (v = 0.5c) straight string, modeled by a planar-symmetric, pure-velocity perturbation, and a slow-moving (v ≪ c) wiggly string, representing a Newtonian line-source potential. Their most interesting finding is an enhancement in the baryon overdensity compared to the dark matter overdensity (i.e. a baryon bias) in the center of the wake. This excess amounted to about 140% in the case of a fast moving string. For early wakes, which are formed around the time of matter radiation equality, the baryon overdensity is initially more spread out and the opposite is true; a sheet of excess dark matter is sandwiched between two baryonic layers. Later, as the matter density begins to dominate, the baryonic matter clumps in the core of the wake, reversing to baryon enhancement in the core.

This beautiful effect is caused by trapping the collisionless dark matter in an oscillatory mode about the wake center, while the baryons shock heat and build a pressure-supported peak in the core of the wake. The oscillatory

1 See Mähönen 1996 for a different, earlier perspective.
mode prevents the full gravitational collapse of the dark matter as long as planar symmetry is maintained.

It is worth stressing that if these results were to be generalized to less restrictive conditions, a sizeable baryon bias would be a signal for the presence of non-Gaussianities in the very early universe. In high-resolution hydrodynamic simulations (Jenkins et al. 1997), none of the tested Gaussian models produced a bias $b > 1$ on scales of $1\text{--}10\ h^{-1}\ \text{Mpc}^3$.

This is particularly intriguing, since earlier work (Robinson & Albrecht 1996) has shown, again in the context of cosmic string theories, that dark matter pancakes, which arise in the nonlinear evolution of Gaussian perturbations, effectively mimic the sheetlike overdensity of the wake. Hence, observing the density field alone will not lead to a detection of primordial non-Gaussianity, even when tailor-made statistics are used. This confusion could be overcome if the interplay between gas dynamics and dark matter produced baryon bias as a remnant of early non-Gaussianity that survives until today. Another exciting possibility is that non-Gaussianity may lead to a natural explanation of the observed high baryon fraction in the center of rich clusters (White et al. 1993; Loewenstein & Mushotzky 1996), as conjectured by Sornborger et al. (1997).

However, Sornborger et al. (1997) simulate sheetlike wakes in a perfectly homogeneous background. Combined with the perfect planar symmetry of their wake model, this induces translational symmetry in the plane parallel to the wake and mirror symmetry about the wake plane, an idealization that limits the phase space available to the system. Here, I consider a “realistic” scenario of wake formation in a gas/HDM mixture at early times ($\eta^* = \eta_{eq}$). To allow the system to explore the full three-dimensional phase space in its evolution, I use the full response of the linearized Einstein equations to a line source to compute the initial wake perturbation and model the effect of early-time wakes in the simulation volume by the addition of a Gaussian field background. In addition, instead of allowing continuous inflow of new material into the simulation volume, I impose periodic boundary conditions on the cube as a crude model of the effects of compensation. Another effect I include is cooling due to bremsstrahlung from hot gas.

The plan of this paper is as follows. In § 2 I discuss some physical background and the assumptions and methods used in my simulations. In § 3 I present the simulation results, and § 4 contains my conclusions.

2. METHODS

2.1. Sheetlike Structure

The reason that different physics may be expected in cosmological sheets formed in the wake of a cosmic string and those generated through the gravitational instability of Gaussian fluctuations is illustrated in Figure 1. It was shown by Zeldovich that most initially overdense regions in a Gaussian scenario have a preferred direction of collapse associated with them, which causes them to evolve into cosmological pancakes. The characteristic scale of these pancakes and their overdensities are given by the shape and amplitude of the power spectrum of initial fluctuations.

In the case of cosmic string wakes, the physical characteristics of the wake are determined by a different dynamical process. Imagine a homogeneous background density and a straight string moving through it. Through the effect of the conical deficit of spacetime around the string, the particles behind the string feel a velocity kick in the direction perpendicular to and toward the plane swept out by the string. In linear theory, the overdensity $\delta$ that is thus created can be written as the response to the positive trace of the string stress energy tensor $\Theta_+ = \Theta_{00} + \Theta_{tt}$,

$$\delta(k) = \frac{4\pi}{1 + \frac{z_{eq}}{z_{hydro}}} \frac{1}{z_{eq}} \int d\eta' \tilde{T}(k; \eta') \tilde{\Theta}_{\pm}(k, \eta') d\eta'. \quad (1)$$

Here the source acts between conformal times $\eta_i$ and $\eta_f$. $\tilde{T}(k; \eta')$ is the HDM transfer function, and $z_{eq} = 1/(1 + a_{eq})$, $a_{eq} = 4.17 \times 10^{-5} \ h^{-2}$. Fourier transforming then yields the real-space perturbation $\delta(x)$.

The parameters that govern the shape and strength of the wake perturbation are the string mass per unit length, the curvature scale of the string, and its bulk velocity. These enter equation (1) through $\tilde{\Theta}_+$. Hence, the physical processes that form cosmic string wakes are very different from the Zeldovich collapse of pancakes described above.

2.2. Initial Conditions

The way I model the initial fluctuations derives from Robinson & Albrecht (1996). The density field consists of a non-Gaussian wake part and a Gaussian part. To understand the appearance of the Gaussian part, consider the following reasoning. A simple scaling argument shows that in a comoving frame and in a matter-dominated era, the string network appears to dilate and stretch with physical time as $t^{1/3}$ (more technically, the curvature scale and the interstring distance increase). This means that at early times, there is a dense tangle of strings inside the box. The integrated perturbations seeded by this tangle are modeled as a Gaussian fluctuation background with a power spectrum given by Albrecht & Stebbins (1992b). Since many strings source this perturbation, the use of a Gaussian model is suggested by the central limit theorem. In this work I focus on the intermediate (1) model of cosmic strings, which broadly agrees with recent determinations of galaxy power spectra (e.g., Albrecht et al. 1997a; Pen et al. 1997). For this purpose, I chose the parameters shown in Table 1.

A single string seeds the non-Gaussian wake part at the time $\eta^* = 6 \eta_{eq}$. Why choose this particular time? This is simply the time at which the string has the best chance of producing a wake that stands out against the Gaussian

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3 Here and throughout this article, $h \equiv H_0/(100 \ \text{km}^{-1} \ \text{s}^{-1} \ \text{Mpc}^{-1})$. 
TABLE 1
SIMULATION PARAMETERS

| Parameter                      | Value            |
|-------------------------------|------------------|
| Box size \((L)\)               | \(20 \ h^{-1} \text{ Mpc}\) |
| Redshift \((z_{\text{hydro}})\) | 100              |
| Wake formation \((\eta_l)\)    | \(6\eta_0\)      |
| Mass per unit length \((\mu_b)\) | 1.1              |
| String bulk velocity \((\beta)\) | 0.3c             |
| Hubble parameter \((h)\)       | 1                |
| Density parameter \((\Omega)\) | 1                |
| Baryon density \((\Omega_b)\)  | 0.05 and 0.1     |

Note.—Parameters used in the hydrodynamics simulations. Three runs were done for each value of the baryon density.

background in an HDM scenario in linear theory.\(^4\) If I prefer a wake that is straight on the scale of the box, picking this time determines the box size used for the curvature scale of the string network at \(\eta^*\). This evaluates to \(20 \ h^{-1} \text{ Mpc}\) in the I-model.

Seeing that the perturbation equations are still linear at this stage, I can obtain the full initial conditions by simply adding together the Gaussian and non-Gaussian parts. Note that \(\sigma_{\text{wake}}(1 \text{ Mpc})\), the fluctuation in a \(1 \ h^{-1} \text{ Mpc}\) ball centered on the wake, is only 20% larger than the fluctuation in the Gaussian field in a ball of the same size. The Gaussian background is therefore expected to have a non-negligible effect. The simulation itself proceeds in two steps. First, the initial conditions are evolved to a redshift \(z_{\text{hydro}}\) in linear theory. Then they are fed into the hydrocode and evolved using the full nonlinear evolution equations until today \((z = 0)\) (see Fig. 2).

Treating much of the history of the wake in linear theory can be justified by numerical and analytical studies of the planar-symmetric, one-dimensional case (Sornborger 1997), showing that HDM wakes formed at conformal time \(\eta_{\text{eq}}\) go nonlinear only at a redshift \(z_{\text{eq}} \approx 30\).

This sets up the formalism for calculating the dark matter perturbations in linear theory. So far, nothing has been said about the gas because a general formalism does not exist for calculating the perturbations in the baryonic fluid. Hence, at this stage one has to decide what initial conditions to assign to the gas. I give it the same initial perturbations as the dark matter fluid, justifying this choice by observing that in Boltzmann calculations of the baryon overdensity, one observes the baryons to flow rapidly into the potential wells created by the dark matter overdensities for a wide range of cosmological parameters (Ma & Bertschinger 1995). For the value of \(\tau_{\text{hydro}}\) used in my simulations, I expect the baryonic density field to follow the dark matter very closely. I have tested this assumption by starting the hydrocode soon after decoupling \((\tau_{\text{hydro}} = 1000)\) and perturbing only the dark matter, leaving the gas completely homogeneous. I find that the results for the baryon bias in the center of the wake, baryon fractions in dense peaks, and wake fragmentation patterns at \(z = 0\) remain largely unchanged under this radical change in initial conditions.

2.3. Nonlinear Evolution

For the nonlinear evolution of the overdensities in both the baryonic and the dark matter fluids, I use the adaptive moving-mesh algorithm developed by Pen (1995, 1998). This code has the following features: (1) good shock resolution, (2) high speed of execution, (3) good resolution of high-density areas, and (4) a grid-based TVD approach that ensures optimal control of artificial viscosity.

The moving-mesh hydrodynamic (MMH) method produces output on an irregular, curvilinear grid. This means that standard statistics, such as the power spectrum, are not easily calculated for these density fields. Fortunately, for this application I am more interested in understanding the dynamics of dark matter and baryons in a cosmic string wake and the processes leading to its fragmentation. This information is not readily extractable from two-point statistics. Instead, I will look at various cuts through the data sets. To use the particular strength of the MMH method, I focus on high-density regions.

While in fact the \(N\)-body solver that computes the dark matter dynamics treats the dark matter as cold, a light massive neutrino of mass \(m_\nu = 93 \ eV\) has free-streamed a distance of \(9 \text{ kpc}\) since \(z = 100\). In the highest density areas, the smallest mesh elements have sizes of \(15.6 \text{ kpc}\), so the dark matter will be slightly too compressed at the small-scale resolution limit. This is not expected to affect my results, since this phenomenon acts on scales that are small compared to the width of the wake.

Cooling from bremsstrahlung is included in this code by computing the amount of energy radiated away from each mesh cell in the simulation volume. The free-free luminosity of the cell is calculated in the standard way (e.g., Peebles 1993 and references therein), except for the inclusion of a helium abundance of \(Y_{\text{He}} = 0.24\) and the Gaunt factor \(g = 1.2\).

3. RESULTS AND DISCUSSION

The code was run on COSMOS, the UK National Cosmology Supercomputer. This is a Silicon Graphics Origin 2000 with 32 R10000 processors and a shared-memory

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\(^4\) As has been noted in Robinson & Albrecht (1996), this is true for all three of the very different models of cosmic strings considered by Albrecht & Stebbins (1992b).
architecture.\textsuperscript{5} I simulated wakes in $\Omega_b = 0.05$ and $\Omega_b = 0.1$ universes and completed 3 production runs each on a mesh with $64^3$ cells.

### 3.1. Checks

Several test runs were performed to check that the results are only weakly dependent on parameter choices and compiler switches for the hydrocode, such as the compression limit on mesh distortions. To avoid overcompression of the baryons, I chose to use the code without enforcing exact energy conservation. Also, I ascertained that runs with the same random seeds for the Gaussian background fluctuations but without wakes showed no significant baryon bias. Tests of the code performance itself are published elsewhere (Pen 1995, 1998).

### 3.2. Simulation Results

As expected, adding a Gaussian background to the theory makes the results considerably richer than in the symmetric case. I find a complex interplay of gravitational and hydrodynamic forces in string wakes. To give a general idea of the output of such simulations, Figures 3, 4, and 5 show the simulation volume for one realization, averaged over one direction along the wake and looking edge-on at the wake. Snapshots of the simulation volume are taken at $z = 8, 4, 2, 1, 0.5,$ and $0$. As the code output is nonuniformly sampled, it has to be smoothed appropriately for display purposes. The density field is computed at the mesh cell centers. These are rebinned onto a cubic lattice with weighting factors determined by the cloud-in-cell scheme (CIC). This prescription will work well in regions of high density where the density of mesh cells is enhanced by a factor of up to several thousand compared to the original cell density, while it will lead to spurious peaks close to the mesh cell centers in highly underdense regions.

The approximate symmetry along the wake can be exploited by averaging through the box to effect further smoothing and to obtain two-dimensional fields for easy visualization. This has the advantage over slices through the volume that all high-density areas will be seen, but it may wash out features on the wake. One-dimensional profiles of the wake are obtained by another average in the direction of cosmic string propagation.

I measure an average of 70% maximum baryon excess at $z = 0$, after CIC binning in the center of the wake. This is about half of what Sornborger et al. (1997) find, but there are significant differences between their simulation setup and mine. Adding a Gaussian background should be expected to reduce the coherence in the dark matter oscillations about the wake plane, and the same is true for the density gradient along the wake. In addition, my crude model of compensation means that the inflow of new material into the wake will subside at late times. Last but not least, the string bulk velocity is smaller in my simulations. All of these effects are expected to reduce the baryon bias. Perhaps it should be stressed that it is surprising that any effect survives at all, which underlines the robustness of this prediction.

Figure 3 shows the evolution of the gas perturbations for one realization. The wake is clearly visible in the first panel. The field is oriented such that the string entered at the bottom center of the plot and propagated through until it reached the top center. The density gradient along the direction of wake propagation is due to the fact that matter has had more time to fall in toward the center where the string entered the box. Subsequent gravitational evolution leads to the fragmentation and collapse of the sheet. It can be clearly seen how the Gaussian background induces the disruption of the initial approximate symmetry. While it is still true that the majority of high-density peaks form on the wake, the sheetiness of the wake is masked by other sheetlike features that form in the final frames. The dark matter distribution in Figure 4 looks visually similar at all times.

The fraction of baryonic mass to dark mass per smoothed cell is shown in Figure 5. Encouragingly, the results broadly agree with the calculations of Sornborger et al. (1997). The apparent doubling of the wake structure after $z = 2$ should be attributed to the distortion of the wake plane in response to the presence of neighboring perturbations (remember that these plots are averages through the volume). Looking at the cut through the simulation volume at $z = 0$ in Figure 6 reveals the baryon enhancement in the center of the wake.\textsuperscript{6} The wake thickness can be read to be about 1.5 $h^{-1}$ Mpc, in very good agreement with earlier determinations.

### 3.3. Baryon Fraction in Density Peaks

Another matter of interest is whether the baryon enhancement I observe in the wake plane carries through to bound objects. The MMH method is particularly suitable for addressing questions about high-density regions, since these are resolved very finely indeed. I select peaks by searching for local maxima and then eliminating all those that are within $2h^{-1}$ Mpc of each other to avoid double counting. I then calculate the excess in the baryon fraction in a sphere of $1h^{-1}$ Mpc radius around each peak over the cosmological value. The population of peaks is separated into those that are within $5h^{-1}$ Mpc of the wake plane and those farther away. The results are summarized in Table 2. I find no significant baryon excess in the highest density peaks in the wake region, independent of the value of $\Omega_b$.

How can this be explained? I suggest that the spherical collapse of high-density peaks erases the initial baryon bias caused by the peculiarities of wake formation. The changing geometry of the density field drives the following mechanism.

At first, the baryons are tightly bound in the Newtonian gravitational potential of the planar wake, $\mathcal{V}_{\text{plane}}(x) \propto x$, where $x$ is the distance from the wake plane. Assuming that at late times the gas is isothermal and in equilibrium in this potential simplifies the hydrodynamic equations, and there is a simple analytic solution for the one-dimensional density

\begin{table}
\centering
\caption{Baryon Fractions in High-Density Regions}
\begin{tabular}{lcc}
\hline
Baryon Excess & $\Omega_b = 0.05$ & $\Omega_b = 0.1$ \\
\hline
Within 5 $h^{-1}$ Mpc of wake plane & 1.03 $\pm$ 0.09 & 1.09 $\pm$ 0.156 \\
Away from wake & 1.08 $\pm$ 0.16 & 1.035 $\pm$ 0.061 \\
\hline
\end{tabular}
\begin{tablenotes}
\item Note.—This table shows the excess of baryon fractions in density peaks over the cosmological value with 1 $\sigma$ standard deviations.
\end{tablenotes}
\end{table}

\textsuperscript{5} For technical information about COSMOS, see http://www.damtp.cam.ac.uk/cosmos/home.html.

\textsuperscript{6} Fluctuations in baryon bias away from the wake are less pronounced and originate from the caustics that form when pancakes collapse.
Fig. 3.—Averaged CIC-smoothed density in the gas. Snapshots are taken at $z = 8$, 4, 2, 1, 0.5, and 0, from top left to bottom right. The wake can be clearly seen in the center of the box at early times, and later structure emerges from the Gaussian perturbation background. The 28 contours are equally spaced in smoothed averaged density from 1.47 to 36.64.
Fig. 4.—Same as Fig. 3, but for the dark matter. Here the 28 contours range from 1.31 to 32.30.
Fig. 5.—Same as Fig. 3, but for the fraction of gas to dark matter densities. At late times, the wake becomes warped as the result of neighboring fluctuations, which causes the apparent doubling of the wake plane (see Fig. 6 for a cut through the $z = 0$ volume). The 28 contours range from 0.50 to 1.97.
profile,
\[
\rho(x) = \frac{M}{2L_{\text{sheet}}} \operatorname{sech}^2 \left( \frac{x}{L_{\text{sheet}}} \right),
\]

where \( M \) is the mass per unit area of the sheet and \( 2L_{\text{sheet}} \) is its thickness (for a detailed derivation, see the Appendix). This density falls off exponentially away from the wake. As long as the one-dimensional symmetry is a good approximation, the dark matter remains in the oscillatory mode found by Sornborger (1997) and is thus prevented from reaching the isothermal configuration.

When a density peak starts to form, the near-planar symmetry is supplanted by a near-spherical symmetry in the vicinity of the peak. This gives rise to a point-charge potential \( \psi_{\text{point}}(r) \propto 1/r \), and the isothermal configuration is now less tightly bound, as the asymptote of the density decays as \( r^{-2} \) (e.g., Peebles 1993). At the same time, the symmetry change allows the dark matter fluid to escape the oscillatory mode and settle down to the isothermal equilibrium distribution. The resulting inflow of dark matter erases the baryon excess in the peak.

This explanation is corroborated when one examines the simulation volume itself. Often baryon excess is seen surrounding bound objects at the end of the simulation, indicating that dark matter flowed from the outside into the center of the object. An example can be seen in Figure 7; a slice through the simulation volume that contains the object
at coordinate positions (12, 7.5) in the final frame of Figure 3 shows that it is close to the center of the wake (warped away from the center of the box by the other fluctuations) and punches a hole into the sheet of baryon bias.

4. CONCLUSIONS AND IMPLICATIONS FOR LARGE-SCALE STRUCTURE

I study a realistic scenario of cosmic string wake formation with a view to understanding the differences in gas and dark matter dynamics that are caused by the peculiarities of wakes. For this purpose, I use a state-of-the-art hydrocode with a view to understanding the differences in gas and dark matter flows at late times.

Even if the thin sheet of baryon bias could lead to an enhancement in galaxy formation, it is difficult to see how this would affect the galaxy power spectrum at 100 h⁻¹ Mpc scales. Two scales are important for the wake: the thickness across it and the size along it. The I-model predicts the size of straight sections of dominant wakes to be 20 h⁻¹ Mpc, and the wake thickness to be 1.5 h⁻¹ Mpc. The power spectrum of even an extreme density field made out of cells with thin walls of this characteristic size covered with “galaxy wallpaper” would cut off above 20 h⁻¹ Mpc, failing to contribute to larger scale power.

In summary, the cosmic string scenario fails to provide a natural explanation for the high baryon fractions observed in rich clusters. While this work is not directly relevant to the b₁₀₀₀ problem posed by Albrecht et al. (1997a), I present heuristic arguments as to why the peculiarities of wakes may be less important to galaxy formation and biasing of the power spectrum on large scales than hitherto conjectured.

I am indebted to U.-L. Pen for making his hydrodynamics code available to me. I would also like to thank A. Albrecht, J. Bartlett, N. Gnedin, U.-L. Pen, A. Sornborger, and A. Stebbins for stimulating discussions and the referee, R. Brandenberger, for fruitful comments. I acknowledge the Knowles Studentship of the University of London and support from the UK High Performance Computing Consortium, who granted me access to COSMOS, the UK National Cosmology Supercomputer.

However, it should be remembered that the size of dominant wakes is model dependent. While detailed simulations are still outstanding, other models of cosmic strings may predict somewhat larger values. Still, even a size of 60 Mpc must be considered extreme.

APPENDIX

THE ISOTHERMAL SHEET

The one-dimensional Boltzmann equation for an ideal gas in terms of the phase space density f(x, v) is

$$\frac{df}{dt} f(x, v) = 0. \quad (A1)$$

Let the density ρ(x) = ∫ f(x, v)dv and the average streaming velocity ⟨v⟩ = ρ⁻¹ ∫ vf(x, v)dv. The zeroth and first moments of equation (A1) are (see, e.g., Binney & Tremaine 1987)

$$\frac{∂}{∂ t} ρ + \frac{∂}{∂ x} (ρ⟨v⟩) = 0 \quad (A2)$$

$$ρ\frac{∂}{∂ x}⟨v⟩ + ρ⟨v⟩\frac{∂}{∂ x}x = -4πGρ \int_0^x \rho(r)dr - \frac{∂}{∂ x} (ρ⟨v^2⟩ - ρ⟨v⟩^2). \quad (A3)$$

Using the ideal gas equation of state identifies ρ⟨v²⟩ as the gas pressure and ⟨v²⟩ = k_B T/m, where m is the mass of a gas particle, T is the gas temperature, and k_B is Boltzmann’s constant.

As the simulations show, close to the center of the wake the baryons sit in a pressure-supported peak until spherical collapse sets in. To approximate the one-dimensional peak profile close to the center, I assume that the baryons have interacted sufficiently to be in thermal equilibrium and that streaming velocities can be neglected compared to thermal effects.
Hence, $\langle v \rangle \approx 0$, and using the isothermal assumption $\partial_z \langle v^2 \rangle = 0$ gives

$$\langle v^2 \rangle \partial_z \rho = -4\pi G \rho \int_0^x \rho(r) \, dr . \quad (A4)$$

Differentiation yields

$$\rho \partial_{xx} \ln \rho = -\frac{4\pi G}{\langle v^2 \rangle} \rho , \quad (A5)$$

which, together with the condition that the maximum density be at the origin, has the solution

$$\rho(x) = C \frac{k_B T}{8\pi G m} \text{sech}^2 \left( \frac{\sqrt{C}x}{2} \right) . \quad (A6)$$

To fix the remaining constant of integration $C$, let the mass per unit area in the sheet be

$$M \equiv \int_{-\infty}^{+\infty} \rho(r) \, dr = \frac{k_B T \sqrt{C}}{2\pi G m} .$$

Then

$$\rho(x) = \frac{M}{2L_{\text{sheet}}} \text{sech}^2 \left( \frac{x}{L_{\text{sheet}}} \right) , \quad (A7)$$

where $2L_{\text{sheet}} = 2k_B T/\pi G m M$ is the thickness of the sheet.

These formulae reproduce the scaling of $M$ with $L_{\text{sheet}}$ I observe in my simulations. Typically, for $\Omega_c = 0.05$ and at $z = 0$, a fifth of the total mass of the box, that is, $8.6 \times 10^{47}$ g, is within the inner wake region. This gives $M \approx 2.4 \times 10^{-4} \text{ g cm}^{-2}$, $T \approx 1.4 \times 10^6$ K, and hence $2L_{\text{sheet}} \approx 4.4 \times 10^{24} \text{ cm} \approx 1.4 \text{ Mpc}$, very close to the observed thickness of the wake.

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These formulae reproduce the scaling of $M$ with $L_{\text{sheet}}$ I observe in my simulations. Typically, for $\Omega_c = 0.05$ and at $z = 0$, a fifth of the total mass of the box, that is, $8.6 \times 10^{47}$ g, is within the inner wake region. This gives $M \approx 2.4 \times 10^{-4} \text{ g cm}^{-2}$, $T \approx 1.4 \times 10^6$ K, and hence $2L_{\text{sheet}} \approx 4.4 \times 10^{24} \text{ cm} \approx 1.4 \text{ Mpc}$, very close to the observed thickness of the wake.