EQUATION OF STATE FOR HYBRID COMPACT STARS WITH A NONLOCAL CHIRAL QUARK MODEL

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We study the thermodynamics of two flavor color superconducting (2SC) quark matter within a nonlocal chiral quark model, using both instantaneous and covariant nonlocal interactions. For applications to compact stars, we impose conditions of electric and color charge neutrality as well as $\beta$ equilibrium and construct a phase transition to the hadronic matter phase described within the Dirac-Brueckner-Hartree-Fock (DBHF) approach. We obtain mass-radius relations for hybrid star configurations which fulfill modern observational constraints, including compact star masses above $2M_\odot$. 

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1. Theoretical framework

Although we understand hadrons as bound states of quarks, there is no unified approach available yet that adequately describes the thermodynamics of the transition from nuclear to quark matter. Here we rely on a two-phase description, in which both states of matter are treated separately, using appropriate effective field theories.

The hadronic matter EoS is obtained within the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) theory. In this ab-initio approach, the nucleon inside the medium is dressed by a self-energy $\Sigma$, obtained from the Bethe-Salpeter equation for the nucleon-nucleon T-matrix in the ladder approximation using the Bonn-A potential in the interaction kernel.

The quark matter phase is described within a nonlocal chiral quark model, which includes scalar and vector quark-antiquark interactions —driven by coupling constants $G_S$ and $G_V$, respectively— and anti-triplet scalar diquark interactions —driven by a coupling constant $H$. We have carried out a systematic analysis considering both instantaneous and fully covariant nonlocal interactions, and study the corresponding parameter ranges. Explicit expressions for the nonlocal Lagrangians can be found in Refs. Our analysis is performed within bosonized versions of these quark models, in which scalar, vector and diquark bosonic fields are introduced. We consider the mean field approximation (MFA), expanding these fields around their respective vacuum expectation values (VEVs) and keep the lowest order relevant contributions (fluctuations are considered in the vacuum case to adjust model parameters from low energy phenomenology). The only nonvanishing mean field values in the scalar and vector sectors correspond to isospin zero fields $\bar{\sigma}$ and $\bar{\omega}$, respectively, while in the diquark sector, owing to the color symmetry, one can perform a rotation in color space ending up with a single nonvanishing VEV $\bar{\Delta}$. In the presence of a baryochemical potential $\mu_B$, the thermodynamical potential per unit volume can be written as

$$\Omega^{\text{MFA}} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\bar{\Delta}^2}{2H} - \frac{\bar{\omega}^2}{2G_V} - \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \ln \det \left[ S^{-1} (\bar{\sigma}, \bar{\Delta}, \bar{\omega}, \mu_{f_c}) \right],$$  

(1)

where the inverse propagator $S^{-1}$ is a $48 \times 48$ matrix in Dirac, flavor, color and Nambu-Gorkov spaces (its explicit form for instantaneous and covariant models can be found in Refs. respectively). Since we require the quark matter to be electric and color charge neutral, we introduce different chemical potentials $\mu_{f_c}$ for each quark flavor $f$ and color $c$. However, if the system is in chemical equilibrium all $\mu_{f_c}$ can be written in terms of three independent quantities, namely the baryon chemical potential $\mu_B$, an electric chemical potential $\mu_Q$ and a color chemical potential $\mu_8$.

The values of $\bar{\sigma}$, $\bar{\Delta}$ and $\bar{\omega}$ can be obtained from a set of coupled gap equations

$$\frac{d\Omega^{\text{MFA}}}{d\Delta} = 0, \quad \frac{d\Omega^{\text{MFA}}}{d\omega} = 0, \quad \frac{d\Omega^{\text{MFA}}}{d\sigma} = 0.$$  

(2)
Due to our nonlocal description, these VEVs come together with momentum-dependent form factors, which generate, e.g., momentum-dependent effective quark masses.

In order to enable neutron stars to be in \( \beta \)-equilibrium, we have to account for electrons and muons. The latter can be treated as free Fermi gas contributions to the grand canonical thermodynamical potential. The electric and lepton chemical potentials turn out to be related by

\[
\mu_{dc} - \mu_{uc} = -\mu_Q = \mu_e = \mu_\mu.
\]  

Finally, \( \mu_Q \) and \( \mu_8 \) become fixed by the conditions of vanishing electric and color densities. In this way, for each value of the temperature \( T \) and chemical potential \( \mu_B \) one can find the values of \( \Delta, \bar{\sigma}, \bar{\omega}, \mu_Q \) and \( \mu_8 \) that solve Eqs. (2).

While the thermodynamics of the three-flavor case with color flavor locking (CFL) phases and selfconsistently determined mean fields could recently be achieved within the instantaneous chiral quark model \(^4\), the extension to the covariant case is a formidable task and deferred to future work. A justification of our restriction to the two flavor case in the present work is given by the following argument. In three flavor models the onset of strange quark matter is found at rather large densities. As a result hybrid star configurations with a strange matter core are found very close to the maximum mass of the corresponding two flavor configuration only \(^3\). Since the corresponding color flavor locking phase in three flavor matter turned out to render compact stars unstable, the maximum compact star masses obtained in the present study might be slightly overestimated compared to a three flavor calculation but should be accurate within a range of 0.1 \( M_\odot \).

2. Numerical results and discussion

In order to obtain numerical predictions from our nonlocal models, we set our model parameters to be in accordance with \( \mu = 0 \) phenomenology. In particular, we reproduce the empirical values of \( m_\pi \) and \( f_\pi \), and obtain a phenomenologically reasonable value of the chiral condensate \( \langle 0|\bar{q}q|0\rangle^{1/3} = -230 \text{ MeV} \) by adjusting the current quark mass, the scalar coupling \( G_S \) and the model interaction range \( \Lambda \). The couplings \( G_V \) and \( H \) as free parameters have been arranged to ensure accordance with phenomenological constraints from flow data analyses of heavy ion collisions \(^5\) and mass and mass-radius constraints from compact star observations. Regarding the flow constraint, it is worth to mention that the hadronic DBHF EoS turns out to be too stiff for densities above \( \sim 0.5 \text{ fm}^{-3} \). This discrepancy is cured in our hybrid model after appropriately tuning the parameters: Since the hadron-to-quark matter transition softens the slope of the high density EoS, it becomes compatible with the data if the transition takes place at a critical density of \( \sim 0.5 \text{ fm}^{-3} \).

Finally, we demonstrate the compatibility of our models with modern compact star observations. In Fig. 1 we show sequences of compact star configurations obtained as solutions of the Tolman-Oppenheimer-Volkoff equations for selfgravitating...
Fig. 1. Mass-radius relations for neutron star configurations (DBHF EoS, solid line) and hybrid star configurations with hadronic shell and a color superconducting quark matter core for covariant (dashed) and instantaneous (dashed-dotted) nonlocal chiral quark models, see also Ref. 5.

hybrid star matter described by the theoretical schemes presented above. It can be seen that both covariant (dashed line in Fig. 1) and instantaneous (dash-dotted line) quark models are in good agreement with observational constraints. In particular, they reproduce the high mass of pulsar PSR J0751+1807 and the mass-radius bounds from the isolated neutron star RX J1856 as well as the debated X-ray burster EXO 0748−676. Thus, hybrid stars possessing quark matter cores appear to be compatible with all present observational constraints.

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