Direct measurements of phonon–phonon scattering in liquid $^4$He

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Abstract. We have measured the scattering between two phonon-beams, at different angles, in HeII. When the angle between their propagation directions is 30°, we detect scattering between the low-energy phonons ($\epsilon/k_B \sim 1$ K) in the two beams. No scattering is seen between low-energy phonon beams at 40° due to the low three phonon scattering rate at large angles. At 30° and 40°, we detect scattering between the high-energy phonons ($\epsilon/k_B > 10$ K) in the two beams for the first time. This is due to four phonon scattering. There is no detectable scattering between the low- and high-energy phonons at these small angles. When beams collide nearly head-on, there is scattering between the low- and high-energy phonons, and also scattering between the high-energy phonons and R$^+$-rotons. The power and pulse length dependencies of these interactions are presented and discussed. The results are analysed to obtain the lifetime of the probe phonons in the scattering beam of phonons.
1. Introduction

Phonon–phonon scattering determines the dynamic behaviour of phonons in liquid $^4$He, and it also determines their quasi-equilibrium energy and momentum distribution if the phonon system is anisotropic [1]–[3]. In this paper, we present direct measurements of phonon–phonon scattering rates for different types of scattering in well-defined geometries. Liquid helium has only longitudinal phonons as it cannot support transverse waves so it is a particularly simple system that is open to detailed theoretical comparisons. It is chemically and, in our case, isotopically very pure and at sufficiently low temperatures there is a negligible number of thermal phonons, so phonon–phonon scattering can be studied in a system where there is no extraneous scattering.

The two most important scattering processes are three phonon processes (3pp) [4, 5] and four phonon processes (4pp) [6]–[8]. In 3pp, one phonon creates two phonons or conversely, two phonons combine into one phonon. In 4pp two phonons combine and create two new phonons. Energy and momentum are conserved in these processes which limits the momentum and energy ranges where they are possible. 3pp can occur in liquid $^4$He because the dispersion curve initially bends upwards from linearity before it bends downwards at higher energies [9, 10]. This is called anomalous dispersion because it is rare in condensed matter systems. The dispersion curve can be expressed as $\epsilon = c_p(1 + \psi(p))$ where $\epsilon$ and $p$ are the energy and momentum of the phonon, $c_p$ is the velocity of sound and $\psi(p)$ determines the deviation from linearity; $|\psi(p)|$ is well below unity for most of the phonon momentum range in liquid $^4$He. At zero pressure, $\psi(p_c) = 0$ at $c_p/k_B = 10$ K. Scattering by 3pp is kinematically allowed in the range $0 \leq p \leq p_3$ where $p_3$ is the momentum where one phonon creates two collinear phonons. At zero pressure $c_p/k_B = 8.7$ K; found from the dispersion curve measured by neutron scattering [10]. As the pressure is increased, $p_c$ decreases and is zero at 19 bar [11, 12].

In the range $p_3 \leq p \leq p_c$ there are higher-order processes which allow spontaneous decay; these are 1-to-$n$ processes where one phonon can decay into $n$ phonons and conversely $n$ phonons combine to form one phonon. As for 3pp, 1-to-$n$ processes are allowed up to certain maximum energies determined by the detailed shape of the dispersion curve. For example, 1-to-3 processes are allowed up to $c_p/k_B = 9.5$ K and are found to have comparable scattering rates to 3pp, [13]; and 1-to-$\infty$ are allowed up to $p_c$.

Scattering by 4pp is allowed at all values of momentum. However, the 4pp scattering rate is slow compared to 3pp and 1-to-$n$ processes [13] and so for $0 < p < p_c$ 4pp is slower than the fastest processes, therefore 4pp processes are ignored in this range. However, for $p > p_c$, 4pp is
the fastest phonon scattering process and plays the dominant role in phonon creation and decay at high momenta.

The interplay between 3pp, 1-to-\textit{n} and 4pp can be seen clearly when a short (100 ns) heat pulse is injected into pure and cold (50 mK) liquid $^4$He. The heat is initially in the form of low-energy phonons which rapidly come into a quasi-equilibrium though 3pp in a time, $\sim 10^{-8}$ s [2]. This is an order of magnitude shorter than the duration of the heat pulse. These phonons, in quasi-equilibrium, occupy a very limited angular segment of momentum space: to a first approximation they occupy a cone with a cone angle typically $10^\circ$ and cone axis along $p_z$, where $z$ is the direction of propagation of the pulse in real space. So these phonons form a highly anisotropic phonon system.

Within this pulse, phonons with momentum $p > p_c$ are readily created by 4pp. The process may be represented as $l_1 + l_2 \rightarrow h + l_3$, where $l_i$ are low momentum phonons with $p < p_c$, and $h$ is a high momentum phonon with $p > p_c$. We shall refer to these types as l-phonons and h-phonons. Once the h-phonon has been created it has a relatively long lifetime within the pulse, typically $\sim 10^{-6}$ s [3], and before it has time to decay, it is left behind by the pulse because its group velocity is considerably less than the velocity of the l-phonon pulse, i.e., 189 and 238 ms$^{-1}$ respectively. Once the h-phonon has been left behind, it is in helium with an insignificant number of thermal phonons and so its lifetime is only limited by the size of the container.

The result of the processes described above means that a single heat pulse creates two propagating phonon pulses in the liquid helium; a fast non dispersed l-phonon pulse that travels at the velocity of sound [14], and a slower dispersed h-phonon pulse [15]. It is dispersed because higher momentum phonons have lower group velocities and are created at all distance in the liquid. Although the behaviour of this system depends on 3pp and 4pp, the details of these processes are difficult to extract from the collective behaviour. This motivated the present study; we wanted to make measurements where the details of the scattering processes could be determined.

Much is already known about 3pp scattering. The fact that phonons spontaneously decay was established by measuring the widening of a collimated beam, as it propagated [16]. There are several good indications that 3pp spontaneous decay rates are very fast. The neutron scattering linewidth was measured using neutron spin echo [17]. It was found that the scattering rate increased rapidly with momentum and reached a peak value of $4 \times 10^9$ s$^{-1}$ at wavevector $q = 0.4$ Å (or $cp/k_B = 7.2$ K). According to theory, the spontaneous decay rate varies as $q^4$ [4, 5], and at $q = 0.4$ Å the scattering rate is $1.45 \times 10^{10}$ s$^{-1}$, which is much higher than the measured value.

When phonons are in a strongly interacting group, the scattering rates are much higher than the spontaneous decay rate. For example, for a phonon with $cp/k_B \sim 1.5$ K in the isotropic system at $T = 1$K, the scattering rate is $\sim 2 \times 10^9$ s$^{-1}$, see figure 3(b) in [2] and in a typical anisotropic system the scattering rate is $\sim 2 \times 10^8$ s$^{-1}$ see figure 5 in [2], compared with the theoretical spontaneous decay rate of $\sim 7 \times 10^6$ s$^{-1}$ [4, 5].

In [18], it was shown that the l-phonon pulse has an area, typically $> 1$ mm$^2$, where the energy density is constant. This is due to h-phonon creation which cools the l-phonons to $\sim 0.7$ K where the h-phonon creation rate is very low. The constant energy area is termed a ‘phonon sheet’ as its transverse dimensions are much greater than its thickness. At typical distances from the heater the sheet is nearly flat.

3pp scattering involves small angles between the two phonons created or annihilated. This was first shown by [16]. Recently, this has been seen in more detail by colliding two phonon sheets together [19]. The strongest interaction between the sheets occurs when the normals to the plane
of the sheets make an angle equal to the angle between typical phonons in a 3pp interaction, i.e., 12° to each other [20]. For two beams crossing at \( \alpha \sim 30° \), the scattering lifetime of the l-phonons is very long compared with the lifetime within the beams, because between the beams, most of the angles between phonons are larger than the maximum angle for three phonon scattering.

4pp scattering rates have been determined by measuring the attenuation of a beam of high-energy phonons in liquid helium at different temperatures [7]. The h-phonons are scattered by the ambient phonons whose energy and density increase with temperature. The measured scattering rates agreed with rates calculated with the corrected theory [7, 6]. In anisotropic phonon systems, the scattering rates have been calculated [21, 22].

A h-phonon is created by 4pp scattering. The rate for h-phonon creation, in a pulse of l-phonons, is \( \gtrsim 2 \times 10^4 \, \text{s}^{-1} \), as we know h-phonons are created in less than the propagation time \( 4.2 \times 10^{-5} \, \text{s} \) in experiments [15]. Theory predicts it to be of order \( 10^4 \, \text{s}^{-1} \) [3], but such rates depend strongly on the h-phonon momentum, and the energy and momentum of the l-phonon pulse [3]. The measured h-phonon signal has been successfully modelled at low pulse energies [23, 24].

Specific predictions for the angular distribution of phonons in equilibrium have been made, it is the Bose distribution which is a function of two parameters, the temperature \( T \) and the drift velocity of the pulse \( u \) [25]. The number of phonons, in unit volume, with energy \( \epsilon(p) \), and momentum \( p \) in the range \( p \rightarrow p + dp \), at an angle \( \theta \) to the symmetry axis in the range \( \theta \rightarrow d\theta \), is given by [25]

\[
N(p, \theta) \, dp \, d\theta = \left[ \exp \left( \frac{\epsilon(p) - pu \cos(\theta)}{k_B T} \right) - 1 \right]^{-1} \frac{p^2 \sin(\theta) 2\pi \, d\theta \, dp}{(2\pi\hbar)^3}.
\]

Equation (1) shows that phonon states are most strongly occupied near \( \theta = 0 \), the propagation direction, and the distribution falls off rapidly as \( \theta \) increases. However, the occupation is not zero at any value of \( \theta \). The data reported here will test the validity this distribution function.

The decay rate of h-phonons in the l-phonon pulse, is strongly curtailed by the l-phonons only occupying a narrow range of angles in momentum space, see equation (1). In contrast, this narrow angular occupation has only a small effect on the creation rate. This asymmetry between creation and decay rates is due to the asymmetry in the angles between phonons in the 4pp; the angles between phonons of similar momentum are smaller than the angles between phonons of dissimilar momentum. For example consider 4pp scattering where phonons with energy 7 and 4 K combine and give two phonons with energy 10 and 1 K; for angles between the 7 and 4 K phonons in the range 0–10°, the angles between the 10 and 1 K phonons are in the range 53–54°. This follows directly from the measured dispersion curve and conservation of momentum and energy. Hence, h-phonon creation involves small angles and h-phonon decay involves large angles. As the l-phonons in the pulse are mostly in a narrow cone, and the h-phonons are mainly along the cone axis, the angles between the h- and l-phonons are mostly small. Hence, there are no l-phonons at the angle needed to scatter the h-phonons and so their decay rate is correspondingly low.

The experiments described in this paper are designed to investigate 3pp and 4pp between phonons at different angles, in a simple and well-defined experimental arrangement. This means that it should be possible to theoretically model the interactions and hence calculate the scattering rates in some detail. The paper is organized as follows. In section 2, the design of the experiment is discussed, in sections 3 and 4 the results of scattering at small and large angles respectively,
Figure 1. Panel (a) is a schematic of the experiment. Panel (b) shows in the top panel, a signal from a pulse which has been through a scattering beam, and in the bottom panel, a signal from an unscattered pulse. There are two cross-talk spikes in the top panel, from the two heater pulses. The first at $t = 0$ is from the scattering pulse and the second at $t = 14.3 \mu s$ is from the probe pulse. In the top panel, the l-phonon signal at $t \sim 65 \mu s$ is clearly smaller than the l-phonon peak in the bottom panel. The peak of the h-phonon signal at $t \sim 78 \mu s$ in the top panel, is smaller than the h-phonon peak in the bottom panel, but this is due to the attenuation of the tail l-phonons, due to the cooling of the substrate, which extends under the h-phonons. There is negligible attenuation of the h-phonons due to h–h scattering at this delay.

are presented. In section 5, expressions for the scattering lifetimes are derived and some values calculated, and in section 6, we draw conclusions.

2. Design of the experiment

The experimental method consists of crossing two beams of phonons and measuring the attenuation of one beam due to the other. The detected beam is called the probe beam and the other is called the scattering beam. A schematic of the experiment is shown in figure 1(a). The cylindrical collimators are machined from one piece of brass. They have internal radii of 3.0
and 6.5 mm and are 0.5 mm thick. The collimation holes are 1 mm diameter. Between heaters H0 and H4 the angle is 30°, and between heaters H1 and H2 the angle is 40°. The support for heaters H1, H2 and H4 has a radius 10.0 mm. The heater substrates are glass cover slips 0.12 mm thick. Heater H0 is glued on the inside of the outer collimator and H5 is glued on the inside of the inner collimator directly below the collimation hole. Bolometer B0 is glued on the inner surface of the outer collimator, and the other bolometers, B1, B2 and B4 are on a 5 mm radius and in line with their heater and collimation.

The attenuation of the probe beam is defined by

\[ A = \frac{s_0 - s_1}{s_0} = \frac{n_0 - n_1}{n_0}, \]

where \( s_0 \) and \( s_1 \) are the phonon signals in the probe beam without any scattering and after passing through the scattering beam, respectively, and \( n_0 \) and \( n_1 \) are the corresponding phonon fluxes. The beams are pulsed so that the times of flight determine which groups of phonons are interacting. The time that a volume element of the probe pulse overlaps the scattering pulse is called the crossing time [25] and it is determined by the lengths and widths of the pulses in the liquid helium. The l-phonon pulse has a length \( c t_p \) where \( t_p \) is the duration of the current pulse in the heater, typically \( 10^{-7} < t_p < 10^{-6} \) s. The h-phonon pulse is much longer and has a typical width at half height of \( 10^{-5} \) s, see figure 1, which is independent of \( t_p \) for \( t_p \lesssim 10^{-6} \). The transverse dimensions of the l- and h-phonon pulses are typically 1 mm, and are determined by the collimation. For there to be scattering, the crossing time must be longer than the inverse of the scattering rate.

As the scattering processes vary with the angle between the propagation directions of the pulses, the experiment is designed to cross beams at several angles. The arrangement is shown figure 1(a). The smallest angle between two beams is 30°; it is limited by the path length and the diameter of the collimating holes. Increasing the path length and decreasing the diameter of the collimating holes, increases the angular resolution but reduces the detected signal, so the geometry in figure 1(a) is a compromise between these effects. The largest angle is \( \sim 160^\circ \), so the beams are then meeting nearly head-on.

At small angles, we delay one pulse with respect to the other so that scattering between different phonons is selected, i.e. l–l, l–h and h–h phonon scattering. Delay is not useful at large angles as the pulses pass through each other over a wide range of delays.

The heaters are thin films of gold evaporated onto glass cover slips with area 1 mm². These are glued onto the brass with GE 7031 varnish. The phonon pulses are detected with superconducting zinc bolometers made from a zinc film 1 \( \times \) 1 mm, cut into a serpentine track. The zinc film was in a constant magnetic field, parallel to its plane, and held at its superconducting transition edge by a feedback circuit [26]–[28], which maintains a constant bolometer resistance at \( \sim 0.1 \) of its normal value at 4.2 K. The corresponding bolometer temperature is \( \sim 0.35 \) K. The cell was cooled to \( \sim 50 \) mK with a dilution refrigerator and filled with isotopically pure \(^4\)He [29].

Heaters were pulsed in pairs with currents from a LeCroy 9210 pulse generator with two independent but synchronous outputs, one of which could be delayed with respect to the other. Heater powers in the range 3.125–25 mW were used. The signals from the bolometer were amplified with an EG&G PAR 5113 preamplifier and digitally recorded with a Tektronix DSA 601A. Many signals were averaged to obtain a good signal to noise ratio.
Table 1. The two types of phonons, which mutually scatter, are shown in column 1; the angular range for the scattering is shown in column 2; the calculated delay time to the probe pulse from H0, for scattering with the pulse from H4, is shown in column 3; and the corresponding results are listed in column 4.

| Scattering type | Angle for scattering | Time with H0 and H4 | Comment |
|-----------------|----------------------|---------------------|---------|
| $l_p$–$l_s$     | $\alpha \lesssim 30^\circ$ | 14.7 $\mu$s | Very strong scattering for $\alpha < 12^\circ$ but orders of magnitude weaker at $\alpha \sim 27^\circ$ [20], examples for $\alpha = 30^\circ$ are shown in figures 2, 4–7 |
| $l_p$–$h_s$     | $50^\circ < \alpha < 180^\circ$ | 25.5 $\mu$s | Not seen at $\alpha \sim 30^\circ$ but can be seen at $\alpha \sim 160^\circ$ examples are shown in figures 8 and 9 |
| $h_p$–$l_s$     | $50^\circ < \alpha < 180^\circ$ | 7.7 $\mu$s | Not seen at $\alpha \sim 30^\circ$, but can be seen at $\alpha \sim 160^\circ$ examples are shown in figures 8 and 9 |
| $h_p$–$h_s$     | All $\alpha$          | 18.5 $\mu$s | Weak scattering at all $\alpha$, examples are shown in figures 3(a)–(c) for $\alpha = 30^\circ$ and 40$^\circ$ |
| $h_p$–$R^+$     | All $\alpha$          | NA                  | Moderate scattering at all $\alpha$, example is shown in figure 10 |

3. Results and discussion at small angles

In the first experiment, the probe beam was collimated with a hole 1 mm diameter in a shield 0.5 mm thick, which was 3 mm in front of the heater H0, see figure 1(a), and a similar collimator 3 mm in front of the bolometer B0. The total path length was 12.8 mm. The scattering beam had two similar collimators 3 and 6.5 mm in front of its heater H4. The two lines through the centres of the collimators crossed at 6.4 mm in front of the probe heater and 9.9 mm in front of the scattering heater. The times for the various phonon interactions were calculated from these distances and the phonon velocities; 238 ms$^{-1}$ for l-phonons and 189 ms$^{-1}$ for h-phonons. The times for scattering between the l- and h-phonon peaks, i.e., $l_p$–$l_s$, $l_p$–$h_s$, $h_p$–$l_s$ and $h_p$–$h_s$ scattering, where the subscripts denote the probe and scattering beams, are respectively 14.7, 25.5, 7.7 and 18.5 $\mu$s. A guide to the results is shown in table 1.

Figure 1(b) shows the probe signals with and without the scattering pulse at a delay of 14.3 $\mu$s. We see that the l-phonon signal is decreased by the scattering beam.

Figure 2 shows the attenuation of the peak of the $l_p$-phonons as a function of the delay of the probe beam relative to the scattering beam. The most significant attenuation of the probe beam is at 14.3 $\mu$s. From this it is clear that the strongest interaction, at an angle of 30$^\circ$, is between the l-phonons of both beams.

There is a weaker attenuation of the $h_p$-phonons at 19$\mu$s which is shown in figure 3(a). This delay indicates that there is an interaction between the $h_p$- and $h_s$-phonons at this angle. The $h_p$-phonon attenuation occurs over a wider range of delays than the $l_p$-phonon attenuation because the h-phonons are more dispersed than the $l_p$-phonons. There is no interaction between the l-phonons of one pulse and the h-phonons of the other pulse at this angle.

The beams can be interchanged by detecting the pulse from heater H4 and scattering it with the pulse from heater H0. The general behaviour is the same as before, there is strong l–l scattering and weak h–h scattering. The attenuation of the $h_p$-phonons is shown as a function of delay in figure 3(b). Using pulses from heaters H1 and H2, at 40$^\circ$ to each other, there is only h–h attenuation, as shown in figure 3(c); there is no measurable l–l attenuation at this angle.
Figure 2. The attenuation of the peak l-phonon signal (H0 to B0), due to the pulse from heater H4, as a function of delay. The attenuation at 14.3 $\mu$s is due to the l-phonons, from the two pulses, interacting. Heaters were pulsed with 12.5 mW and 100 ns and $\alpha = 30^\circ$.

This is to be expected because the 3pp scattering rate is very low at 40$^\circ$ because of the small number of phonons at the right angle for 3pp scattering. Again, there was no sign of l–h or h–l scattering.

Figure 4 shows the effect of varying the pulse length of the scattering pulse in the first arrangement, while the probe pulse is constant at 6.25 mW and 100 ns. The delay is constant at 14.3 $\mu$s, so the scattering is between l-phonons in the two pulses. We see that the attenuation initially increases linearly with pulse length, then increases more slowly and saturates when the scattering pulse is more than $\sim 1\mu$s. The value of the saturation attenuation is only at 0.8 and not 1, which shows that, due to the angular spread of phonon momenta, some of the l-phonons in the probe beam are at too large an angle to scatter with phonons in the other beam.

Figure 5 shows the effect of varying the power in the scattering pulse. The phonon density increases with increasing pulse power which leads to more scattering and a higher attenuation. The power in the probe beam is constant at 12.5 mW, and both pulses are 100 ns duration. The delay is again 14.3 $\mu$s so selecting l–l scattering. The attenuation initially increases linearly with scattering pulse power and then increases more slowly. With the powers used in this experiment, we do not see the attenuation saturate as it did with pulse duration.

When two beams collide, both beams lose the same number of phonons. Usually the probe beam is made much weaker than the scattering beam so the probe beam is significantly attenuated but the scattering beam remains nearly constant. If both beams are equal, then they both are attenuated the same amount, and if the probe beam is stronger than the scattering beam then the attenuation of the probe beam saturates when all the scattering beam has been scattered away. In figure 6, the power in both beams is increased together but, because of the different distances
Figure 3. (a) The attenuation of the peak h-phonons (H0 to B0), due to the h-phonon pulse from heater H4 ($\alpha = 30^\circ$), as a function of delay to H0. The attenuation, centred at 19 $\mu$s, is due to the h-phonons, from the two pulses, interacting. (b) The attenuation of the peak h-phonons (H4 to B4), due to the h-phonon pulse from heater H0 ($\alpha = 30^\circ$), as a function of delay to H0. The attenuation, is due to the h-phonons, from the two pulses, interacting. (c) The attenuation of the peak h-phonons (H2 to B2), due to the h-phonon pulse from heater H1 ($\alpha = 40^\circ$), as a function of delay of H2. The attenuation, centred at zero delay because in this case the path lengths are equal, is due to the h-phonons, from the two pulses, interacting. In all cases the heaters were pulsed with 12.5 mW and 100 ns.

and collimation, the scattering beam is weaker than the probe beam. The pulse duration of both beams is 100 ns and the delay is 14.3 $\mu$s so that the scattering is l–l, and the angle between the beams is 30°. The attenuation of the probe beam is low at low powers because the phonon densities are low which makes the scattering rate low. It rapidly increases with power and at 12.5 mW it has saturated at an attenuation of 0.32. This indicates that the probe beam has $1/0.32 \sim 3$ times the phonon density of the scattering beam.
4. Scattering at large angles

When the angle between the probe and scattering beams is large, the l–l scattering will be zero because the angle is too large for 3pp scattering. Hence, the attenuation of h-phonons can be by either h–l or h–h scattering. In this section, we present results where the angle between the beams is \( \sim 160^\circ \). At such a large angle, the beams overlap for such a long time that different interactions cannot be well separated by times of flight. However, the delay run shows that the attenuation of the \( h_p \)-phonons is constant for delays between 7 and 20 \( \mu s \) and then the attenuation decreases linearly to zero at a delay of \( \sim 47 \mu s \). At this delay, the \( h_p \)-phonons reach the heater H5 at the start of the scattering pulse (9.4 mm/189 ms\(^{-1}\) = 50 \( \mu s \)), so we would expect no attenuation at this delay. A delay of 20 \( \mu s \) is consistent with the \( h_p \)-phonons meeting the \( l_s \)-phonons at 6.4 mm from heater H0: the \( l_s \)-phonons have travelled 3.2 mm from heater H5.

The \( h_s \)-phonons are strongly concentrated in the direction normal to the heater H5 [3, 30] this direction is parallel to, but off-set, from the h-phonons in the probe beam and so little interaction between these h-phonons and the h-phonons of the probe beam is expected. The concentration of the \( h_s \)-phonons along direction normal to heater H5 also explains the observation that the l-phonons of the probe beam are not noticeably attenuated by the h-phonons from heater H5. All this suggests that the attenuation of the h-phonons in the probe beam is due to \( h_p \)-\( l_s \) scattering rather than \( h_p \)-\( h_s \) scattering. Further evidence for this conclusion is given by figure 7.

Figure 7 shows the attenuation of the peak of the h-phonon probe signal as a function of the pulse length of the scattering beam. It is measured at a delay of 17 \( \mu s \) where the attenuation is independent of delay. The probe beam is 12.5 mW and 100 ns, and the scattering beam power is 25 mW, the highest power used in these experiments. The figure shows that the
Figure 5. The attenuation of the peak l-phonon signal (H0 to B0), due to the l-phonon pulse from heater H4 ($\alpha = 30^\circ$), as a function of the H4 pulse power. H0 pulse: 12.5 mW 100 ns, H4 pulse: 100 ns, delay 14.3 µs.

![Graph showing attenuation vs. power to H4](image)

Attenuation rises with pulse length. Previously, we have established that the h-phonon signal saturates at a heater pulse duration $\sim 0.3\mu s$ [31], where the h-phonons are scattered within the l-phonon pulse and do not escape the pulse, i.e., $\sim 0.3\mu s$ is the boundary between short- and long-pulse regimes. In contrast, the l-phonon density increases linearly with pulse length see figure 6 of [18] which is similar to the behaviour of the attenuation shown in figure 7, and there is no sign of the attenuation saturating at a scattering pulse length of 0.3 µs where no more h-phonons are created. This supports the conclusion that the h-phonons of the probe beam scatter with the l-phonons of the scattering beam.

Figure 8 shows the attenuation of the peak of the h-phonon probe signal as a function of the power of the scattering beam. The probe beam is 12.5 mW and 100 ns, and the scattering beam has a pulse length of 1 µs. The attenuation rises with pulse power in a similar way to the rise in the l-phonon signal, see figure 7 of [18]. Again this power dependence supports the idea that these l-phonons are the cause of the scattering. (The attenuation does not appear to go through zero power but this offset is within the random error of $\sim 2\%$, it stems from the reference probe being a little too small when there is no scattering pulse.)

When the probe pulse length is 100 ns and the scattering beam is 25 mW and 500 ns, the attenuation of the h-phonons in the probe beam is 17%, independent of probe power in the measured range $3 < W_p < 25$ mW. As the probe beam is always much smaller than the scattering beam then the scattering beam is unaffected by the probe beam and so the fractional decrease in the h-phonon probe signal is independent of probe beam power, as is found.

Figure 9 shows the effect of varying the pulse width of the probe pulse. The power of the probe beam is 12.5 mW and the scattering beam is 25 mW and 500 ns. The attenuation decreases...
Figure 6. The attenuation of the peak l-phonon signal (H0 to B0), due to the l-phonon pulse from heater H4 (α = 30°), as a function of equal pulse power to H0 and H4. Both pulses 100 ns, delay 14.3 µs.

as the pulse width increases which is to be expected when the probe pulse scatters away a significant part of the scattering beam; the later part of the probe beam is attenuated less than the leading part, and so a long pulse, as a whole, is attenuated less than a short pulse. Figure 9 shows that this happens when the probe pulse is longer than ∼200 ns. For very long probe pulses the attenuation should asymptotically approach zero.

Finally at this angle the scattering beam was set to a low power, 3 mW, and very long-pulse lengths, up to 10 µs. Under these conditions the heater creates mainly R⁺-rotons and very few h-phonons [31]. Figure 10 shows that, for a probe beam of 12.5 mW and 100 ns, the h-phonons are scattered by rotons. The scattering increases with scattering beam pulse length which just increases the number of rotons in the dispersed pulse of rotons.

5. Analysis

A probe beam passing through a scattering beam, where the probe beam density is much smaller than the density of the scattering beam so the scattering beam density is not altered by the scattering, decays exponentially with time $t$. The probe phonon density after scattering, $n(t)$, is given by

$$\frac{n(t)}{n_0} = \exp\left(-\frac{t}{\tau}\right),$$

(3)

where $\tau$ is the lifetime of a probe beam phonon in the scattering beam and $n_0$ is the initial probe density.
Figure 7. The attenuation of the peak h-phonon signal (H0 to B0), due to the l-phonon pulse from heater H5 ($\alpha \sim 160^\circ$), as a function of the H5 pulse length. H0 pulse: 12.5 mW 100 ns, H5 pulse 25 mW, delay 17 $\mu$s.

Figure 8. The attenuation of the peak h-phonon signal (H0 to B0), due to the l-phonon pulse from heater H5 ($\alpha \sim 160^\circ$), as a function of the H5 pulse power. H0 pulse: 12.5 mW 100 ns, H5 pulse length 1000 ns, delay 17 $\mu$s.
Figure 9. The attenuation of the peak h-phonon signal (H0 to B0), due to the l-phonon pulse from heater H5 ($\alpha \sim 160^\circ$), as a function of the H0 pulse length. H0 pulse: 12.5 mW, H5 pulse: 25 mW 500 ns, delay 17 $\mu$s.

Figure 10. The attenuation of the peak h-phonon signal (H0 to B0), due to the $R^+$-rotons pulse from heater H5 ($\alpha \sim 160^\circ$), as a function of the H5 pulse length. H0 pulse: 12.5 mW 100 ns, H5 pulse: 3.125 mW, delay 17 $\mu$s.
The time that an element of the probe pulse is in the scattering pulse is given by the crossing time, $t_x$, [25]. For two sheet-like pulses, where the dimension in the propagation direction is much smaller than the transverse dimensions, this is given by [25]

$$t_x = \frac{t_{ps}}{2\sin^2(\alpha/2)}, \tag{4}$$

where $t_{ps}$ is the duration of the scattering pulse and $\alpha$ is the angle between their propagation directions.

For long and collimated pulses, the phonons are in a box-shaped volume where the dimension of the pulses in their propagation directions is larger than their transverse dimension $w$ in the plane of the heater normals. The crossing time for these box-shaped pulses, at small values of $\alpha$, can be derived from their relative velocity $2v \sin(\alpha/2)$ where $v$ is the speed of the pulses, and the length of the path, $w/\cos(\alpha/2)$, of the probe pulse through the scattering pulse in the direction of their relative velocity. $t_x$ is given by

$$t_x = \frac{w}{v \sin(\alpha)}. \tag{5}$$

For these long pulses, the ends of the pulses experience a shorter crossing path than $w/\cos(\alpha/2)$, so the crossing time will be overestimated by equation (5), as it is also for cylindrically shaped pulses.

From equations (2) and (3) and putting $t = t_x$, the attenuation is given by

$$A = 1 - \exp\left(-\frac{t_x}{\tau}\right). \tag{6}$$

For $t_x \ll \tau$ the attenuation is small and is given by

$$A = \frac{t_x}{\tau}. \tag{7}$$

Combining equations (4) and (7), we obtain an expression for the scattering lifetime, for sheet-like pulses and with $t_x \ll \tau$

$$\tau = \frac{t_{ps}}{2A \sin^2(\alpha/2)}. \tag{8}$$

Now $A/t_{ps}$ is just the initial gradient of the attenuation versus pulse length plot, see for example figure 4, hence equation (8) enables $\tau$ to be calculated.

For box-like pulses when $t_x \ll \tau$, we combine equations (5) and (7) and obtain another expression for the scattering lifetime

$$\tau = \frac{w}{A v \sin(\alpha)}. \tag{9}$$

For l–l scattering with the pulses at $\alpha = 30^\circ$, the initial gradient of the graph in figure 4 is $2.25 \times 10^6$ s$^{-1}$ for the scattering pulse power of 12.5 mW. This gives, using equation (8), the lifetime $\tau = 3.3 \times 10^{-6}$ s. This lifetime is some two orders of magnitude longer than for phonons.
in equilibrium at 1 K [2]. The reason for the long lifetime in the crossed beams experiment is because there are very few phonons in the two beams which have a small enough angle between them, to interact. This is also the reason why a hot line is not formed by two pulses at this angle [20].

For h–h scattering, we use equation (9) because the h-phonon pulses are very dispersed. Taking \( w = 1 \) mm, \( v = 189 \) ms\(^{-1} \) and \( A = 0.04 \) from figure 3(a), we find the lifetime of a h-phonon in the scattering h-phonon pulse, created by a heater pulse of 12.5 mW and 100 ns, is \( \tau = 2.6 \times 10^{-4} \) s.

For h–l scattering, we can use equation (8). From figure 7, we find \( A/t_{\text{ps}} = 3.5 \times 10^5 \) s\(^{-1} \) which gives the lifetime of a h-phonon in the l-phonon pulse, \( \tau = 1.5 \times 10^{-6} \) for scattering pulse power of 25 mW.

6. Conclusions

From the attenuation of various phonon probe pulses as they pass through pulses of other phonons, we have extracted the lifetime of probe phonons due to scattering with phonons in the scattering pulse. This lifetime is proportional to the density of phonons in the scattering pulse, which is determined by the pulse power, the collimation, the transverse expansion of the l-phonon sheet, the temperature and drift velocity of the l-phonons, and the details of the creation of h-phonons.

Further, theoretical study is required to estimate the densities. However, the conditions of the experiment are well defined so we hope that in the future a detailed comparison with theory can be made.

When pulses cross at 30°, l-phonons in the probe pulse scatter with the l-phonons in the scattering pulse. The actual angles between the phonons are given by the angular spread of momenta in each pulse, see equation (1), as well as the angle between the heater normals. The l–l scattering rate at these relatively large angles, which are comparable with the maximum angle of 27° for 3pp [2], is much slower than the l–l scattering rate in liquid helium at 1 K. This is due to the relatively small number of phonons at an angle where 3pp are possible. At 40° between the heater normals there is no sign of l–l scattering, as then there are even fewer phonons at angles for 3pp scattering.

There are h–h interactions at 30° and 40°, see figure 3, but the attenuation is small. However, this is the first time that scattering between phonons, both with energy greater than 10 K, has been detected and there is no theoretical calculation of the 4pp scattering rate for such phonon interactions. No h–l scattering is seen with pulses intersecting at these angles. This is probably because a larger angle between the h- and l-phonon is needed to satisfy both energy and momentum conservation in 4pp scattering.

When the beams intersect nearly head-on, the h-phonons of the probe beam are attenuated. We have argued that this is due to these phonons scattering with the l-phonons of the scattering pulse, because only these phonons intersect the h-phonons in the probe pulse, as the h-phonons of the scattering pulse are narrowly beamed along the heater normal, so they are parallel to, but offset from, the h-phonons in the probe pulse. The lifetime of a h-phonon scattering with low-energy phonons is around \( 10^{-6} \) s. When the pulse to the scattering heater is long and low power, R\(^+\)-rotons are created, and we see the h-phonons of the probe beam attenuated by these R\(^+\)-rotons. This is the first observation of such scattering.

We hope these results will stimulate the development of calculations of 3pp and 4pp lifetimes for the conditions in these experiments.
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