Spectral noise for edge states at the filling factor $\nu = 5/2$

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Abstract. We present a detailed analysis of finite-frequency noise for the $\nu = 5/2$ fractional quantum Hall state in a quantum point contact geometry. The results were obtained within the Pfaffian and anti-Pfaffian models. We show that the behaviour of the coloured noise allows us to unambiguously discriminate among tunnelling excitations with different charges. Optimal values of the external bias are found in order to emphasize the visibility of the noise peak associated with the tunnelling of a 2-agglomerate, namely an excitation with charge that is double the fundamental one. These correspond to the regime in which the bias is larger than the neutral modes cut-off frequency. The dependence on temperature is also investigated to discriminate between the considered models.

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1. Introduction

One of the most challenging and intriguing examples of strongly correlated electron systems is the fractional quantum Hall (FQH) fluid [1]. In recent years, this peculiar state of matter has been the subject of many theoretical and experimental studies, leading to the observation of a large variety of states at different values of the filling factor $\nu$ [2]. Among them, the $\nu = 5/2$ state [3] has recently attracted increasing attention in view of its promising application in topologically protected quantum computation [4]. Several models were proposed in order to describe this peculiar state [5] ranging from Abelian models, already considered for other states [6, 7], to more exotic non-Abelian descriptions such as the Pfaffian model [8, 9] and its particle–hole conjugate, the anti-Pfaffian one [10–12]. Despite the different statistical predictions [13], all these models have the common prediction of a fundamental excitation with charge $e/4$ ($e$ the electron charge), the single-quasiparticle (qp). At the bulk level, the signatures of this value have recently been found in [14]. For edge states, this result was confirmed in a quantum point contact (QPC) geometry [15] allowing tunnelling of excitations between the edge of the Hall bar. Here, measurements of the differential backscattering conductance [16] confirmed the presence of an $e/4$ tunnelling charge. Other experiments focused on the zero-frequency shot noise through the QPC, as already used for different filling factors [17–19], and confirm the $e/4$ charge as the relevant tunnelling excitation [20].

Recent experiments carried out at extremely low temperatures [21] showed a richer phenomenology with evolution of the tunnelling charge from $e/4$ to $e/2$, lowering the temperature [22]. Similar behaviour has been found also for other composite edge states [19, 23], indicating the possible common trend of tunnelling excitations with charge greater than the fundamental one at low energies [24, 25].

Despite these preliminary indications, there is still open debate about which charge is involved in the tunnelling through the QPC. The main purpose of this paper is to shed light on this issue and to unambiguously determine the charge excitation that is participating in tunnelling. In order to answer this question, we analyse the symmetrized finite-frequency noise [26] as a property complementary to and alternative to the current and the zero frequency noise. We will focus on Pfaffian and anti-Pfaffian models.

From the experimental point of view, measurements of coloured noise have been carried out for a QPC in a two-dimensional (2D) electron gas without magnetic field [27]. Nevertheless, great efforts are devoted to extending the observations to more interesting cases including...
the FQH regime. Here, much of the relevant physics is at frequencies around the Josephson frequency \( \omega_0 = e^* V / h \) (\( e^* \) the charge of the elementary excitation) that is in the range of GHz for external bias \( V \) in the range of \( \mu V \). These values, although high, should be experimentally observable.

From a theoretical point of view, the coloured noise was investigated for the Laughlin sequence \([28]\), and predicted to show resonance at the Josephson frequency \([29–32]\). Also the filling factor \( \nu = 5/2 \) has been considered, but only for the fundamental charge contribution in the Pfaffian model, in order to extract signatures of the non-Abelian statistics of the excitations \([33]\).

In this paper, we discriminate among the different tunnelling charges involved in the transport, their contributions being resolved at different frequencies. We demonstrate, for both models, the necessity to consider voltages larger than the neutral modes cut-off frequency, in order to efficiently detect tunnelling of the 2-agglomerate. The Pfaffian and anti-Pfaffian models are also compared regarding their different behaviour in the temperature scaling of the peaks.

The paper is organized as follows. Section 2 introduces the two models for the filling factor \( \nu = 5/2 \) and the tunnelling Hamiltonian for the QPC geometry. The relation between finite-frequency noise and tunnelling rates is discussed with the evaluation of tunnelling scaling properties. Section 3 is devoted to an analysis of the results for the frequency-dependent noise as a function of external bias and temperature, focusing on the possibility to detect tunnelling excitations with charge higher than the fundamental one. In section 4, we sum up the main results of the paper.

2. Transport properties

2.1. Model

We consider the effective field theoretical description of edge states at filling factor \( \nu = 5/2 \) for the Pfaffian \([8, 9]\) and the disorder-dominated anti-Pfaffian models \([10, 11]\). The Lagrangian density \( \mathcal{L} = \mathcal{L}_c + \mathcal{L}_n \), with \( (\hbar = 1) \)

\[
\mathcal{L}_c = -\frac{1}{2\pi} \partial_x \varphi_c \left( \partial_t \varphi_c + v_c \partial_x \varphi_c \right)
\]

(1)

and

\[
\mathcal{L}_n = -i\psi \left( \xi \partial_t \psi + v_n \partial_x \psi \right) - \frac{\alpha}{4\pi} \partial_x \varphi_n \left( \xi \partial_t \varphi_n + v_n \partial_x \varphi_n \right),
\]

(2)

describes the Hall fluid at filling factor \( \nu = 1/2 \) with two completely filled inert Landau levels that represent the vacuum of the theory. The charged bosonic field \( \varphi_c(x) \) is related to the electron number density \( \rho(x) = \partial_x \varphi_c(x) / 2\pi \), while \( \varphi_n(x) \) is a bosonic neutral field and \( \psi(x) \) a neutral Majorana fermion \([34]\). The parameter \( \xi = \pm 1 \) denotes the direction of propagation of neutral modes with respect to the charged one. The two models differ in the neutral sector \( \mathcal{L}_n \) only, with \( \alpha = 0 \) and \( \xi = 1 \) for the Pfaffian model (P) and \( \alpha = 1 \) and \( \xi = -1 \) for the anti-Pfaffian (AP) one. The propagation velocities of the charged and neutral modes are indicated with \( v_c \) and \( v_n \), respectively. Numerical calculations performed for the Pfaffian case suggest that \( v_c > v_n \) \([35]\).

In accordance with the results \([11]\) obtained within the disorder-dominated phase of the anti-Pfaffian model, we assume equal velocity \( v_n \) for both the bosonic and fermionic parts of the neutral sector. We also introduce the energy bandwidths \( \omega_{c/n} = a^{-1} v_{c/n} \), with \( a \) being finite
length cut-off. In the entire paper we will assume $\omega_c$ as the largest energy. The quantization of the bosonic fields is given by the commutators

$$\left[ \varphi_{\nu/n}(x), \varphi_{\nu/n}(y) \right] = i\pi \nu \frac{\text{sgn}(x-y)}{n},$$

with $\nu = 1/2$ and $\nu_n = \xi$, while the Majorana fermion commutes with both.

Operators destroying an excitation along the edge are [9, 11]

$$\Psi_p^{(x,m)}(x) \propto \chi(x) e^{i\pi \varphi(x)},$$

$$\Psi_{AP}^{(x,m,n)}(x) \propto \chi(x) e^{i\pi \varphi(x) + i\pi \varphi(x)},$$

with $m, n \in \mathbb{Z}$ and where $\chi(x)$ belongs to the Ising conformal field theory [36]. The latter can be the identity operator $I$, the Majorana fermion $\psi(x)$ or the spin operator $\sigma(x)$. The electric charge associated with the operators in (4) is $(m/4)e$ and depends only on the charged mode. In the following, we will call an $(m/4)e$-charged excitation an $m$-agglomerate [22, 24]. The single-valuedness properties of $m$-agglomerates with respect to the electrons force $m$ and $n$ to be: even integers for $\chi = I$ or $\psi$ and odd integers for $\chi = \sigma$ [12]. Note that, in the Ising sector, the spin operator $\sigma$, due to its nontrivial operator product expansion $\sigma \times \sigma = I + \psi$ [36], leads to the non-Abelian statistics of the excitations [4, 13].

2.2. Tunnelling through a quantum point contact

Tunnelling of a generic $m$-agglomerate between the two edges of the Hall bar, through the QPC at $x = 0$, is described by $H_T^{(m)} = t_m \Psi_{R}^{(m)}(0) \Psi_{L}^{(m)}(0) + \text{h.c.}$, where $\Psi_{j}^{(m)}$ is the annihilation operator of an $m$-agglomerate (cf equation (4)) on the right $j = R$ and left $j = L$ edges. For simplicity, if not necessity, we will omit all other indices present in (4) treating the two models on the same footing. The tunnelling amplitudes $t_m$ depend on factors such as the geometry of the constriction, the intensity of the gate potential and the details of the edge structure. Although, in general, these amplitudes can be energy dependent [15], for the sake of clarity and to restrict the number of free parameters, in the following we will assume them to be $m$-dependent constants. Once a comparison with experiments becomes possible this assumption will be verified and possibly more complex choices will be considered. The total tunnelling Hamiltonian will consist of the sum over all possible excitations $H_T = \sum_m H_T^{(m)}$.

In this paper we will focus on the weak tunnelling regime. At lowest order in $H_T$ the backscattering current of the $m$-agglomerate $(I_B^{(m)})$ and the symmetrized noise $S_B^{(m)}(\omega)$ are directly written in terms of the tunnelling rates [26]. Here, we quote the final relations only, which have already been discussed in other works where full out of equilibrium approaches, such as the Keldysh Green’s function method, were applied [26, 32, 33]. Using the detailed balance relation, the current is [37] ($k_B = 1$)

$$I_B^{(m)}(\omega_0) = me^{*} \left( 1 - e^{-\omega_0 / T} \right) \Gamma^{(m)}(\omega_0),$$

with the golden rule tunnelling rate at fixed voltage

$$\Gamma^{(m)}(E) = |t_m|^2 \int_{-\infty}^{\infty} d\tau e^{i m E \tau} G_{m,R}^{<}(0, -\tau) G_{m,L}^{>}(0, \tau).$$

Here, $\omega_0 = e^* V$ ($e^* = e/4$) is the Josephson frequency associated with the single-qp with $V$ being the bias. The correlators $G_{m,j}^{<}(0, t) = (\Psi_j^{(m)}(0, t) \Psi_j^{(m)*}(0, 0)) = (G_{m,j}^{<}(0, t))^*$ are the two-point Green’s functions of the $m$-agglomerate operators on the edge $j = R, L$. It is worth noting

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that, at the lowest order in the tunnelling, the detailed balance relation is satisfied. Indeed, for weak tunnelling the time between two tunnelling events is much longer than any other time scale including the time that the collective excitations take to thermalize. In the Keldysh formalism, one can easily show that the detailed balance condition derives directly, considering the contribution of the tunnelling at the lowest order.

Similarly, the noise spectral density

$$S_B^{(m)}(\omega) = \int_{-\infty}^{+\infty} d\tau \, e^{i\omega \tau} S_B^{(m)}(\tau),$$

with

$$S_B^{(m)}(\tau) = \frac{1}{2} \left[ \langle I_B^{(m)}(\tau) I_B^{(m)}(0) \rangle + \langle I_B^{(m)}(0) I_B^{(m)}(\tau) \rangle - 2 \langle I_B^{(m)}(\tau) \rangle \langle I_B^{(m)}(0) \rangle \right],$$

being related to the rate by extending what was developed in [37] to finite frequency. Here, for the sake of simplicity, we focus on the backscattering current noise, whereas in experiments the total current noise is often measured. At zero frequency, to the lowest order in the tunnelling, the backscattering noise can be directly connected with the total current noise [31, 38]. However, at finite frequency, one needs, in general, a four-terminal setup and the backscattering noise is given by an appropriate measurement of the correlators of the different terminals. A complete and exhaustive study of these issues can be found in [32]. Our choice gives us the possibility to keep our discussion as simple as possible, directly in terms of a physically relevant quantity, the backscattering current. All the other possible correlators could be obtained using the techniques developed in [32]. One then has

$$S_B^{(m)}(\omega) = \frac{(me)^2}{2} \sum_{\epsilon = \pm} \left[ \Gamma^{(m)}(\epsilon \omega/m + \omega_0) + \Gamma^{(m)}(\epsilon \omega/m - \omega_0) \right],$$

or equivalently in terms of the backscattering current

$$S_B^{(m)}(\omega) = \frac{1}{2} (me)^2 \sum_{\epsilon = \pm} \coth \left( \frac{\epsilon \omega m + \omega_0}{2T} \right) \langle I_B^{(m)}(\epsilon \omega/m + \omega_0) \rangle.$$

This result is fully consistent with the non-equilibrium fluctuation–dissipation theorem derived some time ago for a single-barrier tunnelling Hamiltonian [26, 31, 32]. From (9) one can easily restore the well-known result for the zero-frequency limit [37, 39, 40].

At the same perturbative order the total noise will be given by the sum

$$S_B(\omega) = \sum_m S_B^{(m)}(\omega)$$

being the contributions of the different $m$-agglomerates statistically independent at the lowest order. Note that the total backscattering current is also given by $\langle I_B \rangle = \sum_m \langle I_B^{(m)} \rangle$.

These simple relations (10) and (11) are due to the Poissonian statistics of the tunnelling processes at lowest order in $|t_m|^2$ and to the independence of the sources of noise.

2.3. Scaling behaviour

Let us now focus on the evaluation of the rate (6), starting with the zero-temperature limit. Here, the Green’s functions are [36, 41]

$$(\chi(0,t)\chi(0,0)) = \frac{1}{(1 + i\omega t)^{1/2} \mathcal{Z}_x}. $$

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\[(\varphi_s(0, t) \varphi_s(0, 0)) = -|\varphi| \ln (1 + i\omega_s t), \quad s = c, n \tag{13}\]

for the Ising sector and for the charged and neutral bosonic fields, respectively. Due to the independence of the Ising and bosonic sectors, the Green functions factorize and the tunneling rate reads

\[
\Gamma^{(m)}(E) = \frac{|t_m|^2}{(2\pi a)^2} \int_{-\infty}^{+\infty} dt' e^{i\omega E t'} \frac{1}{\Gamma(\eta_m + \mu_a)} \Theta(mE),
\]

with \(\delta_i = 0, \delta_\phi = 1\) and \(\delta_\sigma = 1/8\) being the conformal weights of the field in the Ising sector \([4, 13, 36]\). The above integral can be performed leading to

\[
\Gamma^{(m)}(E) = \frac{|t_m|^2}{2\pi a^2} \left( \frac{mE}{\omega_c} \right)^{\eta_m + \mu_a} \left( \frac{mE}{\omega_n} \right)^{1 - \eta_m - \mu_a} \left( \frac{mE}{\omega_c} - \frac{mE}{\omega_n} \right) \Theta(mE),
\]

with \(\Gamma(x)\) the Euler Gamma function. Here, \(\Gamma_1[a; b; z]\) is the Kummer confluent hypergeometric function \([42]\), \(\eta_m = m^2/4\) the charge exponent and \(\mu_a = 2\delta_\chi + \alpha n^2/2\) the model-dependent neutral exponent (\(\alpha = 0\) for the Pfaffian model and \(\alpha = 1\) for the anti-Pfaffian one).

Note that for equal neutral and charge mode velocities, i.e. \(\omega_c = \omega_n\), the above formula reduces to

\[
\Gamma^{(m)}(E) = \frac{|t_m|^2}{2\pi a^2} \left( \frac{mE}{\omega_c} \right)^{\eta_m + \mu_a} \left( \frac{mE}{\omega_n} \right)^{1 - \eta_m - \mu_a} e^{-\frac{mE}{\omega_c}} \Theta(mE),
\]

recovering the standard result for a Luttinger liquid \([30, 33, 43–45]\).

The rate (15) shows different regimes depending on the value of the energy with respect to the cut-off frequency \(\omega_n\) of the neutral modes. Note that the charge cut-off \(\omega_c\) is assumed to be the largest energy scale and does not enter the definition of these regimes. At low energies \(E \ll \omega_n\), the rate scales as

\[
\Gamma^{(m)}(E) \approx E^{4\Delta_{P,A}^{(m)} - 1},
\]

receiving contributions from both the charged and the neutral modes. In the same limit, for frequencies close to the Josephson frequency \(\omega \to m\omega_0\), one has (cf equation (9))

\[
\Delta_{B}^{(m)}(\omega \to m\omega_0) \approx (\omega - m\omega_0)^{4\Delta_{P,A}^{(m)} - 1}.
\]

Here, \(\Delta_{P,A}^{(m)}\) are the \(m\)-agglomerate scaling dimensions of the operators (4), for the Pfaffian and anti-Pfaffian models, respectively. They are deduced by the long-time behaviour at \(T = 0\) of the imaginary time two-point Green’s function \(\langle T_\tau \Psi(\tau) \Psi(0) \rangle \propto |\tau|^{-2\Delta}\) \([46]\)

\[
\Delta_{P}^{(m)} = \frac{1}{2}\delta_\chi + \frac{1}{16} m^2, \quad \Delta_{AP}^{(m)} = \frac{1}{2}\delta_\chi + \frac{1}{16} m^2 + \frac{1}{8} n^2.
\]

For the single-qp fundamental charge they are given by \(m = 1, n = \pm 1\) and \(\chi = \sigma\)

\[
\Delta_{P}^{(1)} = \frac{1}{8}, \quad \Delta_{AP}^{(1)} = \frac{1}{4}.
\]

For the next excitation, the 2-agglomerate, the scaling is driven by the charged mode contribution only with \(m = 2, n = 0\) and \(\chi = I\)

\[
\Delta_{P}^{(2)} = \Delta_{AP}^{(2)} = \frac{1}{4}.
\]

This behaviour indicates that the single-qp particle is the most dominant excitation in the Pfaffian case, while in the anti-Pfaffian case single-qp and 2-agglomerate have equal
relevance with the same scaling dimensions [11]. All other excitations, with higher charges, have higher scaling dimensions and can be safely neglected in the frequency region \( \omega \lesssim 2e^*V \).

Note that the peculiar values of the scaling (20) and (21) imply a divergent power-law behaviour of the total noise only for the Pfaffian case due to the single-qp contribution \( S_B(\omega \rightarrow \omega_0) \approx (\omega - \omega_0)^{-1/2} \). In all other cases, this quantity appears flat due to the power-law cancellation in (18); therefore it will be more difficult to separate the contribution of different excitations. For example, at \( \omega \rightarrow 2\omega_0 \) the 2-agglomerate can be detected only if \( S_B^{(1)}(2\omega_0) < S_B^{(2)}(2\omega_0) \). As we will see, this condition can be more easily achieved by considering energies (bias) higher than the neutral mode cut-off, \( \omega_0 > \omega_n \). Indeed, in this case the neutral modes are saturated modifying the scaling dimension (20) of the single-qp leading to a lower effective dimension

\[
\tilde{\Delta}_p^{(1)} = \tilde{\Delta}_A^{(1)} = \frac{1}{16},
\]

which will be active at the tail of the single-qp peak. On the other hand, they leave unaffected the 2-agglomerate scaling as in (21). Then, the tail of the single-qp near \( \omega = 2\omega_0 \) will drop faster with respect to the case when \( \omega_0 < \omega_n \), giving more visibility to the agglomerate itself. We will see this behaviour in detail in the next section.

At finite temperature the above behaviour will be smoothed with more remarkable changes near the Josephson resonances \( \omega = m\omega_0 \) for \( T > |\omega - m\omega_0| \). To quantitatively determine the temperature influence on the noise, we consider the finite-temperature expressions for the Green’s functions in equations (12)–(13) for both the bosonic [24, 37, 41] and the Ising [9] fields

\[
\langle \chi(0, t)\chi(0, 0) \rangle = \left[ \frac{\Gamma(1 + T/\omega_m - i\nu T)}{\Gamma^2(1 + T/\omega_m)(1 + i\omega_m t)} \right]^{\delta_{\lambda}},
\]

\[
\langle \varphi_s(0, t)\varphi_s(0, 0) \rangle = -|v_s| |\ln \left[ \frac{\Gamma^2(1 + T/\omega_s)(1 + i\omega_s t)}{\Gamma(1 + T/\omega_s - i\nu T)} \right]|, \quad s = c, n.
\]

The tunnelling rate (14) can still be explicitly evaluated for temperatures lower than the bandwidths, namely \( T \ll \omega_n, \omega_c \), leading to

\[
\Gamma^{(m)}(E) = \frac{|t_m|^2}{(2\pi a)^2} \frac{(2\pi)^{\eta_m+\mu_a}}{\omega_m^{\nu_m} \omega_n^{\mu_a}} T^{\eta_m+\mu_a-1} e^{\frac{mE}{2\pi T}} B\left( \eta_m + \mu_a - \frac{mE}{2\pi T} \right) \eta_m + \mu_a + 1 \frac{mE}{2\pi T}.
\]

\( B(a; b) \) being the Euler beta function [42]. At higher temperatures we evaluate the rate numerically.

Around the Josephson frequencies the noise in (10) assumes the simple form

\[
S_B^{(m)}(\omega \rightarrow m\omega_0) \approx TG_B^{(m)}(T)
\]

in terms of the \( m \)-agglomerate linear conductance

\[
G_B^{(m)}(T) = (me^*)^2 \frac{\Gamma^{(m)}(0)}{T}.
\]

The power-law behaviour for the height of the peaks as a function of the temperature is therefore given by \( S_B^{(m)}(\omega \rightarrow m\omega_0) \approx T^{\eta_m+\nu_m-1} \) for \( T \ll \omega_n \) and \( S_B^{(m)}(\omega \rightarrow m\omega_0) \approx T^{\eta_m-1} \) for \( T \gg \omega_n \).
3. Results

In this section, we will discuss the behaviour of the ratio $S_B(\omega)/e\langle I_B \rangle$, which, at zero frequency, corresponds to the standard definition of the Fano factor [24, 37]. We focus the analysis on a frequency regime up to the second Josephson resonance $\omega = 2\omega_0$, although one could extend to higher frequencies. Therefore, we restrict the sum in the total noise (11) to $m = 1, 2$. For notational convenience we introduce $\kappa = |t_2|^2/|t_1|^2$, which represents the relative weight of the two-agglomerate amplitude in comparison with the single qp one. The charge mode bandwidth is assumed to be of the same order as the activation gap for the Hall fluid at $\nu = 5/2$ ($\omega_\kappa = 500 \text{ mK}$) [47].

Let us start by considering the noise as a function of frequency varying the external bias $\omega_0$, for different ratios $\kappa$. This will allow us to find optimal values of $\omega_0$ necessary to increase the visibility of the two-agglomerate. Note that, for higher values of the frequency $\omega$ plotted in the figures, one could expect also additional features, related to the bulk qp dynamics, whose analysis is beyond the scope of this paper. The values of $\omega_0$ (i.e. the bias) must be carefully tuned considering that decreasing this quantity leads to an increase in the relative importance of the thermal noise contribution. In our discussion, we took the bias at quite high values to make the thermal effects on the finite-frequency spectrum as small as possible. A better compromise can be considered in the presence of real experiments. Nevertheless, in such a case, one need also keep the frequency window of the measurement as small as possible and consider energies smaller than the gap.

Figure 1. Noise $S_B(\omega)/e\langle I_B \rangle$ as a function of frequency for the Pfaffian model with (a) $\kappa = 0.5$ and (b) $\kappa = 2$. The different values of the bias are $\omega_0 = 30 \text{ mK}$ (red, solid), $\omega_0 = 100 \text{ mK}$ (green, long dash), $\omega_0 = 200 \text{ mK}$ (blue, short dash) and $\omega_0 = 300 \text{ mK}$ (magenta, dotted). Other parameters are $\omega_c = 500 \text{ mK}$, $\omega_n = 50 \text{ mK}$ and $T = 10 \text{ mK}$. The unit of frequency $\omega_0 = e^* V$. 

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Figures 1 and 2 show this analysis for two different models (figure 1 for the Pfaffian model and figure 2 for the anti-Pfaffian one, respectively).

All figures show a general trend: the single-qp excitation peak at $\omega = \omega_0$ is always well visible and much more pronounced in the Pfaffian model, while the 2-agglomerate peak at $\omega = 2\omega_0$ notably increases its visibility in the regime $\omega_0 > \omega_n$, namely at bias larger than the neutral cut-off $\omega_n$. This result is in agreement with the discussion in the previous section on the scaling dimension of the excitations. Indeed, for both models the two-agglomerate noise contribution is flat around $\omega = 2\omega_0$, as can be verified from (18) and from the scaling dimension $\Delta_{P,AP}^{(2)} = 1/4$, and decreases at higher frequencies as $(\omega - 2\omega_0)/\omega_c$, as a consequence of the exponential cut-off (cf equation (15)). It is then necessary to depress the tail of the single-qp contribution in order to emphasize its presence. This is indeed achieved for $\omega_0 > \omega_n$ (cf section 2.3).

Another common feature is the behaviour as a function of the ratio $\kappa$ between the tunnelling amplitudes. By increasing $\kappa$ the height of the single-qp peak progressively decreases with increasing 2-agglomerate contribution in the high-frequency region. This is connected to the reduction of the relative weight of the single-qp noise with respect to the 2-agglomerate. Indeed, for $\kappa = 2$ (figures 1(b) and 2(b)) a much better resolved peak appears at $\omega \approx 2\omega_0$, whenever $\omega_0 > \omega_n$. 

**Figure 2.** Noise $S_B(\omega)/e\langle I_B \rangle$ as a function of frequency for the anti-Pfaffian model with (a) $\kappa = 0.5$ and (b) $\kappa = 2$. The different values of the bias are $\omega_0 = 30\text{ mK}$ (red, solid), $\omega_0 = 100\text{ mK}$ (green, long dash), $\omega_0 = 200\text{ mK}$ (blue, short dash) and $\omega_0 = 300\text{ mK}$ (magenta, dotted). Other parameters are as in figure 1. The unit of frequency $\omega_0 = e^*V$. 

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Concerning the possible differences between the two models one can easily recognize that in the Pfaffian model the single-qp peak is sharper and more pronounced than that in the other. The reason is again connected to the scaling dimensions (cf equation (20)), which are more relevant in the first case. In order to better characterize this fact, we consider the thermal evolution of the total finite-frequency noise for both the models at $\kappa = 2$. The results are shown in figure 3. In the Pfaffian case (figure 3(a)), the single-qp peak remains visible for a wide range of temperatures despite the fact that the one of the 2-agglomerate disappears quite soon. For the other model (figure 3(b)) the two peaks rounded in an analogous way with increasing temperature. Note that for the higher considered temperature ($T = 100 \text{ mK}$) the described two-peak structure is completely washed out for both models. According to the power-law behaviour described in section 2.3, we expect for the ratio between the heights of the peaks $R = S_B(2\omega_0)/S_B(\omega_0)$ that the power-law dependences on temperature $R_P \approx T^{\frac{1}{2}}$ and $R_{AP} \approx \text{constant}$ for the Pfaffian and the anti-Pfaffian model, respectively. These completely different trends clearly appear in figure 4, where the ratios have been represented as a function of temperature and allow us to better distinguish between the two models.

Before concluding we would like to comment on the role of coupling with possible external environments. As already discussed in several papers for both the Laughlin and Jain series, and more generally for one-dimensional Luttinger systems [24, 48–54], these interactions cause a change in the scaling dimensions which could induce visible effects in transport properties.

Figure 3. Noise $S_B(\omega)/e\langle I_B \rangle$ as a function of frequency for the Pfaffian model (a) and the anti-Pfaffian model (b) at different temperatures: $T = 1 \text{ mK}$ (red, solid), $T = 10 \text{ mK}$ (green, long dashed), $T = 30 \text{ mK}$ (blue, dashed) and $T = 100 \text{ mK}$ (magenta, dotted). Other parameters are $\kappa = 2$, $\omega_c = 500 \text{ mK}$, $\omega_n = 50 \text{ mK}$ and $\omega_0 = 300 \text{ mK}$. The unit of frequency $\omega_0 = e^* V$. 
Figure 4. The ratio $S_B(2\omega_0)/S_B(\omega_0)$ as a function of temperature for the Pfaffian model (black, full) and the anti-Pfaffian model (black, dashed). Other parameters are: $\kappa = 2$, $\omega_c = 500$, $\omega_n = 50$ and $\omega_0 = 300 \text{ mK}$.

In particular, for the $\nu = 5/2$ case the effects of the out of equilibrium $1/f$ noise determined by the coupling of the edge with trapped charges in the substrate [55] have recently been proposed to be a source of interaction for the charged and neutral bosonic modes [22].

Despite the several theoretical predictions, no defined experimental results on the role of interaction have been achieved so far, especially for the $\nu = 5/2$ case. That is why we confined the analysis of the present work to the non-interacting case. However, the inclusion of interactions is quite straightforward according to the results described in the previous section. The main effect is a dressing of the bosonic correlators in equations (13) and (24) $\langle \varphi_s(0, t)\varphi_s(0, 0) \rangle_{\text{int}} = g_s \langle \varphi_s(0, t)\varphi_s(0, 0) \rangle$ ($s = c, n$) with renormalization parameters $g_c$ and $g_n$ leading to the change in scaling dimensions

$$\Delta_1^{(m)} = \frac{1}{2} \delta_x + g_c \frac{1}{T_\text{c}} m^2, \quad \Delta_1^{(n)} = \frac{1}{2} \delta_x + g_c \frac{1}{T_\text{c}} m^2 + g_n \frac{1}{8} n^2. \quad (28)$$

For weak-enough interactions the presented phenomenology is robust and we do not expect qualitative changes with respect to the above results. In the strong renormalized case two distinct regimes are possible. For $g_c$ and $g_n$ leading to scaling dimensions satisfying $1/4 < \Delta_1^{(m)} < 1/2$ the peaks at $\omega = m\omega_0$ turn into dips. Despite this change in the shape of the curve, it is still possible to resolve the contributions due to the different $m$-agglomerates. Conversely, when the interactions are so intense that $\Delta_1^{(m)} > 1/2$ the total finite-frequency noise increases monotonically with frequency and one loses track of the presence of the resonances. In this case, it is more difficult to distinguish the presence of the various excitations. A detailed analysis of the renormalized case will be given elsewhere.

4. Conclusion

In this paper, we have analysed the finite-frequency noise for two credited theories proposed for the FQH state at $\nu = 5/2$, the non-Abelian Pfaffian model and its particle–hole conjugate, the anti-Pfaffian. We considered the presence of the two most dominant excitations: the single-qp, with charge $e/4$, and the 2-agglomerate, with charge $e/2$. The finite-frequency noise has the unique possibility to resolve spectroscopically the contributions of the different
excitations looking at different Josephson resonances. We showed that the peak associated with the two-agglomerate is more evident at Josephson frequency higher than the neutral mode cut-off, where the tail of the single-qp contribution decreases faster. We also considered the different evolutions of the height of the single-qp peak as a function of temperature, driven by the different scaling dimensions, as an important tool for discriminating between the models.

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