Vector, Axial, Tensor and Pseudoscalar Vacuum Susceptibilities

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Abstract

Using a recently developed three-point formalism within the method of QCD Sum Rules we determine the vacuum susceptibilities needed in the two-point formalism for the coupling of axial, vector, tensor and pseudoscalar currents to hadrons. All susceptibilities are determined by the space-time scale of condensates, which is estimated from data for deep inelastic scattering on nucleons.

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1 Introduction

Since the operator product expansion (O.P.E.) fails for low momentum transfer, the coupling of hadrons to currents cannot be treated by the usual method of QCD sum rules extended to three-point functions. Recently, a three-point formalism, the standard starting point for such couplings in field theory, was developed[1] for a QCD sum rule approach to hadronic couplings with particular emphasis on vacuum susceptibilities. It was shown that the vacuum susceptibilities that occur in the external field method with two-point functions are given in the three-point approach by four-quark correlations which can be estimated from the scale of the nonlocality of quark condensates, with the main limitation of accuracy given by the use of vacuum saturation. In Ref.[1] the pion susceptibility was treated. There has been a great deal of other work using three-point functions for hadronic couplings [see Ref.[1] for discussion and references]. In the present note the method is used to determine the susceptibilities for all currents that have been treated previously by QCD Sum Rules with the external field method.

In the early work on electromagnetic coupling to nucleons[2] it was observed that the O.P.E. cannot be used consistently at low momentum transfer and bilocal corrections were introduced. This problem was solved using a two-point function with an external field method[3], with phenomenological vacuum susceptibilities introduced as parameters for nonperturbative propagation in the external field. Although the nucleon magnetic dipole moments were determined in Ref.[3] by sum rules which avoided the use of the vector susceptibility, additional sum rules were used to estimate the value of this parameter. There have been a number of applications of this two-point method for the calculation of the axial coupling constant \((g_A)[4, 5, 6]\) and for the isoscalar axial coupling constant \((g_A^S)[6]\). From the appropriate sum rules the axial vector susceptibility has been estimated. There as also been an estimate of the vacuum tensor susceptibility[7] in work on the tensor charge of the nucleon, however, it has been pointed out[8] that the treatment of the nucleon tensor charge is subtle and that different theoretical treatments can give very different results.
In a study of the parity-violating pion-nucleon interaction the pion-nucleon coupling was shown to depend critically on the pion-induced susceptibility, which was estimated. Recently, the axial-nucleon coupling, which involves the closely-related pseudoscalar coupling constant was also treated in the external field QCD sum rule method. Although in principle the pion susceptibility can be estimated using PCAC, in practice there are inconsistencies with the values needed phenomenologically.

In the present note we show that the method developed in Ref. can be successfully used for predicting the induced vacuum susceptibilities for all of these currents. With the factorization approximation used in our work there is a single scale provided by the space-time size of the nonlocal condensates that determines all of the susceptibilities.

2 Vacuum Susceptibilities

In a field theory the coupling of a current $J_y(y)$ to a hadron is studied by the three-point function:

$$V^\Gamma(p, q) = \int d^4x \int d^4y e^{ix \cdot p} e^{-iy \cdot q} V^\Gamma(x, y)$$

$$V^\Gamma(x, y) = \langle 0| T[\bar{\eta}(x) J^\Gamma(y) \bar{\eta}(0)]|0 \rangle$$

where the quantity $\eta(x)$ is a composite field operator representing the hadron, with quark fields as constituents. By evaluating $V^\Gamma$ at large $p^\mu$ one can make an operator product expansion with respect to the $(0,x)$ variables, and thereby introduce the nonperturbative QCD effects through quark condensates. However, since one wishes to evaluate $V^\Gamma$ for small $q$, the O.P.E. for the $(0,y)$ or $(x,y)$ variable cannot be justified and the QCD sum rule method for treating nonperturbative QCD does not work.

To avoid this difficulty a two-point formulation of the QCD Sum Rule in an external electromagnetic field was introduced. The form of the correlator is

$$\Pi^\Gamma(p) = i \int d^4e^{ix \cdot p} \langle 0| T[\bar{\eta}(x) \bar{\eta}(0)]|0 \rangle_{J^\Gamma},$$

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in which the presence of the external current \( J^\Gamma \) is indicated by the symbol in Eq. (2). By going to large \( p^\mu \) the microscopic evaluation of \( \Pi^\Gamma(p) \) can be done using the O.P.E of the quark propagator in the presence of the the \( J^\Gamma \) current

\[
S^\Gamma_q(x) = <0| [q(x)\bar{q}(0)] |0>^\Gamma,
\]

\[
= S^\Gamma_{q,PT}(x) + S^\Gamma_{q,NP}(x),
\]  

(3)

where \( S^\Gamma_{q,PT}(x) \) is the quark propagator coupled perturbatively to the current and \( S^\Gamma_{q,NP}(x) \) is the nonperturbative quark propagator in the presence of the external current, \( J^\Gamma \).

\[
S^\Gamma_{q,NP}(x) = <0| :\bar{q}(x)q(0): |0>^\Gamma
\]  

(4)

The operator product expansion for \( S^\Gamma_{q,NP}(x) \) is justified as in the ordinary two-point function. The nonperturbative vacuum matrix elements for the quark propagator in the external \( J^\Gamma \) current, given in Eq.(4), are expressed in terms of condensates. For example, the lowest-dimensional term is

\[
<0| :\bar{q}J^\Gamma q : |0>^\Gamma = -\Gamma \chi^\Gamma <0| :\bar{q}q : |0>^\Gamma /12
\]  

(5)

in terms of the vacuum susceptibility \( \chi^\Gamma \). These susceptibilities must be determined in order to predict the coupling constants using QCD sum rules.

In Ref.[1] it was shown that \( S^{cc',\Gamma,NP}_q(x) \) of Eq.(3) with color indices \( c,c' \)is given in the three-point method by the four-quark condensate

\[
S^{cc',\Gamma,NP}_q(x) = -i \int d^4y e^{-iy\cdot q} <0| :q^c(x)\bar{q}^{c'}(y)\Gamma q^{c'}(y)\bar{q}^c(0) : |0>^\Gamma
\]  

(6)

If we assume vacuum saturation for intermediate states \( S^{cc',\Gamma,NP}_q(x) \) is given in terms of nonlocal condensates, which have been used for the pion wave function[11], the pion form factor[12] and deep inelastic scattering from nucleon targets[13]. Taking the \( q^\mu = 0 \) limit we obtain

\[
S^{cc',\Gamma,NP}_q(x) \simeq \Gamma G(x)(<0| :\bar{q}(0)q(0) : |0>^\Gamma /12)^2,
\]

\[
G(x) = (-i) \int d^4y g(y^2)g((x-y)^2),
\]  

(7)
where the function \( g(y^2) \) gives the space-time structure of the nonlocal condensates:

\[
<0| : \bar{q}(0)q(y) : |0> \equiv g(y^2) <0| : \bar{q}(0)q(0) : |>, \tag{8}
\]

In Ref.\[[13]\] it was shown that \( g(y^2) \) can be determined from the experimental sea-quark distribution as measured in deep inelastic scattering on the nucleon. Using the form

\[
g(y^2) = \frac{1}{(1 + \kappa^2y^2/8)^2} \tag{9}
\]

we obtain

\[
S^{c\epsilon\Gamma, NP}_q(x) \simeq \frac{a^2}{6\pi^2\kappa^4} \tag{10}
\]

Finally, from a reanalysis of the QCD sum rule three-point treatment\[[13]\] with the form given in Eq.\(((9)\) it is shown in Ref.\[[1]\] \( \kappa^2 \simeq (0.15-0.2) \text{ GeV}^2 \).

### 3 QCD Sum Rule Three-Pont Method for Known Susceptibilities

In this section we apply the three-point method described in Sec. 2 for the susceptibilities that have been estimated for axial vector, vector and pionic currents and briefly discuss the tensor case.

#### 3.1 Axial Vector Susceptibility

The axial and pseudoscalar coupling constants, \( g_A \) and \( g_P \), are defined by the nucleon matrix element of the axial current,

\[
< N(p')|J_5^\mu|N(p) > = \bar{v}(p')(g_A\gamma_\mu\gamma_5 + g_Pq_\mu\gamma_5)v(p), \tag{11}
\]
In this subsection $g_A$ is discussed, while $g_P$ is discussed below. There are two susceptibilities that appear in the conventional two-point treatment of an external axial current, $Z_\mu$: $\chi_A$ and $\kappa_A$

\begin{align}
&< 0|\bar{q}\gamma_\mu\gamma_5 q|0 >_Z = \chi_A < \bar{q}q > Z_\mu \\
&< 0|\bar{q}g_c\tilde{G}_{\mu\nu}\gamma_\nu q|0 >_Z = \kappa_A < \bar{q}q > Z_\mu,
\end{align}

where $g_c$ is the strong coupling constant and $G_{\mu\nu}$ represents the gluon field in the fixed-point gauge and $\tilde{G}_{\mu\nu} = \epsilon_{\mu\alpha\beta\nu} G^{\alpha\beta}/2$. The susceptibility $\chi_A$ involves the type of four quark matrix elements treated in the present work, while the mixed susceptibility $\kappa_A$ involves some correlations not considered here. In the treatment of the isovector axial coupling constant $(g_A)\[4, 5, 6]$ and the isoscalar axial coupling constant $(g_S^A)\[6]$ the sum rule for the nucleon were used to reduced the dependence on these susceptibilities. We use the formulation of Ref.[6] here.

The two sum rules in the two-point method are

\begin{align}
\beta^2_N (\frac{g_A}{M_B^2} + A)e^{-m^2/M_B^2} &= \frac{M_B^2 E_2}{8L^{4/9}} + \frac{< g_c^2 G^2 > E_0}{M_B^2} + \frac{5a^2 L^{4/9}}{18M_B^2} - \frac{\kappa_A a E_0}{18L^{68/81}} \\
&+ \frac{\chi_A a < g_c^2 G^2 > E_0}{288M_B^2 L^{4/9}},
\end{align}

and

\begin{align}
\beta^2_N g_A (1 - \frac{2m^2}{M_B^2})e^{-m^2/M_B^2} &= \frac{M_B^2 E_2}{8L^{4/9}} + \frac{M_B^2 < g_c^2 G^2 > E_0}{32L^{4/9}} + \frac{a^2 L^{4/9}}{18M_B^2} - \frac{M_B^2 \kappa_A a E_0}{2L^{68/81}} \\
&+ \frac{\chi_A a < g_c^2 G^2 > E_0}{288M_B^2 L^{4/9}},
\end{align}

with $M_B$ the Borel mass, $< g_c^2 G^2 > \approx 0.47$ GeV$^4$, $a = -(2\pi)^2 < \bar{q}q > \approx 0.55$ GeV$^2$, $L = .62$ ln$(10 M_B)$, and $< 0|\eta(x)|N > = (\beta_N / \pi^2) v(x)$, where $v(x)$ is a Dirac spinor. The functions $E_m$, defined in Ref.[6], and the parameter $A$ help treat the continuum.

In the evaluation of these sum rules in Ref.[6] it was shown that $g_A -1$ is mainly given by the quark condensate term proportional to $a^2$ with a value of 0.25, in agreement with experiment and the Goldberger-Trieman relation, and giving the expected result of $g_A \approx$
1.0 when the quark condensates vanish with chiral symmetry restoration. The susceptibility terms were unimportant in that evaluation. Here we eliminate the susceptibility $\kappa_A$ from Eqs.(13,14) to obtain an expression for the susceptibility of interest, $\chi_A$. We find that

$$\chi_A(\text{two-point}) a \simeq -1.24 GeV^2$$

(15)

This value is in agreement within expected errors of the value predicted from the three-point method (with $G(0) = \frac{2^5\pi^2}{3\kappa^4}$):

$$\chi_A(\text{three-point}) a \simeq -G(0) \frac{a^2}{12(2\pi)^2}$$

$$\simeq -(1.7 - 3.0) GeV^2.$$  

(16)

Since the susceptibility gives a rather minor contribution for the isovector axial coupling constant, the value given in Eq.(15) is not very accurate, but shows that the prediction of the three-point method with no free parameters is consistent with the value extracted by the two-point method. The study of $g_A^S$, the isoscalar axial coupling constant[6] adds no new information. The prediction of the present work would be the same for $\chi_A^S$ and in Ref.[6] the same values were used for the isoscalar susceptibilities as for the isovector case.

### 3.2 Vector Susceptibility

Susceptibilities associated with the electromagnetic field were treated by Ioffe and Smilga[3] in their pioneering work on the use of the two-point QCD sum rule method for calculation of nucleon magnetic moments. Using the fixed-point gauge and representing the electromagnetic field by the $F_{\mu\nu}$ electromagnetic tensor, one finds three electromagnetic susceptibilities

$$<0|\bar{q}\sigma_{\mu\nu}q|0>_F = e_q\chi_V <\bar{q}q > F_{\mu\nu},$$

(17)

$$<0|\bar{q}g_{\mu\nu}G_{\nu}q|0>_F = e_q\kappa_V <\bar{q}q > F_{\mu\nu},$$

$$<0|\bar{q}g_{\mu\nu}\tilde{G}_{\nu}q|0>_F = e_q\xi_V <\bar{q}q > F_{\mu\nu}.$$  

As in the axial case, only the susceptibility $\chi_V$ is treated here. There are three sum rules, two of which were used in Ref.[3]. Although it is possible to make estimates of the nucleon
moments by eliminating the susceptibilities, they give quite important effects for the nucleon moments. A detailed analysis resulted in the estimate

$$\chi_{V\text{(two-point)}} \approx -(3.5 - 6.1) GeV^2,$$

while the present three-point sum rule method gives

$$\chi_{V\text{(three-point)}} \approx -(3.4 - 6.0) GeV^2.$$  \hfill (19)

Once more the prediction of the present theory is in satisfactory agreement with the value for the susceptibility extracted from the two-point external field method and experiment.

### 3.3 Pion and Pseudoscalar Susceptibilities

The susceptibility associated with an external pion field was treated in Ref.[1]. We briefly review this here. The strong and parity-violating pion-nucleon coupling depends crucially upon $\chi_{\pi}$, the pion susceptibility

$$<0|\bar{q}\tau_3\gamma_5 q|0>_{\pi} = ig_{\pi}\chi_{\pi} <\bar{q}q>\gamma_5$$ \hfill (20)

From the analysis of the strong pion-nucleon coupling constant, which is known to be $g_{\pi} - N = 13.5$, one finds in the two-point QCD sum rule method

$$\chi_{\pi\text{(two-point)}} \approx 1.88 GeV^2.$$ \hfill (21)

Using the three-point QCD sum rule method one finds[1]

$$\chi_{\pi\text{(three-point)}} \approx \frac{2a^2}{9\kappa^4} \approx (1.7 - 3.0) GeV^2,$$ \hfill (22)

in agreement with the value extracted by the external field method.

The pseudoscalar coupling, $g_P$ is difficult to treat by the two-point external field QCD sum rule method designed for zero momentum transfer, as one can see from Eq. (11). Recently we
have carried out such a study and have shown the crucial dependence on susceptibilities. In order to derive sum rules for $g_P$ one can introduce an external field of the form

$$Z_\mu(x) = Z_{\mu\nu}x^\nu,$$

which in turn leads to the pseudoscalar vacuum susceptibility, $\chi_P$, defined in momentum space at low momentum transfer by

$$<0|\bar{q}\gamma_5q|0>_Z = i\chi_P <\bar{q}q > Z_{\mu\nu}q^\nu\gamma_5.$$

(24)

The three-point formalism must be modified so that Eq.(7) becomes

$$S_{cc'P,NP}(x) \simeq \gamma_5Z^{\mu\nu}G_{\nu}(x)(<0|\bar{q}(0)q(0):|>/12)^2,$$

$$G_{\nu}(0) \simeq \text{Lim}_{q \to 0}q^\nu \int d^4y g^2(y^2)e^{iq\cdot y} = q^\nu4G(0)/\kappa^2.$$

(25)

From Eq.(25) one finds

$$gM\chi_P(three\text{-}point)a \simeq (40 - 53)\text{GeV},$$

(26)

corresponding to the space-time range of the nonlocality given by $0.2 > \kappa^2 > 0.15$. This range of values for $\chi_P$ is found to predict values for $g_P$ consistent with the Goldberger-Trieman relation.

### 3.4 Tensor Susceptibility

The tensor susceptibility is defined through Eq.(7) and

$$<0|\bar{q}\sigma_{\mu\nu}q|0>_Z = e_q\chi_T <\bar{q}q > Z_{\mu\nu},$$

(27)

with $\chi_T$ a factor 6 larger than the definition in Ref.[7]. We obtain

$$\chi_T(three\text{-}point)a/6 \simeq -(0.57 - 1.0)\text{GeV}^2.$$

(28)

Two estimates of $\chi_T$ made in Ref.[7] were about -0.15 GeV$^2$ and -0.29 GeV$^2$, much smaller that the values that we find; however, the authors point out that the estimates are crude and that the susceptibilities do not give large effects for the tensor charge of the nucleon.
4 Conclusions

The three-point QCD sum rule method of Ref. [1] with the factorization approximation has been used to estimate the external-field susceptibilities which appear in the external field two-point method in terms of a parameter giving the space-time scale of the quark condensate. Satisfactory agreement with all known vacuum susceptibilities has been found. There is one scale, the size of the space-time structure of the nonlocal condensates, which determines the susceptibilities within our approximations. This new method, using nonlocal condensates, does not require an operator product expansion of the quark propagators and can be used to study coupling constants and hadronic form factors as a function of momentum transfer.

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