SOLITARY WAVE SOLUTIONS FOR SPACE-TIME FRACTIONAL COUPLED INTEGRABLE DISPERSIONLESS SYSTEM VIA GENERALIZED KUDRYASHOV METHOD

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Abstract. In this article, space-time fractional coupled integrable dispersionless system has been considered, and we have used fractional derivative in the sense of modified Riemann-Liouville. The fractional system has been reduced to an ordinary by fractional transformation and the generalized Kudryashov method is applied to obtain exact solutions. We also testify performance as well as the precision of the applied method by means of numerical tests for obtaining solutions. The obtained results have been graphically presented to show the properties of the solutions.

Keywords. integrable dispersionless system; fractional derivative; differential system.

1. Introduction

In recent years, fractional differential equations have gained much attention from researchers due to their numerous applications in many fields of sciences and engineering. These equations are widely used to describe various phenomena in many fields such as the fluid flow, electro chemistry, scattering theory, transport theory, probability, elasticity, control theory, potential theory, signal processing, image processing, diffusion theory, kinetic theory, systems identification, biology and other areas [1, 2]. The first application of fractional calculus was introduced by Abel [3] in the solution of an integral equation that was arisen in the formulation of the tautochronous problem. This problem deals with the determination of the shape of a frictionless plane curve through the origin in a vertical plane along with a particle of mass m can fall in a time that is independent of the starting position [4].

Travelling wave methods have an important role to obtain solutions that are described and explained these natural phenomena. Most famous of by these effective methods are \((G'/G)\)-expansion method [5 – 6], variational iteration algorithm-I

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[7–8], Exp-function method [9], fractional iteration algorithm [10–11], Generalization of He’s Exp-Function Method [12], reproducing Kernal method [13], a new extended Auxiliary equation method [14], variational iteration algorithm-II [15–16] and Modified Kudryashov method [17–19]. In this paper, we use generalized Kudryashov method for finding the exact solutions of space-time fractional coupled integrable dispersionless system.

2. Properties of fractional derivatives

In this paper, we consider the most common definition named in modified Riemann-Liouville derivative which is defined [20–26]

\[
D_t^\gamma u(t) = \begin{cases} 
\frac{1}{\Gamma(1-\gamma)} \int_0^t (t-\tau)^{-\gamma-1}(u(\tau) - u(t))d\tau, & \gamma < 0, \\
\frac{1}{\Gamma(1-\gamma)} \frac{d}{dt} \int_0^t (t-\tau)^{-\gamma-1}(u(\tau) - u(t))d\tau, & 0 < \gamma \leq 1, \\
(u^{(n-1)}(\tau))^{(\gamma-n-1)}, & n-1 < \gamma \leq n, \ n \geq 2
\end{cases}
\]

where \( u : \mathbb{R} \rightarrow \mathbb{R}, t \rightarrow u(t) \), denotes a continuous function.

Property 1,

\[
D_t^\gamma t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\gamma)} t^{r-\gamma}, \ r > 0
\]

Property 2,

\[
D_t^\gamma (u(t)g(t)) = g(t)D_t^\gamma u(t) + u(t)D_t^\gamma g(t).
\]

Property 3,

\[
D_t^\alpha u(g(t)) = \frac{du(g(t))}{dg(t)} D_t^\alpha g(t)
\]

3. Description of the method for FDEs

Consider a given nonlinear wave equation

\[
N(u, D_t^\alpha u, D_x^\alpha u, D_{xx}^\alpha u, D_t^{2\alpha} u, D_x^{2\alpha} u, D_t^\alpha D_x^\alpha u, ...) = 0,
\]

we seek its wave solutions

\[
u = U(\eta), \quad \eta = \frac{b_i x_i^\alpha}{\Gamma(1+\alpha)} + \frac{g t^\alpha}{\Gamma(1+\alpha)}, \quad i = 1, 2, ...
\]
Consequently, Eq. (5) is reduced to the ordinary differential equation (ODE) by transformation:

\[(7) \quad U(u, gu', hu', g^2u'', h^2u'', ...) = 0.\]

The generalized Kudryashov method (GKM) is based on the assumption that the travelling wave solutions can be expressed in the following form

\[(8) \quad u(\eta) = \sum_{i=0}^{m} \frac{a_i}{(1 + \phi(\eta))^i},\]

where \(m\) is positive integer which are unknown to be further determined, \(a_i\) are unknown constants. In addition, \(\phi(\eta)\) satisfies Riccati equation

\[(9) \quad \phi'(\eta) = A + B\phi(\eta) + C\phi^2(\eta).\]

We obtained a type of solutions of Eq. (9)

**Family 1:** \(A\) and \(B\) are free constants, \(C \neq 0\)

\[\phi(\eta) = -B + \sqrt{4AC - B^2} \tan\left(\frac{1}{2}(\sqrt{4AC - B^2}(\eta + d_0))\right) / 2C.\]

**Family 2:** \(A = 0, B \neq 0,\) and \(C\) is a free constant

\[\phi(\eta) = -B \exp(B\eta + Bd_0) / C\exp(B\eta + Bd_0) - 1.\]

**Family 3:** \(A\) is free constant, \(B \neq 0,\) and \(C = 0\)

\[\phi(\eta) = -A / B + 1 / B \exp(B\eta).\]

**Family 4:** \(A = 0, B = -1\) and \(C = -1\)

\[\phi(\eta) = -d_0 / \exp(\eta) + d_0.\]

4. **Space-time fractional coupled Integrable Dispersionless system**

We consider the space-time fractional coupled Integrable Dispersionless (CID) system

\[\frac{\partial^{2\alpha} u}{\partial t^\alpha \partial x^\alpha} + \frac{\partial^\alpha}{\partial x^\alpha}(vw) = 0,\]

\[\frac{\partial^{2\alpha} v}{\partial t^\alpha \partial x^\alpha} - 2v \frac{\partial^\alpha u}{\partial x^\alpha} = 0,\]
where \( u, v \) and \( w \) are all functions of \( x \) and \( t \). Eqs. (10) describes the current-fed string within an external magnetic field [27, 28]. This equations was presented and solved by the inverse scattering method [29], the exp-function method [30] and residue harmonic balance [31].

We perform the transformation \( \eta = \frac{b e^{\theta}}{1+(1+\alpha)^{\gamma}} + \frac{a e^{\theta}}{1+(1+\alpha)^{\gamma}} \), Eq. (10) can be reduced into an ODEs

\[
\begin{align*}
\frac{\partial^2 w}{\partial t^2} - 2u \frac{\partial w}{\partial x} &= 0, \\
\frac{\partial^2 w}{\partial x^2} - 2v \frac{\partial w}{\partial x} &= 0, \\
\frac{\partial^2 w}{\partial x^2} - 2w \frac{\partial w}{\partial x} &= 0,
\end{align*}
\]

We can freely know that the solution does not depend on the balancing the highest order linear and nonlinear terms [32]. For simplicity, we set \( i=2 \), we have:

\[
\begin{align*}
U(\eta) &= a_0 + \frac{a_1}{1 + \phi(\eta)} + \frac{a_2}{(1 + \phi(\eta))^2}, \\
V(\eta) &= b_0 + \frac{b_1}{1 + \phi(\eta)} + \frac{b_2}{(1 + \phi(\eta))^2}, \\
W(\eta) &= r_0 + \frac{r_1}{1 + \phi(\eta)} + \frac{r_2}{(1 + \phi(\eta))^2}.
\end{align*}
\]

Substituting Eq. (12) into Eq. (11), equating to zero the coefficients of all powers of \( \phi(\eta) \) yields a set of algebraic equations for \( a_i, b_i, r_i \).

\[
\begin{align*}
&h(6BCr_0a_1 + 2Ar_0a_1 + 3gr_1AB + 4Br_0a_2 + gr_1B^2C + 2BCr_1a_1 + 4gr_2B^2) = 0, \\
&h(2Ar_0a_1 + 6Bb_0a_1 + 4gb_2B^2 + 3gb_1AB + gb_1B^2 + 4BCb_1a_1 + 2Bb_1a_1) = 0, \\
&h(2Br_0a_1 + gr_1B^2) = 0, h(gb_1B^2 + 2Bb_0a_1) = 0, h(ga_1B^2 - Bb_0a_1 - Bb_1r_0) = 0, \\
&h(6gr_1A^2 + 6gr_2A^2 + 2Ar_0a_1 + 4Ar_0a_2 + 2Ar_1Ca_1 + 4Ar_1a_2 + 2ACr_2a_1 + 4Ar_2a_2 - 2gr_2AB - gr_1AB) = 0, \\
&h(4Ar_1a_2 + 2Ab_2a_2 + 2Ab_0a_1 + 4Ab_0a_2 + 6gb_2A^2 - gb_1AB - 2gb_2AB + 2Ab_1a_2 + 2gb_1A^2) = 0, \\
&h(-2Ab_1a_2 + 2ga_1A^2 + 6ga_2A^2 - ACb_1r_0 - 4Ab_2r_2 - 3ACb_1r_2 - 2Ab_2r_0
\end{align*}
\]
\[-3Ab_2r_1 - Ab_0r_1 - ga_1AB - 2Ab_0r_2 - 2ga_2AB = 0,\]

(13)

Solving the system of algebraic equations with the help of Maple, we obtain the solutions organized in the following cases:

**Case (1)**

\[
\begin{align*}
a_1 &= -gA - gC + gB, \\
b_0 &= \frac{-1}{4r_0}g^2(4C^2 - 4BC + B^2), \\
b_1 &= \frac{-1}{2r_0}g^2(-2AC - 2C^2 + 3BC + AB - B^2), \\
r_1 &= \frac{2r_0(A + C - B)}{-2C + B}, \quad a_0 \text{ is arbitrary, } b_2 = r_2 = a_2 = 0.
\end{align*}
\]

(14)

Substituting these results into (11) and with the aid of families 1-4, we obtain the following multiple soliton-like and periodic solutions for space-time fractional CID system

\[
\begin{align*}
u(x, t) &= u(x, t) = a_0 + \frac{-gA - gC + gB}{1 + \frac{-B}{2C} + \frac{\sqrt{4AC - B^2}\tan\left(\frac{1}{2}\sqrt{4AC - B^2}(\eta + d_0)\right)}{2C}}, \\
v(x, t) &= v(x, t) = \frac{-1}{4r_0}g^2(4C^2 - 4BC + B^2) + \frac{-2Cg^2(-2AC - 2C^2 + 3BC + AB - B^2)}{2r_0(2C - B + \sqrt{4AC - B^2}\tan\left(\frac{1}{2}\sqrt{4AC - B^2}(\eta + d_0)\right))}, \\
w(x, t) &= w(x, t) = \frac{r_0}{(-2C + B)(2C + -B + \sqrt{4AC - B^2}\tan\left(\frac{1}{2}\sqrt{4AC - B^2}(\eta + d_0)\right))}.
\end{align*}
\]

(15)

where \(\eta = \frac{hx^\alpha}{\Gamma(1+\alpha)} + \frac{gt^\alpha}{\Gamma(1+\alpha)}\).

**Case (2)**

\[
\begin{align*}
a_1 &= gC, \\
b_1 &= \frac{g^2C^2}{r_1}, B = 2C, r_1 \text{ is arbitrary} \\
r_0 &= b_0 = a_2 = b_2 = r_2 = A = 0.
\end{align*}
\]

(16)

Substituting these results into (11) and with the aid of families 1-4, we obtain the following multiple soliton-like and periodic solutions for space-time fractional CID system

\[
\begin{align*}
u(x, t) &= u(x, t) = a_0 - \frac{gC(C \exp(B\left(\frac{hx^\alpha}{\Gamma(1+\alpha)} + \frac{gt^\alpha}{\Gamma(1+\alpha)}\right) + Bd_0) - 1)}{C \exp(B\left(\frac{hx^\alpha}{\Gamma(1+\alpha)} + \frac{gt^\alpha}{\Gamma(1+\alpha)}\right) + Bd_0) + 1}, \\
v(x, t) &= v(x, t) = b_0 - \frac{g^2C^2(C \exp(B\left(\frac{hx^\alpha}{\Gamma(1+\alpha)} + \frac{gt^\alpha}{\Gamma(1+\alpha)}\right) + Bd_0) - 1)}{r_1(C \exp(B\left(\frac{hx^\alpha}{\Gamma(1+\alpha)} + \frac{gt^\alpha}{\Gamma(1+\alpha)}\right) + Bd_0) + 1}).
\end{align*}
\]
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(17) \[ w(x,t) = r_0 - \frac{r_1(C \exp(B(\frac{hx}{\Gamma(1+\alpha)} + \frac{gt}{\Gamma(1+\alpha)}) + Bd_0) - 1)}{C \exp(B(\frac{hx}{\Gamma(1+\alpha)} + \frac{gt}{\Gamma(1+\alpha)}) + B)d_0 + 1}. \]

Case (3)

\[ a_1 = g(B - A), b_0 = \frac{-g^2B^2}{4r_0}, r_1 = \frac{-2r_0(A - B)}{B}, a_0 \text{ is arbitrary} \]

Substituting these results into (11) and with the aid of families 1-4, we obtain the following multiple soliton-like and periodic solutions for space-time fractional CID system

\[ u(x,t) = a_0 + \frac{g(B - A)}{1 - \frac{A}{B} + \frac{1}{B} \exp(B(\frac{hx}{\Gamma(1+\alpha)} + \frac{gt}{\Gamma(1+\alpha)}))}, \]
\[ v(x,t) = -\frac{g^2B^2}{4r_0} - \frac{g^2B(A - B)}{2r_0(1 - \frac{A}{B} + \frac{1}{B} \exp(B(\frac{hx}{\Gamma(1+\alpha)} + \frac{gt}{\Gamma(1+\alpha)})))}, \]
\[ w(x,t) = r_0 - \frac{-2r_0(A - B)}{B(1 - \frac{A}{B} + \frac{1}{B} \exp(B(\frac{hx}{\Gamma(1+\alpha)} + \frac{gt}{\Gamma(1+\alpha)})))}. \]

5. Discussion

In this section, we discuss the physical explanations of the obtained solutions. Note that, the plots of the solutions (15), (17), and (19) are presented in figures 1 to 6 at specific values of the free constants. It appears that the solutions of (15), (17) and (19) depend on the sign of the magnitude $4AC - B^2$. In the case of $4AC - B^2 > 0$, the solution (15) is expressed in terms of the trigonometric tan function and hence an anti-kink wave is produced as shown by Fig. 7.1(a). Similarly the solution (19) in Fig. 7.3(a). On the other side, $4AC - B^2 < 0$, the solution (17) can be expressed in terms of the hyperbolic tan function and accordingly a kink wave is resulted as displayed in Fig. 7.2(b).

On the other hand, the solutions are affected by the fractional derivatives $\alpha$, in Fig. 7.1(b) the anti-kink wave increases with increasing of $\alpha$ but the reverse effect is observed a little near off the plate \((x > 0.5)\). Similarly the solution (19) in Fig. 7.1(b). Finally, Fig. 7.2(b) describes the u-solution in (17), the kink wave increases with increasing of $\alpha$ at $0 < x \leq 1$ and then it stabilizes with different value of $\alpha$ at $x > 1$. Accordingly, this method is capable of producing a different types of wave solutions for partial differential equations.
6. Conclusion

In the present paper, GKM has been successfully employed to obtain the exact solution of space-time fractional coupled integrable dispersionless system. New travelling wave technique is applied to search for the exact solitary solutions. The main advantage of the proposed method over the others is the fact that it can be applied to a wide class of nonlinear evolution equations. The modified Kudryashov [16] is special case of this technique (family 3, take $A=0$ and $B=1$). Finally, the obtained results have been graphically presented to show the properties of the obtained solutions.

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(a) Anti-Kink wave solution of Eq. (17) where $a_0 = a_1 = g = A = B = C = 1, d_0 = 0, h = 0.1$ and $\alpha = \frac{1}{2}$

(b) Anti-Kink wave solution of Eq. (17) where $t=0, a_0 = a_1 = g = A = B = C = 1, d_0 = 0$ and $h = 0.1$

Fig. 7.1: (a) Anti-Kink wave solution of Eq. (17) where $a_0 = a_1 = g = A = B = C = 1, d_0 = 0, h = 0.1$ and $\alpha = \frac{1}{2}$

(b) Anti-Kink wave solution of Eq. (17) where $t=0, a_0 = a_1 = g = A = B = C = 1, d_0 = 0$ and $h = 0.1$
Fig. 7.2: (a) Kink wave solution of Eq. (15) where $a_0 = 1$, $C = 2$, $g = -1$, $d_0 = 0$, $h = 1$ and $\alpha = \frac{3}{4}$.
(b) Kink wave solution of Eq. (15) where $t = 0$, $a_0 = 1$, $C = 1$, $g = -1$, $d_0 = 0$ and $h = 1$.

Fig. 7.3: (a) (5) Anti-Kink wave solution of Eq. (19) where $a_0 = r_0 = 1$, $b_1 = -0.1$, $g = B = 1$, $h = 0.1$ and $\alpha = \frac{1}{2}$.
(b) Anti-Kink wave solution of Eq. (19) where $a_0 = r_0 = 1$, $b_1 = -0.1$, $g = B = 1$, $h = 0.1$.
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Solitary Wave Solutions

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