Constraining $H_0$ from Sunyaev-Zel’dovich effect, Galaxy Clusters X-ray data, and Baryon Oscillations

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ABSTRACT

Estimates of $H_0$ from Sunyaev-Zel’dovich effect (SZE) and X-ray surface brightness of galaxy clusters depends on the underlying cosmology. In the current ΛCDM flat cosmology, a possible technique to broke the degenerescency on the mass density parameter ($\Omega_m$) is to apply a joint analysis involving the baryon acoustic oscillations (BAO). By adopting this technique to the ($H_0, \Omega_m$) parameter space, we obtain new constraints on the Hubble constant $H_0$ from BAO signature as given by the Sloan Digital Sky Survey (SDSS) catalog. Our analysis based on the SZE/X-ray data for a sample of 25 clusters yields $H_0 = 74^{+4}_{-3.5}$ km s$^{-1}$ Mpc$^{-1}$ (1σ, neglecting systematic uncertainties). This result is in good agreement with independent studies from the Hubble Space Telescope key project and the recent estimates of WMAP, thereby suggesting that the combination of these three independent phenomena provides an interesting method to constrain the Hubble constant.

Subject headings: Hubble constant, galaxy clusters, Sunyaev-Zel’dovich effect, X-ray surface brightness, baryon acoustic oscillations
1. INTRODUCTION

Galaxy clusters are one of the most impressive evolving structures from an earlier stage of the Universe. Usually, they congregate thousands of galaxies and are endowed with a hot gas (in the intra cluster medium), emitting X-rays primarily through thermal bremsstrahlung. Several studies in the last decade have suggested that the combination of data from different physical processes in galaxy clusters provides a natural method for estimating some cosmological parameters (Bartlett & Silk 1994, Rephaeli 1995, Kobayashi et al. 1996, Reese et al. 2002; Bartlett 2004; De Filippis et al. 2005; Bonamente et al. 2006). The ultimate goal in the near future is to shed some light on the nature of the dark energy.

An important phenomena occurring in clusters is the Sunyaev-Zel’ dovich Effect (SZE), a small distortion of the Cosmic Microwave Background (CMB) spectrum provoked by the inverse Compton scattering of the CMB photons passing through a population of hot electrons (Sunyaev & Zel’dovich 1972). Since the SZE is insensitive to the redshift of galaxy clusters, it provides a very convenient tool for studies at intermediate redshifts where the abundance of clusters depends strongly on the underlying cosmology (the unique redshift dependence appear in the total SZE flux due to the apparent size of the cluster). Another fundamental process is the X-ray emission from the hot electrons in the intracluster medium. When the X-ray surface brightness is combined with the SZE temperature decrement in the CMB spectrum, the angular diameter distance of galaxy clusters is readily obtained.

The possibility to estimate the galaxy cluster distances trough SZ/X-ray technique was suggested long ago by many authors (Silk & White 1978; Birkinshaw 1979; Cavaliere et al. 1979), but only recently it has been applied for a fairly large number of clusters (for reviews, see Birkinshaw 1999; Carlstrom, Hoder & Reese 2002). Such a method is based on the different dependence of the cluster electron density ($n_e$) and the temperature $T_e$ of the SZE ($\propto n_e T_e$) and X-ray bremsstrahlung ($\propto n_e^2 T_e^{1/2}$). Combining both measurements it is possible to estimate the angular diameter distance and infer the value of the Hubble constant whether the cosmology is fixed. The main advantage of this method for estimating $H_0$ is that it does not rely on extragalactic distance ladder being fully independent of any local calibrator. A basic disadvantage rests on the difficulty of modeling the cluster gas which causes great systematic uncertainties in its determination. In particular, this means that systematic effects on $H_0$ are quite different from the ones presented by other methods, like the traditional distance ladder or gravitational lensing (Reese et al. 2002; Jones et al. 2005; De Filippis et al. 2005).

In order to estimate the distance to the cluster from its X-ray spectroscopy, one needs to add some complementary assumptions about its geometry. The importance of the intrinsic geometry of the cluster has been emphasized by many authors (Fox & Pen 2002; Jing &
The standard spherical geometry has been severely questioned, since Chandra and XMM-Newton observations have shown that clusters usually exhibit an elliptical surface brightness. In a point of fact, the cluster shape estimation problem is a difficult matter since many clusters do not appear in radio, X-ray, or optical. Another source of difficulty is related to the error bars. Assuming that the clusters have an axisymmetric form, different authors introduced a roughly random uncertainty in $H_0$ between 15% − 30% (Hughes & Birkinshaw 1998; Sulkanen 1999; Reese et al. 2002; Jones et al. 2005). The assumed cluster shape also affects considerably the SZE/X-ray distances, and, therefore, the Hubble constant estimates.

Fox and Pen (2002) estimate the Hubble constant by assuming triaxial clusters and measuring the distance to artificial observations corrected for asphericity. De Filippis and collaborators (2005) showed that the spherical hypothesis is strongly rejected for most members of the sample studied. By taking into account such an effect for two samples, a better agreement with the cosmic concordance model ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) was obtained. Triaxial clusters may also be useful for reconciling the observed discrepancies in the total mass of clusters as computed with lensing and X-ray measurements (in this connection see Bonamente et al. 2006).

The determination of $H_0$ has a practical and theoretical importance to many astrophysical properties of galaxies and quasars, and several cosmological calculations, like the age of the Universe, its size and energy density, primordial nucleosynthesis, and others (Freedman 2000; Peacock 1999). Spergel et al. (2006) have shown that CMB studies can not supply strong constraints on the value of $H_0$ on their own. This problem occurs due to the degenerescency on the parameter space (Tegmark et al 2004), and may be circumvented only by using independent measurements of $H_0$ (Hu 2005).

On the other hand, according to cold dark matter (CDM) picture of structure formation, large-scale fluctuations have grown since $z \sim 1000$ by gravitational instability. The cosmological perturbations excite sound waves in the relativistic plasma, producing the acoustic peaks in the early universe. Eisenstein et al. (2005) presented the large scale correlation function from the Sloan Digital Sky Survey (SDSS) showing clear evidence for the baryon acoustic peak at $100h^{-1}$ Mpc scale, which is in excellent agreement with the WMAP prediction from the CMB data. The Baryon Acoustic Oscillations (BAO) method is independent of the Hubble constant $H_0$ which means that we can use BAO signature to break the degenerescency of the mass parameter $\Omega_m$. Hence, combining SZE/X-ray method to obtain $D_A$ with BAO it is possible to improve the limits over $H_0$ (for recent applications of BAO see, Lima et al. 2006).

In this letter, by assuming that the clusters are ellipsoids with one axis parallel to the line
of sight, we derive new constraints on the Hubble constant $H_0$. By considering the sample of 25 triaxial clusters given by De Filippis et al. (2005), we perform a joint analysis combining the data from SZE and X-ray surface brightness with the recent SDSS measurements of the baryon acoustic peak (Eisenstein et al. 2005).

2. Basic equations and Sample

Let us now consider that the Universe is described by a flat Friedman-Robertson-Walker (FRW) geometry driven by cold dark matter plus a cosmological constant. In this case, we have only two free parameters ($H_0, \Omega_m$) and the angular diameter distance, $D_A$ reads (Lima et al. 2003, Alcaniz 2004, De Filippis et al. 2005)

$$D_A(z; h, \Omega_m) = \frac{3000h^{-1}}{(1 + z)} \int_o^z \frac{dz'}{\mathcal{H}(z'; \Omega_m)} \text{ Mpc},$$

where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and the dimensionless function $\mathcal{H}(z'; \Omega_m)$ is given by

$$\mathcal{H} = \left[ \Omega_m(1 + z')^3 + (1 - \Omega_m) \right]^{1/2}.$$

Following De Filippis et al. (2005), a general triaxial morphology it will adopted here. In this case, the intra cluster medium is described by an isothermal triaxial $\beta$-model distribution and the SZE decrement reads

$$\Delta T_{SZ} \equiv T_0 f(\nu, T_e) \frac{\sigma_T k_B T_e}{m_e c^2} n_{e0} \sqrt{\pi}$$

$$\times \frac{D_A \theta_{c,\text{proj}}}{b^{3/4}} \sqrt{\frac{e_1 e_2}{e_{\text{proj}}}} g(\beta),$$

where $T_0 = 2.728K$ is the CMB temperature, $T_e$ is the gas temperature, $\sigma_T$ is the Thompson cross section, the factor $f(\nu, T_e)$ accounts for frequency shift and relativistic corrections, $n_{e0}$ is the central number density of the cluster gas, $b$ is a function of the cluster shape and orientation, $e_{\text{proj}}$ is the axial ratio of the major to the minor axes of the observed projected isophotes, $\theta_{c,\text{proj}}$ is the projection on the plane of the sky of the intrinsic angular core radius, and $g(\beta) = \Gamma(3\beta - 1/2)/\Gamma(3\beta)$ ($\Gamma$ denotes the Gamma function).

Similarly, the X-ray central surface brightness $S_{X0}$ can be written as

$$S_{X0} \equiv \frac{\Lambda e_H \mu_e/\mu_H n_{e0}^2 D_A \theta_{c,\text{proj}}}{4\sqrt{\pi}(1 + z)^4} \frac{e_1 e_2}{e_{\text{proj}}} g(\beta),$$
Fig. 1.— Angular diameter distance as a function of redshift for $\Omega_m = 0.3$ and some selected values of the $h$ parameter. The data points correspond to the the SZE/X-ray distances for 25 clusters from De Filippis et al. (2005). The open diamond indicates the Abell 773 outlier cluster, which has been excluded from our statistical analysis (see section 3).
where $z$ is the redshift of the cluster, $\Lambda_{eH}$ is the X-ray cooling function of the ICM in the cluster rest frame and $\mu$ is the molecular weight ($\mu_i \equiv \rho / n_i m_p$).

De Filippis and collaborators (2005) studied and corrected the $D_A$ measurements for 25 clusters, getting a better agreement with the $\Lambda$CDM models. We used two samples studied by them to investigate the bounds arising from SZE/X-ray observations. One of the samples, compiled by Reese et al. (2002), is a selection of 18 galaxy clusters distributed over the redshift interval $0.14 < z < 0.8$. The other one, the sample of Mason et al. (2001), has 7 clusters from the X-ray limited flux sample of Ebeling et al. (1996). De Filippis et al. (2005) show that the samples turn out slightly biased, with strongly elongated clusters preferentially aligned along the line of sight. Their results suggest that 15 clusters are in fact more elongated along the line of sight, while the remaining 10 clusters are compressed.

In Fig. 1, the galaxy cluster sample is plotted on a residual Hubble diagram using a flat cosmic concordance model ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$). We see that the A773 cluster is the largest outlier, and our statistical analysis confirms that its inclusion leads to the highest $\chi^2$. For that reason we have excluded this data point from the statistical analysis.

3. Analysis and Results

Now, let us perform a $\chi^2$ fit over the $h - \Omega_m$ plane. In our analysis we use a maximum likelihood that can be determined by a $\chi^2$ statistics,

$$\chi^2(z|p) = \sum_i \frac{(D_A(z_i;p) - D_{Ao,i})^2}{\sigma_{D_{Ao,i}}^2},$$

(5)

where $D_{Ao,i}$ is the observational angular diameter distance, $\sigma_{D_{Ao,i}}$ is the uncertainty in the individual distance and the pair, $p \equiv (h, \Omega_m)$, is the complete set of parameters.

In what follows, we first consider the SZE/X-ray distances separately, and, further, we present a joint analysis including the BAO signature from the SDSS catalog. Note that a specific flat cosmology has not been fixed by hand in the analyzes below.

3.1. Limits from SZE/X-ray

We now consider the 24 clusters (without the A773, see Fig.1), which constitutes the SZE/X-ray data from De Filippis et al. (2005). Our analysis indicated that any cosmological model could be accepted by that sample until $3\sigma$ (with 2 free parameters). It also shows that using only the ellipsoidal cluster sample we cannot constrain the energetic components
of the cosmological model. This happens basically because the error bars are large, mainly at intermediate and high redshifts.

In Fig. 2 we show the contours of constant likelihood (68.3%, 95.4% and 99.7%) in the space parameter $h - \Omega_m$ for the SZ/X-ray data discussed earlier. Note that only a small range for the $h$ parameter is allowed, $(0.64 \leq h \leq 0.85)$, at 1$\sigma$ of confidence level. In particular, we found $h = 0.75^{+0.07}_{-0.07}$ and $\Omega_m = 0.15^{+0.57}_{-0.15}$ with $\chi^2_{\text{min}} = 24.4$ at 68.3% c.l. for 1 free parameter. Naturally, such bounds on $h$ are reasonably dependent on the cosmological model adopted. For example, if we fix $\Omega_m = 0.3$ we have $h = 0.74$, for $\Omega_m = 1.0$ we have $h = 0.67$, and both cases are permitted with high degree of confidence. Clearly, we see that an additional cosmological test (fixing $\Omega_m$) is necessary in order to break the degenerescency on the $(\Omega_m, h)$ plane.

Systematic effects still need to be considered. The common errors are: SZ $\pm 8\%$, X-ray $\pm 10\%$, radio halos $-3\%$, 5\% for Galactic $N_H$, 10\% for isothermality, 2\% kinetic SZ, clumping causes $-20\%$, radio source confusion $\pm 12\%$, primary beam $\pm 3\%$ and 1% on the CMB. When we combine the errors in quadrature, we find that the typical error are of 20\% - 30\%, in agreement with others works (Mason et al. 2001; Reese et al. 2002; Reese 2004).

3.2. Joint Analysis for SZE/X-ray and BAO

As remarked earlier, more stringent constraints on the space parameter $(h, \Omega_m)$ can be obtained by combining the SZE/X-ray with the BAO signature (Eisenstein et al. 2005). The peak detected (from a sample of 46748 luminous red galaxies selected from the SDSS Main Sample) is predicted to arise precisely at the measured scale of 100 $h^{-1}$ Mpc. Basically, it happens due to the baryon acoustic oscillations in the primordial baryon-photon plasma prior to recombination. Let us now consider it as an additional cosmological test over the ellipsoidal cluster sample. Such a measurement is characterized by

$$A \equiv \frac{\Omega_m^{1/2}}{H(z_\ast)^{1/3}} \left[ \frac{1}{z_\ast} \Gamma(z_\ast) \right]^{2/3} = 0.469 \pm 0.017, \quad (6)$$

where $z_\ast = 0.35$ is the redshift at which the acoustic scale has been measured, and $\Gamma(z_\ast)$ is the dimensionless comoving distance to $z_\ast$.

Note that the above quantity is independent of the Hubble constant, and, as such, the BAO signature alone constrains only the $\Omega_m$ parameter. This property is very characteristic of the BAO signature, thereby differentiating it from many others classical cosmological tests, like the gas mass fraction (Lima et al. 2003; Allen et al. 2004; Cunha et al. 2006),
Fig. 2.— Confidence regions (68.3%, 95.4% and 99.7%) in the \((\Omega_m, h)\) plane provided by the SZE/X-ray data from De Filippis et al. (2005). The best fit values are \(h = 0.75\) and \(\Omega_m = 0.15\).
Fig. 3.— Contours in the $\Omega_m - h$ plane using the SZE/X-ray and BAO joint analysis. The contours correspond to 68.3%, 95.4% and 99.7% confidence levels. The best-fit model converges to $h = 0.74$ and $\Omega_m = 0.27$. 
luminosity distance (Peebles & Ratra 2003; Cunha et al. 2002), or the age of the universe (Alcaniz et al. 2003; Cunha & Santos 2004).

In Fig. 3, we show the confidence regions for the SZE/X-ray cluster distance and BAO joint analysis. By comparing with Fig. 2, one may see how the BAO signature breaks the degenerescency in the \((\Omega_m, h)\) plane. As it appears, the BAO test presents a striking orthogonality centered at \(\Omega_m = 0.27^{+0.03}_{-0.02}\) with respect to the angular diameter distance data as determined from SZE/X-ray processes. We find \(h = 0.738^{+0.042}_{-0.033}\) and \(\chi^2_{\text{min}} = 24.5\) at 68.3% (c.l.) for 1 free parameter. An important lesson here is that the combination of SZE/X-ray with BAO provides an interesting approach to constrain the Hubble constant.

In Fig. 4, we have plotted the likelihood function for the \(h\) parameter in a flat \(\Lambda\)CDM universe for the SZE/X-ray + BAO data set. The dotted lines are cuts in the regions of 68.3% probability and 95.4%.

Our results are in line with some recent analyzes based on different cosmological observations, like the one provided by the WMAP team \(h = 0.73 \pm 0.03\) (Spergel et al. 2006), and the HST Project \(h = 0.72 \pm 0.08\) (Freedman et al. 2001). Note, however, that it does not agree with the recent determination, \(h = 0.62 \pm 0.013\) (random) \(\pm 0.05\) (systematics), recently advocated by Sandage and collaborators (2006). A result obtained with basis on Type Ia Supernovae, calibrated with Cepheid variables in nearby galaxies that hosted them.

At this point, it is interesting to compare our results with others recent works in which the limits on \(h\) were obtained by fixing the cosmology \((\Omega_m = 0.3, \Omega_\Lambda = 0.7, \text{cosmic concordance})\), and assuming spherical geometry. A measurement using SZ effect was accomplished by Mason et al. (2001), using 5 clusters, and gives \(h = 0.66^{+0.14}_{-0.11}\); Reese and coauthors (2002), using 18 clusters, found \(h = 0.60 \pm 0.04\), and in a posterior analysis Reese (2004), with 41 clusters, obtains \(h \approx 0.61 \pm 0.03\); Jones et al. (2005) derived \(h = 0.66^{+0.11}_{-0.10}\), using a sample of 5 clusters free of any orientation bias. In a recent paper, Bonamente et al. (2006), using 38 clusters, obtained \(h = 0.769^{+0.039}_{-0.034}\). All these results, using SZ/X-ray technique, presented a systematic uncertainty of 10%-30%. In Table 1, we summarize the estimates of \(H_0\) from clusters in the framework of \(\Lambda\)CDM models (the data in round brackets is the number of clusters).

It is worth notice that the best-fit scenario derived here, \(\Omega_m = 0.27^{+0.03}_{-0.02}\) and \(h = 0.738^{+0.042}_{-0.033}\), corresponds to an accelerating Universe with \(q_0 = -0.6\), a total evolutionary age of \(t_o \approx 10h^{-1}\) Gyr, and a transition redshift (from deceleration to acceleration) \(z_t \approx 0.6\). At 95.4% c.l. (2\(\sigma\)) the BAO+SZE/X-ray analysis also provides \(h = 0.74^{+0.08}_{-0.07}\). Hopefully, future developments related to the physics of clusters may shed some light on the nature of the dark energy (for reviews see Peebles & Ratra 2003; Padmanhaban 2003; Lima 2004).
Fig. 4.— Likelihood function for the $h$ parameter in a flat $\Lambda$CDM universe, from SZE/X-ray emission. The shadow lines are cuts in the regions of 68.3% probability and 95.4%. We see that the region permitted is well constrained and in concordance with others studies (Freedman et al. 2001; Spergel et al. 2006).
4. Conclusions

Since the original work of Hubble, the precise determination of the distance scale ($H_0$) has been a recurrent problem in the development of physical cosmology. In this letter we have discussed a new determination of the Hubble constant based on the SZE/X-ray distance technique for a sample of 24 clusters as compiled by De Filippis et al. (2005). The degenerescency on the $\Omega_m$ parameter was broken using the baryon acoustic oscillation signature from the SDSS catalog. The Hubble constant was constrained to be $h = 0.74^{+0.04}_{-0.035}$ and $1.08^{+0.08}_{-0.07}$ for 1$\sigma$ and 2$\sigma$, respectively. These limits were derived assuming elliptical $\beta$-model and a flat $\Lambda$CDM scenario.

As we have seen, the baryon acoustic signature is an interesting tool for constraining directly the mass density parameter, $\Omega_m$, and, indirectly, it also improves the Hubble constant limits acquired from other cosmological techniques (like the SZE/X-ray cluster distance). Our Hubble constant estimation using the joint analysis SZE/X-ray + BAO is largely consistent with some recent cosmological observations, like the third year of the WMAP and the HST Key Project. Implicitly, such an agreement suggests that the elliptical morphology describing the cluster sample and the associated isothermal $\beta$-model is quite realistic. It also reinforces the interest to the observational search of galaxy clusters in the near future, when more and larger samples, smaller statistic and systematic uncertainties will improve the limits on the present value of the Hubble parameter.

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Table 1. Limits to $h$ from galaxy clusters (ΛCDM)

| Reference (data)          | $\Omega_m$ | $h \,(1\sigma)$ | $\chi^2$ |
|---------------------------|------------|------------------|---------|
| Mason et al. 2001 (7)     | 0.3        | $0.66^{+0.14}_{-0.11}$ | ≃ 2    |
| Reese et al. 2002(18)    | 0.3        | $0.60^{+0.04}_{-0.04}$ | 16.5   |
| Reese 2004 (41)          | 0.3        | $0.61^{+0.03}_{-0.03}$ | –      |
| Jones et al. 2005 (5)     | 0.3        | $0.66^{+0.11}_{-0.10}$ | –      |
| Bonamente et al. 2006 (38)| 0.3        | $0.77^{+0.04}_{-0.03}$ | 31.6   |
| Present work (24)        | $0.15^{+0.07}_{-0.15}$ | $0.75^{+0.04}_{-0.04}$ | 24.4   |
| Present work (24)+BAO    | $0.27^{+0.04}_{-0.03}$ | $0.74^{+0.03}_{-0.03}$ | 24.5   |