Effects of Magnetic field on Peristalsis transport of a Carreau Fluid in a tapered asymmetric channel

J Prakash\(^1\), N Balaji\(^2\), E P Siva\(^3\) M Kothandapani\(^4\) and A Govindarajan\(^5\)

\(^1\)Department of Mathematics, Agni College of Technology, Chennai – 130, Tamil Nadu, India.
\(^2,\ 3,\ 5\) Department of Mathematics, SRM Institute of Science and Technology,
\(^4\) Department of Mathematics, University College of Engineering Arni 632 326, Tamil Nadu, India.

Email: siva.e@ktr.srmuniv.ac.in

Abstract: The paper is concerned with effects of a uniform applied magnetic field on a Carreau fluid flow in a tapered asymmetric channel with peristalsis. The channel non-uniform & asymmetry are formed by choosing the peristaltic wave train on the tapered walls to have different amplitude and phase \((\phi)\). The governing equations of the Carreau model in two - dimensional peristaltic flow phenomena are constructed under assumptions of long wave length and low Reynolds number approximations. The simplified non - linear governing equations are solved by regular perturbation method. The expressions for pressure rise, frictional force, velocity and stream function are determined and the effects of different parameters like non-dimensional amplitudes walls \((a\ and\ b)\), non - uniform parameter \((m)\), Hartmann number \((M)\), phase difference \((\phi)\), power law index \((n)\) and Weissenberg numbers \((We)\) on the flow characteristics are discussed. It is viewed that the rheological parameter for large \((We)\), the curves of the pressure rise are not linear but it behaves like a Newtonian fluid for very small Weissenberg number.

1. Introduction

Generally peristaltic pumping is a form of fluid transport from a region of lower to higher pressure, with means of a progressive wave of area contraction or expansion; this circulates along the length of a tube-like structure. In physiology, peristalsis occurs in urine transport from kidney to bladder, the blood pumps in dialysis, movement of chime in the gastrointestinal tract, transport of spermatooza in the ducts efferent’s of the male reproductive tracts and in the cervical canal, in the movement of the ovum in the female fallopian tube and in the vasomotion of small blood vessels and the heart lung machine also involves the mechanism of peristalsis. In addition such a mechanism has numerous applications in engineering and in biomedical systems including roller and finger pumps. In 1966, Latham considered the fluid mechanics of peristaltic pumps and Burns and Parkes (1967) investigated the peristaltic motion of Newtonian fluid through a pipe and channel by considering sinusoidal waves along the walls. Shapiro et al. (1969) made a study on peristaltic pumping through a tube and a channel under the assumptions of low Reynolds number and long wavelength. El Shehawey et al. (1999) studied the peristaltic flow of a Newtonian fluid through a porous medium. Sobh (2008) discussed the peristaltic transport of couple stress fluid in uniform and non-uniform channels by analyzing the slip effect. His study pointed out that the pressure decreases with the Knudsen number.
which represents the slip effect as the friction force increases with it. Hayat et al. (2010) have examined the induced magnetic field effects on the peristaltic flow of a third order fluid in symmetric channel. Rao and Rajagopal (1999) analyzed some simple flows of Johnson–Segalman fluid and brought out some interesting conclusions. Shukla and Gupta (1982) investigated the peristaltic transport of a power-law fluid with variable viscosity.

In the modern humankind, there has been increasing interest in peristaltic flows, especially the peristaltic transport of non-Newtonian flows. Such flows occur in a variety of backgrounds like Blood, polymer solution, emulsion, shampoo, paints, certain oils and drilling mud are some examples of non-Newtonian fluids. The non-Newtonian fluids are studied as more suitable models of fluids in industrial and technological applications than Newtonian fluid. Some freshly motivating studies are dealing the flows of non-Newtonian fluids and they are given in Refs. (Vajravelu et al. 2012; Hayat et al. 2008; Abd-Alla et al. 2013; Javed et al. 2014; Fetecau and Fetecau 2004; Kothandapani and Srinivas 2008a,2008b; Tan and Masuoka 2005; Haroun 2007).

Magnetohydrodynamic (MHD) is the science which deals with the motion of highly electrical conducting fluids in the presence of a magnetic field. The problem of the MHD fluid flow has attracted the attention of many investigators due to its wide range of applications in optimization of the solidification processes of metal and metal alloys, the study of geothermal sources, the treatment of nuclear fuel debris, the control of underground spreading of chemical wastes and pollution, design of MHD power generators, blood pump machines, magnetic wound or cancer tumor treatment causing hyperthermia, bleeding reduction during surgeries and targeted transport of drug using magnetic particles as drug carries. The first investigation of the effect of the induced magnetic field on peristaltic flow was studied by Vishnyakov and Pavlov (1972) therein they considered the peristaltic MHD flow of an electrical conductive Newtonian fluid. Hayat et al. (2008) studied the MHD Jeffery fluid in a channel having compliant walls with porous space. Hayat and Hina (2010) have discussed the peristaltic motion of a MHD Maxwell fluid along with wall effects, heat and mass transfer. The peristaltic transport of a third order fluid under the effects of transverse magnetic field examined by Hayat et al. (2007). The effects of an induced magnetic field on the peristaltic flow of a Carreau fluid in a symmetric channel have studied by Hayat et al. (2010). He has concluded that the decrease in the induced magnetic field is maximum for sinusoidal and triangular waves and minimum for square and trapezoidal waves. Also the extended work of the analysis had given, and the interaction of peristalsis with an induced magnetic field and heat transfer has been studied for the motion of a Carreau fluid in an asymmetric channel by Hayat et al. (2011).

Recently, Physiologists observed that the intrauterine fluid flow due to myometrial contractions is peristaltic type motion and the myometrial contractions may occur in both symmetric and asymmetric directions, De Vries et al. (1990). Eytan et al. (1999) has also observed that the characterization of non-pregnant woman’s uterine contractions is very complicated as they are composed of variable amplitudes, a range of frequencies and different wavelengths. Further it is worthwhile to mention that the intrauterine fluid flow in a sagittal cross-section of the uterus discloses a narrow channel enclosed by two fairly parallel walls with wave trains having different amplitudes and phase difference (Eytan et al. 2001). Keeping view of the above facts in mind, we propose a general mathematical model for asymmetric wall – induced fluid motion in an infinite two – dimensional non-uniform channel. To the best of our knowledge, so far no attempt has been made to analyze the MHD Peristaltic transport of a Carreau Fluid in the tapered asymmetric channel. Therefore, the principal goal of this work is to make such an attempt. The governing problem is solved analytically by employing the regular perturbation technique to obtain the explicit expressions for the axial velocity, stream function and pressure drop. Variation of pertinent parameters on the flow quantities are sketched and discussed in detail.
2. Mathematical Formulation

Let us consider the peristaltic transport of a Carreau fluid through a two dimensional tapered asymmetric channel. Let $Y = H_1$ and $Y = H_2$ be respectively the lower and upper wall boundaries of the tapered asymmetric channel. The magnetic field is taken to be in the $Y$ direction. The medium is considered to be induced by a sinusoidal wave train propagating with a constant speed $c$ along the tapered asymmetric channel wall, such that

$$H_2(x, t) = d + m' \tilde{x} + a_2 \sin \left[ \frac{2\pi}{\lambda} (x - ct) \right],$$  \hfill (1)
$$H_1(x, t) = -d - m' \tilde{x} - a_1 \sin \left[ \frac{2\pi}{\lambda} (x - ct) + \phi \right].$$  \hfill (2)

where $a_1$ and $a_2$ are the amplitudes of wave lower and upper walls respectively, $2d$ is the width of the channel (Fig.1), $\lambda$ is the wavelength, $c$ is the speed of the wave, $m'(m'<<1)$ is the non-uniform parameter, the phase difference $\phi$ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of the phase i.e. both walls move towards the outward or inward simultaneously and further $a_1, a_2, d$ and $\phi$ satisfy the condition for the divergence channel at the inlet of flow

$$a_1^2 + a_2^2 + 2a_1a_2 \cos(\phi) \leq (2d)^2.$$  \hfill (3)

The governing equations of a Carreau fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$  \hfill (4)

$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} - \sigma B_0^2 u,$$  \hfill (5)

$$\rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y},$$  \hfill (6)

Where
The appropriate boundary conditions of this problem are given as below forms:

\[ \psi = \frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \text{ at } y = h_2 = 1 + mx + b \sin(2\pi(x-t)), \]  

\[ \psi = -\frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \text{ at } y = h_1 = -1 - mx - a \sin(2\pi(x-t) + \phi). \]  

Which satisfy
\[ a^2 + b^2 + 2ab \cos(\phi) \leq 4. \]  

3. Perturbation Solution

Here we attempt to find the approximate solution to the boundary-value problem consisting of Eq. (12) by employing the perturbation method for small Weissenberg numbers. For this purpose we expand the flow quantities in powers of the Weissenberg numbers \( \text{We} \) as follows:

\[ \psi = \psi_0 + \text{We}^2 \psi_1 + o(\psi_2), \]  

\[ p = p_0 + \text{We}^2 p_1 + o(p_2), \]  

\[ F = F_0 + \text{We}^2 F_1 + o(F_2). \]  

Substituting the above expressions in Eqs. (10 and 12) and boundary conditions in Eq.(13), we get the following order of systems.

3.1 For the system of order zero \( (\text{We}^0) \),

\[ \frac{\partial^4 \psi_0}{\partial y^4} - M^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0, \]  

\[ \frac{\partial p_0}{\partial x} = \frac{\partial^3 \psi_0}{\partial y^3} - M^2 \frac{\partial^2 \psi_0}{\partial y^2}, \]  

\[ \psi_0 = \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = 0, \text{ at } y = h_2, \]  

\[ \psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = 0, \text{ at } y = h_1, \]  

3.2 For the system of order one \( (\text{We}^1) \),

\[ \frac{\partial^4 \psi_1}{\partial y^4} - M^2 \frac{\partial^2 \psi_1}{\partial y^2} = \left( \frac{n-1}{2} \right) \frac{\partial^2}{\partial y^2} \left[ \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^3 \right], \]  

\[ \frac{\partial p_1}{\partial x} = \frac{\partial}{\partial y} \left[ \frac{\partial^2 \psi_1}{\partial y^2} + \frac{n-1}{2} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^3 \right] - M^2 \frac{\partial \psi_1}{\partial y}, \]  

\[ \psi_1 = \frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0, \text{ at } y = h_2, \]  

\[ \psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0, \text{ at } y = h_1, \]
3.3 Zeroth-Order Solution

Solution of Eq. (17) satisfying the boundary conditions (19) can be written as

\[
\psi_0 = \frac{F_0}{2} - \frac{F_0}{2} \left( \sinh(Mh_2) - \sinh(Mh_1)(\cosh(Mh_2) - \cosh(Mh_1)) (h_2 M \sinh(Mh_1) + \cosh(Mh_2)) \right)
\]

\[
+ \frac{F_0}{2} \left( \sinh(Mh_2) - \sinh(Mh_1)(\cosh(Mh_2) - \cosh(Mh_1)) (h_2 M \sinh(Mh_1) - \sinh(Mh_2)) \right)
\]

\[
+ \frac{M F_0}{2} \sinh(Mh_1)(\cosh(Mh_2) - \sinh(Mh_1)) y
\]

\[
+ \left( 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) (\sinh(Mh_1 - \sinh(Mh_2)) \right)
\]

\[
+ \left( 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) (\sinh(Mh_1 - \sinh(Mh_2)) \right)
\]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
+ \left( 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) (\sinh(Mh_1 - \sinh(Mh_2)) \right)
\]

\[
+ \frac{M^3 F_0}{2} \cosh(Mh_1)(\sinh(Mh_2) - \sinh(Mh_1)) y
\]

\[
+ \frac{M^3 F_0}{2} \cosh(Mh_1)(\sinh(Mh_2) - \sinh(Mh_1)) y
\]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

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- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

\[
- 2 \cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1) M \sinh(M(h_1 - h_2)) \]

3.4 First-Order Solution

Substituting Eq.(23) into Eq. (20) and solving the resulting equation subject to the boundary conditions in Eq.(22), we obtain

\[
\psi_1 = B_1 + B_2 y + B_3 \cosh(My) + B_4 \sinh(My) + \frac{(n-1)M^4}{64} \left( A_3^1 + 3A_3A_4^2 \right) \cosh(3My)
\]

\[
- \frac{(n-1)M^4}{64} \left( A_3^1 + 3A_3A_4^2 \right) \sinh(My) - \frac{(n-1)M^5}{16} \left( 3A_3^3 - 3A_3A_4^2 \right) y \sinh(My)
\]

\[
- \frac{(n-1)M^5}{16} \left( 3A_3^3 - 3A_3A_4^2 \right) y \cosh(My),
\]

\[
\frac{\partial P_1}{\partial x} = -M^2 \left( MB_3 \sinh(Mh_1) - MB_4 \cosh(Mh_1) - A_3 \right).
\]

Defining

\[
F = F_0 + We^2 F_1.
\]

Summarizing the perturbation solutions up to first order for \( \psi, \, dp/dx \) and \( \Delta p \) as

\[
\psi = \psi_0 + We^2 \psi_1, \quad \frac{dp}{dx} = \frac{dp_0}{dx} + We^2 \frac{dp_1}{dx}, \quad \Delta p = \Delta p_0 + We^2 \Delta p_1.
\]

Using \( F_0 = F - o(We^2 F_1) \) and then neglecting the terms greater than \( o(We^2) \) the results given by Eq. (28) expressed up to second order.
The non-dimensional expressions for the pressure rise $\Delta p$, frictional forces on the upper and lower walls $F_{\lambda,1}$ and $F_{\lambda,2}$ per wavelengths are given respectively as follows:

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx dt,$$

$$F_{\lambda,1}(t) = \int_0^1 h_1^2 \left( - \frac{\partial p}{\partial x} \right) dx dt,$$

(29) \hspace{1cm} (30)

4. Numerical Results and Discussion

In order to discuss the above obtained results quantitatively, we assume the instantaneous volume rate of the flow $F(x,t)$, periodic in $(x-t)$, (Srivastava et al. 1983; Srivastava and Srivastav 1988; Gupta and Sheshadri 1976; El Shehawey and Husseny 2000; Vajravelu et al. 2013; Mishra and Rao 2003; Kothandapani and Prakash 2015a, 2015b, 2015c) as

$$F(x,t) = \Theta + a \sin[2\pi(x-t) + \phi] + b \sin[2\pi(x-t)]$$

(31)

Where $\Theta$ is the time-averaged of the flow flux.

In this section, we present the behaviour of solutions of the Carreau fluid flow with peristalsis through graphs. The expression for average rise in pressure $\Delta p$ and frictional force $F_{\lambda,1}$ are calculated numerically using Mathematica software. The effect for various values of non-dimensional amplitudes of lower and upper walls ($a$ and $b$), dimensionless non-uniform parameter ($m$), Hartmann number ($M$), phase difference ($\phi$), power law index ($n$), Weissenberg numbers ($We$) of various parameters on the average rise in pressure $\Delta p$ are shown in Figures.2-6. It is considered that from Figure. 2 that the retrograde ($\Delta p > 0, \Theta < 0$) and peristaltic pumping ($\Delta p > 0, \Theta > 0$) regions and the pumping rate increases with an increase in the amplitudes of the upper wall ($b$), while in the co pumping ($\Delta p < 0, \Theta > 0$) region, the behavior is quite opposite.

Figure 2. The variation of $\Delta p$ with $\Theta$ for different values of $b$ with $a = 0.3$, $m = 0.2$, $\phi = \pi / 4$, $M = 1$, $n = 0.2$ and $We = 0.2$.

Figure 3. The variation of $\Delta p$ with $\Theta$ for different values of $m$ with $a = 0.2$, $b = 0.3$, $\phi = \pi / 3$, $M = 0.25$, $n = 0.5$ and $We = 0.1$. 
To see the effects of non-uniform channel $m$ on $\Delta p$, Figure 3 is delivered and is illustrated that pumping decreases when $m$ increases. A reverse state is examined in the co pumping region where pumping raises with the increase of the non-uniform channel. It is represented from Figure 4 that with an increase in Hartmann number $M$, the pumping rate increases in peristaltic pumping region, as in co pumping region the pumping rate decreases with an increase in Hartmann number. From Figures 5 – 6, it is observed that there is no variation in peristaltic pumping region for the Newtonian and Carreau fluid as the pumping curves coincide. Figure 5 is elucidated that in the retrograde and co pumping regions, pumping rate increase with an increase in power law index $n$, despite the fact that in the peristaltic pumping region over the curve of average rise in pressure versus mean flow rate coincide. Figure 6 shows the variation of average rise in pressure against mean flow rate $\Theta$, for different values of Weinberg number. It is noticed that in the retrograde and co pumping sections the pumping rate
decreases with an increase in Weinberg number and in the peristaltic pumping region, the curves are overlapped. The behavior of frictional force $F_{\lambda,1}$ as a function of $\Theta$, for different values of Weissenberg numbers, dimensionless non-uniform parameter, geometric parameter and power law index are portrayed in Figures. 9 - 11. It is viewed that the frictional force $F_{\lambda,1}$ has reverse behavior compared to average rise in pressure. The distribution of axial velocity ($u$) is plotted against $y$ in Figures. 12-17 for the fixed values of $x = 0.4$ and $t = 0.2$. It is noticed from Figure.12 that the velocity profile is parabolic in nature and it increases with an increase in dimensionless amplitude of lower wall $a$. In Figure. 13, the cause of phase difference $\phi$ on $u$ is captured. It is detected that with an increase in $\phi$ the axial velocity decreases at the centre point of the channel.

Figure.8. The variation of $F_{\lambda,1}$ with $\Theta$ for different values of $a$ with $b = 0.3$, $m = 0.1$, $\phi = \pi/2$, $M = .5$, $n = 0.5$ and $We = 0.5$.

Figure.9. The variation of $F_{\lambda,1}$ with $\Theta$ for different values of $M$ with $a = 0.5$, $b = 0.4$, $\phi = \pi/4$, $m = 0.5$, $n = 0.3$ and $We = 0.05$.

Figure.10. The variation of $F_{\lambda,1}$ with $\Theta$ for different values of $m$ with $b = 0.3$, $a = 0.2$, $\phi = \pi/3$, $M = 1$, $n = 0.2$ and $We = 0.4$.

Figure.11. The variation of $F_{\lambda,1}$ with $\Theta$ for different values of $n$ with $a = 0.3$, $b = 0.4$, $\phi = \pi$, $m = 0.15$, $M = 0.5$, and $We = 0.6$. 
Figure. 14 shows that with an increase in volume flow rate $\Theta$, the axial velocity profile increases. The axial velocity for the Hartmann number $M$ is shown in Figure. 15. It is observed that an increase in $M$ causes an increase in magnitude of axial velocity $u$ at the boundaries and situation revered in the core part of the channel. Figure. 16 reveals that the axial velocity near the channel walls is not similar in view of the non-uniform parameter $m$. The velocity decreases by increasing $m$ in the hub of the channel. Figure. 17 is plotted to seen the influence of $n$ and $We$ on the velocity. We notice that an increase in $We$ and $n$ increases the axial velocity in the core part of the channel. Here $n = 1$ or $We = 0$, which revisionthe axial velocity for the Newtonian fluid.

Figure.12. The variation of $F_{\lambda, F}$ with $\Theta$ for different values of $We$ with $b = 0.4$, $a = 0.5$, $\phi = \pi / 2$, $M = 2$, $n = 0.2$ and $m = 0.4$.

Figure.13. Velocity for various values of $a$ for fixed $b = 0.4$, $m = 0.2$, $\phi = \pi / 3$, $\Theta = 1.5$, $M = 0.75$, $n = 0.5$, $We = 0.05$, $x = 0.4$, $t = 0.2$.

Figure.14. Velocity for various values of $\phi$ for fixed $a = 0.2$, $b = 0.1$, $m = 0.3$, $\Theta = 1.75$, $n = 0.35$, $We = 0.15$, $x = 0.4$, $t = 0.2$.

Figure.15. Velocity for various values of $\Theta$ for fixed $a = 0.4$, $b = 0.3$, $m = 0.2$, $\phi = \pi / 2$, $M = 1.5$, $n = 0.25$, $We = 0.1$, $x = 0.4$, $t = 0.2$. 
An additional attractive phenomenon in peristaltic motion is trapping. It is essentially the formation within circulating bolus of fluid by closed streamlines and this trapped bolus bushes a head along peristaltic waves. Figure 18 shows that by increasing in \( b \), the trapping bolus increases in the upper and lower parts of the channel. The effect of non-uniform parameter \( m \) on the trapping is shown in Figure 19. It is examined that the size of the channel increases with an increase in \( m \). Figure 20 denotes that the size of the bolus decreases with an increase in \( M \). To see the effects of \( n \) and \( We \) on trapping, we have prepared Figure 21, one can note that increase in the \( We \) decreases the size of trapped bolus and the volume of trapped bolus also decreases by \( n \) decreases.

Figure 16. Velocity for various values of \( M \) for fixed \( a = 0.3, b = 0.25, m = 0.25, \Theta = 1.8, \phi = 3\pi/4, n = 0.2, We = 0.25, x = 0.4, t = 0.2 \).

Figure 17. Velocity for various values of \( m \) for fixed \( a = 0.3, b = 0.2, \phi = \pi/4, M = 1, \Theta = 1.6, n = 0.4, We = 0.1, x = 0.4, t = 0.2 \).

Figure 18. Velocity for various values of \( We \) and \( n \) for fixed \( a = 0.3, b = 0.2, m = 0.1, Q = 1.3, \phi = \pi/3, M = 2, x = 0.4, t = 0.2 \).
Figure 19. Streamlines for (a) \( b = 0 \) (b) \( b = 0.3 \).
\[
m = 0.1, \phi = \pi / 2, We = 0.1, t = 0.2, \Theta = 1.2, b = 0.3, M = 1, n = 0.3,
\]

Figure 20. Streamlines for (a) \( m = 0 \) (b) \( m = 0.3 \).
\[
\phi = \pi / 3, We = 0.2, t = 0.2, \Theta = 1.4, a = 0.2, b = 0.3, M = 2, n = 0.5
\]

Figure 21. Streamlines for (a) \( M = 0.5 \) (b) \( M = 1 \) \( \phi = \pi / 4 \), \( We = 0.15 \), \( t = 0.2 \), \( \Theta = 1.6 \),
Figure 22. Streamlines for (a) $\phi = 0, \Theta = 1.4$ (b) $\phi = 0, \Theta = 1.6$ (c) $\phi = \pi / 3, \Theta = 1.6$ (d) $\phi = 2\pi / 3, \Theta = 1.6$ $M = 3$. We = 0.1, $t = 0.2$, $a = 0.3$, $b = 0.2$, $m = 0.3$ $n = 0.5$
6. CONCLUSION

A mathematical model under long wavelength and low Reynolds number approximations is introduced to revision the effects of an applied magnetic field on peristalsis of a Carreau fluid in a non-uniform asymmetric channel. Perturbation technique is used to determine the analytical solution for the axial velocity field and stream function. The pressure rise and frictional forces are calculated by numerical integration. The main notes are pointed out as follows.

- There is no difference in the peristaltic pumping region for the Newtonian and Carreau fluid, when \( n = 1 \) or \( We = 0 \).
- The average rise in pressure increases with the increase of \( b, n \) and \( M \) while it decreases by increasing \( m \) and \( We \).
- Axial velocity increases with an increase of \( M \) and \( m \) near the boundary of channel and decreases by increasing \( M \) and \( m \) at the core of the channel.
- The volume of trapped channel increases with increasing non-uniform parameter \( m \) and shows opposite behavior for \( M \) and \( We \).
- Fig. 24 shows that, when \( n = 1 \) or \( We = 0 \) and \( m = 0 \), our results are found in good agreement with the results produced by Mishra and Rao [38] at \( d = 1 \).

It is trusted that the present analysis of the study of peristaltic flow of a Carreau fluid in a non-uniform asymmetric channel through an uniform applied magnetic field can be used as the basis for many scientific and engineering applications.

**APPENDIX**

\[
A_1 = \frac{F_0(\sinh(Mh_2) - \sinh(Mh_1))(\cosh(Mh_2) - \cosh(Mh_1))}{(2\cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1)M\sinh(M(h_1 - h_2)))}\left(\sinh(Mh_1) - \sinh(Mh_2)\right),
\]
\[
A_2 = \frac{F_0(\sinh(Mh_2) - \sinh(Mh_1))}{2\cosh(M(h_1 - h_2)) - 2 + (h_2 - h_1)M\sinh(M(h_1 - h_2))},
\]
\begin{align*}
A_3 &= -\frac{(n-1)M^4}{64} (A_1^3 + 3A_1A_2^2 \cosh(3Mh_1)) - \frac{(n-1)M^4}{64} (A_2^3 + 3A_2A_1^2 \sinh(3Mh_1)) \\
&\quad - \frac{(n-1)M^5}{16} (3A_1^3 - 3A_1A_2^2) h_1 \sinh(Mh_1) - \frac{(n-1)M^5}{16} (3A_2^3 - 3A_2A_1^2) h_1 \cosh(Mh_1), \\
A_4 &= -\frac{(n-1)M^4}{64} (A_1^3 + 3A_1A_2^2 \cosh(3Mh_2)) + \frac{(n-1)M^4}{64} (A_2^3 + 3A_2A_1^2 \sinh(3Mh_2)) \\
&\quad - \frac{(n-1)M^5}{16} (3A_1^3 - 3A_1A_2^2) h_2 \sinh(Mh_2) - \frac{(n-1)M^5}{16} (3A_2^3 - 3A_2A_1^2) h_2 \cosh(Mh_2), \\
A_5 &= -\frac{3M(n-1)M^4}{64} (A_1^3 + 3A_1A_2^2 \sinh(3Mh_1)) - \frac{3M(n-1)M^4}{64} (A_2^3 + 3A_2A_1^2 \cosh(3Mh_1)) \\
&\quad - \frac{(n-1)M^5}{16} (3A_1^3 - 3A_1A_2^2) (\sinh(Mh_1) + Mh_1 \cosh(Mh_1)) \\
&\quad - \frac{(n-1)M^5}{16} (3A_1^2A_2 - 3A_2^3) (\cosh(Mh_1) + Mh_1 \sinh(Mh_1)), \\
A_6 &= -\frac{3M(n-1)M^4}{64} (A_1^3 + 3A_1A_2^2 \sinh(3Mh_2)) - \frac{3M(n-1)M^4}{64} (A_2^3 + 3A_2A_1^2 \cosh(3Mh_2)) \\
&\quad - \frac{(n-1)M^5}{16} (3A_1^3 - 3A_1A_2^2) (\sinh(Mh_2) + Mh_2 \cosh(Mh_2)) \\
&\quad - \frac{(n-1)M^5}{16} (3A_1^2A_2 - 3A_2^3) (\cosh(Mh_2) + Mh_2 \sinh(Mh_2)), \\
B_4 &= \frac{F_1 M (\sinh(Mh_1) - \sinh(Mh_2)) + (A_5 - A_6) (\cosh(Mh_2) - \cosh(Mh_1))}{M (h_2 - h_1) \sinh(Mh_1)} (A_4 - A_3 - A_5 (h_2 - h_1)) \\
&\quad - M (\cosh(Mh_1) - \cosh(Mh_2)) (\cosh(Mh_2) - \sinh(Mh_1) - M (h_2 - h_1) \cosh(Mh_1)) \\
&\quad - M (\cosh(Mh_1) - \cosh(Mh_2)) (\cosh(Mh_2) - \cosh(Mh_1) - M (h_2 - h_1) \sinh(Mh_1)) \\
B_3 &= \frac{A_6 - A_5 - MB_4 (\cosh(Mh_1) - \cosh(Mh_2))}{M (\sinh(Mh_1) - \sinh(Mh_2))}, \\
B_2 &= -MB_3 \sinh(Mh_1) - MB_4 \cosh(Mh_1) - A_5, \\
B_1 &= \frac{F_1}{2} - B_2 h_2 - B_3 \cosh(Mh_2) - B_4 \sinh(Mh_2) - A_4.
\end{align*}

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