I briefly review several key issues in understanding the cuprate superconductors from the point of view of doped-Mott-insulator. Then I present an effective low-energy theory and show that the phase diagram of such a model includes an antiferromagnetic (AF) phase, a superconducting state, and a pseudogap phase at low doping and low temperature, consistent with the experimental one. The dual topological gauge structure of this model is responsible for unifying these phases within a single framework with correct doping and temperature scales. Some unique predictions in such a model are also discussed.

1. Introduction

High-temperature superconductivity has been discovered in the cuprate materials for more than a decade by now. These materials are widely considered to be doped Mott insulators. They become Mott insulators in the so-called half-filling case where each lattice site in copper-oxide layers is occupied by and only by one conduction electron. The superconducting phase emerges in the doped case as the half-filled electrons in the copper-oxide layers are pumped out (or equivalently, ‘holes’ are injected into the system). The normal state above the superconducting transition temperature also exhibits anomalous phenomena distinctively different from a conventional metal under a Landau-Fermi liquid description.

How to understand the mechanism of superconductivity and other novel properties, which have been experimentally found in almost all probing channels, including the single-particle, magnetic, transport, and pairing channels, etc., has posed a great challenge for the condensed matter physicists. In spite of tremendous experimental and theoretical efforts, with a
lot of theoretical proposals, there still lacks a consensus by far on what is the correct theory or sensible theoretical framework. Nevertheless, a great number of researchers in this field now believe that such materials belong to strongly correlated electron systems and the fundamental physics of a doped Mott insulator may hold the key to the heart of main issues in high-temperature superconductivity.

**Doped Mott insulators.** A Mott insulator is purely an interaction effect as the strong on-site Coulomb repulsion prevents two electrons from staying at the same site, such that the charge degrees of freedom are essentially frozen. In the doped case, as the electron number is reduced to less than one on average at each site, the charge degrees of freedom are no longer completely frozen and electrons can hop to empty sites without a penalty from the Coulomb potential. However, the majority of the charge degrees of freedom of the electrons still remains frozen at small doping and in general electrons cannot be simply viewed as quasiparticles with a long life time near the Fermi surface. In particular, a quasiparticle in conventional Fermi liquids carries both charge and spin. But in the half-filling limit of the Mott insulator, the remaining low-lying degrees of freedom of the electrons only involve spins. Such a separation of two degrees of freedom is also obvious at small doping, where the charge carrier number $x$ is much less than the number of spins, $1 - x$, per site (which is equal to the electron number). The spin-charge separation concept is therefore the most essential basis in understanding the Mott physics.

**Spin-charge separation description.** But an oversimplified spin-charge separation picture, namely, the charge carriers (‘holons’) and neutral spin carriers (‘spinons’) are completely decoupled, are not consistent with experiments as well as theoretical considerations, although the overall counting of charge and spin numbers of the doped Mott insulator are correctly incorporated. It turns out that there still exists a strong coupling between holons and spinons, reflecting the fact that the Hilbert space is restricted due to the on-site Coulomb interaction. Generally such a type of interactions is present in a form of gauge fields (This shows how interestingly a gauge theory can emerge in condensed matter systems with strong correlations of electrons).

The spin-charge separation and the gauge interactions constitute a very basic framework for describing a doped Mott insulator. This has been generally accepted. But it is not yet the whole story. One still needs to identify the (short-range) spin correlations, which will provide a driving force behind the low-lying charge and spin dynamics. This is where various
theories of doped Mott insulators differ from each other, in essence.

RVB states. At half-filling, the low-lying spin degrees of freedom of the cuprates are well described by the Heisenberg model, given by

\[ H_J = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \text{const.} \]  \hspace{1cm} (1)

Here \( \langle ij \rangle \) denotes the nearest-neighbor (NN) sites. This model predicts a long-range Néel order in the ground state and spin-wave excitations in long-wavelength, low-energy regime, consistent with the experiment. However, for the purpose of studying the doped case, strong quantum correlations at short-range are actually more important. This is because the hole hopping is usually quite sensitive to the short-range spin correlations in the background. For example, in the most widely used model of doped Mott insulator, i.e., the \( t - J \) model, besides the superexchange term \( H_J \), the hopping term is given by

\[ H_t = -t \sum_{\langle ij \rangle} c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.} \]  \hspace{1cm} (2)

under the no double occupancy constraint

\[ \sum_i c_{i\sigma}^\dagger c_{i\sigma} \leq 1. \]  \hspace{1cm} (3)

Here the hole hopping involves the NN sites, which is strongly influenced by the short-range spin correlations. To correctly describe the doping effect, therefore, a correct description of short-range spin correlations is essential.

Different kinds of short-range spin correlations can lead to drastically different low-energy physics. These short-range spin correlations will then set a natural ‘ultraviolet’ (high-energy) cutoff for a low-energy effective model of doped Mott insulators, at least in lightly doped regime, and provide the driving force for the system’s low-energy behavior. In contrast, the long-range AF correlations of spins will quickly and easily get suppressed as a small amount of holes are added to the system, and will usually play only a less important and respondent role in the doped regime.

Anderson proposed a resonating-valence-bond (RVB) picture to describe such short-range quantum correlations of spins, in which spins form singlet pairs and superpose coherently to form an RVB background. Once the RVB state is given, the low-energy effective model for doped Mott insulators is
usually fixed through a spin-charge separation and gauge-theory formulation. Therefore, how to correctly describe the short-range spin correlations is the most essential issue in constructing a sensible theory for the cuprates based on the doped Mott insulator scenario.

So far there are roughly two kinds of RVB states being proposed. In the first kind, spins (spinons) are fermions and they form RVB pairs. The theoretical structure is quite similar to the BCS theory, except that the charged Cooper pairs are replaced by neutral spinon pairs. The mathematical advantage of this description is the familiarity to the BCS theory. But such a state notably fails to describe the correct long-range AF correlations at half-filling. Nevertheless, people have hoped that it may capture the right physics of short-ranged correlations and thus could become more relevant in the doped case when the AF long-range order is gone and superconductivity sets in. However, as we shall argue below, this may well not be the case even at finite doping.

The second kind is known as the bosonic RVB state. Here spins (spinons) are bosons. A variational wavefunction based on the bosonic RVB picture can produce an unrivaled accurate ground-state energy ($-0.3344J$ per bond as compared to the exact numerical value of $-0.3346J$ per bond for the Heisenberg model), and a generalized calculation can precisely provide not only the ground-state energy, staggered magnetization, but also spin excitation spectrum in the whole Brillouin zone. Since the energy of the Heisenberg model is directly related to the NN spin-spin correlations, a good variational energy also means a good description of short-range spin correlations. By contrast, a variational energy in the fermionic RVB state gives rise to a number, $-0.319J$ per bond, which suggests that the description of the short-range spin correlations is less accurate. In fact, in the short-range spin-spin correlations of the fermionic RVB state, the AF component is even worse (one finds that the majority contribution to the energy is not from the AF correlations as opposed to the exact numerics and the bosonic RVB theory).

**Doping: the phase string effect.** Due to the absence of the short-range AF correlations, the hopping of holes is not very frustrated in the fermionic RVB description. In contrast, in the bosonic RVB description, the coherent motion of doped holes is much more difficult. For example, it can easily show that the hopping integral vanishes on average in the half-filling bosonic RVB state, and thus the kinetic energy is highly frustrated for a hole to move on such an AF background. This kinetic energy frustration reflects the sharp difference between the fermionic and bosonic RVB states.
in the doped case.

The underlying physics can be traced back to a fundamental property of the $t-J$ model, known as the phase string effect. It has been well known that the ground state wavefunction of the Heisenberg model satisfies the Marshall sign. Such a Marshall sign rule would hold even at arbitrary doping, if holes remain static on lattice sites. But once the holes start to move, the Marshall sign rule will be scrambled by the hopping of a hole on its path. Such a disordered Marshall sign can be described by a product of sequential $+$ and $-$ signs, $(+1) \times (-1) \times (-1) \times ...$, where signs are determined by simply counting the index $\sigma$ of each spin exchanged with the hole during its hopping. Physically a phase string can be regarded as the transverse mismatches of spins created by the motion of the hole. The significance of such a phase string is that it represents the sole source of phase frustrations in the $t-J$ model, which cannot be ‘repaired’ at low energy, and therefore is the most essential doping effect in such a doped Mott insulator.

**Mutual gauge interactions.** The phase string effect indicates an intrinsic interaction between the spin and charge degrees of freedom. At small doping limit, without changing the spin background, which remains to be an antiferromagnet, the doped holes will pick up the phase string effect which strongly frustrates their kinetic energy. Here the total sign $(+1) \times (-1) \times (-1) \times ...$ will become uncertain with the increase of the length of paths, leading to the localization of the holes (see below). At larger doping, the spin background has to be eventually re-adjusted to minimize the frustration of the phase string effect in favor of the kinetic energy of holes. Note that the phase string $(+1) \times (-1) \times (-1) \times ...$ depends on those spins exchanged with the hole during its hopping. So if the background spins are tightly in RVB pairing, the randomness in a phase string will be qualitatively removed and holes will then become delocalized. The corresponding ground state will be shown to be superconducting. On the other hand, the long-range AF order generally has to disappear in such a state.

Therefore, in the following we shall take the point of view that the cuprate superconductors are essentially doped Mott insulators. In order to describe a doped Mott insulator, we introduce a spin-charge separation framework with an intrinsic gauge structure. We adopt the bosonic RVB description to characterize the short-range, high-energy spin correlations, which works extremely well at half-filling and provides an essential underpinning for the low-energy theory at both half-filling and small doping. We
identify a nonlocal mutual interaction between the spin and charge degrees of freedom, i.e., the phase string effect, which will result in a topological gauge structure in the spin-charge separation formulation of the whole problem.

In the remaining part of the talk, I shall first present the low-energy effective model established based on the above-outlined considerations. We call it the phase string theory. I shall then present a series of important results predicted by this model. I shall show that this theory naturally unifies antiferromagnet and superconductivity within a single theoretical framework and its unique topological structure determines a rich, novel phase diagram in a self-consistent fashion, which is also in good agreement with the experimental measurements of the cuprate superconductors.

2. Phase string theory

Phase string model. The low-energy effective Hamiltonian with incorporating the bosonic RVB pairing and the phase string effect induced by doping can be derived from the \( t-J \) model as follows:

\[
H_{\text{string}} = H_h + H_s
\]

with

\[
H_h = -t_h \sum_{\langle ij \rangle} (e^{iA_i^h}) h_i^\dagger h_j + H.c.
\]

\[
H_s = -J_s \sum_{\langle ij \rangle} (e^{i\sigma A_i^s}) b_i^{\dagger \sigma} b_j^{\sigma} + H.c.
\]

Here \( h_i \) and \( b_{i\sigma} \) are bosonic ‘holon’ and ‘spinon’ operators, respectively. An electron operator is composed of such spinless holon and neutral spinon operators by \( c_i^{\sigma} = h_i^\dagger b_{i\sigma} e^{i\hat{\Theta}_{i\sigma}} \). Note that the fermionic commutation relations of \( c_{i\sigma} \)’s are guaranteed by the phase factor \( e^{i\hat{\Theta}_{i\sigma}} \), which is defined by \( e^{i\hat{\Theta}_{i\sigma}} = (-\sigma)^{t_h} e^{\frac{i}{2} [\Phi^h_i - \sigma \Phi^{\dagger h}_i]} \), with \( \Phi^h_i = \Phi^s_i - \Phi^{\dagger h}_i \), where \( \Phi^s_i = \sum_{l \neq i} \theta_i(l) \sum_{\alpha} n_{l\alpha}^{\dagger} n_{l\alpha} \), \( \Phi^h_i = \sum_{l \neq i} \theta_i(l) n_{l\alpha}^{\dagger} \), and \( n_{l\alpha}^{\dagger} \) and \( n_{l\alpha} \) are spinon and holon number operators, respectively, and \( \theta_i(l) \equiv \text{Im} \ln (z_i - z_l) \), where \( z_i = x_i + iy_i \) is the complex coordinate of a lattice site \( l \).

At low doping, \( t_h \sim t \) and \( J_s = 1/2\Delta^* J \sim J \) \((t \text{ and } J \text{ are the parameters in the } t-J \text{ model})\). Here the effective theory is underpinned by the bosonic RVB order parameter \( \Delta^* \), which characterizes the short range (NN) AF correlations as \( \langle S_i \cdot S_j \rangle_{NN} = -3/8|\Delta^*|^2 \). In the following we shall denote the transition temperature for \( \Delta^* \neq 0 \) by \( T_0 \). At small doping, \( \Delta^* \neq 0 \)
Figure 1. The phase string model is underpinned by the bosonic RVB pairing ($\Delta^s \neq 0$), characterizing short-range AF correlations, in the regime below $T_0$, which covers a series of phases including AF, pseudogap, and superconducting states.

covers a temperature regime extended over $T_0 \sim 1,000$K. With the increase of doping, $T_0$ is expected to monotonically decrease as illustrated in Fig. 1, showing an applicable region for the present effective theory. Beyond this region, the short-range AF correlations disappear and the novel properties described by (4) and (5), including the superconductivity, are no longer present.

Topological gauge structure. $H_{\text{string}}$ is invariant under the gauge transformation: $h_j \rightarrow h_j \exp(i\varphi_j)$, $A^s_{ij} \rightarrow A^s_{ij} + i(\varphi_i - \varphi_j)$ and $b_{j\sigma} \rightarrow b_{j\sigma} \exp(i\sigma\phi_j)$, $A^h_{ij} \rightarrow A^h_{ij} + i(\phi_i - \phi_j)$. So this model has a $\text{U}(1) \times \text{U}(1)$ gauge symmetry. The nontriviality of $H_{\text{string}}$ arises from the ‘dual’ gauge fields, $A^s_{ij}$ and $A^h_{ij}$, which satisfy topological conditions: $\sum_{c} A^s_{ij} = \pm \pi \sum_{l \in c} (n^s_{lj} - n^s_{li})$ and $\sum_{c} A^h_{ij} = \pm \pi \sum_{l \in c} n^h_{li}$ for a closed loop $c$.

The holons in (4) feel the presence of the spinons as quantized $\pi$ flux-
Figure 2. Dual topological gauge structure represented by $A^s_{ij}$ and $A^h_{ij}$ in the effective Hamiltonians, $H_h$ and $H_s$.

Oids through $A^s_{ij}$, which reflects the nonlocal frustrations of the spin RVB background on the kinetic energy of the charge degrees of freedom, due to the phase string effect. Vice versa the spinons also perceive the doped holes as $\pi$ flux quanta through $A^h_{ij}$, which represents the dynamic frustrations of the doped holes on the spin degrees of freedom. Such a dual and mutual gauge structure is schematically illustrated in Fig. 2. In the following, the phase diagram determined by such a topological gauge structure will be present, which unifies the AF states, the superconducting phase, and the so-called pseudogap phase within a single framework, as shown in Fig. 3.

3. Phase diagram

3.1. Half-filling and dilute doping

Half-filling. At half-filling, the holon number is zero and $H_{\text{string}}$ reduces to $H_s$. With $A^h_{ij} = 0$, $H_s$ here is equivalent to the Schwinger-boson mean-field Hamiltonian $^{13}$, which describes AF correlations of the cuprates fairly well over a wide temperature range $\sim 1,000 K$. As a matter of fact, using the mean-field wavefunction of $H_s$, we can numerically calculate the ground-state energy and the AF magnetization by the variational Monte Carlo method, which agree with the exact diagonalization results of the Heisenberg model extremely well. All of these indicate that both long-range and short-range spin correlations are well captured by the bosonic RVB theory at half-filling.

Dilute doping. Now consider hole doping into such an AF spin background. As pointed out in the Introduction, the motion of holes will generally induce the phase string effect. Since here we focus on the dilute limit of the hole concentration, the phase string effect will mainly influence the hole
dynamics, without drastically affecting the spin part. The holes will then be self-trapped or localized by the phase string effect in such a dilute doping regime\textsuperscript{10,14,15}, as to be discussed below. Such a localization is purely a consequence of the Mott insulators upon doping, unlike the conventional Anderson localization in the presence of disorders.

**Localization.** The localization of holes can be mathematically determined by the topological gauge structure of the phase string model. Note that the long range AF order is realized by the spinon Bose condensation, i.e., $\langle b_\alpha \rangle \neq 0$, at half-filling. If the spinon condensation persists into a dilute doping regime, each holon will induce a vortex in the spinon condensate through $A^h_{ij}$ in (5), which costs a logarithmically divergent energy. As the result, each holon will have to induce an antivortex from the spinon condensate and be confined to the latter to form a hole-dipole object in the spin ordered (spinon condensed) phase\textsuperscript{14}. In this hole-dipole entity, the holon sits at one of two poles instead of the center of a dipole. It can be shown that the effective mass of the induced antivortex is infinity such that the whole hole-dipole object is self-trapped in space. This hole localized phase is called \textit{holon confined phase}\textsuperscript{14}, in which no free holons will appear in the finite-energy spectrum.

**Competing orders.** In such a localization regime, the kinetic energy of holes is suppressed. Such a suppression of the kinetic energy is one of the most essential features of the holon-confined phase. Without the balance from the kinetic energy, the low-energy physics in this regime will be determined by the potential energies. The latter will then decide various competing orders at such a low-doping insulating phase.

First of all, at sufficiently low doping, the long-range AF order should persist if a weak interlayer coupling is considered. But the freedom in the directions of the hole-dipole moment will lead to the reduction of the Néel temperature $T_N$ as shown in Fig. 3. Based on the hole-dipole picture and the renormalization group (RG) calculation we have determined\textsuperscript{14} the critical doping $x_0 \simeq 0.03$, at which the AF order disappears at $T = 0$.

Beyond $x_0$ or $T_N$, the system is in a cluster spin glass phase with the dipole moments being quenched randomly in space\textsuperscript{14}. This phase has been also observed in experiment. In obtaining such a phase, we have assumed that hole-dipoles are self-trapped in space \textit{uniformly}. This latter condition may be realized in real systems by impurities or disorders.

However, if there is no impurities or disorders, the uniform distribution of the self-trapped hole-dipoles are usually not stable against the formation of the stripes\textsuperscript{15}. This is because the long-range dipole-dipole interaction
Figure 3. The phase diagram of the phase string model at small doping. The dual topological gauge structure decides various phases through confinement-deconfinement procedures. The RG calculations give rise to $x_0 \sim 0.03$ and $x_c \sim 0.043$ (see text).

will dominate the low-energy phase as there is no competition from the kinetic energy. Such a dipole-dipole interaction will lead to the attraction among hole-dipoles with a certain alignment of their dipole moments. Their subsequent collapse together with the ordered dipole moments will lead to various stripe formations. Therefore, in the phase string theory, inhomogeneous distribution of the charges is energetically very competitive at low doping.

_Holon deconfinement._ With the further increase of doping, the sizes of hole-dipoles will get larger and larger, and eventually a deconfinement can occur at a critical doping $x_c$ \(^{14}\), beyond which single holons will be unbound from their anti-vortex partners. This procedure is like a KT transition in the $x$ axis and by the RG method we have determined $x_c \simeq 0.043$ at $T = 0$. In the following section, we will point out that the ground state in the holon deconfined phase is a superconducting state.
3.2. Superconducting state

We now consider the regime beyond the critical doping \( x_c \). The deconfined bosonic holons will experience a Bose condensation at low temperature in \( H_h \). The corresponding ground state is a d-wave superconducting state as to be discussed below.

\textit{d-wave pairing order parameter.} In the phase-string theory, superconducting order parameter can be expressed as \( \Delta_{ij}^{SC} = \Delta_0^{ij} \langle e^{i\frac{1}{2}(\Phi^s_i + \Phi^s_j)} \rangle \),

\begin{equation}
\Delta_0^{ij} = \Delta^s_0 \langle h^+_i \rangle \langle h^+_j \rangle,
\end{equation}

where \( ij \) refer to nearest neighbor sites and the pairing amplitude

\[ \Delta_0^{ij} \propto \Delta^s \langle h^+_i \rangle \langle h^+_j \rangle \]

in which the bosonic RVB pairing \( \Delta^s \neq 0 \) and the holon condensation \( \langle h^+_i \rangle \neq 0 \) usually occurs at relatively higher temperatures, \( T_0 \) and \( T_v \), respectively. The true superconducting transition will happen when \( \langle e^{i\frac{1}{2}(\Phi^s_i + \Phi^s_j)} \rangle \neq 0 \), namely, when the phase coherence is established. The phase factor \( e^{i\frac{1}{2}(\Phi^s_i + \Phi^s_j)} \) also determines the d-wave symmetry of the pairing \( 16 \).

\textit{Spinon as a vortex.} The phase \( \Phi^s_i \) represents vortices bound to spinons \( 17 \). In the ground state, where spinons are all paired up, \( \Phi^s_i \) is canceled out and the phase coherence is realized in (6). Each unpaired spinon, in an excited state, will induce a vortex in the order parameter through \( \Phi^s_i \). Such an effect appears in \( H_h \) through \( A^s_{ij} \), which will cost a logarithmically divergent energy.

\textit{Confinement and elementary excitations.} Thus a single spinon as a vortex cannot appear in the superconducting bulk alone. The excited spinons must appear in vortex-antivortex pairs and be confined to form \( S = 1 \) triplet excitations \( 16, 17 \). In contrast to the holon confinement in the spin ordered phase with spinon condensation, the present spinon confinement occurs in the holon condensed phase, due to the gauge field \( A^s_{ij} \) in \( H_h \).

Besides the spin triplet excitations composed of confined spinons, there is another kind of elementary excitations, namely, a nodal quasiparticle excitation. Such an excitation can be created by the electron operator \( c_{i\sigma} \) as a composite of confined holon and spinon with a phase-string phase factor \( 16 \). Such an excitation is similar to that in a BCS theory.

But there is a clear non-BCS feature here. Namely, the \( S = 1 \) spin excitation composed of two confined spinons, which will form a resonance-like peak at \( E_g \sim xJ \) in the phase-string theory, is \textit{independent} of the
quasiparticle excitations at the zeroth-order approximation.

Superconducting transition. The deconfinement of spinon-vortex pairs occurs at $T = T_c$, which is responsible for destroying the phase coherence of the order parameter (6) as free unbound spinon-vortices emerge. In this way, the superconducting transition is naturally related to both the spin dynamics and the phase coherence within a single unified framework. $T_c$ determined in this description is found to be scaled with the spin characteristic energy scale $E_g$, in good agreement with the cuprate superconductors.

Flux quantization at $hc/2e$ and vortex core. In the fermionic RVB mean-field theory, the superconducting state corresponds to the holon Bose condensation, where the magnetic flux quantization is usually at $hc/e$, as holons carry a charge $+e$. How to get the correct flux quantization at $hc/2e$ has been a challenging issue in a spin-charge separation framework based on the fermionic RVB theory.

In the phase-string theory, although the framework is still based on a spin-charge separation description, the minimal magnetic flux is found to be quantized at $hc/2e$, similar to the BCS theory. However, as a unique prediction of the theory, the flux quantization leads to the trapping of a spinon inside the magnetic vortex core. We expect that the free $S = 1/2$ moment trapped inside a vortex core can be observed in the NMR and other experimental measurements.

Furthermore, the vortex core is different from the one in the BCS theory. Here the pairing amplitude $\Delta^0_{ij}$ can remain finite throughout the vortex core, while the superconducting order parameter $\Delta^{SC}_{ij}$ vanishes as the phase coherence is destroyed inside the core within the a characteristic length scale. Such a ‘normal core state’ resembles the ‘pseudogap’ phase above $T_c$, to be discussed below.

3.3. Pseudogap phase

Mechanism: kinetic energy driven. The spin superexchange energy is favored when spins form bosonic RVB pairs at $T_0 (\sim J_s/k_B \sim 1,000K$ at $x \sim 0)$. With the decrease of temperature, bosonic holons will start to gain kinetic energy and eventually experience a Bose condensation at $T_v$ (in the Kosterlitz-Thouless sense). Such a $T_v$ will represent an onset temperature for the pseudogap phase in the bosonic RVB theory, rather than the superconducting transition temperature in the conventional fermionic RVB theory. The pseudogap temperature in the latter theory is defined as the
onset temperature of the RVB pairing, and thus the pseudogap phase is superexchange-energy-driven there, instead of kinetic energy driven in the present description.

Amplitude forming of the superconducting order parameter. As discussed in the above section, the amplitude of the superconducting order parameter $\Delta_0$ forms at $T = T_v$, whereas the phase coherence of the order parameter can still remain absent until $T = T_c < T_v$.

In the pseudo-gap phase of $T_c < T < T_v$, spin RVB pairs are “charged” and become Cooper pairs, as holons are condensed. In this sense, one may say that preformed Cooper pairs emerge in the pseudogap phase.

Spinon vortices. A unique prediction of the phase string theory in the pseudo-gap phase is the presence of topological vortices which are spinons. This may be easily seen based on (4). According to $H_h$, when holons are condensed, each excited (unpaired) spinon will induce a current vortex through the gauge field $A^s_{ij}$, which also shows up in the pairing order parameter as discussed before. These spinon vortices are free and the pseudo-gap phase has been thus called the spontaneous vortex phase.

Experimentally the presence of free vortices will contribute to the Nernst effect. Indeed, nontrivial Nernst effect has been observed in different cuprate compounds in the pseudogap regime. It is very hard to interpret the experimental results as due to the conventional superconducting fluctuations as the observed Nernst signal can persist over a temperature range ten times higher than $T_c$ in the underdoped regime. The spinon-vortices in the present theory provides a natural explanation for the experiment.

Spin pseudogap. Since the spin degrees of freedom is influenced by the charge dynamics through the gauge field $A^h_{ij}$, the holon condensation below $T_v$ will directly cause the qualitative change of spin dynamics. Namely, it will lead to the suppression of the spin fluctuations at the low-energy end, resembling a spin pseudogap feature, with the spectral weight being pushed upwards to form a resonance-like peak structure around a characteristic energy scale $E_g \sim \delta J$. As discussed before, this feature persists into the superconducting phase.

A pseudogap means that it is more difficult to excite spinons from the RVB pair condensate. Since $A^h_{ij}$ is canceled out in the RVB state, in the pseudogap phase the fluctuations of $A^s_{ij}$ are also reduced at low energy. This in turn favors the holon condensation. Therefore, self-consistently, the holon condensation and spin pseudogap are mutually enforced by each other.
4. Summary

In this talk, I have discussed several fundamental issues of the doped Mott insulator, including spin-charge separation, the gauge theory description of couplings between two degrees of freedom, and short-range quantum correlations which provide the driving force for the low-energy physics.

I have pointed out that the bosonic RVB pairing characterizes the crucial short-range correlations at both half-filling and small doping. Once such a short-range, high-energy correlations are determined, the low-energy, long-range theory is fixed, which is known as the phase string theory, as the motion of holes in such a bosonic RVB background always generates irreparable phase string effect.

I have reviewed some very basic features obtained within the phase string framework. The AF long-range ordered phase persists into a finite doping below $x_0 \sim 0.03$, and then becomes cluster spin glass phase at $x > x_0$. In both phases holes are localized or self-trapped by the phase string effect. We found that the stripe state is also very competitive in this regime. Beyond a critical doping at $x_c \sim 0.043$, the holes will become delocalized, and the system experiences a superconducting transition at low temperature. In such a d-wave superconducting state, spinons are confined to form $S = 1$ triplet excitations. Spinon and holon are confined to form nodal quasiparticles. $T_c$ is related to the phase coherence transition and is scaled with the spin ‘resonance’ energy $E_g \sim xJ$. Above $T_c$, a pseudogap phase is found in which free spinons behave like unbound vortices. Several novel properties are revealed in both superconducting and pseudogap phases, including the prediction that a free moment trapped inside a $hc/2e$ magnetic vortex core and the anomalous Nernst effect. All of these interesting features and competing phases at different doping and temperature are determined self-consistently by the unique dual topological gauge structure in the phase string theory.

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