Gamma-ray free-electron lasers: Quantum fluid model

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Abstract – A quantum fluid model is used to describe the interaction of a nondegenerate cold relativistic electron beam with an intense optical wiggler taking into account the beam space-charge potential and photon recoil effect. A nonlinear set of coupled equations is obtained and solved numerically. The numerical results indicate that intense $\gamma$-ray free-electron laser emission, with intensities approaching the Schwinger limit, can be driven by the strong nonlinear space-charge wave, for feasible values of the electron beam parameters. However, the achievement of this regime of extreme intensities depends rather critically on the choice of the detuning and of the signal initial phase at the entrance of the interaction region.

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It is well known that the free-electron laser (FEL) conceived by Madey [1] is a source of coherent and tunable electromagnetic waves with modulated frequency, which can generate very short wavelengths. In the standard configuration, a magnetostatic wiggler is used to induce transverse motion in the relativistic electron beam, such that the kinetic energy of the beam is released as electromagnetic stimulated radiation. The wavelength of the emitted radiation obeys the approximate resonance condition $\lambda_s = \lambda_w/2\gamma^2$, where $\lambda_s$, $\lambda_w$ and $\gamma$ are the wavelength of the emitted radiation, wiggler period, and normalized relativistic beam energy, respectively. Somewhat later it was realized that an electromagnetic wiggler can alternatively be used to modulate the transverse beam motion [2,3]. In this case, a counterpropagating laser pulse, working as a wiggler, changes the resonance condition to $\lambda_s = \lambda_w/4\gamma^2$, with $\lambda_w$ being now the laser wavelength. Currently, the FELs can produce coherent radiation in the X-ray range of operation [4,5], which allows for new schemes to investigate physical processes at atomic and molecular scales, with possible applications in fundamental and applied physics [6]. The extension of the FEL emission range to ultra-short wavelengths, namely down to the gamma-ray range, will provide access to a wider range of experimental possibilities at sub-atomics scales.

In the microwave regime of operation, the FEL is described by the equations of motion of each single electron in the beam, under the external magnetic field, and the wave equation of the electromagnetic field, with an external beam current. Assuming that all ponderomotive potential wells are identical in steady state, and neglecting beam slippage effects, the coupled system is analyzed within a unit ponderomotive potential well. The electrons in the potential well are modeled by a discrete ensemble of initially uniform macro-particles. The nonlinear interaction between electrons and radiation field generates nonlinear electron bunching inside the potential well, exciting a collective wave emission mode. This is the basic mechanism of a conventional FEL, where the emitted photon momentum recoil is not larger than the electron beam momentum spread. As the radiation frequency gets significantly higher, this standard description of particles in potential wells has to be reconsidered. In the X-ray regime, for instance, the longitudinal dimension of the ponderomotive potential well is so small that, for any reasonable beam density, not all the potential wells are occupied, so that no more than one electron is found per well. However, the progress in the development of FELs and the need to reach new levels of energy and ultra-short wavelengths of the laser radiation, and the beam oscillations, require that the quantum characteristic of the FEL interaction be properly considered. Taking photon recoil into account, when the photon energies become either comparable or larger than the energy of the incoming relativistic beam, i.e., $\hbar \omega_s/\gamma_e m_e c^2 \approx \hbar \omega_s/\gamma_e m_e c \geq 1$, the number of photons per electromagnetic mode becomes very small, thus justifying a quantum description. New interesting and different theoretical quantum models have been
proposed [7–15] to describe this regime. Here, we will follow the quantum fluid model outlined in refs. [12–14] in order to describe the collective interaction of the electromagnetic waves with a nondegenerate cold relativistic electron beam. In this model, the electromagnetic waves are treated classically and the electron beam as a quantum cold-plasma fluid [16,17]. The quantum recoil effect (or quantum diffraction) is given by the Bohm potential term in the Euler-like equation of motion. In the steady-state regime, the quantum FEL dynamics in one-dimension is described by the following set of fluid equations [13]:

\[
\begin{align*}
\frac{\partial \rho}{\partial \xi} &= -\frac{\partial}{\partial \xi} [V - e\phi] + \frac{\hbar^2}{2m_e \gamma_e^2} \frac{\partial}{\partial \xi} \left[ \frac{1}{\sqrt{n}} \frac{\partial^2 \sqrt{n}}{\partial \xi^2} \right], \\
\frac{\partial n}{\partial \xi} &= 0,
\end{align*}
\]

(1)

(2)

where, \( \xi = (z - v_t t) \), \( m_e \) is the electron rest mass, \( v_e \) is the unperturbed electron beam velocity, \( \gamma_e = 1/\sqrt{1 - v_e^2/c^2} = \sqrt{1 + p_e^2/m_e c^2} \) is the electron Lorentz factor, \( V \) is the ponderomotive potential, and \( \phi \) is the space-charge potential, which satisfies the Poisson equation, viz., \( \partial^2 \phi / \partial \xi^2 = 4\pi e (n - n_0) \). Here, \( n_0 \) is the unperturbed beam density and \( n \) is the density of the electron fluid element, which moves with longitudinal velocity (momentum) \( v(p) \). We point out that eq. (1) differs from the relativistic classical momentum equation due to the last term, which results from the quantum Bohm potential. This quantum correction leads to an additional electron current that incorporates the recoil effect (sudden loss of momentum) due to the photon emission. The photon field is connected to the electron quantum field through this additional current, as discussed in our previous publications [18].

Now, let us consider that the electron beam interacts with a scattered radiation propagating in the positive \( \hat{z} \)-direction (the same direction as the electron beam) and with a constant amplitude optical wiggler, which propagates in the opposite direction. The amplitude of the radiation is written as \( \tilde{a}_r = 1/\sqrt{2} [\tilde{e}_a, \xi] \exp(i k_w z - i \omega_w t) + \text{c.c.} \) and that of the optical wiggler as \( \tilde{a}_w = 1/\sqrt{2} [\tilde{e}_w, \xi] \exp(-i k_w z - i \omega_w t) + \text{c.c.} \), where \( \tilde{e} = \hat{x} + i \hat{y} \) is the unitary polarization vector. Here, \( \tilde{a}_w = e \tilde{A}_w / m_e c^2 (a_s = e \tilde{A}_w / m_e c^2) \) is the normalized optical (radiation) wiggler amplitude with \( k_w \) (\( \omega_w \)) being the wave number and frequency of the optical wiggler (radiation). Hence, the ponderomotive potential \( V \) associated with these fields, which acts on the electron beam, can be defined as \( V = \gamma m_e c^2 \), where \( \gamma = \sqrt{1 + \tilde{a}_w \cdot \tilde{a}_w} \) and \( \tilde{a}_w = \tilde{a}_w + \tilde{a}_s \) are the beam relativistic factor during the interaction and total normalized potential vector. To complete the physical model, eqs. (1) and (2) have to be complemented with the electromagnetic wave equation, with the transverse current density \( \tilde{J}_t = -en^2 \tilde{i}_t \) as the driving term, where \( \tilde{i}_t = e (\tilde{A}_t + \tilde{A}_w) / \gamma m_e c, \) as follows from transverse momentum conservation, in the laboratory frame. Assuming that \( a_s \) is a slowly varying envelope amplitude in space and time, the equation for the scattered radiation amplitude, \( a_s \), can then be written as \([14] \)

\[
(\beta_s - \beta_e) \frac{\partial}{\partial \theta} a_s = i \delta a_s - i \frac{\omega_p^2}{2 \gamma k_w \omega_s} \left( 1 - \frac{n}{n_b} \right) a_s = -i \frac{\omega_p^2}{2 \gamma k_w} \frac{n}{n_b} a_s e^{-i \theta},
\]

(3)

where \( \theta = k_l z - \omega t \) is the ponderomotive wave phase with \( k_l = k_s + k_w \) and \( \omega_l = \omega_s - \omega_w \). We note that this phase can be rewritten as \( \theta = k_l (z - v_t t) \) assuming that \( v_e \approx v_t \), where \( v_t = \omega_l / k_l \) is the electron resonant velocity. Here, \( \beta_s = c k_s / \omega_s \) is the normalized radiation group velocity, \( \omega_s^2 = 4\pi c^2 n_b / m_e \) is the plasma frequency square, and \( \delta = \beta/c = \beta_e - \beta_r \) is \( (\gamma_e - \gamma_r) / \gamma_l \) the detuning parameter [10].

Assuming that \( v(\xi = 0) = 0, V(\xi = 0) = \gamma_0 m_e c^2, \gamma_0 = (\xi = 0, \phi(\xi = 0) = 0, \text{and } n(\xi = 0) = n_b, \text{eqs. (1)} \) and (2) can be integrated to give, in a normalized form, the following nonlinear pendulum equation, viz.,

\[
\frac{\partial^2 \phi}{\partial \theta^2} + 2\sigma^2 |\gamma_0 - \gamma + \Phi| P = 0,
\]

(4)

with the space-charge potential, \( \Phi = e \phi / m_e c^2 \), which obeys the normalized Poisson equation, given by

\[
\frac{\partial^2 \Phi}{\partial \theta^2} = \Omega^2_\phi (P^2 - 1),
\]

(5)

where \( P = \sqrt{n/n_b}, \Omega^2_\phi = \omega_p^2 / \omega_s^2, \gamma = (\gamma_e^2 + a_s^2 / 2 + |a_s|^2 / 2 + a_s^* a_w e^{-i \theta} + \text{c.c.})^{1/2}, \) and \( \sigma = m_e c \gamma_e^{3/2} / \hbar k_l \). It should be noticed that this equation is formally identical to the equation obtained in ref. [19] for a quantum plasma at rest, \( \gamma_e = 1, \) in the laboratory frame, and in the absence of an optical wiggler, which gives a relativistic solitonic wave in a cold plasma, in the classical limit \( (h \rightarrow 0) \). Considering that \( (\beta_s - \beta_e) \approx 1/2 \gamma_s \) and redefining \( a_s = a e^{-i \phi} \), eq. (3) can be rewritten as

\[
\frac{\partial a}{\partial \theta} = -i [1 + 2 \gamma^2 \delta + \Omega^2_\phi (1 - P^2)] a = -i \Omega^2_\phi a_w P^2,
\]

(6)

where \( \Omega^2_\phi = \gamma e \omega_p^2 / \omega_s^2 \) and \( \gamma \approx \gamma_e \) have been used in the denominators of eq. (3). Using the polar representation for the scattered radiation amplitude, \( a, \) in such a way that \( a \rightarrow a \exp(i \varphi) \), we obtain the following set of real equations:

\[
\frac{\partial a}{\partial \theta} = -\Omega^2_\phi a_w P^2 \sin \varphi,
\]

(7)

and

\[
\frac{\partial \varphi}{\partial \theta} = 1 + 2 \gamma^2 \delta + \Omega^2_\phi (1 - P^2) - \Omega^2_\phi a_w P^2 / a \cos \varphi.
\]

(8)

Equations (4)–(8) form a set of nonlinear coupled basic equations that describes a quantum FEL pumped by an optical wiggler, where the relativistic electron beam is treated as a cold quantum plasma. These equations
are solved numerically considering an electron beam with normalized energy, $\gamma_c = 60$, and density, $n_b = 1.0 \times 10^9 \text{cm}^{-3}$, propagating in the positive $z$-direction, interacting with a counterpropagating laser wiggler with amplitude $a_w \approx 0.6$, which is equivalent to a laser intensity $I_w \approx 2.3 \times 10^{18} \text{W/cm}^2$ for a wavelength $\lambda_w = 6.50 \times 10^{-5} \text{cm}$ (650 nm). It should be noticed that such intensity is compatible with the existing laser technology. In order to enhance the stimulated radiation, a signal with initial amplitude $a(\theta = 0) = 1.0 \times 10^{-5}$, propagating in the same direction as the beam, is introduced. Due to the FEL resonance condition, $k_s \approx 4k_w^2 \gamma_c^2$, in the interaction region, a very short wavelength, $\lambda_s \approx 4.52 \times 10^{-9} \text{cm}$, is obtained, corresponding to an input signal with intensity of $I_s \approx 1.32 \times 10^{17} \text{W/cm}^2$. We point out that the quantum effect correction on the resonance condition, for a low-energy electron beam, has been neglected [14].

Figure 1 displays the output laser intensities, excited during the FEL interaction, for different values of the detuning and of the initial phase of the signal, at the entrance of the interaction region. We can see that, depending on the values of the detuning and initial phase of the radiation signal, at the entrance...
this process could be used, following the approach developed in our recent publication [23]. Finally, it should be pointed out that intrinsic relativistic quantum effects, such as those associated with spin and with electron-positron coupling, were ignored.

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