An optimization design method for aerodynamic configuration of high aspect ratio wing

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Abstract: The aerodynamic shape optimization design of the aircraft is to combine the aerodynamic performance analysis of the aircraft with the optimization method, by constantly changing the design shape of the aircraft, its aerodynamic performance is improved to achieve optimal aerodynamic performance under certain constraints. Aircraft aerodynamic shape optimization is a comprehensive design platform that integrates geometric parameterization, moving grid, CFD calculation and optimization algorithms. With the development of computational fluid dynamics (CFD) and the maturity of calculation methods, the role of aerodynamic shape optimization in modern aircraft design becomes more and more important. To this end, an aerodynamic shape optimization design platform based on non-uniform rational B-spline (NURBS) was established. In the optimization process, a mesh deformation method based on radial basis functions is used, and an arbitrary Lagrangian Eulerian method (ALE) is used to describe the unsteady process of the wing. The optimization analysis of the large aspect ratio wing under subsonic conditions has certain reference value for the deformation problem in the aerodynamic shape optimization design.

1. Introduction

Aerodynamic shape optimization design includes four aspects: the geometric parameterization of the aerodynamic shape, the generation of the flow field grid, the CFD simulation calculation, and the optimized search algorithm [1-2]. The quality of the geometric parameterization method directly affects the optimization results. The commonly used parameterization methods are: grid point method (point-based deformation), various types of function methods, partial differential equation methods, polynomial and spline methods, geometric parameterization methods with physical meanings in design variables. With the increasing refinement of aerodynamic shape design, parametric methods are required to describe the geometric shape smoothly, and to meet the characteristics of deformation diversity and precise local modification inside the design space. The traditional parameterized method is more mature for curve deformation control, but its ability to deform the surface is not satisfactory. Generally, more design variables are needed to control the deformation, which increases the amount of calculation. The generation and movement of the computational flow grid has become one of the key technologies for optimization design. The quality and rate of the generated computational grid also closely affect the optimization results. The automatic mesh generation methods commonly used in optimization design include spring method, infinite difference method, etc., and methods such as using a commercial software to call a script to generate a mesh. In this article, the radial basis function method is used to generate and update the grid. These mesh generation methods are not directly...
related to the parameterization methods, and need to be performed separately, which is not efficient. In view of the fact that NURBS surfaces are mostly used in engineering research fields such as computer-aided design/computer-aided manufacturing (CAD/CAM) and computer graphics[3-4], this article also chooses it as a parameterization method, and combines the dynamics based on radial basis functions. The grid method and intelligent optimization algorithm are used to optimize the aspect ratio wing under subsonic speed, and the results are analyzed and discussed.

2. Numerical methods

2.1 NURBS parameterization method
Firstly, mesh the wing surface, set the wing surface as a structured grid, and use the difference to transfer between the structured grid and the unstructured grid. Select a NURBS surface assuming p-order in the u direction, n+1 control points, q-order in the v direction, and m+1 control points, then

\[
U(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} G_{i,j}(u,v)P_{i,j}
\]

\[
G_{i,j}(u,v) = \frac{N_{i,p}(u)N_{j,q}(v)w_{i,j}}{\sum_{k=0}^{m} \sum_{l=0}^{n} N_{k,p}(u)N_{l,q}(v)w_{k,l}}
\]

In \( P_{i,j} \) formula, it is the surface control point vector, the total number of control points is \((n+1)\times(m+1)\), \( w_{i,j} \) is the weight coefficient value, \( N_{i,p}(u) \) and \( N_{j,q}(v) \) are the basis functions of the node vectors U (u) and V (v).

\[
N_{i,0}(u) = \begin{cases} 
1, & u_i \leq u < u_{i+1} \\
0, & \text{otherwise}
\end{cases}
\]

\[
N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)
\]

\[
U(u) = \{0...0, u_{p+1}, u_{r-1}, ... 1\}
\]

For complex surfaces, when the shape of the surface changes, the surface superposition method is used to calculate the newly generated surface shape, which is defined as follows:

\[
S_{\text{new}} = S_{\text{initial}} + \Delta S(u,v)
\]

\[
\Delta S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} G_{i,j}(u,v)\mathcal{G}_{i,j}P_{i,j}
\]

From formula (1) to (7), it is known that the NURBS surface is a mesh surface formed by \((n+1)\times(m+1)\) control points of u and v as independent variables, when the position of a control point changes, the shape of the surface near the control point also changes, while the shape of other areas does not change. The NURBS surface mainly controls the free change of the surface through control parameters such as control points, nodes and weight coefficients. The NURBS deformation process of the ONERA M6 wing is shown in the figure 1.
2.2 Radial basis function interpolation

In the process of wing optimization design, because the wing surface nodes adopt a regular grid distribution, and the spatial flow field uses an unstructured grid, the object-surface grid and the spatial grid often do not match at the interface. The interpolation method is used to pass parameters. During the optimization process, the object mesh is updated once each iteration[5]. This article uses radial basis function interpolation to achieve.

Radial basis functions take spatial node distance $\|x\|$ as the basic variable, and are generally divided into three categories: general radial basis functions, part radial basis functions, and compactly supported radial basis functions. The compactly supported radial basis function has a constant value of 0 after the center distance reaches a certain value $r$. $R$ is called the compactly supported radius. The variable $\zeta$ of the compactly supported radial basis function is the ratio of the center distance to the compactly supported radius ($r$), that is, $\zeta = x / r$, when $\zeta > 1$, the function value is 0. Table 1 below shows the commonly used radial basis functions.

| Table 1. Common radial basis functions. |
|-----------------------------------------|
| $\varphi(x)$ and $\varphi(\zeta)$       | Type            |
| $\sqrt{c^2 + \|x\|^2}, c = 10^{-5} \sim 10^{-3}$ | General         |
| $\frac{1}{\sqrt{c^2 + \|x\|^2}}, c = 10^{-5} \sim 10^{-3}$ | Part            |
| $(1 - \zeta)^4(4\zeta + 1)$           | Compactly       |

Suppose a set of center points $X = \{x_1, x_2, \ldots, x_n\}$ in the n-dimensional Euclidean space, the corresponding scalar value is $g = \{g_1, g_2, \ldots, g_n\}$, and the radial basis function $\varphi(x)$ has a continuous function, as follows:

$$f(x) = \sum \gamma_i \varphi(\|x - x_i\|)$$  \hspace{1cm} (8)

The process of determining the interpolation coefficient $\gamma$ through all the central points is the radial basis function interpolation. Where $\|x - x_i\|$ is the Euclidean distance. For three-dimensional space, $s$ can be directly expressed as:
3. Optimize process design

The wing optimization design platform in this paper is mainly composed of fast Fourier transform (FFT), geometric parameterization module, CFD aerodynamic parameter calculation module, and optimized search module. The CFD aerodynamic calculation module performs aerodynamic evaluation calculations on the wing during the optimization design. The flow field is used to solve the RANS governing equations. The implicit time advancement method is used[6]. The spatial discretization is the second-order Roe format. The optimization search module evaluates the wing optimization results and finally finds the best individual. In the optimization design, the NURBS surface method is to parameterize the geometric shape of the wing model, select appropriate control points, adjust the wing shape, and adjust the space calculation grid at the same time, directly output the net required for CFD calculation Grid file. The flow field calculation grid of the three-dimensional wing is actually a three-dimensional space grid including the wing. In the optimization design process of this article, while using the NURBS surface method to change the wing, the corresponding flow field space grid is also used. Simultaneous updates generate a computing grid. When the flow field grid is deformed, based on the initial space grid, the position of the wing space grid points is recalculated according to the NURBS surface, and the full flow field space grid is finally output[7].

The optimization process is shown in Figure 2.

4. Moving grid method based on RBF

The dynamic grid method is one of the key technologies for the optimization design of aerodynamic shapes. Appropriate moving grid method can not only improve the optimization efficiency, but also have an important influence on the optimization effect. The RBF dynamic mesh method has the characteristics of simple operation, strong adaptability and high quality of deformed mesh, so this method is used to update the mesh in the optimization process. The specific process is to construct the interpolation matrix between the spatial flow field grid and the object surface grid through the initial

\[ s = \|x - x_i\| = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \] (9)
grid, and then calculate the deformation amount of the space grid by the displacement amount of the object grid. The specific construction process is as follows.

For the radial basis function, the space distance $||x||$ is used as its basic variable, and $d$ is defined as a set of center points $X = \{x_1, x_2, \cdots, x_n\}$ in space. The continuous function of the radial basis function $\varphi(||x||)$ is:

$$f(x) = \sum \gamma_i \varphi(||x - x_i||)$$  \hspace{1cm} (10)

In the formula, $||x - x_i||$ is the Euclidean distance, and the radial basis function interpolation is the process of solving the interpolation coefficient $\gamma_i$. In two-dimensional space, $r = ||x - x_i|| = \sqrt{(x - x_i)^2 + (y - y_i)^2}$. $d$ is the coordinate of the flow field grid node at the fluid-structure coupling interface, and the displacement components of the corresponding flow field grid points in two directions are $D^x$, $D^y$. The interpolation coefficients are solved by the following equations:

$$\begin{cases} M \gamma^x = D^x \\ M \gamma^y = D^y \end{cases}$$  \hspace{1cm} (11)

$M$ is an $n$-th order interpolation matrix, and its element solving formula is:

$$m_{ij} = \varphi(x_{ij}) = \varphi(||x_i - x_j||), 1 \leq i, j \leq n$$ \hspace{1cm} (12)

Bring the grid node coordinates in the flow field into the RBF interpolation function to get the movement amount of the grid node coordinates:

$$\begin{cases} \Delta x_{ij} = \sum_{i=1}^{n} \gamma_i^x \varphi(||x_{ij} - x_i||) \\ \Delta y_{ij} = \sum_{i=1}^{n} \gamma_i^y \varphi(||x_{ij} - x_i||) \end{cases}$$ \hspace{1cm} (13)

Calculate the change amount of grid nodes in the entire calculation domain by formula (13), and then add the new grid node coordinates to the initial grid to update the grid. This article uses a large aspect ratio wing as an example to test the reliability and applicability of RBF moving grid technology. It can be seen from the mesh before and after the wing optimization that the mesh quality is basically unchanged in the small deformation range, which ensures the accuracy of CFD calculation and the practicality of the RBF dynamic mesh [8-9].

5. Numerical optimization algorithm

This article uses genetic algorithm to optimize the design of the wing. Genetic algorithm is a computational model that simulates the natural selection of genetic evolution and the biological evolution process of genetic mechanism. It is a method of searching the optimal solution to simulate the natural evolution process. It starts by searching for a population that may contain a solution to the problem. The population consists of genetically encoded individuals. Each individual has a chromosome that characterizes it. Each chromosome is a collection of multiple genes that determines the external shape characteristics of its individual. Therefore, the binary coding of the mapping relationship between external shape features and genes must be compiled first. After the initial population is determined, a better approximate solution is generated from generation to generation according to the natural rule of survival of the fittest. In each generation, individuals are selected according to fitness and combined. Cross and mutate to generate a new population, and then decode the optimal individual of the last generation population to get an approximate optimal solution to the problem. The genetic algorithm searches from the set of strings and processes multiple individuals at
6. Case analysis

This article takes a large aspect ratio wing as an example, and uses the optimization platform described above to optimize its wing shape. The k-ω SST turbulence model is employed to close the N-S equations, has a good effect on the boundary layer turbulence, free shear turbulence and the severe separation of flow in this paper. The convection term is in the second-order upwind scheme and coupling algorithm is used.

The design state Mach number $Ma = 0.3$, the angle of attack $\alpha = 2^\circ$, the calculation grid is 5 million, 65 design variables are set under the initial wing plane shape, of which 30 are on the upper and lower wings, 6 are on the leading edge, and control The points are evenly distributed at 0, 20%, 40%, 60%, 80% and wingtips along the span. Under the Windows system, an Intel core i7 processor with 16Gb of memory is used for optimization calculation. During the optimization process, each iteration, the flow field calculation takes about 30 minutes, and the grid update takes about 1 minute. Under the constraint of lift coefficient, the wing is optimized.

From the wing optimization results in Table 1, it can be seen that the drag coefficient of the wing has been greatly reduced, from 0.0163 to 0.0152, the drag reduction has reached 7.6%, and the lift coefficient has changed little from 0.2642 to 0.2684, an increase of 1.5%. The optimization design of the large aspect ratio wing mainly improves the lift-to-drag ratio of the wing by reducing the drag to achieve the goal of optimizing the aerodynamic performance of the wing.

| Table 1. Comparison of wing optimization results. |
|---------------------------------|---|---|---|
| Large aspect ratio wing         | $Cl$ | $Cd$ | $Cl/Cd$ |
| Optimized front lift resistance | 0.2642 | 0.0163 | 16.21 |
| Lift-drag characteristic after optimization | 0.2684 | 0.0152 | 17.66 |

Figure 3 shows the comparison of the wing surface pressure coefficients before and after the optimization. It can be seen from the figure that the wing tip vortex of the initial wing is obvious, the induced resistance is large, and the wing tip vortex after optimization is weakened. Figure 4 shows the airfoil and pressure coefficient distributions at the spans of 40% and 80%. It can be seen that the airfoil's wing has a slightly larger airfoil change, the leading-edge radius of the airfoil is reduced, and the whole is upturned. The maximum thickness increases, and the airfoil changes are mainly concentrated at the leading edge.
Figure 4. Airfoil and pressure coefficient distribution along different spans.

7. Conclusion
(1) The NURBS parameterization method uses a specific basis function and has good local property control capabilities. It also uses control points, orders, nodes, and weight coefficients as design variables to control the free change of the shape of the surface, which has strong operability.

(2) By optimizing the wing of a large aspect ratio wing under low speed conditions, the drag coefficient of the wing is reduced by 7.6%, and the wing tip vortex is significantly weakened; the decrease in resistance mainly comes from the reduction of the wing tip induced resistance and the wing rises. The resistance characteristics are significantly improved.

(3) The effectiveness of the optimization platform based on the NURBS parameterization method is proved by an optimization case, which can be applied to the optimization design of wing and other components, and has certain application value to engineering.

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