A note on the effect of laser field direction in laser-assisted pion decay

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Abstract. We study the pion decay process in the presence of a circularly polarized laser field propagating along an arbitrary general direction. Using the first Born approximation and the Dirac-Volkov states for charged particles, we derive an analytic expression for the decay rate. The direction of the laser field was found to have no significant effect on the nature of the result obtained. This study generalizes the results found for a field with a wave vector along the z-axis by Mouslih et al in a recent paper (Mouslih 2020 Phys. Rev. D 102 073006).

Keywords: laser-assisted, electroweak processes, Dirac-Volkov state, decay rate

1. Introduction

The study of processes in the presence of laser fields is a fertile area of research that has attracted the attention of many scientists [1], both theoretical and experimental, due to the development of laser technology [2]. These studies contribute significantly to the understanding of the matter-radiation interaction. The ultimate goal behind all this is to understand the behavior of particles and to discover their new properties arising in the presence of an external field. The ultrafast processes that occur in the presence of the laser field are varied, depending on the adopted framework of study. In atomic physics, many atomic processes have been studied in the presence of an electromagnetic field, both in relativistic and non-relativistic regimes [3–9]. In the framework of quantum electrodynamics and electroweak theory, many articles have been devoted to the investigation of scattering [10–14] and decay [15–18] processes in the presence of an electromagnetic field. In this context, we have recently studied the pion decay process in the presence of a circularly polarized electromagnetic field with a wave vector along the z-axis direction [15]. We found that the laser contributed to
the increase of the lifetime, and this result was explained by the so-called quantum Zeno effect. In this letter, we will extend this study to the case of general laser field direction. This would lead to a generalization of the results obtained previously. This is done by configuring the wave vector with a general spherical geometry that allows us to cover all particular cases. The main purpose of this letter is to establish a general theoretical formalism that allows us to examine the effect of laser field direction on decay rate, lifetime and branching ratios. More details on the theoretical calculation and the literature on this subject can be found in our previous article. Throughout this letter, we use natural units $c = \hbar = 1$ and work with the metric tensor $g = \text{diag}(1, -1, -1, -1)$.

2. Formalism

The process considered is the decay of a charged pion $\pi^-$ into two leptons,

$$\pi^-(p_1) \longrightarrow \ell^-(p_2) + \bar{\nu}_\ell(k'), \quad (\ell = e, \mu)$$

where the arguments are our labels for the associated four-momenta. The laser field is assumed to be monochromatic and circularly polarized. Its classical four-potential satisfies the Lorentz gauge condition $\partial_\mu A^\mu = 0$, and is given by:

$$A^\mu(\phi) = a_1^\mu \cos(\phi) + a_2^\mu \sin(\phi),$$

where $\phi = (k.x)$ is the phase of the laser field. The wave 4-vector $k$ is introduced theoretically in a general geometry with spherical coordinates as follows:

$$k = (\omega, k) = \omega \left(1, \cos(\varphi_k) \sin(\theta_k), \sin(\varphi_k) \sin(\theta_k), \cos(\theta_k)\right),$$

where $\omega$ is the frequency of the laser field. The Lorentz gauge condition applied to the four-potential $A^\mu$ implies that $k_\mu A^\mu = 0$, meaning that $(k.a_1) = (k.a_2) = 0$. To keep these relationships verified, we set also the polarization 4-vectors $a_1^\mu$ and $a_2^\mu$ in a general spherical geometry as follows:

$$a_1^\mu = (0, a_1) = |\mathbf{a}| \left(0, \cos(\varphi_{a_1}) \sin(\theta_{a_1}), \sin(\varphi_{a_1}) \sin(\theta_{a_1}), \cos(\theta_{a_1})\right),$$

$$a_2^\mu = (0, a_2) = |\mathbf{a}| \left(0, \cos(\varphi_{a_2}) \sin(\theta_{a_2}), \sin(\varphi_{a_2}) \sin(\theta_{a_2}), \cos(\theta_{a_2})\right),$$

with $|\mathbf{a}| = \mathcal{E}_0/\omega$, where $\mathcal{E}_0$ is the laser field strength. The polarization 4-vectors are orthogonal and equal in magnitude, which implies $(a_1.a_2) = 0$ and $a_1^2 = a_2^2 = a^2 = -|\mathbf{a}|^2$.

The wave function of the relativistic lepton $\ell^-$, with four-momentum $p_2$ and spin $s_1$, moving in an electromagnetic field was first presented by Volkov in 1935 [19], and is given, when normalized to the volume $V$, by [20]

$$\psi_\ell(x) = \left[1 + \frac{e k.A}{2(k.p_2)}\right] \frac{u(p_2, s_1)}{\sqrt{2Q_2V}} \times e^{iS(q_2, x)},$$

where $e = -|e|$ is the charge of the electron, and

$$S(q_2, x) = -q_2.x - \frac{e(a_1.p_2)}{k.p_2} \sin(\phi) + \frac{e(a_2.p_2)}{k.p_2} \cos(\phi),$$

with $q_2 = p_2 - k$. The polarization $4\times2$ matrix components of the electromagnetic field are given as

$$e^{iS(q_2, x)} = e^{-iS(q_2, x)}.$$
with the electron’s effective momentum and mass
\[ q_2 = p_2 - \frac{e^2 a^2}{2(k \cdot p_2)} k, \quad m_{\ell}^2 = m_{\ell}^2 - e^2 a^2, \]
where \( m_{\ell} \) is the rest mass of the free lepton.

For the laser-dressed charged pion (spinless particle), its wave function obeys the Klein-Gordon equation for bosons with spin zero. Therefore, the corresponding Volkov solution reads [21]:
\[ \psi_{\pi^-}(x) = \frac{1}{\sqrt{2Q_1 V}} e^{iS(q_1, x)}, \]
with
\[ S(q_1, x) = -q_1 \cdot x - \frac{e(a_1, p_1)}{k \cdot p_1} \sin(\phi) + \frac{e(a_2, p_1)}{k \cdot p_1} \cos(\phi). \]

The outgoing antineutrino \( \bar{\nu}_\ell \) is treated as massless particle with four-momentum \( k' \) and spin \( s_2 \). Its wave function is given by [22]:
\[ \psi_{\bar{\nu}_\ell}(x) = \sqrt{2\pi} e^{i k' \cdot x}, \]
where \( E_2 = k'^0 \) is the total energy of the outgoing antineutrino.

In the first Born approximation, the S-matrix element for the laser-assisted pion decay can be written as [22]:
\[ S_{fi}(\pi^- \rightarrow \ell^- \bar{\nu}_\ell) = \frac{-iG}{\sqrt{2}} \int d^4 x J_{\mu}^{(\pi)}(x) J_{(\ell^-)}(x), \]
where \( G \) is the Fermi coupling constant. \( J_{(\ell^-)}(x) \) and \( J_{\mu}^{(\pi)}(x) \) are, respectively, the leptonic and hadronic currents expressed by:
\[ J_{(\ell^-)}(x) = \bar{\psi}_\ell(x, t) \gamma_\mu (1 - \gamma_5) \psi_{\bar{\nu}_\ell}(x, t), \]
\[ J_{\mu}^{(\pi)}(x) = i \sqrt{2} f_\pi \bar{p}_\mu \frac{1}{\sqrt{2Q_1 V}} e^{-iS(q_1, x)}, \]
where \( f_\pi = 90.8 \text{ MeV} \) is called the pion decay constant [22].

Inserting the expressions of currents and wave functions into equation (11) and after some algebraic manipulations, we get:
\[ S_{fi} = \frac{-G f_\pi}{2\sqrt{2} Q_1 Q_2 E_2 V^2} \sum_{n=-\infty}^{\infty} M^n_{fi}(2\pi)^4 \delta^4(k' + q_2 - q_1 - nk), \]
where \( n \) is the number of exchanged photons, and the quantity \( M^n_{fi} \) is defined by:
\[ M^n_{fi} = \bar{u}(p_2, s_1) \Gamma^n v(k', s_2), \]
where
\[ \Gamma^n = \left\{ B_n(z) + [e/(2(k \cdot p_2))] d_1 k B_1 n(z) + [e/(2(k \cdot p_2))] d_2 k B_2 n(z) \right\} \delta_1 \left(1 - \gamma_5\right). \]

The coefficients \( B_n(z) \), \( B_1 n(z) \) and \( B_2 n(z) \) are expressed in terms of ordinary Bessel functions by:
\[ \begin{bmatrix} B_n(z) \\ B_1 n(z) \\ B_2 n(z) \end{bmatrix} = \begin{bmatrix} J_n(z) e^{i \phi_0} \\ (J_{n+1}(z) e^{i (n+1) \phi_0} + J_{n-1}(z) e^{i (n-1) \phi_0})/2i \\ (J_{n+1}(z) e^{i (n+1) \phi_0} - J_{n-1}(z) e^{i (n-1) \phi_0})/2i \end{bmatrix}, \]
where \( \phi_0 = \arctan(\alpha_2/\alpha_1) \), and the argument of the Bessel function \( z \) is defined by:
\[
z = \sqrt{\alpha_1^2 + \alpha_2^2} \quad \text{with} \quad \alpha_1 = e \left( \frac{a_1.p_1}{k.p_1} - \frac{a_1.p_2}{k.p_2} \right) ; \quad \alpha_2 = e \left( \frac{a_2.p_1}{k.p_1} - \frac{a_2.p_2}{k.p_2} \right).
\]

Following the same procedure as detailed in [15], we obtain for the decay rate:
\[
W'(\pi^- \to \ell^- \bar{\nu}_\ell) = \sum_{n=-\infty}^{+\infty} W_n(\pi^- \to \ell^- \bar{\nu}_\ell),
\]
where the \( n \)-resolved decay rate \( W_n \) is defined by:
\[
W_n = \frac{G^2 f_\pi^2}{(2\pi)^2} \frac{|q_2|^2 d\Omega_\ell}{E_2 Q_2 g'(|q_2|)} |M_{f_1}^n|^2,
\]
with
\[
g'(|q_2|) = \frac{|q_2| - n\omega \cos(\theta)}{\sqrt{(n\omega)^2 + |q_2|^2 - 2n\omega|q_2| \cos(\theta)}} + \frac{|q_2|}{\sqrt{|q_2|^2 + m^2_n}},
\]
where \( \theta \) is the final angle of the emitted lepton. The term \( |M_{f_1}^n|^2 = \sum_{s_1,s_2} |M_{f_1}^{s_1,s_2}|^2 \) in equation (19) reduces down to the following trace:
\[
|M_{f_1}^n|^2 = \text{Tr}[\gamma^0 \gamma_n \Gamma^0 \bar{\Gamma}^n],
\]
where
\[
\Gamma^n = \gamma^0 \Gamma_{n+1} \gamma^0,
\]
\[
= \gamma_1 \left( 1 - \gamma_5 \right) \left\{ B_n^*(z) + \left[ e/(2(k.p_2)) \right] \gamma_2 B_{1n}^*(z) + \left[ e/(2(k.p_2)) \right] \gamma_1 \gamma_2 B_{2n}^*(z) \right\}.
\]
The trace work can be done numerically with the help of FEYNCALC [23]. The result we have obtained is as follows:
\[
|M_{f_1}^n|^2 = \frac{4}{(k.p_2)^2} \left[ \Delta_1 |B_n|^2 + \Delta_2 |B_{1n}|^2 + \Delta_3 |B_{2n}|^2 + \Delta_4 B_n B_{1n}^* + \Delta_5 B_{1n} B_n^* + \Delta_6 B_n B_{2n}^* + \Delta_7 B_{2n} B_n^* + \Delta_8 B_{1n} B_{2n}^* + \Delta_9 B_{2n} B_{1n}^* \right],
\]
where the nine coefficients from \( \Delta_1 \) to \( \Delta_9 \) are explicitly expressed, in terms of different scalar products, by
\[
\Delta_1 = 4(k.p_2)^2(p_1.p_2)(p_1.k') - 2(k.p_2)^2m^2_{\pi^-}(p_2.k'),
\]
\[
\Delta_2 = -e^2(2(a_1.k)(a_1.p_2) - a^2(k.p_2))(k.k')m^2_{\pi^-} - 2(k.p_1)(p_1.k'),
\]
\[
\Delta_3 = -e^2(2(a_2.k)(a_2.p_2) - a^2(k.p_2))(k.k')m^2_{\pi^-} - 2(k.p_1)(p_1.k'),
\]
\[
\Delta_4 = e(k.p_2)(2(a_1.p_2)(p_1.k) + 2(a_1.k)(p_1.p_2)(p_1.k') - (a_1.k)m^2_{\pi^-}(p_2.k')) - (a_1.k)m^2_{\pi^-}(p_2.k'),
\]
\[
\Delta_5 = e(k.p_2)((a_1.k')(k.p_2)m^2_{\pi^-} - (a_1.p_2)(k.k')m^2_{\pi^-} + 2(a_1.p_2)\times (k.p_1)(p_1.k') + 2(a_1.k)(p_1.p_2)(p_1.k') - (a_1.k)m^2_{\pi^-}(p_2.k') - \text{i}(2(p_1.k')e(a_1, k, p_1, p_2) + m^2_{\pi^-}e(a_1, k, p_2, k')),
\]
\[
\Delta_6 = e(k.p_2)((a_2.k')(k.p_2)m^2_{\pi^-} - (a_2.p_2)(k.k')m^2_{\pi^-} + 2(a_2.p_2)(k.p_1)\times (p_1.k') + 2(a_2.k)(p_1.p_2)(p_1.k') - (a_2.k)m^2_{\pi^-}(p_2.k') - \text{i}(2(p_1.k')e(a_2, k, p_1, p_2) + m^2_{\pi^-}e(a_2, k, p_2, k'))) - \text{i}(2(p_1.k')e(a_2, k, p_1, p_2) + m^2_{\pi^-}e(a_2, k, p_2, k')).
\]
\[ \Delta_7 = e(k.p_2)((a_2,k')(k.p_2)n_\pi^2 - (a_2,p_2)(k,k')m_{\pi^-}^2 + 2(a_2.p_2) \]
\[ \times (k.p_1)(p_1,k') + 2(a_2,k)(p_1,p_2)(p_1,k') - m_{\pi^-}^2(a_2,k)(p_2,k') + 2i(p_1,k')\epsilon(a_2,k,p_1,p_2) + im_{\pi^-}^2\epsilon(a_2,k,p_2,k') \]  
\[ \Delta_8 = e^2[ - ((a_1,p_2)(a_2,k) + (a_1,k)(a_2,p_2) - (a_1.a_2)(k.p_2))((k,k')m_{\pi^-}^2 - 2(k.p_1)(p_1,k')) + i(2(a_1,k)(p_1,k')\epsilon(a_2,k,p_1,p_2) - (a_2.k)m_{\pi^-}^2 \]
\[ \times \epsilon(a_1,k,p_2,k') + (a_1.k)m_{\pi^-}^2\epsilon(a_2,k,p_2,k') ] \]
\[ \Delta_9 = e^2[ - ((a_1,p_2)(a_2.k) + (a_1,k)(a_2,p_2) - (a_1.a_2)(k.p_2))((k,k')m_{\pi^-}^2 - 2(k.p_1)(p_1,k')) - (a_1.k)(p_1,k')\epsilon(a_1,a_2,k,p_1) - 2(a_2.k) \]
\[ \times (p_1,k')\epsilon(a_1,k,p_1,p_2) + 2(a_1.k)(p_1,k')\epsilon(a_2,k,p_1,p_2) - (k,p_2)m_{\pi^-}^2\epsilon(a_1,a_2,k,k') - (a_2.k)m_{\pi^-}^2\epsilon(a_1,k,p_2,k') \]
\[ + (a_1,k)m_{\pi^-}^2\epsilon(a_2,k,p_2,k') ] \]
where the different scalar products are evaluated in the rest frame of pion; and for all 4-vectors \(a, b, c\) and \(d\), we have
\[ \epsilon(a, b, c, d) = \epsilon^{\mu\nu\rho\sigma}a_\mu b_\nu c_\rho d_\sigma \],
where \(\epsilon^{\mu\nu\rho\sigma}\) is the antisymmetric tensor with the convention \(\epsilon^{0123} = 1\). The reader may refer to our previous works [15, 16] to see how these tensors are calculated analytically. After the decay rate, it comes the lifetime of the charged pion, which is defined simply as the inverse of the total decay rate
\[ \tau_{\pi^-} = \frac{1}{W_{\text{total}}}, \quad \text{with} \quad W_{\text{total}} = W(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) + W(\pi^- \rightarrow e^- \bar{\nu}_e). \]  
Considering the two decay modes (muonic and electronic) of the charged pion, we define their corresponding branching ratios (Br) as follows:
\[ \text{Br}(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) = \frac{W(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)}{W_{\text{total}}}, \]
\[ \text{Br}(\pi^- \rightarrow e^- \bar{\nu}_e) = \frac{W(\pi^- \rightarrow e^- \bar{\nu}_e)}{W_{\text{total}}}. \]

3. Numerical results and discussion

This section will be devoted to the presentation and analysis of the numerical results obtained. We will see exactly how the direction of the laser field can affect the different quantities calculated in the previous section. But, let us first check the consistency of our theoretical calculation by recovering the results previously obtained where the laser field is along the \(z\)-axis [15]. A laser field propagating in the direction of the \(z\)-axis is theoretically represented by the wave vector \(k\) with a component along only the \(z\)-axis: \(k = (0, 0, \omega)\). This requires the first polarization vector \(a_1\) to be in the \(x\)-axis
direction and the second \(a_2\) in the \(y\)-axis direction, so that the three vectors form a direct orthogonal basis \((a_1, a_2, k)\). Therefore, we have

\[
\begin{align*}
\begin{cases}
\theta_k = 0; \varphi_k = 0 \text{ or any} & \Rightarrow k = (\omega, 0, 0, \omega), \\
n & \Rightarrow k = (\omega, 0, 0, \omega), \\
\theta_{a_1} = \pi/2; \varphi_{a_1} = 0 & \Rightarrow a_1 = (0, |a|, 0, 0), \\
\theta_{a_2} = \pi/2; \varphi_{a_2} = \pi/2 & \Rightarrow a_2 = (0, 0, |a|, 0).
\end{cases}
\end{align*}
\]

(37)

The result of this particular case is shown in figure 1, which illustrates the changes in the \(n\)-resolved decay rate \(dW_n/d\theta (\pi^- \rightarrow \mu^-\bar{\nu}_\mu)\) (in units of \(10^{-8}\)) as a function of the number of photons exchanged \(n\) for \(\theta = 90^\circ\). The laser field strength and frequency are \(E_0 = 10^7 \text{ V cm}^{-1}\) and \(\hbar \omega = 1.17 \text{ eV}\). After checking that our calculation is accurate, let us now discuss the effect of the laser field direction on the decay rate. For the \(x\) and \(y\)-axis directions, we set the spherical angles as follows:

\[
\begin{align*}
\begin{cases}
\theta_k = \pi/2; \varphi_k = 0 & \Rightarrow k = (\omega, \omega, 0, 0), \\
n & \Rightarrow k = (\omega, 0, 0, \omega), \\
\theta_{a_1} = \pi/2; \varphi_{a_1} = \pi/2 & \Rightarrow a_1 = (0, 0, |a|, 0), \\
\theta_{a_2} = 0; \varphi_{a_2} = 0 \text{ or any} & \Rightarrow a_2 = (0, 0, |a|, 0).
\end{cases}
\end{align*}
\]

(38)

**Figure 1.** The behavior of the \(n\)-resolved decay rate \(dW_n/d\theta (\pi^- \rightarrow \mu^-\bar{\nu}_\mu)\) (in units of \(10^{-8}\)) as a function of the number of photons exchanged \(n\) for \(\theta = 90^\circ\). The laser field strength and frequency are \(E_0 = 10^7 \text{ V cm}^{-1}\) and \(\hbar \omega = 1.17 \text{ eV}\). The result of this particular case is shown in figure 1, which illustrates the changes in the \(n\)-resolved decay rate \(dW_n/d\theta (\pi^- \rightarrow \mu^-\bar{\nu}_\mu)\) (in terms of the number of photons exchanged \(n\) at field strength \(10^7 \text{ V cm}^{-1}\) and frequency \(1.17 \text{ eV}\). It is the same envelope obtained in our previous paper (see figure 1(a) in [15]). Thus, the theoretical formalism adopted here is consistent and can lead to all results we have obtained before in [15].

After checking that our calculation is accurate, let us now discuss the effect of the laser field direction on the decay rate. For the \(x\) and \(y\)-axis directions, we set the spherical angles as follows:

\[
\begin{align*}
\begin{cases}
\theta_k = \pi/2; \varphi_k = \pi/2 & \Rightarrow k = (\omega, \omega, 0, 0), \\
n & \Rightarrow k = (\omega, 0, 0, \omega), \\
\theta_{a_1} = 0; \varphi_{a_1} = 0 \text{ or any} & \Rightarrow a_1 = (0, 0, |a|, 0), \\
\theta_{a_2} = \pi/2; \varphi_{a_2} = 0 & \Rightarrow a_2 = (0, |a|, 0, 0).
\end{cases}
\end{align*}
\]

(39)

In figure 2, we show the variations of the total \(n\)-resolved decay rate \(W_n(\pi^- \rightarrow \mu^-\bar{\nu}_\mu)\) (integrated over \(d\Omega_e\)) in terms of the number of photons exchanged \(n\) for three laser field directions. Each subfigure in figure 2 contains the changes in \(W_n\) with respect to a
Figure 2. The behavior of the total $n$-resolved decay rate $W_n(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$ (in units of $10^{-8}$) as a function of the number of photons exchanged $n$ for three laser field directions. The laser field strength and frequency are $E_0 = 10^7$ V cm$^{-1}$ and $\hbar \omega = 1.17$ eV.

given direction and its opposite. We obtain envelopes that are symmetric with respect to the $n = 0$-axis. These envelopes give us information about the photon exchange process (absorption and emission) between the laser field and the decay system. It appears to us through these figures that the order of magnitude varies according to each direction and between each direction and its opposite. Concerning the amount of photons exchanged, we see that it is almost the same ($-40 \leq n \leq 40$) for the three directions, but it differs between each axis and its opposite except for the $y$-axis. It can be seen that the number of photons exchanged when the field is along the $y$-axis is the same as along its opposite direction, since the cutoff number is equal to $\pm 45$ in both cases (see figure 2(b)). As for the $x$ and $z$-axes, there is a slight difference in the number of photons exchanged between the direction and its opposite. For example, along the $x$-axis, the cutoff number is $\pm 35$ and along the opposite direction, it is $\pm 45$.

We will now study the variation of other quantities with respect to the different directions of the laser field. We will sum over a specific range of number of photons that we choose to be from $-10$ to $+10$. First, we show in table 1 the numerical values of the pion lifetime for different field strengths and along three laser field directions. In the absence of the laser, the pion lifetime is equal to $\tau_{\pi^-} = (2.6033 \pm 0.0005) \times 10^{-8}$ sec [24].
Table 1. Values of laser-modified pion lifetime (34) for different field strengths and with respect to three laser field directions. The laser frequency is $\hbar \omega = 1.17$ eV.

| $\mathcal{E}_0$ (V cm$^{-1}$) | $\mathbf{k}$ along $x$-axis | $\mathbf{k}$ along $y$-axis | $\mathbf{k}$ along $z$-axis |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $10^1$                        | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       |
| $10^2$                        | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       |
| $10^3$                        | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       |
| $10^4$                        | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       | $2.5419 \times 10^{-8}$       |
| $10^5$                        | $2.54194 \times 10^{-8}$      | $2.5419 \times 10^{-8}$       | $2.54191 \times 10^{-8}$      |
| $10^6$                        | $2.54215 \times 10^{-8}$      | $2.54207 \times 10^{-8}$      | $2.54204 \times 10^{-8}$      |
| $10^7$                        | $5.88956 \times 10^{-8}$      | $1.46564 \times 10^{-7}$      | $9.539 \times 10^{-8}$        |
| $10^8$                        | $2.46692 \times 10^{-7}$      | $1.46255 \times 10^{-6}$      | $9.88839 \times 10^{-7}$      |

From table 1, it is clear to us that the laser field at its low strengths (10 to $10^6$ V cm$^{-1}$) remains without significant effect on the lifetime. But, as the laser field strength increases to $10^7$ and $10^8$ V cm$^{-1}$, we notice that the lifetime starts to increase depending on the direction of the laser field. Note that the lifetime increases faster on the $y$ and $z$-axes than on the $x$-axis.

Tables 2 and 3 contain the values of the two branching ratios for different field strengths.

Table 2. Values of $\text{Br}(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$ (35) for different field strengths and with respect to three laser field directions. The laser frequency is $\hbar \omega = 1.17$ eV.

| $\mathcal{E}_0$ (V cm$^{-1}$) | $\mathbf{k}$ along $x$-axis | $\mathbf{k}$ along $y$-axis | $\mathbf{k}$ along $z$-axis |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $10^1$                        | 99.9874                       | 99.9874                       | 99.9874                       |
| $10^2$                        | 99.9874                       | 99.9874                       | 99.9874                       |
| $10^3$                        | 99.9874                       | 99.9874                       | 99.9874                       |
| $10^4$                        | 99.9874                       | 99.9874                       | 99.9874                       |
| $10^5$                        | 99.9889                       | 99.9874                       | 99.9875                       |
| $10^6$                        | 99.9973                       | 99.9939                       | 99.9926                       |
| $10^7$                        | 99.9994                       | 99.9966                       | 99.9966                       |
| $10^8$                        | 99.9998                       | 99.9968                       | 99.9965                       |

It is well known that the branching ratio of the muon channel is more favored, in the absence of the laser, compared to that of other channels [24]. From tables 2 and 3, we note that the laser enhanced the muon branching ratio and, on the other hand, suppressed the electronic one, which becomes almost nonexistent at high field strengths.

The direction of the laser field in this case has no effect on the branching ratios.
Table 3. Values of $\text{Br}(\pi^- \rightarrow e^- \bar{\nu}_e)$ (36) for different field strengths and with respect to three laser field directions. The laser frequency is $\hbar \omega = 1.17$ eV.

| $\mathcal{E}_0$ (V cm$^{-1}$) | $\text{Br}(\pi^- \rightarrow e^- \bar{\nu}_e)$ (%) | k along x-axis | k along y-axis | k along z-axis |
|-----------------------------|------------------------------------------|----------------|----------------|----------------|
| $10^1$                      | 0.0126005                               | 0.0126005      | 0.0126005      | 0.0126005      |
| $10^2$                      | 0.0126005                               | 0.0126005      | 0.0126005      | 0.0126005      |
| $10^3$                      | 0.0126005                               | 0.0126005      | 0.0126005      | 0.0126005      |
| $10^4$                      | 0.0126005                               | 0.0126005      | 0.0126005      | 0.0126005      |
| $10^5$                      | 0.0110947                               | 0.0126005      | 0.0126005      | 0.0124843      |
| $10^6$                      | 0.00268799                              | 0.00965144     | 0.00739155     |                |
| $10^7$                      | 0.000586612                             | 0.00484354     | 0.00343227     |                |
| $10^8$                      | 0.000239362                             | 0.00974693     | 0.00349305     |                |

4. Conclusion

In this paper, we dealt with the effect of laser field direction on the decay process of pion. We have extended the study of the laser-assisted pion decay process to the case of a general laser field direction. We have concluded that the direction of the laser field does not play an important role in laser-assisted decay processes in which the decaying particle is at rest. The configuration of the laser field direction does not change the nature of the result obtained even if it slightly affects the photon exchange process. This is due to the fact that the decaying particle is at rest in the initial state. Therefore, the geometry of the laser field will inevitably have a significant effect in the case where the initial particle is in motion as in the scattering processes. However, this idea seems important and it justifies the debate on the choice of the laser field direction.

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