Enhanced Electroweak Radiative Corrections in SUSY: Gluon-free Observables

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Abstract

Large top quark mass is responsible for the enhancement of the oblique radiative corrections in SUSY models. We present the analytical formulas for these corrections to the $W$-boson mass $m_W$ and to $Z l^+l^-$ coupling constants. The comparison with the result of the Standard Model fit is made.
The precision measurements of the $Z$-boson parameters, $W$-boson mass and top-quark mass have demonstrated that a Standard Model perfectly describes this part of Physics [1]. One has no hopes to improve a Standard Model fit by introducing New Physics since it is already perfect. For the data set announced at HEP97 Conference we obtained $\chi^2/d.o.f. = 18/14$ [2].

Nevertheless, the people still believe that there should be physics beyond the Standard Model and the most popular candidate is supersymmetry. In SUSY models there is a natural reason for the success of the Standard Model in describing the data at the energy scale of the order of the intermediate boson masses. This reason is decoupling: the contribution of SUSY particles to the "low energy" observables is suppressed like $(m_{W,Z}/m_{SUSY})^2$. That is why, when comparing the results of the SUSY model calculations with the experimental data, one gets the lower bounds on superpartner masses. To perform such a program one-loop electroweak radiative corrections in SUSY models should be calculated and this was done, see e.g. [3] - [5]. However, since there are a lot of one-loop diagrams and a large number of parameters even in the simplest MSSM the qualitative picture of electroweak radiative corrections did not emerge. The aim of our investigation is two-fold. Firstly, we will present simple analytical formulas which describe the main part of the radiative corrections in a wide class of SUSY models; secondly, we will include SUSY particles in the description of the electroweak radiative corrections developed and reviewed in [6].

In the present paper we will deal with the observables which are less sensitive to the strong interactions, i.e. with the mass of the $W$-boson and with $Z\, l^+l^-$-coupling constants $g_A$ and $g_V$. In future we plan to incorporate $Z$-boson decays to hadrons which will enable us to perform the general fit of the data and to get lower bounds on the masses of Superpartners.

In order to calculate the radiative corrections to $m_W$, $g_A$ and $g_V$ one starts with the corrections to $G_\mu$, $m_Z$ and $\tilde{\alpha} \equiv \alpha(m_Z)$. The corrections to $G_\mu$ are described in SUSY theories by box and vertex diagrams and by the correction to $W$-boson propagator. The corrections to $m_Z$ and $\tilde{\alpha}$ are described by self-energy insertions. Having calculated all the terms one comes to the corrections to $m_W$, $g_A$ and $g_V$. The corrections to $m_W$ contain self energy insertions and those to the coupling constants contain the vertices as well. In general all the above corrections are of the order of $(m_{W,Z}/m_{SUSY})^2$ and there are a lot of contributions from a large number of diagrams. However, the violation of $SU(2)_V$ symmetry by large top quark mass penetrates into the SUSY sector of the theory and leads to huge en-
hancement of the corresponding oblique corrections for any value of $m_{\text{SUSY}}$. To begin with, let us neglect $\tilde{t}_L \tilde{t}_R$ mixing. In this case we have with good accuracy: $m_{\tilde{t}_L}^2 - m_{\tilde{b}_L}^2 = m_t^2$ (in what follows we will often designate $\tilde{b}_L$ as $\tilde{b}$).

For $(\tilde{t}_L, \tilde{b}_L)$ loop insertion into $W$-boson propagator we get the result, proportional to $(m_{\tilde{t}_L}^2 - m_{\tilde{b}_L}^2)/m_W^2 m_{\text{SUSY}}^2 \approx 16(m_W/m_{\text{SUSY}})^2$ since $m_t \approx 2m_W$. That is why the $(\tilde{t}, \tilde{b})$ oblique corrections are responsible for the main part of SUSY corrections to gluon-free observables. (Let us note that the existence of the terms $\sim (m_t)^4$ in the SUSY radiative corrections was observed long ago [7]. Now when we know that top quark is very heavy and from the direct searches it follows that most of SUSY partners should be much heavier than $M_Z/2$ we can claim that the stop-sbottom contribution to the oblique corrections dominates over the other SUSY contributions to gluon-free observables.)

The term $\sim m_t^4/m_W^2 m_{\text{SUSY}}^2$ comes from $\Sigma_W(0)$, while the terms proportional to $\Sigma'(0)$ produce the corrections of the order of $m_t^4/m_{\text{SUSY}}^2$. Higher order derivatives of self energies are suppressed as $(m_W/m_{\text{SUSY}})^2$ and should be omitted. Calculating the vector boson self energies and keeping the first two terms in the $k^2$ expansion, we get the following expressions for the SUSY contributions to the quantities $V_i$, which are directly related to the physical observables ($V_m$ to $m_W$, $V_A$ to $g_A$ and $V_R$ to the ratio $g_V/g_A$, see [3]):

$$\delta_{\text{SUSY}} V_A = \frac{1}{m_Z^2} g(m_{\tilde{t}_L}, m_{\tilde{b}}) \ ,$$

(1)

$$\delta_{\text{SUSY}} V_R = \delta_{\text{SUSY}} V_A + \frac{1}{3} Y_L \ln \left( \frac{m_{\tilde{t}_L}^2}{m_{\tilde{b}}^2} \right) \ ,$$

(2)

$$\delta_{\text{SUSY}} V_m = \delta_{\text{SUSY}} V_A + \frac{2}{3} Y_L s^2 \ln \left( \frac{m_{\tilde{t}_L}^2}{m_{\tilde{b}}^2} \right) + \left( \frac{c^2 - s^2}{3} \right) h(m_{\tilde{t}_L}, m_{\tilde{b}}) \ ,$$

(3)

where $Y_L$ is the left doublet hypercharge, $Y_L = Q_t + Q_b$, $s \equiv \sin \theta$, $c \equiv \cos \theta$, $\sin^2 \theta \cos^2 \theta = \pi \bar{\alpha}(m_Z)/\sqrt{2}G_F m_Z^2$ ($\theta$ is the electroweak mixing angle) and the functions $g$ and $h$ are standard functions which describe the scalar particle contributions to the vector boson self energies at one loop:

$$g(m_1, m_2) = m_1^2 + m_2^2 - 2 \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_1^2}{m_2^2} \right) \ ,$$

(4)
\( h(m_1, m_2) = -\frac{5}{3} + \frac{4m_1^2m_2^2}{(m_1^2 - m_2^2)^2} + \frac{(m_1^2 + m_2^2)(m_1^4 - 4m_1^2m_2^2 + m_2^4)}{(m_1^2 - m_2^2)^3} \ln\left(\frac{m_1^2}{m_2^2}\right) . \)  

(5)

For SUSY partners with masses larger than \( m_t, m_\tilde{b} \gg m_t \), the leading term in (1) – (3) is universal and originates from \( \Sigma_W(0) \) and is given by the function \( g \):

\[
\delta_{SUSY} V_i = \frac{1}{3} \frac{(m_{i L}^2 - m_\tilde{b}^2)^2}{m_Z^4 m_\tilde{b}^4} .
\]

(6)

For light SUSY partners \( m_\tilde{b} \sim m_t \) we should not expand \( \delta_{SUSY} V_i \) over \((m_{i L}^2 - m_\tilde{b}^2)/m_\tilde{b}^2\) but use expressions (1) – (3) to describe the electroweak corrections. In Figures 1 - 3 the dependences of the functions \( \delta_{SUSY} V_i \) on \( m_\tilde{b} \) are shown. For top quark mass we use \( m_t = 175 GeV \). The horizontal lines show the difference of the experimental values of \( V_i \) and their Standard Model fit. The solid lines correspond to the central values while the dotted ones correspond to the one standard deviation corridor. From the Figures we see that for \( m_\tilde{b} \geq 200 \text{ GeV} \) SUSY corrections are not essential and the inclusion of SUSY partners will not spoil the Standard Model fit of the data.

Figure 1: The dependence of the function \( \delta_{SUSY} V_A \) on \( m_\tilde{b} \). The horizontal lines show the difference between experimental value of \( V_A \) and its fit in the Standard Model, \( V_A^{exp} - V_A^{theor} = -0.38(46) \). The solid line corresponds to
the central value, while dotted – to one standard deviation corridor.

Figure 2: The same as Figure 1 for $V_R$. $V_R^{\text{exp}} - V_R^{\text{theor}} = -0.0(4)$ was used.

Figure 3: The same as Figure 1 for $V_M$. $V_m^{\text{exp}} - V_m^{\text{theor}} = 0.45(65)$ was used.
Now let us take into account $\tilde{t}_L \tilde{t}_R$ mixing which in general is not small being of the order of $t$-quark mass times the superpartner mass scale, $m^2_{LR} \sim m_t m_{SUSY}$. As a result of mixing two states with masses $m_1$ and $m_2$ are formed, $\tilde{t}_1 = c_u \tilde{t}_L + s_u \tilde{t}_R$ and $\tilde{t}_2 = -s_u \tilde{t}_L + c_u \tilde{t}_R$, where $c_u \equiv \cos \theta_{LR}, s_u \equiv \sin \theta_{LR}$ and our functions $\delta_{SUSY}^{LR}V_i$ depend on 4 parameters: $m_1, m_2, \theta_{LR}$ and $m_{\tilde{b}}$, three of which are independent (the difference of $m^2_{\tilde{t}_L \tilde{t}_L}$ and $m^2_{\tilde{b}_L \tilde{b}_L}$ is determined by the top quark mass). The numerical values of these parameters depend on the SUSY model. Let us present the formulas for the electroweak radiative corrections which take $\tilde{t}_L \tilde{t}_R$ mixing into account:

$$\delta_{SUSY}^{LR}V_A = \frac{1}{m^2_Z} [c_u^2 g(m_1, m_{\tilde{b}}) + s_u^2 g(m_2, m_{\tilde{b}}) - c_u s_u g(m_1, m_2)] ,$$  (7)

$$\delta_{SUSY}^{LR}V_R = \delta_{SUSY}^{LR}V_A + \frac{1}{3} Y_L [c_u^2 \ln\left(\frac{m^2_1}{m^2_b}\right) + s_u^2 \ln\left(\frac{m^2_2}{m^2_b}\right)] - \frac{1}{3} c_u^2 s_u^2 h(m_1, m_2) ,$$  (8)

$$\delta_{SUSY}^{LR}V_m = \delta_{SUSY}^{LR}V_A + \frac{2}{3} Y_L s^2 [c_u^2 \ln\left(\frac{m^2_1}{m^2_b}\right) + s_u^2 \ln\left(\frac{m^2_2}{m^2_b}\right)] +$$

$$+ \frac{c^2 - s^2}{3} [c_u^2 h(m_1, m_{\tilde{b}}) + s_u^2 h(m_2, m_{\tilde{b}})] - \frac{c_u^2 s_u^2}{3} h(m_1, m_2) .$$  (9)

Our approximation should be good for the case when all superpartners have more or less equal masses. When some spartners are considerably lighter than sbottom their contribution can dominate and our approximation can fails.

It is desirable to compare numerically the formulas for the enhanced radiative corrections with the results of full one loop calculations in SUSY models in order to check how good our approximation is.

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