On the lepton CP violation in a $\nu2$HDM with flavor

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In this work we propose an extension to the Standard Model in which we consider the model 2HDM type-III plus massive neutrinos and the horizontal flavor symmetry $S_3$ ($\nu2$HDM$\otimes$S$_3$). In the above framework and with the explicit breaking of flavor symmetry $S_3$, the Yukawa matrices in the flavor adapted basis are represented by means of a matrix with two texture zeroes. Also, the active neutrinos are considered as Majorana particles and their masses are generated through type-I seesaw mechanism. The unitary matrices that diagonalize the mass matrices, as well as the flavor mixing matrices, are expressed in terms of fermion mass ratios. Consequently, in the mass basis the entries of the Yukawa matrices naturally acquire the form of the so-called Cheng-Sher ansatz. For the leptonic sector of $\nu2$HDM$\otimes$S$_3$, we compare, through a $\chi^2$ likelihood test, the theoretical expressions of the flavor mixing angles with the masses and flavor mixing leptons current experimental data. The results obtained in this $\chi^2$ analysis are in very good agreement with the current experimental data. We also obtained an allowed value ranges for the “Dirac-like” phase factor, as well as for the two Majorana phase factors. Furthermore, we study the phenomenological implications of these numerical values of the CP-violation phases on the neutrinoless double beta decay, and for Long Base-Line neutrino oscillation experiments such as T2K, NO$\nu$A, and DUNE.

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I. INTRODUCTION

According to the most recent neutrino physics literature \cite{1, 2}, there are several issues still unresolved. Among others: whether the neutrinos are Dirac or Majorana fermions; the absolute neutrino mass scale; the possible sources of Charge-Parity [CP] violation [CPV] in leptons. Nowadays, answers to these questions are being searched by means of the experimental results concerning KamLAND reactor neutrinos \cite{3, 4, 5}, in each one of the current high-statistics Short Base-Line (SBL) reactor neutrino experiments RENO \cite{6, 7}, Double Chooz \cite{8} and Daya Bay \cite{9}. Also, one of the most interesting effects related to the neutrino oscillation in matter is that these periodic transformations of neutrinos from one flavor to another, can induce a fake CP violating effect. Therefore, the Long Base-Line (LBL) neutrino oscillation experiments are good candidates for determining the “Dirac-like” CP violation phase as well as resolving the mass hierarchy problem \cite{10}. The recent measures reported by T2K \cite{11, 12, 13}, NO\textsubscript{\nu}A \cite{15, 16} and Super-Kamiokande experiments \cite{17} suggest a nearly maximal CP violation. In these experiments the “Dirac-like” CP phase takes the value $\delta_{CP} \simeq 3\pi/2$, with a statistical significance below 3$\sigma$ level. Moreover, the data obtained in the global fits of neutrino oscillations agree with a nonzero $\delta_{CP}$ phase, whereby the previous value is confirmed \cite{2, 18, 19, 20}.

The flavor and mass generation are two concepts strongly intertwined. In order to know the flavor dynamic in models beyond the Standard Model (SM), we need to understand the mechanism of flavor and mass generation arising in the standard theory. In the later, the Yukawa matrices are of great interest because its eigenvalues define the fermion masses. Moreover, for multi-Higgs models the Flavor Changing Neutral Current (FCNC) arises naturally from impediment to diagonalize simultaneously the mass and Yukawa matrices. In particular, models like the Two Higgs Doublet Model type-III (2HDM-III), in which the two Higgs doublets are coupled to all fermions, allow the presence of FCNC at tree level mediated by Higgs \cite{22, 23, 24, 25, 26, 27}. The 2HDM predicts three neutral states $H_0^{a}$ and a pair of charged states denoted as $H_{1,2}^\pm$ \cite{28, 29}. The Higgs-fermions couplings ($H^0 \bar{f} f$) in the 2HDM are given as \cite{22, 23, 24, 25, 27}:

$$L_Y = \bar{f}_i \left(S_{ij} + \gamma^5 P_{ij}\right) f_j H_0^a, \quad (a = 1, 2, 3). \tag{1}$$

In 2HDM-III the FCNC’s are kept under control by imposing some texture zeroes in the Yukawa matrices. This fact reproduce the observed fermion masses and mixing angles \textcite{30}. Using texture shapes allow a direct relation between the Yukawa matrix entries and the parameters used to compute the decay widths and cross section, without losing the terms proportional to the light fermion masses. Specifically, considering a zero texture Yukawa matrix, one obtains the Cheng-Sher ansatz for flavor mix couplings, widely used in literature, where flavored couplings are considered proportional to the involved fermion masses \textcite{31}.

The matter content in the 2HDM is divided among the quarks and leptons sectors. In turn these sectors are subdivided in two sectors, the up- and down-type for quarks sector, while charged leptons and neutrinos for the leptons sector. The fermions in each one of these subsectors are analogous each other, because they have completely identical couplings to all gauge bosons, although their mass values are not the same. Therefore, before of the spontaneous symmetry breaking (SSB), the Yukawa Lagrangian in the above subsectors is invariant under permutations of flavor indices. In other words, each one of these subsectors is invariant under the action of a $S_3$ symmetry group. This symmetry group has only three irreducible representations that correspond to two singlets and a doublet \textcite{32, 33}.

On the other hand, obtained from the experimental data, the mass spectrum for Dirac fermions obeys the following
strong hierarchy \cite{34}:

\[
\hat{m}_e \sim 10^{-6}, \quad \hat{m}_\mu \sim 10^{-3}, \quad \hat{m}_\tau \sim 1, \\
\hat{m}_u \sim 10^{-5}, \quad \hat{m}_c \sim 10^{-3}, \quad \hat{m}_t \sim 1, \\
\hat{m}_d \sim 10^{-3}, \quad \hat{m}_s \sim 10^{-2}, \quad \hat{m}_b \sim 1. 
\tag{2}
\]

In the above expression, \( \hat{m}_l = m_l/m_\tau \) (\( l = e, \mu, \tau \)) stands for charged leptons, \( \hat{m}_U = m_U/m_t \) (\( U = u, c, t \)) for up-type quarks, while \( \hat{m}_D = m_D/m_b \) (\( D = d, s, b \)) for down-type quarks. The behavior of these mass ratios in terms of irreducible representations of some symmetry group, can be interpreted as follows: the two lighter particles are associated with a doublet representation \( 2 \), whereas for the heaviest particle it is assigned a singlet representation \( 1 \). The smallest non-abelian group with irreducible representations of singlet and doublet, is the group of permutations of three objects. Hence, we expect the hierarchical nature of the Dirac fermion mass matrices to have its origin in the representation structure \( 3 = 2 \oplus 1 \) of \( S_3 \). In the theoretical framework of SM, as well as for 2HDM, the neutrinos are massless particles, fact that is in disagee with the results obtained in the neutrino oscillation experiments.

Therefore, in this work we will study the flavor dynamics through Yukawa matrices in the specific scenario of 2HDM-III plus massive neutrinos and a horizontal flavor symmetry \( S_3 \) (\( \nu2\text{HDM} \otimes S_3 \)). In this context, under the action of \( S_3 \) flavor symmetry group the right-handed neutrinos as well as the two Higgs fields transform as singlets, while the active neutrinos are considered as Majorana particles and their masses are generated through type-I seesaw mechanism. Hence, it is necessary to consider the following hybrid mass term, which involves the Dirac and Majorana neutrino mass terms \cite{35},

\[
\mathcal{L}_{M+D} = -\frac{1}{2} \bar{\eta}_L M_{M+D} (\eta_L)^c + \text{h.c.},
\tag{3}
\]

where \( \eta = (\nu_L, (N_R)^c)^\top \) and

\[
M_{M+D} = \begin{pmatrix} 0 & M_{\nu D} \\ M_{\nu D}^\top & M_R \end{pmatrix}.
\tag{4}
\]

In the above expression, \( M_{\nu D} \) and \( M_R \) are the Dirac and right-handed neutrino mass matrix, respectively. In the special limit \( M_R \gg M_{\nu D} \), the effective mass matrix of left-handed neutrinos is given by the type-I seesaw mechanism whose expression is\(^1\):

\[
M_\nu = M_{\nu D} M_R^{-1} M_{\nu D}^\top.
\tag{5}
\]

If the fermion mass matrices do not have any element equal to zero, on one hand, the mass matrix of active neutrinos has twelve free parameters, since \( M_\nu \) is a complex symmetric matrix because this matrix comes from a Majorana mass term. On the other hand, the Dirac fermion mass matrices do not have any special feature, \textit{i.e.}, these matrices are not Hermitian, nor symmetric. This is mainly due to the fact that the Yukawa matrices are represented through a \( 3 \times 3 \) complex matrix. Hence, for Dirac mass matrices we have eighteen free parameters.

\(^1\) Analysis of heavy Majorana neutrinos implications in LHC is dealt in Refs. \cite{36,37}. 

After the explicit sequential breaking of flavor symmetry according to the chain $S^3_{3L} \otimes S^3_{3R} \supset S^\text{diag}_3 \supset S^\text{diag}_2$, all Yukawa matrices in the flavor adapted basis are represented by means of a matrix with two texture zeroes. Therefore, all fermion mass matrices in the model have the same generic form with two texture zeroes.

The difference between 2HDM and $\nu$2HDM depends on the Yukawa structure, on the symmetries of the Higgs sector and on the possible appearance of new CPV sources. This CPV can arise from the same phase appearing in the Cabibbo-Kobayashi matrix, as in the SM, or some extra phase which arises from the Yukawa field or from the Higgs potential, either explicitly or spontaneously. The Higgs potential preserves CP symmetry, whereby CPV comes from the Yukawa matrices.

In order to validate our hypothesis where the $S_3$ horizontal flavor symmetry is explicitly breaking, hence all fermion mass matrices are represented through a matrix with two texture zeroes, we make a likelihood test where the $\chi^2$ function is defined in terms of leptonic flavor mixing angles. Afterwards, we shall investigate the phenomenological implications of these results on the neutrinoless double beta decay and the CPV in neutrino oscillations in matter.

The organization of this work is as follows. In section II we present the Yukawa Lagrangian in the $\nu$2HDM$\otimes S_3$, the form of the Dirac and Majorana fermion mass matrices in terms of its eigenvalues. In this way, we derive explicit and analytical expressions for the leptonic flavor mixing angles and Higgs-fermions couplings. In section III we present a detailed likelihood test where the $\chi^2$ function is defined in terms of leptonic mixing angles. Also, in section IV we explore the phenomenological implications of the numerical values obtained for the CP violating phase factors, on the neutrinoless double beta decay and the neutrino oscillations in matter. Finally, in the section V we present the conclusions and remarks of the present work.

II. THE YUKAWA LAGRANGIAN IN THE $\nu$2HDM$\otimes S_3$

In the fermion matter content of the SM, which is the same for the 2HDM, there are no right-handed neutrinos, consequently in both models a neutrino mass term is not allowed. This latter fact gainsay the results obtained in the neutrino oscillation experiments which requires neutrinos to have nonzero masses [34]. In order to include a Majorana neutrino mass term to the 2HDM, we need to increase its matter content. For this reason, we consider six neutrino fields: three left-handed $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)^T$ and three right-handed $N_R = (N_{1R}, N_{2R}, N_{3R})^T$. The right-handed neutrinos must be uncharged under the weak and electromagnetic interactions, which means that this kind of neutrinos are singlets under $G_{EW} \equiv SU(2)_L \otimes U(1)_Y$. In other words, only the left-handed neutrinos take part in the electroweak interaction. In this theoretical framework, we have all SM matter content plus massive neutrinos and an extra Higgs boson, whereby it is called as $\nu$2HDM. In the weak basis, the Yukawa interaction Lagrangian for Dirac fermions in the $\nu$2HDM is given by [29, 38, 39]:

$$L^w = \sum_{k=1}^{2} \left( Y^w_{k,u} \bar{Q} \Phi_k u_R + Y^w_{k,d} \bar{Q} \Phi_k d_R + Y^w_{k,\nu} \bar{L} \Phi_k N_R + Y^w_{k,l} \bar{L} \Phi_k l_R \right) + \text{h. c.,} \tag{6}$$

where $Q = (u, d)^T_L$ and $L = (\nu_l, l)^T_L$ are the left-handed doublets of $SU(2)_L$; $u_R$, $d_R$ and $l_R$ are the right-handed singlets of the electroweak gauge group. In this expression, the w superscript indicates that we are working in the weak basis, while the indices $l$, $u$ and $d$ represent the charged leptons, $u$- and $d$-type quarks, respectively. Also, $\Phi_k = (\phi_k^+, \phi_k^0)^T$ denotes the two Higgs fields which are doublets of $SU(2)_L$, with $\Phi_k = i\sigma_2 \Phi_k^\ast$. Finally, the $Y^w_{k,j}$ are
the Yukawa matrices in the weak basis, where the \( j \) superscript denotes the Dirac fermions, \((j = u, d, l, \nu_D)\) \(^{29}\). In general, after the SSB and in the context of \(\nu2HDM\), the Dirac fermion mass matrix in the weak basis can be written as \(^{29, 38, 40}\):

\[
M^w_j = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k Y^w_{jk},
\]

(7)

where \(v_k\) are the vacuum expectation values (vev’s) of the two Higgs bosons \(\Phi_k\), with \(k = 1, 2\).

A. Mass matrices from the \(S_3\) flavor symmetry

To reduce the free parameters in the fermion mass matrices, we will consider a horizontal symmetry which correlates the particle flavor indices each one with the other, thus the Yukawa matrices could be represented by means of a \(3 \times 3\) Hermitian matrix. Consequently, for three families or generations of quarks and leptons, we propose\(^ {2}\) that after the SSB, the Yukawa Lagrangian in the \(\nu2HDM\) presents the permutations group \(S_3\) as horizontal flavor symmetry. In this context, the right-handed neutrinos, as well as the two Higgs bosons, transform as singlets under the action of \(S_3\) flavor symmetry. In other words, the right-handed neutrinos and the two scalar fields \(\Phi_k\) are flavorless particles, whereby these fields are treated as scalars with respect to the \(S_3\) symmetry transformations. The general way to implement the flavor symmetry is considering that under the action of \(S_3\) symmetry, the left- and right-handed spinors transform as\(^ {42}\):

\[
\psi^s_{jL} = g^s_{ja} \psi_{jL} \quad \text{and} \quad \psi^s_{jR} = \tilde{g}^s_{ja} \psi_{jR}, \quad a, b = 1, \ldots, 6.
\]

(8)

At this point, the proposed flavor symmetry for the Yukawa Lagrangian is the \(S^3_{3L} \otimes S^3_{3R}\) group, whose elements are the pairs \(\left(g^s_{ja}, \tilde{g}^s_{ja}\right)\), where \(g^s_{ja} \in S^3_{3L}\) and \(\tilde{g}^s_{ja} \in S^3_{3R}\), while the superscript “s” means that fields are in the flavor symmetry adapted basis. However, the mass terms \(\mathcal{L}_m \sim \bar{\psi}_{jL} M^j_{L} \psi_{jR}\) and charged currents \(\mathcal{J}_\mu \sim \bar{\psi}_{jL} \gamma_\mu \psi_{jL} W^\mu\) in the Lagrangian are not invariant under \(S^3_{3L} \otimes S^3_{3R}\) group. In order to make \(\mathcal{L}_m\) and \(\mathcal{J}_\mu\) invariant under the flavor group transformations, the elements of the \(S^3_{3L} \otimes S^3_{3R}\) group must satisfy the relation \(g^s_{ja} \equiv g^3_{ja} = \tilde{g}^3_{ja}\). The latter condition implies that the flavor group is reduced according to the chain: \(S^3_{3L} \otimes S^3_{3R} \supset S^3_{3}\)\(^ {42}\). This flavor symmetry breaking chain should be interpreted as: all fermions in the model must be transformed with the same flavor group and the same element thereof. In the above, the flavor group is called \(S^3_{3}\) because its elements are the pairs \((g^3_a, g^2_a)\), where \(g^3_a \in S^3_{3L}\)\(^ {42}\).

Finally, it is easy conclude that \(S^3_{3}\) is the horizontal flavor symmetry which conserves the invariance of \(\mathcal{L}^w_v\) under the action of electroweak gauge group. Also, from the invariance of \(\mathcal{L}_m\) Lagrangian under the action of \(S^3_{3}\) group, we obtain that fermion mass matrices commute with all elements of the flavor group.

In the weak basis, the two Yukawa matrices in Eq. (7) are represented by means of a matrix with the exact \(S^3_{3}\) symmetry. Therefore, these Yukawa matrices are expressed as

\[
Y^w_{k}^{\nu,j} \equiv Y^3_{k}^{\nu,j} = \alpha^3_k P_1,
\]

(9)

\(^2\) As other authors have done in the SM \(^{41, 43}\) (and references in there).
where $\alpha^j_k$ are real constants associated with the flavor symmetry. The explicit form of $P_1$ matrix is given by Eq. (A2), and corresponds to the projector associated with the symmetric singlet representation of $S_3$. Hence, the Dirac fermion mass matrix is
\[
M^w_j = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k Y^j_k = m_{3j} P_1, \tag{10}
\]
where $m_{3j} = \frac{1}{\sqrt{2}} \left( v_1 \alpha^1_j + v_2 \alpha^2_j \right)$. In the flavor adapted basis, the Dirac fermion mass matrices are [40]
\[
M^s_j = U^s \ U^\dagger_j M^w_j U_j = m_{3j} U^s P_1 U^s = \text{diag}(0, 0, m_{3j}), \tag{11}
\]
where
\[
U^s = \frac{1}{\sqrt{6}} \begin{pmatrix}
\sqrt{3} & 1 & \sqrt{2} \\
-\sqrt{3} & 1 & \sqrt{2} \\
0 & -2 & \sqrt{2}
\end{pmatrix}. \tag{12}
\]
The interpretation of Eq. (11) is that under an exact $S_{3 \text{diag}}$ symmetry, the mass spectrum for Dirac fermions consists of a massive particle and two massless particles [44]. The only massive particle in each of fermion mass spectrum corresponds to the heaviest fermion. However, this result disagrees with the experimental data on quarks and leptons masses [34].

Since the two Higgs fields are invariant under flavor symmetry transformations, these are naturally assigned to $S_3$ flavor singlets. If the Yukawa Lagrangian is exactly invariant under $S_3$ flavor transformations, the two scalar fields can only couple with the $S_3$-singlet component of fermion fields. Consequently, only the $S_3$-singlet component of fermion fields acquires a mass non zero. As the third family is the heaviest, here we assign the fermion fields in the third family to singlet irreducible representation of $S_3$.

So, with the aim to generate a non zero mass for all fermions in the model, here we will break the flavor symmetry in an explicit sequential way, according to the chain $S^3_{3L} \otimes S^3_{3R} \supset S^\text{diag}_3 \supset S^\text{diag}_2$. Respectively, the first two fermion families and third one are assigned to the doublet and singlet irreducible representations of $S^\text{diag}_3$. The mass of the second fermion family is generated when the $S^\text{diag}_3$ flavor symmetry is explicitly breaks into the $S^\text{diag}_2$ group. This symmetry breaking is carried out when we add the following term to the $Y^{j3}_{k}$ matrix in Eq. [9] :
\[
Y^{j2}_{k} = \beta^j_k \ T^+_{z1} + \gamma^j_k \ T^+_{z2}, \tag{13}
\]
where $\beta^j_k$ and $\gamma^j_k$ are real constant parameters. The explicit form of the tensors $T^+_{z1}$ and $T^+_{z2}$ is given by Eq. (A10). The $Y^{j2}_{k}$ matrix mixes the symmetric component of the doublet with the singlet. Finally, the first fermion family’s mass is generated by adding the term
\[
Y^{j1}_{k} = \epsilon^j_k \ T^+_{x} + \rho^j_k \ T^-_{x} \tag{14}
\]

where $\epsilon^j_k$ and $\rho^j_k$ are real constant parameters. Thus, the explicit form of the tensors $T^+_{x}$ and $T^-_{x}$ is given by Eq. (A4). The $Y^{j1}_{k}$ matrix mixes the components of the doublet representation between each other in the weak basis. So, in the weak basis and under the explicit sequential breaking of flavor symmetry according to the chain; $S^3_{3L} \otimes S^3_{3R} \supset S^\text{diag}_3 \supset S^\text{diag}_2$, we obtain the Yukawa matrices which produce three massive fermions. These
Yukawa matrices are the sum of the three expressions given by Eqs. (9), (13), and (14). Then,

\[
Y^{w,j}_k = Y^{j3}_k + Y^{j2}_k + Y^{j1}_k = \alpha^{j1}_k P_1 + \beta^{j2}_k T^z_1 + \gamma^{j3}_k T^z_2 + \epsilon^{j1}_k T^+_x + \rho^{j2}_k T^+_x ,
\]

where

\[
Y^{w,j}_k = \begin{pmatrix}
e^{w,j}_k & a^{w,j}_k & f^{w,j}_k \\
e^{w,j}_k^* & b^{w,j}_k & c^{w,j}_k \\
f^{w,j}_k^* & c^{w,j}_k & d^{w,j}_k
\end{pmatrix},
\]

\[
e^{w,j}_k = \frac{\alpha^{j1}_k + 3(\beta^{j2}_k + i\rho^{j2}_k)}{3},
\]

\[
b^{w,j}_k = \frac{\alpha^{j1}_k + 3(\beta^{j2}_k + i\rho^{j2}_k)}{3},
\]

\[
c^{w,j}_k = \frac{\alpha^{j1}_k + 3(\gamma^{j3}_k - \epsilon^{j1}_k + i\rho^{j2}_k)}{3},
\]

\[
d^{w,j}_k = \frac{\alpha^{j1}_k + 3(\gamma^{j3}_k + \epsilon^{j1}_k - i\rho^{j2}_k)}{3}.
\]

In this same basis, now the fermion mass matrices \( M^w_j \) take the form:

\[
M^w_j = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k Y^{w,j}_k = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k \left( \alpha^{j1}_k P_1 + \beta^{j2}_k T^z_1 + \gamma^{j3}_k T^z_2 + \epsilon^{j1}_k T^+_x + \rho^{j2}_k T^+_x \right).
\]

Thus, with help of previous expression and Eq. (15), it is easy to conclude that the Dirac fermion mass matrices are Hermitian matrices without any of their elements equal to zero. However, in the flavor adapted basis the mass matrices in Eq. (17) acquire the following form:

\[
M^w_j = U_s^\dagger M^w_j U_s = \frac{1}{\sqrt{2}} \sum_{k=1}^{2} v_k U_s^\dagger Y^{w,j}_k U_s ,
\]

\[
P^j = \begin{pmatrix}
0 & |A_j| & 0 \\
|A_j| & B_j & C_j \\
0 & C_j & D_j
\end{pmatrix}
\]

\[
P^j = \frac{\cos \beta}{\sqrt{2}} \times \begin{pmatrix}
0 & A^j_1 & 0 \\
A^j_1 & B^j_1 & C^j_1 \\
0 & C^j_1 & D^j_1
\end{pmatrix} + \tan \beta \begin{pmatrix}
0 & A^j_2 & 0 \\
A^j_2 & B^j_2 & C^j_2 \\
0 & C^j_2 & D^j_2
\end{pmatrix},
\]

where \( P_j = \text{diag} (1, e^{-i\phi_j}, e^{-i\phi_j}) \) with \( \phi_j = \arg \{A_j\} \), and

\[
A^j_k = -\sqrt{3} \left( \epsilon^j_k - i\rho^j_k \right),
\]

\[
B^j_k = -\frac{3}{2} \left( \beta^j_k + 2\gamma^j_k \right),
\]

\[
C^j_k = \frac{\sqrt{2}}{2} \left( 4\beta^j_k - \gamma^j_k \right),
\]

\[
D^j_k = \alpha^j_k + \frac{3}{2} \left( \beta^j_k + 2\gamma^j_k \right),
\]

with \( k = 1, 2 \). The parameters in Eq. (19) are the Yukawa matrices elements expressed in the flavor adapted basis. Finally, in the Higgs sector \( \tan \beta = \frac{v_2}{v_1} \) with \( v^2 = v_1^2 + v_2^2 = (246.22 \text{ GeV})^2 \).

Here, we consider that the active neutrinos acquire their mass through the type-I seesaw mechanism, Eq. (5), where the Dirac fermion mass matrix is given by Eq. (18), while we suppose that in the flavor adapted basis the right-handed neutrino mass matrix has the form:

\[
M^w_R = \text{diag} (A_R, A_R, D_R) \mathbf{D}^{(3)} (A_1).
\]

In the latter expression \( A_R \) and \( D_R \) are real parameters, and the form of \( \mathbf{D}^{(3)} (A_1) \) matrix is given by Eq. (A1). So, in the flavor adapted basis the active neutrinos mass matrix is

\[
M^w_\nu = P^\dagger_\nu \begin{pmatrix}
0 & a_\nu & 0 \\
a_\nu & |b_\nu| & |c_\nu| \\
0 & |c_\nu| & d_\nu
\end{pmatrix} P^j_\nu ,
\]
where \( P_\nu = e^{i\phi_\nu} \text{diag} \left( 1, e^{-i2\phi_\nu}, e^{-i3\phi_\nu} \right) \) with \( \phi_\nu = \arg \{ C_\nu \} \), and \( \arg \{ C_\nu \} = 2 \arg \{ B_\nu \} \),

\[
a_\nu = \left| \frac{A_{\nu L}}{A_R} \right|^2 , \quad b_\nu = \frac{C^2_{\nu e}}{D_R} + \frac{2B_{\nu e}A^*_{\nu e}}{A_R} , \quad c_\nu = \frac{C_{\nu e}D_{\nu d}}{D_R} + \frac{C_{\nu d}A^*_{\nu d}}{A_R} , \quad \text{and} \quad d_\nu = \frac{D^2_{\nu e}}{D_R} .
\] (22)

In this work, we study the flavor dynamics through the Yukawa matrices in the 2HDM-III plus massive neutrinos and a horizontal flavor symmetry \( S_3 \). This theoretical framework is called \( \nu 2 \text{HDM} \otimes S_3 \).

**B. The mass and mixing matrices as function of fermion masses**

For a normal [inverted] hierarchy\(^3\) in the mass spectrum, the real symmetric matrices in Eqs. (18) and (21), which are associated with the fermion mass matrices, can be reconstructed through the orthogonal transformation\(^4\)

\[
\hat{M}^{n[i]}_f = m_{f3[2]} \Omega^{n[i]}_f \Delta^{n[i]}_f \left( O^{n[i]}_f \right)^T , \quad f = u, d, l, \nu ,
\] (23)

where \( \Delta^{n[i]}_f = \text{diag} \left( \hat{m}_{f1[3]}, -\hat{m}_{f2[1]}, 1 \right) \), in this expression \( \hat{m}_{f1[3]} = m_{f1[3]} / m_{f3[2]} \) and \( \hat{m}_{f2[1]} = |m_{f2[1]}| / m_{f3[2]} \). Here, \( m_{f2[1]} = -|m_{f2[1]}| \) and the \( m_f \)'s are the eigenvalues of fermion mass matrices, i.e., the particle masses. From algebraic invariants of the expression in Eq. (23) we have

\[
a^{n[i]}_f = \frac{(M_f)_{12}}{m_{f3[2]}}, \quad b^{n[i]}_f = \frac{(M_f)_{22}}{m_{f3[2]}} = \hat{m}_{f1[3]} - \hat{m}_{f2[1]} + \delta_f ,
\]

\[
c^{n[i]}_f = \frac{(M_f)_{23}}{m_{f3[2]}}, \quad d^{n[i]}_f = \frac{(M_f)_{33}}{m_{f3[2]}} = 1 - \delta_f ,
\] (24)

where \( \xi_{f1[3]} = 1 - \hat{m}_{f1[3]} - \delta_f \) and \( \xi_{f2[1]} = 1 + \hat{m}_{f2[1]} - \delta_f \). Also, the free parameter \( \delta_f \) must satisfy the relation \( 1 - \hat{m}_{f1[3]} > \delta_f > 0 \). The real orthogonal matrix given in Eq. (23) written in terms of fermion masses has the shape [38, 43]

\[
O^{n[i]}_f = \begin{pmatrix}
\sqrt{\frac{m_{f2[1]} \xi_{f1[3]}}{D_{f1[3]}}} & -\sqrt{\frac{m_{f1[3]} \xi_{f2[1]}}{D_{f2[1]}}} & \sqrt{\frac{m_{f1[3]} \delta_f m_{f2[1]} \xi_{f2[1]}}{D_{f3[2]}}} \\
\sqrt{\frac{m_{f1[3]} (1 - \delta_f) \xi_{f1[3]}}{D_{f1[3]}}} & \sqrt{\frac{m_{f2[1]} (1 - \delta_f) \xi_{f2[1]}}{D_{f2[1]}}} & \sqrt{\frac{m_{f1[3]} (1 - \delta_f) \xi_{f2[1]}}{D_{f3[2]}}} \\
-\sqrt{\frac{m_{f1[3]} \delta_f \xi_{f2[1]}}{D_{f1[3]}}} & -\sqrt{\frac{m_{f2[1]} \delta_f \xi_{f1[3]}}{D_{f2[1]}}} & \sqrt{\frac{\xi_{f1[3]} \xi_{f2[1]}}{D_{f3[2]}}}
\end{pmatrix} .
\] (25)

In this orthogonal matrix we have

\[
D_{f1[3]} = (1 - \delta_f) \left( \hat{m}_{f1[3]} + \hat{m}_{f2[1]} \right) \left( 1 - \hat{m}_{f1[3]} \right) ,
\]

\[
D_{f2[1]} = (1 - \delta_f) \left( \hat{m}_{f1[3]} + \hat{m}_{f2[1]} \right) \left( 1 + \hat{m}_{f2[1]} \right) ,
\]

\[
D_{f3[2]} = (1 - \delta_f) \left( 1 - \hat{m}_{f1[3]} \right) \left( 1 + \hat{m}_{f2[1]} \right) .
\] (26)

In the theoretical framework of \( \nu 2 \text{HDM} \otimes S_3 \), the unitary matrices that diagonalize the mass matrices of charged leptons and active neutrinos are defined as

\[
U^{n[i]}_\ell = U_\ell P_\ell O^{n[i]}_\ell , \quad \ell = l, \nu .
\] (27)

---

\(^3\) The inverted hierarchy is only valid for neutrinos.

\(^4\) The superscript \( n[i] \) denote the normal [inverted] hierarchy in the neutrino mass spectrum.
The matrix $\mathbf{U}_s$ is given by Eq. (12), and the diagonal phase matrices can be found in Eqs. (18) and (21). Finally, the real orthogonal matrix $\mathbf{O}_n^{[i]}$ is given in Eq. (29). From the previous unitary matrix, in the mass states basis, the Dirac fermion mass matrices take the shape

$$\Delta_j = \mathbf{U}_j^\dagger \mathbf{M}_w^j \mathbf{U}_j = \frac{1}{\sqrt{2}} \sum_{k=1}^2 v_k \mathbf{U}_j^\dagger \mathbf{Y}_w^{w,j} \mathbf{U}_j = \frac{1}{\sqrt{2}} \sum_{k=1}^2 v_k \tilde{Y}_k^j, \quad j = u, d, l, \nu_D, \quad (28)$$

where $\tilde{Y}_k^j = \mathbf{U}_j^\dagger \mathbf{Y}_k^{w,j} \mathbf{U}_j$ are the Yukawa matrices in the mass states basis. Now, with the help of the orthogonal matrix given in Eq. (25), and as the mass spectrum of Dirac particles only has the normal hierarchy, it is easy to obtain that the elements of Yukawa matrices $\tilde{Y}_k^j$ can be expressed in terms of geometric mean of Dirac fermion masses normalized with respect of electroweak scale, i.e.,

$$\left( \tilde{Y}_k^j \right)_{rt} = \frac{\sqrt{m_r m_t}}{u} \left( \tilde{\chi}_r^j \right)_{rt} \quad (r, t = 1, 2, 3). \quad (29)$$

Here, $\left( \tilde{\chi}_k^j \right)_{rt}$ are complex parameters, whose the explicit form is given in Appendix B. In the literature, this last relation is called *Cheng-Sher ansatz* [45], which is associated with Higgs-fermions couplings. Also, the result obtained in Eq. (29) can be extended to any Hermitian mass matrix that may be brought to a two zero texture matrix by means of an unitary transformation.

As one knows, the 2HDM-III is a generic description of particle physics at a higher energy scale ($\gtrsim$ TeV), and its imprint at low energies is reflected in Yukawa couplings structure. A detailed study of Yukawa Lagrangian within the 2HDM-III is given in [23, 24, 40, 46, 47], while the phenomenological implications of this model in scalar sector including Lepton Violation (LV) and/or FCNC’s are presented in [48–50]. In these works, the FCNC’s are under control because the authors have been assuming that Yukawa matrices in the weak and mass basis, are represented by means of an Hermitian matrix with two texture zeroes and the *Cheng-Sher ansatz*, respectively. In our model $\nu$2HDM$\otimes S_3$, the two texture zeroes shape for the Yukawa matrices is obtained by imposing a flavor symmetry $S_3$ and from its explicit sequential breaking according to the chain $S_{3L}^2 \otimes S_{3R}^2 \supset S_{3}^{\text{diag}} \supset S_2^{\text{diag}}$. An immediate consequence from the above is that Yukawa matrices in the mass basis naturally take the form of so-called *Cheng-Sher ansatz*. Therefore, we can say that the FCNC’s are under control in the $\nu$2HDM$\otimes S_3$.

The lepton flavor mixing matrix is defined as $\mathbf{U}_{\text{PMNS}} = \mathbf{U}_j^\dagger \mathbf{U}_\nu$. In this context the PMNS matrix takes the expression

$$\mathbf{U}_{\text{PMNS}} = \mathbf{O}_n^{\dagger} \mathbf{D}^{(\nu-l)} \mathbf{O}_n^{\nu[i]}, \quad (30)$$

where $\mathbf{D}^{(\nu-l)} = \text{diag} (1, e^{i\phi_1}, e^{i\phi_2})$ with $\phi_{\ell 1} = \phi_\ell - 2\phi_\nu$ and $\phi_{\ell 2} = \phi_\ell - \phi_\nu$. The explicit form of the entries of the PMNS matrix are given in Appendix C. The first conclusion of this PMNS matrix is that $\mathbf{U}_s$ unitary matrix, the one that allows us to pass from the weak basis to the flavor symmetry adapted basis, is unobservable in the lepton flavor mixing matrix.
C. The mixing angle and CP violation phases

In the symmetric parametrization of lepton flavor mixing matrix, the relation between mixing angles and the entries of PMNS matrix is [51, 52]

$$
\sin^2 \theta_{13} \equiv \frac{|(U_{PMNS})_{13}|^2}{1 - |(U_{PMNS})_{13}|^2}, \quad \sin^2 \theta_{12} = \frac{|(U_{PMNS})_{12}|^2}{1 - |(U_{PMNS})_{13}|^2}, \quad \sin^2 \theta_{23} = \frac{|(U_{PMNS})_{23}|^2}{1 - |(U_{PMNS})_{13}|^2}.
$$

(31)

On the one hand, the lepton Jarlskog invariant which appears in conventional neutrino oscillations is defined as:

$$
\mathcal{J}_{CP} = \Im \left\{ (U_{PMNS})_{12}^* (U_{PMNS})_{21} \right\}, \quad \mathcal{J}_{CP} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \cos \theta_{13} \sin \delta_{CP},
$$

(32)

and in the symmetric parametrization it has the form

$$
\mathcal{J}_{CP} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP},
$$

(33)

where $\delta_{CP} = \phi_{13} - \phi_{23} - \phi_{12}$. Moreover, the invariants

$$
\mathcal{I}_1 = \Im \left\{ (U_{PMNS})_{12}^2 (U_{PMNS})_{11}^2 \right\} \quad \text{and} \quad \mathcal{I}_2 = \Im \left\{ (U_{PMNS})_{13}^2 (U_{PMNS})_{11}^2 \right\},
$$

(34)

associated with the Majorana phases [53, 55] take the expressions

$$
\mathcal{I}_1 = \frac{1}{4} \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin(-2\phi_{12}) \quad \text{and} \quad \mathcal{I}_2 = \frac{1}{4} \sin^2 2\theta_{13} \cos^2 \theta_{12} \sin(-2\phi_{13}).
$$

(35)

Then, the phase factors associated with the CP violation can be written as:

$$
\sin(\delta_{CP}) = \frac{\mathcal{J}_{CP} \left(1 - |(U_{PMNS})_{13}|^2\right)}{|(U_{PMNS})_{11}| |(U_{PMNS})_{12}| |(U_{PMNS})_{13}| |(U_{PMNS})_{23}| |(U_{PMNS})_{33}|},
$$

$$
\sin(-2\phi_{12}) = \frac{\mathcal{I}_1}{|(U_{PMNS})_{11}| |(U_{PMNS})_{12}|^2}, \quad \sin(-2\phi_{13}) = \frac{\mathcal{I}_2}{|(U_{PMNS})_{11}|^2 |(U_{PMNS})_{13}|^2}.
$$

(36)

The equivalence between the PDG and symmetric parameterization may be expressed as $U_{PDG} = KU_{Sym}$, where $K = \text{diag} \left(1, e^{i \frac{\pi}{10}}, e^{i \frac{4\pi}{10}}\right)$ with $\delta_{CP} = \phi_{13} - \phi_{23} - \phi_{12}$, $\alpha_{21} = -2\phi_{12}$ and $\alpha_{31} = -2(\phi_{12} + \phi_{23})$.

III. NUMERICAL ANALYSIS

In the three flavor neutrino scheme there are six independent parameters which rule the behavior of neutrino oscillation phenomena: flavor mixing angles, the “Dirac-like” CP-violating phase and squared-mass splitting. The latest neutrino oscillation parameter is defined as $\Delta m_{ij}^2 \equiv m_{\nu_i}^2 - m_{\nu_j}^2$, in agreement with the results obtained in the global fit reported in Ref. [20]. These neutrino oscillation parameters have the following numerical values (at BFP±1σ and 3σ)\(^5\)

$$
\begin{align*}
\Delta m_{21}^2 \ (10^{-5} \text{eV}^2) &= 7.50^{+0.39}_{-0.17}, \ 7.03 - 8.09, \\
\Delta m_{31}^2 \ (10^{-3} \text{eV}^2) &= 2.524^{+0.039}_{-0.040}, \ 2.407 - 2.643, \text{ for NH}, \\
\Delta m_{23}^2 \ (10^{-3} \text{eV}^2) &= 2.514^{+0.038}_{-0.041}, \ 2.399 - 2.635, \text{ for IH},
\end{align*}
$$

(37)

\(^5\) Here, NH and IH denote the normal and inverted hierarchy in neutrino mass spectrum, respectively.
From the definition of squared-mass splitting $\Delta m^2_{ij}$ two of neutrino masses can be written as:

$$m_{\nu_3[2]} = \sqrt{m_{\nu_1[3]}^2 + \Delta m^2_{31[23]}}, \quad m_{\nu_2[1]} = \sqrt{m_{\nu_1[3]}^2 + \Delta m^2_{21[31]}},$$

where $m_{\nu_1[3]}$ is the lightest neutrino mass. Also, this neutrino mass is considered as the only one free parameter in the above expressions, since the mass-squared differences $\Delta m^2_{ij}$ are determined by experimental means.

From the results reported by Planck Collaboration for cosmological parameters, the upper limit on the active neutrino masses sum is $\sum m_{\nu_i} < 0.23$ eV, for an active neutrinos number equal to $N_{\text{eff}} = 3.15 \pm 0.23$ [56]. This upper bound is independent of hierarchy in the neutrino mass spectrum. So, with all the above experimental information and considering the expressions in Eq. (38), we can obtain the following value range for neutrino masses

$$m_{\nu_1} \ (10^{-2} \text{ eV}) = \begin{cases} [0.00, 7.10], & m_{\nu_2} \ (10^{-2} \text{ eV}) = \begin{cases} [0.84, 7.13], & m_{\nu_3} \ (10^{-2} \text{ eV}) = \begin{cases} [4.80, 8.75], & [0, 6.45]. & \end{cases} \end{cases} \end{cases}$$

Here, the oscillation parameters $\Delta m^2_{ij}$ are taken within the currently allowed 3σ range [20]. The values in the first and second row correspond to a normal and inverted hierarchy in the neutrino mass spectrum, respectively. For both hierarchies there is the possibility that the lightest neutrino mass could be zero. Namely in this case, the lightest neutrino is a massless particle. In the Fig. 1 the behaviour of neutrino masses (right panel), as well as of the neutrino masses sum as function of lightest neutrino mass $m_{\nu_1}$ (left panel), are shown.

The subscript $i[j]$ denote to normal [inverted] hierarchy in the neutrino masses spectrum.
TABLE I: Numerical values obtained in the BFP for the five parameters, the lepton mixing angles and the phase factors associated with the CPV. These results were obtained considering to $\Delta m^2_{ij}$ at BFP, BFP $\pm \sigma$ and $3\sigma$ range [20], and simultaneously $\Phi_{11}$, $\Phi_{12}$, $\delta_l$, $\delta_\nu$, and $m_{\nu_{[3]}}$ are free parameters in the $\chi^2$ function.

| $\Delta m^2_{ij}$ at | $\Phi_{11}$ [°] | $\Phi_{12}$ [°] | $m_{\text{lightest}}$ [eV] | $\delta_l$ | $\delta_\nu$ | $\theta_{12}^{\text{th}}$ [°] | $\theta_{23}^{\text{th}}$ [°] | $\theta_{13}^{\text{th}}$ [°] | $\delta_{\text{CP}}$ [°] | $\phi_{12}$ [°] | $\phi_{13}$ [°] | $\chi^2_{\text{min}}$ |
|----------------------|-----------------|-----------------|--------------------------|---------|---------|-----------------------|---------------------|---------------------|-------------------|----------------|----------------|----------------|
| NH BFP $\pm 1\sigma$ | 270              | 195             | $2.57 \times 10^{-3}$    | 0.20460 | 0.63519 | 33.58                  | 41.60               | 8.47                | $-68.65$          | $-5.86$         | 14.77          | $4.63 \times 10^{-2}$ |
| NH BFP $3\sigma$    | 270              | 195             | $2.57 \times 10^{-3}$    | 0.22256 | 0.64507 | 33.59                  | 41.61               | 8.46                | $-70.74$          | $-5.79$         | 14.67          | $3.13 \times 10^{-4}$ |
| BFP $3\sigma$       | 270              | 195             | $2.57 \times 10^{-3}$    | 0.21492 | 0.64008 | 33.80                  | 41.63               | 8.45                | $-69.85$          | $-5.80$         | 14.73          | $8.75 \times 10^{-2}$ |

FIG. 2: The allowed regions in the parameter space for $\delta_l$ and $\delta_\nu$ at $3\sigma$ C.L., the red point $(\bullet)$ represents the BFP. The left-panel is for normal hierarchy, while the right-panel is for the inverted hierarchy. Here, the $m_{\nu_{[3]}}$, $\Phi_{11}$, and $\Phi_{12}$ parameters are fixed to the values obtained when the $\Delta m^2_{ij}$ neutrino oscillation parameters are given at BFP, see Table I.

A. The likelihood test $\chi^2$

To validate our hypothesis where the $S_3$ horizontal flavor symmetry is explicitly sequential breaking according to the chain $S^2_{3L} \otimes S^2_{3R} \supset S^\text{diag}_3 \supset S^\text{diag}_2$, hence all fermion mass matrices are represented through a matrix with two texture zeroes, we make a likelihood test where the $\chi^2$ function is defined as:

$$\chi^2 = \sum_{i<j} \frac{(\sin^2 \theta_{ij}^{\text{th}} - \sin^2 \theta_{ij}^{\text{exp}})^2}{\sigma_{\theta_{ij}}^2}. \quad (40)$$

In this expression, the “th” superscript is used to denote the theoretical expressions of lepton mixing angles, while the terms with superscript “exp” denote to the experimental data with uncertainty $\sigma_{\theta_{ij}}$ for lepton mixing angles. For
these latter we consider the following values, at BFP±1σ [20]:

\[
\sin^2 \theta_{12}^{\exp} (10^{-1}) = 3.06 \pm 0.12, \quad \sin^2 \theta_{23}^{\exp} (10^{-1}) = \begin{cases} 4.41^{+0.27}_{-0.21}, \\ 5.87^{+0.20}_{-0.24} \end{cases} \quad \sin^2 \theta_{13}^{\exp} (10^{-2}) = \begin{cases} 2.166 \pm 0.0075, \\ 2.179 \pm 0.0076, \end{cases}
\] (41)

the values in the first and second row correspond to a normal and inverted hierarchy in the neutrino mass spectrum, respectively. From expressions in Eqs. (25), (30), (31), and (38), it is easy to conclude that the \( \chi^2 \) function depends of five free parameters \( \chi^2 = \chi^2 (\Phi_{\ell 1}, \Phi_{\ell 2}, \delta_{\ell}, \delta_{\nu}, m_{\nu_{1[3]}}) \). However, the \( \chi^2 \) function depends only on three experimental data which correspond to the leptonic flavor mixing angles. Therefore, if simultaneously we consider \( \Phi_{\ell 1}, \Phi_{\ell 2}, \delta_{\ell}, \delta_{\nu}, \) and \( m_{\nu_{1[3]}} \) as free parameters in the likelihood test, we can only determine the values of these parameters in the best fit point (BFP). In accordance with the above, we first seek the BFP by means of a likelihood test where the \( \chi^2 \) function have all those the five free parameters \( \Phi_{\ell 1}, \Phi_{\ell 2}, \delta_{\ell}, \delta_{\nu}, \) and \( m_{\nu_{1[3]}} \) as free parameters in the likelihood test, we can only determine the values of these parameters in the best fit point (BFP). To minimize the \( \chi^2 \) function we have done a scanning of the parameter space where we considered the following values for the charged lepton masses [34]

\[
m_e = 0.5109998928 \pm 0.000000011, \quad m_\mu = 105.6583715 \pm 0.0000035, \quad m_\tau = 1776.82 \pm 0.16,
\] (42)

while in Eq. (41) are given the experimental values for leptonic mixing angles.

\[\text{FIG. 3: The allowed regions for “Dirac-like” CPV phase } \delta_{\text{CP}}, \text{ and the free parameters } \delta_{\ell} \text{ and } \delta_{\nu}. \text{ The } m_{\nu_{1[3]}}, \Phi_{\ell 1} \text{ and } \Phi_{\ell 2} \text{ parameters are fixed to the values given in Table I for the BFP. The BFP is denoted by red point \bullet, the green and blue regions are for } 1\sigma \text{ and } 3\sigma \text{ C.L., respectively. The upper and lower panels correspond to normal and inverted hierarchy, respectively.} \]

In the Table I we show the numerical values obtained in the BFP for the five parameters, the lepton mixing angles and the phase factors associated with the CPV. All of these results were obtained considering to \( \Delta m_{ij}^2 \) at BFP,
BFP±σ and 3σ range, and simultaneously Φ₁, Φ₂, δℓ, δν, and mν₁[3] are free parameters in the χ² function of the likelihood test.

Now, as we know the numerical values of the five free parameters at BFP, we perform a new χ² analysis for the case when the oscillation parameters Δm² take the values at BFP and where we fix mν₁[3], Φ₁ and Φ₂ parameters to the values given in Table I.

So, χ² = χ²(δℓ, δν) function implies one degree of freedom. In the Fig. 2 we show the allowed regions in the parameter space for δℓ and δν at 3σ C.L., the red point (•) represent the BFP. The left-panel is for normal hierarchy and we can see that δν and δℓ parameters are of the order of 10⁻¹. The right-panel is for the inverted hierarchy and here δν parameter is ∼ 10⁻¹, while the δℓ parameter is of the order of 10⁻².

FIG. 4: The allowed regions for atmospheric mixing angle sin²θ₂₃ and the free parameter δℓ and δν. The mν₁[3], Φ₁ and Φ₂ parameters are fixed to the values given in Table I when the oscillation parameters Δm² take the values at BFP. The BFP is denoted by red point •, the green and blue regions are for 1σ and 3σ C.L., respectively. The upper and lower panels correspond to normal and inverted hierarchy, respectively correspondingly.

Associated to the parameter regions of δν and δℓ given in Fig. 2 for both hierarchies on neutrino mass spectrum, and based on Eq. [36], we found the predicted regions by the ν2HDM ⊗ S₃ for “Dirac-like” phase δCP. These regions are shown in Fig. 3. In concordance with experimental data, plots as function of δℓ are more restricted than the other one as function of δν. These results correspond closely with allowed regions obtained in the global fit reported in Ref. [20].
In the same way, for both hierarchies, we analyze the three leptonic flavor mixing angles, but in Fig. 4 we just show the allowed regions for the atmospheric mixing angle \( \theta_{23} \), at BFP, \( \pm 1 \sigma \) and \( 3 \sigma \) C.L. In order to round the above results, from our analysis we obtain the following values for the three mixing angles, at BFP\( \pm 1 \sigma \) C.L.:

\[
\sin^2 \theta_{12}^{\text{th}}(10^{-1}) = \begin{pmatrix}
3.09^{+0.066}_{-0.065} \\
3.10^{+0.011}_{-0.011}
\end{pmatrix},
\sin^2 \theta_{23}^{\text{th}}(10^{-1}) = \begin{pmatrix}
4.41^{+0.10}_{-0.14} \\
5.87^{+0.224}_{-0.233}
\end{pmatrix},
\sin^2 \theta_{13}^{\text{th}}(10^{-2}) = \begin{pmatrix}
2.160 \pm 0.14, \\
2.177 \pm 0.12.
\end{pmatrix}
\]

We also obtained the following allowed value ranges at BFP\( \pm 1 \sigma \) for the “Dirac-like” phase \( \delta_{CP} \), as well as for the two Majorana phase factors \( \phi_{12} \) and \( \phi_{13} \):

\[
\delta_{CP}(^\circ) = \begin{pmatrix}
-69.8^{+5.08}_{-6.110} \\
-80.83^{+0.652}_{-0.709}
\end{pmatrix},
\phi_{12}(^\circ) = \begin{pmatrix}
-5.800^{+0.170}_{-0.150} \\
-5.24^{+0.153}_{-0.148}
\end{pmatrix},
\phi_{13}(^\circ) = \begin{pmatrix}
14.744^{+1.266}_{-1.366} \\
-2.190^{+0.0030}_{-0.0005}
\end{pmatrix}.
\]

From Eq. (44) and the Table I we can conclude that values for the \( \delta_{CP} \) phase obtained in our scheme are consistent with a maximal CP violation.

Finally, as a immediate result of the above likelihood analysis, the entries magnitude of \( U_{\text{PMNS}} \) mixing matrix can numerically computed. So, at \( 3 \sigma \) C.L., we have that \( U_{\text{PMNS}} \) matrix takes the form:

\[
\begin{pmatrix}
0.822^{+0.0044}_{-0.0045} & 0.550^{+0.0055}_{-0.0054} & 0.147^{+0.0047}_{-0.0048} \\
0.395^{+0.0181}_{-0.0154} & 0.642^{+0.0008}_{-0.0001} & 0.657^{+0.0082}_{-0.0111} \\
0.410^{+0.0056}_{-0.0089} & 0.534^{+0.0045}_{-0.0056} & 0.739^{+0.0088}_{-0.0096}
\end{pmatrix}, \text{ Normal Hierarchy,}
\]

\[
\begin{pmatrix}
0.822^{+0.0012}_{-0.0012} & 0.551^{+0.0007}_{-0.0006} & 0.147^{+0.0041}_{-0.0041} \\
0.355^{+0.0072}_{-0.0071} & 0.547^{+0.0144}_{-0.0149} & 0.758^{+0.0138}_{-0.0141} \\
0.446^{+0.0077}_{-0.0080} & 0.630^{+0.0120}_{-0.0122} & 0.636^{+0.0174}_{-0.0178}
\end{pmatrix}, \text{ Inverted Hierarchy.}
\]

**IV. PHENOMENOLOGICAL IMPLICATIONS**

In the above section we have seen that in our theoretical framework, where the \( S_3 \) flavor symmetry sets up that the fermion mass matrices should have two texture zeroes, we can reproduce the values of oscillation parameters in very good agreement with the last experimental data. In the following, we shall investigate the phenomenological implications of these results for the neutrinoless double beta decay (0\( \nu \beta\beta \)) and the CP violation in neutrino oscillations in matter.

**A. Neutrinoless double beta decay**

The 0\( \nu \beta\beta \) is a rare second-order weak process where a nucleus \((A, Z)\) decays into another one by the emission of two electrons, whose mode decay is \((A, Z) \rightarrow (A, Z + 2) + e^- + e^-\). The observation of this process would establish that neutrino are Majorana particles and that total lepton number is not a conserved symmetry in nature \[60\] [61]. In the most simple version of the process, the amplitude for the decay is proportional to a quantity called the effective mass \( m_{ee} \) \[62\] [63]. In the symmetric parametrization of lepton mixing matrix the effective mass parameter have the shape \[64\] [65]:

\[
|m_{ee}| = |m_{\nu_1} \cos^2 \theta_{12} \cos^2 \theta_{13} + m_{\nu_2} \sin^2 \theta_{12} \cos^2 \theta_{13} e^{-i2\phi_{12}} + m_{\nu_3} \sin^2 \theta_{13} e^{-i2\phi_{13}}|,
\]

(47)
FIG. 5: In the upper panel we show the effective mass $|m_{ee}|$ which is involved in the $0\nu\beta\beta$ decay. The red and blue bands are obtained with the current experimental data on neutrino oscillations, at 3σ [20], for an inverted and normal neutrino mass hierarchy, respectively. On the one hand, from the combination of EXO-200 [57, 58] and KamLAND-ZEN [59] results we have the following upper bound for $|m_{ee}| < 0.120 \text{eV}$. On the other hand, from the results reported by Planck Collaboration we have that $\sum m_{i} < 0.230 \text{eV}$ at 95% level [56], thus an upper bound on the lightest neutrino mass is established. In the left- and right-lower panels we show a zoom in of the allowed regions for $|m_{ee}|$ obtained at 95% C.L. in the context of $\nu^{2}\text{HDM}\otimes S_{3}$ for a normal and inverted hierarchy, respectively.

where $\phi_{12}$ and $\phi_{13}$ are the Majorana phases given in Eq. (36). In the Fig. 5 we show the allowed regions for the magnitude of effective mass parameter $m_{ee}$, which were obtained in the context of $\nu^{2}\text{HDM}\otimes S_{3}$. Each one of these regions was obtained by setting the values of some of the five free parameters in the $\chi^2$ function, Eq. (40), to the values given in the Table I for $\Delta m_{ij}^2$ at BFP. Then, for both hierarchies in the lower panels of Fig. 5 the blue lines were obtained by means a likelihood test where the values of $\phi_{\ell 1}$, $\phi_{\ell 2}$, $\delta_{e}$ and $\delta_{\nu}$ are fixed, while $m_{\nu_{\text{lightest}}}$ is free parameter. The orange bands were obtained by means a likelihood test where the values of $\phi_{\ell 2}$, $\delta_{e}$ and $\delta_{\nu}$ are fixed, while $m_{\nu_{\text{lightest}}}$ and $\phi_{\ell 1}$ are free parameters. The yellow bands were obtained by means of a likelihood test where the values of $\phi_{\ell 1}$, $\delta_{e}$ and $\delta_{\nu}$ are fixed, while $m_{\nu_{\text{lightest}}}$ and $\phi_{\ell 2}$ are free parameters. The sky blue bands were obtained through a likelihood test where the values of $\phi_{\ell 1}$, $\phi_{\ell 2}$ and $\delta_{\nu}$ are fixed, while $m_{\nu_{\text{lightest}}}$ and $\delta_{e}$ are free parameters. Finally, the turquoise bands were obtained via a likelihood test where the values of $\phi_{\ell 1}$, $\phi_{\ell 2}$ and $\delta_{e}$ are fixed, while
Fixed parameters | $m_{\text{lightest}}$ [10^{-2} \text{ eV}] | $|m_{ee}|$ [10^{-2} \text{ eV}]
---|---|---
NH | $\phi_{\ell_1}, \phi_{\ell_2}, \delta_e, \delta_\nu$ | [0.2360, 0.2768] | [0.3204, 0.3608] |
| $\phi_{\ell_2}, \delta_e, \delta_\nu$ | [0.2374, 0.2735] | [0.3215, 0.3583] |
| $\phi_{\ell_1}, \phi_{\ell_2}, \delta_\nu$ | [0.2349, 0.2761] | [0.3229, 0.3577] |
| $\phi_{\ell_1}, \phi_{\ell_2}, \delta_e$ | [0.2164, 0.2908] | [0.3121, 0.3659] |
| $\phi_{\ell_1}, \phi_{\ell_2}, \delta_e, \delta_\nu$ | [2.317, 2.635] | [3.515, 3.694] |
| $\phi_{\ell_2}, \delta_e, \delta_\nu$ | [2.311, 2.650] | [3.515, 3.696] |
| $\phi_{\ell_1}, \phi_{\ell_2}, \delta_e$ | [2.123, 2.787] | [3.416, 3.767] |
| $\phi_{\ell_1}, \phi_{\ell_2}, \delta_e, \delta_\nu$ | [2.268, 2.685] | [3.491, 3.717] |

IH | $\phi_{\ell_1}, \phi_{\ell_2}, \delta_e, \delta_\nu$ | [2.301, 2.648] | [3.512, 3.695] |
| $\phi_{\ell_2}, \delta_e, \delta_\nu$ | [2.311, 2.650] | [3.512, 3.695] |
| $\phi_{\ell_1}, \phi_{\ell_2}, \delta_\nu$ | [2.123, 2.787] | [3.416, 3.767] |

TABLE II: The allowed numerical ranges, at 95\% C.L., for the effective mass parameter magnitude $m_{ee}$ and the lightest neutrino mass $m_{\text{lightest}}$.

$m_{\text{lightest}}$ and $\delta_\nu$ are free parameters.

To round the previous results, in the Table II we show the allowed numerical ranges, at 95\% C.L., for the magnitude of effective mass parameter $m_{ee}$ and the lightest neutrino mass $m_{\text{lightest}}$. From the results in the Table II it is easy to conclude that for the normal hierarchy $m_{\nu_1} \sim 2 \times 10^{-3} \text{ eV}$ and $|m_{ee}| \sim 3 \times 10^{-3} \text{ eV}$, while for the inverted hierarchy $m_{\nu_3} \sim 2 \times 10^{-2} \text{ eV}$ and $|m_{ee}| \sim 3 \times 10^{-2} \text{ eV}$.

### B. CP violation in neutrino oscillations in matter

In the recent years, we have entered into a precision era in the determination of flavor leptonic mixing angles. However, it is not the same situation for the CP violation in this sector, since has yet to be determined experimentally the numerical value of CP violation phase. But we have a hunch of where to look: the neutrino oscillations with matter effects [67]. One of the aims of the LBL neutrino experiments such as T2K [68] and NOνA [69], as well as the proposed experiment DUNE [70], it is determination of the “Dirac-like” CP violation phase and other parameters that rule the neutrino oscillations $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$. The transition probability in matter for the oscillation between electron and muon neutrinos, as well as for the oscillation between electron and muon antineutrinos, have the form [66] [71] [72]:

\[
P(\nu_\mu \to \nu_e) \simeq P_{\text{atm}} + P_{\text{sol}} + 2\sqrt{P_{\text{atm}}P_{\text{sol}}} \cos(\Delta_{32} + \delta_{\text{CP}}),
\]

\[
P(\bar{\nu}_\mu \to \bar{\nu}_e) \simeq P_{\text{atm}} + P_{\text{sol}} + 2\sqrt{P_{\text{atm}}P_{\text{sol}}} \cos(\Delta_{32} - \delta_{\text{CP}}),
\]

(48)

where

\[
\sqrt{P_{\text{sol}}} = \cos \theta_{23} \sin 2\theta_{12} \frac{\sin \alpha L}{\alpha L} \Delta_{21},
\]

\[
\sqrt{P_{\text{atm}}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31}-\alpha L)}{(\Delta_{31}-\alpha L)} \Delta_{31},
\]

\[
\sqrt{P_{\text{atm}}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31}+\alpha L)}{(\Delta_{31}+\alpha L)} \Delta_{31}.
\]

(49)
FIG. 6: The $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ transition probabilities, and the asymmetry $A_{\mu e}$ between them, for a normal hierarchy in the neutrino mass spectrum. The blue, red and yellow bands are obtained for a Base-Line energy $L$ of 295, 810 and 1300 km for left panels. For the right panels these bands belong to a neutrino energy $E$ of 0.3, 2 and 2.8 GeV, which correspond to the T2K, NOvA and DUNE experiment, respectively. Here, the $\delta_{CP}$ phase takes values within 1$\sigma$ C.L. range given in Eq. (44). The remaining parameters are fixed to the values obtained at BFP, which are given in Eq. (37) for $\Delta m_{ij}^2$ and Table I for flavor mixing angles.

In the above expressions, $L$ is the Base-Line,

$$\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}, \quad \Delta m_{ij}^2 = m_i^2 - m_j^2 \quad \text{and} \quad a = \frac{G_F N_e}{\sqrt{2}}.$$ (50)
FIG. 7: The $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ transition probabilities, and the asymmetry $A_{\mu e}$ between them, for an inverted hierarchy in the neutrino mass spectrum. The blue, red and yellow bands are obtained for a Base-Line energy $L$ of 295, 810 and 1300 km for left-panels. For the right-panels these bands belong to a neutrino energy $E$ of 0.3, 2 and 2.8 GeV, which correspond to the T2K, NOνA and DUNE experiment, respectively. Here, the $\delta_{CP}$ phase takes values within 1σ C.L. range given in Eq. (44). The remaining parameters are fixed to the values obtained at BFP, which are given in Eq. (37) for $\Delta m_{ij}^2$ and Table I for flavor mixing angles.
Here, $E$ is the energy of neutrino beam, $G_F$ is the Fermi constant and $N_e$ is the density of electrons. The $a$ parameter is $a \approx (3500 \text{ km})^{-1}$ for the Earth crust [71]. The asymmetry between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in matter is [72]

$$A_{\mu e} = \frac{P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}$$

$$= \frac{(P_{\text{atm}} - P_{\text{atm}}) + 2\sqrt{P_{\text{sol}}}(\sqrt{\frac{\sin(\Delta_{32} + \delta_{\text{CP}})}{\sin(\Delta_{32} - \delta_{\text{CP}})}})\sqrt{\frac{\sin(\Delta_{32} + \delta_{\text{CP}})}{\sin(\Delta_{32} - \delta_{\text{CP}})}} + 2P_{\text{sol}}}{(P_{\text{atm}} + P_{\text{atm}}) + 2\sqrt{P_{\text{sol}}}(\sqrt{\frac{\sin(\Delta_{32} + \delta_{\text{CP}})}{\sin(\Delta_{32} - \delta_{\text{CP}})}} + \sqrt{\frac{\sin(\Delta_{32} - \delta_{\text{CP}})}{\sin(\Delta_{32} + \delta_{\text{CP}})}} + 2P_{\text{sol}}}.$$  (51)

The above asymmetry $A_{\mu e}$ is basically due to the absence of positrons in the journey of neutrino (anti-neutrino) through the earth. Hence, a neutrino experiment with a LBL would be more sensitive to measure this asymmetry.

The T2K neutrino oscillation experiment has a LBL of 295 km, while the energy of its neutrino beam has a peak around to 0.6 GeV and width of $\sim$0.3 GeV [68]. In Fig. 6 we show the transition probability $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$, as well as the asymmetry $A_{\mu e}$ for T2K experiment.

The NOvA neutrino oscillation experiment has a LBL of 810 km, while the energy of its neutrino beam has a peak around to 2 GeV [69]. In Fig. 6 we show the transition probability $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$, as well as the asymmetry $A_{\mu e}$ for NOvA experiment.

Finally, the future neutrino oscillation experiment DUNE will have a LBL of $\sim$ 1300 km, while the energy of its neutrino beam will have a peak around to 2.5 – 3.0 GeV [70]. In Fig. 6 the transition probability $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$, and the asymmetry $A_{\mu e}$ for DUNE experiment are shown.

V. CONCLUSIONS

We have studied the theory of neutrino masses, mixings and CPV as the realization of an $S_3$ flavor symmetry in the framework of the Two Higgs Doublet Model type-III. In this $\nu$2HDM$\otimes S_3$ extension of Standard Model, on the one hand, the active neutrinos acquire their little masses via the type-I seesaw mechanism. On the other hand, the explicit sequential breaking of flavor symmetry according the chain $S^3_{3L} \otimes S^3_{3R} \supset S^3_{3} \supset S^2_{3}$, allow us to represent in the flavor basis the Yukawa matrices with an Hermitian matrix with two texture zeroes. Consequently, we obtained an unified treatment for all fermion mass matrices in the model, which are represented through of a matrix with two texture zeroes.

The unitary matrices that diagonalize the mass matrices are expressed in terms of fermion mass ratios. Then, the entries of the Yukawa matrices in the mass basis naturally acquire the form of the so-called Cheng-Sher ansatz. Also, the lepton flavor mixing matrix PMNS is expressed as function of the masses of charged leptons and neutrinos, two phases associated with the CP-violation, and two parameters associated with the flavor symmetry breaking. The unitary matrix that allows us to pass from the weak basis to the flavor symmetry adapted basis, is unobservable in the Higgs-fermions couplings and lepton flavor mixing matrix.

To validate our hypothesis where the $S_3$ horizontal flavor symmetry is explicitly breaking according to the chain $S^3_{3L} \otimes S^3_{3R} \supset S^3_{3} \supset S^2_{3}$, all fermion mass matrices are represented through a matrix with two texture zeroes. Furthermore, we make a likelihood test where we compare the theoretical expressions of the flavor mixing angles with the current experimental data on masses and flavor mixing of leptons. The results obtained in this $\chi^2$ analysis are in very good agreement with the current experimental data.

We also obtained the following allowed value ranges, at BFP$\pm 1\sigma$ for the “Dirac-like” phase factor, as well as for
In this representation the projection operators take the form:

\[ \begin{align*}
\phi_{12}(\nu) &= \begin{pmatrix} -69.8^{+5.508}_{-6.110} & +5.052 & -0.709 \end{pmatrix}, \\
\phi_{13}(\nu) &= \begin{pmatrix} -5.800^{+0.170}_{-0.150} & +0.153 & -0.148 \end{pmatrix}, \\
\phi_{13}(\nu) &= \begin{pmatrix} 14.744^{+1.266}_{-1.366} & +0.030 & -0.005 \end{pmatrix}. 
\end{align*} \tag{52} \]

The upper (lower) row corresponds to the normal (inverted) hierarchy in the neutrino mass spectrum. These values of the phase factors are in agreement with a maximum CPV in the neutrino oscillation in matter. Finally, we also analyzed the phenomenological implications of the above numerical values of the CP-violation phases on the neutrinoless double beta decay, as well as for LBL neutrino oscillation experiments such as T2K, NOνA, and DUNE.

**Appendix A: Three dimensional representation of \( S_3 \)**

The permutations of symmetry group \( S_3 \) can be represented on the reducible triplet as \( [32, 33] \):

\[ \begin{align*}
D^{(3)}(E) &= \begin{pmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \end{pmatrix}, & D^{(3)}(A_1) &= \begin{pmatrix} 0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \end{pmatrix}, & D^{(3)}(A_2) &= \begin{pmatrix} 0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \end{pmatrix} \\
D^{(3)}(A_3) &= \begin{pmatrix} 1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \end{pmatrix}, & D^{(3)}(A_4) &= \begin{pmatrix} 0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \end{pmatrix}, & D^{(3)}(A_5) &= \begin{pmatrix} 0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \end{pmatrix}. 
\end{align*} \tag{A1} \]

In this representation the projection operators take the form:

- **Symmetric singlet**, \( P_1 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \end{pmatrix} \), where \( |v_1\rangle\langle v_1| \).

- **Anti-symmetric singlet**, \( P_1 = 0 \).

- **Doublet**, \( P_2 = \frac{1}{3} \begin{pmatrix} 2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2 \end{pmatrix} \), where \( |v_{2A}\rangle\langle v_{2A}| + |v_{2S}\rangle\langle v_{2S}| \).

Here, the vector \( |v_1\rangle = \frac{1}{\sqrt{3}} (1, 1, 1)^T \) is associated with the symmetric singlet. In the projection operator \( P_2 \), we have the vectors \( |v_{2A}\rangle = \frac{1}{\sqrt{2}} (-1, 1, 0)^T \) and \( |v_{2S}\rangle = \frac{1}{\sqrt{2}} (1, 1, -2)^T \), which are associated with the doublet.

Correspondingly, the vectors \( |v_{2A}\rangle \) and \( |v_{2S}\rangle \) are antisymmetric and symmetric, under the permutation of first two elements. With the previous three vectors we can construct some tensors that can be helpful. Then,

\[ \begin{align*}
|v_{2S}\rangle\langle v_{2A}| &= \frac{1}{\sqrt{12}} \begin{pmatrix} -1 & 1 & 0 \\
-1 & 1 & 0 \\
2 & -2 & 0 \end{pmatrix} \quad \text{and} \quad |v_{2A}\rangle\langle v_{2S}| = \frac{1}{\sqrt{12}} \begin{pmatrix} -1 & -1 & 2 \\
1 & 1 & -2 \\
0 & 0 & 0 \end{pmatrix}. \tag{A3} \end{align*} \]

If we define the tensors \( T^+_x = |v_{2S}\rangle\langle v_{2A}| + |v_{2A}\rangle\langle v_{2S}| \) and \( T^-_x = i (|v_{2A}\rangle\langle v_{2S}| - |v_{2S}\rangle\langle v_{2A}|) \), we obtain

\[ \begin{align*}
T^+_x &= \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & 0 & 1 \\
0 & 1 & -1 \\
1 & -1 & 0 \end{pmatrix}, & T^-_x &= \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & i & -i \\
-i & 0 & i \\
-i & i & 0 \end{pmatrix}. \tag{A4} \end{align*} \]
The terms proportional to tensors $T^+_x$ and $T^-_x$ mix the components of the doublet representation each other. Now,

$$|v_1\rangle\langle v_{2A}| = \frac{1}{\sqrt{6}}\begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix} \text{ and } |v_{2A}\rangle\langle v_1| = \frac{1}{\sqrt{6}}\begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \quad (A5)$$

If we define the tensors $T^+_y = |v_1\rangle\langle v_{2A}| + |v_{2A}\rangle\langle v_1|$ and $T^-_y = i(|v_{2A}\rangle\langle v_{2A}| - |v_{2A}\rangle\langle v_{2S}|)$, we obtain

$$T^+_y = \frac{1}{\sqrt{6}}\begin{pmatrix} -2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix} \text{ and } T^-_y = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 & 2i & i \\ -2i & 0 & -i \\ -i & i & 0 \end{pmatrix}. \quad (A6)$$

The terms proportional to tensors $T^+_y$ and $T^-_y$ mix the antisymmetric component of doublet with the singlet. Finally,

$$|v_1\rangle\langle v_{2S}| = \frac{1}{3\sqrt{2}}\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 1 & 1 & -2 \end{pmatrix} \text{ and } |v_{2S}\rangle\langle v_1| = \frac{1}{3\sqrt{2}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}. \quad (A7)$$

If we define the tensors $T^+_z = |v_1\rangle\langle v_{2S}| + |v_{2S}\rangle\langle v_1|$ and $T^-_z = i(|v_{2A}\rangle\langle v_{2A}| - |v_{2A}\rangle\langle v_{2S}|)$, then

$$T^+_z = \frac{1}{3\sqrt{2}}\begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & -4 \end{pmatrix} \text{ and } T^-_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & -i \\ i & i & 0 \end{pmatrix}. \quad (A8)$$

The terms proportional to tensors $T^+_z$ and $T^-_z$ mix the symmetric component of doublet with the singlet. The tensor $T^+_z$ can be written as a linear combination of two independent matrices,

$$T^+_z = \sqrt{\frac{2}{3}}T^+_{z1} - \frac{1}{3\sqrt{2}}T^+_{z2}, \quad (A9)$$

where

$$T^+_{z1} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \text{ and } T^+_{z2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad (A10)$$

Appendix B: Cheng-Sher Parameters

$$\tilde{Y}^j_k = U^j_k Y^w_j U^j_k = O^j_P^j U^j_k Y^w_j U^j_k P^j_k O^j, \quad (B1)$$

$$\left(\tilde{Y}^j_k\right)_{rt} = \frac{\sqrt{m_{jr} m_{jt}}}{v} \left(\tilde{\chi}^j_k\right)_{rt}, \quad r, t = 1, 2, 3$$
\[
\begin{align*}
(\tilde{\chi}^2_k)_{11} &= 2 \xi_{j1} \sqrt{\frac{m_{12}}{m_{31}}} (1 - \delta_j) \cos (\phi_k^1 - \phi_j) \tilde{a}_k^1 + \frac{(1 - \delta_j) \xi_{j1} \xi_{j2}}{D_{j1}} \tilde{b}_k^1 - 2 \sqrt{\delta_j (1 - \delta_j) \xi_{j1} \xi_{j2}} \tilde{c}_k^1 + \delta_j \xi_{j2} \tilde{d}_k^1, \\
(\tilde{\chi}^2_k)_{12} &= \sqrt{\frac{(1 - \delta_j) \xi_{j1} \xi_{j2}}{D_{j1} D_{j2}}} \left( \sqrt{\frac{m_{12}}{m_{31}}} e^{i(\phi_k^1 - \phi_j)} - \sqrt{\frac{m_{31}}{m_{12}}} e^{-i(\phi_k^1 - \phi_j)} \right) \tilde{a}_k^1 + (1 - \delta_j) \sqrt{\xi_{j1} \xi_{j2}} \tilde{b}_k^1 \\
&- (\xi_{j1} + \xi_{j2}) \sqrt{\frac{m_{12}}{D_{j1} D_{j2}}} \tilde{c}_k^1 \delta_j \sqrt{\xi_{j1} \xi_{j2}} \tilde{d}_k^1, \\
(\tilde{\chi}^2_k)_{13} &= \sqrt{\frac{m_{12}}{m_{31}}} (1 - \delta_j) \xi_{j1} \left( \tilde{m}_{j1} e^{-i(\phi_k^1 - \phi_j)} + e^{i(\phi_k^1 - \phi_j)} \right) \tilde{a}_k^1 + (1 - \delta_j) \sqrt{\delta_j \xi_{j1} \xi_{j2}} \tilde{b}_k^1 \\
&+ (\xi_{j1} - \delta_j) \sqrt{\frac{m_{12}}{D_{j1} D_{j2}}} \tilde{c}_k^1 - \xi_{j2} \sqrt{\delta_j \xi_{j1} \xi_{j2}} \tilde{d}_k^1, \\
(\tilde{\chi}^2_k)_{22} &= -2 \sqrt{\frac{m_{12}}{m_{31}}} (1 - \delta_j) \xi_{j1} \left( \tilde{m}_{j1} \cos (\phi_k^2 - \phi_j) \tilde{a}_k^2 + \frac{(1 - \delta_j) \xi_{j1} \xi_{j2}}{D_{j2}} \tilde{b}_k^2 - 2 \sqrt{\delta_j (1 - \delta_j) \xi_{j1} \xi_{j2}} \tilde{c}_k^2 + \delta_j \xi_{j2} \tilde{d}_k^2, \\
(\tilde{\chi}^2_k)_{23} &= -\sqrt{\frac{m_{12}}{m_{31}}} \xi_{j1} \left( \epsilon (\phi_k^1 - \phi_j) - \tilde{m}_{j2} e^{-i(\phi_k^1 - \phi_j)} \right) \tilde{a}_k^2 + (1 - \delta_j) \frac{\sqrt{\delta_j \xi_{j1} \xi_{j2}} \tilde{b}_k^2}{D_{j2}} \\
&+ (\xi_{j2} - \delta_j) \sqrt{\frac{m_{12}}{D_{j2} D_{j1}}} \tilde{c}_k^2 - \xi_{j1} \sqrt{\delta_j \xi_{j1} \xi_{j2}} \tilde{d}_k^2, \\
(\tilde{\chi}^2_k)_{33} &= 2 \sqrt{(1 - \delta_j) \tilde{m}_{j1} \tilde{m}_{j2}} \delta_j \sqrt{\frac{m_{12}}{D_{j1} D_{j2}}} \cos (\phi_k^3 - \phi_j) \tilde{a}_k^3 + \frac{(1 - \delta_j) \xi_{j1} \xi_{j2}}{D_{j3}} \tilde{b}_k^3 + 2 \sqrt{\delta_j (1 - \delta_j) \xi_{j1} \xi_{j2}} \tilde{c}_k^3 + \frac{\delta_j \xi_{j2} \xi_{j3}}{D_{j3}} \tilde{d}_k^3,
\end{align*}
\]
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