L – Fuzzy Ordered ‘Γ’- Semi rings

T. Srinivasa Rao, B. Srinivasa Kumar, S. Hanumantha Rao, T. Nageswara Rao

Abstract— ‘Γ’- ring concept was introduced by Nobusawa which is the generalization of a ring. In this paper we studied the concept of L – Fuzzy Ordered ‘Γ’- Semi ring along with non-membership and membership functions whose values are taken from a complete lattice and some properties.

AMS Mathematics Subject Classification: 03B52

Key words : ordered ‘Γ’-semi ring, complete lattice, L-fuzzy sets, L-fuzzy ordered ‘Γ’-semi ring

I. INTRODUCTION
The approach of Fuzzy sets was invented by Zadeh, L.A[1] in the year 1965. Fuzzy sets which are using in different fields such as linguistics and clustering are special cases of L-relations. Goguen J.A [2] introduced L-fuzzy subset from a no empty subset to a complete lattice, L which is the generalization of fuzzy subset. The concept of generalization of a ring in ‘Γ’- ring was invented by Nobusawa [3] in 1961. M.K.Rao [4] studied the nation of a ‘Γ’- semi ring , ternary semiring and semiring. Majority of the researchers deduced many important results on ‘Γ’- Semi rings. Several authors studied fuzzy sets on ordered ‘Γ’- Semi rings, ideals, prime ideals and many more. Bhargavi, Y. et.al, [5] studied fuzzy ideals on ‘Γ’- Semi rings and deduced some important properties related to ‘Γ’- Semi rings. Bhargavi, Y. and Eswaralal, T, Nageswara Rao, B., Ramakrishana, N [6-9&15] studied the translates applications of vague sets in cear decision making and vague set in medical diagnosis. Vasavi, C. H et.al[10,11&14] deliberated fuzzy dynamic equations on time scales under second type hukuhara delta derivative. Bind et.al[12] discussed about the approach of Complete left ideals in ternary semi groups. Nayagam, V. L. G et.al [13] studied fuzzy multi-criteria decision-making. Srinivasa Rao, B et.al [16] introduced some properties and generalization on ‘Γ’-semi ring, R, which is named as an ordered ‘Γ’- Semi rings, ideals, prime ideals and many more. Bhargavi, Y. et.al [17] introduced Vague filters on ‘Γ’- Semi rings. Several authors studied fuzzy sets on ordered ‘Γ’- Semi rings, ideals, prime ideals and many more.

II. PRELIMINARIES
In the present part we discussed some basic concepts and definitions related to this article.

2.1 Definition:
For two additive commutative semi groups R and ‘Γ’, R is called ‘Γ’-semi ring if there exists a mapping RXR→R defined by p,q ∈ R and α,β ∈ ‘Γ’ satisfying the below four conditions
1. p α ( q + r ) = p α q + q α r
2. ( p + q ) α r = p α r + q α r
3. p ( α + β ) r = p α r + q β r
4. p α ( q β r ) = ( p α q ) β r , ∀ p,q,r ∈ R, α, β ∈ ‘Γ’.

2.2 Definition:
For a ‘Γ’-semi ring, R, which is named as an ordered ‘Γ’-semi ring if it satisfying the relation ≤ i.e., ≤ is a partial ordering on R satisfies the below three conditions. If p ≤ q and r ≤ s then
(i) p+r ≤ q+s
(ii) p α r ≤ q α s
(iii) r α p ≤ s α q, ∀ p,q,r ∈ R, α ∈ ‘Γ’.

2.3 Definition:
Suppose a non-empty sub set S, of an ordered ‘Γ’-semi ring, then S is called an ordered sub ‘Γ’-semi ring of R if (S,+), is sub semi ring of (R,+) and p α q ∈ S, α ∈ ‘Γ’.

Definition:
A partially ordered set is a complete lattice in which all sub sets have both supremum (join) and infimun (meet).
2.8 Definition:
Let S be a subset of ordered \( \Gamma' \)-semi ring R. The L-fuzzy set is the characteristic function of S taking values in L which is a given by
\[
\delta_S(g) = \begin{cases} 
1_L & \text{if } g \in M \\
0_L & \text{if } g \text{ is not in } M
\end{cases}
\]
Then \( \delta_S \) is a \( 'L' \)-fuzzy characteristic function of Sin L.

III. L-FUZZY ORDERED \( \Gamma' \)-SEMI RINGS & RESULTS

In this section we study the concept of L-fuzzy ordered \( \Gamma' \)-semi ring along with non-membership and membership functions taking values in a complete lattice. Also we deduce that there is a one-to-one correspondence between L-fuzzy ordered \( \Gamma' \)-semi rings to the crisp ordered \( \Gamma' \)-semi rings. In this section R stands for an ordered \( \Gamma' \)-semi ring.

3.1 Definition:
A fuzzy set \( \mu \) is said to be L-fuzzy ordered \( \Gamma' \)-semi ring of R if for all \( g, h \in R \), \( \forall g, h \in \Gamma' \). The mapping \( R \rightarrow L \).

(i) \( \mu(g+h) \geq \mu(g) \wedge \mu(h) \)
(ii) \( \mu(gy)h) \geq \mu(g) \wedge \mu(h) \)
(iii) \( g \leq h \rightarrow \mu(g) \geq \mu(h) \)

3.2 Example:
Suppose that R is the set of whole numbers (W) and let \( \Gamma' = \{0,1\} \). The mapping R, from \( \Gamma' \) into R by \( \rho \neq q \) usual product of \( \rho, q \). \( \forall p, q \in R, \rho \in \Gamma' \). Then R is ordered \( \Gamma' \)-semi ring.

Consider \( L = \{0,1\} \) and define the mapping \( \mu : R \rightarrow L \) by
\[
\mu(x) = \begin{cases} 
0.7 & \text{if } g = 0 \\
0.6 & \text{if } g \text{ is even} \\
0.4 & \text{if } g \text{ is odd}
\end{cases}
\]

By this definition we can observe clearly \( \mu \) is L-fuzzy ordered \( \Gamma' \)-semi ring.

3.3 Theorem:
An L-fuzzy subset \( \mu \) of R is a L-fuzzy ordered \( \Gamma' \)-semi ring if and only if its level set \( \mu_t \), \( t \in L \) is an ordered sub \( \Gamma' \)-semi ring of R.

Proof:
Let us suppose that \( \mu \) is an L-fuzzy ordered \( \Gamma' \)-semi ring of R. Then \( \mu \) is an L-fuzzy ordered \( \Gamma' \)-semi ring of R.

Let \( g, h \in R \) and \( g, h \in \Gamma' \).

\[
\Rightarrow (\mu(g) \geq t, \mu(h) \geq t) \Rightarrow (\mu(g) \wedge \mu(h)) \geq t \\
\Rightarrow (g+h) \geq t \Rightarrow (g, h) \in \mu_t \\
\Rightarrow (g+h) \geq \mu_t \Rightarrow gyh \geq \mu_t.
\]

Conversely, \( \mu_t \) is an ordered sub \( \Gamma' \)-semi ring of R.

Now we show that \( \mu \) is L-fuzzy ordered \( \Gamma' \)-semi ring of R.

Let \( g, h \in R \) and \( g, h \in \Gamma' \).

Suppose \( \mu(g) = p \) and \( \mu(h) = q \).

Put \( t = a \wedge b \).

Then \( \mu(g) \geq t, \mu(h) \geq t \)

\[
\Rightarrow g, h \in \mu_t \\
\Rightarrow g+h \geq \mu_t \Rightarrow gyh \geq \mu_t.
\]

\[
\Rightarrow \mu(g+h) \geq t \Rightarrow \mu(gy)h \geq \mu_t.
\]

\[
\Rightarrow \mu(g+h) \geq \mu(g) \wedge \mu(h) \text{ and } \mu(gy)h \geq \mu(g) \wedge \mu(h).
\]

Let \( g \leq h \).
If possible, suppose that \( \mu(g) < \mu(h) \).
Then there exists \( t_1 \in L \) such that \( \mu(g) < t_1 < \mu(h) \).
Then \( h \in \mu_{t_1} \) and \( x \) does not belong to \( \mu_{t_1} \).

Which is a contradiction?

The contradiction arises our supposition is wrong.

Therefore \( \mu(g) \geq \mu(h) \).

Thus \( \mu \) is L-fuzzy ordered \( \Gamma' \)-semi ring of R.

3.4 Theorem:
Let S is a non-empty subset of an ordered \( \Gamma' \)-semi ring R.

Then \( \delta_S \) is an L-fuzzy ordered \( \Gamma' \)-semi ring of R if and only if S is an ordered sub \( \Gamma' \)-semi ring of R.

Proof:
Suppose that \( \delta_S \) is an L-fuzzy ordered \( \Gamma' \)-semi ring of R.

Let \( g, h \in R \) and \( g \in \Gamma' \).

By the definition \( \delta_S(g+h) \geq \delta_S(g) \wedge \delta_S(h) = 1_L \Rightarrow g \cdot h \in S \).

Also \( \delta_S(gy)h \geq \delta_S(g) \wedge \delta_S(h) = 1_L \Rightarrow g \cdot h \in S \).

Then S is an ordered sub \( \Gamma' \)-semi ring of R.

Conversely suppose that S is an ordered sub \( \Gamma' \)-semi ring of R.

Let \( g, h \in R \) and \( g, h \in \Gamma' \).

If \( g, h \in S \Rightarrow g+h \in S \) and \( g \cdot h \in S \).

\[
\Rightarrow \delta_S(g+h) = 1_L \Rightarrow \delta_S(gy)h = 0_L \Rightarrow \delta_S(g) \geq \delta_S(h).
\]

Suppose that \( g \leq h \) then we have \( \mu(g) = \mu(h) \).

Therefore \( \mu(g) \geq \mu(h) \).

Thus \( \delta_S \) L-fuzzy ordered \( \Gamma' \)-semi ring of R.

3.5 Theorem
Let \( \mu \) and \( \sigma \) be L-fuzzy ordered \( \Gamma' \)-semi ring of R. Then \( \mu \wedge \sigma \) is a L-fuzzy ordered \( \Gamma' \)-semi ring of R.

Proof:
Let \( g, h \in R \) and \( g, h \in \Gamma' \).

\[
(\mu \wedge \sigma)(g+h) = \mu(g+h) \wedge \sigma(g+h) \]

\[
\geq (\mu(g) \wedge \mu(h)) \wedge (\sigma(g) \wedge \sigma(h)) \\
\Rightarrow (\mu(g) \wedge \sigma(g)) \wedge (\mu(h) \wedge \sigma(h)) \\
\Rightarrow (\mu \wedge \sigma)(g) \wedge (\mu \wedge \sigma)(h)
\]

Also we have
\[
(\mu \wedge \sigma)(gy)h = \mu(gy)h \wedge \sigma(gy)h \]

\[
\geq (\mu(g) \wedge \mu(h)) \wedge (\sigma(g) \wedge \sigma(h)) \\
\Rightarrow (\mu(g) \wedge \sigma(g)) \wedge (\mu(h) \wedge \sigma(h)) \\
\Rightarrow (\mu \wedge \sigma)g)(h) \wedge (\mu \wedge \sigma)(h)
\]

Moreover, let suppose that \( g \leq h \).

\[
(\mu \wedge \sigma)(g) \wedge (\sigma(g) \wedge (\mu(h) \wedge \sigma(h)) \\
\Rightarrow (\mu \wedge \sigma)(g) \wedge (\mu \wedge \sigma)(h)
\]

Thus \( \mu \wedge \sigma \) is an L-fuzzy ordered \( \Gamma' \)-semi ring of R.

3.6 Theorem
Two L-fuzzy ordered \( \Gamma' \)-semi rings \( \mu \) and \( \theta \) of Rsuch that card \( \mu < \infty \) and card \( \mu < \infty \) are equal if and only if card \( \mu \)

\[
= \text{Im} \theta \text{ and } F_{\mu} = F_{\theta}.
\]

Proof:
Suppose \( \mu \) and \( \theta \) are equal.
Let $t \in \text{Im} \mu$

$\iff \mu(g) = t.

$\Rightarrow \theta(g) = t.$ (because $\mu = \theta$).

$\Rightarrow t \in \text{Im} \theta$.

Therefore $\text{Im} \mu = \text{Im} \theta$.

Now

$F_\mu = \{ \mu(t) / t \in \text{Im} \mu \} = \{ \theta(t) / t \in \text{Im} \theta \} = F_\theta$.

Therefore $F_\mu = F_\theta$.

Conversely suppose that $\text{Im} \mu = \text{Im} \theta$ and $F_\mu = F_\theta$.

Let $t \in \text{Im} \mu$

$\iff t \in \text{Im} \theta$

Then $\mu(g) = t$ and $\theta(g) = t$.

i.e., $\mu(g) = t = \theta(g), \forall g \in R$.

$\Rightarrow \mu = \theta$.

**IV. CONCLUSIONS**

In the present work we studied the concept of L – Fuzzy Ordered ‘Γ’- Semi ring with membership and non-membership functions whose values are taken from a complete lattice and some important properties. Also we discussed L-fuzzy ordered ‘Γ’-semi ring with suitable example.

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