Quantum speed meter based on dissipative coupling

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Abstract. We consider dissipative coupling Fabry-Perot cavity, i.e. its input mirror transmittance depends on position of probe mass. We show that dissipative coupling provide possibility to realize quantum speed meter by natural way, without additional setup for subtraction of position $x(t)$ and delayed position $x(t-\tau)$. Quantum speed meter is a quantum nondemolition (QND) variable, suitable for a sensitive detection of a classical force. Dissipative coupling without the dispersive one [13] provides this kind of subtraction automatically, enabling a simple way of measurement of speed of a free test mass, which is a possibility to realize quantum speed meter by natural way, without additional setup for speed meter based on dissipative coupling.

The sensitivity of the mechanical position measurement in an opto-mechanical system usually is limited due to quantum back action by so called standard quantum limit (SQL) [1, 2]. The SQL was studied in various configurations ranging from macroscopic kilometer-sized gravitational wave detectors [3] to microcavities [4, 5]. Detection of a classical force acting on a mechanical degree of freedom of an opto-mechanical system is an example of such a measurement. SQL of force measurement is not a fundamentally unavoidable limit. It can be overcome with variational measurement [3, 6, 7], opto-mechanical speed measurement [8, 9], and measurements using opto-mechanical rigidity [10, 11].

In this report we show that usage of dissipative coupling for detection of small signal force acting on a free mechanical test mass (figure 1) provides a direct possibility to realize speed meter and to beat the SQL [12].

The idea of speed meter enabling the force detection is that output optical field contains information about difference $\sim [x(t)-x(t-\tau)] \simeq \tau \dot{x}(t)$ (where $x$ is the position, $\tau$ is a delay time). In frequency domain it corresponds to $\sim x(\Omega) \left( 1 - e^{-i\Omega \tau} \right) \simeq -i\Omega \tau x(\Omega)$. In proposed earlier speed meters such subtraction is realized by interaction of a mechanical degree of freedom with two coupled optical modes — it increases complexity of experimental realization [8, 9]. We show that the dissipative coupling without the dispersive one [13] provides this kind of subtraction automatically, enabling a simple way of measurement of speed of a free test mass, which is a quantum nondemolition (QND) variable, suitable for a sensitive detection of a classical force.

Our analysis gives formula for test mass position $\dot{x}$ and output field $a_{out}$ in frequency domain:

$$\dot{x}_\Omega = -\frac{\hbar \eta A \sqrt{\kappa_0}}{m \Omega (\kappa_0 - 2i\Omega)} (\hat{b}_+ - \hat{b}_-^\dagger) - \frac{F_\Omega}{m \Omega^2}, \quad a_{out+} = \frac{\kappa_0 + 2i\Omega}{\kappa_0 - 2i\Omega} \hat{b}_+ + \eta A \sqrt{\kappa_0} \frac{-i\Omega x_\Omega}{\kappa_0 - 2i\Omega},$$

where $\dot{x}_\Omega$, $a_{out+}$, $F_\Omega$, and $\hat{b}_\pm = \hat{b}(\omega_0 \pm \Omega)$ are Fourier amplitudes of corresponding operators, $\omega_0$ being the frequency of input optical field.
is resonant frequency of cavity. We see that the output field provides information on speed of the probe mass \(-i\Omega x\) if \(\kappa_0 \gg \Omega\), but not the displacement.

In order to find sensitivity of detection of the classical force \(F_\Omega\) acting on the test mass we find phase and amplitude quadratures \(q_a\), \(q_p\) of output field through similar quadratures \(d_p\), \(d_a\) of input one:

\[
q_p = \frac{\kappa_0 + 2\Omega}{\kappa_0 - 2\Omega} d_p, \quad q_a = \frac{\kappa_0 + 2\Omega}{\kappa_0 - 2\Omega} \left\{ d_a - Q \cdot d_p + \sqrt{2Q} \cdot f_s \right\},
\]

where

\[
Q \equiv \frac{P\kappa_0^2}{\kappa_0^2 + 4\Omega^2}, \quad P \equiv \frac{2|A|^2\eta^2}{m\kappa_0}, \quad f_s \equiv e^{-i\beta} \frac{F_s(\Omega)}{F_{SQL}}, \quad F_{SQL} \equiv \sqrt{2m\Omega^2}.
\]

Here \(f_s\) is the signal force normalized by Standard Quantum Limit (SQL). We see that \(q_a\) contains shot noise term \((\sim d_a)\), back action term \((\sim Qd_p)\) and signal term \((\sim \sqrt{2Q}f_s)\). We assume that output signal is registered by balanced homodyne detector, i.e. we are able to measure arbitrary quadrature amplitudes \(q_\theta\) of the output light: \(q_\theta = q_a \cos \theta + q_p \sin \theta\). One can find minimal \(f_s\) choosing homodyne angle \(\theta\) by optimal way:

\[
f \equiv \frac{e^{-i\beta}}{\sqrt{2}} \left\{ \frac{d_a}{\sqrt{Q}} + \left( -\sqrt{Q} + \frac{\tan \theta}{\sqrt{Q}} \right) d_p \right\}, \quad \tan \theta_{opt} = \frac{P\kappa_0^2}{\kappa_0^2 + 4\Omega^2}.
\]

Single-sided spectral density \(S_f(\Omega)\) of noise \(f\) may be expressed through spectral densities \(S_a\) and \(S_p\) of corresponding quadratures \(d_a\) and \(d_p\). We assume that input light is in coherent state (c.f.[3]), i.e. \(S_a = S_d = 1\). The plots of \(S_f\) presented in figure 2, show that the sensitivity of the

**Figure 1.** Speed meter based on dissipative coupling. Resonantly pumped mode of an optical Fabry-Perot cavity is dissipatively coupled with a mechanical degree of freedom represented by a free mass \(m\) a force of interest, \(F_s\), acts upon. The coupling changes the power transmission coefficient of the cavity front mirror \(T(x)\). In linear approximation this change can be presented as \(T(x) = T_0(1 + \eta x)\), where \(x\) is the mirror displacement, \(\eta\) is a coupling coefficient.
Figure 2. (color online) Normalized spectral density of the noise $f$ for the optimal procedure of the force measurement using the speed meter technique evaluated for the optimal homodyne angle at frequencies $2\Omega/\kappa_0 = 0.5$ for different power parameters $P$. The horizontal line describes SQL.

Figure 3. Michelson-Sagnac Interferometer as a input generalized mirror (GM) of a Fabry-Perot cavity. Perfectly reflecting mirror $M$ represents a free test mass. Distance $L$ between the generalized mirror and the end mirror (EM) is much larger than the size of interferometer.

force detection better than SQL can be achieved in a relatively large frequency band. Realization of pure dissipative coupling was suggested [13] on base of Michelson-Sagnac interferometer (see figure 3).
Figure 4. Illustration of the difference between types of ponderomotive squeezing achieved in the cases of dispersive and dissipative coupling. The blue circles on the both diagrams represent distribution of quantum fluctuations of the input wave prepared in the coherent state, red ellipses describe quantum squeezing of the fluctuations of the output waves.

Let's compare speed meter with displacement meter based on dispersive coupling (end mirror of resonantly pumped cavity is free mass) we rewrite output amplitudes from [3]:

\[ q_{\text{disp}}^a = e^{-2i\alpha} d_a, \quad q_{\text{disp}}^p = e^{-2i\alpha} \left\{ d_p - \kappa d_a + \sqrt{2\kappa} f_{s_{\text{disp}}} \right\}, \tag{4} \]

with

\[ \kappa \equiv \frac{8\kappa_0\omega_0^2 A^2}{mL^2\Omega^2(\kappa_0^2 + 4\Omega^2)}, \quad e^{-2i\alpha} \equiv \frac{\kappa_0 + 2i\Omega}{\kappa_0 - 2i\Omega}. \]

We see that amplitude quadrature amplitude \( q_a \) in (2) is replaced with phase quadrature amplitude \( q_p \) in (4) as well as \( q_p \Rightarrow q_{\text{disp}}^a \). Ponderomotive squeezing takes place for the both cases of coupling at large enough pump power (\(|Q|, |K| \gg 1\)). However, squeezed quadratures are different (figure 4).

Summing up, purely dissipative coupling naturally provides a realization of a quantum speed meter.

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