Abstract—Gaussian filter is a filter with impulse response of Gaussian function. These filters are useful in image processing of 2D signals, as it removes unnecessary noise. Also, they could be helpful for data transmission (e.g., GMSK modulation). In practice, the Gaussian filters could be approximately designed by several methods. One of these methods is to construct Gaussian-like filter with the help of memristors and RLC circuits. Therefore, the objective of this project is to find and design Gaussian-like filter with the help of memristors and RLC circuits. By several methods. One of these methods are to construct Gaussian filters could be approximately designed in practice, the Gaussian filters could be approximately designed.
remembers the current that flowed before through it. To define memristance, equations 4-6 could be written.

\[ d\Phi = Mdq \]  
\[ \phi(t) = \int_{-\infty}^{t} v(\tau)d\tau \]  
\[ q(t) = \int_{-\infty}^{t} i(\tau)d\tau \]

where \( \phi(t) \) is flux linkage, \( q(t) \) is the amount of electric charge flowed through the memristor, \( v(t) \) is voltage and \( i(t) \) is current. Hence, the memristance of the device could be defined as shown in equations 7-8.

\[ M(q) = \frac{d\phi}{dq} \]  
\[ M(q(t)) = \frac{d\phi}{dt} = R_M(q) \]

Finally, it could be concluded for memristance (equation 9): \[ v(t) = M(q(t)) i(t) \]

Hence, I-V plot of memristor could be sketched (refer to figure 1).

In addition, scientists recommend to implement memristors to the design of filters, since they allow to create electronically adjustable filters [9].

III. SIMULATIONS AND MATHEMATICAL ANALYSIS

A. First Approach

The first step was to find appropriate electrical circuit for a filter with transfer function of Gaussian. Initially, lumped-element ladder-network approximation of the complete Gaussian transmission line (refer to figure 2) was considered. This circuit (figure 2) was constructed on Spice (figure 3) and simulated. The values for passive elements were selected approximately as follows: \( R_1 = R_5 = 0.5 \) k\( \Omega \), \( R_2 = R_3 = R_4 = 1 \) k\( \Omega \), \( R_6 = R_7 = R_8 = R_9 = 2 \) k\( \Omega \), \( C_1 = C_5 = 0.6 \) \( \mu \)F, \( C_2 = C_3 = C_4 = 1.2 \) \( \mu \)F, \( L_1 = L_2 = L_3 = L_4 = 0.1 \) H. The result of the simulation is shown in figure 4.

\[ C_1 = C_5 = 0.6 \ \mu F, \ C_2 = C_3 = C_4 = 1.2 \ \mu F, \ L_1 = L_2 = L_3 = L_4 = 0.1 \ H \]

The sketched solid curve in figure 4 is a magnitude response of the designed filter. The behaviour of this curve is close to the behaviour of Gaussian. However, in this electrical circuit inductors were used [8]. It is recommended [7] to avoid the use of inductors for the design of electrical filters due to size, width, lack of isolation from environment, energy losses and magnetic field issues. Therefore, another design for the filter was considered.

As it was mentioned before, the circuit shown in figure 3 was considered as the approximation of Gaussian-like filter.
Here, input signal is $V_{in}$ and output signal - $V_{out}$. The transfer function of the filter could be considered, since it provides an algebraic representation of LTI filter in the frequency domain. The formula for transfer function is given as $H(z) = Y(z)/X(z)$, where $Y(z)$ is output, $X(z)$ is input and $H(z)$ stands for transfer function, which is the $z$-transform of the impulse response $h(n)$. On spice this result could be represented as the ratio of the output voltage to the input voltage, i.e., $V_{out}/V_{in}$ and the result is shown on figure 4.

As can be seen above, the transfer function of the system considered have the properties akin to the properties of an ideal Gaussian filter. As it was expected, this filter operates as low-pass filter, since it attenuates high frequencies. The next step, it to find approximate equation for the obtained transfer function. This could be done via Excel, by inserting all data points and plotting. For simple approximation 20 data points in the of the transfer function is going to be considered. The collected data was reported to table 1 below. According to these data the mathematical equation (equation 10) has been obtained.

$$y = -9.087 \ln(x) + 38.758 \quad (10)$$

In this equation $x$ stands for the frequency and $y$ stands for the magnitude in decibels. However, this approximation is not quite accurate. As the number of points considered increase, the accuracy of the obtained equation also increases.

### B. Second Approach

The electrical circuit designed before was changed to the design, known as an 8-order Sallen-Key filter, which is considered as the approximation to Gaussian filter. Then, all resistors were replaced with memristors and the resultant circuit is shown in figure 5.

### IV. RESULTS

The magnitude-frequency response of this designed filter is shown in figure 6.

The cutoff frequency is $f_c = 4.78\, Hz$ and phase shift is $-135\, deg$. This result satisfies to expected theoretical approaches. Now the transfer function of the filter could be determined according to the equation 3. Hence, the transfer function of this filter is:

$$H(s) = \frac{0.2s^3 + 1.2s^2 - 5s + 40}{2.9s^4 + 12s^3 + 27.4s^2 + 34s + 19} \quad (11)$$

### V. CONCLUSION

In this paper Gaussian analog filter based on memristor was approximated. Firstly, Gaussian-like filter was designed from lumped-element ladder-network. However, due to the presence of inductors, this design was changed to Sallen-Key filter design. Finally, an 8-order memristor-based Gaussian filter was designed and its parameters were determined. The advantages of this design are that this circuits does not contain inductors and could be constructed simply with RLC elements. Furthermore, the use of memristors significantly increases the sensitivity of this filter. Nevertheless, not all aspects of memristors were considered. For instance, since the resistance of a memristor changes according to input voltage, this filter should...
be simulated at different inputs. Hence, it is recommended to focus further studies on memristor’s resistance to input voltage relation and its effect to cutoff frequency, because ability to adjust the cutoff frequency and gain are crucially important.

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