Strong thermal fluctuations in cuprate superconductors in magnetic field above $T_c$

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Recent measurements of fluctuation diamagnetism in high temperature superconductors show distinct features above and below $T_c$, which cannot be explained by simple gaussian fluctuation theory. Self consistent calculation of magnetization in layered high temperature superconductors, based on the Ginzburg-Landau-Lawrence-Doniach model and including all Landau levels is presented. The results agree well with the experimental data in wide region around $T_c$, including both the vortex liquid below $T_c$ and the normal state above $T_c$. The gaussian fluctuation theory significantly overestimates the diamagnetism for strong fluctuations. It is demonstrated that the intersection point of magnetization curves appears in the region where the lowest Landau level contribution dominates.

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Introduction. One of the numerous qualitative differences between high $T_c$ superconductors (HTSC) and low $T_c$ superconductors is dramatic enhancement of thermal fluctuation effects. The thermal fluctuations are much stronger in HTSC not just due to higher critical temperatures, much shorter coherence length and high anisotropy play a major role in the enhancement too. Since thermal fluctuations are strong the effect of superconducting correlations (pairing) can extend into the normal state well above the critical temperature. The normal state properties of the underdoped cuprates exhibit a number of anomalies collectively referred to as the "pseudogap" physics [1] and their physical origin is still poorly understood. It is natural therefore to attempt to associate some of these phenomena with the superconducting thermal fluctuations or "preformed" Cooper pairs [2].

The interest in fluctuations was invigorated after the Nernst effect was observed [3] all the way up to the pseudogap crossover temperature $T^*$ in underdoped $La_{2−x}Sr_xCuO_4$ (LSCO). Assuming that Nernst effect is primarily due to thermal fluctuations, the whole pseudogap region would be associated with preformed Cooper pairs and become a precursor of the superconducting state. The finding motivated additional experiments on Nernst effect in various HTSC [4], as well as renewed study of thermal fluctuations in the temperature region between $T_c$ and $T^*$ by other probes: electric [5] and thermal conductivity [6] and diamagnetism [7]. The main goal was to try to quantify the superconducting fluctuation effects, so they can be either directly linked or separated from the pseudogap physics. This requires a reliable quantitative theory of influence of thermal fluctuations on transport (Nernst effect, thermal and electric conductivity) and thermodynamic (magnetization, specific heat) physical quantities. Since there is no sufficiently simple or/widely accepted microscopic theory of HTSC, one has to rely on a more phenomenological Ginzburg - Landau (GL) theory [8] that, although not sensitive to microscopic details, is accurate and simple enough to describe the fluctuations above $T_c$. While the transport experiments like Nernst effect have some hotly debated experimental [9] and theoretical [10] issues to be addressed, the clearest data come from recent thermodynamical measurements of magnetization [11] in LSCO, $Bi_2Sr_2CaCu_2O_{8+δ}$ (BSCCO) and $YBa_2Cu_3O_7$ (YBCO) [4].

The purpose of this note is to provide a convincing theoretical description of the magnetization data. Our conclusion is that the GL description of the layered materials LSCO, BSCCO and YBCO by the Lawrence - Doniach model within the self consistent fluctuation theory (SCFT, sometimes referred to as Hartree approximation) fits well the fluctuation effects in major families of HTSC materials in wide range of fields and temperatures and demonstrates that the fluctuation effects extend to well above $T_c$, far below $T^*$. This means that there is no evidence that the pseudogap physics influences the diamagnetism and that superconductivity probably plays no role at $T^*$.

Strong diamagnetism of a type II superconductor takes
a form of network of Abrikosov flux lines (vortices) created by magnetic field. Vortices strongly interact with each other creating highly correlated configurations. A generic magnetic phase diagram of HTSC \[2\], Fig.1, contains four phases: two inhomogeneous phases, unpinned crystal and pinned Bragg glass and two homogeneous phases, unpinned vortex liquid and pinned vortex glass. In HTCS thermal fluctuations are strong enough to melt the lattices \[3\] into the vortex liquid over very large portion of the phase diagram. This portion covers the fields and temperatures of the above experiments, both above and below \(T_c\) and for fields up to \(40T\). The glass line separates pinned vortex matter (zero resistivity) from the unpinned one (nonzero resistivity due to flux flow).

Fluctuation diamagnetism in type II superconductors has been studied theoretically\[8\] within both the microscopic theory (starting from the pioneer work of Aslamazov and Larkin) and the GL approach. In all of these calculations (with an exception of the strong field limit that allows the lowest Landau level approximation, see \[14\]) the fluctuations were assumed to be small enough, so they can be taken into account perturbatively. Within the GL approach this is referred to as the gaussian fluctuation theory (GFT)\[8, 13\]. The GFT applied to the recent HTSC magnetization data was criticized \[16\] to fit just a single curve (magnetic field) rather than a significant portion of the magnetic phase diagram near \(T_c\). To determine theoretically fluctuation diamagnetism for strong thermal fluctuations, one therefore must go beyond this simple approximation neglecting the effect of the quartic term in the GL free energy. The effect of the quartic term is taken into account within SCFT, widely used in physics of phase transitions at zero magnetic field and was adapted to transport property in magnetic field \[17, 18\]. Since disorder is not considered, our results are limited to the vortex liquid phase of the magnetic phase diagram of Fig.1, where vortices are depinned.

The GL model of layered superconductor. Layered superconductor is sufficiently accurately described on the mesoscopic scale by the Lawrence-Doniach free energy (incorporating microscopic thermal fluctuation via dependence of parameters on temperature \(T\), but not containing thermal fluctuations of the order parameter on the mesoscopic scale):

\[
F[\psi] = s' \sum_l \int \left[ \frac{\hbar^2}{2m_0} |D\psi_l|^2 + \frac{\hbar^2}{2m_a d_2} |\psi_l - \psi_{l+1}|^2 + \alpha (T - T_\Lambda) |\psi_l|^2 + \frac{\beta}{2} |\psi_l|^4 \right].
\]

Here \(\psi_l (x, y)\) is the order parameter in the \(l\)th layer, \(D \equiv \nabla + \frac{e}{\hbar c} A\), is the covariant derivative \((e^* = 2|e|)\) and \(A\) is the vector potential of magnetic field oriented along the crystallographic \(c\) axis. The (effective) layer thickness is \(s'\) and the distance between the layers - \(d'\). Note that the temperature \(T_\Lambda\), that will be called "mean field" or "bare" transition temperature, is larger than the real transition temperature \(T_c\).

The "bare" coherence length \(\xi = \hbar/\sqrt{2m_0 c T_\Lambda}\) will be used as the unit of length and the upper critical field \(H_{c2} \equiv \hbar c/e^* \xi^2\) as the magnetic field unit. They depend on coarse graining scale (cutoff scale \(\Lambda\)) at which the mesoscopic model is derived (in principle) from The dimensional order parameter is \(\phi = \sqrt{\beta}/2\alpha T_\Lambda \psi\), so that the GL Boltzmann factor in scaled units takes a form,

\[
f = \frac{F}{T} = \frac{1}{2\omega T_\Lambda} \sum_l \int \left[ |D\phi_l|^2 + d^{-2} |\phi_l - \phi_{l+1}|^2 - (1 - t_\Lambda)|\phi_l|^2 + |\phi_l|^4 \right].
\]

Here \(t_\Lambda = T/T_\Lambda, b = B/H_{c2}\) are dimensionless temperature and induction. It is more convenient to use the fluctuation strength parameter \(\omega_\Lambda = \sqrt{2G_\Lambda / \pi s}\), instead of the more customary ("bare") Ginzburg number \(G_\Lambda = 2(e^*/hc)^3 \kappa^4 T_\Lambda^2 H_{c2}/\xi^2\). Since the renormalization by strong thermal fluctuations is central in this work, bare quantities carry index \(\Lambda\), although the results used for fitting experiments will utilize renormalized parameters. The anisotropy \(\gamma = \sqrt{m_c/m_a}, s = \gamma \xi\) and \(d = d' \xi\). In strongly type II superconductors the Ginzburg parameter \(\kappa = \lambda/\xi >> 1\) and magnetic field is nearly homogeneous\[19\], so we choose the Landau gauge \(A = (-by, 0)\) in \(D = \nabla + iA\).

Fluctuation diamagnetism calculated within SCFT. The idea the method \[14\] is as follows\[19\]. Let us divide the GL Boltzmann factor \(f[\phi]\) into an optimized quadratic ("large") part,

\[
K = \frac{1}{2\omega T_\Lambda} \sum_l \int \left[ |D\phi_l|^2 + d^{-2} |\phi_l - \phi_{l+1}|^2 + (2\varepsilon - b)|\phi_l|^2 \right],
\]

and a small perturbation

\[
W = \frac{1}{2\omega T_\Lambda} \sum_l \int \left[(t_\Lambda + b - 1 - 2\varepsilon)|\phi_l|^2 + |\phi_l|^4 \right].
\]

Here, the variational parameter \(\varepsilon\) (that depends on temperature, magnetic field and material parameters) has a physical meaning of the excitation gap in the vortex liquid phase. It is found from minimization of the variational free energy including the fluctuations on the mesoscopic scale. The only nontrivial technical difficulty is the summation over Landau levels in the presence of UV cutoff \(\Lambda\). It is shown\[19\] that to absorb all UV divergences one has to sum over Landau levels till the “maximal” one \(N_{\text{max}} = \Lambda/b - 1\). This results in the vortex liquid gap equation

\[
\varepsilon = \frac{t_\Lambda + b - 1}{2} + \frac{\omega T_\Lambda d}{2m^2} \int_{k=0}^{2\pi/d} \{ \psi (g + \Lambda/b) - \psi (g) \},
\]

\[
g \equiv (1 - \cos (kd)) / (d^2 b) + \varepsilon/b,
\]
FIG. 2. Magnetization data of ref. [7] (dots) and their self consistent approximation fits (solid lines). Three major families of high $T_c$ superconductors are represented: (a) underdoped LSCO, (b) optimally doped BSCCO, (c) optimally doped YBCO. The curve closest to $T_c$ for each sample were used to determine the fitting parameters given in Table I. Each set of curves uses just three fitting parameters.

**TABLE I.** Fitting parameters for LSCO, BSCCO, and YBCO.

| Material | $T_c$ (Kelvin) | $d'$ (Angstrom) | $H_{c2}$ (Tesla) | $T_\Lambda$ (Kelvin) | $\gamma$ | $\Lambda$ | $\kappa$ | $G_i$ |
|----------|----------------|-----------------|-----------------|---------------------|--------|--------|--------|-------|
| LSCO     | 24             | 6.58            | 31              | 33                  | 29     | 0.30   | 66.7   | 0.033 |
| BSCCO    | 88             | 19.6            | 115             | 99                  | 19     | 0.25   | 55.6   | 0.025 |
| YBCO     | 92             | 11.68           | 220             | 100                 | 4.1    | 0.22   | 78.7   | 0.026 |

where $\psi$ is the digamma function. The integration is over the Fourier harmonics $k$ in the $c$ direction.

The SCFT is widely used in GL model without magnetic field, $b = 0$, under the name of "mean field" and in this case simplifies to

$$\varepsilon = \frac{t_\Lambda - 1}{2} + \omega t_\Lambda (h (\Lambda + \varepsilon) - h (\varepsilon));$$

$$h (u) = \ln \left(1 + u d^2 + \sqrt{2 u d^2 + (u d^2)^2}\right)/\pi.$$  \hfill (6)

In this case $\varepsilon$ has a meaning of the "mass" of the field $\phi$ describing the fluctuations in the normal phase. It vanishes at the "renormalized" transition temperature $T_c$ leading to its relation to $T_\Lambda$

$$T_\Lambda^{-1} = T_c^{-1} (1 - 2 \omega h (\Lambda)).$$ \hfill (7)

Here the renormalized coupling $\omega = \sqrt{2 G_i \pi}/s$, this time expressed via renormalized Ginzburg number $G_i = 2 (e^*/c h)^3 \kappa_4 T_2^2 \gamma_2 /H_{c2}$, is used. Expressing $T_\Lambda$ via $T_c$ in Eq. (5), the gap equation becomes,

$$\varepsilon = \frac{\omega s t_\Lambda}{2 \pi^2} \int_{k=0}^{2\pi/d} \left[ \psi (g + \Lambda/b) - \psi (g) \right]$$

$$- \omega h (\Lambda) + \frac{t + b - 1}{2},$$ \hfill (8)

with $t = T/T_c$. Physical quantities are then calculated using numerical solution of this algebraic equation. For $b, \varepsilon \ll \Lambda$ it is cutoff independent and simplifies:

$$\varepsilon = \frac{(t + b - 1)}{2} - \frac{\omega s t_\Lambda}{2 \pi^2} \int_{k=0}^{2\pi/d} \left[ \psi (g) + \ln 2 \right].$$ \hfill (9)

Magnetization is [19],

$$M = \frac{\omega s t_\Lambda H_{c2}}{8 \pi^3 \kappa^2} \int_{k=0}^{2\pi/d} \left[ (g + \Lambda/b - 1/2) \psi (g + \Lambda/b) - (g - 1/2) \psi (g) + \ln (\Gamma (g)/\Gamma (g + \Lambda/b)) - \Lambda/b \right],$$ \hfill (10)

while for $b, \varepsilon \ll \Lambda$ it simplifies to

$$M = \frac{\omega s t_\Lambda H_{c2}}{8 \pi^3 \kappa^2} \int_{k=0}^{2\pi/d} \left[ \ln \left(\frac{\Gamma (g)}{\sqrt{2 \pi}} + g - \left(g - \frac{1}{2}\right) \psi (g)\right) \right].$$ \hfill (11)

In certain portions of the magnetic phase diagrams the strong inequalities $b, \varepsilon \ll \Lambda$ are not obeyed, while SCFT is still valid, so we have used the formula Eq. (10), with weak (logarithmic) cutoff dependence instead of the cutoff independent renormalized formula.

Comparing with experiments and GFT. Recent accurate magnetization data [7] on magnetization of three major families of HTSC materials, including under-doped $La_{2-x}Sr_xCuO_4$ for $x = 0.99$, optimally doped $Bi_2Sr_2CaCu_2O_8 + \delta$, and optimally doped $YBa_2Cu_3O_7$, are fitted in Fig. 2a, 2b, 2c respectively. Measured magnetization curves of LSCO and YBCO in the $0 - 14T$
field range and BSCCO at $0-40T$ show distinct features above and below $T_c$, thus allowing meaningful fitting. The conditions $b, \varepsilon << \Lambda$ are obeyed provided temperature does not deviate too far from $T_c$ and magnetic field is small compared to $H_{c2}$. Several temperatures within 10% of $T_c$ were used to determine three fitting parameters, $H_{c2}$, anisotropy $\gamma$, and $\kappa^2/s$, using simplified formulas Eqs. (9)(11). The interlayer distances $d'$ were taken from [20]. Near $T_c$, the correlation length is large, therefore we take $s = d$, as the maximum value of $s$. The results for each material are given in Table I.

For the rest of the data (higher temperature and higher magnetic field) the theoretical curves shown in Fig. 2 were logarithmically dependent on cutoff and therefore the full formulas, Eqs. (5)(10), were utilized. The two additional parameters, namely mean field critical temperature $T_A$ and $\Lambda$ are constrained via Eq. (7) (with experimentally measured $T_c$, also listed in Table I). The values of $T_A$ and $\Lambda$ in units of $\hbar^*H_{c2}/(m_wc)$ are given in Table I.

To demonstrate the importance of nonperturbative effects the SCFT magnetization, Eq.(10) is compared with GFT within the 2D layered superconductors model [15] in Fig. 3. One observes that The SCFT magnitude is much smaller than the GFT one. One of the reasons is that the vortex liquid gap $\varepsilon$ is larger than the reduced temperature (perturbative gap) $(t_A + b - 1)/2$.

The data of ref. [7] in the region of smaller fields exhibit the so called "intersection point" of the magnetization curves plotted as function of temperature. Our magnetization curves (underdoped LSCO is shown in Fig. 4 as an example) demonstrate the intersection point in this region for all three materials. The intersection points were measured in many high $T_c$ cuprate [21] and explained within the "lowest Landau level" approximation [22] valid for $\varepsilon << b$. It turns out that an additional requirement for the intersection point is $\varepsilon d^2 >> 1$. Our results demonstrate that beyond this approximation the intersection point disappears.

**Conclusions.** We have investigated the fluctuation diamagnetism of HTSC using a self consistent nonperturbative method beyond gaussian fluctuations term within Lawrence - Doniach GL model. The comparison with recent accurate experiments near $T_c$ demonstrate that the effect of quartic terms should to be included due to strong fluctuations. The theory describes well wide class of materials from relatively low anisotropy optimally doped $YBCO$ to highly anisotropic underdoped $LSCO$ and optimally doped $BSCCO$ at temperatures both below and above $T_c$. No input from the microscopic "pseudogap" physics is needed to describe the magnetization data. Dynamical effects like Nernst effect, electrical and thermal conductivity can be in principle approached within the similar SCFT generalized to a time dependent variants of the GL model.

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