Open-circuit sensitivity model based on empirical parameters for a capacitive-type MEMS acoustic sensor

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Abstract. An empirical-based open-circuit sensitivity model for a capacitive-type MEMS acoustic sensor is presented. To intuitively evaluate the characteristic of the open-circuit sensitivity, the empirical-based model is proposed and analysed by using a lumped spring-mass model and a pad test sample without a parallel plate capacitor for the parasitic capacitance. The model is composed of three different parameter groups: empirical, theoretical, and mixed data. The empirical residual stress from the measured pull-in voltage of 16.7 V and the measured surface topology of the diaphragm were extracted as +13 MPa, resulting in the effective spring constant of 110.9 N/m. The parasitic capacitance for two probing pads including the substrate part was 0.25 pF. Furthermore, to verify the proposed model, the modelled open-circuit sensitivity was compared with the measured value. The MEMS acoustic sensor had an open-circuit sensitivity of -43.0 dBV/Pa at 1 kHz with a bias of 10 V, while the modelled open-circuit sensitivity was -42.9 dBV/Pa, which showed good agreement in the range from 100 Hz to 18 kHz. This validates the empirical-based open-circuit sensitivity model for designing capacitive-type MEMS acoustic sensors.

1. Introduction
MEMS microphones based on capacitive-type acoustic sensors compatible with CMOS processes have been rapidly replacing electret condenser microphones (ECMs) in the market of communication systems, especially smart phones, due to their competitive cost and technology. Recently, their application range has been expanded to a variety of monolithic integrated circuits, such as digital MEMS microphone modules and beam-forming modules, because they have monolithic integrated capability with ROIC. As a result, various design issues and fabrication methods related to capacitive-type MEMS acoustic sensors have been reported based on surface and bulk micromachining [1]-[8]. On the other hand, to investigate the characteristics of the MEMS acoustic sensors, an analogous equivalent circuit model has been applied in sensor modules [2], [6], [9]-[12]. Generally, the behaviour of acoustic-electro transducers has been modelled with lumped parameters. However, in spite of theoretically complicated analysis or finite-element method (FEM) simulation, it may be difficult to model some parameters, such as the effective residual spring constant in a diaphragm and parasitic capacitance for a MEMS sensor chip, owing to their inherently non-separability from an intrinsic part. In this paper, an empirical-based model of open-circuit sensitivity at a low frequency for a capacitive-type MEMS acoustic sensor is proposed and investigated. Each parameter was modelled
through an extracted method based on empirical measurement, a theoretical method, or a mixed method of the two. In particular, both the residual stress and the parasitic capacitance are critical for determining the open-circuit sensitivity. To verify the proposed model, the modelled open-circuit sensitivity was compared with the measured sensitivity using an evaluation board for a packaged MEMS acoustic sensor.

2. Empirical based open-circuit sensitivity model

2.1. Lumped spring-mass model

Figure 1 shows a schematic diagram of a conventional lumped spring-mass model for a capacitive-type acoustic sensor with a circular diaphragm. In this model, the movable plate is assumed to act like a piston without any difference between the centre point and an edge of the moving plate, when a uniform force is applied to the diaphragm.

![Schematic diagram of a conventional lumped spring-mass model](image)

Figure 1. Schematic diaphragm of a conventional lumped spring-mass model for a capacitive-type acoustic sensor with a circular diaphragm

Generally, from the large deflection theory, linearized restoring force \( F_{res} \) from a uniform force is expressed by

\[
F_{res} \approx \left( \frac{64 \pi D}{a^2} + 4 \pi \sigma_0 h \right) \cdot z,
\]

where \( D \) is the flexural rigidity, \( a \) is the diaphragm radius, \( \sigma_0 \) is the residual stress, \( h \) is the diaphragm thickness, and \( z \) is the axis direction in a cylindrical coordinate. In addition, the electrical force \( F_{ele} \) between two electrodes is described as

\[
F_{ele} = -\frac{V_b^2}{2} \frac{\varepsilon_0 \pi a^2}{(g_0 - z)^2},
\]

where \( g_0 \) is the height of the air gap, \( V_b \) is the bias voltage, and the permittivity of vacuum. Thus, at an equilibrium condition equal to
the pull-in voltage can be determined by
\[ F_{res} = F_{ele} \text{ at } z = g_0 / 3 \] (3)

From equation (4), it is obvious that the effective residual stress \((\sigma_{eff})\) can be extracted from both the pull-in voltage and the air gap between two electrodes. If its value is more than +5 MPa, the effective residual stress plays an essential role on the spring constant of a diaphragm, resulting in dominant impact on open-circuit sensitivity.

\[ V_p = \sqrt{\frac{8}{27} \left( \frac{64\pi D}{a^2} + 4\pi \sigma_{eff} h \right) \frac{g_0^3}{\varepsilon_0 \pi a^2}} \] (4)

2.2. Each parameter extraction
As shown in figure 2, parameters of open-circuit sensitivity at a low frequency for a capacitive-type acoustic sensor are divided into three parts: empirical, theoretical, and mixed data group. Among the empirical data, the residual stress \((\sigma_0)\) of the diaphragm must be extracted from the measured values due to its strong dependence on the specific structure of the sensor itself. In addition, it is not a simple task to extract the parasitic capacitance \((C_{par})\) from the measured total capacitance owing to the non-linear substrate effect. Thus, on the basis of the empirical data, \(\sigma_0\) and \(C_{par}\) must be determined to reflect some practical impact on open-circuit sensitivity at a low frequency. The modelling process based on empirical data is described in figure 3.

![Figure 2. Three parameter groups of open-circuit sensitivity at a low frequency for a capacitive-type MEMS acoustic sensor](image)
2.3. MEMS acoustic sensor
For a capacitive-type MEMS acoustic sensor with a Al/PECVD-Si$_3$N$_4$/Al diaphragm, each step is carried out with a CMOS compatible process only, except a DRIE process related to back chamber etching, having 7 photo masks. A 2.0 µm low-stress LPCVD Si$_3$N$_4$ was implemented as a rigid back plate with a back plate electrode of 0.5 µm Al. As a sacrificial layer, 2.6 µm of polyimide was adapted for wide material selectivity. Also, the membrane consisted of a 100 nm Al electrode, a 400 nm PECVD Si$_3$N$_4$ membrane, and a 100 nm Al passivation layer, which formed a multilayer sandwich stack. The average mean stress of the diaphragm was measured to be +20 MPa by a curvature method. After the back chamber was etched by DRIE, the sacrificial layer was easily released using O$_2$ gas. Figure 4 shows the fabrication process for the MEMS acoustic sensor.

**Figure 3.** Modelling process of open-circuit sensitivity at a low frequency for a capacitive-type MEMS acoustic sensor
Figure 4. SEM images for the fabricated MEMS acoustic sensor with an Al/PECVD-Si$_3$N$_4$/Al diaphragm

2.4. Residual stress and Parasitic capacitance

To extract the residual stress, the pull-in volatage and surface profile of the diaphragm were measured as shown in figure 5. From the measured pull-in voltage of 16.7 V and the height of air gap of 2.6 um, the effective residual stress was extracted by using equation (4). Here, $\sigma_{\text{eff}}$ was determined to be +13 MPa, showing a difference of +7 MPa from the average mean stress. Thus, it is noted that $\sigma_{\text{eff}}$ must be evaluated by using the practical value rather than the mean stress by the curvature method.

Figure 5. Surface topology of the diaphragm (a) and pull-in characteristic (b) for the MEMS acoustic sensor
On the other hand, the relation of capacitance as a function of bias voltage can be applied in the extraction process of $C_{par}$ with an additional test sample: only a two-pad chip without an active parallel-plate capacitor. The extracted $C_{par}$ from the test sample was 0.25 pF. Also, the measured total capacitance ($C_{mea}$) at a zero bias and the cable parasitic capacitance ($C_{cab}$) were 1.43 pF and 0.05 pF, respectively. Thus, the intrinsic capacitance ($C_{in}$) can be extracted as 1.13 pF by using

$$C_{in} = C_{mea} - C_{par} - C_{cab}.$$  

(5)

In addition to $C_{par}$, the model process of separating $C_{in}$ from $C_{mea}$ is critical because both $C_{in}$ and $C_{par}$ are applied to model the open-circuit sensitivity.

2.5. Open-circuit sensitivity

When a bias voltage ($V_b$) is forced in the sensor, the open-circuit sensitivity at a low frequency can be expressed as

$$S_o = \frac{V_b A_{eff}}{g_b k_{eff} (C_b + C_{par})} C_b,$$  

(6)

where $g_b$ is the height of the air gap at the bias voltage ($V_b$), $C_b$ is the intrinsic capacitance at the bias voltage, $A_{eff}$ is the effective capacitor area excluding the etching hole area, and $k_{eff}$ is the effective spring constant containing the effective residual stress. Here, $A_{eff}$ and $k_{eff}$ were $2.33 \times 10^{-7}$ m$^2$ and 110.9 N/m, respectively. As a result, when $V_b$ was 10 V, $S_o$ was modelled as -42.9 dBV/Pa at 1 kHz, where $g_b$, $C_b$, $C_{par}$ were 2.45 µm, 1.2 pF, and 0.25 pF. The design and modelled parameters are shown in Table 1.

| Parameters                  | Values     |
|------------------------------|------------|
| Diaphragm radius ($r$)       | 325 µm     |
| Diaphragm thickness ($h$)    | 0.6 µm     |
| Flexural rigidity ($D$)      | $4.78 \times 10^{-9}$ N-m |
| Effective residual stress ($\sigma_{eff}$) | +13 MPa |
| Pull-in voltage ($V_p$)      | 16.7 V     |
| Air-gap height ($g_o$)       | 2.6 µm     |
| Effective spring constant ($k_{eff}$) | 110.9 N/m |
| Effective area ($A_{eff}$)   | $2.33 \times 10^{-7}$ m$^2$ |

3. Results and Discussion

To analyze the frequency characteristic of an acoustic-electro transducer, an equivalent circuit model reflecting the influence of all of the acoustic, mechanical, and electrical signal domains [10] was used for open-circuit sensitivity ($S_o$). Figure 6 shows the structure-based model and its equivalent circuit for the dependency of open-circuit sensitivity on the frequency. In the steady-state condition, $S_o$ is defined
as the transfer function of the output voltage as a function of the input pressure. Thus, it can be expressed [10] as

\[ S_0 (\omega) = \frac{V_{\text{out}}(\omega)}{P(\omega)} \]

\[ S_0 (\omega) = \frac{v_{\text{out}}}{P} = \frac{1}{A_{\text{eff}} K_1(\omega) + \left( \frac{1}{A_{\text{eff}}} \right) K_2(\omega)} \cdot \frac{\Gamma}{j\omega(C_0 + C_p)} \]

, (7)

where \( K_1(\omega) \) and \( K_2(\omega) \) are given by

\[ K_1(\omega) = R_r(\omega) + R_h + R_g + j\omega M_r + \frac{1}{j\omega C_{bc}} \]

, (8)

\[ K_2(\omega) = j\omega M_d + \frac{1}{j\omega C_d} + \frac{\Gamma^2}{j\omega(C_0 + C_p)} \]

, (9)

As \( S_0 \) is determined by the complicated interconnection among elements in three signal domains, the proposed equivalent circuit is suitable to accurately design and evaluate microphone performance.
Based on the equivalent circuit model shown in figure 6, each parameter was calculated or extracted by either theoretical calculation or empirical measurement. The modelled $S_o$ was -42.9 dBV/Pa at 1 kHz at the bias voltage of 10 V, and the first resonance frequency was 67 kHz. Furthermore, to compare the modelled values with the measured values, the MEMS acoustic sensor was packaged with a read-out IC (ROIC). Figure 7 shows an integrated MEMS microphone module, where the PCB had an area of 3.4 mm $\times$ 2.8 mm, and the metal lid had the height of 0.6 mm. The measurement was performed in an 8.0 m $\times$ 8.0 m $\times$ 7.0 m anechoic chamber with a commercial analysis set (B&K) in the sweep range of 100 Hz to 18 kHz. Accordingly, the total measured sensitivity ($S_{\text{mea}}$) of the packaged acoustic sensor was -42.0 dBV/Pa at 1 kHz (0 dB = 1 V/Pa), where the ROIC had 0.7 pF input capacitance, 10.0 V output bias, and 6 dB gain. Subsequently, $S_o$ was extracted from $S_{\text{mea}}$ using

$$
S_{\text{mea}} = \frac{S_o}{C_b + C_{\text{par}} + C_{\text{in}}G},
$$

(10)

where $G$ is the gain of the ROIC, and $C_{\text{in}}$ is the input capacitance of the ROIC.

![Figure 7. Images of the packaged MEMS acoustic sensor with ROIC](image)

**Figure 7.** Images of the packaged MEMS acoustic sensor with ROIC

![Figure 8. Modelled and measured $S_o$ as a function of frequency for the MEMS acoustic sensor](image)

**Figure 8.** Modelled and measured $S_o$ as a function of frequency for the MEMS acoustic sensor
From (10), $S_o$ was extracted to be -43.0 dBV/Pa at 1 kHz with the bias point of 10.0 V, while the modelled open-circuit sensitivity was -42.9 dBV/Pa, as shown in figure 8. This validates the structure-based equivalent circuit modelling and the open-circuit sensitivity at a low frequency presented in the previous section. Table II summarizes the measured and modelled data of the proposed MEMS acoustic sensor.

Table II. Measured and modelled data of the proposed MEMS acoustic sensor

| Parameters                                      | Values        |
|------------------------------------------------|---------------|
| Bias voltage ($V_b$)                           | 10.0 V        |
| Aig-gab height at $V_b$ ($g_b$)                | 2.45 µm       |
| Intrinsic capacitance at $V_b$ ($C_b$)         | 1.2 pF        |
| Parasitic capacitance ($C_p$)                  | 0.25 pF       |
| Modelled open-circuit sensitivity at $V_b$ ($S_{o,mod}$) | -42.9 dBV/Pa |
| Measured open-circuit sensitivity at $V_b$ ($S_{o,med}$) | -43.0 dBV/Pa |
| ROIC input capacitance ($C_{in}$)              | 0.7 pF        |
| ROIC gain ($G$)                                | 6 dB          |
| Measured total sensitivity at $V_b$ ($S_{med}$) | -42.0 dBV/Pa  |

4. Conclusions
An empirical-based model of open-circuit sensitivity at a low frequency for a capacitive-type MEMS acoustic sensor was proposed. To model the open-circuit sensitivity, both the residual stress and the parasitic capacitance were extracted from empirical data because it is difficult to analytically obtain both of them due to their structure dependence. The residual stress was modelled from both the pull-in voltage and surface topology of the diaphragm, whereas the parasitic capacitance was determined from an additional test pattern. To evaluate the proposed empirical model, the modelled open-circuit sensitivity was compared with the measured sensitivity by using a packaged MEMS acoustic sensor. The modelled open-circuit sensitivity was in good agreement with the measured value in the range from DC to higher frequency, demonstrating the validity of the modelling.

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