Nucleon-Nucleon Interaction and Isospin Violation

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Abstract. The application of the chiral effective theory to processes with two or more nucleons is discussed. We gain a qualitative understanding of the gross features of nuclear physics and quantitative, testable postdictions and predictions involving photons and pions.

1 Introduction

My topic is not yet part of mainstream chiral perturbation theory (χPT): the low-energy effective field theory (EFT) for systems with more than one nucleon. This subject is fascinating because it not only involves the symmetries of QCD, but also demands an understanding of a non-trivial interplay between non-perturbative and perturbative physics.

I will try to emphasize here the aspects of the problem that are not standard in other applications of χPT. In particular, my main goal is to make sense of nuclear physics, rather than search for tests of chiral dynamics. (Although, as you will see, we are also opening a window on a whole new set of tests of χPT.) By “to make sense” I mean to formulate the problem in terms of an EFT, so that after the relevant degrees of freedom and symmetries are determined we can devise an expansion in powers of $Q/M$, where $Q$ represents the typical momentum of the processes we are interested in and $M$ stands for a characteristic mass scale of the underlying theory. If this is accomplished, then we will have made nuclear physics rooted in QCD (consistent with chiral symmetry), systematic (amenable to a perturbation treatment) and applicable to all processes where $Q \ll M_{QCD} \sim 1$ GeV (a theory, rather than a model).

The questions I would like to address are: Why are nuclei so loosely bound compared to $M_{QCD}$? Why are processes involving nucleons and external probes (pions and photons) generically dominated by two-body interactions? Why is isospin a very good symmetry at low energies? In the first part of the talk I concentrate on shallow bound states in EFTs, a problem that can be discussed without explicit pions. In the second part I briefly review some of the results that have been achieved in few-nucleon problems where pions do play a very important role. In the third part isospin violation is discussed.

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2 Very low energies

In processes with typical momenta $Q$ much smaller than the pion mass $m_\pi$, the only relevant degree of freedom is the nucleon of mass $m_N$, the important symmetries are parity, time-reversal and “small” Lorentz boosts, and the appropriate expansion parameter is $Q/M \sim \partial/(m_\pi, m_N, \ldots)$. (Electromagnetic processes can also be considered by adding the photon, $U(1)_{em}$ gauge invariance, and $\alpha_{em}$ to this list.) Much effort has been spent during the last year in trying to understand issues related to regularization and fine-tuning of this “pionless” theory (Kaplan et al. (1996), Kaplan (1997), Cohen (1997), Phillips and Cohen (1997), Scaldeferri et al. (1997), Luke and Manohar (1997), Lepage (1997), Richardson et al. (1997), Phillips et al. (1997), Beane et al. (1997c), van Kolck (1997b), Bedaque and van Kolck (1997)).

The most general Lagrangian with such input consists of an infinite number of contact terms, which are quadratic, quartic, ..., in the nucleon fields $N$ with increasing number of derivatives:

$$\mathcal{L} = N^\dagger \left( i\partial_0 + \frac{\nabla^2}{2m_N} + \ldots \right) N - \frac{C_0}{2} N^\dagger N N^\dagger N$$

$$+ \frac{C_2}{8} \left[ N^\dagger \nabla N \cdot N^\dagger \nabla N - N^\dagger N N^\dagger N^2 N - N^\dagger N \nabla^2 (N^\dagger N) + \text{h.c.} \right]$$

$$+ \ldots, \quad (1)$$

where $C_{2n}$ are parameters of mass dimension $-2(n+1)$. Here, to avoid cluttering the discussion with unnecessary detail, I omitted spin and isospin combinations, the $Q^4/m_N^3$ relativistic correction, and a two-derivative four-nucleon interaction that contribute to $P$-waves, and lumped terms that only contribute to higher orders in “...”. Nucleons are non-relativistic and the corresponding field theory has nucleon number conservation.

Let me first consider the two-nucleon system in the center-of-mass frame, with total energy denoted by $k^2/m_N$. The two-nucleon amplitude $T_{2N}$ is simply a sum of bubble graphs, whose vertices are the four-nucleon contact terms that appear in the Lagrangian (1). It consists of two different expansions, a loop expansion and an expansion in the number of insertions of derivatives at the vertices or nucleon lines. Compare for example the one-loop graph made out of two $C_0$ vertices to the tree-level graph from $C_0$. Their ratio is

$$m_N C_0 \int \frac{d^3 l}{(2\pi)^3} \frac{1}{l^2 - k^2 - i\epsilon} = m_N C_0 \frac{\theta A + ik + O(k^2/A)}{4\pi} \quad (2)$$

where I introduced a regulator $A$. Both the number $\theta$ and the function $O(k^2/A)$ depend on the regularization scheme chosen. This dependence is mitigated by renormalization. The linear divergence can be absorbed in $C_0$ itself, $1/C_0^{(R)} = 1/C_0 + m_N \theta A/4\pi + \ldots$, while to correctly account for the $k^2$ term, at least one insertion of $C_2$ is necessary. The loop expansion is therefore an expansion
in \( m_N Q C_0^{(R)} / 4\pi \), while the derivative expansion is in \( C_2^{(R)} Q^2 / C_0^{(R)} \) and similar combinations of the higher order coefficients.

In a natural theory, there is no fine-tuning and one scale \( M \). In such case we expect all the parameters to scale with \( M \). If \( C_2^{(R)} = 4\pi \gamma_2 n / m_N M^{2n+1} \) with \( \gamma \)'s all dimensionless parameters of \( O(1) \), then the loop expansion is in \( Q/M \) and the derivative expansion in \( (Q/M)^2 \). \( T_{NN} \) is perturbative and equivalent to an effective range expansion for \( k \ll 1/a \sim 1/\sqrt{\alpha \sigma_0} \sim \ldots \sim M \), where the S-wave scattering length is \( a = m_N C_0^{(R)} / 4\pi \), the S-wave effective range is \( r_0 = 16\pi C_2^{(R)} / m_N (C_0^{(R)})^2 \), and so on. This scaling of effective range parameters is indeed what one gets in simple potential models, like a square well of range \( R \sim 1/M \) and depth \( V_0 \sim M \). Similarly, one can show that in a wave of angular momentum \( l \), the relevant mass scale in the expansion is \( (2l + 1)M \). Since it is perturbative, the amplitude in this EFT can only describe scattering, not bound states.

In nuclear physics, we are interested in shallow bound states. This means that the underlying theory has at least one parameter \( \alpha \) which is fine-tuned to be close to a critical value \( \alpha_c \) at which there is a bound state at zero energy. In this case there exist two distinct scales: \( M \) and \( \mathcal{R} = (\alpha - \alpha_c)M \ll M \). Assume that \( C_2^{(R)} = 4\pi \gamma_2 n / m_N \mathcal{R}^n \) with \( \gamma \)'s again all dimensionless parameters of \( O(1) \). (This can be accomplished with natural-size bare coefficients if they are fine-tuned against a regulator \( \Lambda \sim M \); for example, \( C_0 = -(4\pi/\theta m_N \Lambda)(1 + \mathcal{R}/\gamma_0\theta\Lambda + \ldots) \). In this case the loop expansion is in \( Q/\mathcal{R} \), while the derivative expansion is in \( Q^2/\mathcal{R} M \). We are justified in resumming the larger \( Q/\mathcal{R} \) terms, which can be done analytically. This produces a leading order amplitude \(-C_0^{(R)}/(1 + i m_N C_0^{(R)} k/4\pi) \). With the loop expansion so resummed, the derivative expansion becomes an expansion in \( C_2^{(R)} Q^2 / C_0^{(R)} (1 + i m_N C_0^{(R)} Q/4\pi) \) and similar combinations. That is, an expansion in \( Q^2/M (\mathcal{R} + iQ) \), which fails at momenta \( \sim 1/M \). For \( Q \ll \mathcal{R} \) the theory is still perturbative, but a (virtual or real) bound state appears at \( k = i(4\pi/m_N C_0^{(R)})(1 + \ldots) \sim (\pm)i\mathcal{R} \). The bound state lies in the region of applicability of the EFT, which can thus be used in a non-trivial way.

For momenta \( Q \sim \mathcal{R} \), the dominant sub-leading terms are terms in \( C_2^{(R)} \) which start at \( O(\mathcal{R}/M) \). Interactions that generate higher effective range parameters (such as the shape parameter) only contribute at \( O((\mathcal{R}/M)^3) \) or higher. It simplifies calculations considerably to also resum the \( C_2^{(R)} \) interaction, which is easily done for again they are in a geometric series. Such a resummation generates a four-fermion interaction of the type \( C_0^{(R)}/(1 - 2(C_2^{(R)}/C_0^{(R)})k^2) \). It was noted by Kaplan (1997) that this resembles the exchange of an s-channel particle, provided \( i \) the sign \( \sigma \) of its kinetic term and its coupling to nucleons \( g^{(R)} \) satisfy \( \sigma (g^{(R)})^2 = -(C_0^{(R)})^2 / 2m_N C_2^{(R)} \sim \pm 2\pi M / m_N^2 \); and \( ii \) its mass is \( \Delta^{(R)} = C_0^{(R)}/2m_N C_2^{(R)} \sim \pm MR / 2m_N \). An alternative EFT can then be constructed by writing the most general Lagrangian consistent with the same symmetries as above, but now including an additional light di-baryon field for
each (real or virtual) bound state. Because the symmetries are the same as for
the EFT with nucleons only, the two EFTs give the same result at any given order in
the derivative expansion. The EFT with a di-baryon field has bare poles (ghosts
with negative kinetic energy for $r_0 \geq 0$) which change character upon dressing;
one becomes the (real or virtual) bound state at $k_0 = \pm i/\aleph$, the other being a
ghost at $k_0 = i/M$, outside the range of the EFT. Since in $NN$ scattering there
are two $S$-wave bound states near threshold, one introduces two di-baryon fields,
an $S=1, I=0$ $D$ and an $S=0, I=1$ $T$, and in leading order there are two
masses and two couplings which are fitted to the triplet and (average) singlet
scattering lengths and effective ranges.

Further resummations can be done similarly, and they can likewise be reproduced
by other alternative EFTs with more di-baryon fields that can mix. I will
not pursue this here. It is clear that to any given order the EFT for short-range
interactions is equivalent to an effective range expansion to the same order:

$$T_{NN}(k) = -\left( \frac{1}{C^{(R)}_0} - 2 \frac{C^{(R)}_2}{(C^{(R)}_0)^2} k^2 + \frac{im_N k}{4\pi} + O(Q^4/RM) \right)^{-1},$$

which gives $a \sim \aleph^{-1}$, while other effective range parameters still scale with
$M$. Once more, this is in agreement with potential model results, for example
a square well with $\sqrt{m_N V_0 R}$ close to $\pi/2$. From the measured $NN$ scattering
lengths, we see that fine-tuning is considerable in the $^1S_0$ channel ($^1R \sim 10$
MeV), but less severe in the $^3S_1$ channel ($^3R \sim 40$ MeV).

The same result (3) can be obtained more directly by first summing insertions
of $C_2$ to all orders and then expanding in $Q^2$. If the higher orders are kept,
regularization scheme dependence will remain through the $Q^4$ and higher terms.
However, this dependence will be no greater than the dependence on neglected
higher order interactions if $\Lambda \sim M$. Formally, this is equivalent to solving a
Schrödinger equation with a bare potential consisting, schematically, of a sum
$C_0 \delta_A(r) + C_2 \delta''_A(r) + ...$, where $\delta_A$ is a regulated delta function. Effectively, we
replace the “true”, possibly complicated potential of range $\sim 1/M$ by a multipole
expansion with moments $C_{2n}$. It is not difficult to show (van Kolck (1997b))
that the effect of renormalization is to turn this bare potential into a generalized
pseudo-potential ($C^{(R)}_0 + 2C^{(R)}_2 k^2 + ... \delta(r) \frac{d^n}{dr^n} r$, or equivalently, turn the
problem into a free one with boundary conditions at the origin which are analytic
in the energy. The first, energy-independent term, parametrized by $C^{(R)}_0$, was
considered by Bethe and Peierls (1935).

I now turn to an example of how the application of these ideas to other nuclear
systems can bring non-trivial model-independent predictions. (Sometimes called
low-energy theorems.)

Consider the three-nucleon system. At momenta $Q \sim \aleph$, only scattering
states are accessible, so I confine myself to nucleon-deuteron scattering. For
illustration, take the zero energy case, where $S$-waves dominate. There are two
channels, a quartet of total spin $J = 3/2$ and a doublet of $J = 1/2$. The leading
interactions involve only two-body interactions conveniently written via the dibaryon fields: assuming naive dimensional analysis, three-nucleon forces start only at $O((N/M)^4)$. The $N\bar{d}$ scattering amplitude is the sum of these two-body interactions to all orders; in the quartet only $D$ contributes while in the doublet $T$ also appears. This results in a particularly simple set of Faddeev equations: there is one integral equation in one variable in the quartet and a pair of coupled integral equations in the doublet channel. The quartet $N\bar{d}$ scattering length can then be obtained in a model independent way with no free parameters. Solving the integral equation, Bedaque and van Kolck (1997) found $4a = 6.33\text{ fm}$ with an uncertainty from higher orders of $\sim \pm 0.10\text{ fm}$. This result is in very good agreement with the experimental value of $4a = 6.35\pm0.02\text{ fm}$ (Dilg et al. (1971)). Work on the energy dependence of the quartet amplitude and on the doublet channel is in progress.

In the same vein, we could consider other $Q \ll m_\pi$ processes. Unfortunately, this leaves out most of the interesting aspects of nuclear physics. Let me use $2m_N B/A$ as a measure of a typical momentum $Q$ of a nucleon in a nucleus with $A$ nucleons and binding energy $B$. (Other quantities such as charge radii give similar estimates.) $Q/m_\pi$ is then about 0.3 for $^2\text{H}$, 0.5 for $^3\text{H}$, 0.8 for $^4\text{He}$, ..., and 1.2 for symmetric nuclear matter in equilibrium. The same argument that justified the use of a pionless theory for the deuteron now suggests that understanding the binding of typical nuclei ($^4\text{He}$ and heavier) requires explicit inclusion of pions, but not of heavier mesons such as the rho. Enter $\chi$PT.

### 3 Chiral Effective Theory

The EFT for typical nuclear momenta $Q \ll M_{\text{QCD}}$ can be formulated along the same lines as the pionless case. The extra degrees of freedom—besides non-relativistic nucleons and photons—are obviously pions and also non-relativistic delta isobars, since both the pion mass $m_\pi$ and the delta-nucleon mass difference $m_\Delta - m_N \sim 2m_\pi$ are of the order of the momenta $Q$ we want to consider. The new and very important symmetry is the approximate chiral symmetry $SU(2)_L \times SU(2)_R \sim SO(4)$, assumed to be broken spontaneously down to its diagonal $SU(2)_{L+R} \sim SO(3)$. The expansion parameter is expected to be $Q/M_{\text{QCD}} \sim (\partial, m_\pi, m_\Delta - m_N, 4\pi f_\pi, \ldots) -$besides $\alpha_{em}$.

Were it not for the approximate chiral symmetry, this would be a very difficult EFT to handle, because it would lack any small parameters. If the quark masses were zero ("chiral limit"), the QCD Lagrangian would be invariant under independent rotations of the quarks’ left- and right-handed components, while the hadronic spectrum suggests only one isospin rotation is realized. Goldstone’s theorem assures us there is in the spectrum a scalar boson, the pion $\pi$, which corresponds to excitations in the coset space $SO(4)/SO(3) \sim S^3$. We call the radius of this “circle” $f_\pi \simeq 92\text{ MeV}$. Since an infinitesimal chiral transformation is of the form $\vec{\pi} \rightarrow \vec{\pi} + f_\pi \epsilon \mathbf{e} + \ldots$, the Lagrangian will be $SO(4)$ symmetric if it depends on $\vec{\pi}$ only through covariant derivatives, that is, derivatives on the circle,
which are non-linear: \( \overrightarrow{D}_\mu = (\partial_\mu \overrightarrow{\pi}/2f_\pi)(1 + O(\pi^2)) \), \( \overrightarrow{D}_\mu N = (\partial_\mu + i\overrightarrow{\pi} \times \overrightarrow{D}_\mu/f_\pi)N \), etc. Quark masses generate two terms in the QCD Lagrangian. One term, \( \overline{m}\overline{q}q \) with \( \overline{m} = (m_u + m_d)/2 \), is the fourth component of an \( SO(4) \) vector and therefore breaks \( SO(4) \) explicitly down to \( SO(3) \) of isospin. The effective Lagrangian will acquire then an infinite set of terms that do depend on \( \overrightarrow{\pi} \) in an isospin invariant way and transform as 4-components of (tensor products of) \( SO(4) \) vectors, and have coefficients proportional to (powers of) the small parameter \( \eta = \overline{m}/\Lambda_{QCD} \sim m_\pi^2/\Lambda_{QCD}^2 \). (Unless the last proportionality factor is zero as argued by Stern (1997), a possibility I will not entertain here.) The other quark mass term, \( \epsilon \overline{m}\overline{q}q \tau_3 \) with \( \epsilon = (m_u - m_d)/(m_u + m_d) \simeq 1/3 \), is the third component of another \( SO(4) \) vector and further breaks \( SO(3) \) down to \( U(1) \times U(1) \). This, plus the effects of electromagnetism, will be discussed in Section 4.

The most general Lagrangian consists of adding pions and deltas to Eq. (1), according to the rules above. (See, for example, Gasser (1997), Bernard (1997), and Kambor (1997).) The important point is that these new interactions are weak at low energies because of their derivative nature and/or because of pion mass and delta-nucleon mass difference factors. We can naturally group the interactions in sets \( \mathcal{L}_\Delta \),

\[
\mathcal{L} = \sum_{\Delta=0}^{\infty} \mathcal{L}_\Delta, \tag{4}
\]

of common index \( \Delta \equiv d + q + n + \frac{f}{2} - 2 \), where \( d, q, \) and \( n \) are, respectively, the number of derivatives, powers of \( m_\Delta - m_N \), and powers of \( m_\pi \), and \( f \) is the number of fermion fields. For non-electromagnetic interactions, we find that \( \Delta \geq 0 \) only because of the constraint of chiral symmetry. (Electromagnetic contributions can have negative \( \Delta \), but this is compensated by enough powers of \( \alpha_{em} \).)

Consider now an arbitrary irreducible contribution to a process involving \( A \) nucleons and any number of pions and photons, all with momenta of order \( Q \). It can be represented by a Feynman diagram with \( A \) continuous nucleon lines, \( L \) loops, \( C \) separately connected pieces, and \( V_\Delta \) vertices from \( \mathcal{L}_\Delta \), whose connected pieces cannot be all split by cutting only initial or final lines. (\( C = 1 \) for \( A = 0, 1; C = 1, \ldots, A - 1 \) for \( A \geq 2 \). The reason to consider irreducible diagrams and \( C > 1 \) will be mentioned shortly.) It is easy to show (Weinberg (1990), Weinberg (1991)) that this contribution is typically of \( O(Q/M_{QCD})^{\nu} \), where

\[
\nu = 4 - A + 2(L - C) + \sum_\Delta V_\Delta \Delta \tag{5}
\]

Since \( L \) is bounded from below (0) and \( C \) from above (\( C_{max} \)), the chiral symmetry constraint \( \Delta \geq 0 \) implies that \( \nu \geq \nu_{min} = 4 - A - 2C_{max} \) for strong interactions. Leading contributions come from tree diagrams built out of \( \mathcal{L}_0 \) and coincide with current algebra. Perturbation theory in \( Q/M_{QCD} \) can be carried out by considering contributions from ever increasing \( \nu \).

In the case of strong mesonic processes, \( \nu = 2 + 2L + \sum_\Delta V_\Delta \Delta \geq 2 \) with \( \Delta = d + n - 2 \) increasing in steps of two. For processes where one nucleon is
present, \( \nu = 1 + 2L + \sum_{\Delta} V_{\Delta} \Delta \geq 1 \), where \( \Delta = d + q + n + f - 2 \) (with \( f = 0, 2 \)) increases in steps of one. In both cases, all there is is perturbation theory. Thanks to this power counting, a low-energy nucleon can in a very definite sense be pictured as a static, point-like object (up to corrections in powers of \( Q/m_N \)), surrounded by: (i) an inner cloud which is dense but of short range \( \sim 1/m_{\rho} \); (ii) an outer cloud of long range \( \sim 1/m_{\pi} \) but sparse, so that we can expand in its relative strength \( Q/4\pi f_\pi \).

As we have seen in Section 2, a non-trivial new element enters the theory when we consider systems of more than one nucleon (Weinberg (1990), Weinberg (1991)). Because nucleons are heavy, contributions from intermediate states that differ from the initial state only in the energy of nucleons are enhanced by infrared quasi-divergences: as it can be seen in Eq. (2), small energy denominators of \( O(Q^2/m_N) \) generate contributions \( O(m_N/Q) \) larger than what would be expected from Eq. (5). The latter is still correct for the class of sub-diagrams—called irreducible—that do not contain intermediate states with small energy denominators. For an \( A \)-nucleon system these are \( A \)-nucleon irreducible diagrams, the sum of which we call the potential \( V \). When we consider external probes all with \( Q \sim m_\pi \), the sum of irreducible diagrams forms the kernel \( K \) to which all external particles are attached. A generic diagram contributing to a full amplitude will consist of irreducible diagrams sewed together by states of small energy denominators. These irreducible diagrams might have more than one connected piece, hence the introduction of \( C \) in Eq. (5). One way to deal with the infrared enhancement is to introduce a regulator \( \Lambda \sim M_{QCD} \) and sum irreducible diagrams to all orders in the amplitude, thus creating the possibility of the existence of shallow bound states (nuclei). For an \( A \)-nucleon system, this is equivalent to solving the Schrödinger equation with the bare potential \( V \). The amplitude for a process with external probes is then \( T \sim \langle \psi' | K | \psi \rangle \) where \( | \psi \rangle \) (\( | \psi' \rangle \)) is the wavefunction of the initial (final) nuclear state calculated with the potential \( V \). All bare parameters depend on \( A \) but after these are fit to data, scheme dependence is no greater than higher orders terms in the Lagrangian. Because our \( Q/M_{QCD} \) expansion is still valid for the potential and the kernel, the picture of a nucleon as a mostly static object surrounded by an inner and an outer cloud leads to remarkable nuclear physics properties that we are used to, but would otherwise remain unexplained from the viewpoint of QCD.

If we assemble a few non-relativistic nucleons together, each nucleon will not be able to distinguish details of the others’ inner clouds. The region of the potential associated with distances of \( O(1/m_{\rho}) \) can be expanded in delta-functions and their derivatives as Bethe and Peierls did. The outer cloud of range \( O(1/m_{\pi}) \) yields non-analytic contributions to the potential, but being sparse, it mostly produces the exchange of one pion, with progressively smaller two-, three-, ...- pion exchange contributions.

For the two-nucleon system, \( \nu = 2L + \sum_{\Delta} V_{\Delta} \Delta \), with \( \Delta \) as in the one-nucleon case (but \( f = 0, 2, 4 \)). A calculation of all the contributions up to \( \nu = 3 \) was carried out by Ordóñez and van Kolck (1992), Ordóñez et al. (1994), and Ordóñez et al. (1996). In leading order, \( \nu = 0 \), the potential is simply static one-
pion exchange and momentum-independent contact terms (Weinberg (1990)).\(\nu = 1\) corrections vanish due to parity and time-reversal invariance. \(\nu = 2\) corrections include several two-pion exchange diagrams (including virtual delta isobar contributions), recoil in one-pion exchange, and several contact terms that are quadratic in momenta. At \(\nu = 3\) a few more two-pion exchange diagrams contribute. As in the pionless case, regularization and renormalization are necessary. It is not straightforward to implement dimensional regularization in this non-perturbative context, so we used an overall gaussian cut-off, and performed calculations with the cut-off parameter \(\Lambda\) taking values 500, 780 and 1000 MeV. Cutoff independence means that for each cut-off value a set of bare parameters can be found that fits low-energy data. As mentioned before, in an \(l\) partial wave the onset of non-perturbative effects is at \((2l + 1)m_\pi\); in high partial waves contact interactions of only high order contribute, so these waves are mostly determined by perturbative pion exchange. A sample of the results for the lower, more interesting partial waves is compared to the Nijmegen phase shift analysis (Stoks et al. (1993)) in Fig. 1, and deuteron quantities are presented in Table 1. Ordóñez et al. (1996) have more details and experimental references.

| Deuteron quantities | \(\Lambda\) (MeV) | \(500\) | \(780\) | \(1000\) | Experiment |
|--------------------|-----------------|--------|--------|--------|------------|
| \(B\) (MeV)        |                 | 2.15   | 2.24   | 2.18   | 2.224579(9)|
| \(\mu_d\) (\(\mu_N\)) |                 | 0.863  | 0.863  | 0.866  | 0.857406(1)|
| \(Q_E\) (fm\(^2\)) |                 | 0.246  | 0.249  | 0.237  | 0.2859(3)  |
| \(\eta\)           |                 | 0.0229 | 0.0244 | 0.0230 | 0.0271(4)  |
| \(P_D\) (%)         |                 | 2.98   | 2.86   | 2.40   |            |

This is, in Aron Bernstein’s terms, a Cadillac calculation, but in part because it is to be eventually replaced by more economic and faster machines... The fair agreement of this first calculation and data up to laboratory energies of 100 MeV or so suggests that this may become an alternative to other, more model-dependent approaches to the two-nucleon problem. Further examination of aspects of two-pion exchange in this context can be found in Celenza et al. (1992), Friar and Coon (1994), da Rocha and Robilotta (1994), da Rocha and Robilotta (1995), da Rocha and Robilotta (1997), Ballot et al. (1997), Savage (1997), and Kaiser et al. (1997).

Obviously, in this EFT theory, too, we can simultaneously get some insight into other aspects of nuclear forces. Let us look for the new forces that appear in systems with more than two nucleons. The dominant potential, at \(\nu = 6 − 3A = \nu_{\min}\), is the two-nucleon potential of lowest order that appeared in the two-nucleon case. We can easily verify that a three-body potential will arise at
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Fig. 1. $^1S_0$, $^3S_1$, and $^3D_1$ NN phase shifts and $\epsilon_1$ mixing angle in degrees as functions of the laboratory energy in MeV: chiral expansion up to $\nu = 3$ for cut-offs of 500 (dotted), 780 (dashed), and 1000 MeV (solid line); and Nijmegen phase shift analysis (squares).

$\nu = \nu_{\text{min}} + 2$, a four-body potential at $\nu = \nu_{\text{min}} + 4$, and so on. It is (approximate) chiral symmetry therefore that implies that $n$-nucleon forces $V_{nN}$ are expected to obey a hierarchy of the type $\langle V_{(n+1)N} \rangle / \langle V_{nN} \rangle \sim O((Q/M_{\text{QCD}})^2)$, with $\langle V_{nN} \rangle$ denoting the contribution per $n$-plet. If we estimate $\langle V_{2N} \rangle \sim \frac{g_{\pi\Delta}}{16\pi f_{\pi}^2} m_{\pi}^3 \simeq 10$ MeV, we can guess $\langle V_{3N} \rangle \sim 0.5$ MeV, $\langle V_{4N} \rangle \sim 0.02$ MeV, and so on. This is in accord with detailed few-nucleon calculations using more phenomenological potentials. The explicit three-body potential at $\nu = \nu_{\text{min}} + 2$ (from the delta isobar) and $\nu_{\text{min}} + 3$ was derived by van Kolck (1994). It is dominated by the delta, and bears some resemblance to other, more phenomenological potentials insofar as two-pion exchange is concerned, but shorter-range pieces are new.

Despite these successful fits and insights, the main advantage of the method of EFT lies in its concomitant application to many other processes, which might yield more predictive statements. I now discuss some of these.
As a result of the factor $-2C$ in Eq. (5), we see immediately—in an effect similar to few-nucleon forces—that external low-energy probes ($\pi$’s, $\gamma$’s) with $Q \sim m_\pi$ will tend to interact predominantly with a single nucleon, simultaneous interactions with more than one nucleon being suppressed by powers of $(Q/M_{QCD})^2$. Again, this generic dominance of the impulse approximation is a well-known result that arises naturally here. This is of course what allows extraction, to a certain accuracy, of one-nucleon parameters from nuclear experiments. More interesting from the nuclear dynamics perspective are, however, those processes where the leading single-nucleon contribution vanishes by a particular choice of experimental conditions, for example the threshold region. In this case the two-nucleon contributions, especially in the relatively large deuteron, can become important.

- $\pi d \to \pi d$ at threshold. This is perhaps the most direct way to check the consistency of $\chi$PT in few-nucleon systems and in pion-nucleon scattering. Here the lowest-order, $\nu = -2$ contributions to the kernel vanish because the pion is in an $S$-wave and the target is isoscalar. The $\nu = -1$ term comes from the (small) isoscalar pion-nucleon seagull, related in lowest-order to the pion-nucleon isoscalar amplitude $b_0$. $\nu = 0, +1$ contributions come from corrections to pion-nucleon scattering and two-nucleon diagrams, which involve besides $b_0$ also the much larger isovector amplitude $b_1$. Weinberg (1992) has estimated these various contributions to the pion-deuteron scattering length, finding agreement with previous, more phenomenological calculations, which have been used to extract $b_0$. (See also Beane at al. (1997b).)

- $np \to \gamma d$ at threshold. This offers a chance of a precise postdiction. Here it is the transverse nature of the real outgoing photon that is responsible for the vanishing of the lowest-order, $\nu = -2$ contribution to the kernel. The single-nucleon magnetic contributions come at $\nu = -1$ (tree level), $\nu = +1$ (one loop), etc. The first two-nucleon term is an one-pion exchange at $\nu = 0$, long discovered to give a smaller but non-negligible contribution. There has been a longstanding discrepancy of a few percent between these contributions and experiment. At $\nu = +2$ there are further one-pion exchange, two-pion exchange, and short-range terms. Park at al. (1995) and Park at al. (1996b) calculated the two-pion exchange diagrams in a “deltaless” theory and used resonance saturation to estimate the other $\nu = +2$ terms. With wavefunctions from the Argonne V18 potential and a cut-off $\Lambda = 1000$ MeV, they found the excellent agreement with experiment shown in Table 2. The total cross-section changes by 0.3% if the cut-off is decreased to 500 MeV. (See Park at al. (1996b) where references to experiment can be found.)

- $\gamma d \to \pi^0 d$ at threshold. This reaction offers the possibility to test a prediction arising from a combination of two-nucleon contributions and the neutral pion single-neutron amplitude. Pion photoproduction on the nucleon has been studied up to $\nu = 4$ in the deltaless theory; see Bernard (1997). A fit constrained by resonance saturation is successful in reproducing the measured differential cross-section at threshold $\propto |E_{0+}|^2$ in the channels $\gamma p \to \pi^+ n$, $\gamma n \to \pi^- p$, and $\gamma p \to \pi^0 p$, and makes a prediction $E_{0+}(\gamma n \to \pi^0 n) = 2.13 \cdot 10^{-3}/m_{\pi^+}$. Neglect-
Table 2. Values for various contributions to the total cross-section $\sigma$ in mb for radiative neutron-proton capture: impulse approximation to $\nu = 2$ (imp), impulse plus two-nucleon diagrams at $\nu = 0$ (imp+tn0), impulse plus two-nucleon diagrams up to $\nu = 2$ (imp+tn), and experiment (expt).

|      | imp  | imp+tn0 | imp+tn | expt |
|------|------|---------|--------|------|
| 305.6 | 321.7 | 336.0   | 334±0.5 |

Table 3. Values for $E_d$ in units of $10^{-3}/m_{\pi^+}$ from single scattering up to $\nu = 1$ (ss), two-nucleon diagrams at $\nu = 0$ (tn0), two-nucleon diagrams at $\nu = 1$ (tn1), and their sum (ss+tn).

|     | ss   | tn0   | tn1   | ss+tn |
|-----|------|-------|-------|-------|
| 0.36 | −1.90| −0.25 | −1.79 |

ing isospin violation, the uncertainties in the three measured amplitudes do not allow to pinpoint $E_{0+}(\pi^0n)$ much better than $-0.5 \rightarrow +2.5 \cdot 10^{-3}/m_{\pi^+}$. The lack of neutron targets highlights the advantage of using the deuteron. Here, it is the neutrality of the outgoing $S$-wave pion that ensures that the leading $\nu = -2$ terms vanish. The single-scattering contribution is given by the same $\nu = -1,0,+1,...$ mechanisms described earlier, with due account of $P$-waves and Fermi motion inside the deuteron. The first two-nucleon term enters at $\nu = 0$, a correction appears at $\nu = +1$, and so on. The differential cross-section at threshold, $\propto |E_d|^2$, was obtained at $\nu = 0$ by Beane et al. (1995) and up to $\nu = +1$ by Beane et al. (1997a). They are shown in Table 3, corresponding to the Argonne V18 potential and a cut-off $\Lambda = 1000$ MeV. Other realistic potentials and cut-offs from 650 to 1500 MeV give the same result within 5%, while a model-dependent estimate (Wilhelm (1997)) of some $\nu = +2$ terms suggests a 10% or larger error from the neglected higher orders in the kernel itself. Some sensitivity to $E_{0+}(\pi^0n)$ survives the large two-nucleon contribution. A preliminary result from Saskatoon presented at this Workshop is close to our prediction, while an electroproduction experiment is under analysis at Mainz. For more details, see Bernard (1997), Bernstein and Kaiser (1997), and Merkel (1997).

- $pp \rightarrow pp\pi^0$ close to threshold. This reaction has attracted a lot of interest because of the failure of standard phenomenological mechanisms in reproducing the small cross-section observed near threshold. It involves larger momenta of $O(\sqrt{m_\pi m_N})$, so the relevant expansion parameter here is the not so small $(m_{\pi}/m_N)^{1/2}$. This process is therefore not a good testing ground for the above ideas. But $(m_{\pi}/m_N)^{1/2}$ is still $< 1$, so at least in some formal sense we can perform a low-energy expansion. Cohen et al. (1996) and van Kolck et al. (1996b) have adapted the chiral expansion to this reaction and estimated the first few contributions. Again, the lowest order terms all vanish. The formally leading non-vanishing terms—an impulse term and a similar diagram from the delta isobar— are anomalously small and partly cancel. The bulk of the cross-section
must then arise from contributions that are relatively unimportant in other processes. One is isoscalar pion rescattering for which two sets of $\chi$PT parameters were used: “ste” from a sub-threshold expansion of the $\pi N$ amplitude and “cl” from an one-loop analysis of threshold parameters. Others are two-pion exchange and short-range $\pi NN\pi NN$ terms, which were modeled by heavier-meson exchange: pair diagrams with $\sigma$ and $\omega$ exchange, and a $\pi\rho\omega$ coupling, among other, smaller terms. Two potentials —Argonne V18 and Reid93— were used. Results are shown in Fig. 2 together with IUCF and Uppsala data. Other $\chi$PT studies of this reaction were carried out by Park et al. (1996a), Gedalin et al. (1997), Sato et al. (1997), and Hanhart et al. (1997), while Park et al. (1993) presented a related analysis of the axial-vector current. The situation here is clearly unsatisfactory, and presents therefore a unique window into the nuclear dynamics. Work is in progress, for example, on a similar analysis for the other, not so suppressed channels $\rightarrow d\pi^+, \rightarrow pn\pi^+$ (da Rocha et al. (1997)).

4 Isospin violation

Why is isospin a good symmetry of low-energy hadronic physics? And charge symmetry (a rotation of $\pi$ around the 2-axis in isospin space) even better? A measure of the size of isospin violation compared to explicit chiral symmetry breaking in the QCD Lagrangian is $\epsilon \sim 1/3$. In low-energy observables isospin violation
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is typically much smaller. For example, in the $NN$ system, using Coulomb-corrected scattering lengths and the fact that differences in potentials are amplified by a factor of $\sim 10$ in differences of scattering lengths, the isospin breaking but charge symmetric (or Class II) potential is estimated to be around $1/40$ of the isospin symmetric (or Class I) component, while the charge symmetry breaking (or Class III) potential is about a further $1/4$ smaller.

Part of the answer is of course that the quark masses are small compared to $M_{QCD}$. This in itself does not explain, however, why the isospin splitting in the pion masses squared is so small ($1/15$ of the average pion mass squared). It also seems to exclude the possibility of isospin violation at low energies that is comparable to $\epsilon$. A more complete answer requires the construction of all the operators that break isospin in the chiral Lagrangian. These can be classified into three types: (i) those operators involving hadronic fields only that stem from the quark mass difference, transform as 3-components of (tensor products of) $SO(4)$ vectors, and are proportional to (powers of) $\epsilon m_{\pi}^2$; (ii) hadronic operators that come from exchange of hard photons among quarks, transform as 34- and 34,34-components of (tensor products of) $SO(4)$ antisymmetric tensors, and are proportional to (powers of) $\epsilon^2$; (iii) hadronic operators that mix the above; (iv) operators involving the photon field, which are $U(1)_{em}$ invariant and are proportional to (powers of) $\epsilon$. I lump type (i) operators in $L^{qm}$, type (ii) and (iii) in $L^{hp}$, and type (iv) in $L^{sp}$.

One can order operators in $L^{qm}$ using the index $\Delta$ and the power of $\epsilon$. Since a photon loop typically brings in a power of $\alpha_{em}/\pi \sim \epsilon(m_{\pi}/m_{\rho})^3$ one should count the index of operators in $L^{hp}$ ($L^{sp}$) as $\Delta + 3 (\Delta + 3/2)$. If this is done, we find (van Kolck (1993), van Kolck (1995), van Kolck (1997a)) that isospin is an accidental symmetry, in the sense that its violation does not appear in the EFT in lowest order. In most cases, an isospin violating operator from $L^{qm+hp+sp}_{(n\geq 1)}$ competes with an isospin conserving operator from $L^{(0)}$, so that its effects are suppressed not by $O(\epsilon)$, but by $O(\epsilon(Q/M_{QCD})^n)$.

If one considers the pion, nucleon and delta masses one sees that the above naive power counting works alright. In $\pi\pi$ scattering the specific form of $L^{qm}$ suggests that, when written in terms of Mandelstam variables, the amplitude is mostly sensitive to photon exchange effects. In $\pi N$ scattering we find an example of a potentially large isospin violation. In $\pi^0$ elastic scattering, where the isoscalar scattering length $b_0$ contributes, an isospin violating operator from $L^{qm}_{(1)}$ competes with an isospin conserving operator from $L^{(1)}$, and isospin violation could be potentially of $O(\epsilon)$. Unfortunately this is hard to measure directly.

In the case of nuclear forces, we recover the observed hierarchy among different types of components. In the chiral expansion, one indeed finds (van Kolck (1993), van Kolck (1995), van Kolck (1997a), van Kolck et al. (1996a)) that higher Class forces appear at higher orders: $\langle V_{M+1} \rangle/\langle V_M \rangle \sim O(Q/M_{QCD})$, where $\langle V_M \rangle$ denotes the contribution of the leading Class $M$ potential. This comes about because Class I forces are dominated by static OPE and contact terms, both with $\nu = 0$, a Class II force appears at $\nu = 1$ from one insertion of the pion mass difference in OPE, which is an $\epsilon m_{\pi}/2m_{\rho} \sim 1/30$ effect, and a Class III force...
comes at $\nu = 2$ from isospin violation in the $\pi NN$ coupling and in contact terms, which is an $\epsilon (m_\pi/m_\rho)^2 \sim 1/90$ effect. Precise calculations of simultaneous electromagnetic and strong isospin violation in the nuclear potential have also been carried out. van Kolck et al. (1997) and Friar et al. (1997) have derived the one-pion-range isospin violating two-nucleon potential up to $\nu = 3$, which includes, besides pion and nucleon mass splittings, also isospin violation in the pion-nucleon coupling and simultaneous $\pi\gamma$ exchange. This is the first calculation of this type that is both ultraviolet and infrared finite, and independent of choice of pion field and gauge. This potential is comparable to other components usually considered, and it has been included in the Nijmegen $NN$ partial-wave analysis (van Kolck et al. (1997)).

5 Conclusions

The challenges of a chiral EFT approach are greater in nuclear physics than in the meson and one-baryon sectors, because of the need to go beyond a straightforward perturbative expansion. Despite the amount of information available, only the very initial steps of a systematic chiral treatment of many-nucleon processes have been taken so far.

I have tried to argue that the first results are very auspicious. The long-established picture of light nuclei as a few-body system interacting through a non-relativistic, two-body, isospin-symmetric potential emerges naturally at leading order. Higher order effects—which account for relativistic corrections, short-range structure and non-analytic pionic contributions—provide the other basic ingredients of nuclear forces, as evidenced by a quantitative fit to two-nucleon data and by the qualitative insights into the size of few-nucleon and isospin-violating forces. $\chi$PT also provides a consistent framework for scattering on the nucleon and on light nuclei, which in turn offers a handle on nucleon parameters (as for pion scattering and pion photoproduction), successful quantitative postdictions (such as in radiative neutron-proton capture), and quantitative predictions (such as in pion photoproduction). And best of all, open problems exist (such as pion production in the $pp$ reaction). There is still a lot to be done: consistent potential/kernel calculations, many other processes at and away from threshold, extensions to $SU(3)$ and nuclear matter, to mention just a few topics. Perhaps $\chi$PT will then fulfill the role of the long-lacking theory for nuclear physics based on QCD.

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