Testing the isotropy of the Universe with type Ia supernovae in a model-independent way

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ABSTRACT
In this paper, we study an anisotropic universe model with Bianchi-I metric using Joint Light-curve Analysis (JLA) sample of type Ia supernovae (SNe Ia). Because light-curve parameters of SNe Ia vary with different cosmological models and SNe Ia samples, we fit the SNe Ia light-curve parameters and cosmological parameters simultaneously employing Markov Chain Monte Carlo method. Therefore, the results on the amount of deviation from isotropy of the dark energy equation of state ($\delta$), and the level of anisotropy of the large-scale geometry ($\Sigma_0$) at present, are totally model-independent. The constraints on the skewness and cosmic shear are $-0.101 < \delta < 0.071$ and $-0.007 < \Sigma_0 < 0.008$. This result is consistent with a standard isotropic universe ($\delta = \Sigma_0 = 0$). However, a moderate level of anisotropy in the geometry of the Universe and the equation of state of dark energy, is allowed. Besides, there is no obvious evidence for a preferred direction of anisotropic axis in this model.

Key words: cosmological parameters – type Ia supernovae

1 INTRODUCTION
Astronomical observations revealed that our Universe is undergoing an accelerating expansion (Riess et al. 1998; Perlmutter et al. 1999), which is one of the most surprising astronomical discoveries in recent years. Accelerating expansion implies that the universe is dominated by an unknown form of energy called ‘dark energy’ with negative pressure, or that Einstein’s theory of gravity fails on cosmological scales and requires some modifications.

The standard $\Lambda$CDM model is established based on the cosmological principle and parametrization of the Big Bang cosmological model. It depicts a homogeneous and isotropic universe on large scales with approximately 30% matter (including baryonic matter and dark matter) and 70% dark energy at present time, which is consistent with vast majority of several precise astronomical observations, including cosmic microwave background (CMB) power spectrum (WMAP Collaboration 2011; Planck Collaboration XIII 2016) and baryon acoustic oscillations (Eisenstein et al. 2005).

However, the standard cosmological model is challenged by a few puzzling cosmological observations (Perivolaropoulos 2014), which may require modifications. Evidences for cosmology anisotropy have been obtained by the power asymmetry of CMB perturbation maps (Eriksen et al. 2007; Hoftuft et al. 2009; Paci et al. 2010; Mariano & Perivolaropoulos 2013; Zhao & Santos 2015), the large scale velocity flows (Kashlinsky et al. 2008; Watkins, Feldman & Hudson 2009; Kashlinsky et al. 2010; Lavaux et al. 2010; Feldman, Watkins & Hudson 2010), anisotropy in accelerating expansion rate (Antoniou & Perivolaropoulos 2010; Mariano & Perivolaropoulos 2012; Yang, Wang & Chu 2014; Wang & Wang 2014), spatial dependence of the value of the fine structure constant $\alpha$ (Webb et al. 2011; Moss et al. 2011; King et al. 2012; Mariano & Perivolaropoulos 2012; Pinho et al. 2016) and so on. These puzzles are in favor of preferred cosmological directions, which seem to violate the cosmological principle. The so-called ‘cosmic anomalies’ (Perivolaropoulos 2014) may either be simply large statistical fluctuations or have some physical origins, which could be either geometric or energy-related (Perivolaropoulos 2014).

Here, we focus on an anisotropic universe model that has a plane-symmetric Bianchi-I metric (Taub 1951; Schtucker, Tilquin & Valent 2014), namely ellipsoidal universe (Campanelli, Cea & Tedesco 2006). The ellipsoidal universe model was first proposed in Campanelli, Cea & Tedesco (2006) to solve the CMB quadrupole problem by assuming a plane-symmetric universe with an eccentricity of order $10^{-2}$ at decoupling. Campanelli, Cea & Tedesco (2007) discussed that the anisotropic expansion can be generated by cosmological magnetic fields, cosmic domain walls or cosmic
strings. The cosmic shear $\Sigma_0$ and skewness $\delta$ are introduced (Campanelli et al. 2011a) to describe the anisotropy level of cosmic geometry and dark energy, respectively. Campanelli et al. (2011c) analysed Union and Union2 compilation and concluded that an isotropic universe is consistent with SNe Ia data. However, their analysis directly used $\mu_{\text{obs}}$ and $\sigma$ obtained in $\Lambda$CDM model (Amanullah 2010). The results are model-dependent, because light-curve parameters change with different universe models and SNe Ia sample. Schäcker, Tilquin & Valent (2014) fitted the Bianchi I metric to the Hubble diagram of SNe Ia.

Therefore, we improve the previous research by fitting the SNe Ia light-curve parameters and cosmological parameters simultaneously. This paper is organized as follows. In the next section, we introduce the ellipsoidal universe model and derive the magnitude-redshift relation. In section 3, we use SNe Ia data of JLA sample to constrain all the free parameters simultaneously, including light-curve and cosmological parameters. The fitting results are shown in section 4. Conclusion and discussions are given in section 5.

## 2 Ellipsoidal Universe Model

The Bianchi type I cosmological model is extensively discussed in Campanelli, Cea & Tedesco (2006) and Campanelli et al. (2011a). In this section, we briefly introduce this model. The Bianchi-I metric (Taub 1951; Schucker, Tilquin & Valent 2014) with planar symmetry (Campanelli, Cea & Tedesco 2011a). In this section, we briefly introduce this model. The Bianchi-I metric (Taub 1951; Schucker, Tilquin & Valent 2014) with planar symmetry (Campanelli, Cea & Tedesco 2006; 2007; Campanelli et al. 2011a; 2011b; 2011c) is described by Taub line element (Campanelli et al. 2011a; 2011b)

$$\mathrm{d}t^2 = \mathrm{d}r^2 - a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2) - b(t)^2\mathrm{d}z^2,$$

where $a(t)$ and $b(t)$ are the scale factors which can be normalized as $a(t_0) = b(t_0) = 1$ at the present time $t_0$.

According to the plane-symmetric metric, the ‘mean Hubble parameter’ $H$ can be defined as (Campanelli et al. 2011b; 2011c)

$$H \equiv \frac{\dot{a}}{a},$$

where $A \equiv (a^2b)^{1/3}$ is the ‘mean expansion parameter’ (Campanelli et al. 2011b).

In order to measure the level of anisotropy, we define cosmic shear $\Sigma$ and skewness $\delta$ (Campanelli et al. 2011a; 2011b; 2011c) as

$$\Sigma \equiv \frac{H_\parallel - H}{H}, \quad \text{and} \quad \delta \equiv w_\parallel - w_\perp,$$

respectively. In the above equation, $H_\parallel = \frac{\dot{a}}{a}$ represents the Hubble parameter in the symmetry plane. $w_\parallel$ and $w_\perp$ are parameters of state equation parallel and perpendicular to the symmetry plane respectively. Besides, the mean parameter of state equation is defined as

$$w \equiv \frac{2w_\parallel + w_\perp}{3}.$$

### 2.1 Anisotropy axis

In the Galactic coordinate reference, the direction cosine of the symmetry axis is

$$\hat{n}_A = (\cos b_A \cos l_A, \cos b_A \sin l_A, \sin b_A).$$

For an arbitrary direction

$$\hat{n} = (\cos b \cos l, \cos b \sin l, \sin b),$$

the angle $\theta$ between $\hat{n}$ and $\hat{n}_A$ is

$$\cos \theta \equiv \hat{n} \cdot \hat{n}_A.$$

### 2.2 Redshift-distance relation

We introduce the ‘eccentricity’ $e$ as

$$e^2 \equiv 1 - \frac{b^2}{a^2}.$$

In galactic coordinates system, the luminosity distance is given by (Campanelli et al. 2011c)

$$d_L(z, \theta) = \frac{c(1 + z)}{H_0} \int_{\theta(0)}^{\theta(1)} \frac{(1 - e^2)^{1/2}}{(1 - e^2 \cos^2 \theta)^{1/2}} \mathrm{d}A,$$

where $H$ is the Hubble constant normalized to its actual at $t_0$. Both $H$ and $e$ are functions of $A$.

## 3 JLA Sample and MCMC Fitting

The Joint Light-curve Analysis (JLA) sample (Betoule et al. 2014) is based on Conley et al. (2011) compilation. It includes three-season data from SDSS-II (0.05 $< z <$ 0.4), three-year data from SNLS (0.2 $< z <$ 1), HST data (0.8 $< z <$ 1.4) and several low-redshift samples ($z < 0.1$) such as Calán/Tololo Survey and Carnegie Supernova Project. The JLA sample totals 740 spectroscopically confirmed SNe Ia with high-quality light curves.

### 3.1 Angular position

In order to determine the anisotropy axis in the galactic coordinate system, we need galactic latitude and longitude $(l, b)$ for each supernova. For transformations from equatorial to Galactic coordinates, we have

$$\sin b = \sin \delta_{\text{NGP}} \sin \delta + \cos \delta_{\text{NGP}} \cos \delta \cos(\alpha - \alpha_{\text{NGP}}),$$

$$\cos b \sin(\delta_{\text{NGP}} - l) = \cos \delta \sin(\alpha - \alpha_{\text{NGP}}),$$

$$\cos b \cos(\delta_{\text{NGP}} - l) = \cos \delta_{\text{NGP}} \sin \delta - \sin \delta_{\text{NGP}} \cos \delta \cos(\alpha - \alpha_{\text{NGP}}).$$

The angular position of SNe Ia are shown in Figure 1.
3.2 Distance modulus
The analysis of SNe light curves (Betoule et al. 2014) gives the distance estimation \( \mu_{\text{obs}} \) as

\[
\mu_{\text{obs}} = m_B^\text{obs} - M_B + \alpha x_1 - \beta c,
\]

where \( m_B^\text{obs} \) corresponds to the observed peak magnitude in rest-frame B-band and \( M_B \) is the absolute B-band magnitude. \( x_1 \) and \( c \) are SALT2 shape parameter and color correction respectively. \( \alpha \) and \( \beta \) are nuisance parameters in the distance estimate to be determined.

We correct the effects of host galaxy properties assuming the absolute magnitude is related to the host stellar mass \( (M_{\text{stellar}}) \) by a simple step function (Betoule et al. 2014)

\[
M_B = \begin{cases} 
M_B^\text{cal} + \delta_M & \text{if } M_{\text{stellar}} < 10^{10} M_{\odot} \text{.} \\
M_B^\text{cal} & \text{otherwise.}
\end{cases}
\]

Here, we introduce the theoretical distance modulus \( \mu \) as

\[
\mu = 5 \log \left( \frac{d_L}{\text{Mpc}} \right) + 25.
\]

3.3 The Hubble diagram covariance matrix
Betoule et al. (2014) assemble a 3N_{\text{SN}} × 3N_{\text{SN}} = 2220 × 2220 covariance matrix for the light-curve parameters, which includes statistical and systematic uncertainties.

\[
C_{\eta} = C_{\text{stat}} + (C_{\text{pecvel}} + C_{\text{neda}})_{\text{SNIa}} + (C_{\text{cal}} + C_{\text{model}} + C_{\text{bias}} + C_{\text{host}} + C_{\text{dust}})_{\text{SN}} \text{,}
\]

\( C_{\text{stat}} \) is obtained from error propagation of light-curve fit uncertainties. The systematic uncertainties include seven components, namely the calibration uncertainty \( C_{\text{cal}} \), the light-curve model uncertainty \( C_{\text{model}} \), the bias correction uncertainty \( C_{\text{bias}} \), the mass step uncertainty \( C_{\text{host}} \), the peculiar velocity uncertainty \( C_{\text{pecvel}} \) and the non-Ia uncertainty \( C_{\text{neda}} \).

The covariance matrix (Betoule et al. 2014) of the vector of distance modulus estimate \( \mu_{\text{obs}} \) is

\[
C = AC_{\eta}A^T + \text{diag} \left( \frac{5 \sigma_{\eta}}{\ln 10} \right)^2 + \text{diag}(\sigma_{\text{pecvel}}^2) + \text{diag}(\sigma_{\text{neda}}^2),
\]

where \( A = \alpha A_0 + \alpha A_1 - \beta A_2 \) is a 2220 × 740 matrix with \( (A_{3k}, i) = \delta_{ij+k} \) such that \( \mu = A \eta - M_B \). Here, \( \eta = (m_B^\text{obs} - x_1, 1, \ldots, m_B^\text{obs} - x_1, 1)_{\text{SNIa}} \). The \( \sigma_{\eta}, \sigma_{\text{pecvel}} \) and \( \sigma_{\text{neda}} \) account for the uncertainty in cosmological redshift due to peculiar velocities, the variation of magnitudes caused by gravitational lensing, and the intrinsic variation in SN magnitude not described by the other terms, respectively. Betoule et al. (2014) suggest \( \sigma_{\eta} = 150 \text{ km s}^{-1} \) and \( \sigma_{\text{pecvel}} = 0.055 \times z \), while values of \( \sigma_{\text{neda}} \) are listed in Table 1.

3.4 Markov Chain Monte Carlo fitting
We employ the emcee to carry out parameters fitting (Goodman & Weare 2010; Foreman-Mackey et al. 2013). The emcee is a python module that employs the affine-invariant ensemble sampler for Markov Chain Monte Carlo (MCMC) algorithm. In this package, many samplers (called walkers) run in parallel and periodically exchange states to more efficiently sample the parameter space. The Maximum Likelihood Estimation (MLE) is applied to MCMC algorithm. The likelihood L is sum of many normal distributions

\[
L = \prod_{i=1}^{740} \frac{1}{\sqrt{2\pi} \sigma_i} \exp \left[ -\frac{(\mu_{\text{obs},i} - \mu_{\text{th},i})^2}{2\sigma_i^2} \right],
\]

where

\[
\sigma_i = C_{\eta}^{ii}.
\]

Then the log-likelihood is

\[
\ln L = -\frac{1}{2} \sum_{i=1}^{740} \left[ \frac{(\mu_{\text{obs},i} - \mu_{\text{th},i})^2}{\sigma_i^2} + \ln(\sigma_i^2) + \ln(2\pi) \right].
\]

The observational distance modulus \( \mu_{\text{obs}} \) depends on light-curve parameters \( \{ \alpha, \beta, M_B^\text{cal}, \Delta M \} \), while the theoretical distance modulus \( \mu_{\text{th}} \) depends on cosmological parameters \( \{ H_0, \Omega_m, \Omega_\Lambda, \delta, l_A, b_A \} \) in ellipsoidal universe model. The priors of parameters are listed in Table 2.

Campanelli et al. (2011c) studied an anisotropic Bianchi type I cosmological model using Union2 compilation, in which \( \mu_{\text{obs}} \) and \( \sigma \) are derived in the \( \Lambda \)CDM model (Amanullah 2010). Therefore, the light-curve parameters \( \{ \alpha, \beta, M_B^\text{cal}, \Delta M \} \) are fixed. However, the light-curve parameters vary with different cosmological model. It’s unreasonable to fit just cosmological parameters. Different from Campanelli et al. (2011c), we fit four light-curve parameters and seven cosmological parameters \( \{ H_0, \Omega_m, \Omega_\Lambda, w, \delta, l_A, b_A \} \) simultaneously. So the likelihood function depends on eleven free parameters. The derived results are model-independent.

As a comparison, we carried out another MCMC fitting which constrained parameters \( \{ \alpha, \beta, M_B^\text{cal}, \Delta M, H_0, \Omega_m, w \} \) in a flat \( w \)CDM cosmology with an arbitrary equation of state \( w \).

4 RESULTS
The confidence contours \( (1\sigma, 2\sigma \text{ and } 3\sigma) \) and marginalized likelihood distribution functions for the parameters \( \{ \alpha, \beta, M_B^\text{cal}, \Delta M, H_0, \Omega_m, \Omega_\Lambda, w, \delta \} \) from MCMC fittings are shown in Figure 2. Table 3 lists the best-fitting value and the \( 1\sigma, 2\sigma \text{ and } 3\sigma \) confidence level intervals. Since \( M_B^\text{cal} \) entirely degrades with \( H_0 \), their values can not be constrained well simultaneously. If we take Hubble constant

| Sample | low – z | SDSS-II | SNLS | HST |
|--------|---------|---------|------|-----|
| \( \sigma_{\text{obs}} \) | 0.134 | 0.108 | 0.080 | 0.100 |

| Parameter | Prior | Parameter | Prior |
|-----------|-------|-----------|-------|
| \( \alpha \) | [0.1, 0.2] | \( \Sigma_\Lambda \) | [-0.5, 0.5] |
| \( \beta \) | [2.0, 4.0] | \( w \) | [-2.0, 0.0] |
| \( M_B^\text{cal} \) | [-19.3, -18.6] | \( \delta \) | [-0.5, 0.5] |
| \( \Delta M \) | [-0.1, 0.0] | \( l_A \) | [0, \pi] |
| \( H_0 \) | [60, 80] | \( b_A \) | [-\( \frac{\pi}{2} \), \frac{\pi}{2}] |
| \( \Omega_m \) | [0.0, 0.5] |

Table 1. Values of \( \sigma_{\text{obs}} \) used in the cosmological fits for different samples.

Table 2. The priors of free parameters in MCMC fitting.
\[ H_0 = 73.24 \text{ km s}^{-1} \text{Mpc}^{-1} \] (Riess et al. 2016), the corresponding \( M_V^* \) is 18.95.

The constraints on the anisotropy parameters \( \delta \) and \( \Sigma_0 \) from JLA sample are

\[ -0.101 < \delta < 0.071 \quad (1\sigma) , \tag{20} \]

and

\[ -0.007 < \Sigma_0 < 0.008 \quad (1\sigma) . \tag{21} \]

Compared with \(-0.16 < \delta < 0.12\) and \(-0.012 < \Sigma_0 < 0.012\) (1\sigma) from Union2 sample (Campanelli et al. 2011c), we can see that our result is more tight. The result implies that there is no evidence in favor of either geometric anisotropy \( (\Sigma_0 \neq 0) \) or dark energy anisotropy \( (\delta \neq 0) \).

The direction cosine of anisotropy axis is

\[ n_A = (\cos b_A \cos l_A, \cos b_A \sin l_A, \sin b_A) . \tag{22} \]

The opposite direction cosine \( n'_A \) is then

\[ n'_A = (-\cos b_A \cos l_A, -\cos b_A \sin l_A, -\sin b_A) . \tag{23} \]

corresponding to an opposite direction \((l_A \pm \pi, -b_A)\) of the same axis. Therefore, we just use the parameter space \([0, \pi]\) for \( n_A \). Figure 4 gives confidence contours of preferred direction in the galactic coordinate system. Since the distance modulus depends weakly on both the angular position of SNe Ia, \( n \), and the direction of the symmetry axis \( n_A \) for small values of the shear \( \Sigma_0 \), the SNe Ia data are not able to constrain the anisotropy parameters \( (\Sigma_0 \text{ and } \delta) \) and the preferred direction defined by the anisotropy axis itself at the same time.

The anti-correlation between \( w \) and \( \Omega_m \) comes from the dependence of luminosity distance on \( w \) and \( \Omega_m \)

\[ \Delta = \sqrt{\Omega_m A^{-3} + \Omega_{DE} A^{-3(1+w)}} . \tag{24} \]

Meanwhile, the correlation between \( \delta \) and \( \Sigma_0 \) (Campanelli et al. 2011c) is given by

\[ \Sigma(A) = \frac{\Sigma_0 + (E - E_0) \delta}{A^3 H} , \tag{25} \]

which shows effects of energy distribution on the spatial curvature of the universe. Meanwhile, we find the best-fitting value for matter density parameter \( \Omega_m = 0.314^{+0.069}_{-0.111} \) (1\sigma), and dark energy equation of state \( w = -0.774^{+0.091}_{-0.214} \) (1\sigma).

The best-fitting results of the flat wCDM model are displayed in Figure 3 for comparison. We find that best-fitting values of parameters in ellipsoidal model are quite similar to those in the wCDM model (see Table 4). Considering that the best-fitting values of \( \Sigma_0 \) and \( \delta \) are nearly zero, and wCDM is the limiting case of ellipsoidal universe model, this result can be expected. Figure 5 shows the Hubble diagram for JLA sample in two different cosmological models. One is the best-fitting model with \( (\Omega_m, \Sigma_0, \delta) = (0.314, 0.001, -0.774, -0.008) \) (blue dashed line). The other is the flat wCDM model with \( \Omega_m = 0.281, w = -0.750 \) (black line). In the lower panel, we show the corresponding distance modulus fitting residuals (distance modulus minus the best-fitting wCDM distance modulus) as a function of redshift. The comparison between the best-fitting values of cosmological parameters from Union2 (Campanelli et al. 2011c) and JLA sample is shown in Table 5. The constraints are more tight than those of Campanelli et al. (2011c).

5 CONCLUSIONS

In this paper, we study the ellipsoidal universe model with plane-symmetric metric and constrain the anisotropy level of cosmic geometry and dark energy fluids. By analyzing the magnitude-redshift data of 740 SNe Ia in the JLA sample, we find a more tight constraint on cosmic shear \( \Sigma_0 \) and skewness \( \delta \). The best constraints are

\[ -0.007 < \Sigma_0 < 0.008 \quad (1\sigma) , \]

and

\[ -0.101 < \delta < 0.071 \quad (1\sigma) . \]

In conclusion, the fitting results favor an isotropic universe without a preferred direction at present time. With the progress of astronomical observations such SNe Ia and CMB, we will have better constraints on universe anisotropy in the near future. The question that whether the cosmic anomalies such as dark energy dipole, fine structure constant dipole or dark flow have same physical origin remains to be answered. Cosmological principle is so vital to modern cosmology that much more effort should be made to verify the fundamental postulation.

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Table 3. Best-fitting values, and the 1σ, 2σ and 3σ confidence level intervals derived from the JLA sample.

|       | α     | β     | ∆M   | Ωm   | Σ0   | w     | δ     |
|-------|-------|-------|-------|-------|-------|-------|-------|
| BF    | 0.124 | 2.554 | -0.045| 0.314 | 0.001 | -0.774| -0.008|
| 1σ    | [0.118, 0.131] | [2.475, 2.622] | [-0.059, -0.032] | [0.173, 0.383] | [-0.007, 0.008] | [-1.028, -0.633] | [-0.101, 0.071] |
| 2σ    | [0.111, 0.137] | [2.406, 2.701] | [-0.072, -0.020] | [0.046, 0.443] | [-0.015, 0.015] | [-1.296, -0.535] | [-0.236, 0.185] |
| 3σ    | [0.105, 0.144] | [2.335, 2.777] | [-0.086, -0.007] | [0.003, 0.488] | [-0.024, 0.023] | [-1.527, -0.468] | [-0.451, 0.388] |

Table 4. Comparisons between best-fitting values in wCDM model and ellipsoidal universe model.

|       | α     | β     | ∆M   | Ωm   | w     |
|-------|-------|-------|-------|-------|-------|
| wCDM  | 0.124 | 2.554 | -18.95| -0.045| -0.750|
| Ellipsoidal | 0.124 | 2.554 | -18.95| -0.045| 0.314 | -0.774|

Table 5. Comparisons between best-fitting values of cosmological parameters from Union2 and JLA sample for ellipsoidal universe model.

|       | Σ0   | δ     | Ωm   | w     |
|-------|------|-------|------|-------|
| Union2| -0.004| -0.050| 0.37 | -1.32 |
| JLA   | 0.001| -0.008| 0.314| -0.774|

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Figure 1. Angular positions of SNe Ia in the JLA sample. $b \ (−90° \leq b \leq 90°)$ is the galactic latitude and $l \ (0° \leq l < 360°)$ is the galactic longitude. Supernovae from four subsets are marked with different colors.
Figure 2. Confidence contours (1σ, 2σ and 3σ) and marginalized likelihood distributions for the parameters ($\alpha, \beta, M_B^1, \Delta M, H_0, \Omega_m, \Sigma_0, w, \delta$) in ellipsoidal universe model. True values (shown as dots) of $\{\alpha, \beta, M_B^1, \Delta M, \Omega_m, w\}$ are taken from $\omega$CDM fitting results as comparison.
Figure 3. Confidence contours (1σ, 2σ and 3σ) and marginalized likelihood distribution functions for the parameters ($\alpha, \beta, M_B^1, \Delta M, H_0, \Omega_m, w$) in the $w$CDM model.
Figure 4. Confidence level contours of preferred direction in the galactic coordinate system.
Figure 5. Upper panel. Hubble diagram for the 740 SNe Ia in the JLA compilation for different cosmological models: best-fitting ellipsoidal model \((\Omega_m, \Sigma_0, w, \delta) \approx (0.314, 0.001, -0.774, -0.006)\) (blue dashed line), and wCDM model \((\Omega_m, w) \approx (0.281, -0.750)\) (black line). Lower panel. Residuals (distance modulus minus distance modulus for the wCDM model) for the same models in the upper panel.