The strong coupling Kondo lattice model as a Fermi gas

Stellan Östlund

Göteborg University
Gothenburg 41296, Sweden

(Dated: March 23, 2022)

Abstract

The strong coupling half-filled Kondo lattice model is an important example of a strongly interacting dense Fermi system for which conventional Fermi gas analysis has thus far failed. We remedy this by deriving an exact transformation that maps the model to a dilute gas of weakly interacting electron and hole quasiparticles that can then be analyzed by conventional dilute Fermi gas methods. The quasiparticle vacuum is a singlet Mott insulator for which the quasiparticle dynamics are simple. Since the transformation is exact, the electron spectral weight sum rules are obeyed exactly. Subtleties in understanding the behavior of electrons in the singlet Mott insulator can be reduced to a fairly complicated but precise relation between quasiparticles and bare electrons. The theory of free quasiparticles can be interpreted as an exactly solvable model for a singlet Mott insulator, providing an exact model in which to explore the strong coupling regime of a singlet Kondo insulator.

PACS numbers: 71.10.-w, 71.27.+a, 71.30.+h
Keywords: Mott insulators, Kondo Lattice, Mott transition

*Electronic address: ostlund@physics.gu.se
**Introduction.** A Fermi liquid is described by starting with an ideal non-interacting Fermi gas. We then adiabatically “switch on” the interactions between particles. States of the interacting system are then identified with those of the non-interacting system. Fermi quasiparticles have the same relation to Fermi liquid ground state as the electron operators do to the vacuum; if \( | \Psi_G \rangle \) is the Fermi liquid ground state, there should exist a quasiparticle creation operator \( c^\dagger_\beta \) so that \( 0 = c_\beta | \Psi_G \rangle \), \( \{ c_\alpha, c^\dagger_\beta \} = \delta_{\alpha\beta} \) and \( Hc^\dagger_\alpha | \Psi_G \rangle = e_\alpha c^\dagger_\alpha | \Psi_G \rangle \).

The spectral weight sum-rule follows directly from this.

Although the spectral weight sum rule is so fundamental it has been surprisingly difficult to implement in approximate theories of the Kondo lattice model. The reason is that the ground state is a dense system of strongly interacting Fermions for which it has been difficult to obtain results using traditional weak coupling methods. This work demonstrates an exact transformation that can be utilized to map the dense strongly interacting singlet Mott insulator to a weak coupling dilute Fermi gas. This allows conventional Fermi gas techniques to be applied to this class of models.

We begin by considering a lattice of sites each consisting of conduction band “c” electrons that can hop between neighboring sites and core “f” electrons that are confined. At half filling a total of two electrons tend to be located on the site. A potential favors single occupancy of the “f” electron, forcing the second electron into the conduction band. The extended model consists of a lattice of such sites with only the “c” electrons hopping between neighbors.

There are four states with one “c” and one “f” electron present. The resulting triplet-singlet degeneracy is lifted by a spin exchange term that gives the singlet lowest energy. This behavior is most easily encoded in the Kondo lattice Hamiltonian

\[
H_{KLM} = t \sum_{rr'}(c^\dagger_{c,\sigma}(r)c_{c,\sigma}(r') + CC) + \sum_r (J S_c(r) \cdot S_f(r) + U_f(n_f(r) - 1)^2)
\]

The atomic ground state is given by \( | \Psi_G \rangle = \frac{1}{\sqrt{2}} (c^\dagger_{c,\uparrow} c^\dagger_{f,\downarrow} - c^\dagger_{c,\downarrow} c^\dagger_{f,\uparrow}) | 0 \rangle \). The full ground state is a product of such two-electron singlets at each site and is nondegenerate with finite gap to the excited states. The gap will persist for small values of the hopping \( t \) and the local conduction electron will then simply make virtual excursions to the neighboring sites. The system will remain a singlet Mott insulator.

Before describing quasiparticles for nonzero \( t \) we begin with the limit of zero hopping. For
the Hubbard impurity, there are four charged spin half excitations on a single site, consisting of each of the four states with three electrons. We first attempt to define a quasiparticle operator simply by \( c^\dagger_{c,\sigma} \) which one might hope adds an extra conduction electron to the singlet.

This ansatz immediately leads to problems. First we find

\[
\langle \Psi_G | c^\dagger_{c,\uparrow} c_{f,\downarrow} | 0 \rangle = -\frac{1}{\sqrt{2}} \langle c^\dagger_{c,\uparrow} c^\dagger_{f,\downarrow} | 0 \rangle.
\]

Although the operator indeed creates a state with an extra charge, this state is not normalized.

A second problem is the following inequality:

\[
\langle \Psi_G | c^\dagger_{c,\uparrow} c_{f,\downarrow} | 0 \rangle = \frac{1}{\sqrt{2}} \langle c^\dagger_{c,\downarrow} c^\dagger_{f,\uparrow} | 0 \rangle \neq 0
\]

which shows that the annihilation operator does not annihilate the atomic ground state. Finally, we note that

\[
\langle \Psi_G | c^\dagger_{c,\uparrow} c_{f,\downarrow} | 0 \rangle = -\frac{1}{\sqrt{2}} \langle c^\dagger_{c,\downarrow} c^\dagger_{f,\uparrow} | 0 \rangle \neq | \Psi_G \rangle
\]

so that this simple annihilation operator does not reconstruct the original singlet ground state. We conclude that none of the conditions we require of a proper quasiparticle operator is fulfilled with this ansatz. This simple analysis reveals the source of the difficulty; the ground state is “entangled” in the sense that it cannot be created from the true vacuum by a simple product of bare electron operators.

**Constructing correct local quasiparticles.**

We now discuss how to define proper quasiparticle operators. The local Hilbert space consists of sixteen states which we label from 1 to 16. First we list the eight states with even particle number ordered first by increasing particle number. Degeneracies are split by total spin, and remaining degeneracies are split by \( s_z \). States with odd particle number are similarly assigned labels 9 to 16.

This “canonical” ordering is shown on an energy versus particle number diagram for the Hamiltonian \( h_{\text{canonical}} = n_{\text{tot}} - \frac{2}{3} (n_f - n_c) \) in Figure 1A. On the horizontal axis is plotted \( n_{\text{tot}} \). On the vertical axis is the energy state labelled with the state of minimum \( z \)-component of spin for each degenerate multiplet. In Figure 1B is a hypothetical diagram with an interacting Hamiltonian \( h_{\text{atomic}} \) favoring the Kondo singlet ground state. To avoid cluttering the figure, triplet states are not drawn.

We denote \( | n \rangle_a \equiv | n \rangle_{\text{atomic}} \) states in Fig. 1B and \( | n \rangle_c \equiv | n \rangle_{\text{canonical}} \) states in Fig. 1A. A precise description of these states is needed to continue.

The singlets are given by

\[
| 1 \rangle_c = 1 | 0 \rangle, \quad | 2 \rangle_c = c^\dagger_{f,\uparrow} c^\dagger_{f,\downarrow} | 0 \rangle, \quad | 3 \rangle_c = \frac{1}{\sqrt{2}} (c^\dagger_{c,\uparrow} c^\dagger_{f,\downarrow} - c^\dagger_{c,\downarrow} c^\dagger_{f,\uparrow}) | 0 \rangle, \quad | 4 \rangle_c = c^\dagger_{c,\uparrow} c^\dagger_{c,\downarrow} | 0 \rangle, \quad | 5 \rangle_c = c^\dagger_{c,\uparrow} c^\dagger_{c,\downarrow} c^\dagger_{f,\uparrow} c^\dagger_{f,\downarrow} | 0 \rangle.
\]

The triplets numbered \( | 6 \rangle_c, | 7 \rangle_c \) and \( | 8 \rangle_c \) are not shown in the diagram.
FIG. 1: In (A), the energy $E = n_t o t - \frac{2}{3}(n_f - n_c)$ is plotted vs particle number and labeled with the state that has the smallest value of $S^2$ and $S_z$. A dotted arrow indicates a $c$-type creation operator is used to create the state at the tip of the arrow from the state at the base. A a solid arrow indicates an $f$-type creation operator is used. A solid-dotted combination indicates a singlet combination is used. In (B) a similar construction is used in an interacting model. The dotted arrow here indicates a hole operator.

The four states of total spin $-\frac{1}{2}$ are given by: $|9\rangle_c = c_{f,\uparrow}^\dagger |0\rangle$, $|10\rangle_c = c_{c,\downarrow}^\dagger |0\rangle$, $|11\rangle_c = c_{c,\downarrow}^\dagger c_{f,\downarrow}^\dagger |0\rangle$, $|12\rangle_c = c_{c,\uparrow}^\dagger c_{c,\downarrow}^\dagger c_{f,\downarrow}^\dagger |0\rangle$. The states of total spin $\frac{1}{2}$: are $|13\rangle_c = c_{f,\uparrow}^\dagger |0\rangle$, $|14\rangle_c = c_{c,\uparrow}^\dagger |0\rangle$, $|15\rangle_c = c_{c,\uparrow}^\dagger c_{c,\downarrow}^\dagger c_{f,\downarrow}^\dagger |0\rangle$, $|16\rangle_c = c_{c,\uparrow}^\dagger c_{c,\downarrow}^\dagger c_{f,\uparrow}^\dagger |0\rangle$. The states with $s_z = -\frac{1}{2}$ are not labelled in the figure.

A local spin-symmetric Hamiltonian $H$ is block diagonal in the blocks enclosed by parentheses: $(1, [2, 3, 4], [5]) ([6]), ([7]), ([8]) ([9, 10][11, 12]) ([13, 14][15, 16])$. If particle number is also conserved, $H$ is further block diagonal in the subblocks in square brackets.

We now draw in Fig. 1B a similar diagram for a more complicated Kondo-Lattice like Hamiltonian with identical but permuted eigenvalues. We assume it has the Kondo singlet ground state and lowest energy charge $2 \pm 1$ states consisting of zero or two c-electrons together with a single “f” electron. Eigenstates in Fig. 1B are labelled according to the same ordering scheme.

Our goal is to construct a unitary transformation $U$ that maps the Kondo singlet ground state in Fig. 1B to the vacuum in Fig. 1A and at the same time define quasiparticle operators that create the states of charge $2 \pm 1$ from the Kondo singlet. The arrows in Fig. 1B illustrate the action of the quasiparticle operators. A dotted hole creation operator creates the singly charged states and or a solid electron operator creates the triply charged states. We thus
map the ground state $|3\rangle_a$ to the vacuum $|1\rangle_c \equiv |G\rangle_c$ i.e. $U|3\rangle_a = |1\rangle_c$. The low energy state of one extra spin down electron $|12\rangle_a$ must map to $|9\rangle_c$; $U|12\rangle_a = |9\rangle_c$. Similarly we demand $U|9\rangle_a = |10\rangle_c$.

Symmetries can now be used to completely specify $U$. We define $Q$ to be the combined particle-hole spin flip transformation. This permutation is an inversion of the picture in Fig. II A through the state $|3\rangle_c$. Let $G_{ij}$ be the $16 \times 16$ matrix that has all entries zero except $G_{ij} = 1$. It can be verified that $Q = G_{1,5} + G_{3,3} + G_{5,1} + G_{6,6} + G_{7,7} + G_{8,8} + G_{9,12} + G_{10,11} + G_{13,16} + G_{14,15} - G_{16,13} - G_{15,14} - G_{12,9} - G_{11,10} - G_{4,2} - G_{2,4}$. Signs are chosen to preserve spin.

We next define $K$ to be the transformation that exchanges the two species of Fermions. This corresponds to the permutation obtained by exchanging the high and low energy state for each value of $n_{tot}$ in Fig. II A. Minus signs are inserted to make the transformation an element of the group of continuous rotations. We find $K = G_{1,1} + G_{2,4} + G_{4,2} + G_{5,5} + G_{6,6} + G_{7,7} + G_{8,8} + G_{10,9} + G_{11,12} + G_{14,13} + G_{15,16} - G_{16,15} - G_{13,14} - G_{12,11} - G_{9,10} - G_{3,3}$.

In order for $U$ to preserve spin and also generate particle and hole operators from the original Fermions, we demand $U$ obeys the following: $[U, S] = 0$, $n_{tot} U = U (2 + n_e - n_h)$ and $UK = QU$. It can be verified that these constraints together with the demand that $U$ be real and unitary results in

$$U = G_{1,4} + G_{2,3}/\sqrt{2} - G_{2,5}/\sqrt{2} + G_{3,1} + ,
\quad G_{4,3}/\sqrt{2} + G_{4,5}/\sqrt{2} + G_{5,2} + G_{6,6} +
\quad G_{7,7} + G_{8,8} + G_{9,10} + - G_{10,12} +
\quad G_{11,11} + G_{12,9} + G_{13,14} + - G_{14,16} +
\quad G_{15,15} + G_{16,13}$$

This defines a new Hamiltonian $h'_{atomic} = U_r^\dagger h_{canonical} U_r$ which has the energy diagram shown in Fig II B. The demand that $U^\dagger n_{tot} U = 2 + n_e - n_h$ shows that $U$ cannot be continued to the identity transformation which is presumably relevant for $J = 0$. This observation adds support to the existence of a quantum phase transition for sufficiently large values of $t/J$, a subject which is outside the scope of this investigation.

From unitarity follows that $\hat{c}_{e,\sigma}^\dagger(r) \equiv U_r^\dagger \hat{c}_{e,\sigma}^\dagger(r) U_r$ and $\hat{c}_{h,\sigma}^\dagger(r) \equiv U_r^\dagger \hat{c}_{h,\sigma}^\dagger(r) U_r$ preserve the Fermi anticommutation relations. Inverting this formula and using the fact that $U_r|\Psi_G\rangle_a \equiv U_r|3\rangle_a = |1\rangle_c \equiv |Vac\rangle_c$ we find that $c_{f,\sigma}(r)|Vac\rangle_c = 0$ is equivalent to $\hat{c}_{eh,\sigma}(r)|\Psi_G\rangle_a = 0$.  

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0. We have therefore obtained a local unitary transformation that maps the original “c” and “f” bare electron operators to quasiparticle operators that annihilate the Kondo singlet. By construction the operators $c^\dagger_{e,h}(r)$ either add or remove exactly one charge in each of the states and $\hat{c}_{e,\sigma}(r)$ is identified as an electron quasiparticle operator and $\hat{c}^\dagger_{h,\sigma}(r)$ as a hole.

Using the method in Ref. [4] and Ref. [5] we can explicitly write down the original operators $c^\dagger_{e,f}$ and $c_{e,f}$ as a polynomial of Fermion operators $\hat{c}_{e,h}(r), \hat{c}_{e,\sigma}(r)$. The result is

$$c^\dagger_{e,\uparrow} = \left( \frac{1}{\sqrt{2}} \hat{c}_{h,\downarrow} - \frac{1}{\sqrt{2}} \hat{c}_{e,\uparrow} \right) +$$

$$\quad \hat{c}_{h,\downarrow} \left( \tau_1 n_{h,\uparrow} n_{e,\uparrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\uparrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} \hat{c}_{h,\uparrow} \hat{c}_{e,\downarrow} - \frac{1}{\sqrt{2}} n_{h,\uparrow} n_{h,\uparrow} \right) +$$

$$\quad \hat{c}_{e,\uparrow} \left( \tau_1 n_{h,\downarrow} + \frac{1}{2} \hat{c}_{h,\uparrow} \hat{c}_{e,\downarrow} + \frac{1}{\sqrt{2}} n_{e,\downarrow} - \tau_1 n_{h,\downarrow} n_{e,\downarrow} - \tau_1 n_{h,\downarrow} n_{e,\downarrow} - \tau_1 n_{h,\downarrow} n_{h,\downarrow} - \tau_2 n_{h,\downarrow} \right) +$$

where $\tau_2 = (1 - 1/\sqrt{2})$ and $\tau_1 = \frac{1}{2}(\sqrt{2} - 1)$. For the “f” electrons I find

$$c^\dagger_{f,\uparrow} = \hat{c}_{e,\uparrow} \left( \frac{1}{2} n_{h,\uparrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} \hat{c}_{h,\uparrow} \hat{c}_{e,\downarrow} \right) +$$

$$\quad \hat{c}_{h,\downarrow} \left( \frac{1}{2} n_{e,\uparrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} n_{h,\uparrow} n_{e,\downarrow} + \frac{1}{2} \hat{c}_{h,\uparrow} \hat{c}_{e,\downarrow} - \frac{1}{\sqrt{2}} \hat{c}_{h,\uparrow} \hat{c}_{e,\downarrow} - n_{h,\uparrow} \right)$$

Similar formulas occur for the other spin component. We note that the “f” electron operators is constructed by third and higher order electron and hole operators in contrast to the conduction electron operators that have a linear coupling to quasiparticles. Deriving or even verifying these formulas is formidable without the help of a computer.

**Effective model for quasiparticles.** The transformation $U$ is now used to define a global transformation by $U_{global} = \prod_r U_r$. Since $U_r$ does not mix states of even and odd particle number we obtain $\{c^\dagger_{e,\sigma}(r), c^\dagger_{f,\sigma}(r')\} = \delta_{i,j} \delta_{r,r'}$, verifying that $U$ is indeed a global canonical transformation.

We now express the Kondo lattice model given by Eq. [1] in terms of electron and hole quasiparticle operators by replacing the Fermi operators $c_{e,f,\sigma}(r)$ by their representation as polynomials in $\hat{c}_{e,h,\sigma}(r)$.

By construction, the Kondo singlet is the quasiparticle vacuum in the limit $t \to 0$. For small values of $t$, quasiparticles can hop between adjacent sites. The resultant low energy charged charged states that appear in the interacting ground state are therefore described by single quasiparticle operators and the unitary transformation guarantees these local states also have the correct energy.

For small $t$ the states with local charge fluctuations will have a small amplitude in the interacting ground; the probability that a site is occupied by more than one quasiparticle
will be accordingly less. The system should therefore be well approximated by a dilute fermi gas.

We thus expand $H_{KLM}$ in quasiparticle operators. Inserting the quasiparticle representation for the original fermions, normal ordering then truncating the Hamiltonian to second order we find\[6\] $H_{KLM} = H_{KLM}^{\text{free}} + H_{KLM}^{\text{interacting}}$ with

$$
H_{KLM}^{\text{free}} = \frac{3}{4} J \left( n_e + n_f - 1 \right) + \\
\frac{1}{2} t \sum_s (-1)^{\frac{1}{2} + s} \left( \hat{c}_{e,s}^\dagger(r) \hat{c}_{h,-s}^\dagger(r') - \hat{c}_{e,s}(r') \hat{c}_{h,-s}(r) \right) + \\
\frac{1}{2} t \sum_s \hat{c}_{e,s}^\dagger(r) \hat{c}_{e,s}(r') - \hat{c}_{h,s}(r) \hat{c}_{h,s}(r') + \\
\frac{1}{2} t \sum_s \hat{c}_{e,s}^\dagger(r) \hat{c}_{e,s}(r) - \hat{c}_{h,s}(r) \hat{c}_{h,s}(r) + \\
- \frac{1}{2} t \sum_s (-1)^{\frac{1}{2} + s} \left( \hat{c}_{h,s}(r') \hat{c}_{e,-s}(r) - \hat{c}_{h,s}(r) \hat{c}_{e,-s}(r') \right)
$$

$H_{KLM}^{\text{free}}$ can then be treated exactly by transforming to momentum space and utilizing a conventional Bogoliubov transformation.\[5\] With a chemical potential we compute the eigenvalues exactly. We let $e_k = \sum_i \cos(k \cdot \hat{i})$. The quasiparticle spectrum is then given by $E_k = \frac{3}{4} \Delta_k \pm t e_k \pm \mu$ where $\Delta_k = \sqrt{1 + \left( \frac{4t e_k}{3} \right)^2}$.

It is reassuring that $U_f$ does not appear to quadratic order; a $J$ which is not small is what is crucial in creating the Kondo singlet and low-energy states. For completeness, the entire onsite part of $H_{KLM}$ is given by $H_{KLM}^{\text{interacting}} =

$$
(n_e(n_e - 1)n_h + n_e n_h (n_h - 1)) \left( \frac{3}{16} J + \frac{1}{4} U \right) + \\
(n_e(n_e - 1) + n_h(n_h - 1)) \left( \frac{3}{8} J + \frac{1}{2} U \right) + \\
\left( (S_e \cdot S_h) \left( \frac{3}{4} J - U \right) + n_e n_h \left( \frac{-9}{16} J + \frac{1}{4} U \right) \right)
$$

High order terms in the hopping become too lengthy to write out here, but are unimportant for the small $t$ physics since not only is the prefactor $t$ small, but also, as we shall see, the fourth order Fermion operators become unimportant since the Fermi gas is dilute.

**Spectral weights.** Since the local electron operator is represented in terms of local quasiparticles the spectral weights can be computed for $H_{KLM}^{\text{free}}$. As an example, the results are plotted in in Fig. 2 where the spectral weight for the c-electron $A_c(k, \omega)$ is plotted as a function of $\omega$ several values of $k$ from 0 to $\pi$. The quasiparticle coherent peak is seen as a broadened $\delta$ function. The incoherent structure is comprised of three-quasiparticle contributions. Not shown are the five-quasiparticle contributions that represent less than 1% of the total spectral weight. We observe that $A_c(k + \pi, \omega) = A_c(k, -\omega)$ a consequence of the particle-hole symmetry of $H_{KLM}$. 

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The quality of the wave function as an approximation to the Kondo Lattice ground state can be measured by the overlap of the ground state with the Hilbert space of $H_{KLM}$. Let

$$\alpha = \frac{1}{2} \int_0^\lambda \frac{1}{\Delta_k^{-1}} \left( \frac{d\theta}{2\pi} \right)^d \approx \frac{2d}{9} \lambda^2 + O(\lambda^4)$$

where $\lambda = t/J$ and $d$ is the dimension. The deviation from unity is measured by

$$\delta_P = \langle (1 - n_f)^2 \rangle$$

which given exactly by

$$\delta_P = \alpha^2 (3 - 2\alpha)$$

demonstrating convergence like $(t/J)^4$ for small $t$. We also find

$$\langle n_e + n_h \rangle = 4 \alpha$$

which verifies that the number of quasiparticles remains surprisingly small even as the Kondo lattice model approaches a phase transition.

A more challenging quantity to compute is the expectation value of the complete full Kondo Lattice Hamiltonian with $U_f = 0$ in the ground state of $H_{KLM}^{\text{free}}$. This quantity is important since it is a rigorous upper bound to the true Kondo lattice ground state energy. We define

$$\beta = \int T^d e_k^2 / \Delta_k \left( \frac{d\theta}{2\pi} \right)^d \approx \frac{4}{2} - \frac{4}{3} \lambda^2 + O(t^4)$$

and find

$$\langle H_{KLM} \rangle^{\text{free}} = \frac{3}{4} + 3 \alpha + \frac{15}{4} \alpha^2 + \frac{3}{2} \alpha^3 + \frac{8}{3} \beta \lambda^2 + .$$

$$\alpha^3 \beta \lambda^2 \left( 72 + 48 \sqrt{2} \right) + \alpha \beta^2 d^{-2} \lambda^4 \left( \frac{64}{3} + \frac{128}{9} \sqrt{2} \right) +$$

$$\alpha \beta \lambda^2 \left( 16 - 8 \sqrt{2} \right) + \beta^2 d^{-4} \lambda^6 \left( \frac{64}{27} + \frac{128}{81} \sqrt{2} \right) +$$

$$\beta^3 d^{-2} \lambda^4 \left( -\frac{64}{9} + \frac{32}{9} \sqrt{2} \right) + \alpha^2 \beta^3 d^{-2} \lambda^4 \left( -\frac{64}{3} + \frac{128}{9} \sqrt{2} \right) +$$

$$\alpha^4 \beta \lambda^2 \left( -36 + 24 \sqrt{2} \right) + \alpha^2 \beta \lambda^2 \left( -52 + 32 \sqrt{2} \right)$$
This can be compared to high order series expansions. We find our result is at most about 3% above the series result, converging very rapidly for small values of $\lambda$. Errors are bounded by $0.61\lambda^4$ for $d = 1$, $2.25\lambda^4$ for $d = 2$ and $4.9\lambda^4$ for $d = 3$ in the entire domain of convergence of the power series.

**Conclusions.** An exact canonical transformation is derived that maps bare electron operators to quasiparticle electron and hole operators. This allows us to approximate the Kondo lattice model as a dilute Fermi gas for large values of the Kondo coupling. Properties of the Fermi gas can then be computed exactly. The overlap per site between the free Fermi gas ground state and the Hilbert space of the Kondo Lattice model is $(1 - O(t/J)^4)$ and ground state energies are also accurate to this order. Spectral weights can be computed that exactly obey the sum rules.

Together with the exact relation between quasiparticle operators, the free quasiparticle Hamiltonian represents an exactly solvable model of a singlet Mott insulator. The difficulty of understanding electrons in this Mott insulator is encoded in the complicated relation between electrons and quasiparticles rather than any subtlety in the quasiparticle dynamics.

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[3] We treat the Kondo spin as an extra fermion. In the exact KLM model the $U$ term should not be present and the “f” electron should be strictly represented as a localized spin. Our treatment of particles and holes requires the “f” electron to represent a complete fermionic degree of freedom. This observation is consistent with others who have come to the same conclusion. See M. Oshikawa, Phys. Rev. Lett. 84, 3370 (2000) ; M. Oshikawa, M. Yamanaka, and I. Affleck, Phys. Rev. Lett. 78, 1984 (1997); P. Coleman, I. Paul and J. Rech, Phys. Rev. B. 72, 094430 (2005).
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well as a guide to the computer algebra.

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