Superbubble dynamics in globular cluster infancy

II. Consequences for secondary star formation in the context of self-enrichment via fast rotating massive stars

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1. Introduction

Recent detailed spectroscopic and deep photometric studies have lead to the conclusion that Galactic globular clusters (GCs) host multiple generations of stars; those differ in their chemical abundances and in some cases in their membership to multimodal sequences in the colour-magnitude diagram (e.g. Gratton et al. 2004, 2012; Piotto 2009; Charbonnel 2010, for recent reviews). The self-enrichment scenario to explain the observations is likely complex. It requires that the products of hot hydrogen-burning ejected by fast evolving first generation stars, remain within the GC and are recycled in second generation stars to explain e.g. the ubiquitous O-Na anti-correlation; but on the other hand, helium-burning products and supernova (SNe) ejecta have to be removed in order to keep the constancy of [C+N+O] and the mono-metallicity observed in most GCs.

Two main models have been developed in the literature, according to the nature of the first generation stars potentially responsible for the GC self-enrichment (hereafter the stellar polluters), namely the fast-rotating-massive-stars (FRMS) or the asymptotic-giant-branch stars (AGB). Both the FRMS and the AGB scenarios face a problem in the mass budget between the amount of matter provided by the polluter stars and the mass locked today in first and second generation low-mass stars (about 30 vs 70 % respectively in most GCs, and up to 50-50 in few cases; for more details see Carretta et al. 2009). Solutions require either a top-heavy IMF for the first stellar generation (D’Antona & Catoli 2004; Bekki & Norris 2006; Prantzos & Charbonnel 2006; Decressin et al. 2007), or substantial loss of first generation low-mass stars from the GCs.
which must then have been much more massive initially than today (Deccrassin et al. 2007a; Bekki et al. 2007; D’Ercole et al. 2008; Decressin et al. 2010; Schaerer & Charbonnel 2011), the second option being currently favoured. In the FRMS scenario, the hydrogen-burning ashes would be ejected from their fast rotating parent stars due to repeated super-critical rotations. These ejecta are thought to mix with pristine gas and then form the second generation stars in the immediate surroundings of the massive star polluters. (Prantzos & Charbonnel 2004; D’Ercole et al. 2007a). Initial or very early mass segregation together with quick gas expulsion after the formation of the second stellar population, are invoked to account preferentially first generation low-mass stars and retard the observed proportion of first and second generation stars (Decressin et al. 2010). In the asymptotic-giant-branch (AGB) scenario, the pristine gas left after the formation of the first stellar generation is supposed first to be expelled together with the unwanted SNe ejecta and the bulk of first generation low-mass stars. Later when the SNe would have ceased, the slow winds of AGB stars would accrete in a cooling flow towards the GC centre, where they would form the second generation of stars after dilution with re-collected pristine gas whose sources are still uncertain (D’Ercole et al. 2008; 2010, 2011, 2012; Conroy & Spergel 2011).

In Krause et al. (2012, Paper I) we showed that gas expulsion via SNe, which has long been the prevailing paradigm to change the GC potential well and induce the loss of first generation low-mass stars, does not work in GCs. The reason is that, while the energy produced by SNe usually exceeds the binding energy, it is not delivered fast enough to avoid the Rayleigh-Taylor instability of the escaping shell. We have also shown that a more powerful event, such as the energy released by accretion on to the dark remnants, may lead to successful gas expulsion. These results suggest three distinct chronological phases in typical GCs: Wind bubbles are created during the whole period from about 0 to 8.8 Myr on our global timescale. Core collapse to black holes and neutron stars occurs from about 3.5 Myr until 35 Myr. Where black holes are formed, the collapse may not be accompanied by a strong or even any momentum ejection. If we therefore consider that only stars with initial masses below 25 M⊙ will give birth to an energetic supernova event then energy ejection by supernovae will occur only between 8.8 and 35 Myr on the global timescale. Finally, the dark remnants are activated and expel the gas as well as the majority of the first generation stars.

Here, we explore the implications of these findings for the formation of the second generation stars in the context of the FRMS scenario and develop a detailed timeline for the first ≈ 40 Myrs in the lifetime of a typical GC. We describe the basic model setup in Sect. 2. The wind bubble, supernova and dark remnant accretion phases are described in Sects. 3, 4 and 5 respectively. We discuss our findings in Sect. 6 and summarise and conclude in Sect. 7. In order to describe the sequence of events, we refer to a global timescale throughout this article. The global clock is set to zero at the coeval birth of the first generation of stars.

Table 1. Parameters of model cluster

| Parameter | Value | Description |
|-----------|-------|-------------|
| M tot     | 9 x 10⁸ M⊙ | Total mass |
| r 1/2     | 3 pc | Half-mass radius |
| e sf      | 1/3 | Star formation efficiency |
| ρ 0       | 10⁶ m p cm⁻³ | Average gas density |

2. Basic model setup

The model presented here generally follows the ideas outlined in Decressin et al. (2007b, 2010). We consider an initial dense gas cloud of M tot = 9 x 10⁸ M⊙ with a half-mass radius of r 1/2 = 3 pc which forms the first generation stars at an efficiency of e sf = 1/3 and according to a Salpeter initial mass function (IMF) for first generation stars more massive than 0.8 M⊙ and to a log-normal distribution for less massive long-lived stars of first and second generations. The parameters summarised in Table 1 are inferred from N-body simulations which assume a Plummer distribution for the spatial distribution of gas and stars (i.e., the star formation efficiency is similar at all radii; Decressin et al. 2010). For our model cluster, we find about 5700 first generation massive stars between 25 and 120 M⊙. Mass segregation leads to all massive stars to be located within r 1/2. We use the stellar evolutionary models of FRMS at subsolar metallicity (Z = 0.0005, [Fe/H] = −1.5) presented by Decressin et al. (2007b). From these models, we have extracted the relevant wind parameters, summarised in Table 2. Decressin et al. (2010) use these parameters to describe the GC NGC 6752 with a present day mass of 3 x 10⁹ M⊙, assuming complete recycling of the slow wind released by FRMS and dilution with pristine gas to reproduce the observed Li-Na anti correlation. For this setup, we will now explore the different phases in detail. A sketch of the model is provided in Fig. 1.

3. Hot wind bubbles

We will now argue (Sect. 3.1) that during the first few Myrs of the GC, before the first type II SNe, the fast radiative winds of the massive stars will create large bubbles that will probably also overlap, but they will probably not unite into a single superbubble and will not lift any important amount of gas out of the GC. This situation is depicted in Fig. 1 (second row from top). We also show that during this wind bubble phase (and also the supernova phase), no stars should form in the normal way (Sect. 3.2), the equatorial ejections of the FRMS establish a decretion disc close to the FRMS (Sect. 3.3), and that accretion may be re-established in the shadow of these accretion discs (Sect. 3.4). The inner decretion and the outer accretion discs merge on the viscous timescale (Sect. 3.5), and finally the second generation stars form in the merged discs (Sect. 3.6).

3.1. Bubble expansion

To see that the wind bubbles may not unite into a single superbubble and that they will also not lift much gas out of the GC’s potential well, we consider the energy budget in the cluster. Freyer et al. (2003) derive that the mechanical power output of a star dominates the effect on the intra-cluster medium (ICM) rather than the effect of ionisation. The power output of the fast winds of a FRMS with a given mass M between 40M⊙ and 120M⊙ may be approximated by

\[ Q_0(M) = 4 \times 10^{39} (M/M_\odot)^{2.8} \text{ erg s}^{-1}. \]
This power law follows from interpolation of the values in Table 2. With a Salpeter IMF, the total wind output, which is dominated by the most massive stars, integrates to: \( Q_{0.08} \approx 2 \times 10^{59} \text{ erg s}^{-1} \times 6 \times 10^{52} \text{ erg Myr}^{-1} \).

How much of this energy is actually used to extend the bubbles and not lost to radiation? Frey et al. (2003) derive that for a background density of 20 cm\(^{-3}\), much lower than the density expected in a young GC, about 90 per cent of this power is radiated away, and thus the energy efficiency is only 10%. If the bubble would remain self-similar, a greater energy efficiency is expected, around 70% (Weaver et al. 1977; Frey et al. 2003). Krause et al. (2013) study the energy efficiency of merging wind bubbles, and find that for small separations, there is no additional effect from bubble merging and the efficiency factor is not changed. We expect that in the dense environment of a forming GC radiation losses may increase and the efficiency of energy transfer may be reduced below the value of Frey et al. (2003). On the other hand, the optical depth is higher, and more of the radiation energy may be captured by the gas. In the following, we adopt \( \eta = 0.1 \) as a working hypothesis for the energy efficiency, but the conclusions would be unchanged if \( \eta = 0.01 \).

The gravitational binding energy of the gas is \( E_{\text{grav}} = 0.4(1 - \epsilon_d)G M^2 / R \approx 6 \times 10^{53} \text{ erg} \) (Baumgardt et al. 2008). The combined wind power of all the massive stars amounts to \( \eta Q_{0.08} = 6 \times 10^{51}(\eta/0.1) \text{ erg Myr}^{-1} \). Thus, it is not possible for the stellar winds to lift any noteworthy amount of gas out of the GC on a relevant timescale.

On the other hand, we may integrate the volume fraction occupied by the wind bubbles by integration of the standard wind bubble volume \( 4 \pi r_{\text{bubble}}^3 / 3 \) over the IMF. The bubble radius \( r_{\text{bubble}} \) at time \( t \) is given by:

\[
r_{\text{bubble}} = \alpha \left( \frac{\eta Q_{0.08}}{\rho_0} \right)^{1/5} t^{3/5},
\]

where \( \alpha \approx 0.8 \) for self-similar expansion (Weaver et al. 1977), and the not necessarily self-similar thin shell approximation (Krause 2003). The average gas density in our model cluster is \( \rho_0 = 10^6 m_p \text{ cm}^{-3} \). We include the efficiency factor \( \eta \) in Eq. 2 because the expansion law in this form is consistent with the sizes of observed wind bubbles (e.g. Oey & Garcia-Segura 2004, and references therein). We show the volume fraction occupied by hot bubbles in Fig. 2. The hot wind bubbles fill the GC very quickly on a timescale of 0.1 Myr.

Thus, while the wind bubbles are not able to lift the gas out of the potential well of the GC, they are nevertheless able to quickly fill the entire volume within the half-mass radius, no matter what we assume in detail for the energy efficiency. It follows that the bulk of the cluster gas is compressed into thin filaments or sheets, while most of the volume is occupied by hot wind bubbles. Bubbles of stars of different masses in general do not have the same pressure, and thus many individual interfaces may be expected to be swept away and pushed against some other bubble walls (van Marle et al. 2012; Krause et al. 2013). So, overall it is clear that some individual bubbles will unite to form larger bubbles, but there will not be one overall superbubble which would imply significant lift-up of gas out of the potential well of the GC.

### 3.2. Lyman-Werner flux

Conroy & Spengler (2011) make the point that during the main sequence phase of the massive stars, the Lyman-Werner flux (912 Å – 1100 Å) is so high that molecular hydrogen is dissociated throughout the cluster, and no stars may form in the usual way during this time. We repeat their analysis here, but with the numbers from the FRMS models of [Decressin et al. (2007)]. We have calculated the Lyman-Werner flux from these models and included it in table 2. A reasonable fit to the photon fluxes, which are roughly constant during the main sequence, as a function of mass of the parent star is:

\[
Q_{\text{LW}} = 7 \times 10^{43}(M/M_\odot)^{2.9} \text{ s}^{-1}.
\]

Conroy & Spengler (2011) estimate the radius of the photodissociation region to

\[
R_{\text{LW}} = 1.1 \text{ pc} \left( \frac{Q_{\text{LW}}}{10^{49} \text{ s}^{-1}} \right)^{1/3} \left( \frac{10^6 m_p \text{cm}^{-3}}{\rho} \right)^{1/3} \left( \frac{0.028 Z_\odot}{Z} \right)^{1/3},
\]

adapted to the metallicity of our model stars. Thus, a 120M_\odot star is already able to photo-dissociate a large fraction of our model GC. Integration over a Salpeter IMF gives the factor \( f_{\text{LW}} \), which describes how many times the GC volume could be photodissociated by the ultraviolet radiation:

\[
f_{\text{LW}} = 200 \left( \frac{10^6 m_p \text{cm}^{-3}}{\rho} \right) \left( \frac{0.028 Z_\odot}{Z} \right).
\]

So, even if the metallicity or the gas density would be higher by one or even two orders of magnitude, the gas would still be fully photodissociated.

In addition to the effects of dust considered in the above analysis, molecular hydrogen may also form via electrons, which first combine with neutral hydrogen to \( H^+ \). \( H_2 \) is then formed via the reaction:

\[
H + H^+ \rightarrow H_2 + e^+.
\]

Regions where both, electrons and neutral hydrogen atoms, are abundant are rare. Ricotti et al. (2001) show that the interface between an \( H \) II region and a neutral region is such a promising place, and that molecular hydrogen may form there in a narrow layer. Ricotti et al. (2001) find a strong dependence on the input spectrum and a maximum column density of log\( (N_{H_2}/\text{cm}^{-2}) = 14 - 15 \) of molecular hydrogen formed in this layer. The corresponding gas mass itself would be too small to form any significant amount of stars in the GC context. However, Ricotti et al. (2001) also show that the column is sufficient for some Lyman-Werner bands to become optically thick. Hence, such layers might in principal shield pockets of

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**Fig. 2.** Combined volume of the wind bubbles of all the 5700 massive stars (\( M > 25M_\odot \)) in our model cluster as fraction of the half mass radius. The solid (dotted, dashed) line is for an energy injection efficiency of \( \eta = 0.1 \), (0.01).
dense gas from photo-dissociation. Quantitatively, 

\[ f_{\text{shield}}(N_{\text{H}_2}, T) = \frac{0.965}{(1 + x/b_5)^{3/4}} + \frac{0.035}{(1 + x)^{0.5}} \times \exp \left[ -8.5 \times 10^{-4} (1 + x)^{0.5} \right], \]  

(7)

where \( b_5 \equiv b/(\text{km s}^{-1}) \) is the Doppler broadening parameter. If thermal broadening dominates, \( b_5 \) should be around or below unity. If their is substantial velocity shear in the filaments, it might be higher. A reduction of the Lyman-Werner flux by a factor of 200 (in order to get the photo-dissociated volume fraction below unity) is reached at \( \log(N_{\text{H}_2}/\text{cm}^{-2}) = 17 (16, 18) \) for \( \log(b_5/\text{km s}^{-1}) = 0 (-1, 1) \). This confirms that the aforementioned layers at the interface of ionised and neutral regions with \( \log(N_{\text{H}_2}/\text{cm}^{-2}) = 14 - 15 \) would not provide a significant optical depth which could significantly alter the conclusion above. A combination of this effect together with strong dust absorption might perhaps allow for some molecular hydrogen in the densest parts of the ICM. However, the question would remain, why these dense pockets would not have formed stars right away.

We therefore confirm the result of 

\[ M_{\text{eq}} \approx 9 \left( \frac{M}{60M_\odot} \right)^{-0.8} M_\odot/ \text{Myr}, \]  

(8)

for stars with mass \( M > 32M_\odot \). The latter limit is simply due to the zero-point in Eq. (8). This is however quite realistic, as the 40 \( M_\odot \) model still has a high equatorial mass loss rate, whereas in the 25 \( M_\odot \) model it is close to zero.

3.4. Accretion on to the discs around the massive stars

3.4.1. The discs persist around the FRMS

The off-time between any two equatorial mass ejections is a few times the on-time (Table 2). Also the radiative energy accumulated in the off-time, when no new mass is ejected equatorially, but the radiation pressure keeps pushing, is likely not sufficient to unbind the entire disc from the parent star. Another

Our wind power \( Q_0 \) refers to the terminal wind power of the fast, radiatively driven wind, but the additional power required to lift the material ejected in this wind out of the gravitational potential is less than 50 per cent of \( Q_0 \).

Table 2. Properties of low metallicity fast rotating massive stars.

| \( M_{\text{eq}}/ \) | \( R_0/ \) | \( M_{\text{fast}}/ \) | \( M_{\text{slow}}/ \) | \( t_{\text{eq, start}}/ \) | \( t_{\text{eq, end}}/ \) | \( \Delta M_8/ \) | \( \Delta M_9/ \) | \( Q_0/ \) |
|---|---|---|---|---|---|---|---|---|
| \( M_\odot \) | \( R_\odot \) | \( M_\odot \) | \( M_\odot \) | Myr | Myr | Myr | Myr | 10^{34} \text{erg s}^{-1} |
| 120 | 12 | 1 | 66 | 0.7 | 3.15 | 0.020 | 0.080 | 28 | 9 |
| 60 | 10 | 0.2 | 14 | 0.9 | 4.4 | 0.028 | 0.067 | 3.9 | 1.26 |
| 40 | 8 | 0.06 | 14 | 1.1 | 5.7 | 0.065 | 0.250 | 1.2 | 0.192 |

Notes. (a) Mass of the star. (b) Photospheric radius. (c) Mass loss rate of the steady fast wind. (d) Mass loss rate of the slow equatorial wind. (e) Global time at which the intermittent equatorial ejections start. (f) Global time at which the intermittent equatorial ejections end. (g) Duration of the individual slow wind episodes. (h) Time span in between any two slow wind episodes. (i) Total power of the steady, radiatively driven fast wind on the main sequence. (j) Flux of Lyman-Werner photons (912Å – 1100Å) at 1 Myr after reaching the main sequence; roughly constant during the main sequence.
process to consider is close encounters of other massive stars, which could in principle unbind a disc (Clarke & Pringle 1993; Hall et al. 1996). In the unfavourable prograde encounter, the disc would be stripped down to about half the periastron radius. The timescale for a given massive star to have a close encounter with another massive star is given by \((n\sigma r)^{-1}\), where \(n = 20,000/(4\pi r_{1/2}^3/3) = 192\) \(r^{-3}\) is the density of massive stars, \(\sigma = \pi(2R_{sd})^2\) is the characteristic cross section of the accretion disc, and \(v \approx 100\) km/s is the typical velocity of the massive stars. Using any of the values for the disc radii discussed above, the encounter timescale turns out to be much larger than the Hubble time. Thus, the discs cannot be tidally stripped. We note that, due to the small cross section of the FRMS discs, ram-pressure stripping is equally ineffective.

3.4.2. Re-establishment of accretion

Importantly, the shielding from the radiation of the parent star should allow a re-establishment of the accretion flow providing pristine gas (Fig. 1, second row from top, right). The Bondi accretion rate should set an upper limit for the mass accumulation in the disc. With the sound speed for an atomic gas consisting of 90 per cent hydrogen and 10 per cent helium by number, \(c_s^2 = y k_{\text{B}} T/(1.27 m_p)\), and a solid angle fraction \(f_{sd}\) of the disc usable for accretion, the Bondi accretion rate evaluates to (Frank et al. 2002):

\[
M_{\text{Bondi}} = \frac{4600 M_\odot}{\text{Myr}} \times \frac{f_{sd}}{0.1} \left( \frac{M}{60 M_\odot} \right)^2 \left( \frac{\rho}{10^5 m_p \text{cm}^{-3}} \right) \left( \frac{100 \text{ K}}{T} \right)^{3/2}. \tag{9}
\]

This accretion rate is most likely not sustained throughout the lifetime of the massive stars. One reason is that the dense gas should be compressed in filaments at this time (compare above), so that accretion may only take place when the star passes through a filament. We show below that, for suitable orbit parameters, the total path an FRMS traverses at low velocities is long compared to the typical bubble size. Since the dependence on the density is linear, we therefore do not need to take this into account for the present discussion, as a reduced accretion rate in voids would be compensated by an increased one in the filaments.

3.4.3. Suppression of accretion due to stellar velocities

If a star moves with a velocity \(v\) relative to the local ICM, the accretion rate is reduced by a factor \((v/c_s)^3\) (Bondi & Hoyle 1944). For typical virial velocities of about 100 km/s, this would lead to a suppression of about six orders of magnitude. Orbits of stars in GCs are not expected to be circular, and therefore the accretion rate is a strong function of the position of a given star along its orbit: It accretes more strongly when the velocity relative to the ICM is small, i.e. near the outer turning point and when at the same time it passes through a filament.

We will now estimate the time averaged accretion rate for a given star. In a spherical potential, the orbits are generally expected to be of the rosette type (Binney & Tremaine 2008), with most of the stars having only slightly negative total energies. Adapted to our model cluster, most of the massive stars will have just enough energy to reach the core radius at their outer turning points. As may be seen from Fig. 3 the circular velocities at the core radius (2.3 pc) are around 80 km s\(^{-1}\). If the stars would have no angular momentum, they would reach a similar velocity after a free fall to the centre from this position. Consequently, velocities around 1 km/s, as necessary for efficient accretion may only be reached for very eccentric orbits and only near their outer turning points.

For this estimate, we approximate the orbits by ellipses. The radius \(r\) as a function of azimuth \(\phi\) is then given by

\[
r = \frac{p}{1 + e \cos(\phi)}, \tag{10}
\]

with the eccentricity \(e\) and the orbital size parameter \(p\). The orbital velocity follows by differentiation and may be expressed as

\[
v_c(\phi) = v_0 \sqrt{1 + 2e \cos(\phi) + e^2} \over 1 - e^2, \tag{11}
\]

where \(v_0^2 = GM_{\text{tot}}/a\) with the semi-major axis \(a\). Instead of the eccentricity, we use in the following the ratio of the velocity at the outer turning point to the circular velocity at this location as parameter. It is given by \(v_{az,0}/v_{\text{circ}} = \sqrt{(1-e)/(1+e)} v_0/v_{\text{circ}}\). The orbit-averaged Bondi accretion rate may then be found by integration of Eq. 9 over the appropriate number of orbits, \(n\), multiplied by the suppression factor, \((c_s/v_c)^3\), and dividing by \(n\) times the orbital period \(T\):

\[
< M_{\text{Bondi}} >= M_{\text{Bondi}} \frac{1}{n T} \int_0^{n T} \left[ \min \left( 1, \frac{c_s}{v_c} \right) \right]^3 \, dr. \tag{12}
\]

We remark here that \(M_{\text{Bondi}}\) depends on density and temperature. We have taken this out of the integral in Eq. 12, because the changes in density along the path of a given star – due to filaments and voids – are not correlated to the changes in velocity. Again, since the Bondi accretion rate is linear in the density, we may use the average value for the density as long as we make sure that the total path traversed while the velocity of the star is small (the contribution to the integral is negligible otherwise) is long compared to the average bubble diameter. This is indeed the case for an interesting part of the parameter space in \(v_{az,0}/v_{\text{circ}}\): We parametrise the accretion suppression by \(f = < M_{\text{Bondi}} > / M_{\text{Bondi}}\) and show the path-length per orbit per
Fig. 4. Differential path length over accretion suppression factor $f = M_{\text{Bondi}}/M_{\text{Bondi}}$. The five curves labelled by their respective $v_{xz,0}/v_{\text{circ}}$ value show the path length in parsecs that a star spends at a given accretion suppression factor $f$ per orbital period and per decade in $\log(f)$. The range of $f$-values that our model requires is to the right of the solid vertical bar. The horizontal dotted lines show the limiting path length for statistically uniform accretion. The plot shows that the accumulated path length is sufficient for uniform accretion for all orbits with $v_{xz,0}/v_{\text{circ}} \geq 3\%$. For orbits with smaller values of $v_{xz,0}/v_{\text{circ}}$, accretion varies strongly from star to star, and the average value shown in Fig. 5 would only apply to the ensemble. The plot also shows, that for all orbits with $v_{xz,0}/v_{\text{circ}} < 10\%$ there is always a basic amount of gas that is uniformly accreted on to each FRMS disc. This amount is sufficient for our model. The part of the accreted gas that varies from star to star comes only on top of this basic amount. See section 3.4.3 for details.

Fig. 5. Dependence of the time averaged accretion rate on to the FRMS as a function of the ratio of the azimuthal velocity at the outer turning point, $v_{xz,0}$, over the circular velocity $v_{\text{circ}}$, for the 60$M_\odot$ star (see section 3.4 for details). The solid line shows the elliptical approximation. The pluses show the result of a direct orbit integration in the actual Plummer potential. There is excellent agreement between the two methods.

The orbital period for stars which have the outer turning point near the core radius is 0.16 Myr for all eccentricities. The minimum accretion time is given by the equatorial ejection phase of the 120$M_\odot$ star, 2.45 Myr. The maximum accretion time would be 4.6 Myr, which would correspond to the 40$M_\odot$ star and the case when the massive stars turn into black holes without energetic SNe. Hence, the FRMS complete 15 to 29 orbits in the GC during the time when they have equatorial ejections. In order to accumulate a total path at low velocity of a bubble diameter, they must therefore spend at least 0.008 pc or, respectively, 0.015 pc at low velocities, per orbit. These two limits are shown as dotted horizontal lines in Fig. 4. A vertical solid line marks the minimum accretion rate we must demand to explain the relative contributions of pristine and ejected gas in the presently observed GC stars. It can be seen that, for small values of $v_{xz,0}/v_{\text{circ}}$, the differential path length distribution is well described by a power law,

$$\frac{ds}{d\log(f)} \propto f^{-3/5},$$

with an upturn at high values of $\log(f)$. Figure 4 shows that the disc is sufficient for our model. Only on top of this, there can be a strongly varying component. Within these limitations, we may therefore use Eq. (12) to describe the average accretion rate.

The orbit-averaged accretion rate for the 60$M_\odot$ as a function of $v_{xz,0}/v_{\text{circ}}$ is shown in Fig. 5. We have also derived the orbit-averaged accretion rate by direct numerical integration of the orbit in the Plummer potential, which is shown as pluses in the same plot. The good agreement is expected, as the inner regions, where the elliptical approximation is bad, do not contribute much to the average accretion rate. We note that the time averaged accretion rate may be well approximated by:

$$M_{\text{Bondi}} = 0.7 \left( \frac{v_{xz,0}/v_{\text{circ}}}{0.1} \right)^{-2} M_\odot / \text{Myr} \times \left( \frac{f_{sd}}{0.1} \right) \left( \frac{G M}{60M_\odot} \right)^2 \left( \frac{\rho}{10^6 m_p cm^{-3}} \right) \left( \frac{100 K}{T} \right)^{3/2},$$

which is fitted to the curve in Fig. 5.

While the characteristic velocity $v_0$ leads to a suppression of the accretion rate by about six orders of magnitude, high orbit eccentricity may compensate this to some degree. An accretion rate of about 1.5 $M_\odot$/Myr, as required to get the right dilution (i.e., that requested to explain the observed Li-Na anticorrelation of ejected and pristine gas (compare Eq. (8)), is reached for $v_{xz,0}/v_{\text{circ}} = 7\%$, for the 60$M_\odot$ star. In this case, the gas in the disc around the 60$M_\odot$ star would consist – on average – of 74% ejected gas from the parent star, and 26% of accreted pristine gas. The accretion rate grows quadratically with the mass of the accreting star, whereas for the equatorial mass ejection rate the dependency is linear. The pristine gas fraction varies therefore with the mass of the parent star. At 60$M_\odot$ it is close to minimal, for 40 (120) $M_\odot$ it rises to 35% (31%), respectively, assuming again $v_{xz,0}/v_{\text{circ}} = 7\%$. We discuss this further in section 3.6.2.
3.5. Viscous merging of accretion and decretion discs

Because of the angular momentum that is likely present in the gas, it will settle into the disc at some radius \( r \), which is likely large compared to the typical locus of the equatorial stellar ejecta. Viscous processes are then responsible for transporting the material within the accretion disc. For a steady, thin, Keplerian disc, the viscous timescale is given by (Frank et al. 2002, p 88):

\[
\tau_{\text{visc}} = \frac{\sqrt{GM\Sigma}}{\alpha c_s^2} = 1.6 \text{Myr} \left( \frac{M}{60 M_\odot} \right)^{1/2} \left( \frac{c_s}{1 \text{km s}^{-1}} \right)^{-2} \left( \frac{\alpha}{0.1} \right) \left( \frac{r}{0.1 \text{pc}} \right)^{1/2},
\]

where a typical value for \( \alpha \) is 0.1 and we have taken a radius \( r \) of order the average distance between massive stars.

Thus, the viscous evolution of the disc is happening on a similar timescale as the equatorial mass ejection. The re-formed discs near the massive stars are fed by both, material nuclearly processed in the massive stars and expelled via the equatorial discs near the massive stars are fed by both, material nuclearly of order the average distance between massive stars.

Discs that are comparable in mass to the central object are known as self-gravitating discs (compare e.g. Sect. 3.1 in the review by Armitage 2011). The Toomre criterion (e.g. Shu 1992) readily shows that the inner discs are expected to be gravitationally unstable:

\[
Q_T = 0.7 \left( \frac{c_s}{10 \text{km s}^{-1}} \right) \left( \frac{m}{10 M_\odot} \right)^{-1} \left( \frac{M}{60 M_\odot} \right)^{1/2} \left( \frac{r}{100 R_\odot} \right)^{1/2},
\]

where \( m \) is the disc mass, which we assume to be equally distributed within the scale radius \( r \) for this estimate. We have assumed a higher sound speed in the disc than calculated for the gas on larger scales, because near the massive stars, the radiation will heat the discs (compare e.g. Ahmed & Sigut 2012). However, even for a sound speed of 10 km/s, the inner discs reach the critical mass for gravitational instability on a timescale of 10^6 years.

3.6. Star formation in the discs

Discs that are comparable in mass to the central object are known as self-gravitating discs (compare e.g. Sect. 3.1 in the review by Armitage 2011). The Toomre criterion (e.g. Shu 1992) readily shows that the inner discs are expected to be gravitationally unstable:

\[
Q_T = 0.7 \left( \frac{c_s}{10 \text{km s}^{-1}} \right) \left( \frac{m}{10 M_\odot} \right)^{-1} \left( \frac{M}{60 M_\odot} \right)^{1/2} \left( \frac{r}{100 R_\odot} \right)^{1/2},
\]

where \( m \) is the disc mass, which we assume to be equally distributed within the scale radius \( r \) for this estimate. We have assumed a higher sound speed in the disc than calculated for the gas on larger scales, because near the massive stars, the radiation will heat the discs (compare e.g. Ahmed & Sigut 2012). However, even for a sound speed of 10 km/s, the inner discs reach the critical mass for gravitational instability on a timescale of 10^6 years.

3.6.2. The distribution of orbit eccentricities and the ratio of pristine-to-processed gas in the second generation stars

We have seen in Sect. 3.4.3 above, that the amount of gas a given FRMS disc accretes from the ICM depends strongly on the parameter \( v_{\text{az}}/v_{\text{circ}} \). The distribution function of \( v_{\text{az}}/v_{\text{circ}} \) for the sample of FRMS in the cluster will therefore directly determine the distribution of the amounts of accreted ICM to the FRMS discs.

The actual abundance pattern in each second generation star will however not only be determined by the overall ratio of pristine gas accreted from the ICM over ejected gas from the FRMS. The abundances in each disc will be also a function of time because of the time dependence of the abundances in the equatorial ejecta (Decressin et al. 2007b). The composition of the equatorial ejecta is first very similar to the one of the pristine gas. As time goes on, the composition changes and shows increasing signatures of hydrogen burning. Decressin et al. (2007b) have shown that the observed spread in light element abundances may only be obtained if the time variation of the abundances in the ejecta is used. For example, only at late times do we yield oxygen depletions high enough to be compatible with the oxygen poor end of the abundance distributions in many GCs (Gratton et al. 2012). If we would mix all the ejecta of a given star into a common reservoir, which would be the disc in our case, the average oxygen depletion would already be insufficient to explain many of the observed second generation stars. Further mixing with pristine gas would make the situation even worse. We therefore require that the star formation in the FRMS discs happens sequentially. This means that for each second generation star, its formation is completely on a timescale significantly shorter than the lifetime of its parent FRMS disc, so that it may conserve the current abundance pattern of the disc at the time of its formation. This may for example be achieved via migra-
tion towards outer orbits, where the gas densities in the discs are low, or encounters between second generation stars which might scatter them out of the plane of the disc.

A consequence of this way of star formation is that the distribution of abundances will always be continuous. Even in the extreme case when we assume a population of stars with circular orbits, which therefore do not accrete anything from the ICM, together with a population of stars with low eccentricities, which therefore accrete strongly, we would only expect that for the population which have more ICM accretion, the range of metallicities would be reduced. It is difficult to imagine that this could lead to a double peaked distribution like the one observed by [Marino et al. (2008)] in M4. We could however well explain the more uniform distribution found in NGC 1851 ([Lardo et al. 2012]). Details of abundance distributions will be the subject of future work.

4. Supernovae

It is well established that massive stars ($M \gtrsim 25M_\odot$) end their lives as black holes (e.g. Fryer 1999). Stars initially less massive than this are expected to produce energetic SNe-events or, may be, Gamma ray bursts for fast rotators (e.g. Yoon et al. 2004, Dessart et al. 2012). This is much less clear for the more massive stars (e.g. Fryer 1999, Belczynski et al. 2012), which might turn silently into black holes (compare also the discussion in Decressin et al. 2010 and references therein).

This introduces a considerable uncertainty into our model timeline: The 120 $M_\odot$ star would explode at 3.5 Myr, the 25 $M_\odot$ star at 8.8 Myr. The wind-bubble phase ends and the supernova phase begins at some point within this range with the first energetic supernova. In the following we will use the term supernova for simplicity, always subsuming the possibility of Gamma-ray bursts.

We will now first argue that the SNe will likely not significantly alter the picture of the star formation in the FRMS discs (Sect. 4.1). The SNe will cause substantial turbulence in the ICM (Sect. 4.2), which will likely lead to mixing of the SN-ejecta with the cold phase of the ICM (Sect. 4.3). The SN-ejecta may however not enter the second generation stars, because on the one hand, the ICM does not form stars in this phase, and on the other hand, the ICM can no longer accrete on to the FRMS discs because of the large gas velocities (Sect. 4.3). From the end stages in the lifeces of the FRMS onwards, we expect the second generation stars to be dispersed, first within the half-mass radius, and on the relaxation timescale also throughout the whole cluster (Sect. 4.3).

4.1. The effect of supernovae on the associated star-forming disc

Supernova ejecta are fast and cannot be retained in the gravitational potential of the discs around their parent stars (compare Decressin et al. 2007, 2008). Since the disc occupies only a small solid angle, much of the energy will escape into the other directions, especially if the energy release occurs in the form of jets as might be the case for fast rotating massive stars (e.g. Bisnovatyi-Kogan & Moiseenko 2008, Takiwaki et al. 2009, Dessart et al. 2012), and ablate only the surface of the disc. At this point (4.5 Myrs for the 60 $M_\odot$ star) we might expect that a large fraction of the inner disc gas has already accreted on to the newly formed stars. In this case, some remaining debris might be cleared away by the explosion. Yet, it is also possible that a substantial amount of gas is left. If there would still be a substantial amount of gas in the disc, one would expect, similar as in the simulations of [Gaibler et al. 2012] for galactic scales, a compression of the disc and stronger gravitational instability during this phase.

4.2. The effect of supernovae on the general ICM: turbulence

Supernovae are energetic enough to overcome the gravitational potential of the cluster. They are expected to form a superbubble with an expanding shell. As shown in [Paper I], such a shell would be Rayleigh-Taylor unstable. The situation is similar to the one described by the 2D-axisymmetric simulations of [Kritsuk et al. 2001]: The fragments of the shell are expected to fall back into the GC, and a convective cooling core is expected to form. Now, the hot high entropy gas escapes, while cooling shell fragments are continuously formed at the edges of escaping bubbles, and fall back towards the centre, much like in a boiling pot of water.

We will now estimate, if the typical turbulent velocities could lead to a significant increase of the scale height of the gas. We are particularly interested in the cold gas, since most of the mass is expected to be in this phase. Hydrodynamic energy transfer is expected between the different gas phases (e.g. Krause & Alexander 2007). In equilibrium, the energy input by SNe should be balanced by radiative losses. But it is not clear that such an equilibrium may be reached: In the simulations of [Kritsuk et al. 2001] the convection zone expands slowly, reflecting a steady energy accumulation. [Krumholz et al. 2006] summarise simulations of isothermal turbulence, and find a decay time of $\Delta t_{\rm dyn} = 0.83 v_{\rm los}/v_{\rm rms}$. The injection scale in our case is given by the half-mass radius, since this is roughly the scale where the shells are destroyed by the Rayleigh-Taylor instability. Following [Krumholz et al. 2006], we set the injection wavelength to $\lambda_{\rm inj} = 4 r_{1/2}$, because the gas is first in isotropic infall, and also because this is the maximum possible wavelength. The rms-velocity is given by $E = (1 - \epsilon_{\rm sf}) M_{\rm inj} v_{\rm rms}^2 / 2$, because the cold...
gas is supersonic so that the thermal energy may be neglected. We may thus approximate the change of the energy in the gas by

$$\frac{\partial E}{\partial t} = \dot{E}_{\text{SN}} - \frac{E}{\tau_{\text{dis}}}.$$  

(16)

Once the rms-velocity is calculated by this approach, the characteristic radius out to which a given gas filament may ascend in the potential of the GC follows from:

$$r_{\text{rms}} = \frac{v_{\text{rms}}}{\sqrt{\frac{2}{3} e_0^{-3} r_{\text{rms}}^2}}.$$  

(17)

where the central escape velocity $v_{\text{esc},0} = 183 \text{ km/s}$ for our model GC. We plot the evolution of the rms-velocity and the characteristic radius of the convective flow, for the case when the gas would be supported by turbulence alone, in Fig. 6. Our choice of $\lambda_0$ guarantees the smallest possible dissipation rate. The turbulent velocities reached are large compared to normal interstellar medium (ISM) turbulence, about 50 km/s. Yet, this falls short of the escape speed. However, if the gas would have some other means of support, apart from ram pressure due to turbulence, it would be quite likely, that it would still be supported in this other way. Alternative means of support against gravity could be via magnetic fields or to some degree also radiation pressure (Krumholz & Thompson 2012). If this would be the case, our result might also be interpreted in the way that SNe driven turbulence is not able to increase the scale height of the gas in this phase in any significant way. We note that the characteristic turbulent velocity is mainly determined by the ratio of energy injection to gas mass, and also by the size of the cluster via $\tau_{\text{dis}}$. Thus, since GCs all have a typical radius of a few pc, and since the ratio of energy injection to gas mass depends on the star formation efficiency, which should be always around 1/3 for efficient ejection of the first generation stars (Decressin et al. 2011), we expect essentially always turbulence at 50 km/s. For proto-cluster clouds with initial masses above $10^8 M_\odot$, which should be approximately the minimum initial gas mass to form a GC, this is below the escape velocity. Thus, apart from very low-mass protocluster clouds with high star formation efficiency, these findings should be generally applicable for all GCs.

4.3. Mixing of supernovae ejecta with pristine gas

Turbulence is considered to be a decisive factor in mixing theories (for a review see Scalco & Elmegreen 2004). Mixing of stellar ejecta into ICM that later forms subsequent generations of stars is strongly constrained by observations (e.g. Gratton et al. 2012 and sect. 4 above). Here, we are interested in an order of magnitude estimate and therefore use the mixing length approach. We follow Xie et al. (1993) who apply mixing length theory to interstellar clouds. Within this framework, the mixing timescale is given by $\tau \approx H/v_d$, where $H$ represents a mean of abundance and density scale heights, and $v_d$ is the diffusion velocity, given by $v_d \approx K/H$. The diffusion coefficient $K$ is the mixing length $L$. Let us assume filament thicknesses of order 0.1 pc, in the SNe driven turbulence phase, and set $L = H = 0.1$ pc. Thus, the diffusion velocity equals the turbulent one. We have shown above that the expected turbulent velocity in the SNe driven turbulence phase is approximately 50 km/s, which therefore would be the characteristic diffusion velocity in the cold gas. The sound speed in the hot component will be larger than this. X-ray observations (e.g. Jaskot et al. 2011) generally find hot gas temperatures of order $10^6$ K in superbubbles. The sound speed will consequently be several 100 km/s. Cold gas will therefore more efficiently be mixed into the hot gas than vice versa. Using these assumptions, the mixing timescale is given by:

$$\tau \approx 2000 \text{ yrs} \left( \frac{H}{0.1 \text{ pc}} \right) \left( \frac{V_t}{50 \text{ km/s}} \right)^{-1}.$$  

(18)

This should be compared to the dynamical timescale, i.e. the time needed by bubbles enriched with fast winds and SNe ejecta to escape the GC. At times when the GC is highly turbulent the dynamical timescale will be given by:

$$\tau_{\text{dyn,t}} = \frac{r_1/2}{V_t} \approx 60,000 \text{ yrs} \left( \frac{r_1/2}{3 \text{ pc}} \right) \left( \frac{V_t}{50 \text{ km/s}} \right)^{-1}.$$  

(19)

When the turbulent velocities are small, the gravitational acceleration on the cold gas which needs to refill the bubble volume will be the limiting factor. Therefore, the Rayleigh-Taylor timescale sets the minimum for the dynamical timescale:

$$\tau_{\text{dyn,RT}} = \sqrt{2r_1/2g} \approx 60,000 \text{ yrs} \left( \frac{r_1/2}{3 \text{ pc}} \right)^{1/2} \left( \frac{g}{5 \times 10^{-6} \text{ cm/s}^2} \right)^{-1/2},$$  

where we have scaled the gravitational acceleration $g$ to the average value within the half-mass radius for our model cluster. Thus, the dynamical timescale is always around 60,000 yrs.

This means that in the supernova phase, we expect the SNe ejecta to mix with the cold gas phase of the ICM, because of fast turbulent mixing. On the other hand, in the absence of an efficient driver, any turbulence should decay quickly and the gas velocities are expected to be of order the sound speed, which is expected to be about 1 km/s (compare Sect. 4). The mixing timescale increases therefore to about 100,000 yrs, almost twice the dynamical timescale. Hence, we expect no significant mixing in the wind phase before the onset of the SNe.

We note that the issue of turbulent mixing in the ISM cannot be regarded as settled yet. De Avillez & Mac Low (2002) find in hydrodynamic simulations of SNe-driven turbulence in the Milky Way disk diffusion coefficients which are one to two orders of magnitude smaller than in the mixing length approach. Correspondingly, the mixing timescales we give above would underestimate the true timescales by one or even two orders of magnitude. While an increase of one order of magnitude would not change our result, two orders of magnitude would mean that also in the SNe-driven turbulence phase, no significant mixing would occur.

It has been suggested that stars with masses above $25 M_\odot$ might not form SNe, but would instead collapse to a black hole without noteworthy energy release (compare above). This would delay the onset of strong turbulence until about 8.8 Myr after the birth of the first generation stars, the time when the $25 M_\odot$ stars would explode. The FRMS could in this case accrete from the ICM up to this point. Yet, the lack of turbulence would also prevent mixing, so that the stellar ejecta carried by the fast radiative winds of the more massive stars at this time, which carry He-burning products that are not found in the second generation stars, cannot enter the FRMS-discs. The same argument applies for the end of the wind bubble phase for the case when the high mass FRMS do develop SNe.
4.4. The effect of turbulence on accretion

As discussed in Sect. 3.4, accretion is significantly suppressed, if the relative velocities between the accreting objects and the accreted gas are large. This is the case for the SNe phase, as already the typical gas velocities are around 50 km/s. Thus, accretion to the FRMS discs may only happen during the first few Myr (4 to 9 Myr, depending on the mass limit for energetic SNe events) after birth of the first generation of stars, before the supernova phase. The associated smaller amount of accreted gas may however be compensated by a slight increase of the orbital eccentricities of the FRMS (Sect. 3.5). We have shown in Sect. 3.6 that the FRMS discs have disappeared long before the end of the SNe phase. Once a given FRMS has exploded as a supernova, there is therefore at most a short period of activity, and the dark remnants then remain inactive until the last SN has exploded.

4.5. Dispersal of the second generation stars

The FRMS loose about half of their mass via equatorial ejections loaded with hydrogen-burning products during the main sequence and the luminous blue variable stage, and this is the main mass loss channel in this early phase. We expect that towards its end, each FRMS disc has about the same mass as the FRMS, perhaps even a bit more. A considerable fraction of the disc mass should already have been used up to form the second generation stars. Thus, even towards the end of the equatorial ejections, it is expected that gravitational interactions lead to migration of the second generation stars in the discs, and quite possibly also to some ejections from the discs.

From the end of the equatorial ejection phase, the FRMS go on loosing mass via fast radiative winds and perhaps eventual SNe ejections. The final dark remnants might then have only about ten per cent of the mass that the FRMS had at the end of the equatorial ejection phase. Consequently, the gravitational binding energy, which is to first order simply proportional to the square of the total mass, would have decreased by a factor of a few. Virial equilibrium at the end of the equatorial ejection phase demands that the kinetic energy is half of the binding energy. This would mean that gas expulsion would work for most of the observed clusters, apart from the most massive ones, e.g., ω Centauri and M 22, where also a spread in iron abundances is observed (Gratton et al. 2012). We have also shown in Sect. 4.3 that the FRMS discs have disappeared long before the end of the SNe phase. Once a given FRMS has exploded as a supernova, there is therefore at most a short period of activity, and the dark remnants then remain inactive until the last SN has exploded.

5. Accretion on the dark remnants

After the last type II SNe has exploded, roughly 35 Myr after the formation of the first generation of stars, turbulence decays. With the formula given in Sect. 4.2 above, the decay time evaluates to 0.2 Myr/\(v_{50}\), where \(v_{50}\) is the rms-velocity in units of 50 km/s.

In principle, black holes and neutron stars may receive natal kicks as large as \(\approx 400\) km s\(^{-1}\) (Repetto et al. 2012), large enough for them to escape from any given GC. However, many neutron stars are actually observed in GCs (e.g. Barnard et al. 2012; Barret 2012; Stacey et al. 2012). The recent detection of two black holes in the Milky Way GC M 22 (Strader et al. 2012) has called into question if all black holes receive large natal kicks. Especially if black holes form directly without SN-ejection, no significant natal kicks are expected.

In the following, we assume therefore that the dark remnants have the same orbits as the FRMS have had before. Therefore, accretion should again set in. Assuming 1.4 \(M_\odot\) for the neutron stars, and 3 \(M_\odot\) for the stellar mass black holes and accretion over the full solid angle, Eq. 9 predicts about 10-100 \(M_\odot\)/Myr. If we assume a similar suppression as in Sect. 3.4 above, we end up with 0.003-0.03 \(M_\odot\)/Myr. The dark remnants will start their energy output after about a viscous timescale, which is needed by the gas to get close enough to their surface, or respectively, the horizon. The viscous timescale is about 1 Myr, according to Eq. 14. Therefore, each dark remnant will receive 0.003-0.03 \(M_\odot\)/Myr of gas and then get activated. This compares to the Eddington accretion rate of 0.02 \(M_{DR}\) \(M_\odot\)/Myr, where \(M_{DR}\) is the mass of the dark remnant in solar masses. This means that once loaded with gas, the dark remnants could be active at the Eddington rate for about 0.1-0.5 Myrs (Fig. 1 bottom). We have shown in Paper I that such a sudden onset of accretion on the stellar mass black holes and neutron stars may plausibly release a sufficient amount of energy to eject the ICs in at most 0.06 Myrs, corresponding to about half a crossing time. The mass accreted on to the dark remnants may thus plausibly be sufficient for this to occur. The expulsion timescale is short enough to remove also the outer first generation low-mass stars, which are not to appear in the GCs today. We have also shown that a gradual activation of the dark remnants as they become available, reinforcing the SNe in that phase, is still insufficient to overcome the Rayleigh-Taylor instability. For the efficiency of energy transfer we have adopted in Paper I (20 per cent of the Eddington luminosity), the limiting proto-cluster cloud mass up to which the gas expulsion worked was about \(2 \times 10^7\) \(M_\odot\). This would mean that gas expulsion would work for most of the observed clusters, apart from the most massive ones, e.g. ω Centauri and M 22, where also a spread in iron abundances is observed (Gratton et al. 2012). The repeated star-bursts expected in this case are similar to the predictions for nuclear star clusters without supermassive black holes by Loose et al. (1982).

For GCs below this mass limit, further gas accretion events are expected from about 40 Myrs after the birth of the first generation stars due to the slow AGB-winds (Fereole et al. 2008). Any such events would re-activate the dark-remnants. Both, the accretion rate and the binding energy are approximately linear in the gas density, if the stellar potential dominates. Otherwise, the binding energy drops even stronger with gas density. Therefore, we expect again quick gas expulsion. This would be similar to the case of super-massive black holes in nuclear star clusters and elliptical galaxies, which may also be activated by AGB-winds (e.g. Gaibler et al. 2005; Davies et al. 2007; Schartmann et al. 2012).
6. Discussion

6.1. Orbit eccentricities of the FRMS

The weakest point of the presented scenario is perhaps the fine-tuning required for the orbital eccentricities of the FRMS, in order to obtain high enough accretion rates, both on to the FRMS discs and also the dark remnants (Sect. 5.4). We require velocities near the outer turning points of less then about a tenth of the circular velocity there. The former evaluate to a few km/s. An important point to make in this context is that the requirements are the very same as for the dark remnant accretion, as the FRMS turn into the dark remnants at the end of their lives. If gas expulsion is indeed due to the dark remnants, as we have proposed in Paper I then the orbits are also ok for accretion on to the FRMS, unless they were significantly affected by the transformation (compare sect. 5 above).

6.2. Support of the gas and relation to GC formation scenarios

The FRMS orbits will of course be related to the formation of the GC. This is so far unclear, and different scenarios are considered (compare e.g. Harris et al. 1998, Krause 2002, Kravtsov & Gnedin 2005, Gnedin 2011, Gray & Scannapieco 2011, Harris & Harris 2011). In general, one may consider two classes of physical conditions: First, the first generation stars might be formed while the gas is in free collapse. This is among the scenarios currently discussed for star formation in general (e.g. Zamora-Avilés et al. 2012). Then, by definition, the stars will be born with large radial velocities, and much smaller azimuthal velocities, roughly as required. The second option is to have internal support against gravity in the protocluster cloud. As the decay time for turbulence is short (compare Sect. 4.2) and an unusually high level of turbulence is required (< 100 km/s), it seems unlikely that such a protocluster would be supported by turbulence. If it was, the stars would be expected to form with comparable radial and azimuthal velocities, which would be a serious problem for this scenario. Thermal pressure seems also unlikely, as the cooling times would be very short. The protocluster cloud might also be supported by magnetic fields. In general ISM simulations, the coldest and densest of molecular clouds (Crutcher 2012). In a magnetically supported protocluster cloud, the stars would lose their magnetic support immediately after formation, and one would also expect highly eccentric orbits.

6.3. Effects of magnetic fields

Magnetic support would mean that the proto-cluster cloud could keep the same scale height as the stars until the gas is expelled from the GC. This would be highly beneficial, as for our scenario accretion takes place mainly when the stars are in the vicinity of the core radius. Magnetic fields might however also inhibit accretion. Cunningham et al. (2012) show that Bondi accretion from an isothermal, magnetised gas is suppressed by a factor 0.4c_s/c_A, where c_A refers to the Alfvén speed. This might seem moderate compared to the (c_s/v)^3-factor for bulk motions at velocity v. Yet, to support the protocluster cloud magnetically, this also amounts to two orders of magnitudes, which would significantly restrict the available orbit eccentricities. Ambipolar diffusion should however be important in this context, as the majority of the cold gas would be neutral. Also, Cunningham et al. (2012) assume that the magnetic flux accumulates around the accretor, which is the underlying physical process inhibiting the accretion. In our case, we would expect the flux to be advected into the accretion discs, there to be dissipated by turbulence. Qualitatively, we therefore believe that a magnetically supported protocluster cloud would be a viable alternative in the present context. Radiation pressure might also help to support the gas after the first generation massive stars have formed (Krumholz & Thompson 2012).

6.4. Stronger mass segregation as alternative to supporting the gas on a scale of the half-mass radius

An alternative to supporting the gas on the scale of the half-mass radius would be to assume a stronger concentration of the gas as well as the massive stars towards the centre of the GC, implying stronger mass segregation: In our model setup (Sect. 5), we specified that the massive stars are confined to the sphere inside the half-mass radius. In fact, Leigh et al. (2013) in their model for dark-remnant accretion and dynamical black-hole ejection in GCs, assume a much stronger mass segregation. This might happen, if a GC is formed from merging sub-clumps. In this case, strong mass segregation of the stars is expected to arise in about 10^9 yr (Allison et al. 2012, Girichidis et al. 2012, Pang et al. 2013). Assuming a velocity dispersion inversely proportional to the stellar mass, as in Leigh et al. (2013), would result in our relevant FRMS population having maximum velocities of order the sound speed. In such a scenario, both, the massive stars and the gas could be very much concentrated towards the cluster centre. Hence, accretion would no longer be affected by the orbital parameters, as the FRMS velocities would always be small. Apart from the stronger accretion, the formation of the second-generation stars would in this variant of the scenario proceed in almost the same way as in the standard case (Sects. 3.5 and 3.6). The star forming discs must have some way to prevent complete mixing of the ejected and the accreted gas in order to explain the most oxygen poor stars (compare Sect. 3.6.2). The SN-phase would be essentially unaffected, whereas the accretion on to the dark remnants would also be enhanced. The latter might lead to an observationally interesting collective active phase of order the viscous timescale, i.e. about 1 Myr.

6.5. Limitations of dark-remnant accretion due to local radiative feedback

Radiative feedback has been found to limit accretion on to stellar-mass black holes in spherical and clumpy accretion flows (Milosavljević et al. 2009). This applies to the dark-remnant accretion phase, as we have assumed spherical accretion in Sect. 5.6. Milosavljević et al. (2009) show that the average accretion rate may be limited by a value two or three orders of magnitude below the Eddington limit. They caution however that angular momentum, turbulence, and thermodynamic phase segregation might affect the result. They also find that on short timescales the accretion rate may exceed their limit substantially. We require only a very small active phase of the dark remnants to expel the gas (< 10^7 yr). Further work is clearly required to settle
the case. The strong observational constraints on the present-day gas content of GCs and the Fe-uniformity of the present-day stars put strong constraints on the gas expulsion mechanism (compare Paper I).

6.6. Top-heavy IMF

Gas expulsion could be substantially eased by a non-standard, top-heavy IMF. For the model presented in this paper (Sect. 5), we have used a normal IMF, following other recent work (e.g. Decressin et al. 2010; D’Ercole et al. 2011). However, the first generation low-mass stars are of no importance for our scenario. In fact, there is even an issue with their ejection, which we have found here not to work with SN-feedback alone (compare paper I and Sect. 5 above), and which may therefore require the coherent activity of the dark remnants. The ejected low-mass stars might contribute significantly to the stellar population of the Galactic halo (Schaerer & Charbonnel 2011), but there is no strong constraint on how many stars have to escape. The IMF for the first generation stars might therefore have been top-heavy (e.g. Prantzos & Charbonnel 2006 and references in Sect. 1).

This would not necessarily be in contradiction with recent work on clusters formed under typical present-day Milky Way conditions (Hennebelle 2012; Krumholz et al. 2012), as the conditions in which the Galactic-halo GCs formed are not known. The central disc in the Milky Way’s nuclear star cluster, a massive and concentrated star cluster similar in this respect to GCs, is an example where observational evidence for a top-heavy IMF exists (Bartko et al. 2011). A high star-formation efficiency, as often assumed for GCs, is also discussed for this region (e.g. Silk et al. 2012). A top-heavy IMF has been suggested in the context of GCs on the basis of the energy requirements for the gas expulsion (Decressin et al. 2011).

In the extreme case, assuming that almost no first generation low-mass stars form, the initial mass of our model-protocluster cloud (9 x 10^6 M_☉, Sect. 2) could be reduced by one order of magnitude. Because of the combined effect of lower gas densities and smaller stellar velocities, the accretion suppression factor would be constrained to be larger by a factor of a few, moving the vertical line in Fig. 1 (second row) to the right. This would narrow the allowed orbital parameters, requiring somewhat higher eccentricities (but see Sect. 6.4 for a non-exclusive model variant which has the opposite effect). Depending on the star-formation efficiency and the energy-injection efficiency in the SN-phase, the SNe might already eject the gas. We stress that this refers to the extreme scenario. We have shown in paper I that even tripping the massive stars would not yet suffice to expel the gas via supernova feedback. The argument is still valid that favourable conditions for accretion on to the FRMS imply later accretion on to the dark remnants. A possibly reduced gas density due to successful SNe-feedback would cause less accretion on to the dark remnants (linear dependency, compare Eq. 9), but the binding energy would also be reduced. Hence, the dark-remnant feedback would be expected to keep the GC gas-free.

6.7. Comparison to Be-star decretion disc and “first-star” simulations

A key point in our scenario is the formation of the second generation stars in discs, fed by both, accretion of pristine gas and equatorial ejections of the FRMS. Such equatorial ejections are known from Be-type stars (e.g. Rivinius et al. 2001). Recently, the spreading out of such discs is also attempted to be understood in terms viscous evolution (Haibois et al. 2012), similarly as we propose here for the FRMS discs. The formation of stars as opposed to planets in circumstellar discs might seem unusual. Yet, detailed models have recently been computed in the context of the formation of the first massive stars (e.g. Greif et al. 2012), which also may have been very fast rotators (Stacy et al. 2011). For example, Stacy et al. (2012) simulate the formation of a ≈ 30 M_☉ star with radiative feedback. They find that the radiative feedback does not stop the secondary star formation in the accretion disc. During their simulation time of a few 1000 yrs, they form a few proto-stars of sometimes several solar masses. Clark et al. (2011) form four secondary proto-stars within 110 yrs in a similar simulation. Some of these proto-stars may end up in the primary star. Adapted to the FRMS case, this might then lead to a more rapid sequence of ejections. Clearly, a lot of details need to be worked out, some are not comparable (e.g. the first stars are usually assumed to form in dark matter halos). Yet, we believe that these developments are encouraging.

7. Conclusions

Using FRMS data, we have developed a comprehensive scenario for the formation of second generation GC stars (compare Fig. 1):

- The ICM obtains a spong structure in the wind phase (Fig. 1, second row) due to the space-filling wind bubbles of the massive stars.
- We confirm the analysis of Conroy and Spergel that the Lyman–Werner photon flux density is sufficiently high that most of the hydrogen molecules dissociate and no stars may form in the normal way, during the wind and the supernova phase.
- If the first generation massive stars have however equatorial ejections as expected in the case of fast rotation, we show that accretion on to their discs resumes in the shadow of the equatorial ejecta. Massive discs are formed fed by both, FRMS ejecta from the inside, and accretion of pristine gas from the outside. Within an order of magnitude estimate, both contributions can be comparable, as required by self-enrichment calculations (Decressin et al. 2007b), if the orbits of the FRMS are sufficiently eccentric.
- The second generation stars may then form due to gravitational instability in these discs. (Fig. 1, second row). This second-generation star formation might carry on for a few Myr through the supernova phase, because SNe ejecta are not expected to enter the discs. The increased pressure due to the SN may however also compress its associated disc such that the last stars in the respective disc form at this occasion.
- The second generation stars form sequentially in their parent FRMS discs, with the current light element abundances of the respective discs. We propose this as the physical mechanism underlying the detailed abundance calculations of Decressin et al. (2007b).
- The formation of the second generation stars is completed latest about 10 Myr after the formation of the first generation. The second generation stars are first dispersed within the half-mass radius, and much later, after the gas expulsion, also throughout the entire GC.
- The SNe drive turbulence at about 50 km/s (Fig. 1, third row). The remaining gas outside the discs might then mix with the SN ejecta due to turbulence caused by the SN energy. The gas remains bound to the cluster until the SNe have
ceased, turbulence has decayed and the gas can once more accrete suddenly on to the dark remnants. The gas may not accrete on to the FRMS discs before due to the high turbulent velocities. In this way, the second generation stars are prevented from pollution by SN ejecta and He-burning products in general. These findings should hold for all globular clusters, apart from perhaps the ones at the low mass end if their star formation efficiency significantly exceeds 1/3.

- In the dark remnant accretion phase (Fig. 1 bottom), the gas is efficiently expelled due to the strong energy release associated with the accretion on to the dark remnants. This happens fast enough so that a large fraction of less tightly bound first generation stars are also lost.

Potential problems for the scenario include:

- For efficient accretion, the orbits of the massive stars need to be eccentric, with low angular momenta, and velocities near the outer turning points below about 10 % of the circular velocity. This may relate to the formation scenario. It would pose a serious difficulty for this model, if such low angular momenta could be ruled out observationally and, consequently, stronger mass segregation would have to be assumed (Sect. 6.2).

- Further, the gas scale height has to be comparable to the core radius, as the main gas accretion would occur near the core radius. The necessary support of the gas against gravity may be due to radiation pressure [Krumholz & Thompson (2012)] and magnetic fields, details are however beyond the scope of this work. Stronger mass segregation would again alleviate this problem.

- Star formation in discs around massive stars is only recently being explored. While the results from the “first stars” simulations are encouraging, simulations tuned to the specific conditions in FRMS are not yet available.

- If for some reason gas expulsion by dark remnant accretion would not work (Sect. 6.3), all gas should at some point form stars from the quite possibly SN enriched gas (See Sect. 4.3 for mixing uncertainties), unless some other way of gas expulsion would be found. This would however conflict with observations except for the rare and most massive GCs like ω Centauri and M22 that exhibit Fe spread.

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Fig. 1. Sketch of the proposed model of first 40 Myrs of evolution of a two-populations globular cluster together with the global timeline (top to bottom) and the associated time when the stellar life ends for selected stars on the right. The leftmost column shows the whole cluster. The second column from the left represents a zoom on to a FRMS. Four important stages are depicted.

First row from top: First a mass segregated star cluster is formed with all the FRMS inside the half-mass radius (dashed line) and with the initial gas (light pistachio green shade) remaining after the star formation. Each massive star (the blue interior signifies the convective hydrogen burning core, the bright forest green envelope depicts the convective envelope, which has still the pristine composition at this stage) creates a hot bubble around it. The corresponding wind shell is shown in a darker shade of green than the uncompressed gas.

Second row: All the hot bubbles connect and create a spongy-structure in the centre. At the same time, slow mechanical winds around the FRMS create a disk around them. The outer envelope of the FRMS is now blue to denote that it has been contaminated by hydrogen burning products. In the interaction between the ejected disc (blue) and the accreting interstellar medium (ISM, green), a second generation of chemically different stars is born (blue filled circles).

Third row: SNe (red stars with straight lines) fail to eject the gas but create a highly turbulent convection zone. Further accretion on to the remaining FRMS discs is inhibited. The equatorial ejections also end around this time and the formation of the second generation stars is completed by about 10 Myrs on the global time axis.

Bottom row: Later, rapid gas expulsion takes place and removes all the remaining gas together with the majority of the less tightly bound first generation stars out of the cluster potential well. Such rapid gas expulsion is likely to be due to the activation of black holes by accretion of matter.