On the universality of rank distributions of website popularity

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Abstract

We present an extensive analysis of long-term statistics of the queries to websites using logs collected on several web caches in Russian academic networks and on US IRCache caches. We check the sensitivity of the statistics to several parameters: (1) duration of data collection, (2) geographical location of the cache server collecting data, and (3) the year of data collection. We propose a two-parameter modification of the Zipf law and interpret the parameters. We find that the rank distribution of websites is stable when approximated by the modified Zipf law. We suggest that website popularity may be a universal property of Internet.

Key words: Internet, Web traffic, Rank Distribution, Zipf Law
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1 Introduction

It has been known for a decade that web-document popularity follows the Zipf law [1]. Nevertheless, the exponent values reported by different authors vary significantly, from 0.60 to 1.03 [1,2,3,4] (see Table 1). We believe that the scattering of the reported values is due to the small sample size in some cases and to the details of the fitting procedure used to extract the exponent.

In this paper, we propose that the rank distribution of the websites follows the Zipf law and give arguments supporting our idea. We must note that

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website statistics are more extensive than web-document statistics, and the distribution parameters can be obtained with higher accuracy.

We address the following questions: Is the rank distribution of websites Zipf-like? If yes, what are the conditions under which the “true” exponent can be obtained? Does the exponent depend on the duration of the observation? Or on the geographical position of the observer? And does the exponent vary with time, as the Internet develops?

We report some answers to these questions. We have studied website statistics, which are indeed more stable than web-document statistics. We have analyzed log files accumulated on cache servers of Russian academic networks (FREEnet, RASnet, and RSSI) for about six years. These networks differ by their connectivity topology and bandwidth, both national and international. These cache servers have different geographical locations (Moscow, Moscow region, and Yaroslavl in Russia). In addition, we analyzed some statistics collected during seven weeks in the fall of 2004 at a number of IRCache servers in the United States (see Table 4).

We found that the statistics studied become stable\(^2\) when the number of queries for the given statistics exceeds \(10^5\). It is therefore meaningful to fit only those data for which the number of queries exceeds this value. This simple criterion can be used to estimate the critical window for the rank interval where the distribution is stable and the power law can be observed.

We found that the statistics are independent of the geographical location of the cache server (observer) collecting the data, at least for the analyzed data sets.

We found that the distribution is independent of the different years of data collection and is therefore stable over Internet history and development.

Nevertheless, we found that the Zipf-like law approximation is suitable only in the middle region of several orders of rank magnitude. We propose a modification of the Zipf-like law with two additional parameters and explain its possible meaning. We found that if we fit the equation of the modified law to the data, the website popularity distribution becomes quite stable. The value of the exponent \(\alpha\) is \(1.02 \pm 0.05\) for all datasets studied in this paper. We thus may suggest that website popularity follows the Zipf law.

We verified that the same modification also works perfectly for the web-document ranked distribution.

The paper is organized as follows. In section 2, we present a brief history of

\(\begin{footnotesize}\text{\footnotesize\textsuperscript{2} The accuracy of the exponent becomes a few percent, e.g., 5\%.}\end{footnotesize}\)
the power laws observed in nature and society. We describe the data collection and processing in section 3. We discuss the results in section 4 and present our conclusions in section 5.

2 Power laws in nature and society

More than 100 years ago, Pareto [5] observed that the income distribution \( f \) in all countries can be described by the relation

\[
F(f) = 1 - (m/f)^\alpha,
\]

where the exponent \( \alpha \simeq 1.5 \) and \( m \) is some constant. About 70 years ago, George Zipf [6] discovered a striking regularity in English texts: the relative occurrence frequency \( f \) of the \( r \)th most popular word is inversely proportional to the rank \( r \):

\[
f_r \sim \frac{1}{r}.
\]

A more general form of Zipf law (2) with the exponent \( \alpha \neq 1 \) is often encountered in the literature and is known as a Zipf-like law:

\[
f_r \sim \frac{1}{r^\alpha}.
\]

A Zipf-like law has been found in many areas of human activity and in nature. Among examples are the distribution of words in random texts [7], of nucleotide “words” in DNA [8,9], of bit sequences in UNIX executable files [8], of book popularities in libraries [6,10], of countries’ areas and population sizes [6,14,15], of scientific publication citation indices [16], of forest-fire areas [17]. Many other examples can be found in recent reviews [18,19].

Meanwhile, there are many discussions whether a lognormal or power law is a better fit for some empirical distributions, for example, income distribution, population fluctuations, file size distribution, and some others (for a short review, see [19]). In many cases a lognormal distribution looks like a power law distribution for a several orders of magnitude [19,20]. We leave this question open and analyse our data using a Zipf-like law.

It is widely assumed that web document popularity follows a Zipf-like law. We summarized all published results in Table 1 with the dataset name, the date and period of log files in days (d) or months (m), the number of requests, the number of unique web pages requested, and the reported value of the exponent.
| Dataset       | Date (Period) | # of requests | # of pages | α   | Ref. |
|---------------|---------------|---------------|------------|-----|------|
| DEC           | 1994          | ~ 100k        | 1          | [1] |
| BU            | Jan95 (42d)   | 575775        | 54438      | 0.99| [23] |
| BU            | 1998          | 66988         | 41049      | 0.65| [24] |
| DEC           | Jul96 (6d)    | 3543968       | 1354996    | 0.77| [25] |
| NLANR.RTP     | Jun99 (13d)   | 9113027       | 3249549    | 0.71| [25] |
| NLANR.SD      | Jun99 (13d)   | 9082461       | 3549609    | 0.72| [25] |
| NLANR.UC      | Jun99 (13d)   | 8983585       | 2459366    | 0.66| [25] |
| USASK         | Oct98 (82d)   | 20754720      | 5527667    | 0.76| [26] |
| CANARIE       | Dec98 (26d)   | 35129680      | 1423081    | 0.63| [26] |
| NLANR.UC      | Dec98 (31d)   | 20018680      | 7681214    | 0.65| [26] |
| USASK         | Feb99 (45d)   | 21070330      | 5510561    | 0.84| [28] |
| CANARIE       | Feb99 (45d)   | 7310038       | 4571539    | 0.77| [28] |
| NLANR.UC      | Feb99 (30d)   | 24560611      | 8482661    | 0.74| [28] |
| NLANR.LJ      | 1998          | ~ 500k        | 1          | 0.64| [29] |
| UPisa         | 1998          | ~ 500k        | 0.91       | [29] |
| FUNET         | 1998          | ~ 500k        | 0.70       | [29] |
| SPAIN         | 1998          | ~ 500k        | 0.72       | [29] |
| RMPLC         | 1998          | ~ 500k        | 0.86       | [29] |
| BU-CS         | Oct95 (14d)   | 80518         | 4471       | 0.85| [30] |
| Hitachi       | 1997 (16d)    | 2000000       | 0.75       | [31] |
| DEC           | Aug96 (7d)    | 3543968       | 0.77       | [2] |
| UCB           | Nov96 (18d)   | 1907762       | 0.78       | [2] |
| UPisa         | (3m)          | 2833624       | 0.83       | [2] |
| Questnet      | Jan98 (7d)    | 2885285       | 0.69       | [2] |
| NLANR         | Dec97 (1d)    | 1766409       | 0.73       | [2] |
| FUNET         | Jun98 (10d)   | 4815551       | 0.64       | [2] |
| HGMP          | Jan98 (7m)    | ~ 750k        | 0.60       | [2] |
| WebTV         | Sep00 (16d)   | 347460865     | 32541361   | 1.03| [3] |
\(\alpha.\) It can be seen that exponent values vary from 0.60 to 1.03. A question arises. *Why is the variation of the exponent so large?* Probably, the sample size is important, and the Zipf-like law only fits two decades of ranks well at best. It is quite inapplicable in the “tails” and in small ranks, and the results are sensitive to the choice of the rank window for fitting the data.

We know only two papers where the website popularity issue was addressed. In paper [22], the authors claim that the destination address of web requests can be characterized by two types of Zipf laws. In paper [2], the authors presented results for three sets of user request traces (shown in [2] in Fig. 5, which is similar to our Figs. 3 and 4). In particular, the UCB-trace in their Fig. 5 looks similar to the set 2001-09-03 shown in our Fig. 4, and it is rather impossible to extract any value of the exponent \(\alpha\) using the fit to Zipf-like law (3). To our knowledge, the authors did not publish the announced preprint with the values of exponent \(\alpha\).

### 3 Datasets and methods

We start our analysis with the data collected on several proxies (cache servers) located in different Russian academic networks and in the next section will compare the results with the analysis of data collected in the fall of 2004 on American IRCache servers. Collections of data from Russian servers are presented in Table 2 with the dataset name, proxy server location, starting date of log files, period of log file in days (d), weeks (w), months (m), or years (y), number of requests, and number of unique websites requested. The following abbreviations are used for proxies: \texttt{CHG} for the proxy located in the Chernogolovka network (AS9113), Chernogolovka, Moscow region, Russia; \texttt{IKIA} for the proxy in Space Research Institute RAS (AS3218), Moscow, Russia; \texttt{FREEnet} for the proxy in FREEnet (AS2895), Moscow, Russia; \texttt{RASnet} for the proxy located in RASnet (AS3058), Moscow, Russia; and \texttt{Yars} for the proxy located in Yaroslavl State University (AS8325), Yaroslavl, Russia. Proxy-servers \texttt{CHG} and \texttt{Yars} are typical regional cache servers serving requests from local users. Other servers located in Moscow are a central part of the Russian web-caching hierarchy [32] and serve requests from local users as well as from other (e.g., regional) cache servers.

All proxy-servers run Squid caching software. Figure 1 sketches the process

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3 Some papers do not provide all the information (e.g., the number of unique pages) for the datasets studied.

4 Here we consider document popularity observed at the client (BU dataset) or proxy side only. Values of the exponent \(\alpha\) observed at the web-server side vary from 0.67 to 1.82 [21].
Table 2
Characteristics of Analyzed Web Datasets in Russia

| Dataset     | Proxy | Starting date | Period | # of requests | # of websites |
|-------------|-------|---------------|--------|---------------|--------------|
| 1996        | CHG   | Sep 1996      | 74d    | 155743        | 4360         |
| 1997        | CHG   | Jan 1997      | 1y     | 2642722       | 44881        |
| 2000        | CHG   | Sep 2000      | 3m     | 27130648      | 146693       |
| 2001        | CHG   | Feb 2001      | 8m     | 64577294      | 269868       |
| ikia-2001   | IKIA  | Jul 2001      | 4m     | 29296632      | 177497       |
| ikia-2002   | IKIA  | May 2002      | 1m     | 2067205       | 53747        |
| wc-2001     | FREEnet | Jan 2001 | 4.5m | 16989853      | 152760       |
| wc-2002     | FREEnet | Feb 2002 | 5m    | 26576501      | 239891       |
| yar-2002    | Yars  | Apr 2002      | 1m     | 9639987       | 86611        |
| ras-2002    | RASnet | Feb 2002 | 5m    | 9240289       | 227686       |
| 2001-09     | CHG   | Sep 2001      | 1m     | 7333162       | 68671        |
| 2001-09-1w  | CHG   | Sep 2001      | 1w     | 1382537       | 24103        |
| 2001-09-03  | CHG   | Sep 2001      | 1d     | 273361        | 7854         |

Fig. 1. Sketch of the data collection

of data collection: user queries go to the cache server, which processes user queries to the web servers and keeps traces of user requests as records in log files. We therefore call the cache servers “observers” to stress a possible importance of their displacement in the Internet. Cache servers in Russian academic networks are organized in hierarchy sketched in Figure 2. User queries goes through the local proxy servers to regional cache servers, which may redis-
tribute them to the servers on national research and educational networks, which may send queries to the neighboring caches or directly to the destination. Also some queries may be sent to IRCache servers. We must note that the cache server network is a logical one, programmable, and does not reflect Internet connectivity but is rather some subgraph of the Internet.

We must note here that information in the datasets is private and is subject to a privacy policy agreement. We therefore use all datasets available to us.

Each record contains information on the requested document (URL). A typical URL looks like protocol://web.site.name[:port]/path/to/document. We treat a substring between the ‘://’ and ‘/’ characters (omitting the ‘:port’ field if present\(^5\) ) as the website name. Only successful GET requests with code 200 are included in our analysis.

We counted the number of requests for each website in the log for each dataset. Those numbers divided by the total number of requests in the dataset give us the normalized rank distribution of websites by popularity \(f_r\).

Fitting equations and parameter estimation was done by the nonlinear least square method with Levenberg-Marquardt minimization.

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\(^5\) As a rule, requests with the ‘:port’ field are about 2% of all requests, probably because some Russian websites often use the port value for switching between various Cyrillic encodings.
4 Discussion

Normalized rank distributions (the fraction of requests to a given website as a function of the corresponding rank) are presented on a log-log scale in Figures 3, 4, 5. Figure 3 shows results for four datasets with the names 1996 (squares), 1997 (circles), 2000 (up triangles), and 2001 (down triangles) as defined in Table 2. All of them were collected by the same proxy site CHG. Consulting Table 2, we can conclude from Figure 3 that the rank distribution for all four datasets coincides well in the “middle” straight-line part of about two decades and that the larger the sample size, the larger this middle region is. We can therefore conclude that the rank distribution does not change qualitatively in five years and that the rank distribution comes closer and closer to the ideal Zipf law.

Our goal in Figure 4 is to demonstrate how a rank distribution depends on the period of observation. For that reason, we plot four distributions obtained from the datasets 2001-09-03 (squares), 2001-09-1w (circles), 2001-09 (up triangles), and 2001 (down triangles). Clearly, distribution does not vary in time but becomes more “flat” in the middle part with the longer period (larger sample size).

Finally, Figure 5 demonstrates that rank distributions with nearly equivalent sample sizes are independent of the displacement of the observer (i.e., cache server) in the Internet geography (at least, for the Russian academic networks). We plot seven datasets, 2001 (squares), ikia-2001 (circles), wc-2001 (up-triangles), ikia-2002 (down-triangles), ras-2002 (diamonds), wc-2002 (left-triangles), and yar-2002 (right-triangles). Figure 5 is quite convincing that the rank distribution of websites is independent of the displacement of the web cache in the hierarchy.

Totally, it can be seen that rank distributions corresponding to different datasets coincide well for the middle values of ranks. Therefore, the fraction of user requests coming to “mainstream” websites (which are often encountered in logs but are still less popular than top sites) is stable and does not vary with time (Figure 3), with dataset size (Figure 4), or with proxy location (Figure 5).

One more common feature of all graphs is the divergence of the rank distributions in the “tails”, the rightmost parts of the graph. Rank distribution turns down strongly in tails, where the websites were requested less than about 100 times.

There is an interesting peculiarity seen in Figure 3: the fraction of requests coming to the most popular sites decreases with time. For example, the frequency of occurrences of the most popular website in 1996 was about an
order of magnitude higher than in 2001. Because the most frequent requests come to different kinds of banners, counters, search engines, etc., Figure 3 demonstrates that their relative popularity diminishes with time. One possible reason is the appearance of many different sites with similar contents (as well as mirror sites) or functions (e.g., banner networks or search engines), which leads to equilibrating user interest to different hot sites. Another reason is improvement of web-client software. The internal cache of the web browser can contain more web documents; requests to the most popular documents are then processed using the internal cache. This phenomena is known as the “trickle-down” effect observed by Doyle et al. [4], which is discussed below.

Figure 4 demonstrates that the top sites have a stable fraction of requests during a given year.

Figures 2 and 3 show that Zipf-like law (3) (which must be represented as a straight line) is a very coarse approximation of the actual distribution. The main deviations from the law (3) are in the region of the most popular (top 50) sites and in the tail of the distribution.

Fitting the data to Zipf-like law, expression (3), and its modifications, expressions (4) and (5), is a tricky problem both because of the influence of the rare statistics of the large ranks and because of the high fluctuations of the leading ranks. Which method is best is not yet understood [27]. We use a least-square fit to estimate the parameters and calculate the accuracy of the estimated values by the standard approach and give it in the parentheses as a correction to the last digit.

We can choose a region of ranks of two orders of magnitude where the rank distribution looks like a straight line. But varying the interval boundaries of the rank window strongly affects the fitting parameters (e.g., the exponent $\alpha$). We obtained $\alpha$ in the range from 0.7 to 1.4 depending on the rank window. For example, fitting dataset 2001-09 with Zipf-like law (3) in the window $10 \leq r \leq 1000$ gives $\alpha = 0.78$ and in window $10^3 \leq r \leq 10^5$ gives $\alpha = 1.13$. Other fitting windows give other values in the range from 0.7 to 1.4. We can therefore conclude that the Zipf-like law cannot give us quantitative characteristics of rank distributions of websites in the whole interval of ranks.

Slightly better results can be derived using a modified Zipf-like law, known as the Zipf–Mandelbrot law [10],

$$f_r = \frac{b}{(c + r)^\alpha},$$

which gives a better approximation in the range of small ranks but is still inapplicable in the “tails”. The fit can be appreciably enhanced by introducing
Table 3
Fitting Results for Russian Servers

| Dataset     | $a$              | $c$      | $\alpha$ |
|-------------|------------------|----------|----------|
| 1996        | $-3.0(1) \cdot 10^{-5}$ | 0.45(4) | 0.95(5) |
| 1997        | $-5.77(2) \cdot 10^{-6}$ | 2.96(5) | 0.92(3) |
| 2000        | $-1.01(11) \cdot 10^{-6}$ | 7.33(7) | 1.04(3) |
| 2001        | $-2.48(3) \cdot 10^{-7}$ | 9.10(5) | 1.06(2) |
| 2001-09     | $-1.44(27) \cdot 10^{-6}$ | 15.16(11) | 1.08(7) |
| 2001-09-1w  | $-7.25(6) \cdot 10^{-6}$ | 14.82(20) | 1.03(2) |
| 2001-09-03  | $-2.01(7) \cdot 10^{-5}$ | 17.82(72) | 0.99(6) |
| ikia-2001   | $-5.10(7) \cdot 10^{-7}$ | 13.35(7) | 1.07(3) |
| ikia-2002   | $-1.58(9) \cdot 10^{-6}$ | 4.53(16) | 1.01(1) |
| wc-2001     | $-5.56(9) \cdot 10^{-7}$ | 14.54(9) | 1.09(4) |
| wc-2002     | $-4.43(7) \cdot 10^{-7}$ | 14.02(5) | 1.06(3) |
| ras-2002    | $-9.45(2) \cdot 10^{-7}$ | 9.17(10) | 0.95(5) |
| yar-2002    | $-1.30(3) \cdot 10^{-6}$ | 4.64(4) | 0.99(5) |

one more parameter in (4):

$$f_r = a + \frac{b}{(c + r)^\alpha}.$$  \hspace{1cm} (5)

Figure 6 shows the rank distribution of websites in the coordinates $\log(f_r - a)$, $\log(c + r)$ for the particular dataset 2001-09. The fraction of requests (the vertical axis) is shifted by the value $a = -1.44 \cdot 10^{-6}$ and the rank by $c = 15.16$. This figure clearly demonstrates that function (5) approximates the data distribution well in almost the entire range of ranks.\(^6\) We have fitted expression (5) to all our data and found that the value of $\alpha$ is quite stable; the results are presented in Table 3 for the datasets discussed. The columns in Table 3 are the dataset name as defined in Table 2 and resulting values of $a$, $c$, and $\alpha$ as defined in expression (5). The mean of the exponent $\alpha$ is $1.02 \pm 0.05$, which may be considered 1.0. The statistical error is calculated as the variation of $\alpha$ from the data in Table 3.

The parameter $a$ can be considered a correction for the finite sample size. The

\(^6\) We note that this method for data “straightening” is often applied in statistical physics [11,12]. A similar equation was also proposed in a recent work on rank distribution of publication popularity [13].
larger the sample size, the less $a$ is.

The parameter $c$ in expression (5) has a very clear physical meaning. It is closely connected with the *trickle-down effect* observed by Doyle [4]. Doyle found that proxies disproportionately absorb requests on different levels of the hierarchy. Rank distributions obtained from data collected on proxies at different hierarchical levels differ in the region of small ranks. This effect has a clear explanation in terms of rank distributions.

As a clarifying example, we consider a two-layer hierarchy of proxies. A first-level proxy receives requests from users. If the requested document is found in its cache, then that document is returned to the client; otherwise, the request is submitted to an upper-level proxy. If we assume that a first-level proxy can hold $N$ documents in its cache, then it accordingly filters the $N$ most popular documents from the request stream, i.e., it “cuts” the leftmost $N$ points from the rank distribution. This is equivalent to the change of variables $r \rightarrow r + N$. Therefore, we presume that the parameter $c$ in equation (5) characterizes cache sizes of low-level proxies (which can also be the user’s browser cache).

It can be seen that for all datasets, $\alpha$ is close to unity with an accuracy of a few percent. We therefore suppose that the exponent $\alpha$ in equation (5) is a universal characteristic of web traffic, which is independent of time (for time-scales comparable with the Internet lifetime), is independent of data collection duration (when the sample size is sufficiently large and contains more than $2 \times 10^5$ requests), and is independent of the displacement of the proxy server in the Internet hierarchy.

We found a possibility to check our findings using available statistics. We chose BU web-client traces available from ita.ee.lbl.gov (the full dataset from Nov 94 to May 95 contains 1143842 requests, 104532 unique URLs, and 4970 unique sites). This dataset was used in early work and gives one of the best examples of the Zipf law for web-page popularity ($\alpha = 0.986$) [23]. Fitting equation (5) to the rank distribution of website popularity gives $\alpha = 1.025$, $a = -3.3 \cdot 10^{-5}$, and $c = 1.97$, which coincide well with the values obtained for Russian academic networks. This is an additional argument that website popularity distribution is universal (in other words, is independent of both the observation point in the Internet and Internet history) and follows the Zipf law with an exponent $\alpha$ close to unity.

To check this statement deeper, we also analyze recently available data\(^7\) collected during the period from 11/03/2004 to 12/29/2004 at nine cache-servers of the US national cache-mesh system for science and education built-up within the IRCache project [33]. Table 4 presents data from the following

\(^7\) Thanks to D. Wessels, who kindly gave us access to the data sets collected at the US IRCache servers.
Table 4
Characteristics of Analyzed Web Datasets in USA and Fitting Results

| cache | # of requests | N = # of websites | aN     | c      | α     |
|-------|--------------|------------------|--------|--------|--------|
| bo    | 23935604     | 592679           | -2.89(1) | 8.54(4) | 1.05(2) |
| ny    | 12789266     | 407952           | -3.89(1) | -0.12(1) | 0.94(3) |
| pa    | 3374392      | 229633           | -1.57(1) | 7.17(12) | 0.96(8) |
| pb    | 10018478     | 304049           | -4.47(1) | 18.96(13) | 0.98(4) |
| rtp   | 13221655     | 339918           | -4.35(1) | 23.52(13) | 1.01(4) |
| sd    | 13840665     | 285356           | -3.22(1) | 0.166(7) | 1.04(3) |
| sj    | 26130582     | 264396           | -6.00(1) | 1.935(13) | 1.09(2) |
| sv    | 11119941     | 530731           | -3.20(1) | 16.34(13) | 0.93(4) |
| uc    | 13294408     | 313178           | -5.17(1) | 15.14(9) | 1.01(4) |
| uc-12d| 3236853      | 84360            | -4.37(2) | 7.79(12) | 0.95(8) |
| uc-1d | 463899       | 13752            | -1.77(4) | 4.99(24) | 0.96(3) |
| all   | 127724991    | 1176623          | -8.96(1) | 5.05(1) | 1.03(2) |

locations:

- **bo** – NCAR at Boulder, Colorado
- **ny** – New York, New York
- **pa** – Digital Internet Exchange in Palo Alto, California
- **pb** – PSC at Pittsburgh, Pennsylvania
- **rtp** – Research Triangle Park, North Carolina
- **sd** – SDSC at San Diego, California
- **sj** – MAE West Exchange Point in San Jose, California
- **sv** – NASA-Ames/FIX-West in Silicon Valley, California
- **uc** – NCSA at Urbana-Champaign, Illinois.

The second and third entries from the bottom demonstrate the stability of the fit for two subsets of the data collected at **uc**-location, for 12 days (set name **us-12d**) and for 1 day (set **us-1d**). The last entry represents the fit to the sum of the preceding data sets. Results of the fit by expression (5) are close to unity and quite similar to those for Russian servers presented in Table 3.
5 Conclusions

We have presented modified Zipf law (5), which fits the rank distribution of web sites in the full range of ranks rather well. We found that the value of the exponent \( \alpha \) in expression (5) is stable for the analyzed datasets. It does not vary with (1) the year of data collection, (2) the period of data collection, or (3) the geographical location of the cache server where we collected data. We found that \( \alpha \) is very close to 1. We have reasons to suppose this value of \( \alpha \) is a universal property of web-traffic for the website rank. We have also presented a clear explanation of the “trickle-down effect” based on the properties of our modified Zipf law. We suggest that website popularity is universal property of Internet and follows the Zipf law.

In a similar experiment, fluctuations of the exponent value were checked [34] as a function of the volume of statistics, where cache traces of user requests to different Internet domains were analyzed. User requests were sent to Internet through the cache triangle, namely, they went to the Master Server, which sent each odd request to the left cache and each even request to the right cache. Clearly, the traces should be nearly equal in the limit of a large number of requests. Indeed, it was estimated that exponents extracted separately from the “left” traces and “right” traces were within five per cent for a set volume larger than ten thousand requests, and that those for set volume less than a few hundred fluctuated strongly. Thus, rare statistics may significantly affect the results.

The results in this paper may be useful for building mirror sites and CDNs as well as for improving software for DNS request caching. We also conjecture that fitting with the modified Zipf law is suitable for describing the rank distribution of web-document popularity.

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Fig. 3. Website distribution for different years

Fig. 4. Website distribution for different periods
Fig. 5. Website distribution for different servers

Fig. 6. Website distribution in modified coordinates: dependence of $f_r - a$ from $r + c$ (compare to expression (5)) in double logarithmic scale.
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