Efficient Verification of Pure Quantum States in the Adversarial Scenario

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Efficient verification of pure quantum states in the adversarial scenario is crucial to many applications in quantum information processing, such as blind measurement-based quantum computation and quantum networks. However, little is known about this subject so far. Here we establish a general framework for verifying pure quantum states in the adversarial scenario and clarify the resource cost. Moreover, we propose a simple and general recipe to constructing efficient verification protocols for the adversarial scenario from protocols for the nonadversarial scenario. With this recipe, arbitrary pure states can be verified in the adversarial scenario with almost the same efficiency as in the nonadversarial scenario. Many important quantum states in quantum information processing can be verified in the adversarial scenario with unprecedented high efficiencies.

Introduction.—Bipartite and multipartite entangled states play a central role in quantum information processing and foundational studies \[1,2\]. Accurate preparation and verification of desired quantum states is a key to various applications. However, characterization of quantum states based on traditional tomography is highly inefficient because the resource required grows exponentially with the number of components. Even popular alternatives, including compressed sensing \[3\] and direct fidelity estimation (DFE) \[4\], cannot avoid this scaling behavior in general. Recently, a powerful approach known as quantum state verification \[5,6\] has attracted increasing attention. This approach has led to efficient protocols for verifying bipartite pure states \[7,8,9,10\], stabilizer states (including graph states) \[11,12,13,14\], hypergraph states \[15,16\], weighted graph states \[17,18\], and Dicke states \[19\].

The problem is much more complicated in the adversarial scenario, in which the states to be verified are prepared by a potentially malicious adversary. Efficient verification of quantum states in this scenario is a key to many important quantum information processing tasks, such as blind measurement-based quantum computation (MBQC) \[12,14,19,20\] and quantum networks \[13,21,22\]. However, little is known on how to address the adversarial scenario in general. The approach proposed in Ref. \[2\] does not apply although it is quite successful in the nonadversarial scenario. Other approaches known in the literature only apply to certain special types of states and are highly inefficient. To verify hypergraph states with recent approaches in Refs. \[23,24\] for example, the number of required tests is enormous even in the simplest nontrivial cases. An outstanding problem underlying this deadlock is that, even for a given verification strategy, no efficient method is known for determining the minimal number of tests required to achieve a given precision, as characterized by the infidelity and significance level \[24,25\].

In this paper we establish a general framework of quantum state verification in the the adversarial scenario and settle several fundamental problems. First, we determine the precision achievable with a given strategy and a given number of tests and thereby clarify the resource cost to achieve a given precision. Then we propose a general recipe to constructing efficient verification protocols for the adversarial scenario from verification protocols for the nonadversarial scenario. With this recipe, arbitrary pure states can be verified in the adversarial scenario with almost the same efficiency as in the nonadversarial scenario. For high-precision verification, the overhead in the number of tests is at most three times. In conjunction with recent works, this recipe can be applied immediately to constructing efficient verification protocols for many important quantum states, including but not limited to bipartite pure states, stabilizer states (including graph states), hypergraph states, weighted graph states, and Dicke states.

This paper extracts the key results in Ref. \[24\], which contains complete technical details and additional results.

Verification of a pure state.—Consider a device that is supposed to produce some target state $|\Psi\rangle$ in the Hilbert space $\mathcal{H}$, but actually produces $\sigma_1, \sigma_2, \ldots, \sigma_N$ in $N$ runs. Our task is to verify whether each $\sigma_j$ is sufficiently close to the target state on average. To achieve this task we can perform two-outcome measurements \{ $E_i$, 1 $- E_i$ \} randomly from a set of accessible measurements in each run. Each measurement is specified by a test operator $E_i$, which corresponds to passing the test and satisfies $E_i(\Psi) = |\Psi\rangle$, so that the target state can always pass the
test. After $N$ runs, we accept the source if and only if it passes all tests. Suppose the test $E_l$ is performed with probability $\mu_l$; then the efficiency of the verification strategy is determined by the verification operator $\Omega := \sum_l \mu_l E_l$. If $\langle \Psi | \sigma_j | \Psi \rangle \leq 1 - \epsilon$; then the maximal probability that $\sigma_j$ can pass the test reads
\[
\max_{\langle \Psi | \sigma_j | \Psi \rangle \leq 1 - \epsilon} \text{tr}(\Omega \sigma) = 1 - [1 - \beta(\Omega)]\epsilon = 1 - \nu(\Omega)\epsilon, \quad (1)
\]
where the maximization is taken over all quantum states $\sigma$ that satisfy $\langle \Psi | \sigma | \Psi \rangle \leq 1 - \epsilon$. Here $\beta(\Omega)$ is the second largest eigenvalue of $\Omega$, and $\nu(\Omega) := 1 - \beta(\Omega)$ is the spectral gap from the maximal eigenvalue.

Suppose the outputs $\sigma_1, \sigma_2, \ldots, \sigma_N$ of the device are independent of each other. Let $\epsilon_j = 1 - \langle \Psi | \sigma_j | \Psi \rangle$ be the infidelity between $\sigma_j$ and $|\Psi\rangle$. Then these states can pass $N$ tests with probability at most
\[
\prod_{j=1}^N \langle \Psi | \sigma_j | \Psi \rangle \leq \prod_{j=1}^N (1 - \nu(\Omega)\epsilon_j) \leq (1 - \nu(\Omega)\epsilon)^N, \quad (2)
\]
where $\overline{\epsilon} = \sum_j \epsilon_j/N$ is the average infidelity. The bound in Eq. (2) is saturated when all $\epsilon_j$ are equal and each $\sigma_j$ is supported in the subspace of $\mathcal{H}$ associated with the two largest eigenvalues of $\Omega$. To ensure the condition $\sum_j \langle \Psi | \sigma_j | \Psi \rangle / N > 1 - \epsilon$ with significance level $\delta$, the minimum number of tests reads
\[
N_{NA}(\epsilon, \delta, \Omega) := \left\lceil \frac{1}{\ln(1 - \nu(\Omega)\epsilon)} \ln \delta \right\rceil \leq \left\lceil \frac{1}{\nu(\Omega)\epsilon} \ln \frac{1}{\delta} \right\rceil. \quad (3)
\]
A similar formula was previously derived in Ref. [7]; however, here the underlying assumption and the interpretation are quite different. Notably, we do not require the unnatural assumption that $\langle \Psi | \sigma_j | \Psi \rangle \leq 1$ for all $j$ or $\langle \Psi | \sigma_j | \Psi \rangle \leq 1 - \epsilon$ for all $j$. In addition, our conclusion concerns the average fidelity, which is more relevant than the maximal fidelity addressed in Ref. [7].

In view of Eq. (3), to minimize the number of tests, we need to maximize the spectral gap. If there is no restriction on the accessible measurements, then the optimal strategy consists of the single test $\{ |\Psi\rangle \langle \Psi|, 1 - |\Psi\rangle \langle \Psi| \}$, so that we have $\Omega = |\Psi\rangle \langle \Psi|$, $\beta(\Omega) = 0$, and $\nu(\Omega) = 1$; cf. Ref. [7]. In practice, we need to consider various constraints on measurements. In addition, the situation for the adversarial scenario is quite different as we shall see.

Adversarial scenario.—In the adversarial scenario, the device is controlled by a potentially malicious adversary and can produce an arbitrarily correlated or even entangled state $\rho$ on $\mathcal{H}^{\otimes (N+1)}$, as encountered in blind MBQC. For example, the device can prepare $|\langle \Psi | \Psi \rangle \rangle^{\otimes (N+1)}$ with probability $0 < a < 1$ and $\sigma^{\otimes (N+1)}$ with probability $1 - a$. In this case, the above approach and the approach in Ref. [7] cannot work. Here we shall propose a simple and efficient recipe to address this problem.

To verify the state produced, we randomly choose $N$ systems and apply a certain strategy $\Omega$ to each system chosen. Our goal is to ensure that the reduced state on the remaining system has fidelity at least $1 - \epsilon$ if $N$ tests are passed. Since $N$ systems are chosen randomly, without loss of generality, we may assume that $\rho$ is permutation invariant. Suppose the strategy $\Omega$ is applied to the first $N$ systems, then the probability that $\rho$ can pass $N$ tests reads $p_\rho = \text{tr}[(\Omega^{\otimes N} \otimes I)\rho]$. The reduced state on system $N+1$ (assuming $p_\rho > 0$) is given by $\sigma'_{N+1} = p_\rho^{-1} \text{tr}_{1,2,\ldots,N}[(\Omega^{\otimes N} \otimes I)\rho]$, where $\text{tr}_{1,2,\ldots,N}$ means the partial trace over the systems $1, 2, \ldots, N$. The fidelity between $\sigma'_{N+1}$ and $|\Psi\rangle$ reads $F_\rho = \langle \Psi | \sigma'_{N+1} | \Psi \rangle = p_\rho^{-1} f_\rho$, where $f_\rho = \text{tr}[(\Omega^{\otimes N} \otimes |\Psi\rangle \langle \Psi|)\rho]$. To characterize the performance of the strategy $\Omega$ applied to the adversarial scenario, define
\[
F(N, \delta, \Omega) := \min_{\rho} \{ p_\rho^{-1} f_\rho | p_\rho \geq \delta \}, \quad 0 < \delta \leq 1. \quad (4a)
\]
This figure of merit denotes the minimum fidelity of $\sigma'_{N+1}$ with the target state suppose that $\rho$ can pass $N$ tests with probability at least $\delta$; it is nondecreasing in $\delta$ by definition. Next, define $N(\epsilon, \delta, \Omega)$ as the minimum number of tests required to verify $|\Psi\rangle$ within infidelity $\epsilon$ and significance level $\delta$, that is,
\[
N(\epsilon, \delta, \Omega) := \min \{ N | F(N, \delta, \Omega) \geq 1 - \epsilon \}. \quad (5)
\]

Homogeneous strategies.—A strategy (or verification operator) $\Omega$ for $|\Psi\rangle$ is homogeneous if it has the form
\[
\Omega = |\Psi\rangle \langle \Psi| + \lambda (1 - |\Psi\rangle \langle \Psi|), \quad (6)
\]
where $0 \leq \lambda < 1$. In this case, all eigenvalues of $\Omega$ are equal to $\lambda$ except for the largest one, so we have $\beta = \lambda$ and $\nu = 1 - \lambda$. Given that the homogeneous strategy $\Omega$ is characterized by the parameter $\lambda$, it is natural and more informative to replace $\Omega$ with $\lambda$ in the notations of various figures of merit; for example, we can write $F(N, \delta, \lambda)$ in place of $F(N, \delta, \Omega)$. A homogeneous strategy is the most efficient among all verification strategies with a given spectral gap and so plays a key role in quantum state verification. Here we first clarify its performance for verifying pure quantum states in the adversarial scenario.

When $\lambda = 0$, the verification operator $\Omega$ is singular. For $0 < \delta \leq 1$, we have
\[
F(N, \delta, \lambda = 0) = \max\{0, \frac{(N+1)\delta - 1}{N\delta} \}. \quad (7)
\]
The minimum number of tests required to verify the pure state $|\Psi\rangle$ within infidelity $\epsilon$ and significance level $\delta$ reads
\[
N(\epsilon, \delta, \lambda = 0) = \frac{1 - \delta}{\epsilon \delta}. \quad (8)
\]
The scaling with $1/\delta$ is suboptimal although the strategy is optimal for the nonadversarial scenario by Eq. (8) when there is no restriction on the accessible measurements.
When $0 < \lambda < 1$, $\Omega$ is nonsingular. Let $\mathbb{Z}^{\geq 0}$ denote the set of nonnegative integers. For $k \in \mathbb{Z}^{\geq 0}$, define

$$
\zeta(N, \delta, \lambda, k) := \frac{\lambda([\delta]_1 + (N-k)\nu) - \lambda^k)}{\nu(k\nu + N\lambda)}.
$$

(9)

The following theorem clarifies the precision that can be achieved by a homogeneous strategy given $N$ tests.

**Theorem 1.** Suppose $0 < \lambda < 1$; then $F(N, \delta, \lambda) = 0$ if $0 < \delta \leq \lambda^N$ and $F(N, \delta, \lambda) = \zeta(N, \delta, \lambda, k_*)/\delta$ if instead $\lambda^N < \delta \leq 1$, where $k_*$ is the largest integer $k$ that satisfies $(N + 1 - k)\lambda^k + k\lambda^{k-1} \geq (N + 1)\delta$.

Let $k_+ := \lceil \log_2 \delta \rceil$ and $k_- := \lfloor \log_2 \delta \rfloor$; then $k_*$ equals either $k_+$ or $k_-$ given the assumption $\lambda^N < \delta \leq 1$. Define

$$
\tilde{N}(\epsilon, \delta, \lambda, k) := \frac{k\nu^2 \delta F + \lambda^{k+1} + \lambda \delta(k\nu - 1)}{\nu(k\nu + N\lambda)},
$$

(10)

where $F = 1 - \epsilon$ and $\nu = 1 - \lambda$. The following two theorems provide analytical formula and informative bounds for $N(\epsilon, \delta, \lambda)$, which can be derived from Theorem 1. The results are illustrated in Fig. 1.

**Theorem 2.** Suppose $0 < \epsilon, \delta, \lambda, \Omega < 1$. Then

$$
N(\epsilon, \delta, \lambda) = \left[ \min_{k \in \mathbb{Z}^{\geq 0}} \tilde{N}(\epsilon, \delta, \lambda, k) \right] = \left[ \tilde{N}(\epsilon, \delta, \lambda, k^*) \right],
$$

(11)

where $k^*$ is the largest integer $k$ that satisfies the inequality $\delta \leq \lambda^k/(F\nu + \lambda) = \lambda^k/(F + \epsilon\lambda)$.

**Theorem 3.** Suppose $0 < \epsilon, \delta, \lambda < 1$. Then

$$
k_- + \frac{k_-F}{\lambda\epsilon} \leq N(\epsilon, \delta, \lambda) \leq \frac{\ln \delta}{\lambda\epsilon \ln \lambda} - \frac{\nu k_-}{\lambda}.
$$

(12)

Both the upper and lower bounds are saturated when $\ln \delta / \ln \lambda$ is an integer.

In the high precision limit $\epsilon, \delta \to 0$, $k_\pm \approx \ln \delta / \ln \lambda$, so Theorem 4 implies that

$$
N(\epsilon, \delta, \lambda) \approx (\lambda \ln \lambda)^{-1} \ln \delta.
$$

(13)

The efficiency of the homogeneous strategy is characterized by the factor $(\lambda \ln \lambda)^{-1}$, as reflected in Fig. 1. The number of tests is minimized when $\lambda = 1/e$ (with $\epsilon$ being the base of the natural logarithm), in which case we have $N(\epsilon, \delta, \lambda = e^{-1}) \approx \epsilon \ln \delta^{-1}$, which is comparable to the counterpart $\epsilon^{-1} \ln \delta^{-1}$ for the nonadversarial scenario.

**General verification strategies.**—Now we turn to a general verification strategy $\Omega$; let $\beta = \beta(\Omega)$ and $\nu = \nu(\Omega)$.

**Theorem 4.** Suppose $0 < \delta \leq 1$ and $0 < \nu \leq 1$. Then

$$
F(N, \delta, \Omega) \geq 1 - \frac{1 - \delta}{N\delta \nu},
$$

(14)

and the inequality is saturated when $\frac{1 + N\delta}{N + 1} \leq \delta \leq 1$.

**Theorem 5.** Suppose $0 < \epsilon, \delta, \lambda < 1$ and $\Omega$ is a positive definite verification operator. Then

$$
F(N, \delta, \Omega) \geq \frac{N + 1 - (\ln \beta)^{-1} \ln (\tau \delta)}{N + 1 - (\ln \beta)^{-1} \ln (\tau \delta) - h \ln (\tau \delta)}.
$$

(16)

where $h = (\beta \ln \beta^{-1})^{-1} = \left[\min\{\beta \ln \beta^{-1}, \tau \ln \tau^{-1}\}\right]^{-1}$.

**Theorem 6.** Suppose $0 < \epsilon, \delta < 1$ and $\Omega$ is a positive definite verification operator. Then

$$
k_- (\tilde{\beta}) + \frac{k_- (\tilde{\beta}) F}{\beta \epsilon} \leq N(\epsilon, \delta, \Omega) < \frac{h \ln (F \delta)^{-1}}{\epsilon},
$$

(17)

where $F = 1 - \epsilon$ and $k_- (\tilde{\beta}) = [\ln \delta / \ln \tilde{\beta}]$.

In the limit $\epsilon, \delta \to 0$, the upper and lower bounds in Eq. (17) are tight with respect to the relative deviation. So $N(\epsilon, \delta, \Omega)$ reduces to

$$
N(\epsilon, \delta, \Omega) \approx \frac{h \ln (\delta^{-1})}{\epsilon} = \frac{\ln \delta}{\epsilon \beta \ln \tilde{\beta}}.
$$

(18)
which has the same scaling behaviors with $\epsilon^{-1}$ and $\delta^{-1}$ the counterpart for the nonadversarial scenario in Eq. 19. The overhead is characterized by $p_{\star}(\nu) = h(\nu/e, \nu) \ln(F\delta)^{-1}/\nu e \ln \delta$. This bound decreases monotonically with $\epsilon, \delta$, and nonincreasing in $\nu$. Here we introduce a general recipe to resolving this problem by adding the trivial test. By “trivial test” we mean the test whose test projector $P$ coincides with the identity operator, that is, $P = 1$, so that all quantum states can always pass the test.

Given a verification operator $\Omega$ for the pure state $|\Psi\rangle$, we can construct a hedged verification operator as follows,

$$\Omega_p = p + (1 - p)\Omega, \quad 0 \leq p < 1. \tag{19}$$

It is realized by performing the trivial test and $\Omega$ with probabilities $0 \leq p < 1$ and $1 - p$, respectively. The second largest and smallest eigenvalues of $\Omega_p$ read

$$\beta_p = p + (1 - p)\beta, \quad \tau_p = p + (1 - p)\tau, \tag{20}$$

where $\beta$ and $\tau$ are the second largest and smallest eigenvalues of $\Omega$, respectively. By Eq. (17), to verify $|\Psi\rangle$ within infidelity $\epsilon$ and significance level $\delta$, the number of tests required by the strategy $\Omega_p$ (assuming $\tau_p > 0$) satisfies

$$N(\epsilon, \delta, \Omega_p) < h(p, \nu, \tau) e^{-1} \ln(F\delta)^{-1}, \tag{21}$$

where $F = 1 - \epsilon$ and

$$h(p, \nu, \tau) = \left[\min\{\beta_p \ln \beta_p^{-1}, \tau_p \ln \tau_p^{-1}\}\right]^{-1}. \tag{22}$$

Compared with the nonadversarial scenario, the overhead satisfies

$$N(\epsilon, \delta, \Omega_p)/(N_{\text{NA}}(\epsilon, \delta, \Omega)) < \nu h(p, \nu, \tau) \ln(1 - \nu e)^{-1} \ln(F\delta)^{-1}/\nu e \ln \delta. \tag{23}$$

This bound decreases monotonically with $1/\epsilon, 1/\delta$, and $1/\nu$ [26]; it approaches $\nu h(p, \nu, \tau)$ in the limit $\epsilon, \delta \to 0$, in which case the bound is saturated. Equation (23) reveals the significance of the function $\nu h(p, \nu, \tau)$ for characterizing the overhead of high-precision state verification in the adversarial scenario.

To achieve high performance, we need to minimize $h(p, \nu, \tau)$ over $p$. The optimal probability $p$ reads

$$p_\star(\nu, \tau) = \min\{p \geq 0 | \beta_p \geq e^{-1} \& \tau_p \ln \tau_p^{-1} \geq \beta_p \ln \beta_p^{-1}\}, \tag{24}$$

which is nondecreasing in $\nu$ and nonincreasing in $\tau$. For a homogeneous strategy $\Omega$ with $\tau = \beta = 1 - \nu$, we have $p_\star(\nu, \tau) = (\nu e - e + 1)/(\nu e)$ if $\nu \geq 1 - (1/e)$ and $p_\star(\nu, \tau) = 0$ otherwise. When $\tau = 0$, $p_\star(\nu) := p_\star(\nu, 0)$ can be approximated by $\nu/e$. In general, it is not easy to derive an analytical formula, but it is very easy to determine $p_\star(\nu, \tau)$ numerically.

**Theorem 6.** If $p = \nu/e$ or $p_\star(\nu, \tau) \leq p \leq p_\star(\nu)$, then

$$N(\epsilon, \delta, \Omega_p) < \nu h(\nu/e, \nu, 0) \ln(F\delta)^{-1}/\nu e \ln \delta. \tag{25}$$

Here the number of tests $N(\epsilon, \delta, \Omega_p)$ achieves the optimal scaling behaviors in both $\epsilon$ and $\delta$ as in the nonadversarial scenario, which have never been achieved before. Theorem 6 sets a general upper bound on the overhead of state verification in the adversarial scenario. If $p = \nu/e$ or $p_\star(\nu, \tau) \leq p \leq p_\star(\nu)$ for example, then

$$N(\epsilon, \delta, \Omega_p)/(N_{\text{NA}}(\epsilon, \delta, \Omega)) < \nu h(\nu/e, \nu, 0) \ln(1 - \nu e)^{-1} \ln(F\delta)^{-1}/\nu e \ln \delta. \tag{26}$$

Analysis shows that $h(\nu/e, \nu, 0)$ decreases monotonically in $\nu$ (for $0 < \nu \leq 1$), while $\nu h(\nu/e, \nu, 0)$ increases monotonically and satisfies $1 < \nu h(\nu/e, \nu, 0) \leq \epsilon/26$. Consequently, the bound in Eq. (26) decreases monotonically in $1/\epsilon, 1/\delta$, and $1/\nu$, as illustrated in Fig. 2. The overhead is at most three times when $\epsilon, \delta \leq 1/10$ and is negligible as $\nu, \epsilon, \delta$ approach zero. So pure states can be verified in the adversarial scenario with almost the same efficiency as in the nonadversarial scenario.

It should be emphasized that we can choose the probability $p$ for performing the trivial test without even knowing the value of $\tau$, while achieving a nearly optimal performance. In particular, the choices $p = p_\star(\nu)$ and $p = \nu/e$ are nearly optimal. In addition, the performance of $\Omega_{\star}$ is not sensitive to $\tau$, unlike $\Omega$. These observations are very instructive to devising efficient verification protocols for the adversarial scenario.
Summary.—We established a general framework for verifying pure quantum states in the adversarial scenario and clarified the resource cost of a general verification strategy. Moreover, we proposed a simple but powerful recipe to constructing efficient verification protocols for the adversarial scenario from the counterpart for the nonadversarial scenario. To construct an efficient protocol for the adversarial scenario, it suffices to find an efficient protocol for the nonadversarial scenario and then apply our recipe.

Our study can readily be applied to verify many important quantum states in the adversarial scenario with unprecedented high efficiencies. In conjunction with recent works, optimal protocols can be constructed for all bipartite pure states which only require $[\epsilon e^{-1} \ln \delta^{-1}]$ tests to achieve infidelity $\epsilon$ and significance level $\delta$. Nearly optimal protocols can be constructed for stabilizer states (including graph states) which require $[3e^{-1} \ln \delta^{-1}]$ tests. General hypergraph states, weighted graph states, and Dicke states can be verified efficiently with about $ne^{-1} \ln \delta^{-1}$ tests, where $n$ is the number of qubits. More details can be found in the companion paper [26]. These results are instrumental to many applications in quantum information processing.

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