MONTE CARLO SIMULATIONS OF OPINION DYNAMICS

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We briefly introduce a new promising field of applications of statistical physics, opinion dynamics, where the systems at study are social groups or communities and the atoms/spins are the individuals (or agents) belonging to such groups. The opinion of each agent is modeled by a number, integer or real, and simple rules determine how the opinions vary as a consequence of discussions between people. Monte Carlo simulations of consensus models lead to patterns of self-organization among the agents which fairly well reproduce the trends observed in real social systems.

1. Introduction

Statistical physics teaches us that, even when it is impossible to foresee what a single particle will do, one can often predict how a sufficiently large number of particles will behave, in spite of the eventually large differences between the variables describing the state of the individual particles.

This principle holds, to some extent, for human societies too. It is nearly impossible to predict when one person will die, as the death depends on many factors, most of which are hard to control: nevertheless statistics of the mortality rates of large populations are stable for long times and have been studied for over three centuries. We then come to the crucial question:

Can one describe social behaviour through statistical physics?

The question is tricky, and bound to trigger hot debates within the physics community. On the one hand, society is made of many individuals which interact mostly locally with each other, like in classical statistical mechanical systems. On the other hand, social interactions are not mechanical and are hardly reproducible. However we expect that the aspects of collective behaviour and self-organization in a society may be reasonably well described by means of simple statistical mechanical models and by now several such models have been introduced and analyzed, giving rise to the new field of sociophysics.
In this contribution we shall concentrate on opinion dynamics. The spread and evolution of opinions in a society has always been a central topic in sociology, politics and economics. One is especially interested in understanding the mechanisms which favour (or hinder) the agreement among people of different opinions and/or the diffusion of new ideas.

Early mathematical models of opinion dynamics date back to the 50’s, but the starting point for quantitative investigations in this direction is marked by the theory of social impact proposed by Bibb Latané. The impact is a measure of the influence exerted on a single individual by those agents which interact with him/her (social neighbours). Models based on social impact were among the first microscopic models of opinion dynamics. They are basically cellular automata, where one starts by assigning, usually at random, a set of numbers to any of the $N$ agents of a community. One of these numbers is the opinion, the others describe specific features of the agents, like persuasiveness, supportiveness, tolerance, etc. Society is modeled as a graph, and each agent interacts with its geometric neighbours, which represent friends or close relatives. The procedure is iterative: at each iteration one takes a set of interacting agents and updates their opinions (or just the opinion of a single agent), according to a simple dynamical rule. After many iterations, the system usually reaches a state of static or dynamic equilibrium, where the distribution of the opinions among the agents does not change shape, even if the agents themselves still change their mind. The dynamics usually favours the agreement of groups of agents about the same opinion, so that one ends up with just a few opinions in the final state. In particular it is possible that all agents share the same opinion (consensus), or that they split in two or more factions.

Most results on opinion dynamics derive from Monte Carlo simulations of the corresponding cellular automata. We shall here shortly present two basic consensus models: the Bounded Confidence Model (BCM) and the Sznajd Model (SM). For a complete exposition of the recent results on these models we refer to. Due to lack of space we are forced to omit the discussion of other important classes of opinion dynamics, like the voter models, the majority rule models and the Axelrod model.

2. The Bounded Confidence Model

The BCM is based on the simple consideration that two persons usually discuss with each other about a topic only if their opinions on that topic are quite close to each other, otherwise they quarrel or avoid discussing. This can be easily modeled by introducing a parameter $\epsilon$, called confidence bound, and by checking whether the opinions $s_i$ and $s_j$ of two social neighbours $i$ and $j$ differ from each other by less than $\epsilon$. If this were the case, we say that the opinions of the two agents are compatible and they can start a conversation which may lead to variations of their opinions. Opinions can be integers or real numbers; the opinions are initially distributed at random among the agents. The number of opinion clusters $n_c$ in the
final configuration depends on the confidence bound: if $\epsilon$ is small, $n_c$ is roughly $1/\epsilon$; above some threshold $\epsilon_c$ the system attains consensus. There are two main versions of the BCM, which are characterized by two different dynamical rules of opinion updating: the consensus model of Deffuant et al. (D) and that of Krause-Hegselmann (KH). Here we shall discuss the latter.

2.1. Krause-Hegselmann

![Figure 1](image.png)

Figure 1. Time evolution (in Monte Carlo steps per agent) of the opinion distribution of the KH model for a society where everybody talks to everybody else. The number of agents is 10000, $\epsilon = 0.13$. The agents form three different factions in the final state.

The iteration of the KH model in the case of real-valued opinions consists of the following three steps:

1. An agent $A$ is selected, sequentially or at random;
2. One checks which of the neighbours of $A$ have opinions compatible with that of $A$.
3. The new opinion of $A$ is the average of the opinions of its compatible neighbours.

The dynamics of the model is not trivial because the opinion space is bounded (typically $[0, 1]$): in fact, the inhomogeneities at the edges determine density variations in the opinion distribution, which propagate towards the center (Fig. 1).

For integer opinions, the update rule is even simpler: agent $A$ takes the opinion of one of its compatible neighbours, chosen at random. This rule recalls that of the voter and Axelrod models. In a society where everybody talks to everybody else, if there are $Q$ possible choices for the agents and the condition of compatibility for two opinions $S_i$ and $S_j$ is $|S_i - S_j| \leq 1$, the community always reaches consensus
provided $Q \leq 7$.

3. The Sznajd Model

The SM is probably the most studied consensus model of the last years. The reasons of its success are the intuitive “convincing rule” and the deep relationship with spin models like Ising. One starts with a simple remark: an individual is more easily convinced to change its mind if more than just a single person try to persuade him/her. So, if two or more of our friends share the same view about some issue, it is likely that they will convince us to accept that view, sooner or later.

![Histogram](image)

Figure 2. Histogram of the fraction of candidates receiving a given number of votes for 1998 election in the state of Minas Gerais (Brazil). A simple election model based on Sznajd opinion dynamics reproduces well the central pattern of the data. The data points are indicated by ×, the results of the election model by + (from Ref. 15).

In the most common implementation of the model, a group of neighbouring agents which happen to share the same opinion imposes this opinion to all their neighbours. The “convincing” pool of friends can be a pair of nearest-neighbours on a graph, or groups of three or more neighbours like triads on networks or plaquettes on a lattice. One usually starts from a random distribution of opinions among the agents, with a fraction $p$ of agents sharing the opinion +1 (the rest of the agents having opinion −1). In the absence of perturbing factors like noise, the state of the system always converges towards consensus and a phase transition is observed as a function of the initial concentration $p$: for $p < 1/2$ ($> 1/2$) all agents end up with opinion −1 (+1).

Since the original formulation of the model, for a one-dimensional chain of agents, countless refinements have been made, which concern the type of graph, the updating rule, the introduction of external factors like a social temperature, advertising and ageing, etc. (for more details see 12, 13).
The Sznajd dynamics has been used to devise simple election models which reproduce the bulk behaviour of votes distributions of real elections\(^{15,16}\) (Fig. 2): this is at present the strongest validation of the SM.

4. Conclusions

Sociophysics and in particular opinion dynamics are moving their first steps, and there is still a lot to do. Nevertheless the first results are encouraging and the hope to explain in this way the collective behaviour of social systems is strong. For the future it is necessary to gather more data from real systems and to open collaborations with sociologists.

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