Evolutionary Dynamics of Investors’ Expectations and Market Price Movement

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Abstract

The paper presents a step forward into the development of the theory of meaning. Stock and financial markets are examined from communication-theoretical perspectives on the dynamics of information and meaning. This study focuses on the link between the dynamics of investors’ expectations and market price movement. The model for market asset price dynamics, based on a non-linear evolutionary equation linking investors’ expectations and market asset price movement, is provided. Model predictions are tested on various FX, energy, food, and indices markets along different time frames. The results suggest that the model predicted time series is co-integrated with asset time series which implies that the proposed model can be used to forecast future price movement.

Key words: Stock and financial markets, information, meaning, non-linearity, model

I. Introduction

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The question of whether financial markets can be forecasted is of major concern for investors and policy makers. The same relates to mechanisms which control market assets price movement. At a quick glance at some market asset price graph one can discern periods of directional price movement and periods of price fluctuations. This graph reflects investors’ decisions whether to buy, sell, or stand out of the market at specific moments of time. Investors, as human beings, are driven by logic and psychology in their decisions. Delicate balance between these two factors defines the asset price behavior. In a nutshell, market asset price is driven by investors decisions, decisions are driven by corresponding trading preferences. But what drives preferences? It may be incoming information which is either rationally analyzed or skewed by psychological biases. This analysis provides information with meaning which is the ground for further actions. The question is – whether this incoming information is uniformly processed by different groups of investors. In other words – whether the same information is supplied different meanings by different groups of investors. The paper strives to study the mechanisms which govern investors’ preferences.

The first aspect of the paper relates to the mechanisms of the financial time series formation and the possibility of forecasting the future asset price movement. There are various approaches to the study of market dynamics in the attempt to predict future changes. One idea in assessing market asset price evolution is that past price values may indicate their future values in accordance with observable market trends. This is the cornerstone of technical analysis (Miner, 2002). Elliott wave patterns (Elliott, 1994) and cycles, some of which can be associated with business cycles, are used in a market timing strategy (Millard, 1999).

Another approach states that future price values have no connection with past price values so that asset prices resemble the movements of molecules (Osborn, 1959; Osborn & Murphy, 1984).
Consequently market movements cannot be anticipated. This is also the essence of the random walk hypothesis (e.g. Fama, 1965; Malkiel, 1973).

The efficient-market hypothesis (EMH) (Fama, 1970) and related theories (e.g. Black & Scholes, 1973; Merton, 1973; Gulko, 1997, etc.) assume that asset prices reflect all available information so that future prices cannot be predicted. However there are also examples of predictable price behavior (e.g. Working, 1960; Cowles and Jones, 1937; Kendall, 1953) which in the framework of the random-walk hypothesis are considered as anomalous. Furthermore, machine-learning based studies are reported to provide high accuracy in forecasting financial time series (e.g. Chang et al., 2009; Bitvai & Cohn, 2014; Patel et al., 2015; Hsua et al., 2016).

Behavioral economics (e.g. Kahneman & Tversky, 1979; Thaler, 1980, 1985; Banerjee, 1992), which concentrates on various psychological factors influencing the economic decisions of people, entertains an approach which is different from EMH. It states that the decisions of economic agents can to a large extent be considered irrational and driven by psychological factors so that the economic worldview of rational agents can no longer be supported. A number of publications refer to studies of mechanisms that govern investors’ decisions on financial markets (e.g. Shiller, 1981; Statman, 1995; Olsen, 1998; Barber & Odean, 1999). Since the departures from complete rationality are systematic and can be modelled and studied, knowledge of these mechanisms may be used to improve predictions of future investors’ behavior and market assets’ price movements.

For example in the case of herd behavior the hitherto formed market trends can be extrapolated for some future period. This indicates that the market is to some degree predictable and past prices at times can be used to forecast the direction of price change (Lo & Mackinlay, 2002).
The adaptive market hypothesis (Lo, 2004, 2005) considers the EMH and behavioral approaches opposite sides of the same coin by interpreting market participants’ behavioral biases in an evolutionary aspect, as adaptation to changing market environmental conditions, so that each group of market participants, behaves in its own appropriate manner. In the evolutionary aspect the adaptive market hypothesis is close to the complex systems approach which considers the market a complex evolving system of coupled networks of interacting agents. The domain of complexity economics refers to the formation, emergence, self-organization and change in the economy (e.g. Anderson, P., Arrow, K., and D. Pines, 1988; Kaufmann, 1995; Krugman, 1995; Arthur, Durlauf and Lane, 1997; Farmer, A., Durlauf, S., Kirman, A., Pines, D., and Sornette, D., 2012; Helbing, 2012). Complexity theory with respect to financial markets explores non-linearity, self-organization, emergent dynamics, and develops prediction models, such as stock market crashes (Sornette, 2003).

When behavioral finance primarily focus on individual level biases social finance - the newly emerging paradigm in financial studies – concentrates on cultural traits, including information signals, beliefs, strategies etc., adopted by larger groups of investors. “Analysis of social interactions promises to provide greater insight into where heuristics come from, and to offer a foundation for understanding shifts in investor sentiment. As such, it can potentially offer a deeper basis for understanding the causes and consequences of financial bubbles and crises” (Hirshleifer, 2015, p.44)

A common assumption in the mentioned approaches is that the market asset prices reflect the investors’ reaction to the information. However they miss the mechanisms of processing this information by investors. One may ask the questions: 1) in what way do investors perceive information, 2) does the same information evoke the same meaning for different groups of
investors, 3) does the way investors process the information play a more significant role than the information itself? The motivation of the paper is to find an answer to these questions.

The present paper entertains a novel approach which relies upon information processing by groups of investors that drives their behavior (hereinafter referred to as informational approach). When information is received it should first be processed, i.e. supplied with meaning. However information can be processed differently by different groups of investors, which provide differing criteria with which to supply information with meaning.

These criteria can be specified as selection environments in terms of specific coding rules (or sets of communication codes). Coding rules drive latent structures which organize different meanings into structural components (Leydesdorff, 2010). “Meanings originate from communications and feedback on communications. When selections can operate upon one another, a complex and potentially non-linear dynamics is generated” (Leydesdorff, 2021, at p.15).

Meanings produce expectations about possible system states which are generated with respect to future moments. Expectations operate as a feedback on the current state (i.e. against the arrow of time). In other words the system simultaneously entertains its past, present and future states according Bachelier’s observation that “past, present and even discounted future events are reflected in market price” (Bachelier, 1900). He further added “but (it) often show no apparent relation to price changes”, but as I show below, this is not always the case.

Expectations provide a source of additional options for possible future system states that are available but have not yet been realized. The more options possessed by the system the greater is the likelihood that the system will deviate from the previous state in the process of autocatalytic
self-organization. The measure for additional options is provided by redundancy which is defined as the complement of information to the maximum informational content (Brooks & Wiley, 1986). The concept of redundancy was applied to innovation studies with respect to the synergy in the Triple Helix (TH) model of university-industry-government relations (e.g. Etzkowitz & Leydesdorff, 1995, 1998; Leydesdorff, 2003; Park & Leydesdorff, 2010; Leydesdorff & Strand, 2013 etc.). Interaction among differently shaped investors’ expectations can eventually generate non-linear dynamics in market prices.

This paper’s contribution is two-fold: 1) in a narrow sense it presents a model for the market asset price dynamics which can shed light on mechanisms which govern market asset price time series formation and in some cases allows one to forecast future price movement; 2) in a broad sense it is a step forward in elaborating the theory of meaning which moves this theory to wider practical domains.

The first research aim of this paper is to test the applicability of the general information and meaning communication concept for the description of market price dynamics. The second research goal is to provide a quantitative description of market price movement based on the evolutionary dynamics of investors’ expectations. I show that this non-linear dynamics can be captured by a non-linear evolutionary differential equation whose solutions give recognizable patterns. These patterns can be used to forecast future market price movement. This allows one to make more accurate predictions on market assets’ future price change and market crashes.

II. Literature Review
The approach which utilizes meaning generation in inter-social communications is not rooted in financial literature. However some model entertained basic features can be found in the literature.

Financial time series modelling have been a matter of the debate among academics and practitioners. A big branch of literature is devoted to machine based time series forecasting. Related types of models comprise traditional Machine Learning (ML) models (e.g. Bahrammirzaee, 2010; Mullainathan, S. and Spiess, 2017; etc.) and emerging within the ML field Deep Learning (DL) models, such as Artificial Neural Networks (ANN), Recurrent Neural Network (RNN), Convolutional Neural Network (CNN) and Deep Multilayer Perceptron (DMLP) (e.g. Schmidhuber, 2015; Sokolov et al., 2020).

The advantage of analytical approach in comparison with numerical one is that in analytical models one is able to trace back time series properties to investor behaviors.

The commonly observed and hard to be explained facts, such as excess volatility (LeRoy & Porter, 1981; Shiller, 1981), volatility clustering (Mandelbrot, 1997), etc., gave rise to the growing body of literature referring to heterogeneous agent based models (HAM) (e.g. Hommes, 2006, Chiarella, Dieci, and He, 2009) in which financial market comprise different groups of agents. Agent interaction can generate complicated market dynamics comprising both chaos and stability (e.g. Brock, W.A., and Hommes, 1997; Lux, 1995, Bonabeau, 2002). Kaizoji (2004) showed that intermittent chaos in the asset price dynamics can be observed in a simple model of financial markets with two groups of agents, which can be attributed to heterogeneity in traders’ trading strategies.
The behavior of financial markets in periods of crises can be described by rogue waves (e.g. Jenks, 2020). This type of waves is reported analytically in the nonlinear option pricing model (Yan, 2010). These solutions may be used to describe the possible mechanisms for rogue wave phenomenon in financial markets. Rogue waves can be found in Korteweg de-Vries (KdV) systems if real nonintegrable effects, higher order nonlinearity and nonlinear diffusion are considered (Lou & Lin, 2018).

Dhesi & Ausloos (2016) when studying agent behaviour reacting to time dependent news on the log returns in the framework of Irrational Fractional Brownian Motion model observed a kink-like effect reminiscent of soliton behavior. They further posed a question - what is the differential equation whose solution describes this effect? In some sense the idea of this approach is close to that used in opinion dynamics models (e.g. Zha et al., 2020; Granha et al., 2022).

Binge and Boshoff (2021) introduced a pseudo-repeat sales method for the construction of art prices. They proposed a regression-based methods which produce better estimates of price changes and allows to investigate bubbles in a number of asset markets. The gap between the asset price and the fundamental value will exhibit temporary explosive behavior. The formed bubble is expected to eventually collapse and consequently a trend reversion can be forecasted. In the absence of structural change in the fundamentals, a period of explosive prices has a non-fundamental explanation, i.e. cognitive bias.

Investors’ inability to properly process available information can cause systematic patterns of price movement, such as seasonalities. This inability can be attributed to a cognitive bias on the part of investors (Hirshleifer et al., 2020; Fang et al., 2021), which employ ease-of processing
heuristics in processing information relevant to market asset pricing. The knowledge of seasonalities mechanisms can yields predictions for repo rate movements.

Biased beliefs about future price movements are important driver of market prices. Changes in investors’ expectations about future stock market returns can explain facts about stock market price movements (De la O and Myers, 2021). Jin and Sui, (2021) showed that investors’ beliefs about future market returns can generate excess volatility in stock market returns. These beliefs depend mainly on recent past returns and can produce significant predictability of returns.

Investors group behavior in interpreting the external information also influence stock price movements. Massa, O’Donovan and Zhang (2021), showed that the strategic risk reallocation of business groups in response to news which can be interpreted as “bad” information is an additional factor to shape stock returns.

Duffy, Rabanal, Rud (2021) studied how exchange traded funds (ETFs) affect asset pricing, and turnover in a laboratory asset market comprised of three subsets of traders, wherein each subset entertains its own preferential trading strategy. The three component market structure is also described in the framework of SIR model which is used to explain the spread of investor ideas and bubbles (Shiller, 2019). SIR-related models (Burnside, Eichenbaum, Rebelo, 2016; Hirshleifer, 2020) can explain price overshoooting and bubbles and serial correlation pattern in asset returns at different lags. SIR model, in turn, can be linked to TH model and corresponding non-linear dynamics, described by KdV equation, can be observed (Ivanova, 2022).

Noussair, Popesku (2021) provided evidence that market asset co-movement in the absence of common shock to underlying fundamentals can be explained by behavioral factors caused by asymmetric information between informed and uninformed traders. Co-movement can arise
between two risky assets even if their fundamentals are not correlated also following a dividend shock, the shocked asset exhibits autocorrelation in returns.

An emerging paradigm for understanding financial markets is social finance (Akçay and Hirshleifer, 2021). Social finance posits that behaviors of financial market participants depend upon adopted cultural traits which define models of how markets work. These traits are peculiar to larger groups of investors and provide informational transmission biases among groups of investors. Investors’ financial traits are not stable and can be modified with time.

III. Method

The paper method-relied upon general theory of meaning generation in inter-social communications (e.g. Leydesdorff, 2021). This theory uses -information- theoretical approach to measuring redundancy as an indicator of the synergy in the Triple Helix model of innovations. Figures 1, 2 and 3 illustrate the origin of this approach which forms the ground for an evolutionary equation describing market asset price movement.

The market can be considered a complex social system whose compound dynamics drive market price movement. The market is comprised of various groups of investors – pension funds, banks, hedge funds, publicly traded corporations, individuals, and so on - who behave according to their investment preferences. Instead of utilizing a market microstructure approach grounded on computational agent-based models, which imitate the behavior of individual investors (e.g. O’Hara, 1995), one can concentrate on information processing among larger groups of investors.
With respect to investors’ sentiment toward the direction of market price movement they can be roughly subdivided into two large groups (or market agents) which comprise investors who expect upward price movement and those anticipating downward price movement. The third group of unresolved investors waiting for more favorable conditions for entering the market should also be taken into account.

Agents make their decisions on the basis of communicated information. However this information should first be provided with meaning (as a ‘signal’) or discarded (as noise). Each group entertains different criteria for filtering information in accordance with their behavioral biases in relation to the held market positions. That is the same information is considered from different perspectives (or positions), and can be supplied with different meanings.

Mechanisms of information and meaning processing are different. Whereas information can be communicated via a network of relationships meanings are provided from different positions (Burt, 1982). Meaning cannot be communicated but only shared when the positions overlap. The processing of meaning can enlarge or reduce the number of options that can be measured as redundancy (Leydesdorff & Ivanova, 2014). The calculus of redundancy is complementary to calculus of information.

Shannon (1948) defined information as probabilistic entropy: \( H = -\sum_i p_i \log p_i \) which is always positive and adds to the uncertainty (Krippendorff, 2009). One can consider two overlapping distributions with information contents \( H_1 \) and \( H_2 \) (Figure 1):
Fig. 1: Set-theoretical representation of two overlapping distributions with informational contents $H_1$ and $H_2$

Total distribution is the sum of two distributions minus overlapping area, since it is counted twice:

$$H_{12} = H_1 + H_2 - T_{12}$$  \hspace{1cm} (1)

Overlapping area relates to mutual or configurational (McGill, 1954) information ($T_{12}$). The formula linking aggregate distribution ($H_{12}$) with $H_1$, $H_2$ and $T_{12}$ is:

$$T_{12} = H_1 + H_2 - H_{12}$$  \hspace{1cm} (2)

Analogously configurational information in three dimensions (e.g., Abramson, 1963) is:

$$T_{123} = H_1 + H_2 + H_3 - H_{12} - H_{13} - H_{23} + H_{123}$$  \hspace{1cm} (3)

Figure 2 is a set-theoretical representation of three overlapping distributions.
Fig. 2: Set-theoretical representation of three overlapping distributions with informational contents $H_1$, $H_2$ and $H_3$

However $T_{123}$ is no longer Shannon-type information, since it is negative. The sign alters with each newly added distribution (e.g. Krippendorff, 2009). Technically the problem of sign change can be solved by introducing “positive overlapping” (Leydesdorff & Ivanova, 2014). This time one does not correct for overlap which is counted twice and therefore redundant, but assume other mechanisms with which two distributions influence one another. These mechanisms are different from the relational exchange of information and lead to an increase of redundancy so that overlapping area is added instead of being subtracted. In formula format, this is represented as follows:

$$H_{12} = H_1 + H_2 + R_{12}$$  \hfill (4)

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This is true for the configuration shown in Figure 3. In general the resulting value can be positive, negative or zero depending upon the relative sizes of terms.
It follows that $R_{12}$ is negative ($R_{12} = -T_{12}$) and is hence a redundancy - reduction of uncertainty\(^3\) (analogously: $R_{123} = T_{123}$). That is in measuring configurational information in three (or more) dimensions, one measures not Shannon-type information but mutual redundancy. Since this measure provides a negative amount of information it can be considered an indicator of synergy among three sources of variance (Leydesdorff, 2008b).

Three groups of investors communicate among themselves and form a network of relations. But there is another mechanism on the top of the structural network which drives the system evolution. Communicated information is processed differently by each group of investors in accordance with different sets of coding rules (communication codes) so that groups are (positionally) differentiated with respect to their positions toward processing the information. For example, the same information can be supplied with different meaning and, according to investors’ sentiments, can be accepted as a signal to buy or sell by different agents. The sets of communication codes are latent but can be partly correlated by forming a correlation network on the top of the relational one. The generation of meaning is provided from the perspective of hindsight. The structural differences among the coding and decoding algorithms provide a source of additional options in reflexive and anticipatory communications, meaning that the generating structures act as selection environments (Leydesdorff, 2021). These additional options are the source of variations.

Variations arise when market eventually changes from its previous state. When a system comprises three or more agents each third agent disturbs interaction between the other two. This mechanism is known as “triadic closure” and drives the system’s evolutionary dynamics

\[^3\text{Redundancy is defined as a fraction of uncertainty that is not used for the uncertainty } H \text{ that prevails: } R = 1 - \frac{H}{H_{\max}} = \frac{H_{\max} - H}{H_{\max}}. H_{\max} - \text{maximum possible entropy. Adding new options increases } H_{\max} \text{ and redundancy.}\]
Simmel (1902) pointed toward qualitative difference between dyads and triads. Triads can be transitive or cyclic (Batagelj et al., 2014). Two cycles may emerge – positive (autocatalitic) and negative (stabilizing) ones (Figure 3). Autocatalytic cycle reinforce the change from the previous system state (the system self-organizes) while the stabilising cycle keeps the system from transformation.

**Fig. 3**: Schematic of three-component positive a) and negative b) cycles (Adapted from Ulanovitz, 2009)

The dynamics of information and meaning can be evaluated empirically using the sign of mutual information ($R$) as an indicator. The balance between stabilization and self-organization can be
simulated in terms of the rotations of the two vectors $P$ and $Q$ (Ivanova & Leydesdorff, 2014a, 2014b)

$$R \sim P^2 - Q^2$$  \hspace{1cm} (5)

The first term in Eq. (5) can be considered due to historical realization (which adds to positive entropy) and the second term corresponds to self-organization and augments negative entropy. Historical realization relates to historically realized options which are generated via a recursive mode and self-organization bears on new, not yet realized options, generated via an incursive mode.\footnote{Recursive systems use their past states to modulate the present ones, incursive (or anticipatory) systems employ possible future states to shape their present states (e.g. Rosen, 1985; Dubois, 1998; Leydesdorff & Dubois, 2004)} Comparing two expressions for redundancy provided by Eqs. (1) and (2) one can identify positive and negative terms in Eq. (1) with positive and negative terms in Equation 2.

The trade-off between historical realization and self-organization (Eq. (5)) leads to redundancy cyclical evolution (Ivanova & Leydesdorff, 2014b). It was shown by Dubois (2019) that for temporal cyclic systems probabilities $p_i$ can oscillate around their average values $p_{i0}$ in a harmonic or non-harmonic mode.

For non-harmonic oscillations, one can derive (see Appendix A):

$$\frac{1}{k} \frac{d^2 p_i}{dt^2} = -(p_i - p_{i0}) + \alpha (p_i - p_{i0})^2 + C_i$$  \hspace{1cm} (6)

The probability density function $P$ satisfies the following non-linear evolutionary equation (see Appendix B for the derivation):

$$P_T + 6PP_x + P_{xxx} + C_1 = 0$$  \hspace{1cm} (7)
which is the generalization of the well-known Korteweg-de Vries equation:

\[ U_T + UU_X + U_{XXX} = 0 \]  

Eq. (7) admits soliton solutions. A single soliton solution has the form:

\[ P(X,T) = 2 \left( \frac{\kappa}{2} \right)^2 ch^{-2} \left[ \frac{\kappa}{2} \left( X - 4 \left( \frac{\kappa}{2} \right)^2 T + \frac{C_1}{2} T^2 \right) \right] - C_1 T \]  

Or by setting \( \rho = \frac{\kappa}{2} \), Eq. (9) can be written in a more general form:

\[ P(X,T) = n(n + 1) \rho^2 ch^{-2} \left[ \rho \left( X - 4 \rho^2 T + \frac{C_1}{2} T^2 \right) \right] - C_1 T \]  

An impulse in the form of Eq. (8) eventually evolves in a train of \( n \) solitary waves with amplitudes: \( 2\kappa^2, 8\kappa^2, 18\kappa^2 \ldots 2n^2\kappa^2 \) and the corresponding velocities \( 4\kappa^2, 16\kappa^2, 32\kappa^2, \ldots 4n^2\kappa^2 \) (Miura, 1976).

There is also a direct method for finding soliton solutions for Eq. (8) which incorporate multiple solitons with arbitrary amplitudes so that the \( N \)– soliton solution takes the form:

\[ P = 2 \frac{d^2}{dX^2} \log F_N \]  

where:

\[ F_N = \sum_{\mu=0}^{1} e^{\mu \eta} \left( \sum_{i=1}^{N} \mu_i \eta_i \right. + \left. \sum_{1 \leq i < j}^{N} \mu_i \mu_j A_{ij} \right) \]  

\(^5\) Equation 12 refers to a net soliton solution. In the case of an arbitrary initial perturbation it evolves in a train of solitons moving off to the right and an oscillatory dispersive state moving off to the left [33].
Here $\eta_i = k_i X - k_i^2 T$; $A_{ij}$ are the phase shifts of the solitons: $e^{A_{ij}} = \left(\frac{k_i - k_j}{k_i + k_j}\right)^2$ (Ablowitz and Segur, 1981). It follows from Eq. (10) that the corresponding $N$-soliton solution for Equation 5 is:

$$\Phi_N = \exp\left[-\frac{C}{2} t x^2 + A x + B\right] \cdot \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \eta_i + \sum_{1 \leq i < j}^{N} \mu_i \mu_j A_{ij}\right)$$

(13)

The additional term at the right hand side of Eq. (5) corrects the amplitude of the solitons with the lapse of time. Wave, described by Eq. (7), moves to the right and after time span: $T_1 = 8k_2/C_1$ returns to the origin. In case of a train of solitons there is a relationship between soliton amplitudes and time intervals: $\frac{A_i - A_j}{T_i - T_j} = \text{const}$. There are also periodic solutions of Eq. (6) (Appendix C, cf. Lax, 1974).

Information obtained via informational exchange is processed with communication codes and expectations with respect to future time are generated at a system’s level. These expectations can be considered redundancy density presenting non-realized but possible options, distributed along time interval, which can, not in all but in some cases, trigger subsequent actions. Here expectations are analytical events (options) and actions are historical events, which can be observed over some time as a response to the expectations (as if expectations move against the arrow of time and turn into actions). In other words, there is a dynamic of the actions in historical events at the bottom and a dynamic of expectations at the upper level operating reflexively.

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6 The sum over $\mu = 0,1$ refers to each of the $\mu_i$. E.g. performing the calculation for N=3 yields $F_3 = 1 + e^{\eta_1} + e^{\eta_2} + e^{\eta_3} + e^{\eta_1+\eta_2+A_{12}} + e^{\eta_1+\eta_3+A_{13}} + e^{\eta_2+\eta_3+A_{23}} + e^{\eta_1+\eta_2+\eta_3+A_{12}+A_{13}+A_{23}}$

7 Shannon (1948) defined the proportion of non-realized but possible options as redundancy, and the proportion of realized options as the relative uncertainty or information.
Expectations are eventually transformed into actions and represent new system states.\textsuperscript{8} The moments of time when expectations turn into actions can be considered as the moments when the solitons return to the origin. This way one can change from a moving frame in the x-dimensional to a fixed frame in the t-dimension.

Initial expectations arise as a set of market beliefs represented by an impulse. These expectations are projected to the future and further stratified following the non-linear dynamics of information processing among investors and can be mapped as a train of solitons moving to the right. At the next stage solitons return back to the origin. Finally expectations are realized and turned into observed market asset price changes forming specific wave patterns in t-axes resembling Elliott wave patterns. The described mechanism operates on all price and time scales generating a self-similar fractal structure.

IV. Data

The paper relies on publicly available data sets. The data comprise Australian dollar daily data (2001.09.21 - 2004.05.13), Swiss franc hourly data (2021.01.07 – 2021.04.12), natural gas daily data (2016.03.02 – 2017.02.13), Corn daily data (2020.08.10 - 2021.07.23), and S&P500 index monthly data (2009.02.01 – 2022.01.01). Various data sets are specially chosen in order to test the model applicability across different markets and different time scales. I compare the model results with the empirically observed data on FX, energy, food, and indices markets. Since the S&P 500 index market data take a fairly long time interval I used inflation-adjusted data.

\textsuperscript{8} According to the second law of thermodynamics a system’s entropy increase with time. For isolated systems it can reach thermodynamic equilibrium
V. Results

It follows from the model that when non-linear effects prevail (Eq. (A 10)) asset prices develop in trends which can be described by a non-linear evolutionary equation (7). In other cases, there may be other mechanisms that determine price dynamics.

To answer the question about whether the information and meaning communication concept can be applied to a description of market price dynamics one can try to find samples that can be described by the model.

Figure 4 shows time series for the a) Australian dollar (AUD/USD), b) Swiss franc (USD/CHF), c) Natural gas (NGas), d) Gold (XAU) and e) Standard and Poor's 500 stock market index (S&P500) and their model approximation (f). The data are fitted with one or two series of solitons obtained from Equation 13. The parameters of approximation are listed in Table 1. Here $A_i$ – soliton amplitudes ($i$ - refers to the number of soliton in the series), $k_i$ - $k$ parameters, $T_i$ – time spans (Equation 12), $\beta$ - vertical shift (are introduced in order to equate the beginning of the first wave to zero).

**Table 1** Approximation parameters for empirically observed data on market assets
Australian dollar daily data (2001.09.21–2004.05.13), natural gas daily data (2016.03.02–2017.02.13) and S&P500 index monthly inflation adjusted data (2009.02.01–2022.01.01) were fitted by four solitons. The Swiss franc hourly data (2021.01.07–2021.04.12) and Corn daily data (2020.08.10–2021.07.23) were fitted by a series of three solitons. Approximation parameters satisfy the model predicted ratio, i.e. the uptrend slope for each two solitons doesn’t change: \[ \frac{A_i - A_j}{T_i - T_j} = const. \]
c)
**Fig. 4:** Time series for the a) daily Australian dollar (AUD/USD), b) hourly Swiss franc (USD/CHF), c) daily Natural gas (NG), d) daily Corn data and e) monthly Standard and Poor's 500 stock market index (S&P500) data; $f$ - model fit.

Figure 5 is a scatter chart of the a) Australian dollar (AUD/USD), b) Swiss franc (USD/CHF), Natural gas d), Corn and e) S&P 500 index values vs. model predicted values ($f$). A straight line is linear regression. Regression parameters are listed in Table 2.
$R^2 = 0.9723$
$R^2 = 0.9597$

b)
$R^2 = 0.9449$
Fig. 5: Scatter chart of the a) Australian dollar (AUD/USD), b) Swiss franc (USD/CHF), c) Natural gas (NG), d) Corn, and e) S&P500 index values vs. model predicted values (f). Straight line – OLS fit.

Table 2 OLS regression parameters (t-values are given in the parentheses)
It can be noted that the empirically observed data linearly relate to the values predicted by the model with a sufficient degree of accuracy. Ordinary least square fit allows the model to explain 88 – 96% of the variations. OLS coefficients are close to unity for all the listed market assets which indicates that the fitted model did a "good job". The constants are insignificant from zero for all the assets save Corn and S&P 500. However these constants can be considered small percentage values since Corn and S&P 500 scales are much wider that those for the rest of the assets.

In order to eliminate the problem of spurious correlation between asset’s price and model predicted curve one can perform an Engel - Granger two-step co-integration test (Engel & Granger, 1987) by regressing asset price by approximation function \( f \) and an intercept and then running an Augmented Dickey-Fuller test (ADF) for stationarity. Table 3 presents critical values for the Dickey–Fuller t-distribution. Test results are summarized in Table 4.

| No | AUD/USD | USD/CHF | NG | Corn | S&P 500 |
|----|---------|---------|----|------|---------|
| \( f \) | 1.01**  | 1.00**  | 0.97** | 0.9**  | 0.95**  |
|     | (155.43) | (195.03) | (41.03) | (64.03) | (51.89)  |
| Constant | -0.004*** | -0.002*** | 0.09*** | 52.39** | 190.56** |
|     | (-4.18)  | (-4.0)  | (1.36)  | (7.26)  | (4.04)   |
| Observations | 690 | 1601 | 246 | 241 | 156 |
| Adjusted \( R^2 \) | 0.97 | 0.96 | 0.87 | 0.94 | 0.95 |

*\( p < 0.05; ** p < 0.01; *** p > 0.05\)

Table 3 Critical values for Dickey–Fuller t-distribution
| Sample size | Without trend | With trend |
|-------------|---------------|------------|
| N = 25      | -3.75 - 3.00  | -4.38 - 3.60 |
| N = 50      | -3.58 - 2.93  | -4.15 - 3.50 |
| N = 100     | -3.51 - 2.89  | -4.04 - 3.45 |
| N = 250     | -3.46 - 2.88  | -3.99 - 3.43 |
| N = 500     | -3.44 - 2.87  | -3.98 - 3.42 |
| N = ∞       | -3.43 - 2.86  | -3.96 - 3.41 |

(Source: Fuller, 1976, p.373)

**Table 4** Unit root test results (ADF)
| Variables   | Critical value 1% | Critical value 5% | Computed value |
|------------|------------------|------------------|---------------|
| AUD/USD    | -3.46            | -2.88            | -3.97         |
| USD/CHF    | -3.43            | -2.86            | -4.23         |
| NG         | -3.46            | -2.88            | -3.51         |
| Corn       | -3.46            | -2.88            | -3.92         |
| S&P 500    | -3.46            | -2.88            | -3.20         |

Comparing the computed and critical values one may conclude that a null hypothesis of a unit root can be rejected at the 5% level for S&P 500 and at the 1% level for AUD/USD, USD/CHF, NG and Corn.

**VI. Discussion**

The results suggest that the model can accurately describe some patterns on price charts, such as trends, and may offer an indication of whether a trend is likely to continue or terminate. The model can also be applied to forecast not only future asset prices but also economic crises. For example, Standard and
Poor's 500 stock market index incorporates the 500 largest companies listed on stock exchanges in the United States. It is one of the largest equity indices which largely reflect the state of the US (and global) economy. The model predicting the termination of historical trend third wave may indicate that we are on the edge of the next global recession. One could also test market asset price behaviour in periods of price consolidation since Eq. (7) also possesses periodic solutions in the form of cnoidal waves but, due to the paper volume limitations, this remains a topic of future research.

The present paper entertains non-linear differential equation as a basic technique for the price dynamic description. Differential equations were earlier used by Caginalp & Balenovich to provide foundation to the technical analysis. The authors mentioned that “While economic and financial scholars often ignore or downplay the role of conventional technical analysis, financial experts are often quite eager to implement it without regard to the nature of the economic assumptions inherent in the methods” (Caginalp & Balenovich, 2003, p.7). They showed that some technical analysis patterns can be simulated with help of differential equations when one assumes the interactions between two or more groups of investors with different differing assessments of value and/or different motivational characteristics. However there were provided no reason of what is the origin of investors’ sentiments.

Information approach allows establish one more link with technical analysis. This relates to the Fibonacci retracement which is used in financial market technical trend analysis. Fibonacci retracement is widely used in technical analysis to predict price support and resistance levels. In

\[ \lim_{i \to \infty} \frac{n_{i+k}}{n_i} \text{, where } k = 1, 2, 3, \ldots \]

Corresponding values are: 1.618; 2.618; 4.236. Also the mirror ratios: \( \lim_{i \to \infty} \frac{n_{i-l}}{n_i} \) can be constructed. Here \( l = 1, 2, \ldots i \) and the most often used values are: 0.382, 0.500, 0.618, 1. These ratios are often used in Fibonacci retracement for determining support and resistant levels and comparing market price movements to one another (e.g. Colby, 2003).
an attempt to describe the market’s temporal evolution Miner (2002) applied the Fibonacci ratios for spotting market time cycles. One of the widespread approaches is to take the past waves or swings and use Fibonacci ratios to predict the relationships of past waves to future waves. For example using an alternate price projection (APP) one can project the proportion of a past swing to the next swing, moving in the same direction. The most important ratios of comparison of such swings that are considered to have the highest probability of support or trend termination are: 62%, 100%, 162%, 200%, 262%, and 424%. The other method refers to using price expansion (Exp) which expands the price range of a swing. All the same the most important ratios to use for price expansions are: 62%, 100%, 162%, 200%, 262%, and 424% (Miner, 2002).

However there are not valid foundations for its use from the economic and financial scholars’ viewpoint. Information approach provides one more possible explanation to the origin of Fibonacci ratios. Table 5 lists first seven Fibonacci ratios and corresponding soliton amplitudes obtained from Equation 8. It follows from Table 3 that Fibonacci ratios relatively well suit soliton amplitudes with maximal relative difference of 12%.

Table 5. First seven Fibonacci ratios and corresponding soliton amplitudes
Still there are no logical foundations for using Fibonacci numbers and ratios in dynamic price and time projection analysis which trace large and intractable sets of phenomena. The only reason for dynamic time analysis is that “If time and price are the effects of the same cause, the same techniques used for price analysis should be applicable to time analysis” (Miner, 2002, p.5-8). The correspondence between soliton amplitudes and Fibonacci ratios can shed light on the foundation of Fibonacci number-based relationship with Elliott waves.

Presented model can be used to predict the trend reversal. One can’t tell for sure at the very beginning that trend-like price movement starts to unfold. But when the trend unfolds and wave structure is confirmed one can await certain price levels to be reached at certain time points. If the level is reached by time scheduled point one can expect (temporal) trend reversion. However the presented model should not be considered a universal tool for forecasting financial time

| No | Fibonacci ratios | soliton amplitudes | difference (%) |
|----|------------------|-------------------|----------------|
| 1  | 1                | 1                 | 0              |
| 2  | 1.62             | -                 | -              |
| 3  | 2.62             | 3                 | 12.7           |
| 4  | 4.24             | 4                 | 5.7            |
| 5  | 6.85             | 6                 | 12.4           |
| 6  | 11.09            | 10                | 9.8            |
| 7  | 17.94            | 16                | 10.8           |
series during any periods of trend and price consolidation given that Equation 7 is derived under certain conditions that may not always be met.

At the same time Information approach adds to a growing literature that studies the role of social norms, moral attitudes, religions and ideologies in of imperfect rationality of market participants.

Recently there has been an increasing understanding that investors’ thoughts and behavior depend on adopted cultural traits which can be considered as evolutionary system. This forms the framework of a new paradigm for understanding financial markets – social finance (e.g. Akçay, Hirshleifer, 2020). There is a significant similarity between social finance and informational approaches. Table 6 summarizes some basic features of the two approaches.

| Social finance                                                                 | Information approach                                                                 |
|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| Adopted cultural traits, including information signals, beliefs, strategies,   | Sets of communication codes are the drivers of information processing and investors’  |
| and folk economic models are the drivers of investors’ decisions               | decisions                                                                            |
| Social transmission determine the evolution and mutation of financial traits    | Sets of communication codes shape each other in the process of communication          |
| Cultural traits are subject to different biases in judgments and decisions.     | Different sets of communication codes provide different meanings to the same         |
|                                                                               | informational content                                                                 |
| Cultural traits are not immediately measurable                                  | Meaning can be measured on the base of the extension of Shannon’s mathematical theory |
|                                                                               | of communications                                                                     |
| Social finance encompasses agent-based modeling of transmission biases         | Information approach encompasses modelling the effect of meaning generation with help |
|                                                                               | of non-linear evolutionary equation                                                  |
| Regards shifts in investors’ sentiment as an endogenous outcome of microevolutionary cultural processes | Regards shifts in investors’ sentiment as an endogenous outcome of interaction of communication codes sets, formed in the evolutionary process |
|---|---|
| Investors adopt and modify their financial traits | Agents adopt and modify their sets of communication codes |
| Considers a wider set of applications and range of time scales | Considers a wider set of applications and range of time scales |
| Cultural evolution operates at multiple time scales. The evolutionary dynamics of financial traits play out at multiple time scales | Communication code sets operate at multiple time scales. The evolutionary dynamics of meaning generation play out at multiple time scales |
| Is focused financial sphere | Can be applied a wider range of applications |

In the same vein as social finance information approach adds to the study of how social interaction shapes thought and behavior of market participants. Both social finance and information approach consider agents’ group peculiarities as major reason of social transmission biases at multiple time scales. These peculiarities evolve in the process of communication.

There are also differences between two approaches: 1) social finance is focused explicitly on how social interaction shapes thought and behavior of investors, information approach goes beyond the sphere of finance and may be applied to a wider sphere of inter social communications (innovation studies, infectious decease spread, etc.; 2) cultural traits are not directly measurable, meaning can be measured with help of redundancy in the framework of informational approach; 3) social finance and information approach use different mathematical apparatus. Implementation of non-linear evolutionary equation technique allows adequately
describe some observable price patterns and additional provide foundations to technical analysis, including Fibonacci retracement.

VII. Conclusion

This paper presents the first study of the dynamics of market price movement which considers the role of information processing and meaning generation in social systems. The research builds upon the seminal works of Loet Leydesdorff on the dynamics of expectations and meaning in inter-human communications (Leydesdorff, 2008; Leydesdorff & Dubois, 2004; Leydesdorff, Dolfsma, Van der Panne, 2006; Leydesdorff & Franse, 2009; Leydesdorff & Ivanova, 2014; Leydesdorff, Petersen & Ivanova, 2017). It also incorporates the conceptual framework of the Triple Helix model of university-industry-government relations (Etzkowitz & Leydesdorff, 1995, 1998) and its mathematical formulation (Ivanova & Leydesdorff, 2014a, 2014b).

The main contribution of the paper is that the paper presents a step forward for the development of a theory of meaning that potentially has many more spheres of applicability than financial research. With respect to the study of stock and financial markets market assets’ price movement is analyzed from the viewpoint of information and meaning dynamics among market participants. This paper can also be considered an example of the application of the theory of meaning, which has long been considered an abstract discipline, to the specific domain of financial markets.

Another contribution is the introduction of quantitative model for the study of the dynamics of market asset price movement and forecasting future price movement, based on non-linear evolutionary equation.
Meaning in social communications is processed via specific sets of communication codes which span horizons of meaning acting as selection and coordination mechanisms. It is based on expectations and incurred on events against the arrow of time. Expectations can be measured as redundancy (i.e. additional options) with help of Shannon’s entropy information theory.

Conceptually, much is to be gained from combining and testing these ideas with new methods and approaches. Maybe even more important is the idea that meaning can be measured quantitatively. This is very interesting and promising research area that has great potential to expand our capabilities in understanding the dynamics of social systems and improve our forecast capabilities concerning events that have yet to take place. It can prove to be useful when applied to different domains connected with informational exchange in social systems in general and to behavioral economics and financial markets in particular. The paper findings can also be useful for market practitioners in their everyday activity.

The paper main points are as follows: the market can be considered as an ecosystem with bi- and trilateral relations among the agents. In this respect mechanisms that drive market evolution are similar to mechanisms of the TH model of innovations. Asset price dynamics can be analyzed from an information theory perspective taking into account the relationships between information processing and meaning generation. Agents represent three groups of investors with preferences to hold long and short positions, or temporally abstain from active actions which can be considered distributions spanning the network of relations. Information is communicated via the network of relations. There are other dynamics on the top of this network. Different agents use different communication codes, reflecting their preferences, to assign meaning to the information. Codes can be considered the eigenvectors in a vector-space (von Foerster, 1960) and structure the communications as selection environments. Communicated information is
supplied with different meaning by different agents. Meaning is provided from the perspective of hindsight. Meanings cannot be communicated, as in case of information, but only shared. Providing information with meaning increases the number of options (redundancy). This mechanism can be considered probabilistically using Shannon’s equations (Shannon, 1948). The generation of options (redundancy) is crucial for system change. The trade-off between the evolutionary generation of redundancy and the historical variation providing uncertainty can be measured as negative and positive information, respectively. The dynamics of information, meaning, and redundancy can be evaluated empirically using the sign of mutual information as an indicator. When the dynamics of expectations, generating redundancies, prevail over the historical construction generating entropy, mutual redundancy is negatively signed because the relative uncertainty is reduced by increasing the redundancy. The balance between redundancy and entropy can be mapped in terms of two vectors ($P$ and $Q$) which can also be understood in terms of the generation versus reduction of uncertainty in the communication that results from interactions among the three (bi-lateral) communication channels. Eventually non-linear mechanisms in redundancy evolution may prevail which gives rise to the predictable behavior of market price evolution. This does not imply that market price movement is totally predictable, but in certain periods plausible assessment of price development can be made.

An approach entertained in this paper considers the market an evolving social system and follows Luhmann’s conjecture (Luhmann, 1982) that evolution theory, systems theory, and communication theory can be combined programmatically from a sociological perspective.

The significance of paper findings is provided by not only accurate description of some observed market phenomena but also by the link with technical analysis and social finance. Mentioned
links of information approach with various strands in finance studies and market practitioners adopted practices gives hope to future unification of different approaches in market research.

The subject of future studies comprises the application of the present approach to some other problems and datasets with a varying numbers of features.

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Appendix A

Shannon informational entropy is defined as:

\[ H = - \sum_{i=1}^{S} p_i \log p_i \]  \hspace{1cm} (A1)

Dubois showed (2019) that taking into account temporal cyclic systems:

\[ H = H(t) = - \sum_{i=1}^{S} p_i(t) \log p_i(t) \]  \hspace{1cm} (A2)

with normalization conditions:

\[ \frac{1}{T} \int_{0}^{T} \sum_{i=1}^{S} p_i(t) \, dt = 1, \quad H_0 = \frac{1}{T} \int_{0}^{T} H(t) \, dt \]  \hspace{1cm} (A3)

in case \( S = 2 \) one obtains harmonic oscillator equation:

\[
\begin{align*}
\frac{dp_1}{dt} &= -\frac{F}{p_{1,0}} (p_1 - p_{1,0}) \\
\frac{dp_2}{dt} &= \frac{F}{p_{2,0}} (p_2 - p_{2,0})
\end{align*}
\]  \hspace{1cm} (A4)

where \( p_{i,0} = \frac{1}{T} \int_{0}^{T} p_i(t) \, dt \) and \( F \) is any function of \( p_i, t \). Following Dubois one can define the state of reference:

\[ I_0 = - \sum_{i=1}^{S} p_{i,0} \log p_{i,0} \]  \hspace{1cm} (A5)

and develop informational entropy \( H \) in Taylor’s series around the reference state:

\[ H = I_0 - \sum_{i=1}^{S} [(\log p_{i,0} + 1)(p_i - p_{i,0}) + \frac{(p_i - p_{i,0})^2}{2p_{i,0}} + \cdots O((p_i - p_{i,0})^3)] \]  \hspace{1cm} (A6)

After substituting the Eq. (A5) into Equation A6 and neglecting the terms beyond the second degree one obtains:
\[ H = -\sum_{i=1}^{S}[p_i \log p_{i,0} + (p_i - p_{i,0})] + D^* \]  
\hspace{1cm} (A7)

where:

\[ D^* = \sum_{i=1}^{S} \left( \frac{(p_i - p_{i,0})^2}{2p_{i,0}} \right) \]  
\hspace{1cm} (A8)

The condition for non-asymptotic stability of cyclic system is:

\[ \frac{dD^*}{dt} = 0 \]  
\hspace{1cm} (A9)

In case \( S = 2N \) one of possible solution of Eq. (A7) is:

\[
\begin{aligned}
\frac{dp_{j-1}}{dt} &= -\frac{\gamma}{p_{j,0}}(p_j - p_{j,0}) \\
\frac{dp_j}{dt} &= \frac{\gamma}{p_{j-1,0}}(p_{j-1} - p_{j-1,0})
\end{aligned}
\]  
\hspace{1cm} (A10)

\( j = 2, 4, \ldots 2N \). Upon differentiating the system (A10) by time we obtain:

\[
\begin{aligned}
\frac{d^2p_j}{dt^2} &= \frac{\gamma^2}{p_{j,0}p_{j-1,0}}(p_j - p_{j,0}) \\
\frac{d^2p_{j-1}}{dt^2} &= \frac{\gamma^2}{p_{j,0}p_{j-1,0}}(p_{j-1} - p_{j-1,0})
\end{aligned}
\]  
\hspace{1cm} (A11)

The function \( D^* \) corresponds to the non-linear residue in (A6) which is a truncated version of (A5). Using non-truncated equations (A5) we obtain:

\[
\begin{aligned}
\frac{d^2p_j}{dt^2} &= \frac{\gamma^2}{p_{j,0}p_{j-1,0}}(p_j - p_{j,0}) + C_j \\
\frac{d^2p_{j-1}}{dt^2} &= \frac{\gamma^2}{p_{j,0}p_{j-1,0}}(p_{j-1} - p_{j-1,0}) + C_{j-1}
\end{aligned}
\]  
\hspace{1cm} (A12)

where \( C_j = O(p_j - p_{j,0})^3 \). When \( p_j \) are smaller than \( p_{j,1} \), in order to keep the same order of magnitude one can drop the terms beyond the second degree for the variable \( p_j \) and the terms
beyond the third degree for the variable $p_{j-1}$. In a similar manner this leads to the function $D^{**}$ defined by analogy with $D^*$:

$$D^{**} = \sum_j \left( \frac{(p_{j-1} - p_{j-1,0})^2}{2p_{j-1,0}} + \frac{(p_j - p_{j,0})^2}{2p_{j,0}} - \frac{(p_j - p_{j,0})^3}{6p_{j,0}^2} \right)$$  \hspace{1cm} (A13)

Differentiating $D^{**}$ by time and equating to zero $\frac{dD^{**}}{dt} = 0$ we obtain a system:

$$\begin{aligned}
\frac{dp_{j-1}}{dt} &= \frac{\gamma}{p_{j,0}} (p_j - p_{j,0}) - \frac{\gamma}{2p_{j,0}} (p_j - p_{j,0})^2 \\
\frac{dp_j}{dt} &= -\frac{\gamma}{p_{j-1,0}} (p_{j-1} - p_{j-1,0})
\end{aligned}$$  \hspace{1cm} (A14)

which yields an equation for the non-harmonic oscillator:

$$\frac{d^2p_j}{dt^2} = -\frac{\gamma^2}{p_{j,0}p_{j-1,0}} (p_j - p_{j,0}) + \frac{\gamma^2}{p_{j,0}^2p_{j-1,0}} (p_j - p_{j,0})^2 + C'_j$$  \hspace{1cm} (A15)
Appendix B

Redundancy (Eq. (1)) can be considered a result of a balance between two dynamics - evolutionary self-organization and historical organization (Leydesdorff, 2010). In other words it is a balance between recursion on a previous state on the historical axis as opposed to the meaning provided to the events from the perspective of hindsight (Dubois, 1998). Redundancy dynamics drives corresponding probabilities dynamics with recursive and incursive perspectives. Provided that probabilities oscillate in non-harmonic mode (Eq. (A15)) one can write:

\[
\frac{d^2p_j}{dt^2} = -\frac{\gamma^2}{p_{j,0}p_{j-1,0}}(p_j^- - p_{j,0}^-) + \frac{\gamma^2}{p_{j,0}^2p_{j-1,0}}(p_j^- - p_{j,0}^-)^2 + \frac{\gamma^2}{p_{j,0}p_{j-1,0}}(p_j^+ - p_{j,0}^+)-
\]

\[
\frac{\gamma^2}{p_{j,0}^2p_{j-1,0}}(p_j^- - p_{j,0}^-)^2 + C'_j + C''
\]

(B1)

here \( p_j^- \) and \( p_j^+ \) are defined with respect to past and future states. Then using the trapezoidal rule we can write Eq. (A3) as:

\[
p_{j,0}^- = \frac{1}{2}(p_j^- + p_j); p_{j,0}^+ = \frac{1}{2}(p_j^+ + p_j); \text{ so that } p_j^- - p_{j,0}^- = \frac{1}{2}(p_j - p_j^-); p_j^+ -
\]

\[
p_{j,0}^+ = \frac{1}{2}(p_j^+ - p_j)
\]

Developing \( p_j^+ \) and \( p_j^- \) in Taylor’s series in the state space: \(^{10}\)

\[
p_j^+ = p_j + p'_j h + \frac{1}{2}p''_j h^2 + \frac{1}{6}p'''_j h^3 + \frac{1}{24}p''''_j h^4 + \cdots
\]

\[
p_j^- = p_j - p'_j h + \frac{1}{2}p''_j h^2 - \frac{1}{6}p'''_j h^3 + \frac{1}{24}p''''_j h^4 + \cdots
\]

(B2)

and keeping the terms up to the \( h^4 \) order of magnitude one obtains (Fermi, Pasta, Ulam, 1955):

\[
\frac{1}{k}p_{j,tt} = p''_j h^2 + 2\alpha p'_j p''_j h^3 + \frac{1}{42}p'''_j h^4 + O(h^5)
\]

(B3)

\(^{10}\) The state space is presented by x axis
\[ k = \frac{\gamma^2}{p_{j,0}p_{j-1,0}}; \alpha = \frac{1}{p_{j-1,0}}; C_1 = \frac{1}{k}(C_j' + C_j'') \]

Setting further: \( w = \sqrt{k}; t' = wt; y = \frac{x}{h}; \varepsilon = 2\alpha \) one can rewrite Eq. (B3) in the form:

\[ -p_{j,tt} + p_{j,yy} + \varepsilon p_{j,y}p_{j,yy} + \frac{1}{12} p_{j,yyyy} + C_1 = 0 \]  \hspace{1cm} (B4)

Going to the moving coordinate system: \( X = y - t' \), rescaling time variable \( T = \frac{\varepsilon}{2} \tau \), and keeping terms up to the first order in \( \varepsilon \) one brings Eq. (B3) to the form:

\[ \varepsilon \Sigma_{XT} + \varepsilon \Sigma_X \Sigma_{XX} + \frac{1}{12} \Sigma_{XXX} + C_1 = 0 \]  \hspace{1cm} (B5)

Here \( p_j = \Sigma(X, T) \). Defining further: \( P = \Sigma_X \) and \( \delta = \frac{1}{12\varepsilon} \) we obtain a non-linear evolutionary equation:

\[ P_T + PP_X + \delta P_{XXX} + C_1 = 0 \]  \hspace{1cm} (B6)

which corresponds to Korteweg de Vries (KdV) equation (Gibbon, 1985):

\[ u_T + uu_X + \delta u_{XXX} = 0 \]  \hspace{1cm} (B7)

with additional term \( C_1 \). By substitution: \( u \to 6U; T \to \sqrt{\delta} \tau; X \to \sqrt{\delta} \chi \) Eq. (B7) is reduced to the form:

\[ U_T + 6UU_X + U_{XXX} = 0 \]  \hspace{1cm} (B8)

Eq. (B8) possesses soliton solutions:

\[ U(\chi, \tau) = \frac{k^2}{2} \text{sech}^2 \left[ \frac{k}{2}(\chi - \kappa^2 \tau) \right] \]  \hspace{1cm} (B9)
the soliton solution of Eq. (B6) is:

\[ P(X,T) = \frac{k^2}{12} \text{sech}^2 \left[ \frac{k}{2\sqrt{\delta}} \left( X - k^2 T + \frac{C_1}{2\sqrt{\delta}} T^2 \right) \right] - C_1 T \]  \hspace{1cm} (B10)

It follows that for the train of solitons with amplitudes: \( A_i \), \( A_j \) and \( A_k \) and times \( T_i, T_j, T_k \) the following condition holds:

\[ \frac{A_i - A_j}{T_i - T_j} = \frac{A_j - A_k}{T_j - T_k} \]
Appendix C

In supposition of existence a periodic solution (Gibbon, 1985):

$$U(x, t) = f(x - vt)$$

substitution of expression (C1) into Eq. (B7) yields:

$$-vf' + ff' + f''' = \frac{1}{2}a$$

(C2)

here $a$ is a constant of integration. Multiplying Eq. (C2) by $2f'$ and integrating it one obtains an equation which has periodic solutions in the form of elliptic functions:

$$f''^2 = -\frac{1}{3}f^3 + vf^2 + af + b$$

(C3)

Then the corresponding solution for Eq. (B7) takes the form:

$$P(x, t) = f\left(x - vt + \frac{1}{2}Ct^2\right) - Ct$$

(C4)