How to Ensure Logical Consistency of Quantum Theory

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Predictions of quantum theory have been confirmed experimentally in the microscopic domain with no known exceptions. This success motivates physicists to assume universal validity of the theory. To put the predictions of the quantum theory to the test in the domain of more complex systems researchers like e.g. Eugene Wigner have proposed carefully designed Gedankenexperiments revealing unexpected difficulties of the theory. Daniela Frauchiger and Renato Renner have recently suggested an extension of the Gedankenexperiment commonly known as Wigner’s friend and arrived at a conclusion that one agent, upon observing a particular measurement outcome, must conclude that another agent has predicted the opposite outcome with certainty. Their analysis shows that quantum theory cannot consistently describe the use of itself. Here, we will study the mentioned Gedankenexperiments and introduce an approach leading to consistent predictions of quantum theory, independent of observer using the theory.

I. INTRODUCTION

Quantum theory has been developed and used over one hundred years and is nowadays widely accepted in the scientific community [1–6]. The enormous success of the quantum theory ranges among many distinct fields of physics. Yet since the introduction of quantum theory there have been many discussions about problematic features of the theory questioning its general validity [7–9]. In this paper we will study in detail the Wigner’s friend [7] and the Frauchiger-Renner-Wigner [9] Gedankenexperiments, which point out the problems quantum theory faces. We will address the problems using a unique quantum-mechanical approach.

Our first chosen Gedankenexperiment has been proposed by the theoretical physicist Eugene Wigner in 1961 (famous for his contributions regarding fundamental symmetry principles in atomic physics [10]). In his original work [7] Wigner discusses how the concept of consciousness enters the quantum theory: “The impression which one gains at an interaction, called also the result of an observation, modifies the wave function of the system. The modified wave function is, furthermore, in general unpredictable before the impression gained at the interaction has entered our consciousness: it is the entering of an impression into our consciousness which alters the wave function because it modifies our appraisal of the probabilities for different impressions which we expect to receive in the future. It is at this point that the consciousness enters the theory unavoidably and unalterably. If one speaks in terms of the wave function, its changes are coupled with the entering of impressions into our consciousness. If one formulates the laws of quantum mechanics in terms of probabilities of impressions, these are ipso facto the primary concepts with which one deals.”

Wigner as well analyses how sensations of a consciousness depend on a physical state of the observer: “Let us first specify the question which is outside the province of physics and chemistry but is an obviously meaningful (because operationally defined) question: Given the most complete description of my body (admitting that the concepts used in this description change as physics develops), what are my sensations?” He postulates a thesis about necessary conditions for the consciousness to arise: “It is very likely that, if certain physico-chemical conditions are satisfied, a consciousness, that is, the property of having sensations, arises. This statement will be referred to as our first thesis. The sensations will be simple and undifferentiated if the physico-chemical substrate is simple; it will have the miraculous variety and colour which the poets try to describe if the substrate is as complex and well organized as a human body. The physico-chemical conditions and properties of the substrate not only create the consciousness, they also influence its sensations most profoundly. Does, conversely, the consciousness influence the physicochemical conditions? In other words, does the human body deviate from the laws of physics, as gleaned from the study of inanimate nature?” Wigner will come back to Wigner’s first thesis later on in our discussion.

Wigner bases his Gedankenexperiment on the fact that in the quantum theory the wave function depends on observer’s information. The experiment consists of two observers - Wigner (W) and his friend (F) (see scheme of the experiment in FIG. [1]). The observer F performs a measurement in an isolated lab L, let us say a measurement of a spin of a particle, which is in a normalized state

\[ |s\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle), \]

where the states |\uparrow\rangle and |\downarrow\rangle denote the spin state “up” and “down” respectively. The result of his measurement is either an electron spin state |\uparrow\rangle or |\downarrow\rangle. From his perspective, the information has arrived to his consciousness and the state vector has collapsed to a definite value. From the perspective of the observer W, the situation looks different. The information about the spin of the electron has not arrived to W’s consciousness and therefore the state is after the F’s measurement still a sum of vectors

\[ \frac{1}{\sqrt{2}} (|\uparrow\rangle|F_{\uparrow}\rangle + |\downarrow\rangle|F_{\downarrow}\rangle), \]

where |F_{\uparrow}\rangle and |F_{\downarrow}\rangle describe the state of the Wigner’s
friend after measuring the spin state \(|↑⟩\) or \(|↓⟩\) respectively. This experiment demonstrates that the description of the system is different for both of the observers. Though the situation may appear “crazy”, this result does not rule out quantum mechanics as a universally valid theory. Let us rather conclude at this point that Wigner’s friend experiment emphasizes the importance of the distinct levels of knowledge of the experiment participating agents. We will analyse the experiment introducing a different approach to quantum theory and find out that in reality such experiments should be perceived as observer independent.

An extension of the Wigner’s friend experiment has been discussed in a recent publication by Daniela Frauchiger and Renato Renner (FRW experiment) \[9\]. As you can see in FIG.2 we consider two separate laboratories \(L,\ L\) with two observers \(F,\ F\) performing measurements and two other observers \(W,\ W\) measuring the state from the outside of the isolated laboratories respectively.

The experiment is repeated in a loop and proceeds as follows:

1. Agent \(F\) performs a measurement on a two-level system prepared in initial state

\[\rho = \frac{1}{\sqrt{3}} |h⟩ + \sqrt{\frac{2}{3}} |t⟩, \quad (3)\]

where \(h\) stands for “heads” and \(t\) for “tails”. Based on the resulting state vector either \(|h⟩\) or \(|t⟩\), agent \(F\) sets the particle spin to state \(|↓⟩\) or \(|→⟩\) = \[\frac{1}{\sqrt{2}} (|↑⟩ + |↓⟩) \]

respectively. The particle is then transferred to the observer \(F\).

2. The agent \(F\) measures the spin with respect to the basis \(|↑⟩\) and \(|↓⟩\).

3. In the third step the agent \(W\) measures the state in the laboratory \(L\) obtaining its projection on basis vectors \(|ok⟩ = \frac{1}{\sqrt{2}} (|h⟩ - |t⟩)\) and \(|fail⟩ = \frac{1}{\sqrt{2}} (|h⟩ + |t⟩)\).

4. Finally the observer \(W\) concludes the round of this experiment by measuring the state in the laboratory \(L\) with respect to the basis \(|ok⟩ = \frac{1}{\sqrt{2}} (|↓⟩ - |↑⟩)\) and \(|fail⟩ = \frac{1}{\sqrt{2}} (|↓⟩ + |↑⟩)\). The experimental procedure is repeated until the observers \(W\) and \(W\) both obtain the results \(|ok⟩\) and \(|ok⟩\) respectively.

In the case of Wigner’s experiment we used \(|F⟩\) to denote the combined state of the observer \(F\) and particle spin. We have simpliﬁed the notation for the more complex FRW experiment for brevity. Further composition details of the vectors showing the connection to the agents \(F,\ F,\ W\) and \(W\) in the same way would make our notation look unnecessarily more complicated and interested readers can find it in the original paper if needed.

Similarly as in the case of Wigner’s friend experiment, after every measurement the state vector collapses to a certain basis vector from the perspective of the observer performing the measurement, but is still perceived as a superposition of states from the perspective of the agents not participating the measurement. After a systematic analysis of the experiment, Frauchiger and Renner have concluded that the observers \(W\) and \(W\) will unavoidably come to contradictory predictions using the quantum theory. According to quantum mechanics the experiment will halt with agents \(W\) and \(W\) measuring \(|ok⟩\) and \(|ok⟩\) respectively at one point despite the fact that after \(W\)’s measurement of the state \(|ok⟩\) the agent \(W\)’s measurement can not result in the state \(|ok⟩\) according to the same quantum theory. The theory has consequently been proven to contain contradictory predictions in itself.

II. OBSERVER INDEPENDENT APPROACH TO QUANTUM MECHANICS

Let us come back to Wigner’s first thesis: "It is very likely that, if certain physico-chemical conditions are satisfied, a consciousness, that is, the property of having sensations, arises.". For our discussion we will not attempt to find out the laws of what exact sensations an observer in a system experiences as a function of time. For this article, the observation that a substance can have sensations is sufﬁcient. From reasons going beyond the scope of this paper, let us assume that any physical system can be divided into parts, each being treated as an observer from a quantum theory point of view. The actual sensations of the observers at an arbitrary point of time, if there are any, can be considered irrelevant for our purposes. We will assume two theses as a basis of our considerations:

- **Thesis 1**: Every system at any time consists of observers, completely described by their state vectors - independent wave functions.

- **Thesis 2**: Collapse of a state vector appears only upon interaction between two observers. Interaction within parts (particles) of one observer does not result in a state vector collapse.

Independence of the wave functions means that they are defined by states of distinct particles in the system. We can assume that a particle belonging to two distinct observers at a time does not exist. In order to guarantee clarity of our two universally valid theses and their consequences, let us analyse the two Gedankenexperiments mentioned before.
The experiment differs from those used in [9]. Consequently, the fundamental description of a quantum-mechanical experiment does not start with one wave function of the whole system, but with independent wave functions for each observer. We have to realize, which part of the experiment belongs to which observer. Our relevant observers are Wigner W, his friend F, the measurement device M and the particle P. We may perhaps hesitate to accept that the measurement device and the particle can be effectively regarded as observers. To become more comfortable about the assignment, let us point out several crucial facts. Treating a measurement device as an observer is relatively common in present publications [11] and does not necessarily mean that the observer experience complex sensations comparable to those of a human mind. The sensations depend on a physical state of the observers and hence we can calmly assume that sensations of a measurement device are practically non-existent. What matters for us is the fact that the sensations can potentially arise according to the Thesis 2 and the laboratory L keeps the superposition state $|s\rangle$ until the agent W performs his measurement. His measurement will collapse the wave function, because W and observer device M, particle P and friend F are two distinct observers.

The experiment could be also prepared in such a way that it is a matter of chance whether the particle belongs to the same observer as the rest of the lab or not. In this case we would introduce a probability factor in the description. In our approach, it is irrelevant whether the device M and agent F are macroscopic or microscopic. Macroscopic objects are not likely to contain only one observer so the Wigner’s scenario would be significantly more complicated to prepare, because it would be necessary to overcome this difficulty and prepare such an observer macroscopic object. Nevertheless, it is in principle possible and it would cause no difficulty to the theory.

We have considered two possible distinct experimental set-ups of the Wigner’s friend scenario leading us to another understanding of the subject. Other combinations of “merged” observers are not relevant at the moment. To summarize, the relevant description of the experiment before W’s measurement is $|\uparrow\rangle\langle\uparrow|\langle s\rangle$ or $|\downarrow\rangle\langle\downarrow|\langle s\rangle$ in case of the particles of the experiments are distinct observers. In case of M, P and F being one observer system keeps the superposition state $|s\rangle$ until W performs his measurement. We are now ready to discuss the more complicated case of the FRW experiment.

IV. FRAUCHIGER-RENNER-WIGNER GEDANKENEXPERIMENT

For the FRW scenario we simply have more possibilities of observer assignments to consider. Everything works in an analogous way as in the previous case. No logical inconsistency is possible, because the derivation used in the original paper [9] does not apply in our approach.
We have a quantum theory that does not directly depend on the point of view of an observer.

As in the Wigner’s friend scenario analyzed above, let us first assume all the parts of the experiment belong to distinct observers. Then apart from the initial state $|i\rangle$ (Eq. (3)), there is no newly arising superposition of states due to our Thesis 2. The process of the experiment is defined by action sequence of the four projection operators $A^i$ in TABLE I on the initial state. The projection operators describe the interactions between distinct observers and are therefore denoted as interaction operators. The probabilities of all possible measurement outcomes of all the four observers can be expressed as

$$P_o(x, y, z, w) = |\langle z|\langle w|A^i_w A^i_z A^i_y A^i_t|i\rangle|^2,$$  \hspace{1cm} (4)

where $|i\rangle$ is defined in Eq. (3), the operators $A^i$ in TABLE I and $x, y, z, w$ describe one of the two possible measurement outcomes of each observer (for example $z$ can be either $\text{ok}$ or $\text{fail}$). This formula allows us to calculate the probabilities of the outcomes using the definitions of the relevant vectors and operators.

It is possible to prepare the experiment in analogous way as in the Wigner’s friend experiment considering everything involved in the $F$’s measurement as one observer, which leads us to an isometry

$$U^I = |h\rangle \rightarrow |h\rangle \downarrow, |t\rangle \rightarrow |t\rangle \rightarrow,$$  \hspace{1cm} (5)

which will conserve the superposition of states. The probability distribution for this experiment is then:

$$P_1(y, z, w) = |\langle z|\langle w|A^I_w A^I_z A^I_y A^I_t|i\rangle|^2.$$  \hspace{1cm} (6)

We can assume the same for the observer $F$ using isometry $U^I = |\psi\rangle \rightarrow |\psi\rangle$ (for every state $|\psi\rangle$). Supposing only F’s procedure is done by a single observer we obtain

$$P_2(x, z, w) = |\langle z|\langle w|A^I_w A^I_z U^I A^I_2|i\rangle|^2.$$  \hspace{1cm} (7)

There is also an option that both observers $F$ and $F$ are the only observers involved in their measurement, implying

$$P_{1,2}(z, w) = |\langle z|\langle w|A^I_w A^I_z U^I U^I |i\rangle|^2.$$  \hspace{1cm} (8)

The subscript $o$ in $P_o$ indicates, which measurements do not cause the wave function collapse due to the Thesis 2 and the one-observer nature of the system (measurements 1 and 2 are done by the observers $F$ and $F$ respectively). Let us suppose for simplicity that in the measurements of $W$ and $W$ there are always many observers involved, so we don’t have to consider more superposition conserving transformations. In the more general situation, when the probability of a such a setup $o$ is $P_o$, we arrive at the final probability distribution

$$P(x, y, z, w) = P_o P_1(\cdot) + \frac{P_1}{2} P_1(\cdot) + \frac{P_2}{2} P_2(\cdot) + \frac{P_1^2}{4} P_{1,2}(\cdot),$$  \hspace{1cm} (9)

where the $\cdot$ in $P_o(\cdot)$ is an abbreviation for the variables of the probability distribution. We have included the normalization factors in the fractions $\frac{P_1}{2}$ ($n$ is integer), because for example the probability distribution $P_1$ does not depend on the variable $x$ (because the particular state $|x\rangle$ does not occur in this scenario) and would be then calculated twice (analogously for the other normalization factors).

| Observer | Measurement basis | Interaction operators |
|----------|-------------------|----------------------|
| $F$      | $|h\rangle$, $|t\rangle$ | $A^I_h = |h\rangle \downarrow$, $A^I_t = |t\rangle \rightarrow$ |
| $F$      | $|\downarrow\rangle$, $|\uparrow\rangle$ | $A^I_{\downarrow} = |\downarrow\rangle \downarrow$, $A^I_{\uparrow} = |\uparrow\rangle \rightarrow$ |
| $W$      | $\text{ok}$, $\text{fail}$ | $A^I_{\text{ok}} = \frac{1}{2} (|\text{ok}\rangle - |\text{fail}\rangle)$, $A^I_{\text{fail}} = \frac{1}{2} (|\text{ok}\rangle + |\text{fail}\rangle)$ |
| $W$      | $\text{ok}$, $\text{fail}$ | $A^I_{\text{ok}} = \frac{1}{2} (|\text{ok}\rangle - |\text{fail}\rangle)$, $A^I_{\text{fail}} = \frac{1}{2} (|\text{ok}\rangle + |\text{fail}\rangle)$ |

The quantum mechanical description is the same from the point of view of all observers. Every observer experiences different sensations during the experiment, but at any time during the experiment there exist a set of "the facts of the world" in terms of states of the considered independent wave functions. We obtain the probability $\frac{1}{12}$ of the experiment to halt obtained in [9] using Eq. (8)

$$P_{1,2}(\text{ok}, \text{ok}) = |\langle \text{ok}|\langle \text{ok}|A^I_{\text{ok}} A^I_{\text{ok}} U^I U^I |i\rangle|^2 = \frac{1}{12}.$$  \hspace{1cm} (10)

This probability is different for the other scenarios by the distributions $P_o(x, y, z, w)$, $P_1(y, z, w)$ and $P_2(x, z, w)$. This brings us to an interesting conclusion that experiments of this kind can help us to measure the probabilities $p_o$ and hence studying the observer composition of the experiment. Similar experiment has been recently performed [11].
V. SUMMARY

We have introduced an approach to quantum mechanics unique in the way of defining and treating observers involved in the studied system. The basis of our considerations are the Theses 1 and 2. The Thesis 1 tells us to describe a system as set of independent wave functions belonging to distinct observers. The Thesis 2 then postulates how these observers interact. Our analysis of the Wigner’s friend and the FRW Gedankenexperiments has shown us that we have no difficulties regarding logical consistency of the theory, unlike the approaches discussed in [9]. The experiments of the FRW type can help us studying the observer composition of the experiment due to the probability factors $p_o$. Similar experiment has been recently performed [11] and could be used for further analysis.

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