s-Channel Production of Minimal Supersymmetric Standard
Model Higgs Bosons at a Muon Collider with Explicit CP
Violation

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Abstract

A muon collider with controllable energy resolution and transverse beam
polarization provides a powerful probe of the Higgs sector in the minimal
supersymmetric standard model with explicit CP violation, through s-channel
production of Higgs bosons. The production rates and the CP–even and CP–
odd transverse–polarization asymmetries are complementary in diagnosing
CP violation in the Higgs sector.
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The experimental observation of Higgs bosons and the detailed investigation of their
dfundamental properties are crucial for our understanding of the mechanism responsible
for the electroweak symmetry breaking. This crucial quest in particle physics constitutes the
primary reason for having a $\mu^+\mu^-$ collider (MC) [1], the physics potential of which has been
investigated with a considerable amount of effort. The fact that muons are significantly
heavier than electrons makes a MC very attractive for both practical and theoretical rea-
sions [2]: (i) synchrotron radiation does not limit their circular acceleration and multi-TeV
energies for a MC can be realized; (ii) the muon beam energy distribution is not smeared
out by beamstrahlung; (iii) the larger Yukawa couplings of muons in many cases allow for
copious production of Higgs bosons as $s$-channel resonances, making precision studies of
the Higgs sector possible. In particular, one can search for CP violation in the couplings
of Higgs bosons to muons by measuring the production rates and the polarization asym-
metries constructed with transversely polarized muon beams [3]. In this paper we point
out the possibility of studying loop–induced CP–violating scalar–pseudoscalar mixing [4]
through $s$-channel production of neutral Higgs bosons at a MC in the minimal supersym-
metric standard model (MSSM). The experimental tools assumed to be available for this
investigation are controllable beam energy resolution and beam polarization, for both muons
and anti–muons.

In general, the MSSM has several new CP–violating phases which are absent in the
standard model (SM). Furthermore, it has recently been pointed out [5,6] that these phases
do not have to be suppressed in order to satisfy the constraints from electron and neutron
electric dipole moments. Of particular interest in this context is the so–called effective su-
persymmetry (SUSY) model [7] in which sfermions of the first and second generations are
decoupled[1], but sfermions of the third generation remain relatively light to preserve natu-
ralness. Motivated by scenarios of this type, we concentrate on CP violation in the Higgs
sector induced at one–loop level by CP–violating phases in the stop and sbottom sectors.
Characteristic CP–violating phenomena in this scheme are a possibly large mixing between
the CP–even and CP–odd neutral Higgs bosons and an induced relative phase $\xi$ [4,8] between
the vacuum expectation values of the two Higgs doublets.

The dominant source of one–loop radiative corrections to the Higgs potential of the
MSSM are the stop and sbottom sectors due to the large $t$ and (for large values of $v_2/v_1 \equiv \tan \beta$) $b$ Yukawa couplings. Then, the characteristic size of CP violation induced in the
Higgs sector is determined by the factor $(1/32\pi^2)(|\mu||A_f|Y_f^2/M_{\text{SUSY}}^2)\sin \phi_{\text{CP}}$, where $Y_f$ is the
Yukawa coupling of the fermion $f$, $\phi_{\text{CP}} = \text{Arg}(\mu A_f) + \xi$ for $f = t, b$, and $M_{\text{SUSY}}$ is a typical
SUSY–breaking scale, the square of which might be taken to be the average of two sfermion
($\tilde{f}_1, \tilde{f}_2$) masses squared, i.e. $M_{\text{SUSY}}^2 = (1/2)[m_{\tilde{f}_1}^2 + m_{\tilde{f}_2}^2]$. In the present work, we take a
common phase $\Phi$ for $A_{t,b}$ and present results for the following parameter values:

$$
|A_t| = |A_b| = 1 \text{ TeV} , \quad |\mu| = 2 \text{ TeV} ,
M_{\text{SUSY}} = 0.5 \text{ TeV} , \quad \text{Arg}(\mu) + \xi = 0 .
$$

(1)

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1 The first and second generation sfermions do not have to be completely decoupled. However,
only if they are fairly heavy, $O(1 \text{ TeV})$, the phases can be large without substantial fine–tuning [3].
In addition, noting that the CP-violating phases could weaken the present experimental bounds on the lightest Higgs mass up to about 60 GeV \[1\], we simply apply the lower mass limit \(M_{H_1} \geq 70\) GeV to the lightest Higgs–boson determining the lowest allowed value of \(M_{H^\pm}\) for a given set of SUSY parameters. For the parameter \(\tan \beta\), we take \(\tan \beta = 3\) or 30 as the values representing the small and large \(\tan \beta\) cases, respectively. This representative choice \([1]\) of parameters, (which was considered in Ref. \([4]\) as well), enables us to avoid the possible Barr–Zee type EDM constraints \([9]\) on the phase \(\Phi\). The Higgs boson masses and couplings to the SM particles are then completely determined by three independent parameters \{\(\tan \beta, \Phi, M_{H^\pm}\)\}.

In the MSSM with explicit CP violation, the couplings of the neutral Higgs bosons to the SM fermions are significantly affected by scalar–pseudoscalar mixing \([4]\). Specifically, the interactions of the neutral Higgs bosons with muons are described by the Lagrangian:

\[
\mathcal{L}_{H\mu\mu} = -\frac{e m_\mu M_{H_i}}{2 m_W s_W c_\beta} \bar{\mu} [O_{2,4-i} - i s_\beta O_{1,4-i} \gamma_5] \mu H_i ,
\]

(2)

where the transpose of \(O\) is the \(3 \times 3\) orthogonal matrix rotating three neutral Higgs bosons \(H_\alpha = (a, \phi_d, \phi_u)\) into three mass eigenstates \(H_i (i = 1, 2, 3)\) with the mass ordering \(M_{H_1} \geq M_{H_2} \geq M_{H_3}: H_\alpha = O_{\alpha, A-i} H_i\). The interaction Lagrangian enables one to derive the helicity amplitudes \(M_{\lambda\bar{\lambda}}\) for the s-channel production of the neutral Higgs bosons in \(\mu^+\mu^-\) collisions, which are given by

\[
M_{\lambda\bar{\lambda}}^i = -\frac{e m_\mu M_{H_i}}{2 m_W s_W c_\beta} (\lambda \beta_i O_{2,4-i} + i s_\beta O_{1,4-i}) \delta_{\lambda\bar{\lambda}} ,
\]

(3)

where \(\lambda, \bar{\lambda} = \pm\) and \(\beta_i = \sqrt{1 - 4 m_\mu^2 / M_{H_i}^2}\). Then, the unpolarized cross section \(\sigma_{H_i}^0\) for the s-channel process \(\mu^+\mu^- \to H_i\) is expressed in terms of the decay width \(\Gamma(H_i \to \mu\mu)\) as

\[
\sigma_{H_i}^0(\sqrt{s}) = \frac{4 \pi \Gamma(H_i \to \mu\mu) \Gamma_{H_i}}{(s - M_{H_i}^2)^2 + M_{H_i}^2 \Gamma_{H_i}^2} ,
\]

(4)

and the polarized s-channel cross section is given in a simple form as

\[
\sigma_{H_i} = \sigma_{H_i}^0 \left[ 1 + P_L \overline{P}_L + (P_T \overline{P}_T - P_N \overline{P}_N) \mathcal{Y}_i + (P_T \overline{P}_N + P_N \overline{P}_T) \overline{\mathcal{Y}}_i \right] ,
\]

(5)

where \(P_{L,T,N}\) (\(\overline{P}_{L,T,N}\)) denote the longitudinal, transverse and normal polarizations of the \(\mu^-\) (\(\mu^+\)) beam and the transverse–polarization (TP) asymmetries \(\mathcal{Y}_i\) and \(\overline{\mathcal{Y}}_i\) are defined in terms of the helicity amplitudes \(M_{\lambda\bar{\lambda}}\) as

\[
\mathcal{Y}_i = \frac{2 \text{Re}(M_{\lambda-\bar{\lambda}}^i M_{\lambda+\bar{\lambda}}^i)}{|M_{\lambda-\bar{\lambda}}^i|^2 + |M_{\lambda+\bar{\lambda}}^i|^2} ,
\]

\[
\overline{\mathcal{Y}}_i = \frac{2 \text{Im}(M_{\lambda-\bar{\lambda}}^i M_{\lambda+\bar{\lambda}}^i)}{|M_{\lambda-\bar{\lambda}}^i|^2 + |M_{\lambda+\bar{\lambda}}^i|^2} .
\]

(6)

\[^2\text{For this bound, the charged Higgs–boson mass is found to be well beyond the present experimental lower bound irrespective of the CP–violating phase.}\]
Here, the $\mathcal{Y}_i$ are CP–even, but the $\overline{\mathcal{Y}}_i$ are CP–odd. In the CP–invariant theories, they should satisfy the relations; $\mathcal{Y}_i = \mp 1$ depending on whether $H_i$ is a pure CP–even or CP–odd state, and $\overline{\mathcal{Y}}_i = 0$. In other words, $|\mathcal{Y}_i| < 1$ and $\overline{\mathcal{Y}}_i \neq 0$ imply CP violation directly.

The physical quantities determining the unpolarized cross sections are the total widths of the Higgs bosons, their partial decay widths $\Gamma(H_i \rightarrow \mu\mu)$, as well as their masses. The Higgs masses, in particular, the lightest Higgs boson mass are expected to be measured with an error of less than 100 MeV at $e^+e^-$ or $\mu^+\mu^-$ colliders \cite{10}. So, it is possible to probe Higgs boson physics by setting the beam energy very close to the Higgs mass. In this case, s–channel studies of narrow Higgs resonances depend critically on the beam energy resolution $R = \Delta E_{\text{beam}} / E_{\text{beam}}$ with respect to the resonance width $\Gamma_{H_i}$. The energy spectrum of each beam is expected to be Gaussian to a good approximation with an rms deviation $\sigma_E \sim 2 \text{ MeV} \left( \frac{\sqrt{s}}{100 \text{ GeV}} \right) \left( \frac{R}{0.003 \%} \right)$ and with a capability of changing $R$ from 0.01\% to 1\%. Of crucial importance are then the ratios of the rms error $\sigma_E$ for the given $R$ and the calculated total widths of Higgs bosons. According to a detailed analysis of Higgs boson decays \cite{11}, the widths of the heavier Higgs states $H_{2,3}$ are much larger than the width of the lightest Higgs boson $H_1$, which is of the order of MeV for a small value of $\tan \beta$. It will thus be reasonable to take an energy resolution $R$ such that $\sigma_E \gg \Gamma_{H_i}$ and $\sigma_E \ll \Gamma_{H_{2,3}}$.

The effective cross section $\sigma_{H_i}$ is given by a convolution with the Gaussian beam energy distribution:

$$\sigma_{H_i} = \int \sigma_{H_i}(\sqrt{s}) \frac{\exp[-(\sqrt{s} - \sqrt{s})^2 / 2\sigma_E^2]}{\sqrt{2\pi}\sigma_E} \, d\sqrt{s}. \quad (7)$$

For $\sigma_E$ satisfying $\sigma_E \gg \Gamma_{H_i}$ and $\sigma_E \ll \Gamma_{H_{2,3}}$, the effective unpolarized cross sections $\sigma_{H_i}^0$ at $\sqrt{s} = M_{H_i}$ ($i = 1, 2, 3$) are given by

$$\sigma_{H_1}^0 = \frac{\pi \Gamma_{H_1}}{2\sqrt{2\pi}\sigma_E} \sigma_{H_1}(M_{H_1}) = \frac{2\pi^2}{\sqrt{2\pi}\sigma_E} \frac{\Gamma(H_1 \rightarrow \mu\mu)}{M_{H_1}^2},$$
$$\sigma_{H_{2,3}}^0 = \sigma_{H_{2,3}}(M_{H_{2,3}}) = \frac{4\pi}{M_{H_{2,3}}^2} B(H_{2,3} \rightarrow \mu\mu), \quad (8)$$

to a good approximation. Therefore, measurements of the effective unpolarized cross sections yield $\Gamma(H_1 \rightarrow \mu\mu)$ and $B(H_{2,3} \rightarrow \mu\mu)$. In addition, it is possible to measure the polarization–dependent quantities $\mathcal{Y}_i$ and $\overline{\mathcal{Y}}_i$ by use of transversely–polarized muon and anti–muon beams, giving independent information on the scalar and pseudoscalar couplings of the Higgs bosons to muons.

As shown explicitly in Ref. \cite{11}, $B(H_1 \rightarrow \mu\mu)$ is almost independent of $\Phi$, but $\Gamma(H_1 \rightarrow \mu\mu)$ (as well as $\Gamma_{H_1}$) is strongly dependent on $\Phi$. On the contrary, $B(H_{2,3} \rightarrow \mu\mu)$ depend significantly on $\Phi$, but $\Gamma(H_{2,3} \rightarrow \mu\mu)$ are almost independent of $\Phi$. The effective unpolarized cross sections are thus expected to be very sensitive to $\Phi$ because their measurements correspond to those of the strongly phase–dependent quantities $\Gamma(H_1 \rightarrow \mu\mu)$ and $B(H_{2,3} \rightarrow \mu\mu)$. In the following, we will demonstrate these encouraging aspects in detail.

\footnote{For a large value of $\tan \beta$, $\Gamma_{H_1}$ can be as large as 1 GeV which requires a larger $R$ to satisfy $\sigma_E \gg \Gamma_{H_1}$.}
The lightest Higgs boson— In our numerical analysis, we take \( R = 0.15\% \), which corresponds to \( \sigma_E \sim 0.1 \) GeV for \( M_{H_1} = 100 \) GeV. As shown in Fig. 1, for a fixed CP–violating phase the effective unpolarized cross section \( \sigma_{H_1}^0 \) decreases rapidly with increasing \( M_{H_1} \), approaching the (phase–independent) cross section for the production of the SM Higgs boson. However, for most of the lightest Higgs–boson mass \( M_{H_1} \), the cross section \( \sigma_{H_1}^0 \) depends strongly on the CP–odd phase \( \Phi \), i.e. on the charged Higgs–boson mass, except for small \( M_{H_1} \) for \( \tan \beta = 30 \). However, as far as the unpolarized cross section is concerned, changing the soft stop/sbottom sector parameters could mimic the effects of changing the CP–violating phase. Thus, it is crucial to confirm the existence of the finite CP–violating phase. This strong dependence will be essential in determining the CP phase for large \( \tan \beta \) given Higgs masses. We therefore investigate the dependence of the CP–even and CP–odd asymmetries versus \( M_{H_1} \), the size of which is independent of \( R \). These TP asymmetries are generally very sensitive to the phase \( \Phi \) for both \( \tan \beta = 3 \) and \( \tan \beta = 30 \). The only exception again occurs in the “decoupling limit” \( M_{H^+} \to \infty \), where \( M_{H_1} \) approaches its upper limit and the TP asymmetries take their SM values (\( \mathcal{Y}_{SM} = -1 \), \( \mathcal{Y}_{SM} = 0 \)). Note also that, except in the decoupling limit, the total cross section scales \( \propto \tan^2 \beta \), while the \( \tan \beta \) dependence of the TP asymmetries is relatively mild. In this light, the unpolarized cross section and the polarization asymmetries play a complementary role in extracting the values of MSSM parameters.

The heavier Higgs bosons— The branching ratios \( B(H_{2,3} \to \mu \mu) \) determined independently of \( \sigma_E \) (\( \ll \Gamma_{H_{2,3}} \)) are in general strongly dependent on the phase \( \Phi \) for a small value of \( \tan \beta \), unlike \( B(H_1 \to \mu \mu) \), which remains almost constant. However, they become (almost) independent of \( \Phi \) for large \( \tan \beta \) where fermionic decay modes are by far the dominant ones. We therefore study the effective unpolarized cross sections \( \sigma_{H_{2,3}}^0 \) only for \( \tan \beta = 3 \). Results are presented in Fig. 3, which shows \( \sigma_{H_{2,3}}^0 \) versus \( M_{H_{2,3}} \), respectively, for various values of \( \Phi \). The cross sections without CP violation decrease abruptly with increasing \( M_{H_{2,3}} \), especially when new decay channels (\( ZH_1, H_1H_1, t\bar{t} \)) open up, but the monotonic decrease is not retained for non–vanishing values of the phase \( \Phi \). In particular, for small Higgs boson masses the cross sections are significantly affected by the phase \( \Phi \). For large Higgs boson masses they are not so strongly dependent on the phase, since here the total widths of the unmixed heavy scalar and pseudoscalar Higgs bosons are both dominated by the partial widths into \( t\bar{t} \), which become identical for both modes in the limit of large Higgs boson masses.

Our analysis for the unpolarized cross sections for heavy Higgs bosons clearly shows the need to use other independent observables to determine the effects of the CP phase for large values of Higgs masses. We therefore investigate the dependence of the CP–even and CP–odd TP asymmetries \( \mathcal{Y}_{2,3} \) and \( \overline{\mathcal{Y}}_{2,3} \) on the CP phase. The TP asymmetries \( \mathcal{Y}_2 \) and \( \overline{\mathcal{Y}}_2 \) versus \( M_{H_2} \) are presented in Fig. 4 and the TP asymmetries \( \mathcal{Y}_3 \) and \( \overline{\mathcal{Y}}_3 \) versus \( M_{H_2} \) in Fig. 5, for the parameter set \([\mathbb{P}]\). We find three interesting aspects concerning those TP asymmetries: (i) for both \( \tan \beta = 3 \) and \( \tan \beta = 30 \) all the TP asymmetries are very sensitive to the CP phase. This strong dependence will be essential in determining the CP phase for a large value of \( \tan \beta \) and large \( M_{H^+} \); because in that case all unpolarized cross sections (for given Higgs masses) are nearly independent of \( \Phi \), as mentioned before; (ii) if the CP–even TP asymmetries are small in size, then the CP–odd TP asymmetries are large in size and vice versa. This property stems from the self–evident sum rule \( \mathcal{Y}_i^2 + \overline{\mathcal{Y}}_i^2 = 1 \) for every \( i \); (iii) except for the low mass regime where mixing between the lightest Higgs state and the
heavier Higgs states is enhanced, the CP–even TP asymmetries of the two heavy states are opposite in sign and so are the CP–odd TP asymmetries. This reflects the fact that the lightest Higgs state is (almost) decoupled from the two heavy states, so that the latter undergo typical two–state mixing. As the TP asymmetries have opposite signs for two Higgs bosons, there will be sizable cancellations if two states are degenerate. Therefore, using the TP asymmetries as a powerful probe of the CP phase requires that the mass difference be at least comparable to or larger than their decay widths. Numerically, we have checked that even for $\tan \beta = 30$ the mass difference is indeed comparable to the decay widths in the CP–invariant case and it becomes larger for non–trivial values of the CP phase unless the masses are larger than 500 GeV [4,11].

In summary, CP–violating phases in the stop and sbottom sectors modify the mass spectrum and couplings of the Higgs bosons significantly from those in the CP–invariant theory. We have shown that a controllable energy resolution and beam polarization at a MC offer powerful and independent opportunities for probing the MSSM Higgs sector via $s$–channel resonance production even with loop–induced explicit CP violation. For the very narrow lightest Higgs–boson resonance, a relatively large beam energy resolution allows one to measure the muonic decay width precisely, and the measured width is complementary to the CP–even and CP–odd TP asymmetries in determining the free parameters. However, for a large value of $\tan \beta$ and large $M_{H^+}$, it turned out that measurements of the transverse polarization asymmetries of the heavier Higgs bosons are essential for determining the CP phase. The availability of transversely polarized beams at a Muon Collider would therefore be of crucial importance for precisely probing the characteristics of the Higgs sector in the general CP–noninvariant theories.

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FIG. 1. The effective unpolarized cross section $\sigma^0_{H_1}$ versus $M_{H_1}$ with $\Phi = 0^0, 40^0, 80^0, 120^0,$ and $160^0$ for $\tan \beta = 3$ (left) and $\tan \beta = 30$ (right). Here, $R$ is taken to be 0.15%.
FIG. 2. The CP–even and CP–odd TP asymmetries $Y_1$ and $\bar{Y}_1$ versus $M_{H_1}$ with $\Phi = 0^0, 40^0, 80^0, 120^0$, and $160^0$ for $\tan \beta = 3$ (upper) and $\tan \beta = 30$ (lower).
FIG. 3. The effective unpolarized cross sections $\sigma^0_{H_2}$ versus $M_{H_2}$ (left) and $\sigma^0_{H_3}$ versus $M_{H_3}$ (right) with $\Phi = 0^0, 40^0, 80^0, 120^0$, and $160^0$ for $\tan \beta = 3$. 
FIG. 4. The CP–even and CP–odd TP asymmetries $Y_2$ (left) and $\overline{Y_2}$ (right) versus $M_{H_2}$ with $\Phi = 0^0, 40^0, 80^0, 120^0, \text{ and } 160^0$ for $\tan \beta = 3$ (upper) and $\tan \beta = 30$ (lower).
FIG. 5. The CP–even and CP–odd TP asymmetries $Y_3$ (left) and $\overline{Y}_3$ (right) versus $M_{H_3}$ with $\Phi = 0^\circ, 40^\circ, 80^\circ, 120^\circ$, and $160^\circ$ for $\tan \beta = 3$ (upper) and $\tan \beta = 30$ (lower).