Scattering Vector Mesons in D4/D8 model

Marcus A. C. Torres  C. A. Ballon Bayona  H. Boschi-Filho  N. Braga

Instituto de Física
Universidade Federal do Rio de Janeiro

Light-Cone 2009: Relativistic Hadronic and Particle Physics
Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors

2. D4/D8 Brane Model
   - Holographic QCD

3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius

4. Summary
Outline

1 Motivation
   - Gauge/String duality vs. the Real QCD
     - D4/D8 pion form factors

2 D4/D8 Brane Model
   - Holographic QCD

3 Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius

4 Summary
AdS/CFT provides means to reach strong coupling gauge theory, but it is not a QCD.

To do:
Break SUSY, add quarks, masses, $\chi$, confinement scale.

$\rightarrow$
Holographic QCD
D4-D8 brane model
AdS/CFT provides means to reach strong coupling gauge theory, but it is not a QCD.

To do:
Break SUSY, add quarks, masses, $\chi$, confinement scale.

→ Holographic QCD
D4-D8 brane model
AdS/CFT provides means to reach strong coupling gauge theory, but it is not a QCD.

To do:
Break SUSY, add quarks, masses, $\chi$, confinement scale.

→
Holographic QCD
D4-D8 brane model
Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors

2. D4/D8 Brane Model
   - Holographic QCD

3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius

4. Summary
(a) \( Q^2 \times F_\pi(Q^2) \) plot from D4/D8 model  
(b) \( Q^2 \times F_\pi(Q^2) \) prediction from hardwall (dashed line), softwall (continuous line) [5] models and data
Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors

2. D4/D8 Brane Model
   - Holographic QCD

3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius

4. Summary
Holographic QCD

- $N_c$ (large) D4 branes fills $\mathcal{M}^{1,3}$, $x_4(\tau)$, with $x_4$ compactified in a circle.
  - 4d SU($N_c$) YM on the brane

- $N_f$ probe branes D8, D8 fills $\mathcal{M}^{1,3}$, $\{x_5, x_8\}$ and radial $U(\tau)$
  $\chi$ symm $\xrightarrow{\chi}$ $SU(N_f) \times SU(N_f)$
  dynamically broken.
**Holographic QCD**

- $N_c$ (large) D4 branes fills $\mathcal{M}^{1,3}$, $x_4(\tau)$, with $x_4$ compactified in a circle.
  - 4d SU($N_c$) YM on the brane

- $N_f$ probe branes D8, $\overline{D8}$
  - fills $\mathcal{M}^{1,3}$, $\{x_5, x_8\}$ and radial $U(\tau)$
  \[ \chi \text{ symm} \]
  \[ SU(N_f) \times SU(N_f) \]
  dynamically broken.
Holography: 4d from 5d

- quarks confined
- D8-\overline{D8} gauge fields independent of \(\{x_5, x_8\}\) directions.
- KK modes: vector (\(v^\mu_n\)) and axial vector mesons (\(a^\mu_n\)).
  \[
  A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} v^{(n)}_\mu(x^\mu)\psi_{2n-1}(z) + a^{(n)}_\mu(x^\mu)\psi_{2n}(z) + A_L, A_R(x)
  \]
  \(\psi_n\): eigenfunctions of e.o.m. (orthonormal).
  \(\lambda_n\): eigenvalues \(\sim m^2_n\) of (axial) vector mesons.

\[
S_{D8} \sim \int d^9 x \ e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi \alpha' F_{MN})}
\]

\[
\int d^4 x \mathcal{L}_{kin+mass} - 2g_{v^n} \text{tr}(v^{n\mu}v_\mu) + g_{v^l v^m v^n}(\partial^\mu v^{l\nu}-\partial^\nu v^{l\mu})[v^m_\mu, v^n_\nu] + g_{v^l a^m a^n}(\partial^\mu a^{n\nu}-\partial^\nu a^{n\mu})[v^l_\mu, a^m_\nu] - [v^l_\nu, a^m_\mu]
\]
Holography: 4d from 5d

- quarks confined
- D8-\overline{D8} gauge fields independent of \( \{x_5, x_8\} \) directions.
- KK modes: vector (\( v_\mu^n \)) and axial vector mesons (\( a_\mu^n \)).
  \[
  A_\mu (x^\mu, z) = \sum_{n=1}^{\infty} v_\mu^n (x^\mu) \psi_{2n-1}(z) + a_\mu^n (x^\mu) \psi_{2n}(z) + A_L, A_R (x)
  \]
  \( \psi_n \): eigenfunctions of e.o.m. (orthonormal).
  \( \lambda_n \): eigenvalues \( \sim m_n^2 \) of (axial) vector mesons.

\[
S_{D8} \sim \int d^9 x e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi \alpha' F_{MN})} \longrightarrow
\]

\[
\int d^4 x \mathcal{L}_{\text{kin+mass}} - 2g_v^n \text{tr} (v_\mu^n v^\mu) + g_{vl} v_m v_n \left( \partial_\mu v^{l\nu} - \partial_\nu v^{l\mu} \right) \left[ v_\mu^m, v_\nu^n \right] \\
+ g_{vl} a_m a^n \left( \partial_\mu v^{l\nu} - \partial_\nu v^{l\mu} \right) \left[ a_\mu^m, a_\nu^n \right] + g_{vl} a_m a^n \left( \partial_\mu a^{n\nu} - \partial_\nu a^{n\mu} \right) \left( [v_\mu^l, a_\nu^m] - [v_\nu^l, a_\mu^m] \right)
\]
Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors

2. D4/D8 Brane Model
   - Holographic QCD

3. Work
   - Scattering Vector Mesons
     - Wave functions
     - Quadratic radius

4. Summary
**Figure:** Vector meson EM scattering.

- **VMD**

\[
\langle \nu^{\mu} (k) | J^{(V)}_{\mu} (0) | \nu^\rho (k') \rangle = f_{\mu \nu \rho}^{(k, k')} F_{\nu^\rho \nu^\rho} \left( Q^2 \right),
\]

\[
F_{\nu^\rho \nu^\rho} \left( Q^2 \right) = \sum_{\rho=1}^{\infty} \frac{g_{\nu^\rho} g_{\nu^\rho \nu^\rho}}{Q^2 + m_{\nu^\rho}^2} = \sum_{\rho=1}^{\infty} \frac{g_{\nu^\rho} g_{\nu^\rho \nu^\rho}}{Q^2 + m_{\nu^\rho}^2 + 1} = \frac{1}{Q^2} \sum_{\rho=1}^{\infty} \frac{g_{\nu^\rho} g_{\nu^\rho \nu^\rho}}{1 + \frac{m_{\nu^\rho}^2}{Q^2}}.
\]
Scattering Vector Mesons

Sum Rules

Table: Masses and coupling constants

| n | $\frac{m_{vn}^2}{M_{KK}^2} = \lambda_{2n-1}$ | $\kappa^{-1/2} \frac{g_{vn}}{M_{KK}^2}$ | $\kappa^{1/2} g_{v^1v^1vn}$ |
|---|---|---|---|
| 1 | 0.66931 | 2.10936 | 0.44658 |
| 2 | 2.87432 | 9.10785 | −0.14654 |
| 3 | 6.59118 | 20.7957 | 1.8434 × 10^{-2} |
| 4 | 11.79669 | 37.1502 | −3.6885 × 10^{-4} |
| 5 | 18.48972 | 58.1701 | 2.6953 × 10^{-4} |
| 6 | 26.67017 | 83.834 | 3.0775 × 10^{-5} |
| 7 | 36.33796 | 114.152 | 1.8572 × 10^{-5} |
| 8 | 47.49318 | 148.103 | 6.9961 × 10^{-6} |
| 9 | 53.6285 | 188.695 | 3.5081 × 10^{-6} |

$$F_{v^m v^n}(Q^2 = 0) = \sum_{p=1}^{9} \frac{g_{vp} g_{vn v^1 v^p}}{m_{vp}^2} \sim \delta_{mn}$$
**$v^1$ Elastic form factor**

![Graphs](image.png)

(a) Elastic form factor $F_{v^1}$ vs $Q^2$

(b) $Q^4 \times F_{v^1}$ vs $Q^2$

**Superconvergence (Grigoryan and Radyushkin):**

\[
\sum_{n=1}^{9} g_{v^n} g_{v^n v^1} v^1 \sim -0.000945(M_{KK})^2
\]

\[
\sum_{n=1}^{9} g_{v^n} g_{v^n a^1 a^1} \sim 0.000903(M_{KK})^2
\]
$a^1$ Elastic form factor

(c) Elastic form factor $F_{a^1}$ versus $Q^2$

(d) $Q^4 \times F_{a^1}$ versus $Q^2$

Figure: $a^1$ elastic form factor
Vanishing of higher 3-vertex coupling coupling

\[ g_{vn} = m_{vn}^2 \kappa \int dz (1 + z^2)^{-1/3} \psi_{2n-1}, \quad g_{v^1 v^1 p} = \kappa \int dz (1 + z^2)^{-1/3} \psi_1^2 \psi_{2p-1} \]

\( \psi_1^2 \) selects small z region. In this region \( \psi_{2p-1} \sim \cos(z\lambda_{2p-1}^{1/2}) \)
Regge Trajectory

\[ \lambda_n(*) \text{ and } 0.25n^2(+) \text{ vs } n \]

\[ n^{1.9} \]

\[ n^{1.27} \]
Outline

1 Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors

2 D4/D8 Brane Model
   - Holographic QCD

3 Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius

4 Summary
Using the $\rho$ (vector) meson form factor expression in the low $Q^2$ limit, in terms of the electric form factor:

$$G_c^\rho(Q^2) = \left(1 - \frac{Q^2}{6m_{v_1}^2} F_{v_1}(Q^2)\right)$$

$$\langle r_\rho^2 \rangle = -6 \frac{d}{dQ^2} G_c^\rho(Q^2) \bigg|_{Q^2=0}$$

We found

$$\langle r_\rho^2 \rangle = 0.57\text{fm}^2$$

While results from Dyson-Schwinger equations: $\langle r_\rho^2 \rangle = 0.54\text{fm}^2$
Besides some weaknesses (mass spectrum, UV behavior), D4-D8 model shows adequate results, concerning vector meson EM scattering.

Outlook

- Would our results improve if quarks were massive?
- What are the general features that makes D4-D8 model interesting?
- D4-D8 is not really a Top-down model, since $M_{KK}$ and $\kappa$ are fixed by experimental values. What can be done?
T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843 [arXiv:hep-th/0412141].

T. Sakai and S. Sugimoto, Prog. Theor. Phys. 114 (2005) 1083 [arXiv:hep-th/0507073].

H. R. Grigoryan and A. V. Radyushkin, Phys. Rev. D 76, 095007 (2007) [arXiv:0706.1543 [hep-ph]].

H. R. Grigoryan and A. V. Radyushkin, Phys. Lett. B 650, 421 (2007) [arXiv:hep-ph/0703069].

S. J. Brodsky and G. F. de Teramond, Phys. Rev. D 77, 056007 (2008) [arXiv:0707.3859 [hep-ph]].