RADII AND BINDING ENERGIES OF NUCLEI FROM A MODEL OF THE pn-PAIR INTERACTIONS

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Abstract
New formulas to calculate the nuclear charge radii and the nuclear binding energies have been obtained in the framework of an alpha-cluster model. The particular features of the model are two assumptions that the proton-neutron pair interactions are responsible for adherence between $\alpha$-clusters and that the proton and the neutron from a proton-neutron pair have equal nuclear potentials. These assumptions allow one to determine the charge and Coulomb radii in dependence on the amount of proton-neutron pairs in the nucleus. Unlike the Weizsäcker’s formula to calculate nuclear binding energies, in the model the binding energies of alpha-clusters and excess neutrons are estimated separately. The calculated values are in a good agreement with the experimental data.

keywords: nuclear structure; alpha-cluster model; Coulomb energy; binding energy; charge radius

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1 Introduction
The alpha-cluster model, proposed by Gamov in 1929, soon was developed by the idea that nucleus behaves as a liquid drop. The nuclear liquid drop model was explicitly formulated by Weizsäcker’s formula in 1935 [1] for calculation of the binding energy. Later the model had been developed by taking into account rotation deformation modes and quantum oscillations. It was shown that the large body of data on $\alpha$-decay is described by a simple cluster model with using a notion of $\alpha$-core potential [2]. There are also some approaches considering some regular forms alpha clusters might build [3]. There are several attempts to incorporate the alpha cluster model into the shell model [3,4]. Currently the idea of alpha-clustering is actively used in the microscopic studies of the nuclear structure of the light nuclei [5-7] in the framework of the few-body task, as well as in studies of the effective nucleon-nucleon interaction [8]. There has been also some activity recently to apply an alpha-cluster model to facilitate the description of some characteristics of nuclei like the $\alpha$-particle separation energy [9] or nuclear radii [10] for wide range of nuclei.

It is well known that the experimental charge radii are well described as depending on the mass number due to the formula $R = const(A)^{1/3}$ Fm. The value of $const$ varies within $0.93 \div 1.27$ from the light nuclei to the heavy ones. For the stable nuclei with $6 \leq Z \leq 82$ the average value of $const = 0.973$ Fm obtained from fitting the experimental values of the radii of the most abundant isotopes [10] gives a root mean square deviation $\langle \Delta^2 \rangle^{1/2} = 0.148$Fm of the calculated values of $R$ from the experimental values known with an average error $\sim 0.030$ Fm. At the same time the calculations show that the radii can be described even with a less deviation $\langle \Delta^2 \rangle^{1/2} = 0.078$Fm by another formula, $R = 1.017(2Z)^{1/3}$ Fm. Therefore the number of protons, or one can say the number of $\alpha$-clusters $N_\alpha$, can be considered as being responsible for charge radii disregarding to the number of excess neutrons.

In this connection it is interesting to modify the classic alpha cluster model in accordance with the idea suggesting that $\alpha$-clusters are the ruling elements of the nuclear structure. This work is devoted to the task of obtaining some phenomenological formulas to calculate the radii and the...
binding energies of nuclei in dependence on the number of alpha-clusters and their inter-cluster bonds. Unlike the Weizsäcker’s formula, the binding energy of excess neutrons is to be estimated separately.

In the $\alpha$-cluster model presented in the work a nucleus is supposed to consist of a number of proton-neutron pairs, $pn$-pairs, that are coupled into alpha-clusters [11]. In case of an odd value of $Z$ there is one $pn$-pair on the surface of an alpha-clustered nucleus. The idea that the last proton and the last neutron in the nucleus with an odd value of $Z$ form a $pn$-pair on the surface of alpha-cluster drop can be supported by the fact that the total spin of the nuclei with $N = Z$ in their ground states is equal to the doubled value of the single-particle momentum of the last nucleon. This gives definite evidence for the existence of strong correlation between the last proton and neutron.

A particular feature of the model is an assumption that the proton and the neutron belonging to one pair have equivalent nuclear single-particle bound state potentials (the EPN requirement [12,13]), which looks reasonable from the point of view of the isotopic invariance of nuclear force. As a result, this assumption brings about a definition of the difference $\Delta E_{pn}$ of the single-particle binding energies for the proton and the neutron of one pair as the Coulomb energy of the proton in field of the nucleus. It can help in estimation of the radius of the last proton position in the nucleus. The nuclear charge radii are calculated by means of a sum of the square radii of the core and the peripheral $\alpha$-clusters weighted by the numbers of clusters in the core and the number of clusters on the nucleus periphery. This calculational procedure is similar to that usually used in the framework of single-particle potential approaches assumed in shell model [10,14].

It has been found [15] from analysis of nuclei with the equal numbers of protons and neutrons, $N = Z$, that the number of short-range bonds between $N_{\alpha}$ alpha-clusters is calculated by the formula $3(N_{\alpha} - 2)$. A good agreement between experimental binding energies of the nuclei with $N = Z$ and the values of energies calculated with taking into account only the short-range bonds allows one to suggest that that portion of the Coulomb energy which comes from the long-range interactions is compensated by the surface tension energy. This suggests a formula for calculation of the surface tension energy.

Thus, on the basis of this model the formulas to calculate the Coulomb energy $E^C$, the nuclear force energy $E^{nuc}$ and the surface tension energy $E^{st}$ of the alpha-cluster nuclear matter have been derived in a consistent way.

It is suggested that the binding energy of excess neutrons for the nuclei with $Z > 20$ can be calculated by means of a sum of the binding energies of excess neutron-neutron pairs, $nn$-pairs, as $\sum_{1}^{N_{nn}} E_{nn}$. The experimental values of $E_{nn}$ are approximated by a phenomenological formula in dependence on the number of pairs $N_{nn}$ with using two parameters. In previous work [11] the experimental values of $E_{nn}$ were used to calculate the nuclear binding energies.

As a result, the formulas do not contain fitting parameters. All the quantities used have their own physical meaning. They are obtained from an independent analysis of such values as $\Delta E_{pn}$, the binding energy of the lightest nuclei or by fitting the experimental binding energies of the excess $nn$-pairs.

To test the validity of the formulas for calculation of $E^{nuc}$, $E^C$ and $E^{st}$ independently of the binding energy of the excess neutrons, the $\alpha$-particle separation energy, as well as the deuteron separation energy, have been calculated. Agreement between the calculated values and experimental ones has been shown.

2 Model Of $pn$-pair Interactions For Nuclei With $N = Z$

2.1 Nuclear binding energy and Coulomb energy

A picture given in Fig. 1 shows the $pn$-pair bonds in the nuclei. The internal binding energy of one cluster is taken equal to that of nucleus $^4$He $\varepsilon_{\alpha} = 28.296$MeV. The distance between clusters is supposed to be constant for all nuclei except nucleus $^8$Be. In the case one binding is not enough
to keep two positively charged clusters close. Adding one more pn-pair causes two new bindings with the pairs of two alpha-clusters with the energy $2\varepsilon_{pnpn}$ in the nucleus $^{10}$B, and the clusters get closer with the distance proper for the alpha-cluster liquid. The energy of adherence of two clusters $\varepsilon_{\alpha\alpha}$ is estimated by means of the following equations[11,15] for the binding energy $E^b$ of the lightest nuclei

$$
E^b_{6Li} = \varepsilon_{\alpha} + \varepsilon_{pnpn} + \varepsilon_{pn}
$$
$$
E^b_{10B} = 2\varepsilon_{\alpha} + \varepsilon_{\alpha\alpha} + 2\varepsilon_{pnpn} + \varepsilon_{pn}
$$
$$
E^b_{12C} = 3\varepsilon_{\alpha} + 3\varepsilon_{\alpha\alpha}
$$

So the values of $\varepsilon_{\alpha\alpha} = 2.425\text{MeV}$, $\varepsilon_{pnpn} = 2.037\text{MeV}$, $\varepsilon_{pn} = 1.659\text{MeV}$, where $\varepsilon_{pnpn}$ denotes the energy between the single pn-pair and a pn-pair of the nearby $\alpha$-cluster, $\varepsilon_{pn}$ stands for the binding energy of the proton and the neutron in the single pair.

The binding energy of the other nuclei including $^{12}$C are calculated with a good accuracy by means of the following equations for the nuclei with even $Z$ with mass number $A$

$$
E^b = N_\alpha \varepsilon_{\alpha} + 3(N_\alpha - 2)\varepsilon_{\alpha\alpha},
$$

and for the nuclei with odd $Z_1 = Z + 1$ and the mass number $A_1 = A + 2$

$$
E^b_{12C} = E^b_{12C} + 6\varepsilon_{pnpn} + \varepsilon_{pn},
$$

where $N_\alpha$ denotes the number of alpha-clusters, $3(N_\alpha - 2)$ stands for the number of inter-cluster bonds. Although the idea that every new added cluster causes three new bonds with three other clusters of its close vicinity was discussed before [3], the formula to calculate the number of bonds was first obtained in [15].

Eq. (1) implies that binding energy of a nucleus $A$ is determined by the number of clusters and the number of bonds. The energy of a nucleus $A_1$ is determined by the energy of the nucleus $A$ plus the energy of six bonds between the single pair and six pairs of the three clusters which get
bound with that pair plus the binding energy of the pair itself. The values of $E^b$ are presented in Table 1 in comparison with experimental values $E_{exp}$ [16].

Table 1. Charge radii and binding energies for nuclei with $N = Z$. Radii are given in Fm, energies in MeV.

| Z | $E_{exp}$ | $E^b$ | $\Delta E_{pn}$ | $\Delta E_\alpha$ | $E^C$ | $E^C_W$ | $E^{shl}$ | $E^{shl}_W$ | $R_{exp}$ | $R_{shl}$ | $R$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 28 | 28 | .764 | .764 | .764 | 2 | 29 | 30 | 1.71(4)$^4$He | 1.71 | |
| 3 | 32 | 32 | 1.007 | 1.771 | 4 | 34 | 26 | 2.57(10)$^6$Li | 1.96 | |
| 4 | 56 | 56 | 1.644 | 2.651 | 3.415 | 6 | 60 | 61 | 2.519(12)$^9$Be | 2.15 | |
| 5 | 65 | 65 | 1.850 | 5.303 | 8 | 70 | 68 | 2.45(12)$^{10}$B | 2.32 | |
| 6 | 92 | 92 | 2.764 | 4.614 | 8.067 | 11 | 100 | 100 | 2.470(15)$^{12}$C | 2.47 | |
| 7 | 105 | 106 | 3.003 | 11.070 | 14 | 116 | 112 | 2.560(20)$^{14}$N | 2.60 | |
| 8 | 128 | 128 | 3.536 | 14.606 | 18 | 142 | 143 | 2.730(25)$^{16}$O | 2.73 | 2.71 |
| 9 | 137 | 142 | 3.544 | 18.150 | 22 | 156 | 157 | 2.85$^{19}$F | 2.85 | 2.82 |
| 10 | 161 | 163 | 4.021 | 22.171 | 26 | 183 | 187 | 2.992(8)$^{20}$Ne | 2.91 | 2.92 |
| 11 | 174 | 177 | 4.328 | 26.499 | 31 | 201 | 203 | 2.97$^{23}$Na | 2.97 | 2.96 |
| 12 | 198 | 199 | 4.838 | 31.337 | 35 | 230 | 233 | 3.02(2)$^{24}$Mg | 3.02 | 3.01 |
| 13 | 212 | 212 | 5.042 | 36.379 | 41 | 248 | 250 | 3.06(9)$^{27}$Al | 3.07 | 3.05 |
| 14 | 237 | 234 | 5.953 | 41.972 | 46 | 279 | 279 | 3.138(3)$^{28}$Si | 3.12 | 3.12 |
| 15 | 251 | 248 | 5.731 | 47.703 | 51 | 299 | 298 | 3.24$^{31}$P | 3.16 | 3.16 |
| 16 | 272 | 270 | 6.227 | 53.930 | 57 | 326 | 327 | 3.24(11)$^{32}$S | 3.22 | 3.25 |
| 17 | 285 | 284 | 6.351 | 60.281 | 63 | 346 | 346 | 3.335(18)$^{35}$Cl | 3.26 | 3.29 |
| 18 | 307 | 306 | 6.746 | 67.027 | 70 | 374 | 375 | 3.39$^{36}$Ar | 3.36 | 3.38 |
| 19 | 321 | 320 | 6.923 | 73.950 | 76 | 395 | 395 | 3.40(7)$^{39}$K | 3.39 | 3.41 |
| 20 | 342 | 341 | 7.286 | 81.236 | 83 | 423 | 424 | 3.48(25)$^{40}$Ca | 3.48 | 3.51 |
| 21 | 355 | 355 | 7.277 | 88.513 | 90 | 443 | 444 | 3.55(5)$^{45}$Sc | 3.51 | 3.54 |
| 22 | 376 | 378 | 7.615 | 96.128 | 97 | 472 | 473 | 3.59(4)$^{48}$Ti | 3.55 | 3.64 |
| 23 | 390 | 391 | 7.6 | 103.8 | 104 | 494 | 494 | 3.58(4)$^{51}$V | 3.58 | 3.67 |
| 24 | 412 | 412 | 7.9 | 111.6 | 112 | 523 | 522 | 3.64(5)$^{52}$Cr | 3.64 | 3.62 |
| 25 | 427 | 426 | 7.9 | 119.5 | 120 | 546 | 544 | 3.68(11)$^{55}$Mn | 3.67 | 3.65 |
| 26 | 448 | 448 | 8.3 | 127.8 | 128 | 575 | 572 | 3.73$^{56}$Fe | 3.74 | 3.73 |
| 27 | 463 | 462 | 8.3 | 136.2 | 136 | 599 | 594 | 3.77$^{58}$Co | 3.78 | 3.77 |
| 28 | 484 | 483 | 8.8 | 144.9 | 145 | 629 | 623 | 3.76$^{58}$Ni | 3.86 | 3.84 |
| 29 | 497 | 497 | 8.8 | 153.7 | 154 | 651 | 645 | 3.88$^{63}$Cu | 3.89 | 3.87 |

Hereafter in Tables the values are given for both even $Z$ and odd $Z_1$ nuclei with denoting the charge and mass number as $Z$ and $A$. The values of energies and radii in Tables and formulas are given in MeV and Fm correspondingly.

A consideration of $pn$-pairs together with the EPN requirement gives one an opportunity to estimate the total Coulomb energy of nucleus $E^C$ as a sum of differences $\Delta E_{pn}$ in the binding energies of protons and neutrons of the $pn$-pairs

$$E^C = \sum_1^N \Delta E_\alpha + \delta,$$

where $\Delta E_\alpha$ denotes the Coulomb energy of an $\alpha$-cluster, which is calculated as a sum of Coulomb energies of its two $pn$-pairs $\Delta E_\alpha = \sum_1 \Delta E_{pn}$, $\delta$ stands for some portion of the Coulomb energy added to the sum after the first two clusters have got closer with adding one $pn$-pair to them in the nucleus $^{10}$B. For the nucleus $A_1$ the corresponding equation is.
The values of $\Delta E_{pm}$ for nuclei with $N = Z$ are calculated from experimental binding energies taken mostly from [16], where the values are given with an accuracy of 1 KeV with the measurement energy error in 2-3 KeV. Some of the values of mass deficiency for light nuclei have been used from more recent tables [17]. For the nuclei with $Z \geq 23$ the values of $\Delta E_{pm}$ cannot be obtained by this procedure due to the lack of the experimental data for nuclei (Z, N-1). The values were estimated [15] in framework of this model by means of the value of radius $R = 1.008(2Z)^{1/3}$fm for the nuclei with even $Z \geq 22$. The values $\Delta E_{pm}$, $\Delta E_{a}$ and $E^{C}$ are given in Table 1.

The value of Coulomb repulsion energy between two clusters $\varepsilon_{aa}^{C} = 1.925\text{MeV}$ is found from two equations for the Coulomb energy of one cluster in nuclei $^{12}\text{C}$ and $^{16}\text{O}$. They are $\Delta E_{a} = \varepsilon_{a}^{C} + 2\varepsilon_{aa}^{C}$ for $^{12}\text{C}$ and $\Delta E_{a} = \varepsilon_{a}^{C} + 3\varepsilon_{aa}^{C}$ for $^{16}\text{O}$, where $\varepsilon_{a}^{C}$ denotes the internal Coulomb energy of one cluster $\varepsilon_{a}^{C} = E_{4\text{He}}^{C} = 0.764\text{MeV}$. The value of the energy of the Coulomb repulsion between the single $pn$-pair and one of the nearby alpha-clusters in case of odd $Z_1$ can be found from the data for the nuclei $^{6}\text{Li}$ and $^{14}\text{N}$, $\varepsilon_{pna}^{C} = 1.001(6)\text{MeV}$.

Having obtained the values of the Coulomb energy of the nuclei one can estimate the Coulomb repulsion energy between two alpha clusters in nucleus $^{8}\text{Be}$ as follows $\varepsilon_{aa}^{C}(^{8}\text{Be}) = E_{^{8}\text{Be}}^{C} - 2\varepsilon_{a}^{C} = 1.887\text{MeV}$, which is less than $\varepsilon_{aa}^{C}$ by the value of $\delta = 0.038\text{MeV}$.

Eqs. (3) and (4) give the empirical values of the total Coulomb energy $E^{C}$ of a nucleus obtained on the basis of the values known from the experimental data with an accuracy of a few KeV. Having obtained the total Coulomb energy of nuclei, one can easily test the validity of the hypothesis of the alpha-cluster structure on the nuclei with few clusters $N_{a} \leq 4$, that is, the nuclei where each cluster interacts with every one of the other clusters. In this case the Coulomb energy can be calculated by the number of clusters and their bonds. For the nucleus $^{8}\text{Be}$ according to the Fig. 1 $E^{C}(^{8}\text{Be}) = 2\varepsilon_{a}^{C} + \varepsilon_{aa}^{C}(^{8}\text{Be})$. For nuclei $^{12}\text{C}$ and $^{16}\text{O}$ values $E^{C}$ are $E^{C}(^{12}\text{C}) = 3\varepsilon_{a}^{C} + 3\varepsilon_{aa}^{C}$ and $E^{C}(^{16}\text{O}) = 4\varepsilon_{a}^{C} + 6\varepsilon_{aa}^{C}$. In case of the nucleus $^{6}\text{Li}$ $E^{C}(^{6}\text{Li}) = \varepsilon_{a}^{C} + \varepsilon_{pna}^{C}$ and for $^{10}\text{B}$ $E^{C}(^{10}\text{B}) = 2\varepsilon_{a}^{C} + \varepsilon_{aa}^{C} + \Delta E_{pm}$. For the nucleus $^{14}\text{N}$ it is to be $E^{C}(^{14}\text{N}) = E^{C}(^{12}\text{C}) + 3\varepsilon_{pna}^{C}$. For these nuclei the equations give values of $E^{C}$ in agreement with the values obtained by (3) and (4) (see Table 1.) with a curious accuracy of 1 KeV with one exception for nucleus $^{6}\text{Li}$ (in the case the accuracy of the equation is of 6 KeV). It confirms the validity of the cluster structure of the nuclei, as well as the equations (3) and (4) are confirmed as the physically reliable formulas for obtaining empirical values of the Coulomb energy with an accuracy of several KeV.

The empirical value of the short-range nuclear force energy, which comes from inter-cluster nuclear force bonds and the effect of surface tension, $E^{shr} = E^{nuc} + E^{st}$ for the nuclei with $N = Z$ can be obtained from

$$E_{\text{exp}} = E^{shr} - E^{C}. \quad (5)$$

Correspondingly, the internal nuclear force energy of one alpha-cluster is estimated [3] as $\varepsilon_{a}^{nuc} = \varepsilon_{a}^{C} = 29.060\text{MeV}$. In the same way the value of the nuclear force energy between 2 alpha-clusters can be estimated as $\varepsilon_{aa}^{nuc} = \varepsilon_{aa} + \varepsilon_{aa}^{C} = 2.425\text{MeV} + 1.925\text{MeV} = 4.350\text{MeV}$. The energy of nuclear force interactions between the single $pn$-pair and the six pairs of three nearby clusters is $\varepsilon_{pna}^{nuc} = 3\varepsilon_{pna} + 6\varepsilon_{pm} = 15.225\text{MeV}$. The binding energy coming from the attracting nuclear force and repulsing Coulomb force interaction between the single pair and three nearby clusters is $\varepsilon_{p3a}^{nuc} = 3\varepsilon_{pna} - 3\varepsilon_{pm3a} = 12.222\text{MeV}$.

To compare the obtained values with the values given by Weizsäcker’s formula [19]

$$E_{W}^{b} = \alpha A - \beta A^{2/3} + \frac{\gamma Z^{2}}{A^{1/3}} \pm \frac{\delta}{A^{3/4}} - \frac{\varepsilon(A/2 - Z)^{2}}{A} \quad (6)$$

the values of $E_{W}^{shr} = \alpha A - \beta A^{2/3} \pm \frac{\delta}{A^{3/4}} - \frac{\varepsilon(A/2 - Z)^{2}}{A}$ and $E_{W}^{C} = \frac{Z^{2}}{A}$ are given in Table 1 in comparison with $E^{shr}$ and $E^{C}$. One can see that the values due to Weizsäcker’s formula and the
2.2 Radius of last proton position and charge radii of nuclei

To estimate the value of the radius of last proton position $R_p$ in nucleus $A$ the value of its Coulomb energy in the field of the nucleus is used and it is expected to be equal to the value of $\Delta E_{pm}$. The Coulomb energy of the proton consists of the energy of its interaction with the other proton of its $\alpha$-cluster $\varepsilon^C_\alpha$ and the Coulomb energy of its interaction with the other $Z - 2$ protons of the nucleus

$$\Delta E_{pm} = \varepsilon^C_\alpha + (Z - 2)e^2/R_p, \quad (7)$$

and for odd $Z_1 = Z + 1$

$$\Delta E_{pn} = (\varepsilon^C_{p\alpha} - \varepsilon^C_\alpha) + (Z - 1)e^2/R_{p1}, \quad (8)$$

where $R_{p1}$ stands for the radius of the position of the single $pn$-pair.

It is valid only for the nuclei with $Z/Z_1 \geq 9$, because the last proton Coulomb energy for the lighter nuclei must be calculated with a different function, for example spherical function with the Coulomb radius $R_C$ like the proton Coulomb potential [18] at small radii less than $R_C$ in the Shrödinger equation for single-particle bound state wave function in Distorted Wave Born Approximation.

Another way to estimate $R_p$ comes from an equation for the Coulomb energy for the last $\alpha$-cluster. The Coulomb energy of the cluster consists of the internal Coulomb energy of the cluster $\varepsilon^C_\alpha$, Coulomb energy of its interaction with the three nearby clusters $3\varepsilon^C_{p\alpha}$ and long-range part of the Coulomb energy of its interaction with the other $N_\alpha - 4$ clusters of the nucleus

$$\Delta E_\alpha = \varepsilon^C_\alpha + 3\varepsilon^C_{p\alpha} + 2(Z - 8)e^2/R_{N\alpha-4}, \quad (9)$$

where $R_{N\alpha-4}$ stands for the distance between the mass center of the remote $N_\alpha - 4$ clusters and the cluster under consideration. The value $R_{N\alpha-4}$ is approximated as $R_{N\alpha-4} = 1.2R_p$ by fitting the empirical values of $\Delta E_\alpha$ for the nuclei with $Z \geq 14$. For the nuclei with odd $Z_1$ in accordance with the same logic of taking into account the long range Coulomb interaction one gets

$$\Delta E_{pn} = 3\varepsilon^C_{p\alpha} + (Z - 7)e^2/(1.2R_p + R_{p1} - R_p). \quad (10)$$

The third way to estimate $R_p$ comes from following idea. That fact that binding energy is calculated by (1) with a good accuracy means that the long range Coulomb energy, which increases with $Z$, must be compensated by the surface tension energy $E^{st}_\alpha$. The latter is expected to be proportional to $R_p^2$. Therefore the last member in the sum (9) is taken equal to $E^{st}_\alpha = \gamma_1 R_p^2 = 2(Z - 8)e^2/(1.2R_p)$, and the value of $\gamma_1 = 0.471$ MeV/Fm$^2$ is obtained from fitting the empirical values of $\Delta E_\alpha$ for the nuclei with $Z \geq 16$. As a result one obtains an equation to calculate $R_p$ in dependence on $N_\alpha$ only.

$$R_p = 2.168(N_\alpha - 4)^{1/3}. \quad (11)$$

The value of $R_{p1}$ is calculated by the same formula with the value of $N_{p1} = N_\alpha + 0.5$

Finally, the fourth way to calculate $R_p$ comes from the requirement that the surface tension energy is expected to be proportional to the number of clusters on the surface of the liquid drop $E^{st}_\alpha = \gamma_1 R_p^2 = \gamma_2 N_\alpha^{2/3}$, where $\gamma_2 = 1.645$ MeV is obtained by fitting the values of $\Delta E_\alpha$ for the nuclei with $Z \geq 14$. Further for the larger nuclei with $Z \geq 30$ the values are taken as Coulomb radius $R_C$. The equation therefore is

$$R_p = R_C = 1.869(N_\alpha)^{1/3}. \quad (12)$$
For odd $Z_1$ the value of $R_{p1} = R_{C1}$ is calculated by (12) with $N_{a1} = N_{a} + 0.5$. The values of $R_{p/p1}$ of all four ways are equal within an average deviation of 0.1 Fm.

The calculation of the charge radii of the nuclei has been made by two different ways, with using shell model representation $R_{shl}$ [11,20] and by using an representation for the core $R$. In the shell model representation the values $R_{shl}$ are calculated with the following equation

$$ ZR_{shl}^2 = (Z' R_{A'}^2 + n R_{p}^2), \quad (13) $$

and for odd $Z_1 = Z + 1$

$$ Z_1 R_{shl1}^2 = (Z_1 - 1) R_{A'}^2 + R_{p1}^2, \quad (14) $$

where $R_{A'}$ stands for the radius of the nucleus with completed shell $A'$ with an atomic number $Z'$, $n$ stands for the occupation numbers for protons of the last uncompleted shell. $A'$ denotes the nuclei $^{16}$O, $^{20}$Ne, $^{40}$Ca. In order to start calculation of $R_{shl/shl1}$ the value of radius $R_{16O}$ is taken to be equal to its experimental value. The values of $R_{shl}$ have been calculated with the values of $R_{p}$ (7) and $R_{p1}$ (8).

The other way with using a representation for core is somewhat simpler, but the deviation of calculated values from the experimental ones is even less than for $R_{shl}$. For the light nuclei $1 \leq N_{a} \leq 5$ the value $R = R_{4He}N_{a}^{1/3}$, where $R_{4He} = 1.71$ Fm [21] stands for the experimental radius $R_{exp}$ of the nucleus $^4$He. In the case of a nucleus with odd $Z_1$ the number of clusters in the formula is taken to be $N_{a1} = N_{a} + 0.5$.

For the nuclei with $5 < N_{a} \leq 10$ the root mean square radius $R$ is calculated by following equation

$$ N_{a} R^2 = 5(R_{20Ne})^2 + (N_{a} - 5) R_{p}^2, \quad (15) $$

where $R_{p/p1}$ is calculated by the number of $N_{a}/N_{a1}$ (12).

For the other nuclei with $N_{a} \geq 11$ the nucleus is represented as a core with radius $R_{a}(N_{a} - 4)^{1/3}$Fm, where $R_{a} = 1.60$Fm stands for the radius of $\alpha$-cluster of the core, plus four peripheral $\alpha$-clusters. Then charge radius $R$ is estimated by a sum of the square radius of the core weighted by the number of clusters in it, $N_{a} - 4$, and square radius $R_{p}$ of the position of the last cluster (11) weighted by 4

$$ N_{a} R^2 = (N_{a} - 4)1.60^2(N_{a} - 4)^{2/3} + 4R_{p}^2. \quad (16) $$

For odd $Z_1 = Z + 1$ the charge radius $R_{1}$ is calculated by equation

$$ N_{a1} R_{1}^2 = N_{a} R^2 + 0.5R_{p1}^2, \quad (17) $$

where the radius of the position of the single pn-pair $R_{p1}$ (11) is weighted by factor 0.5. Thus, the values of the radii calculated by the formulas depend only on the number of $\alpha$-clusters.

For the nuclei with $Z \leq 29$ the values of $R_{shl}$ and $R$ are given in Table 1 in comparison with $R_{exp}$ [10,21,22] for the most abundant isotopes. The values of $R$ calculated for three nuclei $^{6}$Li, $^{9}$Be and $^{18}$B are seen not to be in agreement with the experimental data. The nuclei do not have enough number of bonds between the clusters and the single pn-pair to provide a sufficient nuclear density proper for the alpha-cluster liquid.

## 3 Model Of pn-pair Interactions For Nuclei With $Z \geq 30$

### 3.1 Charge radii

The radii $R_{shl}$ [11] are calculated from (13) and (14) with the values of $R_{p/p1}$ (11). Besides the nuclei $^{16}$O, $^{10}$Ne and $^{20}$Ca the nuclei placed in the right end of the Periodical Tables Of Elements $^{28}$Ni, $^{36}$Kr, $^{46}$Pd, $^{54}$Xe, $^{78}$Pt, $^{86}$Rn, $^{110}$E – Pt, $^{118}$E – Rn are taken [11] as the nuclei with the completed shells. The nuclei $^{62}$Sm, $^{70}$Yb, $^{94}$Pt, $^{102}$No were added [20] to the set to make a sense
that four clusters make a completed shell. For the nuclei with the completed shells $28 \leq Z' \leq 70$ the value of the radius $R_{Z'} = 1.60(N'_{\alpha})^{1/3}$, where $N'_{\alpha}$ denotes the number of clusters in nucleus with completed shells, and for the nuclei with $Z' \geq 78$ the value of $R_{Z'}$ is calculated from the equation 

$$Z'R_{Z'}^2 = (Z' - 2)R_{Z'-2}^2 + 2R_p^2.$$ 

The latter means that the fourth cluster is located above the other three clusters completing the molecule $^{16}$O, see Fig. 1.

The root mean square deviation of the calculated radii $R_{shl}$ from the experimental values for nuclei with $9 \leq Z \leq 82$ is found to be $<\Delta^2>^{1/2} = 0.051$ Fm.

The second way to calculate the charge radii $R$ is to make use of (16) and (17) with $R_{p/p1}$ (11). The obtained values of $R_{shl}$ and $R$ are presented in Fig. 2 as solid and dashed lines, correspondingly.

Figure 2: Charge radii $R_{shl}$ and $R$ in comparison with $R_{exp}$ known for the most abundant isotopes and average charge radius of one nucleon $r_{ch}$ of $\alpha$ clusters in dependence on $Z$.

The experimental values $R_{exp}$ [10,21,22] for nuclei with $2 \leq Z \leq 82$ are indicated with crosses. For the nuclei with $6 \leq Z \leq 83$ the deviation of values of $R$ from experimental values $R_{exp}$ $<\Delta^2>^{1/2} = 0.050$ Fm. The deviation between $R_{shl}$ and $R$ for the nuclei with $9 \leq Z \leq 118$ is $<\Delta^2>^{1/2} = 0.034$ Fm. In Fig. 2 the values of charge radius of one nucleon of an $\alpha$-cluster $r_{ch} = R_{shl}/exp/(2Z)^{1/3} \approx 1.01$ Fm are also presented with the indications corresponding those of $R_{shl}$, $R$ and $R_{exp}$. Fig. 2 shows that the values of $R_{shl}$ and $R$ are close to the values of $R_{exp}$. That fact that $r_{ch} \approx$ constant approves that the size of a nucleus is determined by the number of $\alpha$ clusters.

### 3.2 Binding energy

One can write Eqs. (1) and (2) in a form with revealing the Coulomb energy and the surface tension energy. For an alpha-cluster nucleus $A$ the binding energy consists of the nuclear force energy $E^{nuc}$, the surface tension energy $E^{st}$, the binding energy of excess neutrons $E_n$ and the Coulomb energy $E^C$

$$E^b = E^{nuc}_A + E^{st} + E_n - E^C,$$ (18)

and for nucleus $A_1$

$$E^b_1 = (E^{nuc}_A + \varepsilon^{nuc}_{pm3\alpha} + \varepsilon_{pm}) + E^{st}_1 + E_{n1} - E^C_1,$$ (19)

where $E^{nuc}_A = N_{\alpha}e^{nuc}_{\alpha} + 3(N_{\alpha}-2)e^{nuc}_{\alpha\alpha}$ is calculated by the number of $\alpha$-clusters and the short-range bonds between them;
\[ E^{st} = \sum_{\alpha}^{N} E^{st}_{\alpha} = \sum_{\alpha}^{14} E^{st}_{\alpha} + \sum_{\alpha=15}^{N-1} \gamma_1 R_C^2 + \gamma_1 R_p^2, \]  
\[ \text{(20)} \]

where the empirical values of \( E^{st}_{\alpha} = 2(Z - 8)e^2/R_{N\alpha-4} = \Delta E_\alpha - \varepsilon_\alpha^C - 3\varepsilon_{\alpha0}^C \) (9) are obtained for \( N_\alpha \leq 14. \)

In case of the nucleus with \( Z_1 \)
\[ E^{st}_{1} = E^{st} + \gamma_1 R_{pl}^2/2, \]  
\[ \text{(21)} \]

where \( R_{pl} \) is obtained from (11) with \( N_{\alpha1} = N_\alpha + 0.5. \)

The empirical values of \( E^C \) and \( E^C_1 \) for nuclei with \( Z \leq 29 \) have been obtained, see Table 1. For other nuclei the formula to calculate the Coulomb energy of a charge sphere with radius \( R_C \) (12) is used
\[ E^C = \frac{3}{5} \frac{Z^2 e^2}{R_C}. \]  
\[ \text{(22)} \]

For \( E^C_1 \) the value \( R_{C1} \) (12) with \( N_{\alpha1} = N_\alpha + 0.5 \) is used.

The value of \( E_n \) is estimated as \( E_n = \sum_{i=1}^{N_{nn}} E_{nni} \), where \( N_{nn} = N_n/2 \) stands for the number of the excess \( mn \)-pairs and \( N_n \) denotes the number of excess neutrons. The values of \( E_{n1} \) are calculated similarly. The values of \( E_{nni} \) is fitted by an equation
\[ E_{nni} = 22.5 \text{MeV} - 1.358 N_{nn}^{2/3}. \]  
\[ \text{(23)} \]

It is well known that the experimental values of the separation energy of \( nn \)-pairs have deviations within 2-5 Mev determined by what nucleus loses the pair, which certainly refers to the shell effects. Despite the relatively big deviation of the values, the empirical values of all excess \( nn \)-pairs energy \( E_{n(exp)} \), known from the equation
\[ E_{n(exp)} = E_{exp}(Z, N + N_n) - E_{exp}(Z, N) \]  
\[ \text{(24)} \]

only for the nuclei with \( N = Z \), are restricted in relatively narrow corridor. For example, for the nuclei with \( 21 \leq Z \leq 29 \) the value of separation energy of two excess neutrons varies to within \( E_{2(exp)} = 21 \div 23 \text{ MeV} \), for four excess neutrons \( E_{4(exp)} = 42 \div 45 \text{ MeV} \), \( E_{6(exp)} = 61 \div 63 \text{ MeV} \) and \( E_{8(exp)} = 77 \div 80 \text{ MeV} \). Therefore, the parameters in (23) have been obtained by fitting both the experimental values of \( nn \)-pair separation energies known [16] for 27 \( nn \)-pairs and the values of \( E_{n(exp)} \). Eq. (23) has been used here to calculate the binding energies for the nuclei with \( Z > 10 \), although the empirical values of the excess neutron separation energies are known (24) and could have been used for the nuclei with \( 2 \leq Z \leq 29 \).

In Table 2 the calculated values of the nuclear binding energies of some isotopes with an integer number of \( mn \)- and \( nn \)-pairs are presented with their deviations \( \Delta = E_{exp} - E_b^h \). The values calculated by Weizsäcker’s formula (6) are presented in Table 2 with their deviations from experimental values \( \Delta_W = E_{exp} - E_{W}^b \) as well.

| Table 2. Comparison of experimental values of energies and calculated ones. All energies and deviations are given in MeV. |
| --- |
| \( Z \) | \( N_n \) | \( E_{exp} \) | \( \Delta \) | \( \Delta_W \) | \( E_\alpha \) | \( E_{th} \) | \( E_d \) | \( E_{th} \) | \( N_n \) | \( E_{exp} \) | \( \Delta \) | \( N_n \) | \( E_{exp} \) | \( \Delta \) |
| 2 | 0 | 28 | 0 | 0 | 28 | 28 | 26 | 26 | 2 | 29 | 0 | 0 | 28 | 0 |
| 3 | 0 | 32 | 0 | 9 | 30 | 30 | 4 | 4 | 2 | 41 | 0 | 0 | 32 | 0 |
| 4 | 0 | 56 | 0 | 0 | 28 | 28 | 25 | 25 | 2 | 65 | 0 | 0 | 56 | 0 |
| 5 | 0 | 65 | 0 | 4 | 33 | 33 | 8 | 8 | 2 | 80 | 0 | 0 | 65 | 0 |
| 6 | 0 | 92 | 0 | 2 | 36 | 37 | 27 | 27 | 2 | 105 | 0 | 0 | 92 | 0 |
| 7 | 0 | 105 | 0 | 7 | 40 | 36 | 12 | 13 | 2 | 118 | 0 | 0 | 105 | 0 |
| 8 | 0 | 128 | 0 | 2 | 35 | 36 | 23 | 22 | 2 | 139 | 0 | 0 | 128 | 0 |
| 9 | 0 | 137 | -2 | 2 | 33 | 35 | 10 | 13 | 2 | 154 | -2 | 0 | 137 | -2 |
| Z | N_n | $E_{\text{exp}}$ | $\Delta$ | $\Delta W$ | $E_o$ | $E_{\text{th}}$ | $E_d$ | $E_{\text{th}}$ | N_n | $E_{\text{exp}}$ | $\Delta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0 | 161 | -3 | -1 | 33 | 36 | 23 | 22 | 2 | 177 | -3 |
| 11 | 0 | 174 | -3 | 1 | 37 | 35 | 14 | 14 | 2 | 193 | -5 |
| 12 | 0 | 198 | -1 | 1 | 38 | 36 | 24 | 22 | 2 | 216 | -3 |
| 13 | 0 | 212 | -1 | 2 | 38 | 35 | 14 | 14 | 2 | 233 | -1 |
| 14 | 0 | 237 | 2 | 3 | 38 | 36 | 25 | 22 | 2 | 256 | 0 |
| 15 | 0 | 251 | 2 | 4 | 39 | 35 | 14 | 14 | 2 | 271 | 1 |
| 16 | 0 | 272 | 2 | 2 | 36 | 36 | 21 | 22 | 2 | 292 | 1 |
| 17 | 0 | 286 | 2 | 3 | 35 | 35 | 14 | 14 | 2 | 307 | 2 |
| 18 | 4 | 344 | -3 | -3 | 35 | 36 | 20 | 22 | 4 | 344 | -3 |
| 19 | 2 | 342 | 1 | -1 | 35 | 35 | 14 | 14 | 6 | 376 | -5 |
| 20 | 0 | 342 | 1 | 1 | 35 | 36 | 21 | 22 | 8 | 415 | -6 |
| 21 | 2 | 377 | 0 | -2 | 34 | 35 | 13 | 14 | 8 | 432 | -3 |
| 22 | 4 | 419 | 0 | -2 | 38 | 36 | 22 | 22 | 6 | 438 | 0 |
| 23 | 4 | 435 | 3 | 0 | 35 | 35 | 15 | 14 | 6 | 453 | 1 |
| 24 | 4 | 456 | 2 | 0 | 38 | 36 | 22 | 22 | 8 | 489 | -4 |
| 25 | 4 | 472 | 4 | 2 | 37 | 35 | 15 | 14 | 6 | 489 | 2 |
| 26 | 4 | 492 | 3 | 1 | 36 | 36 | 20 | 22 | 6 | 510 | 1 |
| 27 | 4 | 507 | 4 | 2 | 36 | 35 | 15 | 14 | 8 | 541 | -1 |
| 28 | 2 | 507 | 2 | 4 | 36 | 36 | 20 | 22 | 8 | 562 | -2 |
| 29 | 4 | 541 | 2 | 1 | 35 | 35 | 13 | 14 | 8 | 576 | -1 |
| 30 | 4 | 559 | 5 | 0 | 32 | 29 | 19 | 16 | 8 | 625 | 2 |
| 31 | 6 | 592 | 5 | -2 | 31 | 29 | 14 | 13 | 10 | 641 | 1 |
| 32 | 10 | 646 | 0 | -2 | 35 | 34 | 20 | 21 | 14 | 677 | -4 |
| 33 | 10 | 660 | 2 | -3 | 34 | 33 | 14 | 13 | 14 | 692 | -2 |
| 34 | 12 | 697 | 0 | -1 | 34 | 33 | 20 | 20 | 14 | 713 | -1 |
| 35 | 10 | 694 | 3 | -3 | 34 | 33 | 14 | 13 | 14 | 728 | 1 |
| 36 | 12 | 732 | 3 | 0 | 35 | 33 | 20 | 20 | 16 | 762 | -2 |
| 37 | 12 | 748 | 6 | 1 | 36 | 32 | 16 | 13 | 16 | 777 | 0 |
| 38 | 12 | 768 | 7 | 3 | 36 | 32 | 21 | 20 | 16 | 796 | -1 |
| 39 | 12 | 782 | 8 | 2 | 34 | 32 | 14 | 13 | 16 | 811 | 1 |
| 40 | 10 | 784 | 8 | 3 | 31 | 32 | 20 | 20 | 16 | 829 | 0 |
| 41 | 10 | 797 | 8 | 2 | 35 | 32 | 13 | 12 | 16 | 842 | 1 |
| 42 | 14 | 846 | 3 | -1 | 31 | 32 | 18 | 19 | 16 | 860 | 0 |
| 43 | 12 | 844 | 6 | 0 | 32 | 31 | 14 | 12 | 16 | 875 | 2 |
| 44 | 14 | 878 | 3 | -1 | 32 | 31 | 18 | 19 | 16 | 893 | 2 |
| 45 | 14 | 891 | 4 | -2 | 32 | 31 | 13 | 12 | 16 | 907 | 3 |
| 46 | 16 | 925 | 3 | -1 | 31 | 31 | 19 | 19 | 18 | 954 | 2 |
| 47 | 14 | 923 | 5 | -1 | 32 | 31 | 13 | 12 | 18 | 983 | 2 |
| 48 | 18 | 973 | 3 | -1 | 32 | 31 | 19 | 18 | 20 | 987 | 1 |
| 49 | 16 | 970 | 5 | -1 | 32 | 30 | 13 | 12 | 20 | 1002 | 3 |
| 50 | 20 | 1021 | 4 | 2 | 33 | 30 | 19 | 18 | 22 | 1050 | 4 |
| 51 | 20 | 1033 | 5 | 1 | 30 | 30 | 13 | 12 | 22 | 1049 | 4 |
| 52 | 26 | 1096 | 3 | 7 | 30 | 30 | 18 | 18 | 28 | 1110 | 3 |
| 53 | 22 | 1079 | 5 | 2 | 31 | 30 | 13 | 12 | 26 | 1136 | 5 |
| 54 | 24 | 1113 | 5 | 4 | 31 | 30 | 18 | 18 | 26 | 1142 | 5 |
| 55 | 24 | 1126 | 6 | 4 | 31 | 29 | 13 | 12 | 28 | 1155 | 5 |
| 56 | 26 | 1159 | 7 | 6 | 31 | 29 | 17 | 17 | 28 | 1170 | 3 |
| 57 | 24 | 1156 | 7 | 4 | 31 | 29 | 13 | 12 | 26 | 1170 | 6 |
| 58 | 24 | 1173 | 7 | 4 | 30 | 29 | 17 | 17 | 28 | 1209 | 2 |
| $Z$ | $N_n$ | $E_{exp}$ | $\Delta$ | $\Delta_{IV}$ | $E^{sep}_{\alpha}$ | $E_{th}$ | $E^{sep}_d$ | $E_{th}$ | $N_n$ | $E_{exp}$ | $\Delta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 59 | 22 | 1169 | 7 | 2 | 30 | 29 | 13 | 12 | 26 | 1209 | 4 |
| 60 | 22 | 1185 | 6 | 3 | 26 | 29 | 17 | 13 | 28 | 1238 | 1 |
| 61 | 24 | 1210 | 4 | 0 | 26 | 28 | 11 | 12 | 28 | 1237 | 1 |
| 62 | 28 | 1253 | 1 | -1 | 28 | 28 | 17 | 17 | 32 | 1280 | -1 |
| 63 | 26 | 1251 | 1 | -2 | 27 | 28 | 11 | 12 | 30 | 1280 | 1 |
| 64 | 30 | 1296 | 1 | 0 | 29 | 28 | 16 | 16 | 32 | 1309 | 0 |
| 65 | 28 | 1294 | 2 | -1 | 29 | 28 | 13 | 11 | 30 | 1309 | 2 |
| 66 | 32 | 1338 | 1 | 1 | 29 | 28 | 16 | 16 | 34 | 1351 | 1 |
| 67 | 30 | 1336 | 2 | -1 | 28 | 27 | 12 | 11 | 32 | 1350 | 2 |
| 68 | 30 | 1351 | 1 | 0 | 27 | 28 | 15 | 16 | 34 | 1391 | -1 |
| 69 | 32 | 1378 | 2 | 0 | 27 | 27 | 12 | 11 | 34 | 1392 | 2 |
| 70 | 34 | 1407 | 2 | 1 | 28 | 27 | 15 | 16 | 36 | 1419 | 1 |
| 71 | 32 | 1404 | 2 | 0 | 26 | 27 | 12 | 11 | 36 | 1431 | 2 |
| 72 | 36 | 1446 | 1 | 0 | 26 | 27 | 15 | 16 | 38 | 1459 | 1 |
| 73 | 34 | 1445 | 2 | 0 | 27 | 27 | 12 | 11 | 38 | 1471 | 2 |
| 74 | 38 | 1486 | 1 | 1 | 27 | 27 | 15 | 15 | 40 | 1498 | 1 |
| 75 | 36 | 1484 | 2 | 0 | 26 | 27 | 12 | 11 | 40 | 1510 | 2 |
| 76 | 40 | 1526 | 3 | 2 | 28 | 26 | 16 | 15 | 40 | 1526 | 3 |
| 77 | 38 | 1524 | 2 | 1 | 26 | 26 | 11 | 11 | 40 | 1538 | 4 |
| 78 | 38 | 1540 | 3 | 2 | 27 | 26 | 15 | 15 | 42 | 1567 | 6 |
| 79 | 40 | 1566 | 7 | 4 | 28 | 26 | 12 | 11 | 42 | 1580 | 7 |
| 80 | 42 | 1595 | 7 | 8 | 28 | 26 | 15 | 15 | 44 | 1609 | 10 |
| 81 | 42 | 1608 | 9 | 8 | 28 | 25 | 12 | 11 | 46 | 1641 | 11 |
| 82 | 44 | 1636 | 12 | 11 | 28 | 25 | 15 | 14 | 48 | 1663 | 7 |
| 83 | 42 | 1633 | 9 | 9 | 25 | 25 | 10 | 11 | 46 | 1664 | 7 |
| 84 | 40 | 1631 | 5 | 7 | 23 | 25 | 13 | 14 | 44 | 1686 | 6 |
| 85 | 40 | 1641 | 4 | 6 | 23 | 25 | 10 | 11 | 48 | 1685 | 2 |
| 89 | 50 | 1742 | 2 | -4 | 23 | 24 | 10 | 11 | 50 | 1742 | 2 |
| 90 | 52 | 1767 | -1 | -4 | 24 | 24 | 14 | 14 | 54 | 1778 | 0 |
| 91 | 50 | 1765 | -2 | -4 | 24 | 24 | 10 | 11 | 52 | 1777 | -1 |
| 92 | 54 | 1802 | 0 | -4 | 24 | 24 | 14 | 13 | 56 | 1812 | 0 |
| 93 | 52 | 1801 | -2 | -3 | 24 | 24 | 11 | 11 | 54 | 1812 | -1 |
| 94 | 58 | 1847 | 1 | -4 | 24 | 24 | 13 | 13 | 58 | 1847 | 1 |
| 95 | 52 | 1824 | -3 | -3 | 23 | 24 | 10 | 10 | 56 | 1846 | 0 |
| 96 | 52 | 1836 | -3 | -3 | 22 | 24 | 12 | 13 | 54 | 1848 | -2 |
| 97 | 52 | 1846 | -4 | -3 | 23 | 23 | 10 | 10 | 56 | 1869 | -1 |
| 98 | 54 | 1870 | -3 | -3 | 22 | 23 | 12 | 13 | 54 | 1870 | -3 |
| 99 | 52 | 1874 | 1 | 4 | 22 | 23 | 9 | 10 | 56 | 1891 | -2 |
| 100 | 54 | 1891 | -5 | -3 | 21 | 23 | 12 | 13 | 60 | 1930* | 5 |
| 101 | 54 | 1903* | -3 | 0 | 23 | 10 | 60 | 1940* | 5 |
| 102 | 56 | 1927* | -1 | 0 | 23 | 12 | 60 | 1951* | 3 |
| 103 | 56 | 1937* | -2 | 0 | 22 | 10 | 62 | 1972* | 5 |
| 104 | 58 | 1960* | -1 | 0 | 23 | 12 | 62 | 1984* | 5 |
| 105 | 58 | 1969* | -2 | 0 | 22 | 10 | 62 | 1993* | 4 |
| 106 | 60 | 1993* | 0 | 0 | 22 | 12 | 64 | 2016* | 6 |
| 107 | 60 | 2001* | -1 | 0 | 22 | 10 | 64 | 2025* | 5 |
| 108 | 60 | 2012* | -2 | 0 | 22 | 12 | 64 | 2036* | 4 |
| 109 | 64 | 2045* | 3 | 0 | 22 | 10 | 68 | 2068* | 9 |
| 110 | 64 | 2056* | 2 | 0 | 21 | 12 | 68 | 2079* | 8 |
| 111 | 64 | 2064* | 1 | 0 | 21 | 10 | 68 | 2088* | 7 |
Table 2. Continued

| $Z$ | $N_n$ | $E_{exp}$ | $\Delta$ | $\Delta_W$ | $E_{\alpha}^{sep}$ | $E_{th}$ | $E_{d}^{sep}$ | $E_{th}$ | $N_n$ | $E_{exp}$ | $\Delta$ |
|-----|-------|----------|---------|----------|------------------|--------|-------------|--------|-------|----------|---------|
| 112 | 68    | 2099*    | 6       | 0        | 21               | 11     | 1           |        | 70    | 2110*    | 10      | 64  | 2075*    | 0       |
| 113 | 70    | 2119*    | 8       | 0        | 21               | 10     | 1           |        | 72    | 2129*    | 11      | 66  | 2095*    | 1       |
| 114 | 68    | 2117*    | 4       | 0        | 21               | 11     | 1           |        | 70    | 2129*    | 8       | 64  | 2093*    | -4      |
| 115 | 68    | 2126*    | 2       | 0        | 21               | 10     | 1           |        | 72    | 2148*    | 9       | 64  | 2101*    | -5      |
| 116 | 68    | 2136*    | 1       | 0        | 21               | 11     | 1           |        | 72    | 2159*    | 9       | 64  | 2123*    | 4       |
| 117 | 68    | 2143*    | -1      | 0        | 21               | 10     | 1           |        | 72    | 2167*    | 7       | 64  | 2118*    | -9      |
| 118 | 68    | 2153*    | -2      | 0        | 21               | 11     | 1           |        | 72    | 2177*    | 6       | 64  | 2128*    | -10     |
| 119 | 68    | 2161*    | -4      | 0        | 20               | 9      |             |        | 72    | 2185*    | 5       | 64  | 2135*    | -13     |

*The value is calculated by Weizsäcker’s formula (6).

One can see from this Table that the values of $\Delta$ and $\Delta_W$ are comparable. To test the formulas for $E_{nuc}$, $E_{st}$ and $E_C$ independently of the energy of excess neutrons, the energy of the alpha-particle separation energy, as well as the neutron separation energy, have been calculated. In the Table the experimental values $E_{\alpha}$ and $E_d$ are followed by the calculated values $E_{th}$. The left part of the Table contains values for nuclids of the most abundant isotopes for even $Z$ or nearest to them for odd $Z$. The right part contains the values of binding energies for the isotopes with large and small number of $N_n$. Since for the nuclei with $Z \leq 10$ the experimental values of the energy of excess neutrons (24) is used, the values of $\Delta$ in the cases with a larger number of neutrons are the same as for nuclei with $N = Z$.

4 Conclusion

There are two specific assumptions in the alpha-cluster model presented here. Namely, (1) the $pn$-pair interactions are responsible for building alpha-clusters and their adherence and (2) the proton and the neutron from a pair have equivalent bound state nuclear potentials. Formulas to calculate the radii and binding energies of nuclei have been obtained in the framework of the model. The deviation of the calculated radii from the experimental values is $<\Delta^2>^{1/2} = 0.05$ Fm, which is $\sim 1\%$ of the magnitude of radius. The calculated values of binding energies of the nuclei with an integer number of $pn$-pairs and $nn$-pairs have deviations from the experimental values comparable with the values due to Weizsäcker’s formula and they have less than $0.5\%$ on average.

The calculations do not take into account the spin-orbit correlations and consequently shell effects stay beyond this consideration. Therefore, the accuracy obtained for the calculated values may provide some evidence on the influence of the shell effects on the values of the charge radii and binding energies for the nuclei in their ground states. The accuracy of several hundredths of Fm for the radius and of several MeV for the binding energy reveal the possible range of the shell effect influence. It is in an agreement with the range of applicability of shell model in the various tasks of nuclear spectroscopy.

The obtained values of the Coulomb repulsion and the nuclear force energies of the cluster-cluster interaction, which are 1.925 MeV and 4.350 MeV correspondingly, can be useful in a microscopic description of light nuclei $^{12}C$ and $^{16}O$ in the framework of the few-body task.

The obtained formulas to calculate the Coulomb radius $R_C$, the charge radius $R$ and the radius of the last proton position $R_p$ by the number of $\alpha$-clusters show that for light nuclei $R < R_C \approx R_p$ and for heavy nuclei $R < R_C < R_p$.

The empirical values of the Coulomb energy $E_C$ for the nuclei with $N = Z$ have been obtained. These values together with the empirical values of the short-range nuclear force energy, $E_{shr} = E_{exp} + E_C$, can be useful in studying the heavy-ion induced reactions such as fusion or fission reactions.
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