In this paper we consider the Extra-solar Planetary Systems recently discovered in our Galaxy as potential sources of gravitational waves. We estimate the frequency and characteristic amplitude of the radiation they emit due to the orbital motion, using the quadrupole formalism. In addition, we check whether the conditions needed for the resonant excitation of the \( f \)- and \( g \)-modes of the central star can be fulfilled. By a Roche-lobe analysis, we show that there could exist systems in which the low-order \( g \)-modes could be excited, although this does not happen in the systems discovered up to now.

1. Introduction

Since 1992, when the first Extra-solar Planetary System (EPS) was discovered, a number of such systems, composed of one or more planets orbiting around a main sequence solar-type star has been observed, and it is conceivable that many others will be found in the near future. Some of the discovered EPS’s are well characterized, since the mass and radius of the main star, the mass of the observed planets and their orbital parameters can be deduced from observations. With this information it is possible to attempt a first estimate of the characteristics of the gravitational radiation emitted by these systems, that, being at a distance of a few tens of parsecs, are very close to us. We shall select a set of EPS’s for which all needed information is available, and compute the gravitational signal emitted by each couple star-planet due to the orbital motion, using the quadrupole formalism. The estimated frequency and amplitude of the waves impinging on Earth will be
compared with the emission of the binary pulsar PSR 1913+16.  

We shall then consider a further mechanism of gravitational emission which may play an interesting role in these systems. It is known that in general relativity the theory of stellar perturbations can be reformulated as a problem of resonant scattering of gravitational waves incident on the potential barrier generated by the spacetime curvature. In this picture, the planet, considered as a pointlike mass, moves on a geodesic of the spacetime metric generated by the star. It emits gravitational waves because of its time dependent quadrupole moment, at frequencies multiple of the keplerian orbital frequency. These waves are partially reflected and partially absorbed by the potential barrier associated to the perturbed star, and if the wave frequency is close enough to one of the eigenfrequencies of oscillation of the star, the scattering is resonant, and the energy emitted by the system can significantly increase. Thus, a mode of the star is likely to be excited by this mechanism if the corresponding frequency is very close to one of the frequencies of the spectral lines emitted by the planet.

In section 4 we shall verify whether the conditions for the resonant excitation of any of the modes of the stars belonging to the selected EPS’s can be fulfilled. In addition, we shall discuss the possibility for a planet to get sufficiently close to a star to excite the lowest order $g$-modes, and possibly the fundamental one, without being disrupted by tidal forces. We will show that for solar type stars the resonant excitation of the low-order $g$-modes may, in principle, be possible.

2. Main characteristics of the extrasolar planetary systems

Among the EPS’s discovered up to now, we have selected those for which the parameters of the central star and of the planets, which we need to estimate the gravitational emission, have been determined with sufficient accuracy. These parameters are tabulated in table 1 and 2. Here and in the following, data will be given with the corresponding errors, when available in the literature. In the first column of table 1 we list the selected EPS’s with the corresponding bibliography, and in column 2 the spectral class of the central stars. Most of them belong to a class similar to that of the Sun, which is a G2 V star.

In column 3, 4 and 5 we tabulate, respectively, the mass of the central star, $M_\star$, its distance from Earth, $D$, - taken from the Extrasolar Planets Catalog - and the quantity $\sqrt{\frac{GM_\star R_\star^3}{D^5}}$, which will be used in section 4. It should be mentioned that the presence of planets has been discovered mainly by using accelerometric techniques, that is to say, by measuring the variations of the radial velocity of the star caused by its gravitational interaction with the planet. These techniques do not allow to determine the mass of the planet, $M_p$, but only the product $M_p \sin i$, where $i$ is the angle between the line of sight and the normal to the orbital plane. In column 6

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* It should be mentioned that the excitation of the modes in solar type stars with a planet and in neutron star-neutron star close binary systems has been studied in the literature by using different approaches based essentially on the deformation induced by the dynamical tides raised by the companion.
Table 1: The spectral class of the central star, its mass, $M_\star$, its distance from Earth, $D$, the factor $\sqrt{\frac{GM_\star}{R_\star^3}}$ (rad s$^{-1}$), and the ratio between the lower limit of the estimated mass of the planet, $M_p \sin i$, and $M_\star$, are tabulated for a selected set of EPS's. Strictly speaking, the bodies orbiting the stars listed below HD 114762 are classified as brown dwarfs rather than as planets.

| Star     | Spectral Cl. | $M_\star(M_\odot)$ | $D$ (pc) | $\sqrt{\frac{GM_\star}{R_\star^3}} \cdot 10^4$ | $M_p \sin i/M_\star$ |
|----------|--------------|---------------------|----------|-----------------------------------------------|----------------------|
| HD 75289 | G0 V         | 1.05                | 28.94    | 5.4                                           | 3.8 $\cdot 10^{-4}$  |
| 51 Peg   | G5 V         | 1.05 ± 0.09         | 15.36    | 5.1 ± 0.7                                     | 4.2 $\cdot 10^{-4}$  |
| υ And    | F8 V         | 1.34 ± 0.12         | 13.47    | 3.7 ± 0.5                                     | 5.3 $\cdot 10^{-4}$  |
|          |              |                     |          |                                               | 1.5 $\cdot 10^{-3}$  |
|          |              |                     |          |                                               | 3.3 $\cdot 10^{-3}$  |
| 55 Cnc   | G8 V         | 0.95 ± 0.10         | 12.53    | 6.6 ± 1.0                                     | 8.8 $\cdot 10^{-4}$  |
| ρ CrB    | G2 V         | 0.89 ± 0.05         | 17.43    | 3.8 ± 0.5                                     | 1.1 $\cdot 10^{-3}$  |
| HD 21027 | G0 V         | 0.92                | 21.29    | 5.0                                           | 1.3 $\cdot 10^{-3}$  |
| 16 Cyg B | G5 V         | 0.96 ± 0.05         | 21.62    | 5.0 ± 0.6                                     | 2.0 $\cdot 10^{-3}$  |
| Gl 876   | M4 V         | 0.3                 | 4.70     | 13                                            | 6.7 $\cdot 10^{-3}$  |
| 47 Uma   | G0 V         | 1.01 ± 0.05         | 14.08    | 4.5 ± 0.5                                     | 2.2 $\cdot 10^{-3}$  |
| 14 Her   | K0 V         | 0.8                 | 18.15    | 8.0                                           | 3.9 $\cdot 10^{-3}$  |
| Gl 86    | K1 V         | 0.79                | 10.91    | 8.0                                           | 4.4 $\cdot 10^{-3}$  |
| τ Boo    | F7 V         | 1.37 ± 0.09         | 15.60    | 4.7 ± 0.6                                     | 3.3 $\cdot 10^{-3}$  |
| HD 168345 | G5 V        | 0.94                | 37.88    | 7.2                                           | 5.1 $\cdot 10^{-3}$  |
| 70 Vir   | G5 V         | 1.01 ± 0.05         | 18.11    | 2.5 ± 0.2                                     | 6.9 $\cdot 10^{-3}$  |
| HD 114762 | F9 V       | 0.75 ± 0.15         | 40.57    | 4.0 ± 1.0                                     | 1.3 $\cdot 10^{-2}$  |
| HD 110833 | K3 V       | 0.75                | 17       | 8.2                                           | 2.2 $\cdot 10^{-2}$  |
| HD 112758 | K0 V       | 0.8                 | 16.5     | 8.0                                           | 4.2 $\cdot 10^{-2}$  |
| HD 29587 | G2 V         | 1.0                 | 45       | 6.1                                           | 3.8 $\cdot 10^{-2}$  |
| HD 283750 | K2 V       | 0.75                | 16.5     | 8.4                                           | 6.4 $\cdot 10^{-2}$  |
| HD 89707 | G1 V         | 1.2                 | 25       | 6.1                                           | 5.0 $\cdot 10^{-2}$  |
| HD 217580 | K4 V       | 0.7                 | 18       | 9.1                                           | 8.2 $\cdot 10^{-2}$  |
we list the values $M_p \sin i$ for each planet, normalized to the mass of the central star. From the data of table 1 we see that the closest system is at 4.70 pc, the farthest at 45 pc, whereas the mass of the central star ranges within $[0.3, 1.37] M_\odot$.

In table 2 we tabulate the orbital parameters of the planets orbiting around the stars listed in table 1, i.e. the orbital period $P$, the semimajor axis $a$, and the eccentricity $e$. From the data of column 2 we see that in some cases the period is very short, indicating that the planet gets very close to the central star. For instance, it can be as short as 1.79 days for the planet orbiting around HD 283750.

### 3. Gravitational wave emission due to the orbital motion

We shall first evaluate the amount of gravitational radiation that each couple star-planet emits because of the orbital motion. Since we are not interested in the detailed form of the signal, the quadrupole formalism is sufficient to derive the required information. It is known that two pointlike masses revolving around their common center of mass emit gravitational energy because of their time varying quadrupole moment. To describe their motion it is convenient to choose a coordinate
system with the origin located at the center of mass of the system, the orbital plane coincident with the x-y plane and the x-axis oriented along the relative position vector, \( \mathbf{X} \equiv \mathbf{x}_* - \mathbf{x}_p \), when the planet is at the periastron. The vector \( \mathbf{X} \) describes, in general, an ellipse with semi-major axis \( a \) and eccentricity \( e \), and its evolution is described by the parametric equations:

\[
\rho = a(1 - e \cos u)
\]

\[
\alpha = 2 \arctan \left( \frac{(1 + e) \tan \frac{u}{2}}{1 - e} \right),
\]

\( \rho = |\mathbf{X}|, \alpha \) is the angle between \( \mathbf{X} \) and the x-axis, and \( u \) is the eccentric anomaly, related to time by the Kepler equation:

\[
\omega_k t = u - e \sin u,
\]

where \( \omega_k \) is the keplerian orbital frequency

\[
\omega_k = \frac{2 \pi}{P} = \left( \frac{GM}{a^3} \right)^{1/2},
\]

and \( M = M_* + M_p \) is the total mass of the system. From the reduced quadrupole moment \( Q_{kl} \), given by:

\[
Q_{kl} = \mu \left( X^k X^l - \frac{1}{3} \delta^k_l |\mathbf{X}|^2 \right)
\]

where \( \mu = M_* M_p / M \) is the reduced mass, it is straightforward to compute the amplitudes of the metric perturbation, projected on a sphere at distance \( r \) from the source:

\[
h_{ij}(t, r, \theta, \phi) = \frac{2Gc^2M\mu}{c^4ra(1 - e \cos u)^3} \times \left\{ (e^2 - 1)(1 + \cos^2 \theta)(1 - 2 \cos^2 \phi) \right. \\
+ \cos^2 u(e \cos u - 2) \left[ (1 - e^2)(\cos^2 \phi + \cos^2 \theta \cos^2 \phi - \cos^2 \theta) \right. \\
+ \cos^2 \phi + \cos^2 \theta \cos^2 \phi - 1 \right] + e \cos u(\cos^2 \phi + \cos^2 \theta \cos^2 \phi - 1) \\
+ 2\sqrt{1 - e^2 \sin \phi \cos \phi(1 + \cos^2 \theta)} \sin u(e \cos^2 u - 2 \cos u + e) \left\},
\]

\[
h_{\theta\theta}(u, r, \theta, \phi) = \frac{4G^2M\mu \cos \theta}{c^4ra(1 - e \cos u)^3} \times \left\{ \sin \phi \cos \phi \left[ (e^2 - 2) \cos^2 u(e \cos u - 2) - e \cos u + 2(e^2 - 1) \right] \\
+ (2 \cos^2 \phi - 1) \sqrt{1 - e^2 \sin \phi(e \cos^2 u - 2 \cos u + e)} \right\}.
\]
If the orbit is circular the radiation is emitted at twice the orbital frequency ($\omega_{GW} = 2\omega_k$). By Fourier-transforming the wave amplitudes, and by averaging over the solid angle, it is easy to show that if the orbit is eccentric, waves will be emitted at frequencies multiple of $\omega_k$, and the number of equally spaced spectral lines will increase with the eccentricity. The average energy flux $F_n$ relative to the $n$–th harmonic is given by:

$$F_n = \frac{c^3(n\omega_k)^2}{8\pi G} \left[ \langle \tilde{h}^{(n)}_{\theta\theta} \rangle^2 + \langle \tilde{h}^{(n)}_{\theta\phi} \rangle^2 \right]$$

(8)

where $\langle \tilde{h}^{(n)}_{\theta\theta} \rangle^2$ and $\langle \tilde{h}^{(n)}_{\theta\phi} \rangle^2$, are the square of the $n$–th Fourier component of the two independent polarizations, averaged over the solid angle

$$\langle \tilde{h}^{(n)}_{\theta\theta} \rangle^2 = \frac{1}{4\pi} \int d\Omega |h_{\theta\theta}(n\omega_k, r, \theta, \phi)|^2,$$

$$\langle \tilde{h}^{(n)}_{\theta\phi} \rangle^2 = \frac{1}{4\pi} \int d\Omega |h_{\theta\phi}(n\omega_k, r, \theta, \phi)|^2.$$

An estimate of the characteristic amplitude of the gravitational waves emitted by a planetary system can now be given by using the well known formula

$$h_c(n\omega_k, r) = \sqrt{\frac{2}{3}} \left[ \langle \tilde{h}^{(n)}_{\theta\theta} \rangle^2 + \langle \tilde{h}^{(n)}_{\theta\phi} \rangle^2 \right]^{1/2},$$

(9)

where the factor $\sqrt{2/3}$ takes into account the average over orientation. The aforementioned procedure, which is equivalent to that introduced by Peters and Mathews, has been applied to compute the characteristic wave amplitude $h_c$ impinging on Earth, emitted by our set of EPS’s and, for comparison, by the binary system PSR 1913+16. Some results are shown in figure 1, where $h_c$ is plotted as a function of the harmonic index $n$. It should be reminded that PSR 1913+16 is composed of two very compact stars with masses $m_1 = 1.4411 \ M_\odot$ and $m_2 = 1.3874 \ M_\odot$, revolving around their center of mass with an eccentric orbit ($e = 0.617139$), semi-major axis $a = 1.9490 \cdot 10^{12}$ cm, and keplerian frequency $\nu_k = 3.583 \cdot 10^{-5} \ Hz$. The binary system is at a distance $D = 5 \ kpc$ from Earth. In the upper panel of figure 1, we show two systems in which the planet moves in a nearly circular orbit around the central star, HD 283750 ($e = 0.02$) and $\tau$ Boo ($e = 0.018$). As expected, the emission is concentrated at twice the keplerian frequency, and the amplitude is comparable to the maximum wave amplitude reached by PSR 1913+16, which is shown for comparison at the bottom of the figure. However, the frequency is about ten times lower than that of the binary pulsar. In the lower panel we show the characteristic emission of two EPS’s with high eccentricity, 16 Cyg B ($e = 0.634$) and HD 89707 ($e = 0.93$). In this case the gravitational emission is spread over a larger set of frequencies, all multiple of $\nu_k$, as for PSR 1913+16.

In table 3 we tabulate the keplerian frequency, the maximum characteristic amplitude on Earth, and the corresponding frequency, for each EPS. In the last row the same data are listed for PSR 1913+16. For most systems the maximum emission frequency appears to be extremely low, in general smaller than $10^{-6} \ Hz$ except the case of HD 283750, for which $\nu_{max} = 1.3 \cdot 10^{-5} \ Hz$. 


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Figure 1: The characteristic amplitude computed from eqs. (3) is plotted versus the harmonic index $n$ for four selected EPS’s and for the binary system PSR 1913+16. For the two systems in the upper panel, HD 283750 and $\tau$ Boo, the planet moves on a nearly circular orbit, with $e = 0.02$ and $e = 0.018$, respectively. In this case the emission is concentrated at twice the keplerian frequency, and the amplitude is comparable to the maximum wave amplitude emitted by the binary pulsar PSR 1913+16. In the lower part of the figure $h_c$ is plotted for two systems with high eccentricity: 16 Cyg B ($e = 0.634$) and HD 89707 ($e = 0.93$), and for PSR 1913+16.
Table 3: The maximum characteristic amplitude of the waves emitted by the selected set of EPS's and by the binary system PSR 1913+16, due to their orbital motion.

| Star     | $\nu_k$ (Hz) | $h_{e_{\max}}$ | $\nu_{\max}$ (Hz) |
|----------|--------------|-----------------|-------------------|
| HD 75289 | $3.3 \cdot 10^{-6}$ | $2.1 \cdot 10^{-25}$ | $6.0 \cdot 10^{-6}$ |
| 51 Peg   | $2.8 \cdot 10^{-6}$ | $4.0 \cdot 10^{-25}$ | $5.5 \cdot 10^{-6}$ |
| $\upsilon$ And | $2.5 \cdot 10^{-6}$ | $7.9 \cdot 10^{-26}$ | $5.0 \cdot 10^{-6}$ |
| 55 Cnc   | $7.9 \cdot 10^{-7}$ | $3.8 \cdot 10^{-25}$ | $1.6 \cdot 10^{-6}$ |
| $\rho$ CrB | $2.9 \cdot 10^{-7}$ | $1.5 \cdot 10^{-25}$ | $5.8 \cdot 10^{-7}$ |
| HD 210277 | $2.6 \cdot 10^{-8}$ | $1.9 \cdot 10^{-26}$ | $7.9 \cdot 10^{-8}$ |
| 16 Cyg B | $1.4 \cdot 10^{-8}$ | $1.6 \cdot 10^{-26}$ | $5.8 \cdot 10^{-8}$ |
| Gl 876   | $1.9 \cdot 10^{-7}$ | $3.5 \cdot 10^{-25}$ | $3.8 \cdot 10^{-7}$ |
| 47 Uma   | $1.1 \cdot 10^{-8}$ | $5.3 \cdot 10^{-26}$ | $2.1 \cdot 10^{-8}$ |
| 14 Her   | $7.1 \cdot 10^{-9}$ | $2.7 \cdot 10^{-26}$ | $1.4 \cdot 10^{-8}$ |
| Gl 86    | $7.3 \cdot 10^{-7}$ | $1.6 \cdot 10^{-24}$ | $1.5 \cdot 10^{-6}$ |
| $\tau$ Boo | $3.5 \cdot 10^{-6}$ | $6.6 \cdot 10^{-24}$ | $7.0 \cdot 10^{-6}$ |
| HD 168443 | $2.0 \cdot 10^{-7}$ | $1.6 \cdot 10^{-25}$ | $6.0 \cdot 10^{-7}$ |
| 70 Vir   | $9.9 \cdot 10^{-8}$ | $3.6 \cdot 10^{-25}$ | $2.0 \cdot 10^{-7}$ |
| HD 114762 | $1.4 \cdot 10^{-7}$ | $2.0 \cdot 10^{-25}$ | $2.7 \cdot 10^{-7}$ |
| HD 110833 | $4.3 \cdot 10^{-8}$ | $2.6 \cdot 10^{-25}$ | $2.1 \cdot 10^{-7}$ |
| HD 112758 | $1.1 \cdot 10^{-7}$ | $3.0 \cdot 10^{-24}$ | $2.2 \cdot 10^{-7}$ |
| HD 29587 | $1.0 \cdot 10^{-8}$ | $2.3 \cdot 10^{-25}$ | $2.0 \cdot 10^{-8}$ |
| HD 283750 | $6.5 \cdot 10^{-6}$ | $3.7 \cdot 10^{-23}$ | $1.3 \cdot 10^{-5}$ |
| HD 89707 | $3.9 \cdot 10^{-8}$ | $8.4 \cdot 10^{-25}$ | $3.9 \cdot 10^{-8}$ |
| HD 217580 | $2.5 \cdot 10^{-8}$ | $8.5 \cdot 10^{-25}$ | $7.6 \cdot 10^{-8}$ |
| PSR 1913+16 | $3.6 \cdot 10^{-5}$ | $1.0 \cdot 10^{-23}$ | $1.4 \cdot 10^{-4}$ |
4. Excitation of the modes of a star by a resonant scattering process

We shall now consider another mechanism through which gravitational waves can be emitted by a system composed of a star and a planet. Since the masses of planets are much smaller than those of stars, in what follows we shall consider the planet as a pointlike mass which perturbs the central star, assumed to have a structure, and to possess a set of eigenmodes of oscillation, the quasi-normal modes, that are associated to the emission of gravitational radiation. We want to establish whether these modes can be excited by a planet. This question can be answered if we formulate the problem in terms of scattering of gravitational waves by the potential barrier generated by the perturbed star, as described in the introduction: if the radiation emitted by the planet because of its time-varying, orbital quadrupole moment, contains a component at a frequency close to that of a quasi-normal mode of the star, the scattering will be resonant and much energy will be emitted. This has been shown in a recent paper \[16\], where the equations describing a compact star perturbed by a massive object moving in an open orbit around it, have been numerically integrated. It turns out that a pointlike mass can excite both the fundamental and the \(p\)-modes, depending on how close it can get to the star, and that a large amount of energy can be emitted at the frequencies of these modes. The case of a mass orbiting in a circular orbit around a compact star has been considered by Kojima \[17\]. By integrating the perturbed equations he showed that a sharp resonance occurs if the frequency of the wave emitted by the planet equates the frequency of the fundamental mode of the star. In this case, he showed that the characteristic wave amplitude can be up to 100 times larger than that evaluated by the quadrupole formula; however, he did not consider the excitation of other modes beyond the fundamental one. It should be stressed that the equations governing the perturbations of a star excited by a pointlike mass have never been integrated for solar type stars. However, the results obtained for compact stars suggest that resonant scattering processes may enhance the gravitational emission also for non compact stars. In this view, it interesting to check whether the conditions of resonant excitation can be fulfilled in planetary systems, and in particular if this is the case for any of the EPS’s listed in table 1.

We shall first verify whether the maximum quadrupole emission frequency of the planet of our set of EPS’s, \(\nu_{\text{max}}\) (table 3, column 4) is close enough to any of the frequencies of the modes of the central star.

The oscillation frequencies of a star can be computed if we make an assumption on its internal structure, i.e. on the equation of state prevailing in the interior. We shall consider, as an example, a very simple, polytropic model of star, with polytropic index \(n = 2\). The oscillation frequencies of newtonian polytropic stars are known to scale with the mean density of the star \[18\]. In table 4 we tabulate the dimensionless eigenfrequencies of the modes, \(\nu_{\text{mode}}/(GM_*/R_\star^2)^{1/2}\), for the chosen value of \(n\). In order to explicitly compute the frequency of a given mode for a given star, the entries of table 4 have to be multiplied by the entries of table 1, column 5. For instance, for the star HD 89707 the frequency of the \(f\)-mode is
Table 4: The dimensionless eigenfrequencies of the modes of a polytropic stars are tabulated for the polytropic index \( n = 2 \).

| mode | \( \nu_{\text{mode}}/(GM_*/R_*^3)^{1/2} \) | mode | \( \nu_{\text{mode}}/(GM_*/R_*^3)^{1/2} \) |
|------|--------------------------------|------|--------------------------------|
| \( p_{10} \) | 2.58 | \( f \) | 0.28 |
| \( p_9 \) | 2.37 | \( g_1 \) | 0.119 |
| \( p_8 \) | 2.15 | \( g_2 \) | 0.087 |
| \( p_7 \) | 1.93 | \( g_3 \) | 0.068 |
| \( p_6 \) | 1.70 | \( g_4 \) | 0.056 |
| \( p_5 \) | 1.47 | \( g_5 \) | 0.048 |
| \( p_4 \) | 1.24 | \( g_6 \) | 0.042 |
| \( p_3 \) | 1.01 | \( g_7 \) | 0.037 |
| \( p_2 \) | 0.78 | \( g_8 \) | 0.033 |
| \( p_1 \) | 0.54 | \( g_9 \) | 0.030 |
| \( g_{10} \) | 0.028 |

given by \( \nu = 0.28 \times 5.9 \cdot 10^{-4} = 1.7 \cdot 10^{-4} \) Hz. This number has to be compared with \( \nu_{\text{max}} = 2.2 \cdot 10^{-6} \) Hz, given in table 3 for the same star, which is much smaller. This means that the planet cannot excite the \( f \)-mode of the central star. By repeating the same calculation for all systems and all modes, we find that it is unlikely that the stellar modes are excited in the EPS’s we consider. This is because the angular velocities reached by the planets are too low, and consequently the frequencies of the radiation they emit are lower than those of the \( f \)-mode or of the lowest-order \( g \)-modes of the star. Higher order \( g \)-modes could be excited, but the efficiency in producing gravitational radiation by this process would be too low.

However, it should be reminded that according to recent theories on the evolution of planetary systems, there could exist planets moving on orbits even closer to the central star than those observed until now. In view of this possibility, it is interesting to investigate whether it is possible for a planet to approach a star at such a short distance that its angular velocity is high enough to excite the \( f \)-mode or the lowest-order \( g \)-modes, without being disrupted by tidal forces. In addition, we shall impose that the star does not accrete matter onto the planet. These conditions are equivalent to impose that neither the planet nor the star overflow their Roche lobe. We would like to stress that this limit takes into account only the gravitational interaction between the planet and the star. There may exist other processes that would prevent the planet to reach the innermost orbit allowed by the Roche-lobe analysis. However, as far as we know, the present knowledge on the formation of planets and on their possible migration toward the central star, still does not allow to firmly establish what is the minimum distance from a star at which a planet can safely sit.

We shall assume, for simplicity, that the planet flies on a circular orbit with keplerian angular velocity \( \omega_k \), and consequently emits radiation at the frequency \( \omega^{GW} = 2\omega_k \). Let us indicate the dimensionless frequency tabulated in table 4
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multiplied by $2\pi$ as $k_{\text{mode}}$: for instance, $k_{g1} = 0.750$. The condition that a mode of the star is excited by the resonant scattering of the waves emitted by the planet, $\omega_{\text{mode}} = 2\omega_k$, therefore becomes

$$k_{\text{mode}} \cdot \sqrt{\frac{GM*}{R_*^2}}^{1/2} = 2 \sqrt{\frac{G (M_p + M_*)}{a^3}}^{1/2},$$

(10)

which can be written as

$$a = \left( \frac{4}{k_{\text{mode}}^2} \cdot \left( 1 + \frac{M_p}{M_*} \right) \right)^{1/3} R_*.$$

(11)

This equation gives the value the separation star-planet must have for a given mode to be excited.

The further condition that the planet lies inside its Roche lobe, $R_p < R_{RL}$, is equivalent to the following constraint on its density:

$$\rho_p > \rho_{RL},$$

where $\rho_{RL} = \frac{4}{3\pi R_{RL}^3}$ is the critical density. If we now introduce the dimensionless quantity

$$R_{RL} = a \bar{R}_{RL}$$

(12)

i.e. we set to 1 the radius of the orbit, by the use of eq. (11) we find

$$\rho_{RL} = \frac{M_p}{\frac{4}{3}\pi R_*^3} \left( \frac{4}{k_{\text{mode}}^2} \cdot \left( 1 + \frac{M_p}{M_*} \right) \right) \bar{R}_{RL}^3,$$

(13)

which can be rewritten as

$$\frac{\rho_{RL}}{\rho_*} = k_{\text{mode}}^2 \cdot \frac{M_p/M_*}{4 (1 + M_p/M_* \bar{R}_{RL})},$$

(14)

where $\rho_*$ is the mean density of the central star. Thus, a planet can excite a mode of the star corresponding to an assigned $k_{\text{mode}}$, without overflowing its Roche lobe, if the ratio between its mean density and that of the central star exceeds the critical ratio (14).

We have computed the dimensionless radius of the Roche lobe $\bar{R}_{RL}$ and the corresponding critical ratio (14), for assigned values of the ratio $M_p/M_*$, and the results are shown in table 5, which has to be read as follows. Suppose that the ratio between the mass of a planet and that of the central star is $M_p/M_* = 10^{-3}$. The frequency of the quadrupole radiation emitted by the planet will coincide with that of the first $g$-mode of the star, and the planet will not be disrupted, only if its density is higher that $1.49 \rho_*$. We have also checked whether the star overflows its Roche lobe and accretes matter onto the planet, and excluded from table 5 the corresponding cases. It should be reminded that the ratio between the mean density
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Table 5: Ratios of the critical density of the planet $\rho_{RL}$ to the central star density $\rho_*$ for different values of the mass ratio, $M_p/M_*$, and different oscillation modes of a polytropic star with $n = 2$ (see text).

| $M_p/M_*$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $f$       | 7.89      | 7.95      | 8.04      | -         | -         | -         |
| $g_1$     | 1.43      | 1.44      | 1.45      | 1.49      | 1.58      | 1.83      |
| $g_2$     | 0.75      | 0.76      | 0.77      | 0.79      | 0.83      | 0.96      |
| $g_3$     | 0.47      | 0.47      | 0.47      | 0.49      | 0.52      | 0.60      |
| $g_4$     | 0.32      | 0.32      | 0.32      | 0.33      | 0.35      | 0.41      |
| $g_5$     | 0.23      | 0.23      | 0.23      | 0.24      | 0.26      | 0.30      |
| $g_6$     | 0.17      | 0.18      | 0.18      | 0.18      | 0.19      | 0.22      |
| $g_7$     | 0.14      | 0.14      | 0.14      | 0.14      | 0.15      | 0.18      |
| $g_8$     | 0.11      | 0.11      | 0.11      | 0.12      | 0.12      | 0.14      |
| $g_9$     | 0.09      | 0.09      | 0.09      | 0.10      | 0.10      | 0.12      |
| $g_{10}$  | 0.08      | 0.08      | 0.08      | 0.08      | 0.09      | 0.10      |

of the planets of the solar system and that of the Sun is 3.9 for Mercury and the Earth, 3.7 for Venus, 2.8 for Mars, 0.9 for Jupiter, etc. A comparison of these values with the data of table 5 suggests that in principle, there can exist EPS’s in which the first $g$-modes could be excited by a resonant scattering process. For instance, a planet like the Earth could approach a polytropic star $(n = 2)$ with the mass of the sun at a distance close enough to excite the mode $g_1$ without being disrupted by the tidal interaction, whereas a planet like Jupiter could only be at a distance good to excite the second $g$-mode.

By the Roche-lobe analysis we can also deduce another interesting information. Suppose that a planetary system made of a star with the mass of the Sun and a planet in circular orbit, is located at a fiducial distance $D = 10$ pc. We do not make any assumption on the internal structure of the star, but assign the values of the mass and of the mean density of the planet. In particular we consider four planets, with mass and density equal to that of Mercury, of the Earth, of Jupiter and one with a mass equal to 13 times the mass of Jupiter and similar mean density. We want to answer the following questions:

- what is the minimum radius of the orbit?
- what is the corresponding quadrupole emission frequency $\nu^{GW} = 2\nu_k$?
- what is the corresponding characteristic amplitude on Earth?.

The answer is in table 6, where the required data are given for the four planets.

From table 6 we see that planets like Jupiter or bigger could emit quadrupole radiation at a frequency in the bandwidth of space interferometers, and with an amplitude which could be even ten times bigger than that emitted by the binary
Gravitational waves emitted by extrasolar planetary systems...

Table 6: We tabulate the minimum radius of the circular orbit, $a_{\text{min}}$, the quadrupole emission frequency, $\nu_{GW}$, and the characteristic amplitude of the corresponding wave emitted by four planets orbiting around a star with the mass of the Sun. The mass of the planets is equal to that of Mercury, Earth, Jupiter and 13 times the mass of Jupiter, respectively, and the same density of those planets. These data are obtained by imposing that the planets lie inside their Roche lobe (see text), and that the system is located at $D = 10$ pc from Earth.

| Planet    | $a_{\text{min}}$ (cm) | $\nu_{GW} = 2\nu_k$ | $h_c$     |
|-----------|------------------------|----------------------|-----------|
| Mercury   | $9.63 \cdot 10^{10}$   | $1.23 \cdot 10^{-4}$ | $1.8 \cdot 10^{-27}$ |
| Earth     | $1.55 \cdot 10^{11}$   | $6.01 \cdot 10^{-5}$ | $2.0 \cdot 10^{-26}$ |
| Jupiter   | $9.71 \cdot 10^{10}$   | $1.21 \cdot 10^{-4}$ | $1.0 \cdot 10^{-23}$ |
| 13$\times$Jupiter | $9.93 \cdot 10^{10}$   | $1.18 \cdot 10^{-4}$ | $1.3 \cdot 10^{-22}$ |

pulsar PSR 1913+16. In addition, if the quadrupole radiation resonates with a mode of the star, the amount of emitted energy could be even larger.

5. Concluding Remarks

In this paper we have studied some of the EPS’s discovered up to now, with the aim of characterizing their gravitational emission.

As far as the quadrupole emission is concerned, we have shown that among those systems there is one, HD 283750, that emits a signal with a maximum amplitude on Earth even higher than that of the binary pulsar PSR 1913+16, but at a frequency which is six times smaller. Moreover, since the orbit of the planet is nearly circular, the radiation is almost entirely emitted at twice the keplerian frequency, whereas for systems with high eccentricity, as the binary pulsar, the radiation is emitted also at higher multiples of $\omega_k$.

A further mechanism of gravitational emission is the resonant scattering of the radiation emitted by the planet on the potential barrier generated by the perturbed central star. For compact stars, this mechanism has proved to be much more efficient than the quadrupole emission, thus it was interesting to check whether the conditions needed for the resonant excitation of the $f$- and $g$-modes can be fulfilled in any of the observed EPS’s. Although there exist planets that have a very short orbital period (up to 1.79 days), the radiation they emit by the quadrupole mechanism has a frequency which is, at best, as high as $10^{-5}\ Hz$, a value which is too low to be resonant with the $f$-mode or the lowest $g$-modes frequencies of a solar type star.

However, since many new EPS’s will certainly be discovered in the near future, we wanted to understand whether more favourable conditions for the emission of gravitational waves may occur in these systems. In particular, since a higher emission frequency would be desirable both for the excitation of the lowest order $g$-modes of the central star, and for a possible detection by space interferometers, we have investigated how close can a planet move on a circular orbit around a star
Gravitational waves emitted by extrasolar planetary systems... without being tidally disrupted, and without accreting matter from the star.

It should be stressed that the limits we establish by the Roche-lobe analysis do not take into account other processes which may destabilize the orbit of the planets, whose study is beyond the scopes of this paper. Having this caveat in mind, we have established that, in principle, there could exist systems in which the excitability conditions of the lowest order $g$-modes could be fulfilled. In this case radiation would be emitted at frequencies of the order of $\sim 10^{-4}$ Hz. The amplitude of the emitted gravitational signals depends on the ratio $M_p/M_*$, and for a planet like Jupiter or bigger, and located at a distance of 10 pc, it would range between $10^{-21} - 10^{-22}$. The emitted radiation could be even larger, if the system is in a condition of resonant excitation of a mode of the star, and we plan to investigate this problem in detail in a subsequent paper.

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