Optimum irrigation water allocation and crop distribution using combined Pareto multi-objective differential evolution

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Optimum irrigation water allocation and crop distribution using combined Pareto multi-objective differential evolution

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Abstract: This paper presents the application of a new evolutionary algorithm technique called combined Pareto multi-objective differential evolution (CPMDE) to optimize irrigation water allocation and crop distribution under limited water availability with three different crops (maize, potatoes and groundnut) planted on a 100 ha farmland at Vaalharts irrigation scheme, South Africa. The algorithm combines methods of Pareto ranking and Pareto dominance selections to implement a novel selection scheme at each generation. The ability of CPMDE in solving unconstrained, constrained and real-world optimization problems was demonstrated. The two objectives of the model are to maximize total crop net benefit (NB) over a planting season while minimizing total irrigation water allocation. A set of non-dominated solutions with the high NBs at lower irrigation water allocation for three crop types was obtained, and compromise programming approach was used in evaluating the most favourable solution. The best solution shows that maize produced the highest crop yield under limited water allocation in the study area.

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PUBLIC INTEREST STATEMENT
The world is facing great challenges of both water scarcity and food supply shortages. Water scarcity is caused by global population increase and climate change. The available freshwater is contested for among agricultural, domestic and industrial users. Currently, the agricultural sector (via irrigation) uses more than 60% of the global freshwater supply because the agricultural sector is the primary producer of food.

Fresh water supply is essential in order to achieve optimal crop growth, which is necessary in order to achieve maximum yield. Water shortages in crop production will negatively affect crop growth, which in turn will affect harvests, which in turn will affect food supplies. This study therefore optimizes irrigation water allocation and crop distribution under limited water availability considering three different crops planted on a farmland at Vaalharts irrigation scheme (VIS), South Africa, using a new evolutionary algorithm technique called combined Pareto multi-objective differential evolution (CPMDE).
Comparing this result with that of a previous study which adopted a multi-objective optimization algorithm called multi-objective differential evolution algorithm, CPMDE is a good and robust alternative algorithm suitable for resolving crop distribution under limited water availability.

**Subjects:** Environment & Agriculture; Earth Sciences; Environmental Studies & Management; Mathematics & Statistics

**Keywords:** Constraints; crop distribution; differential evolution; evolutionary algorithms; multi-objective optimization; irrigation water allocation

1. Introduction

Water is an important but scarce natural resource because it sustains the lives of living organisms. The scarcity is caused by population growth and changes in climate (Nguyen, Maier, Dandy, & Ascough, 2016). Climate change effects include high temperature, variability of rainfall and drought which dries up both surface and groundwater resources (Mishra & Dehuri, 2011). As the global population increases, the demands for food, fibre and all other needs tend to shrink the available water resources (Fanuel, Mushi, & Kajunguri, 2018). The world population is projected to hit 9.5 billion by year 2050, and this will have a significant effect on food security (Singh, 2014).

The sustainable management of water resource is very important particularly in the arid and semi-arid regions due to low annual average rainfall experienced in such regions (Belaqziz et al., 2014). For instance, South Africa is a country characterized by low average annual rainfall and falls within the semi-arid region (USDA, 2013). The current water demand is greater than the available supply; hence, it is termed a water-stressed country. The available fresh water is used for domestic, irrigation, industrial, recreation and hydropower purposes (Adewumi & Chetty, 2017).

Among the competing users of freshwater in South Africa, irrigation is the largest single user because it accounts for almost 60% of the annual available consumptive water in the country (USDA, 2013). As a result, it is important to optimize the water allocation for agricultural production, so that maximum crop productivity under deficit irrigation can be obtained. The available water must be managed in a way that all wastages due to excessive irrigation are avoided (Olofintoye, 2015). Therefore, policies and strategies that seek to minimize irrigation water allocation and maximize crop yield must be developed in the face of the water-stress challenges experienced by the agricultural sector in the country.

The objective of global optimization techniques in irrigation planning and crop production is to achieve maximum crop yield under limited water allocation within an irrigated area (Fanuel et al., 2018). This involves the use of computer modelling techniques to find a near-optimal solution of the global optimization problem. Optimization methods can be classified into two categories: (1) classical methods and (2) evolutionary or soft computing methods (Peralta, Forghani, & Fayad, 2014). Basic examples of classical methods as outlined by Whitley (2001) include dynamic programming, linear programming (LP) and non-linear programming. Classical methods sometimes have difficulties with extremely non-linear systems and do not directly yield alternative optimal solutions.

On the other hand, evolutionary methods such as genetic algorithms (GAs), differential evolution (DE) algorithm, genetic programming (GP), evolution strategies (ES) and particle swarm optimization (PSO) can solve optimization problems having non-linear, non-differentiable or even discontinuous functions (Ikudayisi & Adeyemo, 2015). The major difference between the classical optimization techniques and soft computing, according to Azamathulla et al. (2008), is that in classical methods, the optimal solution is derived, whereas in the soft computing techniques, it is searched from a randomly generated population of possible solutions. Evolutionary algorithms
(EAs) go for discovery of the optima from a population of solutions rather than from a single point. These gimmicks make them suitable for solving complex design issues (Reddy & Kumar, 2007).

In optimizing irrigation water allocation, the objectives are conflicting in nature with many objectives that must be satisfied simultaneously. Therefore, irrigation water allocation is often handled in multi-objective framework to facilitate the development of suitable and sustainable strategies for practical implementation (Adekunmbi & Olugbara, 2015; Adeyemo, Otieno, & Ndiritu, 2008; Azamathulla et al., 2008; Belaqziz et al., 2013; Casadesús, Mata, Marsal, & Girona, 2012; Haq & Anwar, 2014; Jiang, Xu, Huang, Huo, & Huang, 2016; Kamble, Irmak, Hubbard, & Gowda, 2013; Mathur, Sharma, & Pawde, 2009; Nguyen et al., 2016; Parsinejad, Yazdi, Araghinejad, Nejadhashemi, & Tabrizi, 2013; Wardlaw & Bhaktikul, 2004b, 2004a).

Azamathulla et al. (2008) conducted a study which involves the development and comparison of two models: a GA and LP. These two models were adopted to find the optimum crop pattern in a farm in Madhya Pradesh, India. The aim of the study was to reduce the amount of wasted water due to over-irrigation and surface runoff. This allows for productive irrigation on the farmland. Subjecting these models to a comparative analysis by adopting both GA and LP techniques to solve them, the GA model gives better yields than the LP model. However, GA has proved to be capable of handling diverse irrigation scheduling and water allocation problems effectively. It produced a suitable outcome by generating a population of optimal solutions along the Pareto front.

Sarker and Ray (2009) adopted methodologies of a proposed evolutionary multi-objective constrained algorithm (EMOA) and an existing method of multi-objective GA for resolving multi-objective crop planning models. The new EMOA found quality non-dominated solutions which represent adoptable crop planning policies for application in real-world situations. It was concluded that methods of EMOAs are adoptable as suitable alternatives for solving multi-objective crop planning models.

A stochastic multi-objective EA named multi-objective differential evolution algorithm (MDEA) was developed by Otieno and Adeyemo (2011) and adopted to solve a multi-objective cropping pattern problem on a farmland in Vaalharts irrigation scheme (VIS) in South Africa. The study seeks to maximize the total net benefit (NB) in South African rand, ZAR, as well as minimize irrigation water use. Four different crops were considered on a 77.1 ha farmland. The amount of irrigation supplied to the study area is 9,140 m$^3$ per ha/annum and the algorithm produced a set of non-dominated solutions that converge to Pareto optimal front. MDEA proved to be a good optimizer for multi-objective cropping pattern problems and for generating maximum agricultural output for the farmers in the area with the constraints of land and water availabilities.

In a study conducted by Adekanmbi and Olugbara (2015), a multi-objective optimization of mixed cropping planning was solved. The adopted technique in this study is an EA called generalized differential evolution 3 (GDE3). GDE3 is a technique which modifies the selection rule of the basic DE algorithm. The objectives of the study are to maximize net profit, maximize crop production and minimize planting area. The constraints of the optimization problem include economic demand of crops, land resource, labour cost and investment in crop production. Data retrieved from South African grain information service and the South African abstract of agricultural statistics were used in the optimization problem. The performance of GDE3 was evaluated by adopting NSGA-II to solve the same problem. About 207 crops are grown in South Africa, but the authors grouped these crops into 8 categories. The land for farming is grouped into single-, double- and triple-cropped lands with values as 8, 14 and 3, respectively. The result of the optimization shows that both GDE3 and NSGA-II performed very well, but GDE3 produced a better performance than NSGA-II.
Recently, a new EA technique called combined Pareto multi-objective differential evolution (CPMDE) algorithm was developed by Olofintoye, Adeyemo, and Otieno (2014). The ability of CPMDE in solving unconstrained and constrained optimization problems was demonstrated, and competitive results obtained from the benchmark and application of CPMDE suggest that it is a good alternative for solving real multi-objective optimization problems. This new algorithm was evaluated on tuneable problems by Adeyemo and Olofintoye (2014), and the only study where CPMDE algorithm had been used is the multi-objective optimization of an operating industrial wastewater treatment plant by Enitan, Adeyemo, Olofintoye, Bux, and Swalaha (2014).

The main aim of this paper is to demonstrate the application of this new technique called CPMDE in optimizing irrigation water allocation and crop distribution under limited water availability while planting maize, potatoes and groundnut on a 100 ha farmland at VIS, South Africa. The objectives of the model were formulated to maximize total NB of crops while minimizing irrigation water use. The results of this study will be compared with that of a previous study by Otieno and Adeyemo (2011) which adopted a multi-objective optimization algorithm called MDEA for the same kind of problem in the same study area in order to further prove the efficiency and robustness of CPMDE technique.

The structure of this paper is as follows: Section 2 explores the materials and methods, section 3 presents the results, section 4 presents the discussion, while section 5 contains the conclusion of the study.

2. Materials and methods

2.1. Study area

VIS in South Africa is the study area. VIS is located in the Northern Cape Province of South Africa with latitude \(−28°00′60.00″\) S and longitude \(24°42′59.99″\) E (Ellington, 2003). The scheme is located on a vast land area of about 370 km\(^2\) and majorly used for irrigation. The scheme is supplied with water abstracted from the Vaal River at the Vaal Harts weir about 8 km upstream of Warrenton (Ojo, 2013). VIS has an irregular rainfall of about 442 mm per year (VIS 2013). During summer, the average rainfall is between 9.1 and 9.6 mm per day, while during winter season, almost no rainfall events occur (Annandale, Stirzaker, Singels, van der Laan, & Laker, 2011). Department of Water Affairs (DWA) allocates water annually at the rate of 9,140 m\(^3\)/ha to the scheme, and it is charged at R8.77 cents per cubic metre of water use (Grove, 2011). Figure 1 shows the location of VIS within South Africa, while Figure 2 shows the geographical location of the study area.

2.2. Combined Pareto multi-objective differential evolution

CPMDE is a newly formulated evolutionary multi-objective algorithm which was developed by Olofintoye et al. (2014). CPMDE adopts a combination of Pareto ranking and Pareto dominance selection methods for implementing a new generational selection scheme. The new scheme produces a methodical way to deal with elitism control of the population which creates short solution vectors that are suitable for local search and long vectors suitable for global search. CPMDE is able to adaptively adjust exploitation of non-dominated solutions found with exploration of the search space by adopting combined Pareto procedures. In this way, it is able to escape all local optima and converge to the global Pareto-optimal front. The outcomes of two previous studies on the application of CPMDE conclude that it represents an improvement over existing algorithms (Adeyemo & Olofintoye, 2014; Enitan et al., 2014). This is evident in the fact that CPMDE produced a better result for both convergence metric and diversity metric when compared with other state-of-the-art EAs. Also, the non-dominated solutions obtained by CPMDE are very close to and well distributed on the true Pareto-optimal surface of test problem. Since the main aim of CPMDE is to simultaneously optimize a set of conflicting objectives to obtain a group of alternative trade-off solutions called Pareto optimal or non-inferior solutions, the technique is able to adaptively balance exploitation of non-dominated
Figure 1. Location of VIS in South Africa.

Figure 2. Geographical location of VIS, South Africa.
In CPMDE, boundary constraints are handled using the bounce-back strategy, and this strategy replaces a vector that has exceeded one or more of its bounds by a valid vector that satisfies all boundary constraints (Olofintoye et al., 2014). Major difference between the bounce-back strategy and random re-initialization is that the former takes the progress towards the optimum into account by selecting a parameter value that lies between the base vector parameter value and the bound being violated (Adeyemo & Olofintoye, 2014). Equality and inequality constraints are handled using the constrained-domination technique suggested by Deb (2001). DE/rand/1/bin variant of DE proposed by Adeyemo and Otieno (2010b) is used as the base for CPMDE. The CPMDE algorithm is summarized as follows (Olofintoye et al., 2014):

1. Input the required DE parameters like number of individuals in the population (Np), mutation scale factor (F), crossover probability (Cr), maximum number of iterations/generations (gMax), number of objective functions (k), number of decision variables/parameters (D), upper and lower bounds of each variable, etc.
2. Initialize all solution vectors randomly within the limits of the variable bounds.
3. Set the generation counter, g = 0
4. Generate a trial population of size Np using DE’s mutation and crossover operations (Price, Storn, & Lampinen, 2005).
5. Perform a domination check on the combined trial and target population and mark all non-dominated solutions as “non-dominated” while marking others as “dominated”.
6. Play domination tournament at each population index.
   (i) If the trial solution is marked “non-dominated” and the target is marked “dominated”, then the trial vector replaces the target vector.
   (ii) If the trial solution is marked “dominated” and the target is marked “non-dominated”, then the trial vector is discarded.
   (iii) If both solutions are marked “dominated”, then replace the target vector if it is dominated by the trial vector or if they are non-dominated with respect to each other.
   (iv) If both vectors are marked “non-dominated”, then note down the index and proceed to the next index. When all solutions marked “non-dominated” from steps i–iii above are installed in the next generation, then sort out all solutions noted in step iv one at a time using the harmonic average crowding distance measure (Huang, Li, Chen, & Ma, 2012). The solution with a greater harmonic average distance is selected to proceed to the next generation.
7. Increase the generation counter, g, by 1. i.e. g = g + 1.
8. If g < gMax, then go to step 4 above else go to step 9
9. Remove the dominated solutions in the last generation
10. Output the non-dominated solutions.

*Note domination checks are performed using the naive and slow method suggested by Deb (2001).

2.2.1. Model formulation
The irrigation water allocation and crop yield optimization problem in this study was conducted for a planting season at VIS. A farmland with an area of 1,000,000 m² (100 ha) and maximum water quota of 9,140 m³ per ha/annum was selected as a case study. Three different crops, namely maize, groundnuts and potatoes, are planted on the piece of land. In addition, an assumption that
all the crops are not rainfed but rely solely on irrigation was adopted in this study. Formulation of the constrained multi-objective mathematical optimization problem follows.

2.2.1. Decision variables and objectives. The main aim of the study was to find the corresponding optimal crop mix and planting areas per crop while maximizing total NB (ZAR/m$^2$) and minimizing irrigation water allocation (m$^3$). The decision variable which represents the total NB is denoted by TNB, $i = 1, 2, 3$ for maize, groundnuts and potatoes, respectively. The objectives are formulated as follows:

**Objective 1**: Maximize total net benefits

Total NB (ZAR/m$^2$) is maximized to increase food production and employment on the farm. This has relative importance in terms of job creation and ensuring food security. Total NB is derived by multiplying the selling price (ZAR/ton) by the crop yield (ton/m$^2$).

**Objective 2**: Minimize irrigation water allocation

South Africa has been termed a water-stressed country and irrigation uses almost 60% of the available freshwater resources in the country (Adeyemo & Otieno, 2010b; Nkondo, van Zyl, Keuris, & Schreiner, 2012); it is therefore pertinent to minimize irrigation water allocation. The multi-objective optimization equation for this problem which maximizes the total NB and minimizes total irrigation water allocation (WU) is presented in equation (1):

\[
\begin{align*}
\text{Maximize} & \\
\text{TNB} & = \sum_{i=1}^{n} (Y_i \times P_i \times AR_i) - \sum_{i=1}^{n} (IN_i \times AR_i \times Ic) + \sum_{i=1}^{n} (AR_i \times VC) \ldots \ldots (n = 1, \ldots, 3) \\
\text{Minimize} & \\
\text{WU} & = \sum_{i=1}^{n} (CWR_i \times AR_i) \\
\text{Subject to} & \\
\sum_{i=1}^{n} (AR_i) & \leq 1,000,000 \\
100000 & \leq AR_i \leq 700000 \\
WU & \leq 914000
\end{align*}
\]

where $Y_i$ is the crop yield of the $i$th crop in (ton/m$^2$); $P_i$ is the selling price of the $i$th crop in (ZAR/ton); $AR_i$ is the planting area of the $i$th crop in (m$^2$); $IN_i$ is the irrigation water need for the $i$th crop (ML/m$^2$); $Ic$ is the irrigation or water cost (ZAR/ML) which is 8.77 cents/m$^3$ (Adeyemo & Otieno, 2010a); and $VC$ is the variable cost per m$^2$ for the $i$th crop (fertilizers, herbicides and sowing) (ZAR/m$^2$).

| Table 1. Total annual crop water requirement, yield and price for the three crops under consideration (Department of Agriculture 2013) |
| --- | --- | --- | --- |
| SN | Crop | Yield (ton/ha) | Price (ZAR/ton) | Crop water requirement (mm) |
| --- | --- | --- | --- | --- |
| 1 | Maize | 9.00 | 991.83 | 720 |
| 2 | Ground nuts | 4.50 | 2849.11 | 840 |
| 3 | Potatoes | 35.00 | 1744.00 | 1213 |
2.2.1.2. Problem constraints. The bi-objective mathematical optimization problem is subject to the following constraints:

**Constraint 1:** Total land area available.

The sum of areas $AR_i$, where the crops are grown, must not be greater than the total land area available for farming. This constraint is presented in equation (2):

$$A = \sum_{i=1}^{n} (AR_i) \leq 1,000,000$$

(2)

**Constraint 2:** Minimum and maximum crop planting areas.

The minimum and maximum planting areas for each crop constitute the boundary constraints of the problem. Each crop is planted in at least 100,000 m$^2$ to avoid crop scarcity which may lead to hike in selling prices of food, while the maximum planting areas ensure there will not be excessive surplus so that farmers will not have storage or selling problems (Adeyemo & Otieno, 2010b). To compute the maximum crop planting areas, the following should be known:

Since the minimum planting area for each crop = 100,000 m$^2$, then the other three crops will occupy a minimum of $(100,000 \times 3) = 300,000$ m$^2$. This leaves $(1,000,000 - 300,000) = 700,000$ m$^2$ as the maximum area available for a particular crop. Therefore, 700,000 m$^2$ is the maximum planting area for all the crops. The boundary constraint for the planting area is given in equation (3) as:

$$100000 \leq AR_i \leq 700000$$

(3)

**Constraint 3:** Irrigation water release.

The amount of water available on the farm annually is limited by the amount of water released by the DWA. According to DWA (2013), the volume of water supplied to VIS annually is 0.914 m$^3$/m$^2$ (9,140 m$^3$/ha). Considering the 1,000,000 m$^2$ (100 ha) planting area considered for this study, therefore the maximum irrigation water release is 914,000 m$^3$ of water annually. It is therefore required that total irrigation water use does not exceed the maximum that can be supplied by the feeder canal. This constraint is presented in equation (4):

$$WU \leq 914000$$

(4)

2.2.2. Model solution and experimental setup

The mathematical model equations of the objective functions and the constraints listed in equations (1–4), for the constrained multi-objective irrigation water allocation and crop yield optimization problem in this study, were solved using the CPMDE algorithm. The pseudo code for CPMDE by Olofinto et al. (2014) was encoded using visual basic for applications to facilitate its application in resolving the crop yield optimization problem stated herein.

The population size for the algorithms was set at $N_p = 50$ as advised by Adeyemo, Bux and Otieno (2010) based on a study of the sensitivity analysis of DE algorithms. CPMDE algorithm was iterated for 1,000 generations resulting in 50,000 fitness computations; the crossover rate $C_r$ was set at 0.95, while the mutation scaling factor $F$ was set at 0.5 as advised by Storn and Price (1995) and Adeyemo and Otieno (2010a). DE/rand/1/bin variant of DE was implemented, and the
harmonic average distance for maintaining spread of solutions on the Pareto front of CPMDE was computed using the two-nearest neighbours scheme.

### 2.2.3. Selecting the best compromise solution

The solution of multi-objective optimization problems results in a set of non-inferior solutions which are Pareto optimal solutions. No solution in this set can be considered better than any other in the absence of specialized information about the peculiarities of the problem at hand. However, it is important that the decision maker chooses only one solution for final implementation. Compromise programming approach (CPA) is the recommended technique in making a final decision regarding a suitable operating policy concerning the problem being solved (Deb, Mohan, & Mishra, 2003). CPA picks a solution which is minimally located from a given reference point. In this study, the reference point is chosen as the ideal point which comprises the best of each of the \( m \) objectives. The best compromise solution (BCS) is the solution with a minimum \( l_p \)-metric distance from a virtual reference point \( z \). \( l_p \)-metric is computed using equation (5). When \( p = 2 \), the \( l_2 \) metric specifies the Euclidean distance metric (Deb, 2001; Olofintoye et al., 2014).

\[
\text{\( l_p \)-metric : } d(f, z) = \left( \sum_{m=1}^{M} |f_m(x) - z_m|^p \right)^{\frac{1}{p}}
\]  

(5)

The visual basic coded CPMDE helped in computing the Euclidean distance. The overall results for the 50 population solutions are presented in Table 2.

### 3. Results

The multi-objective irrigation water allocation and crop yield problem of maximizing total NB while minimizing irrigation water allocation in a farmland in VIS was solved using CPMDE. Figure 3 presents the Pareto front obtained by CPMDE and the BCS which represents the 18th solution is indicated on this figure. Table 2 presents the details of the Pareto solutions obtained from the best run of CPMDE. Figure 4 presents the objective values for the final non-dominated solutions obtained in the best run of CPMDE, while Figure 5 presents the corresponding planting areas for the three crops in the non-dominated solutions using CPMDE. Figure 6 presents the total crop planting areas for the three crops which form the BCS (solution 18) obtained by CPMDE. The BCS is marked with boldface in Table 2.

### 4. Discussions

In this study, it was found out that the CPMDE algorithm performed excellently in finding optimal solutions to the crop yield problem at VIS, South Africa. In a single simulation run, CPMDE found quality Pareto solutions that provide trade-off between the conflicting objectives of the crop yield optimization problem. In the Pareto optimal solution set, each solution is not better than the others in all the objectives. In practice, the decision maker ultimately has to select one solution from this set for system implementation. All the solutions converged to Pareto front. Also, from the Pareto optimal set, it is evident that planting the crops within the optimal land area at the BCS will reduce irrigation water allocation, and hence, the total NB will be maximized. From a critical analysis of all the 50 solutions as presented in Table 2, solution 18 has the highest total NB of ZAR 767,961.49 generated from planting the three crops with total volume of irrigation water of 391,061.52 m³ and total planting areas of 937,961.49 m². This solution suggests that maize should be planted in 403,543.44 m² land area, ground nut should be planted in 181,542.00 m² in the farmland, while potatoes should be planted on 352,876.05 m² areas of land, respectively.

The second best non-dominated solution is solution 1 which has a total NB of ZAR 770,996.66 with irrigation water volume of 675,774.56 m³ and total planting area of 720,620.4 m². The third best solution is solution 38 with total NB of ZAR 756,763.86, irrigation water volume of 665,036.46 m³ and total planting area of 706,762.4 m². Since the BCS is the solution which is minimally located from the ideal point which comprises the extremes of all the conflicting
Table 2. Details of Pareto solutions for the crop yield model when maximizing total net benefits and minimizing irrigation water

| Solution | Land area for each crop (m$^2$) | Total land area (m$^2$) | Total net benefits (ZAR) | Total water irrigation (m$^3$) |
|----------|---------------------------------|-------------------------|-------------------------|-------------------------------|
|          | Maize                           | Ground nut              | Potatoes                |                               |
| 1        | 619,253.17                      | 51,324.00               | 50,043.23               | 720,620.4                    | 770,996.6                   | 675,774.56 |
| 2        | 443,809.45                      | 51,250.65               | 50,012.78               | 545,072.88                   | 595,089.41                  | 548,648.09 |
| 3        | 263,288.52                      | 51,384.23               | 50,013.35               | 364,698.31                   | 414,733.47                  | 418,843.99 |
| 4        | 494,619.55                      | 51,237.45               | 50,013.35               | 595,870.35                   | 645,898.45                  | 585,243.51 |
| 5        | 431,572.01                      | 50,000.12               | 50,000.17               | 531,572.3                    | 581,605.39                  | 538,795.78 |
| 6        | 404,934.20                      | 50,000.08               | 50,000.07               | 504,934.35                   | 554,936.06                  | 519,556.11 |
| 7        | 240,989.17                      | 50,000.00               | 50,000.00               | 341,726.91                   | 391,734.24                  | 391,061.52 |
| 8        | 225,491.95                      | 50,000.00               | 50,000.00               | 326,263.36                   | 376,276.58                  | 391,061.52 |
| 9        | 564,983.99                      | 50,000.00               | 50,000.00               | 665,093.7                    | 715,107.29                  | 634,954.97 |
| 10       | 50,000.00                       | 50,000.00               | 50,000.00               | 150,000                      | 200,000                     | 264,000     |
| 11       | 327,374.44                      | 51,208.06               | 50,011.84               | 428,594.34                   | 478,616.34                  | 446,787.92 |
| 12       | 518,901.46                      | 50,081.48               | 50,036.61               | 669,091.34                   | 726,306.35                  | 601,881.23 |
| 13       | 209,470.20                      | 50,805.84               | 50,004.83               | 341,726.91                   | 391,734.24                  | 391,061.52 |
| 14       | 358,162.37                      | 50,079.64               | 50,013.63               | 388,127.26                   | 438,152.61                  | 438,152.61 |
| 15       | 288,033.97                      | 50,079.64               | 50,013.63               | 388,127.26                   | 438,152.61                  | 438,152.61 |

(Continued)
objectives, solution 18 is suggested for final implementation in this study. Among the three crops optimized, maize has the greatest land area, followed by potatoes. The best solution shows that maize produced the highest crop yield under limited water allocation in the study area.

For comparison, a similar study was conducted using MDEA by Otieno and Adeyemo (2011) in the same study area. At the optimum crop yield, MDEA was able to allocate 9.12 m$^3$/ha of irrigation water, while CPMDE allocates 3.91 m$^3$/ha of irrigation water to the farmland. Furthermore, CPMDE was able to reduce irrigation water allocation to only 57.2% of the total available irrigation water on the farmland. This result is consistent with the results of Adeyemo, Bux and Otieno (2010) and Grove (2011).

5. Conclusion
This study has successfully demonstrated the ability of CPMDE algorithm to generate non-dominated solutions along the Pareto front of the selected problem and its ability to solve unconstrained, constrained and real-world optimization problems. From the generated Pareto optimal

Table 2. (Continued)

| Solution | Land area for each crop (m$^2$) | Total land area (m$^2$) | Total net benefits (ZAR) | Total water irrigation water (m$^3$) |
|----------|---------------------------------|-------------------------|--------------------------|-------------------------------------|
|          | Maize | Ground nut | Potatoes | Maize | Ground nut | Potatoes | Maize | Ground nut | Potatoes |
| 43       | 276,306.54 | 50,000.00 | 50,048.03 | 376,354.57 | 426,360.44 | 427,038.43 |
| 44       | 358,162.37 | 50,476.59 | 50,004.83 | 458,643.79 | 508,644 | 486,286.34 |
| 45       | 385,586.77 | 50,000.00 | 50,058.66 | 485,645.43 | 535,687.04 | 505,807.95 |
| 46       | 288,033.99 | 50,079.64 | 50,013.63 | 388,127.26 | 438,152.61 | 435,524.58 |
| 47       | 144,318.15 | 50,002.15 | 50,000.01 | 244,320.31 | 294,323.91 | 331,917.81 |
| 48       | 535,676.52 | 50,052.67 | 50,003.21 | 635,732.4 | 685,738.86 | 613,749.51 |
| 49       | 113,816.22 | 50,061.33 | 50,023.77 | 213,901.32 | 263,902.92 | 310,045.06 |
| 50       | 125,527.36 | 51,177.46 | 50,013.92 | 226,718.74 | 276,727.44 | 319,410.54 |

Note: Represents the non-dominated solution (BCS = 18).
Figure 4. Objective values for the final non-dominated solutions obtained in the best run of CPMDE (BCS = 18).

Figure 5. Different planting areas for the three crops in the non-dominated solutions using CPMDE.

Figure 6. Optimal crop planting areas for maize, ground nut and potatoes in the best non-dominated solution (BCS = 18) using CPMDE.
set, it is evident that planting the crops within the optimal land area at the BCS will reduce irrigation water use, and hence, the total NB will be maximized. The best compromise solution (Figure 3) suggests that maize should be planted in 403,543.44 m² land area, ground nut should be planted in 181,542.00 m² in the farmland, while potatoes should be planted on 352,876.05 m² areas of land, respectively. The cumulative planting area is 937,961.49 m² and a cumulative of 391,061.52 m³ volume of irrigation water allocation out of the total irrigation water volume of 914,000 m³ provided by DWA. It has reduced water allocation to only 57.2% of total available irrigation water. This has proved that CPMDE is suitable and efficient than MDEA for solving multi-objective irrigation water allocation and crop distribution problems on the farm level. When compared with a previous study which adopted MDEA, it was discovered that CPMDE performed better in reducing the water allocation rate while still maximizing crop yield in the same study area. It can be concluded that CPMDE is a good and robust alternative algorithm suitable for resolving crop distribution under limited water availability.

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