TESTING LOGARITHMIC VIOLATIONS TO SCALING IN STRONGLY COUPLED QED

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ABSTRACT

Using very precise measurements of the critical couplings for the chiral transition of non compact $QED_4$ with up to 8 flavours, we analyse the behaviour of the order parameter at the critical point using the equation of state of a logarithmically improved scalar mean field theory, that of the Nambu-Jona Lasinio theory and a pure power law. The first case is definitively excluded by the numerical data. The stability of the fits for the last two cases, as well as the behaviour with the number of flavours of the exponent of the logarithmic violations to the scaling favour clearly a pure power law scaling with non mean field exponents.
Non perturbative renormalizable gauge theories are among the most stimulating and less known subjects in theoretical physics. It is not known if it is possible to construct such a theory in four dimensions and the standard prejudice is that only asymptotically free gauge theories with gaussian fixed points are "good" field theories.

Non compact $QED_4$ or its generalized version, the gauged Nambu-Jona Lasinio ($GNJL$) model, are good candidates to analyze this problem. Both models have the common feature that they couple fermions strongly enough to produce fermion condensates, and therefore a phase, where chiral symmetry is spontaneously broken, appears at sufficiently strong gauge coupling. Since composite scalars are present in the spectrum of these models, the existence of a non trivial continuum limit is strongly related, as discussed in [1], with the balance between the fermion attraction due to the interaction and the zero point repulsion due to the kinetic energy. If the short distance attraction is too strong, composite scalars with vanishing physical size appear in the spectrum, thus giving rise to a non interacting field theory. Even if no rigorous proof exists, this seems to be the case in the Nambu-Jona Lasinio ($NJL$) model where mean field exponents with logarithmic violations to scaling driving to a vanishing renormalized coupling are expected [2].

In a recent investigation of the gauged $NJL$ model [3] we have found strong evidence supporting the fact that the gauge interaction can change qualitatively the trivial scenario of the $NJL$ model. Stimulated by these results and following our investigation on non compact $QED_4$ started several years ago [4], we want to report in this letter some new results concerning the critical behaviour of this last model.

Our analysis is based on very precise determinations of the critical couplings obtained from the computation of the susceptibilities in the Coulomb phase [3] on $10^4$, $12^4$ and $14^4$ lattices. From these results we analyse the behaviour of the order parameter at the critical point and for several number of flavours by fitting it with three different equations of state ($EOS$): i) The $EOS$ of a logarithmically improved scalar mean field theory, ii) a power law scaling without logarithmic violations and iii) a $NJL$ model-like $EOS$.

The case i), as will be shown in what follows, is definitively excluded by the numerical data. This is not surprising at all since as suggested in [1] and corroborated in [2], triviality in a theory with composite scalars should manifest in a different way than in a theory with fundamental scalars. In fact the logarithmic violations to scaling in the first case, as in the $NJL$ model, are expected to have an effective $\delta$ exponent less than 3 whereas in
the second case an effective $\delta$ larger than 3 is obtained.

The difficult task is to distinguish from the numerical data between ii) (power law scaling) and iii) (four fermion $EOS$) since in both cases the effective $\delta$ is less than three. Let us anticipate that even if the behavior of the order parameter at the critical point is well fitted by both equations of state, the stability of the fits with lattice size and the behavior of the exponent which controls the logarithmic violations to scaling with the number of flavours strongly favour a pure power law scaling against logarithmic violations to mean field.

The main ingredient of this analysis is the very precise determination of the critical coupling $\beta_c$ in $QED_4$ which follows from the computation of the susceptibility in the Coulomb phase of this model. A detailed explanation of this computation for lattices up to $10^4$ can be found in [5].

Let us recall the two essential ingredients we have used in this analysis. First, exploiting the potentialities of the Microcanonical Fermionic Average ($MFA$) approach to simulate dynamical fermions in gauge theories we have done simulations in the chiral limit at exactly zero fermion mass, which allows to overcome the ambiguities in the fermion mass extrapolations.

Second, we have also exploited the fact that in the Coulomb phase of this model, characterized by a non degenerate ground state, we can interchange the chiral limit with the thermodynamical limit. Notice that the reason why this exchange is not allowed in general cases ($QCD$ simulations, broken phase of $QED$, etc.) is the degeneration of the vacuum state. If we are in a broken phase where many degenerate vacua connected by symmetry transformations coexist, a permutation of the chiral and thermodynamic limits would imply a path integral over all the Gibbs state. Then several unpleasant facts like the violation of the cluster property for correlation functions, vanishing values for the order parameter or wrong values for the computed susceptibilities will appear. However if calculations are done in the symmetric phase characterized by a non degenerate ground state, there are not, in general cases, physical reasons to prevent from doing such a permutation.

The most general expression for the vacuum expectation value of any operator $O$, after integration of the Grassmann variables, can be written as

\[
\langle O \rangle = \frac{\int dE_n(E) \frac{\overline{O \Delta}}{\det \Delta} e^{-6V_\beta E - S_{eff}(E,m)}}{\int dE_n(E) e^{-6V_\beta E - S_{eff}(E,m)}}
\]

where

\[
\text{(1)}
\]
\[ n(E) = \int [dA_\mu] \delta(6VE - S_G[A_\mu]) \] (2)

is the density of states at fixed energy \( E \) and \( S_{\text{eff}}^E(E, m) \) in (1) is the fermion effective action defined as

\[ S_{\text{eff}}^E(E, m) = -\log \det \Delta(E, m) \] (3)

\( O \det \Delta \) means the mean value of the product of the operator \( O \) times the fermionic determinant, computed over gauge field configurations at fixed energy \( E \).

Since we are interested here in the computation of susceptibilities in the chiral limit, we will use the previous expression for the particular cases in which \( O \) is the longitudinal or transverse susceptibility.

The longitudinal and transverse susceptibilities in the Coulomb phase are equal except a sign. They can be computed by taking for the operator \( O \) the expression

\[ O = \frac{2}{V} \sum_i \frac{1}{\lambda_i^2} \] (4)

where the sum in (4) runs over all positive eigenvalues of the massless Dirac operator.

The transverse susceptibility \( \chi_T \) in the broken phase diverges always because of the Goldstone boson. The longitudinal susceptibility \( \chi_L \) on the other hand, can not be computed in the broken phase using equation (4) since in this phase the ground state is not invariant under chiral transformations.

We refer the reader interested to details on the computation of susceptibilities to [5]. Let us recall that using the MFA approach, computations at several number of flavours \( N_f \) can be done without extra computer cost [6]. In table I we report the critical couplings extracted from the susceptibilities in \( 10^4, 12^4 \) and \( 14^4 \) lattices at \( N_f = 1, 2, 3, 4, 8 \). The most striking fact of this table is the small statistical errors of the critical couplings which follows from the high quality of the susceptibility fits, as reported in [5].

We have fitted the behaviour of the chiral condensate \( \langle \bar{\chi}\chi \rangle \) against the bare fermion mass \( m \) at the critical values of \( \beta \) reported in Table I using different equations of state. In Fig. 1 we plot \( \langle \bar{\chi}\chi \rangle^3/\log\langle \bar{\chi}\chi \rangle \) against \( m \) in the mass interval (0.005, 0.1) for the four flavour theory and in a \( 14^4 \) lattice. Were the critical behaviour of this model described by a gaussian fixed point as in a logarithmically improved scalar mean field theory, the points in the small mass region of this figure should be well fitted by a
straight line crossing the origin. The solid line in this figure is the best fit obtained under the previous assumption. The very bad quality of this fit ($\chi^2_{d.o.f.} = 1267, m \leq 0.1, \chi^2_{d.o.f.} = 358, m \leq 0.05$) disproves definitively such a possibility. Similar results have been obtained in smaller lattices and for different flavour numbers.

Having shown that the critical behaviour of non compact $QED$ cannot be described by a logarithmically improved scalar mean field theory, we will try the same kind of fit for the other two reasonable possibilities, i.e. pure power law behaviour and mean field with logarithmic violations a la Nambu-Jona Lasinio.

Fig. 2 is a plot of the same data reported in Fig. 1 but in the ordinate axis we plot $\langle \bar{\chi}\chi \rangle^{2.80}$. The solid line is the best fit of all the points at small $m$ with a straight line crossing the origin. The high quality of this fit is corroborated by the value $\chi^2_{d.o.f.} = 0.38$, a value which is stable until masses of the order of 0.08 and increases slowly when masses larger than 0.08 are included in the fit. The value 2.80 of the $\delta$ exponent has been chosen as the best one for the linear fit (see Table II and the discussion on the determination of $\delta$ at the end). From the results reported in Fig. 2 we conclude that a pure power with $\delta = 2.80$ describes with high accuracy the behaviour of the chiral condensate at the critical coupling.

Our last plot for the chiral condensate at the critical coupling is reported in Fig. 3. For this plot we use an equation of state a la Nambu-Jona Lasinio

$$\langle \bar{\chi}\chi \rangle^{3\log(p)(1/\langle \bar{\chi}\chi \rangle)} = Cm \tag{5}$$

where $C$ in (5) is a constant and the exponent $p$ of the logarithmic violations to scaling is left as a parameter of the fit. Recall that $p = 1$ in the large $N_f$ limit and that this result does not changes after taking into account the $\frac{1}{N_f}$ and $\frac{1}{N_f^2}$ corrections, this suggesting that a value of $p$ different from 1 is not very reliable. Notwithstanding that we decided to left $p$ as a parameter of the fit for two reasons: i) our results for the chiral condensate does not support a fit with equation (1) and $p = 1$ and ii) there is no rigorous proof that $p$ does not changes with $N_f$.

The best fit of our results in the four flavour model with equation (5) is obtained for $p = 0.28$. In Fig. 3 we plot $\langle \bar{\chi}\chi \rangle^{3\log^{0.28}(1/\langle \bar{\chi}\chi \rangle)}$ against $m$. The solid line in this figure is a fit of all the points at small $m$ with a straight line crossing the origin. Again now, as in the case of the power law behaviour, we get a high quality fit of these points ($\chi^2_{d.o.f.} = 0.89$).
At first sight it seems difficult to decide between the last two cases (power law and Nambu-Jona Lasinio). However we can get some insight on the reliability of the last two fits by analyzing their stability with the lattice size as well as the flavour dependence of the $p$ exponent in equation (5).

Table II contains a summary of the results we obtain for the $\delta$ and $p$ exponents for the three lattice sizes 10, 12, 14 and $N_f = 1, 2, 3, 4, 6, 8$. The values of $\delta$ reported in this table are obtained fitting the chiral condensate with a pure power law (case ii previously, corresponding to non mean-field exponents). Using instead a Nambu-Jona Lasinio like EOS, where $\delta = 3$, we obtain values of the $p$ exponent of equation (5), which we also report in Table II. The errors in this Table take into account both, the errors of the fits and the error in the determination of the critical couplings (see Table I). The high precision of the exponents reported in Table II follows from the very small errors in the critical couplings of Table I.

Looking at the results reported in Table II we can discuss both, the stability of the results with the lattice size and the flavour dependence of $p$. First notice that the values of $\delta$ for the three lattice sizes at each value of $N_f$ are always compatible whereas this does not hold for the values of $p$. We conclude that the pure power law fits are much more stable with lattice size than NJL-like fits.

The second important fact that can be observed in this Table is that the value of the $p$ exponent, contrary to expectations based on the $1/N_f$ expansion of the NJL model, not only is different from 1 but decreases when $N_f$ increases and therefore goes away from the expected $p = 1$ at large $N_f$. As well known [9],[10] the chiral transition of noncompact QED changes from second to first order at large $N_f$ and therefore we can not extrapolate our results for $p$ in Table II to $N_f = \infty$. Notwithstanding that, Kim, Kocic and Kogut have shown in [2] how numerical simulations of the NJL model with discrete $Z_2$ symmetry at $N_f = 12$ reproduce very well the logarithmic violations to mean field scaling given by the EOS of the model at $N_f = \infty$. Therefore, were the critical behaviour of non compact QED described by the NJL model, we would expect a value of $p \sim 1$, at least at $N_f = 8$.

In conclusion, we believe that the features previously discussed strongly favour a pure power law scaling with non mean field exponents against logarithmic violations to mean field a la Nambu-Jona Lasinio. This is a very interesting and important improvement with respect to previous work on this subject [4],[11] and it has been possible as a consequence of the very precise measurements of the critical couplings reported in Table I.

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References

[1] A. Kocic, J.B. Kogut, Nucl. Phys. B422 [FS] (1994) 593.
[2] S. Kim, A. Kocic, J.B. Kogut, Nucl. Phys. B429 (1994) 407.
[3] V. Azcoiti, G. Di Carlo, A. Galante, A.F. Grillo, V. Laliena, C.E. Piedrafita, Phys. Lett. B355 (1995) 270.
[4] V. Azcoiti, G. Di Carlo, A.F. Grillo, Mod. Phys. Lett. A7 (1992) 3561; Int. J. Mod. Phys. A8 (1993) 4235.
[5] V. Azcoiti, G. Di Carlo, A. Galante, A.F. Grillo, V. Laliena and C.E. Piedrafita, Phys. Lett. B353 (1995) 279.
[6] V. Azcoiti, G. Di Carlo and A.F. Grillo, Phys. Rev. Lett. 65 (1990) 2239; V. Azcoiti, A. Cruz, G. Di Carlo, A.F. Grillo and A. Vladikas, Phys. Rev. D43 (1991) 3487; V. Azcoiti, G. Di Carlo, L.A. Fernandez, A. Galante, A.F. Grillo, V. Laliena, X.Q. Luo, C.E. Piedrafita and A. Vladikas; Phys. Rev. D48 (1993) 402.
[7] S. Hands, A. Kocic, J.B. Kogut, Ann. of Phys. 224 (1993) 29.
[8] J.A. Gracey, Phys. Lett. B308 (1993) 65; Phys. Rev. D50 (1994) 2840.
[9] E. Dagotto, A. Kocic, J.B. Kogut, Phys. Lett. B231 (1989) 235.
[10] V. Azcoiti, G. Di Carlo, A.F. Grillo, Phys. Lett. B305 (1993) 275.
[11] A. Kocic, J.B. Kogut, K.C. Wang, Nucl. Phys. B398 (1993) 405.
Figure captions

**Figure 1.** Numerical results for $\langle \bar{\chi}\chi \rangle^3 / \log \langle \bar{\chi}\chi \rangle$ against $m$ for the four flavour theory and in a $14^4$ lattice. The solid line in this figure is the best linear fit crossing the origin.

**Figure 2.** Plot of $\langle \bar{\chi}\chi \rangle^{2.80}$ against $m$. The solid line is the best fit of all the points at small $m$ with a straight line crossing the origin.

**Figure 3.** $\langle \bar{\chi}\chi \rangle^3 \log^0.28(\frac{1}{\langle \bar{\chi}\chi \rangle})$ against $m$. The solid line is a fit of all the points at small $m$ with a straight line crossing the origin.
Table captions

**Table I** Critical couplings extracted from the susceptibilities in $10^4, 12^4$ and $14^4$ lattices at $N_f = 1, 2, 3, 4, 8$.

**Table II** Results for the $\delta$ and $p$ exponents for the three lattice sizes $10, 12, 14$ and $N_f = 1, 2, 3, 4, 6, 8$. The values of $\delta$ reported in this table correspond to the results for the fits of the chiral condensate with a pure power law whereas in the case of the Nambu-Jona Lasinio like EOS, where $\delta = 3$, we report the value of the $p$ exponent in equation (5).
$\chi \chi^{2.80}$

Fig. 2
\bar{\chi} \chi^3 \log^{0.28}(1/\bar{\chi}\chi)
| $L$ | $n_f$ | $\beta_c$ |
|-----|------|--------|
| 10  | 1    | 0.2332(4) |
|     | 2    | 0.2230(3) |
|     | 3    | 0.2127(2) |
|     | 4    | 0.2025(2) |
|     | 6    | 0.1820(2) |
|     | 8    | 0.1616(2) |
| 12  | 1    | 0.2356(3) |
|     | 2    | 0.2255(2) |
|     | 3    | 0.2153(2) |
|     | 4    | 0.2051(2) |
|     | 6    | 0.1847(2) |
|     | 8    | 0.1644(2) |
| 14  | 1    | 0.2391(3) |
|     | 2    | 0.2289(3) |
|     | 3    | 0.2187(2) |
|     | 4    | 0.2086(2) |
|     | 6    | 0.1882(2) |
|     | 8    | 0.1679(2) |

Table I

| $L$ | $n_f$ | $\delta$ | $\chi^2$/dof | $p$ | $\chi^2$/dof |
|-----|------|------|-------------|-----|-------------|
| 10  | 1    | 2.86(8) | 0.09 | 0.39(8) | 0.12 |
|     | 2    | 2.79(6) | 0.09 | 0.46(5) | 0.14 |
|     | 3    | 2.80(5) | 0.13 | 0.33(5) | 0.25 |
|     | 4    | 2.82(5) | 0.20 | 0.26(4) | 0.40 |
|     | 6    | 2.83(4) | 0.25 | 0.17(5) | 0.68 |
|     | 8    | 2.83(4) | 0.27 | 0.12(4) | 1.1  |
| 12  | 1    | 2.81(6) | 0.05 | 0.57(6) | 0.09 |
|     | 2    | 2.79(5) | 0.12 | 0.48(5) | 0.22 |
|     | 3    | 2.78(5) | 0.24 | 0.41(4) | 0.56 |
|     | 4    | 2.78(5) | 0.36 | 0.35(5) | 1.0  |
|     | 6    | 2.78(4) | 0.44 | 0.26(5) | 1.8  |
|     | 8    | 2.78(4) | 0.45 | 0.18(4) | 3.1  |
| 14  | 1    | 2.74(8) | 0.11 | 0.74(4) | 0.55 |
|     | 2    | 2.77(7) | 0.18 | 0.47(4) | 0.60 |
|     | 3    | 2.79(6) | 0.25 | 0.34(3) | 0.75 |
|     | 4    | 2.80(5) | 0.38 | 0.28(3) | 0.89 |
|     | 6    | 2.80(5) | 0.49 | 0.18(2) | 1.3  |
|     | 8    | 2.80(4) | 0.53 | 0.12(1) | 3.3  |

Table II