Quantum Hall Effects at Finite Temperatures

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We study the finite temperature (FT) effects on integer quantum Hall effect (IQHE) and fractional quantum Hall effect (FQHE) as predicted by the composite fermion model. We find that at $T \neq 0$, universality is lost, as is quantization because of a new scale $T_0 = \pi \rho/m^* p$. We find that this loss is not inconsistent with the experimentally observed accuracies. While the model seems to work very well for IQHE, it agrees with the bulk results of FQHE but is shown to require refinement in its account of microscopic properties such as the effective mass. Our analysis also gives a qualitative account of the threshold temperatures at which the FQHE states are seen experimentally. Finally, we extract model independent features of quantum Hall effect at FT, common to all Chern-Simons theories that employ mean field ansatz.

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The purpose of this paper is to study quantum Hall effect (QHE), both integer (IQHE) and fractional (FQHE), in the composite fermion model (CFM) at finite temperatures. The merit of such a study is recognized if we recall that IQHE is used to measure the fine structure constant – to a great accuracy of 0.01 ppm – with a complex ‘dirty system’ and yet without any dependence on the sample properties such as the shape, density and mass of electron etc. It is pertinent to investigate upto what temperature this universality and the precise quantization of the Hall resistivity $\rho_{xx}$ will survive, given a required degree of accuracy. Similarly, the associated phenomenon, FQHE, also exhibits quantization at fractional filling factors; and as it was emphasized by Laughlin13 long back, the experimentalists14 have found it necessary to go to lower temperatures (and higher magnetic fields) to see more and more plateaus. This fact has to be accounted for, not merely qualitatively, and it is of interest to see if the CFM, which appears to be a valid description at $T = 0$, is also reliable at finite temperatures.

We point out that there are two questions here – the dependence of quantization, and the dependence of plateau width on temperature. There is a wealth of experimental data available (although a lot more are needed for a full study) which we shall discuss contextually below. In this paper, we merely address the first question since for simplicity, we choose a pure system – which yields a vanishing diagonal resistivity $\rho_{xx}$, and also does not incorporate the plateau widths. These drawbacks will be remedied in a more detailed presentation of our work.

There has been a considerable gain in our understanding of QHE, thanks to the contributions made by Ando et al.15 Prange,16 Laughlin,17 Halperin,18 and Streda19 for IQHE and by the seminal work of Laughlin20 which was followed by that of Haldane21 and Halperin22 Halperin22 was the first to recognize the anyonic nature of the Laughlin’s wave function which suggests a Chern-Simons (CS) interaction in the system. This idea has been employed by Jain23 in his CFM where, however, the CS term does not transmute the statistics even while playing a dynamical role. A field theoretic version of the Jain picture has been given by Lopez and Fradkin24.

CFM has the attractive feature of treating IQHE and FQHE on an equal footing (and is thus a convenient model to study the finite temperature (FT) properties, atleast in a first approach20). Indeed, when the system is subjected to an external magnetic field $B$, a part $b$ of $B$ gets attached as singular flux tubes to the electrons and becomes the dynamical magnetic field in the CS action. The Hall resistivities are then derived by proceeding with a mean field (MF) ansatz which smears the flux lines to a uniform value of $b$. A study of the fluctuations about this MF state then immediately yields $\rho_{xy}$. We propose to extend this study to FT below. Finally, we shall argue that it is possible to extract those features that are common to all CS based theories which invoke the MF ansatz of which the CFM is but an instance.

Consider then a system of nonrelativistic spinless fermions in an external magnetic field of strength $B$ confined to the direction perpendicular to the plane of the system. The Jain proposal20 consists of introducing an internal CS magnetic field of strength $b$ such that the system sees an effective field $B_{\text{eff}} = |B + b|$. The next step is to attribute a strength $\mp (2\pi \rho/e)(2s)$ to the CS field and a strength $(2\pi \rho/e)(1/p)$ to $B_{\text{eff}}$, where $e$ and $\rho$ are the charge and density of the fermions respectively, and $s$ and $p$ are integers. This assignment immediately leads to a state of filling fraction $\nu = p/(2sp \pm 1)$. The study of such an effective system is accomplished through the following Lagrangian density,

$$\mathcal{L} = \psi^* iD_0 \psi - \frac{1}{2m^*} |D_k \psi|^2 + \frac{\theta}{2} \nu^\lambda \rho^\lambda \partial_\lambda a_\nu + \psi^* \mu \psi$$
\[-eA_{0}^{\text{in}}\bar{\rho} + \frac{1}{2} \int d^{3}x' A_{0}^{\text{in}}(x)V^{-1}(x-x')A_{0}^{\text{in}}(x') \, . \tag{1} \]

Here \( D_{\nu} = \partial_{\nu} - ie(a_{\nu} + A_{\nu} + A_{0}^{\text{in}}) \) (\( a_{\nu} \), \( A_{\nu} \), and \( A_{0}^{\text{in}} \) being the CS, external and internal Maxwell gauge fields respectively), \( \mu \) is the chemical potential, \( m^{*} \) is the effective mass of the fermions, and \( \theta = \pm(e^{2}/2\pi)(1/2\varepsilon) \) is the CS parameter. Finally, \( V^{-1}(x-x') \) represents the inverse of the instantaneous charge charge interaction potential (in the operator sense). The above Lagrangian density is equivalent to the usual four fermion interaction term as is considered by Lopez and Fradkin. Observe that the IQHE corresponds to the choice \( s = 0 \) (i.e., \( \theta = \infty \)) which implies a net mean zero value for the CS field. FQHE follows from the choice \( s = \pm 1 \) which is equivalent to \( \theta = 0 \) or \( \theta = 2\pi \) respectively, \( \mu \) being the inverse temperature.

\[
\rho_{\mu
u} = \frac{1}{2} \int d^{3}x d^{3}x' \left[ e^{-2\int_{0}^{\beta} d\tau \int d^{d}x L^{(E)}} \right] \, .
\]

which on integration over the fermionic fields, in the MF ansatz, factors into \( \mathcal{Z} = \mathcal{Z}_{MF} \mathcal{Z}_{f} \) after the usual saddle point computation. Here \( L^{(E)} \) is the euclidian version of \( L \). The MF part of the partition function is given by \((1/\Lambda) \ln \mathcal{Z}_{MF} = \rho_{t} \sum_{n=0}^{\infty} \sum_{j=-\infty}^{\infty} \ln \left[ e^{-\mu + \omega_{c}} \right] \), where \( \epsilon_{n} = (n + 1/2)\omega_{c}, (\omega_{c} = (e/m^{*})B_{\text{eff}} \) being the effective cyclotron frequency), is the energy corresponding to a th Landau level, \( \omega_{c} = (2j + 1)\pi/\beta \) is the Matsubara frequency, \( \rho_{t} = m^{*}\omega_{c}/2\pi \) is the degeneracy per unit area in each level, and \( A \) is the area of the system. The corresponding thermodynamic potential is obtained as \( \Omega/A = -(\rho_{t}/\beta) \sum_{n=0}^{\infty} \ln \left[ 1 + \exp(-\beta(\epsilon_{n} - \mu)) \right] \), from which all the MF properties can be inferred.

Writing the fluctuating part of the partition function as \( \mathcal{Z}_{f} = \int [da][dA][dA^{\text{in}}] \exp[-S_{\text{eff}}] \), (where we have expanded upto second order in the gauge field fluctuations around the MF configuration), we identify \( S_{\text{eff}} \) with the one-loop effective action which is given by

\[
S_{\text{eff}} = \int d^{3}x a_{\nu} \partial_{\nu} a_{\rho} + \frac{1}{2} \int d^{3}x \left( \partial^{\nu} A_{\mu} + A_{\nu}^{\text{in}} \partial_{\nu} A_{\mu} \right) + \frac{1}{2} \int d^{3}x \left( a_{\mu} + A_{\mu} + A_{\nu}^{\text{in}} \delta_{\mu\nu} \right) \times \Pi^{\mu\nu}(x,x')(a_{\nu} + A_{\nu} + A_{\nu}^{\text{in}} \delta_{\mu\nu} - 1) \int d^{3}x \int d^{3}x' A_{0}^{\text{in}}(x)V^{-1}(x-x')A_{0}^{\text{in}}(x') \, . \tag{3} \]

The current correlation functions \( \Pi^{\mu\nu}(x,x') \equiv (\langle j^{\mu}(x)\delta A_{\nu}(x') \rangle, j^{\mu} \) is the fermionic current, and \( A_{\nu} \) is the sum of all the gauge fields, have to be determined. Using Galilean and gauge invariance, we write (in the momentum space)

\[
\Pi^{\mu\nu}(\omega, q) = \Pi_{0}(\omega, q)(q^2g^{\mu\nu} - q^{\mu}q^{\nu}) + (\Pi_{2} - \Pi_{0})(\omega, q) \times (q^2\delta^{ij} - q^{i}q^{j})\delta^{ij} + i\Pi_{1}(\omega, q)e^{i\mu\lambda}q_{\lambda} \, , \tag{4} \]

with \( \Pi_{0} = \bar{\Pi}_{0} + \Gamma/q^{2} \). In the low \( q \) limit, we find the form factors to be

\[
\bar{\Pi}_{0} = \frac{e^{2}}{2\pi\omega_{c}} \sum_{n=0}^{\infty} f_{n} - \frac{e^{2}}{16\pi} \sum_{n=0}^{\infty} (2n + 1) \text{sech}^{2}\frac{\beta}{2\Omega_{n}} \, , \quad \Gamma = \frac{e^{2}m^{*}\omega_{c}}{8\pi} \sum_{n=0}^{\infty} \text{sech}^{2}\frac{\beta}{2\Omega_{n}} \, , \quad \Pi_{1} = \bar{\Pi}_{0} \omega_{c} \, , \quad \Pi_{2} = \frac{e^{2}}{2\pi m^{*}} \sum_{n=0}^{\infty} (2n + 1) f_{n} - \frac{e^{2}m^{*}\omega_{c}}{32\pi m^{*}} \sum_{n=0}^{\infty} (2n + 1) \text{sech}^{2}\frac{\beta}{2\Omega_{n}} \, , \tag{5} \]

where \( f_{n} = [1 + \exp(\beta\Omega_{n})]^{-1} \) and \( \Omega_{n} = \epsilon_{n} - \mu \). Note that the exclusively thermal form factor \( \Gamma \) has the interesting property that \( \Gamma = 0 \) for \( q^{2} = 0, \omega \to 0 \) and it is nonzero for \( \omega = 0, q^{2} \to 0 \). The FT properties of the system are driven by the temperature behaviour of these form factors. In particular, the parity and time reversal violation caused by the external magnetic field acts through the form factor \( \Pi_{1} \), which indeed controls the behaviour of \( \rho_{xy} \) with changing temperature.

The experiments are performed at temperatures in the range from 20 mK to a few K. We need to evaluate the form factors in this regime. If we are interested in extremely small deviations from the zero temperature value or in estimating whether the lowest temperatures reached or small enough, a low temperature (LT) expansion of the form
factors should suffice. In that case, they are analytically evaluated as a perturbation in \( \exp[-\beta \omega_c/2] \) (see Ref. 18 and 19 for details of calculation) and are found to be

\[
\Pi_0 = \frac{e^2 m^* p^2}{4\pi^2 \rho} (1 - 4y), \quad \Gamma = 4 \frac{e^2 m^*}{\pi},
\]

\[
\Pi_1 = \frac{e^2 p^2}{2\pi} (1 - 4y), \quad \Pi_2 = \frac{e^2 p^2}{2\pi m^*} (1 - 4y),
\]

where \( y \equiv (T_0/T) \exp[-T_0/T] \) with \( T_0 = \pi \rho/m^*p \). At higher temperatures, we need the exact values which can only be obtained numerically.

Given the form factors, a straightforward linear response analysis which involves (see Ref. 19 for procedure) the average over the internal fluctuations as well as a coupling to a weak external electric field, yields the Hall resistivity to be

\[
\rho_{xy}(\omega, \mathbf{q}) = \frac{\Pi_0(\Pi_0\omega^2 - \Pi_2\mathbf{q}^2) - (\Pi_1 + \theta)^2 - \Pi_0 V(q)q^2}{\Pi_0\theta(\Pi_0\omega^2 - \Pi_2\mathbf{q}^2) - \Pi_1\theta(\Pi_1 + \theta)}.\]

Note that the diagonal resistivity vanishes by virtue of the purity of the system. For the same reason, the quantizations occur at specific values of \( B \) which are recognized to be the central values of the plateaus seen experimentally. As remarked earlier, we concentrate on \( \rho_{xy}(T) \) at this central value.

Before we present our results, and compare them with experiments wherever possible, it should be noted that the behaviour of the form factors, and hence the response functions, has a crucial dependence on the choice of \( V(q) \). Recall that the Laughlin wave function which correctly describes the states with filling fractions given by \( \nu = 1/(2k + 1), \ k = 1, 2, \cdots, \) has been shown numerically by Haldane to be exact for a large class of short range repulsive potentials. It is clear from our FT analysis by Eqs. (4) – (7) that if the static conductivity \( \sigma^s = -1/\rho^s \), where \( \rho^s \equiv \rho_{xy}(\omega = 0, \mathbf{q}^2 \rightarrow 0) \), is to survive, then we require \( V(q) \rightarrow C \) (const.) as \( q^2 \rightarrow 0 \). If \( V(q) \) diverges as \( q^2 \rightarrow 0 \) as it could happen for potentials which are long ranged, i.e., \( V(r) \rightarrow 0 \) as \( r \rightarrow \infty \) slower than \( 1/r^2 \), then \( \sigma^s \) would have its support only at \( T = 0 \). Clearly, such interactions are ruled out from this analysis. Further \( V(q) \rightarrow C \neq 0 \) (as it would happen for \( V(r) \sim 1/r^2 \) or \( \delta(r) \)) is a threshold case in the sense that \( \sigma^s(T = 0) \neq 0 \), but is sensitive to the strength of the interaction. This means the universality has its support only at \( T = 0 \) with a strong dependence on strength at \( T \neq 0 \), which is again unphysical. We conclude that \( V(r) \) should be more short ranged, in confirmation with the analysis of Haldane and also Trugman and Kivelson who showed the exactness of Laughlin’s wave function for one such potential. Indeed, \( \sigma^s \) is then independent of \( V(q) \) as it should be by continuity requirement.

Note that if we define \( \rho^d \equiv \rho_{xy}(\omega \rightarrow 0, \mathbf{q}^2 = 0) \), \( \rho^s \neq \rho^d \). It has a different temperature evolution, but is again governed by \( T_0 \), which we shall discuss in the detailed paper.

In order to discuss the limit on accuracy of quantization imposed by temperature, consider \( \rho^s \) at very small values of \( y \). It has the analytic form

\[
\rho^s(T) = \rho^s(0) + 4(2\pi/e^2)p y \left[ 1 - \frac{4sp}{2sp \pm 1} \right],
\]

with \( \rho^s(0) = (2\pi/e^2)(2sp \pm 1)/p \). From the expression (8), it is clear that the temperature dependence is indeed accompanied by a corresponding deviation from universality in virtue of its dependence on the parameter \( T_0 \) which is the only sample specific parameter that enters the analysis. The argument is more robust. Indeed, at any temperature, although we can not evaluate \( \rho^s(T) \) analytically, it is easy to check that the Hall defect

\[
\mathcal{R} \equiv \left| \frac{\rho^s(T) - \rho^s(0)}{\rho^s(0)} \right|
\]

is a function of the dimensionless variable \( T_0/T \). This kind of dependence and the specific form of \( T_0 \) is a reflection of the MF ansatz which introduces the fundamental scale \( \omega_c \), the cyclotron frequency.

Having discussed these general features, we now consider IQHE in more detail. This is of great significance since at \( T = 1.8 \) K, the fine structure constant was measured to an accuracy of 5 ppm by von Klitzing et al. A further lowering of the temperature has led to an accuracy of 0.01 ppm suggesting that the universality is achieved asymptotically as \( \beta \rightarrow \infty \).

Fig. 1 shows how \( \mathcal{R}_0 \) (which is simultaneously a measure of both universality and quantization loss) evolves with temperature for an accuracy range 0.01 ppm to 1% for \( \nu = 1 \). In fact, we find that if we fix the value of \( \mathcal{R}_0 \), the temperature \( T_{\mathcal{R}_0} \) at which \( \mathcal{R}_0 \) is achieved follows a simple expression.
where the const. $C'$ depends only on $R_0$. For example, the values of $C'$ at $R_0 = 10^{-n}$ are approximately given by $0.325 + 0.776(n+1)$ for $3 \leq n \leq 8$. It would be extremely interesting if some or all of these predicted features can be verified experimentally. Any confirmation of a dependence of $T_{R_0}$ on $\rho/m^*$ or of scaling with $p$ would be a striking vindication of the MF ansatz. In any case, we note that a requirement of higher accuracy at any given temperature demands correspondingly higher values of $\rho/m^*$. Further, it is also clear that the temperature scales with $\rho/m^*$ for a fixed $R_0$ and $p$.

Yoshihiro et al.\cite{28} report that $\Delta \rho^s = \rho^s(T) - \rho^s(0)$ has the form $c \rho^s_{\min}$ (with $c = -0.1$) over a temperature range $0.5 \text{ K} - 1.6 \text{ K}$, and for the densities in the range $1 - 3 \times 10^{12} \text{ cm}^{-2}$. We are not in a position to make any comparison with this experimental observation. However, our analysis shows that $\Delta \rho^s \neq 0$ even if $\rho_{xx} = 0$. Further at LT, the accuracy can be expressed in a simple form $R_0 = 4y$ which is obtained by putting $s = 0$ in Eqs. (8 and 9), as is appropriate here. The measured accuracy of $0.2 \text{ ppm}$ by Yoshihiro et al.\cite{28} at the lowest temperature mentioned above, i.e., at $T = 0.5 \text{ K}$ in the above range of densities and for a narrow range of $B = 9 - 10.5 \text{ T}$ for the states $p = 4, 8$ and 12 is consistent in our analysis as it yields a reasonable value of $m^* \approx 0.7 m_e$.

However, we have a later measurement of $\rho^s(T)$ by Cage et al.\cite{21} at $T = 3 \text{ K}$ and $1.2 \text{ K}$. The corresponding experimental values of $R_0$ are $4.2 \text{ ppm}$ and $0.017 \text{ ppm}$ for $p = 4$. The values of $\rho/m^*$ may again be obtained using Eq. (14) and they turn out to be $63.3 \text{ K}$ and $34.2 \text{ K}$ respectively. The rather significant difference in the value of $\rho/m^*$ obtained possibly indicates that the role of impurities becomes more important (as indeed the experimentalists find) at such relatively higher temperatures, especially since $\rho_{xx}$ has a strong dependence on $T$.

Finally, before we go on to discuss FQHE, we observe that the experimental results can be used to place an upper limit on the value of $\rho/m^* p$ within the model considered here. These are summarized in Table I.

We now discuss FQHE for which there are some data available\cite{28} on the slope of the plateau. Both $\rho_{xx}(T)$ and $\rho_{xy}(T)$ have also been measured, the former primarily for the purpose of extracting the gap energy $E_g$. Theoretically, the temperature dependence of $R$ in this case is given by Eqs. (8 and 9) as a functions of $s$ and $p$. Similar to (10), for a fixed $R_{\pm}$, the scaling is now generalized to

$$\frac{T_0}{T} e^{-\frac{T_0}{T}} = \frac{R_{\pm} (2sp \pm 1)^2}{4 - 2sp \pm 1},$$

where $R_+ = (2sp - 1)4y/(2sp + 1)^2$ and $R_- = 4y(2sp + 1)/(2sp - 1)^2$ are the accuracies corresponding to the states with antiparallel (‘+’) and parallel (‘−’) flux attachments. Clearly, for a fixed $s$, and $p$, ‘+’ states will be seen with larger accuracy than ‘−’ states at the same sample. For example, $\nu = 3/7$ state should be seen at a higher accuracy than $\nu = 3/5$ state, which is indeed true as it has been seen experimentally\cite{28}. Fig. 1 shows how the temperatures $T_{R_{\pm}}$, (for which the accuracies are $R_{\pm}$) vary with $R_{\pm}$ for $(2,1)_+ (2,1)_-$ states, where we have used the notation $(p, s)_{\pm}$ to denote the states.

Chang et al.\cite{21} report that at $65 \text{ mK}$ and for $p = 2.1 \times 10^{11} \text{ cm}^{-2}$, the quantization at $\nu = 5/3$ has an accuracy of 1.1 parts in $10^6$ and at $\nu = 2/3$, it has an improved accuracy of 3 parts in $10^4$. Our theoretical estimate at $\nu = 2/3 ((2,1)_-)$ yields $m^* = 3.9 m_e$. Further, the estimates of $m^*$ from CFM for different filling fractions using the results (measured at $T = 90 \text{ mK}$) obtained by Chang et al.\cite{21} are shown in Table II. Clearly, the estimates are several times larger than the realistic value. The over estimations possibly indicate a more decisive role of impurities in FQHE, in contrast to the integral case. In this context, we observe that Du et al.\cite{28} find that $m^*$ for states with parallel flux attachment is different from those otherwise. We speculate that while Eq. (14) is correct in essence, the right hand side (RHS) will possibly get scaled by such an effect, compensating for the overestimation of $m^*$. The situation would become clearer after a proper study is made with the impurities and other effects put in, as has been done by Halperin, Lee and Read\cite{22} at $T = 0$ in vicinity of $\nu = 1/2$.

We now make a few qualitative observations. Although we are not able to calculate the slope of the plateau as a function of temperature, we report that we find a reasonable agreement between $\omega_c$ and $E_g$, which is measured by Du et al.\cite{28}. Recall that $\omega_c$ is otherwise an MF artefact. If we take a rather naive view point that the deviation from the quantization is proportional to the deviation from the zero slope, we are then in a position to compare a host of experimental results with the theoretical prediction. If we assume that the threshold accuracy for a plateau to be seen is a minimum of $0.1\%$, we report here that the temperatures that we estimate are completely consistent with the temperatures at which these levels have been seen experimentally.

Finally, Fig. 2 shows the compressibility as a function of temperature for both the quantum fluids (which are incompressible at $T = 0$) for various values of filling fractions. The generalized expression for the compressibility is given by $k = (1/e^2)\tilde{\rho}^2 \Gamma [\Gamma^2/2 + (\Pi_1 + \theta)^2]$, which, for IQHE (i.e., for $\theta \to \infty$), reduces to $k = \Gamma/e^2 \rho^2$. The smooth behaviour of $k$ with temperature up to a value $5 \text{ K}$ clearly shows that there is no phase transition involving these fluids. The same conclusion has been arrived by Chang et al.\cite{21} by their measurement $\rho_{xx}(T)$ for $\nu = 2/3$. 

\begin{align}
\frac{\rho}{m^* pT_{R_0}} = C',
\end{align}
To conclude, we reiterate that what is at the heart of our analysis is the CS interaction and the MF ansatz. Since all MF arguments yield an appropriate $\omega_c$ as a natural energy scale, it is clear that the dependence on $\rho/m^*$ at FT must be a common feature of these models. In particular, Eqs. (10,11) for $R$ must hold with the constant on RHS depending on the particulars of the model. Thus, any experimental verification of the relations (10,11) would shed light on the validity of the MF ansatz in general. A precise information of the RHS will hopefully allow us to discern amongst various models and would complement measurement of other experimental results.

Note: It has recently come to our notice that Zhang has studied FQHE at finite temperatures with emphasis on a study of the collective excitations. The preprint is duly referred.

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FIG. 1. The temperatures $T_R$ as a function of accuracy shown for (a)$\nu = 1$, (b)$\nu = 2/5$ and (c)$\nu = 2/3$ for a typical value of $\rho/m^* = 20$ cm$^{-1}$.

FIG. 2. compressibilities are shown for (a)$\nu = 1$, (b)$\nu = 2/5$ and (c)$\nu = 2/3$ as function of temperatures for a typical value of $\rho/m^* = 20$ cm$^{-1}$.

| Reference Number | $R_0$ (ppm) | $T$ (K) | $\rho$ (10$^{12}$ cm$^{-2}$) | $p$ | estimated $\rho/m^* p$ (K) | estimated $m^*(m_e)$ |
|------------------|--------------|---------|-----------------------------|-----|---------------------------|---------------------|
| 1                | 5.0          | 1.8     | -                           | -   | 9.39                      | -                   |
| 21               | 0.2          | 0.5     | 1.0                         | 4   | 3.15                      | 0.7                 |
| 22               | 4.2          | 3.0     | -                           | -   | 15.83                     | -                   |
| 22               | 0.017        | 1.2     | -                           | -   | 8.55                      | -                   |

TABLE I. estimated values of $\rho/m^* p$ and hence $m^*$ from experimental data.

| $\nu$ | $(p, s)_\pm$ | $R_\pm$ (10$^{13}$ cm$^{-2}$) | $\rho$ (10$^{11}$ cm$^{-2}$) | $m^*(m_e)$ |
|-------|--------------|-------------------------------|-----------------------------|------------|
| 1/3   | $(1, 1)_+$   | $3.0 \times 10^{-3}$          | 1.53                        | 3.89       |
| 2/3   | $(2, 1)_-$   | $3.0 \times 10^{-5}$          | 2.42                        | 2.69       |
| 2/5   | $(2, 1)_+$   | $2.3 \times 10^{-4}$          | 2.13                        | 2.28       |
| 3/5   | $(3, 1)_-$   | $1.3 \times 10^{-3}$          | 2.13                        | 2.54       |
| 3/7   | $(3, 1)_+$   | $3.3 \times 10^{-3}$          | 2.13                        | 3.25       |

TABLE II. estimated values of $m^*$ for different filling fractions from experimental data of Ref. 24.