Electron transport through a mesoscopic metal-CDW-metal junction

I. V. Krive and A. S. Rozhavsky,
B.I. Verkin Institute for Low Temperature Physics and Engineering, Kharkov, Ukraine

E. R. Mucciolo and L. E. Oxman
Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro,
Caixa Postal 38071, 22452-970 Rio de Janeiro, Brazil
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In this work we study the transport properties of a finite Peierls-Fröhlich dielectric with a charge density wave of the commensurate type. We show that at low temperatures this problem can be mapped onto a problem of fractional charge transport through a finite-length correlated dielectric, recently studied by Ponomarenko and Nagaosa [Phys. Rev. Lett 81, 2304 (1998)]. The temperature dependence of conductance of the charge density wave junction is presented for a wide range of temperatures.

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I. INTRODUCTION

The problem of charge transport through an inhomogeneous system (e.g., a dielectric junction between two metallic leads) continues to attract considerable attention from both purely theoretical and practical points of view. It is especially interesting in the case of a one-dimensional (1D) junction where the interaction (electron-electron or electron-phonon) drastically changes the system properties. The most studied example is a 1D quantum metallic wire connected to 2D electron reservoirs. It is known that the low energy properties of electrons in the wire can be described by the Luttinger liquid (LL) model. Since electrons do not propagate freely in a LL, a nontrivial problem arises: How are conduction electrons converted into propagating charged excitations at the metal-LL boundary?

A few years ago, it was shown that, for adiabatic contacts (i.e., when all the inhomogeneities are smooth on a scale of the electron Fermi length), the dc conductance of a pure quantum wire is perfect, \( G_0 = 2e^2/h \), and temperature independent (the factor of two accounts for spin degeneracy). This result is not surprising since electrons coming from the leads are not backscattered by the effective barrier represented by the LL piece of the wire. The linear conductance, according to the Landauer-Büttiker approach, should then be equal to the conductance quantum \( G_0 \). Therefore, the dc conductance of a pure quantum wire in adiabatic contact with source and drain leads is not renormalized by the interaction.

The electron-electron interaction does affect the current-voltage characteristics of a LL junction if either impurities are present or electrons are backscattered by Umklapp (U)-processes. U-processes lead, as a rule, to the formation of a gap in the spectrum of charged excitations and in this case one may consider the junction as a dielectric wire.

Recently, Ponomarenko and Nagaosa studied electron transport through a finite Mott-Hubbard dielectric. By mapping the problem at low temperatures onto the quantum impurity problem in an infinite LL (the so-called Kane-Fisher problem) they obtained a surprising result: The conductance of a finite-length correlated dielectric, adiabatically connected to metallic leads equals, at \( T = 0 \), the conductance of a perfect metal junction \( G_0 \). However, an exponentially small conductivity, typical of a tunnel junction, is restored already at very small temperatures \( T_i \sim \Delta_L \exp(-2\Delta/\Delta_L) \), where \( \Delta \) is the Mott-Hubbard gap and \( \Delta_L = \hbar v_p/L \) (\( L \) is the length of dielectric and \( v_p \) is the velocity of the charged excitations). This result was later generalized to the transport of fractional charge through a finite-length correlated dielectric.

The purpose of the present paper is to consider the electron transport through a junction formed by a Peierls-Fröhlich dielectric where a charge density wave (CDW) is present. The problem of conversion of electronic current into CDW current at the boundary metal-Peierls dielectric was studied for the first time over a decade ago. In Ref. 3 (see also Ref. 10) the Peierls dielectric was treated in the so-called adiabatic approximation, when the dynamics of the electrons is considered for a fixed phonon field (the order parameter), which is found self-consistently (for a review, see Refs. 11 and 12). For an order parameter with a time-independent phase, the transport problem can be reduced to solving the Bogoliubov-de Gennes equation for the electron wave function in the piecewise constant effective field of the order parameter. The calculations are very close to those for a metal-superconductor junction. There is only one important difference: While in a superconductor the condensate wave function is formed by two particles with opposite momenta, in a Peierls state the corresponding spinor consists of a particle and a hole with momenta differing by \( 2k_F \). As a result, the scattering of particles on the off-diagonal barrier produced by the order parameter is not of the Andreev type. For electrons in
the metallic leads with energies inside the gap there is always a strong backscattering. It is then evident that the quasiparticle contribution to the conductance will be exponentially small, \( G \propto \exp(-2\Delta_0/\Delta_L) \), for temperatures and voltages less than \( \Delta_0 \) (\( \Delta_0 \) is the Peierls gap at \( T = 0 \)).

One should not forget, however, that there exists another channel for charge transport in Peierls-Fröhlich systems. Namely, the electronic current coming from the metallic leads can be converted into a CDW current which freely propagates through the junction. The process of conversion is perfect when it involves an incommensurate CDW and the contacts between the Peierls dielectric and the metallic leads are adiabatic. In other words, when the phase degree of freedom of the order parameter in the Peierls dielectric is correctly treated, one finds that the conductance of an impurity-free adiabatic CDW junction is perfect at low temperatures. This result was proved in Refs. 1 and 3 using different approaches.

In Ref. 4 we related the property of perfect conductance of an incommensurate CDW junction to the existence of an anomalous chiral symmetry in the system. In this paper we focus our attention instead on the commensurate case, when the electron filling factor is a rational number and no chiral symmetry exists.

We show that quantum fluctuations make the conductance of a commensurate, finite Peierls dielectric perfect at \( T = 0 \), much in analogy to the case of a Mott-Hubbard dielectric. As temperature is increased, however, the conductance rapidly decreases (we call this the soliton regime of quantum transport).

The perfect conductance is lost at temperatures \( T \sim \Delta_i \sim \sqrt{\Delta_{CDW}^0 E^*_s \exp(-E^*_s/\Delta_L)} \), where \( \Delta_{CDW}^0 = h\nu_c/L \), \( \nu_c \) is the velocity of the CDW, and \( E^*_s \) is the rest energy of quantum CDW solitons. For higher temperatures, quantum phase fluctuations are suppressed and the conductance scales approximately as \( T^{-2} \), reaching a global minimum value \( G_i \propto G_0 \exp(-2E^*_s/\Delta_{CDW}^0) \) at temperatures \( T \sim \sqrt{\Delta_{CDW}^0 E^*_s \gg \Delta_{CDW}^0} \). A further increase of temperature induces the appearance of fractionally charged solitons which dominate the conduction in the system (we call this the soliton regime of quantum transport). Thus, at temperatures \( T \sim E^*_s \) the conductance sharply increases, saturating at the value \( G_M = G_0/M^2 \) (\( M \) is the commensurability index), which also corresponds to a perfect transport, but of fractionally charged solitons \( (q_s = e/M) \) instead of bare electrons. This step-like behavior in \( G(T) \) terminates at \( T \sim \Delta_0 \), where the conductance rapidly reaches the pure metallic value \( G_0 \).

It is important to remark that the Mott-Hubbard and the Peierls-Fröhlich systems are not identical. There are two energy scales in the latter which are not present in the former. These scales are the phonon energy \( \hbar\omega \) (here \( \omega \equiv \omega(2k_F) \) denotes the frequency of the phonons with momentum \( \pm 2k_F \)) and the rest energy \( E^*_s \) of quantum solitons of the commensurate CDW. In principle, the existence of these additional scales could lead to a more complicated temperature dependence of the conductance than that found in Refs. 1 and 3. However, we will argue that the phonon energy scale, in particular, does not lead to any new features.

This paper is organized as follows. In Sec. II we introduce the effective phase Lagrangian that describes commensurate CDW junction. The conductance and its temperature dependence in the instanton regime are discussed in Secs. IV and V, the soliton regime is explored in Sec. VI. Final remarks and the conclusions are presented in Sec. VII.

II. THE PEIERLS-FRÖHLICH SYSTEM

The low-energy dynamics of a one-dimensional electron-phonon system is normally governed by the Lagrangian:

\[
\mathcal{L} = -\frac{\Delta^2}{g^2} + \frac{\hat{\Delta}^2 + \Delta^4}{\omega^2} + \bar{\psi}(i\hbar\gamma_{\mu}\partial^\mu - \Delta e^{i\gamma_5\varphi})\psi
+ \mathcal{L}_{\text{com}}.
\]

Here, \( u(x,t) = (\Delta/2)\cos(2k_F x + \varphi) \) is the magnitude of the phonon field with momentum \( Q \approx \pm 2k_F \) and \( g \) is the bare constant of the electron-phonon interaction. We have defined \( \psi \equiv \psi^{\dagger}\gamma_0 \), with \( \psi^{\dagger} = (\psi^L, \psi^R) \) comprising left- and right-moving electron fields. The Dirac matrices \( \gamma_\mu (\mu = 0, 1) \) and \( \gamma_5 \) in 1D are represented by the Pauli matrices, whereas the partial derivatives follow the convention \( \partial_\mu \equiv (\partial_x, v_F \partial_x) \). The last term in Eq. (1) represents the commensurability energy that exists for systems with rational electron filling \( \nu = N/M \) with respect to the underlying lattice model \( (N < M) \). The commensurability index is defined as \( M = \pi N/k_F a > 2 \), where \( a \) is the lattice constant. The appearance of the commensurability energy is due to \( M \)-fold Umklapp scattering. This term will be specified later.

The interacting 1D electron-phonon system model presented in Eq. (1) has already been studied by many authors (e.g., see reviews in Refs. 2 and 3). It was shown that a gap \( \Delta_0 \) develops in the electron spectrum at low temperatures. For spin-1/2 fermions and for a vanishingly small phonon frequency \( \omega \rightarrow 0 \), even a weak electron-phonon interaction produces a gap. Let us, for simplicity, consider only spinless electrons. In this case, for a finite \( \omega \), the electron-phonon interaction should be sufficient to produce a gap while, at the same time, the junction can be sufficiently long to make the influence of finite-size effects on the gap negligible.

In the classical Peierls theory, where all fluctuations are neglected, the gap for a weak electron-phonon coupling \( (\lambda = 2g^2/\pi\hbar v_F \ll 1) \) takes the familiar BCS-like form \( \Delta_0 \propto \varepsilon_F \exp(-1/\lambda) \), where \( \varepsilon_F \) is the electron Fermi energy. For a quasi-1D systems, this gap is the modulus of the order parameter \( \Delta_0 \) of the Peierls state which develops at temperatures \( T < T_P \) (the Peierls transition
temperature $T_p$ is determined by the strength of inter-chain interaction and, usually, $T_p \ll \Delta_0$. The fluctuations of the phase field $\varphi$ in a pure 1D system destroy the long-range order. Nevertheless, the fact that $\Delta_0 \neq 0$ minimizes the energy of the system shows that at $T \ll \Delta_0$ the fluctuations of the modulus of the phonon field are strongly suppressed.

In what follows we will assume that the ground state of a sufficiently long ($L \gg \hbar v_F/\Delta_0$) CDW junction is characterized by a gap $\Delta_0$ in the single-particle spectrum. This gap will be the largest relevant energy in the problem ($\hbar \omega \ll \Delta_0$).

To proceed further, it is convenient to bosonize the fermion degrees of freedom in Eq. (1): according to standard rules, the electron-phonon interaction exists only in the region $-L/2 < x < L/2$ and is adiabatically switched off outside the junction. To model the metal-CDW-metal junction one can multiply the last three terms in Eq. (3) by the step function $\Theta(x + L/2)\Theta(L/2 - x)$, confining the electron-phonon interaction to the finite region of length $L$.

For a homogeneous system, the model represented by Eq. (4) was studied in Ref. 7 for two different cases: (i) at a fixed phase of the phonon field, $\varphi = \text{const}$; (ii) for an “adiabatic” CDW motion, when $\varphi(x, t) = \eta(x, t) + \text{const}$. As we will see in what follows the last ansatz leads to the correct quantum description of the CDW. Thus it can be called the quantum regime of the CDW dynamics.

The physical motivation for studying the regimes mentioned above is the following. Both cosine terms in Eq. (3) describe the backward scattering of electrons by the lattice distortions. Notice that the term arising from the multiple Umklapp processes enters into Eq. (3) with a coefficient much smaller than that for the direct electron-phonon interaction. Therefore, in the case of a fixed phase [ansatz (i) above], one can neglect the effects of commensurability and Eq. (3) is reduced to the bosonic form of a Lagrangian describing free massive Dirac fermions (with the gap equal to $\Delta_0$). It gives us the Bogoliubov-de Gennes equations for the electron wave function in the piecewise constant field of the order parameter. So the ansatz (i) corresponds to the situation when the dynamics of CDW is totally ignored. It describes only the quasiparticle contribution to the conductivity which at temperatures $T \ll \Delta_0$ is exponentially small and thus can be neglected.

At low temperatures one cannot neglect the quantum fluctuations of the phonon field $\varphi(x, t)$ and a quantum description of the CDW dynamics has to be adopted. The correct formulation assumes that the electrons are in “adiabatic motion” with the lattice distortion [ansatz (ii)]. In the Appendix we justify this assumption by considering a simplified field-theoretical model for the CDW. For the ansatz (ii) the most troublesome term ($\sim \varepsilon_F$) in Eq. (3), which arises from the direct backscattering of electrons by phonons, disappears already at the classical level. The resulting model is equivalent to the theory of a commensurate CDW alone. For interacting electrons in Eq. (3), the corresponding Lagrangian takes the form

$$L_{CDW} = \frac{\hbar}{8\pi K_F v_c} \left[ (\partial_t \varphi)^2 - v_F^2 (\partial_x \varphi)^2 + \frac{2\Delta_0^2}{g^2 \omega^2 M^2} \cos (\eta - \varphi + M \varphi) \right],$$

where $\varphi$ is the charge velocity and $K_F$ is the bare correlation parameter. These quantities can be expressed in terms of the bare interaction constants (e.g., see Ref. 22).

We will use the Lagrangian density of Eq. (4) to analyze the electron transport through a finite CDW junction of the length $L$ (see Fig. 3). Let us assume that
and
\[
\Omega^2 = \frac{8\pi K_c v_c}{\hbar} \left( \frac{\Delta \omega_0}{g \omega} \right)^2 .
\] (10)

For noninteracting electrons \((K_p = 1\) and \(v_p = v_F)\), Eq. (9), taken in the limit \(v_F \Delta_0^2 \gg g^2 \omega^2 \hbar\), coincides with the well-known Lagrangian for a commensurate CDW. The corresponding parameters (CDW velocity and pinning energy) are exactly the same ones found in the standard approach by integrating out the fermionic degrees of freedom in Eq. (9). Namely,
\[
v_c \simeq v_F \sqrt{\frac{g^2}{8\pi \hbar v_F}} \left( \frac{\hbar \omega}{\Delta_0} \right) \ll v_F \quad \text{with} \quad \Omega \simeq \omega_0 .
\] (11)

From a mathematical point of view, Eq. (9) is a sine-Gordon theory with well-known quantum properties. Notice that in the regime of quantum transport our model [Eq. (9)], coincides (up to the bare parameters \(K_c, v_c\), and \(\Omega\)) with the Lagrangian derived in Ref. 26. Therefore, the low temperature properties of a CDW junction should be similar to those of a correlated dielectric. Our analysis of the conductance of a CDW junction can now be performed in analogy to that in Ref. 26. Thus we will be brief in the mathematical details and will concentrate on the physical interpretations of our main results.

### III. The Instanton Contribution to the Conductance

In an infinite system, the sine-Gordon model of Eq. (9) is characterized by an infinite set of equivalent vacua \([|0\>_n = |2\pi n/M\>_n\) \((n\) is integer). In a finite 1D system the ground state is nondegenerate and if the length of the system is sufficiently long, a unique vacuum can be found in the dilute instanton gas approximation. The instanton approach to the thermodynamics of finite CDW system was put forward for the first time in Refs. 24 and 25, where the partition function for a CDW ring threaded by a magnetic field was calculated. Vacuum-to-vacuum tunneling introduces a new energy scale
\[
\Delta_t \simeq \frac{\hbar v_c}{L} \sqrt{\frac{S_0}{2\pi \hbar}} \exp \left( -\frac{S_0^\circ}{\hbar} \right) .
\] (12)

\((S_0^0\) is the one-instanton tunnel action\), which is nothing but the width of the instanton band. In terms of the bare parameters shown in Eqs. (7), (8), (9), and (10) the one-instanton action reads
\[
S_i^0 = \frac{E_s L}{\hbar v_c} = \frac{1}{K_c M^2} \frac{2 \hbar \Omega}{\pi \Delta_{CDW}} .
\] (13)

where \(E_s\) is the rest energy of the classical kink.

In the full quantum description the bare parameters of the sine-Gordon model are renormalized by the interaction. For the gapped phase the renormalized parameters are \(K_c^*, M^2 = 1\) (see also the review Ref. 26)

\[
K_c^* M^2 = 1 \quad \text{with} \quad E_s \rightarrow E_s^* .
\] (14)

where \(E_s^*\) is the energy of the quantum soliton. As it was noticed in Ref. 27, this renormalization is equivalent to the summation of all multi-loops contributions to the renormalized one-instanton tunnel action \(S_i^0 \rightarrow S_i = E_s^*/\Delta_L\). While the second prescription in Eq. (14) is physically evident, the first one needs some comment. The simplest way to understand this result (so-called Luther-Emery free fermion point\([23]\) is to consider the situation when the chemical potential exceeds the gap. Let us at first rescale the field \(\phi \rightarrow \varphi / M\) in Eq. (9). Then the same Lagrangian with the renormalized parameters \((K_c \rightarrow K_c^*, \Omega \rightarrow \Omega^*)\) has to describe the quantum phase of the sine-Gordon model. In the case when chemical potential exceeds the gap \(E_s^*\), the system can be treated as a gas of weakly interacting solitons at density \(\rho_s\). It is then described by the first two terms of the rescaled Lagrangian
\[
L_s = \frac{1}{8\pi \hbar v_c K_c^*(\rho_s) M^2} \left[ (\partial_t \varphi)^2 - v_c^2 (\partial_x \varphi)^2 \right] .
\] (15)

In the vicinity of a commensurate-incommensurate phase transition \((\rho_s \rightarrow 0)\), the quantum solitons are noninteracting particles. In the bosonic language, it means that \(K_c^*(0) M^2 = 1\) (see also Ref. 26). The model in its gapped phase, at this point, is equivalent to spinless massive Dirac fermions.

The crucial difference between the closed CDW system studied in Refs. 24 and 25 and a finite Peierls-Fröhlich conductor connected to metallic leads is in the boundary conditions imposed by the electron reservoirs on the CDW dynamics. These boundary conditions can be derived by integrating out electrons outside the CDW piece of the junction. As it was shown in Ref. 26 this procedure results in a Caldeira-Leggett (CL) action\([26]\) for the boundary CDW field \(\varphi(x = \pm L/2, t)\). Physically, it means that electrons in the leads induce friction (which appears as a logarithmic interaction between instantons\([23]\) in the vacuum-to-vacuum tunneling. Although, numerically, the logarithmic interaction only changes the exponential prefactor in the tunnel energy shift \([Eq. (12)]\), it drastically affects the instantons trajectories with a nonzero total topological charge. The action taken on these paths diverges, and only trajectories with zero net charge contribute to the partition function\([14]\). This very property allows one to use a dual representation\([26]\) to studying transport properties of CDW junction at low temperatures. In terms of the Josephson-like field \(\Theta(x, t)\), dual to the CDW field \(\varphi(x, t)\), the Lagrangian which yields the same partition function as that calculated in the dilute instanton gas approximation takes the form (see Ref. 26)

\[
\hat{L} = \frac{\hbar}{8\pi \hbar v_c} \left[ (\partial_t \hat{\Theta})^2 - v_c^2 (\partial_x \hat{\Theta})^2 \right]
\]
\[ -\Delta_i^{(R)} \delta(x) \cos \left( \frac{\Theta(x, t)}{M} \right). \] (16)

Here \( \Delta_i^{(R)} \) is the instanton shift of the vacuum energy, Eq. (12), renormalized by the friction. The \( 1/M \) factor in the last term is needed to take into account the fractional topological charge \( q_i = 2\pi/M \) of individual instantons. The induced CL-action does not introduce any new dimensional parameter to the problem and affects only the prefactor in Eq. (12) (corrected by the multiloop contributions, Eq (13)). Thus, up to an irrelevant numerical factor, we have \( \Delta_i^{(R)} \simeq \Delta_i \) (friction, of course, could only diminish the one-instanton vacuum energy shift).

After rescaling the dual field \( \Theta/M \Rightarrow \Theta \), Eq. (16) is transformed into the Lagrangian for a quantum impurity problem in the infinite LL (the Kane-Fisher problem) with a correlation parameter \( K_c = 1/M^2 \). The desired conduction in the initial problem (CDW-junction) is related to the dimensionless conductance \( g_{KF} \) of the dual problem by a simple expression:

\[ G_i(T) = \frac{e^2}{h} \left[ 1 - M^2 g_{KF}(T) \right]. \] (17)

Notice the factor \( M^2 \) in Eq. (17). In order to map the dual model, Eq. (16), onto the known problem, we rescaled the dual field \{\( \Theta(x, t) \)\}. Since, in general, the conductance is proportional to the square of the dynamical field, the dimensionless conductance of the dual problem is \( M^2 g_{KF} \). Physically, the additional factor of \( M^2 \) originates from the correct definition of the Josephson-like current in the model of Eq. (16). Namely, it comes from the \( M \)-fold backscattering current induced by the potential difference \( MV \), where \( V \) is the voltage across the junction.

The quantity \( g_{KF}(T) \) is known exactly (e.g., see Ref. 16, where the current-voltage dependence for the Kane-Fisher problem was obtained by using the dual transformation method). For the CDW system the commensurability index \( M \) is an integer larger than 2 and the corresponding correlation parameter is small, \( K_c \ll 1 \). In this case, the low- and high-temperature asymptotics of the conductance take a simple form (up to irrelevant numerical constants)

\[ M^2 g_{KF}(T, K_c \ll 1) \simeq \begin{cases} \frac{2}{\sqrt{K_c}} \Delta_i^2, & T \ll \sqrt{K_c} \Delta_i, \\ 1 - \frac{2}{\sqrt{K_c}} \Delta_i^2, & T \gg \sqrt{K_c} \Delta_i, \end{cases} \] (18)

According to Eqs. (17) and (18), the conductance of a metal-CDW-metal junction is perfect at \( T \to 0 \). Loosely speaking, very slow (low-energy) electrons from the metal leads, when arriving at the CDW junction, see a strongly fluctuating, translationally invariant electron-phonon system and, consequently, are not backscattered by the lattice distortions. A significant backscattering appears when the Heisenberg time \( t_H \sim \hbar/\varepsilon \) for quasiparticles coming from leads becomes comparable or smaller than the characteristic lifetime of the perturbative vacuum \( (\sim \hbar/\Delta_i) \). It is clear that at \( T > \Delta_i \), the instanton mechanism of charge transport predicts a strongly suppressed conduction.

At finite temperatures, however, there is a mechanism competing with the instanton contributions to charge transport in CDW systems. It is associated with the thermally excited fractionally charged solitons of the commensurate CDW.\[ \text{The soliton contribution to the persistent current in a CDW ring was considered in Refs. 24 and 25.} \] Thus, we now proceed to calculate the soliton part, \( G_s \), of the conductance of a mesoscopic metal-CDW-metal junction.

**IV. THE SOLITON CONTRIBUTION TO THE CONDUCTANCE**

It is physically evident that at sufficiently “high” temperatures, \( T > \Delta_L \), the transport coefficients under consideration are determined by the bulk properties of a Peierls- Fröhlich system. In the quantum regime of transport a commensurate CDW is described by a quantum sine-Gordon model [the Lagrangian in Eq. (7) at the point \( K_c^* M^2 = 1 \)]. It is known\[ \text{that this point corresponds to noninteracting Dirac fermions. Thus, at temperatures } T \gg \Delta_L \text{ the problem of electron transport through a CDW junction can be mapped onto the well-known problem of transport of Dirac fermions.} \] The latter in its turn is mathematically equivalent to that of quasiclassical transport through a CDW junction.\[ \text{The only important distinction is that in our case the electric charge of the Dirac fermions is fractional } q_s = e/M \text{ (they are solitons of the quantum sine-Gordon model).}\]

The soliton contribution to the conductance can be calculated using Landauer formula:

\[ G_s(T) = \frac{\hbar^2}{e^2} \int_{-\infty}^{\infty} d\varepsilon \left( -\frac{\partial f_{FD}}{\partial \varepsilon} \right) T_i(\varepsilon), \] (19)

where \( f_{FD} \) is the Fermi-Dirac distribution function and \( T_i(\varepsilon) \) is the transmission probability of massless fermions through a “gapped” region. The transmission coefficient can be found by matching the wave functions at the boundary points \( x = \pm L/2 \). The result is (see also Ref. 5)

\[ T_i(\varepsilon) = \frac{\varepsilon^2 - E_s^*}{\varepsilon^2 - E_s^* + E_s^* \sin^2 \left( \sqrt{\varepsilon^2 - E_s^*/\Delta_L} \right)}, \] (20)

By carrying out the integral in Eq. (19), it is easy to find the following low- \( T \) \( (T \ll E_s^*) \) and high- \( T \) asymptotes

\[ G_s(T) \simeq \begin{cases} \frac{e^2}{16\pi^2} \exp \left( \frac{-2E_s^*}{\Delta_L} \right), & \Delta_L \ll T \ll E_s^* \\ \frac{e^2}{16\pi^2} \left( 1 - 2\pi \frac{E_s^*}{\Delta_L} \right) T, & T \gg E_s^*/\Delta_L. \end{cases} \] (21)
As one can see from Eq. (21), the soliton conductance at low temperatures is exponentially small and temperature independent. It corresponds to the tunneling of fractionally charged particles through a dielectric region. At high temperatures \((T > E_s^*)\), the thermally excited solitons and antisolitons yield a perfect (in terms of the fractional charge \(q_s\)) conductance \(G_M = q_s^2/h\).

The total conductance can be represented by the interpolative formula \(G(T) \approx G_s(T) + G_0(T)\). This is schematically shown on Fig. 2. From Eqs. (17) and (18) one can readily find that the “instanton” part of the conductance matches the soliton contribution, Eq. (21), at temperatures \(T_m \sim \sqrt{\Delta L E_s^*} \gg \Delta L\). As a result, in a wide temperature interval \(T_m < T < E_s^*\) the conductance is exponentially small and practically temperature independent, namely, \(G_m \sim G_0 \exp(-2E_s^*/\Delta L)\). This trough-like shape of the \(G \times T\) curve changes at \(T \sim E_s^*\) to the step-like form, with \(G \approx G_M = G_0/M^2\) which characterizes the transport of fractional charge along the CDW junction.

V. FINAL REMARKS

It is evident from the above considerations that in the case of an incommensurate CDW, where the last term in Eq. (3) is absent, the temperature dependence of the dc conductance is much simpler than for a commensurate CDW. The quantum theory of an incommensurate CDW is equivalent in many aspects to a theory of a Luttinger liquid. So, for adiabatic contacts the conductance of an incommensurate CDW junction equals \(G_0\) and it is temperature independent. For a commensurate CDW the step in the conductance which corresponds to the soliton mechanism of conductivity will last (for a purely 1D system) up to temperatures of order \(\Delta_0\). At this point, the conductance begins to increase and eventually reaches the pure metallic value, \(e^2/h\), due to the contribution of thermally excited quasiparticles (electrons and holes). Moreover, the temperature suppression of the gap \(\Delta(T)\) in the quasiparticle spectrum also becomes important. For quasi-1D systems, the restoration of metallic conductivity will happen, of course, at much low temperatures, in the vicinity of the Peierls phase transition.

The above consideration holds for the case when electron-phonon interaction in 1D system is strong enough to produce Peierls gap. Otherwise, the interacting electron system falls into the Luttinger liquid class of universality with parameters determined both by electron-electron and electron-phonon interactions (see Refs. 18 and 32).

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APPENDIX A:

Using a simpler version of the Lagrangian of Eq. (1), we will argue in favor of the ansatz (ii) of Sec. VI. Let us thus begin with the following Lagrangian, describing a relativistic, bosonized electron-phonon system of the incommensurate type,

\[
\mathcal{L} = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \frac{\rho_0^2}{2} \partial_\mu \phi \partial^\mu \phi + A \rho_0 \cos \beta (\eta - \phi) - \frac{\beta}{2 \pi} \partial_\mu \eta \epsilon^{\mu \nu} a_\nu,
\]  

(A1)

where \(\eta\) is the bosonic field related to the electronic degrees of freedom, while \(\phi\) and \(\rho_0\) represent the phase and amplitude of the phonon field (we only allow for phase fluctuations). The gauge field \(a_\nu\) is the external source of electric field. Already at the classical level we can see that there exist two distinct modes in the system. Setting the electric field to zero, we find the equations of motion

\[
\partial_\mu \partial^\mu (\eta + \rho_0^2 \phi) = 0
\]  

(A2)

and

\[
\partial_\mu \partial^\mu (\eta - \phi) + A \beta \left( \rho_0 + \frac{1}{\rho_0} \right) \sin \beta (\eta - \phi) = 0.
\]  

(A3)

Equation (A2) describes a massless mode, whereas (A3), a sine-Gordon equation, gives rise to a massive mode. We expect the low-energy physics to be controlled by the former. Indeed, the electrical conductance of this system can be obtained from the expectation value of the current

\[
\langle J^\nu \rangle = -i \left. \frac{\partial \ln Z}{\partial a_\nu} \right|_{a=0},
\]  

(A4)

where the partition function is given by

\[
Z = \int D\eta D\phi \; e^{-i \int d^2 x \mathcal{L}}.
\]  

(A5)

Introducing “relative” and center-of-mass” fields

\[
\chi \equiv \eta - \phi
\]  

(A6)

and

\[
\xi \equiv \phi + \frac{\rho_0^2 \phi}{1 + \rho_0^2},
\]  

(A7)

we can write this partition function as \(Z[a] = Z_{rel}[a] Z_{CM}[a]\), where
\[ Z_{\text{rel}} = \int D\chi \ e^{-i \int d^2x L_{\text{rel}}} \quad (A8) \]

with

\[
L_{\text{rel}} = \frac{\rho_0^2}{2(1 + \rho_0^2)} \partial_\mu \chi \partial^\mu \chi + A \rho_0 \cos \beta \chi - \frac{\beta \rho_0^2}{2\pi(1 + \rho_0^2)} \partial_\mu \chi \epsilon^{\mu\nu} a_\nu \quad (A9)
\]

and

\[ Z_{\text{CM}} = \int D\xi \ e^{-i \int d^2x L_{\text{CM}}} \quad (A10) \]

with

\[
L_{\text{CM}} = \frac{1 + \rho_0^2}{2} \partial_\mu \xi \partial_\nu \xi - \frac{\beta}{2\pi} \partial_\mu \xi \epsilon^{\mu\nu} a_\nu. \quad (A11)
\]

The path integration in Eq. (A10) can be carried out exactly, since the action is quadratic. The modes described by the Lagrangian \( L_{\text{CM}} \) are massless (gapless). On the other hand, the modes in \( L_{\text{rel}} \) are massive due to the presence of the cosine term. The contribution of these modes to the current is strongly suppressed. At low temperatures, the massive modes can be neglected all together in comparison to the massless ones. This, in turn, is equivalent to setting \( \chi = \text{const.} \) in the theory, in accordance to the ansatz (ii) of Sec. 2. As a result, the current is solely determined by the center-of-mass motion,

\[ \langle J^\nu \rangle = -i \left. \frac{\partial \ln Z_{\text{CM}}}{\partial a_\nu} \right|_{a=0}. \quad (A12) \]

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FIG. 1. A mesoscopic metal-CDW-metal junction.

FIG. 2. Conductance as function of temperature.