Geographic Trough Filling for Internet Datacenters

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Abstract—To reduce datacenter energy consumption and cost, current practice has considered demand-proportional resource provisioning schemes, where servers are turned on/off according to the load of requests. Most existing work considers instantaneous (Internet) requests only, which are explicitly or implicitly assumed to be delay-sensitive. On the other hand, in datacenters, there exist a vast amount of delay-tolerant jobs, such as background/maintenance jobs. In this paper, we explicitly differentiate delay-sensitive jobs and delay tolerant jobs. We focus on the problem of using delay-tolerant jobs to fill the extra capacity of datacenters, referred to as trough/valley filling. Giving a higher priority to delay-sensitive jobs, our schemes complement to most existing demand-proportional resource provisioning schemes. Our goal is to design intelligent trough filling mechanisms that are energy efficient and also achieve good delay performance. Specifically, we propose two joint dynamic speed scaling and traffic shifting schemes, one subgradient-based and the other queue-based. Our schemes assume little statistical information of the system, which is usually difficult to obtain in practice. In both schemes, energy cost saving comes from dynamic speed scaling, statistical multiplexing, electricity price diversity, and service efficiency diversity. In addition, good delay performance is achieved in the queue-based scheme via load shifting and capacity allocation based on queue conditions. Practical issues that may arise in datacenter networks are considered, including capacity and bandwidth constraint, service agility constraint, and load shifting cost. We use both artificial and real datacenter traces to evaluate the proposed schemes.

I. INTRODUCTION

The fast proliferation of cloud computing has promoted rapid growth of large-scale commercial datacenters. Major service providers often deploy tens to hundreds of datacenters distributed nationwide or even worldwide, referred to as Internet-scale datacenters (IDC). Because electricity bill contributes to a large portion of IDC operational expenditure, there have been lots of efforts towards reducing IDC energy consumption/cost.

Researchers have considered designing ‘load-aware’ IDCs, e.g., in [1][2][4]. The key idea is to provision servers according to the load of Internet requests. Extra servers are shut down or scheduled in sleeping mode to save energy. In this paradigm, a major challenge is to properly size an IDC, i.e., to determine the number of active servers, and in the meantime guarantee the service requirement. For example, in [2], the authors propose to predict the load of windows live messengers and provision servers accordingly. In [4], the authors estimate the current load, and design online server provisioning schemes to reduce energy and server state transition cost, which is referred to as dynamic “right sizing”.

In the above-mentioned work, service requests are typically delay-sensitive, i.e., requiring a short delay and low drop rate. Such applications include searching or signing in a messenger. When the load is lower, more servers would be turned off to save energy. However, in practice, an IDC operator may be reluctant to turn off servers in a large scale even at a low load of requests. One reason is that turning on/off servers frequently affects QoS and long term system reliability, as considered in [1]. But the foremost reason is that there are also a large amount of background or maintenance jobs in IDCs to process, e.g., searching engine tunes ranking algorithms. Thus, the “extra” capacity can be utilized to process the background analytical jobs. This is referred to as trough/valley filling.

Trough filling has not been studied thoroughly. In this paper, we focus on intelligent trough filling. We assume a given capacity provisioning and scheduling mechanism for delay-sensitive jobs (DSJs), e.g., those proposed in [1][2][4][5][6][8]. We decide how to use load shifting and dynamic speed scaling to control delay tolerant jobs (DTJs), e.g., background analytical jobs. On one hand, DTJ load is high and thus its energy cost is considerable. On the other hand, it is desirable to assure a good delay performance for DTJs. The goal of intelligent trough filling is thus to achieve energy efficiency as well as good delay performance (or at least guarantee the queue stability) for DTJs.

Intelligent trough filling needs to accommodate the following issues. First, the overall capacity of a datacenter is likely to be random, e.g., due to server failure. Second, capacity demand of DSJs, such as Internet requests, varies due to dynamic load. Given the higher priority of DSJs, available capacity for DTJs is random and hard to predict or learn in statistics. Meanwhile, the demand of DTJs is also likely to be dynamic.

Further, in order to consider a set of geographically distributed IDCs, there are additional constraints. First, load shifting is constrained by the bandwidth available between IDCs. In our setting, similar to capacity, bandwidth is prioritized for shifting DSJs, and thus results in a random ‘residual bandwidth’ for DTJs. Second, electricity prices diversity and dynamics bring challenges as well as opportunities, e.g., in price-aware load shifting [2][3], in the context of trough-filling. Third, due to heterogenous service agility, different classes of DTJs may require different sets of IDCs. Moreover, different IDCs maybe heterogenous in service rates and energy consumption for each type of DTJs. We consider these issues and address the above challenges in this paper.

In this paper, our goal is to design intelligent trough filling mechanisms, that achieve both energy efficiency and good delay performance. We design joint dynamic speed scaling and load shifting schemes. Specifically, we make the following
contributions:

- We focus on trough filling in distributed IDCs, which complements the current work on load-aware capacity provisioning, or price-aware load shifting.
- We consider practical issues in IDCs, such as dynamic capacity and bandwidth constraints, dynamic demand, and heterogenous service agility and service rates.
- We first propose a stochastic subgradient based trough filling scheme, named SSTF, with the objective of minimizing energy and shifting cost while stabilizing the DTJ queues. The proposed algorithm does not need underlying probability of system states, which is usually difficult to estimate.
- We further propose a queue-based trough filling algorithm, called QTF, which does not need any statistical system information. We show the QTF achieves desirable performance in terms of cost and queue delay.
- We discuss on how to incorporate capacity provisioning and QoS assurance for DSJs into our proposed SSTF and QTF.
- We use both synthetic traffic trace and real datacenter traffic trace to evaluate our proposed schemes. Simulation results show that QTF outperforms SSTF significantly in both cost and queue delay.

The rest of paper is organized as follows. In Section II, we survey related work. In Section III, we describe the system model. In Section V, we present stochastic subgradient based trough filling scheme. We further propose a queue based trough filling scheme in Section VI. We also discuss how to extend the schemes to DSJs and implementation issues in Section VII. We evaluate our proposed schemes in Section VIII and conclude in Section IX.

II. RELATED WORK

Industry and academic research community have paid much attention to reducing datacenter energy consumption and cost. Solutions are considered in all spectra, including power-efficient chip, cooling system, deployment, and many others.

Our work complements to load-aware server provisioning or power-proportional design [1][7]. Such works focus on server or resource provisioning based on load of Internet requests, with service level agreement SLA or other QoS metrics assured. For example, in [1], the authors propose server provisioning and dynamic speed/voltage scaling schemes for a data center, through load prediction and feedback control. Load prediction-based server provisioning and load dispatch is proposed in [2] for connection-intensive Microsoft datacenter. Online resource or server provisioning schemes are designed in [3][4]. In [4], the authors consider a relative large time interval such that current load of requests can be estimated. Server state transition cost is also considered. Furthermore, the authors also consider the impact of trough filling on energy saving by the proposed scheme through simulations. Queue based server provisioning and Lyapunov optimization based performance establishment is proposed in [6]. Although the Lyapunov optimization technique is also used to show performance of the queue-based scheme, our problem is different, i.e., we consider trough-filling, with cross-datacenter load shifting and capacity provisioning. In [7], the authors propose an economic framework which maximizes the total profit of resource provisioning for all requests.

Many other power management schemes for a datacenter have been proposed, e.g., in [8]-[30]. Dynamic speed/voltage scaling saves power consumption of a processor by adjusting the frequency based on the instantaneous load demand, e.g. in [8]-[10], which can also be considered as load-aware resource provisioning. However, most of the work only considers a single processor. In [13], the authors use MDP to find optimal stationary DVS and load balancing policy to reduce service cost. In this paper, we use DVS as a part of control mechanism for trough filling in IDCs. Another popular scheme is virtualization and server consolidation, e.g., in [20]-[25], which can reduce the traffic dynamics by consolidating applications, and thus reduce the number of active servers. There are also some other works on datacenter-level power management, such as load prediction-based performance establishment is proposed in [6]. optimal power allocation for servers with total power budget [27], model predictive control (MPC) theory based hierarchical power control [28], and other techniques [29][30][31].

Most recently, cross-IDC power and cost optimization that exploits geographic diversity has received significant attention, e.g., in [32]-[39]. The key idea is to shift requests to IDCs with lower electricity prices to reduce cost. The tradeoff is the extra delay caused by traffic shifting. Thus, in [34][38], the authors consider response time as the constraint. In [37][39], the authors consider shifting cost as the revenue loss incurred by extra delay. Our work can also leverage price diversity, i.e., by filling cheap troughs of IDCs. The difference is that since background jobs are delay tolerant, our capacity provisioning and load shifting schemes also exploit the temporal price diversity, in addition to geographic diversity. In a recent work [51], the authors use energy storage systems to leverage the temporal price dynamics to cut the energy cost, but for a single datacenter.

We refer readers to [43] for a survey and [44] for discussions on challenges and issues in IDC power management.

III. SYSTEM MODELS

A. The IDC and server model

We consider one service provider with a set of N IDCs in different locations. An IDC i has K_i max homogenous servers. We consider a time slotted system, where the slot length can be from hundreds of milliseconds to minutes. We assume in each slot t, the number of active servers of an IDC i is fixed and is denoted by K_i. Note that K_i varies over time, due to either dynamic service provisioning (e.g., those proposed in [2][4][34]) or server failure.

An active server operates at a CPU speed of s. Following the models in [11][12][35], we normalize s, i.e., 0 ≤ s ≤ 1, where 0 represents the idle state of an active server, and 1 represents the maximum frequency. We define the capacity of an IDC i as the sum of speed of all active servers. If each
server runs at the same speed \( s \), the total capacity in time slot \( t \) is \( K_i^t s \). Clearly, the maximum capacity with \( K_i^t \) servers is \( K_i^t \). In this paper, we consider CPU resource as the the main bottleneck and focus on CPU capacity scheduling. The impact of other equipments, i.e., memory and I/O, will be considered in heterogenous service rates, as discussed in subsection III-C.

Because scaling up/down the speed \( s \) of an active server only takes several microseconds [12][13], which is negligible, dynamic speed scaling can be conducted instantaneously in each time slot.

B. Workload model

We consider two categories of demand: delay sensitive jobs (DSJs), e.g., searching, email login in, or messenger sign up, and delay tolerant jobs (DTJs), e.g., background analytical jobs. DSJs enjoy a higher priority on capacity allocation. The remaining capacity can be utilized by the DTJs. Since the load of DSJs is usually dynamic, capacity demand of DSJs in an IDC \( i \) in each slot is considered random. We use \( S^t_{i0} \) to denote the capacity allocated to DSJs at IDC \( i \) in slot \( t \). We assume \( S^t_{i0} \) is given, based on some existing load-aware capacity provisioning schemes. Available capacity for DTJs in IDC \( i \) is thus \( K_i^t - S^t_{i0} \).

For DTJs, they can be further divided into different classes to capture their different resource requirements. We consider there are in total \( M \) different classes of DTJs in the \( N \) IDCs. If the same kind of DSJs, e.g., tuning webpage ranking algorithms, originates (first arrives) at different IDCs, we treat them as different classes. This is because they may have different sets of IDCs to be shifted to due to distance constraints. For DTJ \( j \), it first originates at an IDC \( i \). Let \( D^t_{ji} \) denote the traffic or load size of DTJ \( j \) in time slot \( t \). \( D^t_{ji} \) is a random variable. We do not make assumptions on its distribution.

C. Models for load shifting and service

Although a DTJ \( j \) originates at an IDC \( i \), we can shift the traffic to other IDCs, e.g., to exploit their available capacity or lower electricity prices. Note that cross-IDC load shifting is practically feasible due to negligible shifting time delay [30], which has been widely considered, e.g., in [32][42]. Load shifting has practical constraints. First, due to limited service agility of IDCs, a class of DTJ \( j \) can potentially be served by only a subset of IDCs. Let \( \Gamma_j \) denote the set of IDCs that can serve DTJ \( j \), which is different for different classes of DTJs. DTJ \( j \) can only be shifted to IDC \( i' \), where \( i' \in \Gamma_j \). Second, bandwidth between IDCs is limited. Moreover, due to potentially load shifting for DSJs, which also requires a high priority of bandwidth provisioning, available bandwidth for DTJs is limited and dynamic. This consideration is similar to that in a very recent work [41], where the authors develop a system to rescue unutilized network bandwidth for shifting the non-real-time bulk data, e.g., backup data. We use \( B^t_{ii'} \) to denote the available bandwidth from IDC \( i \) to \( i' \) for DTJs in slot \( t \). \( B^t_{ii'} \) varies over time, and can be set in an appropriate value to prevent significant network delay. Note when two IDCs have limited connections or a long distance such that load shifting is not desirable, \( B^t_{ii'} \) can be set as 0 for all time slots. Let \( D^t_{ji} \) denote the traffic of DTJ \( j \) shifted from IDC \( i \) to \( i' \). Further let \( T^t_{ii'} \) denote the set of DTJs that first arrive at IDC \( i \) and can be served by IDC \( i' \). We have \( \sum_{j \in T^t_{ii'}} D^t_{ji} \leq B^t_{ii'} \) as the load shifting constraint.

For an IDC \( i \in \Gamma_j \), it allocates a certain capacity to DTJ \( j \) in time slot \( t \), denoted by \( S^t_{ij} \). We have \( S^t = \{ S^t_{ij} | i = 1, \ldots, M, i \in \Gamma_j \} \), as the capacity allocation matrix, which is our control variable. An IDC \( i \) may serve multiple DTJs. Let \( \Pi_i \) denote the set of all DTJs served by an IDC \( i \). Obviously, we have the capacity allocation constraint as \( \sum_{j \in \Pi_i} S^t_{ij} \leq K_i^t - S^t_{i0} \).

With capacity \( S^t_{ij} \), DTJ \( j \) receives a certain service rate. We use the \( R_{ij}(S^t_{ij}) \) as the service rate function on the capacity. For simplicity, we consider \( R_{ij}(\cdot) \) as a linear function of \( S^t_{ij} \), i.e., \( R_{ij}(S^t_{ij}) = r_{ij} S^t_{ij} \). The unit service rate \( r_{ij} \) is heterogenous for different pairs of DTJ \( j \) and IDC \( i \). This is because different DTJs may require different memory, I/O resource, etc. Load shifting and dynamic speed scaling are coupled. The amount of traffic of DTJ \( j \) shifted from IDC \( i \) to \( i' \) depends on the capacity allocated at IDC \( i' \). Thus we have \( D^t_{ji'} \leq r_{ij} S^t_{ij} \). Since both energy and load shifting cost increase with \( S^t_{ij} \), we have \( D^t_{ji'} = r_{ij} S^t_{ij} \).

The unfinished jobs of a DTJ \( j \) are buffered in a queue at the IDC where DTJ \( j \) originates. Let \( Q_j(t) \) denote the queue in time \( t \), the queue dynamics of DTJ \( j \) can be written as

\[
Q_j(t + 1) = \max \left[ Q_j(t) - \sum_{i \in \Gamma_j} r_{ij} S^t_{ij}, 0 \right] + D^t_{ji},
\]

where \( \sum_{i \in \Gamma_j} r_{ij} S^t_{ij} \) is the total service rate a DTJ \( j \) receives in slot \( t \).

D. Power consumption and cost model

According to [11][12], power consumption of a server (processor) running at a speed \( s \in [0,1] \) is

\[
P(s) = ps^\nu + 1 - \rho,
\]

where the exponent \( \nu \geq 1 \), with a typical value of 2 [12], and \( 1 - \rho \) represents the power consumption in the idle state.
which is around 0.6, and hardly lower than 0.5 [21]. In this paper, we choose \( \nu = 2 \), as in [12]. Note that our schemes can be extended to the cases with other values of \( \nu \).

Consider an IDC \( i \). In a time slot \( t \), there are \( K_i^t \) active servers, and the total capacity demand is \( S_i^t \). It can be shown that the most energy-efficient operation is to let each server evenly share the demand, i.e., each server is running at a speed \( \frac{S_i^t}{K_i^t} \), which results in a total power consumption in slot \( t \) of

\[
P_i^t = (1 - \rho)K_i^t + \rho \frac{S_i^t}{K_i^t},
\]

where \( S_i^t = S_{i0}^t + \sum_{j \in \Pi_i} S_{ij}^t \). Because we focus on trough-filling, we take \( K_i^t \) and \( S_{i0}^t \) as given constants in each time slot. We only control \( S_{ij}^t \). Note that \( P_i^t \) is a convex function of \( S_{ij}^t \).

Besides the power consumption of servers, other components in an IDC, e.g., memory, I/O, hard disk, and non-IT equipment such as cooling systems, also contribute to the total power consumption, which is roughly proportional to that by servers [40]. Thus total power consumption of an IDC can be obtained by scaling up \( P_i^t \) with a constant factor. For notation brevity, we absorb this constant factor into the electricity price at IDC \( i \). Electricity price exhibits significant diversity in both location and time. We use \( \alpha_i^t \) to denote the price at IDC \( i \) in time slot \( t \). Although \( \alpha_i^t \) is a time-varying variable, it varies slowly. Typically, in a wholesale market, \( \alpha_i^t \) is determined by Regional Transmission Organization (RTO) day-ahead based on expected load and changes hourly; or alternatively, \( \alpha_i^t \) is determined in real-time (every 15 min) based on the actual load. We consider energy cost of an IDC as the product of power consumption and its electricity price.

### E. Load shifting cost

We also consider load shifting cost. In practice, datacenter operators may have a lease with ISPs for data traffic among IDCs. Some large operators like Google and Microsoft may even have their own backbone networks to interconnect the IDCs. Either case, shifting cost is usually incurred during the acquisition or construction phase, which depends less on the traffic volume that the internal links carry [43]. However, since DTJs have a lower priority, it is desirable to schedule a limited link bandwidth to them. For example, when the time slot is relatively long, a higher utilization of the link capacity by DTJs will make the system more sensitive to the burst of DSJs, which enjoy a higher priority on load shifting. To prevent the increasing sensitiveness to DSJs, we use a piece-wise linear cost function with increasing rate to model the shifting cost for DTJs. Let \( \phi_{ij}' \) denote the shifting cost in slot \( t \) between IDC \( i \) and \( j \), we have

\[
\phi_{ij}' = \max \left\{ a_{ij}^\theta \frac{D_{ij}'}{B_{ij}'}, b_{ij}^\theta \right\}, \quad \theta = \{1, 2, \ldots, \theta\},
\]

where \( \frac{\sum_{j \in \Pi_i} D_{ij}'}{B_{ij}'} \) is the link capacity occupation ratio by DTJs. We have \( a_{ij}^1 \leq \cdots \leq a_{ij}^\theta \leq a_{ij}^{\theta+1} \) which captures the increasing sensitiveness to capacity occupation ratio by DTJs. \( \phi_{ij}' \) is a convex function on \( D_{ij}' \), and thus on \( S_i'^t \) since it is the pointwise maximum of a set of affine functions, and \( D_{ij}' \) is linear on \( S_i'^t \). The model is also widely considered by previous works, e.g., in [47]. Note that our work can also incorporate other shifting cost models with minor modifications.

### IV. A Benchmark scheme

In this section, we first consider a benchmark scheme, where the goal is to minimize the time average of the total cost of \( N \) IDCs, including energy cost and shifting cost, while stabilizing the \( M \) DTJ queues. We name it stability-assured cost optimal trough-filling (SCOTF). In each time slot, both the energy cost and the shifting cost are functions of \( S_i'^t \). The overall cost in each slot also depends on \( K_i^t, \alpha_i^t \), and \( S_{i0}^t \), \( i = 1, \ldots, N \). Thus the overall cost is a time-varying function on \( S_i'^t \), denoted by \( g_i'(S_i'^t) \). Besides, capacity allocation and shifting constraints, i.e., \( C_i^t \) and \( B_{ij}' \), are also time-varying. Thus \( S_i'^t \) takes values in a time-varying set. Let \( \Lambda^s \) denote the set of \( S_i'^t \) that satisfies capacity allocation and shifting constraints in slot \( t \). SCOTF is formulated as

\[
\min_{S_i'^t} \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} g_i'(S_i'^t) \quad \text{s. t.} \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} Q_j(t) \leq \infty, \quad S_i'^t \in \Lambda^s, \quad j = 1, \ldots, M.
\]

where the first constraint is to guarantee each DTJ queue’s stability. Note we use ‘sup’ (‘inf’) to guarantee the infinity exists.

It is difficult to solve problem (6) directly in practice, because it is hard to obtain prior system information of all time slots. We present the problem of SCOTF here as a cost benchmark. Our proposed schemes, one stochastic subgradient-based and one queue-based, require little system statistical information, and thus are more practical. The objectives of proposed schemes are not limited to guaranteeing DTJ queue stability as in SCOTF. Good delay performance is also desired, especially for the queue-based scheme.

### V. Stochastic subgradient based trough filling

We first consider an ergodic scenario where system state has a steady state distribution. Here a state characterizes a unique set of all variables involved in the system, including \( K_i^t, \alpha_i^t, S_{i0}^t \), and \( B_{ij}' \), \( i, j = 1, \ldots, N \). Let \( \Omega \) denote the set of system states, and \( \omega \) a generic system state, \( \omega \in \Omega \), \( \pi_\omega \) the steady distribution of \( \omega \), \( g_\omega(\cdot) \) is the cost function in state \( \omega \). Let \( S_\omega \) denote the capacity allocation matrix in state \( \omega \), which is in the set \( \Lambda_\omega \). Let \( \bar{\lambda} \) denote the mean of arrival rate vector of DTJs. SCOTF can be rewritten as

\[
\min_{S_\omega} \quad \sum_{\omega \in \Omega} \pi_\omega g_\omega(S_\omega) \quad \text{s. t.} \quad \sum_{\omega \in \Omega} \pi_\omega R_\omega(S_\omega) \geq \bar{\lambda}, \quad S_\omega \in \Lambda_\omega,
\]
We use $g^\omega_*$ to denote the optimal solution to the above problem, i.e., optimal cost in the ergodic system case, with the arrival rate $\lambda$ stabilized. In practice, $\lambda$ can possibly be estimated by historic database or prediction schemes. If the steady state distribution $\pi_\omega$ is available, then (7) is a deterministic convex optimization problem. However, in practice it may be difficult to obtain such statistical knowledge. We thus design a stochastic subgradient-based algorithm that can solve (7), without prior information on $\pi_\omega$. Note the scheme needs the information of the average rate, i.e., $\bar{\lambda}$, or at least an upper bound to guarantee stability.

We first define a Lagrangian function associated with problem (7) as

$$L(\vec{\mu}, \vec{S}) = \sum_{\omega \in \Omega} \pi_\omega g^\omega(S^\omega) - \sum_{j=1}^M \mu_j (\sum_{\omega \in \Omega} \sum_{i \in I_j} r_{ij} S^\omega_{ij} - \lambda_j),$$

where $\vec{S} = \{S^\omega | \omega \in \Omega\}$, $S^\omega \in \Lambda^\omega$, and $\vec{\mu} = (\mu_1, \ldots, \mu_M)$ is the set of the Lagrangian multipliers. Note $\vec{\mu} \geq 0$. The dual problem of (7) is defined as

$$\max_{\vec{\mu} \geq 0} F(\vec{\mu}),$$

where

$$F(\vec{\mu}) = \min_{\vec{S}} L(\vec{\mu}, \vec{S}).$$

To solve the dual problem, we first consider (8). For a given multiplier $\vec{\mu}$, the problem is separable for different states. Thus, we can solve the following problem for a given state $\omega$,

$$\min_{S^\omega} g^\omega(S^\omega) - \sum_{j=1}^M \mu_j \left( \sum_{i \in I_j} r_{ij} S^\omega_{ij} - \lambda_j \right)$$

s.t. $S^\omega \in \Lambda^\omega$. An examination of (11) yields the following optimization problem of joint capacity allocation and load shifting after observing system state in the current slot

$$\min_{S^\omega} \sum_{i=1}^N \sigma^n_i \left[ (1 - \rho) K^{\omega_i}_i + \frac{\rho (S^\omega_{ii} + \sum_{j \in \Omega_j} S^\omega_{ij})^2}{K^{\omega_i}_i} \right] +$$

$$\sum_{i=1}^N \sum_{i' \neq i} \max_{1 \leq \theta \leq \theta} \left\{ \sigma^\theta_i \frac{\sum_{j \in \Omega_j} r_{ij} S^\omega_{ij} - \lambda_j}{B^\theta_{ii}} \right\} +$$

$$\sum_{j=1}^M \mu_j \left( \sum_{i \in I_j} r_{ij} S^\omega_{ij} - \lambda_j \right)$$

s.t. $S^\omega \in \Lambda^\omega$, $\sum_{j \in \Omega_j} r_{ij} S^\omega_{ij} \leq K^{\omega_i}_i - S^\omega_{ii}, i = 1, \ldots, N$, $\sum_{j \in \Omega_j} r_{ij} S^\omega_{ij} \leq B^\omega_{ii}, i, i' = 1, \ldots, N, i \neq i'$. (12)

In (12), the first item is the total energy cost, the second is the shifting cost, the first constraint is the capacity constraint on DTJs in IDC $i$ and the second constraint is bandwidth constraint between IDCs $i$ and $i'$. Clearly, (12) is a convex optimization problem of $S^\omega$. This is because, the objective function is the sum of a set of convex and affine functions of $S^\omega$, and the constraints are both affine and thus convex. We can solve it efficiently for a given state $\omega$ in each time slot. When capacity allocation is determined, load shifting policy is also jointly determined, i.e., shift an amount of $r_{i'j} S^\omega_{i'j}$ for DTJ $j$ from IDC $i$ to $i'$ if $j \in \Omega_i'$. The dual problem can be solved using a stochastic subgradient algorithm [49], which has the following iterative steps

$$\mu_j^{n+1} = [\mu_i + \beta^n \sigma^n_j]^+, \quad \text{(13)}$$

where $n$ denote the $n$th iteration, i.e., $n$th time slots in our case, and $\beta^n = (\sigma^n_1, \ldots, \sigma^n_M)$ is the vector of stochastic subgradient that is chosen as

$$E(\sigma^n_1^2 + \ldots + \sigma^n_M^2) = \partial F(\vec{\mu}^n), \quad \text{(14)}$$

where $\partial F(\vec{\mu}^n)$ is a subgradient of $F(\vec{\mu})$ at $\vec{\mu}^n$. In this case, by updating $\vec{\mu}^n$ using (13), $\vec{\mu}^n$ converges to the optimal solution of the dual problem (9) with probability 1, if the following conditions are satisfied

$$E((\sigma^n_1^2 + \ldots + \sigma^n_M^2) | \vec{\mu}^0, \ldots, \vec{\mu}^n) \leq c, \quad \text{(15)}$$

where $c$ is a constant, and $\sum_{n=0}^{\infty} \sigma^n = \infty, \sum_{n=0}^{\infty} (\beta^n)^2 = \infty$. Note a candidate for $\beta^n$ can be $\frac{1}{\sqrt{n}}$.

The subgradient $\partial F(\vec{\mu})$ can be a set, where by Danskins Theorem [50], we can choose a subgradient as

$$\partial F(\vec{\mu}) = - \sum_{\omega \in \Omega} \pi_\omega \sum_{i \in I_j} r_{ij} S^\omega_{ij} + \lambda_j, \quad j = 1, \ldots, M, \quad (16)$$

where $S^\omega_{ij}$ is the optimal solution to problem (12). Note that $\sigma^n_j$ is a stochastic subgradient if its expectation equals to a subgradient. We can choose $\sigma^n_j$ as

$$\sigma^n_j = - \sum_{i \in I_j} r_{ij} S^\omega_{ij} + \lambda_j, \quad j = 1, \ldots, M, \quad (17)$$

where $\omega_i$ is the index of the system state at iteration $n$. It is satisfied, because $r_{ij} S^\omega_{ij}$ is bounded, $\forall i, j$, which leads to bounded $\sigma^n_j$, $\forall j$. $\sigma^n_j$ defined in (17) is a stochastic subgradient, because we consider an ergodic setting and thus the time average of $\sigma^n_j$ equals to the subgradient of (16). Further, since the original problem (7) is a convex optimization problem that satisfies the Slater’s condition, there is no duality gap.

We name the above algorithm stochastic subgradient-based trough filling (SSTF). SSTF converges to the optimal solution of problem (7). Thus it can achieve the optimal cost given a service rate that assures queue stability. Note that SSTF can work in non-ergodic settings. Lagrangian multiplier $\vec{\mu}$ has practical properties. It can be considered as a price, which increases as service rate being smaller than the average arrival rate, i.e., capacity under-provisioning. In practice, by updating $\vec{\mu}$, SSTF can achieve good cost performance. Moreover, the objective of SSTF is not limited to cost optimality only. One can tune the average service rate of SSTF, i.e., by adjusting $\bar{\lambda}$ in (7), to control the DTJ delay. Thus, SSTF is NOT SCOTF in the ergodic setting. Another benefit of SSTF is that it also exploits temporal diversity of electrical prices. However, SSTF needs the knowledge of the average DTJ arrival rate, which may not be available in practice. Further, it may converge slowly and it is difficult to characterize its delay performance. This motivates us to consider the following...
queue-based algorithm, which leverages queue information so that neither $\bar{\lambda}$ nor system distribution information is required.

VI. QUEUE BASED TROUGH FILLING

A. Algorithm Design

In this section, we present a queue-based algorithm that explicitly considers queue backlog of DTJs. The algorithm takes the instantaneous system state (i.e., queue length, available server capacity and bandwidth, DSJ load demand) as the input. The algorithm also has a parameter to control the tradeoff between cost and queue delay. We will also show that the algorithm achieves bounded average queue backlog such that the system is stabilized, while the cost can be arbitrarily close to the optimal cost achieved by (7).

In each time slot $t$, observe current queue backlog $Q_j(t), j = 1, \ldots, M, \alpha^i_t, S^i_{0t}, C^i_t$, and $B^i_{j}, i = 1, \ldots, N$. Allocate the capacity at each IDC $i$ for each queue $j$ according to the following optimization scheme, named queue-based trough filling (QTF):

$$\min_{S^i} - \sum_{j=1}^M Q_j(t) \sum_{i \in \Gamma_j} r_{ij} S^i_j + V \sum_{i = 1}^N \alpha^i_t \left[ (1 - \rho) K^i_t + \frac{\rho (S^i_{0t} + \sum_{j \in \Pi_i} S^i_j)^2}{K^i_t} \right] + \sum_{i = 1}^N \sum_{j \neq i} \max_{1 \leq \theta \leq 0} \left\{ \frac{a^\theta_{ij}}{a^\theta_{ii}} \sum_{j \in \Pi_i} r_{ij} S^i_j + b^\theta_{ij} \right\}$$

$$\text{s.t. } \sum_{j \in \Pi_i} S^i_j \leq K^i_t - S^i_{0t}, i = 1, \ldots, N$$

$$\sum_{j \in \Pi_i} r_{ij} S^i_j \leq B^i_{j}, i = 1, \ldots, N$$

$$\sum_{i \in \Gamma_j} r_{ij} S^i_j \leq Q_j(t), j \in \{1, \ldots, M\}.$$ (18)

Similar to (12), (19) is a convex optimization problem. Thus at the beginning of each slot, capacity allocation $S^i$ can be determined efficiently.

The intuition of QTF is clear. When queue length $\sum_{j=1}^M Q_j(t)$ is high, QTF has incentive to allocate a larger capacity to reduce the queue length. When the cost is relatively large or queue length is small, QTF is driven to allocate less capacity to reduce the cost. The control variable $V$ is to balance the queue length and cost. If $V$ is large, QTF will result in lower cost but longer average queue delay.

To better illustrate the intuition of the algorithm, we further consider a special case, where there is only one IDC with $M$ delay tolerant queues. In the single IDC case, we can simplify notations by removing subscript $i$. The capacity vector becomes $S^i = \{S^i_1, \ldots, S^i_M\}$. We have the following scheme for capacity allocation, named single-IDC queue-based trough filling (SQTF):

$$\min_{S^i} - \sum_{j=1}^M Q_j(t) r_{ij} S^i_j + V \alpha^t \left[ (1 - \rho) K^t + \frac{\rho (S^i_{0t} + \sum_{j \in \Pi_j} S^i_j)^2}{K^t} \right]$$

$$\text{s.t. } \sum_{j=1}^M S^i_j \leq K^t - S^i_0$$

$$S^i_j \geq 0, j = 1, \ldots, M.$$ (20)

We have the following solution on $S^i$.

Observation 1: SQTF allocates $S^i$ as: in each time slot $t$, choose the queue with the maximum $Q_j(t) r_{ij}$, denote as $j^*$, then

$$S^i_j = \begin{cases} K^t - S^i_0, & \text{if } Q_{j^*}(t) r_{ij^*} \geq 2V \rho \alpha^t \frac{K^t}{2V \rho \alpha^t}, \\ 0, & \text{else if } Q_{j^*}(t) r_{ij^*} \geq 2V \rho \alpha^t \frac{K^t}{2V \rho \alpha^t}, \end{cases}$$ (24)

$$S^i_j = 0 \text{ if } j \neq j^*.$$ (24)

In other words, SQTF is a threshold-based policy, which serves the longest queue and only when its queue length is above a certain threshold.

B. Performance analysis

In this subsection, we analyze the performance of the QTF algorithm in terms of the cost and average delay performance. Our analysis is based on Lyapunov drift optimization [51].

Define $r_i = \max(r_{ij}|j \in \Pi_i)$, i.e., maximum unit service rate for all DTJs in IDC $i$. Let $D^m_j$ denote the upper bound of arrival traffic size of DTJ $j$ in each slot. We have the following proposition.

Proposition 1: Assuming traffic of DTJs is i.i.d in each slot with mean $\lambda$, the QTF algorithm stabilizes the system for a given parameter $V$. In addition, an upper bound on average queue length is

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t = 1}^T \sum_{j = 1}^M E[Q_j(t)] \leq \frac{\sum_{j \in \Gamma_j} r_{ij}^2 K^m_{ij} + \sum_j D^m_j + V g^*_{\epsilon}(\epsilon)}{\epsilon}$$ (25)

Further, average cost achieved by QTF, which has a cost denoted as $g^*_{\epsilon}(S^i)$ in each slot $t$, is upper bounded as

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t = 1}^T E[g^*_{\epsilon}(S^i)] \leq V g^*_{\epsilon} + \frac{\sum_{j \in \Gamma_j} r_{ij}^2 K^m_{ij} + \sum_j D^m_j}{V}$$ (26)

where $g^*_{\epsilon}$ is the optimal solution to problem (7), and $\epsilon$ is a positive value, $g^*_{\epsilon}(\epsilon)$ is the optimal solution to (7) with $\lambda$ replaced by $\lambda + \epsilon t$.

Proof: In the Appendix.

VII. DISCUSSIONS

A. Joint DSJ and DTJ design

Although SSTF and QTF are both proposed for trough-filling, with some modifications, they can be used for joint DSJ and DTJ capacity provisioning. First, $S^i_{0t}$, for DSJs, will become a part of the decision variables, together with $S^i_j$ for DTJs. An important issue is how to guarantee service requirements for DSJs.
For SSTF, we can simply introduce a QoS constraint for DSJs. For example, if the slot length is large, i.e., tens of seconds to minutes, following [24], we can estimate the mean of DSJ rate for IDC $i$ in the beginning of the current slot, denoted by $X_{i0}$. Note that it is possible for $X_{i0}$ to incorporate traffic from other IDCs due to certain traffic shifting schemes. Let $r_{i0}$ denote unit service rate for DSJs in IDC $i$. Following [34], a delay constraint can be imposed, e.g., $\frac{1}{r_{i0}X_{i0}} - X_{i0} \leq \delta_i$, which is a linear constraint on $S_{i0}$, and thus can be easily incorporated to our convex optimization problem. When the time slot length is small, such as hundreds of milliseconds, it is unlikely to estimate mean of DSJ traffic in the current slot. In this case, one may assume DSJ traffic follows certain distributions based on past measurement. One can define outage probability as a QoS constraint. That is, the probability that the load of DSJ in IDC $i$, i.e., $D_{i0}$, exceeds capacity $S_{i0}$. The DSJ QoS constraint can be expressed as $\Pr(D_{i0} > S_{i0}) \leq \delta_i$. Based on the knowledge of traffic distribution, e.g., Gaussian or exponential distribution, one can rewrite the constraint function as a convex function of $S_{i0}$. Since time time slot length is small, outage probability can be easily measured. Adjusting $S_{i0}$ is probably necessary to eliminate the discrepancy between the real distribution of $D_{i0}$ and the assumed one using stochastic approximation schemes.

Similar approaches can be applied to extend QTF. For example, one can use the outage probability as a DSJ QoS constraint. Let $\delta_i$ denote outage probability constraint. To enforce it, we can design a virtual outage queue. Let $I_i(t)$ as an indicator function. We have $I_i(t) = 1$ if there is outage in slot $t$, i.e., $D_{i0} > S_{i0}$, and $I_i(t) = 0$ otherwise. We use $O_i(t)$ to denote the virtual outage queue backlog in slot $t$, which updates as $O_i(t + 1) = O_i(t) - I_i(t)$. It can be shown that the virtual queue is stable if $\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} I_i(t) \leq \delta_i$, i.e., outage probability constraint satisfied. Note that $\delta_i$ can be considered as the service rate of the virtual queue. Using the virtual outage queue, we can modify QTF to provide capacity provisioning for DSJs. It is our future work to further investigate the joint design of capacity provisioning and QoS assurance for DSJs, and trough-filling.

B. Implementation issues and caveats

In our schemes, the decision-maker needs to gather the input in the beginning of each slot. The messaging delay is about tens of milliseconds [36], and each IDC only has a few parameters sent to the decision-maker. Each time slot can be from several seconds to some minutes. Thus the messaging overhead is negligible. Note that the decision overhead is also negligible since the convex optimization problems can be solved efficiently. Load shifting overhead, i.e., network delay, can be easily constrained by controlling the bandwidth for DTJs.

In this paper, we consider homogenous servers for simplicity. However, in an IDC, servers may be different in terms of power consumption, maximum speed, and memory. To apply our schemes, we can further classify the servers to different units. Homogenous or similar servers belong to one unit. The input is no longer IDC-based, but unit-based. In practice, we can simply classify servers according to their ages. Typically, there are three stock-keeping units (SKUs) in an IDC, i.e., latest, one-year-old, and two-year-old.

In practice, some DTJs may need to be finished by a deadline. Different classes of DTJs may have different deadlines. Designing energy-efficient DTJ scheduling algorithms with heterogenous deadlines for IDCs is an interesting open problem. We will consider it in the future.

In this paper, we mainly focus on CPU-intensive DSJs. We will also extend our work to I/O intensive DTJs. Besides, we will also explicitly consider the effect of virtualization, by which performance versus power curve may become more difficult to quantify [22][36].

VIII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of SSTF and QTF, using both synthetic and real traces.

A. Synthetic traces based simulation

1) Simulation setup: We consider five IDCs in different locations. There are totally ten DTJ queues randomly originated in one of the five IDCs. The IDC set $\Gamma_j$ that can serve a DTJ $j$ is chosen randomly. Idle power consumption $1 - \rho$ is set as 0.5. To create an ergodic setting, we set 100 states, in each of which we set different total capacity, load shifting constraint, demand by DSJs, and electricity prices. Capacity of each IDC is uniformly distributed from 10k to 15k. Load shifting constraint is uniformly distributed from 3000 to 4000. Load shifting cost parameters are set the same as in [47]. Electricity price is uniformly distributed from 1 to 10. DSJ demand, set as a ratio of the total capacity, is randomly distributed from 0 to 0.4. Thus average DSJ demand is about 20% of the total capacity. We consider different ratios between the load of DTJ and DSJ, by setting different average arrival rates of DTJs. The ratios are 0.5, 1, 1.5, 2, 2.5, 3, and 3.5, respectively. Thus the percentage of DTJ demand in the total capacity ranges from 10% to 70%. We simulate 100k time slots in each of the 30 simulation settings. In different time slots, a system state is chosen randomly according to a predefined probability.
2) Simulation results: We first compute the Optimal Solution to \( K \) with the System distribution Information, which is difficult to obtain in practice. We name it OSS1 and compare it with SSTF and QTF. First, by Fig. 1 we observe that the cost of SSTF is very close to that of OSS1, under different DTJ load ratios. Their queue delays are also very close. In this paper, since we also consider idle power consumption, i.e., \((1 - \rho)K_i^T\), and DSJ power consumption. When load of DTJs is low, such as with the ratios of 0.5 and 1, costs of different schemes are very close because the impact of DTJs is small. To study the convergence of SSTF, we also consider the DTJ power consumption separately. Results show that SSFT and OSS1 achieve very close performance in terms of cost and delay. We do not plot results here due to the page limit.

In Fig. 1 we consider QTF with \( V = 1 \) and \( V = 1000 \), respectively. For both cases, we see QTF leads to a higher cost, but the queue delay is significantly smaller compared to that by OSS1 and SSTF. QTF with \( V = 1000 \) has a slightly larger cost than OSS1 and SSTF, but much smaller delay, even when DTJ load is high, e.g., with a ratio of 3.5. In this case, QTF with \( V = 1 \) has a very small delay, i.e., almost 1, with a much higher cost. Thus, in practice, one can tune the value of \( V \) to obtain a desirable tradeoff between cost and delay, especially when load of DTJs is high.

In Fig. 1 the queue delay of OSS1 and SSTF is very large, which holds even when we set the average service rate (slightly) larger than the arrival rate. We examine the service rates of a DTJ queue in different time resolutions to find the reasons. We first consider one slot service rate, normalized over the average DTJ arrival rate. We plot 100 slots rate in Fig. 2a and (b), for SSTF and QTF, respectively. (Service rate by OSS1 is very similar to that by SSTF). Note in Fig. 2a the ratio of DTJ load is 1 and \( V \) for QTF is 1000. It is observed that rate assignment by SSTF is quite even for each slot. The DTJs always receive a service rate in each slot. Rate assignment by QTF is much more bursty. Service rate is nonzero only by every several slots. This result is consistent to Observation 1 where we show capacity allocation by QTF for a single IDC is a threshold-based policy based on the queue length. In the time slots without being served, jobs accumulate and queue delay increases. This is the reason that there is a queue delay about 5 in Fig. 1a for QTF (\( V = 1000 \) and DTJ load ratio of 1). Nevertheless, queue stability is guaranteed since service rates are fairly large every several slots such that jobs accumulated can be finished. We also examine a large time resolution rate, i.e., average rate over every 1000 time slots (normalized over average arrival rate). We plot results in Fig. 2c. An interesting observation is that in this case, rate by SSTF is more bursty than that by QTF. Then during the periods that normalized service rates are lower than 1, DTJs accumulate such that queue length is fairly large in most slots. Although jobs can be finished during periods when service rates are large than 1, significant delay cannot be avoided.

One can increase average service rates of SSTF to obtain a smaller delay. But much more capacity needs to be consumed, which results in much higher cost. In many cases when load of DTJ is high, there is little space for SSTF to increase service rates. QTF can lead to arbitrary delay by tuning \( V \). One important property of QTF is that no matter \( V \) is large or small, the average service rate of QTF is always close to arrival rate, because it leverages the queue information. Thus QTF provides a more efficient method in saving cost and reducing delay. There are other findings, such as load shifting also plays an important role in reducing cost and queue delay. Due to the page limit, we omit them here.

B. Real trace based simulation

In this subsection, we use real datacenter traffic trace to study the performance of SSTF and QTF. Our trace comes from a commercial datacenter operated by a large cloud service provider in U.S. We obtain a Hadoop distributed file system (HDFS) log for one datacenter for thirty days. The HDFS log records the information of all received packets, including the packet size and time-stamp. The original data
does not differentiate DSJs and DTJs (In fact, to differentiate such traffic without application-layer information is itself a challenging issue in practical data center operations, which is an active research topic itself.). To address this issue, we simply adopt a threshold-based policy. We assume that a large packet is likely to be delay tolerant, and treat a packet with a size larger than a certain threshold as DTJ. This classification is rational, as authors in [43] indicate that most Internet request such as searching and web browsing are are only a few kb in size. We set threshold as 10, 50, 100, and 150Mb, to obtain different ratios between DSJ load and DTJ load, which results in the percentage of DTJ load in the total load roughly as 90%, 70%, 50%, and 10%, respectively. Note here we assume one unit (Mbit) of DSJs requires one unit of capacity, and one unit of DTJs requires 0.133 unit of capacity on average, by the same rate setting as the above simulations (average unit rate \( r_{ij} \) is roughly 7.5).

To simulate multiple IDCs and multiple DTJ queues, we choose twenty days of large packet traces as ten DTJ traffic traces, so that each of them has a two-day traffic trace. We choose ten days of small packet traces as the demand of DSJ for five IDCs considered. We consider a time slot length as 20 seconds. Therefore we have 8640 time slots for each two-day traffic trace.

Further, we use the electricity data in five wholesale market regions in 02/22/2011. They are California (Hub SP 15-EZ), Louisiana (Entergy), New England (NEPOOL Mass), Pennsylvania (PJM West), and Texas (ERCOT SOUTH). The capacity is uniformly distributed between 1000 and 1200. The bandwidth constraint is uniformly distributed between 1000 and 1500. The other setting is the same as in the synthetic traffic case.

We compare SSTF and QTF to the best effort service scheme (BES). In each slot, BES serves as much demand as possible for DTJ queue, in a best-effort fashion. When the available capacity in an IDC is not enough to finish current jobs, it equally shares the capacity among all DTJ queues. In the simulation, we assume SSTF knows the average DTJ arrival rate. The average service rate of SSTF is set equal to the average DTJ arrival rate. The control variable of QTF is set to 1000. We observe from Fig. 3 that for different percentages of DTJ load, BES always leads to the highest cost, while SSTF always has the lowest cost. The delay of SSTF is large, almost 5 hours. One reason is that it explores temporal electrical price diversity in a large time scale. One may think that BES would result in the lowest delay. But in Fig. 3 average delay of BES is always larger than that of QTF. The reason is that load shifting is not used in BES. Thus queues suffer large delay in an IDC with less available capacity. This illustrates that load shifting is not only necessary in reducing cost, but also important in exploring available capacity to improve delay performance. In summary, in Fig. 3 we observe that QTF is efficient in both saving cost and reducing delay.

It is also observed that as the percentage of DTJ load increases, the total cost decreases and the average DTJ delay also decreases. The reason is that when DSJ load decreases, total load amount decreases as DSJ requires more capacity per unit traffic. More capacity is thus available for DTJ, which leads to a smaller DTJ delay and more space for energy saving.

**IX. Conclusions**

In this paper, we study intelligent trough filling that achieves both energy efficiency and good delay performance. We design joint dynamic speed scaling and load shifting schemes. We first present a stochastic subgradient based trough filling algorithm, named SSTF, which solves a convex optimization problem for capacity allocation and load shifting in each slot. SSTF does not need the information of underlying distribution of system state. The SSTF can converge to optimal cost with a certain service rate constraint. We further propose a queue-based trough filling algorithm, named QTF, which also solves a convex optimization problem for capacity allocation and load shifting in each slot. We show QTF can achieve optimal tradeoff between queue delay and cost. Our extensive simulations based on both synthetic and real datacenter traces show that SSTF achieves optimal cost, but has a large delay. QTF achieves both desirable cost and delay. In practice, SSTF can be applied to the scenario where DTJs can have a large time delay, e.g., half of a day. QTF can be applied to the case where smaller time delay is desirable, e.g., tens of minutes.

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To prove Proposition 1, we first need Lemma 1 as

**Lemma 1:** For the optimization problem (27), with $\bar{S}$ replaced by $\bar{S} + 1\epsilon$, the resulting optimal solution $g^*_\epsilon(\epsilon)$ reaches $g^*_\epsilon$ as $\epsilon$ reaches 0.

**Proof:** We write the Lagrangian of problem (27) as

$$L(\mu_i, \bar{S}) = \sum_{\omega \in \Omega} \pi_\omega g^{\omega}(\bar{S}^\omega) - \sum_{j=1}^{M} \mu_j \left( \sum_{\omega \in \Omega} \sum_{i \in \Gamma_j} r_{ij} S_{ij}^{\omega} - \lambda_j \right) \tag{27}$$

When $\bar{S}$ replaced by $\bar{S} + 1\epsilon$, we have $L(\bar{S} + 1\epsilon, \lambda + 1\mu_i) \to L(\bar{S}, \lambda, \mu_i)$ as $\epsilon \to 0$. Since (27) is a convex optimization problem. We have $g^*_\epsilon(\epsilon)$ reaches $g^*_\epsilon$ as $\epsilon$ reaches 0.

We next present proof to Proposition 1.

**Proposition 1:** Consider the $M$ DTJ queues $\bar{Q}(t) = (Q_1(t), \ldots, Q_M(t))$. We introduce a non-negative Lyapunov function as $L(\bar{Q}(t)) = \sum_{j=1}^{M} Q_j^2(t)$. Define one-slot Lyapunov drift as

$$\Delta(t) = E \left[ L(\bar{Q}(t+1)) - L(\bar{Q}(t)) \right] \tag{28}$$

In terms of the fact that $(\max[a-b,0]+c)^2 \leq a^2 + b^2 + c^2 + 2a(c-b)$, for any $a, b, c \geq 0$, we have

$$Q_j^2(t+1) - Q_j^2(t) \leq \sum_{i \in \Gamma_j} (r_{ij} S_{ij}^t)^2 + D_j^2 + 2Q_j(t)(D_j^t - \sum_{i \in \Gamma_j} r_{ij} S_{ij}^t), \forall j \tag{29}$$

Based on (29), we further have

$$\Delta(t) \leq E \left[ \sum_{j=1}^{M} \sum_{i \in \Gamma_j} (r_{ij} S_{ij}^t)^2 \bar{Q}(t) \right] + E \left[ \sum_{j=1}^{M} D_j^2 |\bar{Q}(t)| \right] + 2E \left[ \sum_{j=1}^{M} Q_j(t) (D_j^t - \sum_{i \in \Gamma_j} r_{ij} S_{ij}^t) |\bar{Q}(t)| \right]. \tag{30}$$

Note $\sum_{j=1}^{M} \sum_{i \in \Gamma_j} r_{ij}^2 S_{ij}^t$ is bounded by $\sum_{i \in \Gamma_j, j \in \Pi_i} r_i^2 K_i^{max^2}$, where $r_i = \max \{ r_{ij} \mid j \in \Pi_i \}$, i.e., the maximum server rate with full server capacity. In each slot, we also have assumed that the arrival traffic size of each DTJ $j$ is bounded by $D_j^{max}$. For brevity, here we define $B = \sum_{i \in \Gamma_j, j \in \Pi_i} r_i^2 K_i^{max^2} + \sum_j D_j^{max^2}$. Since traffic of DTJs in each slot is independent of queue backlog $\bar{Q}(t)$, we can rewrite (30) as

$$\Delta(t) \leq B + 2 \sum_{j=1}^{M} Q_j(t) \lambda_j - 2E \left[ \sum_{j=1}^{M} Q_j(t) \sum_{i \in \Gamma_j} r_{ij} S_{ij}(t) |\bar{Q}(t)| \right]. \tag{31}$$

We consider the drift-plus-cost for the system where cost is resulted by QTF. The cost is the expected cost that is
conditional on queue backlog in time slot $t$, which can be written as $E(g_q^*(\mathbf{S}^t) | \mathbf{Q}(t))$. Note $V$ is a control variable, we have

$$\Delta(t) + VE[g_q^*(\mathbf{S}^t)] \leq B + 2 \sum_{j=1}^{M} Q_j(t) \lambda_j$$

$$- 2E \left[ \sum_{j=1}^{M} Q_j(t) \sum_{i \in I_j} r_{ij} s_{ij}^t | \mathbf{Q}(t) \right] + VE[g_q^*(\mathbf{S}^t)] | \mathbf{Q}(t)].$$

By (32), we can see that QTF minimizes drift-plus-cost in each time slot. Thus we have

$$2 \sum_{j=1}^{M} Q_j(t) \lambda_j + 2E \left[ Vg_q^*(\mathbf{S}^t) - \sum_{j=1}^{M} Q_j(t) \sum_{i \in I_j} r_{ij} s_{ij}^t | \mathbf{Q}(t) \right]$$

$$\leq 2 \sum_{j=1}^{M} Q_j(t) \lambda_j + 2E \left[ Vg_q^*(\epsilon) - \sum_{j=1}^{M} Q_j(t) (\lambda_j + \epsilon) | \mathbf{Q}(t) \right]$$

$$= - 2e \sum_{j=1}^{M} Q_j(t) + Vg_q^*(\epsilon).$$

By (32)(33), we have

$$\Delta(t) \leq B - 2e \sum_{j=1}^{M} Q_j(t) + Vg_q^*(\epsilon) - VE[g_q^*(\mathbf{S}^t)] \mathbf{Q}(t)].$$

Taking expectations of drift $\Delta(t)$ with respect to the distribution of the random queue backlog $\mathbf{Q}(t)$ at time $t$, we have

$$E \left[ L(\mathbf{Q}(t + 1)) - L(\mathbf{Q}(t)) \right] \leq$$

$$B - 2e \sum_{j=1}^{M} E[Q_j(t)] + Vg_q^*(\epsilon) - VE[g_q^*(\mathbf{S}^t)].$$

The above inequity is satisfied for all time slot $t$. Summing the $\Delta(t)$ over time slot $t = 1, 2, \ldots, T$, we have

$$E \left[ L(\mathbf{Q}(T)) - L(\mathbf{Q}(1)) \right] \leq$$

$$TB - 2e \sum_{t=1}^{T} \sum_{j=1}^{M} E[Q_j(t)] + TVg_q^*(\epsilon) - V \sum_{t=1}^{T} E[g_q^*(\mathbf{S}^t)].$$

By (36), we can get

$$\frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{M} E(Q_j(t)) \leq \frac{B + Vg_q^*(\epsilon)}{\epsilon} + \frac{L(\mathbf{Q}(1))}{Te}.$$ 

As $T \to \infty$, we have $\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{M} E(Q_j(t)) \leq \frac{B + Vg_q^*(\epsilon)}{\epsilon}$. Thus the queue backlog is bounded and system stability holds. Further

$$\frac{1}{T} \sum_{t=1}^{T} E[g_q^*(\mathbf{S}^t)] \leq g_q^*(\epsilon) + \frac{B}{V} + \frac{L(\mathbf{Q}(1))}{TV}.$$