Discussing Quantum Aspects of Higher-Derivative 3D-Gravity in the First-Order Formalism

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Abstract

In this paper, we reassess the issue of deriving the propagators and identifying the spectrum of excitations associated to the vielbein and spin connection of (1+2)-D gravity in the presence of dynamical torsion, while working in the first-order formulation. A number of peculiarities is pointed out whenever the Chern-Simons term is taken into account along with a combination of bilinear terms in the torsion tensor. We present a procedure to derive the full set of propagators, based on an algebra of enlarged spin-type operators, and we discuss under which conditions the poles of the tree-level 2-point functions correspond to physical excitations that do not conflict with causality and unitarity.

1 Introduction

The need to better understand gauge fields has lead to an widespread use of local transformations due the natural manner gauge fields appear in it. In the attempt to write (1+2)-D gravity as a gauge theory, the formulation
requires some specific technicalities, by virtue of the possibility of including the so-called (topological) Chern-Simons term. Adopting the Poincaré group as the local gauge group, one naturally obtains the curvature and torsion tensor by means of the Cartan’s structure equations. The translational part of the Poincaré group is represented by the vielbein gauge fields, $e^a_\alpha$, which are also diffeomorphic invariant under general coordinate transformations, and the Lorentzian part — realizing the equivalence principle — given by the spin connection gauge fields, $\omega^{ab}_\alpha$. The vielbein fields associate to each point a locally flat coordinate system and the spin connection relates any two local Lorentz coordinate systems at the given point.

This formalism is believed to be completely equivalent to the formalism that employs affine connections and define curvature and torsion in terms of them. There is a great deal of results that confirm this, mainly at the level of expressions to the curvature and torsion. At the classical level, this equivalence is indeed true. However, when we go over to the quantum field-theoretic version, there appear remarkable differences and we must indeed adopt the vielbein and spin connection as the independent fundamental degrees of freedom [12].

More recently, there has been a considerable raise of interest in (1+2)-dimensional gravity models with higher powers of the curvature. Specially, the New Massive Gravity Theory proposed by Bergshoeff, Hohm and Townsend [1, 2, 3] has triggered a number of contributions to the study of remarkable peculiarities and properties of higher-order $3d$-gravity models [4]-[11].

Motivated by these recent results, we propose here to pursue an investigation of planar gravity actions with propagating tensor, since we have in mind to understand which role torsion degrees of freedom may play in connection with unitary higher-derivative gravitation in $3D$. In order to investigate this further, we begin with the analysis of a traditional (1+2)-D gravity model previously done by two of us [13], where we have studied the inclusion of torsion in three-dimensional Einstein-Chern-Simons gravity and added up higher-derivative terms; we have worked in affine connection formalism. In this work, due to invertibility problems that appear in the theory if we adopt the first-order approach, we are lead to change to another Lagrangian density, where we introduce torsion algebraic terms, yielding dynamical spin-connection.

Due to the importance of the torsion terms, it is worthwhile to remember that torsion was introduced by E. Cartan in 1922, as the antisymmetric part of the affine connection and was recognized by him as a geometric object related to an intrinsic angular moment of matter. After the introduction of the
spin concept, it was suggested that torsion should mediate a contact interaction between spinning particles without propagation in matter-free space \cite{14, 15, 16}. Later, due the fact that at the microscopic level particles are classified by their mass and spin according to the Poincaré group, gauge theories of General Relativity were developed that brings in it dynamic torsion \cite{17}. These theories are motivated by the requirement that the Dirac equation in a gravitational field preserves local invariance under Lorentz transformations which yields, across the minimal coupling, a direct interaction between torsion and fermions. Observational constraints for a propagating torsion and its matter interactions are discussed in \cite{18, 19, 20, 21, 22, 23}.

Our work is organised according to the following outline: in Section 2, we present a quick review of the Einstein-Cartan formalism, with the purpose of fixing notation and setting our conventions. Next, the general model and the decomposition of the action in terms of spin operators is the subject of Section 3, where we point out a serious problem related to a spin-2 excitation. This motivates us to introduce and to analyse a number of torsion terms in the action, which is done in Section 4. In Section 5, we come to the task of computing the propagators and we analyse thereby their poles with the corresponding residues, in order to locate the physically relevant regions in the parameter space. Finally, in Section 6, we present our Concluding Comments, with a critical discussion on our main results and possible issues for future investigation.

2 Well-known Results on the Einstein-Cartan Approach

A Riemann-Cartan space-time \cite{14, 21, 22, 23} is defined as a manifold where the covariant derivative of the metric field exists and is given by:

$$\nabla_\gamma g_{\alpha\beta}(x) = 0,$$

where this equation defines the so called metric-compatible affine connection, $\Gamma_{\alpha\beta}^\gamma$; it allows the presence of torsion, given by the antisymmetric part of the affine connection,

$$T_{\alpha\beta}^\gamma = 2\Gamma_{[\alpha\beta]}^\gamma. \tag{2}$$

We then have:

$$\Gamma_{\alpha\beta}^\gamma = \left\{ \frac{\gamma}{\alpha\beta} \right\} + K_{\alpha\beta}^\gamma, \tag{3}$$
where \( \gamma_{\alpha\beta} \) is the Christoffel symbol, which is completely determined by the metric,

\[
\left\{ \gamma_{\alpha\beta} \right\} = \frac{1}{2} g^{\gamma\lambda} (\partial_{\alpha} g_{\lambda\beta} + \partial_{\beta} g_{\alpha\lambda} - \partial_{\lambda} g_{\alpha\beta})
\]

and

\[
K_{\alpha\beta\gamma} = \frac{1}{2} (T_{\alpha\beta\gamma} + T_{\gamma\alpha\beta} - T_{\beta\gamma\alpha})
\]

is the contortion tensor, antisymmetric in the last two indices.

In order to study local properties one introduces (in our specific (1+2)-D case) the dreibein vector fields, \( e_a^\alpha(x) \), that spans at any given point the local Minkowski space-time, which in this work has metric: \( \eta_{ab} = diag(1, -1, -1) \).

The introduction of the tangent Minkowski space-time allows local Lorentz transformations on geometrical objects (with Latin index). In order to render these objects invariant under local Lorentz rotations, one introduces the spin connection \( \omega_{\gamma}^{\alpha\beta} \). The covariant derivative of the dreibein then reads:

\[
\nabla_\gamma e_\alpha^a = D_\gamma e_\alpha^a - \Gamma_{\gamma\alpha\lambda} e_\lambda^a = 0,
\]

where \( D_\gamma e_\alpha^a = \partial_\gamma e_\alpha^a + \omega_{\alpha}^{\gamma a} e_\alpha^i \) is the Lorentz covariant derivative.

One finds from eq. (6) that the affine connection can then be written as:

\[
\Gamma_{\alpha\beta\gamma} = e_j^\gamma D_\alpha e_\beta^j,
\]

and the torsion tensor, eq. (2), reads

\[
T_{\alpha\beta\gamma} = 2\Gamma_{[\alpha\beta]}^\gamma = e_j^\gamma (\partial_\alpha e_\beta^j - \partial_\beta e_\alpha^j + \omega_{\alpha}^{j\beta} e_\beta^i - \omega_{\beta}^{j\alpha} e_\alpha^i).
\]

As known, the curvature tensors and scalar are given in terms of the affine connection by the expressions:

\[
R_{\mu\alpha\beta}^\nu = \partial_\mu \Gamma_{\alpha\beta}^\nu - \partial_\nu \Gamma_{\alpha\mu}^\beta + \Gamma_{\mu\rho}^\beta \Gamma_{\alpha\rho}^\nu - \Gamma_{\alpha\rho}^\nu \Gamma_{\mu\beta}^\rho,
\]

\[
R_{\alpha\beta} = R_{\mu\alpha\beta}^\mu = \partial_\mu \Gamma_{\mu\beta}^\alpha - \partial_\alpha \Gamma_{\mu\beta\mu} + \Gamma_{\mu\rho}^\rho \Gamma_{\alpha\beta}^\mu - \Gamma_{\alpha\beta}^\mu \Gamma_{\mu\rho}^\rho
\]

and

\[
\mathcal{R} = g^{\alpha\beta} R_{\alpha\beta}.
\]

In terms of the spin connection,
\[
R_{\mu \alpha \beta \nu} = e_\beta^i e_j^\nu (\partial_\mu \omega_{\alpha i}^j - \partial_\alpha \omega_{\mu i}^j + \omega_{\mu k}^i \omega_{\alpha k}^j - \omega_{\alpha k}^i \omega_{\mu k}^j),
\]
and
\[
R_{\alpha \beta} = e_\beta^i e_j^\mu (\partial_\alpha \omega_{\mu i}^j - \partial_\mu \omega_{\alpha i}^j + \omega_{\mu k}^i \omega_{\alpha k}^j - \omega_{\alpha k}^i \omega_{\mu k}^j).
\]

3 A Problem Related to a Spin-2 Excitation

We start off with the three-dimensional action for topologically massive gravity:

\[
S = \int d^3 x \; e \left( a_1 R + a_2 \mathcal{R}^2 + a_3 \mathcal{R}_{\alpha \beta} \mathcal{R}^{\alpha \beta} + a_4 \mathcal{L}_{CS} \right),
\]

where

\[
\mathcal{L}_{CS} = \epsilon^{\alpha \beta \gamma} \Gamma_\gamma \lambda \left( \partial_\alpha \Gamma_\lambda \beta + \frac{2}{3} \Gamma_\alpha \rho \delta \Gamma_\beta \lambda \rho \right),
\]
is the topological Chern-Simons term \[24, 25\] and

\[
\epsilon^{\alpha \beta \gamma} = \frac{\epsilon^{\alpha \beta \gamma}}{e}
\]
is the completely antisymmetric tensor in (1+2)-D, with \(\epsilon^{\alpha \beta \gamma}\) the Levi-Civita tensor density in the flat space and \(e = \sqrt{g}\) where \(g = \det(g_{\alpha \beta}) = \eta e^2\). \(a_1, a_2\) and \(a_3\) are free coefficients, whereas \(a_4\) is the Chern-Simons parameter. For a clear and detailed discussion of theories with Chern-Simons term, see \[26\].

As the Riemann tensor, \(R_{\mu \alpha \beta \nu}\), it has the same number of independent components as the Ricci tensor, \(R_{\alpha \beta}\), in three dimensions, a term squared in \(R_{\mu \alpha \beta \nu}\) is not necessary in the action of eq.(15)

In \[13\], we have written the affine connection as in eq.(3), further decomposing the torsion in its SO(1,2) irreducible components: a scalar from the totally antisymmetric part, a three-vector from the trace and a symmetric traceless rank-2 tensor. With this procedure, we have obtained a particle spectrum where only massive excitations of spin-2 associated with the linearized gravitational field, \(h^{\alpha \beta}\), and with the symmetric part of the torsion field had dynamics that preserved the unitarity of the theory for some values of the action parameters. In view of the results we obtained there we saw
that a ghost-free 3D gravity theory can be formulated once some constraints are imposed on the parameters of the Lagrangians we discuss \[27, 28\]. Thus, the interesting result that arised from our previous discussion is the possibility to write down a higher-derivative model for 3-D gravity with unitarity under control.

In this section, we reconsider the action \(15\) but, contrary to what we have done in \[13\], we propose to adopt in the first-order formulation, dropping the torsion as a fundamental excitation and electing the dreibein and the spin connection as the fundamental quantum fields, which reveals the full gauge structure of gravity.

Now, we consider eqs. \(7, 13\) and \(14\), and we adopt the weak field decomposition of the gravitational field,

\[
\varepsilon_{\alpha}^{a} = \delta_{\alpha}^{a} + \frac{k}{2} H_{\alpha}^{a} \left( \Rightarrow g_{\alpha\beta} = \eta_{\alpha\beta} + k h_{\alpha\beta}, \ h_{\alpha\beta} = \frac{1}{2} (H_{\alpha\beta} + H_{\beta\alpha}) \right),
\]

where \(k\) is the Planck length,

\[
H_{ab} = h_{ab} + \mathcal{H}_{ab}, \ h_{ab} = H_{(ab)} \quad \mathcal{H}_{ab} = H_{[ab]}
\]

and

\[
\mathcal{H}_{ab} = \epsilon_{abc} h^{c} \Rightarrow h_{a} = \frac{1}{2} \epsilon_{abc} \mathcal{H}^{bc}. \tag{20}
\]

The spin connection can be written in terms of its dual as follows:

\[
\omega_{a}^{bc} = \epsilon^{bcd} Y_{ad}, \tag{21}
\]

which can be further split according to,

\[
Y_{ab} = y_{ab} + \mathcal{Y}_{ab}; \ y_{ab} = Y_{(ab)} \quad \mathcal{Y}_{ab} = Y_{[ab]} \tag{22}
\]

and

\[
\mathcal{Y}_{ab} = \epsilon_{abc} y^{c} \Rightarrow y_{a} = \frac{1}{2} \epsilon_{abc} y_{bc}. \tag{23}
\]

We then rewrite the action \(15\), to which we add the gauge-fixing terms,

\[
\mathcal{L}_{GF- \text{diff}} = \lambda F_{a} F^{a}, \ F_{a} = \partial_{b} \left( H_{a}^{b} - \frac{1}{2} \delta_{a}^{b} H_{c}^{c} \right), \tag{24}
\]

and

\[
\mathcal{L}_{GF-LL} = \xi \left( \partial_{\mu} \omega_{\mu}^{ab} \partial^{\nu} \omega_{\nu ab} \right), \tag{25}
\]
in a more suitable (linearized) form:

\[
S = \int d^3 x \frac{1}{2} \Phi^T M \Phi, \quad \Phi = \begin{pmatrix} y^c_{,d} \\ y^c_{,c} \\ h^d_{,c} \\ h^c_{,d} \end{pmatrix}.
\]

(26)

\(\lambda\) e \(\xi\) are the gauge-fixing parameters. The wave operator, \(M\), can be expressed in terms of an extension of the spin-projection operator formalism introduced in \([29, 30, 31, 13]\).

Here, we would like to point out that, if we are concerned just with the excitation spectrum associated to the model under consideration and its unitarity property, we could simply decompose the fields according to their irreducible components, diagonalise the bilinear piece of the action (this would split the physical field components from the gauge compensating ones) and then read off the spectrum. However, the field components so obtained are non-local fields, since a \(\Box^{-1}\) appears in the projectors which act on the fields to separate their physical components. Since we wish to get the propagators for the local fields, having in mind that we can later carry out pertubative loop computations, we choose to keep the full fields and we are then obliged to gauge-fix the action so as to give propagation to the compensating components, and invert the wave operator, \(M\).

In this article, we follow the notations of \([30, 31]\) for the Barnes-Rivers operators, where it refers to the energy-momentum tensor: \((2)\) is the pure spin-2 sector, \((1-m)\) is the part related to the spin-1 momentum vector, \((0-s)\) is the part related to the spin-0 stress scalar and \((0-w)\) is the part related to the spin-0 work (energy) scalar; \((0-sw)\) and \((0-ws)\) are operators that map the spaces with the same spin. Five additional operators coming from the \(y^a\) – and Chern-Simons terms are needed, where the notation \((2a)\) indicates that this operator is a spin-2 operator with commutation relations only with the pure spin-2, in analogy to \((1a)\). Throughout this work, it is supposed that all differential operators that appear in the spin operators are dually replaced by a momentum 3-vector, in Fourier space.

The six operators for a rank-2 symmetric tensor in 3D are then given by:

\[
P^{(2)}_{ab,cd} = \frac{1}{2}(\theta_{ac}\theta_{bd} + \theta_{ad}\theta_{bc}) - \frac{1}{2}\theta_{ab}\theta_{cd},
\]

\[
P^{(1-m)}_{ab,cd} = \frac{1}{2}(\theta_{ac}\omega_{bd} + \theta_{ad}\omega_{bc} + \theta_{bc}\omega_{ad} + \theta_{bc}\omega_{ad}),
\]
\[ P_{ab,cd}^{(0-s)} = \frac{1}{2} \theta_{ab} \theta_{cd}, \quad (27) \]

\[ P_{ab,cd}^{(0-w)} = \omega_{ab} \omega_{cd}, \]

\[ P_{ab,cd}^{(0-sw)} = \frac{1}{\sqrt{2}} \theta_{ab} \omega_{cd} \]

and

\[ P_{ab,cd}^{(0-ws)} = \frac{1}{\sqrt{2}} \omega_{ab} \theta_{cd}, \]

where \( \theta_{ab} = \eta_{ab} - \omega_{ab} \) is the transverse and \( \omega_{ab} = \frac{\partial_{a} \partial_{b}}{2} \) is the longitudinal projector operator that act on vector fields to split their spin-0 and spin-1 components. The others five operators are:

\[ S_{ab,cd}^{(2a)} = (\epsilon_{ace} \theta_{bd} + \epsilon_{ade} \theta_{bc} + \epsilon_{bce} \theta_{ad} + \epsilon_{bde} \theta_{ac}) \partial^{e}, \]

\[ R_{ab,cd}^{(1a)} = (\epsilon_{ace} \omega_{bd} + \epsilon_{ade} \omega_{bc} + \epsilon_{bce} \omega_{ad} + \epsilon_{bde} \omega_{ac}) \partial^{e}, \]

\[ A_{ab} = \epsilon_{abc} \partial^{c}, \quad (28) \]

\[ B_{a,bc} = \eta_{ab} \partial_{c} + \eta_{ac} \partial_{b} \]

and

\[ D_{a,bc} = A_{ab} \partial_{c} + A_{ac} \partial_{b}. \]

We recall that the usual Barnes-Rivers operators obey the algebra:

\[ P_{ab,kl}^{(i-a)} P_{cd}^{(j-b)} kl = \delta^{ij} \delta^{ab} P_{ab,cd}^{(j-b)} \]

\[ P_{ab,kl}^{(i-ab)} P_{cd}^{(j-cd)} kl = \delta^{ij} \delta^{bc} P_{ab,cd}^{(j-a)} \quad (29) \]

\[ P_{ab,kl}^{(i-a)} P_{cd}^{(j-ac)} kl = \delta^{ij} \delta^{bc} P_{ab,cd}^{(j-ac)} \]

\[ P_{ab,kl}^{(i-ab)} P_{cd}^{(j-c)} kl = \delta^{ij} \delta^{bc} P_{ab,cd}^{(j-ac)} \]
and satisfy the tensor identity,

\[ P_{ab,cd}^{(2)} + P_{ab,cd}^{(1m)} + P_{ab,cd}^{(0s)} + P_{ab,cd}^{(0w)} = \frac{1}{2} (\eta_{ac} \eta_{bd} + \eta_{ad} \eta_{bc}) . \]  

(30)

The new set of spin operators that comes about displays, besides the operators \( S_{ab,cd}^{(2a)} \), \( R_{ab,cd}^{(1a)} \), \( A_{ab} \), and \( B_{a,bc} \) (already known from [13]), one new operator, \( D_{a,bc} \), given in (28). These five operators have their own multiplicative table; we quote below only some of the relevant products amongst them:

\[ S_{ab,ef}^{(2a)} S_{cd}^{(2a)} = -16 \Box P_{ab,cd}^{(2)} , \]

\[ R_{ab,ef}^{(1a)} R_{cd}^{(1a)} = -4 \Box P_{ab,cd}^{(1m)} , \]

\[ P_{ab,ef}^{(2)} S_{cd}^{(2a)} = S_{ab,ef}^{(2a)} P_{cd}^{(2)} = S_{ab,cd}^{(2a)} , \]

\[ P_{ab,ef}^{(1m)} R_{cd}^{(1a)} = R_{ab,ef}^{(1a)} P_{cd}^{(1m)} = R_{ab,cd}^{(1m)} , \]  

(31)

\[ A_{ae} A_{eb} = -\Box \theta_{ab} , \]

\[ B_{a,ef} B_{c,ef} = 2 \Box (\theta_{ac} + 2 \omega_{ac}) , \]

\[ B_{e,ab} B_{c,cd} = 2 \Box (P_{ab,cd}^{(1m)} + 2 P_{ab,cd}^{(0w)}) , \]

\[ D_{a,ef} D_{c,ef} = 2 \Box^2 \theta_{ac} \]

and

\[ D_{e,ab} D_{c,cd} = 2 \Box^2 P_{ab,cd}^{(1m)} . \]

Here we have, just for the sake of calculational simplification, omitted the \( h^c \) component, dual of \( H_{ab} = H_{[ab]} \), in \( M \), since we are not going to actually calculate the propagators in this section, and we already can see the problem that is going to appear with the reduced matrix. In the next section, where the propagators for all field components will be derived, the anti-symmetric part will not be left aside.

Thus, the wave operator acquires the form, without the anti-symmetric part of \( H_{ab} \) as commented above:
\[ M = \left( \begin{array}{ccc} M_{\text{yy}}^{ab,cd} & M_{\text{yy}}^{ab,c} & M_{\text{yy}}^{yh} \\ M_{\text{yy}}^{a,cd} & M_{\text{yy}}^{a,c} & M_{\text{yy}}^{h} \\ M_{\text{yy}}^{h} & M_{\text{yy}}^{h} & M_{\text{hh}}^{ab,cd} \end{array} \right), \]  

where

\[ M_{\text{yy}}^{ab,cd} = (2a_1 - 2a_3 \Box) P_{\text{ab,cd}}^{(2)} + (2a_1 - a_3 \Box - 2\xi \Box) P_{\text{ab,cd}}^{(1m)} - (2a_1 + 2a_3 \Box) P_{\text{ab,cd}}^{(0s)} - 4\xi \Box P_{\text{ab,cd}}^{(0w)} - 2\sqrt{2}a_1 (P_{\text{ab,cd}}^{(0sw)} + P_{\text{ab,cd}}^{(0ws)}) + \frac{a_4}{2} (S_{\text{ab,cd}}^{(2a)} + R_{\text{ab,cd}}^{(1a)}), \]

\[ M_{\text{yy}}^{ab,c} = a_4 B_{c,ab} + (2\xi - a_3) D_{c,ab}, \]

\[ M_{\text{yy}}^{h} = \frac{k}{2} a_4 (P_{\text{ab,cd}}^{(2)} - P_{\text{ab,cd}}^{(0s)}) + \frac{k}{4} a_1 (S_{\text{ab,cd}}^{(2a)} + R_{\text{ab,cd}}^{(1a)}), \]

\[ M_{\text{yy}}^{a,cd} = -a_4 B_{a,bc} + (2\xi - a_3) D_{a,bc}, \]

\[ M_{\text{yy}}^{a,c} = -4a_1 + 2a_3 \Box + 4\xi \Box \theta_{a,c} - (4a_1 + 32a_2 \Box + 12a_3 \Box) \omega_{a,c} + 2a_4 A_{a,c}, \]

\[ M_{\text{yy}}^{h} = -\frac{k}{2} a_1 B_{a,bc} + ka_1 (\theta_{bc} + \omega_{bc}) \partial_a, \]

\[ M_{\text{yy}}^{h} = \frac{k}{2} a_4 (P_{\text{ab,cd}}^{(2)} - P_{\text{ab,cd}}^{(0s)}) + \frac{k}{4} a_1 (S_{\text{ab,cd}}^{(2a)} + R_{\text{ab,cd}}^{(1a)}), \]

\[ M_{\text{yy}}^{h} = \frac{k}{2} a_1 B_{a,bc} - ka_1 (\theta_{bc} + \omega_{bc}) \partial_a \]

and

\[ M_{\text{yy}}^{h} = -\lambda \Box \left( P_{\text{ab,cd}}^{(1m)} + P_{\text{ab,cd}}^{(0s)} + \frac{1}{2} P_{\text{ab,cd}}^{(0w)} - \frac{\sqrt{2}}{2} (P_{\text{ab,cd}}^{(0sw)} + P_{\text{ab,cd}}^{(0ws)}) \right). \]

In order to write down the propagators of the model,

\[ \langle 0 | T[\Phi(x)\Phi(y)] | 0 \rangle = iM^{-1} \delta^{(3)}(x - y), \]
we need to calculate the inverse matrix, $M^{-1}$, of the wave operator. But, here we face a problem: the matrix element $M_{ab,cd}^{hh}$ does not have a term in $P^{(2)}_{ab,cd}$, and we cannot find the inverse element of this fundamental term (to compute the inverse, we need to close the relation given in eq. (30), which not occur).

At this point, a comment is worthy: the lack of invertibility of the wave operator, $M$, is understandable and should be expected, once we are now adopting the first-order formalism, where some of the gravity-field components are non-dynamical, and so can rather be replaced in terms of the independent components by means of the classical equations of motion, which actually play the role of constraints. This is a particularity of auxiliary fields appearing in actions with local symmetry. This is indeed the case of gravity. We can see, in this manner, that a completely invertible theory, when decomposed in terms of one gauge field and its torsion tensor components, has difficulties when we adopt the version where the torsion is not taken as the fundamental field, but rather work with the gauge field associated to Lorentz local transformation that incorporates the torsion information (in a Einstein-Cartan theory $\omega_{abc} = \gamma_{abc} - K_{abc}$, where $\gamma_{abc}$ is the "pure Riemannian", without torsion, part and $K_{abc}$ is the contortion term). The missing spin-2 term of the gravitational gauge field is incorporated into the "Riemannian part" of the spin connection gauge field in the first-order formalism.

4 Introducing the Torsion Terms

In order to try to formulate a pure gauge model for planar gravitation, and yet to understand the role torsion plays, we drop from the action (15) to a new action where the curvature terms are replaced by torsion:

$$S = \int d^3x \, e(a_1 R + a_2 T_{\alpha \beta \gamma} T^{\alpha \beta \gamma} + a_3 T_{\alpha \beta \gamma} T^{\beta \gamma \alpha} + a_4 T_{\alpha \beta} T^{\alpha \gamma} + a_5 L_{CS}). \quad (35)$$

$L_{CS}$ being the usual Chern-Simons term, given in eq. (16). $a_1$, $a_2$, $a_3$ and $a_4$ are free coefficients, whereas $a_5$ is the Chern-Simons parameter. See reference [27] for a complete discussion about the specific torsion terms. From now on, all our calculations and results refer to the action (35). In our final section, we shall make a comment on the possibility of introducing a term which is linear in the torsion [26].
We consider equations (14), (8) and (7), and the decompositions (21), (22) and (23), with the weak expansion (18) and equations (19) and (20), we can rewrite the action (35), introducing the gauge-fixing term,

$$\mathcal{L}_{GF-diff} = \lambda F_a F^a, \quad F_a = k \partial^b \left( H_{ba} - \frac{1}{2} \eta_{ba} H^c_c \right),$$

in the linearized form below:

$$S = \int d^3 x \frac{1}{2} \Phi^T M \Phi, \quad \Phi = \begin{pmatrix} h^{cd} \\ h^c \\ y^{cd} \\ y^c \end{pmatrix}. \quad (37)$$

As before, we express the wave operator, $M$, in terms of the extended spin-projection operators. In addition to the operators listed above, there appear two new operators:

$$\theta_{ab} \partial_c \quad \text{and} \quad \omega_{ab} \partial_c, \quad (38)$$

which, together with the old ones, completely close the algebra.

This yields the form below for the wave operator:

$$M = \begin{pmatrix} M_{ab,cd}^{hh} & M_{ab,c}^{hh} & M_{ab,cd}^{hy} & M_{ab,c}^{hy} \\ M_{ab,c}^{hh} & M_{a,cd}^{hh} & M_{a,cd}^{hy} & M_{a,c}^{hy} \\ M_{ab,cd}^{yh} & M_{ab,c}^{yh} & M_{ab,cd}^{yy} & M_{ab,c}^{yy} \\ M_{a,cd}^{yh} & M_{a,c}^{yh} & M_{a,cd}^{yy} & M_{a,c}^{yy} \end{pmatrix}, \quad (39)$$

where

$$M_{ab,cd}^{hh} = \frac{k^2}{2} \Box (a_3 - 2a_2) P^{(2)}_{ab,cd} + \frac{k^2}{4} \Box (a_3 - 2a_2 - a_4 - 4\lambda) P^{(1m)}_{ab,cd}$$

$$+ \frac{k^2}{2} \Box (a_3 - 2a_2 - a_4 - 2\lambda) P^{(0s)}_{ab,cd} - \left( \frac{k^2}{2} \Box \lambda \right) P^{(0w)}_{ab,cd}$$

$$-(\sqrt{2} k^2 \Box \lambda)(P^{(0sw)}_{ab,cd} + P^{(0ws)}_{ab,cd}) - \frac{k^2}{2} a_5 (S^{(2a)}_{ab,cd} + R^{(1a)}_{ab,cd}), \quad (40)$$

$$M_{ab,c}^{hh} = -\left( \frac{k^2}{2} a_5 \right) B_{c,ab} + \frac{k^2}{4} (a_3 - 2a_2 - a_4 + 4\lambda) D_{c,ab}, \quad (41)$$
\[ M^{hy}_{ab,cd} = \frac{k}{2}(\Box^2 a_6 - 2a_5)P^{(2)}_{ab,cd} - (ka_5)P^{(1m)}_{ab,cd} - \frac{k}{2}(\Box a_6 + 2a_5)P^{(0s)}_{ab,cd} \\
- (ka_5)P^{(0w)}_{ab,cd} + \frac{k}{4}(a_1 + 2a_2 - 2a_3)(S^{(2a)}_{ab,cd} + R^{(1a)}_{ab,cd}), \quad (42) \]

\[ M^{hy}_{ab,c} = \frac{k}{2}(a_1 - 2a_2 - 2a_4)B_{c,ab} - k(a_1 - 2a_2 - 2a_4)(\theta_{ab} + \omega_{ab})\partial_c, \quad (43) \]

\[ M^{hh}_{a,cd} = (\frac{k^2}{2}a_5)B_{a,cd} + \frac{k^2}{4}(a_3 - 2a_2 - a_4 + 4\lambda)D_{a,cd}, \quad (44) \]

\[ M^{hh}_{a,c} = \frac{k^2}{2}\Box(a_3 - 2a_2 - a_4 - 4\lambda)\theta_{a,c} - (k^2\Box)(2a_2 + a_3)\omega_{a,c} - (k^2a_5)A_{a,c}, \quad (45) \]

\[ M^{hy}_{a,cd} = -\frac{k}{2}(a_1 + 2a_2)B_{a,bc} + k(a_1 - 2a_2 - 2a_3)(\theta_{bc} + \omega_{bc})\partial_a, \quad (46) \]

\[ M^{hy}_{a,c} = (2ka_5)\theta_{a,c} + k(2a_5 - \Box a_6)\omega_{a,c} + k(a_1 - 2a_2 - 2a_4)A_{a,c}, \quad (47) \]

\[ M^{bh}_{ab,cd} = \frac{k}{2}(\Box a_6 - 2a_5)P^{(2)}_{ab,cd} - (ka_5)P^{(1m)}_{ab,cd} - \frac{k}{2}(\Box a_6 + 2a_5)P^{(0s)}_{ab,cd} \\
- (ka_5)P^{(0w)}_{ab,cd} + \frac{k}{4}(a_1 + 2a_2 - 2a_3)(S^{(2a)}_{ab,cd} + R^{(1a)}_{ab,cd}), \quad (48) \]

\[ M^{bh}_{ab,c} = \frac{k}{2}(a_1 + 2a_2)B_{c,ab} - k(a_1 - 2a_2 - 2a_3)(\theta_{ab} + \omega_{ab})\partial_c, \quad (49) \]

\[ M^{sy}_{ab,cd} = 2(a_1 + 2a_2 - a_3)P^{(2)}_{ab,cd} + 2(a_1 + 2a_2 - a_3)P^{(1m)}_{ab,cd} \\
+ 2(6a_2 + 5a_3 - a_1)P^{(0s)}_{ab,cd} + 4(2a_2 + a_3)P^{(0w)}_{ab,cd} \\
+ 2\sqrt{2}(2a_2 + 3a_3 - a_1)(P^{(0w)}_{ab,cd} + b^{(0w)}_{ab,cd}) + (\frac{a_6}{2})^2(S^{(2a)}_{ab,cd} + R^{(1a)}_{ab,cd}), \quad (50) \]
\[ M^{yy}_{ab,c} = a_6 B_{c,ab}, \]  
\[ M^{yh}_{a,cd} = -\frac{k}{2}(a_1 - 2a_2 - 2a_4)B_{a,bc} + k(a_1 - 2a_2 - 2a_4)(\theta_{bc} + \omega_{bc})\theta_a, \]  
\[ M^{yh}_{a,c} = (2ka_5)\theta_{a,c} + k(2a_5 - \Box a_6)\omega_{a,c} + k(a_1 - 2a_2 - 2a_4)A_{a,c}, \]  
\[ M^{yy}_{a,cd} = -a_6 B_{a,cd} \]  
and  
\[ M^{yy}_{a,c} = 4(2a_2 + 2a_4 - a_1 - a_3)\theta_{a,c} + 4(2a_2 + 2a_4 - a_1 - a_3)\omega_{a,c} + (2a_6)A_{a,c}. \]  

Once all operators have been identified, we finally come to the task of computing the inverses. This is what we shall do next.

5 Propagators and Excitation Modes

In order to calculate the propagators, eq. (34), we use a straightforward, but lengthy, procedure in terms of which we decompose the matrix \( M \) into four sectors, namely:

\[ M = \begin{pmatrix} M^{hh} & M^{hy} \\ M^{yh} & M^{yy} \end{pmatrix}. \]  

Thus the inverse matrix \( M^{-1} \) can be written as:

\[ M^{-1} = \begin{pmatrix} M^{HH} & M^{HY} \\ M^{YH} & M^{YY} \end{pmatrix}, \]  

where its blocks are given by:

\[ M^{HH} = [M^{hh} - M^{hy}(M^{yy})^{-1}M^{yh}]^{-1}. \]  
\[ M^{HY} = -(M^{hh})^{-1}M^{hy}M^{YY}. \]  
\[ M^{YH} = -(M^{yy})^{-1}M^{yh}M^{HH}. \]  
\[ M^{YY} = [M^{yy} - M^{yh}(M^{hh})^{-1}M^{hy}]^{-1}. \]
Once the propagators are read off, we must check the tree-level unitarity of the theory. To this, we have to analyse the residues of the current-current transition amplitude in momentum space, given by the saturated propagator after a Fourier transformation. The sources that saturate the propagators can be expanded in terms of a complete basis in the momentum space as follows:

\[
S_{\mu
u} = c'_1 p_\mu p_\nu + c'_2 p_\mu q_\nu + c'_3 p_\mu \varepsilon_\nu + c'_4 q_\mu p_\nu + c'_5 q_\mu q_\nu + c'_6 q_\mu \varepsilon_\nu + c'_7 \varepsilon_\mu p_\nu + c'_8 \varepsilon_\mu q_\nu + c'_9 \varepsilon_\mu \varepsilon_\nu,
\]

where \( p_\mu = (p_0, -\vec{p}) \), \( q_\mu = (p_0, \vec{p}) \) and \( \varepsilon_\mu = (0, -\vec{\varepsilon}) \) are linearly independent vectors that satisfy the conditions:

\[
\begin{align*}
p_\mu p^\mu &= q_\mu q^\mu = m^2. \\
p_\mu q^\mu &= p_0^2 + \vec{p}^2 \neq 0. \\
p_\mu \varepsilon^\mu &= q_\mu \varepsilon^\mu = 0. \\
\varepsilon_\mu \varepsilon^\mu &= -1.
\end{align*}
\]

These conditions and the symmetry requirements of the theory split the sources, \( S_{\mu\nu} \), in a symmetric and an antisymmetric part:

\[
S_{S\mu\nu} = S_{(\mu\nu)} = c_1 p_\mu p_\nu + c_2 (p_\mu q_\nu + q_\mu p_\nu) + c_3 (p_\mu \varepsilon_\nu + \varepsilon_\mu p_\nu) + c_4 q_\mu q_\nu + c_5 (q_\mu \varepsilon_\nu + \varepsilon_\mu q_\nu) + c_6 \varepsilon_\mu \varepsilon_\nu
\]

and

\[
A_{S\mu\nu} = S_{[\mu\nu]} = d_1 (p_\mu q_\nu - q_\mu p_\nu) + d_2 (p_\mu \varepsilon_\nu - \varepsilon_\mu p_\nu) + d_3 (q_\mu \varepsilon_\nu - \varepsilon_\mu q_\nu),
\]

where \( c_1 = c'_1, c_2 = \frac{c'_1 + c'_2}{2} \), \( c_3 = \frac{c'_3 + c'_4}{2} \), \( c_4 = c'_5 \), \( c_5 = \frac{c'_6 + c'_8}{2} \), \( c_6 = c'_9 \) \( d_1 = \frac{c'_2 - c'_3}{2} \), \( d_2 = \frac{c'_4 - c'_5}{2} \), and \( d_3 = \frac{c'_6 - c'_7}{2} \).

The current-current transition amplitude is written as:

\[
A = \begin{pmatrix} \tau^* & \rho^* \end{pmatrix} \begin{pmatrix} M^{HH} & M^{HY} \\ M^{YH} & M^{YY} \end{pmatrix} \begin{pmatrix} \tau \\ \rho \end{pmatrix} \Rightarrow
\]

\[
A = \tau^* M^{HH} \tau + \tau^* M^{HY} \rho + \rho^* M^{YH} \tau + \rho^* M^{YY} \rho,
\]
where \( \tau \) is the source to the \( h \) fields and \( \rho \) the source to the \( y \) fields.

\( A \) can then be cast into the form below:

\[
A = \frac{1}{2}(H H H(2)) + \frac{1}{6}(H H H(0)) + \frac{1}{2}(H Y H(2)) + \frac{1}{3}(H Y H(0)) + \frac{1}{2}(Y Y Y(2)) + \frac{1}{3}(Y Y Y(0))
\]

where \( H = \text{the symmetric rank-2 field propagator associated to the operator} \) gravitational field propagator associated to the operator \( p_{ab,cd} \), \( p_{ab,cd} \) survives and contributes.

For a massless pole, or for a massive pole in the rest frame (where \( \mu = (m,0), \eta = (m,0), \) only the projectors \( p_{ab,cd} \), \( p_{ab,cd} \) survive and contribute.

With the restrictions above, the amplitude reads:

\[
A = H H H(2) + H Y H(2) + Y Y Y(2)
\]

where \( H H H(2) > \) is the symmetric rank-2 \( H H H(2) \) gravitational field propagator associated to the operator \( p_{ab,cd} \). The other coefficients have analogous meaning. Explicitly writing the sources, we get:

\[
A = \frac{1}{2}(H H H(2)) + \frac{1}{6}(H H H(0)) + \frac{1}{2}(H Y H(2)) + \frac{1}{3}(H Y H(0)) + \frac{1}{2}(Y Y Y(2)) + \frac{1}{3}(Y Y Y(0))
\]

where \( H = \text{the symmetric rank-2 field propagator associated to the operator} \) gravitational field propagator associated to the operator \( p_{ab,cd} \), \( p_{ab,cd} \) survives and contributes.

For a massless pole, or for a massive pole in the rest frame (where \( \mu = (m,0), \eta = (m,0), \) only the projectors \( p_{ab,cd} \), \( p_{ab,cd} \) survive and contribute.

With the restrictions above, the amplitude reads:

\[
A = H H H(2) + H Y H(2) + Y Y Y(2)
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where \( H H H(2) > \) is the symmetric rank-2 \( H H H(2) \) gravitational field propagator associated to the operator \( p_{ab,cd} \). The other coefficients have analogous meaning. Explicitly writing the sources, we get:

\[
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\]

where \( H = \text{the symmetric rank-2 field propagator associated to the operator} \) gravitational field propagator associated to the operator \( p_{ab,cd} \), \( p_{ab,cd} \) survives and contributes.
where $t$ and $r$ in the $c$ mean the source associated to the particular term.

We must now replace the results obtained by the procedure described in (58) into (66). Before, explicitly we put our results, the following comments should be done:

1. With the whole set of action parameters, $a_1, a_2, a_3, a_4, a_5$ and $\lambda$, different from zero, our computational algebraic facilities failed in attaining an analytical result, due to the extension of the resulting expressions.

2. Considering the Chern-Simons term, $a_5$, we obtained the following behaviour in the denominator of the propagator:
   - With $a_1 = 0$, we have terms proportional to $p^{22}$.
   - The lowest power, $p^6$, occurs with $a_1 = a_2 = a_4 = 0$, only $a_3$ and $a_5$ are considered.
   - With $a_3 = 0$, we do not have an invertible case.

3. Without the Chern-Simons term, $a_5 = 0$, we obtain, in all invertible cases, a power $p^2$. This is not a straightforward result; we may justify it by pointing out that Chern-Simons contributes a term quadratic in the spin connection with a space-time derivative, whereas the scalar curvature contributes a term that mixes $H$ with $\omega$. Setting $a_5$ to zero, we suppress $\omega - \omega$ terms with a derivative, and so we unavoidably reduce the powers in the momentum appearing in the propagators.

We then consider in (66) only the cases with $a_5 = 0$.

Anyway, we have a procedure that works out for all possibilities in parameter space (once we keep $a_6 = 0$; we come back to this point in our Concluding Comments). We simply report here the cases with $a_3$, and $a_2$ and $a_3$, different from zero to have an illustration of how our general procedure works.

The least invertible case occurs by considering only $a_3$ different from zero in the action. In this case, the relevant propagators read:
\[ H2H2_{(2)} = \frac{2}{3k^2p^2a_3} i. \]
\[ H2H2_{(0s)} = -\frac{2}{k^2p^2a_3} i. \]
\[ H2Y2_{(2)} = H2Y2_{(0s)} = Y2H2_{(2)} = Y2H2_{(0s)} = 0. \] (67)
\[ Y2Y2_{(2)} = \frac{1}{6a_3} i. \]
\[ Y2Y2_{(0s)} = 0 \]

and the saturated amplitude is as given below,

\[ \mathcal{A} = \left( -\frac{2}{3k^2p^2a_3} |c_6|^2_{tt} + \frac{1}{12a_3} |c_6|^2_{rr} \right) i. \] (68)

We notice in this expression that the massless pole comes from the \( h \)-block and has contributions from the spin-0 and the spin-2 sectors.

Then, by calculating the imaginary part of the residue of the amplitude at the massless pole, we get:

\[ \text{Im}(\text{res} \mathcal{A}) = \text{Im} \left( \lim_{p^2 \to 0} [p^2 \mathcal{A}] \right) = \frac{2|c_6|^2_{tt}}{3k^2a_3}. \] (69)

From the requirement of having positive-definite residue at the pole, we must have \( a_3 < 0 \).

Considering now the addition of the scalar of curvature term \( a_1 \), we get:

\[ H2H2_{(2)} = \frac{2(a_3 - a_1)}{k^2p^2(3a_3^2 + a_1^2 - 3a_3a_1)} i. \]
\[ H2H2_{(0s)} = -\frac{2(a_3 + a_1)}{k^2p^2(a_3^2 - a_1^2 + a_3a_1)} i. \]
\[ H2Y2_{(2)} = H2Y2_{(0s)} = Y2H2_{(2)} = Y2H2_{(0s)} = 0 \] (70)
\[ Y2Y2_{(2)} = \frac{a_3}{2(3a_3^2 + a_1^2 - 3a_3a_1)} i. \]
\[ Y2Y2_{(0s)} = 0 \]

and the amplitude becomes:

\[ \mathcal{A} = \left( -\frac{2}{k^2p^2} \times \frac{a_3^3}{3a_3^4 - 5a_3^2a_1^2 + 4a_3a_1^3 - a_1^4} |c_6|^2_{tt} + \frac{a_3}{2(3a_3^2 + a_1^2 - 3a_3a_1)} |c_6|^2_{rr} \right) i. \]
We can see that the structure of the amplitude is not changed, with the pole having contributions from the same spin sectors. The parameters relations now reads:

\[ \text{Im}(\text{res} A) = \text{Im} \left( \lim_{p^2 \to 0} [p^2 A] \right) = -\frac{2}{k^2} \frac{a_3^2}{3a_3^4 - 5a_3^2a_1^2 + 4a_3a_1^3 - a_1^4} |c_6|^2. \]  

(71)

The denominator in (71) can be written as:

\[ (a_3^2 + a_3a_1 - a_1^2)(3a_3^2 - 3a_3a_1 + a_1^2). \]  

(72)

The binomial \( 3a_3^2 - 3a_3a_1 + a_1^2 \) has complex roots and is greater than zero.

The requirement of having positive-definite residue at the pole implies (with \( a_3 < 0 \)) \( a_3^2 - a_3a_1 - a_1^2 < 0 \). And the scalar term must obey \( \frac{1 + \sqrt{5}}{2} a_3 \approx 1.618a_3 < a_1 < \frac{1 - \sqrt{5}}{2} a_3 \approx -0.618a_3 \).

The case where all parameters (with exception to \( a_5 \)) are different from zero brings only new algebraic corrections to the amplitude, without changing its structure. The relations among the parameters become very cumbersome, due to the considerable number of parameters involved, so that many hypotheses must be done.

### 6 Concluding Comments

In the course of the calculations we report in this work, if we complete the action [35] by adjoining the term \( a_6 \varepsilon^{\mu \nu \lambda} T_{\mu \nu} \eta^a e_\lambda b \eta_{ab} = a_6 \varepsilon^{\mu \nu \lambda} T_{\mu \nu} \eta^a e_\lambda b \eta_{ab} = a_6 \varepsilon^{\mu \nu \lambda} T_{\mu \nu \lambda} \) [28], a problem shows up: though our procedure of introducing the spin operators works, the propagators could not be found in their generality (with all the six coefficients \( a_i \)) even with the help of algebraic computation techniques. However, we found out that, once any of the \( a_i \) are set to zero, we succeed in reading off the propagators, even if they display higher powers in the momentum. It is worthwhile to mention here that this linear term in the torsion combines with the Chern-Simons action to give a rich structure of poles in the propagators. We do not report these results here because this investigation is the matter of a forthcoming publication [32]. The situation gets better when we discovered that, ruling out the Chern-Simons term, we get only simple poles in the terms that contribute to the amplitude. Very surprising was the discovery of the very different role the torsion terms (\( a_2 \) and \( a_3 \)) play, being \( a_3 \) fundamental to compute the inverse matrix, which is not the case for \( a_2 \).
We see that the physical poles are all massless. It is worthy to note that, in [13], we get only physical mass poles. The unitarity condition for the physical poles demand that $a_3 < 0$ and this implies in that the parameter that governs the scalar curvature must obey the condition $\frac{1 + \sqrt{5}}{2} a_3 < a_1 < \frac{1 - \sqrt{5}}{2} a_3$.

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