Optimization of Conventional and Green Vehicles Composition under Carbon Emission Cap

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Abstract: The CO\textsubscript{2} emission of transportation is significantly reduced by the employment of green vehicles to the existing vehicle fleet of the organizations. This paper intends to optimize the composition of conventional and green vehicles for a logistics distribution problem operating under a carbon emission cap imposed by the government. The underlying problem involves product delivery by the vehicles starting from a single depot to geographically distributed customers. The delivery occurs within specified time windows. To solve the proposed problem, we design a hybrid metaheuristic solution based on ant colony optimization (ACO) and variable neighborhood search (VNS) algorithms. Extensive computational experiments have been performed on newly generated problem instances and benchmark problem instances adopted from the literature. The proposed hybrid ACO is proven to be superior to the state-of-the-art algorithms available in the literature. We obtain 21 new best-known solutions out of 56 benchmark instances of vehicle routing problem with time windows (VRPTW). The proposed mixed fleet model obtains the best composition of conventional and green vehicles with a 6.90% reduced amount of CO\textsubscript{2} emissions compared to the case when the fleet consists of conventional vehicles only.

Keywords: green vehicles; CO\textsubscript{2} emission cap; mixed fleet of logistics problem with time window; VRPTW; ant colony optimization; hybrid metaheuristic

1. Introduction

Transportation in business logistics has a two-faced contribution to countries. It generates approximately 5.00% of GDP in countries [1]. It adversely produces nearly one-quarter of global CO\textsubscript{2} emission every year and the amount is estimated to be one-half by 2050 [2], which has many negative effects on the environment. The CO\textsubscript{2} emission is caused by burning fossil fuels such as diesel and gasoline used in conventional vehicles [3]. World leaders have pledged to slash CO\textsubscript{2} emissions in their countries. The CO\textsubscript{2} emission cap (restriction) is underway to be imposed for businesses and organizations by world organizations such as the Conference of Parties (COP) and the European Commission [4]. Transportation sectors are encouraged, in some cases, they are forced, to reduce their CO\textsubscript{2} emissions by adapting green vehicles in their existing vehicle fleets or face the penalties otherwise. However, conventional vehicles from fleets cannot be excluded completely because their operation costs are cheap compared to green vehicles. Eventually, company logistics will become more common with the mixed vehicle fleet for their operations in the near future.

Green vehicles, such as electric vehicles, hybrid vehicles, and hydrogen vehicles, offer the advantage of reduced CO\textsubscript{2} emissions compared to conventional vehicles. Conventional vehicles have the benefits of cheaper operation costs, a higher driving range and less refueling time over green vehicles. For that reason, businesses with mixed fleets need to optimize the usage of green vehicles and conventional vehicles for their operations in addition to
optimizing vehicles’ travelled distance or time. In this work, we study an optimization problem of a mixed fleet of vehicle routing problems with time windows under a CO\textsubscript{2} emission cap. The CO\textsubscript{2} emission cap for the distribution problem, vehicle capacity, and time window of each customer are considered as constraints for this optimization problem. Under the time window constraint, customers in the problem are required to be served within their chosen time interval between the earliest arrival time and the latest arrival time. The time window constraint does not allow vehicles to serve the customers out of the preferred time window. Vehicles are normally permitted to wait at no cost if they arrive earlier, but a later arrival is not allowed. The studied logistics problem consists of a heterogeneous fleet of conventional vehicles and green vehicles characterized by changed carrying capacities and CO\textsubscript{2} emission models. Each customer must be served only once by one vehicle only from mixed fleet vehicles situated in a depot. As a combinatorial optimization problem, the problem in this study is an NP-hard problem and it requires an exponentially high time to solve by exact methods. Therefore, it is aimed to design a hybrid ant colony optimization (ACO) based solution approach to solve the problem. This hybrid ACO includes a standard ACO with a variable neighborhood search (VNS) to achieve a better solution quality of the problem.

This study is motivated by many observations, such as world leaders’ pledge to achieve sustainable transportation, including green vehicles to the companies’ vehicle fleets, and the imposition of a CO\textsubscript{2} emission cap in the distribution networks. The vehicle routing problems with time windows are seen in many real-life situations such as home deliveries (grocery delivery, furniture delivery, internal installation, and parcel distribution systems), rail distribution, mail and newspaper delivery, repair and maintenance services, school bus routing, patient delivery problems, and waste collection problems. The main contribution of this paper is to present a mixed fleet of logistics problem comprising conventional vehicles and green vehicles with CO\textsubscript{2} emission cap in the network. This paper also contributes in terms of the solution method, which can be testified with the scalability of the proposed algorithm in the numerical experiment section. To our best knowledge, the logistics problem model arising in the context of CO\textsubscript{2} emission cap is first time introduced in this paper. The novelty of our model is to optimize the fleet composition to respect the CO\textsubscript{2} emission cap imposed by enforcement agencies.

The rest of the paper is structured as herein described. The literature related to the logistics distribution problem with time window under CO\textsubscript{2} emission cap is reviewed in Section 2. In Section 3, the mixed fleet-based logistics distribution problem is defined and its mathematical formulation is presented. The proposed hybrid ACO is discussed in detail in Section 4. The computational results are reported in Section 5. Finally, the conclusion is stated in Section 6.

Table 1 presents the abbreviations and acronyms used in the paper.

Table 1. Abbreviations and acronyms.

| Acronym/Abbreviation | Description |
|----------------------|-------------|
| VRPTW                | Vehicle Routing Problem with Time Windows |
| ACO                  | Ant Colony Optimization |
| VNS                  | Variable Neighborhood Search |
| COP                  | Conference of Parties |
| ALNS                 | Adaptive Large Neighborhood Search |
| VRPSTW               | Vehicle Routing Problem with Soft Time Windows |
| BKS                  | Best-known Solution |
| CPU                  | Central Processing Unit |
| TS                   | Tabu Search |
| mGA                  | Messy Genetic Algorithm |
| Hybrid GA            | Hybrid Genetic Algorithm |
| CGH                  | Column Generation Heuristic |
Table 1. Cont.

| Acronym/Abbreviation | Description                  |
|----------------------|------------------------------|
| HS                   | Hybrid Search                |
| Hybrid ACS           | Hybrid Ant Colony System     |
| Hybrid ACO           | Hybrid Ant Colony Optimization|

2. Literature Review

Organizations incorporate many effective techniques in their logistics to curb CO$_2$ emissions [5]. These sorts of techniques can be considering CO$_2$ emission caps and heterogeneous vehicles. Qu et al. [6] constructed a supply chain optimization model and emission reduction policies for manufacturing businesses under the CO$_2$ emission cap. Liu et al. [7] identified the CO$_2$ emission cap as the best method to minimize CO$_2$ emission in the delivery routes of logistics problems. Heinold and Meisel [8] addressed a routing problem for freight companies where the CO$_2$ emission cap must be satisfied for each delivery order in a road and rail transportation intermodal network. Li et al. [9] optimized an inventory routing problem for cold chain logistics under the CO$_2$ emission cap. Jharkharia and Das [10] examined the impact of the CO$_2$ emission cap in a vehicle routing problem with integrated delivery and pickup operations. In the problem, a uniform load and fixed vehicle speed are considered for all customers. Qiu et al. [11] considered the CO$_2$ emission cap in their cost minimization objective function of the pollution production–routing problem. Song et al. [12] studied the impact of the CO$_2$ emission limit in a capacity and planning optimization problem. Cheng et al. [13] considered an inbound routing problem where a CO$_2$ emission cap for each period of time was imposed for the vehicles utilized from the depot to pick up products from the customers in the network. Yang et al. [4] studied a heterogeneous routing problem with soft time windows to decide the optimal composition of vehicle routes, vehicle speed, and vehicle types from three categories of vehicles. The objective functions of all problems in the literature are either to minimize a vehicle’s travelled distance or travelled time or to minimize the number of vehicles used in the routes or combination of them. However, in the cases of mixed fleet vehicle logistics problems, it is important to know the best composition of vehicles used in the fleet in order to investigate problems more precisely and to optimize vehicle utilization in the problem. Knowing the best composition of vehicles especially helps decision-makers to identify the effect of vehicle types on the green logistics problem. From this perspective, we consider optimizing the conventional and green vehicles of logistics problem under the CO$_2$ emission cap.

The problem studied in this paper resembles the vehicle routing problem with time windows (VRPTW). The time windows for the customers can either be a hard or soft restriction. Vehicles are not allowed to visit the customers out of the chosen time window in the hard time windows constraints, but vehicles can wait at no cost if they arrive earlier, but later arrival is not permitted. Contrarily, vehicles are allowed for both early and late servicing at customers out of the chosen time window in the soft time windows but are subject to some penalty at the cost of customer inconvenience. Keskin et al. [14] presented an electric VRP with time windows, where waiting time at the recharging station is considered stochastic. A two-stage simulation-based Adaptive Large Neighborhood Search (ALNS) heuristic was designed to solve the problem. Hoogeboom et al. [15] determined the best routes in a VRP with time window assignment with a minimum possibility of time window violation. The VRP with time window was surveyed in studies of Bräysy and Gendreau [16,17] and Toth and Vigo [18]. A time-dependent VRP with time window was investigated by Gmira et al. [19], where variation in travel time among customers was considered to define the road network. The VRPTW was also investigated in many studies such as Baldacci et al. [20], Gendreau and Taillard [21], Kallehauge [22], Vidal et al. [23] and Hashimoto et al. [24]. A VRP with soft time windows (VRPSTW) problem was addressed in a study where two exact algorithms such as standard branch-and-cut-and-price
and bi-objective optimization based on the bisection method were designed to solve the problem [25]. An electric VRP with soft time window constraint is investigated by Tas [26]. The problem is solved by the column generation algorithm. The VRPSTW was also studied in many other works [27–31].

Our study designs a hybrid ACO based solution for the proposed heterogeneous mixed fleet of logistics distribution problem with time window under CO₂ emission cap. The standard ACO is a stochastic-based combinatorial optimization solving technique introduced by Dorigo et al. [32]. The technique has been inspired by the food-seeking behaviors of ant colonies in nature. The ACO has well been evidenced in the literature to be a very effective, powerful, and competitive algorithm for solving various VRPs and their variants [33–39]. Baioletti et al. [40] introduced a new precedence-based ACO algorithm to investigate the permutation based optimization problems. Jia et al. [41] studied a capacitated electric vehicle transportation logistics problem by proposing a bi-level ACO algorithm.

Mladenovic and Hansen [42] introduced the variable neighborhood search (VNS) to solve a travelling salesman problem in 1997. The VNS is used as a local search algorithm usually in optimization problems to attain the local best solution. The VNS is also extensively used as a heuristic search method in optimization problems [43]. Santucci and Ceberio [44] applied an efficient VNS in their proposed metaheuristic for solving a linear ordering problem. Marinakis et al. [45] solved a constrained shortest path problem by adopting VNS in a hybrid PSO. Hore et al. [46] introduced a hybrid VNS algorithm where a standard VNS was coupled with a stochastic approach for solving a travelling salesman problem. Zhao et al. [47] proposed an improved VNS hybridized with simulated annealing to effectively investigate a solid waste collection and transportation problem with split deliveries. Islam et al. [48] combined a neighbourhood search with a standard PSO to study a green clustered logistics problem consisting of delivery customers and pickup customers. Guan and Lin [49] hybridized ACO with VNS for solving a single row facility layout optimization problem.

Overall, the literature shows that large numbers of VRPs were solved using a greater number of heuristics and metaheuristics algorithms. It is also apparent that architecting a framework to integrate multiple algorithms with different characteristics extensively improves the overall performance of a hybrid algorithm. With this observation, this paper designs a hybrid ACO-based solution approach to obtain a good quality solution of the proposed mixed fleet logistics problem under the CO₂ emission cap. The performance of the proposed algorithm is tested by comparing state-of-the-art algorithms for the VRPTW and its variants.

3. Problem Definition and Mathematical Formulation

This section presents problem definition, CO₂ emission calculation and mathematical models of the mixed fleet logistics distribution problem in the study.

3.1. Problem Definition

In this mixed fleet logistics distribution problem under CO₂ emission cap, many geographically located customers are required to be served by the vehicles. Multiple heterogeneous conventional and green vehicles located in a single depot are utilized to serve the customers. Each vehicle starts from the depot and returns back to the depot after serving customers in their routes. Each customer is served during specified time windows once only by one vehicle. The demands of the customers, distances among different nodes (customers and depot), vehicles speed on arcs, and preferred time windows for customers are known in the problem. The CO₂ emission cap (in kg) per kilometer distance travel is also given. Customers can be served by each type of vehicle, but total emissions by the vehicles must satisfy the CO₂ emission cap for the network. Thus, the challenges lie in the problem to obtain the best composition of vehicles in addition to a minimum travel distance for vehicles. Emissions are calculated in kg per kilometer of vehicle travelled.
distances. The time windows of customers, CO$_2$ emission cap for the distribution network, and vehicle capacities are considered as constraints in the problem.

The problem is defined on a complete, directed graph $G = (N, A')$, where $N$ is a set of nodes $N = \{0, 1, 2, \ldots, n\}$, a set of nodes (vertices) including the customers $\{1, 2, \ldots, n\}$ and a depot, 0. The arc set $A'$ denotes all possible connections between the nodes, defined as $\{(i, j) : i, j \in N, i \neq j\}$. Each customer is characterized by a non-negative demand, $a_i$, chosen hard time windows, $[e_i, l_i]$, and service time, $s_i$. A hard time window, where $e_i$ and $l_i$ are the earliest and latest arrival time respectively at a customer (node), specifies that vehicles are not allowed to start their service late, but waiting in case of early arrival at the nodes is possible. Let $k_{cv}$ and $k_{gv}$ be the available number of conventional and green vehicles, respectively, in the fleet. The considered heterogeneity of the vehicles including different vehicle capacities and CO$_2$ emission amounts for conventional and green vehicles. The vehicle capacities of the conventional and green vehicles are $Q_{cv}$ and $Q_{gv}$, respectively, and $E_{cap}$ is the given CO$_2$ emission cap for the logistics network. The objective of this study to find the best composition of conventional and green vehicles and the minimum vehicle travelled distance subject to the total carbon emission is within a specified limit and time window constraints for customers.

The notations used in this study are shown in Table 2.

### Table 2. The notations used in this study.

| Parameters: | |
| --- | --- |
| $d_{ij}$ | Euclidian distance in an arc $(i, j)$ |
| $v_{ij}$ | Vehicle speed in an arc $(i, j)$ |
| $t_{ij}$ | Travel time $= \frac{d_{ij}}{v_{ij}}$ |
| $Q_k$ | Capacity of any vehicle, $k$ |
| $M^*$ | A very large value |
| $T_{max}$ | Maximum allowable driving time for individual vehicle |

| Decision Variables: | |
| --- | --- |
| $x_{ij}^k$ | 1 if arc $(i, j)$ is travelled by vehicle $k$ otherwise 0. |
| $y_i^k$ | 1 if customer $i$ is visited by vehicle $k$ otherwise 0. |
| $T_{ij,k}$ | Service start time of vehicle $k$ for customer, $i$. |

#### 3.2. Calculation of CO$_2$ Emission of Vehicles

The vehicles’ CO$_2$ emissions are calculated from the instantaneous fuel consumption model proposed by Bektas and Laporte [50]. The pragmatic fuel consumption model for the vehicles is considered as a function of travelled distance, cargo load, and speed over the arcs, instead of only a distance function [51,52]. Considering the fuel consumption model as a linear function of the travelled distance of vehicles only is not useful in studying the green logistics of businesses.

The fuel consumption of a green vehicle is calculated from the total mechanical power, $P_t$, needed for the vehicle. The required total mechanical power, $P_t$, on an arc $(i, j)$ can be estimated as:

$$P_{ij} \approx P_t \cdot t_{ij} \approx P_t \cdot \left(\frac{d_{ij}}{v_{ij}}\right)$$

$$= \alpha_{ij} \left(w_0 + u_{ijk}\right)d_{ij} + \beta v_{ij}^2d_{ij}$$

Where, $\alpha_{ij} = \alpha + gs\sin\theta_{ij} + gC_r\cos\theta_{ij}$

$$\beta = 0.5CdA|\rho$$

In the equations, $\alpha_{ij}$ is the arc specific constant in the arc $(i, j)$; $\beta$ is the vehicle specific constant; $w_0$ is the curb weight (empty vehicle weight); $u_{ijk}$ is the vehicle load for the
vehicle, $k$, in the arc; $t_{ij}$ and $v_{ij}$ is the vehicle travelling time and vehicle speed in the arc, respectively; and $d_{ij}$ is the Euclidian distance in an arc. The battery energy ($E_{ij}$) of the vehicles depends on their motor efficiency ($eff_m$) and battery discharging efficiency ($eff_f$). The required $E_{ij}$ on the arc $(i,j)$ is calculated as:

$$E_{ij} = eff_m \cdot eff_f \cdot P_{ij} \approx eff_m \cdot eff_f \cdot \left\{ a_{ij} \left( w_0 + u_{ijk} \right) d_{ij} + \beta v_{ij}^2 d_{ij} \right\}$$  \hspace{1cm} (5)

The battery energy is used to calculate the CO$_2$ emissions of green vehicles. Depending on the power source efficiency ($eff_p$), the battery energy needed by the vehicle is converted to the equivalent electric power amount [53]. The electric power ($EP$) required by the vehicle on the arc $(i,j)$ is estimated as:

$$EP_{ij} \approx eff_m \cdot eff_f \cdot P_{ij} \cdot eff_p \approx eff_m \cdot eff_f \cdot \left\{ a_{ij} \left( w_0 + u_{ijk} \right) d_{ij} + \beta v_{ij}^2 d_{ij} \right\} \cdot eff_p$$  \hspace{1cm} (6)

The electric power obtained in Equation (6) is in joules ($J = kg \cdot m^2 \cdot s^{-2}$), which can be converted into kilowatt-hours (kWh) by a multiplication factor of 207.68. Finally, CO$_2$ emission is obtained from the required electric power (kWh) with the emission rate of $r = 0.64$ kg/kWh [54]. The CO$_2$ emission of green vehicles on the arc $(i,j)$ is estimated as:

$$Cb_{ij} \approx 207.68 \times EP_{ij} \cdot r$$

$$\approx 207.68 \times eff_m \cdot eff_f \cdot \left\{ a_{ij} \left( w_0 + u_{ijk} \right) d_{ij} + \beta v_{ij}^2 d_{ij} \right\} \cdot eff_p \cdot r$$

$$\approx 207.68 \times \text{eff}_f_{\text{total}} \cdot \left\{ a_{ij} \left( w_0 + u_{ijk} \right) d_{ij} + \beta v_{ij}^2 d_{ij} \right\}$$  \hspace{1cm} (7)

where $eff_f_{\text{total}} = eff_m \cdot eff_f \cdot eff_p \cdot r$. In this study, the efficiency values are used as follows: $eff_m = 1.2$; $eff_f = 1.1$; $eff_p = 0.75$ [53,55]. The value of $eff_f_{\text{total}}$ is estimated as 1.00.

The CO$_2$ emission of conventional vehicles on the arc an arc $(i,j)$ is estimated as follows:

$$Cf_{ij} \approx FE_{\text{factor}} \times 207.68 \times \text{eff}_f_{\text{total}} \cdot \left\{ a_{ij} \left( w_0 + u_{ijk} \right) d_{ij} + \beta v_{ij}^2 d_{ij} \right\}$$  \hspace{1cm} (8)

where $FE_{\text{factor}} = 1.44$ is the fuel economy factor used for conventional vehicles compared to green vehicles [56]. The typical values of all parameters in the emission models are shown in Table 3.

**Table 3.** The values of notations used in emission models. Adopted from Demir et al. [57].

| Notation | Description | Typical Value |
|----------|-------------|---------------|
| $w_0$    | Curb weight (empty vehicle weight) (kg) | 6350          |
| $a$      | Acceleration in the $(i,j)$ | 0             |
| $\theta$ | Gradient in the $(i,j)$ | 0             |
| $g$      | Gravitation constant (m/s$^2$) | 9.81          |
| $C_d$    | Coefficient of aerodynamic drag | 0.7           |
| $\rho$   | Air density (kg/m$^3$) | 1.2041        |
| $A$      | Frontal surface area | 3.912         |
| $C_r$    | Coefficient of rolling resistance | 0.01          |
| $v_{ij}$ | Vehicle speed on the arc (meter/second) | 12.5~25 (45~90 km/h) |

### 3.3. Mathematical Formulation

This section describes the mathematical formulation of the studied problem in this study as follows:

**Objective function:**

$$\text{Minimize}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{k_{ij}} d_{ij} x_{ij}^k$$  \hspace{1cm} (9)
Constraints:

\[
\sum_{i=1}^{n} a_i y_i^k \leq Q_{cv} \quad \forall k \in k_{cv} \tag{10}
\]

\[
\sum_{i=1}^{n} a_i y_i^k \leq Q_{gv} \quad \forall k \in k_{gv} \tag{11}
\]

\[
\sum_{k=1}^{k_{cv}+k_{gv}} y_i^k = 1 \quad i = 1, \ldots, n \tag{12}
\]

\[
\sum_{j=1}^{n} x_{0,j}^k \leq k_{cv} \quad \forall k \in k_{cv} \tag{13}
\]

\[
\sum_{j=1}^{n} x_{0,j}^k \leq k_{gv} \quad \forall k \in k_{gv} \tag{14}
\]

\[
\sum_{i=0}^{n} x_{ij}^k = y_j^k \quad j = 1, \ldots, n \quad \forall k \in k_{cv} \tag{15}
\]

\[
\sum_{i=0}^{n} x_{ij}^k = y_i^k \quad i = 0, 1, \ldots, n \quad \forall k \in k_{cv} \tag{16}
\]

\[
\sum_{i=0}^{n} x_{ij}^k = y_j^k \quad j = 1, \ldots, n \quad \forall k \in k_{gv} \tag{17}
\]

\[
\sum_{i=0}^{n} x_{ij}^k = y_j^k \quad i = 0, 1, \ldots, n \quad \forall k \in k_{gv} \tag{18}
\]

\[
T_{j,k} \geq T_{i,k} + (s_i + t_{ij}) \left( x_{ij}^k \right) - M^* \cdot \left( 1 - x_{ij}^k \right) \quad \forall i \in N, \; \forall j \in N, \; \forall k \in k_{cv} + k_{gv} \tag{19}
\]

\[
e_i \leq T_{i,k} \leq l_i \quad \forall i \in N, \; \forall k \in k_{cv} + k_{gv} \tag{20}
\]

\[
0 \leq T_{0,k} \leq T_{\text{Max}} \quad \forall k \in k_{cv} + k_{gv} \tag{21}
\]

\[
\sum_{i=0}^{n} \sum_{k=1}^{k_{cv}+k_{gv}} (C_{fi,j} \cdot x_{ij}^k + C_{bi,j} \cdot x_{ij}^k) \leq E_{cap} \tag{22}
\]

\[
\sum_{j=0,j \neq i}^{n} u_{ijk} - \sum_{j=0,j \neq i}^{n} u_{ijk} = a_i \quad i, j = 1, \ldots, n, \; \forall k \in k_{cv} + k_{gv} \tag{23}
\]

\[
a_{i-1} \cdot x_{ij}^k \leq u_{ijk} \leq (Q_k - a_i) \cdot x_{ij}^k \quad i, j = 1, \ldots, n, \quad k = 1, \ldots, k_{cv} + k_{gv} \tag{24}
\]

\[
x_{ij}^k \in \{0, 1\} \quad \forall i \in N, \; \forall k \in k_{cv} + k_{gv} \tag{25}
\]

\[
T_{i,k} \geq 0; \quad \forall i \in N, \; \forall k \in k_{cv} + k_{gv} \tag{26}
\]

The objective function (9) minimizes the total distance travelled by the vehicles in the routes. Constraints (10) and (11) ensure the capacity of conventional and green vehicles cannot be exceeded while serving customers. Constraint (12) guarantees each customer must be served by exactly one vehicle. Constraints (13) and (14) ensure the maximum number of used vehicles for each type in the route must follow the fleet composition. Constraints (15), (16), (17), and (18) represent the flow conservation ensuring each node must have an incoming number of arcs equal to outgoing arcs for each vehicle. The time window constraints are confirmed by constraints (19) and (20). The constraint (19) becomes \( T_{j,k} \geq T_{i,k} + (s_i + t_{ij}) \left( x_{ij}^k \right) \) if arc \((i,j)\) is traveled by a vehicle of \(k_{cv}\) or \(k_{gv}\) otherwise it remains \( T_{j,k} \geq T_{i,k} + (s_i + t_{ij}) \left( x_{ij}^k \right) - M^* \cdot \left( 1 - x_{ij}^k \right) \). Constraint (21) guarantees the route length restriction for each vehicle. Constraint (22) ensures the total amount of CO₂
emissions in the model must not go beyond the emission cap, $E_{\text{Cap}}$. Constraint (23) and (24) confirm flow balance that denotes the flows as increasing by the amount of each customer demand. Constraint (25) and (26) define the condition of decision variables in the model.

### 4. The Proposed Hybrid Ant Colony Optimization (Hybrid ACO)

A new hybrid ACO is designed to solve the problem in this study. The proposed hybrid ACO approach is a combination of standard ACO and VNS algorithms. In the standard ACO, artificial ants are generated to construct feasible solutions of the problem iteratively. The constructions of the solutions are guided by the collected trail intensity. Each ant deposits pheromones (i.e., updating trail intensity) in the paths (solution components) of a solution after generating a solution. The paths (solution components) of better solutions generated by the ants over the iterations collect higher amount of pheromone (trail intensity). The paths (solution components) with higher trail intensity are most likely to be followed by the ants in subsequent iterations. This way, the best solution is found for the problems. The VNS consists of many renowned local search methods. The VNS is adopted into the ACO to overcome the shortage of premature convergence attributes of standard ACO and to obtain an improved solution quality of the algorithm by avoiding the trap in the local optimum. The pseudo-code of the proposed hybrid ACO for the problem is shown in Algorithm 1.

**Algorithm 1: Pseudo-code of the proposed hybrid ACO**

1: Initialization
2: Initialize Trail intensities and parameters
3: Main phase
4: Do while
5: $S \leftarrow$ Generate AntSolutions for $m$ number of ants
6: $S \leftarrow$ VNS ($S$)
7: Update the best solution
8: Update elitist ants and trail intensities using the best solution
9: End Do
10: Report best solution

#### 4.1. Initialization of Trail Intensities and Parameters

In this paper, trail intensity, $\tau_{ij}$, is defined as the intensity of visiting a customer, $j$, after customer, $i$. The trail intensities collect the information of the ants travelling between customers. The ACO literature uses initial solutions to initialize trail intensities, but in our experiment, we could not find any significant effect when trail intensity is initialized with different numbers varying from 1000 to $\frac{1}{1000}$. Thus, we initialize trail intensities as $\tau_{ij} = 0.01$ for all $i, j = 0, \ldots, n$. The parameter trail persistence factor, $\rho$, is set as 0.95.

#### 4.2. Generation of Ant Solutions ($S$)

The $m$ numbers of ant are used to generate the $m$ number of complete solutions of the problem. In the generated solutions, each ant starts a route from the depot and continuously visits customers until the vehicle capacity allows visiting them. After the first trip, the ants start building a route for the second trip. The process continues until all customers are served. In the routes, ants choose customers based on the trail intensity values between two customers.

The probability of choosing the next customer $j$ from the customer $i$ is calculated as follows:

$$P_{ij} = \frac{\tau_{ij}}{\sum_{l \in N} \tau_{il}} \quad j \in N$$  \hspace{1cm} (27)

Here, $N$ is the list of all feasible customers that have not been visited yet by the ant. In this way, all ants generate complete solutions of the problem. Here, the generated solution’s
quality cannot be a good solution. In this respect, the VNS algorithm is adopted in every iteration of the algorithm to improve the quality of these solutions.

4.3. Variable Neighborhood Search (VNS)

The VNS is employed on each generated solution by ants to find the local optima. The VNS includes many local searches such as customer shift (1,0), shift (2,0), swap (1,1), and swap (2,1) based on the current solution. In shift (1,0) local search, one customer from one route is shifted to another route. In shift (2,0) move, two consecutive customers from a route are shifted from one route and reinserted to another route in a similar sequence. In swap (1,1), one customer is interchanged between two routes. In Swap (2,1), two consecutive customers are interchanged with a customer from another route. In the VNS iterations, local searches are randomly selected one by one. In the VNS, each local search is started with an additional penalty function of three constraints, such as vehicle capacity, time windows, and CO$_2$ emission constraints. In the iterations, the penalty for each constraint is increased if infeasible routes are generated from the constraints, and vice versa. A list of feasible best ant solutions, $E^b$, (named Elitist ants) is generated at the end of VNS iterations.

4.4. Updating Elitist Ants

The elitist ants, defined as $\gamma$, are the best ant solutions generated so far in any iteration. Each ant solution is distinct in terms of solution quality from one another in the elitist solution. They are used to update the trail intensities of the paths. The list of elitist ants (solutions) is updated if any current ant solution is found to be better than the existing elitist solutions.

4.5. Updating Trail Intensities

After the VNS is performed in an iteration, the trail intensities for all paths are updated by the elitist ant solutions, $\gamma$, to ensure that ants can produce improved solutions in the following iterations. The trail intensity of choosing customer $j$ from the customer $i$ is updated as follows:

$$\tau_{ij}^{\text{new}} = \rho \times \tau_{ij}^{\text{old}} + \sum_{\theta=1}^{\gamma} \tau_{ij}^{\theta}, \quad i \neq j \text{ and } i, j = 1, 2, \ldots, n$$

(28)

According to Dorigo et al. [32], the trail intensities, $\rho$, is defined with $0 \leq \rho < 1$. The Equation (28) reveals that the trail intensities are updated by two occurrences such as trail intensities or pheromone evaporate on arcs over time (first term of the equation) and the pheromones grow on the paths of the best solutions reported in elitist ant solutions, $\gamma$, as ants deposit pheromones on the visited paths (second term of the equation). The value of $\tau_{ij}^{\theta}$ is stated as:

$$\tau_{ij}^{\theta} = \begin{cases} 
\frac{1}{L^\theta}, & \text{if the arc } (i,j) \text{ is in the elitist ant route} \\
0, & \text{otherwise}
\end{cases}$$

(29)

Here, $L^\theta$ is the total route length of the $\theta$th elitist ant solution.

5. Computational Experiments

The proposed hybrid ACO metaheuristic is implemented to solve the logistics problem under the CO$_2$ emission cap in this study using the C++ programming language. The experiments are run on a Linux server with four 2.1 GHz processors with 16-core each and a total of 256GB of RAM. The proposed ACO is tested on newly generated instances. The new 15 instances are created from Solomon’s R1 benchmark VRPTW instances with 100 customers for this study. As the studied logistics problem resembles well-known VRPTW, the performance of our metaheuristic is evaluated on Solomon’s benchmark VRPTW instances by comparing the ACO results with the results of several algorithms obtained in the literature. Overall, the performance of the proposed algorithm is measured
by two criteria. The first criterion is that in how many instances does the algorithm finds a better solution than the best-known solution (BKS) found in the literature so far. The second criterion is the relative deviation percentage of the algorithm solution compared to the BKS. The relative deviation percentage of an instance is reported as “%Gap” in the tables. It is measured by Equation (30), where Sol is used to denote the solutions (total distance) found by the other algorithms. In addition, the processing time (CPU time) is reported as $t$ in second. The following formula is used to calculate “%Gap” from the BKS.

$$\text{%Gap} = \frac{\text{Sol} - \text{BKS}}{\text{BKS}} \times 100\%$$ (30)

Hence, a negative value of “%Gap” means improved solution quality, and a positive value of “%Gap” means worse solution quality with respect to existing BKS. Moreover, distances refer to the corresponding Euclidian distances. Double precision distances with no rounding nor truncation are considered in entire computational experiments.

Statistical tests such as the non-parametric Friedman test and post-hoc Bonferroni test are used in order to compare the performance of different algorithms statistically. Friedman’s test is used to check whether the results of different algorithms are statistically different or not. The Bonferroni test is used for pairwise comparisons between two algorithms. The Bonferroni test is performed after Friedman’s test to check whether one algorithm is statistically better than the other algorithm or not [58]. The Friedman and post-hoc Bonferroni test are performed on the statistical software IBM SPSS version 19 using $p$ value = 0.05 as the level of significance.

5.1. Numerical Experiments for the Mixed Fleet Logistics Distribution Problem under Carbon Emission Cap

This section shows the numerical results of the proposed hybrid ACO for the logistic problem studied in this paper. The results specify the best composition of conventional and green vehicles for the problem under the CO$_2$ emissions cap and the comparative results of hybrid ACO on mixed fleet instances.

The instances of the mixed fleet logistics problem are created from the Solomon [59] R1 benchmark VRPTW instances with 100 customers. In the newly generated 15 instances for this study, each customer is labelled with a different vehicle speed that is randomly chosen from a speed range of 45–90 km/hour. Each instance consists of a specific number of conventional and green vehicles, but the fleet size remains the same as the Solomon [59] benchmark instances. The conventional vehicle capacities are kept as 200, similar to the Solomon [59] instances, and the green vehicle capacities are chosen as 150 to obtain the heterogeneous fleet along with different CO$_2$ emissions models for the vehicle types. Additionally, the CO$_2$ emission cap for each instance of the network is generated by uniform distributions, $\text{Min}(E_{all\ \text{cvs}}, E_{all\ \text{gvs}}) + |E_{all\ \text{cvs}} - E_{all\ \text{gvs}}| \times U(0.1, 0.3)$. Here, $E_{all\ \text{cvs}}$ is the total vehicle emissions with all vehicles are conventional and $E_{all\ \text{gvs}}$ is the total emissions with all vehicles are green vehicles in the problem. All remaining attributes of newly generated instances for this study are kept the same in the Solomon [59] instances.

5.1.1. Identification of the Best Composition of Vehicles under CO$_2$ Emissions Cap

Table 4 highlights the effect of the mixed fleet, conventional vehicles only, and green vehicles only on the problem. The vehicle travelled distance (solution), the CPU time (in seconds), number of conventional vehicles and green vehicles, and CO$_2$ emission (in gram) for each problem instance are shown in the table. The table shows that distance, CPU time, and CO$_2$ emission for the hybrid ACO with mixed fleet vehicles are 1154.09, 116.76, and 742.53, respectively. Those results with conventional vehicles only are 1151.87, 127.83, and 794.96, respectively. This indicates that the travelled distance is reduced by 0.16%, but CO$_2$ emission is increased by 6.90% if the vehicle fleet consists of only conventional vehicles. The deviations of those results are expected because of the higher vehicle capacity and higher CO$_2$ emission rate of the conventional vehicles. Distance and CO$_2$ emission
with green vehicles only are 1185.86 and 567.19, respectively. This shows that the travelled distance is increased by 3.10%, but CO$_2$ emission is reduced by 23.67% if the vehicle fleet consists of only green vehicles only. The deviations of those results are also expected because of the lower vehicle capacity and lower CO$_2$ emission rate of the green vehicles.

Table 4 also reveals the best composition of conventional and green vehicles for each instance of the mixed fleet logistics problem under the CO$_2$ emission cap. For example, the best vehicle composition for instance R101MfC2 is seven conventional vehicles and five green vehicles. The instance requires 14 conventional vehicles if the vehicle fleet consists of only conventional vehicles and 13 green vehicles if the vehicle fleet consists of only green vehicles. The mixed fleet vehicles in the distribution problems are common nowadays in the real-life scenarios of logistics problems. In these scenarios, it becomes important to know the vehicle composition of the problem in order to satisfy all problem constraints. In this regard, this study helps to identify the best vehicle composition for the logistics distribution problem.

5.1.2. Effect of Hybridization on ACO’s Performance

The proposed hybrid ACO comprises a standard ACO and the VNS. The effect of hybridization on ACO performance is presented as $\%Gap$ in Table 5. The effect of hybridization on the ACO’s performance for the problem instance is evaluated under two settings: (1) ACO without VNS and (2) ACO with VNS. In the hybridization test, the number of iterations for each setting is kept different to retain approximately the same CPU time, but all other parameters in the ACO are kept the same. Table 5 displays that the solution (distance) of the hybrid ACO setting is 11.75% better than the ACO without the VNS setting. Additionally, the hybrid ACO reduces the CO$_2$ emission by 10.90% compared to the ACO without VNS scheme. This observation justifies the use of VNS with the ACO for solving the logistics distribution problem.
Table 4. Effect of fleet compositions and identification of the best composition of vehicles of logistics problem under CO$_2$ emission cap.

| Instances   | Distance | t (s) | #Conv Vehicle | #Green Vehicle | CO$_2$ Emission | Distance | t (s) | #Conv Vehicle | #Green Vehicle | CO$_2$ Emission | Distance | t (s) | #Conv Vehicle | #Green Vehicle | CO$_2$ Emission |
|-------------|----------|-------|---------------|----------------|-----------------|----------|-------|---------------|----------------|----------------|----------|-------|---------------|----------------|----------------|
| R101MfC1    | 1380.00  | 99.91 | 9             | 4              | 882.95          | 1376.09  | 113.20| 12            | 0              | 921.17         | 1375.72  | 91.96 | 0             | 13             | 676.71         |
| R101MfC2    | 1386.64  | 100.00| 7             | 5              | 845.32          | 1379.50  | 111.66| 14            | 0              | 966.00         | 1390.64  | 91.05 | 0             | 13             | 673.80         |
| R101MfC3    | 1372.01  | 104.06| 8             | 4              | 843.92          | 1366.86  | 118.04| 13            | 0              | 953.51         | 1387.58  | 92.82 | 0             | 13             | 681.69         |
| R102MfC1    | 1239.58  | 108.95| 8             | 3              | 805.92          | 1234.57  | 120.85| 12            | 0              | 841.84         | 1261.84  | 97.64 | 0             | 12             | 643.87         |
| R102MfC2    | 1249.66  | 107.91| 8             | 3              | 827.88          | 1247.51  | 120.49| 11            | 0              | 863.51         | 1267.74  | 95.62 | 0             | 13             | 625.22         |
| R102MfC3    | 1242.91  | 110.70| 9             | 3              | 784.85          | 1239.98  | 124.19| 10            | 0              | 908.74         | 1256.29  | 96.13 | 0             | 12             | 613.66         |
| R103MfC1    | 1047.99  | 123.93| 9             | 1              | 688.21          | 1044.61  | 130.99| 10            | 0              | 712.11         | 1082.48  | 103.88 | 0             | 11             | 506.69         |
| R103MfC2    | 1040.92  | 122.03| 9             | 1              | 692.32          | 1042.64  | 129.32| 10            | 0              | 721.44         | 1075.78  | 102.52 | 0             | 11             | 472.74         |
| R103MfC3    | 1051.86  | 130.65| 8             | 1              | 688.61          | 1035.27  | 148.84| 10            | 0              | 704.20         | 1087.93  | 106.16 | 0             | 11             | 487.24         |
| R104MfC1    | 906.70   | 129.05| 8             | 1              | 632.44          | 906.08   | 134.01| 9             | 0              | 600.81         | 964.68   | 104.05 | 0             | 11             | 404.25         |
| R104MfC2    | 897.72   | 131.33| 8             | 1              | 565.58          | 905.15   | 136.71| 9             | 0              | 622.54         | 976.64   | 104.19 | 0             | 10             | 442.27         |
| R104MfC3    | 900.44   | 135.02| 8             | 1              | 556.70          | 906.89   | 141.22| 9             | 0              | 612.18         | 990.40   | 105.06 | 0             | 11             | 496.55         |
| R105MfC1    | 1195.12  | 117.91| 8             | 2              | 783.48          | 1202.28  | 133.33| 9             | 0              | 855.61         | 1222.51  | 100.96 | 0             | 12             | 592.18         |
| R105MfC2    | 1197.93  | 113.85| 8             | 2              | 758.70          | 1191.45  | 124.78| 9             | 0              | 825.78         | 1219.70  | 98.78 | 0             | 12             | 595.52         |
| R105MfC3    | 1201.80  | 116.11| 7             | 2              | 781.05          | 1199.14  | 129.78| 10            | 0              | 814.95         | 1228.03  | 99.73  | 0             | 12             | 595.45         |

Average 1154.09 116.76 — — 742.53 1151.87 127.83 — — 794.96 1185.86 99.37 — — 567.19
Table 5. Effect of hybridization on the ACO performance for mixed fleet logistics problem instances.

| ACO without VNS | Proposed hybrid ACO (ACO with VNS) |
|-----------------|-----------------------------------|
| Iteration | Distance | t (s) | CO2 Emission(g) | Iteration | Distance | %Gap | t (s) | CO2 Emission |
| 6000 | 1319.20 | 119.02 | 838.39 | 75 | −11.75% | 116.76 | −10.90% |

5.2. Numerical Experiments on VRPTW

The proposed mixed fleet logistics distribution problem resembles the vehicle routing problem with time windows (VRPTW). The purpose of this section is to check the effectiveness of the proposed algorithm for the mixed fleet logistics distribution problem with state-of-the-art algorithms available in the VRPTW literature. The proposed hybrid ACO is evaluated for VRPTW on the mostly used 56 instances of Solomon’s benchmark with 100 customers [59]. The instances have six sets of problems: C1, C2, R1, R2, RC1, and RC2. Existing algorithms (shown in Table 6) are used to compare their results with the proposed hybrid ACO algorithm results. It is important to note that the VRPTW literature aims to minimize the vehicles’ total travelled distances and the total number of vehicles. Our objective function in this study is to obtain the best composition of vehicles while minimizing their total travel distance under the problem constraints. Table 7 exhibits the entire comparison results for VRPTW. It can be noted that the first column refers to instances, the second column refers to the BKS found in the literature, the algorithm used for BKS is stated in parenthesis in the column, and the other columns refer to the algorithm’s results and their %Gaps.

The comparison shows that the proposed hybrid ACO algorithm obtains a total of 21 new BKSs out of 56 instances. The overall results indicate that the hybrid ACO produces an improved solution quality compared to all existing solutions as hybrid ACO obtains an overall %Gap between BKS and the proposed hybrid ACO is −3.28%. In particular, the result of the ACO is 3.82% (from 0.91% to −3.28%), 4.19%, and 4.47% superior to the algorithm of HS, CGH, and hybrid ACS, respectively. The results clearly indicate the superiority of our proposed hybrid ACO over the existing state-of-the-art metaheuristics. The Friedman test reveals a significant statistical difference in the performance of our hybrid ACO compared with all other algorithms (p values = 0.05). Additionally, the post-hoc Bonferroni tests are performed to find the pairwise differences among the algorithms. In the Bonferroni tests, the adjusted p values are used as 0.00625. The p-value for the comparison between our algorithm and TS is 0.0000063; the comparison between our algorithm and mGA is 0.000059, which is less than the Bonferroni adjustment’s significant level of 0.00625. The p-values of comparison between our algorithm and all other algorithms (in the pairwise comparison tests) are also found as less than the Bonferroni adjustment significant level. The p-values reject the null hypotheses and confirm the statistically better performance of hybrid ACO compared to all other algorithms available in the literature for the VRPTW.

Table 6. List of algorithms used in the evaluation of proposed ACO algorithm for VRPTW.

| Notation | Algorithm |
|----------|-----------|
| TS | Tabu search with probabilistic diversification and intensification technique [60] |
| mGA | A messy genetic algorithm by Tan et al. [61] |
| Hybrid GA | Hybrid genetic algorithm by Jung and Moon [62] |
| CGH | Column generation heuristic by Alvarenga et al. [63] |
| HS | Hybrid search that combines simulated annealing with non-monotonic temperature control, random start, and hill-climbing by Oliveira and Vasconcelos [64] |
| Hybrid ACS | A hybrid ant colony system with brainstorm optimization by Shen et al. [65] |
| Hybrid ACO | The algorithm proposed in this paper |
Table 7. Comparison results of Solomon’s 56 instances with 100 customers for VRPTW.

| Instance | BKS | TS (1995) [60] | mGA (2001) [61] | Hybrid GA (2002) [62] | CGH (2007) [63] | HS (2010) [64] | Hybrid ACS (2020) [65] | Hybrid ACO |
|----------|-----|----------------|-----------------|----------------------|-----------------|-----------------|------------------------|-----------|
|          |     | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap |
| C101     | 828.94 | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C102     | 828.94 | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C103     | 828.07 | 859.78 | 3.83% | 828.06 | 0.00% | 828.06 | 0.00% | 828.06 | 0.00% | 828.06 | 0.00% | 828.94 | 0.11% |
| C104     | 824.78 | 893.23 | 8.30% | 824.78 | 0.00% | 824.78 | 0.00% | 824.78 | 0.00% | 824.78 | 0.00% | 828.07 | 0.40% |
| C105     | 828.94 | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C106     | 828.94 | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C107     | 828.94 | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C108     | 828.94 | 830.94 | 0.24% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C109     | 828.94 | 849.03 | 2.42% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% | 828.94 | 0.00% |
| C201     | 591.56 | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% |
| C202     | 591.56 | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% | 591.56 | 0.00% |
| C203     | 591.17 | 618.00 | 4.54% | 591.17 | 0.00% | 591.17 | 0.00% | 591.17 | 0.00% | 591.17 | 0.00% | 591.17 | 0.00% |
| C204     | 590.60 | 609.02 | 3.12% | 590.60 | 0.00% | 590.60 | 0.00% | 590.60 | 0.00% | 590.60 | 0.00% | 590.60 | 0.00% |
| C205     | 588.88 | 616.32 | 4.66% | 588.88 | 0.00% | 588.88 | 0.00% | 588.88 | 0.00% | 588.88 | 0.00% | 588.88 | 0.00% |
| C206     | 588.49 | 615.92 | 4.66% | 588.49 | 0.00% | 588.49 | 0.00% | 588.49 | 0.00% | 588.49 | 0.00% | 588.49 | 0.00% |
| C207     | 588.29 | 636.62 | 8.22% | 588.29 | 0.00% | 588.29 | 0.00% | 588.29 | 0.00% | 588.29 | 0.00% | 588.29 | 0.00% |
Table 7. Cont.

| Instance | BKS | TS (1995) [60] | mGA (2001) [61] | Hybrid GA (2002) [62] | CGH (2007) [63] | HS (2010) [64] | Hybrid ACS (2020) [65] | Hybrid ACO |
|----------|-----|----------------|----------------|----------------------|-----------------|----------------|------------------------|------------|
|          |     | Distance       | %Gap           | Distance             | %Gap            | Distance       | %Gap                   | Distance   |
|          |     | Distance       | %Gap           | Distance             | %Gap            | Distance       | %Gap                   | Distance   |
|          |     | Distance       | %Gap           | Distance             | %Gap            | Distance       | %Gap                   | Distance   |
| C208     | 588.32 | 588.32 (TS)    | 611.29        | 588.32                | 0.00%           | 588.32        | 0.00%                  | 588.32     |
| R101     | 1650.80 | 1642.87 (HGA)  | 1648.86       | 1642.88               | 0.00%           | 1642.88       | 0.00%                  | 1642.88    |
| R102     | 1486.12 | 1472.62 (CGH)  | 1486.71       | 1472.81               | 0.01%           | 1472.62       | 0.00%                  | 1479.55    |
| R103     | 1213.62 | 1213.62 (HGA)  | 1213.62       | 1213.62               | 0.00%           | 1222.68       | 0.75%                  | 1225.31    |
| R104     | 982.01  | 976.61 (HGA)   | 1024.38       | 976.61                | 0.97%           | 990.78        | 1.45%                  | 1002.62    |
| R105     | 1377.11 | 1360.78 (HGA)  | 1372.71       | 1360.78               | 0.00%           | 1363.74       | 0.22%                  | 1365.66    |
| R106     | 1252.03 | 1240.47 (HGA)  | 1271.11       | 1240.47               | 0.08%           | 1244.58       | 0.33%                  | 1249.51    |
| R107     | 1159.85 | 1073.34 (HGA)  | 1106.19       | 1073.34               | 0.26%           | 1081.88       | 0.80%                  | 1091.21    |
| R108     | 980.05  | 947.55 (HGA)   | 992.12        | 947.55                | 0.11%           | 952.37        | 0.51%                  | 960.23     |
| R109     | 1181.76 | 1101.37 (mGA)  | 1101.37       | 1115.84               | 4.58%           | 1153.89       | 4.77%                  | 1165.71    |
| R110     | 1080.36 | 1072.41 (HGA)  | 1119.12       | 1072.41               | 1.86%           | 1087.94       | 1.45%                  | 1090.92    |
| R111     | 1083.05 | 1053.50 (HGA)  | 1083.05       | 1053.50               | 0.00%           | 1053.80       | 0.03%                  | 1063.69    |
| R112     | 953.63  | 953.63 (HGA)   | 1020.52       | 960.68                | 0.74%           | 973.34        | 2.07%                  | 976.28     |
| R201     | 1147.80 | 1147.80 (HGA)  | 1149.87       | 1148.48               | 0.06%           | 1147.80       | 0.00%                  | 1161.20    |
| R202     | 1057.56 | 1034.35 (HGA)  | 1049.74       | 1039.32               | 0.48%           | 1058.83       | 2.31%                  | 1061.90    |
| R203     | 948.74  | 922.38 (HGA)   | 974.87        | 900.08                | 2.88%           | 874.87        | 0.00%                  | 883.42     |

Notes: %Gap = Distance % gap compared to the best known solution.
| Instance | BKS   | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap | Distance | %Gap |
|----------|-------|----------|------|----------|------|----------|------|----------|------|----------|------|----------|------|----------|------|
| R204     | 735.80 (HS) | 869.29 | 18.14% | 791.78 | 7.61% | 736.52 | 0.10% | 772.33 | 4.96% | 735.80 | 0.00% | 756.93 | 2.79% | 762.07 | 3.57% |
| R205     | 954.16 (HS) | 1063.24 | 11.43% | 1015.99 | 6.48% | 955.82 | 0.17% | 970.89 | 1.75% | 954.16 | 0.00% | 978.47 | 2.48% | 1001.16 | 4.93% |
| R206     | 879.89 (HGA) | 912.97 | 3.76% | 884.65 | 0.54% | 879.89 | 0.00% | 898.91 | 2.16% | 884.25 | 0.50% | 906.27 | 2.91% | 921.39 | 4.72% |
| R207     | 797.99 (HS) | 814.78 | 2.10% | 875.76 | 9.75% | 799.86 | 0.23% | 834.93 | 4.63% | 797.99 | 0.00% | 812.35 | 1.77% | 831.85 | 4.24% |
| R208     | 705.45 (HGA) | 738.60 | 4.70% | 778.38 | 10.34% | 705.45 | 0.00% | 723.61 | 2.57% | 705.62 | 0.02% | 725.05 | 2.70% | 723.06 | 2.50% |
| R209     | 859.39 (HGA) | 944.64 | 9.92% | 920.34 | 7.09% | 859.39 | 0.00% | 879.53 | 2.34% | 860.11 | 0.08% | 879.01 | 2.23% | 897.19 | 4.40% |
| R210     | 910.70 (HGA) | 967.50 | 6.24% | 961.18 | 5.54% | 910.70 | 0.00% | 932.89 | 2.44% | 910.98 | 0.03% | 923.43 | 1.38% | 929.68 | 2.08% |
| R211     | 755.82 (HS) | 949.50 | 25.63% | 820.23 | 8.52% | 755.96 | 0.02% | 787.51 | 4.19% | 755.82 | 0.00% | 776.17 | 2.62% | 768.40 | 1.66% |
| RC101    | 1623.58 (TS) | 1623.58 | 0.00% | 1659.68 | 2.22% | 1643.41 | 1.22% | 1639.97 | 1.01% | 1642.83 | 1.19% | 1643.78 | 1.23% | 1360.37 | −16.21% |
| RC102    | 1461.23 (TS) | 1477.54 | 1.12% | 1492.10 | 2.11% | 1461.23 | 0.00% | 1466.84 | 0.38% | 1480.46 | 1.32% | 1464.63 | 0.23% | 1223.47 | −16.27% |
| RC103    | 1249.86 (mGA) | 1262.02 | 0.97% | 1249.86 | 0.00% | 1277.54 | 2.21% | 1264.71 | 1.19% | 1308.64 | 4.70% | 1275.65 | 2.02% | 1124.52 | −10.03% |
| RC104    | 1135.52 (CGH) | 1135.83 | 0.03% | 1202.12 | 5.87% | 1136.81 | 0.11% | 1135.52 | 0.00% | 1162.75 | 2.40% | 1156.92 | 1.85% | 1038.05 | −8.58% |
| RC105    | 1518.58 (HGA) | 1733.56 | 14.16% | 1585.34 | 4.40% | 1518.58 | 0.00% | 1518.60 | 0.00% | 1534.60 | 1.05% | 1535.78 | 1.12% | 1279.85 | −15.72% |
| RC106    | 1377.35 (CGH) | 1384.92 | 0.55% | 1449.30 | 5.22% | 1381.23 | 0.28% | 1377.35 | 0.00% | 1386.82 | 0.69% | 1378.45 | 0.08% | 1148.76 | −16.60% |
| RC107    | 1212.83 (HGA) | 1230.95 | 1.49% | 1303.36 | 7.46% | 1212.83 | 0.00% | 1212.83 | 0.00% | 1247.53 | 2.86% | 1216.65 | 0.31% | 1075.07 | −11.36% |
| RC108    | 1117.53 (HGA) | 1170.70 | 4.76% | 1197.13 | 7.12% | 1117.53 | 0.00% | 1117.53 | 0.00% | 1135.87 | 1.64% | 1134.28 | 1.48% | 1039.33 | −7.00% |
| Instance | BKS | TS (1995) [60] | mGA (2001) [61] | Hybrid GA (2002) [62] | CGH (2007) [63] | HS (2010) [64] | Hybrid ACS (2020) [65] | Hybrid ACO |
|----------|-----|----------------|-----------------|----------------------|-----------------|-----------------|----------------------|------------|
| RC201    | 1265.56 (HGA) 1095.64 (HGA) 926.89 (HGA) 786.38 (HGA) 1157.55 (HGA) 779.31 (HGA) | 1438.89 13.70% 1165.57 6.38% 1079.57 16.47% 806.75 2.59% 1333.71 15.22% 1212.64 14.98% 1085.61 12.57% 833.97 7.01% | 1354.96 7.06% 1151.46 5.09% 1018.09 9.84% 865.51 10.06% 1225.69 5.89% 1122.23 6.41% 1047.86 8.47% 854.75 9.68% | 1265.56 0.00% 1095.64 0.00% 928.51 0.17% 786.38 0.00% 1157.55 0.00% 1054.61 0.00% 966.08 0.00% 779.31 0.00% | 1274.54 0.71% 1113.53 1.63% 945.96 2.06% 799.67 0.00% 1161.81 0.37% 1059.89 0.50% 976.40 0.00% 795.39 0.00% | 1266.11 0.04% 1096.75 0.10% 926.89 0.00% 786.38 0.00% 1157.55 0.00% 1056.21 0.15% 966.08 0.00% 780.72 0.00% | 1284.71 1.49% 1127.02 2.78% 943.13 1.72% 807.71 2.64% 1170.98 1.15% 1093.64 3.57% 986.70 2.09% 785.60 0.80% | 1342.12 6.05% 1123.89 2.58% 966.76 4.30% 785.09 −0.16% 1218.74 5.29% 1091.32 3.48% 973.69 0.79% 791.02 1.50% | 982.51 0.54% | 988.48 1.19% | 930.52 −3.28% |

Table 7. Cont.
6. Conclusions

With the view of adapting to sustainable transportation, this paper studies a mixed fleet logistics distribution problem under the CO\textsubscript{2} emission cap imposed by the enforcement agencies. The study aims to obtain the best composition of conventional and green vehicles used to serve all customers while minimizing vehicle travelled distances in the routes and respecting all constraints of the problem. All customers are designated with time window constraints. The proposed problem is seen in many real-life situations such as home deliveries system (repair and maintenance services, mail and newspaper delivery, furniture delivery, and parcel distribution systems, etc.), school bus routing problems, patient delivery problems, and waste collection problems. Our study supports organizations to design the best composition of vehicles for a mixed fleet logistics distribution problem. A new hybrid ACO metaheuristic is designed to solve the investigated problem. The variable neighborhood search (VNS) is combined with standard ACO to obtain the hybrid ACO. The effectiveness of the proposed problem and metaheuristic solution method is testified by numerical experiments on the newly generated instances and VRPTW benchmark instances. The proposed mixed fleet model reduces significant amount of CO\textsubscript{2} emissions for logistics problems under CO\textsubscript{2} emission cap. The newly designed hybrid ACO generates 21 new best-known solutions out of the 56 benchmark VRPTW instances. Considering both of the features of the problem and the proven efficiency of the proposed algorithm, it can be concluded that this study has great potential in the field of green logistics problems with mixed fleet and CO\textsubscript{2} emission cap.

Many attributes such as backhaul, cluster, time-dependence, refuelling stations, and multi-depot are encountered in many real-world scenarios. These real-life scenarios can be added to the proposed logistics model in future studies. From an algorithm point of view, the proposed hybrid ACO can be easily extended to solve other variants of the logistics model. Future research works can also explore the possibility of combining ACO with other metaheuristics such as particle swarm optimization, genetic algorithm, tabu search, simulated annealing, etc.

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