Bayes Conditional Probability of Fuzzy Damage and Technical Wear of Residential Buildings

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Abstract: The purpose of the research presented in the article is to identify the impact of the processes associated with the broadly understood maintenance of old residential buildings with a traditional construction on the size and intensity of the wear of their elements. The goal was achieved by analyzing the symptoms of the technical wear process, which involved the understanding of the mechanism of the occurrence of the phenomenon of damage, and the identification of the size and intensity of the damage to the elements of the evaluated buildings. The consequence of systematizing the most important processes that influence the loss of functional properties of residential buildings was the creation of the authors' own qualitative model and its transformation into a quantitative model. This, in turn, enabled a multi-criteria quantitative analysis of the cause and effect phenomena—"damage-technical wear"—of the most important elements of downtown tenement buildings to be carried out in fuzzy conditions, i.e., uncertainty concerning the occurrence of damage and the wear process. The following key question was answered in the subjective expert assessment of the technical condition of an evaluated residential building: what is the probability of the wear of an element, which may be more or less correlated with its average maintenance conditions, or more simply, what is the probability that the element is more or less (approximately) worn? It has been proven that the conditional probability of the technical wear of an element in relation to its damage increases with the deterioration of the maintenance conditions of the building, and this increase is very regular, even in the case of different building elements. This probability is characterized by a low standard deviation and a narrow range of the dispersion of results in the case of various elements with regards to each of the considered building maintenance conditions.

Keywords: residential buildings; technical wear; damage; Bayes conditional probability; fuzzy sets

1. Introduction

1.1. Literature Survey

The literature survey was based on the theory of decision-making in the conditions of uncertainty and fuzziness, which is given by Kacprzyk [1], and which defines the following decision situations [2]:

- Certainty: all the information that describes the issue of decision-making is deterministic, i.e., options for choosing a decision, and what each choice gives in terms of certain usefulness (e.g., value analysis) etc., is known. In this case, making decisions comes down to the direct maximization of the utility function;
- Risk: information that describes the decision-making issue is probabilistic, i.e., appropriate probability distributions are provided. In this case, making decisions comes down to maximizing the expected value of the utility function;
- Indeterminacy: even probabilities are not known. Decision-making usually comes down to applying a minimax strategy in order to ensure the highest utility value under the most unfavorable conditions;
• Fuzziness: indeterminacy not only relates to the occurrence of an event, but also to its meaning in general, which can no longer be described using probabilistic methods. Of course, further extensions are possible here, such as adding risk to fuzziness.

When assessing the degree of the technical wear of building elements, apart from measurable (quantitative) criteria, non-measurable (qualitative) criteria are also used. They are expressed in the analysis of symptoms by, i.e., damage that reduces the technical condition and utility value of building elements. Only some of these criteria can be roughly quantified. However, most of these criteria are qualitative. Their value is determined verbally by using terms such as “significant”, “poor”, “strong”, “almost not at all”, “partial”, or “complete”, and always appears in the description of damage phenomena. The interpretation of the effects of these phenomena, which is performed according to subjective and qualitative premises, leads to the indiscriminate categorization of the technical maintenance conditions for buildings and their elements, i.e., good, satisfactory, average, poor, or bad. Striving for a quantification of criteria that are inherently qualitative and immeasurable, and trying to determine the relations between them, led to the use of the category of fuzzy sets (the basis of which were formulated by Zadeh [3,4] and Yager [5,6]) with regards to this issue. Their properties enable damage to building elements, as well as the conditions of their technical maintenance, to be described within an unambiguous measurable quantitative aspect.

In the methodical approach to the technical assessment of residential buildings, research by Nowogonska [7–11] was used, which provides methods and models for the estimation of the degree of the technical wear of buildings. However, it should be remembered that the presented methodical approach of Nowogonska is exclusively deterministic, and therefore simplified and also practical. This approach is confirmed by the research of Lee and Kim [12], who indicated the degree of risk that is associated with damage to a building element. The assessment of the entire service life of a building structure includes a fuzzy calculation, which was presented in the publications of Plebankiewicz, Wieczorek, and Zima [13–16] in order to determine the impact and significance of the risk of the emergency operation of a building. The works by Ibadov [17–20] concerning the building investment process with a fuzzy phase allowed for the practical application of uncertain and subjective events when determining the degree of damage to the tenement houses. The assessment of the risk and costs of maintaining construction facilities, and also the conducting of the construction process in fuzzy conditions, were also presented by Kamal and Jain [21], Andrić, Wang, Zou and Zhang [22], J. Marzouk and Amin [23], Knight, Robinson, and Fayek [24], Sharma and Goyal [25], Al-Humaidi and Hadipriono [26], Ammar, Zayed and Moselhi [27], Chan, Kwong, Dillon and Fung [28], and Naszrzadeh, Afshar, Khanzadi, and Howick [29].

Methods, models, and methodological tools for the assessment of the technical condition of buildings, which are considered in the article with regards to the research sample, were described and summarized by Konior in papers [30–34] with co-authors [35–37] and in a collective study under the supervision of Kapliński, which is entitled “Methods and Models in the Engineering of Building Processes” [38].

1.2. Research Sample

The research sample, which included 102 technically assessed residential buildings from the “Srodmiescie” district of Wrocław, was selected from a group of 160 examined objects [39]. The overriding criterion for sampling involved the obtaining of a comparable group of objects. Mutual comparability of the downtown tenement houses meant:

• age coherence, i.e., a similar period of erection, maintenance, and use with regards to historical and social aspects;
• compactness of the building development in the urban layout that remained unchanged for years;
• similar location along downtown street routes with an urban, but not representative, character;
• construction and material homogeneity, especially with regards to the load-bearing structure of buildings;
• identical functional solutions, understood as the standard of apartment amenities and furnishings (for that time), and a defined standard of living for residents.

The method of selecting the research sample at the level of greater detail was based on the mutual similarity of all the technical solutions of the downtown tenement houses. The selected research sample, according to the criteria presented above, is a representative sample with regards to the concept of representativeness that is specific to the adopted purpose of the study [40,41]. It contains all the values of the variables that could be recreated from the research carried out earlier using a different objective function than the one adopted in the study.

These values and variables were then compiled and processed in such a way that it was possible to make conclusions about the cause–effect relationships between them in the general population.

Therefore, it can be considered as a typologically representative sample that includes the desired types of homogeneous variables. Due to the fact that the structure of the population and its properties were previously well recognized, such a selection of the research sample can also be seen as a deliberate selection. It should be noted that the sample may not be representative in terms of the distributions of the studied variables, which may— for the adopted level of significance—not correspond to analogous distributions in the general population. It is also not known—at this stage of the research—whether the selected sample is representative due to the relationship between its variables and the identically defined variables in the entire set of downtown residential buildings. Therefore, at the very beginning of the research, it was assumed that a specific research sample occurs in the existing population with the fuzzy phase.

Tested buildings have been classified into classes, determined by the degree of the technical wear. The technical wear 0–15% has been classified to the class I, 16–30% to the class II, 31–50% to the class III, 51–70% to the class IV, 71–100% to the class V. Owing to the fact that all considering apartment houses belong to the same group of their age it is possible to assume that the class of the technical wear corresponds to the conditions of building maintenance. Therefore, the equivalence has been defined: a poor maintenance—the class IV, V, an average maintenance—the class III, an above than an average maintenance—the class II, a very well cared maintenance—the class I.

2. Research Method

2.1. Problem Identification

The research methodology at a level of greater detail was prepared in such a way that allowed the previously prepared qualitative model to be transformed into a quantitative model. Therefore, the diagnosis of the impact of the maintenance of the residential buildings on the amount of their technical wear was carried out using quantitative methods in fuzzy set categories, and also by using the authors’ own model that was created in the conditions of fuzziness. The model allowed for the determination of the conditional probabilities of the process of technical wear, and also the set of damage according to both Bayes formulas [40–42] and the combined approach of Zadeh [3,4] and Yager [5,6].

As mentioned in the introduction, when visually assessing the technical wear of building elements, the symptoms of their destruction are taken into account, i.e., individual damage that can be categorized into the following groups (groups) of damage:
• UM—mechanical damage to the structure and texture of building elements;
• UW—damage to building elements caused by water penetration and moisture penetration;
• UD—damage resulting from the loss of the original shape of wooden elements;
• UP—damage to wooden elements attacked by biological pests.

The purpose of such a conceptual and technical systematization of damage is a comprehensive diagnosis of the extent to which a building element is worn. This assessment, in turn, leads to the implication of stating under what technical conditions—good, satisfactory, average, poor, or bad—the building element was (is) maintained. It is difficult to define a fuzzy set with such a broad meaning as “average technical condition of maintenance” using one membership function. In this case, a semantic analysis of the term “technical wear of a building element” was used, which was denoted with the symbol of a fuzzy set “Z”. Let the technical wear of building element Z consist of: mechanical wear of its structure and texture (fuzzy set ZM), its technical wear caused by water penetration and moisture penetration (fuzzy set ZW), technical wear resulting from the loss of its original shape (fuzzy set ZD), and technical wear caused by the attack of biological pests (fuzzy ZP harvest). This sum can then be expressed as follows:

\[ Z = ZM \cup ZW \cup ZD \cup ZP \]  

and when assuming the identity of the degree of technical wear and its visual symptom (Z ⊆ U)—damage to a building element that is integrated into the above-described damage sets—this expression takes the following form:

\[ U = UM \cup UW \cup UD \cup UP \]  

If technical wear was assumed in the observed states with its measure—the degree of wear as a fuzzy set with no crisp membership boundary of \( \{z\} = Z \)—then the visual image of this wear—global damage to a building element—should be treated as a fuzzy set, the fuzzy events of which are arguments—distinguished types of damage \( \{u\} = U \). Therefore, fuzzy random events are fuzzy sets that express the degree of technical wear, for which there is no complete (measurable) certainty of membership to the II, III or IV class of the technical maintenance of an element. The question then arises: what is the probability of an element being worn, which will more or less represent its average maintenance conditions. To put it simply, what is the probability that an element is more or less (approximately) worn?

The approach of Zadeh [3,4], who defined the probabilities of fuzzy events in the form of real numbers from interval [0, 1], was used in the research. Therefore, the probability of a fuzzy event, which is the technical wear of a building element, which corresponds to satisfactory, average, and poor maintenance conditions, was defined as:

\[ P(Z)_{II, \, III, \, IV} = \sum_{i=1}^{n} p(z_i) \mu_a(z_i), \text{ if } Z = \{z\} = \{z_1, \, z_2, \ldots, \, z_n\} \]  

For the global damage of a structural element, which is assumed equivalently to the event of technical wear, the probability of its occurrence is expressed by the following analogous relationship:

\[ P(U)_{II, \, III, \, IV} = \sum_{j=1}^{m} p(u_j) \mu_a(u_j), \text{ gd } U = \{u\} = \{u_1, \, u_2, \ldots, \, u_m\} \]  

The probabilities p(u) of the occurrence of elementary damage u in sets II, III, IV were calculated and then presented in Table 1.
### Table 1. Probability \( p(u) \) of elementary damage \( u_i \) for 10 selected building elements.

| No. of Damage | Name of Damage | Probability of Damage \( u_i \) that Corresponds to the II, III, and IV Maintenance Conditions of an Element |
|---------------|----------------|------------------------------------------------------------------------------------------------------------------|
|               |                | Z2–Foundations \( p(u)_{II} \) | Z3–Basement Walls \( p(u)_{III} \) | Z4–Solid Floors Above Basements \( p(u)_{IV} \) | Z7–Structural Walls \( p(u)_{II} \) |
| u1            | Mechanical damage | 0.71 | 0.77 | 0.91 | 0.50 | 0.63 | 0.79 | 0.61 | 0.76 | 0.65 | 0.81 | 0.98 | 1.00 |
| u2            | Leaks            | 0.50 | 0.63 | 0.88 | 0.63 | 0.95 | 1.00 |
| u3            | Brick losses     | 0.43 | 0.81 | 0.64 | 0.42 | 0.65 | 0.71 | 0.83 | 0.71 | 0.70 | 0.44 | 0.89 | 0.80 |
| u4            | Mortar losses    | 0.25 | 0.53 | 0.58 | 0.50 | 0.79 | 0.76 |
| u5            | Brick decay      | 0.57 | 0.83 | 0.55 | 0.33 | 0.60 | 0.46 | 0.65 | 0.58 | 0.43 | 0.06 | 0.30 | 0.28 |
| u6            | Mortar decay     | 0.25 | 0.44 | 0.29 | 0.75 | 0.64 | 0.76 |
| u7            | Peeling off of paint coatings | 0.00 | 0.00 | 0.73 | 0.00 | 0.16 | 0.25 |
| u8            | Falling off of paint coatings | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u9            | Cracks in bricks | 0.00 | 0.66 | 1.00 | 0.08 | 0.33 | 1.00 | 0.09 | 0.65 | 0.70 | 0.13 | 0.67 | 0.96 |
| u10           | Cracks on plaster | 0.00 | 0.30 | 0.91 | 0.00 | 0.18 | 0.46 | 0.00 | 0.02 | 0.57 | 0.00 | 0.15 | 0.36 |
| u11           | Scratching on walls | 0.00 | 0.00 | 0.73 | 0.00 | 0.16 | 0.25 |
| u12           | Scratching on plaster | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u13           | Loosening of plaster | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u14           | Falling off of plaster sheets | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u15           | Dampness         | 0.00 | 0.30 | 0.91 | 0.00 | 0.18 | 0.46 | 0.00 | 0.02 | 0.57 | 0.00 | 0.15 | 0.36 |
| u16           | Weeping          | 0.00 | 0.00 | 0.73 | 0.00 | 0.16 | 0.25 |
| u17           | Biological corrosion of bricks | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u18           | Fungus           | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u19           | Mold and rot     | 0.00 | 0.00 | 0.18 | 0.00 | 0.00 | 0.13 |
| u20           | Corrosion raid of steel beams | 0.13 | 0.60 | 0.70 |
| u21           | Surface corrosion of steel beams | 0.52 | 0.71 | 0.78 |
| u22           | Deep corrosion of steel beams | 0.00 | 0.22 | 0.43 |
| u23           | Flooding with water | 0.00 | 0.00 | 0.09 |
Dynamic sensitivity of floor beams
Deformations of wooden beams
Skewing of window joinery
Warping of window joinery
Delamination of wooden elements
Partial insect infestation of wooden elements
Complete insect infestation of wooden elements

| No. of Damage | Name of Damage | Probability of Damage (u) that Corresponds to the II, III, and IV Maintenance Conditions of an Element |
|---------------|----------------|---------------------------------------------------------------------------------------------------|
| u1            | Mechanical damage | Z8–Inter-Story Wooden Floors p(u)II 0.82 p(u)III 0.88 p(u)IV 0.84 Z9–Stairs p(u)II 0.88 p(u)III 0.85 p(u)IV 1.00 |
| u2            | Leaks            | Z10–Roof Construction p(u)II 0.88 p(u)III 0.92 p(u)IV 1.00 |
| u3            | Brick losses     | Z13–Window Joinery p(u)II 0.08 p(u)III 0.35 p(u)IV 0.96 |
| u4            | Mortar losses    |                                                                                 |
| u5            | Brick decay      |                                                                                 |
| u6            | Mortar decay     |                                                                                 |
| u7            | Peeling off of paint coatings |                                                                                 |
| u8            | Falling off of paint coatings |                                                                                 |
| u9            | Cracks in bricks |                                                                                 |
| u10           | Cracks on plaster|                                                                                 |
| u11           | Scratching on walls |                                                                                 |
| u12           | Scratching on plaster | 0.74 0.92 0.75                                                                                 |
| u13           | Loosening of plaster | 0.44 0.57 0.63                                                                                 |
| u14           | Falling off of plaster sheets |                                                                                 |
| u15           | Dampness         |                                                                                 |
| u16           | Weeping          |                                                                                 |
| u17           | Biological corrosion of bricks |                                                                                 |
| No. of Damage | Name of Damage                        | Probability of Damage (u) that Corresponds to the II, III, and IV Maintenance Conditions of an Element |
|--------------|--------------------------------------|-----------------------------------------------------------------------------------------------------|
| Z15–Inner Plasters | Z20–Facades                                                                 |
| p(u)II | p(u)III | p(u)IV | p(u)II | p(u)III | p(u)IV |
| u1 | Mechanical damage | 0.80 | 0.69 | 0.68 | 0.65 | 0.73 | 0.78 |
| u2 | Leaks | | | | | |
| u3 | Brick losses | | | | | |
| u4 | Mortar losses | | | | | |
| u5 | Brick decay | | | | | |
| u6 | Mortar decay | 0.40 | 0.81 | 0.95 | 0.59 | 0.86 | 0.96 |
| u7 | Peeling off of paint coatings | 0.90 | 0.76 | 0.86 | 0.59 | 0.91 | 0.96 |
| u8 | Falling off of paint coatings | 0.20 | 0.31 | 0.23 | 0.00 | 0.23 | 0.30 |
| u9 | Cracks in bricks | | | | | |
| u10   | Cracks on plaster | 0.50 | 0.78 | 0.91 | 0.41 | 0.86 | 1.00 |
|-------|-------------------|------|------|------|------|------|------|
| u11   | Scratching on walls |      |      |      |      |      |      |
| u12   | Scratching on plaster | 0.30 | 0.78 | 0.86 | 0.35 | 0.59 | 1.00 |
| u13   | Loosening of plaster | 0.00 | 0.29 | 0.95 | 0.00 | 0.14 | 0.89 |
| u14   | Falling off of plaster sheets | 0.00 | 0.00 | 0.36 | 0.00 | 0.05 | 0.22 |
| u15   | Dampness | 0.00 | 0.00 | 0.23 | 0.00 | 0.00 | 0.93 |
| u16   | Weeping | 0.00 | 0.00 | 0.14 | 0.00 | 0.00 | 0.41 |
| u17   | Biological corrosion of bricks |      |      |      |      |      |      |
| u18   | Fungus | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 |
| u19   | Mold and rot | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| u20   | Corrosion raid of steel beams |      |      |      |      |      |      |
| u21   | Surface corrosion of steel beams |      |      |      |      |      |      |
| u22   | Deep corrosion of steel beams |      |      |      |      |      |      |
| u23   | Flooding with water |      |      |      |      |      |      |
| u24   | Dynamic sensitivity of floor beams |      |      |      |      |      |      |
| u25   | Deformations of wooden beams |      |      |      |      |      |      |
| u26   | Skewing of window joinery |      |      |      |      |      |      |
| u27   | Warping of window joinery |      |      |      |      |      |      |
| u28   | Delamination of wooden elements |      |      |      |      |      |      |
| u29   | Partial insect infestation of wooden elements |      |      |      |      |      |      |
| u30   | Complete insect infestation of wooden elements |      |      |      |      |      |      |
It should be noted that a slightly simplified approach, in which a fuzzy number is assigned to the probability of fuzzy events, was used here. This is opposed to the Yager approach [5,6], in which the probabilities are fuzzy events. It is important that the study did not consider the differences between the concepts of fuzziness and randomness. It was only assumed, although these phenomena are different and described differently, that they may nevertheless occur together as two types of uncertainty.

2.2. Model of Determining the Conditional Probabilities of the Process of Technical Wear in Relation to the Occurrence of Damage

The preliminary assumption: the process of technical wear of building elements occurs when there is identifiable damage: \([ZII, ZIII, ZIV] = Z \in U\)

The technical wear of building elements, determined by a group of experts in the II, III, and IV state of their technical maintenance, takes the following argument values:

- \(ZII = [20, 25, 30]\% = [z1, z2, z3]\), moreover, \(z4 = 0\);
- \(ZIII = [35, 40, 45, 50]\% = [z1, z2, z3, z4]\);
- \(ZIV = [55, 60, 65, 70]\% = [z1, z2, z3, z4]\).

Technical inspections of the residential buildings were executed by a team of experts consisted of:

- 1 architect;
- 1 structural engineer;
- 1 mechanical/sanitary engineer;
- 1 electrical engineer;
- 2 quantity surveyors;
- 1 technician/administrator.

In order to simplify the calculations, it was assumed that the domain of sets defined as fuzzy (ZII, ZIII, and ZIV) is interval \([0.2, 0.7]\), and each of the sets contains a sum of \(N\) arguments of \(z1, z2, z3, z4\). Each of these arguments occurs \(n\) times in the set. Without complicating the method with operations performed on fuzzy sets, it can be assumed that the degree to which arguments \(z1, z2, z3, z4\) belong to fuzzy sets ZII, ZIII, ZIV is equal to the frequency of their occurrence in the sets:

\[\mu_i = n/N_i, \text{ where } i = 1, 2, 3, 4, \text{ and } k = 1, 2, 3 \in II, III, IV\]  

Each of the fuzzy sets ZII, ZIII, ZIV can be written with the use of the membership function as follows:

\[ZII = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3)II\]  
\[ZIII = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3 + \mu_{z4}/z4)III\]  
\[ZIV = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3 + \mu_{z4}/z4)IV\]

and when supplementing the output data with the values of their intersections:

\[ZII \cap ZIII = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3)II, III\]  
\[ZII \cap ZIV = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3)II, IV\]  
\[ZIII \cap ZIV = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3 + \mu_{z4}/z4)III, IV\]  
\[ZII \cap ZIII \cap ZIV = (\mu_{z1}/z1 + \mu_{z2}/z2 + \mu_{z3}/z3 + \mu_{z4}/z4)II, III, IV\]

The probabilities of the occurrence of individual arguments in sets ZII, ZIII, ZIV are as follows:

\[p(z1)\text{II }= 1/3; \, p(z2)\text{II }= 1/3; \, p(z3)\text{II }= 1/3; \, p(z4)\text{II }= 0\]  
\[p(z1)\text{III }= 1/4; \, p(z2)\text{III }= 1/4; \, p(z3)\text{III }= 1/4; \, p(z4)\text{III }= 1/4\]
\[ P(U) = 1/4; \quad P(Z1) = 1/4; \quad P(Z2) = 1/4; \quad P(Z4) = 1/4 \]  
(15)

When using dependence (3), the degrees of membership of arguments \( z1, z2, z3, \) and \( z4 \) (6–12), and the probabilities of the occurrence of particular arguments in sets \( ZII, ZIII, ZIV \) (13–15), the partial probabilities of the occurrence of technical wear processes were calculated as fuzzy events in the satisfactory, average, and poor technical maintenance conditions of the analyzed residential buildings:

\[ P(ZII) = \sum_{i=1}^{3} p(z_i) \mu_a(z_i)II \]  
(16)

\[ P(ZIII) = \sum_{i=1}^{4} p(z_i) \mu_a(z_i)III \]  
(17)

\[ P(ZIV) = \sum_{i=1}^{4} p(z_i) \mu_a(z_i)IV \]  
(18)

and their products:

\[ P(ZII \times ZIII) = \sum_{i=1}^{3} [(p(z_i) \mu_a(z_i)II \times (p(z_i) \mu_a(z_i)III)] \]  
(19)

\[ P(ZII \times ZIV) = \sum_{i=1}^{3} [(p(z_i) \mu_a(z_i)II \times (p(z_i) \mu_a(z_i)IV)] \]  
(20)

\[ P(ZIII \times ZIV) = \sum_{i=1}^{4} [(p(z_i) \mu_a(z_i)III \times (p(z_i) \mu_a(z_i)IV)] \]  
(21)

\[ P(ZII \times ZIII \times ZIV) = \sum_{i=1}^{3} [(p(z_i) \mu_a(z_i)II \times (p(z_i) \mu_a(z_i)III \times (p(z_i) \mu_a(z_i)IV)] \]  
(22)

abilities of the occurrence of a set of damage to residential building elements in relation to the processes of their wear. It was assumed that the conditional probabilities, defined in such a way, correspond to the frequency of the occurrence of all elementary damage related to a single element in the following building maintenance conditions:

- satisfactory—\( P(U/ZII) \);
- average—\( P(U/ZIII) \);
- poor—\( P(U/ZIV) \);
- satisfactory and average—\( P(U/ZII \times ZIII) \);
- satisfactory and poor—\( P(U/ZII \times ZIV) \);
- average and poor—\( P(U/ZIII \times ZIV) \);
- satisfactory, average and poor—\( P(U/ZII \times ZIII \times ZIV) \).

The above calculations of conditional and partial probabilities (16–22) allowed the probability of the occurrence of a group of damage to be determined in the middle, non-acute technical maintenance states of the analyzed residential buildings:

\[ P(U) = P(U/ZII) \times P(ZII) + P(U/ZIII) \times P(ZIII) + P(U/ZIV) \times P(ZIV) - P(U/ZII \times ZIII) \times P(ZII \times ZIII) \]

\[ - P(U/ZII \times ZIV) \times P(ZII \times ZIV) - P(U/ZIII \times ZIV) \times P(ZIII \times ZIV) \]

\[ + P(U/ZII \times ZIII \times ZIV) \times P(ZII \times ZIII \times ZIV) \]  
(23)

In the last stage of the developed model, the Bayes formula [37–39] for a posteriori probabilities was used. It determines the conditional probabilities of fuzzy events (i.e., the processes of the technical wear of building elements) in relation to another fuzzy event,
i.e., the occurrence of their damage. The Bayes formula under satisfactory, average, and poor fuzziness conditions is as follows:

\[
P(Z_{II} / U) = \frac{P(U / Z_{II}) \cdot P(Z_{II})}{P(U)} \quad (24)
\]

\[
P(Z_{III} / U) = \frac{P(U / Z_{III}) \cdot P(Z_{III})}{P(U)} \quad (25)
\]

\[
P(Z_{IV} / U) = \frac{P(U / Z_{IV}) \cdot P(Z_{IV})}{P(U)} \quad (26)
\]

The defined conditional probabilities of fuzzy event \( Z = \{z_1, z_2, z_3, z_4\} \) were supplemented, using the relationships (3) and (13)–(15), with the calculations of its mean value in relation to the probabilistic measure \( P(Z) \) in classes II, III, and IV of the technical maintenance of building elements:

\[
m_p(Z)_{II, III, IV} = 1 / P(Z)_{II, III, IV} \cdot \sum_{i=1}^{4} p(z_i) \mu_{z_i}(z_i) \quad (27)
\]

The values of the conditional probabilities of the technical wear processes \( Z \), which correspond to the II, III, and IV maintenance conditions of 10 selected elements of the analyzed buildings, in relation to the occurrence of their damage \( U \), and with their mean values in relation to the probabilistic measure \( P(Z) \), are given in Table 2.
Table 2. Fuzzy conditional probabilities of the technical wear process in relation to the probabilistic measure and the occurrence of damage, as well as the fuzzy conditional probabilities of the occurrence of a group of damage in relation to the technical wear process.

| Group Number | Building Element | No. of Damage | Name of Damage | Conditional Probabilities of Fuzzy Events According to the Bayes Formula for: |
|--------------|------------------|---------------|----------------|---------------------------------------------------------------------------------|
|              |                  |               |                | the Process of Technical wear (Z) that corresponds to the II, III and IV Conditions of the Maintenance of an Element in Relation to its Damage (U) / Average Value of the Fuzzy Event (Z) in Relation to the Probabilistic Measure P(Z) |
| Z2 Foundation walls | u3 | Brick losses | P(ZII/U)/mp(ZII) | 0.2655/0.300 |
|                  | u5 | Brick decay  | P(ZIII/U)/mp(ZIII) | 0.3957/0.441 |
|                  | u9 | Cracks in bricks | P(ZIV/U)/mp(ZIV) | 0.5755/0.600 |
|                  | u15 | Dampness | P(UII/Z) | 0.0000 |
|                  | u16 | Weeping | P(UIII/Z) | 0.2702 |
|                  | u17 | Biological corrosion of bricks | P(UIV/Z) | 0.9586 |
|                  | u19 | Mold and rot | | |
| Z3 Basement walls | u3 | Brick losses | | |
|                  | u4 | Mortar losses | | |
|                  | u5 | Brick decay | | |
|                  | u6 | Mortar decay | | |
|                  | u9 | Cracks in bricks | | |
|                  | u10 | Cracks on plaster | | |
|                  | u15 | Dampness | | |
|                  | u16 | Weeping | | |
| Group Number | Building Element | No. of Damage | Name of Damage                      | Conditional Probabilities of Fuzzy Events for: |
|--------------|------------------|---------------|-------------------------------------|-----------------------------------------------|
|              |                  |               |                                     | the Process of Technical wear \( (Z) \) that corresponds to the II, III and IV Conditions of the Maintenance of an Element in Relation to its Damage \( (U) \) / Average Value of the Fuzzy Event \( (Z) \) in Relation to the Probabilistic Measure \( P(Z) \) |
|              |                  |               |                                     | the Occurrence of the Group of Damage \( (U) \) that Corresponds to the II, III and IV Conditions of Maintenance of an Element in Relation to the Wear Process \( (Z) \) |
|              |                  |               |                                     | \( P(Z_{II}/U) \)/mp\( (Z_{II}) \) \( 0.3846/0.270 \) \( 0.3778/0.406 \) \( 0.5173/0.613 \) \( 0.2371 \) \( 0.4577 \) \( 0.6330 \) |
|              |                  |               |                                     | \( P(Z_{III}/U)/mp(Z_{III}) \) \( 0.3999/0.431 \) \( 0.5125/0.626 \) \( 0.0000 \) \( 0.4487 \) \( 1.0000 \) |
|              |                  |               |                                     | \( P(Z_{IV}/U)/mp(Z_{IV}) \) \( 0.3464/0.300 \) \( 0.3999/0.431 \) \( 0.5125/0.626 \) \( 0.0000 \) \( 0.4487 \) \( 1.0000 \) |
|              |                  |               |                                     | \( P(U_{II}/Z) \) \( 0.3846/0.270 \) \( 0.3778/0.406 \) \( 0.5173/0.613 \) \( 0.2371 \) \( 0.4577 \) \( 0.6330 \) |
|              |                  |               |                                     | \( P(U_{III}/Z) \) \( 0.3999/0.431 \) \( 0.5125/0.626 \) \( 0.0000 \) \( 0.4487 \) \( 1.0000 \) |
|              |                  |               |                                     | \( P(U_{IV}/Z) \) \( 0.3464/0.300 \) \( 0.3999/0.431 \) \( 0.5125/0.626 \) \( 0.0000 \) \( 0.4487 \) \( 1.0000 \) |
| Group Number | Building Element | No. of Damage | Name of Damage | Conditional Probabilities of Fuzzy Events According to the Bayes Formula for: |
|--------------|------------------|---------------|----------------|--------------------------------------------------------------------------------|
|              |                  |               |                | the Process of Technical wear (Z) that Corresponds to the II, III and IV Conditions of the Maintenance of an Element in Relation to its Damage (U) / Average Value of the Fuzzy Event (Z) in Relation to the Wear Process (Z) |
|              |                  |               |                | **P(UII/Z)** **P(UIII/Z)** **P(UIV/Z)** **P(ZII/U)** **P(ZIII/U)** **P(ZIV/U)** |
| Z8           | Inter-story wooden floors | u12 | Scratching on plaster | 0.3508/0.283 | 0.4401/0.435 | 0.6443/0.636 | 0.0000 | 0.3048 | 0.9615 |
|              |                  | u13 | Loosening of plaster |               |                |                |        |        |        |
|              |                  | u15 | Dampness |               |                |                |        |        |        |
|              |                  | u16 | Weeping |               |                |                |        |        |        |
|              |                  | u18 | Fungus |               |                |                |        |        |        |
|              |                  | u24 | Dynamic sensitivity of floor beams |               |                |                |        |        |        |
|              |                  | u25 | Deformations of wooden beams |               |                |                |        |        |        |
|              |                  | u30 | Complete insect infestation of wooden elements |               |                |                |        |        |        |
| Z9           | Stairs           | u1  | Mechanical damage |               |                |                |        |        |        |
|              |                  | u3  | Brick losses |               |                |                |        |        |        |
|              |                  | u16 | Weeping |               |                |                |        |        |        |
|              |                  | u20 | Corrosion raid of steel beams |               |                |                |        |        |        |
|              |                  | u21 | Surface corrosion of steel beams |               |                |                |        |        |        |
| Group Number | Building Element | No. of Damage | Name of Damage | Conditional Probabilities of Fuzzy Events According to the Bayes Formula for: |
|--------------|-----------------|---------------|----------------|--------------------------------------------------------------------------------|
|              |                 |               |                | the Process of Technical wear (Z) that corresponds to the II, III and IV Conditions of the Maintenance of an Element in Relation to its Damage (U) / Average Value of the Fuzzy Event (Z) in Relation to the Probabilistic Measure P(Z) |
|              |                 |               | Deep corrosion of steel beams | P(ZII/U)/mp(ZII) | P(ZIII/U)/mp(ZIII) | P(ZIV/U)/mp(ZIV) | P(UII/Z) | P(UIII/Z) | P(UIV/Z) |
| u22          |                 |               | Partial insect infestation of wooden elements | 0.2817/0.288 | 0.3510/0.437 | 0.6466/0.630 | 0.0513 | 0.6579 | 0.7999 |
| Z10          | Roof construction | u15           | Dampness       |                           |              |                |                |                |                |
|              |                 | u16           | Weeping        |                           |              |                |                |                |                |
|              |                 | u28           | Delamination of wooden elements |                           |              |                |                |                |                |
|              |                 | u29           | Partial insect infestation of wooden elements |                           |              |                |                |                |                |
|              |                 | u30           | Complete insect infestation of wooden elements |                           |              |                |                |                |                |
| Z13          | Window joinery  | u1            | Mechanical damage |                           |              |                |                |                |                |
|              |                 | u2            | Leaks          |                           |              |                |                |                |                |
|              |                 | u15           | Dampness       |                           |              |                |                |                |                |
|              |                 | u16           | Weeping        |                           |              |                |                |                |                |
|              |                 | u19           | Mold and rot   |                           |              |                |                |                |                |
|              |                 |               |                | 0.3275/0.271 | 0.3717/0.423 | 0.5538/0.648 | 0.2130 | 0.4992 | 0.7592 |
| Group Number | Building Element | No. of Damage | Name of Damage                        |
|--------------|------------------|---------------|--------------------------------------|
| u26          |                  |               | Skewing of window joinery            |
| u27          |                  |               | Warping of window joinery            |
| u29          |                  |               | Partial insect infestation of wooden elements |
| u30          |                  |               | Complete insect infestation of wooden |
| u1           |                  |               | Mechanical damage                    |
| u6           |                  |               | Mortar decay                         |
| u7           |                  |               | Peeling off of paint coatings         |
| u8           |                  |               | Falling off of paint coatings         |
| u10          |                  |               | Cracks on plaster                     |
| u12          |                  |               | Scratching on plaster                 |
| u13          |                  |               | Loosening of plaster                  |
| u14          |                  |               | Falling off of plaster sheets         |
| u15          |                  |               | Dampness                              |
| u16          |                  |               | Weeping                               |

Conditional Probabilities of Fuzzy Events According to the Bayes Formula for:

the Process of Technical wear (Z) that Corresponds to the II, III and IV Conditions of the Maintenance of an Element in Relation to its Damage (U) / Average Value of the Fuzzy Event (Z) in Relation to the Probabilistic Measure P(Z)

P(ZII/U)/mp(ZII) P(ZIII/U)/mp(ZIII) P(ZIV/U)/mp(ZIV)

the Occurrence of the Group of Damage (U) that Corresponds to the II, III and IV Conditions of Maintenance of an Element in Relation to the Wear Process (Z)

P(UII/Z) P(UIII/Z) P(UIV/Z)

| Z15            | Inner plasters   |                     | Conditional Probabilities | 0.3303/0.290 | 0.3579/0.431 | 0.4995/0.648 | 0.1444 | 0.4578 | 0.7792 |
|----------------|------------------|---------------------|---------------------------|---------------|---------------|---------------|--------|--------|--------|
| u10            |                  |                     | Cracks on plaster         | 0.3303/0.290  | 0.3579/0.431  | 0.4995/0.648  | 0.1444 | 0.4578 | 0.7792 |
| u12            |                  |                     | Scratching on plaster     |               |               |               |        |        |        |
| u13            |                  |                     | Loosening of plaster      |               |               |               |        |        |        |
| u14            |                  |                     | Falling off of plaster sheets|             |               |               |        |        |        |
| u15            |                  |                     | Dampness                  |               |               |               |        |        |        |
| u16            |                  |                     | Weeping                   |               |               |               |        |        |        |
| Group Number | Building Element | No. of Damage | Name of Damage                  | Conditional Probabilities of Fuzzy Events According to the Bayes Formula for: |
|--------------|-----------------|---------------|---------------------------------|--------------------------------------------------------------------------------|
|              |                 |               |                                 | the Process of Technical wear \((Z)\) that corresponds to the II, III and IV Conditions of the Maintenance of an Element in Relation to its Damage \((U)\)/ Average Value of the Fuzzy Event \((Z)\) in Relation to the Probabilistic Measure \(P(Z)\) |
|              |                 |               |                                 | \(P(Z_{II}/U)/mp(Z_{II})\)                                                    |
|              |                 |               |                                 | \(P(Z_{III}/U)/mp(Z_{III})\)                                                  |
|              |                 |               |                                 | \(P(Z_{IV}/U)/mp(Z_{IV})\)                                                    |
|              |                 |               |                                 | \(P(U_{II}/Z)\) \(P(U_{III}/Z)\) \(P(U_{IV}/Z)\)                           |
| u18          |                 |               | Fungus                          |                                                                               |
| u19          |                 |               | Mold and rot                    |                                                                               |
| Z20 Facades  |                 |               |                                 | 0.2854/0.253 0.3670/0.450 0.6299/0.641 0.1250 0.3623 0.9012                  |
| u1           |                 |               | Mechanical damage               |                                                                               |
| u6           |                 |               | Mortar decay                    |                                                                               |
| u7           |                 |               | Peeling off of paint coatings   |                                                                               |
| u8           |                 |               | Falling off of paint coatings   |                                                                               |
| u10          |                 |               | Cracks on plaster               |                                                                               |
| u12          |                 |               | Scratching on plaster           |                                                                               |
| u13          |                 |               | Loosening of plaster            |                                                                               |
| u14          |                 |               | Falling off of plaster sheets   |                                                                               |
| u15          |                 |               | Dampness                        |                                                                               |
| u16          |                 |               | Weeping                         |                                                                               |
| u18          |                 |               | Fungus                          |                                                                               |
| u19          |                 |               | Mold and rot                    |                                                                               |
2.3. The Model for Determining the Conditional Probabilities of a Set of Damage in Relation to the Process of Their Technical Wear

The preliminary assumption: damage to building elements occurs when there is a process of their technical wear, which can be estimated within the range of 0–100%: \{\text{UII, UIII, UIV}\} = U \subseteq Z

Damage to building elements, which is identified by experts in classes II, III, and IV of their technical maintenance, is defined as being dichotomous variables that assume values “0” (damage does not occur) or “1” (damage occurs). The domain of the set of damage, defined as fuzzy UII, UIII, UIV, is binary \{0\}, \{1\}.

It was assumed that the measure of the degree of membership of a single damage \(\mu_{\text{uj}}\) to the set of a group of damage \(U\), which is the symptom of the ongoing wear processes \(Z\), is the feature that most fully expresses the correlation between these variables. This can be a point two-series correlation coefficient \(r(Z) \leftrightarrow r(U)\), which is determined in each of the states II, III, and IV of the technical maintenance.

Each of the fuzzy sets UII, UIII, UIV can therefore be written using the membership function as follows:

\[
\text{UII, III, IV} = \left( \sum_{j=1}^{m} \frac{r(u_j)}{r_{\text{uj}}(u_j)} \right)_{\text{II, III, IV}} \text{gдз} j \rightarrow m \in [5, 12] \tag{28}
\]

and when supplementing the output data with the values of their products:

\[
\text{UII} \bullet \text{UIII} = \left( \sum_{j=1}^{m} r(u_j) \bullet r(u_j) \right)_{\text{II, III}} \tag{29}
\]

\[
\text{UII} \bullet \text{UIV} = \left( \sum_{j=1}^{m} r(u_j) \bullet r(u_j) \right)_{\text{II, IV}} \tag{30}
\]

\[
\text{UIII} \bullet \text{UIV} = \left( \sum_{j=1}^{m} r(u_j) \bullet r(u_j) \right)_{\text{III, IV}} \tag{31}
\]

\[
\text{UII} \bullet \text{UIII} \bullet \text{UIV} = \left( \sum_{j=1}^{m} r(u_j) \bullet r(u_j) \bullet r(u_j) \right)_{\text{II, III, IV}} \tag{32}
\]

When using relationship (4), the degrees of memberships of individual damage \(\mu_{\text{uj}} = r(u)\) to sets of groups of damage \(U\) (28)–(32), and by having data concerning the probabilities of individual damage in sets ZII, ZIII, ZIV (13)–(15), the partial probabilities of the damage were calculated as fuzzy events in the satisfactory, average, and poor technical maintenance conditions of the analyzed residential buildings:

\[
P(\text{UII}) = \left( \sum_{j=1}^{m} p(u_j) r(u_j) \right)_{\text{II}} \tag{33}
\]

\[
P(\text{UIII}) = \left( \sum_{j=1}^{m} p(u_j) r(u_j) \right)_{\text{III}} \tag{34}
\]

\[
P(\text{UIV}) = \left( \sum_{j=1}^{m} p(u_j) r(u_j) \right)_{\text{IV}} \tag{35}
\]

and their products:
\[
P(\text{UII} \bullet \text{UIII}) = \sum_{j=1}^{m} [p(u)r(u)]_{\text{II} \bullet p(u)r(u)}_{\text{III}}
\]

(36)

\[
P(\text{UII} \bullet \text{UIV}) = \sum_{j=1}^{m} [p(u)r(u)]_{\text{II} \bullet p(u)r(u)}_{\text{IV}}
\]

(37)

\[
P(\text{UIII} \bullet \text{UIV}) = \sum_{j=1}^{m} [p(u)r(u)]_{\text{III} \bullet p(u)r(u)}_{\text{IV}}
\]

(38)

\[
P(Z \bullet ZIII \bullet ZIV) = \sum_{j=1}^{m} [p(u)r(u)]_{Z \bullet p(u)r(u)}_{ZIII \bullet p(u)r(u)}_{ZIV}
\]

(39)

The next stage of the created model involved the calculation of the conditional probabilities of the wear processes of the residential buildings' elements in relation to the occurrence of their damage. Due to the assumption that damage is an expression of technical wear, it was assumed, as in the case of defining the wear processes, that the conditional probabilities of the technical wear correspond to the frequency of the occurrence of all the elementary damage ([u] = 1) of a selected building element in the II, III, IV conditions of its maintenance: \( P(Z/\text{UII}) \), \( P(Z/\text{UIII}) \), \( P(Z/\text{UIIV}) \), \( P(Z/\text{UIII} \bullet \text{UIV}) \), \( P(Z/\text{UIII} \bullet \text{UIV}) \), \( P(Z/\text{UIII} \bullet \text{UII} \bullet \text{UIV}) \).

The above calculations of the conditional and partial probabilities (33–39) enabled the probability of the occurrence of technical wear processes to be determined in the middle, non-acute technical maintenance states of the analyzed residential buildings:

\[
P(Z) = P(Z/\text{UII}) \bullet P(\text{UII}) + P(Z/\text{UIII}) \bullet P(\text{UIII}) + P(Z/\text{UIIV}) \bullet P(\text{UIIV})
\]

\[
- P(Z/\text{UIII} \bullet \text{UII}) \bullet P(\text{UIII} \bullet \text{UII}) - P(Z/\text{UIII} \bullet \text{UIV}) \bullet P(\text{UIII} \bullet \text{UIV}) - P(Z/\text{UII} \bullet \text{UIIV}) \bullet P(\text{UII} \bullet \text{UIIV})
\]

\[
+ P(Z/\text{UIII} \bullet \text{UII} \bullet \text{UIIV}) \bullet P(\text{UIII} \bullet \text{UII} \bullet \text{UIIV})
\]

(40)

In the last stage of the developed model, the Bayes formula for a posteriori probabilities was used again, which determines the conditional probabilities of fuzzy events (i.e., the occurrence of damage to building elements) in relation to another fuzzy event (i.e., the processes of their technical wear) [38]. The Bayes formula under satisfactory, moderate, and poor fuzziness conditions is as follows:

\[
P(\text{UII} / Z) = \frac{P(Z / \text{UII}) \bullet P(\text{UII})}{P(Z)}
\]

(41)

\[
P(\text{UIII} / Z) = \frac{P(Z / \text{UIII}) \bullet P(\text{UIII})}{P(Z)}
\]

(42)

\[
P(\text{UIIV} / Z) = \frac{P(Z / \text{UIIV}) \bullet P(\text{UIIV})}{P(Z)}
\]

(43)

In this case, the mean value \( m_p(U) \) of fuzzy event \( U = [u] \) in relation to the probabilistic measure \( P(U) \) is a constant value equal to one, because only the cases in which the dichotomous variable occurred were taken into account.

The values of the conditional probabilities of the occurrence of a group of damage, which correspond to the II, III, and IV maintenance conditions of 10 selected elements of the analyzed buildings, in relation to the processes of their technical wear, are presented in Table 2.
3. Results

The results of research concerning the impact of damage to building elements on their technical wear in the Bayes conditional probability domain (for damage and technical wear as fuzzy events) led to the following conclusions (within two aspects A and B—Table 2):

A. the probability of the conditional process of the technical wear, which corresponds to the three middle states of maintenance of the building elements, with regards to damage—\( P(Z/U) \) II, III, IV—is as follows:

- the conditional probability of the technical wear of an element in relation to its damage increases with the deterioration of the maintenance conditions of the building (this is an exceptionally steady increase, even in the case of different building elements);
- the probability of such a conditionally defined fuzzy event indicates the state of the technical wear for which the fuzzy damage occurs with the highest intensity, and it amounts, for the following elements of the tested residential buildings in their average maintenance condition \( P(ZIII/U) \), to:
  - for foundations: dampness of foundations 0.40
  - for basement walls: crack in bricks 0.39
  - for solid floors above basements: dampness of floors 0.38
  - for structural walls: cracks of plaster 0.40
  - for wooden inter-storey floors: weeping on floors 0.44
  - for internal stairs: weeping on stairs 0.46
  - for roof constructions: delamination of beams 0.35
  - for window joinery: mold and rot on windows 0.37
  - for inner plasters: scratches on plaster 0.36
  - for facades: scratches on plaster 0.37

The above values are therefore a fuzzy value of the probability of the degree of the technical wear, which was determined as an average degree, i.e., within the range of 35–50%—in the case of the occurrence of a fuzzy damage to the building element;

- this probability is characterized by a low standard deviation and a narrow range of the results of various elements within each of the considered building maintenance conditions—satisfactory (0.2622–0.3846), average (0.3510–0.4613) and poor (0.4995–0.6466). A similar remark concerns the mean value of this probability in relation to its probabilistic measure;

B. the conditional probability of a group of damage, which corresponds to the three middle states of maintenance of building elements, in relation to the process of their technical wear—\( P(U/Z) \) II, III, IV—is as follows:

- the conditional probability of damage to the element in relation to its technical wear increases with the deterioration of the building maintenance conditions;
- the probability of such a conditionally defined fuzzy event is indicated by the damage that most intensely affects the technical wear of the following elements of the tested residential buildings, and it amounts in their average maintenance condition \( P(UIII/Z) \) to:
  - for foundations: dampness of foundations 0.27
  - for basement walls: crack in bricks 0.55
  - for solid floors above basements: dampness of floors 0.46
  - for structural walls: cracks of plaster 0.45
  - for wooden inter-storey floors: weeping on floors 0.31
  - for internal stairs: weeping on stairs 0.58
  - for roof constructions: delamination of beams 0.66
  - for window joinery: mold and rot on windows 0.50
  - for inner plasters: scratches on plaster 0.46
for facades: scratches on plaster 0.36

The above values are therefore a fuzzy value of the probability of damage to a building element, but only in the case that its fuzzy technical wear is determined to be an average degree, i.e., within the range of 35–50%;

- the irregularity of this increase and the too-high coefficients of variation indicate only a partial identity of the fuzzy event defined within aspect B with the reverse event; the fuzzy event determined within aspect A is characterized by a much greater consistency of the obtained results.

4. Discussion and Conclusions

Quantitative damage analysis, which was carried out using empirical methods of assessing the technical condition of a building, indicates the type and size of damage to the building’s elements, which are characteristic of the appropriate maintenance conditions. Research concerning the cause–effect relationships ("damage-technical wear") in fuzzy calculus allowed for a numerical approach to the impact of building maintenance conditions on the degree of technical wear of its elements. The analysis of fuzzy cause–effect relationships ("damage-technical wear") created the possibility of determining conditional probabilities of these dependencies that are treated as fuzzy events. The fuzzy conditional probabilities of the technical wear process in relation to the occurrence of damage (with a probabilistic measure), as well as conditional probabilities of the occurrence of a group of damage in relation to the process of technical wear, were determined.

The research methodology has been prepared in such a way that allowed the previously prepared qualitative model to be transformed into a quantitative model. Therefore, the diagnosis of the impact of the maintenance of the residential buildings on the amount of their technical wear was executed using quantitative methods in fuzzy set categories, and also by using the authors’ own model that was created in the conditions of fuzziness. The model allowed for the determination of the conditional probabilities of the process of technical wear, and also the set of damage according to both Bayes formulas applied to fuzzy sets operations.

The research procedure was developed in a way that allowed for the transition of a previously prepared qualitative model into a quantitative model. The diagnosis of the impact of the maintenance of residential buildings on the amount of their technical wear was carried out using quantitative methods in the categories of fuzzy sets, and also by using the authors’ own model of determining the mutually dependent probabilities created in the conditions of fuzziness. The model enabled the conditional probabilities of the process of the technical wear, as well as the set of damage, to be determined according to probabilistic Bayes formulas. Moreover, it also allowed the fuzzy approach of Zadeh to be combined with the Yager approach. In such a multi-criteria fuzzy technical assessment of residential buildings, a simplified approach was used. In this approach, the probability of fuzzy events was assigned to a fuzzy measure, as opposed to the Yager approach, in which the probabilities are fuzzy events. The differences between the concepts of fuzziness and randomness were not considered in the study. It was assumed that these phenomena are different and described differently, however, they may—as two types of uncertainty—occur together.

The methods and results of the research presented in the article indicated a way that allows for the transition of the previously prepared qualitative model into a quantitative model. The diagnosis of the impact of the maintenance of residential buildings on the amount of their technical wear was carried out using quantitative methods in the categories of fuzzy sets, and also using the authors’ own models created in fuzzy conditions. The key question from the subjective expert assessment of the technical condition of the evaluated residential buildings was answered: what is the probability of the wear of an element that may be more or less represented by its average maintenance conditions? Therefore, the probability that the element is more or less worn was determined. It was proven
that the conditional probability of the technical wear of an element in relation to its failure increases with the deterioration of the maintenance conditions of the building, and this increase is extremely regular, even in the case of different building elements. This probability is characterized by a low standard deviation and a narrow range of the dispersion of the results in the case of various elements within each of the considered building maintenance conditions.

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