Superconductivity in a Spin Liquid - a One Dimensional Example

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Abstract

We study a one-dimensional model of interacting conduction electrons with a two-fold degenerate band away from half filling. The interaction includes an on-site Coulomb repulsion and Hund’s rule coupling. We show that such one-dimensional system has a divergent Cooper pair susceptibility at $T = 0$, provided the Coulomb interaction $U$ between electrons on the same orbital and the modulus of the Hund’s exchange integral $|J|$ are larger than the interorbital Coulomb interaction. It is remarkable that the superconductivity can be achieved for any sign of $J$. The opening of spectral gaps makes this state stable with respect to direct electron hopping between the orbitals. The scaling dimension of the superconducting order parameter is found to be between $1/4$ (small $U$) and $1/2$ (large $U$). Possible experimental realizations of such systems are discussed.

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1 Introduction

The idea that a doped spin liquid state favours superconductivity (SC) belongs to Anderson [1]. This idea was introduced in the context of quasi-two-dimensional copper oxide superconductors which are believed to be adequately described by the t-J model. However, it has proved very difficult to reliably analyse this model on a square lattice. Recently it has been suggested [2], [3] that SC might appear
in one dimensional doped spin liquids which are much easier to analyse\cite{4}. In this letter we further develop these ideas and give a more detailed description of the SC spin liquid state suggested in these papers.

We study the model of two Hubbard chains with the Hund’s coupling between the chains:

\[
H = H_{\text{Hub}} + \sum_r \left( \frac{u}{2} \rho_{1,r} \rho_{2,r} + 2 JS_{1,r} S_{2,r} \right)
\]

\[
H_{\text{Hub}} = -\frac{D}{2} \sum_{r,a,\alpha} \left( c_{a,\alpha,r+1}^+ c_{a,\alpha,r} + H.c. \right) + U \sum_{r,a} c_{r,a,\uparrow}^+ c_{r,a,\uparrow} c_{r,a,\downarrow}^+ c_{r,a,\downarrow}
\]

(1)

(2)

where \( c_{a,\alpha,r}^+ \) and \( c_{a,\alpha,r} \) are electron creation and annihilation operators on the site \( r \), \( \alpha = \pm 1/2 \) and \( a = 1, 2 \) are electron spin and chain (orbital) indices respectively, and

\[
\rho_a = \sum_\alpha c_{a,\alpha}^+ c_{a,\alpha}; \quad S = \frac{1}{2} \sum_{\alpha,\beta} c_{a,\alpha}^+ \sigma_{\alpha\beta} c_{a,\beta}
\]

(3)

where \( \sigma^a \) are the Pauli matrices. Electronic mechanisms give ferromagnetic exchange (Hund’s rule); an antiferromagnetic \( J \) can be generated by the Jahn-Teller effect (see the discussion later). Later we shall present the arguments that the model (1) may describe quasi-one-dimensional materials based on chains of \( C_{60} \) molecules.

The model (1) does not include a direct hopping between the orbitals because such terms are irrelevant when there are spectral gaps. We shall demonstrate that at \( U >> u \) such gaps exist for all excitations except for the symmetric charge density mode. Since a single electron hopping process inevitably involves modes with spectral gaps, the direct hopping becomes energetically costly and therefore irrelevant. It turns out that if the certain conditions on \( U, u \) and \( J \) are met, the ground state of the model (1,2) has strongly enhanced SC fluctuations. Due to the presence of spectral gaps in a single pair of chains (further we shall sometimes call such pair a “ladder”), the only excitations which can tunnel between the ladders are Cooper pairs. As we shall demonstrate later, the scaling dimension of the SC order parameter is small which makes the pair susceptibility more singular than in BCS superconductors and thus increases the critical temperature of a three dimensional system made of weakly interacting ladders.

Khveshchenko and Rice\cite{3} in their analysis of the model (1,2) have used bosonization in the weak coupling limit combined with the renormalization group approach (RG) (later this model has been also studied by Fujimoto and
Kawakami with the same qualitative results). We suggest an alternative approach which allows us to proceed directly to the strong coupling fixed point avoiding a cumbersome RG analysis of the weak coupling limit. The suggested reformulation of the Hamiltonian enables us to expand around the strong coupling point. The reason this representation is so useful is that it elucidates the inherent symmetries of the Hamiltonian. We demonstrate that in order to realize the mechanism of SC suggested in [2],[3], one needs to have a hierarchy of couplings where the on-chain repulsion must be the strongest one. Together with the interchain exchange interaction it creates spectral gaps for all excitations except for the symmetric charge mode. The interchain Coulomb repulsion $u$ has a potentially damaging effect on SC establishing a lower limit on the magnitude of $|J|$. It is intriguing that the conditions for SC can be met for both signs of $J$. The SC state is stable with respect to direct hopping if the hopping amplitude $t$ is much smaller than a sum of the gaps.

2 Bosonization and Introduction of Majorana Fermions.

As usual, interactions between electrons in one-dimension cannot be treated perturbatively. Hence we have to resort to non-perturbative methods, the most popular among which is bosonization. Since the Hubbard model is exactly solvable for any $U/D$ and doping, the bosonization approach requires only a smallness of the Hund’s rule couplings $u$ and $J$. The behaviour of the model (1) at half filling has been already well studied [6]. In this paper we assume a fair amount of doping.

We begin by bosonizing the charge and the spin density operators (3). According to Frahm and Korepin([7]) the continuous limit of these operators is given by (here we omit the orbital indices)

$$
\rho(x) = J_R + J_L + 2\cos(\sqrt{2\pi}\Phi_s) \left[ a_c e^{2ik_Fx} \exp(i\sqrt{2\pi}\Phi_c) + H.c. \right] \\
+ \left[ a_u e^{4ik_Fx} \exp(i\sqrt{8\pi}\Phi_c) + H.c. \right] \\
S(x) = J_R + J_L + \left[ a_s e^{2ik_Fx} \bar{\sigma}(x) + H.c. \right] \\
\sigma^3 = i \sin(\sqrt{2\pi}\Phi_s) e^{-i\sqrt{2\pi}\Phi_c}; \ \sigma^\pm = \exp[i\sqrt{2\pi}(-\Phi_c \pm \Theta_s)]
$$

(4) (5) (6)
where the corresponding bosonic fields are governed by the following Hamiltonian:

$$H_0 = \frac{1}{2} \int dx \left[ v_c K_c (\partial_x \Theta)^2 + \frac{v_c}{K_c} (\partial_x \Phi)^2 + v_s (\partial_x \Theta)^2 + v_s (\partial_x \Phi)^2 \right]$$  \hspace{1cm} (7)

where the operators satisfy the standard commutation relations:

$$[\Phi_a(x), \Theta_b(y)] = i \theta (x - y) \delta_{ab}$$  \hspace{1cm} (8)

and $a_c$, $a_u$ and $a_s$ are numbers which numerical values are known only in some limiting cases (we shall return to this issue later). In particular, $a_u = 0$ for a free electron gas. The charge and spin currents $J_R, L$ obey the corresponding Kac-Moody algebras (see, for example, [8]). The constant $K_c$ and charge and spin velocities $v_c, v_s$ depend on the interaction and doping such that $1/2 < K_c < 1$. The lowest limit is achieved for $U = \infty$ and $K_c = 1$ corresponds to the non-interacting case. The fact that the corresponding spin constant remains unrenormalized ($K_s = 1$) reflects the SU(2) symmetry of the individual Hubbard chain ([7]).

Substituting expressions (4, 5, 6) into Eq.(1) and keeping only non-oscillatory terms, we get the following expression for the interaction density:

$$V_{int} = V + \frac{u}{2} [J_{R1} + J_{L1}] [J_{R2} + J_{L2}] + 2J [J_{R1} + J_{L1}] [J_{R2} + J_{L2}]$$  \hspace{1cm} (9)

$$V = 2 \cos (\sqrt{4\pi} \Phi_c^+) |\mu| a_c|^2 (\cos \sqrt{4\pi} \Phi_s^+ + \cos \sqrt{4\pi} \Phi_s^-)$$

$$+ J |a_s|^2 (\cos \sqrt{4\pi} \Phi_s^+ - \cos \sqrt{4\pi} \Phi_s^- + 2 \cos \sqrt{4\pi} \Theta_s^-)$$  \hspace{1cm} (10)

The $4k_F$-scattering contributes the irrelevant operator $\cos 4\sqrt{4\pi} \Phi_c^-$ which we omit. Here we have introduced symmetric and antisymmetric combinations of the bosonic fields: $\Phi_{c,s}^\pm = (\Phi_{c,s}^1 \pm \Phi_{c,s}^2)/\sqrt{2}$ This transformation leaves the bosonic Hamiltonian (7) invariant.

The interaction of charge currents can be easily incorporated into the free Hamiltonian (7) by the change of $K_c$’s and velocities for the symmetric and antisymmetric charge modes:

$$K_c^\pm = \left[ 1 \pm \left( \frac{u}{2\pi v_c} \right)^{-1/2} \right] K_c$$  \hspace{1cm} (11)

We shall assume that $K_c^- < 1$, i.e. the interaction in the antisymmetric charge channel remains repulsive. Then the scaling dimension of the operator $V d_V = 1 + K_c^- < 2$ is always smaller than the dimension of the product of spin currents, which is a marginal operator. Therefore we omit the latter term. In the
subsequent analysis we will take advantage of the fact that all bosonic exponents in the square brackets in Eq.(10) have the scaling dimension 1 which means that they can be expressed as fermionic bilinears. The corresponding expressions in terms of Majorana (real) fermions have been derived in Ref.[6]:

\[(2\pi a_0)^{-1} \cos \sqrt{4\pi \Phi^+} = i(R_2 L_2 + R_3 L_3)(12)\]

where \(a_0\) is the short distance cut-off and the fermions satisfy the following anticommutation relations:

\[\{R^a(x), R^b(y)\} = \frac{1}{2} \delta(x-y) \delta_{ab}; \{L^a(x), L^b(y)\} = \frac{1}{2} \delta(x-y) \delta_{ab}, \{R^a(x), L^b(y)\} = 0(13)\]

Being recast in new terms the Hamiltonian (7) becomes

\[H_0 = \frac{1}{2} \sum_{\pm} \int dx \left[ v^\pm_c K^\pm_c (\partial_x \Theta^\pm_c)^2 + \frac{v^\pm_c}{K^\pm_c} (\partial_x \Phi^\pm_c)^2 \right] + i v_s \sum_{a=0}^3 \int dx \left( L^a \partial_x L^a - R^a \partial_x R^a \right) (14)\]

The interaction density acquires the following form:

\[V = i \cos \sqrt{4\pi \Phi^+} [m_1 (1 - \delta_{a,0}) R^a L^a - m_2 R^0 L^0] \]

\[m_1 = 4\pi a_0 (u|a_c|^2 + J|a_s|^2), \quad m_2 = 4\pi a_0 (3J|a_s|^2 - u|a_c|^2) (15)\]

If \(K_c^- < 1\) (that is \(u/2\pi v_c^- < 1 - K_c^2\), which requires the Coulomb repulsion \(U\) to be stronger than \(u!\) the interaction (15) is relevant making the Majorana fermions and the field \(\Phi^+_c\) massive. The modes \(a \neq 0\) and \(a = 0\) acquire different masses which we call \(M_t\) and \(M_0\) respectively with \(t\) standing for “triplet”. The triple degeneracy of three fermion modes reflects the fact that they realize the spin 1 representation of the SU(2) group [6]. The mass of the bosonic field \(\Phi^-_c\) is denoted \(\tilde{M}\). The dimensional analysis gives \(M_t, M_0, \tilde{M} \sim J^{1/(1-K_c^-)}\). One can safely assume that the following averages do not vanish:

\[\langle \cos \sqrt{4\pi \Phi^-_c} \rangle \neq 0; \langle R^a L^a \rangle \neq 0 (16)\]

In the limit \(K_c^- \ll 1\) this can be rigorously proven by the perturbation expansion in \(K_c^-\). Namely, one should rescale the field \(\Phi^-_c\): \(\Phi^-_c = \sqrt{K_c^-} \tilde{\Phi}^-_c\) and expand the cosine term in Eq.(15):\(\cos \sqrt{4\pi K_c^- \tilde{\Phi}^-_c} = 1 - 2\pi K_c^- (\tilde{\Phi}^-_c)^2 + ...\) Then in the first approximation in \(K_c\) one obtains the quadratic effective Hamiltonian (17). One
can check that corrections coming from higher terms are not singular and contain powers of the small parameter $K_c$. The appearance of the averages (16) breaks the discrete $Z_2$ symmetry which is not forbidden in $(1 + 1)$-dimensions. The corresponding order parameter is non-local in terms of the original fermionic fields[6].

3 Superconducting fluctuations.

As follows from the previous discussion, the Hamiltonian for the field $\Phi^+_c$ remains unaffected such that this field is still a free bosonic field. The other fields become massive and their strong coupling limit can be qualitatively described by the effective Hamiltonian:

$$H_{\text{eff}} = i \int dx v_s \sum_{a=0}^3 \left[ -R_a \partial_x R_a + L_a \partial_x L_a + (M_t(1 - \delta_{a,0}) - M_0 \delta_{a,0}) R_a L_a \right]$$

$$+ \frac{1}{2} \int dx \left[ v_c (-\partial_x \Theta_c^-) - \partial_c (-\partial_x \Phi_c^-) \right]^2 + \bar{M} (\Phi_c^-)^2 \right] \quad (17)$$

As we have mentioned above this Hamiltonian becomes exact in the limit $K_c \rightarrow 0$.

We have deliberately put a minus sign at $M_0$ in Eq.(17). We shall see that the correlation length of the SC order parameter is infinite only provided $M_t$ and $M_0$ in Eq.(17) have the same sign. As follows from Eq.(15), the masses are

$$M_t \sim (u|a_c|^2 + J|a_s|^2) \cos \sqrt{2\pi \Phi_c^-}; \quad M_0 \sim (3J|a_s|^2 - u|a_c|^2) \cos \sqrt{2\pi \Phi_c^-} \quad (18)$$

Combining this criterion with $K_c^- < 1$ we obtain the following criterion for the existence of an infinite correlation length for SC fluctuations in the model (1,2):

$$K_c^- < 1; \quad (u|a_c|^2/|a_s|^2 + J)(3J - u|a_c|^2/|a_s|^2) > 0 \quad (19)$$

The coefficients $a_c, a_s$ are known only in the weak coupling limit $U/D << 1$ where $a_c = a_s$. It is also likely that $|a_s| >> |a_c|$ close to half filling where charge fluctuations are suppressed. Thus we can resolve the inequalities (19) explicitly only in the limit of weak interactions where $K_c \approx 1 - U/4\pi v_c$:

$$U > u; \quad J > u/3 \text{ or } J < -u \quad (20)$$

Let $\psi_{R,L}$ be right- and left moving components of the original fermions. Now we shall demonstrate that if the criterion (19) is met, the susceptibility of the SC order parameter defined as

$$\Delta = \psi_{R,1,\uparrow} \psi_{L,2,\downarrow} \pm \psi_{R,2,\uparrow} \psi_{L,1,\downarrow}$$

$$\sim \exp i\sqrt{\pi} (\Phi_s^+ + \Theta_c^+) \{ \exp [i\sqrt{\pi} (\Theta_s^- + \Phi_c^-)] \pm \exp [-i\sqrt{\pi} (\Theta_s^- + \Phi_c^-)] \} \quad (21)$$
is singular. The choice of sign in the above expression depends on the sign of $\langle \cos \sqrt{4\pi} \Phi_c^- \rangle$ (broken $Z_2$ symmetry). The order parameter includes exponents of the fields $\Phi_s^+$ and $\Theta_s^-$ with scaling dimensions $1/4$. In the Majorana approach it is most convenient to express these fields in terms of the order ($\sigma$) and disorder ($\mu$) parameter fields of the Ising model using the fact that the model of massive Majorana fermions describes a two dimensional Ising model off the critical point, where the fermionic mass is related to the deviation from $T_c$: $M \sim (T - T_c)$. The corresponding operators are related as follows:

$$\exp(i\sqrt{\pi} \Phi_s^+) \sim \mu_1 \mu_2 + i\sigma_1 \sigma_2; \exp(-i\sqrt{\pi} \Theta_s^-) \sim \sigma_3 \mu_0 - i\sigma_0 \mu_3$$

Substituting these expressions into Eq.(21) we get

$$\Delta^+ = \exp(i\sqrt{\pi} \Theta_c^+) \cos \sqrt{\pi} \Phi_c^- (\mu_1 \mu_2 + i\sigma_1 \sigma_2) \begin{cases} \sigma_3 \mu_0 \\ -i\sigma_0 \mu_3 \end{cases} \approx \lambda : e^{i\sqrt{\pi} \Theta_c^+}$$

where depending on the sign of $\langle \cos \sqrt{4\pi} \Phi_c^- \rangle$

$$\lambda = \langle \cos \sqrt{4\pi} \Phi_c^- (\sigma_1 \sigma_2 \sigma_3 \mu_0) \rangle$$

In the expression we have omitted the term proportional to $\sin \sqrt{\pi} \Phi_c^-$ since the sinus has a zero average. As is well known, one of the operators $\sigma$ and $\mu$ has a nonzero average off the critical point. Which one does not vanish depends on the sign of $(T - T_c)$, that on the sign of mass in the Majorana representation. In order to have $\lambda \neq 0$, the mass term of the zeroeth Majorana mode should have a sign opposite to the mass term of the other modes which gives as criterion (19).

It is easy to see that if $M_0 M_t < 0$ the order parameter describes the charge density wave:

$$O_{CDW}(x) = \sum_\alpha (\psi_{R,1\alpha}^+ \psi_{L,1\alpha} \pm \psi_{R,2\alpha}^+ \psi_{L,2\alpha})$$

As follows from Eq.(23), at $T = 0$ the correlation function of Cooper pairs has the following asymptotic behaviour at distances $>> M$ ($M$ is the smallest gap):

$$\langle \Delta(x, \tau) \Delta^+(0, 0) \rangle \sim \frac{1}{(M|\tau + \frac{i\pi}{\nu_c}|)^{1/2K_c^+}}$$

with $K_c^+$ defined by Eq.(11). Since $K_c$ varies from 1 at $U = 0$ to $\frac{1}{2}$ at $U = \infty$, the scaling dimension of the order parameter $1/4 < d = \frac{1}{4K_c^+} < 1/2$ and the Fourier transformation of the correlation function (pairing susceptibility) always diverges at $\omega, q = 0$. At finite temperatures there will be finite number of kinks interpolating between vacua with $\langle \cos \sqrt{4\pi} \Phi_c \rangle > 0$ and $< 0$. Thus the $Z_2$ symmetry is
restored with the finite correlation length $\xi \sim \exp(M_k/T)$, where $M_k \sim M_0/K_c$ is the kink’s mass. This exponentially large correlation length will be completely overshadowed by the correlation length of the $\Phi_c^+$-field: $\xi_c \sim 1/T$.

4 Physical Realization

One possible group of experimental systems where the described superconducting mechanism may work are materials based on chains of $C_{60}$ molecules. An isolated $C_{60}$ molecule in a crystal has a rich degeneracy and the Hund’s interaction may be generated by local vibronic modes (the Dynamical Jahn-Teller effect; see Ref. [10] for a review). In a structure where $C_{60}$ molecules are situated on well separated chains a two-fold local degeneracy may survive even in the presence of an intermolecular hopping. Then the one-dimensional electronic band is split into a single degenerate band created by the $s, p_z$ orbitals and a doubly degenerate band created by $p_x, p_y$ orbitals. The measurements performed for RbC$_{60}$ give many indications of a quasi-one-dimensional behaviour [11]. The direct band structure calculations done for $A_nC_{60}$-compounds with $A = K, Rb$ show, however, that the resulting electronic bands have a three dimensional character [12]. Nevertheless, even if these calculations are correct in the given case, one may expect that it would not be too difficult to produce truly one-dimensional $C_{60}$-based materials. The authors of Ref. [12] remark that the interchain hopping is very sensitive to the distance between the chains and therefore the one-dimensional limit “might well be achieved by doping with substantially larger species”. These authors further suggest creating chemical compounds of $C_{60}$ with “spacer molecules” such as NH$_3$.

The model describing a one-dimensional chain of Jahn-Teller molecules with electrons belonging to two degenerate local orbitals hopping along the chain and interacting with local optical (vibronic) modes of the molecules was introduced by Manini et al. [13]. The Hamiltonian of electron-phonon interaction is given by

$$H = \sum_{r,\alpha=x,y} \left[ g^{c^+}_{r,\alpha} \sigma^{n}_{\alpha} c_{r,\beta} \sigma^{n}_{\alpha} u_n(r) - \frac{1}{2M} \frac{\partial^2}{\partial u_n(r)^2} + \frac{k u_n(r)^2}{2} \right]$$ (27)

where $u^x(r), u^y(r)$ describe local vibronic modes. A single $C_{60}$ molecule has a degeneracy higher than 2, but it is assumed that such high degeneracy does not survive in a crystal. Neglecting all retardation effects related to the kinetic energy of vibrons and integrating over the vibronic modes we get the interaction [11] with
$u = J = 2g^2/k$. As in the standard BCS theory, the retardation effects will give us a high energy cut-off of the order of the vibronic frequency $\omega_0 = (k/M)^{1/2}$. Santoro et al.\cite{14} have studied this interaction in the approximation where the Jahn-Teller bound states were considered first, the hopping was treated as a perturbation and the Coulomb interaction was ignored. Thus the dynamical nature of the vibronic modes appeared to be an essential feature of this approach. As a result they got the scaling dimension of the SC order parameter equal to $1/4$ in agreement with our calculations. The fact that the two approaches give the same infrared behaviour is remarkable, but is by no means unusual. This is a standard feature of one-dimensional systems where one almost always reproduces strong coupling results starting from weak coupling. This is due to the fact that the strong coupling appears eventually in the renormalization process.

5 Conclusions

A curious property of our results is that SC requires a strong Coulomb repulsion between electrons on the same orbital. It is this repulsion which leads to formation of the spin gap with subsequent freezing out some electronic degrees of freedom. There is also a charge gap corresponding to antiphase fluctuations of the orbital charge densities. In the situation where the exchange interaction is generated by the Jahn-Teller mechanism, this gap coincides with the spectral gap of vibrons at $q = 2k_F$.

The Coulomb interaction, however, plays a double role since the scaling dimension of the order parameter increases when the Coulomb repulsion grows becoming equal to $1/2$ at $U = \infty$. The small scaling dimension of the SC order parameter increases the probability of having a relatively large transition temperature in a quasi-three-dimensional system built of weakly interacting chains. If $t$ is the interchain Josephson coupling, then the dimensional analysis gives for the temperature of SC transition the estimate

$$T_C \sim M_0(t/M_0)^\eta; \quad \eta = \frac{1}{2 - 2d}$$

(28)

where $M_0 \sim g^{1+1/(1-K_-)}$ is the spin singlet gap. In the conventional case $d \approx 1$ and the transition temperature is exponentially small in $M_0/t$, but for the present model $1/4 < d < 1/2$ and $2/3 < \eta < 1$.

We conclude this latter with a brief description of the physical properties expected for a system described by model (1,2). First, the magnetic excitations are
spin triplets and spin singlets, as at half filling \[3\]. As for a general \(S = 1\) magnet, the triplet excitations have a gap (the Haldane gap) which we call \(M_t\). There is a singlet gap \(M_0\) as well; the corresponding excitations are spin and charge singlets and can be treated as RVB (resonance valence bond) excitations. Remaining gapless mode is the charge free boson which gives a dramatic enhancement to SC fluctuations or, if the criterion \(M_t M_0 > 0\) is not met, to CDW. We expect a real SC transition in the quasi-three-dimensional systems.

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References

[1] P. W. Anderson, Science 235, 1196 (1987).

[2] M. Sigrist, T. M. Rice and F. C. Zhang, preprint ETH-TH/93-35; T. M. Rice et al. in Proc. 16th Tanigushi Symposium, ed. by A. Okiji and N. Kawakami, Springer-Verlag, (1994).

[3] V. Khveshchenko and T. M. Rice, Phys. Rev. B50, 252 (1994); see also references [3],[5] in this paper.

[4] Here and thereafter we define one dimensional SC as a divergence in the pairing susceptibility at \(T = 0\).

[5] S. Fujimoto and N. Kawakami, unpublished.

[6] D. G. Shelton, A. A. Nersesyan and A. M. Tsvelik, sissa preprint, cond-mat/9508047.

[7] H. Frahm and V. Korepin, Phys. Rev. B 42, 10553 (1990).

[8] C. Itzykson and J.-M. Drouffe, Statistical Field Theory, v. 2, Chapter 9C, ed. by Cambridge University Press, 1989.

[9] M. Sato, T. Miwa and M. Jimbo, Holonomic Quantum Fields V, published by RIMS, Kyoto Univ., 16, 531 (1980).
[10] M. C. M. O’Brien and C. C. Chancey, Am. J. Phys. 61, 688 (1993).

[11] O. Chauvet, G. Oszlanyi, L. Forro, P. W. Stephens, M. Tegze, G. Faigel and A. Janossy, Phys. Rev. Lett. 72, 2721 (1994).

[12] S. C. Erwin, G. V. Krishna and E. J. Mele, Phys. Rev. B51, 7345 (1995).

[13] N. Manini, E. Tosatti and S. Doniach, Phys. Rev. B51, 3731 (1995).

[14] G. Santoro, M. Airoldi, E. Tosatti and A. Parola, sissa preprint (1994).