Multiquark states and QCD sum rules

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Abstract

There have been arguments about hadronic molecules, which are weakly-bound states of two or more hadrons. We investigate the possibility of some candidates (f^0(980), a_0(980), f_0(1500), f_0(1710), etc.) using QCD sum rule approach and compare our results with multiquark states in the MIT bag model. We find that f_0(1500), f_0(1710) can be good candidates for vector-vector molecule-type multiquark states.

1 Introduction

One can classify exotics by the number of quarks plus antiquarks they contain; i.e., glueballs, hybrid mesons, hybrid baryons, four quark states, and so on. Four quark states have two quarks and two antiquarks. A special case which we discuss below are hadronic molecules [(q̅q)(q̅q)].

The first serious estimation of the interaction between quarks in the four quark system was done within the MIT bag model by Jaffe[2]. He found that the scalar states O^{++} have lowest mass, and interpreted the f_0 (980) and a_0 (980) as four quark states. Table 1 shows several states of scalar mesons and their masses which are discussed in our study. For the details, see Ref.[2]. The bag model results were confirmed in a Nonrelativistic Quark Model calculation by Weinstein and Isgur[3]. These authors found that the predominant component in the f_0 and the a_0 wave functions was K̅K, as a mesonic molecule. Very interesting proposals were made separately by Törnqvist[4] and by Dooley et al.[5], in suggesting the existence of vector meson molecules. There are several signatures for hadronic molecules[1].

In this paper we investigate the possibility of hadronic molecules for several candidates using QCD sum rule approach[7, 8]. They are f_0(980), a_0(980), f_0(1500), and f_0(1710) are candidates of K̅K molecule, and f_0(1500) is a ρρ or ρω molecular state. f_0(1710) is a K^* K̅ or K^* K̅ + ωφ molecular state. Of course there are another interpretations on f_0(1500) and f_0(1710)[9, 10, 11]; i.e. glueball states or mixed states of a s̅s meson and a digluonium, etc. We predict masses of these particles with appropriate interpolating fields for molecular states (e.g., “molecular-like” interpolating field (q̅Γq)(q̅Γq)). In addition to this, we assume that the four quark states in Jaffe’s notations as hadronic molecules and then calculate their masses, and compare our results with Jaffe’s.

2 K̅K Molecule

Let’s consider the following correlator:

\[ \Pi(q^2) = i \int d^4x e^{iqx} \langle T(J(x)J^\dagger(0)) \rangle, \]

where \( J(x) = (\bar{u}(x)i\gamma^5s(x))(\bar{s}(x)i\gamma^5u(x)) + (\bar{d}(x)i\gamma^5s(x))(\bar{s}(x)i\gamma^5d(x)) \) corresponds to the multiquark interpolating field for f_0(980) state. This is the K^0 K̅^0 + K^+ K̅^- state (isospin I=0). Then, in the OPE side we

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1For a recent review of the hadronic molecules, see Ref.[3].
2The spin state of f_0(1710) is not clarified at present[9].

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Table 1: The predicted $Q^2\bar{Q}^2$ $0^+$ mesons. Masses are quoted to the nearest 50 MeV.

| SU(3) multiplet | State | Mass (MeV) |
|-----------------|-------|------------|
| 9               | $C^0(9)$ | $\sqrt{3}\pi\pi + \frac{1}{2}\eta_0\eta_0$ | 650 |
|                 | $C^*(9)$ | $\frac{1}{\sqrt{2}KK} + \frac{1}{\sqrt{2}}\eta_0\eta_s$ | 1100 |
|                 | $C^\pm(9)$ | $-\frac{1}{\sqrt{2}}KK - \frac{1}{\sqrt{2}}\eta_s\pi$ | 1100 |
| 36              | $C^0(36)$ | $-\frac{1}{2}\pi\pi + \frac{\sqrt{3}}{2}\eta_0\eta_0$ | 1150 |
|                 | $C^*(36)$ | $\frac{1}{\sqrt{2}}KK - \frac{1}{\sqrt{2}}\eta_0\eta_s$ | 1550 |
|                 | $C^\pm(36)$ | $\frac{1}{\sqrt{2}}KK - \frac{1}{\sqrt{2}}\eta_s\pi$ | 1550 |
| 9*              | $C^0(9^*)$ | $\frac{\sqrt{3}}{2}\rho\rho + \frac{1}{2}\omega\omega$ | 1450 |
|                 | $C^*(9^*)$ | $\frac{1}{\sqrt{2}}K^*K^* + \frac{1}{\sqrt{2}}\omega\phi$ | 1800 |
|                 | $C^\pm(9^*)$ | $-\frac{1}{\sqrt{2}}K^*K^* - \frac{1}{\sqrt{2}}\omega\phi$ | 1800 |
| 36*             | $C^0(36^*)$ | $-\frac{1}{2}\rho\rho + \frac{\sqrt{3}}{2}\omega\omega$ | 1800 |
|                 | $C^*(36^*)$ | $\frac{1}{\sqrt{2}}K^*K^* - \frac{1}{\sqrt{2}}\omega\phi$ | 2100 |
|                 | $C^\pm(36^*)$ | $\frac{1}{\sqrt{2}}K^*K^* - \frac{1}{\sqrt{2}}\omega\phi$ | 2100 |

get

$$\Pi_{\text{OPE}}(q^2) = -\frac{1}{\pi^2}q^8\ln(-q^2) + \frac{m_s^2}{\pi^2}q^6\ln(-q^2)$$

$$+ \frac{m_s^2}{\pi^2}2(\bar{q}q) - (\bar{s}s)q^4\ln(-q^2) - \frac{1}{\pi^2}2\overline{3}(24(\bar{q}q)(\bar{s}s) + 2(\bar{q}q)^2 + (\bar{s}s)^2)q^2\ln(-q^2)$$

$$- \frac{m_s^2}{\pi^2}2(\overline{2}\bar{q}q)^2 - 8(\bar{q}q)(\bar{s}s) + (\bar{s}s)^2)\ln(-q^2) + \frac{m_s^2}{\pi^2}3(6(\bar{q}q)^2 (\bar{s}s) - 4(\bar{q}q)(\bar{s}s)^2)\frac{1}{q^2}$$

$$+ \frac{\pi\alpha_s}{3}\frac{1}{q^4}102(\bar{q}q)^2(\bar{s}s)^2 - 16(\bar{q}q)^3(\bar{s}s) - 20(\bar{q}q)(\bar{s}s)^3)\frac{1}{q^4},$$

(2)

where $m_s$ is a strange quark mass. On the other hand, for $a_0(980)$ we take the interpolating field as $K^0\bar{K}^0 - K^+K^-$ state (isospin $I=1$). Then, we have

$$\Pi_{\text{OPE}}(q^2) = -\frac{1}{\pi^2}q^8\ln(-q^2) + \frac{m_s^2}{\pi^2}q^6\ln(-q^2)$$

$$+ \frac{m_s^2}{\pi^2}2(\bar{q}q) - (\bar{s}s)q^4\ln(-q^2) - \frac{1}{\pi^2}2\overline{3}(24(\bar{q}q)(\bar{s}s) + (\bar{s}s)^2)q^2\ln(-q^2)$$

$$- \frac{m_s^2}{\pi^2}2(\overline{2}\bar{q}q)^2 - 8(\bar{q}q)(\bar{s}s) + (\bar{s}s)^2)\ln(-q^2) + \frac{m_s^2}{\pi^2}3(6(\bar{q}q)^2 (\bar{s}s) - 4(\bar{q}q)(\bar{s}s)^2)\frac{1}{q^2}$$

$$+ \frac{\pi\alpha_s}{3}\frac{1}{q^4}178(\bar{q}q)^2(\bar{s}s)^2 - 24(\bar{q}q)^3(\bar{s}s) - 20(\bar{q}q)(\bar{s}s)^3)\frac{1}{q^4},$$

(3)

In the above calculations we use two diagrams: Fig. 1 and Fig. 2. Here, we neglect the contribution of gluon condensates and concentrate on tree diagrams. We assume the vacuum saturation hypothesis to calculate quark condensates of higher dimensions. Similar calculation is found in Kodama et al.’s H-dibaryon sum rule [12].

In Eqs.(2), (3) above the OPE sides have the following form:

$$\Pi_{\text{OPE}}(q^2) = a q^8\ln(-q^2) + b q^6\ln(-q^2) + c q^4\ln(-q^2) + d q^2\ln(-q^2)$$

$$+ e \ln(-q^2) + f \frac{1}{q^2} + g \frac{1}{q^4},$$

(4)

where $a, b, c, \cdots, g$ are constants. Then we parameterize the phenomenological side as

$$\frac{1}{\pi} Im\Pi_{\text{phen}}(s) = \lambda^2 \delta(m^2 - s) + [-a s^4 - b s^3 - c s^2 - d s - e\theta(s - s_0),$$

(5)
\begin{align*}
\text{Figure 1: Diagrams of 2 loop-type. Solid lines are the quark propagators and curly line represents the gluon propagator. Dots denotes the quark condensates and cross represents the mass correction from the strange quark.}
\end{align*}

where \( s_0 \) is a continuum threshold. After Borel transformation we obtain a mass of \( f_0 \) and \( a_0 \) respectively. The mass \( m \) is given by

\begin{align*}
m^2 &= M^2 \times \\
&\{ -120a[1 - e^{-s_0/M^2}(1 + s_0/M^2 + s_0^2/2M^4 + s_0^3/6M^6 + s_0^4/24M^8 + s_0^5/(120M^{10})] \\
&- 24b/M^2[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4 + s_0^3/6M^6 + s_0^4/24M^8)] \\
&- 6c/M^4[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4 + s_0^3/6M^6)] \\
&- 2d/M^6[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4)] \\
&- e/M^8[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4)] - g/M^{12} \} / \\
&\{ -24a[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4 + s_0^3/6M^6 + s_0^4/24M^8)] \\
&- 6b/M^2[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4 + s_0^3/6M^6)] \\
&- 2c/M^4[1 - e^{-s_0/M^2}(1 + s_0^2/2M^4)] \\
&- d/M^6[1 - e^{-s_0/M^2}(1 + s_0^2/M^2)] \}
\end{align*}
where we take $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$, $\langle \bar{s}s \rangle = 0.8\langle \bar{q}q \rangle$, $\alpha_s = 0.5$, and $m_s = 0.150 \text{ GeV}$ throughout this paper.

The continuum contribution is large, so this formula has large uncertainties. We cannot find a plateau for the mass of $f_0$ and $a_0$. Thus, we have to change our strategy. Let’s consider Fig. 1 and Fig. 2 again. The diagrams in Fig. 1 (hereafter we call it 2 loop-type) are proportional to $N_c^2$ (where $N_c$ is a number of color) and the diagrams in Fig. 2 (hereafter 1 loop-type) $N_c$. Hence, the 1 loop-type diagrams are $1/N_c$ corrections to the 2 loop-type diagrams. It means that the OPE side can be written as

$$\Pi_{\text{OPE}}(q^2) = N_c^2(1 \times 2\text{ loop – type} + \frac{1}{N_c} \times 1\text{ loop – type}).$$

According to Witten’s arguments on large $N_c$ dynamics [13], there are no exotics in the leading order of $N_c$. 2 loop-type corresponds to the leading order, and it means that the two kaons are flying without any interaction among themselves. Therefore, our new strategy are as follows: First, consider 2 loop-type only and vary the continuum threshold $s_0$ and Borel interval $M^2$ in order that the mass should be 990 MeV (the sum of two free kaon masses). The Borel interval $M^2$ is restricted by the following conditions as usual: OPE convergence and pole dominance. Second, consider all diagrams (2 loop-type + 1 loop-type) and get a new mass $m'$ with the same $s_0$ and Borel interval $M^2$ which are obtained from the first step. Third, compare $m'$ with 990 MeV. If $m'$ is less than 990 MeV, it can be one signature for molecular-like multiquark states.

Our results are in Table 2. In the Table, one can see that the masses of $f_0$ and $a_0$ are greater than the two kaons’ mass (990 MeV). Even the mass of $K^+K^+$ state is less than 990 MeV. It is worthy to note that our results do not change even though we take another value from the case of “$N_c \to \infty$”. The masses of $f_0$ and $a_0$ are always greater than the threshold, and $K^+K^+$ state is always lower than the threshold.

### 3 Vector-Vector Molecule

In this section we consider vector-vector molecules, such as $\rho\rho$ and $K^*\bar{K}^*$ molecules. For the $\rho\rho$ we take $\rho^+\rho^- + \rho^0\rho^0$ state($f_0(1500)$). There are two types in $K^*\bar{K}^*$ molecules: isospin I=0 and I=1 states. These are...
Table 2: $K\bar{K}$ molecule

| Mass Formulation | $s_0$ (GeV$^2$) | $M^2$ (GeV$^2$) | Mass (GeV) |
|------------------|-----------------|-----------------|------------|
| $K\bar{K}$       | $N_c \to \infty$ | 2.06            | 0.81 – 0.98 | 0.990     |
| $K^0\bar{K}^0 + K^+K^-$ | 1.031            |                 |            |
| $K^0\bar{K}^0 - K^+K^-$ | 1.000            |                 |            |
| $K^+K^-$          |                 | 0.968           |            |

Table 3: $\rho\rho$ and $K^*\bar{K}^*$ molecule

| Mass Formulation | $s_0$ (GeV$^2$) | $M^2$ (GeV$^2$) | Mass (GeV) |
|------------------|-----------------|-----------------|------------|
| $\rho\rho$       | $N_c \to \infty$ | 4.12            | 1.20 – 2.00 | 1.540     |
| $\rho^0\rho^0 + \rho^+\rho^-$ | 1.499            |                 |            |
| $\rho^+\rho^+$   |                 | 1.542           |            |
| $K^*\bar{K}^*$   | $N_c \to \infty$ | 5.36            | 1.25 – 2.85 | 1.784     |
| $K^0\bar{K}^0 + K^{*+}K^{*-}$ | 1.748            |                 |            |
| $K^0\bar{K}^0 - K^{*+}K^{*-}$ | 1.753            |                 |            |
| $K^{*+}K^{*+}$   |                 | 1.791           |            |

$K^{*0}\bar{K}^{*0} + K^{*+}\bar{K}^{*-}$ (I=0, i.e., $f_0(1710)$) and $K^{*0}\bar{K}^{*0} - K^{*+}\bar{K}^{*-}$ (I=1). We use the same procedure as in the case of $K\bar{K}$ molecule. The results are in Table 3.

We see that there is a binding and $f_0(1500)$ is a good candidate of vector-vector molecular-like multiquark state. For the case of $K^*\bar{K}^*$, two states (I=0 and I=1) are lower than the threshold 1.784 GeV. In this case $f_0(1710)$ is also a good candidate of vector-vector molecular-like multiquark state. In addition, as can be seen in the Table, I=1 state ($K^{*0}\bar{K}^{*0} - K^{*+}\bar{K}^{*-}$) can be another good candidate of vector-vector molecular-like multiquark state.

4 Discussion

In Table 2 and Table 3 we compare our results with that of Jaffe. Detail calculations are given in Ref. [4]. First, consider Table 2. We present two cases, $K\bar{K}$ and $\pi\pi$ molecular states. In the case of $\pi\pi$ state when $N_c \to \infty$, we can not set the mass to that of the sum of two pion masses ($\sim$ 280 MeV). So we take the same $s_0$ and $M^2$ as those from the case of $K\bar{K}$, and compare a magnitude; i.e. whether the mass is below or above the threshold. Our results are much different from those of Jaffe. We can not predict mass splittings as in the case of bag model. Next, move on to Table 3. The result of two cases ($K^*\bar{K}^*$ and $\rho\rho$ molecular states) are presented. We can not obtain the mass splitting as that from the bag model. However, the masses of $1/\sqrt{2}K^{*0}\bar{K}^{*+}$ and $1/\sqrt{2}\omega$ ($f_0(1710)$) and $1/\sqrt{2}\rho^0 + 1/\sqrt{2}\omega$ ($f_0(1500)$) are lower than their respective threshold values. Thus, we can think these as vector-vector molecular-like multiquark state. Besides, $-1/\sqrt{2}K^{*+}\bar{K}^{*-} - 1/\sqrt{2}\rho\phi$ state may be another candidate of vector-vector molecular-like multiquark state which was proposed in [3].

In Table 3 we present a result for the case of $\Lambda(1405)$. It has long been considered a candidate of $\bar{K}N$ bound state [12, 10], since it is just below the $\bar{K}N$ threshold. We assume this a $\bar{K}N$ molecular state, and investigate the possibility of hadronic molecule using the previous approach. Because of its dimension the OPE side has two structures:

$$\Pi_{DPE}(q^2) = \Pi_1(q^2) + \Pi_\omega(q^2)$$

We can obtain a mass from $\Pi_1(q^2)$ and $\Pi_\omega(q^2)$ respectively. In the Table a result from $\Pi_\omega(q^2)$ is given. In this case there is a binding also. Note that this is a tentative result.

3We also checked that $f_0(1500)$ with the other normalization, i.e. $1/\sqrt{2}\rho\rho + 1/\sqrt{2}\omega$, has a lower mass than the threshold.
In summary, we showed that $f_0(1500)$, $f_0(1710)$ are good candidates for vector-vector molecular-like multiquark state. As a possible modification to the cases of $f_0(980)$, $a_0(980)$ we can consider the direct instanton effect [17] in our calculation. However, our work is still insufficient to predict whether it is a molecular state or a molecule-type multiquark state or even an ordinary quark-antiquark state with large virtual quark-antiquark pair. So we have to calculate another quantity, such as $\gamma\gamma$ decay widths of these scalar mesons, and compare with the experiments. Recently, Close et al. [18] suggested a related test for the $f_0(980)$ and $a_0(980)$ involving the radiative decays $\phi \to \gamma(f_0, a_0)$, which may be possible at DAΦNE and CEBAF. Our methods may be applied to hadron-hadron scattering because the study of molecules is a subtopic of the problem of determining 2 → 2 hadron-hadron scattering amplitudes near threshold [1].

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Table 6: Λ(1405) (tentative)

| $s_0$ (GeV$^2$) | $M^2$ (GeV$^2$) | mass (GeV) |
|-----------------|-----------------|------------|
| $\sim \frac{\pi}{q N_c \to \infty}$ | 3.553 | 1.409 |
| 1.4 – 2.0 | | 1.335 |

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