Double Beta Decay and the Absolute Neutrino Mass Scale

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Abstract. After a short review of the current status of three-neutrino mixing, the implications for the values of neutrino masses are discussed. The bounds on the absolute scale of neutrino masses from Tritium $\beta$-decay and cosmological data are reviewed. Finally, we discuss the implications of three-neutrino mixing for neutrinoless double-$\beta$ decay.

The marvelous recent results of neutrino oscillation experiments have given us important information on neutrino mixing. The data of solar neutrino experiments and of the KamLAND long-baseline $\bar{\nu}_e$ disappearance experiment show $\nu_e \rightarrow \nu_\mu, \nu_\tau$ transitions generated by the squared-mass difference $\Delta m^2_{\text{SUN}}$ in one of the two ranges [1]

$$5.1 \times 10^{-5} < \Delta m^2_{\text{SUN}} < 9.7 \times 10^{-5} \text{ (LMA-I)}, \quad 1.2 \times 10^{-4} < \Delta m^2_{\text{SUN}} < 1.9 \times 10^{-4} \text{ (LMA-II)},$$

at 99.73% C.L., with best-fit value $\Delta m^2_{\text{SUN}} \simeq 6.9 \times 10^{-5}$ (we measure squared-mass differences in units of eV$^2$). The effective solar mixing angle $\vartheta_{\text{SUN}}$ is constrained at 99.73% C.L. in the interval [1]

$$0.29 < \tan^2 \vartheta_{\text{SUN}} < 0.86,$$

with best-fit value $\tan^2 \vartheta_{\text{SUN}}^{\text{bf}} \simeq 0.46$. The results of atmospheric neutrino experiments and of the K2K long-baseline $\nu_\mu$ disappearance experiment indicate $\nu_\mu \rightarrow \nu_\tau$ transitions generated by the squared-mass difference $\Delta m^2_{\text{ATM}}$ in the 99.73% C.L. range [2]

$$1.4 \times 10^{-3} < \Delta m^2_{\text{ATM}} < 5.1 \times 10^{-3},$$

with best-fit value $\Delta m^2_{\text{ATM}}^{\text{bf}} \simeq 2.6 \times 10^{-3}$. The best-fit effective atmospheric mixing angle $\vartheta_{\text{ATM}}$ is maximal, $\sin^2 2\vartheta_{\text{ATM}}^{\text{bf}} \simeq 1$, with the 99.73% C.L. lower bound [2]

$$\sin^2 2\vartheta_{\text{ATM}} > 0.86.$$

These evidences of neutrino mixing are nicely accommodated in the minimal framework of three-neutrino mixing, in which the three flavor neutrinos $\nu_e, \nu_\mu, \nu_\tau$ are unitary.

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linear combinations of three neutrinos $\nu_1$, $\nu_2$, $\nu_3$ with masses $m_1$, $m_2$, $m_3$, respectively (see Ref. [3]). Figure 1 shows the two three-neutrino schemes allowed by the observed hierarchy $\Delta m_{\text{SUN}}^2 \ll \Delta m_{\text{ATM}}^2$, with the massive neutrinos labeled in order to have

$$
\Delta m_{\text{SUN}}^2 = \Delta m_{21}^2, \quad \Delta m_{\text{ATM}}^2 \simeq |\Delta m_{31}^2| \simeq |\Delta m_{32}^2|.
$$

(5)

The two schemes in Fig. 1 are usually called “normal” and “inverted”, because in the normal scheme the smallest squared-mass difference is generated by the two lightest neutrinos and a natural neutrino mass hierarchy can be realized if $m_1 \ll m_2$, whereas in the inverted scheme the smallest squared-mass difference is generated by the two heaviest neutrinos, which are almost degenerate for any value of the lightest neutrino mass $m_3$.

Solar neutrino oscillations depend only on the first row $U_{e1}, U_{e2}, U_{e3}$ of the mixing matrix, and the hierarchy $\Delta m_{\text{SUN}}^2 \ll \Delta m_{\text{ATM}}^2$ implies that neutrino oscillations generated by $\Delta m_{\text{ATM}}^2$ depend only on the last column $U_{e3}, U_{\mu 3}, U_{\tau 3}$ of the mixing matrix. Hence, the only connection between solar and atmospheric neutrino oscillations is due to the element $U_{e3}$. The negative result of the CHOOZ long-baseline $\bar{\nu}_e$ disappearance experiment implies that electron neutrinos do not oscillate at the atmospheric scale and $|U_{e3}|$ is small: $|U_{e3}|^2 < 5 \times 10^{-2}$ at 99.73% C.L. [4]. Therefore, solar and atmospheric neutrino oscillations are practically decoupled [5] and the effective mixing angles in solar, atmospheric and CHOOZ experiments can be related to the elements of the three-neutrino mixing matrix by (see also Ref. [6])

$$
\sin^2 \theta_{\text{SUN}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \quad \sin^2 \theta_{\text{ATM}} = |U_{\mu 3}|^2, \quad \sin^2 \theta_{\text{CHOOZ}} = |U_{e3}|^2.
$$

(6)

Taking into account all the above experimental constraints, we have reconstructed the best-fit and allowed ranges for the elements of the mixing matrix (see Ref. [7] for a

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**FIGURE 2.** Allowed ranges for the neutrino masses as functions of the lightest mass $m_1$ and $m_3$ in the normal and inverted three-neutrino scheme, respectively.
FIGURE 3. Effective neutrino mass $m_\beta$ in Tritium $\beta$-decay experiments as a function of the lightest mass $m_1$ and $m_3$ in the normal and inverted three-neutrino scheme, respectively.

reconstruction taking into account the correlations among the mixing parameters):

$$U_{\beta f} \simeq \begin{pmatrix} -0.83 & 0.56 & 0.00 \\ 0.40 & 0.59 & 0.71 \\ 0.40 & 0.59 & -0.71 \end{pmatrix}, \quad |U| \simeq \begin{pmatrix} 0.71 & -0.88 & 0.46 \ -0.68 & 0.00 & -0.22 \ 0.08 & -0.66 & 0.26 \ -0.79 & 0.55 & -0.85 \ 0.10 & -0.66 & 0.28 \ -0.80 & 0.51 & -0.83 \end{pmatrix}.$$  \quad \text{(7)}

Such mixing matrix, with all elements large except $U_{e3}$, is called “bilateral”. It is very different from the quark mixing matrix.

The absolute scale of neutrino masses is not determined by the observation of neutrino oscillations, which depend only on the differences of the squares of neutrino masses. Figure 2 shows the allowed ranges (between the dashed and dotted curves) for the neutrino masses obtained from the allowed values of the oscillation parameters in Eqs. (1)–(4), as functions of the lightest mass in the normal and inverted three-neutrino schemes. The solid lines correspond to the best fit values of the oscillation parameters. One can see that at least two neutrinos have masses larger than about $7 \times 10^{-3}$ eV.

The most sensitive known ways to probe the absolute values of neutrino masses are the observation of the end-point part of the electron spectrum in Tritium $\beta$-decay, the observation of large-scale structures in the early universe and the search for neutrinoless double-$\beta$ decay, if neutrinos are Majorana particles (see Ref. [8]; we do not consider here the interesting possibility to determine neutrino masses through the observation of supernova neutrinos).

1: The two three-neutrino schemes allowed by the observed hierarchy $\Delta m_{\text{SUN}}^2 \ll \Delta m_{\text{ATM}}^2$. 

normal \hspace{2cm} inverted
FIGURE 4. Effective Majorana mass $|\langle m \rangle|$ in neutrinoless double-$\beta$ decay experiments as a function of the lightest mass $m_1$ and $m_3$ in the normal and inverted three-neutrino scheme, respectively.

Up to now, no indication of a neutrino mass has been found in Tritium $\beta$-decay experiments, leading to an upper limit on the effective mass

$$m_\beta = \sqrt{\sum_k |U_{ek}|^2 m_k^2}$$

of 2.2 eV at 95% C.L. [9], obtained in the Mainz and Troitsk experiments. After 2007, the KATRIN experiment [10] will explore $m_\beta$ down to about 0.2 – 0.3 eV. Figure 3 shows the allowed range (between the dashed curves) for $m_\beta$ obtained from the allowed values of the oscillation parameters in Eqs. (1)–(4), as a function of the lightest mass in the normal and inverted three-neutrino schemes. The solid line corresponds to the best fit values of the oscillation parameters. One can see that in the normal scheme with a mass hierarchy $m_\beta$ has a value between about $3 \times 10^{-3}$ eV and $2 \times 10^{-2}$ eV, whereas in the inverted scheme $m_\beta$ is larger than about $3 \times 10^{-2}$ eV. Therefore, if in the future it will be possible to constraint $m_\beta$ to be smaller than about $3 \times 10^{-2}$ eV, a normal hierarchy of neutrino masses will be established.

The analysis of recent data on cosmic microwave background radiation and large scale structure in the universe in the framework of the standard cosmological model has allowed to establish an upper bound of about 1 eV for the sum of neutrino masses, which implies an upper limit of about 0.3 eV for the individual masses [11, 12]. This limit is already at the same level as the sensitivity of the future KATRIN experiment. Let us emphasize, however, that the KATRIN experiment is important in order to probe the neutrino masses in a model-independent way.

A very important open problem in neutrino physics is the Dirac or Majorana nature of neutrinos. From the theoretical point of view it is expected that neutrinos are Majorana particles, with masses generated by effective Lagrangian terms in which heavy degrees of freedom have been integrated out (see Ref. [13]). In this case the smallness of neutrino masses is naturally explained by the suppression due to the ratio of the electroweak
symmetry breaking scale and a high energy scale associated with the violation of the total lepton number and new physics beyond the Standard Model.

The best known way to search for Majorana neutrino masses is neutrinoless double-$\beta$ decay, whose amplitude is proportional to the effective Majorana mass

$$|\langle m \rangle| = \left| \sum_k U_{ek}^2 m_k \right|. \quad (9)$$

The present experimental upper limit on $|\langle m \rangle|$ between about 0.3 eV and 1.3 eV has been obtained in the Heidelberg-Moscow and IGEX experiments. The large uncertainty is due to the difficulty of calculating the nuclear matrix element in the decay. Figure 4 shows the allowed range for $|\langle m \rangle|$ obtained from the allowed values of the oscillation parameters in Eqs. (1)–(4), as a function of the lightest mass in the normal and inverted three-neutrino schemes (see also Ref. [14]). If CP is conserved, $|\langle m \rangle|$ is constrained to lie in a shadowed region. Finding $|\langle m \rangle|$ in an unshaded strip would signal CP violation. One can see that in the normal scheme large cancellations between the three mass contributions are possible and $|\langle m \rangle|$ can be arbitrarily small. On the other hand, the cancellations in the inverted scheme are limited, because $\nu_1$ and $\nu_2$, with which the electron neutrino has large mixing, are almost degenerate and much heavier than $\nu_3$. Since the solar mixing angle is less than maximal, a complete cancellation between the contributions of $\nu_1$ and $\nu_2$ is excluded, leading to a lower bound of about $1 \times 10^{-3}$ eV for $|\langle m \rangle|$ in the inverted scheme. If in the future $|\langle m \rangle|$ will be found to be smaller than about $1 \times 10^{-3}$ eV, it will be established that either neutrinos have a mass hierarchy or they are Dirac particles. Many neutrinoless double-$\beta$ decay experiments are planned for the future, but they will unfortunately not be able to probe such small values of $|\langle m \rangle|$, extending their sensitivity at most in the $10^{-2}$ eV range (see Ref. [8]).

In conclusion, we would like to emphasize that, although the recent years have been extraordinarily fruitful for neutrino physics, yielding important information on the neutrino mixing parameters, still several fundamental characteristics of neutrinos are unknown. Among them, the Dirac or Majorana nature of neutrinos, the absolute scale of neutrino masses, the distinction between the normal and inverted schemes and the existence of CP violation in the lepton sector are very important for our understanding of the new physics beyond the Standard Model.

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