THE REIONIZATION HISTORY IN THE LOGNORMAL MODEL

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ABSTRACT

We study the evolution of baryonic gas before reionization in the lognormal (LN) model of cosmic clustering. We show that the thermal history of the universe around reionization can roughly be divided into three epochs: (1) a cold dark age \( z > z_r \), in which the baryon gas is neutral and opaque to Ly\( \alpha \) photons; (2) a hot dark age \( z_r > z > z_{GP} \), in which a predominant part of the baryon gas is ionized and hot but is still opaque to Ly\( \alpha \) photons; and (3) a bright age \( z < z_{GP} \), in which the universe is ionized enough to be transparent to Ly\( \alpha \) photons. In the flat cold dark matter cosmological models given by WMAP and COBE, the difference of the two redshifts \( z_r - z_{GP} \) is found to be as large as \( \sim 10 \), with \( z_r \sim 17 \) and \( z_{GP} \sim 7 \). This reionization history naturally yields a high optical depth of the CMB \( \tau_e \sim 0.12 \sim 0.19 \), which is observed in the TE polarization in the WMAP data, and a low redshift \( z_{GP} \) for the appearance of the Ly\( \alpha \) Gunn-Peterson trough, \( z_{GP} \sim 6-8 \), in QSO absorption spectra. The universe stays so long in an ionized, yet Ly\( \alpha \) opaque, stage because the first photoionization heats the intergalactic gas effectively and balances the gravitational clustering for a long period of time. Therefore, a high \( \tau_e \) and low \( z_{GP} \) is a result of all the considered models. Besides the cosmological parameters, the only free parameter we used in the calculation is \( N_{\text{ion}} \), the mean number of ionization photons produced by each baryon in collapsed objects. We take it to be \( 40-80 \) in the calculations.

Subject headings: cosmology: theory — early universe — large-scale structure of universe

1. INTRODUCTION

The large-scale structures of the universe evolved from initial mass density perturbations seeded by the quantum fluctuations of inflation. The power spectrum of the initial perturbation is a power law, \( P(k) \propto k^n \), with index \( n \sim 1 \), and so the mass density perturbation is stronger on smaller scales. Accordingly, objects on smaller scales generally formed at earlier times. Small objects like primordial black holes could form in the early radiation-dominated era. Objects on the scales of stars could form in the dark age, i.e., between recombination and reionization. These stars play the leading role in the history of reionization.

The study of structure formation in the cosmic dark age was challenged recently by two observations. First, the complete Ly\( \alpha \) Gunn-Peterson trough in the absorption spectra of high-redshift quasars shows that reionization occurred probably at the redshift \( z_r \approx 6-8 \) (Fan et al. 2002, and references therein). Second, the TE polarization of the Wilkinson Microwave Anisotropy Probe (WMAP) yields a model-independent value of the electron scattering optical depth of the cosmic microwave background (CMB), \( \tau_e = 0.17 \pm 0.04 \), which requires the reionization redshift to be \( z \approx 11-30 \), if the reionization happens rapidly (Kogut et al. 2003). Therefore, the problem naturally arises of how to reconcile the two results. Why is the reionization redshift from the Ly\( \alpha \) spectrum significantly lower than that given by the optical depth to the CMB? It has been suggested that the high \( \tau_e \) and low \( z_r \) probably indicate that the reionization is not abrupt but lasts for a long time from \( z = 30-20 \) to \( 6-8 \) (Cen 2003; Haiman & Holder 2003; Wyithe & Loeb 2003; Holder et al. 2003; Hui & Haiman 2003; Onken & Miralda-Escudé 2003). Further questions we might ask are how to describe the long period of reionization and what is the physical mechanism leading to such a long period? We address these problems using a semianalytical approach with the lognormal (LN) model.

The LN model assumes that the mass field of the intergalactic medium (IGM) is given by an exponential mapping from the linear field (Bi 1993; Bi & Davidsen 1997). The probability distribution function (PDF) of the IGM field is then lognormal. The first argument in support of the lognormal PDF is from the isothermal model of the IGM in a gravitational potential, \( \rho(x) \propto \exp \left( -\frac{\mu m_p}{\gamma kT} \phi_{\text{ion}}(x) \right) \), where \( \rho(x) \) is the IGM mass density and \( \phi_{\text{ion}}(x) \) is the gravitational potential. The variables \( T \) and \( \mu \) are, respectively, the temperature and molecular weight of the gas, and \( \gamma \) is the polytropic index. Equation (1) is the well-known IGM model of intracluster gas (Sarazin & Bahcall 1977). Zel’dovich, Ruzmaikin, & Sokoloff (1990) noted that if the gravity potential \( \phi_{\text{ion}}(x) \) is a random field with a Gaussian PDF, the field \( \rho(x) \) of equation (1) is lognormal. It has been shown that the lognormal solution is a reasonable approximation of the dynamical equations of the IGM (e.g., Jones 1999; Matarrese & Mohayaee 2002). Therefore, the LN is dynamically legitimate for modeling the weakly nonlinear properties (Bi & Davidsen 1997) and highly non-Gaussian features (Feng & Fang 2000) of the IGM. Recently, it has also been used to study the collapsing of hydrogen clouds in the IGM (Bi et al. 2003, hereafter BFFJ).

As BFFJ emphasized that the formation of the first generation of collapsed hydrogen clouds arises from rare events, i.e., the tail of the mass field PDF on the high perturbation
side. They noted that the LN PDF is long tailed and predicts a higher probability of rare, high-perturbation events than PDFs without a long tail. From this consideration, BFFJ showed that the reionization lasts a period from redshifts of 15 to about 10. However, they did not consider the reaction of the evolved IGM to reionization. During reionization, the IGM underwent a dramatic evolution from neutral to ionized hydrogen, from cold to hot gas, and from low- to high-entropy states. Consequently, the clustering behavior of the IGM should also undergo a remarkable evolution in the epoch around reionization. On the other hand, the clustering of the underlying dark matter is not affected by reionization. Therefore, we must analyze the impact of the IGM evolution on the reionization history.

The LN model directly provides the spatial distribution of the IGM and reveals the effect on the evolution of the IGM of reionization. We show that a long recombination period is a natural outcome of the LN model if the evolution of the IGM at reionization is considered. A long reionization period generally yields a high optical depth $\tau_e$ for the CMB and a low reionization redshift in the Ly$\alpha$ forests. We calculate these quantities in the cold dark matter cosmological models.

The paper is outlined as follows. In § 2 we describe the LN model of the reionization. The history of hydrogen reionization is discussed in § 3. Finally, § 4 gives the discussion and conclusion.

2. THE LOGNORMAL MODEL

2.1. Mass Field of Baryonic Gas

In the LN model, the mass field of the IGM is given by an exponential mapping of the linear density field of the underlying dark matter as

$$\rho(x) = \bar{\rho} \exp[\delta_0(x) - \sigma_0^2/2],$$

where $\sigma_0$ is the variance of the linear Gaussian field of the dark matter filtered on scales of the Jeans length $\lambda_J$ of the IGM and $\delta_0(x)$ is the linear density contrast of dark matter $\delta_{\text{dm}}(x)$ smoothed by a window on the Jeans scale, i.e., its Fourier counterpart is (Bi & Davidsen 1997)

$$\delta_0(k) = \frac{\delta_{\text{dm}}(k)}{1 + x_b^2 k^2},$$

where $x_b$ is $1/2\pi$ of the comoving Jeans length of the IGM.Obviously, when the fluctuation $\delta_0(x)$ is small, i.e., $\delta_0(x) \ll 1$, equation (2) reduces to approximately $\delta(x) \simeq \delta_0(x)$, where $\delta(x) = [\rho(x) - \bar{\rho}] / \bar{\rho}$ is the density contrast of the IGM. Moreover, when the fluctuation $\delta_0(x)$ is large, combining equations (2) and (3) yields equation (1) (Bi & Davidsen 1997). Since the initial density perturbation is stronger on smaller scales, the perturbations of the dark matter gravitational potential $\phi_{\text{dm}}(x)$ are larger on smaller scales. Thus, even in the early universe, the IGM field on small scales may already be in the state described by equation (1), and it would be inappropriate to assume that the IGM field is always Gaussian on all scales. The LN model provides a uniform description of the IGM distribution in the linear, weakly nonlinear, and lognormal regimes at various scales and redshifts.

Since the random field $\delta_0(x)$ is Gaussian, the PDF of the field $\rho(x)$ is lognormal as

$$p(\rho/\bar{\rho}) = \frac{1}{(\rho/\bar{\rho})\sigma_0\sqrt{2\pi}} \times \exp\left\{ -\frac{1}{2} \left[ \ln(\rho/\bar{\rho}) + \sigma_0^2/2 \right]^2 \right\}, \quad \rho \geq 0,$$

which gives the probability density of the event such that the mass density within an area on the scale of the Jeans length is equal to $\sim \rho/\bar{\rho}$. The tail of the PDF equation (4) is $\sim \exp(-1/2)$ for $\rho/\bar{\rho}$ for a given $\sigma_0$ would be equal to the probability of $\rho/\bar{\rho}$ for $2\sigma_0$. Namely, the change of variance $\sigma_0$ by a factor 2 leads to a change of the possible high density events from $\rho/\bar{\rho}$ to $(\rho/\bar{\rho})^2$.

Obviously, the PDF is normalized as $\int_{-\infty}^{\infty} p(x) \, dx = 1$, and the variance of $\rho/\bar{\rho}$ is given by (Vanmarcke 1983)

$$\sigma = \left( e^{\sigma_0^2} - 1 \right)^{1/2}. \quad (5)$$

The $n$th moment of $\rho$ is

$$\bar{(\rho/\bar{\rho})^n} = \exp\left( n^2 - n \frac{\sigma_0^2}{2} \right). \quad (6)$$

Equation (6) shows $\bar{(\rho/\bar{\rho})^n} < \bar{(\rho/\bar{\rho})^{3/2}}$. That is, the ratio between the high- and low-order moments is divergent in the limit of $n \to \infty$. A field with such a divergent moment ratio, by definition, is intermittent (Zel’dovich et al. 1990). Actually, the LN model is a mathematical model of a typical intermittent field. This property is consistent with the detected intermittency of the transmitted flux of QSO Ly$\alpha$ absorption (Jamkhedkar, Zhan, & Fang 2000; Feng, Pando, & Fang 2003, Jamkhedkar et al. 2003).

2.2. Jeans Length of Baryonic Matter

Equation (2) shows that in the LN model, the clustering features of the baryonic gas are characterized by the variance $\sigma_0$ of the linear density field on the scale of the Jeans length. Primordial baryons created at the time of nucleosynthesis recombine with electrons to become neutral gas at $z \sim 1000$. Thereafter, the gas cools down adiabatically with the expansion of the universe. In a homogeneous universe with mean mass density $\rho_m$, the Jeans length of gaseous baryonic matter or the IGM is defined by $\lambda_J \equiv \nu_e (\pi G \rho_m)^{1/2}$, where $\nu_e$ is the sound speed of gas. The corresponding Jeans mass is $m_J = (4\pi/3) \lambda_J^3 \rho_m$. In a comoving scale of $x_b = \lambda_J/2\pi$, we have (Bi & Davidsen 1997)

$$x_b = \frac{1}{\bar{\rho}_0} \left[ \frac{2\gamma k_B T}{3\mu m_p \Omega_m (1 + z)} \right]^{1/2}. \quad (7)$$

The polytropic index is assumed to be $\gamma = 5/3$, and the hydrogen temperature thus follows $T \propto \rho^{2/3}$, where $\rho$ is the mean mass density of baryonic matter. At $z = 15$, the hydrogen temperature is $\sim 4$ K (Medvigy & Loeb 2001), and the Jeans length $x_b \sim 1.2$ kpc. The redshift evolution of...
the comoving Jeans length $x_b$ is approximately given by $(1+z)^{1/2}$.

During reionization, the baryonic gas will be heated by the UV ionizing background from a few K to $\sim 1.3 \times 10^4$ K; correspondingly, the increase in $x_b$ is about 2 magnitudes, or 6 magnitudes in the Jeans mass (e.g., Ostriker & Gnedin 1996). After reionization, the IGM temperature is maintained at about $\sim 10^4$ K by the UV background photons, and therefore $x_b$ will gradually increase with decreasing $z$ because of the factor $(1+z)$ in equation (7).

For the variance $\sigma_0$ in equation (2), we consider three fiducial cosmological models. Two are inferred from the WMAP data and one from the COBE data. The cosmological parameters of these models are listed in Table 1. We use the linear power spectrum of the dark matter given by Eisenstein & Hu (1999). The linear variance grows with redshift as $\sigma_0(z) \propto 1/(1+z)$. For cosmological models with $\Omega = 1$ and $\Omega_A \neq 0$, the linear growth of density perturbations does not exactly follow the factor $1/(1+z)$ (Lahav et al. 1991). We take account of this effect by replacing the factor $1/(1+z)$ by $1/g(\Omega_m, \Omega_\Lambda)(1+z)$, where the $g$ factor is

$$g(\Omega_m, \Omega_\Lambda) = \frac{2.5\Omega_m^{4/7} - \Omega_\Lambda + (1 + \Omega_m/2)(1 + \Omega_\Lambda/70)}{\Omega_m^{4/7} - \Omega_\Lambda}(1+z),$$

This correction is approximately reasonable for $z > 1$ (Avelino & de Carvalho 1999; Carroll, Press, & Turner 1992). For the models WMAP1, WMAP2, and COBE, the $g$ factor is also shown in Table 1.

Because the powers of the linear perturbations $k^3P(k)$ in all of these models are smaller on larger scales, the 2 order magnitude jump of $\sigma_0$ at reionization generally leads to a drop-off of the variance $\sigma_0$. A typical curve of $\sigma_0$ is displayed in Figure 1 for the model WMAP2, in which the reionization is assumed to happen instantly around redshift $z_r = 17$. The dotted line in Figure 1 is for $\sigma_0$ in the case of no reionization. The drop-off $z_r$ is a common feature of the $\sigma_0$ curve. For instance, Figure 2 of BFFJ also shows a drop-off at $z = 7$. BFFJ assumed that the reionization takes place at redshift $z_r = 7$. In this paper, we do not use a priori a drop-off redshift $z_r$ but determine it by a self-consistent calculation. It should be pointed out that the drop-off in $\sigma_0$ does not affect the evolution of dark matter, since the clustering basically disregards the reionization.

### 2.3. Cumulative Mass Fraction and Clumping Factor

For a field with the PDF of equation (4), the cumulative mass fraction $M(>\rho/\bar{\rho})$, i.e., the fraction of mass in regions having mass density larger than a given $\rho/\bar{\rho}$, is

$$M(>\rho/\bar{\rho}) = \int_{\rho/\bar{\rho}}^{\infty} x P(x) dx = \frac{1}{2} \text{erfc} \left[ \frac{\ln(\rho/\bar{\rho})}{\sqrt{2}\sigma_0} - \frac{\sigma_0}{2\sqrt{2}} \right],$$

which implies that $M(>\rho/\bar{\rho})$ is a strongly varying function of the variance $\sigma_0$, but weakly varies with $\rho/\bar{\rho}$. Figure 2 demonstrates a cumulative mass fraction for $\rho/\bar{\rho} = 2, 6$, and 10, in which $\sigma_0$ is taken from Figure 1 and uses the dotted line for $z < z_r$, i.e., without considering the drop-off of $\sigma_0$ at $z_r$. Thus, Figure 2 can be used only at redshift $z > z_r$.

The decrease of $M(>\rho/\bar{\rho})$ with $\rho/\bar{\rho}$ is much slower than for a Gaussian PDF or PDFs with a Gaussian-like tail. For a Gaussian tail, we have $M(>10)/M(>6) \simeq \exp \left[ (-10^2 + 6^2)/2\sigma_0^2 \right]$, or $M(>10)/M(>6) \simeq \exp (-30)$, when $\sigma_0 \lesssim 1$. However, Figure 2 shows that even when $z$ is as high as 20, $M(>10)$ is less than $M(>6)$ only by a factor less than 10. This is obviously due to the long tail behavior of the lognormal PDF, which gives a larger probability of high-density events at high redshifts when the variance $\sigma_0$ is still small.

The variable $\rho/\bar{\rho}$ in equation (9) is the density within an area on scales of the Jeans length, and therefore, the area with $\rho/\bar{\rho} > 1$ has mass larger than the Jeans mass. However, these regions are not always collapsing, because that the IGM clouds have no time to collapse even when the cloud mass is larger than the Jeans mass. BFFJ found that at redshift $z \simeq 7$, only clouds with density $\rho/\bar{\rho} > 6$ are collapsing. The threshold $\rho/\bar{\rho} \gtrsim 6$ is much larger than the Jeans mass. In other words, clustering on Jeans length scales is significantly suppressed. The suppression of collapse on Jeans length scales has also been noted in studying the so-called filtering scale (Gnedin & Hui 1998).

The mass threshold for the clouds going to collapsing at redshift $z > 7$ is a little lower than 6. Therefore, the mass fraction of collapsed clouds can be estimated by the cumulative mass fraction $M(>\rho)$. One can estimate the velocity dispersion of the collapsed clouds by considering that in the LN model, the collapsed clouds in a potential well are probably

### Table 1

| Model     | $\Omega_m$ | $\Omega_\Lambda$ | $\Omega_b$ | $n_s$ | $\sigma_8$ | $h_0$ | $g$ |
|-----------|------------|------------------|------------|-------|------------|-------|-----|
| WMAP1     | 0.27       | 0.73             | 0.044      | 0.97  | 0.84       | 0.71  | 0.758 |
| WMAP2     | 0.29       | 0.71             | 0.047      | 0.99  | 0.79       | 0.72  | 0.772 |
| COBE      | 0.3        | 0.7              | 0.045      | 1     | 0.85       | 0.7  | 0.797 |
After redshift $z_r$, the mean clumping factor over the whole space is relatively low.

### 3. HISTORY OF THE REIONIZATION

#### 3.1. Clustering of Baryonic Gas around Reionization

The physical reason for the two-phase scenario of structure formation is obvious. Baryonic gas falls into dark matter gravity wells, and the first generation of stars forms before $z_r$. Because the UV ionizing photon background is gradually established by the first generation stars, baryonic gas is heated to a temperature of $10^4$ K, and the Jeans length becomes bigger. The irregularities or clustering within the large Jeans scale would be smeared out by thermal motion. Since the density fluctuations on large Jeans scales would still be in the linear regime, the process of the structure formation is significantly suppressed, even halted. At the end of this smearing out, some ionized hydrogen will move away from the bottom of the gravity wells of dark matter halos. Equation (1) also shows that when the temperature $T$ increases by a factor of $10^4$, many non-linear structures will disappear. The clustering will not recover until the variance of $\sigma_0$ grows again. In the second phase, objects form from the ionized IGM, and their masses range from $10^9$ to $10^{10} M_\odot$.

In the two-phase scenario, the structure formation slows down just after reionization. Consequently, there are not enough UV photons produced by stars to maintain a fully ionized IGM, and a significant fraction of H\textsc{i} remains in the IGM. In this case, the IGM is opaque to Ly$\alpha$ photons. The IGM becomes transparent for Ly$\alpha$ photons until the redshift at which the rate of star formation grows again and can produce enough UV photons to maintain a highly ionized IGM. Therefore, we can define two redshifts: (1) $z_r$, the redshift of reionization, which is the first time when the IGM was fully (or almost fully) ionized and heated up to temperature $\sim 10^4$ K;
and (2) \( z_{\text{GP}} \), the redshift of the Gunn-Peterson transparency, which is when the IGM enters a state such that the volume-averaged fraction of \( \text{H} \) is less than \( 10^{-4} \) and the Gunn-Peterson absorption ceases. In a word, the clustering of the IGM is suppressed during the period from \( z_r \) to \( z_{\text{GP}} \) and thus leads to the reionization epoch, which lasts for a long period, from \( z_r \) to \( z_{\text{GP}} \).

The two-phase scenario is based on the \( z-\sigma_0 \) relation indicated in Figure 1. Obviously, this \( z \)-dependence of \( \sigma_0 \) is approximate, as we assumed that the temperature of the IGM suddenly increases by a factor of \( 10^4 \). More precisely, the sharp decline of \( \sigma_0 \) at \( z_r \) should be replaced by a softer transition from the top branch to the bottom branch in the \( z-\sigma_0 \) plane. Nevertheless, the mechanism for a long reionization period outlined above still works, as it essentially depends only on the decrease of \( \sigma_0 \) with the heating and ionizing of the IGM.

### 3.2. The Redshift of Reionization

We first calculate the reionization redshifts \( z_r \). The reionization of hydrogen clouds is usually characterized by the filling factor of ionized hydrogen \( Q_{\text{HI}}(z) \). The reionization redshift \( z_r \) can be approximately determined by requiring \( Q_{\text{HI}}(z_r) \approx 1 \). The filling factor \( Q_{\text{HI}}(z) \) is given by (e.g., Barkana & Loeb 2001)

\[
\frac{dQ_{\text{HI}}}{dt} = \frac{N_{\text{ion}}(t)}{0.76} \frac{dM(\rho/\bar{\rho})}{dt} - \frac{1}{a^2} \alpha B C_{\text{HI}}(t) \bar{n}_{\text{HI}} Q_{\text{HI}},
\]

where the factor 0.76 is the fraction of hydrogen in the cosmic baryons, \( \bar{n}_{\text{HI}} = 1.88 \times 10^{-7} \) \((\Omega_b h^2/0.022) \text{ cm}^{-3} \) is the present value of the mean number density of hydrogen, and \( N_{\text{ion}}(t) \) is the ionization photons produced by each baryon in collapsed objects, which might be \( t \)-dependent. The first term on the right-hand side of equation (12) accounts for the ionizing source. The ionization photons produced by primordial black holes are negligible because the spectral index \( n \leq 1 \) on small scales (He & Fang, 2002).

The second term on the right-hand side of equation (12) is from recombination, in which \( a \) is the cosmic factor, \( \alpha_{B} \) is the case B recombination coefficient \( \approx 2.6 \times 10^{-13} \text{ cm}^3 \text{s}^{-1} \) for hydrogen at \( T = 10^4 \text{ K} \), and \( C_{\text{HI}}(t) \) is the clumping factor of \( \text{H} \).

Solving equation (12) yields

\[
Q_{\text{HI}}(t) = \int_0^t \frac{N_{\text{ion}}(t')}{{0.76}} \frac{dM(\rho/\bar{\rho})}{dt'} e^{\int_{t'}^t F(t'', t') dt''} dt',
\]

where

\[
F(t', t) = -\alpha_B \bar{n}_{\text{HI}} \int_{t'}^t \frac{C_{\text{HI}}(t'')}{a^3(t'')} dt''.
\]

In equation (13), \( N_{\text{ion}} \) is an uncertain parameter, which depends on the atomic processes in star formation. As usual, one can express this number by \( N_{\text{ion}} = N_s f_{\text{star}} f_{\text{esc}} \), where \( f_{\text{star}} \) is the fraction of baryon that form stars, \( N_s \) is the ionization photon number produced per baryon in stars, and \( f_{\text{esc}} \) is the fraction of photons that escape from the galaxy. Generally, the first generation stars are massive (Abel, Bryan, & Norman 2002; Bromm, Coppi, & Larson 2002), and their lifetimes are much less than the Hubble time at high \( z \). Thus, it would be reasonable to approximate the photons \( N_s \) as being produced at the same redshift as collapsed clouds.

The number \( N_s \) is quite different for stars with different mass and metallicity. With the IMF of Scalo (1998), one has \( N_s \approx 4000 \) for stars with metallicity \( \sim 0.05 \). For the Salpeter IMF, \( N_s \approx 6000 \) on average for stars with masses of 1 to 100 \( M_\odot \). For stars with zero metallicity, \( N_s \) could be as high as \( 10^5 \) (Bromm et al. 2001; Venkatesan, Tumlinson, & Shull 2003). Observational estimations of \( f_{\text{star}} \) and \( f_{\text{esc}} \) generally yield \( f_{\text{star}} f_{\text{esc}} \sim \approx 10^{-2} \). To sketch the reionization history driven by the mechanism discussed in § 3.1, we use the simplest model of \( N_{\text{ion}} \), i.e., let \( N_{\text{ion}} \) to be constant (\( z \)-independent) and take the values of 40, 60, and 80, respectively, which cover the possible range of \( N_{\text{ion}} \).

Figure 4 plots the filling factor \( Q_{\text{HI}} \) versus redshift \( z \) in the three cosmological models with \( N_{\text{ion}} = 40, 60, \) and 80, with \( C_{\text{HI}}(z) = C(z) \). The curve of the model \( \text{WMAP1} \) is similar to \( \text{COBE} \), while \( \text{WMAP1} \) shows a lower filling factor, because both the spectral index \( n \) and the normalization of the power spectrum \( \sigma_0 \) of \( \text{WMAP1} \) are smaller than the other two models. Accordingly, \( \text{WMAP1} \) gives a smaller variance \( \sigma_0 \) on the Jeans length of the IGM. From Figure 4, the redshift of reionization is in the range \( 18 > z_r > 16 \).

We also estimate \( z_r \) by taking \( N_{\text{ion}} = 40, 60, \) and 80, and \( \rho/\bar{\rho} = 6, 8, \) and 10. The results are tabulated in Table 2. In all of the considered parameter range, the redshift of reionization \( z_r \) is greater than 10. This result is consistent with BFFJ, who also found \( z_r > 10 \) but with a different method. BFFJ’s conclusion is based on the evolution of the mass density and velocity profiles of hydrogen clouds. Clearly, both approaches arrive at the same result that the reionization could occur much earlier than the redshift of the appearance of the complete Gunn-Peterson trough of Ly\( \alpha \) absorption.

The assumption of \( C_{\text{HI}}(z) = C(z) \) is correct only if the density distribution of \( \text{H} \) follows the distribution of the total IGM. Since \( \text{H} \) may be preferentially located in nearby collapsed clouds, we should use a biased relation \( C_{\text{HI}}(z) = b C(z) \). The bias parameter \( b \) can be estimated by

\[
b = \left[ \frac{\rho/\bar{\rho}}{(\rho/\bar{\rho})^2} \right]^{1/2} = e^{2\sigma_0^2},
\]

using equation (6). Since \( 0.4 < \sigma_0^2 < 0.8 \) (Fig. 1), we have \( 2 < b < 5 \). These values of \( b \) is consistent with estimates from

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**Fig. 4.—Filling factor \( Q_{\text{HI}} \) vs. redshift for the three cosmological models, WMAP1, WMAP2, and COBE. The variables \( \rho/\bar{\rho} \) and \( N_{\text{ion}} \) are taken to be 6 and 40, respectively.**
These correspond to the two phases of star
N be 1, 1.5, 2, and 4. Therefore this uncertainty will not affect
the conclusions shown below.

3.3. The Redshift of the Gunn-Peterson Transparency

Although \( Q_{\text{HI}} \simeq 1 \) at \( z_r \), it does not mean that \( Q_{\text{HI}} \simeq 1 \) for all \( z < z_r \), because \( \sigma_0 \) decreases at \( z = z_r \). For a self-consistent calculation of \( Q_{\text{HI}} \) in the epoch of \( z < z_r \), we should use the lower branch of \( \sigma_0 \) as shown in Figure 1.

The filling factor \( Q_{\text{HI}} \) at \( z < z_r \) can also be calculated using equation (12). Figure 6 plots the results of \( Q_{\text{HI}} \) versus redshift in the model WMAP2 for the parameters \( (\rho/\dot{\rho}, N_{\text{ion}}) \) of (6, 40), (6, 80), and (8, 40). Other models yield similar curves of \( Q_{\text{HI}} \).

Each curve of Figure 6 shows that there are two solutions to the equation \( Q_{\text{HI}}(z) \simeq 1 \): the first is at \( z_r > 15 \), and the second is at \( z_{\text{GP}} \sim 6-8 \). These correspond to the two phases of star formation. During the period between \( z_r \) and \( z_{\text{GP}} \), the IGM has multiple phases. The IGM consists of neutral and ionized hydrogen, H i and H ii. Both components are comparable and are opaque to Ly\( \alpha \) photons. However, the temperature of such a multiphase IGM can be of the order of \( 10^4 \) K. Therefore, it is self-consistent to apply the bottom branch of \( \sigma_0 \) from Figure 1.

When \( z \rightarrow z_{\text{GP}} \), the filling factor of neutral hydrogen quickly becomes small. The IGM becomes transparent to Ly\( \alpha \) photons when the volume filling factor of H ii is less than \( 10^{-4} \), and thus the redshift \( z_{\text{GP}} \) corresponds to the onset of the Gunn-Peterson transparency.

It should be pointed out that the two-peak curve of \( Q_{\text{HI}} \) actually depends on the sharp drop-off of \( \sigma_0 \) at \( z_r \). As discussed in § 3.1, a more realistic model would replace the sudden increase of IGM temperature by a more gradual process. In this case, \( Q_{\text{HI}} \) probably never reached 1 at \( z_r \), but, at most, showed a peak around \( z_r \). Therefore, before \( z_{\text{GP}} \), the IGM was never ionized to the level of \( 1 - Q_{\text{HI}} < 10^{-4} \). Thus, the redshift \( z_{\text{GP}} \) is the first time when the IGM becomes Gunn-Peterson transparent. Table 3 gives \( z_{\text{GP}} \) for three sets of parameters.

Figure 6 and Table 3 shows that the redshift interval \( z_r - z_{\text{GP}} \), which is always as large as \( \approx 10-12 \), is not sensitive to the adopted parameters, because the long time from \( z_r \) to \( z_{\text{GP}} \) is essentially due to the drop-off of \( \sigma_0 \), which is less sensitive to cosmological parameters, and the details of star formation. The temperature of the IGM increases by a factor of about \( 10^4 \) during reionization, while the entropy parameter \( T/m_{\text{HI}} \) also increases by a few orders of magnitude. Since high temperature and high entropy prevent baryonic gas from collapsing into the gravity well of dark matter, reionization is slowed down by the increase of temperature and entropy of the IGM.

### Table 2

| \( \rho/\dot{\rho} \) | \( N_{\text{ion}} \) | WMAP1 | WMAP2 | COBE |
|-----------------|----------------|--------|-------|------|
| 6................. | 40 | 15.8 | 18.0 | 17.8 |
| 6.................. | 60 | 17.0 | 19.5 | 19.0 |
| 6.................. | 80 | 17.4 | 20.4 | 20.0 |
| 8.................. | 40 | 14.0 | 16.0 | 15.9 |
| 10................ | 40 | 12.9 | 14.9 | 14.6 |

**Fig. 5.**—Filling factor \( Q_{\text{HI}} \) vs. redshift of the model WMAP2, \( \rho/\dot{\rho} = 6 \) and \( N_{\text{ion}} = 40 \). The bias parameter \( b \) of the clumping factor \( C_{\text{HI}} = bC \) is taken to be 1, 1.5, 2, and 4.

**Fig. 6.**—Redshift evolution of the volume filling factor \( Q_{\text{HI}}(z) \) from \( z > z_r \) to \( z_{\text{GP}} \) for the model WMAP2. The variables \( \rho/\dot{\rho} \) and \( N_{\text{ion}} \) are taken to be (8, 40; dotted line), (6, 40; solid line), and (6, 80; dashed line). The reionization redshifts \( z_r \) are given by \( Q_{\text{HI}} \approx 1 \) at \( z > 10 \). The redshifts of the Gunn-Peterson transparency \( z_{\text{GP}} \) are given by \( Q_{\text{HI}} \approx 1 \) at \( z < 10 \).
This behaves like a negative feedback mechanism: once the rate of star formation is higher, the UV background produced by star formation is also higher, and then more baryonic gas will be heated to higher temperature and a higher entropy state, which suppresses the rate of star formation. Therefore, the long duration of reionization is mainly driven by the evolution of the temperature and entropy of the baryonic gas during reionization.

### 3.4. Optical Depth to the CMB

If reionization lasts the long period of $z_r \sim z_{\text{GP}}$, the number density of free electrons is significant through the whole epoch $z_r > z > z_{\text{GP}}$. One can then expect that in this reionization history, the electron scattering optical depth of the CMB would be much larger than in models with $z_r \approx z_{\text{GP}}$. With the solution of $Q_{\text{H II}}$, the electron scattering optical depth of the CMB can be calculated by (e.g., Hui & Haiman 2003)

$$\tau_e = 0.0525\Omega_b h \int_0^z \frac{V_{\text{H II}}(z)(1+z)^2 \, dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}, \quad (16)$$

where $V_{\text{H II}}(z)$ is the volume filling factor of H II. Equation (16) only considers the electrons from ionized hydrogen. To consider the contribution of He II, we had assumed the filling factor of He II to be identical to H II, i.e., the optical depth is increased by a factor of 1.08. The He II → He III ionization occurred at redshift $\sim 3.3$ (Theuns et al. 2002), and, therefore, the correction for He III on $\tau_e$ is no more than 0.001.

The relation between the volume filling factor $V_{\text{H II}}$ and $Q_{\text{H II}}$ is dependent on the statistical behavior of the formation of the ionized region. If the ionized hydrogen region grows, $dQ_{\text{H II}}(t)$ in the duration $t$ to $t + dt$ is without spatial memory, so the formation probability would be uniform in space regardless of whether an ionized region formed at a time before $t$. Thus, we have $dV_{\text{H II}}/dt = (1 - V_{\text{H II}}) dQ_{\text{H II}}/dt$, with the solution is $V_{\text{H II}}(z) = 1 - \exp[-Q_{\text{H II}}(z)]$. However, the formation of the ionized region probably has spatial memory. The probability of forming collapsed clouds at the position where clouds already collapsed should be low. Therefore, the process of ionized hydrogen region formation most likely is self-avoiding, i.e., the process has less probability at the positions where the process has already happened (e.g., Cardy 1996). For a perfectly self-avoiding process, no position can host the formation process more than once (e.g., Cardy 1996). In this case, we have $V_{\text{H II}}(z) = Q_{\text{H II}}(z)$ until $Q_{\text{H II}} \leq 1$.

Table 3 lists the results of $z_r$, $z_{\text{GP}}$, and $\tau_e$ for the model WMAP2 with parameters $(N_{\text{ion}}, \rho/\bar{\rho})$ specified by (6, 40), (8, 40), and (6, 80). The optical depth $\tau_e(1)$ is for the model without any self-avoiding, while $\tau_e(2)$ is for model with perfect self-avoiding. The real number is probably between these two models. All results shown in Table 3 give a low $z_{\text{GP}} \sim 6\text{--}8$ and a high $\tau_e \sim 0.12\text{--}0.19$. These results are in excellent agreement with the observations of the redshift of the Gunn-Peterson trough and the electron optical depth to the CMB. The key of reconciling the two observation is that there exists a long period when $0.1 < y_{\text{H II}} < (10^{-8})$. The free electrons in this period contribute to a high value of $\tau_e$. On the other hand, the Gunn-Peterson transparency occurred at the end of this epoch, and the redshift $z_{\text{GP}}$ is small.

### 4. DISCUSSION AND CONCLUSIONS

We showed with the LN model that the reionization and Gunn-Peterson transparency emerged at very different times. The thermal history of the universe around reionization can roughly be divided into three epochs: (1) cold dark age $z > z_r$, in which baryonic gas is cold and opaque to Lyα photons; (2) hot dark age $z_r > z > z_{\text{GP}}$, in which a substantial part of the baryon gas is ionized and hot but is still opaque for Lyα photons, and (3) bright age $z < z_{\text{GP}}$. For the LCDM models favored by WMAP, we have $z_r \approx 13\text{--}19$ and $z_{\text{GP}} \approx 6\text{--}8$. That is, the hot dark age is in the redshift interval from $13\text{--}19$ to $6\text{--}8$. This thermal history naturally yields a high optical depth $\tau_e = 0.12\text{--}0.19$ and a low $z_{\text{GP}} = 6\text{--}8$, which is in excellent agreement with the observations of $\tau_e$ (WMAP) and $z_{\text{GP}}$ (Gunn-Peterson trough).

The long period from $z_r$ to $z_{\text{GP}}$ is essentially due to the evolution of the temperature and entropy of the baryonic gas during reionization. For all the considered cosmological models, there is always a drop-off of $\sigma_0$ at $z_r$. Therefore, a large difference $z_r - z_{\text{GP}}$ is a common property of these models. The model with $z_r > z_{\text{GP}}$ and a high $\tau_e$ is less sensitive to the cosmological parameters.

This result is also weakly dependent on the details of production of ionization photons from stars, because of the feedback mechanism of star formation (§ 3.3). The higher rate of ionizing photon production will cause a stronger input of entropy to baryonic gas, which then leads to a lower rate of collapse of baryonic clouds, suppressing the rate of star formation and the UV photon production. Therefore, a model, which properly considers the dramatic evolution of the temperature and entropy of the IGM, will generally yield a low $z_{\text{GP}}$ and a high $\tau_e$.

The LN model provides an intuitive and quantitative description of the effect of the IGM temperature evolution on the reionization history, because the LN model directly uses the variance $\sigma_0$ as a key parameter. It has already been recognized that the variance $\sigma_0$ is more effective and successful in modeling the IGM detected by QSO Lyα forests (Bi & Davidsen 1997). We show that the LN model provides a plausible description of photoionized and neutral IGM not only in the redshift range $z \leq 4$ but also to $z \approx 15$.

The hot dark age between the redshifts $z_r$ and $z_{\text{GP}}$ is an interesting issue. During that period, star formation is significant, although the rate of star formation is lower than that at $z_r$. On the other hand, baryonic gas is in a multiphase medium and...
consists of comparable components of H\textsc{i} and H\textsc{ii}. The gas has already been hot. Therefore, we expect to find some observable signatures, such as high-redshift 21 cm emission, of the hot dark age.

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