Numerical Simulation of Haline Convection in Geothermal Reservoirs

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Abstract. Haline-convective flows of two-component fluids inside a porous rock are investigated numerically. Numerical simulations are based on the mathematical model which consists of the continuity, Darcy’s and admixture transport equations. In a geothermal reservoir, the constant admixture concentration is held at the upper boundary leading to diffusive admixture transport into the volume. Near the upper boundary, fluid becomes denser and causes haline convection. Porous domains composed of layers at different properties are considered and their influence on haline-convective flows is investigated.

1. Introduction
In the present time, alternative energy sources which are renewable and environmentally friendly become more popular. One of such sources is deep heat of Earth which is used almost in 80 countries around the world. As known, there is a geothermal gradient directed to the Earth’s core [1] so that the temperature of crust and mantle reaches 450-550 K at a depth of several kilometers. Thermal gravity-driven convection of groundwater can develop under the geothermal gradient. Because of salt containing in groundwater and existence of concentration gradients, haline convection can develop as well. In some investigations, mathematical modeling of double-convective flows in geothermal systems triggered both the temperature and concentration inhomogeneities is carried out [2]. However, as shown further, there are conditions ensuring haline-convective flows to be significantly more intensive than flows developed under the typical geothermal gradient. In this case thermal convection is negligible and only haline convection should be taken into consideration. This case is considered in the present study. We investigate admixture transport in a geothermal reservoir to obtain information on space distribution and quantity of salt in water. This information is important for successful exploitation of facility for extraction of geothermal energy.

We consider a rectangular reservoir (a porous medium) initially filled with pure water. Salt contained over the reservoir enters the domain due to diffusion and triggers gravity-driven haline convection. We study enhancing of mass transfer due to convection in comparison with diffusion, evaluate amount and space distribution of salt in the reservoir. We consider reservoirs at uniform porosity and permeability as well as composed of horizontal layers at different properties and investigate how the properties of porous rocks affect on admixture transport.
2. Equation of state and properties of geothermal systems

We consider fluids saturating a porous rock consisting of water and a dissolved admixture. Let $\rho$, $\rho_c$, and $T$ be the density of solution and density of admixture (salt mass per the volume of solution) and temperature. We suppose that $\rho$, $\rho_c$, and $T$ are related by the linear relation

$$\rho - \rho_0 = -\alpha_T (T - T_0) + \alpha \rho_c$$

Here, $\rho_0$ is the density of pure water at the temperature $T_0$, and $\alpha_T$, $\alpha$ are the thermal and salinity expansion coefficients. The relation (1) is the equation of state. If fluid can be considered as isothermal, the relation (1) is reduced to the form

$$\rho = \rho_{T0} + \alpha \rho_c$$

Here, $\rho_{T0} = \rho_0 - \alpha_T (T - T_0)$ is the density of pure water at the temperature $T$. Geothermal systems at the pressure $P$ in tens-hundreds atmospheres and at the temperature $T$ in several hundreds degrees are considered. To define $\alpha_T$ for water at that conditions, we use the data on the specific volume of pure water depending on $P$ and $T$ [3]. Two pressures $P = 20, 150$ atm and the temperature range $T = 450-550$ K are taken into consideration. The obtained coefficient $\alpha_T$ is shown in figure 1. As clear, $\alpha_T$ increases with $T$ less than twice and depends on $P$ very slightly. The density $\rho_{T0}$ at $P = 150$ atm taken from [3] are put in table 1. One can see slight decreasing in $\rho_{T0}$ with $T$. We have no exact data on the salinity expansion coefficient at the considered conditions therefore choose $\alpha = 0.815$ following from the linear approximation of the sea water equation of state [4]. The value $\alpha$ shows how a solution volume increases with adding a dissolved admixture. The value of $\alpha$ is limited by $\alpha = 1$ corresponding to the solution volume to be constant independently of admixture quantity.

The temperature influences on the ability of water to dissolve salts. The maximal amount of dissolved admixture increases with $T$. The relation

$$c_{sat} = 0.3486 - 2.527 \frac{T}{T_b} + 0.1847 \left( \frac{T}{T_b} \right)^2$$

is usually used. Here, $c_{sat}$ is the concentration (mass fraction) of admixture in saturated brine and $T_b = 373.15$ K.

The temperature of Earth’s crust increases in the direction to the hot core. The geothermal gradient $G$ being about 20-30 K per kilometre can trigger thermal gravity-driven convection. Inhomogeneities

![Figure 1](image1.png)  
![Figure 2](image2.png)  

**Figure 1.** Thermal expansion coefficient $\alpha_T$ versus the temperature $T$.

**Figure 2.** Marginal curves $N = 1$ at $T = 450$ (1), 500 (2), 550 K (3).
in admixture concentration can lead to development of haline convection. One can estimate contributions from two types of convection into summary flows estimating the buoyancy ratio \( N \) [5]. As following from the equation of state (1)

\[
N = \frac{\alpha \Delta \rho_c}{\alpha_T \Delta T}
\]

(4)

The numerator in (4) is the density increment due to changing in admixture amount and the denominator is the density increment defined by thermal expansion. If \( N > 1 \), haline convection is dominant. Thermal convection is more intensive at \( N < 1 \). Marginal curves \( N = 1 \) demarcating zones of haline and thermal convection are shown in figure 2. These curves are plotted at \( P = 150 \text{ atm} \) for three magnitudes of \( T \) and consequently for three sets of \( \alpha_T \) and \( \rho_{T0} \) (see table 1). The concentration \( c \) instead of admixture density \( \rho_c = \rho_c \) are given in figure 2. Subscript “sh” marking \( c_{sh} \) denotes a value at \( N = 1 \). If \( c > c_{sh} \), haline convection is dominant.

Data in figure 2 can be interpreted in terms of the height of reservoir \( h \). One can relate \( h \) with the temperature difference \( \Delta T \) as \( \Delta T = hG \) because the geothermal gradient \( G \) is assumed to lead to existence of the temperature difference \( \Delta T \). We put typical magnitude \( G = 0.03 \text{ K/m} \) and calculate \( h \) corresponding to \( \Delta T \). Some concentrations \( c_{sh} \) at different \( h \) and \( T \) are listed in table 1. As clear, \( c_{sh} \) becomes smaller with decreasing in \( h \). For example, at \( h = 10 \text{ m} \) the concentration \( c_{sh} \) is of the order of \( 10^{-4} \) that is very small. In geothermal systems at that height, haline convection is dominant at all realistic concentrations. Our estimations justify the isothermal approximation employed further to solve mass transfer problems in geothermal systems.

### Table 1. Parameters of geothermal systems.

| \( \rho_{T0} \) (kg m\(^{-3}\)) | \( T = 450 \text{ K} \) | \( T = 500 \text{ K} \) | \( T = 550 \text{ K} \) |
|---|---|---|---|
| \( \alpha_T \) (kg m\(^{-3}\) K\(^{-1}\)) | 899.4 | 842.5 | 769.3 |
| \( \mu_s \) (Pa s) | 1.011 | 1.267 | 1.693 |
| \( c_{sh} \) (h =10 m) | 1.992\( \times \)10\(^{-4}\) | 1.212\( \times \)10\(^{-4}\) | 9.770\( \times \)10\(^{-5}\) |
| \( c_{sh} \) (h =50 m) | 4.137\( \times \)10\(^{-4}\) | 5.535\( \times \)10\(^{-4}\) | 8.093\( \times \)10\(^{-4}\) |
| \( c_{sh} \) (h =250 m) | 2.066\( \times \)10\(^{-3}\) | 2.762\( \times \)10\(^{-3}\) | 4.036\( \times \)10\(^{-3}\) |

3. **Problem statement and mathematical model**

We consider a rectangular porous reservoir at the length \( l \) and height \( h \) filled initially with pure water of density \( \rho_{T0} \). Concentrated brine lies above this reservoir. Dissolved admixture from brine diffuses into considered domain. In mathematical model, it means that the upper boundary of domain is held at the constant admixture concentration (figure 3). Admixture diffusion leads to production of solution which is denser than pure water. Denser fluid tries to fall and causes developing of haline convection.

The permeability of porous medium is in general a second-order tensor because of anisotropic properties [6]. In a simple case considered here, the permeability can be treated as a scalar [5]. A porous reservoir is supposed to be at the permeability \( k \) and porosity \( \phi \) which can vary in space. Thermal effects aren’t taken into account. Fluid motions are described by the continuity and Darcy’s equations [5].
Admixture transport is defined by convective flux occurring due to solvent motions and diffusive flux arising under admixture gradients. If solution moves into a porous medium, diffusive transport generally differs from that in unbounded solution because of hydrodynamic dispersion [6]. The coefficient of molecular diffusion in this case is a second rank symmetric tensor. However, dispersion is more influential in long-time processes and is considered to be negligible in our study. We believe that the diffusion coefficient of salt in water saturated a porous medium is scalar and equal to the similar coefficient $D_c$ in unbounded solution. The admixture transport equation is based on Fick’s law put in terms of the admixture density $\rho_c$. The set of governing equations is written as follows

$$\nabla(\tilde{W}) = 0, \quad \tilde{W} = -\frac{k}{\mu_s} (\nabla P - \rho g \tilde{c}), \quad \phi \frac{\partial \rho_c}{\partial t} + \tilde{W} \nabla \rho_c = \nabla (\phi D_c \nabla \rho_c ) \quad (5)$$

Here, $\tilde{W}$ is the filtration velocity related to the fluid velocity $\tilde{w}$ by the relation $\tilde{W} = \phi \tilde{w}$, and $P$, $\mu_s$, $g$, and $\tilde{c}$ is the pressure, dynamic viscosity, gravity force acceleration, and unit vector directed along the gravity force; $\tilde{c} = (0, -1)$ in the problem sketched out in figure 3. The set of equations (5) is added by the equation of state (2).

We transform the equations (5) and (2) introducing deviations of $P$ and $\rho$ from the reference values $P_0$ and $\rho_{T0}$: $p = P - P_0$, $r = \rho - \rho_{T0}$. Density $\rho_{T0}$ corresponding to pure water at the temperature $T$ is assumed to be constant whereas $P_0$ be hydrostatic pressure increasing with depth: $\nabla P_0 = \rho_{T0} g \tilde{c}$ . One can subtract the last equation from second equation (5) and obtain the form of Darcy’s equation in terms of $p$ and $r$. Equation (2) reduces to the identity $r = \alpha \rho_c$ relating the relative density of solution $r$ with admixture density $\rho_c$. We put obtaining equations in dimensionless form.

The diffusive velocity $D_c/h$ is used as the velocity scale. The other scales are: $h$ for length, $h^2/D_c$ for time, $(\rho_{sat} - \rho_{T0}) g_0 h$ for pressure, $(\rho_{sat} - \rho_{T0})$ for density. Here, $\rho_{sat}$ is the density of saturated brine, $g_0$ is the Earth’s gravity field acceleration. The dimensionless variables are $\tilde{U} = \tilde{W}(D_c/h)^{-1}$, $\Pi = p((\rho_{sat} - \rho_{T0}) g_0 h)^{-1}$, $S = r((\rho_{sat} - \rho_{T0})^{-1}$. We assume the permeability and porosity to vary in space and use the reference values $k_0$ and $\phi_0$ as scales. The dimensionless permeability is $\kappa = k(k_0)^{-1}$. We have the following dimensionless equations

$$\nabla(\tilde{U}) = 0, \quad \tilde{U} = -Rd \phi_0 \kappa (\nabla \Pi - S g \tilde{c}), \quad \phi \frac{\partial S}{\partial t} + \tilde{U} \nabla S = \nabla (\phi \nabla S ) \quad (6)$$

Figure 3. Sketch of the problem.
The set of equations (6) includes the only density variable $S$ resulted from the linear relation between $r$ and $\rho_c$. Here, $g$ is the dimensionless gravity force acceleration to be 1 in Earth’s conditions. The dimensionless parameter in (6) is the Rayleigh-Darcy number corresponding to the maximal difference of fluid density $\left(\rho_{sat} - \rho_T0\right)$

$$Rd = \frac{g_0(\rho_{sat} - \rho_T0)k_0h}{\phi_0\mu_sD_c}$$

(7)

Second parameter is the admixture density at the upper boundary $S^*$ held constant which is responsible for the intensity of mass transfer. Values at the upper boundary are marked with superscript “*”. Combining $Rd$ and $S^*$, one can obtain the actual Rayleigh-Darcy number $Rd^*$ including the real density difference at the upper boundary and being the only criterion of similarity

$$Rd^* = Rd \times S^* = \frac{g_0(\rho^* - \rho_T0)k_0h}{\phi_0\mu_sD_c}$$

(8)

Initially, a porous domain is saturated with pure water at rest. It gives the initial conditions

$$t = 0: \bar{U} = 0, \quad \Pi = 0, \quad S = 0$$

(9)

Vertical and lower boundaries are assumed not to pass admixture which can enter a considered volume only due to diffusion through the upper boundary. It means

$$x = 0, \quad x = A: \quad \bar{n} \cdot \bar{U} = 0, \quad \bar{n} \cdot \nabla \Pi = 0, \quad \bar{n} \cdot \nabla S = 0$$

$$y = 0: \quad \bar{U} = 0, \quad \Pi = 0, \quad \nabla S = 0$$

$$y = 1: \quad \bar{U} = 0, \quad \Pi = 0, \quad S = S^*$$

(10)

Here, $A = l/h$ is the aspect ratio, $\bar{n}$ is the unit vector normal to boundaries.

The dimensionless admixture density $S$ can be expressed through the admixture concentration $c$ as follows

$$S = \frac{(1 - \alpha c_{sat}) \alpha c}{\alpha c_{sat}(1 - \alpha c)}$$

(11)

The concentration $c_{sat}$ in saturated brine depends on the temperature according to (3).

4. Numerical method and parameters

The problem under study is solved numerically. We employ the finite-difference method with the use of a straggled nonuniform space grid. The earlier 2D code designed for the case of porous media at uniform properties [7-9] is augmented to include space-varied porosity and permeability. The continuity and Darcy’s equations are integrated together using the SIMPLE-type algorithm. Then the admixture transport equation is computed. The obtained three-point (along each coordinate) linear systems of equations are solved by a semi-implicit method.

The numerical code has been tested carefully. We have varied a size of space grid and a time step. An accuracy of computations is controlled by a discrepancy in a mass balance (the total admixture mass in the domain compares with the mass entering this domain through the upper boundary) which is less than $10^{-4}$ - $10^{-5}$.
We consider a geothermal reservoir at $T = 500$ K and $P = 150$ atm. Pure water initially filling a porous medium has the density $\rho T_0 = 842.5$ kg m$^{-3}$ and viscosity $\mu_s = 1.212 \times 10^{-4}$ Pa s (see table 1); $\mu_s$ is taken from [3]. Water is able to dissolve admixture of maximal concentration $c_{sat} = 0.3417$ as following from (3). The diffusivity of admixture varies with the temperature very little and is assumed to be $D_c = 3 \times 10^{-9}$ m$^2$/s. A porous domain is of height $h = 50$ m and length $l = 100$ m and, in a typical case, of porosity $\phi = 0.05$ and permeability $k = 10^{-13}$ m$^2$. The salt concentration at the upper boundary of domain is $c^* = 0.1$.

One can check applicability of isothermal approximation. At the height $h$ chosen, the temperature difference $\Delta T$ determined by the typical geothermal gradient $G$ is about 1.5 K that corresponds to the concentration on the marginal curve $c_{sh} \approx 0.003$ according to figure 2. We put $c^*$ much more than $c_{sh}$ and therefore haline convection is much more intensive than thermal convection.

An estimation of the actual Rayleigh-Darcy number $Rd^*$ at parameters above gives $Rd^* = 9.557 \times 10^5$. As $Rd^*$ is so high, we expect a stochastic behavior of fluid in geothermal reservoir and a small-scale structure of haline convection. Our tests give a space grid 4000x2000 to be suitable for accurate simulation. The space step along a vertical direction varies. A small step at the top is needed to

![Figure 4](image-url)

Figure 4. Field of admixture concentration over the time $t = 30$ (a), 60 (b), 90 days (c) from the start.
resolve a thing diffusive boundary layer. Near the upper boundary, space step is 0.227 × 10⁻⁴ and shorter by a factor of 100 than near the lower boundary. The grid along a horizontal direction is uniform. The time step in typical computations is 18 min or 0.130 × 10⁻⁸ part of time scale defined by diffusion. Numerical study has been performed with the use of PC with 4-core processor Intel Core i7-920. Each calculation requires about or less than 25 h of computation time.

5. Results and discussion

Development of haline convection is shown in figure 4 which presents admixture distributions at typical parameters in fragments of domain. The total domain occupies an area at \( x \in [0; 2], \ y \in [0; 1] \). As is clear, small stochastic salt “fingers” are formed near the upper boundary. Moving dawn, “fingers” merge together and become wider. The intensity of flow can be characterized by the module of maximal fluid velocity \( |\vec{w}_{\text{max}}| \). As obtained, \( |\vec{w}_{\text{max}}| = 42.68 \) (a), 38.98 (b), 44.99 cm/day (c).

We study convective flows to be far from the lower boundary of domain and not subjected by effect of this boundary. One can assume that salt “fingers” spread into a semi-infinite domain.

The porosity of solid phase is varied. We put \( \phi = 0.05, 0.10, 0.20 \) and calculate the total admixture mass \( M_c \) per the unit of volume occurring in the domain at the time \( t \) (figure 5). The mass \( M_c \) is defined as

\[
M_c = \frac{1}{hl} \iint \phi( x, y ) S( x, y ) dx \, dy
\]

For comparison, we solve the problem at the same parameters but under zero gravity force to obtain mass transfer only due to diffusion. As shown in figure 5, at the initial time interval convection is negligible and only diffusive transport occurs. At this time interval, solid and dashed lines coincide with each other for every \( \phi \). Further, when solid lines start to deviate from dashed ones, convection becomes significant and enlarges mass flux of admixture into a volume. The most increase in \( M_c \) occurs at \( \phi = 0.05 \) and reaches, for example, a factor of 12.1 at \( t = 120 \) days.

We investigate how convective falling flows move through porous layers of different porosity and permeability. To describe an ability of porous layers pass fluid flows the length-average mass of salt \( m_c( y ) \) is calculated. The value \( m_c( y ) \) is the mean admixture mass \( S( x, y ) \) per a unit of reservoir length defined by the expression

![Figure 5. Total mass \( M_c \) in geothermal reservoirs at different porosity under Earth’s gravity (solid lines) and zero gravity (dashed lines).](image-url)
\[
m_c(y) = \frac{1}{l} \int \phi(x, y) S(x, y) dx
\]

The value \( m_c \) shows the mean admixture mass at the height \( y \) and relates to \( M_c \) as follows:
\[
M_c = 1/l \int m_c(\ y\ ) dy
\]
We consider three porous reservoirs. First one has uniform properties at the porosity \( \phi = 0.2 \) and permeability \( k = 10^{-13} \text{ m}^2 \). Second reservoir contains a horizontal porous layer of lower porosity \( \phi = 0.05 \) and the same \( k \) as above, other parameters being equal. This layer lies 2.5 m below than the upper boundary and has the height 2.5 m. In dimensionless coordinates, a low-porous layer occupies a region at the vertical coordinate \( y = 0.90-0.95 \). In third case, the permeability of low-porous layer varies. For porous media formed by similar particles of fibers, the formulae of Carman-Kozeny [5] gives a dependence of permeability on porosity. If \( \phi \) decreases from 0.2 to 0.05, falling in \( k \) is about of one order according to this formulae. Therefore we put \( k = 10^{-14} \text{ m}^2 \) at \( y = 0.90-0.95 \), other things being the same as in the previous case. The length-average mass \( m_c(y) \) for three considered configurations is given in figure 6. Parts of curves are shown. In a close vicinity of the upper boundary, \( m_c(y) \) has a very high gradient reaching the magnitude \( m_c(1) = 0.0460 \). If \( y \) is lower than \( 0.8 \), \( m_c(y) \approx 0 \).

As obvious from comparison of figures 6 (a) and 6 (b), admixture transport is the same in these two cases up to \( t = 90 \) days until the convective flux reaches the incorporated layer. Further salt enters a porous medium at lower porosity and moves faster due to decreasing in reservoir void space. An acceleration of convective flows in the incorporated layer leads to enhancing mass transfer through this layer into a region below. At \( t = 150 \) days, for example, the mass \( m_c(y) \) in figure 6 (b) reaches 0.005

![Figure 6](image_url)

**Figure 6.** The length-average mass of salt \( m_c(y) \) in porous domains being uniform at \( \phi = 0.2 \) and \( k = 10^{-13} \text{ m}^2 \) (a), and incorporate a layer at \( \phi = 0.05 \) (b), \( \phi = 0.05 \) and \( k = 10^{-14} \text{ m}^2 \) (c) at times \( t = 90 \) (1), 120 (2), 150 (3), 180 days (4). Grey color indicates a position of incorporated layer.
immediately below $y = 0.9$, whereas it is of the order of magnitude smaller in figure 6 (a). Salt does not delay in a low-porous layer and passes it fast so that the mass $m_c$ is several times smaller here than in regions above and below at $\phi = 0.2$.

If permeability of incorporated layer falls as in figure 6 (c), salt transport becomes different. The layer is not able to pass convective flows fast due to blocked permeability. Salt concentrates above this layer so that the mass $m_c$ in the layer and below becomes smaller than in a uniform porous medium (figure 6 (a)).

The total mass $M_c$ is about the same in the first and second cases during all considered time interval. In third case, $M_c$ is in 2.5 % lower due to enlarged amount of admixture near the upper boundary and reduced diffusive flux.

6. Conclusions
Our estimations have shown that there are a wide range of parameters of geothermal systems ensuring haline convection to be more intensive than thermal convection under the typical geothermal gradient. We studied stochastic haline convection. In domains at uniform properties, convection enhances admixture transport to a greater extent if the porosity is lower. A porous medium which consists of layers with different properties exhibits a different ability to pass fluid phase and transfer admixture. If a domain incorporates a layer at lower porosity and the same permeability, this layer accelerates convective flows and promotes mass transfer into a region below. If, in addition, the incorporated layer is of decreased permeability, admixture transport on the contrary may be impaired.

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