\( \Lambda N \rightarrow NN \) EFT potentials and hypertriton non-mesonic weak decay

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Abstract. The potential for the \( \Lambda N \rightarrow NN \) weak transition, the main responsible for the non-mesonic weak decay of hypernuclei, has been developed within the framework of effective field theory (EFT) up to next-to-leading order (NLO). The leading order (LO) and NLO contributions have been calculated in both momentum and coordinate space, and have been organised into the different operators which mediate the \( \Lambda N \rightarrow NN \) transition. We compare the ranges of the one-meson and two-pion exchanges for each operator. The non-mesonic weak decay of the hypertriton has been computed within the plane-wave approximation using the LO weak potential and modern strong EFT NN potentials. Formally, two methods to calculate the final state interactions among the decay products are presented. We briefly comment on the calculation of the \( ^3\Lambda H \rightarrow ^3He + \pi^- \) mesonic weak decay.

1. Introduction

The hypertriton, a bound system of a proton, a neutron and a \( \Lambda \), is the lightest known hypernucleus, and therefore also the simplest to study theoretically. Because it contains only three particles, it is possible to describe its wave function, its weak decay modes, and the corresponding final state interactions with exact few-body techniques and precise strong and weak potentials.

Hypernuclei decay weakly in mainly two modes, mesonically, resembling the weak decay of the \( \Lambda \) in free space, \( \Lambda \rightarrow N\pi \), and non-mesonically, in which the \( \Lambda \) is converted into a nucleon by interacting with a nucleon (or nucleons) from the medium, \( \Lambda N \rightarrow NN, \Lambda NN \rightarrow NNN \), etc. The non-mesonic weak decay mode constitutes a very small part of the total decay rate for the lightest hypernuclei, —the hypertriton, \( ^4\Lambda H \) and \( ^4\Lambda He \)—. However, these decay rates can in principle be measured independently of the mesonic ones, and can offer useful information to constrain the \( \Lambda N \rightarrow NN \) interaction. In particular, the experiment E22 in J-PARC, plans to measure the non-mesonic weak decay partial rates for \( ^4\Lambda H \) and \( ^4\Lambda He \) [1].

The hypertriton non-mesonic and mesonic weak decays were studied previously using one-meson-exchange potentials for the strong and weak forces and the Faddeev few-body scheme to implement all the interactions [2], [3]. In this work we focus on studying the non-mesonic weak decay of the hypertriton by implementing the same few-body framework and using modern strong and weak EFT potentials for the initial and final wave functions. We put special emphasis...
in the non-mesonic weak decay transition, \( \Lambda N \rightarrow NN \), for which we have developed the EFT potentials up to NLO in momentum and coordinate space. For the strong interactions NLO NN and YN EFT potentials are used [4]. At the end, we briefly comment on how to compute the mesonic partial decay rate \( \Gamma(\Lambda N \rightarrow \Lambda N \pi^{-}) \), which might be relevant in understanding the specially short lifetime of the hypertriton [5]. Note that one should, in principle, also consider the role of the \( \Sigma \) baryon in the weak decays, as it contributes to the hypertriton wave function due to \( \Lambda - \Sigma \) mixing. In our work we have focused in the \( \Lambda \) decay modes since they represent the most dominant contribution.

2. \( \Lambda N \rightarrow NN \) potential with EFT

The weak transition \( \Lambda N \rightarrow NN \), driving the non-mesonic weak decay of the hypertriton, has been described within the EFT framework up to next-to-leading order (NLO). As in the EFT description of the NN and YN strong interactions, the idea is to model this transition in a systematic, general way through a separation of the relevant scales. The soft and hard scales of a baryon-baryon low-energy interaction are the three-momenta \( (p, \sigma) \), \( n \) being the initial momentum and the transferred momentum, respectively, and \( (M, m) \) are the masses (\( m \)) of the NN pair and the final NN one. Therefore, in our case we must allow for PV contributions, and consider that the soft scale is determined not by the typical momenta of the hypertriton constituents, but by the higher momenta of the two emerging nucleons, of the order of \( p \sim 400 \text{ MeV} \). The pion mass is well below \( p \), and since the kaon mass is of the same order we consider both mesons as explicit degrees of freedom at leading order (LO). At NLO, for simplicity and to not add more uncertainties coming from the unknown kaon couplings, we have only considered the two-pion exchange.

The EFT potential is thus divided in the different orders in the expansion, each order containing the corresponding meson exchanges and contact interactions. The contact terms consist in all the possible four-point operators of order \( \mathcal{O}(p/M)^n \). They describe the unresolved short range dynamics and their coefficients, or so called low-energy constants (LECs), must be fixed to the data. In momentum space, the contact part of the potential up to \( \mathcal{O}(q^2/M^2) \), consisting of all the PC and PV operational structures, is shown in Tab. 1.

| \( \mathcal{O}(q^0) \) | \( \mathcal{O}(q) \) | \( \mathcal{O}(q^2) \) |
|-----------------|-----------------|-----------------|
| \( \mathcal{O}(q^0) \) | \( 1, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \) | \( \vec{q} \cdot \vec{p}, \vec{q} \cdot \vec{\sigma}_1, \vec{p} \cdot \vec{\sigma}_2, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \vec{\sigma}_1 \times \vec{\sigma}_2, \vec{q} \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \vec{\sigma}_1 \) |
| \( \mathcal{O}(q) \) | \( \vec{q} \cdot \vec{p}, \vec{q} \cdot \vec{\sigma}_2, \vec{p} \cdot \vec{\sigma}_1, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \vec{\sigma}_1 \times \vec{\sigma}_2, \vec{q} \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \vec{\sigma}_1 \) |
| \( \mathcal{O}(q^2) \) | \( \vec{q} \cdot \vec{\sigma}_1, \vec{q} \cdot \vec{\sigma}_2, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \vec{q} \times \vec{\sigma}_1, \vec{\sigma}_2 \cdot \vec{q} \times \vec{\sigma}_2, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \vec{\sigma}_1 \) |

Table 1. All contact operational structures up to next-to-leading order. \( \vec{q} \), \( \vec{p} \) are the average initial momentum and the transferred momentum, respectively, and \( \vec{\sigma}_1 \) and \( \vec{\sigma}_2 \) are the Pauli matrices connecting the spins of the two pairs of particles.

The LO part of the potential was first studied in [6], [7]. Its mesonic exchange contribution consists in the one-pion and one-kaon exchanges at LO, and all the possible two-pion exchanges at NLO. The strong vertices appearing in the diagrams are taken from the strong chiral Lagrangian. The \( YNn \) and \( NNe \) weak vertices are phenomenological, while the \( \Lambda N \) and \( \Lambda N \pi\pi \) ones, entering at NLO diagrams, are given by the weak SU(3)_F Lagrangian [8].
energy and momentum has been used to fix the momenta label the spin and isospin projections of the three nucleons (\( m_1, m_2, m_3 \)). The calculation of the non-mesonic weak decay rate of the hypertriton consists in a matrix element driving the transition between the hypertriton and the final state of three baryons, an average over initial hypertriton spin projections, and a sum over all possible final states (momenta, spins, and isospins). The rates for d+n and 3N can be written as

\[
\Gamma^{d+n} = \frac{32}{9} \pi^2 M_N p_3^{(d+n)} \sum_{m_{j1}, m_{j2}, m_3} \left| \langle \Psi_{m_{j1} m_{j2} m_3} | (1 + P)V_{12}^{\pi} | \psi_H^+ \rangle \right|^2, 
\]

\[
\Gamma^{3N} = \frac{128}{9} \pi^3 M_N \sum_{m_{j1}, m_{j2}, m_3} \int p_{12}^2 p_{13}^2 p_3^{(3N)} \sin(\theta) d\theta \left| \langle \Psi_{m_{j1} m_{j2} m_3} | (1 + P) \left( V_{12}^{\pi} | \psi_H^+ \rangle + | U \rangle \right) \right|^2 .
\]

\[ m_{j1} \text{ and } m_{j2} \text{ are the spin projections of the hypertriton and the deuteron, and } m_i \text{ and } m_{t_i} \text{ label the spin and isospin projections of the three nucleons } (i = 1, 2, 3). \]

\[
\text{The conservation of energy and momentum has been used to fix the momenta } p_3^{(d+n)} = \sqrt{4M_N/3}(\Delta M + \epsilon - \epsilon_d) \text{ and } p_3^{(3N)} = \sqrt{4M_N/3}(\Delta M + \epsilon - \epsilon_d), \text{ where } \Delta M = M_\Lambda - M_N, \text{ with } M_N \text{ and } M_\Lambda \text{ the masses of the nucleon and the } \Lambda. \] \epsilon \text{ and } \epsilon_d \text{ are the binding energies of the hypertriton and the deuteron, and } P = P_{12}P_{23} + P_{13}P_{23} \text{ is an antisymmetry operator, where } P_{ij} \text{ exchanges the quantum numbers and coordinates of particles } i \text{ and } j. V_{12}^{\pi} \text{ is the weak } \Delta N \rightarrow NN \text{ potential and } | \psi_H^+ \rangle, | \psi_{m_{d_1} m_1} \rangle, \text{ and } | \Psi_{m_{j1} m_{j2} m_3} \rangle \text{ are the wave functions for the hypertriton, the outgoing deuteron and neutron, and the three outgoing nucleons.}

Implementing the LO EFT potential into Eqs. (1) and (2) and setting \(| U \rangle = 0 \) we obtain the decay rates to d+n and 3N within the plane-wave approximation. The decay rates obtained by
considering only the OPE in the weak transition are \( \Gamma_{d+n} = 0.16 \cdot 10^8 \) \( \text{s}^{-1} \) and \( \Gamma_{3N} = 0.17 \cdot 10^9 \) \( \text{s}^{-1} \). Including both the OPE and the OKE, the decay rates get reduced by about a factor three, \( \Gamma_{d+n} = 0.45 \cdot 10^7 \) \( \text{s}^{-1} \) and \( \Gamma_{3N} = 0.54 \cdot 10^8 \) \( \text{s}^{-1} \). The low-energy constants appearing in the LO \( \Delta N \to NN \) potential were constrained in [11] by fitting them to the most recent data on hypernuclear non-mesonic weak decay of \( ^5\Lambda\text{He} \), \( ^{11}\Lambda\text{B} \), and \( ^{12}\Lambda\text{C} \). However, the fitting did not give precise enough values, and the LECs were found to depend on the model used for the \( SU(3)_F \) couplings appearing in the weak transition and the final state interactions. We have computed the decay rates for a reasonable range of the values of the LECs to study how they depend on the two constants. We observe that both \( \Gamma_{d+n} \) and \( \Gamma_{3N} \) increase (decrease) when the LECs grow with opposite (same) signs.

The final state interactions are being computed in the present with two different methods. The first one consists in solving the rescattering state \(|U\rangle\) in Eqs. (1) and (2). This is done by solving iteratively the following Lippmann-Schwinger equation,

\[
|U\rangle \equiv V_{12}G(1 + P)V_{12}^\dagger|\psi_3^2\rangle = V_{12}G_0(1 + P)V_{12}^\dagger|\psi_3^2\rangle + V_{12}G_0(1 + P)|U\rangle,
\]

where \( G = \frac{1}{E + \sigma - H}, \ G_0 = \frac{1}{E + \sigma - H_0}, H (H_0) \) being the strong (free) Hamiltonian and \( E \) the energy of the final state, and \( V_{12} \) the two-body strong force.

Alternatively, we use the Stieltjes transformation to compute the total non-mesonic decay rate. In this method, we add a spurious degree of freedom to the decay rate, \( w \), which can be thought of as an extra mass of the hypertriton. Labelling \( |f\rangle \) a general final state, the total decay rate with this new degree of freedom can be written as

\[
\Gamma(w) = \frac{1}{2} \sum_m \int df \ 2\pi \delta(M_{\Lambda^3H} - E_f + w)|\langle f|\hat{O}|\psi_3^2\Lambda^3Hm\rangle|^2.
\]

Applying the Stieltjes transformation to \( \Gamma(w) \), \( \phi(\sigma) = \int_{w_0}^{\infty} dw \frac{\Gamma(w)}{w + \sigma} \), and using completeness, one obtains

\[
\phi(\sigma) = -\frac{1}{2} \sum_m 2\pi \langle \Lambda^3Hm|\hat{O}\left(\frac{1}{\Delta M + E_{\Lambda^3H} - \sigma - H} + \frac{1}{E_f - \Delta M + E_{\Lambda^3H} + \sigma}\right)\hat{O}_m|\psi_3^2\Lambda^3Hm\rangle.
\]

The decay rate in the transformed space, \( \phi(\sigma) \), can be obtained by solving the iterative equation

\[
|\phi\rangle = \frac{1}{\Delta M + E_{\Lambda^3H} - \sigma - H}\left(1 - \langle \Lambda^3H|\hat{O}_m|\psi_3^2\Lambda^3Hm\rangle\right)|\Lambda^3H\rangle + V|\phi\rangle.
\]

In order to obtain the total non-mesonic decay rate, \( \Gamma(0) \), we make the inverse Stieltjes transformation through the singular value decomposition method and set \( w = 0 \). The advantage of this method is that for large enough \( \sigma \), we avoid the pole of the deuteron in solving the iterative equation. The drawback is that we cannot distinguish between both partial decay rates.

4. On the mesonic weak decay \( ^3\Lambda^3H \rightarrow ^3\Lambda^3He + \pi^- \)

Recent experimental results on the lifetime of the hypertriton \( (\tau_{\Lambda^3H} = 216^{+19}_{-16} \text{ ps}) \) [5], seem to indicate that it decays considerably faster than the \( \Lambda \) in free space \( (\tau_{\Lambda} = 263 \pm 2 \text{ ps}) \). In order to understand the relatively fast decay rate of the hypertriton, one should study its mesonic decay mode, since it is the dominant one in the lightest hypernucleus. Previous works have studied this decay, but none of the results seem to be compatible with the short hypertriton lifetime.
Once the non-mesonic weak decay formalism is established, one can apply the same few-body scheme to compute the mesonic decay rates. In this case, the weak one-body transition, $\Lambda \rightarrow p\pi^-$, is implemented by the operator $V^w_3 = i\sqrt{2}G_F m_\pi^2 (A + \vec{B} \cdot \vec{q})$, where the couplings $A = 1.05$ and $B = -7.15$ are known from the weak decay of the $\Lambda$ in free space. $G_F m_\pi^2 = 2.21 \times 10^{-7}$, $\vec{q}$ is the momentum of the pion, and the factor $\sqrt{2}$ appears due to isospin. For the strong part the same EFT potentials can be used. The possible final decay products are not only the three free nucleons or the deuteron and the nucleon, but also $^3H$ or $^3He$. This is because the emerging nucleon in the mesonic decay does not carry as much energy as the outgoing nucleons in the non-mesonic one, allowing the three final nucleons to form a bound state.

Note that the decays with an emerging $\pi^-$ are twice as the ones with an outgoing $\pi^0$ due to isospin, e.g. $\Gamma(\Lambda H \rightarrow 3He + \pi^-) = 2\Gamma(\Lambda H \rightarrow 3H + \pi^0)$. Therefore, there are only three independent channels.

As an example, the rate for the decay to $^3\Lambda H \rightarrow 3He + \pi^-$ is, integrating the deltas of energy and momentum conservation,

$$
\Gamma^{He} = \frac{1}{12\pi M_{3He}} q_{He} \sum_{m_{\Lambda}} \sum_{m_{He}} \left| \langle \Psi_{m_{He}} | (1 + P) V^w_3 | \psi_{\Lambda H} \rangle \right|^2 , \tag{7}
$$

where $q_{He}$ is the modulus of the momentum of the outgoing $\pi^-$,

$$
q_{He} = \sqrt{2M_{3He} \left( M_{3He} - \sqrt{m^2 + 2M_{He} M_{\Lambda}} - M_{He}^2 \right)^2} , \tag{8}
$$

and $E_x = \sqrt{m^2 + q_{He}^2}$ its energy, $M_{He}$ the Helium-3 mass, and $\Psi_{m_{He}}$ its wave function.

This decay was computed previously in [3] using OME potentials for both the strong and weak part and not considering strong interactions between the pion and the rest of the nucleons. The authors of this work obtained, in combination with the non-mesonic decay rate, a value for the hypertriton lifetime of $\tau_{\Lambda H} = 256$ ps, about $3\%$ smaller than $\tau_\Lambda$, still far from the $11 - 24\%$ difference from [5]. As in the non-mesonic case, our aim is to compute it within the same Faddeev scheme but with more modern EFT potentials. Note that in $\Gamma^{He}$, once the wave functions of the hypertriton and Helium-3 are known, the only final strong interactions to be computed are the ones between the pion and the nucleons. In order to study its effect, specially on the hypertriton lifetime, the plane-wave of the pion can be replaced by a distorted wave described through optical potentials. In contrast to the non-mesonic weak decay, which contains LECs that are still not well constrained, the weak part of the mesonic transition is fixed, and in principle one should be able to study, already on the Helium-3 mode, how the more modern EFT wave functions and the final state interactions affect the hypertriton lifetime.

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