On the dynamic properties of metamaterials in civil engineering structures

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Abstract. Metamaterials have a foundation in electromagnetics, and the unique properties of these metamaterials have found wide applications in acoustics and electrodynamics disciplines. By exploring local resonant effect of the included aggregates in the concrete mix with ellipsoidal coating rubber, a new elastic metamaterial called metaconcrete is formed. The metaconcrete with anisotropic effective dynamic mass density is determined as functions of frequency numerically. This metaconcrete has unique properties that can prevent earthquakes, shock and other disturbances at a targeted frequency.

1. Introduction

Civil engineering structures have been under severe attack from dynamic loadings in recent times due to terrorists attack, accidental explosions, new features introduced by clients that impose vibration problems on buildings, and increased vulnerability of structures to seismic, impact and blast loads [1–3]. Therefore, there is an urgent need to protect these structures from these various attacks. Few concepts and engineering methods have been demonstrated in tackling these dynamic problems in structures [2,4,5]; for example, the earthquake resistant design for reinforced concrete structures commonly have been conducted on the basis of the assumption that characteristics under dynamic loading are equal to those under static loading. BS 8110 stipulates that self-weight and dynamic live loading allowance of 5kN/m² be adopted for dancing halls or event centres [6]. But little studies have been carried out on how to block elastic waves in concrete structures.

Here an anisotropic mass density is designed by placing locally resonant inclusion in the form of engineered aggregate in a concrete matrix. When the resonant microstructures are represented by mass and spring systems, analytical solutions are derived for the effective mass densities along the two principal directions. According to the homogenization method, negative effective mass of metamaterials can be achieved by either local resonances of single unit or nonlocal resonances of adjacent units [7].

2. Effective mass densities

2.1. Single mass-with two different springs
The effective mass density principle in metaconcrete has been studied in our earlier paper [3] as shown in figure 1. Using Newton’s law, the two-object system $m_1$ and $m_2$ can be considered as a homogenous one-object system $M_1$ with the resonant frequency $\omega_0$ and an effective mass is obtained as:
By applying the same principles to the analytical model shown in figure 2, we shall engineer another metaconcrete with additional unique properties. In figure 2, the inner mass $m_1$ is connected by two springs $k_1$ and $k_2$. This model was first adopted by Milton et al [8] in explaining the design of elastic cloaking where an object is cloaked from elastic waves for a given frequency.

Denoting the displacements of masses $m$ and $m_1$ in $x$ direction by $u_x$ and $u_{1x}$, respectively. Symbols with subscript $y$ have the same meaning as defined above in $y$ direction. We write the equations of motion for the two masses as

$$m_{eff} = M_1 + \frac{m_2\omega^2_0}{\omega^2_0 - \omega^2}$$

(1)

The forces are presented as

$$m_1\ddot{u}_x = k_1 \left( u_x - u_{1x} \right)$$

$$m_1\ddot{u}_y = k_2 \left( u_y - u_{1y} \right)$$

$$m\ddot{u_x} = k_1 \left( u_{2x} - u_x \right) + F_x$$

$$m\ddot{u_y} = k_2 \left( u_{2y} - u_y \right) + F_y$$

(2)
\[ F_x = F_{0x} e^{-i\omega t}, \quad F_y = F_{0y} e^{-i\omega t} \]  

while the displacements are represented as:

\[ u_x = u_{0x} e^{-i\omega t}, \quad u_y = u_{0y} e^{-i\omega t}, \]
\[ u_{tx} = u_{0tx} e^{-i\omega t}, \quad u_{ty} = u_{0ty} e^{-i\omega t} \]  

The solution for the steady-state harmonic motion can be obtained by substituting Eqs. (3) and (4) into Eq. (2), we can obtain the relationships between the outer mass and the displacements in the two principal directions \(x\) and \(y\):

\[ -m\omega^2 u_{0x} = k_1 \left( \frac{\omega_1^2}{\omega_1^2 - \omega^2} u_{0x} - u_{0x} \right) + F_{0x} \]  
\[ -m\omega^2 u_{0y} = k_2 \left( \frac{\omega_2^2}{\omega_2^2 - \omega^2} u_{0y} - u_{0y} \right) + F_{0y} \]

where \(\omega_1^2 = k_1/m\) and \(\omega_2^2 = k_2/m\) are the local resonant frequencies of the inner mass along the \(x\) and \(y\) directions, respectively. The effective mass can then be defined as:

\[ m_{eff,x} = m + \frac{\omega_1^2}{\omega_1^2 - \omega^2} m_1 \]  
\[ m_{eff,y} = m + \frac{\omega_2^2}{\omega_2^2 - \omega^2} m_1 \]

For the first numerical examples, we select the following material constants: \(m = 0.1 \ kg/m, m_1 = 0.3 \ kg/m, k_1 = 5 \ M/m, k_2 = 1 \). We plot the effective masses given by Eqns (6) and (7) in figure 3. It can be found that the anisotropy between \(m_{eff,x}\) and \(m_{eff,y}\) is mainly caused by the difference between the local resonant frequencies \(\omega_1^2\) and \(\omega_2^2\), which can be tuned through the design of the internal springs \(k_1\) and \(k_2\) along the \(x\) and \(y\) directions, respectively. Specifically, design of the local stiffness anisotropy in the coating layer is the key to achieving the anisotropic effective mass density of the metaconcrete \([9,10]\). It is apparent that the two principal effective masses are generally different for any given frequency. This simple principle will be used to create a metaconcrete with an anisotropic coating material with a circular resonant aggregate. It is easier to create different shapes of coating in concrete than change the resonant aggregate of the mix. Therefore, for this our numerical study the ellipsoidal coating will be used for the metaconcrete.
From the above equations, the approximate effective mass densities for a solid in both $x$ and $y$ directions can be given:

$$
\rho_{\text{eff},x} = \frac{1}{V} \left( m + \frac{\omega_{n,1}^2}{\omega_{n,1}^2 - \omega^2} m_1 \right) 
$$

$$
\rho_{\text{eff},y} = \frac{1}{V} \left( m + \frac{\omega_{n,2}^2}{\omega_{n,2}^2 - \omega^2} m_1 \right) 
$$

in which $V$ is the volume of the unit cell.

### 3. Design procedure for the proposed metaconcrete

From section 2 analysis, our concrete mixture can be modified to display negative effective densities in the two principal directions. The earlier methods used theoretically and experimentally by Mitchell group [11–14], only considered circular coating. This is has been proved to forbid elastic waves propagation in one direction in our earlier paper [3]. Here, ellipsoidal coating will be used instead of circular coating. This will pave the way to having different stiffness in $x$ and $y$ directions. The coating round the resonant aggregate can be made in ellipsoidal form experimentally. Hence, our unit cell model is shown in figure 4 where this can be realized. A reference properties of the structure is detailed in Table 1.

| Geometrical parameters | mm | Material parameters |
|------------------------|----|---------------------|
| a                      | 5.0| Mortar             |
|                        |    | Mass density 2500 kg/m$^3$ |
|                        |    | Rubber coat 900 kg/m$^3$ |
|                        |    | Lead 20,000 kg/m$^3$  |
| b                      | 9.1| Mortar             |
|                        |    | Young’s modulus 30 Gpa |
|                        |    | Rubber coat 0.01 GPa |
|                        |    | Lead 16 Gpa        |
| \( \Phi \)             | 3.0| Mortar             |
|                        |    | Poisson’s ratio 0.2 |
|                        |    | Rubber coat 0.49    |
|                        |    | Lead 0.44          |
| L                      | 20 | Mortar             |
|                        |    | Young’s modulus 30 Gpa |
|                        |    | Rubber coat 0.01 GPa |
|                        |    | Lead 16 Gpa        |

**Figure 3.** Effective mass density $M_{\text{eff}}$ as a function of $\omega$
4. Numerical simulations

In this second numerical example, COMSOL Multiphysics will be used to determine the effective mass density for both principal directions. Figure 5 (a) shows the mesh of the proposed metaconcrete. Due to geometric complexity of the microstructure, analytical-based methods cannot be applied directly for the determination of the effective dynamic mass densities, hence numerical method is used.

4. Numerical simulations

In this second numerical example, COMSOL Multiphysics will be used to determine the effective mass density for both principal directions. Figure 5 (a) shows the mesh of the proposed metaconcrete. Due to geometric complexity of the microstructure, analytical-based methods cannot be applied directly for the determination of the effective dynamic mass densities, hence numerical method is used.
5. Conclusions
This paper presented metaconcrete with varying densities in $x$ and $y$ directions normally called anisotropic density. It is well known that for isotropic materials, wave propagation behaviour is the same for any wave propagation direction. For anisotropic elastic metaconcrete, wave propagation behaviour is different along different directions and is direction-dependent. This designed concrete structures with resonant aggregate and ellipsoidal coating can serve as new methods to preparing concrete in civil engineering to prevent waves for a targeted frequency.

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