g-modes and the Solar Neutrino Problem

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ABSTRACT

We show that low-order g-modes with large enough amplitudes to affect significantly the solar neutrino fluxes would produce surface velocities that are $10^4$ times larger than the observed upper limits and hence are ruled out by existing data. We also demonstrate that any large-amplitude, short-period oscillations that grow on a Kelvin-Helmholtz time scale will require, to affect solar neutrino fluxes, a large amount of energy (for g-modes, $10^9$ times the energy in the observed $p-$mode oscillations) and a tiny amount of dissipation (for g-modes, $10^{-8}$ the dissipation rate of the $p$-modes).

1. INTRODUCTION

Press (1981) and Press & Rybicki (1981) have explored the possibility that hydrodynamic phenomena involving g-modes could change the conventional quasistatic picture of main-sequence evolution. The particular process that Press and Rybicki investigated involved the hydrodynamic coupling, by g-modes of high radial wavenumbers, between turbulent fluid motions at the base of the convective zone and shear fluid motions closer to the solar core. They concluded that the coupling was not sufficient to drive large time-dependent phenomena in the region in which the neutrinos are produced. A later study by Spruit (1987) suggested that the coupling is probably even smaller than estimated by Press (1981).

The effect of low order g-modes on the solar interior has been considered by many authors following a series of stimulating initial discussions by Dilke & Gough (1972), Ulrich (1974), and Christensen-Dalsgaard, Dilke, & Gough (1974) (for recent references, see Merryfield, Toomre, & Gough 1991) of a possible instability associated with the nuclear burning of $^3$He. Most recently, Gough (1991) has considered a schematic model in
which hypothesized very large amplitude $\delta T/T \sim 0.1$ temperature fluctuations in the center of the sun are
driven by low order, low degree (small radial and angular wave numbers) g-mode oscillations. Gough (1991)
and De Rújula & Glashow (1992) have suggested that the hypothesized temperature fluctuations could
lower significantly the neutrino fluxes calculated in the standard solar model and produce an observable
time dependence in the $^8\text{B}$ neutrino flux that would be detected in the second generation solar neutrino
detectors now under construction. As emphasized by Gough (1991) and by De Rújula & Glashow (1992), it
is important to evaluate the possible effects of g-modes on the solar neutrino fluxes since the existing conflict
between predictions made using the standard solar model and the results of the four operating solar neutrino
experiments have important implications for astronomy and physics.

In this paper we concentrate primarily on the constraints on large amplitude g-modes, whose existence
has been hypothesized by Gough (1991) and De Rújula & Glashow (1992) to resolve the solar neutrino
problem. However, in § 2, we first describe some order-of-magnitude considerations that show why it is
difficult for low order g-modes, or any similar short-period oscillations that grows on a Kelvin-Helmholtz
time scale, to influence significantly the neutrino fluxes. In §3, we present the results of detailed calculations
of the g-mode eigenfunctions that show that temperature fluctuations large enough to significantly affect
the solar neutrino fluxes would produce characteristic velocity fluctuations on the solar surface that are $10^4$
times larger than the observed upper limits. We summarize our conclusions in §4.
In order to produce temperature fluctuations that significantly affect the neutrino fluxes, the hypothesized g-modes must have large amplitudes. In the linear regime, the g-mode amplitude is approximately proportional to the fractional temperature change, $\delta T/T$. Gough (1991) estimates that a periodic fluctuation in the central temperature of order $\delta T/T \sim 0.1$ is required to solve the solar neutrino problem. This result is consistent with the fact that different central temperatures with a range of order $\delta T_{\text{central}}/T_{\text{central}} \sim 0.02$ yield calculated neutrino fluxes within the quoted standard theoretical uncertainties (Bahcall & Ulrich 1988).

The energy content of low-order g-modes is approximately

$$E_g \sim M_{\text{core}} V^2 \sim (\delta T/T)^2 M_{\text{core}} R_{\text{core}}^{-2} \tau_{\text{period}}^{-2} \sim (\delta T/T)^2 10^{45}\text{erg},$$

where $V$ is fluid velocity associated with the mode, $M_{\text{core}} \simeq 0.2 M_\odot$ and $R_{\text{core}} \sim 0.15 R_\odot$. In deriving the second part of the above equation, we have made use of the fact that the radial wavelength of low order g-modes is $\sim R_{\text{core}}$. This energy content is much larger than the energy in the well-observed and accurately-studied solar p-modes (Leibacher et al. 1985; Toomre 1986; Brown, Mihalas, & Rhodes 1986; Libbrecht & Woodard 1991; Christensen-Dalsgaard & Berthomieu 1991; Guzik & Cox 1992). Using the result for the total energy in approximately ten million solar p-modes (Libbrecht & Woodard, 1991) one finds that

$$\frac{E_g}{E_p} > (\delta T/T)^2 10^{11}.$$  

Thus, for the required $(\delta T/T) = 0.1$, the energy content of the hypothesized g-modes is more than $10^9$ times the energy content of the p-modes.

The huge amount of energy required in the large-amplitude g-modes severely restricts the possible physical mechanisms for their excitation. The source of this energy must be either the nuclear fuel in the solar core or the turbulence in the convective zone. The energy in g-modes due to convective excitation (e.g. Goldreich & Keeley 1977, Spruit 1987, and Goldreich & Kumar 1988) is estimated to be small – on the order of the kinetic energy in a turbulent eddy of turnover time equal to the mode period. For g-modes of period about an hour, this convective energy is more than ten orders of magnitude smaller than required to
significantly affect the solar neutrino fluxes.

A number of authors have investigated a possible instability associated with the high temperature sensitivity of $^3$He burning in the solar core (Dilke & Gough 1972; Rosenbluth & Bahcall 1973; Christensen-Dalsgaard et al. 1974; Ulrich 1974; Saio 1980 and references quoted therein). It should be noted that the results appear to depend upon the choice of the solar model used as well as the modeling of the interaction of oscillation with radiation and turbulence near the solar surface. Rosenbluth & Bahcall (1973), using an accurate solar model and the full detail of the nuclear reactions, found $\ell=1$ modes to be stable. All authors agree that if the instability is present, it is largest for modes of low spherical harmonic degree ($\ell$) and order $n$. In fact most authors find that g-modes of degree 2 and higher are stable in the present sun (Boury et al. 1975, Shibahashi et al. 1975, and Saio 1980), which is not surprising since the radiative damping increases with the square of the inverse wavelength or $\ell^2$, whereas driving is expected to be roughly constant for low order g-modes.

Can low order g-modes attain the large amplitudes required to modify the solar neutrino fluxes? The following considerations suggest that this is unlikely. The ratio of period to characteristic growth time, $Q^{-1}$, for g-modes is

$$Q^{-1} = \frac{1}{10^7 y} \sim 10^{-11}.$$  

We have used in Eq. (3) the fact that the characteristic growth time for low order g-modes is of order the Kelvin-Helmholtz time (see, e.g., Christensen-Dalsgaard et al. 1974). Thus modes will be stabilized if more than one part in $10^{11}$ of the energy is dissipated per cycle (Press 1986). It is hard to imagine that such a dissipationless instability will be found. We note that the observed p-modes have a fractional dissipational rate of $Q^{-1} \sim 10^{-3}$, eight orders of magnitude larger than the Q-value of g-modes required to change the solar neutrino fluxes.

Press (1986) has suggested that the energy dissipation in the convection zone associated with the interaction with convective eddies will be an important limiting process. In fact, according to our numerical calculations, the dissipation time due to turbulent viscosity for the $g_1$ mode ($n=1$) of $\ell=1$ is about a factor of 40 smaller than its growth time of $10^{7}$ years. The turbulence dissipation rates for $g_2$ and $g_3$ modes of
\(\ell=1\) are about a factor of four greater than their growth rates due to the \(^3\text{He}\) instability, and the dissipation rates for \(\ell=2\) modes are about a factor of 30–100 greater than their growth rates. We have calculated the turbulent dissipation rate, following Goldreich and Keeley (1977), by assuming an effective eddy viscosity \((\lambda V_\lambda\), where \(\lambda\) and \(V_\lambda\) are the size and the velocity of the largest turbulent eddies such that \(V_\lambda/\lambda\) is equal to or greater than the mode frequency \(\omega\)), and using the viscous stress tensor for nonradial oscillations (see Landau and Lifshitz, 1982). Due to uncertainties associated with the modeling of turbulent viscosity our results are uncertain by a factor of a few, but we think it unlikely that the results are off by a factor of 40 (for comparison the computed linewidth of p-modes, around 3mHz, due to turbulent dissipation is within a factor of 2 of their observed linewidths). Therefore \(g_1\) modes of low degree, if subjected to an instability acting on Kelvin-Helmholtz time scale, will be stabilized due to turbulence dissipation. The dissipation rates for modes of \(\ell=1\) and order \((n)\) greater than one are close to their calculated growth rates, and thus, for these modes, we can not conclude that the \(^3\text{He}\) instability in the core, if present, is suppressed. We note that several people have included the perturbation to the convective flux in their linear stability calculations (e.g. Boury et al. 1975, and Saio 1980). However, we are not aware of any previous paper that has considered the turbulence dissipation of g-modes, which we find to be an important process. In the future, the interaction of convection with oscillations should be calculated carefully by proponents of any instability.

Suppose, despite the difficulties, that the low degree g-modes manage to grow to large amplitudes. Could one hide these large amplitude waves in the solar core? Gravity waves propagate only in a stably stratified medium, and are evanescent in the outer third of the sun, which is unstable to convection. One might try to conceal large amplitude g-modes inside the core by assuming that they will be dissipatively damped outside the core and so will go undetected at the solar surface. As is shown in the following section, if the dissipation time, \(\tau_{\text{dissipative}}\), is much longer than the mode period then one would expect to see surface velocities associated with the g-modes that are much larger than the current observational limits. If one tries to avoid the observational limit by assuming \(\tau_{\text{dissipative}}\) to be of order the mode period, then the effective luminosity due to g-modes, \(L_{\text{effective, g}}\), is much larger than the standard radiative luminosity, \(L_{\text{effective, g}} = \frac{E_g}{\tau_{\text{dissipative}}} \sim \left(\frac{\delta T}{T}\right)^2 10^8 L_\odot\). Intuitively, one would expect that such a large perturbation from the standard solar model would show up in some discrepancy between the standard model and the many
available observations of the sun. In any event, the scenario represented by this huge luminosity would not yield a self-consistent solution since the hypothesized damping rate is much greater than the calculated growth rates.

In the next section we present results of the numerical calculation of solar g-mode eigenfunctions which show that the attenuation factor for low degree g-modes in the convection zone is not large, and so they should be easy to detect at the surface. But, in the spirit of the order-of-magnitude estimates of this section, we note that the radial wavenumber for a low frequency high degree gravity wave, which is of course imaginary in the convection zone, is roughly equal to the horizontal wavenumber or \( \ell/r \). Therefore, g-mode amplitude varies approximately as \( 1/r^\ell \) in the convection zone, and so large \( \ell \) g-modes will have small amplitudes at the surface. However, amplitudes of low degree modes at the surface are not much smaller than the value at the bottom of the convection zone. Finally, note that the velocity amplitude, \( V_{\text{core}} \), and the temperature fluctuation amplitude, \( (\delta T)_{\text{core}} \), in the core region for the g-modes of frequency \( \omega \) are related by \( (\delta T/T)_{\text{core}} \sim (dV_{\text{core}}/dr)/\omega \sim V_{\text{core}}/(\omega R_{\text{core}}) \). Thus, \( V_{\text{core}} \sim (\delta T/T)_{\text{core}}R_{\text{core}}\omega_{\text{core}} \sim 10^7(\delta T/T)_{\text{core}} \) cm s\(^{-1}\); these relations are derived by making use of the linearized mass and entropy equations.

3. EIGENMODES AND EIGENVELOCITIES

We have calculated illustrative g-mode eigenfunctions and frequencies for a standard solar model by numerically integrating adiabatic wave equations which includes the perturbation to the gravitational potential (the relevant equations and boundary conditions are described in Christensen-Dalsgaard & Berthomieu 1991 and in Kumar et al. 1992).

Table 1 gives the calculated characteristics of some typical g-modes with relatively small number of nodes. The Table presents, as a function of the radial node, \( n \), and the angular degree, \( \ell \), the period, the mode energy, and the amplitude of the temperature variation. We have normalized the mode energy and the temperature fluctuation to a RMS surface velocity of 1 cm/s (averaged over spherical surface and time). For this normalization, the typical temperature variations in the solar core, \( \delta T/T \), are between \( 10^{-7} \) and \( 10^{-6} \). The energy in each mode scales approximately as \( (\delta T/T)^2 \). If we require that the temperature fluctuation be large enough to change the \(^{8}\)B solar neutrino flux, \( (\delta T/T) = 0.1 \), we obtain mode energies (see the table)
Table 1. Some Characteristics of Typical Low-order Solar g-modes.

| $n$ | $l$ | $\nu$ (µHz) | Period (hr) | Energy† (erg) | $\delta T/T$† |
|-----|-----|-------------|------------|--------------|--------------|
| 1   | 1   | 267.4       | 1.04       | $3.4 \times 10^{31}$ | $6.7 \times 10^{-8}$ |
| 1   | 2   | 297.1       | 0.94       | $2.1 \times 10^{31}$ | $4.4 \times 10^{-8}$ |
| 1   | 3   | 340.2       | 0.82       | $4.3 \times 10^{31}$ | $7.7 \times 10^{-8}$ |
| 2   | 1   | 193.0       | 1.44       | $1.3 \times 10^{33}$ | $4.6 \times 10^{-7}$ |
| 2   | 2   | 259.7       | 1.07       | $3.7 \times 10^{31}$ | $7.2 \times 10^{-8}$ |
| 2   | 3   | 299.2       | 0.93       | $4.4 \times 10^{31}$ | $8.8 \times 10^{-8}$ |
| 3   | 1   | 153.7       | 1.81       | $1.7 \times 10^{33}$ | $6.9 \times 10^{-7}$ |
| 3   | 2   | 224.0       | 1.24       | $8.7 \times 10^{31}$ | $1.3 \times 10^{-7}$ |
| 3   | 3   | 264.7       | 1.05       | $3.3 \times 10^{31}$ | $8.1 \times 10^{-8}$ |
| 4   | 1   | 128.3       | 2.17       | $1.7 \times 10^{33}$ | $7.4 \times 10^{-7}$ |
| 4   | 2   | 195.5       | 1.42       | $1.6 \times 10^{32}$ | $2.0 \times 10^{-7}$ |
| 4   | 3   | 240.5       | 1.15       | $2.9 \times 10^{31}$ | $7.4 \times 10^{-8}$ |
| 5   | 1   | 109.9       | 2.53       | $1.7 \times 10^{33}$ | $8.7 \times 10^{-7}$ |
| 5   | 2   | 171.8       | 1.62       | $3.1 \times 10^{32}$ | $3.1 \times 10^{-7}$ |
| 5   | 3   | 218.9       | 1.27       | $6.8 \times 10^{31}$ | $1.3 \times 10^{-7}$ |

† Normalized to a velocity of 1 cm/s at the solar surface.

in agreement with those estimated in the previous section, $\sim 10^{43}$ erg.

The characteristic velocities at the surface scale proportionally to the hypothesized amplitude of the oscillation, i.e., proportional to $(\delta T/T)$, and hence can be estimated directly from Table 1. We find that the surface velocities for the low degree g-modes with $(\delta T/T) = 0.1$ are typically a km/s, approximately $10^4$ times larger than the typical observed limits of 10 cm/s (see Kuhn, Libbrecht, & Dicke 1986). According to our numerical calculations, g-modes of degree less than about 15 can not simultaneously satisfy both the large energy requirement for the solution to the neutrino problem as well as the observational upper limit on the surface velocity amplitude. From this we conclude that g-modes of degree less than 15, with large enough amplitude to solve the neutrino problem, are ruled out by current observations.

One might try to avoid this contradiction with observation by proposing that the g-modes are damped by dissipative processes that do not occur in the core but instead occur outside the region in which the
neutrinos are produced. However, the damping required to decrease the large surface amplitude to a value below the observational limit is so strong that it will kill the instability responsible for mode excitation.

Another possible way to hide large amplitude g-modes in the solar core is to appeal to modes of degree greater than approximately 15. As discussed earlier, high degree g-modes, with $\delta T/T$ of 0.1 in the core, will have surface velocities that are below the observational limits. However, there are several problems with this proposal. First, no one finds that these high-$\ell$ modes are overstable; so one needs to appeal to an unknown mechanism for their excitation. Moreover, the mechanism that excites $\ell=15$ modes is very likely to excite modes of lower degree as well to similar energies; and the energies of lower degree modes are subject to the tight observational constraints. Also, the requirements on the total mode energy and on the temperature fluctuation required to change the neutrino fluxes are more stringent for higher degree modes than for the previously discussed case of a single, low-order g-mode.

Transferring the energy from low $\ell$ modes to high $\ell$ modes will also not work. The rate at which energy is injected into the high $\ell$ modes would be approximately equal to the total energy in low degree modes, $E_{\text{low-}\ell}$ divided by their characteristic growth time, $\tau_{\text{low-}\ell}$, of order a few million years (see Saio 1980). It is easy to see that the total energy in the high $\ell$ modes must be less than $(\tau_{\text{high-}\ell}/\tau_{\text{low-}\ell}) \times E_{\text{low-}\ell}$. The characteristic dissipation time for the high $\ell$ modes, $\tau_{\text{high-}\ell}$, is $\sim 10^{5.5}$ y (Shibahashi et al. 1975). Therefore, the total energy in the high $\ell$ modes is expected to be an order of magnitude less than the (inadequate) total energy in the low degree modes. As we have seen earlier, the observational upper limits on the surface velocities of low degree g-modes require that their energies be small, $10^{-8}$ times the energy needed to solve the neutrino problem.

4. DISCUSSION AND CONCLUSIONS

The order-of-magnitude considerations discussed in § 2 apply to any hypothesized large-amplitude, short-period oscillations that grow on a Kelvin-Helmholtz time scale in the solar core and that one might consider as a possible solution of the solar neutrino problem. In particular, such solutions require large amounts of energy, Eq. (2) (for $g$-modes, $10^9$ times the energy in the observed $p$-modes), and a tiny amount of dissipation, Eq. (3) (for $g$-modes, $10^{-8}$ the fractional dissipation rate of the $p$-modes). We find that for
low degree $g_1$ modes the turbulence dissipation rate is larger by a factor of about 40 than the growth rate due to the $^3$He instability. The uncertainty in the calculation of the turbulence dissipation rate is unlikely to be as large as a factor of 40, and so we believe that low degree $g_1$ modes are stabilized by turbulence dissipation.

By explicit calculation, we show in § 3 that low order g-mode oscillations with large enough amplitudes to affect the calculated solar neutrino fluxes would produce large periodic velocity shifts at the solar surface that are not observed. We conclude that it is unlikely that the solution of the solar neutrino problem depends upon large amplitude oscillations of the kind explored here.

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