Trajectory tracking control of tracked vehicles considering nonlinearities due to slipping while skid-steering

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ABSTRACT
In many applications, mobile vehicles operate over rough terrains to handle certain tasks and missions. Therefore, they are designed with tracks to deal with such irregular surfaces. Motion control and steering of tracked vehicles under the influence of slipping while skidding are still an interesting topic for many specialists and researchers and need further investigation. In fact, modelling and control of skid-steered tracked vehicles with slipping are very challenging. In this paper, various control schemes are proposed to investigate the closed-loop dynamical performance of tracked vehicles using different desired trajectories. Three control strategies are formulated and tuned to handle the undesirable effect of slipping while skidding: (i) robust nonlinear controller, (ii) speed compensation-based fuzzy logic controller, and (iii) speed compensation-based proportional–integral controller. The proposed control schemes are designed to explore new methods of skid-steering control with slipping. Preliminary simulation results are introduced to verify the effectiveness of the proposed controllers, using different desired trajectories, and to demonstrate the feasibility of utilizing such controllers.

1. Introduction
Mobile vehicles have a vast range of applications where they can autonomously operate in various environments (Adascalitei & Doroftei, 2011). It has been proven that they can be employed to accomplish different tasks in medical, agricultural, industrial, land mining, exploration, defense, security, rescuing, and military applications (Adascalitei & Doroftei, 2011; Zhai et al., 2016; Zhang et al., 2017). Unmanned ground vehicles are widely used in many industries where repetitive tasks or high-risk missions, in an inaccessible environment or within a hostile terrain, are required (Al-Milli et al., 2010; Durmus et al., 2015). Therefore, such vehicles are mostly equipped with tracks (i.e. tracked vehicles) in order to provide high traction even on slippery surfaces in addition to supporting heavy load with high performance and power delivery efficiency (Mossavian & Kalantari, 2008). Moreover, a significant advantage can be observed in performing small radius cornering maneuvers (Zhang et al., 2017) in addition to having a stable locomotion on loose and uneven terrain in comparison with wheeled vehicles (Zou et al., 2018). However, tracked vehicles consume a significant amount of energy to steer or turn due to the skidding they perform or due to the tracks’ slippage.

Skid-steering can be accomplished by varying the tracks thrust individually. Consequently, a turning moment is generated to overcome the moment of turning resistance, which is caused by the tracks skidding on the ground, and rotational inertia of the vehicle (Zhang et al., 2017). In fact, skid-steering depends on varying the relative velocities of the two tracks that, in turn, causes a slippage and soil shearing in order to achieve an effective steering (Zhang et al., 2017). Actually, they cause inaccurate kinematics and dynamics due to the highly nonlinear effect of slippage while skid-steering that is difficult to quantify (Le, 1999; Mossavian & Kalantari, 2008; Wong, 2008; Zhang et al., 2017). The skid-steering in the presence of slipping and soil shearing becomes worse as the vehicle moves over uneven terrains. The nonlinear behaviour varies significantly and thus increases the uncertainties and inaccuracies in the system model. Hence, predicting the exact motion of the tracked vehicle is difficult and designing the motion controller would become a challenging task (Zou et al., 2018).

Since tracked vehicles modelling is crucial, various number of kinematics and dynamics models were developed for tracked vehicles. These models were developed either (i) mathematically based on Newton’s laws that
depend on soil mechanics and vehicle dynamics (Aleksey & Vadym, 2012; Ferretti & Girelli, 1999; Le, 1999; Martinez et al., 2005; Tang et al., 2017; Wang et al., 2014; Wong, 2008) or (ii) depending on empirical formulas, which make them of no use for online predictions (Al-Milli et al., 2010). Wong (2008) derived a general theory for skid-steering that analytically predicts track thrusts as well as longitudinal and lateral moments acting on the tracks. Currently, Wong approach of the track-terrain interaction has been well studied where it is believed to be the most rational formulation of track–terrain interaction than others (Tang et al., 2017).

As for controlling the motion of tracked vehicles, it is not an easy task. The presence of high-order dynamics, system uncertainties, and inherent nonlinearities remain big challenges despite of many proposed control solutions in the literature as in Banihani et al. (2021), Gu et al. (2021), Le (1999), Li (2001), Saeedi et al. (2005), Tian and Sarkar (2014), Zhai et al. (2016) and Zhang et al. (2017) where the authors did not directly tackle the slipping of the vehicle while skid-steering. For instance, the authors of Zhai et al. (2016) proposed an optimization-distribution-based closed-loop control strategy for the lateral stability of tracked vehicles without considering the slipping. Moreover, the authors of Zhang et al. (2017) proposed a dynamic controller for an unmanned skid-steering vehicle where the steering was controlled via a hydraulic braking system on each side of the vehicle to achieve a desired yaw rate. In Gu et al. (2021), a path-tracking control algorithm of tracked mobile robots was proposed. The algorithm was based on preview linear model predictive control, which was used to achieve autonomous driving in the unstructured environment under an emergency rescue scenario. Finally, the authors of Banihani et al. (2021) proposed new control approaches for unmanned ground vehicles that utilize skid-steering system where an energy based variable structure control scheme was utilized.

In this paper, various control techniques are proposed for tracked vehicles to investigate and explore new methods of skid-steering control with slipping. A Robust Nonlinear Controller (RNC), Speed Compensation-based Fuzzy Logic Controller (SCFLC), and Speed Compensation-based Proportional–Integral Controller (SCPIC) are designed and tested to actively force a tracked vehicle to follow a prescribed desired trajectory under the presence of slipping while skid-steering. The performance of the proposed control schemes are tested in simulation and the results demonstrated a satisfactory performance when tracking different desired prescribed trajectories with fixed desired vehicle speed. It should be noted that a preliminary attempt to model the effect of slipping while skidding in tracked vehicles was conducted in Salah and Al-Jarrah (2019). However, the dynamics were completely modified along with the control design in the proposed study to better describe the slipping while skidding.

2. Mathematical model

The mathematical model of tracked vehicles (refer to Figure 1) is very well introduced in the literature. However, embedding the effects of slipping for skid-steering in the original model can be expressed in different ways.

2.1. Tracked vehicle kinematics

By considering the planner motion of the tracked vehicle shown in Figure 2, the kinematics model without the presence of slipping while skid-steering can be expressed as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & V \\
\sin \theta & 0 & \omega \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}
\end{bmatrix}
\]

(1)

where \(x, y, \theta\) are the position components and heading angle of the vehicle, \(\dot{x}, \dot{y}\) are the linear velocity components of the vehicle and \(\omega\) or \(\dot{\theta}\) is the rate of heading angle or vehicle angular velocity, and \(V\) is the vehicle forward linear velocity. It should be noted that the tracked vehicle kinematics, described in (1), are subject to the nonholonomic constraint, \(\dot{y} \cos(\theta) - \dot{x} \sin(\theta) = 0\), which specifies that the vehicle cannot move laterally along the y-axis of the vehicle coordinates shown in Figure 2. The vehicle

Figure 1. An example of a tracked vehicle (Tinger Track 2, 2010).
By combining Equations (2)–(4) and (6), the following expression can be obtained for the path curvature radius while the vehicle is manoeuvering:

\[
\rho = \frac{d(V_{out} + V_{in})}{2(V_{out} - V_{in})} = \frac{d[r_{out} + r_{in}]}{2[r_{out} - r_{in}]}.
\]

Introducing slippage into the vehicle kinematics while skid-steering can be expressed in different ways (Wong, 2008). One way is by utilizing the slip parameters, \(i_{out}\) and \(i_{in}\), in the tracks speeds to mimic the slippage where they represent the longitudinal slips of the outer and inner tracks, respectively. Hence, the radius of curvature can be expressed under the presence of slipping while skid-steering as follows:

\[
\rho_s = \frac{d(V_{out} + V_{in})}{2(V_{out} - V_{in})} = \frac{d[r_{out}(1 - i_{out}) + r_{in}(1 - i_{in})]}{2[r_{out}(1 - i_{out}) - r_{in}(1 - i_{in})]}
\]

where \(V_{out}\) and \(V_{in}\) represent the outer and inner tracks velocity considering the slipping effect. In fact, the slip values, \(i_{out}\) and \(i_{in}\), depend on the nature of the tracks and terrain as well as the thrust forces. They can be determined experimentally as explained in Mossavian and Kalantari (2008) and Wong (2008). Moreover, the slip values can be approximated using the following equations (Le, 1999):

\[
i_{out} = \frac{K}{I \times \ln \left(\frac{F_{out}}{F_{max}}\right)}, \quad i_{in} = \frac{K}{I \times \ln \left(\frac{F_{in}}{F_{max}}\right)}
\]

where \(K\) is the soil shear deformation modulus, \(I\) is the length of the track, \(F_{out}\) and \(F_{in}\) are the thrust forces on the outer and inner tracks, respectively, \(F_{max} = 0.5(A\psi + W\tan(\theta))\), \(A\) is the track contact area, \(\psi\) is a coefficient that depends on the nature of the ground or soil, and \(W\) is the vehicle weight.

### 2.2. Tracked vehicle dynamics

Assuming the vehicle is manoeuvering at low speed, the centrifugal force may be neglected in this case and the dynamics of the tracked vehicle in the translational and rotational motions can be expressed as (refer to Figure 2):

\[
m\ddot{V} = \frac{1}{r} (\tau_{out} + \tau_{in}) - R
\]

\[
J\ddot{\omega} = \frac{d}{2r} (\tau_{out} - \tau_{in}) - M
\]

where \(m\) is the mass and \(J\) is the moment of inertia of the entire tracked vehicle, respectively, and \(\tau_{out}\) and \(\tau_{in}\) are the outer and inner driving torques for each track, respectively. It should be noted that they can be computed from the thrust forces, \(F_{out}\) and \(F_{in}\), such that \(\tau_{out} = rF_{out}\) and \(\tau_{in} = rF_{in}\). The parameters \(R\) and \(M\) are the total longitudinal resisting force and the turning resisting moment exerted on the tracks by the ground, respectively.

Since the ultimate goal is to control the vehicle while turning at high constant speed, the centrifugal force, \(F_c\), would be significant in this case and its effect should be...
tracks required to maintain the vehicle steady while turning at high speed. In order to determine the thrust forces, $M$ and $R$ must be evaluated taking into consideration the lateral resistance forces, centrifugal force, pressure distribution along each track from the ground, and the load transfer between sides of the vehicle. Moreover, $M$ and $R$ depend on the vehicle weight, geometry of contact patch between the track and ground, and coefficient of friction between the track and ground. Since the coefficient of friction and contact patch, for a given track, varies due to the nature of ground and its contour, a few assumptions has to be made. It is assumed that each track carries an equal part of the vehicle weight and the normal pressure is uniformly distributed along each track. With reference to Figures 3 and 4, in addition to the aforementioned assumptions, applying Newton’s second low results in the following resisting forces:

$$f_{resist}^\text{out} = R_{out} + \frac{F_{clg}}{2} + \frac{M}{d},$$

where $R_{out} \approx \left(\frac{mg}{d} + \rho h V^2 \frac{d}{dh}\right)\mu_{lg}$ and $R_{in} \approx \left(\frac{mg}{d} - \rho h V^2 \frac{d}{dh}\right)\mu_{lg}$ are the longitudinal resistance forces of the outer and inner tracks, respectively, $F_{clg} = \frac{\mu_{lg} V^2}{d}$ is the longitudinal component of the centrifugal force with $D = \frac{h^2 V^2}{2\mu_0 g}$, $g$ is the gravitational acceleration, and $\mu_{lg}$, $\mu_{lt}$ are the longitudinal and lateral friction coefficients, respectively. Note that $M = \frac{\mu_{lg} mgl}{4} \left(1 - \frac{V^4}{2g^2 \rho^2 \sigma^2}\right)$ was earlier introduced in Equation (11). The parameter $\mu_{lt}$ depends on the vehicle translational velocity, $V$, and the type of tracks contour. Also, the value of $\mu_{lt}$ depends on the nature of ground and the type of track contour. Wong (2008) proposed an experimental equation to estimate the average values of $\mu_{lg}$ and $\mu_{lt}$ for military tracked vehicles.

Since motors are used to drive the tracked vehicle, their torques must overcome the slipping dynamics illustrated in Equation (12) through the thrust forces $F_{out}$ and $F_{in}$. Assuming the outer and inner driving motors are identical, and their corresponding damping resisting torques are very small in comparison with the vehicle resisting torques due to slipping, the dynamics equations of the outer and inner driving motors can be expressed, respectively, as follows:

$$\tau_{out} = J_m \dot{\omega}_{out} + \tau_{resist}$$

$$\tau_{in} = J_m \dot{\omega}_{in} + \tau_{resist}$$

where $J_m$ is the moment of inertia of the motor, $\dot{\omega}_{out}$, $\dot{\omega}_{in}$ are the outer and inner motors angular accelerations of the tracks, respectively, and $\tau_{resist} = r_{resist}^\text{out}$, $\tau_{resist} = r_{resist}^\text{in}$. 

![Figure 3. A schematic diagram of a tracked vehicle while turning at high speed.](image1)

![Figure 4. Vehicle front view while turning at high speed.](image2)
3. Controllers formulation

In this section, three controllers for tracked vehicles are developed and proposed to explore new methods of skid-steering control with tracks slipping. In addition, the developed control laws, introduced in the following subsections, utilized to force the tracked vehicle to track desired trajectories can be implemented in real-time without any computational burden. In other words, the practical implementation with the current advanced computing technology is easy and doable.

3.1. Robust nonlinear controller design

A Robust Nonlinear Controller (RNC) is designed and formulated to guarantee that the robot is following a desired trajectory in the following sense:

\[
\begin{align*}
x & \rightarrow x_d, \\
y & \rightarrow y_d, \\
\theta & \rightarrow \theta_d \\
& \text{as } t \rightarrow \infty (15)
\end{align*}
\]

where \(x_d, y_d, \theta_d\) are the desired position components and heading angle of the vehicle introduced in Equation (1). It should be noted that the tracks’ desired speeds, \(\omega_{\text{out}}^d\) and \(\omega_{\text{in}}^d\), are tracked by the RNC in the following sense:

\[
\begin{align*}
\omega_{\text{out}} & \rightarrow \omega_{\text{out}}^d, \\
\omega_{\text{in}} & \rightarrow \omega_{\text{in}}^d \\
& \text{as } t \rightarrow \infty (16)
\end{align*}
\]

where \(\omega_{\text{out}}^d\) and \(\omega_{\text{in}}^d\) are generated by the vehicle inverse kinematics based on \(x_d, y_d, \theta_d\) (refer to Equation (5)). In fact, control laws are designed to generate appropriate motor torques in the outer and inner tracks (i.e. \(\tau_{\text{out}}\) and \(\tau_{\text{in}}\)) in order to ensure achieving the control objective stated in (16) and hence in (15). Figure 5 depicts the vehicle control system using the proposed RNC. From Figure 5, it is clear that the RNC generates the torques of tracks motors (i.e. control inputs) needed to force the tracked vehicle to follow desired trajectories. In fact, the controller acquires the motors rotational speed errors of the outer and inner tracks to compute the appropriate tracks thrust. Hence, to facilitate the development of the proposed controller, the following error signals are defined:

\[
\begin{align*}
e_{\text{out}} & \triangleq \omega_{\text{out}}^d - \omega_{\text{out}}, \\
e_{\text{in}} & \triangleq \omega_{\text{in}}^d - \omega_{\text{in}} (17)
\end{align*}
\]

Hence, if \(e_{\text{out}}, e_{\text{in}} \to 0\) as \(t \to \infty\) then the statements in (16) are satisfied.

3.1.1. Open-loop error system dynamics

By taking the first time derivative of Equation (17) and utilizing the dynamics introduced in Equations (13) and (14), the open-loop error system dynamics can be written as:

\[
\begin{align*}
J e_{\text{out}} & = J \omega_{\text{out}}^d + \tau_{\text{resist}} - \tau_{\text{out}} (18) \\
J e_{\text{in}} & = J \omega_{\text{in}}^d + \tau_{\text{resist}} - \tau_{\text{in}} (19)
\end{align*}
\]

where the following inequalities can be satisfied:

\[
\begin{align*}
|J \omega_{\text{out}}^d + \tau_{\text{resist}}| & \leq \rho_{\text{out}} (20) \\
|J \omega_{\text{in}}^d + \tau_{\text{resist}}| & \leq \rho_{\text{in}} (21)
\end{align*}
\]

and \(\rho_{\text{out}}, \rho_{\text{in}} \in \mathbb{R}^+\) are bounding constants.

3.1.2. Closed-loop error system dynamics

In order to ensure the stability of the proposed RNC, a Lyapunov-based stability analysis is introduced (Marquez, 2003). Let \(P(z, t) \in \mathbb{R}\) denotes the following non-negative function:

\[
P \triangleq \frac{1}{2} J e_{\text{out}}^2 + \frac{1}{2} J e_{\text{in}}^2 (22)
\]

Note that by designing the appropriate control laws (to be shown in the following analysis), Equation (22) can be proven to be bounded as (refer to Theorem 2.14 of Qu [1998]):

\[
\lambda_1 ||z(t)||^2 \leq P(z, t) \leq \lambda_2 ||z(t)||^2 (23)
\]

where \(\lambda_1, \lambda_2 \in \mathbb{R}^+\) are constants and \(z \triangleq [e_{\text{out}}, e_{\text{in}}]^T \in \mathbb{R}\). By taking the first-time derivative of Equation (22), the following expression can be obtained:

\[
\dot{P} = e_{\text{out}} (J \omega_{\text{out}}^d + \tau_{\text{resist}} - \tau_{\text{out}}) + e_{\text{in}} (J \omega_{\text{in}}^d + \tau_{\text{resist}} - \tau_{\text{in}}) (24)
\]
where Equations (18) and (19) were utilized. The control inputs in Equation (24) can be designed as:

\[
\tau_{\text{out}} = k_{\text{out}} e_{\text{out}} + \rho_{\text{out}} \tag{25}
\]

\[
\tau_{\text{in}} = k_{\text{in}} e_{\text{in}} + \rho_{\text{in}} \tag{26}
\]

where \( k_{\text{out}}, k_{\text{in}} \in \mathbb{R}^+ \) are control gains, \( e_{\text{out}}, e_{\text{in}} \) were introduced in Equation (17), and \( \rho_{\text{out}}, \rho_{\text{in}} \) were introduced in Equations (20) and (21). It is clear from Equations (25) and (26) that the designed control inputs do not depend on any uncertain or unknown parameters. In fact, the bounding constants, introduced in (20) and (21), shall compensate for them.

By substituting the control inputs, designed in Equations (25) and (26), the expression in (24) can be upper bounded as:

\[
\dot{P} \leq \left| e_{\text{out}} \right| (|J_\omega_d^d + \tau_{\text{resist}}^\text{out} | - k_{\text{out}} |e_{\text{out}}| - \rho_{\text{out}}) \\
+ \left| e_{\text{in}} \right| (|J_\omega_d^d + \tau_{\text{resist}}^\text{in} | - k_{\text{in}} |e_{\text{in}}| - \rho_{\text{in}}) \tag{27}
\]

and by utilizing the bounding expressions in (20) and (21), the bounding statement in (27) becomes:

\[
\dot{P} \leq -k_{\text{out}} |e_{\text{out}}|^2 - k_{\text{in}} |e_{\text{in}}|^2, \tag{28}
\]

and then

\[
\dot{P} \leq -\lambda_3 |z|^2 \tag{29}
\]

where \( \lambda_3 = \min \{k_{\text{out}}, k_{\text{in}}\} \) and \( z(t) \) was introduced in (23). Hence, satisfying the globally asymptotically stability result. From the bounding statements in (23) and (29), it is clear that \( z(t) \) is bounded. Since the tracks’ desired speeds, \( \omega_{\text{out}}^d \) and \( \omega_{\text{in}}^d \) are assumed to be bounded, hence, \( \omega_{\text{out}} \) and \( \omega_{\text{in}} \) are bounded, and then \( x_d, y_d, \) and \( \theta_d \) are bounded using the inverse kinematics. At the end, the expression in (29) can be rewritten as:

\[
\dot{P} \leq -\frac{\lambda_3}{\lambda_1} \rho \tag{30}
\]

where the bounding expression in (23) was utilized.

### 3.2. Speed compensation-based FL controller design

In this control technique, the desired speeds (i.e. angular velocities) of tracks motors are adjusted to compensate for the slipping while skid-steering via a high-level controller. Figure 6 shows the vehicle tracking system for SCFLC and SCPIC, which they have the same structure except for the algorithm used to adjust the desired speeds of the tracks motors. The position and orientation errors are utilized to adjust the desired tracks motors angular speeds affected by the slipping while skid-steering. The adjusted desired angular speeds, \( \omega_{\text{out}}^{adj} \) and \( \omega_{\text{in}}^{adj} \), are then utilized in the vehicle dynamics by a low-level controller to overcome the total longitudinal resisting force and the turning resisting moment exerted on the tracks by the ground, M and R, and to ensure that the tracked vehicle tracks a prescribed desired trajectory with a constant desired vehicle speed as shown in Figure 7. It should be noted that the vehicle inverse kinematics are utilized to generate the desired angular speeds, \( \omega_{\text{out}}^d \) and \( \omega_{\text{in}}^d \) from the desired trajectory represented by \( x_d, y_d, \) and \( \theta_d \) (refer to Equation (5)). Due to slipping, those speeds must be adjusted using a high-level controller as mentioned earlier.

Due to slipping while skid-steering, the vehicle will be shifted out of trajectory although its driving motors are controlled using a low-level controller. As a result, a pose shift error, \( e_{\text{pose}} = [e_p, e_\theta]^T \in \mathbb{R} \), is generated. Hence, high-level controller is required to adjust the pose error and force the vehicle to track the desired trajectory. To guarantee the performance of the controller, the pose errors are considered in the design, which include the actual position and orientation of the vehicle. Hence, the following error signals are defined to facilitate the design of the high-level controller:

\[
e_p \triangleq \sqrt{e_x^2 + e_y^2}, \quad e_x \triangleq x_d - x, \quad e_y \triangleq y_d - y \tag{31}
\]

\[
e_\theta \triangleq \theta_d - \theta \tag{32}
\]
where $e_p$ is the position error (i.e. coordinates error) and $e_\theta$ is the orientation error (i.e. heading error). Hence, a SCFLC can be designed. The outputs of the SCFLC, denoted by the signals, $u_1$ and $u_2$, are the adjustment values for the desired outer and inner angular velocities of the vehicle tracks, to compensate for the slipping while skid-steering, where:

$$\omega_{\text{adj}}^{\text{out}} = \omega^{d}_{\text{out}} + u_1 \quad (33)$$

$$\omega_{\text{adj}}^{\text{in}} = \omega^{d}_{\text{in}} + u_2 \quad (34)$$

Figure 8 shows the inputs and outputs of the SCFLC.

3.3. Speed compensation-based PI controller design

In this control technique, the position and orientation error signals, $e_p$ and $e_\theta$, introduced in subsection 3.2 are utilized in the SCPIC. Hence, a SCPIC can be designed, as a high-level controller, as:

$$\omega_{\text{adj}}^{\text{out}} = \omega^{d}_{\text{out}} + k_1 e_p + k_2 e_\theta + k_3 \int_0^t e_p \, dt + k_4 \int_0^t e_\theta \, dt \quad (35)$$

$$\omega_{\text{adj}}^{\text{in}} = \omega^{d}_{\text{in}} + k_5 e_p + k_6 e_\theta + k_7 \int_0^t e_p \, dt + k_8 \int_0^t e_\theta \, dt \quad (36)$$

where $\omega_{\text{adj}}^{\text{out}}$ and $\omega_{\text{adj}}^{\text{in}}$ are the adjusted desired speeds that represent the control signals of the outer and inner tracks motors, respectively, $k_1$, $k_2$, $k_3$, $k_4$ are the controller gains of the outer track motor, and $k_5$, $k_6$, $k_7$, $k_8$ are the controller gains of the inner track motor. The same methodology, utilized for the SCFLC, is implemented for the SCPIC.

4. Numerical simulations

In this section, numerical simulations are introduced to verify the proposed controllers design and demonstrate their effectiveness in forcing the tracked vehicle to follow certain trajectories. Table 1 shows the proposed values of the tracked vehicle parameters used in the numerical simulations. The SCPIC and RNC parameter values are introduced in Table 2 and the control surfaces generated from the fuzzy rules in the SCFLC (i.e. if–then statements) are shown in Figures 9 and 10. Matlab/Simulink© is used to test the performance of the tracked vehicle with different trajectories. To verify the proposed controllers performance, four cases are introduced as shown in Table 3.

The cases introduced in Table 3 are chosen to demonstrate the effect of the slip on the tracked vehicle motion. The initial vehicle position was chosen to be $x = -0.2$ m and $y = -0.1$ m for cases I and II, and $x = -0.1$ m and $y = -0.2$ m for cases III and IV. The turning radius was
chosen to be 5 m for cases I and II, and the translational velocity of the vehicle was chosen to be high at 10 km/h for all cases to more challenge the controllers.

Figures 11 and 12 show the tracked vehicle response under the effect of 5% and 10% slip, respectively, according to cases I and II, introduced in Table 3, for all proposed controllers. One can notice the increase of the vehicle shift out of the trajectory while maneuvering as the slip increases from 5% to 10%. It is clear that the proposed controllers performed satisfactorily with a small margin of error. However, the SCFLC could not handle a long turn. In Figures 13 and 14, the outer and inner generated torques (i.e. control inputs) of tracks motors are shown for cases I and II, respectively, for all proposed controllers. It is clear that the generated torques are relatively smooth but exhibits a sudden spike at the beginning, which is normal for starting. However, when the slip is increased to 10% as stated in Case II (refer to Figure 14), more efforts are exerted from the outer track motor to force the tracked vehicle to follow the trajectory. As a comparison between the controllers performance in cases I and II, it is noted that the RNC tends to exert more torque from the tracks motors at the end of the ‘part of a circle’ trajectory. That is due the nature of RNC algorithm that depends on boundedness statement in (20) and (21).
To more challenge the proposed controllers, cases III and IV are introduced. The ‘winding’ trajectory is designed to have four consecutive turns (i.e. left, right, left, then right at the end). Figures 15 and 16 show the tracked vehicle response under the effect of 5% and 10% slip, respectively, according to cases III and IV, introduced in Table 3, for all proposed controllers. It is clear that the proposed controllers performed satisfactorily except for the SCPIC. Although SCPIC performed satisfactorily in cases I and II, it could not handle the consecutive hard turns along with the unknown system parameters and uncertainties. This could eliminate the use of SCPIC in maneuvering within a hard maneuvering. However, some improvement to the SCPIC design can be done such as adding an online estimator for the unknown parameters. In Figures 17 and 18, the outer and inner generated torques (i.e. control inputs) of tracks motors are shown for cases III and IV, respectively, for all proposed controllers. The generated torques of tracks motors are relatively...
smooth. However, it is clear that there are some spikes each time the tracked vehicle turns right or left in addition to the sudden spike at the beginning of motion, which is normal for starting. It is also observed that when the slip is increased to 10% as stated in Case IV (refer to Figure 18), more efforts are exerted from the outer track motor to force the tracked vehicle to follow the desire trajectory. It is also noted that since the SCPLIC does not perform well, it exerts less torques from the tracks motors and that is normal. More efforts are usually needed to provide better performance.

For the sake of quantifying the performance among the proposed controllers, an error and control effort measures (i.e. Position Error Measure (PEM), Orientation Error Measure (OEM), Outer Torque Effort Measure (OTEM), and Inner Torque Effort Measure (ITEM)) are defined as follows:

Figure 16. Tracked vehicle response for Case IV.

Figure 17. Generated torques in the tracks motors for Case III for all controllers.

Figure 18. Generated torques in the tracks motors for Case IV for all controllers.
its performance was very satisfactory when tested using the other side, the SCPIC performed the worst although uncertainties when tested using the ‘winding’ trajectory. On satisfying for the unknown system parameters and uncertainties, the RNC performed the best among the proposed controllers in terms of compensating the unknown parameters and uncertainties. However, the SCFLC sometimes compete with the SCPIC performed the worst among the proposed controllers. However, the SCFLC sometimes compete with the SCPIC performed the worst among the proposed controllers. However, the SCFLC sometimes compete with the SCPIC performed the worst among the proposed controllers.

5. Conclusions

Various control designs, namely RNC, SCFLC, and SCPIC are proposed and tested in simulation for tracked vehicles. The proposed controllers are designed and formulated to operate tracked vehicles under the effect of slipping while skid-steering. The results showed satisfactory performance in general. However, the RNC performed the best among the proposed controllers in terms of compensating for the unknown system parameters and uncertainties when tested using the ‘winding’ trajectory. On the other side, the SCPIC performed the worst although its performance was very satisfactory when tested using ‘part of a circle’ trajectory. It is intended, in the future, to implement the control schemes practically in addition to develop an online estimator for the uncertainties and unknown parameters.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Table 4. Error and effort measures for all controllers for ‘part of a circle’ trajectory.

| Controller | Slip [%] | PEM      | OEM       | OTEM       | ITEM     |
|------------|----------|----------|-----------|------------|----------|
| RNC        | 5        | 0.03662  | 7.878e-7  | 0.1549     | 0.07796  |
|            | 10       | 0.052   | 9.629e-5  | 0.2387     | 2.803    |
| SCFLC      | 5        | 0.04811  | 1.307e-5  | 0.2541     | 0.02704  |
|            | 10       | 0.08303  | 0.0003536 | 0.1992     | 2.285    |
| SCPIC      | 5        | 0.1038   | 0.00562   | 0.6856     | 2.395    |
|            | 10       | 0.088    | 0.001761  | 0.1386     | 0.1017   |

Table 5. Error and effort measures for all controllers for the ‘winding’ trajectory.

| Controller | Slip [%] | PEM      | OEM       | OTEM       | ITEM     |
|------------|----------|----------|-----------|------------|----------|
| RNC        | 5        | 0.03158  | 6.837e-6  | 0.09912    | 0.1184   |
|            | 10       | 0.02936  | 4.596e-5  | 0.2931     | 1.546    |
| SCFLC      | 5        | 0.05874  | 3.542e-5  | 0.1323     | 0.06057  |
|            | 10       | 0.05679  | 5.364e-5  | 0.3192     | 1.362    |
| SCPIC      | 5        | 0.1831   | 0.004402  | 0.5939     | 2.879    |
|            | 10       | 0.1575   | 0.003231  | 0.6961     | 2.09     |

where T is the entire simulation time. Tables 4 and 5 show the quantified controllers performance (i.e. error and effort measures) for both introduced trajectories in Table 3.

From Tables 4 and 5, it is clearly shown that the RNC performed the best, in all cases, in terms of the PEM and OEM. In addition, it exerted the least effort (i.e. motors power) in the outer and inner tracks motors. Obviously, the SCFLC performed the worst among the proposed controllers. However, the SCFLC sometimes compete with the RNC in terms of the OTEM and ITEM.

References

Adascalitei, F., & Doroftei, I. (2011). Practical applications for mobile robots based on Mecanum wheels - A systematic survey. The Romanian Review Precision Mechanics, Optics & Mechatronics, 40, 21–29.

Aleksy, K., & Vadym, K. (2012). Modeling and control of tracked mobile robot. Third International Scientific Congress, 50th Anniversary Technical University of Varna, Bulgaria.

Al-Milli, S., Seneviratne, L. D., & Althoefer, K. (2010). Track–terrain modelling and traversability prediction for tracked vehicles on soft terrain. Journal of Terramechanics, 47(3), 151–160. https://doi.org/10.1016/j.jterra.2010.02.001

Banihani, S., Hayajneh, M., Al-Jarrah, A., & Mutawe, S. (2021). New control approaches for trajectory tracking and motion planning of unmanned tracked robot. Advances in Electrical and Electronic Engineering, Mechatronics, 19(1), 42–56. https://doi.org/10.15598/ae ee.v19i1.4006

Durmus, H., Gunes, E., Kirk, M., & Ustundag, B. B. (2015). The design of general purpose autonomous agricultural mobile robot: AGROBOT. 4th International Conference on Agro-Geoinformatics, Turkey.

Ferretti, G., & Girelli, R. (1999). Modelling and simulation of an agricultural tracked vehicle. Journal of Terramechanics, 36(3), 139–158. https://doi.org/10.1016/S0022-4898(99)00004-X

Gu, Q., Bi, G., Meng, Y., Wang, G., Zhang, J., & Zhou, L. (2021). Efficient path tracking control for autonomous driving of tracked emergency rescue robot under 6G network. Wireless Communications and Mobile Computing, 1–9. https://doi.org/10.11155/2021/5593033

Le, A. T. (1999). Modeling and control of tracked vehicles [Doctoral thesis]. Department of Mechanical and Mechatronics Engineering, Australian Centre for Field Robotics, The University of Sydney.

Li, Y.-F. (2001). High precision motion control based on discrete-time sliding mode approach [Doctoral thesis]. Royal Institute of Technology, Stockholm, Sweden.

Marquez, H. (2003). Nonlinear control systems – analysis and design. John Wiley & Sons.

Martinez, J. L., Mandow, A., Morales, J., Pedraza, S., & Garcia-Cerezo, A. (2005). Approximating kinematics for tracked mobile robots. The International Journal of Robotics Research, 24(10), 867–878. https://doi.org/10.1177/0278364905058239

Mossavian, S. A., & Kalantari, A. (2008). Experimental slip estimation for exact kinematics modeling and control of a tracked mobile robot. IEEE/RSJ International Conference on Intelligent Robots and Systems, France.

Qu, Z. (1998). Robust control of nonlinear uncertain systems (1st ed.). John Wiley & Sons.

Saeedi, P., Lawrence, P. D., Lowe, D. G., Jacobsen, P., Kuslovic, D., Ardron, K., & Sorensen, P. H. (2005). An autonomous excavator with vision-based track-slippage control. IEEE...
Salah, M., & Al-Jarrah, A. (2019). Robust backstepping control for tracked vehicles under the influence of slipping and skidding. International Conference on Research and Education in Mechatronics, Wels, Austria, pp. 1–6.

Tang, S., Yuan, S., Hua, J., Li, X., Zhou, J., & Guo, J. (2017). Modeling of steady-state performance of skid-steering for high-speed tracked vehicles. *Journal of Terramechanics*, 73, 25–35. https://doi.org/10.1016/j.jterra.2017.06.003.

Zhai, L., Sun, T. M., Wang, Q. N., & Wang, J. (2016). Lateral stability control of dynamic steering for dual motor drive high speed tracked vehicle. *International Journal of Automotive Technology*, 17(6), 1079–1090. https://doi.org/10.1007/s12239-016-0105-y