Dynamical determination of the top quark and Higgs masses in the Standard Model

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Abstract

We consider the application of the multiple point criticality principle to the pure Standard Model, with a desert up to the Planck scale. According to this principle, Nature should choose coupling constant values such that the vacuum can exist in degenerate phases. Furthermore we require a strongly first order phase transition between the two vacua, in order that the dynamical mechanism be relevant. Thus we impose the constraint that the effective Higgs potential should have two degenerate minima, one of which should have a vacuum expectation value of order unity in Planck units. In this way we predict a top quark mass of $173 \pm 4$ GeV and a Higgs particle mass of $135 \pm 9$ GeV. A possible model to explain the multiple point criticality principle is the lack of locality in quantum gravity and the effects of baby universes which, on general grounds, are expected to make coupling constants dynamical.

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1 Introduction

There have been a number of attempts to determine the top quark mass $M_t$ dynamically. Some time ago, Veltman suggested that the fermionic and bosonic quadratic divergences in the Standard Model (SM) should cancel, in order that the ultra-violet cut-off for the theory could be very high. It is not clear at what scale this one-loop relation should be valid; but if applied at the electroweak scale it gives a SM Higgs particle mass of $M_H \approx 330$ GeV.

More recently, in a toy model, Nambu combined the Veltman condition for the cancellation of quadratic divergences with the idea that the vacuum energy density be minimised with respect to the Yukawa couplings, keeping all the other parameters fixed. This interesting idea of making the Yukawa couplings dynamical, constrained by the Veltman condition, naturally generates one Yukawa coupling (identified with the top quark) much larger than all the other ones.

The Nambu model immediately raises the question of whether there is a physical justification for treating the Yukawa couplings as dynamical variables rather than numerical parameters. This may be possible in superstring models, in which the Yukawa couplings depend on the vacuum expectation values (VEVs) of some gauge singlet scalar fields called moduli, corresponding to flat directions of the scalar potential. Thus their VEVs are determined by quantum corrections, with the possibility that some survive to the electroweak scale. In this case the low energy effective potential of the Minimal Supersymmetric Standard Model should be minimised with respect to the Yukawa couplings, possibly subjected to constraints depending on the number of moduli fields remaining at low energy. This minimisation tends to drive the top quark mass to its largest allowed value and hence close to its infra-red quasi-fixed point value.

In fact we expect, on rather general grounds, that all coupling constants become dynamical in quantum gravity due to the non-local effects of baby universes. However, except at very short distances of the order of the Planck scale, locality is only broken in the mild way that there is an effect which is the same at all points in the space-time manifold, and this effect depends on an averaging over all space-time. This translational invariant non-local effect is much less easy to reveal empirically, if it existed, than an effect that could be switched on and off. It will namely be conceived of as a modification of the parameters in the laws of Nature, the “coupling constants” we may call them, and non-locality in this mild form only means that the coupling constants depend on what has happened in the past and what will happen in the future (in the sense of an average over all of space-time). We have argued that this mild form of non-locality in quantum gravity, respecting reparameterisation invariance, leads to the realisation in Nature of the “multiple point criticality principle”. According to this principle, Nature should choose coupling constant values such that the vacuum can exist in degenerate phases, in a very similar way to the stable coexistence of ice, water and vapour (in a thermos flask for example) in a mixture with fixed energy and number of molecules.
Now it is well-known that the pure Standard Model, with one loop corrections say, can have two minima in the effective Higgs field potential. If really there were some reason for Nature to require phase coexistence, it would be expected that the “vacua” corresponding to these minima should be able to energetically coexist, which means that they should be degenerate. That is to say the effective Higgs potential should take the same value in the two minima: $V_{\text{eff}}(\phi_{\text{min} 1}) = V_{\text{eff}}(\phi_{\text{min} 2})$. This condition really means that the vacuum in which we live is just barely stable; we are just on the border of being killed by vacuum decay. With this assumption and the Fermilab top quark mass of $180 \pm 12$ GeV, it is easily read off from the vacuum stability curve that we predict the Higgs pole mass to be $149 \pm 26$ GeV from this degeneracy of the minima. Below we consider a prediction for both the top and Higgs masses, without using either the Fermilab or LEP results as phenomenological input.

In the analogy of the ice, water and vapour system, the important point for us is that by enforcing fixed values of the extensive quantities, such as energy, the number of moles and the volume, you can very likely come to make such a choice of these values that a mixture has to occur. In that case then the temperature and pressure (i.e. the intensive quantities) take very specific values, namely the values at the triple point. We want to stress that this phenomenon of thus getting specific intensive quantities only happens for first order phase transitions, and it is only likely to happen for rather strongly first order phase transitions. By strongly first order, we here mean that the interval of values for the extensive quantities which do not allow the existence of a single phase is rather large. Because the phase transition between water and ice is first order, one very often finds slush (partially melted snow or ice) in winter at just zero degree celsius. And conversely you may guess with justification that if the temperature happens to be suspiciously close to zero, it is because of the existence of such a mixture: slush. But for a very weakly first order or second order phase transition, the connection with a mixture is not so likely.

In the analogy considered in this paper the coupling constants, such as the Higgs self coupling and the top quark Yukawa coupling, correspond to intensive quantities like temperature and pressure. If the vacuum degeneracy requirement should have a good chance of being relevant, the “phase transition” between the two vacua should be strongly first order. That is to say there should be an appreciable interval of extensive variable values leading to a necessity for the presence of the two phases in the Universe. Such an extensive variable might be e.g. $\int d^4x |\phi(x)|^2$. If, as we shall assume, Planck units reflect the fundamental physics, it would be natural to interpret this strongly first order transition requirement to mean that, in Planck units, the extensive variable densities $\frac{\int d^4x |\phi(x)|^2}{\int d^4x} = < |\phi|^2 >$ for the two vacua should differ by a quantity of order unity. Phenomenologically we know that $|\phi|^2$ is very small in Planck units for the vacuum in which we live, and thus the only way to get the difference of order unity (or larger) is to have the other vacuum have $|\phi|^2$ of the order of unity in Planck units (or larger). From the philosophy that Planck units are the fundamental ones, we should really expect the average $|\phi|^2$ in the other phase.
Figure 1: This symbolic graph of the effective potential $V_{eff}(\phi)$ for the Standard Model Higgs field illustrates the two assumptions which lead to our prediction of the top quark and Higgs boson masses: 1) Two equally deep minima, 2) achieved for $|\phi|$ values differing, order of magnitudewise, by unity in Planck units.

just to be of Planck order of magnitude.

It is the main point of the present article to compute the implications of the following two assumptions, which could naturally be satisfied according to the above fixed extensive quantity argument:

a) The two minima in the Standard Model effective Higgs potential are degenerate: $V_{eff}(\phi_{min 1}) = V_{eff}(\phi_{min 2})$.

b) The second minimum, which is not the one in which we live, has a Higgs field or Higgs field squared of the order of unity in Planck units: $<|\phi_{min 2}|^2> = O(M_{Planck}^2)$.

In section 2 we show that these assumptions, illustrated in Fig. 1, lead to precise predictions of the top quark mass and Higgs particle mass. In section 3 we take up the discussion of the assumptions. Finally we present our conclusions in section 4.
2 Calculation of the Higgs and Top Masses

We use the approximation of the renormalisation group improved effective potential \([15]\), meaning that we use the form of the polynomial classical potential but with running coefficients taken at the renormalisation point identified with the field strength \(\phi\):

\[
V_{\text{eff}}(\phi) = \frac{1}{2} m_h^2(\mu = |\phi|) |\phi|^2 + \frac{1}{8} \lambda(\mu = |\phi|) |\phi|^4
\]  

(1)

We also do not distinguish between the field \(\phi\) renormalised, say, at the electroweak scale and the renormalised running field \(\phi(t) = \phi(\xi(t))\) at another scale \(\mu(t) = M_Z \exp(t)\), where \(\xi(t) = \exp(-\int_0^t dt' \gamma_1)\). The reason is that, due to the Planck scale being only used in order of magnitude, we shall get uncertainties of the same order as this correction. In fact the anomalous dimension \(\gamma_1\) is of the order of 1/100, making the difference at most of the order of our uncertainty.

Now the vacuum degeneracy condition is the requirement that the Standard Model renormalisation group improved effective Higgs potential should take the same value in two minima:

\[
V_{\text{eff}}(\phi_{\text{min} 1}) = V_{\text{eff}}(\phi_{\text{min} 2})
\]  

(2)

One of the minima corresponds to our vacuum with \(\phi_{\text{min} 1} = 246\) GeV and eq. (2) defines the vacuum stability curve. We are interested in the situation when \(\phi_{\text{min} 2} \gg \phi_{\text{min} 1}\). In this case the energy density in our vacuum 1 is exceedingly small compared to \(\phi_{\text{min} 2}\). Also, in order that \(\phi_{\text{min} 1} = 246\) GeV, the coefficient \(m_h^2(\mu)\) of \(|\phi|^2\) in the effective Higgs potential has to be of order the electroweak scale (we ignore quadratic divergences as causing any scale change). Thus, in the other vacuum 2, the \(|\phi|^4\) term will a priori strongly dominate the \(|\phi|^2\) term. So we basically get the degeneracy condition eq. (2) to mean that, at the vacuum 2 minimum, the effective coefficient \(\lambda(\phi_{\text{min} 2})\) must be zero with high accuracy. At the same \(\phi\)-value the derivative of the effective potential \(V_{\text{eff}}(\phi)\) should be zero, because it has a minimum there.

In the approximation \(V_{\text{eff}}(\phi) \approx \frac{1}{8} \lambda(\phi) \phi^4\) the derivative of \(V_{\text{eff}}(\phi)\) with respect to \(\phi\) becomes

\[
\frac{dV_{\text{eff}}(\phi)}{d\phi}|_{\phi_{\text{min} 2}} = \frac{1}{2} \lambda(\phi) \phi^3 + \frac{1}{8} \frac{d\lambda(\phi)}{d\phi} \phi^4 = \frac{1}{8} \beta_\lambda \phi^3
\]  

(3)

and thus at the second minimum the beta-function (given to first order by the right hand side of eq. (4))

\[
\beta_\lambda = \beta_\lambda(\lambda(\phi), g_t(\phi), g_3(\phi), g_2(\phi), g_1(\phi))
\]  

(4)

vanishes, as well as \(\lambda(\phi)\).

The running top and Higgs masses are related to the running top Yukawa coupling constant \(g_t(\mu) = \sqrt{2} m_t(\mu)/\phi_{\text{min} 1}\) and the Higgs self coupling \(\lambda(\mu) = m_H^2(\mu)/\phi_{\text{min} 1}^2\), evaluated when the renormalisation point \(\mu\) is put equal to the masses themselves. For the top quark we have the relation \([16]\)

\[
\frac{M_t}{m_t(M_t)} = 1 + \frac{4}{3} \frac{\alpha_S(M_t)}{\pi} + 10.95(\frac{\alpha_S(M_t)}{\pi})^2,
\]  

(5)
between the pole mass $M_t$ (usually identified as the physical mass) and the running mass $m_t(\mu)$.

So we need to use the renormalisation group to relate the couplings at the scale of vacuum 2, i.e. at $\mu = \phi_{\text{min} 2}$, to their values at the scale of the masses themselves, or roughly at the electroweak scale $\mu \approx \phi_{\text{min} 1}$. The running $\lambda(\mu)$ is easily computed by means of the (first order) renormalisation group equations:

$$16\pi^2 \frac{d\lambda}{d\ln \mu} = 12\lambda^2 + 3 \left(4g_t^2 - 3g_2^2 - g_1^2\right)\lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 \quad (6)$$

Here the $g_i(\mu)$ are the three Standard Model running gauge coupling constants, which satisfy the renormalisation group equations:

$$16\pi^2 \frac{dg_i}{d\ln \mu} = b_i g_i^3; \quad \text{where} \quad b_1 = \frac{41}{6}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \quad (7)$$

The top quark running Yukawa coupling constant $g_t(\mu)$ satisfies the renormalisation group equation:

$$16\pi^2 \frac{dg_t}{d\ln \mu} = g_t \left(\frac{9}{2}g_t^2 - 8g_2^2 - \frac{9}{4}g_1^2 - \frac{17}{12}g_t^2\right) \quad (8)$$

Because the top quark Yukawa coupling is of order unity, while the other Yukawa couplings are very small, it is only the top quark Yukawa coupling that is significant for the renormalisation group development of the Higgs self-coupling. So we ignored the other quark and lepton Yukawa couplings, including transition Yukawa couplings (quark mixing angles).

The degenerate minima condition eq. (6) and the associated vacuum stability curve have been studied for the Standard Model in three recent publications [12, 13, 14]. Their results are slightly different but, within errors, are each consistent with the linear fit

$$M_H = 135 + 2(M_t - 173) - 4\alpha_S - 0.117 \frac{0.006}{(9)}$$

to the vacuum stability curve, in GeV units. When this degenerate minima condition eq. (9) is combined with the experimental value [10] of the top quark pole mass, $M_t = 180 \pm 12$ GeV, we obtain a rather clean prediction [17] for the Higgs pole mass:

$$M_H = 149 \pm 26 \text{ GeV} \quad (10)$$

If we now also impose the strong first order transition requirement, discussed in the previous section, which takes the form:

$$\phi_{\text{min} 2} = \mathcal{O}(M_{\text{Planck}}) \quad (11)$$

we no longer need the experimental top quark mass as an input, but rather obtain a prediction for both $M_H$ and $M_t$. So we imposed the conditions $\beta_\lambda = \lambda = 0$ near the Planck scale, $\phi_{\text{min} 2} \simeq M_{\text{Planck}}$. We then evaluated the renormalisation
group development numerically, using two loop beta functions, to obtain \( g_\lambda(\mu) \) and \( \lambda(\mu) \) at the electroweak scale \( \mu = \phi_{\text{min 1}} \). Figures 2a-2d show the running \( \lambda(\phi) \), i.e. approximately the effective potential divided by \( \phi^4/8 \), as a function of \( \log(\phi) \) computed for various values of \( \phi_{\text{min 2}} \) (where we impose the conditions \( \beta_\lambda = \lambda = 0 \)). According to our strong first order phase transition argument, we expect Nature to have \( \phi_{\text{min 2}} \approx M_{\text{Planck}} = 2 \times 10^{19} \text{ GeV} \); so we see from Fig. 2b that our predicted combination of top and Higgs pole masses becomes \( M_t = 173 \text{ GeV}, \ M_H = 135 \text{ GeV} \).

From comparing the Figures 2 a-c, we see that a change in the scale of the minimum \( \phi_{\text{min 2}} \) by an order of magnitude from \( 10^{19} \text{ GeV} \) to \( 10^{18} \text{ GeV} \) or \( 10^{20} \text{ GeV} \) gives a shift in the top quark mass of ca. 2.5 GeV. Since the concept of Planck units only makes physical sense w.r.t. order of magnitudes, this means that we cannot, without new assumptions, get a more accurate prediction than of this order of magnitude of 2.5 GeV uncertainty in \( M_t \) and 5 GeV in \( M_H \).

The uncertainty at present in the strong fine structure constant \( \alpha_S(M_Z) = 0.117 \pm 0.006 \) leads to an uncertainty in our predictions of \( \sim \pm 2\% \) meaning \( \pm 3.5 \text{ GeV} \) in the top quark mass. So our overall result for the top quark mass is \( M_t = 173 \pm 5 \text{ GeV} \). For the Higgs mass the \( \alpha_S \)-dependence also leads to an uncertainty of \( \pm 4 \text{ GeV} \), \( \Delta M_H \approx \alpha_S^{-0.117} 4 \text{ GeV} \). Given the value of \( M_t \), say 173 GeV, the Higgs pole mass corresponding to the degeneracy of minima is given by the vacuum stability curve. Three recent articles \[12, 13, 14\] give slightly different calculations of the vacuum stability curve; for \( \alpha_S = 0.117 \) and \( M_t = 173 \text{ GeV} \), the corresponding Higgs pole masses are \( M_H = 139 \text{ GeV} \), \( M_H = 134 \text{ GeV} \) and \( M_H = 131 \text{ GeV} \) respectively. According to Ref. \[13\], when also the difference between first and second order calculations is included, an error in the calculations of order 5 to 10 GeV (we take it as 7 GeV) is suggested. Combining the uncertainty from the Planck scale only being known in order of magnitude and the \( \alpha_S \) uncertainty with the calculational uncertainty of \( \pm 7 \text{ GeV} \), we get an overall uncertainty in the Higgs boson mass of \( \pm 9 \text{ GeV} \). So our Standard Model criticality prediction for both the top quark and Higgs boson pole masses is:

\[
M_t = 173 \pm 4 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}.
\] (12)

3 Discussion

The first absolutely crucial ingredient, in addition to just the pure Standard Model, in obtaining the above results for the top quark and Higgs masses was the requirement (a) of the two minima being degenerate, essentially suggesting somehow a coexistence of two phases (=vacua) corresponding to these two minima. The second assumption, that (b) the vacuum 2 minimum has a Higgs field VEV of the order of the Planck scale, was what we called the “strong first orderness of the phase transition”.

Maybe the simplest would be just to take these two assumptions as our basic principle, but we think it adds to their credibility to suggest that they can somewhat naturally arise from very abstract assumptions, or better from a rather
Figure 2: Plot of $\lambda$ as a function of the scale of the Higgs field $\phi$ for degenerate vacua with the second Higgs VEV at the scale (a) $\phi_{\text{min}2} = 10^{20}$ GeV, (b) $\phi_{\text{min}2} = 10^{19}$ GeV, (c) $\phi_{\text{min}2} = 10^{18}$ GeV and (d) $\phi_{\text{min}2} = 10^{10}$ GeV. We formally apply the SM renormalisation group equations up to a scale of $10^{25}$ GeV.
large class of scenarios. How, in the high energy physics vacuum discussion, are we going to have an analogy to the extensive quantities being fixed at the outset? In Ref. [18] we suggested that this should be achieved by giving up the principle of “locality” (or we could say causality essentially) at the fundamental level.

Really the easiest way to formally bring our analogy to the water, vapour and ice system into play would be to use the well-known analogy between the Feynman path integral and the statistical mechanics partition function. In the Feynman path formalism the development of the quantum field theory—in our case of interest the Standard Model—is given by a functional integral (the integral over the paths):

\[
\int D[A] D[\psi] D[\phi] \exp(iS[A, \psi, \phi])
\]

where we have used the very condensed notation of letting \( A \) symbolize all the Yang-Mills fields, \( \psi \) all the fermion fields and \( \phi \) all components of the Higgs field. If we are only interested in vacuum 1, we can extract its energy (density) by use of the functional integral describing formally a development in imaginary time rather than real time; this is the euclideanised functional integral. In the analogy such functional integrals correspond to the canonical partition function with fixed temperature rather than fixed energy, i.e. with a fixed intensive parameter. The analogy with a fixed extensive variable—for instance the microcanonical ensemble of fixed energy—would correspond to replacing, in the integrand of the Feynman path functional integral, \( \exp(iS[A, \psi, \phi]) \) by a (or several) delta-function(s):

\[
\int D[A] D[\psi] D[\phi] \delta(I[A, \psi, \phi] - I_0)
\]

Here \( I \) is taken to be of the form

\[
I[A, \psi, \phi] = \int d^4x \mathcal{L}(x)
\]

and is the extensive quantity that is fixed, to the value \( I_0 \). For instance we think here of taking

\[
\mathcal{L}(x) = \text{const.} \, |\phi(x)|^2.
\]

This is analogous to the microcanonical statistical mechanics integral

\[
\int dq dp \delta(H(q, p) - E_0)
\]

where \( E_0 \) is the prescribed energy for the microcanonical ensemble and \( H \) is the hamiltonian.

As is well-known, it is usually possible to approximate a microcanonical ensemble by an appropriate canonical one in statistical mechanics. In a similar way we can also approximate our integral [14] by a “canonical one”, meaning here one with an action which apart from a constant factor will be \( I \). But we
are essentially free to add a usual exponentiated action as a factor in addition to the delta-function, so as to really start from a path integral—still essentially a microcanonical one—of the form:

\[ \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\phi \exp(iS_{\text{extr.}}[A,\psi,\phi])\delta(I[A,\psi,\phi] - I_0) \]  

(18)

Although the “extra” action \( S_{\text{extr.}}[A,\psi,\phi] \) can be chosen as freely as a usual full action, it should be clear that adding to it a term which is a function of \( I[A,\psi,\phi] \) would make no difference, since it could be replaced by the same function of \( I_0 \). This means that taking \( S_{\text{extr.}} \) to be the usual Standard Model action, the value of the bare Higgs mass squared, i. e. the coefficient \( m^2_{h_0} \) in the term \( \int d^4x m^2_{h_0}|\phi(x)|^2 \) in \( S_{\text{extr.}} \), is immaterial, except for the overall normalisation of the functional integral.

A standard technology for approximating the microcanonical ensemble by a canonical one consists in replacing the delta-function by its Fourier representation - say in our analogy:

\[ \delta(I - I_0) = \frac{1}{2\pi} \int d(m^2_{h_0}) \exp(im^2_{h_0}(I - I_0)) \]  

(19)

One then observes that—in the complex plane—the resulting integral for the whole partition function, after this insertion, is dominated by a very small range (saddle) w.r.t. the Lagrange multiplier variable \( m^2_{h_0} \). So we can just take this dominant value, provided we adjust it to give the correct average value of \( I \), i.e. \( \langle I \rangle = I_0 \). The Lagrange multiplier \( m^2_{h_0} \) (plus a possible term from \( S_{\text{extr.}} \)) functions as a bare Higgs mass squared and, for a given value of it, we have just the Standard Model action: now \( S_{\text{extr.}} + \int d^4x m^2_{h_0}|\phi(x)|^2 \) w.r.t. form. When one-loop corrections or, better, renormalisation group improvement to the Standard Model Higgs field effective potential \( V_{\text{eff}} \) is calculated, it turns out that formally there are usually two minima of \( V_{\text{eff}} \) as a function of \( |\phi|^2 \). Really the effective potential is defined so as to become the convex closure of what is obtained formally (see Appendix of Ref. [15]), leading to a linear piece of \( \phi \) dependence between the two formal minima. But we shall here talk as if we use the formal renormalisation group improved potential, which then has two minima corresponding to two phases, one of which will though usually be unstable. The only new point in our delta function model is that, provided a mixture of two phases is needed to obtain \( \langle I \rangle = I_0 \), we must adjust the Lagrange multiplier, i. e. the bare Higgs mass squared \( m^2_{h_1} \), so as to make the two phases appear in appropriate amounts and get the right value for \( \langle I \rangle \); this will only occur if their energy densities are very closely equal. We can imagine all this to have been done for a fixed set of all the other parameters (coupling constants) of the Standard Model, such as \( g_t \) and \( \lambda \). We are thus in much the usual situation as having to fit the Standard Model parameters to data, except that the Higgs bare mass (squared) has to be adjusted so as to make the two minima in the (formal) renormalisation group improved effective potential be degenerate.

The above degeneracy argumentation presupposed that the \( I_0 \)-value chosen by Nature happened to fall in the interval between what could be achieved
with one or the other minima all over the space-time. Thus if this interval is very narrow that choice is unlikely to occur. We speculatively estimate that Nature chooses \( I_0/V_4 \) randomly with a distribution of the order of \( M^2_{\text{Planck}} \), where \( V_4 \) is the quantization four volume of space-time. So, if the difference in the average values of \( |\phi|^2 \) for the two phases is much smaller than \( M^2_{\text{Planck}} \), then the situation with two vacua is very unlikely to occur. Thus if we should at all find the degenerate vacua, it should be with

\[
<|\phi|^2>_{\text{min} \ 2} - <|\phi|^2>_{\text{min} \ 1} \sim M^2_{\text{Planck}}
\]  (20)

So we may as well assume, in investigating the degeneracy prediction, that this difference is of the Planck scale \( M^2_{\text{Planck}} \). Since the phenomenologically known vacuum 1 has, compared to Planck units, a negligible \( |\phi|^2 \) the vacuum 2 must have its VEV of the Planck scale order of magnitude or larger. Assuming the fundamental scale being the Planck scale, it is though suggested that the VEV of vacuum 2 be just of that order.

The above “explanation” for our two main assumptions would not have been disturbed (much) had we, instead of inserting a delta-function in the functional integral formula, used some other nonexponential function. We could in fact Fourier resolve it—like any function can be—and would then usually find a sufficiently mildly varying Fourier transformed function that it would not spoil the property of a rather narrow dominating region of the Lagrange multiplier \( m^2_{hl} \). So the argumentation above would only fail in rather exceptional cases, such as the case of the inserted function being a constant, in which case we would just have the completely usual Standard Model.

It should be remarked that whatever nonexponential function we insert—like the used \( \delta(I-I_0) \)—it strictly speaking means violation of the principle of locality in space and time. That is to say that with such a term our effective coupling constant(s)—we here really think of \( m^2_{hl} \) the bare Higgs mass squared—gets possibly influenced, for instance, from the future or from very far away places and times. This is really the same effect as in baby universe theory [7]. The baby universes quite obviously cause connections between far separated space time points, without any restriction as to whether that may allow the future to influence us (especially the value of the coupling constants). In fact the considerations of this section can be considered as a derivation of our prediction for the top and Higgs masses from a class of models containing baby universe theory as a special case.

We originally hoped that the multiple point assumption would help explain why the electroweak scale is so exceedingly low compared to the Planck scale [8]. However it seems that, in the above picture with the second vacuum having \( \phi \) of the order of the Planck scale, we lose the potential for solving this fine tuning problem. If the top mass had been so small as to allow a Linde-Weinberg scenario [9], the requirement of degeneracy of two phases in the Standard Model could have led to an exponential expression for the electroweak scale in terms of the cut-off (identified naturally with the Planck scale); but once one minimum is assumed to be at the Planck scale itself the corresponding argument no longer functions.
4 Conclusion

We have observed that the top quark mass fits very well with the requirements of the effective Higgs potential having two degenerate minima and having one of them at the Planck scale. These requirements are derivable from a rather general insertion of delta functions under the Feynman path integral; very analogous to the restriction to a fixed total energy in statistical mechanics leading to a microcanonical ensemble, by the insertion of a delta function, $\delta(H - E)$, into the integrand of the partition function. It must be admitted that this violates locality but only mildly. This violation is like the one in baby universe theory and only means that coupling constants feel an average over all of spacetime. We have argued in Ref. [9] that assuming reparameterisation invariance, any fundamental violation of locality becomes of this mild form. It might, of course, be possible to invent some different physical mechanism that could give the effects of a fixed integral I, similarly to what the inserted delta function does, but without violating locality.

Our scheme predicts the pole masses: $(M_t, M_H) = (173 \pm 4, 135 \pm 9)$ GeV. If we take it that the top mass agreement and the acceptable Higgs mass prediction are not accidental, then we must accept the crucial content of the assumptions:

1. The pure Standard Model is valid up to the Planck scale, at least as far as the top quark and Higgs interactions are concerned. This would mean that no new physics interacts significantly with the top or Higgs particles before the Planck scale; in particular supersymmetry would not be allowed.

2. There is a need for some physical explanation of the principle of degenerate phases. This means that we either have some coexistence of phases in space or more likely in spacetime (the latter threatening locality, e. g. baby universes) or we need another mechanism doing the same job.

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