Research Article

Overall Efficiency of Four-Stage Structure with Undesirable Outputs: A New SBM Network DEA Model

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Benchmarking is the major reason for the widespread use of DEA models for efficiency analysis. Determining the closest targets for DMUs, DEA models play a key role in benchmarking their best performance. In fact, these models help develop certain performance enhancement plans that need fewer attempts made by DMUs. Therefore, this study proposes a novel method based on the network DEA to determine the most appropriate target for every stage in addition to benchmarking the DMUs. The proposed model differs from those proposed by other studies in the fact that all DEA models of benchmarking consider input and output values to be linear. However, in real-world problems, many DMU inputs and outputs have nonlinear values (values are the coefficients of inputs or outputs in modeling and can be the price of desirable outputs or the cost of inputs and undesirable outputs), something which was taken into account in the modeling process in this study. The proposed model was employed to benchmark cement factories listed on the Tehran Stock Exchange.

1. Introduction

Data envelopment analysis (DEA) was first proposed by Charnes et al. [1] and is a mathematical programming technique for evaluating the relative efficiency of a set of decision-making units (DMUs) with multi-inputs and multi-outputs. Because of using mathematical programming, DEA will be able to consider a lot of variables and constraints in evaluation, and consequently, it will decrease obligations posed by selecting a limited number of inputs and outputs [2]. In conventional DEA models, DMUs are regarded as black-boxes, whereas the internal structure and operation of DMUs are ignored. Therefore, they cannot provide information on the performance of internal stages, and the resultant efficiency can be misleading [2, 3]. Hence, the network DEA (proposed by Färe and Grosskopf [4]), considering the internal processes of systems, can determine the relative performance of network structure systems and yield more meaningful and informative results than the traditional DEA [5].

One of the most important applications of DEA for correcting and improving organizational efficiency is benchmark selection. Benchmarking necessitates an efficient method for determining the best performance, which is based on the relative efficiency of all DMUs. Benchmark selection is among the most important applications of DEA efficiency correction and enhancement in organizations. Benchmarking requires an effective method for identifying the best performance, a process that necessitates evaluating the relative efficiency of all DMUs. Usually performed through DEA, a major benchmarking step is to identify the best performance. The DEA provides a reference set of benchmark DMUs for inefficient DMUs and recommends the extent to which DMUs should be improved in order to become efficient. Various methods have been proposed for benchmarking in the literature. In most papers, the DMU efficiency is mainly measured through different DEA models, and the distance between efficiency and the efficient frontier is then determined.
In the conventional DEA models, the efficiency score of each unit is determined by projecting the unit on the efficient frontier, and therefore the projected (virtual) unit can be considered as the corresponding target. However, identifying a single efficient unit as a target may be inappropriate in practice because it might be very different from the unit under consideration, or it is too far to reach in the short term. Evidently, the decision-maker (DM) prefers a stepwise approach to reach the far efficient benchmark through a sequence of (intermediate) targets. Most DEA models for target settings were subject to some constraints. There have been a few studies on benchmark selection and target setting for network structure. All of them face some limitations as follows: (1) Network structures were traditionally thought to have two stages [6–10], whereas most production processes have more than two stages, while the two-stage structure widely exists in areas, such as supply chains and bank productions, which is a basic production structure [11]. So, the two-stage network DEA models can be directly applied to industries and regarded as a basis to solve general network evaluation problems [12]. (2) Neither of these studies considered the nonlinear behavior of variables in efficiency evaluation and benchmarking, although the real-world reactions of variables are not linear, and modeling them as linear will result in unreliable results. It may be argued that, in a number of situations, certain analysis variables (outputs or inputs) behave in a nonlinear rather than linear manner. Since the DEA models are intended to increase output, they are unable to distinguish between desirable and undesirable outputs and will associate a higher value to undesirable outputs. For instance, highly polluted air will make people more allergic; therefore, different degrees of importance should be attributed to different levels of pollution. In efficiency measurements, it is essential to attribute different levels of importance to different values of undesirable outputs. Hence, in order to analyze the effects of undesirable outputs on performance evaluation and obtain accurate and useful results, the nonlinear valuation (values are the coefficients of inputs or outputs in modeling and can be the price of desirable outputs or the cost of inputs and undesirable outputs) and behavior of outputs should also be considered in modeling. In the current study, we assume that, on the inputs and desirable outputs, all variables have a linear impact on efficiency. We thus restrict attention to undesirable outputs.

Thus, a benchmarking model that takes into account the nonlinear values of data must be introduced. This study aimed to propose a model capable of considering an appropriate structure (with all stages in the DMUs) and the nonlinear behavior of the variables for efficiency evaluation in addition to proposing benchmarks for each inefficient DMU.

The rest of the paper consists of the following sections. Section 2 presents a brief review of literature related to our work. In Section 3, a novel network SBM-DEA model is presented along with the piecewise linear to determine the most appropriate target for every stage in addition to benchmarking the inefficient DMUs. The model is employed in Section 4 to evaluate the efficiency of 42 cement facilities in Iran. The empirical results are also presented in this section. The conclusion is finally drawn in Section 5.

2. Literature Review

In this section, a brief introduction to network SBM-DEA, PLDEA, benchmarking, and target setting is presented so that the reader becomes familiar with the basic concepts that will be used in the next section for modeling and solving the problem under study. Some related studies are presented at the end of the subsection as a literature review.

2.1. Network Data Envelopment Analysis (NDEA): A Slack-Based Approach

Charnes et al. developed a fractional programming model, commonly known as the CCR model. In this model, the efficiency measure of any DMU is determined as the maximum ratio of weighted outputs to weighted inputs. The only restriction is that productivity ratios of all DMUs must be smaller than or equal to one [1]. A flaw of the radial methods was that they did not differentiate between inefficient and poorly efficient DMUs. Hence, there were issues with ranking and comparing the DMUs [13]. One approach to solve these problems is the use of slacks-based measures. In addition to radial measures of efficiency, there is the slacks-based measure (SBM) proposed by Despotis and Sotiros [14]. The SBM is not based on the ray from the origin along the DMU being evaluated, so it is nonradial. The slacks-based approach uses the slacks to measure performance and is suitable for measuring efficiencies when inputs and outputs may change nonproportionally.

In the conventional DEA, DMUs are usually formulated as a single process transforming inputs into outputs. They are treated as a black box in which internal structures are generally ignored [4]. Therefore, they cannot provide information on the performance of internal stages, and the resultant efficiency can be misleading [2, 3]. Hence, the network DEA (proposed by Färe and Grosskopf [4]), considering the internal processes of systems, can determine the relative performance of network structure systems and yield more meaningful and informative results than the traditional DEA [5]. The NDEA consists of two basic structures, named serial and parallel [15, 16]. For the sake of simplicity, a serial structure is considered with a two-stage process according to Figure 1. Assume that there are $n$ DMU, $j = 1, \ldots, n$, and let DMUO, $O \in \{1, \ldots, n\}$, refer to a unit from a total of $n$ units, the relative efficiency of which is being evaluated. Define $x_o \in R_m$, as the inputs, $z_o \in R_d$, as the intermediate products, and $y_o \in R^2$, as the outputs of DMUO. The slacks associated with the inputs and output are $S_i^-$ and $S_i^+$, respectively. If all slack variables have a value of zero, then the DMU being evaluated is efficient. The nonzero slacks show the amount of the corresponding factor that can be improved to become efficient. The proposed network SBM model for measuring the system efficiency can be formulated as follows [17]:
Every phrase in the objective function’s numerator or denominator represented the distance between the DMU and its target. The objective function can be utilized to benchmark inefficient DMUs and set the target for them [18]. $S_i^r$ are slack variables that represent the gap between inefficient DMU $j$ and its benchmark point. On the basis of optimal slacks ($S_i^r^*$, $S_j^r^*$) obtained from models equation (1), the target values are defined as follows:

$$
\begin{align*}
\bar{x}_i &= x_i - s_i^* \\
\bar{y}_r &= y_r - s_r^* 
\end{align*}
$$

where $\bar{x}_i$ and $\bar{y}_r$ as target values applied to improve the efficiency of the inefficient DMUs and project them on the efficient frontier [19].

In the existing literature, many models for evaluating the efficiency of network systems have been proposed [15–17]. One of the most important of these is the slacks-based measure model [15–17, 20, 21]. Tone proposed the slacks-based measure integrating the data envelopment analysis (SBM-DEA) [19], which fully takes undesirable outputs and slack variables into account. The SBM-DEA may have network structures. Two- or three-stage slacks-based network DEA has also been applied for performance evaluation of systems. These kinds of DMUs have inputs, outputs, and also intermediate amounts that flow from one stage to another. The stages may also have their own inputs and outputs.

Tone and Tsutsui proposed a slacks-based NDEA model that can deal with intermediate products formally. The model can measure both individual and network performances. They apply the proposed model to vertically integrated electric power companies as a network structure. In this model, the generation plants and the transmission facilities can be taken as Stage 1 and Stage 2, respectively [17]. Tone and Tsutsui have developed this model to a dynamic DEA model involving network structure in each period within the framework of a slacks-based measure approach [27]. Lozano and Adenso-Diaz considered a multiproduct supply network, in which losses (e.g., spoilage of perishable products) can occur at either the nodes or the arcs. They proposed an NDEA to assess the efficiency of the product flows in varying periods. The results indicated that the proposed approach can identify and remove the inefficiencies in the observed data and that the potential spoilage reduction increases with the variability in the losses observed in the different periods [28]. Mahmoudabadi and Emrouznejad used a network slacks-based measure (SBM) DEA model in which the efficiency of the overall system is equal to the weighted average of the efficiency of the individual stages. The main advantage of this model is its ability to provide better efficiency criteria, calculate the weight of each stage.
separately, and simultaneously evaluate the mediator variables as both input and output [29]. Li et al. evaluated the environmental performance of Chinese industrial systems by introducing a new network slacks-based measure (SBM) DEA model [30]. Yang et al. combined DEA and multiobjective programming to solve the resource allocation and target setting problem in organizations with a centralized decision-making environment [31]. Badiezadeh et al. evaluated optimistic and pessimistic efficiencies using NDEA in the presence of undesirable outputs [32]. Su and Sun introduced the dual-role factors into the model of Badiezadeh and evaluated the sustainability of supply chains [33]. Kao said that NDEA is a relatively new subject, with a short history of no more than 20 years since the term first appeared in 2000. In the last two decades, dozens of models have been proposed, and new models are still being developed [34]. Keshin proposed the NDEA-SBM method to answer the questions of whether the OPEC members used their oil wealth effectively to increase the social prosperity in their countries [35]. Zhu et al. proposed a Mixed Integer Linear Program (MILP) to determine the closest efficient targets on the extended facet production possibility set in data envelopment analysis [36]. Zhu et al. measured sustainability efficiency in terms of energy usage and environmental impact by using a new generalized equilibrium efficient frontier DEA (EEFDEA) approach. The equilibrium efficient frontier is based on minimum satisfaction degree maximization, considering both minimum and maximum adjustment strategy [37]. Chen et al. adopted a four-stage NDDF-DEA approach with considering undesirable output and environmental impact factors to measure the environmental adjusted dynamic energy efficiency of China’s transportation sector [38]. Shi et al. developed a network slacks-based DEA model to measure the overall efficiency of the operational process with undesirable outputs containing both series and parallel processes [39]. Li et al. evaluated the total factor waste gas treatment efficiency (TFWGTGE) of China’s iron and steel enterprises and its influencing factors based on a four-stage SBM-DEA model [40]. We should develop a method to model the network structure. The slacks-based measure network data envelopment analysis model is a nonradial approach, which could obtain all the slacks of inputs, intermediates, desirable outputs, and undesirable outputs when optimizing DMU’s efficiency [27, 41, 42].

The approaches that have so far been proposed to consider undesirable outputs in efficiency evaluation through the DEA method can be divided into two main categories:

1. The first category includes the approaches that have directly analyzed the effects of undesirable outputs on efficiency measurement through the DEA method. Some of them are as follows: (1) slack-based measure (SBM) model, (2) additive model, and (3) Range-Adjusted Measure (RAM) model

2. The second category includes the approaches that evaluate efficiency indirectly by converting undesirable outputs into desirable outputs. Some of them are as follows: (1) the additive inverse method, (2) undesirable outputs as inputs, (3) translation invariance, (4) multiplicative inverse, (5) nonseparating outputs model, and (6) linear monotone decreasing transformation [43].

Although indirect methods have been used in the bulk of studies to consider the undesirable outputs [44], these studies have faced many limitations [24]. Therefore, the undesirable outputs are directly considered in the network SBM-DEA model in this study, and their nonlinear behaviors will be modeled through the piecewise linear function.

The NDEA modeling technique was also employed in this paper. In addition, since a large number of real-world problems entail different inputs and outputs with nonlinear values, the results will not be accurate, reliable, and useful if the linear valuation is used in performance evaluation (values are the coefficients of inputs or outputs in modeling and can be the price of desirable outputs or the cost of inputs and undesirable outputs). The piecewise linear function was hence employed to distinctly evaluate the variables of a DMU with those of another [45].

2.2. Piecewise Linear Models in DEA (PLDEA). Cook and Zhu stated that, in the standard DEA model, the aggregate output (input) was a pure linear function of each output (input). This means that if DMU \(_j\) generates twice as much of an output as another DMU \(_j'\), then DMU \(_j\) is credited with having created twice as much value [45]. In many situations, however, linear pricing \((\mu, y_r)\) may not sufficiently show differences in values obtained from one DMU to another. Therefore, they solved this problem and proposed the PLDEA. They established that certain factors, previously treated as behaving linearly, should be looked upon as having a nonlinear effect on efficiency. Thus, they have considered the theory of piecewise linear programming, where the scale of a variable with nonlinear behavior could be divided into \(k\) segments with each variable supposed to behave linearly in those segments. Clearly, the more the segments, the closer the piecewise linear estimation to the actual nonlinear function. Accordingly, the scale of the variable indicating the diminishing marginal value (DMV) behavior should be divided into \(k\) ranges \([0, L_1], (L_1, L_2], \ldots, (L_{k-1}, L_k]\). Let \(u_{rj}\) be the value assigned to the portion of \(y_{rj}\) which lies within the \(k\)th range [46].

If \(y_{rj} \in (L_{k-1}, L_k]\), then the parameters \(y^k_{rj}\) are defined as follows:

\[
y^k_{rj} \begin{cases} 
L_k, & \text{if } k = 1, \\
L_k - L_{k-1}, & \text{if } k = 2, \ldots, k_j - 1, \\
y_{rj} - L_{k-1}, & \text{if } k = k_j, \\
0, & \text{if } k > k_j.
\end{cases}
\]

The PLDEA model developed by Cook and Zhu [45] is as follows:
sets of regular and DMV outputs, they considered this issue in the piecewise linear CCR PLDEA model would fail to create acceptable targets [46]. Cook et al. proposed a modified DEA model and improved the model to facilitate the production of Pareto-efficient targets. Ji et al. combined the DEA algorithm with classification information and presented a novel DEA-based classifier to construct a piecewise linear discriminant function. In this classifier, class information is added, and the nonnegative conditions of the DEA model are lost [48].

2.3. Benchmarking. Efficiency analysis is performed not only to estimate the current level of efficiency but also to provide information on how to remove inefficiency, that is, to obtain benchmarking information. Data envelopment analysis (DEA) was developed in order to satisfy both objectives, and the strength of its benchmarking analysis gives DEA a unique advantage over other methodologies of efficiency analysis. This section gives an overview of a few benchmarking studies.

The benchmark, according to Gonzalez and Alvarez, was the most similar efficient DMU. They proposed the concept of input-specific contractions to determine the shortest path to efficiency [49]. Baek and Lee calculated the shortest distance and provided the smallest projection from the DMU under evaluation [50]. Alirezaee and Afsharian proposed a DEA-based model for evaluating and ranking DMUs. The efficiency evaluation in this model was performed in several layers so that the inefficient DMU could be moved to a better layer. A flaw of this model was that it did not specify information about the method of selecting reference DMU in each layer [51]. Estrada et al. proposed the method of stepwise benchmarking for inefficient DMUs based on the proximity-based target selection. They provided the best path for obtaining the efficiency frontier of DMUs based on a self-organizing map (SOM) and reinforcement learning. This method disregarded the set of DMU references; however, it did emphasize the similarity of input patterns for the benchmarking path on practical target DMUs [52]. Park et al. proposed a method for benchmarking DMU performance levels and clustering them based on the similarity of input patterns [53]. Lim et al. proposed selecting the benchmarking path in the DEA based on DMU clustering, that is, classifying the DMUs as distinct layers based on their efficiency status and then setting the
benchmark path in accordance with a layer sequence. The resultant benchmark can improve inefficient DMU performance in a real and executable way [54]. Park et al. introduced a novel stepwise benchmarking method that relied on three criteria: preference, direction, and similarity [55]. Ruiz and Sirvent argued that DEA benchmarking models should be incorporated into their objectives criteria for the selection of suitable benchmarks, in addition to considering the setting of appropriate targets [56]. Ramón et al. extended the popular benchmarking method to identify the most reasonable practices for planning the learning and developing improvements [57].

2.4. Target Setting. Generally, the targets are of great importance since they indicate how an inefficient unit could improve its performance. The DEA models estimated the DMU efficiency and provided useful information for resolving the DMUs inefficiency by comparing each DMU to its corresponding target. As a result, since they provide targets for inefficient DMUs, DEA models are known as powerful benchmarking tools [52]. This section gives an overview of a few target setting studies.

Ravelojaona proposed a CCR model to evaluate the efficiency of DMUs based on the input-output ratio and then set targets based on the projecting of the evaluated DMUs on the efficient frontier. The distance measurement methods of setting targets include the directional distance function, slacks-based efficiency measure, and range-adjusted efficiency measure [58–60]. According to Portela et al., traditional DEA models are incapable of revealing the closest target for inefficient DMUs. Therefore, they sought to find the closest target in two states: free disposal hull (FDH) and convex [61]. Lozano and Villa established a sequence of targets, each of which was closer to the respective DMU than the previous target. Finally, the last target was placed on the efficient frontier. The resultant efficient target would be closer than in the method in which the DMUs were projected on the efficient frontier [62]. Aparicio et al. proposed a method for determining the closest target for DMUs. For making efficient DMU, this approach used the closest targets, fewer inputs, and more outputs [63]. Jahan-shahloo et al. proposed a linear bilevel programming problem to provide the closest target while minimizing DMU’s distance from the strong efficient frontier. Accordingly, the closest targets, fewer inputs, and more outputs were set for making efficient DMUs [64]. According to a few studies, target setting based on disparate models for measuring DEA efficiency can lead to optimal responses, for they reflect the greatest improvement potential. As a result, Wu proposed the closest target method, which allows DMUs to gradually improve their efficiency in accordance with the context-dependent DEA. Furthermore, Lv et al. proposed a bargaining method with DEA and acceptable targets for all DMUs. However, none of these methods considered that the inputs and outputs weights for each DMU would be obtained from common DEA models by maximizing their efficiency. Thus, DMUs cannot guarantee their results, although the majority of DMUs are obtained in order to be efficient. They cannot be separated [59]. In the presence of many efficient DMUs, decision-makers might be more aware of the constraints of target setting than of the targets being ideal [65]. Cook et al. employed a common set of weights DEA to solve this problem. Although this method reduced the number of DMUs, it required nonlinear planning, which is difficult to implement [66]. Lozano and Calzada-Infante proposed a new benchmarking approach based on the concept of efficiency field potential given by a continuous and differentiable function [67]. Nasrabadi et al. proposed an algorithm that results in a path of targets for each inefficient unit. All units on this path are better than the unit under evaluation in terms of efficiency scores defined for interval scale data [68].

The DEA models described in the preceding papers for target settings were subject to some constraints, one of which indicated that they failed to consider the internal structure of DMUs and treated them as a black box. As a result, the sub-DMU target setting would be ambiguous. Bi et al. proposed a target setting model for parallel production lines in manufacturing units. The proposed model managed to maximize the performance of the worst production line in the unit while maintaining total efficiency [69]. Azadi et al. proposed two DEA models for establishing targets for two-stage networks. These methods were proposed as a way to provide a feasible response to ensure that the targets obtained from the models could be placed within the current operational capacity [70]. Sarah and Halili-Damghani adopted fuzzy de novo programming to find the best resources (inputs) and optimal targets (outputs) for DMUs in a network DEA [71]. Borghiepour et al. used the DEA to evaluate the structural efficiency of a five-stage network and also proposed some formulas for setting targets at each stage and achieving high efficiency. The findings indicated the amount of change that the outputs of each stage had to undergo in order to achieve an efficient status [72].

This paper draws attention to the fact that traditional data envelopment analysis (DEA) models do not provide the closest possible targets to inefficient units and presents a procedure to obtain such targets.

Compared to previous studies, this paper has implemented the following innovations:

(i) Proposing a new idea for considering the effects of undesirable outputs on performance evaluation while taking different weights into account in order to justify the nonlinear behavior of undesirable outputs.

(ii) Developing a network SBM-DEA model with the piecewise linear function to account for undesirable outputs in performance evaluation.

(iii) Proposing a general network structure designed through NDEA modeling for the first time to evaluate the efficiency of cement companies by analyzing the nonlinear behavior of undesirable
outputs in the modeling process since undesirable outputs (e.g., greenhouse gas emissions) significantly impact the efficiency of cement companies in production and consumption processes.

(iv) Benchmarking inefficient cement companies and setting a suitable target for each of them by considering the nonlinear behavior of undesirable outputs in the modeling process.

3. Methodology

This section describes steps for the proposed method before explaining its network structure and modeling.

3.1. Steps of the Proposed Model. Figure 2 demonstrates that the input and output variables were initially set. The undesirable outputs were then identified, and since the values of the various undesirable outputs vary, this critical point must be considered in modeling in order to provide reliable results for decision-makers. Thus, after an appropriate network structure is designed, the new slack-based network DEA will be proposed with nonlinear pricing for undesirable outputs modeled through the piecewise linear. Following the resolution of the problem, a model of appropriate benchmarks for the inefficient DMUs will be introduced. The proposed model will provide more supplementary information for decision-makers and help them formulate strategies for improving the efficiency of inefficient DMUs.

Few papers have evaluated the efficiency of a general network structure with respect to undesirable outputs while suggesting that considering undesirable outputs in a general network structure could affect efficiency and make the results more accurate. This paper aims to address the following questions: “Which model can properly account for the effects of undesirable outputs on a general network structure and determine efficiency?” What is the best benchmark for each inefficient DMU to improve the efficiency of them with multiple stages?

As mentioned earlier, this paper is intended for developing a DEA-based model to evaluate sustainable performance in different industries. So, the present study sought to propose a model capable of considering an appropriate structure (with all targets in DMUs) and the nonlinear behavior of undesirable variables for efficiency evaluation as well as proposing targets for each inefficient DMU.

3.2. Structure of General Network. Assume that there are n DMUs and \( j = 1, \ldots, n \). According to Figure 3, \( X_j = (x_{1j}, \ldots, x_{ij})^T \) is a set of inputs consumed by Stage 1, whereas \( Z_j = (z_{1j}, \ldots, z_{dj})^T \) is a set of intermediate products consumed by Stage 2 and produced by Stage 1. Furthermore, \( Z_j' = (z_{1j}', \ldots, z_{dj}')^T \) is a set of intermediate products consumed by Stage 3 and produced by Stage 1, whereas \( M_j = (m_{1j}, \ldots, m_{mj})^T \) is a set of inputs consumed by Stage 2, and \( F_j = (f_{1j}, \ldots, f_{dj})^T \) is a set of intermediate products consumed by Stage 3 and produced by Stage 2. In addition, \( K_j = (k_{1j}, \ldots, k_{kj})^T \) is a set of intermediate products consumed by Stage 4 and produced by Stage 2, whereas \( L_j = (l_{1j}, \ldots, l_{dj})^T \) is a set of intermediate products consumed by Stage 4 and produced by Stage 3, and \( Y_j = (y_{1j}, \ldots, y_{rj})^T \) is a set of outputs produced by Stage 4. Here, \( i_k \) and \( i_m \) are the number of independent inputs of \( x \) and \( m \) variables, respectively, whereas \( d_0, d_1, d_2, d_3, \) and \( d_z \) represent the number of intermediate variables \( l, f, K', Z', \) and \( z \), respectively, and \( r_y \) is the number of final output \( y \).

3.3. Modeling. The modeling process to evaluate the efficiency of network-structured systems and benchmarking for inefficient DMUs is carried out using a piecewise linear function while considering undesirable outputs in modeling.

The designed SBM model will then be presented for the previous section’s design structure. The undesirable nonlinear behavior will then be added to the model using a piecewise linear function, and the results from the proposed model will be used as a benchmark for inefficient DMUs.
3.3.1. Network Slack-Based Measure DEA.

\[
\begin{align*}
\max & \sum_{i=1}^{n} (S_{i}^+) + \sum_{i=1}^{n} (S_{i}^-) + \sum_{d_{x}} (S_{d_{x}}^+) + \sum_{d_{z}} (S_{d_{z}}^+) + \sum_{d_{r}} (S_{d_{r}}^-) + \sum_{d_{k}} (S_{d_{k}}^-) \\
\text{s.t} & \sum_{j=1}^{n} \lambda_{j} X_{j}^+ + S_{i}^- = X_{O}, \\
& \sum_{j=1}^{n} \lambda_{j} Y_{j} - S_{r}^+ = Y_{r, O}, \quad r_{Y} \in R_{Y}, \\
& \sum_{j=1}^{n} \lambda_{j} Y_{j} + S_{r}^- = Y_{r, O}, \quad r_{Y} \in R_{Y}', \\
& \sum_{j=1}^{n} \lambda_{j} Z_{j}^{12} - S_{d_{z}}^+ = Z_{d_{z}, O}^{12}, \quad d_{Z} \in R_{Z}, \\
& \sum_{j=1}^{n} \lambda_{j} Z_{j}^{12} + S_{d_{z}}^- = Z_{d_{z}, O}^{12}, \quad d_{Z} \in R_{Z}', \\
& \sum_{j=1}^{n} \lambda_{j} Z_{j}^{12} - S_{d_{z}}^+ = Z_{d_{z}, O}^{12}, \quad d_{Z} \in R_{Z}, \\
& \sum_{j=1}^{n} \lambda_{j} Z_{j}^{13} + S_{d_{z}}^- = Z_{d_{z}, O}^{13}, \quad d_{Z} \in R_{Z}', \\
& \sum_{j=1}^{n} \lambda_{j} Z_{j}^{13} - S_{d_{z}}^- = Z_{d_{z}, O}^{13}, \quad d_{Z} \in R_{Z}, \\
& \sum_{j=1}^{n} \lambda_{j} K_{j}^{24} + S_{d_{k}}^+ = K_{O}^{24}, \\
& \sum_{j=1}^{n} \lambda_{j} K_{j}^{24} - S_{d_{k}}^- = K_{O}^{24}, \\
& \sum_{j=1}^{n} \lambda_{j} F_{j} + S_{d_{k}}^- = F_{O}^{23}, \\
& \sum_{j=1}^{n} \lambda_{j} F_{j} - S_{d_{k}}^+ = F_{d_{k}, O}^{23}, \quad d_{F} \in R_{F}, \\
& \sum_{j=1}^{n} \lambda_{j} F_{j} + S_{d_{k}}^- = F_{d_{k}, O}^{23}, \quad d_{F} \in R_{F}'.
\end{align*}
\]
In model equation (5), \( R \) and \( R' \) are two sets of ordinary and nonlinear valuation variables, respectively. The slacks in the objective function represent distances of DMUs under evaluation from the efficient frontier, according to this model, which is the maximization of the input slacks, that is, an input-oriented model. Hence, if the above model is solved, the optimal value of the objective function for the DMUs under evaluation will be zero, a value that indicates that DMU is on the efficient frontier and thus efficient. Furthermore, if the objective function’s value is positive, at least one of the input slacks is nonzero. As a result, the DMUs under evaluation are far from the efficiency frontier, according to this model equation (5) is always feasible.

**Theorem 1.** Model 5 is always feasible.

**Proof.** Where the value of the objective function is signified by \( \theta \), the following assumptions will be considered as the feasible response equation (6) for model equation (5):

\[
\theta = \lambda_j^1 = \lambda_j^2 = \lambda_j^3 = \lambda_j^4 = 0, \quad \forall j, j \neq 0, \lambda_j^0 = \lambda_j^0 = \lambda_j^0 = 1.
\]

Relation 6 is a feasible response for model equation (5), and it is true in all model constraints. In other words, where \( \theta = 0 \), all slack variables pertaining to DMU under evaluation will be equal to zero. Thus, by placing the values of relation equation (6) in model equation (5), the underevaluation variables will be obtained. Then, without considering the quantitative value of the inputs, the middle and output of the model, there is a feasible response for model equation (5), and it will be identified by \( \theta \); therefore, the model equation (5) is always feasible. \( \Box \)

**Definition 1.** Using the following formula, the corresponding efficiency of the DMUs under evaluation can be calculated based on the optimal resultant variables, which represent the distance of the DMUs under evaluation from the efficiency frontier.

\[
\text{EF} = \frac{1}{1 + \sum_{i} (S_i^+)}.
\]

(EF.Stage 1 shows Stage 1 efficiency, which is the reverse of one plus sum of slacks input into Stage 1.

\[
\text{EF.Stage 2} = \frac{1}{1 + (\sum_{i} (S_i^+)) + \sum_{d} (S_d^-)}.
\]

(EF.Stage 2 shows Stage 2 efficiency, which is the reverse of one plus sum of slacks input into Stage 2.

\[
\text{EF.Stage 3} = \frac{1}{1 + \sum_{d} (S_d^-) + \sum_{d} (S_d^-)}.
\]

(EF.Stage 3 shows Stage 3 efficiency, which is the reverse of one plus sum of slacks input into Stage 3.

**3.3.2. Modeling Undesirable Variable with Nonlinear Valuing Based on Piecewise Linear Function.** A piecewise linear function was employed to determine the intervals of undesirable outputs with nonlinear behavior and nonlinear valuation in the following manner:
In the above model, $s^{-k}_d$ is the surplus input variable pertinent to the $k$ interval from the piecewise linear variables.

$$s^{-k}_d \geq \sum_{i=1}^{k} (1 - \nu_{k-1}), \quad \nu_i \in R^+_r (a_i)$$

$$\nu_k \geq (\sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{l=1}^{k} \nu_{k-1}) \cdot M, \quad \nu_k \in R^+_r (b_1)$$

$$\left( \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{l=1}^{k} \nu_{k-1} \right) \geq \nu_k \cdot M, \quad \nu_k \in R^+_r (c_1)$$

The model aims to allocate lower values to highly undesirable outputs (if the production rate increases due to undesirable outputs, the maximization objective function of production rate will increase the amount of undesirable outputs, and since larger amounts of raw materials will be used for producing fewer desirable outputs, an increasing production rate will lead to a reduced value).

### 3.3.3. Benchmarking for Inefficient DMUs

Benchmarking is a process through which every inefficient DMU is assigned a DMU on the efficient frontier that most closely resembles that DMU as its benchmark to improve the performance of the inefficient DMU [1].

The obtained $\lambda^*_j$ for each inefficient DMU can be employed to introduce benchmarks for each of them. In other words, when $\lambda^*_j$ pertaining to each inefficient DMU is greater than zero, the DMU can be served as a benchmark to use the obtained benchmark. Column 2 in Table 1 summarizes the method of benchmark selection for inefficient DMUs [1].

### 3.3.4. Target Setting for Inefficient DMUs

To suggest target values, first of all, it is required to identify the inefficient DMUs. Let DMU0 be an inefficient DMU that can be identified using. By solving an LP problem, the improvement in the values of each input and output for DMU0 can be determined [19]. Optimal slacks were obtained from model equations (5)–(11) to improve the efficiency of the inefficient DMUs. Table 2 summarizes the method of efficiency improvement for inefficient DMUs.
obtained from the above table; that is, the resultant on the efficient frontier that has fewer inputs than those is not placed on the efficiency frontier, there will be a point optimalslacksizeobtainedfromthemodelandincaseDMU

ures2and3, the input and output variables are classified into thoroughly be analyzed and evaluated. According to Fig-
ncementindustrydependonmanyoperational,executive,

and technical factors, the qualities and effects of which must thoroughly be analyzed and evaluated. According to Fig-

damental industry in developing Iran’s economic
development for every country and has a key role as a fun-
drate of all other industries in the modern era. Some argue that cement production is an indicator of growth and de-

s| Variable | Minimum | Maximum | Std. deviation | Variance |
|---------|---------|---------|---------------|----------|
| X1      | 3.67    | 4.67    | 0.34764       | 0.121   |
| X2      | 3743.65 | 28651.32| 6665.90269    | 44434258.660 |
| X3      | 55000.00| 140000.0| 410903.06560  | 168430676.300 |
| X4      | 51418.79| 393524.12| 92278.87003  | 8515389854 |
| X5      | 22461.89| 171907.91| 40156.70943   | 1612561312 |
| X6      | 27920.15| 144926.34| 40292.41565   | 1623478759 |
| X7      | 43976.59| 215747.88| 480137.8300   | 2305323358 |
| X8      | 4318    | 659846  | 200278.34510  | 40111415510 |
| X9      | 80      | 1276    | 33784908      | 114142   |
| X10     | 116312  | 3648000 | 1158841.62600 | 134291394000 |

| DMUs | Benchmarks | Stages | Inputs | Slacks | Targets |
|------|------------|--------|--------|--------|---------|
| 1    |            |        |        |        |         |
| 2    |            |        |        |        |         |
| 3    |            |        |        |        |         |
| 4    |            |        |        |        |         |

Table 2: Benchmark selection for inefficient DMUs and target setting through the optimal slacks.

**Table 1**: Statistical description of $X$ variable related to DMU1.

**Theorem 3.** According to column 6 in Table 2, target values for the inputs project the inefficient DMUs on the efficient frontier.

**Proof.** If the DMU input under evaluation is reduced to optimal slack size obtained from the model and in case DMU is not placed on the efficiency frontier, there will be a point on the efficient frontier that has fewer inputs than those obtained from the above table; that is, the resultant of model equation (5) is not optimal. Since the responses from model equation (5) are optimal, the improvements to the inputs using Table 2 will project the inefficient DMUs on the efficient frontier.

**4. Results and Discussion**

The proposed model is able to evaluate sustainable performance in different industries. The cement industry has a strategic and significant role in a nation’s civil and economic development and is characterized by the largest production rate of all other industries in the modern era. Some argue that cement production is an indicator of growth and development for every country and has a key role as a fundamental industry in developing Iran’s economic infrastructures, so the proposed model is adopted to evaluate the cement industry. It was tested by collecting actual data from 10 Iranian cement companies listed on the Tehran Stock Exchange in 2016. The durability and survivability of the cement industry depend on many operational, executive, and technical factors, the qualities and effects of which must thoroughly be analyzed and evaluated. According to Figures 2 and 3, the input and output variables are classified into four stages. The network structure modeling criterion is based on the same classification:

### Variables of Stage 1: these variables are considered the input entrances or suppliers, the most important examples of which are introduced as mines, mining, initial capital, and the status quos of suppliers of usable instruments. In this stage, processing yields the variables that are identified as the outputs of this stage. Preparation of materials and equipment required for the supply chain stage is among the most important outputs of the first stage, which are divided into two classes: (1) the outputs that are used as the inputs of the second stage and (2) the outputs that will directly be used as the inputs of the third stage.

### Variables of Stage 2: the input variables of this stage are actually the very outputs of the first stage used in the process of production and operations. Research and development activities are used as the input in this stage. The output variables of this stage include products and produced goods that will be transferred to the next stage along with the undesirable outputs yielded by the process of production and operations. This stage also results in greenhouse gases, particles, and environmental pollution that endanger the supply chain. Moreover, this stage generates some outputs that are transferred directly as the input variables to the fourth stage. Due to the presence of undesirable outputs in this stage and the higher probability of generating these outputs than the national standard (determined by the Iranian Department of Environment) and adversely affecting the supply chain, certain moderating variables will be employed. In fact, if a chain has

...
undesirable performance in one index and violates the determined standard, it will be fined. The fine is considered an independent variable in this stage.

Variables of Stage 3: the main activities of this stage of the four-stage supply chain include preparation of products for distribution, sale, and inventory. The input variables of this stage are divided into two categories: (1) the inputs that are the very outputs obtained from the second stage and (2) some output variables of the first stage that were directly transferred to the third stage. The outputs of the internal processes of this stage are used as the inputs of the fourth stage.

Variables Stage 4: considering the final stage of the supply chain, this stage yields the final outputs of the chain. The main activities of this stage include selling products, supplying the demands of customers and consumers, and considering social responsibility. The input variables of this stage are the outputs of the third stage as well as the outputs of the second stage that entered the fourth stage directly. The output variables of this stage are identified as the final outputs of the supply chain.

The key performance indicators were selected according to four important levels, namely, environmental sustainability, strategic, process, and operational. They are as follows: X: (1) quality of suppliers in terms of sustainable supply of minerals and consumables, (2) cost of green and sustainability education for following relevant rules in the supply chain, (3) total initial investment in mine exploitation and factory processes, (4) total salaries and wages, (5) total costs of purchasing minerals, chemicals, and other consumable substances, (6) the sum of money paid to contractors for mining, (7) total transportation cost, (8) total financial costs, (9) total number of employees, and (10) total debts of factories; Z: (1) total available mineral reserves, (2) total tonnage of extracted minerals, (3) total tonnage of chemical and mineral raw materials added to the production process, (4) total mineral raw materials in storage depots for use in cold seasons, (5) the quality of training programs for suppliers and employees for sustainable production and TQM, (6) annual mazut energy fuel consumption in liter, (7) total research and development expenditures, (8) total costs of purchasing minerals, chemicals, and other consumable substances, (9) the quality of suppliers in terms of sustainable supply of minerals and consumables, (10) total growth rate according to performance, (11) return on assets (ROA), (12) return on equity (ROE), (13) effectiveness of factory in a particular area of activity, (14) customer satisfaction, (15) implementing quality working conditions for personnel, (16) social responsibility, (17) ton of waste (bags) disposed in the environment, and (18) ton of pollution caused by emitting unrecyclable substances.

Then we determine inputs and desirable and undesirable outputs for each stage. The general network structure includes 10 homogenous DMUs, all of which consist of four stages and also have similar internal structures and internal relations presented in Figure 3.

Table 1 indicates the statistical data for the X variable pertaining to DMU1. The statistical information regarding the variables Z, F, L, M, K′, Z′, and Y was obtained based on Table 1 for each DMU under evaluation in SPSS. Due to the large amount of data and the limited number of pages in papers, their titles were omitted, and only the statistical information of variable X pertaining to DMU1 in Table 1 will be discussed.

The piecewise linear function was used to determine the intervals of undesirable outputs with nonlinear effect and nonlinear valuation in the following manner:

\[
y_{y'j}^{kk}\left\{ \begin{array}{ll}
L_k & \text{if } k = 1 \\
L_k - L_{k-1} & \text{if } k = 2, \ldots, k_j - 1 \\
y_{y'j}^r - L_{k-1} & \text{if } k = k_j \\
0 & \text{if } k > k_j \\
\end{array} \right.
\]

Out of 14 variables, the first 12 are considered desirable outputs, whereas the thirteenth and fourteenth variables are undesirable. \( y_{y'j}^{kk} \), \( k = 1, 2, 3 \), \( r_{y'} = 13 \), refers to the scale of the thirteenth variable in the \( k \) th interval. Accordingly, the scale of the thirteenth output was divided into three intervals \( (k = 1, 2, 3) \), in each of which the output indicated linear behavior and the intervals had a different value.
Accordingly, the scale of the fourteenth output was divided into four intervals, in each of which the output indicated linear behavior and the intervals had a different value. Similarly, for undesirable intermediate products \((f_4, f_5, f_6, f_7, \text{ and } z_{12})\), the amount of the given variable is divided into intervals in which the behavior of the variable is linear.

\[ z_{23}^{12} k = \begin{cases} L_k & \text{if } k = 1 \\ L_k - L_{k-1} & \text{if } k = 2, \ldots, k_j - 1 \\ z_{dj} - L_{k-1} & \text{if } k = k_j \\ 0 & \text{if } k > k_j \end{cases} \]

\[ z_{23}^{12} k = \begin{cases} [0, 1.5], (1.5, 2.5], (2.5, 3], & \text{if } k = 1 \\ 0, 400], (400, 450], (450, 550], & \text{if } k = 2, \ldots, k_j - 1 \\ 0, 2060], (2060, 2180], (2180, 2220], & \text{if } k = k_j \\ 0, 650], (650, 680], (680, 710], & \text{if } k > k_j \end{cases} \]

\[ f_{23}^{23} k = \begin{cases} L_k & \text{if } k = 1 \\ L_k - L_{k-1} & \text{if } k = 2, \ldots, k_j - 1 \\ f_{dj} - L_{k-1} & \text{if } k = k_j \\ 0 & \text{if } k > k_j \end{cases} \]

\[ f_{23}^{23} k = \begin{cases} [0, 650], (650, 680], (680, 710], & \text{if } k = 1 \\ 0, 126], (126, 129], (129, 131] \end{cases} \]

According to the indicators presented above and the general network structure model proposed previously, undesirable outputs and their nonlinear behavior were taken into account and given nonlinear values for the efficiency evaluation of companies in GAMS. Table 3 presents the results.

The efficiency and benchmarks of each stage can be calculated through the formulas proposed in this study. Due to the abundance of results, only the efficiency and benchmarks for Stage 1 were presented in this section.

Table 3 shows the results of applying the proposed model to data from ten cement-manufacturing companies in GAMS. Furthermore, the efficiency values of DMUs 4, 5, and 6 are smaller than one, whereas those of the remaining DMUs are equal to one. Based on Table 4, since the sum of slack variables values resulting from model equation (5) for DMUs 4, 5, and 6 was not zero, these three DMUs were inefficient, and a benchmark for each of them must be introduced to improve them. As a result, the appropriate benchmarks for inefficient DMUs will be presented here.

According to Table 5, the obtained nonzero \(\lambda^*\) can be employed to select an appropriate benchmark for each of the inefficient DMUs. In other words, the resultant nonzero \(\lambda^*\) corresponding to each efficient DMU was selected as a benchmark for the inefficient DMUs. For DMU4, nonzero \(\lambda_1^*\) and \(\lambda_2^*\) were obtained. Therefore, the DMUs corresponding to \(\lambda^*\) include DMU1 and DMU9 that can be introduced as a benchmark to improve and modify the efficiency of DMU4.

According to Table 2, in order to improve the efficiency of inefficient DMUs, the values of input variables pertaining to them can be changed based on the obtained benchmarks. Therefore, the respective DMU is placed on the efficient frontier. As a result, the optimal slacks will be utilized to specify the reduction of the values of the input variables or the increase of the values of the output variables. Due to the abundance of data, this section only discussed the table of \(X^*\) input modification and improvement for each inefficient DMU (Table 6). For instance, for DMU4, \(x_1^* = 4\) and \(s_{x_1}^* = 0.333\); thus, in order to increase the efficiency of

| Table 3: Efficiency measurement using model equation (8) for 10 cement companies. |
| DMU | Efficiency | DMU | Efficiency |
|-----|------------|-----|------------|
| 1   | 1          | 6   | 0.7517     |
| 2   | 1          | 7   | 1          |
| 3   | 1          | 8   | 1          |
| 4   | 0.9987     | 9   | 1          |
| 5   | 0.9946     | 10  | 1.1        |

| Table 4: Results of objective function from model equation (5) for DMUs. |
| DMU | Sum of slacks |
|-----|---------------|
| 1   | 0             |
| 2   | 0             |
| 3   | 0             |
| 4   | 309260.8      |
| 5   | 787765.7      |
| 6   | 4136733       |
| 7   | 0             |
| 8   | 0             |
| 9   | 0             |

| Table 5: The benchmarks obtained for inefficient DMUs. |
| DMU | \(\lambda^* > 0\) | The benchmarks |
|-----|-------------------|----------------|
| 4   | \(\lambda_1^*, \lambda_2^* > 0\) | DMU1; DMU9 |
| 5   | \(\lambda_3^*, \lambda_4^* > 0\) | DMU1; DMU9 |
| 6   | \(\lambda_5^*, \lambda_6^*, \lambda_7^* > 0\) | DMU1; DMU9; DMU10 |

| Table 6: Target values of inefficient DMUs for achieving the benchmark. |
| \(x^*\) | DMUs |
|--------|------|
| 3.67   | 4    |
| 3.67   | 5    |
| 3.67   | 6    |
| 3743.649 | 4    |
| 3743.649 | 5    |
| 3743.649 | 6    |
| 88000  | 4    |
| 88000  | 5    |
| 88000  | 6    |
| 51418.79 | 4    |
| 51418.79 | 5    |
| 51418.79 | 6    |
| 22461.89 | 4    |
| 22461.89 | 5    |
| 22461.89 | 6    |
| 27920.15 | 4    |
| 27920.15 | 5    |
| 27920.15 | 6    |
| 43976.6 | 4    |
| 43976.6 | 5    |
| 43976.6 | 6    |
| 12004  | 4    |
| 12004  | 5    |
| 12004  | 6    |
| 80     | 4    |
| 80     | 5    |
| 80     | 6    |
| 351205 | 4    |
| 351205 | 5    |
| 351205 | 6    |
DMU4, the first input \(x_1\) should be reduced to 0.333, so \(x_1^* = 3.67\) will then be obtained. All modifications such as reducing input consumption or increasing output production can increase efficiency and transform inefficient DMUs into efficient ones.

5. Conclusion

Since managers and decision-makers are seeking ways to improve the performance of their companies, complete information about corporate efficiency is required. The undesirable outputs are among the factors that have negative impacts on the efficiency of a company; thus, decision-makers are attempting to measure efficiency by considering the effect of the undesirable factors in order to identify DMUs with better performance under the same conditions and regard them as a benchmark for inefficient DMUs. Therefore, this study proposed the SBM network DEA model to analyze the performance of cement-manufacturing companies with a 4-stage structure while considering undesirable outputs. The piecewise linear theory was employed to consider fewer values for quantities greater than the undesirable outputs. Consequently, the DMUs that produced fewer undesirable outputs were distinguished from DMUs that produced more undesirable outputs. Finally, the proposed model was solved to introduce an appropriate benchmark for inefficient DMUs.

The current study faces the following limitations:

(i) Structural complexity of the analyzed supply chain and development of mathematical equations and constraints.

(ii) Inaccessibility of exclusive websites and databanks in the cement industry of Iran.

(iii) Lengthy bureaucratic procedures for acquiring permits or relative information.

(iv) Unavailability of sufficient classified information on undesirable outputs in the Iranian Department of Environment.

(v) Presence of certain constraints on the proposed mathematical modeling framework for the research problem, including the use of binary values or the large-scale M variable.

Future studies can use the proposed method for more accurate ranking of units under evaluation, examining density, and evaluating of progress, regression, and efficiency, as well as pricing appropriate to the amount of production in different areas.

Data Availability

Data from the published literature were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] A. Charnes, W. W. Cooper, and E. Rhodes, “Measuring the efficiency of decision making units,” European Journal of Operational Research, vol. 2, no. 6, pp. 429–444, 1978.

[2] S. Ang and C.-M. Chen, “Pitfalls of decomposition weights in the additive multi-stage DEA model,” Omega, vol. 58, pp. 139–153, 2016.

[3] A. Emrouznejad and M. Tavano, Performance measurement with fuzzy data envelopment analysis, Vol. 309, Springer, New York, US, 2014.

[4] R. Färe and S. Grosskopf, “Network DEA,” Socio-Economic Planning Sciences, vol. 1, no. 34, pp. 35–49, 2000.

[5] Q. An, Y. Wen, T. Ding, and Y. Li, “Resource sharing and payoff allocation in a three-stage system: integrating network DEA with the Shapley value method,” Omega, vol. 85, pp. 16–25, 2019.

[6] L. Liang, F. Yang, W. D. Cook, and J. Zhu, “DEA models for supply chain efficiency evaluation,” Annals of Operations Research, vol. 145, no. 1, pp. 35–49, 2006.

[7] C. Kao and S.-N. Hwang, “Efficiency decomposition in two-stage data envelopment analysis: an application to non-life insurance companies in Taiwan,” European Journal of Operational Research, vol. 185, no. 1, pp. 418–429, 2008.

[8] Y. Chen, W. D. Cook, N. Li, and J. Zhu, “Additive efficiency decomposition in two-stage DEA,” European Journal of Operational Research, vol. 196, no. 3, pp. 1170–1176, 2009.

[9] Y. Li, Y. Chen, L. Liang, and J. Xie, “DEA models for extended two-stage network structures,” Omega, vol. 40, no. 5, pp. 611–618, 2012.

[10] D. K. Despotis, G. Koronakos, and D. Sotiros, “The “”,” European Journal of Operational Research, vol. 254, no. 2, pp. 481–492, 2016a.

[11] W. D. Cook, L. Liang, and J. Zhu, “Measuring performance of two-stage network structures by DEA: a review and future perspective,” Omega, vol. 38, no. 6, pp. 423–430, 2010.

[12] J. Xie, X. Zhu, and L. Liang, “A multiplicative method for estimating the potential gains from two-stage production system mergers,” Annals of Operations Research, vol. 288, no. 1, pp. 475–493, 2020.

[13] D. K. Despotis, L. V. Stamatii, and Y. G. Smirlis, “Data envelopment analysis with nonlinear virtual inputs and outputs,” European Journal of Operational Research, vol. 202, no. 2, pp. 604–613, 2010.

[14] D. K. Despotis and D. Sotiros, “Value-based data envelopment analysis: a piece-wise linear programming approach,” International Journal of Multicriteria Decision Making, vol. 4, no. 1, pp. 47–68, 2014.

[15] C. Kao, “Efficiency decomposition in network data envelopment analysis with slacks-based measures,” European Journal of Operational Research, vol. 192, no. 3, pp. 949–962, 2004a.

[16] C. Kao, “Network data envelopment analysis: a review,” European Journal of Operational Research, vol. 239, no. 1, pp. 1–16, 2014b.

[17] K. Tone, M. Tsutsui, and D. E. A. Network, “Network DEA: a slacks-based measure approach,” European Journal of Operational Research, vol. 197, no. 1, pp. 243–252, 2009.

[18] X. Ji, J. Wu, Q. Zhu, and J. Sun, “Using a hybrid heterogeneous DEA method to benchmark China’s sustainable urbanization: an empirical study,” Annals of Operations Research, vol. 278, no. 1, pp. 281–335, 2019.

[19] K. Tone, “A slacks-based measure of efficiency in data envelopment analysis,” European Journal of Operational Research, vol. 130, no. 3, pp. 498–509, 2001.
[20] M. Zarei, A. Azar, and A. Emrouznejad, “A novel multilevel network slacks-based measure with an application in electric utility companies,” *Energy*, vol. 158, pp. 1120–1129, 2018.

[21] F. Yang, D. Wang, L. Zhao, and F. Wei, “Efficiency evaluation for regional industrial water use and wastewater treatment systems in China: a dynamic interactive network slacks-based measure model,” *Journal of Environmental Management*, vol. 279, pp. 1–11, 2021.

[22] X. Guo, Q. Zhu, L. Lv, J. Chu, and J. Wu, “Efficiency evaluation of regional energy saving and emission reduction in China: a modified slacks-based measure approach,” *Journal of Cleaner Production*, vol. 140, pp. 1313–1321, 2017.

[23] Y. Shang, H. Liu, and Y. Lv, “Total factor energy efficiency in regions of China: an empirical analysis on SBM-DEA model with undesired generation.” *Journal of King Saud University Science*, vol. 32, no. 3, pp. 1925–1931, 2020.

[24] L. Cecchin, S. Venanzini, A. Pierri, and M. Chiorri, “Environmental efficiency analysis and estimation of CO2 abatement costs in dairy cattle farms in Umbria (Italy): a SBM-DEA model with undesirable output,” *Journal of Cleaner Production*, vol. 197, pp. 895–907, 2018.

[25] S. H. Pishgar-Komleh, T. Zylowski, S. Rozakis, and J. Kozyra, “Efficiency under different methods for incorporating undesirable outputs in an LCA+DEA framework: a case study of winter wheat production in Poland,” *Journal of Environmental Management*, vol. 260, p. 110138, 2020.

[26] Y.-T. Chang, H.-s. Park, J.-b. Jeong, and J.-w. Lee, “Evaluating economic and environmental efficiency of global airlines: a SBM-DEA approach,” *Transportation Research Part D: Transport and Environment*, vol. 27, pp. 46–50, 2014.

[27] K. Tone and M. Tsutsui, “Dynamic DEA with network structure: a slacks-based measure approach,” *Omega*, vol. 42, no. 1, pp. 124–131, 2014.

[28] S. Lozano and B. Adenso-Díaz, “Increasing Sustainability of Logistic Networks by Reducing Product Losses: A Network DEA Approach,” *Mathematical Problems in Engineering*, vol. 2018, Article ID 3479251, 21 pages, 2018.

[29] M. Z. Mahmoudabadi and A. Emrouznejad, “Comprehensive performance evaluation of banking branches: a three-stage slacks-based measure (SBM) data envelopment analysis,” *International Review of Economics & Finance*, vol. 64, pp. 359–376, 2019.

[30] Y. Li, X. Shi, A. Emrouznejad, and L. Liang, “Environmental performance evaluation of Chinese industrial systems: a network SBM approach,” *Journal of the Operational Research Society*, vol. 69, no. 6, pp. 825–839, 2018.

[31] T. Yang, P. Wang, and F. Li, “Centralized resource allocation and target setting based on data envelopment analysis model,” *Mathematical Problems in Engineering*, vol. 2018, Article ID 3826096, 10 pages, 2018.

[32] T. Badiezadeh, R. F. Saen, and T. Samavati, “Assessing sustainability of supply chains by double Frontier network DEA: a big data approach,” *Computers & Operations Research*, vol. 98, pp. 284–290, 2018.

[33] Y. Su and W. Sun, “Sustainability evaluation of the supply chain with undesired outputs and dual-role factors based on double Frontier network DEA,” *Soft Computing*, vol. 22, no. 16, pp. 5525–5533, 2018.

[34] C. Kao, *Network data envelopment analysis*, vol. 10, pp. 26–33, Springer, New York, US, 2017.

[35] B. Keskin, “An efficiency analysis on social prosperity: OPEC case under network DEA slack-based measure approach,” *Energy*, vol. 231, p. 120832, 2021.

[36] Q. Zhu, J. Aparicio, F. Li, J. Wu, and G. Kou, “Determining closest targets on the extended facet production possibility set in data envelopment analysis: modeling and computational aspects,” *European Journal of Operational Research*, vol. 296, no. 3, pp. 927–939, 2022.

[37] Q. Zhu, X. Li, F. Li, J. Wu, and D. Zhou, “Energy and environmental efficiency of China’s transportation sectors under the constraints of energy consumption and environmental pollution,” *Energy Economics*, vol. 89, Article ID 104817, 2020.

[38] Y. Chen, S. Cheng, and Z. Zhu, “Measuring environmental-adjusted dynamic energy efficiency of China’s transportation sector: a four-stage NDDF-DEA approach,” *Energy Efficiency*, vol. 14, no. 3, pp. 1–14, 2021.

[39] X. Shi, A. Emrouznejad, and W. Yu, “Overall efficiency of operational process with undesirable outputs containing both series and parallel processes: a SBM network DEA model,” *Expert Systems with Applications*, vol. 178, Article ID 115062, 2021.

[40] H. Li, X. Zhu, and J. Chen, “Total factor waste gas treatment efficiency of China’s iron and steel enterprises and its influencing factors: an empirical analysis based on the four-stage SBM-DEA model,” *Ecological Indicators*, vol. 119, Article ID 106812, 2020.

[41] X. Zhou, R. Luo, L. Yao, S. Cao, S. Wang, and B. Lev, “Assessing integrated water use and wastewater treatment systems in China: a mixed network structure two-stage SBM DEA model,” *Journal of Cleaner Production*, vol. 185, pp. 533–546, 2018.

[42] Q. An, H. Chen, J. Wu, and L. Liang, “Measuring slacks-based efficiency for commercial banks in China by using a two-stage DEA model with undesirable output,” *Annals of Operations Research*, vol. 235, no. 1, pp. 13–35, 2015.

[43] N. A. Ramli and S. Munisamy, “Modeling undesirable factors in efficiency measurement using data envelopment analysis: A review,” 2013.

[44] L. Cherchye, B. D. Rock, and B. Walheer, “Multi-output efficiency with good and bad outputs,” *European Journal of Operational Research*, vol. 240, no. 3, pp. 872–881, 2015.

[45] W. D. Cook and J. Zhu, “Piecewise linear output measures in DEA (third revision),” *European Journal of Operational Research*, vol. 197, no. 1, pp. 312–319, 2009.

[46] F. H. Lotfi, M. Rostami-Malkhalifeh, and Z. Moghaddas, “Modified piecewise linear DEA model,” *European Journal of Operational Research*, vol. 205, no. 3, pp. 729–733, 2010.

[47] W. D. Cook, F. Yang, and J. Zhu, “Nonlinear inputs and diminishing marginal value in DEA,” *Journal of the Operational Research Society*, vol. 60, no. 11, pp. 1567–1574, 2009.

[48] A.-b. Ji, Y. Ji, and Y. Qiao, “DEA-based piecewise linear discriminant analysis,” *Computational Economics*, vol. 51, no. 4, pp. 809–820, 2018.

[49] E. González and A. Álvarez, “From efficiency measurement to efficiency improvement: the choice of a relevant benchmark,” *European Journal of Operational Research*, vol. 133, no. 3, pp. 512–520, 2001.

[50] C. Baek and J. D. Lee, “The relevance of DEA benchmarking information and the least-distance measure,” *Mathematical and Computer Modelling*, vol. 49, no. 1-2, pp. 265–275, 2009.

[51] M. R. Aliirezaee and M. Afshar, “Model improvement for computational difficulties of DEA technique in the presence of special DMUs,” *Applied Mathematics and Computation*, vol. 186, no. 2, pp. 1600–1611, 2007.

[52] S. A. Estrada, H. S. Song, Y. A. Kim, S. H. Namn, and S. C. Kang, “A method of stepwise benchmarking for inefficient DMUs based on the proximity-based target selection,”
Expert Systems with Applications, vol. 36, no. 9, pp. 11595–11604, 2009.

[53] J. H. Park, H. R. Bae, and S. M. Lim, “Method of benchmarking route choice based on the input-similarity using DEA and SOM,” Journal of Korean Institute of Industrial Engineers, vol. 36, no. 1, pp. 32–41, 2010.

[54] S. Lim, H. Bae, and L. H. Lee, “A study on the selection of benchmarking paths in DEA,” Expert Systems with Applications, vol. 38, no. 6, pp. 7665–7673, 2011.

[55] J. Park, H. Bae, and S. Lim, “A DEA-based method of stepwise benchmark target selection with preference, direction and similarity criteria,” International Journal of Innovative Computing, Information and Control, vol. 8, no. 8, pp. 5821–5834, 2012.

[56] J. L. Ruiz and I. Sirvent, “Benchmarking within a DEA framework: setting the closest targets and identifying peer groups with the most similar performances,” International Transactions in Operational Research, vol. 29, no. 1, pp. 554–573, 2022.

[57] N. Ramon, J. L. Ruiz, and I. Sirvent, “Cross-benchmarking for performance evaluation: looking across best practices of different peer groups using DEA,” Omega, vol. 92, Article ID 102169, 2020.

[58] P. Ravelojaona, “On constant elasticity of substitution - c,” European Journal of Operational Research, vol. 272, no. 2, pp. 780–791, 2019.

[59] K. Ly, A. Yu, and Y. Bian, “Regional energy efficiency and its determinants in China during 2001–2010: a slacks-based measure and spatial econometric analysis,” Journal of Productivity Analysis, vol. 47, no. 1, pp. 65–81, 2017.

[60] M. Meng, Y. Fu, and L. Wang, “Low-carbon economy efficiency analysis of China’s provinces based on a range-adjusted measure and data envelopment analysis model,” Journal of Cleaner Production, vol. 199, pp. 643–650, 2018.

[61] M. C. A. S. Portela, P. C. Borges, and E. Thanassoulis, “Finding closest targets in non-oriented DEA models: the case of convex and non-convex technologies,” Journal of Productivity Analysis, vol. 19, no. 2, pp. 251–269, 2003.

[62] S. Lozano and G. Villa, “Determining a sequence of targets in DEA,” Journal of the Operational Research Society, vol. 56, no. 12, pp. 1439–1447, 2005.

[63] J. Aparicio, J. L. Ruiz, and I. Sirvent, “Closest targets and minimum distance to the Pareto-efficient Frontier in DEA,” Journal of Productivity Analysis, vol. 28, no. 3, pp. 209–218, 2007.

[64] G. R. Jahanshahloo, J. Vakili, and M. Zarepisheh, “A linear bilevel programming problem for obtaining the closest targets and minimum distance of a unit from the strong efficient Frontier,” Asia Pacific Journal of Operational Research, vol. 29, no. 02, Article ID 1250011, 2012.

[65] L. Chen and Y. M. Wang, “DEA target setting approach within the cross efficiency framework,” Omega, vol. 96, Article ID 102072, 2019.

[66] W. D. Cook, J. L. Ruiz, I. Sirvent, and J. Zhu, “Within-group common benchmarking using DEA,” European Journal of Operational Research, vol. 256, no. 3, pp. 901–910, 2017.

[67] S. Lozano and L. Calzada-Infante, “Computing gradient-based stepwise benchmarking paths,” Omega, vol. 81, pp. 195–207, 2018.

[68] N. Nasrabadi, A. Dehnokhalaji, P. Korhonen, and J. Wallenius, “A stepwise benchmarking approach to DEA with interval scale data,” Journal of the Operational Research Society, vol. 70, no. 6, pp. 954–961, 2019.