Probing Noise in Flux Qubits via Macroscopic Resonant Tunneling

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Macroscopic resonant tunneling between the two lowest lying states of a bistable RF-SQUID is used to characterize noise in a flux qubit. Measurements of the incoherent decay rate as a function of flux bias revealed a Gaussian shaped profile that is not peaked at the resonance point, but is shifted to a bias at which the initial well is higher than the target well. The r.m.s. amplitude of the noise, which is proportional to the decoherence rate 1/T2 eff, was observed to be weakly dependent on temperature below 70 mK. Analysis of these results indicates that the dominant source of low frequency (1/f) flux noise in this device is a quantum mechanical environment in thermal equilibrium.

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The viability of any scalable quantum computing architecture is highly dependent upon its performance in the presence of noise. In the case of superconducting qubits it has been shown that low frequency (1/f) noise is of particular concern [1]. Furthermore, evidence suggests that such devices generically couple to an ensemble of effective 2-level systems (TLS) that may be material defects [2]. A number of theories exist that attempt to correlate these two observations [3], however it is not certain whether the TLS observed in spectroscopy experiments are the dominant source of low frequency noise in these devices [4]. The development of additional experimental probes of low frequency noise will prove critical in the quest to build reliable superconductor-based quantum computing hardware. In this article we demonstrate a new experimental procedure for quantifying low frequency flux noise in RF-SQUID qubits. The procedure developed herein complements other approaches to using qubits as spectrometers for studying noise [5].

Macroscopic resonant tunneling (MRT) [6, 7] is an important probe of quantum effects in Josephson junction-based devices. In an MRT experiment flux tunnels between two wells of a double well potential when energy levels are aligned. The tunneling rate and width of the tunneling region are strongly influenced by the environment. The effect of flux noise on the two lowest energy levels can be described using an effective Hamiltonian

$$H_{\text{eff}} = -\frac{1}{2} [\sigma_z + \Delta \sigma_z] - \frac{1}{2} Q_z \sigma_z,$$  (1)

where ε is the bias energy between the wells, Δ is the tunneling amplitude, Qz is an operator that acts on the low frequency modes of the environment and σ±(z) are Pauli matrices. In general, a transverse coupling to the environment should also exist but it is believed to be subdominant to the longitudinal coupling in flux qubits [8]. It is demonstrated that the experimental results presented herein are consistent with this expectation.

A theoretical analysis of MRT in the presence of low frequency (non-Markovian) flux noise was reported in Ref. [9]. The transition rate Γ01 from state \( |0\rangle \) to state \( |1\rangle \) (both eigenfunctions of \( \sigma_z \) with eigenvalues \( \mp 1 \), respectively) was found to be

$$\Gamma_{01}(\epsilon) = \frac{7}{8} \Delta^2 \exp \left[ -\frac{(\epsilon - \epsilon_p)^2}{2W^2} \right],$$  (2)

$$\epsilon_p = \mathcal{P} \int_{-\infty}^{\infty} d\omega S(\omega) \left[ \int_{-\infty}^{\infty} d\omega S(\omega) \right]^{1/2},$$

where \( S(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \langle Q_z(t)Q_z(0) \rangle \) is the unsymmetrized noise spectral density [10]. Γ10 is obtained by substituting \( \epsilon_p \rightarrow -\epsilon_p \). Equation (2) is valid provided the noise spectrum is peaked at low frequency and the integrals defining \( \epsilon_p \) and \( W \) are finite [9]. The latter constraint may require low and high frequency cutoffs.

The quantities \( \epsilon_p \) and \( W \) represent the energy shift and width of a Gaussian shaped tunneling rate, respectively. \( W \) is the r.m.s. amplitude of the noise and \( \epsilon_p \) is a measure of the asymmetry of \( S(\omega) \). For a classical noise source \( S(\omega) \approx S(-\omega) \), hence \( \epsilon_p = 0 \). For a quantum source \( S(\omega) \) need not be symmetric and therefore \( \epsilon_p \neq 0 \). In thermal equilibrium at temperature \( T_{\text{eff}} \) the fluctuation-dissipation theorem dictates that \( S(\omega) \) can be expressed as a sum of symmetric and antisymmetric components \( S(\omega) = S_\sigma(\omega) + S_a(\omega) \), where \( S_\sigma(\omega) = S_\sigma(\omega) \coth(\omega/2T_{\text{eff}}) \) \((\hbar/k_B = 1)\). If \( S_\sigma(\omega) \) is sharply peaked near \( \omega = 0 \) then it can be shown that

$$W^2 \approx \mathcal{P} \int_{-\infty}^{\infty} d\omega S_\sigma(\omega)(2T_{\text{eff}}/\omega) = 2T_{\text{eff}} \epsilon_p.$$  (3)

The width \( W \) is an important parameter in adiabatic quantum computation [11] as it defines the precision to which a target Hamiltonian can be specified in the logical basis defined by the eigenstates of \( \sigma_z \). From the perspective of gate model quantum computation \( W \) is closely
related to the decoherence time $T^*_2$ in the energy basis defined by the eigenstates of Eq. (1). For a low frequency environment the decay due to dephasing has the form $e^{-t^2/2T^*_2}$, where $1/T^*_2 = \cos(\eta) W$ and $\eta = \arctan(\Delta/\epsilon)$.

To demonstrate the MRT method for characterizing low frequency noise we employed an RF-SQUID qubit [12, 18]. Previous experimental observations of MRT in RF-SQUIDs have been limited to tunneling into higher energy levels with and without the help of microwave activation [7, 13]. In this paper we examine MRT between the two lowest energy levels of an RF-SQUID. Measurements of directional tunneling rates $\Gamma_{01}$ and $\Gamma_{10}$ are used to extract quantitative information regarding $S(\omega)$.

A schematic of a compound Josephson junction (CJJ) RF-SQUID is shown in the inset of Fig. 1(a). It consists of a main loop and CJJ loop subjected to external flux biases $\Phi^x$ and $\Phi^{\text{cj}}$, respectively. The CJJ loop is interrupted by two nominally identical Josephson junctions connected in parallel with total capacitance $C^q$ and critical current $I^q$. The CJJ and main loop possess inductances $L^{x\text{ij}}$ and $L^q$, respectively. If $L^{x\text{ij}} \ll L^q$ then the RF-SQUID Hamiltonian can be written as

$$\mathcal{H}_t(\Phi^q, \Pi^q) = \frac{1}{2C^q}(\Pi^q)^2 + U(\Phi^q),$$

where $\Phi^q$ represents the total flux threading the main loop, $\Pi^q$ is the conjugate momentum, $E_J \equiv \Phi^x_0 I^q_0/2\pi$ and $\Phi_0 = h/2e$. This device can be operated as a qubit for $\Phi^{\text{cj}}_x \in [0.5, 1] \Phi^x_0$ and $\Phi^{\text{cj}}_z \approx 0$. Denoting the ground and first excited state of $\mathcal{H}_t$ at $\Phi^x_0 = 0$ by $|g\rangle$ and $|e\rangle$, respectively, the qubit states can be expressed as $|0\rangle = (|g\rangle + |e\rangle)/\sqrt{2}$ and $|1\rangle = (|g\rangle - |e\rangle)/\sqrt{2}$. The bias energy of Eq. (1) is given by $\epsilon = 2 I^q_0 \Phi^{\text{cj}}_x$, where the persistent current $|I^q_0| \equiv |\langle 0| \Phi^q/L^q |0\rangle| \equiv |\langle 1| \Phi^q/L^q |1\rangle|$. The tunneling amplitude of Eq. (1) is given by $\Delta = \langle e| \mathcal{H}_t | e \rangle - (|g| \mathcal{H}_t | g \rangle)$. Both $|I^q_0|$ and $\Delta$ are controlled by $\Phi^{\text{cj}}_x$. Maximum $\Delta \sim \omega_p$, where $\omega_p$ is the plasma frequency of the RF-SQUID, is obtained at $\Phi^{\text{cj}}_x = \Phi_0/2$. For $\Phi^{\text{cj}}_x \approx \Phi_0$ one expects $\Delta \rightarrow 0$ and the system becomes localized in $|0\rangle$ or $|1\rangle$. In this latter regime $|I^q_0|$ generates a measurable amount of flux that can be detected via an inductively coupled DC-SQUID read-out (not shown) as described in Ref. [15, 16].

The CJJ RF-SQUID from which the data presented herein were obtained was fabricated on an oxidized Si wafer using a Nb trilayer process with wiring layers isolated by sputtered SiO$_2$. The device parameters were $L^q = 661 \pm 6\text{ pH}$, $C^q = 146 \pm 3\text{ fF}$ and $I^q_0 = 1.95 \pm 0.05\text{ mA}$. $L^q$ was measured using a breakout structure and $C^q$ was inferred from the value of $L^q$ and measurements of the MRT peak spacing [7]. From the MRT spacing we also determined $\omega_p \sim 10\text{ GHz}$. The DC-SQUID was observed to have a maximum switching current $I^\text{DC}_{\text{sw}} = 1.9 \pm 0.1\text{ mA}$ and the readout-qubit mutual inductance was $M_{\text{ro-q}} = 16.7 \pm 0.2\text{ pH}$. The device was mounted in an Al box in a dilution refrigerator and all on-chip cross couplings were calibrated in-situ as described in Ref. [16].

Our experimental procedure is a variant of the MRT technique first developed by Rouse, Han, and Lukens [7]. We exploit $\Phi^{\text{cj}}_x$ to modulate $\Delta$ using a bias line with bandwidth $\sim 5\text{ MHz}$. A depiction of the control sequence and the evolution of the qubit potential $U(\Phi^q)$ are shown in Fig. 1. The state of the qubit is initialized by setting $\Phi^{\text{cj}}_x \approx \Phi_0/2$ (i) and then tilting $U(\Phi^q)$ via $\Phi^{\text{cj}}_x$ to an initial value of $\Phi^q_1$ (ii). Slowly raising $\Phi^{\text{cj}}_x$ to $\Phi_0$ in the presence of the tilt traps the system in its groundstate (iii). With tunneling suppressed, $\Phi^q_x$ is reset to a target value $\Phi^q_2$ (iv). Thereafter $\Phi^{\text{cj}}_x$ is lowered (v) to $\Phi^{\text{cj}}_x$ for a prescribed amount of time $t$ during which the system can tunnel from the initial state to the lowest lying state in the opposite well (vi). Raising $\Phi^{\text{cj}}_x$ (vii) to $\Phi_0$ then localizes the qubit state in $|0\rangle$ or $|1\rangle$ (viii) which can then be distinguished by a single shot readout. The probability of tunneling from the initial to final state is then measured as a function of $t$ and $\Phi^q_x$. For a given $\Phi^q_x$ the probability of the system being found in $|0\rangle$ can be calculated from

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\text{FIG. 1: (Color online) (a) MRT flux control bias sequence as a function of time. Inset depicts a CJJ RF-SQUID subjected to the external biases $\Phi^{\text{cj}}_x$ and $\Phi^x$. Sense of the macroscopic persistent current states $|0\rangle$ (counterclockwise) and $|1\rangle$ (clockwise) are noted. (b) Evolution of the qubit potential $U(\Phi^q)$.}
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Using the calibrated device parameters and Eq. (4), we determined that the above mentioned value of $|I_f^p|$ could be achieved for $\Phi_f^{(0)} = 0.606 \pm 0.001 \Phi_0$. The tunnel splitting was then estimated to be $\Delta_0 = 0.10^{+0.028}_{-0.028} \text{mK}$. The asymmetric error bars on $\Delta_0$ are a consequence of its exponential sensitivity to errors in $L^0$, $C^0$, and $I_f^0$.

We have measured $P_0(\Phi_f^p, t)$ for $\Phi_f^p \in [-3, 3] \text{ m} \Phi_0$ and for the qubit initialized with $P_0(0) = 1$ and $P_1(0) = 1$. Example decay data are shown in Fig. 2(a) for $P_0(0) = 1$ at three nominal target values of $\Phi_f^p$ and the device thermalized at $T_{\text{th}} = 28 \text{ mK}$. A summary of initial decay rates as a function of $\Phi_f^p$ for both initializations is shown in Fig. 2(b). We have independently calibrated $\Phi_f^p = 0$ by measuring the population statistics in the limit $t \rightarrow \infty$ and observing that the center of the resultant thermal distribution is independent of initial state. Fitting of the distribution to $P_0(t)=\frac{1}{2}[1 - \tanh(\epsilon/2T_{\text{th}})]$ yields $T_{\text{th}}$. Note that $\Gamma_{01}$ and $\Gamma_{10}$ are mirror images across $\Phi_f^p = 0$ but both are asymmetric about this point. The data suggest that $\Gamma_{01}$ and $\Gamma_{10}$ consist of broad peaks that are displaced away from $\Phi_f^p = 0$ in the direction opposite to in which the system was initialized. These peaks can be attributed to splitting by noise of a single MRT peak corresponding to tunneling between the two lowest lying states of the bistable RF-SQUID. This splitting indicates that $S_n(\omega) \neq 0$. Beyond $|\Phi_f^p| > 2 \Phi_0$, the decay rates increase due to MRT between the initialized state and the first excited state in the opposing well.

The data in Fig. 2(b) have been fit to Eq. (2) using $W$, $\epsilon_p$, and $\Delta$ as free parameters. The fact that the tunneling rate can be fit to a Gaussian lineshape indicates that the noise is dominated by low frequency components. We re-
peated the measurements for five different temperatures and the resultant fit values of $\epsilon_p$ and $W$ are summarized in Fig. 3(a). Here it can be seen that $W$ is only weakly dependent upon $T$. Fitting these results to a line indicates that $W(T \to 0) \approx 80$ mK. This conclusion corroborates the observations presented in Ref. [17] from phase qubits in which the Rabi decay time is observed to have a weak $T$ dependence as $T \to 0$. In contrast, $\epsilon_p$ varies strongly with $T$, behaving as $\sim 1/T_{th}^{0.80 \pm 0.05} \approx 1/T$. Finally, an explicit demonstration of the self-consistency of our analysis is shown in Fig. 3(b) where $T_{th}$ obtained from Eq. (3) is plotted versus $T_{th}$. Agreement between $T_{th}$ and $T_{th}$ demonstrates the equilibrium nature of the low frequency flux noise in our system.

Figure 4 shows the variation of $\Delta$ with temperature. Note that the results are comparable to $\Delta_0$ as estimated from qubit parameters, but that the uncertainty in $\Delta_0$ makes it difficult to draw any quantitative conclusions from such a comparison. Nonetheless, the data suggest that $\Delta \propto T_{th}$. It is known that high frequency flux noise can lead to renormalization of $\Delta$ [10] and that a polynomial dependence of $\Delta$ on $T$ results if the environment possesses an ohmic density of states at high frequency [18]. A similar combination of sharply peaked low frequency noise, as required for Eq. (2), and ohmic high frequency noise has been reported for other qubits [1, 19].

In addition to studying MRT between the two lowest lying states, we have measured MRT from the lowest state in the initial well to higher levels in the final well. It was observed that the tunneling rate versus $\Phi_f^0$ could be fit to a lineshape whose width increased monotonically with the number of levels below the target level, as anticipated in Refs. [6] and [9]. The width of the peak for tunneling to the 50-th level in the final well was roughly twice that for the lowest order MRT peak. This weak dependence on the number of states below the target level indicates the relative weakness of $S(\omega)$ at high frequency.

**Conclusions:** Measurements of MRT between the two lowest energy states of a bistable RF-SQUID have been used to characterize flux noise. Analysis indicates that the noise source is a quantum mechanical environment in thermal equilibrium whose spectral density is sharply peaked at low frequency.

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