Trajectory Planning for Periodic Operation of Autonomous Excavating Robot Based on Time-Jerk Synthetic Optimum

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Abstract. Considering the problem of trajectory planning of redundant degree of freedom in autonomous operation for excavating robot, a method of trajectory planning for periodic operation of autonomous excavating robot based on time-jerk synthetic optimum was proposed in this paper. Through a kinematic model establishment and forward/inverse kinematics analysis for working mechanism, the trajectory planning based on Hermite interpolation method was carried out. Taking the time-jerk as objective function, a mathematical model of nonlinear programming under the inequality constraints was established, and Genetic Algorithm was used to find out the globally optimal solution. The rapidity and stability of working process of excavating robot is effectively enhanced and it has an important significance to realize the intelligentization of construction machinery.

1. Introduction
In order to realize periodically autonomous operation of the excavating robot, research on the trajectory planning is essential. At recent, there are many optimization methods for the robot trajectory planning\cite{1-4}. Excavating robot is a robot with three degrees of freedom in a planar form, can realize the autonomous motion, obstacle avoidance and other special tasks, but its trajectory planning is more complicated than non-redundant structure. For the common foundation pit digging operation, the robot need to complete excavation point fixing from the start position, then full bucket lifting, rotations, unloading, empty bucket returning and posture adjustment, and at last return to the start position for the next cycle\cite{5}. To enhance at the same time of the work efficiency enhancement of periodic excavation, sudden changes of velocity and acceleration will be inevitable, and so as the joint impact, which eventually decrease service life of the component. The approach to improve the work efficiency effectively and reduce the impact of each joint and further prolong the service life is critical to enhance the comprehensive performance of excavator. As mentioned above, a method of trajectory planning for periodic operation of autonomous excavating robot with redundant degree of freedom based on time-jerk synthetic optimum was proposed in this paper. Then the dual factors of the time and the impact after optimization are analyzed, the result shows that rapidity and stability of the working process of excavating robot is effectively enhanced.

2. Kinematic analysis of excavating robot
2.1. The establishment of D-H model of excavating robot’s working device

Figure 1(a) is the photograph of backhoe excavator. The working device consists of boom, arm and bucket joints[6]. To realize autonomous operation, the bucket should keep a certain angle with the ground to realize the horizon digging or autonomous excavation ditch. The tip trajectory of the working device is composed of the movement of three joints when the rotation is not considered. It can be regarded as a planar three bar manipulator. The manipulator, which is made up of four connecting rods with three rotating joints in series, is fixed to the base of excavating robot. In this paper, the trajectory planning of the excavator is carried out by using the combined movement mode. The boom hydraulic cylinder, arm hydraulic cylinder and bucket hydraulic cylinder are cooperated with each other. The digging action is driven by the arm rotating around the hinge point of the boom and the boom hydraulic cylinder is also conducted to control the motion trajectory.

![Figure 1(a) Photograph of Backhoe Excavator](image)

Figure 1. Joints Mechanism of Excavating Robot.

Figure 1(b) is the schematic diagram of the three joints mechanism of excavating robot. The kinematic model of the working device is established according to the Denavit-Hartenberg method[7]. Reference coordinate system of the base is fixed in the connection between the boom and the base of the excavator, and this coordinate system and the coordinate of the connecting rod 1 is set to be coincident. The coordinates, and of connecting rod 2, connecting rod 3 and a bucket tip are established respectively.

2.2. Analysis of kinematics

In this paper, a planar robotic excavator with three degrees of freedom is studied, which is one of the important basic models of robot with redundant degree of freedom. Firstly, the kinematic model of the working device is established, and the forward and inverse kinematics analysis is carried out. In forward kinematics solution, working device of each joint angle and the length of the connecting rod are known, and the position and posture of bucket tip relative to the base coordinate system can be solved[8]. According to the transformation matrix between the two adjacent coordinate. So the solution of the inverse kinematics is obtained

\[
\theta_1 = \arctan \frac{P_y - l_3 \sin \theta_2}{P_x - l_1 \cos \theta_2} - \arcsin \frac{l_3 \sin \theta_2}{l_4}, \quad \theta_2 = \arccos \left( \frac{l_2^2 + l_4^2 - l_1^2}{2l_1l_2} \right), \quad \theta_3 = \theta_1 - \theta_1 - \theta_2.
\] ...

3. Analysis of Working space

In order to ensure the expect target workspace of the bucket tip is completely enveloped the actual movement space, analysis of working space for excavating robot is carried out. The actual movement space of the bucket tip is drawn by the Monte Carlo method[9]. The three joint angles \((\theta_1, \theta_2, \theta_3)\) of the boom, arm and bucket contain a large number of random samples. Position and posture of the bucket tip are deduced by the forward kinematics. Then the approximate solution of this problem is obtained.
by using the matlab program. Then the corresponding random points of the bucket tip are obtained. Figure 2 is the working space of excavating robot.

Figure 2. Working Space of Excavating Robot.

According to the map of excavator workspace, it can be convenience to select the target position within the reach of bucket and improve work efficiency of excavating robot. In general, when the Monte Carlo method is used to analyze the working space of excavator, the more random points are given and the results are more close to the truth. However, due to the large amount of computation in the general engineering practice, it will be acceptable if the working space of excavator is solved within a certain error range.

4. Trajectory planning based on time-jerk synthetic optimum

4.1. Trajectory planning based on the spline curve function

The trajectory planning for excavating robot is carried out in the joint space[10]. Of course, cubic polynomials, quintic polynomials or other curves, all can be applied on the proposed the optimization method. But we just show the process of the time-jerk synthetic optimization. So the five groups of cubic spline functions have been used[11], which can make excavator complete the expected action through the set target point in a cycle. Six points of the bucket tip in the working space are given for the spline curve fitting. The method is to represent the joint angle as the cubic functions of time. Joint velocity expression is obtained by solving first order derivative of cubic polynomial, the expression of joint acceleration is achieved by solving the two order derivative, and joint impact function is obtained by solving the three order derivative. So the expression of the joint trajectory for excavator is

\[
Q(t) = \theta_1 + \theta_i t + \left\{ \frac{3(\theta_2 - \theta_1)}{t^2} - \frac{2}{t^2} \dot{\theta}_1 - \frac{1}{t^2} \ddot{\theta}_1 \right\} t^2 + \left\{ -\frac{2}{t^2} (\theta_2 - \theta_1) + \frac{1}{t^2} (\dot{\theta}_2 + \dot{\theta}_1) \right\} t^3.
\]

4.2. Mathematical model based on time-jerk synthetic optimum

Normally, the excavating robot must meet the requirements of rapidity and stability in the working process. The travelling times between work points affect its working efficiency and the joint impacts affect the vibration amplitude and frequency of the excavator. Considering the influence of the two factors on the performance of the excavator, a mathematical model based on time-jerk characteristic is established. Five groups of cubic polynomials are used in this paper. Excavator completes a work cycle and the bucket tip travels from the beginning to the end through 6 via-points in total. Specified time interval sequences are \([t_{i1}, t_{i2}], [t_{i2}, t_{i3}], [t_{i3}, t_{i4}], [t_{i4}, t_{i5}], [t_{i5}, t_{i6}],\) the length of time \(h_i\) between the two adjacent time points and the total travelling time are

\[
h_i = t_{i+1} - t_i \quad (i = 1, 2, 3, 4, 5), \quad T = \sum_{i=1}^{5} h_i = h_1 + h_2 + h_3 + h_4 + h_5.
\]

Since the cubic polynomials cannot guarantee continuous of acceleration at the middle point, the joints of excavator will produce flutters and impacts in the transition points of the adjacent trajectories in the digging process. The maximum-minimum impact method is used to satisfy the dynamic
constraint conditions, and make the sum of maximum impact values of each joint minimum. The expression of impact function is

\[ J = \min \left[ \sum_{j=1}^{3} \max_{t \in [0,h_{j}]} |\ddot{Q}_{ij}(t)| \right] \quad (i = 1,2,3,4,5), \tag{4} \]

where \( t \) is the time length of adjacent two segments, \( \ddot{Q}_{ij}(t) \) is the impact function of robot joint. Its expression is

\[ \ddot{Q}_{ij}(t) = 6\left(\frac{2}{h_{j}^2}(\theta_{i+1} - \theta_{i}) + \frac{1}{h_{j}}(\theta_{i+2} + \theta_{i})\right). \tag{5} \]

To consider operation efficiency and jerk restriction simultaneously for excavating robot, the weighted sum function is introduced in this paper. The time weight \( K_{T} \) and the impact weight \( K_{J} \) are set in the different working conditions. Taking these into account, the mathematical model of trajectory planning based on time-jerk synthetic optimum for excavating robot is given

\[ \min \left[ K_{T} \sum_{i=1}^{5} h_{i} + K_{J} \sum_{j=1}^{3} \max_{t \in [0,h_{j}]} |\ddot{Q}_{ij}(t)| \right], \tag{6} \]

among them, \( i=1,2,...5; j=1,2,3; K_{T} \) is the weight of the movement time; \( K_{J} \) is the weight of the impact value. Different optimization models can be obtained by selecting different \( K_{T} \) and \( K_{J} \). The parameters are optimized by Genetic Algorithm.

4.3. Constraint condition

In this paper, constraints of the excavating robot are established by limiting the maximum rotational speed and angular acceleration of each joint. Such as velocity expressions

\[ \max |\dot{Q}_{ij}(t)| \leq VC_{j} \quad (i = 1,2,...5; j = 1,2,3), \tag{7} \]

where \( t \in [0,h_{j}] \), \( VC_{j} \) represents the maximum speed of the joint. \( \dot{Q}_{ij}(t) \) is a quadratic curve which appears the maximum speed at \( t = 0, t = h_{j}, t = t' \) \((t' \in [0,h_{j}]) \). \( t' \) is the point, where the two derivative is zero, that is \( \ddot{Q}_{ij}(t') = 0 \).

Similarly, it can get acceleration constraints, such as the equation

\[ \max |\ddot{Q}_{ij}(t)| \leq AC_{j} \quad (i = 1,2,...5; j = 1,2,3), \tag{8} \]

where \( t \in [0,h_{j}] \), \( AC_{j} \) represents the maximum acceleration of the joint. \( \ddot{Q}_{ij}(t) \) is a linear function and thus the maximum value of acceleration occurs at \( t = 0 \) or \( t = h_{j} \).

Here, Genetic Algorithm is used to solves the optimization problem of excavating robot trajectory. The population size \( n \) is set to 100 and the initial population is generated. Objective function

\[ f = K_{T} \ast f_{1} + K_{J} \ast f_{2} \]

is used as the fitness function \(( f_{1} = \sum_{i=1}^{5} h_{i}, f_{2} = \min \sum_{j=1}^{3} \max_{t \in [0,h_{j}]} |\ddot{Q}_{ij}(t)| \)) . The probability of initial crossover \( \xi \) is 0.5, the number of elite preservation \( m \) is 4, the penalty factor \( \rho \) is 2, the fitness of each individual in the next generation is calculated, and the genetic operation is carried out on the chromosome. According to the genetic strategy, the selection, crossover and mutation are used to the group to form the next generation. The lower bound of the boundary condition is set to \([0.01,0.01,0.01,0.01,0.01] \), and the upper bound is \([5,5,5,5,5] \). The iteration number is 100, and the stagnation algebra is 80. The performance of groups will be judged to show whether to the results meet the predetermined value. If not, return to modify the genetic parameters. By genetic algorithm, the fitness function value is less than the optimal \( 10^{-6} \) and stops about 42 generations.
Different optimizing results can be obtained by using different time weights and impact weights. The time optimal trajectory planning mathematical model is obtained when time weight is zero. After optimization, \( h_t = [4.724, 5.3, 3.931, 3.531, 5] \), \( f = 0.6606 \). Although the maximum impact value of each joint is reduced and smaller fitness results are achieved, but the travelling time of excavator is too long, which cannot reach the expected requirements. The impact optimal trajectory planning mathematical model is obtained when impact weight is zero and here \( h_i = [1.933, 0.697, 1.047, 0.737, 0.892] \), \( f = 6.3067 \). It can be seen that the time interval is very short, but does not have the impact optimization. To sum up, in order to guarantee that the excavating robot is working smoothly and efficiently, it should be adequately taken the dual influence of time and impact into consideration by selecting the appropriate weights. The weight factor and are different under different working conditions, but this paper mainly focuses on the optimization method of the excavator in a certain condition. Thus relatively reasonable trajectory planning mathematical model of excavating robot can be obtained.

![Diagram](image)

(a) Angle Curve of Joint Rotation

(b) Angular Velocity Curve of Joint Rotation

(c) Angular Acceleration Curve of Joint Rotation

5. Analysis of results

Figure 3 is the optimal trajectory of excavating robot. The joint angle, angular velocity and angular acceleration of each joint at each moment are shown. The rotation angle curves of impact optimization are gentle and smooth, and the impact value of the joint in the working process is about 0.6606, and the total time of a work cycle needs 22.186s. But the rotation angle of each joint has exceeded the determination of boundary value, and the cycle time, which is close to the maximum value of 25s, is too long for the efficiency of the excavator in a cycle and cannot meet the requirements in the periodically independent digging. The operate time of the angle curves of time optimization in a working cycle is about 6s, and the joint impact value is about 6.3067 after optimization. It can be seen that the total operation time of this model is the shortest, but it generates a larger mutation for the angular velocity and angular acceleration of each joint. Therefore, the impact value of the joint for the
excavator working mechanism is increased, and the service life is reduced, and the working requirement is difficult to meet. The total time of the synthetic optimization curve is about 9s, the rotation angle of each joint is in the normal range and joint impact value is about 11.6977. Through the comprehensive comparison of the parameters of the two curves, the optimization process of the latter approach not only improves the working efficiency of the excavator, but also the angle curve, angular velocity and angular acceleration curve are all within the set boundary. The maximum impact value is also within the allowable range and it can realize periodically efficient operation in a certain working condition.

6. Conclusions
This method contributes to control the motion trajectory of the working mechanism in the periodically autonomous operation both in the time and impact. In a certain sense, it not only improves the speed of movement, but also reduces the joint impacts. Therefore, the work efficiency and the motion stability of the excavating robot are significantly improved. Of course, the proposed method still have the problems about the regulation of weight coefficients about time and impact, the incomplete convergence and slow convergence rate in the simple Genetic Algorithm, but it also has important significance to promote the overall performance for the excavating robot.

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