The effect of plasmon mass on spin light of neutrino in dense matter

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Abstract

We develop the theory of spin light of neutrino in matter ($SL\nu$) and include the effect of plasma influence on the emitted photon. We use the special technique based on exact solutions of particles wave equations in matter to perform all the relevant calculations, and track how the plasmon mass enters the process characteristics including the neutrino energy spectrum, $SL\nu$ rate and power. The new feature it induces is the existence of the process threshold for which we have found the exact expression and the dependence of the rate and power on this threshold condition. The $SL\nu$ spatial distribution accounting for the above effects has been also obtained. These results might be of interest in connection with the recently reported hints of ultra-high energy neutrinos $E = 1 \div 10$ PeV observed by IceCube.

1. Introduction

Neutrino physics in matter and external electromagnetic fields is a rather longstanding research field nevertheless still having advances and providing some interesting predictions for various phenomena. A broad spectrum of issues here are connected with possible electromagnetic properties of neutrino (for more details refer to [1]). The recent studies of neutrino electromagnetic properties revealed a new mechanism of electromagnetic radiation by a neutrino propagating in dense matter that has been proposed in [2]. This type of electromagnetic radiation was called the spin light of neutrino in matter ($SL\nu$). In a quasi-classical treatment this radiation originates due to neutrino electromagnetic moment precession in dense background matter. The quantum theory of this phenomena has been developed in [3, 4].

A new convenient and elegant way of description of neutrino interaction processes in matter has been proposed and developed in a series of papers [3, 5] (see also [4]). The elaborated method is based on the use of the exact solutions of the modified Dirac equation

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for a neutrino (the method of exact solutions) in matter. The method was used to describe
the phenomenon of the spin light of neutrino in matter within quantum approach. Note
that the most recent development of the method in application to studies of neutrino
motion in different extended environments can be found in [6].

The discussed method is based on the use of the modified Dirac equations for the
particles wave functions, in which the correspondent effective potentials accounting for
matter influence on the particles are included. It is similar to the Furry representation
[7] in quantum electrodynamics, widely used for description of particles interactions in
the presence of external electromagnetic fields. In this technique, the evolution operator
$U_F(t_1, t_2)$, which determines the matrix element of the process, is represented in the usual
form

$$
U_F(t_1, t_2) = T \exp \left[ -i \int_{t_1}^{t_2} j_\mu(x) A^\mu dx \right],
$$

(1)

where $A_\mu(x)$ is the quantized part of the potential corresponding to the radiation field,
which is accounted within the perturbation-series techniques. At the same time, the elec-
tron (a charged particle) current is represented in the form

$$
\dot{j}_\mu(x) = \frac{e}{2} [\Psi_e \gamma_\mu, \Psi_e],
$$

(2)

where $\Psi_e$ are the exact solutions of the Dirac equation for the electron in the presence of
external electromagnetic field given by the classical non-quantized potential $A^\text{ext}_\mu(x)$:

$$
\{ \gamma^\mu (i \partial_\mu - e A^{cl}_\mu(x)) - m_e \} \Psi_e(x) = 0.
$$

(3)

By analogy with the above-mentioned Furry representation one can introduce effective
potentials for the impact of the background matter on the propagation of the particles.
In the presence of matter a neutrino dispersion relation is modified [8], in particular, it
has a minimum at nonzero momentum [9]. As it was shown in [8], the standard result
for the MSW effect can be derived using a modified Dirac equation for the neutrino wave
function with a matter potential proportional to the density being added. The problem of
a neutrino mass generation in different media was studied [10] with use of modified Dirac
equations for neutrinos. On the same basis spontaneous neutrino-pair creation in matter
was also studied [11].

In this letter we make a reasonable step towards the completeness of the physical picture
and consider the question of the plasmon mass influence on $SL\nu$. The $SL\nu$ is a process
of photon emission in neutrino transition between different helicity states in matter. As
it has been shown [2], in the relativistic regime $SL\nu$ mechanism could provide up to one
half of the initial neutrino energy transition to the emitted radiation. It was also shown
that the $SL\nu$ provides the spin polarization effect of neutrino beam moving in matter
(similarly to the well-known effect of the electron spin self-polarization in synchrotron
radiation [12]). The estimations already performed in [13] indicated that the plasmon has
a considerable mass that can affect the physics of the process. To see how the plasmon
mass enters the $SL\nu$ quantities we appeal to the method of exact solutions and carry out all the computations relevant to $SL\nu$ in an exact way and track how the plasmon mass enters the process characteristics including the neutrino energy spectrum, $SL\nu$ rate and power with particular focus on the spatial distribution.

2. Kinematics of the process

First we briefly consider the external conditions in which the $SL\nu$ process is possible. Suppose the matter density is close to that of nuclear matter. In astrophysics it could be found, for example, in neutron stars with density span up to $10^{38-41} cm^{-3}$ [14]). As it has been shown in (3) in the case when neutrons are the main component in the background matter, in fact, an antineutrino (not neutrino) produce the $SL\nu$. Nevertheless we shall call it neutrino for convenience (the relevant formulae remain the same).

In turn, the plasmon effects emerge only due to the electron matter component which is considerably less dense in typical matter of a neutron star. For definiteness, in what follows, in respect of densities we assume the condition

$$n_e \simeq 0.1 n_n.$$  \hfill (4)

Due to the smallness of the charged fraction we neglect its influence on neutrinos and account below only for the neutron matter component in any quantities associated with neutrino.

For the description of neutrino propagation in matter we use the exact solutions of the modified Dirac equation for the neutrino field in matter [3]

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma^5) f^\mu - m_\nu \right\} \Psi(x) = 0,$$ \hfill (5)

where in the case of the particle motion through the non-moving and unpolarized neutron matter $f^\mu = -G_F/\sqrt{2} (n_n, 0)$. With that equation (5) has plane-wave solution determined by the 4-momentum $p^\mu = (E, \mathbf{p})$ and quantum numbers of the helicity $s = \pm 1$ and the sign of energy $\varepsilon = \pm 1$. The corresponding neutrino energy is given by

$$E = \sqrt{(p + s\hat{n})^2 + m_\nu^2} + \hat{n}, \quad \hat{n} = \frac{1}{2\sqrt{2}} G_F n_n.$$ \hfill (6)

The neutrino-photon coupling is described by the usual dipole electromagnetic vertex $\Gamma = i\omega\left\{ \left[ \Sigma \times \mathbf{x} \right] + i\gamma^5 \Sigma \right\}$, for details see [3, 5]. From the energy-momentum conservation one obtains

$$E = E' + \omega; \quad \mathbf{p} = \mathbf{p}' + \mathbf{k},$$ \hfill (7)

where the primed values correspond to the outgoing neutrino. One has to account for the plasma dispersion law in the form of

$$\omega = \sqrt{k^2 + m_\gamma^2}, \quad m_\gamma = \sqrt{2\alpha(3\sqrt{\pi n_e})^{1/3}}.$$ \hfill (8)
We consider the relativistic energy limit and $s = 1, s' = -1$ that is the most relevant case for applications. From the energy-momentum conservation (4) we obtained that

$$\omega \sqrt{(p + \tilde{n})^2 + m_\nu^2} = \tilde{n}(p' + p) + pk \cos \theta + \frac{m_\gamma^2}{2}, \tag{9}$$

where $\theta$ is the angle between the directions of the initial neutrino propagation and that of the radiated photon. Note that the structure of eq. (9) is very much similar to the corresponding equation for the case of the $SL\nu$ in matter with different masses of neutrinos in the initial and final state [15].

To obtain the threshold condition for the $SL\nu$ process one should consider the system (7) relative to the vector $p'$. The corresponding exact equation for $p'$ is quadratic and thus can be resolved exactly [16]. The resulting threshold condition can be written in the form

$$\frac{m_\gamma^2 + 2m_\nu m_\nu}{4\tilde{n}p} < 1. \tag{10}$$

Note that the condition nontrivially depends on matter density through $\tilde{n}$ and $m_\gamma$ given by (6) and (8) respectively.

To meet the requirements on the parameters imposed by the threshold condition (10) it is worth to begin with considering the neutrino mass as its value is very restricted experimentally. To date, the most conservative upper limit for the neutrino mass which agrees with existing experiments is set at the order of 1 eV (for the most recent result from the direct search for neutrino mass refer to [17], for a major review of the field see for instance [18]). On the other hand, as it was shown in previous investigations [3, 5] that $SL\nu$ is more efficient for high the matter density. Therefore we consider the highest densities of matter discussed in the literature, that are of the order $10^{38-41} \text{ cm}^{-3}$ for nuclear matter [14, 19]. Indicated values correspond to the range of the “density parameter” $\tilde{n} \simeq 1 - 10^3$ eV. Taking also into account the condition (4), from (8) we estimate the value of plasmon mass as $m_\gamma \simeq 10^{7-8}$ eV.

With the above scales of the parameters the term that contains neutrino mass in the numerator in Eq.(10) is not important and the threshold condition (7) is reduced to

$$m_\gamma^2/4\tilde{n}p < 1. \tag{11}$$

That leads, together with the scales of the parameters mentioned above, to the allowable range for the neutrino momentum $p \gtrsim 10^{12}$ eV. Thus, even in the case the threshold condition is accounted for, the room for $SL\nu$ is opened for high energy neutrinos (see also [20]). The lower the density, the higher values for the neutrino momentum are needed for the $SL\nu$ process to be opened. The last relation corresponds to the case of highest matter density among considered, $n \simeq 10^{41} \text{ cm}^{-3}$ (the dependance of the plasmon mass on the matter density is rather weak).

Note that in spite of the considered ultra-high neutrino energies the need for accounting for non-local contributions in the neutrino-matter interactions (emphasized earlier, for instance, in [13]) is not justified (see also [20]).
3. The rate and power of the $SL\nu$ process

Performing calculations similar to those described in [3] with including additionally the effects of plasma discussed above we arrive at the following expression for the $SL\nu$ transition rate

\[ \Gamma = \frac{\mu^2}{2\pi} \int \frac{k^2}{\omega} S(k) \delta(E - E' - \omega) \, dk \, d\Omega, \]  

(12)

\[ S(k) = (k^2 + \omega^2) \left( \frac{k - p \cos \theta}{p'} \cos \theta \right) - 2k \omega \left( \cos \theta - \frac{k - p \cos \theta}{p'} \right). \]  

(13)

Here $\mu$ is the neutrino magnetic moment and $\theta$ is the angle between the direction of the initial neutrino and the plasmon momentum, $p$ and $k$ respectively.

The exact evaluation of the integral in (12) accounting for the composite delta function in the presented form is a rather involved procedure (also has been performed by the authors) and resulting expression is cumbersome to be presented here. Therefore it is reasonable to consider several ranges of parameters and obtain much simpler though less general expressions, that cover the whole parameters range of interest for astrophysical applications. Accounting for the threshold condition (11) it is convenient to consider the following three ranges of parameters and the corresponding three cases.

First we single out the “near-threshold” case,

\[ \frac{m_{\gamma}^2}{4\tilde{n}p} \lesssim 1. \]  

(14)

Introducing notation $a = \frac{m_{\gamma}^2}{4\tilde{n}p}$, the expression for the process transition rate, that follows from (14), can be written as

\[ \Gamma = 4\mu^2 \tilde{n}^2 p((1 - a)(1 + 7a) + 4a(1 + a) \ln a), \]  

(15)

in which one can recognize the estimation from [13]. For the corresponding expression for the total radiation power ($I = \int \omega d\Gamma$) our calculations yield

\[ I = \frac{4}{3} \mu^2 \tilde{n}^2 p^2 ((1 - a)(1 - 5a - 8a^2) - 12a^2 \ln a). \]  

(16)

The next case we consider is the “far-from-threshold” regime, realized when

\[ \frac{m_{\gamma}^2}{4\tilde{n}p} \ll 1, \quad \text{or} \quad a \to 0. \]  

(17)

Since the value for $\tilde{n}$ is restricted by $\tilde{n} \simeq 1 - 10^3$ eV, the validity of condition (17) can be provided by the increase of the neutrino momentum alone, i.e. $p > 10^{12}$ eV. Thus this particular case, settled by the condition (17) can be referred to as the case of “ultra-relativistic” neutrinos. In this case the threshold effect is negligible so that the corresponding result of calculations without account for plasmon mass should be relevant. Indeed the expressions (15) and (16) under the condition (17) are transformed respectively to

\[ \Gamma = 4\mu^2 \tilde{n}^2 p, \]  

(18)
which are just results for the $SL\nu$ rate and power at high neutrino momentum $p$ without inclusion of plasmon mass $m_\gamma$ (see [3, 5]).

Now let us consider the case of the $SL\nu$ at the threshold, i.e. very close to it. In the case the following condition is realized

$$1 - m_\gamma^2 / 4 \tilde{n} p = 1 - a \ll 1, \quad \text{or} \quad a \to 1,$$

and the transition rate and radiation power are respectively given by

$$\Gamma = 4 \mu^2 \tilde{n}^2 (1 - a) ((1 - a)p + 2 \tilde{n}), \quad (21)$$

$$I = 4 \mu^2 \tilde{n}^2 p (1 - a) ((1 - a)p + 2 \tilde{n}). \quad (22)$$

The obtained expressions enable us to follow these two characteristics dependance on the neutrino energy as the parameter $a$ reaches the threshold, $a \to 1$. As is expected the rate and power given by (21), (22) go to zero at the threshold.

The performed above analysis of the $SL\nu$ rate and power enable us to obtain a fine description for the angular distribution of the radiation. As follows from (9), (10) the radiation is confined within the narrow solid angle defined by the relation

$$\sin \theta \approx \theta \leq \frac{4 \tilde{n}(p + \tilde{n}) - m_\gamma^2}{2p m_\gamma}. \quad (23)$$

The angular distribution of the $SL\nu$ radiation is presented on Fig.1 and Fig.2.

Far from the threshold (at high values of the neutrino momentum $p$) the angular distribution of the $SL\nu$ exhibit the characteristic that are typical for the radiation produced by a relativistic particle. As is clearly seen on Fig.1 and Fig.2, the $SL\nu$ radiation is focused in the direction of the initial neutrino propagation in a very narrow solid angle.
\[ \theta \lesssim 2\bar{n}/m_\gamma. \] The radiation distribution has a cup-like shape at the directions closest to the initial neutrino momentum. A remarkable feature that one can observe on the base of the \( SL\nu \) consideration accounting for the plasma effects is the presence of the plasmon mass in the angular distribution. Note that the presence of plasmon mass makes the \( SL\nu \) angular distribution analogous to the angular distribution of on-flight decay of a massive particle. As it follows from the exact expressions that govern the angular distribution on Fig.1 and Fig.2, the \( SL\nu \) angular distribution goes to infinity as the angle reaches its boundary value [23] that corresponds to the outer cone on Fig.1. However the distribution is integrable and the contribution of the divergent areas into the total radiation power is negligible as it is common for kinematics of nuclear reactions (see [21]).

4. Conclusions

We developed a detailed evaluation of the spin light of neutrino in matter accounting for effects of the emitted plasmon mass. Based on the exact solution of the modified Dirac equation for the neutrino wave function in the presence of the background matter the appearance of the threshold for the considered process is confirmed. The obtained exact and explicit threshold condition relation exhibit a rather complicated dependance on the matter density and neutrino mass. The dependance of the rate and power on the neutrino energy, matter density and the angular distribution of the \( SL\nu \) is investigated in details. It is shown how the rate and power wash out when the threshold parameter \( a = m_\gamma^2/4\bar{n}p \) approaching unity [11]. Within the performed detailed analysis it is shown that the \( SL\nu \) mechanism is practically insensitive to the emitted plasmon mass for very high densities of matter (even up to \( n = 10^{41} cm^{-3} \)) in the case of ultra-high energy neutrinos for a wide range of energies starting from \( E = 1 \) TeV. One may expect that the conclusion is of interest for astrophysical applications of \( SL\nu \) radiation mechanism in light of the recently reported hints of \( 1 \div 10 \) PeV neutrinos observed by IceCube [22].

Although the neutrino fluxes in different astrophysical environments can be huge, considering possible applications of the \( SL\nu \) one should account for the fact that the mean free path of a neutrino in neutron matter with densities above \( 10^{14} g/cm^3 \) becomes rather short (\( \lambda_\nu \sim 10^{-5} cm \)) and matter becomes opaque for neutrinos. Since the neutrino-nucleons cross section increases with the increasing neutrino energy, the higher the energy of neutrino the lower should be the density of matter for the mean free path of neutrino to exceed some characteristic length (for example, the radius of a neutron star). This is necessary for the process of the spin light of neutrino to be open.

In the considered case the high density of matter (\( n \sim 10^{41} cm^{-3} \)) forbids the propagation of the most energetic neutrinos with energies above 1 TeV. On the other hand the mass of plasmon in the matter is high enough to require the neutrino energies within the TeV scale to get over the threshold. Therefore, these two conditions (the sufficiently long free path of neutrino and the threshold) strictly limit the possible parameter space where the \( SL\nu \) process can be effective.

It is worth noting that one can consider the case of a neutrino moving through dense medium consisting of neutrinos. The studies of neutrino propagation and oscillation in
dense neutrino gases has started in \cite{23} and up to now is under focus of research (see, for instance, one of the recent papers \cite{24}) The neutrino-neutrino scattering cross-section is of the same order of magnitude as the neutrino-nucleon one considered above. Besides the plasmon mass here becomes irrelevant since the interaction of the emitted photons with neutrinos of the medium is governed by the neutrino magnetic moment. In the discussed case the interaction is suppressed as the ratio ($\frac{\mu^2}{\omega^2}$), where $\alpha$ is the fine structure constant. Therefore, the threshold condition (10) nearly disappears and even the neutrino with rather low energy (GeV scale or lower) can effectively emit the $SL\nu$. Such situation could be found during the supernova collapse, for instance on the edge of neutrinosphere.

In addition there could be other effects that should be considered within the theory of $SL\nu$. For instance, under certain conditions the neutrino mixing and oscillations can be important. This situation can be treated within the approach that was developed for general kinetic description of relativistic mixed neutrinos that was first proposed in the pioneering paper \cite{25} and further studied in \cite{26,27,28} and adopted for the neutrino oscillations in the early Universe (see also \cite{29}). For this case the neutrino oscillations should be treated using the neutrino density matrix in flavour space.

However, the latter mentioned effect as well as the newly proposed possibility for the $SL\nu$ in dense neutrino gases require further additional detailed investigations.

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