Extracting $\sigma_{\text{eff}}$ from the LHCb double-charm measurement

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Abstract: We discuss various issues related to the definition of single and double open charm cross sections. We conclude that LHCb’s extraction of $\sigma_{\text{eff}}$, the effective cross section for double-parton scattering, is too large by a factor of two. This correction brings the data from open-charm pairs closer to that from $J/\psi$ plus open charm and jet production.
1 Introduction

Multi-parton interactions are firmly established as the primary source of underlying event activity in high energy hadronic collisions (see for example Ref. [1] and references therein). However, attempts to study their properties by cleanly identifying multi- (initially double-) parton scattering events have proved difficult.

The general principle behind all such measurements is to assume that there is little correlation between the two scatters, so that using final state observables one can separate the signal for a given process into a correlated contribution coming from one scatter and two uncorrelated components each coming from a different scatter. The earliest searches for and measurements of double-parton scattering used four jet production (AFS [2], UA2 [3] and
CDF [4]), in which the double-parton signal consists of two back-to-back dijet pairs, uncorrelated in azimuth, whereas the single-parton background consists of a back-to-back dijet pair, with two additional jets produced by initial- or final-state bremsstrahlung; the former giving little correlation between the two emissions, the latter giving a strong correlation with the primary jet pair.

The most precise measurements of double-parton scattering to date come from the Tevatron measurements of $\gamma + 3$ jets production (CDF [5] and D0 [6]), based on the same idea, but with one of the jets replaced by a photon, which has a considerably smaller single-parton background.

The LHC measurements have focussed on channels that have even smaller single-parton backgrounds, for example $W + \text{dijets}$ (ATLAS [7], with preliminary work towards such a measurement also by CMS [8]), in which again the dijet pair should be back-to-back, with little recoil from the $W$. The ultimate channel in this direction would be like-sign $W$ pair production [9], for which not only is the single-parton background small, but it is also very distinctive since by charge conservation the $W$ pair must be accompanied by at least two high-$p_t$ jets.

LHCb have studied double-parton scattering by measuring double-charm cross sections. In particular, they have measured double-charmonium production in [10], charmonium + open-charm production in [11] and double-open-charm production, also in [11]. The channels involving charmonia, as well as double-open-charm production in the case that both of the measured charmed hadrons contain a charm quark (or both an anticharm), are expected to be dominated by double-charm production. On the other hand, double-open-charm production in which one charmed hadron contains a charm quark and the other an anticharm is expected to be dominated by single-charm production, although with a significant contribution from double-charm production that could perhaps be separated using correlations in their phase space distributions.

The amount of double-parton scattering is typically parametrized by the effective cross section, $\sigma_{\text{eff}}$, through:

$$
\sigma_{ii} = \frac{\sigma_i^2}{2\sigma_{\text{eff}}}, \quad \sigma_{ij} = \frac{\sigma_i \sigma_j}{\sigma_{\text{eff}}},
$$

where $\sigma_i$ and $\sigma_j$, and $\sigma_{ii}$ and $\sigma_{ij}$, are suitably defined cross sections for single- or double-parton scatters of types $i$ and $j$. We discuss their definitions in more detail in the next section, but when they are defined properly, we take Eq. (1.1) as defining $\sigma_{\text{eff}}$. The factor of 2 appearing in Eq. (1.1) is a simple symmetry factor.

In this paper, we discuss LHCb’s extraction of $\sigma_{\text{eff}}$ from their measurements of single- and double-open-charm production cross sections. We wish to stress that we do not question their measurements of the cross sections themselves, only the way they combine them to extract $\sigma_{\text{eff}}$, which has the potential to act as a strong constraint on models of multiple parton scattering and, in particular, their models of the transverse-space distribution of partons in hadrons [12].

The remainder of the paper is as follows. In Section 2 we discuss the cross section definitions for single- and double-inclusive cross sections. We show that, in the eikonal model of multi-parton interactions, only if the cross sections are defined in the correct,
inclusive, sense, is the effective cross section defined by Eq. (1.1) a process-independent quantity. Technical details are deferred to Appendix A. In Section 3, we specialize these to the cases of charm quark and charmed hadron production. This section already contains our main result: LHCb’s extraction of $\sigma_{\text{eff}}$ from their single- and double-open-charm cross sections is too large by a factor of 2. In Section 4 we summarize our re-extraction of $\sigma_{\text{eff}}$ from the LHCb data. In Section 5 we briefly summarize our paper. Finally, in Appendix B, we make comparisons with a theory paper by two LHCb authors and others[13], which appears to agree with the LHCb results.

### 2 Single- and double-inclusive cross sections

Considerable confusion has arisen concerning cross section definitions for use in studies of double-parton scattering. The classic measurement of CDF[5], for example, used an exclusive definition and therefore the $\sigma_{\text{eff}}$ they extracted was not the usual one, but a process-dependent approximation to it, which was pointed out and corrected in [14, 15].

In this section we define precisely what we mean by the inclusive cross sections and draw comparisons with results using another common definition of the word “inclusive” and with exclusive cross sections. In order to motivate our definition, we discuss an eikonal model of multi-parton interactions, but we stress that our definition is completely independent of that model. However, we will see that in that model, our definition of inclusive cross sections leads to an effective cross section that is a property only of the colliding hadrons and not of the process by which it is measured.

We therefore begin by defining this eikonal model. We assume that parton distribution functions factorize into a longitudinal momentum part and transverse space part, and further that multi-parton distribution functions factorize into products of single-parton distribution functions. In the details of exclusive final states, this approximation must fail, but it is believed to be a good approximation for the distribution of number of hard scatters, at least, and is the basis of all current multi-parton interaction models that describe LHC underlying event data. In this approximation, the cross section for $n$ partonic scatters of a type $i$ can be written as a convolution over impact parameter, $b$, of a factor that represents the Poisson-distributed probability of having $n$ independent collisions, with $b$-dependent average value:

$$\sigma_{ni} = \int d^2b \left( \frac{\sigma_i A(b))}{n!} \right)^n e^{-\sigma_i A(b)} ,$$

(2.1)

where $\sigma_i$ is the cross section for a single-parton scatter of type $i$, calculated with the conventional (inclusive) parton distribution functions, and $A(b)$ is referred to as the matter distribution, normalized according to

$$\int d^2b A(b) = 1 .$$

(2.2)

### 2.1 Single-particle cross sections

#### 2.1.1 Inclusive definition

The conventional definition of the inclusive production of some state $i$ is to imagine a hypothetical detector that counts $i$’s in some fiducial volume of phase space. The counter is
assumed to be perfect, in the sense that every i is counted, independently of the structure of the event it appears in. In particular, independently of whether the event contains additional i’s and, if so, how many. The number of i’s counted, \( N_i \), in a run with integrated luminosity \( \mathcal{L} \) then defines the inclusive i cross section:

\[
\sigma_{\text{incl}} \equiv \frac{N_i}{\mathcal{L}}. 
\]  

(2.3)

The requirement that every i is counted, independently of how many there are, is what is usually meant by inclusive. Note that this definition does not require any reference to the number of events measured, only the number of i’s.

As shown in Appendix A, this is equivalent to writing

\[
\sigma_{\text{incl}} = \sum_n n \sigma_{ni},
\]  

(2.4)

where \( \sigma_{ni} \) is the exclusive cross section to produce \( n \) i’s. Note that an event that contains \( n \) i’s contributes \( n \) times to the inclusive cross section. This may seem like a simple result, but a surprising number of papers in both the theory and experiment of multi-parton interactions do not make the step from Eq. (2.3) to Eq. (2.4).

The cross section for events in which there are exactly \( n \) scatters of type i is precisely what we defined in the eikonal model in Eq. (2.1). We can therefore write the inclusive cross section in the eikonal model as

\[
\sigma_{\text{incl}} = \sum_n n \int d^2b \frac{(\sigma_i A(b))^n}{n!} e^{-\sigma_i A(b)} = \int d^2b \left[ \sum_n \frac{(\sigma_i A(b))^n}{n!} \right] e^{-\sigma_i A(b)} = \int d^2b \sigma_i A(b) = \sigma_i. 
\]  

(2.5)

That is, the inclusive cross section is equal to the partonic cross section. This deceptively simple result is a non-trivial test of the self-consistency of the eikonal model: since the partonic cross section is calculated from the inclusive parton distribution functions, and these are defined as operators for the production of a single parton with all other information integrated out, it must be that this partonic cross section is fully inclusive. That is, each parton that a hadron produces is described by the parton distribution function and each collision that that parton initiates contributes to the inclusive cross section.

2.1.2 Alternative inclusive definition

Another definition of the word “inclusive” appears frequently in the literature. In this definition, the inclusive i cross section is the cross section for events that contain one or more i,

\[
\sigma_{\geq 1i} = \sum_{n=1} \sigma_{ni}. 
\]  

(2.6)

In the eikonal model, this is given by

\[
\sigma_{\geq 1i} = \sum_{n=1} \int d^2b \frac{(\sigma_i A(b))^n}{n!} e^{-\sigma_i A(b)} = \int d^2b \left[ \sum_{n=1} \frac{(\sigma_i A(b))^n}{n!} \right] e^{-\sigma_i A(b)} = \int d^2b \left( 1 - e^{-\sigma_i A(b)} \right). 
\]  

(2.7)
2.1.3 Exclusive definition

The exclusive single-\(i\) cross section has already been defined, \(\sigma_{1i} = \sigma_{ni}\), with \(n = 1\), which in the eikonal model is given by

\[
\sigma_{1i} = \int d^2 b \sigma_i A(b) e^{-\sigma_i A(b)}. \tag{2.8}
\]

2.2 Double-particle cross sections

2.2.1 Inclusive definition

As we again motivate in more detail in Appendix A, the double-particle inclusive cross section is given by

\[
\sigma_{incl} = \sum n_1 n_2 n (n-1) \sigma_{ni}. \tag{2.9}
\]

An event that contains \(n\) \(i\)'s contains \(\frac{1}{2}n(n-1)\) different \(ii\) pairs and contributes that many times to the inclusive cross section.

It is worth noting that this definition imposes a specific requirement on the experimental measurement. Since the reconstruction efficiency is typically small, but not infinitesimal, it can happen that more than two \(i\)'s are reconstructed in the same event. The inclusive cross section definition requires that in such events each pair contributes to the cross section. Thus, a three-\(i\) event contributes three times to the double-\(i\) cross section. The event sample used for LHCb’s measurement is not large enough for this to be an issue\cite{16}, but it could be in future.

In the eikonal model, the double-particle inclusive cross section is therefore given by

\[
\sigma_{incl} = \sum_{n} \frac{1}{2} n(n-1) \sigma_{ni}. \tag{2.10}
\]

In a completely analogous way, the double-particle cross sections for two different particle types \(i\) and \(j\) can be defined in terms of the exclusive cross section for events containing \(n\) \(i\)'s and \(m\) \(j\)'s as

\[
\sigma_{inclij} = \sum_{n,m} n m \sigma_{ni,mj}. \tag{2.11}
\]

In the eikonal model this gives

\[
\sigma_{inclij} = \sigma_i \sigma_j \int d^2 b A(b)^2. \tag{2.12}
\]

2.2.2 Alternative inclusive definition

The alternative inclusive definition is the cross section for events containing two or more \(i\)'s,

\[
\sigma_{\geq 2i} \equiv \sum_{n=2} \sigma_{ni}. \tag{2.13}
\]

In the eikonal model, this is given by

\[
\sigma_{\geq 2i} = \int d^2 b \left( 1 - e^{-\sigma_i A(b)} - \sigma_i A(b) e^{-\sigma_i A(b)} \right). \tag{2.14}
\]
By analogy, we have
\[ \sigma_{\geq i,i\geq j} = \sum_{n,m=1} \sigma_{ni,mj}, \]  
(2.15)
and
\[ \sigma_{\geq i,i\geq j} = \int d^2b \left( 1 - e^{-\sigma_i A(b)} \right) \left( 1 - e^{-\sigma_j A(b)} \right). \]  
(2.16)

### 2.2.3 Exclusive definition

The exclusive double-\(i\) cross section has already been defined, and in the eikonal model is given by
\[ \sigma_{2i} = \int d^2b \frac{1}{2} (\sigma_i A(b))^2 e^{-\sigma_i A(b)}, \]  
(2.17)
and
\[ \sigma_{1i,1j} = \int d^2b \sigma_i \sigma_j A(b)^2 e^{-(\sigma_i + \sigma_j)A(b)}. \]  
(2.18)

### 2.3 The effective cross section

In all cases, we take Eq. (1.1) as the definition of \(\sigma_{\text{eff}}\).

#### 2.3.1 Inclusive definition

\[ \sigma_{\text{eff}} = \frac{\sigma_{\text{incl}}^2}{2\sigma_{\text{incl}}^2} = \frac{\sigma_{\text{incl}} \sigma_{i\text{incl}}}{\sigma_{i\text{incl}}} = \frac{1}{\int d^2b A(b)^2}. \]  
(2.19)

Note that in the eikonal model, the effective cross section is independent of the cross section \(\sigma_i\), and hence the choice of \(i\).

#### 2.3.2 Alternative inclusive definition

\[ \sigma_{\text{eff}} = \frac{\sigma_{\geq i,i\geq j}^2}{2\sigma_{\geq i,i\geq j}^2} = \frac{\left[ \int d^2b \left( 1 - e^{-\sigma_i A(b)} \right) \right]^2}{2 \int d^2b \left( 1 - e^{-\sigma_i A(b)} - \sigma_i A(b) e^{-\sigma_i A(b)} \right)}. \]  
(2.20)

With this definition, \(\sigma_{\text{eff}}\) is process-dependent, since its value depends on \(\sigma_i\). The two different definitions in Eq. (1.1) give different results,

\[ \sigma_{\text{eff}} = \frac{\sigma_{\geq i,i\geq j}}{\sigma_{\geq i,i\geq j}} = \frac{\left[ \int d^2b \left( 1 - e^{-\sigma_i A(b)} \right) \right] \left[ \int d^2b \left( 1 - e^{-\sigma_j A(b)} \right) \right]}{\int d^2b \left( 1 - e^{-\sigma_i A(b)} \right) \left( 1 - e^{-\sigma_j A(b)} \right)}. \]  
(2.21)

#### 2.3.3 Exclusive definition

\[ \sigma_{\text{eff}} = \frac{\sigma_{1i}^2}{2\sigma_{2i}} = \frac{\int d^2b A(b) e^{-\sigma_i A(b)} \right)^2}{\int d^2b A(b)^2 e^{-\sigma_i A(b)}}, \]  
(2.22)

\[ \sigma_{\text{eff}} = \frac{\sigma_{1i} \sigma_{1j}^2}{\sigma_{1i,1j}} = \frac{\left[ \int d^2b A(b) e^{-\sigma_i A(b)} \right] \left[ \int d^2b A(b) e^{-\sigma_j A(b)} \right]}{\int d^2b A(b)^2 e^{-(\sigma_i + \sigma_j)A(b)}}. \]  
(2.23)
2.3.4 Conclusion

The conclusion of this study is that, of the different cross section definitions used in the literature, only the conventional inclusive one gives an expression for $\sigma_{\text{eff}}$ that is independent of the chosen process(es), in the eikonal model. That is, for the single-particle inclusive cross section, particles of type $i$ should be counted, so that an event that contains $n$ $i$'s contributes $n$ times. For the double-particle inclusive cross section, pairs of particles of type $i$ should be counted, so that an event that contains $n$ $i$'s contributes $\frac{1}{2}n(n-1)$ times. For the double-particle inclusive cross section, pairs of particles of type $i$ and $j$ should be counted, so that an event that contains $n$ $i$'s and $m$ $j$'s contributes $nm$ times. We use these definitions for the remainder of the paper and drop the subscript $\text{incl}$ from them.

3 Charm quark and charmed hadron cross sections

The discussion of the previous section can be applied directly to the partonic cross sections to produce charm quark pairs. We include all pair production mechanisms, whether directly through $q\bar{q} \rightarrow c\bar{c}$ and $gg \rightarrow c\bar{c}$ or through flavour excitation, e.g., $qc \rightarrow qc$, with an accompanying $\bar{c}$ produced in the corresponding initial state shower. For simplicity we neglect the possibility that a single partonic collision produces more than one $c\bar{c}$ pair.

However, experiments do not directly observe charm quarks, but rather the hadrons they fragment to, whether charmonium, e.g. $J/\psi$, or open charm, e.g. the set of $D$ mesons or $\Lambda_c$ baryons. We concentrate on the case of open charm.

3.1 Open charm cross sections

In QCD, the production of charmed hadrons can be factorized into a hard process, which produces a charm quark, and a perturbative evolution followed by the non-perturbative confinement of the charm quark into a charmed hadron. The last two processes are collectively called fragmentation. In the present discussion we neglect the possibility that the evolution of a gluon or quark could produce a charm-anticharm pair and hence the fragmentation process preserves the charm quantum number: a charm quark produces a charmed hadron with unit probability. We further assume that the hadronization stage is a local process and, hence, the probability distributions of which charmed hadron a given charm quark produces are independent.

The fragmentation process typically degrades the energy of the charm quark so that the produced charmed hadron has less energy than it (although not necessarily in the laboratory frame, a point that we shall return to in the next sub-section), without a significant change in direction. Thus, the kinematic distributions of the produced charmed hadrons are related to those of the initiating charm quarks, but folded with fragmentation functions. For the present analysis, we will assume that the kinematic distributions of $c\bar{c}$ pairs produced in different partonic collisions are independent. Thus, the probability that a given charm quark produces a charmed hadron of a given species within the fiducial region of an experiment is a fixed number, $p_D$. (We use a generic $D$ label for a charmed hadron, although it could also be a charmed baryon. Specifically, we are interested in the cases $D = \{D^0, D^+, D_s^+, \Lambda_c\}$). We assume that $p_D = 0$. 

\[ \text{– 7 –} \]
Within these assumptions, and using the single- and double-inclusive charm quark cross sections, it is straightforward to calculate the single- and double-inclusive charmed hadron cross sections. An event containing \( n \) charm quarks has an independent probability \( p_{\text{c}} \) for each of them to produce a \( D \) hadron and hence

\[
\sigma_D = p_D^c \sigma_c, \quad (3.1)
\]

\[
\sigma_{DD} = (p_D^c)^2 \sigma_{cc}, \quad (3.2)
\]

(it is worth noting that these relations are not true for the alternative inclusive or exclusive cross section definitions). Likewise, a pair of charmed hadrons of different species, \( D_1 \) and \( D_2 \), can be produced by a pair of charm quarks in either of two ways, and we have

\[
\sigma_{D_1D_2} = 2p_{D_1}^c p_{D_2}^c \sigma_{cc}. \quad (3.3)
\]

Thus, we can use measurements of single- and double-charmed hadron production to extract \( \sigma_{\text{eff}} \), independent of the unknown charm quark cross section and fragmentation probabilities:

\[
\frac{\sigma_D^2}{2\sigma_{DD}} = \frac{(p_D^c)^2 \sigma_c^2}{2(p_D^c)^2 \sigma_{cc}} = \sigma_{\text{eff}}, \quad (3.4)
\]

\[
\frac{\sigma_{D_1D_2}}{\sigma_{D_1D_2}} = \frac{2p_{D_1}^c p_{D_2}^c \sigma_{cc}^2}{2p_{D_1}^c p_{D_2}^c \sigma_{cc}} = \sigma_{\text{eff}}. \quad (3.5)
\]

### 3.2 Charge conjugate modes

Since QCD is charge-conjugation-symmetric, one might expect that \( p_{\overline{D}}^c = p_{D}^c \). However, the fact that the LHC collides particles, rather than antiparticles, can in principle induce an asymmetry. One expects that the primary production distributions of charm and anticharm are the same and likewise, because it is local, the probability distribution of which charmed hadron is produced. However, the kinematic distributions of produced charmed and anticharmed hadrons are not necessarily the same, because the colour structure of their production is different. The colour partner of a charm quark is more likely to be the proton remnant, or lie towards the proton remnant direction, whereas the colour partner of an anticharm quark is more likely to be in the final state of the hard process, and therefore towards the centre of the event. The hadronization phase is more properly thought of as being largely longitudinal in the rest frame of colour-connected pairs and hence, on average, charmed hadrons are expected to be produced at slightly higher rapidities than their parent charm quarks, while anticharmed hadrons are expected to be produced at the same rapidity as their parent anticharm quarks. This effect was called “string drag” in Ref. [17, 18], but as it occurs in other hadronization models, we prefer to call it a “colour drag”. Thus, while the total number of \( D \) hadrons of a given species is expected to be the same as the number of \( \overline{D} \) hadrons, the numbers within a given fiducial volume are not necessarily the same.

In the analysis we are comparing to, LHCb did not notice any difference between charge conjugate modes and hence did not explicitly extract a measurement for this asymmetry[16]. They have made dedicated analyses of \( D^+_s \) production asymmetries[19] and found no effect (at the < 1% level) and \( D^\pm[20] \) and found 3\( \sigma \) evidence for an asymmetry at the \( \sim 1\% \) level.
It would be interesting to take this effect into account explicitly in any future measurement, but it appears to be small enough that in the remainder, we follow LHCb[11] in assuming
\[ p^c_D = p^c_{\bar{D}}, \] (3.6)
and therefore that the cross sections for D and \( \bar{D} \) are equal,
\[ \sigma_D = \sigma_{\bar{D}}, \] (3.7)
\[ \sigma_{DD} = \sigma_{\bar{D}\bar{D}}, \] (3.8)
\[ \sigma_{D_1D_2} = \sigma_{\bar{D}_1\bar{D}_2}. \] (3.9)
LHCb used these results to effectively double their data set and defined
\[ \sigma_{D, \text{LHCb}} \equiv \sigma_D + \sigma_{\bar{D}}, \] (3.10)
\[ \sigma_{DD, \text{LHCb}} \equiv \sigma_{DD} + \sigma_{\bar{D}\bar{D}}, \] (3.11)
\[ \sigma_{D_1D_2, \text{LHCb}} \equiv \sigma_{D_1D_2} + \sigma_{\bar{D}_1\bar{D}_2}. \] (3.12)
It is still possible to use these charge-conjugation-summed cross sections to extract \( \sigma_{\text{eff}} \), at least under the assumption (3.6), but one must be careful to include an additional factor of 2:
\[ \frac{\sigma^2_{D, \text{LHCb}}}{2 \times 2 \sigma_{DD, \text{LHCb}}} = \frac{(\sigma_D + \sigma_{\bar{D}})^2}{4(\sigma_{DD} + \sigma_{\bar{D}\bar{D}})} = \frac{4\sigma^2_D}{8\sigma_{DD}} = \sigma_{\text{eff}}, \] (3.13)
\[ \frac{\sigma_{D_1D_2, \text{LHCb}}}{2 \times \sigma_{D_1D_2, \text{LHCb}}} = \frac{(\sigma_{D_1} + \sigma_{\bar{D}_1})(\sigma_{D_2} + \sigma_{\bar{D}_2})}{2(\sigma_{D_1D_2} + \sigma_{\bar{D}_1\bar{D}_2})} = \frac{4\sigma_{D_1}\sigma_{D_2}}{4\sigma_{D_1D_2}} = \sigma_{\text{eff}}. \] (3.14)
It appears to us that LHCb have not included this factor of two and hence that their extracted values of \( \sigma_{\text{eff}} \) are too large by a factor of two.

It is worth mentioning that charmonium channels are not subject to this factor of two. Since charmonium is self-conjugate, there is no summation to be done. The double-charmonium channel therefore does not contain any additional factors of two. The single-charmonium, single-open charm channel contains a factor of two in the numerator, from the sum over D and \( \bar{D} \), but also in the denominator, from the sum over \( J/\psi + D \) and \( J/\psi + \bar{D} \). Thus the two factors of two cancel.

We summarize LHCb’s results, corrected by this factor of 2 in Sect. 4.

3.3 Opposite-sign charmed hadron pairs

While the main focus of this paper is double- (and single-) inclusive production of charmed hadrons both containing a charm quark (or both an anticharm), we briefly mention the channels in which a charmed and anticharmed hadron pair are detected, which have also been measured by LHCb. Even within the assumption \( p^c_D = p^c_{\bar{D}} = 0 \), \( D\bar{D} \) and \( D_1\bar{D}_2 \) pairs can come from a single \( c\bar{c} \) pair, which have equal and opposite transverse momenta and correlated rapidities. Therefore we cannot consider the probabilities of charm quarks to produce charmed hadrons within the fiducial region as uncorrelated\(^1\).

\(^1\) Rapidity correlations were proposed as a means to separate single- and double-charm production in Ref. [21].
We introduce a correlation coefficient $C$, such that the probabilities that a $c\bar{c}$ pair from a single partonic scattering produces a $D\bar{D}$ or $D_1\bar{D}_2$ pair within the fiducial region are $C(p^c_D)^2$ and $Cp^c_Dp^\bar{c}_{D_2}$ respectively. The corresponding probabilities that a $c$ and a $\bar{c}$ from different partonic scatterings produces a $D\bar{D}$ or $D_1\bar{D}_2$ pair within the fiducial region are still $(p^c_D)^2$ and $p^c_Dp^c_{D_2}$ respectively.

We can then show that
\begin{align}
\sigma_{D\bar{D}} &= (p^c_D)^2(C\sigma_c + 2\sigma_{cc}), \\
\sigma_{D_1\bar{D}_2} &= p^c_Dp^\bar{c}_{D_2}(C\sigma_c + 2\sigma_{cc}).
\end{align}

Then, forming the same ratios as in the like-sign case, we obtain:
\begin{align}
\frac{\sigma^2_D}{2\sigma_{D\bar{D}}} &= \frac{\sigma_{eff}}{2(\sigma_{eff}/\sigma_c + 1)}, \\
\frac{\sigma_{D_1\bar{D}_2}}{\sigma_{D_1\bar{D}_2}} &= \frac{\sigma_{eff}}{C\sigma_{eff}/\sigma_c + 1}.
\end{align}

Note that both $C$ and $\sigma_{eff}/\sigma_c$ are expected to be larger than 1. Thus, these results are expected to be significantly smaller than $\sigma_{eff}$, but without further studies to extract the values of these constants, we cannot quantify the expected size. It is important to note however that they are independent of the specific flavours of the charmed hadrons.

Finally, we note again that LHCb sum over charge conjugate modes,
\begin{align}
\sigma_{D\bar{D},\text{LHCb}} &\equiv \sigma_{D\bar{D}}, \\
\sigma_{D_1\bar{D}_2,\text{LHCb}} &\equiv \sigma_{D_1\bar{D}_2} + \sigma_{D_1\bar{D}_2}.
\end{align}

Therefore when they form the ratios in Eqs. (3.17) and (3.18), they obtain
\begin{align}
\frac{\sigma^2_{D,\text{LHCb}}}{2\sigma_{D\bar{D},\text{LHCb}}} &= \frac{4\sigma^2_D}{2\sigma_{D\bar{D}}} = \frac{2\sigma_{eff}}{C\sigma_{eff}/\sigma_c + 1}, \\
\frac{\sigma_{D_1\bar{D}_2,\text{LHCb}}}{\sigma_{D_1\bar{D}_2,\text{LHCb}}} &= \frac{4\sigma_{D_1\bar{D}_2}}{2\sigma_{D_1\bar{D}_2,\text{LHCb}}} = \frac{2\sigma_{eff}}{C\sigma_{eff}/\sigma_c + 1}.
\end{align}

That is, defined in this way, the ratios are flavour-independent, as LHCb noted, and still smaller than $\sigma_{eff}$. Since we do not have reliable estimates for $C$ and $\sigma_{eff}/\sigma_c$, we do not consider these opposite-sign cases further.

4 Results

Our main result is the fact that the LHCb extraction of $\sigma_{eff}$ from their double-inclusive open-charm data is too large by a factor of 2. Applying this factor of 2, we summarize in Fig. 1 their results both for the open-charm channels and the channels with a $J/\psi$, together with the average of the D0 and corrected\cite{15} CDF results quoted in Ref. [12].

It is clear that while the additional factor of 2 has brought the results closer together, the results for double open charm are still significantly higher than for the other processes.

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5 Summary

We have discussed various issues related to the measurement of double-charm cross sections and the extraction of the effective cross section for double-parton scattering, $\sigma_{\text{eff}}$, from them. We have emphasized the importance of properly-defined inclusive cross sections, in which each final state particle, or particle pair, of the given type is counted. With this definition, the effective cross section can be extracted directly from charmed hadron data without needing further information from theory, and is also a process-independent quantity in the commonly-used eikonal model.

Figure 1. The values of $\sigma_{\text{eff}}$ extracted from various LHCb measurements (data points, with statistical and systematic errors added in quadrature) compared with the value extracted from CDF and D0 data (yellow band).
We have noticed that LHCb use both single- and double-open charm cross section definitions that are summed over charge conjugate modes and that, therefore, an additional factor of 2 needs to be applied to the extraction of $\sigma_{\text{eff}}$ from these cross sections. This brings the data from open-charm pairs closer to that from $J/\psi$ plus open charm and jet production, but it still lies considerably higher.

We have mentioned several issues that could be worthy of further study: triple-charm production; differences between charmed and anticharmed hadron distributions; and correlations between charm-anticharm pairs.

In Appendix B below, we make comparisons with another theory paper that has compared with the same LHCb measurements.

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A Cross section definitions in more detail

In this Appendix we prove some of the results quoted in Sect. 2 and illustrate their discussion with their approximations in the limit of small cross sections. There is inevitably some overlap with the discussion of Sect. 2, but we try to keep this to a minimum, resulting in a rather terse presentation. Nevertheless, we hope that interested readers will be able to follow this discussion and reconstruct our argument if necessary.

A.1 Single-particle cross sections

A.1.1 Inclusive definition

The number of $i$'s counted, $N_i$, in a run with integrated luminosity $\mathcal{L}$ defines the inclusive $i$ cross section:

$$\sigma_{\text{incl}} \equiv \frac{N_i}{\mathcal{L}}. \quad (A.1)$$

There is an alternative way of writing this cross section that will help us in generalizing to multi-particle cross sections. As a first step, one can imagine that our hypothetical counter has a vanishingly small reconstruction efficiency $\epsilon_{\text{rec}}$, but that the reconstruction probability of a given $i$ is independent of any other $i$'s in the event. Thus the inclusive cross section is now written as

$$\sigma_{\text{incl}} = \lim_{\epsilon_{\text{rec}} \to 0} \frac{N_{i\text{rec}}}{\epsilon_{\text{rec}} \mathcal{L}}. \quad (A.2)$$

In this limit, it never happens that more than one $i$ is reconstructed in the same event. In an event in which there are $n$ $i$'s, the probability that one of them is reconstructed is $n\epsilon_{\text{rec}}$. 

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We therefore have

\[ \sigma_{\text{incl}} = \lim_{\epsilon_{\text{rec}} \to 0} \sum_n \frac{n \epsilon_{\text{rec}} N_{ni}}{\epsilon_{\text{rec}} \mathcal{L}} \]

\[ = \sum_n \frac{n N_{ni}}{\mathcal{L}}, \]  

where \( N_{ni} \) is the number of events in which there are exactly \( n \) \( i \)'s. Finally, we define the exclusive cross section for events in which there are exactly \( n \) \( i \)'s,

\[ \sigma_{ni} \equiv \frac{N_{ni}}{\mathcal{L}}, \]  

and hence

\[ \sigma_{\text{incl}} = \sum_n n \sigma_{ni}. \]  

That is, an event that contains \( n \) \( i \)'s contributes \( n \) times to the inclusive cross section.

The inclusive cross section in the eikonal model is then

\[ \sigma_{\text{incl}} = \sum_n n \int d^2b \left( \frac{\sigma_i A(b)}{n!} \right)^n e^{-\sigma_i A(b)} = \int d^2b \left[ \sum_n n \left( \frac{\sigma_i A(b)}{n!} \right)^n \right] e^{-\sigma_i A(b)} = \int d^2b \sigma_i A(b) \]

\[ = \sigma_1. \]  

A.1.2 Alternative inclusive definition

In the alternative inclusive definition, the inclusive \( i \) cross section is the cross section for events that contain one or more \( i \),

\[ \sigma_{\geq 1i} \equiv \sum_{n=1}^{\infty} \sigma_{ni}. \]  

In the eikonal model, this is given by

\[ \sigma_{\geq 1i} = \sum_{n=1}^{\infty} \int d^2b \left( \frac{\sigma_i A(b)}{n!} \right)^n e^{-\sigma_i A(b)} = \int d^2b \left[ \sum_{n=1}^{\infty} \frac{\sigma_i A(b)}{n!} \right]^n e^{-\sigma_i A(b)} = \int d^2b \left( 1 - e^{-\sigma_i A(b)} \right). \]  

For small \( \sigma_1 \) this agrees with the conventional definition, but for larger \( \sigma_1 \) it clearly differs,

\[ \sigma_{\geq 1i} \approx \sigma_1 - \frac{1}{2} \sigma_1^2 \int d^2b A(b)^2 + \mathcal{O}(\sigma_1^3). \]  

A.1.3 Exclusive definition

\[ \sigma_{1i} = \int d^2b \sigma_i A(b) e^{-\sigma_i A(b)}. \]  

Again, for small \( \sigma_1 \) this agrees with the inclusive definition, but for larger \( \sigma_1 \) it differs,

\[ \sigma_{1i} \approx \sigma_1 - \sigma_1^2 \int d^2b A(b)^2 + \mathcal{O}(\sigma_1^3). \]
A.2 Double-particle cross sections

A.2.1 Inclusive definition

For the formal definition of the two-particle inclusive cross section, we return to our hypothetical particle counter. We count the number of times in which it counts two i’s in the same event:

$$\sigma_{\text{incl}ii} = \lim_{\epsilon_{\text{rec}} \to 0} \frac{N_{\text{incl}ii}}{\epsilon_{\text{rec}}^2 L}.$$  \hfill (A.13)

In this limit, it never happens that more than two i’s are reconstructed in the same event. In an event in which there are \(n\) i’s, the probability that two of them are reconstructed is given by the binomial probability \(\binom{n}{2} \epsilon_{\text{rec}}^2 = \frac{1}{2} n(n-1) \epsilon_{\text{rec}}^2\). We therefore have

$$\sigma_{\text{incl}ii} = \sum_n \frac{1}{2} n(n-1) \sigma_{ni}.$$  \hfill (A.14)

That is, an event that contains \(n\) i’s contains \(\frac{1}{2} n(n-1)\) different ii pairs and hence contributes that many times to the inclusive cross section.

In the eikonal model, the double-particle inclusive cross section is given by

$$\sigma_{\text{incl}ii} = \sum_n \frac{1}{2} n(n-1) \sigma_{ni} \int d^2b \frac{(\sigma_i A(b))^n}{n!} e^{-\sigma_i A(b)} = \frac{1}{2} \sigma_i^2 \int d^2b A(b)^2.$$  \hfill (A.15)

For two different particle types i and j,

$$\sigma_{\text{incl}ij} = \sum_{n,m} n m \sigma_{ni,mj},$$  \hfill (A.16)

$$\sigma_{\text{incl}ij} = \sigma_i \sigma_j \int d^2b A(b)^2.$$  \hfill (A.17)

A.2.2 Alternative inclusive definition

$$\sigma_{\geq 2i} \equiv \sum_{n=2} \sigma_{ni},$$  \hfill (A.18)

$$\sigma_{\geq 2i} = \int d^2b \left( 1 - e^{-\sigma_i A(b)} - \sigma_i A(b)e^{-\sigma_i A(b)} \right).$$  \hfill (A.19)

For small \(\sigma_i\) this agrees with the conventional definition, but for larger \(\sigma_i\) it clearly differs,

$$\sigma_{\geq 2i} \approx \frac{1}{2} \sigma_i^2 \int d^2b A(b)^2 - \frac{1}{3} \sigma_i^3 \int d^2b A(b)^3 + O(\sigma_i^4).$$  \hfill (A.20)

By analogy, we have

$$\sigma_{\geq 1i,\geq 1j} \equiv \sum_{n,m=1} \sigma_{ni,mj},$$  \hfill (A.21)

$$\sigma_{\geq 1i,\geq 1j} = \int d^2b \left( 1 - e^{-\sigma_i A(b)} \right) \left( 1 - e^{-\sigma_j A(b)} \right),$$  \hfill (A.22)

and

$$\sigma_{\geq 1i,\geq 1j} \approx \sigma_i \sigma_j \int d^2b A(b)^2 - \frac{1}{2} \sigma_i \sigma_j (\sigma_i + \sigma_j) \int d^2b A(b)^3 + O(\sigma_i^4).$$  \hfill (A.23)
A.2.3 Exclusive definition

In the eikonal model the exclusive double-i and ij cross sections are given by

\[
\sigma_{2i} = \int d^2 b \frac{1}{2} (\sigma_i A(b))^2 e^{-\sigma_i A(b)} \approx \frac{1}{2} \sigma_i^2 \int d^2 b A(b)^2 - \frac{1}{2} \sigma_i^3 \int d^2 b A(b)^3 + \mathcal{O}(\sigma_i^4), \quad (A.24)
\]

and

\[
\sigma_{i1,1j} = \int d^2 b \sigma_i \sigma_j A(b)^2 e^{-(\sigma_i + \sigma_j) A(b)} \approx \sigma_i \sigma_j \int d^2 b A(b)^2 - \sigma_i \sigma_j (\sigma_i + \sigma_j) \int d^2 b A(b)^3 + \mathcal{O}(\sigma_{i,j}^4). \quad (A.25)
\]

A.3 The effective cross section

In all cases, we take Eq. (1.1) as the definition of \( \sigma_{\text{eff}} \).

A.3.1 Inclusive definition

\[
\sigma_{\text{eff}} = \frac{\sigma_{i1,i1}^2}{2 \sigma_{i1,i1}} = \frac{\sigma_{i1,i1} \sigma_{i1,i1}}{\sigma_{i1,i1}} = \frac{1}{\int d^2 b A(b)^2}. \quad (A.26)
\]

A.3.2 Alternative inclusive definition

\[
\sigma_{\text{eff}} = \frac{\sigma_{i1,i1}^2}{2 \sigma_{i1,i1}} = \frac{\int d^2 b b (1 - \sigma_i A(b))^2}{\int 2 d^2 b (1 - \sigma_i A(b) - \sigma_i A(b) e^{-\sigma_i A(b)})}. \quad (A.27)
\]

With this definition, \( \sigma_{\text{eff}} \) is process-dependent, since its value depends on \( \sigma_i \). The two different definitions in Eq. (1.1) give different results,

\[
\sigma_{\text{eff}} = \frac{\sigma_{i1,i1}^2}{2 \sigma_{i1,i1}} = \frac{\int d^2 b b (1 - \sigma_i A(b)) \int d^2 b (1 - e^{-\sigma_i A(b)})}{\int d^2 b (1 - e^{-\sigma_i A(b)}) (1 - e^{-\sigma_i A(b)})}. \quad (A.28)
\]

Both cases become process independent in the limit of small cross sections, but with different process-dependent corrections,

\[
\sigma_{\text{eff}} \approx \frac{1}{\int d^2 b A(b)^2} - \left(2 - \frac{2}{3} \int d^2 b A(b)^3 \right) \sigma_i + \mathcal{O}(\sigma_i^2), \quad (A.29)
\]

\[
\sigma_{\text{eff}} \approx \frac{1}{\int d^2 b A(b)^2} - \left(1 - \frac{1}{2} \int d^2 b A(b)^3 \right) \sigma_i + \mathcal{O}(\sigma_i^2). \quad (A.30)
\]

A.3.3 Exclusive definition

\[
\sigma_{\text{eff}} = \frac{\sigma_{i1,i1}^2}{2 \sigma_{i1,i1}} = \frac{\int d^2 b b A(b) e^{-\sigma_i A(b)}}{\int d^2 b b A(b)^2 e^{-\sigma_i A(b)}}. \quad (A.31)
\]

\[
\sigma_{\text{eff}} = \frac{\sigma_{i1,i1} \sigma_{i1,i1}}{\sigma_{i1,i1}} = \frac{\int d^2 b b A(b) e^{-\sigma_i A(b)}}{\int d^2 b b A(b)^2 e^{-(\sigma_i + \sigma_j) A(b)}}. \quad (A.32)
\]

\[
\sigma_{\text{eff}} \approx \frac{1}{\int d^2 b A(b)^2} - \left(2 - \frac{2}{3} \int d^2 b A(b)^3 \right) \sigma_i + \mathcal{O}(\sigma_i^2), \quad (A.33)
\]
\[ \sigma_{\text{eff}} \approx \frac{1}{\int d^2b A(b)^2} - \left(1 - \frac{\int d^2b A(b)^3}{\left[\int d^2b A(b)^2\right]^2}\right)(\sigma_i + \sigma_j) + \mathcal{O}(\sigma_{ij}^2). \] (A.34)

It is worth noting that the ratio of integrals of the matter distribution appearing in these expressions is a dimensionless feature of a given model that does not depend on the proton-radius-like parameter of the model, varying between about 1.25 for a ‘black disc’ model to about 1.75 for an exponential model, so is always between 1 and 2. Therefore the sign of the correction between the standard definition of \( \sigma_{\text{eff}} \) and the other definitions is different in different cases.

B Comparison with Berezhnoy et al.

Another paper, Ref. [13], has also made comparisons with the LHCb data of Ref. [11]. In their Table I, they show good agreement between their predictions and the LHCb data. Since they use a value of \( \sigma_{\text{eff}} = 14.5 \text{ mb} \) to make these predictions this is surprising, since we have seen in Fig. 1 that the LHCb data are consistent with a value of \( \sigma_{\text{eff}} \) a factor of 2 higher, even after taking into account the factor of 2 coming from the sum over charge conjugate modes.

In this Appendix, we consider the analysis of Ref. [13] and highlight the causes of their apparent agreement with data.

B.1 The single-inclusive cross section

To set the notation, we begin with Eq. (15) of [13]:

\[ "\sigma^\text{incl}_i = \sigma_1 p_i^{c\bar{c}} + \sigma_2(2p_i^{c\bar{c}} - (p_i^{c\bar{c}})^2)". \] (B.1)

Although they do not precisely define \( \sigma^\text{incl}_i \), from this equation we can infer that it is what we call the alternative inclusive definition – the cross section for one or more \( i \)s, and that \( \sigma_1, \sigma_2 \) are the exclusive cross sections for 1 and 2 \( c\bar{c} \) pairs respectively. Despite this, [13] (Eq. (20)) uses the inclusive formula,

\[ "\sigma_2 = \frac{\sigma_2^2}{2\sigma_{\text{eff}}} = 1.3 \pm 0.4 \text{ mb}". \] (B.2)

and sets \( \sigma_1 \) equal to the theoretical prediction for the inclusive charm cross section, \( 6.1 \pm 0.9 \text{ mb} \) to obtain that value of \( \sigma_2 \). It is evident that it is the values of \( \sigma_{1,2} \) that are used in the remainder of the analysis. In fact, in a given model for the matter density, it is possible to obtain the values of \( \sigma^\text{incl} \) and \( \sigma_{\text{eff}} \) from the values of \( \sigma_{1,2} \) and, in the form factor model used by [12] for example, these values correspond to \( \sigma^\text{incl} \approx 10 \text{ mb} \) and \( \sigma_{\text{eff}} \approx 18 \text{ mb} \). On the other hand, using the values that [13] quotes, \( \sigma^\text{incl} = 6.1 \text{ mb} \) and \( \sigma_{\text{eff}} = 14.5 \text{ mb} \), in the same form factor model, they should have used \( \sigma_1 = 4.2 \text{ mb} \) and \( \sigma_2 = 0.72 \text{ mb} \) in their calculation.

Like LHCb, [13] includes the sum over charge conjugate modes throughout and therefore \( p_D^{c\bar{c}} \) is the probability that a \( c\bar{c} \) pair produces one or more \( i \) or \( \bar{i} \)s. Thus, in our notation, and assuming \( p_D = p_D^{c\bar{c}} \),

\[ p_i^{c\bar{c}} = p_D^{c} + p_D^{\bar{c}} - p_D^{c}p_D^{\bar{c}} = 2p_D^{c} - (p_D^{c})^2. \] (B.3)
It is evident that Eq. (B.1) is a truncation at two scatters of a sum that should extend over all numbers of scatters,

\[
\sigma_{i}^{incl} = \sum_{n} \sigma_n \left( 1 - (1 - p_{i}^{c\bar{c}})^n \right),
\]  

(B.4)

Using the same form factor model again, and \( \sigma_{incl} = 6.1 \text{ mb} \) and \( \sigma_{eff} = 14.5 \text{ mb} \), one obtains \( \sigma_3 = 0.13 \text{ mb} \). Although this gives a negligible correction to the single-inclusive cross section, we will see below that neglecting \( \sigma_3 \) from the double-inclusive cross section results in a more significant error.

Finally, we can note that in practice the probabilities \( p_{i}^{c\bar{c}} \) are small (on the percent level) and hence it is actually a good approximation to neglect terms suppressed by factors of \( p_{i}^{c\bar{c}} \). Hence we can approximate Eq. (B.4) as

\[
\sigma_{i}^{incl} \approx \sum_{n} \sigma_n \left( n p_{i}^{c\bar{c}} \right) = p_{i}^{c\bar{c}} \sigma_{incl}.
\]  

(B.5)

We summarize the main points of our analysis of Eq. (15) of [13] as:

- \( \sigma_{i}^{incl} \) is defined as the alternative inclusive cross section, but the difference between this and the conventional definition is small in practice.

- \( \sigma_{1,2} \) are defined as the exclusive cross sections for 1 and 2 partonic scatters to produce \( c\bar{c} \) pairs respectively, but the inclusive formula is used to calculate them. This results in a significant error in the final result.

- The sum over number of scatters is truncated at 2. This makes a small difference in practice.

B.2 The double-inclusive cross section for same-sign pairs

Eqs. (17) and (19) of [13] read:

\[
\sigma_{i,i}^{same} = \sigma_2 \left( (p_{i}^{c\bar{c}})^2 + 2(p_{i}^{c\bar{c}})(p_{i}^{c\bar{c}} - p_{i}^{c\bar{c}}) + (p_{i}^{c\bar{c}} - p_{i}^{c\bar{c}})^2 / 2 \right),
\]  

(B.6)

\[
\sigma_{i,j}^{same} = \sigma_2 \left( 0.5(p_{i}^{c\bar{c}})^2 + 2(p_{i}^{c\bar{c}})(p_{j}^{c\bar{c}} - p_{j}^{c\bar{c}}) + 2(p_{j}^{c\bar{c}})(p_{j}^{c\bar{c}} - p_{j}^{c\bar{c}}) + 2(p_{j}^{c\bar{c}})(p_{j}^{c\bar{c}} - p_{j}^{c\bar{c}}) + 2(p_{j}^{c\bar{c}})(p_{j}^{c\bar{c}} - p_{j}^{c\bar{c}}) \right).
\]  

(B.7)

Because they use the alternative inclusive definition (the cross sections for two or more is and one or more \( i \) and one or more \( j \), their structure is complicated, but if we take the leading terms for small probabilities, we see the structure more clearly:

\[
\sigma_{i,i}^{same} \approx \frac{1}{2}(p_{i}^{c\bar{c}})^2 \sigma_2 = 2(p_{D})^2 \sigma_2,
\]  

(B.8)

\[
\sigma_{i,j}^{same} \approx p_{i}^{c\bar{c}} p_{j}^{c\bar{c}} \sigma_2 = 4p_{D_1} p_{D_2} \sigma_2.
\]  

(B.9)

It is evident that these expressions are truncations at two scatters. They agree with our expectations, since they include sums over charge conjugate modes, so the first is \( DD \) or \( \overline{D}_1 \overline{D}_2 \), each of which can only come from two \( c\bar{c} \) pairs in one way, while the second is \( D_1 D_2 \) or \( \overline{D}_1 \overline{D}_2 \), each of which can come from two \( c\bar{c} \) pairs in two possible ways.
However, for this double-inclusive cross section, the neglect of the three-c¯c cross section is more significant. One expects
\[
\sigma_{i,i}^{\text{same}} \approx 2(p_D^i)^2(\sigma_2 + 3\sigma_3 + \ldots), \\
\sigma_{i,j}^{\text{same}} \approx 4p_D^i p_D^j (\sigma_2 + 3\sigma_3 + \ldots), 
\]
since there are three c pairs within a three-c¯c event. Since our estimate of \(\sigma_3\) is approximately six times smaller than \(\sigma_2\), \(3\sigma_3\) is an \(\sim 50\%\) correction to \(\sigma_2\).

We summarize the main points of our analysis of Eqs. (17) and (19) of [13] as:

- \(\sigma_{i,i}^{\text{same}}\) and \(\sigma_{i,j}^{\text{same}}\) are defined as alternative inclusive cross sections, but the difference between these and the conventionally defined ones are small in practice.
- \(\sigma_2\) continues to be defined as the exclusive cross section for 2 partonic scatters, but the inclusive formula is used to calculate it. This results in a significant error in the final result.
- The sum over number of scatters is truncated at 2. This results in a significant error in the final result.

### B.3 The double-inclusive cross section for opposite-sign pairs

Eqs. (16) and (18) of [13] read:
\[
\begin{align*}
\bar{\sigma}_{i,i}^{\text{diff}} &= \sigma_1 p_{i,i}^{\ell\bar{\ell}} + \sigma_2 (2p_{i,i}^{\ell\bar{\ell}} - (p_{i,i}^{\ell\bar{\ell}})^2 + (p_{i,i}^{\ell\bar{\ell}} - p_{i,j}^{\ell\bar{\ell}})^2/2), \\
\bar{\sigma}_{i,j}^{\text{diff}} &= \sigma_1 p_{i,j}^{\ell\bar{\ell}} + \sigma_2 (2p_{i,j}^{\ell\bar{\ell}} - (p_{i,j}^{\ell\bar{\ell}})^2 + 2p_{i,j}^{\ell\bar{\ell}} p_{j,j}^{\ell\bar{\ell}} + 2p_{i,j}^{\ell\bar{\ell}} (p_{j,j}^{\ell\bar{\ell}} - p_{i,j}^{\ell\bar{\ell}}) + 2p_{j,j}^{\ell\bar{\ell}} (p_{j,j}^{\ell\bar{\ell}} - p_{i,j}^{\ell\bar{\ell}}) (p_{j,j}^{\ell\bar{\ell}} - p_{j,j}^{\ell\bar{\ell}})).
\end{align*}
\]

Again, because they use the alternative inclusive definition, their structure is complicated. To find the leading terms, we have to consider the additional probabilities \(p_{i,i}^{\ell\bar{\ell}}\) and \(p_{i,j}^{\ell\bar{\ell}}\) appearing in these equations. These are defined as the probabilities for a \(\ell\bar{\ell}\) pair produced in a single partonic collision to produce an \(i\) and an \(\bar{i}\), and an \(i\) and a \(\bar{j}\) or an \(i\) and a \(j\), respectively. Since these involve a correlation between the \(c\) and the \(\bar{c}\), in our notation they are
\[
\begin{align*}
p_{i,i}^{\ell\bar{\ell}} &= C(p_D^i)^2, & p_{i,j}^{\ell\bar{\ell}} &= 2Cp_D^i p_D^j.
\end{align*}
\]
On the other hand, Eq. (21) of [13] states:
\[
\bar{\sigma}_{i,i}^{\ell\bar{\ell}} \approx (p_{i,i}^{\ell\bar{\ell}})^2, \quad \bar{\sigma}_{i,j}^{\ell\bar{\ell}} = 2p_{i,j}^{\ell\bar{\ell}} p_{j,j}^{\ell\bar{\ell}}.
\]
This implies that they are taking the value of the correlation coefficient, \(C\), to be equal to 4, without explicitly saying so and without backing up this choice with a Monte Carlo study or experimental measurement.

Taking the leading terms for small probabilities, we then obtain:
\[
\begin{align*}
\sigma_{i,i}^{\text{diff}} &\approx (p_{i,i}^{\ell\bar{\ell}})^2 (\sigma_1 + \frac{5}{2}\sigma_2), \\
\sigma_{i,j}^{\text{diff}} &\approx 2p_{i,j}^{\ell\bar{\ell}} p_{j,j}^{\ell\bar{\ell}} (\sigma_1 + \frac{5}{2}\sigma_2),
\end{align*}
\]
which agrees with our expectation, provided $C = 4$ is assumed. Although these expressions are again truncations at two scatters, since they start already at one scatter, the neglect of three or more scatters is a small correction.

We summarize the main points of our analysis of Eqs. (16) and (18) of [13] as:

- $\sigma_{i,i}^{\text{diff}}$ and $\sigma_{i,j}^{\text{diff}}$ are defined as alternative inclusive cross sections, but the difference between these and the conventionally defined ones are small in practice.

- $\sigma_{1,2}$ continue to be defined as the exclusive cross sections for 1 and 2 partonic scatters respectively, but the inclusive formula is used to calculate them. This results in a significant error in the final result.

- The sum over number of scatters is truncated at 2. This makes a small difference in practice.

- A correlation of $C = 4$ between the fragmentation products of the charm and anticharm in a single $c\bar{c}$ pair is assumed. This is crucial for fitting data, but is not justified in the paper.

### B.4 Summary

The LHCb data on double open charm production are incompatible with $\sigma_{\text{eff}} = 14.5\,\text{mb}$. The apparent agreement between the data and the analysis of Ref. [13], which uses $\sigma_{\text{eff}} = 14.5\,\text{mb}$, is a coincidence caused by:

- Using the formula that defines $\sigma_{\text{eff}}$ in terms of single- and double-inclusive charm cross sections to calculate the single- and double-exclusive charm cross sections. This results in a factor $\sim 1.5$ error in $\sigma_1$ and $\sim 1.8$ in $\sigma_2$.

- Truncating the formula for the double-inclusive cross section for same-sign pairs at two scatters. This results in a factor of $\sim 1.5$ error in $\sigma_{\text{same}}^{i,i}$ and $\sigma_{\text{same}}^{j,j}$.

- Assuming without explicit justification a correlation of $C = 4$ between the fragmentation products of the charm and anticharm in a single $c\bar{c}$ pair.

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