Comment on Quantum teleportation via GHZ-like state

Anindita Banerjee, Kamal Patel and Anirban Pathak

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Jaypee Institute of Information Technology University, Noida, India

Abstract

Recently Yang et al. [Int. J. Theo. Phys. 48 (2009) 516] have shown that an unknown qubit can be teleported by using a particular GHZ-like state as quantum channel. However, there are several errors in the calculation which lead to incorrect conclusions. The errors have been indicated and corrected. It is also noted that their scheme and the independently proposed teleportation scheme of Zhang et al. [Int. J. Theo. Phys. 48 (2009) 3331] uses quantum channel from the same family and any state of that family may be used for teleportation.

In a recent paper Yang et al. [1] have shown that an unknown qubit can be teleported by using GHZ-like state as quantum channel. In [1] Yang et al. have made some mistakes in calculation. For example, in section 2.1 of [1] they start preparing their channel with a quantum state $|\phi\rangle_1 = |0\rangle_1 + |1\rangle_1$ which is not normalized. Ideally it should read as $|\phi\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)$. This is important because without this normalization constant the third line of equation (1) of [1] will not follow from the second line of the same equation. As $|\phi\rangle_1$ can be produced by operating Hadamard transformation $H$ on the state $|0\rangle$, the quantum circuit that prepares the GHZ-like quantum channel used by Yang et al. is essentially two EPR circuits attached in sequence in the cascaded manner (see Fig. 1).

After preparing the GHZ-like quantum channel they use that channel to teleport an unknown qubit $|\alpha0\rangle + |\beta1\rangle$. To do so Alice keeps the first two qubits of the channel with herself (i.e. particle 1 and 2) and particle 3 and 4 are sent to Bob and Charlie respectively. Now the product state of the unknown qubit and the channel is

$$\psi_{1234} = (|\alpha0\rangle + |\beta1\rangle) \otimes \frac{1}{\sqrt{2}}(|001\rangle + |100\rangle + |111\rangle).$$

This equation coincides with equation (5) of [1]. After this step they have tried to expand $|A\rangle_{123}$ and $|B\rangle_{123}$ as sum of product of Bell states (involving particle 1 and 2) and one qubit state so that if Charlie measures his qubit in computational basis and Alice does a Bell measurement on her qubits, then the necessary information about the unknown qubit is transferred to Bob. The outcome of the measurements are sent to Bob via classical channel and after receiving those information Bob chooses a suitable unitary operation to recreate the unknown state. Yang et al. did simple mistakes in the expansion step. The last two lines of both equations (6) and (7) of [1] are incorrect and consequently the unitary operations reported in Table 1 of [1] which are supposed to be performed by Bob are also incorrect. Correct expansion of product state would be

$$\psi_{1234} = \frac{1}{2\sqrt{2}} (|\psi^\pm\rangle_{12}(\alpha|0\rangle + \beta|1\rangle)_3 + |\psi^\mp\rangle_{12}(\alpha|0\rangle - \beta|1\rangle)_3 + |\phi^\pm\rangle_{12}(\alpha|1\rangle + \beta|0\rangle)_3 + |\phi^\mp\rangle_{12}(\alpha|1\rangle - \beta|0\rangle)_3).$$

Using (2) one can easily generate the Table 1 and this table considerably differs from the Table 1 of [1].

The erroneous calculation continues to the fidelity calculation. In this section they assume

1. Charlie has not measured the particle 4 but Alice has done the Bell measurement.
2. After the Bell measurement the state of particle 3 and 4 are in the state (see equation (9) of [1]).

$$|\phi\rangle_{34} = (|\alpha0\rangle + |\beta1\rangle)_3 = (|\alpha0\rangle + |\beta1\rangle)_3.$$  

Figure 1: A circuit to create GHZ-like states $|u\rangle|v\rangle|w\rangle$ are the input bits to the circuit and the output is a GHZ-like state.
A systematic study \cite{3} has revealed that the quantum channel chosen by Yang \textit{et al.} are member of a family of states denoted by $|\phi\rangle_{Zhang} = \frac{1}{\sqrt{2}} ((000) + (110) + (101) + (011))$ as quantum channel. A systematic study \cite{3} has revealed that the quantum channels described by Yang \textit{et al.} and Zhang \textit{et al.} are member of a family of states denoted by $\frac{1}{\sqrt{2}} (|\psi\rangle + |\phi\rangle)$ respectively. Altogether there are 12 such channels and essentially the Bell measurement done by Alice swaps the entanglement and its natural that the Yang’s consideration of separable state after the Bell measurement lead to incorrect conclusions. Further GHZ-like states are capable to work as quantum channel for controlled teleportation of $n$-qubit non-maximally entangled quantum state of the form $\psi = \alpha \left| x \right\rangle \pm \beta \left| x \right\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$, $x$ varies from 0 to $2^n - 1$ and $\bar{x} = 1^\otimes n \oplus x$ in modulo 2 arithmetic \cite{3}. Thus Yang’s scheme only considers a special case of more generalized scheme proposed in \cite{3}.

This is impossible and one can not bring particle 3 and 4 in a separable state without doing any measurement on any one of them. Further, since in the equation (9) of \cite{1}, the state of Bob (particle 3) is already in the desired state $(\alpha|0\rangle + \beta|1\rangle)$. The Fidelity should be 1. They have really got it 1 as in their calculation $F_3 = (|\alpha|^2 + |\beta|^2) = 1$ (see equation (12) of \cite{1}). But for some unclear reasons they could not recognize the fidelity as unity and they choose non physical condition like $|\alpha| = |\beta| = \frac{1}{\sqrt{2}}$ which does not satisfy the fundamental relation $|\alpha|^2 + |\beta|^2 = 1$. These non physical considerations lead to the conclusion that Fidelity varies between $\frac{1}{2}$ and 1 and rest of the conclusions of Yang \textit{et al.} follows from this incorrect conclusion. Actually if we assume that Charlie is not cooperating in this controlled teleportation scheme and he has not measured his bit and Alice has done Bell measurement on her qubit then the corrected scenario will be as described in Table 2.

Further we wish to add that the quantum channel chosen by Yang \textit{et al.} are neither robust nor special. For example, Zhang \textit{et al.} have recently reported a scheme for controlled teleportation by using another tripartite state $(|\phi\rangle_{Zhang} = \frac{1}{\sqrt{2}} ((000) + (110) + (101) + (011)))$ as quantum channel. A systematic study \cite{3} has revealed that the quantum channels described by Yang \textit{et al.} and Zhang \textit{et al.} are member of a family of states denoted by $\frac{1}{\sqrt{2}} (|\psi\rangle + |\phi\rangle)$ and $\frac{1}{\sqrt{2}} (|\psi\rangle + |\phi\rangle)$ respectively. Altogether there are 12 such channels and essentially the Bell measurement done by Alice swaps the entanglement and its natural that the Yang’s consideration of separable state after the Bell measurement lead to incorrect conclusions. Further GHZ-like states are capable to work as quantum channel for controlled teleportation of $n$-qubit non-maximally entangled quantum state of the form $\psi = \alpha \left| x \right\rangle \pm \beta \left| x \right\rangle$ where $|\alpha|^2 + |\beta|^2 = 1$, $x$ varies from 0 to $2^n - 1$ and $\bar{x} = 1^\otimes n \oplus x$ in modulo 2 arithmetic \cite{3}. Thus Yang’s scheme only considers a special case of more generalized scheme proposed in \cite{3}.

\textbf{References}

\begin{itemize}
  \item [1] K. Yang \textit{et al.}, Int. J. Theo. Phys. \textbf{48}, 516 (2009)
  \item [2] Q. Y. Zhang \textit{et al.}, Int J Theo. Phys. \textbf{48}, 3331 (2009)
  \item [3] A. Banerjee and A. Pathak, arXiv, quant-ph\1006.1042 (2010)
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