Implications of the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states

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Recently, the hidden charm tetraquark states $Z_{cs}(3985)$ and $Z_{cs}(4000)$ with strangeness were observed by the BESIII and LHCb collaborations, respectively, which are great breakthroughs for exploring exotic QCD structures. The first and foremost question is whether they are the same state. In this work, we explore the implications of the narrower state $Z_{cs}(3985)$ in BESIII and the wider one $Z_{cs}(4000)$ in LHCb as two different states. Within a solvable nonrelativistic effective field theory, we include the possible violations of heavy quark vector di-meson state as the heavy quark spin symmetry in a comprehensive approach. If $Z_{cs}(3985)$ and $Z_{cs}(4000)$ are two different states, our results show that $Z_{cs}(4000)/Z_{cs}(3985)$ is the pure $(D_{s}^{*}D)/(|D_{s}^{*}D|+|D_{s}D|)/\sqrt{2}$ state, and the SU(3) flavor partner of $Z_{cs}(3900)$ is $Z_{cs}(4000)$ rather than the $Z_{cs}(3985)$. Another two important consequences are the existence of a tensor $D_{s}^{*}D^{*}$ resonance with mass about 4126 MeV and width 13 MeV, and the suppression of the decay mode $Z_{cs}(3985) \rightarrow J/\psi K$. The two consequences can be tested in experiments and distinguish the two-state interpretation from the one-state scheme.

Keywords: tetraquark; strangeness; heavy quark spin symmetry; $Z_{cs}$ states;

I. INTRODUCTION

The quark model provides a very successful classification scheme for the conventional hadrons in terms of the valence quarks, the mesons ($qq$) and baryons ($qqq$). It has been experimentally verified during the past time. However, the observation of the $X(3872)$ [1], which does not fit into the quark model makes the hadron spectroscopy become again a challenging problem. Since then, the pace of exploring the QCD exota beyond the simple quark model, the so-called XYZ exotic states, has never stopped [2-10]. The observations of the $Z_{c}$ states [11–13] and the $P_{s}$ states [14, 15] are the milestones in the exploration of the exotic states. According to their charges and decay channels, their minimum quark constituents are $c\bar{c}q\bar{q}$ and $c\bar{c}qqq$ rather than the $c\bar{c}$ in the conventional charmonium. This makes them to be the multiquark states undoubtedly. Recently, the LHCb and BESIII collaborations made great breakthroughs in searching for hidden charmed multiquark states with strangeness [16–18], which starts a new epoch of exotic hadrons. Before these experimental observations, the hidden charm tetra- and pentaquarks with strangeness were predicted [19–21]. LHCb collaboration obtained the first evidence of the pentaquark state with strangeness [16]. BESIII collaboration reported a structure $Z_{cs}(3985)^{-}$ in the $K^{+}$ recoil-mass spectra in the $e^{+}e^{-} \rightarrow K^{+}(D_{s}^{+}D_{s}^{0})$ with $5.3 \sigma$ [17]. Several months later, LHCb collaboration announced the observation of two strange hidden charm states, $Z_{cs}(4000)^{+}$ and $Z_{cs}(4220)^{+}$ in the invariant mass spectrum of the $J/\psi K^{+}$ channel in the $B^{+} \rightarrow J/\psi K^{+}$ process with the significance of $15 \sigma$ and $5.9 \sigma$, respectively [18]. The observation of $Z_{cs}(3985)^{-}$ in BESIII inspired amounts of theoretical works to explore its nature [22–42], which had been released before the observation of $Z_{cs}(4000)$ and $Z_{cs}(4220)$ in LHCb collaboration. Since the $Z_{cs}(3985)$ lies close to the mass threshold of the $D_{s}^{*}D^{*}/D_{s}^{*}D$ channel, a natural and popular interpretation of the $Z_{cs}(3985)$ is the loosely bound molecule composed of the two mesons, $D_{s}^{*}D^{*}/D_{s}^{*}D$, which is the SU(3) flavor [SU(3)$_{F}$] partner of the $Z_{c}(3900)$ [11, 12]. In Refs. [22, 26], the hadronic components of $Z_{c}(3900)$ and $Z_{cs}(3985)$ are given

$$\langle PV/V\bar{P}, G_{1/U/V}\rangle = \frac{1}{\sqrt{2}} (\langle PV\rangle + G_{1/U/V}\langle V\bar{P}\rangle),$$

where $P$ and $V$ are the pseudoscalar and vector heavy-light mesons, respectively. $G_{1/U/V} = \pm 1$ is the generalized $G$-parity, where conventional parity is extended to $U$-spin and $V$-spin sectors. $G_{1/U/V}$ of $Z_{c}(3900)$ and $Z_{cs}(3985)$ are both +1 in Refs. [22, 26]. The $Z_{cs}(3985)$ was assigned as the partner state of $Z_{c}(3900)$ due to their consistent masses with respect to thresholds, widths and line shapes. A natural prediction is a vector $D_{s}^{*}D^{*}$ di-meson state as the heavy quark spin symmetry (HQSS) partner of the $Z_{cs}(3985)$ and the SU(3)$_{F}$ partner of the $Z_{c}(4020)$ [13]. The typical features or assumptions of above interpretations are the good HQSS and SU(3)$_{F}$ symmetry as shown in Fig. 2 (a).

The $Z_{cs}(4000)$ was observed in LHCb in the proximity of the $D_{s}^{*}D^{*}/D_{s}^{*}D$ thresholds like the $Z_{cs}(3985)$. However, the width of $Z_{cs}(4000)$ is over 100 MeV, which is about ten times larger than that of $Z_{cs}(3985)$ as shown in Fig. 1. Moreover, the $Z_{cs}(4220)$ state with higher mass is also a much broader resonance than the theoretical predictions of the HQSS partner of $Z_{cs}(3985)$ [22–26]. Thus, theorists resorted to considering the $Z_{cs}(4000)$ and $Z_{cs}(3985)$ as two different states in the hadronic molecular [45] and the compact tetraquark schemes [46]. The observations in LHCb also triggered a new round of theoretical investigations on the $Z_{cs}$ states [47–50].

Up to now, most theoretical works only focused on the $Z_{cs}(3985)$ states, or neglected the large width difference between the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ and treated them as
The Hamiltonian of the $\bar{D}^{(*)} D^{(*)}$ di-meson systems can be divided into the free part $\hat{H}_0$ and the interacting part $\hat{V}$. The SU(3)$_F$ symmetry and HQSS are both approximate symmetries of the Hamiltonian. For the near-threshold states, the interaction is weak. In Ref. [51], it was shown that the leading violations of the two symmetries arise from the mass terms in $\hat{H}_0$. There are two large mass splittings compared to the interaction $\hat{V}$,

$$m_{D^{(*)}} - m_{\bar{D}^{(*)}} \simeq 100 \text{ MeV},$$

$$m_{D^{(*)}} - m_{\bar{D}^{(*)}} \simeq 140 \text{ MeV},$$

which will break the SU(3)$_F$ symmetry and HQSS, respectively. In this work we will consider the symmetry breaking effects from both the mass term and interaction.

In the spin space, the eigenvectors of $\hat{H}_0$ are $|\bar{P}P\rangle$ ($J^P = 0^+$), $|\bar{V}V\rangle$ ($J^P = 1^+$), and $|\bar{W}\rangle$ ($J^P = 0^+ / 1^+ / 2^+$) states. The mixing effect among them with the same quantum number would be suppressed by the mass splittings in Eq. (3). The $|\bar{P}V\rangle$ and $|\bar{V}P\rangle$ states are (almost) degenerate for the free Hamiltonian and will mix with each other (The mass splitting between the $D^*_s D^{*0}$ and $D^{*}_s D^{0}$ channels is only 2 MeV). The mixing angle will be determined by the interaction $\hat{V}$.

The interaction in the spin space could be introduced as [52, 53],

$$V^{\pm} = V^{\pm}_{qq} + \text{HQSS breaking terms},$$

$$V^{\pm}_{qq} = c_1^s + c_2^s s_q \cdot s_{\bar{q}},$$

where $V^{\pm}_{qq}$ is the interaction between the light degrees of freedom, which satisfies the HQSS. The $s_q / s_{\bar{q}}$ is the spin operator of the light (anti)-quark. $c_1^s$ and $c_2^s$ are two independent coupling constants.

For the $|\bar{P}V / \bar{V}P\rangle$ states, the hadronic interactions in the $\{\bar{P}V, \bar{V}P\}$ basis read,

$$\langle V^+ \rangle_{\{\bar{P}V, \bar{V}P\}} = \begin{pmatrix}
    c_1^s & -\frac{1}{4}c_2^s \\
    -\frac{1}{4}c_2^s & c_1^s + \delta c^s
\end{pmatrix},$$

where we introduce the $\delta c^s$ in the diagonal term to account for the HQSS violation effect. In the HQSS limit ($\delta c^s = 0$), the interactions in the $|\bar{P}V / \bar{V}P\rangle$ channels are the same, and the eigenvectors of the Hamiltonian read,

$$|\bar{P}V / \bar{V}P, \pm \rangle = \frac{1}{\sqrt{2}} (|\bar{P}V\rangle \pm |\bar{V}P\rangle),$$

where the relative signs of two hadronic components are used to label the states. For the $[D^*D^*/D^*D^*]_{J=1}$ and $D_s^* D^*/D_s^* D$ states, the sign is that of the $G_{1/2}/V$-parity as shown in Eq. (1) (see Ref. [22] for details).

Another two related channels are the $|\bar{W}, 1^+\rangle$ and $|\bar{W}, 2^+\rangle$. Their interactions in the HQSS limit read,

$$\langle V^+ \rangle_{\{|\bar{W}, 1^+\rangle\}} = \langle V^+_{++} \rangle_{\{|\bar{W}, 1^+\rangle\}} = c_1^s - \frac{1}{4}c_2^s, \quad (8)$$

$$\langle V^+ \rangle_{\{|\bar{W}, 2^+\rangle\}} = \langle V^+_{-+} \rangle_{\{|\bar{W}, 2^+\rangle\}} = c_1^s + \frac{1}{4}c_2^s. \quad (9)$$

The two equalities indicate the existence of HQSS partners [54, 55]. For example, $Z_c(3900)$ and $Z_c(4020)$ have similar masses with respect to the corresponding thresholds and decay widths, which stems from the Eq. (8). The existence of $Z_c(3900)$ and $Z_c(4020)$ as the partner states indicates that the HQSS is a good approximation. It is reasonable to infer a similar property for the $D_s^{(*)} D^{(*)}$ systems. We will prove this conclusion numerically and predict a tensor $D_s^* D^*$ state with the $Z_c(3985)$ and $Z_c(4000)$ as two different states.

We can rewrite the above four states on the basis of the heavy quark symmetry $|S_{cc}^{P}, S_{qq}^{P}, J^P\rangle$,

$$|\bar{P}V / \bar{V}P, + \rangle = \frac{1}{\sqrt{2}} (|0^+_{cc}, 1^+_{qq}, 1^+\rangle - |1^-_{cc}, 0^-_{qq}, 1^+\rangle), \quad (10)$$

$$|\bar{P}V / \bar{V}P, - \rangle = |1^-_{cc}, 1^-_{qq}, 1^+\rangle, \quad (11)$$

$$|\bar{W}, 1^+\rangle = \frac{1}{\sqrt{2}} (|0^+_{cc}, 1^+_{qq}, 1^+\rangle + |1^-_{cc}, 0^-_{qq}, 1^+\rangle), \quad (12)$$

$$|\bar{W}, 2^+\rangle = |1^-_{cc}, 1^-_{qq}, 2^+\rangle. \quad (13)$$

The wave function decomposition would give rise to a series of selection rules for the decay modes of the $\bar{D}^{(*)}_{(s)} D^{(*)}$ di-meson systems.
**FIG. 2.** Comparison of two assignments of $Z_{cs}(3985)$ and $Z_{cs}(4000)$ states. The information in subfigure (a) is extracted from Refs. [22, 26] and subfigure (b) illustrates the main conclusions of this work treating the two $Z_s$ as different states. The particles in green and yellow cards are observations and our predictions, respectively. The quantum numbers $I^G/ I^P/ I^C$ of the states are given in the first row of each card. $\Gamma$ is the decay width. The hadronic components are listed in the third row of each card. In the last row, the wave function on the basis of the heavy quark symmetry [$(c\bar{c})S_H(q_1q_2)S_L$] $(S_H$ and $S_L$ are the heavy and light degrees of freedom, respectively) is given after quark rearrangement according to Eqs. (10)-(13). We omit the possible $1/\sqrt{2}$ factor for conciseness. The corresponding ground charmonium and the light mesons in the kets represent the $S_H$ and $S_L$ in the last row, respectively. They are the possible hidden charmed decay modes of the $Z_c/Z_{cs}$ states if allowed by the phase space in the heavy quark symmetry. We use the double head arrows to denote partner states in HQSS and SU(3)$_F$ symmetry. In subfigure (b), we use the oblique lines to represent the breaking of SU(3)$_F$ symmetry. For completeness, we list the information of two partner channels in white cards, which are neither the experimental resonances nor their theoretical predictions.

In the flavor space, the leading SU(3)$_F$ breaking effect stems from the mass splitting in Eq. (2), which will suppress the mixing effect between $|\bar{d}d\rangle + |\bar{u}u\rangle)/\sqrt{2}$ and $|ss\rangle$ systems (We omit the heavy quark part for conciseness.) [51]. For the isovector and open strange systems, the SU(3)$_F$ violation effect starts to appear at the interaction terms. Unlike Ref. [22], we neither presume the SU(3)$_F$ symmetry is a good approximation nor relate the interaction of $[\bar{D}^+(s)D^+(s)]_{I=1}^P$ to those of $\bar{D}^+_sD^+(s)$. We will show that the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states imply the large SU(3)$_F$ violation effect.

### III. SOLVABLE NREFT FOR RESONANCES

We adopt a solvable NREFT to investigate the $Z_{c}$ and $Z_{cs}$ states as the $S$-wave di-meson resonances [56]. We construct the general contact $S$-wave interaction to the next-to-leading order,

$$V(p, p') = \frac{c_a}{\Lambda^2} + \frac{c_b}{\Lambda^4} (p^2 + p'^2)^2,$$

where $c_a$ and $c_b$ are the low energy constants (LECs) for the leading order (LO) and the next-to-leading order (NLO). $\Lambda$ is the cutoff scale. The possible $p \cdot p'$ term becomes vanishing after the partial wave expansion for the $S$-wave channel. Similar interaction was adopted in Refs. [22, 57].

We first adopt the single-channel formalism. The $Z_{c}(Z_{cs})$ states are molecule-type states barely above the thresholds. The full-apart decaying into their constituent hadrons would be dominant. $Z_c(Z_{cs})$ couples to the $J/\psi\pi(K)$ channel through recluster of the heavy quarks, which is very weak. In Ref. [22], a coupled-channel calculation considering the $J/\psi\pi(J/\psi K)$ also shown that the such channels play minor roles, which neither changes the property of the pole nor contributes to the large partial decay with. Therefore, we only include the open-charmed $\bar{D}^{(*)}_sD^{(*)}$ channels. The $PV/VP$ and $WW$ thresholds are different by about 100MeV as shown in Eq. (3), which would suppressed their mixing effect. Therefore, we deal with $PV/VP$ and $WW$ channels separately. For the $Z_{c}(3900)$, $Z_{c}(4020)$, $Z_{cs}(3985)$ and $Z_{cs}(4000)$ states, we adopt four sets of independent LECs in Eq. (14), which will be determined by the mass and width of the corresponding state.

The single-channel Lippmann-Schwinger equation (LSE) reads,

$$T(p, p') = V(p, p') + \int \frac{d^3q}{(2\pi)^3} \frac{V(p, q)T(q, p')}{E - \frac{\mu^2}{2}\mu + i\epsilon},$$

where $E$ is the energy with respect to the mass threshold and $\mu$ is the reduced mass of the two heavy meson. The resonance corresponds to a pole of the $T$-matrix in the nonphysical Riemann sheet. A general numerical approach to solving the LSEs is the matrix-inversion method.

For the specific separable interaction in Eq. (14), the LSEs can be solved analytically [58]. For the single-channel LSE, the inverse of the $T$-matrix reads,

$$\frac{1}{T(k)} = -G_0A^5c_a - 2G_2A^3c_b + \left(\frac{G_2 - G_0G_4}{2}\right) c_b^2 + A^6,$$

$$\Lambda G_0c_a + c_b \left[ G_0G_4k^4 - 2G_2k^2 + G_4 \right] + 2k^2A^4,$$

where $k \equiv \sqrt{2}E$ and $G_n(k)$ is defined as

$$G_n = \int \frac{d^3q}{(2\pi)^3} \frac{q^n}{E - \frac{\mu^2}{2}\mu + i\epsilon}.$$

Here, we adopt the cutoff regularization ($\Lambda$ is the cutoff parameter). In the cutoff regularization scheme, we obtain the
TABLE I. The scattering lengths $a_s$ and effective ranges $r_0$ extracted from $Z_c$ and $Z_{cs}$ states within the single-channel NREFT (in units of fm). The results in Ref. [60] are listed for comparison.

|            | $Z_c$ (3900) | $Z_c$ (4020) | $Z_{cs}$ (3985) | $Z_{cs}$ (4000) |
|------------|---------------|---------------|-----------------|-----------------|
| $a_s$      | $-0.96 \pm 0.09$ | $-0.74 \pm 0.24$ | $-0.76 \pm 0.26$ | $-0.32 \pm 0.07$ |
| $r_0$      | $-2.88 \pm 0.37$ | $-3.95 \pm 2.70$ | $-6.70 \pm 5.54$ | $-2.08 \pm 0.80$ |
| [60]       | $a_s = -0.85 \pm 0.13$ | $-1.04 \pm 0.30$ | $-1.00 \pm 0.74$ | - |
| $r_0$      | $-2.52 \pm 0.25$ | $-3.90 \pm 1.35$ | $-4.04 \pm 1.82$ | - |

$G_n$, $G_0 = \frac{4\pi}{(2\pi)^3} 2\mu \left[k \tanh^{-1} \left(\frac{k}{\Lambda}\right) - \frac{\pi}{2} k\right]$, \hspace{1cm} (18)

$G_n = k^2 G_{n-2} - \frac{\mu}{\pi^2 n + 1}$, \hspace{1cm} (19)

The recursive relation relates the $G_n$ to $G_{n-2}$. In our calculation, only the even numbered $n$ cases are involved in. Thus, we only give the $G_0$ explicitly here and obtain the $G_2$ and $G_4$ in Eq. (16) by the recursive relation.

There are two unknown LECs $c_a$ and $c_b$ in each $D_s^{(*)} D_s^{(*)}$ channel. We treat $Z_c$ and $Z_{cs}$ states as resonances and their poles correspond to $1/\Gamma = 0$ in Eq. (16). We use their masses and widths (as shown in Fig. 1) to obtain two LECs for interactions in the corresponding channels. The solution of the LSE is cutoff dependent, which will be canceled out by the cutoff-dependence of the LECs (practical explicit examples can be seen in Refs. [51, 59]). Therefore, varying the cutoff parameter in a reasonable range will not change our results qualitatively. In this work, we fix the cutoff scale $\Lambda = 1.0$ GeV. We obtain the scattering lengths and effective ranges with the effective range expansion,

$T^{-1}(k) = \frac{\mu}{2\pi} \left(-\frac{1}{a_s} - ik + \frac{1}{2} r_0 k^2 + \ldots \right)$. \hspace{1cm} (20)

In Fig. 1, we compare our results with those in Ref. [60].

The $Z_{cs}$ (3985) and $Z_{cs}$ (4000) are the candidates of $|\bar{P}V/\bar{V}P, +\rangle$ and $|\bar{P}V/\bar{V}P, -\rangle$. We do not assign the specific corresponding relation for now. In general, the $|\bar{P}V/\bar{V}P, +\rangle$ and $|\bar{P}V/\bar{V}P, -\rangle$ states will couple with each other. The mixing effect arises from the HQSS breaking effect. We will evaluate the mixing effect in the coupled-channel formalism.

The coupled-channel potential for $Z_{cs}$ (3985) and $Z_{cs}$ (4000) systems can be parameterized as,

$V(p, p')_{(\bar{P}V/\bar{V}P, \pm)} = \begin{pmatrix}
\frac{c_a^+ + \delta c_a}{\Lambda} & \frac{\delta c_a}{\Lambda} \\
\frac{\delta c_a}{\Lambda} & \frac{c_a^- + \delta c_a}{\Lambda} \\
\frac{c_a^+ (p^2 + p'^2)}{\Lambda^3} & \frac{c_a^- (p^2 + p'^2)}{\Lambda^3}
\end{pmatrix}$, \hspace{1cm} (21)

where $c_{a/b}^+/-$ and $\delta c_a$ are the LECs. The subscripts $a$ and $b$ represent the LO and NLO, respectively. The superscript $+/-$ labels the two channels. For the off-diagonal term, we only keep the LO interaction. For the coupled-channel systems with four available inputs (masses and widths of two resonances), we have five unknown LECs to be determined. Thus, a rigorous fitting to determine the mixing effect is not feasible.

The second best approach is to fix the $c_a^+/-$, $c_b^+/-$, $c_b^-$ and $\delta c_a$ from the single-channel calculation and then vary $\delta c_a$ to check whether the coupled-channel effect is ignorable or not. The $\delta c_a$ is actually the HQSS breaking term and its meaning becomes clear when we change the coupled-channel potential into the $\{\bar{P}V, \bar{V}P\}$ basis,

$V^{\perp}_{\{\bar{P}V, \bar{V}P\}} = \frac{1}{2\Lambda} \begin{pmatrix}
c_a^+ + c_a^- & c_a^+ - c_a^- \\
c_a^- - c_a^+ & c_a^- + c_a^+ + 4\delta c_a
\end{pmatrix} + \text{NLO term}$. \hspace{1cm} (22)

We also define a ratio $R_{\text{HQSSB}}$ as a reflection of the HQSS breaking effect $R_{\text{HQSSB}} = 4\delta c_a/c_a^+ + c_a^-$. The nonzero $R_{\text{HQSSB}}$ indicates the nonvanishing off-diagonal terms in Eq. (21). Then, the bases $|\bar{P}V/\bar{V}P, +\rangle$ and $|\bar{P}V/\bar{V}P, -\rangle$ will mix with each other, in which mixing angle $\theta$ is defined to reflect the significance of the coupled-channel effect.

We present the pole trajectories and the mixing angle with varying $R_{\text{HQSSB}}$ in Fig. 3. When we vary the $R_{\text{HQSSB}}$ from $-0.4$ to $0.4$, the mixing angle changes quite slightly, less than three degree and thus negligible. We find that the 40% HQSS breaking effect does not change the qualitative properties of two resonances (relative orders for the masses and widths).

The NLO momentum-dependent interaction in Eq. (14) is very important and necessary in our framework, which would contribute to the effect range $r_0$ term in ERE and be able to generate a resonance pole above two-meson threshold even in a single-channel formalism. The $G_0$ in eq. (18) can be expanded as,

$G_0 = \frac{\mu}{\pi^2} \left(-\lambda - i\frac{\pi}{2} k + \frac{k^2}{\lambda} + O(k^4)\right)$. \hspace{1cm} (23)

If one only includes the LO constant contact potential, from Eqs. (16)-(20), one can obtain $r_0 = \frac{\Lambda}{2\pi}$, which is suppressed by the cutoff parameter. With only the LO constant contact term potential, the effective range $r_0$ is small and only the bound or virtual states can be produced. An alternative way to make the resonance solution possible in the LO contact interaction is to introduce the coupled channels (See [61] for a general discussion.).

In principle, one could estimate the LECs in Eq. (14) through the meson-exchange model. For $Z_{cs}$ states, it was shown that the exchange of ground light vector mesons is not allowed [35, 62, 63]. In contrast, the exchange of pseudoscalar $\eta$ meson could be introduced from the chiral Lagrangian [26]. If we follow the path of Bonn meson-exchange model [64], the scalar-meson-exchange [29] and two-kaon-exchange [26] interactions also contribute to the $D_s^* D/D_s D^*$ potential. Moreover, the heavy-meson-exchange interaction in Ref. [35] might also have contribution. However, in practice, estimating the contact LECs from the short-range part of the meson-exchange interaction is very involved. First, some of the coupling constants in the meson-exchange model for...
$\bar{D}^*_s D / \bar{D}_s^* D^*$ are not available. Meanwhile, in Ref. [65], it was shown that the contact interaction of EFT agrees with short-range part of meson-exchange interaction only when the EFT and the meson-exchange model were calculated to a high precision. Thus, it is hard to give a very reasonable estimation of the LECs through a low order (NLO) calculation from a rough meson-exchange model.

In Refs. [35, 62, 63], the authors adopted a coupled-channel formalism to investigate $Z_c$ and $Z_{cs}$ states. It was found that though the $Z_{cs}$ state couples to the $J/\psi K$ and $\eta K^*$ channels weakly, the two channels play important roles [35]. In Ref [22], we concluded differently that the $J/\psi \pi(K)$ channel neither changes the property of the pole nor contributes to the large partial decay width. From our perspective, the interacting mechanism and physical interpretation of $Z_{cs}$ in Ref. [35] are different with those in this manuscript and Ref [22]. In Ref. [35], the interactions are determined within the local hidden gauge approach in SU(4) flavor symmetry through heavy vector meson exchange. The diagonal potentials are small, then the off-diagonal potentials arising from the coupled-channel effect with the $J/\psi \pi$, etc., becomes important. The authors explained the $Z_{cs}$ as a cusp effect. In our work, we presume $Z_c$ and $Z_{cs}$ are resonances considering their masses are above the related thresholds. We choose to determine the low energy constants by fitting the experimental data. Given different premises, it is not strange to obtain different significance of the coupled-channel effect in two framework.

**IV. PHENOMENOLOGICAL ANALYSIS AND CONCLUSION**

We combine the experimental results [17, 18, 66] and our calculations to analyze the implications of $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states. The main conclusions are illustrated in Fig. 2 (b) and explained as follows,

1. **HQSS is a good approximation for the $\bar{D}^*_s D^*$ systems.** The consistent properties of $Z_c(3900)$ and $Z_c(4020)$ indicate that HQSS works well for $\bar{D}^*_s D^*$ systems. Meanwhile, the HQSS violation effect for the $|\bar{D}_s^* D^*/D_s^* D^*|$ and $|\bar{D}_s^* D^*/D_s^* D^*|$ systems is negligible. In Fig. 3, the 40% HQSS breaking effect will neither change the qualitative properties of two resonances, nor give rise to considerable mixing effect.

2. $Z_{cs}(4000)$ and $Z_{cs}(4220)$ are the HQSS partners. $Z_{cs}(4220)$ is a wide resonance above the $D_s^* D^*$ threshold. It is natural to interpret it as $D_s^* D^*$ resonances with $J^P = 1^+$. Its HQSS partner state $|\bar{D}_s^* D / D_s^* D^*, +>$ should be a wide resonance as $Z_{cs}(4220)$ because of the same interactions in the heavy quark limit in Eq. (8). Thus, it is reasonable to infer that the wider resonance $Z_{cs}(4000)$ rather than the narrower resonance $Z_{cs}(3985)$ is the partner of the $Z_{cs}(4220)$ state.

3. $Z_{cs}(4000)$ and $Z_{cs}(3985)$ are almost pure $|\bar{D}_s^* D / D_s^* D^*, +>$ and $|\bar{D}_s^* D / D_s^* D^*, ->$ states, respectively. As the HQSS partner of $Z_{cs}(4220)$, the hadronic component of the $Z_{cs}(4000)$ is $|\bar{D}_s^* D / D_s^* D^*, +>$. If the $Z_{cs}(3985)$ is the second state near the $D_s^* D / D_s^* D^*$ threshold, it should be dominated by the $|\bar{D}_s^* D / D_s^* D^*, ->$ component. In Fig. 3, the mixing effect of two different components is very tiny.

4. **There should exist a tensor $\bar{D}^*_s D^*$ state as the HQSS partner of $Z_{cs}(3985)$.** As illustrated in Eq. (9), the interaction in the tensor state $|V_s^*\bar{q}_g^T(\bar{q})>$ is the same as that of $Z_{cs}(3985)$. With the interaction extracted from the $Z_{cs}(3985)$, we give a prediction of the mass and decay width of the tensor state,

$$M = 4126 \pm 3 \text{ MeV}, \quad \Gamma = 13 \pm 6 \text{ MeV. (24)}$$

5. **The branch ratio $R(Z_{cs} \to \bar{D}^*_s D / Z_{cs} \to \bar{D}_s^* D^*) \approx 0.5$ for $Z_{cs}(3985)$ or $Z_{cs}(4000)$ states.** It is the consequence of that $Z_{cs}(4000)$ and $Z_{cs}(3985)$ are almost pure $|\bar{D}_s^* D / D_s^* D^*, +>$ and $|\bar{D}_s^* D / D_s^* D^*, ->$ states, respectively.

6. **The decay mode $Z_{cs}(3985) \to J/\psi K$ is suppressed in the HQSS limit.** In Eqs. (10)-(13), we decompose the $Z_{cs}(3985)$, $Z_{cs}(4000)$, $Z_{cs}(4220)$ and our prediction $Z_{cs}(4126)$ into the light and heavy degrees of freedom, which are separately conserved in the heavy quark limit. For example, the flavor-spin wave function of $|D^*_s D^*, 2^+>$ is $|1_{\bar{q}_s}, 1_{\bar{q}_g}^T, 2^+>$. In the heavy quark spin symmetry, we have $|1_{\bar{q}_s}, 1_{\bar{q}_g}^T, 2^+> = |1_{\bar{q}_s}, 1_{\bar{n}_s} q_g^T, 2^+>$. In Fig. 2 (b), we use $|J/\psi K>$ with total spin $J = 2$ to label the wave function after rearrangement. This leads to a series of selection rules [67-69] as displayed in Fig. 2 (b). For instance, the decay process $Z_{cs}(3985) \to J/\psi K$ is forbidden within the HQSS. Another way to derive these selection rules is the conservation of the $G_{U/V}$ parity. For example, the $G_{U/V}$ parities of $Z_{cs}(3985)$, $J/\psi$ and $K$ are $-1, -1$ and $-1$, respectively. The decay $Z_{cs}(3985) \to J/\psi K$ is suppressed by the $G_{U/V}$ parity conservation. The $G_{U/V}$-parity encodes the spin information into its charge con-
jugation part. Thus, it is natural to obtain the same results from the HQSS decomposition and $G_{I/U/V}$ analysis. As illustrated in Fig. 2 (b), the $Z_{cs}(3985)$ state is more likely to be observed in the $D_s^* D/\bar{D}_s^* D^*$ channel, while it is hard to be observed in the $J/\psi K$ channel, which is consistent with the experimental results [17]. Similar results have also been obtained in Ref. [45]. Finally, one should note that the above selection rules of the $G_{I/U/V}$ parity may be violated by the SU(3)$_F$ flavor symmetry and heavy quark symmetry breaking effects.

7. The violation effect of the SU(3)$_F$ symmetry might be significant. As shown in Fig. 2, the widths of the $Z_{cs}(3900)$ and $Z_{cs}(4020)$ states are much smaller than those of their SU(3)$_F$ partners $Z_{cs}(4000)$ and $Z_{cs}(4220)$. Moreover, the $I^G(J^{PC}) = 1^-(1^{++})$ state, as the partner state of $Z_{cs}(3985)$ in the SU(3)$_F$ limit, is missing in experiments. Therefore, the assignment of the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states implies the large SU(3)$_F$ breaking effect.

The implications of $Z_{cs}(3985)$ and $Z_{cs}(4000)$ in Fig. 2 (b) are quite different from the consequences of there existing only one $[D_s^* D^*/D_s^* D^*]^{1+}$ state in Fig. 2 (a). In the theoretical aspects, the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states implies that the $Z_{cs}(3985)$ is the $[D_s^* D/\bar{D}_s^* D^*, -]$ state, which is different from the assignment as the $[D_s^* D/\bar{D}_s^* D^*, +]$ in Fig. 2 (a). Meanwhile, the significant SU(3)$_F$ violation implied by $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two states contradicts the presuming SU(3)$_F$ symmetry in Fig. 2 (a).

With $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states, the decay $Z_{cs}(3985) \rightarrow J/\psi K$ is forbidden in the heavy quark limit. Meanwhile, an additional tensor state $D_s^* D^*$ is predicted as the HQSS partner of the $Z_{cs}(4000)$ in the two-state framework. If one assumes that there only exists one $[D_s^* D^*/D_s^* D^*]^{1+}$ state as shown in Fig. 2 (a), no tensor partner is predicted and the $Z_{cs}(3985) \rightarrow J/\psi K$ decay mode is allowed in the HQSS limit. In the experimental aspects, searching for the tensor $D_s^* D^*$ state and the $Z_{cs}(3985) \rightarrow J/\psi K$ decay mode can be used to distinguish the interpretation of the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ as two different states from there only existing one $D_s^* D/\bar{D}_s^* D^*$ resonance.

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