Gravitational Wave Bursts from Collisions of Primordial Black
Holes in Clusters

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The rate of gravitational wave bursts from the mergers of massive primordial black holes in clusters is calculated. Such clusters of black holes can be formed through phase transitions in the early Universe. The central black holes in clusters can serve as the seeds of supermassive black holes in galactic nuclei. The expected burst detection rate by the LISA gravitational wave detector is estimated.

Key words: black holes, gravitational waves, cosmology, galaxies.

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INTRODUCTION

The Laser Interferometer Space Antenna (LISA) designed to detect gravitational waves is scheduled to be launched in 2015 (for details, see the official LISA site). In comparison with ground-based detectors, LISA will be able to record signals with relatively low frequencies, $10^{-4} - 1$ Hz. The mergers of massive black holes (BHs) in galaxies can be the sources of such events. This process is very likely, since many of the suggested models for the formation of supermassive BHs include multiple mergers of intermediate-mass BHs as a necessary stage (Cattaneo et al. 2005; Volonteri et al. 2007; Di Matteo et al. 2007). The suggested scenarios predict different BH distributions and, accordingly, different BH merger rates. It is hoped that future observations with LISA will help narrow the range of possible models and identify the most likely ones by characteristic features in the distribution of gravitational wave bursts.

The origin of supermassive BHs is an especially challenging problem. In particular, the existence of supermassive BHs in quasars at high redshifts, $z \simeq 6 - 7$, requires an explanation. These supermassive BHs appeared very early and, hence, there existed an efficient mechanism of their birth and subsequent rapid growth in a relatively short cosmological time interval. Several models for the formation of supermassive BHs have been suggested (for a review of the various astrophysical and cosmological scenarios, see, e.g., Dokuchaev et al. 2007). One of the most interesting research directions is based on cosmological formation mechanisms of massive primordial BHs at very early pregalactic epochs, which can explain the existence of BHs at high redshifts following from observations.

Our calculations are based on the formation mechanism of massive primordial BHs at very early radiation-dominated epochs. This mechanism was suggested and described in detail by Rubin et al. (2000, 2001), Khlopov and Rubin (2004), and Khlopov et al.
Briefly, its essence is as follows. In inflationary models using potentials with more than one minimum, second-order phase transitions occur even at the inflationary stage. Spatial regions that will be at a minimum of the potential at the end of inflation different from the minimum to which the surrounding space evolves are formed. In contrast, at the inflationary stage, the size of these regions increases exponentially. The fundamental point is that after the completion of inflation, the exponentially expanded region has a size much larger than that of the horizon size (at that time). In the above papers, these regions were also shown to be surrounded by a closed field wall with some surface energy density. The energy contained in the wall is proportional to the square of its size and can vary over a wide range, since its size is not limited by the horizon size. After crossing the horizon to the post-inflationary stage, the wall evolves rapidly and collapses into a primordial black hole (PBH). The PBH mass depends on the parameters of the inflaton potential and initial conditions. We chose the conditions under which the number and mass of PBHs were consistent with the observational data. More specifically, the mass of the most massive PBHs formed via the collapse of the field wall should be $10^5 M_{\odot}$ and the energy density in the PBH does not exceed 1% of the dark energy density. Rubin et al. (2001) showed that the mass spectrum of the most massive PBHs falls off rapidly with mass and we chose $10^5 M_{\odot}$ as a typical value, which is sufficient for the subsequent estimations.

Based on the formalism of these papers, we constructed a model for the formation of protogalaxies (from dwarf to supermassive ones) induced by massive PBHs (Dokuchaev et al. 2005, 2008). This paper, in which we consider the gravitational wave bursts from the mergers of PBHs in clusters, is directly based on the results of our previous works on the formation of BH clusters (Dokuchaev et al. 2005, 2008). The protogalaxy formed around a BH cluster turns out to have a number of realistic properties that correspond to the observed properties of galaxies: the presence of a central supermassive BH, the existence of intermediate-mass BHs in an extended halo, and a sufficient amount of dark and baryonic matter to produce the observed number of stars in galaxies. An important characteristic feature of the suggested scenario is a very early formation of quasars (Dokuchaev et al. 2005).

LISA provides a unique opportunity to test the scenarios for the formation of supermassive BHs in galactic nuclei. The possibility of such testing is related to the inevitable BH mergers that generate bursts of gravitational radiation. The merger rate of BHs in the cosmological PBH formation model under consideration differs significantly from that of BHs formed through astrophysical mechanisms. As a typical case, we consider a cluster that we calculated based on the papers of Khlopov et al. (2005) and Dokuchaev et al. (2005, 2008). There is a supermassive BH with a mass $M_0 \sim 10^5 M_{\odot}$ surrounded by lower-mass BHs at the center of such a characteristic cluster. The distributions of intermediate-mass BHs obtained from this formalism (Klopov et al. 2005) will be used below as the initial data. For convenience, we found a fit to the BH mass distribution. More specifically, after the virialization of the PBH cluster at the radiation-dominated
epoch, the differential BH number density in the cluster can be represented as
\[
\frac{dn}{dM} = 1.6 \times 10^3 \left( \frac{r}{1 \text{ pc}} \right)^{-3} \left( \frac{M}{M_\odot} \right)^{-2} M_\odot^{-1} \text{pc}^{-3}.
\]  
(1)

Another dynamical component of the clusters that should be taken into account when the structure of protogalaxies is investigated is dark matter. The total mass of the dark matter is higher than that of the PBHs at distances larger than \( r \simeq 1.6 \) pc from the cluster center. However, in this paper, we will restrict our analysis only to the central region of the cluster where the BH mass dominates over the dark matter mass.

**COLLISIONS OF BLACK HOLES IN CLUSTERS**

Dynamical relaxation takes place in the densest central region of the cluster under the effect of twobody gravitational BH scatterings. As a result of this relaxation, the intermediate-mass BHs surrounding the most massive central BH occasionally fall into the loss cone and merge with the central BH. The intermediate stage of these mergers is the formation of short-lived close BH pairs. The cross section for the gravitational capture of two BHs with masses \( M_0 \) and \( M \) into a close pair (Mouri and Taniguchi 2002) is
\[
\sigma_{\text{mer}} = 2\pi \left( \frac{85\pi}{6\sqrt{2}} \right)^{2/7} \frac{G^2(M_0 + M)^{10/7}M_0^{2/7}M^{2/7}}{c^{10/7}v_{\text{rel}}^{18/7}},
\]  
(2)

where \( v_{\text{rel}} \) is the relative BH velocity; \( G \) and \( c \) are the gravitational constant and the speed of light, respectively. After their capture, the BHs merge together relatively fast through the radiation of gravitational waves. A powerful gravitational wave burst is generated at the time of the merger. The cross section for the merger of two BHs with the formation of an intermediate close pair in a wide range of BH masses and velocities exceeds significantly the cross section for direct BH collisions with gravitational focusing.

Let us analyze the main parameters of the collisions between a BH with a mass \( M \) and a central BH at rest with a mass \( M_0 > M \) in the cluster under consideration. The differential BH merger rate is
\[
d\dot{N} = \sigma_{\text{mer}} v_{\text{rel}} dn,
\]  
(3)

where the number density \( dn \) of BHs with masses in the interval \( M-M + dM \) depends on the distance to the cluster center and the time. The pair relaxation through distant Coulomb scatterings distorts the initial spatial distribution. The pair relaxation time for BHs with mass \( M \) in a multicomponent system is (Spitzer and Saslaw 1966)
\[
t_{\text{rel}} \simeq \frac{1}{4\pi} \frac{v^3}{G^2MA_c\rho_h},
\]  
(4)

where \( \rho_h \) is the total density of the BHs of all masses, \( v \) is the velocity dispersion in the cluster, and \( A_c \simeq 15 \) is the Coulomb logarithm. The relaxed spherical subsystem of BHs persists in the cluster for \( \sim 40t_{\text{rel}} \) until its dynamical evaporation. Some of the BHs merge with the central most massive BH. The subsystems of an increasingly large
scale with a lower density relax in the course of time. Consequently, the BH number density in the central region, where the most massive BH is located, falls and the merger rate gradually decreases. During our calculations, we first find the size of the completely relaxed cluster region for each cosmological time \( t \), where \( t \approx 40 t_{\text{rel}} \). We use the average BH number density in the relaxed region (the initial number density (1) is averaged) as the number density \( d_n \) in Eq. (3) and assume for our estimation that the density in this region is uniform. We perform our calculation with a logarithmic accuracy by neglecting the change in the Coulomb logarithms compared to the power-law dependences. In this approximation, we obtain the following expression for the BH merger rate in one cluster:

\[
\frac{d\dot{N}(z)}{dM} \approx 5.1 \times 10^{-7} \left( \frac{M}{M_\odot} \right)^{-67/21} (t(z)/t_0)^{-31/21} M_\odot^{-1} \text{ yr}^{-1},
\]

where \( t(z) \) is the redshift dependence of the cosmological time calculated from well-known formulas and \( t_0 \) is the present time. In reality, mass segregation takes place in the cluster: high-mass BHs settle toward the cluster center and their merger rate will be slightly higher than the calculated one. Therefore, our calculations should be considered as a lower limit.

A gravitational detector records bursts based on an optimal filtering technique with an efficiency dependent on the signal-to-noise ratio (Grishchuk et al. 2001). Let us find the signal-to-noise ratio \( \rho_{\text{SN}} \) for the detection of BH mergers in the clusters under consideration by LISA. Our approach is similar to that used by Will (2004), but we apply cosmological corrections, since the redshift can be much higher than unity. The signal-to-noise ratio as a function of the BH mass and the cluster redshift is

\[
\rho_{\text{SN}}(z, M)^2 = 4 \int_{f_i}^{f_f} \frac{|\tilde{h}(f)|^2 df}{S_h(f)},
\]

where \( f_i \) and \( f_f \) are, respectively, the initial (minimum) and final (maximum) frequencies of the observed gravitational radiation, \( \tilde{h}(f) \) is the Fourier transform of the observed signal, and \( S_h(f) \) is the detector noise spectral density.

Denote the sum of the merging BH masses by \( \tilde{M} = M_0 + M \). The orbit of the BH pair decays due to energy losses through gravitational radiation and the orbital revolution frequency increases. The minimum frequency of the gravitational signal corresponds to the initial time of observations, while the maximum frequency is twice the revolution frequency in the innermost stable orbit \( f_f = f_m/(1 + z) \), where (Will 2004)

\[
f_m \simeq \frac{c^3}{6^{3/2} \pi G \tilde{M}}.
\]

For example, \( f_m = 0.04 \text{ Hz for } \tilde{M} = 10^5 M_\odot \). Over the observing time \( T = 1 \text{ yr} \) (and, accordingly, over the proper time of the source \( T/(1 + z) \)), the orbital revolution frequency and the gravitational radiation frequency increased from the initial value of (Will 2004; without the factor \( 1 + z \))

\[
f_i = f_f \left( 1 + \frac{256 \pi^{8/3}}{5} G^{5/3} M_0 M \frac{f_m^{8/3}}{c^3 M^{1/3}} \frac{T}{1 + z} \right)^{-3/8}.
\]
Figure 1: Predicted distribution of gravitational wave bursts in redshift $z$ at which the signal sources, the clusters of primordial BHs, are located. The solid curves correspond to the total merger rates for BHs from the mass intervals $M = 10^2 - 5 \times 10^2 M_\odot$, $5 \times 10^2 - 10^3 M_\odot$, $10^3 - 5 \times 10^3 M_\odot$, $5 \times 10^3 - 10^4 M_\odot$, $10^4 - 5 \times 10^4 M_\odot$ and $5 \times 10^4 - 10^5 M_\odot$ (from the top down). The value of $z$ at the right boundaries of the curves is the maximum redshift from which the detection of bursts by LISA is possible. The enveloping dashdotted curve indicates the total rate of bursts from the mergers of BHs with masses in the range $10^{-1} - 10^5 M_\odot$. For comparison, the dashed line indicates the results of calculations from Sesana et al. (2005) for the mergers of BHs formed through the collapse of gaseous clouds.

The observed gravitational wave spectrum is well fitted by a power law $\tilde{h}(f) = Af^{-7/6}$ for $f < f_f$ and $\tilde{h}(f) = 0$ for $f > f_f$ (Will 2004), where

$$A = \frac{1}{\sqrt{30\pi^{2/3}}} \frac{G^{5/6}(M_\odot M)^{1/2}(1 + z)^{1/3}}{d_L c^{3/2} M^{1/6}}. \quad (9)$$

Here, $d_L$ is the luminosity distance. We will use an analytical fit to the noise spectrum from Finn and Thorne (2000):

$$S_h(f) = 6.12 \times 10^{-51} / f^4 + 1.06 \times 10^{-40} + 6.12 \times 10^{-37} f^2 \text{ Hz}^{-1}. \quad (10)$$

The observed rate of bursts from redshifts $< z$ is

$$d\dot{N}_{\text{GB}}(z) = \int_0^z dz (dl_c/dz) 4\pi l_c^2 n_{cl} d\dot{N}(z)(1 + z)^{-1}\theta(\rho_{\text{SN}}(z, M) - 5), \quad (11)$$

where $n_{cl}$ is the cluster number density, $\theta$ is the Heaviside function, and, as is commonly done for LISA, we chose $\rho_{\text{SN}}(z, M) = 5$ as the threshold of reliable detection. For each
specific mass $M$, the detection threshold implicitly defines the maximum redshift $z$ from which the signal can be received. The quantity $d\dot{N}(z)$ in Eq. (11) corresponds to some small mass interval $dM$ and we will present the final results for $\dot{N}_{\text{GB}}(z)$ integrated over finite mass intervals. In the above formulas, the cosmological distance in comoving coordinates and the luminosity distance are, respectively,

$$l_c(z) = \int_0^z c \, dz / H(z), \quad d_L(z) = (1 + z) l_c(z),$$

(12)

where $H(z) = \dot{a}/a$ is the Hubble constant; $a$ and $\dot{a}$ are the scale factor of the Universe and its time derivative, respectively.

To determine the cluster number density $n_{cl}$ in a unit comoving volume, we will use the following estimate. Suppose that the BH clusters have settled toward the galactic centers by now and that their central BHs have merged into supermassive BHs with masses $\sim 10^8 M_\odot$. We will then find that there were initially $N_{cl} \sim 500$ clusters in each structured galaxy. Since the number density of structured galaxies containing supermassive BHs is $n_g \sim 0.01 \text{ Mpc}^{-3}$, the cluster number density is $n_{cl} \sim 500 \times 0.01 = 5 \text{ Mpc}^{-3}$.

Figure 1 shows the distribution of gravitational wave bursts in redshift $z$ that we derived in comparison with the results of Sesana et al. (2005), who considered the mergers of intermediate-mass BHs formed according to a standard astrophysical scenario (see Fig. 3 from the above paper). As we see from the figure, the rate of BH mergers in the lower mass range is high in our model, because they are numerous. In one-year-long observations, the detection of several mergers of BHs with masses in the range $10^2 - 10^3 M_\odot$ with the central BHs of the clusters is probable. The bends on the plot are attributable to the presence of a $\theta$-function in (11). These bends correspond to the maximum redshifts from which the mergers of minimum-mass BHs from each range are seen.

Note the significant difference, obvious from Fig. 1, in the shapes of the burst distribution in our scenario and the distribution in Fig. 3 from Sesana et al. (2005). According to this paper, BHs are formed through dissipative collapse of the central regions of gaseous clouds; subsequently, the protogalaxies coalesce and the BHs settle toward the new dynamical center and merger together. Obviously, the BH merger rate in this model increases with time, at least at the first stage. In our model, primordial BHs were formed at the radiation-dominated epoch of the Universe and the dynamical evolution of dense BH clusters began at the same epoch. Therefore, the BH merger rate in our model decreases with time. The significant merger rate in our model compared to astrophysical scenarios is attributable to the high BH number density at the cluster centers. Thus, the mergers in clusters and the pair mergers in the model by Sesana et al. (2005) occur under fundamentally different conditions, which leads to the difference between the distributions in Fig. 1. This difference during LISA observations can help in choosing between the various scenarios for the BH origin.

To interpret Fig. 1, it is also useful to plot the observed dimensionless gravitational wave amplitude $h_c$ against the observed frequency $f$. According to Sesana et al. (2005), $h_c = h \sqrt{n}$ for $n < fT$ and $h_c = h \sqrt{fT}$ for $n > fT$, where $h$ and $n$ depend on $f$, the
Let us now consider the recoil impulse gained by the central BH as it collides with lower-mass BHs. This effect can lead to the BH ejection from the cluster if BHs of comparable masses with a high angular momentum collide. Damour and Gopakumar (2006) found the recoil velocity due to asymmetric gravitational radiation to be at a maximum at a BH mass ratio $\eta \simeq 0.38$ and to decrease approximately as $\eta^2$. Note, however, that the probability of such collisions in a separate cluster in our model is very low and that the high rate of gravitational wave bursts is related to the large number of clusters in the observable Universe. Indeed, it is easy to find from Eq. (5) that the collision rate between BHs with masses $10^4 - 10^5 M_\odot$ and the central BH is $\sim 2 \times 10^{-19}$ yr$^{-1}$. The collision rate between the central BH and lower-mass BHs is higher, but the collision probability in the Hubble time is $\sim 0.005$ even for $M \sim 100 M_\odot$ and the recoil velocity is approximately four orders of magnitude lower than the velocity dispersion in the cluster. Thus, the recoil through gravitational radiation has virtually no effect on the rate of gravitational wave bursts in our model. Another source of the recoil velocity is the impulse of an impinging BH. Through this effect, the central BH will also gain a very low recoil velocity.
\( \sim v(M/M_0) \), where \( v \) is of the order of the velocity dispersion in the cluster \( \sim 50 \text{ km s}^{-1} \), and such collisions do not lead to the ejection of the central BH from the cluster.

We calculated the rate of BH mergers with the central, most massive BH in the cluster. Let us now show that the contribution from these mergers is dominant compared to that from the mergers of BHs of lower masses \( M_1 < M_0 \) and \( M_2 < M_0 \) with one another. Indeed, the merger rate in a volume element \( dV \) is

\[
\frac{d\dot{N}}{dM_1 dM_2} = \sigma_{\text{mer}} v_{\text{rel}} \frac{dn_1}{dM_1} \frac{dn_2}{dM_2} dV. \tag{13}
\]

Using distribution (11) and integrating Eq. (13) over the cluster volume and the BH masses, we will obtain

\[
\int \int \int 4\pi r^2 dr dM_1 dM_2 \frac{d\dot{N}}{dM_1 dM_2} \propto r_{\text{min}}^{-3} \left( \frac{M_{1,\text{max}}}{M_{2,\text{min}}} \right)^{5/7}, \tag{14}
\]

where \( r_{\text{min}} \) is the radius of influence of the central BH, while \( M_{2,\text{min}} \) and \( M_{1,\text{max}} = M_0 \) are the minimum and maximum BH masses from the distribution function. For a typical case where \( M_2 \ll M_1 \), the merger rate of small BHs is lower than the rate of mergers with the central BH and tends to it in the limit \( M_{1,\text{max}} \rightarrow M_0 \). Thus, the mergers of relatively low-mass BHs (the mass boundary is defined by the condition \( \rho_{\text{SN}}(z, M) > 5 \)) with the central BH near the cluster center \( r \approx r_{\text{min}} \) make the largest contribution to the gravitational radiation, while the rate of such mergers was calculated above.

As has already been noted above, the present-day structured galaxies could include \( \sim 500 \) BH clusters. These clusters sink to the galactic centers under dynamical friction and their central BHs merger together. These mergers occur simultaneously with the BH mergers inside the clusters that we considered above. Distinctive signatures of the mergers of central BHs are their high masses, \( M_0 \sim 1.3 \times 10^5 M_\odot \), and, accordingly, large amplitudes and low frequencies of the gravitational signals by which these mergers can be separated from the mergers of lower-mass BHs in observations. An accurate calculation of the rate of gravitational wave bursts requires cumbersome numerical simulations of the galaxy merging process (allowance for the merger tree) and the secular BH settlement toward the galactic centers. Here, we give a simple estimate for the merger rate of central BHs with masses higher than \( 10^5 M_\odot \) from the observable Universe:

\[
\frac{d^2N_5}{dtdz} \sim \frac{4\pi}{3} \frac{N_{\text{t}}}{t_0 \Delta z} (ct_0)^3 \frac{n_g}{10^{-2} \text{Mpc}^{-3}} \left( \frac{n_g}{10^{-2} \text{Mpc}^{-3}} \right) \left( \frac{t_0}{1.3 \times 10^{10} \text{yrs}} \right)^2 \left( \frac{\Delta z}{10} \right)^{-1} \left( \frac{N}{500} \right) \text{ yr}^{-1}, \tag{15}
\]

where \( n_g \) is the mean number density of structured galaxies \( N \sim 500 \) is the mean number of mergers per galaxy, and \( \Delta z \sim 10 \) is the characteristic duration of the merger epoch in redshifts. Thus, the merger rate of massive central BHs with \( M > 10^5 M_\odot \) is also fairly high and the gravitational wave bursts from these mergers can be detected by LISA.
DISCUSSION

At present, there is no single, fully justified scenario for the formation of supermassive BHs. Many different scenarios, both astrophysical and cosmological ones, that could give rise to supermassive BHs have been developed and identifying the most likely mechanisms is of paramount importance. Future information from the LISA gravitational wave detector will provide one of the possibilities for solving this problem, since existing models predict different rates and shapes of gravitational wave bursts.

Our paper is based on the cosmological formation mechanism of the clusters of massive primordial BHs at a pregalactic expansion phase of the Universe developed by Rubin et al. (2000, 2001), Khlopov and Rubin (2004), and Khlopov et al. (2005). We considered two possibilities. First, gravitational waves are generated during the mergers of primordial BHs in clusters. Second, the BH clusters themselves are involved in the hierarchical clustering of dark matter, along with ordinary protogalaxies, being members of large galaxies. Under dynamical friction, the clusters of massive primordial BHs settle toward the galactic centers, where they merge together to produce supermassive BHs, which is also accompanied by gravitational wave bursts.

Based on this model, we found the rate of gravitational wave bursts generated during the mergers of primordial BHs in clusters. The hypothesis that this process is the main formation mechanism of supermassive BHs at the galactic centers allows the possible number of primordial BH clusters in a typical galaxy to be estimated. This hypothesis is of fundamental importance in calculating the possible rate of gravitational wave bursts from BH mergers detectable by LISA. The rate of gravitational wave bursts turned out to be high enough for their detection over the LISA operation time.

Our main result is the conclusion that the redshift distribution of bursts based on the cosmological scenario differs significantly from the distribution given by the astrophysical models of late BH formation in protogalaxies. Therefore, future observations of gravitational wave bursts will allow the dilemma of whether the massive BHs are galactic or cosmological in origin to be solved in principle. In particular, for the cosmological origin, massive BHs could affect significantly the formation of large-scale structure.

Note also that the formation mechanism of BH clusters suggested by Rubin et al. (2000, 2001), Khlopov and Rubin (2004), and Khlopov et al. (2005) can also be tested in principle based on ordinary optical observations. A number of BH clusters remained outside galaxies should appear as dwarf galaxies (spheroids) with a sharp central density spike. Searching for such objects is of great interest for this model. The dwarf spheroidal galaxies known to date have no central density spikes and, hence, could not be produced by this mechanism (Dokuchaev et al. 2008) but originated from ordinary density perturbations. It may also well be that BH clusters can be among the observed, but not yet identified powerful X-ray sources (Gao et al. 2004). The X-ray emission can be generated when baryons are accreted onto intermediate-mass BHs (Mapelli et al. 2008).
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