Power-Law Decay of Standing Waves on the Surface of Topological Insulators

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The discovery of topological insulators (TIs) has attracted a great deal of attention [1,2]. The three-dimensional TIs are characterized by the gapped bulk states and gapless surface states (SSs), which are protected by time-reversal symmetry (TRS) and consist of an odd number of spin-helical Dirac cones. Exotic effects such as Majorana fermions [3, 4] and magnetic monopole [5] are predicted to exist as results of the topological SSs.

The low-temperature scanning tunneling microscope (STM) and spectroscopy (STS) provide a direct way to study the SSs through probing the local density of states (LDOS) oscillations in the vicinity of impurities or step edges [6]. The quasiparticle interference (QPI) patterns induced by non-magnetic impurities on the surface of Bi₂Te₃ [7] and Bi₂Se₃ [8], together with subsequent theoretical analysis [19, 21], demonstrated the absence of backscattering for the topological SSs. Meanwhile, the LDOS oscillations of SSs near step edges on Bi₂Te₃ showed a power-law decay with index −1 in a certain energy range [25], compared to −1/2 for the conventional two-dimensional electron system (2DES) [13]. The faster decay of QPI once again indicates the suppression of backscattering in TIs.

Despite the intensive investigation, a complete understanding of QPI on the surface of TIs remain elusive partially due to the warping effect of the Dirac cone [8, 26]. The warping effect of the SSs results from not only the anisotropic surface band dispersion, but also the coupling between the surface and bulk bands. In this Letter, we present a general formalism to account for the complex scattering geometry. We propose the interference patterns are dominated by the extremal points on the constant energy contour (CEC) of 2D electron band. In applying the theory to Bi₂Te₃ with strong warping effect, we show that the decay index varies from −3/2 to −1/2 and finally to −1 as the energy increases. As for TIs with nearly isotropic Dirac cones, such as Bi₂Se₃ [27], the decay index is simply −3/2. Moreover, the theory can be extended to QPI induced by point defects and readily elucidate the missing of ∂ₓ and the deviation of Fermi velocity in Ref. [18]. To confirm the predictions, we have performed STM study on both Bi₂Te₃ and Bi₂Se₃ films and found excellent agreement with theory. In particular, we have been able to obtain the decay index on Bi₂Se₃, whose interference pattern is usually too weak to extract the information.

We start with a general 2D surface band with a single Fermi surface within the surface Brillouin zone (SBZ). Because of elastic scattering, the incoming surface wave with a wave vector kₓ must be scattered into the outgoing one with kᵧ on the same CEC. Assuming a step edge along the y-direction, the kᵧ component of the wave vectors should be conserved in a scattering process, i.e. kᵧ = kᵧ' ≡ kᵧ. The interference between the incoming and outgoing waves gives rise to the standing wave oscillation in the x-direction. The total LDOS is the sum of contributions from all these oscillations from the SSs on a CEC. For a given energy E, we can integrate over kᵧ on the entire CEC and express the LDOS explicitly as

$$\delta \rho(E, x) = \mathbb{R} \left[ \oint_E 2r/(1 + |r|^2) \xi f e^{i(k_x' - k_x) x} dk_y \right],$$

where r is the reflection coefficient. Here the scattering wave function is of the form $\xi e^{i(\mathbf{k}'_x x + ik_y y)}$ ($\xi$ denotes the spin wave function).

A pair of scattering states $\mathbf{k}'_x$ and $\mathbf{k}'_y$ lead to a standing wave with a spatial period of $2\pi/(k_x' - k_x)$. Since the
The effective model describing such oscillations appears at different bias voltages. (b) In Bi$_2$Te$_3$, as energy increases, CEC evolves from hexagon, concave hexagon to concave hexagram, and the pairs of EPs are first at (k$_1$, -k$_1$); then at (k$_2$, -k$_2$), (k$_3$, -k$_3$); then at (k$_4$, -k$_4$); finally at (k$_5$, -k$_5$) and (k$_5$, -k$_5$). Different types of oscillation pattern appear at different bias voltages.

period is different for different value of k$_y$, only the pairs whose periods are stationary with respect to small variation in k$_y$ can make dominant contribution to the LDOS oscillations. We call such pair of points (with identical k$_y$) on CEC the extremal points (EPs) [28]. Other standing waves interfere with other one and cancel at large x. The spatial dependence of LDOS oscillations in Eq. (1) can be evaluated by expanding the relevant quantities around each EP, namely, let k$_y = k_{y0} + \delta k_y$, then $k'_f - k'_s = \Delta k_{x0} + \sum_n \Delta k_{xn} \delta k_y, r = \sum m \eta \delta k_y^m$, and $\xi'_f \xi'_s = \sum_n \chi_n \delta k_y^m$. Here $\Delta k_{x0}$ is the characteristic wave vector depending on the geometry of CEC. To the leading order of k$_y$, the LDOS varies at long distance as

$$\delta \rho(E, x) \approx R \int_{E_{f0}} \int_{E} 2r/(1 + |r|^2) \xi'_f \xi'_s e^{ik'_f - k'_s x} dk_y,$$

$$\approx \sum_{\text{EPs}} \sum_{\text{energies}} [g \chi_m \phi_x x^{-a+b+1}].$$

where a = min(l), b = min(m), c = min(n), and $\phi_x$ is the initial phase of each pair of EPs. $g$ is given by $\sum_{\text{energies}} \sum_{\text{EPs}} dk' k'^{a+b+1} / c e^{i \Delta k_{x0} k'}$. The decay behavior of LDOS in Eq. (2) is valid as long as $x \gg \Delta k_{x0}^{-1}$. The decay index associated with a pair of EPs is given by $(a + b + 1) / c$, which is solely determined by the properties of the scattering wave function around the EPs.

Now we turn to the topological SSs on Bi$_2$Te$_3$ and Bi$_2$Se$_3$ with a single Dirac cone near \( \Gamma \) point in the SBZ on each surface. The effective model describing such topological SSs reads [26, 29]

$$\mathcal{H}(k) = v (\sigma_x k_y - \sigma_y k_x) + \frac{\lambda}{2} (k^3_+ + k^3_-) \sigma_z,$$

where $h = 1, k^3_\pm \equiv k_y \pm ik_z, v$ is the Dirac velocity, $\lambda$ is the warping parameter, and $\sigma_i$ are Pauli matrices acting on spin space. For simplicity, here we ignore the particle-hole asymmetry as it affects the shape of Fermi surface little. The surface band dispersion is

$$\varepsilon_\pm(k_x, k_y) = \pm \sqrt{v^2 k^2 + \lambda^2 k^3 \sin^2(3\theta)},$$

where $\varepsilon_\pm$ denotes respectively the upper and the lower energy bands touching at the Dirac point, and $k \equiv (k_\theta, \theta)$ with $\theta$ as the angle between the wave vector $k$ and $k_y$-axis ($\Gamma-M$). The step edge is always along the close packed $\bar{\Gamma}-\bar{M}$ direction. Defining the characteristic energy $\varepsilon_* \equiv v \sqrt{v/\lambda}$ and length $\sqrt{\lambda/v}$, in Fig. 1 we plot a set of CEC of the upper band in momentum space for Bi$_2$Se$_3$ and Bi$_2$Te$_3$, respectively, in Bi$_2$Se$_3$ $\lambda = 128$ eV$\cdot$A$^3$ and $\varepsilon_* = 0.59$ eV, so the CEC is nearly a circle from 0 to 0.42$\varepsilon_*$ (0.25 eV) [27]. We plot four representative CEC shown in Fig. 1(a). When the Fermi energy increases, the shape of CEC evolves from a circle ($E_1 = 0.31\varepsilon_*$), more hexagon-like ($E_2 = 0.55\varepsilon_*$), hexagon ($E_3 = 0.7\varepsilon_*$) and to concave hexagon ($E_4 = 0.83\varepsilon_*$). In a wide range of energy only a single pair of EPs exists at $(k, -k)$, so the characteristic wave vector is always equal to $2k$ and $c = 2$. In Bi$_2$Te$_3$ the warping effect is stronger with $\lambda = 250$ eV$\cdot$A$^3$ and $\varepsilon_* = 0.23$ eV [30]. As shown in Fig. 1(b), EPs evolve with the energy as follows: Single pair of EPs $(k_1, -k_1)$ at $E_1 = 0.7e\varepsilon*$; Multiple pairs of EPs $(k_1, -k_1), (k_2, -k_2)$ and $(k_3, -k_3)$ at $E_2 = 1.46e\varepsilon*$, $E_3 = 3^1/3 \sqrt{11/9} \varepsilon_* \approx 1.45\varepsilon*$; Two pairs of EPs $(k_2, -k_2)$ and $(k_3, -k_3)$ survive at $E_3 = 1.91\varepsilon*$, as the SSs along the $\bar{G}-\bar{M}$ direction merge into the bulk conduction band; No EPs at all at $E_4 = 2.4e\varepsilon*$, because the SSs separate from bulk one only in the very vicinity along $\bar{G}-\bar{K}$ direction on the Fermi surface as observed in the ARPES experiment [30]. In this case, scattering between states around $(k_1, -k_1)$ and $(k_3, -k_3)$ will be dominant for LDOS oscillations. Thus in Bi$_2$Te$_3$ the characteristic wave vector and the LDOS oscillation period critically depend on the bias. In most cases we have parameter $c = 2$ except for Fermi energy as high as $E_4$ (c = 1).

For the incoming state with wave vector $k^i = (k^i, \theta^i)$ and energy $\varepsilon^i(k^i)$, the inner product of two spin wave functions $\int f_x^i f_x^j e^{i(k^i_x - k^j_x) x} d\phi = \int g \chi_m \phi_x x^{-a+b+1}$.

$$r(\theta^i) = \frac{e^{-i(\theta^i - \theta^f)/2} + \beta(\theta^i) e^{i(\theta^i - \theta^f)/2}}{e^{-i(\theta^i + \theta^f)/2} + \beta(\theta^i) e^{i(\theta^i + \theta^f)/2}};$$

where $\beta(\theta^i) = (\varepsilon^i + k^i d/k^i + \lambda(k^i)^2 \sin(3\theta^i))/((\varepsilon^i + k^i d/k^i + \lambda(k^i)^2 \sin(3\theta^i)), \varepsilon^i, \varepsilon^i = \varepsilon^i + k^i d/k^i$ and $\theta^i = \theta^i + \theta^f$. The decay behavior of LDOS oscillations is

$$\delta \rho(E, x) \approx R \int_{E_{f0}} \int_{E} 2r/(1 + |r|^2) \xi'_f \xi'_s e^{i k'_f - k'_s x} dk_y.$$
FIG. 2: (color online) LDOS oscillations due to the step edge scattering on the surface of Bi2Se3 and Bi2Te3. (a) The STM topograph of the Bi2Se3(111) film, in which a step edge on the surface is observed. (b) Energy dispersion deduced from the standing waves at the step edge. (c)-(j) dI/dV maps and LDOS oscillations for several bias voltages obtained on the upper terrace in film shown in (a). (k)-(n) The LDOS oscillations on Bi2Te3 at 4 bias voltages. The LDOS is averaged over the width of the step and represented by red dots, and the fitting lines are in blue.

To experimentally confirm the above predictions, we analyzed the interference fringes at the step edges on Bi2Se3 and Bi2Te3 surfaces. All experiments were carried out at 4.2 K in an ultrahigh-vacuum low temperature STM system (Unisoku) equipped with molecular beam epitaxy (MBE) for film growth. The stoichiometric films of Bi2Se3 and Bi2Te3 were respectively prepared on graphene and Si(111) substrates by MBE [30, 31]. A typical STM image of Bi2Se3 film with a thickness of 50 quintuple layers is shown in Fig. 2(a). We can clearly see the atomically flat morphology of the film and the step of the height of a quintuple layer. The steps are preferentially oriented along the three close packing (\(\Gamma-\bar{K}\)) directions. The LDOS of electrons at energy eV is measured through the differential tunneling conductance dI/dV maps by STS. The Fermi velocity along \(\Gamma-M\) is deduced to be \(4.8 \times 10^5\) m/s by fitting the linear dispersion curve [Fig. 2(b), in good agreement with the first-principles calculation and the ARPES data [30, 31].]

\[
\delta \rho(x) = \frac{\cos(\Delta \theta f) - \cos(\Delta \theta)}{1 - \cos(\Delta \theta)}
\]

Due to the constraint by TRS, \(\Delta \theta f = -\Delta \theta\), and \(\Delta \theta = 0\) = 0. Thus \(\Delta = 1\) for \((k, -k)\) pair in Bi2Se3 and \((k_1, -k_1)\) pair in Bi2Te3, and \(\Delta = 0\) for other pairs in Bi2Te3.

In short, the algebraical decay index is 3/2 for \((k, -k)\) and \((k_1, -k_1)\) pairs, 1/2 for \((k_2, -k_2)\) and \((k_3, -k_2)\) pairs, and 1 for \((k_4, -k_5)\) and \((k_5, -k_6)\) pairs. Therefore, the LDOS oscillations of the SSs in Bi2Se3 should decay as \(x^{-3/2}\) in a wide range of energy (as long as \(E < 0.85\) eV), much faster than \(x^{-1/2}\) as in 2DES [16]. On Bi2Te3 surfaces, as the bias increases, LDOS oscillations decay first as \(x^{-3/2}\) (\(E < 0.33\) eV), then as a combination of \(x^{-3/2}\) and \(x^{-1/2}\), and finally \(a^{-1}\).

To experimentally confirm the above predictions, we analyzed the interference fringes at the step edges on Bi2Se3 and Bi2Te3 surfaces. All experiments were carried out at 4.2 K in an ultrahigh-vacuum low temperature STM system (Unisoku) equipped with molecular beam epitaxy (MBE) for film growth. The stoichiometric films of Bi2Se3 and Bi2Te3 were respectively prepared on graphene and Si(111) substrates by MBE [30, 31]. A typical STM image of Bi2Se3 film with a thickness of 50 quintuple layers is shown in Fig. 2(a). We can clearly see the atomically flat morphology of the film and the step of the height of a quintuple layer. The steps are preferentially oriented along the three close packing (\(\Gamma-\bar{K}\)) directions. The LDOS of electrons at energy eV is measured through the differential tunneling conductance dI/dV maps by STS. The Fermi velocity along \(\Gamma-M\) is deduced to be \(4.8 \times 10^5\) m/s by fitting the linear dispersion curve [Fig. 2(b), in good agreement with the first-principles calculation and the ARPES data [30, 31].] Figs. 2(c)-(j) exhibit the dI/dV maps on the upper terrace by the step shown in Fig. 2(a) at various bias voltages, and the LDOS as a function of the distance \(x\) from the step. The Dirac point is at about 0.18 eV in STS, so the energy of the surface electron counted from the Dirac point in Fig. 2(j) is 0.38 eV (0.64\(\epsilon_n\)). The best fit to the LDOS oscillations is given by \(\delta \rho(x) \propto \cos(\Delta k_{\epsilon_n} x + \phi) x^{-3/2}\) as predicted. The suppression of backscattering and the circle-like shape of CEC lead to a much faster decay of LDOS in Bi2Se3 than that in 2DES [16].

The case of Bi2Te3 is even more interesting because of the strong warping effect. The LDOS oscillations at 4 different bias voltages on Bi2Te3 film are shown in Figs. 2(k)-(n), and the corresponding dI/dV maps were reported in Ref. [18] already. The Dirac point is estimated to be at \(-0.21\) eV in STS fitting, thus the energy of the surface electron counted from the Dirac point are respectively \(0.20\) eV (0.87\(\epsilon_n\)), \(0.41\) eV (1.78\(\epsilon_n\)), \(0.51\) eV (2.18\(\epsilon_n\)), and \(0.61\) eV (2.60\(\epsilon_n\)). To extract the decay behavior from the LDOS data, for each energy we have compared the fittings with different decay indices. The best fitted LDOS oscillations clearly demonstrate the crossover of the decay from \(x^{-3/2}\) to a combination of \(x^{-3/2}\) and \(x^{-1/2}\), and then to \(x^{-1}\) as the bias voltage increases, which agree well with our theoretical predictions. Besides, as predicted theoretically and
the energy and the scattering is forbidden. The surface states around the extremal points on Fermi surface is generally determined by the scattering between surface states around the extremal points on Fermi surface, either by step edges or by non-magnetic impurities. We have directly observed different standing wave patterns caused by scattering off a step on Bi$_2$Te$_3$ and Bi$_2$Se$_3$ with various warped surface bands, which, together with the decay indices at different bias voltages, clearly demonstrate the 2D Dirac nature of topological surface states.

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FIG. 3: (color online) The scattering geometry due to a non-magnetic impurity on Bi$_2$Te$_3$. When the CEC is a concave hexagram, the high DOS regions are denoted by bold lines along the direction of $\bar{\Gamma}-\bar{K}$. Two kinds of characteristic scattering wave vectors include the forbidden $\vec{q}_I$ (dashed arrow) and the allowed $\vec{q}_2$ (solid arrow) which connects a pair of EPs.

shown in Figs. 2(a)-2(n), the LDOS oscillates with shorter period as increasing the bias. It is noticeable that due to the strong warping effect of the CEC, the decay rate of LDOS oscillations on Bi$_2$Te$_3$ is not always faster than in 2DES.

In the last part, we extend our general formalism for the step edge scattering to the non-magnetic impurity scattering on Bi$_2$Te$_3$. Suppose an incident wave $\psi_i \propto e^{ik_{\parallel} \cdot r}/\sqrt{r}$ is scattered into $\psi_f \propto f(\theta)e^{i(k_{\parallel} - k')\cdot r}/\sqrt{r}$ by the impurity potential of $U = U_0\delta(r)$, then the LDOS oscillates as $\delta \rho \propto f(\theta)e^{i(k_{\parallel} - k')\cdot r}/r$ with $f(\theta)$ denoting the scattering amplitude. The characteristic wave vector of the QPI pattern at large angle of scattering is stationary for certain direction $\hat{r}$. With the concave hexagram Fermi surface of Bi$_2$Te$_3$ as shown in Fig. 3 there exist only two kinds of characteristic wave vectors: $\vec{q}_I$ along $\bar{\Gamma}-\bar{K}$ and $\vec{q}_2$ along $\bar{\Gamma}-\bar{M}$ direction. Obviously, the $\vec{q}_I$ connects a pair of TRS states whose scattering is forbidden. The $\vec{q}_2$ connects a pair of states at EP, which dominates the spatial decay. In such scattering geometry, numerically we find $\vec{q}_2$ vary linearly with the energy and $\vec{q}_2 = 1.5\vec{k}$ where $\vec{k}$ is the length of $\bar{\Gamma}-\bar{K}$. Together with the STM data in Ref. [13] we obtain the Dirac velocity along $\bar{\Gamma}-\bar{K}$ as $v = 4.15 \times 10^5$ m/s, in good agreement with ARPES result ($v = 4.05 \times 10^5$ m/s) [8]. In Bi$_2$Se$_3$, the CEC is circle-like up to 0.25 eV and the characteristic scattering wave vector is always along the diameter. Therefore, we expect the Fourier transformation of LDOS on Bi$_2$Se$_3$ surface is ring-like.

In conclusion, our theoretical and experimental investigations indicate that the LDOS oscillation on the surface of TIs is generally determined by the scattering between surface states around the extremal points on Fermi surface.