Boson Pair Productions

In $e^+ e^-$ Annihilation

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Abstract

We examine the processes $e^+ e^- \rightarrow W^+ W^-$ and $Z^0 Z^0$ in the context of the $SP(6)_L \otimes U(1)_Y$ model. We find that there are significant deviations in the total cross sections $\sigma(s)$ from the standard model results due to the presence of additional gauge bosons $Z'$ and $W'$ in the model. These deviations could be detected at LEP.
1. Introduction

A major target experiment at the CERN $e^+e^-$ collider LEPII ($\sqrt{s} = 200$ GeV) is undoubtedly $W$-boson pair productions. It should thus be possible to examine electroweak (EW) theories quantitatively. The process\(^1\) $e^+e^- \rightarrow W^+W^-$ allows us not only to determine various properties of the $W$ boson, but to measure the trilinear couplings\(^2\) $VW^+W^-$ ($V = Z^0, \gamma$). Another boson pair production process $e^+e^- \rightarrow Z^0Z^0$ is physically less important than the $W$-pair and $Z\gamma$ pair production because of smaller cross section but it has to be studied in order to provide a crucial background for high mass Higgs searches\(^3\). It is still an important test for the standard model (SM) to examine trilinear couplings.

The aim of this paper is to investigate the possibility that there could be significant changes in the behavior of the process $e^+e^- \rightarrow W^+W^-$ and $Z^0Z^0$ in terms of the total cross sections $\sigma(s)$ for $\sqrt{s} \sim 200$ GeV, leading to considerable deviations from the SM result if a new neutral gauge boson ($Z^0$) and a new charged gauge boson ($W'\pm$) such as the ones in the $SP(6)_L \otimes U(1)_Y$ model were present. There are a number of authors who have investigated this problem within models with only one extra neutral gauge boson\(^4\). In fact, two of us (T. K and G. P) presented in a previous paper a similar work on $e^+e^- \rightarrow W^+W^-$ where the contribution from $W'\pm$ was neglected for simplicity in the analysis. Inclusion of the $W'\pm$ turns out to enhance the deviation considerably, as we will show in this paper.

The standard model (SM) has been spectacularly successful in describing the data that are available from recent experiments\(^5\). The agreements between theory and experiments include not just tree-level results, but also radiative corrections. Nevertheless, there are still a few places where room for new physics exists. Further, from the theoretical point of view, there is a consensus that the SM can only be the low energy limit of a more complete theory. Extensions of the SM usually add extra gauge bosons, or extra fermions, or both, to the known particle spectrum. In this paper we consider the $SP(6)_L \otimes U(1)_Y$ family...
model, in which there is a larger flavor gauge group with additional gauge bosons, keeping
the fermion spectrum intact.

The $SP(6)_L \otimes U(1)_Y$ model, proposed some time ago [6], is the simplest extension of the
standard model of three generations that unifies the standard $SU(2)_L$ with the horizontal
gauge group $G_H(= SU(3)_H)$ into an anomaly free, simple, Lie group. In this model,
the six left-handed quarks (or leptons) belong to a 6 of $SP(6)_L$, while the right-handed
fermions are all singlets. It is thus a straightforward generalization of $SU(2)_L$ into $SP(6)_L$,
with the three doublets of $SU(2)_L$ coalescing into a sextet of $SP(6)_L$. Most of the new
gauge bosons are arranged to be heavy ($\geq 10^2$–$10^3$ TeV) so as to avoid sizable FCNC.

$SP(6)_L$ can be naturally broken into $SU(2)_L$ through a chain of symmetry breakings. The
breakdown $SP(6)_L \rightarrow [SU(2)]^3 \rightarrow SU(2)_L$ can be induced by two antisymmetric Higgs
which transform as $(1, 14, 0)$ under $SU(3)_C \otimes SP(6)_L \otimes U(1)_Y$. The standard $SU(2)_L$ is to
be identified with the diagonal $SU(2)$ subgroup of $[SU(2)]^3 = SU(2)_1 \otimes SU(2)_2 \otimes SU(2)_3$,
where $SU(2)_i$ operates on the $i$th generation exclusively. In terms of the $SU(2)_i$ gauge
boson $A_i$, the $SU(2)_L$ gauge bosons are given by $A = \frac{1}{\sqrt{3}}(A_1 + A_2 + A_3)$. Of the other
orthogonal combinations of $A_i$, $A' = \frac{1}{\sqrt{6}}(A_1 + A_2 - 2A_3)$, which exhibits unversality only
among the first two generations, can have a mass scale in the TeV range [7]. The three
gauge bosons $A'$ will be denoted as $Z'$ and $W^{\prime \pm}$.

2. The $SP(6)_L \otimes U(1)_Y$ family model and the cross sections

We now turn to a detailed analysis of the effects of the extra bosons from $SP(6)_L \otimes U(1)_Y$
model. The dominant effects of new heavier gauge boson $Z'(W^{\prime \pm})$ show up in its mixing
with the standard $Z(W^\pm)$ to form the mass eigenstates $Z_{1,2}(W_{1,2})$:

$$
Z_1 = Z \cos \phi_Z + Z' \sin \phi_Z , \quad Z_2 = -Z \sin \phi_Z + Z' \cos \phi_Z ,
$$

$$
W_1 = W \cos \phi_W + W' \sin \phi_W , \quad W_2 = -W \sin \phi_W + W' \cos \phi_W ,
$$

where $Z_1(W_1)$ is identified with the physical $Z(W)$.

With the additional gauge boson $Z'$, the neutral-current Lagrangian is generalized to contain an additional term

$$
L_{NC} = g_Z J^\mu_Z Z_\mu + g_{Z'} J'^\mu_{Z'} Z'_\mu ,
$$

where $g_{Z'} = \sqrt{1 - x^2_W/g_Z} = g/\sqrt{2}$, $x_W = \sin^2 \theta_W$, and $g = e/\sin \theta_W$. The neutral currents $J_Z$ and $J_{Z'}$ are given by

$$
J^\mu_Z = \sum_f \bar{\psi}_f \gamma^\mu \left( g^f_V + g^f_A \gamma_5 \right) \psi_f ,
$$

$$
J'^\mu_{Z'} = \sum_f \bar{\psi}_f \gamma^\mu \left( g'^f_V + g'^f_A \gamma_5 \right) \psi_f ,
$$

where $g^f_V = \frac{1}{2} (I_{3L} - 2x_W q)_f$, $g^f_A = \frac{1}{2} (I_{3L})_f$ as in SM, $g^f_V = g'^f_V = \frac{1}{2} (I_{3L})_f$ for the first two generations and $g^f_V = g'^f_V = - (I_{3L})_f$ for the third. Here $(I_{3L})_f$ and $q_f$ are the third component of weak isospin and electric charge of fermion $f$, respectively. And the neutral-current Lagrangian reads in terms of $Z_{1,2}$

$$
L_{NC} = g_Z \sum_{i=1}^2 \sum_f \bar{\psi}_f \gamma^\mu \left( g^f_{V_i} + g^f_{A_i} \gamma_5 \right) \psi_f Z^\mu_i ,
$$

where $g^f_{V_i}$ and $g^f_{A_i}$ are the vector and axial-vector couplings of fermion $f$ to physical gauge boson $Z_i$, respectively. They are given by

$$
g^f_{V1,A1} = g^f_{V,A} \cos \phi_Z + \frac{g_{Z'}}{g_Z} g^f_{V,A} \sin \phi_Z ,
$$

$$
g^f_{V2,A2} = -g^f_{V,A} \sin \phi_Z + \frac{g_{Z'}}{g_Z} g^f_{V,A} \cos \phi_Z .
$$

Similar analysis can be carried out in the charged sector.
In order to see visible effects of the presence of $Z'$ and $W'$, the mixing angles $\phi_Z$ and $\phi_W$ should not be too small. According to an analysis\cite{ref} using the latest LEP data the present constraint on the mixing angles is $|\phi_Z|, |\phi_Z| \leq 0.01$. This constraint will be used in choosing the mixing angles in the following analysis.

Let’s first consider the process $e^+e^- \rightarrow W_1^+W_1^-$

$$e^+(p_+,\sigma_+) + e^-(p_-,\sigma_-) \rightarrow W_1^+(k_+,\lambda_+) + W_1^-(k_-,\lambda_-)$$

Neglecting the electron mass for high energies, only two initial helicity configurations $\Delta \sigma = \sigma_--\sigma_+ = \pm 1$ are allowed. All polarized differential cross sections are given by

$$\frac{d\sigma_{\Delta\sigma,\lambda_+\lambda_-}}{d \cos \theta} = \frac{x}{16\pi s} |M(\Delta\sigma, \lambda_+\lambda_-)|^2$$

where $\theta$ is the angle between the $e^-$ and the $W_1^-$ momenta, $k_-$ = $(E_-, k \sin \theta, 0, k \cos \theta)$, $x = \frac{k}{\sqrt{s}}$ and

$$M(\Delta\sigma, \lambda_+\lambda_-) = -\frac{e^2}{\sqrt{2}} d_{\Delta\sigma,\lambda_+\lambda_-}^{J_0}(\theta) \left[ \frac{2X_{\Delta\sigma}^{\lambda_+\lambda_-}}{A + 4x \cos \theta} M_l(\Delta\sigma, \lambda_+\lambda_-) - M_s(\Delta\sigma, \lambda_+\lambda_-) \right]$$

where $\Delta \lambda = \lambda_- - \lambda_+$, $J_0 = \max(|\Delta\sigma|, |\Delta\lambda|)$, $d_{\Delta\sigma,\Delta\lambda}^{J_0}(\theta)$ being an ordinary Wigner function, and

$$A = -(1 + 4x^2), \quad X_{\gamma}^{\Delta\sigma} = -1, \quad X_{t}^{\Delta\sigma=-1} = 2B_L^2, \quad X_{t}^{\Delta\sigma=+1} = 0, \quad X_{Z_1}^{\Delta\sigma=-1} = A_{t_e}^1 L_{Z_1}, \quad X_{Z_1}^{\Delta\sigma=+1} = A_{t_e}^1 R_{Z_1}, \quad X_{Z_2}^{\Delta\sigma=-1} = A_{t_e}^2 L_{Z_2}, \quad X_{Z_2}^{\Delta\sigma=+1} = A_{t_e}^2 R_{Z_2}.$$
\[ B_L = \frac{1}{\sqrt{2}} (\cos \phi_W + \frac{1}{\sqrt{2}} \sin \phi_W) , \] (15)

\[ A_{Le}^1 = \frac{1}{2sc} \left[ (-1 + 2s^2) \cos \phi_Z - \frac{c}{\sqrt{2}} \sin \phi_Z \right] , \] (16)

\[ A_{Le}^2 = \frac{1}{2sc} \left[ (1 - 2s^2) \sin \phi_Z - \frac{c}{\sqrt{2}} \cos \phi_Z \right] , \] (17)

\[ A_{Re}^1 = (\frac{s}{c}) \cos \phi_Z , \] (18)

\[ A_{Re}^2 = (\frac{s}{c}) \sin \phi_Z , \] (19)

\[ \Lambda_{Z_1} = \frac{1}{s} (c \cos \phi_Z - \frac{1}{\sqrt{2}} \sin \phi_Z \sin^2 \phi_W) , \] (20)

\[ \Lambda_{Z_2} = -\frac{1}{s} (c \sin \phi_Z + \frac{1}{\sqrt{2}} \cos \phi_Z \sin^2 \phi_W) , \] (20)

with \( s \equiv \sin \theta_W, c \equiv \cos \theta_W \). Similarly, for the process \( e^+e^- \rightarrow Z^0Z_1^0 \),

\[ M(\Delta \sigma, \lambda_+, \lambda_-) = -2\sqrt{2} e^2 d_{\Delta \sigma, \Delta \lambda}(\theta) X^{\Delta \sigma} \left[ M_t(\Delta \sigma, \lambda_+ \lambda_-) \frac{A}{A + 4x \cos \theta} \right] \] (21)

where \( X^{\Delta \sigma = -1} = (A_{Le}^1)^2 \) and \( X^{\Delta \sigma = +1} = (A_{Re}^1)^2 \). For brevity we omit the explicit expressions for \( M_t, M_u, \) and \( M_s \) which are t-, u- and s- channel amplitudes, respectively. Using the above formulas and \( \sin^2 \theta_W = 0.23 \), \( M_{Z_1} = 91.17 \text{ GeV}, M_{W_1} = 80.11 \text{ GeV}, \Gamma_{Z_1} = 2.5 \text{ GeV} \) and the calculated value for \( \Gamma_{Z_2} [9], \) we calculate the total cross sections for \( e^+e^- \rightarrow W_1^+W_1^- \), \( Z^0Z_1^0 \).

Now let us turn to our numerical results. Figures 1 and 2 show \( \sigma(e^+e^- \rightarrow W_1^+W_1^-) \) for four different sets of mixing angles and a fixed \( M_{Z_2} \) in comparison with the SM results.

We see that the effects of the extra gauge bosons is more pronounced for \( \phi_Z = -\phi_W \). The deviations of \( \sigma \) from the SM result at \( \sqrt{s} = 200 \text{ GeV} \) are \( 2.94 - 3.34\% \) for \( |\phi_Z| = |\phi_W| = 0.01 \) and \( M_{Z_2} = 1 \text{ TeV} \). Considering the fact that the deviations were found to be less than 1.1\% for \( |\phi_Z| = 0.05 \) and \( M_{Z_2} = 500 \text{ GeV} \) neglecting \( W' \) contribution[8], it is very interesting to see that there is considerable contribution from the charged sector. Therefore, an accurate measurement (with statistical error \( \leq 1\% \)) of \( \sigma \) at \( \sqrt{s} = 200 \text{ GeV} \) at LEP II will be able to test the \( SP(6)_L \otimes U(1)_Y \) model at the level of \( |\phi_Z| \simeq |\phi_W| \simeq 0.01 \). Figure 3 shows...
\[ \sigma(e^+e^- \rightarrow Z_1^0Z_1^0) \] for \( \phi_Z = \pm 0.01 \) and \( M_{Z_2} = 1 \text{ TeV} \). The deviations from the SM result at \( \sqrt{s} = 200 \text{ GeV} \) are \( \sim 3.0\% \).

3. Summary and Conclusions

We have examined the processes \( e^+e^- \rightarrow W^+W^- \) and \( Z^0Z^0 \) in the context of the \( SP(6)_L \otimes U(1)_Y \) model. Owing to the presence of the additional gauge bosons \( Z' \) and \( W' \) in this model, total cross section \( \sigma(s) \) can be significantly different from that of the standard model. This effect is dependent on the mixing angles between \( Z(W) \) and \( Z'(W') \). For mixing angles at the level of 1\%, these deviations are roughly 3\%, which should be detectable at LEPII. Thus, the production of \( W \) and \( Z \) pairs should provide a sensitive test of possible new physics beyond the SM.

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References

[1] O.P. Sushkov, V.V. Flambaum, and I.B. Kriplovich, Yad. Fiz. 20, 1016 (1975); W. Alles, Ch. Boyer, and A.J. Buras, Nucl. Phys. B119, 125 (1977); D.A. Dicus and K. Kallianpur, Phys. Rev. D32, 35 (1985); M.J. Duncan, G.L. Kane, and W.W. Repko, Phys. Rev. Lett. 55, 773 (1985); K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B274, 1 (1986); K. Hagiwara et al., ibid. B282, 253 (1987).

[2] See, for example, R.W. Brown and K.O. Mikaelian, Phys. Rev. D19, 922 (1979); C.L. Bilchak and J.D. Strouhair, ibid. 30, 1881 (1984); J. Maalampi, D. Schildknecht, and K.H. Schwarzer, Phys. Lett. B166, 361 (1986).

[3] See, for example, G. Barbiellini et al., in Physics at LEP, edited by J. Ellis and R. Peccei (CERN Report No. 86-02, Geneva, Switzerland, 1986), Vol. 2, P. 1.

[4] See, for example, P. Kalyniak and M.K. Sundaresan, Phys. Rev. D35, 75 (1987); T.G. Rizzo, ibid. 36, 713 (1987); P. Comas and A. Mendez, Phys. Lett. B260, 211 (1991); A.A. Bagneid, T.K. Kuo and G.T. Park, Phys. Rev. D44, 2188 (1991); A.A. Pankov and N. Paver, ICTP Report Number IC/91/164 (Unpublished).

[5] For a recent review see, G. Altarelli, in Neutrino 90, edited by J. Panman and K. Winter (North Holland, Amsterdam, 1991).

[6] T.K. Kuo and N. Nakagawa, Phys. Rev. D30, 2011 (1984); Nucl. Phys. B250, 641 (1985); A. Bagneid, T.K. Kuo and N. Nakagawa, Int. J. Mod. Phys. A2, 1351 (1987).

[7] V. Barger et al., Int. J. Mod. Phys. A2, 1327 (1987).

[8] A.A. Bagneid et al. in Ref. [4].
[9] G.T. Park and T.K. Kuo, Purdue University Report Number PURD-TH-92-15 (December 1992).
**Figure Captions**

1. The total cross section $\sigma(e^+e^- \rightarrow W^+_1W^-_1)$ as a function of $\sqrt{s}$ for $SP(6)_L \otimes U(1)_Y$ model in comparison with the SM value for $M_{Z_2} = 1$ TeV. Solid line: SM; long dashed: $\phi_Z = \phi_W = 0.01$; short dashed: $\phi_Z = \phi_W = -0.01$.

2. Same as in Figure 2 except for $\phi_Z = -\phi_W = \pm 0.01$ used instead.

3. The total cross section $\sigma(e^+e^- \rightarrow Z^0_1Z^0_1)$ as a function of $\sqrt{s}$ for $SP(6)_L \otimes U(1)_Y$ model in comparison with the SM value for $M_{Z_2} = 1$ TeV. Solid line: SM; long dashed: $\phi_Z = 0.01$; short dashed: $\phi_Z = -0.01$. 