Magnetic moments of $J^P = \frac{3}{2}^+$ decuplet baryons using the statistical model

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Abstract. A suitable wave function for the baryon decuplet is framed with the inclusion of the sea containing quark-gluon Fock states. Relevant operator formalism is applied to calculate the magnetic moments of $J^P = \frac{3}{2}^+$ baryon decuplet. The statistical model assumes the decomposition of the baryonic state in various quark-gluon Fock states and is used in combination with the detailed balance principle to find the relative probabilities of these Fock states in flavor, spin and color space. The upper limit to the gluon is restricted to three with the possibility of emission of quark-antiquark pairs. We study the importance of strangeness in the sea (scalar, vector and tensor) and its contribution to the magnetic moments. Our approach has confirmed the scalar-tensor sea dominancy over the vector sea. Various modifications in the model are used to check the validity of the statistical approach. The results are matched with the available theoretical data. A good consistency with the experimental data has been achieved for $\Delta^{++}$, $\Delta^+$ and $\Omega^-$.

1 Introduction and motivation

Recently, a new state of matter called “pentaquarks” ($uudc\bar{c}$) has been observed at LHCb bringing a revolution in the study of baryon spectroscopy. The observations from the LHCb was motivated by the prediction made by theoretical approaches. Besides this the prediction of lifetime of the spin-3/2 heavy baryon state at CMS [1, 2] helps us to explore their properties in a better way. A lot of progress has been observed in both theoretical and experimental approaches for the study of hadron properties since octet magnetic moments were predicted by Coleman and Glashow [3] about fifty years ago. These predictions motivated theorists and experimentalists to measure baryon octet magnetic moments [4]. The experimental information about decuplet baryons is limited because they have short lifetimes so till now the experimental data of $\Delta^{++}$, $\Delta^+$, $\Omega^-$ are available [5–8]. The study about the properties of baryon constitute an important role for the investigation of the baryon structure. The advancements in the experimental facilities at CDF [9] etc. have become a subject of motivation to study baryon properties and hence its structure in the non-perturbative regime of Quantum Chromodynamics (QCD). The well-known experiments like EMC (Electron-Muon Collaboration) and the SMC (Spin-Muon Collaboration) [10, 11] studied the static properties of the hadrons.

The magnetic moments of baryon decuplet have been theoretically investigated using different approaches, such as: the simple additive quark model in the non-relativistic limit which calculates the magnetic moments of the baryons as the sum of its constituent quark magnetic moments. Further improvements were made by including effects such as sea quark contributions [12], quark orbital momentum effects [13], $SU(3)$ symmetry breaking effects [14–17]. Sogami and Oh’yamaguchi [18] presented a concept of effective mass to calculate magnetic moments of baryons and later, Bains and Verma [19] used the concept of effective mass and screened charge of quarks to calculate magnetic moments. The subject of magnetic moments is a bit difficult to explain or understand because this phenomenon of baryons is contributed from the magnetic moments of valence quarks as well as from various other complicated effects such as relativistic effects, contributions from pion cloud, confinement effect on quark masses, etc.

Recently, predictions based on a number of theoretical formalisms have been made to calculate the magnetic moment of decuplet baryons. The relativistic quark model (RQM) [20, 21], QCD-based quark model (QCDQM) [22, 23], effective mass scheme (EMS) [24, 25], light cone QCD sum rule (LCQSR) [26], QCD sum rule (QCDSR) [27–29], Skyrme model [30, 31], chiral quark soliton model (CQSM) [32–34], chiral perturbation theory ($\chi$PT) [35,
The principle of detail balance and the statistical approach to the total wave function on flavor and spin space. i.e. $−$ antiquark pair is very less (−0.0004$q_N$) with respect to the magnetic moment of the nucleon.

To calculate the magnetic moments of baryon decuplet particles, we assume the baryon to be comprising of a valence part and a virtual sea consisting of quark-antiquark pairs multiconnected by gluons. In sect. 2, a suitable wave function is framed for the baryon decuplet having color, flavor and spin space. Here, valence $q^2$ and a sea combines in a way to reproduce the desired quantum numbers of the decuplets, i.e. spin 3/2, color singlet and flavor 10. Section 3 shows the application of the magnetic moment operator to the total wave function on flavor and spin space. The principle of detail balance and the statistical approach are applied in combination to find the relative probabilities in spin, flavor and color states in sect. 4. Here, the detailed balance is used to put a constraint (1 − $C_j$) on sea to be taking up the $\pi$ pair due to their heavy masses in terms of respective baryons. Various modifications in the statistical model studied in sect. 4.1. Numerical results are analysed in sect. 5, followed by conclusions in sect. 6.

### 2 Decuplet wave function with a sea component

The structure of the hadron consists of two parts, i.e. a valence part (qqq) and a sea part which consists of quark-antiquark pairs multiconnected by gluons $(g, gq)$ [46–51]. A $q^2$ state in the baryon is in the 1, 8 and 10 color states which means the sea should also be in the corresponding states to form a color singlet baryon. The valence part of the hadronic wave function can be written as

$$\Psi = \Phi(\phi|\chi|\psi)(|\xi\rangle),$$

(1)

where $|\phi\rangle, |\chi\rangle, |\psi\rangle$ and $|\xi\rangle$ denote flavor, spin, color and space $q^2$ wave functions and their contributions make the total wave function antisymmetric in nature. Here, the spatial part $(|\xi\rangle)$ is symmetric under the exchange of any two quarks for the lowest-lying hadrons and therefore the flavor-spin-color part $\Phi(\phi|\chi|\psi)$ should be antisymmetric in nature such that when combined with $(|\xi\rangle)$ it gives antisymmetry of the total wave function.

The sea considered here is in the $S$-wave state with spin (0,1,2) and color $(1,8,10)$ and is assumed to be flavorless. Let $H_{0,1,2}$ and $G_{1,8,10}$ denote spin and color sea wave functions, which satisfy $(H_j|\psi\rangle = \delta_{ij} \langle G_k|G_l\rangle = \delta_{kl})$. In this approach we have assumed a sea to be composed of two gluons or $qar{q}q\bar{q}$ pairs and different possible states for them can be written as

Spin: \text{udd: } 1/2 \otimes 1/2 \otimes 1/2 = 2(1/2) \otimes 3/2, \quad gg: 1 \otimes 1 = 0_8 \oplus 1_4 \oplus 2_8,

$q\bar{q}q\bar{q}$: \text{(1/2 \otimes 1/2) \otimes (1/2 \otimes 1/2) = (0_8 \oplus 1_8) \otimes (0_A \oplus 1_S)} = 2(0_S) \oplus 1_S \oplus 2(1_A) \oplus 2_S; \quad \text{gg: } 1 \otimes 1 = 0_8 \oplus 1_4 \oplus 2_8.$

Color: \text{udd: } 3 \otimes 3 \otimes 3 = 1_A \oplus 8_{MS} \oplus 8_{MA} \oplus 10_S, \quad gg: 8 \otimes 8 = 1_S \oplus 8_S \oplus 8_A \oplus 10_A \oplus 10_U \oplus 27_S,

$q\bar{q}q\bar{q}$: \text{(3 \otimes 3) \otimes (3 \otimes 3) = (1_A \oplus 8_S) \oplus (1_A \oplus 8_S)} = 2(1_S) \oplus 2(8_S) \oplus 2(8_A) \oplus 10_S \oplus 10_U \oplus 27_S.$

Subscripts $S$ and $A$ denote symmetry and antisymmetry on combining the states. We have assumed in our model that gluon and $q\bar{q}$ carry the same quantum numbers. The total antisymmetry of the baryon should be kept in mind while combining the valence and sea parts. In general, the symmetry property arises when $(S + S), (A + A)$ combine while antisymmetry comes into play when the $(S + A)$ combination is formed.

So, the possible combinations of valence $q^2$ and sea wave functions which can yield spin 3/2, flavor decuplet and color singlet state thereby maintaining the antisymmetrization of the total baryonic wave function are

$$\Phi^{(3/2)}_1 H_0 G_1, \quad \Phi^{(3/2)}_1 H_1 G_1, \quad \Phi^{(3/2)}_1 H_1 G_8, \quad \Phi^{(3/2)}_1 H_2 G_1, \quad \Phi^{(3/2)}_1 H_2 G_8.$$

(2)

The total flavor-spin-color wave function of a spin-up baryon decuplet consisting of three valence quarks and a sea component can be written as

$$|\Phi^{(1)}_{3/2}\rangle = \frac{1}{N}|a_0 \Phi^{(3/2)}_1 H_0 G_1 + b_1 \Phi^{(3/2)}_1 H_1 G_1 + b_2 \Phi^{(3/2)}_1 H_2 G_1 + b_3 |H_1 G_8 + d_1 |H_2 G_8 | + d_2 |H_2 G_8 | \rangle,$$

(3)

$$N^2 = a^2_0 + b^2_1 + b^2_2 + d^2_1 + d^2_2,$$

where $N$ is the normalization constant. The first term in eq. (3) is obtained by combining the $q^2$ wave function with spin 0 (scalar sea) and the next two terms are obtained by coupling $q^2$ with spin 1 (vector sea) such that

$$\Phi^{(3/2)}_1 H_1 \equiv \Phi_{b_1}^{(3/2)} \Phi_1^{A}, \quad \Phi^{(1/2)}_1 H_1 \equiv \Phi_{b_0}^{(1/2)} \Phi_1^{MS},$$

(5)

(6)

where

$$\Phi_{b_1}^{(3/2)} = \sqrt{\frac{3}{5}} H_{1,0} F_{S}^{(3/2)} - \sqrt{\frac{2}{5}} H_{1,1} F_{S}^{(1/2)}, \quad \Phi_{b_0}^{(1/2)} = \sqrt{\frac{3}{5}} H_{1,0} F_{S}^{(3/2)} - \sqrt{\frac{2}{5}} H_{1,1} F_{S}^{(1/2)},$$

(7)

(8)
The $q^3$ wave functions in eqs. (5), (6) for a flavor decuplet baryon can be written as
\[ \Phi^{(3/2)}_i \equiv \Phi(10, 3/2, 1) = F_S \Phi^A_i, \]
where
\[ F_S = \phi^A \chi^A \]
and
\[ \Phi^{(1/2)}_i \equiv \Phi(10, 1/2, 1) = F_{MS} \Phi^A_i, \]
\[ \Phi^{(1/2)}_s \equiv \Phi(10, 1/2, 8) = F_A \phi^S, \]
where
\[ F_{MS} = \phi^A \chi^{MS}, \]
\[ F_A = \frac{1}{\sqrt{2}}(\phi^A \chi^S - \phi^S \chi^A). \]

Here, indices $S$ and $A$ denote total symmetry and antisymmetry and $\lambda$, $MS$ denotes mixed symmetry under quark permutations $q_1 \leftrightarrow q_2$. $\Phi_i^{(3/2)}$ denotes a function with spin 3/2, color singlet and 10 represents the flavor part. This function can be written as a combination of $F_S$ (denotes flavor and spin) and $\Phi^A_i$ (represents the color of baryons and is antisymmetric). For $F_S$ to be symmetric, $\phi$ and $\psi$ should be symmetric in nature. Similarly, other functions like $\Phi_i^{(1/2)}$ and $\Phi_s^{(1/2)}$ denotes a function with spin 1/2 and flavor 10 with color singlet and octet, respectively. Each wave function is a combination of a symmetric and an antisymmetric term such that the total wave function becomes antisymmetric in nature.

The final two terms are the result of the coupling with spin 2 (tensor sea). Their expressions can be written as
\[ \left( \Phi^{(3/2)}_i \otimes H_2 \right)^i \equiv \phi^{(3/2)}_{d1} \phi^A_i, \]
\[ \left( \Phi^{(1/2)}_s \otimes H_2 \right)^i \equiv \phi^{(1/2)}_{d8} \phi^S_i, \]
where
\[ \phi^{(3/2)}_{d1} = \sqrt{\frac{5}{2}} H_{2,0} F_S^{(3/2)} - \sqrt{\frac{2}{5}} H_{2,1} F_S^{(1/2)}, \]
\[ \phi^{(1/2)}_{d8} = \sqrt{\frac{5}{2}} H_{2,0} F_A^{(1/2)} - \sqrt{\frac{2}{5}} H_{2,1} F_A^{(1/2)}. \]

Wave functions $\phi^{(3/2)}_{d1}$, $\phi^{(1/2)}_{d8}$, $\phi^{(3/2)}_{d8}$, $\phi^{(1/2)}_{d8}$ are written by taking the coupling between the spin of the sea part and the flavor part of the $q^3$ wave function. The coefficients associated with each term contain the information about magnetic moments, spin distribution among valence quarks and need to be determined statistically. The parameter $a_0$ comes from a spin 3/2 of a $q^3$ state coupled to the spin 0 (scalar) of the sea, $b_1, b_8$ comes when spins 3/2 and 1/2 of the $q^3$ state are coupled to the spin 1 (vector) of the sea and $d_1, d_8$ corresponds to the coupling of spin 3/2 to spin 2 (tensor) of the sea. The idea of different coefficients for each baryon function of the baryon decuplet comes from the fact that each baryon has different mass and quark content.

### 3 Magnetic moments

Magnetic moments are a property of hadrons observed at low energies and long distances. Magnetic moments are composed of all the constituents of the baryon (valence+sea) by experiencing the same magnetic field. Thus, for baryons at the ground state, the magnetic moments are a vector sum of quark magnetic moments,
\[ \mu_{baryon} = \sum_{i=1,2,3} \mu_i \sigma_i, \]
where $\sigma_i$ is the Pauli matrix representing the spin term of the $i$-th quark and $\mu_i$ represents the magnitude of quark magnetic moments and therefore the values of magnetic moments are different for different baryons.

\[ \mu_{baryon} = \mu = \frac{e_i}{2m_i} \]
for $i = u, d, s$ and $e_i$ represents the quark charge. The present work shows the calculation of magnetic moments of $J^P = \frac{3}{2}^+$ by applying the magnetic moment operator ($\hat{\mathcal{O}} = \mu_i \sigma_i$) which depends on the flavor and spin of the $i$-th quark that can be applied on the baryon wave function in the following way (see table 1):
\[ \langle \Phi^{(3/2)}_i | \hat{\mathcal{O}} | \Phi^{(3/2)}_i \rangle = \frac{1}{\sqrt{2}} \left[ \phi^{(3/2)}_{d1} \phi^A_i \right] \]
\[ \langle \Phi^{(1/2)}_i | \hat{\mathcal{O}} | \Phi^{(3/2)}_i \rangle = \frac{1}{\sqrt{2}} \left[ \phi^{(1/2)}_{d8} \phi^S_i \right]. \]

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\[ \langle \Phi^{(1/2)}_i | \hat{\mathcal{O}} | \Phi^{(3/2)}_i \rangle = \frac{1}{\sqrt{2}} \left[ \phi^{(1/2)}_{d8} \phi^S_i \right]. \]

Wave functions $\phi^{(3/2)}_{d1}$, $\phi^{(1/2)}_{d8}$, $\phi^{(3/2)}_{d8}$, $\phi^{(1/2)}_{d8}$ are written by taking the coupling between the spin of the sea part and the flavor part of the $q^3$ wave function. The coefficients associated with each term contain the information about magnetic moments, spin distribution among valence quarks and need to be determined statistically. The parameter $a_0$ comes from a spin 3/2 of a $q^3$ state coupled to the spin 0 (scalar) of the sea, $b_1, b_8$ comes when spins 3/2 and 1/2 of the $q^3$ state are coupled to the spin 1 (vector) of the sea and $d_1, d_8$ corresponds to the coupling of spin 3/2 to spin 2 (tensor) of the sea. The idea of different coefficients for each baryon function of the baryon decuplet comes from the fact that each baryon has different mass and quark content.
Table 1. Expressions obtained after applying the magnetic moment operator to the baryon decuplet are shown.

| Baryon | $\langle \Phi_{3/2}^3 | \hat{O} | \Phi_{3/2}^3 \rangle N^2$ |
|--------|-------------------------------------------------|
| $\Delta^{++}$ | $a_0^2(15\mu_a) + b_2^2(11\mu_a) + b_1^2(7\mu_a) + d_2^2(3\mu_a) + d_1^2(\frac{7}{2}\mu_a)$ |
| $\Delta^+$ | $a_0^2(30\mu_a + 15\mu_d + b_2^2(22\mu_d + 11\mu_a) + b_1^2(22\mu_d + 11\mu_a) + d_2^2(8\mu_d + \mu_a) + d_1^2(4\mu_a + \frac{7}{2}\mu_d)$ |
| $\Delta^0$ | $a_0^2(30\mu_d + 15\mu_a + b_2^2(22\mu_d + 11\mu_a) + b_1^2(22\mu_d + 11\mu_a) + d_2^2(8\mu_d + \mu_a) + d_1^2(4\mu_a + \frac{7}{2}\mu_d)$ |
| $\Delta^-$ | $a_0^2(15\mu_d + b_2^2(11\mu_d) + b_1^2(7\mu_d) + d_2^2(3\mu_d) + d_1^2(\frac{7}{2}\mu_d)$ |
| $\Sigma^{++}$ | $a_0^2(5\mu_a + 15\mu_d + b_2^2(22\mu_d + 11\mu_a) + b_1^2(22\mu_d + 11\mu_a) + d_2^2(8\mu_d + \mu_a) + d_1^2(4\mu_a + \frac{7}{2}\mu_d)$ |
| $\Sigma^+$ | $a_0^2(30\mu_a + 15\mu_d + b_2^2(22\mu_d + 11\mu_a) + b_1^2(22\mu_d + 11\mu_a) + d_2^2(8\mu_d + \mu_a) + d_1^2(4\mu_a + \frac{7}{2}\mu_d)$ |
| $\Sigma^0$ | $a_0^2(30\mu_a + 15\mu_d + b_2^2(22\mu_d + 11\mu_a) + b_1^2(22\mu_d + 11\mu_a) + d_2^2(8\mu_d + \mu_a) + d_1^2(4\mu_a + \frac{7}{2}\mu_d)$ |
| $\Sigma^-$ | $a_0^2(30\mu_d + 15\mu_a + b_2^2(22\mu_d + 11\mu_a) + b_1^2(22\mu_d + 11\mu_a) + d_2^2(8\mu_d + \mu_a) + d_1^2(4\mu_a + \frac{7}{2}\mu_d)$ |
| $\Omega^+$ | $a_0^2(15\mu_d) + b_2^2(11\mu_d) + b_1^2(7\mu_d) + d_2^2(3\mu_d) + d_1^2(\frac{7}{2}\mu_d)$ |

4 Principle of detailed balance and statistical model

The principle of the detailed balance proposed by Zhang et al. [52–54] calculates the probability of Fock states present inside hadrons. The detailed balance principle demands equality between arriving from one substate and leaving it. Hadron is treated to be consisting of a complete set of quark gluon Fock states and can be expressed in expanded form as

$$\langle B \rangle = \sum_{i,j,l,k} C_{i,j,l,k}(q), (i, j, l, k),$$  \hspace{1cm} (24)

where $q$ represents the valence quarks of the baryon, $i$ is the number of quark-antiquark $u\bar{s}$ pairs, $j$ is the number of quark-antiquark $d\bar{u}$ pairs, $l$ is the number of $s\bar{s}$ pairs and $k$ is the number of gluons in sea. The probability to find a quark-gluon Fock state is

$$\rho_{i,j,l,k} = |C_{i,j,l,k}|^2,$$  \hspace{1cm} (25)

and $\rho_{i,j,l,k}$ satisfies the normalization condition

$$\sum_{i,j,l,k} \rho_{i,j,l,k} = 1.$$  \hspace{1cm} (26)

The assumption of the detailed balance principle is that every two subensembles balance with each other in a way

$$\rho_{i,j,l,k}(q), (i, j, l, k), \langle \text{b}al\text{a}nce \rangle = \rho_{i', j', l', k'}(q), (i', j', l', k').$$  \hspace{1cm} (27)

The transfer between two Fock states has two ways: go-out rate and come-in rate which are proportional to the number of partons that may split and the number of partons recombining, respectively. The calculation of probability distributions includes various subprocesses like $g \leftrightarrow q\bar{g}$, $g \leftrightarrow gg$, $q \leftrightarrow gq$. The detailed balance principle is applied to $\Sigma^{0+}$ to calculate probabilities and can be written as

1) When $q \leftrightarrow gg$ is considered the general expression of probability can be written as

$$|uds, i, j, l, k - 1\rangle \rightleftharpoons |uds, i, j, l, k\rangle,$$  \hspace{1cm} (28)

$$\frac{\rho_{i,j,l,k}}{\rho_{i,j,l,k-1}} = \frac{1}{k}.$$  \hspace{1cm} (29)

Here $\rho_{i,j,l,k-1}$ represents the probability ratios of two processes.

2) When both the processes $g \leftrightarrow gg$ and $q \leftrightarrow gg$ are included

$$|uds, i, j, l, k - 1\rangle \rightleftharpoons |uds, i, j, l, k\rangle,$$  \hspace{1cm} (30)

$$\rho_{i,j,l,k} = \frac{3 + 2i + 2j + 2l + k - 1}{3 + 2i + 2j + 2l + k - 1}.$$  \hspace{1cm} (31)

3) When $g \leftrightarrow g\bar{q}$ is considered the processes $g \leftrightarrow u\bar{q}$, $g \leftrightarrow d\bar{q}$, $g \leftrightarrow s\bar{s}$ are involved here:

$$\rho_{i,j,l,k} = \frac{1}{k(k-1)},$$  \hspace{1cm} (32)

The details of the above calculations can be found in ref. [55]. The subprocess $g \leftrightarrow s\bar{s}$ is active only when it satisfies the condition that gluons should have energy larger than at least two times the mass of the strange quark because the strange quark has non-negligible mass for gluons to undergo the process. The subprocess $g \leftrightarrow s\bar{s}$ is restricted by applying the constraint defined as $k(1 - C_1)n^{-1}$ [54] which is introduced from gluon free energy distribution. Here, $n$ is the number of partons present in the Fock state, i.e. $n = 3 + 2i + 2j + l + 2k$. So, taking $(1 - C_1)n^{-1}$ as the suppressing factor for generating
Table 2. Expressions for probabilities in terms of \( \rho_{0,0,0,0} \) for the \( J^P = \frac{3}{2}^+ \) decuplet.

| Baryon | \( \rho_{i,j,k,l} \) |
|--------|------------------|
| \( \Delta^+ \) | \( \frac{1}{i!j!(j+1)!(l+k)!(l+k+1)!} \) |
| \( \Delta^0 \) | \( \frac{1}{i!(j+1)!(l+k)!(l+k+1)!} \) |
| \( \Delta^- \) | \( \frac{1}{i!(j+1)!(l+k+1)!(l+k+2)!} \) |
| \( \Sigma^+ \) | \( \frac{1}{i!(j+1)!(l+k)!} \) |
| \( \Sigma^0 \) | \( \frac{1}{i!(l+k)!} \) |
| \( \Sigma^- \) | \( \frac{1}{i!(l+1)!(l+k)!} \) |
| \( \Omega^− \) | \( \frac{1}{i!(l+1)!(l+k)!} \) |

\( s\bar{s} \) pairs from a gluon and by using the detailed balance model, the strange quark contribution to the baryon can be calculated. The value of the number of \( s\bar{s} \) pairs has been restricted to two due to their large mass and to the limited free energy of the gluon undergoing the subprocess \( g \leftrightarrow s\bar{s} \). The detailed balance principle when applied to different baryons gives different results because the sea content will split and recombine with the quark content which is different for every baryon.

The expressions of probabilities in terms of \( \rho_{0,0,0,0} \) for \( \Sigma^{*0} \), are

\[ \frac{\rho_{i,j,k,l}}{\rho_{0,0,0,0}} = \frac{1}{i!(i+1)!(j+1)!(l+k)!(l+k+1)!}. \]  \( (33) \)

Similar expressions of probabilities for other decuplet particles can be written in terms of \( \rho_{0,0,0,0} \) and are shown in table 2.

The normalization condition \( \sum_{i,j,k,l} \rho_{i,j,k,l} = 1 \) gives the individual probabilities of baryon decuplets. The entire list of probabilities of various Fock states, i.e. \( \rho_{i,j,k,l} \)'s, is shown in table 3 for other decuplet members as well.

From table 3, it is well observed that lesser contributions from higher mass Fock states have suppressed the whole Fock states with higher mass and \( SU(2) \) symmetry is well obeyed by Fock states with \( u \) and \( d \) quarks. The number of \( s\bar{s} \) pairs have been limited to two because of the heavy strange quark mass and limited free energy of the gluon as strange-antistrange pairs are generated from the subprocess \( g \leftrightarrow s\bar{s} \).

The statistical model [56] is used in our formalism to calculate magnetic moments of decuplet members by assuming hadrons as an ensemble of quark-gluon Fock states. We statistically decompose quark-gluon Fock states \( |q^3, i, j, l, k \rangle \) of a baryon in a set of states in which the valence part and sea have definite spin and color quantum number. The statistical model earlier applied to proton [56] assumes the baryon to be comprised of valence quarks plus a virtual sea consisting of strange/non-strange quark antiquark pairs multi-connected through gluons in the form of subprocesses like \( g \leftrightarrow q\bar{q}, g \leftrightarrow gg, q \leftrightarrow gg \).

The five coefficients \((a_0, b_1, b_8, d_1, d_8)\) in eq. (3) are to be determined statistically. The wave function in eq. (3) can also be written in the form of \( \Phi_{val}\Phi_{sea} \) and the unknown parameters \((a_0, b_1, b_8, d_1, d_8)\) by a factor \( \sum_{\mu,\nu} n_{\mu,\nu} c_{\nu,\mu} \) such that the total wave function becomes \( \Phi^+_f = \sum_{\mu,\nu} (n_{\mu,\nu} c_{\nu,\mu}) \Phi_{val}\Phi_{sea} \) where \( \mu \) and \( \nu \) have values 0, 1, 2 and 1, 8, 10, respectively. All \( n_{\mu,\nu} \)'s are calculated from multiplicities of each Fock state in spin and color space. These multiplicities are expressed in the form of \( \rho_{p,q} \) where the relative probability for the core part should have spin \( p \) and the sea should have spin \( q \) such that the resultant should come out as 3/2. Similar probabilities could be calculated for the color space which yields the color singlet state. The calculation of these probabilities helps to find a common factor \( (\zeta^n) \) for every combination of valence and sea which is multiplied by the multiplicity factor \( (n) \) for each Fock state. The common parameter \( \zeta^n \) can be calculated from table 2 of various Fock states derived from the principle of detailed balance. The procedure to compute \( \zeta^n \) is shown in later calculations. Each unknown parameter in the equation of the wave function will have a particular value of \( \sum_{\mu,\nu} n_{\mu,\nu} c_{\nu,\mu} \) depending on the Fock state [57]:

\[
\begin{align*}
a_0^2 &= (n_{01} c_{sea})(gg) + (n_{01} c_{sea})(u\bar{q}g) + (n_{01} c_{sea})(d\bar{q}g) \\
&+ (n_{01} c_{sea})(s\bar{q}g) + \ldots .
\end{align*}
\]  \( (34) \)
\[ b_t^2 = (n_{11}c_{sea})(gg) + (n_{11}c_{sea})(u\pi)_g + (n_{11}c_{sea})(d\bar{g}_g) + (n_{11}c_{sea})(s\pi)_g + \ldots, \]  
\[ d_t^2 = (n_{21}c_{sea})(gg) + (n_{21}c_{sea})(u\pi)_g + (n_{21}c_{sea})(d\bar{g}_g) + (n_{21}c_{sea})(s\pi)_g + \ldots \]  

Combinations for other unknown parameters can be written in a similar way. Though the set of different Fock states \((gg), (u\pi)_g, (d\bar{g}_g)\) etc. is the same for all baryon decuplet members but the probability distribution is different for different baryons due to the mass inherited from the flavor leading to different values of unknown parameters. These calculations will give the value of a factor “nc” for Fock states which has a significant role in determining the magnetic moments (see table 8). The subscripts \(S\) and \(A\) denote symmetry and antisymmetry conditions. To find the ratio of probabilities in spin and color space, various decompositions are carried out in the following way.

Consider the decomposition of state \(|\rho^3, 0, 0, 0, 2\rangle\) or \(|gg\rangle\) sea. Different cases of probability ratios for spin of valence and sea can be written as

\begin{align*}
\rho_{3/2, 1}^1 &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1, \\
\rho_{3/2, 2}^1 &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1, \\
\rho_{3/2, 0}^1 &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1, \\
\rho_{3/2, 2}^0 &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1, \\
\rho_{3/2, 1}^0 &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2, \\
\rho_{3/2, 1}^{-1} &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2, \\
\rho_{3/2, 0}^{-1} &= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2,
\end{align*}

where all the terms on the r.h.s. are multiplicities expressed in the form of \(\rho_{p, q}\) where the valence quark part has spin \(p\) and this “sea” carries spin \(q\) such that the resultant spin is \(3/2\). The multiplicities in spin and color states are calculated for all Fock states in valence and sea and are defined in the form of suitable ratios. The first term in the numerator, or the denominator in the r.h.s., is the relative probability for the valence quarks to have spin \(p\), the second term is for the Fock state of gluons to have spin \(q\) and the third term is the same for \(p\) and \(q\) to have resultant \(3/2\). Similar probability ratios can be calculated for color spaces finally giving a color singlet baryon and can be written as

\[ \frac{\rho_{3/2, 1}}{\rho_{8, 8, 8}} = \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1 \quad \text{and} \quad \frac{\rho_{1, 1}}{\rho_{8, 8, 8}} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1, \]  
\[ \frac{\rho_{1, 1}}{\rho_{10, 10}} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1. \]  

These are the probability distributions to find the valence quarks in spin \(3/2\) and color singlet states with sea. To compute the common parameter “c” the product of probabilities in spin and color spaces can be written in terms of common factor “c” as

\[ \rho_{3/2, 1}^1[\rho_{8, 8, 8}, \rho_{10, 10}] = c(2, 1), \]  
\[ \rho_{3/2, 2}^1[\rho_{1, 1}, \rho_{8, 8, 8}] = c(1, 2), \]  
\[ \rho_{3/2, 0}^1[\rho_{1, 1}, \rho_{8, 8, 8}] = c(1, 2), \]  
\[ \rho_{3/2, 1}^{-1}[\rho_{8, 8, 8}] = 2c, \]  
\[ \rho_{3/2, 2}^{-1}[\rho_{1, 1}, \rho_{8, 8, 8}] = c(1, 2). \]

The values present on the r.h.s. of the above equations are the multiplicities for a particular Fock state. There is no contribution from \(H_0G_{\Sigma}\), \(H_1G_1\) and \(H_2G_{\Pi}\) as they form an antisymmetric sea under the exchange of two gluons which makes these wave functions antisymmetric and therefore unacceptable for a bosonic system \((gg)\). Equating the sum of all these partial probabilities to the value of probabilities \(\rho_{1, 1, k}\), i.e., \(\rho_{0002}, \rho_{2000}, \rho_{0200}\) taken from table 3 for \(\Sigma^0\), gives the unknown parameter \(c\) as

\[ 2c + c + 2c + c + 2c + 2c + c + 2c = 14c = 0.00599579 \]  
\[ \Rightarrow c_{0002} = 0.000428271 \]

and the other values can be computed and written as \(c_{2000} = 0.000919879\), \(c_{0200} = 0.000919879\). The value 14c is computed from eqs. (42)–(46). Similar decompositions can be done for other Fock states as well, i.e. by taking the sea up to three gluons as shown in tables 4, 5.

### 4.1 Modifications in statistical model

Statistical model and detailed balance principle are used in combination to study properties like magnetic moment and \(\bar{\tau} - \tau\) asymmetry, scalar-tensor sea dominancy and strange quark importance to the magnetic moment of decuplets. A significant role is played by the confining forces among the constituents in determining the properties of the decuplet and this leads to check the consistency of the statistical model in the form of certain modifications. There are three modifications in specific that we have focussed upon, and they are discussed below.

Model D assumes that a sea containing a large number of gluons has relatively smaller probabilities and hence their multiplicities have been suppressed over the rest of valence particles with limited quarks. Model D is assumed to be a special case of Model C. In order to check the predicting power of the statistical approach, we have modified the relative probabilities by suppressing the contribution of states coming from higher multiplicities. Here relative probabilities are divided with their respective spin and color multiplicities to achieve the suppression. This modification is based on the phenomenological ground [57] stating that the higher the multiplicities, the lower will be the associated probabilities. The decomposition of Fock states with this new input is shown as follows.
Table 4. Computed probability ratios for various Fock states in spin and color space.

| Probability Ratio → | $\rho_{\uparrow,1}^{1/2}$ | $\rho_{\uparrow,2}^{1/2}$ | $\rho_{\uparrow,0}^{1/2}$ | $\rho_{\uparrow,1}^{3/2}$ | $\rho_{\uparrow,0}^{1/2}$ | $\rho_{\uparrow,1}^{3/2}$ | $\rho_{\uparrow,1}^{5/2}$ | $\rho_{\uparrow,0}^{1/2}$ | $\rho_{\uparrow,1}^{3/2}$ |
|---------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| States ↓            | S, A                     | S, A                     | S, A                     | S, A                     | S, A                     | S, A                     | S, A                     | S, A                     | S, A                     |
| $|gg\rangle$        | 1                         | 1                         | 1                         | 2                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         |
| $|u\pi\eta\rangle$  | 1                         | 1                         | 1                         | 2                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         |
| $|d\pi\eta\rangle$  | 1                         | 1                         | 1                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         | 1                         |
| $|s\pi\eta\rangle$  | 1                         | 1                         | 1                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         | 1                         |
| $|u\pi\eta\pi\rangle$ | 1                       | 1                         | 1                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         | 1                         |
| $|d\pi\eta\pi\rangle$ | 1                       | 1                         | 1                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         | 1                         |
| $|u\pi\eta\pi\pi\rangle$ | 1                     | 1                         | 1                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         | 1                         |
| $|u\pi\eta\pi\pi\rangle$ | 1                     | 1                         | 1                         | 2                         | 2                         | $\frac{1}{2}$             | 1                         | 1                         | 1                         |

| $|\pi\eta\rangle$, $|\pi\eta\pi\rangle$, $|\pi\pi\rangle$, $|\pi\pi\rangle$ sea, symmetry consideration is not needed: |

$$
\rho_{\uparrow,1}^{1/2} = c(1, 4, 1) = d \left( \frac{1}{2}, \frac{1}{48}, \frac{1}{300} \right),
$$

$$
\rho_{\uparrow,2}^{1/2} = c(1, 4, 1) = d \left( \frac{1}{2}, \frac{1}{80}, \frac{1}{500} \right),
$$

$$
\rho_{\uparrow,0}^{1/2} = c(1, 4, 1) = d \left( \frac{1}{2}, \frac{1}{32}, \frac{1}{200} \right),
$$

$$
\rho_{\uparrow,1}^{3/2} = 4c = \frac{d}{96},
$$

$$
\rho_{\uparrow,2}^{3/2} = c(1, 4, 1) = d \left( \frac{1}{10}, \frac{1}{160}, \frac{1}{1000} \right).
$$

Summing and equating all the partial probabilities to $\rho_{1001}$, $\rho_{1001}$, $\rho_{0111}$, $\rho_{1010}$, $\rho_{1010}$, $\rho_{0110}$ we get values of $d$ as 0.030115253, 0.030115253, 0.026488907, 0.031513051, 0.031513051, 0.031513051 respectively. All the calculations of the relative probability $\rho_{p,q}$ for spin and color spaces with the possibilities arising from gluons are shown in tables 4, 5. Similar numbers can be obtained for other Fock states as well. Details of the calculations are given in ref. [57].

The detailed balance principle is applied to put a limit on the number of $\pi\eta$ pairs in the sea (due to the fixed mass of decuplets), in terms of the constraint as $(1 - C_l)^{n-1}$, where $C_l = \frac{2M_B - 2M_{\pi\pi}}{M_B}$ (mass of the baryon, $M_B$=mass of the strange quark, $n$ is the total number of partons). This kind of constraint is proven to be helpful to understand the strange behavior of the sea in various decuplets. It has been noticed in general that the strange sea dominates over non-strange sea quarks for strange baryon particles ($\Sigma^+, \Sigma^{0}, \Sigma^{-}, \Xi^{0}, \Xi^{-}, \Omega^{-}$) as compared to non-strange decuplet members ($\Delta^+, \Delta^+, \Delta^0, \Delta^-$).
To appreciate the importance of the sea with spin, modifications in the model are done by choosing the sea to be contributing through scalar, vector or tensor coefficients. Here the sea with spin 0, 1, 2 is called scalar, vector and tensor sea, respectively. These coefficients are directly related to probabilities of quark-ghon Fock states in spin color and flavor space. Here the sea is found to be dynamic for the scalar and tensor part unlike baryon octets where the tensor appears to be less dominating due to quark spin flip processes [58]. In specific, magnetic moments get influenced by suppressing any of the parameter/coefficient in the total wave function.

The importance of $SU(3)$ symmetry and its breaking has been discussed for baryon octets [58]. Due to the limited experimental information on $SU(3)$ symmetry breaking in decuplets, we have restricted ourselves to analyse the $\bar{d} - \bar{\pi}$ asymmetry contributing to the magnetic moment of decuplets. These data may be useful for experimentalists to investigate further. Due to the variation in the values observed experimentally. Also, Zhang et al. have used the principle of detailed balance to calculate flavor asymmetry and predicted the value of 0.118 [64] matching well with the experimental value of 0.124 [63].

5 Results and discussion

Magnetic moments of baryon decuplets are calculated in the statistical model, where the baryonic structure is considered to be consisting of valence quarks and a sea limited to a few number of quark-antiquark pairs multiconnected non-perturbatively through gluons. The statistical approach and detailed balance principle are used in combination to compute the coefficients in eq. (3) leading to a scalar, a vector and a tensor contribution to the magnetic moments of the decuplet. A relevant operator is applied on the wave function with $(a_0, b_1, b_8, d_1, d_8)$ coefficients to calculate their magnetic moments. Further, to check the validity of our approach few modifications were made. Magnetic moments were calculated in two approaches i.e. Model C and Model D, where Model C aims at finding relative probabilities of the Fock states in color, spin and flavor space, whereas Model D finds the probabilities of Fock states by suppressing the contribution of states with higher multiplicities. The magnetic moments obtained from the C Model seem to deviate upto 5% from the simple quark model and for the D Model upto 25% when compared with SQM.

Our aim is to see the importance of the sea in the relative probabilities of the Fock states having strange and non-strange quark contents. Here we study strangeness in the sea, its scalar, vector and tensor contributions to magnetic moments individually and the importance of two approaches, i.e. C and D Models. We put the mass constraint to the decuplet to accommodate strange quark-antiquark pairs as condensates in the sea. With the various above-mentioned assumptions, relative probabilities have been computed in flavor, spin and color space using a statistical approach. Here the larger are the multiplicities of a group of particles, the larger will be the probability of their interaction with the rest of the particles.

With these studies, in general, we conclude that strangeness in sea gives information about the internal structure of the baryon. In particular the significance of the strange sea over non-strange sea, more specifically for strange decuplets, is important. Strange quark sea contributions are less in non-strange decuplets. Here, the importance of the constraint $(1 - C_l)^{n-1}$, where $C_l = \frac{n_0}{M_0 - 2M_l}$, can be seen with the data shown in the following table, which clearly distinguishes between doubly strange baryon and single strange baryon to accommodate $s\bar{s}$ pairs in the sea. The same trend can be observed for similar particles with higher $s\bar{s}$ pairs in the sea.

| Fock state | Value of the constraint $(1 - C_l)^{n-1}$ |
|------------|---------------------------------------|
| $uds\bar{u}\bar{t}s\bar{g}$ | 0.271 |
| $uss\bar{u}\bar{t}s\bar{g}$ | 0.317 |
| $uds\bar{s}s\bar{g}$ | 0.2504 |
| $uss\bar{s}s\bar{g}$ | 0.3060 |

The effect of suppressing the contribution from scalar (spin 0), vector (spin 1), tensor (spin 2) and scalar plus tensor sea has also been estimated separately. A numerical analysis is performed to study the contribution from the sea. For calculating the contribution from the pure scalar sea, the following assumptions were made: $a_0 \neq 0$ and $b_1, b_8, d_1, d_8 = 0$, for the vector: $b_1, b_8 \neq 0$ and $a_0, d_1, d_8 = 0$ and similarly for the tensor $d_1, d_8 \neq 0$ and $a_0, b_1, b_8 = 0$. For the case of the scalar plus tensor sea: $a_0, d_1, d_8 \neq 0$ and $b_1, b_8 = 0$. Table 6 shows the extent to which sea contributions from scalar-tensor and vector sea affect the magnetic moments of decuplet members. For instance, when the vector sea is neglected in the statistical model (C Model), the magnetic moments deviate by 10–20% from the experimental data shown in table 6, whereas the data deviate by 10–30% when the calculation is done only with the vector sea and not with the scalar-tensor sea. The contribution to the spin of the decuplet from sea plus valence quarks comes from $1 \otimes 1 = 0 \oplus 1 \oplus 2$ combining with $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = 2(\frac{1}{2}) \oplus \frac{3}{2}$. Multiplicities cal-
Table 6. Various modifications in the statistical model.

| Particle | C Model (scalar+vector +tensor) | With vector sea | With scalar-tensor sea | Non-strange sea (only $g \rightarrow u \bar{u}, d \bar{d}$) | $\bar{d} - \pi$ asymmetry | Exp. results |
|----------|-------------------------------|----------------|-----------------------|------------------------------------------|----------------------|-------------|
| $\Delta^{++}$ | 5.47 | 4.09 | 5.47 | 5.48 | 0.4 | 4.52 ± 0.50 |
| $\Delta^+$ | 2.68 | 2.007 | 2.69 | 2.70 | 0.12 | 2.71^{+0.3}_{-1.3} ± 1.5 ± 3 |
| $\Delta^0$ | −0.09 | −0.068 | −0.097 | −0.091 | −0.12 | − |
| $\Delta^-$ | −2.87 | −2.13 | −2.88 | −2.88 | −0.4 | − |
| $\Sigma^{++}$ | 3.03 | 2.28 | 3.04 | 3.02 | 0 | − |
| $\Sigma^{+}$ | 0.26 | 0.19 | 0.26 | 0.26 | 0 | − |
| $\Sigma^- - \Sigma^+$ | −2.52 | −1.87 | −2.53 | −2.51 | −0.37 | − |
| $\Xi^{++}$ | 0.61 | 0.45 | 0.61 | 0.59 | 0.26 | − |
| $\Xi^{+}$ | −2.15 | −1.34 | −2.15 | −2.12 | −0.26 | − |
| $\Omega^-$ | −1.82 | −1.34 | −1.82 | −1.72 | 0 | −2.02 ± 0.05 |

Table 7. Comparison of computed magnetic moments (in terms of $\mu_N$) of the baryon decuplet with other models and experimental data.

| Particle | C Model | D Model | SQM | QCDQM [22,23] | $\chi$QM [16,17] | CQSM [32–34] | CBM [65] | Data |
|----------|---------|---------|-----|--------------|---------------|-------------|----------|------|
| $\Delta^{++}$ | 5.47 | 4.70 | 5.56 | 5.689 | 5.30 | 4.85 | 4.52 | 4.52 ± 0.50 [5] |
| $\Delta^+$ | 2.68 | 2.32 | 2.73 | 2.778 | 2.58 | 2.35 | 2.12 | 2.71^{+0.3}_{-1.3} ± 1.5 ± 3 [8] |
| $\Delta^0$ | −0.09 | −0.14 | −0.09 | −0.134 | −0.13 | −0.14 | −0.29 | − |
| $\Delta^-$ | −2.87 | −2.47 | −2.92 | −3.045 | −2.85 | −2.63 | −2.69 | − |
| $\Sigma^{++}$ | 3.03 | 2.62 | 3.09 | 2.933 | 2.88 | 2.47 | 2.63 | − |
| $\Sigma^{+}$ | 0.26 | 0.22 | 0.27 | 0.137 | 0.17 | −0.02 | 0.08 | − |
| $\Sigma^-$ | −2.52 | −2.19 | −2.56 | −2.659 | −2.55 | −2.52 | −2.48 | − |
| $\Xi^{++}$ | 0.61 | 0.46 | 0.63 | 0.424 | 0.47 | 0.49 | 0.44 | − |
| $\Xi^{+}$ | −2.15 | −1.81 | −2.31 | −2.307 | −2.25 | −2.40 | −2.27 | − |
| $\Omega^-$ | −1.82 | −1.63 | −1.84 | −1.970 | −1.95 | −2.29 | −2.06 | −2.02 ± 0.05 [4] |
| $\mu_{\Delta^{++}}/\mu_{\bar{p}}$ | 1.96 | 1.68 | 2.0 | − | − | − | 1.62 [5] |
| $\mu_{\Delta^+}/\mu_{\bar{p}}$ | 2.96 | 2.65 | 3.0 | − | − | − | 3.16 [6] |

culated for the pure vector sea come out to be 2 whereas for the scalar plus tensor sea the multiplicity is $\frac{5}{3} = 1.66$ which confirms the dominance of the scalar-tensor sea by 17% in relation to the probabilities for $J^P = \frac{3}{2}^+$ decuplet baryons. These multiplicities are contributing to the coefficients ($a_0, b_1, b_2, d_1, d_2$) as can be seen in eqs. (33)–(36) and are used in the wave function of the decuplet baryons to calculate their magnetic moments. Here, suppressing any of the coefficients in the total wave function leads to the change in the magnetic moments. It is well observed that only the scalar-tensor sea is enough to retrieve the experimentally known magnetic moments and therefore we conclude that there is a scalar-tensor dominance over the vector sea. The available experimental information on the magnetic moment of $\Delta^{++}, \Delta^+, \Omega^-$ [4,5,8] is preserved with our model and other magnetic moments are compared with other theoretical models matching well within the error bar 10–20%.

The magnetic moment ratios $\frac{\mu_{\Delta^{++}}}{\mu_{\bar{p}}}$ and $\frac{\mu_{\bar{p}}}{\mu_{\Delta^+}}$ have been experimentally determined [5,6]. To check the validity of our model we calculate these ratios with values of $\mu_{\Delta^{++}}$ and $\mu_{\Delta^+}$ of our model as shown in table 7. Our model has been able to produce the results which are verified by the sum rules given by Soon-Tae Hong [66] for baryon decuplet magnetic moments, i.e.

$$\mu_{\Sigma^+, \Sigma^0} = \frac{1}{2} \mu_{\Sigma^+} + \frac{1}{2} \mu_{\Sigma^0},$$

$$\mu_{\Delta^-} + \mu_{\Delta^{++}} = \mu_{\Delta^+} + \mu_{\Delta^0}.$$
In our model

\[
\mu_{\Sigma^{+\omega}} = 0.26, \quad \frac{1}{2}\mu_{\Sigma^{++}} + \frac{1}{2}\mu_{\Sigma^+} = 0.26, \quad (54)
\]

\[
\mu_{\Delta^{-}} + \mu_{\Delta^{++}} = 2.6, \quad \mu_{\Delta^{0}} + \mu_{\Delta^{+}} = 2.6. \quad (55)
\]

6 Conclusion

A wave function for the baryon decuplet with the inclusion of a sea containing an admixture of quark-gluon Fock states is studied for magnetic moments of \(J^P = \frac{3}{2}^+\) baryons. Valence and sea states are considered to be in the \(S\) state. The statistical model based on the principle of detailed balance is able to find relative probabilities in flavor, spin and color space. Using \(M_s = 101\) MeV, a proper mass constraint is applied to the sea, to see the importance of strange sea vs. non-strange sea. Our results of magnetic moments for different cases are compared with experimental data and other theoretical models so that although the strange contribution in sea is negligible for non-strange particles, yet its effect can be seen for strange particles. The strange contribution to magnetic moments is also mentioned in table 6. The uniqueness of our model lies in the fact that our framework is working well for all \(J^P = \frac{3}{2}^+\) baryons. It is worth to mention that our calculations are performed for the scale of order 1 GeV² and are in non-relativistic frame.

Appendix A.

We substitute the values of coefficients from table 8 to the expression of \(\Sigma^{+\omega}\) from table 1:

\[
\mu_{\Sigma^{+\omega}} = a_s^2[5(\mu_u + \mu_d + \mu_s)] + b_s^2 \left[ \frac{11}{3} (\mu_u + \mu_d + \mu_s) \right]
\]

\[
+ b_s^2 \left[ \frac{11}{3} (\mu_u + \mu_d + \mu_s) \right] + d_s^2 (\mu_u + \mu_d + \mu_s)
\]

\[
+ d_s^2 \left[ \frac{1}{2} (\mu_u + \mu_d + \mu_s) \right], \quad (A.1)
\]

\[
\mu_{\Sigma^{+\omega}} = \left( \frac{1}{N} \right) [(0.161989536)[5(\mu_u + \mu_d + \mu_s)]
\]

\[
+ (0.000510899) \left[ \frac{11}{3} (\mu_u + \mu_d + \mu_s) \right] + (0.000241126)
\]

\[
\times \left[ \frac{11}{3} (\mu_u + \mu_d + \mu_s) \right] + (0.002516263)(\mu_u + \mu_d + \mu_s)
\]

\[
+ (0.000274684) \left[ \frac{1}{2} (\mu_u + \mu_d + \mu_s) \right]. \quad (A.2)
\]
Putting the values of $\mu_a$, $\mu_b$, $\mu_c$ from eq. (23), we can compute magnetic moments for $\Sigma^{20}$. Magnetic moments of other decuplet particles can be found in the same way.

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