A study on the capture of a droplet impact on a fiber

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Abstract-The phenomenon of droplets captured by fibers is ubiquitous and plays an important role in natural and industrial applications. This study focused on the impact process during a droplet impact on a fiber and established a physical model to describe this process. The critical velocity of capture above which the droplet might not be captured after the impact was deduced according to this model and the influence factors were investigated. These findings will be helpful to elucidate the mechanism of the impact between droplets and fibers and optimize the design of coalescence applications.

1. Introduction

The phenomena of droplets impact on fibers are ubiquitous and commonly observed in natural and industrial applications: rain droplets impact on spider webs on rainy days [1], collect water through fiber meshes in deserts [2], filtrations applied to separate liquid in aerosol [3,4], and coalescence separation devices [5,6]. These impact phenomena attracted intensive research and the captured phenomenon has been a vibrant topic of engineering and science due to the extensive applications [7].

The study of droplets impact on fibers was derived from the droplets impact on surfaces, and the geometry of surfaces reduced from plane to fiber eventually [8-10]. The first research of droplets impact on fibers was performed by Hung et al [11], and they focused on the impact phenomena of large amounts of droplets impact on a fiber. The typical phenomena after the impact were classified as droplets disintegration and dripping, and a relationship between the phenomena and dimensionless numbers such as Weber number and Bound number was established. The influence of fiber wettability and the phenomena of droplets impact on fiber meshes were investigated in further research [12]. The dynamics of a droplet impact on a fiber were investigated by Lorenceau et al [13] and the study showed that the droplet could not be captured by the fiber when the impact velocity exceeded a critical value which was termed as the critical velocity of capture. This velocity was studied both experimentally and theoretically through forces analysis, and further studies showed that off-center impact and the inclined fibers could significantly increase this critical velocity which could enhance the capture ability of fibers [14, 15]. The impact between fibers and wetted fibers was investigated by Liang et al [16, 17], and some special phenomena such as rebound and spreading were observed. Comtet et al [18] and Dressaire et al [19] studied a droplet impact on a flexible fiber respectively and found that the capture ability of fibers could be improved by choosing an optimal fiber length. Kim et al [20] extended the impact phenomena by changing the impact velocity in a more wide range and the typical outcomes could be classified as capturing, single droplet falling, and splitting. Besides, the residual volume of droplets left on the fiber after falling was also investigated.

This research investigated the impact process of droplets impact on fibers and primarily focused on the capture phenomenon during the impact which is of vertical importance to the industrial applications.
Some models were established to describe the impact process and have good accuracy with the experiment results under certain conditions, however, they were not suitable for universal situations. The existing deviations could be quite obvious and some new models were needed to describe the impact process. Therefore, we studied the impact process based on the experiment phenomena and developed a new model aiming at calculating the critical velocity of capture more precisely. Innovation through the experiment data available in the literature and the influence of droplets and fiber properties on the critical velocity was researched through numerical calculations. These studies will be helpful to elucidate the mechanism of the impact between a droplet and a fiber and optimize the design of industrial applications.

2. Theoretical Model
The phenomena of a droplet impact on a fiber could be briefly classified as capture phenomenon and detach phenomenon shown in Fig. 1 according to previous studies [11, 20]. The droplet kept moving after impact on the fiber and the capture phenomenon occurred when the impact velocity was smaller than a critical value which was regarded as the critical velocity of capture $v_c$ which was first proposed by Lorenceau et al [13] and many factors such as surface wettability [20], impact position (the eccentric of droplets and fibers [14]) and the impact angles [15] could have a significant influence on it. The impact process we investigated in this study began at the contact of the droplet and the fibers and ended at the status that the droplet was just about to leave the fiber and these two situations correspond to Status A and Status B in Fig. 1 respectively.

![Fig. 1 The schematic of capture phenomenon and detach phenomenon of a droplet impact on a fiber](image)

According to previous studies, a theoretical model was established to describe the impact process and this model was established through the droplet dynamics [13]: the movement of the droplet subjects to the Newton’s Second Law and the forces acting on the moving droplet were gravity, drag force caused by the fiber and the capillary force caused by the surface tension. Hence, the critical velocity could be solved by the differential equation mathematically, and this model was a significant innovation and important research for subsequent studies. However, this model simplified the forces during the impact process by neglecting the change of drag force and capillary force which could bring in significant deviations. In this section, we analyze the forces during the impact process more precisely and establish a new model to describe the movement of the droplet after the impact.

When a droplet impact on a fiber, the droplet moves due to inertance and forces such as gravity, drag force, and capillary force. The gravity forces the droplet to leave the fiber while other forces resist the droplet from detaching. We established a coordinate system to investigate the impact process quantitatively, and the forces were shown in Fig. 2.
We donated $x$ as the axis pointing downward and set the original point at the center of the fiber as shown in Fig. 2(a). The droplet was assumed spherical during the impact process and much larger than the fiber during this study and moved downwards. Hence, the impact starts at the initial position of $x = -R_d$ while the final position was $x = R_d$. Besides, the droplet was regarded as detached from the fiber entirely which meant the gravity of the droplet kept constant during the impact process and can be calculated as:

$$F_g = \frac{4}{3} \pi R_d^3 \rho g$$

Where $R_d$ is the droplet radius; $g$ is the local gravitational acceleration; $\rho$ is the density of the droplet. However, the other two forces were difficult to calculate due to the movement of the droplet.

Drag force was primarily determined by velocity and the projected areas of contact area between droplet and fiber according to the previous study [13], and can be written as:

$$F_d = C_d \rho v^2 A_p$$

Where $C_d$ is the drag force coefficient; $v$ is the droplet velocity during the impact process; $A_p$ is the projected area in the moving direction and is exhibited as the red line in Fig. 2(b). Due to $v$ and $A_p$ being ever-changing during the impact, the drag force is very difficult to calculate. To solve this problem, we import angle $\alpha$ shown in Fig. 2(b) to calculate the projected area. This angle changes from $0^\circ$ to $180^\circ$ during the impact process. Therefore, the projected area $A_p$ can be calculated as:

$$A_p = 4R_d R_f \sin \alpha$$

Considering the geometrical relationship and the coordinate system in Fig. 2(b), the cosine of angle $\alpha$ can be described as $x/R_d$. Hence, the sine of angle can be expressed as:

$$\sin \alpha = \sqrt{\frac{1 - \left(-\frac{x}{R_d}\right)^2}{}}$$

Where $x$ is the coordinate of the droplet center, and $R_d$ is the droplet radius. It is difficult to obtain an exact analytical solution by utilizing Eq. (4) and some simplifications are indispensable to get an explicit solution.

Considering $\sin \alpha$ increases from 0 to 1 and decreases to 0 again during the impact process, we assume $\sin \alpha$ changes linearly and we simplify the equation as:

$$\sin \alpha = 1 - \frac{-x}{R_d}$$

The comparison between the simplified expression and the original relationship is shown in Fig. 3. The red line represents Eq. (4) while the blue line represents Eq. (5).
Although there are differences between the two expressions, the changing tendency and the values at the threshold points are the same which means this simplification is quite acceptable. Hence, the drag force can be written as:

\[ F_d = 2C_d \rho v^2 R_f R_d \left(1 - \frac{x}{R_d}\right) \]  

(6)

The capillary force is another crucial factor during the impact process and is caused by the surface tension due to the contact of the droplet and the fiber. The maximum value of this force is \(4\pi\sigma R_f\) when the droplet is about to detach according to previous research \[13\]. As noted in Fig. 2(c), the capillary force can be decomposed to horizontal component and vertical component which has a significant influence on the droplet movement and the total capillary force in a vertical direction can be written as:

\[ F_c = 4\pi\sigma R_f \cos \alpha \]  

(7)

Besides, the cosine of \(\alpha\) can be expressed as \(\cos \alpha = -x/R_d\) according to the geometrical relationship between the droplet and the fiber, and this force:

\[ F_c = -4\pi\sigma R_f x/R_d \]  

(8)

During the impact, the droplet moves under the coefficient of forces above and the impulse cause the momentum to change:

\[ M (v_B - v_A) = (F_g - F_d - F_c) \Delta t \]  

(9)

Where \(v_B\) is the velocity at Status B and \(v_A\) is the velocity at Status A. For the differential form of Eq. (9):

\[ \frac{dv^2}{dx} - \frac{2}{M} \left(\frac{4}{3}\pi R_d^3 \rho g - 2C_d \rho v^2 R_f \left(R_d - |x|\right) - 4\pi\sigma R_f x/R_d\right) = 0 \]  

(10)

This equation is the description of the droplet velocity changes with displacement after impact. When the droplet exactly detaches from the fiber, the droplet velocity at the maximum displacement position \(x=k\) reduces to \(0 \text{m/s}\) which means \(v(x=k) = 0\). Under this condition, the droplet velocity can be solved and can be written as:

\[ v^2 = e^{-\frac{b(2R_d-x)^2}{2}} \left( \frac{c+aR_d}{b\sqrt{e^{bx^2}}} \right) + \frac{1}{2\sqrt{b}} \left( \frac{\sqrt{2b}}{e^{bx^2}} \right) \]

(11)

Where \(a = \frac{6\sigma R_f}{\rho R_d^3}, b = \frac{6C_d R_f}{\pi R_d^3}, c = -2g\), are parameters related to the system properties and \(\text{erf}(x)\) is the Gauss error function and the expression is:

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\eta^2} d\eta \]  

(12)
The Eq. (12) is the droplet velocity change law that the droplet exactly detach from the fiber and the impact velocity under this condition can be calculated by calculating the velocity at the position \(x=k\) and this is the critical velocity of capture \(v_c\) that can be calculated as:

\[
v_c = \left[ \frac{a}{b} \left( 1 - \sqrt{e^{4aR_d}} \right) + \frac{\left( c + aR_d \right) \sqrt{2\pi e^{aR_d}} \text{erf} \left( \sqrt{2bR_d^2} \right)}{2\sqrt{b}} \right]^{1/2}
\]  

(13)

Thus, Eq. (13) is the critical impact velocity above which the droplet may not be captured by the fiber and can be obtained for the given droplet radius, fiber radius, droplet surface tension, and droplet density. This velocity is of significance to the design and optimization of industrial filter facilities.

3. Result and Discussion

3.1 Numerical validation

The accuracy of the above model is verified in this section by comparing the computational results with experimental data in the reference [13]. The experimental data were performed under a certain condition and the parameters were shown in Tab. 1.

| Physical Parameters       | Unit   | Water  |
|---------------------------|--------|--------|
| Density \(\rho\)          | kg/m\(^3\) | 990    |
| Viscosity \(\mu\)         | Pa·s   | 0.897×10\(^{-3}\) |
| Surface tension \(\sigma\) | N/m   | 72.1×10\(^{-3}\) |
| Gravitational acceleration \(g\) | m/s\(^2\) | 9.81   |

Based on the above experiment condition, we made a simplifying assumption that the surface morphologies of fiber were smooth and neglected the influence of fiber surface roughness to facilitate comparisons with the experiment data. The comparisons of numerical results with experimental data are shown in Fig. 4.

Fig. 4 The comparison of numerical results and experiment data

The abscissa of Fig. 4 is the ratio of droplet size \(R_d\) to the maximum droplet size \(R_M\) which is calculated by Eq. (13), and the ordinate is the ratio of the critical velocity of capture \(v_c\) to the capillary characteristic velocity \(v_M\) which is defined by \(v_M = \sqrt{4gR_0}\) that \(R_0\) is a characteristic length related to fiber radius \(R_f\) and the capillary length \(\kappa^{-1} = \sqrt{\sigma/\rho g}\), and the relationship is \(R_0 = 3^{1/3} R_f^{1/3} \kappa^{-2/3}\) [13].
As noted in Fig. 5, the prediction of this physical model (solid line) is consistent with the experimental data obviously and has better accuracy than the model in references (dash line) [13]. The scatters are the experimental data obtained from the reference. The predicted velocity decreases with the increase of droplet size which is in accord to the experiment data. The predicted value decreases sharply at first and then experiences a relatively smooth decrease stage and then decreases sharply again. The predicted results exhibit good agreement with the experimental data. Furthermore, with the increase of droplet size, the critical velocity gradually decreases to 0 which means when the droplet may not be captured by the fiber when the droplet size is larger than a critical size.

Besides, as shown in the graph, there still exist errors between the predicted values and experiment values. The main reason for the deviation may be our simplified assumptions in calculating the drag force and capillary force and the experiment data may be imprecise due to the experiment and test limit.

Although there are some deviations between the predicted results of the current model and the experimental data, the predictions are completely acceptable. In particular, the predictions under the situation $R_d/R_M>0.8$ are very consistent with the experimental results which means this model is much more suitable for larger droplets. Therefore, this model can be used to analyze the detaching behavior qualitatively and is very useful for many practical applications, such as providing a useful guideline for the design of coalescence separation, fibrous filters, and fog harvesting equipment.

### 3.2 The influence of Oh number on the behavior of droplet impact

The impact process is influenced by the droplet properties according to experimental results and theoretical analysis. Under this condition, the droplet properties such as viscosity, density, and surface tension, and size (radius) can be described through a dimensionless parameter Ohnesorge Number which is the description of the relative influence of viscous versus capillary-inertial effects.

To better understand how Oh influences the impact process, the variations in the critical velocity of capture $v_c$ with Oh on four different fibers were investigated here through numerical calculation according to Eq. (13). In this work, Oh was changed by changing the droplet size $R_d$ and varied from 0.003-0.010 and shown in Fig. 5.

The dependence of the critical velocity on Oh number is plotted in Fig. 5. The variations of critical velocity of capture $v_c$ with Oh number on four different fibers are significantly different, and the critical velocity increases apparently with the increase of fiber size. Under the condition Oh is 0.004, the critical velocity changes from 0.3m/s to about 2.3m/s when the fiber changes from 50μm to 250μm. Besides, $v_c$ increases more quickly when the fiber is larger. When Oh changes from 0.0035 to 0.004, $v_c$ changes from 0.9m/s to 2.1m/s when $R_f=350μm$ and changes from 0.4m/s to 1.0m/s when $R_f=250μm$. 

![Fig. 5 Variations in Oh number with the critical velocity of capture $v_c$](image)
Moreover, the critical velocity may decrease to 0 with the decrease of $Oh$ which means the droplet may not be captured by fibers when $Oh$ is less than a critical value, and this value increases with the decrease of fiber size. As $Oh$ reflects the relative influence of viscous versus capillary-inertial effects, decreasing the droplet viscosity and increasing the surface tension or droplet size all will lead to the decrease of $Oh$ and further influence the critical velocity.

In a word, the $v_c$ increases with the increase of $Oh$ and fiber size which means increasing the droplet viscosity and decreasing the surface tension, droplet size all will lead to $v_c$ increase and benefit to the droplet captured by fibers. These will be useful to the design of filters and other industrial applications.

### 3.3 The influence of fiber properties

The fiber size $R_f$ has a significant influence on the forces during the impact process and then affects the movement of the droplet. Accordingly, the change of fiber sizes needs to be studied and to lucubrate the influence of $R_f$, the variations in the $v_c/v_M$ with $R_d/R_f$ is also investigated under four different conditions and shown in Fig. 6.

![Fig. 6](image)

**Fig. 6 (a)** Variations in radius ratio with velocity ratio. **(b)** Variations in fiber radius with the critical velocity of capture

As shown in Fig. 6(a), the critical velocity increases with the increase of fiber size, and the increasing velocity is larger when $Oh$ is larger. These findings attributed to the fact that larger fibers are more easily to capture larger droplets and larger impact velocities. This means increasing the fiber size is good in enhancing the capture of a droplet.

As shown in Fig. 6(b), the dimensionless velocity decreases with the increase of the ratio of droplets to fiber, and eventually decreases to zero. Besides, the dimensionless velocity is much larger when the fiber is small under the same droplet ratio. All these indicate that the small fiber is a benefit to the capture of droplets, the droplets range is large and the relative velocity is small which is in favor of detaching after being captured by fiber in industrial applications.

To show the influences of $Oh$ and $R_f$ on the velocity more intuitively and understand these factors preferably, the variations of threshold capture velocity with $Oh$ and $R_f$ were plotted in Fig. 7.
As noted in Fig. 7, the changing law of the critical velocity and the fiber radius and Oh can be observed obviously. The velocity increases with the increase of fiber radius and Oh. The velocity decreases to zero when the fiber is less than a value or Oh is less than a value which means this droplet cannot be captured by fiber under this condition. Accordingly, to enhance the capture of a droplet, we can consider it from fiber and droplets respectively. Increasing the fiber radius properly may augment the critical velocity and then enhance the capture ability; increasing the droplet viscosity and decreasing the surface tension or droplet radius all will increase Oh and then enhance the capture ability of fiber to some degree. These are the theoretical directions and the specific operations during the industrial applications need to consider the real operation condition.

4. Conclusion
In this work, the impact process between a droplet and a fiber was investigated and a new theoretical model was established to study the critical velocity of capture. This model takes the specific change of forces during the process and makes reasonable simplifications that the theoretical values have good accuracy with the experiment results. This model will be helpful to investigate the mechanism of a droplet impact on a fiber.

In addition, the factors that influence the threshold velocity of capture were also discussed, such as the effects of fiber and droplets properties on the critical velocity of capture. The critical velocity increases with the increase of fiber size which could increase the capture of a droplet when impact. And the droplet properties influence the critical velocity very much: when increasing the droplet size and decreasing the droplet viscosity and increasing the droplet surface tension all will lead to the increase of critical velocity of capture which may be a new thought to increasing the critical velocity of capture. These investigations are of great importance to industrial applications.

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