A Minimal Inflation Scenario

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ABSTRACT: We elaborate on a minimal inflation scenario based entirely on the general properties of supersymmetry breaking in supergravity models. We identify the inflaton as the scalar component of the Goldstino superfield. We write plausible candidates for the effective action describing this chiral superfield. In particular the theory depends (apart from parameters of $O(1)$) on a single free parameter: the scale of supersymmetry breaking. This can be fixed using the amplitude of CMB cosmological perturbations and we therefore obtain the scale of supersymmetry breaking to be $10^{12-14}$ GeV. The model also incorporates explicit R-symmetry breaking in order to satisfy the slow roll conditions. In our model the $\eta-$problem is solved without extra fine-tuning. We try to obtain as much information as possible in a model independent way using general symmetry properties of the theory’s effective action, this leads to a new proposal on how to exit the inflationary phase and reheat the Universe.
1. Introduction and summary of results

The inflationary paradigm provides a robust framework to explain the size, flatness, homogeneity of the universe and its perturbations [1–9]. It postulates that the universe went through a quasi de-Sitter phase in the past leading to an exponential expansion of spacetime. If this quasi de-Sitter phase is associated to a scalar field, then it can be shown, rather generally, that vacuum fluctuations of the field are stretched by inflation to produce a nearly scale invariant power spectrum and with sufficient strength to produce the currently observed large scale structure in the universe [2,9]. In spite of its successes, we still face several questions to be answered in inflation, namely: what is the inflaton? how do we bring the universe out of the inflationary phase? how can we stop inflation and transit to the decelerating/accelerating universe we live in today? what sets the energy scale of inflation? At a more fundamental level we have to deal with the problem of explaining the rich dynamics of the inflaton without too many free parameters and fine tuning.

In a previous letter [10], guided by the idea that the inflaton should be found naturally among the fields of any fundamental physics model, we have exploited some generic features of the supersymmetry (supergravity) breaking mechanism to design a model of inflation. In this model we identified the inflaton with the order parameter of supersymmetry breaking and associated the supersymmetry breaking scale with that generating cosmological perturbations. Under these conditions, and imposing explicit R-symmetry breaking, we showed that one can obtain enough number of e-foldings (> 70) to explain the observed universe.
Supersymmetry and inflation have a long history [11–16], however in our approach we try to avoid making specific and concrete models and try to see to what extent generic features of supersymmetry breaking are enough to provide a good inflationary scenario.

Our main motivation to propose to identify the inflaton field with the order parameter of supersymmetry breaking is guided by the fact that, independently of the particular microscopic mechanism driving supersymmetry breaking (in what follows we will restrict ourselves to $F$-breaking) we can, whenever we have violation of conformal invariance in the UV, define a superfield $X$ whose $\theta$ component at large distances becomes the "Goldstino" (see [17, 18]). This superfield is the chiral superfield that appears in the divergence of the superfield of currents, the Ferrara-Zumino (FZ) super-multiplet [19] with universal properties at low energy shared by large classes of models with supersymmetry breaking. In the UV the scalar component $x$ of $X$ is well defined as a fundamental field while in the IR, once supersymmetry is spontaneously broken, this scalar field becomes the superpartner of the Goldstino i.e a two Goldstino state. The realization of $x$ as $GG$ can be implemented by imposing a non linear constraint in the IR for the $X$ field of the type $X^2 = 0$. In our previous approach to inflation we used one real component of the UV $x$ field as the inflaton. We assumed the existence of a $F$-breaking effective superpotential for the $X$-superfield and we induced a potential for $x$ from gravitational corrections to the Kähler potential$^1$.

In this paper we refine our model [10] by exploring a family of Kähler potentials depending on few parameters and which can lead to a reasonable cosmology. In particular we consider a simple class of models that apart from a few parameters of $O(1)$, depend on the scale of supersymmetry breaking $f$. The Kähler potential explicitly breaks the $R$-symmetry as is needed in supergravity in order to have a slow-roll phase where the universe inflates. By fitting these models to cosmological data, we can read a supersymmetry breaking scale of $O(10^{12−13})$ leading to a fairly heavy gravitino. Given the scarcity of parameters in the model presented, it is quite remarkable that many cosmological constraints can be satisfied in such an economical manner. The minimal choice we make has to also provide a graceful exit from inflation without invoking a “waterfall” field that will bring the theory out of exponential expansion. In our case, the end of inflation is reached when the $X$-field begins to enter the nonlinear phase ($X^2 ∼ 0$), the scalar component of the FZ chiral superfield is

$^1$Like most inflationary theories containing supersymmetry, we present a simple model of multifield inflation (sometimes called hybrid) [20]
converted into a pair of Goldstinos and the state of the universe can in principle be viewed as some weakly interacting Fermi liquid. The Fermi nature of the elementary components of the liquid creates the necessary pressure to exit the inflationary period. This is admittedly a far-off idea, and we are currently exploring its microscopic properties in detail. We expect to report on our results in a future publication [21]. This detailed physical description is not necessary in order to explore some of the phenomenological properties of our model. This is what we will do in the rest of the paper.

Let us insist once more that the key feature of our philosophy is to obtain the basic properties of the inflation scenario (inflation, graceful exit and reheating) out of a single superfield $X$ and the general properties of supersymmetry breaking.

2. Some motivation and the minimal model

Our inspiration for the effective model we describe in this paper was motivated by some novel approaches to inflation based on Higgs field and non-minimal supergravity couplings. In reference [22] (see also [23]) an approach to inflation was suggested based on the standard model Higgs potential with the Higgs field coupled to gravity in a suitable Jordan frame. In this approach the role of the inflaton is played by the Higgs field and the essential features of inflation appear as a consequence of the chosen frame. The lagrangian for the Higgs field in the Jordan frame is

$$L = \sqrt{-g_J}(R\Phi(h) - L(h; v))$$

(2.1)

with $L(h; v)$ the standard Higgs lagrangian with $v$ the vev and $\Phi(h)$ a function of the Higgs field defining the frame. In Einstein frame the potential is given by

$$V_E(h) = \frac{V(h)}{\Phi(h)^2}$$

(2.2)

with $V(h) = \lambda(h^2 - v^2)^2$ the standard Higgs potential. For $\Phi(h) \sim (M_P^2 + \alpha h^2)$ we observe that the net effect of working in Jordan frame is to create an inflationary plateau for values of the Higgs field $h > \frac{1}{\alpha}$ in Planck units. In spite of its beauty and simplicity this approach suffers from several problems: fine tuning for $\alpha$, non unitary higher order corrections etc. The study of “Higgs inflation” was considered in supergravity with non-minimal couplings in [24]. The general approach using the full $N = 1$ supergravity theory in the Jordan frame was worked out in [25,26]. See also in this context [27–29].
We were inspired in the construction of our model by the Jordan frame formulation, and by some vague similarities with no-scale supergravity theories [30]. This is what led us to consider together with the simplest form of a Kähler potential, a logarithmic correction. This is suggested by the form of the scalar potential in the Jordan frame (see [25] for details). In the simplest case of $N = 1$ supergravity coupled to a single chiral superfield $X$, the scalar potential is determined completely by the Kähler potential $K(X, \bar{X})$ and the superpotential $W(X)$ [31]

$$V_E = e^{\frac{\kappa}{M_P}} (-\frac{3}{M_P} W \bar{W} + G^{X,\bar{X}} D_X W D_X \bar{W}),$$

where the Kähler metric and the Kähler covariant derivatives are given by:

$$G_{X,\bar{X}} = \partial_X \bar{\partial}_X K(X, \bar{X}) \quad DW(X) = \partial_X W(X) + \frac{1}{M_P} \partial_X K W(X)$$

We now make an explicit choice for $K$ and $W$.

For us the inflaton superfield is the FZ-chiral superfield $X = z + \sqrt{2} \theta \psi + \theta^2 F$, the order parameter of supersymmetry breaking. We will consider the simplest superpotential implementing F-breaking of supersymmetry. More elaborate superpotentials often reduce to this one once heavy fields are integrated out. In the following $M_P$ is the Planck mass.

$$W = fX + f_0 M_P$$

with $f_0$ some constant to be fixed later by imposing the existence of a global minimum with vanishing cosmological constant and with $f$ the supersymmetry breaking scale $f = \mu_{susy}^2$.

As Kähler potential $K$ we consider

$$K = X \bar{X} + \frac{a}{2M_P} (X^2 \bar{X} + \text{c.c.}) - \frac{b}{6M_P^2} (X \bar{X})^2 - \frac{c}{9M_P^3} (X^3 \bar{X} + \text{c.c.}) + \ldots - 2M_P^2 \log(1 + \frac{X + \bar{X}}{M_P})$$

The coefficient $a$ will be determined by the condition that the minimum of the scalar potential is at $z = 0$. With this choice, we can write explicitly (2.3) as a function of $z, \bar{z}$ the scalar components of $X, \bar{X}$. After $f_0, a$ are determined, the condition that the potential is a minimum at the origin imposes some constraints on the matrix of second derivatives. After some algebra we obtain the following results:

$$a = 0 \quad f_0 = -f M_P \quad b + c - 1 \geq 0 \quad b - c - 1 \geq 0$$
Using the real components of the $z$-field:

$$z = \frac{1}{\sqrt{2}}(\alpha + i \beta), \quad (2.8)$$

we can obtain their masses:

$$m_{\alpha}^2 = \frac{f^2(b + c - 1)}{3M_P^2}, \quad m_{\beta}^2 = \frac{f^2(b - c - 1)}{3M_P^2}. \quad (2.9)$$

The potential is plotted in figure I for the limiting values $b = 3$, $c = 1$. It is easy to observe that the effect of the logarithm is to induce a waterfall at the end of inflation where reheating could take place. Before entering to analyze the region where the slow roll conditions are achieved it is interesting to observe the form of the potential near the minimum $z = 0$ in the limiting case when $b \sim 1$ and $c \sim 0$. In this case both scalar fields are nearly massless at the origin, the potential is very flat there, and these fields could be used as a way of realising a quintessence scenario.

Let us now compute the slow roll parameters $\epsilon$ and $\eta$. What we find, shown in the upper panels of figure II, is that slow roll inflation starts in a subplanckian region with fields of the order $0.1M_P$ and ends for values of order $0.01M_P$.

Pushing backwards from the end of inflation 70 e-foldings we can compute the fluctuation spectrum in terms of the supersymmetry breaking scale $f$. This allows us to fix the value of $f$ in terms of the WMAP experimental data for the spectrum of fluctuations. This WMAP graph is presented in figure III. The value of $\mu_{\text{susy}}$ we get is $O(10^{13})$ GeV. The dependence of $f$ on $b, c$ is rather mild for $b, c$ values of $O(1)$.

Note that in the particular limiting case $b = 1, c = 0$, the Kähler potential is:

$$K = z\bar{z} + \frac{1}{M_P^2}(z\bar{z})^2 - 2M_P^2 \log(1 + \frac{z + \bar{z}}{M_P}) \quad (2.10)$$

and the R-symmetry breaking is all due to the logarithmic term. This explicit breaking is important to obtain the desired form of the potential and to satisfy the slow roll conditions.

If we ignore the polynomial terms in (2.10) and consider only the logarithmic part, then the Kähler potential is invariant under imaginary shifts of $z$, this is analogous to what happens in no-scale models and helps in generating the inflationary properties of our potential.

### 3. Phenomenology of the model

For the above model, leaving only as a free parameter the scale of supersymmetry breaking, we are now in a position to study its phenomenology. Recall that current observations of
Figure I: The inflationary potential. Left panel: the shape of the potential, in logarithmic units of $M_{pl}^4$, for the two real fields $\alpha$ and $\beta$ and $b = 3, c = 1$. Inflationary trajectories are obtained for all initial conditions of the field as it will always slow roll toward the steep throat located at $\alpha = \beta \sim 0$.

Right panel: the minimum of the potential. This time the potential is shown in linear scale. Note that the potential is very flat and can therefore provide a natural candidate for quintessence.

the cosmic microwave background (CMB) provide constraints on the amplitude of the fluctuations, the slope of the primordial power spectrum, upper limits to the amplitude of gravitational waves and the minimum number of required e-foldings.

For simplicity we will focus in this section on particular values of $b, c$, although any other allowed values of $b, c$ are equally easy to analyse and also provide acceptable fits to cosmological data. Actually it is easy to check that the value of the supersymmetry breaking scale determined by the experimental data on the spectrum of fluctuations is almost independent of the value of $b$ and $c$. Around the line $b = 1 + c$ one of the two scalar components of $z$ is very light giving rise to some potential form of quintessence. Generically the region of initial conditions leading to inflation reduces when we go deep inside the allowed region of values in the $b, c$ plane. We can determine the range of values for $f$ by matching the predicted level of the fluctuation 70 e-foldings before the slow roll parameters $\epsilon, \eta$ are $\sim 1$. This will provide a value for $\alpha, \beta$ (recall that $z = \alpha + i\beta$) at which the level of fluctuations will have to match the ones measured by WMAP7 [32]. As we show below this only happens for a small range of values of $f$.

First, we look at the overall shape of the inflaton potential. This is shown in Fig. I for the case $b = 3, c = 1$. The left panel shows the overall shape of the inflaton for ranges of the real fields $\alpha, \beta$ below $M_{pl}$. Note the flatness of the potential. The deep throat at $\alpha = \beta \sim 0$ is an attractor and for any initial value of the field, it will end up the slow-roll
Figure II: In the two upper panels we plot the slow-roll parameters $\epsilon, \eta$ as a function of both fields, for the case $b = 1, c = 0$, to show that there are large regions where both $\epsilon$ and $\eta$ are smaller than 1 and inflation can take place. Note also that inflation ends at $\alpha, \beta \sim 0.01$ and that because of $\eta$ inflation is always sub-Planckian. In the two lower panels we plot $\epsilon$ and $\eta$ as functions of the two parameters $b, c$ in our potential for some typical values of $\alpha, \beta$ to show that slow-roll conditions are natural in our model.

This throat provides the graceful exit from inflation. The right panel, shows the minimum of the potential, note that it is definite positive and naturally provides a very flat direction, which could be a good candidate for quintessence.

Fig. II shows the value of the slow-roll parameters as a function of the fields $\alpha, \beta$. Note that inflation ends for $\alpha, \beta = 0.01$ and that 70 e-foldings occur for $\alpha, \beta \sim 0.1$. At this point the value of the spectral slope of the primordial spectrum of fluctuations is in the range $n_s = 0.95 - 1.0$. The predicted value of the scalar-to-tensor ratio is $r < 2 \times 10^{-3}$, thus it could be measurable by future experiments [33].
Figure III: Value of the supersymmetry breaking scale $\mu$ for which the amplitude of the scalar fluctuations, as measured in the CMB, are matched for our minimal model with $b = 1, c = 0$.

The constraint on the scale of supersymmetry breaking using the amplitude of fluctuations of the CMB is shown in Fig. III.

In our scenario reheating will take place as a consequence of the existence of a waterfall that opens at the end of the inflationary period. The peculiar features of our choice of the inflaton field as the bosonic component of the supersymmetry breaking order parameter comes from the natural conversion of this field in the IR into pairs of Goldstinos $z \sim \frac{GG}{f}$ [17, 18]. This transmutation of the inflaton field at low energies into pairs of Goldstinos allows us to model the final state of the universe at the end of inflation as some sort of Fermi liquid with Fermi momentum $p_F \sim \mu_{\text{susy}}$ in the simplest Fermi gas approximation. This Fermi gas gives us the first approximation to the positive pressure FRW phase after inflation. The reheating temperature we get in this "barebone" approximation is $T_{rh} \sim 10^9 - 10^{11}$ Gev, and entropy production of the order of one. More details on the graceful exit provided by the transmutation of the inflaton field into Goldstino pairs will be given in a separate publication [21].

Another interesting consequence of the Goldstino transmutation of the inflaton appears when we consider fluctuations at the end of the slow roll period of inflation. In this regime the spectrum of fluctuations for the composite $GG$ field goes as $k^3$ that is quite irrelevant in the IR [34]. Therefore, fluctuations produced during this phase, and any isocurvature mode, will be at very small scales and thus swamped by non-linearities caused by gravitational collapse in the decelerating FRW stage.
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