Optimization Decision Method based on Nonlinear Analytic Hierarchy Process for Active Distribution Network Planning

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Abstract. Aiming at the investment environment of the distribution network, this paper deeply analyses the investment conditions of the distribution network, introduces the analytic hierarchy process and modern investment theory, establishes the analytic hierarchy of the investment conditions in the distribution network project, and proposes an optimization decision-making method of investment plan in distribution network. This paper also establishes two new concepts: condition score and correlation score, uses the improving analytic hierarchy process to calculate the weights, proposes a priority ranking method for distribution network projects, and gives detailed process to formulate distribution network investment plans. The application of the proposed method to optimize the distribution network planning scheme shows that the optimization investment scheme given has better investment benefits under the condition of the power supply reliability requirements of the distribution network and investment capital constraints.

1. Introduction

In recent years, the investment in the construction of distribution network has been continuously increased. How to make a reasonable investment decision of the distribution network to improve the investment efficiency of the distribution network construction has become an urgent problem.

At present, the investment decision-making methods are mainly used by power grid companies. The main idea is to maximize the reliability of power supply and minimize the investment and operation cost of the distribution network [1-2]. In the decision-making process, the analytic hierarchy process and its improvement methods are generally used to construct a comprehensive evaluation index system, analyse an investment planning and select the best from the results. Literature [3] builds a project evaluation system, but the project comes from the grid rolling planning library of the grid company and the literature ignores the project access analysis and technical feasibility demonstration. Literature [4] discusses the access system of construction projects, analyses individual projects at the district bureau level and the project portfolio plan at the municipal bureau levels, but lacks economic benefit analysis. Literature [5-8] establish program evaluation systems from the aspects of reliability and economics.

Judging from the current research results, it has not yet formed a comprehensive evaluation index system for the investment plan of the distribution network project portfolio and has not considered the mutual influence between the projects. In addition, the current evaluation index system does not pay enough attention to economic benefit analysis.
In response to the problems above, this article proposes a comprehensive optimization decision-making method for the distribution network investment plan: after in-depth analysis of the distribution network investment conditions, introduces the analytic hierarchy process and modern investment theory to conduct hierarchical analysis of investment conditions, gives a method to calculate the weights in the framework. Combined with the concept of condition score and project correlation score method, sort the priority of project, and select an optimization distribution network investment scheme.

2. Decision-making model based on nonlinear analytic hierarchy process

2.1. Structure of decision model

As shown in Figure 1, there are three important evaluation aspects in the decision-making model of distribution network planning: boundary condition evaluation, dominant condition evaluation and correlation evaluation. These three assessments, combined with the logical check of the correlation between the projects, jointly determine the comprehensive score for the project \( x \) to be put into operation in year \( t \). Calculate the comprehensive score of each year, and the year of the highest score is the solution to project operation.

![Figure 1](image_url)

**Figure 1.** Decision model of planning and construction project in the distribution network.

2.2. Boundary condition evaluation

Boundary conditions are a set of mandatory conditions, including the tasks that must be completed, the requirements that must be met, and the provisions that must not be violated. It can directly determine whether a project is approved, whether it must be implemented, or whether it cannot be approved.

2.3. Dominant condition evaluation
2.3.1. Dominant condition evaluation index system. This paper proposes a method based on hierarchical analysis [9] that can guarantee the independence of investment conditions and establish the dominant condition evaluation index system:

1) Classify dominant conditions, and select the initial first-level conditions, second-level conditions and the corresponding subset of bottom investment conditions.
2) Fully consider comprehensiveness and relevance, supplement and modify the subset of secondary conditions and the subset of bottom investment conditions.
3) Find the investment conditions that appear in all the bottom investment condition subsets, upgrade them to the first-level conditions, and delete the conditions in the corresponding subsets.
4) Find the elements of the same first-level that appear in all the subsets of the bottom conditions, upgrade them to the second-level conditions, and modify the corresponding subsets.
5) Find the same second-level conditions different first-level conditions, upgrade them to first-level conditions, and modify the corresponding subset of second-level conditions.
6) Other conditions not only appearing in a subset of investment conditions remain unchanged.
7) Use formula (1) to find the correlation coefficient $R$ between the remaining bottom-level indicators.

If the correlation coefficient of two indicators is greater than 0.85, delete one of them, and if the correlation coefficient of any indicator and more than two indicators is greater than 0.5, delete it.

$$R = \frac{\sum(x_i - \bar{X})(y_i - \bar{Y})}{\sqrt{\sum(x_i - \bar{X})^2 \sum(y_i - \bar{Y})^2}}$$

where $x_i$, $y_i$ are the $i$-th sampled values of index $x$ and index $y$ respectively; $\bar{X}$, $\bar{Y}$ are the average of $x$ and $y$ respectively.

2.3.2. Improved method on weight calculation. Each lower-level element belongs to a higher-level element, and its contribution to the higher-level element is represented by a corresponding weight. The expert compares the relative importance of $n$ lower-level elements of the same higher-level element pair by pair to obtain a judgment matrix: If the judgment matrix passes the consistency check, the normalized eigenvector corresponding to its largest eigenvalue is the weight vector of the lower-level elements. To reduce subjectivity, this paper introduces the Delphi method and the concept of interference matrix. The specific process is as follows:

Request $m$ experts to construct $m$ judgment matrices $(m \geq 5)$, and the judgment matrix given by the $k$th expert $(k=1, 2, \ldots, m)$ is $A^{(k)} = \{a_{ij}^{(k)}\}_{n \times n}$, and the mean matrix $\bar{A} = \{\bar{a}_{ij}\}_{n \times n}$ is as follows:

$$\bar{a}_{ij} = \frac{1}{m} \sum_{k=1}^{m} a_{ij}^{(k)}$$

Compare the elements of the $m$ judgment matrices with the corresponding elements of the mean matrix. If the degree of deviation between a certain element $a_{ij}^{(k)}$ of $A^{(k)}$ and $\bar{a}_{ij}$ exceeds the threshold, the opinions should be summarized and fed back to the experts for modification. Repeat the above process until the degree of dispersion meets the requirements and record the mean matrix as $A' = \{\bar{a}'_{ij}\}_{n \times n}$.

Amend to $A'$ and calculate out the judgment matrix of reciprocity $A = \{a_{ij}\}_{n \times n}$:

$$\begin{cases} a_{ij} = \bar{a}'_{ij} & \bar{a}'_{ij} \geq 1 \\ a_{ij} = 1/\bar{a}'_{ij} & \bar{a}'_{ij} < 1 \end{cases}$$

From this, an optimized judgment matrix that integrates the opinions of $m$ experts is obtained. Check the consistency of this optimized judgment matrix. If it passes the check, the weight of each element can be obtained according to the eigenvalue method shown in formula (4); if it fails the check, consider rebuilding the judgment matrix $A$ and calculate the weights again.

$$Aw = \lambda_{\text{max}}w$$

(4)
Where \( w_i \) and \( w_j \) are elements of \( w = (w_1, w_2, \ldots, w_n)^T \).

### 2.3.3. Calculation of condition score

Require \( l(l \geq 5) \) analysts of distribution network investment to grade and score the bottom investment conditions according to the comment set of \{excellent: 100-85, good: 84-70, passing: 69-55, failing: 54 and below\}. Use formula (5) to calculate the score value of the upper element.

\[
IN = \sum_{i=1}^{q} IN_i w_i
\]

Where \( IN \) is the score of an element in the project investment hierarchy analysis framework; \( q(q \geq 1) \) represents the number of lower-level elements contained in the element; \( IN_i \) represents the \( i \)-th of the element (\( 1 \leq i \leq q \)) the score of the lower element; \( w_i \) represents the weight coefficient of the \( i \)-th lower element.

Calculate up layer by layer with formula (4), and the condition scores of all projects can be obtained. Assuming that the set of projects participating in the scoring is \( \phi \), the condition score of project \( x \) in year \( t \) is \( IS_{x,t} \ (x \in \phi) \).

### 2.4. Correlation evaluation

This paper proposes the concept of correlation to measure the degree of relevance between projects. Correlation includes dependency and contribution. Dependency refers to the degree of dependence between the implementation of a project and its role on the implementation of other projects; contribution refers to the degree of effects of the implementation of one project on the implementation of other projects.

In order to calculate the correlation of projects, the PageRank method is introduced [10-11].

The calculation of the correlation of the project is as follows:

1) For the projects in the set of all projects \( \phi \), the initial correlation value is as \( 1/N \) generally [12].

2) Perform iterative calculation of correlation according to formula (6), until the \( CS \) change value between the \( k \)-th and \( (k-1) \)-times is less than the given threshold.

\[
CS_{x,t}^k = \frac{(1-\sigma) N}{\sigma} + \sigma \sum_{y \in \gamma(x,t)} \frac{CS_{y,t}^{k-1}}{P_{out}(y,t)}
\]

Where \( CS_{x,t} \) is the correlation value of project \( x \) in planning year \( t \); \( \gamma(x,t) \) is the set of all the projects that have a direct contribution to project \( x \) in planning year \( t \); \( CS_{y,t} \) is the correlation value of projects that directly contribute to the project in planning year \( t \), and \( P_{out}(y,t) \) is the number of projects directly contributed by project \( y \) in planning year \( t \). Considering that some projects have no direct objects to contribute to, so the damping coefficient \( \sigma \) is retained.

### 3. Optimization decision-making process

#### 3.1. Condition score of construction project

1) Calculate the condition score of a single judge

Construct a hierarchy analysis framework that meets the investment conditions of the distribution network. Solicit expert opinions and calculate the weight of each element. After that, investment analysts archive and score the bottom conditions, and calculate the condition scores given by each judge.

2) Count the correlation between projects and calculate the correlation score

3) Calculate the comprehensive score of the project
For the $i$-th project, the comprehensive score of the judges is the sum of condition score and correlation score: $S_{i,t} = IS_{i,t} + (-CS_{i,t})$. Archive the comprehensive score of the judges according to the five-level comment set and list the fuzzy relationship matrix $T = (t_{ij})_{m \times n}$:

$$t_{ij} = l_{ij} / l$$  \hspace{1cm} (7)

Where: $t_{ij}$ represents the subordination of the $i$-th investment project to the $j$-th level of the comment set; $l_{ij}$ represents the number of investment analysts who rate the $i$-th investment project as the $j$-th level of the comment set; the elements in the $i$-th row $T_i$ of the matrix $T$ sequentially reflect the subordination of each level in the $i$-th project.

At the same time, the average score of project $i$ in each level is calculated as the matrix $\overline{S_i} = (\overline{S_{i1}}, \overline{S_{i2}}, \ldots, \overline{S_{i5}})$.

The comprehensive score of project $i$ is as follows:

$$S_i' = T_i (\overline{S_i})^T$$  \hspace{1cm} (8)

### 3.2. Prioritization and associated logic check of construction projects

The priority is determined by the comprehensive score and the mandatory start time of the project. According to the comprehensive scores of the projects, the projects are sorted in descending order of the scores. Taking the investment amount as a constraint and taking the latest start time requirements of projects into account, collate the projects according to the comprehensive score of the project.

A project cannot be started earlier than the project it depends on, and no later than the project that depends on it. Check whether the collation scheme satisfies this logical order of relevance. Figure 2 shows the flow chart of formulating the optimization investment plan for the distribution network. Due to the limitation of the total investment and the amount of work, some projects that can be implemented but not compulsory may not appear. For the projects that are subject to the boundary conditions analysis are mandatory, regardless of the comprehensive score, they will inevitably appear in the project investment schedule.
**Figure 2.** Flow chart of formulating the optimal investment plan for the distribution network.

### 4. Case analysis
Taking the "13th Five-Year" Distribution Network Planning of Xiantao City as the case, use the method of this paper to analyse the projects and obtain optimization investment scheme.

#### 4.1. Analysis of investment conditions
1) Analysis of necessary conditions
According to the project attributes, there are eight necessary secondary conditions, namely: ① Meeting power supply requirements of the new load; ② Improving situations of equipment overload ③ Regulations on "low voltage" area; ④ Strengthening the grid structure; ⑤ Solving the "stuck neck" problem; ⑥ Modifying distribution transformers of high-loss; ⑦ Replacing the old equipment; ⑧ Others.
2) Analysis of feasible investment conditions
This article lists six secondary conditions, namely:① Funding conditions ② Substation site selection ③ Transmission line path selection ④ Impact on the environment ⑤ Resilience ⑥ Technology maturity
3) Analysis of economic investment conditions
① Financial benefits ② National economic benefits
4) Calculate the weight of investment conditions
It is necessary to set the weight of each condition in the hierarchy analysis.

#### 4.2. Decision-making model of investment and its solution
Statistics and classification of the projects are conducted according to voltage levels and engineering attributes. The 110 kV Zaba Bay transformation and transmission project in 2014 continues to be invested and constructed in 2015, and the mandatory commissioning time of it is 2015. Based on the investment budget, calculate the annual investment quota $IC_t (t=2015-2020)$ and total investment quota.

1) Calculate the investment condition score of the project
Take the 110kV Louhe power transformation and transmission project (project number 1) as an example to illustrate the scoring process of judge 1.

① Necessity condition score:
There is one 35kV substation and two main transformers with a capacity of 11,300 kVA in Yanhe. In order to solve the heavy overload problem of equipment, the construction of the 110kV Louhe Substation should be accelerated. The necessity level is "excellent" with a score of 97 points.

② Feasibility condition score:
The project adds a 1x50MVA main transformer, a new 110kV single-circuit line from Maozui to Louhe, which length is 12.2km. The substation site and transmission line path can be rated as "excellent" with a score of 90 points; the project funds in the project library are all supported by the state and the score is 100 points; the environmental impacts and resilience meet the basic level of projects, and the score is 80 points; the technology maturity level is "excellent", and a score of 97 points is given.

③ Economic criteria score:
The total investment reached 42 million yuan. After the investment, it will alleviate the heavy overload problem of Yanhe Town. Therefore, both financial benefits and national economic benefits are rated as "excellent", with a score of 95 points.

After judge 1 fills the bottom-level condition scores into the scoring table, calculate the weights of conditions at all levels to obtain the score given by judge 1 for project 1.

In fact, the scoring results of the five judges are: 95.57, 96.85, 94, 93.6, 97.48. According to D, the scores of all judges are counted. Calculate the average score within each level. In this way, the statistical results of the judges’ condition scores for all 110kV projects can calculate out.

2) Construct the incidence matrix of the project and calculate the correlation score of the project
3) calculate the comprehensive score of the project
Table 1. Statistics of judges’ comprehensive scores for 110kV project investment conditions

| Project | Project name                          | Project rating level | Number | Average score |
|---------|--------------------------------------|----------------------|--------|---------------|
| 1       | 110kV Louhe Project                  | excellent            | 5      | 96.5          |
| 2       | 110kV Xianfeng Substation Expansion Project | excellent         | 4      | 90.5          |
| 3       | 110kV Xiliuhe Project                | good                 | 1      | 84.5          |

The fuzzy relation matrix $T$ is as follow:

Table 2. Fuzzy relationship matrix $T$ of judges’ comprehensive scores for investment conditions of 110kV project

| Probability | Excellent | Good | Medium | Poor | Very Poor |
|-------------|-----------|------|--------|------|-----------|
| Project 1   | 1         | 0    | 0      | 0    | 0         |
| Project 2   | 0.8       | 0.2  | 0      | 0    | 0         |
| Project 3   | 1         | 0    | 0      | 0    | 0         |

Judges’ condition scoring matrix $IS'$ is as follows:

Table 3. Judges’ scoring matrix $IS'$ for investment conditions of 110kV project

| average score | Excellent | Good | Medium | Poor | Very Poor |
|---------------|-----------|------|--------|------|-----------|
| Project 1     | 96.5      | 0    | 0      | 0    | 0         |
| Project 2     | 90.5      | 84.5 | 0      | 0    | 0         |
| Project 3     | 94.9      | 0    | 0      | 0    | 0         |

Conducting weighted average calculation with the fuzzy relationship matrix $T$ and the judge’s condition score matrix $IS'$, the condition score of the 110kV project is as follows:

Table 4. Comprehensive score of 110kV project

| project | Project name                          | Condition score |
|---------|--------------------------------------|-----------------|
| 1       | 110kV Louhe Project                  | 96.5            |
| 2       | 110kV Xianfeng Substation Expansion Project | 94.9            |
| 3       | 110kV Xiliuhe Project                | 89.3            |

Follow the steps to find the comprehensive scores of all projects to be analysed and record them in the set $S$.

From the step 1) to 3), the comprehensive score of all the projects can be obtained. Take the investment amount as the constraint condition, and make sure that the annual investment is reasonably arranged, is sort the investment time sequence of projects according to the comprehensive scores, and finally suggest investment year of each project

4.3. Evaluation of investment plan
The optimization plan delays some projects with large investment but not urgent. The delay has an influence on the 35kV substation capacity ratio, medium voltage line load and other indicators, technical indicators score slightly lower than “13th Five-Year” distribution network planning. Meanwhile, the expenditure of the distribution network optimization investment plan is 10.1% lower than the “13th
Five-Year” distribution network planning. The scores of social benefit and environmental indicators of the two plans are the same. The specific evaluation results are shown in Table 5. Obviously according to the comprehensive optimization decision-making method proposed in this chapter, the optimizing investment scheme is reasonable and feasible and can achieve better economic benefits.

Table 5. Comparison of evaluation results

|                          | Optimization investment scheme | “13th Five-Year Plan” |
|--------------------------|-------------------------------|-----------------------|
| Technical indicators     | 88.13                         | 88.72                 |
| Economic indicators      | 86.89                         | 80.41                 |
| Social benefit indicators| 96.48                         | 96.48                 |
| Environmental indicators | 88.5                          | 88.5                  |
| Comprehensive score      | 89.59                         | 87.67                 |

According to the comparison result, the project priority ranking method can basically meets the planning requirements in terms of technical and social benefits. Besides adjusting the timing of some projects is conducive to saving funds and good for economic conditions.

5. Conclusion

Based on the in-depth analysis of the key investment conditions of projects in distribution network, this paper puts forward the concepts of boundary conditions, dominant conditions, correlation, contribution, and dependence, and gives a method for optimization investment decision-making scheme based on Analytic Hierarchy Process (AHP), Delphi and fuzzy comprehensive evaluation. The comparison of the evaluation results shows that the decision-making method can obtain greater economic benefits under the premise of the reliability index of the construction and transformation of the distribution network. By combining AHP with Delphi method and modified matrix, the method greatly reduces the subjective influence of traditional AHP method. The research results can provide reference for the investment decision-making work of power companies. In addition, this method only needs to judge the relative importance of the conditions in the hierarchy analysis framework and give a score for the bottom conditions. The rest of the work can be automatically completed by the software. If there is enough data support, machine learning can also be used to give reference values of relative importance and scoring, which can be transformed into a “data + experience” multi-dimensional decision-making mechanism to further reduce manual workload and enhances the objectivity of the result.

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