ABSTRACT This paper illustrates the reference model-based dead-time compensator (RM-DTC) recently developed for first-order time-delayed systems using a real-time example, and extends this novel approach for a time-delayed double integrator system. RM-DTC enables fully decoupled setpoint tracking and disturbance reconstruction and compensation. The overall structure of RM-DTC includes feedforward control using either a transfer function based implementation or the primary loop which produces a filtered inverse of the model transfer function. The setpoint feedforward is complemented by a disturbance-observer-based input disturbance reconstruction that also uses the filtered inverse of the plant model. The reference models of setpoint and input disturbance feedforward control allow the introduction of a stabilising PD controller without compromising the nominally independent dynamics of setpoint tracking and disturbance rejection. The main advantage of retaining the full functionality of disturbance reconstruction in RM-DTC is that it enables monitoring and diagnostics of the controlled process, which is extremely important in terms of full automation of running processes representing the fundamental goal of the Industry 4.0 campaign. This surpasses the approaches based on internal model control, which have been modified for unstable systems to achieve internal stability by eliminating the reconstructed disturbance signal from the control loop. The excellent properties of RM-DTCs are illustrated by simulation and real-time control of thermal process.

INDEX TERMS Reference model, time-delayed system, double integrator, dead-time compensator, PID control

I. INTRODUCTION

In mechatronics, the model of a double integrator representing the motion of physical bodies plays a central role in position control, whether in high-speed positioning control, robotic arms and manipulators, high-performance servo systems, pneumatic muscle actuators, control of electrical vehicles, robot-aided upper limb rehabilitation, flying vehicles, magnetic levitation, etc. (see, for example [1]–[7]). Chains of integrators also appear in the design of nonlinear systems by feedback linearization [8]. In order to obtain an optimal tuning that fully exploits the capabilities of the control loop, it must be supplemented by the time delays, which are usually represented by a dead time element. This then leads to a double integrator plus dead time (DIPDT) model.

Even today, due to the specifics of the applications, such as nonlinear properties (control constraints, friction, hysteresis, quantization), measurement noise, robustness, and computational complexity, we need a variety of different control approaches. The latter is particularly important due to the growing number of applications based on embedded controllers. Therefore, it is still important to search for new approaches and compare them with the traditional ones.

Recently, Grimhold and Skogestad [9] discovered that by constructing a PID controller according to the DIPDT model, it can be successfully used for a variety of dynamical systems. However, such observations are far from unique and do not only concern PID controllers. Among the large number of controllers based on DIPDT models, the discrete-time solutions with dead-beat performance can be highlighted as examples of “time-optimal” control, first described and applied in [10], to design constrained controllers with anti-windup
integral action for stable and unstable, possibly higher-order plants. They were obtained by combining one of the first dead-time compensators (DTCs), based on the reconstruction and compensation of input disturbances by an extended state observer (ESO), with a generalization of the famous method for controller tuning by Ziegler and Nichols [11]. Similar in several aspects to [10], the constrained control of a double integrator was the basis of a special \(f_{\text{han}}\) function developed by Han [12] and used in combination with the reconstruction and compensation of equivalent disturbances by a state approach with an extended state observer (ESO). This approach, proposed as an alternative to the traditional PID controllers, has been called active disturbance rejection control (ADRC) and in the linear setup denoted also as LADRC has been widely popularized by Gao [13]–[16]. The need to consider the influence of dead time was later incorporated in the LADRC solutions suitable for time-delayed systems [17], [18]. Another similar approach, referred to as model-free control (MFC), based on finite impulse response filters (FIR), was developed by Flies [19], [20]. However, the double integrator models were used even earlier. The time-optimal control algorithms applied to the double integrator for controlling some nonlinear systems were already used by Feldbaum [21], who cites a 1935 patent for rolling mill control with quadratic feedback, which is a typical example of a relay time optimal double integrator control.

It is therefore not surprising that in addition to the aforementioned ADRC and MFC, DIPDT models also play a central role in various PID [22]–[24], or disturbance observer (DOB) control structures [25]–[28]. The slowest penetration of integrating models for modelling and control is seen in the area of Internal Model Control (IMC), which relies on the properties of the Smith predictor (SP) [29]. SP represents the best known structure for the control of time-delayed systems by combining the advantage of an independent design of dynamic feedforward control and of disturbance reconstruction and compensation. However, the use of integrating process models in SP leads to unobservable and unconstrained output disturbances [30], [31]. Moreover, equivalent output disturbances may increase beyond all limits in the presence of constant input disturbances [32]. Therefore, several authors have tried to avoid the problem of diverging output disturbances by modifying the SP for integrating process models by taking into account the input disturbances [33]–[37]. However, in ensuring internal stability they only succeeded after reducing the overall functionality by eliminating the signal of the reconstructed disturbance. This, finally, severely limited the applicability of the SP control structure.

Only the control structures using the set point and input disturbance reference models [32] can eliminate the hidden internal instability of structures with disturbance observers for unstable plants while maintaining the full functionality of the circuit in terms of disturbance reconstruction. Since the disturbance signal can be critically important in several control, monitoring and diagnostic applications, the development of the reference model control has proven to be important also for systems using time-delayed integrator models.

The novelty of the manuscript thus relates to the overall design of a dead-time compensator for time-delayed chains of first and second order integrators. In particular, the proposed solution allows (1) a decoupled design with (2) decoupled control branches used for separate setpoint tracking and disturbance rejection. The master controller, which ensures the internal stability of the control loop, (3) has a nominal input signal of zero so that it does not affect the transients, and (4) preserves the reconstructed input disturbance signal that can be used for monitoring, diagnostics and optimisation of the control loop. Since the design is based on an ultra-local integrative model, (5) the proposed design can be applied to a wide range of process models that can be approximated by second-order dead-time processes.

To satisfy the requirements to work with disturbance signals even in the case of marginally stable integrator-plus-dead-time (IPDT) and DIPDT models, this work provides a generalization of the control approach with a decoupled setpoint feedforward and disturbance rejection dynamics proposed in [32] for IPDT process models. Thereby, the paper is structured as follows: Section II discusses two different implementations of setpoint feedforward for second-order systems: the transfer-function-based implementation and the primary-control-loop-based implementation. The requirements for the design of the input disturbance observer based on DIPDT models, as well as the design of the

| TABLE 1: List of acronyms and abbreviations |
|--------------------------------------------|
| ADRC | Active Disturbance Rejection Control |
| DIPDT | Double Integrator Plus Dead-Time |
| DOB | Disturbance Observer |
| DTC | Dead-Time Compensator |
| ESO | Extended State Observer |
| FIR | Finite Impulse Response |
| FOTD | First Order Time Delayed |
| IAE | Integral Absolute Error |
| IMC | Internal Model Control |
| IPDT | Integrator Plus Dead-Time |
| LADRC | Linear Active Disturbance Rejection Control |
| MFC | Model-Free Control |
| M_{\text{1}},M_{\text{2}} | Maximal Sensitivity, Maximal Complementary Sensitivity |
| P | Proportional |
| PD | Proportional-Derivative |
| PI | Proportional-Integral |
| PID | Proportional-Integral-Derivative |
| RM | Reference Model |
| RMC | Reference Model Control |
| RM-DTC | Reference Model-based Dead-Time Compensator |
| SDOB | Stabilized Disturbance Observer |
| SE | Speed-Effort |
| SOTD | Second-Order Time-Delayed |
| SP | Smith Predictor |
| SW | Speed-Wobbling |
| TV | Total Variation |
| TV_0 | Deviation from monotonicity (0P shape) |
| TV_1 | Deviation from 1P shape |
| TV_2 | Deviation from 2P shape |
| TOM1A | Arduino-based Thermo-Opto-Mechanical system |
| QRDP | Quadruple Real Dominant Pole |

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higher-level stabilizing controller and the necessary reference models for setpoint tracking and disturbance rejection are discussed in Section III. Section IV compares the newly designed RM-DTC with PID controllers in terms of achievable performance and robustness. Section V illustrates the developed controller by a real-time example. The results of the simulation verification are summarized in the Conclusions.

II. SETPOINT FEEDFORWARD DESIGN

As commonly applied in motion control [38], the control structure is composed of a feedforward controller and a disturbance observer. These key control structures will be extended by setpoint and disturbance reference models [32] and a stabilizing PD controller [39].

Remark 1 (Basic loop modelling constraints): RM-DTCs have potential to increase the transient quality of high-end applications. As different constraints on the usual plant modelling apply in such situations, such limitations, as the deadtime, nonlinearities, or constraints on the process input and state, need to be given due consideration in the design from the outset.

A. THE SIMPLEST PROCESS MODELLING

The advantage of feedforward over feedback is that any transport delays do not affect the shape of the controlled system transients, but merely delay the time responses. The primary disadvantage of feedforward control is that, without a stabilizing controller, it can only be applied to stable systems. This fundamental limitation can be circumvented by reference model control (RMC). In terms of setpoint tracking, RMC is already part of standard textbooks (see, e.g., [40]). However, its use in disturbance reconstruction and compensation is much less known [32].

From the point of view of modeling more complex and often nonlinear processes, it may be advantageous to use models as simple as possible, such as the DIPDT model. For the process model \( F(s) \), which relates the Laplace transform \( Y(s) \) of the output \( y(t) \) to \( U(s) \) of the DIPDT input \( u(t) \)

\[
F(s) = \frac{Y(s)}{U(s)} = \frac{K_s e^{-T_a s}}{s^2} \tag{1}
\]

it is necessary to identify only two model parameters \( K_s \) and \( T_d \). Thus, the models (1) represent the simplest possible and thereby realistic second-order process approximation. To express differences between the abstraction of the process model \( F(s) \) and its estimate used in the controller design, we will introduce the transfer function

\[
F_m(s) = \frac{K_m e^{-T_m s}}{s^2} \tag{2}
\]

Here \( K_m \) represents the estimate of the process gain \( K_s \), which is always unknown. So, by considering \( K_s \neq K_m \) when designing the controller, we introduce some uncertainty into the control calculations.

From an identification process that can be carried out in open-loop conditions we get also some estimate of the plant delay \( T_m \). \( T_m \) can represent a composition of different loop delays, including the dominant process delay \( T_p \), an actuator dead-time \( T_a \), a communication delay \( T_c \), or a measurement sensor delay \( T_s \), yielding together \( T_m = T_p + T_a + T_c + T_s \). In principle, different, or additional delays may occur when implementing closed-loop control. Then, if we still want the controller settings derived for the transfer function (1) to accurately reflect the needs of the real circuit, we usually have to supplement \( T_m \) with estimates of the newly introduced delays. These will almost always include the delay of the low-pass filters needed to obtain the feasible controller transfer function. Filter delay can be represented separately by an equivalent dead-time \( T_c \) [39]. Then, the total loop delay considered in the design will be calculated as

\[
T_d = T_m + T_c \tag{3}
\]

Both (1) filtered inversion of the process transfer functions and (2) primary control loops can be used to implement feedforward control. The simulation schemes used to compare both approaches in terms of robustness are shown in Fig. 1. The transfer function of the “real” controlled system, which is extended by an unknown internal feedback parameter \( a \), is assumed in the form

\[
F(s) = \frac{Y(s)}{U(s)} = \frac{K_s}{s^2 + as} \tag{4}
\]

We will use the subscript “\( m \)” to indicate the parameter estimate of the model used for the control design.

B. PRIMARY-LOOP-BASED FEEDFORWARD CONTROL

For the extended second-order process models (4), the stabilizing controller can always be written as

\[
C(s) = \frac{U(s)}{E(s)} = K_P \frac{1 + sT_D}{1 + T_{fd}s} = \frac{K_P + K_Ds}{1 + T_{fd}s}, \tag{5}
\]

where \( K_P \) is the controller gain and \( T_D \) is the derivative time constant, which both give the derivative gain \( K_D \). For the implementation of the primary loop with a measurable state shown in Fig. 1, the PD control can ideally be implemented without the derivative filter, i.e., with \( T_{fd} = 0 \) in (5). For the nominal process model with \( a_m = a; K_m = K_s \), when from the requirement of a double closed loop time constant \( T_c \) one gets

\[
K_P = \frac{1}{T_c^2 K_m}; K_D = \frac{2/T_c - a_m}{K_m}; \tag{6}
\]

and when there are no input constraints on the process, such feedforward control leads to the closed-loop transfer functions

\[
F_c(s) = \frac{Y(s)}{W(s)} = \frac{1}{(1 + T_c s)^2}, \tag{7}
\]

\[
F_f(s) = \frac{U(s)}{W(s)} = \frac{K_m(1 + T_c s)^2}{s^2 + a_m s}. \tag{7}
\]

As discussed in detail in [32], the advantages of primary-loop-based feedforward control will be evident in unstable
systems with input constraints. As shown in Fig. 2 by two simulations with parameters

\[
T_c = 0.5;\quad K_s = 1;\quad K_m = K_s; \\
a = -0.2;\quad a_m = -0.205; \\
a = 0.05;\quad a_m = 0; \\
U_{\text{min}} = -0.5;\quad U_{\text{max}} = 0.5; \\
\]

(8)
corresponding to parameter uncertainty \(\Delta a = a - a_m = 0.05\), the effect of inaccurate parameters is much less pronounced than the effect of constraints, at least in the short run. Surprisingly, the parametric inaccuracy of feedforward has more profound consequences in the simplified position control with a stable speed subsystem \(a > 0\). Of course, the uncontrolled feedforward does not guarantee a longer stable setpoint tracking for unstable systems. But when the primary closed-loop feedforward control is used, the results are much better than for the open-loop feedforward control based on transfer function. The differences are mainly to be seen in the course of the output variables.

Remark 2 (Primary DTC Loop Mission): The use of the primary DTC loop (typical for SP) is not related to the transport delay itself, but to the feedforward generation and to the control constraints, which are among the fundamental nonlinearities of the circuit.

As for the effect of the constraints themselves on the shape of the transients of the primary loop, the linear PD controller can handle smaller constraints without disturbing the smooth shapes of the transients. For more significant constraints, a constrained design must already be used (see, e.g., [5], [41]–[43]).

C. SETPOINT REFERENCE MODEL

It is well known that open-loop feedforward control cannot be used to control unstable and marginally stable processes. In combination with a higher-level stabilization controller and a reference model, the advantages of feedforward control (e.g., in terms of time-delayed process control) can be exploited [32], [40]. The reference model ensures the hierarchical division of activity between the stabilizing controller and feedforward control by providing the higher-level controller with information about the correct course of the setpoint tracking initiated by the feedforward control. For example, for the closed-loop (setpoint to process output) transfer function \(F_c(s)\) (7), the control error, defined as

\[
E_w(s) = F_c(s)W(s) - Y(s) \\
\]

(9)
will be nominally zero. The processing of the control error by the PD controller (5)-(6) (with \(a_m = 0\)) does not cause any effect, but ensures the stability of the closed loop even in unstable systems. For primary control, the signal \(F_c(s)W(s)\) can be taken directly from the output of the plant model output. For a delay-free plant, such a controller could be proposed by omitting all the transport delay blocks \(T_m\) and \(T_d\) in Fig. 3 (e.g., by setting \(T_d = T_m1 = T_m2 = 0\)). However, later we will also deal with the time-delayed systems, while also focusing on the reconstruction and compensation of disturbances.
III. DISTURBANCE RECONSTRUCTION AND FEEDFORWARD

The basic idea of the input disturbance observer is to evaluate the difference between the estimated input of the process and the output of the controller. The estimate of the process input is obtained from the measured process output using its inverse model. Low-pass filters with sufficiently high relative degree must also be introduced to perform the given operation. The presence of a transport delay (as in Fig. 3) naturally complicates the whole control process, since there is no inversion to the delay in causal systems and its influence must be mitigated in other ways. The first DTCs for DIPDT processes reduced the loop delay by reconstructing less delayed outputs and disturbance signals [10]. Such an approach was no longer a violation of causality, but a replacement of the transport delay by the delays of the observer filters. In turn, such solutions increased the speed of the transients and improved disturbance rejection performance. Works [33], [44] based on independent setpoint and disturbance feedforward loops without a stabilizing controller encountered the problems with internal instability in unstable and marginally stable systems, which they could solve only by eliminating the disturbance signal from the control structure. The mentioned works did not respect the fundamental property of disturbance observers [32], [45], which is not only reconstructing disturbance, but at the same time, the controlled object is forced to follow the nominal dynamics of the chosen model. Thereby, in terms of external disturbance reconstruction, the nominal model must be chosen as close as possible to the controlled process. However, the unstable nominal model does not then guarantee the stability of the solution in the long run. The conflicting requirements on precise plant modelling with respect to disturbance reconstruction and compensation and internal stability have been solved just by introducing stabilizing controller and disturbance reference models [32]. In RM-DTCs, the higher-level stabilization controller ensures the stability of the overall loop, but does not interfere with the nominal transients specified at the slave level with setpoint and disturbance feedforward loops. For better clarity, we divide the overall disturbance feedforward design into two steps, the first describing the observer design with a reference model for a system with negligible dead time and just then including also the dead-time term.

A. DISTURBANCE REFERENCE MODEL FOR DELAY-FREE MODEL

To make the inverse of the process model (2) feasible, the disturbance observer must use the low-pass filter $Q_l(s)$:

$$Q_l(s) = \frac{1}{(1 + T_o s)^2}.$$  \hspace{1cm} (10)

For an independent application of disturbance reconstruction and compensation, the disturbance response must be stable. Two unstable plant poles can be eliminated from the disturbance response by the disturbance feedforward

$$C_l(s) = \frac{1 + b_1 s + b_2 s^2}{(1 + T_o s)^n}; \; n \geq 4,$$  \hspace{1cm} (11)

with $n$ denoting the total filter order. With the lowest possible value $n = 4$ and designation

$$S_{uu}(s) = C_l(s)Q_l(s)$$  \hspace{1cm} (12)

and considering the reconstruction of the actual plant input

$$S_{yu}(s) = \frac{U_{af}(s)}{Y(s)} = \frac{S_{uu}(s)}{F_m(s)} = \frac{s^2(1 + b_1 s + b_2 s^2)}{K_m(T_o s + 1)^4},$$  \hspace{1cm} (13)

the disturbance compensation signal can be calculated as

$$U_{af}(s) = S_{yu}(s)Y(s) - S_{uu}(s)U(s).$$  \hspace{1cm} (14)

In the nominal case with $K_s = K_m$ and $F(s) = K_s/s^2 = F_m(s)$, we get a “stabilized” disturbance response

$$F_{iy}(s) = \frac{Y(s)}{D_i(s)} = \frac{F_{iy}(s)}{1 + F_u S_{yd} F}; \; U_u = \frac{1}{1 - S_{uu}}.$$  \hspace{1cm} (15)

Note that to cancel the double plant pole $s = 0$ and to get the zero steady-state disturbance response characterized with $F_{iy}(0) = 0$, the numerator in

$$F_{iy}(s) = K_s (1 + T_o s)^4 - (1 + b_1 s + b_2 s^2)$$  \hspace{1cm} (16)

has to guarantee the triple pole at $s = 0$. Let us use the new variable $p = T_o s, \beta_1 = b_1 / T_o, \beta_2 = b_2 / T_o^2$, to simplify the calculation. Then

$$F_{iy}(p) = K_s T_o^2 (1 + p)^4 - (1 + \beta_1 p + \beta_2 p^2)$$  \hspace{1cm} (17)

By comparing the coefficients at the first and second power of $p$ we get the required values $\beta_1, \beta_2$ and subsequently also $b_1, b_2$, which yields

$$\beta_1 = 4; \; \beta_2 = 6; \; \alpha_0 = 4;$$  \hspace{1cm} (18)

$$F_{iy}(p) = K_s T_o^2 \frac{p(1 + \alpha_0)}{(1 + p)^2}.$$  \hspace{1cm} (19)

In the “$s$” domain with the same value of $\alpha_0 = 4$,

$$b_1 = 4T_o; \; b_2 = 6T_o^2;$$  \hspace{1cm} (20)

$$F_{iy}(s) = Y(s) D_i(s) = K_s T_o^3 s(T_o s + \alpha_0)$$  \hspace{1cm} (21)

At this point, however, it should be emphasized that a stabilized disturbance response does not imply a stabilized system state. Through reconstruction and disturbance compensation, the system behaves according to the chosen model $F_m(s)$, i.e., as a marginally stable double integrator with gain $K_m$. To obtain a system with stabilized states, an additional stabilizing controller is required. Such a controller can be designed according to expressions (5)-(6) with $\alpha_0 = 4$, where a suitable derivative filter time constant $T_{fd}$ should be chosen. Since the nominal disturbance response is given by the transfer function $F_{iy}(s)$ (18) and the DOB gives...
the disturbance signal filtered by the second-order $Q_i(s)$, the wanted disturbance reference model $R_i$ is

$$F_i(s) = \frac{F_{yy}(s)}{Q_i(s)} = K_p T^3 \left( T_o s + \alpha_0 \right) \left( 1 + T_o s \right)^2.$$  

(20)  

The examples of the setpoint and disturbance step responses obtained by the controller scheme in Fig. 3 for $T_d = T_m = 0$ and the parameters

$$T_c = T_o = 0.8; \quad K_s = 1; \quad K_m \in \{0, 0.1, 1, 2\};$$

$$K_p = K_f; \quad K_d = K_D; \quad T_jd = 0.2$$

(21)  

are presented in Fig. 4. The choice of these parameters was motivated by the possibility to compare the obtained transients with the further analysed loop with dead time. They show that the plant gain perturbation produced by $K_m \neq K_s$ corresponds to an input disturbance that increases as $T_e$ and $T_d$ decrease.

Remark 3 (The main advantage of RM-DTCs): Besides the generalization shown for the double integrator, the main advantage of the newly introduced reference model control over the solutions in [33]–[37] is the availability of the equivalent input disturbance signal, which can be of great use for finding the optimal model process and for its further diagnosis. From the opposite signs of the reconstructed disturbance signal $(d_i f)$ in the initial interval without external disturbances for the non-nominal values $K_m = 0.8$ and $K_m = 1.2$ in Fig. 4, it is evident that the actual value of the process gain $K_m = 1$ should lie between these two values. Thus, the advantage of keeping the full functionality of the RM-DTC is that it allows monitoring and diagnostics of the controlled process, which is extremely important in terms of full automation of running processes, representing the fundamental goal of the Industry 4.0 campaign [46], [47].

B. REFERENCE MODEL FOR TIME-DELAYED SYSTEM

The obtained solution can be further extended with nonzero dead time values. From the point of view of setpoint tracking, the extension can be easily achieved by including the dead time model $T_m$ in the reference model of setpoint signal tracking (Fig. 3). While the feedforward transfer function remains the same as in (7), the reference-to-output transfer function $F_{ic}(s)$ corresponding to a double real-time constant $T_c$ becomes

$$F_{ic}(s) = \frac{Y(s)}{W(s)} = e^{-T_d s} \left( 1 + T_c s \right)^2.$$  

(22)  

To equate both the process delay $T_d$ and the DOB delay, the time delay $T_{m1}$ must be included in the DOB path from the controller output. In the nominal case with $T_{m1} = T_d$, $S_{uu}(s)$ (12) is changing to

$$S_{uu}(s) = C_i(s) Q_i(s) e^{-T_d s}.$$  

(23)  

For $p = T_o s$, $\tau_d = T_d / T_o$, the disturbance response becomes

$$F_{yy}(p) = K_p T^3 e^{-\tau_d p} \left( 1 + p \right)^4 - \left( 1 + \beta_1 p + \beta_2 p^2 \right) e^{-\tau_d p}.$$  

(24)  

A direct comparison of expressions (24) and (17) is not possible because of the time delay. However, in (17), the parameter $\alpha_0$ could also be obtained by triple derivation.
of the $F_{iyo}(p)$ numerator $K_s T_o^2 p^3(p + \alpha_0)$, by substituting $p = 0$ and dividing by $3! = 6$. Similarly, by evolving the numerator of expression (24) we get
\[ \alpha_0 = 4 + 6d + 2T_d^2 + \frac{3}{2} T_d^3. \]  
(25)

For $T_{m2} = T_d$ in Fig. 3, (24) and (18) become equivalent when
\[ b_1 = 4T_0 + T_d; \quad b_2 = 6T_0^2 + 4T_0 T_d + T_d^2/2; \]
\[ F_{iyo}(s) = K_s T_o^3 e^{-T_d s} \frac{s(T_0 s + \alpha_0)}{(1 + T_0 s)^2}. \]
(26)

From $F_{iyo}(s)$ it is then possible to calculate $F_i(s)$ according to (20).

Design of a stabilizing PD controller for the DIPDT plant model based on the multiple real dominant pole [6], [48], [49], results in parameters
\[ K_P = 0.079/(K_s T_o^2); \quad K_D = 0.461/(K_s T_d). \]
(27)

The low-pass filter with time constant $T_{fd}$, which is necessary to realize a stabilizing PD controller of the form (5) has no major influence in the considered controller structure. In Fig. 5, transients corresponding to the parameters
\[ a = a_m = 0; \quad T_d = T_m = 0.4; \quad T_c = T_o = 0.8; \]
\[ K_s = 1; \quad K_m \in \{0.8, 1, 1.2\}; \quad T_{fd} = T_m/2 = 0.2, \]
(28)

with $K_P$, $K_D$ (6) and $K_P$, $K_D$ (27), have similar shapes as for $T_d = 0$. Again, note that the choice of $T_c$ and $T_o$ values is taken into account to show the differences that arise due to the influence of $T_d$ and in relation to the model uncertainty considered. In the illustrative example, we will show their significance in terms of taking into account the measurement noise. However, $d_i$ is now reconstructed with a time delay $T_m$, so the effect on the disturbance response is much stronger. Again, in the case of non-nominal setpoint changes with $K_m \neq K_s$, the equivalent disturbances can already be observed during the periods without external disturbances.

IV. COMPARISON WITH 2DOF PID CONTROL

Comparing the proposed solution with alternative methods based on a PID controller is not straightforward. For example, Grimholt and Skogestad [9] choose the tradeoff between servo and regulatory by optimizing a weighted average of the integral of the absolute error
\[ IAE = \int_0^\infty |e(t)| \, dt; \quad e = w - y, \]
(29)
during process input and output disturbance steps, but they do not consider the design of a setpoint prefilter to ensure monotonic tracking of the reference steps. The aforementioned solution also does not deal with a suitable controller low-pass filter design that would result in a feasible, fully implementable transfer function of the controller to attenuate a measurement noise and to minimize the excessive signal increments at the plant input.
At the plant output, a modification of the total output variation criterion (TV) [50]

\[ TV_0(y_s) = \sum_i (|y_{i+1} - y_i|) - |y_\infty - y_0| \]  

(30)

can be interpreted as the output’s deviation from monotonicity. At the plant input, the excessive control effort can be defined in terms of the deviations from the two-pulse (2P) control signal. For the double integrator, the expected control signal consists of the two extremes \( u_{m1} \) and \( u_{m2} \), which lie at some time instants between the initial value \( u_0 \) and the final \( u_\infty \) of the control signal and have the amplitudes \( u_{mi} \neq [u_0, u_\infty]; i = 1, 2; (u_{m1} - u_\infty)/(u_{m2} - u_\infty) < 0 \). Ideally, such a signal has three monotonic intervals [39], [51], [52]. The plant input deviation from 2P shape can be calculated as follows

\[ TV_2(u) = \sum_i \left[ |u_{i+1} - u_i| - 2u_{m1} + 2u_{m2} + (u_\infty - u_0) \right] \operatorname{sign}(u_{m1} - u_\infty). \]  

(31)

**Remark 4 (Reason for changed evaluation excessive control effort):** In contrast to the definition of TV given in [50], the application of (31) does not penalize active changes in the controller output forced by the necessary acceleration and deceleration of the process. Such shortcomings in the evaluation of the control effort are encountered in the majority of recent publications and the separation of useful control actions from redundant ones is only slowly being promoted [23], [53], [54].

### A. THE MULTIPLE REAL DOMINANT POLE PID CONTROLLER TUNING

Unlike [55], which is dominantly dealing with control constraints, without considering the transport delays and noise attenuation, the limitations are only briefly mentioned here, without a detailed evaluation. The considered analytically derived optimal PID controller is based on a generalization of the approach used in [56], [57].

As shown in Theorem 1 in [39], for the parameters \( T_d > 0 \), \( K_s \neq 0 \) of model (1), for ideal PID controller with parameters \( K_s \) (the controller gain), \( T_i \) (the integral time constant) and \( T_D \) (the derivative time constant)

\[ C(s) = \frac{U(s)}{E(s)} = K_c \left( 1 + \frac{1}{sT_i} + sT_D \right) \]  

(32)

the “optimal” values \( K_{c_o}, T_{io} \) and \( T_{Do} \) can be determined analytically to ensure a quadruple real dominant pole (QRDP) \( s_o \) of the characteristic quasi-polynomial

\[ P(s) = T_i s^3 e^{T_D s} + K_c K_s (1 + T_i s + T_i T_D s^2) \]  

(33)

such that

\[ s_o = -0.416/T_D. \]  

(34)

These values may be expressed by dimensionless (normalized) parameters \( \kappa_o, \tau_{io} \) and \( \tau_{Do} \) as

\[ \kappa_o = K_{c_o} K_s T_d^2 = 0.125, \]  

\[ \tau_{io} = T_{io}/T_d = 10.324, \]  

\[ \tau_{Do} = T_{Do}/T_d = 4.043. \]  

(35)

For zero compensation of the closed loop transfer function

\[ F_w(s) = \frac{Y(s)}{W(s)} = \frac{K_c K_s (1 + T_i s + T_i T_D s^2)}{T_i s^2 e^{T_D s} + K_c K_s (1 + T_i s + T_i T_D s^2)} \]  

(36)

leading to overshoot after reference setpoint steps, [56], [57] proposed the two-degree-of-freedom (2DOF) PID using a setpoint prefilter [51]

\[ F_p(s) = s T_i T_D s^2 + b T_i s + 1 \]  

(37)

In the simplest case, \( F_p(s) \) is used with

\[ b_0 = c_0 = 0. \]  

(38)

To accelerate the transients, the weighting parameters \( b \) and \( c \) can also be set to cancel one of the quadruple closed loop poles \( s_o \) (34)

\[ b_1 = \frac{1}{|s_o|} \]  

(39)

The fastest possible setpoint step responses correspond to the numerator of (37), which is equal to the double pole \( s_o \):\n
\[ (s - s_o)^2 = s^2 - 2s s_o + s_o^2 = s^2 + bs/(cT_D) + 1/(cT_i T_D). \]  

(40)

This gives the prefilter coefficients

\[ b_2 = -2/(c T_i s_o) = 0.466, c_2 = 1/(c T_i T_D s_o^2) = 0.150. \]  

(41)

However, such a design increases the required control signal amplitudes (see Fig. 6) and also the overshoot due to gain uncertainty.

Finally, to obtain a feasible controller transfer function, \( C(s) \) must be combined with a first-order series filter [51]

\[ Q(s) = 1/(T_i s + 1) \]  

(42)

with the time constant \( T_1 = 0.2717 T_e \) expressed by means of an equivalent dead-time \( T_e \). It has been considered in the controller tuning as an additional delay \( T_e = 0.4 \), added to the total dead time of the control loop.

Since it is generally difficult to obtain good performance for a double integrating process, when the time delay \( T_d \) is large, cascade control is used in practice whenever possible for controlling double integrating processes [9], [51].

### B. ROBUSTNESS TESTS BY NEW PERFORMANCE SENSITIVITY MEASURES

Traditional robustness analysis is mostly based on sensitivity functions [58], which are defined for the open-loop transfer function \( L(s) \) as

\[ M_s = \max \left\{ \left| \frac{1}{1 + L(j \omega)} \right| : \omega \geq 0 \right\} ; M_I = \max \left\{ \left| \frac{L(j \omega)}{1 + L(j \omega)} \right| : \omega \geq 0 \right\}. \]  

(43)

\( M_s \) is primarily defined for the nominal loop and is related to the loop stability. The recommended values of the sensitivity function are usually less than 2 and are generally not suitable for unstable systems, where the required sensitivity values
may increase to more than 20 [59]. Therefore, instead of using sensitivity functions, we preferred to implement the robustness test similar to the approach proposed in [60]. In such a test, the controller based on model (2) is applied to the plant (4) extended by a dead time

\[ S(s) = \frac{Y(s)}{U(s)} = \frac{e^{-0.4s}}{s(s+a)}; \quad a \in [-0.2, 0.2]; \quad \Delta a = 0.1. \]

(44)

Its internal feedback, quantified by the pole \( s = -a \), transforms the DIPDT plant into the second-order time-delayed system (SOTD) (44). By changing \( a \) in (44), the performance measures corresponding to the setpoint steps under DIPDT-based controller draw trajectories in the speed-effort (SE) plane (\( IAE - TV_2(u) \)) and speed-wobbling (SW) plane (\( IAE - TV_0(y) \)), as shown in Fig. 7. Here, longer trajectories correspond to stronger performance changes and hence higher sensitivity (lower robustness) of the control loop. Denote the individual uncertain parameter values as

\[ a_i = a_{\text{min}} + (i - 1)\Delta a; \quad i = 1, 2, ..., N; \]

\[ \Delta a = (a_{\text{max}} - a_{\text{min}})/(N - 1), \]

(45)

and the coordinates of the performance measures vector \((\xi, \eta)\) as

\[ \xi = TV_2(u), \quad \text{or} \quad \xi = TV_0(y); \quad \eta = IAE. \]

(46)

The corresponding sensitivity measures for the setpoint responses at the plant input, or output, reflecting the length of the trajectory traced out by the change in position of the operating point (46), can then be defined as

\[ S = \sum_{i=1}^{N-1} \sqrt{(\xi_i - \xi_{i+1})^2 + (\eta_i - \eta_{i+1})^2}. \]

(47)

The introduction of these new sensitivity measures \( S(u) \) and \( S(y) \), in contrast to the traditional \( M_s \) and \( M_t \) sensitivity peaks, brings the differentiation of the achieved sensitivity levels with respect to the input and output of the system, as well as the consideration of the controller effort required to maintain the required performance at the input or output.

The results in Fig. 7 were evaluated taking into account the measurement noise with a maximum amplitude of 0.001 generated in Matlab-Simulink by the Uniform Random Number block. Corresponding to PID control with different prefilters (38)-(41) they show that by using more complex prefilters we can speed up the transients while keeping nearly the same excessive control effort (Fig. 7 left). However, at the cost of larger output fluctuations in performance when changing the internal system feedback parameter \( a \) (Fig. 7 right). For RM-DTC, the results vary to a greater extent. The smallest value of the tuning parameter \( T_c = T_o = T_d/2 \) leads to the fastest transients (minimal \( IAE \) with the lowest dependence on the perturbed parameter (reflected by low \( S(u) \) and \( S(y) \) values). However, this is achieved on the costs of the highest excessive controller effort \( TV_2(u) \), reminiscent of robust systems using sliding mode control [61], [62].

The \( TV_2(u) \) values can be reduced to by increasing \( T_c \) and \( T_o \). In general, we achieve optimal values of \( S(u) \), \( S(y) \) and \( S(u)S(y) \) (see Fig. 8) with different settings, which challenges the design according to \( M_s \) and \( M_t \) criteria.

Of course, the impact of measurement noise can also be reduced by using higher order filters in DOB, or in the stabilizing controller.

Remark 5 (Impact of the feedback parameter \( a \), ADRC, PID and DTC design): The characteristics in Fig. 7 also show that the influence of the parameter \( a \) on the excessive controller effort and the speed of transients is negligible compared to the effect of the measurement noise. This makes it possible to avoid identification of this parameter and so to simplify the controller design. Similar feature represents one of the key reasons for the popularity of ADRC. However, such simplification is clearly also relevant in designing RM-DTCs and can also be shown in PI and PID control [63].

V. ILLUSTRATIVE EXAMPLE: TEMPERATURE CONTROL

The decoupled setpoint tracking and disturbance rejection, together with the use of a superior stabilizing controllers, have created new degrees of freedom in tuning of RM-DTCs compared to traditional PID controllers. Therefore, it will be useful to start clarifying the increased tuning complexity by explaining particular tuning steps, preferably from the simplest tasks. To illustrate the use of the RM-DTC design and practical problems associated with its application, we will consider the thermal process control discussed already.
FIGURE 7. Robustness characteristics expressing IAE changes due to uncertainty of the internal plant feedback coefficient $a$ versus the shape related deviations at the input of the plant (44), $\Delta a = 0.1$ and different PID controllers tuned with $T_e = T_d = 0.4$ and prefilters (38)-(41) and the RM-DTCs with the parameters $T_c = T_o \in \{1/2, 1, 2, 4\}$; $K_m = K_s = 1$; measurement noise amplitude $|\delta| \leq 0.001$.

FIGURE 8. Sensitivities (46)-(47) defined for the plant (44) with uncertain $a$ and PID controllers tuned with $T_e = T_d = 0.4$ and prefilters (38)-(41) and the RM-DTCs with the parameters $T_f = T_o \in \{1/2, 1, 2, 4\}$; $K_m = K_s = 1$.

in [31], [60], [64]. The choice of this process is motivated by several aspects - from a physical point of view, it is a highly nonlinear and time-variable higher-order process posing a challenge for robust control - only due to the fact that the concept of a nominal dynamics is strongly questionable. The existing time delays and the resulting measurement noises also require due attention to the appropriate filtering of the measured signals. In addition, it is a process that is clearly not integrative. So, the use of integrative models is not a meaningful step for many users and needs to be shown that it can still be beneficial. Nevertheless, it makes it possible to clarify several aspects of RM-DTC control based on IPDT models and subsequently to highlight the specifics of control of systems with DIPDT models.

A. SIMPLIFIED PLANT MODELLING

The essence of thermal process control is to vary the amount of heat released by the actuator (bulb) so that the temperature measured at the desired point (by a sensor $pt1000$) reaches in the shortest time possible the setpoint reference value. Among the different possible ways of heat transfer participating in heating the temperature sensor (such as advection, conduction convection, radiation, boiling, condensation, or melting), occur in the case of the thermo-optical-mechanical laboratory system TOM1A [65] mainly the fastest heat transfer by radiation and conduction. So, although a physically more accurate modeling of the dynamics of the system under consideration would require the use of higher-order models, limiting to the fastest process mode it is usually enough to work with the IPDT model

$$F(s) = \frac{Y(s)}{U(s)} = \frac{K_s}{s} e^{-T_d s}. \quad (48)$$

Parameters of the plant model identified in the vicinity of the selected operating point and applied in works [31], [64] can be given as

$$K_s = 0.01; \quad T_d = 0.3s. \quad (49)$$

The identified internal feedback coefficient $a = 0.05$s$^{-1}$, corresponding to a time constant $T = 1/a = 20s$, will be neglected. (From this moment on, we stop emphasizing that these are the parameters of the model and we simply write $K_s$ and $T_d$ instead of $K_m$ and $T_m$.)

B. IPDT-BASED RM-DTCs

Firstly, since the RM-DTC designed in [32] with a second-order low-pass filter (used in the disturbance observer and the disturbance feedforward) proved to be insufficient for a noisy environment, its extension had to be developed to more effectively reduce noise. To illustrate the practical aspects of such a design continuing from simpler to more complex
setup, let us first consider controller design based on a first-order integral model (i.e., $T_d = 0$)

$$F(s) = \frac{Y(s)}{U(s)} = \frac{K_s}{s} \quad (50)$$

In this case, use just the P controller

$$u_w = K_P (w - y); \quad K_P = 1/(T_c K_s)$$

to generate the setpoint feedforward control. For the disturbance feedback the parameters $b^n$ corresponding to the total filter order $n$ with

$$Q_1(s) = \frac{1}{1 + T_o s}; \quad C_i(s) = \frac{1 + b^n s}{(1 + T_o s)^{n-1}}; \quad n \geq 2, \quad (52)$$

the disturbance compensation signal can be calculated as

$$U_{if}(s) = S_{yu}(s) Y(s) - S_{uu}(s) U(s);$$

$$S_{yu}(s) = \frac{Y(s)}{1 + b^n s};$$

$$S_{uu}(s) = \frac{1}{(T_o s + 1)^n}. \quad (53)$$

From (53) we get the “stabilized” disturbance response

$$F_u = \frac{1}{1 - S_{uu}} F,$$

$$F^n_{iy}(s) = \frac{1}{1 + F_u S_{yu} F} = \frac{K_s (T_o s + 1)^n - (1 + b^n s)}{s (T_o s + 1)^n} \quad (54)$$

To simplify the calculation of a stable $F^n_{iy}$ let us again use the variable $p = T_o s$ and parameters $\beta^n = b^n / T_o, \quad \kappa = K_s T_o$, when

$$F^n_{iy}(p) = \kappa \frac{(p + 1)^n - (1 + \beta^n p)}{p (p + 1)^n} \quad (55)$$

Since an elimination of the numerator coefficient at $p$ yields

$$\beta^n = n; \quad b^n = n T_o \quad (56)$$

and due to this choice the lowest two numerator coefficients in $F^n_{iy}(p)$ disappear,

$$F^n_{iy}(p) = \kappa \frac{\sum_{j=2}^{n} \left( \begin{array}{c} n \\ j \end{array} \right) p^{j-1}}{(p + 1)^n} \quad (57)$$

Therefore, by increasing $n$ we get gradually transfer functions

$$F^2_{iy}(p) = \kappa p \frac{(p + 1)^2}{p + 1} \quad \text{(58)}$$

$$F^3_{iy}(p) = \kappa p \frac{(p + 1)^3}{p + 3} \quad \text{(59)}$$

$$F^4_{iy}(p) = \kappa p \frac{p^2 + 4p + 6}{(p + 1)^4} \quad \text{(60)}$$

Instead of calculating the $F^n_{iy}(p)$ numerator by comparing the coefficients at individual powers of $p$, the tuning $\beta^n$ and coefficients $\alpha^n_p$ could also be evaluated to get zero values of the numerator (55) derivatives according to $p = 0$, when from

$$N^0_n(p) = (p + 1)^n - (1 + \beta^n p) = 1 + \alpha^n_1 p + \alpha^n_2 p^2 + \alpha^n_3 p^3 + \ldots + \alpha^n_n p^n - (1 + \beta^n p),$$

$$N^j_n(p) = \frac{d N^n_{j-1}(p)}{dp}; \quad j = 1, 2, \ldots, n, \quad (60)$$

follows

$$\beta^n = N^0_n(0); \quad \alpha^n_j = \frac{N^j_n(0)}{j (j - 1) \ldots 1}; \quad j = 2, 3, \ldots, n. \quad (61)$$

The calculation according to (61) is preferred in the case of time-delayed integrator, when the numerator of $F^n_{iy}(p)$ is

$$N^0_n(p) = (p + 1)^n - (1 + \beta^n p) e^{-\tau d p}. \quad (62)$$

With the help of computer algebra system we can then easily get

$$b^n = n T_f + T_d; \quad F^n_{iy}(s) = K_s \frac{N^n_{iy}(s)}{(T_o s + 1)^n} \quad (63)$$

where

$$n = 2; \quad N^2_{iy}(s) = T_d^2 + 2 T_o T_d + T_d^2 / 2;$$

$$n = 3; \quad N^3_{iy}(s) = A_{31} s + A_{30};$$

$$A_{31} = T_d^3 - 3 T_d^2 T_o - \frac{9}{2} T_o T_d^2 - \frac{5}{6} T_o^3;$$

$$A_{30} = 3 T_d^3 + 3 T_d T_d + T_d^2 / 2;$$

$$n = 4; \quad N^4_{iy}(s) = A_{42} s + A_{41} s + A_{40};$$

$$A_{42} = 4 T_d^4 - 6 T_d^3 T_o - 6 T_d^2 T_o^2 - \frac{5}{3} T_o^3;$$

$$A_{41} = 6 T_d^4 + 4 T_d T_d + T_d^2 / 2;$$

$$n = 5; \quad N^5_{iy}(s) = A_{53} s^3 + A_{52} s^2 + A_{51} s + A_{50};$$

$$A_{53} = T_d^5 - 5 T_d^4 T_o + 5 T_d^3 T_o^2 - 5 T_d^2 T_o^3 - 25 T_d T_o^4 + 49 / 120 T_o^5;$$

$$A_{52} = 5 T_d^5 - 10 T_d^4 T_o + 5 T_d^3 T_o^2 + 35 T_d^2 T_o^3 + 17 / 24 T_o^4;$$

$$A_{51} = 10 T_d^5 - 10 T_d^4 T_o - 15 / 2 T_d^3 T_o^2 + 5 T_d^2 T_o^3 - 6 / 3 T_o^4;$$

$$A_{50} = 10 T_d^5 + 5 T_d^4 T_o + T_d^2 / 2. \quad (64)$$

From $F^n_{iy}(s)$ it is possible to calculate $F_i(s)$ according to (20).

**C. EXPERIMENTS BASED ON IPDT MODELS**

Returning now to the experiments on a system with parameters (49) and a minimum filter order $n = 2$, with the time constant chosen for simplicity as $T_o = T_d$ and the gain of the feedforward and the stabilization controllers $K_P = K_P$

$$K_P = \frac{1}{e K_s T_d} \quad (65)$$

corresponding to the double real dominant closed loop pole.

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The first aspect we will notice is the consideration of the constraints of the control signal. If we design the whole scheme in the linear domain and include the output limitation on the range \( u \in [0, 100] \) only at the output of the controller, the transients will be with a typical over-regulation at the output (see Figure 9). To avoid overshooting due to the control saturation, according to Figure 12 in [32], the scheme with feedforward loop feedback from the output of Dynamic Saturation block might be used. However, this scheme works correctly only for \( K_p = K_p \), which in our case may not be suitable, because with regard to the transmission of measurement noise we will try to reduce the value of \( K_p \) (while maintaining the loop stability) as much as possible. Because when using such a scheme with \( K_p < K_p \), a permanent error occurs at the output, we prefer to modify the scheme with Dynamic Saturation block according to Figure 10. In this scheme, the output from the stabilizing controller has the highest priority, which ensures monotonic responses after setpoint step changes.

Responses corresponding for \( K_p = K_p \) (65) to the setpoint change from \( w = 31 \) to \( w = 37^\circ\text{C} \) at \( t = 150\text{s} \) and to the fan voltage step from \( d_i = 5 \) to \( d_i = 15 \) at \( t = 250\text{s} \) in Figure 11 show a monotonic output transient to the new setpoint variable. The disturbance step caused by the fan voltage change from \( d_i = 5 \) to \( d_i = 15 \) at \( t = 250\text{s} \) is practically not visible in the output, just in the control signal values. Due to the inclusion of internal plant feedback in the equivalent disturbance, the reconstructed disturbance significantly changes its value after the setpoint change. Its values change even after reaching the required output, which is a manifestation of slow heat transfer by convection. Significant changes in the reconstructed disturbance occur after a change in fan power.

As it is clear from the course of the controller output \( u \), such a nearly ideal output response was achieved by a strongly noisy controller output. It is also evident that the stabilizing controller output \( u_d \) does not completely remain at zero and, especially during the transition to the new setpoint value, it acquires considerable values. However, given the significant difference between the IPDT model used and the physical nature of the controlled process, this may not be surprising. In other words, with significant differences of the plant and model dynamics, the RM-DTC behaves the same as traditional DOB-based solutions with a stabilizing controller. In such situations RM-DTC forces, with the help of stabilizing controller, the controlled system dynamics to cope the selected model [45]. However, the RM-DTC allows the deviations between expected and actual behavior to be evaluated separately with respect to setpoint tracking, disturbance reconstruction and compensation, and overall stabilization, which can be used for more detailed analysis of system behavior and optimization of controller design.

Transients corresponding to a slightly decreased value \( T_o = 0.25 \text{s} \), \( K_p = K_p \) (65), \( n \in [2, 5] \) and with the disturbance observer and reference model tuning (63)-(64) are in Figure 12. Due to the decoupling of the setpoint and disturbance responses, the DOB filter does not significantly influence the shapes of setpoint step responses. As a result of a constant relative order of the disturbance reference model \( F_i(s) \), the increase in DOB filter order \( n \) does not immediately reduce the impact of noise. Therefore, we will look for other ways to reduce the noise impact. In a nominal circuit with an integrative plant, it is sufficient for stability to work with any small positive value \( K_p \). In the next experiment, we therefore reduce the gain of the stabilizing controller to the value \( K_p = K_p / 10 \). The corresponding transients on Figure 13 show that the rise of the output to the required setpoint value slowed down a bit by decreasing \( K_p \). However, it still depends only slightly on the order of disturbance compensation filters \( n \). A comparison of performance measures in Figure 14 shows that with the reduction of \( K_p \), the values of \( IAE_p \) increased slightly, but all other performance values decreased significantly. A similar effect could be achieved by simplifying the disturbance reference model, in which the coefficients at higher powers would be neglected.

**Remark 6 (RM-DTCs and noise elimination):** Decoupled setpoint tracking and disturbance rejection enabled by the use of RM-DTCs focus on reducing the stabilizing controller activity by zeroing its input. This is achieved by decreasing the measured output impact by opposite expected deterministic signals added to the controller input. This is partly reminiscent of the methods known from acoustics as active noise control (ANC), noise cancellation (NC), or active noise reduction (ANR). They are reducing an unwanted permanent, or periodic sound by the addition of a second sound produced in antiphase. Although it might be interesting, we did not deal with the compensation of the effect of steady periodic signals in this work.
FIGURE 10. IPDT: Reference model control scheme for the thermo-opto-mechanical systems TOM1A proposed by modification of the scheme in Figure 12 in [32], by considering higher order noise attenuation filters in the disturbance feedforward (52) and in the disturbance response $F_n(s)$ (63) applied in Matlab/Simulink with $K_m = K_s$, $T_m = T_d$, $a_m = 0$ (49); $F_n(s) = N_f(s)/P_{n1}(s)$; $P_{n1}(s) = (1 + T_o s)^{-1}$

D. EXPERIMENTS BASED ON DIPDT MODELS

The TOM1A system can be extended to a system with dominant second-order dynamics by including an additional integrator to the input. This, when controlled by a limited-range signal $u_2(t)$, produces a control signal $u(t)$ with the limited rate of change at its output. Of course, $u(t)$ must meet the limits of the admissible 0-100 TOM1A input range. To (at least partially) avoid the windup problem, we will use the Integrator Limited block (see Figure 15) in the Matlab/Simulink program for this purpose. We will assume the system model (1) with parameters (49).

Because the output $u(t)$ of the input integrator will obviously not be affected by external influences, we will expect the values of the reconstructed disturbance to be zero at least at steady states. Thanks to the use of controllers with a derivative action, we can also expect an increased effect of measurement noise. Therefore we choose $T_o = 0.5s$.

An example of measured responses of individual variables is shown in the Figure 16. In their brief evaluation, it is necessary to mention:

1) Thanks to the use of an integrator with limited output, the course of the controlled output is overshooting. Although its shape resembles the situation in Figure 9, we now need significantly different approaches to eliminate this overshoot: limiting the signal $u$ is actually a limitation of the state variable of the controlled second-order system. In such a situation, the parallel work of two controllers can be used by interconnecting them using the selector based “lowest wins” strategy [66].

In addition, when wishing to control with a limited input $u_2$, non-linear algorithms described e.g. in works [5], [10], [41]–[43] should be used. However, a more detailed discussion of this issue will require a separate contribution.

2) From the point of view of the design of the controller with decoupled setpoint tracking and disturbance rejection, it is important to check the output of the stabilization controller. Although it is not completely zero (due to the deviations of the model used and the actual process and measurement noise), the stabilization signal $u_s$ is relatively small compared to the carrier signals of the design $u_{wf}$ and $u_{if}$. Experiments show that $u_s$ can be further reduced without compromising the overall stability by reducing the gains of the stabilizing regulator, which also reduces the noise level of the signals $u_2$ and $u$.

3) As we assumed, in steady states, the reconstructed disturbance is really zero and its course is completely different from the reconstructed disturbance based on the IPDT model. It is also worth noting that switching on the fan represents now, in terms of the reconstructed disturbance, a far smaller intervention than changing the setpoint.

4) In addition to the already mentioned issues concerning the elimination of output overshooting and the design of transients with rate-limited transients, it would also be interesting to test the use of higher order filters and their impact on noise attenuation and closed-loop
VI. CONCLUSIONS

The generalisation of the RM-DTC design methodology from the work [32], based on the IPDT models, (1) by the case of disturbance feedforward with higher order filters, (2) the case of DIPDT models and (3) the addition of experiments with real-time temperature control allowed to show the advantages and the current limitations of the methodology.

The advantages of RM-DTCs include (1) the extension of the number of degrees of freedom in controller design and (2) the decoupled design of setpoint tracking and disturbance rejection dynamics, together with (3) the modified controller structure for separate implementation of setpoint tracking and disturbance rejection control.

The introduction of a superior stabilising controller, complemented by reference models for setpoint tracking and disturbance rejection, enabled (4) the generalisation of the use of IMC structures to control unstable circuits while maintaining the reconstructed disturbance signal.

The separate evaluation of setpoint tracking and disturbance rejection together with the change of the controller signal (controller noise) opens up (5) new possibilities in terms of diagnosis, monitoring and optimisation of control loops, which is extremely important especially with regard to fulfilling the objectives of Industry 4.0 and 5.0.

RM-DTCs are particularly suitable for (6) high-end applications with high performance and robustness requirements due to their nature. On the other hand, the structure of the RM-DTC controller is more complex than the structure of common IMC controllers and therefore also requires a more complex control implementation for constrained processes (with input, state and output constraints), for processes with periodic disturbances, nonlinearities, etc.

(7) It is yet worth noting that the closed loop analysis based on the RM-DTC methodology can also be beneficial when finally results in using simpler controller structures.

Compared to 2DOF structures SP, which are based on the reconstruction of the output disturbance by a parallel model recalculated for an input disturbance and modified for internal stability by eliminating the reconstructed disturbance, RM-DTC seems to be more complex. Its main advantage...
is that the reconstructed disturbance signal is preserved. Another advantage of the proposed approach is the separation of setpoint tracking and disturbance rejection to two different branches, which are supplemented by the reference models and the stabilising controller, which simplifies the debugging and parallelization of the controller program, which can be beneficial when programming fast embedded-control based applications.

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FIGURE 16. DiPDT: Transients corresponding to the Simulink model in Figure 3 with the controlled plant according to Figure 15 for the setpoint change from \( w = 31 \) to \( w = 37 \) °C at \( t = 150 \% \) for the fan voltage step from \( d_1 = 0 \) to \( d_1 = 15 \) at \( t = 250 \% \); \( T_{dc} = 0.1\%; T_{d} = 0.3\%; n = 4; K_c = 0.01; \) both PD controllers tuned according to (27), \( T_p = T_a/2 \). 

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