Remarks on possible local parity violation in heavy ion collisions

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Abstract

In this note we discuss some observations concerning the possible local parity violation in heavy ion collisions recently announced by the STAR Collaboration. Our results can be summarized as follows (i) the measured correlations for same charge pairs are mainly in-plane and not out of plane, (ii) if there is a parity violating component it is large and, surprisingly, of the same magnitude as the background, and (iii) the observed dependence of the signal on the transverse momentum ($p_t$) is consistent with a soft boost in $p_t$ and thus in line with expectations from the proposed chiral magnetic effect.

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1 Introduction

Recently the STAR collaboration announced [1] the results on possible local parity violation in heavy ion collisions. In Ref. [2, 3] it was argued that in the hot dense matter created in heavy ion collisions local, instanton or sphaleron, transitions to QCD vacua with different topological charge may result in metastable domains, where parity is locally violated.

In this paper we will solely concentrate on an analysis of the experimental results. We will neither attempt to provide alternative explanations for the observed correlations, such as e.g. given in Ref. [4] nor will we discuss the likelihood that the proposed effect may occur in a heavy ion collision. For detailed discussion of the underlying mechanism and the latest theoretical review of this problem we refer the reader to Ref. [3].

The phenomenon due to local parity violation, which is of relevance for the discussion here, is the so called chiral magnetic effect [2, 3]. It leads to the separation of negatively and positively charged particles along the system’s angular momentum (or equivalently the direction of the magnetic field) into two hemispheres separated by the reaction plane. As a result, the system exhibits a electric current along the direction of the angular momentum, and thus breaks parity locally in a given event. However, since instanton (sphaleron) and anti-instanton (anti-sphaleron) transitions occur equally likely, the chiral magnetic current is either aligned or anti-aligned with the angular momentum. As a result, the expectation value of any parity odd observable, such as \( \langle \vec{j}_{\text{CM}} \vec{I} \rangle \) vanishes. Here \( \vec{j}_{\text{CM}} \) is the chiral-magnetic current and \( \vec{I} \) is the angular momentum. Consequently, a direct measurement of parity violation even in a small subsystem is impossible. However, one may attempt to identify the existence of these parity violation domains by studying the fluctuations or the variance of a parity-odd observable. Since the variance of a parity-odd observable is parity even, in principle other, genuinely parity-even, effects may contribute, and one needs to separate those carefully before being able to draw any conclusions about the existence of local, parity violating domains.

In Ref. [5] Voloshin proposed a method to measure the variance of a parity odd observables. He suggested to measure the following correlator \( \langle \cos(\phi_{\alpha} + \phi_{\beta} - 2\Psi_{RP}) \rangle \), where \( \Psi_{RP}, \phi_{\alpha} \) and \( \phi_{\beta} \) denote the azimuthal angles of the reaction plane and produced charged particles respectively, see Fig. 1.

As we will discuss in more detail in Section 2 this rather involved correlation function has the advantage that correlations which are independent of the reaction plane do not contribute. As a result, a large fraction of the expected background should cancel. Recently the STAR collaboration has reported the measurement of the above correlation function [1], both integrated over the entire acceptance as well
Figure 1: The transverse plain in a collision of two heavy ions. $\Psi_{RP}$, $\phi_\alpha$ and $\phi_\beta$ denote the azimuthal angles of the reaction plane and produced charged particles, respectively.

as differential in transverse momentum and pseudo-rapidity.

This paper is organized as follows. In the following section we will analyze the integrated STAR result and will suggest additional measurements necessary to further clarify the situation. In the subsequent section, we will concentrate on the $p_t$ differential results and explore to which extent they are consistent with the expected soft phenomena due to the chiral magnetic effect.

2 The integrated signal

In Ref. [1] the details of the STAR measurement are given. Among other things STAR shows the results for $\langle \cos(\phi_\alpha - \phi_\beta) \rangle$ and for $\langle \cos(\phi_\alpha + \phi_\beta - 2\phi_c) \rangle$, where $\phi_{\alpha,\beta,c}$ are the azimuthal angles of the produced charged particles. The paper gives reasonable arguments that

$$\langle \cos(\phi_\alpha + \phi_\beta - 2\phi_c) \rangle = \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle v_{2,c},$$

where $\Psi_{RP}$ is the angle of the reaction plane, and $v_{2,c}$ characterizes the elliptic anisotropy for the particle with angle $\phi_c$.

For the rest of the discussion we will assume that the relation (1) is correct. As a consequence, we will work in a frame where the reaction plane is defined by the $x$–$z$ coordinates and where the $y$ direction is perpendicular to the reaction plane. In other word we work in a frame where $\Psi_{RP} = 0$, see Fig. 1. Furthermore, since $\langle \cos(\phi_\alpha - \phi_\beta) \rangle$ is independent of the direction of the reaction plane, it will be the same also in the frame where the reaction plane is specified e.g., $\Psi_{RP} = 0$. Thus

\footnote{Indeed $\langle \cos(\phi_\alpha - \phi_\beta) \rangle \equiv \langle \cos([\phi_\alpha - \Psi_{RP}] - [\phi_\beta - \Psi_{RP}]) \rangle$}
within our frame we have to consider the following two-particle correlations:

\[
\langle \cos(\phi_\alpha - \phi_\beta) \rangle = \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle + \langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle,
\]
\[
\langle \cos(\phi_\alpha + \phi_\beta) \rangle = \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle - \langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle.
\] (2)

STAR has measured both these correlation functions for same sign, \((+, +), (-, -),\) and opposite sign, \((+, -),\) pairs of charged particles. Qualitatively the data for \(Au + Au\) collisions can be characterized as follows.

- For same sign pairs:

\[
\langle \cos(\phi_\alpha + \phi_\beta) \rangle_{\text{same}} \simeq \langle \cos(\phi_\alpha - \phi_\beta) \rangle_{\text{same}} < 0.
\] (3)

Using Eq. (2) this implies

\[
\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{\text{same}} \simeq 0,
\]
\[
\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{\text{same}} < 0.
\] (4)

- For opposite sign pairs we find that

\[
\langle \cos(\phi_\alpha + \phi_\beta) \rangle_{\text{opposite}} \simeq 0
\]
\[
\langle \cos(\phi_\alpha - \phi_\beta) \rangle_{\text{opposite}} > 0.
\] (5)

Again, using Eq. (2), this means

\[
\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{\text{opposite}} \simeq \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{\text{opposite}} > 0.
\] (6)

The actual data decomposed into the above components are shown in Fig. 2.

The fact that for same charge pairs the sinus-term in Eq. (4) (see Fig. 2) is essentially zero whereas the cosine term is finite, tells us that the observed correlations are actually in plane rather than out of plane. This is contrary to the expectation from the chiral magnetic effect, which results in same charge correlation out of plane. In addition, since the cosine term is negative, the in-plane correlations are stronger for back-to-back pairs than for small angle pairs. Second, we see that for opposite charge pairs the in- and out-of-plane correlations are virtually identical. This is hard to comprehend, given that there is a sizable elliptic flow in these collisions. At present, there is no simple explanation for neither of these observations. However, they may be explained by a cluster model, which requires several, not unreasonable, assumptions [4].

One may ask if there is room for a parity violating component if for the same sign \(\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{\text{same}} \simeq 0,\) i.e. the signal is in-plane rather than out of plane. Following the argument of Ref. [1, 5], we can always write

\[
\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{\text{same}} = B_{\text{out}} + P,
\] (7)
Figure 2: Correlations in-plane \( \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle \) and out of plane \( \langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle \) for same and opposite charge pairs in Au + Au collisions. As can be seen the correlations for same charge pairs are mainly in-plane.

where \( P \) is the part of the correlation which is caused by the parity violation (at this stage we do not claim that \( P \neq 0 \)) and \( B_{\text{out}} \) represents all other contributions by correlations projected on the direction perpendicular to the reaction plane. Denoting the correlations in-plane \( \langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{\text{same}} \) by \( B_{\text{in}} \) we obtain:

\[
\langle \cos(\phi_\alpha + \phi_\beta) \rangle_{\text{same}} = [B_{\text{in}} - B_{\text{out}}] - P,
\]

\[
\langle \cos(\phi_\alpha - \phi_\beta) \rangle_{\text{same}} = [B_{\text{in}} + B_{\text{out}}] + P.
\]

(8)

The advantage of \( \langle \cos(\phi_\alpha + \phi_\beta) \rangle \) is obvious. The background is \( B_{\text{in}} - B_{\text{out}} \), meaning that all correlations that do not depend on the reaction plane orientation
cancel. The STAR collaboration studied many known sources of reaction plane
dependent correlations and all effects produce $B_{in} - B_{out}$ which is much smaller
than the observed signal. We note, however, that at present the background is
not understood since none of the present models is able to explain the value of
$\langle \cos(\phi_\alpha - \phi_\beta) \rangle$.

Following the above argument, however, immediately implies that (using Eq. (4)
and Eq. (7))

$$ P \simeq -B_{out} \simeq -B_{in}, $$

i.e., the parity violating effect has to be precisely of the same magnitude as all other,
standard correlations. This relation is quite an unexpected coincidence. It means
that the parity signal is quite strong, and consequently should also be visible in
$\langle \cos(\phi_\alpha - \phi_\beta) \rangle_{\text{same}}$ if the background is well understood.

In our view, it is mandatory to explore if the relation, Eq. (9), is just a co-
incidence or an indication of potential problems with the present interpretation of
the data. To answer this question it is essential to analyze the correlation function
$\langle \cos(\phi_\alpha - \phi_\beta) \rangle_{\text{same}}$ differentially in transverse momentum and pseudo-rapidity as it
has been already done for $\langle \cos(\phi_\alpha + \phi_\beta) \rangle_{\text{same}}$. Should relation (9) persist also for
the differential correlations, one would have to conclude that the proposed parity
violating effect is not seen in the data.

3 Transverse momentum dependence

The STAR collaboration has also presented [1] the measurement of $\langle \cos(\phi_\alpha + \phi_\beta) \rangle$
in mid-central $Au + Au$ collisions as a function of
$p_+ = (p_{t,\alpha} + p_{t,\beta})/2$ and $p_- = |p_{t,\alpha} - p_{t,\beta}|$, where $p_{t,\alpha}$ and $p_{t,\beta}$ are the absolute values of the particles momenta.
Qualitatively the data can be characterized as follows:

- For same sign pairs in the range $0 < p_+, p_- < 2.2$ GeV:

$$ \langle \cos(\phi_\alpha + \phi_\beta) \rangle_{p_+, \text{same}} \propto p_+ $$
$$ \langle \cos(\phi_\alpha + \phi_\beta) \rangle_{p_-, \text{same}} \approx \text{const.} $$

- For opposite sign pairs the signal vs $p_+$ and $p_-$ is consistent with zero.

One would expect [1, 2] that the parity violating signal should be a soft, low $p_t$
phenomenon. Thus the observed increase of the signal for same sign pairs with $p_+$
seems to be inconsistent with the chiral magnetic effect. As we will show such a

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\[^2\text{In the present Section we are only interested in the } p_t \text{ dependence of the signal, not in the}\]

overall normalization.
conclusion is not necessarily correct and the true signal may indeed be consistent with the expected low \( p_t \) dynamics.

Indeed, by definition

\[
\langle \cos(\phi_\alpha + \phi_\beta) \rangle = \frac{N_{\text{corr}}}{N_{\text{all}}},
\]

where \( N_{\text{corr}} \) is the number of correlated pairs \([\text{via } \cos(\phi_\alpha + \phi_\beta)]\) and \( N_{\text{all}} \) is the number of all pairs. The latter can be easily approximated by \( |p_+ - [p_{t,\alpha} + p_{t,\beta}]| \):

\begin{align*}
N_{\text{all}}(p_+ & ) \propto \int d^2p_{t,\alpha}d^2p_{t,\beta} \exp \left( -\frac{p_{t,\alpha}}{T} \right) \exp \left( -\frac{p_{t,\beta}}{T} \right) \delta \left( 2p_+ - [p_{t,\alpha} + p_{t,\beta}] \right) \\
& \propto p_+^3 e^{-2p_+/T} \tag{12}
\end{align*}

and

\begin{align*}
N_{\text{all}}(p_- & ) \propto \int d^2p_{t,\alpha}d^2p_{t,\beta} \exp \left( -\frac{p_{t,\alpha}}{T} \right) \exp \left( -\frac{p_{t,\beta}}{T} \right) \delta \left( p_- - |p_{t,\alpha} - p_{t,\beta}| \right) \\
& \propto T^2 e^{-|p_-/T|} (p_- + T), \tag{13}
\end{align*}

where in the following calculations we take \( T = 0.22 \text{ GeV}^3 \).

The calculated distributions of all pairs vs \((p_{t,\alpha} + p_{t,\beta})/2\) and \(|p_{t,\alpha} - p_{t,\beta}|\) are presented in Fig. 3. It is worth noticing that both functions are concentrated in the small \( p_t \) region, reflecting typical thermal distributions for \( p_- \) and \( p_+ \). Due to the soft nature of chiral magnetic effect, one expects that the distributions in \( p_- \) and \( p_+ \) for the correlated particles should not differ much from the underlying thermal distributions. This is indeed the case as we will demonstrate next.

In order to estimate the distribution of correlated same sign pairs it is sufficient to multiply Eq. (10) by the expressions (12) and (13), respectively. Consequently we obtain

\begin{align*}
N_{\text{corr}}(p_-) & \propto N_{\text{all}}(p_-), \\
N_{\text{corr}}(p_+) & \propto p_+ N_{\text{all}}(p_+). \tag{14}
\end{align*}

As can be seen the dependence of the number of correlated same pairs vs \(|p_{t,\alpha} - p_{t,\beta}|\) is identical to the dependence of all pairs presented in Fig. 3. Clearly the signal is concentrated in the low \( p_t \) region and indeed is unchanged from a thermal distribution. In Fig. 4 the dependence of the number of same sign pairs vs \((p_{t,\alpha} + p_{t,\beta})/2\) is compared with the dependence of all pairs (previously shown in Fig. 3). We find that the momenta of correlated particles are slightly shifted to the higher \( p_t \) and the shape is roughly similar. The momentum shift required by the

\[ \text{It corresponds to the average transverse momentum of the pions } \langle p_t \rangle = 0.45 \text{ GeV}. \]
data is $\delta p_+ \simeq 150$ MeV which could conceivably be due to the large magnetic field, although it is somewhat on the high end of what one would naively expect from electromagnetic phenomena.

### 4 Conclusions

In this note we have discussed several aspects of the recent measurement of possible local parity violation in $Au + Au$ collisions by the STAR Collaboration. We made the following three observations:

(i) For particles with the same charge STAR sees large negative correlations in-plane $\langle \cos(\phi_\alpha) \cos(\phi_\beta) \rangle_{\text{same}}$ and very small correlations out of plane $\langle \sin(\phi_\alpha) \sin(\phi_\beta) \rangle_{\text{same}}$. For opposite sign correlations in-plane and out-plane are both positive and of the same magnitude.
(ii) If there is indeed a parity violating component in the STAR data it has to be of the same magnitude as all other, “trivial” correlations projected on the direction perpendicular to the reaction plane. This may be a pure coincidence or an indication that the present interpretation of the data as a signal for local parity violation needs to be revised. To investigate this problem in more detail we need differential distribution (vs pseudo-rapidity or transverse momenta) of $\langle \cos(\phi_\alpha + \phi_\beta) \rangle$ and $\langle \cos(\phi_\alpha - \phi_\beta) \rangle$ at the same time.

(iii) We have also argued that the distribution of the number of correlated pairs is concentrated in the low $p_t$ region i.e. $p_t < 1$ GeV. It is not inconsistent with the predictions of the chiral magnetic effect.

At present, the data from the STAR Collaboration does not allow for a definitive conclusion about the presence of local parity violation. The measurement of the correlation function $\langle \cos(\phi_\alpha - \phi_\beta) \rangle$ differential in transverse momentum and pseudo-rapidity is absolutely essential to further distinguish between trivial correlations and those due to the chiral magnetic effect.

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