Calculated model of wedge-shaped sliding supports in turbulent friction regime

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Abstract—The solution of the problem has been found on the basis of a system of equations describing the motion of an incompressible liquid electrically conductive lubricant for the case of a "thin layer", the continuity equation and the expression for the dissipation rate of mechanical energies to determine the function caused by the melt of the guide surface coated with a melt of a fusible coating. Asymptotic solution of a system of differential equations taking into account the boundary conditions on the surface of the guide, slider and contour are found in the form of series in powers of the small parameter $K$, caused by the melt and the rate of dissipation of mechanical energy. To determine the velocity and pressure fields in the lubricating and molten layer, a precise self-similar solution for the zero and first approximations is found. As a result of finding the exact self-similar solution, the value of the function, caused by the melt of the guide is found.

The influence of the following parameters has been estimated: caused by the presence of an electric field, Hartmann number, caused by the melt and the rate of dissipation of mechanical energy on the main operating characteristics of the thrust sliding bearing (bearing capacity and frictional force).

Difference from the existing design models of sliding supports operating on the melt of a low-melting coating is the consideration of a whole range of variable factors allowing increasing the accuracy of models approximating to the real needs of practice.

The results of numerical analysis show that the design models of thrust sliding bearings have been significantly refined as a result of additional simultaneous consideration in their development of the dependence on the hydrodynamic pressure and temperature of such important factors as electrical conductivity, as well as the effects of magnetic induction and electric field strength.

Keywords—melt; low-melting coating; wedge-shaped support; electrically conductive lubricant; energy dissipation rate

1. INTRODUCTION

The main friction knot of modern machines is sliding bearings, lubricated with liquid lubricants. For modern engineering practice, sliding bearings are designed taking into account the increase in static and dynamic loads. At high load and speed regimes, the use of mineral oils as lubricants causes certain difficulties. They are realized in the form of scratching and seizures of I and II kind. To avoid these drawbacks, especially when leaking or evaporation of the lubricant from the bearing, a backup lubricant may be the coating of the bearing surface of a low-melting metal bearing sleeve. Lubrication with liquid metals is used at temperatures at which conventional lubricating media undergo irreversible physicochemical properties. The advantage of lubrication with the melt is that the lubricant is formed in the contact area where it is necessary. Melting delivers a sufficient amount of lubricant material to the friction zone, there are no mechanical and structural difficulties associated with its feeding. Lubrication with a melt was studied in many applied problems, in particular, in the processes of forming and cutting metals [1-12]. A large number of works have been devoted to hydrodynamic calculation of sliding bearings in the absence of a lubricant and taking into account the dependence of the viscosity of the lubricant on pressure. A significant drawback of the friction pair operating on melt lubrication is a low load-bearing capacity. In addition, the lubrication process with the lubricant is not self-sustaining.

The works are devoted to theoretical analysis of the work of radial and thrust sliding bearings in the presence of a lubricant of a tribological system caused by the melt of a surface covered with a low-melting metallic melt when taking into account the dependence of the viscosity of the lubricant on pressure [13-16].

A number of works are dedicated to the development of the design model of radial and thrust sliding bearings, taking...
into account the rheological properties of micropolar, viscoelastic lubricant, taking into account the dependence of the viscosity of the lubricant on the pressure and melt of the fusible coating [17-24].

In this paper, the solution of the problem is given for the case when the lubricant has the properties of the electrically conductive lubricant.

Thus, the development of theoretical bases for the calculation of tribosystems lubricated by a metallic melt, taking into account the rheological properties of an electrically conductive lubricant under the condition of the action of an electromagnetic field, the justification of a dynamics and lubrication model that outstrips the real processes occurring in the lubricating layer, and the creation of algorithms and software for solving practical tasks, increasing the reliability of machines and mechanisms determines the relevance of this article.

II. TASK SETTING

The turbulent flow of a viscous electrically conductive lubricant between an inclined slider and a guide is considered. It is assumed that the surfaces of the slider and the guide are separated by a layer of a lubricant having electrically conductive properties, the slider is stationary, and the guide made of the material with a low melting point moves toward the narrowing of the gap at a speed $u^*$ (Fig. 1).

![Fig. 1. Working scheme](image)

The dependence of the viscosity and electrical conductivity of a liquid lubricant is given in the form:

$$
\mu^* = e^{\alpha^*\rho^* - \beta^* T}, \quad \sigma^* = e^{\alpha^*\rho^* - \beta^* T},
$$

(1)

where $\mu^*$ is the coefficient of a dynamic viscosity of the lubricant; $\rho^*$ is hydrodynamic pressure in the lubricating layer; $\alpha^*, \beta^*$ are experimental constant values; $\sigma^*$ is electric conductivity of the lubricating material, $T$ is temperature in lubricating layer.

The conditions for the motion of an infinitely wide slider are considered under the following assumptions:

1. The liquid medium is a viscous incompressible fluid.
2. All the heat released in the lubricating film goes to the melting surface of the material of the guide.
3. The effect of turbulence can be reflected using the coefficient $j > 1$, to which the viscosity should be multiplied in order to obtain the effective viscosity value. In addition, it is assumed that this coefficient can be expressed as the following function of the Reynolds number $j = 0,0139 \text{Re}^{0.655}$, where $\text{Re} = \rho u^* h^* / \mu^*$ is the Reynolds number, $\mu^*$ is the dynamic viscosity, $h^*$ is the film thickness in the initial section, $\rho$ is density, $u^*$ is the movement speed slider, $l$ is the length of the fixed working surface of the bearing (slider).

III. INITIAL EQUATIONS AND BOUNDARY CONDITIONS

As initial equations, we consider the dimensionless equations of motion of a lubricant possessing electrically conductive properties for the case of a "thin layer" with allowance for (1), the continuity equation, and the formula for the rate of dissipation of mechanical energy for determining the function $\Phi(x)$, caused by the molten contour of the surface of the guide:

$$
\frac{\partial^2 v}{\partial y^2} + \frac{1}{\rho e^{\alpha^* - \beta^* T}} \frac{dp}{dx} + N v - A \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0,
$$

$$
\frac{d\Phi(x)}{dx} = K \int_{-\Phi(x)}^{h(x)} \left( \frac{\partial v}{\partial y} \right)^2 dy,
$$

(2)

where $u, v$ are components of the velocity vector of the lubricating medium; $N = \frac{\sigma_0 B h_0^2}{\mu_0}$ is Hartmann number, $B = \{0; B_z; 0\}$ is magnetic induction vector, $E = \{0; 0; E_z\}$ is the electric field strength vector, $B_z$ is the component of the magnetic induction vector, $E_z$ is the component of the electric field strength vector, $A = \frac{\sigma_0 B L' h_0^2}{\mu_0 u^*}$ is the magnitude caused by presence of an electric field, $K = \frac{2 \mu_0 u^* l}{h_0 L'}$ is the parameter due to the melt and the rate of dissipation of mechanical energy, $L'$ is the specific heat of fusion per unit volume.

At that, the values $B = \{0; B_z; 0\}$ and $E = \{0; 0; E_z\}$ are assumed to be given and satisfy the Maxwell equations:

$$
\text{div} \, B = 0, \quad \text{rot} \, E = 0.
$$

(3)
Values $B$, $E$ and the flow velocity of the electrically conductive lubricant are such that the magnitude of the current strength can be neglected on the electric and magnetic fields.

In a Cartesian coordinate system $x'y'$ the equation of the contour of the slider and the molten surface of the guide can be written in the form.

\[ y' = h_0 + x' \tan \alpha', \quad y' = -\Phi(x'), \quad (4) \]

where $\alpha'$ is the angle of slope to the axis $Ox'$.

The boundary conditions in this case are written in the form:

\[ u = 0, \quad v = 0 \quad \text{at} \quad y = 1 + \eta x = h(x), \]

\[ u = 0, \quad v = -1 \quad \text{at} \quad y = -\Phi(x), \]

\[ \Phi(x) = \bar{g}_0 = Kg_0 = 0 \quad \text{at} \quad x = 0, \quad (5) \]

\[ p(0) = p(1) = \frac{p_u}{p}, \]

where $\eta = \frac{h \tan \alpha'}{h_0}$.

The relations between dimensionless and dimensional quantities are given in the form:

\[ u' = u^* u, \quad v' = u^* v, \quad x' = lx, \quad y' = h_0 y, \]

\[ p' = p^* p, \quad T' = T^* T, \quad \beta = T^* \beta', \quad \varepsilon = \frac{h_0}{l}, \quad \mu' = \mu_0 \mu, \]

\[ \sigma' = \sigma_0 \sigma, \quad \bar{a} = \frac{a}{p'}, \quad p^* = \frac{h_0 u^*}{l}. \quad (6) \]

Taking $K$, caused by the melt and the energy dissipation rate as a small parameter, we seek the function $\Phi(x)$ as:

\[ \Phi(x) = -K \Phi_1(x) - K^2 \Phi_2(x) - K^3 \Phi_3(x) - ... = H. \quad (7) \]

Boundary conditions for the dimensionless velocity components $u$ and $v$ on the contour $y = 0 - \Phi(x)$ can be written as:

\[ v(0-H(x)) = v(0) - \left( \frac{\partial v}{\partial y} \right)_{y=0} H(x) - \left( \frac{\partial^2 v}{\partial y^2} \right)_{y=0} H^2(x) - ... = -1; \]

\[ u(0-H(x)) = u(0) - \left( \frac{\partial u}{\partial y} \right)_{y=0} H(x) - \left( \frac{\partial^2 u}{\partial y^2} \right)_{y=0} H^2(x) - ... = 0. \quad (8) \]

We seek the asymptotic solution of the system of differential equations (2) with allowance for the boundary conditions (5) and (8) in the form of series in powers of the small parameter $K$:

\[ v = v_0(x, y) + K v_1(x, y) + K^2 v_2(x, y) + ..., \]

\[ u = u_0(x, y) + K u_1(x, y) + K^2 u_2(x, y) + ..., \]

\[ \Phi(x) = -K \Phi_1(x) - K^2 \Phi_2(x) - K^3 \Phi_3(x) - ..., \quad (9) \]

\[ p = p_0(x) + K p_1(x) + K^2 p_2(x) + K^3 p_3(x) + ..., \]

\[ T = T_0(x) + K T_1(x) + K^2 T_2(x) + K^3 T_3(x) + ..., \]

\[ \mu = \mu_0(x) + K \mu_1(x) + K^2 \mu_2(x) + K^3 \mu_3(x) + ... \]

Adding (9) into the system of differential equations (2), taking into account the boundary conditions (5), we obtain the following equations:

\[ \frac{\partial^2 v_0}{\partial y^2} - N v_0 + A = \frac{1}{\mu_0} \frac{dp_0}{dx}, \quad \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} = 0, \quad (10) \]

With boundary conditions:

\[ v_0 = 0, \quad u_0 = 0, \quad \text{at} \quad y = 1 + \eta x, \]

\[ v_0 = 0, \quad u_0 = 0 \quad \text{at} \quad y = 0, \quad (11) \]

\[ p_0(0) = p_0(1) = \frac{p_u}{p}, \quad K \Phi_0(0) = Kg_0 = 0. \]

The work was has been made under the grant of Russian Railways No. 2210370/22.12.2016 for the development of scientific and pedagogical schools in the field of railway transport.
– for the first approximation:

\[ \frac{\partial^2 v_i}{\partial y^2} = -\frac{\mu_1(x)}{j\mu_0(x)} \frac{dp_0}{dx} + \frac{1}{j\mu_0(x)} \frac{dp_1}{dx} \frac{\partial v_i}{\partial x} + \frac{\partial u_j}{\partial y} = 0, \]

\[ \frac{1}{j\mu_0(x)} \frac{d\Phi_1(x)}{dx} = \int_0^{1-x} \left( \frac{\partial \nu_0}{\partial y} \right)^2 dy, \]  

(12)

With boundary conditions:

\[ v_i = \left( \frac{\partial \nu_0}{\partial y} \right) \cdot \Phi_1(x), \quad u_i = \left( \frac{\partial \mu_0}{\partial y} \right) \cdot \Phi_1(x), \]

\[ v_1 = 0, \quad u_1 = 0 \text{ at } h(x) = 1 + \eta x, \]

(13)

\[ p_i (0) = p_i (1) = 0, \quad K\Phi_1 (0) = K\tilde{a}, \quad \Phi_1 (0) = \Phi_1 (1) = \tilde{a}. \]

Replacing in the first equation of system (2) the velocity v by its maximum value equal to –1, the exact solution of the problem for the zero approximation will be sought:

\[ u_0 = -\frac{\partial \psi_0}{\partial x} + U_0(x,y), \quad v_0 = \frac{\partial \psi_0}{\partial y} + V_0(x,y), \]

\[ \psi_0 (x,y) = \tilde{\psi}_0 (\xi), \quad \xi = \frac{y}{h(x)}, \]

(14)

\[ V_0(x,y) = \tilde{v} (\xi), \quad U_0(x,y) = -\tilde{u}_0 (\xi) \cdot h'(x). \]

Adding (14) into the system of differential equations (10), taking into account the boundary conditions (11), we obtain the following system of differential equations:

\[ \tilde{\psi}_0 = \tilde{C}_2, \quad \tilde{v}_0 = \tilde{C}_1, \quad \tilde{u}_0 + \tilde{\psi}_0' = 0, \]

\[ \frac{1}{j\mu_0(x)} \frac{dp_0}{dx} = \tilde{C}_1 + \frac{\tilde{C}_2}{h^2(x)} + N + A \]

(15)

And boundary conditions:

\[ \tilde{\psi}_0'(0) = 0, \quad \tilde{\psi}_0'(1) = 0, \quad \tilde{u}_0(0) = 0, \quad \tilde{v}_0(1) = 0. \]

\[ \tilde{u}_0(0) = 0, \quad \tilde{v}_0(0) = -1, \quad \int_0^1 \tilde{v}_0(\xi) d\xi = 0, \quad p_0(0) = p_0(1) = \frac{p_0}{p}. \]

By the direct integration we obtain:

\[ \tilde{\psi}_0(\xi) = \frac{C_2}{2}(\xi^2 - \xi), \quad \tilde{v}_0(\xi) = \tilde{C}_1 - \frac{C_2}{2} \left( 1 + \frac{C_1}{2} \right) \xi + 1. \]

\[ \tilde{C}_1 = 6 \]  

(17)

From the condition \( p_0(0) = p_0(1) = \frac{p_0}{p} \) with an accuracy up to the terms of the second order of smallness \( O(\eta^2) \) for \( \tilde{C}_2 \) we obtain the expression:

\[ \tilde{C}_2 = -6 \left( 1 + \frac{1}{2} \eta \right) \left( N \right) - \left( 1 + \frac{3}{2} \eta \right). \]  

(18)

**IV. DEFINITION OF HYDRODYNAMIC PRESSURE**

The dimensionless hydrodynamic pressure in the lubricating layer is determined from equation:

\[ \frac{1}{j\mu_0(x)} \frac{dp_0}{dx} = \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} + N + A. \]  

(19)

To solve equations (19), we first define \( \mu_0(x) \). For this, differentiating \( \mu_0(x) = e^{a_0} - p_0 \) we obtain:

\[ \frac{d\mu_0}{dx}(x) = \mu_0(x) \left( a\frac{dp_0}{dx} - \beta \frac{dT_0}{dx} \right). \]  

(20)

In order to define \( \frac{dT_0}{dx} \) we use the formula for the energy dissipation rate:

\[ \frac{dT_0}{dx} = \frac{4\mu_0(x) a_0 l(x)}{T_c h_0 C_2} \int \left( \tilde{\psi}_0(\xi) + \tilde{v}_0(\xi) \right)^2 d\xi. \]  

(21)

Adding (21) into (20) and making a series of transformations, we obtain:
\[ \frac{1}{\mu_0(x)} \frac{d\mu_0(x)}{dx} = \frac{\alpha C_v}{h^2(x)} + \frac{\alpha C_p}{h'(x)} + \frac{24\mu_0\mu'\beta h(x)}{T_c p_c h C_2} \int_0^1 \left( \frac{\psi'_0(\xi)}{h'(x)} + \frac{\psi''_0(\xi)}{h(x)} \right)^2 d\xi, \]  

where \( c_p \) is heat capacity at constant pressure.

Integrating (22), we obtain:

\[ \frac{1}{\mu_0(x)} = 1 - \alpha \left[ \tilde{C}_J J_1(x) + \tilde{C}_J J_1(x) \right] - \frac{D}{C_2} \left[ \Delta J_1(x) + \Delta J_2(x) + \Delta J_3(x) \right], \]  

where \( D = \frac{24\mu_0\beta u'\gamma}{T_c p_c h'} \); \( \Delta J_1 = \int_0^1 \left( \frac{\psi_0(\xi)}{h'(x)} \right)^2 d\xi = \frac{\tilde{C}_2}{12}; \) \( \Delta J_2 = 2\int_0^1 \left( \frac{\psi_0(\xi)}{h'(x)} \right) d\xi = -\frac{1}{6} \tilde{C}_J \tilde{C}_2; \) \( \Delta J_3 = \int_0^1 \left( \frac{\psi_0'(\xi)}{h'(x)} \right)^2 d\xi = 4; \) \( J_1(x) = \int_0^x \frac{dx}{h'(x)}. \)

Substitute the function \( \mu_0(x) \) by its averaged integral value:

\[ \bar{\mu}_0 = \frac{1}{x-\alpha} \left[ \bar{C}_J J_1(x) + \bar{C}_J J_1(x) \right] - \frac{D}{C_2} \left[ \Delta J_1(x) + \Delta J_2(x) + \Delta J_3(x) \right]. \]  

Solving the obtained equations \( \Delta_1, \Delta_2, \Delta_3, J_1(x), J_2(x), J_3(x) \) within an accuracy of \( O(\eta^3) \) for \( \bar{\mu} \) obtain the following expression:

\[ \bar{\mu}_0 = \frac{1}{x-\alpha} \left[ \bar{C}_J J_1(x) + \bar{C}_J J_1(x) \right] - \frac{D}{C_2} \left[ \Delta J_1(x) + \Delta J_2(x) + \Delta J_3(x) \right] \]

\[ + \alpha \left[ \frac{(N+A)^3}{2} - \eta \left( \frac{(N+A)^2}{2} \right) \right]. \]  

Thus,

\[ \frac{d\phi_0}{dx} = j\mu_0 3 \eta \left( 1 + \frac{(N+A)}{2} \right) \left( x^2 - x \right) + \frac{p_c}{\rho}. \]  

In order to define \( \Phi_1(\theta) \) taking into account equations (17) and (25), we come to the following equation:

\[ \frac{d\Phi_1}{dx} = j\mu_0 3 \eta \left( 1 + \frac{(N+A)}{2} \right) \left( \frac{\psi_0'(\xi)}{h'(x)} + \frac{\psi_0''(\xi)}{h(x)} \right)^2 d\xi. \]  

Integrating equation (27), we obtain:

\[ \Phi_1(x) = j\mu_0 \left[ \int_0^x \frac{\Delta_1 dx}{h^3(x)} + \int_0^x \frac{\Delta_2 dx}{h^2(x)} + \int_0^x \frac{\Delta_3 dx}{h(x)} \right]. \]  

Solving the equation (28) at the condition \( K \Phi_1(0) = K \tilde{a} \), we obtain:

\[ \Phi_1(x) = j\mu_0 \left[ \int_0^x \frac{\tilde{C}_2}{12} \left( x - \frac{3}{2} \eta^2 \right) + \tilde{C}_J (x - \eta^2) + 4 \left( x - \frac{3}{2} \eta^2 \right) + \tilde{a} \right]. \]  

The exact self-similar solution for the first approximation will be sought in the form:

\[ u_1 = -\frac{\partial \psi_1}{\partial x} + U_1(x, y), \quad v_1 = \frac{\partial \psi_1}{\partial y} + V_1(x, y), \]

\[ \psi_1(x, y) = \tilde{\psi}_1(\xi), \quad \xi = \frac{y}{h(x)}, \]

\[ V_1(x, y) = \tilde{v}(\xi), \quad U_1(x, y) = -\tilde{u}_1(\xi) \cdot h(x). \]

Adding (30) into the system of differential equations (12), taking into account the boundary conditions (13), we obtain the following system of differential equations:

\[ \tilde{\psi}_1(\xi) = \tilde{C}_2, \quad \psi_1 = \tilde{C}_1, \quad \tilde{u}_1 + \xi \tilde{v}_1 = 0, \]

\[ \frac{1}{j\mu_0} \frac{dp_c}{dx} - \frac{\mu_0(x)}{j\mu_0^2} \frac{dp_c}{dx} = \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h'(x)}. \]  

And boundary conditions:

\[ \tilde{\psi}_1(0) = 0, \quad \tilde{\psi}_1(1) = 0, \quad \tilde{u}_1(1) = 0, \quad \tilde{v}_1(1) = 0. \]
\[ \bar{v}_1(0) = M, \; \dot{u}_1(0) = 0, \; \int_0^1 \bar{v}_1(\xi) d\xi = 0, \; p_1(0) = p_1(1) = 0. \quad (32) \]

By the direct integration we obtain:

\[
\bar{\psi}_1(\xi) = \frac{C_2}{2}(\xi^2 - \xi), \quad \bar{v}_1(\xi) = \frac{C_1}{2} + M, \quad \bar{\xi} = 6M \quad (33)
\]

where

\[
M = \sup_{\omega^{(1)}} \left( \frac{d\nu}{dy} \right) \Phi_1(x) = \\
= \sup_{\omega^{(1)}} \left[ 1 - \eta \left( 1 + \frac{N + A}{2} \right) - \frac{N + A}{2} \eta \left( 1 + \eta \right) \right] \\
\times \int_0^1 \left[ \frac{C_1}{2} \left( x - \frac{3}{2} \eta^2 \right) + \bar{C}_1(x - \eta^2) + 4 \left( x - \frac{1}{2} \eta^2 \right) + \bar{u} \right] dx.
\]

In order to find the value \( \bar{C}_2 \) and to solve the equation for hydrodynamic pressure:

\[
\frac{1}{j\eta} \frac{d\mu_1}{dx} - \frac{\mu_1(x)}{j\eta^2} \frac{d\eta^2}{dx} = \frac{\bar{C}_1}{h^2(x)} + \frac{\bar{C}_2}{h^3(x)},
\]

we define \( \mu_1(x) \) first. For this we differentiate the expression \( \mu_1(x) = e^{\eta_1 - \beta_1} \):

\[
\frac{d\mu_1}{dx} = \alpha \mu_1(x) \frac{d\eta_1}{dx} + \beta \mu_1(x) \frac{d\beta_1}{dx} - \beta \mu_1(x) \frac{d\beta_1}{dx} - \beta \mu_1(x) \frac{d\beta_1}{dx}.
\]

In order to define \( \frac{d\mu_1}{dx} \) we use the formula for the energy dissipation rate:

\[
\frac{d\mu_1}{dx} = 2 \mu_1(\eta) u^* h(\eta) \left( \frac{\bar{\psi}_1(\xi)}{h(\xi)} + \bar{v}_1(\xi) \right) \left( \frac{\bar{\psi}_1(\xi)}{h(\xi)} + \bar{v}_1(\xi) \right) \frac{d\xi}{dx}. \quad (35)
\]

Adding (35) to (34) and making a series of transformations up to terms \( O(Ka) \), we obtain:

\[
\frac{1}{\mu_1(x)} = 1 - 2 \frac{D}{C^2} \left[ \tilde{\Delta}_1 \tilde{J}_1(x) + \tilde{\Delta}_2 \tilde{J}_2(x) + \tilde{\Delta}_3 \tilde{J}_3(x) + \tilde{\Delta}_4 \tilde{J}_4(x) \right]. \quad (36)
\]

where \( \tilde{\Delta}_1 = \int_0^1 \bar{\psi} \tilde{v}(\xi) \bar{\psi}_1(\xi) d\xi \), \( \tilde{\Delta}_2 = \int_0^1 \bar{\psi}_1(\xi) \bar{v}_1(\xi) d\xi \), \( \tilde{\Delta}_3 = \int_0^1 \bar{\psi}_1(\xi) \bar{v}_1(\xi) d\xi \), \( \tilde{\Delta}_4 = \int_0^1 \bar{\psi}_1(\xi) \bar{v}_1(\xi) d\xi \).

By its averaged integral value:

\[
\tilde{\mu}_1 = 1 + D \left[ \frac{1}{2} \frac{5n}{12} + \frac{N + A}{6} \left( \frac{1}{2} \frac{\eta}{4} \right) \right] + \\
+ M \left[ \frac{1}{2} \frac{10 - \eta}{3} + (N + A) \left( \frac{1}{2} \frac{5n}{6} \right) \right]. \quad (37)
\]

Then for \( p_1 \) we obtain:

\[
p_1 = \dot{\bar{J}}_2 \left[ 6M(x - \eta^2) + \bar{C}_1 \left( x - \frac{3}{2} \eta^2 \right) \right] + \\
+ 3\eta \dot{\bar{J}}_2 \left( 1 + \frac{N + A}{2} \right)(x^2 - x). \quad (38)
\]

From the condition \( p_1(0) = p_1(1) = 0 \) we obtain:

\[
\bar{C}_2 = -6M \left( 1 + \frac{\eta}{2} \right). \quad (39)
\]

Adding \( \bar{C}_2 \) to (37) for \( \tilde{\mu}_1 \) we eventually obtain:

\[
\tilde{\mu}_1 = 1 + D \left[ \frac{1}{2} \frac{5n}{12} + \frac{N + A}{6} \left( \frac{1}{2} \frac{\eta}{4} \right) \right] - \\
- \frac{1}{3} \left[ \frac{5}{3} \frac{8}{3} \eta + (N + A) \left( \frac{1}{2} \frac{\eta}{4} \right) \right]. \quad (40)
\]

V. RESULTS OF RESEARCH AND THEIR DISCUSSION

For the bearing capacity and the frictional force, we obtain:
The work was has been made under the grant of Russian Railways No. 2210370/22.12.2016 for the development of scientific and pedagogical schools in the field of railway transport.

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