Cosmological Implication of Antisymmetric Tensor Field on D-brane

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Abstract

We discuss the cosmological implications of an antisymmetric tensor field when it experiences a kind of “Higgs mechanism,” including nonlocal interaction terms, on D-branes. Even when a huge magnetic-condensation-inducing anisotropy is assumed in the early universe, the expansion of the universe leads to an isotropic B-matter-dominated universe.

PACS numbers: 11.15.Uv, 04.60.-m

Keywords: D-brane, Antisymmmetric tensor field, Cosmology

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I. INTRODUCTION

If we are interested in the very early universe near or slightly below the Planck scale, the cosmological model driven by string theory should be taken into account. The first signal of string cosmology is given by dilaton gravity instead of Einstein gravity \(^1\). In string theory, another indispensable bosonic degree is the antisymmetric tensor field of rank two \(^2, 3\). When the ten-dimensional bulk is compactified to a (1+3)-dimensional spacetime, the cosmological effect of the massless antisymmetric tensor field has been taken into account. The homogeneous, but anisotropic, Bianchi-type universe is supported due to the antisymmetric tensor field \(^4\). Once the anisotropy is generated, its effect can usually survive in the present universe, despite the sufficient expansion of the universe. The observed data of cosmic microwave background radiation (CMBR) threatens the viability of this string-driven cosmological model with the antisymmetric tensor field.

The development of D-branes and related topics for the last ten years has opened another possibility in cosmology that our universe may be identified with a D3-brane, or parts of higher-dimensional D-branes in a nine-dimensional spatial bulk. A clear distinction between string cosmology \(^5\) in the bulk without D-brane and that on the D-brane appears through the mass generation of the antisymmetric tensor field on the D-brane \(^6\). In the context of Einstein gravity, the cosmological effect of this massive antisymmetric field was studied in detail \(^7\). Specifically, the anisotropy induced by condensation of the antisymmetric tensor field is diluted during the expansion of the universe \(^8\). If the expansion is sufficient, the resultant present universe becomes isotropic and is free from the constraint of the CMBR.

In this paper, we will investigate the effect of the massive interacting antisymmetric tensor field. In Section II, we introduce the model of our interest, including graviton, dilaton, and antisymmetric tensor field in both the string frame and the Einstein frame. In Section III, we consider only a single magnetic component of the homogeneous antisymmetric tensor field and study its oscillation without damping in a flat spacetime. In Section IV, we consider another flat system of the antisymmetric tensor field with a dilaton. For the case of negative cosmological constant, the stabilization of the dilaton is achieved for some initial values. In Section V, the cosmological evolution of the antisymmetric tensor field is studied without and with a D3-brane. In the case of a D-brane universe, solutions of the isotropic $B$-matter-dominated universe are obtained. We conclude in Section VI with a summary and
II. STRING EFFECTIVE THEORY IN A D3-BRANE UNIVERSE

When a ten-dimensional string theory is compactified to a four-dimensional theory, the spectrum of particles depends on a specific pattern of the compactification, and the presence of (D-)branes generates various models, including a model that resembles the standard model. In order to understand the cosmological evolution, we take into account closed strings containing the graviton. The universal nature of cosmology based on four-dimensional effective field theory requires bosonic particles at the lowest energy level be produced irrespective of the compactification pattern. Then, we mainly examine the cosmological effect due to the presence of a D3-brane whose world-volume is identified with our spacetime.

Bosonic degrees from the closed strings of string theory include the graviton $\tilde{g}_{\mu\nu}$, the dilaton $\Phi$, and the antisymmetric tensor field $B_{\mu\nu}$. On the D3-brane forming our universe, the sum of the antisymmetric tensor field $B_{\mu\nu}$ and the field strength tensor $F_{\mu\nu}$ of a U(1) gauge field $A_\mu$ defines a gauge invariant $B_{\mu\nu} = B_{\mu\nu} + 2\pi\alpha'F_{\mu\nu}$. Their four-dimensional effective action in the string frame is given by the sum of the bulk action and the Dirac-Born-Infeld (DBI)-type brane action:

$$S_4 = \frac{1}{2\kappa_4^2} \int d^4\tilde{x} \sqrt{-\tilde{g}} \left[ e^{-2\Phi} \left( \tilde{R} - 2\Lambda \right) + 4\tilde{\nabla}_\mu \Phi \tilde{\nabla}^\mu \Phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right]$$

(II.1)

where $B^*_{\mu\nu} = \sqrt{-g} \epsilon_{\mu\nu\alpha\beta} B^{\alpha\beta}/2$ with $\epsilon_{0123} = 1$ and $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$. $m_B$ is a parameter defined by $m_B^2 = 2\kappa_4^2T_3$ where $T_3$ is the tension of the D3-brane. The action in Eq. (II.1) leads to the following equations of motion for the antisymmetric tensor field, the dilaton, and the graviton, respectively:

$$\tilde{\nabla}^\lambda H_{\lambda\mu} - 2H_{\lambda\mu} \tilde{\nabla}^\lambda \Phi$$

$$- m_B^2 e^{-\Phi} \frac{B_{\mu\nu} - \frac{1}{2}B_{\mu\nu}^* (BB^*)}{\sqrt{1 + \frac{1}{2}B^2 - \frac{1}{16} (BB^*)^2}} = 0,$$

(II.2)
\[4\tilde{\nabla}^2\Phi - 4\left(\tilde{\nabla}\Phi\right)^2 = -\tilde{R} + 2\Lambda + \frac{1}{12}H^2 \]
\[+ \frac{1}{2}m_B^2 e^{\Phi} \sqrt{1 + \frac{1}{2}B^2 - \frac{1}{16} (BB^*)^2}, \quad (\text{II.3})\]

\[\tilde{G}_{\mu\nu} = -\tilde{g}_{\mu\nu}\Lambda + \kappa_4^2 \tilde{T}_{\mu\nu}, \quad (\text{II.4})\]

where the energy-momentum tensor is
\[\tilde{T}_{\mu\nu} = \tilde{T}_\Phi + \tilde{T}_B, \quad (\text{II.5})\]

\[\kappa_4^2 \tilde{T}_\Phi = 2\tilde{g}_{\mu\nu} \left[\tilde{\nabla}^2\Phi - \left(\tilde{\nabla}\Phi\right)^2\right] - 2\tilde{\nabla}_\mu \tilde{\nabla}_\nu \Phi, \quad (\text{II.6})\]

\[\kappa_4^2 \tilde{T}_B = \frac{1}{12} \left(3H_{\mu\lambda\rho}H^\lambda_{\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}H^2\right) \]
\[+ \frac{1}{2}m_B^2 e^{\Phi} \left[-\tilde{g}_{\mu\nu} + \frac{1}{2}\tilde{g}_{\mu\nu}B^2 + B_{\mu\lambda}B^\lambda_{\nu}\right] \sqrt{1 + \frac{1}{2}B^2 - \frac{1}{16} (BB^*)^2}. \quad (\text{II.7})\]

Since we are interested in the time evolution, specifically the expansion of the universe, a natural scheme may require the reproduction of Einstein gravity at late times. In the classical level, the model of our interest in the string frame of Eq. (II.1) does not exhibit stabilization of the dilaton \(\Phi\), so one option is to study this topic in the Einstein frame obtained by a coordinate transformation from the string frame,
\[g_{\mu\nu} = e^{-2\Phi}\tilde{g}_{\mu\nu}. \quad (\text{II.8})\]

Then, from Eq. (II.1), we obtain the action
\[S_E = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\tilde{g}} \left[R - 2\Lambda e^{2\Phi}\right] \]
\[- 2(\nabla \Phi)^2 - \frac{1}{12} e^{-4\Phi} H^2 \]
\[- m_B^2 e^{3\phi} \sqrt{1 + \frac{1}{2} e^{-4\phi} B^2 - \frac{1}{16} e^{-8\phi} (BB^*)^2}. \quad (\text{II.9})\]

In the Einstein frame, we read again the equations of motion for the antisymmetric field \(B_{\mu\nu}\), the dilaton \(\Phi\), and the graviton \(g_{\mu\nu}\):
\[\nabla_\lambda H_{\lambda\mu\nu} = 4H_{\lambda\mu\nu} \nabla^\lambda \Phi \quad (\text{II.10})\]
\[- m_B^2 e^{3\phi} \frac{B_{\mu\nu} - \frac{1}{4} e^{-4\phi} B_{\mu\nu} (BB^*)}{\sqrt{1 + \frac{1}{2} e^{-4\phi} B^2 - \frac{1}{16} e^{-8\phi} (BB^*)^2}} = 0, \]
\[ \nabla^2 \Phi = \frac{\partial V(\Phi)}{\partial \Phi}, \quad \text{(II.11)} \]

where the potential is
\[ V(\Phi) = \frac{1}{4} \left[ 2\Lambda e^{2\Phi} + \frac{1}{12} e^{-4\Phi} H^2 \right. \]
\[ + \left. m_B^2 e^{3\Phi} \sqrt{1 + \frac{1}{2} e^{-4\Phi} B^2 - \frac{1}{16} e^{-8\Phi} (B^* B)^2} \right], \quad \text{(II.12)} \]

and
\[ G_{\mu\nu} = -g_{\mu\nu} \Lambda e^{2\Phi} + \kappa_4^2 T_{\mu\nu}, \quad \text{(II.13)} \]

where the energy-momentum tensor is
\[ T_{\mu\nu} = T_{\mu\nu}^\Phi + T_{\mu\nu}^B, \quad \text{(II.14)} \]
\[ \kappa_4^2 T_{\mu\nu}^\Phi = 2 \nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} (\nabla \Phi)^2, \quad \text{(II.15)} \]
\[ \kappa_4^2 T_{\mu\nu}^B = \frac{1}{12} e^{-4\Phi} \left( 3H_{\mu\lambda\rho} H_{\nu}^{\lambda\rho} - \frac{1}{2} g_{\mu\nu} H^2 \right) \]
\[ + \frac{1}{2} m_B^2 e^{3\Phi} \frac{-g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} e^{-4\Phi} B^2 + e^{-8\Phi} B_{\mu\lambda} B_{\nu}^{\lambda}}{\sqrt{1 + \frac{1}{2} e^{-4\Phi} B^2 - \frac{1}{16} e^{-8\Phi} (B B^*)^2}}, \quad \text{(II.16)} \]

Note that conservation of energy-momentum is consistent with the equations of motion in each frame, (II.2)–(II.3) or (II.10)–(II.13), respectively.

A few topics will be discussed at the quantum level in the subsequent sections, but most of the cosmological applications will be made at the classical level. For the antisymmetric tensor field of second rank \( B_{\mu\nu} \), we employ the terminology of the electric components for \( B_{0i} \equiv (E)^i \) and of the magnetic components for \( B_{ij} \equiv 2\epsilon_{0ijk} (B)^k \) in what follows. In the subsequent sections, we only consider the homogeneous magnetic component, \( E = 0 \) and \( B = B(t) \neq 0 \), with a fixed direction
\[ B = B(t) \hat{k}, \quad \text{(II.17)} \]

and its time evolution in a flat spacetime.

**III. HOMOGENEOUS B-MATTER WITHOUT DILATON IN FLAT SPACETIME**

Since our main goal is to understand the effect of the antisymmetric tensor field in the early universe near the Planck scale, we begin this section with studying it in flat spacetime.
with a stabilized dilaton. Substituting the magnetic field ansatz in Eq. (II.17) with the flat metric $g_{\mu\nu} = \eta_{\mu\nu}$ and the stabilized dilaton $\Phi = 0$ into the Euler-Lagrange equation of the antisymmetric tensor field, Eq. (II.2) in the string frame or Eq. (II.10) in the Einstein frame, we obtain the single dynamical equation of the $B$ field

$$\partial_0^2 B = -m_B^2 \frac{B}{\sqrt{1 + B^2}}.$$  \hspace{1cm} (III.18)

Integration of Eq. (III.18) gives

$$E = \frac{1}{2} \dot{B}^2 + V_{\text{eff}}(B),$$ \hspace{1cm} (III.19)

where $E$ is an integration constant, $V_{\text{eff}}(B) = \sqrt{1 + B^2}$, and the overdot represents differentiation with respect to the rescaled time variable $\tilde{t} = m_B t$. Equation (III.19) can be rewritten as an integral equation

$$\tilde{t} - \tilde{t}_0 = \pm \int_{B_0}^{B} \frac{dB}{\sqrt{2(E - \sqrt{1 + B^2})}}.$$ \hspace{1cm} (III.20)

The only pattern of nontrivial solutions is an oscillating one with the amplitude $\sqrt{E^2 - 1}$ for $E > 1$. This oscillating solution with a fixed amplitude is natural at the classical level in a flat spacetime when a homogeneous condensation of the magnetic field is given by an initial condition. In expanding universes, the solution is expected to change to that with an oscillation and a damping due to the growing of a spatial scale factor, which means a dilution of $B$-matter in the expanding D3-brane universe.

**IV. HOMOGENEOUS $B$-MATTER WITH LINEAR DILATON IN FLAT SPACETIME**

If we turn on the linear dilaton $\Phi$ in flat spacetime, the equations of motion for the antisymmetric tensor field $B_{\mu\nu}$ and the dilaton $\Phi$ become different in the string frame, Eqs. (II.2)–(II.3), and the Einstein frame, Eqs. (II.10)–(II.11). In this section, we again discuss the evolution of the magnetic field with a fixed direction for both frames.

In the string frame, plugging the ansatz for the magnetic field, Eq. (II.17), and the homogeneous dilaton

$$\Phi = \Phi(t)$$ \hspace{1cm} (IV.21)
into the equations of motion, Eqs. (II.2)–(II.3), gives

\[ \ddot{B} - 2\dot{B}\dot{\Phi} = -e^{\Phi}\frac{B}{\sqrt{1 + B^2}}, \]  
(IV.22)

\[ \ddot{\Phi} - \dot{\Phi}^2 - \frac{1}{8}\dot{B}^2 = -\frac{1}{8}e^\Phi\sqrt{1 + B^2} - \frac{\tilde{\Lambda}}{2}, \]  
(IV.23)

where we rescaled \( \tilde{\Lambda} \equiv \Lambda/m^2_B \). Plugging the ansatz for the magnetic field, Eq. (II.17), and the homogeneous dilaton, Eq. (IV.21), into the equations of motion, Eqs. (II.10)–(II.11), in the Einstein frame, we have

\[ \ddot{B} - 4\dot{B}\dot{\Phi} = -e^{3\Phi}\frac{B}{\sqrt{1 + B^2e^{-4\Phi}}}, \]  
(IV.24)

\[ \ddot{\Phi} + \frac{1}{2}e^{-4\Phi}\dot{B}^2 = -\frac{1}{4}e^{3\Phi}\sqrt{1 + B^2e^{-4\Phi}} - \frac{1}{2}\frac{e^{3\Phi}}{\sqrt{1 + B^2e^{-4\Phi}}} - \tilde{\Lambda}e^{2\Phi}. \]  
(IV.25)

We solve the field equations for \( B \) and \( \Phi \) in both frames with different values of \( \tilde{\Lambda} \). In the string frame, for \( \tilde{\Lambda} = 0 \), an initial oscillation in \( B \) discussed in the previous section is observed. Later, \( B \) settles down to a constant. The dilaton \( \Phi \) decreases to negative infinity.

For \( \tilde{\Lambda} > 0 \), the \( \tilde{\Lambda} \)-term in the \( \Phi \)-equation, Eq. (IV.23), effectively provides a potential term which is linear in \( \Phi \). Therefore, the dilaton is pushed to a negative value more rapidly, and the oscillation in \( B \) is suppressed. The numerical solutions are plotted in Fig. 1.

In the Einstein frame, in order to analyze the dilaton behavior, let us assume \( B = 0 \). Then, from Eq. (IV.25), we see that the system is frictionless (no \( \dot{\Phi} \)-term) and that the dilaton field \( \Phi \) has an effective potential

\[ V_e = \frac{1}{4}e^{2\Phi}(e^\Phi + 2\tilde{\Lambda}). \]  
(IV.26)

For \( \tilde{\Lambda} \geq 0 \), \( V_e \) is a monotonically increasing function of \( \Phi \), which pushes \( \Phi \) to negative infinity. Once \( \Phi \) achieves sufficiently large negative values, the \( \tilde{\Lambda} \)-term in the dilaton equation, Eq. (IV.25), in the Einstein frame becomes negligible.

For \( \tilde{\Lambda} < 0 \), \( V_e \) possesses a global minimum at \( \Phi_s = \ln(-4\tilde{\Lambda}/3) \) and approaches zero from below as \( t \to -\infty \). If the initial \( \Phi(0) \equiv \Phi_0 \) is imposed such that \( V(\Phi_0) \geq 0 \), the dilaton \( \Phi \) rolls down to the minimum and is pushed to negative infinity eventually. However, if \( \Phi_0 \) is taken such that \( V(\Phi_0) < 0 \), \( \Phi \) will oscillate about \( \Phi_s \), and the dilaton is stabilized. Even after we turn on the \( B \)-field, the story is not altered very much. We observe this with the
FIG. 1: Φ and B in the string frame for $\tilde{\Lambda} = 0$ and $\tilde{\Lambda} = 1/2$. The initial conditions are $\Phi(0) = 1$ and $B(0) = 0.1$. For $\tilde{\Lambda} > 0$, $\Phi$ drops very rapidly, and the oscillation in $B$ is suppressed.

FIG. 2: Φ and B in the Einstein frame for $\tilde{\Lambda} = 0$ and $\tilde{\Lambda} = 1/2$. The initial conditions are $\Phi(0) = 1$ and $B(0) = 0.1$. The configurations are very similar regardless of $\tilde{\Lambda}$.

$B$-field included, from the numerical solutions plotted in Fig. 2 for $\tilde{\Lambda} \geq 0$ and Fig. 3 for $\tilde{\Lambda} < 0$.

V. HOMOGENEOUS UNIVERSE DOMINATED BY $B$-MATTER

The main topic of our interest is cosmological implications of large-scale fluxes of the antisymmetric tensor field which might have existed in the early universe of a string theory scale. When the early universe is dominated by the massive antisymmetric tensor field in appropriate initial configurations, its cosmological evolution will be traced.

We take into account the same homogeneous magnetic ansatz as in Eq. (II.17), i.e., the dilaton is stabilized $\Phi = 0$ and the single component of $B_{ij}$ is nonzero:

$$B_{0i} = 0, \quad B_{23}(t) = B_{31}(t) = 0, \quad B_{12}(t) \equiv B(t) \neq 0. \quad (V.27)$$

The metric consistent with the matter, Eq. (V.27), should keep isotropy on the $(x^1 - x^2)$ plane and, thus, is of Bianchi type I:

$$ds^2 = -dt^2 + a_1(t)^2[(dx^1)^2 + (dx^2)^2] + a_3(t)^2(dx^3)^2. \quad (V.28)$$

The energy-momentum tensor $T_{\mu\nu}$ of the antisymmetric tensor field, Eq. (V.27), is written
FIG. 3: \( \Phi \) and \( B \) in the Einstein frame for \( \tilde{\Lambda} = -1/2 \) and \( B(0) = 0.1 \). For large \( \Phi(0) = 0.202733 \) (upper panel), the configurations are very similar to those for \( \tilde{\Lambda} \geq 0 \). For small \( \Phi(0) = -0.202733 \) (lower panel), the dilaton stabilization is observed.

in the form

\[
(T_B)_{\mu}{}^\nu = (T_{\text{bulk}})_{\mu}{}^\nu + (T_{\text{brane}})_{\mu}{}^\nu
\]

\[
= \left(\tilde{\Lambda}/\kappa_4^2\right)\delta_{\mu}{}^\nu + \text{diag} \left[-\rho_B, -\rho_B, -\rho_B, +\rho_B\right]
\]

\[
+ \text{diag} \left[-\rho_b, -\tilde{\rho}_b, -\tilde{\rho}_b, -\rho_b\right],
\]

where \( \tilde{\Lambda} = m_B^2\Lambda \), \( \kappa_4^2 = \kappa_4^2/m_B^2 \), \( \rho_B = \dot{B}^2/4\kappa_4^2a_1^4 \), \( \rho_b = (1/2\kappa_4^2)\sqrt{1+B^2/a_1^2} \), and \( \tilde{\rho}_b = (1/2\kappa_4^2)/(1+B^2/a_1^2) \). Substituting Eqs. (V.27), (V.28), and (V.29) into the Einstein equations, Eq. (II.4) or (II.13) for the tensor-field-dominated case are

\[
\left(\frac{\dot{a}_1}{a_1}\right)^2 + \frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} = \tilde{\kappa}_4^2(\rho_B + \rho_b) + \tilde{\Lambda},
\]

\[
\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} = \tilde{\kappa}_4^2(\rho_B + \tilde{\rho}_b) + \tilde{\Lambda},
\]

\[
2\frac{\ddot{a}_1}{a_1} + \left(\frac{\dot{a}_1}{a_1}\right)^2 = -\tilde{\kappa}_4^2(\rho_B - \rho_b) + \tilde{\Lambda}.
\]

The equation of motion for \( B(t) \), Eq. (II.2) or (II.10), is

\[
\ddot{B} + \left(2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\right)\dot{B} + \frac{B}{\sqrt{1+B^2/a_1^2}} = 0.
\]
Introducing \( \alpha_i = \ln a_i \), we can rewrite the Einstein equations, Eqs. (V.31)–(V.33), as

\[
\begin{align*}
\dot{\alpha}_1^2 + 2\dot{\alpha}_3\dot{\alpha}_1 &= \kappa_4^2 (\rho_B + \rho_b) + \tilde{\Lambda}, \\
\ddot{\alpha}_1 + 2\dot{\alpha}_1^2 + \dot{\alpha}_3^2 &= \kappa_4^2 \rho_b + \tilde{\Lambda}, \\
\ddot{\alpha}_3 + \dot{\alpha}_3^2 + 2\dot{\alpha}_1\dot{\alpha}_3 &= \kappa_4^2 (2\rho_B + \tilde{\rho}_b) + \tilde{\Lambda}.
\end{align*}
\] (V.35) (V.36) (V.37)

It is also convenient to introduce a new magnetic variable \( b \equiv B/a_1^2 \) by which we can rewrite the equation of the magnetic field, Eq. (V.34), as

\[
\ddot{b} + (2\dot{\alpha}_1 + \dot{\alpha}_3) \dot{b} + (2\ddot{\alpha}_1 + 2\dot{\alpha}_1\dot{\alpha}_3) b + \frac{b}{\sqrt{1 + b^2}} = 0.
\] (V.38)

Now let us examine Eqs. (V.35)–(V.38) with and without the D3-brane and find cosmological solutions under appropriate initial conditions of \( b \).

A. Anisotropy in the Bulk (\( m_B = 0 \))

In the absence of the D3-brane or in the limit of vanishing string coupling \( g_s \), we recapitulate possible cosmological solutions. This limit corresponds to the massless limit \( m_B = 0 \) of the antisymmetric tensor field so that the derivative terms with the rescaled time variable \( \tilde{t} = m_B t \to 0 \) dominate in Eqs. (V.34)–(V.37). In this limit, the \( B \)-equation, Eq. (V.34), before the rescaling is easily integrated to yield a constant of the motion,

\[
\frac{a_3\dot{B}}{a_1^2} \equiv L_3 \ (= \text{constant}).
\] (V.39)

With vanishing potential, \( B \) manifests itself by nonvanishing time derivatives. In the dual variable, it corresponds to the homogeneous gradient along the \( x^3 \)-direction. The spacetime evolution with the dilaton rolling in this case was studied in Ref. [5]. Here, we have assumed that the dilaton is stabilized by some mechanism. Following Ref. [5], we introduce a new time coordinate \( \lambda \) via the relation \( d\lambda = L_3 dt/a_1^2 a_3 \). Then, Eqs. (V.35)–(V.37) can be written as

\[
\begin{align*}
\dot{\alpha}_1' \dot{\alpha}_2' + \dot{\alpha}_2' \dot{\alpha}_3' + \dot{\alpha}_3' \dot{\alpha}_1' &= \frac{1}{4} a_1^4, \\
\alpha_1'' &= \alpha_2'' = 0, \\
\alpha_3'' &= \frac{1}{2} a_1^4,
\end{align*}
\] (V.40) (V.41) (V.42)

where the prime denotes the differentiation with respect to \( \lambda \).
The solution for $\alpha_1$ is trivial; from Eq. (V.41),

$$\alpha_1 = C_1 \lambda, \quad (V.43)$$

where $C_1$ is constant, and we have omitted the integration constant corresponding simply to rescaling of the scale factor. The $\alpha_3$-equation, Eq. (V.42), is also easily integrated to give

$$\alpha_3 = \frac{e^{4C_1 \lambda}}{32C_1^2} + C_3 \lambda. \quad (V.44)$$

The constraint equation, Eq. (V.40), relates $C_1$ and $C_3$ by $C_3 = -C_1/2$. Then, the relation between $\lambda$ and $L_3 t$ is explicitly given by

$$L_3 t = \int^{\lambda} d\lambda \frac{a_1(\lambda)^2 a_3(\lambda)}{(2C_1 + C_3)\lambda + \frac{e^{4C_1 \lambda}}{32C_1^2}}$$

$$= (16C_1)^{2C_1+C_3} \int_0^x dy \ y^{-P} e^y, \quad (V.45)$$

where $x = e^{4C_1 \lambda}/32C_1^2$ and $P = (2C_1 - C_3)/4C_1$. The evolution of the scale factors for large $L_3 t$ is given by

$$a_1 \propto (\log L_3 t)^{q_1}, \quad a_3 \propto L_3 t, \quad (V.46)$$

where $q_1 = 1/4$. Therefore, with non-vanishing $B_{12} = B(t)$, only $a_3$ grows significantly, and a spatial anisotropy develops. This can be seen clearly by considering the ratio between Hubble parameters $H_1$ and $H_3$, $H_3/H_1$, where $H_i \equiv \dot{a}_i/a_i$

$$\frac{H_3}{H_1} = 4 \log L_3 t, \quad (V.47)$$

which grows as time elapses. Finally, we remark that such anisotropy cannot be overcome by some other type of isotropic energy density in an expanding universe. If one assumes the isotropic universe ($a_i = a$) is driven by, for example, the radiation energy density $\rho_R$, then one finds $\rho_B \propto 1/a^2$ from Eqs. (V.29) and (V.39); thus, $\rho_B/\rho_R \propto a^2$, which implies that the late-time isotropic solution can be realized only in a contracting universe.

B. Isotropy on the D3-brane

Let us now take into consideration the effect of a space-filling D-brane. To get sensible solutions, we fine-tune the bulk cosmological constant term to cancel the brane tension; that is, $\Lambda = -m_B^2/2$, so that the effective four-dimensional cosmological constant vanishes.
If we read the $B$-matter part explicitly, then the full Einstein equations, Eqs. (V.35)–(V.37), are rewritten as follows:

\[
\dot{\alpha}_1^2 + 2\dot{\alpha}_1\dot{\alpha}_3 = \frac{1}{4} \left( \dot{b} + 2\dot{\alpha}_1 b \right)^2 + \frac{1}{2} \left( \sqrt{1 + b^2} - 1 \right), \quad (V.48)
\]

\[
\dot{\alpha}_1 + \dot{\alpha}_1 (2\dot{\alpha}_1 + \dot{\alpha}_3) = \frac{1}{2} \left( \sqrt{1 + b^2} - 1 \right), \quad (V.49)
\]

\[
\dot{\alpha}_3 + \dot{\alpha}_3 (2\dot{\alpha}_1 + \dot{\alpha}_3) = \frac{1}{2} \left( \dot{b} + 2\dot{\alpha}_1 b \right)^2 + \frac{1}{2} \left( \frac{1}{\sqrt{1 + b^2}} - 1 \right). \quad (V.50)
\]

With the help of Eq. (V.48), Eq. (V.38) can be written as

\[
\ddot{b} + (2\dot{\alpha}_1 + \dot{\alpha}_3) \dot{b} + \left[ -4\dot{\alpha}_1^2 + \left( \sqrt{1 + b^2} - 1 + \frac{1}{\sqrt{1 + b^2}} \right) \right] b = 0, \quad (V.51)
\]

which is valid only for the $B$-dominated case, but is more convenient for numerical analysis. Here, we have set \( \tilde{\kappa}_4 = 1 \).

First, we examine the evolution of $b(t)$ and the scale factors qualitatively. Suppose $b$ starts to roll from an initial value $b_0$ while the universe is isotropic in the sense that $\dot{\alpha}_{10} = \dot{\alpha}_{30}$. We assume initially $a_{10} = a_{30} = 1$ ($\alpha_{10} = \alpha_{30} = 0$) and $\dot{B}_0 = 0$ so that $b_0 = B_0$ and $\dot{b}_0 = -2\dot{\alpha}_{10}B_0$. While $b$ is much larger than unity, the rapid expansion of $\alpha_1$ due to the large potential proportional to $m_B^2b$ drives $b$, in feedback, to drop very quickly to a small value of order one. Our numerical analysis in Fig. 4 shows that this happens within $m_B^2t < 2$ up to reasonably large value of $b_0$ for which the numerical solution is working. The behavior of $b(t)$ after this point is almost universal irrespective of the initial value $b_0$ if it is much larger than unity.

Once $b$ becomes smaller than unity, the quadratic mass term dominates over the expansion, and $b$ begins to oscillate about $b = 0$. Then, the expansion of the universe provides a slow decrease in the oscillation amplitude. The situation is the same as that of a coherently oscillating scalar field, such as the axion, or the moduli in the expanding universe. For small $b$, the energy-momentum tensor of the oscillating $B$ field is given by $T_{\mu\nu} = \text{diag}[-\rho, p_1, p_2, p_3]$. 

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With the expansion of the universe neglected, the equation of motion for $b$ is then approximated by

$$\ddot{b} + b \approx 0. \quad (V.55)$$

Since the oscillation is much faster than the expansion, we can use the time-averaged quantities over one period of oscillation for the evolution of spacetime. Equation (V.55) gives the relation $\langle \dot{b}^2 \rangle = \langle b^2 \rangle$. Thus, the oscillating $B$-field has the property $p_1, p_2, p_3 \approx 0$ and behaves like homogeneous, isotropic matter. This justifies the name of $B$-matter. Therefore, after $b$ begins to oscillate, the isotropy of the universe is recovered.

To quantify how the universe recovers isotropy, let us define the parameter

$$s \equiv \sqrt{2 \frac{H_1 - H_3}{2H_1 + H_3}}. \quad (V.56)$$

When $H_1 \approx H_3$, the evolution of the quantity $s(t)$ is determined by

$$\dot{s} = \frac{1}{6} \left( \frac{4p_1 - p_3 - 9H^2}{H} \right) s + \frac{1}{3} \frac{p_1 - p_3}{H} + O(s^2), \quad (V.56)$$

where $H \equiv \sum_i H_i/3$. At late time, Eq. (V.56) is approximated as $\dot{s} \approx -(3/2)Hs$; thus, one finds $s \propto 1/t$. Since $H$ is also proportional to $1/t$, the initial and the final anisotropies $s_{i,f}$ follow the relation

$$s_f = s_i \frac{H_f}{H_i}, \quad (V.57)$$

where $H_{i,f}$ denotes the initial and the final Hubble parameters, respectively. To get an idea of how fast the anisotropy disappears, let us consider $H_f \sim 10^{-15}$ GeV, which corresponds to the Hubble parameter at the electroweak symmetry-breaking scale of temperature, $T \sim 100$
GeV. Taking the initial condition $s_i \sim 1$ around the beginning of the $B$ oscillation $H_i \sim m_B$, we find that the final anisotropy can be completely neglected for reasonable values of $m_B$.

For $m_B t \gg 1$ with $H_1 \simeq H_2 \simeq H_3$ and $b \ll 1$, the asymptotic solution of Eqs. (V.48)–(V.51) can be explicitly found to yield

$$H \propto t^{2/3}, \quad b \propto 1/t, \quad \rho_{B,b} \propto 1/t^2,$$

which show the usual matter-like evolution. That is, the total energy density of the $B$-matter diminishes like $\rho \propto 1/a^3$ even though the amplitude $B(t)$ itself grows like $B(t) \propto a^{1/2}$.

Our qualitative results can be confirmed by solving Eqs. (V.49), (V.50), and (V.51) numerically. Figure 4 shows the numerical solutions for the initial value $b_0 = 100$. For a large value of $b_0$, $a_1$ grows very fast while $a_3$ is frozen until $b$ becomes smaller than unity. Then, $b$ begins to oscillate, and the universe becomes isotropic again in that the expansion rates $\dot{a}_1$ and $\dot{a}_3$ converge; finally, the universe becomes $B$-matter dominated.

### VI. CONCLUSION

We considered the antisymmetric tensor field on a D3-brane. In a flat spacetime with a stabilized dilaton, the condensed homogeneous magnetic field oscillated without damping. When the linear dilaton was turned on, we analyzed the difference between the string frame and the Einstein frame. Particularly, the effect of the cosmological constant from (1+9)-dimensions was drastic; i.e., the dilaton was stabilized for some initial values and a negative cosmological constant [9].

In the early universe with gravitation, the existence of the D3-brane is significant. In the bulk without the D-brane, the anisotropy induced by the massless antisymmetric tensor field cannot be washed out through the cosmological evolution, but it remains as an observable fossil, which goes against the observed fact in the present universe. Once the mass and the self-interaction terms are included due to the presence of D3-brane in our universe, even the huge condensation of the homogeneous magnetic component is diluted as the universe expands. When the cosmological expansion is sufficiently large, the universe becomes isotropic and $B$-matter dominated.

It is intriguing to study cosmological evolution when all the closed string degrees in the bulk are included. Such a string cosmology of the graviton, the dilaton, and the antisym-
FIG. 4: Numerical solutions for $b_0 = 100$: from the top down, the evolution of $b(t)$, the scale factors $a_1(t)$ (solid line) and $a_3(t)$ (dotted line), and the expansion rates $H_1(t)$ (solid line) and $H_3(t)$ (dotted line).

metric tensor field or that of unstable D-brane need further study.

Acknowledgments

This work was supported by the BK 21 project of the Ministry of Education and Human Resources Development, Korea (I.C.), the grant KRF-2002-070-C00022 (E.J.C.), the research fund of Hanyang University (HY-2004-S) (H.B.K.), and the Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCSEC) supported by the KOSEF
and by Samsung Research Fund, Sungkyunkwan University, 2005 (Y.K.).

[1] For example, see Chapters 3, 8, and 13 of String Theory by J. Polchinski, (Cambridge University Press, Cambridge, 1998).
[2] M. Kalb and P. Ramond, Phys. Rev. D 9, 2273 (1974).
[3] E. Cremmer and J. Scherk, Nucl. Phys. B 72, 117 (1974).
[4] D. S. Goldwirth and M. J. Perry, Phys. Rev. D 49, 5019 (1994) arXiv:hep-th/9308023; K. Behrndt and S. Forste, Nucl. Phys. B 430, 441 (1994) arXiv:hep-th/9403179; E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D 50, 4868 (1994) arXiv:hep-th/9406216.
[5] E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D 51, 1569 (1995) arXiv:hep-th/9410136.
[6] E. Witten, Nucl. Phys. B 460, 335 (1996) arXiv:hep-th/9510135.
[7] E. J. Chun, H. B. Kim, and Y. Kim, JHEP 0503, 036 (2005) arXiv:hep-ph/0502051.
[8] C. Kim, H. B. Kim, Y. Kim, O. K. Kwon and C. O. Lee, J. Korean Phys. Soc. 45, S181 (2004) arXiv:hep-th/0404242.
[9] H. B. Kim, Mod. Phys. Lett. A 21, 363 (2006) arXiv:hep-th/0505128.
[10] J. H. Chung and W. S. l’Yi, J. Korean Phys. Soc. 45, 318 (2004).