The four-dimensional Kerr–Schild geometry contains two stringy structures. The first is the closed string formed by the Kerr singular ring, and the second is an open complex string obtained in the complex structure of the Kerr–Schild geometry. The real and complex Kerr strings together form a membrane source of the over-rotating Kerr–Newman solution without a horizon, $a = J/m \gg m$. It was also recently found that the principal null congruence of the Kerr geometry is determined by the Kerr theorem as a quartic in the projective twistor space, which corresponds to an embedding of the Calabi–Yau twofold into the bulk of the Kerr geometry. We describe this embedding in detail and show that the four sheets of the twistorial K3 surface represent an analytic extension of the Kerr congruence created by antipodal involution.

Keywords: Kerr–Schild geometry, complex shift, Kerr theorem, twistor, K3 surface, $N=2$ superstring

1. Introduction

It is now commonly accepted that black holes (BHs) are to be associated with elementary particles. The physics of BHs is based on complex analyticity and conformal field theory, which unites BH theory with superstring theory and particle physics. In our previous paper [1], we considered the emergence of stringlike structures in the four-dimensional Kerr–Schild (KS) geometry without horizons. In the dimensionless units $G = c = \hbar = 1$, this occurs when the Kerr rotational parameter $a = J/m$ exceeds the mass parameter $m$. This case of the over-rotating Kerr geometry is very important for application to elementary particles. Because elementary particle masses are very small and the spin may be extremely high, we find that $a$ exceeds $m$ by many orders for spinning particles. In particular, the electron mass in dimensionless units is $m \sim 10^{-22}$, while the spin $J = \hbar/2 = 1/2$. This yields $a = J/m \sim 10^{22}$, and $a$ therefore exceeds $m$ by about 44 orders.

The principal peculiarity of the over-rotating Kerr geometry is the presence of the naked Kerr singular ring, which is a branch line of the Kerr space into two sheets, forming a stringlike topological defect in the space–time. The singular metric should be regularized to an almost flat space–time to obtain a correspondence with quantum theory and the experimentally established negligibly small role of gravity in particle phenomenology. This procedure was specified step by step by many authors over more than four decades and finally led to a smooth, regular source in the form of a relativistically rotating, highly oblate vacuum bubble. The structure of this source and the corresponding references were discussed in [1] and [2].

The vacuum bubble covers the Kerr singular ring, and the singular interior of the bubble is replaced with a flat space. In this case, the Kerr–Newman (KN) electromagnetic field is regularized, attaining a finite maximum value at the edge rim of the oblate bubble and forming a stringlike loop around the bubble. The very old proposal that the Kerr singular ring plays the role of a closed string [3] was later supported...
by systematic investigations of singular string solutions of the low-energy string theory as solitons [4]. In particular, it was shown in [5] that the structure of fields around the Kerr singular ring is similar to the structure of the fundamental string solutions obtained in [6], [7].

A second stringy structure was obtained in the complex representation of the Kerr geometry, in which the KN gravitational and electromagnetic fields arise as a retarded-time field of a particle propagating along a complex world line (CWL) [8]–[10]. But the CWL from the reality standpoint is a world sheet [11] and is therefore equivalent to a string [12], [13]. The complex world sheet of this string is embedded in the real Kerr geometry, and the real and complex Kerr strings, as discussed in [1], together form a membrane by analogy with the passage from string theory to M-theory [14]. Recently, the two Kerr strings were mentioned in [15], and to describe the reaction of the authors, it is best to present a quotation from that work: “It would have been a cruel god to have layed [sic] down such a pretty scheme and not have it mean something deep.”

In addition to this stringy system, one more remarkable fact was obtained in [1]: it was shown that the Calabi–Yau complex twofold (K3 surface) appears in the projective twistor space of the Kerr geometry. This manifold arises as a quartic on the projective twistor space CP³ and determines the Kerr principal null congruence (PNC) in accordance with the Kerr theorem. In [1], we concluded that the arising close analogy to superstring theory is not accidental and may be a consequence of the presence of some fundamental structure lying beyond these relations. In particular, we proposed that this correspondence may be related to a mysterious complex N=2 string with the critical real dimension four [16] and closely related to twistors [11], [17].

In this paper, we extend and specify the treatment of the Calabi–Yau space (K3 surface) embedded in the four-dimensional KS geometry and give additional arguments supporting the proposition that the source of the striking analogy with superstring theory is the N=2 critical superstring whose structure is consistent with the twistorial structure of KS geometry. We work in the KS formalism [18], which is similar to the Newman–Penrose formalism but has an advantage related to the fact that the Kerr theorem is applicable in the KS formalism and allows describing the geodesic and shear-free congruences on an auxiliary Minkowski background in terms of twistors.

2. Basics of the Kerr–Schild formalism

Structure of the Kerr–Newman solution. The KN metric is represented in the KS form [18]

\[ g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_{\mu}e^3_{\nu}, \]

where \( \eta_{\mu\nu} \) is an auxiliary Minkowski background in the Cartesian coordinates \( x = x^\mu = (t, x, y, z) \) with the signature \((- + + +)\). In this expression,

\[ h = P^2 \frac{mx - e^2/2}{r^2 + a^2 \cos^2 \theta}, \quad P = \frac{1 + Y\bar{Y}}{\sqrt{2}}, \]

and \( e^3(x) \) is the tangent direction to the PNC determined by the form

\[ e^3_{\mu} dx^\mu = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv, \]

where

\[ \zeta = \frac{x + iy}{\sqrt{2}}, \quad \bar{\zeta} = \frac{x - iy}{\sqrt{2}}, \quad u = \frac{z + t}{\sqrt{2}}, \quad v = \frac{z - t}{\sqrt{2}} \]