Reconstructing the inflaton potential for an almost flat COBE spectrum

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Abstract

Using the Hubble parameter as new ‘inverse time’ coordinate ($H$-formalism), a new method of reconstructing the inflaton potential is developed also using older results which, in principle, is applicable to any order of the slow-roll approximation. In first and second order, we need three observational data as inputs: the scalar spectral index $n_s$ and the amplitudes of the scalar and the tensor spectrum. We find constraints between the values of $n_s$ and the corresponding values for the wavelength $\lambda$. By imposing a dependence $\lambda(n_s)$, we were able to reconstruct and visualize inflationary potentials which are compatible with recent COBE and other astrophysical observations. From the reconstructed potentials, it becomes clear that one cannot find only one special value of the scalar spectral index $n_s$.

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1. INTRODUCTION

The observations by the Cosmic Background Explorer (COBE) may have shown some hints on the nature of the inflation driven by the vacuum energy \[ \text{[1,2]} \]. In theoretical models this vacuum energy is simulated by the self-interaction potential of a scalar inflaton field. The amplitudes of the scalar and the tensor perturbations and the scalar and the gravitational spectral index of the background radiation are astrophysically observable. Inflationary models \[ \text{[3–5]} \] show that the scalar spectral index \( n_s \) takes values both between zero and one and, for some models, beyond 1. Recent data \[ \text{[6]} \] provide now a value of \( n_s \) between 1.1 and 1.59 which is slightly beyond the Harrison-Zel’dovich spectrum; cf. recently values in \[ \text{[7]} \].

The previous reconstructions of inflationary potentials \[ \text{[8–10]} \] have used both approximations and exact potentials depending on the wavelength \( \lambda \). The value of the potential at a special wavelength \( \lambda_0 \) or at a special value of the scalar field together with its first and second derivatives could be reconstructed. Hence, experimental data at different wavelengths determine, in this way, the form of the inflationary potential.

In \[ \text{[11]} \] we have found the general exact inflationary solution depending on the Hubble constant \( H \), the ‘inverse time’, and were able to classify a regime in which inflationary potentials are viable. In this paper, we apply this \( H \)-formalism to the first and second order perturbation formalism and reconstructed a phenomenologically viable inflationary potential. The construction of the graceful exit function \( g(H) \) is the essential point of our new method. Our function \( g(H) \), parametrized by \( n_s \), determines the inflaton potential and the exact Friedman type solution. From this differential equation we are able to present three different type of potentials for \( n_s = 1 \), \( n_s > 1 \), and \( 0 < n_s < 1 \). Moreover, for each regime of \( n_s \), we find a different dependence on the wavelength \( \lambda(n_s) \). This can be important for future observations.
For a rather general class of inflationary models the Lagrangian density reads
\[
\mathcal{L} = \frac{1}{2\kappa} \sqrt{|g|} \left( R + \kappa \left[ g^{\mu\nu} (\partial_{\mu} \phi)(\partial_{\nu} \phi) - 2U(\phi) \right] \right),
\]
where \( \phi \) is the scalar field and \( U(\phi) \) the self-interaction potential. We use natural units with \( c = \hbar = 1 \). A constant potential \( U_0 = \Lambda/\kappa \) would simulate the cosmological constant \( \Lambda \).

For the flat \((k = 0)\) Robertson-Walker metric
\[
ds^2 = dt^2 - a^2(t) \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right],
\]
the evolution of the generic inflationary model \((2.1)\) is determined by the autonomous first order equations
\[
\dot{H} = \kappa U(\phi) - 3H^2, \quad (2.3)
\]
\[
\dot{\phi} = \pm \sqrt{\frac{2}{\kappa}} \sqrt{3H^2 - \kappa U(\phi)}. \quad (2.4)
\]
This system corresponds to the Hamilton-Jacobi Eqs. \((2.1)\) and \((2.2)\) of Ref. \cite{10}. However, by introducing the Hubble expansion rate \( H := \dot{a}(t)/a(t) \) as the new ‘inverse time’ coordinate we \cite{11} found the general solution:
\[
t = t(H) = \int \frac{dH}{\kappa \tilde{U} - 3H^2}, \quad (2.5)
\]
\[
a = a(H) = a_0 \exp \left( \int \frac{HdH}{\kappa \tilde{U} - 3H^2} \right). \quad (2.6)
\]
\[
ds^2 = \left( \kappa \tilde{U} - 3H^2 \right)^2 a_0^2 \exp \left( 2 \int \frac{HdH}{\kappa \tilde{U} - 3H^2} \right) \times\]
\[
\left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right], \quad (2.7)
\]
\[
\phi = \phi(H) = \mp \sqrt{\frac{2}{\kappa}} \int \frac{dH}{\sqrt{3H^2 - \kappa \tilde{U}}}, \quad (2.8)
\]
where \( \tilde{U} = \tilde{U}(H) := U(\phi(t(H))) \) is the reparametrized inflationary potential.

Since the singular case \( \tilde{U} = 3H^2/\kappa \) leads to the de Sitter inflation, we use in explicit models the ansatz
\[ \bar{U}(H) = \frac{3}{\kappa} H^2 + \frac{g(H)}{\kappa} \]  
(2.9) 

for the potential, where \( g(H) \) is a nonzero function for the graceful exit. Our \( H \)-formalism will facilitate considerably the reconstruction of the inflaton potential to any order, as we will demonstrate in the following.

A classification of all allowed inflationary potentials and scenarios has recently been achieved by Kusmartsev et al. [12] via the application of catastrophe theory to the Hamilton-Jacobi type equations (2.3) and (2.4).

3. DENSITY PERTURBATIONS

For a long time one thought that the spectrum of density perturbations is described by the scale-invariant Harrison-Zel’dovich form [13–15]. But new observations by COBE [1] show the possibility of small deviations. In Ref. [9,10] the slow-roll approximation is specified by the three parameters \( \epsilon, \eta, \) and \( \xi \) defined as follows:

\[ \epsilon := 2 \left( \frac{H'}{H} \right)^2 = - \frac{g}{H^2} , \quad (3.1) \]
\[ \eta := 2 \frac{H''}{\kappa} H = 3 \frac{d\bar{U}}{2HdH} = - \frac{dg}{dH^2} , \quad (3.2) \]
\[ \xi := 2 \frac{H'''}{\kappa} H' = \eta - \sqrt{2\kappa \epsilon} = - \frac{dg}{dH^2} - 2H^2 \frac{d^2 g}{(dH^2)^2} . \quad (3.3) \]

From our \( H \)-formalism [11], the relations for the graceful exit function \( g(H) \) on the right hand side of (3.1)-(3.3) follow quite generally. These slow-roll parameters effectively provide a Taylor expansion of the graceful exit function. The minus sign in \( \sqrt{\epsilon} = -\sqrt{2/\kappa H'/H} \) is necessary in order to be consistent with the choice \( \dot{\phi} > 0 \), cf. [10].

In general, these parameters are scale-dependent and have to be evaluated at the horizon. The parameter \( \epsilon \) describes the relation between the kinetic and the total energy, whereas \( \eta \) is a measure for the relation between the “acceleration” of the scalar field and its “curvature-depending velocity”. In the slow-roll approximation, all three parameters are
small quantities. Actually, the phase of acceleration ($\ddot{a} > 0$) is now equivalently to the condition $\epsilon < 1$.

The amplitudes of scalar and transverse-traceless tensor perturbations [16,3] are given in first order slow-roll approximation [3] by

$$P_R^\frac{1}{2} (k) = \left( \frac{H^2}{4\pi |H'|} \right) \bigg|_{aH = \hat{k}}, \quad \text{(3.4)}$$

$$P_g^\frac{1}{2} (k) = \left( \frac{H}{2\pi} \right) \bigg|_{aH = \hat{k}}, \quad \text{(3.5)}$$

where $R$ denotes the perturbation in the spatial curvature, $H' = dH/d\phi$, and $\hat{k} := 2\pi/\lambda$ the wave number. The expressions on the right hand side have to be evaluated at that comoving scale $\hat{k}$ which is leaving the horizon during the inflationary phase. The scalar and the gravitational spectral indices in first order approximation read

$$n_s := 1 + \frac{d\ln P_R}{d\ln k} = 1 - 4\epsilon + 2\eta, \quad \text{(3.6)}$$

$$n_g := \frac{d\ln P_g}{d\ln k} = -2\epsilon \leq 0. \quad \text{(3.7)}$$

The last condition is fulfilled, because $\epsilon > 0$ or $g < 0$, respectively.

The relation of the wavelength $\lambda$ to the scalar field [9,10]

$$\frac{d\lambda}{d\phi} = \lambda \frac{H}{H'} \kappa (1 - \epsilon) \quad \text{(3.8)}$$

converts, in the $H$-formalism, exactly to

$$\frac{d\ln \lambda}{dH^2} = \frac{1}{2H^2} \left( \frac{1}{\epsilon} - 1 \right) = -\frac{1}{2} \left( \frac{1}{H^2} + \frac{1}{g} \right). \quad \text{(3.9)}$$

By using the slow-roll condition $\epsilon << 1$, the $H$-dependence simplifies to [8]

$$\frac{d\lambda}{dH} \approx \frac{\lambda}{H\epsilon}. \quad \text{(3.10)}$$

4. RECONSTRUCTING THE INFLATON POTENTIAL IN FIRST ORDER

From Eq. (3.6) we find the differential equation for the graceful exit function
\[
\frac{dg}{dH} = (1 - n_s)H + \frac{4g}{H},
\]  
(4.1)

which has the solution

\[
g(H) = \Delta H^2 - AH^4,
\]  
(4.2)

where we abbreviated the deviation from the flat spectrum, i.e., \(n_s = 1\), by \(\Delta := (n_s - 1)/2\).

For the integration constant \(A\) we find the \(n_s\)-dependent reality condition

\[
0 \leq \frac{\Delta}{H^2} < A,
\]  
(4.3)

provided \(n_s \geq 1\). In second order perturbation, the condition

\[
\xi = \Delta - 2AH^2 << 1
\]  
(4.4)

would arise.

We can distinguish an inflationary and a Friedmann era of spacetime for the potential

\[
\tilde{U}(H) = \frac{1}{\kappa}[(\Delta + 3)H^2 - AH^4].
\]  
(4.5)

The inflation starts at the ‘inverse time’ \(H_1 = \sqrt{\Delta/A}\) (which means \(\kappa\tilde{U} = 3H^2\)), it ends at \(H_2 = \sqrt{(\Delta + 1)/A}\) (which corresponds to \(\kappa\tilde{U} = 2H^2\)), whereas for \(H_3 = \sqrt{(\Delta + 3)/A}\) we have \(\tilde{U}(H) = 0 = U(\phi)\). Of course, only the inflationary and the beginning of the Friedmann parts are physically relevant. After this, the approximation is no longer valid. For \(0 \leq H \leq H_1\), the dilaton field would become a ghost (or one could construct gravitational instantons, following [17]). The duration of inflation determines a range for \(A\)

\[
\Delta < AH^2 < \Delta + 1.
\]  
(4.6)

If \(\Delta < 0\), i.e., \(n_s < 1\), and \(A > 0\), the inflationary phase exists but does no longer start at the point \(\kappa\tilde{U} = 3H^2\). If \(\Delta + 1\) is negative (i.e., \(n_s \leq -1\)), the constant \(A\) has to become negative, too, in order to allow an inflationary phase. Hence, in all such cases, we find an inflationary phase.
Via \( g =: -d\tilde{W}/dH \), the solution \(^{(4.2)}\) corresponds to the non-Morse function

\[
\tilde{W}_{\text{deform}} = \frac{A}{5} H^5 - \frac{\Delta}{3} H^3 + B ,
\] (4.7)

\( (B \) is an integration constant\) and therefore belongs to the 4th Arnold class \( A_4 \), see our recent bifurcation analysis of inflation in Ref. \(^{[12]}\).

**A. The Harrison-Zel’довich potential**

The flat spectrum of Harrison-Zel’dovich is obtained for \( n_s = 1 \). From \(^{(4.2)}, (2.9),\) and \(^{(2.3)}\) we get the Hubble expansion rate

\[
H = \left[3A(t + C_1)\right]^{-1/3} ,
\] (4.8)

whereas the scale factor reads

\[
a(t) = a_0 \exp \left[ (3A)^{-1/3} \frac{3}{2}(t + C_1)^{2/3} \right] .
\] (4.9)

The scalar field is then given by

\[
\phi(t) + C_3 = \pm \sqrt{\frac{2}{A\kappa}} \left[3A(t + C_1)^{1/3} = \pm \sqrt{\frac{2}{A\kappa}} \frac{1}{H} .
\] (4.10)

The corresponding potential

\[
U(\phi) = \frac{6}{A\kappa^2} (\phi + C_3)^{-4} \left[ (\phi + C_3)^2 - \frac{2}{3\kappa} \right] 
\] (4.11)

describes the flat Harrison-Zel’dovich spectrum.

The more general ansatz

\[
g(H) = -AH^n ,
\] (4.12)

where \( n \) is real and \( A \) a positive constant of dimension \( \text{length}^{n-2} \), leads to several known and new solutions, cf. \(^{[13]}\).

\(^{1}\)Eq. (7.14) in \(^{[13]}\) is misprinted; the correct potential reads:

\[
U(\phi) = \frac{1}{\kappa} \left[ \frac{A\kappa}{8(2 - n)^2(\phi + C_3)^2} \right]^{2/(2-n)} \left( 3 - \frac{8}{\kappa(2 - n)^2(\phi + C_3)^2} \right) .
\]
B. Inflationary potential with an almost flat spectrum: \( n_s > 1 \)

For \( n_s > 1 \), i.e., \( \Delta := (n_s - 1)/2 > 0 \), and \( A > 0 \), we find from (4.2) and (2.5)-(2.8) the solution

\[
t = -\frac{1}{\Delta H} + \sqrt{\frac{A}{\Delta^3}} \text{arcoth} \left[ \sqrt{\frac{A}{\Delta}} H \right], \quad (4.13)
\]

\[
a(H) = a_0 \left( \frac{AH^2}{AH^2 - \Delta} \right)^{1/(2\Delta)}, \quad (4.14)
\]

\[
\phi(H) = \mp \sqrt{\frac{2}{\kappa \Delta}} \arcsin \left( -\sqrt{\frac{\Delta}{A \, H}} \right), \quad (4.15)
\]

\[
H(\phi) = -\sqrt{\frac{\Delta}{A \sin^2 \left( \mp \sqrt{\frac{\Delta}{2 \phi}} \right)}}. \quad (4.16)
\]

The potential \( U(\phi) \) reads

\[
U(\phi) = \frac{\Delta}{\kappa A \sin^2 \left( \mp \sqrt{\frac{\Delta}{2 \phi}} \right)} \left( \Delta + 3 - \frac{\Delta}{\sin^2 \left( \mp \sqrt{\frac{\Delta}{2 \phi}} \right)} \right). \quad (4.17)
\]

We recognize that the limit \( \Delta \to 0 \) is not singular for \( a, \phi, H, U \); for \( t(H) \) the limit is indeterminate (confirmed by MATHEMATICA and MAPLE), because of the restricted definition range of arcoth.

We investigate now the case where \( n_s = \text{const.} \). The inflaton starts in an extremum of the potential \( U(\phi) \) (\( \tilde{\kappa} \tilde{U} = 3H^2 \)). Then, depending on \( n_s \) we find two types of behavior of the potential. The local \textit{extrema} of the potential occur at

\[
\phi_1 = \sqrt{\frac{2}{\kappa \Delta}} (m + 1)_2 \pi, \quad m = 0, 1, 2, \ldots \quad (4.18)
\]

\[
\phi_2 = \sqrt{\frac{2}{\kappa \Delta}} \arcsin \left( \sqrt{\frac{2\Delta}{3 + \Delta}} \right). \quad (4.19)
\]

For \( \Delta < 3 \) or \( n_s < 7 \), respectively, \( \phi_1 \) is always a minimum at the beginning of inflation followed by a maximum at \( \phi_2 \). But for \( n_s \geq 7 \), the maximum at \( \phi_2 \) disappears and \( \phi_1 \), the previous minimum, becomes a maximum. The potentials for \( n_s < 7 \) belongs to the “old” inflationary theory, whereas for \( n_s \geq 7 \) we would find the “new” inflationary potentials. Fig. [II] shows the possible inflationary potentials for \( n_s = 1.01 \ldots 2.1 \). The inflationary
potential which will be measured is then a “way on this rug”. Fig. 2 and Fig. 3 show two cross-sections within the “rug”.

C. Inflationary potential with an almost flat spectrum: $n_s < 1$

For $0 < n_s < 1$, we have to distinguish two cases: a) $\Delta < 0$ and $A < 0$, b) $\Delta < 0$ and $A > 0$. In both cases, the reality condition $g < 0$ can be satisfied. The solution of case a) is given by Eqs. (4.13)-(4.17). For case b), we find

$$t = -\frac{1}{\Delta H} + \sqrt{-\frac{A}{\Delta^3}} \arctan \left[ \sqrt{-\frac{A}{\Delta}} \right],$$

$$\phi(H) = \mp \sqrt{-\frac{2}{\kappa \Delta}} \ln \left( \frac{1}{H} \left[ \sqrt{-\frac{\Delta}{A}} + \sqrt{-\frac{\Delta}{A} + H^2} \right] \right),$$

$$H(\phi) = \sqrt{-\frac{\Delta}{A}} \exp \left( \mp \sqrt{\frac{2\Delta}{\kappa \Delta}} \phi \right),$$

$$U(\phi) = -\frac{4\Delta}{A \kappa} \frac{\exp \left( \mp \sqrt{-2\kappa \Delta} \phi \right)}{\left( \exp \left( \mp \sqrt{-2\kappa \Delta} \phi \right) - 1 \right)} \left( \Delta + 3 + \frac{4\Delta \exp \left( \mp \sqrt{-2\kappa \Delta} \phi \right)}{\left( \exp \left( \mp \sqrt{-2\kappa \Delta} \phi \right) - 1 \right)^2} \right),$$

and $a(H)$ is given by (4.14). Related inflationary solutions parametrized by $n_s$ below and above one have been obtained with a different method in Ref. [18].

5. INFLATIONARY POTENTIAL PARAMETRIZED BY THE WAVELENGTH

Because the scalar field is not observable, we also specify the dependence of the potential on the wavelength $\lambda$. From (3.10) together with (3.1) and (4.2), we get

$$\frac{\lambda}{\lambda_0} \approx \left( 1 - \frac{\Delta}{AH^2} \right)^{1/(2\Delta)}, \quad H^2 \approx \frac{\Delta}{A} \left( 1 - \left[ \frac{\lambda}{\lambda_0} \right]^{2\Delta} \right)^{-1},$$

where $\lambda_0$ is an integration constant. The value of the inflationary potential (4.3) is determined by
For \( n_s = 1 \) and \( A > 0 \), we obtain the solution

\[
H^2 \simeq -\frac{1}{2A \ln[\lambda/\lambda_0]},
\]

\[
U \simeq -\frac{1}{2\kappa A \ln[\lambda/\lambda_0]} \left( 3 + \frac{1}{2 \ln[\lambda/\lambda_0]} \right).
\]
where \( C = -2 + \ln(2) + \gamma \sim -0.73 \) and \( \gamma \sim 0.577 \) is the Euler constant \[19\]. In the \( H \)-formalism, Eq. (6.1) converts into the nonlinear second order equation

\[
\Delta = 2 \frac{g}{y} - g' - 4(1 + C) \left( \frac{g}{y} \right)^2 + (3 + 4C) \frac{g}{y} g' - 2C gg''
\]  

(6.2)

for the graceful exit function \( g \), where \( y := H^2 \) and \( t = d/d(H^2) \). In terms of \( \epsilon = -g/y \), cf. (3.1), we can rewrite this condition as

\[
2C \epsilon \dddot{\epsilon} - (2C + 3) \epsilon \ddot{\epsilon} - \epsilon + \epsilon^2 + \epsilon + \Delta = 0 ,
\]  

(6.3)

where \( \dot{\epsilon} = d/d \ln y \). Equation (6.2) has the exact \( \Delta \)-dependent solution

\[
g = \left( \frac{1}{2} \pm \sqrt{\frac{1}{4} - \Delta} \right) y .
\]  

(6.4)

The solution with the plus sign can be ruled out because we require \( g < 0 \), while the solution with the minus sign possesses an inflationary part if and only if \( n_s < 1 \). The potential for this \( g \) was already constructed \[11\], it belongs to the class of power-law models:

\[
U(\phi) = \frac{3 - A}{\kappa} C_3 \exp(\pm \sqrt{2\kappa A} \phi) .
\]  

(6.5)

Note, however, that in second order the integration constant \( A = \frac{1}{2} - \sqrt{\frac{1}{4} - \Delta} \) is now fixed by the observational data for the scalar spectral index \( n_s \). Because of the nonlinearity of (6.2), further solutions exist. In order to obtain more inflaton potentials \( U(\phi) \) in second order, we rewrite (2.8) into the form

\[
\frac{d\phi}{dy} = \mp \sqrt{\frac{2}{\kappa}} \frac{1}{\sqrt{-4gy}} .
\]  

(6.6)

This equation together with (6.2) combines to a coupled system which can be solved numerically, for example by using MATHEMATICA. The potential \( U(\phi) \) is given by the parametric solution \( \{\phi(H), U(H) = (3H^2 + g)/\kappa\} \); see Fig. 4.

7. REMARKS

Using the \( H \)-formalism, in first order, we were able to present three alternative dependences of the potential: \( U(H), U(\phi), \) and \( U(\lambda) \). In order to have no arbitrary integration
constant, we need three observables as input: the two amplitudes for the tensor and the scalar spectrum and the scalar spectral index \(n_s\), each time at the wavelength \(\lambda_0\) under consideration. From \(P_g^{\frac{1}{2}}\) at \(\lambda_0\), one finds the Hubble parameter

\[
H^2(\lambda_0) = 4\pi^2 P_g
\]  

(7.1)

and, from \(P_R^{\frac{1}{2}}\) at \(\lambda_0\), the Harrison-Zel’dovich constant

\[
A(\lambda_0) = \frac{\sqrt{2}}{4\pi\sqrt{\kappa}} \frac{1}{P_R} + \frac{\Delta}{4\pi^2 P_g}
\]  

(7.2)

follows. From (1.13) for \(n_s > 1\) or from (1.21) for \(n_s < 1\), respectively, the scalar field \(\phi(\lambda_0)\) is obtained and, hence, \(U(H(\phi(\lambda_0)))\). Equation (5.1) is the consistency condition for the observed quantities. Equations (5.5)-(5.7) give, up to first order, some restrictions for the \(\lambda-n_s\) relation.

In second order, we were able to present an exact solution for \(n_s < 1\). Nevertheless, there exist some more solutions because of the nonlinearity and the singularities of the differential equation (6.2).

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FIGURES

FIG. 1. Case: \( n_s = 1.01 \) up to \( n_s = 2.1 \). The inflaton potential \( U(\phi) \) is presented in units of \([1/(\kappa A)]\) and the scalar field \( \phi \) in units of \([\sqrt{2/\kappa}]\).

FIG. 2. Case: \( n_s = 1.1 \). It has the feature of the old inflationary theory. One would get this potential if one finds the same value \( n_s \) for each wavelength \( \lambda \). The physical units are the same as in Fig. [1].

FIG. 3. Case: \( n_s = 10 \). It describes a potential of new inflationary theory. Again, this potential is valid, if \( n_s \) is independent of the wavelength \( \lambda \). The physical units are the same as in Fig. [1].

FIG. 4. Case: \( 0.01 < n_s < 0.95 \) and \( \lambda/\lambda_0 = n_s^2 \). The physical units are the same as in Fig. [1].

FIG. 5. Case: \( 1.01 < n_s < 2.1 \) and \( \lambda/\lambda_0 = 1/n_s^2 \). The physical units are the same as in Fig. [1].

FIG. 6. Case: \( 1.01 < n_s < 2.1 \) and \( \lambda/\lambda_0 = 1/\sqrt{n_s} \). The physical units are the same as in Fig. [1].

FIG. 7. Several potentials \( U(\phi) \) in units of \([1/(\kappa)]\) depending on the scalar field \( \phi \) in units of \([\sqrt{2/\kappa}]\). We have chosen the initial conditions: a) \( g(5) = -0.1, \ g'(5) = 1, \) and \( \phi(5) = 1 \) for the drawn curve, b) \( g(5) = -0.1, \ g'(5) = 0.1, \) and \( \phi(5) = 1 \) for the dashed curve, and c) \( g(5) = -1, \ g'(5) = 1, \) and \( \phi(5) = 1 \) for the dotted curve. The solutions were found within the \( H \)-interval \([1, 5]\).
For \( \phi > 1 \), the minus sign was used in (6.6), whereas for \( \phi < 1 \), the plus sign has produced the graphs. One recognizes no smooth transition at \( \phi = 1 \).
Fig. 1:
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Fig. 2:

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Fig. 3:

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Fig. 4:

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Fig. 5:

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Fig. 6:

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Fig. 7:

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