On gspsg-homeomorphisms and sggsp-homeomorphisms in topological spaces

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Abstract

In the present paper we introduce two new types of mappings called gspsg-homeomorphism and sggsp-homeomorphism and then shown that one of these mapping has a group structure. Further we investigate some properties of these two homeomorphisms.

Key words: Homeomorphism; gspsg-homeomorphism; sggsp-homeomorphism.

1. Introduction

The notion homeomorphisms play a very important role in topology. A homeomorphisms between two topological spaces X and Y is a bijective map f : X → Y when both f and f⁻¹ are continuous. Devi, Balachandran and Maki in 1995 defined two new classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two new classes of maps called sgc-homeomorphisms and gsc-homeomorphisms. Ahmed and Narli in 2007 defined two classes of maps called gsg-homeomorphisms and sgs-homeomorphisms. Garg, Chauhan and Agarwal in 2007 introduced two new classes of maps namely gsϕ-homeomorphisms and ϕgs-homeomorphisms. Garg et al. again in 2007 introduced two classes of maps called gsϕ-homeomorphisms and ϕgs-homeomorphisms. In this paper we introduce two new classes of maps called gprsg-homeomorphisms and sggpr-homeomorphisms and then study some of their properties.

Throughout the present paper, (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ) the cl(A), int(A) and A^C denote the closure of A, the interior of A and the complement of A in X respectively.

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2. Preliminaries:

In this section we recall the following definitions.

**Definition 2.01**: A subset $A$ of a topological space $(X, \tau)$ is called semi-open\(^6\) (resp. pre-open, regular open) set if $A \subseteq \text{cl}(\text{int}(A))$ (resp. $A \subseteq \text{int}(\text{cl}(A))$, $A = \text{int}((\text{cl}(A))$). The complement of semi-open, pre-open, regular open set is called semi-closed, pre-closed, regular-closed respectively.

**Definition 2.02**: A subset $A$ of a topological space $(X, \tau)$ is called semi-generalized closed\(^7\) (briefly sg-closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open. The complement of sg-closed set is called sg-open set. Every semi-closed set is sg-closed set. The family of all sg-closed sets of any topological space $(X, \tau)$ is denoted by $\text{sgc}(X, \tau)$.

**Definition 2.03**: A subset $A$ of a topological space $(X, \tau)$ is called generalized semi closed \(^8\) (briefly gsc-closed) if $\text{sgc}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open. The family of all gsc-closed set is called gsc-open set. Every closed (semi-closed, g-closed and sg-closed) set is gsc-closed set. The family of all gsc-closed sets of any topological space $(X, \tau)$ is denoted by $\text{gsc}(X, \tau)$.

**Definition 2.04**: A subset $A$ of a topological space $(X, \tau)$ is called $\psi$-closed\(^12\) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open. The complement of $\psi$-closed set is called $\psi$-open set. Every closed (semi-closed) set is $\psi$-closed set and every $\psi$-closed set is sg-closed (gs-closed) set. The family of all $\psi$-closed sets of any topological space $(X, \tau)$ is denoted by $\text{gsc}(X, \tau)$.

**Definition 2.05**: A subset $A$ of a topological space $(X, \tau)$ is called generalized pre-regular closed\(^13\) (briefly gpr-closed) set if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular-open in $(X, \tau)$. The complement of gpr-closed set is called gpr-open set. Every closed set, sg-closed set and gs-closed set is gpr-closed set. The family of all gpr-closed sets of any topological space $(X, \tau)$ is denoted by $\text{gprc}(X, \tau)$.

**Definition 2.06**: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $\text{sgc}$-continuous\(^14\) (resp. $\text{gs}$-continuous\(^11\), $\psi$-continuous\(^12\), gpr-continuous\(^4\), $\text{gs}$-irresolute\(^14\), $\text{gs}$-irresolute\(^12\), gpr-irresolute\(^11\)) if the inverse image of every closed (resp. closed, closed, closed, closed, $\psi$-closed, gpr-closed) set in $Y$ is sgc-closed (resp. $\text{gs}$-closed, $\psi$-closed, gpr-closed, sgc-closed, gs-closed, $\psi$-closed, gpr-closed) set in $(X, \tau)$.

**Definition 2.07**: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $\text{gs}$-irresolute\(^14\) (resp. $\text{gs}$-irresolute\(^4\), $\text{gs}$-irresolute\(^12\), $\text{gs}$-irresolute\(^11\), $\text{gs}$-irresolute\(^12\)) if the inverse image of every closed (resp. closed, closed, closed, gpr-closed, $\psi$-closed, gpr-closed) set in $Y$ is sg-closed (resp. $\text{gs}$-closed, $\psi$-closed, gpr-closed, sgc-closed, gs-closed, $\psi$-closed, gpr-closed) set in $(X, \tau)$.

**Definition 2.08**: A bijective map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $\text{sgc}$-homeomorphism\(^11\) (resp. $\text{gs}$-homeomorphism\(^11\), $\text{gs}$-homeomorphism\(^15\), $\text{gs}$-homeomorphism\(^15\), $\text{gs}$-homeomorphism\(^17\), $\text{gs}$-homeomorphism\(^11\), $\text{gs}$-homeomorphism\(^18\), $\text{gs}$-homeomorphism\(^19\)) if $f$ and $f^{-1}$ are sg-irresolute (resp. $\text{gs}$-irresolute, $\text{gs}$-irresolute, $\text{gs}$-irresolute, $\text{gs}$-irresolute, $\text{gs}$-irresolute, $\text{gs}$-irresolute).

3. GSPSG-homeomorphisms:

In this section we introduce gspsg-homeomorphisms and then investigate the group structure of the set of all gspsg-homeomorphisms.

**Definition 3.01**: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a gspsg-irresolute map if the set $f^{-1}(A)$ is sg-closed in $(X, \tau)$ for every gspsg-closed set $A$ of $(Y, \sigma)$.

**Definition 3.02**: A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a gspsg-homeomorphisms if the function $f$ and the inverse function $f^{-1}$ are both gspsg-irresolute maps. If there exists a gspsg-homeomorphism from $X$ to $Y$, then the spaces $(X, \tau)$ and $(Y, \sigma)$ are called gspsg-homeomorphic. The family of all gspsg-homeomorphism of any topological space $(X, \tau)$ is denoted by $\text{gspsg}(X, \tau)$. 
Remark 3.03: The following examples show that the concepts of homeomorphism and gspsg-homeomorphism are independent of each other.

Example 3.04: Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, [a], [a, b], X\} \). Define \( f : (X, \tau) \rightarrow (X, \tau) \) by identity mapping then \( f \) is a homeomorphism but not a gspsg-homeomorphism.

Example 3.05: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\phi, [a], [a, b], X\} \) and \( \sigma = \{\phi, [a], [a, b], [a, c], Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is gspsg-homeomorphism but not homeomorphism.

Theorem 3.06: Every gspsg-homeomorphism is (i) sgc-homeomorphism (ii) gsg-homeomorphism (iii) \( \psi \)sg-homeomorphism (iv) \( \hat{g} \)sg-homeomorphism (v) \( \hat{g} \)gs-homeomorphism (vi) sgs-homeomorphism.

The converse of the above proposition is not true as it can be seen from the following examples.

Example 3.07: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\phi, [a], [a, b], X\} \) and \( \sigma = \{\phi, [a], [a, b], [a, c], Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is sgs-homeomorphism but not gspsg-homeomorphism.

Example 3.08: Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, [a], [c], [a, c], X\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is gsg-homeomorphism but not gspsg-homeomorphism.

Example 3.09: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\phi, [a], [a, b], X\} \) and \( \sigma = \{\phi, [a], [a, b], Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is \( \psi \)sg-homeomorphism but not gspsg-homeomorphism.

Example 3.10: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\phi, [a], [a, b], X\} \) and \( \sigma = \{\phi, [a], [a, b], [a, c], Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is gsg-homeomorphism but not gspsg-homeomorphism.

Example 3.11: In example (3.07), map \( f \) is \( \psi \)sg-homeomorphism but not gspsg-homeomorphism.

Example 3.12: In example (3.07), map \( f \) is sgs-homeomorphism but not gspsg-homeomorphism.

Remark 3.13: gspsg-homeomorphism is independent from sg\( \psi \)-homeomorphism and gsg-homeomorphism as it can be seen from the following examples.

Example 3.14: Let \( X = Y = \{a, b, c\} \), \( \tau = \{\phi, [a], [b, c], X\} \) and \( \sigma = \{\phi, [a], [a, b], Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is gspsg-homeomorphism but not sgs\( \psi \)-homeomorphism.

Example 3.15: In example (3.07), map \( f \) is sgs-homeomorphism but not gspsg-homeomorphism.

Example 3.16: Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, [a], [b, c], X\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is gspsg-homeomorphism but not \( \hat{g} \)sg-homeomorphism.

Example 3.17: Let \( X = \{a, b, c\} \) and \( \tau = \{\phi, [a], [b], X\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by identity mapping then \( f \) is gsg-homeomorphism but not gspsg-homeomorphism.

Theorem 3.18: Every \( \psi \)gs-homeomorphism and so gsg-homeomorphism and sgc-homeomorphism from a topological space \( X \) onto itself is gspsg-homeomorphism if every gsp-closed set is \( \psi \)-closed in \( X \).

Theorem 3.19: Every \( \hat{g} \)sg-homeomorphism and \( \hat{g} \)gs-homeomorphism from a topological space \( X \) onto itself is gspsg-homeomorphism if every gsp-closed set is \( \hat{g} \)-closed in \( X \).

Theorem 3.20: If \( f : (X, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) are gspsg-homeomorphism then their composition \( g \circ f : (X, \tau) \rightarrow (Z, \eta) \) is also gspsg-homeomorphism.

Theorem 3.21: If gspsgh\( (X, \tau) \) is non-empty then the set gspsgh\( (X, \tau) \) is a group under the composition of maps.

Theorem 3.22: If \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a gspsg-homeomorphism then \( f \) induces an isomorphism from the group gspsgh\( (X, \tau) \) onto the group gspsgh\( (Y, \sigma) \).

Proof: Define \( \theta_h : \text{gspsgh}(X, \tau) \rightarrow \text{gspsgh}(Y, \sigma) \) by \( \theta_h(h) = foh^{-1} \) for every \( h \in \text{gspsgh}(X, \tau) \). Then \( \theta_h \) is a bijection. Further, for all \( h_1, h_2 \in \text{gspsgh}(X, \tau) \), \( \theta_h(h_1h_2) = fo(h_1o(h_2)f^{-1} = (foh_1o(f^{-1})o(h_2o(f^{-1}) = \theta_h(h_1)h_2) \).
o $\theta_{h_2}$. So $\theta_1$ is a homeomorphism and so it is an isomorphism induced by $f$.

4. SGGSP-homeomorphisms:

In this section we introduce sggsp-homeomorphism and investigate its properties.

**Definition 4.01:** A map $f : (X, \tau) \to (Y, \sigma)$ is called sggsp-irresolute map if the set $f^{-1}(A)$ is sgp-closed in $(X, \tau)$ for every sg-closed set $A$ of $(Y, \sigma)$.

**Definition 4.02:** A bijection $f : (X, \tau) \to (Y, \sigma)$ is called a sggsp-homeomorphism if the function $f$ and the inverse function $f^{-1}$ are both sggsp-irresolute maps. If there exists a sggsp-homeomorphism from $X$ to $Y$, then the spaces $(X, \tau)$ and $(Y, \sigma)$ are called sggsp-homeomorphic.

The family of all sggsp-homeomorphism of any topological space is denoted by sggsp $(X, \tau)$.

**Theorem 4.03:** Every (i) homeomorphism (ii) sg-homeomorphism (iii) sgs-homeomorphism (iv) gsc-homeomorphism (v) gsg-homeomorphism (vi) gs-$\hat{\psi}$-homeomorphism (vii) gsg-$\hat{\psi}$-homeomorphism (viii) sgg-$\hat{\psi}$-homeomorphism (ix) gsgsp-homeomorphism (x) gs-$\psi$-homeomorphism (xi) gsc-homeomorphism is sggsp-homeomorphism.

The following examples show that the converse of the above proposition is not true.

**Example 4.04:** In example (3.14), map $f$ is sggsp-homeomorphism but not homeomorphism.

**Example 4.05:** Let $X = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not sggc-homeomorphism.

**Example 4.06:** Let $X = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not sgc-homeomorphism.

**Example 4.07:** Let $X = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not gsg-homeomorphism.

**Example 4.08:** In example (3.10), map $f$ is sggsp-homeomorphism but not gsg-homeomorphism.

**Example 4.09:** Let $X = \{a, b, c, d, \}, \tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$. Then sgc$(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, X\})$ and sgc$(Y, \sigma) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not gsgsp-homeomorphism.

**Example 4.10:** Let $X = \{a, b, c, \}, \tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not gs-$\hat{\psi}$-homeomorphism.

**Example 4.11:** Let $X = \{a, b, c, d, \}, \tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not sg-$\hat{\psi}$-homeomorphism.

**Example 4.12:** Let $X = \{a, b, c, d, \}, \tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not gsg-$\psi$-homeomorphism.

**Example 4.13:** In example (3.05), map $f$ is sggsp-homeomorphism but not g-$\psi$-homeomorphism.

**Remark 4.14:** sggsp-homeomorphisms is independent from $\psi$g-$\psi$-homeomorphism, $\hat{ggs}$-homeomorphism, $\hat{gs}$-homeomorphism, $\psi$g-$\hat{\psi}$-homeomorphism as it can be seen from the following examples.

**Example 4.15:** Let $X = \{a, b, c, \}, \tau = \{\phi, X\}$ and $\sigma = \{\phi, \{b\}, \{c\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is $\psi$g-$\psi$-homeomorphism but not sggsp-homeomorphism.

**Example 4.16:** In example (4.05), map $f$ is sggsp-homeomorphism but not $\psi$g-$\psi$-homeomorphism.

**Example 4.17:** Let $X = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then $f$ is sggsp-homeomorphism but not $\hat{ggs}$-homeomorphism.
Example 4.18: In example (4.11), map f is \(\hat{ggs}\)-homeomorphism but not sggsp-homeomorphism.

Example 4.19: Let \(X = Y = \{a, b, c,\}, \tau = \{\phi, \{a\}, X\} \) and \(\sigma = \{\phi, \{a, b\}, Y\}\). Define \(f : (X, \tau) \to (Y, \sigma)\) by identity mapping then f is sggsp-homeomorphism but not \(\hat{gs}\)-homeomorphism and \(\psi gs\)-homeomorphism.

Example 4.20: In example (4.11), map f is \(\hat{gs}\)-homeomorphism but not sggsp-homeomorphism.

Example 4.21: Let \(X = Y = \{a, b, c,\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}\) and \(\sigma = \{\phi, \{a\}, \{a, c\}, Y\}\). Define \(f : (X, \tau) \to (Y, \sigma)\) by identity mapping then f is \(\psi gs\)-homeomorphism but not sggsp-homeomorphism.

All the above discussions can be represented by the following diagram.

Theorem 4.22: Every sggsp-homeomorphism from X onto itself is gspsg-homeomorphism if every gsp-closed set is sg-closed in X.

Theorem 4.23: Every sggsp-homeomorphism from X onto itself is homeomorphism and so gsc-homeomorphism, sgc-homeomorphism, sgs-homeomorphism, gs\(\psi\)-homeomorphism, g\(\hat{s}\)g-homeomorphism and g\(\hat{g}\)g-homeomorphism if every gsp-closed set is closed in X.

Remark 4.24: The composition of two sggsp-homeomorphisms is not necessarily sggsp-homeomorphisms as it can be seen from the following example.

Example 4.25: Let \(X = Y = Z = \{a, b, c,\}, \tau = \{\phi, \{a\}, X\}, \sigma = \{\phi, \{a, b\}, Y\} \) and \(\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}\). Define \(f : (X, \tau) \to (Y, \sigma)\) and \(g : (Y, \sigma) \to (Z, \eta)\) by identity mapping then f and g are sggsp-homeomorphism but gof : \((X, \tau) \to (Z, \eta)\) is not sggsp-homeomorphism.

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