Abstract. Prajapati HR. 2021. Use of duality theory in organic farming: evidence from India. Asian J Agric 5: 45-52. The duality analysis provides an alternative way for solving the problem of cost minimization. In this method, a specified suitable cost function has been used as an objective function with certain constraints, rather than using production functions. In duality theory, both cost and profit functions are used as an objective function under a well-defined production technology along with related behavioral assumptions. This paper has applied translog cost function as an objective function under certain input constraints for the estimation of five input parameters viz. Land, Labor, Capital, Machinery, and Irrigation based on field survey data of 284 organic and non-organic wheat producers in four rain-fed districts of Gujarat. Based on these parameters, all inputs price elasticity and elasticity of input substitution have been calculated using Iterated Smilingly Unrelated Regression Equations (ISURE) method of estimation. The results of the estimation, for both organic and non-organic farms, have found both positive and negative signs as was expected theoretically but the value is not significant.

Keywords: Conventional farming, duality theory, elasticity of substitution, input price elasticity, organic farming

INTRODUCTION

In economic analysis, the aim of the producer is to maximize objective function under given constraints. The problem of producers, especially in the agriculture sector, is to optimally allocate the resources, such as land, labor, capital, technology, and irrigation in such a way that his output or profit is maximized. Thus, the optimum and efficient use of resources is a challenging task for the producer with specific objective of maximization of production or minimization of cost. In economic theory, it has been recognized that the producer is motivated by the desire to maximize his/her utility or satisfaction. Many studies have been conducted on farm household decision-making behavior based on the classical theory of firm. All these studies assumed a single objective of profit maximization as the motivation for farmers' decision-making behavior. Thus, these studies have ignored the role of other factors that influence the decision of farm households and are usually motivated by multiple, often conflicting goals, rather than only profit maximization (Romero and Rehman 1989).

The duality approach was developed by Shephard (1953), while its empirical applications became popular from the 1970s onwards. The first empirical study which exploited duality theory was conducted by Nerlove (1961). He used Cobb-Douglas type cost function as an indirect way for estimation of the parameters of the production function in electricity consumption. After that, the concept of flexible functions was invented and later they were used for the derivation of probable dual cost and profit functions in the early literature of 1970s (Diewert 1971; Christensen et al. 1973). It was an important step that led to the proliferation of empirical application of duality theory. There are several studies that are concerned about the agricultural sector. Of these, the study by Binswanger (1974) using U.S.A. data appears to be one of the earliest. Recently two studies were conducted by Roas and Lence (2017, 2019) using pseudo-data of U.S. agriculture, they found that parameters are not correctly resulting in the expected sign of elasticity as per economic theory. Thus, the results were dependent on data source and sample size.

Why is the use of duality theory contentious increasing? The reason behind the growing popularity of duality theory in production economics is that it allows a greater degree of flexibility in factor demand specification and output supply response equations along with showing very close relationship between economic theory and practice. For example, suppose transformation or production function depends on several input factors, the specified production technology, and a vector of output levels for empirical investigation factors equation can be derived through first-order condition of cost minimization problem. If the producer assumed the profit maximization, then the output supply response equation can also be derived from first-order condition of profit function. Unfortunately, in duality analysis very simple restrictive functions are used for the function transformation such as Cobb-Douglas and Constant Elasticity Substitution (Lopez 1982). Thus, the use of duality theory permits, to side-step problems by solving first-order conditions through either directly specifying minimization of appropriate cost function or profit maximization function rather than production function.
THEORETICAL ADVANCES

The theoretical advances of duality have passed through various phases, from hypothetical understanding, logical reasoning, and mathematical model formulation to empirical testing. As discussed above, the empirical application of duality was popular during 1970s, after the Nerlove work in 1961 and later continued by Diewert (1971) and Christensen et al. (1973). Both Binswanger (1974) and Rosas and Lence (2017) have applied duality theory in agriculture sector using a U.S.A. production data set with actual and pseudo-data and found contradictory results.

In duality mechanism, a set of essential properties of profit or cost functions are implied under a ‘well behaved’ production technology along with related behavioral assumptions. The application of duality theory has several advantages by specifying profit or cost function rather than transformation of production function. To derive the estimation factors, demand and output supply responses, there is no need to solve any complex production system of the first-order condition. The behavioral response equations can be obtained through differentiation of the dual function with respect to input or output prices. Another advantage is its application, as it needs less algebraic implications along with the flexibility to specify complex functions. It does not impose restrictions on the value of elasticity of substitution, separability, homotheticity, etc. (Lopez 1982). During the last four decades, the cost approach was more popular. It is used to estimate Hicksian input demand in addition to obtain information regarding properties of the underlying production technology. On the other hand, profit function approach allowed estimating Marshallian factors demand jointly with multi-output supply responses.

THE COST FUNCTION APPROACH

The cost function approach is the most popular and is applied for measuring the inputs/factor’s demand elasticity, elasticity of substitution and technical changes in agriculture production. In early literature, Binswanger (1974) and Kako (1978) specified a translog cost function that estimates inputs/factor shares in log-linear form. Both have applied the cost function, which is further adopted by Lopez (1982):

\[
\ln C = a_0 + a_i \ln Y + \sum_j \ln P_j + \gamma_i \ln Y \ln P_i + \gamma_{tt} \ln t (1)
\]

Where, \( C \) is the cost of production or cultivation and \( Y \) is output, \( P_i \) is the price of input factors \( i \), and \( t \) is used for a time trend variable as a proxy for technical change. Factor share specification can be obtained from equation (1) where factors/inputs share \( (S_i) \) is calculated by using logarithmic differentiation of Shephard’s lemma.

\[
S_i = v_i + \sum_j y_{ij} \ln P_j + \gamma_{tt} \ln t \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (2)
\]

Where, \( y_{ij} = y_{ji} \) and \( i = 1, 2, 3 \ldots N \)

Both Binswanger (1974) and Kako (1978) have measured the elasticity of agriculture production inputs like; land, labor, machinery, fertilizers, and other intermediate inputs. By using the above specification of cost function (1) and share/input equations (2), we can separate the effect of biased such as technical change \( (y_{tt}) \) parameters) of factor/input share from the effect of ordinary factor substitution due to change in factor/input price \( (y_{ij}) \) parameters) in equation (2). The result of both studies shows that technical change is very important and explains ample of the observed changes in factor shares in the U.S.A. and Japan. Though, both studies were based on the rigid assumption of homothetic production technology, with linear expansion paths, changes in the scale of production would not affect factor’s share. In other words, factors/inputs shares in equation (2) are assumed to be independent of the level of output. It means that all changes in factor/input share are attributed to substitution or factors/inputs augmenting in technological change. If the production technology is not homothetic, a risk of overestimating the effect of factor/input substitution or, more likely, technical change, exists. It happens because the time trend variable is used as a proxy for technical change and positively correlated with output levels.

Another similar study conducted by Lopez (1980), applied a more general specified cost function using Canadian agricultural data. This specification allowed for a non-homothetic production function under some degree of flexibility. In this study, he had applied a flexible cost function known as generalized Leontief cost function and written as:

\[
C = Y \sum_i b_{ij} p_i^{1/2} p_i^{1/2} + Y^2 \sum_i a_i p_i + Y \sum_i y_{ij} p_i \ldots (3)
\]

In equation (3) applying Shepard’s lemma, the factor/input demand equations in input/output ratio can be obtained in following forms:

\[
\frac{\gamma_{ij}}{Y} = \sum_j b_{ij} \left( \frac{p_i}{p_j} \right)^{1/2} a_i Y_t + \gamma_{tt} t \ \ \ \ \ \ \ \ (4)
\]

Where, the coefficient \( b_{ij} = b_{ji} \) and \( i = 1, 2, 3 \ldots N \)

In equation (4) Lopez (1980) analysis allowed separating the effect of relative factor price substitution, factors augmenting technical change and the scale of production on the input-output ratios. In equation (4) it allows, as a special case, for homothetic. This occurs if \( a_i = 0 \) for all \( i \), that is, when the input-output ratios are independent of the output. By estimating a function of four factors (labor, capital, land and structures and other intermediate inputs), input-output ratios showed that the hypothesis of homotheticity is rejected by a wide margin and that changes in the scale of production explain a very important proportion of changes in the input-output or share equations. The effect of non-neutral technical change was found to be insignificant, which was a rather surprising result. However, a recent more disaggregated study by Lopez and Tung (1982) using combined cross-section and
time-series data for Canadian agriculture, considered the inputs of: energy, energy-based, labor, capital, and other intermediate inputs. They showed that the factors augmenting technical change parameters (it) were jointly significant. Though, the technical change effect was substantially less dramatic than those obtained by Binswanger (1974) and Kako (1978), while the output scale effect is very strong and significant.

Table 1 shows that the own factor price elasticity of Hicksian input factors demand are quite similar for the four studies, despite using different data and models. The results concluded that factor demands are inelastic; where land \((L)\) demand elasticity ranges from -0.35 and -0.50, the demand of labor \((L_a)\) elasticity ranges between -0.40 and -0.50, but the Binswanger's result presents an outlier. The demand for fertilizers and chemicals \((F + Ch)\) tends to be more elastic at least in the studies using North American data (-0.9) and farm capital \((K)\) demand also exhibits somewhat lower values than the former. It means the estimated demand elasticity may provide some guidance to policymakers with several notions of the various degrees of price responsiveness of the inputs used in agricultural production.

These studies have applied different cost functions and found that the inputs demand are moderately responsive to prices. There exists a significant substitution possibility among several input pairs of which energy-based inputs and land appear to exhibit the greatest potential. The aggregate agricultural technology is not homothetic and the based inputs appear to exhibit the greatest potential. The Cobb-Douglas or Leontief are not appropriate specifications as shown by the studies of Binswanger (1974a) and Lopez (1980), respectively.

![Image](Image)

\[
\ln C = \alpha_0 + \alpha_y (\ln Y) + \sum_j n_j \alpha_j (\ln P_j) + \gamma \sum_t \sum_j \beta_{ij} (\ln P_j)(\ln P_t) + \sum_j \beta_{ij} (\ln P_j)
\]

............... (5)

Where, \(C, Y,\) and \(P_i\) are in the form of natural logarithm values, respectively, total cultivation cost, value of output and price of input \(i\). The input includes land \((N)\), labor \((L)\), capital \((K)\), fertilizer \((F)\), and others \((O)\). This cost function is an estimate of an arbitrary analytical function.

Table 1. Hicksian Input Demand Elasticity.

| Study          | Data                                      | Production function | Finding (input DE) |
|----------------|-------------------------------------------|---------------------|--------------------|
| Binswanger (1974) | U.S. Ag; cross sec + time series          | Translog            | -0.34 (L), -0.91 (La), -0.95 (Fer +Ch), -1.09 (K) |
| Kako (1978)    | Japan rice farm, cross-section + time series (1953-1970) | Translog            | -0.49 (L), -0.46 (La), -0.32 (Fer +Ch), -0.59 (K) |
| Lopez (1980)   | Canada Ag; time series (1946-1977)        | Gen Leontief        | -0.42 (L), -0.52 (La), -0.41 (Fer +Ch), -0.35(K) |
| Lopez and Tung (1982) | Canada Ag; cross sec + time series (1961-1979) | Gen Leontief        | -0.42 (L), -0.39 (La), -0.89 (Fer +Ch), -0.63(K) |

Note: L: Land; La: Labour; Fer+Ch: Fertiliser and chemical, K: Farm capital. Source: Lopez (1982).
Here we assume that farmer is a rational producer, who acts as the sole decision-maker to choose farming methods and factors of production with given factors/inputs price and other constraints. The farmer’s aim is to grow a certain level of output at the minimum cost at a given expenditure outlay, factors/inputs prices, and perfect competition in both product and input/factor market. It assumes symmetry across the price effect, which implies that further, it follows homogeneity in prices, which is defined as \( \beta_{ij} = \beta_{ji} \). It requires following restrictions on the parameters:

\[
\begin{align*}
\sum \alpha_i &= 1; \\
\sum \beta_{ij} &= 0; \\
\sum \beta_{ij} &= \beta_{ji} \\
\sum \beta_{ij} &= \beta_{ji}.
\end{align*}
\]

The estimation of this cost function can go two ways: either it may be estimated directly or through cost-share equations or simultaneously both through cost function and inputs/factors share equations jointly. The estimation through cost-share equation needs to derive first share equation by using Shepherd’s Lamma, which ensures that the cost minimization level of any input, \( X_i \) is equal to the derivatives of the cost function with respect to its price. Using first derivatives and applying Shepherd’s Lamma as:

\[
\ln TC = a_0 + a_N \ln P_N + a_L \ln P_L + a_K \ln P_K + a_F \ln P_F + a_I \ln P_I + a_{NL} \ln P_N \ln P_L + a_{NK} \ln P_N \ln P_K + a_{NF} \ln P_N \ln P_F + a_{NI} \ln P_N \ln P_I + \ldots
\]

The log of TC equation contains the log of all input factors, level of output, their squares, and their cross-products of one to another input. Partial differentiations of the total cost function with respect to log input price, get the shares equations for all inputs. These share equations are expressed as elasticity of the cost function with respect to the factor prices. The input share equations are derived by logarithmic differentiation of the cost function and applying Shephard’s lemma (McFadden 1978). Assuming that there is competition among input/factor providers in upstream markets and input prices are determined by the input market. The input demand functions are derived applying cost-minimizing procedure at a certain level of output by logarithmically differentiation of equation (8):

\[
\frac{\partial \ln TC}{\partial P_j} = \frac{\partial TC}{\partial P_j} = \alpha_i + \sum_j \ln P_j, \quad \text{i.e.} \quad j = N, L, K, F, I
\]

Then by applying Shephard’s Lemma it has found that,

\[
S_i = \frac{\partial TC}{\partial P_i}, \quad i = N, L, K, F, I
\]

Here, using above procedure, the share equations of inputs are obtained as:

\[
\begin{align*}
S_N &= \frac{P_N}{TC} = a_N + \gamma_{NN} \ln P_N + \gamma_{NL} \ln P_L + \gamma_{NK} \ln P_K + \gamma_{NF} \ln P_F + \gamma_{NI} \ln P_I + \gamma_{NY} \ln P_N \ln P_Y \\
S_L &= \frac{P_L}{TC} = a_L + \gamma_{LN} \ln P_N + \gamma_{LL} \ln P_L + \gamma_{LK} \ln P_K + \gamma_{LF} \ln P_F + \gamma_{LY} \ln P_Y \\
S_K &= \frac{P_K}{TC} = a_K + \gamma_{KN} \ln P_N + \gamma_{KL} \ln P_L + \gamma_{KK} \ln P_K + \gamma_{KF} \ln P_F + \gamma_{KY} \ln P_Y \\
S_F &= \frac{P_F}{TC} = a_F + \gamma_{FK} \ln P_N + \gamma_{FL} \ln P_L + \gamma_{KF} \ln P_F + \gamma_{FY} \ln P_Y \\
S_I &= \frac{P_I}{TC} = a_I + \gamma_{IN} \ln P_N + \gamma_{IL} \ln P_L + \gamma_{IK} \ln P_K + \gamma_{IF} \ln P_F + \gamma_{IY} \ln P_Y
\end{align*}
\]
The estimation procedure described above, and its analysis may proceed by estimating cost function directly (8) or by estimating the cost-share equations (9), or by estimating both together. While the direct estimation facilitates the determination of returns to scale embodied with underlying technology and the characteristics of farm input demands, this estimation has risk of reduction in the degree of freedom which adversely affects the statistical significance of the estimates. The alternative, the cost-share estimation does not lend itself to the determination of three parameters \( a_0, a_n \) and \( a_{xy} \). The estimation of cost-share equations allows for the estimation of input demand characteristics, elasticity, elasticity of substitution, but does not permit examining the nature of returns to scale of using underlying technology for crop production.

The joint estimation of translog cost function and associated cost share equations in literature were used by Zellner (1962, 1963), Kemeny and Gilbert (1968),Binswanger (1974), Lopez (1982) and Chaudhary and Mufti (1999). It has a dual advantage, as it increases the degree of freedom on the one hand and on the other hand, it provides more information. Here the general form of the translog non-homothetic cost function along with share equations is estimated by applying Iterated Smilingly Unrelated Regression Equations (ISURE) approach. This estimation approach has some restrictions such as the values of the factor shares must be equal to one. In this estimation process, only \((n - 1)\) of the share equations are estimated and the parameters of \( n \)th omitted equation are recovered by adding up restrictions. It can be written as:

\[
\sum_{i=1}^{n} \frac{P_{i/k}}{x} = 1 \quad \text{or} \quad \sum_{i} s_i = 1 \quad \text{...............}(10)
\]

**Elasticity of substitution estimation and price elasticity**

In theory, the elasticity of substitution measures the degree of substitution between the inputs. The elasticity of substitution can be estimated by using both cost and production function and researchers have applied both methods as per available data. For the estimation of elasticity of substitution through cost function, different methods can be used either to estimate directly or through using Allen partial elasticity as well as Allen-Uzawa elasticity. The elasticity of substitution estimation is made by using a well-behaved second differentiation of production function. The direct estimates of elasticity of substitution can be obtained by the following partial derivatives:

\[
C_{iK} = \frac{\partial \ln (L/K)}{\partial \ln (P_i/P_K)} \quad \text{...............}(11)
\]

However, the Allen partial elasticity substitution of inputs is obtained by following estimates:

\[
C_{ij} = C_{ij} / C_{i} \quad \text{...............}(12)
\]

Where, the subscripts \( i \) and \( j \) of \( C \) indicate partial differentiation of cost function with respect to the factor/input price of \( i \) and \( j \).

Here the expression (12) provides information of the cross-demand elasticity for inputs but does not directly show the behavior of relative share (McFedden 1978). The Allen partial elasticity is characterized by symmetry across the two inputs \( i \) and \( j \), that is, \( \alpha_{ij} = \alpha_{ji} \), it can be calculated from translog cost function at the mean value of the share of the inputs. It can be obtained from following share equations:

\[
\sigma_{ij} = \frac{f_{ij} + s_i s_j}{s_i s_j} \quad i \neq j \quad \text{...............}(13)
\]

\[
\sigma_{ii} = \frac{b_{ii} + s_i^2 - s_i}{s_i^2} \quad \text{...............}(14)
\]

The price elasticity of input demand (\( E_{ij} \)) is obtained by using following method:

\[
E_{ij} = \frac{\partial \ln x_i}{\partial \ln p_i} \quad \text{...............}(15)
\]

Where, quantity of output and all other inputs prices are constant, Allen (1938) has illustrated that Allen Elasticity of Substitution (AES) is analytically related to the price of elasticity of demand for factors of production, therefore;

\[
E_{ij} = S_i \sigma_{ij} \quad \text{...............}(16)
\]

Thus, even though \( \sigma_{ij} = \sigma_{ji} \) in general but \( E_{ij} \neq E_{ji} \)

**Data and variables description**

This study is based on sample of organic and conventional farm survey of rain-fed four districts of Gujarat. The total sample was 284 farmers who had been personally interviewed, from more than 20 villages and 11 talukas of four districts. The entire sample consisted of equal number of organic and conventional farms for the purpose of making comparison. The structured schedule had been used for accessing the information. The variables used for the analysis were; land (N), labor (L), capital (K), fertilizer (F), and irrigation (I). The price of land (\( P_n \)) Rs/acre, was calculated on the rent paid on lease land and rented value of own land existed in the village.

Here the price of labor (\( P_l \)) force included bullock cost also, which equals three-men power, calculated as Rs per day paid to both types of labor force. Capital price (\( P_k \)) is calculated as cost incurred on the use of hired or own machine and cost of interest paid to work and fixed capital Rs per hour paid to machinery. The price of fertilizer (\( P_f \)) was the sum of price of pesticides per packet and price of per bag 50 kg of chemical fertilizer or price of manure Rs per quintal, while the price of irrigation (\( P_i \)) included the cost of irrigation charges per hour.
PARAMETER ESTIMATES AND DISCUSSION

The parameter coefficients were estimated from the restricted translog cost function and share equations for organic and conventional farms and the results are presented in Tables 2 and 3. Most of the estimated coefficients showed desired statistical properties. Specifically, the expected regression coefficients are commenced with theoretically consistent algebraic signs and statistically significant with at least 5% level of significance. The Breusch-Pagan test was applied to test the contemporaneous correlation across the equations. During the process of estimation, absence of autocorrelation was found among observations in the data. Therefore, the validity of the derived estimates is as per regression axioms. While the estimates of the cost function and share equations have desirable statistical properties, such as expected sign, appropriate p-value, etc. The values of $R^2$ are not much important in ISUR application, they were, 0.719 for (TC) equation and 0.865 for (N), 0.395 (L), 0.632 (K), and 0.847 (F) share equations for conventional farm. Similarly, the value of $R^2$ were 0.565 (TC) for and 0.811 for (N), 0.633 (L), 0.600 (K), and 0.367 (F) in organic farm.

The statistical significance is estimated by p-value or t statistic of respective variables with appropriate sign but the power of prediction of the function is equally important for correct decisions. The prediction power of the function and the validity of the restrictions imposed on the estimating procedure depends on the significance of its F-value. The F-value is computed using usual formula as; the weighted residuals sum of square divided by number of restrictions and then divided by the ratio of the weighted sum of the square residuals with number of independent variables. Without imposing restriction, the number of the residual degrees of freedom is less than its tabulated value at 5% level of significance for both types of Farms.

Tables 2 and 3 revealed that the coefficients of the equation on the share of land (N) for conventional farms, log of prices of all inputs are statistically highly significant and the same for organic farms at 5% level of significance. Similarly, share of labor (L) equation coefficients were again highly significant for both these categories of farms. For the share of capital (K) in conventional farms prices log labor (L), fertilizer (F) and irrigation (I) were not statistically significant. The share equation of Fertilizer (F), log prices of all input were significant in both farms. The coefficient of irrigation (I) share equation is derived from the cross-equation restriction and symmetry constraints.

| Input | $a_i$ | N ($\beta_1$) | L ($\beta_2$) | K ($\beta_3$) | F ($\beta_4$) | I ($\beta_5$) | Y |
|-------|-------|---------------|---------------|--------------|--------------|---------------|-----|
| $S_N$ | -0.334 | 0.174* | -0.056* | -0.034* | -0.067* | -0.017* | -0.021 |
|       | (-11.65) | (223.79) | (-77.14) | (-43.40) | (-111.68) | (-25.19) | (-2.53) |
| $S_L$ | 0.625 | -0.056* | 0.157* | -0.029* | -0.056* | -0.017* | 0.006 |
|       | (7.75) | (-77.14) | (16.16) | (-4.38) | (-11.49) | (-2.97) | (0.26) |
| $S_K$ | -0.023 | -0.034* | -0.029* | 0.123* | -0.048* | -0.012* | 0.012 |
|       | (-0.49) | (-43.40) | (-4.38) | (15.83) | (-10.19) | (2.38) | (0.91) |
| $S_F$ | 0.769 | -0.067* | -0.056* | -0.048* | 0.199* | -0.028* | -0.007 |
|       | (16.98) | (-111.68) | (-11.49) | (-10.19) | (38.70) | (-6.89) | (-0.56) |
| $S_I$ | 4.353 | -103.267 | 65.343 | 31.938 | 133.332 | 30.614 | 1.371 |

Source: Derived from Field Sample data, * at 5% level of significance, t statistics in brackets

| Input | $a_i$ | N ($\beta_1$) | L ($\beta_2$) | K ($\beta_3$) | F ($\beta_4$) | I ($\beta_5$) | Y |
|-------|-------|---------------|---------------|--------------|--------------|---------------|-----|
| $S_N$ | -0.42 | 0.164* | -0.068* | -0.029* | -0.049 | -0.019* | 0.018 |
|       | (-5.86) | (156.13) | (-74.45) | (-21.36) | (-45.02) | (-16.39) | (0.81) |
| $S_L$ | 0.92 | -0.068* | 0.199* | -0.049* | -0.054* | -0.028* | -0.014 |
|       | (5.47) | (-74.45) | (16.97) | (-8.73) | (-5.13) | (-5.61) | (-0.36) |
| $S_K$ | 0.05 | -0.029* | 0.125* | -0.043* | 0.160* | -0.015* | 0.057 |
|       | (0.57) | (-21.36) | (-8.73) | (15.68) | (-6.18) | (-0.84) | (-0.85) |
| $S_F$ | 0.31 | -0.049* | -0.054* | -0.043* | 0.160* | -0.015* | 0.057 |
|       | (1.71) | (-45.02) | (-5.13) | (-6.18) | (11.49) | (-2.43) | (1.04) |
| $S_I$ | -0.05 | -60.339 | 66.182 | 14.405 | 56.315 | 22.907 | 0.365 |

Source: Derived from Field Sample data, * at 5% level of significance, t statistics in brackets
Input elasticity of substitution
The Allen elasticity of substitution (AES) is calculated using pairs of inputs and the estimates of parameters of share equations with the mean values of shares. The coefficients of $\beta_i$'s are used for calculation of AES, in the manner shown by equations (14) and (15) presented in Tables 4 and 5. Theoretically, a negative (positive) value of AES shows that input of pair is complement (substitute) to each other. According to this criterion, N and L are substitutes to each other in conventional farms and complimentary in organic farms. Here, labor (L) (both manual and animal) and land (N) are essential for the production activity.

One probable reason for such a complementary between land (N) and labor (L) in organic farming, that this method is labor-intensive and depends more on labor rather than machinery or capital (K). But the value of input elasticity of substitution $\sigma_{LK}$ is negative in case of organic farm and positive for conventional, having different meanings in economic explanation. However, the inputs substitution have negative sign e.g. N to K (-0.001), K to F (-0.12), K to I (-0.41) and F to I (-0.17) in conventional farm and N to L (-0.03), N to I (-0.05), L to K (-0.02), L to I (-0.03) and K to F (-0.18) in organic farms means there is some degree of complementary, but not so much strength because the value of elasticity substitution is less than 0.5. The substitutions between K to L (0.22) and K to F (0.22) and K to I (0.17) inputs in conventional farm and L to F (0.27) are also not strong in both types of farms, except K to I (.064) in organic farms because these five inputs are combination of a bundle of inputs and with single input/factor production is not possible.

The usage of fertilizer and pesticides may reduce labor use in nurturing the crops in conventional farming, thereby enabling farmers to save time (hours) but increasing the cost on the other hand with use of chemicals and fertilizers. While the use of organic compost or manure and traditional methods of weed control increases the use of labor that leads to increase in time (hours) and cost, it also reduces the cost of use of chemicals and pesticides at the same time. That's why the differences in cost of production do not differ in both types of farming.

Price elasticity of inputs demand
The price elasticity of input demand serves much useful theoretical and practical information as elasticity of substitution. Theoretically, own-price elasticity are negative (−), whereas the cross-price elasticity of inputs are positive (+) when inputs are substitutes and negative (−) when inputs are complementary in production. Theoretically, similar sign holds for the elasticity of substitution also. The price elasticity demands for five inputs are presented in Tables 6 and 7 respectively. The own-price elasticity of all inputs in both types of farms is negative (−). While the cross-price elasticity of all five inputs has positive (+) sign as was expected theoretically in case of conventional farm. But in case of organic farms only land (N) and labor (L) cross-price elasticity has negative sign. This means that the prices of land and labor have some degree of complementary to each other.

The positive own price elasticity for irrigation is contrary to the theoretical expectations. These may be some of the possible reasons for the noted contradictions. As irrigation or water charges had a fixed lump-sum amount per acre per crop. Once in each season or charges are fixed per hour basis, water charges vary among crops depending on their consumptive water requirements e.g., lower for crops like cereals and fodder crops and higher for those like rice with higher water requirements. Irrigation water charges are statutorily fixed in water-rich regions and vary in response to the irrigation water demand in scare regions like Patan, Surendranagar and Banskantha in Gujarat. A rise in the water rate may induce a shift from high water consumptive to low water consumptive crops. This will reduce the overall demand for water, indicating a negative response as per the theoretical expectation.

| Table 4. Allen elasticity of substitution of inputs for conventional farms. |
|-----------------------------------------------|
| \( \sigma_N \) | \( \sigma_L \) | \( \sigma_K \) | \( \sigma_F \) | \( \sigma_I \) |
| --- | --- | --- | --- | --- |
| -0.066 | 0.025 | -0.469 | symmetric | |

Source: Derived from Field Survey Data (2015)

| Table 5. Allen elasticity of substitution of inputs for organic farms. |
|-----------------------------------------------|
| \( \sigma_N \) | \( \sigma_L \) | \( \sigma_K \) | \( \sigma_F \) | \( \sigma_I \) |
| --- | --- | --- | --- | --- |
| -0.034 | -0.029 | -0.169 | symmetric | |

Source: Derived from Field Survey Data (2015)

| Table 6. Inputs price elasticity for conventional farms. |
|-----------------------------------------------|
| Elasticity | \( E_N \) | \( E_L \) | \( E_K \) | \( E_F \) | \( E_I \) |
|---|---|---|---|---|---|
| \( E_N \) | -0.015 | 0.006 | 0.001 | 0.001 | 0.009 |
| \( E_L \) | 0.006 | -0.116 | 0.033 | 0.064 | 0.014 |
| \( E_K \) | 0.001 | 0.054 | -0.022 | -0.031 | -0.001 |
| \( E_F \) | 0.001 | 0.054 | -0.016 | -0.025 | -0.014 |
| \( E_I \) | 0.025 | 0.042 | -0.001 | -0.048 | -0.018 |

Source: Derived from Field Survey Data (2015)

| Table 7. Inputs price elasticity for organic farms. |
|-----------------------------------------------|
| Elasticity | \( E_N \) | \( E_L \) | \( E_K \) | \( E_F \) | \( E_I \) |
|---|---|---|---|---|---|
| \( E_N \) | -0.007 | -0.009 | 0.015 | 0.006 | -0.005 |
| \( E_L \) | -0.006 | -0.053 | -0.003 | 0.065 | -0.003 |
| \( E_K \) | 0.021 | -0.006 | -0.027 | -0.043 | 0.055 |
| \( E_F \) | 0.005 | 0.086 | -0.027 | -0.087 | 0.023 |
| \( E_I \) | -0.011 | -0.010 | 0.097 | 0.064 | 0.002 |

Source: Derived from field survey data (2015)
Similarly, a variety of chemical fertilizers and pesticides are used in conventional farming. Their prices varied from farmer to farmer due to asymmetric information, but not much. Further, excessive advertising may have influenced their demand positively even in the presence of their rising prices. The own-price elasticity of demand for labor is significantly lower than unity in both types of farms and maybe inelastic. This can be explained by the nature of labor force supply in particular sample districts. As most of the farms, especially organic farms receive labor from their family members usually with very low opportunity cost in rural areas; its use typically changes little in response to changes in wage rate in another sector. Most of the estimates of the cross-price elasticity are less than one and have positive algebraic signs except land and labor. Further, their magnitudes for organic farms are smaller than those for conventional farms. Certain consistency in the results to a large extent indicates farmer’s behavior conforming to the postulates of minimizing cost in producing farm output in sample area.

In conclusion, theoretically, a negative (positive) value of the partial elasticity of substitution indicates that the inputs/factors of a given pair are complements (substitutes) to each other. Under this criterion, land (N) and labor (L) are complements to each other in conventional farms and organic farms. Own-price elasticity is negative, whereas the cross-price elasticity of inputs is positive when given inputs are substitutes and negative when they are complementary in production. Similar sign holds for the elasticity of substitution. The own-price elasticity of all inputs in both types of farms is negative. While the cross-price elasticity of all five inputs has positive sign as theoretically expected. But in case of organic farms, only land (N) and labor (L) cross prices elasticity have negative signs. It means that the prices of land and labor are complementary to each other. Finally, the results of elasticity estimation and theoretical validity depend on the calculated value of inputs/factors parameters, data source, and sample size.

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