A new criterion for zero quantum discord

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Abstract. We propose a new criterion for judging zero quantum discord for arbitrary bipartite states. A bipartite quantum state has zero quantum discord if and only if all the blocks of its density matrix are normal matrices and commute with each other. Given a bipartite state with zero quantum discord, the question of how to find the set of local projectors that do not disturb the whole state after being imposed on one subsystem is also presented. A class of two-qubit X-state is used to test the criterion, and an experimental scheme is proposed for realizing it. Consequently, we prove that the positive operator-valued measurement cannot extinguish the quantum correlation of a bipartite state with nonzero quantum discord.

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1. Introduction

For a bipartite system prepared in an entangled state, a local measurement on one of the two subsystems will affect the other subsystem owing to the ‘nonlocal features’ of the entanglement [1]. However, entanglement is not always necessary for illustrating the nonlocalities in a quantum system [2]. In 1998, a model of deterministic quantum computation with one qubit (DQC1) was proposed for quantum computing by using highly mixed states [3, 4], and this was experimentally implemented in 2008 [5]. It is a good example for illustrating that some highly mixed states, even fully separable, contain intrinsic quantum correlations and have potential applications in quantum computing. Furthermore, quantum correlation is found to be more robust than entanglement in a noisy environment, which makes quantum algorithms based solely on quantum correlation more robust than those based on entanglement [6–8].

If a bipartite quantum state is in a product state, \( \rho = \rho_A \otimes \rho_B \), with \( \rho_A \) (\( \rho_B \)) being the reduced density matrix of subsystem A (B), the state has no quantum correlation. However, a state with zero quantum correlation is not always a product state. The quantum correlation of a bipartite state is usually measured by quantum discord, introduced by Ollivier and Zurek in [9]. The question of how to find out whether a quantum state has zero quantum discord or not is fundamentally important; it is the first step to distinguish the quantum features of a bipartite state from the classical. For example, it is shown that zero quantum discord between a quantum system and its environment is necessary and sufficient for describing the evolution of the system through a completely positive map [10, 11]. In addition, a quantum state can be locally broadcast if and only if it has zero quantum discord [12, 13]. Recently, a necessary and sufficient condition for nonzero quantum discord was proposed in [14] with the help of a correlation matrix, derived from the density matrix, and its singular value decomposition. In this paper, we present a simpler method for judging zero quantum discord, where we only need to partition the density matrix into \( N^2 \) block matrices (\( N \) is the dimension of one subsystem) and check some properties of these block matrices. This method is valid for arbitrary bipartite states and is easy to implement. An example with a scheme to experimentally realize it is proposed in order to test this criterion. Based on this new criterion, we also prove that the positive operator-valued measurement (POVM) [15, 16] cannot extinguish the quantum correlation of a bipartite state with nonzero quantum discord.

2. A new criterion for zero quantum discord

Ollivier and Zurek introduced the concept of quantum discord to quantify the quantum correlation of a bipartite state, which is defined as the difference between two conditional entropies (classically equivalent quantities) [9],

\[
\delta(\rho_{AB})_{|\{|k_B\}\}} = H(A|\{|k_B\}\}) - [H(\rho_{AB}) - H(\rho_B)],
\]

where \( H(A|\{|k_B\}\}) \) is calculated by \( \sum_k p_{kB} H(\rho_{kB}) \) with \( \rho_{kB} = \frac{1}{p_{kB}} (k_B|\rho_{AB}|k_B) \) and \( p_{kB} = Tr_A((k_B|\rho_{AB}|k_B)) \), and \( H(\rho) \) is the von Neumann entropy of the quantum state \( \rho \) [17–19]. Here the subsystem A is regarded as the system and B as the apparatus. \( \{|k_B\}\} \) represents a set of local projectors on B, rather than the POVM used in [20]. In the calculation, different sets of projectors will give out different values of quantum discord for the same quantum state. Solving the question of how to find the set of local projectors that yield minimum quantum discord is very difficult [21–26].

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In a given basis, \( \{ |i_A k_B \rangle \} \) \((i = 1, 2, \ldots, N \) and \( k = 1, 2, \ldots, M \)) arranged as \( \{ |1_A 1_B \rangle, \ldots, |1_A M_B \rangle, |2_A 1_B \rangle, \ldots, |N_A M_B \rangle \)\), an AB bipartite quantum state can be described by the following density matrix:

\[
\rho_{AB} = \begin{pmatrix}
\rho_{11} & \cdots & \rho_{1(NM)} \\
\vdots & \ddots & \vdots \\
\rho_{(NM)1} & \cdots & \rho_{(NM)(NM)}
\end{pmatrix},
\]

(2)

which has zero quantum discord if and only if it can be also written as [9]

\[
\rho_{AB} = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} C_{iA,jA,k_B'} (|i_A \rangle \langle j_A|)(|k_B' \rangle \langle k_B|),
\]

(3)

with \( C_{iA,jA,k_B'} \) being real or complex numbers and \( \{ |k_B' \rangle \} \) \((k' = 1, 2, \ldots, M \) being a particular set of local projectors on B. The quantum state in the form of equation (3) is called pointer state [9], in which one can locally access the information in the system without changing the whole density matrix. Since the evaluation of quantum discord is asymmetric and depends on which subsystem is chosen as the system and which as the apparatus, the zero quantum discord of the quantum state (3) with the subsystem B being the apparatus does not guarantee zero quantum discord for A being the apparatus.

The \((NM) \times (NM)\) matrix in equation (2) can be partitioned into \(N^2\) blocks,

\[
\rho_{AB} = \begin{pmatrix}
\rho^{(1A1A)} & \cdots & \rho^{(1A N A)} \\
\vdots & \ddots & \vdots \\
\rho^{(N A1A)} & \cdots & \rho^{(N A N A)}
\end{pmatrix},
\]

(4)

with each block being an \( M \times M \) matrix,

\[
\rho^{(iA,jA)} = \begin{pmatrix}
\rho_{((i-1)M+1)((j-1)M+1)} & \cdots & \rho_{((i-1)M+1)(jM)} \\
\vdots & \ddots & \vdots \\
\rho_{(iM)((j-1)M+1)} & \cdots & \rho_{(iM)(jM)}
\end{pmatrix},
\]

(5)

which means that the state (2) or (4) is equivalent to

\[
\rho_{AB} = \sum_{i=1}^{N} \sum_{j=1}^{N} (|i_A \rangle \langle j_A|) \rho^{(iA,jA)}.
\]

(6)

We rewrite the quantum state equation (3) in the basis \( \{ |i_A k_B \rangle \} \),

\[
\rho_{AB} = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} C_{iA,jA,k_B} (|i_A \rangle \langle j_A|) U (|k_B \rangle \langle k_B|) U^\dagger,
\]

(7)

where the local unitary transformation \( U \) connects \( \{ |k_B \rangle \} \) and \( \{ |k_B' \rangle \} \) through the relation \( |k_B' \rangle = U |k_B \rangle \). In order to make equation (6) have the same form as equation (7), all block matrices \( \rho^{(iA,jA)} \) must be such that they are able to be diagonalized by the same unitary transformation \( U \),

\[
\rho^{(iA,jA)} = U \left[ \sum_{k=1}^{M} C_{iA,jA,k_B} (|k_B \rangle \langle k_B|) \right] U^\dagger,
\]

(8)
which gives us the following relation:

\[ [\rho^{(i_A j_A)}, (\rho^{(i_A j_A)})^\dagger] = 0. \] (9)

The matrix satisfying equation (8) or (9) is called the normal matrix [27]. In addition, since all \( \rho^{(i_A j_A)} \) are diagonalized by the same unitary matrix \( U \),

\[ \rho^{(i_A j_A)} = U \Lambda^{(i_A j_A)} U^\dagger, \] (10)

they have the same eigenvectors. Any two normal matrices have the same eigenvectors if and only if they commute with each other [27]. Consequently, we can conclude the criterion for zero quantum discord now: all \( N^2 \) blocks \( \rho^{(i_A j_A)} \) in equation (4) are normal matrices (satisfying equation (9)), and must commute with each other. This criterion has the advantage that we can work directly on the matrix equation (2) in any tensor product basis without the need to find the particular basis, \( \{|k_B\rangle\} \), required in the criterion of equation (3). As we all know, a bipartite state with zero quantum discord (see equation (3)) must be a separable state. Based on the new criterion, we can conclude that if a density matrix is composed of diagonal block matrices, it represents a separable state, which is valid for bipartite systems in any dimension. However, the inverse is not true. A separable state does not necessarily have a density matrix composed of diagonal block matrices. Given a high-dimensional bipartite or multipartite quantum state, how to efficiently verify its separability or entanglement is still an open question.

We stress here that although commutation relations are used to describe the criterion of zero quantum discord, as in [14], the present criterion has no direct connection with that in [14]. The number of commuting matrices used in [14], denoted by \( L \), is equal to the rank of the correlation matrix, which is smaller than or equal to the minimal one between the two squared dimension degrees of the two subsystems A and B, i.e. \( L \leq \min\{N^2, M^2\} \). However, the number of commuting matrices used in our criterion is fixed at \( N^2 \), with \( N \) being the dimension of subsystem A. Furthermore, all the commuting matrices used in [14] are Hermitian operators, while the commuting matrices in our criterion can be non-Hermitian or Hermitian, depending on the density matrix itself.

If a quantum state \( \rho_{AB} \) has been verified to have zero quantum discord, we can obtain \( U \) and \( \{|k_B\rangle\} \) by diagonalizing any nonzero one of the block matrices \( \rho^{(i_A j_A)} \) in equation (5). From equations (6) and (10), we have

\[ \rho_B = \text{Tr}_A(\rho_{AB}) = \sum_{i=1}^{N} \rho^{(i_A j_A)} = \sum_{i=1}^{N} U \Lambda^{(i_A j_A)} U^\dagger = U D U^\dagger, \] (11)

with the diagonal matrix \( D = \sum_{i=1}^{N} \Lambda^{(i_A j_A)} \), which tells us that the reduced matrix \( \rho_B \) can also be diagonalized by the unitary matrix \( U \).

Let us now consider an example. Given a class of two-qubit X-state in the basis of \( \{|1_A 1_B\rangle, |1_A 2_B\rangle, |2_A 1_B\rangle, |2_A 2_B\rangle\} \),

\[ \rho_x = \begin{pmatrix}
  x & 0 & 0 & \sqrt{x(0.5-x)} \\
  0 & 0.5 - x & \sqrt{x(0.5-x)} & 0 \\
  0 & \sqrt{x(0.5-x)} & x & 0 \\
  \sqrt{x(0.5-x)} & 0 & 0 & 0.5 - x
\end{pmatrix}, \quad (x \in [0, 0.5]), \] (12)
we can check whether the above states have zero quantum discord through the following three steps.

1. Partition the density matrix (12) into four blocks:
   \[
   \rho^{(1\Lambda 1\Lambda)} = \rho^{(2\Lambda 2\Lambda)} = \begin{pmatrix} x & 0 \\ 0 & 0.5 - x \end{pmatrix}, \quad \rho^{(1\Lambda 2\Lambda)} = \rho^{(2\Lambda 1\Lambda)} = \begin{pmatrix} 0 & \sqrt{x(0.5 - x)} \\ \sqrt{x(0.5 - x)} & 0 \end{pmatrix}.
   \] (13)

2. Check whether the four blocks are normal matrices (satisfying equation (9)): yes, here they are.

3. Check whether all of them commute with each other: since
   \[
   \rho^{(1\Lambda 1\Lambda)} \rho^{(1\Lambda 2\Lambda)} = \begin{pmatrix} 0 & x \sqrt{x(0.5 - x)} \\ (0.5 - x) \sqrt{x(0.5 - x)} & 0 \end{pmatrix}, \quad (14a)
   \]
   and
   \[
   \rho^{(1\Lambda 2\Lambda)} \rho^{(1\Lambda 1\Lambda)} = \begin{pmatrix} 0 & (0.5 - x) \sqrt{x(0.5 - x)} \\ x \sqrt{x(0.5 - x)} & 0 \end{pmatrix}, \quad (14b)
   \]
   the equality \(\rho^{(1\Lambda 1\Lambda)} \rho^{(1\Lambda 2\Lambda)} = \rho^{(1\Lambda 2\Lambda)} \rho^{(1\Lambda 1\Lambda)}\) holds true only when \(x = 0, 0.25\) or 0.5. In the case of \(x = 0.25\), the unitary transformation
   \[
   U = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}
   \]
diagonalizes the four matrices in equation (13), and the local projectors \(\{|k' B\}\) in the pointer state are \(|1' B\rangle = \frac{\sqrt{2}}{2} (|1 B\rangle + |2 B\rangle)\) and \(|2' B\rangle = \frac{\sqrt{2}}{2} (|1 B\rangle - |2 B\rangle)\). For \(x = 0\) and 0.5, the four matrices in equation (13) are already diagonal (or zero matrix), and \(\{|1 B\rangle, |2 B\rangle\}\) is just the set of local projectors used in the pointer state.

The quantum discord of this state can be directly calculated using the results in [21–26]:
   \[
   \delta(\rho_A) = -1 - (2x) \log_2(2x) - (1 - 2x) \log_2(1 - 2x) \\
   - \sum_{k=1}^{2} \left[ 0.5 + (-1)^k \sqrt{2x(1 - 2x)} \right] \log_2 \left[ 0.5 + (-1)^k \sqrt{2x(1 - 2x)} \right].
   \] (15)

The zero quantum discord occurs only when \(x = 0, 0.25\) or 0.5, which is the same as predicted using our criterion.

3. Proposed experiment and discussions

Now we propose an experimental scheme to test the criterion based on the above X-state (12), which can be generated through the following procedure. The entangled photon pairs from the type-I parametric downconversion are in the state
   \[
   |\psi_1\rangle = \cos \theta |H_A H_B\rangle + \sin \theta |V_A V_B\rangle,
   \] (16)
where \(\theta\) is the angle of the pump polarization direction with respect to the vertical orientation, and the two optical axes of the nonlinear crystals are arranged in the horizontal and vertical orientations (H and V) [28, 29], respectively; see figure 1. An electro-optical modulator (EOM)
Figure 1. The proposed experimental setup. One of the two entangled photons (B), produced from nonlinear crystals via type-I parametric downconversion, is sent to two single-photon detectors, $D_1$ and $D_2$, after its polarization is rotated by a half-wave plate (HWP). The polarization beam splitter (PBS) is used to distinguish between the two types (H or V) of polarization of photon B and separate them. The polarization of the other photon (A) is inverted, with probability 50%, by the EOM and then measured through single-qubit tomography. The quantum state of the two photons after using the EOM can be measured through two-qubit tomography.

in path A, switched on (acting as an HWP) or off (performing nothing) through the control of a random number generator (RNG), converts, with probability 50%, the polarization of the photon in path A from V to H, or vice versa [30]. The quantum state after the EOM is

$$\rho_{AB} = 0.5|\psi_1\rangle\langle\psi_1| + 0.5|\psi_2\rangle\langle\psi_3|,$$

with

$$|\psi_2\rangle = \cos \theta |V_A H_B\rangle + \sin \theta |H_A V_B\rangle. \quad (17b)$$

The density matrix of the above state $\rho_{AB}$ in the basis \{|$H_A H_B\rangle$, |$H_A V_B\rangle$, |$V_A H_B\rangle$, |$V_A V_B\rangle$\} is just the X-state (12) with $x = 0.5\cos^2\theta$, which can be experimentally measured through two-qubit tomography [31]. Now we have all the four block matrices, and we can apply them in our criterion to tell whether the quantum discord is zero or not. In order to verify the zero quantum discord for $x = 0.25$ and nonzero quantum discord for $x \neq 0, 0.25$ and 0.5 experimentally, we use the following procedure. An HWP in path B rotates the polarization of the photon in this path by the angle of 45°, which corresponds to the unitary transformation

$$U = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

as mentioned above. The H and V photons in this path will be separated by the PBS and then register on the two detectors $D_1$ and $D_2$, respectively, which corresponds to two local orthogonal projectors on photon B. Once the photon B is detected by $D_1$ (with probability $p_1 = 2x$) or by $D_2$ (with probability $p_2 = 1 - 2x$), the other photon in path A will turn into the state $\rho_1^A$ or $\rho_2^A$, accordingly, which can be found out through single-qubit tomography [31]. With the measured $\rho_1^A$ and $\rho_2^A$, we can construct a density matrix for the AB system,

$$\rho_{\text{measure}} = p_1 \rho_1^A \otimes (|\phi_1\rangle\langle\phi_1|) + p_2 \rho_2^A \otimes (|\phi_2\rangle\langle\phi_2|), \quad (18)$$
where $|\phi_1\rangle = \frac{\sqrt{2}}{2}(|H_B\rangle + |V_B\rangle)$ and $|\phi_2\rangle = \frac{\sqrt{2}}{2}(|H_B\rangle - |V_B\rangle)$ are the two eigenvectors associated with the measurement in the current experimental setup. For $x = 0.25$, equation (18) is equal to equation (12), which means zero quantum discord. For $x \neq 0, 0.25$ and 0.5, equation (18) will not be equal to the state of equation (12), no matter what kind of local projectors on photon B are chosen, which means nonzero quantum discord. For the two trivial cases of $x = 0$ or 0.5, the zero quantum correlation of the state (12) can be verified via the same method as above by removing the HWP in path B.

A useful result can be derived from our criterion: any POVM cannot extinguish the quantum correlation of the bipartite state with nonzero quantum discord. For a bipartite quantum state $\rho_{AB}$, we perform a POVM on subsystem B by attaching an ancillary system $\rho_C$ on it and carrying out a projective measurement on the extended BC system. The new bipartite state, composed of one subsystem A and another subsystem B plus C, is $\rho_{A(BC)} = \rho_{AB} \otimes \rho_C$, which has zero quantum discord if and only if the quantum discord of the original bipartite quantum state $\rho_{AB}$ is zero, no matter what kind of ancillary system is chosen. The matrix product rule [32] directly gives us the following equality,

$$\left[ \rho^{(iA,\lambda)} \otimes \rho_C, \rho^{(i',A,\lambda')} \otimes \rho_C \right] = \left[ \rho^{(iA,\lambda)}, \rho^{(i',A,\lambda')} \right] \otimes (\rho_C \rho_C),$$

and thus the above statement can be easily proved through the commutation relations among the blocks of the original density matrix $\rho_{AB}$ and also those of the density matrix $\rho_{A(BC)}$. Therefore, the above criterion for zero quantum discord is valid for all types of local measurement, including POVM.

4. Conclusions

To summarize, we derive a new criterion for zero quantum discord of arbitrary bipartite states, which is easy to implement in three steps: (i) partition the density matrix of the $N \otimes M$ quantum state into $N^2$ block matrices; (ii) check whether every block is a normal matrix (commuting with its Hermitian transpose); and (iii) check whether all block matrices commute with each other. For a bipartite state with zero quantum discord, we can find the set of projectors that do not change the whole state after being imposed on one of the subsystems by diagonalizing any nonzero block matrix of its density matrix. This set of projectors provides a way to locally access the information in the system without disturbing the whole state. A class of two-qubit X-state is used to test the criterion, which can be implemented experimentally. It is also shown that POVM cannot extinguish the quantum correlation of a bipartite state with nonzero quantum discord, although it may have an effect on the evaluation of nonzero quantum correlation.

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