Neutrino dark matter in clusters of galaxies

R A Treumann†‡, A Kull and H Böhringer

Center for Interdisciplinary Plasma Studies, Max-Planck-Institute for Extraterrestrial Physics, PO Box 1312, D-85741 Garching, Germany
E-mail: tre@mpe.mpg.de

New Journal of Physics 2 (2000) 11.1–11.13 (http://www.njp.org/)
Received 12 April 2000; online 13 July 2000

Abstract. We present a model calculation for the radial matter density and mass distribution in two clusters of galaxies (Coma and A119) including cold dark matter, massive though light ($\simeq 2$ eV) neutrino dark matter and collisional intra-cluster gas which emits x-ray radiation. The calculation uses an extension of the Lynden-Bell statistics to the choice of constant masses instead of constant volume. This allows proper inclusion of mixtures of particles of various masses in the gravitational interaction. When it is applied to the matter in the galaxy cluster the radial ROSAT x-ray luminosity profiles can be nicely accounted for. The result is that the statistics identifies the neutrino dark matter in the cluster centre as being degenerate in the sense of Lynden-Bell’s spatial degeneracy. This implies that it is distributed in a way different from the classical assumption. The best fits are obtained for the $\simeq 2$ eV neutrinos. The fraction of these and their spatial distribution are of interest for understanding cluster dynamics and may have cosmological implications.

1. Introduction

The recent thrilling first successful laboratory indication that the neutrino has a finite rest mass (Fukuda et al 1998a, b, 1999, Levi 1998, Schwarzschild 1998, Schewe 1999) can be taken as a starting point for re-thinking the effect of massive though light neutrinos as dark matter candidates in clusters of galaxies. Dark matter in astrophysics is not only a mystery but also plays a central role in the expected gross dynamics of the universe, its large scale structure and the dynamics of its giant constituents, the galaxies, clusters of galaxies and superclusters. Silk (1988) claimed, for instance, that the primary distinction between astrophysical and laboratory particle physics evidence for dark matter is ‘that the existence of (some of) the’ former ‘is not in question’, but ‘there is virtually no means of estimating their contribution to $\Omega$ with any confidence, whereas’,
for the latter, there are available ‘elegant techniques for computing their abundances although their very existence is largely a question of faith at present’. The present paper tries to contribute to a remote measure of the contribution of dark matter to the matter trapped inside clusters of galaxies.

The indication that the neutrino has a finite mass may have substantially changed Silk’s (1988) claim. In cosmology massive neutrino dark matter, because of the early decoupling of neutrinos from radiation, had always been taken as hot in the sense that the structures produced by the hot dark matter must be very large scale because any small scale structure will readily have been wiped out by the ultra-relativistic thermal motions of neutrinos. Since observation of cosmological patchwork structure on small to intermediate scales is common, cold dark matter has become of more interest. However, if neutrinos actually have a finite rest mass then they may have been captured in appreciable amounts in clusters of galaxies making up a non-vanishing amount of the dark matter component of clusters. Cosmological considerations have set a limit on the required neutrino mass in the range $< 100$ eV (assuming a normalized Hubble constant $h_{100} = 0.5$ and a cosmological density $\Omega \leq 2$). Such light neutrinos combined with an additional component of cold dark matter have become known as hot-and-cold-dark-matter (HCDM) or simply mixed cosmological models.

Two variants of neutrino mass schemes have been proposed by Primack et al (1995). The first follows the rule $m(\nu_\mu) \simeq m(\nu_\tau) \approx 2.45$ eV and $m(\nu_e) \approx 0$ eV. The second satisfies $m(\nu_e) \simeq m(\nu_\mu) \simeq m(\nu_\tau) \approx 1.6$ eV. A cosmological decision between these schemes is expected from the behaviour of the cosmological fluctuation spectra, which has not yet been made possible because of the intrinsic uncertainties and biases in the measurements. However, it seems that the former scheme may be closer to reality because it leads to a mass difference of $\Delta m^2_{e\mu} \approx 6$ eV which seems to be probable (Caldwell 1995). One should note that the recent experimental indications that the neutrino has a finite mass also point in this direction. Both of these schemes are, however, still reasonable and will therefore be used in what follows.

In the present paper we use observations of two clusters of galaxies in the ROSAT soft x-ray regime in order to estimate the effect of a neutrino mixed dark matter model on the distribution of the various components of the dark matter throughout the clusters. What is interesting in this approach is that we apply a fitting procedure which is based on a modified version of Lynden-Bell’s (1967) statistics of collisionless gravitating matter. This kind of statistics has been described by Kull et al (1996, 1997) and is based on the observation that in a statistical mechanical description of non-interacting (collisionless) massive many-particle systems under only gravitational interaction, the equations of motion become independent of the interacting particle mass. Therefore the mass is an invariant even in a many-particle system and the relevant phase space cells must be redefined as constant mass cells. Lynden-Bell’s ‘exclusion principle’, which is a purely spatial exclusion principle, since in collisionless systems no two particles can occupy the same phase space element, in this case applies to cells of equal masses. Formally the phase space distribution function which derives from this procedure then becomes similar to the Fermi–Dirac one. For non-distinguishable species, which are the only interesting ones in our cases, it reads

$$\langle f_j(v, x) \rangle = \frac{\eta_j}{\exp[\beta_j \eta_j \omega(\epsilon - \mu) + 1]}$$

where $\epsilon(v, x) = v^2/2 + \Phi(x)$ is the total energy of the particles per mass, $\beta$ is the inverse temperature and $\mu$ is the chemical potential (as defined in a gravitational field for non-interacting particles), $\eta_j$ are the mass densities and $\omega_j = m/\eta_j$ is the phase space volume element which
11.3

is not constant in this case. This removes the velocity dispersion problem still contained in Lynden-Bell’s (1967) analysis.

In this expression the mass $m$ is a common normalizing reference mass and is thus a constant, however, since, in agreement with the above discussion, the differences in mass densities of the various components are taken into account by the variation of the phase space elements (Kull et al 1997). It can thus be chosen arbitrarily. It is then reasonable to select the smallest participating mass as the reference mass $m$, which in this case is the neutrino mass. This then specifies the phase space volume element for known densities as the multiple of a reference element $\omega = \omega_j/g_j$.

The gravitational potential satisfies the Newton–Poisson equation for self-gravitating systems. Clearly, it is itself a functional of the distribution functions $f_j$ of the various matter components through the mass density $\rho$:

$$\rho_j = 4\pi \eta \int_0^\infty dv v^2 f_j(v)$$

written here in spherical symmetry. For hydrostatic equilibria, in particular, one has the defining equation

$$\rho \nabla \Phi = -\nabla P$$

where $P$ is the thermal pressure.

The above form of the distribution function is one which resembles the Fermi–Dirac statistics valid here for classical systems. It must be kept in mind that it has only macroscopic meaning and does not result from anti-symmetry of the wavefunction. The exclusion principle used is a macroscopic principle which simply accounts for the impossibility of having two particles in the same place, whence they do not collide in any observation time or are non-interacting in the sense of the dark matter doctrine. As a general remark we note, however, that a theory of this kind is inherently approximate. Gravitational systems cannot be in equilibrium, however violent their interaction may be, as long as this violent interaction does not generate an internal pressure which keeps the system in equilibrium against the action of the gravitation which must ultimately lead to collapse. Hence this kind of violent relaxation causes an intermediate state which lasts for a while until gravitation overcomes the repulsion of the particles caused by the violent relaxation process. It is usually believed that this process causes an elevated equivalent temperature which balances gravitation. However, it is not clear whether such a picture can ever be strictly justified from first principles. It is more reasonable that, in equilibrium, a certain large mass accumulates in the centre of the gravitating gas, which heats up to a very high temperature, becomes somehow collisional (whatever this might mean in the language of non-interacting particles) and will remain in equilibrium while the really non-interacting matter will form a gas cloud of some temperature outside this central mass like a very extended atmosphere. We do not concern ourselves with this difficult philosophy here, but turn to the more pragmatic issue of looking for the effect of such a kind of violent relaxation as that described by the above distribution on the matter in clusters of galaxies.

2. Massive neutrinos in clusters of galaxies

Typical clusters of galaxies have masses of the order of $M \simeq 10^{15} M_\odot$, radii $R \simeq 3$ Mpc and velocity dispersions $\sigma \simeq 10^3$ km s$^{-1}$. They consist of $N \simeq 1000$ galaxies of average mass...
$M_G \simeq 10^{10} M_\odot$. The binary collisional relaxation time is thus
\[
\tau_c = (\sigma R)^3 \left[ 24G^2 M_G M \ln(N/2) \right]^{-1} \approx 10^{12} \text{years}
\] (4)
which is about two orders of magnitude longer than the age of the universe $t_0 \approx 2 \times 10^{10}$ years. Violent relaxation is in much better agreement with any reasonable formation and relaxation time. It leads to relaxation within a time
\[
\tau_{vr} = \left( \frac{3R^3}{8GM} \right)^{1/2} \approx 1.5 \times 10^9 \text{years.}
\] (5)

For massive neutrinos it becomes evident from these expressions that due to their small mass of only a few electron-volts, their binary relaxation time $\tau_c$ would be enormous. However, when they are participating in violent relaxation it is independent of the neutrino mass and thus of the same order for all components in the cluster.

Cowsik and McClelland (1973) were the first to investigate the effect of massive neutrinos in clusters. They tried to explain the dark matter halo in clusters by assuming that it consisted entirely of neutrinos in the mass range $m_\nu \simeq 10^{-20} \text{eV}$. In this approach they simply assumed a fully degenerate state at $T_\nu = 0 \text{K}$. Subsequently Tremaine and Gunn (1979) estimated from Lynden-Bell’s violent relaxation statistics that the possible mass of a neutrino in clusters should always be larger than $3.6 \text{eV}$. The modified statistics mentioned above improves this limit by reducing it to $\approx 2.5 \text{eV}$ (Kull et al. 1996).

In applying the violent relaxation statistics to neutrinos it is reasonable to fix the phase space element as $\omega = \left( \frac{2\pi\hbar}{m_\nu} \right)^3$. Hence the maximum mass density in phase space is $(g_\nu/2) [m_\nu^4/16(\pi\hbar)^3]$. Following Madsen and Epstein (1984), the population density of a phase space element for the relevant families of neutrinos is less than 6%. Most of the cells are thus occupied at best by only one neutrino. The normalized mass density $\omega$ thus has the value either 0 or $m_\nu^4/[16(\pi\hbar)^3]$ and the phase space distribution assumes the isotropic form
\[
\langle f_\nu(v,x) \rangle = \eta \left\{ \exp\left[ \beta \eta \omega (v^2 + \Phi - \mu) \right] + 1 \right\}^{-1}.
\] (6)
This distribution maximizes at $v = 0$ and leads to the condition that
\[
\left\{ \exp\left[ \beta \eta \omega (v^2 + \Phi - \mu) \right] + 1 \right\}^{-1} \leq 1.
\] (7)
For non-degenerate (very dilute) states one has $\langle f_\nu(v,x) \rangle \ll \eta$ and thus $\beta m_\nu (\Phi - \mu) > 0$, in which case the distribution becomes simply a Boltzmann one. The mass density $\rho_\nu$ in this case, which is the moment of the distribution multiplied by $\eta$ satisfies (with $\sigma = (\beta m_\nu)^{-1/2}$) the condition
\[
\rho_\nu \ll m_\nu^4 \sigma^3/(4\pi^{3/2}\hbar^3).
\] (8)
Solving for the neutrino mass yields the condition
\[
m_\nu \gg \left( 4\rho_\nu \pi^{3/2} \hbar^3/\sigma^3 \right)^{1/4}.
\] (9)
Hence, in order to have a Boltzmann state, the neutrino mass has to be sufficiently large for a given mass density. If these limits are violated the distribution generates degeneration effects and the distribution function can be written as
\[
\langle f_\nu \rangle = \eta \quad \text{for} \quad v \leq (|2\Phi|)^{1/2}
\] (10)
and vanishes for larger velocities. Under these conditions of degeneracy the limit for the mass density maximum is defined as

\[ \rho_{\nu, \text{max}} = 4\pi \eta (|2\Phi|^{1/2}) \int_0^\infty \mathrm{d}v \, v^2 \]

which immediately gives an upper limit on the mass density of the bound neutrinos of

\[ \rho_{\nu} \leq m_{\nu}^4 (|2\Phi|^{3/2}) / (12\pi^2 \bar{h}^3) \]

which can be used to derive a lower limit on the neutrino mass in the degenerate case:

\[ m_{\nu} > (12\rho_{\nu} \pi^2 \bar{h}^3) / (|2\Phi|^{3/2})^{1/4} . \]

This is the limit mentioned above. For isothermal spheres one may use \( \sigma^2 \approx \Phi / 3 \) everywhere.

Figure 1 shows the dependences of the limiting mass on the velocity dispersion for the two neutrino mass schemes. The contours in figure 1 indicate the upper limit on the total mass density in the cluster for given neutrino densities under the assumption of hydrostatic equilibrium. The larger the total mass the more degenerate the neutrino component. This is of course reasonable since the probability that two neutrinos want to occupy the same phase space element increases. One may immediately conclude that, for dense or very heavy clusters, there is a finite probability that the neutrino fluid will be in a degenerate state close to the centre of the cluster while in the outer regions degeneracy will be inhibited. This picture then corrects the approach of Cowsik and McClelland (1973), who assumed total degeneracy and thus overestimated the effect of neutrinos in the cluster dark matter problem.

3. Two real clusters: A1656 and A119

An application of the above statistics to the calculation of the distributions of a mixture of ordinary baryonic matter, cold dark matter of unspecified nature, and massive neutrino dark matter can be given for clusters of known nearly spherical shape and known mass distributions. Two particular clusters of this kind have been investigated recently by Kull (1997), who analysed...
Table 1. Basic parameters of the clusters A1656 (Coma) and A119. $z$ is the red shift, $N_H$ is the hydrogen column density, $R_x$ is the x-ray radius of the cluster and $T_x$ is the x-ray temperature of the intra-cluster gas.

| Cluster | $z$ | $N_H$ ($10^{20}$ cm$^{-2}$) | $R_x$ (Mpc) | $k_B T_x$ (keV) |
|---------|-----|-----------------------------|-------------|-----------------|
| A1656   | 0.023 | 0.89                        | 3.89        | 8.3$_{7.8}^{8.9}$ |
| A119    | 0.044 | 3.10                        | 2.37        | 5.9$_{5.3}^{6.5}$ |

the ROSAT x-ray emission profiles and spectra of the relatively symmetrical clusters A1656 (the Coma cluster) and A119 under the assumption of a hydrostatic equilibrium configuration. They possess nearly centrosymmetrical optical and x-ray configurations which to some extent justify the assumption of central symmetry. They are strong x-ray emitters, have comparable red shifts and there is no indication of strong cooling flows in them. The latter would indicate that they were still in a state of poor relaxation. Table 1 gives the main parameters for both clusters.

In the HCDM model we assume that the distribution function of these clusters is composed of the distributions $f_\nu$, $f_{CDM}$ and $f_g$ of the three matter components, neutrinos ($\nu$), cold dark matter (CDM) and baryonic gas ($g$). Moreover, the mass of the cold dark matter constituent $m_{CDM} \gg m_\nu$ should be much larger than the neutrino mass. We thus write for the superposition of the distributions

$$f_{\text{tot}} = f_\nu + f_{CDM} + f_g = \sum_{j=\nu,CDM} \eta_j \left( 1 + g_j^{-1} \exp\left[ \beta m_j (\epsilon - \mu_j) \right] \right)^{-1} + f_g. \quad (14)$$

Here $\epsilon(\nu, x)$ is the particle energy, $\mu_j$ the chemical potential and $g_j$ the degeneracy coefficient. (We neglect the error made by mixing a collisional ($f_g$) distribution with the non-collisional distributions of the two dark matter components because of the minute contribution of the former.) We also define the kinetic temperatures of the dark matter components by $\langle T_j \rangle = m_j / m_{CDM} k_B$ and put $\bar{\mu}_j = \mu_j + \ln(g_j)/\langle m \beta \rangle$. The intra-cluster gas temperature is defined as $T_g = m_m m_p / (m k_B \beta)$ where $m_m$ is the average molecular mass (in amu) of the gas, and $m_p$ is the proton mass. This allows us to write for the neutrino component

$$\langle f_\nu \rangle = \eta \left( 1 + \exp\left[ m_\nu / (k_B \langle T_\nu \rangle) (\epsilon - \mu) \right] \right)^{-1} \quad (16)$$

while the distributions of the other components are Maxwellians

$$\langle f_{CDM} \rangle = f_{0,CDM} \exp \left[ -m_{CDM} \epsilon / (k_B \langle T_{CDM} \rangle) \right] \quad (17)$$
$$\langle f_g \rangle = f_{0,g} \exp \left[ -m_g \epsilon / (k_B \langle T_g \rangle) \right]. \quad (18)$$

Here $\epsilon \equiv v^2 / 2 + \Phi$. These definitions allow us to express the densities and pressures in hydrostatic equilibrium in terms of the gravitational self-potential $\Phi$, the average densities $\rho_{0,j}$ and the above-mentioned relative masses and temperatures. In addition we have for each component the hydrostatic equation (2) and the Newton–Poisson equation

$$\nabla^2 \Phi(r) = 4\pi G \left( \rho_\nu(r) + \rho_{CDM}(r) + \rho_g(r) \right) \lim_{r \to \infty} \Phi(r) \to 0. \quad (19)$$

The model calculation is based on this set of equations and the parameters given in table 1. Several ratios have been assumed in the models for the neutrino-to-total-mass and gas-to-dark-matter

New Journal of Physics 2 (2000) 11.1–11.13 (http://www.njp.org/)
Figure 2. A contour map of the ratio of velocity dispersions of the two dark matter components $\sigma_{\text{CDM}}/\sigma_{\nu}$ as a function of the neutrino mass density $\rho_{\nu}$ and temperature $\langle T_{\nu} \rangle$ for clusters of galaxies. The potential depth in the centre has been assumed to be $\Delta \Phi = -5 \times 10^{12}$ J kg$^{-1}$. Large relative neutrino mass concentrations in the cluster require low neutrino temperatures.

In hydrostatic equilibrium the temperature of each component is a constant. However, because the neutrino dark matter has a different distribution function, its velocity dispersion can still have a radial dependence. This is in strict contrast to ordinary models. Indeed, $\sigma_{\nu}$ is calculated from

$$\sigma_{\nu}^2 = \frac{2}{3} \frac{k_B \langle T_{\nu} \rangle}{m_{\nu}} \frac{I_{3/2}}{I_{1/2}}$$

where the integrals $I_\alpha(z)$ with $z \equiv \mu/(k_B\langle T_{\nu} \rangle)$ are defined as

$$I_\alpha(z) = \int_0^\infty dx \, x^\alpha \left[ 1 + \exp(x - z) \right]^{-1}.$$  \hfill (21)

Since $\sigma_{\nu}$ is a functional of the gravitational potential it is also a function of the radius.

Figure 2 shows the dependence of the velocity dispersion ratio $\sigma_{\text{CDM}}/\sigma_{\nu}$ on the neutrino mass density $\rho_{\nu}$ and temperature $\langle T_{\nu} \rangle$. It is clear from figure 2 that large relative neutrino concentrations in the centre require low neutrino temperatures. Of course this could have been expected.

The procedure for estimating the most probable density models of the three components of the intra-cluster matter (neutrinos, CDM and gas) is to numerically solve the hydrostatic model for a set of parameters for the gravitational potential profile extracted from the x-ray measurements, calculate the x-ray emission from the intracluster gas for each model and fit it to the radial x-ray profile. The best $\chi^2$ fit is then chosen as the most probable model. This procedure requires some preparatory steps such as symmetrizing the x-ray contours and extracting contributions from infalling galaxies.
3.1. Coma: A1656

Figure 3 shows a synopsis of nine of the models which fitted particularly well. The total mass density profiles are given by the full lines. Their splitting into components changes from model to model. The very best fit is obtained for model 7 in the lower left-hand corner ($\chi^2 = 3.2$). In most models the CDM provides the main component of matter in the cluster, with the neutrinos...
Figure 4. Ranges of the relative densities $\rho_j/\rho_{\text{tot}}$ and masses $M_j/M_{\text{tot}}$ for the best fitting models of Coma as a function of the radius.

Figure 5. The profile of the velocity dispersion $\sigma_\nu$ of massive neutrinos in A1656 (Coma) based on ROSAT x-ray observations. The central densities amount to $\rho_{0,\nu}/\rho_{0,\text{tot}} = 0.2$ and $\rho_{0,g}/\rho_{0,\text{DM}} = 0.3$.

contributing least and being concentrated at the centre of the cluster. This is most interesting because the usual assumption that neutrinos are hot dark matter leads to the naive impression that they could not be captured by the cluster potential. This is obviously incorrect even for light massive neutrinos.

Some of the models with still very good fits even show that the neutrino mass contribution at the centre can exceed that of the intra-cluster gas. This is the case for models 6, 8 and 9. In some cases the intra-cluster gas extends considerably farther out than one or both of the dark matter components (models 1–3, 5 and 6), which is interesting insofar as sometimes gravitational lensing has suggested that the potential trough is not extended farther out than the luminous or x-ray luminous matter. In this case the dark matter must be strongly concentrated at the centre of the cluster.
A summary of the ranges of the relative densities and masses as functions of the radius for the best fitting models is given in figure 4. In the inner part of Coma the CDM contributes by far the most to the density and mass of the cluster. Beyond 1 Mpc radius, however, the neutrino and gas contributions contribute appreciable fractions of the total density and mass.

Finally, the velocity dispersion profile calculated for the Coma cluster from ROSAT x-ray observations is shown in figure 5. Even though the temperatures are constant, it turns out that the velocity dispersion obeys a very particular profile with highest ‘thermal’ velocities in the cluster core and nearly constant dispersion outside a radius of 1 Mpc. Thus naive interpretation of a direct measurement of the velocity dispersion would lead one to conclude that the neutrino gas...
in the centre of the cluster would be hotter than that towards its boundaries, whereas in reality the temperature is constant by definition for the hydrostatic thermal equilibrium model. The dispersive increase is caused solely by the degeneration of the neutrino component in the inner part of Coma.

3.2. The cluster A119

The cluster A119 has been the subject of a similar investigation. A119 is known to be still in the phase of weak merging. It is one of the brightest x-ray clusters and is nearly centrosymmetrical. Figure 6 for A119 corresponds to figure 3 for Coma. It shows the radial mass density profiles for the nine best-fit models, including the fits on the measured x-ray profile. It is interesting that, in all models for this cluster, the x-ray emitting cluster gas extends farther out than any of the dark matter components. In addition, in some of the models the neutrino component contributes more than does the cold dark matter component to the total mass density in the cluster. Figure 7 corresponds to figure 4. Figure 7 shows the ranges of contributions of the various components to the mass density and mass in A119. The density of CDM clearly follows the well expected profile of this cluster, with the CDM being more concentrated in the centre and contributing most of the mass to the cluster while both neutrino hot dark matter and gas take over at larger radial distances. In particular the range of the neutrino mass increases when reaching the outskirts of A119 where neutrinos dominate and theoretically may constitute the main extra-cluster component.

4. Discussion and conclusions

In the present paper we presented model calculations for two clusters of galaxies. It was our intention to make inferences about the relative contributions of cold dark matter, intra-cluster gas and massive neutrino (hot) dark matter to the mass and density distribution in clusters of galaxies. It was also our intention to make inferences about the respective spatial profiles of the three different matter components: neutrino and cold dark matter and x-ray luminous matter. It has been assumed that the relevant neutrino mass is of the order of $\sim 2$ eV, which is in rough agreement with the recent experimental indications from observation of neutrino
oscillations for a finite neutrino mass. The cold dark matter in our model remained unspecified, however. The model calculations have been based on the assumption of a collisionless (for the dark matter) hydrostatic equilibrium in the clusters taken as examples. Selection of the sample clusters was guided not only by their availability in x-ray emission but also by their proximity to spherical shapes which allowed us to impose perfect spherical symmetry in the calculation. In order to remain as close to reality as possible we used measured x-ray profiles in order to fit the x-ray emission profiles resulting from the model calculations. We listed the best fitting models and found that, in addition to the cold dark matter component, light massive neutrinos can substantially contribute to the dark matter in the two sample clusters. Radial profiles were calculated for the mass and density distributions. One of the most interesting results is that the radial extension of the clusters as visible in the x-ray images indeed matches the true radial extension as found previously in gravitational lensing observations (Tyson et al 1990, Tyson 1991; for a comprehensive account of these observations see Tyson 1992).

The important new ingredient of this theory is that it uses a statistical model for the distribution functions of the matter components resulting from the so-called violent relaxation theory. This theory has been used in an extended version whereby one avoids the velocity dispersion problem in a multi-component system. It was thus possible to calculate velocity dispersion profiles for the neutrino component in the cluster. Interestingly, though the system is in thermal equilibrium with given common temperature, the velocity dispersion of the neutrinos increases towards the centre of the cluster while the temperature of the constituents is constant. This effect is due to the degeneracy of the light but massive neutrino dark matter component. Though this degeneracy is also present in the cold dark matter it is less important than it is for neutrinos because, at the virial temperatures of the cluster, the cold dark matter still behaves like an ordinary gas (in a statistical mechanical sense) which is described by the Boltzmann distribution. Hence degeneracy of cold dark matter can be neglected and becomes important only for the neutrinos.

We finally point out the deficiencies of the present theory. First, it only applies to strictly spherically symmetric clusters. Most clusters do not satisfy this condition sufficiently well but are believed to still remain in their evolutionary phase or interacting with the extra-cluster medium or other clusters. Second, the assumption of virialization may be violated in many cases as well. Third, isothermality may be too strong an assumption for application under real conditions. Fourth, radial infall of matter entirely neglects the Keplerian motion of the matter particles under the self-action of their gravitational fields. Taking into account this motion the angular momenta of the rotating fluids should be taken into account. One expects that this motion will heavily distort the spherical symmetry of the cluster. Finally, an important point which is still missing in the present calculation is the relation of the cluster dark matter components to the extra-cluster dark matter background. This sets a boundary condition on the trapped dark matter component at the external boundary of the cluster. We have not yet attempted to satisfy this condition nor to merge the cluster material at its outskirts with the extra-cluster matter. The present calculation still overestimates the amount of matter at the outer boundary. This results from the method of integrating the equations outwards. Adjusting to more realistic external conditions as imposed by current knowledge about the extra-cluster cosmological matter density will be the subject of future investigation. The possibility that the merging of the dark matter profiles may cause problems, however, in our simplified one-dimensional spherically symmetric model cannot be excluded, nor the possibility that there may be no smooth transition from the interior of the cluster to its voided environment.
Acknowledgments

The authors thank K Pretzl for the invitation to publish this paper and for his continuous encouragement of this work. We thank J Trümper for access to the ROSAT galaxy cluster data as well as for his interest in the progress of this work. We also thank G Morfill for his support. Part of the work by RT was performed during a period as a senior visiting scientist at the International Space Science Institute, Bern. The hospitality of its directors J Geiss, G Paschmann and R von Steiger, as well as the support of the ISSI staff, is gratefully acknowledged.

References

Caldwell D O 1995 Proc. 16th Int. Conf. on Neutrino Physics and Astrophysics ed A Dar et al (Amsterdam: North-Holland) p 122
Cowsik R and McClelland J 1973 Astrophys. J. 180 7
Fukuda Y et al 1998a Phys. Rev. Lett. 81 1562
Fukuda Y et al 1998b Phys. Lett. B 436 33
Fukuda Y et al 1999 Phys. Rev. Lett. 82 1810
Kull A 1997 Statistisch-mechanische Untersuchungen zur Violent Relaxation von HCDM-Galaxienhaufen PhD Thesis Ludwig-Maximilians-Universität München
Kull A, Treumann R A and Böhringer H 1996 Astrophys. J. 466 L1
Kull A, Treumann R A and Böhringer H 1997 Astrophys. J. 484 58
Levi B G 1998 Physics Today August p 9
Lynden-Bell D 1967 Mon. Not. R. Astron. Soc. 136 101
Madsen J and Epstein R I 1984 Astrophys. J. 282 11
Primack J R, Holtzman J, Klypin A and Caldwell D O 1995 Phys. Rev. Lett. 74 2160
Schewe P F 1999 Physics Today August p 17
Schwarzschild B 1998 Physics Today August p 17
Silk J 1988 Dark Matter ed J Audouze and J Tran Than Van (Gif-sur-Yvette: Frontières) p 3
Tyson J A 1991 After the First Three Minutes (AIP Conf. Proc. 222) ed S Holt et al (New York: AIP) p 437
Tyson J A 1992 Physics Today June issue 24
Tyson J A Valdes F and Wenk R A 1990 Astrophys. J. 349 L1