I. INTRODUCTION

One-dimensional (1D) electron systems have attracted much attention over the past decade(s). While 3D systems are well described by the Fermi liquid theory, for 1D systems this is not the case. The low-energy physics is rather the one of a Luttinger liquid (LL) \([4,5]\), where the excitations are collective zero sound modes. The Hubbard- and the \(t-J\) chains provide so far the most reasonable description of 1D (super)conductors \([6,7]\).

The extension to quasi-one-dimensional systems is obtained by coupling \(N\) (Hubbard, \(t-J\)) chains to form a \(N\)-leg ladder. The interest in doped ladders started, when the possibility of superconductivity in two-leg ladder materials was proposed \([8,9]\). For the Hubbard- and the \(t-J\) two-leg ladders different numerical methods have shown that \(d\)-wave like superconducting correlations indeed dominate \([10,11]\). Experimentally, superconductivity has up to now only been observed in a two-leg ladder material under high pressure \([12]\).

The Hubbard chain is exactly solvable for all values of the hopping parameter \(t\) and the on-site repulsion \(U\) \([13]\), and the \(t-J\) chain at the supersymmetric point, \(t = J\) \([14]\). Analytic works for ladders were made for particular limits: For weakly coupled chains (small interchain hopping \(t_\perp < U, t\) \([15,16]\), and for stronger coupled chains, but small interactions (small on-site repulsion \(U < t_\perp, t\), such that the \(N\)-leg ladder is equivalent to a \(N\)-band model) \([17,18]\). Finally, at- and very close to half filling, where recent works have shown how to map the (weakly interacting) two-leg ladder on the Gross-Neveu model \([19]\) (for an application, see Ref. \([20]\)) and the \(t-J-J_\perp\) model on the \(XXZ\) spin chain \([21]\).

In order to determine whether the system exhibits superconductivity or not, it is necessary to calculate the fluctuations (correlation functions) for the charge-, and spin-density and for the superconducting (SC) pairing. In the two-leg ladder, the decay of the charge density correlation function is \(\propto x^{-2K}\) and of the pairing correlation function \(\propto x^{-1/(2K)}\), where \(K\) is the Luttinger liquid parameter (LLP) being one in the noninteracting case, \(K = 1\). For sufficiently weak interactions pairing correlations thus dominate which is interpreted as the appearance of superconductivity \([22,23]\). \(\Box\)

Spinless fermions are usually studied as a model for the physical more interesting (but more difficult) case of spin-1/2 fermions — in particular when focusing on the interchain hopping \(t_\perp\) \([14,22,23]\) or the metal-insulator transition \([24]\). Physically, spinless fermions can be considered as completely polarized spin-1/2 fermions in a (high) magnetic field. For weak interactions, a two-leg ladder of spinless fermions is conveniently mapped on a two-band model. Without making the link to ladders, such a model has been treated in Ref. \([24]\). More recent works have studied two weakly coupled chains, \(t_\perp \ll t\) \([14,22,23]\). The authors found that the LL fixed point of the two separated chains is unstable upon weak coupling. But in contrast to the spin-1/2 case, in a two-leg ladder of spinless fermions, the decay of the charge density correlation function is \(\propto x^{-K}\) and that of the SC pairing is \(\propto x^{-1/K}\), and superconductivity does not occur for purely repulsive interactions (in the \(t-V-U\) model \(V > 0\) and \(U > 0\)) and \(t_\perp \ll t\) \([22,23]\).

In this paper, we study the low-energy physics of the spinless two-leg ladder for weak repulsive interactions and finite interchain hopping \(t_\perp \sim t\). We obtain a non-universal analytic exponent for the power-law decays \(\propto x^{-\gamma}\) of the charge density- and \(\propto x^{1-\gamma}\) of the SC pairing correlation functions. The phases as function of the temperature \(T\) and the doping away from half filling \(\delta\) (for spinless fermions, \(0 \leq \delta \leq 0.5\)) are shown in Fig. 1. At half filling the ladder is insulating; upon doping away from half filling, we come to a phase with one gapless mode, which exhibits due to the finite \(t_\perp\) dominant SC correlations with interchain pairing. When the doping is increased, this phase undergoes a transition to
a phase with two gapless modes, where first SC correlations coexist with charge density correlations. The SC pairing correlations finally disappear completely at high doping, where the ladder becomes a 1D Luttinger metal. We thus have a nontrivial crossover from the SC state to the normal conducting one.

where $t$ and $t_{\perp}$ are the hopping amplitudes along- and between the chains and $d^j_i(x)$ creates a fermion in the chain $j$ at the rung $x$. We are going to consider small repulsive interactions $0 < U < t, t_{\perp}$. In this limit, it is a good approach to diagonalize first $H_0$ by a canonical transformation,

$$\Psi_j(x) = \frac{1}{\sqrt{2}} (d_1(x) \pm d_2(x)). \tag{2}$$

Going over to the momentum space, we find a decoupling into two bands (we set the lattice parameter equal one),

$$H_0 = \sum_{j=1,2} \int dkd\epsilon_j(k)\Psi_j^\dagger(k)\Psi_j(k), \tag{3}$$

where the dispersion relations are

$$\epsilon_j(k) = \mp t_{\perp} - 2t \cos(k). \tag{4}$$

Band 1 is the bonding- and band 2 the antibonding band. By analogy with the 2D case, the associated transverse momenta are denoted as $k_{\perp} = 0, \pi$. Since we are discussing only the low-energy physics, we linearize $\epsilon_j$ around the Fermi momenta $\pm k_F$, resulting in Fermi velocities $v_j = 2t \sin(k_F j)$. For the operator $\Psi_j$ at $\pm k_F$, we write $\Psi_R/L_j(k) = \Psi_j(\pm k_F + k)$.

In the small $U$ limit, it is a good approximation to take for $k_F$ the values obtained in the noninteracting system $17$. The definitions of the band filling $n = (k_{F1} + k_{F2})/(2\pi)$ and the chemical potential, $\mu = \epsilon_1(k_{F1}) = \epsilon_2(k_{F2})$ allow to calculate $k_{Fj}$ and $v_j$ as a function of $n$, $t$, and $t_{\perp}$,

$$v_{1,2} = 2t \sin \left[ \pi n \pm \arcsin \left( \frac{t_{\perp}}{2t \sin(\pi n)} \right) \right]. \tag{5}$$

For spinless ladders, the band filling is $0 \leq n \leq 1$ and the hole doping away from half filling is $\delta = 0.5 - n$. The effect of the interchain hopping $t_{\perp}$ is thus included in the velocities $v_j$ $17$. We like to point out that the velocities are not equal for finite $t_{\perp}$,

$$v_1 - v_2 = \frac{2t_{\perp}}{\tan(\pi n)}. \tag{6}$$

This is different to previous treatments of the spinless two-leg ladder 22. We will see that this difference in the velocities has the remarkable effect of driving the system to a SC state for repulsive interactions, $U > 0$. We note that both bands are partially filled, when $t_{\perp} < 2t \sin(\pi n)^2$.

We do not consider the half-filled case, $k_{F1} + k_{F2} = \pi$ ($v_1 = v_2$), where the ladder is insulating, allowing us to neglect umklapp processes (we also exclude the particular points $k_{Fj} = \pi/2$). Including all interactions allowed by symmetry (leaving away completely chiral...
one’s), in momentum space, the Hamiltonian is given by

\[ H_0 = \sum_{j=1,2} v_j \int dk \left[ \Psi_{Rj}^\dagger(k) \Psi_{Rj}(k) - \Psi_{Lj}^\dagger(k) \Psi_{Lj}(k) \right], \]

(7)

and

\[ H_{\text{Int}} = \int dk_1 dk_2 dk_3 dk_4 \delta(k_1 + k_3 - k_2 - k_4) \times \left[ g_1 \Psi_{R1}^\dagger(k_1) \Psi_{R1}(k_2) \Psi_{L1}^\dagger(k_3) \Psi_{L1}(k_4) + g_2(1 \leftrightarrow 2) \right. \]

\[ + g_x \left( \Psi_{R1}^\dagger(k_1) \Psi_{R1}(k_2) \Psi_{L2}^\dagger(k_3) \Psi_{L2}(k_4) + 1 \leftrightarrow 2 \right) \]

\[ + g_t \left( \Psi_{R1}^\dagger(k_1) \Psi_{R2}(k_2) \Psi_{L1}^\dagger(k_3) \Psi_{L2}(k_4) + 1 \leftrightarrow 2 \right). \]

(8)

The bare values of the couplings are chosen as \( g_1 = g_2 = g_x = g_t = U > 0 \). We will see, that the \( g_t \) interaction (pair hopping of left/right going quasiparticles from band one to band two) is the most relevant in determining the low-energy physics.

Next, we apply the generalized LSM theorem (see Refs. 24, 33), to the Hamiltonian \( H \) in order to show the existence of gapless modes. The particular symmetry of the Hamiltonian \( H \) allows to define “twist operators” for left/right going fermions separately, i.e.,

\[ U_R = \exp \left( 2\pi i \sum_{x,j} x \Psi_{Rj}^\dagger(x) \Psi_{Rj}(x) \right), \]

(9)

where \( L \) is the length of the ladder (and similarly \( U_L \) for the left going fermions \( \Psi_{Lj} \)). Commuting \( U_{R/L} \) with the translation operator \( T \) in real space, we obtain \( T U_R T^{-1} = e^{-4\pi i v_R} U_R \). The chiral symmetry \( n = 2v_R = 2v_L \) proves then the existence of gapless (charge density) excitations at a wavevector \( 2m = k_{F1} + k_{F2} \), if \( n \) is not an integer (remember that we left away umklapp interactions in \( H \) present at half filling: they break the individual \( U_{R/L} \) “quasisymmetry” 33). We show, that this is in agreement with the results obtained by bosonization (see section IV). However, the LSM theorem does neither tell us the number of gapless modes nor whether finally charge density- or superconducting excitations dominate.

III. THE RG-EQUATIONS

We give the RGEs resulting from the Hamiltonian \( H \) and the flow of the couplings depending on the ratio of the velocities \( v_1/v_2 \). We find that for \( v_1/v_2 < 7 \), all couplings diverge, while for \( v_1/v_2 > 7 \), \( g_t \to 0 \) and the rest remains of the order of \( U \).

The model with the couplings \( g_1, g_2 \), and \( g_x \) alone is exactly solvable (by bosonization, see below) and in particular at a \( RG \) fixed point. Products of couplings without at least one \( g_i \) do therefore not appear in the RGEs for \( g_1, g_2, \) and \( g_x \). Including the one-loop exact results, the particular form of the \( g_t \) interaction then implies that the RGEs (to all orders) have the following form,

\[ \frac{dg_1}{dt} = -\frac{1}{2\pi v_2} g_t^2 \left[ 1 + O(g_\alpha/t) \right] \]

\[ \frac{dg_2}{dt} = -\frac{1}{2\pi v_1} g_t^2 \left[ 1 + O(g_\alpha/t) \right] \]

\[ \frac{dg_x}{dt} = \frac{1}{\pi(v_1 + v_2)} g_t^2 \left[ 1 + O(g_\alpha/t) \right] \]

\[ \frac{dg_t}{dt} = \frac{g_t}{\pi} \left[ \frac{2g_x}{v_1 + v_2} - \frac{g_1}{2v_1} - \frac{g_2}{2v_2} + O(g_t^2/t^2) \right]. \]

(10)

The energy (temperature) scale is related to \( l \) by \( T \sim t e^{-l} \) and \( O(g_\alpha^2/t^n) \) denotes higher order terms in the couplings, \( g_\alpha^2/t^n \) \( (\alpha = 1, 2, x, t) \). The plus and minus signs result from particle-hole respectively particle-particle diagrams. The one-loop RGEs have been derived in Refs. 24, 33, but the authors have not noted the particular form to all orders.

The exact solution of (10) to one-loop order is given in Appendix A. We obtain for comparable ratios of the velocities, \( 1/7 < v_1/v_2 < 7 \), all couplings diverge at the same scale, while for other ratios, \( g_1 \) scales to zero and \( g_2 \) and \( g_x \) stay of the order of \( U \). The stability of the \( g_t \) fixed point follows from the particular form of the RGEs to all orders (10). Since all couplings (to all orders) are multiplied at least once with \( g_t \), this fixed point is stable for \( g_t \to 0 \) (any higher order term of the form \( g_t^2 \) would drive the system to another fixed point).

IV. BOSONIZATION AND PHASES

Using bosonization and the above RG results, we derive the low-energy phases (for an overview, see Fig. 1). When doping the half filled ladder, a SC phase with interchain pairing arises (up to \( v_1/v_2 < 7 \), see section IV.A. In section IV.B, we discuss the \( g_t = 0 \) phase, present well away from half filling, \( v_1/v_2 > 7 \), and exhibiting two gapless modes. In section IV.C, we finally study the transitions and crossovers between the different phases.

Bosonizing the Hamiltonian \( H \), we obtain (see Appendix B),

\[ H = \int dx \sum_{j=1,2} \left[ \frac{v_j}{2} + \frac{g_j}{4\pi} (\partial_x \Phi_j)^2 + \frac{g_j}{4\pi} \Pi_j^2 \right] \]

\[ + \frac{g_x}{2\pi} [\partial_x \Phi_1 \partial_x \Phi_2 - \Pi_1 \Pi_2] - \frac{g_t}{(2\pi\alpha)^2} \cos \left( \sqrt{4\pi}(\theta_1 - \theta_2) \right). \]

(11)

A flow to strong coupling of \( g_t \) results (classically) in a “pinning” of \( \theta_1 - \theta_2 = 0 \) in order to minimize the energy, and a single gapless mode, while for \( g_t \to 0 \), two gapless modes are present.
A. The interchain-pairing SC phase

We show that as a result of a finite interchain hopping $t_{\perp}$, doping the half filled ladder, we obtain a phase, where SC correlations dominate. The pairing takes place between left (right) going particles in chain one and right (left) going particles in chain two. For $t_{\perp} \ll t$, we recover previous results [22].

When doping the half filled ladder (up to a ratio $v_1/v_2 < 7$), the couplings grow and eventually diverge. However, one should not overinterpret this divergence. For small $U$, the combination $\theta_1 - \theta_2$ in (12) can fluctuate at high temperatures, but for growing $g_t$ (i.e., decreasing temperature), these fluctuations become suppressed at the scale where $g_t(T_c) \sim t$. The $\theta_1 - \theta_2 = 0$ then leads to a coherence between the two bands and is interpreted as a crossover to a new phase. Since $T_c \sim t e^{-t/U} \ll t$ is a very low temperature ($t_c \propto t/U$), the change of the couplings as a function of temperature for $T < T_c$ is weak as long as we keep away from the point of divergence, i.e., within the validity of the one-loop RGEs. It is then a common practice to set $g_t \sim t$ as a strong coupling value (and similarly, for the other couplings).

Using the canonical transformation $\Phi_\pm = (\Phi_1 \pm \Phi_2)/\sqrt{2}$ and $\Pi_\pm = (\Pi_1 \pm \Pi_2)/\sqrt{2}$, the Hamiltonian (11) takes the form

$$H = H_B + H_{SG} + H_{mix},$$

(12)

where $H_B$ is the Hamiltonian of a massless boson,

$$H_B = \int dx \frac{u_+}{2} \left[ \frac{1}{K_+}(\partial_x \Phi_+)^2 + K_+ \Pi_+^2 \right],$$

(13)

and $H_{SG}$ is the sine-Gordon Hamiltonian,

$$H_{SG} = \int dx \left\{ \frac{u_-}{2} \left[ \frac{1}{K_-}(\partial_x \Phi_-)^2 + K_- \Pi_-^2 \right] \right. + \left. \frac{g_t}{(2\pi\alpha)^2} \cos \left( \sqrt{8\pi\theta_-} \right) \right\},$$

(14)

and finally, $H_{mix}$ is a mixing term,

$$H_{mix} = \int dx \left( v_{c,p}^c \partial_x \Phi_+ \partial_x \Phi_- + v_p^p \Pi_+ \Pi_- \right).$$

(15)

The velocities $u_\pm$, $v_{c,p}^c$, and the LLP $K_\pm$ are given by

$$u_\pm = \sqrt{\left( \frac{v_1 + v_2}{2} \right)^2 - \left( \frac{g_1 + g_2 \pm 2g_x}{4\pi} \right)^2},$$

(16)

$$v_{c,p}^c = \frac{v_1 - v_2}{2} \pm \frac{g_1 - g_2}{4\pi},$$

(17)

and

$$K_\pm = \frac{2\pi(v_1 + v_2) - (g_1 + g_2 \pm 2g_x)}{2\pi(v_1 + v_2) + (g_1 + g_2 \pm 2g_x)},$$

(18)

The mixing term hinders for $v_1 - v_2 \neq 0$ a (simple) analytic solution of the (classical) equations of motion. Since the $\theta_\pm$ field is pinned, the current density takes the form $j = u_+ K_+ \Pi_+$. At half filling the $\Phi_+$ field is also pinned resulting in $j = 0$ and an insulating phase [24].

Next, we discuss the correlation functions. For repulsive interactions, the charge density- and SC pairing fluctuations with the most divergent susceptibilities are the following ones: The CDW (at a wavevector $k_{F1} + k_{F2}$, for a comparison, see section II) is given by the operator

$$O_{CDW} = d_{R1}d_{L1} - d_{R2}d_{L2} = \Psi_{R1}^\dagger \Psi_{L1} + \Psi_{R2}^\dagger \Psi_{L2} \propto \exp \left( i\sqrt{2\pi\Phi_+} \cos \left( \sqrt{2\pi\theta_-} \right) \right),$$

(19)

and the SC fluctuations by

$$O_{SC} = d_{R1}d_{L2} + d_{R2}d_{L1} = \Psi_{R1}^\dagger \Psi_{L1} - \Psi_{R2}^\dagger \Psi_{L2} \propto \exp \left( i\sqrt{2\pi\theta_+} \cos \left( \sqrt{2\pi\theta_-} \right) \right),$$

(20)

where the $d_{R/L_j}$ are the annihilation operators for the fermions in chain $j$ [22]. The operator $O_{CDW}$ represents an antisymmetric charge density wave and $O_{SC}$ superconductivity with interchain pairing — previously called $d$-wave like due to the antisymmetry with respect to the bonding- and antibonding band [24]. However, the operator $O_{SC}$ has odd parity, $O_{SC}(-x) = -O_{SC}(x)$, which one associates rather with $p$-wave like superconductivity in each band (see Appendix C.1).

We obtain for the charge density correlation function (for a derivation, see Appendix C),

$$\langle O_{CDW}^\dagger(x)O_{CDW}(0) \rangle \propto x^{-\gamma}$$

(21)

and for the SC pairing correlation function

$$\langle O_{SC}^\dagger(x)O_{SC}(0) \rangle \propto x^{-1/\gamma},$$

(22)

where the exponent is

$$\gamma = \frac{K_+}{1 - K_+ K_- / \left( \frac{v_{c,p}^c}{2\pi\alpha} \right)^2}.\quad (23)$$

As a result, the finite $t_{\perp} \sim v_{c,p}^c$ leads to $\gamma > 1$, implying that superconducting pairing correlations dominate.

The increase of $\gamma$ can be understood as follows. The pinning of $\theta_-\pm$ allows us to set $\Pi_- = 0$. The only coupling is then between the $\Phi_\pm$ fields. The $\Phi_-$ field fluctuates strongly and affects the $\Phi_+$ correlation function with additional fluctuations thus increasing $\gamma$ and stabilizing the superconductivity.

We furthermore find that in the low-energy regime, $T \to 0$ ($l \to \infty$), the LLP $K_+$ becomes bigger than one. In detail, whether $K_+$ is bigger or smaller than one depends on the following sum (see (15))
\[ g_1 + g_2 + 2g_x = \left( 3 + \frac{v_1^2 + v_2^2}{2v_1v_2} \right) U - \frac{(v_1 - v_2)^2}{2v_1v_2} g_x. \] (24)

Since \( g_x \) increases for decreasing temperature, the above sum becomes negative and \( K_+ > 1 \).

FIG. 2. The exponent \( \gamma \) of the charge density correlation function, \( \propto x^{-\gamma} \), is shown for \( U/t = 0.2 \) and different ratios of \( t_\perp/t \). While for small ratios \( t_\perp \ll t \), CDW dominate for almost all physical values of the doping \( \delta \), comparable ratios \( t_\perp \sim t \) favor superconductivity already at lower doping. The SC correlations are strongest, when approaching the transition to the mixed SC+CDW phase, where the chemical potential lies near the bottom of the antibonding band (\( v_1/v_2 \approx 7 \)). A comparison of Fig. 2 with Fig. 1 shows that the SC correlations are largest where the crossover temperature is highest.

For small \( t_\perp/t \) we recover previous results: the bosonized Hamiltonian goes over to the one studied in Refs. [22,26] and the exponent \( \gamma \) stays smaller than one, \( \gamma \approx K_+ \leq 1 \), and charge density correlations dominate (for almost all physical values of the doping \( \delta \)).

B. The mixed SC+CDW phase

For \( v_1/v_2 > 7 \), the fixed points of the couplings are such that \( g_t = 0 \), and \( g_1, g_2, \) and \( g_x \) are of the order of \( U \) (see Appendix A). We diagonalize the Hamiltonian [11] for \( g_t = 0 \) using a current representation of the fields (for a comparison, see Appendix B). Defining \( J = (J_{R1}, J_{L1}, J_{R2}, J_{L2}) \) and

\[
M = \begin{pmatrix}
  v_1 & g_1/2\pi & 0 & g_x/2\pi \\
  g_1/2\pi & v_1 & g_x/2\pi & 0 \\
g_x/2\pi & 0 & v_2 & g_2/2\pi \\
g_2/2\pi & 0 & g_2/2\pi & v_2
\end{pmatrix}, \tag{28}
\]

the Hamiltonian [11] can be written as \( H = \pi \int dx J^T M J \) and the new LLP and velocities are determined by the eigenvalues of \( M \), which are

\[
\frac{v_1 + v_2}{2} + \frac{g_1 + g_2}{4\pi} \pm D_+ \frac{D_+}{2},
\]

\[
\frac{v_1 + v_2}{2} - \frac{g_1 + g_2}{4\pi} \pm D_- \frac{D_-}{2}, \tag{29}
\]

where

\[
D_\pm = \sqrt{\left( v_1 - v_2 \pm \frac{g_1 - g_2}{2\pi} \right)^2 + \frac{\left( g_x \right)^2}{2\pi}}. \tag{30}
\]

Since \( v_1 - v_2 \gtrsim g_x \), the coupling \( g_x \) does not contribute to \( D_\pm \) leading order in \( U \). The velocities and the LLP have thus the same form as in two decoupled LL,

\[
u_j = \sqrt{v_j^2 - \frac{g_j^2}{(2\pi)^2}} \quad \text{and} \quad K_j = \sqrt{\frac{2\pi v_j - g_j}{2\pi v_j + g_j}} \approx 1 - \frac{g_j}{2\pi v_j}, \tag{31}
\]

where in our case the \( g_j \) are scale dependent. The new basis is “almost” the old one, \( \Pi_1 \approx \Pi_1 + \epsilon \Pi_2 \), where \( \epsilon \sim U/t \) (and similarly for \( \bar{\Pi}_2 \) and the fields \( \Phi_j \)) and the current density takes the usual form,

\[
j = u_1 K_1 \bar{\Pi}_1 + u_2 K_2 \bar{\Pi}_2. \tag{32}
\]
where the $\tilde{\Pi}_j$ obey the equations of motion $\partial_t^2 \tilde{\Pi}_j = \nu_j^2 \tilde{\Pi}_j$. It should be noted that here the Drude coefficient $u_j K_j$ is not equal to that of the free Fermi gas \cite{28}.

In a short range of ratios, $7 < v_1/v_2 < 8$, the low temperature fixed point of the couplings is such that $g_1 < 0$ and $g_2 > 0$ (for $v_1/v_2 = 7$, we find $g_1 = -0.57 U$ and $g_2 = 0.78 U$), implying $K_1 > 1$ and $K_2 < 1$ — a coexistence of pairing correlations in the bonding- with charge density correlations in the antibonding band. Rewriting the SC pairing operator $O_{SC1} = \Psi_{R1} \Psi_{L1}$ in terms of the chain operators $d_{R/L,j}$ shows, that there are as well intra- and inter-chain SC pairs. We interpret this coexistence as a precursor of the SC phase at $v_1/v_2 < 7$, i.e., pairing correlations first become established in the bonding band (at a momentum $k_\perp = 0$), and then also in the antibonding (at a momentum $k_\perp = \pi$) and phase coherence between the bands sets in for increasing $g_1$. In Fig. 1, the dashed line shows, where $K_1 > 1$.

For ratios $v_1/v_2 > 8$, we find $g_j > 0$ and in both bands charge density fluctuations dominate, $K_j < 1$ (for $v_1/v_2 \gg 8$, the fixed points are $g_1 \approx g_2 \approx U$). This corresponds to the usual 1D metallic (LL) phase.

C. Phase Transitions and Crossovers

Next, we discuss the transitions (at $T = 0$) and the crossovers between the different phases as a function of doping and temperature (see Fig. 1). The transitions are accompanied by non-analyticities in the compressibility $\kappa$ respectively in the coefficient $\lambda$ of the specific heat, $C = \lambda T$.

We interpret the energy-scale $T_c \sim e^{-x_{-}}$ ($l_c \propto t/U$) at which the couplings become of the order of $t$ as the temperature, where the crossover to the interchain SC phase takes place, see Fig. 1 (the high temperature cutoff $T_0$ is fixed but arbitrary). The crossover temperature $T_c$ increases upon doping, reaching a maximum at a finite, $t_\perp/t$ dependent doping, and then decreases to zero when approaching the transition point to the mixed SC+CDW phase, $v_1/v_2 \approx 7$.

The compressibility $\kappa$ and the coefficient $\lambda$ of the specific heat in the mixed phase have the same form as in two decoupled LL \cite{28}, e.g.,

$$\frac{\lambda}{\lambda_0} = 1 + \frac{1}{2} \left( \frac{v_1}{u_1} + \frac{v_2}{u_2} \right) \approx 1 + \frac{1}{4} \left( \frac{g_1}{2\pi v_1} \right)^2 + \frac{1}{4} \left( \frac{g_2}{2\pi v_2} \right)^2,$$

(33)

where $\lambda_0$ is the specific heat coefficient of the noninteracting system. In the interchain SC phase, the main dependencies of $\kappa$ and $\lambda$ come from $K_+$ and $u_+$, but similarly as for the correlation functions, there are corrections in $v_\perp^2$. In any case, the values of $\lambda$ and $\kappa$ “jump” at $v_1/v_2 = 7$.

The SC+CDW phase then evolves into a CDW+CDW phase, where the crossover takes place at $v_1/v_2 \approx 8$ (at $T = 0$). The band edge of the antibonding band ($v_1/v_2 \rightarrow \infty$) cannot be treated by the present method, since the dispersion relation is there quadratic, $\sim k^2$. However, choosing a sufficiently small $U$, we can approach this point arbitrarily close. We thus conjecture that the CDW+CDW phase can be extended up to this band edge.

When only one band is occupied, i.e., $v_2 = 0$, the phase consists of a single LL (with CDW excitations): at the band edge of the antibonding band, we then have a transition from a one-band CDW to a two-band CDW+CDW phase, where for $U/t \rightarrow 0$, the coefficient $\lambda$ and the compressibility $\kappa$ diverge as $\lambda, \kappa \sim 1/k_F^2$.

V. CONCLUSIONS

We have shown that a finite interchain hopping $t_\perp$ has the effect of driving the weakly interacting spinless two-leg ladder to a SC phase, where left (right) going particles in chain one are paired with right (left) going particles in chain two. The SC correlations are largest when the chemical potential is close to the bottom of the antibonding band. Between the SC phase at lower- and the CDW phase at higher doping, we have found a (new) phase with coexistence of SC and CDW excitations. Two phase transitions take place (for $T \rightarrow 0$): one from the interchain SC phase to the mixed SC+CDW phase and one from the CDW+CDW to the (one band) CDW phase.

On the one hand, SC correlations are enhanced by the velocity difference $v_1 - v_2 = 2t_\perp/\tan(\pi n)$, but one the other hand, this difference suppresses the coherence between the two bands, when the chemical potential is too close to the bottom of the antibonding band. We argue that this sort of competition also takes place in the spin-1/2 case \cite{17,34} and generally for (possible) superconductivity in $N$-band models.

ACKNOWLEDGMENTS

We thank T.M. Rice for fruitful discussions and comments throughout this work.

APPENDIX A: RENORMALIZATION GROUP

The renormalization group (RG) method is a controlled way of subsequently eliminating (integrating out) high-energy modes in a given Hamiltonian. While the noninteracting part $H_0$ is (usually) at a \textit{RG fixed point}, couplings of the interacting part $H_{\text{int}}$ may grow or decrease under a RG transformation (i.e., when lowering the energy).
A subsequent (perturbative) elimination of high-energy modes in $H_{\text{int}}$ then results in RG equations, which give the change of the coupling when lowering the energy, see Ref. [27].

Next, we solve analytically the one-loop RGEs given in equation (10). Keeping only the terms quadratic in $g$, we transform the set of the four differential equations (10) into one differential equation for $g_x$,

$$\frac{1}{B} \frac{dg_x}{dl} = (g_x - CU)^2 + DU^2; \quad (A1)$$

where

$$B = \frac{4v_1v_2 + (v_1 + v_2)^2}{2\pi v_1v_2(v_1 + v_2)}, \quad (A2)$$

$$C = \frac{2(v_1 + v_2)^2}{4v_1v_2 + (v_1 + v_2)^2}, \quad (A3)$$

and

$$D = \frac{-v_1^4 + 6v_1^3v_2 + 6v_1^2v_2^2 + 6v_1v_2^3 - v_2^4}{[4v_1v_2 + (v_1 + v_2)^2]^2}. \quad (A4)$$

The solution of equation (A1) is qualitatively different for $D < 0$ and $D > 0$. For $D > 0$, all couplings diverge, while for $D < 0$, $g_t$ scales to zero and the others remain of the order of $U$.

Solving the equation

$$x^4 - 6x^3 - 6x^2 - 6x + 1 = 0, \quad (A5)$$

for $x = v_1/v_2$, we obtain the exact transition ratio $v_1/v_2$. For comparable velocities,

$$1/7 \approx 0.14327 \ldots < v_1/v_2 < 6.9798 \ldots \approx 7, \quad (A6)$$

we find $D > 0$ resulting in

$$g_x(l) = U \left\{ C + \sqrt{D} \tan \left[ B\sqrt{DU}l - \arctan \left( \frac{C-1}{\sqrt{D}} \right) \right] \right\} \quad (A7)$$

and

$$g_t(l) = U \frac{\sqrt{\pi(v_1 + v_2)BD}}{\sqrt{\pi} \sqrt{\pi/2}} \times \left\{ 1 + \tan \left[ B\sqrt{DU}l - \arctan \left( \frac{C-1}{\sqrt{D}} \right) \right] \right\}^{1/2}. \quad (A8)$$

For ratios $v_1/v_2 > 7(<1/7)$, we find $g_t \to 0$ and

$$g_x \to (C - \sqrt{-D})U, \quad (A9)$$

for $l \to \infty$. Similar as $g_x$, the coupling constants $g_1$ and $g_2$ stay of the order of $U$ (e.g., $U < g_x < 1.4U$). Since $g_t$ flows to zero and the form of the RGEs to all orders [11] is such that couplings whatever the order is, are always multiplied at least once with $g_t$, the fixed point is stable, $dg_t/dl \to 0 \ (\alpha = 1, 2, x, t)$ for $l \to \infty$. For finite $U < v_1$, higher order terms lead to a change of the transition ratio $v_1/v_2$.

The one-loop RGEs (10) have been used in Ref. [14] to treat spin-1/2 fermions and the author has noted the two different flow regimes, but he has not given the solutions of the RGEs.

**APPENDIX B: BOSONIZATION**

Bosonization is a method of rewriting Dirac-fermion operators $\Psi$ in terms of bosonic fields $\Phi$ and $\Pi$ satisfying the commutation relation

$$[\Phi(x), \Pi(y)] = i\delta(x-y). \quad (B1)$$

For fermions on a chain or ladder, bosonization applies in the continuum limit. A “complete” treatment of bosonization is given in Refs. [2,28].

For convenience, we introduce the dual field of $\Phi$,

$$\theta(x) = \int_\infty^x dx' \Pi(x'). \quad (B2)$$

For spinless fermions, the bosonization scheme is then as follows. We first decompose the fermionic operator $\Psi$ into right- and left-movers $\Psi_R$ and $\Psi_L$,

$$\Psi(x) = e^{ik_F x} \Psi_R(x) + e^{-ik_F x} \Psi_L(x). \quad (B3)$$

In terms of bosonic operators, the $\Psi_{R/L}$ take the form

$$\Psi_{R/L}(x) = \frac{\eta_{R/L}}{\sqrt{2\pi\alpha}} \exp \left[ i\sqrt{\pi}(\mp \Phi(x) + \theta(x)) \right], \quad (B4)$$

where $\alpha$ is a cutoff parameter of the order of the lattice constant and the $\eta_{R/L}$ are Majorana (“real”) fermionic operators (usually called “Klein factors”), which are necessary to fulfill the anticommutation relations of the $\Psi_{R/L}$ fields. Currents $J_{R/L} = \Psi_{R/L} \partial_x \Psi_{R/L}$ then become

$$J_L + J_R = \frac{1}{\sqrt{\pi}} \partial_x \Phi \quad \text{and} \quad J_L - J_R = \frac{1}{\sqrt{\pi}} \Pi. \quad (B5)$$

The generalization to $N$ species of spinless fermions is straightforward.

Fourier transforming the Hamiltonian $H$ of the spinless two-leg ladder (two-band model) back to the $x$ space and using the above rules, we simply obtain the bosonized Hamiltonian $[11]$. 

APPENDIX C: CORRELATION FUNCTIONS

For the spinless two-leg ladder in the bosonized representation (12), we calculate (equal time) charge density fluctuations
\[
\left\langle \left( e^{i \beta \Phi_+ (x)} - e^{i \beta \Phi_+ (0)} \right) \left( e^{-i \beta \Phi_+ (0)} - e^{-i \beta \Phi_+ (x)} \right) \right\rangle = \left\langle e^{i \beta \Phi_+ (x)} e^{-i \beta \Phi_+ (0)} \right\rangle^2 \tag{C1}
\]
and similarly, fluctuations of the SC pairing operator parameterized by the \( \theta_+ \) field (for simplicity, we drop the pinned \( \theta_- \) field). We make use of the path integral formalism, where expectations values of operators are calculated by integration over all field configurations weighted with the exponential of the classical action.

We first revisit the correlation functions of the free massless boson and of the SG-model (see e.g. [25]).

1. Massless boson and SG-model

The nature of the phases strongly depends on the long-range behavior of charge- and SC pairing correlation functions. In a single chain, such correlation functions are determined by a single (interaction dependent) parameter, usually denoted by \( K \) (Luttinger liquid parameter, LLP).

The bosonized action of a single chain of spinless fermions is in the continuum limit the one of a massless boson, i.e., it is gaussian in the field \( \Phi \) allowing for an analytic calculation of correlation functions. It is convenient to perform a Wick rotation to imaginary time, \( \tau = i t \). The action then reads,
\[
S = \frac{1}{2K} \int dx d\tau \left( \frac{1}{v} \partial_x^2 + \partial_\tau^2 \right) \Phi, \tag{C2}
\]
where \( K \) is the interaction dependent LLP and \( v \) the Fermi velocity. The Green’s function \( G \) satisfying,
\[
- \left( \frac{1}{v} \partial_x^2 + \partial_\tau^2 \right) G(x, \tau) = \delta(x) \delta(\tau), \tag{C3}
\]
is given by
\[
G(x, \tau) = \frac{1}{4\pi} \ln \left( \frac{R^2}{x^2 + v^2 \tau^2 + \alpha^2} \right), \tag{C4}
\]
where \( \alpha \) is a short distance cutoff and \( R \) the radius of the integration boundary in the complex plane (we finally take \( R \to \infty \), corresponding to the usual thermodynamic limit). The one-point correlation function is then equal zero \( \left\langle e^{i \beta \Phi_+(x)} \right\rangle = 0 \). Defining \( O_{\text{CDW}} = \Psi_R^\dagger \Psi_L \), the long-range behavior of the (equal time) charge density correlation function at \( 2k_F \) is
\[
\left\langle O_{\text{CDW}}^\dagger (x) O_{\text{CDW}} (0) \right\rangle \propto x^{-2K}. \tag{C5}
\]
Here, \( \beta^2 = 4\pi \) (see above). The SC pairing operator is \( O_{\text{SC}} = \Psi_R^\dagger \Psi_L \) and its correlation function
\[
\left\langle O_{\text{SC}}^\dagger (x) O_{\text{SC}} (0) \right\rangle \propto x^{-2/K}. \tag{C6}
\]
A \( K > 1 \) therefore implies dominant pairings, while a \( K < 1 \) results in dominant charge density correlations. Note that the SC pairing operator has odd parity, \( O_{\text{SC}}(x) = -O_{\text{SC}}(-x) \) (for spin-1/2 fermions, the parity is even). For chiral fermions, the parity transformation is \( \Psi_L (x) \to \pm \Psi_R (x) \) for periodic (antiperiodic) boundary conditions.

The action of the SG-model has the form
\[
S = \frac{1}{2} \int dx d\tau \left( \partial_x^2 + \partial_\tau^2 \right) \Phi + g \cos(\beta \Phi), \tag{C7}
\]
where we take \( \beta^2 < 8\pi \). The “pinning” term \( g \cos(\beta \Phi) \) renders the one-point correlation function of the \( \Phi \) field a constant, \( \left\langle e^{i \beta \Phi (x)} \right\rangle = \text{const} \neq 0 \). The two-point correlation function becomes a constant at large distances (cluster decomposition principle),
\[
\left\langle e^{i \beta \Phi (x)} e^{-i \beta \Phi (0)} \right\rangle \to \left\langle e^{i \beta \Phi (x)} \right\rangle \left\langle e^{-i \beta \Phi (0)} \right\rangle = \text{const}, \tag{C8}
\]
for \( x \to \infty \). The one-point correlation functions of the (unpinned) dual field \( \theta \) is equal zero and the two-point correlation function decays exponentially.

2. The spinless two-leg ladder

Next, we calculate the correlation functions of the spinless two-leg ladder making use of the above results. In our case, the \( \theta_- \) field is pinned.

The action of the spinless two-leg ladder (resulting from the Hamiltonian (12)) is invariant under the shift \( \Phi_+(x) \to \Phi_+(x) + c \), where \( c \) is any constant. Similarly as it is the case for the action of the free massless boson (C2), we then obtain (for a comparison, see Ref. [25])
\[
\left\langle e^{i \sum_j \beta_j \Phi_+ (x_j)} \right\rangle = 0, \tag{C9}
\]
if \( \sum_j \beta_j \neq 0 \). In particular, the one-point correlation function of the \( \Phi_+ \) field is equal zero, \( \left\langle e^{i \beta \Phi_+ (x)} \right\rangle = 0 \), i.e., the \( \Phi_+ \) field is indeed a free (unpinned) field. The same holds for the dual field \( \theta_+ \). We conjecture that the mixing term \( \text{[13]} \) for \( v_1 \neq v_2 \) is an analytic perturbation.

Using the Green’s function (C4), we carry out the integration over the fields \( \Pi_+ \) and \( \Phi_+ \) and obtain for the equal time charge density correlation function
\[
\left\langle e^{i \beta \Phi_+ (x)} e^{-i \beta \Phi_+ (0)} \right\rangle \propto x^{-2K \beta^2 / 4\pi} \left\langle e^{S_{\text{mix}}^\dagger (x)} \right\rangle_{\text{SG}} \tag{C10}
\]
for \( x \to \infty \).
The expression $S_{\text{mix}}^\Phi$ depends (nonlocally) on the fields $\Phi_-, \Pi_-$, vanishing for $v_1 = v_2$ (for simplicity, we drop in the following the short-distance cutoff $\alpha$),

$$S_{\text{mix}}^\Phi = -i \int dz_1 \left( \nu^- \partial_{x_1} \Phi_- + i \frac{v^p}{K_+ u_+} \partial_{\tau} \Pi_- \right) \times \frac{\beta K_-}{4\pi} \ln \left[ \frac{(x-x_1)^2 + u_+^2 \tau_1^2}{x_1^2 + u_+^2 \tau_1^2} \right]$$

$$- \int dz_1 dz_2 \left( \nu^- \partial_{x_1} \Phi_- (1) + i \frac{v^p}{K_+ u_+} \partial_{\tau} \Pi_- (1) \right) \times \frac{K_+}{8\pi} \ln \left[ x_{12}^2 + u_+^2 \tau_{12}^2 \right] (1 \leftrightarrow 2), \quad (C11)$$

where $(1) = (x_1, \tau_1), \quad x_{12} = x_1 - x_2, \quad \tau_{12} = \tau_1 - \tau_2,$ and $dz_1 = dx_1 d\tau_1$. The average is taken with the SG action resulting from $[4]$. Since the $\Phi_-$ field is unpinned, integration over $\Phi_-$ gives corrections $\propto (\nu^-)^2$ (and higher order) to the exponent of $x$.

Next, we carry out a canonical transformation from $(\Phi_-, \Pi_-)$ to $(\Phi_-, \Pi_-) = (\theta_-, \partial_{\theta} \Phi_-)$ (the transformation is canonical, since it preserves the commutation relations and the integration measure is not changed, because the Jacobian of the transformation has determinate one). Rewriting $S_{\text{mix}}^\Phi$ in terms of the new fields then allows us to carry out the integration over $\Pi_-$. The $\Pi_-$ part has the form,

$$- \frac{u_-}{2K_-} \int dz_1 \Pi_-(1) \left\{ \Pi_- (1) + \frac{K_-}{4\pi u_-} (\nu^-)^2 \right\}$$

$$\times \int dz_2 \Pi_-(2) \partial_{x_1} \partial_{x_2} \ln \left[ x_{12}^2 + u_+^2 \tau_{12}^2 \right]. \quad (C12)$$

The inverse of the above operator on $\Pi_-$ can be expanded in a power series in $(\nu^-)^2$. The linear part in $\Pi_-$ reads

$$i \int dz_1 \Pi_-(1) \left\{ \frac{\beta K_+ \nu^c}{4\pi} \partial_{\tau_1} \ln \left[ \frac{(x-x_1)^2 + u_+^2 \tau_1^2}{x_1^2 + u_+^2 \tau_1^2} \right] \right.$$  \begin{align*}
  &+ \partial_{\tau_1} \Phi_- (1) \\
  &+ \frac{v^p v^c}{4\pi u_+} \int dz_2 \partial_{x_1} \ln \left[ x_{12}^2 + u_+^2 \tau_{12}^2 \right] \partial_{x_2} \partial_{\tau_2} \Phi_- (2). \quad (C13)
\end{align*}

Carrying out the integration over $\Pi_-$ we obtain the following contribution to the correlation function in $(\nu^-)^2$,

$$- \frac{K_-}{2u_-} \left( \frac{\beta K_+ \nu^c}{4\pi} \right)^2 \int dz_1 \left\{ \partial_{x_1} \ln \left[ \frac{(x-x_1)^2 + u_+^2 \tau_1^2}{x_1^2 + u_+^2 \tau_1^2} \right] \right.$$  \begin{align*}
  &\times \left[ \partial_{\tau_1} \ln \left[ \frac{(x-x_1)^2 + u_+^2 \tau_1^2}{x_1^2 + u_+^2 \tau_1^2} \right] \right] \right\}^2 \right). \quad (C14)$$

The integral is equal $8\pi (\ln x)/u_+$. Including all orders in $\nu^c$, the correlation function finally decays $\propto x^{-\gamma_c}$, where

$$\gamma_c = \frac{\beta^2 K_+}{2\pi} \frac{1}{1 - \frac{K_+}{2u_+} (\nu^-)^2}. \quad (C15)$$

Similarly, the pairing correlation function takes the form

$$\left\langle e^{i\theta_+ (x)} e^{-i\theta_+ (0)} \right\rangle \propto x^{-2\beta^2/(4\pi K_+)} \left\langle e^{S_{\text{mix}}^\theta (x)} \right\rangle_{\text{SG}}, \quad (C16)$$

where again $S_{\text{mix}}^\theta$ depends (nonlocally) on the fields $\Phi_-, \Pi_-$, also vanishing for $v_1 = v_2$.

$$S_{\text{mix}}^\theta = \frac{\beta}{2\pi} \int dz_1 \left( \nu^- \partial_{x_1} \Phi_- + i \frac{v^p}{K_+ u_+} \partial_{\tau_1} \Pi_- \right) \times \left[ \frac{x_1}{u_+ \tau_1} \right] \right) - \arctan \left( \frac{x_1}{u_+ \tau_1} \right) \right.$$  \begin{align*}
  &\times \int dz_2 \left( \nu^- \partial_{x_1} \Phi_- (1) + i \frac{v^p}{K_+ u_+} \partial_{\tau_1} \Pi_- (1) \right) \times \frac{K_+}{8\pi} \ln \left[ x_{12}^2 + u_+^2 \tau_{12}^2 \right] (1 \leftrightarrow 2). \quad (C17)$$

The logarithm in $[C11]$ and the arclength in $[C17]$ give after partial integration a similar leading order contribution. The difference comes from the $iK_+$ present in $[C11]$ but not in $[C17]$. Here, there are only terms $\propto (\nu^-)^2$. The combination $\partial_{x_1} \arctan (iK_+) \ln (iK_+)$ does not give logarithmic contributions after integration. A similar calculation as above leads to a decay $\propto x^{-\gamma}$, where

$$\gamma = \frac{\beta^2}{2\pi K_+} \left[ \frac{K_+}{2u_+ u_-} (\nu^-)^2 \right]. \quad (C18)$$

In both cases, the remaining part in the $\Phi_-$ fields is either real (and does therefore pin the field) or it is imaginary but multiplied with derivatives of logarithmes being strongly peaked at $\tau_1 = 0$ and $x_1 = x$ or $x_1 = 0$, resulting in a effective contribution $\propto i(\Phi_-(x) - \Phi_-(0))$. In both cases, the remaining part then becomes a constant ($\neq 0$) for large $x$.

Comparing $[C15]$ with $[C18]$ we see that one can express both correlation function in terms of a single exponent,

$$\gamma = \frac{K_+}{1 - \frac{K_+ K_-}{2u_+ u_-} (\nu^-)^2}, \quad (C19)$$

such that the density charge correlation function decays $\propto x^{-\beta^2/2\pi}$ and the pairing correlation function $\propto x^{-\beta^2/(2\pi)}$. In our case, $\beta^2 = 2\pi$.

[1] J.M. Luttinger, J. Math. Phys. 4, 1154 (1963).
[2] F.D.M. Haldane, J. Phys. C14, 2585 (1981).
[3] D. Jérome and H.J. Schulz, Adv. Phys. 31, 299 (1982).
[4] P.W. Anderson, Science 235, 1196 (1987).
[5] E. Dagotto, J. Riera, and D.J. Scalapino, Phys. Rev. B 45, 5744 (1992).
[6] T.M. Rice, S. Gopalan, and M. Sigrist, Europhys. Lett. 23, 445 (1993).
[7] For a review, see E. Dagotto and T.M. Rice, Science 271, 618 (1996) and E. Dagotto, cond-mat/9908250.
[8] R.M. Noack, S.R. White, and D.J. Scalapino, Phys. Rev. Lett. 73, 882 (1994).
[9] C.A. Hayward et al., Phys. Rev. Lett. 75, 926 (1995).
[10] K. Kuroki, T. Kimura, and H. Aoki, Phys. Rev. B 54, R15641 (1996).
[11] M. Uehara et al., J. Phys. Soc. Jpn 65, 2764 (1996).
[12] E.H. Lieb and F.Y. Wu, Phys. Rev. Lett. 20, 1445 (1968).
[13] P.A. Bares and G. Blatter, Phys. Rev. Lett. 64, 2567 (1990).
[14] M. Fabrizio, Phys. Rev. B 48, 15838 (1993).
[15] D.V. Khveshchenko and T.M. Rice, Phys. Rev. B 50, 252 (1994).
[16] H.J. Schulz, Phys. Rev. B 53, R2959 (1996).
[17] L. Balents and M. Fisher, Phys. Rev. B 53, 12133 (1996).
[18] H. Lin, L. Balents, and M. Fisher, Phys. Rev. B 56, 6569 (1997).
[19] H. Lin, L. Balents, and M. Fisher, Phys. Rev. B 58, 1794 (1998).
[20] R. Konik et al., cond-mat/9806334 and cond-mat/980332.
[21] H.J. Schulz, Phys. Rev. B 59, R2472 (1999).
[22] A.A. Nersesyan, A. Luther, and F.V. Kusmartsev, Phys. Lett. A 176, 363 (1993).
[23] The metal-insulator transition in a spinless two-band model in infinite dimensions has been studied in Q. Si et al., Phys. Rev. Lett. 72, 2761 (1994) and L. Craco, Phys. Rev. B 59, 14837 (1999).
[24] K.A. Muttalib and V.J. Emery, Phys. Rev. Lett. 57, 1370 (1986).
[25] A.O. Gogolin, A.A. Nersesyan, and A.M. Tsvelik, in Bosonization and Strongly Correlated Systems (Cambridge University Press, 1998).
[26] The effect of disorder on the different phases has been studied in E. Orignac and T. Giamarchi, Phys. Rev. B 53, R10453 (1996) and Phys. Rev. B 56, 7167 (1997).
[27] R. Shankar, Rev. Mod. Phys. 66, 129 (1994).
[28] H.J. Schulz, in Proceedings of Les Houches Summer School LXI, ed. by E. Akkermans et al. (Elsevier, Amsterdam, 1995).
[29] E.H. Lieb, T. Schultz, and D.C. Mattis, Ann. Phys. (N.Y.) 16, 407 (1961).
[30] M. Yamanaka, M. Oshikawa, and I. Affleck, Phys. Rev. Lett. 79, 1110 (1997).
[31] P. Gagliardini, S. Haas, and T.M. Rice, Phys. Rev. B 58, 9603 (1998).
[32] The difference of $O_{\text{SC}}$ to the bosonized form of $O_{\text{SC}}$ in Ref. 22 is due to an error in Ref. 22 (already noted in Ref. 21). The difference to the fermionic form of $O_{\text{SC}}$ in Ref. 24 is due to a mistake in Ref. 21 (private communication).
[33] When choosing $g_x \gg t$, we obtain $\gamma \to \infty$ and $O_{\text{SC}} \sim \text{const}$ (at $T = 0$).
[34] A similar finding has been made in numeric calculations (for spinful fermions) in R.M. Noack et al., Phys. Rev. B 56, 7162 (1997).