Topological transitions in spin interferometers

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We establish the possibility of topological transitions in electronic spin transport by a controlled manipulation of spin-guiding fields within experimental reach. The transitions are determined by an effective Berry phase related to the topology of the field texture rather than the spin-state structure, irrespective of the actual complexity of the spin dynamics. This manifests as a distinct dislocation of the interference pattern in the quantum conductance of mesoscopic loops. The phenomenon is robust against disorder, and can be exploited to determine the magnitude of inner spin-orbit fields.

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In the early 1980s Berry showed that quantum states in a cyclic motion may acquire a phase component of geometric nature [1]. This opened a door to a class of topological quantum phenomena in optical and material systems [2]. With the development of quantum electronics in semiconducting nanostructures a possibility emerged to manipulate electronic quantum states via the control of spin geometric phases driven by magnetic field textures [3]. After several experimental attempts [4–8] indisputable signatures of spin geometric phases in conducting electrons were found in 2012 [9] in agreement with the theory [10]. This paved the way for the development of a topological spin engineering [11].

An early proposal for the topological manipulation of electron spins by Lyanda-Geller involved the abrupt switching of Berry phases in phase interferometers [12]. These are conducting rings of mesoscopic size subject to Rashba spin-orbit (SO) coupling, where a radial magnetic field $\mathbf{B}_{SO}$ steers the electronic spin (Fig. 1a). For relatively large field strengths (or, alternatively, slow orbital motion) the electronic spins follow the local field direction adiabatically during transport, acquiring a Berry phase factor $\pi$ of geometric origin (equal to half the solid field strength subtended by the spins in a roundtrip) leading to destructive interference effects (Fig. 1b, left). By introducing an additional in-plane uniform field $\mathbf{B}$, it was assumed that the spin geometric phase undergoes a sharp transition at the critical point beyond which the corresponding solid angle vanishes together with the Berry phase and interference turns constructive (Fig. 1b, right). The transition from destructive to constructive interference should manifest as a steplike characteristic in the ring’s conductance as a function of the coupling fields (so far unreported). However, this reasoning appears to be oversimplified: the adiabatic condition can not be satisfied in the vicinity of the transition point, since the local steering field vanishes and reverses direction abruptly at the rim of the ring. Moreover, typical experimental conditions correspond to moderate field strengths, resulting in nonadiabatic effects and mixing of spin-split bands in analogy to the case of spin transport in helical magnetic fields [13]. Hence, a more sophisticated approach is required. This includes identifying the role played by nonadiabatic Aharonov-Anandan (AA) geometric phases [14], smaller than adiabatic Berry phases but with similar geometric characteristics.

Here, we predict the presence of a topological phase transition in loop-shaped spin interferometers away from the limit of adiabatic spin transport. Strikingly, despite the complexity of the emerging AA geometric phases, the topological transition is determined by an effective Berry phase associated with the field topology as in the adiabatic case rather than being characterized by the topology of the spin states.

We consider a two-dimensional electron gas (2DEG) confined at the interface of a semiconducting heterostructure ($xy$ plane in Fig. 1). The 2DEG is subject to SO interaction due to structure inversion asymmetry, which can be tuned by gate electrodes [15]. The SO field $\mathbf{B}_{SO}$ couples to conduction electron spin as described by the Bychkov-Rashba Hamiltonian [16]

$$H_{SO} = \frac{\alpha}{\hbar}(\sigma \times \mathbf{p}) \cdot \hat{z} \equiv \mathbf{B}_{SO} \cdot \sigma, \quad (1)$$

with $\mathbf{B}_{SO} = B_{SO}(\hat{k} \times \hat{z})$, $\alpha$ the SO strength, $\mathbf{p}$ the electronic momentum, $\sigma$ the vector of Pauli spin matrices, $\hat{k}$ the unit vector along the electron wave vector $\mathbf{k}$, and $\hat{z}$ the unit vector perpendicular to the 2DEG. This SO term gives rise to the Aharonov-Casher (AC) interference patterns appearing in the conductance of ring ensembles [9, 11]. For a ring, both geometric and dynamical phases developed by electrons moving in circular orbits have been identified as distinct contributions to the AC phase [10]. Moreover, spin eigenstates describe a regular cone shape in the Bloch’s sphere with solid angle $\Omega = -2\pi(1 - 1/\sqrt{Q^2 + 1})$ where $Q = 2m^*\alpha r/\hbar^2$ is the adiabaticity parameter [10], $m^*$ the effective electron mass and $r$ the ring radius. This corresponds to a geometric AA phase $\phi_g = -\Omega/2$ acquired by the spins in a
roundtrip in addition to the dynamical phase \[ \Phi \]. The
spin states are radial only in the adiabatic limit \[ \gamma \gg 1 \],
developing a Berry phase \( \phi_k = \pi \).

We add now a homogeneous in-plane Zeeman field in the \( xy \) plane
\[
H_Z = B \cdot \sigma = B (\cos \gamma \sigma_x + \sin \gamma \sigma_y),
\]
where \( \gamma \) is the angle with respect to the axis of the wire. In geometries where the contact leads are symmetrically coupled to the rings, electron spins traveling along symmetric interference paths acquire equal Zeeman phases resulting in constructive interference for \( B > B_{SO} \). Interference of paths with unpaired windings is possible but more sensitive to disorder. In rings coupled tangentially to conducting wires to form loops \[ 13 \] both constructive and destructive interference of Zeeman phases are possible due to interference of straight paths along the wire and (counter)clockwise winding paths around the loop (Fig. 3).

We adopt here the loop geometry to study the interplay between Zeeman and AC phases. The former is more susceptible to temperature and disorder since it is proportional to \( 1/k \), in contrast to the AC phase which is independent of \( k \). In the presence of SO coupling the in-plane magnetic field manifests as a pure geometrical effect at the lowest order in \( B \), without affecting the dynamical phase \[ 11 \]. The perturbation approach fails as \( B \) exceeds \( B_{SO} \), requiring alternative techniques. We introduce two different approaches: i) one-dimensional (1D) calculations based on semiclassical methods, providing access to local spin dynamics and geometric phases in the ballistic regime, and ii) two-dimensional (2D) numerical simulations suitable for multi-mode systems with or without disorder. We assume that the leads are spin-compensated and that the largest energy scale is the

Fermi energy \( E_F \), so that the SO and Zeeman energies can be considered small when compared to the kinetic term. Minor anisotropies arise as a function of \( \gamma \) but these are not crucial for our conclusions and we do not discuss them here.

In the 1D semiclassical model we assume three possible and equally probable paths for transmitting spin carriers: a direct path along the wire and (counter)clockwise paths around the loop (Fig. 1h). The \( 2 \times 2 \) transmission amplitude matrix for spins then reads \( \Gamma \sim I + \Gamma_+ + \Gamma_- \), where \( \Gamma_\pm \) are the (counter)clockwise transmission amplitude matrices. These are calculated by approximating the circular loop as a regular polygon with a large number of vertices following the method used in \[ 19 \], which is extended here to include in-plane magnetic fields. The conductance is obtained from the transmission probabilities (Landauer formula), given by the trace of \( \Gamma^\dagger \Gamma \).

The 2D numerical calculations of electron transport are based on a tight-binding system of transport equations which was solved using the recursive Green’s function method (RGFM) \[ 20 \] as well as the Kwant code \[ 21 \]. Disorder in the system is introduced by a lattice disorder model \[ 22 \]. To simulate typical experimental conditions, we use the material parameters of InGaAs \( (m^* = 0.05 m_0 \) with \( m_0 \) the bare electron mass) with \( 0 \leq Q < 10 \) which corresponds to a nonadiabatic regime.

Figure 2 shows the conductance in a single-mode ballistic loop calculated with both methods. It displays an interference pattern with two main characteristics: (i) radial wavefronts starting from the origin and (ii) a distinct phase dislocation along the critical
The rich pattern results from complex nonadiabatic spin dynamics near $B_{SO} = B$. Lower panel: A complementary complexity arises in the cosine of the AA geometric phase component $\phi_g$. Correlations between $\phi_d$ and $\phi_g$ lead to a smooth topological transition in the total spin phase $\phi = \phi_d + \phi_g$. The dots are selected to elaborate Fig. 4.

where $B_{SO} = B + B_{SO}$ and $E(\varphi, m)$ are elliptic integrals of the 2nd kind. Lines of constant adiabatic $\phi_d$ are plotted in Fig. 2. The fit with the calculated wavefronts is very good despite the fact that actual spin dynamics is nonadiabatic (some deviations are visible for $\Delta \ll 1$, where wavefronts are best described by geometric phase shifts). The critical line corresponds to the frontier where the field changes topology, see Fig. 2, which coincides with the spin-eigenstate texture only in the adiabatic regime. These results are intriguing, since the observed pattern presents properties recalling adiabatic dynamics in a nonadiabatic scenario. The RGFM gives results qualitatively similar to those obtained with the 1D model, indicating that the semiclassical approach captures the essential features.

The main contribution to the 1D results in Fig. 2 is given by terms of the form $\Gamma_{\pm} + \Gamma^*$ (solid line) and the total phase $\phi$ (dashed line). Cosine of the effective Berry phase $\phi_B$ sustained by the spin states only under hypothetical adiabatic conditions is shown with dotted line. A–D panels: spin-eigenmode textures at the points indicated in Fig. 4 illustrating the complexity of the spin dynamics.

where $\Delta \equiv B/B_{SO} = 1$. The wavefronts correspond to Zeeman oscillations of period about $2m^*rB/\hbar^2k = 2.0$ along the vertical axis. In the adiabatic regime the dynamical spin phase $\phi_d$ is proportional to the average field $\int^1_0 \sqrt{(B_{SO} \sin \theta + B)^2 + (B_{SO} \cos \theta)^2} \, d\theta$ acting on a spin, where $\theta$ is the angle shown Fig. 1 giving

$$\phi_d \propto 2B^* \left[ E \left( \frac{\pi}{4}, \frac{4B_{SO}}{B^*} \right) + E \left( \frac{3\pi}{4}, \frac{4B_{SO}}{B^*} \right) \right],$$

where $B^* = B_{SO} + B$, and $E(\varphi, m)$ are elliptic integrals of the 2nd kind. Lines of constant adiabatic $\phi_d$ are plotted in Fig. 2. The fit with the calculated wavefronts is very good despite the fact that actual spin dynamics is nonadiabatic (some deviations are visible for $\Delta \ll 1$, where wavefronts are best described by geometric phase shifts). The critical line corresponds to the frontier where the field changes topology, see Fig. 2, which coincides with the spin-eigenstate texture only in the adiabatic regime. These results are intriguing, since the observed pattern presents properties recalling adiabatic dynamics in a nonadiabatic scenario. The RGFM gives results qualitatively similar to those obtained with the 1D model, indicating that the semiclassical approach captures the essential features.

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in a radial SO texture) and vanishing along the vertical axis corresponding to a uniform Zeeman field. In the vicinity of the critical line \( \Delta = 1 \) no clear-cut topological transition is visible in the geometric AA phase \( \phi_g \).

In Fig. 4, we depict the evolution of \( \phi_g \) along the dashed lines of Fig. 3 and compare it with Lyanda-Geller’s prediction for adiabatic spin dynamics [12]. We find that the assumption of adiabatic spin dynamics close to \( \Delta = 1 \) is not valid even in the limit of strong fields. However, a clear-cut topological transition is visible in the total phase at \( \Delta = 1 \) (dashed line in Fig. 4), recalling adiabatic spin transport (dotted line in Fig. 4). This gives rise to the characteristic dislocation in the interference pattern in the conductance of Fig. 2.

The emergence of a topological transition in the nonadiabatic case can be understood as follows. The geometric AA phase can be decomposed as \( \phi_g = \phi_B + \Delta \phi_g \), where \( \phi_B \) is an effective Berry phase taking discrete values on different sides of the critical line: \( \phi_B = 0 \) for \( \Delta > 1 \) and \( \phi_B = \pi \) for \( \Delta < 1 \). Hence, the simplified dimensionless conductance writes

\[
\mathcal{G} = 1 + \cos(\phi_B) \cos(\phi_d + \Delta \phi_g). \tag{4}
\]

Lines of constant conductance in Fig. 2 correspond to lines with constant phase \( \phi' \equiv \phi_d + \Delta \phi_g \). The pattern presents a phase dislocation at \( \Delta = 1 \) as \( \cos(\phi_B) \) jumps from 1 to –1. The dislocation is a signature of a topological phase transition in the field texture, but not in the spin-eigenstate texture. Dynamic and geometric phases are correlated in such a way that the complex spin dynamics depicted in Figs. 3 and 4 are hidden.

Experiments are often performed in ensembles of multi-mode rings where the interference signal is strengthened and nongeneric features from individual structures are averaged out [24]. Figure 5 shows simulated interference patterns in the conductance of multimode InGaAs loops calculated with the RGFM at low temperatures. The in-plane field leads to dephasing of the AC oscillations [25]. However, in loop geometries the Zeeman phases are relevant and the interference pattern persists at \( \Delta > 1 \) for the lowest mode. The AC oscillation frequency doubles when the mean free path decreases due to interference from counter-rotating Altshuler-Aronov-Spivak (AAS) paths [26] in the loop with the geometric phase \( 2 \pi r \) in the adiabatic limit [9]. The Zeeman phases for AAS paths are equal and do not produce a double-frequency pattern. Since the \( \Delta_{SO} \) is proportional to the propagating velocity of a mode, multiple critical lines may arise. Nevertheless, the interference pattern for a triple-mode loop with the mean free path of the order of the loop circumference in Fig. 3 fits remarkably well the single-mode results for the lowest transport mode (Fig. 2). The higher modes traverse through the loop at slower speed and therefore they are more prone to scattering and decoherence. These results show that the topological transition is robust, and could be detected in ensembles of multichannel loops in the presence of moderate disorder.

Our findings open possible lines of future research. Alternative ring geometries with asymmetric interference paths could be implemented, with the benefit of higher signal strength in experiments due to stronger wire-to-ring coupling in comparison to loop geometries. Besides, asymmetries could be introduced by Aharonov-Bohm fluxes in symmetric ring geometries. The robustness of the topological transition turns loop device appropriate as magnetometers measuring the in-situ intensity of the Rashba spin-orbit fields. Deviations from the critical line \( \Delta = 1 \) may be used to estimate the strength of the Dresselhaus SO interaction [27]. Furthermore, signatures of complex AA geometric phases may be revealed by studying transport of spin-polarized carriers [28].

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