Comment on arXiv:1007.0718 by Lee Smolin

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Abstract

In a recent paper it was suggested a novel interpretation of deformed special relativity. In that new approach, nonlocal effects that had previously been shown to occur and be incompatible with experiment to high precision, are interpreted as coordinate artifacts that do not lead to real physical consequences. It is argued here that if one follows through the consequences of this thought, one finds that the theory one is dealing with needs to be ordinary special relativity to precision even better than the bound on nonlocal effect already requires. Consequently, the new approach cannot be understood as a version of deformed special relativity that circumvents the bound.

1 Introduction

Deformations of special relativity (DSR) \[1, 2, 3, 4\] allegedly make it possible to introduce an energy-dependent speed of light in position space while still preserving observer-independence. In \[5, 6\] it has been shown that such an energy-dependent and observer-independent speed of light is in conflict with observations, at least to first order in energy over Planck mass, \(E/m_p\). The reason for this conflict is that the model causes macroscopic violations of locality that are incompatible with well-confirmed particle interaction processes of the standard model.

It was then claimed by Smolin in \[7\] that quantum uncertainty may be a possibility to hide the nonlocality despite the fact that the contrary was shown in \[5, 6\]. In \[8\] it was demonstrated that Smolin’s ansatz is inconsistent and, when corrected, just reproduces the problem. In \[9\], Jacob et al found the same nonlocality as \[5, 6\] and correctly identified it as a glaring inconsistency, unfortunately without acknowledging they were just reproducing the earlier result from \[5, 6\]. In \[10\] Amelino-Camelia et al then shifted the nonlocality from the detector (Earth) to the source (gamma ray burst) and thus moved the original setup by some Gpc. This is a possibility that was already commented on in \[5\]. Moving the nonlocality does of course not remove it. It was explained in \[11\] that Amelino-Camelia et al’s claim that moving the nonlocality changes the bound by more than 20 orders of magnitude is due to them inaccurately using the duration of a gamma ray burst (of the

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1 Note that the reply \[8\] referred to the first version of \[7\], which has since been significantly revised.
order seconds) for a single elementary particle collision (of the order fm/c). The location at which the nonlocality becomes non-negligible depends on the exact scenario one considers as will become clear in the following. The bound itself however does not significantly depend on where the non-locality becomes noticeable.  

Now Smolin has put forward a new preprint [12] with another attempted circumvention of the problem of nonlocality in DSR. The ansatz he presents overlaps partly with the one of Amelino-Camelia et al in [10], but it differs in the interpretation and adds some interesting new layers to the argument that I will discuss here.

In the following we use units in which $c = \hbar = 1$.

## 2 Brief summary of the problem with nonlocality in DSR

Let us start with explaining briefly how the bound in [5, 6] was derived and introduce some terminology. Consider a source in the far distance (in the original example, a gamma ray burst) that emits two photons. In DSR the speed of the photon $c(E)$ depends on its energy such that deviations from the speed that we have measured become larger with larger energy. One photon is assumed to have arbitrarily low energy and thus moves with the ordinary speed of light. This photon is only there as a reference and represents the special relativistic limit. The other photon has a higher energy and is slowed down due to the energy-dependence of the speed of light. It travels a long distance till it collides with an electron in a detector (in the original example, on Earth). The scattering process causes a macroscopic event. The delay between the arrival time of the low and the high energetic photon is for a first order modification of the speed of light

$$\Delta T = \frac{E_\gamma}{m_p} L, \tag{1}$$

where $E_\gamma$ is the energy of the highly energetic photon, $m_p$ is the Planck scale, $L$ is the distance to the source, and $\alpha$ is a dimensionless parameter of the model characterizing the strength of the effect. The above formula has to be corrected when one takes into account the cosmological expansion [13, 14] but this will not matter for the following. The numbers we will use in the following for these quantities are $E_\gamma \approx 10$ GeV and $L \approx 4$ Gpc, such that $\Delta T \approx \alpha 1$ s.

The question is now, how does the same process look like for an observer who moves relative to the first restframe? To avoid having to carry around too many different parameters, the energy of the electron has been assumed to me much smaller than that of the electron. It does slightly depend on the location because one could argue that experimental tests of elementary particle collisions in distant astrophysical sources are not as precise as those done on Earth. However, we have so far no evidence that quantum field theory has worked any different during the evolution of the universe than it does today on our planet. Thus, if these bounds are weaker after shifting the nonlocality elsewhere, they cannot be much weaker.

It could also speed up, depending on the function $c(E)$. The following arguments however do not depend on whether the speed decreases or increases with energy. We will thus stick with one case, that in which the speed decreases, to improve readability.
highly energetic photon and the detector represents another macroscopic and special relativistic limit. Thus, we know how the worldlines of the electron and the detector have to be transformed. It’s just a normal Lorentz transformation.

To transform the worldline of the highly energetic photon, we cannot use the normal Lorentz transformation. We can however derive its transformation by making use of the invariance of the speed of light. We know how the energy of the photon transforms in DSR because the Lorentz-transformations are well known in momentum space. The speed of the photon is then just the value of the speed of light $c(E'_\gamma)$ at the value of the energy $E'_\gamma$ that is the result of the DSR-transformation of $E_\gamma$. With that we know the angle of the worldline. To have the full transformation, we need to fix one point on the worldline. This can most easily been done by using the notion of the fixed point for this DSR transformation, previously introduced in [11].

Consider making a transformation of the worldline of the particle into another restframe twice, once by use of the standard special relativistic transformation, and once by use of the DSR-transformation. Since after the transformation the angles of both lines differ, they will meet (in 1+1 dimensions and flat space) in exactly one point. The fixed point is this one point on the worldline the transformation of which does not depend on whether one uses the special relativistic or the DSR transformation. This point does always exist in 1+1 dimensions, and in 3+1 it can always be chosen to exist, but we will in the following restrict our attention to the 1+1-dimensional case.

Choosing the fixed point then entirely determines the transformation of the worldline of the highly energetic photon. In [5,6] the fixed point has been in the gamma ray burst.

Since the DSR transformation differs from the usual special relativistic transformation, in the second restframe the highly energetic photon misses the electron and only catches up with it well outside the detector. That is the problematic nonlocality: What is a point-interaction in one frame becomes a highly nonlocal interaction in another frame. Since all frames are equivalent we can use well-tested particle interactions to constrain the possibility of there being such nonlocality. This results in a bound of approximately $\alpha < 10^{-23}$. As already pointed out in [5,6], one should not take the last 5 orders of magnitude of this bound too seriously because from $\alpha \approx 10^{-18}$, effects of second order in $E_\gamma/m_p$ would become non-negligible and those were not taken into account in the derivation of the bound. One can however safely conclude that a first order modification is ruled out.

It should be clear from the above explanation that if one chooses a different fixed point, then the problem with nonlocality one obtains is just a translation of the originally considered one. The nonlocality always becomes macroscopic at large distance (here $\approx 4 \text{ Gpc}$) from the fixed point. By definition the fixed point is the point close by which deviations from special relativity are small and thus problems with nonlocality negligible.

In summary, we see that two assumptions have been made to arrive at the scenario considered [5,6]: That $c(E)$ is observer-independent and that the fixed point of the transformation is in the gamma ray burst. We also see that only one of these assumptions, the observer-independence of $c(E)$, is relevant to arrive at the bound. The location of the fixed point

Note that the term ‘fixed point’ in the context of DSR is in [12] used differently.
point merely determines where the nonlocality becomes relevant, but it does not change its disastrous consequences for elementary particle physics.

3 Smolin’s paper

For his argument, in addition to the observer-invariance of \( c(E) \), Smolin needs to make the following assumptions

1. The fixed point of the transformation is always the origin of the coordinate system.
2. Any detector is always at the origin of the coordinate system.
3. Any observer is always at the origin of the coordinate system.

The first assumption is a consequence of the particular framework that Smolin is using to derive the DSR-transformation. This assumption/use of the framework is of course possible, just that one then needs to keep in mind with this identification one can no longer independently chose the location of the fixed point and the origin. Consequently, to reproduce the scenario in [5, 6], the origin of the coordinate system would have to be put at the source \( S \). This first assumption is actually unnecessary for Smolin’s argument. One can drop it and replace ‘origin of the coordinate system’ with ‘fixed point’ in the second and third assumption, but for better comparison to his paper, we will stick with it.

With the first, the second assumption is necessary to prevent nonlocality at the detector. The third assumption is necessary so the observer is not sitting in the region where nonlocal effects are sizeable. The motivations Smolin puts forward for assumption two and three is that it is natural to locate the detector at the origin of the coordinate system, and that in the usual Einsteinian synchronization procedure one commonly locates the observer at the origin. That is indeed an often used and convenient choice, but in ordinary special relativity it is not necessary, and thus there is no good reason why this choice in particular should be carried over to DSR where it then gains additional relevance. The assumptions are in combination not even very realistic: An observer is in most cases not in the detector, they can in fact be arbitrarily far away from the detector, even though it might then take a while to receive the signal. Anyway, while Smolin’s assumptions seem artificial, we will in the following just accept them and see where they take us.

With these assumptions being made, Smolin still has a problem with nonlocality. There is now no nonlocality at the detector where an observer, \( D \), sits, but the DSR transformation brings up the nonlocality instead at the source. By virtue of assumptions 1-3, we are prohibited from just putting an observer, \( S \), in the same coordinate system as \( D \) but located at the source, where he would sit directly in the middle of the nonlocality. Instead, all we can do is consider another observer at the source, \( S' \), that one differing from \( S \) by the location of the origin of their coordinate systems. Since \( S' \) has his fixed point at the

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\(^5\)The scenario in [5, 6] does not depend on the choice of the origin of coordinates, it merely depends on the location of the fixed point.
origin and thus at the source, he sees no nonlocality there. This is in conflict with what D’s
DSR transformation says though. Since the observer $S’$ at the source sees no nonlocality
at the source and $S$ does not exist by assumption 3, Smolin argues that the nonlocality at
the source that appears in D’s transformation is an unphysical coordinate artifact.

So far so good. But at this point one is left to wonder about two things.

First, if the nonlocality is a coordinate artifact of the transformation and the transformation thus only reliable close by the origin, then what is the physically relevant transformation? Basically, Smolin is saying that we can only consider the DSR-transformation to be a local transformation. But then what is the global transformation? Well, the only global transformation that has no nonlocality neither at the detector nor at the source is one that has a fixed point both at the detector and the source, and is thus just the usual special relativistic transformation. This necessitates together with the low-energy limit of the speed of light that we are dealing with an energy-independent speed of light in ordinary special relativity.\(^6\)

Second, Smolin hints at there being just no global transformation, but only a set of local patches. To begin with, this is unsatisfactory as an offer for a novel approach to DSR since it would be necessary to know how to describe the propagation of a particle in the full spacetime, and not just locally, to even be able to tell whether or not the new approach circumvents the bound while preserving the prediction about the time delay. But besides this, it also begs the question why not the observer D can just construct the physically relevant global transformation from synchronization with the observer $S’$. The argument Smolin offers against this is that there is a limit to how accurately the both observers can synchronize their clocks, respectively compare their coordinates. If that was true, then it might indeed not be meaningful to ask for the global transformation and one could not conclude that one has to get back just special relativity.

So let us look closer at the synchronization procedure. We note that Smolin is talking in his paper about a purely classical setup. In this case, while an energy-dependent speed of light alters the synchronization procedure, it does not add ambiguity to it. So the additional limitations that he is talking about were probably meant to be quantum effects (as discussed in Version 2.1. of the box problem in \([5]\)). This does indeed cause an uncertainty to the sending back and forth of light signals that becomes more relevant at high energies and is an excellent point to make. One might think that to prevent this uncertainty, one could just take the low energy limit, get rid of the DSR-effects, and recover special relativity. However, in the low energy limit the precision of the synchronization procedure is limited by the usual uncertainty. Taken together, the total uncertainty in the synchronization procedure with photons of energy $E$ is:

$$
\Delta x_{\text{tot}} = \frac{1}{E} + \frac{E}{m_p L}.
$$

\(^6\) Without using the low-energy limit of the speed of light, one can still have an energy-dependent speed of light that is compatible with special relativity, but it corresponds to a massive (possibly tachyonic) photon and has nothing to do with DSR.
The most precise synchronization can then be achieved with energy

\[ E_{\text{best}} = \sqrt{\frac{m_p}{\alpha L}}, \]

and the corresponding uncertainty is

\[ \Delta x_{\text{best}} = \frac{2}{\sqrt{\alpha L/m_p}}. \]

We note that this is equal to \( \sqrt{\Delta T/E_T} \). One can now compute the maximal possible precision of the synchronization for the case of \( \alpha = 1 \). One finds that the best energy is \( \approx 10^{-2} \) eV and the corresponding precision \( \approx 10^{-4} \) m. This has to be contrasted to the nonlocality, which in this case for a small boost to a relative velocity of \( v = 10^{-5} \) already amounts to approx \( 1 \) km. Thus, clearly the observer D can find out that the DSR-transformation he has been using is in conflict with reality, there is nothing preventing him from this insight.

However, we note that the uncertainty is only proportional to \( \sqrt{\alpha} \) while the nonlocality is proportional to \( \alpha \). This means, if we decrease \( \alpha \), we will reach a value where the uncertainty indeed prevents the synchronization procedure from conflicting with the result of the DSR-transformation. In the following, we use the results from [5], just translated to the source to see what value of \( \alpha \) is necessary to achieve this. The nonlocality is caused by a mismatch between the transformation behavior of a worldline under ordinary special relativistic transformation and the modified DSR-transformation. This mismatch has in [5] been denoted as \( \Delta T' - t_a' \). The primes indicate that the quantities are those of a restframe in relative motion to the first (Earth) restframe. The mismatch vanishes for zero relative velocity, since \( \Delta T = t_a \) by construction. (For details and figures, please refer to [5].)

The requirement that Smolin’s argument holds and the observers cannot use the synchronization to discover the DSR coordinate artifact then means the mismatch has to be smaller than the maximally possible resolution in the second restframe, ie:

\[ |\Delta T' - t_a'| \lesssim \Delta x_{\text{best}}'. \]

With the moderate boost to a relative velocity of \( 10^{-5} \) used for the original example with the satellite, ie. \( v \ll 1 \), one gets

\[ 10^{-5} \Delta T \lesssim \sqrt{\frac{\Delta T'}{E_T'}}. \]

From the transformation of \( \Delta T \) [5,10]

\[ \Delta T' = \frac{1 - v}{1 + v \Delta T} \].
(neglecting terms of second order in $E_γ/m_p$) and the transformation of $E$ being (also to first order in $E_γ/m_p$) just the usual relativistic redshift, one has

$$\frac{\Delta T'}{E_γ'} = \sqrt{\frac{1 - v \Delta T}{1 + v E_γ}} + \text{higher order} \quad (8)$$

We remind the reader that in the original example $v < 0$. Since the transformation-factor in Eq. (8) is for small $v$ approximately 1, we find with $E_γ\Delta T \approx \alpha 10^{24}$ from Eq. (6) that

$$\alpha \lesssim 10^{-14} \quad (9)$$

That is already quite a tight constraint on Smolin’s scenario. But, as argued in [5, 6], we have tested boosts up to $\gamma = 30$ and found no disagreement with ordinary special relativity. Translation invariance allows us to apply these at the source as well as the detector, and we will thus have to look at the constraint on $\alpha$ for $\gamma$ up to 30. Then, one has $\gamma \approx 1/\sqrt{2\varepsilon}$ with $\varepsilon = 1 + v \approx 10^{-3}$ and the relevant uncertainty (cmp to Eq. (18) in [5]) is expressed through

$$\Delta T \approx \Delta x_{\text{best}} = \sqrt{\frac{\Delta T'}{E_γ'}} \quad (10)$$

Since

$$\frac{\Delta T'}{E_γ'} \approx \sqrt{\frac{2}{\varepsilon} \frac{\Delta T}{E_γ}} \quad (11)$$

and one can neglect the term in the inequality proportional to $\varepsilon$, one obtains

$$\frac{2}{\varepsilon} \lesssim \left(\frac{2}{\varepsilon}\right)^{1/4} \sqrt{\frac{1}{E_γ\Delta T}} \quad (12)$$

Inserting $\varepsilon$ and $E_γ\Delta T$ one finds

$$\alpha \lesssim 10^{-28} \quad (13)$$

Which again rules out a first order modifications of DSR, even in Smolin’s new interpretation. However, to be very clear here: this bound is of a fundamentally different nature than the originally derived one. The originally derived bound resulted from constraints on nonlocal effects that appear in the usual DSR scenario. The bound derived here is a bound necessary for Smolin’s scenario to be consistent, such that nonlocal effects can be interpreted as coordinate artifacts.

The reason for the constraints that we have arrived at here is that we have studied a synchronization procedure that allows a better precision than the one considered by Smolin in section 4 in his paper, leading to his Eq. (30). The difference is that we have first calculated the energy at which photons yield the best possible precision for the synchronization.
It remains to be said that while we have restricted ourselves here to distances, energies, and boosts that are accessible by today’s experiments, the conceptual problem is far worse. If the theory was to apply to all boosts, then the nonlocality would become arbitrarily large in arbitrary vicinity of all observers and detectors. Or, to put it differently, the requirement that no two observers under no circumstances are able to make a synchronization in conflict with the DSR-transformation would just result in $\alpha = 0$, i.e., special relativity.

4 Summary

Smolin suggests to interpret the nonlocality in deformed special relativity as coordinate artifacts, arising from applying a local coordinate transform out of its range of applicability. First, we have summarized which additional assumptions are necessary to make this interpretation possible. Then, we have then seen that if one takes this interpretation seriously, one must require that no two observers are able to use a synchronization procedure whose results are in conflict with the (now local) DSR-transformation, because otherwise they had a way to physically construct a better transformation. Since the uncertainty in the synchronization procedure does not scale the same way as the coordinate artifacts, there exist parameter ranges in which Smolin’s scenario is consistent. However, these parameter ranges do again not allow a modification to first order in energy over Planck mass.

Thus, the statement made in [5, 6] still holds: DSR effects, if they exist, cannot be the source of an observable time delay in the arrival times of highly energetic photons from distant gamma ray bursts. It should be emphasized that this of course does not mean there is no observable signature in the signal from the gamma ray bursts and that one should not look for it. It just that means that, at the present status of discussion, any such signature is very likely to be of astrophysical rather than of quantum gravitational origin.

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