Evading $1/m_b$-suppressed IR divergencies in QCDF: $B_s \to KK$ Decays and $B_{d,s}$ mixing

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We analyze the deviations of the mixing induced CP asymmetry in $B \to \phi K_s$ from sin $2\beta$, as well as the deviations of the asymmetries in $B_s \to K^{*0}K^{*0}$, $B_s \to \phi K^{*0}$ and $B_s \to \phi\phi$ from sin $2\beta_s$, that arise in SM due to penguin pollution. We use a theoretical input which is short-distance dominated in QCD-factorization and thus free of IR-divergencies. We also provide alternative ways to extract angles of the unitarity triangle from penguin-mediated decays, and give predictions for $B_s \to K^{*0}K^{*0}$ observables.

I. INTRODUCTION

The phenomenology of hadronic $B_d$-decays has been a matter of intensive research in the past 15 years, in part due to the large amount of data collected at the B-factories Babar and Belle, and at CDF and D0. This research has led to the consensus that the CKM mechanism for CP and flavor violation is accurate. However, some puzzles have survived up to this day [1, 2, 3, 4, 5].

A new era in B-physics has been triggered by the experimental precision and by the emergent exploration of the $B_s$ system. The future is marked by the starting of LHCb and the possibility of a super-B factory. Therefore, the evolution of the $B_s$ system is of interest for the exploration of the physics program.

On the theoretical side, the study of non-leptonic $B$-decays is difficult because of the presence of important long distance strong interaction effects. The correct computation of these contributions is crucial to be able to resolve small NP contributions. The amplitude of a B meson decaying into two light mesons can be written as

$$A(B \to M_1M_2) = \lambda^{(D)*}_{q/p} T_{M_1,M_2} + \lambda^{(D)}_{q/p} P_{M_1,M_2}$$  \hspace{1cm} (1)

were $\lambda^{(D)}_{q/p} \equiv V_{qb}V_{qD}^*$, and $T$ and $P$ are called “tree” and “penguin”. These hadronic parameters can be extracted from data, or can be predicted using symmetries (such as flavor). The direct computations from QCD are much more involved and are based on factorization and the $1/m_b$ expansion. They appear in the context of QCDF, pQCD or SCET [6]. While the methods based on flavor symmetries include naturally all kinds of long distance physics, they suffer from big uncertainties due to bad data an poorly estimated SU(3) breaking. On the other hand, methods based on factorization suffer from uncertainties due to non-factorizable chirally enhanced $1/m_b$ corrections, and long distance charm loops (charming penguins).

These proceedings review a recent proposal to improve (at a phenomenological level) on some of the weak points of the approaches mentioned above [10, 11]. We also include a straightforward application to $B \to \phi K_s$ and comment on the limitations and the applicability of the method.

II. EXPRESS REVIEW OF $B_q - \bar{B}_q$ MIXING

The time evolution of a $B_q$ meson ($q = d, s$) can be easily described by changing to the mass eigenbasis. The relationship between the flavor basis and the physical mesons is specified by a mixing parameter usually denoted by $q/p$.

$$|B_L\rangle = \frac{1}{\sqrt{1 + |q/p|^2}} \left( |B^0\rangle + \frac{q}{p} |\bar{B}^0\rangle \right)$$  \hspace{1cm} (2)

$$|B_H\rangle = \frac{1}{\sqrt{1 + |q/p|^2}} \left( |B^0\rangle - \frac{q}{p} |\bar{B}^0\rangle \right).$$

The time evolution of the mass eigenstates is straightforward and depends only on the masses and the widths of the physical B-mesons. Therefore, the evolution of the flavor eigenstates (which describes the oscillations) is specified by the masses and widths of the physical mesons and on the mixing parameter $q/p$.

Then one can define the mixing angle $\phi_M$ as

$$\phi_M \equiv - \arg (q/p)$$ \hspace{1cm} (3)

In terms of the entries of the effective hamiltonian,

$$\frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2} \Gamma_{12}}{M_{12} - \frac{i}{2} \Gamma_{12}}} \approx \sqrt{\frac{M_{12}^2}{M_{12}^2}}$$ \hspace{1cm} (4)

where we have used that $|\Gamma_{12}| \ll |M_{12}|$. This means that the above definition of the mixing angle is equivalent to

$$\phi_M = \arg (M_{12})$$ \hspace{1cm} (5)

One should be aware, though, that these quantities are not convention independent and are sensitive to unphysical phase redefinitions. However, once a convention for the weak phases is chosen everywhere, this definition is meaningful. In the Wolfenstein parametrization

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In order to have under control the smallness of the correction $\Delta S_{\phi K_s}$, one should be able to bound the size of $\text{Re}(T_{\phi K_s}/P_{\phi K_s})$. In fact, latest data gives \[ \Delta S_{\phi K_s}^{\text{exp}} = -0.39 \pm 0.20 \] so if the uncertainties are reduced around this central value, the claim of a NP signal will have to rely on a solid bound for the SM penguin pollution.

A first approach to bound this tree-to-penguin ratio was taken in the context of flavour SU(3) symmetry \cite{17, 18}. The argument is that if there was a large hierarchy between $T$ and $P$ in a $b \to s$ mode, the hierarchy would persist when one moves to the SU(3)-related $b \to d$ modes. But in these modes the tree is not suppressed with respect to the penguin, because $|\lambda_u^{(d)}| \sim |\lambda_c^{(d)}|$, so this hierarchy would have a clear impact in the observables of the $b \to d$ modes. In this way one can write the following bound
\[ |\Delta S_{\phi K_s}| < \sqrt{2} \lambda \left( \sqrt{\frac{BR_{\phi \tau^+}}{BR_{\phi K_s}}} + \sqrt{\frac{BR_{K^+ \phi K}}{BR_{\phi K_s}}} \right) + O(\lambda^2) \] valid in the SU(3) limit and under a non-cancellation assumption between $B_d \to \phi K_s$ and $B^+ \to \phi K^+$. With the present data the bound is \[ |\Delta S_{\phi K_s}^{\text{SU(3)}}| < 0.4 \] The same kind of analysis can be applied to other penguin-dominated decays \cite{18, 19}.

A second approach has been followed in the framework of QCD-factorization \cite{20}, which gives a much more competitive bound,
\[ 0.01 < \Delta S_{\phi K_s}^{\text{QCD}} < 0.05 \] Related analyses have been carried out in SCET \cite{21} and pQCD \cite{22}. For a recent review see also \cite{23}.

Concerning the $B_s - \bar{B}_s$ mixing angle, the clean tree-level determination comes from $B_s \to J/\psi \phi$. The related penguin-mediated decays are, for example, $B_s \to K^{*0} K^{*0}$, $B_s \to \phi K^{*0}$ or $B_s \to \phi \phi$. Again, one can write
\[ \eta_f A_{\text{mix}}(B_s \to f) = \sin 2\beta_s + \Delta S_f \] A study of the amounts by which their mixing induced CP asymmetries deviate from $\sin 2\beta_s$ in the SM can be found in \cite{11, 24}.

In the next pages we follow the approach in \cite{11}, based on a theoretical input that we call $\Delta$.

### III. THEORETICAL INPUT

Consider the quantity $\Delta \equiv T - P$. This quantity is a hadronic, process-dependent, intrinsically non-perturbative object, and thus difficult to compute theoretically. Such hadronic quantities are usually either extracted from data or computed using some factorization-based approach. In the latter case, $\Delta$ could suffer from
the usual problems related to the factorization ansatz and in particular long-distance effects.

However, for a certain class of decays, T and P share the same long-distance dynamics: the difference comes from the (u or c) quark running in the loop, which is dominated by short-distance physics \[10\]. Indeed, in such decays, $\Delta = T - P$ is not affected by the breakdown of factorization that affects annihilation and hard-spectator contributions, and it can be computed in a well-controlled way leading to safer predictions and smaller uncertainties.

Table I shows the values of $\Delta$ for our cases of interest. This quantity was used to predict branching ratios and asymmetries in $B_d \to K\bar{K}$ modes, and the outcome was promising \[10, 25\]. In \[26\] this quantity was used to extract the angle $\alpha$ of the unitarity triangle from $B_d \to K^0\bar{K}^0$. In the following section we review the formulae that allow to extract $T$ and $P$ from data and the theoretical input $\Delta$.

### IV. TREE AND PENGUIN CONTRIBUTIONS

We begin writing the two self-conjugated amplitudes in terms of tree and penguin contributions,

$$A = \lambda_u^{(D)*}T + \lambda_c^{(D)*}P, \quad \bar{A} = \lambda_u^{(D)}T + \lambda_c^{(D)}P$$

Now we put $T = P - \Delta$ and we square the amplitudes,

$$|A|^2 = \left|\lambda_u^{(D)*} + \lambda_c^{(D)*}\right|^2 \left|P + \frac{\lambda_u^{(D)*}}{\lambda_u^{(D)*} + \lambda_c^{(D)*}}\right|^2$$

$$|ar{A}|^2 = \left|\lambda_u^{(D)} + \lambda_c^{(D)}\right|^2 \left|P + \frac{\lambda_u^{(D)}}{\lambda_u^{(D)} + \lambda_c^{(D)}}\right|^2$$

But the squared amplitudes are directly related to observables,

$$|A|^2 = \frac{BR(1 + \mathcal{A}_{\text{dir}})}{g_{PS}}$$

$$|ar{A}|^2 = \frac{BR(1 - \mathcal{A}_{\text{dir}})}{g_{PS}}$$

where $g_{PS}$ is the usual phase-space factor. Neglecting the masses of the light mesons with respect to the $B$ mesons,

$$g_{PS}(B_d) = 8.8 \times 10^9 \text{ GeV}^{-2}$$

$$g_{PS}(B_u) = 8.2 \times 10^9 \text{ GeV}^{-2}$$

For two non-identical particles in the final state. The resulting expressions are

$$\frac{BR(1 + \mathcal{A}_{\text{dir}})}{g_{PS}} = \left|P + \frac{\lambda_u^{(D)*}}{\lambda_u^{(D)*} + \lambda_c^{(D)*}}\Delta\right|^2$$

$$\frac{BR(1 - \mathcal{A}_{\text{dir}})}{g_{PS}} = \left|P + \frac{\lambda_u^{(D)}}{\lambda_u^{(D)} + \lambda_c^{(D)}}\Delta\right|^2$$

These are the equations for two circles in the complex $P$ plane, whose solutions are the two points of intersection. This will result in a two-fold ambiguity in the determination of $P$ (and $T$). Before writing down the analytical solutions, notice that in order for solutions to exist, the separation between the centers of these circles must be smaller than the sum of the radii but bigger than the difference. This translates into a consistency condition between $BR$, $\mathcal{A}_{\text{dir}}$ and $\Delta$:

$$|\mathcal{A}_{\text{dir}}| \leq \sqrt{\frac{R_D^2 \Delta^2}{2BR} \left(2 - \frac{R_D^2 \Delta^2}{2BR}\right)}$$

where $\bar{BR} \equiv BR/g_{PS}$ and $R_D$ is a specific combination of CKM elements (see Table II). This condition turns out to be highly nontrivial. For example, Fig.4 shows the allowed values for the longitudinal direct CP asymmetry of $B_d \to K^{*0}\bar{K}^{*0}$ in terms of its longitudinal branching ratio. It can be seen that for $BR \gtrsim 3 \times 10^{-6}$ the direct CP asymmetry is quite constrained.

The hadronic quantities $P$ and $T$ are then given by

$$\text{Im}[P] = \frac{\bar{BR} \mathcal{A}_{\text{dir}}}{2c_0^{(D)}\Delta}$$

$$\text{Re}[P] = -c_1^{(D)}\Delta \pm \sqrt{-\text{Im}[P]^2 - \left(\frac{c_1^{(D)}}{c_2^{(D)}}\right)^2 + \frac{\bar{BR}}{c_2^{(D)}}}$$

$$T = P + \Delta$$

### TABLE I: Values of $\Delta$ for the various decays of interest. In the case of two vector mesons these numbers correspond to longitudinal polarizations.

| Decay                  | $\Delta$ Value | $\text{GeV}$     |
|------------------------|----------------|-----------------|
| $\Delta_{A_{0}K^{*}}$  | $(2.29 \pm 0.67) \times 10^{-7}$ |
| $\Delta_{K^0K^{*}}$    | $(1.85 \pm 0.79) \times 10^{-7}$ |
| $\Delta_{K^{*}K^{*}}$  | $(1.62 \pm 0.69) \times 10^{-7}$ |
| $\Delta_{\phi K^{*}}$  | $(1.16 \pm 1.05) \times 10^{-7}$ |
| $\Delta_{\phi\phi}$    | $(2.06 \pm 2.24) \times 10^{-7}$ |
where the coefficients $c_i^{(D)}$ are again some specific combinations of CKM elements (see Table II).

Equations (24) allow to extract the hadronic parameters $T$ and $P$ from experimental data on $BR$ and $A_{\text{dir}}$, information on sides of the unitarity triangle and the weak phase $\gamma$, and the theoretical value for $\Delta$. This method is also powerful because if no experimental information is available for $A_{\text{dir}}$, one can just vary $A_{\text{dir}}$ over its allowed range in eq. (23). So in fact $T$ and $P$ can be extracted from $BR$, $\Delta$ and CKM elements.

### V. $\sin 2\beta$ FROM $B \to \phi K_s$

Following the discussion in the previous section, the bounds for $\text{Re}(T_{\phi K_s}/P_{\phi K_s})$ are given by

$$ \text{Re} \left( \frac{T_{\phi K_s}}{P_{\phi K_s}} \right) \leq 1 + \left( -c_1^{(s)} + C(BR_{\phi K_s}, \Delta_{\phi K_s}^d) \right)^{-1}$$

$$ \text{Re} \left( \frac{T_{\phi K_s}}{P_{\phi K_s}} \right) \geq 1 + \left( -c_1^{(s)} - C(BR_{\phi K_s}, \Delta_{\phi K_s}^d) \right)^{-1}$$

$$ C(BR, \Delta) \equiv \sqrt{-\left( c_0^{(s)} / c_2^{(s)} \right)^2 + (1 / c_2^{(s)}) \overline{BR}/\Delta^2} \quad (25) $$

Introducing the numbers for the coefficients from Table II the value or $\Delta_{\phi K_s}^d$ in Table II and the latest experimental value for the branching ratio $|10|$

$$ BR(B_d \to \phi K_s)_{\text{exp}} = 8.3^{+1.2}_{-1.0} \times 10^{-6} \quad (26) $$

we get the following bounds for $\Delta S_{\phi K_s}$,

$$ 0.03 < \Delta S_{\phi K_s} < 0.06 \quad (27) $$

### VI. $\sin 2\beta_s$ FROM $B_s \to VV$

Equations (25) apply as well to $B_s \to VV$. Here we focus on longitudinal polarizations for which the numerical values of $\Delta$ are under control. As mentioned above, penguin mediated $B_s \to VV$ decays measure $\sin 2\beta_s$, but no experimental information is available yet for CP asymmetries in $B_s$ decays. There is, though, an experimental number for the $B_s \to \phi \phi$ branching ratio $|10|$

$$ BR(B_s \to \phi \phi)_{\text{exp}} = 14^{+8}_{-7} \times 10^{-6} \quad (28) $$

If we suppose that the longitudinal polarization fraction is $f_L^{\phi \phi} \sim 50\%$ as QCDF suggests $|20|$, then we find

$$ 0.006 \leq \Delta S_{\phi \phi} \leq 0.072 \quad (29) $$

The decay $B_s \to K^{*0} \bar{K}^{*0}$ is more appropriate because $\Delta_{K^*K^*}$ is under a much better numerical control, but there is no experimental value for the branching ratio. For the sake of illustration, we just mention that for $BR_{\text{long}}(B_s \to K^{*0} \bar{K}^{*0}) \sim (30 - 40) \times 10^{-6}$, one gets

$$ 0.037 \leq \Delta S_{K^*K^*} \leq 0.051 \quad (30) $$

### VII. OTHER ANGLES FROM DATA AND $\Delta$

There are also some expressions that can be written that relate directly branching ratios and asymmetries to other angles of the unitarity triangle through the quantity $\Delta$ $|11|$. These expressions do not require any CKM angle as an input, just sides of the unitarity triangle. The experimental input is minimized by measuring $A_{\Delta\gamma}$, which can be extracted from the time-dependent untagged rate $|28, 29|$. Then, in the case of a $B_d$ meson decaying through a $b \to D$ process ($D = d, s$),

$$ \sin^2 \alpha = \frac{\overline{BR}(1 - A_{\Delta\gamma})}{2|\lambda_u^{(D)}|^2|\Delta|^2} ; \sin^2 \beta = \frac{\overline{BR}(1 - A_{\Delta\gamma})}{2|\lambda_c^{(D)}|^2|\Delta|^2} \quad (31) $$

and in the case of a $B_s$ meson decaying through a $b \to D$ process ($D = d, s$),

$$ \sin^2 \beta_s = \frac{\overline{BR}(1 - A_{\Delta\gamma})}{2|\lambda_c^{(D)}|^2|\Delta|^2} \quad (32) $$

### VIII. PREDICTIONS FOR $B_s \to K^*K^*$

Assuming no new physics contributing to $B_d \to K^{*0} \bar{K}^{*0}$, we can also give SM predictions for the branching ratios and CP asymmetries of its U-spin partner $B_s \to K^{*0} \bar{K}^{*0}$ $|11|$. The inputs are $\Delta_{K^*K^*}^d$, SU(3) breaking from QCDF, and the experimental value for $BR(B_d \to K^{*0} \bar{K}^{*0})$. While there is still no experimental information on this branching ratio, it is remarkable that for $BR(B_d \to K^{*0} \bar{K}^{*0}) \gtrsim 5 \times 10^{-7}$ the results are almost insensitive to it. Taking $\gamma = (62 \pm 6)^\circ$ and $2\beta_s = -2^\circ$, we find

$$ \left( \frac{BR_{\text{long}}(B_s \to K^{*0} \bar{K}^{*0})}{BR_{\text{long}}(B_d \to K^{*0} \bar{K}^{*0})} \right)_{SM} = 17 \pm 6 \quad (33) $$

$$ A_{\text{dir}}(B_s \to K^{*0} \bar{K}^{*0})_{SM} = 0.000 \pm 0.014 \quad (34) $$

$$ A_{\text{mix}}(B_s \to K^{*0} \bar{K}^{*0})_{SM} = 0.004 \pm 0.018. \quad (35) $$

### IX. CONCLUSIONS

To conclude, we would like to comment on the relevance of the proposal. First, the applicability of the approach has to be checked individually for each mode: it can only be applied to those decays for which $\Delta$ receives no contributions from annihilation or hard spectator

| $c_i^{(d)}$ | $c_i^{(s)}$ | $c_i^{(D)}$ | $\mathcal{R}_d$ |
|------------|------------|------------|-------------|
| $-3.15 \cdot 10^{-5}$ | $-0.034$ | $6.93 \cdot 10^{-5}$ | $7.58 \cdot 10^{-3}$ |
| $3.11 \cdot 10^{-5}$ | $0.011$ | $1.63 \cdot 10^{-3}$ | $1.54 \cdot 10^{-3}$ |
scattering graphs, such as $B \to K^+(*)K^(*)$, $B_d \to \phi K^(*)$, $B^+ \to \pi^+\phi$, etc. The predictions derived in this way include most of the long distance physics, which is contained inside the experimental input. The used theoretical input is minimal, and is the most reliable input that QCDF can offer, free from the troublesome IR-divergencies. Moreover, the theoretical error is under control and is likely to be reduced in the near future due to, for example, the fast progress that is taking place in lattice simulations.

There is, however, an honest criticism due to the fact that some long distance effects that are controversial in lattice simulations.

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