Stable heavy pentaquarks in constituent models

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(Dated: October 24, 2017)

It is shown that standard constituent quark models produce $\bar{c}cqqq$ hidden-charm pentaquarks, where $c$ denotes the charmed quark and $q$ a light quark, which lie below the lowest threshold for spontaneous dissociation and thus are stable in the limit where the internal $\bar{c}c$ annihilation is neglected. The binding is a cooperative effect of the chromoelectric and chromomagnetic components of the interaction, and it disappears in the static limit with a pure chromoelectric potential. Their wave function contains color sextet and color octet configurations for the subsystems and can hardly be reduced to a molecular state made of two interacting hadrons. These pentaquark states could be searched for in the experiments having discovered or confirmed the hidden-charm meson and baryon resonances.

I. INTRODUCTION

In recent years, many new hadrons have been discovered in the hidden-charm sector, leading to a flurry of theoretical works. For a review, see e.g., [1]. Except for $X(3872)$, most states have been seen only in one experiment, or in one type of experiment, e.g., $B$ decay or production in $e^+e^-$ collisions, and thus await firm confirmation. Among the recent findings, the two states seen by the LHCb collaboration [2] have attracted much attention and inspired many interesting studies. Several approaches have been proposed to describe the LHCb states, and some of them even anticipated the discovery. In the molecular approach, these states consist of coupled $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Sigma_c^*$, and there are some predictions for strange partners. See, e.g., Refs. [3, 4] and references therein. A variant is the so-called hadroquarkonium [5], in which a compact charmonium is trapped inside an ordinary hadron. This idea has been tested in lattice simulations [6]. Another method relies on QCD sum rules, see, e.g., [7]. Note that neither lattice simulations nor QCD sum rules are fully ab-initio yet, due to the complexity of the computations, and use the guidance of specific models, such as diquarks, to select the operators. In the quark model approaches, the attention is often focused on the chromomagnetic part of the interaction [8]. Among the exceptions, we can mention [9].

For sure, this physics has not been exhausted yet. Several experimental searches are limited by the small production cross section, and the restriction to specific triggers, e.g., $J/\psi$ in the final state, while flavor exotic configurations are also awaited [10, 11]. But, as shown in this letter, even the hidden-charm sector has not yet been fully explored, and some states have perhaps escaped our scrutiny.

Most multiquarks, so far, are resonances, and necessarily involve an interplay between re-scattering effects and collective dynamics. Bound states are more easily described with normalizable wave functions, and when the binding becomes deeper and deeper one expects a transition from a dominant long-range hadron-hadron interaction towards a dominance of interquark dynamics.

The constituent quark model has often been used for exploratory studies whose results have been refined and confirmed by more rigorous treatments of QCD. For instance, the prediction of flavor-exotic mesons ($QQ'q\bar{q}$) has been made first by many potential-model calculations and later reinforced by lattice simulations and QCD sum rules [1, 10, 12].
In this letter, we revisit the hidden-charm configurations \((\bar{c}cqqq)\), where \(c\) is the charmed quark and \(q = u\) or \(d\), using a standard potential model and estimate consistently the mass of the pentaquark states and of the ordinary hadrons constituting their dissociation threshold, to identify states which are stable if the internal annihilation of the \(\bar{c}c\) pair is neglected, i.e., lie below their lowest dissociation threshold. The model and the method used for the calculations are described in Sec. III. Our results are displayed in Sec. III and commented upon in Sec. IV. Finally, some other promising configurations are listed in Sec. V which will be the subject of further studies, possibly with an improved modeling of the confining interaction.

### II. THE MODEL

We use a simple quark model consisting of non-relativistic kinetic energy and a color-additive interaction corresponding to pairwise forces mediated by color-octet exchanges. The validity of this picture has been discussed at length in many papers and review articles, and hardly needs further comments. The model is used for exploratory studies that could stimulate investigations within more sophisticated pictures. To be more specific, we choose the so-called AL1 model of Senay and Silvestre-Brac [13], which has been already used for multiquark calculations, see, e.g., [14] [15]. It reads

\[
V(r) = -\frac{3}{16} \lambda_i \lambda_j \left[ \lambda r - \kappa \frac{r}{r} - \Lambda + \frac{V_{SS}(r)}{m_i m_j} \hat{\sigma}_i \cdot \hat{\sigma}_j \right],
\]

\[
V_{SS} = \frac{2\pi \kappa'}{3} \frac{1}{\pi^{3/2} r_0^3} \exp(-r^2 / r_0^2), \quad r_0(m_i, m_j) = A \left( \frac{2m_i m_j}{m_i + m_j} \right)^{-B},
\]

where \(\lambda = 0.1653\) GeV\(^2\), \(\Lambda = 0.8321\) GeV, \(\kappa = 0.5069\), \(\kappa' = 1.8609\), \(A = 1.6553\) GeV\(^{-1}\), \(B = 0.2204\), \(m_u = m_d = 0.315\) GeV, and \(m_c = 1.836\) GeV. Here, \(\lambda_i \lambda_j\) is a color factor, suitably modified for the quark-antiquark pairs. We disregard the small three-body term of this model, introduced to fine-tune the baryon masses vs. the meson masses. We also use a variant with slightly different quark masses and parameters \(\lambda\) and \(\kappa\), and a fixed smearing radius for the spin-spin potential, that has already been utilized to study the six-quark problem [16].

Note that our aim is to show whether the constituent quark model, if taken seriously, may lead to binding of some \((\bar{c}cqqq)\) configurations. In quark models, mass differences are usually better predicted than the masses themselves. In our case, what is best predicted is \(m(h) - m(h_1) - m(h_2)\) for a multiquark \(h\) with threshold \(h_1 + h_2\), so changing the parameters of the potential will change the masses but barely the binding energy of the multiquark \(h\).

To solve the 5-body problem, we introduce the Jacobi coordinates

\[
\vec{x} = \vec{r}_2 - \vec{r}_1, \quad \vec{y} = \vec{r}_4 - \vec{r}_3, \quad \vec{t} = \vec{r}_5 - \frac{\vec{r}_3 + \vec{r}_4}{2},
\]

\[
\vec{z} = \frac{\sum_{i=1}^2 m_i \vec{r}_i}{\sum_{i=1}^2 m_i}, \quad \vec{R} = \frac{\sum_{i=3}^5 m_i \vec{r}_i}{\sum_{i=3}^5 m_i},
\]

with different numbering of the \((\bar{Q}Qqqq)\) constituents. Namely, three different quark arrangements, shown in Fig. IV, generate three sets of Jacobi coordinates. The arrangements \((a)\) and \((b)\) simulate the asymptotic thresholds that contribute to the five-quark state and are explicitly reached when the range parameters, of the trial wave function defined below, associated to the Jacobi coordinate \(\vec{z}\) vanish. The asymptotic state is made of a charmonium and a light baryon for \((a)\) and of an anticharmed meson and a charmed baryon for \((b)\). In \((c)\), the thresholds are somewhat hidden and, instead, a confined diquark-diquark-antiquark configuration is put forward. Clearly, these choices of Jacobi coordinates should in principle lead to the same spectrum. However, the convergence of the variational calculation turns out easier if one adopts a quark arrangement rather than another one, depending on the pentaquark state. In particular, for each set of quantum numbers we have used the three Jacobi coordinate arrangements to improve and speed convergence. Note that this strategy is not identical to the one promoted by Kamimura et al., who astutely mix in the wave functions configurations corresponding to different sets of Jacobi coordinates [17].

In a simple model such as [1], color, which enters through the \(\lambda_i \lambda_j\) factor and the statistics, is treated as a global degree of freedom, similar to isospin in nuclear physics. There are three independent color states for the pentaquarks, which can be chosen for arrangements \((a)\) and \((b)\) as:

1. \((\bar{c}c)\) singlet coupled to \((qqq)\) singlet,
2. \((\bar{c}c)\) octet coupled to the first \((qqq)\) octet, in which the quarks 3 and 4 are in a \(\bar{3}\) state,
3. \((\bar{c}c)\) octet associated to the second \((qqq)\) octet, in which the quarks 3 and 4 form a sextet,
(a) (b) (c)

FIG. 1: The three sets of quark rearrangements and the relative Jacobi coordinates.

and for arrangement (c), as

1. \((Qq)\) in a \(\bar{3}\) state coupled to the first \((qq\bar{Q})\) triplet, in which quarks 3 and 4 are in a \(\bar{3}\) state,

2. \((Qq)\) in a \(\bar{3}\) state coupled to the second \((qq\bar{Q})\) triplet, in which the quarks 3 and 4 are in a \(6\) state,

3. \((Qq)\) sextet associated to the \((qq\bar{Q})\) \(\bar{6}\) state, in which quarks 3 and 4 form a \(\bar{3}\) state.

The color states are explicitly spelled out in a computer code, using the SU(3) Clebsch-Gordan coefficients provided in [19] and the associated web site. In this basis the matrix elements of the color coefficients 

\[ C_{ij} = -\frac{3}{16} \lambda_i \cdot \lambda_j \]

are obvious for the pairs \((1,2)\) and \((3,4)\). For the other pairs, new color states are generated, deduced by permutations in the quark sector, and their overlap (crossing matrices) with the initial color states are calculated. The same pedestrian method is used for the spin-states, to estimate the matrix elements of the spin-color operators \(C_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j\). For a total spin \(S = 5/2\), there is only one spin state, and thus 3 color-spin states \(|\alpha\rangle\). For \(S = 3/2\), there are 4 spin states, and thus 12 \(|\alpha\rangle\) color-spin states. For \(S = 1/2\), there are 5 independent spin configurations, and then 15 color-spin vectors \(|\alpha\rangle\) in the wave function.

A similar situation is found with the isospin basis (note that for the spin coupling all arrangements are identical, because we are dealing with five particles of spin 1/2) that depending on the particle arrangement used the isospin vectors, and thus their symmetry properties, are different. There are two linearly independent isospin 1/2 vectors and one isospin 3/2 vector. As the interaction is isospin independent, the two allowed isospin 1/2 vectors are orthogonal and when the Pauli principle is imposed, the number of vectors in each arrangement may be different.

For each choice of Jacobi coordinates, the wave function is expanded as

\[ \Psi = \sum_\alpha \psi_\alpha(x, y, t, z) |\alpha\rangle, \quad \psi_\alpha(x, y, t, z) = \sum_i \gamma_{\alpha,i} \exp(-\tilde{X}^\dagger \cdot A_{\alpha,i} \cdot X/2), \]

where the \(A_{\alpha,i}\) are \(4 \times 4\) positive-definite matrices whose elements are the range parameters, and \(\tilde{X}^\dagger = \{x, y, t, z\}\). All spatial matrix elements can be calculated analytically, using standard techniques of Gaussian integration [20]. The range parameters entering the \(A_{\alpha,i}\) are optimized numerically to minimize the variational energy. No particular Anstaz has been used for these parameters, but the minimization MINUIT code tries random sets whenever a minimum is reached to check that it is not a local minimum. The number of generalized Gaussians is increased till the result is

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1 This a drawback in the analysis of Ref. [18] together with the choice of a set of Jacobi coordinates that does not take into account the existence of particles with different masses.
TABLE I: Masses (in GeV) of the hadrons entering the thresholds, calculated in the potential model of Eq. (1) and compared to the experimental values.

| Baryons | Mesons |
|---------|--------|
| State  | Calc. | Exp. | State  | Calc. | Exp. |
| N      | 0.996 | 0.940 | D      | 1.862 | 1.868 |
| ∆      | 1.307 | 1.232 | D*     | 2.016 | 2.008 |
| Λ      | 2.292 | 2.286 | ηc     | 3.005 | 2.989 |
| Σc     | 2.467 | 2.455 | J/ψ    | 3.101 | 3.097 |

TABLE II: Relevant thresholds (in GeV) for $I = 1/2$, 3/2 and 5/2 states, and angular momentum between the meson and the baryon for isospin $I = 1/2$ and $I = 3/2$ states.

III. RESULTS

The masses of the hadrons involved in the thresholds, calculated with the potential model of Eq. (1) are compared to the experimental ones in Table I. The thresholds of the channels made out of these hadrons are listed in Table II. We restrict ourselves to odd overall parity, thus the relative angular momentum between the meson and the baryon has to be even, $S$- or $D$-wave in practice.

The masses obtained for the pentaquark states are listed in Table III together with the lowest threshold in relative $S$- and $D$-wave. As already mentioned, if a mass is estimated as being larger than the lowest threshold, it just means that the state is unbound. Any conclusion about a possible resonance, and the estimate of the width of the resonance, would require dedicated further calculations, for instance by applying the method of real scaling [21].

Let us first note the degeneracy between $I = 1/2$ and $I = 3/2$ states, as could have been expected a priori due to the isospin independence of the potential model in Eq. (1), although the result is not trivial due to the requirements of the Pauli principle. It is worth to emphasize that the quark arrangement schemes shown in Fig. [1] are basic to get the lowest energy. In fact, the isospin 1/2 states do easily converge to their lowest energy in arrangement (a) and (b), while isospin 3/2 states converge better in arrangement (b). Arrangement (c), that was used to look for explicitly exotic states with a tightly bound diquark substructure, is not favored by any set of quantum numbers. It is seen that the lowest state for $(J, I) = (1/2, 3/2)$ and $(3/2, 3/2)$ are found below their lowest $S$- and $D$-wave thresholds: $\Delta\eta_c$ and $D\Sigma_c$. In fact they are substantially lower, so that they remain stable or metastable if one accounts for the width of the $\Delta$ and considers that the actual lowest threshold is $N\pi\eta_c$. As can be seen in Table III, the vicinity of the two lowest thresholds would also contribute to enhance the potential attractive character of the interaction for these quantum numbers. The lowest state for $(J, I) = (5/2, 1/2)$ is above its lowest $D$-wave threshold while it is below the lowest $S$-wave threshold. In this case, as highlighted long ago in Ref. [22], the contribution of color vectors different...
TABLE III: Variational estimate of the pentaquark masses (in GeV) in various spin and isospin channels, compared to the lowest $S$-wave and $D$-wave threshold.

| $(J,I)$     | $(ccqqq)$ | Lowest threshold               |
|------------|-----------|--------------------------------|
| $(1/2,1/2)$| 4.077     | 4.001 (S) / 4.408 (D)          |
| $(3/2,1/2)$| 4.161     | 4.001 (D) / 4.097 (S)          |
| $(5/2,1/2)$| 4.429     | 4.001 (D) / 4.562 (S)          |
| $(1/2,3/2)$| 4.077     | 4.312 (D) / 4.329 (S)          |
| $(3/2,3/2)$| 4.161     | 4.312 (S) / 4.329 (D)          |
| $(5/2,3/2)$| 4.429     | 4.312 (D) / 4.408 (S)          |

from the singlet-singlet combination would prevent by the centrifugal barrier the tunneling of the quarks to combine in a colorless object, enhancing in this way the stability of this state. The other states are found above the lowest $S$-wave threshold.

IV. DISCUSSION

The stability of the $(J,I) = (1/2,3/2)$ and $(3/2,3/2)$ ground state with parity $P = -1$ does not depend on the detailed values of the parameters of the potential. We have tried several variants, without qualitative change of the conclusions. For instance, we have modified the parameters into $\lambda = 0.2$ GeV$^2$, $\kappa = 0.4$ and adopted a simplified spin-spin interaction with a smearing radius frozen at $r_0 = 1$ GeV$^{-1}$ for all pairs [10], and the binding remains. Within the model of Eq. (1), we have studied the influence of the smearing parameter $A$. It is known, indeed, that variational calculations become harder with a superposition of long- and short-range potentials. We show in Fig. 2 the evolution of the masses of the five-quark state and the lowest $S$- and $D$-wave thresholds as the smearing parameter of the spin-spin interaction is gradually increased. As it can be seen, for the $(J,I) = (1/2,3/2)$ case the five-quark state remains below the lowest $S$-wave threshold, although for large values of $r_0$ it goes above the lowest $D$-wave threshold. For such large values of $r_0$ the chromomagnetic potential is rather weak, what indicates that such state might appear in nature as a narrow resonance as explained above [22]. Regarding the $(J,I) = (3/2,3/2)$ state, it also survives very weak chromomagnetic potentials, going above the lowest $S$-wave threshold in the chromoelectric limit.

It is interesting to analyze the role of the spin-spin interaction vs. the spin-independent one. At the time of the pioneers in color chemistry [23], the eigenvalues of the chromomagnetic interaction were calculated for a variety of

FIG. 2: Evolution of the masses of the five-quark state and the lowest $S$– and $D$–wave thresholds as the smearing parameter of the spin-spin interaction is gradually increased.
clusters, including \((qqq)\) in color 8. See in particular [22, 24]. One can see in this literature, that the color-octet \((qqq)\) configuration receives a more favorable chromomagnetic energy than the color-singlet \(\Delta\) in the threshold. However, at the time of color chemistry, attention was paid mainly to states made of two colored clusters separated by some angular-momentum barrier, while our study deals with an overall S-wave multiquark.

In Fig. 3, it is seen that the binding starts already with a small fraction of the spin-spin interaction. This means that there is a favorable interplay of chromoelectric and chromomagnetic effects, although the binding disappears in the pure chromoelectric limit.

V. OUTLOOK

Our finding opens interesting perspectives, as some similar configurations are also bound in this approach. A detailed study will be done in a forthcoming paper. Let us just mention that variations in the heavy quark sector generally maintains the binding for spin 1/2 and isospin 3/2, as it is shown in Table IV.

When moving to the strange sector, the spin-spin interaction is magnified due to its \(1/(m_im_j)\) dependence and the the smaller value of regularization radius \(r_0\), as per Eq. (1), which is able to maintain a large amount of binding.

It is interesting to note how the binding energy decreases when going from the charm to the bottom sector: the interaction is still attractive, but the degeneracy of the thresholds is lost, and thus the cooperative effect of the coupled-channels is less effective. In atomic physics, for instance, the binding of the \((m_1^+,m_2^-,m_3^-)\) ions is improved as \(m_2 \rightarrow m_3\), as the two thresholds made of an atom and an isolated charge become degenerate [25]. When the \(H = (uuddss)\) was first calculated in the SU(3) limit [26], the thresholds \(\Lambda\Lambda, \Sigma\Sigma\) and \(N\Xi\) were artificially degenerate. It was later shown that with SU(3) breaking, i.e., with the thresholds at different masses, the binding is weakened [27].

Except for the deuteron, which is mainly a nucleon-nucleon system, and the doubly-heavy mesons \((QQ\bar{q}\bar{q})\) which are not yet seen, there are very few examples of stable multiquarks in the literature. Most experimental candidates, and most states predicted in various models, deal with resonances. It is shown in this paper that standard potential models generate new bound states in the sector of baryons with hidden-charm. If this is confirmed in other approaches, then one of the LHCb pentaquarks could be a kind of radial excitation, or, say, a collective excitation of a bound pentaquark and the other might correspond to the first orbital excitation, with the hope that the orbital barrier in the latter case or the nodal structure in the former case will somewhat inhibit the decay, and make the excited state not too broad in spite of its high mass. Note there are still uncertainties about the quantum numbers of the LHCb pentaquarks. Proposals to study the LHCb pentaquarks in photoproduction of \(J/\Psi\) on the proton to verify their existence would also help in determining their spin and parity [28].
Other states could be bound as well and would deserve some further study:

- the \((bbqqq)\) analogs, or even better, the \((bcqqq)\) ones, which are free of internal annihilation,
- the naked-anticharm \((\bar{c}uuds)\) and its analogs obtained by permuting \(u, d\) and \(s\) have never been submitted to a detailed 5-body calculation since the first study by Gignoux et al.\cite{29}, and independently by Lipkin\cite{30},
- Any \((\bar{Q}cuds)\) is free of restrictions due to the Pauli principle. Thus the confinement can take benefit of the flip-flop interaction and some connected diagrams, instead of the color-additive \(\lambda_i \cdot \lambda_j \cdot r_{ij}\). This provides more attraction\cite{31, 32}.

On the experimental side, it would be interesting to analyze \(J/\psi p\pi^-\) final state. If the pentaquark lies below the \(J/\psi p\pi^-\) threshold, one could look at the isospin violating decay \(J/\psi N\) channel, or to any final state corresponding to internal \(c\bar{c}\) annihilation, such a \(\mu^+\mu^-\pi^\pm\pi^\mp\) or \(p\bar{p}\pi\pi\) or a proton and pions with a \((p,\pi)\) subset in the \(\Delta\) region, and the remaining pions in the charmonium region.

VI. ACKNOWLEDGMENTS

This work has been partially funded by Ministerio de Economía, Industria y Competitividad and EU FEDER under Contracts No. FPA2016-77177 and FPA2015-69714-REDT, by Junta de Castilla y León under Contract No. SA041U16, by Generalitat Valenciana PrometeoII/2014/066. Interesting discussions with Emiko Hiyama, Makoto Oka, and Atsushi Hosaka are gratefully acknowledged.

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