A Multi-strategy Improved Ant Colony Algorithm for Solving Traveling Salesman Problem

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Abstract. A multi-strategy improved ant colony algorithm is proposed. In order to solve the problem of solving the TSP problem in the ant colony algorithm, it has the problems of low solution accuracy, easy fall into local optimum, and low solution efficiency. The nearest neighbor method is used to influence the distribution of the initial pheromone to reduce the pheromone concentration on the short path in the initial stage of the algorithm. Based on the mutation adjustment of the transfer rule, a mean cross-evolution strategy is combined with the mean of the path to enhance the global solution space of the algorithm. Ability and ability to avoid falling into a local optimum. Then, the iterative and elitist strategies are combined to improve the pheromone update mechanism to further improve the solution algorithm's solution performance and solution efficiency. Finally, the eight instances selected from the TSP LIB database are solved and compared with other algorithms. The experimental results show that the improved algorithm is efficient when solving the traveling salesman problem and has high computing performance.

1. Introduction

Traveling Salesman problem [1] is a NP-complete problem in combinatorial optimization problems. At present, the enumeration method has been an effective method for solving smaller scale TSP problem, but it is difficult to obtain the exact solution of larger scale TSP problem. As a result, Scholars put forward an optimal algorithm for solving TSP problem with approximate satisfactory solution. In the early stage, there were mainly branch and bound method, heuristic nearest neighbor method, improved loop method, etc. However, with the increasing of the scale of TSP problem, the difficulty of solving the problem increases rapidly. Groups intelligent optimization algorithms [2] gradually emerged in the 1980s, such as Ant Colony Algorithm (ACA), Genetic Algorithm(GA), Particle Swarm Optimization (PSO), and so on [1]. ACA is an algorithm with strong robustness, parallel distributed computing and easy to combine with other algorithms. But owning to the basic ACA has low spatial search ability, and it is easy to fall into the local optimum and the search efficiency of finding the optimal path is low, so it can not solve the optimization problem efficiently and accurately.

Many researchers have improved the ACA from different aspects in order to compensate for the shortage of ACA. In improving the initial global search capability and convergence speed of the algorithm: Jianli Ding et al. [3] used the rapid global random searching ability of genetic algorithm to generate the initial pheromone of related problems to make up for the deficiency of pheromone in the
early stage of the algorithm. Weide Ren et al. [4] proposed an ACA that combines genetic algorithm to adjust mutation factor and uniform design optimization strategy in the initial stage. In the algorithm selection path: Jingle Zhang et al. [5] introduced the hybrid and mutation mechanism of genetic algorithm in order to improve the precocity and slow convergence of the algorithm, so that a new improved ACA with mutation characteristics was proposed. Qin Haisheng et al. [6] proposed an ACA based on dynamic local search, which enables all ants in the algorithm to have the ability of local search. It can be used to improve the quality of the search solution and the stability of the algorithm according to the real-time situation. In terms of pheromone updating strategy of the algorithm: Yimeng Yue et al. [7] proposed an improved ACA for pheromone dynamic evaporation probability strategy, that is, an adaptive dynamic factor was introduced into the pheromone updating rules to control the updating ratio of pheromone concentration. Thus the optimal solution can be obtained in a single iteration. Yuxian Zhang et al. [8] proposed an pheromone update strategy based on experimental rational acquisition of residual factors with iterative thought information, which could improve the convergence rate of ACA. In terms of the parameters of the algorithm: Lijuan Sun [9] optimized the information heuristic factor, expected heuristic factor, pheromone residue factor and the selection threshold of search space by genetic algorithm. Zaifu Yang [10] aimed at the phenomenon that the basic ACA is easy to fall into local optimum, using the sorting rules of genetic algorithm, the ant's position in the algorithm is initialized before visiting the city position every time.

In this paper, a multi-strategy improved ACA is proposed in order to improve the ability of the algorithm to jump out of the local optimum, the search space is expanded and the efficiency of the solution is improved. First, the nearest neighbor method is adopted to construct an initial travel path, which can weaken the initial pheromone concentration and forms the initial pheromone distribution of the algorithm. And a variation factor was established to adjust the transition probability based on the evolutionary theory of biology. In order to take into account the overall situation on the basis of improving the probability, the principle of genetic algorithm crossover is used to mean cross-evolution of paths. The updated pheromone with iterative idea and elite strategy is used to update the pheromone to further improve the solution ability of the algorithm.

2. The Principle of Solving TSP with Basic ACA
ACA is a random search algorithm proposed by Italian scholar Marco Dorigo. In his doctoral thesis in the 1990s. It can make full use of the feedback information in the optimization process. For example, the process of solving the TSP combinatorial optimization problem with ACA is as follows. It can make full use of the feedback information in the optimization process when solving problems. For example, the process of solving the TSP combinatorial optimization problem with ACA is as follows:

Step1: Initialization of the algorithm. The given m ants are randomly assigned to n vertex positions, and the pheromone of the path of the city was initialized with $\tau_i(0)$.

Step2: Each ant finds its own access path through an iterative loop. The ant $k$ ($k=1, 2, ..., m$) determines the next city to be visited based on the pheromone concentration value on the path, and records the currently visited city number using a tabulation table. In the search process, the state transition probability of ant $k$ transitioning from city $i$ to city $j$ at time $t$ is set to $p^k_{ij}(t)$. The calculation of the transition probability is related to the amount of information and the path length on each path. The state transition probability is expressed by the following formula (1).

\[
p^k_{ij}(t) = \begin{cases} 
\frac{[\tau_i(t)]^{p} \cdot [\eta_i(t)]^{p}}{\sum_{j \in \text{allowed}_k} [\tau_i(t)]^{p} \cdot [\eta_i(t)]^{p}}, & j \in \text{allowed}_k \\
0, & j \notin \text{allowed}_k
\end{cases}
\]  

(1)

In formula (1), allowed$_k$=$\{S$-tabu$_k\}$ represents the set of cities that can be selected by the ants next time. The heuristic factor $\alpha$ denotes the relative importance of trajectories. The expectation heuristic
factor $\beta$ visibility is relative importance. The $\tau_{ij}(t)$ denotes the pheromone concentration on the path between cities $i, j$ at $t$ moment. And $\eta_{ij}(t) = 1/d_{ij}$ denotes the distance between city $i$ and city $j$.

Step3: Pheromone update. In order to solve the excessive accumulation of residual information, thus affecting the role of heuristic information. Therefore, the pheromone on the path needs to be updated after each ant individual has completed each step or visited all the cities. The pheromone update rule at the time $t+n$ from city $i$ to city $j$ follows the formula (2):

$$
\tau_{ij}(t+n) = \begin{cases} 
(1-\rho)\cdot \tau_{ij}(t) + \Delta \tau_{ij}(t) \\
\Delta \tau_{ij}(t) = \sum_{k=1}^{m} \Delta \tau_{ij}^k(t)
\end{cases} \tag{2}
$$

In equation (2), $\rho$ and $1-\rho$ denote the volatile and residual factors of the pheromone. $\tau_{ij}(t)$ denotes the amount of pheromone on the path $i, j$ in this cycle. The initial time $\tau_{ij}(t) = 0$. The $k_{th}$ ant is present the amount of pheromone remaining in the $i, j$ path of the city in the secondary loop is applied, because the Ant-Cycle model has better performance in solving the TSP problem, which is shown in the following formula(3):

$$
\Delta \tau_{ij}^k = \begin{cases} 
Q/L_k, & \text{if the } k_{th} \text{ ant passes through the path } (i, j) \text{ in this cycle} \\
0, & \text{otherwise}
\end{cases} \tag{3}
$$

In formula (3), $Q$ represents the intensity of pheromone. $L_k$ represents the total length of the path of the $k$ ant in an iterative process. The shortest path of $m$ ant paths in each iteration is obtained and recorded.

Step4: Determines whether the algorithm termination condition is satisfied. When the termination condition is satisfied, the shortest path of the final iteration is the optimal solution. Otherwise, clear the list of tabu and go to step step2.

Although ACA has a good effect in solving TSP problem, there are some disadvantages of searching space and searching time, and it is easy to appear local convergence and calculation time too long.

3. Organization of the Text

3.1. Weighted Initial Pheromone Distribution Based on Nearest Neighbor Method

Nearest Neighbor (NN) [11] is one of the simplest classification algorithms for data mining. Because of its intuitionistic and fast solving speed, this paper uses the NN method to construct an initial travel path $l_N$ of the TSP problem in the early stage of the algorithm. The basic idea of NN method for solving TSP problem, and its essence is the practical application of greedy strategy in solving circuit problems, which can be described as: First, the source city is given as the starting city of the tour path. Then, the city closest to the source city is found, and the city is added to the constructed route as the next source point. Finally, repeat until all the city coordinate points have been added to the constructed route. And add the starting point to the end of the tour route that is to build the initial tour route $l_N$.

The initial pheromone is always a fixed value when the basic ACA is used to solve the TSP problem. As a result, when the initial iteration of the algorithm is performed for the first time, the shorter the path distance between cities is, the larger the pheromone concentration value is, and the selected probability of the path is greater. As a result, the algorithm may eventually choose to use a longer route as the optimal solution. In order to improve the deficiency of the basic ACA in the initial stage for solving the TSP problem, the path distance between any two cities is transformed by using the initial tour path obtained by the nearest neighbor method, which is regarded as the initial Pheromone distribution matrix $\tau_{ij}(0)$. Therefore, to solve the problem that the algorithm easily enters
the local optimal solution when selecting the path with high concentration of pheromone in the initial stage. The initial distribution matrix of pheromone is obtained by formula (4):

$$\tau_{ij}(0) = \begin{cases} n/(d_{ij} \cdot l_{i}), & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}$$  

(4)

3.2. Path Evolutionary Strategy Based on Improved Transition Probability

A variation parameter based on biological evolution theory is introduced to affect ant state transition probability (random variation factor $r$ of probability). $q_0$ indicates that the ant selects a probability selection threshold for the next city and it is an attribute given to the ant itself. First, the probability of each ant in an ant colony moving to the next city according to the rules set by the basic ACA. When $q_0 > p$, the probability that the ant selects the next city needs to be adjusted. The probability of the ant transferring to the next city in the algorithm is shown in formula (5):

$$p_{ij}^{t}(t) = \begin{cases} \arg \max_{i \in \text{allowed}} \{\tau_{i} \cdot \eta_{i}^{r}\}, & r \cdot p \leq q_0 \\ p_{ij}^{t}(t), & \text{otherwise} \end{cases}$$  

(5)

In formula (5), $r$ and $p$ are a random number uniformly in an interval $[0, 1]$ and $q_0$ is a number in the interval $[0, 1]$. The existence of the argument will reduce the transition probability of the ant when it moves to the next city, so that the information dependence of other ant feedback can be weakened and the local optimum solution can be avoided. Thus searching for the better solution in the unknown space effectively. Because it is a secondary revision of the algorithm transfer rule. It is not a completely random search for all unknown spaces, but it is for the local optimal solution that the algorithm has found, and when the path transition probability that meets the mutation strategy is adjusted. It effectively improve the disorder of the algorithm.

A mean value crossover operator is proposed based on the principle of crossover operation of genetic algorithm, in order to reduce the influence of the existing local optimal solutions on the next iteration cycle and improve the ability of jumping out of the local search for other new solutions in the whole world. The implementation flow of path evolution is as follows:

Step1: When $m$ ants complete once visit to the city in the TSP problem, a set $P$ consist of $m$ access routes is obtained. The length $p_i$ and average path length of each access route is calculated.

Step2: Select the optimal access route $\tilde{P}_{\text{best}}$ as the sample individual, and select a random segment of the optimal route as the crossover object. In order to ensure the randomness of the position and length of the selected fragment, the position and length of the segment are selected on the sample by roulette selection mechanism.

Step3: Place the selected segment at the front end of each access route that is less than average value $\bar{l}$ in the set $\tilde{P}$. And keep the segment unchanged, delete the same city number in the access route as in in the fragment, that is, the set $\tilde{P}_{\text{new}}$ of the new route is obtained.

Step4: Calculate and compare the path length of access routes in a new collection. The minimum path length in the new set is compared with the minimum value obtained by this iteration, and the smaller path length value is kept as the final iterative optimal solution. The process is shown in Figure 1:
Figure 1. Mean crossover algorithm operation flow

After each iteration is completed in turn, the path evolves once according to the principle of the mean crossover operator until the iteration cycle stops, and the final solution is the global optimal solution to the TSP problem. The introduction of the improved state transition rule and the concept of mean crossover operator can help to jump out of the local optimum and improve the precision of the ACA.

3.3. An Improved Strategy for Pheromone Updating

The pheromone update rules are improved from both global pheromone and local pheromone. First, in order to avoid the delay of the effect of the global pheromone update strategy of the basic ACA on the ant behavior, and to make effective use of the generated initial pheromone, the local pheromone update rule is determined as following formula (6):

$$\tau_{ij}(t+1) = (1 - \rho_1) \cdot \tau_{ij}(t) + \rho_1 \cdot \frac{n}{l_{ij}}$$  \hspace{1cm} (6)

In formula (6), $\rho_1$ is a value in the interval [0, 1] representing the local pheromone volatility coefficient.

Then, the importance of pheromone updating in ACA to the whole algorithm is considered. Pheromones volatile factors make old pheromone fade as time goes by and avoid the accumulation of residual pheromone overwhelming heuristic information. From formula (1), it can be seen that the original information update rule only uses the original information at the time $t$, which is not conducive to the synergy among the ants.

Finally, in order to make up for the deficiency of ACA, a pheromone updating strategy to enhance the cooperative ability of ants is proposed based on the pheromone updating rule with iterative thought in reference[8], which can make full use of the original information at different times. In order to obtain the optimal solution after each iteration, it has some feedback effect on the next iteration, and improves the rule of pheromone update with the elitist strategy [12]. That is, the rules for updating the global pheromone in this paper are shown as formula (7):

$$\tau_{ij}(t+n) = \left( \frac{\rho_2}{1 - \rho_2} - \rho_2 \right) \cdot \tau_{ij}(t) + \left( \frac{1}{1 - \rho_2} - \rho_2 \right) \cdot \Delta \tau_{ij}(t) + \rho_2 \cdot \frac{1}{d_{ij}^{best}}$$  \hspace{1cm} (7)

In formula (7), $\rho_2$ is the global pheromone volatility coefficient; the distance $d_{ij}^{best}$ between any connected cities in the optimal route for the current iteration.
4. Experimental Simulation and Result Analysis

In order to prove the performance of the improved ACA proposed in this paper. The author selected att48, eil51, st70, eil76, kroD100, ch130, rat195, and ts225 from the TSPLIB standard database as examples of improved ACA simulation experiments, and used Euclidean distance to calculate the distance between any two cities. The initial parameters of the algorithm were set to: The number of ants \( N_{C_{\text{max}}} = 1000 \), the heuristic factor \( \alpha = 1 \), the expected heuristic factor \( \beta = 2 \), the local volatilization coefficient \( \rho = 0.1 \), the global volatilization coefficient \( \rho = 0.1 \), the parameter \( q_0 = 0.75 \), probability random evolution factor \( r \) and random number \( p \) are interval \([0, 1]\). The experiment was carried out on a computer platform equipped with 3.20 GHz / 16GB RAM CPU of RXeon RAM, and tested with MATLAB R2014b under Win7.

By using the improved algorithm, 20 independent simulation experiments are carried out on the selected 8 instances, and the experimental results are shown in Table 1. The improved ACA was applied to perform 20 independent simulation experiments on the selected eight instances. The Table 1 also includes the experimental results of the basic ACA, the adaptive ACA and the IWSMACO algorithm (its data source reference [13]) to solve the corresponding TSP problem. By comparing the simulation results of different algorithms, the performance of the improved ACA proposed in this paper was verified.

Table 1. The improved algorithm, ACA, Q-ACA, IWSMACO algorithm to solve the optimal value and the average value of the TSP instance

| instance | Known optimal solution | improved algorithm | ACA | Q-ACA | IWSMACO algorithm |
|----------|------------------------|--------------------|-----|-------|-------------------|
|          |                        | optimal value      | average value | optimal value | average value | optimal value | average value |
| att48    | 33522                  | 33522              | 33624 | 34078 | 34356 | 33804 | 34158 | 33524 | 33737 |
| eil51    | 426                    | 426                | 429  | 445   | 466   | 433   | 449   | 426   | 441   |
| st70     | 675                    | 679                | 688  | 696   | 721   | 686   | 710   | 680   | 704   |
| eil76    | 538                    | 538                | 546  | 578   | 591   | 551   | 565   | 546   | 559   |
| kroD100  | 21294                  | 21305              | 21847 | 21964 | 23392 | 22006 | 22406 | 21394 | 22319 |
| ch130    | 6110                   | 6118               | 6165 | 6156  | 6191  | 6129  | 6179  | 6118  | 6145  |
| rat195   | 2323                   | 2335               | 2367 | 2407  | 2562  | 2395  | 2489  | 2346  | 2451  |
| ts225    | 126643                 | 128453             | 128935 | 131435 | 132328 | 131388 | 132040 | —     | —     |

That the quality of the optimal solution and average value obtained by the improved algorithm is higher than that of other algorithms (except that the optimal solution of ch130 is the same as that of IWSMACO algorithm and the average value is larger than that of IWSMACO algorithm) obtained in Table 1. The optimal solution of att48, eil51 and eil76 is the same as the known historical optimal solution, that is, the deviation of its optimal solution is 33522, 426 and 538, respectively. It is showed that this algorithm could effectively explore the high quality solution in unknown space and had a strong ability to jump out of the local optimal solution, So that the algorithm can get more accurate optimal solution. In this paper, the partial optimal solution roadmap of 8 TSP examples selected in this paper is obtained by simulation as shown in Figure 2.
Figure 2. Partial optimal roadmap for improved algorithms

Figure 3. The optimal value and average value of the iterative process of the improved algorithm

Figure 4. Comparison of algorithm deviation before and after improvement

The iterative process of eil51 and rat195 for their optimal values and averages are showed in Figure 3. From the graph, we can see that the algorithm can quickly find the optimal solution near the solution space, but will not immediately enter the phase of complete stability. The average value is stable quickly and fluctuates near a certain value, which shows that the improved initial pheromone distribution strategy is helpful to avoid some wrong solutions. The improved pheromone updating strategy can make full use of the initial pheromone to stabilize the performance of the algorithm and improve the accuracy of the algorithm to solve the TSP problem.

The deviation of the algorithm, the basic ACA, the adaptive ACA and the IWSMACO algorithm for solving the optimal solutions of TSP instances with different city sizes are showed in Figure 4. It can be seen from the graph that the deviation of the optimal solution of the improved algorithm in this paper is smaller than that of the other three algorithms, which shows that the proposed algorithm has a strong ability to jump out of the local optimum and is beneficial to the improvement of the overall performance of the ACA.

5. Conclusion

In this paper, the improved ACA makes used of the initial pheromone distribution matrix established by the nearest neighbor method. In the initial stage, it effectively filtered out the optimal solution with
large deviation, and quickly entered the space range near the optimal solution for optimization. It provided the basis for the updating of pheromone and the choice of path for ants when the path was optimized in the later stage of the algorithm. The path evolution strategy with random mutation adjusted state transition probability and genetic algorithm crossover ability was advantageous to jump out of the local optimum. In the later stage of the algorithm, the precision of the optimal solution was further improved by combining the iterative strategy of elite pheromone updating. The simulation results showed that the multi-strategy improved ACA was effective and the efficiency of the algorithm to solve the TSP problem was improved. However, with the increase of TSP scale, the degree of deviation of optimal solution and the number of iterations increase gradually. How to improve the accuracy and fast convergence of large-scale TSP problem will need further research.

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