Dephasing time of composite fermions

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Abstract

We study the dephasing of fermions interacting with a fluctuating transverse gauge field. The divergence of the imaginary part of the fermion self energy at finite temperatures is shown to result from a breakdown of Fermi’s golden rule due to a faster than exponential decay in time. The strong dephasing affects experiments where phase coherence is probed. This result is used to describe the suppression of Shubnikov-de Haas (SdH) oscillations of composite fermions (oscillations in the conductivity near the half-filled Landau level). We find that it is important to take into account both the effect of dephasing and the mass renormalization. We conclude that while it is possible to use the conventional theory to extract an effective mass from the temperature dependence of the SdH oscillations, the resulting effective mass differs from the $m^*$ of the quasiparticle in Fermi liquid theory.

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I. INTRODUCTION

It has been known for some time that fermions coupled to transverse electromagnetic field fluctuations lead to singular corrections to the specific heat and self energy. While these effects are small and so far unobserved in metals, they play a prominent role in the study of strong correlation models, such as the $t$-$J$ model and more recently, in the composite fermion description of the half-filled Landau level. In the latter case, flux tubes are attached to electrons forming composite fermions and the mean field theory leads to a Fermi sea. Corrections to the mean field theory are due to space- and time-dependent density fluctuations, resulting in gauge fluctuations which directly affect physical observables. Indeed, the fluctuations are so strong that quasiparticles in the Landau sense may not be well defined, and the question arises as to why the composite fermion description appears to work so well. The question was addressed recently by Stern and Halperin and by Kim et al. In the case of Coulomb repulsion between the electrons, the self-energy $\Sigma = \Sigma' + i\Sigma''$ takes the form $\Sigma''(\omega) \sim \omega$ and $\Sigma'(\omega) \sim \omega \ln \omega$, and Stern and Halperin showed that the quasiparticle concept remains marginally valid, albeit with a logarithmically divergent effective mass. They further showed that in the quantum Boltzmann equation, the effect of the divergent mass is cancelled by a singular Landau function. Kim et al. treated the more singular case when the Coulomb interaction is screened to become short ranged (by a nearby metallic gate, for instance). They showed that even though the quasiparticles are not well defined, a quantum Boltzmann equation can be derived, and the cancellation between the divergent mass and the Landau function occurs for all measurements involving a smooth distortion of the Fermi surface. Most of the experiments, including surface acoustic wave (SAW) resonances, magnetic focusing, and anti-dot resonances belong to this category. It is expected that the divergent mass will show up only in the activation gap of the effective Landau levels of the composite fermions.

The issue of the effective mass is related to the real part of the self energy. In this paper we address the issue of how the imaginary part of the self energy will affect experiments. The experiments we have in mind are those which require phase coherence of the composite fermions, such as mesoscopic phenomena and quantum oscillations [de Haas-van Alphen (dHvA) and SdH effects]. Such phenomena are outside the purview of the quantum Boltzmann equation. The difficulty which we immediately encounter is that, at finite temperatures, $\Sigma''(T)$ is infinite. This is because the thermal factor $k_B T / \omega$ for the soft gauge fluctuations gives rise to a further singularity at small $\omega$ which has no obvious cutoff. Here we first analyze this problem and show that the difficulty results from a breakdown of Fermi’s golden rule (Sec. II). We then use a semiclassical approach to discuss the suppression of the SdH oscillations due to the dephasing of composite fermions (Sec. III). The effect of mass renormalization is considered in Sec. IV. Our theoretical results are compared with experimental data in Sec. V and conclusions are drawn in Sec. VI.

II. BREAKDOWN OF FERMI’S GOLDEN RULE

We begin by treating the Green’s function of a particle in a space- and time-dependent gauge field $a(r, t)$ and potential $a_0(r, t)$ using the semiclassical (Gorkov) approximation
\[ G(\mathbf{r}, t) = G_0(\mathbf{r}, t)e^{i\phi(t)}, \quad (2.1) \]

where \( G_0 \) is the free fermion Green’s function and

\[ \phi(t) = \int_0^t [\mathbf{a}(\mathbf{r}(t'), t') \cdot \mathbf{v}_F + a_0(\mathbf{r}(t'), t')] \, dt' \quad (2.2) \]

(our units are such that \( \hbar = c = k_B = 1 \)). We assume that the particle travels in a straight line with velocity \( \mathbf{v}_F \), so that \( \mathbf{r}(t') = \mathbf{r}_0 + t' \mathbf{v}_F \). Since the gauge fluctuations scatter mainly in the forward direction, the velocity is affected only on a long time scale (the transport time) so that this assumption is justified. Equation (2.2) has the property that under a gauge transformation, \( \mathbf{a} \rightarrow \mathbf{a} + \nabla \Lambda, \ a_0 \rightarrow a_0 + \dot{\Lambda}, \ \phi(t) \rightarrow \phi(t) + \Lambda(\mathbf{r}(t), t) - \Lambda(\mathbf{r}(0), 0) \), as is required for the gauge transformation of \( G(\mathbf{r}, t) \) given by Eq. (2.1). We shall work in the transverse gauge, \( \nabla \cdot \mathbf{a} = 0 \), where it can be shown that the contribution from \( a_0 \) fluctuations are unimportant, and will be ignored from here on. The fluctuations of \( \mathbf{a} \) are Gaussian and controlled by the correlation function

\[ \langle \tilde{a}_\alpha(\mathbf{q}, \omega) \tilde{a}_\beta(-\mathbf{q}, -\omega) \rangle = \coth \left( \frac{\omega}{2T} \right) \left( \delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right) \text{Im} D_{11}(q, \omega), \quad (2.3) \]

where, for \( \omega < qv_F \) and \( q \ll k_F \), the retarded transverse gauge propagator is given by

\[ D_{11}(q, \omega) = \frac{1}{i\omega \gamma_q - q^2 \tilde{\chi}'(q)}, \quad (2.4) \]

with

\[ \gamma_q^{-1} = \frac{2\pi q}{k_F} \quad \text{and} \quad \tilde{\chi}'(q) = \tilde{\chi}'_0 + \frac{\tilde{v}(q)}{(4\pi)^2}. \]

In the expressions above \( \tilde{\chi}'_0 \) represents the diamagnetic susceptibility of the composite fermions at the half-filled state, \( k_F = mv_F \) is the Fermi momentum, and \( \tilde{v}(q) \) is the Fourier transform of the two-body interaction potential \( v(r) \). We refer to \( m \) as the mean-field mass of the composite fermion which arises from the interaction energy and differs substantially from the band mass of an electron. For convenience, we shall simply write

\[ D_{11}(q, \omega) = \frac{(2\pi q/k_F)}{i\omega - C_\eta q^{n+1}}. \quad (2.5) \]

In the Coulomb case \( v(r) = e^2/(4\pi\varepsilon r) \), where \( \varepsilon \) is the medium dielectric constant, therefore \( \eta = 1 \) and \( C_1 = e^2/(4\pi k_F) \). For the short-range, \( \delta \)-function interaction \( \eta = 2 \) and \( C_2 = 2\pi \tilde{\chi}'(0)/k_F \).

From Eq. (2.1) we see that the dephasing of the fermion is given by the factor

\[ \langle \exp[i\phi(t)] \rangle = e^{-F_\eta(t)}, \quad \text{where} \]

\[ F_\eta(t) = \frac{1}{2} \langle \phi^2(t) \rangle \]

\[ = -\int \frac{d^2 q}{(2\pi)^2} \int_0^\infty \frac{d\omega}{\pi} |\mathbf{v}_F \times \mathbf{q}|^2 \coth \left( \frac{\omega}{2T} \right) \text{Im} D_{11}(q, \omega) \left\{ 1 - \cos[(\mathbf{v}_F \cdot \mathbf{q} - \omega)t] \right\} \frac{1 - \cos[(\mathbf{v}_F \cdot \mathbf{q} - \omega)t]}{(\mathbf{v}_F \cdot \mathbf{q} - \omega)^2}. \quad (2.6) \]
The term appearing in \{\ldots\} is well known in elementary quantum mechanics. When \( t \) is large, it is a strongly peaked function of \((v_F \cdot q - \omega)\) with height \( \sim t^2 \) and width \( \sim t^{-1} \) and is usually approximated by \( 2\pi t \delta(v_F \cdot q - \omega) \). The \( \delta \)-function is recognized as enforcing energy conservation (after ignoring the \( q^2/2m \) term) and \( F_{\eta}(t) \) takes the form \( t\Sigma'' \), where \( \Sigma''(T) \) is the usual formula for the imaginary part of the self energy. The exponential decay \( \exp[-F(t)] \) then leads to the interpretation of \( \Sigma'' \) as the inverse lifetime. This is the standard derivation of Fermi’s golden rule. However, for our problem \( \Sigma''(T) \) is infra-red divergent for both \( \eta = 1 \) and 2. On the other hand, \( F_{\eta}(t) \) is finite for all \( t \) when \( \eta < 2 \). \( \Sigma'(t) \) is logarithmically divergent, as is easily seen by a small \( t \) expansion. We therefore directly evaluate the integral in Eq. (2.6) by the following steps. In order to eliminate the singular denominator in Eq. (2.6) we first consider the second derivative \( \ddot{F}_{\eta}(t) \equiv d^2F_{\eta}(t)/dt^2 \). Then we perform the \( q \) integration (details are shown in Appendix A). We find (for \( \eta = 1 \))

\[
\ddot{F}_1(t) = \left( \frac{v_F^2}{2k_F C_1^{3/2}} \right) \int_0^\infty d\omega \sqrt{\omega} \coth \left( \frac{\omega}{2T} \right) \cos(\omega t) f \left( v_F t \sqrt{\frac{\omega}{C_1}} \right), \quad (2.7)
\]

where \( C_1 \) is defined below Eq. (2.3) and

\[
f(x) = \frac{2}{\pi} \int_0^{\pi/2} d\theta \cos^2 \theta \exp \left( -\frac{x \sin \theta}{\sqrt{2}} \right) \cos \left( \frac{x \sin \theta}{\sqrt{2}} + \frac{\pi}{4} \right). \quad (2.8)
\]

We conclude that \( \ddot{F}_1(t) \) has two regimes. For \( t \ll t_T \), where \( t_T \equiv (1/v_F)\sqrt{C_1/T} \), it is given by \( 1/(2^{3/2}E_F t_T^3)[1 - 2^{3/2}t/(3\pi t_T)] \), whereas for \( t \gg t_T \), it is \( 1/(2E_F t_T^2 t) \). Upon integration, we find that

\[
F_1(t) \simeq \begin{cases} 
\frac{1}{2^{3/2} 2E_F t_T} \left( \frac{1}{t_T} \right)^2 + O(t/t_T)^3 & t \ll t_T \\
\frac{1}{2E_F t_T} \ln \left( \frac{t}{t_T} \right) & t \gg t_T
\end{cases}, \quad (2.9)
\]

where \( E_F = k_F^2/2m \) is the Fermi energy. Thus we see that the dephasing of the fermion is faster than exponential in time, which explains why the self energy \( \Sigma'' \) is infinite, because this amounts to force-fitting to an exponential decay. The technical reason is that the replacement of the \{\ldots\} factor in Eq. (2.6) by a \( \delta \)-function is invalid if the rest of the integrand is singular in the small \( q \) and \( \omega \) limit. This explains the breakdown of Fermi’s golden rule. Similar considerations show that the long-time behavior for \( F_{\eta}(t) \) is proportional to \( t^\eta/(\eta - 1) \) for \( 1 < \eta < 2 \).

**III. SUPPRESSION OF QUANTUM OSCILLATIONS**

The Green’s function in Eq. (2.7) is not gauge invariant and is not a directly measurable quantity. Nevertheless, the strong dephasing found in the previous section does manifest itself physically by suppressing quantum oscillations. Perhaps the clearest example is the strong damping of oscillations in the longitudinal conductivity \( \sigma_{xx}(B) \) near the half-filled state. The damping is mainly due to the thermal fluctuations of the field \( a(r,t) \). This phenomenon is essentially the SdH effect for composite fermions.
The approach outlined in the previous section will now be used to discuss physical measurements such as the SdH effect. We shall employ the formulation in terms of semiclassical paths, which can describe quantum oscillations in unmodulated electron gases, as well as in anti-dot arrays.

The oscillatory part of the conductivity measured in a SdH experiment, \( \sigma_{\text{osc}}^{xx} \), comes mostly from the interference between repetitions of the same cyclotron trajectory [represented by the off-diagonal term in Eq. (15) of Ref. 13]. As a consequence of that, \( \sigma_{\text{osc}}^{xx} \) can be expressed in a form very similar to the oscillations in the density of states, which are proportional to the integral

\[
\int_{-\infty}^{\infty} d\varepsilon \ n_F(\varepsilon) \ \text{Re} \sum_{p=1}^{\infty} (-1)^p \left< e^{iS_p(\varepsilon)+i\varphi_0} \right>,
\]

where \( n_F(\varepsilon) = (e^{\varepsilon/T} + 1)^{-1} \) and \( \varphi_0 \) is a constant. We call \( S_p(\varepsilon) \) the action of a classical path with energy \( \varepsilon \) which traverses a cyclotron orbit \( p \) times. In the presence of a fluctuating gauge field we have

\[
\left< e^{iS_p(\varepsilon)} \right> = e^{2\pi \varepsilon \omega_c / \omega_c} \left< e^{i \oint_a (r_p(t), t) \cdot \mathbf{v}_F dt} \right>,
\]

where \( r_p(t) \) is the classical orbit and \( \omega_c = eB/m \) is the cyclotron frequency. \( (B \) denotes the external magnetic field felt by the composite fermions.) More generally, the phase factor in Eq. (3.2) should include the \( a_0 \) term, as in Eq. (2.3), so that under a gauge transformation \( S_p(\varepsilon) \) will be shifted by \( \chi(\mathbf{r}(t), t) - \chi(\mathbf{r}(0), 0) \). In a gauge-invariant formulation of the SdH oscillations, this phase factor should be cancelled by a corresponding gauge transformation of \( \varphi_0 \). For our purposes it suffices to note that a gauge transformation introduces only a fixed phase shift in \( \left< e^{iS_p(\varepsilon)} \right> \), so that as far as the damping factor is concerned, it can be evaluated in any gauge. In particular, in the transverse gauge the dominant contributions come from the transverse gauge fluctuations, and Eq. (3.2) can be used. In any other gauge, \( a_0 \) fluctuations must be included, which complicates the calculation.

There is one further complication we must overcome before we can proceed with the evaluation of Eq. (3.2). If we evaluate Eq. (3.2) using Eq. (2.6), it was noted before that \( F_\eta(t) \) diverges logarithmically for any path in the short-range case \( (\eta = 2) \). This is because the fluctuations in \( \mathbf{a} \) can get arbitrarily large, even though the fluctuations in \( \mathbf{h} = \nabla \times \mathbf{a} \) are bounded. The divergence is avoided for closed paths in the case when the gauge field \( \mathbf{a} \) is not explicitly time dependent. In this case, \( S_p(\varepsilon) \) can be written explicitly in terms of the fluctuations of the field \( \mathbf{h} \), which are bounded. On the other hand, for dynamical fluctuations with frequency \( \omega > \omega_c/p \), the flux enclosed by the orbit has changed by the time the particle completes \( p \) cycles, and \( S_p \) can no longer be written in terms of \( \mathbf{h}(\mathbf{r}, t) \). For these paths we can use the Gorkov approximation discussed earlier. Thus our strategy is the following. We divide the frequency spectrum for the gauge fluctuations into two parts: the part with \( \omega < \omega_c/p \) will be considered static and treated in an explicitly gauge invariant way in terms of the flux fluctuations; the remaining part will be treated in the way discussed earlier, except that now a low-frequency cutoff \( \omega_c/p \) is introduced. This part of the gauge fluctuation is the same for either closed or open paths and our discussion in the previous section is applicable.
The dephasing due to static gauge fluctuations was first discussed by Aronov and coworkers\(^\text{13}\) for \(\delta\)-correlated flux fluctuations and their treatment can be easily extended to the more general case presented here. It yields a dephasing factor \(e^{-W_s}\), where

\[
W_s = \frac{1}{2} \left\langle \left( \oint a \cdot dl \right)^2 \right\rangle = \frac{p^2}{2} \int d^2r_1 \int d^2r_2 \langle h(r_1)h(r_2) \rangle , \tag{3.3}
\]

with \(\langle |h(q,\omega)|^2 \rangle = q^2 \langle |a(q,\omega)|^2 \rangle\). (The line integration is performed over a cyclotron trajectory of winding number \(p\) and the surface integrations encompass a circle of radius \(R_c = \frac{v_F}{\omega_c}\).) Note that the \(q^2\) factor gives additional convergence at small \(q\), making all integrals finite. For the Coulomb case we find that (see Appendix B)

\[
W_s = \frac{\pi p^2 T}{mC_1 \omega_c} \ln \left( \frac{E_F}{p\omega_c} \right) \tag{3.4}
\]

for \(E_F \gg \omega_c\).

For the dynamical part, we proceed as in Sec. II, the only new feature being that the \(\omega_c\) cutoff introduces a new time scale \(t_{\omega_c} \equiv (1/v_F)\sqrt{C_1/\omega_c}\). We find that \(\ddot{F}_1(t)\) decreases as \(t^{-3}\) for \(t \gg t_{\omega_c}\) (see Appendix A). This decrease is fast enough so that \(F_1(t)\) is now linear in \(t\) for \(t \gg t_{\omega_c}\). From now on we shall restrict ourselves to \(T \gg \omega_c\), so that only the \(p=1\) orbit is important. We are interested in evaluating \(F_1(t)\) at \(t = 2\pi/\omega_c\), which is the linear regime. We conclude that the spectrum for \(\omega > \omega_c\) contributes to a dephasing factor \(e^{-W_d}\), where

\[
W_d = \frac{\pi T}{mC_1 \omega_c} \ln \left( \frac{T}{\omega_c} \right) . \tag{3.5}
\]

Note that \(W_d\) is smaller than \(W_s\) if \(T < E_F\).

For short-range repulsion \((\eta = 2)\) we can carry out a similar analysis. We find that

\[
F_2(t) \simeq \begin{cases} \frac{v_F T^2}{12mC_2} \ln \left( \frac{T}{\omega_c} \right) , & t \ll \frac{1}{v_F} \left( \frac{C_2}{T} \right)^{1/3} , \\ \frac{v_F T^2}{12mC_2} \ln \left( \frac{C_2}{\omega_c T^{1/3}} \right) , & \frac{1}{v_F} \left( \frac{C_2}{T} \right)^{1/3} \ll t \ll \frac{1}{v_F} \left( \frac{C_2}{\omega_c} \right)^{1/3} , \\ \frac{2Tt}{\sqrt{3\pi mC_2^{2/3} \omega_c^{1/3}}} , & t \gg \frac{1}{v_F} \left( \frac{C_2}{\omega_c} \right)^{1/3} . \end{cases} \tag{3.6}
\]

Consequently, the spectrum for \(\omega > \omega_c\) contributes a dephasing factor with argument

\[
W_d = \frac{4\pi T}{\sqrt{3mC_2^{2/3} \omega_c^{1/3}}} . \tag{3.7}
\]

This is to be compared with

\[
W_s = \frac{\pi^2 v_F T}{2mC_2 \omega_c^2} , \tag{3.8}
\]

a result first obtained by Mirlin et al.\(^\text{16}\) We note that the introduction of an upper cutoff \(\omega_c\) in the \(\omega\) integral did not change the result for \(W_s\). Comparing Eq. (3.8) with (3.7) we
conclude that for short-range interactions, quasi-static fluctuations dominate the dephasing, so that $W_d$ can be neglected.

We note in passing that in carrying out the integration, we find that the factor $\cos(\omega t)$ in Eq. (2.7) can be set to unity for all values of $t$. This means that our result would be the same if we had treated the $a(r,t)$ field as static from the beginning, and written the phase factor as $\int a(r) \cdot dl$. This shows that the suppression factor is purely geometrical, depending only on the orbit circumference and not on the transit time. Thus any renormalization of the velocity of the particle will not affect our estimate. Furthermore, in the Coulomb case ($\eta = 1$), if we do not introduce the separation into static and dynamical part, and treat the gauge field as purely static, we see (from Appendix B) that the suppression factor is correctly given as $W_s + W_d$. Alternatively, we could also treat all fluctuations as “dynamical”, using the Gorkov approximation. The suppression factor would then be given by $F_1(t = 2\pi/\omega_c)$, where $F_1$ is given by Eq. (2.9). We can see that the result $W_s + W_d$ is correctly reproduced.

However, for the general $\eta$, it is necessary to separate the spectrum of the gauge fluctuations into high and low frequency components. Nevertheless, the final answer is that the dominant dephasing factor is correctly given by the static approximation evaluated in terms of the flux fluctuations, as was done in Refs. 13 and 16. Thus, the results of this section can be viewed as a justification of this intuitively appealing approach. With the understanding we have gained, we can now proceed to include the effect of mass renormalization, where dynamical gauge fluctuations are essential.

IV. MASS RENORMALIZATION

The semiclassical method yields a dephasing time for the composite fermion, but it cannot address the issue of the renormalization of the mass (or the energy gap) due to virtual dynamical gauge fluctuations. In the case of SdH oscillations, thermal broadening of the Landau levels plays an important role and it is necessary to include mass renormalization on the same footing as dephasing. To do this we return to a many-body diagrammatic treatment. For the Coulomb case, the work of Stern and Halperin suggests that Fermi liquid theory remains valid. We shall therefore use the standard treatment, such as that given by Engelsberg and Simpson for the dHvA magnetization in the presence of a strong electron-phonon coupling. From semiclassical arguments we expect similar density-of-state oscillations to appear in the SdH effect as well. For spinless particles moving in two dimensions, the oscillatory part of the dHvA magnetization is given by

$$\frac{M_{osc}(B)}{\Omega} = \frac{m E_F}{\pi B} \Re \left\{ \sum_{k=1}^{\infty} (-1)^k e^{2\pi i k E_F/\omega_c} \int_{-\infty}^{\infty} d\varepsilon \ n_F(\varepsilon) \ e^{(2\pi i k/\omega_c)(\varepsilon - \Sigma'(\varepsilon) - i\Sigma''(\varepsilon))} \right\}$$

$$+ \frac{2mT E_F}{B} \sum_{k=1}^{\infty} (-1)^k \sin \left( \frac{2\pi k E_F}{\omega_c} \right) \sum_{n=0}^{\infty} e^{-2\pi k/\omega_c \omega_n} \cos(2\pi k/\omega_c \omega_n),$$

where $\Omega$ is the sample area and $\omega_n = \pi T(2n + 1)$, while $\Sigma(i\omega_n) = \Sigma'(i\omega_n) + i\Sigma''(i\omega_n)$. Equation (4.1) is suggestive of the semiclassical formulas of the previous section, with $2\pi k \Sigma''/\omega_c$ playing the role of the dephasing factor. In addition, we now have a modification of the energy due to $\Sigma'$. Equations (4.1) and (4.2) cannot be used directly because $\Sigma$ is divergent.
at finite $T$. We now adopt the same strategy as before and separate the frequency of the gauge fluctuations into $\omega < \omega_c$ and $\omega > \omega_c$. The $\omega < \omega_c$ part we treat as static, while the $\omega > \omega_c$ part is treated using Eq. (4.1) with a low-energy cutoff. Therefore, to lowest order in the gauge field, we use the following formula for the (retarded) self energy:

$$\Sigma(k, \varepsilon) = -\int \frac{d^2q}{(2\pi)^2} \int_{\omega_c}^{\omega_f} \frac{d\omega}{\pi} \frac{|k \times q|^2}{m^2} \text{Im} D_{11}(q, \omega) \left[1 + n_B(\omega) - n_F(\xi_{k+q}) \right] \left[\frac{1}{\varepsilon + i0^+ - \xi_{k+q} - \omega} \right] + \frac{n_B(\omega) + n_F(\xi_{k+q})}{\varepsilon + i0^+ - \xi_{k+q} + \omega}, \quad (4.3)$$

where $n_B(\varepsilon) = (e^{\varepsilon/T} - 1)^{-1}$ and $\xi_k = k^2/(2m) - E_F$.

For Coulomb interactions and $\varepsilon \ll T$, we find that

$$\Sigma'(k_F, \varepsilon) = -\frac{\varepsilon}{2\pi m C_1} \ln \left(\frac{E_F}{T}\right) \quad (4.4)$$

and

$$\Sigma''(k_F, \varepsilon) = -\frac{T}{2m C_1} \ln \left(\frac{T}{\omega_c}\right). \quad (4.5)$$

The contribution coming from $\Sigma''$ is identical to the semiclassical result given by Eq. (3.5). On the other hand, the real part $\Sigma'$ leads to a renormalization of the mass given by

$$m^* = m \left[1 + \frac{1}{2\pi m C_1} \ln \left(\frac{E_F}{T}\right)\right], \quad (4.6)$$

which, as stated before, cannot be obtained in a semiclassical treatment. Upon performing the $\varepsilon$ integration in Eq. (4.1) in the standard way, we find for $T > \omega_c$ that it is sufficient to keep the $k = 1$ and $n = 0$ term and the dHvA oscillations are suppressed by the factor

$$A_{\text{dynamic}} = \exp \left(-\frac{2\pi^2 T}{\omega_c^*}\right) \exp \left[-\frac{2\pi}{\omega_c} \Sigma''(T)\right], \quad (4.7)$$

where $\omega_c^* = eB/m^*$. The first term in Eq. (4.7) comes from the thermal smearing of the renormalized Landau levels, while the second term comes from the dephasing of the composite fermion. In metals this second term is usually ignored relatively to the first one because the electron scattering rate scales as $T^2$. Here the two terms are comparable. In fact, while $\omega_c^*$ and $\Sigma''$ separately have a $\ln T$ dependence, we find that this is cancelled when we combine the two factors together, yielding

$$A_{\text{dynamic}} = \exp \left\{ -\frac{2\pi^2 T}{\omega_c} \left[1 + \frac{1}{2\pi m C_1} \ln \left(\frac{E_F}{\omega_c}\right)\right]\right\}. \quad (4.8)$$

Remarkably, as the temperature is raised, the dephasing of the fermion is exactly balanced by the reduction of the mass enhancement. This cancellation of the $T$ dependence also occurs in the electron-phonon problem, as $T$ goes from below to above the Debye temperature, as was shown in Ref. 17 using Eq. (4.2). We can check that in our case Eq. (4.8) also follows from Eq. (4.2) because
\[ \Sigma(i\omega_n) = -\frac{iT}{2mC_1} \ln \left( \frac{E_F}{\omega_c} \right). \] (4.9)

Finally, to obtain the total suppression factor, we have to include the contribution from \( \omega < \omega_c \), which is given by \( e^{-W_s} \). For the case of long-range Coulomb interactions, combining all contributions, we find that the suppression of the amplitude of the conductivity oscillations is given by

\[ \delta\sigma^{\text{osc}}_{xx}(B) \propto \exp \left\{ -\frac{2\pi^2 T}{\omega_c} \left[ 1 + \frac{1}{\pi mC_1} \ln \left( \frac{E_F}{\omega_c} \right) \right] \right\}. \] (4.10)

This expression implies that it is possible to analyze the \( T \)-dependence of the SdH amplitude using the conventional theory, as was done in Ref. [18], and obtain a temperature-independent effective mass \( m_{\text{eff}} \) from the slope of \( \ln(\delta\sigma^{\text{osc}}_{xx}) \) versus \( T \). This effective mass is predicted to show a logarithmic divergence near \( B = 0 \), i.e.,

\[ m_{\text{eff}}(B) = m \left[ 1 + \frac{1}{\pi mC_1} \ln \left( \frac{E_F}{\omega_c} \right) \right]. \] (4.11)

Notice that the prefactor of the logarithm is twice that given by Eq. (1.8), which is the same as \( m^* \) given by Eq. (3.7) with \( T \) replaced by \( \omega_c \) as a cutoff. Thus one cannot identify \( m_{\text{eff}} \) with the \( m^* \) from Fermi liquid theory. The difference comes from the essential contribution to the dephasing due to the static component of the gauge fluctuation.

A similar analysis can be carried out for short-range interactions. Note, however, that the use of results obtained in Ref. [17] is much more questionable in this case, since Fermi liquid theory does not apply for the short-range interactions. We find that, close to the Fermi surface,

\[ \Sigma'(k_F, \varepsilon) \approx -\frac{\alpha\varepsilon}{mC_2^{2/3}T^{1/3}} \] (4.12)

and

\[ \Sigma''(k_F, \varepsilon) = -\frac{2T}{\sqrt{3mC_2^{2/3}\omega_c^{1/3}}}, \] (4.13)

where \( \alpha \) is a number of order \( 10^{-1} \). The contribution of the mass renormalization to the damping factor is now negligible in comparison with \( \Sigma''/\omega_c \). The latter accounts exactly for the dynamical term [Eq. (3.7)]. Hence the damping of the conductivity oscillations is dominated by the static term of Eq. (3.8). It leads to an effective mass of the form

\[ m_{\text{eff}}(B) = m\left[ 1 + \frac{(2mk_F^2)}{E_F/\omega_c} \right]. \]

**V. COMPARISON WITH EXPERIMENTS**

Several recent experiments [18, 19, 20, 22] have measured the oscillations in the longitudinal conductance near half-filling. The data were fitted using the conventional theory of SdH oscillations for non-interacting electrons, by allowing their effective mass \( m_{\text{eff}}(B) \) to depend
on magnetic field. With the exception of Ref. 19, these studies indicate that $m_{\text{eff}}(B)$ is enhanced as the magnetic field felt by the composite fermions $B = B_{\text{ext}} - B_{1/2}$ approaches zero.

In the case of Coulomb interactions ($\eta = 1$), our results are consistent with a logarithmically divergent effective mass. For $1 < \eta \leq 2$ we find that $m_{\text{eff}}(B)$ should diverge as $B^{1-\eta}$. This enhancement, however, is not as strong as that observed in the experiments. To quantify this, we have attempted to fit Eq. (4.11) to the high effective-field part of the data obtained by Du et al.21 Following Ref. 4, we have estimated the mean-field mass by assuming that it is related to the strength of the Coulomb interaction through $m^{-1} = 4\beta C_1$, where $C_1$ is given below Eq. (2.5) and $\beta$ is a numerical constant. Our fit [see Fig. 1] indicates that $\beta \approx 0.15$, yielding $m \approx 0.76m_0$, with $m_0$ being the free electron mass. The values are reasonably close to those obtained from numerical simulations of finite-size systems, such as those carried out recently by Morf and d’Ambrumenil.23 These authors found that $\beta \approx 0.20$, which implies $m \approx 0.55m_0$.

The reason for the discrepancy at small $B$ may be due to the fact that disorder has not been taken into account in our treatment. Close to half-filling, where the apparent increase in the effective mass is large, the activation gap approaches zero and it is likely that disorder effects are playing a dominant role.

### VI. SUMMARY AND CONCLUSIONS

In this work we have demonstrated how the divergence, due to gauge fluctuations, of the imaginary part of the fermion self energy is related to the non-exponential time dependence of the dephasing factor. We have calculated the dephasing factor for both Coulomb and short-range interactions by separating the spectrum of gauge fluctuations into static and dynamic parts. We have considered the effect of dephasing and mass renormalization on the oscillating component of the density of states in a magnetic field by relating our approach to that of Engelsberg and Simpson for the case of electron-phonon interactions. Our calculated oscillations in the density of states are compared to the observed Shubnikov-de Haas oscillations in order to extract an effective mass of the composite fermions. While we can account for part of the apparent increase in the mass as the $\nu = 1/2$ state is approached, we are not able to explain the strong divergence observed in the immediate vicinity of the $\nu = 1/2$ state. This may be due to the effect of disorder, which is not included in the present description.

### VII. ACKNOWLEDGEMENTS

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In this appendix we derive Eq. (2.9) in detail and point out the important steps in the derivation of Eq. (3.6). First we notice that upon differentiating twice with respect to time, Eq. (2.6) becomes

$$\ddot{F}_\eta(t) = \int_0^\infty \frac{d^2 q}{(2\pi)^2} \int_0^\infty \frac{d\omega}{\pi} |v_F \times \dot{q}|^2 \coth \left( \frac{\omega}{2T} \right) \text{Im}D_{11}(q, \omega) \cos(v_F \cdot q t) \cos(\omega t),$$  \hspace{1cm} (A1)

since terms which are odd in $q$ do not contribute. Introducing the gauge field propagator from Eq. (2.5), we then have

$$\ddot{F}_\eta(t) = v_F^2 \int_0^\infty \frac{d\omega}{\pi} \coth \left( \frac{\omega}{2T} \right) \cos(\omega t) \int_0^{2k_F} \frac{dq}{q^2} \cos(qv_F t \cos \varphi)$$

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} \sin^2 \varphi \int_0^{2k_F} \frac{dq}{q^2} \cos(qv_F t \cos \varphi)\frac{\omega^2 + C_\eta^2 q^2 (\eta + 1)}{\omega^2},$$  \hspace{1cm} (A2)

where $\varphi$ is the angle between $v_F$ and $q$. To proceed, we notice that the $q$ integration can be carried out by contour in the complex plane when $\omega \ll C_\eta k_F^{\eta + 1}$, which is satisfied provided that $\omega \ll E_F$. Specializing our results to $\eta = 1$ and 2, we need to calculate two integrals, namely,

$$\int_0^\infty dz \frac{z^2 \cos(zx)}{z^4 + 1} = 2\pi e^{-x/\sqrt{2}} \cos \left( \frac{x}{\sqrt{2}} + \frac{\pi}{4} \right),$$  \hspace{1cm} (A3)

and

$$\int_0^\infty dz \frac{z^2 \cos(zx)}{z^6 + 1} = \frac{4\pi}{3} \left[ e^{-x/2} \cos \left( \frac{x\sqrt{3}}{2} \right) - \frac{1}{2} e^{-x} \right],$$  \hspace{1cm} (A4)

with $x > 0$. Introducing Eq. (A3) into Eq. (A2), we arrive at Eq. (2.7). Analogously, for the short-range case we find that

$$\ddot{F}_2(t) = \frac{v_F^2}{3k_F C_2} \int_0^\infty d\omega \coth \left( \frac{\omega}{2T} \right) \cos(\omega t) g(v_F t (\omega/C_2)^{1/3}),$$  \hspace{1cm} (A5)

where

$$g(x) = \frac{2}{\pi} \int_0^{\pi/2} d\theta \cos^2 \theta \exp \left( -x \sin \theta \right) \left[ \cos \left( \frac{x\sqrt{3}}{2} \sin \theta \right) - \frac{1}{2} \exp \left( -x \sin \theta \right) \right],$$  \hspace{1cm} (A6)

with $\theta = \pi - \varphi$. The functions $f(x)$ and $g(x)$ cannot be represented in terms of elementary functions; however, it is not difficult to find their asymptotic properties:

$$f(x) \rightarrow \begin{cases} 2^{-3/2} - 2x/(3\pi) & x \rightarrow 0 \\ 1/(\pi \sqrt{2x}^3) & x \rightarrow \infty \end{cases},$$  \hspace{1cm} (A7)

and

$$g(x) \rightarrow \begin{cases} 1/4 - x^2/(4\pi) & x \rightarrow 0 \\ 3/(\pi x^3) & x \rightarrow \infty \end{cases},$$  \hspace{1cm} (A8)
Moreover, it is readily shown from Eq. (2.8) that

\[ \int_{0}^{\infty} dx \, f(x) = 1/2 . \]  

(A9)

These properties are important because they allow us to obtain \( F_{1,2}(t) \) in certain regimes.

We now discuss the Coulomb case. We are interested in the thermal contribution, which means that we approximate \( \coth(\omega/2T) \) by \( 2T/\omega \). Then, for \( t << t_{T} \), the integral in Eq. (2.7) is dominated by \( \sqrt{\omega} \) and we can use the first line in Eq. (A7) to get

\[ \ddot{F}_{1}(t) \approx \frac{1}{2 E_{F} t_{T}^{3}} \left[ 1 - \frac{2^{3/2}t}{3\pi t_{T}} \right] . \]  

(A10)

For \( t >> t_{T} \) the convergence in Eq. (2.7) at high frequencies is due to the fast decay of \( f(x) \), allowing us to use Eq. (A9) and obtain

\[ \ddot{F}_{1}(t) \approx \frac{1}{2 E_{F} t_{T}^{3}t} . \]  

(A11)

When we introduce a lower cutoff in frequency, like \( \omega_{c} \ll T \), there is a third regime, namely \( t >> t_{\omega_{c}} \), where we can replace \( f(x) \) by its large-\( x \) asymptotic form. We then get

\[ \ddot{F}_{1}(t) \approx \frac{1}{2^{5/2} \pi E_{F} \omega_{c} t^{3}} . \]  

(A12)

Notice that in all regimes it is legitimate to neglect the frequency dependence coming from the \( \cos(\omega t) \) factor. This means that the explicit \( t \) dependence of \( a(r, t) \) given by Eq. (2.2) can be neglected. Integrating Eqs. (A10), (A11), and (A12) twice over \( t \) and recalling that \( F_{1}(t) \sim t^{2} \) for \( t \to 0 \), one finds Eq. (2.9), as well as Eq. (B4). The calculations are analogous for the short-range case, with the exception that in all three regimes a lower cutoff in frequency, \( \omega_{c} \), is required to achieve convergence.

**APPENDIX B**

Equation (3.4) can be derived in the following way. We first notice that, by going to Fourier space, we can rewrite Eq. (3.3) in terms of separate frequency and spatial integrations,

\[ W_{s} = p^{2} \int \frac{d^{2}q}{(2\pi)^{2}} \int_{0}^{\omega_{c}/p} d\omega \left[ \int_{R_{c}} d^{2}r \, e^{i\mathbf{q} \cdot \mathbf{r}} \right]^{2} \langle |h(\mathbf{q}, \omega)|^{2} \rangle , \]  

(B1)

where we have only kept the static part (\( \omega < \omega_{c}/p \)) of the gauge fluctuation spectrum. The spatial integration gives \( 2\pi R_{c} J_{1}(qR_{c})/q \), where \( R_{c} = v_{F}/\omega_{c} \) is the cyclotron radius and \( J_{1}(z) \) is the first-order Bessel function. Recalling that \( \langle |h(\mathbf{q}, \omega)|^{2} \rangle = q^{2} \langle |a(\mathbf{q}, \omega)|^{2} \rangle \), we can carry out the integration over frequency,

\[ \int_{0}^{\omega_{c}/p} d\omega \, \coth \left( \frac{\omega}{2T} \right) \frac{2\pi q^{3} \omega/k_{F}}{\omega^{2} + C_{1}^{2} q^{4}} = \frac{\pi T q}{k_{F} C_{1}} . \]  

(B2)
for \( q \ll \sqrt{\omega_c/(pC_1)} \). Therefore, we have that

\[
W_s = \frac{2\pi^2 p^2 T R_c^2}{k_F C_1} \int_0^{\sqrt{\omega_c/(pC_1)}} dq \ [J_1(qR_c)]^2 \\
\simeq \frac{\pi p^2 T}{mC_1 \omega_c} \ln \left( \frac{E_F}{p\omega_c} \right)
\]

(B3)

for \( E_F \gg \omega_c \).

Notice that if we remove the upper limit of the frequency integral in Eq. (B1) and treat the gauge field as purely static, we would still obtain Eq. (B2), but with the modified restriction \( q \ll \sqrt{T/C_1} \). As a result, the momentum integration would lead instead to \( (p = 1) \)

\[
W_s^{\text{pure}} \simeq \frac{\pi T}{mC_1 \omega_c} \ln \left( \frac{T E_F}{\omega_c^2} \right)
\]

(B4)

which is equal to \( W_s + W_d \).
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FIG. 1. The effective mass of composite fermions as extracted from Shubnikov-de Haas measurements near the $\nu = 1/2$ state. The circles indicate data from Ref. [21]. The dashed line is a fit of Eq. (4.10) to the $|B| \geq 2T$ part of the data, resulting in $m = 0.76m_0$. The dotted line corresponds to the same expression evaluated with $m = 0.55m_0$. 