UNRESOLVED ISSUES IN THE SEARCH FOR ETA-MESIC NUCLEI

N.G. Kelkar

Departamento de Física, Universidad de los Andes
Cra 1E, 18A-10, Bogota, Colombia

(Received December 8, 2014)

Even if the theoretical definition of an unstable state is straightforward, its experimental identification often depends on the method used in the analysis and extraction of data. A good example is the case of eta-mesic nuclei where strong hints of their existence led to about three decades of extensive theoretical and experimental searches. Considering the still undecided status of these states and the limitations in the understanding of the eta–nucleon as well as the eta–nucleus interaction, the present article tries to look back at some unresolved problems in the production mechanism and final state interaction of the eta mesons and nuclei. An unconventional perspective which provides a physical insight into the nature of the eta–nucleus interaction is also presented using quantum time concepts.

DOI:10.5506/APhysPolB.46.113
PACS numbers: 21.85.+d, 13.60.Le, 13.75.–n, 03.65.Xp

1. Introduction

The strong nuclear interaction which is usually understood as the interaction between two nucleons is by now quite well known. However, the interaction is mediated by mesons which are also strongly interacting objects and it makes sense to have a good understanding of the meson–nucleon and especially the meson–nucleus interaction too. Hadron–nuclear interactions, in general, can be probed either via scattering experiments or the study of bound states. It has been possible to investigate the pion–nucleus interaction through elastic scattering experiments, but the same is not true of the heavier eta meson due to its being extremely short lived. Due to the non-availability of eta beams, the eta–nucleon ($\eta-N$) interaction can only be deduced via the eta–nucleon or eta–nucleus interaction in the final state. Fortunately, its interaction with the nucleon in the $s$-wave (which proceeds through the formation of an $N^*(1535)$ resonance) is attractive and leads to
the possibility of forming and studying unstable bound states of eta mesons and nuclei [1] (see also [2–4] for an extensive list of references). After about 25 years of investigations in this field, a lot of progress has been made and some evidence for the existence of such states exists, however, there still does not exist a general agreement and a final word on the strength of the eta–nucleon and eta–nucleus interaction.

The eta meson being heavy, with a mass around 547 MeV, reactions producing eta mesons on nuclei involve large momentum transfers to the nuclei involved. This fact gives rise to the possibility of reaction mechanisms [5] which go beyond the one step process where a single nucleon gets excited to the \( N^*_1 \) resonance which eventually decays into an \( \eta \) and a nucleon [6]. The \( \eta–N \) and, hence, the \( \eta–nucleus \) interaction can be deduced only from reactions where the \( \eta \) is produced in the final state. Such a deduction then depends on the theoretical model used for the reaction mechanism for production as well as the final state interaction of the \( \eta \) meson with the nucleus. A theoretical prediction for the existence of an eta-mesic unstable nuclear state which requires the strength of the \( \eta–N \) interaction as an input, thus depends indirectly on the correctness of the reaction mechanisms describing the production reactions. Besides this, the strong effects of the \( \eta–nucleus \) interaction are prominent near the threshold of the eta producing reactions where the off-shell rescattering of the eta and the nucleus becomes important. Methods based on the extraction of the eta–nucleus scattering length (which is further used to comment on the possible existence of eta-mesic nuclei) involving on-shell approximations can, indeed, lead to quite different conclusions as compared to few body calculations of the same. Here, an attempt to compare such discrepancies and look back at the deficiencies in explaining the meson production data will be made. The next section begins with a brief introduction to the methods of identifying unstable states focusing especially on the application of a quantum time concept introduced by Wigner and Eisenbud [7], and modified recently for locating eta-mesic nuclear states [8]. The sections which follow, discuss the experimental searches, the limitations of some of the theoretical approaches and possibilities for the future.

2. Identification of unstable mesic states

Baryon resonances are usually identified by performing a partial wave analysis of the elastic meson baryon scattering data and obtaining the energy dependent amplitude (or transition matrix) by fitting cross section data. Resonances are then determined by locating the poles of the S-matrix on the unphysical sheet and studying the Argand diagrams of this complex transition matrix. With the eta meson being extremely short-lived (\( \tau \sim 10^{-18} \) s),
the possibility of performing such analyses with elastic $\eta$–N or eta–nucleus elastic scattering data does not exist. Hence, the next best thing to do in a theoretical search is to use the information on the $\eta$–N interaction obtained from models fitting the $\eta$-meson producing reaction data and use it as an input to theoretically construct a complex eta–nucleus elastic scattering matrix. The poles of this matrix in the complex energy/momentum plane can then be used to infer on the existence of unstable states. The complex energy $E$ is related to the complex momentum $p$ as $E = p^2/2\mu$, where $\mu$ is the $\eta$–nucleus reduced mass. The physical and unphysical sheets correspond to $\text{Im } p > 0$ and $< 0$ respectively. Denoting $\text{Re } p = p_R$ and $\text{Im } p = p_I$, 

$$E = \frac{1}{2\mu} \left( p_R^2 - p_I^2 + 2i p_R p_I \right)$$

and hence, $\text{Re } E = (p_R^2 - p_I^2)/2\mu$ and $\text{Im } E = p_R p_I/\mu$. For the existence of a bound (or quasibound) state, the requirement is $\text{Re } E < 0$. This means that for a quasibound state to exist, $p_R^2 < p_I^2$ and for $p_I > 0$, the pole, of the type $-|E| - i\Gamma/2$ should lie in the second quadrant (see figure 5 of Ref. [4]) of the complex $p$ plane above the diagonal which divides this quadrant into two. As $p_R \to 0$, the pole lies on the positive imaginary $p$ axis and corresponds to a bound state. The virtual state pole lies on the negative imaginary $p$ axis. Resonances are defined as the states on the unphysical sheet ($\text{Im } p < 0$) with $\text{Re } E > 0$, i.e. a pole of the type $|E| + i\Gamma/2$. Quasivirtual states lie on the unphysical sheet too, but with a pole like $-|E| + i\Gamma/2$, they lead to an exponential growth and not decay, and hence are not physical unstable states.

### 2.1. Wigner’s time delay and the dwell time method

The $\eta$–nucleus transition matrix can also be used to evaluate the so-called time delay (a concept initially introduced by Wigner and Eisenbud [7], and elaborated by Smith and many others later [9, 10]) which is large and positive with a typical Lorentzian (Breit–Wigner type) shape at positive and negative energies for resonances and quasibound states respectively. Virtual and quasivirtual states lead to negative delay times [11]. A variation of this concept, namely, the dwell time delay which is useful for identifying unstable states near threshold was introduced in [8]. The relation between the dwell and phase time delay, $\tau_D(E)$ and $\tau_\phi(E)$ respectively, in scattering was found to be

$$\tau_D(E) = \tau_\phi(E) + \hbar \mu [t_R/\pi] \frac{dk}{dE},$$

where the phase time delay is given in terms of the S-matrix as $\tau_\phi(E) = \text{Re}[-i\hbar (S^{-1}dS/dE)]$, with, $S = 1 - i\mu k(t_R + it_I)/\pi$. 


Since time delay is the difference in the time spent by the scattering objects in a given region with and without interaction, an attractive interaction would be expected to “delay” the process, whereas a repulsive one would cause the time spent with interaction to be smaller and hence cause the difference to be negative. Thus a large positive time delay signals the existence of an unstable bound (quasibound) state or a resonance which is formed, propagates and decays delaying the process.

2.2. $\eta^3\text{He}$ and $\eta^4\text{He}$ mesic states

A time delay analysis in [11] led the authors to conclude that only small $\eta-N$ scattering lengths favour the formation of light quasibound eta-mesic nuclei. A large scattering length such as $a_{\eta N} = (0.88, 0.41)$ fm which, in principle, is the value which reproduces the $pd \rightarrow ^3\text{He} \eta$ reaction data well [12] leads to a (positive energy) $\eta^3\text{He}$ resonance and a strong repulsion near the threshold for $\eta^4\text{He}$ states (in agreement with the absence of quasibound states as found in [13]). A quasibound state such as that claimed in [14] and later withdrawn [15] can be formed only with a small $\eta-N$ scattering length. In fact, the authors [8, 11] noticed a movement of the poles from a relatively deep quasibound state at $(-5 - i8)$ MeV to one near threshold at $(0 - i1.95)$ MeV and then a resonance at $(0.5 + i0.65)$ MeV, for increasing values of the scattering length, $a_{\eta N} = (0.28 + i0.19)$ fm, $(0.51 + i0.26)$ fm and $(0.88 + i0.41)$ fm, respectively.

One of the hints for the possible existence of eta-mesic $^3\text{He}$ was the rapid increase of the magnitude of the s-wave amplitude in the $pd \rightarrow ^3\text{He} \eta$ reaction near threshold. In [16], the authors showed that the phase of the s-wave amplitude varies strongly near threshold too. Considering contributions of the s- and p-waves, the authors found that the angular distribution of this reaction is sensitive to the s–p interference. They associated the sharp rise in the total cross sections with the existence of a pole corresponding to either a quasibound or a quasivirtual state very close to threshold ($Q_0 = (-0.30 \pm 0.15_{\text{stat}} \pm 0.04_{\text{syst}}) \pm i(0.21 \pm 0.29_{\text{stat}} \pm 0.06_{\text{syst}})$ MeV).

In a much earlier work [17], within a Watson-like multiple scattering formalism, the authors tried to simultaneously analyse the $pd \rightarrow ^3\text{He} \eta$ as well as the $dd \rightarrow ^4\text{He} \eta$ data in order to investigate the existence of etamesic helium nuclei. With the $\eta-N$ interaction not being well-known, they studied the existence of such states for various values of the $\eta-N$ scattering length. The authors concluded that though $\eta^3\text{He}$ quasibound states would be less likely to exist, $\eta^4\text{He}$ states could possibly exist. In passing, we also mention Ref. [18], where, based on a scattering length approximation for the final state interaction and with a simultaneous fit to the $dd \rightarrow ^4\text{He} \eta$ and $pd \rightarrow ^3\text{He} \eta$ data, the authors determined $a_{\eta^4\text{He}} \simeq (-2.3 + i3.2)$ fm,
Unresolved Issues in the Search for Eta-mesic Nuclei

and $a_{\eta^4\text{He}} \simeq (-2.2 + i1.1)$ fm indicating the existence of both $\eta^3\text{He}$ and $\eta^4\text{He}$ states. Conclusions about the existence of eta-mesic states, based on scattering length approaches for the final state should, however, be taken with some caution as will be discussed in the next section. Some hope of shedding light on the existence of these light nuclear states lies in the ongoing efforts made by the WASA Collaboration at COSY [19–23].

3. Analyses of $\eta$ producing reactions

The basic ingredients of any theoretical analysis of an $\eta$ meson producing reaction on nuclei are: (i) a model for the reaction mechanism to produce an $\eta$ in the final state, (ii) a framework to incorporate the $\eta$–nucleus final state interaction (FSI), and (iii) information on the interaction of the $\eta$ meson with the nucleons within the nucleus. The large momentum transfer to the nucleus involved in these reactions rules out the possibility of using a simple model, where the $\eta$ is produced directly from the decay of the $N^*$ resonance in a one-step process. Since data on $\eta$ production have confirmed the existence of strong FSI effects near threshold, it is also important to notice that the $\eta$ meson can, in principle, be produced off-shell and brought on-shell (after several rescatterings) by its FSI with the nucleus. Below, we try to analyse the deficiencies in literature in the treatment of these three main ingredients in the search of eta mesic nuclei.

3.1. Two-step models of $\eta$–helium production

With the eta mass being around 547 MeV, the momentum transferred to the nucleus is large. Laget and LeColley noticed the need for a three-body mechanism where the momentum is shared by the nucleons in the nucleus. Indeed, they found that [5] a one-body mechanism underestimated the cross sections by 2–3 orders of magnitude. Faeldt and Wilkin [24] found good agreement with the threshold data on the $pd \to ^3\text{He} \eta$ reaction using the so-called two step model where one nucleon in the deuteron first interacts with the incident proton to produce an intermediate pion which eventually produces the eta meson via the $\pi N \to \eta N$ reaction. The transition amplitude for the two-step process can be then written as [12]

$$\langle |T_{pd \to ^3\text{He} \eta}| \rangle = i \int \frac{dp_1}{(2\pi)^3} \frac{dp_2}{(2\pi)^3} \sum_{int m's} \langle pn|d \rangle \left\langle \pi d | T_{pp \to \pi d} | pp \right\rangle \times \frac{1}{k_\pi^2 - m_\pi^2 + i\epsilon} \left\langle \eta p | T_{\pi N \to \eta p} | \pi N \right\rangle \langle ^3\text{He}|pd \rangle , \quad (2)$$

where the sum runs over the spin projections of the intermediate off-shell particles and $k_\pi$ is the four momentum of the intermediate pion. Motivated
by the success of this model, in [12] the final state $\eta^{-3}$He interaction was incorporated using few body equations which took into account the off-shell rescattering of the $\eta$ from the nucleus. Thus,

$$\langle \Psi_f \left( k_{\eta}^e \right) | T_{pd \rightarrow ^3\text{He} \eta} | \Psi_i \left( k_p^e \right) \rangle = \langle k_{\eta}^e; m_3 | T_{pd \rightarrow ^3\text{He} \eta} | k_p^e; m_1 m_2 \rangle$$

$$+ \sum_{m_3'} \int \frac{d q}{(2\pi)^3} \frac{\langle k_{\eta}^e; m_3 | T_{\eta ^3\text{He} | \vec{q}; m_3'} \rangle}{E(k_{\eta}) - E(q) + i\epsilon} \langle \vec{q}; m_3' | T_{pd \rightarrow ^3\text{He} \eta} | k_p^e; m_1 m_2 \rangle . \quad (3)$$

The intermediate $\pi N \rightarrow \eta N$ process was described by a coupled channel off-shell $t$-matrix. Though such a model could reproduce the threshold total cross sections as well as the isotropic angular distributions, it was later found [25] to produce backward peaked angular distributions at high energies in complete disagreement with data. Indeed, backward peaked angular distributions seem to be a common feature of high momentum transfer reactions (like for example, $pd \rightarrow ^3\text{He} \omega$, $pd \rightarrow ^3\text{H} \Lambda K^+$) described within a two-step model [26]. Improved calculations including coupled channel effects arising, for example, from the $pd \rightarrow ^3\text{He} \pi^0$ reaction, the interaction of the off-shell intermediate particles with the inclusion of higher partial waves could lead to a better understanding of the problem. The latter is supported by [16] where the authors found the angular distributions sensitive to the $s-p$ wave interference.

### 3.2. On-shell approximations

In a proper description of the FSI between the $\eta$ and the nucleus, one must consider the fact that the $\eta$ meson can also be produced off the mass shell and eventually brought on-shell due to its interaction with the nucleus. The half off-shell transition matrix can be written using few body equations. In [12, 27, 28], the off-shell FSI included using few body equations was found to be important for the $pd \rightarrow ^3\text{He} \eta$, $p^6\text{Li} \rightarrow ^7\text{Be} \eta$ and $pn \rightarrow d \eta$ reactions. The eta–nucleus scattering lengths deduced from such off-shell $t$-matrices are also found to be quite different [4] from those using a rather simple on-shell approximation with the FSI amplitude given by

$$f \approx \frac{f_B}{1 - ika_{\eta N}} \quad (4)$$

with $f_B$ fitted to reproduce the right magnitude of the data. It is surely tempting to use a simple approximation as the above expression to extract eta–nucleus scattering lengths from eta production data and comment on the existence of eta-mesic states based on the magnitude and sign of the scattering length. However, such conclusions could, indeed, be way away from reality.
3.3. The $\eta$–nucleon interaction

We started off with the aim of understanding the eta meson–nucleon interaction and having obtained indications of its attractive nature, we set out on a search of its exotic nuclear states, namely, the eta-mesic nuclei. However, extensive investigations of hadron and photon induced $\eta$ producing reactions over the past decades have left us with a less clear understanding of the strength of the $\eta$–nucleon interaction than what we started with. The very first prediction [29] was that of a small scattering length, $a_{\eta N}$, of $0.28 + i0.19$ fm. Interestingly, a more recent calculation [30] involving nine baryon resonances and the $\pi N$, $\eta N$, $\rho N$ and $\sigma N$ coupled channels finds $a_{\eta N} = 0.3 + i0.18$ fm in close agreement to the very first prediction. The latest coupled channel analysis [31] of the $\pi N \rightarrow \pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$ reactions, however, yielded $a_{\eta N} = 0.49 + i0.24$ fm and $0.55 + i0.24$ fm from different fits. Based on the theoretical and phenomenological predictions in literature, one finds that the $\eta N$ scattering length varies over a wide range of values given by, $0.18 \leq \text{Re} a_{\eta N} \leq 1.03$ fm and $0.16 \leq \text{Im} a_{\eta N} \leq 0.49$ fm.

To summarize briefly, one can say that a crucial step forward for eta-mesic physics lies in determining more accurately, the strength of the $\eta N$ interaction, from data on elementary $\eta$–meson producing reactions. This would help in improving the theoretical predictions for eta-mesic states and focussing the experimental searches on to specific nuclei and reactions.

REFERENCES

[1] Q. Haider, L.C. Liu, Phys. Rev. C66, 045208 (2002).
[2] B. Krusche, C. Wilkin, Prog. Part. Nucl. Phys. 80, 43 (2015).
[3] H. Machner, arXiv:1410.6023 (nucl-th], J. Phys. G (2015, in press).
[4] N.G. Kelkar, K.P. Khemchandani, N.J. Upadhyay, B.K. Jain, Rep. Prog. Phys. 76, 066301 (2013).
[5] J.M. Laget, J.F. Lecolley, Phys. Rev. Lett. 61, 2069 (1988).
[6] A.B. Santra, B.K. Jain, Phys. Rev. C64, 025201 (2001).
[7] E.P. Wigner, Phys. Rev. 98, 145 (1955).
[8] N.G. Kelkar, Phys. Rev. Lett. 99, 210403 (2007).
[9] F.T. Smith, Phys. Rev. 118, 349 (1960).
[10] N.G. Kelkar, M. Nowakowski, Phys. Rev. A78, 012709 (2008).
[11] N.G. Kelkar, K.P. Khemchandani, B.K. Jain, J. Phys. G 32, 1157 (2006).
[12] K.P. Khemchandani, N.G. Kelkar, B.K. Jain, Nucl. Phys. A708, 312 (2002).
[13] W. Krzemien, P. Moskal, J. Smyrski, M. Skurzok, Eur. Phys. J. Web Conf. 66, 09009 (2014).
[14] M. Pfeiffer et al., Phys. Rev. Lett. 92, 252001 (2004).
[15] F. Pheron et al., *Phys. Lett.* **B709**, 21 (2012).
[16] C. Wilkin et al., *Phys. Lett.* **B654**, 92 (2007).
[17] S. Wycech, A.M. Green, J.A. Niskanen, *Phys. Rev.* **C52**, 544 (1995).
[18] N. Willis et al., *Phys. Lett.* **B406**, 14 (1997).
[19] M. Skurzok, W. Krzemien, P. Moskal, arXiv:1408.0893 [nucl-ex].
[20] W. Krzemien, P. Moskal, M. Skurzok, *Few Body Syst.* **55**, 795 (2014).
[21] P. Adlarson et al., *Eur. Phys. J.* **A50**, 100 (2014).
[22] M. Hodana et al., *Acta Phys. Pol. B* **45**, 697 (2014).
[23] P. Moskal, *Few Body Syst.* **55**, 667 (2014).
[24] G. Faeldt, C. Wilkin, *Nucl. Phys.* **A587**, 769 (1995).
[25] K.P. Khemchandani, N.G. Kelkar, B.K. Jain, *Phys. Rev.* **C68**, 064610 (2003).
[26] V.I. Komarov, A.V. Lado, Y.N. Uzikov, *J. Phys. G* **21**, L69 (1995); L.A. Kondratyuk, Yu.N. Uzikov, *Phys. Atom. Nucl.* **60**, 468 (1997); *Acta Phys. Pol. B* **27**, 2977 (1996); K. Schöning et al., [CELSIUS/WASA Collaboration], *Phys. Rev.* **C79**, 044002 (2009).
[27] N.J. Upadhyay, N.G. Kelkar, B.K. Jain, *Nucl. Phys.* **A824**, 70 (2009).
[28] H. Garcilazo, M.T. Peña, *Eur. Phys. J.* **A38**, 209 (2008).
[29] R.S. Bhalerao, L.C. Liu, *Phys. Rev. Lett.* **54**, 865 (1985).
[30] J. Durand et al., *Phys. Rev.* **C78**, 025204 (2008).
[31] D. Rönchen et al., *Eur. Phys. J.* **A49**, 44 (2013).