Radion Mediated Supersymmetry Breaking as a Scherk-Schwarz Theory

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(Dated: November 2, 2018)

Recently, it has been demonstrated that radion mediated supersymmetry breaking gives the same spectrum as Scherk-Schwarz supersymmetry breaking, and can be interpreted as a dynamical realization of it. We make this connection explicit by exhibiting the direct transformation from one theory to the other. We then use the extreme UV softness of Scherk-Schwarz theories to calculate the one-loop soft masses of matter fields. We do not find any cutoff sensitive “Kaluza-Klein mediated” contributions.

I. INTRODUCTION

One of the most elegant solutions of the gauge hierarchy problem is supersymmetry, in which fields have partners with opposite statistics. However, superpartners have not been observed. If supersymmetry is realized in nature, it must be broken by some means.

To this end, many models have been put forward. Recently there has been great interest in Scherk-Schwarz supersymmetry breaking [8, 7]. In particular, by maintaining the manifest UV softness of Scherk-Schwarz theories, we do not find the contributions to soft scalar masses in radion mediation [7]. We use the extreme UV softness of Scherk-Schwarz theories to calculate the one-loop soft masses of matter fields. We do not find any cutoff sensitive “Kaluza-Klein mediated” contributions.

II. RADION MEDIATION AND A FIELD REDEFINITION

In this section we show, with a simple field redefinition, that a theory with a compact extra dimension in which supersymmetry is broken by the auxiliary component of the radion superfield is equivalent to a theory in which supersymmetry is broken by a non-trivial winding of some fields (i.e., the Scherk-Schwarz mechanism). We first concentrate on bulk gauge fields as they are relevant to the example in the section 3 and then we discuss

hypermultiplets. Throughout, we will closely follow the notation of [5, 4, 3, 6].

A. Vector Multiplets

We use four-dimensional \( N = 1 \) superfield notation for extra dimensional theories. This elegant tool was developed in [6] with the five-dimensional case generalized to included the radion superfield and curved backgrounds in [2]. We will be working in flat space with one dimension compactified on a circle with a radius described by the radius modulus \( R \) and parameterized by the coordinate \(- \pi \leq \varphi < \pi\). In addition, we impose an orbifold projection with the identification \( \varphi \to -\varphi \). We use the angular coordinate \( \varphi \) to emphasize that it has canonical dimension zero. Later, when we set the radion field to its vev we will work with the dimension \(-1\) coordinate \( y = \varphi R \).

To use the above described notation we incorporate \( R \) into a chiral superfield:

\[
T = R + iB_5 + \theta \Psi_R^5 + \theta^2 F_T,
\]

where \( B_5 \) is the fifth component of the graviphoton, \( \Psi_R^5 \) is the fifth component of the right-handed gravitino and \( F_T \) is the radion’s auxiliary component.

The minimal vector multiplet in five dimensions consists of a vector superfield \( V \) and a chiral superfield \( \chi \):

\[
V = -\theta \sigma^\mu \bar{\theta} A_\mu - i \partial^2 \theta \lambda_1 + i \theta^2 \bar{\theta} \lambda_1 + \frac{1}{2} \bar{\theta} \theta^2 D, \\
\chi = \frac{1}{\sqrt{2}} (\Sigma + i A_5) + \sqrt{2} \theta \lambda_2 + \theta^2 F_\chi.
\]

We can now write down the action of a five-dimensional Abelian gauge multiplet coupled to the radion [2]:

\[
S_5 = \int d^5x d\varphi \left[ \frac{1}{4g_5^2} \int d^2\theta TW^\alpha W_\alpha + \text{h.c.} \right] + \frac{2}{g_5^2} \int d^4\theta \left( \frac{1}{T + T^\dagger} \left( \partial_\varphi V - \frac{1}{\sqrt{2}} (\chi + \chi^\dagger) \right)^2 \right)
\]
Note, this action is invariant under the full five-dimensional gauge transformation \( V \to V + \Lambda + \Lambda^\dagger, \chi \to \chi + \sqrt{2} \partial \Lambda \) and can be shown to give the correct component-field action.

Now let us assume the radion has a non-zero auxiliary component such that \( (T) = R + \theta^2 F_T \). After eliminating the other auxiliary fields via their equations of motion, replacing the radion with its vacuum expectation value and rescaling the fields \( \Sigma \to R \Sigma, \lambda_2 \to -i R \lambda_2 \), and replacing the coordinate \( \varphi \) with \( y/R \), we have:

\[
S_5 = \frac{1}{g_5^2} \int d^2 y \sqrt{-g} \left[ -\frac{1}{2} \partial M \Sigma M \Sigma - \frac{1}{4} F_{MN} F^{MN} \right. \]
\[
- i \lambda_1 \sigma^m \partial_\mu \lambda_1 + \frac{1}{2} \lambda_1 \epsilon_{ij} \partial_\mu \lambda_j + \text{h.c.} \\
- \left( \frac{F_T}{4 R} \lambda_1 \lambda_2 + \frac{F_T^\dagger}{4 R} \lambda_2 \lambda_1 \right) + \text{h.c.} ,
\]

where \( \sigma^m = (1, \sigma), \ \sigma^m = (1, -\sigma) \). We use two-component spinor for simplicity of notation when we couple the theory to boundary fields below.

The action for the non-Abelian theory appears in \( [3] \) and will not be presented here. The arguments below are presented for the case of an Abelian theory and apply equally well to the non-Abelian case.

Now we show that a theory with supersymmetry breaking by a radion is equivalent to a theory where supersymmetry is broken explicitly by boundary conditions in the fifth dimension (i.e., the Scherk-Schwarz mechanism). This can be done simply by a field redefinition equivalent to an \( y \)-dependent \( \text{SU}(2)_R \) transformation.

We perform the following field redefinition on the action in \( [4] \) \( [11] \):

\[
\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \to e^{-i F_T y \sigma^2/2 R} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} ,
\]

where we take \( F_T \) to be real. This transformation is similar to the \( \text{SU}(2)_R \) component of the field redefinition in \( [12] \) (for generalizations, see \( [13] \)). Note that the only term in \( [4] \) which is not invariant under this transformation is the fifth component of the gaugino kinetic term. This term transforms as

\[
\frac{1}{2 g_5^2} \lambda_1 \epsilon_{ij} \partial_\mu \lambda_j \to \frac{1}{2 g_5^2} \lambda_1 \epsilon_{ij} \partial_\mu \lambda_j + \frac{F_T}{4 g_5^2 R} \lambda_1 \lambda_2 .
\]

Here it is chosen to completely remove the Majorana mass terms for the gaugino fields at the expense of giving them a non-trivial winding around the compact dimension. The winding is precisely the Scherk-Schwarz mechanism for breaking supersymmetry.

The above transformation can be generalized to complex \( F_T \) by replacing the phase as

\[
F_T \sigma^2 y/2 R \to F_T - \sigma y/2 R
\]

where \( F_T = \{ \text{Im} F_T, \text{Re} F_T, 0 \} \).

The winding of the gauginos effects the gaugino coupling to charged matter living at the boundary \( y = 0 \) and their identified points \( y = 2 \pi R n \). The coupling of the bulk gauge multiplets to charged fields \( Q \) living at the boundary \( y = 0 \) appears as

\[
S_{\text{boundary}} = \int d^3 x \left[ \int d^4 \theta Q^i e^V \delta(y) \right] .
\]

These couplings, and the field content at the boundary, need only preserve an \( N = 1 \) component of the supersymmetry. Now instead of a compact direction, let us treat the fifth dimension as infinite and impose a periodicity condition on the Lagrangian, in which case the above delta function would become a sum of delta functions \( \sum_n \delta(y - 2 \pi R n) \). The fields at \( y = 0 \) couple to the gaugino \( \lambda_1 \) as one would expect in a normal four-dimensional \( N = 1 \) theory. At \( y = 2 \pi R n \) however, the boundary fields couple to the linear combination \( \cos \omega n \lambda_1 + \sin \omega n \lambda_2 \) where

\[
\omega = \pi |F_T| .
\]

Because supersymmetry breaking is due to non-trivial winding of fields, loop corrections to soft parameters will be physically cut off by the compactification scale and thus rendered finite. This will be important in Section 3 when we do explicit calculations in one picture to gain information about the other.

### B. Hypermultiplets

We now turn our attention to hypermultiplets. We again use the formalism developed by \( [2] \). We will not consider fields transforming under gauge symmetries, but to include them does not change the features of the arguments here.

The coupled radion-hypermultiplet action is given by

\[
S = \int d^4 x \partial_\mu \varphi \left( \int d^4 \theta \left( \frac{T + T^\dagger}{2} \right) (\Phi^\dagger \Phi + \Phi \Phi^\dagger) \right) \\
+ \int d^2 \theta \Phi^\dagger \partial_\mu \Phi .
\]

If we include a radion F-term, the \( F \) terms of the hypermultiplet fields are

\[
F_\phi = \frac{1}{R} \left( \partial_\mu \phi^* c^\dagger - \frac{F_T}{2} \phi \right) \\
F_{\phi^* c^\dagger} = -\frac{1}{R} \left( \partial_\mu \phi - \frac{F_T}{2} \phi^* \right) .
\]

Defining \( \tilde{\phi} = (\phi \phi^\dagger) \), rescaling \( \phi \to \tilde{\phi} / \sqrt{R} \) and \( \varphi \to y/R \) and taking \( F_T \) to be real i gives a scalar potential of

\[
V(\tilde{\phi}) = \tilde{\phi} \left( -\partial_\mu^2 + \frac{F_T^2}{4 R^2} - \frac{F_T}{R \partial_\mu} \right. \\
\left. - \partial_\mu^2 + \frac{F_T^2}{4 R^2} \right) \tilde{\phi}
\]

\[
= \tilde{\phi} \left( -\partial_\mu^2 + \frac{F_T^2}{4} \right) 1 + i F_T \partial_\mu \sigma_2 \tilde{\phi}
\]
We now redefine \( \Phi \) by rotating it

\[
\Phi \rightarrow e^{-i\gamma_5 k_4 y/2R} \Phi.
\]  

(14)

The potential is now

\[
V(\Phi) = 9 \left( -\partial_5^2 \Phi + \frac{1}{\Phi} - \frac{1}{\Phi} \right) \Phi.
\]  

(15)

With the same rotation as was required by the gauge sector, the mass terms vanish. Note the same generalization to complex \( F_T \) works here as well.

### III. RADION MEDIATION WITH BULK GAUGE FIELDS

If gauge fields propagate in the bulk, a radion \( F \)-term can generate tree-level gaugino masses in realistic theories, i.e., those in which the radius is stabilized \[14, 15\]. It is thus a natural framework in which to realize the gaugino mediated supersymmetry breaking scenario \[14, 15\]. However, in \[14\], it was argued that a radion \( F \)-term gave rise to cutoff dependent contribution to scalar masses.

Recently, there has been much discussion about the softness of Scherk-Schwarz supersymmetry breaking \[16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\]. Hard softness of Scherk-Schwarz supersymmetry breaking \[14, 15\]. However, in \[8\], it was argued that a radion \( F \)-term couples to a different linear combination of \( \lambda_1 \) and \( \lambda_2 \), given by the \( SU(2)_R \) transformation. Thus the true amplitude to propagate from a brane to itself, summing all windings is

\[
G(k, 0) = \sum_n G(k, 2n \pi R) \cos (\omega n).
\]  

(17)

\[
= \frac{k \sinh(2 \pi k R)}{4k \sinh^2(\pi k R) \cosh^2(\pi R)}
\]

Notice that if we analytically continue \( k \) back to Minkowski momentum, the propagator has poles for

\[
k = \frac{\omega}{2 \pi R} \pm \frac{n}{R}.
\]  

(18)

Thus the poles in the propagator are precisely those of the spectrum in section 2. By summing over winding modes for an infinite space propagator, we have summed over the discrete spectrum of the compact space Kaluza-Klein tower.

#### A. One loop contributions to soft masses

With our approach explicit, we can calculate the contributions to soft masses. The gaugino loop contributions are given by:

\[
(-1) \times (\sqrt{2}g^2) C_2(G) \int \frac{dk}{(2 \pi)^4} Tr [P_L \frac{1}{k} G(k, 0)].
\]  

(19)

There are other diagrams contributing to the soft masses at one loop. The gaugino does not appear in any of these diagrams, however, and thus these diagrams are not aware of the supersymmetry breaking parameter \( \omega \). Nonetheless, these must cancel the contribution in \[19\] in the limit of no supersymmetry breaking. No supersymmetry breaking corresponds to \( F_T = 0 \), or, equivalently, \( \omega = 0 \). Thus, we can reevaluate (the negative of) \[19\] with \( \omega \) set to 0, and this gives the contribution of the remaining diagrams. Integrating the total contribution to the soft masses is up to a scale \( \Lambda \) is

\[
m_{\phi^2} = \int_0^\Lambda \frac{d^4 k}{4 \pi^2} C_2(G) g^2 k^2 \coth(k \pi R)
\]

\[
\frac{\cosh^2(k \pi R)}{(\sinh^2(k \pi R) \cot^2(k \pi R) + \cosh^2(k \pi R))}.
\]

Because of the exponential suppression in the \( k \gg R^{-1} \) limit, this is finite in the UV, giving a total contribution

\[
m_{\phi^2} = \frac{q^2 C_2(G)}{16 \pi^5 R^3} \left( 2 \zeta(3) - Li_3(e^{i \omega}) - Li_3(e^{-i \omega}) \right)
\]  

(21)
Since we are assuming the dimension is small and supersymmetry breaking is weak, we expand about small $\omega$ making the identification $M_{1/2} = \omega/2\pi R \equiv \omega/M_c$, and find

$$m_{\phi^2} \approx \frac{g_5^2 C_2(G) M_{1/2}^2}{4\pi^2} (3 + 2\log(M_c/M_{1/2})). \quad (22)$$

While this is a complete one-loop calculation, we have not included the fact that both $g_5$ and $M_{1/2}$ run as functions of energy as well (note, $M_{1/2}$ does not run above the compactification scale). It would have been more accurate, in fact, to replace $g_5^2$ in (21) with $g_5^2(k)$ and $M_{1/2}(k)$. We can study the running of $m_{\phi^2}$ by differentiating with respect to the cutoff. For $\omega R \ll \Lambda < R^{-1}$, we have

$$\frac{\Lambda}{d\Lambda} \frac{dm_{\phi^2}}{d\Lambda} = \frac{g_5^2 C_2(G) \omega^2}{16\pi^5 R^3} = \frac{g_5^2 C_2(G) M_{1/2}^2}{2\pi^2}, \quad (23)$$

we we immediately recognize this as the renormalization group equation for the running of the soft mass squared.

Given this, we can identify the $a \log(R^{-1}/M_W)$ piece of (23) as the ordinary gaugino mediated contribution. All other pieces are both finite and small.

One might be concerned that if $g_5$ began power law running it could change this and introduce cutoff dependence. There are two simple reasons why this is not the case: first, quantum corrections actually drive $g_5$ down, thus this is an upper limit on the UV contributions. Secondly, even if there were a positive power law piece, the integrand in (21) goes as $e^{-k^2 2\pi R}$ for $k > R$, and thus even with power law growth, these contributions damp off exponentially.

There is a simple physical interpretation for this: divergences are associated with contracting loops to a point. Here, supersymmetry violating contributions must sample the entire space, that is, they must wind at least once around the extra dimensions. These contributions cannot be contracted to a point and hence will not be divergent.

IV. CONCLUSION

We have shown that a five-dimensional theory with a radion $F$ term is equivalent to a theory with Scherk-Schwarz boundary conditions up to a field redefinition. This makes more concrete the statement by Martí and Pomarol that radion mediated supersymmetry breaking is simply a dynamical realization of Scherk-Schwarz breaking. The equivalence makes the calculation of scalar masses easier and we find contributions from Kaluza-Klein modes to be unimportant.

It is interesting to note that a radion $F$ term translates into a specific Scherk-Schwarz theory. For example, the boundary conditions in [2] require an additional twist along the direction of a different symmetry SU(2)$_H$. It would be interesting to see if dynamical versions of such theories could be realized.

Acknowledgements

The authors thank Gia Dvali, Ann Nelson and Alex Pomarol for reading a draft of the paper and for useful discussions and the Aspen Center for Physics where this work was completed. This work was partially supported by the DOE under contracts DE-FG03-96-ER40956, DE-FG02-90ER-40560 and W-31-109-ENG-38.

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$$\lambda_i \rightarrow \left( e^{F \gamma^a \gamma^5 / 2 R} \right)^{ij} \lambda_j,$$

with the basis for gamma matrices defined in [1].
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