Mathematical Modelling of Spatial Position of 4-High Mill Housing

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Abstract. Nowadays with the rise of laser measure technique, the laser tracker is applied to measure and calibrate the spatial position of mill equipment in cold and hot rolling mills. How to analyze the spatial state of mill housing with the amount measured data via laser tracker is the key to the problem. However, there is still no effective model to study the spatial position of mill housing. In this paper, a comprehensive mathematical model of the spatial position of 4-high mill housing was established. The least square method was used to fit the liner plates. It can be used to calculate the spatial distance and inclination relationship of each liner plate. The model was validated by the measured data of the 4-high mill housing via laser tracker. The calculated results of the model were consistent with those of the spatial analysis software, and the relative deviation was less than 0.01%. The median and opening value and inclination state of the mill liner plates were further studied. The ability and efficiency of data analysis could be improved based on this model, which has good prospect in on-site application.

1. Introduction

In recent years, models and algorithms of spatial measurement of large-scale equipment have made some progress in aerospace, heavy industry and other fields [1]-[3]. Due to the obvious advantages of laser tracker in detection accuracy, measurement efficiency and effect [4]-[5], it is gradually used in metallurgical industry for spatial accuracy detection, such as spatial position detection of mill housing and roll liner plate for cold and hot rolling lines [6]-[7]. The least square method is normally used to fitting planes or transformation with point clouds [8]-[9]. At present, the laser tracker basically depends on import in China [10]. The existing three-dimensional industrial measurement software such as SpatialAnalyzer (SA) is a kind of general commercial measurement visualization software [11]-[13], which demands high level operating skills [14]. Meanwhile, the data analysis for the spatial position of mill housing via SA is cumbersome and time-consuming. However, the survey persons usually wish to use little time and human manipulation on data processing to obtain the spatial position results of mill.

The accuracy of spatial position of mill housing has an important influence on roll deformation, mill stiffness and even strip shape [15]-[17]. The unreasonable position of mill liner plate easily leads to poor assembly accuracy, which is adverse to the mill stiffness and production stability [18]-[19]. The laser tracker could be utilized to measure and calibrate the spatial position of mill equipment in cold and hot
rolling lines[20]. Generally, the spatial position of each liner plate is measured by surveyors to determine whether the assembly position of mill housing are appropriate. However, due to without unified algorithm at present, the processed data and analyzed results may be different from person to person. Therefore, how to analyze the spatial state of mill housing with these measured data is the key to the problem. There is still no effective mathematical model to guide the analysis of spatial position of mill housing. This paper is focus on how to use model to obtain the spatial position of mill liner plates fast and accurately with the measuring points data.

In this paper, aiming at the spatial position of the mill housing, a mathematical model of the spatial position of the mill liner plates is proposed and established, which can realize the quantitative calculation of the spatial position and inclination state of the liner plates of the rolling mill housing. It can provide data support for on-site maintenance and dimension checking of mill assembly.

2. Mathematical model

2.1. plane fitting of mill liner plates

According to roll system in a 4-high mill, there are 16 liner plates on the mill housing, as shown in figure 1. These are namely the entry-operating-side liner plate for upper backup roll (En-OS-UBR), the exit-operating-side liner plate for upper backup roll (Ex-OS-UBR), the entry-driving-side liner plate for upper backup roll (En-DS-UBR), the exit-driving-side liner plate for upper backup roll (Ex-DS-UBR), the entry-operating-side liner plate for upper work roll (En-OS-UWR), the exit-operating-side liner plate for upper work roll (Ex-OS-UWR), the exit-driving-side liner plate for upper work roll (Ex-DS-UWR), the entry-operating-side liner plate for lower work roll (En-OS-LWR), the exit-operating-side liner plate for lower work roll (Ex-OS-LWR), the entry-driving-side liner plate for lower work roll (En-DS-LWR), the exit-driving-side liner plate for lower work roll (Ex-DS-LWR), the entry-operating-side liner plate for lower backup roll (En-OS-LBR), the exit-operating-side liner plate for lower backup roll (Ex-OS-LBR), the entry-driving-side liner plate for lower backup roll (En-DS-LBR) and the exit-driving-side liner plate for lower backup roll (Ex-DS-LBR).

Usually, if the liner plate of mill housing was measured, the rolls need to be removed. A lots of spatial points on liner plates can be measured via the laser tracker. The least square method is used to fit the liner plate, and then the plane equation of each liner plate can be obtained. It is stipulated that the \( x \) component of the normal vector of the entry-side liner plate is negative and the \( x \) component of the normal vector of the exit-side liner plate is positive.

![Figure 1. Physical model of mill housing and liner plate.](image)

It is assumed that there are \( n \) (\( n \geq 3 \)) points through measuring a liner plate. The coordinates of point \( i \) is expressed as \((x_i, y_i, z_i)\), where \( i = 1, 2, ..., n \). The plane equation of liner plate \( a_1 x + a_2 y + a_3 z + a_4 = 0 \) is fitted and calculated by least square method, so that
$S = \sum_{i=1}^{n} \left( a_i x_i + 2a_j y_i + 2a_k z_i + a_4 \right)^2$ should be minimized. Then the partial derivative of this function with respect to each component is 0, that is
\[
\frac{\partial S}{\partial a_l} = 0, \quad l = 1, 2, 3
\]
Substituting each variable into Eq. (1), then
\[
\begin{bmatrix}
\sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i z_i \\
\sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & \sum_{i=1}^{n} y_i z_i \\
\sum_{i=1}^{n} x_i z_i & \sum_{i=1}^{n} y_i z_i & \sum_{i=1}^{n} z_i^2
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
= a_4 \begin{bmatrix}
\sum_{i=1}^{n} x_i \\
\sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} z_i
\end{bmatrix}
\tag{2}
\]

The coefficients $(a_1, a_2, a_3)$ of plane equation can be obtained by solving the system of ternary non-homogeneous $(a_4 \neq 0)$ or homogeneous $(a_4 = 0)$ linear equations of Eq. (2).

2.2. Spatial distance of mill liner plate
Let $a_1 x + a_2 y + a_3 z + a_4 = 0$ to be the equation of the mill central symmetrical plane between entry-side and exit side, as shown in figure 1. Then, $(a_1, a_2, a_3)$ is the normal vector of the central symmetrical plane. The central point of an arbitrary liner plate is expressed as $P_{ij} = (x_{ij}, y_{ij}, z_{ij})$, in which $i = 1, 2, 3, 4$ refer to upper backup roll (UBR), upper work roll (UWR), lower work roll (LWR) and lower backup roll (LBR) respectively. $j = 1, 2$ represent the operating-side and the driving-side respectively and $k = 1, 2$ indicate the entry-side and the exit-side respectively. Therefore, the distance from the central point of an arbitrary liner plate to the central symmetrical plane can be obtained by the Eq. (3).
\[
d_{ij} = \frac{|a_1 x_{ij} + a_2 y_{ij} + a_3 z_{ij} + a_4|}{\sqrt{a_1^2 + a_2^2 + a_3^2}}
\tag{3}
\]

The median value of the liner plates is defined as the distance difference between the central point of the exit-side liner plate and the central point of the corresponding entry-side liner plate to the central symmetrical plane.
\[
M_{ij} = d_{ij} - d_{ij}
\tag{4}
\]

Where $i = 1, 2, 3, 4$ indicate UBR, UWR, LWR and LBR respectively. $j = 1, 2$ mean the operating-side and the driving-side respectively.

The median value of the liner plates reflects the symmetrical relationship between the entry-side liner plate and the exit-side liner plate with respect to the central symmetrical plane. When $M_{ij} = 0$, the entry-side and exit-side liner plate for roll $i$ distribute symmetrically along the central symmetrical plane. When $M_{ij} > 0$, the distance from the center of the exit-side liner plate to the central symmetrical plane is larger than that of entry-side liner plate for roll $i$. On the contrary, the distance from the center of the exit-side liner plate to the central symmetrical plane is larger than that of exit-side liner plate for roll $i$ when $M_{ij} < 0$.

The opening value of the liner plates is defined as the distance sum between the center point of the exit-side liner plate and the center point of the corresponding entry-side liner plate to the central symmetrical plane.
\[
O_{ij} = d_{ij} + d_{ij}
\tag{5}
\]
Where $i=1,2,3,4$ refer to UBR, UWR, LWR and LBR respectively. $j=1,2$ indicate the operating-side and the driving-side respectively.

The opening value of liner plate reflects the distance relationship between the center of the entry-side liner plate and the center of the corresponding exit-side liner plate, and there always is $O_j > 0$.

### 2.3. Inclination relation of liner plate

The inclination of liner plate in mill housing can be divided into two types: the inclination along the height direction and the inclination to the rolling center line. The normal vector of the rolling center line is defined as the position direction of $y$-axis. The inclination direction of each liner plate is judged by the positive and negative of the unit normal vector component $n_i$ or $n_z$.

The inclination of the liner plate along the height direction is described as “larger at top-side” or “larger at bottom-side”, as shown in figure 2. When the liner plate inclines downward along the height direction, the unit normal vector component $n_z$ of the liner plate (including the entry-operating-side, the entry-driving-side, the exit-operating-side and the exit-driving-side) is negative, and the opening value at top-side of the liner plate will be larger than that at the bottom-side, called as “larger at top-side”. When the liner plate inclines upward along the height direction, the unit normal vector component $n_z$ of the liner plate (including the entry-operating-side and the entry-driving-side, exit-operating-side and the exit-driving-side) is positive, and the opening value at bottom part of the liner plate will be larger than that at the top part, called as “larger at bottom-side”.

![Diagram of inclination of mill liner plate at top-side or bottom-side.](image)

**Figure 2.** Diagram of inclination of mill liner plate at top-side or bottom-side.

Let the normal vector of a liner plate be $\mathbf{V}_i = (x, y, z)$. The inclination degree of lining plate at top or bottom side ($\gamma_i$) can be calculated by Eq. (6).

$$\gamma_i = -\frac{|z| \cdot 1000}{\sqrt{x^2 + y^2 + z^2}} \quad (6)$$

The inclination degree at top or bottom side indicates the deviation of $z$ component of normal vector in unit length, and the unit is mm/m.

The liner plate inclines to the rolling center line, which is described as “larger at inside” or “larger at outside”, as shown in figure 3. When the liner plate is far from the rolling center line, the $y$ component ($n_y$) of the unit normal vector of the operating-side liner plate (including the entry and exit side) is negative, or the $y$ component ($n_y$) of the unit normal vector of the driving-side liner plate (including the entry and exit side) is positive. The opening value of liner plates near the rolling center line will be larger than that far away from the rolling center line, and this is called “larger at inside”. When the liner plate inclines toward to the rolling center line, the $y$ component ($n_y$) of the unit normal vector of the operating-side liner plate (including the entry and exit side) is positive, or the $y$ component ($n_y$) of the unit normal vector of the driving-side liner plate (including the entry and exit side) is negative. The opening of the liner plates far away from the rolling center line will be larger than that of near the rolling center line.
The inclination degree of lining plate at inside or outside ($\beta$) can be calculated by Eq. (7).

$$\beta_i = \frac{|y_i| \cdot 1000}{\sqrt{x_i^2 + y_i^2 + z_i^2}}$$

The inclination degree at inside or outside indicates the deviation of y component of the normal vector in unit length, and the unit is mm/m.

![Figure 3. Diagram of inclination of mill liner plate at inside or outside.](https://via.placeholder.com/150)

3. Validation

The spatial position of the liner plates of a hot rolling mill housing was measured via the laser tracker, as shown in figure 4. The measured discrete points were processed and calculated by SA software and this model respectively. Figure 5 shows a comparison between SA results and model results. It can be seen from the figure that the distance between the central point of the liner plate and the central symmetrical plane (including the entry-operating-side, the exit-operating-side, the entry-driving-side and the exit-driving-side) calculated by this model were basically equal to the results of SA, and the relative deviation was less than 0.01%.

![Figure 4. Picture of on-site measurement via laser tracker.](https://via.placeholder.com/150)

![Figure 5. Comparison of SA results and model results.](https://via.placeholder.com/150)

In the past, it needed to go through certain processing steps and operation procedures to analyze measurement data of mill housing with SA or other software. It took a long time (about 1 hour) to process the data and had low flexibility. However, the establishment of the spatial position model of mill housing greatly reduces the difficulty of data analysis and obviously improves the calculation efficiency. The developed software based on this model can realize one-button calculation, which takes
only 1–2 seconds to analyze the same data. It can be seen that this model can significantly save time, reduce manpower costs, and improve the efficiency and ability of spatial position analysis of mill housing. Therefore, it can completely replace SA software to process spatial accuracy data of mill housing.

4. Results and discussion

4.1. Analysis of distance relation of liner plate

The measured spatial data of above hot rolling mill housing is analyzed by the model. Figure 6 shows the distance from the measuring points to the corresponding fitting liner plates. It can be seen from the figure that the distance between the measuring points of each liner and the liner plate is within 0.1 mm. The liner plane fitted by least square method has a good fitting degree, and it also shows that the accuracy of the measured data via laser tracker is high and reliable.

![Distance from the measuring points to the corresponding fitting liner plates for (a) BR and (b) WR.](image)

**Figure 6.** Distance from the measuring points to the corresponding fitting liner plates for (a) BR and (b) WR.

![Median value and opening value of mill liner plates.](image)

**Figure 7.** Median value and opening value of mill liner plates.

Figure 7 shows the median and opening values of the operating-side and the driving-side liner plates. The median values at OS for UBR, UWR, LWR and LBR are 0.003, -0.016, 0.010 and 0.001 mm respectively. While the median values at DS for UBR, UWR, LWR and LBR are 0.040, -0.018, -0.052
and 0.031 mm respectively. All of the median values are acceptable and reasonable. The opening values of operating-side and driving-side are consistent with the actual opening values.

4.2. Analysis of inclination relation of liner plate
Aiming at the two inclination modes of “inclination at top-side or bottom-side” and “inclination at inside or outside”, the spatial position model of 4-high mill can be used to judge the inclination direction and degree of each mill liner plate, as shown in Table 1. The inclination of each liner plate is small, and all of them are at a reasonable level. When the inclination of some liner plates exceeds the allowable deviation (might be more than 1 mm/m), it is necessary to correct the inclination degree in time.

**Table 1. Inclination status of mill liner plates.**

| Roll | Liner plate | Tilt at top or bottom side | Tilt at inside or outside |
|------|-------------|----------------------------|--------------------------|
|      |             | tilt direction (larger at) | tilt value/(mm/m)        | tilt direction (larger at) | tilt value/(mm/m) |
| UBR  | En-OS       | Top-side                   | 0.152                    | Outside                   | 0.135             |
|      | Ex-OS       | Bottom-side                | 0.016                    | Outside                   | 0.020             |
|      | En-DS       | Top-side                   | 0.093                    | Inside                    | 0.003             |
|      | Ex-DS       | Top-side                   | 0.209                    | Inside                    | 0.026             |
|      | En-OS       | Bottom-side                | 0.146                    | Outside                   | 0.156             |
|      | Ex-OS       | Bottom-side                | 0.062                    | Outside                   | 0.139             |
|      | En-DS       | Top-side                   | 0.188                    | Inside                    | 0.008             |
|      | Ex-DS       | Top-side                   | 0.209                    | Inside                    | 0.030             |
|      | En-OS       | Top-side                   | 0.059                    | Inside                    | 0.065             |
|      | Ex-OS       | Top-side                   | 0.285                    | Outside                   | 0.019             |
| UWR  | En-OS       | Bottom-side                | 0.429                    | Inside                    | 0.160             |
|      | Ex-OS       | Top-side                   | 0.107                    | Outside                   | 0.078             |
|      | En-DS       | Top-side                   | 0.094                    | Inside                    | 0.241             |
|      | Ex-OS       | Bottom-side                | 0.408                    | Outside                   | 0.112             |
|      | En-DS       | Top-side                   | 0.224                    | Outside                   | 0.067             |
|      | Ex-DS       | Bottom-side                | 0.091                    | Outside                   | 0.128             |

Due to wear, deformation and rust of liner plate during the long-term use process, the inclination of liner plate might occur and the assembly accuracy of mill liner might be reduced. The liner plate should be adjusted slightly by means of grinding, padding or directly replacing the liner plate. Therefore, the inclination of liner plate can be eliminated, and the assembly accuracy of rolls can be improved. The
quantitative results of the spatial position of liner plates provide necessary data support for the reasonable adjustment of the mill housing liner plates.

5. Conclusions
The mathematical model of spatial position of 4-high mill housing has been established. The least square method was used to fit the liner plate of each roll system of the mill, and the quantitative analysis of the spatial distance and inclination relationship of the mill housing was realized. Based on the measured data of a hot rolling mill, the analysis results of SA software and this model were compared. The relative deviation between them was less than 0.03% and the rationality and accuracy of the model were verified. This model could improve the efficiency and ability of spatial position analysis of mill housing.

Using the mathematical model of spatial position of the mill liner plate, the median value and opening value of each liner plate can be calculated, and the inclination direction and degree (“inclination at inside or outside” and “inclination at top-side or bottom-side”) of each liner plate can be judged. It provides data support for the maintenance of rolling mill housing and has good application prospect in the field.

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