Role of T-odd functions in high energy hadronic collisions

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Abstract
I propose a simple model for predicting the energy behavior of T-odd, chiral odd function $h_\perp^\perp$. Furthermore I illustrate a method for extracting $h_\perp^\perp$ and the transversity function from Drell-Yan. The method may be applied also to other reactions.

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1 Introduction

The T-odd functions[1-3] have become important in the last ten years, since when high energy physicists realized that such functions could be used as polarimeters for extracting chiral odd functions, especially transversity[4-6]. Here I consider the T-odd, chiral odd function $h_1^{\perp}[7]$ of a quark inside the proton. In particular I propose a simple model, which allows to predict the behavior of this function at varying proton momentum. Moreover I am concerned with asymmetries relative to unpolarized and singly polarized Drell-Yan (DY), i. e.,

$$pp \rightarrow \mu^+\mu^- X.$$  \hspace{1cm} (1)

I show how to extract $h_1^{\perp}$ from this reaction and I suggest an alternative method for determining transversity.

2 General formulae

The single transverse spin asymmetry for reaction (1) is defined as

$$A = \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow},$$  \hspace{1cm} (2)

where $d\sigma_{\uparrow(\downarrow)}$ refer to cross sections with opposite polarizations of one of the proton beams. In one-photon approximation,

$$d\sigma_\uparrow - d\sigma_\downarrow \propto d\Gamma L^{\mu\nu} H^a_{\mu\nu},$$  \hspace{1cm} (3)

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k',$$  \hspace{1cm} (4)

$$H^a_{\mu\nu} = \int d^2 p_{1\perp} Tr \left[ \gamma_\mu \Phi_{X.o}(x_1, p_{1\perp}) \gamma_\nu \Phi_{\chi.o}(x_2, p_{2\perp}) + (1 \leftrightarrow 2) \right].$$  \hspace{1cm} (5)

Here $d\Gamma$ is the phase space element. $k$ and $k'$ are the four-momenta of the muons, $x_1$ and $x_2$ are the longitudinal fractional momenta of the annihilating quark and antiquark, $p_{1\perp}$ and $p_{2\perp}$ their transverse momenta with respect to the initial beams and $\Phi_{X.o}$ and $\Phi_{\chi.o}$ the chiral odd components of their correlation matrices. The index 1 in $x$ and $p_\perp$ refers to the transversely polarized proton, the index 2 to the
unpolarized one. \( p_{2\perp} \) is chosen in such a way that the transverse momentum of the muon pair with respect to the proton beam in the laboratory frame, i.e.,

\[
Q_\perp = p_{1\perp} + p_{2\perp},
\]

is kept fixed. Lastly the sum over flavors has been omitted.

### 3 Parametrization of the T-odd correlation matrix

In the laboratory frame, at sufficiently high energies, the chiral odd component of the correlation matrix of the transversely polarized proton can be parametrized as[7]

\[
\Phi_{\chi.o.} = \frac{1}{4} x_1 \mathcal{P} \gamma_5 \left\{ [\hat{S}, \hat{p}_+^\perp] h_{1T} + \frac{1}{\mu} [\hat{r}_\perp^\perp, \hat{p}_+^\perp] h_1^\perp \right\}.
\]

(7)

Here \( h_{1T} \) is the transverse momentum dependent transversity distribution, while \( h_1^\perp \) will be illustrated in a moment. Moreover

\[
r_\perp = p_{1a} S - p_{1b} n_a \equiv (0, -p_{1b}, p_{1a}, 0),
\]

(8)

\[
p_{1a} = p_{1\perp} \cdot S \times n, \quad p_{1b} = p_{1\perp} \cdot S,
\]

(9)

\[
n_+ \equiv (1, \mathbf{n}), \quad n_a \equiv (0, S \times \mathbf{n}).
\]

(10)

\( \mathcal{P} \mathbf{n} \) and \( S \equiv (0, \mathbf{S}) \) are respectively the momentum and the Pauli-Lubanski four-vector of proton 1, \( \mathbf{S} \) and \( \mathbf{n} \) being unit vectors such that \( \mathbf{S} \cdot \mathbf{n} = 0 \). Lastly \( \mu \) is an undetermined mass scale, which was set equal to the proton mass by various authors[7-9]; as I shall show, this is not the most suitable choice.

The second term of parametrization (7) is T-odd and gives a nonvanishing contribution also when the proton is unpolarized. In this case, given a unit vector \( \mathbf{s} \) not parallel to \( \mathbf{n} \), the density of quarks whose spin component along \( \mathbf{s} \) is positive, minus the density of quarks for which this spin component is negative, amounts to

\[
\delta q_\perp = -\frac{r_\perp \cdot S_0}{\mu} h_1^\perp,
\]

(11)
where \( s_0 \equiv (0, s) \). Eq. (11) is a consequence of eq. (10) for an unpolarized proton. The two equations exhibit the meaning of the function \( h_1^\perp \): in an unpolarized proton, a quark with nonzero transverse momentum is polarized perpendicularly to its momentum and to the proton momentum, in agreement with parity conservation.

4 A model for T-odd functions

A proton may be viewed as a bound state of the active quark with a set \( X \) of spectator partons. In order to take into account coherence effects, I project the bound state onto scattering states with a fixed third component of the total angular momentum with respect to the proton momentum, \( J_z \), and with a spin component \( s = \pm 1/2 \) of the quark along the unit vector \( s \) introduced in the previous section. For the sake of simplicity, I assume \( X \) to have spin zero, moreover I choose a state with \( J_z = \frac{1}{2} \).

Then

\[
|J_z = 1/2; s; X\rangle = \alpha |\to, L_z = 0; s; X\rangle + \beta |\leftarrow, L_z = 1; s; X\rangle.
\]

(12)

Here \( \to (\leftarrow) \) and \( L_z \) denote the components along \( n \), respectively, of the quark spin and orbital angular momentum, while \( \alpha \) and \( \beta \) are Clebsch-Gordan coefficients. Then the probability of finding a quark with \( J_z = 1/2 \) and spin component \( s \) along \( s \), in a longitudinally polarized proton with a positive helicity, is

\[
|\langle P, \Lambda = 1/2|J_z = 1/2; s; X\rangle|^2 = \alpha^2|\langle P, \Lambda = 1/2|\to, L_z = 0; s; X\rangle|^2
+ \beta^2|\langle P, \Lambda = 1/2|\leftarrow, L_z = 1; s; X\rangle|^2 + I,
\]

(13)

\[
I = 2\alpha\beta Re \left[ \langle P, \Lambda = 1/2|\to, L_z = 0; s; X\rangle\langle\leftarrow, L_z = 1; s; X\rangle|P, \Lambda = 1/2\rangle \right].
\]

(14)

Expanding the amplitudes in partial waves yields

\[
I = 2 \sum_{l,l'}^\infty Re \left[ i e^{-i\phi} A_l B_{l'}^* \right] P_l(\cos\theta)P_{l'}^1(\cos\theta).
\]

(15)

Here \( A_l \) and \( B_l \) are related to partial wave amplitudes; moreover \( \theta \) and \( \phi \) are respectively the polar and the azimuthal angle of the quark momentum, assuming \( n \) as the polar axis and, as the azimuthal plane, the one through \( n \) and \( s \). In the Breit frame
one has
\[ P_l(cos\theta) \sim 1, \quad P^l_1(cos\theta) \sim \frac{|P_{1\perp}|}{xP}. \quad (16) \]

Then eq. (13) yields
\[ I \sim \frac{|P_{1\perp}|}{xP} (Acos\phi + Bsin\phi), \quad (17) \]

where \( A \) and \( B \) are real functions made up with \( A_l \) and \( B_l \). Since \( s \) is an axial vector, parity conservation implies \( A = 0 \). Therefore eqs. (13) and (17) imply that the interference term \( I \) is T-odd and that the final quark is polarized perpendicularly to the proton momentum and to the quark momentum, independent of the proton polarization. Comparing eq. (17) with eq. (11) yields
\[ \mu = xP. \quad (18) \]

Eqs. (18) predicts that the quark transverse polarization in an unpolarized (or spinless) hadron decreases as \( P^{-1} \).

5 Extracting chiral odd functions from DY

5.1 The transversity function

Eqs. (3), (5) and (7) imply that the numerator of the DY asymmetry (2) is of the form
\[ d\sigma_{\uparrow} - d\sigma_{\downarrow} \propto \int d^2p_{1\perp} \left[ \left( \frac{p_{2a}}{x_2P} h_{1T} + \frac{P_{1\perp} \cdot p_{2\perp}}{x_1x_2P^2} h^\perp_1 \right) h^\perp_1 + (1 \leftrightarrow 2) \right], \quad (19) \]

assuming the constraint (3). Here
\[ p_{2a} = p_{2\perp} \cdot S \times n. \quad (20) \]

In order to extract the transversity, i.e., \( h_1 = \int d^2p_{\perp} h_{1T} \), from DY, I define the following weighted asymmetry[10,11]:
\[ \langle A_1 \rangle = \frac{\sum_n d\sigma^{(n)} Q_a^{(n)}}{M_P \sum_n d\sigma^{(n)}}, \quad Q_a = p_{1a} + p_{2a}. \quad (21) \]
Here $M_P$ is the proton rest mass and $d\sigma^{(n)}$ the differential cross section at a fixed transverse momentum, the sum running over the data. Eq. (19) implies

$$\sum_n d\sigma^{(n)} Q_{a}^{(n)} \propto h_1(x_1) \bar{h}_{1(1)}^\perp(x_2) + \bar{h}_1(x_1) h_{1(1)}^\perp(x_2), \quad (22)$$

$$h_{1(1)}^\perp(x_2) = \int d^2 p_\perp p_\perp^2 h_1^\perp \quad (23)$$

This allows to extract $h_1$ and $\bar{h}_1$, provided $h_{1(1)}^\perp$ and $\bar{h}_1^\perp$ are known. These functions have to be inferred from an independent analysis, for example with the method I exhibit in the next subsection. According to my model, one has $\langle A_1 \rangle \propto P^{-1}$.

5.2 The function $h_1^\perp$

$h_1^\perp$, which is washed out by the weighted asymmetry (21), can be singled out by using an alternative weight function. Indeed, defining

$$\langle A'_1 \rangle = \frac{\sum_n (d\sigma^{(n)}_+ - d\sigma^{(n)}_-) (Q_{a}^{(n)})^2}{M_P^2 \sum_n d\sigma^{(n)}}, \quad (24)$$

formula (19) yields

$$\langle A'_1 \rangle \propto h_{1(1)}^\perp(x_1) \bar{h}_{1(1)}^\perp(x_2) + \bar{h}_{1(1)}^\perp(x_1) h_{1(1)}^\perp(x_2), \quad (25)$$

where $h_{1(1)}^\perp(x_1)$ is defined analogously to eq. (23). Formula (19) implies that the weight functions $Q_\perp^2$ and $(Q_\perp \cdot S)^2$ could be used instead of $Q_{a}^2$. Moreover $h_1^\perp$ may be extracted also from unpolarized DY; in this case the weighted asymmetry is defined as

$$\langle A''_1 \rangle = \frac{\sum_{n_+} d\sigma^{(n_+)} [Q_\perp^{(n_+)}]^2 - \sum_{n_-} d\sigma^{(n_-)} [Q_\perp^{(n_-)}]^2}{M_P^2 \sum_n d\sigma^{(n)}}. \quad (26)$$

Here the symbols $\pm$ refer to events whose transverse momenta $Q_\perp$ are at the right (+) or at the left (-) of the plane through $n$ and the unit vector $s$. The model I have elaborated predicts that both $\langle A'_1 \rangle$ and $\langle A''_1 \rangle$ decrease as $P^{-2}$.

6 Discussion

I have suggested how to extract $h_1$ and $h_1^\perp$ from DY. In particular, the T-odd function $h_1^\perp$ can be inferred from unpolarized proton-proton collisions and it can be used as a
quark polarimeter in order to get $h_1$ from single transverse spin asymmetry. Similar methods could be elaborated for semi-inclusive deep inelastic scattering (SIDIS)\cite{12}, for $e^+e^- \rightarrow \pi X$ and, with some assumptions, also for $pp \rightarrow \pi X$. In these reactions, in order to infer the transversity, the Collins function\cite{13} is needed, which can be deduced, for example, from $e^+e^-$ collisions. Moreover in such kinds of experiments, as well as in DY, one is faced with the problem of disentangling the contributions of quarks and antiquarks of any flavor, as pointed out by Boglione and Leader\cite{14}. In this connection, a comparison between SIDIS and DY results is particularly helpful, since the two cross sections depend on the quark and antiquark functions according to different combinations.

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