The didactical and epistemological obstacles on the topic of geometry transformation

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Abstract. This study intends to investigate didactical and epistemological obstacles that occur when students accomplish the problem on the topic of geometry transformation. The data were collected through the students' answer and interviews of 16 ninth graders and 16 twelfth graders when solving the test instrument used is eight problems related to geometry transformation where there are five of them which are contextual problems. The data analysis reveals that most students have difficulty in identifying contextual problems related to geometry transformation; not quite firm to know the characters from each type of geometry transformation (reflection, translation, rotation, and dilation); use procedural knowledge rather than conceptual understanding; and erroneous in applying previous material concepts related to solving geometry transformation problems. These findings are expected to overcome didactical and epistemological obstacles in teaching and learning of geometry transformation.

1. Introduction
Some student naturally experiences a situation called a learning obstacle. Student difficulties arise as part of the way students adapt their knowledge to a milieu or environment [1]. Refers to the obstacle term of the theory delivered by Bachelard and Piaget regarding the "errors" [2]. According to him, mistakes and failures play a role that is not simple, uncertain and unpredictable and is an important part of the process of acquiring knowledge, which is called the obstacles. Even [1] sees it as one process for knowing. In other words, the obstacles cannot be avoided because it is an important part of the learning process as one way to find out.

Two of the obstacles that the teachers need to consider in mathematics teaching and learning are didactical and epistemological obstacles. The didactical obstacle which can emerge from didactical practices in the class. It can be identified as the results of the methods or approaches used by the teacher [3]. The didactical obstacle is the difficulty that occurs because of the inaccuracy in the selection of instructional materials or learning designs prepared by the teacher in carrying out his teaching practices in certain contexts. It becomes a reflection process that the students’ understandings also depend on the quality of teachers’ instruction. It indicates the importance of finding the appropriate methods, a good understanding of materials and instructions. Didactical obstacles are more easily recognizable and probably easier to overcome rather than epistemological obstacles [4].

Whereas, epistemological obstacles are the obstacles that arise from the learning approach derived from the concept itself. Epistemological obstacles can be construed as faulty ways of thinking but such a perspective ignores their importance, their developmental necessity, and their productivity in specific settings [1]. Contrary to the 'didactical obstacle' (which is caused by the way of teaching), the idea of
'epistemological obstacle' to obstacles that are rooted in the structure of the mathematical content itself, in the history and development of the application [2]. In line with [5] that obstacle is not related to the learning used by the teacher but is the result of the mathematical concept itself.

Previous study about learning obstacle found that the epistemological obstacle is a conception or knowledge which is correct in a particular context, and this conception becomes invalid in another context, this obstacle relates to the truth of knowledge that is limited to a particular concept [2]. This difficulty is related to students erroneous in applying previous material concepts related to solving problems in different contexts that they did not find at the time of learning. This has happened because the students are not facilitated with a variety of contexts for developing their experiences and understanding, while it is important to construct new understanding.

In addition to the above, another factor that participates in the classroom learning process is the curriculum. According to the changes in the educational curriculum that occurred in Indonesia was not the first time, but had undergone 11 curriculum changes, starting from the 1947 Curriculum (Lesson Plan 1947) until 2006 Education Unit Level Curriculum (KTSP), then changing to 2013 Curriculum, and 2013 Curriculum which has undergone revised editions three times, namely the revised 2014 edition, 2016 revised edition, and the latest is the 2013 Curriculum revised edition of 2018.

Changes in the curriculum from year to year are policies taken by the government. The reason the government has made changes to the new education curriculum is to improve the quality of education in Indonesia. However, the objectives of the government are not always in line with reality in the field. The frequent changes in the curriculum are considered to have less influence on the progress of education, especially for teachers, because the teacher's teaching methods have not changed (not adjusting to curriculum changes), so the government is expected to make little changes to the curriculum, but strengthen teacher competencies due to the success of education is largely influenced by teacher factors [6].

Ideally, a curriculum is patented and used for several years to obtain and analyse the results. If you look at more advanced countries, for example, Japan, both teachers, and their students are not confused by the rapid changes in the curriculum. Based on curriculum changes that impressed the marathon, starting from the Education Level Curriculum Unit (KTSP) in 2006 then changed to the 2013 Curriculum, then quickly changed to the 2014 revised edition of the 2013 Curriculum, then continued to the revised 2016 edition, and in the interim period long ago it changed back to the revised 2013 edition of Curriculum 2018, so there was material on mathematics subjects in the geometry domain which experienced significant transfer of material delivery at different levels of education.

Geometry is the domain of mathematics that plays an important role in developing students' thinking skills. Suggests there are five reasons why geometry is very important to learn [7]: (a) geometry helps humans have a complete appreciation of their world, (b) exploration of geometry can help develop problem-solving skills, (c) geometry plays a major role in mathematics others, (d) geometry is used by many people in their daily lives, (e) geometry is full of puzzles and fun.

The above illustrates the importance of geometry to be taught at the secondary school level. Nevertheless, empirical evidence in the field shows that there are still many students who have not understood the concept of geometry. Learning obstacles found in senior high school related to concepts in geometry transformation material, including students cannot distinguish terms that appear in each concept in geometry transformation material, do not understand how the displacement process that occurs in each concept contained in geometry transformation, has difficulty when faced with non-routine problems, have not mastered the full prerequisite material which is a condition for following geometry transformation material [8]. Besides, students of grade 7 have some misconceptions and lack of knowledge on the topic of geometry, because the geometry topics are more complex than numeral or algebraic operations [9]. Thus, geometry becomes the most common teaching and learning problems in mathematics.

In the geometry domain, one of the materials studied by students in secondary schools is material about geometry transformation. Referring to the KTSP, geometry transformation material is given to 12th grade high school students in semester 1, while in 2013 Curriculum is given to high school
students in grade 11 semester 2. Submission of this material experienced a significant shift in the 2014 revised edition of the 2013 Curriculum, which is given to students in junior high school level 7 semester 2. Then in the 2016 revised edition of the 2013 Curriculum, this material moved back to grade 9 semester 2 students. And in the 2013 revised edition of 2018, this material was given to 9th grade students in semester 1. Because of the change in the position of material delivery at different levels of education, of course, the teacher will experience new experiences in delivering geometry transformation material at lower levels of education and provide quite a challenge for developing teaching materials.

Based on the above description, this paper examines the student’s learning obstacles in learning mathematics on geometry transformation topic, especially in the light of didactical and epistemological obstacles which are existence. The findings are expected not only to show an understanding of didactical and epistemological obstacles in geometry transformation topic but as a first step in developing didactical designs. This is important for teachers because the teachers play a very important role in designing didactical situations so that the teaching and learning process occurs within the students [5]. Thus, this study is expected to overcome the emergence of didactical and epistemological obstacles in teaching and learning of geometry transformation.

2. Methods
This research used an interpretive paradigm which was part of Didactical Design Research. The interpretive paradigm is used to view the world through the perceptions and experiences of participants [10, 11] to identify didactical and epistemological obstacles in the geometry transformation topic.

The data were collected from the students’ answers on their tests and interviews. There were 16 students of grade 12 in 9 of senior high school and 16 students of grade 9 in two junior high school in Bandung, West Java, Indonesia, as the research subjects who have learned geometry transformation. Each student was given a test of respondent’s ability consisting of eight problems related to geometry transformation where there are five of them which are contextual problems. The approval time was around 60 minutes.

After they completed the test, the researcher observed the student’s answers and selects some students for the interview. The interview activity was needed to attain a deeper students’ understanding of the problems and to find out the detail of obstacles or difficulties experienced by students in answering the test. This is a series of mutually reinforcing learning obstacles that arise in the geometry transformation topic. Furthermore, the data was analysed qualitatively.

3. Result and Discussion
Based on the results of students’ answers on the test and interviews were given, the researchers found didactical and epistemological obstacles identified from students’ strategies in solving problems on geometry transformation topics. The results show that most students have difficulty identifying contextual problems related to geometry transformation, not quite firm to know the characters from each type of geometry transformation (reflection, translation, rotation, and dilation), use procedural knowledge rather than conceptual understanding, and erroneous in applying previous material concepts related to solving geometry transformation problems.

3.1. Didactical Obstacle
The didactical obstacle was identified from the results of student answers that focused on procedural knowledge rather than conceptual understanding. This can be seen when students are faced with problems in question number 1 as in Figure 1, students confidently assume that the problems are by the procedures they have received before. Students do not understand the type of problem situation so that a formula can be used. Students assume that when they find a problem in the form of rotation, students can solve it using the matrix method and estimating the angular size of 90°. For example, student answer on problem number 1 is “when younger sister Nayla is in first position (1) then older sister Nawra is now in position (6), while the translation is shifting 7x from the starting position.
Students try to remember the formula that the teacher has taught, and try to substitute only known numbers in the problem. Students believe that the formula used is correct and accurate to solve the first problem, even though the results and conclusions that are wrong are not even logical. The student's answer about “translation” also shows that they only understand the first problem as a shift in position without considering the whole information, in the form of a clearly defined frame of rotation. Students try to remember and associate with the type of geometry transformation that has a sense of position shift, finally students mistakenly do it with translation, even though the problem presented is a rotation.

The didactical obstacle is also seen from the way students solve problems in question number 3 (Figure 2). The problem is given to determine the level of student understanding in identifying the characteristics of each type of geometric transformation. It was found few of students answer the problem number 3 by writing down all types of geometry transformation, as if the answer is given is getting closer to the truth, without considering the existing image on the terrace tiling problems. This indicates that students are not quite firm to know the characters from each type of geometry transformation (reflection, translation, rotation, and dilation).
3. Ali was working on group projects at a friend's house. He was interested in the appearance of the terrace floor outside the room (as shown in Figure 4). What types of transformations are there in the tessellation of the terrace? Explain!

![Figure 2](image)

**Figure 2. Problem No.3**

Another examples of learning obstacle were also found in many students when answering the problem number 6 (Figure 3), the answer given is incomplete because it does not come with the direction and distance of translation, the line of reflection, or a centre and a large angle of rotation. There are also answers from students who are still mistaken in identifying and placing the characteristics of each type. For instance, student answered problem number 6 are “(a) This is a rotation in a clockwise direction, (b) this is a translation in the opposite direction, and the angle or centre is $180^\circ$, (c) this is a reflection, and the angle is $360^\circ$. From this problem, it shown that Students understand that translation and reflection also have a centre and a large angle as in rotation.

![Figure 3](image)

**Figure 3. Problem No.3**

The above conditions can occur due to learning which prioritizes procedural knowledge rather than conceptual understanding. The learning only emphasizes the memorization process so that the students get less meaningful. This is as stated by Ausubel that learning by memorizing will be fast and easily lost and will not be forced at all [12].
3.2. Epistemological Obstacle

The results of student answers are categorized into an epistemological obstacle when there is a limited understanding of students in certain contexts [2]. Students find it difficult to apply the understanding of the concept of geometry transformation into other contexts because they feel they never get that context in classroom learning. This condition can be seen from student answers in solving problems 1. The answer in Figure 4 (a) shows that students forget about how to obtain a large angle between two adjacent swings. Therefore, the algorithm that is done by students is only a known number operation on the question without understanding the algorithm is performed. Students also only try to estimate the angle size (Figure 4 (b)). This indicates the limitations of students in relating the concept of angles that have been studied with the concept of rotation. Other finding from student answer also showed that students forget about the symmetry order material that is closely related to the rotational symmetry material in the figure plan that has been studied so mistaken when answering the question about the symmetry order of the twist.

The epistemological obstacle also arises from the way students apply other mathematical concepts. Mathematical concepts are interrelated or not partial [13] so students must apply them appropriately. The results obtained are students trying to associate concepts that have been studied to solve problems but do not use correct understanding. This indicates the limited knowledge of students to apply mathematical concepts that are interrelated with the material being studied.

**Student answer:**

(a) known: 30 swings
   older sister (Nawra) = number 7
   younger sister (Nayla) = number 1
(b) around 50°

**Figure 4.** Example of student answer to problem 1

The epistemological obstacle is also seen from the way students solve problems in question number 3. Students rarely get the kind of contextual problems like question number 3 so that it seems difficult to analyse the images given. They are wrong in giving answers, that dilation, because as if the image given has a different size of the motif. Even though in actual circumstances (not in the form of pictures/photos), tessellation on the terrace has the same size.

4. Conclusion

This study examines the student’s learning obstacles in learning mathematics on geometry transformation topic, especially in the light of didactical and epistemological obstacles which are existence. The findings are expected not only to show an understanding of didactical and epistemological obstacles in geometry transformation topic but as guidance of designing teaching and learning, understanding what mathematical skills can be developed, and a first step in developing didactical designs to overcome didactical and epistemological obstacles in teaching and learning of geometry transformation.

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