Effects of the reflective scattering in hadron production at high energies

S.M. Troshin, N.E. Tyurin

Institute for High Energy Physics,
Protvino, Moscow Region, 142281, Russia

Abstract

A gradual transition to the reflecting scattering mode developing already at the LHC energies is affecting multiparticle production dynamics, in particular, relation of the centrality with the impact parameter values of $pp$-collisions. We discuss the issues in the framework of the geometrical picture for the multiparticle production processes proposed by Chou and Yang. We consider effects of reflective scattering mode presence for the inclusive cross-sections.
Introduction.

Various aspects of the multiparticle production processes are nowadays under studies at the LHC (cf. e.g. [1, 2, 3]). Those being aimed to the searches of the solution of the fundamental problems of QCD and obtaining the observables which could help to understand the mechanisms of confinement, formation of the quark-gluon plasma and properties of the nonperturbative QCD. There is a common opinion that these observables are related to and reflect properties of the deconfined transient state existing in the hadron and nucleus collisions for a very short transient time. A number of distinctive features of multiparticle production processes can be understood using the most simple geometrical picture of hadron interactions which is based on the properties of collisions in the impact parameter space. The geometrical approaches were proposed by many authors, for the most comprehensive considerations one can be referred to the Chou and Yang paper [4].

In this note we discuss how the geometry and characteristic features of the multiparticle production processes would be affected by the gradual transition to the reflective scattering mode. Such transition is expected to start at the LHC energies.

The elastic scattering amplitude in the impact parameter representation can cross the “black disk limit” at small impact parameters and very high energies. Such possibility was considered many years ago in the paper [5] where transition to the “black ring picture” has been anticipated in the general framework of the rational form of unitarization on the base of the CDF data obtained at Tevatron. The value of the energy where the amplitude reaches “black disk limit” was estimated later in [6] and the approximate correspondence between the two estimations performed in [5] and [6] has been established. The energy dependence of the amplitude resulting in the crossing of the black disk limitation is a manifestation of a gradual transition to the reflective scattering mode introduced and discussed in [7]. Recent measurements of the differential cross-section of the elastic pp–scattering at the LHC [8] have led to reopening (cf. e.g. [9, 10]) of the theoretical discussion on the above mentioned regime in elastic scattering and the recent straightforward and model-independent reconstruction of the impact-parameter dependent elastic amplitude has clearly indicated that transition to the reflective scattering mode has already taken place at the LHC energy of 7 TeV [11].

It has been shown that the proton interaction region evolved with growing energy from the BEL (Blacker, Edgier, Larger) [12] picture at lower energies to the REL (Reflective, Edgier, Larger) picture at the LHC energy of 7 TeV. The appearance of the latter mode can be explained by the fact that opening of the new inelastic channels with growing energy, does not lead to saturation
of the total probability of the inelastic collisions at small transverse distance, but instead, results in the destructive interference and the self-damping of the inelastic channels \[7, 13\]. The natural question is how the LHC data can be affected, namely, what are direct or indirect results of this reflective scattering mode at the LHC energies and beyond.

We consider here the implications of the above mentioned issues for the observables related to many-particle production dynamics, in particular, multiplicity distribution of the secondary particles and centrality in \( pp \)-collisions.

1 Reflective scattering mode and multiparticle production.

The most essential feature of the reflective scattering mode is the peripheral impact profile of the inelastic overlap function

\[
h_{\text{inel}}(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{\text{inel}}}{db^2} \tag{1}
\]

which enters the unitarity equation for the elastic scattering amplitude \( f(s, b) \):

\[
\text{Im} f(s, b) = h_{\text{el}}(s, b) + h_{\text{inel}}(s, b). \tag{2}
\]

The function \( S(s, b) = 1 + 2i f(s, b) \) is the \( 2 \to 2 \) elastic scattering matrix element. Since the following discussion is the most qualitative one, we take the amplitude \( f(s, b) \) to be a pure imaginary function, replacing \( f \to if \). The function \( S(s, b) \) is a real one, but can change its sign. The maximum value of \( h_{\text{inel}}(s, b) = 1/4 \) can be reached at high energies at the positive non-zero values of the impact parameter at \( b = R(s) \). The numerical analysis \[11\] clearly indicates that it happens at \( R(s) \simeq 0.2 \ \text{fm} \) at the energy \( \sqrt{s} = 7 \ \text{TeV} \). Energy dependence of the function \( R(s) \) is usually described by the logarithmic function of energy, i.e. \( R(s) \sim \frac{1}{\mu} \ln s \). Such a dependence takes place in the most phenomenological models. It is consistent with the analytical properties of the elastic scattering amplitude and mass \( \mu \) might be related to the mass of pion. Since the derivatives of \( h_{\text{inel}}(s, b) \) have the forms:

\[
\frac{\partial h_{\text{inel}}(s, b)}{\partial s} = S(s, b) \frac{\partial f(s, b)}{\partial s} \quad \text{and} \quad \frac{\partial h_{\text{inel}}(s, b)}{\partial b} = S(s, b) \frac{\partial f(s, b)}{\partial b}, \tag{3}
\]

it is evident that

\[
\frac{\partial h_{\text{inel}}(s, b)}{\partial b} = 0 \quad \tag{4}
\]
at $b = R(s)$. $S(s, b) = 0$ at $b = R(s)$ by the definition of the function $R(s)$, i.e. the complete absorption of the initial elastic channel takes place at the value of the impact parameter $b = R(s)$. This transition from a central to a peripheral form would be slowing the energy dependence of the observables related to the multiparticle production processes. In particular, slow down of the mean multiplicity is expected at the highest values of the LHC energies [14]. Evidently, the derivative of the inelastic overlap function has the sign opposite to the sign of $\partial f(s, b)/\partial b$ in the region where $S(s, b) < 0$. It is the region of the $s$ and $b$ variables where the function $S(s, b)$ is negative (the phase of $S(s, b)$ is such that $\cos 2\delta(s, b) = -1$) is responsible for the transformation of the central impact profile of the function $f(s, b)$ into a peripheral profile of the inelastic overlap function $h_{\text{inel}}(s, b)$ (Fig.1). Thus, at the sufficiently high energies ($s > s_0$), the two separate regions of impact parameter distances can be anticipated, i.e. the outer region of peripheral collisions with scattering of a typical absorptive origin, i.e. $S(s, b)|_{b > R(s)} > 0$ and the inner region of central collisions where the scattering has a combined reflective and absorptive nature, $S(s, b)|_{b < R(s)} < 0$. The function $S(s, b)$ can be rewritten in the form

$$S(s, b) = \kappa(s, b) \exp[2i\delta(s, b)],$$

where $\kappa(s, b)$ and $\delta(s, b)$ are the real functions. The function $\kappa(s, b)$ ($0 \leq \kappa(s, b) \leq 1$) is called an absorption factor which is related to the contribution of the inelastic channels into unitarity relation, its zero value, $\kappa = 0$, corresponds to a complete absorption of the incoming channel. The interpretation of this factor, in fact, depends on the value of the phase $\delta(s, b)$. The transition to the negative values of $S$ leads to appearance of the real phase shift, i.e. $\delta(s, b)|_{b < R(s)} = \pi/2$ [5]. The value of this factor is determined by

![Figure 1: Transition of the inelastic overlap function with growing energy from a central to a peripheral profile.](image-url)
the inelastic channels contribution to the unitarity equation for the elastic scattering amplitude \( f(s, b) \), i.e.

\[
\kappa^2(s, b) = 1 - 4h_{\text{inel}}(s, b),
\]

(6)

It can be easily seen by expressing the function \( h_{\text{inel}}(s, b) \) as a product, i.e

\[
h_{\text{inel}}(s, b) = f(s, b)[1 - f(s, b)].
\]

(7)

When \( f(s, b) \to 1 \) the inelastic overlap function \( h_{\text{inel}}(s, b) \) becomes the most peripheral one while the elastic overlap function \( h_{\text{el}}(s, b) \) remains to be central. Therefore, elastic scattering at large values of \(-t\) is dominated by the pure elastic reflective scattering, while at the large impact parameters (note, that the amplitude \( f(s, b) \) is small in this limit) the following approximate relation is valid

\[
f(s, b) \approx h_{\text{inel}}(s, b).
\]

(8)

This relation means that the diffraction peak in elastic scattering at small values of \(-t\) results from the multiparticle production processes dynamics in this region. We have used here a qualitative kinematical correspondence between small values of \(-t\) and large values of \(b\) and vice versa. It is not surprising therefore that elastic scattering at small and large values of \(-t\) would have different dependencies on the transferred momentums since the scattering processes in these two regions are determined by the different dynamical mechanisms, namely, one is determined by absorption and another one — by reflection.

Typical asymptotic patterns of elastic and inelastic overlap functions are depicted on Fig. 2. It should be noted that the probability of an inelastic process in the hadron collision at the energy \(s\) and impact parameter \(b\) is the following

\[
P_{\text{inel}}(s, b) = 4h_{\text{inel}}(s, b) = \frac{d\sigma_{\text{inel}}}{2\pi bdb}
\]

(9)

and

\[
\sigma_{\text{inel}}(s) = 2\pi \int_0^{\infty} P_{\text{inel}}(s, b)bdb.
\]

(10)

Therefore, any observable, which describe a multiparticle production process, \(A(s, \xi)\) (\(\xi\) is a variable or a set of variables), can be obtained from the corresponding impact-parameter dependent function \(A(s, b, \xi)\) by integrating it with the weight function \(h_{\text{inel}}(s, b)\), e.g.

\[
A(s, \xi) = \frac{\int_0^{\infty} A(s, b, \xi)h_{\text{inel}}(s, b)bdb}{\int_0^{\infty} h_{\text{inel}}(s, b)bdb}.
\]

(11)
Thus, on the grounds of the prominent peripheral dependence of \( h_{\text{inel}}(s, b) \) (it has a peak at \( b = R(s) \)) at asymptotically high energies (cf. Fig. 2) one can obtain the following approximate relation valid in the limit \( s \to \infty \):

\[
A(s, \xi) \simeq A(s, b, \xi)|_{b=R(s)}. \tag{12}
\]

This relation is applicable for many observables associated with particle production processes when the reflective scattering dominates at very high energies. In particular, it can be applicable to the mean multiplicity, average transverse momentum, anisotropic flows \( v_n \) and multiplicity distribution \( P_n(s) \). In general, it means that in the multiparticle production processes the relative range of the variations of the impact parameter is decreasing with energy and the typical inelastic event at very high energy is the event with the non-zero value of the impact parameter in the region around \( b = R(s) \) while the inelastic events at small and large impact parameter values are strongly suppressed. However, it is not the case for elastic collisions, where impact parameter profile being a central one (Fig. 2).

It should be noted that the average values of the impact parameters \( \langle b^2 \rangle_{\text{el}} \) and \( \langle b^2 \rangle_{\text{inel}} \) for elastic and inelastic collisions have the similar asymptotic energy dependencies at \( s \to \infty \), i.e.

\[
\langle b^2 \rangle_{\text{el}}(s), \, \langle b^2 \rangle_{\text{inel}}(s) \sim R^2(s), \tag{13}
\]

while the ratios

\[
\sigma_{\text{el}}(s)/\langle b^2 \rangle_{\text{el}}(s) \sim \text{const.}, \quad \sigma_{\text{inel}}(s)/\langle b^2 \rangle_{\text{inel}}(s) \sim 1/\ln(s) \tag{14}
\]
and as it is evident, behave differently at \( s \to \infty \). The second relation corresponds to decrease with energy of the relative range of the impact parameter fluctuations in the inelastic processes, namely \( \Delta b/\langle b \rangle \) decreases as \( 1/\ln s \) for such processes.

We consider now the multiplicity distribution function \( P_n(s) \),

\[
P_n(s) \equiv \frac{\sigma_n(s)}{\sigma_{\text{inel}}(s)},
\]

where \( \sigma_n(s) \) is the \( n \)-particle production cross-section. According to Eq. (12) \( P_n(s) \) can be written in the form

\[
P_n(s) \simeq \left. P_n(s, b) \right|_{b=R(s)} = \frac{d\sigma_n}{2\pi bdb} \left|_{b=R(s)} \right.
\]

since

\[
\frac{d\sigma_{\text{inel}}}{2\pi bdb} \left|_{b=R(s)} \right. = 1.
\]

### 2 What is the centrality in \( pp \)-collisions?

In nuclear reactions, the results of the measurements are presented for various centralities. This variable is determined as a sum (cf. [15] for the recent discussion of this quantity).

\[
c_N(s) = \sum_{n=N}^{\infty} P_n(s).
\]

If one can correlate centrality with the impact parameter, this variable can be used for description of the collision geometry. The relation of centrality with impact parameter has been obtained for the nuclear reactions in [16]:

\[
c_N(s) \simeq \frac{\pi b^2(N)}{\sigma_{\text{inel}}(s)},
\]

and the same relation was considered to be valid in case of \( pp \)-interactions in [17]. Here \( b(N) \) is the impact parameter value where the mean multiplicity \( \langle n \rangle(s, b) \) is equal to \( N \). In both cases it was supposed that the black disk geometrical picture of the collisions is valid.

This assumption ceases to be true when a gradual transition to the reflective scattering mode starts. As it was already mentioned, it is already experimentally observed at \( \sqrt{s} = 7 \text{ TeV} \) [11]. To obtain an expression for
centrality in case of the reflective scattering domination, one can apply Eq. (16) for $P_n(s)$, i.e.

$$c_N(s) \simeq \sum_{n=N}^{\infty} \frac{d\sigma_n}{2\pi b db} \bigg|_{b=R(s)} = 1 - \sum_{n=3}^{N} \frac{d\sigma_n}{2\pi b db} \bigg|_{b=R(s)},$$

(19)
esince

$$\sum_{n=3}^{\infty} \frac{d\sigma_n}{2\pi b db} = \frac{d\sigma_{inel}}{2\pi b db}.$$  (20)

Thus, when the reflective scattering dominates at $s \to \infty$, centrality $c_N$ is not a measure of the impact parameter, but it is associated with the value of $b = R(s)$, where absorption is maximal. Centrality is then related to the dynamics of the multiparticle production processes when impact parameter can varivate in the relatively narrow range around the value $b = R(s)$.

To proceed further and to get an additional information on centrality we need information on the impact parameter dependence of the cross–sections in multiparticle production and we would like to apply for that purpose the geometrical picture of hadron production which has been mentioned in the Introduction.

### 3 Geometrical picture and multiparticle production.

As it is evident from Fig. 2, elastic scattering occurs in the wide range of the impact parameter variatons. The same is true and for the inelastic processes, but at lower energies only (cf. Fig. 1). It was shown [18] that a wide range of angular momentum produces a coherent superposition and results in the high forward elastic peak observed in the data from the low energies till the LHC ones. In [4] it was proposed that this wide range of impact parameter variations is responsible for the nonstochastic aspects of the multiparticle production dynamics leading to the KNO scaling [19] or negative-binomial distribution of the multiplicity distribution function.

The following geometric mechanism has been proposed in [4]: the broad multiplicity distribution in hadron production has been related to the result of the incoherent superposition of the collisions with the different impact parameter values. In contrast with above, collisions at fixed impact parameter value were supposed to lead a narrow multiplicity distribution. However, as it was shown in [11] the reflective scattering mode has already been detected at $\sqrt{s} = 7$ TeV. Starting from the indicated energy value this mode is expected to become more and more prominent leading asymptotically to Eq.
The relative range of the impact parameter variations will be decreasing with the energy increase and gradual transition from the nonstochastic multiplicity distribution to the stochastic one should be expected.

Combining reflective scattering with the mechanism of Chou and Yang [4], one can conclude that asymptotically the multiplicity distribution would be a product of two Poisson–like distributions one in $n_F$ and another one in $n_B$ ($n_F$ and $n_B$-the multiplicities of the secondary particles in forward and backward hemispheres) both with the average multiplicity $\langle n \rangle_{b=R(s)}$. It should be also noted that since particle production in forward and backward hemispheres occurs at the same value of the impact parameter, it is predicted that $n_F \simeq n_B$ and the correlation parameter $b_c$ defined as

$$b_c = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle n_F^2 \rangle - \langle n_F \rangle^2}$$  \hspace{1cm} (21)$$

tends to unity at $s \to \infty$. This prediction remains valid when reflective mode is present. It seems interesting to check this prediction at the LHC energies.

It should be noted that the reflective scattering mode dominance would affect inclusive cross–section which has the following form (cf. [20] for details and notions) in the rational form of unitarization:

$$E \frac{d\sigma}{d^3q} = 8\pi \int_0^\infty bdb \frac{I(s, b, q)}{|1 - iU(s, b)|^2} \equiv 8\pi \int_0^\infty bdb \frac{I(s, b, q)}{\text{Im}U(s, b)} h_{\text{inel}}(s, b)$$ \hspace{1cm} (22)$$

and

$$\int d^3q \frac{E}{I(s, b, q)} = \langle n \rangle_{s, b} \text{Im}U(s, b).$$ \hspace{1cm} (23)$$

The reflective scattering appears naturally in the $U$–matrix (rational) form of unitarization which has been used above for the inclusive cross–section. In the $U$–matrix approach, the $2 \rightarrow 2$ scattering matrix element in the impact parameter representation is the following linear fractional transform:

$$S(s, b) = \frac{1 + iU(s, b)}{1 - iU(s, b)}.$$  \hspace{1cm} (24)$$

$U(s, b)$ is the generalized reaction matrix, which is considered to be an input dynamical quantity. This relation (24) is one-to-one transform and can easily be invertible. Such form of unitarization can result from from the cofinement condition for the colored degrees of freedom[21]. Inelastic overlap function $h_{\text{inel}}(s, b)$ can be expressed through the function $U(s, b)$ by the following relation

$$h_{\text{inel}}(s, b) = \frac{\text{Im}U(s, b)}{|1 - iU(s, b)|^2},$$  \hspace{1cm} (25)$$
and the only condition to obey unitarity by the elastic scattering amplitude is \( \text{Im}U(s, b) \geq 0 \). Elastic overlap function is related to the function \( U(s, b) \) as follows

\[
h_{\text{el}}(s, b) = \frac{|U(s, b)|^2}{|1 - iU(s, b)|^2}.
\]

The form of \( U(s, b) \) depends on the particular model chosen, for our qualitative purposes it is taken to be an increasing function with energy in a power-like way and decreases with impact parameter like a linear exponent or Gaussian. To simplify the qualitative picture, we consider also the function \( U(s, b) \) as a pure imaginary.

Again, starting from the prominent peripheral dependence of \( h_{\text{inel}}(s, b) \) at asymptotically high energies, one can obtain the following approximate relation for the inclusive cross-section (we have supposed that \( U(s, b) \) is pure imaginary function and have taken into account that \( U(s, b)|_{b=R(s)} = 1 \) valid in the limit \( s \to \infty \):

\[
E \frac{d\sigma}{d^3q} \simeq I(s, b, q)|_{b=R(s)} \sigma_{\text{inel}}(s),
\]

where

\[
\int \frac{d^3q}{E} I(s, b, q)|_{b=R(s)} \simeq \langle n \rangle(s).
\]

Thus, to obtain inclusive cross-section at very high energies, one should perform modelling of inelastic hadron interactions at the value of the impact parameter of the colliding hadrons around \( b = R(s) \), where the complete absorption occurs. The impact parameter \( b \) is a weighted sum of the impact parameters of the final particles

\[
b = \sum_{i=1}^{n} x_i b_i,
\]

where \( x_i \) is a Feynman \( x \) of a final particle \( i \).

To proceed further in this way of modelling we consider one particularly interesting class of the multiple production processes, i.e. the double–pomeron exchange (dpe) processes of the type

\[
pp \to p + X + p,
\]

where plus corresponds to a gap in rapidity and \( X \) is a system (or a particle) of particles deprived of a rapidity gap. According to Eq. (27), the inclusive cross-section of the dpe–process can be written in the form

\[
E \frac{d\sigma_{\text{dpe}}}{d^3q} \simeq I_{\text{dpe}}(s, b, q)|_{b=R(s)} \sigma_{\text{inel}}(s).
\]
This cross-section corresponds to the selection from the set of all final states $|n\rangle$ of the states $|n\rangle_{dpe}$ relevant for the double-pomeron exchange processes. To construct the expression for the function $I_{dpe}(s, b, q)$ we use notions described in [20] and write it as a convolution of the two distributions of the condensates in the colliding protons and condensate (or parton) cross-section:

$$I_{dpe}(s, b, q) = \int D_c(b_1)\sigma_0(s, b - b_1 - b_2, q)D_c(b_2)db_1db_2,$$

(30)

where $\sigma_0(s, b - b_1 - b_2, q)$ is the (point-like) cross-section of condensate interaction written in the form

$$\sigma_0(s, b - b_1 - b_2, q) = \tilde{\sigma}_0(s, q)\delta(b - b_1 - b_2).$$

(31)

Thus, $I_{dpe}$ can be written as a convolution

$$I_{dpe}(s, b, q) = \tilde{\sigma}_0(s, q)\int D_c(b - b_1)D_c(b_1)db_1$$

(32)

We can assume that the condensate distribution in the hadron is controlled by the pion mass and $I_{dpe}$ has therefore the form

$$I_{dpe}(s, b, q) \simeq \tilde{\sigma}_0(s, q)e^{-m_\pi b}.$$  

(33)

Since $R(s) = \frac{1}{\mu} \ln s$, we will have a suppression factor decreasing with the energy

$$I_{dpe}(s, b, q)|_{b = R(s)} \simeq \tilde{\sigma}_0(s, q)s^{-m_\pi/\mu}.$$  

(34)

This factor arises due to the reflective scattering mode dominating at $s \to \infty$ and enters to the expression for the $dpe$ inclusive cross-section affecting its energy dependence given by a product of the functions $\tilde{\sigma}_0(s, q)$ and $\sigma_{inel}(s)$, i.e.

$$E \frac{d\sigma_{dpe}}{d^3q} \simeq \tilde{\sigma}_0(s, q)\sigma_{inel}(s)s^{-m_\pi/\mu},$$

(35)

where $\sigma_{inel}(s) \sim \ln s$ at $s \to \infty$ when the reflective mode becomes dominating one. Thus, in this energy limit the double-pomeron exchange processes would not survive unless $\tilde{\sigma}_0(s, q)$ is growing with energy at least as a power of it, i.e. $\tilde{\sigma}_0(s, q) \sim s^\alpha$ with $\alpha \geq -m_\pi/\mu$ (we neglect here the logarithmic-type dependencies).

**Conclusion.**

In general, the reflective scattering mode would change nonstochastic dynamics of multiparticle production at the available energies to an almost
stochastic one at the asymptotic energies. Transition to the reflective dynamics starts at the LHC energies and this fact makes searches of its signatures in multiparticle production, which were discussed above to be rather interesting. As it was noted in [7] transition to reflective scattering mode would provide a faster decrease of the energy spectrum reconstructed from extensive air showers in the cosmic rays measurements, i.e. it will result in appearance of the knee in this spectrum. This knee is correlated with the slowing-down of the mean multiplicity growth discussed in [14] and should occur in the same region of energy.

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