Multi GNSS Precise Point Positioning Using Adaptive Kalman Filter

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Abstract

In recent years, satellite navigation systems have been actively developed in the world, and they are called GNSS (Global Navigation Satellite System) which includes Global Positioning System (GPS) of the United States, Russian Global Navigation Satellite System (GLONASS), European Galileo (GAL), Chinese BeiDou Navigation Satellite System (BDSS) and Japanese Quasi-Zenith Satellite System (QZSS). In this paper, we focus on the so-called multi GNSS precise point positioning (PPP), and propose a positioning method to improve its performance.

1 Introduction

Currently, many GNSSs (Global Navigation Satellite Systems) are being developed and can be used. In October 2018, over 100 satellites are available. Also, PPP (Precise Point Positioning) has been widely used for research and various applications, such as GNSS meteorology, automatic driving agriculture and robotics. In the GNSS PPP, generally, the user position is estimated by the Kalman filter based on the observables. The code pseudorange and carrier phase are the measurements of the distance between satellites and the receiver, and the accuracy of each measurement depends on the signal characteristic such as signal transmission power, antenna gain, frequency of the carrier wave and structure of code signal. They depend on the satellite system applied, in other words, the accuracy of the measurement differs depending on the satellite system. Therefore, in order to apply the Kalman filter, the covariance matrix of the measurement errors are estimated from the nominal signal specifications provided by developers of systems. However, it is a common problem that the accuracy of the measurement depends on not only the signal specification (satellite system) but also the individual difference of the satellite, elevation angle of the satellite, degradation of the signal to noise ratio due to obstacles etc. Therefore, it is necessary to determine the covariance matrix of the measurement noise adaptively for highly accurate positioning methods, and several methods have been proposed [1],[2]. For example, in [1], the weight for each measurement is determined as the function of the elevation angle of the satellite. In this paper, we apply and extend the concept of the adaptive Kalman filter [3],[4],[5] to multi GNSS PPP, and propose the method which can provide improved positioning performance. In this paper, we introduce the model of the observables of L1 and L2 pseudoranges and carrier phases in GNSS signal, such that we have developed the PPP algorithms [6]. Then, we explain the normal Kalman filter for GNSS PPP and the adaptive Kalman filter used in this paper. Finally, we show the result of a comparative experiment between the conventional method and the proposed method.

2 Measurement Model

At first, based on [6]–[8], the measurement models for PPP are shown. We consider the following fundamental measurements of the pseudoranges $\rho_{C1,u}$, $\rho_{C2,u}$ based on L1 and L2 code, and L1 and L2 bands carrier phases $\varphi_{L1,u}$, $\varphi_{L2,u}$ as follows:

$$\rho_{C1,u} = r_u + c[\delta t_u - \delta t^p] + \delta I_u + \delta T^p_u + \delta b^p_{C1,u} + \delta b^p_{C2,u}$$  \hspace{1cm} (1)

$$\rho_{C2,u} = r_u + c[\delta t_u - \delta t^p] + \frac{f_2^2}{f_1^2} \delta I_u + \delta T^p_u + \delta b^p_{C2,u}$$ \hspace{1cm} (2)

$$\varphi_{L1,u} = \lambda_1 \varphi_{L1,u}$$

$$\varphi_{L2,u} = \lambda_2 \varphi_{L2,u}$$

where $p$ is the satellite, $u$ is the receiver, $r_u$ is the geometric distance between the receiver $u$ and satellite $p$, $c$ ($\approx 2.99792458 \times 10^8$[m]) denotes the speed of light, $\delta t_u$ and $\delta t^p$ are the receiver and satellite clock errors respectively, $\delta T^p_u$ is the tropospheric delay, $f_1$ and $\lambda_1$ are the central frequency and the wave length of the $L_1$ carrier wave, $\delta b^p_{C1,u}$, $\delta b^p_{C2,u}$, $\delta b^p_{L1,u}$ and $\delta b^p_{L2,u}$ are
the so-called hardware biases, \( N_u^p \) denotes integer ambiguity between the satellite \( p \) and the receiver \( u \) and \( p, \phi_{wind} \) is the phase wind up correction and \( e^p \) and \( e^p \) denote measurement errors. Also \( \delta T_u^p \) is the ionosphere delay. Since the ionosphere delay depends on the frequency, it is possible to eliminate it with linear combination in observations of two or more frequencies. It is called Ionosphere-free combination and as follows [9]:

\[
\rho_{IF,u}^p = \frac{f_2^p \cdot \rho_{C1,u}^p + f_2^p \cdot \rho_{C2,u}^p}{f_2^p - f_1^p} = r_u^p + c [\delta t_u - \delta t^p] + \delta T_u^p + \delta I_{IF,C,u}^p + e^p_{IF,u}
\]

(5)

\[
\Phi_{IF,u}^p = \frac{f_2^p \cdot \lambda_1 \varphi_{l1,u}^p + f_2^p \cdot \lambda_2 \varphi_{l2,u}^p}{f_2^p - f_1^p} = r_u^p + c [\delta t_u - \delta t^p] + \delta T_u^p + \delta I_{IF,L,u}^p + \lambda_{IF} N_u^p + \lambda_{IF} \varepsilon_{IF,u}^p
\]

(6)

where

\[
\lambda_{IF} = \frac{f_2^p \cdot \lambda_1 + f_2^p \cdot \lambda_2}{f_2^p - f_1^p}
\]

(7)

Because the measurement models (5)–(6) are nonlinear due to the geometric distance term \( r_u^p \), it is linearized by the first order Taylor series approximation around the initial estimate \( \hat{u} \) and \( \hat{s} \) as follows:

\[
r_u^p \approx r_u^p + \frac{\partial r_u^p}{\partial u} [u - \hat{s} - (\hat{u} - \hat{s}^p)]
\]

(8)

where \( u \equiv [x_u, y_u, z_u]^T \) and \( s^p \equiv [x^p, y^p, z^p]^T \) are the user (unknown) and satellite positions, respectively and for \( p = 1, 2, \ldots, n_s \), where

\[
\hat{s}^p \equiv \left[ \frac{\partial r_u^p}{\partial u} \right]_{u=\hat{u},s=\hat{s}^p} (\hat{u} - \hat{s}^p) \quad \frac{\partial r_u^p}{\partial s} (\hat{u} - \hat{s})
\]

(9)

Also the estimate of the satellite position \( \hat{s} \) can be obtained from the precise ephemerides of all satellites provided by the International GNSS Service (IGS) [10]. From (5)–(6), therefore, we have the approximations:

\[
\rho_{IF,u}^p \approx \hat{\rho}_u^p + c [\delta t_u - \delta t^p] + \delta T_u^p + \delta I_{IF,C,u}^p + e^p_{IF,u}
\]

(10)

\[
\Phi_{IF,u}^p \approx \hat{\Phi}_u^p + c [\delta t_u - \delta t^p] + \delta T_u^p + \delta I_{IF,L,u}^p + \lambda_{IF} N_u^p + \lambda_{IF} \varepsilon_{IF,u}^p
\]

(11)

The following parameters in (10)–(11) are estimated by GNSS PPP models. The satellite position \( \hat{s}^p \) and clock errors \( \delta t^p \) are obtained from the precise orbit and clock error information provided by GFZ (GeoforschungsZentrum) [11], tropospheric dry delay which is a part of \( T_u^p \) is obtained from the zenith delay model and mapping function, phase wind up correction \( \phi_{wind} \) is by the model[12]. The receiver and satellite phase center offset (PCO) and phase center variation (PCV) [10] from IGS, Earth rotation [10], Differential Code Bias, tidal loading are from International Earth Rotation and Reference Systems Service (IERS) model [13] are taken into account in the positioning calculations. Also, the unknown terms are receiver position \( u \), receiver clock error \( \delta t_u \), tropospheric wet delay \( T_u \) and the integer ambiguity \( N_u^p \).

Then from (10)–(11), we have the following multi GNSS (G: GPS, R: GLONASS, E: Galileo, C: BeiDou, J: QZSS) measurement equations from PPP as follows:

\[
\rho_{IF,u}^{p,G} = \hat{\rho}_u^{p,G} + c [\delta t_u^G - \delta t^{p,G}] + \delta T_u^G + \delta I_{IF,C,u}^G + e^{p,G}_{IF,u}
\]

(12)

\[
\rho_{IF,u}^{p,R} = \hat{\rho}_u^{p,R} + c [\delta t_u^R - \delta t^{p,R}] + \delta T_u^R + \delta I_{IF,C,u}^R + e^{p,R}_{IF,u}
\]

\[
\rho_{IF,u}^{p,E} = \hat{\rho}_u^{p,E} + c [\delta t_u^E - \delta t^{p,E}] + \delta T_u^E + \delta I_{IF,C,u}^E + e^{p,E}_{IF,u}
\]

\[
\rho_{IF,u}^{p,C} = \hat{\rho}_u^{p,C} + c [\delta t_u^C - \delta t^{p,C}] + \delta T_u^C + \delta I_{IF,C,u}^C + e^{p,C}_{IF,u}
\]

\[
\rho_{IF,u}^{p,J} = \hat{\rho}_u^{p,J} + c [\delta t_u^J - \delta t^{p,J}] + \delta T_u^J + \delta I_{IF,C,u}^J + e^{p,J}_{IF,u}
\]

(12)

where \( \delta s \), denotes the number of satellites of the GNSS system indicated by the subscript "S". For example, if the number of GPS satellites \( p_G \) is \( n_G \), it becomes \( \rho_{IF,u}^{p,G} \) as follow:

\[
\rho_{IF,u}^{p,G} = \begin{bmatrix} \rho_{IF,u}^{p,G} \\ \rho_{IF,u}^{p,G} \\ \rho_{IF,u}^{p,G} \end{bmatrix}
\]

(12)
GPS, GLONASS and QZSS, E1 and E5a for Galileo and B1 and B2 for BeiDou. So that, $\lambda_{G}\approx 0.1070$[m], $\lambda_{R}\approx 0.1051$[m], $\lambda_{E}\approx 0.1089$[m], $\lambda_{F}\approx 0.1083$[m] and $\lambda_{I}\approx 0.1070$[m] are wave length of each satellite Ionosphere-free combination.

Also, we have the following vector regression equation:

$$y_n = H_ux_u + v_u$$  \hspace{1cm} (13)

where

$$y_n = \begin{bmatrix} p_{G,R} \\ p_{R,R} \\ p_{E,E} \\ p_{E,E} \\ p_{E,C} \\ p_{E,C} \\ p_{F,R} \\ p_{F,R} \\ p_{F,E} \\ p_{F,E} \\ p_{F,C} \\ p_{F,C} \end{bmatrix}, \quad x_u = \begin{bmatrix} u \\ \delta T^G_u \\ \lambda^G_F \Phi^G_F \Phi^G_F \\ \lambda^G_R \Phi^G_R \Phi^G_R \\ \lambda^G_E \Phi^G_E \Phi^G_E \\ \lambda^G_C \Phi^G_C \Phi^G_C \\ \lambda^F_R \Phi^F_R \Phi^F_R \\ \lambda^F_E \Phi^F_E \Phi^F_E \\ \lambda^F_C \Phi^F_C \Phi^F_C \end{bmatrix}, \quad v_u = \begin{bmatrix} \epsilon_{p_{G,R}} \\ \epsilon_{p_{R,R}} \\ \epsilon_{p_{E,E}} \\ \epsilon_{p_{E,E}} \\ \epsilon_{p_{E,C}} \\ \epsilon_{p_{E,C}} \\ \epsilon_{p_{F,R}} \\ \epsilon_{p_{F,R}} \\ \epsilon_{p_{F,E}} \\ \epsilon_{p_{F,E}} \end{bmatrix}$$

$$H_u = \begin{bmatrix} G_{G,R} & 0 & 0 & 0 & 0 & M^p_{F} & 0 & 0 & 0 & 0 \\ G_{R,R} & 0 & 0 & 0 & 0 & M^p_{R} & 0 & 0 & 0 & 0 \\ G_{E,E} & 0 & 0 & 1 & 0 & M^p_{E} & 0 & 0 & 0 & 0 \\ G_{E,E} & 0 & 0 & 0 & 1 & M^p_{E} & 0 & 0 & 0 & 0 \\ G_{C,C} & 0 & 0 & 0 & 0 & M^p_{C} & 0 & 0 & 0 & 0 \\ G_{G,R} & 1 & 0 & 0 & 0 & M^p_{G} & 0 & 0 & 0 & 0 \\ G_{R,R} & 0 & 0 & 1 & 0 & M^p_{R} & 0 & 0 & 0 & 0 \\ G_{E,E} & 0 & 0 & 1 & 0 & M^p_{E} & 0 & 0 & 0 & 0 \\ G_{C,C} & 0 & 0 & 0 & 1 & M^p_{C} & 0 & 0 & 0 & 0 \\ G_{G,R} & 0 & 0 & 0 & 0 & M^p_{G} & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $M^p$ is the known matrix which comes from the Mapping function by using Global Mapping Function [14], $I \equiv [1, \ldots, 1]^T$: $n_s \times 1$ vector, $0 \equiv [0, \ldots, 0]^T$: $n_s \times 1$ vector and $O \equiv [0, \ldots, 0]^T$: $n_s \times n_s$ matrix.

where $n_s$ is the number of satellites corresponding to the focusing satellite system. For example, focusing on the first row of $H_u$, the size of $G_{u,G} = p_G \times 3$, therefore $n_s$ for the first row is $p_G$, and $I$ is the $n_s \times n_s$ matrix identity as follows:

$$G_u^G = \begin{bmatrix} g_{1}^G \\ g_{2}^G \\ \vdots \\ g_{n_s}^G \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{bmatrix}$$  \hspace{1cm} (14)

And then, Kalman filter is applied to the observation equation (13) for multi GNSS PPP implementation.

3 Kalman filter

Hereafter, in order to simplify the notation, the subscripts "u"s is equation (13) will be dropped, and sub-

script "t" which shows the observation epoch time is added to each component is equation (13). The GNSS state and observation model of multi GNSS PPP can be expressed with appropriate linearization as follows:

$$x_{t+1} = F_tx_t + w_t$$

$$y_t = H_tx_t + v_t$$

where $x_t \in R^n$ is state vector, $y_t \in R^p$ is observation vector, $F_t \in R^{n \times n}$, $H_t \in R^{p \times n}$ are coefficient matrix and $w_t \in R$, $v_t \in R^p$ are Gaussian white noise with system noise and observation noise vector, respectively.

$$E\{w_t\} = E\{v_t\} = 0$$

$$E\{w_tw_T\} = Q_t$$

$$E\{v_tv_T\} = R_t$$

where $E\{\}$ is the expectation function. Then calculating unknown parameters $x_t$ by applying Kalman filter to the state and observation model. The predicted state vector $\hat{x}_{t|t-1}$ and its covariance matrix $P_t$ are as follows:

$$\hat{x}_{t|t-1} = F_{t-1}\hat{x}_{t-1|t-1} + Q_{t-1}$$

$$P_{t|t-1} = F_{t-1}P_{t-1|t-1}F_{t-1}^T + Q_{t-1}$$

$$F_t = \begin{bmatrix} I_{3 \times 3} \\ O \\ O \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_N^2 \end{bmatrix}$$

where $n_m$ is the number of receiver clock errors, $n_s$ is the number of satellites, $\sigma_x$, $\sigma_y$ and $\sigma_N$ are standard deviations of the system noise of receiver position, $\sigma_N$ is standard deviation of the system noise of multi GNSS receiver clock error, $\sigma_T$ is standard deviation of the system noise of tropospheric delay and $\sigma_N$ is standard deviation of the system noise of integer ambiguity. Also measurement update of the state vector and covariance matrix are represented by measurement vector $y_t$ at epoch $t$ as follows:

$$z_t = y_t - H_t\hat{x}_{t|t-1}$$

$$x_{t|t} = \hat{x}_{t|t-1} + K_tz_t$$

$$P_{t|t-1} = K_tP_{t|t-1}H_t^T + R_t$$

$$K_t = P_{t|t-1}H_t^T(H_tP_{t|t-1}H_t^T + R_t)^{-1}$$

where $z_t$ is the innovation, $K_t$ is the Kalman gain.

4 Adaptive Kalman filter

An adaptive Kalman filter provides estimates of covariance matrices of system and observation noise.
One of the most important techniques of the adaptive Kalman filter is innovation-based algorithm. The innovation-based adaptive Kalman filter used to adapt the observation noise covariance matrix $R_t$ [3] [4] [5]. The matrix can be adapted based on the innovation process $z_t$ as follows:

$$\hat{R}_t = \hat{C}_{\nu_t} - H_t P_{t|t-1} H_t^T$$  \hspace{1cm} (27)

where $\hat{C}_{\nu_t}$ is the estimated variance-covariance matrix of the innovation and it can be computed by averaging in the moving estimation window of size $N$ as follows:

$$\hat{C}_{\nu_t} = \frac{1}{N} \sum_{i=1}^{N} z_{t-i} z_{t-i}^T$$  \hspace{1cm} (28)

### 4.1 Residual based Adaptive Kalman filter

Another adaptive estimation of covariance matrix is the residual-based algorithm. The residual $\nu_t$, which is the difference between the present observation and its estimated value, can be expressed using equations (23)-(24) as follows:

$$\nu_t = y_t - H_t \hat{x}_{t|t}$$

$$\nu_t = y_t - H_t (\hat{x}_{t|t-1} + K_t z_t)$$

$$= (I - H_t K_t) z_t$$  \hspace{1cm} (29)

In this case the covariance matrix $R$ takes the following from (27) as follow:

$$\hat{R}_t = \hat{C}_{\nu_t} + H_t P_{t|t-1} H_t^T$$  \hspace{1cm} (30)

where $\hat{C}_{\nu_t}$ is calculated as follows:

$$\hat{C}_{\nu_t} = \frac{1}{N} \sum_{i=1}^{N} \nu_{t-i} \nu_{t-i}^T$$  \hspace{1cm} (31)

The residual can be used not only adaptive estimation but also unknown error of measurement data. The same strategy used for $R$ will also be used to obtain an estimate of the covariance matrix of system noise $Q$.

## 5 Experimental Results

The following subsections show configurations and results of experiments that have been carried out to evaluate the proposed method.

### 5.1 Experimental configuration

In the experiment, the GNSS data were obtained from the reference point in JAXA MADOCOA on January 1, 2018, and the true position is shown in Table 1. The data and experimental conditions are shown in Table 2.

By using the true position provided by the JAXA, we compared the performance by evaluating the error from the true value.

### Table 1: True position

| X[m] | Y[m] | Z[m] |
|------|------|------|
| -3961263.4128 | 3308913.7043 | 3734564.5919 |

### Table 2: Experimental configuration

| Data | Tsukuba, Japan (TKSC from MADOCOA product) |
|------|-------------------------------------------|
| Time | 2018/01/01 00:00:00 - 2018/01/01 23:59:30 (GPST) |
| Epoch | data 7 30 sec |
| Elevation mask | 10 deg |
| GNSS System Usage | GPS(C1C,C2W,L1C,L2W), GLONASS(C1C,C2C,L1C,L2C), Galileo(C1X,C5X,L1X,L5X), Beidou(C2I,C7I,L2I,L5I), QZSS(C1C,C2X,L1C,L2X) |
| Orbit position | Precise orbit(GPS) |
| Orbit clock error | Precise clock(GPS) |
| Estimation | Kalman filter |
| window size | 5/10/15/20 |
| Receiver | TRIMBLE NETR9 |
| Antenna | SEPCHOKE,MC(Trimble) |

### 5.2 Kalman filter configuration

The covariance matrix of the process noise is shown in Table 3 and the initial covariance matrix of the state vector $P_0^{-1}$ is shown in Table 4.

As the conventional methods, we have conducted the experiment, with the following two methods:

(i) the covariance matrix $R$ (18) is set based on the value in Table 5.

(ii) The variance of each measurement is weighted depending on the satellite’s elevation angle $\theta$ as follows:

$$\hat{R} = \text{diag}(\sigma_{E,c,IF}^2, \sigma_{E,c,IF}^2, \sigma_{E,c,IF}^2, \sigma_{G,p,IF}^2, \sigma_{G,p,IF}^2, \sigma_{G,p,IF}^2, \sigma_{C,p,IF}^2, \sigma_{C,p,IF}^2, \sigma_{C,p,IF}^2)$$  \hspace{1cm} (32)

$$R = \frac{1}{\sin^2 \theta} \times \hat{R}$$  \hspace{1cm} (33)

### 5.3 Result of position error

Fig. 1 shows the east, north and up positioning errors of Kalman filter of conventional methods and adaptive Kalman filter. In Fig. 1, the red line shows the error of the conventional methods of (i), the blue line shows the error of the conventional methods of (ii) and the green line shows the error of the adaptive Kalman filter with window size is 5. Also Table 6 shows the result of East, North, Upper and 3D RMS errors.

Next, Fig. 2 shows the positioning errors with changing window size of adaptive Kalman filter. In Fig. 2, the blue line shows the error of the adaptive Kalman filter with window size 5, the red line shows the error of the adaptive Kalman filter with window size 10, the
Table 3: Process noise in \( Q \)

| name          | value [m] |
|---------------|-----------|
| Position X \( \sigma_x \) | 0         |
| Position Y \( \sigma_y \) | 0         |
| Position Z \( \sigma_z \) | 0         |
| Receiver Clock error \( \sigma_{\delta_t} \) | \( 1 \times 10^{-5} \) |
| Toroposphic error \( \sigma_{T_a} \) | \( 1 \times 10^{-7} \) |
| Integer ambiguity \( \sigma_N \) | \( 1 \times 10^{-7} \) |

Table 4: Initial covariance matrix in \( P_{0|-1} \)

| name          | value [m] |
|---------------|-----------|
| Position X    | 1         |
| Position Y    | 1         |
| Position Z    | 1         |
| Receiver Clock error | 5         |
| Toroposphic error | 0.03     |
| Integer ambiguity | 5        |

Table 5: Measurement noise

| name          | value [m] |
|---------------|-----------|
| GPS IF Code \( \sigma_{G,e,IF} \) | 0.9       |
| GLONASS IF Code \( \sigma_{R,e,IF} \) | 2.7       |
| Galileo IF Code \( \sigma_{E,e,IF} \) | 0.9       |
| BeiDou IF Code \( \sigma_{C,e,IF} \) | 0.9       |
| QZSS IF Code \( \sigma_{J,e,IF} \) | 0.9       |
| GPS IF phase \( \sigma_{G,p,IF} \) | 0.009     |
| GLONASS IF phase \( \sigma_{R,p,IF} \) | 0.027     |
| Galileo IF phase \( \sigma_{E,p,IF} \) | 0.009     |
| BeiDou IF phase \( \sigma_{C,p,IF} \) | 0.009     |
| QZSS IF phase \( \sigma_{J,p,IF} \) | 0.009     |

Fig. 1: ENU Position error Kalman filter and adaptive Kalman filter

green line shows the error of the adaptive Kalman filter

Table 6: ENU Position error Kalman filter and adaptive Kalman filter

| Method                          | Direction | RMS [m] |
|---------------------------------|-----------|---------|
| Conventional method             | East      | 0.0696  |
| No weight i)                    | North     | 0.1561  |
|                                 | Up        | 0.3921  |
|                                 | 3D        | 0.2059  |
| Conventional method             | East      | 0.0427  |
| Elevation angle ii)             | North     | 0.0948  |
|                                 | Up        | 0.2211  |
|                                 | 3D        | 0.1193  |
| Adaptive Kalman filter          | East      | 0.0310  |
|                                 | North     | 0.0581  |
|                                 | Up        | 0.1385  |
|                                 | 3D        | 0.0759  |

Fig. 2: ENU Position error adaptive Kalman filter by changing window size

We can observe from Fig. 1 and Table 6 that the multi GNSS PPP performance is improved by using the adaptive Kalman filter. The unknown error is eliminated by using adaptive Kalman filter. So, it seems to be more effective when used in appropriate cases such as when the large differences of the measurement noises are large. Also, there is a possibility that convergence will be fast by using the adaptive Kalman filter. However, we can observe from Fig. 2 and Table 7 that, multi GNSS PPP performance have no large differences with respect to the window size of adaptive Kalman filter.

6 Conclusions

In this paper, the multi GNSS PPP technique and the adaptive Kalman filter method has been applied to PPP
Table 7: ENU Position error adaptive Kalman filter by changing window size

| Method       | Direction | RMS[m] |
|--------------|-----------|--------|
| adaptive     | East      | 0.0310 |
| Window size 5| North     | 0.0581 |
|              | Up        | 0.1385 |
|              | 3D        | 0.0759 |
| adaptive     | East      | 0.0345 |
| Window size 10| North   | 0.0561 |
|              | Up        | 0.1539 |
|              | 3D        | 0.0815 |
| adaptive     | East      | 0.0345 |
| Window size 15| North   | 0.0547 |
|              | Up        | 0.1399 |
|              | 3D        | 0.0764 |
| adaptive     | East      | 0.0398 |
| Window size 20| North   | 0.0613 |
|              | Up        | 0.1391 |
|              | 3D        | 0.0801 |

in order to improve the performance of the positioning. From the experimental results, we can conclude that the performance of the multi GNSS PPP can be performed by using adaptive Kalman filter method.

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