Superradiance on the Landau levels and the problem of power of decameter radiation of Jupiter

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Abstract

We determine the conditions of formation of spontaneous polarization phase transition to the superradiance regime in the inverted system of nonrelativistic electrons on equidistant Landau levels in rarefied magnetized plasma. The possibility of realization of such conditions in the lower Jupiter magnetosphere is shown. The effect of cyclotron superradiance on the Landau levels gives a key to interpretation of the nature of superpower radioemission of the Jupiter-Io system.

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1 Introduction.

The phenomenon of superradiance (SR) was considered for the first time in the known paper by Dicke [1] in the example of a two-level model. At present, a significant number of publications are devoted to its study (see for example, reviews [2,3,4]), but, as noted by many authors, the theme is far from being exhausted, and many interesting and physically important questions and situations still remain to be investigated. One of them is the question of the possibility of realization of SR in systems with equidistant levels in the presence of inversion. The system of fast electrons in a homogeneous magnetic field, for which the transversal movement (rotation) energy spectrum is described by the known Landau levels

\[ E_\perp = n\hbar\omega, \quad n = 0, 1, 2, \ldots, \]

\[ \omega = \frac{eH}{mc}, \]

is the most easily realizable and important of such systems [5]. The question on the possibility and conditions of SR formation in such systems, besides general physical interest, is also of big astrophysical interest because the inversion on the Landau levels is easily achieved in rarefied magnetized plasma of active space objects in the presence of bunches of accelerated electrons if their initial velocities are directed to some angle to the magnetic field, and so, besides the longitudinal energy \( E_\parallel \), they also possess significant transversal energy \( E_\perp \). For example, in the Jupiter magnetosphere [6,7], the observed superpower sporadic nonthermal decameter radiation of the Jupiter-Io system with brightness temperature up to \( 10^{15} - 10^{17} \) K can serve as an evidence of the possible realization of such situation and generation of SR.

In the present work, we investigate the question about the possibility and conditions of SR formation in the inverted system of electrons on high Landau levels (1) with \( n \gg 1 \) and the possibility of realization of these conditions in the lower Jupiter magnetosphere. As it is known, for the generation of the induced coherent radiation in systems like masers, the equidistance of energy levels is an obstacle because of the specific competition of radiation and absorption processes in this case. In the case of SR, we deal not with induced but with spontaneous radiation, and here, as we shall see, equidistance of energy levels appears to be of advantage. This is, first of all, because the SR regime is usually realized in open finite systems without mirrors, where radiation leaves the active volume of generation quickly enough, having practically no time to get in the absorption regime [3]. Second, in this case, all the inverted electrons occupy, as a rule, not one level, but some significant interval of high levels \( \Delta n \ (n \gg \Delta n \gg 1) \), and because of equidistance, all of them radiate the same mode on frequency (2), and also, as we shall see, at the same rate independent of the initial energy.

The phenomenon of SR arises when in “coherence domains”, with sizes \( R_0 \) smaller then the wavelength \( \lambda \), all \( N_0 \) radiating dipoles gradually during radiation become aligned in one direction due to the dipole-dipole interaction between them in the “near zone” \( (R_0 \ll \lambda) \), so that, as a result, the total dipole of the domain \( \vec{D} \) turns out to be \( N_0 \) times larger then the elementary dipole \( \vec{d} \). Therefore, the intensity of the collective dipole radiation becomes proportional to \( N_0^2 \), and not \( N_0 \) as in the case of radiation of uncorrelated dipoles. This is described in Sec. 2.

The transition to such a correlated polarized state is similar to the phase transition in magnetics or ferroelectrics, and for its description it is convenient to use the Weiss method of mean self-consistent field [8]. This theory is developed in Sec. 3. We note that the phase
transition under consideration is a nonequilibrium one, and it has all the features of the self-organization phenomena in dissipative systems. In Sec. 4, we discuss the key role of the SR effect for the interpretation of the observable power of the decameter radiation of the Jupiter-Io system.

2 Cyclotron superradiance on Landau levels.

For large \( n \), levels (1) correspond to quantum states with wave functions localized near classical Larmor orbits with radii \( r_L = V_\perp/\omega \):  

\[
    r_n = \sqrt{\frac{\hbar}{eH}}(2n + 1) = \sqrt{\frac{2n\hbar\omega}{m\omega^2}} \left(1 + \frac{1}{2n}\right) \approx \sqrt{\frac{2E_\perp}{m\omega^2}} = \frac{V_\perp}{\omega} = r_L.  
\]

In this connection, we can proceed to (quasi)classical description of such states and transitions between them. In a classical limit, in a coordinate system in which the longitudinal movement is absent, these orbits are given by

\[
    \mathbf{r}_\perp(t) = r_L \{\cos(\omega t + \alpha), \sin(\omega t + \alpha), 0\},  
\]

where the cartesian components of the vector \( \mathbf{r}_\perp \) are written in braces.

Being initially inverted on high levels, electrons start to fall down step by step on the ladder of states (1), radiating quanta with frequency (2). The differential (angular) and integral intensities of the dipole radiation of one electron in the classical limit are described by the known formulas [9]

\[
    \frac{dI}{d\Omega} = \frac{\mathbf{d} \times \mathbf{n}_k}{4\pi c^3}, \quad \mathbf{n}_k = \frac{\mathbf{k}}{k};  
\]

\[
    I = \frac{2e^2\omega^2V_\perp^2}{3c^3} = \frac{4e^2\omega^2}{3mc^3} E_\perp.  
\]

Equation (6) implies the following evolution law of the electron energy:

\[
    \frac{dE_\perp(t)}{dt} = -I = -\frac{E_\perp}{\tau}, \quad \tau = \frac{3mc^3}{4e^2\omega^2};  
\]

\[
    E_\perp(t) = E_\perp(0) \exp(-t/\tau).  
\]

We see that the radiation time \( \tau \) does not depend on \( E_\perp \), i.e., electrons with different initial energies [within the limits of dispersion \( \Delta E_\perp(0) \ll E_\perp(0) \)] will fall down at the same rate. It is easy to see that the dispersion of energy will also decrease at the same rate,

\[
    \Delta E_\perp(t) = \Delta E_\perp(0) \exp(-t/\tau),  
\]

so that the following relation is satisfied:

\[
    \Delta E_\perp(t)/E_\perp(t) = \text{const} \ll 1.  
\]

This allows one to judge about the evolution of the whole collective of electrons by the evolution of their average energy and other quantities. Therefore, for simplicity, in what follows, by the symbols \( E_\perp, V_\perp, \) and \( d_0 = e\tau_L = eV_\perp/\omega \) we denote the corresponding values averaged over the ensemble of inverted electrons.
with the mutual aligning of dipoles. If correlations are absent, i.e., if the contributions from all electrons. The second term in the square brackets describes the correlation effects connected we get

Thus, a large enough number of $N_0$ radiating dipoles will be in one volume $V_{coh}$:

$$n_e V = N \gg N_0 = n_e V_{coh} \gg 1.$$  

Let us consider the total dipole moment of such a subsystem

$$\mathbf{D}(t) = \sum_{j=1}^{N_0} \mathbf{d}_j(t) = e \sum_{j=1}^{N_0} \mathbf{r}_j(t).$$  

Similarly to (5) for one electron, the collective dipole radiation of our subsystem will be described by the formula

$$dI = \left( \frac{\mathbf{D} \times \mathbf{n}_k}{c^3} \right)^2 \frac{d\Omega}{4\pi} = \frac{\omega^4}{c^3} \left[ \mathbf{D}^2 - (\mathbf{D} \cdot \mathbf{n}_k)^2 \right] \frac{d\Omega}{4\pi}.$$  

By substituting (13) and (4) into this formula and averaging over the period $T = 2\pi/\omega$, we have

$$\langle \cos(\omega t + \alpha_i) \cos(\omega t + \alpha_j) \rangle = \cos(\alpha_i - \alpha_j)/2,$$

$$\langle \sin(\omega t + \alpha_i) \sin(\omega t + \alpha_j) \rangle = \cos(\alpha_i - \alpha_j)/2,$$

$$\langle \cos(\omega t + \alpha_i) \sin(\omega t + \alpha_j) \rangle + \langle \sin(\omega t + \alpha_i) \cos(\omega t + \alpha_j) \rangle = 0.$$  

Furthermore,

$$\langle \mathbf{D}^2(t) \rangle = \sum_{i,j} \langle \mathbf{d}_i(t) \cdot \mathbf{d}_j(t) \rangle = \sum_{i=1}^{N_0} \langle \mathbf{d}_i(t) \cdot \mathbf{d}_i(t) \rangle +$$

$$+ \sum_{i \neq j}^{N_0(N_0-1)} \langle \mathbf{d}_i(t) \cdot \mathbf{d}_j(t) \rangle = e^2 r_L^2 \left[ N_0 + \sum_{i \neq j}^{N_0(N_0-1)} \cos(\alpha_i - \alpha_j) \right]$$  

and

$$\langle (\mathbf{D} \cdot \mathbf{n})^2 \rangle = e^2 r_L^2 \left[ N_0 + \sum_{i \neq j}^{N_0(N_0-1)} \cos(\alpha_i - \alpha_j) \right] (n_x^2 + n_y^2)/2.$$  

Using the relations

$$n_x^2 + n_y^2 + n_z^2 = 1, \quad 1 - \frac{n_x^2 + n_y^2}{2} = \frac{1 + n_z^2}{2},$$

we get

$$\langle dI \rangle = \frac{e^2 \omega^2 V_e^2}{c^3} \left[ N_0 + \sum_{i \neq j}^{N_0(N_0-1)} \cos(\alpha_i - \alpha_j) \right] \frac{1 + n_z^2}{2} \frac{d\Omega}{4\pi}.$$  

The factor $(1 + n_z^2)/2$ reflects the anisotropy properties of the dipole radiation of nonrelativistic electrons. The second term in the square brackets describes the correlation effects connected with the mutual aligning of dipoles. If correlations are absent, i.e., if the contributions from all
cos(\alpha_i - \alpha_j) are mutually compensated and give zero in the sum, then only the first term in (18) works, which corresponds to the total radiation of \( N_0 \) independent elementary dipoles. It is easy to describe situations where the second term in (18) completely compensates the first one so that dipole radiation does not arise. These are, of course, the situations where the total dipole moment \( \overrightarrow{\mathcal{D}} \) becomes equal to zero. Consider, for example, the case where \( N_0/2 \) dipoles are oriented precisely in one direction, and the rest \( N_0/2 \) dipoles are oriented in the opposite direction. Then the correlations of pairs in each of these groups are constructive and give \( \cos(\Delta \alpha) = 1 \); there are \( N_0(N_0/2 - 1)/2 \) such pairs in one group and as many in the other, i.e., their common positive contribution is equal to \( N_0(N_0/2 - 1) \). Correlations between pairs formed by dipoles of different groups are destructive and give \( \cos \pi = -1 \) so that the contribution of such pairs is, obviously, equal to \( -2(N_0/2)^2 \). Adding these contributions together with the first term in (18), we obtain: \( N_0 + N_0(N_0/2 - 1) - 2(N_0/2)^2 = 0 \), as it should be from the physical viewpoint because we consider the case with \( \overrightarrow{\mathcal{D}} = 0 \).

In the case of total correlation, where all \( \cos(\alpha_i - \alpha_j) = 1 \), formula (18) gives

\[
\langle dI \rangle_{\text{corr}} = N_0^2 \frac{e^2 \omega^2 V_\perp^2}{c^3} \frac{1 + n_z^2}{2} \frac{d\Omega}{4\pi},
\]

i.e., the intensity grows in \( N_0 \) times as compared with the radiation of \( N_0 \) uncorrelated dipoles. This is precisely the SR effect [1].

The radiation time of such correlated dipoles radiating coherently will decrease \( N_0 \) times as compared with time (7):

\[
\tau_{\text{coh}} = \tau / N_0.
\]

In reality, it is possible to expect only partial positive correlation of phases, i.e., partial aligning of all dipoles, for which the average over ensemble value of cosine is positive

\[
\langle \cos(\alpha_i - \alpha_j) \rangle \equiv \langle \cos \Delta \alpha \rangle > 0.
\]

Having replaced all \( \cos(\alpha_i - \alpha_j) \) in (18) by their average values, we obtain

\[
\langle dI \rangle = \frac{e^2 \omega^2 V_\perp^2}{c^3} [N_0 + N_0(N_0 - 1) \langle \cos \Delta \alpha \rangle] \frac{1 + n_z^2}{2} \frac{d\Omega}{4\pi}.
\]

In this case, the intensity of coherent radiation is proportional to \( N_0^2 \langle \cos \Delta \alpha \rangle \).

Now we proceed to the consideration of the mechanism of spontaneous aligning of the dipoles giving rise to the SR regime.

### 3 Polarization phase transition in “coherence domains”.

To solve the problem of the phase transition, we apply here the Weiss method of self-consistent mean field [8] confirmed in the theory of spontaneous magnetization. Consider the potential energy of a trial dipole \( \overrightarrow{d}_0(\overrightarrow{r}_0, t) \) with the electric field \( \overrightarrow{E}(\overrightarrow{r}_0, t) \) induced at a point \( \overrightarrow{r}_0 \) by the rest \( (N_0 - 1) \) dipoles:

\[
U(d_0) = -\overrightarrow{d}_0(\overrightarrow{r}_0, t) \cdot \overrightarrow{E}(\overrightarrow{r}_0, t),
\]

(23)
where
\[ \vec{E}(\vec{r}_0, t) = \sum_{j}^{N_0-1} \frac{3\vec{r}_j \cdot \vec{d}_j(t) - \vec{d}_j(t)}{|\vec{r}_0 - \vec{r}_j|^3}, \] (24)
\[ \vec{n}_j = (\vec{r}_0 - \vec{r}_j)/|\vec{r}_0 - \vec{r}_j|. \]

All dipoles are rotating according to law (4) and radiate, and their electric field is not static but also is rotating with frequency \( \omega \). Therefore, the use of expression (24) for the field \( \vec{E}(t) \) requires explanation. The point is that conditions (11) that determine the “coherence volume” imply that different dipoles of a domain are in the so-called near zone \( (R_0 \ll \lambda) \) with respect to each other, where the main term in the decomposition of the retarded potentials and fields in powers of the small parameters \( (r_L/R_0) \) and \( (r_L/\lambda) \) turns out to be precisely expression (24) (in this respect, see, e.g., [9]).

Averaging Eq. (23) over the rotation period, we notice that the points \( \vec{r}_0 \) and \( \vec{r}_j \) characterize not the instantaneous positions of the rotating electrons but the positions of the static centers of rotation and, consequently, do not depend on time. Substituting (24) into (23) and averaging over the period, we obtain
\[ \langle U(r_0) \rangle = -\frac{d_0^2}{2} \sum_{j=1}^{N_0-1} \frac{1 - 3(n_{jz})^2}{|\vec{r}_0 - \vec{r}_j|^3} \cos(\alpha_0 - \alpha_j), \] (25)

where \( d_0 = \epsilon r_L, n_{jz} = (z_0 - z_j)/|\vec{r}_0 - \vec{r}_j| \), and the relation \( 3[n_{jx}^2 + n_{jy}^2] - 2 = 1 - 3n_{jz}^2 \) is used. One can approximate the sum in (25) by the integral with respect to the coordinates \( r_j \) over the “coherence volume” \( V_{coh} \) with the obvious measure \( n_e dV_j \) representing the mean number of dipoles in the volume element \( dV_j \equiv (d\vec{r}_j) \) in the neighborhood of the point \( \vec{r}_j \). But, prior to writing this integral, we notice that, because the position of our trial dipole \( \vec{d}_0(\vec{r}_0, t) \) can be arbitrary, it is necessary to average (25) over this parameter, i.e., to introduce the additional integration \( (d\vec{r}_0)/V_{coh} \). Moreover, in the spirit of the mean-field method, we replace \( \cos(\alpha_0 - \alpha_j) \) in (25) by its value \( \langle \cos \Delta \alpha \rangle \) averaged over the ensemble. After all this averaging, we get the expression
\[ \langle U \rangle = -\frac{d_0^2}{2} n_e \langle \cos \Delta \alpha \rangle \int (d\vec{r}_0)/V_{coh} \int d\vec{r}_j \frac{1 - 3(n_{jz})^2}{|\vec{r}_0 - \vec{r}_j|^3}. \] (26)

By making the change of variables \( \{\vec{r}_0, \vec{r}_j\} \rightarrow \{\vec{r} = \vec{r}_0 - \vec{r}_j, \vec{R} = (\vec{r}_0 + \vec{r}_j)/2\} \) and integrating over \( d\vec{R} \), we get
\[ \langle U \rangle = -\frac{d_0^2}{2} n_e \langle \cos \Delta \alpha \rangle \int_{V_{coh}} \frac{r^2 - 3z^2}{r^5} (d\vec{r}). \] (27)

Aligning of the dipoles in the same direction is energetically favourable because of reduction of the negative contribution to potential energy \( \langle U \rangle \). Therefore, the energetically preferable correlations will occur only in that part of the “coherence volume” in which the region of integration over the relative coordinates satisfies the condition
\[ r^2 - 3z^2 > 0, \] (28)

i.e., in the region similar to a flattened circular cylinder. We call this part of the coherence region by “coherence domain”, or by “domain of self-polarization”. In a similar neighboring domain, the direction of the average vector of polarization should be close to the opposite one.
to minimize the positive energy of the electric field of polarization in the system as a whole. It is known that, by similar reasoning, the macroscopic volumes of magnetics and ferroelectrics are also divided into domains. Adjacent domains are separated by transition regions ("domain walls") within the limits of which the turning of the polarization vector takes place.

We return now to the estimation of the integral in (27) with constraint (28). It is convenient to calculate it in the cylindrical coordinates $(\rho, z, \varphi)$, in which

$$r^2 = \rho^2 + z^2, \quad r^2 - 3z^2 = \rho^2 - 2z^2 > 0.$$  \hspace{1cm} (29)

We consider this integral separately:

$$I(\rho_1, \rho_2) = \frac{2\pi}{\rho_1} \int_0^{\rho_2} d\rho \int_0^{\rho/\sqrt{3}} \rho^2 \left( \rho^2 - 2z^2 \right) \left( \rho^2 + z^2 \right)^{5/2}.$$

(30)

Integration over $z$ in the specified limits gives $2/3\sqrt{3}\rho^2$ and, as a result,

$$I(\rho_1, \rho_2) = \frac{4\pi}{3\sqrt{3}} \int_{\rho_1}^{\rho_2} \frac{d\rho^2}{\rho^2} = \frac{4\pi}{3\sqrt{3}} \ln \left( \frac{\rho_2}{\rho_1} \right)^2,$$

(31)

$$\langle U \rangle = -\frac{2\pi}{3\sqrt{3}} \ln \left( \frac{\rho_2}{\rho_1} \right)^2 \cdot d^2 n_e \langle \cos \alpha \rangle.$$

(32)

Now we consider the question about the minimal and maximal limits $(\rho_1, \rho_2)$ of the relative coordinate $\rho = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ in the plane between two dipoles in the coherence domain. We remind, that the characteristic sizes of the initial "coherence volume" were determined by conditions (11): $r_L \ll R_0 \ll \lambda$. Inequality (29) means that, in relative coordinates, "the coherence domain" has the form of a flattened circular cylinder with radius $2R_0$. Hence, the maximal value of $\rho$ is $\rho_2 = 2R_0$, and the minimal value of $\rho_1$ should be taken to be about twice the Larmor radius: $\rho_1 \sim 2r_L$, because, at smaller distances between dipoles centers, the interaction between a pair of electrons is not of dipole character any more, and it is impossible to use the dipole formulas. It must be noted that, under the conditions considered here, the radius of the Debye screening is a little bit smaller than $r_L$. Thus, we can write $\ln(\rho_2/\rho_1) \approx \ln(R_0/r_L)$. Because the ratio $R_0/r_L$ enters under a sign of logarithm, the result is weakly sensitive to the exact value of this ratio. Thus, taking into account also the condition $R_0 \ll \lambda$, we can replace $R_0$ here by the quantity of the order of $\lambda/10$. As a result, we can write

$$\ln \left( \frac{\rho_2}{\rho_1} \right)^2 \approx \ln \left( \frac{\lambda \omega}{10V_\perp} \right)^2 = \ln \left( \frac{2\pi c}{10V_\perp} \right)^2 = \ln \left( \frac{4\pi^2 mc^2}{10^2 mV_\perp^2} \right) \approx \ln \left( \frac{mc^2}{5E_\perp} \right).$$

(33)

Hence, at this stage, inequalities (11) turn into the condition

$$E_\perp \ll mc^2/5 \approx 100 \text{ keV}.$$  \hspace{1cm} (34)

With the account of (33), expression (32) takes the following form

$$\langle U \rangle \approx -\frac{2\pi}{3\sqrt{3}} \ln \left( \frac{mc^2}{5E_\perp} \right) d^2 n_e \langle \cos \alpha \rangle.$$  \hspace{1cm} (35)
To find \( \langle \cos \triangle \alpha \rangle \), we turn to the Weiss method [8]. For this purpose, in the beginning it is necessary to consider the response of our system of rotating dipoles in the \((x, y)\) plane

\[
\vec{d}_j = d_0 \{ \cos(\omega t + \alpha_j), \sin(\omega t + \alpha_j), 0 \}
\]

to the external homogeneous electric field

\[
\vec{E}_e = E_e \{ \cos(\omega t + \alpha_0), \sin(\omega t + \alpha_0), 0 \}
\]

rotating synchronously with dipoles. The potential energy of a dipole \(\vec{d}_j\) in this field, averaged over the period of rotation, is

\[
\bar{U} = -\vec{d}_j(t) \cdot \vec{E}_e(t) = -d_0 E_e \cos(\alpha_j - \alpha_0).
\]

Thus, one can see that the aligning of dipoles along the field with the radiation of released energy is energetically favourable. Thermal fluctuations suppress this tendency. They occur in rarefied magnetized plasma mainly in the form of plasma fluctuations and Alfvén waves. The distribution over the phase differences \(\triangle \alpha = \alpha_j - \alpha_0\) is thus given by the Boltzmann formula

\[
\rho(\triangle \alpha) = C \exp \left( \frac{-\bar{U}(\triangle \alpha)}{kT} \right) = C \exp \left( \frac{d_0 E_e \cos \triangle \alpha}{kT} \right)
\]

with the normalization factor

\[
C^{-1} = \int_0^{2\pi} d(\triangle \alpha) \exp \left( \frac{d_0 E_e}{kT} \cos \triangle \alpha \right) = 2 I_0 \left( \frac{d_0 E_e}{kT} \right),
\]

where \(I_0(x)\) is the modified Bessel function of zero order [10]. The mean value of \(\langle \cos \triangle \alpha \rangle\) is determined by the integral

\[
\langle \cos \triangle \alpha \rangle = \int_0^{2\pi} \cos \triangle \alpha \rho(\triangle \alpha) d(\triangle \alpha) = \frac{d}{dx} \ln \int_0^{2\pi} d(\triangle \alpha) \exp \{ x(\cos \triangle \alpha) \}
\]

\[
= \frac{d}{dx} \ln I_0(x) = \frac{I_0'(x)}{I_0(x)} = \frac{I_1(x)}{I_0(x)},
\]

where

\[
x = d_0 E_e / kT.
\]

So, the external field \(E_e\) induces the nonzero correlator (41), i.e., in other terms, polarizes the system. The measure of polarization is the mean dipole moment of unit volume

\[
P = n_e d_0 \langle \cos \triangle \alpha \rangle.
\]

Polarization generates the additional internal electric field

\[
E_p = \nu \cdot P,
\]

where \(\nu\) is some dimensionless parameter which will be defined below. The field \(E_p\), in turn, strengthens the polarization. This feedback effect will be taken into account if in (42) we replace \(E_e\) by the sum \(E_e + E_p = E_e + \nu P\) in correspondence with the ideology of the self-consistent
mean field developed by Weiss. As a result, we obtain the following nonlinear equation for the
determination of the polarization $P$:

$$
P = n_e d_0 \langle \cos \Delta \alpha \rangle = n_e d_0 F \left( \frac{d_0(E_e + \nu P)}{kT} \right),
$$

(45)

where

$$
F(x) = \frac{I_1(x)}{I_0(x)}.
$$

(46)

By excluding the external field ($E_e \to 0$), we obtain “the self-consistency equation” for $P$:

$$
P = n_e d_0 F \left( \frac{d_0 \nu}{kT} P \right).
$$

(47)

We consider now the conditions of existence of its nontrivial solutions. Introducing the
variable $z = (P d_0 \nu / kT)$, we rewrite equation (47) in the following form:

$$
z = \frac{d_0^2 \nu n_e}{kT} F(z).
$$

(48)

The function $F(z)$ has the asymptotics [10]

$$
F(z) = \left[ 1 - \frac{1}{2z} - \frac{1}{8z^2} - \cdots \right], \quad z \gg 1,
$$

(49)

and

$$
F(z) = \frac{z}{2} \left[ 1 - \frac{z}{8} + \cdots \right], \quad z \ll 1.
$$

(50)

From (48) and (49) it is follows that, at large $z$, the solution exists and corresponds to the
polarization of saturation

$$
z = \frac{\nu n_e d_0^2}{kT} \gg 1,
$$

(51)

and

$$
P_{\text{max}} = n_e d_0 = \frac{kT}{\nu d_0} z \gg \frac{kT}{\nu d_0}.
$$

(52)

To determine the threshold value of the electron density above which there arises a nontrivial
solution of equations (47), (48), it is necessary to consider asymptotic (50). Restricting ourselves
to the first term, we obtain from (48)

$$
z = \frac{\nu n_e d_0^2}{2kT} \gg 1,
$$

(53)

It follows from this equation that the critical (threshold) value of the density $n_e$ is determined
by the condition

$$
\left( \frac{\nu n_e d_0^2}{2kT} \right)|_c = 1.
$$

(54)

For $n_e \geq n_{ec}$, nontrivial domain self-polarization appears.

To determine the factor $\nu$, it is necessary to compare the expression for potential energy
following from (43), (44), and (38),

$$
\langle U \rangle = -\nu n_e d_0^2 \langle \cos \Delta \alpha \rangle,
$$

(55)
with the previously obtained expression (35). Requiring the equality between them, we obtain

\[ \nu = \frac{2\pi}{3\sqrt{3}} \ln \left( \frac{mc^2}{5E_L} \right), \quad \left( E_L \ll \frac{mc^2}{5} \right). \]  

(56)

As an estimate, for example, taking \( E_L = 1 \) keV, we obtain \( \nu \approx 5.56 \).

It is useful to express \( d_0^2 \) through the energy \( E_L \) and magnetic field \( H \):

\[ d_0^2 = (e r_L)^2 = \left( \frac{eV_L}{\omega} \right)^2 = \left( \frac{mcV_L}{H} \right)^2 = \frac{2mc^2E_L}{H^2}. \]  

(57)

As a result, the criterion of the occurrence of domain self-polarization of the inverted electron system on high Landau levels leading to SR takes the form

\[ \frac{2\pi}{3\sqrt{3}} \ln \left( \frac{mc^2}{5E_L} \right) \frac{mc^2 n_e E_L}{H^2 kT} \geq 1. \]  

(58)

4 Problem of the power of the decameter radiation of Jupiter.

As an illustration of the application of criterion (58), we consider the values of parameters in (58) characteristic for the lower magnetosphere of the active system Jupiter-Io at the base of the so-called “Io flux tube”, where, according to the observations, one of the sources of powerful decameter radiation is located [7]. The characteristic values of the parameters in this case are: \( H \sim 10 \text{ Gs}, T \sim 10^3 \text{ K}, \) and \( E_L \sim 1 \) keV. Substituting these values into (58), we find the critical density of inverted electrons \( (n_e)_c \), above which the criterion (58) will be satisfied: \( n_e \geq (n_e)_c = 2 \cdot 10^3 \text{ cm}^{-3} \). This is quite a modest requirement to the density of inverted fast electrons at the base of the Io flux tube which can easily be satisfied, so there are all grounds to believe that the sporadic super-power decameter radiation of the Jupiter-Io system can be connected with the generation of SR.

Let us consider this question in more detail. We notice at once that, as in the SR regime, the energy \( E_L(t) \) entering (58) quickly decreases, and the density \( n_e \) satisfying criterion (58) at \( E_L = E_L(0) \) can cease to satisfy it with the decrease in \( E_L(t) \). Therefore, for an effective and sufficiently long operating SR regime, one obviously requires a large enough excess of \( n_e \) over \( n_{ee} \sim 2 \cdot 10^3 \text{ cm}^{-3} \). Thus, we conclude that, at the transition to SR regime, electrons at the base of the Io flux tube near Jupiter’s magnetic pole must have density \( n_e \geq 10^4 \text{ cm}^{-3} \).

According to the basic assumption [6,7] now accepted, the streams of energetic electrons appear near Jupiter magnetic poles due to their acceleration up to several keV in the Io ionosphere and further movement to the Jupiter along the lines of the magnetic dipole fields. Acceleration of electrons is accounted by the induction of electromotive force \( \sim 400 \text{ kV} \) in the Io body and ionosphere due to the Io motion through the Jupiter’s magnetic field. The lines of the magnetic field of the Io flux tube converge to Jupiter’s magnetic poles. So, the initial area of acceleration of electrons in the Io ionosphere \( S_0 \sim 10^{16} - 10^{17} \text{ cm}^2 \) will be “projected” by the magnetic lines to an area smaller approximately by two orders of magnitude, \( S_1 \sim 10^{-2} \cdot S_0 \sim 10^{14} - 10^{15} \text{ cm}^2 \) near Jupiter’s poles. Therefore, the supercritical density of inverted fast electrons \( n_e \geq 10^4 \text{ cm}^{-3} \), necessary for the occurrence of SR, will be provided by the initial density \( n_0 \geq 10^2 \text{ cm}^{-3} \) near Io. It is quite an acceptable value in view of the known data about Io and its ionosphere [11].
Electrons moving along the Io flux tube initially have density below the critical value and, therefore, radiate rather weakly. But, near the Jupiter magnetic poles, the density of electrons reaches the supercritical value, and they pass to the SR regime. Thus, in this region, the intensity of collective cyclotron radiation grows approximately by ten orders of magnitude ($N_0 = n_e \cdot V_{coh} \sim 10^{10}$), what accounts for the observable huge power of decametric radiation from a rather small volume of the circumpolar area occupied by the source [7]. A separate work will be devoted to more detailed calculation of this power and also to the interpretation of the rather unusual dynamical spectra of the decameter radiation of the Jupiter-Io system within the framework of the present model.

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References

[1] R.H.Dicke, Phys.Rev. 93, 99 (1954).

[2] V.V.Zheleznyakov, V.V.Kocharovskiy, and Vl.V.Kocharovskiy, Uspekhi Fizicheskikh Nauk, 159, 193 (1989).

[3] A.V.Andreev, Uspekhi Fizicheskikh Nauk, 160,1 (1990).

[4] L.I.Men’shikov, Uspekhi Fizicheskikh Nauk, 42,2 (1999).

[5] L.D.Landau and E.M.Lifshitz, Quantum mechanics (Moscow: Nauka, 1974).

[6] Jupiter, edited by T.Gehrels(University of Arizona Press, Tucson, Arizona, 1976).

[7] B.P.Ryabov and N.N.Gerasimova, Jupiter decameter sporadic radioemission(Kiev: Naukova Dumka,1990).

[8] J.S.Smart, Effective field theoris of magnetism (W.Saunders Company, Philadelphia-London, 1966).

[9] L.D.Landau and E.M.Lifshitz, Field theory (Moscow: Nauka, 1973).

[10] Handbook of Mathematical Functions, Eds.M.Abramovitz and I.Stegun (Nat.Bureau of Stand., 1964).

[11] Satellites of Jupiter, edited by D.Morrison (University of Arizona Press, Tucson, Arizona, 1982).