Energy loss of a heavy quark produced in a finite-size quark-gluon plasma

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Abstract. We study the energy loss of an energetic heavy quark produced in a high temperature quark-gluon plasma and travelling a finite distance before emerging in the vacuum. While the retardation time of purely collisional energy loss is found to be of the order of the Debye screening length, we find that the contributions from transition radiation and the Ter-Mikayelian effect do not compensate, leading to an energy loss reduction of the zeroth order (in an opacity expansion) energy loss.

1. Introduction

Collisional energy loss has recently attracted some attention [1] and might be an explanation [2] for the single electron puzzle observed in ultrarelativistic heavy ion collisions [3]. When incorporating collisional loss into quenching scenarios, one usually assumes that a heavy quark produced in a QGP immediately undergoes stationary energy loss. In [4], it was however argued that the reaction of the medium on the heavy quark (of mass $M$ and energy $E \gg M$) can only set in after a retardation time $t_{\text{ret}} \sim \gamma/m_D$, where $m_D$ is the Debye mass and $\gamma = E/M \gg 1$. A calculation performed in a semi-classical framework based on the collective response of the medium (previously used in Ref. [5] to study the stationary collisional loss $-\Delta E_\infty$), indeed showed a large reduction (scaling as $\gamma$) of $-\Delta E_\infty$. While half of the effect could be attributed to a modification of initial bremsstrahlung in medium as compared to vacuum, the other half was interpreted as a retardation of purely collisional loss. In the present work we show that this latter interpretation is incorrect, and we instead have $t_{\text{ret}} \sim 1/m_D$ for purely collisional loss, in agreement with the diagrammatic approach of Ref. [6]. We also recall the study of Ref. [7], which does not suffer from the misinterpretation of [4], and presents for the first time a consistent calculation of heavy quark energy loss at zeroth order in an opacity expansion [8], i.e. including initial bremsstrahlung, transition radiation, as well as purely collisional processes.

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∥ Gluon radiation induced by quark rescattering in the QGP is not considered here, meaning that we focus on the zeroth order of the energy loss in an opacity expansion [8].
2. Theoretical framework and critical discussion

We assume the heavy quark to be produced in a static QGP of high temperature $T$. The latter hypothesis implies the hierarchy $1/T \ll r_D \ll \lambda$, where $1/T$ is the average distance between two constituents of the QGP, $r_D = 1/m_D \sim 1/(gT)$ is the Debye radius and $\lambda \sim 1/(g^2 T)$ is the mean free path of the heavy quark. Under these hypotheses, we can describe the QGP via its collective response to the current $\mathbf{J}$, where the dielectric functions are obtained from the gluon polarization tensor.

Let us first consider the simple case of a heavy quark produced in the past (thus associated to a stationary classical current) in an infinite plasma. We first solve Maxwell’s equations in Fourier space and then evaluate the work $W$ done on the charge by the induced electric field, i.e. the field generated by the polarization of the medium. Identifying the quark energy loss travelling the distance $L$ in the QGP as $-\Delta E_\infty(L) = -W$ we have:

$$
- \frac{\Delta E_\infty(L)}{C_F \alpha_s} = \frac{L}{v} \int \frac{d^3 \mathbf{k}}{4\pi^2} \int_{-\infty}^{+\infty} \frac{d\omega}{\omega} \left[ k^2 \rho_L v_L^2 + \omega^2 \rho_T v_T^2 \right]_{\text{ind}} \delta(\omega - \mathbf{k} \cdot \mathbf{v})
$$

(1)

where $v_L$ ($v_T$) is the component of the quark velocity $\mathbf{v}$ parallel (orthogonal) to $\mathbf{k}$, and $\rho_s(\omega, k)$ (with $s = L$ or $T$) is the gluon spectral density

$$
\rho_s(\omega, k) \equiv 2 \text{Im}\Delta_s(\omega + i \eta, k) = 2\pi \text{sgn}(\omega) z_s(k) \delta(\omega^2 - \omega_s^2(k)) + \beta_s(\omega, k) \theta(k^2 - \omega^2)
$$

(2)

where the first term corresponds to the (time-like) pole of the thermal gluon propagator $\Delta_s$, while the second is the magnitude of its cut in the space-like region and is the only term contributing to $-\Delta E_\infty$.

We now turn to the case of a heavy quark produced at initial time $t_{\text{prod}} \sim 1/E$ in a hard subprocess (still in an infinite QGP). When $E$ is much larger than all other scales (in particular $1/E \ll 1/T$), the production of the bare quark factorizes from subsequent evolution. The kinetic energy loss of the quark travelling the distance $L$ reads:

$$
- \Delta E_{\text{m,med}}(L) = E_{\text{kin}}(t_{\text{prod}}) - E_{\text{kin}}(t = L/v) = - \int_{t_{\text{prod}}}^{L/v} dt \int d^3 \mathbf{x} \mathbf{j}_1 \cdot \mathbf{E}_{\text{ind}}
$$

(3)

where the subscript “m” denotes the mechanical work done on the quark. In fact, this work already differs from zero for a quark produced in vacuum since initial bremsstrahlung, induced by the sudden acceleration of the quark, leads to the reduction of $E_{\text{kin}}$ after $t_{\text{prod}}$. This vacuum contribution should be subtracted from (3). When discussing jet quenching observables such as nuclear attenuation factors $R_{AA}$, only the vacuum-subtracted medium-induced energy loss matters. Indeed, schematically $R_{AA}(E) \sim d\sigma_{\text{vac}}(E - [\Delta E_{\text{m,med}}(\infty) - \Delta E_{\text{m,vac}}(\infty)])/d\sigma_{\text{vac}}(E)$, since the heavy quark vacuum production cross-section $d\sigma_{\text{vac}}(E)$ already includes radiative corrections. For the medium-induced energy loss we get:

$$
- \Delta E_m \equiv -\Delta E_{\text{m,med}} + \Delta E_{\text{m,vac}} \equiv -W = - \int_{t_{\text{prod}}}^{L/v} dt \int d^3 \mathbf{x} \mathbf{j}_1 \cdot \mathbf{E}_{\text{ind}}
$$

(4)
where it is again the induced electric field which enters the expression of \( W \). The calculation of (4) is performed similarly to the stationary case\(^4\) and we obtain \([4]\):

\[
-\frac{\Delta E_m(L)}{C_F\alpha_s} = \int \frac{d^3k}{2\pi^2} \left\{ 1 - \frac{\cos(kL\cos\theta)}{k^2 + m_D^2} \right\}
+ v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[ k^2 \cos^2\theta \rho_L + \omega^2 \sin^2\theta \rho_T \right] \times \frac{2\sin^2((\omega - kv\cos\theta)L/(2v))}{(\omega - kv\cos\theta)^2}
\]  

\((5)\)

where \( \theta \) is the angle between \( \vec{k} \) and \( \vec{v} \). Using \( \frac{2\sin^2(uL/(2v))}{u^2} \xrightarrow{L\to\infty} \frac{L}{v}\delta(u) \), we find that the dominant contribution for large \( L \) is \(-\Delta E_\infty(L)\). Of particular phenomenological importance is the quantity \( d_\infty = \lim_{L\to\infty} (-\Delta E_m + \Delta E_\infty) \), characterizing the shift of the asymptote of \(-\Delta E_m\) with respect to the stationary regime \(-\Delta E_\infty\). A good approximation of \( d_\infty \) is found to be \( d_\infty \approx -C_F\alpha_s m_D(1 + \sqrt{2}\gamma - 1) \). For \( \gamma \gg 1 \), we thus obtain a fractional energy gain \(-d_\infty/E \approx \sqrt{2}C_F\alpha_s m_D/M \approx 10 - 15\% \) for a charm quark (with \( M = 1.5 \text{ GeV} \)), as compared to the stationary result \(-\Delta E_\infty\).

In Fig. 1 (left, dashed line), we present the energy loss \([5]\) for a charm quark of momentum \( p = 10 \text{ GeV} \). Although we observe a significant delay before the onset of stationary (linear) energy loss, this is not a genuine retardation of purely collisional loss, since various physical effects contribute to this delay. First, part of the work done on the charge from \( t_{\text{prod}} \) to \( t = L/v \) is due to initial radiation. Due to the difference between the gluon dispersion relations in medium and in vacuum, the energy radiated in QGP differs from that in vacuum. This is the QCD equivalent of the Ter-Mikayelian (TM) effect, studied in \([9]\) and also properly identified in \([4]\). In order to single out contributions specific to collisional loss, we thus subtract from \([5]\) the TM contribution. This is easily done by subtracting \( \pi \sgn(\omega) z_\alpha(k)\delta(\omega^2 - \omega_\alpha^2(k)) \) - which sets the gluons or plasmons on mass shell - from the spectral functions \([2]\). Denoting \(-\Delta \tilde{E}\) and \( \tilde{d}_\infty \) the

\(\footnote{In \([3]\) and \([4]\) \( j_1 \) denotes the spatial component of the quark current, which is part of the total \textit{conserved} current used in \([4]\). We refer to \([4]\) for more details.}

Figure 1. Energy loss as a function of the path-length in an infinite (left) or finite (right) QGP; dotted lines illustrate the stationary collisional energy loss; dashed lines correspond to the mechanical work \(-\Delta E_m\) done on the charge while in QGP, part of it being due to the initial radiation. Left: this contribution is taken out on the dash-dotted line; further subtracting the self-energy contribution, one obtains the genuine collisional energy loss (plain line). Right: one incorporates the mechanical work done by the transition field on the charge with \(-\Delta E_m\) to obtain the “total” (coll. + init. rad. + trans. rad.) energy loss in the case of a finite-size QGP (plain line).
quantities of interest after this subtraction, we find \( \tilde{d}_\infty \simeq \frac{d_\infty}{2} \) for \( \gamma \gg 1 \), i.e. about half of the apparent “retardation” is due to the TM effect. In Fig. 1 (left), the dash-dotted line represents \( -\Delta \tilde{E} \).

As can already be seen in the case of a charge with \( v \simeq 1 \) produced in vacuum, the radiated energy represents only \( \sim \text{half} \) of the mechanical work done on the charge after its production. Using Poynting’s theorem we can show that the other half is associated to the creation of the charge’s proper field. Obviously, the self-energy contribution should not be counted as “energy loss” as it is part of the charge asymptotic state. Therefore, an accurate definition of energy loss - to be used for values of \( L \) large enough so that the proper field can be disentangled from other components - is \(-\Delta E(L) = -\Delta E_{\text{in}}(L) - \Delta E_{\text{self}}\), where \(-\Delta E_{\text{self}} = E_{\text{self,vac}} - E_{\text{self,med}}\). Although the self-energies - defined as the integral of the energy density \((\epsilon E^2 + \mu B^2)/2\) - are separately UV divergent, \(-\Delta E_{\text{self}}\) is UV convergent and \(\simeq -d_\infty/2\) for \( \gamma \gg 1 \). In Fig. 1 (left), the plain line represents \(-\Delta E(L)\) (after subtraction of the TM effect), i.e. the genuine collisional energy loss in the case of an infinite QGP. The shift w.r.t. \(-\Delta E_\infty\) appears to be \(\sim \alpha_s m_D\), showing that the retardation time due to purely collisional processes is \(t_{\text{ret}} \sim r_D\), not \(t_{\text{ret}} \sim \gamma r_D\) as was argued in [4].

We finally consider the most realistic situation, where the quark is produced in a hard subprocess in a finite size QGP and travels the distance \(L_p\) before escaping the medium. In this case the “asymptotic” (we assume the quark to hadronize long after escaping the medium) quark self-energy is the same as in vacuum, i.e. \(\Delta E_{\text{self}} = 0\). It is thus legitimate to evaluate the (induced) energy loss using (4), provided one includes the transition radiation field in the electric field \(\vec{E}\). In Fig. 1 (right), the plain line presents the final result for the energy loss [7], incorporating all contributions of our model (collisional energy loss + initial radiation + transition radiation). The shift with respect to \(-\Delta E_\infty\) is \(\simeq d_\infty/3\), in particular it scales as \(\gamma\). This is due to a non-compensation - already noted in [10] - between the TM effect and transition radiation. Numerically \((|d_\infty|/3)/\gamma \simeq 3 - 5\%\) (in agreement with [10]) for \(L_p > \gamma r_D\), implying an effective retardation time \(t_{\text{ret}} \simeq 4\text{ fm before the linear (stationary) regime.}\)

3. Conclusion

Here we have shown that the results of [4] (for an infinite QGP) are modified when defining the energy loss in terms of proper asymptotic states. The retardation time of purely collisional loss turns out to be \(t_{\text{ret}} \sim r_D\). However, the results of [7] for a finite QGP do not suffer from any misinterpretation and suggest a quite large retardation time \(t_{\text{ret}} \sim \gamma r_D\) of the “zeroth-order” energy loss, including purely collisional processes, initial bremsstrahlung and transition radiation.

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