Location of Automated External Defibrillators

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Abstract

In this paper, the problem of the allocation and location of automated external defibrillators (AEDs) is considered. An AED is a portable device that has been developed to allow for emergency treatment against sudden heart seizures such as myocardial infarction by electric shock in the absence of healthcare workers. This emergency treatment by AED must be administered as soon as possible following heart seizure. Therefore, it should be easily accessible. We discuss the problem of placing a fixed number of AEDs in a given set of buildings. For this problem, we determine both the number of AEDs that should be allocated to each building and the location of each AED in the building. We formulate this combination problem by considering the rectilinear distance and developing an algorithm to obtain a good approximation solution.

Keywords: Automated external defibrillator (AED), Location problem, Rectilinear distance

1. Introduction

An automated external defibrillator (AED) is a portable electronic device that automatically diagnoses the potentially life-threatening cardiac arrhythmias of ventricular fibrillation and ventricular tachycardia in a patient and is able to treat them through defibrillation, the application of electrical therapy that stops the arrhythmia, allowing the heart to reestablish an effective rhythm. An AED can be operated with simple audio and visual commands either by trained personnel or by a layman. Being portable, these public access units can be installed corporate and government offices, shopping centers, airports, airplanes, restaurants, casinos, hotels, sports stadiums, schools and universities, community centers, fitness centers, and even at the bedside. Public access AEDs should be placed such that they can cater to large groups of people, regardless of their age or activity, because children as well as adults may experience sudden cardiac arrests (SCAs). Since 2003, AEDs have become increasingly popular at public institutions in Japan, because even a layman is able to operate them [1,2]. Decreasing the time interval between the onset of an SCA and treatment is the most crucial factor to improve the efficacy of the treatment. This study focuses on the problem of determining the location of AEDs to optimize this time interval. Accordingly, in this study, we assume the following:

(i) There is a group of buildings, and there are AEDs.
(ii) The number of AEDs is fixed.
(iii) At least one AED should be allocated to each building.
(iv) For each building, the stairs are located at the edges of the floors; each location is evaluated based on the rectilinear distance corresponding to the height and horizontal distance of the stairs.

The problem is to determine both the allocation (i.e., the number of AEDs for each building) and location of the AEDs in each building. The allocation problem is similar to an apportionment problem (e.g., allocating parliamentary seats to districts based on population) and has been studied extensively [3]. As this study does not require strict impartiality like in an apportionment problem, this study has developed a simple approximation algorithm. The problem of determining the location of AEDs in each building is very similar to determining the rectilinear distance in a minimax problem under rectangular barriers [4].

In the approximation algorithm, the tentative location of each AED is determined first. Then, the exact location of each AED is determined. The
accuracy of the solution is improved by evaluating the worst AED location.

2. Mathematical Formulation

The mathematical formulations for the abovementioned problem are defined as follows.

2.1 Assumptions for allocation

(i) A group of buildings \( B \) includes \( m \) buildings \( B_1, B_2, \ldots, B_m \).

(ii) Each building \( B_i \) should have at least one AED.

(iii) Let \( u_i \) be the proportion of usage for building \( B_i \).

\[ \sum_{i=1}^{m} u_i = 1. \]

(iv) The total number of AEDs allowed in a group is fixed, \( X \), due to budget limit.

2.2 Assumptions for location

(i) Building \( B_i \) is composed of \( n_{f_i} \) floors and stairs.

(ii) Let \( d_{h_i} \) be the horizontal distance, or the width, of the floor. Let \( d_{v_i} \) be the vertical distance, or the height, of a flight of stairs.

(iii) AED installation is not permitted in the stairways.

(iv) Let \( n_i \) be the number of AEDs installed in building \( B_i \). \( n_i \) is less than the number of floors \( f_{B_i} \) (i.e., \( 1 \leq n_i < f_{B_i} \)).

(v) Distance is rectilinear (i.e., vertical and horizontal). Let \( d_i \) be the rectilinear distance (i.e., \( d_i = d_{v_i} + d_{h_i} \)).

2.3 Objective function

(i) Let \( d_i^{\text{worst}} \) be the worst-case distance of building \( B_i \).

(ii) Let \( d_i^{\text{worst}} \) be the maximum value of \( d_i^{\text{worst}} \) for \( i = 1, \ldots, m \).

(iii) The objective is to determine the allocation and location of all AEDs to minimize \( d_i^{\text{worst}} \).

3. Proposed Algorithm

In this section, the approximating algorithm used to solve the problem of the allocation and location of AEDs is discussed. This algorithm includes two parts, one for determining the initial allocation and one for determining the location through allocation iterations. The first part is a simple version of the largest remainder method in the apportionment problem. The second part involves improving the solution by repeatedly allocating the remaining AEDs on the basis of the evaluation result of tentative allocation and location.

Algorithm \( \text{ALOC}(X, B) \) determines the number of AEDs allocated to each building and then determines the location of each AED in the buildings. Inputs are the number of AEDs and the set of buildings. Outputs are the allocation and location results. Algorithm \( \text{loc}(B_i, n_i) \) determines the locations of the AEDs and gives the set of buildings that should be improved. The details of this algorithm are given in the following section.

3.1 Algorithm \( \text{ALOC}(X, B) \)

**Step 1:** Tentatively allocate \( n_i \) to building \( B_i \) as follows: \( n_i' = X \times u_i, \ n_i^{\text{int}} = \lfloor n_i' \rfloor \) \( (\lfloor n \rfloor \text{: integral part of } n) \)

**Step 2:** Initial allocation of AED.

(Case 1): All \( n_i^{\text{int}} \geq 1 \) for \( i = 1, 2, \ldots, m \). Then, for each building \( B_i \), set the place for \( n_i^{\text{int}} \) AED. Let \( R_{\text{AED}} = X - \sum_{i=1}^{m} n_i^{\text{int}} \).

(Case 2): Otherwise \( \exists n_i^{\text{int}} \in \{ n_i^{\text{int}} | n_i^{\text{int}} = 0, i = 1, 2, \ldots, m \} \). Then, we set one AED to each building.

Let \( u_{\text{min}} \) be the minimum value of \( u_i \) for \( i = 1, 2, \ldots, m \) and redefine \( u_i \) as \( u_i - u_{\text{min}} \) and \( X \) as \( X - m \). Calculate \( n_i' = X \times u_i, n_i^{\text{int}} = \lfloor n_i' \rfloor \). Then, for each building \( B_i \), we add \( n_i^{\text{int}} \) AED to the place. Let \( R_{\text{AED}} = X - (m + \sum_{i=1}^{m} n_i^{\text{int}}) \).

**Step 3:** Allocate the remaining \( R_{\text{AED}} \) AEDs. Determine and evaluate the location using \( \text{loc}(B_i, n_i) \).

Let \( B' \) be the output of \( \text{loc}(B_i, n_i) \). This indicates the buildings that should be improved.

(Case 1): If \( B' \) is greater than or equal to the number of remaining AEDs \( R_{\text{AED}} \), allocate the remaining AEDs adequately. Output the allocation and location of AEDs as the result. \( \text{ALOC}(X, B) \) is terminated.

(Case 2): If \( B' \) is less than the number of remaining AEDs \( R_{\text{AED}} \), allocate the AEDs to each building in set \( B' \) individually. Reset the number of remaining AEDs \( R_{\text{AED}} \) and repeat Step 3.

3.2 Determination of location

(1) Single stairway model

In this model, the following assumptions are made concerning the structure of the buildings:

(i) The building consists of multiple floors.

(ii) Each floor consists of rooms and a hallway in one straight line.

(iii) The stairs are located on only one side of the building.

Let \( p_1, p_2 \) be points on floor \( f_1, f_2 \) (\( f_1 \neq f_2 \)). The maximal distance between \( p_1 \) and \( p_2 \) is \( d_{h_1} + 2d_{v_1} \). If point \( p_2 \) is at the front of the stairs, then the maximal distance between \( p_1 \) and \( p_2 \) is \( d_{h_1} + d_{v_1} \). This is the optimal distance for installing an AED. Thus, AEDs should be installed in front of the stairs.

The next step is to determine the floor on which the AEDs should be installed.

If \( n_i = 1 \), then AED is installed on floor \( |f_{B_i}/2| \). (If \( n_{f_i} \) is odd, then it is similar to \( |f_{B_i}/2| + 1 \).

If \( n_i > 1 \), let \( q_{p_i} r_i \) be the quotient and remainder, respectively, when \( n_{f_i} \) divided by \( n_i \) (i.e., \( f_{B_i} = n_i r_i \)).
This study assumes that building \( B_i \) is separated into \( n_i \) blocks. Furthermore, \( n_i \) blocks contain \( r_i \) blocks \( B_i^{q_i+j} \) \((j = 1, ..., r_i)\) with \( q_i \) floors and \( n_i - r_i \) blocks \( B_i^{q_i} \) \((j = 1, ..., n_i - r_i)\) with \( q_i \) floors. Building \( B_i \) is separated as \( B_i^{q_1}, B_i^{q_1+1}, ..., B_i^{q_i}, B_i^{q_i+1}, ..., B_i^{n_i-r_i} \) from the ground (Figure 1). Thus, one AED is allocated to each block.

![Diagram](image)

**Figure 1**

**Algorithm \( \text{loc}_s(B_i, n_i) \)**

**Step 1**: Evaluate the distance of the worst case.

**Step 2**: Output the set of buildings.

Let \( B' \) be the buildings that should be improved. Let \( d_{\text{worst}} \) be the maximum value of \( d_i^w \) that satisfies \( d_i^w < d_i^{\text{threshold}} \). \( d_i^{\text{threshold}} \) is the threshold of distance for building \( B_i \) and is equal to building width \( d_{wi} \).

There is no case where \( d_i^w \) is less than \( d_{wi} \) because of the constraint that AEDs must be installed in front of the stairs. The elements of \( B' \) are the sets of buildings \( B_i \) where the worst-case distance \( d_i^w \) is equal to \( d_{\text{worst}} \) (i.e., \( B' = \{B_i | d_i^w = d_{\text{worst}}, d_i^w < d_i^{\text{threshold}} \} \)).

### 4. Numerical Instance

Here, the application of the proposed algorithm is discussed in detail.

#### 4.1 Problem instance (10 Buildings, 20 AEDs)

The building specifications are listed in Table 1. In this instance, all buildings have only a single stairway. The number of AEDs, \( X \), is 20.

Tentative allocation is obtained by applying Step 2 of \( \text{ALOC}(X, B) \). In this instance, it applies to case 2 in Step 2. Therefore, six AEDs are allocated based on \( n_i^{\text{int}} \), after which one AED is allocated to each building. Thus, a total of 16 AEDs are allocated. Based on the tentative allocation obtained by applying algorithm \( \text{loc}_s(B_i, n_i) \), the solution is improved by repeatedly allocating the AEDs. Table 3 shows this sequence. In this instance, the desired result is obtained at the fourth iteration of the allocation procedure.

#### Table 1: 10 buildings with single stairways and 20 AEDs

| AED X | Building \( B_i \) | Floor \( f_{B_i} \) | Height \( d_{hi} \) | Width \( d_{wi} \) | Proportion of usage \( u_i(\%) \) |
|-------|-----------------|----------------|----------------|----------------|-----------------|
| 20    | 1               | 3              | 10.5           | 50             | 1.0204          |
| 2     | 3               | 3              | 10.5           | 50             | 1.0204          |
| 3     | 4               | 3              | 10.5           | 40             | 1.0204          |
| 4     | 4               | 4              | 10.5           | 40             | 1.0204          |
| 5     | 6               | 6              | 10.5           | 50             | 4.0816          |
| 6     | 6               | 6              | 10.5           | 70             | 5.102           |
| 7     | 6               | 6              | 10.5           | 70             | 10.2041         |
| 8     | 10              | 12             | 110            | 110            | 15.3061         |
| 9     | 10              | 12             | 110            | 110            | 20.4082         |
| 10    | 15              | 12             | 120            | 120            | 40.8163         |

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Table 2: Tentative allocation by $\text{ALLOC}(X, B)$ Step 2

| Building $B_i$ | $n_i$ | $u'_i$ | $n'_i = u'_i \times 10^{-2}$ | $n_{int}$ | Tentative allocation |
|---------------|-------|--------|----------------------------|----------|---------------------|
| 1             | 20.4082 | 0      | 0                          | 0        | 1                   |
| 2             | 20.4082 | 0      | 0                          | 0        | 1                   |
| 3             | 20.4082 | 0      | 0                          | 0        | 1                   |
| 4             | 20.4082 | 0      | 0                          | 0        | 1                   |
| 5             | 0.816327 | 0.030612 | 0.306122                  | 0        | 1                   |
| 6             | 1.020408 | 0.040816 | 0.408163                  | 0        | 1                   |
| 7             | 2.040816 | 0.091837 | 0.918367                  | 0        | 1                   |
| 8             | 3.061224 | 0.142857 | 1.428571                  | 1        | 2                   |
| 9             | 4.081633 | 0.193878 | 1.938776                  | 1        | 2                   |
| 10            | 8.163265 | 0.397959 | 3.979592                  | 3        | 4                   |

Total Remaining AEDs 5

Table 3: Result for four iterations of the allocation procedure

| Building | 1st iteration | 2nd iteration | 3rd iteration | 4th iteration |
|----------|---------------|---------------|---------------|---------------|
|          | Allocation   | Distance      | Allocation   | Distance      | Allocation   | Distance      | Allocation   | Distance      |
| 1        | 1            | 60.5          | 1            | 60.5          | 1            | 60.5          | 1            | 60.5          |
| 2        | 1            | 60.5          | 1            | 60.5          | 1            | 60.5          | 1            | 60.5          |
| 3        | 1            | 61            | 1            | 61            | 1            | 61            | 1            | 61            |
| 4        | 1            | 61            | 1            | 61            | 1            | 61            | 1            | 61            |
| 5        | 1            | 81.5          | 1            | 81.5          | 1            | 81.5          | 1            | 81.5          |
| 6        | 1            | 101.5         | 1            | 101.5         | 1            | 101.5         | 1            | 101.5         |
| 7        | 1            | 101.5         | 1            | 101.5         | 1            | 101.5         | 1            | 101.5         |
| 8        | 2            | 134           | 2            | 134           | 4            | 134           | 5            | 122           |
| 9        | 2            | 134           | 2            | 134           | 4            | 134           | 5            | 122           |
| 10       | 4            | 144           | 5            | 132           | 5            | 132           | 5            | 132           |

Maximum Distance 144

Total AEDs 15

Remaining AEDs 5

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5. Analysis of the Number of AEDs

If the number of AEDs is fixed and the location algorithm is applied strictly, there are a case that it may be useless costs. Assume that building $B_i$ has $f_{B_i}$ floors and is allocated $X_{B_i}$. When $\frac{f_{B_i}}{X_{B_i}} \geq 3$, increasing the number of allocations decreases the maximal distance. However, when $1 \leq \frac{f_{B_i}}{X_{B_i}} < 3$, increasing the number of allocations does not change the maximal distance.

For example, Building No. 10 has 15 floors, and for each floor the weighted height and width are $d_{h_i} = 12$ and $d_{w_i} = 120$. For this structure, Figure 3 shows the relation between the number of AEDs and the maximal distance. When the number of AEDs ranges from 1 to 5, the maximal distance decreases. When the number of AEDs is greater than 5, the maximal distance does not decrease until the AEDs are allocated to all floors. Thus, the number of AEDs should be more than 10 to decrease the maximal distance. It is not appropriate in cost. Thus, the number of AEDs $X_{B_i}$ should be more than $\frac{f_{B_i}}{3}$.

6. Conclusion

In this study, we considered the combined problem of the allocation and location of AEDs in a set of buildings and developed an algorithm to obtain the solution. Furthermore, we considered the appropriate number of AEDs.

This problem can be extended to other facilities such as restrooms and vending machines. Developing an algorithm for different types of building structures is a question for future research.

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