TENSOR-TENSOR THEORY OF GRAVITATION

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Abstract

We consider the standard gauge theory of Poincaré group, realizing as a subgroup of $GL(5,R)$. The main problem of this theory was appearing of the fields connected with non-Lorentz symmetries, whose physical sense was unclear. In this paper we treat the gravitation as a Higgs-Goldstone field, and the translation gauge field as a new tensor field. The effective metric tensor in this case is hybrid of two tensor fields.

In the linear approximation the massive translation gauge field can give the Yukava type correction to the Newtons potential. Also outer potentials of a sphere and ball of the same mass are different in this case.

Corrections to the standard Einshtein post Newtonian formulas of the light deflection and radar echo delay is obtained.

The string like solution of the nonlinear equations of the translation gauge fields is found. This objects can results a Aharonov-Bohm type effect even for the spinless particles. They can provide density fluctuations in the early universe, necessary for galaxy formations.

The spherically symmetric solution of the theory is found. The translation gauge field lead to existence of a impenetrable for the matter singular surface inside the Schwarzschild sphere, which can prevent gravitational collapse of a massive body.

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1 Introduction

Most variational principles of current interest in physics are manifestly invariant under the ten-parameter Poincaré group

\[ P_{10} = L_6 \triangleright T_4, \]

where \( L_6 \) is the six-parameter Lorentz group, \( T_4 \) is the four-parameter translation group, and \( \triangleright \) denotes the semidirect product. In addition \( P_{10} \) is the maximal group of isometries of the Minkowski space-time. Soon after establishing of the gauge formalism for the groups of internal symmetries gauge theories based on the \( P_{10} \) appear (see [1] and references therein). The intrinsic difficulties in gauging the \( P_{10} \) were arise immediately. The reason is that \( P_{10} \) is not a semisimple group, and because it acts both on the matter fields and on the underlying space-time manifold.

The gauge theories has most elegant form in the fibre bundle formalism [2]. In this formalism two kinds of the gauge transformations generally are considered:

1. Passive gauge transformations which remain the matter fields constant, but change atlas of the fibre.
2. Active gauge transformations, which transform the matter fields themself.

The standard gauge formalism and the principle of the local invariance is formulated for the gauge transformations of the first kind. In case of the internal symmetries to the gauge transformations of the first kind correspond gauge transformations of the second kind with the same matrix form. So for the Yang-Millse fields gauge transformations usually are not differentiated.

As was mentioned above, difficulties with the gauge transformations arises in the case of space-time transformations, which acts on the derivative operators \( \partial_\nu \), as well as on the vectors of the tangent space on a space-time manifold. Here we have two types of the gauge transformation of the first kind.

a). Transformation of a atlas of the only tangent bundle.

The principle of relativity is formulated as the requirement of symmetry of the atlas of the tangent bundle with respect to the general linear group. The principle of equivalence reduces this group to the \( L_6 \). This invariance of the theory besides of gauge fields - Lorentz connection \( \Gamma_\nu \), is provided by introduction of the metric field \( g_{\mu\nu} \), or isomorphic to him tetrad field \( h^A_\mu \). The gauge transformation of this kind act on \( h^A_\mu \) from the left, transforming index \( \mu \), which corresponds to the atlas of the tangent bundle. They also act on \( \Gamma_\nu \), and on the indexes of the covariant derivative operators. The invariance of the Lagrangian leads to the conservation law for the energy-momentum tensor. This kind of transformations can be connected with the gauge theory of gravity, but still remains open the question about gauge status of the gravitational field \( h^A_\mu \).

In this paper we consider the theory connected to the second type of the gauge transformations of the first kind:

b). Transformation of a atlas of the only matter fields.

Invariance of the theory is provided by introduction of the only gauge fields. Bundle of the matter fields is associated with the tangent bundle, so the principle of equivalence contracts it structural group too. For the correlation of the symmetries of both bundles the structural group of matter fields bundle reduces to the \( P_{10} \) immediately, in difference with the tangent bundle, where one must introduce the metric tensor \( g_{\mu\nu} \). So, except of the gauge fields of \( L_6 \), this theory contains gauge fields of translations \( \theta^\nu_\mu \). Gauge transformations of this type
acted on matter fields, on the tetrad functions $h^A_\mu$ from the right, changing index $A$, which corresponded to the atlas were fields are defined, and on $\Gamma_\nu$. They provides conservation laws, usual for gauge theories. For this type of symmetries the physical mining of gauge fields $\theta^\nu_\mu$ was unclear.

Because of importance of the question we claim again that tetrad gravitational fields $h^A_\mu$ and gauge field of translations $\theta^\nu_\mu$ can not be identified. Main difference is that $\theta^\nu_\mu$ has both coordinate indexes and $h^A_\mu$ has one coordinate and one tetrad indexes. This results different transformation laws of this fields under transformations of space-time groups. Also $h^A_\mu$ can be zero, but $\theta^\nu_\mu$ not; $\theta^\nu_\mu$ is single valued, $h^A_\mu$ - with accuracy of Lorentz transformations and so on.

The conventional gauge technique can be applied to the $P_{10}$ if one ignore its physical role as a dynamical group and looks at it as an abstract structural group of a bundle, providing gauge transformations of first kind, of type b). In this paper we follow the formalism of direct gauging of the abstract Poincaré group. In difference with this works gauge fields of $P_{10}$ we don’t connect with the gravity, but treat as a new fields, describing new fundamental interaction in the space-time.

The gravity in our scheme appear not as the gauge, but Higgs-Goldstone type field, usually considered in gauge theories. In the gauge gravitational theory the equivalence principle leads to contraction of the structural group of the tangent bundle to the $P_{10}$, imitating the situation, which is analogous to the spontaneous symmetry breaking. Then we can look upon the Minkowski metric field as being the vacuum Higgs field, and small perturbations may play role of Goldstone fields. This metric perturbations can be identified with the presence of a gravitational field.

So our approach is hybrid of the two gravitational theories. The effective metric tensor in our case depends on two tensor fields - gravitational field and Poincaré gauge fields. So we generalize known scalar-tensor and vector-tensor theories of the gravitation (see references in [3]).

In Section 2 we present a brief review in gauge theory of dislocations and disclinations in a elastic medium. Section 3 develops the formalism of the gauge theory of the group $GL(5, R)$ and general form of equations of Poincaré gauge fields are obtained. Then we consider only the case when Lorentz gauge fields are zero. In Section 4 we express the theory of translation gauge field. In rest Sections 5, 6, 7 and 8 the solutions of translation gauge field equations and possible effects of this field are considered.
The gauge theory of dislocations and disclinations

The question about the physical sense of gauge fields corresponding to 3-rotations and 3-translations had been discussed also in the gauge theory of dislocations and disclinations for the elastic medium \[7\]. Here we remind the main basis of this theory.

The gauge theory of dislocations and disclinations is based on the fact that diffeomorphism

\[ \chi^i = u^i + \delta^i_j x^j, \quad i, j = 1, 2, 3, \]  

(2.1)

which characterizing displacement \( u^i \) of the point \( x^i \) of the elastic medium in the case of a small deformation is determined only with the accuracy to local gauge transformations

\[ \chi^i \rightarrow \chi^i + \varepsilon_{kl}^i \chi^k A^l(x) + B^i(x), \]  

(2.2)

where \( A^i(x) \) and \( B^i(x) \) are local rotations and local translations. Corresponding gauge fields \( W_{ij}^l \) and \( \Theta_{ij}^l \) are forming plastic part \( \beta_{ij}^P \) of total distortion

\[ \beta_{ij}^T = \partial_j \chi_i = \beta_{ij} + \beta_{ij}^P. \]  

(2.3)

The "covariant" derivative

\[ D_j \chi_i = \beta_{ij}^T - \beta_{ij}^P = \partial_j \chi_i - \varepsilon_{ikl} \chi^k W_{jl}^i - \Theta_{ij}, \]  

(2.4)

coincides with the elastic distortion \( \beta_{ij} \). Gauge fields strengths

\[ F_{ikl}^j = \partial_k W_{il}^j - \partial_l W_{ik}^j + C_{mn}^i W_{kl}^m W_{ln}^n, \]  

\[ \alpha_{ikl}^j = \partial_k \Theta_{ij}^l - \partial_l \Theta_{ij}^k + \varepsilon_{ikm} (W_{ln}^m \Theta_{jl}^n - W_{ln}^m \Theta_{kl}^n + F_{kl}^m \chi^n), \]  

(2.5)

are densities of dislocations and disclinations respectively. In this case \( \beta_{ij}^T, F_{kl}^i \) and \( \alpha_{ikl}^j \) appear to be gauge invariant quantities.

The Lagrangian of the gauge theory of continuous defects is choose in the following form:

\[ L = \frac{1}{4} (2 \mu e_{ij} \varepsilon_{ij}^j + \lambda e_{ij} \varepsilon_{ij}^j) - \frac{a}{2} \alpha_{kl}^i \alpha_{ij}^{kl} - \frac{b}{2} F_{kl}^i F_{ikl}^j, \]  

(2.6)

where

\[ e_{ij} = D_i \chi^k D_j \chi_k - \delta_{ij} \]  

(2.7)

is nonlinear tensor of deformations. In \( \mu, \lambda \) coupling constants \( \mu, \lambda \) corresponded to Lamé parameters, describing elastical properties of the media. Parameters \( a \) and \( b \) characterizing the energies required in order to create "unit" dislocation and disclination, respectively.

Equations of theory has the following general structure:

\[ (\delta_{mj}^i \partial_j - \varepsilon_{km}^i W_{lj}^k) \frac{\partial L}{\partial (D_j \chi^i)} = \varepsilon_{km}^i F_{jl}^k \frac{\partial L}{\partial \alpha_{jl}^i}, \]  

(2.8)

\[ (\delta_{mj}^i \partial_j - \varepsilon_{km}^i W_{lj}^k) \frac{\partial L}{\partial \alpha_{jl}^i} = - \frac{1}{2} \frac{\partial L}{\partial (D_l \chi^m)}, \]  

(2.9)

\[ (\delta_{mj}^i \partial_j - C_{km}^i W_{lj}^k) \frac{\partial L}{\partial F_{jl}^i} = \varepsilon_{mj}^i \left( \frac{\partial L}{\partial (D_l \chi^m)} \chi^l + 2 \frac{\partial L}{\partial \alpha_{kl}^i} \Theta_{jk}^l \right). \]  

(2.10)

From the experiments is known, that disclination energy is large compared with the dislocation energy and this is very large compared with the elastic energy. So, \( \lambda/a \sim a/b \ll \).
1. Then in the first order of this scaling parameters from (2.8) we get well known equation of equilibrium of the continuous media

$$\partial_j \frac{\partial L}{\partial (D_j \chi^i)} = \partial_j \sigma^j_i = 0, \quad (2.11)$$

where strength tensor $\sigma^j_i$ is expressed by elastic deformation (in the linear approximation)

$$\tau_{ij} = D_i u_j + D_j u_i, \quad (2.12)$$

in correspondence of the Hooke’s law for the isotropic media:

$$\sigma^j_i = \mu \tau^j_i + \lambda \delta^j_i \tau^k_k. \quad (2.13)$$

Lamé coefficients $\mu$ and $\lambda$ are positive and provided positiveness of the elastic energy.

Equations (2.9) and (2.10) are equations for dislocations and disclinations respectively. The source of dislocations $\Theta^j_i$ is nonlinear tensor of strength

$$\sigma^j_i = \frac{\partial L}{\partial(D_j \chi^i)} \quad (2.14)$$

and the source of disclination, besides of $\sigma^j_i$ is the torsion of media also.

Dislocations appear in the second order approximation and only in the third and higher order approximations do the disclination field $W^j_i$ enter the equations. So in case of a weak deformation of the media disclinations are not created.
3 General theory

Success of the gauge theory of continuous defects [4], where a tangent vector corresponds to the vector of medium displacement and gauge fields of 3-translations and 3-rotations interpreted as dislocations and disclinations (linear defects of a crystal lattice), allows us to interpret gauge fields of the nondynamical Poincaré group as the "defects" of the space-time manifold [4]. The gravity appeared in the our scheme as the Higgs-Goldstone field, as in papers [3].

The standard Yang-Millse gauge theory of the group $P_{10}$ can be constructed by presenting of $P_{10}$ as a matrix subgroup of $GL(5, R)$ [5]

$$P_{10} = \left( \begin{array}{cc} L_6 & T_4 \\ 0 & 1 \end{array} \right),$$

where $L_6$ and $T_4$ denote transformation matrices of the Lorentz group and translation group respectively. This matrices induce diffeomorphisms of the five dimensional space - gauge transformations of the second kind:

$$x \rightarrow P_{10}x = \left\{ L_6x + T_4 \right\}.$$  \hspace{1cm} (3.2)

Let $I_A$ ($A = 0, \ldots, 5$) be the basis of the algebra of group $L_6$ and $e_\mu$ ($\mu = 0, \ldots, 3$) be the generators of group $T_4$. Then the standard Yung-Millse connection has the form:

$$\Gamma = \left( \begin{array}{cc} W^A I_A & \theta^\mu e_\mu \\ 0 & 0 \end{array} \right),$$

where $W^A_{\mu}$ and $\theta^\mu_\nu$ are gauge fields of groups $L_6$ and $T_4$ respectively. Covariant derivatives and differentials can be written in the following form:

$$\tilde{\partial}_\mu = (h^{-1})^\alpha_\mu \partial_\alpha,$$

$$\tilde{d}x^\mu = h^\mu_\nu dx^\nu,$$  \hspace{1cm} (3.4)

where

$$h^\mu_\nu = \delta^\mu_\nu + I_{A\gamma}x^\gamma W^A_{\nu} + \theta^\mu_\nu$$

(3.5)

denotes distorsion for Poincaré group, and $(h^{-1})^\alpha_\mu$ is it reverse matrix. Then elements of length and volume in the Minkowski space can be transform to:

$$ds^2 \rightarrow \tilde{ds}^2 = \eta_{\mu\nu} h^\mu_\alpha h^\nu_\beta dx^\alpha dx^\beta = g_{\alpha\beta} dx^\alpha dx^\beta,$$

$$dV \rightarrow d\tilde{V} = hdV,$$  \hspace{1cm} (3.6)

where $h$ is the determinant of $h^\mu_\nu$. In this case arbitrary 4-vector is transforming as a 4-vector of velocity

$$\tilde{v}^\mu = \frac{dx^\mu}{ds} = h^\mu_\nu v^\nu,$$

$$\tilde{v}_\mu = (h^{-1})^\nu_\mu v_\nu,$$  \hspace{1cm} (3.7)
i.e. in the form:

\begin{align*}
\tilde{B}^\mu &= h^{\mu\nu}B^\nu, \\
\tilde{B}_\mu &= (h^{-1})^\mu_\nu B^\nu,
\end{align*}

(3.8)

By the connection (3.3) one can build up a Yung-Millse curvature for this case

\[ R = d\gamma + \gamma \wedge \gamma. \]

(3.9)

Then we can obtain curvatures corresponding to the \( L_6 \) and \( T_4 \) groups respectively:

\begin{align*}
F^A &= dW^A + \frac{1}{2} C^{A}_{BC} W^B \wedge W^C, \\
\alpha^\mu &= d\theta^\mu + C^\mu_{\alpha\nu} W^\alpha \wedge \theta^\nu.
\end{align*}

(3.10)

Here we have used properties of structural constants

\[ C^\mu_{\alpha\beta} = C^\nu_{\alpha\beta} = C^A_{\alpha\beta} = 0, \]

\[(A, B, ... = 0, ..., 5, \mu, \alpha, \beta, ... = 0, ..., 3) \]

(3.11)

of the Poincaré group.

In the ordinary way we can get also the expression of the torsion tensor

\[ S^\mu = \alpha^\mu + F^A P^\mu_{A\nu} x^\nu. \]

(3.12)

So localization of the nondinamical Poincaré group lifts Minkowski space into the nonholonomic space and now in the every point we have two spaces, as in the bimetric theories of gravitation (see references in [6]). But we assume, that the effective metric

\[ \tilde{g}_{\mu\nu} = \eta_{\alpha\beta} h^\alpha_{\mu} h^\beta_{\nu} \]

(3.13)

is not contain the gravitational field. It constructed by means of the distorsion (3.5), both indexes of which are coordinate and not with the tetrad gravitational field \( h^A_{\mu} \). So our theory differs from other gauge models of the Poincaré group [1] by the fact that in our case the distorsion \( h^{\mu}_{\nu} \) and torsion \( S^\mu_{\alpha\beta} \) depends on the translation gauge fields, as well as on the Lorentz gauge fields \( W^A_{\mu} \), and differs from the model [5], where distorsion (3.3) is treated as the gravitational field. Gauge theory of \( GL(5, R) \) considered in this section must be supplied various kind of Goldstone and Higgs field appearing, which has not place in scheme of theory [3]. In our model this fields connected to the gravitational field \( g_{\mu\nu} \). In the presence of gravitational field background Minkowski space becomes the Riemann space. Then the effective metric tensor and effective connection in this case -

\begin{align*}
\tilde{g}_{\mu\nu} &= g_{\alpha\beta} h^\alpha_{\mu} h^\beta_{\nu} = \eta_{AB} H^A_{\mu} H^B_{\nu}, \\
\tilde{\Gamma}^\alpha_{\mu\nu} &= h^\gamma_{\mu} h^\delta_{\nu} (h^{-1})^\alpha_{\beta} \Gamma^{\beta}_{\gamma\delta} + (h^{-1})^\delta_{\beta} \partial^\gamma_{\mu} h^\beta_{\nu}.
\end{align*}

(3.14) (3.15)

where \( \Gamma^{\beta}_{\gamma\delta} \) is ordinary Christoffels symbols, appear to be hybrid of two fields - classical gravitation field and Poincaré gauge fields. Thus our theory appears to be tensor-tensor and modifies known scalar-tensor and vector-tensor theories of gravity (see references in [3]).

A Poincaré gauge fields is inserted into the Lagrangian of matter fields only via the effective metric (3.14) and the effective connection (3.15).
Let us consider that total Lagrangian $L_{TOT}$ of matter fields $\varphi^a$ and Poincaré gauge fields $\theta^\mu, W^A_\nu$ has the standard Yang-Mills structure

$$L_{TOT} = h L_{mat}(\varphi^a, \tilde{\partial}_\mu \varphi^a) + h L_{\theta,W}(\theta^\mu, W^A_\nu, S^\mu_{\alpha\beta}, F^A_{\alpha\beta}).$$  \hspace{1cm} (3.16)

We put for the convenience, that the gauge fields Lagrangian instead of $\alpha^\mu_{\alpha\beta}$ depends on the torsion tensor $S^\mu_{\alpha\beta}$, which is expressed by $\alpha^\mu_{\alpha\beta}$ and $F^A_{\alpha\beta}$ by the formula (3.12).

By variation with respect of fields $\varphi^a$ and $\theta^\mu$ we obtain field equations

$$\left(\delta^b_a \partial_\mu - W^A_\mu M^b_{Aa}\right) h(h^{-1})^\mu_\nu \frac{\partial L_{mat}}{\partial \varphi^b} = h \frac{\partial L_{mat}}{\partial \varphi^a}, \hspace{1cm} (3.17)$$

$$2(\delta^\beta_\alpha \partial_\mu - W^A_\mu C^\beta_{Aa}) \frac{\partial (hL_{\theta,W})}{\partial S^\beta_{\mu\nu}} = -h(h^{-1})^\mu_\nu T^\beta_\alpha + \frac{\partial (hL_{\theta,W})}{\partial \theta^\mu} \vert_{S,F}, \hspace{1cm} (3.18)$$

$$2(\delta^B_\mu \partial_\mu - W^C_\mu C^B_CA) \frac{\partial (hL_{\theta,W})}{\partial F^\beta_{\mu\nu}} = h(h^{-1})^\mu_\nu M^a_{A\alpha} \frac{\partial L_{mat}}{\partial (\partial_\mu \varphi^a)} - h^\alpha_\beta I^{\alpha}_A \frac{\partial (hL_{\theta,W})}{\partial S^\mu_{\beta\nu}}, \hspace{1cm} (3.19)$$

where $M^b_{Aa}$ are matrices of representation of fields $\varphi^a$ induced by infinitesimal transformations of the Poincaré group, and $T^\alpha_\mu$ is the canonical energy-momentum tensor of matter fields $\varphi^a$. From equations (3.18) we see that sources of the translation gauge field $\theta^\mu$ are the matter energy-momentum tensor (first term on the right side) and energy-momentum tensor of the field $h^\mu_\nu$ itself (second term on the right side). From (3.19) we see, that sources of fields $W^A_\nu$ are the spin tensor of the matter (first term on right side) and the tensor of spin of the field $h^\mu_\nu$ (second term on the right side).
4 Translation gauge fields

In the previous section the general form of equations of Poincaré gauge fields was constructed. Now we need to choose the Lagrangian of the theory. In the elastic media disclinations appear only in the case of strong deformations (see the Section 2), so it is reasonable to consider the case when the Lorentz gauge fields are zero

$$W^\mu_\nu = 0.$$  (4.1)

The distorsion (3.3) and the torsion (3.12) tensors now has the form:

$$h^\mu_\nu = \delta^\mu_\nu + \theta^\mu_\nu,$$

$$S^\alpha_\mu_\nu = \alpha^\alpha_\mu_\nu = \tilde{\partial}_\mu \theta^\alpha_\nu - \tilde{\partial}_\nu \theta^\alpha_\mu.$$  (4.2)

By analogy with the Lagrangian of the dislocation gauge theory (2.6) the Lagrangian of the gauge translation field $\theta^\mu_\nu$ is proposed to consist the Lagrangian of displacement field $L_u$, under the gauge $u^\nu = 0$, and the Lagrangian $L_\theta$ of the field $\theta^\mu_\nu$ itself. Since, under the gauge $u^\nu = 0,$

$$D_\mu u^\nu = -\theta^\nu_\mu,$$  (4.3)

the Lagrangian $L_u$ is reduced to algebraic combinations of fields $\theta^\mu_\nu$. By analogy with the Lagrangian (2.6) we choose $L_u$ in the form [4]:

$$L_u = \frac{\mu}{2} e^\mu_\nu e^\nu_\mu + \frac{\lambda}{4} e^\alpha_\mu e^\beta_\nu,$$  (4.4)

where

$$e^\mu_\nu = h^\alpha_\mu h^\alpha_\nu - \eta^\mu_\nu.$$  (4.5)

The Lagrangian $L_\theta$ represents all possible combinations from components of the strength tensor $S^\mu_\nu$, and we choose it in the form [4]:

$$L_\theta = -\left\{ \frac{1}{2a} S^\mu_\nu S^\nu_\alpha + \frac{1}{2b} S^\nu_\mu S^\nu_\alpha + \frac{1}{2c} S^\nu_\mu S^\nu_\alpha + d \varepsilon^{\mu_\nu_\alpha_\beta} S^\gamma_\mu S^\gamma_\nu \right\},$$  (4.6)

Here $a, b, c, d$ are coupling constants.

To obtain constraints on the constants in the Lagrangian (4.6), now we consider the linear approximation in $\theta^\mu_\nu$ and assume the matter to be spinless. Then, variation of the total Lagrangian

$$L = L_{\text{matt}} + L_u + L_\theta$$  (4.7)

over $\theta^\mu_\nu$ results in the equations

$$\frac{1}{a} (\partial_\mu S^\gamma_\nu - \eta_\mu_\nu \partial_\gamma S^\gamma_\nu) + \frac{1}{b} (\partial_\nu S^\gamma_\mu - \partial_\gamma S^\gamma_\nu) + \frac{2}{c} \partial_\nu S^\nu_\mu +$$

$$2d (\varepsilon_\mu_\alpha_\gamma \eta_\mu_\nu \partial_\alpha S^\gamma_\sigma + \varepsilon_\nu_\sigma_\gamma \partial_\alpha S^\gamma_\sigma) + 2\mu \theta^\mu_\nu + \lambda \eta_\mu_\nu \theta^\gamma_\nu = -T^\mu_\nu,$$  (4.8)

where $T^\mu_\nu$ is the energy-momentum tensor of the matter fields.

The divergence of this equations with respect to the index $\nu$ takes on the form:

$$2\mu \partial_\nu \theta^\mu_\nu + \lambda \partial_\mu \theta^\gamma_\nu = 0.$$  (4.9)
It seems naturally to require that the divergence of the equation (4.8) with respect to
the second index $\mu$ in the spinless case takes on the same form as equation (4.9) -

$$2\mu \partial^\mu \theta_{\mu\nu} + \lambda \partial_\nu \theta_\gamma^\gamma = 0.$$  \hspace{1cm} (4.10)

This requirement imposes the following constraints

$$d = 0, \quad \frac{1}{a} + \frac{1}{b} + \frac{2}{c} = 0$$  \hspace{1cm} (4.11)

on the constants in the Lagrangian $L_\theta$.

With the constraints (4.11) nonlinear equations of translation gauge fields (3.18) has the
form:

$$\frac{1}{a}(\partial_\mu S_\gamma^\gamma - \eta_{\mu\nu} \partial^\xi S^\gamma_\xi) + \frac{1}{b}(\partial^\xi S_{\xi\mu\nu} - \partial^\xi S_{\nu\mu\xi}) - \left(\frac{1}{a} + \frac{1}{b}\right) \partial^\xi S_{\mu\nu\xi} +$$

$$+ 2\mu e_{\gamma\nu} h^{\gamma}_\nu + \lambda e^{\gamma}_\gamma h_{\mu\nu} + \eta_{\mu\nu} L_{\theta,W} = -(h^{-1})_{\nu\beta} T^\beta_{\mu}.$$  \hspace{1cm} (4.12)

The equation (4.9) has the form of the gauge condition. It’s analogous to the equilibrium
equation of the elastic media (2.11). This equation can be obtained from the Lagrangian
by variation with respect of the field $u^\nu$, which is a “displacement” in the bundle. Similar
to (4.9) conditions are considered in bimetric theories of gravitation [6, 8], but without any
connections to the variational principle.

If we take

$$\lambda = -\mu,$$  \hspace{1cm} (4.13)

then in the linear approximation, one can derive the equations for the symmetrical part,

$$e_{\mu\nu} = \theta_{\mu\nu} + \theta_{\nu\mu}$$  \hspace{1cm} (4.14)

of the free translation gauge field in the form:

$$(\Box + m^2)(e_\alpha^\alpha - \frac{1}{2} e_\beta^\alpha e^\beta_\nu) = 0,$$

$$\partial_\alpha(e_\beta^\alpha - \frac{1}{2} e_\beta^\beta e^\nu_\nu) = 0.$$  \hspace{1cm} (4.15)

This equations describe fields with the mass

$$m^2 = \frac{a\lambda}{2} = -\frac{a\mu}{2},$$  \hspace{1cm} (4.16)

and with the spin 2 and 0, as in the theory [8].
5 Newton’s approximation

In the case of joint action of the gravitational field \( g_{\mu\nu} \) and the translation gauge field \( \theta^\mu_\nu \) the effective curvature tensor has the form:

\[
\bar{R}^\alpha_\beta\mu\nu = h^\gamma_\mu (h^{-1})^\alpha_\delta R^\delta_\beta\gamma\nu, \tag{5.1}
\]

where \( R^\delta_\beta\gamma\nu \) is the ordinary Riemann tensor. Then the modified Hilbert Lagrangian has the form:

\[
L_g = -h \sqrt{-g} R. \tag{5.2}
\]

In the linear approximation, when

\[
g_{\mu\nu} = \eta_{\mu\nu} + \Phi^\mu_\nu, \tag{5.3}
\]

and in the case of the spinless source, equations of the gravitational field and translation gauge field are:

\[
\Box \Phi^\alpha_\beta - \partial^\beta \partial^\nu \Phi^\nu_\beta - \partial^\alpha \partial^\nu \Phi^\nu_\beta + \partial^\alpha \partial^\beta \Phi^\nu_\nu - \delta^\alpha_\beta \Box \Phi^\nu_\nu +
+ \delta^\alpha_\beta \partial^\mu \partial^\nu \Phi^\mu_\nu = -8\pi G T^\alpha_\beta, \tag{5.4}
\]

\[
\Box e^\alpha_\beta - \partial^\beta \partial^\nu e^\nu_\alpha - \partial^\alpha \partial^\nu e^\nu_\beta + \partial^\alpha \partial^\beta e^\nu_\nu - \delta^\alpha_\beta \Box e^\nu_\nu + \delta^\alpha_\beta \partial^\mu \partial^\nu e^\mu_\nu +
+ m^2 (e^\alpha_\beta - \frac{1}{2} \delta^\alpha_\beta e^\nu_\nu) = - \frac{a}{2} T^\alpha_\beta. \tag{5.5}
\]

Here (5.4) are Einshtein equations in the Newton’s approximation, (5.5) are translation field equations (4.12) in the linear approximation for the symmetrical part (4.14), and \( a \) is the coupling constant of the translation gauge field.

For the energy-momentum tensor of the point mass \( M \) in the rest, i.e.

\[
T^0_0 = M \delta(r), \quad T^i_\mu = 0. \tag{5.6}
\]

equations (5.4) and (5.5) possesses the static solutions

\[
\Phi^0_0 = -\frac{GM}{r}, \quad e^0_0 = -\frac{aM}{4\pi r} e^{-mr}. \tag{5.7}
\]

From the effective metric tensor

\[
\bar{g}_{00} = 1 + 2(\Phi_{00} + e_{00}) \tag{5.8}
\]

we obtain the modification of the Newtonian gravitational potential

\[
\Phi_{00} + e_{00} = -\frac{GM}{r}(1 + \frac{a}{8\pi G} e^{-mr}). \tag{5.9}
\]

Such a modification of the Newton’s potential (whose experimental verification received much attention in the 80s) is usually related to the hypothetical “fifth” fundamental force \[9\]. This interaction must be described by massive classical field. The mass expressed by means of the constants \( \mu \) and \( \lambda \), having the sense of coefficients of ”elasticity” of a space-time, and can be unusually small. On the experimental side, existing laboratory, geophysical and
astronomical data (see references in [9]) make restrictions on the value of coupling constants of the translation gauge field

\[ a < G, \quad m < 10^{-8} ev, \]  

(5.10)

where \( G \) is the gravitational constant.

Note that the potential (5.8) with parameters \( a/8\pi G \sim -1 \) and \( 1/m \sim 10kpc \) may contribute to the problem of mass discrepancies in galaxies [10].

Another effect of the massive translation gauge field in the Newtons approximation can be the difference of the fields of a ball and sphere of the same mass.

For the ball of the uniform density

\[ T = T_0^0 = \rho, \]  

(5.11)

with the radius \( R \), one can obtain by standard way [11] the potential for the outside region

\[ (\Phi_0^0 + e_0^0)_{r>R} = -\frac{4}{3} \pi R^3 G \rho \{ 1 + \frac{3ae^{-mr}}{8\pi Gm^2 R^3} [R \cosh(mR) - \frac{1}{m} \sinh(mR)] \}. \]  

(5.12)

For the inside region we have:

\[ (\Phi_0^0 + e_0^0)_{r<R} = -2\pi G \rho \{ R^2 - \frac{r^2}{3} + \frac{ae^{-mr}}{4\pi Gm^2} [\cosh(mr) - e^{-m(R-r)} \sinh(mr)(R + \frac{1}{m}) + \sinh(mr)] \}. \]  

(5.13)

The similar expression for the outer region of the sphere has the form:

\[ (\Phi_0^0 + e_0^0)^{\text{sph}}_{r>R} = -4\pi R^2 G \rho \{ 1 + \frac{ae^{-mr}}{4\pi GmR} \sinh(mR) \}. \]  

(5.14)

So outer potentials for the uniform density ball and sphere are differ from each other in the case of the massive translation gauge field.
Post Newtonian approximation

To find post Newtonian approximation of our theory we must solve nonlinear equations of the gravitational field and Poincaré gauge fields and then divide them in the line with respect of the post Newtonian parameter \( \sim 10^{-6} \). It is difficult way. We can use the different method using the fact, that Newton’s potential \( V = MG/r \) in the Sun system has the order of the post Newtonian parameter and can be used as the parameter of division. In the calculations of the light deflection and radar echo delay we can restrict ourself by the first order of \( V \), what corresponds to the linear approximation. Thus in calculations of the post Newtonian approximation we can use linear equations of translation gauge fields (5.5). We had used this equation when finding corrections to the Newton potential. In differ of the that case we must use all and not only zero-zero components of this equations. We shell consider the spherically symmetrical case when \( e_{\mu \nu} \) and \( T^\mu_{\nu} \) depends only on \( \vec{r} \), and regarding the source as 'pointlike', i. e. of all the components \( T^\mu_{\nu} \), assume different from zero only the component
\[
T^0_0 = -M \delta(r). \tag{6.1}
\]

To find spherical form of equations (5.3) is not easy without using there nonlinear form. Because we at first find two identities from this equation, which are not depended on coordinate systems
\[
\partial_\mu [m^2(e^\mu_\nu - \frac{1}{2} \delta^\mu_\nu e^\alpha_\alpha) + \frac{a}{2}(T^\mu_\nu - \frac{1}{2} \delta^\mu_\nu T^\alpha_\alpha)] = 0,
\]
\[
\Box e^\nu_\nu - \partial_\alpha \partial^\beta e^\alpha_\beta + \frac{m^2}{2} e^\nu_\nu = -\frac{a}{4} T^\nu_\nu. \tag{6.2}
\]

Using this identities and the equation of conservation of the energy-momentum one can get two equivalent forms of equations (5.3):
\[
(\Box + m^2)(e^\nu_\nu - \frac{1}{2} \delta^\nu_\nu e^\alpha_\alpha) = -\frac{a}{2}(T^\nu_\nu - \frac{1}{2} \delta^\nu_\nu T^\alpha_\alpha) + \frac{a}{2m^2}(\partial^\nu \partial_\nu - \frac{1}{2} \delta^\nu_\nu \Box) T^\alpha_\alpha, \tag{6.3}
\]
\[
\Box e^\mu_\nu + \partial^\mu \partial_\nu e^\alpha_\alpha - \partial^\mu \partial_\alpha e^\alpha_\nu - \partial_\alpha \partial^\nu e^\alpha_\alpha + m^2 e^\nu_\nu = -\frac{a}{2} T^\mu_\nu. \tag{6.4}
\]

From the equation (6.3) we noticed that in the spherically symmetrical case (6.1) non-diagonal components of the field \( e^\mu_\nu \) are zero. From equation (6.4) we see, that components \( e^\theta_\theta, e^\phi_\phi \) in this case also are zero.

Using the only nonzero components \( e^t_t, e^r_r \) we get
\[
e^\alpha_\alpha = e^t_t + e^r_r,
\]
\[
e^t_t - \frac{1}{2} e^\alpha_\alpha = \frac{1}{2}(e^t_t - e^r_r), \tag{6.5}
\]
\[
e^r_r - \frac{1}{2} e^\alpha_\alpha = -\frac{1}{2}(e^t_t - e^r_r).
\]

Then zero-zero components of equations (5.3) and (6.4), are:
\[
(-\Delta + m^2)(e^t_t - e^r_r) = -\frac{aM}{2m^2} (-\Delta + m^2) \delta(r), \tag{6.6}
\]
\[
(-\Delta + m^2)e^t_t = -\frac{aM}{2} \delta(r). \tag{6.7}
\]
From (6.6) we get:

$$e^t_t - e^r_r = -\frac{aM}{8\pi m^2} \delta(r),$$  \hspace{1cm} (6.8) 

(this relation can be obtained from (5.2) immediately using (6.1) and (6.5)), and from (6.7) we have:

$$e^t_t = -\frac{aM e^{-mr}}{8\pi r}.$$  \hspace{1cm} (6.9) 

Finally solution of equations of translation gauge field in the linear approximation is:

$$e^t_t = -\frac{aM e^{-mr}}{8\pi r}, \hspace{1cm} e^r_r = \frac{aM}{8\pi} \left[ \frac{\delta(r)}{m^2} - \frac{e^{-mr}}{r} \right],$$  \hspace{1cm} (6.10) 

$$e^\theta_\theta = e^\phi_\phi = 0, \hspace{1cm} e^\mu_\nu = 0 \hspace{0.1cm} (\mu \neq \nu).$$

Using solutions (6.10) and solution of the Einstein equations in linear approximation (5.3) -

$$\Phi^t_t = -V = -\frac{GM}{r}, \hspace{1cm} \Phi^r_r = V = \frac{GM}{r}$$

$$\Phi^\theta_\theta = \Phi^\phi_\phi = 0, \hspace{1cm} \Phi^\mu_\nu = 0 \hspace{0.1cm} (\mu \neq \nu),$$  \hspace{1cm} (6.11) 

we can belt the effective metric tensor of the theory in the linear approximation

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + 2(e_{\mu\nu} + \Phi_{\mu\nu}),$$  \hspace{1cm} (6.12) 

in the form:

$$\tilde{g}_{tt} = 1 - 2V(1 + \frac{a}{8\pi G} e^{-mr}),$$

$$\tilde{g}_{rr} = -1 - 2V(1 - \frac{a}{8\pi G} e^{-mr}),$$

$$\tilde{g}_{\theta\theta} = -r^2, \hspace{1cm} \tilde{g}_{\phi\phi} = -r^2 \sin^2 \theta.$$  \hspace{1cm} (6.13) 

Equation for particles moving on geodesic lines has the form:

$$\frac{d^2x^\mu}{ds^2} + \tilde{\Gamma}^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0.$$  \hspace{1cm} (6.14) 

Here $\tilde{\Gamma}^\mu_{\alpha\beta}$ are effective Christoffel symbols built by effective metric (6.12). Then by standard way [12] we can get formulae for the light deflection angle

$$\phi = \int_{r_1}^{r_2} (-\tilde{g}_{rr})^{1/2} \left[ \frac{\tilde{g}_{tt}}{r_2} \right] \tilde{g}_{tt} - \frac{1}{r_2} \right]^{-1/2} dr.$$  \hspace{1cm} (6.15) 

and for the radar echo delay time

$$t = \int_{r_1}^{r_2} (-\tilde{g}_{rr}\tilde{g}_{tt})^{1/2} \left[ \frac{\tilde{g}_{tt}}{r_2} \right] \tilde{g}_{tt} - \frac{1}{r_2} \right]^{-1/2} dr,$$  \hspace{1cm} (6.16) 

in fields of the spherical mass. Putting the solution (6.13) in these equations, in the linear approximation by the Newtons potential we obtain

$$\phi = \left( \frac{\pi}{2} + 2 \frac{MG}{r'} \right) + \frac{aM}{8\pi} \left[ \frac{r'}{2} - \int_0^{mr'} K_0(mr')d(mr') \right],$$

$$t = (\sqrt{r_2^2 - r_1^2} + MG \sqrt{\frac{r_2 - r_1}{r_1 + r_2}} + 2MG \ln \left| \frac{r_2 + \sqrt{r_2^2 - r_1^2}}{r_1} \right|) -$$

$$- \frac{aMr_1^2}{8\pi} \left\{ \frac{e^{-mr_1}}{\sqrt{r_1^2 - r_1^2} + m[K_0(mr_2) - K_0(mr_1)]} \right\},$$  \hspace{1cm} (6.17)
where $K_0(x) = \lim_{\nu \to 0} K_\nu(x)$ is a MacDonald’s function of the zero order. First terms in this expressions are values of in the Einshtein theory and second terms are corrections by the translation gauge field.
7 The string like solution

The analogy with the linear defect in crystals leads to the search of the string like solution of the theory. Two-dimensional objects, for example cosmic strings, are considered in field theories. They are solutions of the hypothetical charged scalar field equations and suggested to appear in early stages of the universe expansion. Due to specific form of the string core energy-momentum tensor

\[ T_t^t = T_z^z = \frac{1}{2} T_\nu^\nu, \]  

(7.1)

strings does not act gravitationally with the surrounding matter. For the cosmic string the solution of Einshtein equations in cylindrical coordinates are:

\[ ds^2 = (dt - \alpha d\theta)^2 - dr^2 - r^2 b^2 d\theta^2 - dz^2. \]  

(7.2)

This solution globally corresponds to the cone, but is locally flat [13].

It can be checked, that in the vacuum \((T_{\mu\nu} = 0)\) nonlinear translation gauge field equations (4.12) have the solution

\[ \theta_\nu^t = \theta_\nu^z = \frac{\alpha y}{x^2 + y^2}, \]

\[ \theta_\nu^y = \theta_\nu^z = -\frac{\alpha x}{x^2 + y^2}, \]  

(7.3)

where \(\alpha\) is arbitrary constant. Indeed, in this case components of the torsion tensor (4.2) and, thus the Lagrangian (4.6) are zero. Nonzero components of (4.5) are:

\[ e_{xx} = -e_{xt} = -\frac{\alpha y}{x^2 + y^2}, \]

\[ e_{yz} = -e_{yt} = -\frac{\alpha x}{x^2 + y^2}, \]  

(7.4)

Thus the Lagrangian (4.4), and combinations \(e_\nu^\nu\) and \(e_{\gamma\mu} h_\mu^\nu\) are also zero, and equations (4.12) are satisfied identically.

The components of the effective metric tensor (3.14), without inserting there gravity, for the solution (7.3) in cylindrical coordinates has the form:

\[ \tilde{g}_{tt} = \tilde{g}_{rr} = \tilde{g}_{zz} = 1, \]

\[ \tilde{g}_{\theta\theta} = -r^2, \]

\[ \tilde{g}_{\theta z} = \tilde{g}_{\theta t} = \alpha, \]  

(7.5)

The reverse tensor has components:

\[ \tilde{g}^{tt} = 1 - \frac{\alpha^2}{r^2}, \quad \tilde{g}^{rr} = -1, \]

\[ \tilde{g}^{\theta\theta} = -\frac{1}{r^2}, \quad \tilde{g}^{zz} = -1 - \frac{\alpha^2}{r^2}, \]  

\[ \tilde{g}^{\theta z} = \tilde{g}^{\theta t} = \frac{\alpha}{r^2}, \quad \tilde{g}^{tz} = -\frac{\alpha^2}{r^2}. \]  

(7.6)
Calculation of the energy-momentum tensor and effective interval for the solution (7.3) gives:

\[
T_{tt} = -T_{zz} = T_{zt} = \frac{\alpha^2}{r^2},
\]

\[
\tilde{ds}^2 = \left( dt - \frac{\alpha}{2} d\theta \right)^2 - dr^2 - r^2 d\theta^2 - \left( dz + \frac{\alpha}{2} d\theta \right)^2.
\] (7.7)

We see that this result is similar south for the cosmic string (7.1) and (7.2). It means that solution (7.3) corresponds to the string like object in the space-time.

Now let’s consider motion of particles in the string like solution (7.3). The scalar field \(\psi\) motion equation in the external translation gauge field, expressed by the effective metric tensor (7.6) is:

\[
(\tilde{g}^{\mu\nu} \partial_\mu \partial_\nu + \mu^2) \psi(x) = 0.
\] (7.8)

We search the solution in the form:

\[
\psi(t, r, \theta, z) = \int dk d\omega e^{i(kz + \omega t)} \sum_{m=-\infty}^{+\infty} e^{im\theta} \varphi(r).
\] (7.9)

For the radial wave function we get the Bessel equation

\[
\left\{ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{[m + \alpha(k + \omega)]^2}{r^2} + P^2 \right\} \varphi(r) = 0,
\] (7.10)

where \(P^2 = \omega^2 - k^2 - \mu^2\). This is a Aharonov-Bohm equation, having the solution:

\[
\varphi(r) \longrightarrow e^{-i(\alpha \theta + Pr \cos \theta)} \pm \frac{e^{iPr}}{\sqrt{2\pi i Pr}} \sin(\pi \alpha) \frac{e^{\pi i \theta/2}}{\cos(\theta/2)}.
\] (7.11)

In this expression signs \(\pm\) correspond to particles going around the string from different sides. We get analogy of the Aharonov-Bohm effect: in spite of absence of the string field strength outside the core, particles feel this field (the interference pattern changes behind the string). Here the effect exists even for spinless particles.

Another possible application of the string like translation gauge field can be galaxy formation. So far in the cosmology existed two mechanisms of generation of density fluctuations on which afterwards were formed galaxies. First of them basing on an inflation scenario connects density fluctuations with quantum fluctuations of the Higgs field in the early universe. Such approach requires the introduction of a dark matter of vague nature. Alternatively the second mechanism considers galaxy formation on strings loops, might be born during phase transition in the early universe and were evaporated due to the gravitational radiation [13]. But in this approach must be allowed existence of the some scalar field forming the cosmic string. The translation gauge field can serve as the new approach which leads to the stable density fluctuations in the early universe. In the difference of other models the translation gauge field appears in the theory from fundamental principles and does not require the introduction of supplementary fields.
8 Spherically symmetrical solution

To the end of this paper we want to find a spherical-symmetrical solution of nonlinear system of equations of the gravitational field and translation gauge field \([4.12]\) in outer region of the spherically symmetrical source, i.e. for the case

\[
T^\mu_\nu = 0. \tag{8.1}
\]

In this section we consider that Lorentz gauge fields are zero \(- W^A_\mu = 0\), and the translation gauge field has not mass, so constants \(\mu\) and \(\lambda\) also are zero.

In the equations of translation gauge fields \([4.12]\) we put

\[
\theta^0_0 = A(r), \quad \theta^i_j = \delta^i_j B(r),
\]

\[
\theta^0_i = \theta^i_0 = 0 \quad (i, j = 1, 2, 3). \tag{8.2}
\]

From the formula \([4.2]\) we can find all nonzero components of field strength

\[
S^0_{zo} = \partial_z A, \quad S^y_{xy} = F^z_{xz} = \partial_x B,
\]

\[
S^0_{yo} = \partial_y A, \quad S^x_{yx} = F^y_{yz} = \partial_y B,
\]

\[
S^0_{zo} = \partial_z A, \quad S^x_{zx} = F^y_{zy} = \partial_z B. \tag{8.3}
\]

So equations \([4.12]\) takes the form:

\[
2g^{00}\triangle B = 0,
\]

\[
g_{xx}[\triangle(A + 2B) - \partial_x^2(A + 2B) - (\partial_y^2 + \partial_z^2)B] = 0,
\]

\[
g_{yy}[\triangle(A + 2B) - \partial_y^2(A + 2B) - (\partial_x^2 + \partial_z^2)B] = 0,
\]

\[
g_{zz}[\triangle(A + 2B) - \partial_z^2(A + 2B) - (\partial_x^2 + \partial_y^2)B] = 0,
\]

\[
\partial_x\partial_y(A + B) = 0, \quad \partial_x\partial_z(A + B) = 0, \quad \partial_y\partial_z(A + B) = 0. \tag{8.4}
\]

From the latter three equations we have:

\[
A(r) = -B(r). \tag{8.5}
\]

Then all of equations \([8.4]\) are satisfied but first, which gets the form:

\[
\triangle A(r) = 0. \tag{8.6}
\]

Thus the spherical-symmetrical solution of translation gauge field equations \([4.12]\) is:

\[
\theta^0_0 = \frac{c}{r}, \quad \theta^i_j = -\delta^i_j \frac{c}{r},
\]

\[
\theta^i_0 = \theta^0_i = 0 \quad (i, j = 1, 2, 3). \tag{8.7}
\]

Here \(c\) is intagrable constant which must be fixed by boundary conditions. In spherical coordinates solution \([8.7]\) has the form:

\[
\theta^0_0 = \frac{c}{r}, \quad \theta^r_0 = -\frac{c}{r},
\]

\[
\theta^\phi_\phi = \theta^\phi_\phi = 0. \tag{8.8}
\]
Now let’s remain the spherical-symmetrical solution of Einshteins equations - the Schwarzschild solution

\[
\begin{align*}
g_{00} &= -g_{rr}^{-1} = 1 + \frac{E}{r}, \\
g_{\theta\theta} &= -r^2, \\
g_{\phi\phi} &= -r^2 \sin^2 \theta,
\end{align*}
\]

where \( E \) also is a intagrable constant.

Using the definition of the effective metric (3.14) we have:

\[
\begin{align*}
\tilde{g}_{00} &= (1 + \frac{E}{r})(1 + \frac{c}{r})^2, \\
\tilde{g}_{rr} &= -(1 + \frac{E}{r})^{-1}(1 - \frac{c}{r})^2, \\
\tilde{g}_{\theta\theta} &= -r^2, \\
\tilde{g}_{\phi\phi} &= -r^2 \sin^2 \theta.
\end{align*}
\]

Values for constants \( c, E \) in this expressions can be obtained from the Newton’s approximation. For the massless translation gauge field in the Newton’s approximation we have:

\[
\tilde{g}_{00} \simeq 1 + 2 \frac{M(G - a)}{r},
\]

where \( M \) is mass of the source, \( G \) is gravitational constant and \( a \) is the gauge field constant. So we have

\[
E = -2MG, \quad c = 2Ma.
\]

Finally the spherical-symmetrical solution in the tensor-tensor theory of gravity has the following form:

\[
\begin{align*}
\tilde{g}_{00} &= (1 - \frac{2MG}{r})(1 + \frac{2Ma}{r})^2, \\
\tilde{g}_{rr} &= -(1 - \frac{2MG}{r})^{-1}(1 - \frac{2Ma}{r})^2, \\
\tilde{g}_{\theta\theta} &= -r^2, \\
\tilde{g}_{\phi\phi} &= -r^2 \sin^2 \theta.
\end{align*}
\]

At the end of this section we want to consider a gravitational collapse problem. From the solution (8.13) one noticed, that, since we have relations (5.10), inside the Schwarzschild sphere

\[
r = 2MG,
\]

which is half-penetrable for the matter to the singular point

\[
r = 0,
\]

there exists second, the real singular surface

\[
r = 2Ma,
\]

which is impenetrable for the matter. Indeed equations of the translation gauge field don’t let Kruskal like coordinate transformations, because of the translation gauge field is single.
valued and it can’t be removed from the effective metric by coordinate transformations. Also
determinant of the gauge field

$$det(\delta^\mu_\nu + \theta^\mu_\nu) = (1 + \frac{2Ma}{r})(1 - \frac{2Ma}{r})$$ \hspace{1cm} (8.17)

on the sphere (8.16) is singular. So existence of the translation gauge field avoid the collapse
problem.

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