In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

Boaz Katz and Eli Waxman

Physics Faculty, Weizmann Institute, Rehovot 76100, Israel
E-mail: boazka@wizemail.weizmann.ac.il and waxman@wicc.weizmann.ac.il

Received 4 July 2007
Accepted 20 December 2007
Published 18 January 2008

Abstract. We present a simple analytic model for the various contributions to the non-thermal emission from shell-type SNRs and show that this model’s results reproduce well the results of previous detailed calculations. We show that the \( \geq 1 \) TeV gamma ray emission from the shell type SNRs RX J1713.7-3946 and RX J0852.0-4622 is dominated by inverse-Compton scattering of CMB photons (and possibly infrared ambient photons) by accelerated electrons. Pion decay (due to proton–proton collisions) is shown to account for only a small fraction, \( \lesssim 10^{-2} \), of the observed flux, as assuming a larger fractional contribution would imply non-thermal radio and x-ray synchrotron emission and thermal x-ray bremsstrahlung emission that far exceed the observed radio and x-ray fluxes. Models where pion decay dominates the \( \geq 1 \) TeV flux avoid the implied excessive synchrotron emission (but not the implied excessive thermal x-ray bremsstrahlung emission) by assuming an extremely low efficiency of electron acceleration, \( K_{ep} \lesssim 10^{-4} \) (where \( K_{ep} \) is the ratio of the number of accelerated electrons to the number of accelerated protons at a given energy). We argue that observations of SNRs in nearby galaxies imply a lower limit of \( K_{ep} \gtrsim 10^{-3} \), and thus rule out \( K_{ep} \) values \( \lesssim 10^{-4} \) (assuming that SNRs share a common typical value of \( K_{ep} \)). It is suggested that SNRs with strong thermal x-ray emission, rather than strong non-thermal x-ray emission, are more suitable candidates for searches of gamma rays and neutrinos resulting from proton–proton collisions. In particular, it is shown that the neutrino flux from the SNRs above is probably too low to be detected by current and planned neutrino observatories. Finally, we note that the magnetic field value implied by the comparison of x-ray to gamma-ray emission, \( \sim 10 \) \( \mu \)G, can be used to constrain magnetic field amplification.
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

**Keywords:** cosmic rays, supernovas, magnetic fields

**ArXiv ePrint:** 0706.3485

**Contents**

1. **Introduction** 2
2. **PP emission versus thermal and non-thermal electronic emission** 4
   2.1. Emission mechanisms ........................................ 4
   2.2. IC to PP emission ratio .................................... 6
   2.3. PP and IC to TB x-rays and non-thermal radio Syn ............ 7
   2.4. Comparison with previous studies ............................ 8
3. **Energy cutoffs** ................................................. 9
   3.1. Energy cutoffs ............................................. 10
   3.2. SNR dynamics ............................................. 12
   3.3. SNRs with observable non-thermal x-rays ..................... 12
   3.4. Suppression of IC due to radiative cooling .................. 14
4. **Lower limit on \( K_{ep} \) from extragalactic SNRs** ............. 15
5. **Application to RX J1713.7-3946 and RX J0852.0-4622** .......... 17
   5.1. Characteristics of RX J1713.7-3946 and RX J0852.0-4622 .......... 17
   5.2. Upper bounds on PP emission ................................ 18
   5.3. IC scenario ............................................... 20
   5.4. Comparison with previous studies ............................ 21
      5.4.1. Claims against IC for RX J1713.7-3946 and RX J0852.0-4622. ... 21
6. **Discussion** ................................................................ 22
7. **Acknowledgments** .................................................. 25

**Appendix. Emission mechanisms** .................................. 25
   A.1. Thermal bremsstrahlung ..................................... 25
   A.2. Gamma rays from proton–proton collisions ..................... 26
   A.3. IC radiation of CMB photons ................................... 27
   A.4. Synchrotron radiation ........................................ 27
8. **References** ......................................................... 28

1. **Introduction**

For a long time it was believed that the galactic cosmic rays observed up to the ‘knee’ energy (\( \sim 10^{15} \) eV) are accelerated in supernova remnants (SNRs, see e.g. [8]). The relativistic protons (and electrons) are believed to be accelerated by the diffusive (Fermi) shock acceleration (DSA) mechanism (for reviews see [18, 14, 32]). Strong evidence for electron acceleration to high energies in SNRs was established by observations of non-thermal x-ray emission which was attributed to synchrotron radiation of multi-TeV
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

Recently, unambiguous detection of \( \gtrsim 1 \text{ TeV} \) \( \gamma \)-rays has been made from the shell-type SNRs RX J1713.7-3946 \cite{36,20,2,4} and RX J0852.0-4622 \cite{28,3,5}, providing the first direct proof for the acceleration of particles at SNRs to multi-TeV energies. There are two candidate emission processes that can account for this radiation, namely inverse Compton (IC) of radio and infrared photons by multi-TeV accelerated electrons or pion decay as a consequence of proton–proton (PP) interactions of multi-TeV accelerated protons with ambient target protons (e.g. \cite{19}).

The distinction between the two mechanisms has important consequences for understanding particle acceleration and magnetic field amplification in SNRs. If it would turn out that the source is PP emission, this would be the first direct evidence for proton acceleration in SNRs. An IC source would allow a rather accurate estimate of the downstream magnetic field value by comparing the x-ray to gamma ray fluxes (see, e.g., \cite{4}).

Broad-band emission models with different levels of sophistication were applied in order to analyze the observed non-thermal radiation from these SNRs, reaching different conclusions as to the dominant \( \gtrsim 1 \text{ TeV} \) \( \gamma \)-ray emission mechanism. For RX J1713.7-3946 Aharonian \textit{et al} \cite{4}, Berezhko and Völk \cite{13} and Moraitis and Mastichiadis \cite{34} claimed that PP emission is favorable and IC is unlikely, Porter \textit{et al} \cite{41} claimed that IC emission is consistent. For RX J0852.0-4622 Enomoto \textit{et al} \cite{21}, Aharonian \textit{et al} \cite{5} did not rule out either mechanism.

The only well-understood non-thermal emission mechanism is currently synchrotron radiation of accelerated electrons. Synchrotron radiation is primarily observed in radio frequencies and is observed in x-rays in a few known SNRs (see, e.g., \cite{10} and references within). Non-thermal radio and x-ray observations are crucial for studying the non-thermal electron population. In the few known examples of SNRs emitting x-ray synchrotron radiation, the x-ray flux (per logarithmic frequency) is decreasing, indicating that the flux peaks at lower, unresolved photon energies. The radio and x-ray luminosities, and the implied position of the cutoff in the x-ray spectrum, may be used (and have been used in the models discussed above) for constraining the accelerated electron distribution, and thus for constraining the expected IC emission.

In this paper we derive simple analytic relations between the dominant radio, x-ray and \( \gamma \)-ray emission mechanisms of SNRs which can be used for distinguishing between the IC and PP origins of the \( \gamma \)-rays observed, and for predicting the \( \gamma \)-ray and neutrino flux values for SNRs where only radio and/or x-ray emissions were detected. Whenever possible, the simple analytic approximations we find are compared to, and shown to agree with, previous detailed calculations, with the advantage of being easier to follow and maintaining the explicit dependence on the unknown parameters.

First we compare in section 2 the expected IC, PP, synchrotron and thermal bremsstrahlung (TB) fluxes in a simple one-zone model of shocked ISM plasma. The plasma is assumed to consist of thermal and accelerated electron and proton components, with the later consisting of relativistic particles having a power law distribution in energy. In this section, the high energy cutoffs of the accelerated electron and proton distributions are ignored. Next, we discuss in section 3 the maximal energy attainable by electrons and protons in an SNR due to cooling and limited SNR age, and the implications for the non-
thermal emitted spectra, assuming diffusive shock acceleration (DSA) as the acceleration mechanism. We then find in section 4 an upper limit to the value of $K_{ep}$, the ratio of the number of accelerated electrons to the number of accelerated protons at a given energy, $K_{ep} > 10^{-3}$, by studying the radio observations of SNRs in M33. This parameter enters into the ratios of IC and synchrotron emission to PP emission. This lower limit is used to rule out previously suggested SNR broad-band emission models that used considerably lower values. In section 5 we apply the results of earlier sections to show that the broadband spectrum of the SNRs RX J1713.7-3946 and RX J0852.0-4622 is inconsistent with a PP origin and is consistent with an IC origin of the $\gtrsim 1$ TeV emission. We discuss previous claims that this emission cannot be due to IC and argue against them. The results are summarized and discussed in section 6.

2. PP emission versus thermal and non-thermal electronic emission

In this section we compare thermal and non-thermal continuum emission mechanisms in the shocked plasma behind SNR blastwaves. The non-thermal emission is assumed to be emitted by relativistic, accelerated electrons and protons with power law distributions in energy. In this section we ignore the energy cutoffs of the accelerated particle distributions. This issue is discussed in section 3. We focus on ratios of the expected fluxes, which are weakly dependent on unknown parameters such as distance to the remnant and total energy.

First we write down in section 2.1 simple expressions for the luminosities due to the different processes in simple forms that allow easy comparison with each other (a derivation of these equations is given in the appendix). Next, we compare in section 2.2 the two $\sim 1$ TeV $\gamma$-ray emission mechanisms, IC and PP. We then derive in section 2.3 constraints on $\gamma$-ray PP and IC emission by comparing them to thermal x-ray bremsstrahlung and to radio synchrotron emission. Finally, the results of sections 2.1–2.3 are compared in section 2.4 to earlier studies of the SNRs RX J1713.7-3946 and RX J0852.0-4622 (the only shell-type SNRs that are known to emit $\gtrsim 1$ TeV $\gamma$-rays).

We note that most of the results presented in this section are not restricted to SNRs and are applicable to any system that efficiently accelerates protons and/or electrons to relativistic energies with power law energy distributions.

2.1. Emission mechanisms

Consider the shocked plasma in the downstream of the blastwave of a SNR. Here we consider radiation emitted by four distinct particle populations:

1. Thermal electron and proton components with similar number densities, $n_e \sim n_p \equiv n$, which we assume consist of most of the particles. For simplicity we assume that the electron and proton energy distributions are given by Maxwellians with temperatures $T_e$ and $T_p$, respectively, with $T_e = \zeta_e T_p$. The total number of protons or electrons is $N$ and the total thermal energy is $E_{th} \approx (3/2)NT_p$.

2. Power law distributions of relativistic accelerated electrons and protons with an electron:proton ratio $K_{ep}$:

$$\frac{dN_e}{d\varepsilon_e} |_{\varepsilon_e = \varepsilon_p} = K_{ep} \frac{dN_p}{d\varepsilon_p} = K_{ep} \frac{E_p}{\varepsilon_p^{2\Lambda_p}} \left( \frac{\varepsilon_p}{\varepsilon_{p,\min}} \right)^{-p},$$  \hspace{1cm} (1)
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

where \( \varepsilon_e, \varepsilon_p \) are the electron and proton energies, respectively, \( p \) is the power law index assumed to be \( p \approx 2 \), \( E_p \) is the total energy in accelerated protons and

\[
\Lambda_p \approx \frac{1}{p-2} \left[ 1 - \left( \frac{\varepsilon_{p,\text{max}}}{\varepsilon_{p,\text{min}}} \right)^{-(p-2)} \right] \frac{\varepsilon_{p,\text{max}}}{p-2} \log \left( \frac{\varepsilon_{p,\text{max}}}{\varepsilon_{p,\text{min}}} \right). \tag{2}
\]

The distribution of the protons is described by (1) for proton energies \( \varepsilon_{p,\text{min}} < \varepsilon_p < \varepsilon_{p,\text{max}} \) with \( \varepsilon_{p,\text{min}} \sim m_p c^2 \). The value of \( \varepsilon_{\text{max}} \) depends on the SNR parameters and acceleration mechanism. Estimates of \( \varepsilon_{\text{max}} \) assuming DSA will be derived in section 3.

We study the following radiation emission mechanisms:

(1) \( \gamma \)-rays and neutrinos emitted as a result of proton–proton collisions (PP) between the relativistic protons and the thermal protons. The PP gamma-ray luminosity per logarithmic photon energy is given by (cf equation (A.8))

\[
\nu L_{\nu,\text{PP}} = C_{\text{PP}}(p) 2 \varepsilon_{\nu} \frac{dN_\nu}{d\varepsilon_\nu} \varepsilon_{\nu,\text{PP}} n_{\text{ch} \nu}, \tag{3}
\]

where \( \varepsilon_{\nu} dN_\nu/d\varepsilon_\nu \) is to be evaluated at \( \varepsilon_{\nu,\text{PP}}(\nu) = 10h\nu \) (photons energies are referred to through the photon frequency throughout the paper), the typical proton energy for which photons with energy \( h\nu \) are emitted. For \( p = 2.2.2 \) we have \( C_{\text{PP}}(2) \approx 0.85, C_{\text{PP}}(2.2) \approx 0.66 \). The neutrino luminosity is similar to the \( \gamma \)-ray luminosity at equal photon and neutrino energies.

(2) \( \gamma \)-rays emitted by inverse Compton (IC) resulting from the interaction of the relativistic electrons with CMB photons. The IC gamma-ray luminosity per logarithmic photon energy is given by (cf equation (A.12))

\[
\nu L_{\nu,\text{IC}} = C_{\text{IC}}(p) \frac{1}{2} \varepsilon_{\nu} \frac{dN_\nu}{d\varepsilon_\nu} 4 \sigma_T \gamma_e^2(\nu) U_{\text{CMB}} c, \tag{4}
\]

where \( T_{\text{CMB}}, U_{\text{CMB}} = a T_{\text{CMB}}^4 \) are the temperature and energy density of the CMB photons. \( \varepsilon_{\nu} dN_\nu/d\varepsilon_\nu \) is to be evaluated at \( \varepsilon_{\nu,\text{IC}}(\nu) = \gamma_e(\nu) m_e c^2 \equiv m_e c^2 (h\nu/3T_{\text{CMB}})^{1/2} \), the typical electron energy for which electrons up-scatter CMB photons to energy \( h\nu \). The correction factor, \( C_{\text{IC}}(p) \), is approximately \( C_{\text{IC}}(p) \approx 0.8 \) (to within 5%) for \( 2 < p < 2.2 \). It is useful to note that \( \gamma_e^2(\nu) U_{\text{CMB}} = [U_{\text{CMB}}/(3T_{\text{CMB}})] h\nu \approx 0.9 n_{\text{CMB}} h\nu \) where \( n_{\text{CMB}} \) is the number density of CMB photons.

(3) Radio and x-ray synchrotron (Syn) emission of the relativistic electrons in an assumed magnetic field \( B \). The synchrotron luminosity per logarithmic frequency is given by (cf equation (A.18))

\[
\nu L_{\nu,\text{Syn}} = C_{\text{Syn}}(p) \frac{1}{2} \varepsilon_{\nu} \frac{dN_\nu}{d\varepsilon_\nu} 4 \sigma_T \gamma_e^2(\nu) U_B c \tag{5}
\]

where \( U_B = B^2/(8 \pi) \). \( \varepsilon_{\nu} dN_\nu/d\varepsilon_\nu \) is to be evaluated at \( \varepsilon_{\nu}(\nu) = \gamma_e(\nu) m_e c^2 \equiv (2\nu/\nu_B)^{1/2} m_e c^2 \), the typical energy of electrons emitting photons with frequency \( \nu \), where \( \nu_B \equiv qB/(2\pi m_e c) \). The correction factor, \( C_{\text{Syn}}(p) \), is approximately \( C_{\text{Syn}}(p) \approx 0.8 \) (to within 5%) for \( 2 \leq p \leq 2.2 \).
(4) Thermal bremsstrahlung (TB) emission of the thermal electrons interacting with the thermal protons. The maximal TB luminosity per logarithmic frequency is emitted at the photon energy $\nu = T_e$ and is given by (cf equation (A.3))

$$
\nu L_{TB_{\nu,\nu=T_e}} = \sqrt{\frac{8}{3\pi}} e^{-1}\alpha_e \bar{g}_e N \sigma_T n_c \sqrt{m_e c^2 T_e}
$$

where $e$ is the natural logarithm, $\alpha_e \approx 1/137$ is the fine structure constant and $\bar{g}_e$ is the thermal Gaunt factor. For 100 eV $< T_e < 10$ keV, the value of $\bar{g}_e$ (for $\nu = T_e$) is in the range, 0.8 $< \bar{g}_e < 1.2$ (e.g. [27]).

We note that the amount of secondary electrons and positrons resulting from PP interactions is most likely negligible compared to the primary population of accelerated electrons. The energy output in electrons and positrons per logarithmic particle energies is roughly equal to the $\gamma$-ray emission given by equation (3). The ratio of secondary electrons + positrons to protons for an SNR of age $t = 1000\,\text{kyr}$ yr evolving into a medium with proton density $n = n_0$ cm$^{-3}$ is thus roughly given by (ignoring cooling, which affects both primary and secondary populations in the same way) $\varepsilon^2 dN_{e+e^-}/d\varepsilon \sim 0.2\varepsilon^2 dN_p/d\varepsilon \approx 10^{-6} dN_p/d\varepsilon_{\text{pp}} t_{\text{kyr}} n_0$. As long as $K_{ep} \gg 10^{-6} t_{\text{kyr}} n_0$, the contribution of the secondary electrons to the broad-band emission is negligible. Henceforth we ignore this contribution.

2.2. IC to PP emission ratio

Here we directly compare the two competing TeV $\gamma$-ray emission mechanisms. Ignoring the possible cutoffs of the spectrum of both species, the ratio of expected IC to PP gamma-ray luminosities per photon frequency can be approximated by (compare equations (3) and (4))

$$
\frac{L_{\nu,\text{IC}}}{L_{\nu,\text{PP}}} \approx 0.3 \frac{\varepsilon_{e,\text{IC}}(\nu) dN_e/d\varepsilon}{\varepsilon_p(\nu) dN_p/d\varepsilon} \frac{\sigma_T}{\sigma_{\text{pp}}^{\text{inel}}} \frac{n_{\text{CMB}}}{n} \approx 10 K_{ep, -2} \nu_{\text{TeV}}^{(p-1)/2} n_0^{-1}, \quad (7)
$$

where $K_{ep} = 10^{-2} K_{ep, -2}$ and $n = 1n_0$ cm$^{-3}$. It is useful to note that

$$
\frac{\varepsilon_{e,\text{IC}}(\nu) dN_e/d\varepsilon}{\varepsilon_p(\nu) dN_p/d\varepsilon} = K_{ep} \left[ \frac{\varepsilon_{e,\text{IC}}(\nu)}{\varepsilon_p(\nu)} \right]^{-(p-1)} \quad (8)
$$

and

$$
\frac{\varepsilon_{e,\text{IC}}(\nu)}{\varepsilon_p(\nu)} = \frac{1}{10} \sqrt{\frac{m_e c^2}{3 T_\text{CMB} h \nu}} \approx 2 \nu_{\text{TeV}}^{-1/2}. \quad (9)
$$

Comparison of equation (7) with the results of previous studies is presented in section 2.4.

An electron to proton ratio of the order of $K_{ep} \approx 10^{-2}$ is commonly assumed based on the measured electron:proton ratio in the cosmic rays (see, e.g., [31]) under the assumption that SNRs are the main source of proton and electron cosmic rays. In section 4 we find a lower limit of $K_{ep} \gtrsim 10^{-3}$ based on radio observations of SNRs in M33.

Using equation (7), we see that, as long as electron cooling does not suppress the IC flux, IC dominates PP emission as long as

$$
n \lesssim 10 K_{ep, -2} \nu_{\text{TeV}}^{(p-1)/2} \text{cm}^{-3}. \quad (10)
$$

The effect of electron cooling is addressed in section 3.
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

2.3. PP and IC to TB x-rays and non-thermal radio Syn

Here we compare the expected PP and IC γ-ray emission to thermal and synchrotron emission. This is useful for constraining the expected gamma-ray and neutrino fluxes based on observed radio and x-ray fluxes.

By comparing equations (3)–(6) we see that the ratio of PP γ-ray luminosity at photon energies $h\nu_\gamma$ to the x-ray TB luminosity at photon energies $h\nu_X = T_e$ (the photon energy of maximal emission per logarithmic photon energy) can be written as

$$\frac{\nu_\gamma L_{\nu_\gamma}^{PP}}{\nu_X L_{\nu_X}^{TB}|_{h\nu=T_e}} = \frac{3e}{10} \sqrt{\frac{\pi}{8}} \frac{\alpha e^{-1}}{g_{\text{eff}}} \frac{C_{pp}(p)}{\epsilon_{p}} \frac{\epsilon_p^2(\nu_\gamma)}{\epsilon_{p}} \frac{dN_p/d\epsilon_p}{\epsilon_{p}} \frac{T_p}{E_{\text{th}}} \frac{\sigma_{\text{pp}}^{\text{inel}}}{\sqrt{m_e c^2 T_e}} \sigma_T$$

(11)

where $E_{\text{th}} \approx (3/2)NT_p$ is the total thermal energy. Using equation (1) this can be written as

$$\frac{\nu_\gamma L_{\nu_\gamma}^{PP}}{\nu_X L_{\nu_X}^{TB}|_{h\nu=T_e}} \approx 3 \times 10^{-3} \epsilon_{p, -1} \epsilon_{e, -1}^{1/2} T_p^{1/2} \Lambda_{p, 1}^{1/2} (10^3 \nu_{\text{TeV}})^{-(p - 2)}$$

(12)

where $\epsilon_p = 0.1\epsilon_{p, -1} = E_p/E_{\text{th}}, \Lambda_p = 10\Lambda_{p, 1}, T_p = T_{p, \text{keV}}$ keV, $h\nu_\gamma = \nu_{\text{TeV}}$ TeV and we substituted $\epsilon_{\text{min}} \sim m_p c^2$. Assuming $p \geq 2$, $\epsilon_{\text{max}} > 10$ TeV and $h\nu_\gamma > 10$ GeV the factor in the second line of equation (12) is smaller than 1.1.

Temperatures $T_p \gtrsim$ keV of the shocked plasma in young SNRs with blastwaves propagating at velocities $v_s \gtrsim 1000$ km s$^{-1}$. In fact, the proton temperature behind a strong shock propagating with velocity $v_s$ is given by

$$T_p = \frac{1}{10} m_p v_s^2 = 2v_s^2 \text{keV},$$

(13)

where $v_s = 1000v_8$ km s$^{-1}$ and an adiabatic index equal to $\gamma = 5/3$ was assumed. A lower limit to the shock velocity, and thus to $T_p$ for SNRs where non-thermal x-rays are observed, is discussed in section 3.

The ratio $\zeta_e$ of electron to proton temperatures depends on the amount of collisionless heating in the shock and the following heating through Coulomb scattering. The amount of collisionless heating for high Mach shocks is not really known (for a recent review see [42]). A lower limit to $\zeta_e$ can be derived by assuming that there is no collisionless heating. After a time $t = t_{\text{kryr}}$ kyr the ratio would be (cf equation (A.5))

$$\zeta_e \gtrsim 0.6 (\lambda_{\text{ep}, 1.5} t_{\text{kryr}})^{2/5} T_p^{-3/5}$$

(14)

where $\lambda_{\text{ep}} = 30\lambda_{\text{ep}, 1.5}$ is the Coulomb logarithm. Equation (14) was derived assuming $m_e/m_p \ll \zeta_e < 1$ and is valid as long as the resultant value is in this range. Substituting equation (14) in equation (12) we find

$$\frac{\nu_\gamma L_{\nu_\gamma}^{PP}}{\nu_X L_{\nu_X}^{TB}|_{h\nu=T_e}} \lesssim 4 \times 10^{-3} \epsilon_{p, -1} T_p^{4/5} \lambda_{\text{ep}, 1.5}^{1/5} T_p^{1/5} (10^3 \nu_{\text{TeV}})^{-(p - 2)}.$$  

(15)

By comparing equations (3) and (5) we see that the expected ratio of γ-ray PP luminosity at photon energies $h\nu_\gamma$ to the radio synchrotron luminosity at frequency $\nu_R$ can be approximated by

$$\frac{\nu_\gamma L_{\nu_\gamma}^{PP}}{\nu_R L_{\nu_R}^{\text{Syn}}} \approx 3 \epsilon_p(\nu_\gamma) dN_p/d\epsilon_p \epsilon_{p}^{\text{inel}} \frac{n h\nu_\gamma}{\epsilon_{e, \text{Syn}}(\nu_R) dN_e/d\epsilon_e \sigma_T \gamma_{e, \text{Syn}}(\nu_R)^2 U_B}$$

(16)

$$\approx 50K_{\text{ep}, -1} B_{-5}^{-3/2} n_0^{1/2} \nu_{\text{GHz}}^{1/2} \left(2 \times 10^3 B_{-5}^{1/2} \nu_{\text{GHz}}^{1/2} \nu_{\text{pp}}^{1/2}\right)^{-(p - 2)}$$

In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

Journal of Cosmology and Astroparticle Physics 01 (2008) 018 (stacks.iop.org/JCAP/2008/i=01/a=018)
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

where \( \gamma_{e,\text{Syn}}(\nu_R) = \frac{\varepsilon_{e,\text{Syn}}(\nu_R)}{m_e c^2} = \left(\frac{4 \pi m_e c^2}{qB}\right)^{1/2} \) is the typical gamma factor of electrons emitting radiation with \( \nu_R = \nu_{\text{GHz}} \) GHz frequency and \( B = 10 B_{-5} \mu \text{G} \). Assuming \( p \geq 2 \), the factor in the second line of equation (16) is smaller than or equal to 1. It is useful to note that

\[
\frac{\varepsilon_p(\nu)}{\varepsilon_{e,\text{Syn}}(\nu_R)} = \frac{10 \sqrt{\hbar \nu B/2}}{m_e c^2} \approx 2 \times 10^3 B_{-5}^{1/2} \nu_{\text{GHz}}^{-1/2} \nu_{\text{TeV}}^{-1/2} \nu_{\text{GHz}}.
\]

Comparison of equation (16) with the results of previous studies is presented in section 2.4.

Ignoring the possible cutoff of the IC spectrum, the expected ratio of TeV IC emission to GHz synchrotron emission is approximately given by (compare equations (4) and (5))

\[
\frac{\nu_{\text{IC}} L_{\text{IC}}}{\nu_{\text{Syn}} L_{\text{Syn}}} \approx \left(\frac{\varepsilon_{e,\text{IC}}(\nu_{\text{IC}})}{\varepsilon_{e,\text{Syn}}(\nu_{\text{Syn}})}\right)^{3-p} \frac{U_{\text{CMB}}}{U_B} \approx 500 B_{-5}^{3/2} \nu_{\text{TeV}}^{-1/2} \nu_{\text{GHz}}^{-1/2} \times \left(4 \times 10^3 \nu_{\text{TeV}}^{1/2} \nu_{\text{GHz}}^{-1/2} B_{-5}^{-1/2}\right)^{-(p-2)}.
\]

The luminosity ratios of the different radio, x-ray and gamma-ray emission mechanisms are given by equations (7), (12), (16) and (18). Flux normalization is obtained by noting that the expected radio flux per logarithmic frequency for a SNR with total energy \( E = 10^{51} E_{51} \) erg and a fraction \( \eta_p = 0.1 \eta_{p-1} \) of the total energy carried by accelerated protons \( E_p = \eta_p E \) \( \eta_p \sim \epsilon_p/2 \) located at a distance \( d = d_{\text{kpc}} \) kpc is approximately given by (using equation (3))

\[
\nu f_{\nu,\text{Syn}} |_{\nu = 1 \text{GHz}} \sim 4 \times 10^{-13} K_{\nu,\text{p-1}} E_{51} B_{-5}^{3/2} d_{\text{kpc}}^{-2} \erg \cm^{-2} \s^{-1}.
\]

2.4. Comparison with previous studies

Next we compare the results presented in this section to previous studies of the broad-band emission of SNRs. We focus on studies that were published following the discovery of the \( \gtrsim 1 \) TeV gamma-rays from the shell-type SNRs RX J1713.7-3946 and RX J0852.0-4622 (the broad-band emission of these SNRs is discussed in section 5). Recent broad-band studies of RX J1713.7-3946 were done in [4, 13, 41, 33] while studies of RX J0852.0-4622 include [21, 5]. These studies differ in the way the particle distributions are obtained, in the assumptions regarding the magnetic field value and in the assumptions regarding the ambient IR radiation field. Berezhko and Volk [13] numerically solved time-dependent CR transport equations, coupled nonlinearly with the hydrodynamic equations for the thermal component. Aharonian et al [4], Porter et al [41] and Aharonian et al [5] assumed a constant injection of particles with a power law spectrum that is cut off exponentially over a fixed period of time. They calculated numerically the effects of cooling on the particle spectrum (taking in addition particle escape into consideration [5]). Moraitis and Mastichiadis [33] found an analytic solution to `two-zone’ (acceleration zone and escape zone) spatially averaged kinetic equations that include cooling. Enomoto et al [21] assume a power law spectrum that is cut off exponentially. Berezhko et al [13] estimated the value of the magnetic field based on observation of thin x-ray filaments (see discussion in section 5.4.1) while the other authors allowed for different magnetic field values. Porter et al [41] included a detailed model of the galactic radiation field with IR and CMB dominating in different places, while the other authors assumed ‘standard’ averaged values.
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

Table 1. Ratio of IC to PP emission for negligible suppression of IC due to cooling. (Note: the value of $L_{\nu IC}/L_{\nu PP}$(TeV) that results from equation (7) for the values of $K_{ep}$ and $n$ (cm$^{-3}$) used in each reference is shown in the last column next to that obtained in each reference (fourth column). References: (1) [33]; (2a) [5], figure 17a; (2b) [5], figure 17b; (3) [21]; the line distinguishes between PP and IC dominated models of SNRs RX J1713.7-3946 and RX J0852.0-4622.)

| Ref. | $K_{ep}$ | $n$ (cm$^{-3}$) | $L_{\nu IC}/L_{\nu PP}$(TeV) | Equation (7) |
|------|----------|----------------|-----------------------------|--------------|
| 1    | $\approx5\times10^{-4}$ | 1               | $\approx0.3$ | 0.5          |
| 2a   | $1.7\times10^{-3}$ | 0.008 | $\approx200$ | 200          |
| 2b   | $3.5\times10^{-2}$ | 0.01  | $\approx7000$ | 3500         |
| 3    | $10^{-2}$  | 0.2    | $\approx30$  | 50           |

- $K_{ep}$ was calculated by $K_{ep} \approx Q_0p_0^2(\bar{Q}_1-p_0)^{-1}(s_2-1)$ assuming the escape zone dominates and that $p, p_0 \ll p_{max}$, using the authors’ notations.

Table 2. Ratio of PP to synchrotron emission. (Note: the value of $\nu L_{\nu PP}$(TeV)/$\nu L_{\nu Syn}$(GHz) that results from equation (16) for the values of $K_{ep}$, $n$ (cm$^{-3}$), $B$ (µG) and $p$ used in each reference is shown in the last column next to that obtained in each reference (sixth column). References: (1) [13]; (2) [33]; (3a) [5], figure 18a; (3b) [5], figure 18b; (3c) [5], figure 17a; (3d) [5], figure 17b; (4) [21]. The line distinguishes between PP and IC dominated models of SNRs RX J1713.7-3946 and RX J0852.0-4622.)

| Ref. | $K_{ep}$ | $n$ (cm$^{-3}$) | $B$ (µG) | $p$ | $PP/Syn^a$ | Equation (16) |
|------|----------|----------------|---------|----|------------|--------------|
| 1    | $\approx10^{-4}$ | 1$^b$ | 130  | $\approx200$ | $\approx200$ (p = 2) |
| 2    | $\approx5\times10^{-4}$ | 1    | 15    | 2.07 | $\approx250$ | 300          |
| 3a   | $2.4\times10^{-6}$ | 0.2  | 120   | 2.1  | $\approx300$ | 400          |
| 3b   | $4.5\times10^{-4}$ | 2    | 85    | 2    | $\approx70$ | 100          |
| 3c   | $1.7\times10^{-3}$ | 0.008| 6     | 2.4  | $\approx0.5$ | 0.3          |
| 3d   | $3.5\times10^{-2}$ | 0.01 | 6.5   | 2.4  | $\approx0.3$ | 0.15         |
| 4    | $10^{-2}$  | 0.2  | $\approx6$ | 2.1  | $\approx2$ | 10           |

- $\nu L_{\nu PP}$(TeV)/$\nu L_{\nu Syn}$(GHz).

- In this reference the ambient density is nonuniform. $n$ is taken as the value of the ambient number density currently encountered by the shock.

All the above studies focused on the non-thermal emission mechanisms only. Comparisons of equations (7) and (16) with the results of these studies are presented in tables 1 and 2 respectively. As can be seen, there is good agreement (up to a factor $\sim 2$) between our analytic expressions, equations (7) and (16), and the results of earlier detailed numerical calculations of the remnants RX J1713.7-3946 and RX J0852.0-4622.

3. Energy cutoffs

In section 2 we discussed the radiation emitted by power-law-distributed electrons and protons. In reality, the particle distribution functions can be approximated by a power law function only over a limited range of particle energies. The maximal energies and
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

corresponding cutoff frequencies in the emitted spectrum were extensively studied before (see, e.g., [19, 43]). For completeness we write down in this section the expressions for the maximal particle energies attainable by DSA in a simple SNR model and the possible spectral cooling break in the electron spectrum and discuss the implications for the spectrum of the emitted radiation.

First we write down in section 3.1 the maximal energies attainable by DSA as a function of the SNR radius, age and energy, ignoring the dynamical relation between these quantities. Next, we focus in section 3.2 on the Sedov–Taylor (ST) SNR evolution phase. We then discuss in section 3.3 SNRs in which non-thermal x-rays are observed. We find a lower limit to the shock velocity and post-shock temperature in such SNRs. In addition we derive constraints on the $\gamma$-ray and x-ray spectral cutoffs that must be satisfied by an IC model for the $\gamma$-rays emitted by such SNRs. Finally, we find in section 3.4 an upper limit to the PP emission for SNRs in which the IC TeV emission is suppressed by synchrotron cooling of the energetic electrons. This is done by comparing the PP emission to the radio synchrotron emission with the implied minimal value of the magnetic field that is required to cool the electrons in times shorter than the SNR age $t$.

### 3.1. Energy cutoffs

Consider an SNR with the following parameters: energy in shocked matter $E = 10^{51}E_{51}$ erg, radius $R = 10R_1$ pc, age $t = t_{kyr}$ kyr, shock velocity $v_s = 1000v_8$ km s$^{-1}$, ambient medium density of $n = n_0$ cm$^{-3}$ at a distance of $d = d_{kpc}$ kpc. The distance to the SNR is related to the radius by $d_{kpc} \approx R_1/\theta^2$ where $\theta^2$ is the angular diameter of the SNR on the sky in degrees. The shock velocity, age and radius are related by $v_s = \alpha R/t \approx 10^9\alpha R_1 t_{kyr}^{-1}$ cm s$^{-1}$ with $0.4 < \alpha < 1$, the lower limit obtained for Sedov–Taylor (ST) expansion and the upper limit for free expansion (FE).

We assume that electrons and protons are accelerated to power law spectra (cf. equation (1)) with $p \approx 2$ up to cutoff energies, $\varepsilon_{e,max}$ and $\varepsilon_{p,max}$, respectively, with a possible cooling break in the electron spectrum at $m_p c^2 < \varepsilon_{e,break} < \varepsilon_{e,max}$ beyond which the power law index is $p + 1$.

Both electron and proton energies are limited by the finite available acceleration time due to the finite SNR age. The maximal proton or electron energy due to the finite time satisfies

$$t_{\text{acc}}(\varepsilon_{\text{max}}) = t,$$

where $t_{\text{acc}}(\varepsilon)$ is the time it takes electrons or protons to reach energy $\varepsilon$ (assumed to be equal for protons and electrons). The maximal energy of accelerated electrons can also be limited by cooling, in which case we have

$$t_{\text{acc}}(\varepsilon_{\text{e,max}}) = t_{\text{cool}}(\varepsilon_{\text{e,max}}),$$

where $t_{\text{cool}}(\varepsilon)$ is the cooling time of electrons with energy $\varepsilon_e$. In the latter case, a cooling break is expected at an energy $\varepsilon_{e,break}$ satisfying

$$t_{\text{cool}}(\varepsilon_{e,break}) = t.$$

The acceleration time, $t_{\text{acc}}(\varepsilon)$, can be approximated by

$$t_{\text{acc}} \approx \frac{t_{\text{cycle}}}{(4/3)\Delta \beta}.$$
where $\Delta \beta = (v_s - u_d)/c$, $u_d \approx v_s/4$ is the downstream velocity and $t_{\text{cycle}}$ is the shock crossing cycle time. Assuming the downstream residence time dominates the cycle time, we have

$$t_{\text{cycle}} = \frac{4D_d}{u_d c},$$

(24)

where $D_d$ is the downstream diffusion coefficient. The diffusion coefficient can be expressed as

$$D_d = \frac{\xi \varepsilon c}{3qB},$$

(25)

where $\xi$ is a dimensionless coefficient that satisfies $\xi \geq 1$ with $\xi = 1$ obtained in the Bhom diffusion limit. Using equations (23)–(25), we can write the acceleration time as

$$t_{\text{acc}} \approx \frac{16}{3} \frac{\varepsilon}{qB\beta_s v_s}.$$  

(26)

Using equations (20) and (26), the maximal energy due to the limited SNR age can be expressed as

$$\varepsilon_{e,p}(t_{\text{acc}} = t) \approx \frac{3}{16} \xi^{-1} \alpha qB\beta_s R \approx 60 \xi^{-1} \alpha B_{-5} v_8 R_1 \text{ TeV}.$$  

(27)

The electron cooling time due to synchrotron emission is given by

$$t_{\text{cool}} = \frac{\varepsilon_e}{(4/3)^2 \gamma_e^2 \sigma_T c U_B} \approx 10 \left( \frac{\varepsilon_e}{10 \text{ TeV}} \right)^{-1} B_{-5}^{-2} \text{ kyr}.$$  

(28)

(we neglect the effect of IC which corresponds to a cooling time of $\approx 100(\varepsilon_e/10 \text{ TeV})^{-1}$ kyr which we assume is much larger than the SNR age). We thus expect a cutoff to the synchrotron and IC spectra at photon energies given by

$$\nu_{\text{Syn}}(t_{\text{cool}} = t) \sim 3B_{-5}^3 v_8^2 R_1^{-2} \xi^{-2} \alpha^{-2} \text{ keV},$$  

(29)

$$\nu_{\text{IC}}(t_{\text{cool}} = t) \sim 10B_{-5}^2 v_8^2 R_1^{-2} \xi^{-2} \alpha^{-2} \text{ TeV},$$  

and a cutoff to the PP spectrum at photon energies given by

$$\nu_{\text{PP}}(t_{\text{cool}} = t) \sim 10B_{-5} v_8 R_1 \xi^{-1} \alpha \text{ TeV}.$$  

(30)

Electron cooling will be relevant at the energy given by equation (27) for strong enough magnetic fields:

$$B > 10 \left( \frac{\xi}{\alpha^2 R_1^2} \right)^{1/3} \mu \text{G}.$$  

(31)

If equation (31) is satisfied, there would be a spectral break at synchrotron and IC photon energies given by

$$\nu_{\text{Syn}}(t_{\text{cool}} = t) \approx 3B_{-5} v_8 R_{1,\text{kyr}}^{-2} \text{ keV},$$  

(32)

$$\nu_{\text{IC}}(t_{\text{cool}} = t) \approx 40B_{-5} v_8 R_{1,\text{kyr}}^{-2} \text{ TeV},$$  

(33)
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

and a cutoff at photon energies of

\[ h\nu_{\text{Syn}}(t_{\text{cool}} = t_{\text{acc}}) \approx \frac{3^3}{2^7} \xi^{-1} \alpha^{-1} m_e v_s^2 \sim \xi^{-1} 0.15 v_s^2 \text{ keV}, \]  

(33)

\[ h\nu_{\text{IC}}(t_{\text{cool}} = t_{\text{acc}}) \sim \xi^{-1} 2 v_s^2 B_{-5}^{-1} \text{ TeV}. \]

The synchrotron and IC flux per logarithmic photon energy at photon energies above the break and below the cutoff would be suppressed by a factor of

\[ \frac{\nu L_\nu \text{ (with cooling break)}}{\nu L_\nu \text{ (no break)}} \sim \left[ \frac{h\nu}{h\nu(t_{\text{cool}} = t)} \right]^{-1/2} \]  

(34)

compared to the flux that would be emitted by a power law without a break.

3.2. SNR dynamics

We now focus on the Sedov–Taylor (ST) phase. The SNR enters the ST phase when the mass of the swept-up ambient medium,

\[ M_{\text{swept}} \sim 100 R_1^3 n_0 M_\odot, \]  

(35)

is larger than the ejecta’s mass. In this case we have

\[ \alpha = 0.4, \]

\[ t \approx \sqrt{\frac{0.5 \rho R^5}{E}} \sim 5 R_1^{5/2} n_0^{1/2} E_5^{-1/2} \text{ kyr}, \]  

(36)

\[ v_s = 0.4 R/t \sim 10^8 E_5^{1/2} R_1^{-3/2} n_0^{-1/2} \text{ cm s}^{-1}. \]

Substituting equation (36) in equations (29)–(33) we obtain

\[ h\nu_{\text{Syn}}(t_{\text{cool}} = t)_{\text{ST}} \sim 0.15 E_5 B_{-5}^{-3} R_1^{-5} n_0^{-1} \text{ keV}, \]

(37)

\[ h\nu_{\text{Syn}}(t_{\text{cool}} = t_{\text{acc}})_{\text{ST}} \sim 0.1 \xi^{-1} E_5 R_1^{-3} n_0^{-1} \text{ keV}, \]

\[ h\nu_{\text{Syn}}(t_{\text{acc}} = t)_{\text{ST}} \sim 0.1 \xi^{-2} B_{-5}^{-3} E_5 R_1^{-1} n_0^{-1} \text{ keV}, \]

\[ h\nu_{\text{IC}}(t_{\text{cool}} = t)_{\text{ST}} \sim 2 B_{-5}^{-4} E_5 R_1^{-5} n_0^{-1} \text{ TeV}, \]

(38)

\[ h\nu_{\text{IC}}(t_{\text{cool}} = t_{\text{acc}})_{\text{ST}} \sim 1.5 \xi^{-1} B_{-5}^{-3} E_5 R_1^{-3} n_0^{-1} \text{ TeV}, \]

\[ h\nu_{\text{IC}}(t_{\text{acc}} = t)_{\text{ST}} \sim 1 \xi^{-2} B_{-5}^{-4} E_5 R_1^{-1} n_0^{-1} \text{ TeV}, \]

and

\[ h\nu_{\text{PP}}(t_{\text{acc}} = t)_{\text{ST}} \sim 3 \xi^{-1} B_{-5}^{-4} E_5 R_1^{-1} n_0^{-1/2} \text{ TeV}. \]  

(39)

3.3. SNRs with observable non-thermal x-rays

For SNRs with observable non-thermal synchrotron x-rays, we can find a lower limit to the shock velocity by demanding that there will be no cooling cutoff for photons with energies smaller than \( h\nu_X = \nu_{keV} \text{ keV}, \) i.e. \( h\nu_{\text{Syn}}(t_{\text{cool}} = t_{\text{acc}}) > h\nu_X. \) Using equation (33), this can be written as

\[ v_s > 3 \times 10^8 \xi^{1/2} \nu_{keV}^{1/2} \text{ cm s}^{-1}. \]  

(40)
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

The minimal velocity constraint has several implications. First, this can be used to obtain a minimal value for the proton temperature in the downstream. Comparing equations (33) and (13), we find

\[ T_p > \frac{8}{9} \xi \alpha_e \frac{m_n}{m_e} h \nu_X \sim 10 \xi \nu_X. \]  

(41)

Second, assuming that the shock velocity is not much larger than \( \beta = 3000v_8 \) km s\(^{-1}\), the diffusion coefficient cannot be much larger than the Bohm limit (\( \xi = 1 \)):

\[ \xi \lesssim 1 \nu_{8.5}^{-1}. \]  

(42)

Under this assumption, the proton temperature is constrained by

\[ 10 \xi \nu_X \lesssim T_p \lesssim 20 \nu_{8.5} \text{ keV}. \]  

(43)

Third, using \( E \gtrsim 3 \rho v_8^2 R^3 \) (which is valid for both the ST and FE phases), we find a lower limit to the ambient medium density of

\[ n < 0.1 \frac{E_{51}}{R_{3}^3} \nu_{8.5}^{-1} \text{ cm}^{-3}. \]  

(44)

Next we compare the cutoff in the IC emission to the cutoff in the synchrotron radiation. The energies of photons emitted by electrons through IC and synchrotron are both proportional to the square of the Lorentz factor of the emitting electrons. The ratio of photon energies emitted through IC by electrons to the photon energies emitted through synchrotron by the same electrons is approximately given by

\[ \frac{h \nu_{IC}}{h \nu_{Syn}} \approx 3 \frac{4 \pi m_e c}{q B} \approx 10^{10} B^{-1/5}. \]  

(45)

The ratio of IC to synchrotron power, emitted by the same electrons, is approximately given by

\[ \frac{\nu_{IC} L_{\nu_{IC}}}{\nu_{Syn} L_{\nu_{Syn}}} \approx \frac{U_{CMB}}{U_B} \approx 0.1 B^{-2/5}. \]  

(46)

In particular, the photon energies where the IC and the synchrotron luminosities are cut off should satisfy equation (45) and the luminosity values at these photon energies should satisfy equation (46) (this is true in principle for any feature in the spectrum). We can use both equations to write a constraint that does not depend on the value of the magnetic field (or the acceleration mechanism):

\[ \frac{h \nu_{IC, \text{cutoff}}}{h \nu_{Syn, \text{cutoff}}} \sim 3 \times 10^{10} \sqrt{\frac{\nu_{IC} L_{\nu_{IC}} |_{\nu_{IC, \text{cutoff}}}}{\nu_{Syn} L_{\nu_{Syn}} |_{\nu_{X, \text{cutoff}}}}}. \]  

(47)

A note of caution is in order regarding equation (47). A `cutoff’ frequency is not a well-defined quantity in general. For known functional forms, prescriptions for defining a specific frequency can be given. The precise value of the numerical coefficient in (47) may be somewhat different for different prescriptions. In addition it should be noted that, while the IC spectrum of a single electron has a sharp cutoff (photons with energies larger than the initial electron energy cannot be generated), the synchrotron spectrum cuts off exponentially, resulting in different photon spectra for given cutoff forms. Taking this into consideration, and since the precise electron spectrum is not known, the cutoff frequencies are defined only to within an order of magnitude.
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

3.4. Suppression of IC due to radiative cooling

The calculated expected ratio given by equation (7) is valid as long as there is no significant suppression of the electron population due to cooling. Electrons responsible for TeV IC emission have Lorentz factors of approximately

\[ \gamma \sim \left( \frac{\text{TeV}}{\nu_{\text{TeV}} / 3 T_{\text{CMB}}} \right)^{1/2} \sim 4 \times 10^7 \nu_{\text{TeV}}^{1/2} \]

and a corresponding cooling time of (cf. equation (28))

\[ t_{\text{cool}} = 6 \nu_{\text{TeV}}^{-1/2} B_{-5}^{-2} \text{ kyr}. \]  

(48)

Cooling will affect these electrons only if the cooling time is shorter than the lifetime \( t \) of the SNR which would be true only if the typical magnetic field is large enough:

\[ B \gtrsim 30 \nu_{\text{TeV}}^{-1/4} t_{\text{kyr}}^{-1/2} \mu \text{G}. \]  

(49)

A larger magnetic field would imply stronger synchrotron emission. We can use this to write a constraint on the TeV PP emission in case the IC emission is suppressed. Assuming that the electrons responsible for the IC TeV emission were suppressed by cooling, we can use equations (49) and (16) to obtain

\[ \frac{\nu_{\gamma} L_{\nu_{\gamma}}^{\text{PP}}}{\nu_{X} L_{\nu_{X}}^{\text{Syn}}} < 10 K_{ep,-2}^{-1/4} t_{\text{kyr}}^{-1/2} t_{0} K_{\text{TeV}}^{3/8}. \]  

(50)

Another constraint can be derived by comparing the PP emission to the synchrotron radiation at x-ray frequencies assuming that the electrons emitting the x-rays are also affected by cooling. This assumption is reasonable since we assume that electrons responsible for TeV IC emission are affected by cooling, and these electrons are responsible for synchrotron radiation of photons with energies \( h\nu \gtrsim 100 B_{-5} \text{ eV} \) (cf. equation (45)). Using equations (16) and (34), the ratio of the PP flux to the x-ray flux in the frequency range between the cooling break given by equation (32) and the cooling cutoff given by equation (33) (for a spectrum with \( p \geq 2 \) this is the maximum value of \( \nu L_{\nu_{\text{syn}}} \)) is given by

\[ \frac{\nu_{\gamma} L_{\nu_{\gamma}}^{\text{PP}}}{\nu_{X} L_{\nu_{X}}^{\text{Syn, max}}} \sim 0.4 n \sigma_{\text{pp}}^{\text{inel}} c t_{\text{ep}}^{-1} K_{ep,-2} t_{\text{kyr}} n_{0} \sim 1.5 \times 10^{-3} K_{ep,-2} t_{\text{kyr}} n_{0}, \]  

(51)

where we assumed \( p = 2 \). This equation has a weak dependence on \( p \) since the x-ray-emitting electrons have energies that are similar to the TeV \( \gamma \)-ray-emitting protons. Equation (51) has the following simple interpretation. Suppose that the amount of protons per unit energy and unit time that are being accelerated by the shock is given by \( Q(\varepsilon) \). The amount of protons per unit energy at an age \( t \) is roughly \( dN/d\varepsilon \sim Q(\varepsilon) t \) and so the PP luminosity per logarithmic frequency is roughly given by (cf. equation (3))

\[ \nu L_{\nu_{pp}} \sim 0.2 Q(\varepsilon) \varepsilon^2 t_{\text{ep}} n c \sigma_{\text{pp}}^{\text{inel}}. \]  

The electron injection rate at electron energies of \( \varepsilon \) is \( K_{ep} Q(\varepsilon) \). As the electrons are constantly being cooled, the energy input in accelerated electrons is equal to the energy emitted in synchrotron radiation. The x-ray synchrotron luminosity per logarithmic frequency is thus roughly \( \nu L_{\nu_{\text{syn}}^{\text{cooled}}} \sim 0.5 K_{ep} Q(\varepsilon) \varepsilon^2 \) (the factor of 0.5 comes from the fact that the logarithmic interval in photon energies is twice that of the emitting electrons due to the \( \nu \propto \gamma^2 \) dependence). The ratio of these expressions is equal to the result in equation (51).
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

4. Lower limit on $K_{ep}$ from extragalactic SNRs

In this section we find a lower limit for $K_{ep}$ using the observed radio fluxes from large SNRs in M33 assuming that the value of $K_{ep}$ does not vary significantly between SNRs.

One way to estimate the amount of accelerated electrons is through the radio synchrotron emission. The radio luminosity is determined by the energy in accelerated electrons and by the magnetic field value. The amount of energy in accelerated electrons cannot be deduced if the value of the magnetic field is not known. An upper limit to the magnetic field is given by the requirement that the magnetic field does not exceed equipartition. Here we assume that the fraction $\eta \approx \eta_{p, -1}$ of the total energy carried by relativistic protons does not significantly exceed $\eta_{p} \sim 0.1$. Using equations (1) and (5) we can approximate the expected luminosity at 1 GHz by

$$L_{\nu, Syn}(\text{GHz}) \approx 4 \times 10^{22} K_{ep, -2} \eta_{p, -1} E_{51} B_{-5}^{-3/2} \Lambda_{p, 1}^{-1} (5 B_{-5}^{-1/2})^{-p} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$$

where $\eta_{p} = 0.1 \eta_{p, -1}$. For an assumed maximal Lorentz factor $\gamma_{p, \text{max}} \sim 10^5$, the factor in the second line of equation (52) equals $\approx 0.9$ for $p = 2$ and $\approx 1.6$ for $p = 2.2$, and will be ignored henceforth.

The ratio of the magnetic to thermal energies behind the shock in the Sedov–Taylor phase can be approximated by

$$\epsilon_{B} \approx \frac{B^2}{8 \pi \rho v_s^2} \approx 3 \frac{B^2}{8 \pi} R_3 E_1^{-1} \approx 4 \times 10^{-4} B_{-5}^2 R_1^{-3} E_{51}^{-1}.$$  

(53)

Extracting the magnetic field from equation (53) and substituting it in equation (52) we have

$$L_{\nu, Syn}(\text{GHz}) \approx 3 \times 10^{24} K_{ep, -2} \eta_{p, -1} E_{51}^{7/4} \epsilon_{B, -1}^{3/4} R_{1}^{-9/4} \text{ erg s}^{-1} \text{ Hz}^{-1}.$$  

(54)

Radio luminosities of SNRs with known distances in nearby galaxies (including the Milky Way) are summarized by Arbutina et al [6], and virtually all have luminosities greatly exceeding $3 \times 10^{22} R_1^{-9/4} \text{ erg s}^{-1} \text{ Hz}^{-1}$, the typical value expected from equation (54) for $K_{ep} = 10^{-4}$. However, we should stress that it is dangerous to reach conclusions based on such comparisons, since the observed luminosities are limited from below by the detectors’ sensitivities. Here we focus on a sample of SNRs in M33 which is perhaps the most complete sample of radio SNRs with known distances in a single galaxy [24].

Using equation (35) we see that SNRs with radii larger than

$$R \gtrsim 2 \left( \frac{M_{ej}}{10 M_{\odot}} \right)^{1/3} \eta_{0}^{-1/3} \text{ pc}$$

(55)

are in the ST phase. The smallest SNR in the sample has a radius of $R \sim 5$ pc and most SNRs in the sample have radii $R > 10$ pc. It is thus reasonable to assume that the SNRs in the sample are in the ST expansion phase. In fact, Gordon et al [23] have shown that the radii distribution function of a larger optical SNR sample that includes the radio SNR sample is consistent with ST expansion (and is inconsistent with free expansion).

The luminosities of the observed SNRs in M33 are shown in figure 1 along with the observational threshold (dashed line) and the expected limits according to equation (54).
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

![Figure 1. Radio 20 cm luminosity of SNRs in M33 [24]. The dashed line is the observation flux (total and density) limit. The full lines are given by equation (54) with $K_{ep} = 10^{-4}$ (lower, green) and $K_{ep} = 10^{-3}$ (higher, red).](image)

corresponding to $K_{ep} = 10^{-4}$ (lower, green) and $K_{ep} = 10^{-3}$ (higher, red), adopting a distance of $d = 840$ kpc to M33.

As stressed by Gordon et al [24], there are probably unobserved SNRs with luminosities that fall beneath the observational threshold. In fact, there are about twice as many SNRs seen in optical wavelengths (Gordon et al [24], the factor being roughly radius-independent, e.g. 8, 23 and 34 SNRs in the radio sample with radii $R < 10, 15, 20$ pc, respectively, compared to 15,42 and 67 SNRs, respectively, in the optical sample). Assuming the optical sample is not far from completeness, it is reasonable that roughly half of the SNRs are missed in the radio sample (this is true for $R \lesssim 20$ pc, while for $R \gtrsim 20$ pc the optical sample is probably incomplete, [23]). Still, it is quite clear from figure 1 that the luminosity implied by a value $K_{ep} = 10^{-4}$ is lower than the typical luminosity of large remnants by at least an order of magnitude. As an illustration, in order to reconcile a value of $K_{ep} \sim 10^{-4}$ with the four SNRs observed with radius $R \approx 50$ pc and luminosity $L_\nu(\text{GHz}) \sim 10^{24}$ erg s$^{-1}$ Hz$^{-1}$, their energies would have to be unreasonably high:

$$E \sim 5 \times 10^{52} \left( \frac{R}{50 \text{ pc}} \right)^{9/7} K_{ep}^{-4/7} \eta_{p,-1}^{-4/7} \epsilon_{B,-1}^{-3/7} \left( \frac{L_\nu(\text{GHz})}{10^{24} \text{ erg s}^{-1} \text{ Hz}^{-1}} \right)^{4/7} \text{ erg}. \quad (56)$$

Note that SNe with energies that are larger than $10^{52}$ erg (termed hypernovae) have been detected (see, e.g., [37] and references therein). However, the estimated fraction of core-collapse SNe that belong to this group is of the order of $10^{-3}$ [39] and thus having four SNRs with energies exceeding $10^{52}$ erg among the $\sim 100$ SNRs in M33 is unlikely.

Using equation (54), we conclude that $K_{ep} \gtrsim 10^{-3}$ is a reasonable lower limit and that $K_{ep} \sim 10^{-4}$ can be conservatively ruled out.

A possible caveat in the arguments in this section comes from the fact that it is possible that the ambient CR electrons that have been swept up by the shock have a considerable contribution to the synchrotron emission [1]. The arguments in this section
will nevertheless remain valid in this case too, provided the ratio of the accelerated electron and proton populations, including the CR contributions, are similar for different SNRs. We note that, assuming that the cosmic rays that enter the shock are reaccelerated by DSA, the shape of the spectrum of the population of relativistic particles will approach a power law and will not be affected by the distribution of the CRs in the ISM [19].

5. Application to RX J1713.7-3946 and RX J0852.0-4622

Here we apply the results of sections 2–4 to show that the broad-band spectrum of the SNRs, RX J1713.7-3946 and RX J0852.0-4622, is inconsistent with a PP origin and is consistent with an IC origin of the ≳ TeV emission. First we summarize in section 5.1 the broad-band observations of these SNRs. Next, we show in section 5.2 that a PP source for the observed γ-ray flux is inconsistent with the broad-band emission in these SNRs. We then show in section 5.3 that an IC source for the observed γ-ray flux is consistent with all observations. We show that the contribution of the PP γ-ray emission is negligible and argue that the neutrino emission from these SNRs is probably too low to be detected by current and planned neutrino telescopes. Finally, we compare in section 5.4 the results presented here to previous studies. In particular we discuss previous claims against an IC source of the γ-rays.

5.1. Characteristics of RX J1713.7-3946 and RX J0852.0-4622

The observations of these SNRs are described by Aharonian et al [4,5] and references therein. Some of the main features are summarized below. In many ways these two shell-type SNRs are similar. Both have comparable radio and TeV fluxes:

\[ \nu f_\nu|_{\text{GHz}} \approx \text{few } 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}, \]
\[ \nu f_\nu|_{\text{TeV}} \approx \text{few } 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}, \]  

span similar angles on the sky (\( \theta \approx 1^\circ, 2^\circ \), respectively) and have non-thermal x-ray emission, which is consistent with a cutoff frequency of the order of \( h\nu_{\text{cutoff}} \lesssim \text{keV} \). The gamma-ray energy flux is consistent in both SNRs with a power law \( \nu f_\nu \propto \nu^0 \) and an exponential cutoff at photon energies of \( \sim 10 \text{ TeV} \) (for RX J0852.0-4622 the detection of the cutoff is less clear [5]).

Perhaps the main difference is in the \( h\nu \sim 1 \text{ keV} \) x-ray flux which is larger for RX J1713.7-3946 by a factor of about 5, \( \nu f_\nu|_{\text{keV}} \approx \text{few } 10^{-10}, \text{few } 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}, \) respectively.

One of the main characteristics of these SNRs is a non-thermal-dominated x-ray emission. The lack of observable thermal radiation can be used to obtain an upper bound on the value of the ambient density. Number densities considerably smaller than 1 cm\(^{-3}\) were obtained [47,48,38,15], which in turn constrain the amount of proton–proton collisions. For RX J1713.7-3946, limits on \( n \) from the lack of thermal radiation of \( n < 0.3 \text{ cm}^{-3}(d_{\text{kpc}/6})^{-1/2}, \) \( n \approx 0.05\text{–}0.07 \text{ cm}^{-3}(d_{\text{kpc}/6})^{-1/2} \) and \( n < 0.02d_{\text{kpc}}^{-1/2} \text{ cm}^{-3} \) were obtained by Slane et al [47], Pannuti et al [38] and Cassam–Chenaï et al [15], respectively. For RX J0852.0-4622 a limit on \( n \) from the lack of thermal radiation (for temperatures greater than 1 keV) of \( n < 0.03d_{\text{kpc}}^{-1/2} \text{ cm}^{-3} \) was obtained by Slane et al [48].
There have been claims that RX J1713.7-3946 is interacting with molecular clouds ([47, 22] at 6 kpc and 1 kpc, respectively). Interaction with molecular clouds of both SNRs is unlikely given the low densities implied from lack of thermal radiation and the observed roughly homogeneous emission [4, 5]. The positive TeV to CO line emission correlation that was claimed for RX J1713.7-3946 is not convincing since the CO intensity changes by some two orders of magnitude while the TeV changes by a factor of two (average to peak, [4, 5]). In any case, interaction with molecular clouds cannot account for the entire emission and we will ignore this possibility henceforth.

Distance and age estimates for these remnants are inconclusive ([4, 5] and references within). We think that it is worth mentioning that claims that the distance to these SNRs is $d \lesssim 1$ kpc (e.g. [22, 7]) require some coincidence since the galactic latitude of both SNRs is $b \lesssim 1^\circ$ ($b = 0.5, 1.2$ for RX J1713.7-3946 and RX J0852.0-4622, respectively) whereas the SNRs at this distance should be distributed in the range $|b| \lesssim 10^\circ$, assuming SNRs are distributed homogeneously throughout the galactic gaseous disc height. For RX J1713.7-3946 the coincidence that is required is more extreme since this SNR lies in the direction of the galactic center, $b = 0.5^\circ, l = 347^\circ$, close to a ‘hole’ in the galactic CO line emission [47, 35]. These positions on the sky may not be coincidental if these SNRs are further away–a few kpcs from us (as most SNRs are). On the other hand, we note that such a coincidence is certainly possible and we do not assume in what follows that the distance to these remnants is larger than 1 kpc.

5.2. Upper bounds on PP emission

We first consider the constraints on the PP emission resulting from the comparison of the PP emission to the IC and synchrotron non-thermal emission. By inserting $K_{ep} \sim 10^{-2}$ and $n \lesssim 0.1 \text{ cm}^{-3}$ in equation (7) we see that, unless the IC emission is suppressed by cooling, the $\gtrsim 1$ TeV emission in these SNRs is completely dominated by IC. Irrespective of cooling, the ratio of PP $\gtrsim 1$ TeV emission to the synchrotron radio emission is given by equation (16). A lower limit for the magnetic field is given by (49) for the case where IC emission is suppressed by cooling. Alternatively, a lower limit of $B \gtrsim 10 \mu \text{G}$ can be derived by demanding that the IC emission generated by the electrons that emit the observed x-ray synchrotron emission does not exceed the observed gamma-ray emission (using equations (45) and (46), see, e.g., [4]). By inserting $n = 0.1 n_{-1} \text{ cm}^{-3}$ and $B \gtrsim 10 B_{-5} \mu \text{G}$ in equations (16) and (50), and assuming $p \geq 2$, we find that

$$\frac{\nu_{\gamma} L_{\nu_{\gamma}} \text{PP}}{\nu_{\text{GHz}} L_{\nu_{\text{GHz}}} \text{ Syn}} \lesssim 5 K_{ep, -2}^{-1} B_{-5}^{-3/2} n_{-1}$$

and

$$\frac{\nu_{\gamma} L_{\nu_{\gamma}} \text{PP}}{\nu_{\text{GHz}} L_{\nu_{\text{GHz}}} \text{ Syn}} \lesssim 1 K_{ep, -2}^{-1} t_{3/4}^{3/4} n_{-1} B_{-5}^{3/8} \nu_{\text{TeV}},$$

with the latter equation applicable if the IC emission of photons with energy $h\nu = \nu_{\text{TeV}}$ TeV is suppressed by cooling. Comparing this to the observed ratio of fluxes per logarithmic frequency at $h\nu = 1 \text{ TeV}$ and $\nu = 1 \text{ GHz}$, which for RX J1713.7-3946 and RX J0852.0-4622 is $\nu f_{\nu}(\text{TeV})/\nu f_{\nu}(\text{GHz}) \sim 100$, we see that the contribution of the PP TeV emission is negligible compared to the total $\gtrsim 1$ TeV emission.
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

In the case that the synchrotron x-rays are also affected by cooling (this is likely if the IC emission is suppressed by cooling), by using equation (51) we find that

\[ \frac{\nu_\gamma L_{\nu_\gamma, PP}}{\nu_X L_{\nu_X, \text{Syn,max}}} \sim 1.5 \times 10^{-4} K_{ep, -2} t_{kyr} n_{-1}. \]  

Comparing this to the observed ratio of fluxes per logarithmic frequency at \( h\nu = 1 \text{ TeV} \) and \( h\nu \sim 1 \text{ keV} \), which for RX J1713.7-3946 and RX J0852.0-4622 are \( \nu f_{\nu}(\text{TeV})/\nu f_{\nu}(\text{keV}) \sim 10 \) and 2, respectively, and assuming that the maximum synchrotron luminosity cannot be much higher at lower frequencies, we see again that the contribution of the PP TeV emission is negligible compared to the total \( \gtrsim 1 \text{ TeV} \) emission.

We have assumed above that \( K_{ep} \sim 10^{-2} \), in accordance with the local ratio of CR electrons to protons and with section 4. It should be emphasized that a direct estimation of the value of \( K_{ep} \) using the arguments of section 4 is not possible for these SNRs, since their radio luminosity is not known (due to the uncertain distances) and since the value of \( \epsilon_B \) is not known for these remnants (the expected value of \( \epsilon_B \) is discussed in section 6).

In section 5.3 it is shown using equation (19) that a magnetic field value of \( B \sim 10 \mu \text{G} \), required in the IC scenario, and a value for \( K_{ep} \) of \( K_{ep} \sim 10^{-2} \) are consistent with the observed radio flux for a distance of \( \sim 1 \text{kpc} \).

We next consider the constraint on the PP emission resulting from the comparison of the PP emission to the TB x-ray emission. The ratio of \( \gtrsim 1 \text{ TeV} \) PP luminosity to TB luminosity is given by equation (12).

Constraints on the shock velocity and, more importantly, the post-shock proton temperature for SNRs with observable non-thermal x-ray radiation are given by equations (40) and (43):

\[ v_s > 3 \times 10^8 \xi^{1/2} \nu_{1/2} \text{keV} \text{ cm s}^{-1}, \]  

and

\[ 10 \xi \nu_{\text{keV}} \lesssim T_p \lesssim 20 v_{8.5}^2 \text{ keV} \]  

respectively (the upper limit to \( T_p \) results from the assumption \( v_s \lesssim 3000 v_{8.5}^2 \text{ km s}^{-1} \)), where we assumed that the cutoff in the x-ray spectrum is at \( h\nu_{\text{cutoff}} = \nu_{\text{keV}} \text{keV} \). \( \xi \) is the inverse of the ratio of the diffusion coefficient to the maximal allowable, Bhom diffusion coefficient and is always larger than 1. The same arguments led Berezhko and Völk [13] to the conclusion that \( v_s > 1.5 \times 10^8 \text{ cm s}^{-1} \) for RX J1713.7-3946 (the value they obtained from the broad-band fit is \( v_s \approx 1.8 \times 10^8 \text{ cm s}^{-1} \)).

Substituting \( T_p = 10 T_{p,1} \text{ keV} \) (following equation (62)) in equation (12) and assuming \( p \geq 2 \) we find

\[ \frac{\nu_\gamma L_{\nu_\gamma, PP}}{\nu_X L_{\nu_X, \text{TB}}|_{h\nu = \zeta e T_p}} \lesssim 10^{-2} \epsilon_{p, -1} \zeta e^{-1/2} T_{p,1}^{1/2}. \]  

\( \zeta e \) is the ratio of post-shock electron and proton temperatures and \( \epsilon_p = 0.1 \epsilon_{p, -1} \) is the fraction of the thermal energy in accelerated protons. Comparing equation (63) to the observed ratio of fluxes per logarithmic frequency at \( h\nu = 1 \text{ TeV} \) and \( h\nu = 1 \text{ keV} \), which for RX J1713.7-3946 and RX J0852.0-4622 is \( \nu f_{\nu}(\text{TeV})/\nu f_{\nu}(\text{keV}) \sim 10 \) and 2, respectively, we see that a PP origin of the \( \gtrsim 1 \text{ TeV} \) is unlikely for RX J1713.7-3946 and not possible.
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

for RX J0852.0-4622 (since a TB flux greatly exceeding the observed x-ray flux would be implied) as long as there is significant collisionless electron heating $\zeta_e \sim 1$.

If there is no collisionless electron heating, we can use equation (15) (with $T_p = 10T_{p,1}$ keV and $n = 0.1n_{-1}$ cm$^{-3}$):

$$
\frac{\nu_{\gamma} L_{\nu_{\gamma}}^{PP}}{\nu_{X} L_{\nu_{X}}^{TB}|_{\nu=T_e}} \lesssim 0.04\epsilon_{p,-1}T_{p,1}^{A/5}(\lambda_{ep,1.5}n_{-1}^{-1}t_{kyr})^{-1/5}.
$$

(64)

The electron temperature will be (see equation (14)):

$$
T_e \gtrsim 0.6(\lambda_{ep,1.5}n_{-1}^{-1}T_{p,1})^{2/5}\text{keV}.
$$

(65)

For a temperature of $T_e \gtrsim 0.6$ keV the thermal emission implied from equation (64) for a PP model would likely be detectable in RX J1713.7-3946, especially if we take into account that there would be line emissions that would have higher luminosities. In RX J0852.0-4622, emission at frequencies below 1 keV might be hard to detect due to the high background of thermal radiation coming from the Vela SNR [48]. We note that, if the proton acceleration is very efficient $\epsilon_p \sim 1$, there is no collisionless heating and the TB emission is not considerably lower than the observed non-thermal x-rays, a PP origin cannot be ruled out based on this argument alone for either SNR. We conclude that the $\gtrsim 1$ TeV photons from RX J1713.7-3946 and RX J0852.0-4622 are unlikely to be emitted by PP interactions and thus are likely emitted by IC scattering.

5.3. IC scenario

We next ask whether the broad-band spectrum of these SNRs is consistent with an IC source of the $\gamma$-rays.

First note that, for both SNRs, the inferred cutoff in the synchrotron at $\sim 1$ keV is consistent with the cutoff observed in the $\sim 10$ TeV emission (we should note that for RX J0852.0-4622 there is only a sign of a cutoff, the uncertainties do not allow a firm conclusion) if we assume a magnetic field of the order of 10 $\mu$G ([4, 5, 41], somewhat less for RX J0852.0-4622). In particular, equation (47) is satisfied (up to the uncertainties in the cutoff frequencies). This by itself can be considered as an indication of an IC source.

As the $\gamma$-ray observations extend somewhat below the cutoff, down to $\approx 0.3$ TeV, it is reasonable to compare the gamma-ray emission directly with the radio emission, ignoring the possible suppression of the gamma-ray flux due to cooling. Comparing equation (18) with the observed ratio of $\gamma$-ray to radio flux, $\nu f_{\nu}(\text{TeV})/\nu f_{\nu}(\text{GHz}) \sim 100$, we see that the expected ratio (for $p = 2$) is 5–10 times larger than observed in these SNRs (larger values corresponding to RX J0852.0-4622). This apparent discrepancy can be due to cooling suppression of the IC flux or due to a value of $p$ slightly larger than 2 (e.g. $p = 2.2$ would result in a factor of 5) consistent with the assumptions made here (a lower observed ratio would be inconsistent).

We next note that for $n \sim 0.1$ cm$^{-3}$, $R \sim 10$ pc and $E \sim 10^{51}$ the expected cutoffs in the radio and $\gamma$-ray spectrum, equations (37) and (38), are consistent with the observed cutoffs and cooling may or may not be important. The expected radio flux according to (19) is consistent with the observed $\sim 1$ GHz flux for the corresponding distance $d \sim 1$ kpc. We would like to emphasize that there are more free parameters than constraints and these values are not the only ones allowable by these constraints.
We conclude that the PP contribution to the $\gtrsim 1$ TeV flux is negligible and that an IC source for the $\gtrsim 1$ TeV flux is consistent with the observed broad-band spectrum.

Using equations (58) and (63), we see that the expected neutrino flux (being roughly equal to the $\gamma$-ray flux) is constrained for these SNRs to values

$$\varepsilon_\nu f_{\varepsilon_\nu} \lesssim 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}.$$  \hspace{1cm} (66)

The neutrino detection rate per logarithmic neutrino energy by a neutrino detector with an area $A = A_{\text{km}^2}$ is given by

$$\varepsilon_\nu \frac{dN_\nu}{d\varepsilon_\nu} = f_{\varepsilon_\nu} P_{\nu\mu, \text{water}} A \sim 0.2 \frac{\varepsilon_\nu f_{\varepsilon_\nu}}{10^{-12} \text{ erg s}^{-1} \text{ cm}^{-2}} A_{\text{km}^2} \text{ yr}^{-1},$$  \hspace{1cm} (67)

where $P_{\nu\mu, \text{water}}$ is the probability that a neutrino will interact with the water and produce a muon within a distance from the detector that is smaller than the muon cooling distance, and is approximately given by $P_{\nu\mu, \text{water}} \sim 10^{-6} \varepsilon_\nu^{1/2}$. This flux is probably too low to be detected by current and planned neutrino observatories.

### 5.4. Comparison with previous studies

Next we compare the results presented in sections 5.2 and 5.3 to previous studies of these SNRs.

In section 2.4 it was shown that equations (7) and (16) agree with the results of studies of RX J1713.7-3946 [4,13,41,33] and RX J0852.0-4622 [21,5] to within a factor of $\sim 2$. We note that all models in which the $\gamma$-ray emission is dominated by PP avoided the implied excessive synchrotron emission (but not the implied excessive thermal x-ray bremsstrahlung emission, see section 5.2) by assuming an extremely low value of $n^{-1}K_{ep}$, of $n^{-1}K_{ep} \lesssim 10^{-3}$. Such low values of $n^{-1}K_{ep}$ are not plausible since a high density $n \gg 0.1$ is inconsistent with the lack of observed thermal x-ray emission and a low value of $K_{ep} \lesssim 10^{-4}$ is inconsistent as shown in section 4.

#### 5.4.1. Claims against IC for RX J1713.7-3946 and RX J0852.0-4622

We next discuss the main claims that were raised against an IC source for the gamma-ray emission in RX J1713.7-3946 and RX J0852.0-4622.

**Low magnetic field:** As discussed in section 5.3, a magnetic field of $B \sim 10$ $\mu$G is implied if the gamma-ray emission is due to IC. The value of the magnetic field was estimated to be much higher, of the order of 100 $\mu$G [13,50,11] by interpreting thin filaments observed in the x-ray images as the result of small cooling lengths of the emitting electrons. If true, this would rule out IC as the source of the gamma-ray emission. The thin filaments could alternatively be interpreted as thin regions of enhanced magnetic field (e.g. [40]) in which case the magnetic field cannot be estimated directly. One way to distinguish between the interpretations is by comparing high resolution radio and x-ray images. The interpretation that the filaments are due to cooling of multi-TeV electrons implies that similar features should not be seen in the radio image since the electrons responsible for the radio emission hardly suffer from radiative cooling [49]. An x-ray to radio comparison was done in [30]; however, the low resolution radio images do not allow a decisive conclusion. We should note that the same arguments were used to deduce high magnetic fields in Tycho’s SNR and the remnant of SN1006 (for which a high resolution
radio image exists [17, 45]), while some of the thin filaments in the x-ray emission are clearly seen also in the radio images (compare [17] figure 1 to [10] figure 1, and [45] figure 1 to [9] figure 1, see also [16]), a fact that was ignored by [10] and [50].

**Detailed spectral shape:** Berezhko and Völk [13] claim that the observed x-ray flux cannot be properly fitted for a magnetic field of the order of $\sim 10 \mu G$. Aharonian et al [4] claim that the shape of the gamma-ray spectrum in RX J1713.7-3946 does not coincide with IC since an electron spectrum chosen to fit the radio and x-ray observations produces a narrow peak in $\nu f_\nu$, in disagreement with the flat gamma-ray spectrum observed. We do not see these claims as an inconsistency as the physics of the cutoff in the particle spectrum is not really known. For example, the assumed diffusion coefficient value is not known for all energies. If the magnetic field disturbances are generated by the accelerated particles, the spectrum at scales relevant to the particles with energies close to the cutoff scale may be different than for intermediate scales. In addition, if the high energy end of the electron energy distribution is affected by synchrotron energy losses, a flat $\nu f_\nu \propto \nu^0$ IC spectrum would be expected.

We note that, if the synchrotron peak was resolved, a more trustable comparison of IC and synchrotron spectra could have been done as long as the effect of the interstellar infrared radiation is negligible (see [41]).

We also note that there is some inconsistency in the model parameters assumed by Aharonian et al [4]. They assume an age of 1000 yr, a distance of 1 kpc and an ambient density of $n = 1 \text{ cm}^{-3}$. For such a distance and density, the swept-up mass is $M \approx (4\pi/3)R^3nm_p \sim 100M_\odot$, which is clearly in the ST regime and implies an energy in the swept-up material of $E \approx 0.5nm_pR^2t^{-2} \approx 10^{52}$ erg, which is rather large. Demanding an energy of $10^{51}$ erg, for example, would imply an age of about 3000 yr, for which cooling in a magnetic field of 10 $\mu G$ may be important (the effect of cooling would be to flatten the IC and synchrotron peaks).

### 6. Discussion

In this paper we derived simple analytic tools for analyzing the radio, x-ray and $\gtrsim 1 \text{ TeV } \gamma$-ray continuum emission mechanisms in shell-type SNRs. The emission mechanisms considered were synchrotron, IC of CMB photons by accelerated electrons, proton–proton collisions of accelerated protons with ambient protons and thermal bremsstrahlung. In section 2 we wrote down the luminosity ratios of these emission mechanisms (ignoring the energy cutoffs), equations (7), (12), (16) and (18). These ratios are independent of the SNR energy and of the distance to the SNR. In section 3 we wrote down the (energy- and distance-dependent) expected cutoffs in the non-thermal radiation spectra, equations (37)–(39), due to cooling and limited SNR age assuming DSA as the acceleration mechanism and Sedov–Taylor evolution. In addition we obtained an energy-and distance-independent constraint, equation (50), for the PP flux in the case the IC spectrum is suppressed, and an energy- and distance-independent lower limit for the proton temperature $T_p$ for SNRs in which non-thermal x-rays are observed, equation (43). We note that the synchrotron cutoff due to cooling given by equation (33) (a similar expression was derived by Berezhko and Völk [12]) naturally explains the fact that synchrotron emission does not extend to photon energies greatly exceeding $\sim $ keV in known SNRs (see, e.g., [44, 25]). This is simply because the shock velocities in SNRs do not greatly exceed a few thousand $\text{ km s}^{-1}$. 

*Journal of Cosmology and Astroparticle Physics 01 (2008) 018 (stacks.iop.org/JCAP/2008/i=01/a=018)*
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

In section 4 we derived a lower limit to the value of $K_{\text{ep}}$, the ratio of the number of accelerated electrons to the number of accelerated protons at a given energy, $K_{\text{ep}} > 10^{-3}$, by studying the radio observations of SNRs in M33. Here we assumed that the value of $K_{\text{ep}}$ (including the possible contributions from the ISM CRs) does not vary considerably between SNRs. This parameter enters into the ratios between IC and synchrotron emissions to PP emissions.

In section 5 we applied the results of the earlier sections to show that the broadband spectra of the SNRs, RX J1713.7-3946 and RX J0852.0-4622, are inconsistent with a PP origin and are consistent with an IC origin of the $\gtrsim$TeV emission. A PP-dominated TeV emission would imply radio synchrotron and probably thermal x-ray bremsstrahlung fluxes that would greatly exceed the observed x-ray flux.

The neutrino flux from these SNRs is expected to be lower than $\varepsilon_\nu f_\nu \lesssim 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ and is probably too low to be detected by current and planned neutrino observatories.

We compared our main results with previous studies of these SNRs (tables 1 and 2) and showed that our simple analytical expressions are in good agreement with more detailed calculations. All models, in which the $\gamma$-ray emission is dominated by PP, avoided the implied excessive synchrotron emission (but not the implied excessive thermal x-ray bremsstrahlung emission, see section 5.2) by assuming an extremely low value of $n^{-1}K_{\text{ep}}$, $n^{-1}K_{\text{ep}} \lesssim 10^{-3}$. Such low values of $n^{-1}K_{\text{ep}}$ are not plausible since a high density $n \gg 0.1$ is inconsistent with the lack of observed thermal x-ray emission and a low value of $K_{\text{ep}} \lesssim 10^{-4}$ is inconsistent with radio observations of SNRs in nearby galaxies as shown in section 4. Previous claims, that the $\gamma$-ray emission in SNRs RX J1713.7-3946 and RX J0852.0-4622 is not IC, were discussed in section 5.4.1.

Interpretation of the narrow filaments seen in the x-ray pictures as the cooling width of the emitting electrons was used to obtain magnetic field estimates of the order of $\sim 100 \mu$G [13, 50, 11]. This would rule out an IC source and thus seem implausible. As an illustration, this would require a value of the electron:proton ratio of $K_{\text{ep}} \sim 10^{-5}n^{-1}B_{\mu G}^{3/2}$ to explain the $\sim 100$ ratio of TeV to GHz fluxes per logarithmic frequency (without solving the thermal bremsstrahlung problem). The interpretation of the narrow filaments as the cooling width of the multi-TeV x-ray emitting electrons implies that similar filaments are not expected in the radio observations. There are at least two examples (Tycho’s SNR and the remnant of SN1006) where similar filaments are observed in both radio and x-rays. This puts into question the high $B$ interpretation of the x-ray filaments (see the discussion in section 5.4.1).

We note that the synchrotron to PP ratios would be affected if the magnetic field is enhanced in a small region behind the shock as suggested above but that the conclusion that a PP model requires low values of $K_{\text{ep}}$ would not change. To see the effect of thin enhancement regions, assume an extreme case where there is a strong magnetic field $B$ in a small region $d \ll R$ behind the shock, and a negligible magnetic field elsewhere. Assuming that the accelerated electrons are not confined to this region, the radio emission would be proportional to $dB^{3/2}$ and it would be possible to allow for a higher value of $K_{\text{ep}}$ in a PP model for a given value of the magnetic field. Note, however, that in order to cool the electrons emitting the TeV IC for a given SNR age (see the discussion in section 3.4), the magnetic field would have to be larger in order to cool the electrons in the time they reside in the high magnetic field region and thus will have to be larger by...
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

A factor of $d^{-1/2}$ compared to a homogeneous case. So the suppression of the radio flux due to the small emitting region, given that the IC emitting electrons are cooled, will be roughly equal to $(d/R)^{-1/4}$ where $R$ is the remnant radius. The thin filaments observed by Chandra have widths of $2'$ and $1'$ for RX J1713.7-3946 and RX J0852.0-4622, respectively \cite{13, 50, 11}. Taking into account a projection factor of $\approx 7$ \cite{13} the emission region widths are fractions $d/R \approx 10^{-2}$ and $3 \times 10^{-3}$ of the SNR radii, respectively. This would require a correction factor of $(d/R)^{1/4} \sim 3-5$ to equation \eqref{59} and will not change the conclusions. Furthermore, assuming that the x-ray synchrotron emitting electrons are also effectively cooled in this region, equation \eqref{60} will remain valid.

Using the magnetic field value $B \sim 10 \mu G$, the ratio of magnetic field energy to thermal energy of swept-up material is roughly given by

$$\epsilon_B \sim \frac{3 B^2}{8 \pi} R^3 E_{\text{swept}}^{-1} \sim 4 \times 10^{-4} B_{-5}^2 R_{1.5}^3 E_{\text{swept},51}^{-1}, \quad \text{(68)}$$

where $E_{\text{swept}} = 10^{51} E_{\text{swept},51} \text{erg}$ is the total energy in swept-up material. We note that, if the distances to these SNRs are a few kpcs, $\epsilon_B$ would equal a few per cent. For example, a radius of $R = 30 R_{1.5} \text{ pc}$, implying distances of $3 R_{1.5} \text{ kpc}$ and $1.5 R_{1.5} \text{ kpc}$ to RX J1713.7-3946 and RX J0852.0-4622, respectively, implies $\epsilon_B \sim 0.01 B_{-5}^2 R_{1.5}^3 E_{\text{swept},51}^{-1}$ and is consistent with all observations. We note that larger distances imply smaller densities since the velocity is limited from below by equation \eqref{61}, $v_s \geq 3 \times 10^8 v_{8.5} \text{ km s}^{-1}$, and the number density can roughly be expressed as $n \approx 2 \times 10^{-3} E_{\text{IC},51} v^{1/2}_{8.5} m_p^{-1} R_{1.5}^3 \text{ cm}^{-3}$ where $v_s = 3000 v_{8.5} \text{ km s}^{-1}$. Such low densities are expected if these shocks are propagating into progenitor winds \cite{13} and references within.

$\gamma$-ray observations in the GeV to sub-TeV range by the GLAST experiment will hopefully allow a clear direct distinction between the IC predicted spectrum, $\nu f_{\nu} \propto \nu^{1/2}$ (which is thus predicted for the SNRs RX J1713.7-3946 and RX J0852.0-4622) and the PP predicted spectrum $\nu f_{\nu} \propto \nu^{\delta}$ (with a cutoff at $\sim 100 \text{ MeV}$ energies). Using equation \eqref{7}, the expected IC to PP flux ratio for GeV photon energies is approximately

$$L_{\nu, \text{IC}}/L_{\nu, \text{PP}}(\text{GeV}) \approx 3 K_{\text{ep},-2} n_{-1}^{-1}. \quad \text{(69)}$$

A non-negligible contribution of the PP emission cannot be ruled out (for smaller photon energies the PP emission is strongly suppressed). However, it is certainly possible that PP emission is masked out by IC at all photon energies for these SNRs.

An interesting question is what kind of SNR parameters are required in order to have an observable gamma-ray emission dominated by PP collisions. Higher densities would result in higher PP emission, albeit with lower maximal proton energy. The maximal proton energy is proportional to $\epsilon_{p,\text{max}} \propto B R v_s \propto E B R^{-1/2} n^{-1/2}$. Based on the observation that electrons are accelerated to $\sim 60 \text{ TeV}$ energies in these SNRs we assume protons are accelerated to similar energies \cite{13} (probably somewhat higher if the electrons are limited by cooling). Therefore, comparing to these SNRs, we have the freedom to increase the density by a factor of $\sim 100$ (fixing the energy, radius and magnetic field), while keeping protons energetic enough to produce $\sim 1 \text{ TeV}$ photons. The cutoff photon energies in the IC spectrum and the synchrotron spectrum are both proportional to $\propto v_{5}^2 B^{-1} \propto E R^{-3} B_{-5}^{-1} n^{-1}$ and $\propto v_{5}^2 B^2 R^2 \propto E R^{-1} B_{-5}^{-1} n^{-1}$, for cooling and age limits, respectively (see equations \eqref{37} and \eqref{38}). A factor of $\sim 100$ in the density (for fixed energy, radius and magnetic field) would shift the IC and synchrotron cutoff energies by a
factor of 1/100, strongly suppressing the TeV IC and keV synchrotron emissions. At the same time, a larger density would increase the thermal x-ray emission (as long as the post-shock temperature does not fall below the x-ray observable energies). It is therefore likely that SNRs with considerably higher ambient densities have observable PP-dominated TeV emission. Such SNRs will have thermal or no observable x-ray radiation rather than non-thermal x-ray radiation. At $\sim 1$ GeV photon energies, densities exceeding $n \gtrsim 0.3 K_{\text{ep},-2}$ (cf. (69)) are enough for PP emission to dominate the IC emission.

Neutrino emission from PP collisions is similarly expected to be higher in SNRs evolving in high density environments (the neutrino flux roughly equals the PP gamma-ray flux) and are likely to be better observed in SNRs with strong thermal x-ray emission (or no x-ray emission). For SNRs with observed thermal x-ray emission, the expected neutrino flux can be estimated directly using (12).

We conclude that there is need for a detailed analysis using the x-ray and radio data of SNRs in order to find suitable candidates for PP $\gamma$-ray and neutrino emission. The analytical tools developed in this paper may be used to estimate the expected $\gamma$-ray and neutrino fluxes and to determine the dominant $\gamma$-ray emission process based on existing radio and x-ray observations of SNRs.

Acknowledgments

We thank M Fukugita for discussions that triggered this work. This research was partially supported by ISF, AEC and Minerva grants.

Appendix. Emission mechanisms

A.1. Thermal bremsstrahlung

The thermal bremsstrahlung emissivity per unit frequency of an optically thin plasma with temperature $T_e$ is given by [46]

$$\epsilon_{\text{ff}} \nu = \frac{2^3 \pi q^6}{3m_e c^3} \left( \frac{2 \pi}{3m_e} \right)^{1/2} T_e^{-1/2} \bar{g}_{\text{ff}} Z^2 n_e n_i e^{-h\nu/T_e} \tilde{g}_{\text{ff}},$$

(A.1)

where $n_e$, $T_e$ are the electron number density and temperature, respectively, $n_i$, $Z$ are the ions’ number density and charge, respectively, and $\bar{g}_{\text{ff}}$ is the thermal Gaunt factor. For a plasma consisting of electrons and protons with equal number density $n$ we have

$$\nu \epsilon_{\nu} = \sqrt{\frac{8}{3\pi}} \sigma_T \alpha_e c \left( \frac{m_e c^2}{T_e} \right)^{1/2} n \frac{h\nu}{T_e} n e^{-h\nu/T_e} \tilde{g}_{\text{ff}}.$$

(A.2)

The function $x e^{-x}$ attains its maximal value ($e^{-1}$) at $x = 1$. The maximal luminosity per logarithmic frequency is thus

$$\nu L_{\nu, h\nu = T_e} = \sqrt{\frac{8}{3\pi}} e^{-1} \alpha_e \tilde{g}_{\text{ff}} N \sigma_T n c \sqrt{m_e c^2 T_e^{4/2}}.$$

(A.3)

For $h\nu = T_e$, 100 eV $< T_e < 10$ keV, the value of $\tilde{g}_{\text{ff}}$ is in the range $0.8 < \tilde{g}_{\text{ff}} < 1.2$ (e.g. [27]).
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

We next consider the expected value of the electron temperature due to Coulomb heating by protons. The equation for the change in the electron temperature due to Coulomb collisions with protons is given by (e.g. [26])

\[
\frac{dT_e}{dt} = (T_p - T_e) \frac{8\sqrt{2\pi nq^4}}{3m_e m_p} \left( \frac{T_e}{m_e} + \frac{T_p}{m_p} \right)^{-3/2} \lambda_{ep},
\]

(A.4)

where \( \lambda_{ep} \) is the Coulomb logarithm. Assuming that \( m_e/m_p \ll T_e/T_p \ll 1 \) the electron temperature after a time \( t = t_{k\text{yr}} \text{ kyr} \) will be

\[
T_e \sim 0.6 \left( \lambda_{ep,1.5} n_0 t_{k\text{yr}} T_p, \text{keV} \right)^{2/5} \text{keV},
\]

(A.5)

where \( \lambda_{ep} = 30 \lambda_{ep,1.5} \). For \( 10^{-2} < T_e/T_p < 0.6 \) the correction to this expression is smaller than 20%.

Next consider power law distributions of accelerated electrons or protons:

\[
\frac{dN_i}{d\gamma_i} = A_i \gamma_i^{-p},
\]

(A.6)

where \( i = e, p \).

A.2. Gamma rays from proton–proton collisions

The spectrum of emitted photons is given by [19]

\[
\frac{1}{h} \nu L_{\gamma \gamma}^{\text{pp}} = \langle mx \rangle^p \sigma_{\text{pp}}^{\text{inel}} n c \frac{dN_p}{d\varepsilon_p} \bigg|_{h\nu},
\]

(A.7)

\[
\langle mx \rangle^p \gamma \approx \frac{2}{p} \langle mx \rangle^p_{\gamma_0},
\]

where \( \sigma_{\text{pp}}^{\text{inel}} \) is the inelastic proton–proton cross section and \( \langle mx \rangle^p_S \) is the spectrum-weighted moment for particles of type \( S \). This can be written as

\[
\nu L_{\gamma \gamma}^{\text{pp}} = C_{\text{pp}}(p) 2\varepsilon_p \frac{dN_p}{d\varepsilon_p} \sigma_{\text{pp}}^{\text{inel}} n c h\nu,
\]

(A.8)

where \( \varepsilon_p dN_p/d\varepsilon_p \) is to be evaluated at \( \varepsilon_p(\nu) = 10h\nu \). Ignoring the correction factor \( C_{\text{pp}}(p) \), this is equivalent to assuming that each inelastic p–p collision produces two photons with energy \( h\nu = \varepsilon_p/10 \). The correction factor is given by

\[
C_{\text{pp}}(p) = \frac{(2/p) \langle mx \rangle^p_{\gamma_0}}{2 \times 10^{-(\nu-1)} t}. \]

(A.9)

For \( p = 2, 2.2 \) we have \( C_{\text{pp}}(2) \approx 0.85, C_{\text{pp}}(2.2) \approx 0.66 \) (values of \( \langle mx \rangle^p_{\gamma_0} \) were taken from [19]).
A.3. IC radiation of CMB photons

The spectrum of IC scattered photons of a black-body target with temperature $T$ in the Thompson regime is given by [46]

$$
\frac{1}{\hbar} L_\nu \ IC = A_e \frac{8\pi^2 r_e^2}{h^3 c^2} (T)^{(p+5)/2} F(p)(\hbar \nu)^{-(p-1)/2},
$$

(A.10)

where

$$
F(p) = 2^{p+3} \frac{p^2 + 4p + 11}{(p+3)^2(p+5)(p+1)} \Gamma \left( \frac{p+5}{2} \right) \zeta \left( \frac{p+5}{2} \right).
$$

(A.11)

This can be written as,

$$
\nu L_\nu \ IC = C_{IC}(p) \frac{1}{2} \frac{\varepsilon_e(\nu)}{\varepsilon_e} dN_e \frac{4}{3} \sigma_T \gamma_e^2(\nu) U_T c,
$$

(A.12)

where $\varepsilon_e dN_e/\varepsilon_e$ is to be evaluated at $\varepsilon_e(\nu) = \gamma_e(\nu) m_e c^2 \equiv m_e c^2 (h \nu/3T)^{1/2}$ and $U_T = aT^4$ is the energy density in the black-body photons. Ignoring the correction factor $C_{IC}(p)$, this is equivalent to assuming that each electron emits all the power $\frac{4}{3} \sigma_T \gamma_e^2 U_T c$, in photons of energy $h \nu = \gamma_e^2 3T$. The correction factor is given by

$$
C_{IC}(p) = 3^{-{(p-7)/2}} \frac{15}{16} \pi^4 F(p).
$$

(A.13)

For $2 < p < 2.2$ we have $C_{IC}(p) \approx 0.8$ (to within 5%). It is useful to note that $\gamma_e^2(\nu) U_T = [U_T/(3T)]\nu \approx 0.9 n_T h \nu$ where $n_T$ is the number density of black-body photons.

A.4. Synchrotron radiation

The spectrum of synchrotron radiation is [46]

$$
L_\nu = \frac{1}{p+1} \Gamma \left( \frac{p}{4} + \frac{19}{12} \right) \Gamma \left( \frac{p}{4} - \frac{1}{12} \right) \frac{\sqrt{3} q^4 B \sin \alpha}{mc^2} \left( \frac{2\pi mc \nu}{3qB \sin \alpha} \right)^{-(p-1)/2},
$$

(A.14)

where $\alpha$ is the angle between the electrons’ velocity and the magnetic field direction. Using

$$
\frac{1}{4\pi} \int_0^\pi 2\pi \sin \alpha \ d\alpha (\sin \alpha)^{(p+1)/2} = \sqrt{\pi} \Gamma((p + 5)/4) \frac{2\Gamma((p + 7)/4)}{2\Gamma(p/4 - 1/12) \Gamma(p/4 + 19/12) \sqrt{3} \left( \frac{2\pi}{3} \right)^{-(p-1)/2}},
$$

(A.15)

the spectrum for an isotropic distribution is

$$
L_\nu = A_e D(p) \frac{q^4 B}{mc^2} \left( \frac{mc \nu}{qB} \right)^{-(p-1)/2}
$$

(A.16)

with

$$
D(p) = \frac{1}{p+1} \sqrt{\pi} \Gamma ((p + 5)/4) \frac{\Gamma(p/4 + 19/12) \Gamma(p/4 - 1/12)}{2\Gamma(p/4 + 7/4)} \sqrt{3} \left( \frac{2\pi}{3} \right)^{-(p-1)/2}.
$$

(A.17)

This can be written as

$$
\nu L_\nu = C_{Syn}(p) \frac{1}{2} \frac{\varepsilon_e(\nu)}{\varepsilon_e} dN_e \frac{4}{3} \sigma_T \gamma_e^2(\nu) U_B c,
$$

(A.18)
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

where \( \varepsilon_e dN_e/\varepsilon_e \) is to be evaluated at \( \varepsilon_e(\nu) = \gamma_e(\nu) m_e c^2 \equiv (2\nu/\nu_B)^{1/2} m_e c^2 \), \( \nu_B \equiv qB/(2\pi n_e c) \) and \( U_B = B^2/(8\pi) \). Ignoring the correction factor \( C_{\text{Syn}}(p) \), this is equivalent to assuming that each electron emits all its power of \((4/3)\sigma_T \gamma^2 U_B c \) in photons of energy

\[
\nu = \gamma^2 \nu_B/2.
\]

The correction factor is given by

\[
C_{\text{Syn}}(p) = \frac{3}{2}(4\pi)^{(n-3)/2} D(p)
\]

(A.19)

and is approximately \( C_{\text{Syn}}(p) \approx 0.8 \) (to within 5%) for \( 2 \leq p < 2.2 \).

References

[1] Anderson M C and Rudnick L, 1993 Astrophys. J. 408 514 [SPIRES]

[2] Aharonian F A et al, 2004 Nature 432 75 [SPIRES]

[3] Aharonian F et al, 2005 Astron. Astrophys. 437 135 [SPIRES]

[4] Aharonian F et al, 2006 Astron. Astrophys. 449 223 [SPIRES]

[5] Aharonian F et al, 2007 Astrophys. J. 661 236 [SPIRES]

[6] Arbutina B, Urosević D, Stanković M and Těsić L, 2004 Mon. Not. R. Astron. Soc. 350 346

[7] Bamba A, Yamazaki R, Ueno M and Koyama K, 2003 Astrophys. J. 589 827 [SPIRES]

[8] Bamba A, Yamazaki R, Yoshida T, Terasawa T and Koyama K, 2005 Astrophys. J. 621 793 [SPIRES]

[9] Bamba A, Yamazaki R and Hiraga J S, 2005 Astrophys. J. 632 294 [SPIRES]

[10] Berezhko E G and Völk H J, 2004 Astron. Astrophys. 427 525 [SPIRES]

[11] Berezhko E G and Völk H J, 2006 Astron. Astrophys. 451 981 [SPIRES]

[12] Blanford R and Eichler D, 1987 Phys. Rep. 154 1 [SPIRES]

[13] Cassam-Chenaï G, Decourchelle A, Ballet J, Sauvageot J-L, Dubner G and Giacani E, 2004 Astron. Astrophys. 427 199 [SPIRES]

[14] Cassam-Chenaï G, Hughes J P, Ballet J and Decourchelle A, 2007 Preprint astro-ph/0703239

[15] Dickel J R, van Breugel W J M and Strom R G, 1991 Astrophys. J. 101 2151 [SPIRES]

[16] Drury L O, 1983 Rep. Prog. Phys. 46 973

[17] Drury L O, Aharonian F A and Voelk H J, 1994 Astron. Astrophys. 287 959 [SPIRES]

[18] Enomoto R et al, 2002 Nature 416 823 [SPIRES]

[19] Enomoto R et al, 2006 Astrophys. J. 652 1268 [SPIRES]

[20] Fukui Y et al, 2003 Publ. Astron. Soc. Jpn. 55 L61

[21] Gordon S M, Kirshner R P, Long K S, Blair W P, Duric N and Smith R C, 1998 Astrophys. J. Suppl. 117 89

[22] Gordon S M, Duric N, Kirshner R P, Goss W M and Viallefond F, 1999 Astrophys. J. Suppl. 120 247

[23] Hendrick S P and Reynolds S P, 2001 Astrophys. J. 559 903 [SPIRES]

[24] Ichimaru S, Statistical Plasma Physics vol 1 (Boulder, CO: Westview Press) p 307

[25] Karzas W J and Latter R, 1961 Astrophys. J. Suppl. 99 127 [SPIRES]

[26] Karzas W J and Latter R, 1961 Astrophys. J. 99 127 [SPIRES]

[27] Koyama K, Petre R, Gotthelf E V, Hwang U, Matsuura M, Ozaki M and Holt S S, 1995 Nature 378 255 [SPIRES]

[28] Lazendic J S, Slane P O, Gaensler B M, Reynolds S P, Phucinsky P P and Hughes J P, 2004 Astrophys. J. 607 271 [SPIRES]

[29] Longair M S, 1994 High Energy Astrophysics (Cambridge: Cambridge University Press)

[30] Moraitis K and Mastichiadis A, 2007 Astron. Astrophys. 462 173 [SPIRES]

[31] Muraishi H et al, 2000 Astron. Astrophys. 354 L57 [SPIRES]

[32] Nomoto K, Tominaga N, Umeda H, Kobayashi C and Maeda K, 2006 Nucl. Phys. A 777 424 [SPIRES]

[33] Podsiadlowski P, Mazzioli A P, Nomoto K, Lazzati D and Cappellaro E, 2004 Astrophys. J. 607 L17 [SPIRES]

[34] Pohl M, Yan H and Lazarian A, 2005 Astrophys. J. 626 L101 [SPIRES]
In which shell-type SNRs should we look for gamma-rays and neutrinos from P–P collisions?

[41] Porter T A, Moskalenko I V and Strong A W, 2006 Astrophys. J. 648 L29 [SPIRES]
[42] Rakowski C E, 2005 Adv. Space Res. 35 1017
[43] Reynolds S P, 1998 Astrophys. J. 493 375 [SPIRES]
[44] Reynolds S P and Keohane J W, 1999 Astrophys. J. 525 368 [SPIRES]
[45] Rothenflug R, Ballet J, Dubner G, Giacani E, Decourchelle A and Ferrando P, 2004 Astron. Astrophys. 425 121 [SPIRES]
[46] Rybicki G B and Lightman A P, 1979 Radiative Processes in Astrophysics (New York: Wiley)
[47] Slane P, Gaensler B M, Dame T M, Hughes J P, Plucinsky P P and Green A, 1999 Astrophys. J. 525 357 [SPIRES]
[48] Slane P, Hughes J P, Edgar R J, Plucinsky P P, Miyata E, Tsunemi H and Aschenbach B, 2001 Astrophys. J. 548 814 [SPIRES]
[49] Vink J and Laming J M, 2003 Astrophys. J. 584 758 [SPIRES]
[50] Völk H J, Berezhko E G and Ksenofontov L T, 2005 Astron. Astrophys. 433 229 [SPIRES]