Flavor structure, Higgs boson mass, and dark matter in a supersymmetric model with vector-like generations

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We study a supersymmetric model in which the Higgs mass, the muon anomalous magnetic moment, and the dark matter are simultaneously explained with extra vector-like generation multiplets. For the explanations, non-trivial flavor structures and a singlet field are required. In this paper, we study the flavor texture by using the Froggatt–Nielsen mechanism, and then find realistic flavor structures that reproduce the Cabibbo–Kobayashi–Maskawa matrix and fermion masses at low energy. Furthermore, we find that the fermion component of the singlet field becomes a good candidate for dark matter. In our model, flavor physics and dark matter are explained with moderate-size couplings through renormalization group flows, and the presence of dark matter supports the existence of just 3 generations in low-energy scales. We analyze the parameter region where the current thermal relic abundance of dark matter, the Higgs boson mass, and the muon $g-2$ can be explained simultaneously.

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1. Introduction

The discovery of the Higgs boson by the ATLAS and CMS Collaborations of the LHC has a big impact on particle physics (Ref. [1,2]). At the experiment, the Higgs mass is confirmed to be $m_h^2 = 125.09 \pm 0.21 \text{(stat.)} \pm 0.11 \text{(syst.)} \text{GeV}$ (Ref. [3]). By the discovery, the particles predicted by the Standard Model (SM) are experimentally confirmed. However, there still remain various problems to be solved beyond the SM. Among them, we focus on the muon anomalous magnetic moment ($\muon g-2$) and dark matter (DM) in this paper. We reveal that flavor structures are important for the issues.

Experimentally, the muon $g-2$ is reported with $\Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}$ (Refs. [4,5]). For the explanation of this result, the extension of the SM is a possibility. In Ref. [6], one of the present authors (M.N.) showed that the supersymmetric model with vector-like generations explains experimental results. In the analysis, the authors solved the renormalization group (RG) flow of the Yukawa matrices, and then found that a certain flavor structure of the quark and lepton sectors is required for the explanation of the muon $g-2$ and the Higgs mass. Further, in the model, a singlet scalar field is required to give large masses to the vector-like generations enough to avoid electroweak precise measurements (Ref. [7]). In this paper, we show that the Yukawa structure is determined by the Froggatt–Nielsen mechanism, and the single field is a good candidate for the DM.
By astrophysical observations such as the Galaxy rotation curves (Ref. [8]), collisions of bullet clusters (Ref. [9]), or cosmic microwave background (Ref. [10]), it is confirmed that the matter contents of the present Universe are mainly dominated by DM with the abundance $\Omega_{DM}h^2 = 0.1198 \pm 0.0015$ (Ref. [10]). The formation of large-scale structures also requires DM since it is a significant source of gravitational potential. However, there does not exist a natural candidate for DM in the SM. Thus, the extension of particle contents is needed. In our model, the superpartner of the singlet scalar field, which is called a singlino, is the lightest supersymmetric particle (LSP), and it could be DM in the presence of $R$-parity. We show that the thermal relic abundance of the singlino field explains the DM abundance.

Another issue relevant to this paper is the origin of the flavor structure in the quark and lepton sectors. As in the case of the Cabbibo–Kobayashi–Maskawa (CKM) matrix in the SM, our model (Ref. [6]) needs Yukawa structures for the mass matrices of the quark and lepton sectors extended with vector-like generations. In particular, there should exist an appropriate flavor structure for explaining experimental values of both the Higgs mass and the muon $g-2$ through quantum corrections simultaneously. However, as in the SM, Yukawa couplings are just free parameters. We try to explain the structure by the Froggatt–Nielsen (FN) mechanism.

Froggatt and Nielsen explained the structure by assuming additional U(1) symmetry called flavor symmetry (Ref. [11]). The mechanism is realized also in SUSY models (Ref. [12,13]). In this work, we reproduce the flavor structure of the model (Ref. [6]) by the FN mechanism. Then we show a charge assignment to the chiral superfields for realizing a realistic Yukawa hierarchy, which explains the CKM matrix and fermion masses at low energy with the parameter $\epsilon \simeq 0.33$. Here, $\epsilon$ is the breaking scale of FN U(1) symmetry normalized by the cutoff scale.

With an appropriate assignment of the FN charge, an SM singlet superfield $\Phi$ plays two important roles. One is to fix the mass scales of vector-like generations with the vacuum expectation value (VEV) of the scalar field $\langle \Phi \rangle$. Another is that its fermion component becomes a candidate for DM with $R$-parity. In this sense, the presence of the DM supports the existence of 3 generations in low-energy scales within our model. As seen later, we show a parameter space where one obtains the right amount of the Higgs boson mass, the muon $g-2$, and the observed relic abundance of DM in our model.

The organization of this paper is as follows. In Sect. 2, we introduce our model with the flavor symmetric superpotential. We assign the U(1) charge to each field and give possible Yukawa structure in both quark and lepton sectors. Then the observed CKM matrix and fermion masses can be reproduced at the $M_Z$ scale. Here, $M_Z \simeq 91$ GeV (Ref. [14]) is the Z boson mass. In Sect. 3, we shall explain a candidate for DM in our model. Next, we give an analytic equation for calculating the thermal relic abundance of the DM. In Sect. 4, we show the parameter region where the DM abundance, Higgs boson mass, and the muon $g-2$ within $2\sigma$ level are simultaneously explained. The final section is devoted to the conclusion and discussion.

2. Model

In this section, we first give an explanation of the model proposed in Ref. [6]. Second, we extend the model by adding a U(1) flavor symmetry with the FN mechanism (Refs. [11–13]), and then show that a nontrivial flavor structure is obtained through the symmetry breaking with an appropriate charge assignment. Sizable couplings between the supersymmetric SM sector and vector-like generations significantly contribute to the Higgs boson mass and the muon $g-2$. 

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Table 1. The chiral superfields and their quantum number under the SM gauge group.

| Superfield | (SU(3), SU(2), U(1)) |
|-----------|----------------------|
| \(Q_4\)   | \((3, 2, \frac{1}{6})\) |
| \(u_4\)   | \((3^*, 1, -\frac{2}{3})\) |
| \(d_4\)   | \((3^*, 1, \frac{1}{3})\) |
| \(L_4\)   | \((1, 2, -\frac{1}{2})\) |
| \(e_4\)   | \((1, 1, 1)\) |
| \(\tilde{Q}\) \(\equiv\) \((u_{5R}, d_{5R})^C\) | \((3^*, 2, -\frac{1}{6})\) |
| \(\tilde{u}\) \(\equiv\) \(u_{5L}\) | \((3, 1, \frac{2}{3})\) |
| \(\tilde{d}\) \(\equiv\) \(d_{5L}\) | \((3, 1, -\frac{1}{3})\) |
| \(\tilde{L}\) \(\equiv\) \((\nu_{5R})^C\) | \((1, 2, \frac{1}{2})\) |
| \(\tilde{e}\) \(\equiv\) \(e_{5L}\) | \((1, 1, -1)\) |
| \(\Phi\)   | \((1, 1, 0)\) |
| \(\Theta\) | \((1, 1, 0)\) |

The original model is an extension of the minimal supersymmetric standard model (MSSM) by adding a pair of vector-like generations and an SM singlet field \(\Phi\) (Ref. [6]). We assume that the Kähler potential is canonical. As usual, the superfields of the MSSM sector are given by

\[
Q_i, u_i, d_i, L_i, e_i, \quad (i = 1, \ldots, 3),
\]

\[
H_u, H_d,
\]

where \(Q_i\) and \(L_i\) are the SU(2) doublets of quarks and leptons, \(u_i, d_i,\) and \(e_i\) are the SU(2) singlets of up-type, down-type quarks, and charged leptons, respectively. The Higgs doublets are denoted by \(H_u\) and \(H_d\). In addition to these, we have other superfields of the vector-like generations and a singlet such as

\[
Q_4, u_4, d_4, L_4, e_4,
\]

\[
\tilde{Q}, \tilde{u}, \tilde{d}, \tilde{L}, \tilde{e},
\]

\[
\Phi.
\]

The quantum charges of these superfields are summarized in Table 1. The superfields of fourth generation in Eq. (2.3) have the same charges as those of matters in the MSSM, while those of the fifth in Eq. (2.4) have the opposite ones. These pairs with opposite charges are called vector-like generations. The field \(\Phi\) is an SM gauge singlet field, and its VEV gives mass scales of the vector-like generations. With these superfields, the Yukawa sector of the superpotential is written as (Ref. [6])

\[
W = \sum_{i,j=1,\ldots,4} \left( (y_u)_{ij} u_i Q_j H_u + (y_d)_{ij} d_i Q_j H_d + (y_e)_{ij} e_i L_j H_d \right)
\]

\[
+ y_u \tilde{u} \tilde{Q} H_d + y_d \tilde{d} \tilde{Q} H_u + y_e \tilde{e} \tilde{L} H_u
\]

\[
+ \sum_{i=1,\ldots,4} \left( y_{Q_i} \Phi Q_i \tilde{Q} + y_{u_i} \Phi u_i \tilde{u} + y_{d_i} \Phi d_i \tilde{d} + y_{L_i} \Phi L_i \tilde{L} + y_{e_i} \Phi e_i \tilde{e} \right)
\]

\[
+ y\Phi^3,
\]
where \( y \)'s are dimensionless couplings for each generation of quark and lepton sectors.\(^1\) Each term of the interactions contributes to experimental results of the muon \( g - 2 \), the Higgs mass, and DM abundance as explained below. The Yukawa couplings in the first line give the flavor structure in the SM sector on top of the fourth generation matter, and they largely contribute to the flavor physics, i.e., the muon \( g - 2 \) (Ref. [6]). The second line shows the coupling of the fifth generations to the Higgs fields, and these interactions do not crucially contribute to the flavor structure in the SM sector but do to the muon \( g - 2 \) in our model. The third line shows the coupling of vector-like generations to the SM singlet field \( \Phi \). After the symmetry breaking of the scalar component of \( \Phi \), the vector-like generations obtain each mass from these terms. Further, owing to these terms, the lower experimental bounds on the mass of vector-like generations are avoided. In our model, the fermion component of \( \Phi \) is a DM candidate whose mass is given by the last term \( y \Phi^3 \). We explain such structures by the FN mechanism.

The FN mechanism requires an additional SM singlet scalar field and it is supposed that the singlet field has interactions with quark and lepton sectors (Ref. [11]). Then, the effective Yukawa couplings are determined through the interactions by the singlet VEV. The magnitudes of the couplings are controlled by an assignment of the FN charge to the quark and lepton sectors. We apply this mechanism to our model by introducing an SM singlet superfield \( \Theta \).

Let us consider an example of the interaction based on FN charge with \( W = ( \Theta / \Lambda )^n u_1 Q_1 H_u \), where \( n \) is an integer and \( \Lambda \) is a cutoff scale. Here we take the scale to \( \Lambda \simeq 10^{16} \) GeV. Under an assignment of the FN U(1) charge, the integer \( n \) is determined to satisfy \( n q(\Theta) + q(u_1) + q(Q_1) + q(H_u) = 0 \), where \( q(\Theta), q(Q_1), q(u_1), \) and \( q(H_u) \) are U(1) charges of respective fields. With this integer, the VEV such that \( \langle \Theta \rangle \neq 0 \) makes effective Yukawa couplings as \( (y_u)_{11} \propto ((\Theta) / \Lambda)^n \). In this way, the Yukawa structures of the quarks and leptons are determined.

Using the FN mechanism, we extend the Yukawa interaction part (2.6). We assign the FN U(1) charge to the FN field \( \Theta \) as

\[
q(\Theta) = -1. \tag{2.7}
\]

Together with the charge assignment exhibited in Table 1, the Yukawa sector of the superpotential is written as

\[
W = \sum_{i,j=1,\ldots,4} \left( Y_{ui} \right)_{ij} \left( \frac{\Theta}{\Lambda} \right)^n u_i Q_j H_u + \left( Y_{di} \right)_{ij} \left( \frac{\Theta}{\Lambda} \right)^n d_i Q_j H_d + \left( Y_{ei} \right)_{ij} \left( \frac{\Theta}{\Lambda} \right)^n e_i L_j H_d
\]

\[
+ Y_{\bar{u}_i} \left( \frac{\Theta}{\Lambda} \right)^n \bar{u}_i \bar{Q}_d H_d + Y_{\bar{d}_i} \left( \frac{\Theta}{\Lambda} \right)^n \bar{d}_i \bar{Q}_d H_d + Y_{\bar{e}_i} \left( \frac{\Theta}{\Lambda} \right)^n \bar{e}_i \bar{L}_d H_d
\]

\[
+ \sum_{i=1,\ldots,4} \left( Y_{Q_i} \right) \left( \frac{\Theta}{\Lambda} \right)^n Q_{i} \Phi Q_{i} \bar{Q} + Y_{u_i} \left( \frac{\Theta}{\Lambda} \right)^n u_i \bar{u} + Y_{d_i} \left( \frac{\Theta}{\Lambda} \right)^n d_i \bar{d} + Y_{e_i} \left( \frac{\Theta}{\Lambda} \right)^n e_i \bar{e} + Y \left( \frac{\Theta}{\Lambda} \right)^n \Phi^3, \tag{2.8}
\]

\(^1\) The superpotential (2.8) evokes for us that the potential has \( Z_3 \) symmetry with respect to \( \Phi \), but we can not assign a discrete charge. Thus, there is no domain wall problem.
where the magnitude of all the Yukawa couplings is assumed to be $\mathcal{O}(1)$. As explained, each power of $\Theta/\Lambda$ is determined from the charge assignment as

\[
\begin{align*}
  n_{ui}^{ij} &= q(u_i) + q(Q_j) + q(H_u), & n_{dij}^{ij} &= q(d_i) + q(Q_j) + q(H_d), & n_{eij}^{ij} &= q(e_i) + q(L_j) + q(H_d), \\
  n_{ui} &= q(\bar{u}) + q(\bar{Q}) + q(H_d), & n_d &= q(\bar{d}) + q(\bar{Q}) + q(H_d), & n_e &= q(\bar{e}) + q(\bar{L}) + q(H_u), \\
  n_{Qe} &= q(Q_i) + q(\bar{Q}) + q(\Phi), & n_{u_i} &= q(u_i) + q(\bar{u}) + q(\Phi), & n_{di} &= q(d_i) + q(\bar{d}) + q(\Phi), \\
  n_{Le} &= q(L_i) + q(\bar{L}) + q(\Phi), & n_{ei} &= q(e_i) + q(\bar{e}) + q(\Phi), & n_\Phi &= 3q(\Phi).
\end{align*}
\]

(2.9)

Then, by the superpotential (2.8), the effective Yukawa couplings are given by

\[
y_x \equiv Y_x \left( \frac{\langle \Theta \rangle}{\Lambda} \right)^{n_x},
\]

(2.10)

where $x$ represents each generation of quark or lepton sectors. Note that with the present charge assignment the cubic coefficient is the same as $y = Y$ because $q(\Phi) = 0$. In this paper, we assume that $U(1)_{FN}$ is a gauged symmetry. With the D-term, the FN field obtains VEV, but in this case, anomalies due to the symmetry could exist. Let us comment here about these issues. The case of global symmetry is discussed at the end of this section.

Now, the VEV of the FN field is given by the FI D-term of the anomalous $U(1)_{FN}$. Under the charge assignment, $U(1)_{FN}$ becomes anomalous in our model because $TR(q) > 0$, if there are no additional chiral multiplets with negative $U(1)_{FN}$ charges. In such cases, theory is ill defined. Based on string theory, however, such anomalies can be canceled by the gauged shift of string-theoretic axions (or p-from potentials) (Ref. [15]), and the Fayet–Iliopoulos term is naturally induced in the $U(1)_{FN}$ D-term at the one-loop level with $\xi \sim (TR(q)M^2) / (16\pi^2)$ (Refs. [16–19]). (See also Ref. [20] for a review.) Here $M_*$ is the string scale. In this paper, we assume that only $\Theta$ develops VEV (Refs. [21,22]) in the presence of $U(1)_{FN}$ D-term potential by $D \sim \xi - |\Theta|^2 \sim 0$ and that $U(1)_{FN}$ anomalies are canceled by shifts of (multiple) axions that are coupled to the SM gauge fields from the viewpoint of the generalized Green–Schwarz mechanism. Then, the chiral superfield $\Theta$ will be eaten by the anomalous $U(1)_{FN}$ vector superfield in a supersymmetric manner. They become massive around the cutoff scale, and hence we will neglect them and the $U(1)_{FN}$ D-term contribution to the SUSY breaking, and focus only on the VEV in the following.

As explained above, the effective Yukawa couplings are determined by the charge assignment of FN $U(1)$ and $\langle \Theta \rangle$. In particular, the charge assignment determines the flavor structure of our model. We explain the strategy to determine the assignment. First, we require the superpotential homomorphic for $\Theta$ at a perturbative level. To satisfy this requirement, the power of $\Theta/\Lambda$ needs to be positive. Under this constraint, we determine the FN charge, paying attention to two points: mixing between second and fourth generations and masses of fourth and fifth generations. In our vector-like generations model, low-energy flavor structure in the SM sector is finally determined by RG equations. By solving RG equations, the authors of Ref. [6] found that for the appropriate flavor structure of the SM, the mixing between the second and fourth generations needs to be large. Thus we have to determine the FN charge such that this large coupling is reproduced. Another point is

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\[2\] In other words, a different gauge sector in the SM may come from a stack of D-branes that are wrapping on a different cycle on the internal extra dimension or contain different world volume fluxes on such a cycle. Then, a GUT-like relation between gauge couplings, which is led by geometric properties, can be found (Ref. [23]) through moduli stabilization (Refs. [24, 25]).
Table 2. The list for FN charge of each field.

| Superfield | \( Q_1 \) | \( Q_2 \) | \( Q_3 \) | \( Q_4 \) | \( \bar{Q} \) | \( u_1 \) | \( u_2 \) | \( u_3 \) | \( u_4 \) | \( \bar{u} \) | \( d_1 \) | \( d_2 \) | \( d_3 \) | \( d_4 \) | \( d \) |
|------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| U(1)FN     | 5    | 2    | 0    | -2   | 2    | 4    | 2    | 0    | -2   | 2    | 4    | 4    | 1    | -2   | -2   |

| Superfield | \( L_1 \) | \( L_2 \) | \( L_3 \) | \( L_4 \) | \( L \) | \( e_1 \) | \( e_2 \) | \( e_3 \) | \( e_4 \) | \( \tilde{e} \) | \( H_u \) | \( H_d \) | \( \Phi \) | \( \Theta \) |
|------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| U(1)FN     | 4    | 1    | 0    | -1   | 0    | 5    | 1    | 1    | 0    | 2    | 0    | 0    | 0    | -1   |

about the couplings between fourth and fifth generations. From the experimental constraint at the Large Hadron Collider (LHC), the masses of vector-like generations are required to be relatively large, as \( m_{q_4} \gtrsim 800 \text{ GeV} \) and \( m_{l_4} \gtrsim 100 \text{ GeV} \) (Ref. [14]). Thus the FN charge needs to be chosen to achieve these large masses.

Paying attention to these points, we have determined the assignment of the FN charge shown in Table 2. We explicitly show the mass matrix of each sector under the assignment. Let us here define a parameter for the convenience of explanation as

\[
\epsilon = \frac{\langle \Theta \rangle}{\Lambda}. \tag{2.11}
\]

With this parameter, the matrix elements of up-type quark mass \( M_u \), down-type quark mass \( M_d \), and charged lepton mass \( M_e \) are given as

\[
M_u \approx \begin{pmatrix}
  u_{1R} & u_{2R} & u_{3R} & u_{4R} & u_{5R} \\
  u_{1L} & e^9 v_u & e^7 v_u & e^5 v_u & e^3 v_u & e^7 V \\
  u_{2L} & e^9 v_u & e^7 v_u & e^5 v_u & v_u & e^4 V \\
  u_{3L} & e^4 v_u & e^2 v_u & v_u & 0 & e^2 V \\
  u_{4L} & e^2 v_u & v_u & 0 & 0 & V \\
  u_{5L} & e^6 V & e^4 V & e^2 V & V & e^4 v_d
\end{pmatrix}, \tag{2.12}
\]

\[
M_d \approx \begin{pmatrix}
  d_{1R} & d_{2R} & d_{3R} & d_{4R} & d_{5R} \\
  d_{1L} & e^9 v_d & e^9 v_d & e^6 v_d & e^3 v_d & e^7 V \\
  d_{2L} & e^6 v_d & e^6 v_d & e^3 v_d & v_d & e^4 V \\
  d_{3L} & e^4 v_d & e^4 v_d & e^1 v_d & 0 & e^2 V \\
  d_{4L} & e^2 v_d & e^2 v_d & 0 & 0 & V \\
  d_{5L} & e^6 V & e^6 V & e^3 V & V & e^4 v_u
\end{pmatrix}, \tag{2.13}
\]

\[
M_e \approx \begin{pmatrix}
  e_{1R} & e_{2R} & e_{3R} & e_{4R} & e_{5R} \\
  e_{1L} & e^9 v_d & e^9 v_d & e^5 v_d & e^4 v_d & e^6 V \\
  e_{2L} & e^6 v_d & e^2 v_d & e^2 v_d & e^1 v_d & e^3 V \\
  e_{3L} & e^5 v_d & e^1 v_d & e^1 v_d & v_d & e^2 V \\
  e_{4L} & e^4 v_d & v_d & v_d & 0 & e^1 V \\
  e_{5L} & e^5 V & e^1 V & e^1 V & V & e^2 v_u
\end{pmatrix}. \tag{2.14}
\]
Table 3. The set of input values.

| $\epsilon$ | $\alpha_{\text{GUT}}$ | $M_{\text{GUT}} = \Lambda$ | $M_{\text{SUSY}}$ | $V$ | $\tan \beta$ |
|------------|------------------------|-----------------------------|--------------------|-----|----------------|
| 0.33       | 0.10                   | $6.0 \times 10^{16}$ GeV    | 5.0 TeV            | 2.0 TeV | 40             |

where we have neglected $O(1)$ bare Yukawa couplings in the superpotential. Here we have defined the VEVs of $H_u$, $H_d$, and $\Phi$ as

$$\langle H_u \rangle \equiv v_u, \quad \langle H_d \rangle \equiv v_d, \quad \langle \Phi \rangle \equiv V,$$

and defined the fermion components of $\tilde{Q}$, $\tilde{u}$, and $\tilde{d}$ as

$$\tilde{Q}|_{\text{fermion}} \equiv \left( \begin{array}{c} (u_{SR})^c \\ (d_{LR})^c \end{array} \right), \quad \tilde{u}|_{\text{fermion}} \equiv (u_{SL}), \quad \tilde{d}|_{\text{fermion}} \equiv (d_{SL}).$$

As for 3 generations of the SM and fourth generations, the up-type Higgs $H_u$ couples to up-type quarks, and the down-type Higgs $H_d$ couples to down-type quarks and leptons in the Yukawa sector. (See the first line in Eq. (2.8).) These terms correspond to the $4 \times 4$ parts of the mass matrices (2.12), (2.13), and (2.14). As for the fifth generations, the Higgs couples in the opposite way, i.e., $H_u$ couples to down-type quarks of fifth generation, and $H_d$ couples to up-type quarks as shown in the fourth line in Eq. (2.8), which corresponds to the $(5, 5)$ entry in the mass matrices. The other elements of the mass matrices are given by the gauge singlet field $\Phi$. (See the third and fourth lines in Eq. (2.8).)

Now we discuss the observables of the CKM matrix and fermion masses in both quark and lepton sectors. In our model, the CKM matrix is extracted from the $5 \times 5$ unitary matrix. Thus we need to check carefully that the unitarity of the CKM matrix is satisfied so that the CKM matrix elements other than the ordinary $3 \times 3$ matrix are suppressed by the vector-like mass. The CKM matrix is defined by the part of the upper-left $3 \times 3$ matrix of the product of $5 \times 5$ unitary matrices:

$$(V_{\text{CKM}})_{ij} = (V_{ul}^\dagger V_{dl})_{ij} \quad (i,j = 1, 2, 3),$$

where $i,j$ are generation labels.

These $5 \times 5$ unitary matrices diagonalize the up-type and down-type quark mass matrices:

$$V_{uR} M_u V_{uL}^\dagger, \quad V_{dR} M_d V_{dL}^\dagger.$$ 

With the structures of the mass matrices (2.12), (2.13), and (2.14), we solve the RG equations. The input parameters for the equation are summarized in Tables 3 and 4. By the calculation, we have confirmed that the CKM matrix and the fermion masses at the $M_Z$ scale reproduce the observed ones. The RG equations in our model are listed in Appendix D. In Table 3, $\alpha_{\text{GUT}}$ and $\Lambda$ are the initial condition of the RG equations for the MSSM gauge couplings obtained in Ref. [6]. It is found that the MSSM gauge couplings unify at a certain scale $M_{\text{GUT}}$ that is slightly high compared to the MSSM without vector-like generations. Thus, we use these values as boundary conditions for RG running. The scale $M_{\text{SUSY}}$ is a typical threshold for supersymmetric particles, and the ratio of the VEVs of Higgs doublets is defined as $\tan \beta \equiv v_u/v_d$. In Sect. 4, we use the same input values for the numerical analysis of the DM abundance, the Higgs boson mass, and the muon $g-2$. Here, as an example, we show the result of RG running for the Yukawa couplings of the third generation in Fig. 1. The
Table 4. The set of input values for coupling constants in Eq. (2.8). Other coupling constants that are not written in this table are set to unity. Note that all couplings are $O(1)$.

| Up-type quark Yukawa | Down-type quark Yukawa | Charged lepton Yukawa |
|----------------------|------------------------|-----------------------|
| $(Y_u)_{11} = 2.000$ | $(Y_d)_{11} = 0.500$  | $(Y_e)_{14} = 2.000$  |
| $(Y_u)_{23} = 2.000$ | $(Y_d)_{21} = 1.930$  | $(Y_e)_{12} = 2.000$  |
| $(Y_u)_{33} = 2.000$ | $(Y_d)_{22} = 1.200$  | $(Y_e)_{24} = 2.000$  |
| $(Y_u)_{41} = 0.500$ | $(Y_d)_{23} = 0.900$  | $(Y_e)_{22} = 0.500$  |
| $Y_u = 2.000$        | $(Y_d)_{31} = 0.632$  | $(Y_e)_{34} = 2.000$  |
| $(Y_d)_{32} = 0.700$ | $Y_e = 2.000$        | $(Y_e)_{33} = 0.500$  |
| $(Y_d)_{33} = 2.000$ | $Y_d = 2.000$        | $Y_e = 2.000$        |
| $(Y_d)_{41} = 2.000$ | $Y_{d_2} = 1.100$    | $Y_{e_2} = 0.500$    |
| $Y_d = 1.930$        | $Y_{d_3} = 0.500$    | $Y_{e_3} = 0.500$    |
| $Y_d = 1.930$        | $Y_d = 0.900$        | $Y_e = 2.000$        |
| $Y_d = 2.000$        | $Y_e = 2.000$        | $Y_e = 2.000$        |
| $Y_d = 2.000$        | $Y_d = 2.000$        | $Y_e = 2.000$        |

Fig. 1. The result of RG running for the Yukawa couplings of the third generation. The horizontal axis is the energy scale and the vertical axis is the strength of the Yukawa couplings. The blue, black, and red lines correspond to the RG running of $(y_u)_{33}$, $(y_d)_{33}$ and $(y_e)_{33}$, respectively.

In Fig. 1, each Yukawa coupling converges to a certain value at low energy, where these couplings do not depend on the initial values at $M_{GUT}$. The detailed analysis for the convergence of Yukawa couplings is performed in Refs. [26–29]. As for $(y_d)_{33}$, it seems that this Yukawa coupling does not evolve, but the initial value of $(y_d)_{33}$ just coincides with the infrared value. Such RG runnings reproduce the fermion masses and the CKM matrix at the scale of $M_Z$:

$$
m_t \sim 170 \text{ GeV}, \quad m_c \sim 0.7 \text{ GeV}, \quad m_\mu \sim 5.0 \text{ MeV},
$$
$$
m_b \sim 3.0 \text{ GeV}, \quad m_s \sim 0.032 \text{ GeV}, \quad m_d \sim 1.0 \text{ MeV},
$$
$$
m_t \sim 1.6 \text{ GeV}, \quad m_\mu \sim 0.10 \text{ GeV}, \quad m_e \sim 0.2 \text{ MeV}, \quad (2.20)
$$
\[
|V_{\text{CKM}}| \sim \begin{pmatrix}
0.974 & 0.226 & 0.0035 \\
0.225 & 0.973 & 0.040 \\
0.0089 & 0.041 & 0.999
\end{pmatrix}.
\] (2.21)

Fermion masses at the \(M_Z\) scale are studied in Ref. [30], and the CKM matrix takes its value within the \(2\sigma\) level of the observed CKM matrix (Ref. [14]) as
\[
\begin{pmatrix}
0.97403 - 0.97449 & 0.22406 - 0.22606 & 0.00327 - 0.00387 \\
0.22392 - 0.22592 & 0.97325 - 0.97377 & 0.04084 - 0.04136 \\
0.00815 - 0.00939 & 0.0377 - 0.0429 & 0.9991 - 0.9992
\end{pmatrix}.
\] (2.22)

These are consistent with Eqs. (2.20) and (2.21).

Here let us discuss the case that the FN symmetry is global.\(^3\) In this case, there exists a (pseudo) NG boson associated with the spontaneous symmetry breaking of \(U(1)_{\text{FN}}\). We call it an FN axion. As in the case of the QCD-axion, this FN axion could be another DM candidate. Further, it has interactions with the quarks and leptons, and through the anomaly effects it couples to gluons and electromagnetic fields with effective decay rates. The interactions give experimental and cosmological constraints on the VEV of the FN field (Ref. [31]). On the basis that the mass matrices of quarks and leptons are diagonalized, the Yukawa interactions are written
\[
-\mathcal{L} = \sum_{f=u,d,l} \left[ m^f_{ij} \bar{f}^i \gamma_5 f^j + \kappa^f_{ij} \bar{s} + ia \bar{f}^i \gamma_5 f^j \right],
\] (2.23)

where the coupling \(\kappa^f_{ij}\) is determined by the mass matrix of fermions and the FN charge as
\[
\kappa^f_{ij} \equiv \left( V^f_{iL} \right) \left( m^f_{kn} \right) \frac{1}{\sqrt{2}} \left( V^f_{kn} \right),
\] (2.24)

Here we have expanded the FN field around the VEV as
\[
\Theta = \nu_\Theta + \frac{s + ia}{\sqrt{2}}.
\] (2.25)

Thus, the interactions of the axion with quarks and leptons are given by
\[
-\mathcal{L}_{\text{int}} = \frac{ia}{\sqrt{2}v_\Theta} \sum_{f=u',d',l'} \left[ \left( \kappa^f_H \right)_{ij} \bar{s} + i\gamma_5 fj + \left( \kappa^A_H \right)_{ij} \bar{s} + i\gamma_5 fj \right],
\] (2.26)

where we have redefined the coupling of axions as
\[
\begin{align*}
\left( \kappa^f_H \right)_{ij} &\equiv \frac{1}{2} \left( \kappa^f + \kappa^{f*} \right)_{ij}, \\
\left( \kappa^A_H \right)_{ij} &\equiv \frac{1}{2} \left( \kappa^f - \kappa^{f*} \right)_{ij}.
\end{align*}
\] (2.27)

\(^3\) In the case of global \(U(1)_{\text{FN}}\), we can realize the symmetry breaking by the F-term of the superpotential such as \(W = S(\Theta \bar{\Theta} - (\Theta)^2)\), where \(S\) and \(\bar{\Theta}\) are singlet fields but \(q(\bar{\Theta}) = 1\) (Ref. [31]).

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Further, through the critical rotation as in the case of the QCD-axion, the axial interaction of the FN axion with gluons is given by

\[-L_{\text{gluon-axion}} = \frac{g_s^2}{32\pi^2} \sum_{i=1}^{5f = u,d} \left( \frac{\kappa_f^i}{m_i} \right) a \sqrt{2v_\Theta} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} = \frac{g_s^2}{32\pi^2} \sum_{i=1}^{5f = u,d} \left( \frac{\kappa_f^i}{m_i} \right) a \sqrt{2v_\Theta} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}, \tag{2.28}\]

where we have defined the domain wall number $N_{\text{DW}}$ and the effective decay constant $f_a$ as

\[N_{\text{DW}} \equiv \sum_{i=1}^{5f = u,d} \left( \frac{\kappa_f^i}{m_i} \right) = \text{TR} \left( n_{uu} + n_{dd} + n_{\bar{u}u} + n_{\bar{d}d} + n_{Qi} + n_{ui} + n_{di} \right) \tag{2.29}\]

and

\[f_a = \frac{\sqrt{2v_\Theta}}{N_{\text{DW}}}. \tag{2.30}\]

Now, the domain wall number for our FN charge assignment is calculated as $N_{\text{DW}} = 54$. In our model, the effective decay constant is typically $f_a \simeq 10^{14}$ GeV. Among them, the interactions of the FN axion to the up-type and down-type quarks give a sizable contribution to the decay of the charged kaon via $K^+ \rightarrow \pi^+ a$. The decay rate for this process is evaluated by

\[\Gamma (K^+ \rightarrow \pi^+ a) = \frac{m_K^3}{32\pi v_\Theta^2} \left( 1 - \frac{m_K^2}{m_K^2} \right)^3 \left( \frac{\kappa_A^d}{m_s - m_d} \right). \tag{2.31}\]

The last term is of order unity as $\left( \kappa_A^d \right)_{12} = (3/2) (m_s - m_d)$. Thus, the branching ratio for this process is given by

\[\text{Br} (K^+ \rightarrow \pi^+ a) \simeq 10^{-11} \left( \frac{10^{11} \text{GeV}}{v_\Theta} \right)^2. \tag{2.32}\]

With experimental bound $\text{Br}(K^+ \rightarrow \pi^+ a) \lesssim 7.3 \times 10^{-11}$ (Ref. [32]), we obtain a constraint

\[v_\Theta \gtrsim 10^{11} \text{GeV}. \tag{2.33}\]

Through the interaction of the axion with quarks, the FN axion takes away the energy of the supernovae explosion. From the observation of the SM1987A by Kamiokande, this interaction is constrained in terms of the effective decay rate as $f_a \gtrsim 10^9$ GeV. With our definition for the effective decay rate, this constraint is reduced to

\[v_\Theta \gtrsim \frac{N_{\text{DW}}}{\sqrt{2}} 10^9 \text{GeV} \sim 10^{10} \text{GeV}. \tag{2.34}\]

Therefore, in the case that the FN U(1) symmetry is global, the VEV of the FN field needs to be $v_\Theta \gtrsim 10^{11}$ GeV. As in the case of the QCD-axion, the energy density of the coherent oscillation of this FN axion could explain the DM abundance given by (Ref. [33])

\[\Omega_{a_0} h^2 = 0.18 \delta_i^2 \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{1.19}, \tag{2.35}\]
where $\theta_i$ is the initial misalignment of the FN axion $\theta_i \equiv a_{\Theta i}/f_a$. When the FN symmetry breaks, the formation of domain walls could occur. We can assume that the breaking takes place before inflation. In this case, the domain wall problem is avoided, while in this case the iso-curvature problem occurs. If the energy scale of inflation, i.e., the Hubble parameter is small, this iso-curvature problem is avoided. The detailed analysis and constraints on the parameters are studied in Ref. [31].

3. **Dark matter**

In this section, we calculate the abundance of thermal relics of a DM candidate, which is the fermion component of the singlet $\Phi$. The singlet superfield $\Phi$ is expanded as

$$\Phi = V + \theta \chi_{\Phi} + \cdots,$$

where $V$ is the VEV of the scalar component, $\chi_{\Phi}$ is the fermion component, and $\theta$ is the fermionic coordinate on the superspace. In our model, the VEV of the singlet field $V$ gives masses to vector-like generations, and then experimental constraints on them at LHC are avoided.

First, we discuss the scale of the DM mass $m_{\chi_{\Phi}}$. In our model, the mass scale is determined by the VEV of $V$ and the coupling constant of the cubic term $y$ as $m_{\chi_{\Phi}} = yV$. The RG running of $y$, whose RG equation is given in Eq. (D.32), is only governed by the Yukawa couplings related to $\Phi$ and it is insensitive to gauge sector. Thus, $y$ is pushed down to $O(10^{-2})$ at low energy. With $V$ of a few TeV, $m_{\chi_{\Phi}}$ takes around 100 GeV. This mass scale is smaller than other neutralino masses: bino and Higgsino masses. As for bino, its mass was studied in the previous study Ref. [6], and then it was revealed that the bino-like mass should be around 200 GeV to accord with the Higgs mass confirmed by the LHC. As for Higgsino-like neutralinos, those masses are determined from the electroweak symmetry breaking. In our model, we have assigned no FN charge to Higgs fields. Thus, in the superpotential, $H_u$ and $H_d$ have the so-called $\mu$ term as

$$W = \mu H_u H_d,$$

where $\mu_H$ is a constant of dimension unity. For the electroweak symmetry breaking, the mass parameter is required to be $\mu_H \simeq 2$ TeV. Thus, the Higgsino-like neutralinos are heavier than the singlino. Therefore, $\chi_{\Phi}$ is the LSP in our model and could be a DM candidate because $R$-parity symmetry is imposed on our model. Let us comment on an operator $\Phi H_u H_d$, which can be written as gauge invariant in the superpotential. In this paper, we focus on the $\Phi^3$ term in the superpotential because we would like to discuss the minimal model of $\chi_{\Phi}$, which does not couple to the Higgs sector. Such a situation might be realized by imposing some symmetry.

In order to evaluate the DM abundance, we use the mass eigenstate basis for squarks, sleptons, quarks, and charged leptons. For quark and lepton sectors, we diagonalize their mass matrices as

$$V_{M_{u}}M_{u}^\dagger_{M_{u}}(i,j = 1, \ldots, 5) \text{ and } (m_{U_i} < m_{U_j} \text{ if } i < j),$$

$$V_{M_{d}}M_{d}^\dagger_{M_{d}}(i,j = 1, \ldots, 5) \text{ and } (m_{D_i} < m_{D_j} \text{ if } i < j),$$

$$V_{M_{e}}M_{e}^\dagger_{M_{e}}(i,j = 1, \ldots, 5) \text{ and } (m_{E_i} < m_{E_j} \text{ if } i < j).$$

The definition of Eqs. (2.12), (2.13), and (2.14) are written without the $O(1)$ couplings in Eq. 2.8. However, as for the diagonalization, we use the mass matrices including $O(1)$ couplings.
We denote the mass eigenvalues $m_{U_i}$, $m_{D_j}$, and $m_{E_i}$ for the mass eigenstates of up-type quarks ($U_i$), down-type quark ($D_j$), and charged leptons ($E_i$), respectively. As for squarks and charged slepton sectors, we also diagonalize their mass matrices as

\begin{align}
(U_i^2 M_u^2 U_i^\dagger)_{\alpha\beta} &= m_{U_{\alpha}}^2 \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \ldots, 10) \text{ and } (m_{U_{\alpha}}^2 < m_{U_{\beta}}^2 \text{ if } \alpha < \beta), \\
(U_d^2 M_d^2 U_d^\dagger)_{\alpha\beta} &= m_{D_{\alpha}}^2 \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \ldots, 10) \text{ and } (m_{D_{\alpha}}^2 < m_{D_{\beta}}^2 \text{ if } \alpha < \beta), \\
(U_e^2 M_e^2 U_e^\dagger)_{\alpha\beta} &= m_{E_{\alpha}}^2 \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \ldots, 10) \text{ and } (m_{E_{\alpha}}^2 < m_{E_{\beta}}^2 \text{ if } \alpha < \beta),
\end{align}

where $M_u^2$, $M_d^2$, and $M_e^2$ are up-type squark, down-type squark, and charged slepton mass matrices defined in Ref. [6]. We denote the mass eigenvalues $m_{U_{\alpha}}^2$, $m_{D_{\alpha}}^2$, and $m_{E_{\alpha}}^2$ for the mass eigenstates of up-type squark ($\tilde{U}_\alpha$), down-type squark ($\tilde{D}_\alpha$) and charged slepton ($\tilde{E}_\alpha$), respectively.

The interaction terms of $\chi_\phi$ can be read from the third to the fourth line of Eq. (2.8). With the diagonalized basis in Eqs. (3.3)–(3.8), the interaction terms of $\chi_\phi$ are given by

\begin{align}
\mathcal{L} &= \tilde{\chi}_\phi (O_{\alpha R j} P_R + O_{\alpha L j} P_L) U_j \tilde{U}_\alpha^* + \tilde{\chi}_\phi (O_{\alpha R j} P_R + O_{\alpha L j} P_L) D_j \tilde{D}_\alpha^* \\
&\quad + \tilde{\chi}_\phi (O_{\alpha L j} P_R + O_{\alpha L j} P_L) E_j \tilde{E}_\alpha^* + \text{h.c.},
\end{align}

where $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, and the coefficients are

\begin{align}
O_{\alpha R j} &= (y_{\alpha} i)(V_{C R})_{ji}(U_{\alpha})_{a 5}, & O_{\alpha L j} &= (y_{\alpha} i)(V_{C L})_{ji}(U_{\alpha})_{a 1 0}, \\
O_{\alpha d R j} &= (y_{u} i)(V_{d R})_{ji}(U_{\alpha})_{a 5}, & O_{\alpha d L j} &= (y_{u} i)(V_{d L})_{ji}(U_{\alpha})_{a 1 0}, \\
O_{\alpha d R j} &= (y_{d} i)(V_{d R})_{ji}(U_{\alpha})_{a 5}, & O_{\alpha d L j} &= (y_{d} i)(V_{d L})_{ji}(U_{\alpha})_{a 1 0}.
\end{align}

The thermal abundance of the singlino DM is determined by their pair annihilation into the SM particles as shown in Fig. 2. This process ceases when the cosmic expansion rate drops below the annihilation rate:

\begin{align}
\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle n_{\chi_\phi} &\simeq H(T_F),
\end{align}

where $\sigma_{\text{ann}}$ is the annihilation cross section, $v_{\text{rel}}$ is their relative velocity, $n_{\chi_\phi}$ is the number density of $\chi_\phi$, and $\langle \ldots \rangle$ represents thermal-averaged cross section. We defined $T_F$ as the freeze-out temperature. After the freeze out, the number density of singlinos drops at the same rate as the entropy density by
the cosmic expansion. Thus, from Eq. (3.13), we can estimate the ratio of the number density to the entropy density at $T_F$ as\footnote{In order to calculate the abundance of DM accurately, we have to solve Boltzmann equations. However, it is sufficient for the estimation of the order of the DM abundance.}

$$\frac{n_{\chi}\phi}{s} \bigg|_{T_F} \simeq \frac{H(T_F)}{(\sigma_{\text{ann}}v_{\text{rel}}) s} \bigg|_{T_F} = \frac{1}{4} \left( \frac{90}{\pi^2 g_*(T_F)} \right)^{1/2} \frac{1}{(\sigma_{\text{ann}}v_{\text{rel}}) T_F M_{\text{pl}}},$$  

(3.14)

where $s$ is the entropy density, $g_*(T_F)$ is the effective degrees of freedom of the radiation at the freeze-out, and $M_{\text{pl}} = 2.43 \times 10^{18}$ GeV is the Planck mass.

The diagrams to calculate the annihilation cross section are shown in Fig. 2, where $f_i$ is the SM fermion and $\tilde{f}_i$ is the sfermion exchanged in the process. Since the singlino is the SM gauge singlet and does not couple to the Higgs boson, the bosons do not appear in the process. The SM fermions entering in the process are the ones whose masses are below the freeze-out temperature $T_F$. In the calculation, one can expand $\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle = a + b/x_F + O((1/x_F)^2)$ in the inverse of the power of $x_F \equiv m_{\chi}/T_F$ and approximates $\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle$ by the coefficients $a$ and $b$. We take $x_F = 20$ for the numerical analysis; $T_F$ becomes around 5 GeV for $m_{\chi} = 100$ GeV, and hence the top quark contribution is neglected in the external lines. We calculate the coefficients $a$ and $b$ for the case of Fig. 2 referring to Refs. [34,35]. The explicit forms of $a$ and $b$ are given in Appendix A. With the values of $x_F$, $a$, and $b$, the analytic form of the DM relic abundance is given by

$$\Omega_{\chi\phi} h^2 \equiv \frac{\rho_{\chi\phi}}{\rho_c/h^2} = \frac{m_{\chi}\phi n_{\chi\phi}/s}{\rho_c/(h^2 s)} = \frac{1.07 \times 10^9/\text{GeV} x_F}{\sqrt{g_* M_{\text{pl}} (a + b/x_F)}} \approx 0.1 \times \left( \frac{8.0 \times 10^{-9} \text{GeV}^{-2}}{\langle \sigma_{\text{ann}}v_{\text{rel}} \rangle} \right),$$  

(3.15)

where $h$ is the rescaled Hubble constant, $\rho_c$ is the critical density of the Universe, the ratio of critical density to the entropy density today is $\rho_c/s \simeq 1.8 \times 10^{-9}$ GeV, and we take $g_*(T_F) = 100$. In our model, the $p$-wave contribution is dominant to the thermal-averaged cross section.

4. Higgs boson mass, muon $g−2$, and DM abundance

In this section, we show the results of numerical calculations for the Higgs boson mass, the muon $g−2$, and the DM abundance. We have evaluated the Higgs boson mass and the muon $g−2$ at the one-loop level. In this analysis, we determine the $\mu_H$ and $b$ terms in order that electroweak symmetry breaking is triggered at the Fermi scale. For the Higgs boson mass, we use the effective potential method (Ref. [36]) as usual for the MSSM case (Ref. [37]). The quantum corrections to the Higgs mass from vector-like generations are calculated in the literature (Refs. [38–42]). In our model, the lightest Higgs boson mass $m_h^2$ is evaluated by

$$m_h^2 = m_{h\text{tree}}^2 + \Delta m_h^2,$$  

(4.1)

where $m_{h\text{tree}}^2 = M_2^2 \cos^2(2\beta)$ and $\Delta m_h^2$ are one-loop corrections that are defined in Eq. (B.1). In the case of MSSM, the mass correction is mainly given by the stop field as $\Delta m_{h,MSSM}^2 \simeq (3/4\pi^2)\gamma_t^2 m_{\tilde{t}_1}^2 \log(m_{\tilde{t}_1}^2/m_t^2)$, but in our model the squark fields of the vector-like generation also give the correction, and it is dominant. For the muon $g−2$, in our model, the one-loop contributions to the muon $g−2$ are given by

$$\Delta a_\mu = \Delta a_\mu^{\text{Susy}} + \Delta a_\mu^{\text{non-Susy}},$$  

(4.2)
where $\Delta a_\mu^{\text{SUSY}}$ and $\Delta a_\mu^{\text{non-SUSY}}$ are SUSY contributions including both the MSSM (Refs. [43–48]) and vector-like (Ref. [6]) sectors, and non-SUSY contributions including only vector-like sectors (Ref. [49]), respectively. The terms $\Delta a_\mu^{\text{SUSY}}$ and $\Delta a_\mu^{\text{non-SUSY}}$ are defined in Eqs. (B.10) and (B.28), respectively. The non-SUSY term is given approximately by the ratio of the muon mass to the charged lepton mass of the vector-like generations as $\Delta a_\mu^{\text{non-SUSY}} \approx (\alpha/4\pi) \frac{m_\mu^2}{m_L'^2}$, where $\alpha$ is the fine structure constant of SU(2) as $\alpha = g^2/(4\pi)$, and $m_L'$ is the charged lepton mass of the vector-like generations. In addition to this term, in our model, the smuon, charged sleptons of the vector-like generations and their mixing in the mass matrix gives corrections to the muon $g-2$. Among of them, the charged leptons and sleptons of the vector-like generations give sizable contributions. For the numerical calculation of the Higgs mass and the muon $g-2$, we have used Eqs. (4.1) and (4.2).

As for the experiments, the current situation of the Higgs mass and muon $g-2$ are shown in the following. The recent combined result of the Higgs boson mass $m_h^{\text{Exp}}$ reported by the ATLAS and CMS Collaborations (Ref. [3]) is given by

$$m_h^{\text{Exp}} = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.}) \text{GeV.} \quad (4.3)$$

The discrepancy in the muon $g-2$ between the SM predictions and experimental value is above $3\sigma$ and quantified as (Refs. [4,5])

$$\Delta a_\mu \equiv a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (26.1 \pm 8.0) \times 10^{-10}. \quad (4.4)$$

In the analysis, we assume the minimal gravity mediation (Refs. [50–52]) as the boundary condition of the SUSY-breaking scenario. In this scenario, there are 5 parameters: $m_{1/2}$, $m_0$, $A_0$, tan $\beta$, and sigh of $\mu_H$. In this mediation model, we assume that the mass scale of gaugino, soft scalar, and trilinear scalar coupling are universal respectively $m_{1/2}$, $m_0$, $A_0$ at the unification scale; tan $\beta$ is fixed to reproduce the fermion masses at low energy and the sigh of $\mu_H$ is fixed plus so that the contributions to the muon $g-2$ become positive. Thus, between the 5 parameters, there remain 3 free SUSY-breaking parameters: $m_{1/2}$, $m_0$, and $A_0$. In the following analysis, for the sake of simplicity, $A_0$ is fixed to be 0 GeV. In addition to the parameters of the minimal gravity mediation, our model has another parameter $y (= Y)$, which is exhibited in the superpotential (2.8) and is related to the DM mass. Yukawa couplings except for $y$ are determined so that the observed CKM matrix and fermion mass are reproduced. Since $y$ is insensitive to these observables, $y$ can be treated as a free parameter. Now there are 3 parameters:

$$m_{1/2}, \ m_0, \ y. \quad (4.5)$$

In the analysis, we have calculated the Higgs boson mass at the one-loop level. Inclusion of higher effects gives slight corrections for the mass. At the two-loop level, the Higgs mass correction in MSSM is studied such as in Refs. [53–62]. The dominant contributions of the two-loop effects are from stop fields in loops by the superpotential

$$W = (y_u)_{33} u_3 Q_3 H_u. \quad (4.6)$$

---

6 Let us comment on $A_0 \neq 0$. In general, the Higgs boson mass, the muon $g-2$, and the DM abundance depend on the $A_0$ parameter. However, since these values are determined by mass spectrum at low energy, the dependence of these values does not drastically change. Thus, it is sufficient that the analysis $A_0$ is zero.
The amount of $\mathcal{O}(\alpha_s \alpha_t)$ and $\mathcal{O}(\alpha_t \alpha_t)$-contributions is typically estimated as

$$\Delta m_h^2 \simeq -3 \frac{G_F \sqrt{2}}{\pi^2} \frac{\alpha_t}{\pi} m_t^4 \ln^2 \left( \frac{m_t^2}{M_S^2} \right) + 3 \frac{G_F \sqrt{2}}{16\pi^2} \frac{\alpha_t}{\pi} m_t^4 \ln^2 \left( \frac{m_t^2}{M_S^2} \right),$$

where $G_F$ is the Fermi coupling constant, $\alpha_s = g_s^2/(4\pi)$, $\alpha_t = (y_u)^2/(4\pi)$, $m_t$ is the top mass, and $M_S$ is the stop mass. In the case of the vector-like generations, scalar components of the fourth, fifth generations, and singlet field mediating in the loops additionally contribute to the two-loop corrections. In the assignment of the FN charge shown in Table 2, the superpotential of the Yukawa interaction is given by

$$W_{\text{Yukawa}} \ni (y_u)_{24} u_2 Q_4 H_u + (y_u)_{42} u_4 Q_2 H_u + (y_d)_{24} d_2 Q_4 H_d + (y_e)_{34} e_3 L_4 H_d + (y_e)_{43} e_4 L_3 H_d + (y_e)_{42} e_4 L_2 H_d + y_{Q_4} \Phi Q_4 \bar{Q} + y_{u_4} \Phi u_4 \bar{u} + y_{d_4} \Phi d_4 \bar{d} + y_{e_4} \Phi e_4 \bar{e},$$

where the Yukawa couplings smaller than order $\epsilon$ are neglected. Among them, the up-type Higgs interactions together with the interaction of the singlet field

$$W_{\text{Yukawa}} \ni (y_u)_{24} u_2 Q_4 H_u + (y_u)_{42} u_4 Q_2 H_u + y_{Q_4} \Phi Q_4 \bar{Q} + y_{u_4} \Phi u_4 \bar{u}$$

give comparable amplitude. It turns out that the down-type Higgs interactions produce relatively small corrections to the Higgs mass in our numerical computations with a large $\tan \beta$ and $\mu_H \simeq 2$ TeV. Some of these are similar to the sbottom contribution (Ref. [57]), whereas the remaining ones are suppressed by $1/\tan \beta$. With interactions (4.9), we roughly evaluate the contributions from vector-like generations in a diagrammatic way. As for gluon exchange diagrams, we obtain new contributions comparable to $\mathcal{O}(\alpha_s \alpha_t)$ in the Higgs mass by replacing stops with the up-type scalar components of second or fourth generations. It is noted that top Yukawa coupling is also replaced with appropriate Yukawas for vector-like generations in Eq. (4.9). By these replacements, it turns out that the number of new diagrams becomes twice as many as for MSSM. As for the diagrams given only by Yukawa interactions, we obtain new contributions comparable to $\mathcal{O}(\alpha_t \alpha_t)$ in the Higgs mass by replacing stops and up-type Higgs with the up-type scalar components of second, fourth, fifth generations, up-type Higgs, and the scalar component of $\Phi$. Similarly to gluon exchange contributions, top Yukawa coupling is replaced with appropriate Yukawas in Eq. (4.9). By these replacements, we can see that the number of new diagrams is 5 times larger than for MSSM. For a rough estimation, we set all squark masses to be of order 1 TeV and the mass of the scalar component of $\Phi$ to be the same order as the mass of the sleptons. In our model, the slepton masses are a few hundred GeV.

Since we calculate the Higgs mass at the one-loop level, we treat the two-loop correction as a theoretical uncertainty and show parameter space with this uncertainty for the Higgs mass as $m_h = 122–129$ GeV. (The loop correction with extension to the vector-like generations is also studied in Ref. [63].) For instance, in the case that these new diagrams produce a similar sign to the MSSM case (minus sign for the gluino exchange diagrams and plus sign for Yukawa interaction diagrams), the Higgs mass at the one-loop level can be estimated as 127 GeV.

Let us comment on the lepton flavor violation processes. In the mass matrix (2.14), the muon has sizable couplings to the vector-like generations, which may induce flavor-changing rare processes.
Among them, we focus on the tau decay $\tau \rightarrow \mu \gamma$ and the muon decay $\mu \rightarrow e\gamma$. Experimental bounds on the branching ratios are given by (Refs. [64,65])

$$\text{Br}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8} \text{ (EXP)},$$

$$\text{Br}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13} \text{ (EXP)}.$$  \hspace{2cm} (4.10) (4.11)

In a model with vector-like generations, the lepton flavor violating processes are studied in Refs. [66–68]. We calculate the branching ratio $\text{Br}(\tau \rightarrow \mu \gamma)$ and $\text{Br}(\mu \rightarrow e\gamma)$, following Refs. [67,68]. The decay amplitudes of $l_i \rightarrow l_j\gamma$ are generally written as

$$\langle |l_j(p')\rangle |l_i(p)\rangle = \bar{u}_e(p') \Gamma_\alpha u_\mu(p),$$  \hspace{2cm} (4.12)

where indices $i,j (=1,2,3)$ denote the generation, $J_\alpha$ is an electromagnetic current for leptons, and the corrected vertex $\Gamma_\alpha$ is given by

$$\Gamma_\alpha(q) = \frac{F_{ij}^{ll}(q)i\sigma_{\alpha\beta}(p' - p)\beta}{m_i + m_j} + \frac{F_{23}^{ll}(q)\sigma_{\alpha\beta}\gamma_5(p' - p)\beta}{m_i + m_j} + \cdots.$$  \hspace{2cm} (4.13)

With these, the branching ratios are given by

$$\text{Br}(\tau \rightarrow \mu\gamma) = \frac{24\pi^2}{5G_Fm_\tau^2(m_\tau + m_\mu)^2}(|F_{2}^{\tau\mu}(0)|^2 + |F_{3}^{\tau\mu}(0)|^2),$$  \hspace{2cm} (4.14)

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{24\pi^2}{G_Fm_\mu^2(m_\mu + m_e)^2}(|F_{2}^{\mu\epsilon}(0)|^2 + |F_{3}^{\mu\epsilon}(0)|^2),$$  \hspace{2cm} (4.15)

where $G_F$ is the Fermi coupling constant and hadronic decay modes are included in $\tau$ decay. The form factors $F_{i,j}^{\tau\mu}(0)$ are defined in Eqs. (C.1) and (C.2). In our model, both the non-SUSY and the SUSY sectors contribute to $\text{Br}(l_i \rightarrow l_j\gamma)$. The former contributions come from one-loop diagrams mediated by W-boson and vector-like neutrinos, and by Z-boson and vector-like charged leptons. The latter contributions come from one-loop diagrams mediated by charginos (charged winos and higgsinos) and sneutrinos, and by neutralinos (neutral wino and higgsinos) and charged sleptons. We take into account these lepton flavor violating processes as constraints on our model.

In Fig. 3, we plot the contours of the Higgs boson mass, the muon $g-2$, the DM abundance, and the branching ratio of $\mu \rightarrow e\gamma$ in the $m_{\chi^0}-m_{l_1/2}$ plane. We fix $m_0$ to be $m_{l_1/2}$/20 because the small ratio of $m_0$ to $m_{l_1/2}$ is preferred to explain the muon $g-2$ (Ref. [6]). It is known that such mass spectra can be obtained in gaugino mediation scenarios (Ref. [69]). The orange region shows the Higgs boson mass in the range from 122 to 129 GeV. The blue region explains the muon $g-2$ anomaly within the $2\sigma$ level. The 3 black contours show $\Omega_{\chi^0}h^2 = 0.01, 0.1, 1$ from left to right. We find also that our model numerically satisfies experimental upper bounds of both $\text{Br}(\tau \rightarrow \mu \gamma)$ and $\text{Br}(\mu \rightarrow e\gamma)$ in all parameter regions of Figs. 3 and 4. For $\mu \rightarrow e\gamma$, a red line shows $\text{Br}(\mu \rightarrow e\gamma) = 1.0 \times 10^{-14}$ as a reference value in both figures. Above the line, the fraction becomes smaller. For $\tau \rightarrow \mu \gamma$, no values of the fraction are shown because they are too small to reach the current experimental sensitivity. As seen in the figure, there exist regions where the Higgs boson mass, the muon $g-2$, and the DM abundance can be explained at the same time. It is noted that the DM abundance highly depends on the DM mass. In our model, the DM mass is mostly determined with only $O(1)$ parameters, whereas realistic values of the CKM matrix and fermion masses are reproduced. On top of these, the masses
Fig. 3. The DM abundance, the Higgs boson mass, and the muon $g-2$ in the $m_{\chi\Phi}-m_{1/2}$ plane with $m_0 = m_{1/2}/20$. The 3 black contours correspond to values of $\Omega_{\chi\Phi} h^2 = 0.01, 0.10, 1.0$ from left to right. The Higgs boson with a mass between 122 and 129 GeV is shown in the orange region. The discrepancy of the muon $g-2$ within the 2$\sigma$ level is explained in the blue region. The red line shows $\text{Br}(\mu \to e\gamma) = 1.0 \times 10^{-14}$.

Fig. 4. The DM abundance, the Higgs boson mass, and the muon $g-2$ in the $m_0-m_{1/2}$ plane. The DM mass is fixed at 111 GeV ($y(= Y) = 2.1$). The Higgs boson with a mass between 122 and 129 GeV is shown in the orange region. The discrepancy of the muon $g-2$ within the 2$\sigma$ level is explained in the blue region. The black contours show values of $\Omega_{\chi\Phi} h^2 = 0.08, 0.11, 0.30$ from left to right. The red line shows $\text{Br}(\mu \to e\gamma) = 1.0 \times 10^{-14}$. The red circle is the sample point of our model.

of the vector-like generations are determined by the VEV of the DM (singlet) multiplet $\Phi$. In this sense, the presence of the DM supports the existence of 3 generations in low-energy scales.

In Fig. 4 we similarly show the Higgs boson mass, the muon $g-2$, the contours of the DM abundance, and the branching ratio of $\mu \to e\gamma$ in the $m_0-m_{1/2}$ plane. We set the DM mass $m_{\chi\Phi}$ to be 111 GeV, which means that $y = 2.1$. The orange and blue regions are the same as in Fig. 3. The black contours show values of $\Omega_{\chi\Phi} h^2 = 0.08, 0.11, 0.3$ from left to right. As seen in Fig. 4, the Higgs boson mass, the muon $g-2$, and $\Omega_{\chi\Phi} h^2 = 0.11$ can be explained simultaneously, where $m_{1/2}$ lies in the range between 1500 and 1800 GeV and $m_0$ is around 300 GeV. The red circle in Fig. 4 is
Table 5. The sample points in our model. All the mass parameters are given in units of GeV: \( m_{\tilde{N}_1} \) is the lightest neutralino in the MSSM sector, \( m_{\tilde{C}_1} \) is the lightest chargino, \( m_t \) is the mass of fourth generations for up-type quarks, \( m_L \) is the mass of fourth generations for charged leptons; the \( \mu_H \) \((\sim 2 \text{ TeV})\) and \( b \) terms are determined so that electroweak symmetry breaking occurs.

| Parameters          | Sample point (1) |
|---------------------|------------------|
| \( m_{1/2} \)       | 1780             |
| \( m_0 \)           | 250              |
| \( y(= Y) \)        | 2.1              |
| \( M_3 \)           | 1202             |
| \( m_{\tilde{N}_1} \) | 347.4           |
| \( m_{\tilde{C}_1} \) | 587.5           |
| \( m_{\text{stop}} \) | 1394             |
| \( m_t \)           | 1157             |
| \( m_{\text{charged slepton}} \) | 324.0     |
| \( m_L \)           | 274.4             |
| \( m_{h^0} \)       | 125.2            |
| \( \Delta a_\mu \)  | \( 10.3 \times 10^{-10} \) |
| \( \Omega_{\chi, \phi} h^2 \) | 0.116            |
| \( \text{Br}(\tau \rightarrow \mu\gamma) \) | \( 5.5 \times 10^{-13} \) |
| \( \text{Br}(\mu \rightarrow e\gamma) \) | \( 6.9 \times 10^{-15} \) |

the sample point of our model: \((m_{1/2}, m_0, y) = (1780, 250, 2.1)\).\(^7\) We show a typical mass spectrum, the Higgs boson mass, the muon \( g-2 \), the DM abundance and the branching ratio of \( \tau \rightarrow \mu\gamma \), and \( \mu \rightarrow e\gamma \) in Table 5. In Table 5, \( M_3 \) is the gluino mass, \( m_{\tilde{N}_1} \) is the lightest neutralino in the MSSM sector, \( m_{\tilde{C}_1} \) is the lightest chargino, \( m_{\text{stop}} \) is a lighter stop mass, \( m_t \) is the mass of vector-like generations for up-type quarks, \( m_{\text{charged slepton}} \) is the mass of the lightest charged slepton, which is the vector-like generations one, and \( m_L \) is the mass of vector-like generations for charged lepton.

Mixings of the vector-like generations with the 3 generations make an electric dipole moment (EDM). We show typical values of the electron EDM \( d_e \) and then discuss the constraints from experiments. The contributions are mainly from 4 sources: the chargino exchange, the neutralino exchange, the \( W \) boson exchange, and the \( Z \) boson exchange. Each contribution is estimated as follows (Ref. [70]):

- **Chargino contribution:**
  \[
  |d_e^+| \sim 8.7 \times 10^{-29} \text{e cm} \left( \frac{m_{\tilde{W}}}{600 \text{ GeV}} \right) \left( \frac{900 \text{ GeV}}{m_{\tilde{\chi}_e}} \right)^2 \left| c_{L\tilde{\nu}_e \tilde{W}} c_{R\tilde{\nu}_e \tilde{W}} \right| \left( \frac{\arg \left(c_{L\tilde{\nu}_e \tilde{W}} c_{R\tilde{\nu}_e \tilde{W}} \right)}{1.1 \times 10^{-4}} \right) \]
  \[\tag{4.16}\]

  where \( m_{\tilde{\chi}_e} \) is the mass of snutrino, \( m_{\tilde{W}} \) is the wino mass, and \( c_{L\tilde{\nu}_e \tilde{W}} \) and \( c_{R\tilde{\nu}_e \tilde{W}} \) are couplings of mixings defined in Appendix B.2.

\(^7\) In the region where \( m_{1/2} \) is below 1500 GeV, there is likely to be a region where the muon \( g-2 \) anomaly is explained within 1\( \sigma \), but the gluino mass in such regions is below 1 TeV, which is a region excluded by the LHC (Ref. [14]).
Neutralino contribution:

\[
|d_{e}^{\chi_0}| \sim 8.7 \times 10^{-29} e\text{ cm} \left(\frac{m_{\tilde{B}}}{300 \text{ GeV}}\right) \left(\frac{1000 \text{ GeV}}{m_{\tilde{\chi}_1}}\right)^2 \left(\frac{|n_{L\tilde{e}_1} n_{R\tilde{e}_1}|}{10^{-5}}\right) \left(\frac{\arg(n_{L\tilde{e}_1} n_{R\tilde{e}_1})}{2.2 \times 10^{-4}}\right),
\]

(4.17)

\[
\times \left(\frac{c_{\tilde{e}_1 \tilde{B}}}{1000 \text{ GeV}}\right)^2 \left(\frac{n_{L\tilde{e}_1} n_{R\tilde{e}_1}}{10^{-5}}\right)\]

(4.18)

where \(\tilde{e}_1\) is the mass of a lighter selectron, \(m_{\tilde{B}}\) is the bino mass, and \(n_{L\tilde{e}_1 \tilde{B}}\) and \(c_{\tilde{e}_1 \tilde{B}}\) are couplings of mixings.

W boson contribution:

\[
|d_{e}^{W}| \sim 5.9 \times 10^{-30} e\text{ cm} \left(\frac{m_{L'}}{300 \text{ GeV}}\right) \left(\frac{80 \text{ GeV}}{m_{W}}\right)^2 \left(\frac{|g_{W1e} g_{W1e}^{WR}|}{10^{-11}}\right) \left(\frac{\arg(g_{W1e} g_{W1e}^{WR})}{10^{-1}}\right),
\]

(4.19)

where \(m_{L'}\) is the vector-like lepton mass, and \(g_{W1e}^{WL}\) and \(g_{W1e}^{WR}\) are couplings of mixings.

Z boson contribution:

\[
|d_{e}^{Z}| \sim 4.6 \times 10^{-30} e\text{ cm} \left(\frac{m_{L'}}{300 \text{ GeV}}\right) \left(\frac{90 \text{ GeV}}{m_{Z}}\right)^2 \left(\frac{|g_{Z1e} g_{Z1e}^{ZR}|}{10^{-11}}\right) \left(\frac{\arg(g_{Z1e} g_{Z1e}^{ZR})}{10^{-1}}\right),
\]

(4.20)

where \(g_{Z1e}^{WL}\) and \(g_{Z1e}^{WR}\) are couplings of mixings.

Among them, the chargino and neutralino exchanges are the main contributions to \(d_e\). Since the \(\mu\) term of our model is around 2 TeV, the Higgsinos decouple from the mixing and its couplings are smaller than those of the gauginos. Thus, \(d_e\) is manly given by the exchange of the gauginos. From experiments, its value is constrained as (Ref. [71])

\[
|d_e| < 8.7 \times 10^{-29} e\text{ cm} \text{ (EXP).}
\]

(4.21)

Therefore, the CP phase of the couplings should satisfy

\[
\theta < 1.1 \times 10^{-4},
\]

(4.22)

where \(\theta\) represents each phase.

5. Conclusion and discussion

In this paper we have studied the flavor structure in a model with vector-like generations by using the Froggatt–Nielsen mechanism. It is notable that the assignment of FN charges can be determined so that the CKM matrix and fermion masses at the \(M_Z\) scale are reproduced. Furthermore, under such FN charge assignments, the fermion component of the gauge singlet superfield becomes a candidate for DM. The DM mass is induced through the RG flow including the flavor textures, which can explain observed flavor physics of muon \(g-2\). With such flavor textures, it is found that there exist parameter regions where we can explain the Higgs boson mass, the muon \(g-2\), and the DM abundance simultaneously. The singlet plays two roles: one is to fix the vector-like mass by the VEV, and another is that its fermion component is a DM candidate. In this sense, the presence of the DM supports the existence of 3 generations in low-energy scales.
Let us here discuss the prospects for discovering the DM particle $\chi$ at direct/indirect detection experiments. In our model, the singlino $\chi$ couples to the first generation of the quark field mediated by the scalar component of the vector-like particle. By integrating out the mediator field, we obtain the effective interaction between DM and first generation as $\mathcal{L}_{\text{int}} \simeq (y_1^2/m_1^2) \bar{\chi} X_1 O_1^\dagger$. Here $m$ is the mass of the mediator field of order $\mathcal{O}$(TeV), and $y_1$ is the Yukawa coupling constant. With this interaction, $\chi$ scatters nucleons independently of spin with the cross section $\sigma_N^N = y_1^4 \times O(10^2)$ pb $(\mu_X/10^2 \text{GeV})^2/(m/\text{TeV})^4$ (Ref. [72]), where $\mu_X$ is the reduced mass defined as $\mu_X = m_N m_X/(m_X + m_N)$ with nuclei mass $m_N$. Since the Yukawa coupling of the singlino to the first generation of the quark is set to the small value by the FN mechanism as $y \simeq 10^{-3}(\epsilon/0.33)^6$, we see that the magnitude of the spin-independent cross section is reduced to $\sigma_N^N \simeq 10^{-10}$ pb. Thus, in future direct detection experiments such as XENON1T (Ref. [73]) or DARWIN (Ref. [74]), our DM model will be tested. In the lepton sector, $\chi$ interacts with third generations by larger coupling $y^2 \simeq 10^{-1}(\epsilon/0.33)^2$ mediated by the vector-like particle. Thus, through the annihilation of the DM particles in the Galactic Center or in dwarf spheroidal galaxies, a significant excess of gamma rays might be produced. The excess of the energetic gamma rays might be detected in indirect detection experiments such as Fermi-LAT (Refs. [75,76]) or CTA (Ref. [77]).

In the present paper, we have studied the thermal production of the DM through the interactions with quarks and leptons, respecting the original model (Ref. [6]) shown in Eq. (2.6), but from the arguments based on symmetries, other terms such as $W \propto \Phi, \Phi^2$, or $H_u H_d$ can be allowed. In this paper, we have just dropped these terms following Ref. [6], but the terms might affect the DM abundance. We will investigate the effects in future work, but here briefly discuss the issues. Among the interactions, the coupling of the singlet fields with the Higgs fields

$$W = \lambda H_u H_d$$

might give large contributions to the experimental results. This interaction makes the effective $\mu$-term, and contributes to the cross section of the dark matter with nucleons. Thus, to reproduce the result of our analysis and to avoid the bound from direct detection experiments, we need to appropriately choose the coupling constants $\lambda$, trilinear coupling $Y$, and the VEV of the singlet field $\langle \Phi \rangle = V$, but on the other hand it affects muon $g - 2$ and Higgs mass corrections. Here we discuss these prospects. With the interaction, the DM components $\chi_\Phi$ could have a sizable interaction with nucleons through the t-channel process of the neutral Higgs boson, and it affects the scattering process of the dark matter and nucleons (Refs. [78,79]). Particularly interactions with the strange-quark are main contributors to the scattering, and its cross section is given in Ref. [79] by

$$\sigma_N^N \simeq \mu_X Y^2 h_s^2 \frac{M_H^4}{M_{H_2}} S_{a3}^2 S_{a1}^2,$$

where $\mu_X$ is the reduced mass, $h_s$ is the Yukawa coupling of the strange quark, $M_{H_2}$ is the Higgs mass, $S_{a3}$ and $S_{a1}$ are diagonalization matrix elements for CP-even neutral Higgs bosons (see Ref. [79] for the definition), and the subscript $a$ runs from 1 to 3. The subscript represents the mass eigenstates of the neutral Higgs with order $M_{H_1} < M_{H_2} < M_{H_3}$. Here the lightest Higgs $H_1$ is identified as the detected one of 125 GeV mass (Ref. [80]), and its contribution would be largest. To avoid the current bound from direct detection, we need to choose a small value of $Y$, but to reproduce the DM mass $m_{\chi_\Phi} = Y V \simeq 10^2 \text{GeV}$, $V$ is required to be larger than $10^2 \text{GeV}$. On the other hand, this singlet VEV also gives the effective $\mu$-term as $\mu_{\text{eff}} = \lambda V$. From the prospect of the Landau pole, $\lambda$ needs to be smaller than unity, and it requires $V$ to be larger than $10^2 \text{GeV}$ to satisfy $\mu_{\text{eff}} = 2 \text{TeV}$. Further,
since the scattering of the DM with the SM quarks is mediated through the interaction (5.1), the smaller value of $\lambda$ is also required to avoid the experimental bounds on the spin-independent cross section (5.2). This large value of $V$ leads to the decoupling limit of vector-like generations (i.e., the MSSM-like limit), and then the contributions from the vector-like generations to the Higgs mass and the muon $g-2$ might be too small to explain the experimental values simultaneously. Moreover, the Higgs mass has to be evaluated by taking into account corrections from a coupling $\lambda$. As for the thermal production process, the DM becomes more likely to annihilate into the SM particles mediated by the Higgs fields due to interaction with the Higgs fields. Thus, we can expect that a larger DM mass is required to explain the observed abundance. On the other hand, the interaction might be forbidden by some symmetry such as R-symmetry. However, at the same time, several terms of the Yukawa interactions (2.8) would also inevitably be absent by the assignment, and the absence would change the muon $g-2$. We will investigate these issues in future work.

Within the FN charge assignment in Table 2, flavor violating processes induced by the mixing between $\tau_R \leftrightarrow \mu_R$ or $d_R \leftrightarrow s_R$ could be allowed. In this case, the SUSY flavor problem is not avoided by the assignment without a high-scale SUSY breaking or a flavor-blind mediation mechanism, but there would be other choices such that the off-diagonal elements of the squark mass matrix are suppressed and we avoid the problem. In our model, since the mSUGRA scenario is assumed, the Yukawa matrices of the SUSY breaking sector are diagonal. Thus, the SUSY flavor problem is avoided.

We have not taken into account the presence of right-handed neutrinos, the relevant flavor textures, and collider physics in this paper. These are important to test our model. We will reveal these prospects in future work.

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Appendix A. Annihilation cross section

In this appendix, we give expressions for $a$ and $b$ in Eq. (3.15), which are needed to calculate the DM abundance. We calculate $a$ and $b$ in the following way. First, as in Fig. 2, we consider DM annihilation into SM fermions, which is denoted by $f_i$ (except for the top quark) by exchanging sfermions, which is denoted by $\tilde{f}_\alpha$. Next, we evaluate the square of the scattering amplitude, which is given by t-channel and u-channel processes, as the annihilation cross section $\sigma_{\text{ann}}$. Last, we derive the thermal-averaged cross section $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle = a + b/x_T$ following Refs. [34,35]. In the limit where the mass of the final state in DM annihilation can be ignored, compared with the DM mass, the coefficients $a$ and $b$ are defined by

$$a = a^0, \quad b = -3a_0 + b_0.$$
After straightforward calculations, \(a_0\) and \(b_0\) are given by

\[
a_0 = \frac{m_{\chi^0}^2}{25\pi} \left[ \sum_{\alpha, \beta, i, j} \frac{(O_{R\alpha} O_{R\beta} + O_{L\alpha} O_{L\beta})^2}{\Delta_{j\alpha} \Delta_{j\beta}} - \sum_{\alpha, \beta, i, j} \frac{O_{R\alpha}^2 O_{R\beta}^2 + O_{L\alpha}^2 O_{L\beta}^2}{\Delta_{j\alpha} \Delta_{j\beta}} \right],
\]

(A.2)

\[
b_0 = \frac{m_{\chi^0}^2}{25\pi} \sum_{\alpha, \beta, i, j} \frac{O_{R\alpha} O_{R\beta} O_{L\alpha} O_{L\beta}}{\Delta_{j\alpha}^3} \delta_{j\beta}^3 \times \left\{ \Delta_{j\alpha} \Delta_{j\beta} \left( 2m_{\chi^0}^4 + 10m_{\chi^0}^2 m_{j\alpha}^2 - 30m_{\chi^0}^2 m_{j\beta}^2 - 30m_{\chi^0}^2 m_{j\beta}^2 \right) + 12(m_{\chi^0}^8 - 2m_{\chi^0}^6 m_{j\alpha}^2) + \left( m_{\chi^0}^6 m_{j\alpha}^2 - 16m_{\chi^0}^4 m_{j\alpha}^2 + m_{\chi^0}^4 m_{j\beta}^2 - 4m_{\chi^0}^4 m_{j\beta}^2 - 13m_{\chi^0}^2 m_{j\alpha}^2 + 3m_{\chi^0}^2 m_{j\beta}^2 m_{j\beta}^2 + 2m_{\chi^0}^4 m_{j\beta}^4 \right) \right\} \times \left\{ \Delta_{j\alpha} \Delta_{j\beta} \left( 2m_{\chi^0}^4 - m_{\chi^0}^2 m_{j\alpha}^2 + \left( m_{\chi^0}^4 m_{j\alpha}^2 - 2m_{\chi^0}^2 m_{j\beta}^2 m_{j\beta}^2 \right) \right) \right\},
\]

(A.3)

where \(\Delta_{j,\alpha,\beta} = m_{\chi^0}^2 + m_{j,\alpha,\beta}^2\), the subscripts \(i, j\) correspond to the generation of the SM particles whose mass is below the freeze-out temperature, and the subscripts \(\alpha, \beta\) correspond to the generations of sfermions that are regarded as the mediators between the DM and the SM particles in our model.

The dependence of \(a\) and \(b\) on the DM and sfermion masses is roughly given by

\[
a, b \sim \frac{O^4 m_{\chi^0}^2}{m_j^2 m_j^2} \sim \frac{y^4 m_{\chi^0}^2}{m_j^2 m_j^2},
\]

(A.4)

where \(O\) means the coupling constants between the DM and sfermions and \(y\) is a Yukawa coupling between them.

**Appendix B. Analytic formulae for Higgs mass and the muon \(g-2\)**

**B.1. Higgs mass**

The one-loop correction to the lightest neutral Higgs mass \(\Delta m_h^2\) is given by (Ref. [42])

\[
\Delta m_h^2 = \left[ \sin^2 \beta \left( \frac{\partial^2}{\partial v_u^2} - \frac{1}{v_u} \frac{\partial}{\partial v_u} \right) + \cos^2 \beta \left( \frac{\partial^2}{\partial v_d^2} - \frac{1}{v_d} \frac{\partial}{\partial v_d} \right) + \sin \beta \cos \beta \frac{\partial^2}{\partial v_u \partial v_d} \right] \Delta V_H,
\]

(B.1)
where $\Delta V_H$ are the one-loop corrections to the Higgs potential and are defined as

$$
\Delta V_H = \sum_{X=u,d,e} \sum_{i=1}^{10} 2N_c \left[ F(m^2_{\tilde{X}_i}) - F(m^2_{\tilde{X}_i}) \right], \quad N_c = \begin{cases} 
3 & (X = u, d), \\
1 & (X = e), 
\end{cases}
$$

(B.2)

where $m^2_{\tilde{X}_i}$ and $m^2_{\tilde{X}_i}$ are the squared-mass eigenvalues of fermions and scalars, respectively, which are obtained by diagonalizing Eqs. (2.12)–(2.14) for fermions ($M_u^a M_u$ and $M_d^a M_d$ etc.). For diagonalization of scalars mass matrices, we use the scalar mass matrices that are defined in Eqs. (B.1)–(B.3) in Ref. [6]. The function $F$ is defined as (Ref. [42])

$$
F(x) = \frac{x^2}{64\pi^2} \left[ \ln \left( \frac{x}{\mu^2} \right) - \frac{3}{2} \right],
$$

(B.3)

where $\mu$ represents the renormalization scale, which is set to be $M_{\text{SUSY}}$ in evaluating the Higgs mass.

B.2. Muon $g-2$

We show the SUSY contributions $\Delta a_{\mu}^{\text{SUSY}}$ and non-SUSY contributions $\Delta a_{\mu}^{\text{non-SUSY}}$ in Eq. (4.2). First, we consider the SUSY contributions. In order to calculate the SUSY contributions, we use the mass eigenstate basis for gauginos, charged leptons, charged sleptons, and neutral sleptons. The analytic formula for SUSY contributions is the same as the previous paper Ref. [6]. Let us define the diagonalization matrix for neutralinos, charginos, sneutrinos, in order to evaluate the SUSY contributions $\Delta a_{\mu}^{\text{SUSY}}$ of the muon $g-2$. In the basis $\{\tilde{\nu}, \tilde{W}^0, \tilde{H}^0_d, \tilde{H}^0_u\}$, the neutralino mass matrix $M_{\chi^0}$ is given by

$$
M_{\chi^0} = \begin{pmatrix}
M_1 & 0 & -g_1 v_d/\sqrt{2} & g_1 v_u/\sqrt{2} \\
0 & M_2 & g_2 v_d/\sqrt{2} & -g_2 v_u/\sqrt{2} \\
-g_1 v_d/\sqrt{2} & g_2 v_d/\sqrt{2} & 0 & -\mu_H \\
g_1 v_u/\sqrt{2} & -g_2 v_u/\sqrt{2} & -\mu_H & 0
\end{pmatrix}.
$$

(B.4)

In the basis $\{\tilde{W}^-, \tilde{H}_d^+\}$ and $\{\tilde{W}^+, \tilde{H}_u^+\}$, the chargino mass matrix $M_{\chi^\pm}$ is given by

$$
M_{\chi^\pm} = \begin{pmatrix}
M_2 & \sqrt{2} g v_d/\mu_H \\
\sqrt{2} g v_d & \mu_H
\end{pmatrix},
$$

(B.5)

where the charged winos $\tilde{W}^\pm$ are defined as

$$
\tilde{W}^\pm = \frac{i}{\sqrt{2}} (\tilde{W}^1 \mp i\tilde{W}^2).
$$

(B.6)

By using the neutralino mixing matrix $N$ and the chargino mixing matrices $J$, $K$, the mass matrices in Eqs. (B.4) and (B.5) are diagonalized by

$$
NM_{\chi^0} N^\dagger = \text{diag} \left( m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0} \right),
$$

(B.7)

$$
JM_{\chi^\pm} K^\dagger = \text{diag} \left( m_{\chi_1^\pm}, m_{\chi_2^\pm} \right),
$$

(B.8)
where \( m_{\chi_0} (x = 1, \ldots, 4) \) are the positive mass eigenvalues \( (m_{\chi_0} < m_{\chi_0} \text{ if } x < y) \), and \( m_{\chi_\pm} (x = 1, 2) \) are the positive mass eigenvalues \( (m_{\chi_+} < m_{\chi_\pm}). \) The diagonalization of neutral sleptons are defined by

\[
(U_{\tilde{v}} M_{\tilde{v}}^2 U_{\tilde{v}}^T)_{\alpha \beta} = m_{\tilde{N}_\alpha}^2 \delta_{\alpha \beta} \quad (\alpha, \beta = 1, \ldots, 5),
\]  

(B.9)

where \( M_{\tilde{v}}^2 \) is the neutral slepton mass matrix defined in Ref. [6].

The SUSY contributions to the muon \( g - 2 \) are divided into 3 parts: neutralinos \( (\chi^0) \), charginos \( (\chi^\pm) \), and singlino \( (\chi_\Phi) \). The singlino contribution is calculated by the replacement of \( \chi^0 \) with \( \chi_\Phi \) in the neutralino diagram (with appropriate replacement of coefficients). The SUSY contributions are given by

\[
\Delta a^{\mu \Phi}_\mu = \Delta a^{\chi^0}_\mu + \Delta a^{\chi^\pm}_\mu + \Delta a^{\chi_\Phi}_\mu,
\]  

(B.10)

where

\[
\Delta a^{\chi^0}_\mu = \sum_{a,x} \frac{1}{16\pi^2} \left[ \frac{m_\mu m_{\chi_0} L_{2ax} L_{2ax}^R} {m_{\tilde{N}_a}^2} F_2^N (r_{1ax}) - \frac{m_\mu^2} {6m_{\tilde{E}_a}^2} (n_{2ax}^L n_{2ax}^L + n_{2ax}^R n_{2ax}^R) F_1^N (r_{1ax}) \right],
\]  

(B.11)

\[
\Delta a^{\chi^\pm}_\mu = \sum_{a,x} \frac{1}{16\pi^2} \left[ -\frac{3m_\mu m_{\chi^\pm} L_{2ax} L_{2ax}^R} {m_{\tilde{\nu}_a}^2} F_2^C (r_{2ax}) + \frac{m_\mu^2} {3m_{\tilde{E}_a}^2} (c_{2ax}^L c_{2ax}^L + c_{2ax}^R c_{2ax}^R) F_1^C (r_{2ax}) \right],
\]  

(B.12)

\[
\Delta a^{\chi_\Phi}_\mu = \sum_a \frac{1}{16\pi^2} \left[ \frac{m_\mu m_{\chi_\Phi} L_{2ax} L_{2ax}^R} {m_{\tilde{N}_a}^2} F_2^N (r_{3a}) - \frac{m_\mu^2} {6m_{\tilde{E}_a}^2} (s_{2ax}^L s_{2ax}^L + s_{2ax}^R s_{2ax}^R) F_1^N (r_{3a}) \right],
\]  

(B.13)

with \( r_{1ax} = m_{\tilde{\nu}_a}^2 / m_{\tilde{N}_a}^2 \), \( r_{2ax} = m_{\tilde{\nu}_a}^2 / m_{\tilde{E}_a}^2 \), \( r_{3a} = m_{\tilde{\nu}_a}^2 / m_{\tilde{N}_a}^2 \), and \( m_\mu \) is the muon mass, the functions \( F_1^N, F_2^C, F_3^C, F_4^C \) are defined by

\[
F_1^N(x) = \frac{2}{(1 - x)^4} (1 - 6x^2 + 3x^3 + 2x^3 - 6x^2 \ln x),
\]  

(B.14)

\[
F_2^N(x) = \frac{3}{(1 - x)^3} (1 - x^2 + 2x \ln x),
\]  

(B.15)

\[
F_1^C(x) = \frac{2}{(1 - x)^4} (2 + 3x - 6x^2 + x^3 + 6x \ln x),
\]  

(B.16)

\[
F_2^C(x) = \frac{-3}{(1 - x)^3} (3 - 4x + x^2 + 2 \ln x),
\]  

(B.17)

and by using diagonalization matrices \( (3.5), (3.8), (B.7), (B.8) \), and \( (B.9) \), the coefficients \( n_{2ax}^L, c_{2ax}^L, s_{2ax}^L, s_{2ax}^R \) in Eqs. (B.11)–(B.13) are defined by

\[
n_{2ax}^L = - \sum_{i,j=1}^4 (y_\tilde{e})_{ij} (V_{eR})_{ij} (U_{\tilde{v})_{ai} N_{x3} + y_\tilde{e} (V_{eR})_{ij} (U_{\tilde{v})_{ai} N_{x4}}
\]  

\[
- \sum_{i=1}^4 \sqrt{2} g_1 (V_{eR})_{ij} (U_{\tilde{v})_{ai} N_{x1} - \frac{g_2}{\sqrt{2}} (V_{eR})_{ij} (U_{\tilde{v})_{ai} N_{x2}}
\]
where blank elements mean zero. The diagonalization of this matrix is defined by in order to evaluate the non-SUSY contributions of the muon Higgs bosons. Let us define the diagonalization matrix for neutral leptons and CP-even Higgs bosons Ref. [49]. We use the mass eigenstate basis for charged leptons, neutral leptons, and CP-even neutral Higgs mass matrix where only $m_{\nu_5}$ has a finite value and the other masses $(i = 1, \ldots, 4)$ are zero. The CP-even neutral Higgs mass matrix $M^2_{h^0}$ is given by

\[
M^2_{h^0} = \begin{pmatrix}
M^2_{\nu_5} \sin^2 \beta + M^2_Z \cos^2 \beta \\
-M^2_{\nu_5} \sin \beta \cos \beta & M^2_Z \cos^2 \beta + M^2_Z \sin^2 \beta
\end{pmatrix},
\]
where $M_\Delta = 2b / \sin(2\beta)$ is the CP-odd neutral Higgs boson mass as in the MSSM. The diagonalization of this matrix is defined by

$$(U_{\lambda 0}^\dagger M_{\lambda 0}^2 U_{\lambda 0}^\dagger)^X_Y = m_{h_X}^2 \delta_{XY} \quad (X, Y = 1, 2),$$

(B.27)

where the mass eigenvalues are ordered as $m_{h_0}^2 < m_{h_2}^2$.

The non-SUSY contributions are divided into 3 parts: $W$-boson, $Z$-boson, and Higgs bosons. Then, $\Delta a_{\mu}^{\text{non-SUSY}}$ is given by

$$\Delta a_{\mu}^{\text{non-SUSY}} = \Delta a_{\mu}^Z + \Delta a_{\mu}^W + \Delta a_{\mu}^h,$$

(B.28)

where

$$\Delta a_{\mu}^Z = - \frac{m_{\mu}}{8\pi^2 M_Z^2} \sum_{a=4,5} \left[ (g_{2a}^{ZL})^2 + (g_{2a}^{ZR})^2 \right] m_{\mu} F_Z(x_{Za}) + g_{2a}^{ZL} g_{2a}^{ZR} m_{Ea} G_Z(x_{Za}),$$

(B.29)

$$\Delta a_{\mu}^W = - \frac{m_{\mu}}{16\pi^2 M_W^2} \sum_{a=4,5} \left[ (g_{52}^{WL})^2 + (g_{52}^{WR})^2 \right] m_{\mu} F_W(x_{W}) + g_{52}^{WL} g_{52}^{WR} m_{Ea} G_W(x_{W}),$$

(B.30)

$$\Delta a_{\mu}^h = \sum_{X=1,2} \sum_{a=4,5} \frac{m_{\mu}}{32\pi^2 m_{h_X}^2} \left[ (\lambda_{2aX})^2 + (\lambda_{a2X})^2 \right] m_{\mu} F_h(x_{h_0aX})$$

$$+ \lambda_{2aX} \lambda_{a2X} m_{Ea} G_h(x_{h_0aX}),$$

(B.31)

with $x_{Za} = m_{Ea}^2 / M_Z^2$, $x_{W} = m_{Ea}^2 / M_W^2$, $x_{h_0aX} = m_{Ea}^2 / m_{h_X}^2$, and $M_W$ is the $W$-boson mass, $m_{Ea}$ are the mass eigenvalues of charged leptons defined in Eq. (3.5), the functions $F_Z$, $F_W$, $F_h$, $G_Z$, $G_W$, $G_h$ are defined by (Ref. [49])

$$F_Z(x) = \frac{12}{(1-x)^4} \left( 8 - 38x + 39x^2 - 14x^3 + 5x^4 - 18x^2 \ln x \right),$$

(B.32)

$$F_W(x) = \frac{-6}{(1-x)^4} \left( 10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln x \right),$$

(B.33)

$$F_h(x) = \frac{12}{(1-x)^4} \left( 8 - 38x + 39x^2 - 14x^3 + 5x^4 - 18x^2 \ln x \right),$$

(B.34)

$$G_Z(x) = \frac{2}{(1-x)^3} \left( -4 + 3x + x^3 - 6x \ln x \right),$$

(B.35)

$$G_W(x) = \frac{-1}{(1-x)^3} \left( -4 + 15x - 12x^2 + x^3 + 6x^2 \ln x \right),$$

(B.36)

$$G_h(x) = \frac{1}{(1-x)^3} \left( 3 - 4x + x^2 + 2 \ln x \right),$$

(B.37)

and by using mixing matrices Eqs. (3.5), (B.25), and (B.27), the coefficients $g_{2a}^{ZL, ZR}$, $g_{52}^{WL, WR}$, $\lambda_{2a}$, $\lambda_{a2}$ are defined by

$$g_{XY}^{ZL} = \sum_{i=1}^{4} \frac{g_2}{\cos \theta_W} \left( -\frac{1}{2} + \sin^2 \theta_W (V_{E_L})_{xi} (V_{E_L})_{yi} \right) + \frac{g_2}{\cos \theta_W} \sin^2 \theta_W (V_{E_L})_{x5} (V_{E_L})_{y5},$$

(B.38)

$$g_{XY}^{ZR} = \sum_{i=1}^{4} \frac{-g_2}{\cos \theta_W} \sin^2 \theta_W (V_{E_L})_{xi} (V_{E_L})_{yi} + \frac{g_2}{\cos \theta_W} \left( -\frac{1}{2} + \sin^2 \theta_W \right) (V_{E_L})_{x5} (V_{E_L})_{y5},$$

(B.39)
Appendix C. Form factors for lepton flavor violation

Following Refs. [67,68], we summarize the form factors that are necessary to evaluate the branching ratio of lepton flavor violating processes as shown in Eqs. (4.14) and (4.15). The form factors are divided into 4 parts: neutralinos, charginos, Z-boson, and W-boson parts as

\begin{align}
F_{2}^{l|i|j}(0) &= F_{2}^{l|i|j} + F_{2}^{l|i|j} + F_{2}^{l|i|j} + F_{2}^{l|i|j}, \\
F_{3}^{l|i|j}(0) &= F_{3}^{l|i|j} + F_{3}^{l|i|j} + F_{3}^{l|i|j} + F_{3}^{l|i|j}.
\end{align}

The neutralino contributions are given by

\begin{align}
F_{2}^{l|i|j} &= \sum_{a=1}^{10} \sum_{i=1}^{4} \left\{ -\frac{m_{l_{i}}(m_{l_{i}} + m_{l_{j}})}{192\pi^{2}m_{\chi_{a}^{0}}^{2}} \left( n_{lax}^{L}n_{lax}^{L} + n_{lax}^{R}n_{lax}^{R} \right) \right\} F_{1} \left( \frac{M_{\chi_{a}^{0}}^{2}}{m_{\chi_{a}^{0}}^{2}} \right), \\
F_{3}^{l|i|j} &= \sum_{a=1}^{10} \sum_{i=1}^{4} \left\{ \frac{m_{l_{i}} + m_{l_{j}}}{64\pi^{2}m_{\chi_{a}^{0}}^{2}} \left( n_{lax}^{L}n_{lax}^{L} + n_{lax}^{R}n_{lax}^{R} \right) \right\} F_{2} \left( \frac{M_{\chi_{a}^{0}}^{2}}{m_{\chi_{a}^{0}}^{2}} \right),
\end{align}

where the functions are defined by

\begin{align}
F_{1}(x) &= \frac{1}{(x-1)^{4}}(-x^{3} + 6x^{2} - 3x - 2 - 6x \ln x), \\
F_{2}(x) &= \frac{1}{(x-1)^{3}}(-x^{2} + 1 + 2x \ln x), \\
F_{3}(x) &= \frac{1}{2(x-1)^{2}} \left\{ x + 1 + \frac{2x \ln x}{1-x} \right\},
\end{align}

and \( n_{lax}^{L,R} \) are given in Eqs. (B.18) and (B.19).

The chargino contributions are given by

\begin{align}
F_{2}^{l|i|j} &= \sum_{a=1}^{5} \sum_{i=1}^{2} \left\{ \frac{m_{l_{i}}(m_{l_{i}} + m_{l_{j}})}{64\pi^{2}m_{\chi_{a}^{+}}^{2}} \left( c_{lax}^{L}c_{lax}^{L} + c_{lax}^{R}c_{lax}^{R} \right) \right\} F_{4} \left( \frac{M_{\chi_{a}^{+}}^{2}}{m_{\chi_{a}^{+}}^{2}} \right), \\
F_{3}^{l|i|j} &= \sum_{a=1}^{5} \sum_{i=1}^{2} \left\{ \frac{m_{l_{i}} + m_{l_{j}}}{64\pi^{2}m_{\chi_{a}^{+}}^{2}} \left( c_{lax}^{R}c_{lax}^{R} + c_{lax}^{L}c_{lax}^{L} \right) \right\} F_{5} \left( \frac{M_{\chi_{a}^{+}}^{2}}{m_{\chi_{a}^{+}}^{2}} \right),
\end{align}

where \( \theta_{W} \) is the Weinberg angle.
\[ F_{3X^+}^{l_i l_j} = \sum_{\alpha=1}^{5} \sum_{\lambda=1}^{2} \frac{(m_i + m_j) m_{X^+}}{32\pi^2 M_{N_a}^2} \left( c_{\lambda \alpha x}^{L} c_{\lambda \alpha x}^{R} - c_{\lambda \alpha x}^{L} c_{\lambda \alpha x}^{R} \right) F_6 \left( \frac{m_{X^+}^2}{M_{N_a}^2} \right), \]  

(C.9)

where the functions are defined by

\[ F_4(x) = \frac{1}{3(x - 1)^4} \left\{ -2x^3 - 3x^2 + 6x - 1 + 6x^2 \ln x \right\}, \]  

(C.10)

\[ F_5(x) = \frac{1}{(x - 1)^3} \left\{ 3x^2 - 4x + 1 - 2x^2 \ln x \right\}, \]  

(C.11)

\[ F_6(x) = \frac{1}{2(x - 1)^2} \left\{ -x + 3 + \frac{2 \ln x}{1 - x} \right\}, \]  

(C.12)

and \( c_{\lambda \alpha x}^{L,R} \) are given in Eqs. (B.20) and (B.21).

The contributions from the Z-boson exchange are given by

\[ F_{2Z}^{l_i l_j} = \sum_{\alpha=1}^{5} \frac{m_i (m_i + m_j)}{64\pi^2 m_Z^2} \left\{ \frac{g_{l_i a}^{ZL} g_{l_j a}^{ZL} + g_{l_i a}^{ZR} g_{l_j a}^{ZR}}{m_{E_a}^2} \right\} F_Z \left( \frac{m_{E_a}^2}{m_Z^2} \right), \]  

\[ + \frac{m_{E_a} (m_i + m_j)}{64\pi^2 m_Z^2} \left\{ g_{l_i a}^{ZL} g_{l_j a}^{ZR} + g_{l_i a}^{ZR} g_{l_j a}^{ZL} \right\} F_Z \left( \frac{m_{E_a}^2}{m_Z^2} \right), \]  

(C.13)

\[ F_{3Z}^{l_i l_j} = \sum_{\alpha=1}^{5} \frac{(m_i + m_j) m_{E_a}}{32\pi^2 m_Z^2} \left\{ \frac{g_{l_i a}^{ZL} g_{l_j a}^{ZR} - g_{l_i a}^{ZR} g_{l_j a}^{ZL}}{m_{E_a}^2} \right\} I_1 \left( \frac{m_{E_a}^2}{m_Z^2} \right), \]  

(C.14)

where the functions are defined by

\[ F_Z(x) = \frac{1}{3(x - 1)^4} \left\{ -5x^4 + 14x^3 - 39x^2 + 18x^2 \ln x + 38x - 8 \right\}, \]  

(C.15)

\[ G_Z(x) = \frac{2}{(x - 1)^3} \left\{ x^3 + 3x - 6x \ln x - 9 \right\}, \]  

(C.16)

\[ I_1(x) = \frac{2}{(1 - x)^2} \left\{ 1 + \frac{1}{4} x + \frac{1}{4} x^2 + \frac{3x \ln x}{2(1 - x)} \right\}, \]  

(C.17)

and \( g_{l_j a}^{ZL,ZR} \) are given in Eqs. (B.38) and (B.39).

The contributions from the W-boson exchange are given by

\[ F_{2W}^{l_i l_j} = \frac{m_i (m_i + m_j)}{32\pi^2 m_W^2} \left\{ \frac{g_{l_i l_j}^{WL} g_{l_i l_j}^{WL} + g_{l_i l_j}^{WR} g_{l_i l_j}^{WR}}{m_{V_S}^2} \right\} F_W \left( \frac{m_{V_S}^2}{m_W^2} \right), \]  

\[ + \frac{m_{V_S} (m_i + m_j)}{32\pi^2 m_W^2} \left\{ g_{l_i l_j}^{WL} g_{l_i l_j}^{WR} + g_{l_i l_j}^{WR} g_{l_i l_j}^{WL} \right\} F_W \left( \frac{m_{V_S}^2}{m_W^2} \right), \]  

(C.18)

\[ F_{3W}^{l_i l_j} = -\frac{m_{V_S} (m_i + m_j)}{32\pi^2 m_W^2} \left\{ g_{l_i l_j}^{WL} g_{l_i l_j}^{WR} - g_{l_i l_j}^{WR} g_{l_i l_j}^{WL} \right\} I_2 \left( \frac{m_{V_S}^2}{m_W^2} \right), \]  

(C.19)

where the functions are defined by

\[ F_W(x) = \frac{1}{6(x - 1)^4} \left\{ 4x^4 - 49x^3 + 18x^2 \ln x + 78x^2 - 43x + 10 \right\}, \]  

(C.20)
\[ G_W(x) = \frac{1}{(x-1)^3} \left[ 4 - 15x + 12x^2 - x^3 - 6x^2 \ln x \right], \quad \text{(C.21)} \]

\[ I_2(x) = \frac{2}{(1-x)^2} \left[ 1 - \frac{11}{4}x + \frac{1}{4}x^2 - \frac{3x^2 \ln x}{2(1-x)} \right], \quad \text{(C.22)} \]

and \( g_{xy}^{WL,WR} \) are given in Eqs. (B.40) and (B.41).

### Appendix D. RG Equations

We present the RG equations of model parameters in our model. Due to the asymptotically non-free nature of the gauge sector, the two-loop RG equations are used for gauge coupling constants and gaugino masses.

#### D.1. Gauge couplings and gaugino masses

The two-loop RG equations of gauge coupling constants \( g_i \) and gaugino masses \( M_i \) \((i = 1, 2, 3)\) are given by

\[
\frac{d g_i}{d (\log \mu)} = \frac{g_i^3}{16\pi^2} + \frac{g_i^3}{(16\pi^2)^2} \left[ \sum_j b_{ij} g_j^2 - \sum_{a=u,d,e} c_{ia} \left[ \text{Tr}(y_a^\dagger y_a) + y_a^\dagger y_a \right] \right. \\
\left. - \sum_{k=1}^4 \sum_{x=Q,u,d,L} \sum_{x_k} d_{ix} y_{x_k}^a y_{x_k} \right], \quad \text{(D.1)}
\]

\[
\frac{d M_i}{d (\log \mu)} = 2b_i g_i^2 M_i + \frac{2g_i^2}{(16\pi^2)^2} \left[ \sum_j b_{ij} g_j^2 (M_i + M_j) + \sum_{a=u,d,e} c_{ia} \left[ \text{Tr}(y_a^\dagger a_a) + y_a^\dagger a_a \right] \right. \\
\left. - M_i \left[ \text{Tr}(y_a^\dagger y_a) + y_a^\dagger y_a \right] \right] + \sum_{k=1}^4 \sum_{x=Q,u,d,L} \sum_{x_k} d_{ix} (y_{x_k}^a A_{x_k} - M_i y_{x_k}^a y_{x_k}) \right], \quad \text{(D.2)}
\]

where the one-loop beta function coefficients are \( b_i = (53/5, 5, 1) \), and the coefficient matrices \( b_{ij}, c_{ia}, d_{ix} \) are

\[
b_{ij} = \begin{pmatrix}
977/75 & 39/5 & 88/3 \\
13/5 & 53 & 40 \\
11/3 & 15 & 178/3
\end{pmatrix}, \quad \text{(D.3)}
\]

\[
c_{ia} = \begin{pmatrix}
26/5 & 14/5 & 18/5 \\
6 & 6 & 2 \\
4 & 4 & 0
\end{pmatrix}, \quad \text{(D.4)}
\]

\[
d_{ix} = \begin{pmatrix}
2/5 & 16/5 & 4/5 & 6/5 & 12/5 \\
6 & 0 & 0 & 2 & 0 \\
4 & 2 & 2 & 0 & 0
\end{pmatrix}. \quad \text{(D.5)}
\]
D.2. Yukawa couplings and bilinear terms

The RG equations of Yukawa couplings and the bilinear terms are given by

\[
\frac{d(y_u)_{ij}}{d(\log \mu)} = (\gamma_u y_u)_{ij} + (y_u \gamma_u)_{ij} + \gamma H_u (y_u)_{ij},
\]
(D.6)

\[
\frac{d(y_d)_{ij}}{d(\log \mu)} = (\gamma_d y_d)_{ij} + (y_d \gamma_d)_{ij} + \gamma H_d (y_d)_{ij},
\]
(D.7)

\[
\frac{d(y_e)_{ij}}{d(\log \mu)} = (\gamma_e y_e)_{ij} + (y_e \gamma_e)_{ij} + \gamma H_d (y_e)_{ij},
\]
(D.8)

\[
\frac{d(y_{\bar{u}})}{d(\log \mu)} = (\gamma_{\bar{u}} + \gamma \bar{Q} + \gamma H_u) y_{\bar{u}},
\]
(D.9)

\[
\frac{d(y_{\bar{d}})}{d(\log \mu)} = (\gamma_{\bar{d}} + \gamma \bar{Q} + \gamma H_d) y_{\bar{d}},
\]
(D.10)

\[
\frac{d(y_{\bar{e}})}{d(\log \mu)} = (\gamma_{\bar{e}} + \gamma \bar{L} + \gamma H_d) y_{\bar{e}},
\]
(D.11)

\[
\frac{d(y_Q)}{d(\log \mu)} = (y_Q y_Q)_{ij} + (y_Q \gamma_Q)_{ij},
\]
(D.12)

\[
\frac{d(y_{ui})}{d(\log \mu)} = (y_{ui} y_{ui})_{ij} + (y_{ui} \gamma_{ui})_{ij},
\]
(D.13)

\[
\frac{d(y_{di})}{d(\log \mu)} = (y_{di} y_{di})_{ij} + (y_{di} \gamma_{di})_{ij},
\]
(D.14)

\[
\frac{d(y_{Li})}{d(\log \mu)} = (y_{Li} y_{Li})_{ij} + (y_{Li} \gamma_{Li})_{ij},
\]
(D.15)

\[
\frac{d(y_{ei})}{d(\log \mu)} = (y_{ei} y_{ei})_{ij} + (y_{ei} \gamma_{ei})_{ij},
\]
(D.16)

\[
\frac{d(y)}{d(\log \mu)} = 3 \gamma \Phi y,
\]
(D.17)

\[
\frac{d(\mu H)}{d(\log \mu)} = (\gamma H_u + \gamma H_d) \mu H,
\]
(D.18)

\[
\frac{d(M)}{d(\log \mu)} = 2 \gamma \Phi M.
\]
(D.19)

The anomalous dimensions \(\gamma\) are

\[
(\gamma_Q)_{ij} = \frac{1}{16\pi^2} \left[ \left( y_{\bar{u}}^* y_{\bar{u}} + y_d^* y_d \right)_{ij} + y_{Q_i}^* y_{Q_j} - \left( \frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{30} g_1^2 \right) \delta_{ij} \right],
\]
(D.20)

\[
(\gamma_u)_{ij} = \frac{1}{16\pi^2} \left[ 2 \left( y_d y_u^* \right)_{ij} + y_{ui} y_{ui}^* - \left( \frac{8}{3} g_3^2 + \frac{8}{15} g_1^2 \right) \delta_{ij} \right],
\]
(D.21)

\[
(\gamma_d)_{ij} = \frac{1}{16\pi^2} \left[ 2 \left( y_d y_d^* \right)_{ij} + y_{di} y_{di}^* - \left( \frac{8}{3} g_3^2 + \frac{2}{15} g_1^2 \right) \delta_{ij} \right],
\]
(D.22)

\[
(\gamma_L)_{ij} = \frac{1}{16\pi^2} \left[ \left( y_{\bar{e}}^* y_{\bar{e}} \right)_{ij} + y_{L_i}^* y_{L_j} - \left( \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right) \delta_{ij} \right],
\]
(D.23)
\begin{align}
(\gamma_c)_{ij} &= \frac{1}{16\pi^2} \left[ 2 \left( y^i_c y^j_c \right)_{ij} + y^i_c y^j_c - \frac{6}{5} g_1^2 \delta_{ij} \right], \\
(\gamma_Q)_{ij} &= \frac{1}{16\pi^2} \left[ \sum_i y^i_Q y^j_Q + y^i_Q y^j_Q + y^i_Q y^j_Q - \left( \frac{8}{3} g_2^2 + \frac{2}{15} g_1^2 \right) \right], \\
(\gamma_u)_{ij} &= \frac{1}{16\pi^2} \left[ \sum_i y^i_u y^j_u + y^i_u y^j_u - \left( \frac{8}{3} g_2^2 + \frac{8}{15} g_1^2 \right) \right], \\
(\gamma_d)_{ij} &= \frac{1}{16\pi^2} \left[ \sum_i y^i_d y^j_d + y^i_d y^j_d - \left( \frac{8}{3} g_2^2 + \frac{2}{15} g_1^2 \right) \right], \\
(\gamma_L)_{ij} &= \frac{1}{16\pi^2} \left[ \sum_i y^i_L y^j_L + y^i_L y^j_L - \left( \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right) \right], \\
(\gamma_e)_{ij} &= \frac{1}{16\pi^2} \left[ \sum_i y^i_e y^j_e + y^i_e y^j_e - \frac{6}{5} g_1^2 \right], \\
(\gamma_H_u) &= \frac{1}{16\pi^2} \left[ 3 \text{Tr} \left( y^i_u y^{i*}_u \right) + 3 y^i_u y^j_u + y^i_u y^j_u - \left( \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right) \right], \\
(\gamma_H_d) &= \frac{1}{16\pi^2} \left[ \text{Tr} \left( y^i_d y^{i*}_d \right) + 3 y^i_u y^j_d + y^i_u y^j_d - \left( \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 \right) \right], \\
(\gamma_H_e) &= \frac{1}{16\pi^2} \left[ \sum_i \left( 6y^i_u y^j_u + 3 y^i_u y^j_u + 3 y^i_d y^j_d + 2 y^i_L y^j_L + y^i_y y^j_y \right) + y^y y \right].
\end{align}

**D.3. A and B terms**

The RG equations of SUSY-breaking A and B terms are given by

\begin{align}
\frac{d(a_u)}{d(\log \mu)} &= (\gamma_u a_u)_{ij} + (a_u \gamma_Q)_{ij} + (\gamma_H_u)_{ij} + 2(\gamma_Q a_u)_{ij} + 2(\gamma_a a_u)_{ij} + 2(\gamma_a q_u)_{ij} + 2(\gamma_e a_u)_{ij}, \\
\frac{d(a_d)}{d(\log \mu)} &= (\gamma_d a_d)_{ij} + (a_d \gamma_Q)_{ij} + (\gamma_H_d)_{ij} + 2(\gamma_Q a_d)_{ij} + 2(\gamma_d q_d)_{ij} + 2(\gamma_e a_d)_{ij}, \\
\frac{d(a_e)}{d(\log \mu)} &= (\gamma_e a_e)_{ij} + (a_e \gamma_Q)_{ij} + (\gamma_H_e)_{ij} + 2(\gamma_Q a_e)_{ij} + 2(\gamma_d q_d)_{ij} + 2(\gamma_e q_e)_{ij} + 2(\gamma_e q_e)_{ij}, \\
\frac{d(a_{\tilde{a}})}{d(\log \mu)} &= (\gamma_{\tilde{a}} a_u + \gamma_{\tilde{Q}} a_u)_{ij} + 2(\gamma_{\tilde{a}} q_u)_{ij} + 2(\gamma_{\tilde{a}} q_u)_{ij}, \\
\frac{d(a_{\tilde{d}})}{d(\log \mu)} &= (\gamma_{\tilde{d}} a_u + \gamma_{\tilde{Q}} a_u)_{ij} + 2(\gamma_{\tilde{d}} q_d)_{ij} + 2(\gamma_{\tilde{d}} q_d)_{ij}, \\
\frac{d(a_{\tilde{e}})}{d(\log \mu)} &= (\gamma_{\tilde{e}} a_u + \gamma_{\tilde{Q}} a_u)_{ij} + 2(\gamma_{\tilde{e}} q_e)_{ij} + 2(\gamma_{\tilde{e}} q_e)_{ij} + 2(\gamma_{\tilde{e}} q_e)_{ij}, \\
\frac{d(A_Q)}{d(\log \mu)} &= (A_Q \gamma_Q)_{ij} + (\gamma_Q a_u)_{ij} + 2(\gamma_Q a_o)_{ij} + 2(\gamma_{\tilde{Q}} q_o)_{ij} + 2(\gamma_{\tilde{Q}} q_o)_{ij}, \\
\frac{d(A_{w})}{d(\log \mu)} &= (A_w a_u)_{ij} + (\gamma_w a_u)_{ij} + 2(\gamma_w q_u)_{ij} + 2(\gamma_{\tilde{w}} q_u)_{ij}.
\end{align}
\[
\frac{dA_{d_{i}}}{d(\log \mu)} = (\gamma_{d}A_{Q})_{i} + (\gamma_{d} + \gamma_{\Phi})A_{d_{i}} + 2(\tilde{\gamma}_{d} y_{d})_{i} + 2(\tilde{\gamma}_{d} + \tilde{\gamma}_{\Phi})y_{d_{i}}, \tag{D.41}
\]

\[
\frac{dA_{L_{i}}}{d(\log \mu)} = (A_{Q}y_{L})_{i} + (\gamma_{L} + \gamma_{\Phi})A_{L_{i}} + 2(\tilde{\gamma}_{L} y_{L})_{i} + 2(\tilde{\gamma}_{L} + \tilde{\gamma}_{\Phi})y_{L_{i}}, \tag{D.42}
\]

\[
\frac{dA_{e_{i}}}{d(\log \mu)} = (\gamma_{e}A_{Q})_{i} + (\gamma_{e} + \gamma_{\Phi})A_{e_{i}} + 2(\tilde{\gamma}_{e} y_{e})_{i} + 2(\tilde{\gamma}_{e} + \tilde{\gamma}_{\Phi})y_{e_{i}}, \tag{D.43}
\]

\[
\frac{dA_{y}}{d(\log \mu)} = 3\gamma_{\Phi}y + 6\tilde{\gamma}_{\Phi}y, \tag{D.44}
\]

\[
\frac{db_{H}}{d(\log \mu)} = (\gamma_{H_{u}} + \gamma_{H_{u}})b_{H} + 2(\tilde{\gamma}_{H_{u}} + \tilde{\gamma}_{H_{u}})\mu_{H}, \tag{D.45}
\]

\[
\frac{db_{M}}{d(\log \mu)} = 2\gamma_{\Phi}b_{M} + 4\tilde{\gamma}_{\Phi}M, \tag{D.46}
\]

where the definitions of \(\tilde{\gamma}\) are

\[
(\tilde{\gamma}_{Q})_{ij} = \frac{1}{16\pi^{2}} \left[ \left( y_{u}^{\dagger}a_{u} + y_{d}^{\dagger}a_{d} \right)_{ij} + y_{Q}^{\dagger}A_{Q} \right] + \left( \frac{8}{3}g_{3}^{2}M_{3} + \frac{8}{15}g_{1}^{2}M_{1} \right) \delta_{ij}, \tag{D.47}
\]

\[
(\tilde{\gamma}_{u})_{ij} = \frac{1}{16\pi^{2}} \left[ 2 \left( a_{u}y_{u}^{\dagger} \right)_{ij} + A_{u}y_{u}^{\dagger} + \left( \frac{8}{3}g_{3}^{2}M_{3} + \frac{8}{15}g_{1}^{2}M_{1} \right) \delta_{ij} \right], \tag{D.48}
\]

\[
(\tilde{\gamma}_{d})_{ij} = \frac{1}{16\pi^{2}} \left[ 2 \left( a_{d}y_{d}^{\dagger} \right)_{ij} + A_{d}y_{d}^{\dagger} + \left( \frac{8}{3}g_{3}^{2}M_{3} + \frac{2}{15}g_{1}^{2}M_{1} \right) \delta_{ij} \right], \tag{D.49}
\]

\[
(\tilde{\gamma}_{L})_{ij} = \frac{1}{16\pi^{2}} \left[ y_{L}^{\dagger}A_{L} \right] + \left( \frac{3}{2}g_{2}^{2}M_{2} + \frac{3}{10}g_{1}^{2}M_{1} \right) \delta_{ij}, \tag{D.50}
\]

\[
(\tilde{\gamma}_{e})_{ij} = \frac{1}{16\pi^{2}} \left[ 2 \left( a_{e}y_{e}^{\dagger} \right)_{ij} + A_{e}y_{e}^{\dagger} + \frac{6}{5}g_{1}^{2}M_{1} \delta_{ij} \right], \tag{D.51}
\]

\[
\tilde{\gamma}_{Q} = \frac{1}{16\pi^{2}} \left( \sum_{i} y_{Q}^{\dagger}A_{Q} + y_{u}^{\dagger}a_{u} + y_{d}^{\dagger}a_{d} + \frac{8}{3}g_{3}^{2}M_{3} + \frac{3}{2}g_{2}^{2}M_{2} + \frac{1}{30}g_{1}^{2}M_{1} \right), \tag{D.52}
\]

\[
\tilde{\gamma}_{u} = \frac{1}{16\pi^{2}} \left( \sum_{i} y_{u}^{\dagger}A_{u} + 2y_{u}^{\dagger}a_{u} + \frac{8}{3}g_{3}^{2}M_{3} + \frac{8}{15}g_{1}^{2}M_{1} \right), \tag{D.53}
\]

\[
\tilde{\gamma}_{d} = \frac{1}{16\pi^{2}} \left( \sum_{i} y_{d}^{\dagger}A_{d} + 2y_{d}^{\dagger}a_{d} + \frac{8}{3}g_{3}^{2}M_{3} + \frac{2}{15}g_{1}^{2}M_{1} \right), \tag{D.54}
\]

\[
\tilde{\gamma}_{L} = \frac{1}{16\pi^{2}} \left( \sum_{i} y_{L}^{\dagger}A_{L} + y_{e}^{\dagger}a_{e} + \frac{3}{2}g_{2}^{2}M_{2} + \frac{3}{10}g_{1}^{2}M_{1} \right), \tag{D.55}
\]

\[
\tilde{\gamma}_{e} = \frac{1}{16\pi^{2}} \left( \sum_{i} y_{e}^{\dagger}A_{e} + 2y_{e}^{\dagger}a_{e} + \frac{6}{5}g_{1}^{2}M_{1} \right), \tag{D.56}
\]

\[
\tilde{\gamma}_{H_{u}} = \frac{1}{16\pi^{2}} \left[ 3 \text{Tr} \left( a_{u}y_{u}^{\dagger} \right) + 3y_{d}^{\dagger}a_{d} + y_{e}^{\dagger}a_{e} + \frac{3}{2}g_{2}^{2}M_{2} + \frac{3}{10}g_{1}^{2}M_{1} \right], \tag{D.57}
\]
\[
\tilde{\gamma}_{H_d} = \frac{1}{16\pi^2} \left[ \text{Tr} \left( 3a_d y_d^\dagger + a_e y_e^\dagger \right) + 3y_{ui}^* a_{ui} + \frac{3}{2} \epsilon_2^2 M_2 + \frac{3}{10} g_1^2 M_1 \right], \\
\tilde{\gamma}_\Phi = \frac{1}{16\pi^2} \left[ \sum_i \left( 6y_{Q_i}^* A_{Q_i} + 3y_{ui}^* A_{ui} + 3y_{di}^* A_{di} + 2y_{L_i}^* A_{Li} + y_{e_i}^* A_{e_i} \right) + y^* A_y \right].
\]

(D.58)  
(D.59)

D.4. Soft scalar masses

We define the following functions to write down the RG equations of soft scalar masses:

\[
f(x_1, x_2, x_3; y, z) = \frac{1}{16\pi^2} \left( x_1 y^\dagger + y y^\dagger x_1 + y x_2 y^\dagger + x_3 y y^\dagger + z z^\dagger \right),
\]

(D.60)

\[
g(a, b, c) = \frac{1}{16\pi^2} \left( \frac{32a}{3} g_2^2 |M_3|^2 + 6b g_2^2 |M_2|^2 + \frac{2c^2}{15} g_1^2 |M_1|^2 \right) - \frac{c}{80\pi^2} g_1^2 S,
\]

(D.61)

\[
S = \text{Tr} \left( m_Q^2 - 2m_u^2 + m_d^2 - m_L^2 + m_e^2 \right) + m_H_u^2 - m_H_d^2
\]

(D.62)

where \(x_1, x_2, x_3\) are generally soft scalar masses in generation space and \(y, z\) are Yukawa couplings and \(A\) parameters with generation indices. The RG equations of soft scalar masses are given by

\[
\frac{d m_Q^2}{d (\log \mu)} = \sum_{x=0,1,3} f(m_Q^2, x, m_{H_d}; y_{ui}^* a_{ui}^*), \quad f(m_Q^2, m_{\Phi}; y_{Q_i}; A_{Q_i}) - g(1, 1, 1),
\]

(D.63)

\[
\frac{d m_u^2}{d (\log \mu)} = 2f(m_u^2, m_{H_u}; y_{ui}^* a_{ui}^*), \quad f(m_u^2, m_{\Phi}; y_{ui}; A_{ui}) - g(1, 0, -4),
\]

(D.64)

\[
\frac{d m_d^2}{d (\log \mu)} = 2f(m_d^2, m_{H_d}; y_d^* a_d^*), \quad f(m_d^2, m_{\Phi}; y_d; A_d) - g(1, 0, 2),
\]

(D.65)

\[
\frac{d m_L^2}{d (\log \mu)} = f(m_L^2, m_{H_L}; y_{e_i}^* a_{e_i}^*), \quad f(m_L^2, m_{\Phi}; y_{e_i}; A_{e_i}) - g(0, 1, -3),
\]

(D.66)

\[
\frac{d m_e^2}{d (\log \mu)} = 2f(m_e^2, m_{H_e}; y_e^* a_e^*), \quad f(m_e^2, m_{\Phi}; y_e; A_e) - g(0, 0, 6),
\]

(D.67)

\[
\frac{d m_Q^2}{d (\log \mu)} = f(m_Q^2, m_{H_d}; y_{ui}^* a_{ui}^*), \quad f(m_Q^2, m_{\Phi}; y_{ui}; A_{ui}) - g(1, 1, -1),
\]

(D.68)

\[
\frac{d m_u^2}{d (\log \mu)} = 2f(m_u^2, m_{H_u}; y_{ui}^* a_{ui}^*), \quad f(m_u^2, m_{\Phi}; y_{ui}; A_{ui}) - g(1, 0, 4),
\]

(D.69)

\[
\frac{d m_d^2}{d (\log \mu)} = 2f(m_d^2, m_{H_d}; y_d^* a_d^*), \quad f(m_d^2, m_{\Phi}; y_d; A_d) - g(1, 0, -2),
\]

(D.70)

\[
\frac{d m_L^2}{d (\log \mu)} = f(m_L^2, m_{H_L}; y_{e_i}^* a_{e_i}^*), \quad f(m_L^2, m_{\Phi}; y_{e_i}; A_{e_i}) - g(0, 1, 3),
\]

(D.71)

\[
\frac{d m_e^2}{d (\log \mu)} = 2f(m_e^2, m_{H_e}; y_e^* a_e^*), \quad f(m_e^2, m_{\Phi}; y_e; A_e) - g(0, 0, -6),
\]

(D.72)
\[
\frac{dm_{H_u}^2}{d(\log \mu)} = \text{Tr} \left[ 3f(m_Q^2, m_{u}, m_{H_u}^2; y_h^u/a_h^u) + 3f(m_Q^2, m_{d}^2, m_{H_u}^2; y_h^d/a_h^d) \right] + f(m_Q^2, m_{d}^2, m_{H_u}^2; y_h^d/a_h^d) - g(0, 1, 3), \tag{D.73}
\]

\[
\frac{dm_{H_d}^2}{d(\log \mu)} = \text{Tr} \left[ 3f(m_Q^2, m_{d}, m_{H_d}^2; y_h^d/a_h^d) + f(m_Q^2, m_{e}^2, m_{H_d}^2; y_h^e/a_h^e) \right] + 3f(m_Q^2, m_{e}^2, m_{H_d}^2; y_h^e/a_h^e) - g(0, 1, -3), \tag{D.74}
\]

\[
\frac{dm_{\phi}^2}{d(\log \mu)} = 12f(m_Q^2, m_{\phi}, y_{\phi}; A_Q) + 6f(m_{u}^2, m_{\phi}, y_{u}; A_u) + 6f(m_{d}^2, m_{\phi}, y_{d}; A_d) + 4f(m_{L}^2, m_{\phi}, y_{L}; A_L) + 2f(m_{e}^2, m_{\phi}, y_{e}; A_e) + f(m_{d}^2, m_{\phi}, m_{\phi}, y; A_\phi). \tag{D.75}
\]

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