Hybrid Adaptive Fault-Tolerant Control for Compound Faults of Hypersonic Vehicle

KAI-YU HU,1,2 WENHAO LI,2 AND ZIAN CHENG1

1College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China
2School of Engineering Science, University of Chinese Academy of Sciences, Beijing 100101, China

Corresponding author: Kai-Yu Hu (hkywuyue@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61533009, and in part by the National Basic Research Program of China (973 Program) under Grant 2015CB857100.

ABSTRACT

This study investigates the hybrid fault-tolerant control (FTC) for hypersonic flight vehicles (HFVs) under the rudder structure fault (RSF) and rudder angle deviation fault (RADF). The nonlinear equations of the model are linearised by fuzzy logic, the external nonlinear disturbance is approximated by radial basis functions, which transforms the classical HFV into a linear system. In the case of only a RSF, the state feedback system can passively shield the fault using a compensation function and design a threshold to create conditions for RADF detection. During the RSF and RADF compound faults, the system with an adaptive observer implements angle deviation fault isolation/estimation and shields the structural fault. By using the estimation results, active-passive hybrid compensation completes passive FTC of the structural fault and active FTC of the angle deviation fault. The adaptive learning rates that mimic animal predatory behaviour increase the sensitivity to incipient deviation of RADF and improve compensation. Lyapunov method proves the stability and semi-physical simulation shows the control efficiency.

INDEX TERMS

Lyapunov method, fuzzy logic, fault detection, fault-tolerant control, nonlinear equations.

I. INTRODUCTION

Hypersonic flight vehicles (HFVs) are important in military applications, which indicate broad prospects for their development [1], [2]. Unlike traditional aircrafts, HFVs possess the following characteristics: multiple working patterns, multivariate coupling, and multi-source nonlinearities. As their status changes rapidly, they are prone to failure and signal lag [3]–[5]. This study aims to improve the fault self-healing capabilities of HFVs. To this end, we investigate multi-source nonlinear control, compound fault self-repairing, and adaptive control for time-varying step faults.

Nonlinear problems have multiple control schemes. In [6], the range of a nonlinear input function was set between two linear function values, converted the nonlinear stability proof into a linear proof. In [7], the Takagi-Sugeno (T-S) fuzzy system is a strong linearisation method; by “blending” local linear system models, it has the advantage of universal approximation for any nonlinear function [8]–[10]. The simplification of HFV models with strong. Nonlinearity via T-S fuzzy theory has been proven feasible many times in previous studies [11], [12]. Given the advantages of the

T-S fuzzy theory, it is used to describe the HFV attitude model of this study. HFV dynamics usually have multiple nonlinear sources. Multi-source nonlinear systems should adopt different solutions for different nonlinear functions. Additio-nal to fuzzy theory, in [13], a series of radial basis functions (RBFs) were used to generate a nonlinear output of stochastic systems. This article applies two approximation methods with different physical variables for two types of nonlinear functions.

Fault-tolerant control (FTC) allows performing online self-repair. In [14], an adaptive fault-tolerant boundary control scheme was proposed for a flexible aircraft described by the distributed parameter model. Based on iterative learning methods, in [15], a high-order sliding mode controller was proposed to stabilise the satellite attitudes under unknown disturbances and actuator faults. In harsh environments, systems have complicated compound faults. The corresponding fault diagnosis and FTC have progressed in recent years. In [16], the problem of diagnosing compound faults was solved by using a coupled factorial Markov model-based framework. In [17], a frequency blind deconvolution algorithm was designed using an adaptive generalised morphological filter for extracting useful signals from the signals contaminated by compound faults. In [18], a nonlinear
FTC and multiple sensor diagnosis was designed for the longitudinal dynamics of hypersonic vehicles. In [19], a composite loop for FTC under compound faults was developed by integrating a newly multi-variable integral sliding-mode control. In [20], a fast kurtosis method was used combined with variational mode decomposition to improve the tracking accuracy for compound faults.

The fault-tolerant design approach can be mainly classified into passive and active types [21], [22]. And active-passive hybrid compensation is a novel attempt to deal with compound faults. In [23], a kind of hybrid compensation scheme was achieved in stochastic systems: passive shielding of sensor fault and active FTC of actuator fault for the first time. In the passive approach, the same controller is used throughout the normal and fault cases; therefore, this passive fault-tolerant controller can be implemented [24], [25]. Passive FTC is suitable for systems with simple fault expressions and small amplitudes, thereby simplifying the HFV control algorithm and improving the response speed [26], [27]. In the active approach, the observer is first designed to estimate faults, and then systems use estimated values to reconstruct the control algorithms for tracking the ideal output [28]. Active FTC can compensate for complex and large amplitude faults and improve the environmental adaptability of HFVs [29]. The parameter adjustment law enhances the system’s sensitivity to incipient faults [30], [31]. Therefore, the hybrid active-passive FTC adopts the advantages of the two approaches and eliminates the corresponding defects to optimise the self-healing performance for compound faults.

The main contributions of this study are as follows:

1) A multi-approximation method with fuzzy logic and RBF is used to achieve linear simplification and tracking control of HFV in the multi-source nonlinear environment.

2) A hybrid active-passive FTC method is proposed to solve compound faults with rudder structural fault (RSF) and rudder angle deviation fault (RADF) to estimate and repair RADF online while shielding RSF.

3) A variable parameter algorithm that mimics the animal predation process is designed to ensure the sensitivity of the controller to the incipient deviations of RADF.

Section II introduces the HFV models, fuzzy and RBF approximations for multi-source nonlinear functions, and compound faults. Section III.A proposes the FTC scheme when only the structural fault occurs. Section III.B isolates the RADF. Section III.C estimates the RADF. In Section III.D, it is shown the hybrid FTC with compound faults. Section III.E explains the prey algorithm. Section IV verifies the effectiveness of the control scheme via simulation.

II. MODELS AND PROBLEMS

The reentry HFV attitude dynamics in the entry phase is expressed as follows [21]:

\[
\begin{align*}
\dot{\omega} &= J^{-1}\Omega_\omega J\omega + J^{-1}G_0u + H(\omega) \\
\ddot{\xi} &= \Xi_\xi \omega
\end{align*}
\]  

where \(\omega = [p_0, q_0, r_0]^T\) refers to the angular rate; \(J\) denotes the inertia; \(\xi = [\varphi_0, \beta_0, \alpha_0]^T\) represents the attitude angle; \(u = [\delta_e, \delta_a, \delta_\gamma]^T\) is the control surface deflection; \(p_0, q_0, r_0, \varphi_0, \beta_0, \) and \(\alpha_0\) refer to the pitch rate, roll rate, yaw rate, bank angle, sideslip angle, and attack angle, respectively; \(\delta_e, \delta_a,\) and \(\delta_\gamma\) denote the elevator, aileron, and rudder deflections, respectively; \(H(\omega)\) is the bounded disturbance vector; and

\[
G_0 = \begin{bmatrix}
g_{p,\delta_e} & g_{p,\delta_a} & g_{p,\delta_\gamma} \\
g_{q,\delta_e} & g_{q,\delta_a} & g_{q,\delta_\gamma} \\
g_{r,\delta_e} & g_{r,\delta_a} & g_{r,\delta_\gamma}
\end{bmatrix}, \quad \Omega(\omega) = \begin{bmatrix} 0 & r_0 & -q_0 \\ -r_0 & 0 & p_0 \\ q_0 & -p_0 & 0 \end{bmatrix},
\]

\[
\Xi(\xi) = \begin{bmatrix} \cos \alpha_0 & 0 & \sin \alpha_0 \\ \sin \alpha_0 & \cos \alpha_0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.
\]

\(G\) is the control allocation matrix from the control instruction to the control rudder surface.

In the augmented scheme, the variables of HFV (1) is defined as: \(x_1 = p_0, x_2 = q_0, x_3 = r_0, x_4 = \varphi_0, x_5 = \beta_0,\) and \(x_6 = \alpha_0\). This study selects roll rate \(q_0\), attack angle \(\alpha_0\), and sideslip angle \(\beta_0\) as the outputs of HFV dynamics. Then HFV described by (1) can be rewritten as a general augmented multiple input multiple output nonlinear system:

\[
\begin{align*}
\dot{x}(t) &= f(x, t) + \sum_{k=1}^{m} g_k(x)u_k(t) + H(\omega, t) \\
y(t) &= Cx(t)
\end{align*}
\]  

Remark 1: Intrinsic nonlinearity is caused by the characteristics of the system itself. External nonlinearity is caused by environmental factors. It is difficult to solve two types of nonlinear problems uniformly. So in this paper, fuzzy theory approximates intrinsic nonlinear functions, RBF approximates external nonlinear disturbance function. The premise variables of the two methods are different.

The T-S fuzzy theory approximates the intrinsic nonlinear models with linear modalities and solves nonlinear problems using a linear method. System (2) is represented by blending the following augmented T-S fuzzy models:

\[
\begin{align*}
\dot{x}(t) &= \sum_{j=1}^{l} h_j(z)[A_jx(t) + (B_j + F_B)u(t)] + H(x) \\
y(t) &= \sum_{j=1}^{l} h_j(z)C_jx(t)
\end{align*}
\]  

where \(A_j, B_j,\) and \(C_j\) are constant matrices with appropriate dimensions. \(l\) is the fuzzy rule number, and

\[
\begin{align*}
\mu_j(z) &= \prod_{i=1}^{k} M_{j_i}(z) \\
0 < h_j(z) &= \mu_j(z) / \sum_{j=1}^{l} \mu_j(z) \\
\sum_{j=1}^{l} h_j(z) &= 1
\end{align*}
\]  

where fuzzy rules can be expressed as if \(z_1(t)\) is \(M_{j_1}, z_2(t)\) is \(M_{j_2}\) and \(\ldots \) \(z_k(t)\) is \(M_{j_k}\), then, \(z(t) = [z_1(t), \ldots, z_k(t)]\) are the premise variables, \(M_{j_1}, \ldots, M_{j_k}\) are the fuzzy sets. \(x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n, u(t) = [u_1(t), \ldots, u_m(t)]^T \in \mathbb{R}^m\) and \(y(t) = [y_1(t), \ldots, y_p(t)]^T \in \mathbb{R}^p\) denote system measurable state vector, control input and system output, respectively;
The highest priority in its equation, which satisfies caused by the breakage and deformation of the rudder wings. is an smooth nonlinear disturbance function. $H(x) = [h_1(x), \ldots, h_n(x)]^T \in \mathbb{R}^n$, $h_i(x) \in \mathbb{R}$ ($i = 1, \ldots, n$) is an smooth nonlinear disturbance function. $F_B$ is the RSF caused by the breakage and deformation of the rudder wings.

**Remark 2:** Fuzzy superposition operator $\sum_{j=1}^{l} h_j(z)(\bullet)$ has the highest priority in its equation, which satisfies:

$$
\begin{align*}
\sum_{j=1}^{l} h_j(z)(O_j \ast X_1) &= \left( \sum_{j=1}^{l} h_j(z)O_j \right) \ast \left( \sum_{j=1}^{l} h_j(z)X_1 \right) \\
\sum_{j=1}^{l} h_j(z)(O_j \ast X_2) &= \left( \sum_{j=1}^{l} h_j(z)O_j \right) \ast X_2 \\
\sum_{j=1}^{l} h_j(z)(X_1 \ast X_2) &= \left( \sum_{j=1}^{l} h_j(z)X_1 \right) \ast X_2
\end{align*}
$$

where $O_j$ is a linear variable participating in the fuzzy superposition, $X_1$ is a compound variable containing both participating and not participating in the fuzzy superposition variables, $X_2$ is a variable not participating in fuzzy superposition, $\ast$ is an arbitrary operator.

The rudder wings make a deflection movement to control HFV attitude, but the actual deflection angles will not match the command due to the intermittent unstable airflow, that is, RADF, the model is described as:

$$
\begin{align*}
\dot{u}(t) &= u_i(t) + f_{i, tvs}(t), \quad t > t_F \\
\dot{f}(t) &= u_i(t) + f_{i, tvs}(t)
\end{align*}
$$

where $i = 1, \ldots, m, f_{i, tvs}(t)$ is the unknown fault signal, $f_{i, tvs}(t) = [f_{1, tvs}(t), \ldots, f_{m, tvs}(t)]^T \in \mathbb{R}^m$, $t_F$ is an unknown fault occurrence time. Other research on incipient faults can guide the definition of RADF in this paper [31]. The time step fault of rudder deflection is set as follows:

$$
\begin{align*}
f_{i, tvs}(t) &= \begin{cases} 
\tilde{f}_{i}(i) \left( \tilde{t}_1, \tilde{t}_2, \tilde{t}_3 \right), & t \in (\tilde{t}_1, \tilde{t}_2, \tilde{t}_3) \\
0, & \text{otherwise}
\end{cases}
\end{align*}
$$

$$
\begin{align*}
f_{i, \sigma}(t) &= \begin{cases} 
\tilde{f}_{\sigma}(i) \left( \tilde{t}_1, \tilde{t}_2, \tilde{t}_3 \right), & t \in (\tilde{t}_1, \tilde{t}_2, \tilde{t}_3) \\
0, & \text{otherwise}
\end{cases}
\end{align*}
$$

$$
\begin{align*}
\left[ f_{\sigma, inc}(t) / u(t) \right] &\leq 10\% \\
\left[ f_{\sigma, non-inc}(t) / u(t) \right] &> 10\%
\end{align*}
$$

where $\tilde{f}_{i, tvs}(t)$ is the intermittent fault, $\tilde{f}_{i, \sigma}(t)$ is the value in the fault interval, $f_{\sigma, inc}(t)$ is the fault in the incipient amplitude interval, $f_{\sigma, non-inc}(t)$ is the fault in the residual amplitude interval, $\sigma(i) = 1, \ldots, \sigma_0$ is the number of fault windows. The general expression of RSF in augmented model (3) is:

$$
F_B = \begin{bmatrix} \tilde{f}_{rate} & \tilde{f}_{ra} & \tilde{f}_{angle} \end{bmatrix}
$$

where $\tilde{f}_{rate} \in \mathbb{R}^{d \times 1}$ is the angular rate allocation structural fault, $\tilde{f}_{angle} \in \mathbb{R}^{(n-d) \times (m-v)}$ is the angle allocation structural fault, $\tilde{f}_{ra}$ and $f_{ar}$ are the coupling fault space, $v$ is the angular rate control dimension, $d$ is the angle control dimension.

The task of this study is (i) to propose a hybrid compensation to enable the system to stably track the reference model outputs under the rudder compound faults (RSF and RADF); (ii) to design a variable parameter scheme to optimise the estimation and FTC curves for incipient and large value RADF; (iii) to satisfy the requirements of HFV model nonlinearity and external disturbance nonlinearity. The control structure is Figure 1.

![FIGURE 1. Structure diagram of hybrid compensation in HFV.](image)

The feedback is designed by the reference model and the controlled object HFV model. The reference model is

$$
\begin{align*}
\dot{x}_m(t) &= \sum_{j=1}^{l} h_j(z)(A_{jm}x_m(t) + (B_j + F_B^0)r(t)) \\
y_m(t) &= \sum_{j=1}^{l} h_j(z)c_jx_m(t)
\end{align*}
$$

where $x_m = [x_{m1}(t), \ldots, x_{mm}(t)]^T \in \mathbb{R}^n$, $r(t) \in \mathbb{R}^n$ are the state vector, input of the reference model and output of the reference model, respectively; $A_{jm}, B_j$ and $C_j$ are the known matrices with appropriate dimensions. $F_B^0$ is the fitted value of RSF.

The feedback residual contains fault information, when the RSF and RSF-RASF compound faults occur in sequence, the residual norm is changed. Assumption 3 and the detection threshold are presented in Section 2. Therefore, the fitting of a structural fault is expressed as:

$$
\zeta(F_1^T\tilde{y}(t))F_2^n = F_B^n
$$

where $\tilde{y}(t) = y(t) - y_m(t), F_1^T \in \mathbb{R}^{m \times q}$ and $F_2^n \in \mathbb{R}^{m \times m}$ are the interpolation fitting matrices based on extended RBFs:

$$
\begin{align*}
F_1^T &= \begin{bmatrix} B_{11}(t, y(t)) & \cdots & B_{1q}(t, y(t)) \\
\vdots & \ddots & \vdots \\
B_{n1}(t, y(t)) & \cdots & B_{nq}(t, y(t)) \end{bmatrix} \\
\zeta(F_1^T\tilde{y}(t)) &= [F_1^T\tilde{y}(t), \ldots, F_1^T\tilde{y}(t)]_{1 \times m}
\end{align*}
$$

$$
\begin{align*}
F_2^n &= \begin{bmatrix} b_{11}(t, y(t)) & \cdots & b_{1m}(t, y(t)) \\
\vdots & \ddots & \vdots \\
b_{n1}(t, y(t)) & \cdots & b_{nm}(t, y(t)) \end{bmatrix} \\
\left[ B_{n0}(t, y(t)) \\
b_{\psi, \phi}(t, y(t)) \right] &= \exp[-\sum_{u=1}^{q} (y_u(t) - c_{n0})^2 / 2\sigma_{n0}^2]
\end{align*}
$$

and $n_0 = 1, \ldots, n, \nu = 1, \ldots, q, \psi^0 = 1, \ldots, m$, and $\sigma^0 = 1, \ldots, m, \psi$ and $\sigma$ are the centre and width of each basis function, $X$ is the arbitrary integer and its combination. The external nonlinear disturbance function $h_i(x(t))$ is approximated by the RBF neural network as

$$
h_i(x(t), \theta_i) = \theta_i^T \tilde{x}_i(x(t))
$$
where \( \theta^* \) denotes the optimal parameter vector and \( \xi_i(x(t)) \) is the basis function like reference \[27\] and satisfies:
\[
\xi_i(t, x(t)) = [\xi_i(x(t)), \ldots, \xi_{im}(x(t))]^T
\]
(18)
\[
\xi_i(t, x(t)) = \exp[-\sum_{i=1}^m (x_i(t) - c_{i1})^2 / 2\sigma_i^2]
\]
(19)
where \( i = 1, \ldots, w \). The optimal approximation error is:
\[
e_i = h_i(x(t)) - \theta_i^T \xi_i(x(t))
\]
(20)

**Remark 3:** In HFV, the intrinsic nonlinear approximation error is avoided by trial and error. The structural fault expression is simple, and extended RBF has more adjustable items, so the approximation error is ignored. External nonlinear disturbance function is complex and fast-changing, therefore its approximation error is not negligible.

**Assumption 1:** According to the definition of actuator incipient fault, set
\[
|\tilde{f}_{ia, inc}(t)| < 0.1 |\tilde{u}_i(t)|_{min} < |\tilde{f}_{ia, non-inc}(t)|,
\]
where \( |\tilde{u}_i(t)|_{min} \) is the minimum absolute value of \( i \)th control input. All faults are bounded, in particular the actuator fault satisfies \( |\tilde{f}_{ia, inc}(t)| \leq f_1, | df_{ia, inc}(t)/dt | \leq f_2 \), where \( f_1 > 0 \) and \( f_2 > 0 \) are known real constants.

**Assumption 2:** \( e_i \) and \( \theta_i^* \) are bounded, i.e., \( |e_i| \leq e_i^* \), \( e_i^* \leq M_{e_i} \), and \( |\theta_i^*| \leq M_{\theta_i} \), where \( e_i^* > 0 \) is an unknown real constant, \( M_{e_i} > 0 \) and \( M_{\theta_i} > 0 \) are known real constants.

According to the setting of multiple approximations and errors in Remark 3, subtracting the reference model from the operating conditions of HFV, the RSF is considered as in sequence that can increase the residual norm. Depending on the operating conditions of HFV, the RSF is considered as a necessary inadequate condition for RADF.

**Assumption 3:** If \( t < t_0 \), there is no fault in the rudders. The residual contains fault information, and two faults appear in sequence that can increase the residual norm. Depending on the operating conditions of HFV, the RSF is considered as a necessary inadequate condition for RADF.

**Remark 4:** \( \text{prey(●)} \) is a predator algorithm. If there is no RADF, this algorithm does not work and the related item degenerates to ●. For the purpose of unifying symbols, we express as \( \text{prey(●)} \) under all conditions, and the specific structure is uniformly designed in Section 3.5.

**Theorem 1:** If there exist real matrices \( K_{j, comp} \), \( P_j = P_j^T > 0, F \) and \( Q > 0 \) with appropriate dimensions such that condition (22) holds:
\[
P_j[A_{jm} + (B_j + F_B)K_{j, comp}] + [A_{jm} + (B_j + F_B)K_{j, comp}]^T P_j \leq -Q
\]
(22)
The adaptive control laws (23), (24) and (25) are applied,
\[
\dot{\theta}_i = 2\pi_p |e_i(x)| - \text{prey}(\eta_0)\dot{\theta}_i
\]
(23)
\[
\hat{e}_i = 2\pi_p \text{sgn}(\tilde{e}_i) - \text{prey}(\eta_0)\hat{e}_i
\]
(24)
\[
u = \sum_{j=1}^l h_j(\tilde{z})(K_{j, comp}\tilde{x} + r - (B_j + F_B)^+(A_j - A_{jm})x + \tilde{H} + \text{sgn}(\tilde{x}^TP_j\hat{e}))
\]
(25)

then (21) is stable, all signals in the closed-loop system are asymptotically bounded to a neighbourhood of the origin \( \Omega \) defined as
\[
\Omega = \{(\tilde{x}, \tilde{\theta}_i, \tilde{\hat{e}_i}) | \| \tilde{x} \| \leq \sqrt{\alpha / \lambda_{min}(P)}, \| \tilde{\theta}_i \| \leq 2\sqrt{\alpha} |\tilde{e}_i|, i = 1, \ldots, n \}
\]
(26)
where \( \tilde{H} = [\tilde{h}_1, \ldots, \tilde{h}_n]^T, \tilde{\hat{e}} = [\tilde{\hat{e}}_1, \ldots, \tilde{\hat{e}}_n]^T \) and
\[
\hat{h}_i = \hat{\theta}_i^T \xi_i(x)
\]
(27)
\( \hat{\theta}_i \) and \( \tilde{\hat{e}}_i \) are the estimate values of \( \theta_i^* \) and \( e_i^* \), symbol function satisfies
\[
\text{sgn}(\tilde{x}^T \sum_{j=1}^l h_j(\tilde{z}P_j) = \text{diag}(\text{sgn}(\tilde{x}_{p1}), \ldots, \text{sgn}(\tilde{x}_{pn}))
\]
(28)
where \( \tilde{x}_{pi} \) is the \( i \)th element of the vector \( \tilde{x}^TP_j, \text{prey}(\eta_0) > 0 \in R \) and \( \text{prey}(\eta_0) > 0 \in R \) are design parameters, \( (B_j + F_B)^+ \) is the generalised inverse matrix of \( B_j + F_B \).

**Proof:** Define the Lyapunov function as
\[
V = \sum_{j=1}^l h_j(\tilde{z})^T \tilde{P}_j \tilde{x} + \sum_{i=1}^n (\tilde{\theta}_i^T \tilde{\theta}_i + \tilde{\hat{e}}_i)
\]
(29)
\( \tilde{\hat{e}} \) differentiating \( V \) and considering (25), one has
\[
\dot{V} = \sum_{j=1}^l h_j(\tilde{z})^T \{ P_j[A_{jm} + (B_j + F_B)K_{j, comp}] + [A_{jm} + (B_j + F_B)K_{j, comp}]^T P_j \}
\]
\[
- \sum_{i=1}^n \text{prey}(\eta_0)\tilde{\hat{e}}_i^T \tilde{\hat{e}}_i + \text{prey}(\eta_0)\tilde{\hat{e}}_i
\]
(30)
\[
\sum_{j=1}^l h_j(\tilde{z})^T \{ 2\tilde{x}^T \tilde{P}_j \tilde{H} - 2\tilde{x}^T \tilde{P}_j (\tilde{H} + \text{sgn}(\tilde{x}^T \tilde{P}_j \hat{e})) \}
\]
\[
\leq 2 \sum_{i=1}^n \tilde{x}_{pi} \text{sgn}(\tilde{x}_{pi}) \text{sgn}(\tilde{x}_{pi}) \text{sgn}(\tilde{x}_{pi})
\]
the following inequality can be obtained:
\[
\dot{V} \leq \sum_{i=1}^l h_j(\tilde{z})^T \{ [A_{jm} + (B_j + F_B)K_{j, comp}] + [A_{jm} + (B_j + F_B)K_{j, comp}]^T \}
\]
(31)
\[ V(t) = -\ddot{\xi}^T Q \ddot{\xi} + \sum_{i=1}^{n} (\text{prey}(\eta_0) \dot{\theta}_i + \text{prey}(\eta_0) \dot{\xi}_i). \]

From (22), (23) and (24), one further has
\[ \dot{V} \leq -\ddot{\xi}^T Q \ddot{\xi} + \sum_{i=1}^{n} (\text{prey}(\eta_0) \dot{\theta}_i + \text{prey}(\eta_0) \dot{\xi}_i) - \sum_{i=1}^{n} (\ddot{\theta}_i^T \ddot{\theta}_i + \ddot{\xi}_i). \]

Since
\[ \sum_{i=1}^{n} \ddot{\theta}_i \ddot{\theta}_i \leq \sum_{i=1}^{n} (-\ddot{\theta}_i^T \ddot{\theta}_i/2 + \ddot{\theta}_i^T \ddot{\theta}_i^*)/2 \]
\[ \sum_{i=1}^{n} \ddot{\xi}_i \ddot{\xi}_i \leq \sum_{i=1}^{n} (-\ddot{\xi}_i^T \ddot{\xi}_i/2 + \ddot{\xi}_i^T \ddot{\xi}_i^*)/2. \]

From Assumption 2, one has
\[ \dot{V} \leq -\ddot{\xi}^T Q \ddot{\xi} - \sum_{i=1}^{n} (\text{prey}(\eta_0) \dot{\theta}_i + \text{prey}(\eta_0) \dot{\xi}_i^2)/2 \]
\[ + \sum_{i=1}^{n} (\text{prey}(\eta_0) \dot{\theta}_i^T \dot{\theta}_i^* + \text{prey}(\eta_0) \dot{\xi}_i^2)/2 \]
\[ \leq -\ddot{\xi}^T Q \ddot{\xi} + \sum_{i=1}^{n} (\text{prey}(\eta_0) \dot{\theta}_i^T \dot{\theta}_i + \text{prey}(\eta_0) \dot{\xi}_i^2)/2 \]
\[ + \mu = -\lambda V + \mu, \]

where
\[ \mu = \sum_{i=1}^{n} (\text{prey}(\eta_0) M_{\theta i}^2 + \text{prey}(\eta_0) M_{\xi i}^2)/2 \]
\[ \lambda = \left\{ \frac{\lambda_{\min}(Q)}{2}, \frac{\lambda_{\max}(P)}{2}, \frac{\text{prey}(\eta_0)}{2} \right\}. \]

Since \( d(V(t)e^{\lambda t})/dt \leq e^{\lambda t} \mu, \) one has
\[ 0 \leq V(t) \leq \mu/\lambda + (V(0) - \mu/\lambda)e^{-\lambda t} \leq \mu/\lambda + V(0) \]

Let \( \alpha = \mu/\lambda + V(0), \) one has
\[ \left\{ \begin{array}{l} \|\ddot{\xi}\| \leq \sqrt{\alpha/\lambda_{\min}(P)} \\ \|\dot{\theta}_i\|, \|\ddot{\xi}\| \leq \sqrt{2\alpha} \end{array} \right. \]

which implies that all signals in the closed-loop system are asymptotically bounded to the compact set \( \Omega. \) So passive FTC can stabilise the controlled object under the condition of only a structural fault (RSF).

RAFD detection is designed on this basis. From (33), it is easy to find that \( V(t) \) has the following property: It decreases while \( t \) increases. Since
\[ \lambda_{\min}(P_j) \ddot{\xi}^T \ddot{\xi} + \ddot{\xi}^T P \ddot{\xi} \leq V(t) \]

one further has
\[ \|\ddot{\xi}\| \leq \sqrt{(\mu/\lambda + [V(0) - \mu/\lambda]e^{-\lambda t})/\lambda_{\min}(P_j)}. \]

The residual is now chosen as
\[ \left\{ \begin{array}{l} J_x = \|\ddot{\xi}\| = \|x - x_{m}\| \\ J_y = \|\ddot{\eta}\| = \|y - y_{m}\| = \|C\ddot{\xi}\| \end{array} \right. \]

Select a design parameter \( \delta > 0 \in R, \) and set
\[ \sqrt{\mu/\lambda + [V(0) - \mu/\lambda]e^{-\lambda t}} = \delta \]

then one has
\[ t_0 = \frac{-\ln \left( \lambda_{\min}(P_j)\delta - \mu/\lambda \right)}{[V(0) - \mu/\lambda]}/\lambda. \]

Therefore we set the output residual threshold value reflecting the system fault is \( \tau, \) and the state error threshold reflecting the RADF is \( \delta. \) \( K_{j,\text{comp}} \) is the passive FTC function for RSF satisfies:
\[ K_{j,\text{comp}} = \text{prey}(K_j) + \Gamma_1(\tilde{\eta} - \Gamma_3 \ddot{\xi}) \Gamma_2 \]

where \( \Gamma_1, \Gamma_2, \Gamma_3, \) and \( \Gamma_4 \) are the information response parameters with matching dimensions. According to Rema-rk 4, when compound faults occur, RADF is detected using \( \delta \) and \( \Gamma_3 \dot{\xi} \Gamma_4 \) cannot be ignored, \( K_{j,\text{comp}} \) uses this function to internally remove the RADF effects, allowing the passive FTC more focus on compensation for RSF: when only RSF occurs, RADF is not detected and the state error norm is small, \( \Gamma_3 \dot{\xi} \Gamma_4 \) can be ignored, the expression degenerates to:
\[ K_{j,\text{comp}} \approx \text{prey}(K_j) + \Gamma_1 \dot{\xi} \Gamma_2 \]

where \( \Gamma_1 \dot{\xi} \Gamma_2 \) is the passive compensation function. According to Assumption 3, this function can use the residual with fault information to shield RSF, ensuring the stable control of the controlled objects.

In summary, RADF detection can be performed through the following scheme: when \( t > t_0, \) if \( J_x > \delta, \) if \( J_x > \tau, \) RSF-RADF compound faults are detected; if \( J_x < \delta, \) if \( J_x > \tau, \) no RADF; if \( J_x < \delta, J_x < \tau, \) no fault.

### B. RADF Isolation

One rudder only has one single deviation failure at one time. When the \( s \text{th} \) \( (1 \leq s \leq m) \) rudder has a deviation fault, the faulty model will be described as
\[ \begin{align*}
\dot{x}_s &= \sum_{j=1}^{m} h_j(z)[A_j x_s + (B_j + F_B) u] \\
&\quad + \theta_{1j} F_B + \theta_{1j} \mu x_j + H + \text{sgn}(s) \mu_j x_j \\
y_s &= \sum_{j=1}^{m} h_j(z) C_j x_s
\end{align*} \]

where \( B_j = \{b_{j1}, \ldots, b_{jm}\}, b_{ji} \in R^{s \times 1}, 1 \leq i \leq m, f_s \text{ is the function of the } s \text{th RADF in (6). The observer (42) are designed to isolate RADF. For } r = 1, \ldots, m, \)
\[ \begin{align*}
\dot{\hat{x}}_r &= \sum_{j=1}^{m} h_j(z)[A_j \hat{x}_r + L_j(y_r - \hat{y}_r) + (B_j + F_B) u] \\
&\quad + (b_{jr} + F_{br}) \mu_j \hat{f}_1 + H + \text{sgn}(s) \mu_j C_{s} \hat{x}_r
\end{align*} \]

where \( \hat{x}_r, \hat{y}_r \) are the \( r \text{th} \) observer’s state and output, respectively, \( \mu_j \) satisfies
\[ \mu_j = -\sum_{j=1}^{m} h_j(z)(s e_{s r} P_j/\|s e_{s r} P_j\|) \]

and \( L_j \in R^{s \times m} \) is selected such that \( A_j - L_j C_j \) is Hurwitz, \( \hat{M}_{s} \) \( \dot{\theta}_i \) are the estimated values of \( H, h_i \) satisfy: \( \dot{H} = \{h_1, \ldots, h_{m}\} \hat{\theta}_i \hat{\xi}(\hat{x}), \hat{M}_{s} = [\hat{M}_{s1}, \ldots, \hat{M}_{s m}]^T, \hat{M}_{s} \) \( \dot{\theta}_i \) will be defined later.

---

**K.-Y. Hu et al.: Hybrid Adaptive FTC for Compound Faults of Hypersonic Vehicle**

VOLUME 9, 2021

56931
\[\text{sgn}(esr^T P_j) = \text{diag}\{\text{sgn}(e_{p1}), \ldots, \text{sgn}(e_{pm})\}, e_{pi}\text{ is the }i\text{th element of the vector }esr^T P_j, esr = x_s - \hat{x}_r\text{ is the state error function between the faulty plant (41) and the }r\text{th observer (42), }P_j = P_j^T > 0 \text{ denotes matrix, which will be defined later.}\]

From (41) and (42), the error dynamics is obtained: If \(s = r\),
\[
\begin{align*}
\dot{es}_r &= \sum_{j=1}^l h_j(z)((A_j - L_j C_j)esr + (b_{js} + F_{bs})(f_{s, tvs} - \mu_s f_1) + H - \hat{H} - \text{sgn}(esr^T P_j)\hat{M}_e) \quad (44)
\end{align*}
\]
If \(s \neq r\),
\[
\begin{align*}
\dot{es}_r &= \sum_{j=1}^l h_j(z)((A_j - L_j C_j)esr + (b_{js} + F_{bs})(f_{s, tvs} - \mu_s f_1) + H - \hat{H} - \text{sgn}(esr^T P_j)\hat{M}_e) \quad (45)
\end{align*}
\]
\[\text{Theorem 2: Under Assumptions 1 and 2, if there exist real matrices }L_j, Q > 0 \text{ and } P_j = P_j^T > 0 \text{ with appropriate dimensions, such that the condition (46) holds,}\]
\[
P_j(A_j - L_j C_j) + (A_j - L_j C_j)^T P_j \leq -Q
\]
and adaptive laws (47) and (48) are applied,
\[
\begin{align*}
\hat{\lambda}_i &= 2e_{pi}\xi_i - \text{prey}(\eta_i)\hat{\theta}_i & (47) \\
\hat{M}_e &= 2e_{pi}\text{sgn}(e_{pi}) - \text{prey}(\eta_M)\hat{M}_e & (48)
\end{align*}
\]
then when the \(r\)th rudder has RADF, for \(s = r\), \(\lim_{t \to \infty}es_r \in \Omega_f\), and for \(s \neq r\), \(\lim_{t \to \infty}es_r \notin \Omega_f\), where \(\text{prey}(\eta_M) > 0 \in R\) and
\[\Omega_f = \{(esr^T, \hat{\theta}_r, \hat{M}_e) \parallel es_r \parallel \leq \sqrt{\lambda_1/\lambda_{\text{min}}(P)}, \parallel \hat{\theta}_r \parallel \leq \sqrt{2\alpha_1}, i = 1, \ldots, n\} \quad (49)
\]
\[\text{Proof: Define a Lyapunov function}\]
\[V_l = \sum_{j=1}^l h_j(z)esr^T P_j esr + \sum_{i=1}^n (\hat{\theta}_i e_i^T \hat{\theta}_i + \hat{M}_e^2) / 2 \quad (50)\]
\[\begin{align*}
\dot{V}_l &\leq -esr^T Q esr + \sum_{j=1}^l h_j(z)[2esr^T P_j(b_{js} + F_{bs})(f_{s, tvs} - \mu_s f_1) + e_{pi} \hat{M}_e] \\
&\quad + 2esr^T P_j(H - \hat{H} - \hat{\epsilon}) - \sum_{i=1}^n (\hat{\theta}_i e_i^T \hat{\theta}_i + \hat{M}_e^2) / 2
\end{align*}
\]
\[
\begin{align*}
\mu_s &= -\sum_{j=1}^l h_j(z)(esr^T P_j / \parallel esr^T P_j \parallel) \\
\end{align*}
\]
and Assumption 2, one has
\[
\sum_{j=1}^l h_j(z)[2esr^T P_j(b_{js} + F_{bs})(f_{s, tvs} - \mu_s f_1) + e_{pi} \hat{M}_e] \leq 0.
\]
Since the abstract value of each entry of \(\xi_i^T (x)\) is less than 1, one has
\[
\sum_{j=1}^l h_j(z)[2esr^T P_j(H - \hat{H} - \hat{\epsilon})] \\
\leq \sum_{j=1}^n 2e_{pi}(\hat{\epsilon}_i (x) \hat{\theta}_i + \text{sgn}(e_{pi})\hat{M}_e).
\]
where
\[M_{\xi} = \sqrt{2N} \parallel \hat{\theta}_r \parallel + \sum_{i=1}^n e_{pi}^2 \quad (52)\]
\[N \text{ is the dimension of the weight vector. Further, one has }\]
\[
\begin{align*}
\dot{V}_l &\leq -esr^T Q esr + 2\sum_{i=1}^n e_{pi}(\xi_i^T (x) \hat{\theta}_i + \text{sgn}(e_{pi})\hat{M}_e) \\
&\quad - \sum_{i=1}^n (\hat{\theta}_i e_i^T \hat{\theta}_i + \hat{M}_e^2) / 2
\end{align*}
\]
\[\text{Similar to Theorem 1, substituting (47) and (48) into the above inequality, it yields}\]
\[
\begin{align*}
\dot{V}_l &\leq -esr^T Q esr - \sum_{i=1}^n (\hat{\theta}_i e_i^T \hat{\theta}_i + \hat{M}_e^2) / 2 \\
&\quad + \sum_{i=1}^n (\text{prey}(\eta_i)\hat{\theta}_i e_i^T \hat{\theta}_i + \text{prey}(\eta_M)(\hat{M}_e)^2) / 2.
\end{align*}
\]
\[\text{From Assumption 2, one has}\]
\[
\dot{V}_l \leq -\lambda V_l + \mu,
\]
where \(\lambda\) is defined in (32) and
\[
\mu = \sum_{i=1}^n (\text{prey}(\eta_i)\hat{M}_e^2 + \text{prey}(\eta_M)(\sqrt{2N}\hat{M}_e + M_{\hat{\theta}})^2) / 2
\]
\[\text{Since }d(V(t)e^{\lambda t}) / dt \leq e^{\lambda t} \mu, \text{ one has}\]
\[
\begin{align*}
0 &\leq V_l(t) \leq \lambda / \mu + [V_l(t_D) - \lambda / \mu]e^{-\lambda t} \leq \mu / \lambda + V_l(t_D)
\end{align*}
\]
\[\text{Let }\alpha_1 = \mu / \lambda + V_l(t_D), \text{ one has}\]
\[
\begin{align*}
\parallel es_r \parallel &\leq \sqrt{\alpha_1 / \lambda_{\text{min}}(P)} \\
\parallel \hat{\theta}_r \parallel, \parallel \hat{M}_e \parallel &\leq \sqrt{2\alpha_1},
\end{align*}
\]
which shows that all signals of the closed-loop system is asymptotically bounded belonging to set \(\Omega_f\).

2) For \(s \neq r\), according to (45), equation (55) is obtained:
\[
\begin{align*}
\dot{es}_r &= \sum_{j=1}^l h_j(z)((A_j - L_j C_j)esr + (b_{js} + F_{bs})(f_{s, tvs} - \mu_s f_1) + H - \hat{H} - \text{sgn}(esr^T P_j)\hat{M}_e) \\
&\quad + 2esr^T P_j(H - \hat{H} - \hat{\epsilon}) - \sum_{i=1}^n (\hat{\theta}_i e_i^T \hat{\theta}_i + \hat{M}_e^2) / 2.
\end{align*}
\]
\[\text{Because matrix }B_j \text{ is of full column rank (Assumption 1), }b_{js} \text{ and } b_{jr} \text{ are linearly independent. Therefore the following inequality does not always hold}\]
\[
\sum_{j=1}^l h_j(z)(2esr^T P_j[-(b_{jr} + F_{br})\mu_s f_1 + (b_{js} + F_{bs})(f_{s, tvs} - \mu_s f_1)]) \leq 0 \quad (56)
\]
What’s worst, inequality’s left varies infinitely since \(b_r\) and \(b_r\) are linearly independent, which further cause that \(V_l(t)\) varies infinitely. Thus, \(\lim_{t \to \infty}es_r \notin \Omega_f\). From 1) and 2), we obtain the conclusion.

A threshold can be set to help isolate the faulty loop if occurs and RADF has been detected. Denote the residuals between (41) and (42) as:
\[
\begin{align*}
J_{sr} = \parallel es_r \parallel = \parallel x_s - \hat{x}_s \parallel, \quad 1 \leq r \leq m
\end{align*}
\]
From Theorem 2, if \(s = r\), then one has
\[
\lambda_{\text{min}}(P) \parallel es_r(t) \parallel^2 \leq e^{-\lambda t} \lambda_{\text{max}}(P) \parallel es_r(t_D) \parallel^2
\]

\[ J_{sr} \leq \sqrt{\lambda_{\max}(P_j)/\lambda_{\min}(P_j)} \|e_{sr}(t_D)\| e^{-\lambda t/2} \]  
\begin{equation}
J_{sr} > \sqrt{\lambda_{\max}(P_j)/\lambda_{\min}(P_j)} \|e_{sr}(t_D)\| e^{-\lambda t/2}
\end{equation}

If \( s \neq r \), one has
\begin{equation}
J_{sr} \geq \sqrt{\lambda_{\max}(P_j)/\lambda_{\min}(P_j)} \|e_{sr}(t_D)\| e^{-\lambda t/2}
\end{equation}

Hence fault isolation is performed by using (61):
\begin{equation}
\begin{cases}
J_{sr} < T_1, r = s \Rightarrow \text{the } r\text{th actuator is faulty} \\
J_{sr} > T_1, r \neq s
\end{cases}
\end{equation}

where the threshold \( T_I \) is defined as
\[ T_I = \sqrt{\lambda_{\max}(P_j)/\lambda_{\min}(P_j)} \|e_{sr}(t_D)\| e^{-\lambda t/2} \]

According to Assumption 3, the working principle of isolation process is that the RADF is detected, and the necessary condition for RADF to occur is the structural fault, so the isolation threshold is set only considering the occurrence of compound faults. \( T_I > 0 \) is the design threshold.

\section{C. ESTIMATION FOR RADF}

Assuming the \( j \text{th} \) (\( 1 \leq j \leq m \)) rudder has RADF. The faulty system can be described as:
\begin{equation}
\begin{aligned}
\dot{x} &= \sum_{j=1}^{l} h_j(z) [A_j x + (B_j + F_B) u + (b_{js} + F_{bs}) f_{s,rvs}] + H \\
y &= \sum_{j=1}^{l} h_j(z) C_j x 
\end{aligned}
\end{equation}

To estimate the RADF, an observer is presented as
\begin{equation}
\begin{aligned}
\dot{\hat{x}} &= \sum_{j=1}^{l} h_j(z) [A_j x + (B_j + F_B) u + (b_{js}) + F_{bs} \hat{f}_{s,rvs} + L_j (y - \hat{y})] + \hat{H} + sgn(e^T P_j) \hat{M}_\xi \\
\hat{y} &= \sum_{j=1}^{l} h_j(z) C_j \hat{x}
\end{aligned}
\end{equation}

where \( \hat{f}_{s,rvs} \) is the estimation of the fault \( f_{s,rvs}(t) \). Define,
\begin{equation}
\begin{aligned}
e &= x - \hat{x} \\
\hat{f}_{s,rvs} &= f_{s,rvs} - \hat{f}_{s,rvs}
\end{aligned}
\end{equation}

using (63) and (64), the error dynamics is obtained:
\begin{equation}
\begin{aligned}
\dot{e} &= \sum_{j=1}^{l} h_j(z) [A_j - L_j C_j] e + (b_{js} + F_{bs}) \hat{f}_{s,rvs} \\
&\quad + H - \hat{H} - sgn(e^T P_j) \hat{M}_\xi 
\end{aligned}
\end{equation}

\textbf{Theorem 3:} Under Assumptions 1 and 2, if real matrices exist \( P_j = P_j^T > 0 \), \( L_j F \) and \( Q > 0 \) with appropriate dimensions, such that the following condition holds
\begin{equation}
P_j (A_j - L_j C_j) + (A_j - L_j C_j)^T P_j < -Q
\end{equation}

(47), (48), and the following algorithms are adopted
\begin{equation}
f_{\text{adapt}} = \sum_{j=1}^{l} h_j(z) \text{prey}(\eta_1) \Gamma_{\text{FD}} e^T P_j (b_{js} + F_{bs}) = \sum_{j=1}^{l} h_j(z) \text{prey}(\eta_1) (\Gamma \tilde{y} \Gamma_6) e^T P_j (b_{js} + F_{bs})
\end{equation}

where \( b_{js} \) is the \( j \text{th} \) column of \( B_j \), \( \Gamma_{\text{FD}} \) is the compensation function of the RSF in the diagnosis process, \( \Gamma_5 \), \( \Gamma_6 \) is the suitable dimension adjustment parameter. \( \text{prey}(\eta_M) > 0 \), \( \text{prey}(\eta_i) > 0 \) and \( \text{prey}(\eta_1) > 0 \), then, error dynamics (66) is stable and all signals are semi-globally uniformly bounded, converging to a small neighbourhood of the origin, i.e., \( \Omega_E \) is defined as
\begin{equation}
\begin{aligned}
\Omega_E &= \{(e, \hat{\theta}_i, \hat{M}_\xi, \hat{f}_{s,rvs}) \|e\| \leq \sqrt{\alpha/\lambda_{\min}(P_j)}, \\
&\quad \|\hat{f}_{s,rvs}\| \leq 2\text{prey} \|\eta_1\| \alpha, \|\hat{\theta}_i\| \leq \sqrt{2\alpha}, \|\hat{M}_\xi\| \leq \sqrt{2\alpha} \}
\end{aligned}
\end{equation}

where \( \alpha = \mu_E / \lambda_E + \mathcal{E}(t_i) \), and
\begin{equation}
\lambda_E = \min \{ \lambda_{\min}(Q) \prey(\eta_0), \prey(\eta_e), 1 \}
\end{equation}

\begin{equation}
\mu_E = 2f_1(2f_1 + f_2) / \prey(\eta_1) + \mu
\end{equation}

\begin{equation}
\mu = \sum_{i=1}^{n} \text{prey}(\eta_0) \hat{\theta}_i^T \hat{\theta}_i + \text{prey}(\eta_M) (\sqrt{2NM_0} + M_0^2 \hat{\theta}_i^2) / 2
\end{equation}

\textbf{Proof:} Define the Lyapunov function as
\begin{equation}
\begin{aligned}
\mathcal{V}_E &= \sum_{j=1}^{l} h_j(z) e^T P_j e + 2\hat{f}_{s,rvs}^T / \prey(\eta_1) \\
&\quad + \frac{1}{2} \sum_{i=1}^{n} \hat{\theta}_i^T \hat{\theta}_i + \hat{M}_\xi^2
\end{aligned}
\end{equation}

Differentiating \( \mathcal{V}_E \) and considering (67) \sim (69), similar to Theorem 2, we obtain
\begin{equation}
\begin{aligned}
\dot{\mathcal{V}}_E &\leq -\sum_{j=1}^{l} h_j(z) e^T Q e + \hat{f}_{s,rvs} \hat{f}_{s,rvs} / \prey(\eta_1) \\
&\quad - \sum_{i=1}^{n} \prey(\eta_0) \hat{\theta}_i^T \hat{\theta}_i + \prey(\eta_e) \hat{\theta}_i^2 / 2 + \mu,
\end{aligned}
\end{equation}

where \( \mu \) is set in (73). As \( \hat{f}_{s,rvs}^2 \leq f_1 \), which is guaranteed by laws (69), and Assumption 2 is satisfied, one has
\begin{equation}
\hat{f}_{s,rvs} \hat{f}_{s,rvs} / \prey(\eta_1) \leq -\hat{f}_{s,rvs}^2 / \prey(\eta_1) + 2f_1(2f_1 + f_2) / \prey(\eta_1).
\end{equation}

Hence, (74) is rewritten as follows:
\begin{equation}
\dot{\mathcal{V}}_E \leq -\lambda_E \mathcal{V}_E(t) + \mu_E,
\end{equation}

where
\begin{equation}
\lambda_E = \min \{ \lambda_{\min}(Q) \prey(\eta_0), \prey(\eta_e), 1, \prey(\eta_M) \}
\end{equation}

\begin{equation}
\mu_E = 2f_1(2f_1 + f_2) / \prey(\eta_1) + \mu
\end{equation}

Then one obtains \( d(\mathcal{V}_E(t)) e^{\lambda_E t} / dt \leq e^{\lambda_E t} \mu_E \), furthermore,
\begin{equation}
0 \leq \mathcal{V}_E(t) \leq \mu_E / \lambda_E + \mathcal{V}_E(t) - \mu_E / \lambda_E e^{-\lambda_E t}
\end{equation}

\begin{equation}
\leq \mu_E / \lambda_E + \mathcal{V}_E(t)
\end{equation}
Let $\alpha = \mu_E/\lambda_E + V_E(0)$, one has
\[
\|e(t)\| \leq \sqrt{\alpha/\lambda_{\min}(P)} \left\| \delta \right\|, \left\| M_R \right\| \leq \sqrt{2\alpha}
\]
(78)

From Theorem 3, one has
\[
\lambda_{\min}(P_f) \|e(t)\|^2 \leq V_E(t) \leq \mu_E/\lambda_E + (V_E(t))
\]
\[
-\mu_E/\lambda_E e^{-\lambda_{E}t}
\]
(79)

\[
\|e(t)\| \leq \left\{ \left\| \mu_E/\lambda_E + (\lambda_{\max}(P_i)) \|e_s(t)\| \right\|^2
\]
\[
-\mu_E/\lambda_E e^{-\lambda_{E}t}/\lambda_{\min}(P_i) \right\}^{1/2}
\]
(80)

The stability is proven.

In engineering, the estimation objective is achieved if $\|e(t)\| \leq \delta_{E}.\delta_{E} > 0$ is a threshold. And
\[
\delta_{E} = \left\{ \left\| \mu_E/\lambda_E + (\lambda_{\max}(P_i)) \|e_s(t)\| \right\|^2
\]
\[
-\mu_E/\lambda_E e^{-\lambda_{E}t}/\lambda_{\min}(P_i) \right\}^{1/2}
\]
(81)

D. HYBRID FTC

Based on the estimated RADF and Theorem 1, the active-passive hybrid FTC is constructed as
\[
u_i(t) = u^p_i(t) - \hat{f}_{i,\text{tvs}}
\]
(82)

where $u^p_i(t)$ is the $i$th control input only with passive FTC, the estimations of $\hat{f}_{i,\text{tvs}}$ are used to active FTC. Setting $u_p(t)$ is the control input vector without RADF estimation. When only RSF occurs, according to (25), the controller satisfies:
\[
u_i(t) = u_i(t) = u^p_i(t) = \sum_{j=1}^{l} h_j(z)(K_{\text{comp}}\hat{x} + r - 2(B_j + F_B)^{+}[A_j - A_{jm}]x
\]
\[
+ \hat{H} + \text{sgn}(x^T P_j)\hat{e})
\]
(83)

If the rudder compound faults occur and estimation error is ignored, according to (83) and (6), the controller satisfies:
\[
u_i(t) = u_i(t) + f_{\text{tvs}} = u^p_i(t) - \hat{f}_{i,\text{tvs}} + f_{\text{tvs}} = u^p_i(t)
\]
\[
= \sum_{j=1}^{l} h_j(z)[K_{\text{comp}}\hat{x} + r - 2(B_j + F_B)^{+}[A_j - A_{jm}]x
\]
\[
+ \hat{H} + \text{sgn}(x^T P_j)\hat{e})
\]
(84)

Therefore, when the compound faults occur, the stability proof of the reconstructed system is the same as Section 3.1.

To consider the RSF, passive FTC, prey algorithm, and active FTC, substituting (12) and (39) into (84), the hybrid fault-tolerant controller under rudder compound faults is:
\[
u_i = \sum_{j=1}^{l} h_j(z)(K_{\text{comp}}\hat{x} + r - 2(B_j + F_B)^{+}[A_j - A_{jm}]x
\]
\[
+ \hat{H} + \text{sgn}(x^T P_j)\hat{e}) - \hat{f}_{i,\text{tvs}}
\]
(85)

The meanings of all symbols in (85) are explained in the preceding text, they are mainly distributed in the symbol explanations of formulas (4)~(8), (11)~(16), and (39).

Controller (85) uses the estimated value to actively cancel the RADF and uses the compensation function to shield the RSF. The prey algorithm enhances the performance for the RADF’s incipient fault. Section 3.5 introduces how the controlled object schedules a subset from the parameter set online based on the estimated RADF amplitude to obtain the best tracking performance.

E. PREY ALGORITHM

The complex active link in hybrid FTC should be protected from the external nonlinear approximation error $\varepsilon_i$, the parameters can switch according to the estimated value to optimise the FTC performance under different fault amplitudes. The prey algorithm is considered a suitable switching method. Figure 2 is the positional relationship.

An antelope’s response links to the cheetah and birds [22]. The correspondence of sensitive/insensitive strategies is shown in Tables 1 and 2.

Replace cheetah, antelope, and alert birds with $f_{\text{tvs}}$, $u(t)$, and $\chi(t) = \|\varepsilon\|$ respectively where $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_i, \ldots, \varepsilon_N]$. Tables 3 and 4 can provide the corresponding relationship of the prey algorithm, where $\eta_{1,1}, \eta_{1,2}, \eta_{1,3}, \eta_{1,4}, \eta_{M1}, \eta_{M2}, \eta_{M3}, \eta_{M4}, \eta_{M5}$, and $K_2$ are the LMI-compliant adaptive learning rates and parameters.

### TABLE 1. Insensitive prey strategy.

| Cheetah | Antelope |
|--------|---------|
| **Status** | **Distance** | **Judgment** | **Response** |
| Progressive | Far | Safe | Static |
| Progressive | Close | Safe | Static |
| Run | Close | Threat | Run |

### TABLE 2. Sensitive prey strategy.

| Cheetah | Antelope |
|--------|---------|
| **Status** | **Distance** | **Judgment** | **Response** |
| Progressive | Far | Safe | Static |
| Progressive | Close | Threat | Run |
| Run | Close | Threat | Run |
TABLE 3. FTC insensitive prey Algorithm.

| Distance | Status   | Judgment          | Response                          |
|----------|----------|-------------------|-----------------------------------|
| $A \in (0, \kappa)$ | Fault free | Small impact | $\eta_0/\eta_1/\eta_1/\eta_M/K_1$ |
| $A \in (\kappa, 0.1)$ | $f_{in,inc}$ | Small impact | $\eta_0/\eta_1/\eta_1/\eta_M/K_1$ |
| $A \in (0.1, 0.2)$ | $f_{in,non-inc}$ | Big impact | $\eta_0/\eta_2/\eta_2/\eta_M/K_2$ |

$\kappa \in \mathbb{R}^+$ is a threshold that is approximately equal to 0 satisfies $\kappa \ll 0.1$, and

$$\Lambda = \max \{|f_{1,inc}/u_1|, \ldots, |f_{n,inc}/u_n|, \ldots, |f_{n,non-inc}/u_n|\}$$

(86)

$$\Lambda_{\text{max}} = f_1/|u_{\text{min}}| = f_1/\min(|u_1|, \ldots, |u_n|, \ldots)$$

(87)

The algorithm steps are as follows:

**Step 1:** $\Lambda$ is less than or equal to $\kappa$; no fault; if the approximation error integral is less than $\tau_0$, then parameters are insensitive and set to $\eta_0/\eta_1/\eta_1/\eta_M/K_1$.

**Step 2:** $\Lambda$ is less than or equal to $\kappa$; no fault; if the approximation error integral is larger than or equal to $\tau_0$, then parameters are sensitive and set to $\eta_0/\eta_1/\eta_1/\eta_M/K_1$.

**Step 3:** $\Lambda$ is larger than $\kappa$ and less than or equal to 0.1; incipient fault; if the approximation error integral is larger than or equal to $\tau_0$, then the parameters are sensitive and set to $\eta_0/\eta_2/\eta_2/\eta_M/K_2$.

**Step 4:** $\Lambda$ is larger than 0.1; large value fault; parameters are set to $\eta_0/\eta_2/\eta_2/\eta_2/\eta_M/K_2$.

**Step 5:** Return to Step 1.

Large than $\tau_0$ means RSF occurs and the non-sensitive state is switched to the sensitive state, which means that the controller is ready to compensate for the faults (RSF or RSF+RDAF). Based on prey algorithm, (23), (24), (25), (48) and (68) are improved by switching laws which are set as

$$\Upsilon = \{\eta_0, \eta_\kappa, \eta_1, \eta_M, K_1\}$$

(88)

$$\text{prey}(\Upsilon) = \begin{cases} 
\eta_0/\eta_1/\eta_1/\eta_M/K_1, \text{if } \hat{\Lambda} \in (0, \kappa) \\
\eta_0/\eta_1/\eta_1/\eta_M/K_1, \text{if } \hat{\Lambda} \in (\kappa, 0.1] \\
\int_0^{T_s} \chi(t)dt \in [0, \tau_0) \\
\eta_0/\eta_2/\eta_2/\eta_M/K_2, \text{if } \hat{\Lambda} \in (0.1u_1, f_1] \\
\eta_0/\eta_2/\eta_2/\eta_M/K_2, \text{if } \hat{\Lambda} \in (0.1u_1, f_1] \\
\int_0^{T_s} \chi(t)dt \in [\tau_0, +\infty) \\
\eta_0/\eta_2/\eta_2/\eta_M/K_2, \text{if } \hat{\Lambda} \in (0.1u_1, f_1] \\
\eta_0/\eta_2/\eta_2/\eta_M/K_2, \text{if } \hat{\Lambda} \in (0.1u_1, f_1] \\
\int_0^{T_s} \chi(t)dt \in [\tau_0, +\infty)
\end{cases}$$

(89)

where $T_s$ is the total simulation time.

The controlled object can use the prey algorithm to achieve the high-performance estimation/compensation of RA-DF’s different amplitude especially in incipient deviation, while avoiding the disturbance from $\varepsilon$ to active FTC.

IV. SIMULATION VERIFICATION EXPERIMENT

Links-Box semi-physical simulator in Figure 3 is used to evaluate the effectiveness of the proposed hybrid compensation. Vibration table can simulate disturbance to test the robustness of aircraft. The features of software package Links-BOX are: 1. Adapt to the models built in MATLAB; 2. Provide input/output hardware to enable users to integrate the hardware environment into the simulation models; 3. Automatic conversion of MATLAB model codes to VxWorks codes; 4. Provide communication, storage, scheduling and other services in VxWorks. Semi-physical simulator can simulate a more realistic industrial environment and enhance the usability of the algorithm.

FIGURE 3. Hypersonic vehicle semi-physical platform.

For the test purpose, aerodynamic coefficients are considered the nominal cruising flight (i.e., $V_t = 2700$ m/s, and $h_t = 30$ km). We consider the nonlinearity mainly comes from attack angle $\alpha_0$ and angular rate $\omega_0$. $\alpha_0$ has two fuzzy sets ($\alpha_0 = 0$ rad) and ($\alpha_0 = \pi/3$ rad), then the corresponding membership functions are defined as

$$M_{\alpha_0=0} = \frac{1 - \frac{1}{1 + \exp(-4 - 11\alpha_0)}}{1 + \exp(-4 - 11\alpha_0)}$$

(90)

$$M_{\alpha_0=\pi/3} = 1 - M_{\alpha_0=0}$$

and angular rate $\omega \in [-0.4\text{rad/s}, 0.4\text{rad/s}]$. $\omega$ has the fuzzy sets: $\omega = -0.4\text{rad/s}$, $\omega = 0\text{rad/s}$, and $\omega = 0.4\text{rad/s}$, then the corresponding membership functions are chosen as:

$$M_{\omega=-0.4} = \frac{1 - \frac{1}{1 + \exp(3 + 22\omega)}}{1 + \exp(3 + 22\omega)}$$

(91)

$$M_{\omega=0.4} = \frac{1 - \frac{1}{1 + \exp(-3 - 22\omega)}}{1 + \exp(-3 - 22\omega)}$$

We select the following operating points: $[\alpha_0, \omega] = [0 - 0.4], [00], [0.4], [\pi/3 - 0.4], [\pi/3 0], [\pi/3 0.4]$. Under the membership functions and the six operating points, six plant and control rules are defined. The correspondence between operating points and linear modes is:

**Rule 1:** If $\omega$ is approximately -0.4 rad/s, $\alpha_0$ is approximately 0 rad, then $\{A_1, B_1, C_1, A_{1m}\}$. 

VOLUME 9, 2021
Rule 2: If \( \omega \) is approximately 0 rad/s, \( \alpha_0 \) is approximately 0 rad, then \( \{A_2, B_2, C_2, A_{2m}\} \).

Rule 3: If \( \omega \) is approximately 0.4 rad/s, \( \alpha_0 \) is approximately 0 rad, then \( \{A_3, B_3, C_3, A_{3m}\} \).

Rule 4: If \( \omega \) is approximately -0.4 rad/s, \( \alpha_0 \) is approximately \( \pi/3 \) rad, then \( \{A_4, B_4, C_4, A_{4m}\} \).

Rule 5: If \( \omega \) is approximately 0 rad/s, \( \alpha_0 \) is approximately \( \pi/3 \) rad, then \( \{A_5, B_5, C_5, A_{5m}\} \).

Rule 6: If \( \omega \) is approximately 0.4 rad/s, \( \alpha_0 \) is approximately \( \pi/3 \) rad, then \( \{A_6, B_6, C_6, A_{6m}\} \).

We consider the external nonlinear function \( H(t) = [0.3\sin(0.3t - 2), 0, 0, 0, 0, 0]^T \). Let \( \kappa, \tau = 0.01, 2 < u_i < 10 \), under the premise of satisfying inequalities (22), (46), (68), (86), we can obtain matrices \( P_f, Q, S \), controller gains \( \eta_f, \eta_e, \eta_i, \eta_M \), and \( K_f, \) by using MATLAB.

According to the reentry HFV (3), rudder allocation structural fault (RSF) \( F_B \in \mathbb{R}^{6 \times 6} \) are set as follows:

\[
F_B = \begin{bmatrix}
    f_{rate} & f_{ra} & f_{far} & f_{angle}
\end{bmatrix}.
\]

And \( f_{rate} \) is the angular rate allocation structural fault, \( f_{angle} \) is the attitude angle allocation structural fault, \( f_{ra} \) and \( f_{far} \) are the coupling structural fault. They satisfy:

\[
f_{rate} = \begin{cases}
f_{rate,1}, & t \in (0, 20s] \\
f_{rate,2}, & t \in (20, 100s],
\end{cases}
\]

\[
f_{angle} = \begin{cases}
f_{angle,1}, & t \in (0, 20s] \\
f_{angle,2}, & t \in (20, 100s],
\end{cases}
\]

\[
f_{rate,2} = \begin{bmatrix}
    0.1 & 0 & 0.3 \\
    0 & 1.6 & 0 \\
    0.3 & 0.2 & 0
\end{bmatrix},
\]

\[
f_{angle,2} = \begin{bmatrix}
    0 & 0.7 & 0 \\
    0.3 & 0 & 0.4 \\
    0 & 1.4 & 0
\end{bmatrix}.
\]

\( f_{rate,1}, f_{angle,1}, f_{ra}, \) and \( f_{far} \) are zero matrices.

According to the definition of the intermittent time-varying RADF, the value of \( f_{4,tvs}(t) \) is:

\[
f_{4,tvs}(t) = f_{5,tvs}(t) = f_{6,tvs}(t) = \frac{1}{2} \sigma(1) = \frac{1}{2} \sigma(2) = \frac{1}{2} \sigma(3)
\]

\[
= f_{4,tvs}(t) \equiv \begin{cases}
f_{4o}, & t \in (t_{o1}, t_{o2}) \\
0, & otherwise.
\end{cases}
\]

Let \( \sigma_0 = 1 \), and set the time interval, the fault assignment is given as:

\[
f_{4,tvs}(t) = \begin{cases}
f_{41}(t), & t \in (20s, 80s] \\
0, & otherwise,
\end{cases}
\]

\[
f_{41}(t) = \begin{cases}
f_{41,inc}(t), & t \in (20s, 40s] \\
f_{41,non-inc}(t), & t \in (40s, 80s],
\end{cases}
\]

\[
f_{41,inc}(t) = f_{42,inc}(t) = 0.2, & t \in (20s, 40s]
\]

\[
f_{41,non-inc}(t) = f_{42,non-inc}(t) = 1.2, & t \in (40s, 60s]
\]

\[
f_{41,non-inc}(t) = f_{42,non-inc}(t) = 1, & t \in (60s, 80s].
\]

The superiority of hybrid FTC is clearly demonstrated by the comparison with the method in [28]. Table 5 shows the simulation tracking comparison of traditional method (TM) in [28] and our new method (NM) in the rudder compound fault condition, where \( t_{r,\text{max}} \) and \( e_{s,\text{max}} \) represent the longest response time (sec) and the maximum steady state error, respectively. The units of \( e_{s,\text{max}} \) are ‘rad’ and ‘rad/s’. Traditional method cannot accurately estimate RADF, \( t_{r,\text{max}} \) at this time represents the response time when the estimated curves stabilize at the error values. ‘–’ indicates that the trace trajectories are divergent.

The simulation result is noisy, but due to the algorithm design and the robustness of the HFV system itself, the noise is small, most of the time is in the fourth place after the decimal point, which can be ignored.

Table 6 is a statistical table of the improved amplitude of fault estimation, NM comparing with TM indicators are significantly improved. The reduction in error produces a qualitative change, because TM estimation error is larger than incipient fault and is worthless. NM is the only way to meet the requirements.

The FTC method improvement does not require depth analysis because Table 5 clearly shows that the TM curve is divergent. So NM is the only available method.

Based on our method, Figure 4 shows that the estimation can effectively track \( f_{4,tvs}(t), f_{5,tvs}(t) \) and \( f_{6,tvs}(t) \) in the rudders. Both the incipient and large deviations of RADF can be accurately and quickly estimated.

Figure 5 reveals the system’s estimation result of the nonlinear function vector \( H(x) \). The experiment sets that there is only one non-zero nonlinear function \( h_1(x) \) in \( H(x) \), which can indicate an external disturbance.

Figure 5 shows that the estimator can accurately estimate and calculate the nonlinear disturbance function.
Accurate estimation of disturbance is the premise of robust FTC.

Figure 6 show the robust FTC tracking results when only RSF occurs. Figure 7 shows the robust FTC tracking results when RSF-RADF compound faults occur simultaneously. The tracking curves diverge first and then converge within algorithm reconstruction time. This phenomenon is similar to overshoot, but it is normal in FTC and will not affect the high quality control result.

The statistical experiment result of FTC performance is in Table 7. There is no significant fluctuation in performance parameters under compound faults. Therefore, it is not necessary to distinguish between the maximum and minimum in the steady-state interval, $t_r$ and $e_s$ are the response time and steady-state error if there is no fault, $t_{r, rep}$ and $e_{s, rep}$ are the repair time and error under fault condition. The units are consistent with Table 5. T1: 20 $\sim$ 40 s, T2: 40 $\sim$ 60 s, T3: 60 $\sim$ 80 s.
In normal operation, HFV can complete the attitude change in about 3s with an error of 0.01; when the RSF occurs, HFV can repair the fault in 3 ∼ 5s with an error of about 0.02; when the compound faults occur, the repair time of roll rate is about 4.7s, the repair time of attack angle is about 1.5s, the repair time of sideslip angle is 4.7 ∼ 5s, and the repair errors are in 0.012 ∼ 0.02, which meet the requirements of HFV, so the verification experiment is successful. After repeated experiments, the response time of the fault detection is 0.0021s, the response time of the fault isolation is 0.0007s, the response time of the fault estimation is 0.0026s, and the prey parameter switching time is 0.0003s. The system packet loss rate for performing a compound fault compensation process is less than 0.01%, and the space used for calculation is less than 0.1%.

**V. CONCLUSION**

Based on the multiple approximation strategy and adaptive control, the robust hybrid active-passive FTC problem for compound faults in HFV attitude dynamics was investigated in this study. Improved adaptive FTC laws, which consisted of both fusion adaptive learning rates and a novel prey architecture, were introduced to reduce the effect of the RSF-RADF compound faults under multiple source nonlinearities conditions. The designed hybrid FTC scheme achieved the predetermined goals: passive compensation for a RSF, adaptive observers that accurately estimated the RADF and proactively repaired the RADF. Finally, the simulation results of HFV dynamics revealed the effectiveness of the developed FTC method of this study. This results provided a theoretical basis for HFV reliability design under complex faults, helped develop the theory of hybrid self-repair control. For the unconsidered issues, follow-up research would focus on sensor fault, fuselage fault, and more complex hybrid FTC deployment algorithms.

**REFERENCES**

[1] A. Abdelmawgoud, M. Jamshidi, and P. Benavidez, “Distributed estimation in multimissile cyber-physical systems with time delay,” IEEE Syst. J., vol. 14, no. 1, pp. 1491–1502, Mar. 2020.

[2] P. Gutiérrez León, J. García-Morales, R. F. Escobar-Jiménez, J. F. Gómez-Aguilar, G. López-López, and L. Torres, “Implementation of a fault tolerant system for the internal combustion engine’s MAF sensor,” Measurement, vol. 122, pp. 91–99, Jul. 2018.

[3] A. A. Amin and K. Mahmood-Ul-Hasan, “Advanced fault tolerant air-fuel ratio control of internal combustion gas engine for sensor and actuator faults,” IEEE Access, vol. 7, pp. 17634–17643, 2019.

[4] Y. Yuan, X. Liu, S. Ding, and B. Pan, “Fault detection and location system for diagnosis of multiple faults in aeroengines,” IEEE Access, vol. 5, pp. 17671–17677, 2017.

[5] F. Hao, D. Zhang, L. Cao, and S. Tang, “Disturbance decoupling control for flexible air-breathing hypersonic vehicles with mismatched condition,” Asian J. Control, vol. 21, no. 3, pp. 1100–1110, May 2019.

[6] M. Zolfaghari, M. Abedi, and G. B. Gharehpetian, “Robust nonlinear state feedback control of bidirectional interlink power converters in grid-connected hybrid microgrids,” IEEE J. Control Syst., vol. 14, no. 1, pp. 1117–1124, Mar. 2020.

[7] H. Hassani, J. Zarei, R. Razavi-Far, and M. Saif, “Robust interval type-2 fuzzy observer for fault detection of networked control systems subject to immeasurable premise variables,” IEEE J. Control Syst., vol. 13, no. 3, pp. 2954–2965, Sep. 2019.

[8] N. Rong, Z. Wang, and H. Zhang, “Finite-time stabilization for discontinuous interconnected delayed systems via interval type-2 T-S fuzzy model approach,” IEEE Trans. Fuzzy Syst., vol. 27, no. 2, pp. 249–261, Feb. 2019.
[9] X.-H. Chang and Y. Liu, “Robust $H_{\infty}$ filtering for vehicle sideslip angle with quantization and data dropouts,” IEEE Trans. Veh. Technol., vol. 69, no. 10, pp. 10435–10445, Oct. 2020.

[10] Y. Gao, F. Xiao, J. Liu, and R. Wang, “Distributed soft fault detection for interval type-2 fuzzy-model-based stochastic systems with wireless sensor networks,” IEEE Trans. Ind. Informat., vol. 15, no. 1, pp. 334–347, Jan. 2019.

[11] Y. Lu, “Adaptive-fuzzy control compensation design for direct adaptive fuzzy control,” IEEE Trans. Fuzzy Syst., vol. 26, no. 6, pp. 3222–3231, Dec. 2018.

[12] H.-N. Wu, S. Feng, Z.-Y. Liu, and L. Guo, “Disturbance observer based robust mixed $H_{2}/H_{\infty}$ fuzzy tracking control for hypersonic vehicles,” Fuzzy Sets Syst., vol. 306, pp. 118–136, Jan. 2017.

[13] M. Yu, Z. Feng, J. Huang, and Y. Yu, “Characteristic extraction of rolling bearing compound faults of aero-engine,” J. Vibroengineering, vol. 19, no. 6, pp. 4285–4299, Sep. 2017.

[14] X. Y. Zhu and D. D. Li, “Robust fault estimation for a 3-DOF helicopter considering actuator saturation,” Mech. Syst. Signal Process., vol. 155, pp. 1–13, 2021, Art. no. 107624.

[15] Z. Zhang, D. Ye, B. Xiao, and Z. Sun, “Third-order sliding mode fault-tolerant control for satellites based on iterative learning observer,” Asian J. Control, vol. 21, no. 1, pp. 43–51, Jan. 2019.

[16] A. Kodali, K. Pattipati, and S. Singh, “Coupled factorial hidden Markov models (CFHMM) for diagnosing multiple and coupled faults,” IEEE Trans. Syst., Man, Cybern. Syst., vol. 43, no. 3, pp. 522–534, May 2013.

[17] M. Yi, N. Pan, and Y. Guo, “Mechanical compound faults extraction based on improved frequency domain blind deconvolution algorithm,” Mech. Syst. Signal Process., vol. 113, pp. 180–188, Dec. 2018.

[18] Q. Shen, B. Jiang, and P. Shi, “Adaptive fault diagnosis for T–S fuzzy systems with sensor faults and system performance analysis,” IEEE Trans. Fuzzy Syst., vol. 22, no. 2, pp. 274–285, Apr. 2014.

[19] V. Reppa, M. M. Polycarpou, and C. G. Panayiotou, “Decentralized isolation of multiple sensor faults in large-scale interconnected nonlinear systems,” IEEE Trans. Autom. Control, vol. 60, no. 6, pp. 1582–1596, Jun. 2016.

[20] P. Cui, D. Zhang, S. Yang, and H. Li, “Friction compensation based on time-delay control and internal model control for a gimbal system in magnetically suspended CMG,” IEEE Trans. Ind. Electron., vol. 64, no. 5, pp. 3798–3807, May 2017.

[21] J. Xiong, X. H. Chang, J. H. Park, and Z. M. Li, “Fault-tolerant control of suspension systems subject to input quantization and actuator fault,” Int. J. Robust Nonlinear Control, vol. 30, pp. 6720–6743, Nov. 2020.

[22] A. A. Amin and K. M. Hasan, “A review of fault tolerant control systems: Advancements and applications,” Measurement, vol. 143, pp. 58–68, Sep. 2019.

[23] K. Hu, C. Wen, and A. Yusup, “Improved adaptive hybrid compensation for compound faults of non-Gaussian stochastic systems,” IEEE Access, vol. 7, pp. 51284–51294, 2019.

[24] G. H. Yang and D. Ye, “Adaptive reliable $H_{\infty}$ filtering against sensor failures,” IEEE Trans. Signal Process., vol. 55, no. 7, pp. 3161–3171, Jun. 2007.

[25] T. Wang, H. Gao, and J. Qiu, “A combined fault-tolerant and predictive control for network-based industrial processes,” IEEE Trans. Ind. Electron., vol. 63, no. 4, pp. 2529–2536, Apr. 2016.

[26] A. A. Amin and K. M. Ui-Hasan, “Hybrid fault tolerant control for air-fuel ratio control of internal combustion gasoline engine using Kalman filters with advanced redundancy,” Meas. Control, vol. 52, nos. 5–6, pp. 1–20, Feb. 2019.

[27] Z. Xu and A. A. Julius, “Robust temporal logic inference for provably correct fault detection and privacy preservation of switched systems,” IEEE Syst. J., vol. 13, no. 3, pp. 3010–3021, Sep. 2019.

[28] A. A. Amin and K. M. Ui-Hasan, “Robust active fault-tolerant control for internal combustion gas engine for air-fuel ratio control with statistical regression-based observer model,” Meas. Control, vol. 52, nos. 9–10, pp. 1–16, 2019.

[29] G. Liu, Y.-Y. Cao, and X.-H. Chang, “Fault detection observer design for fuzzy systems with local nonlinear models via fuzzy Lyapunov function,” Int. J. Control, Autom. Syst., vol. 15, no. 5, pp. 2233–2242, Oct. 2017.

[30] K. Hu, F. Chen, Z. Cheng, and C. Wen, “Adaptive minimum-entropy hybrid compensation for compound faults of non-Gaussian stochastic systems,” IEEE Access, vol. 7, pp. 120695–120707, 2019.

[31] H. Chen, B. Jiang, N. Lu, and Z. Mao, “Deep PCA based real-time incipient fault detection and diagnosis methodology for electrical drive in high-speed trains,” IEEE Trans. Veh. Technol., vol. 67, no. 6, pp. 4819–4830, Jun. 2018.