ABSTRACT: We discuss the dependence of running couplings on the choice of regularization method in a general softly-broken $N = 1$ supersymmetric theory. Regularization by dimensional reduction respects supersymmetry, but standard dimensional regularization does not. We find expressions for the differences between running couplings in the modified minimal subtraction schemes of these two regularization methods, to one loop order. We also find the two-loop renormalization group equations for gaugino masses in both schemes, and discuss the application of these results to the Minimal Supersymmetric Standard Model.
1. Introduction

Low-energy $N = 1$ supersymmetry [1] provides an elegant solution to the naturalness problem associated with the origin of the electroweak scale. If supersymmetry proves to be correct, it is quite possible that the first superpartner to be discovered will be the gluino, because of its high production rate at hadron colliders. Since the gluino mass is a physical parameter of the supersymmetry-breaking sector, its determination will provide an important clue as to the rest of the sparticle spectrum. Measurements of the masses of the other superpartners will then yield tests of the various specific extensions of the Minimal Supersymmetric Standard Model (MSSM) which have been proposed over the years. For example, in models with soft supersymmetry-breaking terms which are “universal” in the sense usually associated with spontaneously broken supergravity [2], the squark masses cannot be less than about 0.8 of the gluino mass. The details of the supersymmetric spectrum have been studied under a variety of assumptions and constraints on the other unknown parameters of the model in [3,4,5] among many others. Further, there are sum rules [6] which relate, for example, the masses of the neutralinos and charginos to the mass of the gluino, without involving any other unknown input parameters. It is therefore useful to work out the predictions for sparticle masses in terms of the running parameters of supersymmetric models, including radiative corrections.

There is a problem of principle, however, in discussing radiative corrections in supersymmetric models. The most popular regularization scheme for discussing radiative corrections within the Standard Model, dimensional regularization [7] (DREG), violates supersymmetry explicitly because it introduces a mismatch between the numbers of gauge boson and gaugino degrees of freedom. The modified scheme known as dimensional reduction [8] (DRED) does not violate supersymmetry, and thus maintains the supersymmetric Ward identities. In DREG, supersymmetry is violated in the finite parts of one-loop graphs, and in the divergent parts of two-loop graphs. This means that the $\beta$-functions for a supersymmetric model will be different for the two schemes starting at the two-loop level. Also, the running couplings computed in DREG with modified minimal subtraction [9] ($\overline{MS}$) will differ from those computed in DRED with modified minimal subtraction ($\overline{DR}$) by virtue of finite one-loop effects.

In this paper, we give the difference between running couplings in the $\overline{MS}$ and $\overline{DR}$
schemes, to lowest non-trivial order, for a general $N = 1$ supersymmetric model with soft breaking. As an application, we then consider the two-loop $\beta$-function for a gaugino mass in both schemes. The $\overline{\text{MS}}$ version is obtained by specializing the results of [10,11] for a general renormalizable theory to the case of a general $N = 1$ supersymmetric model. The two-loop $\beta$-function in the $\overline{\text{DR}}$ scheme is then derived by simply translating all $\overline{\text{MS}}$ couplings into their $\overline{\text{DR}}$ counterparts. We do not enter here into the question of whether a fully consistent $\overline{\text{DR}}$ calculation at the two-loop level actually exists; although some results have been obtained for supersymmetric theories [12], the use of dimensional reduction in non-supersymmetric theories is problematic. However, if such a consistent scheme exists, it must reproduce the results obtained here to two-loop order.

We consider a general $N = 1$ supersymmetric Yang-Mills model. The chiral superfields $\Phi_i$ contain a complex scalar $\phi_i$ and a two-component fermion $\psi_i$ which transform as a (possibly reducible) representation $R$ of the gauge group $G$. The superpotential is

$$W = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k$$

(1.1)

with $Y^{ijk} = (Y_{ijk})^*$. In addition, the Lagrangian contains soft supersymmetry-breaking terms of the form

$$-\mathcal{L}_{SB} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}M\lambda\lambda + \text{h.c.}$$

(1.2)

where $M$ is the mass of the gaugino $\lambda$. Strictly speaking, the most general renormalizable softly-broken $N = 1$ supersymmetric model also contains couplings in the superpotential with dimensions of (mass) and (mass)$^2$, and soft supersymmetry-breaking scalar couplings with dimensions of (mass)$^2$ and (mass)$^3$. However, such terms are not relevant to the present discussion.

For simplicity we first give our results for the special case of a simple [or $U(1)$] gauge group. We then explain the modifications required if the gauge group is a direct product in Section 4, where we will also discuss the MSSM. We let $t^A \equiv (t^A)_i^j$ denote the representation matrices for the gauge group $G$. Then

$$(t^At^A)_i^j = C(R)\delta_i^j$$

$$\text{Tr}_R(t^At^B) = S(R)\delta^{AB}$$

define the quadratic Casimir invariant $C(R)$ and the Dynkin index $S(R)$ for a representation $R$. For the adjoint representation [of dimension denoted by $d(G)$], $C(G)\delta^{AB} = f^{ACD}f^{BCD}$ with $f^{ABC}$ the structure constants of the group.
2. Dependence of Running Couplings on the Regularization Method

The strategy for determining the relationship between couplings in the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ schemes is to relate each running parameter to a physical quantity which cannot depend on the choice of scheme.

First, consider the running gaugino mass parameter. By standard techniques, one can compute the physical pole mass of the gaugino to one-loop order as a function of the running mass parameter in each scheme. In the $\overline{\text{MS}}$ scheme, one finds

$$M_{\text{pole}} = M_{\overline{\text{MS}}} (\mu) \left[ 1 + \frac{g_{\overline{\text{MS}}}}{16\pi^2} \left( 4C(G) + 3C(G) \ln \frac{\mu^2}{M_{\overline{\text{MS}}}^2} + \sum_{\Phi} A_{\Phi} \right) \right],$$

while in the $\overline{\text{DR}}$ scheme, the result is

$$M_{\text{pole}} = M_{\overline{\text{DR}}} (\mu) \left[ 1 + \frac{g_{\overline{\text{DR}}}}{16\pi^2} \left( 5C(G) + 3C(G) \ln \frac{\mu^2}{M_{\overline{\text{DR}}}^2} + \sum_{\Phi} A_{\Phi} \right) \right].$$

In both cases

$$A_{\Phi} \equiv 2S(R_{\Phi}) \int_0^1 dx \ln \left( [xm_{\phi}^2 + (1-x)m_{\psi}^2 - x(1-x)M_{\text{pole}}^2]/\mu^2 \right)$$

and the sum is over all chiral supermultiplets $\Phi = (\phi, \psi)$ which couple to the gaugino. Comparing (2.1) and (2.2), we obtain (to one-loop order)

$$M_{\overline{\text{MS}}} = M_{\overline{\text{DR}}} \left[ 1 + \frac{g^2}{16\pi^2} C(G) \right].$$

Similarly, we can compute the relationship between the gauge couplings in the two schemes. Here we need to make an important distinction between the gauge coupling $g$ which appears in the interactions of the vector gauge bosons, and the coupling $\hat{g}$ which occurs in the Yukawa interaction

$$\mathcal{L}_{\hat{g}} = \sqrt{2} \hat{g} \phi^i (t^A)_{ij} (\psi_j \lambda^A) + \text{H. c.}$$

of the gaugino and the chiral superpartners. Gauge invariance guarantees that $g$ is the same everywhere it appears, but only supersymmetry guarantees that $\hat{g} = g$. Hence we have $\hat{g} = g$ in DRED, but we expect $\hat{g}_{\overline{\text{MS}}} \neq g_{\overline{\text{MS}}}$ in DREG, since the radiative corrections in DREG violate supersymmetry. By computing the relevant one-loop graphs in the effective
action, and demanding that the physical scattering amplitudes computed in each scheme are the same, we find
\[ g_{\text{MS}} = g_{\text{DR}} \left[ 1 - \frac{g^2}{96\pi^2} C(G) \right] \]  
(2.4)
and
\[ \hat{g}_{\text{MS}} = g_{\text{DR}} \left\{ 1 + \frac{g^2}{32\pi^2} [C(G) - C(r)] \right\} . \]  
(2.5)
Thus there is a different coupling \( \hat{g}_{\text{MS}} \) of the gaugino to each distinct irreducible representation \( r \) of the chiral supermultiplets. Note that we consistently neglect the distinction between \( g_{\text{MS}}, \hat{g}_{\text{MS}}, \) and \( g_{\text{DR}} \) in the one-loop correction parts of (2.3), (2.4), and (2.5), and in the two-loop correction parts of formulas below, since all of the incarnations of the gauge coupling are the same to zeroth order. The relations (2.3) and (2.4) have been discussed by Yamada [13] (see also [14,15]) for a theory with only gauge vector supermultiplets. Here we note that these formulas are not modified by the presence of chiral superfields, which is not surprising in view of the fact that DRED and DREG only differ for graphs in which there is at least one internal gauge boson line which does not terminate (at either end) in a scalar-scalar-gauge boson vertex.

As a non-trivial consistency check, we consider the two-loop running of both \( g_{\text{MS}} \) and \( \hat{g}_{\text{MS}} \), by specializing the results of [10,11] for a general renormalizable theory to the case of \( N = 1 \) supersymmetry. The gauge coupling satisfies
\[ \mu \frac{d}{d\mu} g_{\text{MS}} = \frac{1}{16\pi^2} b^{(1)} + \frac{1}{(16\pi^2)^2} b^{(2)} \]  
(2.6)
\[ b^{(1)} = g^3 \left[ S(R) - 3C(G) \right] \]
\[ b^{(2)} = g^5 \left\{ -6[C(G)]^2 + 2C(G)S(R) + 4S(R)C(R) \right\} - g^3 Y^{ijk} Y_{ijk} C(k)/d(G) \]
(with all \( \overline{\text{MS}} \) couplings on the RHS) as is already known [16]. Here \( S(R) \) is the Dynkin index summed over all chiral multiplets and \( S(R)C(R) \) is the sum of the Dynkin indices weighted by the quadratic Casimir invariant. Note that the gauge coupling \( g_{\text{DR}} \) also satisfies eq. (2.6) with \( g_{\text{MS}} \) replaced everywhere by \( g_{\text{DR}} \) according to (2.4); this is the well-known result that the \( \beta \)-function for the gauge coupling is scheme-independent through two loops.

On the other hand, for the coupling of a gaugino to a chiral supermultiplet in the irreducible representation \( r \), we find from [11]
\[ \mu \frac{d}{d\mu} \hat{g}_{\text{MS}} = \frac{1}{16\pi^2} \hat{b}^{(1)} + \frac{1}{(16\pi^2)^2} \hat{b}^{(2)} \]  
(2.7)
\[ \hat{b}^{(1)} = \hat{g}_{\text{MS}}^2 S(R) - 3 \hat{g}_{\text{MS}}^2 C(G) + 3(\hat{g}_{\text{MS}}^2 - \hat{g}_{\text{MS}}^2) C(r) \]
\[ \hat{b}^{(2)} = g^5 \left\{-10[C(G)]^2 + 2C(G)S(R) + 5S(R)C(R) - S(R)C(r) - C(G)C(r) + 3[\hat{C}(r)]^2 \right\} - g^3 \tilde{Y}^{ijk} Y_{ijk} C(k) /d(G) \]

Here \( \hat{g}_{\text{MS}}^2 S(R) \) means a sum over all chiral multiplets of the Dynkin index times the appropriate \( \hat{g}_{\text{MS}}^2 \). It is easy to check that (2.7) is indeed consistent with (2.4), (2.5), and (2.6).

We now consider the Yukawa coupling \( Y^{ijk} \) between a scalar \( \phi_i \) and two chiral fermions \( \psi_j, \psi_k \). By again demanding that physical scattering amplitudes computed from the one-loop effective action in each scheme are the same, we find

\[ \left[ Y_{ij} \right]_{\text{MS}}^{jk} = Y_{ijm} Y_{klm} + g^2 \left( t^A_i t^A_j + t^A_k t^A_l \right) \]  

Note that the \( Y^{ijk} \) are totally symmetric in the supersymmetry-respecting \( \overline{\text{DR}} \) scheme (because of the way they appear in the superpotential), but not in the \( \overline{\text{MS}} \) scheme due to the radiative corrections. In \( N = 1 \) supersymmetry, the scalar quartic interaction

\[ \mathcal{L}_{\text{quartic}} = -\frac{1}{4} \lambda_{ij}^{kl} \phi^i \phi^j \phi_k \phi_l \]  

depends entirely on the Yukawa and gauge couplings. In a supersymmetry-respecting scheme like \( \overline{\text{DR}} \), this dependence is given simply by

\[ \lambda_{ij}^{kl} = Y_{ijm} Y^{klm} + g^2 \left( t^A_i t^A_j + t^A_k t^A_l \right) \]  

However, in \( \overline{\text{MS}} \) this relation is modified by radiative corrections. Indeed, we find

\[ [\lambda_{\overline{\text{DR}}}]_{ij}^{kl} - [\lambda_{\overline{\text{MS}}}]_{ij}^{kl} = \frac{g^4}{16\pi^2} \{ t^A_i, t^B \}_j^k \{ t^A_i, t^B \}_j^l + (i \leftrightarrow j) \]  

again by comparing physical scattering amplitudes derived from the one-loop effective actions in the two schemes. This completes the “dictionary” for translating between \( \overline{\text{MS}} \) and \( \overline{\text{DR}} \) couplings at one loop order. A gauge-invariant, supersymmetric fermion mass coming from a quadratic term in the superpotential will also differ between the two schemes, but that difference is trivially derived from (2.8) by taking the scalar with index \( i \) to be a dummy field with \( C(r_i) = 0 \) and \( C(r_j) = C(r_k) \). None of the other couplings (written in component language rather than superfield language) of a general softly-broken
3. Two-Loop Running of Gaugino Masses

In this section we will consider the two-loop $\beta$-functions for the gaugino mass computed in both DREG and DRED. The results of [11] provide the 2-loop $\overline{\text{MS}}$ $\beta$-function for a scalar-fermion-fermion coupling in a general renormalizable theory. From this, it is trivial to extract the corresponding result for a fermion mass term, by treating the external scalar in eqs. (3.3) and (3.4) of [11] as a dummy field (with no gauge or other Yukawa interactions).

After further specializing to the case of a gaugino mass in $N = 1$ supersymmetry, and using (2.4) and (2.5) to eliminate $\hat{g}_{\overline{\text{MS}}}$ in favor of $g_{\overline{\text{MS}}}$, we obtain:

$$\mu \frac{d}{d\mu} M_{\overline{\text{MS}}} = \frac{1}{16\pi^2} \gamma_{\overline{\text{MS}}}^{(1)} + \frac{1}{(16\pi^2)^2} \gamma_{\overline{\text{MS}}}^{(2)} \left( \frac{1}{16} \pi^2 \right)^2 \gamma_{\overline{\text{MS}}}^{(2)}$$

(3.1)

$$\gamma_{\overline{\text{MS}}}^{(1)} = g_{\overline{\text{MS}}}^2 \left[ 2S(R) - 6C(G) \right] M_{\overline{\text{MS}}}$$

$$\gamma_{\overline{\text{MS}}}^{(2)} = g_{\overline{\text{MS}}}^4 \left\{ -32[C(G)]^2 + \frac{22}{3} C(G) S(R) + 16 S(R) C(R) \right\} M$$

$$+ 2 g_{\overline{\text{MS}}}^2 \left[ h_{ijk} - MY_{ijk} \right] Y_{ijk} C(k)/d(G) \right.$$

$$The simplest way to find the corresponding equation in DRED is to plug eqs. (2.3) and (2.4) into (3.1). This gives$$

$$\mu \frac{d}{d\mu} M_{\text{DR}} = \frac{1}{16\pi^2} \gamma_{\text{DR}}^{(1)} + \frac{1}{(16\pi^2)^2} \gamma_{\text{DR}}^{(2)} \left( \frac{1}{16} \pi^2 \right)^2 \gamma_{\text{DR}}^{(2)}$$

(3.2)

$$\gamma_{\text{DR}}^{(1)} = g_{\text{DR}}^2 \left[ 2S(R) - 6C(G) \right] M_{\text{DR}}$$

$$\gamma_{\text{DR}}^{(2)} = g_{\text{DR}}^4 \left\{ -24[C(G)]^2 + 8 C(G) S(R) + 16 S(R) C(R) \right\} M$$

$$+ 2 g_{\text{DR}}^2 \left[ h_{ijk} - MY_{ijk} \right] Y_{ijk} C(k)/d(G) \right.$$}

The method used here avoids potential ambiguities and conceptual problems [12] associated with doing two-loop calculations directly in DRED. We will return to these issues elsewhere.

Eqs. (3.1) and (3.2) generalize the expressions given by Yamada[13], who pointed out that the one-loop relation $\beta_M = (2M/g)\beta_g$ fails to hold at the two-loop level.

4. Direct Product Gauge Groups and the MSSM

As promised, we now point out the modifications which must be made to the preceding
formulas if the gauge group is a product of simple [or \( U(1) \)] subgroups \( G_a \). The formulas involving radiative corrections for a gauge coupling \( g_a \) [(2.4)-(2.7)] or gaugino mass \( M_a \) [(2.1)-(2.3) and (3.1)-(3.2)] for each subgroup \( G_a \) may be obtained by the following set of rules [10]. First, each term which does not involve the quadratic Casimir invariant of a non-adjoint representation is diagonal in subgroups; i.e., obtained by \( g \to g_a, S(R) \to S_a(R), \) and \( C(G) \to C(G_a) \). For the other terms, one has \( g^2C(r) \to \sum_b g_b^2 C_b(r) \) in (2.5) and (2.8); \( C(k)/d(G) \to C_a(k)/d(G_a) \) in (2.6), (2.7), (3.1), and (3.2); and

\[
g^5 S(R)C(R) \to g_a^3 S_a(R) \sum_b g_b^2 C_b(R)
\]

in eq. (2.6). [Consistency then determines the replacement rules for the \( C(r) \)-dependent terms in (2.7).] The correction term in (2.11) becomes a double sum \( \sum_b \sum_c g_b^2 g_c^2 \ldots \) over subgroups. Finally, in eqs. (3.1) and (3.2) we need the replacement

\[
16 g^4 C(R) S(R) M \to 8 g_a^2 S_a(R) \sum_b g_b^2 C_b(R) (M_a + M_b).
\]

In the MSSM, the gauge group is \( SU(3)_c \times SU(2)_L \times U(1)_Y \), with chiral superfields \( Q \) and \( L \) for the \( SU(2)_L \)-doublet quarks and leptons, and \( u, d, e \) for the \( SU(2)_L \)-singlet quarks and leptons, and two Higgs doublet chiral superfields \( H_u \) and \( H_d \). The relevant part of the superpotential is

\[
W = H_u Q Y_u u + H_d Q Y_d d + H_d L Y_e e
\]

where \( Y_u, Y_d, Y_e \) are \( 3 \times 3 \) Yukawa matrices. The soft supersymmetry-breaking Lagrangian includes trilinear scalar couplings

\[
-L = H_u Q h_{u} u + H_d Q h_{d} d + H_d L h_{e} e + \text{h.c.}
\]

where \( h_{u,d,e} \) are again \( 3 \times 3 \) matrices in family space, and we use the same symbol for scalar components as for the chiral superfield. Then in the \( \overline{\text{DR}} \) scheme the 2-loop running of the three gauge couplings are given by

\[
\mu \frac{d}{d\mu} g_a = \frac{g_a^3}{16\pi^2} B_a^{(1)} + \frac{g_a^3}{(16\pi^2)^2} \left[ B_a^{(2)} g_a^2 + \sum_{b=1}^{3} B_{ab}^{(2)} g_b^2 - \sum_{x=u,d,e} C_a^x \text{Tr} Y_x Y_x^\dagger \right].
\]

Here \( B_a^{(1)} = (33/5, 1, -3) \) for \( U(1)_Y \) (in a GUT normalization), \( SU(2)_L \), and \( SU(3)_c \) respectively; \( B_a^{(2)} = (0, 4, -18) \), and

\[
B_{ab}^{(2)} = \begin{pmatrix}
199/25 & 27/5 & 88/5 \\
9/5 & 21 & 24 \\
11/5 & 9 & 32
\end{pmatrix}
\]

and

\[
C_a^{u,d,e} = \begin{pmatrix}
26/5 & 14/5 & 18/5 \\
6 & 6 & 2 \\
4 & 4 & 0
\end{pmatrix}.
\]
The renormalization group equation for the three gaugino mass parameters in \( \overline{\text{DR}} \) can then be written easily in terms of the same coefficients:

\[
\mu \frac{d}{d\mu} M_a = \frac{2g_a^2}{16\pi^2} B_a^{(1)} M_a + \frac{2g_a^2}{(16\pi^2)^2} \left[ 2B_a^{(2)} g_a^2 M_a + \sum_{b=1}^{3} B_{ab}^{(2)} g_b^2 (M_a + M_b) + \sum_{x=u,d,e} C_x^{(2)} \left( \text{Tr} Y_x h_x^\dagger - M_a \text{Tr} Y_x Y_x^\dagger \right) \right]
\]

(4.4)

In the \( \overline{\text{MS}} \) scheme, the two-loop renormalization group equations for the gauge couplings are exactly the same as in \( \overline{\text{DR}} \) (provided, of course, that one consistently uses \( g_{\overline{\text{MS}}} \) rather than \( \hat{g}_{\overline{\text{MS}}} \)). The \( \overline{\text{MS}} \) two-loop renormalization group equations for the gaugino mass parameters in the MSSM are given by an equation exactly like (4.4), with the only difference that \( B_a^{(2)} \) must be replaced in (4.4) by \( B_a^{(2)\overline{\text{MS}}} = (0, 16/3, -24) \).

5. Conclusion

In this paper, we have presented analytical expressions for the differences between couplings in the \( \overline{\text{MS}} \) and \( \overline{\text{DR}} \) schemes, to one loop order. The \( \overline{\text{MS}} \) scheme, while convenient for analyzing the predictions of the standard model, is slightly inconvenient for the MSSM because it violates supersymmetry, in the sense that the tree-level supersymmetric relations between coupling constants are modified by finite radiative corrections even at one loop. We were able to exploit this fact to derive the two-loop \( \beta \)-function for a gaugino mass in \( \overline{\text{DR}} \) from knowledge of the corresponding formula in the \( \overline{\text{MS}} \) scheme, which in turn followed from the results for a general renormalizable theory (using \( \overline{\text{MS}} \)) given in [10,11]. The same method can in fact be used to derive the renormalization group equations for all of the other soft supersymmetry-breaking parameters of the MSSM. We will report on this in a future publication.

Let us close by remarking on the significance of our results for predictions in supergravity-inspired models. These are typically obtained by choosing a set of input parameters, including a common gaugino mass, at some very high scale and running the parameters down to the electroweak scale. The boundary conditions at the very high scale should presumably be applied in a supersymmetry-respecting scheme like \( \overline{\text{DR}} \), and threshold effects are simpler [14] in \( \overline{\text{DR}} \). However, it may be more convenient at low energies to work in \( \overline{\text{MS}} \). In the MSSM, the two-loop contributions can lower the predicted value of the gluino
mass by several percent compared to the one-loop predictions, while having a much smaller
effect on the other gaugino masses. (In extensions of the MSSM with non-minimal particle
content, these corrections are potentially much larger.) Another effect which is at least
as important is the proper treatment of the gluino pole mass, as given by eqs. (2.1) and
(2.2). One might choose to evaluate the running mass at a scale equal to the running mass
at that scale (i.e., solve the equation $M_{\overline{\text{MS}}}^\mu(\mu) = \mu$ or $M_{\overline{\text{DR}}}^\mu(\mu) = \mu$, depending on which
scheme one is working in); however, this will always underestimates the true pole mass of
the gluino, and often by more than 5 per cent. To roughly estimate the size of the effect,
suppose the squarks are taken to be degenerate with the gluino and much heavier than all
quarks; then from (2.2) one finds $M_{\text{pole}} \approx M_{\overline{\text{DR}}}(M_{\overline{\text{DR}}})[1 + 9\alpha_3/4\pi]$. Such considerations
may have an effect on future efforts to understand the parameters of the MSSM.

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