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On study of fractional order epidemic model of COVID-19 under non-singular Mittag–Leffler kernel

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ARTICLE INFO

Keywords:
Fractional mathematical model
COVID-19
Qualitative analysis
Adams–Bashforth method
Numerical simulations

Mathematics Subject Classification 2020:
37A25
34D20
37M01

ABSTRACT

This paper investigates the analysis of the fraction mathematical model of the novel coronavirus (COVID-19), which is indeed a source of threat all over the globe. This paper deals with the transmission mechanism by some affected parameters in the problem. The said study is carried out by the consideration of a fractional-order epidemic model describing the dynamics of COVID-19 under a non-singular kernel type of derivative. The concerned model examine via non-singular fractional-order derivative known as Atangana-Baleanu derivative in Caputo sense (ABC). The problem analyzes for qualitative analysis and determines at least one solution by applying the approach of fixed point theory. The uniqueness of the solution is derived by the Banach contraction theorem. For iterative solution, the technique of iterative fractional-order Adams-Bashforth scheme is applied. Numerical simulation for the proposed scheme is performed at various fractional-order lying between 0, 1 and for integer-order 1. We also compare the compartmental quantities of the said model at two different effective contact rates of $\beta$. All the compartments show convergence and stability with growing time. The simulation of the iterative techniques is also compared with the Laplace Adomian decomposition method (LADM). Good comparative results for the whole density have been achieved by different fractional orders and obtain the stability faster at the low fractional orders while slowly at higher-order.

Introduction

Recently, a dangerous pandemic called COVID-19 has spread extensively all over the globe. Scientists named it COVID-19 because of the seventh-generation coronavirus. Almost all over the world, 140 million people have been affected by the COVID-19 and 3 million people have been forced to die in 216 countries. Many countries ordered their citizens to follow the sop’s, blockade and stopped air and plane traffic to control the infection’s from furthermore transmission. WHO declared it a worldwide pandemic [1]. The economic conditions and health systems of many countries have collapsed. Historically, the outbreak started at the seafood market in Wuhan at the end of 2019, and the virus infected the entire city within a month. The laws and forcemeat agencies of China have sealed the entire cities in the crucial time, while the infectious populations were collected to quarantine areas. In this way, the countries mentioned above will be able to control their infections within two months. Due to immigration and travel, the infection spread to almost the world within two months. Thus, scientists, doctors, and policymakers are continuously controlling the further spread of this killer infection. Every country has taken all the precautions [2–5].

For the analysis of this outbreak, different technologies, procedures and tools were used to understand and to take some preventive or safety measures. Therefore, the tools of mathematical modeling were also used as powerful tools for understanding the spread and planning of infectious diseases. In this regard, many infectious models of various diseases in the past have been constructed. We are referring to some [6–9]. Infected diseases constitute significant news to humans and can seriously affect the economy of any state. A correct sympathetic to disease dynamics may play an essential role in eliminating community infections [10–12]. Similarly, implementing appropriate disease transmission control strategies is also considered a huge challenge. Mathematical modeling methods are one of the key tools to deal with these and other challenges. Several general models and epidemic problems have been studied in the pre-existing books to find better and control infectious viruses spread [13–15] as follows.

https://doi.org/10.1016/j.rinp.2021.104402

Received 5 May 2021; Received in revised form 23 May 2021; Accepted 27 May 2021
Available online 12 June 2021
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Parametric values for our model (2).

| Parameter value Parameter | Value |
|--------------------------|-------|

| Parameter value | Value |
|-----------------|-------|

\[
\begin{align*}
\frac{dS}{dt} & = \Pi - \beta \frac{SE}{N} - \zeta S, \\
\frac{dE}{dt} & = \beta \frac{SE}{N} - (\zeta + \sigma + \psi) E, \\
\frac{dI}{dt} & = \sigma E - (\zeta + \sigma + \eta_1) I, \\
\frac{dH}{dt} & = \chi I - (\zeta + \chi + \eta_2) H, \\
\frac{dR}{dt} & = \zeta H + \psi E - \zeta R.
\end{align*}
\]

In model (1), we develop a coronavirus model to describe the interaction between the population where the COVID-19 exists. The whole population is divided into five sub-classes namely the susceptible class \( S \), asymptotic infected class \( E \), infected individual \( I \), hospitalized class \( H \) and the recovered class \( R \). The used parametre in system (1) with whole description are given below.

In recent years, using fractional differential equations (FDE), people can model common phenomena with greater degrees of freedom [16]. Many subjects have realized this idea, such as engineering, economics, control theory, and finance and have achieved remarkable results. Modern calculus is an extension of classic calculus. FDE has multiple attributes, people are increasingly interested in using FDE to model complex and practical problems, while integer attributes are not more realistic differential equations (IDE). On comparing local IDE, FDE is non-local, has a storage effect, and has superior performance. For analysis of different cases, the coming situation of the model will change not only based on the recent time but also based on past historical records. These characteristics enable FDE to not only simulate non-Gaussian phenomena virtually but also to simulate non-Markov phenomena effectively. Besides, traditional IDEs cannot provide information between two dissimilar integer values, nor can they clarify FDE’s help. Different kinds of non-integer order derivative operators have been proposed in the various existing sources of research to affect this limitation of differentiation only integer values. The uses of the fractional-order differential operators may be seen in various fields [17–20].

During the 18th century, Fourier, Euler, Liouville, and Riemann worked hard to obtain excellent general calculus results. At that time, famous work was provided in the area of modern calculus. The uses of non-integer order calculus in mathematical formulation make it interesting as compare to ordinary calculus which cannot clearly explain several genetic materials and storage processes. Modern calculus (including classical calculus) in a particular situation, the degree of freedom of its differential operator is more important than that of the local ordinary differential operator. The main application of calculation can be found in [21–26]. Therefore, researchers attach great importance to the study of non-integer order differentiation and integrations. The arbitrary order derivatives are mostly the definite integrals, explaining the entire function’s accumulation from a geometric perspective or the entire global integration range. Researchers have made significant contributions to the optimization of qualitative, numerical research, and differential equation research. We are referring to [27–34]. It is also worth noting that modern derivative operators were formulated in some definite integrals techniques. It is a familiar fact that there is no core rule for determining this. Therefore, both types of kernels involve various definitions. In 2016, the fundamental formulation that accepts more attraction is the Atangana, Baleanu (ABC) fractional-order derivative [35]. Mostly non-linear equations are not easy to compute for exact or analytical solutions. Therefore, many numerical techniques have found to evaluate the said equations. Many researchers, have recently investigated numerical methods to study FDE under ABC derivatives [36–44].

The mathematical formulation has a vital part in studying the dynamics of diseases and their control, especially when there is no vaccination or the early disease stage. This field is dedicated to studying biological models of infectious diseases in recent research hot spots.
Many mathematical models can be found in stability theory, existing solutions and optimizing of epidemiological systems. Inspired by the characteristics of FDEs particularly applying the formula of ABC fractional-order derivative and from [45], we take the transmission of the COVID-19 model as follows

\[ \begin{align*}
\text{ABC} D^\alpha_S t = \Pi - \beta \frac{SE}{N} - \zeta S, \\
\text{ABC} D^\alpha E = \beta \frac{SE}{N} - \left( \zeta + \sigma + \psi \right) E,
\end{align*} \]

(2)

To analyse (2), we use the following initial conditions

\[ S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \quad H(0) = H_0, \quad R(0) = R_0. \]

This manuscript is different sections as given: In part “Preliminaries” we provide basic definitions from modern calculus. In part “Qualitative Analysis of model (2)” we develop some significant proofs for qualitative analysis using the “Banach fixed point theory” approach along with Ulam-Hyers stability. In part “Numerical scheme”, by AB technique, we determine the approximate solution of the proposed model. We also perform numerical simulations to illustrate and validate the approximate solution of the part “Qualitative Analysis of model (2)”. The obtained results are briefly discussed in the same section as well. Finally, Section “Conclusions” concludes our work.

Preliminaries

For a better understanding of the manuscript, we consider it advantageous to provide some useful and important results related to fractional calculus and non-linear analysis [35,46,47].

**Definition 1.** The ABC in the Caputo sense derivative of order \( \delta \in (0,1] \) for a mapping \( \mathcal{H}(t) \in \mathcal{H}^1[0,T] \) is given as

\[ \text{ABC} D^\alpha \mathcal{H}(t) = \frac{-\delta}{\Gamma(1-\delta)} \int_0^t (t-s)^{-\delta-1} \mathcal{H}'(s) ds, \]

(3)

where \( \Gamma(\cdot) \) is the Gamma function and \( \delta > 0 \) and it is a Gamma function designated by \( \delta(\cdot) \).

**Definition 2.** Let a function \( \mathcal{H} \in L^1(0,T) \) and the left hand side of fractional integral of order \( \delta \in (0,1] \) in \( \text{ABC} \) sense is defined as

\[ \text{ABC} D^\alpha \mathcal{H}(t) = \frac{1-\delta}{\Gamma(1-\delta)} \mathcal{H}'(t) + \frac{\delta}{\Gamma(1-\delta)} \int_0^t (t-s)^{-\delta-1} \mathcal{H}'(s) ds, \quad t > 0. \]

(4)

**Lemma 1.** ([47]) The solution for the following problem is given as and for \( \delta \in [0,1] \)

\[ \text{ABC} D^\alpha \mathcal{H}(t) = \Psi(t), \quad \mathcal{H}(0) = \mathcal{H}_0, \]

\[ \mathcal{H}(t) = \mathcal{H}_0 + \frac{1-\delta}{\Gamma(1-\delta)} \int_0^t (t-s)^{-\delta-1} \Psi(s) ds, \]

(5)

assumed as

**Theorem 1.** Assume \( \mathcal{S} \) be a closed norm linear space and \( D \subset \mathcal{S} \) be the convex, limits existing set. If a continuous operator \( \psi : D \rightarrow D \) such that \( \psi D \subset \mathcal{S} \) and \( \psi D \) is compact relatively, then the operator hasone or greater than one fixed point in \( D \).

Feasibility and boundedness of solution

**Lemma 2.** All feasible solutions \( S(t), E(t), I(t), H(t) \) and \( R(t) \) for Eq. (2) are bounded in the region

\[ \Omega = \left\{ (S,E,I,H,R) \in R^5 : S + E + I + H + R < \frac{N}{\zeta} \right\}. \]

Proof. For proof we have to add all the five equations of (2), and get as

\[ \frac{dN}{dt} = \Pi - \frac{\zeta N(t)}{H(t)} - \eta_1 I - \eta_2 H. \]

(7)

**Implies that**

\[ \frac{dN}{dt} = \Pi - \frac{\zeta N(t)}{H(t)}. \]

(8)

It follows that

\[ \frac{dN}{dt} + \frac{1}{\zeta} N(t) + N(t) e^{-\zeta t}. \]

(9)

Here \( N(0) \) is constant initial population. In (9), as \( t \) tends to \( \infty \), then \( N(t) \leq \frac{\Pi}{\sigma} \).

The last result proved the boundedness and feasibility of solution.

Qualitative Analysis of model (2)

Existence results

In this section of the paper, we show the existence, uniqueness of solution and stability results of the considered model (2) using the approach of fixed point theory. To obtain the desired results, we reformulate the model under investigation in the following form

\[ \begin{align*}
\text{ABC} D^\alpha S(t) &= G_1(t,S,E,I,H,R), \quad \\
\text{ABC} D^\alpha E(t) &= G_2(t,S,E,I,H,R), \quad \\
\text{ABC} D^\alpha I(t) &= G_3(t,S,E,I,H,R), \quad \\
\text{ABC} D^\alpha H(t) &= G_4(t,S,E,I,H,R), \quad \\
\text{ABC} D^\alpha R(t) &= G_5(t,S,E,I,H,R),
\end{align*} \]

(10)

where

\[ \begin{align*}
G_1(t,S,E,I,H,R) &= \Pi - \beta \frac{SE}{N} - \zeta S, \quad \\
G_2(t,S,E,I,H,R) &= \beta \frac{SE}{N} - \left( \zeta + \sigma + \psi \right) E, \quad \\
G_3(t,S,E,I,H,R) &= \sigma E - \left( \zeta + \sigma + \psi \right) I, \quad \\
G_4(t,S,E,I,H,R) &= \delta E - \left( \zeta + \xi + \eta_1 \right) H, \quad \\
G_5(t,S,E,I,H,R) &= \delta H + \psi E - \zeta R.
\end{align*} \]

(11)

Next, we express the model (2) in the following form

\[ \begin{align*}
\text{ABC} D^\alpha \mathcal{X}(t) &= \Omega(t, \mathcal{X}(t)), \quad \mathcal{X}(0) = \mathcal{X}_0,
\end{align*} \]

(12)

where

\[ \mathcal{X} := (S,E,I,H,R)^T, \quad \mathcal{X}(0) := (S(0),E(0),I(0),H(0),R(0))^T, \quad \Omega(t, \mathcal{X}_0) := G(t,S,E,I,H,R)^T, \quad i = 1,2,3,9. \]

(13)
Note that $(\cdot)^T$ presents the transpose of vector. Using Lemma 1, the Eq. (12) is same as the given equation

\[ X(t) = X_0 + \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t, X(t))} + \frac{\delta}{M(\delta) I(\delta)} \int_0^t (t - \beta)^{k-1} \Omega \left( \beta, X(\beta) \right) d\beta. \]  

(14)

Further, let us prescribe closed norm space \( Y = C([0, T], \mathbb{R}) \) having norm \( \| Y \| = \sup_{t \in [0, T]} | Y(t) | \). We give the close norm space \( z = (Y^*, \| Y \|) \) having norm \( \| Y \| \).

Let, find the existence results for the considered system (2) by applying the theorem of Schauder’s.

**Theorem 2.** Suppose a continuous function \( \Omega \in z \) and \( \exists \delta > 0, \exists | \Omega(t, X(t)) | \leq \frac{1}{2} \| Y \| + 1, \forall t \in [0, T], X(t) \in z \). For the considered model (2) there exists at least one solution as

\[ \nabla_1 = \left( 1 - \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t, X(t))} + \frac{\delta}{M(\delta) I(\delta)} \right)^{1/2}. \]

(15)

**Proof.** It is clear that solution of the considered model (2) is equivalent to Eq. (14). Suppose the function \( Y \) is given as

\[ (Y X)(t) = X_0 + \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t, X(t))} + \frac{\delta}{M(\delta) I(\delta)} \int_0^t (t - \beta)^{k-1} \Omega \left( \beta, X(\beta) \right) d\beta. \]

(16)

Let the bounded closed convex ball is defined as \( B_m = \{ X \in \Omega : \| X \| \leq m, \ m > 0 \} \) with \( m \geq \frac{1}{\nabla_1} \), where

\[ \nabla_2 = \left( X_0 + \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t, X(t))} + \frac{\delta}{M(\delta) I(\delta)} X \right). \]

(17)

Now, we need to show that \( (Y B_m) \subset B_m, \forall t \in [0, T] \). We have

\[ \| (Y X)(t) \| \leq \| X_0 \| + \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t, X(t))} + \frac{\delta}{M(\delta) I(\delta)} \int_0^t (t - \beta)^{k-1} \Omega \left( \beta, X(\beta) \right) d\beta. \]

(18)

Again since \( X \in B_m \), one may write

\[ \| (Y X)(t_2) - (Y X)(t_1) \| \leq \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t_2 - t_1)} + \frac{\delta}{M(\delta) I(\delta)} \left( 1 + \frac{\| X \|}{M(\delta)} \right) \left( t_2 - t_1 \right). \]

For this let, \( X \in B_m \) and there is \( t_1, t_2 < 0 \) with \( t_1 < t_2 \). Then we have

\[ \| (Y X)(t_2) - (Y X)(t_1) \| \leq \frac{1 - \frac{\delta}{M(\delta)}}{\Omega(t_2 - t_1)} + \frac{\delta}{M(\delta) I(\delta)} \left( 1 + \frac{\| X \|}{M(\delta)} \right) \left( t_2 - t_1 \right). \]
Apparently, the right side $\|Y(t_2) - Y(t_1)\|$ goes to zero as $t_2 \to t_1$. By the well known “Arzela-Ascoli theorem”, the operator $(Y_{m})$ is relatively compact and thus the operator $Y$ is completely continuous. We deduce from Theorem 1 that the proposed system (2) has at least one solution. Hence, we finished the proof.

**Stability result**

In the next theorem, we discuss the stability results for the considered model (2) due to a small perturbation in the starting.

**Theorem 3.** Let we have continuous operator $\Omega \in \mathbb{C}$ and there exist $J > 0$
\[ \exists [\Omega(t, \mathcal{X}) - \Omega(t, \mathcal{X})] \leq J ||\mathcal{X} - \mathcal{X}||, \forall \ t \in [0, T] \text{ and } \mathcal{X} \in \mathbb{C} \]

Suppose $\mathcal{X}$ and $\mathcal{X}$ be the solution for problem (12) and
\[ \begin{align*}
\mathcal{X} = (S(0) + \epsilon, E(0) + \epsilon, I(0) + \epsilon, R(0))^{T}, \Omega (1, \mathcal{X}) = G_{i}(S, E, I, R), \quad i = 1, 2, 3, 4, 5.
\end{align*} \tag{19} \]
respectively, where
\[ \mathcal{X} = (S, E, I, R, \mathcal{X})^{T}, \mathcal{X} + \epsilon = (S(0) + \epsilon, E(0) + \epsilon, I(0) + \epsilon, R(0))^{T}, \Omega (1, \mathcal{X}) = G_{i}(S, E, I, R), \quad i = 1, 2, 3, 4, 5. \tag{20} \]

Then the following inequality holds
\[ \|\mathcal{X} - \mathcal{X}\| \leq J \left(1 - \frac{(1 - \delta)\Gamma(\delta) J + JT^\delta}{M(\delta)\Gamma(\delta)}\right)^{-1} |\epsilon|. \tag{21} \]

**Proof.** We note that the solutions of the systems (12) and (19) equal to Eq. (14) and
\[ \begin{align*}
\mathcal{X}(t) &= \mathcal{X}_{0} + \epsilon + \frac{1 - \delta}{M(\delta)} \Omega (1, \mathcal{X}) + \frac{\delta}{M(\delta)\Gamma(\delta)} \int_{0}^{t} (t - \beta)^{\delta - 1} \Omega (\beta, \mathcal{X}(\beta)) d\beta, \quad t \in [0, T].
\end{align*} \tag{22} \]

respectively. It follows that for every $t \in [0, T]$ we may have
\[ \begin{align*}
&\|\mathcal{X}(t) - \mathcal{X}(t)\| \leq \frac{1 - \delta}{M(\delta)} \Omega (1, \mathcal{X}) + \frac{\delta}{M(\delta)\Gamma(\delta)} \int_{0}^{t} (t - \beta)^{\delta - 1} \Omega (\beta, \mathcal{X}(\beta)) d\beta, \|\mathcal{X} - \mathcal{X}\| \leq J \left(1 - \frac{(1 - \delta)\Gamma(\delta) J + JT^\delta}{M(\delta)\Gamma(\delta)}\right)^{-1} |\epsilon|.
\end{align*} \tag{23} \]

One may then write
\[ \|\mathcal{X} - \mathcal{X}\| \leq J \left(1 - \frac{(1 - \delta)\Gamma(\delta) J + JT^\delta}{M(\delta)\Gamma(\delta)}\right)^{-1} |\epsilon|. \tag{24} \]

Thus, we proved the theorem.

**Remark 1.** By inserting $\epsilon = 0$ in derived theorem, we obtain the unique solution for the considered problem (2).

**Numerical scheme**

In this part of the manuscript, we investigate the numerical solutions of the proposed system (2). Using the proposed scheme the numerical results are obtained. The famous fractional Adam-Bashforth technique [48] has been used to evaluate the arbitrary-order integration. With the help of starting values along with the application of $\frac{\partial^{\alpha} |\epsilon|}{\partial t^\alpha}$, we change the problem (2) into fractional type integration as
\[ S(t) - S(0) = \frac{1 - \delta}{M(\delta)} \mathcal{K}_1(S(t), t) + \frac{\delta}{M(\delta) \Gamma(\delta)} \int_0^t (1 - B)^{t-1} \mathcal{K}_2(S(\beta), \beta) d\beta, \]
\[ E(t) - E(0) = \frac{1 - \delta}{M(\delta)} \mathcal{K}_3(E(t), t) + \frac{\delta}{M(\delta) \Gamma(\delta)} \int_0^t (1 - B)^{t-1} \mathcal{K}_4(E(\beta), \beta) d\beta, \]
\[ I(t) - I(0) = \frac{1 - \delta}{M(\delta)} \mathcal{K}_5(I(t), t) + \frac{\delta}{M(\delta) \Gamma(\delta)} \int_0^t (1 - B)^{t-1} \mathcal{K}_6(I(\beta), \beta) d\beta, \]
\[ H(t) - H(0) = \frac{1 - \delta}{M(\delta)} \mathcal{K}_7(H(t), t) + \frac{\delta}{M(\delta) \Gamma(\delta)} \int_0^t (1 - B)^{t-1} \mathcal{K}_8(H(\beta), \beta) d\beta, \]
\[ R(t) - R(0) = \frac{1 - \delta}{M(\delta)} \mathcal{K}_9(R(t), t) + \frac{\delta}{M(\delta) \Gamma(\delta)} \int_0^t (1 - B)^{t-1} \mathcal{K}_10(R(\beta), \beta) d\beta. \] 

Fig. 2. Dynamics of $E(t)$ having two different values of $\beta = 0.025, 0.25$ in the model under investigation \(2\) at different arbitrary orders.

Fig. 3. Dynamics of $I(t)$ having two different values of $\beta = 0.025, 0.25$ in the model under investigation \(2\) at different arbitrary orders.
For producing an iterative scheme, we set \( \epsilon = \epsilon_{n+1} \) for \( \delta = 0.1, 2, \ldots \), into the system (25) which results the following

\[
\mathcal{J}_{i-1,j} = \int_{\epsilon_i}^{\epsilon_{i+1}} (\epsilon - \epsilon_i)(\epsilon_{i+1} - \epsilon_i) d\epsilon
\]

Using the two points interpolation polynomial for the approximate functions \( \mathcal{K}_1(S(\epsilon), \beta), \mathcal{K}_2(E(\epsilon), \beta), \mathcal{K}_3(I(\epsilon), \beta), \mathcal{K}_4(I(\epsilon), \beta) \) and \( \mathcal{K}_5(R(\epsilon), \beta) \) which are inside the interval (26) on the interval \([\epsilon_i, \epsilon_{i+1}]\), we obtain

\[
\begin{align*}
\mathcal{K}_1(S(\epsilon), \beta) &= \frac{\mathcal{K}_1^{(\epsilon_{i+1})}}{h} (\epsilon - \epsilon_{i+1}) + \mathcal{K}_1^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{K}_2(E(\epsilon), \beta) &= \frac{\mathcal{K}_2^{(\epsilon_{i+1})}}{h} (\epsilon - \epsilon_{i+1}) + \mathcal{K}_2^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{K}_3(I(\epsilon), \beta) &= \frac{\mathcal{K}_3^{(\epsilon_{i+1})}}{h} (\epsilon - \epsilon_{i+1}) + \mathcal{K}_3^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{K}_4(I(\epsilon), \beta) &= \frac{\mathcal{K}_4^{(\epsilon_{i+1})}}{h} (\epsilon - \epsilon_{i+1}) + \mathcal{K}_4^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{K}_5(R(\epsilon), \beta) &= \frac{\mathcal{K}_5^{(\epsilon_{i+1})}}{h} (\epsilon - \epsilon_{i+1}) + \mathcal{K}_5^{(\epsilon_{i+1})} (\epsilon_i - \epsilon).
\end{align*}
\]

This system further gives

\[
\begin{align*}
\mathcal{S}(\epsilon_{i+1}) &= \mathcal{S}(0) + \frac{1 - \delta}{h} \mathcal{K}_1(\epsilon_i, \epsilon_i) + \frac{\delta}{h} \mathcal{K}_1(\epsilon_i, \epsilon_i) \sum_{i=0}^{i+1} \mathcal{K}_1^{(\epsilon_{i+1})} (\epsilon_{i+1} - \epsilon_i) + \mathcal{K}_1^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{E}(\epsilon_{i+1}) &= \mathcal{E}(0) + \frac{1 - \delta}{h} \mathcal{K}_2(\epsilon_i, \epsilon_i) + \frac{\delta}{h} \mathcal{K}_2(\epsilon_i, \epsilon_i) \sum_{i=0}^{i+1} \mathcal{K}_2^{(\epsilon_{i+1})} (\epsilon_{i+1} - \epsilon_i) + \mathcal{K}_2^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{I}(\epsilon_{i+1}) &= \mathcal{I}(0) + \frac{1 - \delta}{h} \mathcal{K}_3(\epsilon_i, \epsilon_i) + \frac{\delta}{h} \mathcal{K}_3(\epsilon_i, \epsilon_i) \sum_{i=0}^{i+1} \mathcal{K}_3^{(\epsilon_{i+1})} (\epsilon_{i+1} - \epsilon_i) + \mathcal{K}_3^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{H}(\epsilon_{i+1}) &= \mathcal{H}(0) + \frac{1 - \delta}{h} \mathcal{K}_4(\epsilon_i, \epsilon_i) + \frac{\delta}{h} \mathcal{K}_4(\epsilon_i, \epsilon_i) \sum_{i=0}^{i+1} \mathcal{K}_4^{(\epsilon_{i+1})} (\epsilon_{i+1} - \epsilon_i) + \mathcal{K}_4^{(\epsilon_{i+1})} (\epsilon_i - \epsilon), \\
\mathcal{R}(\epsilon_{i+1}) &= \mathcal{R}(0) + \frac{1 - \delta}{h} \mathcal{K}_5(\epsilon_i, \epsilon_i) + \frac{\delta}{h} \mathcal{K}_5(\epsilon_i, \epsilon_i) \sum_{i=0}^{i+1} \mathcal{K}_5^{(\epsilon_{i+1})} (\epsilon_{i+1} - \epsilon_i) + \mathcal{K}_5^{(\epsilon_{i+1})} (\epsilon_i - \epsilon).
\end{align*}
\]

where

\[
\begin{align*}
\mathcal{J}_{i-1,j} &= \int_{\epsilon_i}^{\epsilon_{i+1}} (\epsilon - \epsilon_i)(\epsilon_{i+1} - \epsilon_i) d\epsilon, \\
\mathcal{J}_{i,j} &= \int_{\epsilon_i}^{\epsilon_{i+1}} (\epsilon - \epsilon_i)(\epsilon_{i+1} - \epsilon_i) d\epsilon.
\end{align*}
\]

Now, we simplify the integrals \( \mathcal{J}_{i-1,j} \) and \( \mathcal{J}_{i,j} \) as follows

\[
\mathcal{J}_{i-1,j} = -\frac{1}{\delta}[(\epsilon_{i+1} - \epsilon_{i+1})^3 - (\epsilon_i - \epsilon_i)(\epsilon_{i+1} - \epsilon_i)\delta],
\]

\[
\mathcal{J}_{i,j} = -\frac{1}{\delta}[(\epsilon_{i+1} - \epsilon_i)(\epsilon_{i+1} - \epsilon_i)\delta] - \frac{1}{\delta (\delta - 1)}[(\epsilon_{i+1} - \epsilon_{i+1})(\delta^{i+1} - (\epsilon_{i+1} - \epsilon_i)\delta^{i+1})].
\]

By putting \( \epsilon_i = \delta h \) one can easily deduce that

\[
\mathcal{J}_{i-1,j} = -\frac{\delta^{i+1}}{\delta (\delta + 1)}[(\epsilon_{i+1} - \epsilon_i)^3 - (\epsilon_{i+1} - \epsilon_i)(\epsilon_{i+1} - \epsilon_i)\delta],
\]

(29)
\[ i \delta t + z \delta \hbar + E = \frac{(z + 1 - i)^4 (z - i + 1 + \delta)}{(z + i + 1)} \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]

\[ \mathcal{S}(\mathcal{R}_{\text{eff}}) = \mathcal{S}(\mathcal{R}_0) + \frac{(1 - \delta)}{\mathcal{H}(\mathcal{R}_0)} + \delta \sum_{i=0}^{\infty} \mathcal{R}_i(\mathcal{S}(\mathcal{R}_0), \mathcal{R}_1) \]
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R\left( t_{n+1} \right) = R\left( t_n \right) + \frac{1 - \delta}{M(\delta)} \left[ \mathcal{S}_t (R(t_n), \ell) \right] + \sum_{i=0}^{n} \left[ \mathcal{S}_t (R(t_n), \ell) \right] \delta \left( \frac{1 - \delta}{\delta + 2} \right) \times R^\prime \left[ (z + 1 - i)^{\delta} (z - i + 2 + \delta) - (z - i)^{\delta} (z - i + 2 + 2\delta) \right] - \frac{\mathcal{S}_t (R(t_n), \ell)}{\delta (\delta + 2)} \times R^\prime \left[ (z + 1 - i)^{\delta} - (z - i)^{\delta} \right] \left( z - i + 1 + \delta \right) \right).

(35)

Numerical simulations and discussion

In the section of numerical simulation we provide the graphical representation of our considered model (2) by using different parameters given in Table 1-2 for verification of the obtained scheme. We have simulated twice the compartments of the said model by changing only the value of \( \beta \) (the effective contact rate) at various fractional order of \( \delta \). At \( \beta = 0.025 \) the disease free situation can be seen in all the infectious related compartments except susceptible class. While at \( \beta = 0.25 \) the pandemic can be seen in the infectious class but after some time the situation is controlled and stable. Figs. 1a and 1b represents the dynamics of susceptible class \( S(t) \) at various arbitrary order of \( \delta \) at two different effective contact rate \( \beta = 0.025, 0.25 \). At the first value the contact rate is low so susceptible class increases and tends to zero, secondly as the contact rate is high susceptible class grows but then become stable. The stability is high at low fractional order and vice versa.

Plots Fig. 2a and b show the behavior of exposed population \( E(t) \) at various arbitrary order of \( \delta \) having two different effective contact rates \( \beta = 0.025, 0.25 \). Firstly value the contact rate is low so exposed individuals decrease and then become stable, secondly as the contact rate is high exposed compartment grows but then become stable. The stability is high at low fractional order and vice versa.

Graphs Fig. 3a and b have shown the behavior of infected class \( I(t) \) at various arbitrary order of \( \delta \) having two different effective contact rates \( \beta = 0.025, 0.25 \). As the contact rate is low the infection declines and at high rate the said class after the beginning increases and then moves towards the disease equilibrium of the problem. In Fig. 4a and b shows the hospitalized peoples \( H(t) \) at various arbitrary order of \( \delta \) having two different effective contact rates \( \beta = 0.025, 0.25 \). Here again at low contact rate the hospitalized peoples are declines and tends to the disease free equilibrium of the problem but at high rate the said peoples increases and then become stable after decaying at the starting. Fig. 5a and b are the representation of recovered population \( R(t) \) at various arbitrary order of \( \delta \) having two different effective contact rates \( \beta = 0.025, 0.25 \). Firstly as the contact rate is low the recovered individuals decrease and than and tends to zero, secondly as the contact rate is high the recovered compartment is high and become stable.

Figs. 6a-8a are the comparison of five compartments of two methods, one is Adams-Bash-forth numerical iterative method and the other one is semi-analytical Laplace Adomian decomposition method (LADM) of the proposed model (2) for \( \beta = 0.025 \) and order 1. By both techniques we get a comparable converging and stable results.

Fig. 6. Comparison of Adams-Bash-forth (AB) and Laplace Adomian Decomposition techniques (LADM) at \( \beta = 0.025 \) for \( S(t) \) and \( E(t) \) on integer order.

Fig. 7. Comparison of Adams-Bash-forth (AB) and Laplace Adomian Decomposition techniques (LADM) at \( \beta = 0.025 \) for \( I(t) \) and \( H(t) \) on integer order.
Fig. 8. Comparison of Adams-Bashforth (AB) and Laplace Adomian Decomposition techniques (LADM) at $\beta = 0.025$ for $R(t)$ on integer order.

Conclusions

In this manuscript, we studied qualitative analysis for the existence and uniqueness of solution together with some approximate results of a mathematical fractional-order model that describes the dynamics of the novel corona-virus Covid-19. The analysis of the model has been carried out under the fractional operator of Atangana-Baleanu (ABC). The model has five compartments namely; susceptible, asymptotic infected, infected class, hospitalized class and recovered class. First of all, we have established an existence theory that ensures the physical existence of the proposed model. By using the fixed point theory of Banach and Krassnoselskii, we proved that model (1) exists and has at least one solution. The considered problem is also proved for stability by making small perturbation. The obtained results were plotted graphically which explains the transmission dynamics of the novel corona-virus. We have also achieved good comparable results at two different effective contact rates, which shows that decreasing the contact rate will decline the infection. We have compared our iterative scheme with that of the already existing scheme of LADM. On the other hand, keeping a social distance is also a self-quarantine situation that can cause a decrease in the infection and also in the future. In this way, we can control this infection from rapidly spreading in the community. The techniques of the fractional differential equation may also be applied to many other dynamical problems so that the information of their dynamical behavior for fractional orders may be obtained. see Fig. 7.

CRediT authorship contribution statement

Sara Salem Alzaid: Conceptualization, Data curation, Formal analysis, Investigation, Project administration, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing. Badr Saad T. Alkahtani: Conceptualization, Methodology, Supervision, Validation, Writing - review & editing.

Declaration of Competing Interest

The authors have declared no conflict of interest.

Acknowledgments

The authors extend their appreciation to the Deanship for Research and Innovation, “Ministry of Education” in Saudi Arabia for funding this research work through the project number IFKSURG-1441–420.

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