I. INTRODUCTION

Thomas-Ehrman shift (TES) is the effect of the isobaric symmetry violation initially introduced for single-particle states of sd-shell nuclei in Refs. 1, 2. In such nuclei the \( l = 0 \) and \( l = 2 \) states are quite close to degeneracy. Because of their different radial extent the relative positions of the s- and d-wave configurations are strongly affected by the presence (absence) of the Coulomb interaction as their Coulomb displacement energies (CDE) are quite different. These differences provide e.g. a simple way for \( l \) identification. Later the studies of the TES effect have also extended to the nuclei with even number of “valence” nucleons having in mind e.g. understanding of configuration mixing.

There is no solid definition of TES, so, let’s discuss those typically used in the literature. One possibility is a kind of “theoretical” definition which considers a difference between actual CDE and the one expected from isobaric symmetry

\[
\Delta_J = \Delta_{\text{Coul}}(\text{calc}) - \Delta_{\text{Coul}}(\text{pert}) . \tag{1}
\]

The \( \Delta_{\text{Coul}}(\text{calc}) \) is CDE obtained by solving Schrödinger equation (SE) both on the proton and neutron sides of the isobar and \( \Delta_{\text{Coul}}(\text{pert}) \) is perturbative CDE obtained by solving SE on the neutron side of the isobar and then using the obtained wave function (WF) to calculate CDE on the proton side perturbatively by assuming complete isobaric symmetry:

\[
\Delta_{\text{Coul}}(\text{pert}) = \langle \Psi_n | V_{\text{Coul}} | \Psi_n \rangle .
\]

Such a definition was used e.g. in Refs. 3-5. A phenomenological analogue of the value \( \Delta \) was analyzed in Ref. 6. This work compared experimental masses \( M_{\text{exp}} \) with masses \( M_{\text{cg}} \) provided by charge-symmetric mass relationship

\[
\Delta_J = M_{\text{exp}} - M_{\text{cg}} .
\]

A systematic increase in the value of TES was demonstrated in Ref. 7 for the systems beyond the proton dripline with increase of the proton (or two-proton) Q-value.

The other opportunity is to use a pure “phenomenological” definition which rely only on the experimental relative shifts of the energy levels with different \( J \) values in proton-rich and neutron-rich mirror systems:

\[
\Delta_{J_2,J_1} = [E(J_2) - E(J_1)]_{\text{pro}} - [E(J_2) - E(J_1)]_{\text{neu}} . \tag{2}
\]

The analysis of such a definition of TES in comparison with one of Eq. (1) can be found in Ref. 8.

The interpretation of TES for the systems with one valence nucleon is very simple as we have already mentioned. The energies of single particle states with \( l = 2 \) can be found around the values defined by the perturbative Coulomb displacement, and \( \Delta_J \) for them should be relatively small. The energies of single particle states with \( l = 0 \) are shifted to considerably lower energies than perturbative values and \( \Delta_J \) for them should be large. As a result the distance between levels (and sometimes even the level ordering) is changing inducing a sizable \( \Delta_{J_2,J_1} \).

The situation is more complicated for systems with two valence nucleons. A simple estimate illustrating that follows. For sd-shell nuclei the \( 0^+ \) and \( 2^+ \) state’s WFs can be schematically approximated as

\[
\Psi_0 = \alpha_0 [s^2]_0 + \beta_0 [d^2]_0 , \quad \Psi_2 = \alpha_2 [sd]_2 + \beta_2 [d^2]_2 , \tag{3}
\]

If we think in terms of the independent particle model and typical value of TES associated with one s-wave nucleon is \( \Delta \) and with \( d \)-wave nucleon is zero then e.g. for \( \alpha_0 = 1 \) and \( \alpha_2 = 0 \) we can expect \( \Delta_{20} = 2\Delta \) and for \( \alpha_0 = 0 \) and \( \alpha_2 = 1 \) we can expect \( \Delta_{20} = -\Delta \). Thus, in principle, the important structure information is encoded in \( \Delta_{J_2,J_1} \) value, but it still can not be extracted without a considerable theoretical work in a contrast with one valence nucleon case.
The existence of a specific mechanism of TES, characterized as a "three-body mechanism" of TES, was demonstrated in Ref. 4. The $^{12}$O, $^{16}$Ne, and their isobaric mirror partners $^{12}$Be, $^{16}$C were considered in a three-cluster core+$N+N$ model. It was shown that in such systems not only conventional (let’s call it “static”) TES, connected with different radial extent of the $[s^2]$ and $[d^2]$ configurations, exists. Also there arises a specific TES of the three-body nature (or a “dynamic” TES); the relative weights of the $[s^2]$ and $[d^2]$ configurations appeared to be strongly different in the neutron-rich and proton-rich mirror partners. A strong increase (tens percent) was predicted for the weight of the $[s^2]$ configuration on the proton side of the isobar caused by its variational preference in the presence of the Coulomb interaction.

Recently the $^{16}$Ne was studied in three experiments using the neutron knockout off $^{17}$Ne beam 7–9, providing data mainly with better statistics and quality than in the previous works. Still these results are not free of contradictions. This inspired us to revisit the issue and consider the TES effect in $^{16}$Ne also in the broader context including the first excited 2$^+$ states. We demonstrate in this work that TES can be used as a very precise tool to check consistency of three-body core+$N+N$ and two-body core+$N$ state properties.

II. THEORETICAL MODEL

Theoretical model of this work is the same as was used for the discrete spectrum 10 and continuum 11 studies in a three-cluster approach. It was applied to $^{16}$Ne and its isobaric mirror partner $^{16}$C in Refs. 4, 5. Here we describe the model mainly to clarify details connected with accurate TES treatment.

For studies of $^{16}$Ne discrete spectrum states we solve a homogeneous three-body Schrödinger equation

$$\hat{H}_3 - E_T \Psi_3(\rho, \Omega_5) = 0, \tag{4}$$

where the energy $E_T$ is counted with respect to the core+$N+N$ threshold. For studies of the $^{16}$Ne continuum the inhomogeneous three-body Schrödinger equation

$$\hat{H}_3 - E_T \Psi_3^{(+)}(\rho, \Omega_5) = \Phi_q^{(q)}(\rho, \Omega_5) \tag{5}$$

is solved for different energies $E_T$ searching for the resonance peak position. The source function $\Phi_q^{(q)}$ for the $^{16}$Ne g.s. was approximated assuming the sudden removal of a neutron from the $^{15}$O core of $^{17}$Ne,

$$\Phi_q^{(q)} = \int d^3r_n e^{i\mathbf{q} \cdot \mathbf{r}_n} \langle \Psi_{14O} | \Psi_{17Ne} \rangle, \tag{6}$$

where $\mathbf{r}_n$ is the radius vector of the removed neutron. The $^{17}$Ne g.s. WF $\Psi_{17Ne}$ was obtained in 12 in a three-body $^{15}$O+$p+p$ model and different aspects of nuclear dynamics for this system were investigated in 13. For the 2$^+$ excitations of $^{16}$Ne we do not have some simple dynamically motivated model and $\Phi_q^{(2)}$ was provided by additionally acting on the valence protons of the $^{17}$Ne g.s. WF by the quadrupole operator:

$$\Phi_q^{(2)} = \int d^3r_n e^{i\mathbf{q} \cdot \mathbf{r}_n} \langle \Psi_{14O} | \sum_{i=1,2} r_i Y_{2m_i}(\hat{r}_i) | \Psi_{17Ne} \rangle. \tag{7}$$

The formulation provided by Eq. (3) is a reasonable approximation for the neutron knockout reaction mechanism used to populate $^{16}$Ne states in the recent experimental studies 7–9.

The three-body WF $\Psi_3$ depends on the set of hyperspherical variables: the hyperradius $\rho$ and the five-dimensional hyperangle $\Omega_5$. The hyperspherical decomposition of the discrete spectrum WF is

$$\Psi_3(\rho, \Omega_5) = \rho^{-5/2} \sum_{K\gamma} \chi_{K\gamma}(\rho) J_{K\gamma}(\Omega_5). \tag{8}$$

The value $K$ is hypermoment (the principal quantum number of the hyperspherical method) and the “multi-index” $\gamma = \{L, S, l_x, l_y, l_z\}$ stands for the complete set of quantum numbers for the specific three-body WF component: total orbital momentum $L$, total spin $S$, and orbital momenta $l_x$ and $l_y$ for the Jacobi subsystems. The boundary conditions for functions $\chi_{K\gamma}$ in the discrete spectrum are

$$\chi_{K\gamma}(\rho) \rho^{-2\infty} \exp[-\kappa \rho], \tag{9}$$

where $\kappa = \sqrt{2ME_T}$ and $M$ is the average mass of the nucleon in the considered system. The decomposition of the continuum WF $\Psi_3^{(+)}$ is analogous to that of Eq. (8). The boundary conditions for the partial continuum functions $\chi_{K\gamma}^{(+)}$ at the extreme remote asymptotic should be provided by diverging waves:

$$\chi_{K\gamma}^{(+)}(\rho) \rho^{-2\infty} \exp[i\kappa \rho]. \tag{10}$$

However, for realistic distances of actual calculations (e.g. $\rho \sim 1000$ fm) we use complicated approximate boundary conditions of the three-body Coulomb problem obtained by diagonalization of the Coulomb potential terms in the hyperspherical representation on the finite hyperspherical basis 11.

The three-body Hamiltonian consists of the kinetic term, three pairwise interactions, and the phenomenological potential $V_3$ depending only on the hyperradius $\rho$:

$$\hat{H}_3 = \hat{T} + V_{\text{core-}N_1} + V_{\text{core-}N_2} + V_{N_1-N_2} + V_3(\rho). \tag{11}$$

The $V_3$ term technically aims fine correction of the state energies when we need to adjust them exactly to experimental values, and physically aims to account for many-body effects which are beyond the three-cluster dynamics 11. In this work we use for this term the Woods-Saxon formfactor

$$V_3(\rho) = V_3^{(J)}/(1 + \exp[(\rho - \rho_0)/a_\rho]), \tag{12}$$
with the radius $\rho_0 = 6$ fm and the diffuseness $a_\rho = 0.6$ fm. The $V_3$ potential depth parameter $V_3^{(J)}$ is adjusted individually for each total momentum $J$ (see Table I for example).

For the $^{14}Z+N$ channel we use the potential very similar to that from Refs. [4, 9], but with minor variations connected with the procedure of the TES treatment discussed in the next paragraph. This is Woods-Saxon potential with derivative (ls) term

$$V_{\text{core-N}} = V_c^{(l)} + (ls)V_{ls}^{(l)} \frac{b}{1 + f(r)} + \frac{f(r)}{1 + f(r)} \frac{b}{r} [1 + f(r)/a],$$

where $b = 2.01532$ fm$^2$. Each component of potential with different orbital momentum is treated separately. The following parameters are used for $s$, $p$, and $d$ orbitals: $a = 0.53$ fm, $V_c^{(1)} = -12$ MeV, $V_{ls}^{(1)} = 11$ MeV, $r_0^{(1)} = 2.89$ fm, $V_c^{(2)} = -11.12$ MeV, $r_0^{(2)} = 3$ fm. The other central potential parameters for $s$- and $d$-waves are provided in Table I. There is also additional repulsive component in the $s$-wave with Woods-Saxon formfactor with the repulsion 144 MeV, the width 1.7 fm and the diffuseness 0.53 fm. This is required to simulate the effect of the occupied deep $s$ orbital in the $^{14}Z$ core cluster in our three-body model.

In this work we employ for the nucleon-nucleon channel a quasirealistic potential Ref. [14] including central, spin-orbit, tensor, and parity splitting terms.

The Coulomb potentials of a homogeneously charged sphere were used in this work with sphere radius adjusted to reproduce the specific charge radius. We also estimated the influence of the charge distribution on the effect using the Gaussian and Fermi type formfactors. The impact of such a modification on energies is on the level $10 - 15$ keV, which is much smaller than the scale of energy uncertainty connected with uncertainty of charge radius itself.

III. COMPUTATION PROCEDURE FOR TES

There are two uncertain ingredients in the three-cluster model computation of TES in the $^{16}$Ne/$^{16}$C mirror partner pair: the (i) charge radius $r_{ch}$ of $^{14}$O, which is experimentally unknown and (ii) the $1/2^+$ ground state energy $E_r$ of the $^{15}$F, which strongly affects the results of the calculations, but on which there exists an experimental controversy, see, e.g. Ref. [15]. The level schemes for $^{16}$Ne-$^{16}$C and $^{15}$F-$^{15}$C isobaric mirror partner pairs are provided in Fig. 1.

Preparing the potential sets we fix the positions of $1/2^+$ and $5/2^+$ states of $^{15}$C-$^{14}$C+$n$ to be exactly experimental by using potentials of somewhat different radii $r_0$ in the corresponding partial wave. Then we switch to $^{15}$F getting different resonant state positions $E_r(J^\pi)$ depending on the potential radius and also on $r_{ch}$ of $^{14}$O. The $d$-wave potential radius is thus fixed basing on the well known position of the $5/2^+$ state in $^{15}$F. The $1/2^+$ state position in $^{15}$F is varied depending on specific $r_0$ and $r_{ch}$. For relatively broad states, to which the $^{15}$F g.s. belongs, there is always some uncertainty in the definition of the state position. In this work we always imply that $E_r$ is the energy at which the phase shift passes $\pi/2$.

With the obtained potential sets we run three-body model calculations for $^{16}$C. The phenomenological three-cluster potential parameters $V_3^{(J)}$ are adjusted to provide exactly experimental energies of $0^+$ and $2^+$ states of $^{16}$C, $E_T(0^+) = -5.469$ MeV and $E_T(2^+) = -3.703$ MeV. With $V_3^{(J)}$ parameters adjusted in such a way we then perform the calculations of the $^{16}$Ne. The results for several potential sets (P1, P2, P3) giving different $^{15}$F g.s. positions are provided in the Table I. In the Table we show only the calculation inputs and results for the charge radius $r_{ch}(^{14}$O) = 2.7 fm as the values obtained with different charge radii differ insignificantly.

It can be seen in the Table I that for realistic potentials P1-P3 the values of $V_3^{(J)}$ are quite small, typically below 1 MeV. They are also quite similar for both $0^+$ and $2^+$ states. This could be a good indication that three-cluster picture of the inert $^{14}$O/$^{14}$C core plus two nucleons is an adequate approximation to the structure of these low-lying states in $^{16}$Ne/$^{16}$C.

To test the sensitivity of TES to the structure we varied the $s/d$ ratio of the WF by the following procedure: we increased the $|s|^2$ content of the WF by multiplying the $d$-wave potential depth parameter $V_d^{(2)}$ by a factor smaller than unity and vice versa, to increase the $|d|^2$ content of the WF we multiply the $s$-wave potential depth parameter $V_c^{(0)}$ by a factor smaller than unity. Two limiting cases of such potential sets (P4, P5) are illustrates in Table I. The P4 results for the $2^+$ state are missing as it is not possible to construct a low-lying $2^+$ only on the $s$-wave orbitals.
IV. CALCULATION RESULTS

A. Charge radius dependence

We can see in Fig. 2 that calculations with potentials providing different positions of the $^{15}$F g.s. produce very different positions of both $0^+$ and $2^+$ states of $^{16}$Ne. For variation of $E_r(1/2^+)$ within the range $1.23 - 1.56$ MeV “allowed” by an uncertainty in existing experimental data the three-body resonance energies $E_r$ variation in $^{16}$Ne is $250 - 300$ keV. In contrast, the predicted curves for different $^{14}$O charge radii practically overlaps (the deviations are less than 15 keV). Thus the influence of the specific value of the unknown charge radius of $^{14}$O on the physically motivated calculation results (those with fixed $^{15}$F g.s. position) is practically negligible and can not considerably affect the conclusions of this work concerning TES.

| TABLE I. Potential sets in the core+N channel adjusted for $r_{ch}(^{14}$O$) = 2.7$ fm. Radii are in fm, energies in MeV, and probabilities in percent. The position $E_r$ of the two-body resonance is defined here by the phase shift equals $\pi/2$. The energy of the first excited state of $^{15}$F is $E_r(5/2^+) = 2.8$ MeV. |
|---|---|---|---|---|---|
| | P1 | P2 | P3 | P4 | P5 |
| $E_r(1/2^+)$ | 1.147 | 1.287 | 1.467 | 1.287 | 1.287 |
| $r_{ch}^{(0)}$ | 3.5 | 3.1 | 2.7 | 3.1 | 3.1 |
| $V_{s}^{(0)}$ | -34.085 | -47.45 | -67.9 | -47.45 | 0 |
| $V_{d}^{(2)}$ | -49.587 | -49.587 | -49.587 | 0 | -49.587 |
| $V_{d}^{(0)}$ | 0.255 | 0.647 | 1.051 | -2.461 | -1.918 |
| $\Delta_{\text{eoul}}$ | 7.017 | 7.130 | 7.301 | 6.517 | 7.661 |
| $W(s^2)$ | 44.2 | 47.8 | 51.4 | 93.5 | 0.82 |
| $W(p^2)$ | 0.88 | 0.86 | 0.83 | 5.47 | 9.01 |
| $W(d^2)$ | 46.5 | 43.1 | 39.9 | 90.8 |
| $E_r$ | 1.136 | 1.303 | 1.514 | 0.821 | 1.972 |
| $\Gamma$ (MeV) | 0.323 | 1.09 | 3.67 | 0.003 | 0.313 |
| $W(s^2)$ | 71.7 | 70.2 | 69.0 | 95.4 | 1.16 |
| $W(p^2)$ | 5.88 | 5.84 | 5.98 | 3.67 | 9.59 |
| $W(d^2)$ | 22.2 | 23.4 | 24.6 | 0.62 | 88.8 |

B. Three-body mechanism of TES

The paper \[4\] demonstrated that there are two major sources of TES in the $0^+$ ground states of even sd-shell nuclei considered in the three-body core+N+N approximation. (i) Conventional “static” TES connected with larger spatial extent of the s-wave orbitals compared to the d-wave orbitals, and (ii) “dynamic” three-body mechanism of TES leading to a relative increase of the $|s^2\rangle$ component in $^{15}$F configuration weight compared to that of the $|d^2\rangle$ configuration.

Both points are well illustrated by Fig. 3 which shows the radial density dependence for two dominant components of $^{16}$Ne and $^{16}$C WFs. The $K = 0$ component weight is very close to that of the $|s^2\rangle$ configuration and the selected $K = 4$ component well corresponds to the $|d^2\rangle$. The densities of $^{16}$C WF’s components at large radii demonstrate behavior which is close to exponential decrease. The densities $^{16}$Ne WF components tend to constant at large radii, which corresponds to the $\sim \exp\{i\kappa\rho\}$ asymptotic of the WF $\Psi_{f}^{(+)}$. The radial extent of the $|d^2\rangle$ component in $^{16}$Ne is a bit larger, but close to that in $^{16}$C. In contrast, the $|s^2\rangle$ component is drastically broader in $^{16}$Ne. Also the weight of the $|s^2\rangle$ component in $^{16}$Ne is evidently larger than in $^{16}$C, while the weight of $|d^2\rangle$ component is smaller.

The relative scale of “static” and “dynamic” TES effects can be understood from Table I. The calculations with potential sets P4 and P5 provide limiting cases (practically pure s-wave or pure d-wave) of $^{16}$Ne/$^{16}$C structure which are very “rigid” and are not altered by imposing the Coulomb interaction. Thus the TES value
for the 0+ state $\Delta_0$ associated solely with radial size increase of orbitals from $^{16}\text{C}$ to $^{16}\text{Ne}$ is $\sim 230$ keV for pure $[s^2]$ and $\sim 120$ keV for pure $[d^2]$ configurations. In contrast, the predicted TES for the realistic structure of $^{16}\text{Ne}/^{16}\text{C}$ also including dynamic effect of structure modification varies between $\sim 300 - 400$ keV. We can thus estimate the scale of the “dynamic” contribution as $45 - 60\%$ of the whole TES.

C. The $^{15}\text{F}$ ground state issue

The calculated positions of the 0+ and 2+ states in $^{16}\text{Ne}$ as function of the $^{15}\text{F}$ g.s. energy are shown in Fig. 4. In this figure we also compare dynamical and perturbative results.

“Theoretical” TES values $\Delta_J$ are always large in our calculations both for 0+ state (varies between $\sim 300$ and $\sim 400$ keV) and for 2+ state (stable at about $300$ keV) in $^{16}\text{Ne}$. In contrast, the “phenomenological” TES value $\Delta_{J1,J2}$ is small and even changes sign, being sensitive to the particular value of $E_r$ in $^{15}\text{F}$. The latter result can probably be qualitatively understood as follows. If we look in the Table 4 the weight $W(s^2)$ varies as 44 - 51% for 0+ in $^{16}\text{C}$, while the $W(sd)$ is more stable around 79% for 2+. So, there are two s-wave nucleons in 0+, which are subject to strong TES, but the weight of this configuration is mainly under 50%. There is only one s-wave nucleon in [sd] configuration of 2+, however, the weight of this configuration is about twice larger in 2+ than $W(s^2)$ in 0+. So, for such structures of 0+ and 2+ the TES modifications of CDE are about to equal.

It can be seen in Fig. 3 that consistent theoretical description simultaneously for both 0+ and 2+ resonances is achieved at $E_r = 1.39 - 1.42$ MeV. For theoretical calculations in 3 we used the potential providing $E_r = 1.45$ MeV. We see now that this value is a bit too large considering the TES results of this work. However, modification of this parameter on such a level do not lead to any modification of conclusions of Ref. 9 related to theory.

Considering high uncertainty of the current experimental situation with $^{15}\text{F}$ g.s. (see, e.g., 15), it is not impossible that the form of the consistency check found in this work can provide now the most reliable information on the $^{15}\text{F}$ g.s. energy.

The positions of the 0+ and 2+ states in $^{16}\text{Ne}$ as a function of the internal structure are shown in Fig. 5. Such curves allow to fix the configuration mixing values in the case when the decay energy $E_r$ is known. However, in our calculations we demonstrate that this can be made differently, depending on the particular 1/2+ resonance energy in $^{15}\text{F}$. We can see that for 0+ state the consistency of the experiment 9 is achieved for broad range (from 50% to 75%) of possible configuration mixing $W(s^2)$ values, depending on the specific g.s. energy $E_r$ of $^{15}\text{F}$. The consistency with experiment 9 for 2+ state can be achieved from 40% to 100% of possible $W(sd)$ values. However, for 2+ state the situation is more restrictive as for $E_r \geq 1.45$ MeV consistency of TES with the experimental energy cannot be achieved at all.

If we fix the three-body state energies $E_r$ to be exactly experimental 9 we can get the region on the planes $\{W(s^2), E_r\}$ or $\{W(sd), E_r\}$ where mutually consistent values of $W$ and $E_r$ are located, see Fig. 6. The mean-
modifications in the structure of the 
configurations and making simple estimates we can suggest
example, the admixture of configurations like
should lead to CDE increase (these configurations should
be more compact than the main one) and thus to shift
work for the three-cluster formula-
certainty of working for the three-cluster
on possible properties of the core+
for 0+
MeV. Thus we see that simultaneous studies of the TES
limiting a possible E
principle can be achieved for the broad range
of the [s sd] F g.s. looks very precise. It should be
considering of these plots is that precise fixing the g.s. properties
of 15 F fixes the structure of the valence configurations
both for 0+ and 2+ states of 16 Ne simultaneously. We
note again that while for the 0+ state the consistency in
principle can be achieved for the broad range Er, which is
much broader than existing experimental uncertainty,
for 2+ state this kind of plot becomes quite restrictive,
limited a possible Er value to be less than 1.43 – 1.45
MeV. Thus we see that simultaneous studies of the TES
for 0+ and 2+ states may impose stringent limitations
on possible properties of the core+N+N systems and its
core+N subsystems.

The value Er = 1.39 – 1.42 MeV deduced in this work for the
15 F g.s. looks very precise. It should be
clearly understood that such a high declared precision
is based just on two uncertainties: (i) experimental uncertainty of ET and (ii) theoretical uncertainties of the
three-cluster 14 O+p+p model for 16 Ne. Theoretical
uncertainties which are beyond the three-cluster formulation for 16 Ne should further increase this uncertainty. For example, the admixture of configurations like
14 O'+p+p should lead to CDE increase (these configurations should be
more compact than the main one) and thus to shift the
consistency range to somewhat lower Er values.
Considering good description of energies of 16 Ne/16 C in our
model we do not expect large admixture of such
configurations and making simple estimates we can suggest a
15 – 30 keV decrease of the lower boundary of Er for
10 – 20% admixture of configurations with excited (in the
2+ state) 14 O core. This would extend the boundaries for the
“TES based” F g.s. position to Er = 1.36 – 1.42
MeV.

D. Structure of 
0+ and 2+ states

The variation of 15 F g.s. energy in the range allowed by the current experimental uncertainty produces certain
modifications in the structure of 16 Ne and 16 C states
calculated in three-cluster model. These modifications were
found to be the largest for the 16 C ground state, where W(s sd) varies on the level 6 – 8%. As we turn to its
isobaric mirror partner, this uncertainty is significantly diminished. In 16 Ne the typical level of structure variation
is 2 – 3% and can be regarded as insignificant. We
can guess that for 16 Ne the peripheral dynamic, associated
with the long-range Coulomb interaction rather than with the short-range nuclear dynamics, is more important
and this leads to the relative stabilization of the
calculations results for the 0+ state in 16 Ne.

For the 2+ state the predictions are much more stable
than for the 0+ state and also follow the trend discussed above: there is ~ 2.5% variation of W(s sd) in the 16 C WF
and just ~ 0.5% variation in the 16 Ne WF. Both values can be regarded as insignificant and predicted structure as stable.

Quite a paradoxical output of our studies is that the presence of the Coulomb interaction drastically increases the
reliability of theoretical predictions for such a class
of systems as 16 Ne/16 C on the proton side of the isobar.

E. Widths of the 0+ and 2+ states

The results of width calculations from Table II are visualized in Figure.

They are accompanied with the
calculation results from Ref. [11]. The later work was one of the first our works on the topic and the results
provided there suffered from technical issues, which were
later overcome [11].

It can be seen that our results for the 0+ state (red
diamonds in Fig. 7a) are around a factor four larger than those from Ref. [3] (solid curves in Fig. 7). However, they
follow the trend provided by the old prediction very well.
So, the difference is a pure convergence issue, connected
to the small basis calculations decade ago.
Different situation can be found for the 2+ state
widths. The new calculation results differs from the old ones even more and also they clearly follow a different energy trend. We have to conclude here that the three-body width increase connected with decrease of the 1/2+ state energy in the 15F subsystem “outweights” the three-body width decrease connected with corresponding shift to lower ET energies.

The width results for the pure [d2] configuration are always more than order of the magnitude lower than value expected continuing the trend for realistic structure calculations or calculations with [s2] dominance. This is an indication of the uncertainty range for the two-proton decay widths, which, in principle, can be associated with unknown nuclear structure.

V. EXPERIMENTAL DATA ON 16Ne 0+ AND 2+ STATES

In Fig. 4 we demonstrated consistency of our theoretical TES values to the most recent experimental data 9 on 16Ne 0+ and 2+ states. The situation with such a consistency is, however, different for the other data.

The available experimental data on 16Ne 0+ and 2+ are listed in the Table II. The first thing which should be noted is that already the ground state data are quite uncertain, spanning from 1.33 to 1.47 MeV (we omit here one of the early and imprecise results). This uncertainty is often larger than the provided errors of particular experiments. Even the most recent experiments 8,9,16,17 both declared to have the best precisions ever, disagree with each other for ET values of 0+ state beyond the provided errors. Thus the overall experimental situation is unsatisfactory already for the 16Ne ground state energies.

Among the experimental data shown in Table II four experiments enlisted first 8,9,16,17 provide both the 0+ and 2+ positions and thus allow to consider a consistency of these data with theoretical TES results as it was done for 9 in Fig. 4. Such a comparison is provided in Fig. 8. The consistency range for experiment 17 ET = 1.27 − 1.4 MeV is quite broad and somehow overlaps with that found for the data 9. The consistency in the terms of TES exists for the data 16. However, the obtained range of ET = 1.25 − 1.35 MeV is not compatible with that of 9. Finally, the results of 8 are not compatible in the TES terms as calculated in our work (the theoretical curves are crossed by the experimental ranges at somewhat different ET ranges). It should be noted that if a larger uncertainty is assumed for the g.s. energy in this experiment, than the consistency with theoretical results would be achieved at ET ∼ 1.39 MeV, also in agreement with 9.

We demonstrated above in Figs. 5 and 6 that the increase in precision of experimental data on state positions is required to make a better use of the TES results even for the most recent data of Ref. 9. The Fig. 8 shows that overall situation is even worse and we see that from experimental side also a broad controversy with the older data should be resolved in general before definitive conclusions on actual TES behavior would become possible.

VI. THEORETICAL DISCUSSION

The evident way to use TES for nuclear structure studies in the sd-shell systems with even number of the valence nucleons is to apply it to derivation of the configuration mixing rates. The basic idea was discussed in the Introduction around Eq. 33. This way of the reasoning about the nuclear structure was elaborated in a number of papers for various mirror pairs of sd-shell systems: 12O−11Be 21, 15Ne−15B 22, 16Ne−16C 23, 17Ne−17N 23, 18Ne−18O 23, 19.

It is necessary to note that the connection between TES and the configuration mixing is straightforward and simple only in the case of an independent particle model.

| Ref. | ET(0+) | Γ(0+) | ET(2+) | Γ(2+) | ET(1/2+) |
|------|--------|-------|--------|-------|----------|
| 16   | 1.33(8) | 0.1(1) | 3.02(11)|    | 1.25 − 1.35 |
| 17   | 1.35(8) | 3.2(2) | 0.2(2) |    | 1.27 − 1.4 |
| 8    | 1.388(15)| 0.082(15) | 3.220(46) | < 0.05 | none |
| 9    | 1.466(15)| < 0.08 | 3.16(2) | 0.02(1) | 1.39 − 1.42 |
| 18   | 1.8(5) |       |        |       |          |
| 19   | 1.466(45)|       |        |       |          |
| 20   | 1.399(24)| 0.11(4) |        |       |          |
with well defined orbital characteristics. Even in the independent particle model fixing of orbital sizes requires precise knowledge of excitation energies of the single particle states for \( A - 1 \) “subsystem” on the proton side of the isobar. Among the mentioned systems this is not the case for \(^{12}\text{O},\thinspace^{15}\text{Ne},\thinspace\text{and} \thinspace^{16}\text{Ne} \), which “subsystems” \(^{11}\text{N},\thinspace^{14}\text{F},\thinspace^{15}\text{F} \) possess quite broad \( s_{1/2} \) states making the precise experimental determination of CDE problematic. In this work we demonstrate by the example of \(^{16}\text{Ne} \) that this issue could have a major impact on conclusions about the configuration mixing, see Fig. 6. On top of this issue we also insist on the existence of the three-body mechanism of TES leading to a strong modification of the configuration mixing rates when we move from neutron to the proton side of the isobar.

Paper [22] provided the impressive prediction of the \(^{15}\text{Ne} \) g.s. energy \( E_{T} = 2.68(24) \) MeV which appeared to be in a very good agreement with the later measured value \( E_{T} = 2.522(66) \) MeV [8]. This prediction is based on two ingredients. (i) Phenomenological linear dependence of “scaled CDE” \( (S_{2n} - S_{2p})A^{1/3}/Z_{c} \) on \( W(s^{2}) \) was derived in [22] based on the data for several \( Z = 8,10 \) isobaric mirror partner pairs. (ii) The plausible value \( W(s^{2}) = 66\% \) for \(^{15}\text{B} \) was assumed in [22] just between the \( W(s^{2}) = 86\% \) deduced for \(^{14}\text{Be} \) and \( W(s^{2}) = 46\% \) for \(^{16}\text{C} \). As a prospect of our studies we point to the need to reconcile this type of phenomenology with more complicated dependencies obtained in this work. The dependence (i) is in principle analogous to the dependence of Fig. 6(a). However, we obtain a set of such dependencies even for one single nuclide \(^{16}\text{Ne} \) depending on \( E_{r} \) in \(^{15}\text{F} \). Studies of the configuration mixing in \(^{15}\text{Ne} \) in three-body model could also elucidate the issue (ii).

**VII. CONCLUSIONS**

We should outline the following main results obtained in this work.

(i) Large isospin symmetry breaking on the level of the nuclear structure associated with TES was predicted in Ref. [4] for \( 0^{+} \) states and further elaborated in this work also in the case of \( 2^{+} \) states. We have found that in the \(^{16}\text{Ne}/^{16}\text{C} \) mirror pair the “dynamic” component of TES, connected to the structure modification, is responsible for about a half of the whole TES effect. The scale of the structure modification in these mirror nuclei is 20 − 25\% for the \( 0^{+} \) ground states and 6 − 10\% for the first \( 2^{+} \) states.

(ii) In this work we study carefully the stability of such predictions to theoretical inputs to the calculations. In our predictions the structure of the \(^{16}\text{Ne} \) states appears to be very stable to the admissible variation of parameters. Quite unexpectedly, the stability of predictions for \(^{16}\text{Ne} \) is much better than for \(^{16}\text{C} \), presumably due to more peripheral character of its WF.

(iii) Accurate studies of the Coulomb displacement energies indicate that the consistency among three parameters should be requested: the decay energy \( E_{r} \), the \(^{15}\text{F} \) g.s. energy \( E_{r} \), and the configuration mixing parameters \( W(s^{2})/W(d^{2}) \) for \( 0^{+} \) and \( W(sd)/W(d^{2}) \) for \( 2^{+} \) states. This is a more complicated dependence than it is ordinarily assumed. Typically the TES is correlated with the configuration mixing only to provide predictions about nuclear structure [21, 22].

(iv) The energy of the \(^{15}\text{F} \) \( 1/2^{+} \) g.s. extracted from our analysis is \( E_{r} = 1.39 − 1.42 \) MeV. Some shift to the lower energies is possible due to the WF configurations which are beyond our model. The \( E_{r} \) values above 1.43 − 1.45 MeV are practically excluded by our analysis.

(v) The current experimental situation is too uncertain to establish fully the validity of the provided results. Even the precise values of the \(^{16}\text{Ne} \) experimental energies of Ref. [9] allow a consistency with theoretical predictions of TES in a broad range of other parameters. Further increase in the precision of the measured energies in \(^{15}\text{F} \) and \(^{16}\text{Ne} \) would impose very stringent limits in the parameter space in which the consistency with theory is possible. Thus TES could become a sensitive tool for the extraction of fine structural information about \(^{16}\text{Ne}/^{16}\text{C} \) mirror nuclei as well as \( sd \)-shell nuclei with analogous dynamics.

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