Hybrid plasma modes in transistors: linear and non-linear responses

H. Marinchio¹, V. Korotyeyev², C. Palermo¹, and L. Varani¹

¹ Institute of Electronics and Systems, UMR CNRS 5214, University of Montpellier, France
² Institute of Semiconductor Physics, Kiev 03650, Ukraine

E-mail: hugues.marinchio@ies.univ-montp2.fr

Abstract. We present an analytical model based on hydrodynamic equations and a pseudo-two-dimensional Poisson equation to study the response of a nanometric field-effect transistor channel in the THz domain. This model allows to study different kinds of external excitations of plasma modes and different geometries. We calculate the two first-order responses of the drain voltage or current, which are of peculiar interest in the perspective of THz wave generation and detection and THz electronics. Even at room temperature, each responses present resonances at the eigenfrequencies of the hybrid plasma modes sustained in the channel.

1. Introduction

The electron collective oscillations phenomena in solid-state devices received a close attention since the 90’s and the theoretical works of Dyakonov and Shur [1, 2] which predict the instability, or at least the resonant behavior, of a gated two-dimensional (2D) electron gas constituting the channel of a field-effect transistor (FET). Such instabilities/resonances, occurring in the terahertz (THz) range for nanometric devices, can be used in view of the development of solid-state emitters or detectors of THz radiation. These possibilities have been confirmed experimentally by measurements of plasma instabilities at low temperature [3, 4] and plasma resonances to external excitations (THz wave or THz optical beating) at room-temperature [5].

Since then, analytical models have been developed to describe the stimulation of plasma modes in FETs and these are usually limited to strictly 2D plasma because they are based on the gradual channel approximation [2, 6, 7] which neglects the effect of the longitudinal electric field in the Poisson equation. We recently proposed an original analytical model based on hydrodynamic equations and a pseudo-2D (P2D) Poisson equation to overtake these limitations [8]. This model allows to study the different kinds of external excitations of plasma modes (THz, optical or both) and different geometries. In particular, we demonstrate the dispersivity of the plasma in the FET channel, the electron gas shifting progressively from a 2D plasma dynamic behavior at low frequency to a 3D plasma at high-frequency, that is why we introduced the concept of hybrid plasma modes [8].

Here we present the calculation of current and voltage responses for different boundary conditions and geometries. We will focus on the first-order harmonic response, interesting in the perspective of THz emission or amplification, the (second-order) average and second-harmonic responses, of interest in terms of THz detection or rectification and THz frequency multiplication, respectively.
2. Theoretical model

Our analytical approach consists in coupling the 3D carrier density $n$ and mean velocity $v$ conservation equations with the P2D Poisson equation [9]:

\[
\begin{align*}
\frac{\partial n}{\partial t} &= -\frac{\partial (nv)}{\partial x} + G \\
\frac{\partial G}{\partial t} &= -v \frac{\partial V}{\partial x} + q \frac{\partial V}{\partial x} - vv \\
\varepsilon_c \frac{\partial^2 V}{\partial x^2} + \varepsilon_s \frac{V_g - V_{th}}{d\delta} &= qn \\
\end{align*}
\]  

(1)

where $n$ is the electron volumic density. $d$ is the effective gate-to-channel distance and $\delta$ the channel thickness. $d$ is related to the gate-to-channel capacitance $C_{gc} = \varepsilon_c L/d$ with $L$ the gate length and $\varepsilon_s$ the permittivity of the insulator. $\varepsilon_c$ is the dielectric constant, $\nu$ the velocity relaxation rate and $m$ the electron effective mass which will be taken equal to their typic values in InGaAs at room-temperature [8]. $V_g$ and $V_{th}$ are the gate and threshold voltages, respectively.

A sketch of the device is proposed in Fig. 1.

Two kinds of THz excitations can be treated within this framework: (i) an electrical stimulation on the gate described by the harmonic part of the gate voltage $V_g(t) = V_{g0} + V_{g1} \cos(\omega t)$ and (ii) an optical beating excitation through the generation coefficient $G(t) = G_0 + G_1 \cos(\omega t)$.

As in Ref. [2], we assume that the excitations are sufficiently weak to only have to look for solutions of the form: $u(x, t) = u_0 + \delta u_1(x, t) + \delta \bar{u}(x) + \delta u_2(x, t)$, with $u = n, v, V$ or $j$ (volumic current density defined below), where $u_0$ is the value in the absence of excitation, $\delta u_1(x, t)$ the first-order solution at the same pulsation $\omega$ than the stimulation, $\delta \bar{u}$ and $\delta u_2(x, t)$ are the average and second harmonic components. We introduce the complex quantities $u_1(x)$ and $u_2(x)$ verifying $\delta u_1(x, t) = \Re[u_1(x) \exp(i\omega t)]$ and $\delta u_2(x, t) = \Re[u_2(x) \exp(i2\omega t)]$. We additionally assume that the device is not or weakly biased implying that $v_0 = 0$, $V_0 = V_s = 0$ and other $u_0$ quantities are uniform.

The volumic current density can be expressed summing its conduction and displacement contributions: $j(x, t) = -qnv - \varepsilon_c \frac{\partial^2 V}{\partial x^2}$.

3. Linear response

Rewriting the system (1) at first-order, we obtain:

\[
\begin{align*}
i\omega n_1 &= -n_0 v'_1 + G_1 : i\omega v_1 = \frac{q}{m} V'_1 - \nu v_1 : \varepsilon_c V''_1 + \varepsilon_s k^2 G_1 (V_{g1} - V_1) = qn_1 \quad (2)
\end{align*}
\]

where $k^2 = 1/d\delta$ and the prime notation stands for the derivative with respect to $x$.

Then, combing Eqs. (2) we get:

\[
\delta V''_1 + \beta(\omega)^2 \delta V_1 = \beta(\omega)^2 e_1 \quad \text{where } \beta(\omega)^2 = \frac{\varepsilon_s k^2}{\varepsilon_c} \left( \frac{\omega^2 - i\nu \omega}{\omega^2 - i\nu \omega + \omega^2} \right) \quad \text{and } e_1 = V_{g1} - \frac{q}{i\omega\varepsilon_s k^2} G_1. \quad (3)
\]
$\omega_{3D} = \sqrt{\frac{q^2 n_0}{m \varepsilon_s}}$ is the bulk plasma frequency of the material constituting the channel.

The solution has the form: $V_1 = A \cos(\beta x) + B \sin(\beta x) + e_1$ where $A$ and $B$ depend on the chosen boundaries conditions (BC). We consider two cases: i) asymmetrical BC, i.e. the drain current is fixed ($j(L) = 0$); ii) symmetrical BC, i.e. $V(L) = 0$. In the former case, the quantity of interest is the amplitude of the drain-to-source voltage potential $V_{ds1} = |V_1(L)|$ while in the latter case, it is the drain current amplitude $j_{d1} = |j_1(L)|$. Both quantities are plotted as functions of the stimulation frequency $f$ on Fig. 2.

The sharp resonances can be associated with the excitation of hybrid plasma modes. Asymmetrical BC allow the stimulation of odd modes and symmetrical ones the sustainment of even modes. In both cases, the resonances frequencies $f_p$ cannot overreach the 3D plasma frequency [see Ref.[8] for further details].

![Figure 2](image.png)

**Figure 2.** (a) Amplitude of the drain-to-source voltage oscillation and (b) amplitude of the drain current density oscillation as functions of the excitation frequency for the reported gate lengths. Each structure is stimulated by a harmonic oscillation of the gate voltage ($V_g1 = 1$ mV) and presents $d_l = 1/k_l = 14$ nm.

4. Non-linear response

At second order, the system (1) becomes:

$$\begin{align*}
\frac{\partial \delta n_2}{\partial t} &= -\frac{\partial(\delta n_1 \delta v_1 + n_0 \delta v_2 + n_0 \delta \pi)}{m} + \frac{\partial^2 V_2 + \delta V)}{m} - \nu(\delta v_2 + \delta \pi) \\
\frac{\partial \delta v_2}{\partial x} &= -\frac{\partial(\delta v_1^2/2)}{m} + \frac{\partial(\delta v_2 + \delta V)}{m} - \nu(\delta v_2 + \delta \pi) \\
\frac{\partial^2}{\partial x^2} (\delta V_2 + \delta V) &= \frac{\partial^2}{\partial x^2} \left(\delta V_2 + \delta V\right)
\end{align*} \tag{4}$$

4.1. Average response

The carrier conservation implies that $\delta \mathbf{J}$ is uniform: i.e. $\delta \mathbf{J} = 0$, obviously for an open drain but also for a shortened device because of its symmetry and bias conditions. As a consequence, no DC response to an external stimulation can be seen under symmetrical BC (at least at zero applied drain-to-source voltage).

We obtain the stationary responses under asymmetrical BC by time-averaging (4) and using the property $\langle \delta x_1 \delta y_1 \rangle = \frac{1}{2} \Re[x_1 y_1^*]$ with $y_1^*$ the complex conjugate of $y_1$:

$$\delta \overline{v} = \frac{1}{2n_0} \Re[n_1 v_1^*]$$

The average drain-to-source response $\Delta V_{ds} = \delta \overline{V}(L)$ is calculated using the expression obtained in the previous section and presented on Fig. 3(a). Peaks at the different plasma
frequencies $f_p$ are seen. Let us remark that this average bias is here slightly negative; nevertheless, Ref. [9] shows that, when a moderate-to-high drain current is imposed, $\Delta V_{ds}$ becomes positive and of much greater amplitude since the overall non-linearity of the system is increased.

4.2. Second harmonic response
By only considering the time-dependent part of the quantities appearing in Eq. (4), one can obtain the differential equation verified by $V_2$:

$$V''_2 + \beta (2\omega)^2 V_2 = \beta (2\omega)^2 e_2$$ with $e_2 = \frac{q}{2i\epsilon k^2_L} \left( \frac{1}{2} (n_1 v_1')' + \frac{1}{2i\omega + \nu} (|v_1|^2/2)' \right)$ (6)

$V_2$ has consequently the same form than $V_1$, substituting $\omega$ with $2\omega$ and $e_1$ with $e_2$. Consequently, resonances at frequencies equal to $f_p/2$ appear in the frequency response of $V_{ds2} = |V_2(L)|$ presented on Fig. 3(b).

Figure 3. (a) Average value and (b) amplitude of the doubled-frequency part of the drain-to-source voltage. $V_g = 1$ mV, $d_t = 14$ nm and $L = 100$ nm.

5. Conclusion
A model allowing the calculation of the first and second order parts of the voltage and current responses of a nanometric FET channel to a THz stimulation has been presented. Such a stimulation not only induced significant drain voltage or current oscillations at the frequency of the excitation $f$ but also a variation of the DC quantities (only for symmetrical BC) and also oscillations at $2f$ when a hybrid stationary plasma mode is stimulated. The amplitudes and frequencies of the corresponding resonances highly depend on the geometry.

References
[1] Dyakonov M and Shur M 1993 Phys. Rev. Lett. 71 2465–2468
[2] Dyakonov M and Shur M S 1996 IEEE Trans. Electron Devices 43 380–387
[3] Knap W, Deng Y, Rumyantsev S, Lu J Q, Shur M S, Saylor C A and Brunel L C 2002 Appl. Phys. Lett. 80 3433–3435
[4] Nouvel P, Torres J, Blin S, Marinchio H, Laurent T, Palermo C, Varani L, Shiktorov P, Starikov E, Gruzinik V, Teppe F, Roelens Y, Shchepetov A and Bollaert S 2012 J. of Appl. Phys. 111 103707
[5] Nouvel P, Marinchio H, Torres J, Palermo C, Chusseau L, Gasquet D, Varani L, Shiktorov P, Starikov E and Gruzinik V 2009 J. Appl. Phys. 106 013717
[6] Veksler D, Teppe F, Dmitriev A P, Kachorovskii V Y, Knap W and Shur M S 2006 Phys. Rev. B 73 125328
[7] Ryzhii V, Khnyrova I, Satou A, Vaccaro P O, Aida T and Shur M 2002 J. Appl. Phys. 92 5756–5760
[8] Marinchio H, Palermo C, Mahi A, Varani L and Korotyeyev V 2014 J. Appl. Phys. 116 013707
[9] Marinchio H, Sabatini G, Pousset J, Palermo C, Chusseau L, Varani L, Starikov E and Shiktorov P 2009 Appl. Phys. Lett. 94 192109