Fence Synthesis under the C11 Memory Model

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Abstract. The C/C++11 (C11) standard offers a spectrum of ordering guarantees on memory access operations. The combinations of such orderings pose a challenge in developing correct and efficient weak memory programs. A common solution to preclude those program outcomes that violate the correctness specification is using C11 synchronization-fences, which establish ordering on program events. The challenge is in choosing a combination of fences that (i) restores the correctness of the input program, with (ii) as little impact on efficiency as possible (\ie, the smallest set of weakest fences). This problem is the optimal fence synthesis problem and is NP-hard for straight-line programs. In this work, we propose the first fence synthesis technique for C11 programs called FenSying and show its optimality. We additionally propose a near-optimal efficient alternative called fFenSying. We prove the optimality of FenSying and the soundness of fFenSying and present an implementation of both techniques. Finally, we contrast the performance of the two techniques and empirically demonstrate fFenSying’s effectiveness.

Keywords: C11, fence-synthesis, optimal

1 Introduction

Developing weak memory programs requires careful placement of fences and memory barriers to preserve ordering between program instructions and exclude undesirable program outcomes. However, computing the correct combination of the type and location of fences is challenging. Too few or incorrectly placed fences may not preserve the necessary ordering, while too many fences can negatively impact the performance. Striking a balance between preserving the correctness and obtaining performance is highly non-trivial even for expert programmers.

This paper presents an automated fence synthesis solution for weak memory programs developed using the C/C++11 standard (C11). C11 provides a spectrum of ordering guarantees called memory orders. In a program, a memory access operation is associated with a memory order which specifies how other memory accesses are ordered with respect to the operation. The memory orders range from relaxed (\texttt{rlx}) (that imposes no ordering restriction) to sequentially-consistent (\texttt{sc}) (that may restore sequential consistency). Understanding all the subtle complexities of C11 orderings and predicting the program outcomes can quickly become exacting. Consider the program (\texttt{RWRW}) (\S2), where the orders
are shown as subscripts. When all the memory accesses are ordered \texttt{rlx}, there exists a program outcome that violates the correctness specification (specified as an \texttt{assert} statement). However, when all accesses are ordered \texttt{sc}, the program is provably correct.

In addition, the C11 memory model supports C11 fences that serve as tools for imposing ordering restrictions. Notably, C11 associates fences with memory orders, thus, supporting various degrees of ordering guarantees through fences.

This work proposes an \textit{optimal} fence synthesis technique for C11 called \texttt{FenSying}. It involves finding solutions to two problems: (i) computing an optimal (minimal) set of locations to synthesize fences and (ii) computing an optimal (weakest) memory order to be associated with the fences (formally defined in §3). \texttt{FenSying} takes as input all program runs that violate user-specified assertions and attempts optimal C11 fence synthesis to stop the violating outcomes. \texttt{FenSying} reports when C11 fences alone cannot fix a violation. In general, computing a minimal number of fences with multiple types of fences is shown to be NP-hard for straight-line programs [23]. We note, rather unsurprisingly, that this hardness manifests in the proposed optimal fence synthesis solution even for the simplest C11 programs. Our experiments (§7) show an exponential increase in the analysis time with the increase in the program size.

Further, to address scalability, this paper proposes a \textit{near-optimal} fence synthesis technique called \texttt{fFenSying} (fast \texttt{FenSying}) that fixes one violating outcome at a time optimally. Note that fixing one outcome optimally may not guarantee optimality across all violating outcomes. In the process, this technique may add a small number of extra fences than what an optimal solution would compute. Our experiments reveal that \texttt{fFenSying} performs exponentially better than \texttt{FenSying} in terms of the analysis time while adding no extra fences in over 99.5% of the experiments.

Both \texttt{FenSying} and \texttt{fFenSying}, compute the solution from a set of combinations of fences that can stop the violating outcomes, also called \textit{candidate solutions}. The candidate solutions are encoded in a head-cycle-free CNF SAT query [8]. Computing an optimal solution from candidates then becomes finding a solution to a \textit{min-model finding problem}.

Many prior works have focused on automating fence synthesis (discussed in §8). However, the techniques presented in this paper are distinct from prior works in the following two ways: (i) prior techniques do not support C11 memory orders, and (ii) the proposed techniques in this paper synthesize fences that are portable and not architecture-specific.

\textbf{Contributions.} To summarize, this work makes the following contributions:

\begin{itemize}
  \item The paper presents \texttt{FenSying} and \texttt{fFenSying} (§6). To the best of our knowledge, these are the first fence synthesis techniques for C11.
  \item The paper shows (using Theorems 1 and 2) that the techniques are sound, \textit{i.e.}, if the input program can be fixed by C11 fences, then the techniques will indeed find a solution. The paper also shows (using Theorem 3) that \texttt{FenSying} produces an optimal result in the number and type of fences.
\end{itemize}
Initially:
\[ x = 0, y = 0 \]
\[ a := x \]
\[ b := y \]
\[ x := y \]
\[ y := x \]
assert\((\neg (a = 1 \land b = 1))\)

Finally, the paper presents an implementation of the said techniques and presents an empirical validation using a set of 1389 litmus tests. Further, the paper empirically shows the effectiveness of fFenSying on a set of challenging benchmarks from prior works. fFenSying performs on an average 67x faster than FenSying.

2 Overview of FenSying and fFenSying

Given a program \( P \), a trace \( \tau \) of \( P \) (formally defined in §3); is considered buggy if it violates an assertion of \( P \). FenSying takes all buggy traces of \( P \) as input. The difference in fFenSying is that the input is a single buggy trace of \( P \).

Consider the input program \( \text{RWRW} \), where \( x \) and \( y \) are shared objects with initial values 0, and \( a \) and \( b \) are local objects. Let \( W^m(o, v) \) and \( R^m(o, v) \) represent the write and read of object \( o \) and value \( v \) with the memory order \( m \). Let \( I(o, v) \) represent the initialization event for object \( o \) with value \( v \). The parallel bars (\( || \)) represent the parallel composition of events from separate threads. Figure \( \text{RWRW-bt} \) represents a buggy trace \( \tau \) of \( \text{RWRW} \) under C11 semantics. For convenience, the relations — sequenced-before (\( \rightarrow_{\text{sb}} \)), reads-from (\( \rightarrow_{\text{rf}} \)), modification-order (\( \rightarrow_{\text{mo}} \)) (formally defined in §3, §4) — among the events of \( \tau \) are shown. The
assert condition of \((\text{RWRW})\) is violated as the read events are not ordered before the write events of the same object, allowing reads from later writes.

Consider the following three sets of fences that can invalidate the trace \((\text{RWRW-bt})\): \(c_1 = \{F_{12}^{sc}, F_{22}^{sc}\}\), \(c_2 = \{F_{12}^{rel}, F_{22}^{aq}\}\) and \(c_3 = \{F_{12}^{sc}\}\) (the superscripts indicate the memory orders and the subscripts represent the synthesis locations for fences). The solutions are depicted in Figures \((\text{RWRW-inv-to})\), \((\text{RWRW-inv-sync})\) and \((\text{RWRW-inv-sync-opt})\) for \(c_1\), \(c_2\) and \(c_3\), respectively. The candidate solution \(c_1\) prevents a total order on the \(sc\) ordered events \([9,12]\), thus, invalidating \((\text{RWRW-inv-to})\) under \(C11\) semantics. With candidate solution \(c_2\), a happen-before \((\rightarrow_{bb})\) ordering is formed (refer \(\S4\)) between \(R^{sc}(y,1)\) and \(W^{sc}(y,1)\). This forbids a read from an ordered-later write, thus, invalidating \((\text{RWRW-inv-sync})\). Candidate solution \(c_3\) establishes a similar \(\rightarrow_{bb}\) ordering by exploiting the strong memory order of \(R^{sc}(x,1)\) and invalidates \((\text{RWRW-inv-sync-opt})\).

The candidate solution \(c_2\) is preferred over \(c_1\) as it contains weaker fences. On the other hand, candidate \(c_3\) represents an optimal solution as it uses the smallest number of weakest fences. We formally define the optimality of fence synthesis in \(\S3\). While \text{FenSying} will compute the solution \(c_3\), \text{fFenSying} may compute one from the many candidate solutions.

Both \text{FenSying} and \text{fFenSying} start by transforming each buggy trace \(\tau\) to an intermediate version, \(\tau^{imm}\), by inserting untyped \(C11\) fences (called candidate fences) above and below the trace events. \((\text{RWRW-Imm})\) shows an intermediate version corresponding to \((\text{RWRW-bt})\). The addition of fences (assuming they are of the strongest variety) leads to the creation of new \(\rightarrow_{bb}\) ordering edges. This may result in cycles in the dependency graph under the \(C11\) semantics (refer \(\S4\)). The set of fences in a cycle constitutes a candidate solution. For example, an ordering from \(F_{12}\) to \(F_{22}\) in \((\text{RWRW-imm})\) induces a cyclic relation \(W^{fix}(y,1) \rightarrow R^{sc}(y,1) \rightarrow_{bb} W^{fix}(y,1)\) violating the \(\rightarrow_{rel} \rightarrow_{bb}\) irreflexivity (refer \(\S4\)).

The candidate solutions are collected in a SAT query \((\Phi)\). Assuming \(c_1\), \(c_2\) and \(c_3\) are the only candidate solutions for \((\text{RWRW-bt})\), then \(\Phi = (F_{12} \land F_{22}) \lor (F_{12} \land F_{22}) \lor (F_{12})\), where for a fence \(F_{12}^{sc}\), \(F_{12}\) represents the same fence with unassigned memory order. \text{fFenSying} uses a SAT solver to compute the min-model of \(\Phi\), \(\text{min}\Phi = \{F_{12}\}\). Further, \text{fFenSying} applies the \(C11\) ordering rules on fences to determine the weakest memory order for the fences in \(\text{min}\Phi\). For instance, \(F_{12}\) in \(\text{min}\Phi\) is computed to have the order \(rel\) (refer \(\S6\)). \text{fFenSying} then inserts \(F_{12}\) with memory order \(rel\) in \((\text{RWRW})\) at the location depicted in \((\text{RWRW-inv-sync-opt})\). This process repeats for the next buggy trace.

In contrast, since \text{FenSying} works with all buggy traces at once, it requires the conjunction of the SAT queries \(\Phi_i\) corresponding to each buggy trace \(\tau_i\). The min-model of the conjunction is computed, which provides optimality.

### 3 Preliminaries

Consider a multi-threaded \(C11\) program \((P)\). Each thread of \(P\) performs a sequence of \textit{events} that are runtime instances of memory access operations (reads, writes, and rmws) on shared objects and \(C11\) fences. Note that an event is
uniquely identified in a trace; however, multiple events may be associated with the same program location. The events may be atomic or non-atomic. Given the set $O$ of shared objects accessed by $P$ and the set $M$ of C11 memory orders, an event is formally defined as:

**Definition 1 (Event).** An event $e$ is a tuple $\langle \text{thr}(e), \text{idx}(e) \rangle \text{act}(e), \text{obj}(e), \text{ord}(e), \text{loc}(e) \rangle$ where,
- $\text{thr}(e) \in P$, represents the thread of $e$;
- $\text{idx}(e)$ represents the identifier of $e$ unique to $\text{thr}(e)$;
- $\text{act}(e) \in \{\text{read}, \text{write}, \text{rmw (read-modify-write)}, \text{fence}\}$, is the event action;
- $\text{obj}(e) \subseteq O$, is the set of memory objects accessed by $e$;
- $\text{ord}(e) \subseteq M$, is the C11 memory order associated with $e$; and
- $\text{loc}(e)$ is the program location corresponding to $e$.

**C11 memory orders.** The atomic events and fence operations are associated with memory orders that define the ordering restriction on atomic and non-atomic events around them. Let $M = \{\text{na, rlx, rel, acq, ar, sc}\}$, represent the orders relaxed ($\text{rlx}$), release ($\text{rel}$), acquire/consume ($\text{acq}$), acquire-release ($\text{ar}$) and sequentially consistent ($\text{sc}$) for atomic events. A non-atomic event is recognized by the $\text{na}$ memory order. Let $\sqsubseteq \subseteq M \times M$ represent the relation weaker such that $m_1 \sqsubseteq m_2$ represents that the $m_1$ is weaker than $m_2$. As a consequence, annotating an event with $m_2$ may order two events that remain unordered with $m_1$. The orders in $M$ are related as $\text{na} \sqsubseteq \text{rlx} \sqsubseteq \{\text{rel, acq}\} \sqsubseteq \text{ar} \sqsubseteq \text{sc}$. We also define the relation $\sqsubseteq$ to represent weaker or equally weak. Similarly, we define $\sqsupseteq$ to represent stronger and $\sqsupset$ to represent stronger or equally strong.

We use $E^W \subseteq E$ to denote the set of events that perform write to shared memory objects i.e., events $e$ s.t. $\text{act}(e) = \text{write or rmw}$. Similarly, we use $E^R \subseteq E$ to denote events $e$ that read from a shared memory object i.e., $\text{act}(e) = \text{read or rmw}$, and $E^F$ to denote the fence events $e$ s.t. $\text{act}(e) = \text{fence}$. We also use $E^{(m)} \in E$ (and accordingly $E^W^{(m)}$, $E^R^{(m)}$ and $E^F^{(m)}$) to represent the events with the memory order $m \in M$; as an example $E^{(sc)}$ represents the set of fences with the memory order $\text{sc}$.

**Definition 2 (Trace).** A trace, $\tau$, of $P$ is a tuple $(E_\tau, \rightarrow_{\tau^{\text{hb}}}, \rightarrow_{\tau}^{\text{mo}}, \rightarrow_{\tau}^{\text{rf}})$, where
- $E_\tau \subseteq E$ represents the set of events in the trace $\tau$;
- $\rightarrow_{\tau}^{\text{hb}}$ (Happens-before) $\subseteq E_\tau \times E_\tau$ is a partial order which captures the event interactions and inter-thread synchronizations, discussed in §4;
- $\rightarrow_{\tau}^{\text{mo}}$ (Modification-order) $\subseteq E^W_\tau \times E^W_\tau$ is a total order on the writes of an object;
- $\rightarrow_{\tau}^{\text{rf}}$ (Reads-from) $\subseteq E^W_\tau \times E^R_\tau$ is a relation from a write event to a read event signifying that the read event takes its value from the write event in $\tau$.

Note that, we use $E^W_\tau$, $E^R_\tau$ and $E^F_\tau$ (and also $E^W^{(m)}$, $E^R^{(m)}$ and $E^F^{(m)}$ where $m \in M$) for the respective sets of events for a trace $\tau$.

**Relational Operators.** $R^{-1}$ represents the inverse and $R^+$ represents the transitive closure of a relation $R$. Further, $R_1; R_2$ represents the composition of relations $R_1$ and $R_2$. Let $R_{\text{sc}}$ represent a subset of a relation $R$ on $\text{sc}$ ordered events: i.e. $(e_1, e_2) \in R_{\text{sc}} \iff (e_1, e_2) \in R \land e_1, e_2 \in E^{(sc)}$. Note that we also use the infix notation $e_1 R e_2$ for $(e_1, e_2) \in R$. Lastly, a relation $R$ has a cycle (or is cyclic) if $\exists e_1, e_2 \in E$ s.t. $e_1 R e_2 \land e_2 R e_1$.
A note on optimality. The notion of optimality may vary with context. Consider two candidate solutions \( \{ \mathcal{P}_i \} \) and \( \{ \mathcal{P}_j \} \) where the superscripts represent the memory orders. The two solutions are incomparable under \( C_{11} \), and their performance efficiency is subject to the input program and the underlying architecture. \textit{FenSying} chooses a candidate solution \( c \) as an optimal solution if:

(i) \( c \) has the smallest number of candidate fences, and
(ii) each fence of \( c \) has the weakest memory order compared to other candidate solutions that satisfy (i).

Let \( sz(c) \) represent the size of the candidate solution \( c \) and given the set of all candidate solutions \( \{ c_1, ..., c_n \} \) to fix \( P \), let \( sz(P) = \min(sz(c_1), ..., sz(c_n)) \). Further, we assign weights \( wt(c) \) to each candidate solution \( c \), computed as the summation of the weights of its fences where a fence ordered \( \text{rel} \) or \( \text{acq} \) is assigned the weight 1, a fence ordered \( \text{ar} \) is assigned 2, and a fence ordered \( \text{sc} \) is assigned 3. Optimality for \( \text{FenSying} \) is formally defined as:

**Definition 3. Optimality of fence synthesis.** Consider a set of candidate solutions \( c_1, ..., c_n \). A solution \( c_i \) (for \( i \in [1,n] \)) is considered optimal if:

(i) \( sz(c_i) = sz(P) \) and (ii) \( \forall j \in [1,n] \) s.t. \( sz(c_j) = sz(P) \), \( wt(c_i) \leq wt(c_j) \).

4 Background: C11 Memory Model

The \( C_{11} \) memory model defines a trace using a set of event relations, described in Definition 2. The most significant relation that defines a \( C_{11} \) trace \( \tau \) is the irreflexive and acyclic happens-before relation, \( \rightarrow_{h} \subseteq \mathcal{E}_\tau \times \mathcal{E}_\tau \). The \( \rightarrow_{h} \) relation is composed of the following relations [12].

- \( \rightarrow_{wh} \) (Sequenced-before): total occurrence order on the events of a thread; i.e. \( \forall e_1, e_2 \in \mathcal{E}_\tau \) s.t. \( thr(e_1) = thr(e_2) \) and \( e_1 \) occurs before \( e_2 \) in their thread \( \implies e_1 \rightarrow_{wh} e_2 \).

- \( \rightarrow_{aw} \) (Synchronizes-with) Inter-thread synchronization between a write \( e_w \) (ordered \( \subseteq \text{re} \)) and a read \( e_r \) (ordered \( \subseteq \text{ac} \)) when \( e_w \rightarrow_{tau} e_r \); i.e. \( \forall e_w \in \mathcal{E}_{\tau}^{\text{w}}, e_r \in \mathcal{E}_{\tau}^{\text{r}} \) s.t. \( \text{ord}(e_w) \supseteq \text{re} \) and \( \text{ord}(e_r) \supseteq \text{ac} \), \( e_w \rightarrow_{aw} e_r \implies e_w \rightarrow_{h} e_r \).

- \( \rightarrow_{do} \) (Dependency-ordered-before): Inter-thread synchronization between a write \( e_w \) (ordered \( \supseteq \text{re} \)) and a read \( e_r \) (ordered \( \supseteq \text{ac} \)) when \( e'_w \rightarrow_{tau} e_r \) for \( e'_w \in \text{release-sequence} \) of \( e_w \) in \( \tau \) [9,12]; i.e. \( \forall e_w, e'_w \in \mathcal{E}_{\tau}^{\text{w}}, e_r \in \mathcal{E}_{\tau}^{\text{r}} \) s.t. \( \text{ord}(e_w) \supseteq \text{re} \) and \( \text{ord}(e_r) \supseteq \text{ac} \), \( e'_w, e_w \in \text{release-sequence} \) of \( e_w \) and \( e'_w \rightarrow_{aw} e_r \implies e_w \rightarrow_{do} e_r \).

- \( \rightarrow_{ih} \) (Inter-thread-hb): Inter-thread relation computed by extending \( \rightarrow_{aw} \) and \( \rightarrow_{do} \) with \( \rightarrow_{hi} \); i.e.

\[
\forall e_1, e_2, e_3 \in \mathcal{E}_\tau,
\begin{align*}
1. & e_1 \rightarrow_{aw} e_2, \text{ or } \\
2. & e_1 \rightarrow_{do} e_2, \text{ or } \\
3. & e_1 \rightarrow_{aw} e_3 \land e_3 \rightarrow_{hi} e_2, \text{ or } \\
4. & e_1 \rightarrow_{aw} e_3 \land e_3 \rightarrow_{hi} e_2, \text{ or }
\end{align*}
\]

\( ^3 \) release-sequence of \( e_w \) in \( \tau \): maximal contiguous sub-sequence of \( \rightarrow_{wo} \) that starts at \( e_w \) and contains: (i) write events of \( thr(e_w) \), (ii) rmw events of other threads [9,12].
5. \( e_1 \rightarrow^\text{ithb} e_4 \land e_3 \rightarrow^\text{ithb} e_2 \)  \( \Rightarrow e_1 \rightarrow^\text{ithb} e_2 \).

\( \rightarrow^\text{hb} \) \text{ (Happens-before): Inter-thread relation defined as } \rightarrow^\text{hb} \cup \rightarrow^\text{ithb}.

The \( \rightarrow^\text{hb} \) relation along with the \( \rightarrow^\text{mo} \) and \( \rightarrow^\text{rf} \) relations (Definition 2) is used in specifying the set of six coherence conditions \([12,17]\):

\( \rightarrow^\text{hb} \) is irreflexive. \hspace{2cm} (co-h)

\( \rightarrow^\text{rf}; \rightarrow^\text{hb} \) is irreflexive. \hspace{2cm} (co-rh)

\( \rightarrow^\text{mo}; \rightarrow^\text{hb} \) is irreflexive. \hspace{2cm} (co-mh)

\( \rightarrow^\text{mo}; \rightarrow^\text{rf}; \rightarrow^\text{hb} \) is irreflexive. \hspace{2cm} (co-mrh)

\( \rightarrow^\text{mo}; \rightarrow^\text{rf}; \rightarrow^\text{hb}; \rightarrow^\text{rf} \) is irreflexive. \hspace{2cm} (co-mrh)

Additionally, all \( sc \) ordered events in a trace \( \tau \) must be related by a total order \( \rightarrow^\text{to} \) that concurs with the coherence conditions. We use an irreflexive relation called \textit{from-reads} \( \left( \rightarrow^\text{fr} \right) \) for ordering reads with \textit{later} writes. Consequently, \( \rightarrow^\text{to} \) must satisfy the following condition \([12,24]\):

\( \text{order}(P,R) \triangleq (\exists a \ R(a,a)) \land (R^+ \subseteq R) \land (R \subseteq P \times P) \) and;

\( \text{total}(P,R) \triangleq \forall a,b \in P \Rightarrow a = b \lor R(a,b) \lor R(b,a). \)

All \( sc \) ordered events must form a total order \( \rightarrow^\text{to} \) s.t. the following conditions are satisfied:

1. \( \text{order}(\mathcal{E}^{\text{sc}}, \rightarrow^\text{to}) \land \text{total}(\mathcal{E}^{\text{sc}}, \rightarrow^\text{to}) \land \rightarrow^\text{hb} \subseteq \tau^{\text{mo}} \subseteq \rightarrow^\text{to} \) (coto)

2. \( \forall e_w \rightarrow^\text{fr} e_r \) s.t. \( e_r \in \mathcal{E}^{\tau^{\text{to}}} \)

   - either, \( e_w \in \mathcal{E}^{\tau^{\text{sc}}} \land \text{imm-scr}(\tau, e_w, e_r) \)  \hspace{2cm} (rfto1)

   - or, \( e_w \notin \mathcal{E}^{\tau^{\text{sc}}} \land \not\exists e'_w \in \mathcal{E}^{\tau^{\text{sc}}} \) s.t. \( e_w \rightarrow^\text{hb} e'_w \land \text{imm-scr}(\tau, e'_w, e_r) \). \hspace{2cm} (rfto2)

where, \( \text{imm-scr}(\tau, a, b) \triangleq a \in \mathcal{E}^{\tau^{\text{to}}} \land a \rightarrow^\tau \text{fr} b \land \text{obj}(a) = \text{obj}(b) \)

\( \land \not\exists c \in \mathcal{E}^{\tau^{\text{to}}} \) s.t. \( \text{obj}(c) = \text{obj}(a) \land a \rightarrow^\tau \text{fr} c \).

3. \( \forall e_w \rightarrow^\text{fr} e_r \) s.t. \( e_r \in \mathcal{E}^{\tau^{\text{to}}} \), \( \exists \mathcal{F} \in \mathcal{E}^{\tau^{\text{to}}} \) s.t. \( \mathcal{F} \rightarrow^\text{fr} e_r \land e_w \rightarrow^\text{to} \mathcal{F} \land \not\exists e'_w \in \mathcal{E}^{\tau^{\text{sc}}} \) \hspace{2cm} (frfto)

We represent the conjunction of the four conditions by \( \rightarrow^\text{to} \) intuitively.

- \( \forall e_{c1} e_{c2} \in \mathcal{E}^{\tau^{\text{sc}}} \) if \( e_{c1} \rightarrow^\text{to} e_{c2} \) then \( e_{c1} \rightarrow^\text{hb} \cup \rightarrow^\text{to} \cup \rightarrow^\text{fr} \cup \rightarrow^\text{fr} \) and,

- an \( sc \) read (or any read with an \( sc \) fence \( \rightarrow^\text{fr} \) ordered before it) must not read from an \( sc \) write that is not immediately \( \rightarrow^\text{to} \) ordered before it.

Conjunction of (coherence conditions) and \( \rightarrow^\text{to} \) forms the sufficient condition to determine if a trace \( \tau \) is valid under \textit{C11}.

\textbf{HB with C11 fences.} \textit{C11} fences form \( \rightarrow^\text{ithb} \) with other events \([9,12]\). A fence can be associated with the memory orders \textit{rel} and \textit{acq}. An appropriately placed fence can form \( \rightarrow^\text{sw} \) and \( \rightarrow^\text{dib} \) relation from an \( \rightarrow^\text{fr} \) relation between events of different threads, formally:

The \( \rightarrow^\text{sw} \) relation is formed with \textit{C11} fences as follows:

\( \forall e_1, e_2 \in \mathcal{E}^{\tau^{\text{fr}}} \) s.t. \( e_1 \rightarrow^\tau e_2 \),

- if \( \text{ord}(e_1) \not\subseteq \textit{rel}, \exists \mathcal{F}^{\textit{acq}} \in \mathcal{E}^{\tau^{\text{sw}}} \) s.t. \( \text{ord}(\mathcal{F}^{\textit{acq}}) \subseteq \textit{acq} \) and \( e_2 \rightarrow^\tau \mathcal{F}^{\textit{acq}} \) then \( e_1 \rightarrow^\tau^\text{sw} \mathcal{F}^{\textit{acq}} \);

- if \( \text{ord}(e_2) \not\subseteq \textit{acq}, \exists \mathcal{F}^{\textit{rel}} \in \mathcal{E}^{\tau^{\text{sw}}} \) s.t. \( \text{ord}(\mathcal{F}^{\textit{rel}}) \subseteq \textit{rel} \) and \( \mathcal{F}^{\textit{rel}} \rightarrow^\tau^\text{sw} e_1 \) then \( \mathcal{F}^{\textit{rel}} \rightarrow^\tau^\text{sw} e_2 \);

\( \rightarrow^\text{dib} \) relation is defined similarly.
The conditions described above, leading to a→swτ between program events, are diagrammatically represented in Figure 1(a-d).

Similarly, the→dobτ relation is formed with C11 fences as follows:

∀e₁, e₂ ∈ Eτ s.t. e₁→rfτ e₂, if ∃e′₁ ∈ E_Wτ s.t. e₁ is in release-sequence of e′₁; and ∃F_acq ∈ E_Fτ s.t. ord(F_acq)⊒acq and e₂→sbτ F_acq then e′₁→dobτ F_acq.

The conditions leading to a→dobτ between program events, are diagrammatically represented in Figure 1(e-f).

5 Invalidating buggy traces with C11 fences

The key idea behind the proposed techniques is to introduce fences such that either (coherence conditions) or (to-sc) are violated. This section introduces two approaches for determining if the trace is rendered invalid with fences.

Consider τ_imm of a buggy trace τ. The candidate fences of τ_imm inflate →sbτ, →swτ, →dobτ and →ithbτ relations (fences do not contribute to →moτ, →rfτ and →frτ). The inflated relations are denoted as →sbτ_imm, →swτ_imm, →dobτ_imm and →ithbτ_imm. We propose Weak-FenSyng and Strong-FenSyng to detect the invalidity of τ_imm.

Weak-FenSyng. Weak-FenSyng computes compositions of relations that correspond to the (coherence conditions). It then checks if there exist cycles in the compositions (using Johnson’s algorithm [13]). The approach assumes the memory order ar for all candidate fences. Consider a buggy trace (WRIR) where x and y have 0 as initial values. Weak-FenSyng detects a cycle in →moτ, →rfτ, →hbτ, →rf⁻¹ with the addition of candidate fences F_ar₁ and F_ar₂ as shown in (WRIR-invalidated). This violates the condition (co-mrhi), thus, invalidating (WRIR). Lemma 1 shows the correctness of Weak-FenSyng.

Strong-FenSyng. This technique works with the assumption that all candidate fences have the order sc. Strong-FenSyng detects the infeasibility in constructing a→ioτ that adheres to (to-sc). In order to detect violation of (to-sc),
Strong-FenSying introduces a possibly reflexive relation on sc-ordered events of $\tau_{\text{imm}}$, called sc-order ($\rightarrow_{\tau_{\text{imm}}}$). The $\rightarrow_{\tau_{\text{imm}}}$ relation is such that a total order cannot be formed on the sc events of $\tau_{\text{imm}}$ if a cycle exists in $\rightarrow_{\tau_{\text{imm}}}$, i.e., all sc event pairs ordered by $\rightarrow_{\tau_{\text{imm}}}$ are contained in $\rightarrow_{\tau_{\text{imm}}}$.

Notably, pairs of sc events that do not have a definite order are not ordered by $\rightarrow_{\tau_{\text{imm}}}$. This is because if such a pair of events is involved in a cycle then we can freely flip their order and eliminate the cycle. Consider the buggy trace (SB), $W^{\text{sc}}(x, 1) \rightarrow_{\tau_{\text{imm}}} W^{\text{sc}}(y, 1)$ and $W^{\text{sc}}(y, 1) \rightarrow_{\tau_{\text{imm}}} W^{\text{sc}}(x, 1)$ are both valid total-orders on the sc events of the trace. The set $\rightarrow_{\tau_{\text{imm}}}$ does not contain either of the two event pairs and would be empty for this example.

As a consequence, pairs of events that do not have definite total order cannot contribute to the reflexivity of $\rightarrow_{\tau_{\text{imm}}}$ and can be safely ignored. Thus, $\rightarrow_{\tau_{\text{imm}}}^{\text{so}} \subseteq \rightarrow_{\tau_{\text{imm}}}$ for a trace $\tau$. Further, if a total order cannot be formed on sc ordered events then a corresponding cycle exists in $\rightarrow_{\tau_{\text{imm}}}$. The observations are formally presented with supporting proofs in Lemmas 2, 3. Lemma 3 further proves that Strong-FenSying is sound.

Definition 4 formally presents $\rightarrow_{\tau_{\text{imm}}}^{\text{so}}$ based on the above stated considerations.

**Definition 4.** sc-order ($\rightarrow_{\tau_{\text{imm}}}^{\text{so}}$)

$\forall e_1, e_2 \in E_{\tau}$ s.t. $(e_1, e_2) \in R$, where $R = \rightarrow_{\tau} \cup \rightarrow_{\tau_{\text{imm}}}^{\text{mo}} \cup \rightarrow_{\tau_{\text{imm}}}^{\text{fr}} \cup \rightarrow_{\tau_{\text{imm}}}^{\text{rf}}$

- if $e_1, e_2 \in E_{\tau}^{(\text{sc})}$ then $e_1 \rightarrow_{\tau_{\text{imm}}}^{\text{so}} e_2$; 
- if $e_1 \in E_{\tau}^{(\text{sc})}$, $\exists F_{\text{sc}} \in E_{\tau_{\text{imm}}}^{(\text{sc})}$ s.t. $e_2 \rightarrow_{\tau_{\text{imm}}}^{\text{so}} F_{\text{sc}}$ then $e_1 \rightarrow_{\tau_{\text{imm}}}^{\text{so}} F_{\text{sc}}$; 
- if $e_2 \in E_{\tau}^{(\text{sc})}$, $\exists F_{\text{sc}} \in E_{\tau_{\text{imm}}}^{(\text{sc})}$ s.t. $F_{\text{sc}} \rightarrow_{\tau_{\text{imm}}} e_1$ then $F_{\text{sc}} \rightarrow_{\tau_{\text{imm}}}^{\text{so}} e_2$; 
- if $\exists F_{\text{sc}}^{(\text{sc})} \subseteq E_{\tau_{\text{imm}}}^{(\text{sc})}$ s.t. $F_{\text{sc}}^{(\text{sc})} \rightarrow_{\tau_{\text{imm}}}^{\text{so}} e_1$ and $e_2 \rightarrow_{\tau_{\text{imm}}}^{\text{so}} F_{\text{sc}}^{(\text{sc})}$ then $F_{\text{sc}}^{(\text{sc})} \rightarrow_{\tau_{\text{imm}}}^{\text{so}} e_2$.

The trace depicted in (SB) can be invalidated with strong fences as shown in (SB-inv). The sc events of (SB-inv) cannot be totally ordered and Strong-FenSying detects the same through a cycle in $\rightarrow_{\tau_{\text{imm}}}^{\text{so}}$ (formed by (soee) and (sofe)).

**Scope of FenSying/F FenSying.**
Initially, $x = 0, y = 0$

This work synthesizes $C11$ fences and stands fundamentally different from techniques that modify the memory orders of program events. $\text{sc} C11$ fences cannot restore sequential consistency [17], hence, strengthening memory orders may invalidate buggy traces that the strongest $C11$ fences cannot.

The examples (iriw-invalid) and (iriw-valid) highlight the difference, where $\rightarrow$ depicts the total-order on $\text{sc}$ events. The fences of (iriw-valid) do not introduce an ordering between the write events and the corresponding read events thereby allowing reads from initial events. Thus, the trace shown in (iriw-valid) cannot be invalidated by $C11$ fences. However, changing the memory order of read and write events achieve the desired ordering and invalidate the outcome, as shown in (iriw-invalid).

FenSying and fFenSying invalidate traces by synthesizing $C11$ fences, thus, (iriw-valid) cannot be stopped by FenSying or fFenSying. However, architectures translate the strong memory access events to memory access operation and supporting barriers. The translation of strongly ordered event to barriers may be sub-optimal.

The examples (SB-inv-mo) and (SB-inv-fen) highlight the difference. The trace shown in the examples is invalidated by strengthening the memory orders of writes and reads in (SB-inv-mo) and by synthesizing $\text{sc}$ fences in (SB-inv-fen). The translation of (SB-inv-mo) to Power and ARMv7 is depicted in (SB-inv-mo-power) and (SB-inv-mo-ARM) respectively. The translation of (SB-inv-fen) to Power and ARMv7 is depicted in (SB-inv-fen-power) and (SB-inv-fen-ARM) respectively. Clearly each of the two architectures place additional (and unnecessary) barriers on interpreting barrier requirement from memory orders.

\footnote{isync+ refers to cmp; bc; isync}
6 Methodology

Buggy traces and candidate fences. Algorithms 1 and 2 present FenSying and fFenSying, respectively. The algorithms rely on an external buggy trace generator (BTG) for the buggy trace (s) of \( P \) (lines 2,8). The candidate fences are inserted (to obtain \( \tau^{imm} \)), and the event relations are updated (lines 16-17).

Detecting violation of trace coherence. The algorithms detect possible violations of trace coherence conditions resulting from the candidate fences at lines 18-19 of the function synthesisCore. Figures (\( RW\)-inv-sync) and (\( RW\)-inv-sync-opt) represent two instances of violations of \( (co-rh) \) detected by \( \text{Weak-FenSying} \) (through a cycle \( \text{WR}\)-inv-sync). The candidate fences corresponding to these cycles (which include only candidate fences) are \( \{ F_{12}, F_{22} \} \) and \( \{ F_{12} \} \). Further, for the same example, (\( RW\)-inv-to) represents a violation detected by \( \text{Strong-FenSying} \) with the candidate solution \( \{ F_{12}, F_{22} \} \). The algorithms discard all candidate fences other than \( F_{12} \) and \( F_{22} \) from future considerations (assuming no other violations were detected). Now \( \tau \) can be invalidated as the set of cycles is nonempty (line 20).

The complexity of detecting all cycles for a trace is \( O((|E|+|C|)(C+1)) \) where \( C \) represents the number of cycles of \( \tau \) and \( E \) represents the number of pairs of events in \( E \). Note that \( E \) is in \( O(|E|) \) and \( C \) is in \( O(|E|!) \). Thus, Weak- and \( \text{Strong-FenSying} \) have exponential complexities in the number of traces and the number of events per trace.

Reduction for optimality. The algorithms use a SAT solver to determine the optimal number of candidate fences. The candidate fences from each candidate solution of \( \tau \) are conjuncted to form a SAT query. Further, to retain at least one solution corresponding to \( \tau \) the algorithms take a disjunction of the conjuncts. The SAT query is represented in the algorithm as \( \Phi_{\tau} := \bigvee (\text{weakCycles}_{\tau} \cup \text{strongCycles}_{\tau}) \) (line 22) and presented in Equation 1 (where \( W_{\tau} \) and \( S_{\tau} \) represent weakCycles and strongCycles, and \( W \) and \( S \) represent the set of candidate fences in cycles \( W \) and \( S \) respectively). Further, FenSying combines the SAT formulas corresponding to each buggy trace via conjunction (line 4),

\[
\Phi_{\tau} := \bigvee (\text{weakCycles}_{\tau} \cup \text{strongCycles}_{\tau})
\]
shown in Equation 2. However, note that for \( \text{ffFenSy}ng \) \( \Phi = \Phi_\tau \).

\[
\Phi_\tau = ( \bigvee_{\nu \in \nu_\tau} \bigwedge_{F_\nu \in \mathbb{F}} F_\nu ) \vee ( \bigvee_{\sigma \in \sigma_\tau} \bigwedge_{F_\sigma \in \mathbb{F}} F_\sigma ) \quad (1)
\]

\[
\Phi = \bigwedge_{\tau \in \mathbb{B}} \Phi_\tau \quad (2)
\]

We use a SAT solver to compute the min-model \( \langle \text{min}\Phi \rangle \) of the query \( \Phi \) (lines 5,10). For instance, the query for \( RWR\text{-bt} \) is \( \Phi = (\mathbb{F}_{12}) \vee (\mathbb{F}_{12} \land \mathbb{F}_{22}) \vee (\mathbb{F}_{12} \land \mathbb{F}_{22}) \) and min-model, \( \text{min}\Phi = \{\mathbb{F}_{12}\} \). The solution to the SAT query returns the smallest set of fences to be synthesized.

The complexity of constructing the query \( \Phi_\tau \) is \( O(C,F) \), where \( C \) is the number of cycles per trace and \( F \) is the number of fences per cycle. The structure of the query \( \Phi \) corresponds to the Head-cycle-free (HCF) class of CNF theories; hence, the min-model computation falls in the FP complexity class [8].

**Determining optimal memory orders of fences.** The set \( \text{min}\Phi \) gives a sound solution that is optimal only in the number of fences. The function \( \text{assignMO} \) (lines 6,11) assigns the weakest memory order to the fences in \( \text{min}\Phi \) that is sound. Let \( \text{min-cycles} \) represent a set of cycles such that every candidate fence in the cycles belongs to \( \text{min}\Phi \). The \( \text{assignMO} \) function computes memory order for fences of \( \text{min-cycles} \) of each trace as follows: If a cycle \( c \in \text{min-cycles} \) is detected, then its fences must form a \( \rightarrow_{\text{sw}} \) or \( \rightarrow_{\text{dob}} \) with an event of \( \tau_{\text{imm}} \) (since, candidate fences only modify \( \rightarrow_{\text{sw}} \) and \( \rightarrow_{\text{dob}} \)). Let \( R = \rightarrow_{\text{sw}} \cup \rightarrow_{\text{dob}} \). The scheme to compute fence types is as follows:

- If a fence \( \mathbb{F} \) in a weak cycle \( c \) is related to an event \( e \) of \( c \) by \( R \) as \( e\mathbb{F} \), then \( \mathbb{F} \) is assigned the memory order \( \text{acq} \);
- if an event \( e \) in \( c \) is related to \( \mathbb{F} \) as \( \mathbb{F}Re \) then \( \mathbb{F} \) is assigned \( \text{rel} \);
- if events \( e, e' \) of \( c \) are related to \( \mathbb{F} \) as \( e\mathbb{F} e' \) then \( \mathbb{F} \) is assigned \( \text{ar} \).
shown with superscripts and the weights of the cycle $\tau$

Let $\tau$ and $c$ in two buggy traces $rel$ 

The solution $\tau$ $\tau$ are combined with $\tau$ with the lower weight. 

Further, assignMO discusses above, the fences $\tau$ and $\ar$ respectively and $wt(c) = 4$ (refer §3).

Further, assignMO iterates over all buggy traces and detects the sound weakest memory order for each fence across all traces as follows. Assume a cycle $c_1$ in $\tau_{1}^{\text{im}}$ and a cycle $c_2$ in $\tau_{2}^{\text{im}}$. The function computes a union of the fences of $\tau_1$ and $\tau_2$ while choosing the stronger memory order for each fence that is present in both the cycles. In doing so, both $\tau_1$ and $\tau_2$ are invalidated. Further, when two candidate solutions have the same set of fences, the function selects the one with the lower weight.

Consider the cycles of buggy traces $\tau_{1}$ and $\tau_{2}$ shown in (candidate-fences). Let $\text{minF} = \{F_1, F_2, F_3\}$. The memory orders of the fences for each trace are shown with superscripts and the weights of the cycle $\tau_{1}c_1, \tau_{1}c_2$ and $\tau_{2}c_1$ are written against the name of the cycles. The candidate solutions $\tau_{1}c_1$ and $\tau_{1}c_2$ are combined with $\tau_{2}c_1$ to form $\tau_{12}c_{11}$ and $\tau_{12}c_{21}$ of weights 5 and 4, respectively. The solution $\tau_{12}c_{11}$ is of higher weight and is discarded. In $\tau_{12}c_{21}$, the optimal memory orders $\text{rel}$, $\text{acq}$ and $\text{ar}$ are assigned to fences $F_1$, $F_2$ and $F_3$, respectively. It is possible that min-cycles may contain fences originally in $P$. If the process discussed above computes a stronger memory order for a program fence than its original order in $P$, then the technique strengthens the memory order of the fence to the computed order. Note that this reasoning across traces does not occur in $\text{fFenSying}$ as it considers only one trace at a time.

Determining the optimal memory orders has a complexity in $O(BT.C.F + M^{BT})$, where $BT$ if the number of buggy traces of $P$, $C$ and $F$ are defined as before, and $M$ is the number of min-cycles per trace.

In our experimental observation (refer to §7), the number of buggy traces analyzed by $\text{fFenSying}$ is significantly less than $|BT|$. Therefore, in practice, the complexity of various steps of $\text{fFenSying}$ that are dependent on $BT$ reduces exponentially by a factor of $|BT|$.

**Nonoptimality of $\text{fFenSying}$**. Consider the example (3-fence). It shows cycles in two buggy traces $\tau_{1}$ and $\tau_{2}$ of an input program. $\text{FenSying}$ provides the formula $\Phi_{\tau_{1}}$ and $\Phi_{\tau_{2}}$ to the SAT solver and the optimal solution obtained is $(F_1 \land F_2)$. However, $\text{fFenSying}$ considers the formula $\Phi_{\tau_{1}}$ and $\Phi_{\tau_{2}}$ in separate iterations and may return a nonoptimal result $(F_1 \land F_2)$. 

| $\tau_{1}c_{1}(4)$ | $F_1^{\text{sc}} \land F_2^{\text{sc}}$ |
|-------------------|-----------------|
| $\tau_{1}c_{2}(4)$ | $F_1^{\text{sc}} \land F_2^{\text{sc}} \land F_3^{\text{sc}}$ |

| $\tau_{2}c_{l}(3)$ | $F_1^{\text{sc}} \land F_2^{\text{sc}} \land F_3^{\text{sc}}$ |
|-------------------|-----------------|
| $\tau_{12}c_{11}(5)$ | $F_1^{\text{sc}} \land F_2^{\text{sc}} \land F_3^{\text{sc}}$ |
| $\tau_{12}c_{21}(4)$ | $F_1^{\text{sc}} \land F_2^{\text{sc}} \land F_3^{\text{sc}}$ |

- All the fences in a strong cycle are assigned the memory order $\text{sc}$.

Consider a cycle $c : e \rightarrow \text{sw}_{t_{1}}$ $F_1 \rightarrow \text{sw}_{t_{2}}$ $F_2 \rightarrow \text{sw}_{t_{3}}$ $F_3 \rightarrow \text{sw}_{t_{4}} e'$ representing a violation of $\rightarrow_{t_{1}}^{\text{sw}} \rightarrow_{t_{2}}^{\text{sw}} \rightarrow_{t_{3}}^{\text{sw}} \rightarrow_{t_{4}}^{\text{sw}} e$ representing a violation of $\rightarrow_{t_{1}}^{\text{im}} \rightarrow_{t_{2}}^{\text{im}}$ irreversibility (condition (co-rh)). According to the scheme discussed above, the fences $F_1$, $F_2$ and $F_3$ are assigned the memory orders $\text{rel}$, $\text{ar}$ and $\text{acq}$ respectively and $wt(c) = 4$ (refer §3).

- Consider the example (3-fence). It shows cycles in $\tau_{1}$ $(\text{candidate-fences})$ $\Phi_{\tau_{1}} = (F_1 \land F_2) \lor (F_1 \land F_3 \land F_4)$ cycles in $\tau_{2}$ $(\text{candidate-fences})$ $\Phi_{\tau_{2}} = (F_3 \land F_4)$
We prove the soundness of \textsc{fFenSying} and \textsc{FenSying} with Theorems 1 and 2 respectively and the optimality of \textsc{FenSying} with Theorem 3.

### Notations for Theorem proofs.
For an input program \( P \), let \( P' = prg(P, E') \), represent a transformation of \( P \) constructed by adding the events \( E' \) to the original events of \( P \), where \( E' \) is a set of fence events i.e. \( \forall e \in E' \ act(e) = \text{fence} \). Further, let \( bt(P) \) represent the set of buggy traces of \( P \). Let \( \tau^{inv} \) represent the invalidated version of \( \tau \) with synthesized fences of a candidate solution and let \( P^{fix} \) represent the fixed version of \( P \) with no more buggy traces. Let \( \mathbb{C} \) represent the set of relation compositions corresponding to the (coherence conditions) i.e.
\[
\mathbb{C} = \{(\rightarrow_{\tau}^{hb}), (\rightarrow_{\tau}^{rf}; \rightarrow_{\tau}^{mo}), (\rightarrow_{\tau}^{mo}; \rightarrow_{\tau}^{rf}; \rightarrow_{\tau}^{mb}), (\rightarrow_{\tau}^{mo}; \rightarrow_{\tau}^{rf}; \rightarrow_{\tau}^{m1}), (\rightarrow_{\tau}^{mo}; \rightarrow_{\tau}^{rf}; \rightarrow_{\tau}^{rf-1})\}. 
\]
For a fence \( F \in \text{min} \Phi \), let \( F^{m} \) represent the same fence with memory order \( m \) assigned by the \text{assignMO} routine (line 14 of Algorithm 1).

#### Lemma 1. Weak-FenSying is sound:
Given an input program \( P \), \( \forall \tau \in bt(P) \), \( \exists \text{cond} \in \mathbb{C} \) s.t. \( \text{cond} \) is reflexive \( \iff \text{weak-cycles}_{\tau} \neq \emptyset \).

There exists a violation of a coherence condition if-and-only-if Weak-FenSying detects a cycle in the corresponding relation compositions.

**Proof.** Case \( \implies : \exists \text{cond} \in \mathbb{C} \) s.t. \( \text{cond} \) is reflexive \( \implies \text{weak-cycles}_{\tau} \neq \emptyset \).

Weak-FenSying checks the validity of coherence conditions on event relations between program events and synthesized fences. The validity is checked by detecting cycles in relation compositions of coherence conditions using Johnson’s algorithm (refer §6). Since, Johnson’s algorithm soundly detects all cycles, Weak-FenSying is sound if the event relations \( \rightarrow_{\tau}^{hb} \rightarrow_{\tau}^{rf} \rightarrow_{\tau}^{mo} \) and \( \rightarrow_{\tau}^{rf-1} \) are correctly computed (i.e. \( \not\exists e_1, e_2 \in E_{\tau}^{imm} \) s.t. a cycle would be formed containing an ordering of \( e_1, e_2 \) but the pair is not in the corresponding relation \( \rightarrow_{\tau}^{hb} \) or \( \rightarrow_{\tau}^{rf} \) or \( \rightarrow_{\tau}^{mo} \) or \( \rightarrow_{\tau}^{rf-1} \). (Note that \( \rightarrow_{\tau}^{rf} \) relation is not invoked by any coherence condition.)

Given a buggy trace \( \tau \), we get the relations \( \rightarrow_{\tau}^{hb} \), \( \rightarrow_{\tau}^{rf} \), \( \rightarrow_{\tau}^{mo} \) and \( \rightarrow_{\tau}^{rf-1} \) from the buggy trace generator. We compute the \( \rightarrow_{\tau}^{hb} \) relation after introducing the synthesized fences in the intermediate trace \( \tau^{imm} \), hence, the soundness condition can be defined as:

\textbf{FenSying} soundly detects all weak cycle without recomputing \( \rightarrow_{\tau}^{rf} \), \( \rightarrow_{\tau}^{mo} \) and \( \rightarrow_{\tau}^{rf-1} \) relations for the events of \( \tau^{imm} \) (i.e. \( \rightarrow_{\tau}^{rf} = \rightarrow_{\tau}^{rf}, \rightarrow_{\tau}^{mo} = \rightarrow_{\tau}^{mo} \) and \( \rightarrow_{\tau}^{rf-1} = \rightarrow_{\tau}^{rf-1} \)).
The relation is formed from write (or rmw) events to read (or rmw) events, since fences cannot be both, the \( \rightarrow \text{rf} \) relations remains unchanged i.e. \( \rightarrow \text{rf} \mathcal{S} = \rightarrow \text{rf} \).

The relation remains unchanged as \( \rightarrow \text{rf} \) remains unchanged i.e. \( \rightarrow \text{rf}^{-1} \mathcal{S} = \rightarrow \text{rf}^{-1} \).

Assume \( \exists \tau, w', \epsilon \) s.t. as a consequence of synthesizing fences in the buggy trace \( \tau \) to form \( \tau^{\text{im}} \), \( w \) is modification-ordered before \( w' \). However, \( (\tau, w') \not\in \rightarrow \text{mo} \mathcal{S} \) since we consider \( \rightarrow \text{mo} \mathcal{S} = \rightarrow \text{mo} \).

We show by case analysis on the coherence conditions involving \( \rightarrow \text{mo} \mathcal{S} \) that \textbf{FenSying} does not miss a weak cycle by not expanding the modification-order after synthesizing fences.

Consider the following four cases of coherence involving \( \rightarrow \text{mo} \mathcal{S} \) (borrowed from [9]):

**CoWW:** \( \forall C11 \) traces \( \tau^{c11}, \exists w_1, w_2 \in \mathcal{E}^{\text{ww}} \mathcal{S}, \text{ s.t. } w_1 \rightarrow^\text{wb} \tau^{c11}, r_1 \) and \( w_2 \rightarrow^\text{mo} \tau^{c11}, r_1 \).

Now, let \( \exists w_1, w_2 \in \mathcal{E}^{\text{ww}} \mathcal{S}, \text{ s.t. } w_1 \rightarrow^\text{bw} \tau^{c11}, r_1 \) and \( w_2 \rightarrow^\text{mo} \tau^{c11}, r_1 \).

If \( w_1 \rightarrow^\text{mo} w_2 \) then there does not exist a violation.

However, if \( w_2 \rightarrow^\text{mo} w_1 \) then we will detect the violation as a cycle in \( \rightarrow \text{mo} \mathcal{S}; \rightarrow \text{bw} \mathcal{S} \) (depicted diagrammatically in \( \text{(coWW)} \)).

**CoWR:** \( \forall C11 \) traces \( \tau^{c11}, \exists w_1, w_2 \in \mathcal{E}^{\text{ww}} \mathcal{S}, \text{ s.t. } w_1 \rightarrow^\text{bw} \tau^{c11}, r_1 \) and \( w_2 \rightarrow^\text{mo} \tau^{c11}, r_1 \).

Now, let \( \exists \exists r_1 \in \mathcal{E}^{\text{ww}}, w_1, w_2 \in \mathcal{E}^{\text{ww}} \mathcal{S}, \text{ s.t. } w_1 \rightarrow^\text{bw} r_1 \) and \( w_2 \rightarrow^\text{rf} \tau^{c11}, r_1 \).

If \( w_1 \rightarrow^\text{mo} w_2 \) then there does not exist a violation.

However, if \( w_2 \rightarrow^\text{mo} w_1 \) then we will detect the violation as a cycle in \( \rightarrow \text{mo} \mathcal{S}; \rightarrow \text{bw} \mathcal{S}; \rightarrow \text{rf}^{-1} \mathcal{S} \) (depicted diagrammatically in \( \text{(CoWR)} \)).

**CoRW:** \( \forall C11 \) traces \( \tau^{c11}, \exists w_1, w_2 \in \mathcal{E}^{\text{ww}} \mathcal{S}, \text{ s.t. } r_1 \rightarrow^\text{bw} \tau^{c11}, r_2 \) and \( w_2 \rightarrow^\text{mo} \tau^{c11}, r_1 \) and \( w_1 \rightarrow^\text{rf} \tau^{c11}, r_1 \).

Now, let \( \exists \exists r_1 \in \mathcal{E}^{\text{ww}}, w_1, w_2 \in \mathcal{E}^{\text{ww}} \mathcal{S}, \text{ s.t. } r_1 \rightarrow^\text{bw} r_2 \) and \( w_2 \rightarrow^\text{mo} \tau^{c11}, r_1 \) and \( w_1 \rightarrow^\text{rf} \tau^{c11}, r_2 \).

If \( w_1 \rightarrow^\text{mo} w_2 \) then there does not exist a violation.

However, if \( w_2 \rightarrow^\text{mo} w_1 \) then we will detect the violation as a cycle in \( \rightarrow \text{mo} \mathcal{S}; \rightarrow \text{rf} \mathcal{S}; \rightarrow \text{bw} \mathcal{S}; \rightarrow \text{rf}^{-1} \mathcal{S} \) (depicted diagrammatically in \( \text{(CoRR)} \)).

Thus, \textbf{FenSying} does not miss a cycle in any coherence rule that is violated.

Case \( \iff \exists \text{cond} \in \mathcal{C} \) s.t. \text{cond} is reflexive \( \iff \text{weak-cycles} \). \textbf{FenSying} expands \( \rightarrow^\text{hb} \mathcal{S} \) to \( \rightarrow^\text{hb} \mathcal{S} \), while \( \rightarrow^\text{rf} \mathcal{S} = \rightarrow^\text{rf} \mathcal{S} = \rightarrow^\text{mo} \mathcal{S} = \rightarrow^\text{mo} \mathcal{S} = \rightarrow^\text{rf}^{-1} \mathcal{S} = \rightarrow^\text{rf}^{-1} \mathcal{S} \). The computation of \( \rightarrow^\text{hb} \mathcal{S} \) is borrowed from C11 standard [12,9] and thus, is sound.
Further, Johnson’s algorithm (used for detecting cycles in the computed event relations) is sound. Thus, if the algorithm returns a cycle \( eCe \) where \( e \in E_{\tau_{imm}} \) and \( C \in C \) then the condition \( C \) is reflexive.

Hence, if FenSying detects a weak cycle then a corresponding coherence condition is violated.

**Lemma 2.** \( \tau \to^{so+}_{\tau} \subseteq \tau \to^{lo}_{\tau} \)

For any valid C11 trace \( \tau \), each pair of events related by \( \tau \to^{so}_{\tau} \) are ordered by \( \tau \to^{lo}_{\tau} \). In other words, \( \tau \to^{so}_{\tau} \) does not order events if the ordering violates \((to-sc)\).

**Proof.** By definition of \( \tau \to^{so}_{\tau} \), \( e_1^{sc} \to^{so}_{\tau} e_2^{sc} \) where \( (e_1^{sc}, e_2^{sc}) \in \to^{hb}_{\tau} \cup \to^{mo}_{\tau} \cup \to^{rf}_{\tau} \cup \to^{fr}_{\tau} \), and

soe: \( e_1^{sc}, e_2^{sc} \in E^{(sc)}_{\tau} \) \( \Rightarrow e_1^{sc} \to^{to}_{\tau} e_2^{sc} \) (since \( e_2^{sc} \to^{to}_{\tau} e_1^{sc} \) violates \((coto)\) or \((rfto1)\)).

soe: \( e_1^{sc}, e_2^{sc} \in E^{(sc)}_{\tau} \) \( \Rightarrow e_1^{sc} \to^{to}_{\tau} e_2^{sc} \) (since \( e_2^{sc} \to^{to}_{\tau} e_1^{sc} \) violates \((coto)\) or \((frfto1)\)).

Soe: \( e_1^{sc}, e_2^{sc} \in E^{(sc)}_{\tau} \) \( \Rightarrow e_1^{sc} \to^{to}_{\tau} e_2^{sc} \) (since \( e_2^{sc} \to^{to}_{\tau} e_1^{sc} \) violates \((coto)\) or \((frfto1)\)).

Since, \( \tau \to^{lo}_{\tau} \) is total, thus, \( \tau \to^{so+}_{\tau} \subseteq \tau \to^{lo}_{\tau} \).

**Lemma 3.** Strong-FenSying is sound:

\( \neg(total(E^{(sc)}_{\tau}, \to^{lo}_{\tau_{imm}}) \land order(E^{(sc)}_{\tau}, \to^{lo}_{\tau_{imm}})) \iff \text{strong-cycles}_{\tau} \neq \emptyset. \)

There does not exist a total order on the \( sc \) ordered events of an intermediate trace \( \tau_{imm} \) if-and-only-if there exists a cycle in \( \tau_{imm} \).

**Proof.** Case \( \Rightarrow \): \( \neg(total(E^{(sc)}_{\tau}, \to^{lo}_{\tau_{imm}}) \land order(E^{(sc)}_{\tau}, \to^{lo}_{\tau_{imm}})) \Rightarrow \text{strong-cycles}_{\tau} \neq \emptyset. \)

Consider \( e_1, e_2 \in E^{(sc)}_{\tau} \) s.t. both \( \tau \to^{to}_{\tau} e_2 \) and \( \tau \to^{to}_{\tau} e_1 \) do not violate the conditions \((coto)\), \((frfto1)\), \((rfto2)\) and \((frfto)\). To form the total order we can assume either one of the two orders [12]. Assume \( \tau \to^{lo}_{\tau} e_2 \).

Further, consider a total order cannot be formed on \( sc \) events of \( \tau_{imm} \) s.t. \( e \to^{to}_{\tau_{imm}} \tau_{imm} \resolution{e_1} \to^{to}_{\tau_{imm}} \tau_{imm} e_2 \to^{to}_{\tau_{imm}} \tau_{imm} e_1 \) then we simply flip \( e_1 \to^{to}_{\tau_{imm}} e_2 \) to \( e_2 \to^{to}_{\tau_{imm}} e_1 \) and eliminate the cycle.

Further, if a cycle in \( \tau_{imm} \) includes \( e_1 \to^{to}_{\tau_{imm}} e_2 \) and another cycle includes \( e_2 \to^{to}_{\tau_{imm}} e_1 \) then there exists a cycle \( e_1 \to^{to}_{\tau_{imm}} \ldots \to^{to}_{\tau_{imm}} e_2 \to^{to}_{\tau_{imm}} \ldots \to^{to}_{\tau_{imm}} e_1 \) (by definition of \( \tau_{imm} \), as shown in the figure below). Thus, pairs of \( sc \) ordered events that don’t have a fixed \( \to^{lo}_{\tau} \) order cannot contribute to a strong cycle. \( \inf(i). \)
Now, if there does not exist a total order on the $sc$ ordered events of $\tau^{im}$ then
\[ -(total(\mathcal{E}(sc), r_{\tau^{im}})) \land order(\mathcal{E}(sc), r_{\tau^{im}}), \text{ i.e.} \]
\[ -(total(\mathcal{E}(sc), r_{\tau^{im}})) \lor \exists e \in \mathcal{E}_{\tau^{im}}(sc) \text{ s.t. } e \xrightarrow{r_{\tau^{im}}} e \lor -(\to_{\tau^{im}}^{+} \subseteq \to_{\tau^{im}}) \]
(by definition of order(\mathcal{E}(sc), r_{\tau^{im}})).

By definition of $r_{\tau^{im}}$, $-total(\mathcal{E}(sc), r_{\tau^{im}})$ and $-(\to_{\tau^{im}}^{+} \subseteq \to_{\tau^{im}})$ are not feasible.

Thus, $\exists e \in \mathcal{E}_{\tau^{im}}(sc)$ s.t. $e \xrightarrow{r_{\tau^{im}}} e$.

$\implies sc$ events of $\tau^{im}$ violate (coto), (rfto1), (rfto2) or (frfto).

[coto] Let $\exists w_{e} \in \mathcal{E}_{\tau^{im}}(sc)$ s.t. $(e, e) \in (\to_{\tau^{im}}^{mo} \cup \to_{\tau^{im}}^{hb})$. Thus, (coto) is violated by $e^{sc}$.

As we know that, $\to_{\tau^{im}}^{mo} \cup \to_{\tau^{im}}^{hb} \subseteq \to_{\tau^{im}}^{so} +$ thus we have a cycle $e^{sc} \to_{\tau^{im}}^{so} \to_{\tau^{im}}^{so} \to_{\tau^{im}}^{so} e^{sc}$.

[rfto1] Let $\exists u_{1}^{sc}, u_{2}^{sc} \in \mathcal{E}_{\tau^{im}}(\omega_{\tau^{im}}), r_{1}^{sc} \in \mathcal{E}_{\tau^{im}}(r_{\tau^{im}})$ s.t. $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \xrightarrow{r_{\tau^{im}}}$ and $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}}$.

Thus, (rfto1) is violated by $u_{1}^{sc}, u_{2}^{sc}$ and $r_{1}^{sc}$.

Since, $\tau$ is a valid trace, $\neg \neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \lor \neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$.

Further, since inserting fences only modifies the $\to_{\tau^{im}}^{hb}$ relation, if $\neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$ then $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$ (because $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$). Similarly, if $\neg u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} u_{1}^{sc}$ then $u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} u_{1}^{sc}$.

Also, $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \implies u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$ (assuming (co-mh) is not violated).

As we know that, $\neg \neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \lor \neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$ thus we have a cycle $r_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} u_{1}^{sc}$.

[rfto2] Let $\exists w_{1}^{sc}, w_{2}^{sc} \in \mathcal{E}_{\tau^{im}}(\omega_{\tau^{im}}), r_{1}^{sc} \in \mathcal{E}_{\tau^{im}}(r_{\tau^{im}})$ s.t. $\text{ord}(w_{2}^{sc})$ is $sc$, $w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} r_{1}^{sc}$, $w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} w_{2}^{sc}$ and $w_{2}^{sc} \xrightarrow{r_{\tau^{im}}}$.

Thus, (rfto2) is violated by $w_{1}^{sc}, w_{2}^{sc}$ and $r_{1}^{sc}$.

Since, $\tau$ is a valid trace, $\neg w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc} \lor \neg w_{2}^{sc} \xrightarrow{r_{\tau^{im}}} r_{1}^{sc}$.

Further, since inserting fences only modifies the $\to_{\tau^{im}}^{hb}$ relation, if $\neg w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} r_{1}^{sc}$ then $w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$ (because $w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$). Similarly, if $\neg w_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$ then $w_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$.

Also, $w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc} \implies w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$ (assuming (co-mh) is not violated).

As we know that, $w_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc} \lor w_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$ thus we have a cycle $r_{1}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$.

[frfto] Let $\exists u_{1}^{sc}, u_{2}^{sc} \in \mathcal{E}_{\tau^{im}}(\omega_{\tau^{im}}), r_{1}^{sc} \in \mathcal{E}_{\tau^{im}}(r_{\tau^{im}})$ s.t. $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} = \tau^{sc}$, $\tau^{sc} \xrightarrow{r_{\tau^{im}}}$.

Thus, (frfto) is violated by $u_{1}^{sc}, u_{2}^{sc}, r_{1}^{sc}$ and $\tau^{sc}$.

Since, $\tau$ is a valid trace, $\neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} \neg u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} \neg \tau^{sc}$.

Further, since inserting fences only modifies the $\to_{\tau^{im}}^{hb}$ relation, if $\neg u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$ then $u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$ (because $u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$). Similarly, if $\neg u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$ then $u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} h_{\tau^{im}}^{sc}$.

Also, $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \implies u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc}$ (assuming (co-mh) is not violated).

As we know that, $u_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \lor u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} u_{1}^{sc}$ thus we have a cycle $r_{1}^{sc} \xrightarrow{r_{\tau^{im}}} u_{2}^{sc} \xrightarrow{r_{\tau^{im}}} u_{1}^{sc}$.
τ

The intermediate trace by definition of Proof. buggy trace. FenSying does not assign a memory order to a fence that is too weak to stop the hold; i.e.

Thus, \( \neg (\text{order}(\mathcal{E}^{\text{sc}}), \rightarrow_{\text{prg}}^{\text{to}}) \) is stronger than \( m \) after assigning a memory order for either of the violations exist in the final memory order of FenSying can construct \( \tau^{\text{inv}} \). Since we know that \( m \) was sufficiently strong (using inf(i)) then the final memory order \( m' \) is also sufficiently strong.

Lemma 5. FenSying is sound for 1 trace. Given an input program \( P \) s.t. \( \text{bt}(P) = \{\tau\} \). \( \exists \mathcal{E}' \) s.t. \( \text{bt}(\text{prg}(P, \mathcal{E}')) = \emptyset \) \( \implies \) FenSying can construct \( \tau^{\text{inv}} \).

Theorem 1. \( \text{ffFenSying} \) is sound. Given an input program \( P \), \( \exists \tau \in \text{bt}(P) \). s.t. \( \exists \mathcal{E}' \) s.t. \( \text{bt}(\text{prg}(P, \mathcal{E}')) = \emptyset \) \( \implies \) \( \text{ffFenSying} \) can construct \( \tau^{\text{inv}} \).

Proof. Firstly, using Lemma 1 and Lemma 3 we can state that (i) a violation in (coherence conditions) or SC total order is not missed by FenSying/ffFenSying, and (ii) a violation detected by FenSying/ffFenSying is indeed a true violation of either one of the (coherence conditions) or (tosc).

Secondly, the fences introduced for at least 1 of the violations exist in the final solution (by construction of \( \text{SAT} \) query). i.e.

\[ \exists e \in \text{weak-cycles}_\tau \cup \text{strong-cycles}_\tau \text{ s.t. } \forall F \text{ (fences included in } e \text{) } \in \mathcal{E}_{\text{prg}} \setminus \mathcal{E}_{\text{prg}}^{F} \text{ } F \in \text{minF}. \]

Thirdly, The memory order assigned to the fences in \( \text{minF} \) is sufficiently strong...
to stop the buggy trace (Lemma 4(inf(i))).

Hence, FenSying is sound for 1 trace, and fFenSying is sound.

**Theorem 2.** FenSying is sound.

Given an input program $P$ s.t. $bt(P) \neq \emptyset$. $\exists E'$ s.t. $bt(prg(P,E')) = \emptyset \implies \forall \tau \in bt(P)$ we can construct $\tau^{inv}$. If $P$ can be fixed by synthesizing or strengthening C11 fences then FenSying fixes $P$.

**Proof.** Let $BT = bt(P)$. Consider induction on $|BT|$.

**Base Case:** Consider $|BT| = 1$. Let $BT = \{\tau\}$

Using Lemma 5, FenSying is sound for 1 trace.

**Induction Hypothesis:** Assume that FenSying is sound for $|BT| = N$.

**Induction Step:** Consider $|BT| = N+1$.

Since, we take a conjunction on the SAT formulas from various traces thus at least 1 cycle from each trace exists in the min-model (by construction of SAT query).

Further, we know from Lemma 4 that FenSying assigns memory orders that can stop all the corresponding traces.

Thus, FenSying is sound for N+1 buggy traces.

**Lemma 6.** min-model returns the optimal number of fences. Let $E^{fix}$ represent the set of synthesized fences of $P^{fix}$ (i.e. $E^{fix} = E^{pr} \setminus E^{fix}$, where $E^{pr}$ represents the set of fences of $P^{pr}$) then $\exists E^o$ s.t. $|E^o| < |E^{fix}|$ and $bt(prg(P,E^o)) = \emptyset$.

**Proof.** Let $F$ represent the set of fences returned by min-model and let $F^o$ represent the optimal set of fences. Assume $|F^o| < |F|$.

The min-model of the nonoptimal solution is computed using a SAT solver (Z3) and the computation is assumed to be correct. As the consequence, $|F^o| < |F| \implies$ the optimal result was not a part of the SAT query formula.

Using Lemma 1 and Lemma 3 we know that FenSying does not miss any weak or strong cycle $\implies$ every set of fences that forms a correct solution, including the optimal solution, is contained in the SAT query formula.

Thus, by contradiction, $|F| = |F^o|$ i.e. min-model returns the optimal number of fences.

**Theorem 3.** FenSying synthesizes the optimal number of fences with the optimal memory orders. Let $E^{fix}$ represent the set of synthesized fences of $P^{fix}$ (i.e. $E^{fix} = E^{pr} \setminus E^{fix}$, where $E^{pr}$ represents the set of fences of $P^{pr}$) then $\exists E^o$ s.t. $|E^o| < |E^{fix}|$ or $\exists E^o \in E^o, e \in E^{fix}$ s.t. $thr(e^o) = thr(e), idx(e^o) = idx(e), act(e^o) = act(e), obj(e^o) = obj(e)$ and $loc(e^o) = loc(e)$ but $ord(e^o) \sqsubseteq ord(e)$ and $bt(prg(P,E^o)) = \emptyset$.

**Proof.** We know that AssignMO iterates over cycles in min-cycles and takes union over fences of cycles from min-cycles. As min$\Phi$ consists of the optimal
number of fences (Lemma 6) then union over cycles of \textit{min-cycles} has the same set of fences as \textit{minΦ}.

Thus, \textbf{FenSying} is optimal in the number of fences.

Let \( \mathbb{B}T \) represent the set of buggy traces. Consider induction on \(|\mathbb{B}T|\).

\textit{Base Case:} Consider \(|\mathbb{B}T| = 1\). By definition of \textbf{AssignMO}, each fence is locally assigned the weakest memory order that is sound (Lemma 4).

Thus, \textbf{FenSying} is optimal in the memory order of fences for 1 buggy trace.

\textit{Induction Hypothesis:} Assume, \textbf{FenSying} is optimal in the memory order of fences when \(|\mathbb{B}T| = N\).

\textit{Induction Step:} Consider \(|\mathbb{B}T| = N+1\).

Let \( s_1, ..., s_M \) represent the \( M \) coalesced solutions for buggy traces \( τ_1, ..., τ_N \) and \( τ_{N+1}c_i \) for \( i \in \{1, ..., q\} \) represent the \( q \) cycles of \((N+1)\text{th} \) trace.

Every coalesced solution \( τ_{N+1}s_j \) has the same number of fences = fences of \textit{minΦ} because \textit{minΦ} returns the minimum number of fences required to stop \( τ_1, ..., τ_{N+1} \).

If there exists a fence \( F \) with memory order \( m \) in a cycle \( τ_{N+1}c_i \) but the final solution of \textbf{FenSying} assigns memory order \( m' \) to \( F \) s.t. \( m' \) is stronger than \( m \) then, \( ∃s_j \) where memory order of \( F \) is \( m' \) (by construction of coalesced solutions), further, \( ∄s_k \) where memory order of \( F \) is \( m \) s.t. \( wt(τ_{N+1}c_i - s_k) < wt(τ_{N+1}c_i - s_j) \) (where \( wt(x-y) \) represents the weight of the solution formed by coalescing cycle \( x \) with candidate solution \( y \)).

Thus, \textbf{FenSying} is optimal in the memory order of fences for \( N+1 \) buggy traces.

\section{Implementation and Results}

\textbf{Implementation details.} The techniques are implemented in \texttt{Python}. \textit{Weak-FenSying} and \textit{Strong-FenSying} use \textit{Johnson’s} cycle detection algorithm in the \texttt{networkx} library. We use \texttt{Z3} theorem prover to find the \textit{min-model} of \textit{SAT} queries. As a \texttt{BTG}, we use CDSChecker \cite{20}, an open-source model checker, for the following reasons:

1. CDSChecker supports the \textit{C11} semantics. Most other techniques are designed for a variant \cite{15} or subset \cite{1,3,22} of \textit{C11}.
2. CDSChecker returns buggy traces along with the corresponding \( →_{hb} \), \( →_{rf} \) and \( →_{mo} \) relations.
3. CDSChecker does not halt at the detection of the first buggy trace; instead, it continues to provide all buggy traces as required by \textbf{FenSying}.

To bridge the gap between CDSChecker’s output and our requirements, we modify CDSChecker’s code to accept program location as an attribute of the program events and to halt at the first buggy trace when specified. \textbf{FenSying} and
Table 1. Litmus Testing Summary

| Tests | min-BT | max-BT | avg-BT | min-syn | max-syn | avg-syn | min-str | max-str | avg-str |
|-------|--------|--------|--------|---------|---------|---------|---------|---------|---------|
| 1389  | 1      | 9      | 1.05   | 1       | 4       | 2.25    | 0       | 0       | 0       |

BT: # buggy traces, syn: # fences synthesized, str: # fences strengthened
min: minimum, max: maximum, avg: average

Results Summary

|                | completed (syn+no fix) | TO | NO | Tbtg (total) | TF (total) | Ttotal |
|----------------|------------------------|----|----|--------------|------------|--------|
| FenSying       | 1333 (1185+148)        | 56 | 0  | 50453.19     | 36896.06   | 87266.09 |
| fFenSying      | 1355 (1207+148)        | 34 | 0  | 30703.71     | 49068.61   | 79772.32 |

Times in seconds.

TO: 15 min for BTG + 15 min for technique
Tbtg: Time of BTG, TF: Time of FenSying or fFenSying, Ttotal: Tbtg+TF

fFenSying are available as an open-source tool that performs fence synthesis for C11 programs at: https://github.com/singhsanjana/fensying.

Experimental setup. The experiments were performed on an Intel(R) Xeon(R) CPU E5-1650 v4 @ 3.60GHz with 32GB RAM and 32 cores. We collected a set of 1389 litmus tests of buggy C11 input programs (borrowed from Tracer [3]) to validate the correctness of FenSying and fFenSying experimentally. We study the performance of FenSying and fFenSying on a set of benchmarks borrowed from previous works on model checking under the C11 memory model and its variants [1,3,20,22].

Experimental validation. The summary of the 1389 litmus tests is shown under Litmus Tests Summary, Table 1. The number of buggy traces for the litmus tests ranged between 1-9 with an average of 1.05, while the number of fences synthesized ranged between 2-4. None of the litmus tests contained fences in the input program. Hence, no fences were strengthened in any of the tests.

We present the results of FenSying and fFenSying under Result Summary, Table 1. The results have been averaged over five runs for each test. fFenSying timed out (column ‘TO’) on a fewer number of tests (34 tests) in comparison to FenSying (56 tests). The techniques could not fix 148 tests with C11 fences (‘no fix’). The column ‘NO’ represents the number of tests where the fences synthesized or strengthened is nonoptimal. To report the values of ‘NO’, we conducted a sanity test on the fixed program as follows: we create versions $P_1, ..., P_k$ of the fixed program $P_{fx}$ s.t. in each version, one of the fences of $P_{fx}$ is either weakened or eliminated. Each version is then tested separately on BTG. The sanity check is successful if a buggy trace is returned for each version.

Performance analysis. We contrast the performance of the techniques using a set of benchmarks that produce buggy traces under C11. The results are averaged over five runs. Table 2 reports the results where ‘#BT’ shows the number of buggy traces, ‘iter’ shows the minimum:maximum number of iterations performed by fFenSying over the five runs and, ‘FTO’ and ‘BTO’ represent FenSying/fFenSying time-out and BTG time-out, respectively (set to 15 minutes each). A ‘?’ in ‘#BT’ signifies that BTG could not scale for the test, so the number of buggy traces is unknown. The column (‘syn+str’) under fFenSying reports
### Table 2. Comparative performance analysis

| Id | Name         | #BT | syn+str | FenSying | iter | syn+str | FenSying |
|----|--------------|-----|---------|----------|------|---------|----------|
|    |              |     | Tbtg    | TF       | Ttotal| Tbtg    | TF       |
| 1  | peterson(2,2)|  30 | 1+0     | 2.63     | 54.31 | 56.94   | 1:1+0    |
| 2  | peterson(2,3)|  198| 1+0     | 29.96    | 594.34| 624.3   | 1:1+0    |
| 3  |              |     | −       | FTo      | −    | −       | BTo      |
| 4  | peterson(5,5)| ?   | −       | BTo      | −    | −       | −        |
| 5  | barrier(5)   | 136 | 1+0     | 1.09     | 207.74| 208.83  | 1:1+0    |
| 6  | barrier(10)  |  416| 1+0     | 1.37     | 565.44| 568.81  | 1:1+0    |
| 7  | barrier(100) |  3106|  −      | −       | FTo   | −       | BTo      |
| 8  | barrier(150) | ?   | −       | FTo      | −    | −       | −        |
| 9  | barrier(200) | −   | −       | −       | −    | −       | −        |
| 10 | store-buffer(2)|  6  | 2+0     | 0.08     | 0.91  | 0.99    | 1:2+0    |
| 11 | store-buffer(4)|  20 | 2+0     | 1.61    | 195.35| 196.96  | 1:2+0    |
| 12 | store-buffer(5)|  30 | −       | −       | FTo   | −       | FTo      |
| 13 | store-buffer(6)|  42 | −       | −       | FTo   | −       | −        |
| 14 | store-buffer(10)| ?  | −       | −       | −    | −       | −        |
| 15 | dekker(2)    |  54 | 2+0     | 0.17     | 0.27  | 0.44    | 1:2+0    |
| 16 | dekker(3)    | 1596| −       | −       | FTo   | −       | −        |
| 17 | dekker-fen(2,3)| 54  | 1+1     | 0.15     | 0.29  | 0.44    | 1:1+1    |
| 18 | dekker-fen(3,2)| 730 | −       | −       | FTo   | −       | −        |
| 19 | dekker-fen(3,4)| 3076| −       | −       | BTo   | −       | −        |
| 20 | burns(1)     |  46 | −       | −       | −     | −       | 7.8×10+2,2| 6.14 | 4.69 | 5.33 |
| 21 | burns(2)     | 10150| −      | −       | FTo   | −       | 6.7×10+0.| 71.53| 554.6| 626.13|
| 22 | burns(3)     | ?   | −       | −       | BTo   | −       | −        |
| 23 | burns-fen(2) | 100760| −     | FTo | −    | −       | 5.7×10+3.| 329.41| 43.96| 373.37|
| 24 | burns-fen(3) | ?   | −       | BTong   | BTo   | FTo     | −        |
| 25 | linuxrlocks(2,1)| 10 | −       | −       | −     | −       | 1:1+0    |
| 26 | linuxrlocks(3,8)| 353 | −       | −       | −     | −       | 2:2+0    |
| 27 | seqlock(1,2)| 500 | −       | −       | −     | −       | 1:1+0    |
| 28 | seqlock(1,2,2)| 592 | −       | −       | −     | −       | 1:2+0    |
| 29 | seqlock(2,2,3)| ?   | −       | −       | −     | −       | 1:2+0    |
| 30 | bakery(2,1)  |  6  | 1+0     | 0.25     | 25.42 | 25.88   | 1:1+0    |
| 31 | bakery(4,3)  |  7272| −       | −       | −     | −       | FTo      |
| 32 | bakery(4,4)  |  50400 | −    | −       | −     | −       | BTo      |
| 33 | lamport(1,1,2)| 1  | −       | −       | −     | −       | 1:1+0    |
| 34 | lamport(2,2,1)|  1  | −       | −       | −     | −       | 1:1+0    |
| 35 | lamport(2,2,3)|  1  | −       | −       | −     | −       | 1:1+0    |
| 36 | flipper(5)   |  297| 2+0     | 6.22    | 254.18| 260.40  | 1:2+0    |
| 37 | flipper(7)   |  4493| −      | −       | −     | −       | −        |
| 38 | flipper(10)  | ?   | −       | −       | −     | −       | −        |

Tbg: Time of BTG, TF: Time of technique (FenSying or fFenSying), Total: Tbg+TF

The performance of FenSying and fFenSying is diagrammatically contrasted in Figure 2. It is notable that fFenSying significantly outperforms FenSying in terms of the time of execution and scalability and adds extra fences in only 7 tests with an average of 1.57 additional fences. With the increase in the number of buggy traces, an exponential rise in FenSying’s time leading to FTo was observed; except in cases 12, 13, 20, and 25, where FenSying times out with as low as 10 traces. The tests time-out in Johnson’s cycle detection due to a high density of the number of related events or the number of cycles.

fFenSying analyzes a remarkably smaller number of buggy traces (‘iter’) in comparison with ‘#BT’ (≤2 traces for ~85% of tests). We conclude that a solution corresponding to a single buggy trace fixes more than one buggy traces.
As a result, \texttt{fFenSying} can scale to tests with thousands of buggy traces and we witness an average speedup of over 67x, with over 100x speedup in \textasciitilde41% of tests, against \texttt{FenSying}.

\textit{Interesting cases.} Consider test 16, where \texttt{BTG} times out in 3/5 runs and completes in \textasciitilde100s in the remaining 2 runs. A fence is synthesized between two events, $e_1$ and $e_2$, that are inside a loop. Additionally, $e_1$ is within a condition. Depending on where the fence is synthesized (within the condition or outside it), \texttt{BTG} either runs out of time or finishes quickly. Similarly, \texttt{BTG} for test 26 times out in 3/5 runs. However, the reason here is the additional nonoptimal fences synthesized that increase the analysis overhead of the chosen \texttt{BTG} (CDSChecker).

Note that, for most benchmarks, \texttt{fFenSying}’s scalability is limited by \texttt{BTo} and observably \texttt{fFenSying}’s time is much lesser than \texttt{FTo} for such cases. Therefore, an alternative \texttt{BTG} would significantly improve \texttt{fFenSying}’s performance.

8 Related Work

The literature on fence synthesis is rich with techniques targeting the x86-TSO \cite{2,5,6,7,10} and sparc-PSO \cite{4,18} memory models or both \cite{14,16,19}. The work in \cite{10} and \cite{16} perform fence synthesis for ARMv7 and RMO memory models. The works in \cite{4,7,11} are proposed for Power memory model, where \cite{11} also supports IA-32 memory model.

Most fence synthesis techniques introduce additional ordering in the program events with the help of fences \cite{5,6,7,10,11,14,18,19,23}. However, the axiomatic definition of ordering varies with memory models. As a consequence, most existing techniques (such as those for TSO and PSO) may not detect \texttt{C11} buggy traces due to a strong implicit ordering. While the techniques \cite{6,7,23} are parametric in or oblivious to the memory model, they introduce ordering between \textit{pairs} of events that is \textit{globally visible} (to all threads). Such an ordering constraint is restrictive for weaker models such as \texttt{C11} that may require ordering on a \textit{set} of events that may be \textit{conditionally visible} to a thread. Similarly, \cite{14} proposes a bounded technique applicable to any memory model that supports interleaving with reordering. Program outcomes under \texttt{C11} may not be feasible under such a model. Moreover, any existing technique, cannot fix a \texttt{C11} input program while conserving its portability.
Some earlier works such as [2,7,11] synthesize fences to restrict outcomes to SC or its variant for store-buffering [5]. Most fence synthesis techniques [4,14,16,18,19] attempt to remove traces violating a safety property specification under their respective axiomatic definition of memory model. Various works [4,5,10,14,16,19,23] perform optimal fence synthesis where the optimality (in the absence of types of fences) is simply defined as the smallest set of fences. Technique [6] assigns weights to various types of fences (similar to our work) and defines optimality on the summation of fence weights of candidate solutions. However, their definition of optimality is incomparable with ours, and no prior work establishes the advantage of one definition over the other.

Lastly, a recent technique [21] fixes a buggy C11 program by strengthening memory access events instead of synthesizing fences.

9 Conclusion and Future Work

This paper proposed the first fence synthesis techniques for C11 programs: an optimal (FenSying) and a near-optimal (fFenSying). The work also presented theoretical arguments that showed the correctness of the synthesis techniques. The experimental validation demonstrated the effectiveness of fFenSying vis-à-vis optimal FenSying. As part of future work, we will investigate extending the presented methods (i) to support richer constructs such as locks and (ii) to include strengthening memory accesses to fix buggy traces.

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