Uncertainties on Atmospheric Neutrino Flux Calculations

G. Battistoni

March 25, 2022

I.N.F.N., Sezione di Milano, via Celoria 16, I-20133 Milano, Italy

Abstract

The strong evidence of new physics coming from atmospheric neutrino experiments has motivated a series of critical studies to test the robustness of the available flux calculations. In view of a more precise determination of the parameters of new physics, new and more refined flux calculations are in progress. Here we review the most important sources of theoretical uncertainties which affect these computations, and the attempts currently under way to improve them.

1 Introduction

The evidence for new neutrino physics beyond the standard model, as is emerging from the results of Super–Kamiokande\textsuperscript{[1]} and other atmospheric neutrino experiments\textsuperscript{[2, 3]}, is now considered robust. Soon after the first announcement of Super–Kamiokande in 1998, many efforts have been devoted to the examinations of theoretical uncertainties in the knowledge of atmospheric neutrino fluxes. None of these sources of uncertainty resulted so critical to vanish the crucial qualitative feature of atmospheric neutrinos: the up–down symmetry of fluxes in absence of oscillations (or other possible mechanisms invoked to explain the observed “anomaly”). Even in absence of oscillations, there exist recognized violations of this absolute symmetry, as those due to the geomagnetic cutoff of primary cosmic rays, but they can be treated as additional corrections. These perturbations are found to be significant mostly in the Sub-GeV region. A new phase of the experimentation on atmospheric $\nu$’s has started, with the primary goal to improve and constraint as much as possible the parameters of the proposed new physics (essentially $\sin^2 2\Theta_{atm}$ and $\Delta m^2_{atm}$ for the 2–family oscillation scenario). This is not only a goal for the existing experiments, but also the motivation for the proposal of new generation high precision detectors, such as ICARUS\textsuperscript{[4]}. For this purpose, the theoretical error has to be reduced as much as possible and new refined calculations are necessary. As a first step we need to improve
our quantitative understanding of all the factors which affect fundamental quantities like symmetry, flavor ratio, the absolute flux value, the spectral index and the details of angular distributions. In the following sections we intend to review the major sources of uncertainties, with some emphasis on the hadronic interaction sector. The work is still in progress, as outlined in the conclusions.

2 The status of present flux calculations

The present relevant experiments are making reference, for their analyses, mainly to the neutrino flux calculations from Honda et al. (HKKM)\cite{5} and the Bartol group\cite{6}. These works have been recognized as the most accurate and their authors introduced for the first time important ingredients in the simulation. For instance the back-tracing technique for the evaluation of geomagnetic cutoff\cite{5} and the effects of muon polarization\cite{6}. These calculations have in common the 1-dimensional calculation approach, in which all secondary particles in the showers, neutrinos included, are considered collinear with the primary cosmic rays. This has been found to be a non correct approximation, at least in principle. A new set of calculation, based on the full FLUKA simulation code\cite{7}, has been recently presented\cite{8}. There it has been realized that a correct 3-dimensional approach in the earth’s spherical geometry leads to different results for the angular distribution at low energy. A more didactic explanation of this is given in ref.\cite{9}. However, the new FLUKA calculation was essentially motivated by the emerging need of a more accurate description of particle production in hadron and nuclear interactions. In fact, there are reasons to consider more critically the standard references. For instance, it has been noticed how they obtain very close final results for the $\nu$-fluxes, although starting from different particle production models and different primary spectra. Another calculation appeared with a reference to FLUKA\cite{10}, but there the authors made use of one the hadronic interfaces extracted from the GEANT package v.3.21 called FLUKA. Actually, that package is only a limited and obsolete part of the hadronic model contained in the real FLUKA code used for the present work. It gives results which can well be different and less reliable when compared with experimental data with respect to FLUKA. Significant differences exist at all energies, but they are particularly striking for hadron energies below a few GeV.

3 Sources of uncertainty

Attempting a review of all possible sources of systematic uncertainties in flux calculations, the following items must be considered: primary spectra (fluxes, nuclear component, isotropy and its breaking), geomagnetic description, atmosphere models, the geometry of calculations, other minor details in the modeling (detector altitude, mountain profiles, etc.) and particle production in hadronic interactions. In the recent past there have been other discussion (at least in part) of these topics. Beyond the already quoted references \cite{8,9}, important discussions are also in \cite{11,12,13,14}.
3.1 Primary Spectrum

On of the most relevant achievements in the measurement of primary cosmic ray spectra is the fact that BESS[15] and AMS[16] particularly succeeded in producing results in very good agreement one to the other, in particular for the proton component. The scientific community has the attitude of considering these last results as the most reliable and therefore the uncertainty of the primary flux value is probably smaller with respect to the estimates of few years ago, although one should not forget other different data sets, like those of CAPRICE[17] until the topic is definitely settled down. A summary of some of the recent proton measurements is shown in Fig. 1, together with the lines showing the solar minimum fits used in [5] and [6]. It has to be noticed how the input primary spectrum used by Bartol (and later by FLUKA) is in very good agreement with BESS data, while HKKM made use of a parameterization (based on the old compilation of ref.[18]) which has a significantly higher normalization above 20÷30 GeV. It is therefore natural to ask how the eventual result from the HKKM calculation would change if they used the same input spectrum as Bartol. This is one of the reasons why it is important to analyze in depth the relevance of the particle production model.

Figure 1: Review of some of the most recent data on primary protons. The continuous line represent the input model adopted in [5] and [6]. The last one was also used in [8, 23].
As far as the Helium component is concerned, the latest AMS and BESS measurements are now converging\[19, 20\]. The heavier nuclei have less relevance for low energy neutrinos; in any case, further results from AMS will hopefully clarify the picture.

The arguments exposed here are relevant for the energy range contributing to contained events in Super–Kamiokande. In case of higher energy neutrinos, like those measured through the detection of up-going muons in MACRO and Super–Kamiokande, the uncertainties on the relevant energy of the spectrum (up to tens of TeV) are still large (up to 20%).

### 3.2 Geomagnetic description

The effect of geomagnetic field is recognized as the most important source of up–down symmetry breaking in the atmospheric neutrino flux. At present, the confidence in the accuracy of IGRF models is rather strong\[21\], and the technique of anti-proton back–tracing is now accepted as the standard procedure to be adopted to evaluate the correct cutoff for primary cosmic rays arriving to the earth. Solar modulation has to be considered as well. Algorithm relating primary flux to data from neutron monitors exist, and at present their quality is considered satisfactory. Recently, there has been discussion about two items: i) the role of recirculating sub-cutoff particles as pointed out by AMS data, and ii) the anisotropy (far from earth) related to solar wind effects at the GeV scale. In both cases it can be demonstrated (see \[22\]) that both phenomena are of small relevance, since at most they affect neutrino rates by less than 1%.

### 3.3 Geometry of calculations

As already reported in the introduction, one of the most interesting outcomes in last two years is the realization of the importance of 3–Dimensional computations in a spherical geometry\[8, 9\]. In summary, the net eventual result of the correct geometrical description of neutrino production around the earth is a modification of both angular distribution and of normalization in the Sub-GeV region (where. \(< \theta_{\nu-p} >\) is significant), with respect to the collinear approximation. The impact of this on the determination of oscillation parameters is still under study. In fact, a final reliable 3–Dimensional calculation of atmospheric neutrino flux is still missing. The computations by the FLUKA group, reported in \[23\], must still be considered as preliminary, since bending of charged particles in the geomagnetic fields has not yet been introduced. As discussed in \[12\], this effect should play a non negligible role. However, a detailed calculation on the whole earth sphere taking into account also the whole B–field map introduces technical complexities in the computation, since spherical symmetry is lost. In fact, in absence of B–field, any point on the earth’s surface is equivalent to another, and this allows to make use all generated events even for a specific detector location, provided that a proper rotation of trajectory parameters to the geografical coordinates of interest is performed. This problem has not yet been solved completely. In \[12\] a simplified solution was proposed for the first studies, while the FLUKA group is designing a new dedicated simulation in which specific
weighting algorithms have to be introduced.

3.4 Atmosphere Description and other details

It is practically impossible to introduce a realistic description of the atmosphere all around the earth, valid at all altitudes and for all weather conditions. This remains an irreducible source of systematics, although probably small. The fact that neutrino experiments last a considerable time and detect neutrinos produced all over the earth positions, gives some confidence on the essential validity of average atmosphere models. A comparison between the performance of different codes, carried on by the Bartol and FLUKA groups, is showing that having considered, or not, ingredients in particle transport like energy loss fluctuation, multiple scattering, etc., may affect the final results, in normalization and flavor ratio, at the level of percent. Other important inputs in the simulation for a specific detector site concern the introduction of detector altitude and possible rock overburden. This last element, for instance, can introduce a difference in the $\nu_e/\nu_\mu$ ratio, again at the level of some percent.

3.5 Particle Production in Hadronic Interactions

For given shower model and primary spectrum, the most important source of theoretical uncertainty comes from the hadronic interaction model. Since QCD does not allow to compute the bulk properties of particle production in the non perturbative regime, the available models are based either on phenomenological models, possibly inspired by partonic concepts, constrained by accelerator data, or directly on the parameterizations of these experimental results. As an example, the Bartol and FLUKA groups have used the same input primary spectrum and, with different models, have obtained different fluxes. The current comparison of angle integrated fluxes is reported (for the Super–Kamiokande site) in Fig.2, where also the differences between the 1D and 3D approach are shown for FLUKA.

The two groups have started a comparison of their hadronic models (FLUKA and TARGET), both in single interaction and within the same shower code, in order to understand the impact of different choices. For the time being, this comparison is limited to an energy region useful for contained and partially contained events in Super–Kamiokande. The two codes are constructed in very different ways, and the net final result is that there is a $\sim 20\%$ asymptotic difference in the neutrino flux normalization from the two models. In reality the difference is energy dependent: it is larger and reversed at low energy. The resulting spectral index is somewhat different, FLUKA being a little harder than the Bartol one. A comprehensive discussion of the matter should requires a dedicated paper, and here we can only summarize a few of the crucial conclusions.

As far as neutrinos up to few tens of GeV are concerned, pion production in nucleon–Nucleus interaction is the process that mostly contributes to the yield. For known kinematical reasons, Kaon production becomes relevant at higher energy, for instance in the region from which upgoing through-muons as detected by MACRO and Super–Kamiokande
are produced. At first order, we are interested in both total multiplicity and in the shape of the energy fraction distribution of these secondary particles, or of Feynman–$X$ and similar longitudinal variables. Unfortunately, although there exist reports on the total charged multiplicity, mostly obtained in emulsion experiments\cite{24}, there are not enough data on the $X$ distribution of pions in the interactions with light nuclei. At present the most valuable data set are those of ref. \cite{25} and \cite{26}, covering different region of phase space. A direct comparison of these data with the predictions from FLUKA and TARGET shows that, very probably, the Bartol model produces too many pions at low $X$. This difference directly reflect in the neutrino yield. The average number of muon neutrinos for vertical proton showers, as a function of primary energy, is shown in Fig.3 for the two models. It must be remembered that from the experimental point of view, event rates should be considered, that is after the convolution with neutrino interaction cross sections.

Although the present data do not allow to give a reliable estimate of the overall systematic error associated to hadronic interactions, in the author’s opinion the existing difference between FLUKA and TARGET ($\sim 20\%$) should not be considered as a measurement of the real error. The different capability of TARGET and FLUKA model to

---

**Figure 2:** *Comparison of angle integrated neutrino fluxes for the Super–Kamiokande site between Bartol and FLUKA (both 1D and 3D)*
Figure 3: Average no. of atmospheric muon neutrinos for primary vertical protons as a function of energy for the Bartol and FLUKA models. The probability of neutrino interaction has not been considered in this plot.

reproduce accelerator data suggests that the actual theoretical uncertainty is likely to be definitively smaller than 20%.

A preliminary comparison, at the level of single interaction features, with the models adopted in HKKM calculations, gives indications that, if they had used the same primary spectrum of Bartol and FLUKA, also in their case the normalization would have been lower. The yield difference between FLUKA and TARGET is larger at lower nucleon energy. This has some direct consequence in the prediction of up-down symmetry of fluxes in different geographical sites. For instance, at Super–Kamiokande, where the cutoff from the above direction is rather high (around 10 GeV), the yield enhancement of TARGET is ineffective, and the FLUKA/TARGET ratio in the Sub-GeV region is reversed (see also Fig.2) with respect to the situation of Soudan, which is instead a low–cutoff site. Therefore, since the analysis of Soudan data are based upon the Bartol predictions, they should expect a lower asymmetry in the Sub-GeV region: see Fig.3.

Another way of looking at this is given in Fig.4, where the ratio of FLUKA (1D) to TARGET fluxes are shown as a function of energy for the different laboratories.

The two interaction models also differ in the $\pi^+ / \pi^-$ ratio, which affects the $\nu / \bar{\nu}$ ratio.
Figure 4: Up–Down asymmetry of $\nu_{\mu}$ fluxes, in absence of oscillations, at 3 different geomagnetic latitudes as calculated with the Bartol and FLUKA (1D and 3D) models. The probability of neutrino interaction has not been considered in this plot. The differences in asymmetry of event rates, after the convolution with neutrino interaction cross sections, are smaller.

Due to the different interaction cross section this is a significant parameter. Considering the proper weighting with the $X$ distribution, FLUKA has a larger $\pi^+ / \pi^-$ ratio by an amount which is around 7% for protons at 10 GeV. For increasing energy the difference becomes smaller, as expected. FLUKA is able to satisfactorily reproduce the charge ratio measured by many experiments\cite{27}.

We have found instead that the $(\nu_e + \bar{\nu}_e)/(\nu_{\mu} + \bar{\nu}_{\mu})$ ratio has not a relevant dependence on the hadronic interaction model.

Oscillation analysis of neutrino events requires the knowledge of production height, which depends on the longitudinal development of showers. This, on turn, depends on the point of first interaction, determined by total inelastic cross sections, and by the following development driven by the energy fraction carried away by leading nucleons in each interaction. Total cross sections are eventually determined by nucleon–nucleon cross sections, which are well constrained by existing data compilation, to which all groups make strong reference. Of course, the energy fraction taken by pions (including $\pi_0$, feeding the e.m. component of showers) is not independent from that taken by nucleons. FLUKA and TARGET have different elasticities and therefore give rise to some differences in the
Figure 5: Ratio of FLUKA (1D) to TARGET (inside FLUKA 1D shower code) calculated $\nu_\mu$ and $\nu_e$ fluxes, in absence of oscillations, as a function of neutrino energy, for 3 different geomagnetic latitudes.
longitudinal development of cascades. Again, the comparison with existing data reinforces
some confidence on the FLUKA model. More complete experimental data on particle
production on light nuclei would be fundamental to minimize the theoretical uncertainties
and constraint the existing models. In order to be significant for this purpose, a new
experiment must explore a range of beam energies from few GeV up to at least $30 \div
50$ GeV, with targets of different atomic number, from Be up to at least Al, in order
to study the dependence on the number of elementary collisions (which in the Glauber
approach scales as $A^{1/3}$). Secondary particles must be measured in a wide solid angle
to cover as much as possible the available phase space. For these reasons the scientific
community welcomes the HARP experiment, proposed to perform a dedicated study
on these subjects.

4 Conclusions

The current understanding of the sources of uncertainty in atmospheric neutrino flux
calculations is still improving. A better confidence on the primary cosmic ray spectra
is the first necessary condition. After that, the most important source of uncertainty
is that due to the particle production model. There is discussion upon which kind of
experiment or study can help in achieving better constraining of simulations. Data on
muon fluxes in atmosphere can help, and they are a useful benchmark tool. However, the
connection between muon and neutrino yield at different altitudes and energy is still a
rather indirect process, and it is not yet clear if muon balloon experiments can guarantee
a level of systematics below $\sim 10\%$. In the author’s opinion, the impact of new data
from accelerators could be more relevant on model building, provided that in this case
systematics is kept under better control. In this will be the case, there are reason to
believe that, although the theoretical error cannot be erased, it could be reduced to the
10\% level or even less. However, one of the most serious problem could stay not only in
the flux calculations, but also in the knowledge of absolute neutrino cross sections with
nuclei, especially at low energy for quasi-elastic scattering and resonance production. As
a matter of fact, the experimentalists are more interested in the eventual event rates than
in the flux itself. There is the serious risk that there will be no experimental answer to
clarify this aspect.

Other improvements in these computations are needed, but they will require more
and more efforts. As an example, the FLUKA group is planning to make use of further
developments in particle production models. In particular we are aiming to study the
effect of nuclear projectiles instead of recurring to the usual incoherent addition of nucleons
(superposition model) adopted so far also in the other standard references. The impact of
more precise calculations on the measurement of the parameters of “new physics” is still
to be understood in detail. In our opinion, this can be reliably done only introducing the
correct simulations of the actual experiments, since detector sources of systematics and
resolutions are likely to be of an importance comparable to that of theoretical factors.

10
5 Acknowledgments

The author is indebted to the other authors of the FLUKA–neutrino calculation, A. Ferrari, T. Montaruli, and P.R. Sala, for the help received in preparing this review. This work has been made possible thanks to the collaboration with the Bartol group: R. Engel, T.K. Gaisser, P. Lipari and T. Stanev. They have provided the TARGET model and a lot of essential data and suggestions.

References

[1] Y. Fukuda et al. (Super–Kamiokande Coll.), Phys. Rev. Lett. 81 (1998) 1562; T. Kajita, Proc. of NOW2000, Sept. 2000, Otranto, Italy.

[2] M. Ambrosio et al. (MACRO Coll.), Phys. Lett. B434 (1998) 451; F. Ronga, Proc. of NOW2000, Sept. 2000, Otranto, Italy.

[3] W.W.M. Allison et al. (Soudan-2 Coll.), Phys. Lett. B449 (1998) 137; A. Mann (Soudan-2 Collab.), Proc. of Neutrino 2000 Conference, Sudbury (Canada), June 16, 2000.

[4] F. Arneodo et al. (ICARUS and NOE Coll.), LNGS-P21/99, INFN/AE-99-17, CERN/SPSC 99-25, SPSC/P314; A. Rubbia (ICARUS Coll.). hep-ex/0001052.

[5] M. Honda et al., Phys. Lett. B 248 (1990) 193.

[6] G. Barr, T.K. Gaisser and T. Stanev, Phys. Rev. D 39 (1989) 3532; V. Agrawal, et al., Phys.Rev. D53, 1314 (1996).

[7] A. Fassó, A. Ferrari, J. Ranft and P.R. Sala. See http://www.cern.ch/fluka and references therein.

[8] G. Battistoni et al., Astrop. Phys. 12 (2000) 315.

[9] P. Lipari, Astrop. Phys. 14 (2000) 153.

[10] Y. Tserkovnyak et al., hep-ph/9907450.

[11] P. Lipari, Proc. of the VIII Int. Workshop on Neutrino Telescopes, Venezia, February 1999; also in hep-ph/9905506.

[12] P. Lipari, Astrop. Phys. 14 (2000) 188.

[13] R. Engel et al., Phys. Lett. 472 (2000) 113.

[14] T.K. Gaisser et al., Phys. Rev. D54 (1996) 5578.

[15] T. Sanuky, astro-ph/0002481 to appear in Ap.J.
[16] J. Alcaraz et al, Phys. Lett. B490 (2000) 27, Phys. Lett. 472 (2000) 215.

[17] M. Boezio at al., Astrophys. Journal 429 (1994) 736.

[18] W.R. Webber, R.L. Golden & S.A. Stephens, Proc. 20th ICRC (Moscow) vol. 1 (1987) 325.

[19] T. Sanuki (BESS Coll.), Proc. of NOW2000, Sept. 2000, Otranto, Italy.

[20] B. Bertucci (AMS Coll.), Proc. of NOW2000, Sept. 2000, Otranto, Italy.

[21] Fundamental references and models can be found in http://nssdc.gsfc.nasa.gov/.

[22] P. Lipari, Proc. of Neutrino 2000 Conference, Sudbury (Canada), June 16, 2000.

[23] FLUKA flux tables are available in http://www.mi.infn.it/˜battist/neutrino.html.

[24] S. Fredriksson et al., Phys. Rep. 144 (1987) 107; A. Tufail et al., Phys. Rev. D42 (1990) 2187.

[25] T. Eichten et al., Nucl. Phys. B44 (1972) 333.

[26] T. Abbott et al., Phys, Rev D45 No. 11 (1992) 3906.

[27] Proc. of the Workshop on Calorimetry, Annecy, October 2000.

[28] M.G. Catanesi et al., (HARP Collaboration), CERN-SPSC/99-35, SPSC/P315.

[29] R.J. Glauber, Phys. Rev. 100 (1955) 242; R.J. Glauber and G. Matthiae, Nucl. Phys. B21 (1970) 135.