Evolution of a Kerr-Newman black hole in a dark energy universe

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This paper deals with the study of the accretion of dark energy with equation of state $p = w \rho$ onto Kerr-Newman black holes. We have obtained that when $w > -1$ the mass and specific angular momentum increase, and that whereas the specific angular momentum increases up to a given plateau, the mass grows up unboundedly. On the regime where the dominant energy condition is violated our model predicts a steady decreasing of mass and angular momentum of black holes as phantom energy is being accreted. Masses and angular momenta of all black holes tend to zero when one approaches the big rip. The results that cosmic censorship is violated and that the black hole size increases beyond the universe size itself are discussed in terms of considering the used models as approximations to a more general descriptions where the metric is time-dependent.

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I. INTRODUCTION

Several astronomical and cosmological observations, ranging from observations of distant supernovae Ia\textsuperscript{1} to the cosmic microwave background anisotropy\textsuperscript{2}, indicate that the universe is currently undergoing an accelerating stage. It is assumed that this acceleration is due to some unknown stuff usually dubbed dark energy, with a positive energy density $\rho > 0$ and with negative pressure $p < -(1/3)\rho$. There are several candidate models for describing the dark energy, being the cosmological constant, $\Lambda$, by far the simplest and most popular candidate\textsuperscript{3}. Other interesting models are based on considering a perfect fluid with given equation of state like in quintessence\textsuperscript{4}, K-essence\textsuperscript{5} or generalized Chaplygin gas models\textsuperscript{6, 7, 8, 9, 10}. Note that there are also other candidates for dark energy based on brane-world models\textsuperscript{11} and modified 4-dimensional Einstein-Hilbert actions\textsuperscript{12}, where a late time acceleration of the universe may be achieved, too.

One of the peculiar properties of the resulting cosmological models is the possibility of occurrence of a cosmic doomsday, also dubbed big rip\textsuperscript{13, 14, 15, 16, 17, 18}. The big rip appears in models where dark energy parameterizes as the so-called phantom energy for which $p + \rho < 0$. In these models the scale factor blows up in a finite time because its cosmic acceleration is even larger than that induced by a positive cosmological constant. In these models every component of the universe goes beyond the horizon of all other universe components in finite cosmic time. It should be noted, that the condition $p + \rho < 0$ is not enough for the occurrence of a big rip\textsuperscript{19}. In recent papers\textsuperscript{20, 21}, it has been shown that the mass of a Schwarzschild black hole decreases with accretion of phantom energy, in such a way that the black hole disappears at the time of the big rip. Therefore, it is interesting to study how dark energy is accreted by more general black holes, that is to say, black holes bearing charge and angular momentum. The interest of this study is enhanced by the eventual competition or joint contribution that may arise between the dark energy accretion process and super-radiance which tends to decrease the rotational (or charge) energy of the hole, so lowering its spin (or charge), such as one would expect phantom energy induced as well. For this reason, in the present paper we shall investigate how distinct forms of dark energy can be accreted onto Kerr-Newman black holes. We in fact obtain that Kerr-Newman black holes progressively increase their mass and angular momentum as a result from dark energy accretion when the equation of state $p + \rho > 0$. That increase of mass and angular momentum is either unbounded or tends to a given plateau, depending on the dark energy model being considered. If $p + \rho < 0$ then both the mass and the angular momentum of black hole rapidly decrease until disappearing at the big rip, or tend to constant values in the absence of a future singularity. It is seen that the latter process prevails over both the Hawking evaporation process and spin super-radiance. Our quantitative results appear to indicate, on the other hand, that whereas phantom energy does not violate cosmic censorship conjecture\textsuperscript{20}, dark energy with $w > -1$ does.

The paper can be outlined as follows. In the next section, we will generalize the solution obtained by Babichev, Dokuchaev and Eroshenko\textsuperscript{20, 21} to the case of dark energy accretion onto a charged, rotating black hole, and present the general equations for the rate of mass and momentum. In the next section we apply such a formalism to quintessence and K-essence cosmological
fields, so as to the generalized Chaplygin gas model, analyzing the corresponding evolution of the black hole. In section IV we discuss the results that cosmic censorship is violated and that the black hole size grows up boundlessly beyond the universe size in terms of considering the used models as approximations to a more general description where the metric is not static. Finally, we briefly summarize and discuss our results in section V.

II. GENERAL ACCRETION FORMALISM FOR KERR-NEWMAN BLACK HOLES

In this section we shall follow the accretion formalism, first considered by Babichev, Dokuchaev and Eroshenko [20, 21], generalizing it to the case in which the black hole has an angular momentum and charge. First of all, we notice that, even though we shall use a static Kerr-Newman metric, the time evolution induced by accretion will be taken into account by the time dependence of the scale factor entering the integrated conservation laws and the rate equations for mass and angular momentum.

The procedure is based on integrating the conservation laws for energy-momentum tensor and its projection onto the four-velocity, using as general definition of energy-momentum tensor a perfect fluid where the properties of the dark energy and those of the black hole metric are both contained. By combining the results from these integrations with assumed rate equations for black hole mass, angular momentum and specific angular momentum, we can derive final rate equations for these quantities in terms of the dark pressure, and its state equation.

Now, since the conservation of dark energy law

\[ \rho \dot{\rho} + 3 H \rho = 0 \]

where \( \rho \) is the energy density, \( \rho \) the pressure and \( \dot{\rho} \) is the time derivative, we can derive final rate equations for these quantities in terms of the dark pressure, and its state equation.

Using a static metric nevertheless restrict in principle ourselves to deal with small accretion rates as the mixed component of the energy-momentum tensor a perfect fluid where the properties of the dark energy and those of the black hole metric are both contained. By combining the results from these integrations with assumed rate equations for black hole mass, angular momentum and specific angular momentum, we can derive final rate equations for these quantities in terms of the dark pressure, and its state equation.

The metric in this case can be given by

\[ ds^2 = \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) dt^2 + \frac{2a (2Mr - Q^2) \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} dtd\phi \]

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Using the general expression for a derivative of the energy-momentum conservation law [22] applied to this case, we get that the zeroth component of the energy-momentum tensor a perfect fluid where the properties of the dark energy and those of the black hole metric are both contained. By combining the results from these integrations with assumed rate equations for black hole mass, angular momentum and specific angular momentum, we can derive final rate equations for these quantities in terms of the dark pressure, and its state equation.

\[ T_{\mu\nu} = (p + \rho) u_{\mu} u_{\nu} - pg_{\mu\nu}, \]

where \( p \) is the pressure, \( \rho \) is the energy density, and \( u^\mu = dx^\mu/ds \) is the 4-velocity with \( u^\mu u_\mu = 1 \). There is no loss of generality in a restricting consideration to this form, as it is actually the most general form that \( T_{\mu\nu} \) can take consistent with homogeneity and isotropy.

Using the general expression for a derivative of the energy-momentum conservation law \( T_{\mu\nu,\nu} = 0 \) can then generally be written as

\[
\frac{d}{dt} \left[ (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \frac{dt}{ds} \right] + \frac{2r}{r^2 + a^2 \cos^2 \theta} (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \frac{dt}{ds} \frac{dr}{ds} + \frac{w^r}{r^2 + a^2 \cos^2 \theta} \left( \frac{\cos \theta}{\sin \theta} - \frac{2a \sin \theta}{r^2 + a^2 \cos^2 \theta} \right) \frac{d\theta}{ds} + \frac{w^\phi}{r^2 + a^2 \cos^2 \theta} \frac{d\phi}{ds} \right. \\
\left. \times (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \frac{dt}{ds} \frac{d\theta}{ds} \frac{d\phi}{ds} = 0. \right) \]

This expression should now be integrated. We consider two cases. First, we take \( \theta \) as a constant. The integration of Eq. (3) gives then,

\[ C_M = \frac{\mu}{M} \left( r^2 + a^2 \cos^2 \theta \right) \frac{(p + \rho)}{2} \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \left( \frac{2a^2 \sin^2 \theta + Q^2 - 2Mr}{2Mr} u^2 \right)^{1/2} \]

where \( u = dr/ds \), and \( C_M \) is an integration constant.

Another integral of motion can be derived by using the projection of the conservation law for energy-momentum tensor along the four-velocity, i.e. the flux equation

\[ u_\mu T^{\mu\nu,\nu} = 0. \]

For a perfect fluid, this equation reduces to

\[ w^\mu \rho_{,\mu} + (p + \rho) w_{,\mu} = 0. \]
The integration of Eq. (6) gives the second integral of motion that we shall use in what follows

\[
\frac{u}{M^2} (r^2 + a^2 \cos^2 \theta) \exp \int_{\rho_\infty}^{\rho} \frac{\frac{d\rho'}{p(\rho') + \rho'}}{r_0} = -A_M, \tag {7}
\]

where \( u < 0 \) in the case of a fluid flow directed toward the black hole, and \( A_M \) is a positive dimensionless constant. Eq. (7) gives us the energy flux induced in the accretion process. From Eqs. (4) and (7) one can easily get:

\[
\left(1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} + \frac{r^2 + a^2 \cos^2 \theta + Q^2 - 2Mr}{r^2 + a^2 + Q^2 - 2Mr} \right)^{\frac{1}{2}} \times (p + \rho) \exp \left[-\int_{\rho_\infty}^{\rho} \frac{d\rho'}{p(\rho') + \rho'}\right] = C_{2M}, \tag {8}
\]

where \( C_{2M} = -C_M/A_M = p(\rho_\infty) + \rho_\infty \).

The rate of change of the black hole mass due to accretion of dark energy can be derived by integrating over the surface area the density of momentum \( T_0^r \), that is\[23\]

\[
\dot{M} = -\int T_0^r \, dA, \tag {9}
\]

with \( dA = r^2 \sin \theta d\theta d\phi \), and \( r \) constant. Using Eqs. (2), (7) and (8) this can be rewritten as

\[
\dot{M} = \frac{4\pi A_M M^3 r}{J} \arctan \left( \frac{J}{Mr} \right) [p(\rho_\infty) + \rho_\infty], \tag {10}
\]

with \( r \) and \( J \) constants. It is worth noticing that Eq. (10) consistently reduces to the corresponding rate equation for a Schwarzschild black hole derived by Babichev, Dokuchaev and Eroshenko in Refs. [20] and [21] when one lets \( J \) to become very small. One has the following integral expression that governs the evolution of the mass of the Kerr-Newman black hole

\[
\int_{M_0}^{M} \frac{J\,dM}{M^3 r \arctan \left( \frac{J}{Mr} \right)} = 4\pi A_M \int_{t_0}^{t} [p(\rho_\infty) + \rho_\infty] \, dt. \tag {11}
\]

Now, the integration in the left-hand-side of Eq. (11) gives

\[
I(M) = \int_{M_0}^{M} \frac{J\,dM}{M^3 r \arctan \left( \frac{J}{Mr} \right)} = \frac{r}{2J} \times \left[ 1 + \frac{J^2}{Mr^2} \arctan \left( \frac{J}{Mr} \right) + \frac{J}{Mr \arctan^2 \left( \frac{J}{Mr} \right)} \right.

+ \left. \frac{4}{\arctan^3 \left( \frac{J}{Mr} \right)} \times \sum_{k=1}^{\infty} \frac{(2k - 1) \arctan^{2k} \left( \frac{J}{Mr} \right)}{\pi^{2k} (2k - 3)} \zeta (2k) \right]_{M_0}^{M}, \tag {12}
\]

where \( \zeta \) is the Riemann zeta function. The integration of the right-hand-side of Eq. (11) will be performed in the next section. We turn now to consider \( r \), instead of \( \theta \), as a constant with which the integration of Eq. (6) yields

\[
C_a = \frac{\omega}{a} \sin \theta (r^2 + a^2 \cos^2 \theta) \left( p + \rho \right) \times \left[ 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right.

+ \left. (r^2 + a^2 \cos^2 \theta + Q^2 - 2Mr) \omega^2 \right]^{\frac{1}{2}}, \tag {13}
\]

where \( \omega = d\theta/\,ds \), and \( C_a \) is another integration constant.

The second integral of motion for the energy flux in this case is also obtained from the projection of the energy-momentum tensor conservation law along the four-velocity; then the integration of Eq. (6) gives that second integral of motion

\[
\frac{1}{a} \omega \sin \theta (r^2 + a^2 \cos^2 \theta) \exp \left[ \int_{\rho_\infty}^{\rho} \frac{d\rho'}{p(\rho') + \rho'} \right] = -A_a, \tag {14}
\]

where \( \omega < 0 \) in the case of a fluid flow directed toward the black hole, and \( A_a \) is a positive dimensionless constant. From Eqs. (13) and (14) one can easily get:

\[
\left[ 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} + \frac{r^2 + a^2 \cos^2 \theta + Q^2 - 2Mr}{r^2 + a^2 + Q^2 - 2Mr} \right]^{\frac{1}{2}} \times (p + \rho) \exp \left[-\int_{\rho_\infty}^{\rho} \frac{d\rho'}{p(\rho') + \rho'}\right] = C_{2a}, \tag {15}
\]

where \( C_{2a} = -C_a/A_a = p(\rho_\infty) + \rho_\infty \).

We take the rate of change of the specific angular momentum of the Kerr-Newman black hole originating from accretion of dark energy to be now given by\[23\]

\[
\dot{a} = -\int rT_0^\theta \, dA, \tag {16}
\]

with \( dA = r^2 \sin \theta \, d\theta d\phi \), and \( \theta \) constant. Using Eqs. (14) and (15) this can be rewritten as

\[
\dot{a} = \frac{2\pi^2 A_a a r^2 [p(\rho_\infty) + p(\rho_\infty)]}{\sqrt{r^2 + a^2}} \tag {17}
\]

with \( r \) constant. Therefore, one has the following integral expression that reports about the evolution of the specific angular momentum

\[
\int_{a_0}^{a} \frac{\sqrt{r^2 + a^2}}{ar^2} \, da = 2\pi^2 A_a \int_{t_0}^{t} [p(\rho_\infty) + \rho_\infty] \, dt. \tag {18}
\]

Then, the integration in the left-hand-side of Eq. (18) gives rise to the following expression
The angular momentum $J$ in dark energy models will again be calculated in the next section for the distinct dark energy models. Now, we study the influence of dark energy accretion in the angular momentum $J$. Using $J = Ma$ and Eqs. \ref{eq:16} and \ref{eq:18} we can obtain the rate of change of the angular momentum of the black hole performing the following integral

$$ J = - \int (M a t_0^g + a t_0^r) dA, $$

(20)

with $dA = r^2 \sin \theta d\theta d\phi$, and $r$ constant. So, we obtain

$$ J = \pi \left[ p \left( \rho_\infty + \rho_\infty \right) \right] \times \left[ \frac{2 \pi A_r r^2}{\sqrt{1 + \frac{2 A_r r}{M}} + 4 A M^2 r \arctan \left( \frac{r}{M} \right)} \right], $$

(21)

with $M$ and $r$ constants. Therefore, one has the following integral expression that governs the evolution of the angular momentum of a black hole

$$ \int_{J_0}^{J} \frac{dJ}{J_0} \frac{2 \pi A_r r^2}{\sqrt{1 + \frac{2 A_r r}{M}} + 4 A M^2 r \arctan \left( \frac{r}{M} \right)} = \pi \int_{t_0}^{t} [p (\rho_\infty + \rho_\infty)] dt. $$

(22)

Once again the integration of the right-hand-side of the equation will be carried out in the next section. Here, we have been however unable to perform the integration of the left-hand side (L) in closed form. Thus, we have proceed as follows. The integral $L$, in Eq. \ref{eq:22} can be recast in the form

$$ L = \int_{x_0}^{x} \frac{dx}{\cos^2 x} \frac{1}{2 \pi A_r r + 4 A M M x}, $$

(23)

where $0 \leq x \leq \pi/2$ and $x = \arctan (J/Mr)$. It can be noticed that, since $0 \leq \sin x \leq x$, we have

$$ L \geq \int_{x_0}^{x} \frac{dx}{2 \pi A_r r + 4 A M M x} \cos^2 x $$

$$ = \frac{1}{2 \pi A_r r + 4 A M M} \times \left[ \tan x \frac{x}{x} + \ln x + \frac{1}{x^2} \sum_{k=2}^{\infty} \frac{(2k-1)x^{2k}}{k-1} \pi^{2k} \zeta(2k) \right]_{x_0}^{x}, $$

where $\zeta$ is again the Riemann zeta function. Thus, $L \geq I(J)$ which in turn implies that if we use $I(J)$ for studying the evolution of the Kerr-Newman black hole and the cosmic censorship is taken to be physically preserved, then $L$ should respect this conjecture, too. This argument entitles us to use $I(J)$ as a suitable expression to study the evolution of $J$ during accretion of dark energy.

### III. COSMOLOGICAL MODELS

In order to obtain exact integrated expressions for the right-hand-side of Eqs. \ref{eq:11}, \ref{eq:18} and \ref{eq:22}, we shall use in this section two different models for dark energy, namely, quintessence and generalized Chaplygin gas models. It can be seen that the results obtained by using the quintessence model are the same as those derived if one used the so-called K-essence model for dark energy.

#### A. Quintessence models

Starting with the equation of state $p = w \rho$, where $w$ is assumed constant, we can use the conservation of the energy-momentum tensor to get

$$ \rho = \rho_0 \left( \frac{R_0}{R} \right)^{3(1+w)}, $$

(25)

where $R \equiv R(t)$ is the scale factor, with $\rho_0$ and $R_0$ constants. Hence

$$ \int_{t_0}^{t} [p (\rho_\infty + \rho_\infty)] dt $$

$$ = (1 + w) \rho_0 R_0^{3(1+w)} \int_{t_0}^{t} R^{-3(1+w)} dt. $$

(26)

We then have for the scale factor \ref{eq:25} corresponding to a general flat quintessence universe

$$ R(t) = R_0 \left( 1 + \frac{3}{2} (1 + w) C^{1/2} (t - t_0) \right)^{2/[3(1+w)]}, $$

(27)

where $C = 8 \pi G \rho_0/3$. Integration of the right-hand-side of Eqs. \ref{eq:11}, \ref{eq:18} and \ref{eq:22}, can then be performed using Eq. \ref{eq:27}. We respectively get

$$ t = t_0 + \frac{I(M)}{(1+w) \left( 4 A M \rho_0 - \frac{1}{2} C^{1/2} I(M) \right)}. $$

(28)
\[ t = t_0 + \frac{I(a)}{(1 + w)\left(2\pi^2A_0\rho_0 - \frac{1}{2}C^{1/2}I(a)\right)}, \]

\[ t = t_0 + \frac{I(J)}{(1 + w)\left(\pi\rho_0 - \frac{1}{2}C^{1/2}I(J)\right)}, \]

where \( I(M), I(a) \) and \( I(J) \) are defined in Eqs. (12), (20) and (26), respectively. These are three parametric equations from which one can obtain how the mass, specific angular momentum and angular momentum evolve in the accelerating universe. Thus, if \( w > -1 \) we see that \( M, a, \) and \( J \) will all progressively increase with time from their initial values, with \( \alpha \) tending to a finite constant value as \( t \to \infty \), showing that the increase of \( M \) tends to become proportional to the increase of \( J \); however \( M \) goes to infinity in a finite time, but \( J \) tends to a finite constant value as \( t \to \infty \). Notice that there is no contradiction between the results of Figs. (2) and (3) as the plot in Fig. (3) is obtained relative to a constant value of mass. The larger \( w \) the quicker the increase of these parameters [see Figs. (1), (2) and (3)].

If \( w < -1 \) we can observe that \( M, a, \) and \( J \) will all progressively decrease from their initial values, tending to zero as one approaches the big rip, where the black holes will disappear independently of the initial values of their mass and angular momentum [see Figs. (1), (5) and (6)]. This generalizes the result obtained by Babichev, Dokuchaev and Eroshenko [20, 21].

In the case of a charged black hole, the process of super-radiance of charge allows the black hole to emit the charge before it disappears. It has been checked as well that the larger \( |w| < -1 \) the quicker is the decrease of \( M \) and \( J \), and that for large \( r \) the evolution of the mass nearly matches the evolution that was derived for the Schwarzschild case. Also remarkable are the features that the larger \( J \), or the smaller \( r \), the smaller the rate of mass decrease. Accretion of phantom energy leads also to a decreasing of \( a \) which becomes zero quickly, so that \( J \) must decrease quite more rapidly than \( M \) does. For any \( w \), it has been finally seen that the rate of variation (increase for \( w > -1 \) and decrease for \( w < -1 \)) of \( J \) speeds up as one makes \( r \) or \( M \) larger.

**B. Generalized Chaplygin gas**

We shall derive now the expression for the rates \( \dot{M}, \dot{a}, \dot{J} \) in the case of a generalized Chaplygin gas. This can be described as a perfect fluid with the equation of state [3]:

\[ p = -\frac{A_{ch}}{\rho^\alpha}, \]

where \( A_{ch} \) is a positive constant and \( \alpha \) is a parameter. In the particular case \( \alpha = 1 \), the equation of state [31] corresponds to a Chaplygin gas. The conservation of the energy-momentum tensor implies

\[ \rho = \left(A_{ch} + \frac{B}{R^{3(1+\alpha)}}\right)^{1/(1+\alpha)}, \]

with \( B \equiv \left(\rho_0^{\alpha+1} - A_{ch}\right)R_0^{\alpha+1} \). Now, from the Friedmann equation we can get

\[ \dot{R} = \sqrt{\frac{8\pi G}{3}} R(t) \left(A_{ch} + \frac{B}{R^{3(1+\alpha)}}\right)^{1/[2(1+\alpha)]}. \]

Hence,

\[ R^{2(1+\alpha)} = \frac{B}{\left(\sqrt{\rho_0} - \sqrt{\frac{3\pi}{8\pi A_{ch}^2}} I(M)\right)^{2(1+\alpha)} - A_{ch}}, \]

\[ R^{3(1+\alpha)} = \frac{B}{\left(\sqrt{\rho_0} - \sqrt{\frac{3\pi}{2\pi A_{ch}^2}} I(a)\right)^{2(1+\alpha)} - A_{ch}}. \]
Figure 2: This figure shows the behaviour of the specific angular momentum of a Kerr-Newman black hole as a function of the cosmic time in presence of dark energy with $w = -0.8$ and $w = -0.9$. One can also observe on the figure that the larger $w$ or $r$, the quicker the increase of specific angular momentum. In the inset we can see that $a$ tends to a constant value for large times in all studied cases.

$$R^{3(1+\alpha)} = \frac{B}{\left(\sqrt{\rho_0} - \sqrt{\frac{6G}{\pi}I(J)}\right)^{2(1+\alpha)}} - A_{ch},$$

for $M$, $a$ and $J$, respectively. Again for the case where the dominant energy condition is preserved, i.e. $B > 0$, we obtain that $M$, $a$ and $J$ all increase with time, $M$ and $a$ tending to constant values for moderately large $B$. If $B$ is large enough, then whereas $M$ tends to infinity, $a$ approaches a larger but still finite constant value. On the other hand, $M$ and $a$ are both seen to increase more rapidly as parameter $\alpha$ is made smaller, with $M$ tending once again to infinity, if $\alpha$ is taken to be sufficiently small. As to the accretion dependence on $r$ for $B > 0$, it has been checked that as $r$ is made very small, $M$ and $a$ are nearly frozen into their original values. On the contrary, for large $r$, the evolution of $M$ will tend to match that in the Schwarzschild case, while $a$ increases now again up to a given constant value. If the dominant energy condition is assumed to be violated, i.e. $B < 0$, then $M$, $a$ and $J$ all decrease with time, with $M$ and $a$ always tending to minimum, nonzero constant values. Making $|B|$ larger, or $\alpha$ smaller, makes the evolution quicker and the final minima values for $M$ and $a$ smaller but still nonzero. The dependence of the evolution process on $r$ in this case is quite similar to what we already described for $B > 0$, that is to say, $M$ and $a$ nearly keep their initial values for very small $r$, but both decrease each time quicker as $r$ is increased. Also common for $B > 0$ and $B < 0$ is the feature that the evolution of $M$ is damped as we choose larger values of the angular momentum $J$.

All these behaviours have been checked by numerical calculations which provides plots that are actually quite the same those corresponding to the quintessence case.
Figure 4: This figure shows the behaviour of the mass of a Kerr-Newman black hole as a function of the cosmic time in presence of phantom energy with \( w = -1.1 \) and \( w = -1.2 \). One can also see on the figure that the larger \(|w| < 1|\) or smaller \(J\), the quicker the decrease of mass.

Figure 5: This figure shows the behaviour of the specific angular momentum of a Kerr-Newman black hole as a function of the cosmic time in the presence of phantom energy with \( w = -1.1 \) and \( w = -1.2 \). One can also observe on the figure that the larger \(|w| < 1|\) or \(r\), the quicker the decrease of specific angular momentum.

C. Super-radiance and cosmic censorship

In the case of a Kerr-Newman metric, the cosmic censorship conjecture [26] holds provided that

\[ Q^2 + a^2 \leq M^2. \]  \hspace{1cm} (37)

Otherwise, the Kerr-Newman black hole will show a naked singularity. It is interesting to study if dark energy accretion can produce a naked singularity in this case. Since accretion of dark energy is a gravitatorial process, whereas angular momentum is affected by it, electric charge is invariant under accretion. We have pointed out above that when \( P + \rho > 0 \), \( a \) and \( M \) increase with time during accretion of dark energy. Even though we have not a formal proof for the violation of cosmic censorship in this case, a numerical analysis performed for most reasonable values of \( M \) and \( a \) appears to indicate that the dark energy accretion process violates the inequality in (37) for most reasonable situations. Actually, there always is a very small initial time interval where the conjecture holds, except at the extreme case where \( Q^2 + a^2 = M^2 \) [see Fig. (9)], but as soon as the initial value of \( a \) is taken to be significantly comparable with that of \( M \), the conjecture is almost immediately violated [see Fig. (10)]. In the next section we shall discuss and interpret the reason for that violation.

If accretion involves phantom energy, then \( a \) and \( M \) both decrease. In this case, since accretion does not affect the value of electric charge, at first sight, it could be thought that when sufficiently small values of \( a \) and
Figure 6: This figure shows the behaviour of the angular momentum of a Kerr-Newman black hole as a function of the cosmic time in the presence of phantom energy with \( w = -1.1 \) and \( w = -1.2 \). One can also see on the figure that the larger \(|w| < -1|\), \( r \) or \( M \), the quicker the decrease of angular momentum.

\( M \) are reached, Eq. (37) would no longer hold too, and cosmic censorship would be violated as well. However, it may also be expected that super-radiance of charge would act upon its value in such way that it decreased charge during accretion of phantom energy so that Eq. (37) would still be satisfied. Moreover, as \( M \) progressively decreases the black hole temperature should rise up and the charge super-radiance would correspondingly speed up. Our numerical calculations appear to indicate that this is actually the case as all the simultaneous effects on \( M \), \( a \) [see Fig. (11)] and \( Q \) due to dark energy accretion and \( Q \)-super-radiance seem to be mutually concited in such way that the cosmic censorship is preserved indeed. Obtaining an explicit, accurate expression for the relation between mass or temperature and electric charge, however is a task that contains some subtleties and therefore requires further elaboration which is left for a future consideration. We do not consider in this paper the process of super-radiance of spin because phantom energy clearly prevails over it.

IV. AN APPROXIMATED ACCRETION MODEL

Violation of cosmic censorship in black holes dark energy accretion \((w > -1)\) is a very surprising result actually, but it is perhaps not so surprising as the features coming about when both rotating and non-rotating black holes continue accreting such type of dark energy at sufficiently large times, according to the accretion model used by Babichev, Dokuchaev and Eroshenko[20, 21] and generalized in section I. One of such features results e.g. from Eq. (28) where it can be seen that the black hole mass blows up at a finite time in the future, when the size of the universe is still finite. It follows that the grown-up black hole will engulf the entire universe in a finite time in the future, an implication which is rather bizarre indeed,
Figure 8: This figure shows the behaviour of the mass and the specific angular momentum of a Kerr-Newman black hole as a function of the cosmic time in presence of a (phantom) generalized Chaplygin gas with $B = -0.5$ and $\alpha = 0.5$.

Figure 9: Evolution of an extreme Kerr-Newman black hole with dark energy. This figure shows the behaviour of the mass and specific angular momentum of a extreme Kerr-Newman black hole as a function of the cosmic time in the presence of dark energy with $w = -0.9$. One can see on the figure that the cosmic censorship conjecture is violated.

and that it is also present when non-rotating black holes are considered [20, 21]. Nevertheless, all the predictions that have been derived for large time could be regarded as artifacts coming from the fact that the black hole metric used in our accretion procedure is static. Really, that procedure becomes in such a case just an approximate description which can only be valid for a sufficiently short initial time interval. Therefore the results obtained in the present paper would just mark the tendency of the different involved parameters once the initial evolution has been overcome, but cannot be taken for granted for large times.

Even in this case the result that cosmic censorship is violated when dark energy with $w > -1$ is being accreted cannot be justified, as that violation takes place from the very beginning of the evolution for extreme black holes. According to the results displayed in Figs. (1) and (2), there appears to be a possibility to avoid incompatibility of a simultaneous violation of cosmic censorship and a black hole engulfing of the universe. Indeed, a black hole might have the following bizarre evolution when accretes dark energy with $w > -1$, according to these figures. It could violate cosmic censorship at the beginning of its evolution and become a naked singularity. In this stage, accretion of dark energy produces a bigger increase of mass than specific angular momentum. Let us remember now that specific angular momentum grows until a constant value as $t \to \infty$, whereas the mass blow up at finite time. So, in a finite time the naked singularity becomes again a black hole. Next, black hole can continue its evolution ending in an universe engulfed by the black hole. Thus, whereas the evolution of black holes induced by accretion of phantom energy appears to be quite reasonable at least on the early periods, in the case of satisfying the dominant energy condition, the accretion onto black holes seems to produce rather unexpected results along the entire subsequent evolution.
Figure 10: Evolution of a Kerr-Newman black hole with dark energy. This figure shows the behaviour of the mass and specific angular momentum of a Kerr-Newman black hole as a function of the cosmic time in the presence of dark energy with $w = -0.8$. The separation between the two curves diminishes as the initial value of $a$ is increased. One can also see on the figure that there exists a small initial time interval (running up to nearly $t = 0.05s$ in this case) where the cosmic censorship conjecture still holds.

Figure 11: Evolution of an extremal Kerr-Newman black hole with phantom energy. This figure shows the behaviour of the mass and specific angular momentum of an extreme Kerr-Newman black hole as a function of the cosmic time in presence of phantom energy with $w = -1.2$. The separation between the two curves increases as the initial value of $a$ is diminished or $r$ is made larger.

V. CONCLUSIONS

In this paper we have studied the behaviour of accretion of dark energy onto a Kerr-Newman black hole. First, we have generalized the accretion formalism originally considered by Babichev, Dokuchaev and Eroshenko \cite{20, 21} for the case in which the black hole has angular momentum and electric charge. We have applied such a formalism to quintessence and K-essence cosmological fields, so as to the generalized Chaplygin gas model. The evolution of mass, specific angular momentum and angular momentum when dark energy with $w > -1$ has been considered. It has been seen that all of these parameters ($M, a$ and $J$) increase with cosmic time. The specific angular momentum $a$ grows up to reaching a constant value whereas $M$ is not bounded from above. It is also checked, in this case, that the accretion of dark energy verifying dominant energy condition usually leads to a situation where the cosmic censorship is violated. There is another feature even more surprising, i.e., the mass of black hole blows up in a finite time and therefore black holes will engulf the entire universe in a finite time. These two predictions could be however regarded as artifacts coming from the fact that the black hole metric used in our formalism is static. Really, the used procedure could be seen just as an approximate description which is valid only for a sufficiently short initial time interval. Therefore the results obtained only mark the
tendency of the considered parameters, and could well be not valid for large times. Even in this case the result that cosmic censorship is violated when black hole accrete dark energy with \( w > -1 \), cannot be justified, since that violation occurs in the very beginning of the evolution for extreme black holes. Thus, the accretion of dark energy verifying \( \rho + \rho > 0 \) onto black holes seems to produce rather surprising and unexpected results.

If accretion involves phantom energy, then \( a \) and \( M \) both decrease from their initial values, tending to zero as one approaches the big rip, where the black holes will disappear, independently of the initial values of their mass and angular momentum. In this case (\( P + \rho < 0 \)), cosmic censorship conjecture is preserved, since super-radiance of charge and phantom energy accretion mutually interrelated.

Whether or not the above features can be taken to imply that phantom energy is a more consistent component than normal dark energy with \( w > -1 \) is a matter that will depend on both the intrinsic consistency of the models and the current observational data and those that can be expected in the future.

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