Application Research of Design Geometry in the Design of High Speed Trains

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Abstract: The visual characteristics of high-speed trains in China are comparatively not at an advantage, while vehicle serialization and product identification remains to be a major problem. From the perspective of design geometry, This paper analyzes the mathematical composition of modeling curves in foreign excellent models, summarizes the geometric proportional relationship of the vehicle system, innovates a design and operation method of the vehicle body shape, and establishes a new generation of model design based on the geometric law of the line shape of vehicle body so as to enhance the viability of vehicle serialization, and provide a basic premise for design strategy research.

1. Introduction of design current situation

Due to the continuous innovation of technology and the dynamic evolution of the aesthetic concept, the social environment and development strategy put forward higher aesthetic requirements for the design of high-speed trains. The new train is not only a high-tech carrier for the strong development of the national economy, but also an aesthetic materialization that highlights the soft power of culture. [1] After years of technology introduction and absorption innovation, China has realized the technical standardization of independent intellectual property rights of high-speed trains. However, in terms of the status quo of design, China’s high-speed trains are still in a state of one shape for one car: The original models of CRH380 series (He’xie)-the largest number of operations-were imported from abroad, and its technical parameters and modeling structure are from three different countries. From the perspective of intuitive visual perception, the contours of each model are obviously different, and the commonality of form recognition is found lacking (Fig.1). The newly-launched “Fu’xing” series of CRRC Group comes from the original model designed by CSR and CNR, while CRH400A and CRH400B can’t constitute brand recognition with different styles, different shapes, and lack of visual form uniformity. The design framework of the existing car system can’t fully meet the image demand of a big country of high-speed train design and manufacture, which hinders the implementation of the national macro policy.
In response to the problem of vehicle serialization and product identification, domestic academic circles have proposed design innovations and ideas from multiple research perspectives. On the whole, various research institutes focus on the design of high-speed trains on three major themes: the arrangement of design process, the metaphor of national culture, and the guidance of humanistic feelings. [2] Within the framework of this research, many design concepts of top-down qualitative studies are based on the theory of aesthetics, sociology, and psychology. [3] As mentioned above, the line shape of the existing models is quite different, and the visual identification system of the series is not formed. The overall qualitative research lacks the basis for conducting common analysis. Therefore, this paper starts with the design case study, analyzes the infrastructure composition of the line shape of foreign excellent models (series) from the top-down research route, explores the line shape rule of the model and the continuity of the serialized matrix features, with a view to making a contribution and forming the material basis of visual recognition of domestic models.

2. The theoretical concept of design geometry

2.1 Academic origins of design geometry

The research and development of design geometry is a process of exploration along with human transformation of the natural world. From the birth of civilization, people’s artistic behaviors continue to accumulate aesthetic experience, and gradually form certain mathematical laws. The ancient Greek philosopher Pythagoras discovered that there is a very harmonious geometric proportional relationship in nature. He defines the geometric relationship between natural elements and proposes three kinds of qualitative differences: shape, order, position; If the relationship between the various parts of the object conforms to this law of division, then it has a strict proportionality, which can make people have a harmonious and pleasing visual experience.

In view of the aesthetic pleasure generated by the proportional order method, A large number of in-depth researches were conducted in the field of psychology research, from the perspective of visual balance and psychological balance to answer the psychological reasons for human beings to obtain sense of beauty. The Gestalt Psychology School regards the sensory response to order as the main research field. Gestalt psychologists believe that one of the most basic forms of our sense of order is the sense of balance. Our perception prefers simple structure, straight line, rounds and other simple orders. Such regular patterns are often the easiest ones to see in the complicated outside world, rather than cluttered shapes. The concept put forward by Fechner Gustav Theodora, a psychologist, is more concise: “Beauty is the order in complexity.” Adolph Zeising, an esthetician, standardized mathematics of geometric proportion, and he defined the dominant ratio of “21:34” over the art and nature as the golden section. Later, Lalo C. conducted several cross-cultural experiments in a systematic approach to demonstrate the advantages of geometric proportion in human morphology judgment.
2.2 Research category of design geometry
Karl Heinrich Marx put forward in the Economic and Philosophical Manuscripts of 1844: “People know how to produce according to any kind of scale, and know how to apply scales to objects everywhere; therefore, people build following the laws of beauty.” [4] Materialistic aesthetics regards scale as the materialized relationship of beauty, and scale is a universal and effective beauty rule formed by human creation of objects. The human creation activity is the exploration process of the scale law and the aesthetic law, and the design geometry is the discipline that studies the universal principle of the visual experience accumulated by people in the aesthetic activities.

Design geometry is not purely geometric in mathematics. Geometry studies mathematical relations and logical relationships; while design geometry focuses on finding visual aesthetics such as equilibrium, coordination, and rhythm. [5] Le Corbusier said in the book Towards New Architecture: “Geometry is the language of mankind... The geometry of art is not a sophisticated mathematics, but the morphological law everyone knows, and it is a standard of scale division that conforms to the form of human beings and nature. The design products that conform to this law will produce a harmonious aesthetic perception.” [6] The design geometry research Covers three aspects: the scale of human and nature with the law of order, the basic principle of geometric proportion and the principle of segmentation of spatial vision. The main body of research is the visual relationship between proportion and composition. [7] Therefore, it is a combination of design art and geometry. Its purpose is to use geometric mathematical logic to achieve spatial separation to present harmonious perfect scales and sequences.

2.3 The application of geometry design
Based on the academic research category, design geometry is not simply a means of drawing that describes the visual composition, but an applied theory that makes objects that have geometric relations in space. It contains two-dimensional, three-dimensional spatial mathematical principles and segmentation principles, and standardizes the composition of scales in the morphological design, with the guiding principle of the basic principles. [8] In view of the combination of scientific rationality and intuitive image, the principle of proportional division of design geometry has played a positive role in the construction of modern practical aesthetics.

Today, design geometry has infiltrated into all areas of art design, and the construction of geometric language provides a guidance of many mathematical relationships for design activities. For example, irrational rectangles such as $\sqrt{2}, \sqrt{3}, \sqrt{5}$, and $\Phi$ (golden section) can be divided by horizontal lines and vertical lines to form a derivative rectangle, and a plurality of spiral curves can be generated by auxiliary lines such as tangents and extension lines. After a series of segmentation reconstruction, the various segmentation forms inside the rectangle are converted into new proportional relationships with each other to form a dynamic rectangle of a geometric cycle. Mathematical forms such as these operate the law of geometric relations of order aesthetics and become the paradigm of modern design. [9]

3. The application of design geometry in high-speed trains
The static image of the appearance of a high-speed train is an aesthetic form consisting of linear extension, surface articulation, and functional segmentation. This paper starts with the analysis of the scale law of the linear frame, and studies the most original structural relationship of the linear surface of the vehicle, which is conducive to grasping the basic principles of visual aesthetics.

Compared to other categories of product design, the geometric design elements in high-speed train design are difficult to be visually intuitive. The technical factors of aerodynamics and the conspicuousness of high-precision mechanized production conditions make the geometric design language obscured in large-area of fluid surfaces. Therefore, the geometric relationship analysis of the design elements can only be deduced by the method of extracting the lines of the base surface, and the mathematical scale study can be carried out on the basis of simplifying the surface.
3.1 The geometric progression in the Korean KTX series

As a representative of the Pan-East Asian cultural circle, the segmentation characteristics of blue and white in the Korean KTX series of trains are very obvious. Take the Sea fog train as an example. It is the third generation of the KTX series, in which there are a lot of arc modeling elements, and large-span arc curvature is used in the cockpit and the color segmentation of vehicle body (Fig. 2). If we deducted the partial arcs, a plurality of standard circles could be obtained. These circular circles of different sizes are arranged in order of small radius, the arc of the headlight is R1, the lower part of the opening and closing mechanism is R2, the upper part of the opening and closing mechanism is R3, and the contour of the window is R4. The blue and white coating split arc is R5. When the radius of the arc R1 is set to one length unit, R2 is 9 length units, R3 is 18 length units, R4 is 27 length units, and R5 is 36 length units; each arc radius constitutes geometric progression.

![Figure 2. Views of sea fog](image)

The front view of the sea fog leaves the visual impression of the cab windshield as an irregular shape. Following the simplification of the complex form, the contour curve is decomposed into four segments according to the law of curvature tangency. The standard circle of R1, R3, R4, etc. is obtained by the completion of the arc, and the inscribed circle R2 is obtained by using the two end points of the upper edge of the front opening and closing mechanism and the vertex of the front end of the vehicle locomotives and the inner edge of the light as the point of contact; The circumscribed circle R5 is obtained by using the four endpoints of the contour as point of contact. When the radius of the arc R1 is set to 1 length unit, R2 is 2 length units, R3 is 3 length units, R4 is 4 length units, and R5 is 5 length units; the radius of each arc also constitutes geometric progression. Although the dimensions of the various functional components of the train vary, and the curvature of the arc and the length of the arc are different, this geometric line that emphasizes scale and proportion can produce an effective visual response between the various modeling elements, forming a unified modeling paradigm.

3.2 Proportional division of the Bombardier Zefiro series

The Zefiro model is a prototype of the CRH380D model introduced in the process of upstream technology. The contour curve of the vehicle has a large curvature and changes significantly (Fig. 3). In view of the fact that the curvature of the front part of the headline is relatively prominent, the turning points in the side view can directly form a rectangular division of the geometrical progression according to the positional relationship of the vector coordinates. In this matrix, the first curvature change point A and the second curvature change point B are respectively located at 1/3 and 1/2 of the vertical height of the vehicle; the second curvature change is made from the end point of the vehicle locomotives to the third curvature change point C as the horizontal distance length. The point is located at 1/3 of the line in the horizontal coordinate direction.

In addition to the physical division of the geometric progression, the model uses more proportions of the \( \sqrt{2} \) segmentation. The arc of the front opening and closing mechanism is divided into upper and lower curvature portions by the vertices of the front vertex D, and the vertical positioning points of the two sections are the lower curvature starting point E and the end point F of upper curvature respectively, and the two forms a rectangle together with the locomotives apex. Similarly, the \( \sqrt{2} \) rectangle whose starting position of the light is coplanar with the apex of the front is divided by the
upper end of the curvature of the opening and closing mechanism. The two $\sqrt{2}$ rectangles together establish the frame of front curve trend, which provides a basis for the curvature extension of subsequent lineshape such as cab windows and escape window contours.

In addition, in the horizontal direction of the vehicle body, the front end point A of the window is located on the $\sqrt{2}$ proportional dividing line between the curvature change point B of the roof and the end point C of the front end of the light coating; in the vertical direction, after the end of the vehicle light is coated with the end point D, the car body height is divided into $\sqrt{2}$ proportions (Fig.4).

The research perspective is transferred to the front view, and the upper edge contour of the front opening and closing mechanism is at a $\sqrt{2}$ proportional division position of the vehicle body height; the lower edge contour of the driving window coating is located at a $\sqrt{2}$ proportional division position of the vertical height line from the roof to the apex of the front (Fig.5).

### 3.3 Golden section in German ICE series models

The ICE series train (or intercity multiple unit) is the fastest and highest train category in Germany. Its curve shape partially hides the golden section relationship. Observed and analyzed in the front view, the chamfering radius $r_1$ of the upper edge contour of the second generation model is in golden section.
with the lower edge chamfering radius r2. This feature has been derived in the third and fourth generation models; the model radius r3 of the angle of chamfer of the lower edge and the upper edge chamfering radius r4 also constitute a golden section relationship (Fig.6).

![Figure 6. The front view of ICE series train](image)

After reconstructing the streamline contour of ICE third-generation trains using the NURBS curve, four curvature control points on the curve can be obtained; the rectangles are formed according to the position of each point, and the ratio of the coordinate distance between each control point is the golden section. In addition, as the low-speed evolution version of the ICE three-generation model, the geometric proportional relationship of the modeling curves of the ICE four-generation model is more subtle and needs to be analyzed by means of auxiliary geometry. The specific method is as follows: on the side view of the vehicle body, the horizontal line (the extension line of the roof contour) is made with the vanishing point of the curvature as the end point, and the base is the edge of the train guard, and the parallelogram is constructed with the oblique parting line as the long side. The length ratio of the short side b to the long side a+b is a golden section, and the curvature change point is a golden point of the long side (Fig.7).

![Figure 7. The right view of ICE series train](image)

As the representative model of the German train, although the four trains of the ICE series have a quite different time-to-market, the unified light-colored body is decorated with long and thin red lines and has a strong overall visual recognition feature. From the geometric relationship, the height of the vehicle is divided by the red line into two parts according to the golden section. Its third-generation and fourth-generation models further expand the range of geometric ratios, from the height of the roof to the centerline of the window, and the ratio of the height of the vehicle to the centerline of the window is also the golden section. In the span of more than 20 years, the ICE series has always adhered to the geometric proportional division relationship, forming the distinctive style characteristics of the vehicle.
3.4 Research Summary

Based on the analysis of the above-mentioned models, the design of foreign high-speed trains uses a large number of rational induction methods to form geometric proportional relationship. All models have uniformity in order division and application. In the model change design, the geometric proportional feature was actively inherited, and finally formed the model gene of the model series. The main points of design of guiding significance are:

In the specific operation process of the design and implementation, the model designer optimizes the geometric relationship between the curvature nodes by enhancing the coordinate parameters of the curve control points to improve the smoothness of the vehicle body.

The functional parting lines and color dividing lines in all kinds of functional parts are all in accordance with the geometric proportional mode; these geometric lines with strict laws are used to derive the mathematical ratio by dividing the scale, and realize the visual equilibrium derivation in the constituent relationship.

In the design of the model evolution, the original proportional body is actively retained to form a visual continuity and eventually form a series.

4. The experimental argumentation

We try to verify the feasibility of the above three design elements through the virtual model, and comprehensively consider the application prospect of geometric design ideas in high-speed train design. The specific steps are: linear design - surface generation - structural line optimization - scale division - coating ratio - serial development - wind resistance calculation.

4.1 surface generation

We set the benchmark curve of the model according to geometric progression. Firstly, we establish the rational ratio of each CV control point of the NURBS curve, and then adjust the linear structure, the curve control point distribution, the order of the curve, and then rationally analyze changes of the curve slope. So the feasible line type is obtained, which is used as the modeling foundation of the newly designed train (Fig.8).

![Figure 8. CV control point of the basic curve](image)

In the design software, the closed surface is generated by curve lofting, and the original model of the new train model is obtained. These original surfaces are upgraded, the surface is adjusted, and the improved model is further optimized. As the order of the surface increases, the visual feature points of the shape gradually form, but the integrity of the reference curve must be guaranteed at any time, and any CV control points of the reference curve are not adjusted (Fig.9).
4.2 Proportional segmentation

After the solid model is generated, we divide the functional parts of the vehicle body according to the setting of the design target. In accordance with the technical requirements of mechanical design and ergonomics, these functional components must ensure the rationality of the scale (not to do technical analysis here). We use the vertices of the front as the starting point of the geometric segmentation. Based on the horizontal distance between the apex A of the front and the end B of the damper, as well as the positioning point C in the lower coating edge of the window in the cab; and then based on the horizontal distance between the point C and the end point D of the escape window, in turn, we establishes the front end point E of the cab window and the escape window coating position point F; finally, based on the horizontal distance between the point B and the vane curvature vanishing point G, a diagonal rectangle is formed, and we establishes the end point H in the cab window, lower edge end I in escape window, end point J of side window, and vehicle lower edge point K in proper order. After dividing the positional relationship of the front part of the vehicle, we keep the curvature of the curve in a smooth transition, and draw the golden section to make the body coating division (Fig.10).

Simulating the three-dimensional effect graph of the above-mentioned linear segmentation, we get the final design effect of the new model, (Fig.11). We reserve the proportional relationship between the window, the escape window, the driving window and the headlights, improve the geometric proportional relationship of the coating division, and adjust the coating connection of the window and the escape window to obtain the middle and right models in the picture. Due to their consistent division ratio, the three models effectively form a visual continuity under the changing form and color, forming a series of designs.
4.3 Simulation calculation
The air resistance analysis of the optimized virtual model is carried out in the analysis software. The wind resistance coefficient of newly designed train locomotives is 0.2087, which is a good aerodynamic performance category (Fig.12). In view of the visual and data results of the above experiments, we believe that design geometry can be effectively applied to the design of high-speed trains.

5. Conclusion
Design geometry studies the relationship between geometric composition and visual perception, and studies the universal visual principle. It uses the mathematical law as the mediation of the order of the visual form, and interprets the morphological law originating from nature and the reason that its derivative style as a question of the human being’s unconscious aesthetic choice [10]. With design geometry as an application guide, orderly structural relationships can effectively organize, design and express. Therefore, the design of high-speed trains requires reasonable data analysis, summarizing the linear rules behind perceptual thinking, and rationally planning the free curve by designers. This design method based on geometric and mathematical angle of view can rigorously establish the characteristics of the new generation of vehicle models; in addition, the geometrical visual proportion and rhythm law extends with the iterative process of the vehicle system, strengthening the homogenization identification system of the series models in the formation of the basis, and providing premise objects for design strategy research.

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References
[1] Innovative Design Strategy Research Group. (2016) China Innovation Design Roadmap, China Science and Technology Press, Beijing, pp. 233-238.
[2] Xu, F. (2015) China’s High-Speed Train “Going out” Strategy: the main Theme, Strategy, Measures, China Engineering Science, 2015, No. 4, pp. 4.
[3] Innovative Design Strategy Research Group. (2016) China Innovation Design Roadmap, China Science and Technology Press, Beijing, pp. 231-232.
[4] Marx: The Complete Works of Marx and Engels. (2008) People’s Publishing House, Beijing, pp. 95-97.
[5] Kimberly, I. (2003) Design Geometry, translated by Li Leshan, China Water Resources and Hydropower Press, Beijing, pp. 76-78.

[6] Le Corbusier. (2014) Towards New Architecture, translated by Yang Zhide, Jiangsu Science and Technology Press, Nanjing, pp. 65-66.

[7] Wang W. (2010) Research on the History, Theory and Application of Art Design Geometry, Journal of Nanjing University of the Arts (Art and Design Edition), No. 11, pp. 168-171.

[8] Gail H. (2003) Design Elements, translated by Li Leshan, Han Qi, Chen Zhonghua, China Water Resources and Hydropower Press, Beijing, pp. 72-74.

[9] Wang W. (2010) Research on the History, Theory and Application of Art Design Geometry, Journal of Nanjing University of the Arts (Art and Design Edition), No. 11, pp. 168-171.

[10] Kimberly, I. (2003) Design Geometry, translated by Li Leshan, China Water Resources and Hydropower Press, Beijing, pp. 32-35.