Extra Dimensions in Superstring Theory

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It was earlier shown that an SO(9,1) $\theta^\alpha$ spinor variable can be constructed from RNS matter and ghost fields. $\theta^\alpha$ has a bosonic worldsheet super-partner $\lambda^\alpha$ which plays the role of a twistor variable, satisfying $\lambda \Gamma^\mu \lambda = \partial x^\mu + i \theta \Gamma^\mu \partial \theta$. For Type IIA superstrings, the left-moving $[\theta_L^\alpha, \lambda_L^\alpha]$ and right-moving $[\theta_R^\alpha, \lambda_R^\alpha]$ can be combined into 32-component SO(10,1) spinors $[\theta^A, \lambda^A]$. This suggests that $\lambda^A \Gamma^{11}_{AB} \lambda^B = 2 \lambda^\alpha_L \lambda^\alpha_R$ can be interpreted as momentum in the eleventh direction. Evidence for this interpretation comes from the zero-momentum vertex operators of the Type IIA superstring and from consideration of $D_0$-branes. As in the work of Bars, one finds an SO(10,2) structure for the Type IIA superstring and an SO(9,1) x SO(2,1) structure for the Type IIB superstring.
1. Introduction

There is accumulating evidence that ten-dimensional superstring theory is related to a theory in eleven dimensions. Since most information about this eleven-dimensional theory comes from compactification or from low-energy analysis of supergravity, little is known about its fundamental nature. Most proposals for understanding the extra dimension introduce a new fundamental object, the supermembrane, whose double-dimensional reduction gives the Type IIA superstring.

In this paper, it will be proposed that the extra dimension can be obtained from the usual superstring theory without introducing new fundamental objects. (Although D-branes are present in the non-perturbative superstring spectrum, they are not fundamental objects in the sense that superstrings do not come from their dimensional reduction.) Since superstring theory only contains ten $x$’s, it is natural to ask where the extra dimension comes from.

In the RNS description of superstrings, one has super-worldsheet ghosts, $[b,c]$ and $[\beta,\gamma]$, which are crucial for constructing Ramond vertex operators and spacetime-supersymmetry generators. In this paper, it will be proposed that the bosonic variable for the extra dimension comes from a particular Ramond-Ramond combination of RNS matter and ghost fields. The appropriate Ramond-Ramond combination of fields is found by constructing twistor-like variables for the superstring. These twistor-like variables first appeared in the GS description of the superstring.

The standard GS description of the superstring contains fermionic Siegel symmetries rather than worldsheet supersymmetries, which has prevented a successful quantization except in light-cone gauge. However, there exists a modified GS superstring which can be quantized (although not with manifest SO(9,1) invariance) and which contains bosonic spinor variables, $\lambda^\alpha$ and $\bar{\lambda}^\alpha$, in addition to the usual GS variables, $x^\mu$ and $\theta^\alpha$. These bosonic spinors are not independent fields, but satisfy the twistor-like constraint

$$\lambda^\alpha \Gamma^\mu_{\alpha\beta} \bar{\lambda}^\beta = \partial x^\mu + \frac{i}{2} \theta^\alpha \Gamma^\mu_{\alpha\beta} \partial \theta^\beta,$$

as well as the pure spinor constraint

$$\lambda^\alpha \Gamma^\mu_{\alpha\beta} \lambda^\beta = \bar{\lambda}^\alpha \Gamma^\mu_{\alpha\beta} \bar{\lambda}^\beta = 0.$$

In this twistor version of the GS superstring, two of the eight Siegel symmetries are replaced with N=2 worldsheet supersymmetries. (Although there is also a twistor version

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1 The unusual factor of $\frac{i}{2}$ is used so that $\{q_\alpha, q_\beta\} = P_\mu \Gamma^\mu_{\alpha\beta}$ rather than $2P_\mu \Gamma^\mu_{\alpha\beta}$. 

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of the GS superstring where all eight Siegel symmetries are replaced with worldsheet supersymmetries, this N=8 twistor version of the GS superstring has not yet been quantized.) Under the N=2 worldsheet supersymmetry transformations, the $\theta^\alpha, \lambda^\alpha, \bar{\lambda}^\alpha$, and $x^\mu$ fields transform as components of the N=2 superfields

$$\Theta^\alpha = \theta^\alpha + \kappa \lambda^\alpha + \bar{\kappa} \bar{\lambda}^\alpha + \kappa \bar{\kappa} h^\alpha,$$

$$X^\mu = x^\mu + i \kappa m^\mu + i \bar{\kappa} \bar{m}^\mu + \kappa \bar{\kappa} n^\mu,$$

satisfying the twistor and pure spinor constraints:

$$\bar{i}/2 \Theta^\alpha \Gamma^\mu_{\alpha \beta} D \Theta^\beta = DX^\mu, \quad \bar{i}/2 \Theta^\alpha \Gamma^\mu_{\alpha \beta} \bar{D} \Theta^\beta = \bar{D} X^\mu, \quad (1.3)$$

where $D = d/d\kappa + \frac{i}{2} \bar{\kappa} \partial_z$, $\bar{D} = d/d\bar{\kappa} + \frac{i}{2} \kappa \partial_z$, and $f^\alpha, m^\mu, n^\mu$ are auxiliary fields.

For the Type IIA superstring, the left-moving $\Theta^\alpha_L$ carry SO(9,1) Weyl spinor indices while the right-moving $\Theta^\alpha_R$ carry SO(9,1) anti-Weyl spinor indices. This allows them to be combined into a 32-component SO(10,1) spinor superfield $\Theta^A$.

The natural higher-dimensional generalization of the twistor constraint is

$$\lambda^A \Gamma^M_{AB} \bar{\lambda}^B = P^M \quad (1.4)$$

where $P^{11}$ is defined by this constraint, i.e.

$$P^{11} = \lambda^A \Gamma^{11}_{AB} \bar{\lambda}^B = \lambda^A_L \bar{\lambda}^B_R + \lambda^A_R \bar{\lambda}^B_L. \quad (1.5)$$

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2 In this paper, Greek letters are SO(9,1) indices, capitalized Latin letters are SO(10,1) or SO(10,2) indices, and uncapitalized Latin letters are SO(2,1) indices. Letters from the first half of the alphabet denote spinor indices and letters from the second half of the alphabet denote vector indices. The SO(9,1) vector indices will take the values 0 ... 9, the SO(10,1) vector indices will take the values 0, ..., 9, 11, 12, and the SO(2,1) vector indices will take the values 0,1,2. The flat metric is $\eta_{00} = \eta_{1212} = 1$ and $\eta_{MM} = -1$ for $M = 1...9, 11$, SO(9,1) $\Gamma^\mu$ matrices are 16 × 16 and satisfy $\Gamma^\mu_{\alpha \beta} \Gamma^\nu_{\beta \gamma} = 2 \eta^\mu_\nu \delta^\alpha_\gamma$. SO(10,1) and SO(10,2) $\Gamma^M$ matrices are 32 × 32 and satisfy $\Gamma^M_{AB} \Gamma^N_{BC} = 2 \eta^{MN}_{AB} \delta^C_A$. For SO(9,1) and SO(10,2), these $\Gamma$ matrices are related to the usual 32×32 and 64×64 $\gamma$ matrices by multiplication with $\gamma^0$ and by taking the upper diagonal quadrant. For SO(10,1), they are related to the usual 32×32 $\gamma$ matrix by multiplication with $\gamma^0$. In other words, $\Gamma^M_{AB} = (\gamma^0 \gamma^M)_{AB}^B$ and $\Gamma^M_{AB} = (\gamma^M \gamma^0)_{AB}^A$. The explicit representation for $\Gamma^M$ will be $\Gamma^0_{AB} = \delta_{AB}, \Gamma^M_{AB} = \sigma_3 \times \Gamma^M_{\alpha \beta}$ for $M = 1...9, \Gamma^{11}_{AB} = \sigma_1 \times 1_{16}$, and $\Gamma^{12}_{AB} = i \sigma_2 \times 1_{16}$ where $\sigma^j$ are the Pauli matrices and $1_{16}$ is the 16×16 identity matrix. Note that $\Gamma^M_{AB} = \Gamma^M_{BA}$ except when $M = 12$, and $\Gamma^{12}_{AB} = -\Gamma^{12}_{BA}$. 

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In fact, one can also interpret $\lambda^A$ and $\bar{\lambda}^B$ as SO(10,2) Majorana Weyl and Majorana anti-Weyl spinors, in which case $P^{12} = \lambda^\alpha_L \bar{\lambda}_{R\alpha} - \lambda_{R\alpha} \bar{\lambda}^\alpha_L$.

To translate this into RNS language (where covariant quantization is known), one needs to find the combination of RNS matter and ghost fields which corresponds to $\lambda^\alpha$ and $\bar{\lambda}^\alpha$. Fortunately, the dictionary between RNS fields and twistor-GS fields was found in reference [9] where it was shown how to explicitly construct $\lambda^\alpha$ and $\bar{\lambda}^\alpha$ in terms of the RNS matter and ghost fields. It was also shown in this reference that the fermionic $N=2$ superconformal generators of the twistor-GS superstring are mapped in RNS language into the RNS BRST current and the $b$ ghost. So the twistor variables, $\lambda^\alpha$ and $\bar{\lambda}^\alpha$, are obtained in RNS language by anticommuting the $\theta^\alpha$ variable with the BRST charge and with the $b$ ghost.

In section 2, the dictionary between the RNS and twistor-GS variables is reviewed. The twistor-like variables, $\lambda^A$ and $\bar{\lambda}^A$, are explicitly constructed in terms of RNS matter and ghost fields.

In section 3, the identification of $P^{11}$ with $\frac{1}{32}(\lambda^\alpha_L \bar{\lambda}_{R\alpha} + \lambda_{R\alpha} \bar{\lambda}^\alpha_L)$ is justified by analyzing zero-momentum vertex operators for massless states of the Type IIA superstring, which correspond to the zero-momentum spectrum of D=11 supergravity. For NS-NS states, these vertex operators are well-known, but for R-R states, these vertex operators are new and are constructed using the R-R sector of closed superstring field theory. [12] Although it is often stated that the R-R vertex operator vanishes at zero momentum, this is not completely true. It will be shown that the zero-momentum R-R vertex operator is BRST-equivalent to an operator of ghost-number $(1+2n, 1-2n)$ where $n$ is arbitrarily large. This allows the construction of a field theory action for the massless R-R string fields [12] [13] and also implies that all vertex operators with finite ghost-number must decouple from the zero-momentum R-R vertex operators. The structure of the zero-momentum R-R

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3 Recently, Dimitri Polyakov has expressed related ideas. [10] [11] However, there are some crucial differences between our approaches. Firstly, he defines his twistor variable, $\lambda^\alpha$, as the anticommutator of $\theta^\alpha$ with the $N=1$ RNS superconformal generator. Therefore, his definition of $\lambda^\alpha$ is not GSO-projected, i.e. it has square-root cuts with the spacetime-supersymmetry generators. Secondly, he only considers left-moving twistor variables so there is no analog of $\lambda^\alpha_L \lambda_{R\alpha}$. Although he claims in [11] that the anticommutator of left-moving spacetime-supersymmetry generators in the +1/2 picture has a five-form central charge proportional to $\Gamma^{\mu_1 \cdots \mu_5} \psi_{\mu_1} \cdots \psi_{\mu_5}$, his calculation appears to be incorrect. My calculation of the five-form term in this anticommutator gives something proportional to $\Gamma^\nu \Gamma^{\mu_1 \cdots \mu_5} \Gamma^\nu$, which vanishes in ten dimensions.
vertex operator suggests that the general $p$-brane R-R charge can be constructed from RNS variables in a manner similar to the zero-brane charge $\frac{1}{32} \oint d\sigma \lambda \Gamma^{11} \bar{\lambda}$.

Actually, sigma model arguments imply that it is the R-R gauge field times the exponential of the dilaton, $e^\phi$, which couples to these zero-momentum R-R vertex operators so $P_{11}$ should really be identified with $\frac{1}{32} e^\phi (\lambda \Gamma^{11} \bar{\lambda})$. In section 4, $D_0$-branes are shown to be massless in eleven dimensions if the zero-brane charge, $P_{11}$, is identified with $\frac{1}{32} e^\phi (\lambda \Gamma^{11} \bar{\lambda})$.

In section 5, these techniques are generalized to the Type IIB superstring where the SO(10,2) structure is replaced by an SO(9,1)$\times$SO(2,1) structure. These SO(10,2) and SO(9,1)$\times$SO(2,1) structures were also found by Bars in reference [14].

Finally, in section 6, some connections are made with other proposals to understand the eleventh dimension.

2. Construction of twistor variables

2.1. Review of GS - RNS dictionary

In the RNS description of the D=10 superstring, the spacetime supersymmetry generator

$$q_\alpha = \oint dz \, e^{-\phi/2} \Sigma_\alpha$$

satisfies the algebra \{\(q_\alpha\), \(q_\beta\)\} = $\oint dz \, e^{-\phi} \psi_\mu \Gamma^\mu_{\alpha\beta}$ where $\Sigma_\alpha$ is the Ramond spin field of weight 5/8, and the $\beta$ and $\gamma$ worldsheet ghosts have been fermionized as $\beta = \partial \xi e^{-\phi}$ and $\gamma = \eta e^\phi$. Although $e^{-\phi} \psi_\mu$ is related by picture-changing to the momentum operator $\partial x^\mu$, this is not good enough for manifest spacetime supersymmetry since picture-changing is only an on-shell operation.

One therefore needs to introduce a second spacetime-supersymmetry generator

$$\bar{q}_\alpha = \oint dz (e^{\phi/2} \Sigma^\beta \partial x_\mu \Gamma^\mu_{\alpha\beta} + b \eta e^{3\phi/2} \Sigma_\alpha)$$

which is BRST invariant and is related to $q_\alpha$ by picture-changing. (Note that $\bar{q}_\alpha$ is Majorana-Weyl and is not the complex conjugate of $q_\alpha$.) It is easy to check that \{\(q_\alpha\), \(\bar{q}_\beta\)\} = $\Gamma^\mu_{\alpha\beta} \oint dz \, \partial x_\mu$ as desired.

However, since \{\(q_\alpha\), \(q_\beta\)\} does not vanish, this is not a standard N=2 D=10 supersymmetry algebra. Nevertheless, it will be useful to define two spinor variables, $\theta^\alpha$ and

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4 For compactifications to four dimensions which preserve N=1 D=4 supersymmetry, one can choose the two chiral N=1 D=4 supersymmetry generators to come from $q_\alpha$ and the two anti-chiral supersymmetry generators to come from $\bar{q}_\alpha$. In this case, \{\(q_\alpha\), \(q_\beta\)\} = 0 which allows a formulation of the superstring with manifest SO(3,1) super-Poincaré invariance.
$\bar{\theta}^{\alpha}$, which satisfy the anti-commutation relations $\{q_{\alpha} , \theta^{\beta}\} = \{\bar{q}_{\alpha} , \bar{\theta}^{\beta}\} = \delta_{\alpha}^{\beta}$.

These are easily found to be

$$\theta^{\alpha} = e^{\phi/2} \Sigma^{\alpha}, \quad \bar{\theta}^{\alpha} = c \xi e^{-3\phi/2} \Sigma^{\alpha}. \quad (2.3)$$

Note that $\bar{\theta}^{\alpha}$ involves the $\xi$ zero mode, which is necessary for preserving manifest spacetime supersymmetry.

As shown in reference [9], the N=2 worldsheet superconformal generators in the twistor-GS formalism are mapped into the following RNS expressions:

$$T = \frac{1}{2} \partial_{x_{\mu}} \partial x^{\mu} + \frac{i}{2} \psi_{\mu} \partial \psi^{\mu} + 2ib\partial c - ic\partial b + i\eta\partial \xi + \frac{1}{2} \partial \phi \partial \phi + \partial^{2} \phi - \frac{i}{2} \partial (bc + \xi \eta),$$

$$G = \eta e^{\phi} \psi^{\mu} \partial x_{\mu} + i\eta \partial \eta e^{2\phi} b + \partial (ic \xi \eta + \partial c)$$

$$+ c(\frac{1}{2} \partial_{x_{\mu}} \partial x^{\mu} + \frac{i}{2} \psi_{\mu} \partial \psi^{\mu} + ib\partial c + i\eta\partial \xi + \frac{1}{2} \partial \phi \partial \phi + \partial^{2} \phi),$$

$$\bar{G} = b, \quad (2.4)$$

$$J = cb + \eta \xi.$$  

These generate a $c = 6$ N=2 superconformal algebra and, after redefining $T \rightarrow T - \frac{i}{2} \partial J$, form a set of twisted N=2 generators whose $T$ is the standard RNS stress-energy tensor, $G$ is the BRST current, $\bar{G}$ is the $b$ ghost, and $J$ is the RNS ghost-number current. (Although the RNS ghost-number charge is usually defined by $\oint dz (cb - i\partial \phi)$, this agrees with $\oint dz J$ at zero picture, i.e. when $\oint dz (\eta \xi + i\partial \phi) = 0$.)

It is natural to ask how the $x^{\mu}$, $\theta^{\alpha}$ and $\bar{\theta}^{\alpha}$ variables transform under commutation with the above generators. One finds that $\{\theta^{\alpha} , \oint dz \bar{G}\} = 0$ so $\theta^{\alpha}$ is the lowest component of an N=2 chiral superfield, $\Theta^{\alpha} = \theta^{\alpha} + \kappa \lambda^{\alpha} + \frac{i}{2} \kappa \bar{k} \partial \theta^{\alpha}$ where

$$\lambda^{\alpha} = \oint dz \ G , \ \theta^{\alpha} = \eta e^{3\phi/2} \partial x^{\mu} \Sigma^{\beta}_{\mu} \Gamma^{\alpha \beta} + b \eta \partial \eta e^{5\phi/2} \Sigma^{\alpha} + c \partial (e^{\phi/2} \Sigma^{\alpha}), \quad (2.5)$$

$D = d/d\kappa + \frac{i}{2} \kappa \bar{k} \partial_{\zeta}$ and $D = d/d\bar{k} + \frac{i}{2} \kappa \partial_{\zeta}$. Similarly, $\{\bar{\theta}^{\alpha} , \oint dz G\} = 0$ so $\bar{\theta}^{\alpha}$ is the lowest component of an N=2 anti-chiral superfield, $\bar{\Theta}^{\alpha} = \bar{\theta}^{\alpha} + \bar{k} \bar{\lambda}^{\alpha} - \frac{i}{2} \kappa \bar{k} \partial \bar{\theta}^{\alpha}$ where

$$\bar{\lambda}^{\alpha} = \oint dz \ G , \ \bar{\theta}^{\alpha} = \xi e^{-3\phi/2} \Sigma^{\alpha}. \quad (2.6)$$

Finally, $[x^{\mu} , \oint dz \bar{G}] = 0$ implies that $x^{\mu}$ is the lowest component of an N=2 chiral superfield

$$X^{\mu} = x^{\mu} + \kappa \lambda^{\mu} + \frac{i}{2} \kappa \bar{k} \partial x^{\mu}$$

where $\chi^{\mu} = [\oint dz \ G , \ x^{\mu}] = \eta e^{\phi} \psi^{\mu}$. 

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Using the usual RNS OPE’s, these superfields can be shown to satisfy the constraint

\[ i(D\Theta^\alpha)\bar{\Theta}^\beta = \Gamma^\alpha_\beta DX^\mu, \quad (2.7) \]

which implies the twistor-like condition\[16\][9]

\[ \lambda^\alpha \bar{\lambda}^\beta = \Gamma^\alpha_\beta \partial x^\mu - i(\partial \theta^\alpha)\bar{\theta}^\beta. \quad (2.8) \]

After defining \( \hat{X}^\mu = X^\mu - \frac{i}{32} \Theta \Gamma^\mu \bar{\Theta} \), the constraint of (2.7) can be rewritten as

\[ \frac{i}{2} (D\Theta^\alpha)\bar{\Theta}^\beta = \Gamma^\alpha_\beta D\hat{X}^\mu, \quad \frac{i}{2} (\bar{D}\bar{\Theta}^\alpha)\Theta^\beta = \Gamma^\alpha_\beta \bar{D}\hat{X}^\mu, \quad (2.9) \]

which resembles the twistor-GS constraint of (1.3).

However, the RNS constraint of (2.8) has 256 components, rather than the 10 components of (1.1), and there is no pure spinor constraint. Furthermore, there are two spinor variables, \( \theta^\alpha \) and \( \bar{\theta}^\alpha \), in the RNS approach while there is only one spinor variable in the twistor-GS approach. These differences come from the fact that the twistor-GS superstring has six Siegel symmetries in addition to the two worldsheet supersymmetries. The equivalence between the RNS and twistor-GS formalisms has only been proven after gauge-fixing these six Siegel symmetries by setting six of the components of \( \Theta^\alpha_{GS} \) to zero. In this non-covariant gauge, the remaining ten components of \( \Theta^\alpha_{GS} \) split into two pure spinors, one of which is an N=2 chiral superfield identified with \( \Theta^\alpha_{RNS} \), and the other is an N=2 anti-chiral superfield which is identified with \( \bar{\Theta}^\alpha_{RNS} \). It is then straightforward to prove the equivalence of the two formalisms.[9]

For the rest of this paper, only the RNS formalism will be discussed.

2.2. Construction of the extra dimension

For the Type IIA superstring, one can construct left and right-moving superfields, \( (\Theta^\alpha_L, \bar{\Theta}^\alpha_L) \) and \( (\Theta^\alpha_R, \bar{\Theta}^\alpha_R) \), which carry Weyl and anti-Weyl SO(9,1) spinor indices. They can therefore be combined into 32-component superfields \( (\Theta^A, \bar{\Theta}^A) \) which transform as SO(10,1) Majorana spinors. In fact, one can also interpret them as 32-component SO(10,2) spinors where \( \Theta^A \) transforms as a Majorana Weyl spinor and \( \bar{\Theta}^A \) transforms as a Majorana anti-Weyl spinor. With this choice of SO(10,2) chirality, \( \Theta^A \Gamma^M_{AB} \bar{\Theta}^B \) transforms as an SO(10,2) vector.
So an obvious generalization of (2.4) is

\[ \frac{i}{16} (D\Theta^A) \Gamma_{AB}^M \bar{\Theta}^B = DX^M, \]  

(2.10)

where \( D = d/d\kappa + i\bar{\kappa} \partial_\tau \) and \( \bar{D} = d/d\bar{\kappa} + i\kappa \partial_\tau, \partial_\tau = \frac{1}{2}(d/dz_L + d/dz_R) = \frac{1}{2}(\partial_L + \partial_R), \partial_\sigma = \frac{1}{2}(\partial_L - \partial_R), \Theta^A = \theta^A + \kappa \lambda^A + i\kappa \bar{\kappa} \partial_\tau \theta^A, \bar{\Theta}^A = \bar{\theta}^A + \bar{\kappa} \lambda^A - i\kappa \bar{\kappa} \partial_\tau \bar{\theta}^A, \) and \( X^M \) is a chiral \( N=2 \) superfield defined by (2.10), i.e.

\[ \partial_\tau (x^{11} + x^{12}) = \frac{1}{16} (\lambda_L^\alpha \bar{\lambda}_{R\alpha} + 2i(\partial_\tau \theta_L^\alpha) \bar{\theta}_{R\alpha}), \]  

(2.11)

\[ \partial_\tau (x^{11} - x^{12}) = \frac{1}{16} (\lambda_{R\alpha} \bar{\lambda}_L^\alpha + 2i(\partial_\tau \theta_{R\alpha}) \bar{\theta}_L^\alpha). \]  

The \( N=2 \) worldsheet supersymmetry generators are now the sum of the left-moving and right-moving \( N=2 \) superconformal generators of (2.4). (This is consistent with the definition of \( D \) and \( \bar{D} \) since \( \partial_\tau \Theta^\alpha_L = \frac{1}{2} \partial_L \Theta^\alpha_L \) and \( \partial_\tau \Theta^\alpha_R = \frac{1}{2} \partial_R \Theta^\alpha_R \).) For \( M = 0 \) to 9, \( x^M \) is easily seen to be defined in the same way as in (2.8), i.e.

\[ \partial_\tau x^\mu = \frac{1}{32} (\lambda_L^{\mu} \bar{\lambda}_L + \lambda_R^{\mu} \bar{\lambda}_R + i(\partial_L \theta_L) \Gamma^{\mu} \bar{\theta}_L + i(\partial_R \theta_R) \Gamma^{\mu} \bar{\theta}_R). \]  

(2.12)

Note that the stronger condition, \( i(D\Theta^A) \bar{\Theta}^B = \Gamma_{AB}^M DX^M \) cannot be correct since \( \lambda_L^\alpha \bar{\lambda}_{R\beta} \) is not proportional to \( \delta^\alpha_\beta \). Also note that the RNS definition of

\[ P_M = \frac{1}{32} \lambda^A \Gamma_{AB}^M \bar{\lambda}^B \]  

(2.13)

differs by a factor of 32 from the GS definition of (1.4).

In the following two sections, the above identification of \( \partial_\tau x^{11} \) will be justified using arguments based on superstring vertex operators and on \( D_0 \)-branes.

3. Justification based on Type IIA zero-momentum vertex operators

3.1. Zero-momentum NS-NS vertex operators

The zero-momentum states of the \( D=10 \) Type IIA superstring match the zero-momentum states of \( D=11 \) supergravity. Under compactification on a circle, the \( D=11 \) graviton decomposes into a \( D=10 \) graviton, dilaton, and graviphoton, and the \( D=11 \) three-form decomposes into a \( D=10 \) three-form and two-form. Since the zero-momentum graviton, \( g_{\mu\nu} \), has vertex operator \( \int d^2z \partial_\tau x^\mu \partial_\tau x^\nu \) (ignoring the \( \partial_\tau x^\mu \) dependence), one might
expect the vertex operators of the zero-momentum dilaton and graviphoton, $\phi$ and $A_\mu$, to be related to $\int d^2z \, (\partial_\tau x^{11})^2$ and $\int d^2z \, \partial_\tau x^{11} \partial_\tau x^\mu$. 

Plugging in the definition of (2.11) for $\partial_\tau x^{11}$ (and ignoring the $\theta$ dependence), one finds

$$\int d^2z \, (\partial_\tau x^{11})^2 = \frac{1}{20} \int d^2z \, (\lambda^\alpha_L \bar{\lambda}_R^\alpha + \lambda_R^\alpha \bar{\lambda}_L^\alpha)^2, \quad (3.1)$$

which is not a simple expression in terms of RNS free fields. However, consider instead

$$\int d^2z \, [(\partial_\tau x^{11})^2 - (\partial_\tau x^{12})^2] = \frac{1}{28} \int d^2z \, \lambda^\alpha_L \bar{\lambda}_R^\alpha \lambda_R^\beta \bar{\lambda}_L^\beta. \quad (3.2)$$

Using the identity of (2.8) (and ignoring the $\theta$ dependence), one finds

$$\int d^2z \, [(\partial_\tau x^{11})^2 - (\partial_\tau x^{12})^2] = \frac{1}{16} \int d^2z \, [\partial_L x^\mu \partial_R x_\mu], \quad (3.3)$$

which is proportional to the zero-momentum dilaton vertex operator in integrated form.\footnote{In unintegrated form, there are two physical zero-momentum dilaton vertex operators, $c_L \partial_L x^\mu c_R \partial_R x_\mu$ and $c_L \partial^2_L c_L + (\eta_L e^{\phi_L}) \partial_L (\eta_L e^{\phi_L}) = \{Q_L, \partial_L c_L\}$.\cite{17} The integrated form is obtained by anti-commuting the unintegrated form with $\int dz_L b_L$ and $\int dz_R b_R$, so the second type of “ghost” dilaton vertex operator decouples in the absence of worldsheet curvature (worldsheet curvature can mix $b_L$ with $b_R$). Note that $\int d^2z \, [(\partial_\tau x^{11})^2 - (\partial_\tau x^{12})^2]$ needs to be normal-ordered in the presence of worldsheet curvature. It would be interesting to see if this normal-ordering procedure is somehow related to the “ghost” dilaton vertex operator.}

So with the twistor definition of $\partial_\tau x^{11}$ and $\partial_\tau x^{12}$, the dilaton appears to be related to the $(11,11) - (12,12)$ components of a twelve-dimensional graviton. (The $(11,11)+(12,12)$ component appears not to have a simple string interpretation.) This suggests that the Type IIA dilaton measures the volume of the torus which compactifies from $10 + 2$ to $9 + 1$ dimensions.

### 3.2. Zero-momentum R-R vertex operators

It is commonly stated that R-R gauge fields decouple from strings at zero momentum. This statement is based on three arguments: 1) The standard massless R-R vertex operator vanishes at zero momentum; 2) There are no coupling terms of the appropriate dimension in the standard GS sigma model; 3) No perturbative superstring states carry R-R charge.

However, if the above statement were true, it would be impossible to construct a superstring field theory action in the R-R sector since there is no Maxwell action without
gauge fields. In recent papers \[12\][13], such an action was constructed, and it will now be explained how superstring field theory solves this problem without violating the above three arguments.

The superstring field theory action comes from a $<\Phi Q\Phi>$ action where $\Phi$ is the superstring field and, for the massless Type IIA R-R sector, $\Phi$ contains infinite copies of four bispinor fields: $C^\beta_{(n)\alpha}, D_{(n)\alpha\beta}, E_{(n)\alpha\beta}, F^\alpha_{(n)\beta}$ for $n=0$ to $\infty$. (The infinite copies come from the dependence of $\Phi$ on the $\beta, \gamma$ zero modes.) The action for these fields can be found in \[12\][13], and it was shown that all the $C^\beta_{(n)\alpha}$ fields and all but one of the $D_{(n)\alpha\beta}$ and $E^\alpha_{(n)\beta}$ fields can be gauged away. The equations of motion in this gauge are

$$F^\alpha_{(0)\beta} = \partial_\mu (D_{\gamma\beta} \Gamma^\mu_{\alpha\gamma} - E^{\alpha\gamma} \Gamma^\mu_{\gamma\beta}),$$  \hspace{1cm} (3.4)

$$\Gamma^\mu_{\alpha\gamma} \partial_\mu F^\alpha_{(0)\beta} = -\Gamma^\gamma_{\alpha\beta} \partial^\mu F^\alpha_{(0)\beta} = 0, \quad F^\alpha_{(n)\beta} = 0 \text{ for } n > 0.$$  

Note that $F^\alpha_{(0)\beta}$ is an auxiliary field which satisfies Bianchi identities only on-shell.

Although this superstring field theory action was constructed using “non-minimal” RNS fields\[18], one can analyze the vertex operators for the gauge fields, $D_{\alpha\beta}$ and $E_{\alpha\beta}$, using the usual minimal set of RNS fields. For simplicity, these vertex operators will be analyzed at zero momentum.

Consider the following R-R vertex operator in unintegrated form:

$$V^{\alpha\beta}_{(0)} = c_L e^{-3\phi_L/2} \Sigma^\alpha_L c_R e^{-\phi_R/2} \Sigma^\beta_R.$$  \hspace{1cm} (3.5)

This operator is naively BRST-trivial since $V^{\alpha\beta}_{(0)} = [Q_L + Q_R , (\partial_L c_L) W^{\alpha\beta}_{(0)}]$ where

$$W^{\alpha\beta}_{(0)} = c_L \partial_L \xi_L \partial^2_L \xi_L e^{-7\phi_L/2} \Sigma^\alpha_L c_R e^{-\phi_R/2} \Sigma^\beta_R.$$  \hspace{1cm} (3.6)

However, since $(b^0_L - b^0_R) \partial_L c_L W^{\alpha\beta}_{(0)} \neq 0$, $V_{(0)}$ is not BRST-trivial but is in the same semi-relative cohomology class as $[Q_L + Q_R , (\partial_R c_R) W^{\alpha\beta}_{(0)}] = V^{\alpha\beta}_{(1)}$ where

$$V^{\alpha\beta}_{(1)} = c_L \partial_L \xi_L \partial^2_L \xi_L e^{-7\phi_L/2} \Sigma^\alpha_L c_R e^{3\phi_R/2} \eta_R \partial_R \eta_R \Sigma^\beta_R.$$  \hspace{1cm} (3.7)

$(b^0_L - b^0_R$ signifies the zero mode of $b_L - b_R$ and semi-relative cohomology is defined in \[19\].)

Similarly, $V^{\alpha\beta}_{(1)}$ is not BRST-trivial, but is in the same cohomology class as

$$V^{\alpha\beta}_{(2)} = c_L \partial_L \xi_L \partial^2_L \xi_L \partial^3_L \xi_L \partial^4_L \xi_L e^{-11\phi_L/2} \Sigma^\alpha_L c_R e^{7\phi_R/2} \eta_R \partial_R \eta_R \partial^2_R \eta_R \partial^2_R \eta_R \Sigma^\beta_R.$$  

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6 I would like to thank Sanjaye Ramgoolam for pointing this out to me.
This chain continues forever, so \( V_{(0)}^{\alpha\beta} \) is in the same BRST cohomology class as \( V_{(n)}^{\alpha\beta} \) for arbitrarily large \( n \) where \( V_{(n)}^{\alpha\beta} \) carries ghost number \((1 - 2n, 1 + 2n)\). (The ghost-number is defined by commuting with \([J_L, J_R]\) of (2.4).) Also,

\[
V_{(0)}^{\alpha\beta} = c_L e^{-\phi_L/2} \Sigma_L c_R e^{-3\phi_R/2} \Sigma_R
\]

(3.8)
is in the same BRST cohomology class as \( V_{(n)}^{\alpha\beta} \) for arbitrarily large \( n \) where \( V_{(n)}^{\alpha\beta} \) carries ghost number \((1 + 2n, 1 - 2n)\).

Since these vertex operators, \( V_{(n)}^{\alpha\beta} \) and \( V_{(n)}^{\alpha\beta} \), have the same BRST structure as the string field for \( D_{(n)}^{\alpha\beta} \) and \( E_{(n)}^{\alpha\beta} \) in [12] [13], they will be conjectured to be equivalent. (This is a conjecture since it is not yet known how to construct a superstring field theory action without introducing “non-minimal” RNS fields.)

So zero-momentum R-R vertex operators are present in superstring field theory and avoid violating the above three arguments for the following three reasons: 1) The standard R-R vertex operator, \( c_L e^{-\phi_L/2} \Sigma_L c_R e^{-3\phi_R/2} \Sigma_R \), is actually the vertex operator for the auxiliary field \( F_{(0)}^{\alpha\beta} \) and not for the gauge field. Only on-shell, this auxiliary field is the field-strength for the gauge field; 2) Because \( V_{(0)}^{\alpha\beta} \) is BRST-equivalent with vertex operators of non-vanishing \( J_L - J_R \) ghost-number, it appears BRST-trivial in the standard GS formalism where ghosts are not yet understood. The confusion about ghosts in the standard GS formalism is probably related to the absence of a Fradkin-Tseytlin term in the standard GS sigma model since the zero-momentum “ghost” dilaton is also described by a vertex operator of non-vanishing \( J_L - J_R \) ghost number; 3) Since all perturbative superstring states can be described by vertex operators of finite \( J_L - J_R \) ghost number, they do not couple to the zero-momentum R-R fields. However, \( D \)-branes are described by boundary states which contain all possible \( J_L - J_R \) ghost numbers, allowing them to couple to zero-momentum R-R fields.

The next step in the analysis is to write the zero-momentum vertex operators, \( V_{(n)}^{\alpha\beta} \) and \( V_{(n)}^{\alpha\beta} \), in integrated form. RNS unintegrated vertex operators carry negative picture (e.g., \( V = c_L e^{-\phi_L} \psi_L^{\mu} c_R e^{-\phi_R} \psi_R^{\mu} \) for the graviton), so one first needs to perform a picture-raising operation before anti-commuting with \( \int dz_L b_L \) and \( \int dz_R b_R \). Because the rule for picture-raising comes from a complicated closed superstring field theory argument, its justification will be left for a separate paper. In this paper, it will be enough to know that the “picture-raised” version of \( V_{(n)}^{\alpha\beta} \) is given by multiplying \( V_{(n)}^{\alpha\beta} \) with \( (\partial_L c_L - \partial_R c_R) \), then multiplying with \( \xi_L \) and taking the second-order pole with \( b_R \xi_R \), and finally commuting
with $Q_R$ and with $Q_L$. $(c_L^R - c_R^L$ removes the $b_L^0 - b_R^0$ constraint, $Q_L \xi_L^0$ is the picture-raising operator for fields, and $Q_R (b_R \xi_R)^0$ is the picture-raising operator for anti-fields.) For the usual states of ghost-number $(1,1)$, this is equivalent to multiplying with $\xi_L \xi_R$ and then commuting with $Q_L$ and $Q_R$.

For $V_{(0)}^{\alpha\beta}$ of (3.3), multiplication with $\xi_L \xi_R$ gives

$$\xi_L \xi_R V_{(0)}^{\alpha\beta} = c_L \xi_L e^{-3\phi_L/2} \xi_R c_R e^{-\phi_R/2} \Sigma_R = \frac{1}{160} \bar{\theta}_L^\alpha (\theta_R^\mu \bar{\theta}_R^\gamma) \Gamma_{\mu}^{\beta,\gamma} \theta_R$$

(3.9)

where $\theta^\alpha$ and $\bar{\theta}^\alpha$ are defined in (2.3). Since $\{Q_L, \bar{\theta}_L^\alpha\}=0$, there is no integrated vertex operator associated with $V_{(0)}^{\alpha\beta}$. For this reason, there is no candidate for a zero-momentum R-R vertex operator in the standard GS sigma model.

However, one can also ask what is the integrated form of the zero-momentum R-R vertex operators $V_{(n)}^{\alpha\beta}$ for $n > 0$. In fact, just as the “ghost” dilaton is necessary for preserving manifest reparameterization invariance, the “ghost” version of the R-R field is necessary for preserving manifest spacetime supersymmetry. This is easiest to see in D=4 Type II superspace effective actions$^{[21]}$ where manifest N=2 D=4 supersymmetry requires the graviphoton field to appear both in the supergravity multiplet (whose vertex operator is $V_{(0)}^{\alpha\beta}$) and in the vector compensator multiplet (whose vertex operator is $V_{(1)}^{\alpha\beta}$).

In other words, fixing $D_{(1)}^{\alpha\beta} = E_{(1)}^{\alpha\beta} = 0$ gauge-fixes part of the super-reparameterization invariances, so one needs to keep $D_{(0)}^{\alpha\beta}, E_{(0)}^{\alpha\beta}$ and $D_{(1)}^{\alpha\beta}, E_{(1)}^{\alpha\beta}$ in the action if one wants to preserve manifest spacetime supersymmetry. It is unclear at the moment if one can gauge away the $D_{(n)}^{\alpha\beta}$ and $E_{(n)}^{\alpha\beta}$ fields for $n > 1$ without breaking manifest spacetime supersymmetry.

After performing the picture-raising operation and anti-commuting with $\int dz_L b_L$ and $\int dz_R b_R$, the integrated form of $V_{(1)}^{\alpha\beta}$ is

$$\{ \int dz_L b_L , c_L \xi_L e^{-3\phi_L/2} \Sigma_L \} \{ \int dz_R b_R , (b_R c_R + \partial R \eta_R) e^{3\phi_R/2} \Sigma_R \}$$

$$= \int dz_L \xi_L e^{-3\phi_L/2} \Sigma_L \{ Q_R , (b_R c_R + \partial R \eta_R) e^{3\phi_R/2} \Sigma_R \}$$

$$= \int dz_L \xi_L e^{-3\phi_L/2} \Sigma_L \{ Q_R , (b_R e^{3\phi_R/2} \Sigma_R \partial_R x^\mu \Gamma^{\beta,\gamma}_R \}$$

7 Upon compactification to D=4, this vertex operator becomes $\bar{\theta}_L^\alpha (\theta_R^\mu \bar{\theta}_R^\gamma) \theta_R^\beta$, which is the vertex operator for the graviphoton in the N=2 D=4 supergravity multiplet.$^{[15][21]}$
\[ = \int d^2z \tilde{\lambda}_L^\alpha \{ Q_R, \theta_R \gamma \partial_R x^\mu \Gamma_\mu^{\beta \gamma} \} \]
\[ = \int d^2z \tilde{\lambda}_L^\alpha \lambda_{R\gamma} \partial_R x^\mu \Gamma_\mu^{\beta \gamma} \]

plus terms which are independent of \( x^\mu \), where it was used that \( Q_R \) anti-commutes with \( \tilde{q}_R^\beta \) of (2.2).

This integrated vertex operator is BRST-trivial since it can be written as the anti-commutator of \( Q_L + Q_R \) with \( \int d^2z \tilde{\lambda}_L^\alpha \theta_R \gamma \partial_R x^\mu \Gamma_\mu^{\beta \gamma} \). (Note that \([Q_L, \int dz \tilde{\lambda}_L^\alpha] = \int dz \partial_L \tilde{\theta}_L^\alpha = 0\).) Nevertheless, it is not identically zero, which means there should be a term in the 2D sigma model of the form

\[ N \int d^2z (D_{(1)}\alpha \beta \tilde{\lambda}_L^\alpha \lambda_{R\gamma} \partial_R x^\mu \Gamma_\mu^{\beta \gamma} + E^{\alpha \beta}_{(1)} \lambda_L^\gamma \partial_L x^\mu \Gamma_\mu^{\alpha \gamma} \tilde{\lambda}_R^\beta) \]

(3.10)

where \( N \) is an as yet undetermined normalization factor.

As discussed in [12][13] and as implied by the equations of motion for \( F_{(0)\beta}^\alpha \) in (3.2), the graviphoton gauge field \( A_\mu \) is given by \( A_\mu = D_{\alpha \beta} \Gamma_\mu^{\alpha \beta} + E^{\alpha \beta} \Gamma_\mu^{\alpha \beta} \). So its vertex operator has a term \( N \int d^2z (\lambda^A \Gamma_{AB}^{11} \tilde{\lambda}_B) \partial_T x^\mu \) suggesting that \( N \int d\sigma \lambda^A \Gamma_{AB}^{11} \tilde{\lambda}_B \) can be associated with the zero-brane R-R charge. This agrees with the proposal of (2.13) if \( N = \frac{1}{32} \).

Furthermore, the R-R three-form gauge field \( A_{\mu \nu \rho} \) is given by \( A_{\mu \nu \rho} = D_{\alpha \beta} \Gamma_{\mu \nu \rho}^{\alpha \beta} + E^{\alpha \beta} \Gamma_{\mu \nu \rho}^{\alpha \beta} \), so its vertex operator contains the term \( \frac{1}{32} \int d^2z (\lambda^A \Gamma_{AB}^{12} \tilde{\lambda}_B) \partial_T x^\rho \) suggesting that the two-brane R-R charge can be identified with \( \int d\sigma \lambda^A \Gamma_{AB}^{12} \tilde{\lambda}_B \). This fits beautifully with the D=11 interpretation since the NS-NS two-form gauge field \( B_{\mu \nu} \) has the vertex operator \( \int d^2z \theta_T x^{[\mu} \partial_T x^{\nu]} = \frac{1}{32} \int d^2z (\lambda^A \Gamma_{AB}^{12} \tilde{\lambda}_B) \partial_T x^\rho \), and the one-brane NS-NS charge is \( \frac{1}{32} \int d\sigma \theta_T x^\mu = \int d\sigma \lambda^A \Gamma_{AB}^{12} \tilde{\lambda}_B \). (\( \theta^\alpha \) dependence is being ignored in this comparison.)

Also, one can identify the four-brane R-R charge with \( \int d\sigma \lambda^A \Gamma_{AB}^{11} \tilde{\lambda}_B \). Note that all of these R-R charges are BRST-trivial, but are not identically zero.

### 3.3. 2D sigma model and \( e^\phi \) dependence

Actually, in the 2D sigma model, \( \frac{1}{32} \int d^2z (\lambda \Gamma^{11} \tilde{\lambda}) \partial_T x^\mu \) must couple to \( e^\phi A_\mu \), rather than simply \( A_\mu \). (\( e^\phi \) is the string coupling constant.) The reason is that the tree-level effective action for \( A_\mu \) is \( \frac{1}{4} \int d^{10}x \partial_{[\mu} A_{\nu]} \partial^{\mu} A_{\nu]} \) with no \( e^{-2\phi} \) factor, so \( e^\phi \) needs to appear

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\[ \text{8 The vertex operator for } A_\mu \text{ also has a term proportional to } \int d^2z (\lambda^A \Gamma^{11}_{AB} \tilde{\lambda}_B) \partial_T x^\mu, \text{ but this term does not seem to have a higher-dimensional interpretation.} \]
with $A^\mu$ in order to cancel the overall $e^{-2\phi}$ which comes at string tree-level from the Fradkin-Tseytlin term $\int d^2z \phi R$. Note that this $e^\phi$ dependence is invisible when expanding the 2D sigma model to first order around a flat D=11 supergravity background (since $e^\phi = (g_{1111})^{3/4} = 1$ in a flat D=11 background).

Since the zero-brane charge (or equivalently $P_{11}$) is given by $\delta S/\delta \Lambda$ where $S$ is the sigma model action and $\delta A_\mu = \partial_\mu \Lambda$, this suggests that $P_{11} = \frac{1}{32} e^\phi (\lambda \Gamma^{11} \bar{\lambda})$.

### 4. Justification based on $D_0$-brane analysis

One of the most compelling arguments for an eleven-dimensional origin of superstring theory is that $D_0$-branes can be interpreted as Kaluza-Klein states coming from compactification on a circle of D=11 supergravity states. As the radius of the circle goes to zero (which corresponds to the string coupling constant going to zero), these Kaluza-Klein states become infinitely massive in the D=10 metric. However, in terms of the eleven-dimensional metric, the $D_0$-branes are massless states. It will now be shown that identification of $P_{11}$ with $\frac{1}{32} e^\phi (\lambda \Gamma^{11} \bar{\lambda})$ agrees with this picture.

The first step is to define the $D$-brane boundary conditions for the twistor variables. This is easy since the spinor superfields $\Theta^\alpha_L$, $\bar{\Theta}^\alpha_L$ and $\Theta^\alpha_R$, $\bar{\Theta}^\alpha_R$ satisfy the same $D$-brane boundary conditions as their lowest component $\theta$ variable. Therefore, at the end of an open string with Neumann boundary conditions in directions $0...P$ and Dirichlet boundary conditions in directions $(P + 1)...9$, the boundary conditions on $\theta^\alpha$ and $\lambda^\alpha$ are given by

$$
\Theta^\alpha_L = (\Gamma^0 ... \Gamma^P)^{\alpha\beta} \Theta^\beta_R, \quad \bar{\Theta}^\alpha_L = (\Gamma^0 ... \Gamma^P)^{\alpha\beta} \bar{\Theta}^\beta_R
$$

where $P$ is assumed to be even.

It is easy to check this implies that

$$
\partial_L x^\mu_L = \frac{1}{16} (\lambda_L \Gamma^\mu \bar{\lambda}_L + i(\partial_L \theta_L) \Gamma^\mu \bar{\theta}_L) = \pm \frac{1}{16} (\lambda_R \Gamma^\mu \bar{\lambda}_R + i(\partial_R \theta_R) \Gamma^\mu \bar{\theta}_R) = \pm \partial_R x^\mu_R
$$

where the plus sign is if $\mu \leq P$ and the minus sign is if $\mu > P$.

So if the end of the open string lies on a $D_0$ brane at rest,

$$
P_{11} = \frac{1}{32} e^\phi (\lambda_L^0 \bar{\lambda}_{R\alpha} + \lambda_{R\alpha} \bar{\lambda}_L^0) = \frac{1}{16} e^\phi (\lambda_R \Gamma^0 \lambda_R) = e^\phi P_0.
$$

Also, $P_\mu = \partial_\tau x_\mu = 0$ for $\mu = 1...9$. 

13
Therefore, the (mass)\(^2\) of the \(D_0\)-brane in the eleven-dimensional metric \(G^{MN}\) is
\[
M^2 = G^{MN} P_M P_N = G^{00}(P_0)^2 + G^{1111}(P_{11})^2
\]
(4.3)
\[
= e^{2\phi/3}(P_0)^2 - e^{-4\phi/3}(e^\phi P_0)^2 = 0
\]
where the ten-dimensional metric has been assumed to be flat, so using the conventions of \(\mathbb{R}\), \(G^{\mu\nu} = e^{2\phi/3}\eta^{\mu\nu}\) and \(G^{1111} = -e^{-4\phi/3}\). Furthermore, the (mass)\(^2\) in the ten-dimensional metric, \((P_0)^2\), can be computed using standard \(D\)-brane techniques\(\mathbb{3}\), and diverges like \(e^{-2\phi}\) as the string coupling constant goes to zero.

So the identification of \(P_{11}\) with \(\frac{1}{32}e^\phi(\lambda^\alpha_L \lambda^\alpha_R + \lambda^\alpha_R \lambda^\alpha_L)\) is supported by the Kaluza-Klein picture of the \(D_0\)-brane.

A further check on this identification of \(P_{11}\) comes from the \(N=2\) \(D=10\) SUSY algebra of the superstring. As discussed in \(\mathbb{23}\), the \(N=2\) \(D=10\) SUSY algebra contains Ramond-Ramond central charge terms of the form
\[
\{q^\alpha_L, q^\beta_R\} = C^\alpha_{(0)\beta}
\]
(4.4)
where \(C^\alpha_{(0)\alpha}\) is the zero-brane Ramond-Ramond central charge which will be identified with \(\oint d\sigma (\lambda^\alpha_L \lambda^\alpha_R + \lambda^\alpha_R \lambda^\alpha_L)\). This can be compared with the \(N=1\) \(D=11\) SUSY algebra,
\[
\{\hat{q}^A, \hat{q}^B\} = \Gamma^{AB}_m E^m MP_M
\]
(4.5)
where \(A = 1\) to 32 are SO(10,1) spinor indices, \(m = 0, ..., 9, 11\) are flat vector indices, \(M = 0, ..., 9, 11\) are curved vector indices, \(\hat{q}^A\) are the \(N=1\) \(D=11\) SUSY generators, and \(E^m MP_M\) is the D=11 vierbein. When the \(D=10\) metric is flat, \(E^m MP_M = e^{\frac{1}{2}\phi}\delta^m MP_M\) for \(m=0\) to 9 and \(E^{11} MP = e^{-\frac{2}{3}\phi}\delta^{11} MP\).\(\mathbb{2}\) So if \(P_M\) is the ten-dimensional momentum for \(M = 0\) to 9, then \(\hat{q}^A\) should be identified with \(e^{\frac{1}{2}\phi}q^\alpha_L\) for \(A=1\) to 16 and with \(e^{\frac{1}{2}\phi}q^\alpha_R\) for \(A=17\) to 32. Comparing (4.5) with (4.4), this implies that \(P_{11} = \frac{1}{32}e^\phi C^\alpha_{(0)\alpha}\).

5. Extra dimensions in the Type IIB superstring

All of the previous constructions for the Type IIA superstring can be repeated for the Type IIB superstring, however because \(\Theta^\alpha_L, \bar{\Theta}^\alpha_L\) and \(\Theta^\alpha_R, \bar{\Theta}^\alpha_R\) now have the same SO(9,1) chirality, they cannot be combined into 32-component SO(10,1) spinor superfields.

But they can be combined into \(16 \times 2\)-component SO(9,1)\(\times\)SO(2,1) spinor superfields, \(\Theta^\alpha_b\) and \(\bar{\Theta}^\alpha_b\), where \(b = 1\) for the left-moving superfield and \(b = 2\) for the right-moving
superfield. In this notation, the momentum and one-brane NS-NS charge can be written as

$$\frac{1}{32} \int d\sigma \lambda_0^a \tau_0^{bc} \Gamma_{\alpha\beta} \bar{\chi}_c^\beta, \quad \frac{1}{32} \int d\sigma \lambda_1^a \tau_1^{bc} \Gamma_{\alpha\beta} \bar{\chi}_c^\beta,$$

(5.1)

where $\tau_q^{bc}$ are $2 \times 2$ SO(2,1) $\Gamma_q$ matrices which are related to the usual SO(2,1) $\gamma_q$ matrices by multiplication with $\gamma_0$ (i.e. $\tau_0^{bc} = \delta^{bc}$, $\tau_1^{bc} = \sigma_3^{bc}$, $\tau_2^{bc} = \sigma_1^{bc}$).

This suggests that the one-brane R-R charge should be identified with $\frac{1}{32} \int d\sigma \lambda_0^a \tau_2^{bc} \Gamma_{\mu} \bar{\chi}_c$, and such an identification is supported by arguments based on zero-momentum Type IIB R-R vertex operators which are similar to those of subsection (3.2). Furthermore, the three-brane R-R charge is naturally identified with $\frac{1}{32} \int d\sigma \lambda_0^a \epsilon^{bc} \Gamma_{\mu\nu\rho} \bar{\chi}_c$, and the five-brane R-R charge with $\frac{1}{32} \int d\sigma \lambda_0^a \tau_2^{bc} \Gamma_{\mu\nu\rho\phi} \bar{\chi}_c$.

6. Conclusions

There is increasing evidence that superstring theory is part of an eleven-dimensional structure. In this paper, it was proposed that superstring theory itself can be used to understand the eleventh dimension. If this proposal turns out to be correct, one should be able to understand $M$-theory compactifications using superstring language. Since the extra dimension is built out of RNS matter and ghost fields, perhaps one will need to consider superstring compactifications which treat the RNS matter and ghost degrees of freedom on an equal footing. Note that using the N=0 $\rightarrow$ N=1 embedding of reference [24], heterotic and Type II superstring backgrounds can be treated equivalently if the distinction between matter and ghost fields is removed. This suggests that invariants which transform the RNS matter and ghost fields into each other might be related to superstring dualities.

An alternative proposal for understanding the eleven-dimensional structure is M(atrix) theory [25], which is closely related to a light-cone GS approach in eleven dimensions. It is interesting to note that covariantization of the light-cone GS superstring in ten dimensions was one of the main motivations for studying the twistor-GS formalism.[7] Perhaps the introduction of twistor variables into M(atrix) theory will allow a more SO(10,1)-covariant description of the theory.

Another proposal for understanding the eleventh dimension uses an N=(2,1) heterotic string to generate target spaces which are either the supermembrane worldvolume or the superstring worldsheet. [20] This formalism shares with the twistor-GS superstring the
property of having \( N=2 \) worldsheet supersymmetry. In the \( N=(2,1) \) heterotic approach, the difference between the supermembrane and superstring comes from the choice of a null superconformal constraint. If this null constraint could be interpreted as a gauge-fixing condition, it would mean that the \( M \)-theory variables and superstring variables were related by a field redefinition which connects the two different gauge choices. This sounds similar to the proposal of this paper, however it is difficult to verify since only the static-gauge \( M \)-theory and superstring variables are easily obtainable in the \( N=(2,1) \) heterotic approach.

**Acknowledgements:** I would like to thank S. Ramgoolam, W. Siegel and B. Zwiebach for useful conversations. I would also like thank the IAS at Princeton and the ITP of SUNY at Stony Brook where part of this work was done. This work was financially supported by FAPESP grant number 96/05524-0.

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\(^9\) This fact was recently used by DeBoer and Skenderis\(^{[27]}\) to construct a hybrid twistor-heterotic string theory which describes self-dual supergravity.
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