Pulse compression and dispersion compensation for high-resolution Lamb wave inspection

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Abstract. The dispersion of ultrasonic guided waves causes the energy of a signal to spread out in space and time as it propagates, which decreases the performance for damage detection significantly. A lot of signal processing methods have been proposed to reduce the effect of dispersion for this reason. In this paper, with the aim of developing an efficient methodology for high resolution Lamb wave inspection, a pulse compression and dispersion compensation method is established. In this method, broadband excitation and pulse compression technique are introduced to reconstruct the transform function with a high SNR. Subsequently, a scheme is established to alleviate the dispersion effects by performing compensation on the original narrowband excitation signals, and thus the time duration of received wave packet can be compressed during the extracting process. Finally, Numerical simulation and experiment are carried on aluminum specimens to investigate the behavior of the proposed method.

1. Introduction
Lamb waves are guided elastic waves that propagate in solid plates or layers with free boundaries, which are also called plate waves [1]. As a promising method for structural damage detection and evaluation, Lamb wave method has attracted considerable attentions.

Dispersion is a big challenge encountered when working with guided waves. It is manifested as an increase in received signal duration and a decrease in amplitude [2]. The former effect is especially undesirable for situations where a defect is located in close proximity to another structural feature. Since both the defect and the structural feature cause the incident Lamb wave to be reflected, the wave packets may overlap with each other at the received position. This interference worsens resolution and makes experimental data hard to interpret. A lot of methods have been developed to avoid the effect of dispersion: optimal design of the excitation waveform [3], time reversal [4], time-frequency representation [5] and dispersion removal [6,7]. Using these methods, the effect of dispersion could be reduced or even eliminated completely.

Different from above methods, the aim of this paper is to develop a rapid methodology for high resolution Lamb wave inspection. For this purpose, broadband excitation is applied to enable simultaneous acquisition of information of multiple distinct frequency ranges, and further, with the application of pulse compression technique, the transform function of the whole system could be reconstructed with a high SNR. On this basis, a new dispersion compensation algorithm is established. This algorithm is performed by applying time-reversed dispersion effects on the original excitation

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signal for a given compensation distance. As a result, the response extraction and the dispersion compensation can be made simultaneously. Numerical simulation is then conducted to illustrate the influence of the compensation distance to the resolvable resolution. Finally, the effectiveness of the proposed algorithm is verified experimentally, in which the results show that the resolvable resolution is improved obviously.

2. Scheme of pulse compression and dispersion compensation

2.1 Transform function estimation via broadband excitation and pulse compression

Considering that two guided wave transducers are positioned on a plate (or layer), which act as the transmitter and receiver respectively, the entire system comprising the instrumentation, transducers and structures may be well-modeled as a linear system [8]. If the guided wave is excited with a broadband and long-duration temporal signal (e.g. chirp signal),

$$s(t) = \int S(\omega)e^{j\omega t}d\omega,$$

(1)

the response captured by the receiver satisfies,

$$y(t) = \int H(\omega)S(\omega)e^{j\omega t}d\omega.$$

(2)

Here, the corresponding transfer function $H(\omega)$ includes the transfer functions of the transmitter and receiver, all instrumentation effects, and the Green’s function(s) needed to describe wave propagation between transmitter and receiver [8].

To estimate this transform function, the pulse compression technique is introduced. In the frequency domain, it is defined as,

$$R(\omega) = S^*(\omega)Y(\omega) = S^*(\omega)S(\omega)H(\omega) = |S(\omega)|^2H(\omega),$$

(3)

where the mark ‘*’ means the complex conjugate. This formula can also be expressed in the time domain as,

$$r(t) = h(t) \text{ conv.} \{s(t) \text{ corr.} s(t)\},$$

(4)

where ‘corr.’ and ‘conv.’ mean the correlation and the convolution process, respectively. The advantages of pulse-compression method are mainly twofold. Firstly, it improves the signal to noise ratios (SNRs) of the raw received signals. Secondly, it reconstructs the accurate phase of the transform function, and further, since the frequency range of the excitation signal is broad, the auto-correlation function, $s(t) \text{ corr.} s(t)$, become pulse-like, and the pulse compression signal, $r(t)$, become closer to the pulse response, $h(t)$.

To recover the amplitude of the transform function, we divide the term $|S(\omega)|^2$ at both sides of equation (3), and thus

$$H(\omega) = R(\omega)/|S(\omega)|^2.$$  

(5)

The broadband excitation Lamb wave signal carries abundant and comprehensive information associating with the structure. Combined with the pulse compression technique, the transform function over a wideband frequency range could be reconstructed with a high SNR. On this basis, multiple narrowband responses could be readily computed provided that the frequency ranges of the corresponding excitations fall within that of the broadband excitation. For instance, the response to a narrowband excitation, $s_d(t)$, can be obtained as,

$$Y_d(\omega) = H(\omega)S_d(\omega).$$

(6)

Hence, it can be concluded that the broadband excitation and the pulse compression constitute a methodology for efficiently optimizing a guided wave inspection.

2.2. Dispersion compensation for resolution improvement
In this transform function, the effect of dispersion manifests itself as a phase adjustment term, \( D(x, \omega) \) \[2\].
\[
D(x, \omega) = e^{\text{i}k(x)}
\]
where \( x \) represents the distance measured from the actuator in the direction of propagation, \( k(\omega) \) is the circular wave number of the guided wave mode, which may be obtained from the phase velocity \( v_{ph}(\omega) \) dispersion curve data by
\[
k(\omega) = \frac{\omega}{v_{ph}(\omega)}.
\]

If the actuator is located at the position of \( x=0 \) while the sensor is located at the position of \( x=L \), dispersion compensation can be accomplished by multiplying \( Y_d(\omega) \), the spectrum of \( y_d(t) \), with a phase correction term \( D^{-1}(L, \omega) \),
\[
F(\omega) = H(\omega)S_d(\omega)D^{-1}(L, \omega)
\]
where,
\[
D^{-1}(L, \omega) = e^{-\text{i}k(x)L}.
\]
Here, the correction term covers the same frequency range as the excitation \( S_d(\omega) \). Hence, a new excitation signal, \( S_n(\omega) \), is gained, with its dispersion effects are pre-compensated for a special propagating distance, \( L \).
\[
S_n(\omega) = S_d(\omega) \cdot D^{-1}(L, \omega).
\]
Accordingly, equation (9) could be rewritten as,
\[
F(\omega) = H(\omega)S_n(\omega) = H(\omega)\left[ S_d(\omega)D^{-1}(L, \omega) \right].
\]

Similar to equation (6), the new response could also be easily computed from the transform function. More importantly, for \( x \in [L-\delta, L+\delta] \), where \( \delta \leq L/2 \) is a threshold controlling the range, it may be easily concluded from equation (12) that the dispersion will be partially compensated. As a result, the resolution of the wave packets will be improved. This method is especially desirable for long range inspections.

3. The vital role of compensation distance

For a wave packet propagating through a particular distance, \( x \), the resolvable resolution is dependent on the parameters of the new excitation waveform, \( s_d(t) \). As shown in equation (11), this excitation consists of two terms, i.e. the original narrowband excitation, \( S_d(\omega) \) and the phase correction term, \( D^{-1}(L, \omega) \). As a routine, the former one is a windowed tone burst with a precise centre frequency and a limited bandwidth, and much research has been published on the optimal design of its waveform \[3\]. In this paper, our study focuses on the latter one, i.e. the compensation distance, \( L \).

In practice, the compensation distance \( L \) dominates the dispersion compensation degree of the output signal. If \( x<L \), the residual distance \( \Delta = -(x-L) \) suffers from time reversed dispersion, as a result, the frequency components with slower group velocity appear at the front of the receive signal. Under completely compensation condition \( (x=L) \), all the frequency components arrive in phase at the receiver position, and the best resolution is achieved. When \( x>L \), dispersion still acts on the residual distance \( \Delta=x-L \), which makes the frequency component corresponding to the maximum group velocity lead the received wave, and the slower components trail.

With the application of broadband excitation and pulse compression, we can select it efficiently for high resolution inspections. For illustration, a numerical simulation is considered. Figure 1 shows the geometry of the aluminum plate used in the investigation. The damage is simulated as a rectangular slot close to the edge of the plate. A surface bonded PZT is used as both the actuator and the sensor. The broadband excitation signal is a chirp with its frequency sweeps from 4 kHz to 152 kHz over a 1ms window. It is assumed that the transducer is ideal and excites only the guided wave
mode of interest. Referring to [9], if there is no mode conversion when the outgoing wave is reflected at a feature, the received time-trace,

\[ g(t) = \sum_j \int_{-\infty}^{\infty} A_j(\omega) F(\omega) e^{i(\omega d_j - \omega t)} d\omega \]  \hspace{1cm} (13)

where \( d_j \) is the round trip propagation distance to each reflector, and the reflection coefficient of each reflector, \( A_j(\omega) \), is taken to be a constant (i.e. \( A_j(\omega) = 0.5 \)). Figure 2(a) shows the response as the A0 Lamb wave mode propagating in this specimen, and Figure 2(b) shows the corresponding pulse compression signal.

Figure 1. Schematic diagram of the aluminum plate (1-mm thick) with a damage slot.

Figure 2. (a) Simulated chirp response; (b) the corresponding pulse compression signal.

It is noted that the quality of the results obtained from the proposed dispersion pre-compensation algorithm is dependent on the accuracy with which the estimated dispersion curves represent the actual characteristics of the system (see equation (10)). Referring to Wilcox’s study [9], the effects of two different types of inaccuracies in the dispersion curves must be considered. As a result, besides the ideal case where the computed dispersion characteristics exactly match the actual ones (i.e. Case I), another two cases for the existence of these two kinds of inaccuracies (i.e. Case II and Case III) are also discussed in this section. For comparison, a 2 cycle Hanning windowed tone burst centered at 65kHz is used as the original narrowband excitation signal, \( s_d(t) \), in these cases. In addition, all responses corresponding to different compensation distances are extracted from the broadband response shown in figure 2(a) through the transform function.

**Case I**

In this ideal condition, the dispersion pre-compensation algorithm can restore the reflected wave packet to the exact shape of the original narrowband excitation signal, \( s_d(t) \), when the compensation
distance coincides with the corresponding propagating distance. Hence, the dispersion distance is always selected as follows. ① In a single actuator-sensor pair, it is often conducted as the path length between the actuator and the sensor. ② In a transducers array, it is often considered as the average of the largest path length and the least path length between the actuator and the sensor elements or the path length of certain actuator-sensor pair which has the greatest demand of resolution. ③ If the size of the array is too large for the resolvable resolution to accommodate all the paths, the array may be divided into a few segments.

In the numerical simulation, the associated output signals as the compensation distance changes from 0.28m to 0.52m with a step of 0.04m are shown in figure 3. According to the schematic diagram shown in figure 1, the first two reflected wave packets are caused by reflections of energy in the incident guided wave from the damage and the surrounding edge respectively. Hence, their propagation distances are 0.4m and 0.44m. Applying the former wave packet for example, the time duration decreases as the increase of the compensation distance first, till the optimal point where the compensation distance is identical to the propagation distance (i.e. \( L = 0.4m \)), the minimum value is obtained, and then it increases as the further increase of the compensation distance.

![Figure 3. Responses to the new excitation signals compensated by different distances (from 0.28m to 0.52m with a step of 0.04m) in the ideal condition.](image)

**Case II**

\[
v_{ph}^{(0)}(f) = (1 + \alpha)v_{ph}(f)
\]

(14)

where \( v_{ph}^{(0)}(f) \) is the actual phase velocity, \( v_{ph}(f) \) is the phase velocity in dispersion curves, and \( \alpha \) is the perturbation. This is representative of the cases as incorrect material properties were used to compute the dispersion curve [9].

In this condition, if the propagation distance is identical to the compensation distance (e.g. the first wave packet), the dispersion effects of the Lamb wave propagating are over compensated, completely compensated and under compensated corresponding to \( \alpha < 0, \alpha = 0, \alpha > 0 \), respectively. Since the calculated phase velocity is exactly proportional to the actual one, the temporal duration of the received signals could be scaled by the same amount in the time domain though optimal design of compensation distance. Figure 4 shows a series of new excitation responses computed from the broadband response by varying the compensation distance from 0.28mm to 0.52m with the increment of 0.04m when the perturbation \( \alpha = -0.2 \). The wave packets as the compensation distance consists with the propagating distance of the first wave packet (i.e. \( L = 0.4m \)) are enclosed by a rectangle in this figure. It can be seen that the perturbation worsens the resolution of these wave packets. Fortunately, with a proper choice of the compensation distance, the resolution will be improved. For example, when the compensation distance \( L = 0.32m \), the wave packets (enclosed by the circle in figure 4) have the same shape as the ones in the ideal conditions when the compensation distance \( L = 0.4m \) (enclosed by a rectangle in figure 3). It means that the first wave packet also meets its best resolution.
Case III

\[
v_{ph}^{(0)}(f) = v_{ph}(f(1 + \beta))
\]  \hspace{1cm} (15)

where \( \beta \) is the perturbation. This perturbation corresponds to the physical thickness of the plate not being the same as the thickness used in the dispersion curves calculation [9].

Different with type I, there is not an exact proportion relation between the calculated dispersion data and the actual one. As a result, the shape of the received signals cannot be compressed to the ideal one through optimal design of the compensation distance. However, the optimization of the compensation distance can also alleviate the effects of this perturbation. Figure 5 shows a series of new excitation responses computed from the broadband response by varying the compensation distance from 0.28mm to 0.52m with the increment of 0.04m when the perturbation \( \beta = 0.2 \). It can be seen that the two wave packets are completely separated in time when the compensation distance \( L = 0.48m \) (enclosed by the circle), though 20% errors are contained in the dispersion data.

In summary, the proposed dispersion pre-compensation algorithm is a stable technique in general. It is of good robustness to inaccuracies in calculated dispersion data. More importantly, there is a great choice of waveform parameters, with which the received wave packets are completely separated in time and thus the structural features can be separately identified (as shown in figures 3-5). This makes the proposed method efficient and effective for high-resolution Lamb wave inspection.

4. Experimental investigation

In this experiment, the excitation signal is generated by an Agilent 33220A function/arbitrary waveform generator, after being amplified with the peak-to-peak voltage of 50 V (by a Piezo Systems
EPA-104 voltage amplifier), which is applied to the actuator (PZT) of the sensor network. The Lamb wave signals pass through the monitoring area and are captured by the sensors (PZT). After being amplified by the AVANT NI-2000 conditioning amplifiers, they are acquired at a sampling rate of 10MHz by a NI PXIe-1082 data acquisition. Figure 6 shows the schematic diagram of the aluminum specimen (1200mm×500mm×1.39mm) and the PZT configuration. An artificial defect was introduced in the form of through-thickness notch with dimensions 50mm×5mm×1.39mm in the specimen.

![Schematic diagram of the PZT sensors placement in the aluminum plate.](image)

A broadband chirp signal, sweeping from 4 kHz to 152 kHz over a 500μs window is applied to actuate Sensor A. Similar to the numerical simulation, the choice of the proper waveform parameters is performed through the broadband response. Here a 2 cycle Hanning windowed tone burst centered at 80 kHz with the dispersive effects being compensated by a distance of 400mm (equal to the distance between Actuator A and Sensor S3) is an appropriate excitation signal. Figure 7 shows the responses of the sensor array, which are calculated from the broadband response. It can be seen that the first three wave packets, i.e. the incident A0 mode, the damage scattered A0 mode and the edge reflected A0 mode in turn, are completely separated in time domain, which means the three structural features could be unambiguously identified.

![Responses to the new excitation signals, which are derived from 2 cycle 80 kHz tone burst being compensated by a distance of 0.4m, as generated from the measured chirp response.](image)

Without dispersion compensation, the responses of the sensor array corresponding to the original tone burst are also extracted from the same broadband response. As shown in Figure 8, the received signals are much more complicated because all the wave packets resulting from different structural features overlap with each other.
5. Conclusions
A pulse compression and dispersion compensation method is established for high-resolution Lamb wave inspection. Some conclusions are obtained as follows.
(i) Combined with broadband excitation and pulse compression, the proposed dispersion compensation algorithm is efficient, since it can be performed during the process of the narrowband response extracting.
(ii) The effects of inaccuracies in calculated dispersion curves would possible to be alleviated by optimal design of the compensation distance.

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Figure 8. Responses to 2 cycle Hanning windowed tone burst centered at 80kHz as computed from the measured chirp responses.