Relativistic corrections to the electric polarizability of the neutron

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Abstract. We demonstrate in a solvable model the connection between the intrinsic electric polarizability $\bar{\alpha}$ and the value $\alpha_{\text{Sch}}$ obtained from neutron-atom scattering.

1 Introduction

According to the Low Energy Theorems of Compton scattering, the electric $\bar{\alpha}$ and magnetic $\bar{\beta}$ polarizabilities are fundamental quantities that characterize the neutron. Given the lack of free neutron targets and the much smaller Compton cross-section (compared to the charged proton), the two current experimental approaches are low energy neutron-atom scattering $[1, 2]$ and quasi-free Compton scattering from the neutron bound in a deuteron $[3, 4]$. The tool used to analyze the data in neutron-atom scattering is the Foldy-Wouthuysen reduction of the Dirac equation $[5]$. For a neutron of magnetic moment $\mu$ and mass $m$, it has been shown $[6]$ that $\bar{\alpha}$ is not the coefficient (labeled $\alpha_{\text{Sch}}$) of $-\frac{1}{2}E^2$ in this wave equation. Instead, $\bar{\alpha} = \alpha_{\text{Sch}} + \frac{\mu^2}{m}$.

The purpose of this communication is to illustrate these results by solving exactly the the Foldy-Wouthuysen-Dirac equation for a neutron in a constant $E$ field. Actually, this is how electric polarizability is defined $[7]$. Our main result is the following: the energy eigenvalues are independent of $\mu^2$ in the neutron rest frame defined by

$$\langle \mathbf{mv} \rangle = \langle \mathbf{p} - (\mathbf{E} \times \mathbf{\mu}) \rangle = 0,$$

where $\mathbf{p}$ is the canonical momentum, $\mathbf{v}$ the velocity operator, and $\mathbf{\mu} = \mu \mathbf{\sigma}$. Since the components of $\mathbf{v}$ implicitly include non-commuting spin operators, the rest frame can only be defined in an average sense. Given this definition, the
rest frame energy eigenvalues do not include the $-\mu^2/m$ coefficient multiplying $-\frac{1}{2}E^2$ present in the Foldy-Wouthuysen-Dirac equation. Thus we confirm by an explicit calculation our earlier identification $\bar{\alpha} = \alpha_{Sch} + \mu^2/m$ which was of a somewhat formal nature.

2 Calculation

The Foldy-Wouthuysen-Dirac equation for a neutron of magnetic moment $\mu = \mu \sigma$ and mass $m$ in a constant external $E$ field is [6]:

$$H\psi = \left(\frac{p^2}{2m} - \frac{p}{m} \cdot (E \times \mu) + \frac{\mu^2E^2}{2m} - \frac{1}{2}\bar{\alpha}E^2\right)\psi = \varepsilon\psi$$ (2)

where $\bar{\alpha}$ is the intrinsic neutron electric polarizability and $\varepsilon$ the energy eigenvalue. Equation (2) can be solved exactly to give:

$$\left[\varepsilon - \frac{p^2}{2m} - \frac{\mu^2E^2}{2m} + \frac{1}{2}\bar{\alpha}E^2\right] = \frac{E\mu}{m} \lambda ,$$ (3)

where $\lambda$ is given by:

$$p \cdot (\sigma \times E)\psi = E\lambda\psi ,$$ (4)

and we have taken $E$ to be along the $x$-axis, ie.:

$$E = (E, 0, 0) .$$ (5)

Writing

$$p = (p_1, p_\perp)$$ (6)

one finds for (4):

$$\lambda = \pm p_\perp , \quad \text{with} \quad p_\perp = |p_\perp| .$$ (7)

With the aid of the explicit eigenstates of Eq. (4), one finds that the average value of the particle velocity operator

$$v \equiv \frac{\partial H}{\partial p} = \frac{1}{m}p - (E \times \mu)$$ (8)

for $\lambda = -p_\perp$ is

$$\langle v_\perp \rangle = \frac{p_\perp}{m} \left(1 - \frac{\mu E}{p_\perp}\right)$$ (9)

and

$$\langle v_1 \rangle = v_1 = \frac{p_1}{m}$$ (10)

Formulae (9) and (10) imply that

$$\langle v \rangle = 0 \quad \text{for} \quad \mu E = p_\perp \text{ and } p_1 = 0 .$$ (11)

This finally leads, from Eqs. (3), (11), and $\lambda = -p_\perp$ to:

$$\varepsilon = -\frac{1}{2}\bar{\alpha}E^2$$ (12)
Thus, in the neutron rest frame ($\langle v \rangle = 0$), all terms quadratic in $\mu$ cancel exactly. This implies, as stated in the Introduction, that the rest frame energy eigenvalues do not include the $-\mu^2/m$ coefficient multiplying $-\frac{1}{2}E^2$ present in Eq. (2). It is this Eq. (2), however, which is used to analyze neutron-atom scattering, so that the quantity $\mu^2/m$ should be added to the quoted polarizability result ($\alpha_{SA}$) of the experiment, thus leading to an increase of the electric polarizability $\bar{\alpha}$ (in the Compton sense), in complete agreement with [6].

3 Discussion

The explicit calculation just described sharpens the argument of Ref. [6]. In that earlier paper we asserted “that the rest frame of a neutron in an external electric field is defined by a vanishing value of the velocity operator, as confirmed by experimental measurements of the Aharonov-Casher effect.” As we have seen in Section 2, the velocity operator cannot vanish in a three dimensional geometry because it contains non-commuting spin operators, and the neutron rest frame is defined by the expectation value of the velocity operator. The observation of the Aharonov-Casher effect was made in a two dimensional geometry (a neutron diffracting around a line of electric charge) where the velocity operator does indeed vanish.

Finally, our result that the rest frame energy eigenvalues do not include the $-\mu^2/m$ coefficient multiplying $-\frac{1}{2}E^2$ is also consistent with Foldy’s well known result. Foldy solved exactly the problem of a structureless neutral Dirac particle with an anomalous magnetic moment $\mu$ in a homogeneous static electric field $E$ [9]. He did find a term quadratic in $\mu^2$ in the non-relativistic limit of the energy eigenvalues (implying a negative polarizability of magnitude $\mu^2/m$ from this Dirac-Foldy term). Foldy’s solution was obtained in the frame $p = 0$ which is not the rest frame of the neutron.

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