Torsion Frequency Response Function of a Train Driving Wheelset

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Abstract. The driven wheelset is used in traction rail vehicles (locomotives, electric trains, trams, etc.) to support part of the weight of the vehicle and to drive and brake the vehicle. The wheelset consists of the axle on which two wheels and the drive gear are mounted. The torsion vibrations of the driven wheelset may be induced by the variation of the adhesion force at the level of the contact between the wheels and the track but may also result from electromechanical interactions between the electric traction motor and the motor axle. In this paper, a mechanical model of the driven wheelset consisting of a circular shaft free at both ends and three rigid discs representing the two wheels and the drive gear is considered. The modal analysis method is used to solve vibrational motion equations in steady state harmonic behavior. The frequency response function of the system and the impact of the positioning of the drive gear on the wheelset shall be highlighted. For the verification of the proposed mechanical model, the numerical results are compared with the results obtained by experimental determinations carried out on a laboratory stand.

1. Introduction
The driven wheelset of a locomotive is an interesting structure subjected to complex stress, including torsional vibrations, especially in the start-up regime of the vehicle, due to the occurrence of the stick-slip phenomenon. Those vibrations lead to intense stress in the drive system and especially the driven wheelset [1]. Torsional vibrations are induced by the drive system for the wheelset, leading to a negative influence of the adhesion coefficient with a negative impact on dynamic rolling performance. Furthermore, torsional vibrations lead to the development of wheel-rail corrugation and noise affecting the environment and passenger comfort in the case of railway vehicles intended for passenger transport [2].

The driven wheelset consists of two wheels fixed rigidly on the axle itself. The loads’ transmission from the vehicle body to the driven wheelset shall be carried out through axle boxes and from the driven wheelset to the rails shall be carried out through the wheels rolling surfaces. The power is transmitted to the wheelset by the gear set which may be with cylindrical or conical gears. The most widespread constructive solution of the driven wheelset is with the gear set located between the wheels, most often located near one of them [3-4]. The gear set located in the middle of the wheelset is not a common constructive solution, being present on some two-wheelset rail cars or on trams with wheelsets driven by the transversely disposed of electric motor.

Many researches into the wheel-rail interaction highlight that torsion vibrations can be self-maintained by the adhesion forces at the contact surface [2].
In this paper, the torsional vibrations of the driven wheelset for the electric LE 5100 kW locomotive are studied. The equation of motion is resolved using the modal analysis method, taking into account the rigid vibration mode along with the first three mode shapes. The results of the numerical simulations are presented in the form of the system receptance (Frequency Response Function) and the influence of the different constructive parameters of the driven wheelset are highlighted. Results obtained based on the theoretical mechanical model are verified by experimental determinations on a laboratory rig and satisfactory agreement has been obtained.

2. Mechanical model of the driven wheelset and the equation of motion
The mechanical model of a driven wheelset is shown in figure 2. The model consists of the wheelset axis considered a straight cylindrical shaft of a constant section with the length equal to the wheelset itself, free at both ends. The wheels and the drive gear are placed taking into account only their inertia effect. On the driven gear acts a concentrated torque variable in time.

![Figure 2. Schematic representation of a driven wheelset.](image)

The parameters of the mechanical model of the driven wheelset are axle length $l$, axle diameter $d$, the density of the material $\rho$, shear modulus $G$, the moment of inertia of the wheel $J_r$, the moment of inertia of the drive gear $J_c$. The distance between the end of the axle and wheel is $a$ and the distance between the end of the axle and the drive gear is $b$.

The equation of motion for the driven wheelset is:
Applying the modal analysis method, solution for equation (1) has the form:

$$\theta(x,t) = T_0(t) + \sum_{n=1}^{\infty} \Theta_n(x) T_n(t)$$

where $\Theta_n(x)$ is the mode shape function, $T_n(t)$ represent the time coordinates of the elastic vibration modes and $T_0(t)$ represents the time coordinate of the rigid vibration mode.

The mode shape function for torsion vibrations for a free axle at both ends is given by:

$$\Theta_n(x) = \cos\left(\frac{n\pi x}{l}\right)$$

The representation of the torsional mode function for the first three vibration modes is found in figure 3. It can be observed that the odd vibration modes of the axle are antisymmetric ($n = 1, 3$) and the even vibration modes are symmetrical vibration modes ($n = 2$).

Introducing (2) in (1), the following equations are obtained:

- for the rigid mode:

$$\left(\rho I_p l + 2J_r + J_c\right) \ddot{T}_0(t) + J_r \sum_{n=1}^{\infty} \ddot{T}_n(t) [\Theta_n(a) + \Theta_n(l-a)] + J_c \sum_{n=1}^{\infty} \ddot{T}_n(t) \Theta_n(b) = C(t),$$

- for the elastic vibration modes:

$$\frac{\rho I_p l}{2} \dddot{T}_j(t) + J_r \dddot{T}_0(t) [\Theta_j(a) + \Theta_j(l-a)] + J_c \dddot{T}_0(t) \Theta_j(b) + J_c \sum_{n=1}^{\infty} \dddot{T}_n(t) \Theta_n(b) \Theta_j(b) +$$

$$J_r \sum_{n=1}^{\infty} \dddot{T}_n(t) [\Theta_n(a) \Theta_j(a) + \Theta_n(l-a) \Theta_j(l-a)] + GI_p \left(\frac{j\pi}{2l}\right)^2 T_j(t) = C(t) \Theta_j(b), \quad j = 1, 2, \ldots$$

In the steady state harmonic behaviour, the form of the excitation torque is $C(t) = C\sin(\omega t)$, with $C$, the amplitude and $\omega$, is the angular frequency, and time coordinates $T_n(t) = T_n\sin(\omega t)$ with $n \in \mathbb{N}$, where $|T_n|$ is the amplitude of the time coordinate.

Solution (2) is in the form:
\[ \theta(x,t) = \Theta(x) \sin \omega t, \]  

where \( \Theta(x) \) is the amplitude in section \( x \) of the driven wheelset:

\[ \Theta(x) = T_0 + \sum_{n=1}^{\infty} \Theta_n(x) T_n. \]  

Next, the rigid vibration mode, together with the first three elastic vibration modes are considered. Equations (4) and (5) can be written in matrix form:

\[ \left( -\omega^2M + K \right) \cdot \mathbf{T} = \mathbf{C} \]  

where

\[ M = (d_{ij})_{i,j=1,\ldots,4} \]  

represents the inertia matrix, and:

\[ K = \text{diag}(0, k_1, k_2, k_3) \]  

represents the rigidity matrix, the elements of which are:

\[ a_{11} = \rho l_p l + 2J_r + J_c \]
\[ a_{12} = a_{21} = J_r \left( \Theta_1(a) + \Theta_1(l-a) \right) + J_c \Theta_1(b) \]
\[ a_{13} = a_{31} = J_r \left( \Theta_2(a) + \Theta_2(l-a) \right) + J_c \Theta_2(b) \]
\[ a_{14} = a_{41} = J_r \left( \Theta_3(a) + \Theta_3(l-a) \right) + J_c \Theta_3(b) \]
\[ a_{nn} = \frac{\rho l_p l}{2} + J \left( \Theta_n^2(a) + \Theta_n^2(l-a) \right) + J_c \Theta_n^2(b), \ n = 2, 3, 4 \]
\[ a_{23} = a_{32} = J_r \left( \Theta_1(a) \Theta_2(a) + \Theta_1(l-a) \Theta_2(l-a) \right) + J_c \Theta_1(b) \Theta_2(b) \]
\[ a_{24} = a_{42} = J_r \left( \Theta_1(a) \Theta_3(a) + \Theta_1(l-a) \Theta_3(l-a) \right) + J_c \Theta_1(b) \Theta_3(b) \]
\[ a_{34} = a_{43} = J_r \left( \Theta_2(a) \Theta_3(a) + \Theta_2(l-a) \Theta_3(l-a) \right) + J_c \Theta_2(b) \Theta_3(b) \]
\[ k_n = \frac{G l_p (n\pi)^2}{2l}, \ n = 1, 2, 3 \]

The column matrix of the excitation torque is:

\[ \mathbf{C} = C \begin{bmatrix} 1 & \Theta_1(b) & \Theta_2(b) & \Theta_3(b) \end{bmatrix}^T \]

The solution of the equation (7) is:

\[ \mathbf{T} = \mathbf{Z}^{-1} \mathbf{C} \]  

where \( \mathbf{Z} = -\omega^2\mathbf{M} + \mathbf{K} \) represents the dynamic rigidity matrix.

The system response is given in the form of receptance:

\[ R(x) = \frac{\Theta(x)}{C}, \]  

specifying that the interest is to determine the receptance next to the wheels.
3. Numerical results

This section presents the numerical results deriving from the model presented above, in which the following driven wheelset parameters were considered: axle length \( l = 2.11 \text{ m} \), the moment of inertia of the wheel \( J_r = 97.6 \text{ kg} \cdot \text{m}^2 \), the moment of inertia of the drive gear \( J_c = 35.5 \text{ kg} \cdot \text{m}^2 \), axle diameter \( d = 0.1991 \text{ m} \), the density of the material is \( \rho = 7850 \text{ kg/m}^3 \), shear modulus \( G = 81 \text{ GPa} \), the distance between the end of the axle and wheel is \( a = 0.305 \text{ m} \) and the distance between the end of the axle and the drive gear is \( b = 0.5205 \text{ m} \). The parameters values correspond to a driving wheelset equipping the 060EA electric locomotive of the Romanian Railways. The frequency range considered is between 10 and 3000 Hz.

The receptance of the drive wheelset determined next to the first wheel is shown in figure 4. Resonance frequencies are 65 Hz, 389 Hz and 1484 Hz respectively, and antiresonance frequencies are 51 Hz, 1304 Hz and 2417 Hz.

Figure 5 shows the receptance of the drive wheelset considering different values of the wheel mass. The value taken as a reference is 500 kg, representing the mass of the wheel in a semi-worn state. In new condition, the wheel mass is 580 kg and fully worn 420 kg. For the three masses of the wheel considered, noticeable differences occur in the area of small and medium frequencies. In the case of a new wheel, the first resonance frequency will be lower (61 Hz), and for the wheel in a fully worn state, the first resonance frequency will be 70 Hz. The anti-resonance frequencies for the three cases are 47 Hz, 51 Hz and 55 Hz. For the second vibration mode, the resonance frequencies are 383 Hz, 388 Hz and 397 Hz, respectively. For the third vibration mode, the resonance and anti-resonance frequencies are very close. The receptance can be found to have higher values when using a lighter wheel.

![Figure 4. Drive wheelset receptance.](image)

![Figure 5. Drive wheelset receptance for different wheel masses.](image)

![Figure 6. The receptance of the wheelset near both wheels.](image)

![Figure 7. The receptance of the wheelset for different positioning of the drive gear.](image)
Figure 6 shows the system receptance calculated next to both wheels. The equality of resonance frequencies is observed next to the two wheels (65 Hz, 389 Hz, and 1484 Hz respectively), the differences consist of different values of the antiresonance frequencies, for the wheel further away from the position of the drive gear has a single antiresonance frequency at 142.3 Hz.

Taking into account the different positions of the drive gear (at 0.5205 m from the end of the axle and in the middle of it, at 1.055 m), figure 7 shows the receptance determined next to the first wheel of the driven wheelset. A difference can be found when positioning the drive gear in the middle of it. In this case, both resonance and antiresonance frequencies are different. The resonance frequencies obtained are 68 Hz, 193 Hz and 2232 Hz respectively.

4. Experimental determinations

The experimental measurements were carried out on a stand located in the laboratory of Dynamics and Vehicle structures of the Railway Vehicle Department of the Transport Faculty, Politehnica University of Bucharest (figure 8). The stand consists of a drive wheelset driven by a longitudinal-arranged asynchronous three-phase motor, the torque is transmitted to the axle by a conical gear with straight teeth and a transmission shaft connecting the motor and the gear. The drive speed of the wheelset can be varied in two steps by modifying the motor phase connection (star-delta connection). The loading of the wheelset can be done using two movable rails fitted with a friction element which is put in contact with the surface of the wheels. The pressing force shall be modified by a hand-operated screw tightening system. The tightening force can be determined by the strain gauge.

The torsion moment of the drive wheelset is measured by a full bridge strain gauge transducer consisting of four strain gauges arranged in directions that make an angle of 45° with the median axis of the axle. Powering and taking over of the bridge signal is done by a collector solidary with the body of the axle, and the connection between the data acquisition system and the bridge is made through the collecting brushes.

![Figure 8. Stand for determining the drive wheelset torsion.](image)

The measuring system consists of a NI cDAQ 9176 chassis with four channels for connecting a various serial modules, a chassis intended to measure a wide range of analog and digital signals. The 9176 chassis is connected to PC via a USB 2.0 data bus.
The powering and actual measurement of the strain gauge bridge transducer is done by the NI 9219 module for voltage and current measurements in resistive electrical applications. The module can operate with transducers connected in quarter, half, and full bridge configuration. Control of the DAQ system, recording of the data and their processing is carried out in the Matlab program (figure 9).

Data recorded from the strain gauge bridge for one of the measurement sequence is shown in figure 10. The determination was made for a fixed motor speed and a constant tightening force of the rails.

![Figure 10. Recorded signal.](image)

The processing of the signal is done by Fourier analysis and the spectrum of its amplitude is found in figure 13. A spectral component is observed next to the value of 16,08 Hz, which represents the first natural torsion frequency of the driven wheelset of the experimental stand. It is specified that the spectral analysis was carried out on a sequence of data where the signal is stationary, i.e. the signal recorded from 105 to 120 seconds was chosen for analysis.

Validation of the presented mechanical model is achieved through numerical simulation having as input data the geometric parameters and masses of the components of the driven wheelset of the stand: the length of the axle \( l = 1.288 \) m, the inertia module of the wheel \( J_r = 4.8 \) kg·m², the inertia module of the drive gear \( J_c = 0.09 \) kg·m², the diameter of the axle \( d = 0.04 \) m, the distance between the end of the axle and the wheel \( a = 0.227 \) m and the distance between the end of the axel and the drive gear \( b = 0.337 \) m. The receptance of the driven wheelset of the stand is shown in figure 12 and the resonance frequency is 17.6 Hz. This value is close to the experimentally obtained frequency value of 16.08 Hz.
By comparing the results obtained by numerical simulation and experimental determination, it can be concluded that the mechanical model of the driven wheelset can be used in the study of the torsional vibrations of the railway vehicle wheelset.

5. Conclusion
The driven wheelset is responsible for supporting the rail vehicle and guiding it safely along the track, representing the interface between the vehicle and the running track.

For the study of torsional vibrations, a mechanical model consisting of an axle of constant diameter and finite length and three concentrated masses representing the two wheels and the drive gear was considered. The motor torque acting next to the gear is represented by a concentrated torque expressed by a harmonic function.

The system receptance is obtained numerically, using as input data the constructive parameters of the driven wheelset of the electric locomotive 060EA. The receptance of the driven wheelset has a sequence of resonance and antiresonance frequencies. Due to the asymmetry of the wheelset (because of the drive gear), the receptance on the two wheels are not equal, which can amplify the vibration regime of the wheelset.

The influence of wheel wear was studied using receptance. Wheel wear has the effect of reducing the mass and the moment of inertia. Calculations have shown that the wear of the wheels affects virtually only the first frequency of resonance. In the case studied, there was a 6% decrease in the first resonance frequency for a wheelset with a worn out wheel.
Using an experimental stand, the first torsional mode frequency of a drive wheelset was determined by the frequency analysis of the signal recorded from a strain gauge bridge transducer. The difference between experimental and numerical results is acceptable, i.e. 9.4% in terms of the first resonance frequency.

6. References
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