Heating of Two-Dimensional Holes in SiGe and the $B = 0$ Metal-Insulator Transition

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We study the resistivity vs. electric field dependence $\rho(E)$ of a 2D hole system in SiGe close to the $B = 0$ metal-insulator transition. Using $\rho$ as a “thermometer” to obtain the effective temperature of the holes $T_c(E)$, we find that the $\rho(E)$ dependence can be attributed to hole heating. The hole-phonon coupling involves weakly screened piezoelectric and deformation potentials compatible with previous measurements. The damping of the Shubnikov-de Haas oscillations gives the same $T_c$ values. Thus the $\rho(E)$ dependence and the $E$-field “scaling” do not provide additional evidence for a quantum phase transition (QPT). We discuss how to study, in general, true $E$-field scaling and extract the ratio of the QPT characteristic lengths.

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The recent observations of a metal-insulator transition (MIT) in two-dimensional (2D) electron or hole systems in high mobility silicon metal-oxide-semiconductor field effect transistors (Si-MOSFETS) and in certain heterostructures have triggered important experimental and theoretical efforts to understand the unexpected metallic behavior. This consists in a decrease of the resistivity $\rho$ for decreasing temperature $T$, at densities $p_s$ larger than the critical density $p_c$. In contrast, $d\rho/dT < 0$ on the insulating side of the MIT ($p_s < p_c$). The extrapolated finite resistivity at $T=0$ is in conflict with the scaling theory of localization for non-interacting particles which predicts localized states in 2D systems. The MIT occurs for values of $r_s$ (the ratio of a carrier pair Coulomb energy to the Fermi energy) much larger than one, suggesting the existence of a non-Fermi liquid due to strong interactions. The MIT could thus be the signature of a $T=0$ quantum phase transition (QPT) between new metallic and insulating phases. However, for some systems, e.g. $p$-SiGe, it seems that the positive $d\rho/dT$ is due to quasi-classical processes masking the usual weak localization which should appear again at low enough temperatures. At present, the nature of the MIT is still a subject of ongoing discussions.

A striking feature of the MIT is that at low temperature, the dependence of $\rho$ as a function of the electric field $E$ is similar to $\rho(T)$, i.e. $d\rho/dE < 0$ for $p_s < p_c$ and $d\rho/dE > 0$ for $p_s > p_c$. The physical origin of this observation is an open question. Irrespective of the possible microscopic explanations of the $\rho(E,p_s)$ dependence, a QPT implies that $\rho$ scales with $E$ if it scales with $T$ close to the critical point, two characteristic length scales are associated with $T$ and $E$, $L_q(T) \sim T^{-1/2}$ and $L_E(E) \sim E^{-1/(z+1)}$, and $\rho$ depends only on the ratio of the smallest of them to the correlation length $\xi \sim |\delta_n|^{-\nu}$ ($z$ is the dynamical exponent, $\nu$ the correlation length exponent, and $\delta_n = (p_s - p_c)/p_c$). Thus $\rho(T,p_s)$ (for $E \to 0$) depends only on $|\delta_n|/T^{1/z}$ on each side of the transition, and $\rho(E,p_s)$ (at low temperature) depends only on $|\delta_n|/E^{1/(z+1)}$. The scaling analysis of $\rho(T,p_s)$ and $\rho(E,p_s)$ allows a separate extraction of $z$ and $\nu$. $E$ scaling has been observed in several systems exhibiting $T$ scaling. However, a natural question is whether the $\rho(E)$ dependence results from carrier heating. At low temperature, the weak carrier-phonon coupling can lead to an effective temperature of the carriers, $T_c(E)$, larger than the lattice temperature $T_l$. Thus, the $\rho(E)$ dependence can be due to the $T_c(E)$ dependence of $\rho$.

In this paper, we demonstrate experimentally that for a 2D hole system in $p$-SiGe exhibiting the MIT features with $T$ and $E$ scaling, the resistivity vs. $E$-field dependence can be attributed to hole heating. We find $\rho(E,p_s) = \rho(T = T_c(E),p_s)$ with a law $T_c(E)$ compatible both in shape and magnitude with known electron-phonon coupling models and data. The same $T_c(E)$ is obtained from the damping of the Shubnikov-de Haas (SdH) oscillations. Thus, in the present case, the $\rho(E,p_s)$ dependencies and the $E$-field “scaling” are not arguments in favor of the QPT interpretation of the “MIT” and the separate extraction of $\nu$ and $z$ is not possible. As heating effects depend on the semiconductor structure, while $E$ scaling is a crucial issue for the QPT, we discuss the conditions required to study it for various systems in spite of carrier heating. We show how the ratio $L_q/L_E$ can be obtained experimentally.

The experiments were performed on Si$_{0.85}$Ge$_{0.15}$ quantum wells sandwiched between undoped Si layers. The 2D hole gas was formed in the triangular potential well at the Si/SiGe interface located on the boron doped side. It occupies the lowest heavy-hole subband, with an effective mass of about 0.25m$_0$. Two gated Hall bars (S1
and S2) were used. Their width $W$ is 100 $\mu$m, and their length $L$ (between voltage probes) 233 $\mu$m for S1, and 125 $\mu$m for S2. By varying the gate voltage, their densities can be tuned between 0.50 and $1.75 \times 10^{11}$ cm$^{-2}$. The mobilities at 200 mK increase with $p_s$ from 600 to 5500 cm$^2$/Vs (resp. 1000 to 7400 cm$^2$/Vs) for S1 (resp. S2). An ungated sample, with a density of $3.9 \times 10^{11}$ cm$^{-2}$ and a mobility of 7800 cm$^2$/Vs has also been used. The temperature range was 70 mK - 1.4 K. To check that the temperature $T$ given by the thermometer fixed in the copper sample holder was that of the lattice $T_l$, the low current resistance of another Hall bar etched onto the same substrate was used as a thermometer. Its temperature remained close to $T$ whatever the current in our samples. $\rho = (V/I)(W/L)$ and $E = V/L$ were obtained from the current $I$ and the voltage drop $V$ between the voltage probes, using a four point DC technique, with a current (10 - 300 pA for low $E$ measurements) periodically reversed at a frequency 0.03 - 0.3 Hz.

The MIT features appear in the $\rho(T,p_s)$ dependence for $E \to 0$ [Fig. 1(a)-(b)]. $d\rho/dT < 0$ is found for $p_s > p_s \approx 1.3 \times 10^{11}$ cm$^{-2}$ ($r_s \approx 6$), while $d\rho/dT > 0$ for $p_s < p_c$, at temperatures above a threshold which decreases when $p_s$ increases. At the largest densities $d\rho/dT$ is positive. The $\rho(T,p_s)$ data plotted as a function of $|\delta_n|/E^b$ for $T > 0.3 - 0.5$ K collapse on two branches (not shown), demonstrating a scaling behavior with $b = 0.45 \pm 0.04$. The values 0.35 and 0.62 were found in p-SiGe. The MIT characteristics and $E$-field scaling appear also in the $\rho(E,p_s)$ curves (Fig. 1), $p_c$ remaining unchanged. When $\rho$ is plotted as a function of $|\delta_n|/E^a$ ($a = 0.19 \pm 0.02$), the curves fall on two branches (inset of Fig. 1). They contain only a part of the data (symbols on Fig. 1): removing the low $E$ points is justified by $L_E < L_R$ (assuming a real QPT), while for large $E$, it can be related to a microscopic limit on $L_E$.

Figure 1 shows that the general shape and the minima of $\rho(E)$ are the same as those of $\rho(T)$, suggesting hot hole effects. Their contribution is studied as follows: (i) the $\rho(E)$ dependence is assumed to result from the hole temperature rise from $T_l$ to $T_s$, due to Joule heating; (ii) for each value of $E$, $\rho$ is used as a “thermometer” giving $T_c$ as the temperature $T$ at which the same value of $\rho$ is measured for $E \to 0$; (iii) the relationship between $T_c$ and the power per carrier $P_E = V(I/p_sW_L) = E^2/(p_sW_L)$ is used to study the validity of the hot carrier assumption (i).

The power loss between a degenerate 2D electron system and the lattice 3D acoustic phonons is given by

$$P_E = A(T_c^{\alpha} - T_l^{\alpha}) + A'(T_c^{\alpha'} - T_l^{\alpha'}).$$

(1)

The first term corresponds to the deformation potential coupling, with $\alpha = 5$ (resp. 7) for weak (resp. strong) screening. The second term is piezoelectric coupling, to be considered for our coherently strained SiGe samples, with $\alpha' = 3$ (resp. 5) for weak (resp. strong) screening. The exponents may be decreased by one in disordered systems because of “dynamic” rather than “static” screening. $A$ and $A'$ are prefactors related to the deformation ($\Xi_0$) and piezoelectric ($\epsilon_{pz}$) coupling constants. To test the validity of Eq. (1) for our measurements, we write it: $P_E + (AT_l^{\alpha} + A'T_l^{\alpha'}) = \cdots$
(\(AT^2 + AT' T^2\)). Figure 3(b) shows that by adding to \(P_E\) a constant \(P_0(T)\) chosen separately for each \(T_i\), the whole set of curves for a given density falls on the same master curve. Hence \(P_E(T_c, T_i) + P_0(T_i)\) depends only on \(T_c\), and this dependence is the sum of two power laws. We have verified that the \(P_0(T_i)\) dependence is the same, thus proving that our data are in agreement with Eq. (6). Similar results are obtained for all densities and samples. For \(p_e\) close to \(p_c\), a power law with \(\alpha' \approx 3\) (resp. \(\alpha \approx 5 - 6\)) dominates at low (resp. large) \(T_c\). These power laws are confirmed by fitting the \(P_E(T_c)\) data with Eq. (6). Fig. 3(a) shows the good quality of the fit when \(\alpha = 5\) and \(\alpha' = 3\) are imposed. For \(p_e > 1.35 \times 10^{11}\) cm\(^{-2}\), a term \(A'(T_c^2 - T_i^2)\) is added in the fit, corresponding to the cooling through the contacts which increases when \(\rho\) decreases \(\approx 24\). \(A'\) has the same order of magnitude as the value given by the Wiedeman-Franz law. Our data are thus compatible with Eq. (6), the two main cooling processes being hole-phonon coupling through weakly screened deformation and piezoelectric potentials. Weak screening at low temperatures has been pointed out in \(p\)-SiGe \([21,26]\) and Si-MOSFETs \([24,25]\). \(A\) and \(A'\) are obtained from a fit with Eq. (6) assuming \(\alpha = 5\) and \(\alpha' = 3\) [Fig. 3(a)]. Following Ref. [21] we then obtain \(\Xi_u = 2.7 \pm 0.3\) eV and \(\epsilon_{pz} = (3.4 \pm 0.9) \times 10^{-3}\) C/m\(^2\) for all the densities. Our \(\Xi_u\) value is compatible with the \(\Xi_u \approx 3.0\) eV found in \(p\)-SiGe at \(p_e = (3.5 - 13) \times 10^{11}\) cm\(^{-2}\) \([21,26]\). Our \(\epsilon_{pz}\) is somewhat lower than the \(\epsilon_{pz} \approx 1.6 \times 10^{-2}\) C/m\(^2\) obtained in \(p\)-Si\(_{0.8}\)Ge\(_{0.2}\) \([21]\), however in Ref. [21] \(\epsilon_{pz}\) was found to be much smaller than in Ref. [22].

In order to further prove the validity of the heating analysis, the \(T_c\) extracted at \(B = 0\) is compared to the temperature obtained using the damping of the SdH oscillations \([21,22,24,27]\). To get rid of the \(\rho_{xx}(T)\) dependence due to carrier scattering \([27]\), the “thermometer” is the difference between a minimum of \(\rho_{xx}(B)\) and the previous maximum. Its \(T\) dependence results from a different physical situation (the density of states oscillations) than at \(B = 0\). The magnetic field values (\(B \approx 1\) T) are low to ensure that the electron-phonon coupling laws extracted are close to those at \(B = 0\) [27]. As shown in the inset of Fig. 3(a) for \(p_e = 1.19 \times 10^{11}\) cm\(^{-2}\) and \(T_i = 150\) mK, the two methods are in very good agreement. A similar agreement is obtained for other \(T_i\) values and for \(p_e = 3.9 \times 10^{11}\) cm\(^{-2}\). Thus, the \(\rho(E)\) dependence can be attributed to hole heating, implying that it does not bring new physical information on the possible MIT when compared to the \(\rho(T)\) dependence.

To study \(E\)-field scaling in spite of heating effects, the condition \(L_E(E) < L_\Phi[T_c(E)]\) must be fulfilled. Figure 4 shows how \(L_\Phi\) and \(L_E\) depend on \(E\). The vertical scale is arbitrary because \(L_\Phi\) and \(L_E\) are not given by the QPT theory, but assuming \(z = 1\) for strongly interacting particles [19], \(L_\Phi \sim T^{-1}\) and \(L_E \sim E^{-1/2}\). Although the “metallic” \(\rho(T)\) in our samples is compatible with a Fermi liquid description [12], it is instructive to consider the possibility of a QPT: the full line corresponds to \(L_\Phi[T_c(E)]\) in our case, for \(p_e \approx p_c\), using the experimental \(T_c(E)\), obtained in the \(E\) interval indicated by the symbols. An upper limit for \(E\) arises since the longitudinal potential drop \(V = EL\) has to be kept small compared to the gate voltage. As we attributed the measured \(\rho(E)\) dependence to heating, the \(L_E(E)\) line (long dashed) crosses the \(L_\Phi[T_c(E)]\) curve beyond this interval. The \(E\)-field scaling can be investigated only beyond this crossing point, (“E scaling” interval in Fig. 4), provided \(L_\Phi\) and \(L_E\) do not reach their microscopic limit.
The short dashed line in Fig. 4 is \( L_\Phi[T_c(E)] \) for Si-MOSFETs. Again, the \( L(E) < L_\Phi[T_c(E)] \) prescription leads to a lower limit for \( E \). This agrees with the results of Ref. [7] where the quality of the E-field scaling is improved when \( E \) is larger than a minimum value (the grey area in Fig. 4 corresponds to their cut-offs of 50 to 500 pW). For p-GaAs, the \( \alpha = 5 \) exponent quoted in Ref. [22] leads to a similar situation, but the thermal coupling should be larger than in Si-MOSFETs. Unscreened piezoelectric coupling would lead to an upper E-field limit. A better knowledge of the power loss laws would allow a quantitative use of the \( L(E) < L_\Phi[T_c(E)] \) prescription. The field \( E_c \) defined by \( L(E_c) = L_\Phi[T_c(E_c)] \) could be extracted as the limit beyond which the experimental \( P_E(T_c) \) law differs from the power loss law, thus yielding the important physical result \( L_\Phi(T)/L_E(E) = [T_c(E_c)/T]^{1/z}(E/E_c)^{1/(z+1)} \).

The ratio of the exponents in the experimental scaling laws \( \rho(\delta_n/T^0) \) and \( \rho(\delta_n/E^0) \) has been proposed as an indicator of the \( \rho(E) \) dependence origin [14]. For the QPT scaling, \( a/b = z/(z+1) = 0.5 \). For carrier heating, Eq. (3) gives \( T_c \sim (E^{2/\alpha})\rho^{-1/\alpha} \), thus \( a/b = 2/\alpha \) neglecting \( A' \), \( T_i \) and the \( \rho(T) \) dependence. We find experimentally \( a/b = 0.42 \pm 0.08 \), while 0.4 and 0.67 are expected for our \( \alpha \) and \( \alpha' \), thus the \( a/b \) criterion can hardly be used [31].

In summary, we have shown that in a 2D hole system exhibiting the \( B = 0 \) MIT characteristics, with \( T \) and \( E \) scaling, the \( \rho(E) \) dependence close to the MIT could be interpreted as being due to hole heating. Thus, in our case, the experimental \( \rho(E, p_s) \) and \( E \)-field scaling are not an indication that the MIT features can be attributed to a QPT, and the separate extraction of \( \nu \) and \( z \) is not possible. However, there is an \( E \) interval defined by \( L(E) < L_\Phi[T_c(E)] \) where \( E \)-field scaling can be investigated independent of hot carrier effects.

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