Extremal Kerr black hole/CFT correspondence in the five-dimensional Gödel universe

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Abstract

We extend the method of Kerr/CFT correspondence recently proposed in [arXiv:0809.4266 [hep-th]] to the extremal (charged) Kerr black hole embedded in the five-dimensional Gödel universe. With the aid of the central charges in the Virasoro algebra and the Frolov-Thorne temperatures, together with the use of the Cardy formula, we have obtained the microscopic entropies that precisely agree with the ones macroscopically calculated by Bekenstein-Hawking area law.
1 Introduction

During the past decades, a lot of efforts have been devoted to studying the origin of Bekenstein-Hawking entropy for the black holes. Great progress has been made in their statistical interpretation thanks to Strominger and Vafa’s remarkable work [1] on the investigation of the microscopic origin of the five-dimensional, supersymmetric (extremal) black hole entropy by using the holographic duality in the context of string theory. When the near-horizon limit has been taken, their work can be viewed as a typical example of the AdS/CFT correspondence [2, 3, 4], which shows that there exists a duality between the higher dimensional gravity and the CFT living on the boundary in less dimensions, providing a powerful tool to study the microscopic statistical mechanics of the black holes. By contrast, without using any supersymmetry, Strominger [5] had successfully evaluated the Bekenstein-Hawking entropy of the three-dimensional BTZ black hole by counting the number of the microstates in the two-dimensional CFT induced on the boundary of spatial infinity [6]. Besides, when the boundary is restricted to the horizon, the statistical entropy of the general black hole was derived via studying the relationship between black hole thermodynamics and the two-dimensional near-horizon CFT [7, 8, 9, 10].

Quite recently, Guica, Hartman, Song and Strominger [11] put forward a new method called as Kerr/CFT correspondence to derive the microscopic entropy of the four-dimensional extremal Kerr black hole by identifying the quantum states in its near-horizon region with the two-dimensional chiral CFT on the spatially infinite boundary. The main ideas of their method go as follows: On the basis of the near-horizon geometry found in [12, 13], one can construct diffeomorphisms that preserve a properly chosen boundary condition at the infinity. These diffeomorphisms generate one copy of the Virasoro algebra and contribute to the conserved charges. By computing the Dirac brackets of these charges, the central charge relative to the angular momentum of the extremal black hole can be obtained. Making use of the Frolov and Thorne temperature [14], the microscopic entropy in the dual CFT can be reproduced via the Cardy formula. Following this work, the microscopic entropies of the three-dimensional black hole and Kerr-AdS black holes in diverse dimensions were derived [15, 16]. In Refs. [17] [18], the Kerr/CFT correspondence was applied to the black holes with $U(1)$ gauge symmetry. Further extensions [19] [20] [21] [22] [23] has been made in supergravity theory and string theory.

In this paper, we shall apply the Kerr/CFT correspondence to the extremal (charged) Kerr black hole embedded in the five-dimensional Gödel universe [24, 25], which is dubbed as a (charged) Kerr-Gödel black hole for shortness. These black hole metrics are exact solutions
in the five-dimensional minimal supergravity. They possess some peculiar features such as the presence of closed time-like curves, and the absence of globally spatial-like Cauchy surface. Our paper is organized as follows. In Section 2, we simply review the Kerr-Gödel black hole and obtain its near-horizon metric under the extremal condition. Based upon the near-horizon metric, we then calculate the central charge and microscopic entropy of the extremal Kerr-Gödel black hole in the chiral dual CFT. In Section 3, we extend the analysis to the extremal charged Kerr-Gödel black hole. Finally, in Section 4, we end up with our conclusions.

2 Extremal Kerr-Gödel black hole and the dual CFT

In this section, we will generalize the method developed in [11, 18] to explore the duality between the extremal Kerr-Gödel black hole [24] and the chiral CFT by showing the equality of the microscopic CFT entropy and the Bekenstein-Hawking entropy. We first give a brief review of the Kerr-Gödel black hole solution [24] and then study its near-horizon geometry. Let’s start with the metric

\[
d s^2 = -(1 - \frac{2\mu}{\hat{r}^2})d\hat{t}^2 + \frac{d\hat{r}^2}{\Delta_{\hat{r}}} - 4(j\hat{r}^2 + \frac{\mu a}{\hat{r}^2})(\cos^2 \theta d\hat{\phi} + \sin^2 \theta d\hat{\psi})d\hat{t} - 4\hat{r}^2(j^2\hat{r}^2 + 2j^2\mu - \frac{\mu a^2}{2\hat{r}^4})(\cos^2 \theta d\hat{\phi} + \sin^2 \theta d\hat{\psi})^2 + \hat{r}^2(d\theta^2 + \cos^2 \theta d\hat{\phi}^2 + \sin^2 \theta d\hat{\psi}^2), \\
\]

(2.1)

and the gauge potential which takes the form

\[
A = \sqrt{3}j\hat{r}^2(\cos^2 \theta d\hat{\phi} + \sin^2 \theta d\hat{\psi}), \\
\]

(2.2)

where

\[
\Delta_{\hat{r}} = 1 - \frac{2\mu}{\hat{r}^2} + \frac{16j^2\mu^2}{\hat{r}^2} + \frac{8j\mu a}{\hat{r}^2} + \frac{2\mu a^2}{\hat{r}^4}. \\
\]

(2.3)

In the above, the parameters \( \mu \) and \( a \) are related to the mass and the angular momenta, respectively, while \( j \) defines the scale of the Gödel background and is responsible for the rotation of the universe. Without loss of generality, we assume \( \mu, a \) and \( j \) are all positive. The metric (2.1) describes the rotating black hole with two equal angular velocities in the five-dimensional Gödel universe. The angular velocities and the electro-static potential on the horizon are given by

\[
\Omega_{\hat{\psi}}^H = \Omega_{\hat{\phi}}^H = 2(j\hat{r}_H^4 + \mu a)/\eta, \quad \Phi_H = -2\sqrt{3}j\hat{r}_H^2(j\hat{r}_H^4 + \mu a)/\eta. \\
\]

(2.4)

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where the event horizon \( \hat{r}_H \) and constant \( \eta \) read
\[
\hat{r}_H^2 = \mu - 4\mu ja - 8j^2 \mu^2 + \mu \sqrt{(1 - 8\mu j^2)(1 - 8\mu j^2 - 8ja - 2\mu^{-1}a^2)},
\]
\[
\eta = \hat{r}_H^4 + 2\mu a^2 - 4j^2 \hat{r}_H^6 - 8\mu j^2 \hat{r}_H^4.
\]

The temperature and the entropy are
\[
S = 2\pi \hat{r}_H \sqrt{\eta}, \quad T_H = \frac{\hat{r}_H^2 - \mu + 4\mu ja + 8j^2 \mu^2}{\pi \hat{r}_H \sqrt{\eta}}.
\] (2.5)

In [26], the conserved quantities such as the mass, the angular momenta and the electrical charge have been computed as
\[
M = \frac{3}{4} \pi \mu - 8\pi j^2 \mu^2 - \pi j\mu a,
\]
\[
J_\phi = J_\psi = \frac{1}{2} \pi \mu a - \pi j\mu a^2 - 4\pi a j^2 \mu^2,
\] (2.6)
\[
Q = 2\sqrt{3}\pi j\mu a.
\]

Treating the Gödel parameter \( j \) as a fixed constant, we find that the variation of the mass, the angular momenta and the electrical charge satisfy the differential form of the first law
\[
dM = T_H dS + \Omega^H_\phi dJ_\phi + \Omega^H_\psi dJ_\psi + \Phi^H dQ.
\] (2.7)

However, if we take \( j \) as a thermodynamical variable, a conjugate generalized force should be introduced [25] to fulfill the first law of the black hole thermodynamics.

Now we turn our attention to the analysis of the near-horizon geometry of the extremal Kerr-Gödel black hole. The extremity condition is
\[
j = \frac{\left(\mu - r_0^2\right) \sqrt{\lambda}}{4\mu^{3/2}}, \quad a = \frac{r_0^2}{\sqrt{2\mu}},
\] (2.8)
where \( r_0 \) is the horizon radius of the extremal black hole, which makes the temperature \( T_H \) vanish. Under the extremal condition (2.8), to obtain the near-horizon geometry of the Kerr-Gödel black hole, we perform the coordinate transformation as follows
\[
\hat{r} = r_0 + r_0 \lambda r, \quad \hat{t} = \frac{r_0 \left(\mu + r_0^2\right) \sqrt{2(2\mu - r_0^2)}}{8\mu^{3/2}\lambda} t,
\]
\[
\hat{\phi} = \phi + \frac{\sqrt{2\mu - r_0^2}}{4r_0 \lambda} t, \quad \hat{\psi} = \psi + \frac{\sqrt{2\mu - r_0^2}}{4r_0 \lambda} t,
\] (2.9)
and then take the scaling parameter \( \lambda \) to zero, thus sending the metric (2.1) to the form
\[
ds^2 = \frac{1}{4r_0^2} \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + 4d\theta^2 - \frac{r_0^4 (3\mu^2 - r_0^4)}{2\mu^3} \cos^2 \theta \sin^2 \theta (d\phi - d\psi)^2 + \frac{r_0^2 (2\mu - r_0^2) (\mu + r_0^2)^2}{2\mu^3} \left[ \cos^2 \theta (d\phi + \alpha r dt)^2 + \sin^2 \theta (d\psi + \alpha r dt)^2 \right] \right],
\] (2.10)
where

\[ \alpha = \frac{r_0(3\mu - r_0^2)}{2(\mu + r_0^2)\sqrt{2\mu - r_0^2}}. \] (2.11)

The near-horizon metric (2.10) depicts a 3-sphere bundle over the \( AdS_2 \) space. It only partially covers the near-horizon geometry of the extremal Kerr-Gödel black hole (2.1). One can perform global coordinate transformation to the coordinates \((t, r)\) in order to make the metric (2.10) overlay the whole space in a single patch [13, 19].

By virtue of the conformal structure of the near-horizon metric (2.10), it is possible for us to compute the central charges in the chiral CFT. Since there exist two rotations corresponding to \( \phi \) and \( \psi \), respectively, when the near-horizon metric (2.10) is assumed to have a certain suitable boundary, it can be shown as did in [11] that this metric can possess two commuting diffeomorphisms

\[
\begin{align*}
\zeta^{(1)}_n &= -e^{-in\phi}\partial_\phi - ir e^{-in\phi}\partial_r, \\
\zeta^{(2)}_n &= -e^{-in\psi}\partial_\psi - ir e^{-in\psi}\partial_r, \quad (n = 0, \pm 1, \pm 2, \ldots)
\end{align*}
\] (2.12)

which preserve the chosen boundary and generate two copies of commuting Virasoro algebra

\[
i[\zeta^{(i)}_m, \zeta^{(j)}_n] = (m - n)\delta_{ij}\zeta^{(i)}_{m+n} \quad (i, j = 1, 2). \] (2.13)

Each diffeomorphism \( \zeta^{(i)}_m \) is associated to a conserved charge defined by [27, 28, 29, 30]

\[
Q_{\zeta^{(i)}_n} = \frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta^{(i)}}[h, g],
\] (2.14)

where \( \partial\Sigma \) is a spatial slice that extends to the infinity and the 3-form \( k_{\zeta^{(i)}}[h, g] \) is written as

\[
k_{\zeta}[h, g] = -\frac{1}{12} \epsilon_{\alpha\beta\gamma\rho\sigma} [\zeta^\rho \nabla^\sigma h - \zeta^\sigma \nabla^\rho h + \zeta_\nu \nabla^\rho h^{\sigma\nu} + \frac{1}{2} h \nabla^\rho \zeta^\sigma \\
- h^{\rho\nu} \nabla_\nu \zeta^\sigma + \frac{1}{2} h^{\rho\nu} (\nabla^\sigma \zeta_\nu + \nabla_\nu \zeta^\sigma) dx^\alpha \wedge dx^\beta \wedge dx^\gamma].
\] (2.15)

In the above equation, \( \zeta = \zeta^{(i)}_n \), and \( h_{\rho\sigma} \) denotes the deviation from the background metric (2.10). The Dirac brackets of the conserved charges corresponding to the diffeomorphisms \( \zeta^{(i)}_m \) and \( \zeta^{(i)}_n \) yield the central term in the Virasoro algebra

\[
\frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta^{(i)}}[\mathcal{L}_{\zeta^{(i)}}g, g] = -\frac{i}{12} \epsilon_{(m^3 + \beta m)d_{m+n,0}},
\] (2.16)

where

\[
\mathcal{L}_{\zeta^{(i)}} g_{\rho\sigma} = \zeta^{(i)\nu}_{\rho} \partial_{\nu} g_{\rho\sigma} + g_{\nu\sigma} \partial_{\rho} \zeta^{(i)\nu}_{\rho} + g_{\nu\rho} \partial_{\sigma} \zeta^{(i)\nu}_{\nu}.
\] (2.17)
is the Lie derivative of the background metric (2.10) relative to the vector field $\zeta^{(i)}$, $c_i$ denote the central charges in the Virasoro algebra, while the constant $\beta$ is trivial since it can be absorbed by a shift in $Q(0)$. In terms of the background metric (2.10), the quantities associated with the central charges $c_i$ are

$$\frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta^{(i)}} [L_{\zeta^{(j)}} g, g] = \frac{-\pi r_0^3 (\mu + r_0^2) \sqrt{2(2\mu - r_0^2)} \alpha \mu^3 m^3}{8\mu^{9/2}} + (2\mu - r_0^2)(\mu + r_0^2)^2 m] \delta_{m+n,0}.$$  (2.18)

Comparing Eq. (2.18) with (2.16), we get the central charges in the chiral Virasoro algebra

$$c_1 = c_2 = \frac{3\pi r_0^3 (\mu + r_0^2) \sqrt{2(2\mu - r_0^2)}}{2\mu^{3/2} \alpha}. \quad (2.19)$$

With the central charges (2.19) in hand, we have to determine the Frolov-Thorne temperatures to calculate the chiral CFT entropy via the Cardy formula. Let $T_1$ and $T_2$ represent the Frolov-Thorne temperatures associated with the azimuthal angles $\phi$ and $\psi$, respectively. Using the definition of the Frolov-Thorne temperature in [19], we obtain

$$T_1 = T_2 = - \lim_{r_H \to r_0} \frac{T_H}{\Omega_H - \Omega_0^H} = \frac{1}{2\pi \alpha}, \quad (2.20)$$

where

$$\Omega_0^H = \Omega_0^\psi (T_H = 0) = \frac{\sqrt{2\mu^3}}{r_0^3 (\mu + r_0^2)} \quad (2.21)$$

is the angular velocity of the extremal Kerr-Gödel black hole. Finally, with the aid of the Cardy formula for the microscopic entropy in the chiral CFT, we get

$$S_1 = S_2 = \frac{\pi^2}{3} c_2 T_2 = \frac{\pi^2 r_0^3 (\mu + r_0^2) \sqrt{2(2\mu - r_0^2)}}{4\mu^{3/2}}, \quad (2.22)$$

which precisely agree with the macroscopic Bekenstein-Hawking entropy of the extremal Kerr-Gödel back hole

$$S(T_H = 0) = \frac{\pi^2 r_0^3 (\mu + r_0^2) \sqrt{2(2\mu - r_0^2)}}{4\mu^{3/2}}. \quad (2.23)$$

3 Extremal charged Kerr-Gödel black hole and CFT duality

In this section, we will extend the above analysis to the extremal charged Kerr black hole in the Gödel universe, which is an exact solution [25] in the five-dimensional Einstein-Maxwell-Chern-Simons supergravity theory. Parameterized by four constants $(\mu, a, q, j)$,
which correspond to the mass, the angular momenta, the electric charge and the scale of the G"{o}del background, respectively, the metric has the form

\[ ds^2 = - \frac{\dot{\hat{r}}^2 V(\hat{r})}{4B(\hat{r})} dt^2 + \frac{d\hat{r}^2}{V(\hat{r})} + \dot{\hat{r}}^2 [d\theta^2 + \cos^2 \theta \sin^2 \theta (d\hat{\phi} - d\hat{\psi})^2] \\
+ 4B(\hat{r}) \left( \cos^2 \theta d\hat{\phi} + \sin^2 \theta d\hat{\psi} - \frac{1}{2} G(\hat{r}) d\hat{t} \right)^2 , \]  

(3.1)

and the gauge potential is given as

\[ A = \sqrt{3} q^2 \hat{r}^2 d\hat{t} + \sqrt{3} (j \hat{r}^2 + 2jq - \frac{aq}{2\hat{r}^2}) (cos^2 \theta d\hat{\phi} + sin^2 \theta d\hat{\psi}) , \]  

(3.2)

where

\[ G(\hat{r}) = j\hat{r}^2 + 3jq + \frac{(2\mu - q)a}{2\hat{r}^2} - \frac{q^2 a}{2\hat{r}^4} ; \]
\[ B(\hat{r}) = -j\hat{r}^2 (\hat{r}^2 + 2\mu + 6q) + 3jq a + \frac{(\mu - q)a^2}{2\hat{r}^2} - \frac{q^2 a^2}{4\hat{r}^4} + \frac{\hat{r}^2}{4} ; \]
\[ V(\hat{r}) = 1 - \frac{2\mu}{\hat{r}^2} + \frac{8j(\mu + q)[a + 2j(\mu + 2q)]}{\hat{r}^2} + \frac{2(\mu - q)a^2 + q^2[1 - 16ja - 8j^2(\mu + 3q)]}{\hat{r}^4} . \]  

(3.3)

The line element (3.1) is the charged generalization of the metric (2.1). On the choice of proper parameters, it can cover other solutions. For example, when \( j = 0 \), it becomes the Kerr-Newman black hole solution with two equal rotations in the five-dimensional minimal supergravity. If we assume \( j = 0 \) and \( q = m \), the metric (3.1) reduces to the BMPV black hole [31, 32], whose microscopic entropy has been recently obtained [21, 22, 23] via the method of Kerr/correspondence. Now we present some useful quantities related to the charged Kerr-G"{o}del black hole (3.1), such as the angular velocities, the temperature and the Bekenstein-Hawking entropy. We have

\[ \Omega_\hat{\phi} = \Omega_\hat{\psi} = \frac{G(\hat{r}^+_\pm)}{2B(\hat{r}^+_\pm)} , \quad T_H = \frac{\hat{r}^+_\pm V(\hat{r}^+_\pm)}{8\pi \sqrt{B(\hat{r}^+_\pm)}} , \quad S = \pi^2 \hat{r}^2 \sqrt{B(\hat{r}^+_\pm)} , \]  

(3.4)

in which and what follows, the prime \( ' \) denotes the derivative with respect to the coordinate \( \hat{r} \). The event horizon \( \hat{r}^+_\pm \) is determined by the equation \( V(\hat{r}^+_\pm) = 0 \) and reads

\[ \hat{r}^2_{\pm} = \mu - 4j(\mu + q)a - 8j^2(\mu + q)(\mu + 2q) + \sqrt{\delta} , \]
\[ \delta = [\mu - q - 8j^2(\mu + q)^2] \times [\mu + q - 2a^2 - 8j(\mu + 2q)a - 8j^2(\mu + 2q)^2] . \]  

(3.5)

Obviously, \( \delta > 0 \) guarantees that the event horizon is well defined. However, when \( \delta = 0 \), the event horizon degenerates to the extremal case, in which the charged Kerr-G"{o}del black
hole has zero temperature but finite entropy

\[ S(T_H = 0) = \pi^2 r_0^2 \sqrt{B(r_0)}, \quad (3.6) \]

where \( r_0 \) is the event horizon of the extremal charged Kerr-Gödel black hole.

As before, we are interested in studying the equivalence property between the Bekenstein-Hawking entropy and the one of the extremal charged Kerr-Gödel black hole in the chiral CFT. To do so, we first perform the coordinate transformation

\[
\hat{t} = t + \frac{2\sqrt{B(r_0)}}{\omega r_0^2} t, \quad \hat{\phi} = \phi + \frac{G(r_0)}{2B(r_0)} \hat{t}, \quad \hat{\psi} = \psi + \frac{G(r_0)}{2B(r_0)} \hat{t}, \quad (3.7)
\]

where \( \omega = V''(r_0)/2 \), and then take \( \lambda \to 0 \) to get the near-horizon metric

\[
d\hat{s}^2 = \frac{1}{\omega} \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right) + r_0^2 \left[ d\theta^2 + \cos^2 \theta \sin^2 \theta (d\phi - d\psi)^2 \right] + 4B(r_0)(\cos^2 \theta d\phi + \sin^2 \theta d\psi + kr dt)^2, \quad (3.8)
\]

in which the constant \( k \) is defined by

\[
k = \frac{B'(r_0)G(r_0) - G'(r_0)B(r_0)}{\omega r_0 B^{3/2}(r_0)}. \quad (3.9)
\]

Next, for the background metric (3.8), we directly calculate the central charges in the chiral Virasoro algebra following the procedure in the previous section. The central terms associated with the diffeomorphisms \( \xi^{(1)} \) and \( \xi^{(2)} \) can be read off as

\[
\frac{1}{8\pi} \int_{\partial\Sigma} k_{\xi^{(i)}} [\mathcal{L}_{\xi^{(i)}}, g, g] = -\frac{i}{2} \pi kr_0^2 \sqrt{B(r_0)}[m^3 + 2\omega B(r_0)m] \delta_{m+n,0}. \quad (3.10)
\]

By comparison with Eq. (2.16), we obtain the central charges

\[
c_1 = c_2 = 6k \pi r_0^2 \sqrt{B(r_0)}. \quad (3.11)
\]

The Frolov-Thorne temperatures of the chiral CFT can be expressed in the forms

\[
T_1 = T_2 = -\frac{\partial_r T_H}{\partial_r \Omega^\phi} \bigg|_{r_+ = r_0} = \frac{1}{2\pi k}. \quad (3.12)
\]

Finally, substituting Eqs. (3.11) and (3.12) into the Cardy formula, we can compute the microscopic entropies in the chiral CFT as

\[
S_1 = S_2 = \frac{\pi^2}{3} c_2 T_2 = \pi^2 r_0^2 \sqrt{B(r_0)}. \quad (3.13)
\]
Clearly, $S_1$ and $S_2$ are consistent with the one of the macroscopically calculated entropy (3.6). Note that the derivation of Eq. (3.13) heavily relies on the non-vanishing angular velocities of the horizon. For example, if we choose a special but nontrivial relation of the parameters, namely [25]

\[(\mu - a)a^2 + 4j(\mu - q)(\mu + 2q)a - 4j^2(3\mu + 5q)q^2 = 0, \tag{3.14}\]

which leads to $V(r_+) = g(r_+) = 0$, the Kerr/CFT correspondence fails to derive the microscopic entropies in Eq. (3.13). Besides, when $j = 0$ and $m = q$, carrying out the coordinate transformation

\[
\hat{\phi} \to \phi + \psi, \quad \hat{\psi} \to \phi - \psi, \quad \theta \to \theta/2, \tag{3.15}
\]

to the metric (3.1), we get the BMPV black hole metric that takes the same form as the one in [21]. Our derivation reproduces the microscopic entropy there.

## 4 Summary

In this paper, we have derived the microscopic entropy of a five-dimensional extremal (charged) Kerr-Gödel black hole by utilizing the Kerr/CFT correspondence recently suggested in [11]. Making use of the near-horizon limit procedure [13], we got the expected near-horizon metrics of the extremal black holes. They possess the topological structure of a 3-sphere bundle over $AdS_2$. Choosing proper boundaries due to the near-horizon metrics, we have two diffeomorphisms $(\zeta_n^{(1)}, \zeta_n^{(2)})$ that preserve these boundaries. Both the diffeomorphisms generate two copies of commuting Virasoro algebra but the Dirac brackets of their corresponding conserved charges induce non zero central terms in the Virasoro algebra. From the central terms, we can obtain the central charges. In favor of the Frolov-Thorne temperatures, the entropies of the extremal (charged) Kerr-Gödel black hole were calculated via the Cardy formula. The entropies got in the chiral dual CFT take the same value as the ones of the Hawking-Bekenstein entropy. Our work implies that the Kerr/CFT correspondence is valid in the background of the five-dimensional Gödel universe, and recovers the previous results [21, 22, 23] in some special cases.
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