A new approach to determining fractal dimension of soil pore space from experimental data on moisture filtration

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Abstract. The value of the fractal dimension of the soil structure is usually calculated by the geometric method based on particle size distributions. There is a hope to express soil-hydrophysical functions through the fractal dimension of the soil pore space found in some more direct, simple and physically independent way.

1. Introduction

Relating to the growing shortage of fresh water throughout the world, the task of saving water resources is urgent. In a number of countries in the Middle East, decision task of saving water resources is already extremely necessary. There is a number of regions in Russia where the solution of water resources saving problem is acutely related to the issue of the region survival. One possible solution lies in water savings in agriculture. For this purpose, it is necessary to obtain accurate estimations of water consumption, for example, for irrigation or water transportation through canals. Accurate estimation of irrigation rates is possible if we learn how to calculate the main hydrophysical functions of the soil: water-retention function (WRF) and hydraulic conductivity (HC). Works of the early 90s of the previous century [1-4] laid the foundation of a completely new approach - modeling and calculation of WRF & HC, which continues to evolve to the present. The new approach lies in the development of the concept of structural-functional hydrophysics of soils [5], the description of the soil pore space for estimation of hydrophysical functions using the theory of fractals [4]. Fractal models of the soil structure, developed based on the generalization of classical fractals of the Sierpinski carpet, Menger sponge, are widely proposed.

Fractal models conceptually divide the volume of soil into fragments. Some models consider the fragmentation of the solid phase of the soil, others - fragmentation of the pore space [6, 7]. The models are working with the total discrete length of fragments (the pore size summary or the size mineral particles of solid phase). The usual fractal generator procedure is applied and the total length of the fragments is iteratively reduced. During the procedure of changing the scale of length of the fragments (iteration) we numerically reduce the pore size, the soil is formally drained. Change of the iterative scale during the modeling of the drying branch of the soil water retention curve is similar to the physical process of drainage. First the water leaves pores of smaller size. Two types of theoretical models were used: in the first case, it is assumed that all solid particles have the same size; the fractal
size of the mass is used to scale the solid phase and pores’ space \((D_m)\). In the second case, the pores have the same size, which is used for scaling (the fractal dimension of the volume \(D_f\)).

The mass fractal dimension of the soil can be calculated from the ratio: \(P = 1 - \left(\frac{V}{\rho}\right)^{3-D_m}\), where 
\[
\left(\frac{V}{\rho}\right) = V^p - \text{solid phase volume, } 'P' - \text{total soil porosity. The fractal dimension of the pore volume is calculated in the same way [1]: } D_f = \frac{\log P}{\log b'} + E \ 'E' - \text{size of Euclidean space, } E=2; 3). If the pore space of the soil is modeled by an ensemble of cylindrical pores, the scale factor could be calculated by the ratio \(b' = (r \cdot \sqrt{\pi})^{-1}\), where \(r\) – capillary radius. Assuming that the ratio of the maximum and minimum particle radii \(R_{max}/R_{min}\) is equal to the ratio of the maximum and minimum pore radii \(R_{max}/R_{min}\), the fractal dimension of the pore space \(D_f\) could be calculated using three parameters [8]:

\[
D_f = 3 - \frac{\log(1-P)}{\log(R_{min}/R_{max})} \tag{1}
\]

PSF model [9-11] provides a direct method for the estimation of fractal dimension of the pore and solid boundaries from the measured size distribution of elementary soil particles (ESP):

\[
\frac{M(R < R_i)}{M_i} = \alpha^{0.5} \left(\frac{R_i}{L}\right)^{E-D_f}, R_{min} \leq R_i \leq R_{max}, \tag{2}
\]

where: \(\alpha <1\) – iteration factor; \(L\) – size indicator \((\alpha R_{max} = L)\); \(M_i\) – total mass of fragments; \(E\) – Euclidean dimension, equal to \(3\); \(D_f\) – fractal dimension with incomplete fragmentation (presence of aggregates in the soil); \(R_{max}\) and \(R_{min}\) – maximum and minimum sizes of fragments. In the case of complete fragmentation of the soil, equations 1 and 2 could be converted to the known expression [6, 7]:

\[
\frac{M(< R_i)}{M_i} = \left(\frac{R}{R_{max}}\right)^{3-D_f} \tag{3}
\]

In practical determination of the magnitude of the fractal dimension of the soil, there is a number of issues that complicate the solution of this problem.

Determination of the boundary conditions, search of the maximum and minimum pore radii of a capillary-porous medium remains open question.

The mass of water is almost equal to its volume and, therefore, the volume and mass fractal dimensions of the soil structure in the simulation of its water-retention capacity coincide with a certain degree of accuracy. For the practical estimation of the fractal dimension, it is necessary to have data on the ESP size distribution and aggregate composition of the soils, to determine the bulk density and particle density, to estimate the porosity of the soils. Calculating the porosity of the soils is not difficult; but the measurements of particle size distribution, aggregative analysis and the determination of the particle density require considerable efforts from the researcher. In addition, calculation methods have an error in calculating the fractal dimension. The error accumulates with an increase in the number computational procedures. Summarily this leads to the errors in the modeling and estimation of the soil water-retention capacity, for example, by the PSF model.

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Therefore, the aim of our research was to propose the concept of a direct, physically sound, experimental method for determining the fractal dimension of the pore space of soils.

The theoretical prerequisites for the development of the presented method are as follows. According to the theory of percolation, the medium “suddenly” acquires the properties of moisture conductivity when all its pores and voids are filled with liquid [12]. This means that when entering the
soil, the water accumulates until all the pores are filled. Then a process of dumping of all volume of moisture, accumulated in a porous medium (soil), occurs. The case, when all voids of the soil could be filled with water, is possible in two ways: the constant influx of moisture or deformation of the compression of the pore space of soils. In the first case, when a small additional amount of water gets into fractal porous medium, the water, accumulated in the pores, is released suddenly, as it occurs in the siphon [13]. This phenomenon is called the Hurst effect. The Hurst effect [14] is well known in hydrology in the studies of the river flow [15].

Since the distribution of pores size and soil volume is random [16-18] and the pores often do not form a single connected pore space, the filling of the pores with water or drying process are random processes. Therefore, solution should be sought in the theory of random processes. The drainage and moistening of the soil are pulsed [13].

There is a special term "flicker process" referred to the processes occurring as a pulse - a short flash with subsequent attenuation of the frequency of the periodic function of the process over time. Such a process could be single or consist of a series of “flashes”. The standard deviation of such a random process (with zero average) is defined as $T^{H}5$ (where 'T' is period), value variance $X_{r}$, accordingly, must be equal to the entire time period $S_{X_{r}} = T$, function of increments $X_{r}$; this is the Dirac function, which means there are no correlations in successive values of increments of magnitude ‘$X_{r}$’ and the constancy of the spectrum at all frequencies ($f(\nu) = const$, $\nu$ - frequency) [13, 15].

In the number of hydrological processes in nature, this proportionality is disturbed. Dispersion is equal: $S_{X_{r}} = T^{H}$, $0 < H < 1$, and the value $H$ is called the Hurst index (in honor of the British climatologist and hydrologist, who first calculated this index for the flood situation on r. Nile) [13, 15, 19]. Hurst index is a measure used in the analysis of time series, for example: during the analysis of the frequency of floods on a river over a long period of time. Its value decreases when delay between two identical pairs of values in the time series increases. Hurst index is related to Hausdorff-Bezikovich dimension ($D$) of volume by this ratio: $D = 2 + H$.

2. Materials and methods

Objects: soils Albeluvisols Umdric classification WRB; FAO.

In laboratory studies of moisture filtration through the undisturbed soil monolith, the value of the flow volume of moisture according standard test methods for measurement of hydraulic conductivity of saturated porous materials was determined [20]. Studies were conducted in the area of initial, non-stationary filtration. Moisture filtration is associated with the structure of the pore space of the soil. We investigated the filtration of moisture at its initial stages in the non-stationary area. Assuming that the soil has a fractal structure, then the filtration of moisture in the non-stationary area will be a flicker process. A fractal analysis of the time series, the dependences of moisture volumetric flow rate on time, measured for non-stationary filtration area allow us to estimate the fractal dimension of the pore space of soils.

The experimental curves of moisture filtration through the soil were smoothed out using the Moving-Average Method. Preliminary Outlier Analysis was carried out. Dependences of moisture volumetric flow on time, which could not be approximated by smooth functions, were taken to further determination of the fractal dimension of the pore space of soils. The experimentally obtained smooth, exponential dependences of moisture volumetric flow rate on time show the absence of fractal properties of the pore space of soils.

3. Results and discussions

According to the results of our laboratory studies of moisture filtration through the undisturbed soil monoliths, two types of curves were obtained for 15 samples of Albeluvisols Umdric soils (figure 1 & 2). The characteristic signal of the flicker process for the soil under conditions of unsteady filtering of moisture through a sample of natural composition is shown in figure 1. The classical exponential dependence of the moisture volume flow rate on time in the filtration process is shown in figure 2. In
the first case (figure 1), the moisture volumetric flow rate is not proportional to the square root of the period, the value of the Hurst index is greater than 0.5. Hurst effect is observed. Here, the statement of the problem of determination of the frequency spectrum of a given random flicker-process is legitimate.

Figure 1. The curve flicker-process of moisture filtering through a soil sample. Frequencies spectrum of process: $f(\nu) \to \infty$.

If the spectrum of the process: $f(\nu) \to \infty$ (frequencies move apart) then the fractal structure of the soil pore space exists. We believe it is quite possible to obtain fractal dimension of soil pores from data presented in figure 1. The fractal dimension of time series (the form of which is revealed in figure 1) is easily determined by the method of linear systems of fractal geometry [19, 21]. In the absence of spectrum, the obtained experimental dependence of the volume flow in time when filtering moisture is smooth: this is a case of a regular soil structure and inseparable pore space (figure 2). In this case, there is no mode of fractal moisture transfer. Soil structure should be described by regular models [22-24]. The presented method for determination of the fractal dimension has several advantages according to the PSD method. This is a direct, physical method for determination of the fractal dimension of the pore space of soils. For this approach, it is not necessary to have data on the grain size distribution and aggregate composition of soils. According to preliminary data, there is a limitation on the applicability of this method. Not all soils have a fractal structure; it is a valuable result of this research.

4. Conclusion
The experimental method for determination of the fractal dimension of the pore space of soil when filtering moisture through the soil monolith of undisturbed addition solves a number of issues. Firstly, not all investigated soils have a fractal structure of pore space. When the process of filtration spectrum of frequencies is constant ($f(\nu) = \text{const}$) the structure of soil pore space is not fractal. Otherwise, the soil structure could be modeled by the methods of fractal geometry and the fractal dimension could be easily determined by the method of linear systems of fractal geometry from experimentally obtained dependences in the process of filtering (which is shown in figure 1).

Secondly, the suggested method is non-destructive; soil with undisturbed structure could be tested which improves the accuracy of determination of the fractal dimension.

The suggested approach includes the following steps: (1) The filtration of moisture through the undisturbed soil monolith is investigated with a special attention to the first stages of filtering process. The time dependences of the volume moisture flow rate are obtained. According to the results of numerous experiments, two cases are observed, i.e. smooth dependences and sawtooth dependences. In the first case, the soil structure has no fractal character. In the second case, the soil structure is a
fractal one. (2) In the second case, we can determine fractal dimension of soil pore space. To do this, the canonical method of analysis of fractal dimension of time series for linear systems (not once described in the literature) can be applied. (3) The obtained value of fractal dimension can be used in fractal models of water retention function.

Acknowledgements
The reported study was funded by RFBR according to the research project No 19-04-00939-a

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