Hydraulic fracturing of cylindrical concrete bodies in a non-uniform stress field

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Abstract. An experimental study of the hydraulic fracture of thick-walled cylinders with the application of a diametrical load was carried out on specimens in the form of cylinders with a central circular hole and a circular hole located at a distance of half the radius from the center of the cylinder at an angle of 45 degrees to the load application line. Specimens were made of concrete based on aluminous cement. Ultimate stresses in the material were determined for two types of stress state: under tension in bending conditions and under a compound stress state on the surface of the hole. The modeling of the fracturing process taking into account the inhomogeneity of the stress state near the hole was carried out using the boundary elements method (in the variant of the fictitious load method) and the gradient fracture criterion. Calculation results of the ultimate pressure were compared with experimental data. Experimental study showed scale effect in concrete specimens with stress concentrators.

1. Introduction

The technology of hydraulic fracturing is widely used in oil and gas production. However, the analysis of fracture under inhomogeneous stress state around the borehole is problematic. In this study, we compared numerical calculations for different fracture criteria with experimental data on hydraulic fracturing of cylindrical specimens made of concrete in an inhomogeneous stress field, in order to determine the most suitable for this purpose.

Under laboratory conditions, experiments can only be carried out with models of wells in rocks. Real wells have a significantly larger diameter than laboratory models. Therefore, it is important to investigate the scale factor, which, according to the literature, takes place during hydraulic fracturing. Haimson [1] conducted laboratory tests on hydraulic fracturing of two rocks: limestone and granite. The diameter of the cylindrical hole imitating the borehole varied from 3 to 32 mm. With this increase in diameter, the critical pressure for limestone fell by a factor of 7, and for granite by a factor of 1.5. The results of similar experiments carried out on the same rocks are presented in [2, 3]. The diameter of the hole was changed over a wider range: in limestone – up to 50 mm, and in granite – up to 100 mm. With increasing diameter, the critical pressure decreased, asymptotically approaching a constant value.

It is known that in the presence of stress concentration, the scale factor manifests itself at a much greater degree than under a homogenous stress state. Since the borehole is a stress concentrator, the inhomogeneity of the stress state near the well has a significant effect on the strength of the geomaterials. Therefore, in order to estimate the ultimate value of pressure during fracturing of rocks, nonlocal fracture criteria can be used to take into account the inhomogeneity of the stress state.
In the present study, the simulation of the fracturing process, taking into account the inhomogeneity of the stress state near the hole, was carried out using the boundary element method (in the variant of the fictitious load method) and the gradient fracture criterion.

2. Hydraulic fracturing experiments on cylindrical concrete specimens
An experimental study of the hydraulic fracture of thick-walled cylinders with the application of a diametrical load was carried out on specimens in the form of cylinders with a central circular hole (Figure 1a) and a circular hole located at a distance of half the radius from the center of the cylinder at an angle of 45 degrees to the load application line (Figure 1b). Cylinders of different diameters with different hole diameters were tested to study the scale effect (outer diameters: 103.5 mm, 235 mm, 300 mm; corresponding hole diameters are 10.5 mm, 24 mm, 31 mm). Experiments on hydraulic fracturing were carried out using the machine that creates a high oil pressure, which can be used to experimentally investigate the fracturing process of samples made of brittle materials, in particular rocks and materials based on cement or gypsum. The maximum pressure can reach 40 MPa. Compressor oil was used as a fracturing fluid. Before applying the oil pressure, the cylinders were compressed in diameter using a lever system with suspended loads.

Specimens were made of concrete based on aluminous cement mark 40. Preparation of the solution for casting samples was carried out by mixing sand, cement and water in proportion 3/2/1, respectively. The solution was poured into cylindrical molds located on the shaker, which are used to remove air bubbles from the solution. The ultimate stresses in the material were determined for two types of stress state: under tension in conditions of bending and under complex stress on the surface of the hole.

Figure 1. Schemes of specimens with central circular hole (a), and with a circular hole located at a distance of half the radius from the center of the cylinder at an angle of 45 degrees to the load application line (b).

Ultimate stresses in the material were determined for four types of stress state: under tension in bending conditions, under compression, under a pure shear on the surface of the hole and under a compound stress state under conditions of diametrical compression of a solid cylinder.

The value of ultimate tensile stress $\sigma_p = 2.11$ MPa was obtained from tests on the three-point bending of a concrete beam using gradient fracture criterion.
The value of the critical stress intensity factor of concrete $K_{lc} = 0.221 \text{MPa} \cdot \text{m}^{1/2}$, which was used to calculate the parameter $L_1$ in the gradient fracture criterion, was obtained from experiments on the diametrical compression of cylindrical specimens with a central notch [4].

The diametrical load was proportional to the sample diameter and was equal to 9800 N, 7676 N, 3381 N for diameters of 300 mm, 235 mm and 103.5 mm, respectively. The process of hydraulic fracturing, taking into account the heterogeneity of the stress state near the hole, was simulated using the boundary element method (in the variant of the fictitious load method) using a program written in Fortran [5]. An algorithm for the joint use of the boundary element method and the gradient fracture criterion [6, 7], modified for the problem of hydraulic fracture, was applied for numerical calculations of the breakdown pressure, as well as for determining the location and direction of fracture.

3. Gradient fracture criterion
In the gradient criterion for the determination of the load at the instant of fracture initiation, the effective stress $\sigma_e$ rather than the maximum stress is compared with the ultimate strength of the material $\sigma_b$. The effective stress at the considered point of the body is proportional to the maximum tensile stress $\sigma_1$, which is taken as equivalent. Besides, $\sigma_e$ depends on the local inhomogeneity of the stress field in the neighborhood of the considered points and representative size of the material inhomogeneity. The local inhomogeneity of the stress distribution is characterized by the relative gradient $g_v = \left| \text{grade} \sigma_1/\sigma_v \right|$ of the positive normal stress $\sigma_v$ acting on the plane, including the area of the first principal stress at the considered point of the body where the plane and the area have a common normal $v$. Calculation of $\left| \text{grade} \sigma_1 \right|$ in some problems is easier than that of $\left| \text{grade} \sigma_v \right|$ used earlier [8]. The relative gradient is found as a result of solving the corresponding problem of the elasticity theory. The expression for the effective stress is written in the form

$$\sigma_e = \sigma_1 \left( 1 - \beta + \sqrt{\beta^2 + L_1 g_v} \right).$$

Here, $L_1$ is a parameter having the dimension of length and characterizing the heterogeneity (defectiveness) of the material; $\beta$ is a non-negative dimensionless parameter, which can be considered as an approximation parameter.

The parameter $L_1$ is found in [9] by matching the gradient criterion with linear fracture mechanics and is expressed in terms of known material characteristics – the ultimate strength $\sigma_b$ and the critical stress intensity factor $K_{lc}$ by the formula $L_1 = \frac{(2/\pi)K_{lc}^2}{\sigma_b^2}$.

We will assume that the fracture in the vicinity of the point under consideration begins when the effective tensile stress $\sigma_e$ reaches the ultimate tensile stress $\sigma_b$ and initially spreads along the plane of the maximum tensile stress. In the case of hydraulic fracturing, this is the plane normal to the circumferential direction.

4. Numerical algorithm
Based on the gradient criterion and boundary element method (in the version of the fictitious stress method), a numerical algorithm for the calculation of strength was developed. Its characteristic feature is that not only the components of the stress state but also their derivatives with respect to the spatial coordinates must be determined in the course of the calculations.

When using the boundary element method, there appears a problem that the stresses at inner points can be found with a satisfactory accuracy only provided that the distance between these points and the contour is larger than the length of a single element [5]. In connection with this, an algorithm allowing us to accurately calculate stresses at body points situated near the boundary was required. The proposed numerical algorithm for the determination of stresses near the boundary of the body includes
two stages. At the first stage, we find stresses $\sigma_i'$ at the middle points of the boundary elements and the tangential derivatives along the contour $\partial \sigma_i'/\partial s$ at these points. At the second stage, we draw a new equidistant boundary-element broken line forming an auxiliary contour in the body at a short distance $[\Delta_1]$ from the boundary elements of the main contour. Using the equilibrium equations for an infinitely small element on the body contour, we approximately obtain boundary conditions for the auxiliary contour in terms of the stresses $\sigma_i'$ on the main contour and derivatives $\partial \sigma_i'/\partial n$ obtained earlier. Applying the boundary element method to the problem with the given boundary conditions on the auxiliary contour and calculating the stresses in the center of each boundary element of this contour, we now find the stresses in the considered inner points of the initial problem with a higher accuracy.

The normal stress derivatives which are necessary for calculating the gradient modulus are determined according to the finite-difference formulas of numerical differentiation. The tangential derivative $\partial \sigma_i'/\partial n$ of the normal stress along the contour is calculated using a three-point stencil of numerical differentiation with uneven steps; and the normal derivative $\partial \sigma_i'/\partial \alpha$ of the normal stress along the contour is calculated using a two-point stencil. Substituting the values $\sigma_i$ and $g$, (which are now known at each middle point of the boundary elements) into expression (1) for $\sigma_e$, we find the point where the effective stress reaches its maximum; i.e., we establish the place of the fracture’s initiation.

In the general case, the curvature of the concentrator’s contour can be a variable quantity rather than a constant. Therefore, to clarify the form of the boundary conditions on the auxiliary contour, we use differential equations of equilibrium from [9] for a plane problem in an arbitrary curvilinear orthogonal coordinate system $(\alpha_1, \alpha_2)$ rather than in a cylindrical system as in [10]. Let us consider the first equation:

$$\frac{\partial}{\partial \alpha_1}(H_2\sigma_{12}) + \frac{\partial}{\partial \alpha_2}(H_1\sigma_{22}) + \frac{\partial H_2}{\partial \alpha_1}\sigma_{12} - \frac{\partial H_1}{\partial \alpha_2}\sigma_{11} + H_1H_2F_2 = 0. \tag{2}$$

Here, $H_1$ and $H_2$ are the Lame coefficients which are the ratios of the length increments of the coordinate lines $\alpha_2 = \text{const}$ or $\alpha_1 = \text{const}$ to the corresponding increments and of the curvilinear coordinates

$$H_1 = \sqrt{(\partial x/\partial \alpha_1)^2 + (\partial y/\partial \alpha_1)^2},$$

$$H_2 = \sqrt{(\partial x/\partial \alpha_2)^2 + (\partial y/\partial \alpha_2)^2},$$

where $\sigma_{11}$, $\sigma_{12}$, $\sigma_{22}$ are components of the stress tensor and $F_1$ is the projection of the volume force on the coordinate line $\alpha_1$.

Let us take a coordinate system in which the concentrator contour is described by the equation $\alpha_1 = \text{const}$. Assume that volume forces in the problems under consideration are absent and tangential stresses $\sigma_{12}$ on the concentrator contour are zero. Then equation (2) on a free contour is written in the form

$$\frac{\partial H_2}{\partial \alpha_1}\sigma_{11} + H_2\frac{\partial \sigma_{11}}{\partial \alpha_1} - \frac{\partial H_2}{\partial \alpha_1}\sigma_{22} = 0,$$

where $\sigma_{11}$ is the normal stress and $\sigma_{22}$ is the tangential stress.

Taking into account that $\alpha_1 = \text{const}$, $\alpha_2 = \text{const}$, $\sigma_{11}' = \sigma_e'$, where $\sigma_e$ is the normal stress on the main contour, we find at $\sigma_{22}' = \sigma_e' > 0$ the increment $\Delta \sigma_n$ when passing to the auxiliary contour:
\[
\Delta \sigma'_n = \frac{\Delta d'}{a'} \left( \sigma'_v - \sigma'_s \right).
\]

Since on the main contour \( \sigma'_n \big|_2 = -p \), one of the two boundary conditions on the auxiliary contour takes the final form

\[
\sigma'_n \big|_2 = -p + \frac{\Delta d'}{a'} \left( \sigma'_v + p \right).
\]

The second boundary condition is the same as in [5, 6].

5. **Results comparison**

The experimental and calculated values of the breakdown pressure \( p^* \) (for the local maximum stress criterion and the gradient fracture criterion) are given in the Table 1 for comparison. Experimental data and calculation results are presented graphically in Figures 2, 3.

| specimen's diameters | hole location | number of experiments | Standard deviation, MPa | experimental value \( <p^*> \), MPa | Calc. result \( p^* \), MPa | Criterion |
|----------------------|---------------|-----------------------|--------------------------|-------------------------------|--------------------------|-----------|
| outer 300 mm         | center        | 3                     | 0.03                     | 3.069                         | 2.97                     | gradient max stress |
| inner 31 mm          |               |                       |                          |                               | 0.771                    |           |
| diametrical load     |               |                       |                          |                               | 3.11                     |           |
| P=9800 N             | 45° R/2       | 4                     | 0.1491                   | 2.973                         | 1.035                    |           |
| outer 235 mm         | center        | 3                     | 0.3135                   | 3.344                         | 3.253                    | gradient max stress |
| inner 24 mm          |               |                       |                          |                               | 0.773                    |           |
| diametrical load     |               |                       |                          |                               | 3.385                    |           |
| P=7676 N             | 45° R/2       | 5                     | 0.541                    | 3.354                         | 1.04                     | gradient max stress |
| outer 103.5 mm       | center        | 5                     | 0.5248                   | 3.658                         | 4.46                     | gradient max stress |
| inner 10.5 mm        |               |                       |                          |                               | 0.773                    |           |
| diametrical load     |               |                       |                          |                               | 4.53                     |           |
| P=3381 N             | 45° R/2       | 3                     | 0.329                    | 3.329                         | 1.045                    |           |

In samples with a central hole, the hydraulic fracture cracks propagated from the hole contour along the load application line. In samples with a hole offset from the load application line, fracture cracks spread from the hole contour in the direction of the linear contact area of the cylinder with the loading system and in the opposite direction.

From the obtained results follows that when a cylindrical specimen is fractured by hydraulic fracturing, a scale effect takes place (as the diameter of the hole increases, the breakdown pressure decreases). It is shown that the local maximum stress criterion gives significantly lower values of the critical pressure \( p^* \). From Figures 2 and 3 it can be seen that for samples with a diameter of 103.5 mm, the gradient criterion gives overestimated values of the critical pressure. This is due to the fact that the hole diameter of 10.5 mm is close to the value of the parameter \( L = 7 \text{ mm} \). In cases where the hole diameter is several times greater than the value of the \( L \) parameter, good agreement is reached between the experimental results and the numerical estimates using the gradient fracture criterion.
6. Conclusion

An experimental study of hydraulic fracturing of thick-walled concrete cylinders compressed along the diameter with holes located in the center of the cylinder and displaced from the center was carried out on a modified installation. The cylinders were made of concrete based on sand and high alumina cement grade 40. Zwick / Roell Z100 universal testing machine was used for experiments on the fracture of concrete under conditions of bending. With the help of the boundary element method, the local maximum stress criterion and the gradient fracture criterion, fracture simulation in non-uniform stress fields was carried out. It is established that the local maximum stress criterion gives significantly lower values of the breakdown pressure during hydraulic fracturing. It is shown that using the gradient criterion, it is possible to obtain a satisfactory agreement between the results of calculations and experimental data on hydraulic fracturing in non-uniform stress fields. Experimental study showed scale effect in concrete specimens with stress concentrators.
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References
[1] Haimson B C 1990 Scale Effects in Rock Masses: Proc. 1st Int. Workshop on Scale Effects in Rock Masses (Rotterdam; Brookfield: Balkema) 89-101
[2] Haimson B C and Zhao Z 1991 Rock Mechanics as a Multidisciplinary Science: Proc. 32nd US Symp. (Rotterdam: Balkema) 191-9
[3] Cuisiat F D and Haimson B C 1992 Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 29 99-117
[4] Novikov N V and Maistrenko A L 1984 Mater. Sci. 19 298-304
[5] Crouch S L and Starfield A M 1983 Boundary Element Methods in Solid Mechanics (London: Jeorge Allen & Unwin) p 322
[6] Legan M A and Blinov V A 2017 Comp. Continuum Mech. 894 332-40
[7] Legan M A and Blinov V A 2018 J. Appl. Mech. Tech. Phys. 59 1227-34
[8] Legan M A 1994 J. Appl. Mech. Tech. Phys. 35 750-6
[9] Legan M A 1993 J. Appl. Mech. Tech. Phys. 34 585–92
[10] Sheremet A S and Legan M A 1999 J. Appl. Mech. Tech. Phys. 40 744-50