We demonstrate how to use gamma-ray burst supernovae (GRB-SNe) to measure the Hubble constant, $H_0$. Our method is identical to those performed in historical and current analyses of type Ia SNe, which involves correcting the peak apparent magnitudes of the GRB-SNe via a luminosity–decline relationship. Then, using SN 1998bw and a fiducial distance to its host galaxy, we determine a range of values for $H_0$ that agree well with those found via other techniques. We employ two statistical data-fitting procedures: a linear-least squares (LLS) method, and a bootstrap (BOOS) algorithm that uses Monte-Carlo sampling and linear-regression data fitting. We adopt conservative errors of 20% in the SN magnitudes. The errors in $H_0$ derived from the BOOS algorithm are of order 2–4 km s$^{-1}$ Mpc$^{-1}$, and roughly ten times larger using the LLS method. We stress that it is not currently possible to determine a true value for $H_0$ simply because no independent distance measurement yet exists for a GRB-SN host galaxy. However, the results of our pilot study show that it is indeed possible to constrain $H_0$ in principle, using GRB-SNe. With the launch of the next generation of space instrumentation, such as the James Webb Space Telescope at the end of the current decade, it should be possible to independently determine the distance to very nearby GRB-SNe using extragalactic distance estimators such as Cepheid variable stars.

1. Introduction

It was recently shown by Cano (2014; C14 here after) and Li & Hjorth (2014) that gamma-ray burst supernovae (GRB-SNe) have the potential to be used as standardizable candles. In
it was shown that a statistically significant correlation was present in the luminosity \((k)\) and stretch \((s)\) factors of nine GRB-SNe relative to a template supernova: SN 1998bw, which was associated with GRB 980425 (e.g. Galama et al. 1998; Patat et al. 2001). In a complementary, but independent analysis, a statistically significant luminosity–decline relationship was seen in the K-corrected \(V\)-band light curves (LCs) of eight GRB-SNe investigated by Li & Hjorth (2014). These results unambiguously demonstrated that GRB-SNe have the potential to be used as standardizable candles.

In the cosmic distance ladder, type Ia SN occupy one of the top-most rungs, helping to determine the distance to objects up to redshifts of almost \(z = 2\) (Jones et al. 2013). Before the luminosity–decline relationship was observed for nine nearby \((\log(cz) < 4.1)\) SNe Ia by Phillips (1993), several papers had estimated the Hubble constant \((H_0)\) by assuming a single peak luminosity for SNe Ia in a given filter (e.g. Kowal 1968; Branch & Tammann 1992; Sandage & Tammann 1993). The luminosity–decline relationship was used thereafter to calibrate SN data in Hubble diagrams to estimate \(H_0\) (e.g. Hamuy et al., 1995; Perlmutter et al. 1997, who also estimated the mass budget of the universe). The use of SNe Ia as standardizable candles reached a crescendo in the late 1990s and early 2000s, when Riess et al. (1998), Perlmutter et al. (1999) and Freedmann et al. (2001) constrained not only the Hubble constant, but also the mass and energy content of the universe, and demonstrated for the first time the existence of “dark energy”, which is responsible for the accelerated expansion of the cosmos, and which makes up almost three-quarters of everything in the universe.

Building upon the preliminary result in C14, in this current paper we demonstrate that GRB-SNe can be used as cosmological probes in much the same fashion as SNe Ia, and provide an estimate of the Hubble constant using our dataset. Note that the value of \(H_0\) determined here is not quoted as a true value, but rather our focus is to demonstrate the applicability of using GRB-SNe in this context. In Section 2 we outline our method for determining the peak magnitudes and decline rate, given as the \(\Delta m_{15}\) parameter (which is the amount the LC fades from peak light to fifteen days later). In Section 3 we perform two statistical analyses (a linear-least squares method and a bootstrap algorithm) to (1) look for a luminosity–decline relation in all \(UBVRI\) filters (Section 3.1), and (2) demonstrate how the Hubble constant can be determined, in principle, using GRB-SNe (Section 3.2). In Section 4 we discuss our results and summarize our conclusions.

2. Method

In this paper, we have returned to the nine GRB-SNe used in the analysis of C14. This time, instead of determining the luminosity and stretch factors relative to a K-corrected template SN, we have used the SN LCs obtained from the decomposition method. To the isolated SN LCs, we fit a variety of models to determine the (1) peak apparent magnitudes, and (2) \(\Delta m_{15}\) in every interpolated rest-frame filter. The peak absolute magnitudes and \(\Delta m_{15}\) values are presented in Table 1 where we have used a \(\Lambda CD M\) cosmology constrained by Planck (Planck Collaboration et
Fig. 1.— An example of fitting the Bazin function (equation [1] shown as solid blue line) to the interpolated $B$-band (blue points) LC of GRB 090618 from C14. Note that the parameter $t_0$ shown on the $x$-axis is the trigger time of the GRB, rather than one of the free parameters in the Bazin function (equation [1]).
al. 2013) to calculate the former, where $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.315$, $\Omega_\Lambda = 0.685$. The magnitudes are corrected for observer- and rest-frame extinction, as well as being K-corrected via spectral energy distribution (SED) interpolation, as discussed in C14.

We fit three different functions to the SN LCs to determine these observational properties: (1) the Bazin function (Bazin et al. 2011; equation 1 in this work), (2) high-order polynomials, and (3) linear splines. All functions were fit to the data using PYTHON scripts. A linear-least-squares (LLS) Levenberg-Marquardt algorithm was used via scipy.optimize to fit the Bazin function to the data, while numpy.polyfit was used to fit the polynomial to the data. Finally, scipy.interpolate.interp1d was used to fit the linear spline to the data.

For most of the GRB-SNe, the LCs were not sampled well enough to use methods (2) and (3), where only the LCs of SN 1998bw were adequately sampled enough to effectively use these fitting techniques. When fitting the LCs of SN 1998bw with the three different functions, the peak magnitude in each filter did not vary by more than 0.01 m, while the time of peak light varied by no more than 0.15 d.

The remaining GRB-SNe were fitted with the Bazin function:

$$f(t) = \Lambda \left[ \frac{\exp \left( \frac{-(t-t_0)}{\tau_{\text{fall}}} \right)}{1 + \exp \left( \frac{-(t-t_0)}{\tau_{\text{rise}}} \right)} \right] + \Phi$$  \hspace{1cm} (1)

The Bazin function is a more elegant empirical model, that possesses fewer free parameters, than the empirical model used in Cano et al. (2011) and Cano (2013) to describe the LC shapes of GRB-SNe and SNe Ibc. It has five free parameters: \Lambda is a normalization constant, \Phi an offset constant, and \tau_{\text{rise}} & \tau_{\text{fall}} are the rise and fall times of the SN LC. \tau_0 is related to the rise and fall times, as well as the time of maximum light by $t_0 = t_{\text{max}} - \tau_{\text{rise}} \times \ln \left( \frac{\tau_{\text{fall}}}{\tau_{\text{rise}} - 1} \right)$.

Note that we adopted the conservative 20% error estimated from C14 for the peak magnitudes and $\Delta m_{15}$ values. This conservative error arises from the fact that the isolation of the SN light involves a complicated decomposition technique (C14), with several sources of error that arise from GRB afterglow modelling and subtraction, the host-galaxy subtraction, the uncertainties in the observer-frame and rest-frame extinction, and the SED interpolation. The 20% error adopted here is much larger than the systematic error associated with the different fitting functions described above.
3. Analysis & Results

3.1. \( M_\nu \) vs. \( \Delta m_{15,\nu} \)

Figure 2 shows the peak absolute magnitudes of the nine GRB-SNe from C14 versus \( \Delta m_{15} \) in filters \( UBVRI \). A linear relation was fitted to the data:

\[
M_\nu = m \times \Delta m_{15,\nu} + b
\]

(2)

to determine the slope \( (m) \) and \( y \)-intercept \( (b) \). We have used two different approaches to the data fitting: (1) a LLS method using PYXPL\textsuperscript{1}T\textsuperscript{2} and (2) a bootstrap (BOOS) algorithm that uses Monte-Carlo sampling and replacement and non-weighted linear regression data-fitting, which was written using PYTHON.

In approach (1), the LLS fitting is performed using the generic fitting algorithm\textsuperscript{2} employed by PYXPL\textsuperscript{1}, and the fit is “weighted” and performed twice, using first the \( y \), and secondly the \( x \) errors. We have conservatively adopted the largest errors from each fit. In the BOOS approach, a new dataset is created from the original by choosing a new value from a Gaussian that is centered on each original datapoint, and whose standard deviation in each direction is equal to the error in that variable. The simulation was performed 10,000 times, each time fitting the straight line to the new dataset using a non-weighted LLS optimization. We have used the standard deviation of the distribution as an estimate of the standard error of each free parameter . Our results are shown in Table 2. On average, the errors from the BOOS approach are of order to, but slightly larger than, those from the double-weighted LLS method.

For each dataset we also calculated the Pearson’s correlation coefficient \( (r) \) and the two-point probability of a chance correlation \( (p) \). The largest sample sizes are in \( B \) and \( V \) \( (N = 7) \), while there are only four datapoints in \( R \), and three each in \( U \) and \( I \). It is seen, both visually and

\textsuperscript{1}http://pyxplot.org.uk

\textsuperscript{2}verbatim from the PYXPL\textsuperscript{1}T user manual, which can be found on their website.
statistically, that the correlation in the $B$-band is the weakest ($r = 0.716$). Conversely, a strong
correlation is seen in the other filters ($r > 0.92$ for each of the remaining filters), though we must
consider the caveat of small sample sizes in $U$, $R$ and $I$. A strong correlation is seen in the $V$-band,
which supports the statistically significant relation seen in the K-corrected, $V$-band LCs of the
GRB-SNe investigated by Li & Hjorth (2014).

We also investigated what the effect of using different values of the input cosmological param-
eters, namely $H_0$, will have on the fitted values of $m$ and $b$ in Fig. 2. In each case, the same slope
($m$) was obtained, but with different $y$-intercept ($b$) values. For example, when using $H_0 = 80$ km
s$^{-1}$ Mpc$^{-1}$ and the LLS method, the $U$-band data had an identical slope ($m = 1.04 \pm 0.66$), but a
different intercept ($b = -20.53 \pm 1.19$). This arises from the fact that the peak absolute magnitudes
of the three GRB-SNe in the $U$-band are all fainter due to the larger value of $H_0$ used to calculate
the distance modulus. As we will show in the following section, using different values for $H_0$ in
the luminosity–decline relationship has a minimal effect the resultant values of the derived Hubble
constant.

3.2. The Hubble Constant

Plotted in Fig. 3 are log($cz$) vs. apparent magnitude of the nine GRB-SNe in filters $UBVRI$.
Equation 3 (e.g. Tammann et al. 2002; Sandage & Tammann 1993), which is valid for nearby SNe
($z \ll 1$), was fitted to the data for each filter, and only included those events whose redshift\footnote{This means that GRBs 011121, 090618, 120422A and 130831A have been omitted from the fits as their redshifts are all greater than $z = 0.2$. It is important to omit these events from the fit as redshifts exceeding $z \approx 0.2$ are no longer in the local Hubble flow, and their inclusion in the fit can result in miscalculating the local value of the Hubble constant. Our procedure follows the examples used in SNe Ia studies (e.g. Riess et al. 2004; 2007) that also impose an upper limit to the redshift when fitting to obtain $H_0$. At larger redshifts the matter and dark energy density parameters need to also be included.} is $z < 0.2$:

$$\log (cz) = 0.2 \times m_{\text{app}} + \delta$$

(3)

to determine the $y$-intercept ($\delta$) of the line, which in turn is used to determine $H_0$ via:

$$\log (H_0) = 0.2 \times M_{\text{abs}} + \delta + 5$$

(4)

The best-fitting values of $\delta$ are displayed in Table 3. As in the preceding section, we used both
the LLS and BOOS methods to estimate $\delta$ and its associated error. It is seen that the errors in $\delta$
determined via the LLS method are almost ten times larger than those determined via the BOOS
method. We have also calculated the dispersions of each fit to each dataset in the corrected and
uncorrected magnitudes, which is calculated as the rms deviation of the datapoints to the fitted line. It is seen that the average dispersion of all filters is approximately 0.1 mag.

In Fig. 3, we determined $\delta$ for both the original and “corrected” peak apparent magnitudes, where the corrected magnitudes are determined using the best-fitting values of $m$ and $b$ (Table 2), and the value of $\Delta m_{15}$ for a given GRB-SN in a given filter. Strictly speaking, the correction should be performed using an independent dataset, as noted by Hamuy et al. (1995), who used the independent set of SNe Ia from Phillips (1993) to correct their additional SNe Ia against. However, as we are merely demonstrating the principle and applicability of our method, rather than attempting to actually determine $H_0$, we have corrected our data using the luminosity–decline relation determined with our dataset. Finally, it is seen that the errors in $\delta$ are much larger for the corrected magnitudes due to the large errors in $m$ and $b$ (which are used to calculate the errors in the corrected magnitudes).

To determine the Hubble constant, all ones needs to do is use the value of $\delta$ in a given filter, and the peak absolute magnitude of a GRB-SN whose distance has been independently determined (here we are using SN 1998bw). Our analysis is now limited by the fact that we do not have an independent distance measurement to the host galaxy of SN 1998bw (and thus the true absolute magnitudes in $UBVRI$). However, at $z = 0.00866$ (Li & Hjorth 2014), flat $\Lambda$CDM cosmological models that incorporate typical values for the Hubble constant and the mass and energy densities, give distances in the range 36–39 Mpc. For the example here, we have adopted a distance of 37 Mpc, which corresponds to a distance modulus of $\mu = 32.84$ mag.

Using the “ultimate” LCs from Clocchiati et al. (2011), which are de-reddened and K-corrected, the rest-frame, peak apparent magnitudes of SN 1998bw in filters $UBVRI$ are: $m_U = 13.75$, $m_B = 14.01$, $m_V = 13.50$, $m_R = 13.49$ and $m_I = 13.63$. We have determined these by fitting the three functions from Section 3 to the $UBVRI$ LCs of SN 1998bw. Each of these is transformed into an absolute magnitude, and plugged into equation 4, along with the corresponding value of $\delta$ in that filter, to give $H_0$. The error in $H_0$ is determined using the error in $\delta$, and the final values in each filter are shown in Table 3.

At this stage, with the small sample sizes and large uncertainties in $m$ and $b$ (equation 2; Table 2), correcting the magnitudes does not lead to a value of $H_0$ with smaller errorbars. Indeed, as the errors in the corrected magnitudes are necessarily larger due to the uncertainties in $m$ and $b$, this results in larger errors in $\delta$ and in turn $H_0$.

Comparing the value of $H_0$ among the five filters of original magnitudes, and for both statistical methods, gives a Hubble constant in the range of 61–65 km s$^{-1}$ Mpc$^{-1}$, with an average of $H_0 = 63$ km s$^{-1}$ Mpc$^{-1}$. The standard deviation of the five filters is $\sim 2$ km s$^{-1}$ Mpc$^{-1}$, which is of order the size of the error for each filter determined from the ST method, but is roughly 10 times smaller than the errors determined via the LLS method. This trend in relative error sizes is also seen when the corrected magnitudes (below) are used.

For the corrected magnitudes, a wider range of values is seen: 62–72 km s$^{-1}$ Mpc$^{-1}$, with an
Fig. 3.— Hubble diagrams of the nine GRB-SNe from C14 in filters $UBVRI$. The coloured solid line in each color is the best-fitting line to equation [3] to the uncorrected apparent peak magnitudes (open circles, solid-lined errorbars) in that filter, while the solid black line in each subplot is the fit to the corrected apparent peak magnitudes (black stars, dotted-lined errorbars). The quoted dispersion ($\sigma$) is the simple rms deviation of the points about the fit. Note that there are errorbars in $\log(cz)$, however they are much smaller than the size of the datapoints.
average value of $H_0 = 67$ km s$^{-1}$ Mpc$^{-1}$. The standard deviation is again $\sim 4$ km s$^{-1}$ Mpc$^{-1}$. In $BVRI$, the errors from the BOOS method is of order 4–8 km s$^{-1}$ Mpc$^{-1}$, but is larger in the $U$-band, with an error of $\approx 25$ km s$^{-1}$ Mpc$^{-1}$. This large error can be attributed to the large errorbars of the corrected $U$-band magnitudes, which ultimately results from the errors of $m$ and $b$ in this filter.

Finally, using a different value of $H_0$ to calculate the peak absolute magnitude of the GRB-SNe using cosmological models in Section 3.1 has a very small effect on the values of the Hubble constant derived here. For example, if we corrected our data using a luminosity–decline relationship that was obtained using $H_0 = 80$ km s$^{-1}$ Mpc$^{-1}$ (instead of 67.3 km s$^{-1}$ Mpc$^{-1}$), we find a difference in the Hubble constant in the $U$-band of only $\Delta H_0 = 0.05$ km s$^{-1}$ Mpc$^{-1}$, while the largest deviation is in the $B$-band, with $\Delta H_0 = 0.53$ km s$^{-1}$ Mpc$^{-1}$. Therefore, the derived value of $H_0$ obtained using GRB-SNe is insensitive (to within the errorbars determined via both fitting methods) to the values of the cosmological parameters that are used to calibrate the luminosity–decline relationship in Section 3.1.

4. Discussion & Conclusions

In this paper we demonstrated that GRB-SNe can, in principle, be used to determine one of the most fundamental cosmological parameters, the Hubble constant. Our result joins the ranks of other investigations using different aspects of the GRB phenomenon (e.g. Amati et al. 2008; Liu & Wei 2014) and other types of SNe Ic (e.g. type Ic super-luminous supernovae, Inserra & Smartt 2014) as alternative cosmological distance indicators.

In order to determine a value of $H_0$ from our dataset we had to assume a distance to SN 1998bw of 37 Mpc, which is a good educated guess based on distances calculated using a redshift $z = 0.00866$ and $\Lambda$CDM models with standard values. Using this distance, we constrained $H_0$ to the range 62–72 km s$^{-1}$ Mpc$^{-1}$. The actual value we determine for $H_0$ is only an estimate, and should be treated with caution at this point as we do not have an independently determined distance to SN 1998bw. However, the fact that we have constrained $H_0$ to a range that is similar to those determined via other methods ($H_0 \approx 60–75$ km s$^{-1}$ Mpc$^{-1}$ from the cosmic background radiation e.g. Planck Collaboration et al. 2013; SN 1a: e.g. Tamman et al. 2001; Riess et al. 2005; Sandage et al. 2006; the Tully-Fisher relation: e.g. Tully & Fisher 1997; Mould et al. 2000, who also include distances determined via the fundamental plane of elliptical galaxies and surface brightness fluctuations) indicates that we are on the right track, and that our estimate of the distance to the host of SN 1998bw is likely not wildly different from its actual value.

We used two statistical data-fitting procedures in our analysis, a LLS method and a BOOS method to first determine the luminosity–decline relationship for GRB-SNe (Fig. 2 and Table 2), and constrain the $y$-intercept ($\delta$) of the Hubble line (equation 3) to ultimately determine $H_0$.

The errors in $m$ and $b$ in Section 3.1 and $\delta$ and in $H_0$ in Section 3.2 were determined via the
two methods. In the LLS method, the errors are statistical, and determined by the fitting algorithm used by PYXPL. In the BOOS method, the 1-σ errors are determined from the standard deviation of the distribution of values of each free parameter from 10,000 simulations. It was seen that the errors in m and b from the BOOS method were larger than the LLS method, while the errors in δ, and thus H0, were smaller in the BOOS method.

Currently, the propagated errors from m and b in equation 2 through to the peak apparent magnitudes that are fit with the Hubble line, result in larger errors in the Hubble constant than when the uncorrected magnitudes are used. It may prove in the future however, with much larger datasets, that tighter luminosity–decline relations are obtained, thus reducing the scatter and errors in the fitted parameters, and ultimately reducing the error in H0. It has been pointed out however by Sandage et al. (2006) that SNe Ia do not necessarily need to be calibrated by a luminosity–decline relationship in order to recover a well-constrained value for H0. Whether this is also applicable to GRB-SNe will only be determined with much larger datasets.

We also investigated what the effect would be if we used different values for the cosmological parameters to determine the distance moduli of the GRB-SNe (and their resultant peak absolute magnitudes) on the luminosity–decline relations, and the final derived value of the Hubble constant. In each case, the slope of the luminosity–decline relationship remained unchanged, while the y-intercept (b) changed depending on the value of the Hubble constant that is used in the cosmology models to calculate the distance modulus. The propagated effect on the final value of H0 is minimal, with a difference of ΔH0 = 0.05–0.53 km s−1 Mpc−1, with the largest deviation seen in the B-band. These differences are much smaller than the errorbars derived from the two fitting procedures.

Our pilot study has demonstrated the suitability of using GRB-SNe as extragalactic distance indicators, in much the same fashion as SNe Ia. Indeed our method is essentially identical to those used in historical and current analyses of SNe Ia. The only hindrance now for obtaining a value for H0 using GRB-SNe is an independent distance measurement to one of their host galaxies. To date there are several methods for determining the distances to objects at distances >30 Mpc, namely surface-brightness fluctuations, the Tully-Fisher relation for spiral galaxies and long-period Cepheid variable stars, the latter of which can in theory be used for distances up to 100 Mpc (Bird et al. 2009). At ≈ 40 Mpc, detecting and monitoring Cepheid variable stars is likely to be quite challenging if attempted with current technology, including the Hubble Space Telescope (HST). However, HST’s successor, the James Webb Space Telescope (JWST), will be much better suited for this task. Indeed, the NIR and IR capabilities of JWST will be perfectly suited for observing Cepheid variables at IJK wavelengths, where the Period–Luminosity and Period–Wesenheit relations have been accurately determined (e.g. Freedman et al. 2001; Inno et al. 2014), albeit with smaller amplitudes in brightness than bluer optical filters, and which include corrections for the metallicity (e.g. Zaritsky et al. 1994; Scowcroft et al. 2009). Thus JWST offers the perfect opportunity for exploiting GRB-SNe as standardizable candles, perhaps up to redshifts of z = 3–5.
5. Acknowledgements

We thank Gulli Björnsson for his very helpful comments to the original manuscript. ZC and PJ gratefully acknowledge support from a Project Grant from the Icelandic Research Fund.

REFERENCES

Amati, L., Guidorzi, C., Frontera, F., et al. 2008, MNRAS, 391, 577
Bazin, G., Ruhlmann-Kleider, V., Palanque-Delabrouille, N., et al. 2011, A&A, 534, A43
Bird, J. C., Stanek, K. Z., & Prieto, J. L. 2009, ApJ, 695, 874
Branch, D., & Tammann, G. A. 1992, ARA&A, 30, 359
Cano, Z., et al. 2011, ApJ, 740, 41C
Cano, Z. 2013, MNRAS, 434, 1098
Cano, Z. 2014 (C14), ApJ, 794, 121
Cenko, S. B., Perley, D. A., Junkkarinen, V., Burbidge, M., Diego, U. S., & Miller, K. 2009, GRB Coordinates Network, 9518, 1
Cucchiara, A., & Perley, D. 2013, GRB Coordinates Network, 15144, 1
Galama, T. J., et al. 1998, Nature, 395, 670
Freedman, W. L., Madore, B. F., Gibson, B. K., et al. 2001, ApJ, 553, 47
Garnavich, P. M., Stanek, K. Z., Wyrzykowski, L., et al. 2003, ApJ, 582, 924
Gorosabel, J., Pérez-Ramírez, D., Sollerman, J., et al. 2005, A&A, 444, 711
Inno, L., Matsunaga, N., Romaniello, M., et al. 2014, arXiv:1410.5460
Inserra, C., & Smartt, S. J. 2014, arXiv:1409.4429
Jones, D. O., Rodney, S. A., Riess, A. G., et al. 2013, ApJ, 768, 166
Kowal, C. T. 1968, AJ, 73, 1021
Li, X., & Hjorth, J. 2014, arXiv:1407.3506
Liu, J., & Wei, H. 2014, arXiv:1410.3960
Margutti, R., et al. 2007, A&A, 474, 815
Mould, J. R., Huchra, J. P., Freedman, W. L., et al. 2000, ApJ, 529, 786
Patat, F., et al. 2001, ApJ, 555, 900

Perlmutter, S., Gabi, S., Goldhaber, G., et al. 1997, ApJ, 483, 565

Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565

Pian, E., et al. 2006, Nature, 442, 1011

Phillips, M. M. 1993, ApJL, 413, L105

Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2013, arXiv:1303.5076

Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009

Riess, A. G., Strolger, L.-G., Tonry, J., et al. 2004, ApJ, 607, 665

Riess, A. G., Li, W., Stetson, P. B., et al. 2005, ApJ, 627, 579

Riess, A. G., Strolger, L.-G., Casertano, S., et al. 2007, ApJ, 659, 98

Sandage, A., & Tammann, G. A. 1993, ApJ, 415, 1

Sandage, A., Tammann, G. A., Saha, A., et al. 2006, ApJ, 653, 843

Schulze, S., Malesani, D., Cucchiara, A., et al. 2014, A&A, 566, A102

Scowcroft, V., Bersier, D., Mould, J. R., & Wood, P. R. 2009, MNRAS, 396, 1287

Starling, R. L. C., Wiersema, K., Levan, A. J., et al. 2011, MNRAS, 411, 2792

Tammann, G. A., Reindl, B., Thim, F., Saha, A., & Sandage, A. 2002, A New Era in Cosmology, 283, 258

Tully, R. B., & Fisher, J. R. 1977, A&A, 54, 661

Zaritsky, D., Kennicutt, R. C., Jr., & Huchra, J. P. 1994, ApJ, 420, 87

This preprint was prepared with the AAS LaTeX macros v5.2.
Table 1: Observational Properties of GRB-SNe

| GRB   | SN   | log(cz)       | Ref. | Filter | $\Delta m_{15,\nu}$ | $M_V$  |
|-------|------|---------------|------|--------|----------------------|--------|
| 980425| 1998bw | 3.4146 ± 0.0119 | 1    | $U$    | 1.62 ± 0.32          | −19.16 ± 0.24 |
|       |       |               |      | $B$    | 1.18 ± 0.24          | −18.90 ± 0.24 |
|       |       |               |      | $V$    | 0.84 ± 0.17          | −19.41 ± 0.24 |
|       |       |               |      | $R$    | 0.63 ± 0.13          | −19.42 ± 0.24 |
|       |       |               |      | $I$    | 0.44 ± 0.09          | −19.28 ± 0.24 |
| 011121| 2001fe | 5.0358 ± 0.0012 | 2    | $B$    | 1.33 ± 0.27          | −19.30 ± 0.28 |
|       |       |               |      | $V$    | 0.71 ± 0.14          | −19.34 ± 0.24 |
| 030329| 2003dh | 4.7029 ± 0.0005 | 3    | $U$    | 1.30 ± 0.26          | −19.56 ± 0.88 |
|       |       |               |      | $B$    | 1.01 ± 0.20          | −19.26 ± 0.61 |
|       |       |               |      | $V$    | 0.63 ± 0.14          | −19.60 ± 0.52 |
| 031203| 2003lw | 4.4998 ± 0.0003 | 4    | $V$    | 0.51 ± 0.10          | −19.96 ± 0.24 |
|       |       |               |      | $R$    | 0.44 ± 0.09          | −19.93 ± 0.24 |
|       |       |               |      | $I$    | 0.36 ± 0.07          | −19.83 ± 0.24 |
| 060218| 2006aj | 4.0011 ± 0.0003 | 5    | $U$    | 2.23 ± 0.46          | −18.61 ± 0.36 |
|       |       |               |      | $B$    | 1.69 ± 0.34          | −18.35 ± 0.34 |
|       |       |               |      | $V$    | 1.28 ± 0.36          | −18.70 ± 0.33 |
|       |       |               |      | $R$    | 0.82 ± 0.16          | −18.85 ± 0.29 |
| 090618|       | 5.2095 ± 0.0008* | 6    | $B$    | 0.66 ± 0.13          | −19.57 ± 0.24 |
| 100316D| 2010bh | 4.2487 ± 0.0007 | 7    | $V$    | 1.10 ± 0.22          | −18.55 ± 0.24 |
|       |       |               |      | $R$    | 0.75 ± 0.15          | −18.55 ± 0.24 |
|       |       |               |      | $I$    | 0.66 ± 0.13          | −18.52 ± 0.24 |
| 120422A| 2012bz | 4.9282 ± 0.0001 | 8    | $B$    | 0.71 ± 0.14          | −19.19 ± 0.24 |
|       |       |               |      | $V$    | 0.59 ± 0.12          | −19.64 ± 0.24 |
| 130831A| 2013fu | 5.1575 ± 0.0009* | 9,10 | $B$    | 1.19 ± 0.24          | −19.49 ± 0.28 |

† Absolute magnitudes are observer-frame and rest-frame extinction corrected, and are calculated using a flat ΛCDM cosmology with $H_0 = 67.3$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_M = 0.315$, $\Omega_\Lambda = 0.685$.

* No error in the redshift quoted in the literature, so an error of $\Delta z = 0.001$ is assumed.

References: (1) Li & Hjorth (2014); (2) Garnavich et al. (2003); (3) Gorosabel et al. (2005); (4) Margutti et al. (2007); (5) Pian et al. (2006); (6) Cenko et al. (2009); (7) Starling et al. (2011); (8) Schulze et al. (2014); (9) Cucchiara & Perley (2013)
Table 2: Luminosity–decline relation: best-fitting parameters

| Filter | $m$         | $b$           | $m$         | $b$           | $r$       | $p$       |
|--------|-------------|---------------|-------------|---------------|-----------|-----------|
| $U$    | $1.036 \pm 0.658$ | $-20.886 \pm 1.190$ | $1.039 \pm 0.816$ | $-20.891 \pm 1.473$ | $0.996$   | $0.056$   |
| $B$    | $0.634 \pm 0.319$ | $-19.863 \pm 0.353$ | $0.584 \pm 0.239$ | $-19.817 \pm 0.244$ | $0.716$   | $0.070$   |
| $V$    | $1.960 \pm 0.402$ | $-20.872 \pm 0.335$ | $1.739 \pm 0.421$ | $-20.724 \pm 0.312$ | $0.944$   | $0.001$   |
| $R$    | $3.415 \pm 0.880$ | $-21.442 \pm 0.582$ | $3.238 \pm 0.994$ | $-21.338 \pm 0.608$ | $0.922$   | $0.078$   |
| $I$    | $4.418 \pm 1.093$ | $-21.360 \pm 0.550$ | $4.404 \pm 1.595$ | $-21.356 \pm 0.735$ | $0.985$   | $0.109$   |

LLS = Weighted, linear-least squares regression analysis.
BOOS = Bootstrap algorithm.

$r$ = Pearson’s correlation coefficient.
$p$ = Two-point probability of a chance correlation.

Table 3: Hubble Constant, $H_0$ (km s$^{-1}$ Mpc$^{-1}$), calculations

| Filter | uncorr. | LLS | uncorr. | BOOS | corr. | LLS | corr. | BOOS | corr. | corr. |
|--------|---------|-----|---------|------|-------|-----|-------|------|-------|-------|
|        |         | LLS |         |      |       | LLS |       |      |       |       |
| $\delta$ | $H_0$   | $\delta$ | $H_0$   | $\delta$ | $H_0$ | $\delta$ | $H_0$ | $\delta$ | $H_0$ |
| $U$    | $0.629 \pm 0.195$ | $64.71 \pm 36.61$ | $0.629 \pm 0.023$ | $64.68 \pm 3.52$ | $0.641 \pm 1.115$ | $66.55 \pm 800.25$ | $0.643 \pm 0.129$ | $66.80 \pm 23.10$ |
| $B$    | $0.576 \pm 0.187$ | $64.61 \pm 34.69$ | $0.576 \pm 0.022$ | $64.54 \pm 3.35$ | $0.625 \pm 0.394$ | $72.23 \pm 106.70$ | $0.624 \pm 0.046$ | $72.08 \pm 8.05$   |
| $V$    | $0.659 \pm 0.124$ | $61.80 \pm 20.43$ | $0.659 \pm 0.014$ | $61.77 \pm 2.02$ | $0.664 \pm 0.248$ | $62.49 \pm 48.25$  | $0.664 \pm 0.029$ | $62.49 \pm 4.32$   |
| $R$    | $0.657 \pm 0.125$ | $61.22 \pm 20.42$ | $0.657 \pm 0.015$ | $61.21 \pm 2.15$ | $0.672 \pm 0.283$ | $63.29 \pm 58.10$  | $0.671 \pm 0.033$ | $63.21 \pm 4.99$   |
| $I$    | $0.655 \pm 0.139$ | $64.99 \pm 24.42$ | $0.655 \pm 0.016$ | $64.98 \pm 2.44$ | $0.676 \pm 0.294$ | $68.23 \pm 65.92$  | $0.676 \pm 0.034$ | $68.20 \pm 5.55$   |

LLS = Weighted linear-least squares regression analysis.
BOOS = Bootstrap algorithm.

$\delta$ = $y$-intercept of Hubble line (Fig. 3).