Assessment of evolutionary status of eclipsing binaries using light-curve parameters and spectral classification

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ABSTRACT

We have developed a procedure for the classification of eclipsing binaries from their light-curve parameters and spectral type. The procedure was tested on more than 1000 systems with known classification, and its efficiency was estimated for every evolutionary status we use. The procedure was applied to about 4700 binaries with no classification, and the vast majority of them were classified successfully. Systems of relatively rare evolutionary classes were detected in that process, as well as systems with unusual and/or contradictory parameters. Also, for 50 previously unclassified cluster binaries evolutionary classes were identified. These stars can serve as tracers for age and distance estimation of their parent stellar systems. The procedure proved itself as fast, flexible and effective enough to be applied to large ground-based and space-borne surveys, containing tens of thousands of eclipsing binaries.

Key words: binaries: eclipsing – stars: evolution.

1 INTRODUCTION

Binary stars are numerous (from 50 to even 90 per cent in the Local Group). Double-lined eclipsing binaries provide the only method by which fundamental stellar parameters (such as mass, radius, luminosity, etc.) can be independently estimated without having to resolve spatially the binary or rely on astrophysical assumptions. Unfortunately, only a small fraction of all binaries eclipse, and spectroscopy, with sufficient resolution, can only be performed for bright stars. The intersection of these two groups leaves only several hundred stars, an amount that is not growing significantly.

Meanwhile recent major advances in CCD detectors and the implementation of image-difference analysis techniques enable simultaneous photometric measurements of tens of thousands of stars in a single exposure, leading to a dramatic growth in the number of stars with high-quality, multi-epoch, photometric data. There are many millions of light curves available from a variety of surveys, such as the ground-based ASAS (Pojmański 2002), MACHO (Alcock et al. 1998), OGLE (Rucinski & Maceroni 2001), EROS (Grison et al. 1995), TrES (Alonso et al. 2004), HAT (Bakos et al. 2004) and the space-borne Kepler (Matijević et al. 2012) and CoRoT (Loeillet et al. 2008) projects. Thus, eclipsing binaries represent the most numerous type of binaries with known orbital period, because it can be easily determined from the not very long photometric observational sets. However, the number of fully characterized eclipsing binaries has not grown significantly, as there has not been a corresponding growth in the quantity of spectroscopic data. Therefore, it would be advisable to develop a procedure for estimation of the fundamental parameter values for eclipsing variables with unknown spectroscopic elements. Obviously, an assessment of eclipsing binary evolutionary status should be performed prior to the start of the fundamental parameter estimation, as the set of rules for parametrization varies from one evolutionary status to another.

A procedure for determination of the evolutionary class from the rest of the observational data was first proposed by Svechnikov, Istomin & Grekhova (1980). The procedure is based on a restricted number of systems with known classes contained in the old Svechnikov (1969) catalogue which, as our analysis has shown (Malkov et al. 2006), is not accurate enough. Useful ideas for classification of eclipsing binaries can also be found in a statistical study conducted by Giuricin, Mardirossian & Mezzetti (1983a); however, they mostly dealt with only three classes of systems (detached, semidetached and contact).

In this paper, we present a novel procedure, which utilizes the most comprehensive set of rules for the classification of eclipsing binaries, while requiring only light-curve parameters and an estimate of the binary’s spectral type or colour index. This procedure can be used to quickly characterize large numbers of eclipsing binaries (which can be advisable e.g. for statistical investigations), and allows the user to categorize them, even if the set of the mentioned parameters is incomplete. The procedure was tested with the catalogue of eclipsing variables (CEV; Avvakumova, Malkov & Kniazev 2013), which is the world’s principal data base of eclipsing binary systems with available classification.

The scheme of classification is presented in Section 2. A testing and application of the procedure is described in Section 3. Discussion of systems with ambiguous or contradictory
classifications, as well as systems belonging to extreme and unusual stages of evolution, can also be found in that section. In Section 4, we draw our conclusions. Appendix A contains discussion of selected binaries. In Appendix B, we give an example of application of our classification procedure, while cluster binaries are listed in Appendix C.

2 CLASSIFICATION SCHEME

The main goal of our work is to develop a fast and effective procedure for determination of the evolutionary status of eclipsing binaries. Since 2004 (Malkov et al. 2004), we have collected information on light-curve parameters and other observational parameters of these variables, on one hand, and recent published information about evolutionary status of eclipsing binaries, on the other hand. The second version of the CEV\(^1\) (Avvakumova et al. 2013) contains about 7200 eclipsing binaries, and the evolutionary status is available for about 1300 of them. The collected data allow us to make a preliminary statistical analysis and find relations between the different parameters for various evolutionary classes of eclipsing binaries. Such an analysis is presented in this section.

Detailed description of the evolutionary classes used in the current study can be found in Avvakumova et al. (2013), while meaning of the light-curve parameter designations, thought generally accepted, is given in Malkov et al. (2007). We have used the following data from CEV in the analysis:

(i) depth of primary minima \(A_1\), mag;
(ii) depth of secondary minima \(A_2\), mag;
(iii) depth difference \(\Delta A = A_1 - A_2\), mag;
(iv) morphological type of the light curve (EA, EB, EW; as in the General Catalog of Variable Stars (GCVS; Samus et al. 2007–2013);
(v) period of the eclipsing variable star, \(P\), days;
(vi) spectral type of the primary star, \(\text{Sp}_1\);
(vii) luminosity class of the primary star;
(viii) spectral type of the secondary star, \(\text{Sp}_2\);
(ix) luminosity class of the secondary star;
(x) the components’ spectral type difference \(\Delta \text{Sp} = \text{Sp}_1 - \text{Sp}_2\).

Unlike Malkov et al. (2007), we did not use in our analysis the information about variability of the period, data on duration of the eclipses and phase of secondary minimum. All these parameters are included in CEV, when available from the literature. However, the number of such systems is relatively small, and additional observations are required to enlarge that number. So we did not include these parameters in the current version of our procedure.

An example of the analysis is shown in Fig. 1. A distribution of two different evolutionary classes of binaries in the \(A_1\) (depth of primary minimum) – \(A_2\) (depth of secondary minimum) is presented in Fig. 1 (bottom-left panel). Hot semidetached binary class (SH, filled circles in Fig. 1) was introduced by Popper (1980) in his review to designate binary with the spectra of both components earlier than classical algols spectra. About 30 such systems are known. Empty squares in Fig. 1 indicate detached subgiant systems (DR). All of these binaries are chromospheric active RS CVn systems with the spectrum of the primary of F-G IV-V and with a strong H and K emission in the spectrum outside the eclipse (Hall 1976). Stellar activity is caused by the magnetic field on a star which is produced by the star’s rapid rotation. The active component rotates faster than usual because of a spin-up by its close companion. According to Hall (1981), the activity phenomena seen in the well-detached RS CVn binaries are fundamentally different from those seen in the semidetached post-main-sequence (MS) binaries, although a few semidetached RS CVn binaries are known (see e.g. Montesinos, Gimenez & Fernandez-Figueroa 1988). There are about 20 such systems in the catalogue.

As can be seen in Fig. 1, the depth of the primary minima \(A_1\) of both SH and DR binaries is usually not larger than 1.5 mag. The value of the depth of secondary minima \(A_2\) is generally not larger than 0.4 mag for DR systems, while \(A_2\) of hot semidetached systems is not larger than 0.6 mag. Four exceptions are DR systems RW UMa (\(A_1 = 1.56\) mag), TY Pyx (\(A_1 = 0.63\) mag), and SH systems TT Lyr (\(A_1 = 2.09\) mag) and Z Vul (\(A_1 = 1.65\) mag). We have studied the literature available on these binaries and their nature is discussed in Appendix A.

The distribution of the DR and SH systems in the parameter space log \(P-A_1\) is shown in the top-left panel of Fig. 1. We have found only three DR binaries with orbital periods being larger than 10 d. All of them belong to the long-period RS CVn group. One DR binary (ES Cnc) has unusual observational parameters, namely short period and small \(A_1\) value. We placed short description of this system in Appendix A.

The left-hand panels of Fig. 1 clearly demonstrate that there is no difference between the two evolutionary classes in the sense of values of depth of minima and the orbital period. Thus, it is necessary to use additional observational data (such as information about chromospheric activity of RS CVn systems, photometric distortion waves or/and variability of orbital period or information about the orbit eccentricity), if available, in order to attribute a system to one or the other class.

Spectral type, which is known from observations or can be estimated from the colour indexes, can also serve as an additional parameter. Secondary spectral type, when unknown, can be drawn from the components’ effective temperature ratio, which can be

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\(^1\) Online live version can be downloaded from http://www.inasan.ru/~malkov/CEV/
estimated from the observed depths of minima $A_1, A_2$, if limb darkening is neglected:

$$T_2 = \sqrt[4]{\frac{j_2}{j_1}} = \sqrt[4]{\frac{1 + 0.4A_2}{1 + 0.4A_1}}.$$

(1)

Here, $j_i$ is surface brightness of $i$th component (see Branczewicz & Dworak 1980; Malkov 2012 for details).

Distributions of primary and secondary spectral types for the hotter and cooler components of DR and SH systems are shown in the right-hand panels of Fig. 1. It can be seen that even roughly estimated (e.g. from colour indices) spectral type allows us to separate detached subgiant systems from the hot semidetached systems.

We have performed such an analysis for every evolutionary class, and the results are given in Table 1. An example for a detached MS (DM) binary, where the primary is larger, hotter and more massive than the secondary, was discussed in Malkov et al. (2007), where the following relation between $A_1$ and $A_2$ upper limits was found:

$$A_1 = -2.5 \log \left( 1 - \frac{r^2/\alpha}{1 + r} \right) + \sigma A_1,$$

(2)

where $\alpha = 5.5$ for MS stars from late O to early M, $t = 10^{0.4A_2} - 1$ and $\sigma A_1$ is an observational error estimated to be about 0.3 mag.

Not all parameters are equally useful for the assessment of the evolutionary status of the eclipsing binaries. CW, CE, CG and majority of observable DM systems comprise similar components, so the value of the depth difference $\Delta A$ should not exceed some limit, and, consequently, $\Delta A$ value can serve as a good indicator of the evolutionary class. In contrast, for DR, DG, DW, S and CB systems, mostly comprising two quite different components, we indicate a major difference of observable DM systems.

Table 1. The limits for observational parameters, used for the classification, for systems of different evolutionary class.

| Classa | Descriptionb | $A_1^c$ (mag) | $(A_2/\Delta A)^{−d}$ (mag) | $P$ (d) | $Sp_1$ | $Sp_2$ | $\Delta Sp^e$ | MTf |
|--------|--------------|---------------|----------------------|--------|--------|--------|------------|-----|
| DM (190) | MS stars | 1.10 | 0.81 | [0.4; 36] | O5–M4.5; IV–VI | O5–M4.5; IV–VI | Up to 1.5 | EA, EB, E |
| DR (25) | With subgiants | 1.56 | 0.35 | [19; 26] | A8–G6; IV–V | G8–K3; IV–V | From 0.6 to 1.6 | EA |
| DGE (8) | With OB giant, supergiant or WR star | 0.65 | 0.34 | [1.6; 35] | WR3–B2; I, III | O4–B3; III–V | Up to 1.7 | EA, EB, E |
| DGL (16) | With late-type giant or supergiant | 2.32 | 0.20 | [69; 7465] | B0–F7; I–V | G3–M2; I–III | From 1 to 4.5 | EA, EB, E |
| DW (14) | With WD | 6.00 | 0.20 | [0.09; 10] | WR8–B0; VI, wd | G8–M5; VVI | From 4.8 to 6.5 | EA, EB, E |
| D2S (5) | Symbiotic | 6.22 | $<8$ | [603; 6310] | WD, OB; V, wd | G5–M6; III | From 4.5 to 7.3 | EA, E |
| SA (376) | Classical Algols | 3.70 | 0.60 | [2.1; 45] | B4–G0; I–V | A2–M7; II–V | Up to 3.8 | EA, EB |
| SC (5) | Both late-type stars | 1.36 | 0.55 | [2.9; 22] | G8–K4; III–V | K1–K5; III–V | From 0.1 to 0.5 | EA, EB |
| SH (34) | Both early type stars | 1.65 | 0.57 | [1.1; 16] | O8–B4; I, III–V | O9–A5; I–V | Up to 1.2 | EA, EB |
| S2C (33) | Cataclysmic | 6.00 | 0.20 | [0.05; 0.33] | WR5–B0; V, wd | G5–M9; V | From 4.5 to 6.9 | EA, EB, E |
| CB (103) | Near-contact | 1.22 | 0.81 | [0.2; 1.5] | B8–K0; III–V | A0–M0; IV–V | Up to 2.8 | EA, EB, EW |
| CBF (11) | F subclass of CB | 1.00 | 0.30 | [0.5; 0.8] | A2–F4; V | G0–K3; III–V | From 1 to 2.5 | EA, EB |
| CBV (13) | V subclass of CB | 0.91 | 0.38 | [0.39; 1.0] | A0–F8; V | F3–K5; V | From 0.8 to 2.6 | EA, EB |
| CE (19) | Early-type | 0.97 | 0.28 | [0.49; 1.9] | O8–B8.5; IV–V | F3–K5; V | From 0.8 to 2.6 | EA, EB |
| CWA (15) | Late-type, A subclass | 0.81 | 0.15 | [0.26; 1.2] | A7–K5; V | F8–K5.5; V | Up to 0.5 | EA, EB, EW |
| CWW (123) | Late-type, W subclass | 1.00 | 0.22 | [0.02; 0.78] | A7–K5; V | F8–K5.5; V | Up to 0.5 | EA, EB, EW |
| CG (4) | With early-type giants or supergiants | 0.69 | 0.12 | [3.9; 6.6] | O8–B8.5; IV–V | F3–K5; V | From 0.8 to 2.6 | EA, EB, EW |

aThe evolutionary status and the number of such systems in CEV.
bFor detailed description, see Avvakumova et al. (2013). cMaximum value. d$A_2$ value is given for DR, DG, DW, S and CB, and $\Delta A$ value is given for DM, CE, CW and CG (see the text for details). eThe components’ spectral type difference $\Delta Sp = Sp_1 – Sp_2$ is given in units of a spectral class. fMorphological type of the light curve. gData on secondary minimum are given in CEV for only one D2S system.

3 PROCEDURE TESTING AND APPLICATION

A large number of the newly discovered eclipsing variables have an incomplete set of observational parameters. We have studied the efficiency of our procedure and will discuss the main results in the section below.
3.1 Membership probability

Our procedure should be effective and stable with respect to the absence of some observational parameter values. In particular, a lack of parameters leads to a condition when a system resides in an area of the parameter space, covered by two or more evolutionary classes. One example is described in Appendix B. Another case is illustrated in two left-hand panels of Fig. 1 where both DR and SH classes can be assigned to binaries without known spectral types.

To solve this problem, we calculate a membership probability (hereafter MP) for each class that can be assigned to the binary based on data of Table 1 by the classification procedure. The probability that a given system belongs to a class \( t \) (\( MP_t \)) is the ratio of binaries with available \( t \)-classification \( (N_t) \) to the total number of binaries \( \sum N_i \) with the available classification in the 3\( \sigma \) radius around the examined system in the parameter space \( S \):

\[
MP_t = \frac{N_t}{\sum N_i}, \quad i \in S_{3\sigma}.
\]

We estimate \( \sigma \) to

(i) 0.1 mag for depth of minima. It is a typical photometric error for photographic photometry, and at least half of magnitudes presented in CEV are photographic ones. Other (mostly photoelectric) catalogued photometric data have a better accuracy;

(ii) about 25 per cent of period value itself. This \( \sigma \) leads to interval \([0.25P:1.75P]\). Although individual periods can in some cases be determined with very high precision, the choice of our \( \sigma \) is driven by the large range of periods in our training set data. Our testing has shown that adopting such a large \( \sigma \) does not degrade the performance of our results;

(iii) five spectral subclasses, which is an approximate accuracy of spectral type, estimated from stellar colour index.

This approach is illustrated in Fig. 2, which represents a 2D box \((A_1-A_2)\) of the multidimensional space, where the number of dimensions is the number of observational parameters used for the classification.

The filled star is TX Nor, the system of unknown evolutionary status, while binaries with available classification are represented by empty circles (semidetached algols, SA), filled circle (hot semidetached systems, SH), filled square (detached MS systems, DM) and empty square (detached system with subgiants, DR).

Thus, the membership probability for TX Nor to be an algol-like system is

\[
MP_{SA} = \frac{N_{SA}}{N_{SA} + N_{SH} + N_{DM} + N_{DR}} = \frac{32}{35} = 0.91.
\]

MP_{SH}, MP_{DM} and MP_{DR} can be calculated similarly. We consider the binary to be categorized successfully if the MP for one of the evolutionary classes exceeds 0.5.

The described procedure is simple, quick and can be implemented in an automated program analysing published catalogues/lists of eclipsing binaries. However, we should draw user’s attention to the following features.

The majority of the catalogued systems with available classification (i.e. out training set) have an incomplete parameter set. For example, spectral classification of both components is available only for 28 per cent of categorized CEV binaries. As a result, the smaller the parameter set that is available, the larger the number of systems that are located within the given parameter space, and vice versa. If there are no systems in the 3\( \sigma \) vicinity, the MP value cannot be calculated because \( \sum N_i \) in equation (3) is equal to zero. In such cases, we increase the size of the parameter space by one or more sigmas, while this number becomes greater than 1. However, the minimum size of \( \pm \sigma \) was sufficient to calculate MP for about 90 per cent of the investigated systems, and the maximum size of \( \pm 6\sigma \) was applied to only two binaries.

The calculated MP values depend on the available parameter set (i.e. the number of dimensions of the examined parameter space); thus, we test our procedure on the systems with different parameter sets separately. This means that if a catalogued system has less or more parameters than examined unclassified binary, it is not included in the parameter space.

Another feature of the procedure is that the MP value directly depends on the number of different evolutionary class representatives in the vicinity of an examined system. For example, an area in the parameter space, occupied by SH systems, is populated also by DM systems (see Table 1). However, the former evolutionary class is much poorly represented among observed systems due to the relatively small number of (high-mass) objects and a rather rapid stage of stellar evolution. Consequently, DM systems are more numerous in the parameter space, and an examined system will be categorized as a DM system with higher probability. We believe that it is a correct solution, as the examined system will more likely belong to an evolutionary class of frequent occurrence.

Nevertheless, we have published at CEV all the predicted classes to each of the considered binary and not only the predicted class with the higher MP value.

3.2 Efficiency of the procedure

To estimate the efficiency of the procedure, we have applied it to CEV binaries with already available classification. The results are given in the second column of Table 2. Data are presented separately for every parameter set, used for classification (the first column). The second column contains the total number of CEV systems with a given parameter set followed by the percentage of correctly classified binaries and, in parentheses, the percentage of unclassified binaries whose evolutionary class remained unknown (i.e. we cannot assign any of the classes to the binary based on its observational parameters).
Data for systems with no available classification (see Section 3.3) are presented in the third column in a similar format, but numbers of successfully classified (instead of correctly classified) systems are given here. The binary has been considered to be successfully classified if the MP for one of the possible classes was larger than 0.5.

As can be seen from the second and third columns of Table 2, if spectra and period are known, then efficiency of our procedure exceeds 80 per cent.

We have also estimated the efficiency of the procedure for each evolutionary state. Results of the application of the same procedure to CEV systems with available classification are given in the error matrix (Table 3). The first column contains CEV evolutionary classes; each row of the table gives the result of the classification. For example, the first row indicates that among all 190 CEV DM systems (see Table 1), 174 were correctly categorized as DM, one was wrongly categorized as DR, etc.

So the matrix diagonal contains numbers of correctly categorized systems, while the other cells of the matrix contain numbers of wrongly categorized systems. The last column contains false negative (type II) error values, and the bottom row contains false positive (type I) error values.

Our analysis of the results, presented in Table 3, shows that the availability of different observational parameters can be crucial for the classification of binaries of various evolutionary classes. The following conclusions can be made.

In the case of detached systems with subgiants (DR), data about secondary spectra are required for correct classification, so only about 40 per cent of these systems have been classified correctly. A reliable classification of detached systems with OB giants (DGE) is only possible when the luminosity class is known because all other observational parameter values are virtually the same for DGE and DM systems. Detached systems with white dwarfs (DW) are close to cataclysmic semidetached binaries except the more longer periods. Therefore, short-period DW systems may be misclassified as S2C or as detached systems with OB giant, if secondary spectra are unavailable. Our procedure is efficient for detached symbiotic systems (D2S) and detached systems with late-type giants (DGL) because of the large luminosity difference between the components and their long orbital periods.

Among semidetached systems of different classes, the highest efficiency is for semidetached algol-like binaries (SA) and the lowest one is for hot semidetached systems (SH). The percentage of correct classification for cataclysmic semidetached systems is independent of the parameter set and is about 80 per cent.

Our procedure exhibits the lowest efficiency for all classes of contact binaries because their observational parameters are close to parameters of detached MS systems and semidetached classical

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### Table 2. The efficiency of the classification procedure for the CEV systems with different parameter sets.

| Parameter set | Class is known | Class is unknown |
|---------------|----------------|-----------------|
|               | Total number in CEV; correctly classified (unclassified) | Total number in CEV; successfully classified (unclassified) |
| $A_1, A_2, P, S_{p1}, S_{p2}$ | 327; 81 per cent (1.5 per cent) | 59; 88 per cent (12 per cent) |
| $A_1, A_2, S_{p1}, S_{p2}$ | 437; 65 per cent (4.6 per cent) | 868; 85 per cent (6 per cent) |
| $A_1, A_2, P$ | 278; 56 per cent (3.6 per cent) | 1359; 79 per cent (2 per cent) |
| $A_1, A_2, S_{p1}, S_{p2}$ | 75; 84 per cent (2 per cent) | 40; 73 per cent (25 per cent) |
| $A_1, S_{p1}, S_{p2}$ | 0; 0 per cent (0 per cent) | 75; 84 per cent (5 per cent) |
| $A_1, S_{p1}$ | 12; 67 per cent (0 per cent) | 15; 80 per cent (7 per cent) |

### Table 3. The confusion matrix of classification procedure.

| Results of classification | DM | DR | DGE | DGL | DW | D2S | SA | SC | SH | S2C | CB | CE | CWA | CWW | CG | Type II err., per cent |
|---------------------------|----|----|-----|-----|----|-----|----|----|----|-----|----|----|-----|-----|----|-----------------------|
| CEV class                 |    |    |     |     |    |     |    |    |    |     |    |    |     |     |    |          |
| DM                        | 174| 1  | 11  | 1   | 3  | 7   |    |    |    |     |    |    |     |     |    | 0.07               |
| DR                        | 8  | 10 | 6   | 1   | 1  | 60  |    |    |    |     |    |    |     |     |    | 0.36               |
| DGE                       | 3  | 6  | 6   | 2   | 1  | 25  |    |    |    |     |    |    |     |     |    | 0.18               |
| DGL                       | 1  | 13 | 13  | 1   | 1  | 19  |    |    |    |     |    |    |     |     |    | 0.09               |
| DW                        | 1  | 10 | 10  | 1   | 1  | 29  |    |    |    |     |    |    |     |     |    | 0.06               |
| D2S                       | 19 | 330| 330 | 16  | 1  | 0   |    |    |    |     |    |    |     |     |    | 0.02               |
| SA                        | 2  | 3  | 3   | 16  | 3  | 40  |    |    |    |     |    |    |     |     |    | 0.18               |
| SC                        | 15 | 1  | 1   | 16  | 1  | 53  |    |    |    |     |    |    |     |     |    | 0.08               |
| SH                        | 3  | 9  | 9   | 27  | 1  | 18  |    |    |    |     |    |    |     |     |    | 0.07               |
| S2C                       | 3  | 87 | 87  | 4   | 4  | 31  |    |    |    |     |    |    |     |     |    | 0.08               |
| CB                        | 2  | 4  | 4   | 80  | 4  | 58  |    |    |    |     |    |    |     |     |    | 0.08               |
| CE                        | 1  | 8  | 8   | 91  | 1  | 26  |    |    |    |     |    |    |     |     |    | 0.05               |
| CWW                       | 1  | 28 | 28  | 91  | 2  | 50  |    |    |    |     |    |    |     |     |    | 0.03               |
| CG                        | 1  | 0  | 0   | 0   | 0  |    |    |    |    |     |    |    |     |     |    | 0.00               |

Type I err., per cent

The efficiency of the classification procedure for the CEV systems with different parameter sets.
algol-like systems. Also, the procedure cannot separate CBF systems from CBV ones; however, it correctly identifies most of them as near-contact CB binaries. Our procedure allows us to separate the two subclasses of W UMa systems since 70 per cent of CWA and 76 per cent of CWW binaries have been classified successfully.

3.3 Application of the procedure

After the testing, the procedure was applied to CEV systems with no available classification. Before application, we have checked the overlapping of training and prediction sets and found that density distributions of the parameters of both sets are the same.

The resulting statistics of application of the procedure is given in the third column of Table 2.

We have detected a large number of candidates for interesting evolutionary classes, requiring further observations and studies. In particular, we have indicated a number of candidates for detached systems: 74 of them are suspected to consist of a white dwarf and an OB companion, 36 of them are presumably MS systems with at least one OB massive component and 30 others are presumably MS systems with a late-K or M star. Three new candidates for cataclysmic systems (S2C) were also found.

Determination of the basic stellar parameters of the components of cluster or nearby galaxy binaries allows us to measure the ages and distances of their parent stellar system, and to test stellar evolutionary models (see Rucinski 2005; Graczyk et al. 2014). Based on our results and data from SIMBAD data base, we have compiled a list of cluster binaries with known evolutionary status (see Table C1). The list includes only previously unstudied binaries, with the evolutionary class determined via our procedure.

We have checked all systems with unusual parameter values during the application of our procedure. For most of them, those values are obsolete or unconfirmed. New observations of those binaries are needed. Another reason of unsuccessful classification is a marginal (usually well-known) evolutionary status of a system. SX Cas (‘active algol’) can serve as a good example. The third reason of unsuccessful classification is the contradictory parameter values, i.e. some observational parameters point to one class, while others point to another. One of such binary RT Lac is described in Appendix A. However, for the majority of such systems we failed to find in the literature a reasonable explanation for contradictory parameters’ values, and their nature remains unclear.

We have compiled and published lists of systems, belonging to these three categories, in the recent version of CEV.

In some cases, an either too small or too large period value prevents successful classification of the system. Period of an eclipsing binary with negligible secondary minimum can erroneously be determined (and catalogued) to be twice longer than the real one. In contrast, catalogued period of a binary with an equal or similar minimum can be twice shorter than the real value. Our procedure can detect such cases. For example, binary VW Hya has orbital period \( P = 2.69 \) d and depth of the primary minimum \( A_1 = 3.12 \) mag in our catalogue. We have taken photometric data from Burki, Barbàn & Carrier (2005) and period was given by Kreiner (2004). With these values of period and \( A_1 \), value of depth of secondary minimum \( A_2 \) is equal to zero and binary can be classified as classical algol-like (SA) system. But Pojmański (2002) has given twice longer period. In this case, \( A_2 \) is equal to \( A_1 \) and our method cannot classify binary as semidetached algol-like system.

Such detection of half/double-period confusion is only possible in the cases where the wrong period does not produce a predicted evolutionary class.

The analysis also shows that the procedure can indicate errors in catalogued data. In particular, we have detected and removed from CEV (after confirmation from the literature) about 30 non-eclipsing variables. All these 30 objects were not classified with our procedure.

CEV and the results of the classification are available in CDS VizieR service. Live version of the data can be downloaded from http://www.inasan.ru/~malov/CEV/

3.4 Systems with uncertain or tentative classification

CEV contains a number of eclipsing binaries with an uncertain or tentative evolutionary class. Examples are MU Aqr (CB:) and TU Boo (CWW and CWA, according to different sources). Most of such binaries were taken from lists of Shaw (1994) and Pribulla, Kreiner & Tremko (2003), and we have not found any other confirmation of the assigned evolutionary class(es).

We have applied our procedure to these binaries, and presented the results in Table 4. The second and third columns list evolutionary class from CEV and one, determined with our procedure, respectively. The fourth column contains the MP value, or a \(^\ast\) flag, if the system was classified unambiguously. In the fifth column, we have used letter ‘L’ to refer to binaries without available light and radial curve analysis. Letter ‘M’ denotes binaries with contradictory classification. Reference to the source of CEV evolutionary class is given in the last column.

As can be seen from Table 4, three systems were classified unambiguously, namely CN And, RV CVn and AL Cas. For the last two systems, there is no confirmation of our results in the literature because both systems have never been properly studied. Our evolutionary class may be helpful for such investigations. The near-contact evolutionary class for CN And was confirmed by light-curve properties (e.g. asymmetry of maxima and unequal depth of minima) and by the solution of light curves which have been derived by van Hamme et al. (2001).

For 15 binaries in Table 4, tentative evolutionary class was confirmed by our procedure. For EE Aqr, RS Ind and V525 Sgr, near-contact evolutionary class (CB) was confirmed while a subclass (CBV or CBF) remained unknown. The MP value for VV Lac, RT LM1 and V Lep of the calculated evolutionary class is smaller than 50 per cent, but our classification is correct.

VV Lac, besides near-contact (CB) system, may also be a semidetached algol-like system (MP = 34 per cent) or a detached MS system (MP = 21 per cent). The uncertainty in RT LM1 evolutionary class actually remains as the MP value for CWA class is only 2 per cent larger than the one for CWW class. V Lep, besides CB system, may also be classified as a detached MS system (MP = 38 per cent).

For seven binaries in Table 4, the determined evolutionary class differs from the CEV (tentative) one. All of these binaries have never been studied carefully; thus, our classification can be considered as a proper one until new observational data are obtained. For example, KZ Vir was classified as an A-type W UMa contact system by Rucinski et al. (2001), but they have noted that the system may be a close detached binary. Later Pribulla et al. (2003) denoted the system as CB. So our DM class is probably correct, as the MP value for DM class is close to 100 per cent, and neither CWA nor CB class was assigned to this system by our method.

The remaining seven systems were also classified but MP values were smaller than 50 per cent. DU Boo may be classified as a near-contact system with MP = 42 per cent and as a detached MS system with MP = 41 per cent but not as a contact W UMa
binary. CW CMi was denoted as a near-contact by Pribulla et al. (2003), but it is rather a contact W UMa of W-subtype (with MP = 50 per cent) or A-subtype (with MP = 45 per cent). The same situation applies to EE Psc, which is a CWA or CWW system with an MP value of 50 and 45 per cent, respectively, whereas the MP value for the system to be a near-contact, as Pribulla et al. (2003) have supposed, is only 5 per cent. We have also found that RS Ser is a CWA or CWW system with corresponding MP values much larger than those for the near-contact configuration. GS Cep appears to be a semidetached SA system while an MP value for near-contact class is smaller and equal to 23 per cent. In contrast, V1034 Cyg seems to be a semidetached SA system while an MP value for near-contact class is smaller and equal to 23 per cent. For TZ Dra we have derived two possible classes, namely the near-contact (MP = 42 per cent) and the semidetached SA with an almost equal probability MP = 41 per cent.

All the systems except DU Boo were not previously studied, so our results can be useful for future investigations.

4 CONCLUSIONS

We constructed a procedure for the classification of eclipsing binaries, based on light-curve parameters and spectral classification (or colour indices). The procedure uses relations between different observational parameters and allows us to attribute a binary to one of the 15 evolutionary classes and estimate a probability of a correct classification. We tested the procedure, using about 1000 binaries with available classification, estimated its efficiency for different evolutionary classes and applied it to 4700 systems with no classification, listed in the CEV. About 3800 systems were successfully classified. About 100 of them happened to belong to some relatively rare evolutionary classes and could be interesting for a further study. Other 50 binaries, with newly determined evolutionary classes, are members of stellar clusters and can be used as additional tracers for age and distance estimation of their parent stellar systems.

At the same time, observational parameters of about 360 systems are too unusual and/or contradictory to provide successful classification. Published data for the most of such systems are obsolete or unconfirmed, and new observations of these objects are needed. Some other binaries are well known to belong to a marginal evolutionary status, while the nature of the rest 50 stars remains unknown. About 40 catalogued systems with uncertain or tentative classification were successfully classified with our procedure. Errors in catalogued data can also be indicated: in particular, some 30 non-eclipsing variables were found and, after confirmation from the literature, removed from CEV.

The procedure is fast, effective and can be applied to eclipsing binaries even if a set of observational parameters is incomplete. It can be extremely useful for the classification of a huge number of objects in large ground-based (MACHO, OGLE, etc.) and space-borne (Kepler, CoRoT, Gaia) surveys.

CEV and the results of the classification are available in CDS VizieR service. Live version of the data can be downloaded from http://www.inasn.ru/~malkov/CEV/.

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APPENDIX A: DISCUSSION OF SELECTED BINARIES

TU Boo may be classified as a contact W UMa of A-subtype because the primary minimum is a transit. The asymmetry of light curves was detected by several authors (Niarchos et al. 1996; Coughlin et al. 2008). Niarchos et al. (1996) found the light-curve solution with Wilson-Devinney code (Wilson & Devinney 1971) using spotted model for contact configuration. They stressed that some physical characteristics of the binary (e.g. mass ratio) are typical for a W-subtype system. Moreover, their solution with spots shows that the less massive and smaller secondary is hotter than the primary. According to Coughlin et al. (2008), TU Boo is a marginal contact system with both components almost filling their critical lobes. They supposed the mass transfer from secondary to primary which is supported by an increased period. Physical parameters (temperature ratio, radii ratio and masses), derived by Coughlin et al. (2008), point to A-subtype, but the small percentage of overcontact (which leads to marginal contact only) together with $q \approx 0.5$ appropriately to W-subtype.

ES Cnc is the only one detached binary with two subgiants which has such small values of $A_2$ and period. According to Yakut et al. (2009), eclipses are partial. Additionally, system is a hierarchical triple in which all three stars are blue stragglers.

RT LMi was classified by Niarchos et al. (1994) as W-subtype of W UMa systems based on the solution of observed light curves while Rucinski et al. (2000) assigned it A-subtype based on derived radial curves. Recently, Qian, He & Xiang (2008) have shown that the primary minimum changed from occultation to transit and concluded that for RT LMi a subtype based on Binnewijki’s classification could not be uniquely assigned.

RT Lac is one of the promising examples of the contradictory classification. The observed value of the secondary minimum depth and the primary spectral type are not typical compared to other SA systems. We have tried to classify it as a detached RS CVn system, but $A_2$ value is not typical for DR class too. Moreover, the evolutionary state of binary is not known exactly because RT Lac is among the most peculiar stars of RS CVn-type systems. While most of RS CVn binaries have equal-mass components, the components of RT Lac do not. Ibanoglu et al. (2001) reported that the brightness of the system at three phases, i.e. mid-primary and quadratures, shows quasi-periodic changes which are caused by a chromospheric activity of a more massive, smaller
and hotter component. Moreover, İbanoğlu et al. (1997) showed that the less massive, larger star fills its critical lobe. Therefore, a gas stream from the larger, less massive star to the more massive one will be expected. The binary may also belong to cool semidetached systems (SC), but its period is smaller than for other SC systems.

TT Lyr was classified as a hot semidetached system because of the spectral type of the primary, but the secondary (cooler) spectral type is K0, according to Liao & Qian (2010). There is no comprehensive analysis of the photometric and spectroscopic data for TT Lyr in the literature.

TY Pyx is a unique active binary of RS CVn type because as Andersen & Popper (1975) have shown, it consists of two almost identical components. Rao & Sarma (1981) have supposed that both components are on the pre-MS contraction phase.

RW UMa is a detached system with an evolved subgiant component according to Popper & Ulrich (1977). The value of $A_1$ is confirmed by Northern Sky Variability Survey (NSVS) data (Woźniak et al. 2004). Thus, we have used the value 1.56 mag as the upper limit for the depth of the primary minimum for DB binaries.

Z Vul is a hot semidetached binary (SH) with two components of almost equal radii according to Lazarz, Arevalo & Almenara (2009), so we have used its $A_1$ value as an upper limit for the depth of SH systems’ primary minimum.

**APPENDIX B: APPLICATION OF THE CLASSIFICATION ALGORITHM**

Application of the classification method is performed with two main stages. First stage is the determination of possible evolutionary classes based on data from Table 1. We illustrate this stage for unclassified system DP CMa.

In the third row of Table B1, we show values of its parameters: $A_1 = 0.90$ mag, $A_2 = 0.30$ mag, $\Delta A = 0.60$ mag, orbital period $P = 3.388$ d, spectral type and luminosity class for primary (more hotter) component $Sp_1 = K2V$, spectral type and luminosity class for secondary component $Sp_2 = M2V$, the components’ spectral type difference (which is given in units of a spectral class) $\Delta Sp = Sp_1 - Sp_2 = 1$, and morphological type of the light curve $MT = 'EA'$. The first step of this stage is to determine what evolutionary classes we may (or, strictly speaking, we may not) assign to DP CMa, basing on its $A_1$ value. In the second column of Table B1, maximum values of $A_1$ for each of our classes are listed. Depth of the primary minimum of DP CMa (0.9 mag) is larger than maximum possible value of $A_1$ of CWA class (0.81 mag). We mark that cell with the grey colour, and we remove CWA class from further consideration (other cells of ‘CWA’ row are empty).

In the second and third steps, we determine what classes cannot be assigned, comparing DP CMa’s $A_2$ and $\Delta A$ values with the corresponding maximum values of remaining evolutionary classes. The reason why we use one of these values for different classes is explained in the text (see Section 2).

The second step is to determine what classes are unsuitable for DP CMa basing on its $A_2$ value. We compare $A_2 = 0.30$ mag of DP CMa with maximum value of $A_2$ of DR, DW, DG*, S*, CB* classes which are listed in third column of Table B1. We find that we can exclude from further analysis DGL, DW, S2C classes because $A_2$ value of DP CMa is larger than maximum possible values of $A_2$ of these classes. We mark cells with these $A_2$ values with the grey colour. In the fourth step, we determine what classes cannot be assigned by comparing the $\Delta A$ value of DP CMa with the maximum possible value of $\Delta A$ of SH systems’ primary minimum.

**Table B1. Performance of the classification method for unclassified binary DP CMa.**

| Class* | $A_1$ (mag) | $A_2$ (mag) | $\Delta A$ (mag) | $P$ (d) | $Sp_1$ | $Sp_2$ | $\Delta Sp$ | MT* |
|--------|-------------|-------------|------------------|--------|--------|--------|------------|-----|
| Detached binaries | | | | | | | | |
| DM (190) | 1.10 | 0.81 | [0.4; 36] | O5–M4.5; IV-VI | EA |
| DR (25) | 1.56 | 0.35 | [1.9; 26] | A8–G6; IV-V | OE |
| DGE (8) | 0.65 | 0.34 | [1.6; 35] | WR3–B2; I, III |
| DGL (16) | 2.32 | 0.20 | | |
| DW (14) | 6.00 | 0.20 | | |
| D2S (5) | 6.22 | | [603; 6310] | |
| Semidetached binaries | | | | | |
| SA (376) | 3.70 | 0.60 | [2.1; 45] | B4–G0; I-V |
| SC (5) | 1.36 | 0.55 | [2.9; 22] | G8–K4; III-V | |
| SH (34) | 1.65 | 0.57 | [1.1; 16] | O8–B4; I, III-V |
| S2C (33) | 6.00 | 0.20 | | |
| Contact binaries | | | | | |
| CB (103) | 1.22 | 0.81 | [0.2; 1.5] | |
| CBF (11) | 1.00 | 0.30 | [0.5; 0.8] | |
| CBV (13) | 0.91 | 0.38 | [0.39; 1.0] | |
| CE (19) | 0.97 | 0.28 | | |
| CWA (115) | 0.81 | | | |
| CWW (123) | 1.00 | 0.22 | | |
| CG (4) | 0.69 | 0.12 | | |

aThe evolutionary status and the number of such systems in CEV.

bMaximum value.

cThe components spectral type difference $\Delta Sp = Sp_1 - Sp_2$ is given in units of a spectral class.

dMorphological type of the light curve.

eData on secondary minimum are given in CEV for only one D2S system.
with the grey colour again and delete DW, DGL and S2C classes from consideration (other cells in rows ‘DW’, ‘DGL’ and ‘S2C’ are empty).

In the third step, we compare \(\Delta \alpha\) of our binary with maximum possible value of \(\Delta \alpha\) of DM, CE, CW and CG classes. As can be seen from Table B1, CE, CW and CG classes should be removed from next steps. We mark corresponding cells with the grey colour.

In the fourth step, we determine the possible classes based on period value. We compare orbital period of DP CMa with intervals of possible periods for each of the remaining classes. It can be clearly seen that \(P = 3.388\) d is longer than the upper limit of interval of periods of CB* classes. We mark these unsuitable periods with grey colour and delete CB* classes from our analysis. D2S class is also impossible for DP CMa because period of this binary is much shorter than lower limit of interval of periods of D2S class.

After these four steps, we see that the following evolutionary classes can be assigned to DP CMa system: DM, DR, DGE, SA, SC or SH.

In the fifth step, we check what classes can be assigned, basing on value of spectral type of the more hotter component. It can be seen from Table B1 that only two classes remain, namely DM and SC. Intervals of values of \(Sp_1\) for DR, DGE, SA and SH classes are all unsuitable for DP CMa. We mark the unsuitable intervals with the grey and delete these classes from the next steps.

In the sixth step, we compare spectral type of the secondary of DP CMa with interval of possible spectral types of DM and SC classes. Only DM class remains.

Then, we check \(\Delta Sp\) (step 7) of DP CMa with one that is possible for DM class and also compare values of morphological type in step 8.

At the end of this procedure, we can classify DP CMa as a DM binary. We derive only one possible class so our classification is unique. There is no necessity in MP calculation.

However, if, as a result of the first stage, more than one class can be assigned to a system, we must estimate the MP value for each of the possible classes. For example, let us imagine that there is no information in the literature about spectral class of the secondary of DP UMa. In this case, our classification procedure (first stage, see above) misses steps 6 and 7. As can be seen from Table B1, we would have two possibilities: DM and SC classes. To choose one of them, we should execute the second stage and calculate the MP value.

In the first stage, we consider binary as a point in the \(N\)-dimensional space (here \(N\) is the number of parameters used for classification in the first stage) and compare its location with location of areas, populated with systems of known evolutionary classes. We do not take into account any of the possible observational errors for each of the parameter that we use for classification of the binary.

The second stage is the estimation of MP value for those binaries which were classified ambiguously, i.e. when we derived more than one class in the first stage.

### APPENDIX C: LIST OF CLUSTER BINARIES

| GCVS name | Cluster | Predicted class | Notes | GCVS name | Cluster | Predicted class | Notes |
|-----------|---------|----------------|-------|-----------|---------|----------------|-------|
| V426 Aur  | NGC 1907 | DM             |       | CN Cru    | NGC 4755 | DM             |       |
| EV Cnc    | NGC 2682 | CWA            | Possibly CB (Yakut et al. 2009) | DP Cru    | NGC 4609 | DM             |       |
| HS Cnc    | NGC 2682 | CWA            |       | V2031 Cyg | NGC 6913 | SA             |       |
| RV CVn    | NGC 5273 | CB             |       | V2108 Cyg | Roslund 5 | SA             |       |
| FF CMa    | Collinder 132 | DM |       | V2388 Cyg | NGC 6819 | CW | Field star? |
| MS CMa    | Collinder 132 | DM |       | TZ Lac    | NGC 7243 | SA             |       |
| MX CMa    | NGC 2362 | DM             |       | V684 Mon  | NGC 2264 | DM             |       |
| QU CMa    | NGC 2354 | DM             | Blue straggler? | V396 Nor | NGC 6025 | DM |       |
| V422 CMa  | NGC 2362 | DM             |       | V405 Nor  | Loden 2158 | SA |       |
| tau CMa   | NGC 2362 | DGE            | Multiple, see short description in Zasche et al. (2009) | AY Per    | Melotte 20 | SA |       |
| tau CMa   | NGC 2362 | DGE            | Multiple, see short description in Zasche et al. (2009) | AY Per    | Melotte 20 | SA |       |
| GV Car    | NGC 3532 | DM             |       | V578 Per  | Melotte 20 | DM             |       |
| QZ Car    | Collinder 228 | DGE | Rare object | BP Per    | Melotte 20 | DM |       |
| V356 Car  | NGC 2516 | DM             |       | V572 Per  | Melotte 20 | DM             |       |
| V661 Car  | Trumpler 16 | DGE |       | V620 Per  | NGC 884 | DM             |       |
| V546 Cas  | NGC 103 | DM             |       | V621 Per  | NGC 884 | DGE | Detached MS+giant binary according to Southworth et al. (2004) |
| V765 Cas  | NGC 457 | DM             |       | V732 Per  | Melotte 20 | DM |       |
| V969 Cas  | NGC 654 | DM             |       | V888 Per  | Melotte 20 | DM |       |
| V1123 Cas | NGC 581 | DM             |       | V607 Pup  | NGC 2422 | DM |       |
| V1130 Cas | NGC 581 | DM             |       | V792 Sgr  | NGC 6514 | DM |       |
| V1133 Cas | NGC 581 | DM             |       | V5563 Sgr | NGC 6530 | CE |       |
| Al Cep    | Trumpler 37 | DM |       | V861 Sco  | Trumpler 24 | DGE | Studied but not classified |
### Table C1 – continued

| GCVS name | Cluster | Predicted class | Notes | GCVS name | Cluster | Predicted class | Notes |
|-----------|---------|-----------------|-------|-----------|---------|-----------------|-------|
| IO Cep    | Trumpler 37 | SA              |       | V1069 Sco | NGC 6242 | DGL             |       |
| SU Cep    | Trumpler 37 | CB              |       | V1290 Sco | NGC 6231 | DM              |       |
| V427 Cep  | Trumpler 37 | DM              |       | V1292 Sco | NGC 6231 | DGE             | Classified as detached by Sana, Gosset & Rauw (2006) |
| V467 Cep  | NGC 6939  | DM              |       | V1293 Sco | Trumpler 24 | DM             |       |
| V470 Cep  | NGC 6939  | DM              |       | V1295 Sco | Trumpler 24 | DM             |       |
| V735 Cep  | Trumpler 37 | SA              |       | V1297 Sco | NGC 6231 | DM              |       |
| V738 Cep  | Trumpler 37 | DM              |       | MY Ser    | NGC 6604 | DM              |       |
| V747 Cep  | NGC 7822  | DM              |       | QR Ser    | NGC 6611 | DGE             |       |
| V767 Cep  | NGC 188   | DM              |       | V343 Vel  | NGC 3228 | SA              |       |
| MZ Com    | Melotte 111 | DM              |       | V451 Vel  | Pismis 4 | DM              |       |

### SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

Appendix
(http://mnras.oxfordjournals.org/lookup/suppl/doi:10.1093/mnras/stu1572/-/DC1).

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