21 cm radiation - a new probe of variation in the fine structure constant

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We investigate the effect of variation in the value of the fine structure constant ($\alpha$) at high redshifts (recombination $> z > 30$) on the absorption of the cosmic microwave background (CMB) at 21 cm hyperfine transition of the neutral atomic hydrogen. We find that the 21 cm signal is very sensitive to the variations in $\alpha$ and it is so far the only probe of the fine structure constant in this redshift range. A change in the value of $\alpha$ by 1% changes the mean brightness temperature decrement of the CMB due to 21 cm absorption by $> 5\%$ over the redshift range $z < 50$. There is an effect of similar magnitude on the amplitude of the fluctuations in the brightness temperature. The redshift of maximum absorption also changes by $\sim 5\%$.

Introduction. — Why do the fundamental constants of nature take the values that we measure? This question was posed as a fundamental problem of physics in the last century. Dirac [1,2] first considered the question of variation of these constants with time. The standard model does not explain the values of these constants, especially the constants determining the strength of the four fundamental forces. These constants can indeed vary naturally, though not necessarily, in space as well as time in Grand Unified Theories (GUTs) and theories of quantum gravity [3]. Thus a measurement of variation or a constraint on non-variation of the fundamental constants is an important probe of new physics beyond the standard model and general relativity. Testing the variation of constants has become more important in light of current data indicating the presence of dark energy. Dark energy could be a cosmological constant, which fits current data [4], but could also be evidence for physics beyond the standard model. It is therefore important to find new ways to distinguish different models of dark energy. If dark energy couples to the standard particle physics it could cause variations in the fundamental constants of the standard model [5]. Testing for variations in the fundamental constants thus also probes the properties of dark energy [6, 7] and hence is important for any attempt to explain dark energy theoretically.

In this Letter we propose a new probe of variations in the fine structure constant on cosmological time scales. The most stringent existing constraints on the value of the fine structure constant in the early Universe are from the measurements of quasar spectra involving fine structure transitions. These measurements suggest a change $[(\alpha_t - \alpha)/\alpha] \approx [\delta \alpha/\alpha] \lesssim 10^{-5}$, where $\alpha_t$ is the value of $\alpha$ at time $t$ [8, 9, 10]. For these quasar measurements $t$ corresponds to a $z < 3.5$. If the variation is monotonic $|\delta \alpha|$ would be larger at higher redshifts. If the variation is not monotonic, then measurements at many different redshifts would be required to trace the features in its evolution. The constraints at very early times, $z > 10$, are not very stringent and come from the CMB ($z \sim 10^3$) and big bang nucleosyntheis (BBN, $z \sim 10^9-10^{10}$). From CMB [11, 12, 13] $|\delta \alpha/\alpha| < 3 - 9\%$ and from BBN [14] $|\delta \alpha/\alpha| < 6\%$. There are no constraints for redshifts in the range $1000 > z > 10$.

The 21 cm absorption of CMB provides an opportunity to constrain $\alpha$ during these “dark ages.” Also the 21 cm absorption signal is available for a range of redshifts during this period and thus can be a useful probe for tracing the evolution of $\alpha$. One advantage of this method is that we are measuring the amount of absorption of the CMB radiation which depends on the Einstein emission/absorption coefficients. As we will see below the Einstein coefficients are more sensitive to changes in $\alpha$ ($A_{10} \propto \alpha^{13}$) than the fine structure/hyperfine structure splitting itself.

21 cm radiation from the dark ages. — The 21 cm signal from neutral hydrogen after recombination and before re-ionization has been investigated by many authors [15, 16, 17, 18]. We refer to [19] for detailed review. After recombination the radiation temperature goes down as $1 + z$. The baryons however are not prevented from cooling adiabatically due to the small amount of residual electrons which couple the gas to the radiation through Thomson scattering. At $z \sim 200$ this process becomes inefficient and the matter decouples thermally from the radiation. The hydrogen atoms after recombination are in the ground state which is split into a singlet and a triplet state due to hyperfine splitting. The occupation of the excited triplet state and the lower energy singlet state can be described by defining the spin temperature by the relation $n_{s}/n_{t} = g_{s}/g_{t} e^{-T_{s}/T_{r}}$, where $n_{s}$ and $n_{t}$ are the number densities of atoms in triplet and singlet states respectively, $g_{t}$ and $g_{s}$ are the corresponding statistical weights with $g_{t}/g_{s} = 3$, $T_{r} = 0.068K$ is the energy difference between the two states and equals 21 cm in wavelength units and $T_{s}$ is the spin temperature [20].

The evolution equations for the gas temperature $T_{g}$,
radiation temperature $T_r = 2.726(1 + z)K$, ionization fraction $x = n_i/n_{H^+}$, where $n_i$ is the number density of electrons and $n_{H^+}$ the total number density of hydrogen nuclei, and spin temperature $T_s$ can be written as \[ 22 \]

$$
\frac{dT_s}{dz} = \frac{4T_s^2}{H(1+z)} \left( \frac{1}{T_s} - \frac{1}{T_g} \right) C_{10} + \left( \frac{1}{T_s} - \frac{1}{T_g} \right) T_s A_{10} T_g
$$

(1)

$$
\frac{dT_g}{dz} = \frac{2T_g}{1+z} \frac{8\pi T_g}{3m_e c} \frac{4\sigma_{SB}}{x} \frac{T_s^4}{1 + x_{He} + x H(1+z)} (T_g - T_s)
$$

(2)

$$
\frac{dx}{d\bar{z}} = \frac{-C_r}{H(1+z)} (\beta (1-x) - n_H \alpha_2 x^2),
$$

(3)

where $H$ is the Hubble parameter at redshift $z$, $C_{10} = (\kappa_{10}^H n_H + \kappa_{10}^{H+} x_{H^+})$, $\alpha_{10}^H$ is the collisional de-excitation rate from triplet to singlet state for H-H collisions $23, 24$, $\kappa_{10}^H$ is the corresponding cross section for e-H collisions $24$, $A_{10}$ is the Einstein A coefficient for spontaneous transition, $\sigma_{SB}$ is the Stephan-Boltzmann constant, $m_e$ is the mass of electron and $c$ is the speed of light in vacuum. $x_{He} = n_{He}/n_{He}$, $n_{He}$ being the number density of helium nuclei. $C_r$, $\beta$ and $\alpha_2$ are defined by the equations $24, 22, 26$

$$
\alpha_2 = F^{10^{-13}} \frac{4.309(T_s/10^4)^{0.6166}}{1+0.6703(T_s/10^4)^{0.8366}} \text{cm}^3 \text{s}^{-1}
$$

(4)

$$
\beta = \left( \frac{m_e k_B T_g}{2\pi \hbar^2} \right)^{3/2} e^{-B_1/k_B T_g} \alpha_2
$$

(5)

$$
C_r = \frac{\Lambda_0 + \Lambda_2 x_{He} (1-x)n_H}{\Lambda_0 + (1-x) n_H (\Lambda_2 - x_{He} + \beta e^{hc/k_B T_g} \lambda_0)}
$$

(6)

where $\Lambda_0 = 8\pi H (1+z)/\lambda_3^2$, $\lambda_3 = 8\pi hc/3B_1$ and $B_1 = \alpha_2 m_e c^2/2$. $F = 1.14$ is the fudge factor to take into account the non-equilibrium among higher energy levels of hydrogen $23$. Helium recombination can be ignored since it has no effect on the 21 cm signal. Solving equations $13$ in a given cosmology gives the evolution of spin temperature with redshift. The change in the brightness temperature of the CMB at the corresponding wavelength is then given by $20, 27$

$$
T_b = \frac{(T_s - T_g) \tau}{(1+z)}, \quad \tau = \frac{3c^2 \hbar A_{10} n_H}{16 \kappa_{10}^H \nu_{21}^2 H T_s},
$$

(7)

where $\nu_{21} = \nu_{21} T_s / h \sim 1420 MHz z$. The brightness temperature is related to the observed intensity by the Rayleigh-Jeans formula $T_b = I_0 e^{\nu / 2 k_B T_s}$, where $I_0$ is the specific intensity and $\nu$ is the frequency of observation.

There will also be fluctuations in $T_s$ and $T_g$ due to inhomogeneities in $n_H$ and $T_g$ which are related to the primordial inhomogeneities in the gravitational potential $17, 18$. The angular power spectrum is then given by $C_l(z) = \langle a_m a_m^* \rangle$, where $a_m$ are the expansion coefficients in the spherical harmonic expansion of $\delta T_b = T_b - \bar{T}_b$, where $\bar{T}_b$ is the mean brightness temperature. We follow $18$ in our calculations. The baryon power spectrum is calculated using CMBFAST $28$.

**Effect of change in $\alpha$.** — A different $\alpha$ during recombination affects the CMB power spectrum due to Thomson scattering of photons through equation (3) and the Thomson scattering cross section $29, 30$. As with CMB we ignore the variation in the fudge factor, $F$, with $\alpha$ in Eq. $9$ since its effect is negligible. The crucial point is that for the $21$ cm transition, there is the following additional dependence on $\alpha$ in equations $1$, $2$ and $7$. The Einstein A Coefficient is given by $A_{10} = 64\pi^4 \nu_{21}^2 S_{21}/3h c^3 g_2$, where $g_2$ is the statistical weight of excited state, $S_{21} = 3\beta_M^2$, $\beta_M = eh/4\pi m_e c$ being the Bohr magneton $31$. Now $\nu_{21} \propto \alpha^2 R_{\infty} \propto \alpha^4$ and $T_s \propto \nu_{21}$, where $R_{\infty}$ is the Rydberg’s constant. Thus we see that the spontaneous emission coefficient, $A_{10} \propto \nu_{21}^3 \beta_M^2 \propto \alpha^3$, is a very sensitive function of $\alpha$.

The dependence of the collisional de-excitation rate is more complicated. We use ab-initio calculations and asymptotic formulae for large separations of potential energy curves of the ground state and the first excited triplet state of hydrogen molecule to calculate the spin change collision cross sections $32, 33, 34, 35, 36, 37$. These are nothing but the total energy at a given separation in the clamped nuclei approximation or the expectation values of the electronic Hamiltonian, $H_e$, which has a kinetic energy term ($T$) and a Coulomb potential term ($V$):

$$
H_e = T + V
$$

(8)

A change in the fine structure constant can be treated as a perturbation in the Coulomb potential ($V \propto \alpha$). Therefore if $\delta$ is the fractional change in alpha, so that $\alpha_{new} = \alpha (1 + \delta)$, then $V_{new} = V (1 + \delta)$. Now to first order in perturbation theory the expectation value of the new Hamiltonian is given by

$$
\langle H_{new} \rangle = \langle T + V_{new} \rangle = \langle H_e \rangle + \delta \langle V \rangle.
$$

(9)

In above the $\langle \rangle$ denote the expectation value over the unperturbed wavefunction everywhere. Also the change in the derivative to first order is given by $d\delta V/dR$. We estimate $dV/dR$ by fitting a polynomial function to the $V$-$R$ curve from the ab-initio calculations $32, 33, 34, 35, 36, 37$. Thus from the ab initio potential energy curve at unperturbed $\alpha$, we can construct the first order corrections when $\alpha$ changes by a small amount. We first calculate the scattering phase shifts by integrating the partial wave equations and use them to calculate the scattering cross sections and the rate coefficients ($\kappa_{10}^H$) using the standard scattering theory $23, 24, 38, 39$.

We checked our code by comparing our cross sections for unperturbed $\alpha$ with those calculated by Zygelman and Allison & Dalgarno $22, 24$. They agree to better than $1\%$ for $T_g > 40K$ and to $2\%$ for $15K < T_g < 40K$. The small error that we make at low temperatures is due to our ignoring the higher order effects which have been
unperturbed value.

Fig. 2: Upper panel shows T with different values of α. Bottom panel shows the fractional change (T/α - T)/T.

Fig. 3: Upper panel shows the angular power spectrum √(1/(1+1))C/2/2π at several redshifts. Bottom panel shows (√(C/δα = -2%) - √(C))/√(C) for the same redshifts.

Variations in α also affects the power spectrum of the spatial fluctuations. This is shown in fig. 6 for δα = -2%. The amplitude of the fluctuations is proportional to T. Thus we expect the amplitude of the power spectrum to have similar dependence on α as T. There is however also contribution due to fluctuations in T due to inhomogeneities in n_H and T_g. This additional effect causes an increase in the sensitivity to variations in α compared to T at z_α > 100 and a decrease in sensitivity at z_α < 100. There is also a change in the sign of ∆C/5% at z_α < 100 which is different from T, where this occurs at z_α ≈ 280. This change in sign is characteristic of the α dependence.

The detectability of the signal is limited by noise due to foregrounds. The noise can be expressed in temperature units for a single aperture telescope [40] as T_N = T_Nsys/ε/√t/1/2 where ε is the aperture efficiency which is close to unity, Δν is the bandwidth, t is the integration time and Tsys is the system temperature which at low frequencies is the temperature of the galactic foregrounds ~ 19000K at 22 MHz in a quiet portion of sky [41]. T_N ~ 1.4mK for Δν ~ 4 MHz and t = 2000 hrs. This means that the sensitivity of a single station of a telescope like LWA [42] or LOFAR [43] can give a constraint on ∆α of ~ 0.85%, improving to ~ 0.3% for the full LWA. The fundamental challenge to realizing this measurement is the required precision to which foregrounds have to be subtracted.

Although the foreground removal from the mean signal may prove to be impossible, this problem may be overcome by using the effect of δα on the angular power spectrum of fluctuations in the 21 cm absorption. Several promising foreground removal strategies have been proposed in the literature [44, 45, 46, 47]. To ob-
tain a similar sub-percent constraint from the angular power spectrum requires the higher sensitivity at small angular scales \( (\theta \sim 0.1\text{mK}) \) of a larger telescope (for example a low frequency equivalent of SKA [http://www.skatelescope.org] or much longer observation times [17]. This is likely to be feasible with future technological advancement.

This new probe is complementary to CMB and BBN since it is in a different redshift range and has the potential to provide constraints comparable to the CMB experiment Planck [12]. The 21 cm signal is of course also sensitive to the cosmological parameters. A 1% change in the baryon density, \( \Omega_b \), has a \( \sim 2\% \) effect on \( T_b \) while a 1% change in the Hubble parameter changes \( T_b \) by \( \sim 3\% \). Similar change in the HeII fraction from BBN and the matter density, \( \Omega_m \), change \( T_b \) by < 0.5%. Future CMB [48] and large scale structure experiments [49, 50] will be able to determine these parameters to \( \sim 1\% \). As seen earlier, the variation in the 21 cm signal due to variation in \( \alpha \) shows a characteristic dependence on \( z \). This is difficult to mimic by changing the cosmological parameters and provide a way to disentangle the two. In the power spectrum there will be additional degeneracy due to the initial conditions which might be difficult to separate. The 21 cm signal is also affected by variations in the gravitational constant (G) and the electron to proton mass ratio (\( \mu \)). A complete treatment should consider variations in all the constants simultaneously.

We note that the 21 cm observations can also provide a test for the spatial variations of \( \alpha \). This can then be used, in principle, to probe the spatial perturbations in the dark energy in the framework of theories of quantum gravity which relate \( \alpha \) to dark energy.

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APPENDIX

Plots corresponding to figures 2 and 3 but plotted and compared at actual redshift \( z \). They are also available as EPAPS Document No. E-PRLTAO-98-004710, [http://www.aip.org/pubservs/epaps.html](http://www.aip.org/pubservs/epaps.html).

**FIG. 4:** Upper panel shows \( T_b \) with different values of \( \alpha \) at actual redshift \( z \). Bottom panel shows the fractional change \( (T_b(\alpha) - T_b)/T_b \).

**FIG. 5:** Upper panel shows the angular power spectrum \( \sqrt{l(l+1)C_l/2\pi} \) at several redshifts. Bottom panel shows \( (\sqrt{C_l(\delta\alpha = -2\%) - \sqrt{C_l}})/\sqrt{C_l} \) for the same redshifts, each compared at actual redshift \( z \).