Determination of Asymptotic Normalization Coefficients by Analytic Continuation of Differential Cross Sections

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1. INTRODUCTION

Asymptotic normalization coefficients (ANC) determine the asymptotics of the wave functions of bound nuclear states in binary channels and are expressed in terms of the residue in energy of the partial-wave amplitude of elastic scattering at the pole corresponding to the binding energy of the system [1]. ANCs are fundamental nuclear characteristics that are important both in the physics of nuclear reactions and in the physics of nuclear structure. ANCs are on-shell observables, in contrast to commonly used spectroscopic factors, which are off-shell quantities and cannot be reliably extracted from experimental data. The role of ANCs is especially great in determining the cross sections for nuclear reactions with charged particles at low energies, which are inaccessible for direct measurement due to the large Coulomb barrier. The most important class of such processes is astrophysical nuclear reactions occurring in the cores of stars, including the Sun. The important role of ANCs in nuclear astrophysics was first noted in the work [2], in which it was shown that ANCs determine the general normalization of the cross sections for peripheral radiative capture reactions (see also [3–5]). The results of ANC calculations within the framework of various approaches depend significantly on the theoretical model used. ANCs turn out to be noticeably more sensitive to the model than such quantities as binding energies or root-mean-square radii. This circumstance makes it possible to use a comparison of the calculated and experimental ANC values to assess the quality of theoretical models.

From the above, it follows the importance of knowing the ANCs and including them in the number of important nuclear characteristics along with such quantities as binding energies, the probabilities of electromagnetic transitions, etc. ANCs cannot be directly measured in the experiment; to determine them, a special analysis of experimental data is required.

ANCs can be determined from the analysis of cross sections of peripheral transfer reactions within the framework of the distorted waves Born approximation (DWBA). This method of determining ANCs is based on the statement that the cross sections of peripheral reactions should be parameterized in terms of ANCs, rather than spectroscopic factors [4]. ANC values can also be extracted from data on radiative capture reactions at low energies [2, 6].

There are approaches specifically designed to determine ANCs without making assumptions based on specific nuclear models. One of such approaches is the method based on the analytic continuation in energy of the partial-wave amplitudes of elastic scattering, determined from the phase-shift analysis of experimental data, to the pole point located in the nonphysical region of negative energies (see, for example, [7, 8] and references therein). However, there are not so many sufficiently accurate results of phase-shift analyses in published sources, which limits the field of applicability of this method. In this work, to determine the ANCs, a method is used, which is also based on the analytic continuation, but not the partial-wave amplitudes in energy, but the experimental differential cross sections (DCS) of nuclear
transfer reactions in the variable \( z = \cos \theta \), where \( \theta \) is
the scattering angle in the center-of-mass system. In
what follows, we will use the notation \( \sigma(E, z) \) for the
DCS.

The paper is organized as follows. Section 2
describes the general formalism of the method used.
Section 3 is devoted to the application of this method
to the determination of ANC for a particular nuclear
system. The results obtained are briefly discussed in
Section 4.

Throughout the paper we use a system of units in
which \( \hbar = c = 1 \).

2. GENERAL FORMALISM
2.1. Formalism of the Method
for Short-Range Interaction

For a better understanding of the approach we use,
let us first outline its formalism in the absence of
the long-range Coulomb interaction. The Coulomb
effects, which can be essential, will be considered in
the following sections.

The idea of the method goes back to the work
of G.F. Chew [9], in which it was stated that the
extrapolation in \( \cos \theta \) of the DCS of the elastic \( NN \)
scattering to the pole corresponding to the virtual
pion can be used to determine the pion-nucleon cou-
ping constant. In our case, we consider the binary
nuclear reaction

\[ A + x \rightarrow B + y \]  

(1)

with transfer of particle \( c \), the contribution to the
amplitude of which is given by the Feynman pole
diagram (Fig. 1a), which we will call diagram 1a. At
a fixed impact energy, the amplitude of this diagram
has a pole in \( z = \cos \theta \), where \( \theta \) is the angle between
the directions of the momenta of particles \( x \) and \( y \)
in the center-of-mass system. The pole \( z = z_p \) lies in
the unphysical region \( z_p > 1 \) and is determined by the
expression [1]:

\[ z_p = [(1 + g^2)E + \epsilon_{BAc} - g^2\epsilon_{xyc}]/(2g\sqrt{EE'}) \].  

(2)

Here \( E(E') \) is the relative kinetic energy in the ini-
tial (final) state, \( E' = E + Q, Q = m_A + m_x - m_B -
m_y, \epsilon_{ijk} = m_j + m_k - m_i \) is the binding energy of
the nucleus \( i \) in the channel \( j + k, g^2 = m_ym_A/(m_xm_B) \),
and \( m_i \) is the mass of nucleus \( i \).

Due to the contribution of the mechanism corre-
sponding to diagram 1a, the DCS \( \sigma(E, z) \) of reaction
(1) has a second-order pole at \( z = z_p \). Let us intro-
duce the quantity

\[ \varphi(E, z) = (z - z_p)^2\sigma(E, z) \].  

(3)

\( \varphi(E, z) \) has a finite limit at \( z \rightarrow z_p \) and can therefore
be approximated in the form:

\[ \varphi(E, z) = \sum_{n=0}^{N} c_n(E)P_n(z) \],  

(4)

where \( P_n(z) \) are some polynomials. In particular, we
can take just \( z^n \) as \( P_n(z) \). The idea of the method,
which we will call the method of continuation to the
pole (MCP), is that the coefficients \( c_n(E) \) in (4) are
fit by the experimental values of \( \varphi(E, z) \) in the
physical region \(-1 \leq z \leq 1 \) using the \( \chi^2 \) method,
after which the approximated value \( \varphi(E, z) \) is con-
tinued analytically into the non-physical region of \( z \)
values up to \( z = z_p \). At the point \( z = z_p \), the contri-
bution of non-pole mechanisms to \( \varphi(E, z) \) vanishes
due to the presence of the factor \((z - z_p)^2\) in the
definition (3), and the remaining contribution of the
pole mechanism of Fig. 1a is expressed through
ANCs corresponding to vertices \( x \rightarrow y + c \) and \( A +
c \rightarrow B \). For these ANC in this paper we will use the
notations \( C_{xyc}(L_xS_x) \) and \( C_{BAc}(L_BS_B) \), where
\( L_x (L_B) \) and \( S_x (S_B) \) are orbital angular momentum
and spin in the channel \( y + c (A + c) \).

The general expression for the contribution of the
pole mechanism to the DCS of the reaction (1) for an
arbitrary diagram 1a, expressed in terms of the so-
called vertex constants, is given in [1]. It is simpli-
ﬁed in the case when the spin of the transferred particle \( c \)
is 0 or 1/2. In this case, using the formulas of [1] and
the known relationship between the vertex constants
and ANCs, the following relation can be obtained:

\[ \sum_{L_x,S_x} C^2(L_xS_x) \sum_{L_B,S_B} C^2(L_BS_B) \]

\[ = 16(2J_A + 1)(2J_B + 1)g^4m_c^2p_p^2EE'R(E). \]  

(5)

In (5) \( J_i \) is the spin of particle \( i, p (p') \) is the relative
momentum of particles in the initial (final) state, \( p^2 =
2\mu_xAE, p'^2 = 2\mu_yBE' \), and \( \mu_{ij} = m_im_j/(m_i + m_j) \)
is the reduced mass of particles \( i \) and \( j \). The factor \( R(E) \)

\[ R(E) = \lim_{z \rightarrow z_p} \varphi(E, z) \]

\[ = \lim_{z \rightarrow z_p} [(z - z_p)^2\sigma(E, z)]. \]  

(6)

By ﬁnding the value of \( R(E) \) from the experimental
data, we can determine the product of ANCs squared
standing on the left-hand side of (5). If the ANC
values for one of the vertices are known (for example,
if it is the vertex \( d \rightarrow n + p \)), then from (5) and (6)
information about ANC for the second vertex will be
obtained.

We should note one circumstance that may com-
plicate the practical application of the above MCP in
the case when the main (or the only) contribution to
at least one vertex of diagram 1a is given by ANCs
with nonzero values of the angular momentum \( L \). The
point is that the vertex form factors for the three-
ray vertex \( a \leftrightarrow b + c \) contain the factor \( g_{bc}^{L}\), where
\( q_{bc} \) is the relative momentum of particles \( b \) and \( c \). Therefore, if \( L > 0 \), then at \( q_{bc} = 0 \) the corresponding form factor is zero. Consider the particular situation where, in diagram 1a, in the vertex \( x \rightarrow y + c \) \( L_{bc} = 0 \) and in the vertex \( A + c \rightarrow B \) the contribution gives a single value \( L_{Ac} > 0 \). At \( q_{Ac} = 0 \) the vertex form factor at the vertex \( A + c \rightarrow B \) is zero and hence the contribution of the pole mechanism to the DCS \( \sigma(E, z) \) and to \( \varphi(E, z) \) is zero. It follows from the momentum conservation at the vertices of Feynman diagrams that \( q_{Ac} = -p + (m_A/m_B)p' \), hence

\[
q_{Ac}^2 = p^2 + (m_A/m_B)^2p'^2 - 2(m_A/m_B)p'z,
\]

where

\[
z = \frac{pp'/p'}{z}. \tag{7}
\]

It follows from (7) that \( q_{Ac} = 0 \) corresponds to \( z = z_0 \),

\[
z_0 = \frac{p^2 + (m_A/m_B)^2p'^2}{2(m_A/m_B)p'}. \tag{8}
\]

On the other hand, at the pole point the vertex form factors in diagram 1a go to the mass surface, i.e., at the pole \( q_{Ac}^2 = -x_{BAc}^2, \quad x_{BAc} = \sqrt{2\mu_{Ac}^2E_{BAc}} \). Therefore, it follows from (7) that the pole value \( z = z_p \) can be written in the form

\[
z_p = \frac{p^2 + (m_A/m_B)^2p'^2 + x_{BAc}^2}{2(m_A/m_B)p'}. \tag{9}
\]

From the comparison of (8) and (9) it is easy to see that \( z_p > z_0 > 1 \). Hence, when function \( \varphi(E, z) \) (3) is continued from the physical region to the point \( z = z_p \), the pole mechanism contribution to \( \varphi(E, z) \) first turns to zero at \( z = z_0 \) and only then completely determines \( \varphi(E, z) \) at \( z = z_p \). Such a behavior of the pole contribution of interest is extremely difficult to reproduce by analytical approximation of the function \( \varphi(E, z) \) in the physical region. A possible way to solve this problem is discussed in the Appendix.

2.2. Accounting for Coulomb Effects

The long-range Coulomb interaction significantly affects the analytic properties of the amplitudes of the processes calculated with only short-range forces. The changes introduced by the Coulomb interaction into the characteristics of the singular points of the amplitudes of Feynman diagrams in the variable \( z = \cos \theta \) for the binary reaction (1) have been considered in [10–12]. In these works it is shown that the inclusion of the Coulomb interaction between particles in the diagram, in contrast to short-range forces, does not change the position of the proper singularities of the diagrams, but it changes the type and power of these singularities. In the case of the pole mechanism (diagram 1a) the account of the Coulomb interaction in the initial, final and intermediate states and in the vertices of the diagram leads to the fact that the pole singularity turns into a branch point. For the three-ray vertices in diagram 1a, the inclusion of the Coulomb interaction between \( y \) and \( c \) and between \( A \) and \( c \) leads to additional factors \((z - z_p)^{n_{yc}}\) and \((z - z_p)^{n_{Ba}}\) in the vertex form factors, where \( n_{ijk} \) is the Coulomb (Sommerfeld) parameter for the bound state:

\[
n_{ijk} = Z_j Z_k e^2 \mu_{jik} / \kappa_{ijk}, \quad \kappa_{ijk} = \sqrt{2\mu_{jk} \epsilon_{ijk}}. \tag{10}
\]

As a result, in the expression for the contribution of diagram 1a to the DCS of reaction (1), the factor \((z - z_p)^{-2}\) turns into \((z - z_p)^{-2(1 - n_{yc} - n_{Ba})}\) Since the values of \( n_{ijk} \) are positive, this substitution weakens the singular behavior of the pole contribution to the DCS at \( z \rightarrow z_p \); at \( n_{yc} + n_{Ba} > 1 \) this contribution turns to zero at \( z = z_p \) at all. This circumstance complicates appreciably the application of the MCP to charged-particle transfer reactions. Below we will consider the \((d, p)\) reaction in which the transferred particle \( c \) in diagram 1a is a neutron, so there are no Coulomb effects in the vertices of diagram 1a. The remaining Coulomb corrections, which we will consider, do not change the factor \((z - z_p)^{-2}\) in the expression for the reaction cross section. In fact, we consider the corrections due to the Coulomb interaction of the \( A + x (B + y) \) particles in the initial (final) state, as well as the Coulomb interaction in the intermediate state. The interaction in the intermediate state is determined by the sum of the diagrams of the infinite series of rescattering in the three-body system \( A, y \) and \( c \), in which the 4-ray vertices correspond to pure Coulomb scattering. The first of the diagrams of this series is shown in Fig. 1b. The amplitudes of all the diagrams of this series have a pole at the same point \( z = z_p \) as the diagram 1a, and in fact lead to a renormalization of the residue at this pole [10–12].

The Coulomb corrections to the cross section for the pole mechanism were found in [10–12] by inves-
Table 1. Singular point positions of the amplitude of the reaction \( d + ^{12}\text{C} \rightarrow p + ^{13}\text{C}^* \)

| \( E_d, \text{MeV} \) | 3.7 | 5.03 | 9.0 | 12.0 | 30.0 |
|-----------------|-----|------|-----|------|------|
| \( z_p \)      | 1.34 | 1.27 | 1.18 | 1.16 | 1.11 |
| \( z_t \)      | 14.88 | 11.15 | 6.54 | 5.15 | 2.69 |
| \( \eta_t + \eta_f \) | 1.24 | 1.06 | 0.78 | 0.69 | 0.43 |

Table 2. Calculated values of ANC

| \( E_d, \text{MeV} \) | \( \beta \) | \( \beta_t \) | \( C_0, \text{fm}^{-1/2} \) | \( C_1, \text{fm}^{-1/2} \) | Reference |
|-----------------|-----|-----|-----------------|-----------------|----------|
| 3.7             | 0.0822 | 0.0853 | 0.543 | 1.858 | [14] |
| 5.03            | 0.156 | 0.162 | 0.783 | 1.948 | [15] |
| 9.0             | 0.357 | 0.369 | 1.320 | 2.174 | [16] |
| 12.0            | 0.469 | 0.484 | 1.238 | 1.780 | [17] |
| 30.0            | 0.776 | 0.797 | 1.379 | 1.543 | [18] |

3. APPLYING THE MCP TO THE REACTION \( ^{12}\text{C}(0^+; 0 \text{ MeV})(d, p)^{13}\text{C}(1/2^+; 3.09 \text{ MeV}) \)

In this section, the MCP is applied to determine the ANC for the vertex \( ^{13}\text{C}(1/2^+; 3.09 \text{ MeV}) \rightarrow ^{12}\text{C}(0^+; 0 \text{ MeV}) + n \) by analytic continuation of the DCS of the reaction \( ^{12}\text{C}(0^+; 0 \text{ MeV})(d, p) \times ^{13}\text{C}(1/2^+; 3.09 \text{ MeV}) \). In diagram 1a for this reaction \( A = ^{12}\text{C}, x = d, B = ^{13}\text{C}, y = p, c = n \). The angular momenta \( L_{yc} \) and \( L_{Ac} \) are zero at both vertices in diagram 1a, so that the non-zero angular momentum problem described above is absent. The small admixture of the \( D \)-state in the deuteron is inessential. Note that the ANC under consideration is important for astrophysics.

The experimental DCS from the works [14–18], corresponding to the energies of the incoming deuteron in the laboratory system \( E_d = 3.7, 5.03, 9.0, 12.0, \) and \( 30.0 \text{ MeV} \), were used for the analysis. The values of the DCS measured in these works were taken from the database [19]. The experimental values of \( \varphi(E, z) \) in the front hemisphere were approximated according to (4), where Chebyshev polynomials were taken as \( P_n(z) \). The coefficients \( c_n \) were fitted to the experimental data by the \( \chi^2 \) method. The obtained expressions were continued analytically to the point \( z = z_p \). After that, the ANC values \( C \) for the vertex \( ^{13}\text{C}(1/2^+; 3.09 \text{ MeV}) \rightarrow ^{12}\text{C}(0^+; 0 \text{ MeV}) + n \) were found using the formulas (5) and (6). For the quantity \( \sum_{L_d S_d} C^2(L_d S_d) \) characterizing the vertex \( d \rightarrow n + p \), the value 0.7715 \( \text{fm}^{-1} \) [20] was taken.

The calculation results are presented in Tables 1 and 2. Table 1 is of an auxiliary nature. In the first line of this table the values of the deuteron energy in the lab system \( E_d \) are given, corresponding to the experimental data used, and in the second, the values of the pole position \( z_p \) corresponding to these energies are presented. As the energy increases, the values of \( z_p \) decrease and approach the boundary of the physical region \( z = 1 \); at \( E \rightarrow \infty \) the limiting value of \( z_p \) is 1.05. For comparison, the third line indicates the position of the nearest to \( z_p \) singularity \( z_t \) of the amplitude of the reaction under consideration. In our case \( z_t \) is the proper singularity of the diagram of Fig. 1c. The values of \( z_t \) were calculated using general formulas for the position of the proper singularity of an arbitrary triangle Feynman diagram [21]. Information about the nearest to \( z_p \) singularity is important because if this singularity is close to \( z_p \), the polynomial approximation (4) may be unjustified. Note that if there are two closely spaced singularities, in order to increase the accuracy of the analytic continuation of the function from the physical region to the nearest singularity, the optimal conformal mapping of the variable in which the continuation is carried out is used [22, 23]. In our case, as can be seen from the Table 1, the value \( z_t - 1 \), which characterizes the distance from \( z_t \) to the boundary of the physical region, for all energies is more than an order of magnitude greater than the corresponding value \( z_p - 1 \), so there is no need for a conformal mapping. Since the discussed Coulomb corrections are determined mainly by the values of the Coulomb parameters in the initial \( \eta_t \) and final \( \eta_f \) states, the last row of Table 1 shows for reference the value \( \eta_t + \eta_f \), which decreases with increasing the collision energy.

The main results of the calculations are presented in Table 2. The second and third columns of Table 2 show the calculated Coulomb corrections \( \beta \) and \( \beta_t \). \( \beta \) was calculated taking into account only the Coulomb interaction in the initial and final states, and \( \beta_t \) additionally takes into account the Coulomb interaction in the intermediate state. The values of \( \beta \) and \( \beta_t \) are close, so we can conclude that the main contribution to the corrections is made by the Coulomb interaction in the initial and final states. In reality, accounting
for Coulomb corrections when determining ANC is expressed in the fact that the coefficient $\beta_1$ (or $\beta$) should be added as a multiplier to the left-hand side of (5). Columns 4 and 5 of Table 2 show the ANC values calculated without ($C_0$) and with ($C$) Coulomb corrections. The ANC $C$ is related to $C_0$ as $C = C_0/\sqrt{\beta}$. For each value of deuteron energy, the number of terms in the expansion (4) and the coefficients $c_i$ were found by the $\chi^2$ method. The last column of Table 2 contains references to the experimental works, the results of which were used to determine the ANC.

It is seen from a comparison of $C$ and $C_0$ values that the Coulomb corrections under consideration significantly affect the ANC values obtained within the MCP. This influence is especially noticeable at low energies. It follows from the ANC values presented in Table 2, obtained from the analysis of various experimental data, that the mean value of the ANC obtained by us is $C = 1.86 \pm 0.16 \text{ fm}^{-1/2}$.

4. CONCLUSION

In this paper, the MCP is used to determine the ANC for the $^{13}$C($1/2^+; 3.09 \text{ MeV}) \to ^{12}$C($0^+; 0 \text{ MeV}) + n$ based on experimental data on the differential cross sections of the reaction $^{12}$C($0^+; 0 \text{ MeV}) (d, p)^{13}$C($1/2^+; 3.09 \text{ MeV})$ at different energies of the incident deuteron. An essentially new element is taking into account the corrections due to the Coulomb interaction in the initial, final, and intermediate states of the reaction. It is shown that these corrections strongly affect the extracted values of the ANC, especially at low energies. Taking these corrections into account, the ANC value $C = 1.86 \pm 0.16 \text{ fm}^{-1/2}$ was obtained. In [6], $C = 1.61 \text{ fm}^{-1/2}$ was found from the analysis of the $^{12}$C($n, \gamma)^{13}$C radiation capture data. An analytical continuation in energy of the phase-shift analysis data of elastic neutron scattering on $^{12}$C gives a significantly higher value $C = 2.07 \pm 0.13 \text{ fm}^{-1/2}$ [8]. Thus, the ANC value $C$ obtained in the present paper lies between the values given in [6] and [8].

The MCP has some fundamental advantages over other methods of determining ANCs from experimental data. It does not rely on the use of any nuclear models, as when determining ANCs from transfer reaction analysis within the DWBA. In contrast to the method based on analytic continuation in energy of the phase-shift analysis data, MCP uses only the directly measured value—the reaction differential cross section. The accuracy of the ANC determination within the MCP is determined mainly by the availability of sufficiently accurate experimental data on the differential cross sections at small scattering angles.

The MCP and a similar method for subtracting the singularity [24] was used in a number of works [24–27] to determine nuclear vertex constants expressed in terms of ANCs for several lightest and light nuclei. However, these works did not take into account the Coulomb corrections discussed in the present paper, and also ignored the problems associated with the nonzero orbital angular momenta at the vertices of the pole diagram. These circumstances make us cautious about the results obtained in these papers.

Note that if more than one ANC contributes to the vertex functions of the pole diagram $1a$, information about the ratios of these ANCs can in principle be obtained by applying the MCP to the polarization observables.

APPENDIX

As noted above in Section 2, when the function $\varphi(E, z)$ (3) is continued to the pole in the case where at least one vertex of diagram $1a$ has an orbital momentum $L > 0$, a problem arises, since the function has in this case the form (the argument $E$ is for brevity omitted)

$$\varphi(z) \equiv (z - z_p)^2 \sigma(z) = (z - z_0)^{2L} A(z) + (z - z_0)^L (z - z_p) B(z) + (z - z_p)^2 D(z). \quad (A.1)$$

Information about the ANC is contained in the function $A(z)$, but it is practically impossible to obtain any reliable value $A(z_0)$ by direct continuation $\varphi(z)$ from the physical region to the point $z = z_0$, since $1 < z_0 < z_p$ and therefore the term in (A.1) containing $A(z)$ turns to zero during continuation. It is possible, however, to propose an approach that allows to solve this problem, at least in principle. Let us consider this approach on the example of $L = 1$. In this case

$$\varphi(z) = (z - z_0)^2 A(z) + (z - z_0)(z - z_p) B(z) + (z - z_p)^2 D(z). \quad (A.2)$$

If we put $z = z_0$, we get

$$D(z_0) = \frac{\varphi(z_0)}{(z_0 - z_p)^2}. \quad (A.3)$$

Given (A.3), it is not difficult to obtain the relation:

$$\varphi(z) - \frac{(z - z_p)^2}{(z_0 - z_p)^2} \varphi(z_0) = (z - z_0) A(z) + (z - z_0)(z - z_p) B(z) + (z - z_p)^2 \frac{D(z) - D(z_0)}{z - z_0}. \quad (A.4)$$

Note that the left-hand side of (A.4) has no singularity at $z = z_0$. For the sake of brevity, we denote the left-hand side of (A.4) by $\varphi_1(z)$ and rewrite (A.4) as

$$\varphi_1(z) = (z - z_0) A(z) + (z - z_p) B_1(z),$$

$$B_1(z) \equiv B(z) + (z - z_p) \frac{D(z) - D(z_0)}{z - z_0}. \quad (A.5)$$
Let us perform another transformation. By putting $z = z_0$ in (A.5), we get

$$B_1(z_0) = \frac{\varphi_1(z_0)}{z_0 - z_p}.$$  \hspace{1cm} (A.6)

Hence, by analogy with (A.4), the relation follows:

$$\varphi_1(z) - \frac{z - z_p}{z_0 - z_p} \varphi_1(z_0)$$

$$\frac{z - z_0}{z - z_0} = A(z) + (z - z_p) \frac{B_1(z) - B_1(z_0)}{z - z_0}.$$ \hspace{1cm} (A.7)

Denoting the left-hand side of (A.7) by $\varphi_2(z)$, we rewrite (A.7) as

$$\varphi_2(z) = A(z) + (z - z_p)B_2(z),$$

$$B_2(z) \equiv (z - z_p)\frac{B_1(z) - B_1(z_0)}{z - z_0}.$$ \hspace{1cm} (A.8)

By putting $z = z_p$ in (A.8), we get the value $A(z_p)$ we need, which is expressed through ANCs. In principle, this procedure can also be done for $L > 1$ by repeating it as many times as necessary.

The above procedure allows us to formally solve the problem of analytic continuation to the pole for $L > 0$. However, for its practical application it is necessary to establish with good accuracy the behavior of $\varphi(z)$ in the neighborhood of $z = z_0$. As can be seen from (A.4) and (A.7), for the example considered with $L = 1$, to determine $A(z_p)$ actually requires knowledge not only of $\varphi(z_0)$ but also of the first ($\varphi^{(1)}(z_0)$) and second ($\varphi^{(2)}(z_0)$) derivatives of $\varphi(z)$ at $z = z_0$. For $L > 1$, even more derivatives $\varphi^{(n)}(z_0)$ are required. The possibility of determining these values accurately enough places very high demands on the completeness and accuracy of the experimental DCS. The experimental data currently available in the literature hardly satisfy these requirements.

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