Analysis of forced gripping of material in a two-roll module

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Abstract. The process of gripping material with two driven rollers in an asymmetric two-roll module is analyzed in the article. Analytical expressions for the minimum and maximum pushing force to grip material with two driven rollers in an asymmetric two-roll module were determined. Analytical expressions were obtained for calculating the contact angles of the free and forced grip of material with two driven rollers, which are the basic quantities that determine the boundary conditions of the contact interaction problems. A condition for forced gripping in an asymmetric two-roll module was found. From this condition, the conditions for forced gripping of the process of symmetric rolling were obtained.

1. Introduction
Roller machines are widely used in many industries. The main working body of roller machines is a roller pair. The roller pair and the material to be processed form a two-roll module. Technological processes in two-roll modules are performed as a result of the contact interaction of rollers with the processed material.

The problems of contact interaction in two-roll modules were investigated and simulated in [1-6]. The solution of contact interaction problems without contact angles is not feasible, since they determine the boundary conditions of these problems. Therefore, contact angles are the main quantities of a two-roll module. They are evaluated by the conditions of material gripping by a pair of rolls [7].

In many machines, two-roll modules are asymmetric. At present, there are many publications [8-11] devoted to the theoretical analysis of contact angles in an asymmetric two-roll module, in which one or several types of asymmetry are investigated. To further develop theoretical concepts, we have investigated the forced gripping of material in an asymmetric two-roll module.

2. Theoretical solution of the problem
Consider the two-roll module shown in figure 1, with the following parameters: the angle of roller inclination - $\beta$, the radii of the rolls - $R_1, R_2$, the initial thickness of the material layer - $\delta$, the angle of inclination of the material layer to the centerline (axis) - $\gamma$, the pushing force - $U$, the distance between the rollers - $h$, both rollers are driven.
First, we analyze the free grip of the material layer. In free grip, force \( U \) does not act on the material layer, that is, \( U = 0 \).

At the moment of gripping at points \( B_1 \) and \( B_2 \) from the side of the rollers, normal pressure forces \( N_1 \), \( N_2 \) and friction forces \( T_1 \), \( T_2 \) act on the material layer (figure 1). Gripping ability depends on the ratio of pulling-in force and repulsive force, and is written as:

\[
N_{1x} + N_{2x} \leq T_{1x} + T_{2x}.
\]

In addition to condition (1), we compose the force balance equation along the axis \( Oy \)

\[
N_{1y} - N_{2y} + T_{1y} - T_{2y} = 0.
\]

From the diagram of forces in figure 1 we obtain

\[
N_{1x} = N_1 \sin \varphi_1, \quad T_{1x} = T_1 \cos \varphi_1, \quad N_{1y} = N_1 \cos \varphi_1, \quad T_{1y} = T_1 \sin \varphi_1,
\]

\[
N_{2x} = N_2 \sin \varphi_2, \quad T_{2x} = T_2 \cos \varphi_2, \quad N_{2y} = N_2 \cos \varphi_2, \quad T_{2y} = T_2 \sin \varphi_2.
\]

With expressions (3), inequality (1) and equality (2) can be rewritten in the following form

\[
N_1 \sin \varphi_1 - T_1 \cos \varphi_1 \leq -(N_2 \sin \varphi_2 - T_2 \cos \varphi_2),
\]

or according to Amonton's law of friction

\[
N_1 \sin \varphi_1 + T_1 \sin \varphi_1 = N_2 \cos \varphi_2 + T_2 \sin \varphi_2.
\]

where \( f_1 \) and \( f_2 \) are the coefficients of friction of the rollers on the material layer at points \( B_1 \) and \( B_2 \), respectively. From equation (5) we define force \( N_2 \).
\[ N_2 = N_1 \cos \phi_1 + f_1 \sin \phi_1. \]  

(6)

Substituting the value of force \( N_2 \) into condition (6) and after a series of transformations we obtain

\[ \tan(\phi_1 + \phi_2) \leq \frac{f_1 + f_2}{1 - f_1 f_2}. \]  

(7)

Let us take into account that \( f_1 + f_2 = \tan \nu_1 + \tan \nu_2 \), where \( \nu_1 \) and are the angles of friction at points \( B_1 \) and \( B_2 \), respectively.

Then we finally get

\[ \phi_1 + \phi_2 \leq \nu_1 + \nu_2. \]  

(8)

This inequality determines the condition of free gripping of the material with two driven rollers in an asymmetric two-roll module, shown in figure 1.

If condition (8) is not met, then the material layer is not gripped by the rollers. In this case, the gripping of the material layer is performed using pushing force \( U \). The force \( U \), like the friction forces \( T_1 \) and \( T_2 \), is directed along the course of processing the material layer.

Let us compose the balance equations of forces acting on the material layer in the steady-state gripping process, taking into account force \( U \):

\[
\begin{align*}
\sum X &= -N_{1x} - N_{2x} + T_{1x} + T_{2x} + U_x = 0, \\
\sum Y &= N_{1y} - N_{2y} + T_{1y} - T_{2y} - U_y = 0.
\end{align*}
\]  

(9)

From the diagram of forces in figure 1 we obtain

\[ U_x = U \cos \gamma, \quad U_y = U \sin \gamma. \]  

(10)

Taking into account expression (10), we rewrite system (9) in the following form

\[
\begin{align*}
N_1 \sin \phi_1 - T_1 \cos \phi_1 + N_2 \sin \phi_2 - T_2 \cos \phi_2 &= U \cos \gamma, \\
N_1 \cos \phi_1 + T_1 \sin \phi_1 - N_2 \cos \phi_2 - T_2 \sin \phi_2 &= U \sin \gamma.
\end{align*}
\]  

or according to Amonton’s law of friction

\[
\begin{align*}
N_1 \frac{\sin(\phi_1 - \nu_1)}{\cos \nu_1} + N_2 \frac{\sin(\phi_2 - \nu_2)}{\cos \nu_2} &= U \cos \gamma, \\
N_1 \frac{\cos(\phi_1 - \nu_1)}{\cos \nu_1} - N_2 \frac{\cos(\phi_2 - \nu_2)}{\cos \nu_2} &= U \sin \gamma.
\end{align*}
\]  

(11)

Multiplying the first equation of system (11) by \( \cos(\phi_2 - \nu_2) \), the second equation by \( \sin(\phi_2 - \nu_2) \) and summing them, after transformations we obtain

\[ \sin(\phi_1 - \nu_1 + \phi_2 - \nu_2) = \frac{U}{N_1} \cos \nu_1 \cos(\phi_2 - \nu_2) - \gamma \]

or using the formula for the cosine of the angular difference, we have

\[ \sin(\phi_1 - \nu_1 + \phi_2 - \nu_2) = \frac{U}{N_1} \cos \nu_1 (\cos(\phi_2 - \nu_2) \cos \gamma + \sin(\phi_2 - \nu_2) \sin \gamma). \]  

(12)

Due to the small value of difference \( (\phi_1 - \nu_1) \) and \( (\phi_2 - \nu_2) \), we can assume that
\[
\sin(\varphi_1 - \nu_1 + \varphi_2 - \nu_2) \approx \varphi_1 - \nu_1 + \varphi_2 - \nu_2, \quad \sin(\varphi_2 - \nu_2) \approx \varphi_2 - \nu_2, \quad \cos(\varphi_2 - \nu_2) \approx 1
\]

and formula (12) can be taken in the following form:

\[
\varphi_1 - \nu_1 + \varphi_2 - \nu_2 = \frac{U}{\gamma} \cos \gamma \left[ \cos \gamma + (\varphi_2 - \nu_2) \sin \gamma \right]. \quad (13)
\]

Multiplying the first equation of system (11) by \( \cos(\varphi_1 - \nu_1) \), and the second equation - by \( \sin(\varphi_1 - \nu_1) \) and subtracting the second equation from the first one, after similar transformations we obtain

\[
\varphi_1 - \nu_1 + \varphi_2 - \nu_2 = \frac{U}{N_2} \cos \nu_2 \left[ \cos \gamma - (\varphi_1 - \nu_1) \sin \gamma \right]. \quad (14)
\]

Multiplying equation (13) by \( \frac{1}{N_2} (\varphi_1 - \nu_1) \cos \nu_2 \), and equation (14) by \( \frac{1}{N_1} (\varphi_2 - \nu_2) \cos \nu_1 \) and summing them, we obtain

\[
(\varphi_1 - \nu_1 + \varphi_2 - \nu_2) \left( \frac{1}{N_2} (\varphi_1 - \nu_1) \cos \nu_2 + \frac{1}{N_1} (\varphi_2 - \nu_2) \cos \nu_1 - \frac{U}{N_1 N_2} \cos \nu_1 \cos \nu_2 \cos \gamma \right) = 0.
\]

Since \( (\varphi_1 - \nu_1 + \varphi_2 - \nu_2) > 0 \), the expression in the second parenthesis is equated to zero and after simple transformations we obtain

\[
U = \frac{N_1}{\cos \gamma \cos \nu_1} (\varphi_1 - \nu_1) + \frac{N_2}{\cos \gamma \cos \nu_2} (\varphi_2 - \nu_2). \quad (15)
\]

Expression (15) shows that any increase in the pushing force \( U \) causes a reciprocal increase in the repulsive forces \( N_1 \) and \( N_2 \). Thus, force \( U \) cannot create an advantage of the pulling forces over the repulsive ones.

Let us assume that under the force \( U \) the layer of material begins to be pressed into the roller bite. At some moment, the front face of the layer is in the section \( B'B'_2 \) determined by the angles \( \varphi_1' \) and \( \varphi_2' \), while the points of application of normal and frictional forces are determined by the angles \( \nu_1' \) and \( \nu_2' \), (figure 2).

\[\text{Figure 2. Forces acting on the material layer in forced gripping.}\]
In accordance with formula (15), we have:

\[ U = \frac{N_1}{\cos \gamma \cos \psi_1} (\psi_1 - \nu_1) + \frac{N_2}{\cos \gamma \cos \nu_2} (\nu_2 - \nu_2). \]  

(16)

Let us assume that normal and frictional forces are applied in the middle of the sections \( B_1B_2 \) and \( B_1'B_2' \), \( \psi_1 = \frac{\phi_1 + \phi_1'}{2} \) and \( \nu_2 = \frac{\phi_2 + \phi_2'}{2} \).

With this, formula (16) will take the form:

\[ U = \frac{N_1}{2 \cos \gamma \cos \psi_1} (\phi_1 + \phi_1' - 2\nu_1) + \frac{N_2}{2 \cos \gamma \cos \nu_2} (\nu_2 + \phi_2' - 2\nu_2). \]  

(17)

Let us express the forces \( N_1 \) and \( N_2 \) in terms of the average contact pressures \( p_{1ay} \) and \( p_{2ay} \) in the sections \( B_1B_2 \) and \( B_1'B_2' \) [7]:

\[ N_1 = p_{1cp} BR_1 (\phi_1 - \phi_1'), \quad N_2 = p_{2cp} BR_2 (\phi_2 - \phi_2'), \]  

(18)

where \( B \) is the length of the rollers.

Substituting the values of \( N_1 \) and \( N_2 \) into formula (17), we obtain

\[ U = \frac{p_{1cp} BR_1}{2 \cos \gamma \cos \psi_1} (\phi_1 - \phi_1')(\phi_1 + \phi_1' - 2\nu_1) + \frac{p_{2cp} BR_2}{2 \cos \gamma \cos \nu_2} (\phi_2 - \phi_2')(\phi_2 + \phi_2' - 2\nu_2). \]  

(19)

Formula (19) shows how the value of the required pushing force changes as the front end of the layer moves in a steady state.

Let us find the values of angles \( \phi_1' \) and \( \phi_2' \), at which the value of the required force \( U \) is maximum. According to the condition for the maximum of a function of two variables:

\[
\begin{align*}
\frac{\partial U}{\partial \phi_1'} &= \frac{p_{1cp} BR_1}{2 \cos \gamma \cos \psi_1} (-2\phi_1' + 2\nu_1) = 0, \\
\frac{\partial U}{\partial \phi_2'} &= \frac{p_{2cp} BR_2}{2 \cos \gamma \cos \nu_2} (-2\phi_2' + 2\nu_2) = 0,
\end{align*}
\]

hence \( \phi_1' = \nu_1, \phi_2' = \nu_2 \).

Let us find the maximum value of the pushing force:

\[ U_{\text{max}} = \frac{N_1}{2 \cos \gamma \cos \psi_1} (\phi_1 - \nu_1)^2 + \frac{N_2}{2 \cos \gamma \cos \nu_2} (\phi_2 - \nu_2)^2. \]  

(20)

From the above calculations, it follows that the material layer by the rollers is gripped in cases when the sum of the angles of contact is greater than the sum of the angles of friction. For this, a force must be applied to the material layer, the maximum value of which can be determined by formula (20).

We raise to the square the first and second equations of system (11) and sum them up:

\[ N_1^2 \cos^2 \nu_2 - 2N_1N_2 \cos \psi_1 \cos \nu_2 \cos (\phi_1 - \nu_1 + \phi_2 - \nu_2) + N_2^2 \cos^2 \nu_1 = U^2 \cos^2 \nu_1 \cos^2 \nu_2. \]

We transform these equalities

\[ N_1^2 \cos^2 \nu_2 - 2N_1N_2 \cos \psi_1 \cos \nu_2 \left(1 - \frac{(\phi_1 - \nu_1 + \phi_2 - \nu_2)^2}{2}\right) + N_2^2 \cos^2 \nu_1 = U^2 \cos^2 \nu_1 \cos^2 \nu_2. \]
or
\[
(N_1 \cos \nu_2 - N_2 \cos \nu_1)^2 + N_1 N_2 \cos \nu_1 \cos \nu_2 (\phi_1 - \nu_1 + \phi_2 - \nu_2) = U^2 \cos^2 \nu_1 \cos^2 \nu_2.
\]

Hence
\[
\phi_1 - \nu_1 + \phi_2 - \nu_2 = \sqrt{\frac{U^2 \cos^2 \nu_1 \cos^2 \nu_2 - (N_1 \cos \nu_2 - N_2 \cos \nu_1)^2}{N_1 N_2 \cos \nu_1 \cos \nu_2}}.
\]

Equations make sense when the following conditions are met
\[
U^2 \cos^2 \nu_1 \cos^2 \nu_2 - (N_1 \cos \nu_2 - N_2 \cos \nu_1)^2 \geq 0
\]
or
\[
U \geq \left| \frac{N_1 \cos \nu_2 - N_2 \cos \nu_1}{\cos \nu_1 \cos \nu_2} \right|.
\]

Hence, the minimum value of the pushing force is determined
\[
U_{\min} = \left| \frac{N_1 \cos \nu_2 - N_2 \cos \nu_1}{\cos \nu_1 \cos \nu_2} \right|.
\]

If we assume that \( U_{\min} = 0 \), then \( N_1 \cos \nu_2 - N_2 \cos \nu_1 = 0 \).

Let us substitute the values of \( N_1 \) and \( N_2 \) from equalities (18) into this equation:
\[
p_{lcp BR_1} \cos \nu_2 (\phi_1 - \nu_1) - p_{2cp BR_2} \cos \nu_1 (\phi_2 - \nu_2) = 0.
\]

Hence
\[
\phi_2 - \nu_2 = \frac{p_{lcp BR_1} \cos \nu_2}{p_{2cp BR_2} \cos \nu_1} (\phi_1 - \nu_1).
\]

Substituting \((\phi_2 - \nu_2)\) into formula (20) and solving it with respect to \( \phi_1 \), we obtain:
\[
\phi_1 = \nu_1 + \sqrt{\frac{2U_{\max} p_{2cp BR_2} \cos^2 \nu_1 \cos \gamma}{p_{lcp BR_1} (p_{lcp BR_1} \cos \nu_2 + p_{2cp BR_2} \cos \nu_1)}}.
\]

Taking into account expression (22) from equality (21), we find a formula for determining \( \phi_2 \):
\[
\phi_2 = \nu_2 + \sqrt{\frac{2U_{\max} p_{lcp BR_1} \cos^2 \nu_2 \cos \gamma}{p_{2cp BR_2} (p_{lcp BR_1} \cos \nu_2 + p_{2cp BR_2} \cos \nu_1)}}.
\]

Summing up expressions (22) and (23), formula (24) can be given a form similar to the condition of free grip, namely
\[
\phi_1 + \phi_2 \leq \nu_1 + \nu_2 + \sqrt{\frac{2U_{\max} (p_{lcp BR_1} \cos \nu_2 + p_{2cp BR_2} \cos \nu_1) \cos \gamma}{p_{lcp BR_1} p_{2cp BR_2} R_1 R_2}}.
\]

Condition (24) can be called the condition of forced gripping of material with two driven rollers of an asymmetric two-roll module shown in figure 1.
Let the two-roll module represent the rolling process with parameters \( R_1 = R_2 = R \), \( \gamma = 0 \), \( \phi_1 = \phi_2 = \phi \), \( \nu_1 = \nu_2 = \nu \), \( p_{1c} = p_{2c} = p_{cp} \).

Substituting these parameters into inequality (24), we obtain the forced gripping condition in a steady state of symmetric two-roll module [7]

\[
\alpha \leq \nu + \frac{U_{\text{max}} \cos \nu}{p_{cp} BR}.
\]

3. Results

Thus, the conditions for forced gripping of material with two driven rollers in an asymmetric two-roll module were determined. The resulting conditions are general in the sense that they are applicable for particular cases of interaction (for example, from this condition follows the conditions for the forced gripping of the symmetric rolling process). The obtained formulas (22) and (23) make it possible to determine the contact angles, which are the basic quantities when solving problems of contact interaction in a two-roll module.

4. Conclusions

The process of gripping material with two driven rollers in an asymmetric two-roll module is analyzed in the article.

Analytical expressions are obtained for calculating the contact angles of free and forced gripping of material with two driven rollers.

A condition for forced gripping in an asymmetric two-roll module was determined. From this condition follow the conditions for forced gripping of a symmetric rolling process.

Analytical expressions were derived for the minimum and maximum values of pushing force for gripping material with two driven rollers in an asymmetric two-roll module.

A formula was obtained that determines the change pattern in the pushing force as the front end of the layer moves in a steady state.

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