$p$-Values for Credibility

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**Abstract:** Analysis of credibility is a reverse-Bayes technique that has been proposed by Matthews (2001) to overcome some of the shortcomings of significance tests. A significant result is deemed credible if current knowledge about the effect size is in conflict with any sceptical prior that would make the effect non-significant. In this paper I formalize the approach and propose to use Bayesian predictive tail probabilities to quantify the evidence for credibility. This gives rise to a \( p \)-value for extrinsic credibility, taking into account both the internal and the external evidence for an effect. The assessment of intrinsic credibility leads to a new threshold for ordinary significance that is remarkably close to the recently proposed 0.005 level. Finally, a \( p \)-value for intrinsic credibility is proposed that is a simple function of the ordinary \( p \)-value for significance and has a direct frequentist interpretation in terms of the replication probability that a future study under identical conditions will give an estimated effect in the same direction as the first study.

**Key Words:** Analysis of Credibility; Confidence Interval; \( p \)-value; Replication Probability; Significance Test

1 Introduction

“\( p \)-values are just too familiar and useful to ditch”

David Spiegelhalter (2017)

Standard \( P \)-values for point null hypotheses still dominate most of the applied literature (Greenland and Poole, 2013), despite the fact that they are commonly misused and misunderstood (Wasserstein and Lazar, 2016; Matthews et al., 2017). Although being criticised intensively in the literature, the dichotomisation of \( P \)-values into “significant” and “non-significant” is still commonplace in practice.
In a series of papers, Robert Matthews Matthews (2001a,b, 2017) has developed the Analysis of Credibility, a specific reverse-Bayes method to assess the credibility of a significant finding. Reverse-Bayes approaches allow to study properties of the prior distribution needed to achieve a certain posterior statement for the data at hand. The idea to use Bayes’s theorem in reverse originates in the work by IJ Good (Good, 1983) and is increasingly used to assess the plausibility of scientific findings (Greenland, 2006, 2011; Held, 2013; Colquhoun, 2017).

Analysis of credibility is based on a conventional confidence interval for an unknown effect size $\theta$ with lower limit $L$ and upper limit $U$, say. In the following I assume that both $L$ and $U$ are symmetric around the point estimate $\hat{\theta}$ (assumed to be normally distributed) and that both are either positive or negative, i.e. the effect is significant at significance level $\alpha$. Matthews (2001a,b) proposed to compute a sceptical prior distribution for the effect size $\theta$, normal with mean zero, that - combined with the information given in the confidence interval for $\theta$ - results in a posterior distribution which is just non-significant at level $\alpha$, i.e. either the $\alpha/2$ or the $1-\alpha/2$ posterior quantile is zero. He has derived a formula for the limits $\pm S$ of the corresponding equi-tailed prior credible interval at the same level $1-\alpha$:

$$S = \frac{(U - L)^2}{4\sqrt{UL}},$$

where $S$ is called the sceptical limit and the interval $[-S, S]$ is called the critical prior interval. Note that (1) holds for any value of $\alpha$, not just for the traditional 5% level.

Equivalently, the variance $\tau^2$ of the sceptical prior can be expressed as a function of the variance $\sigma^2$ (the squared standard error, assumed to be known) of the estimate $\hat{\theta}$, the corresponding test statistic $t = \hat{\theta}/\sigma$ and $z_{\alpha/2}$, the $1-\alpha/2$ quantile of the standard normal distribution:

$$\tau^2 = \frac{\sigma^2}{t^2/z_{\alpha/2}^2 - 1},$$

$$3$$
where $t^2 > z^2_{\alpha/2}$ is required for significance at level $\alpha$. Equation (2) shows that the prior variance $\tau^2$ can be both smaller or larger than $\sigma^2$, depending on the value of $t^2$. If $t^2$ is close to $z^2_{\alpha/2}$ (i.e. the effect is “borderline significant”), then the prior variance will be relatively large. If $t^2$ is substantially larger than $z^2_{\alpha/2}$, then the prior variance will be relatively small.

For illustration, consider results from a recently published randomized placebo-controlled clinical trial (Hayward et al., 2017) on the efficacy of corticosteroids in the treatment of sore throat. There were 102/288 events in the intervention group and 75/277 events in the control group for the outcome complete resolution of pain at 48 hours. A relative risk of $\exp(\hat{\theta}) = 1.31$ can easily be calculated (95% CI from 1.02 to 1.68, $p = 0.034$). Analysis of credibility for the log relative risk $\theta$ gives the sceptical limit $0.60 = \log(1.83)$, so the critical prior interval is from $1/1.83 = 0.55$ to 1.83 on the relative risk scale. The associated standard deviation $\tau = 0.31$ of the sceptical prior is considerably larger than the standard error $\sigma = 0.13$. Figure 1 displays the sceptical prior together with the confidence interval for the data and the associated posterior.

Matthews (2001b) proposed to compare external knowledge about effect sizes with the sceptical limit $S$:

\[
\text{If previous evidence indicates that plausible values for the parameter in question exist outside the critical prior interval (CPI), the reality of the stated effect may be deemed credible [...].}
\]

In the above example, extrinsic credibility of the Hayward et al. (2017) results can be investigated in the light of preceding trials on the same clinical research question. Sadeghirad et al. (2017) performed a systematic review and identified three preceding studies relevant for the Hayward et al. (2017) analysis. A random-effects meta-analysis of the three trials preceding the Hayward (2017) study gives a relative risk estimate of 1.63 (95% CI from 1.31 to 2.02, $p < 0.0001$), also shown in Figure 1. Since the external point estimate is smaller than the sceptical limit 1.83, the results from the Hayward
Figure 1: Analysis of credibility for the relative risk of complete resolution of pain at 48 hours as reported in the Hayward et al. (2017) trial. The sceptical prior for the relative risk $\theta$ has 95% credible interval from 0.55 to 1.83. Combined with the data represented by the 95% confidence interval from 1.02 to 1.68, the resulting equi-tailed 95% posterior credible interval has lower limit 1. The external data is based on a meta-analysis of preceding trials. The $p$-value $p_E$ for extrinsic credibility is discussed in Section 2.1. The $p$-value $p_I$ for intrinsic credibility is discussed in Section 3.2.

... study are not considered credible (at the 95% confidence level) using the Matthews (2001b) approach.

This approach seems somewhat unsatisfactory, since the dichotomisation into credible or not credible seems too simplistic, just as the dichotomisation into significant and non-significant. Instead, a quantitative measure of credibility seems warranted. To this end I propose a more formal assessment of credibility based on the Box (1980) proposal to quantify prior-data conflict with a Bayesian tail probability (Section 2). This gives rise to a $p$-value for extrinsic credibility, taking into account both the internal and the external evidence for a significant effect, see Section 2.1. The assessment of
intrinsic credibility is described in Section 3, where a new justification for the recently proposed 0.005 threshold for significance is given (Section 3.1). A new \( p \)-value for intrinsic credibility is introduced in Section 3.2 and interpreted as a replication probability in Section 3.3. I investigate the distribution of \( p \)-values for credibility under the assumption of no effect (Section 4), and close with some comments in Section 5.

2 Assessing evidence for extrinsic credibility

In the present context the prior is the sceptical prior and the data are derived from the external study used to assess credibility. The sceptical prior is normal with mean zero and variance \( \tau^2 \). The likelihood of the external information on the effect size \( \theta \) can usually be described as normal with mean \( \hat{\theta}_0 \) and variance \( \sigma^2_0 \), say. The Box (1980) approach is now based on the prior-predictive distribution, which is normal with mean zero and variance \( \tau^2 + \sigma^2_0 \) (Spiegelhalter et al., 2004, Section 5.8). The procedure computes the test statistic

\[
t_{\text{Box}} = \frac{\hat{\theta}_0}{\sqrt{\tau^2 + \sigma^2_0}}
\]

and finally the (two-sided) tail probability

\[
p_{\text{Box}} = \Pr(\chi^2(1) \geq t_{\text{Box}}^2)
\]

as the corresponding upper tail of a \( \chi^2 \)-distribution with one degree of freedom. Small values of \( p_{\text{Box}} \) indicate a conflict between the sceptical prior and the external data. Note that this procedure is different from the assessment of compatibility of the current study with the external study - then we would use the test statistic \((\hat{\theta}_0 - \hat{\theta})/\sqrt{\sigma^2 + \sigma^2_0}\), which will give small tail probabilities whenever the results from the two studies are incompatible, independent of whether the trials show evidence for an effect or not.

The proposed assessment of conflict between the sceptical prior derived from the
95% confidence interval and the meta-analytic summary estimates from the preceding three trials gives $t_{\text{Box}} = 1.49$ with Box’s tail probability $p_{\text{Box}} = 0.14$. However, this tail probability depends on $\alpha$ through $\tau^2$ as given in (2), here 5%. For example, for $\alpha = 10\%$, Box’s tail probability is considerably smaller ($p_{\text{Box}} = 0.004$). This is because the lower limit of the 90% confidence interval is further away from the null than the lower limit of the 95% confidence interval, so we need a smaller prior variance $\tau^2$ to push it to zero. A smaller $\tau^2$ results in a larger test statistic (3), hence a smaller tail probability (4). In practice, however, it is difficult to interpret tail probabilities for credibility that depend on the confidence level $1 - \alpha$ of the underlying confidence interval. Furthermore, computation of $p_{\text{Box}}$ is not possible if the result from the underlying study is not significant at level $\alpha$. These issues motivate the work described in the next section where I define a $p$-value for extrinsic credibility, independent of the level $\alpha$.

### 2.1 A $p$-value for extrinsic credibility

There are two disadvantages of the procedure described in the previous section: First, the tail probability $p_{\text{Box}}$ for extrinsic credibility depends on the confidence level $1 - \alpha$ of the underlying confidence interval. Secondly, $p_{\text{Box}}$ only exists if the confidence interval does not include zero, i.e. is significant at level $\alpha$. To address both problems, I suggest to determine the largest confidence level $1 - \alpha$, where the stated effect is (just) extrinsically credible at level $1 - \alpha$, in analogy to the usual assessment of significance based on confidence intervals. The required level $\alpha$ will be called the $p$-value for extrinsic credibility, denoted by $p_E$.

To determine $p_E$, let $c = \sigma^2 / \sigma_0^2$ denote the ratio of the variances of the internal and external effect estimates. With (2) we then have

\[
\tau^2 + \sigma_0^2 = \sigma_0^2 \left( \frac{c}{t^2/z_{\alpha/2}^2} - 1 + 1 \right).
\]
Using (3), the requirement \( t^2_{\text{Box}} \geq z^2_{\alpha/2} \) for extrinsic credibility is then equivalent to

\[
\left( \frac{t^2_0}{z^2_{\alpha/2}} - 1 \right) \left( \frac{t^2}{z^2_{\alpha/2}} - 1 \right) \geq c, \tag{5}
\]

here \( t_0 = \hat{\theta}_0 / \sigma_0 \) is the test statistic of the external study. The required level \( \alpha = p_E \) to obtain equality in (5) can easily be computed numerically. Note that (2) requires \( t^2 > z^2_{pE/2} \) to hold, so also \( t^2_0 > z^2_{pE/2} \) must hold to have the left side of (5) being positive. In other words, \( p_E \) will be larger than both ordinary (two-sided) \( p \)-values for significance \( p = 2 \left[ 1 - \Phi(t) \right] \) and \( p_0 = 2 \left[ 1 - \Phi(t_0) \right] \), say, from the internal and the external data, respectively, here \( \Phi(.) \) denotes the cumulative standard normal distribution function.

Figure 2 shows the dependence of \( p_E \) on \( p \), \( p_0 \) and \( c \). Note that \( p_E \) remains unchanged if we switch \( t_0 \) and \( t \) but keep \( c \) fixed. This does not mean that switching the role of the external and internal data will not change \( p_E \) as then \( c \) would also change, except for the case \( c = 1 \), where \( \sigma^2 = \sigma^2_0 \) holds. For fixed \( t \) and \( t_0 \), \( p_E \) is increasing with \( c \), so the evidence for extrinsic credibility decreases with increasing variance ratio \( \sigma^2 / \sigma^2_0 \) if the two test statistics \( t \) and \( t_0 \) (and the associated ordinary \( p \)-values) are kept constant. If \( c \) is small, then \( p_E \) will be close to \( \max \{ p_0, p \} \), see Figure 2. If \( p_0 \ll p \), then \( p_E \) will be close to \( p \).

In the above example we obtain \( p_E = 0.062 \) by numerical computation. If we think of \( p \)-values as indicators of the strength of evidence and adopt the “rough and ready guide” by Bland (2015, Section 9.4), then \( p_E = 0.062 \) indicates weak evidence for credibility of the results from the Hayward et al. (2017) trial in the light of the three preceding trials. A more technical interpretation is that for a confidence level of \( 1 - p_E = 93.8\% \) for the log relative risk obtained from the Hayward et al. (2017) trial, Box’s tail probability (4), quantifying the conflict between the corresponding sceptical prior and the external summary estimate, is equal to \( p_E = 0.062 \).
Figure 2: The $p$-value for extrinsic credibility as a function of the ordinary $p$-values $p$ and $p_0$ of the internal and external studies. Shown are contour plots for different values of the variance ratio $c$.

Note that the check $\theta_0 > S$ for credibility by Matthews (2001b) can be re-written as $\frac{\theta_0^2}{\tau^2} \geq z_{\alpha/2}^2$ and leads to the requirement

$$\frac{t_0^2}{z_{\alpha/2}^2} \left( \frac{t^2}{z_{\alpha/2}^2} - 1 \right) \geq c.$$  \hspace{1cm} (6)

This is different from (5) and no longer requires $t_0^2 > z_{\alpha/2}^2$. Specifically, if $c < 1$, it may happen that Matthews’ check will declare a significant result as extrinsically credible, although the external study was not conventionally significant on its own right.
3 Assessing evidence for intrinsic credibility

A significant effect is intrinsically credible, if it is in conflict with any sceptical prior that would make the effect non-significant. This can be thought of as a double-check to ensure that a significant effect is not spurious. Matthews (2017) proposed to declare an effect as intrinsically credible, if the internal estimate $\hat{\theta}$ is outside the sceptical prior interval $-S$ to $S$. He shows that, for $\alpha = 0.05$, this is equivalent to the conventional two-sided $p$-value being smaller than 0.0127.

However, Matthews’ check for intrinsic credibility does not take the uncertainty of $\hat{\theta}$ directly into account. One could argue that the sceptical prior variance (2) is already a function of the variance $\sigma^2$, so the uncertainty of $\hat{\theta}$ is already implicitly taken into account in the calculation of the sceptical limit. This is true, but - for fixed $\sigma^2$ - the point estimate $\hat{\theta}$ also affects the sceptical limit, so Matthews’ check does use $\hat{\theta}$ twice. It is not clear why $\hat{\theta}$ can be used twice, but not $\sigma^2$. In what follows I will use both $\hat{\theta}$ and $\sigma^2$ directly in the assessment of intrinsic credibility, following the approach by Box (1980) for the assessment of prior-data conflict, with the perhaps unusual feature that the prior has been derived from the data. I argue that there is nothing intrinsically inconsistent in investigating the compatibility of a prior, defined through the data, and the data itself, extending an argument made by Cox (2006, Section 5.10) to the reverse-Bayes setting.

3.1 Another justification for the 0.005 threshold

If we use the Box (1980) check for prior-data conflict between the sceptical prior and the original (internal) data with point estimate $\hat{\theta}$ and variance $\sigma^2$, the test statistic (3) reads

$$t_{\text{Box}} = \frac{\hat{\theta}}{\sqrt{\tau^2 + \sigma^2}}. \quad (7)$$

Intrinsic credibility at the 5% level (i.e. $|t_{\text{Box}}| > 1.96$) can then be shown to be equi-
valent to the conventional two-sided p-value $p < 0.0056$. This is remarkably close to
the recently proposed new threshold of 0.005 for statistical significance (Johnson, 2013;
Benjamin et al., 2017).

To derive the new significance threshold, set $c = 1$ and $t_0 = t$ in (5) to obtain the
simple requirement

$$t^2 \geq 2 z_{\alpha/2}^2$$

for intrinsic credibility at level $\alpha$. In (8), $t$ is the ordinary test statistic $t = \hat{\theta}/\sigma$, so the
intrinsic credibility threshold $\alpha_I$ for the conventional two-sided p-value is

$$\alpha_I = 2 \left\{ 1 - \Phi \left( t = \sqrt{2} z_{\alpha/2} \right) \right\}. \quad (9)$$

For $\alpha = 0.05$ we have $t = \sqrt{2} \cdot 1.96 = 2.77$ and the credibility threshold (9) turns out to
be $\alpha_I = 0.0056$, as claimed above. Figure 3 compares the new threshold with the one
obtained by Matthews (2017, Appendix D) (using $t = 1.272 z_{\alpha/2}$) for values of $\alpha$ below
10%.

The squared test statistic (7) can also be written in terms of $L$ and $U$,

$$t_{\Box}^2 = z_{\alpha/2}^2 \left\{ \frac{4UL}{(U-L)^2} \right\},$$

and the requirement $t_{\Box}^2 \geq z_{\alpha/2}^2$ for intrinsic credibility can be shown to be equivalent
to require that the credibility ratio $U/L$ (or $L/U$ if both $L$ and $U$ are negative) fulfills

$$\frac{U}{L} \leq d = 3 + 2 \sqrt{2} \approx 5.828. \quad (10)$$

Thus, there is a simple way to assess intrinsic credibility based on the ratio of the
Figure 3: The threshold for intrinsic credibility of significant results as a function of the conventional $\alpha$ level. The blue line corresponds to the proposal by Matthews (2017). The red line is the proposed new threshold.

limits of a confidence interval at any level $1 - \alpha$, without using the ordinary $p$-value for significance: if the credibility ratio is smaller than $d = 5.828$ than the result is credible at level $1 - \alpha$. Note that this is a stronger requirement than the check of confidence intervals for significance, where it is only required that the credibility ratio is positive. To derive the cut-point $d$ in (10), set $U = L \, d$ so the requirement $t_{\text{Box}}^2 = z_{\alpha/2}^2$ is equivalent to

$$1 = \frac{4 \, U \, L}{(U - L)^2} = \frac{4 \, d}{(d - 1)^2},$$

a quadratic equation in $d$ with $d = 3 + 2 \sqrt{2}$ as solution.

If the sceptical prior distribution is already available, then another way to assess
intrinsic credibility is to compare the prior variance $\tau^2$ to the data variance $\sigma^2$. Comparing (2) with (8) it is easy to see that intrinsic credibility is achieved if and only if the sceptical prior variance is not larger than the variance of the estimate, i.e. $\tau^2 \leq \sigma^2$.

### 3.2 A $p$-value for intrinsic credibility

As for the $p$-value for extrinsic credibility introduced in Section 2.1, I now derive the $\alpha$ value that just achieves intrinsic credibility, i.e. where equality holds in (8). This gives the $p$-value for intrinsic credibility, denoted by $p_I$. The $p$-value for credibility will always be larger than the $p$-value for significance, which is based on the confidence level $1 - \alpha$ such that the lower limit $L$ is exactly zero.

The $p$-value $p_I$ for intrinsic credibility can be written as a function of the ordinary $p$-value $p$ for significance:

$$p_I = 2 \left[ 1 - \Phi \left( \frac{t}{\sqrt{2}} \right) \right],$$

(11)

here $t = t(p) = \Phi^{-1}(1 - p/2)$ is the standard test statistic for significance. Equation (11) can be derived by solving equation (9) for $\alpha = p_I$ and replacing $\alpha$ with $p$. Note that the corresponding test statistic $t_I = t/\sqrt{2}$ for intrinsic credibility is just a shrunken version of the test statistic $t$ for significance.

Figure 4 shows that the $p$-value $p_I$ for intrinsic credibility is considerably larger than the $p$-value $p$ for significance, particularly for small values of $p$. For example, the $p$-value for intrinsic credibility of the Hayward et al. (2017) study is $p_I = 0.13$, whereas the conventional $p$-value for significance is $p = 0.034$. 
3.3 Interpretation as replication probability

The p-value for intrinsic credibility has a direct and useful interpretation in terms of the replication probability that an identical study will give an estimated effect $\hat{\theta}_2$ in the same direction as the estimate $\hat{\theta}_1 = \hat{\theta}$ from the current (first) study. To see this, note that under an initial uniform prior the posterior for $\theta | \hat{\theta}_1 \sim N(\hat{\theta}_1, \sigma^2)$. This posterior now serves as the prior for the mean of the (unobserved) estimate $\hat{\theta}_2 | \theta \sim N(\theta, \sigma^2)$ from the second study, where we assumed the two studies to be identically designed, so with equal variances $\sigma^2$. This leads to the prior-predictive distribution $\hat{\theta}_2 | \hat{\theta}_1 \sim N(\hat{\theta}_1, 2\sigma^2)$ and the p-value for intrinsic credibility (11) is then twice the probability that the second study will give an estimate $\hat{\theta}_2$ in the opposite direction.
as the estimate $\hat{\theta}_1$ of the first study:

$$2 \Pr(\hat{\theta}_2 \leq 0 \mid \hat{\theta}_1 > 0) = 2\Phi\left(\frac{0 - \hat{\theta}_1}{\sqrt{2\sigma}}\right) = 2\Phi\left(-\frac{t}{\sqrt{2}}\right) = 2 \left[1 - \Phi\left(\frac{t}{\sqrt{2}}\right)\right] = p_I,$$

and vice versa if $\hat{\theta}_1 < 0$. The probability $\Pr(\hat{\theta}_2 \leq 0 \mid \hat{\theta}_1 > 0)$ is one of the three replication probabilities that have been considered by Senn (2002). A related, but different quantity has been calculated by Goodman (1992), the probability that the result from the second study is significant.

4 The distribution under the null

It is of interest to study the distribution of $p_I$ and $p_E$ under the assumption of no effect. Then $p$ is uniformly distributed and we can derive the density of $p_I$ with a change-of-variables using (11):

$$f(p_I) = 2\sqrt{\pi} \varphi\{t(p_I)\},$$

here $\varphi(.)$ is the standard normal density function and $t(p_I) = \Phi^{-1}(1 - p_I/2)$. The density functions of $p$ and $p_I$ are compared in Figure 5. Under the assumption of no effect, the risk of small $p$-values for intrinsic credibility is drastically reduced, compared to standard $p$-values for significance.

The distribution of $p_E$ can be easily studied via simulation. Histograms of 50 000 samples is displayed in Figure 6 for the same values of $d$ as in Figure 6. The distribu-
Figure 5: The density function of the $p$-value for significance and the $p$-value for intrinsic credibility under the assumption of no effect.

...tion of $p_E$ is shifted to the right with increasing $c$. For $c \to 0$ we have $p_E \to \min\{p, p_0\}$, which follows a rectangular Be($2, 1$) distribution under the assumption that $p$ and $p_0$ are independently uniform. For comparison, Figure 6 also gives the density function of the $p$-value for significance and for intrinsic credibility. Under the assumption of no effect, the risk of small “false positive” $p$-values for credibility is drastically reduced, compared to standard $p$-values for significance. This is already to see for $p_I$, but even more pronounced for $p_E$. The rectangular distribution (for $c \to 0$) gives an upper bound for the tail probability $\Pr(p_E < \alpha \mid H_0) \leq \alpha^2$ for any threshold $\alpha < 0.5$ and for any value of the variance ratio $c$. For example, for $\alpha = 0.05$ we obtain $\Pr(p_E < 0.05 \mid H_0) \leq 0.0025$. For comparison, $\Pr(p_I < 0.05 \mid H_0) = 0.0056$. 

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Figure 6: Histograms of the distribution of the $p$-value for extrinsic credibility for different values of $c$ under the assumption of no effect. The density function of the $p$-value for extrinsic credibility for the limiting case $c = 0$ is superimposed (blue line), as well as the density function of the $p$-value for intrinsic credibility (red line) and for significance (black line).

5 Discussion

In this paper I have introduced $p$-values to assess extrinsic and intrinsic credibility. The $p$-value for extrinsic credibility is a function of the two ordinary $p$-values from the
internal and external data and the ratio of the squared standard errors of the internal and external effect estimates only. The $p$-value for intrinsic credibility has been shown to be function of the ordinary $p$-value for significance from the internal data.

The proposed $p$-values for credibility are always larger than the corresponding $p$-values for significance. As such, they provide a new calibration of ordinary $P$-values. For the conventional $\alpha = 0.05$ level, the threshold for intrinsic credibility turns out to be remarkably close to the recently proposed 0.005 significance level, which has been proposed based on different arguments.

Although derived using a Bayesian approach, $p$-values for credibility do not require specification of a prior probability of the null hypothesis of no effect. In practice it is therefore easy to accompany $p$-values for significance with the corresponding $p$-values for credibility. This is in contrast to the calibration of $p$-values to lower bounds on posterior probability of the null, which require specification of a prior probability. However, it is noteworthy that the $p$-value for intrinsic credibility is surprisingly close to a commonly used bound on the posterior probability of the null hypothesis (of no effect) (Sellke et al., 2001) under the condition of equipoise (Johnson, 2013). Minimum Bayes factors have also been proposed to calibrate $p$-values, see Held and Ott (2018) for a recent review. They have the advantage that they do not require specification of a prior probability of the null hypothesis, but they cannot be interpreted as probabilities.

The analysis of credibility assumes a simple mathematical framework, where likelihood, prior and posterior are all normally distributed. It will be of interest to extend this approach to other settings, for example to the $t$-distribution.

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