Response of Vertical Velocities in Extratropical Precipitation Extremes to Climate Change

ZIWEI LI* AND PAUL O’GORMAN

Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA

ABSTRACT

Changes in vertical velocities affect the intensity of precipitation extremes but remain poorly understood. We find that mid-tropospheric vertical velocities in extratropical precipitation extremes strengthen in the zonal mean in simulations of 21st-century climate change. For each extreme event, we solve the quasi-geostrophic omega equation to decompose this strengthening behavior into different physical contributions. Much of the positive contribution to upward motion from increased latent heating is offset by negative contributions from changes in dry static stability and horizontal length scale. However, taking the latent heating as given is a limitation for understanding strongly-precipitating events in which the vertical velocity and latent heat release are closely related. Therefore, we also perform a moist decomposition of the changes in vertical velocities in which latent heating is represented through a moist static stability rather than being treated as an external forcing of the omega equation. In the moist decomposition, decreases in moist static stability and increases in the depth of the circulation make important contributions to the strengthening of the vertical velocities.

1. Introduction

Projected changes in the intensity of precipitation extremes in response to climate warming may be decomposed into a positive thermodynamic contribution (roughly 6% K$^{-1}$ in the extratropics) from increased humidity and a dynamical contribution from changes in vertical velocities (Emori and Brown 2005; O’Gorman and Schneider 2009). The dynamical contribution is responsible for most of the geographical and seasonal variation of the projected response of precipitation extremes, and it is large enough to cause decreases in the intensity of precipitation extremes over parts of the subtropical oceans (Pfahl et al. 2017). We focus on the dynamical contribution in the extratropics where it is relatively robust across coupled general circulation models (GCMs) (Pfahl et al. 2017) but remains challenging to understand given the importance of latent heat release and convection in extreme precipitation events (Nie et al. 2018).

Extreme precipitation events in coupled GCM simulations have been found to be of order 700km in horizontal extent and 12 hours in duration (Dwyer and O’Gorman 2017), such that the quasi-geostrophic omega (QG-$\omega$) equation is a useful approximate tool to better understand these events (e.g., O’Gorman 2015; Nie et al. 2018). According to the QG-$\omega$ equation, large-scale ascent is forced by horizontal balanced flow and a feedback from diabatic heating (Nie and Sobel 2016). Tandon et al. (2018b) (hereafter T18) and Tandon et al. (2018a) performed scaling analyses of the terms in the QG-$\omega$ equation for extreme precipitation events in GCM simulations under climate change. Here, we take an important further step by numerically solving the QG-$\omega$ equation in domains centered on such events.

We decompose the projected changes in vertical velocities into different physical contributions. We begin with a dry decomposition with diabatic heating (which is dominated by latent heating) treated as an external forcing. Consistent with the analysis of T18, we find that increased diabatic heating tends to amplify the changes in vertical velocities. However, the static stability term in the QG-$\omega$ equation that T18 found was small is actually a dominant term in our numerical inversions, and as a result, we find that changes in horizontal length scale are less important than was suggested by the analysis of T18.

The dry decomposition is useful as a first step and follows the approach used in previous work, but it treats diabatic forcing as an external forcing when it is really part of the internal dynamics of the atmosphere (Emanuel et al. 1994). In particular, changes in latent heating should not be taken as given if the aim is to understand changes in precipitation since the surface precipitation rate is closely related to the column-integrated latent heating. To mitigate this problem, we also introduce a moist dynamical decomposition of changes in vertical velocities in which latent heating is represented using a moist static stability. In the moist decomposition, factors such as increases in

* Corresponding author address: Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139.
E-mail: ziweili@mit.edu
the vertical extent of the circulation and decreases in the moist stability play an important role.

We first describe the simulations used and the changes in vertical velocities associated with 6-hourly precipitation extremes in response to climate change (section 2). We then describe the numerical inversion of the QG-\(\omega\) equation in the extreme precipitation events (section 3), and the physical contributions to the changes in vertical velocities in the dry decomposition (section 4) and moist decomposition (section 5). We briefly describe the results for a second GCM and for daily precipitation extremes (section 6) before giving our conclusions (section 7).

2. Simulations and vertical velocities associated with precipitation extremes

We use coupled model output from the Community Earth System Model Large-Ensemble Project (CESM-LE) (Kay et al. 2015) and the contribution of GFDL-CM3 (Donner et al. 2011) to CMIP5 (Taylor et al. 2012). Climate change is defined as the difference between the historical and RCP8.5 scenario simulations, and percentage changes of physical quantities are reported normalized by the historical values and the change in global-mean surface air temperature. For CESM-LE, the data are on a 1.25° latitude by 0.94° longitude grid, and we use 1991-2000 for the historical climate and 2071-2080 for RCP8.5. Because of storage constraints and the computational expense of solving the QG-\(\omega\) equation for many events, we are able to analyze only 6 out of 40 ensemble members of CESM-LE. For GFDL-CM3, there is only one ensemble member, the data are on a coarser 2.5° latitude by 2° longitude grid, and we use 1980-1999 for the historical simulation and 2081-2100 for RCP8.5. We focus on 6-hourly precipitation extremes in CESM-LE, but we also describe results for GFDL-CM3 and daily precipitation extremes (Section 6).

We define an extreme precipitation event at a grid point in a given climate as a 6-hourly period over which the average precipitation rate exceeds its 99.9th percentile for that grid point and climate (Fig. 1a). Instances with zero precipitation are included when calculating percentiles (Schür et al. 2016). Visual inspection of individual extratropical events suggests that they are typically associated with precipitation structures in extratropical cyclones rather than grid-point storms.

The pressure vertical velocity \(\omega\) is not directly available and is instead calculated using the continuity equation following equations (3.11)-(3.13) of Simmons and Burridge (1981). For consistency, the precipitation rates are linearly interpolated in time such that the centers of precipitation accumulation periods correspond to the times of the dynamical fields. The QG-\(\omega\) equation is solved to give \(\omega_{QG}\) for each event as described in detail in the next section.

6-hourly instantaneous horizontal winds \((u, v)\) and temperature \((T)\) are needed as inputs when solving the QG-\(\omega\) equation, and these are linearly interpolated from a hybrid sigma coordinate to a pressure coordinate.

The instantaneous vertical velocity averaged over all extreme precipitation events at each grid point is denoted as \(\overline{\omega}\) and is interpreted as the vertical velocity associated with precipitation extremes at that grid point. We focus on \(\overline{\omega}\) at 500hPa over the course of the paper for simplicity. To be consistent with our analysis of changes in \(\omega_{QG}\) in section 4, we evaluate \(\overline{\omega}\) using \(\omega\) at the location of the local maximum in \(-\omega_{QG}\) at 500hPa that is closest in horizontal distance to the extreme precipitation event. The event-means for other variables are also denoted by an overbar and evaluated at the same locations. Taking the 6 ensemble members of CESM-LE together, there are roughly 85 6-hourly extreme precipitation events to be analyzed at each grid point in each climate. We find that \(\omega\) and \(\omega_{QG}\) are generally negative in extreme precipitation events, consistent with upward motion. High-elevation regions with mean surface pressure lower than 550hPa are excluded from the analysis. A small fraction of events are also excluded due to issues such as numerical instability of the QG-\(\omega\) inversions or positive \(\omega_{QG}\) at 500hPa (see appendix for details of event selection).

The vertical velocity associated with precipitation extremes, \(\overline{\omega}\), maximizes in strength in regions such as the extratropical storm tracks (Supplemental Fig. 1a). The response of \(\overline{\omega}\) to climate change is a strengthening or little change in most extratropical regions, with weakening primarily confined to parts of the subtropical oceans and nearby land regions (Fig. 2a). The zonal-mean response of \(\overline{\omega}\) shows a strengthening at all extratropical latitudes (Fig. 3a), and the extratropical-average response is relatively modest at 2.0% K\(^{-1}\). This extratropical-average response is calculated by taking the percentage change of the zonal mean at each latitude, averaging between 30° and 70° latitude with area weighting in both hemispheres, and normalizing by the increase in global-mean surface air temperature. Extratropical-average responses calculated in this way are summarized in tables 1 and 2.

3. Numerical inversion of the QG-\(\omega\) equation

To understand the strengthening of the vertical velocities in the extreme precipitation events, we numerically solve the QG-\(\omega\) equation for all such events at grid points between 70°S and 70°N. For each event, we expand a three-dimensional domain centered around the location of the event (Fig. 1b). For CESM-LE, the domain ideally extends 29 grid points in each horizontal direction and from 1000hPa to 100hPa in the vertical. However, the domain can shrink to a minimum of 15 grid points simultaneously in both horizontal directions to avoid missing values where the surface pressure is below 1000hPa. If
FIG. 1. A typical extreme precipitation event from the historical climate in CESM-LE featuring strong upward motion in the center of the domain. (a) The extreme precipitation event (red dot) is defined as an exceedance of the 6-hourly precipitation rate at a given grid point (blue line with squares) relative to the 99.9th percentile of the distribution at that grid point (yellow dashed line). (b) The precipitation rate for the event is shown by the contours at 1000hPa with contour interval 20 mm day$^{-1}$, and $\omega_{\text{QG}}$ is shown by the shading at 500hPa and above. The two red dots indicate the horizontal location of the extreme precipitation event at the surface, and the red star indicates the location at which we evaluate $\omega$ and $\omega_{\text{QG}}$ at 500hPa.

This horizontal shrinking is not sufficient to avoid missing values, the lower boundary is then moved up to levels as high as 550hPa. For GFDL-CM3, the domain is chosen following the same approach except that it varies between 15 grid points and 9 grid points as necessary in each horizontal direction. We impose Dirichlet boundary conditions on all boundaries: $\omega_{\text{QG}}$ is set to climatological means on the lateral boundaries and to zero at both the top and bottom boundaries. This bottom boundary condition is a simplification that neglects topographic forcing and Ekman pumping, but the results at 500hPa are nonetheless reasonably accurate, and the impact of instead taking the exact boundary values from the GCM simulations is discussed in Section 6.

The QG-$\omega$ equation is written as

$$\left(\nabla^2 \sigma + f_0^2 \frac{\partial^2}{\partial p^2}\right) \omega_{\text{QG}} = \text{Adv} - \frac{\kappa}{p} \nabla^2 J, \quad (1)$$

where $\omega_{\text{QG}}$ is the quasi-geostrophic vertical velocity satisfying this equation, $f_0$ is the Coriolis parameter evaluated at the center of the domain, $p$ is pressure, $\kappa$ is the ratio of the gas constant to specific heat capacity at constant pressure, and $J$ is the diabatic heating. The static stability parameter $\sigma$ is given by $\sigma = -\frac{RT}{\rho W} \frac{\partial W}{\partial p}$, where $R$ is the gas constant for dry air, $T$ is temperature, and $\theta$ is potential temperature. The advective forcing is given by

$$\text{Adv} = -2\nabla_h \cdot \mathbf{Q} + f_0 \beta \frac{\partial v_g}{\partial p}, \quad (2)$$

where $\beta$ is the meridional derivative of the Coriolis parameter and $\nabla_h$ is the horizontal gradient. The Q-vector in spherical coordinates is given by

$$\mathbf{Q} = -f_0 \left[ \frac{\partial u_g}{\partial p} a \cos \phi \left( \frac{\partial v_g}{\partial \lambda} + u_g \sin \phi \right) + \frac{\partial v_g}{\partial p} \frac{1}{a} \frac{\partial u_g}{\partial \phi} \right] \hat{i} +$$

$$-f_0 \left[ \frac{\partial u_g}{\partial p} a \cos \phi \left( -\frac{\partial u_g}{\partial \lambda} + v_g \sin \phi \right) - \frac{\partial v_g}{\partial p} \frac{1}{a} \frac{\partial u_g}{\partial \phi} \right] \hat{j}, \quad (3)$$

where $a$ is Earth’s radius, $u_g, v_g$ are the zonal and meridional geostrophic winds, and $\hat{i}, \hat{j}$ are the zonal and meridional unit vectors, respectively. This Q vector is the same as that given by equation (19) in Dostalek et al. (2017), except that we use a beta plane within the domain of each event. We choose to use the Q-vector form of the QG-$\omega$ equation (Hoskins et al. 1978) because it yields a smoother advective forcing ($\text{Adv}$) than the traditional form, and this is likely because it avoids a cancellation between terms in the traditional form that can lead to substantial errors when the derivatives are approximated numerically. To minimize the influence of gravity waves, the geostrophic winds are calculated as the rotational component of the horizontal wind (Nielsen-Gammon and Gold 2008), and the rotational wind is obtained through inverting the relative vorticity on a spherical grid.

To reduce numerical noise, the input temperature ($T$) at each level (used in calculating $J$ and $\sigma$) is smoothed by a 3-by-3 running-mean filter. Furthermore, the anomaly of the smoothed temperature field from its horizontal mean over the domain is rescaled so that it preserves the second moment of the un-smoothed field. The diabatic heating ($J$) is then calculated from the thermodynamic equation without quasi-geostrophic approximations.
We allow the static stability $\sigma$ to vary in the horizontal to increase the accuracy of the inversion, noting that this does not compromise the derivation of the QG-$\omega$ equation as long as $\sigma$ is kept inside the Laplacian operator. However, horizontal variations in $\sigma$ can decrease the stability of numerical solutions. To minimize this instability, we set the spatially-varying $\sigma$ to 20% of the $\sigma$ which is calculated from the horizontal-mean temperature over the domain whenever the spatially-varying $\sigma$ falls below this value. The resulting $\sigma$ field is also smoothed according to the same procedure as the temperature discussed in the previous paragraph.

The QG-$\omega$ equation is inverted in each domain in spherical coordinates using a 3D variant (Zedan and Schneider 1983; Ferziger and Peric 2002) of the strongly implicit method (Stone 1968), similar to the approach of Shaevitz et al, 2016 (arXiv:1603.01317). The Laplacian term $\nabla^2 (\sigma \omega_{QG})$ and the diabatic heating term $-\frac{\kappa}{p} \nabla^2 J$ are dominant and are of similar magnitudes (Fig. 4). This similarity is expected since $\sigma + (J \kappa)/(p \omega)$ may be viewed as a measure of the moist stability which will be small for a stratification that is close to moist adiabatic in extreme precipitation events (O’Gorman 2015).

The spatial pattern of $\omega_{QG}$ at 500hPa in the historical climate closely resembles that of $\bar{\omega}$, although the magnitude is underestimated by roughly 16% (Fig. S1), mainly because we do not use the exact lateral- and bottom-boundary $\omega$ values. The response of $\bar{\omega}$ to climate change is also well captured by $\omega_{QG}$ (Fig. 2b and 3a), and thus we will analyze $\omega_{QG}$ to better understand this response.
4. Dry Decomposition

We decompose changes in $\overline{\omega}_{QG}$ at 500hPa into different physical contributions according to the QG-$\omega$ equation. We begin with a dry decomposition in which the diabatic heating ($J$), dominated by latent heating, is considered as an external forcing. For each event, we focus on the local maximum of $-\omega_{QG}$ field at 500hPa that is horizontally closest to the extreme precipitation event at the surface. Averaging the QG-$\omega$ equation across all extreme precipitation events at a given location gives

$$-k^2 \overline{\delta \omega}_{QG} - f_0^2 \overline{m^2 \omega}_{QG} = \overline{Adv} + \frac{k}{p} \overline{k^2 J},$$

(4)
FIG. 5. Contributions in the dry decomposition from changes in (a) $k$ and (b) $k_J$ to the change in $\bar{\omega}_{QG}$ at 500 hPa associated with 6-hourly precipitation extremes in CESM-LE. Percentage changes are shown relative to the historical climate and normalized by the change in global mean surface air temperature. Masking is as in Fig. 2.

where $k$ and $k_J$ are composite effective horizontal wavenumbers defined through $k^2 = -\nabla^2(\sigma \omega_{QG})/\sigma \omega_{QG}$ and $k_J^2 = -\nabla^2 J/\sigma \omega_{QG}$, respectively, the composite effective vertical wavenumber is defined as $m^2 = -\nabla^2 \omega_{QG}/\sigma \omega_{QG}$, and the composite static stability is defined as $\delta = \sigma \omega_{QG}/\bar{\omega}_{QG}$. The composite variables resemble what would be obtained from a simple average, but they have the advantage that they satisfy the QG-\(\omega\) equation to the same level of accuracy as the numerical inversions. Our focus on the local maximum of $-\omega_{QG}$ helps to ensure that $k^2$ and $k_J^2$ are positive. Note that $k^2$ accounts for the combined spatial structure of $\omega_{QG}$ and $\sigma$.

Equation (4) can then be solved for $\bar{\omega}_{QG}$ as

$$\bar{\omega}_{QG} = -\frac{\Delta dv + \xi k_J^2 J}{k_J^2 + f_0^2 m^2}, \quad (5)$$

We use a linear expansion (first-order Taylor expansion) of equation (5) about the historical values to decompose the response of $\bar{\omega}_{QG}$ to climate change into contributions from changes in static stability ($\sigma$), horizontal wavenumbers ($k$, $k_J$), vertical wavenumber ($m$), QG forcing ($\Delta dv$) and diabatic heating ($J$). The effects of changes in the horizontal wavenumbers are combined because they offset one another as discussed below. The addition of all the contributions approximately reconstructs the change in $\bar{\omega}_{QG}$ (Fig. 2c and 3a).

The largest contributions are from changes in diabatic heating ($J$) and static stability ($\sigma$) (Fig. 2d,e and 3a). The contribution of changes in $J$ is mostly a strengthening of upward motion because of stronger latent heating in a warmer and moister atmosphere for a given upward velocity, but this contribution can be negative where there is a sufficiently large weakening of upward motion. The contribution of changes in $\sigma$ is almost uniformly a weakening, consistent with the projected increase of tropospheric dry static stability with warming (Frierson 2006). In the extratropical average, the contribution from increases in static stability (-3.6\%K$^{-1}$) offsets much of the effect of increased diabatic heating (6.7\%K$^{-1}$).

The combined changes in the horizontal wavenumbers weaken the upward velocity at higher latitudes (Figs. 2f and 3a), but averaged over the extratropics the combined contribution is only -0.9\%K$^{-1}$. Both $k$ and $k_J$ increase substantially with climate warming which implies a decrease in the horizontal length scale of the vertical velocity field. This consistent decrease in length scale differs from the more mixed response of the length scale of ascent (T18) or of precipitation (Dwyer and O’Gorman 2017) found in previous studies of changes in extratropical precipitation extremes, likely because different models and measures of length scale were used. However, the contributions from the increases in $k$ and $k_J$ partially cancel each other in the QG-\(\omega\) equation (Fig. 5). This partial cancellation arises because $k$ and $k_J$ behave similarly, which results from latent heating being associated with ascent, but they appear on opposite sides of equation (4). At higher latitudes (poleward of 50$^\circ$) the increase in $k$ is larger than the increase in $k_J$, yielding a net weakening contribution to upward motion at these latitudes.

We find that the contribution of changes in horizontal wavenumbers is relatively unimportant except at higher latitudes, and it is of opposite sign to the change in upward motion. By contrast, T18 found that the contribution from changes in eddy length plays a key role and is consistent in sign with the change in upward motion, particularly in the subtropics. T18 motivated this finding using a simplified balance of the QG-\(\omega\) equation (their equation 3) of the QG-\(\omega\) equation that neglected the static stability term $\nabla^2 (\sigma \omega_{QG})$, but we find this to be a dominant term at all latitudes (see Fig. 4). Thus, while there is some resemblance in the spatial pattern of changes in eddy length scale and changes in vertical velocity, the simplified balance of the QG-\(\omega\) equation used to explain this in T18 does not hold in our inversions. T18 may have underestimated the static stability term because they scaled the Laplacian operator as $\nabla^2 \sim -1/L^2$ with the eddy length $L$ defined by $L^2 = L_x^2 + L_y^2$ where $L_x$ and $L_y$ are the e-folding distances in x and y directions. We argue that the Laplacian should instead be scaled as $\nabla^2 \sim \partial_x^2 + \partial_y^2 \sim -(1/L_x^2 + 1/L_y^2)$ which is larger by a factor of 4 when $L_x = L_y$, and this factor would be even greater when $L_x \neq L_y$.

The vertical wavenumber decreases because of an upward stretching of the vertical velocity profile. This decrease is consistent with an increase in the vertical extent of the circulation as the climate warms (Singh and
O’Gorman 2012; Fildier et al. 2017). However, changes in both vertical wavenumber and advective forcing contribute little in the dry decomposition, with extratropical-average contributions of 0.2%K\(^{-1}\) and 0.1%K\(^{-1}\), respectively (Figs. 2g,h and 3a).

Overall, the dry decomposition shows a dominant role of increases in diabatic heating and dry static stability which tend to offset each other in their effect on the vertical velocities. We will see in the next section that when this partial cancellation is taken into account by introducing a moist static stability, other factors such as the increase in vertical extent of the circulation become more important.

5. Moist decomposition

We introduce a moist decomposition of the QG-\(\omega\) equation that links diabatic heating to the vertical velocity and thus avoids treating it as an external forcing. The diabatic heating in extreme precipitation events is dominated by latent heating, and here we approximate it as the latent heating associated with saturated moist-adiabatic ascent,

\[
J = -\frac{P}{\kappa} \omega \sigma^* + \varepsilon, \tag{6}
\]

where \(\sigma^*\) is the static stability parameter for a moist-adiabatic lapse rate and \(\varepsilon\) is the error of the approximation. Equation (6) follows from equation (1) of O’Gorman (2011), and similar parameterizations of condensational heating have been used previously (Emanuel et al. 1987).

For convectively-unstable events in which the stratification is close to moist adiabatic, equation (6) may also be viewed as a simple quasi-equilibrium convective parameterization that maintains a moist-adiabatic vertical temperature profile when convection is forced by large-scale ascent. The extreme precipitation events in our analysis are generally close to saturation, and equation (6) is a good approximation for the diabatic heating in these events as shown in Fig. 6. Equation (6) also faithfully captures the contribution of changes in diabatic heating to the changes in \(\overline{\omega}_{QG}\) in response to climate change (compare the red solid and dash-dotted lines in Fig. 3a).

|               | 6-hourly | daily     |
|---------------|----------|-----------|
|               | CESM     | GFDL      | CESM | GFDL  |
| \(\varpi\)    | 2.0      | 2.0       | 0.9  | 0.6   |
| \(\overline{\omega}_{QG}\) | 2.2      | 2.0       | 1.5  | 0.7   |
| \(\mathcal{J}\) | 6.7      | 7.4       | 5.8  | 6.4   |
| \(\hat{\sigma}\) | -3.6     | -3.9      | -3.2 | -3.5  |
| \(\delta \Delta v\) | 0.1      | -0.4      | -0.1 | -1.1  |
| \(k, k_j\)    | -0.9     | -1.2      | -0.9 | -1.3  |
| \(m\)         | 0.2      | 0.5       | 0.1  | 0.5   |

Table 1. Changes (% K\(^{-1}\)) in vertical velocities and contributions in the dry decomposition averaged over the extratropics (30° to 70° in both hemispheres) for 6-hourly and daily precipitation extremes with CESM-LE and GFDL-CM3. All events are calculated at the 99.9th percentile except for the daily events for GFDL-CM3 which are at the 99.5th percentile.

Using equation (6), we rewrite the diabatic term in equation (4) as

\[
\frac{\kappa}{P} k^2 \mathcal{J} = -k^2 \overline{\omega}_{QG} \hat{\sigma}^* + \frac{\kappa}{P} k^2 \mathcal{J}_{res} \tag{7}
\]

where \(-k^2 \overline{\omega}_{QG} \hat{\sigma}^*\) accounts for the latent heating induced by \(\overline{\omega}_{QG}\) and \(\hat{\sigma}^* = \overline{\omega}_{QG} \sigma^2 / \overline{\omega}_{QG}\) is the composite static stability parameter for a moist-adiabatic lapse rate. The residual diabatic heating term,

\[
\mathcal{J}_{res} = -\frac{P}{\kappa} \left( \overline{\omega} \sigma^* - \frac{k^2 \overline{\omega}_{QG} \sigma^2}{k_j^2} \right) + \varepsilon, \tag{8}
\]

is positive because the magnitude of \(\overline{\omega}\) is usually underestimated by \(\overline{\omega}_{QG}\), and it is also modified by differences between \(k_j\) and \(k\) and the error \(\varepsilon\) in equation (6).

Substituting equation (7) into (4) yields

\[
\overline{\omega}_{QG} = -\frac{\delta \Delta v + \kappa}{k^2 \sigma_m + f_0^2 m^2} k^2 \mathcal{J}_{res}, \tag{9}
\]

where we have defined a moist static stability \(\sigma_m = \hat{\sigma} - \hat{\sigma}^*\). The moist static stability is smaller than the dry static stability and would be zero if the stratification in the extreme precipitation events was exactly moist-adiabatic. A linear expansion of equation (9) about the historical values
gives the moist decomposition of changes in $\overline{\omega}_{QG}$ into contributions from changes in moist static stability ($\delta_m$), horizontal wavenumbers ($k, k_j$), vertical wavenumber ($m$), advective forcing ($\overline{Adv}$) and residual diabatic heating ($J_{res}$).

The moist decomposition is noisier than the dry decomposition, and so we focus on the zonal-mean results (Fig. 3b). The sum of the contributions approximately reconstructs the total response at all latitudes. Decreases in the moist static stability strengthen the upward motion at all latitudes with an extratropical average contribution of 1.8% K$^{-1}$, in contrast to the weakening effect of increases in dry stability in the dry decomposition. The decrease in moist static stability corresponds to the stratification becoming closer to moist adiabatic with warming, an effect that has also been found for the mean stratification as the climate warms over a wide range in an idealized GCM (see Fig. 9 in O’Gorman 2011). The contribution from changes in residual diabatic heating is relatively small with an extratropical average of -0.3%K$^{-1}$.

Replacing the dry static stability with the smaller moist static stability and the diabatic heating with the smaller residual diabatic heating increases the importance of other terms in the moist decomposition compared to the dry decomposition. The contribution from decreases in vertical wavenumber is larger in the moist decomposition, with an extratropical-average contribution of 0.5%K$^{-1}$ as compared to 0.2%K$^{-1}$ in the dry decomposition. Similarly, the contribution from changes in advective forcing is larger in magnitude in the moist decomposition, with an extratropical-average contribution of 0.5%K$^{-1}$ as compared to 0.1%K$^{-1}$ in the dry decomposition. However, $k^2$ is multiplied by the smaller moist static stability and $k_j^2$ is multiplied by a smaller residual diabatic heating in equation (9), and the combined contribution from changes in $k$ and $k_j$ in the moist decomposition is smaller in magnitude in the extratropical average in the moist decomposition (-0.5%K$^{-1}$) than in the dry decomposition (-0.9%K$^{-1}$).

Overall, the moist decomposition for CESM-LE suggests that increased upward motion as the climate warms results from factors such as decreased moist static stability and increased vertical extent of the circulation. However, changes in residual diabatic heating play a greater role for GFDL-CM3 as compared to CESM-LE as discussed in the next section.

6. Results for GFDL-CM3 and for daily precipitation extremes

Changes in $\overline{\omega}$ are similar in magnitude for GFDL-CM3 as for CESM-LE (Figs. S2 and S3). However, because GFDL-CM3 has coarser horizontal resolution, the horizontal Laplacian terms in the QG-$\omega$ equation are smaller in magnitude, and thus there is a relatively greater role for the vertical derivative and advection terms as compared to CESM-LE (Fig. S4). As a result, the contributions from changes in vertical wavenumber and advective forcing are of larger magnitude in the dry and moist decompositions for GFDL-CM3 (Fig. S3). One other notable difference is that the moist decomposition for GFDL-CM3 has a more positive contribution from increased residual diabatic heating.

The importance of increases in residual diabatic heating for the response in GFDL-CM3 suggests that differences between $\omega$ and $\omega_{QG}$ as well as diabatic heating not captured by the approximation for latent heating (the error $\epsilon$ in equation 6) are more important for the response in this GCM. Differences between $\omega$ and $\omega_{QG}$ are caused by unbalanced dynamics but also the boundary conditions that we use when inverting the QG-$\omega$ equation (climatological means at the lateral boundaries and zeroes at the top and bottom). We further carry out another analysis for GFDL-CM3 in which for each event, the bottom and lateral boundary values for the inversions are set to $\omega$ from the GCM output. This new setup leads to a 500hPa-$\overline{\omega}_{QG}$ at that more accurately reproduces $\overline{\omega}$, in that the underestimation of $\overline{\omega}$ by $\omega_{QG}$ is 8% in the full-boundary case compared to 20% in the default case. However, the moist decomposition remains broadly similar (compare Figs. S3 and S5), which suggests that the boundary conditions are not a key factor for our overall results.

Daily extreme precipitation events are analyzed similarly to the 6-hourly events with some modifications. We calculate the 99.9th-percentile daily events for CESM-LE but the 99.5th-percentile daily events for GFDL-CM3 because there are fewer events with daily resolution compared to 6-hourly, and GFDL-CM3 has only one ensemble member. With these choices, there are roughly 20 daily events per grid point for CESM-LE and 30 for GFDL-CM3. The precipitation rate for a given day is calculated by averaging the four 6-hourly interpolated precipitation rates for that day. The static stability parameter ($\sigma$) is calculated using the smoothed and time-averaged temperature over the day. The vertical velocity ($\omega$) shown in figures, advective forcing ($\overline{Adv}$) and diabatic heating ($J$) are computed at each 6-hourly instance and then averaged to a daily value.

|                | 6-hourly CESM | 6-hourly GFDL | daily CESM | daily GFDL |
|----------------|--------------|--------------|-----------|------------|
| $\overline{\omega}$ | 2.0          | 2.0          | 0.9       | 0.6        |
| $\overline{\omega}_{QG}$ | 2.2          | 2.0          | 1.5       | 0.7        |
| $J_{res}$        | -0.3         | 0.9          | -0.8      | 1.4        |
| $\delta_m$       | 1.8          | 1.2          | 2.4       | 1.7        |
| $\overline{Adv}$ | 0.5          | -0.8         | -0.2      | -2.5       |
| $k, k_j$         | -0.5         | -0.6         | -0.6      | -0.7       |
| $m$              | 0.5          | 1.1          | 0.3       | 0.9        |

Table 2. As in table 1, but for the moist decomposition.
For daily precipitation extremes in CESM-LE (Figs. S6-S7), the strengthening of upward motion in the extratropics is smaller in magnitude (0.9%K$^{-1}$ in the extratropical average) than for 6-hourly precipitation extremes (2.0%K$^{-1}$). For daily precipitation extremes in GFDL-CM3 (Figs. S8-S9), the strengthening of upward motion is even less pronounced (0.6%K$^{-1}$) as compared to the 6-hourly extremes (2.0%K$^{-1}$). The vertical velocity responses for daily precipitation extremes in both GCMs have mixed positive and negative changes in the extratropics (Figs. S6 and S8), consistent with the behavior of the dynamical contribution to changes in daily precipitation extremes in the ensemble mean of CMIP5 (Pfahl et al. 2017). In the extratropical average, the terms in the dry and moist decompositions are of the same sign for daily extremes as for 6-hourly extremes (Tables 1 and 2), with the exception of the contribution of changes in advective forcing which changes sign in CESM-LE.

In the moist decomposition for both GCMs, more negative contributions from changes in advective forcing help to explain why the vertical velocities strengthen less for daily precipitation extremes as compared to 6-hourly precipitation extremes. However, the weaker responses in vertical velocities at 500hPa at the daily time scale does not translate to equivalently weaker changes in precipitation extremes at the daily time scale, perhaps due to our sole focus on 500hPa instead of the whole column, or differences in the thermodynamic response. For example, in CESM-LE the extratropical-average response of precipitation extremes is 6.1%K$^{-1}$ for 6-hourly events and 5.7%K$^{-1}$ for daily events, which shows less of a difference than the responses in vertical velocities at 500hPa (2.0% for 6-hourly events and 0.9% for daily events).

### 7. Conclusions

We have analyzed changes in vertical velocities associated with extratropical precipitation extremes in simulations of 21st-century climate change with two coupled GCMs. For each extreme-precipitation event, we solved the QG-ω equation in a local domain, and the resulting vertical velocities at 500hPa were shown to be in agreement with the vertical velocities from the GCMs. Upward motion in the extreme precipitation events is strengthened in response to climate warming, and this was first explained by a dry decomposition of the QG-ω equation in which diabatic heating was treated as an external forcing. According to the dry decomposition, strengthening of upward motion by increased diabatic heating is partly offset by increased dry static stability and, to a lesser extent, changes in the horizontal extent of the extreme events. Changes in horizontal extent contribute little except at higher latitudes, in contrast to previous results based on a scaling analysis of the QG-ω equation (T18).

The treatment of diabatic heating as an external forcing is a major limitation of the dry decomposition, especially when the overall aim is to understand changes in surface precipitation rates which are directly related to the column-integrated latent heating. Therefore, we also performed a moist decomposition of the QG-ω equation in which diabatic heating is approximated by the latent heating in moist-adiabatic saturated ascent. In the moist decomposition, the upward velocity is generally strengthened by decreases in moist static stability and increases in vertical extent of the circulation under climate warming. In the coarser GFDL-CM3 GCM, changes in advective forcing, vertical extent and residual diabatic heating play a greater role.

In future work, the role of the residual diabatic heating (i.e., diabatic heating that is not captured by moist-adiabatic saturated ascent driven by quasi-geostrophic dynamics) could be investigated using high-order equations for the vertical velocity (Muraki et al. 1999; Davies 2015) or by including convective-scale dynamics as in the approach of Nie et al. (2018). In addition, changes in advective forcing could be better understood by relating them to changes in eddy kinetic energy (O’Gorman 2010) and changes in the horizontal length scales of the geostrophic winds (Kidston et al. 2010). It would also be interesting to solve the QG-ω equation and analyze the dry and moist decompositions for precipitation extremes in a wider range of GCMs and in different seasons.

### Acknowledgments

We thank Matthieu Kohl, Ji Nie, and Neil Tandon for helpful discussions. We acknowledge the CESM Large Ensemble Community Project and supercomputing resources provided by NSF/CISL/Yellowstone. We also acknowledge the World Climate Research Programme’s Working Group on Coupled Modeling, which is responsible for CMIP, and we thank the NOAA Geophysical Fluid Dynamics Laboratory for producing and making available the output of GFDL-CM3. The datasets used in this paper are available at https://www.cesm.ucar.edu/projects/community-projects/LENS/data-sets.html and ftp://nomads.gfdl.noaa.gov/CMIP5. This work is supported by NSF AGS 1552195 and the MIT Environmental Solutions Initiative. Data, and code for the QG-ω inversion and figures are available at: https://github.com/dante831/QG-omega.git.

### APPENDIX

#### Exclusion of events from the analysis

A small fraction of extreme precipitation events are excluded from all of our results if any of the following conditions holds:

1. The climatological mean surface pressure is lower than 550hPa.
2. The domain for the QG-ω inversion still includes grid points below the surface even when it is shrunk to the smallest allowed size as described in section 3.

3. The closest local extremum of \( \omega_{QG} \) at 500hPa is more than 3 grid points away in CESM-LE (2 grid points in GFDL-CM3) from the horizontal location of the extreme precipitation event in either the zonal or meridional direction.

4. The percentage of grid points with negative \( \sigma \) in the (3-D) domain for the QG-ω inversion exceeds 10%.

5. The numerical inversion of the QG-ω equation is erroneous as manifested by NaN’s due to numerical instability or unphysically large \( \omega_{QG} \) (vertically averaged absolute value larger than 10Pa s\(^{-1}\)) at the closest local extremum at 500hPa.

6. The closest local extremum of \( \omega_{QG} \) at 500hPa is positive, since this implies downward motion at 500hPa which is not contributing to the extreme precipitation at the surface.

Fewer than 9% of the total events in the extratropics (between 30° and 70° latitude in both hemispheres) are discarded in a given climate and GCM. Therefore, the omission of these events is not expected to strongly affect our results.

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Supporting Information for “Response of Vertical Velocities in Extratropical Precipitation Extremes to Climate Change"

Ziwei Li *, Paul O’Gorman

Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA
*ziweili@mit.edu

Figure S1. (a) $\bar{\omega}$ and (b) $\bar{\omega}_{QG}$ in Pa s$^{-1}$ at 500hPa for precipitation extremes in the historical simulations with CESM-LE. Masking is as in Fig. 2.
Figure S2. As in Fig. 2, but for GFDL-CM3.
**Figure S3.** As in Fig. 3, but for GFDL-CM3. (Note the change of scale of the vertical axis)

**Figure S4.** As in Fig. 4, but for GFDL-CM3. (Note the change of scale of the vertical axis)
Figure S5. As in Fig. S3, but the QG-ω equation is solved for GFDL-CM3 using ω taken from the GCM simulations for the lateral- and lower-boundary conditions.
Figure S6. As in Fig. 2, but for daily precipitation extremes in CESM-LE.
Figure S7. As in Fig. 3, but for daily precipitation extremes in CESM-LE.
Figure S8. As in Fig. 2, but for daily precipitation extremes at the 99.5-percentile in GFDL-CM3.
Figure S9. As in Fig. 3, but for daily precipitation extremes at the 99.5-percentile in GFDL-CM3.