The relationships between permeability and geological entropy of fracture networks

Zuyang Ye1,2,*, Sanqi Li1,2

1 School of Resource and Environmental Engineering, Wuhan University of Science and Technology, Wuhan, 430081, P.R. China
2 Hubei Key Laboratory for Efficient Utilization and Agglomeration of Metallurgical Mineral Resources, Wuhan University of Science and Technology, Wuhan 430081, P.R.China

Corresponding author: Zuyang Ye (yezuyang@wust.edu.cn)

Abstract. A systematic approach has been proposed to estimate the relationship between the permeability and geological entropy of two-dimensional fracture networks. The relative entropy index is developed as a measure of multiple properties of two-dimensional fracture networks, including fracture density, length, orientation and spatial distribution. In order to understand geological entropy dependence of the permeability, a computational method combining fracture network generation and steady-state flow simulation is employed. Based on the results of detailed numerical simulations, the relative entropy index and permeability increase simultaneously as the fracture density and length are independently increased. A simple closed-form empirical expression of the logarithmic dependence of the normalized permeability on the relative entropy index is proposed.

1. Introduction
Permeability is generally used to characterize the hydraulic properties of fractured rock. Due to the high permeability of fractures, the water flow is dominated by the spatial distribution of fracture networks. The spatial distribution of fracture networks has been quantified by several comprehensive indicators, such as the ratio of the number of intersections to the number of fractures [1], a combination of the number of fractures, the number of intersection nodes and fracture length [2]. The influence of fracture networks on permeability are investigated by the equivalent continuous porous media model [3-4], in which the equivalent permeability is obtained from the individual fracture parameters such as fracture density (the number of fractures per domain area), length and aperture [5-6]. However, the effect of spatial distribution of fracture networks on the permeability is not completely estimated.

Because information entropy [7] is a measure of the average amount of information required to predict the outcome of a random process, it has been extended to a wide range of many fields such as hydrogeology, geostatistics [8-11]. Thus, a new method called geological entropy and a new metric called relative entropy index are deduced by Bianchi and Pedretti [12-13] to quantify the degree of spatial order in the heterogeneous media based on the assumption that the flow and transport in the
aquifer is dependent on its degree of spatial order/disorder. At present, geological entropy has been applied to saturated alluvial aquifers \([12-13]\), unsaturated flow systems \([14]\) and transport in fractured media \([15]\). Therefore, the main objective of this research is to perform a more comprehensive study on the relationship between the permeability and spatial distribution of fracture networks using geological entropy.

2. Geological entropy

According to the concept of information entropy \([7]\), Bianchi et al. \([12-13]\) firstly developed the concept of geological entropy to describe the spatial disorder of heterogeneous porous media and then estimate the solute transport behavior subjected to stochastic permeability. In this study, geological entropy is applied to describe the spatial distribution of fracture networks.

![Fig. 1. Illustration of fracture network with a set of \(N\) grids](image)

The discrete random fracture system is divided into a set of \(N\) grids as shown in Fig. 1, the geological entropy of such fracture networks can be expressed with the following equations

\[
H_R = \frac{H_L}{H_G} \tag{1}
\]

\[
H_G = -\sum_{n=1}^{N} p_{G,n} \ln p_{G,n} \tag{2}
\]

\[
H_L = \frac{1}{I \times J} \sum_{j=1}^{J} \sum_{i=1}^{I} H_L(i,j) \tag{3}
\]

\[
H_L(i) = -\sum_{n=1}^{N} p_{l,n} \ln p_{l,n} \tag{4}
\]

where \(H_R\) is the relative entropy index, which is a quantitative measure of the spatial disorder in a discrete random system; \(H_G\) is the global entropy; \(H_L\) is the local entropy of each grid; \(p_G\) are defined as the area proportions of the fractures or matrix over the whole domain of interest; \(p_l\) are defined as the area proportions of the fractures or matrix over each grid; \(H_L\) is the average of the local entropy with \(I\) and \(J\) numbers of elements along Cartesian axes for a two-dimensional grid.

3. Results and analysis

3.1. Scenarios of two-dimensional networks

In order to establish the relationship between the permeability and spatial distribution of fracture networks, the fracture networks are generated by a Monte Carlo method proposed by Long et al. \([3]\) and the normalized permeability is calculated by the finite element method by Jiang et al. \([16]\). For each fracture set, fracture centers are generated randomly in the domain of interest and the lengths and
orientations are assigned to each center subsequently. The flow within each fracture is characterized by the cubic law and the finite element procedure can be derived based on the continuity equations and its variational inequality formulations. When a hydraulic head difference through the inflow and outflow boundaries, the normalized permeability can be obtained by solving a series of mass balance equations at each node.

Two-dimensional fracture networks are realized inside a square domain with 50m×50m. Four groups of scenarios of fracture networks are presented in Table 1. The apertures of all fractures are equal to $1 \times 10^{-3}$m. The hydraulic head difference $\phi_1 - \phi_2$ between the inflow and outflow boundaries of each fracture networks is specified as 10m and the residual boundaries are impermeable as shown in Fig. 1. It should be emphasized that the generation of fracture networks with respect to the same geometry parameters may have different spatial distributions, so each data point is based on the average results from 50 simulations with the same geometry parameters for spatial distributions of fracture networks.

| Fracture set | Orientation(°) | Case | Length(m) | Density (#/m²) |
|--------------|---------------|------|------------|---------------|
| Group A     | 1 0 30 45 60  | 1    | 20         | 50            |
| Group B     | 2 20 100     | 2    | 20         | 100           |
| Group C     | 3 20 150     | 3    | 20         | 150           |
| Group D     | 4 20 200     | 4    | 20         | 200           |
|             |               | 5    | 30         | 50            |
|             |               | 6    | 30         | 100           |
|             |               | 7    | 30         | 150           |
|             |               | 8    | 30         | 200           |
|             |               | 9    | 40         | 50            |
|             |               | 10   | 40         | 100           |
|             |               | 11   | 40         | 150           |
|             |               | 12   | 40         | 200           |

3.2. Relationships between permeability, geological entropy and geometry characteristics

Owing to space constraints, the ensemble of relationships between density, length, the relative entropy index $H_R$ and the normalized permeability $k'$ for all cases of Group A are shown in Fig.2 and 3. It can be seen that for a given fracture length in each group, both $H_R$ and $k'$ are increasing functions of fracture density. Also for a given fracture density in each group, $H_R$ and $k'$ will increase with increasing fracture length. The increase in the normalized permeability with increasing density is more substantial than that is with increasing fracture length, which is consistent with the numerical results in two-dimensional models of Jafari and Babadagli [5].

Figs.2 and 3 also demonstrate that the relationships between the normalized permeability and relative entropy index are positively correlated. This is because that for a larger fracture density and length, the interconnection between fractures turn to be more sufficient and large-scale flow paths are more easily formed, the fracture system should be more permeable. Simultaneously, the area proportions occupied by fractures in each grid will get larger with increasing fracture density and length, which leads to the enlargement of the relative entropy index. However, the main difference between the normalized permeability and relative entropy index curves are that the normalized
permeability is approximately a linear function of fracture density or length, while the relative entropy index increases about nonlinearly with fracture density or length.

**Fig. 2.** The relationships between $H_R$, $k'$ and density for Group A. Blue dash lines indicate the normalized permeability with linear fit. Red dash lines indicate the relative entropy index with nonlinear fit.

**Fig. 3.** The relationships between $H_R$, $k'$ and length for Group A. Blue dash lines indicate the normalized permeability with linear fit. Red dash lines indicate the relative entropy index with nonlinear fit.

The relationships between the relative entropy index $H_R$ and the normalized permeability $k'$ for all fracture networks within five groups are shown in Fig. 4. In general, the relative entropy index increases nonlinearly with the normalized permeability, according to the relationship that can be best approximated by a logarithmic function in the form
\[ H_r = A + B \ln k' \]  

where \( A \) and \( B \) are empirical parameters. The fitted values of parameters \( A \) and \( B \) in each group are presented in Fig. 5 and the values for the correlation coefficient of curve fitting \( (R^2) \) are above 0.98. When the orientation of set 1 ranges from 0° to 60°, the value of \( A \) decreases from 0.056 to 0.050 and \( B \) increases from 0.774 to 0.794, respectively. Therefore, Eq. 26 can be used to quantify the dependence of the permeability on the geometry characteristics of fracture networks including fracture density, length, orientation and spatial distribution.

Fig. 4. The relative entropy index versus the normalized permeability for all fracture networks.

Fig. 5. The trend of parameters \( A \) and \( B \) with the fracture orientation increment.

4. Conclusions

A Monte Carlo method for generation of fracture network and a finite element method for permeability calculation are combined to quantify the relationship between permeability and spatial distribution. A new index \( H_r \) based on geological entropy is employed to describe the spatial distribution of fracture networks including fracture density, length and orientation. The numerical results show that for a given fracture length, the normalized permeability and relative entropy index both increase with the fracture density. Similarly, for a given fracture density, the normalized
permeability and relative entropy index both increase with the fracture length. In addition, an empirical expression relating the normalized permeability to the relative entropy index is proposed in a logarithmic form, which can provide reasonable prediction between the spatial distribution and permeability of two-dimensional fracture networks.

References
[1] Jafari, A., Babadagli, T. (2011). Effective fracture network permeability of geothermal reservoirs. Geothermics, 40(1), 25-38.
[2] Leung, C., Zimmerman, R. (2012). Estimating the hydraulic conductivity of two-dimensional fracture networks using network geometric properties. Transport in porous media, 93(3), 777-797.
[3] Long, J., Remer, J., Wilson, C., Witherspoon, P. (1982). Porous media equivalents for networks of discontinuous fractures. Water Resources Research, 18(3), 645-658.
[4] Rong, G., Peng, J., Wang, X., Liu, G., Hou, D. (2013). Permeability tensor and representative elementary volume of fractured rock masses. Hydrogeology Journal, 21(7), 1655-1671.
[5] Jafari, A., Babadagli, T. (2012). Estimation of equivalent fracture network permeability using fractal and statistical network properties. Journal of Petroleum Science and Engineering, 92, 110-123.
[6] Maillot, J., Davy, P., Le Goc, R., Darcel, C., De Dreuzy, J. (2016). Connectivity, permeability, and channeling in randomly distributed and kinematically defined discrete fracture network models. Water Resources Research, 52(11), 8526-8545.
[7] Shannon, C. (1948). A mathematical theory of communication. Bell System Technical Journal, 27(3), 379-423.
[8] Ye, Y., Chiogna, G., Lu, C., Rolle, M. (2018). Effect of anisotropy structure on plume entropy and reactive mixing in helical flows. Transport in Porous Media, 121(2), 315-332.
[9] Castillo, A., Castelli, F., Entekhabi, D. (2015). An entropy - based measure of hydrologic complexity and its applications. Water Resources Research, 51(7), 5145-5160.
[10] Zhang, F., Zhang, X., Li, Y., Tao, Z., Liu, W., He, M. (2018). Quantitative description theory of water migration in rock sites based on infrared radiation temperature. Engineering Geology, 241, 64-75.
[11] Vranken, I., Baudry, J., Aubinet, M., Visser, M., Bogaert, J. (2015). A review on the use of entropy in landscape ecology: heterogeneity, unpredictability, scale dependence and their links with thermodynamics. Landscape Ecology, 30(1), 51-65.
[12] Bianchi, M., Pedretti, D. (2017). Geological entropy and solute transport in heterogeneous porous media. Water Resources Research, 53(6), 4691-4708.
[13] Bianchi, M., Pedretti, D. (2018). An entrogram-based approach to describe spatial heterogeneity with applications to solute transport in porous media. Water Resources Research, 54, 4432-4448.
[14] Ye, Z., Jiang, Q., Yao, C., Liu, Y., Cheng, A., Huang, S., Liu, Y., (2018). The parabolic variational inequalities for variably saturated water flow in heterogeneous fracture networks. GeoFluids, 9062569, 1-16.
[15] Pedretti, D., Bianchi, M. (2019). Preliminary results from the use of entrograms to describe transport in fractured media. Acque Sotterranee-Italian Journal of Groundwater, AS31(421):7-11.
[16] Jiang, Q., Yao, C., Ye, Z., Zhou, C. (2013). Seepage flow with free surface in fracture networks. Water resources research, 49(1), 176-186.

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