Misconception in Linear Equation System: The Case of Students Using Imitative Reasoning

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Abstract. Imitative reasoning is one of the strategies students used in solving mathematical problems. The imitative reasoning links to the ways in which students using their prior knowledge on solving a similar problem. Those using imitative reasoning, in fact, could only comprehend a problem on its surface level without deeply conceptualise it. The issue leads to errors and misconception in answering linear equation problems. This study aims to analyse students’ misconception when solving linear equation problems, specifically in those using imitative reasoning. This is a qualitative study with 64 students participated in the study. The study reveals that the misconceptions can be found in erroneous calculation, the application of problem solving strategies, the correlation between procedure and concept as well as the solutions of linear equations in intersecting, parallel, and coinciding lines.

Keywords: Misconception, Linear Equation, Imitative Reasoning

1. Introduction

Imitative reasoning (IR) is part of mathematical reasoning [1]. Further, it is explained that IR is a strategy to imitate the given problem-solving strategies without any efforts to construct a new idea. There are two types of imitation in IR namely memorised reasoning (MR) and Algorithmic Reasoning (AR). The MR is indicated by memorising a fact or a result which then followed by writing down the memorised fact as part of a student’s applying MR. Meanwhile, the AR is indicated by picking up a strategy by memorising a certain algorithm [2]. Part of MR application is by using the algorithm to solve a certain problem. One of the sharp differences between MR and AR is the former is used to solve relatively simple problems such as memorising a certain fact while the latter is used to solve problems requiring calculation procedures [3].

Mainly, the application of IR in mathematic lessons is not complicated as the use of IR is indeed can lead to a better learning outcome. However, the application is deteriorated if it is solely used to memorise a procedure or fact with the absence of an in depth concept understanding [4]. With the dominant use of IR, students could hardly develop their ideas to
solve problems [5]. It is reflected from difficulties faced by 29 grade X students in the previous study where the students using IR in solving linear equation problems. The difficulties varied in procedural and conceptual stages.

The above problem contradicts the school’s learning objectives that the students were hoped to understand linear equation system considering how important good comprehension of the topic is [6]. The advanced linear equation system will be studied in the next stage of studying math, science, technology engineering and economics. Additionally, the comprehension is needed for the development of STEM education [7]. Unfortunately, many problems occur due to students’ difficulties in comprehending linear equation system. Specifically, the difficulties occur when students learning about abstract mathematical topics one of which is linear equation comprehension [8]. Still, many of the students are unable to generate solutions regarding solving linear equation problems [9].

Students had better be encouraged to generate solution to a certain problem. Motivating students to develop ideas to solve the problem can increase their mathematic competence [9]. Also, students should be encouraged to improve their mathematical reasoning skill as the mastering skill is an indicator to track the students’ success in learning math [10].

The ability to understand concepts and using them to problem solving procedures can also be part of a successful mathematic learning [11]. The ability to remember concepts, analyse facts and use a proper procedure are needed to be able to solve math problems [12]. Lack of comprehension in fundamental concepts exhibited by students, reflected through errors in solving mathematical problems, is part of the students’ misconception [11]. The studies of misconception in learning linear equation have been conducted by many [13][14][15]. However, the studies have yet to link the misconception with students’ imitative reasoning. Therefore, this study aims to analyse the misconception among students using imitative reasoning in solving linear equation problems. The results of this study can be further used to decide what learning strategies to use to avoid misconception.

2. Methods

This study involves 64 grade X students from one of the selected schools in Magetan, East Java. The participants are in their 15 to 16. The participants were selected as they had studied linear equation system and their abilities were varied. Data gathering was conducted through giving problem solving questions to the participants. The questions were about linear equation system as shown in Figure 1. Next, the questions were developed, as shown in Figure 2 and Figure 3, to determine whether or not misconceptions occurred. The questions were selected to measure the participants’ conceptual and procedural understanding. The selected questions were already validated.

This study is a qualitative one. The qualitative approach is aimed to gather misconceptions in detail. The data analysis is conducted to analyse the answers the students writing down with the focus is on the numbers of errors occurred. The errors analysed are those resulted from the use of imitative reasoning. Then, interviews were conducted to further examine the imitative reasoning and misconception among the students. Later, the data is obtained descriptively. To ease the data analysis basic statistic methods are obtained, namely presentation and frequency.

![Figure 1. Student Problem](image-url)
3. Results and Discussion

3.1 Imitative Reasoning

Students' work on the problem in figure 1 can be seen in the following table:

### Table 1. Strategies obtained in solving problems in figure 2

| Strategies                        | f | %  | T | %  | F | %  |
|-----------------------------------|---|----|---|----|---|----|
| Substitution                      | 10| 15.6| - | 10 | 15.6|
| Elimination                       | 14| 21.9| - | 14 | 21.9|
| Elimination & Substitution        | 32| 50  | 7 | 10.9| 25 | 39.1|
| Adopting a different point of view| 5 | 7.8 | 5 | 7.8| -  |    |
| Others                            | - | -   | - | -  | -  | -  |
| Not answering                     | 3 | 4.7 | - | -  | -  | -  |

According to the table, it can be seen that only 14 of the 64 participants (19%) answering the questions correctly while 76.6% had a wrong answer. The reason 57 students used substitution, elimination or the combination of both are because the strategies were found in their book as well as were previously explained by their teachers. Therefore, when being exposed to such a problem, the strategies would come first in their minds so that they will spontaneously answer the question without deeply analyzing the problem. It is indicative that the students used their imitative reasoning where they imitated familiar strategies studied in previous lessons [3][4][16]. Such a memorized procedure is often used in solving linear equation problems.

It can also be seen through the table that there are only three students used less-common strategy in the problem solving which eventually led them to give a correct answer. During the interview, the students explained that they previously examined the question to obtain as much information as they could. Specifically, the three focused on finding clues to determine the value of x+y. Then, they tried to find correlation between the two given equations. If the two equations are added, variables x and y will have the same coefficient so that they could simplify the equation to determine the value of x+y. The strategy used by the three students is called problem solving strategy by “adopting a different point of view”. Those obtaining the strategy were able to highlight important points in the question and were able to pick up correct procedures, which in fact simpler, in solving the problem [17].

Further, to examine the correlation between imitative reasoning and misconception, the participants were given the problems stated in figure 2. Before answering the questions, the participants were given information about problem solving with the strategy of adopting a different point of view. The results are shown in the table below:

### Table 2. Students’ answers to problems in figure 2

| Answer                   | f | With reasons | Without reasons |
|--------------------------|---|--------------|-----------------|
| Could find the solution  | 37| 32           | 5               |
| Could not find the solution | 27| 25           | 2               |

The above table indicates that 32 of the 37 answering correctly were giving their reasons which all of which are obtaining the strategy of adopting a point of view. It indicates that the students still use imitative reasoning. The students imitated a simpler step which they were just exposed to. They could barely use their conceptual understanding as to solely did so through their procedural understanding. The students were unfamiliar with both conceptual and procedural understandings in solving the problem [18].
3.2 Errors and misconception in Solving Linear Equation Problems

The errors produced by the students in solving the problem in figure 1 is erroneous calculation as can be seen in figure 2 below.

![Figure 2. Error in S1 Calculation](image)

The students produced errors when solving multiplication operation. The errors made in the beginning of the calculation leads to errors in problem solving. According to the interview, the errors occurred because S1 did not thoroughly examine the question. Assuming that the coefficients on the two equations were too big led the students to mistakenly answer the question. In general, errors in solving this type of problems are caused by careless calculation, lack of conceptual understanding and ability to gather information as well as errors in calculation procedure [12]. When explaining the reason behind choosing 38,483 as common multiplication of the two equations, S1 mentioned that “this is the common procedure of directly multiplying coefficients in two equations”.

According to the explanation, it is clear that S1 used his AR in which he imitated steps from a familiar procedure without understanding the aim of him undertaking every calculation. The selection of procedures when using AR is only relied upon the memorized familiar procedure without examining whether or not the selected procedure is logic [4].

Furthermore, building on the participants’ problem solving of the question in figure 2, the data gathered are as followed:

| Problem | Answer | \( f \) | % |
|---------|--------|---------|---|
| 1. Solution of two parallel linear equations | Correct reason | 7 | 10.9 |
| | False reason | | |
| | The students’ errors and misconception | | |
| | By applying the problem solving strategy namely “adopting a different point of view” the value of \( x+y \) could be found | 32 | 50 |
| | Could not differentiate between zero solution and no solution | 13 | 20.3 |
| | Could not differentiate between zero solution and infinitely many solutions. | 13 | 20.3 |
| | Misperception that any strategies could be employed for all types of problems. | 37 | 57.8 |
| | Could not differentiate between parallel and coinciding linear equations | 5 | 7.8 |
| | Giving no reason | 7 | 10.9 |
| 2. Solution of two coinciding linear equations | | | |
The above table reveals the forms of errors and misconception produced by the students. A student could generally make at least two errors. The following figures show errors students made due to the use of “adopting a different point of view” strategy.

Based on the interview conducted, the reason behind S2 and S3 selecting the strategy was because it is easier to apply such strategy in determining the value of x+y. S2 and S3 also explained that the strategy could be used to solve every problem related to linear equation system. This is a false perception which occurred in 57.8% of the participants.

The two did aware of the concepts regarding solutions of two parallel linear equations. However, the S2 made a mistake as he did not remember the concept while the S3 could remember the concept but did not understand differences between parallel and coinciding linear equations. The S3 explained “I do remember Mam. Two coinciding linear equations have infinite solutions. Meanwhile two parallel linear equations have no solution”. Later the S3 mentioned “Each of them are intersecting because the value of x+y could be determined”. Therefore, it is concluded that the reasons mentioned by S2 and S3 are merely based on the value of x+y which was determined through the undertaken procedures. They did not use their conceptual understanding to analyze the problem due to the use of AR hindering them to recall their conceptual understanding. It is examined that 50% of the participants showed the same error since they only relied on their ‘adopting a different point of view” strategy.

Another misconception can be seen through a student’s work using elimination strategy, as follows:

Based on the interview, the S4 chose the strategy as it is a general strategy to solve linear equation system. The S4 explained that the subtraction of variable x and y’s coefficients resulted 0 (zero) assuming then that the values of x=0 and y=0. The S4 failed to understand that such a linear equation has no solution. There are 20.3% of the students had misconception in regard to solutions of two parallel linear equations. It is clear that the S4’s assumption is based on AR as the S4’s understanding is only based on procedural understanding as opposed to the conceptual one. One of the ways to improve students’ conceptual ability is through stimulations such as by asking “how” and “why” [19].
The same explanation was given by S5 on solving linear equation problems of two coinciding linear equations. The S5 mentioned that the 0=0 means that x=0 and y=0. Specifically, the S5 explained “the values of x and y are both zero since the subtraction of the two is 0”. This explanation indicates that there is a misconception that the solution of two coinciding linear equations is 0.

Based on the above figure, it is clear that the S6 failed to differentiate parallel and coinciding linear equations. Meanwhile, 7.8% of the participants did the same mistake. The S6 only paid attention on the removal of x and y coefficients when undertaking elimination procedure. When further asked about the removal of the coefficient, the S6 explained “the value of x+y could not be determined”. It is then indicative that the S6 has lack of understanding on parallel and coinciding linear equations. On of the reasons behind this misconception is because the students are not familiar with exploring non routine and counter example problems. The exposure to monotonous or incomplete examples can lead to impartial concept understanding [20] whereas in fact giving them such examples will help them to use their imitative reasoning. Imitative reasoning for a certain procedure, but with the absence of in-depth concept understanding, can lead to misconception. Therefore, it is imperative that teachers giving their students advance problem solving through practice. Apart from helping students to deeply comprehend a concept, giving them more challenging problems to solve can also train the students to use their creative thinking [21][22].

4. Conclusion

Based on this study, it is found that misconceptions in solving linear equation system are evident. The misconceptions are found in the students using their imitative reasoning. Imitative reasoning is performed through following strategies which have already been given by teachers. Being familiar with a new strategy to solve a certain problem, students can falsely use the same strategy to solve an entirely different problem. The students fail to link appropriate procedures with appropriate concepts. This failure leads to students having misconception. Building on the issue, it is important that students are trained to explore various problem solving and counter examples of a certain concept. Whereas the errors and misconceptions generally consist of erroneous calculation, problem solving strategies, linkages between procedure and concept as well as different solutions for intersecting, parallel and coinciding linear equations.

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