RESEARCH ARTICLE

A Novel Tent-Levy Fireworks Algorithm for the UAV Task Allocation Problem Under Uncertain Environment

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ABSTRACT
Recently, unmanned aerial vehicle (UAV) task allocation is a hot topic both in the civilian and military, while the research of considering uncertainty and multi-objective is still in its infancy. Firstly, based on the uncertainty theory, a mathematical model of the uncertain multi-objective UAV task allocation problem with uncertain variables in both objective function and constraint conditions is established. The expected value criterion and opportunity constraint are introduced to transform the model into a deterministic optimization model. Furthermore, because traditional fireworks algorithm (FWA) has the shortcomings of low solution accuracy and slow convergence speed in solving the UAV task allocation problem, a novel Tent-Levy FWA (TLFWA) based on discrete update process is designed by introducing integer coding, Tent chaotic mapping and Levy variation. Experimental results show that the mean cost calculated by TLFWA is 8.17% and 13.73% lower than that of FWA and particle swarm optimization algorithm respectively, which proves the effectiveness of TLFWA. This study provides a new way to solve multi-objective and uncertain decision-making problems.

INDEX TERMS
Levy variation, tent chaotic mapping, TLFWA, UAV task allocation, uncertainty theory.

I. INTRODUCTION
Recently, unmanned aerial vehicle (UAV) is widely used in the military field due to its characteristics of fast speed, all-weather, non-contact and zero casualties, which has a revolutionary impact on the air combat mode [1], [2]. Relative researches about UAV task allocation problem have been carried out in the decade [3], [4], [5]. The UAV task allocation problem under complex battlefield environment and uncertainty condition has become a hot spot in the current UAV application field.

At present, most researches on the UAV task allocation problem are based on the assumption of parameter determination. However, affected by false intelligence information, bad weather conditions and other uncertain factors, the fuel consumption, flight time and threat in the process of UAV task allocation are unmeasurable. Those uncertain variables will result in the loss of optimality or even infeasibility of the task allocation scheme under certain conditions. Traditional methods to solve uncertain problems include probability theory, fuzzy set theory and robust optimization theory. However, probability theory cannot solve the uncertain problem when probability distribution of variables cannot estimate [6], [7]. Fuzzy set theory is not self-consistent in mathematics [8], and cannot solve uncertain problems in some specific situations [9], [10]. The results of robust optimization theory are relatively conservative [11]. Liu indicated that probability theory, fuzzy set theory and robust optimization theory may lead to counterintuitive results when dealing with these uncertainties [12]. Therefore, Liu [13], [14] proposed a complete theoretical system with normativity, subadditivity and self-duality--uncertainty theory to solve uncertain problems.

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Uncertainty theory and its applications have been researched by many scholars, producing uncertain finance model [15], uncertain regression model [16], uncertain risk model [17] and other models [18], [19]. But research progress on the UAV task allocation problem is still low. To solve the problem in which both objective function and constraint have uncertain variables, a new uncertain UAV task allocation model is established based on uncertainty theory. This model with uncertain variables is transformed into deterministic optimization model by introducing expected value criterion and opportunity constraint.

At present, there are mainly exhaustive methods, including integer programming method, constrained programming method, graph theory method, list method and clustering method to solve the UAV task allocation problem. However, these methods have the shortcomings of weak real-time computing and low accuracy to varying degrees. Fireworks algorithm (FWA) was a swarm intelligence algorithm proposed by Tan Ying in 2010, which is mainly used to solve continuous space optimization problems [20]. FWA is widely used in filter design, image recognition, vehicle routing problem and other problems. Pholdee et al. [21] systematically and comprehensively compared 24 meta-heuristic algorithms, and the results showed that FWA had better optimization performance. However, it also has problems of high iteration cost, premature convergence and poor searching effect of the optimal solution far from the far-point function [22]. Zheng et al. [23], [24], [25] proposed enhanced fireworks algorithm, dynamic search fireworks algorithm and adaptive fireworks algorithm with adaptive radius. By analysing and improving each operator of basic fireworks one by one, premature convergence of traditional FWA has been overcome to some extent. Zhao et al. [26] proposed updating information guided adaptive fireworks algorithm based on the best firework, which improved the search speed of FWA. However, none of the above algorithms can deal with discrete domain optimization problems, and there are still some shortcomings in improving the solution accuracy and speed of the algorithm. Considering the inherent defects of FWA, Tent-Levy fireworks algorithm (TLFWA) is proposed to better solve the UAV task allocation problem. First, integer coding is adopted, which makes sure that TLFWA can solve the discrete domain optimization problem. Furthermore, Tent chaotic mapping is introduced to generate initial task allocation schemes, which avoids falling into local optimal solution due to local search excessively. Finally, Levy variation with a larger variation range replaces Gaussian variation to accelerate the search speed of the algorithm.

The main contributions of this paper are as follows.

1. Establishing an uncertain UAV task allocation model based on uncertainty theory, which solves the problem that both objective function and constraint conditions contain uncertain variables.

2. A novel Tent-Levy fireworks algorithm is proposed to solve the UAV task allocation problem. Compared with FWA, TLFWA can solve discrete domain optimization problems. Besides, TLFWA has fast search speed and high solution accuracy.

3. The performance of TLFWA is tested by six benchmark functions. And the effectiveness of the proposed model and method in this paper are verified by UAV task allocation examples.

The paper is organized as follows. Section 2 introduces the basic definitions and theorems of uncertainty theory. The uncertain UAV task allocation model is established in Sect. 3. In order to solve the model, a novel improved fireworks algorithm is designed in Sect. 4. In section 5, the effectiveness of the model and algorithm proposed in this paper is verified by experiments. Section 6 concludes this work in this paper.

II. PRELIMINARY

Definition 1 [13]: Let be a nonempty set, and is a -algebra over . Each element in is called a measurable set, which is renamed event in uncertainty theory. A set function is from to that satisfies axioms 1, 2, 3, and the triplet is called an uncertain measure in product space. Axiom 1 (Normality Axiom): 

\[ M(\{\Omega\}) = 1 \]

Axiom 2 (Duality Axiom): \[ M(\Lambda) + M(\Lambda^c) = 1 \]

Axiom 3 (Subadditivity Axiom): For each countable sequence of events , , ..., we have

\[ M\left(\bigcup_{i=1}^{\infty} \Lambda_i\right) \leq \sum_{i=1}^{\infty} M(\Sigma_i) \]

Axiom 4 (Product Axiom): For a series of uncertainty spaces , , ..., product uncertain measure is an uncertain measure satisfying

\[ M\left(\prod_{i=1}^{\infty} \Lambda_i\right) \leq \prod_{i=1}^{\infty} M_i(\Lambda_i) \]

where are arbitrary events in the set for .

Definition 2 [27]: An uncertain variable is a measurable function from the uncertainty space to the real numbers set , such that for any Borel set of real numbers, the set

\[ \{x \in B\} = \{y \in \Gamma| x(y) \in B\} \]

represents an event.

Definition 3 [8]: The uncertainty distribution of the uncertain variable is defined by

\[ \Phi(x) = M[\xi \leq x], \]

where is any real number.

Definition 4 [13]: An uncertain variable is called normal if it has a normal uncertainty distribution

\[ \Phi(x) = \left(1 + \exp\left(\frac{\pi (e - x)}{\sqrt{3\sigma}}\right)\right)^{-1}, \quad x \in R \]

\[ \Phi(x) = \left(1 + \exp\left(\frac{3\sigma (e - x)}{\pi}\right)\right)^{-1}, \quad x \in R \]
denoted by \( N(e,\sigma) \) where \( e \) and \( \sigma \) are real numbers with \( \sigma > 0 \).

**Theorem 1 [19]**: If the inverse distribution of uncertain variable \( \xi \) is \( \Phi_{-1}(\alpha) \), then \( M[\xi \leq c] \geq \alpha \) is equivalent to \( \Phi_{-1}(\alpha) \leq c \), where \( \alpha \) and \( c \) are constant and \( 0 \leq \alpha \leq 1 \).

**Theorem 2 [19]**: Let \( \xi \) be an uncertain variable with regular distribution \( \Phi \). If the expected value exists, then

\[
E[\xi] = \int_{0}^{1} \Phi_{-1}(\alpha) \, d\alpha. \quad (6)
\]

**Theorem 3 [8]**: Let \( \xi_1, \xi_2, \ldots, \xi_n \) be a series of independent uncertain variables with regular distribution \( \Phi_1(\alpha), \; \Phi_2(\alpha), \ldots, \Phi_n(\alpha) \), respectively. If constraint function \( f(x, \xi_1, \xi_2, \ldots, \xi_n) \) is strictly monotonically increasing with respect to \( \xi_1, \xi_2, \ldots, \xi_n \), and strictly decreasing with respect to \( \xi_m, \xi_{m+1}, \ldots, \xi_n \) \( (m < n) \), then the chance constraint

\[
M[f(x, \xi_1, \xi_2, \ldots, \xi_n) \leq 0] \geq \alpha. \quad (7)
\]

is equivalent to

\[
f\left(x, \Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\right) \leq 0. \quad (8)
\]

## III. EXPECTED VALUE MODEL-BASED TASK ALLOCATION PROBLEM

### A. TASK ALLOCATION PROBLEM MODELING

The UAV attack task allocation problem refers to the process that UAVs depart from the base, attack all task targets, and then return to the base with the minimum cost under relevant constraints. It can be expressed as an uncertain integer programming problem on undirected complete graph \( G = (V^+, E^+) \). The following notations are used to describe the problem.

- \( m \): the number of UAVs.
- \( n \): the number of task targets.
- \( V^+ \): \( V^+ = V \cup O \), the set of both task targets and UAV base. The task targets \( V \) are numbered as \( 1, 2, 3, \ldots, n \), which can express as \( V = \{1, 2, 3, \ldots, n\} \). The UAV base \( O \) are numbered as \( 0 \).
- \( E^+ \): the flight path set of adjacent points. The flight path between \( i \) and \( j \) is \( (i,j) \in E^+ \) for \( i, j \in V^+ \).
- \( Y^k \): the \( k \)-th UAV attack task allocation sequence \( (l \leq m) \). \( Y^k \) indicates that the \( k \)-th UAV attacks task targets \( y_1, y_2, \ldots, y_l \) in turn and then returns to the base.
- \( T^k \): \( T^k = \{T_{y_1}^k, T_{y_2}^k, \ldots, T_{y_l}^k\} \) is the time of attacking target \( y_i \) by the \( k \)-th UAV in \( Y^k (l \leq m) \).
- \( D \): \( D = \{d_{ij}\}_{(l+1) \times (n+1)} \) is the distance matrix of both task targets and UAV base. \( d_{ij} \) is the distance from \( i \) to \( j (i,j \in V^+) \). If \( i = 1, d_{ij} \) is the distance from UAV base to target \( j - 1 \), otherwise \( d_{ij} \) is the distance from task target \( i - 1 \) to task target \( j \). The distance \( d_{ij} \) is related to the actual battlefield environment, enemy radar threat and other factors. And it can be calculated by route planning. Since route planning is not the focus of this paper, the distance \( d_{ij} \) is replaced by Euclidean distance between \( i \) and \( j \) for \( i, j \in V^+ \).

#### 1) DECISION VARIABLES

The task allocation problem is essentially to allocate a task attack sequence to each UAV. The objective decision variable is denoted as \( X \). If UAV attacks the target \( j \) directly after attacking task target \( i, x_{ij} = 1 \), otherwise \( x_{ij} = 0 \).

#### 2) OBJECTIVE FUNCTION

The objective function of the UAV task allocation problem includes fuel consumption cost and delay penalty. Due to the influence of flight altitude, speed, atmospheric disturbance and other uncertain factors, both the fuel consumption per unit flight distance \( e_{ij} \) and the flight time \( t_{ij} \) in the actual flight process are regarded as uncertain variables. Let \( e_{ij} \) and \( t_{ij} \) be a series of independent uncertain variables satisfying normal distribution \( N(\epsilon_1, \sigma_1) \) and \( N(\epsilon_2, \sigma_2) \), respectively. Therefore, the fuel consumption cost \( F_1 \) can be expressed by

\[
F_1 = \sum_{k=1}^{m} \sum_{(i,j) \in E} e_{ij}^k d_{ij} x_{ij} \quad (9)
\]

And the delay penalty \( F_2 \) can be expressed by

\[
F_2 = \sum_{k=1}^{m} \sum_{j \in Y^k} C_j^k, \quad (10)
\]

\[
C_j^k = \begin{cases} 0 & 0 \leq T_j^k \leq Time_j \\ \lambda (T_j^k - Time_j) x_{ij} & Time_j < T_j^k \end{cases}, \quad (11)
\]

\[
t_j^k = t_{0y_1} y_{0y_1} + \sum_{i=2}^{j} t_{y(i-1)y(i-1)y_{i}}, \quad (12)
\]

where \( C_j^k \) is the delay penalty of the \( k \)-th UAV, \( \lambda \) is the coefficient of the delay penalty, \( T_j^k \) is the time of attacking task target \( j \) by the \( k \)-th UAV, \( Time_j \) is the time window of the task target \( j \).

In conclusion, the objective function of the UAV attack task allocation problem is

\[
\min F = \lambda_1 F_1 + \lambda_2 F_2 = \lambda_1 \left( \sum_{k=1}^{m} \sum_{(i,j) \in E} e_{ij}^k d_{ij} x_{ij}^k \right) + \lambda_2 \left( \sum_{k=1}^{m} \sum_{i \in Y^k} C_j^k \right), \quad (13)
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the weight coefficient of fuel consumption cost and delay penalty respectively, and satisfy \( \lambda_1 + \lambda_2 = 1 \).
3) RELATIVE CONSTRAINT
One of the key points of solving the UAV attack task allocation problem is to reasonably convert constraints into mathematical expressions. Constraints mainly include the following three categories.

a: LIMITED FUEL
Limited by UAV performance, the amount of fuel carried by each UAV is certain. To ensure the UAV’s safe return to base, it must be satisfying

\[ \sum_{(i,j) \in E^+} \varepsilon_{ij}^k d_{ij} x_{ij}^k \leq A^k, \quad (14) \]

where \( A^k \) is the total amount of fuel carried by the \( k \)th UAV.

b: TASK COORDINATION
Each task target can be attacked only once in the UAV attack task allocation problem. The task coordination constraint can be expressed by

\[ \sum_{m} \sum_{k=1} \sum_{i \in V^+, i \neq j} x_{ij}^k = \sum_{m} \sum_{k=1} \sum_{i \in V^+, i \neq j} x_{ij}^k \leq 1, \quad \forall j \in V^+. \quad (15) \]

c: ROUND-TRIP CONSTRAINT
UAV starts its task from the base and eventually returns to the base, which can be expressed by

\[ \sum_{i \in V} x_{0i} = \sum_{j \in V} x_{j0} = 1. \quad (16) \]

Remark 1: In the objective function and constraints, the uncertainty distribution of uncertain variables \( \varepsilon_{ij} \) and \( t_{ij} \) can be obtained from expert’s experimental data.

B. MODEL TRANSFORMATION
Obviously, both objective function and inequality constraints contain uncertain variables. Therefore, the feasible solution set of the model cannot be determined and the optimization cannot be realized, which belongs to the uncertain programming problem.

Expected value is an important statistical feature of uncertain variable. In this paper, the expected value model is established to solve the UAV attack task allocation problem by introducing uncertain opportunity constraint and minimizing the expected value of objective function. The expected value model is defined by

\[
\begin{align*}
\min_x & \quad E \{ F(x, \xi) \} \\
\text{s.t.} & \quad M \{ G_i(x, \xi) \leq 0 \} \geq \alpha_i, \quad i = 1, 2, \ldots
\end{align*}
\]

(17)

where \( M \{ G_i(x, \xi) \leq 0 \} \geq \alpha_i \) is the uncertain opportunity constraint with \( \alpha_i \) reliability level.

Therefore, the expected value model of the UAV attack task allocation problem can be expressed as

\[
\begin{align*}
\min_x & \quad E \{ F(\xi) \} = E \left( \lambda_1 \left( \sum_{k=1}^m \sum_{(i,j) \in E} \varepsilon_{ij}^k d_{ij} x_{ij}^k \right) \\
& \quad + \lambda_2 \left( \sum_{k=1}^m \sum_{i \in V^+} C_i^k \right) \right) \\
\text{s.t.} & \quad \sum_{(i,j) \in E^+} \varepsilon_{ij}^k d_{ij} x_{ij}^k \leq A^k, \quad k = 1, 2, \ldots, m \\
& \quad \sum_{k=1}^m \sum_{i \in V^+, i \neq j} x_{ij}^k = \sum_{k=1}^m \sum_{i \in V^+, i \neq j} x_{ij}^k \leq 1, \quad \forall j \in V^+ \\
& \quad \sum_{i \in V} x_{0i} = \sum_{j \in V} x_{j0} = 1 \\
& \quad x_{ij} \in \{0, 1\}, \quad (i, j) \in E.
\end{align*}
\]

(18)

IV. A NOVEL IMPROVED FIREWORKS ALGORITHM
A. TRADITIONAL FIREWORKS ALGORITHM
Tan Ying [23] proposed fireworks algorithm to solve continuous space optimization problems, which inspired by the emergent swarm behaviour of fireworks. In the FWA, each fireworks is regarded as a feasible solution to optimization problem, searching by the sparks from firework explosion. FWA consists of explosion operator, variation operator, mapping rule and selection strategy. The specific steps of FWA are as follows.

1) INITIAL SOLUTION SPACE
Each fireworks represents a feasible task allocation scheme in the UAV attack task allocation problem. Let \( x \) is a fireworks, \( M \) is number of fireworks, and \( D \) is the number of task targets to ensure that each task target can be attacked only once. The initial solution space is initialized according to the number of fireworks, and the \( i \)th fireworks \( x_i \) can be calculated by

\[ x_i = x_{\min} + (x_{\max} - x_{\min}) \text{rand} (1, D), \quad (19) \]

where \( \text{rand} \) is random number generator.

2) EXPLOSION
Explosion is the core step in the FWA. The explosion amplitude and the spark number depend on the fitness of fireworks. The good fitness of fireworks makes amplitude smaller and generate more sparks. The optimal value can be found more quickly by strengthening local search. In contrast, the global search is strengthened, which enhances the diversity of solution space. The fireworks explosion amplitude and spark number are calculated by

\[ R_i = R \times \frac{f(x_i) - f_{\min} + \varepsilon}{\sum_{i=1}^N (f(x_i) - f_{\min}) + \varepsilon}, \quad (20) \]
Algorithm 1 Tent-Levy Fireworks Algorithm

(1) Initialize fireworks based on Tent chaotic mapping
(2) iter = 1
(3) if (iter < max_iter)
(4) for i = 1: N
(5) Calculate the explosion amplitude and the spark number by formula (20) and (20)
(6) Generate explosion sparks by formula (22)
(7) end for
(8) Generate variation sparks based on Levy variation by formula (32)
(9) Map sparks according to mapping rules which is out of bounds
(10) Calculate all fireworks and sparks fitness
(11) Selected next generation fireworks according to the selection strategy
(12) end if

$$S_{i} = \text{round} \left[ S \times \frac{f_{\text{max}} - f(x_{i}) + \varepsilon}{\sum_{i=1}^{N}(f_{\text{max}} - f(x_{i})) + \varepsilon} \right], \quad (21)$$

where $R$ and $S$ are the coefficient of explosion amplitude and explosion spark respectively. $R$ and $S$ are used to adjust the explosion radius and the spark number. $f_{\text{max}}$ and $f_{\text{min}}$ are the maximum and minimum fitness of all fireworks currently. $f(x_{i})$ is the fitness of firework $i$. $\varepsilon$ is a very small number, which avoids formulas with 0 in the denominator.

The sparks will shift within amplitude by the explosion, which can be calculated by

$$x'_{ik} = \text{round} \left[ x_{ik} + R \times \text{rand} \times (-1, 1) \right], \quad (22)$$

where $x_{ik}$ is the value of the $i$th firework in the $k$th dimension. To avoid overwhelming effects of extreme sparks, the number of sparks generated by each firework explosion is corrected as follows.

$$S_{i} = \begin{cases} S_{\text{min}} & S_{i} < S_{\text{min}} \\ S_{i} & S_{\text{min}} \leq S_{i} \leq S_{\text{max}} \\ S_{\text{max}} & S_{i} > S_{\text{max}}. \end{cases} \quad (23)$$

3) VARIATION
In order to increase the diversity of fireworks, the Gaussian variation is carried out fireworks, which can be expressed by

$$x'_{ik} = \text{round} \left( \alpha \times x_{ik} \right), \quad (24)$$

where $\alpha$ is a random number subject to Gaussian distribution $N(\varepsilon, \sigma)$.

4) MAPPING
In order to ensure that sparks generated by firework explosion and variation are in the feasible region, the modular operation is adopted in this paper. The formula is expressed by

$$x'_{j} = x_{\text{min}} + \text{mod}((x'_{j} - x_{\text{min}}). \quad (25)$$

where $x_{\text{max}}$ and $x_{\text{min}}$ are upper and lower bounds of sparks, respectively, and mod are absolute value operation and modular operation, respectively.

### TABLE 1. The expression and domain of chaotic mapping functions.

| Chaotic mapping | Expression | Domain |
|-----------------|------------|--------|
| ICMC            | $x_{ii} = \sin(a \times x_{i})$, $a \in (0, \infty)$ | $[-1, 1]$ |
| Chebyshev       | $x_{ii} = \cos(k \times \text{gcd} \times x_{i})$ | $[-1, 1]$ |
| Logistic        | $x_{ii} = \lambda x_{i} \times (1 - x_{i})$, $\lambda \in [0, 4]$ | $[0, 1]$ |
| Tent            | $x_{ii} = \left\{ \begin{array}{ll} x_{i}\times \beta & x_{i} \in [0, \beta] \\ (1-x_{i})/(1-\beta) & x_{i} \in (\beta, 1] \end{array} \right.$ | $[0, 1]$ |

5) SELECTION
The fireworks and sparks which produced by exploding and varying are the candidate set $K$ for the next generation of fireworks. The convergence speed of the algorithm can be accelerated theoretically by choosing optimal individual directly, whereas it may also cause the algorithm to fall into local optimal solution. To avoid this problem, roulette wheel strategy is introduced on the basis of elite strategy, which select the best firework by elite strategy. The remaining fireworks will be determined by roulette wheel strategy, together being the next generation of fireworks. This method not only ensures that high fitness individuals can be selected with a high probability, but also ensures that low fitness individuals have chance to be selected, which effectively avoids the algorithm falling into local optimal solution. The selection probability of candidate individual $x_{i}$ can be calculated by

$$P(x_{i}) = \frac{D(x_{i})}{\sum_{x_{i} \in K} D(x_{i})}, \quad (26)$$

$$D(x_{i}) = \sum_{x_{i}, x_{j} \in K} d(x_{i} - x_{j}) = \sum_{x_{i}, x_{j} \in K} \|x_{i} - x_{j}\|, \quad (27)$$

where $D(x_{i})$ is the sum of distances from the firework $x_{i}$ to all fireworks in the candidate set except itself.

### B. TENT-LEYE FIREWORKS ALGORITHM
Pholdee et al. [25] have proved that FWA has good optimization performance. However, there are problems of fireworks gathering and random search in the process of solving the UAV task allocation problem, which results in
slow convergence speed and trapped local optimal solution and so on. Therefore, TLFWA is proposed in this paper (Algorithm 1). The specific improvements of the algorithm are as follows.

1) INTEGER CODING
Traditional FWA can only solve the continuous space optimization problem due to its real number coding. However, the UAV task allocation problem has a discrete space optimization problem. To solve this problem by FWA, integer coding is used in this paper. Taking two UAVs attacking five task targets for example, the code (2, 1, 1, 2, 1) represents that UAV 1 attacks task target 2, 3, 5 and UAV 2 attacks task target 1, 4.

2) CHAOTIC MAPPING STRATEGY
The search efficiency and effectiveness of FWA depend on the diversity of initial task allocation schemes to some extent. The initial schemes have high diversity when they evenly distributed in the whole space. It can avoid the algorithm falling into local optimal solution and conducive to finding the optimal scheme quickly. However, the diversity of initial task allocation schemes generated by “rand” function in FWA is poor. Chaotic optimization is an optimization method with ergodicity, regularity and other characteristics. The diversity of the initial schemes can be enhanced by adding chaos into algorithm. Table 1 shows the frequently used chaotic mapping functions.

In order to display the characteristics of the four chaotic mapping functions more intuitively, chaotic mapping is iterated for 10000 times to generate chaotic mapping values. The range of each chaotic mapping function is divided into 100 intervals. The probability of chaotic mapping values falling into each interval is counted. Fig. 1 shows the numerical graph of the first 100 iteration values, and Fig. 2 shows the probability distribution.

It can be seen from Fig. 2 that the probability distribution curves of Logistic, Chebyshev and ICMIC chaotic mapping are multi-peak. The initial solutions after chaotic mapping will gather at the peak point, which is not conducive to optimization. However, the probability distribution curve of Tent chaotic mapping is almost a straight line. The initial solution after chaotic mapping will be evenly distributed in the range, which avoids falling into local optimization caused by excessive search in some local areas.

Therefore, Tent chaotic mapping is adopted in this paper to generate initial task allocation schemes, which can be calculated by

\[ x_i = x_{\text{min}} + (x_{\text{max}} - x_{\text{min}}) \cdot z_{n+1}, \]

\[ z_{n+1} = \begin{cases} z_n/0.6 & z_n \in (0, 0.6] \\ (1 - z_n) / (1 - 0.6) & z_n \in (0.6, 1]. \end{cases} \]

3) LEVY VARIATION OPERATOR
FWA carries out the explosion operation by Gaussian variation. However, Gaussian variation greatly reduces the variation efficiency due to its small perturbation and the variation of all real number range. It is not applicable to the task allocation problem. Fig. 3 shows the probability density function and distribution function of Gaussian distribution, Cauchy distribution and Levy distribution. Compared with Gaussian distribution and Cauchy distribution, Levy distribution has a higher wave peak, wider tail, and a positive real number range of variation. Therefore, the Levy variation with a large variation range and strong disturbance ability is used in this paper (see Fig. 3), which can increase the search scope and search efficiency. This is more conducive to finding the optimal task allocation scheme. The probability density function \( f (x, c) \) and distribution function \( F (x, c) \) of Levy distribution are as follows.

\[ f (x, c) = \sqrt{\frac{c}{2\pi x^2}} \exp \left( -\frac{c}{x} \right), \]

\[ F (x, c) = P \{ X \leq x \} = \int_0^x \sqrt{\frac{c}{2\pi x^2}} \exp \left( -\frac{c}{x} \right) dx, \]

where \( u \) is the position parameter that affects the left and right translation of the distribution curve. To ensure the domain of...
Levy distribution function is $[0, +\infty]$, $u = 0$. $c$ is the scale parameter, and the larger value is, the more scattered the distribution is. The probability density function and distribution function of different $c$ are shown in Fig. 4.

As shown in Fig. 4, when $c = 0.5$, Levy distribution has the highest wave peak and the widest tail, and the range of its variation is the biggest. However, the Levy distribution function is not easy to solve. The main methods to solve the random number subject to arbitrary distribution function are inverse transformation method and rounding method. The exact expression of the distribution function and its inverse function is the precondition for inverse transformation method. Therefore, rounding method, which is more widely applicable, is adopted in this paper. Based on the method of sampling rejection, rounding method achieves the goal of approaching the target distribution $q(x)$ by setting sampling distribution $p(x)$ with uniform, normal distribution and other distributions and rejecting some samples according to the specified rules. The specific steps for solving random numbers subject to Levy distribution $f(x)$ are as follows.
Step 1: Generate independent random numbers $X \sim U(x_{\min}, x_{\max})$ and $Y \sim U(y_{\min}, y_{\max})$ subject to uniform distribution.

Step 2: $X$ is a random number subject to Levy distribution if $Y \leq f(X)$; Otherwise, repeat step 1.

The spark variation formula of Levy variation operator is

$$x_{ik}' = \text{round} \left( \beta \times x_{ik} \right), \quad (32)$$

where $\beta$ is a random number subject to the Levy distribution.

V. RESULTS AND DISCUSSION

In this section, the classical benchmark functions in CEC2010 are selected to test the performance of TLFWA. Then the effectiveness and feasibility of the proposed model and TLFWA are verified by UAV task allocation simulation results in different scales.

A. VERIFICATION TESTS BY BENCHMARK FUNCTIONS

Table 2 shows the mathematical expressions and related properties of the six benchmark functions. Among them, Sphere, Schwefel_1.2 and Rosenbrock functions are unimodal functions with only one minimum, which mainly test the convergence speed and optimization accuracy. Ackley, Griewank and Rastrigin functions are multimodal functions, including multiple local minimums, which mainly test the global optimization performance and ability of jumping out the local minimum. Therefore, the optimization ability of TLFWA can be comprehensively inspected by the above six benchmark functions.
To better evaluate the effectiveness of TLFWA, this paper uses the above six benchmark functions to test it on different scales and compare it with traditional FWA and particle swarm optimization (PSO) algorithm. The parameter settings of PSO algorithm are as follows: the number of particles is 100, the individual learning factor is 1.5, the social learning factor is 1.5, the inertia weight is 0.7 and the maximum iteration number is 1000. The parameter settings of FWA and TLFWA are as follows: the number of fireworks is 5, the maximum explosive spark number is 40, the minimum explosive spark number is 2, the explosion radius coefficient is 40, the explosive spark coefficient is 50, the variable spark number is 5, and the maximum iteration number is 1000.

MATLAB R2018a software is used for simulation in this paper. In order to reduce errors, the optimal fitness obtained by running 50 times is taken as the test result. The fitness...
TABLE 3. Comparison of benchmark functions’ fitness and variances under different algorithms.

| Function    | Dimension | PSO mean | PSO standard deviation | FWA mean | FWA standard deviation | TLFWA mean | TLFWA standard deviation |
|-------------|-----------|----------|------------------------|----------|------------------------|------------|-------------------------|
| Sphere      | 10        | 7.7E-5   | 3.33E-5                | 0        | 0                      | 0          | 0                       |
|             | 30        | 1.34E-2  | 3.04E-3                | 0        | 0                      | 0          | 0                       |
|             | 50        | 1.03E-1  | 5.63E-2                | 0        | 0                      | 0          | 0                       |
| Schwefel_1.2| 10        | 5.09E-4  | 2.14E-4                | 0        | 0                      | 0          | 0                       |
|             | 30        | 8.34E-1  | 2.09E-1                | 0        | 0                      | 0          | 0                       |
|             | 50        | 18.39    | 3.66                   | 0        | 0                      | 0          | 0                       |
| Rosenbrock  | 10        | 7.11     | 1.82                   | 4.35     | 1.21E1                 | 3.97       | 1.21E1                  |
|             | 30        | 4.99E1   | 3.32E1                 | 1.65E1   | 1.26E1                 | 1.41E1     | 1.23E1                  |
|             | 50        | 1.41E2   | 5.79E1                 | 3.07E1   | 2.25E1                 | 2.77E1     | 2.06E1                  |
| Ackley      | 10        | 1.12E-2  | 2.19E-3                | 8.88E-16 | 0                      | 8.88E-16   | 0                       |
|             | 30        | 2.84     | 3.19E-1                | 8.88E-16 | 0                      | 8.88E-16   | 0                       |
|             | 50        | 4.3      | 3.61E-1                | 8.88E-16 | 0                      | 8.88E-16   | 0                       |
| Griewank    | 10        | 2.9E-1   | 6.7E-2                 | 2.39E-16 | 1.57E-16               | 5.07E-16   | 6.77E-16                |
|             | 30        | 5.98E-2  | 3.32E-2                | 3.37E-16 | 3.29E-16               | 3.13E-16   | 2.44E-16                |
|             | 50        | 9.3E-1   | 2.18E-1                | 6.63E-16 | 1.32E-15               | 3.33E-16   | 3.14E-16                |
| Rastrigin   | 10        | 4.07     | 7.94E-1                | 4.42E-15 | 4.25E-15               | 3.59E-15   | 2.92E-15                |
|             | 30        | 3.63E1   | 4.63                   | 4.53E-15 | 5.09E-15               | 3.1E-15    | 2.13E-15                |
|             | 50        | 8.24E1   | 1.03E1                 | 4.11E-15 | 4.72E-15               | 3.94E-15   | 3.35E-15                |

TABLE 4. The parameters of targets.

| Target number | Time window /min | Coordinate /km |
|---------------|------------------|----------------|
| 1             | [0,60]           | [8,47]         |
| 2             | [0,50]           | [22,40]        |
| 3             | [0,100]          | [45,75]        |
| 4             | [0,40]           | [90,60]        |
| 5             | [0,50]           | [17,82]        |
| 6             | [0,70]           | [120,55]       |
| 7             | [0,50]           | [107,42]       |
| 8             | [0,70]           | [100,75]       |
| 9             | [0,30]           | [60,30]        |
| 10            | [0,30]           | [75,20]        |
| 11            | [0,80]           | [110,23]       |
| 12            | [0,60]           | [50,5]         |
| 13            | [0,100]          | [105,13]       |
| 14            | [0,100]          | [50,22]        |

TABLE 5. Allocation result of different scenarios.

| Scenario | UAV number | Task sequence |
|----------|------------|---------------|
| 1        | 1          | 0→2→1→5→3→0 |
|          | 2          | 0→4→7→6→0   |
| 2        | 1          | 0→2→1→5→3→0 |
|          | 2          | 0→4→8→6→0   |
|          | 3          | 0→9→14→12→0 |
|          | 4          | 0→10→7→11→13→0 |

Remark 2: The mean of Sphere and Schwefel_1.2 function’s fitness obtaining by FWA and TLFWA are less than 1E-16, which may be caused by value rounding error or truncation error, so the value is approximated to zero.

Remark 3: E is scientific notation.

In summary, TLFWA, which introduced integer coding, Tent chaotic mapping and Levy variation operator, improves its convergence speed and the ability to jump out of local optimal solution. TLFWA is superior to the traditional FWA and PSO algorithm.

B. UAV TASK ALLOCATION TEST

In order to verify the effectiveness and feasibility of the proposed model and TLFWA, UAV task allocation scenarios with different scales are designed in this paper. Scenario 1: two UAVs attack seven task targets. Scenario 2: four UAVs attack fourteen task targets. The uncertain variables $\epsilon_{ij}$ and $t_{ij}$ satisfy normal distribution $N(1, 2)$ and $N(1, 1)$, respectively. $\lambda_1 = 0.6$, $\lambda_2 = 0.4$, $\alpha = 0.85$. To facilitate the analysis, UAVs and task targets are regarded as particle, and communication constraints between UAVs are not considered in the task allocation process. In the Fig. 11, the first and second coordinate of task target are horizontal coordinate and vertical coordinate, which represent the position information of task target. And the third coordinate of task target is the time limit. TLFWA is used to solve the two UAV task allocation scenarios, and the results are as shown in Table 5, Fig. 12-13.
TABLE 6. The cost of each algorithm in different scenarios.

| Scenario | Algorithm | Minimum | Mean  | Variance | Optimal number | Optimal rate | Convergence algebra |
|----------|-----------|---------|-------|----------|----------------|--------------|---------------------|
| scenario 1 | PSO       | 287.7   | 287.7 | 0        | 30             | 100%         | 10                  |
|          | FWA       | 287.7   | 289.4 | 27.9     | 27             | 90%          | 20                  |
|          | TLFWA     | 287.7   | 287.7 | 0        | 30             | 100%         | 9                   |
| scenario 2 | PSO       | 424.3   | 487.2 | 1791.5   | 0              | 0            | 92                  |
|          | FWA       | 414.1   | 457.7 | 1335.3   | 6              | 20%          | 158                 |
|          | TLFWA     | 414.1   | 420.3 | 160.1    | 22             | 73.3%        | 64                  |

FIGURE 11. Spatial distribution of targets and UAV base.

FIGURE 12. Schematic diagram of allocation results for Scenario 1.

UAVs may abandon the relatively closest task targets due to the time limit. As shown in Fig. 12, UAV 2 directly attacks the task target 7 rather than the closest task target 6 after attacking task target 4. This is because the delay penalty is higher than fuel consumption cost if UAV attacks the closest task target 6. Therefore, UAV task allocation results obtained by TLFWA are feasible. To further illustrate the rationality and advantages of TLFWA in solving the UAV task allocation problem, compare it with traditional FWA and PSO algorithm. The results are analyzed from the aspects of stability and convergence.

1) STABILITY ANALYSIS
Parameter settings of the three algorithms are the same as section 5.1. Each algorithm runs independently for 30 times. The maximum iteration is 200. The task allocation results are shown in Table 6.

Table 6 shows that the minimum cost, mean cost and variance of different task allocation scenarios calculated by TLFWA are less than or equal to the traditional FWA and PSO algorithm. Taking scenario 2 for example, the mean cost calculated by TLFWA is 8.17% lower than that of FWA and 13.73% lower than that of PSO algorithm. FWA and PSO algorithm basically fall into the local optimal solution. TLFWA successfully jumps out of local optimal solution and obtains the optimal task allocation scheme. In conclusion, TLFWA can stably obtain feasible task allocation scheme with a small cost, and its stability is superior to FWA and PSO algorithm.

2) CONVERGENCE ANALYSIS
Parameter settings of the three algorithms are the same as section 5.1. The three algorithms are iterated 200 times in the same scene, and the convergence curves is shown in Fig. 14.

It can be seen from Fig. 14 that the costs obtained by the three algorithms can converge to a stable value. When the problem scale is small (scenario 1), all three algorithms can obtain the optimal task allocation scheme. With the increase of problem scale (scenario 2), TLFWA and FWA can obtain the optimal solution, whereas PSO algorithm falls into local
optimal solution. In addition, TLFWA has the fastest convergence speed. This is the inherent advantage of TLFWA. On the one hand, TLFWA uses Tent chaotic mapping, which increases the diversity of initial task allocation schemes. It greatly improves the search speed of the algorithm. On the other hand, Levy variation operator avoids the premature convergence of the algorithm by expanding the search range. Therefore, TLFWA has better convergence performance than PSO algorithm and FWA.

VI. CONCLUSION
This paper solves the Multi-objective UAV task allocation problem under uncertain environment, and the innovations are as follows.

(1) Based on the uncertainty theory, a new UAV task allocation model is constructed. The model effectively solves the UAV task allocation problem with uncertain variables in both objective function and constraints.

(2) Three improvements are proposed to make the algorithm better applied to the task allocation model. Firstly, integer coding is designed so that TLFWA can solve discrete domain optimization problems such as the UAV task allocation problem. Then, Tent chaotic mapping increases the diversity of initial task allocation schemes, which avoids falling into local optimal solution. Finally, Levy variation replaces Gaussian variation in the explosion operator, which improves the efficiency of solving the UAV task allocation problem by avoiding algorithm’s repeated search.

Experimental results show that the mean cost calculated by TLFWA is 8.17% and 13.73% lower than that of FWA and PSO algorithm respectively, which proves the effectiveness of TLFWA.

In the future research, the following three aspects would be concerned mainly. From the theoretical perspective, we consider using the optimistic value and pessimistic value criteria of uncertainty theory to solve the multi-objective problem directly. From the model perspective, the model is refined by considering the time-varying value of mission objectives, the change of battlefield threat, and the battle loss of UAV to make it closer to the battlefield reality. From the algorithm perspective, it is considered to design appropriate coding method and optimization mechanism algorithm to solve the task allocation problem more efficiently.

APPENDIX

TABLE 7. Abbreviated comparison table.

| Abbreviation | The full name                  |
|--------------|--------------------------------|
| UAV          | unmanned aerial vehicle        |
| FWA          | fireworks algorithm             |
| TLFWA        | Tent-Levy fireworks algorithm   |
| PSO          | particle swarm optimization     |
| CEC2010      | 2010 IEEE Congress on Evolutionary Computation |

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