Analytical estimations for statistical characteristics of Casimir field detected by indirect measurement scheme

Sheremetyev V.O., Triefanov A.I.
ITMO University, Kronverksky, d9, Saint-Petersburg, 197101, Russia
E-mail: vlad.sheremetiev@gmail.com, alextrifanov@gmail.com

Abstract. We investigate the process of Casimir field indirect photodetection. The protocol of measurement is implemented via interaction between two-level atom and cavity. We evaluated basic statistical characteristics of conditional state of Casimir field.

1. Introduction
Dynamical Casimir Effect (DCE) is the process of photon creation from vacuum due to the cavity boundary movement or medium physical parameters changing (e.g. dielectric constant). This effect is based on the parametric process of vacuum fluctuations amplification [1, 2, 3].

Earlier the main difficulty of DCE experimental verification was the very high frequency variation of moving boundaries [4]. Experimental confirmations of DCE was obtained by means of the superconductive scheme [5]. Here authors realized the idea of imitation of boundary moving [6, 7]).

In this paper, we investigate the process of electromagnetic field photodetection, generated from DCE. In particular, we are interested in indirect measurement scheme implemented via interaction between two-level atom and cavity electromagnetic field. To analyze statistical properties of DCE field we use a two-level atom-pointer passing through the cavity. After the interaction atomic state is detected in ionizing chamber. Examination of detection result allows us to determine properties of cavity field conditional state. Through repeating the interaction many times we can investigate the quantum trajectory for the field state using atomic state detection statistics. Using this ideas we evaluated analytically basic statistical characteristics of conditional state of DCE field.

2. The model
A composite mirror changes its reflection properties (under intermittent laser light irradiation) in empty cavity for periodic movement imitation. It is used for field generation in GHz frequency range [7]. Nonadiabatic change of cavity spectrum produces real photons with frequencies determined by parametric resonance conditions. We assume that the main contribution to photon production is given by squeezing process and relativistic effect of intermode interaction due to mirror’s accelerated movement is neglected. We investigate the generate regime of photon creation which allows us to reduce the problem to interaction of atom with single mode of intracavity electromagnetic field. We describe the interaction between the atom and the field by the following...
Hamiltonian [8]:
\[ \hat{H} = x\hat{a}^\dagger \hat{a} + \frac{\Delta + x}{2} \hat{\sigma}_z + i \frac{\epsilon \nu}{4} (\hat{a}^\dagger^2 - \hat{a}^2) + g \left( \hat{a}^\dagger \hat{\sigma}_+ + \hat{a} \hat{\sigma}_- \right), \]  
(1)

where \( \hat{a} \) is the field annihilation operator; \( g \) is the atom-field coupling constant (assumed real); \( \epsilon \nu \) is the effectiveness of generation process; \( x \) is the resonance shift between modulation frequency and cavity frequency; \( \Delta \) is the atom-field detuning; \( \hat{\sigma}_- = |g\rangle \langle e| = \hat{\sigma}_+^\dagger \) are Pauli operators, \( |g\rangle \) and \( |e\rangle \) are atomic ground and excited states; \( \hat{\sigma}_z = |e\rangle \langle e| - |g\rangle \langle g| \).

Applying Bogolyubov transformation of (1) with squeezing operator \( \hat{S} = \exp \left[ \frac{1}{2} (\beta^* \hat{a}^2 - \beta \hat{a}^2) \right] \),
where
\[ \beta = r e^{i\theta}, \quad r = \frac{1}{2} \arctan \frac{\epsilon \nu}{2x}, \quad \theta = \pi/2, \]  
(2)
we get the following form of Hamiltonian \( \hat{H}_B = \hat{S}^\dagger \hat{H} \hat{S} \):
\[ \hat{H}_B = \mu \left( \hat{a}^\dagger \hat{a} + 1 \right) + \frac{\alpha}{2} \hat{\sigma}_z + g \left[ (\hat{a} \cosh(r) - i\hat{a}^\dagger \sinh(r)) \hat{\sigma}_+ + (\hat{a}^\dagger \cosh(r) + i\hat{a} \sinh(r)) \hat{\sigma}_- \right]. \]  
(3)

Here \( \alpha = \Delta + x \) and
\[ \mu = \sqrt{\frac{4x^2 - (\epsilon \nu)^2}{2}}. \]

From (2) condition of validity of this unitary transformation was obtained: \( |\epsilon \nu / 2x| < 1 \).

Also we assume resonance regime \( |\mu - \alpha| \ll \mu, \alpha \) in system (3). Writing (3) in interaction picture with operators \( \exp \left( i\mu (\hat{a}^\dagger \hat{a} + 1/2)t \right) \) and \( \exp \left( i\alpha \hat{\sigma}_z t \right) \) and applying RWA we reach the following Hamiltonian in the form of Jaynes-Cummings:
\[ H_{JC} = g \cosh(r) (\hat{a}^\dagger \hat{\sigma}_+ e^{it(\mu-\alpha)} + \hat{\sigma}_+ \hat{a} e^{-i(t(\mu+\alpha))}). \]  
(4)

From here we can obtain analytical expressions for operator of vacuum state conditional evolution. It can be done by expanding evolution operator in basis of atomic states. Then we solve system of operator-valued differential equation and get expressions for case of the atom prepared in its ground state and detected in excited state. This result for operator of vacuum state conditional evolution is given by
\[ U_{ge}(t) = -i \frac{(g \cosh(r)) e^{-i\delta t/2}}{\sqrt{\delta^2/4 + (g \cosh(r))^2}} \sin \left( t\sqrt{\delta^2/4 + (g \cosh(r))^2} a a^\dagger \right) a, \]  
(5)
where \( \delta = \mu - \alpha \).

Operators \( U_{ge} \) have the following physical interpretation: they map initial state \( |\phi(0)\rangle \) of intracavity field onto the final state \( |\phi(t)\rangle \), conditioned by detection result \( e \). This final state however is not normalized and full transformation is
\[ |\varphi(t)\rangle = \frac{U_{ge}(t) |\phi(0)\rangle}{P_e(t)}, \quad P_e(t) = \langle \varphi_e(t) | \varphi_e(t) \rangle. \]  
(6)

Here \( P_e(t) \) is the probability to detect the atom in state \( |e\rangle \) after interaction lasted time \( t \).

we will consider the system under resonance conditions \( \delta = 0 \).
3. Analytical results

From (3-5) we can evaluate basic statistical properties of DCE field quantum state, specifically:

probability to detect atom in its excited state

\[ P_e(t) = \langle \tilde{\phi}_e(t) | \tilde{\phi}_e(t) \rangle = \sum_m |\langle 0 | S|m \rangle|^2 \sin^2 \left( g t \sqrt{m} \right) ; \quad (7) \]

mean number of produced photons

\[ \langle \hat{n}(t) \rangle = \langle \tilde{\phi}_e(t) | \tilde{a}^\dagger \tilde{a} | \tilde{\phi}_e(t) \rangle = \sum_m |\langle 0 | S|m \rangle|^2 \sin^2 \left( g t \sqrt{m} \right) (m-1) ; \quad (8) \]

probability distribution for mean photon number

\[ P_e(n) = |\langle 0 | \hat{S}^\dagger U_{ge} \hat{S} | n \rangle|^2 = \sum_m \langle 0 | S^\dagger | m \rangle \frac{\sin^2 \left( g t \sqrt{m} \right)}{\sqrt{m}} \sqrt{m+1} \langle m+1 | S | n \rangle^2 ; \quad (9) \]

dispersions of field quadratures

\[ \langle (\Delta \hat{X})^2 \rangle = \frac{1}{2} \left[ \sum_m \langle 0 | S^\dagger | m \rangle M \langle m | S | 0 \rangle + \sum_m |\langle 0 | S|m \rangle|^2 \sin^2 \left( g t \sqrt{m} \right) (2m) \right] , \quad (10) \]

where

\[ M = \frac{\sin \left( g t \sqrt{m+2} \right) \sin \left( g t \sqrt{m} \right)}{\sqrt{m+2}} \frac{\sin \left( g t \sqrt{m} \right)}{\sqrt{m}} \left( m \sqrt{(m+1)(m+2)} + (m-2) \sqrt{m(m-1)} \right) ; \]

\[ \langle (\Delta \hat{P})^2 \rangle = -\frac{1}{2} \left[ \sum_m \langle 0 | S^\dagger | m \rangle M \langle m | S | 0 \rangle - \sum_m |\langle 0 | S|m \rangle|^2 \sin^2 \left( g t \sqrt{m} \right) (2m) \right] , \quad (11) \]

where

\[ \langle n | S | m \rangle = \left( \frac{e^{i \theta} \tanh r}{\sqrt{2} \sqrt{n! \cosh r}} \right)^n \exp \frac{e^{-i \theta} m^2 \tanh r - m^2}{2} H \left( n, \frac{me^{-i \theta}}{\sqrt{2} \cosh r \sinh r} \right) . \quad (12) \]

Then we present the results followed from the numerical analysis and comparison of evolution generated by Hamiltonian (1) and expressions (7 – 11). The system parameters were chosen in such a way to meet conditions which gives by transformations leading from (1) to (4). Also we have chosen parameters so as to reach the full resonance (|\mu - \alpha| = 0). Namely, the following values of parameters are used:

\[ \epsilon \nu = 1.37, \quad \Delta = -0.29, \quad g = 0.7 , \quad (13) \]

then \( x = -((\epsilon \nu)^2 + 4 \Delta^2)) / 8 / \Delta = 0.954 \).

In first figure the probability to detect atom in excited state is shown: we see what probabilities corresponding with (1) and (4) are close in start of interaction between atom and field. In second figure we show the mean cavity photon number. These quantities, conditioned by detection result, are close in the same period of time. For the instant of time \( gt \approx 1 \) the mean photon number probability distributions are presented in Fig.2. These distributions contain the nonzero probability only for odd numbers of photons. The matter is that in the absence of atom the intracavity field is in squeezed vacuum state (which has the distribution with only even photons). When atom is found in its excited state, the photon number in cavity decreases by one. We can see that probabilities to find the certain number of photons are almost identical.

In Fig.3 field quadratures are shown. These quantities describe the nonclassicality of the generated intracavity field interacted with atom during certain time interval. From here it is easy to see that statistical properties of the field change dramatically [9] due to the interaction with atom. In the origin of this process quadrature of momentum decreases to the values, corresponded to squeezed state. Quadratures of momentums and quadratures of coordinates are well coincide in the certain period of time.
Figure 1. Probability (a) and mean photon number (b) as functions of dimensionless time $gt$; dash line corresponds to analytical analysis, solid line corresponds to numerical analysis.

Figure 2. Photon number probability distributions for $gt \approx 1$; (a) corresponds to numerical analysis, (b) corresponds to analytical analysis.

Figure 3. Field quadratures: $\langle (\Delta X)^2 \rangle$ (a), $\langle (\Delta P)^2 \rangle$ (b); dash line corresponds to analytical analysis, solid line corresponds to numerical analysis.

4. Conclusion
As a result, we have evaluated basic statistical properties of DCE field quantum state. These expressions are used to research evolution of electromagnetic field state and, specifically, to demonstrate non-classical properties of Casimir field.
Acknowledgments
This work was partially financially supported by the Government of the Russian Federation (grant 074-U01), by Ministry of Education and Science of the Russian Federation (GOSZADANIE 2014/190, Project 14.Z50.31.0031 and ZADANIE No. 1.754:2014/K), by grant of Russian Foundation for Basic Researches and grant of the President of Russia (MK-2736.2015.2).

References
[1] Law C K 1994 Phys. Rev. A 49 433-537
[2] Dodonov V V and Klimov A B 1996 Phys. Rev. A 53 2664-2682
[3] Schötzhold R, Plunien G and Soff G 1998 Phys. Rev. A 57 2311-2318
[4] Dodonov V V 2010 Physica Scripta 82 038105
[5] Wilson C M et al. 2011 Nature 479 376-379
[6] Yablonovitch E 1989 Phys. Rev. Lett. 62 1742-1745
[7] Braggio C et al. 2005 Europhys. Lett. 70 (6) 754-760
[8] Dodonov V V 2012 Phys. Rev. A 85 2311-2318
[9] Dodonov V V 2011 Physics Letters A 375 4261-4267
[10] Trifanov A I and Miroshnichenko G P 2013 Nanosystems: Phys., Chem., Math. 4 (5) 635-647
[11] Clerk A A 2010 Rev. Mod. Phys. 82 1155-1208
[12] Nation P D 2012 Rev. Mod. Phys. 84
[13] Lamoreaux S K 1997 Phys. Rev. Lett. 78 5-8
[14] Sheremetyev V O, Trifanov A I and Trifanova E S 2014 J. Phys.: Conf. Series 541 012105