Radiation-induced magnetoresistance oscillation in a two-dimensional electron gas in Faraday geometry

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Microwave-radiation induced giant magnetoresistance oscillations recently discovered in high-mobility two-dimensional electron systems are analyzed theoretically. Multiphoton-assisted impurity scatterings are shown to be the primary origin of the oscillation. Based on a theory which considers the interaction of electrons with electromagnetic fields and the effect of the cyclotron resonance in Faraday geometry, we are able not only to reproduce the correct period, phase and the negative resistivity of the main oscillation, but also to predict the secondary peaks and additional maxima and minima observed in the experiments. These peak-valley structures are identified to relate respectively to single-, double- and triple-photon processes.

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The discovery of a new type of giant magnetoresistance oscillations in a high mobility two-dimensional (2D) electron gas (EG) subject to crossed microwave (MW) radiation and a magnetic field, especially the observation of "zero-resistance" states developed from the oscillation minima, has revived tremendous interest in magneto-transport in 2D electron systems. These radiation-induced oscillations of longitudinal resistivity $R_{xx}$ are accurately periodical in inverse magnetic field $1/B$ with period determined by the MW frequency $\omega$ rather than the electron density $N_e$. The observed $R_{xx}$ oscillations exhibit a smooth magnetic-field variation with resistivity maxima at $\omega/\omega_c = j + \delta_+$ and minima at $\omega/\omega_c = j - \delta_-$ ($\omega_c$ is the cyclotron frequency, $j = 1, 2, 3, ...$) having positive $\delta_+$ around $1/4$. The resistivity minimum goes downward with increasing sample mobility and/or increasing radiation intensity until a "zero-resistance" state shows up, while the Hall resistivity keeps the classical form $R_{xy} = B/N_e e$ with no sign of quantum Hall plateau over the whole magnetic field range exhibiting $R_{xx}$ oscillation.

To explore the origin of the peculiar "zero-resistance" states, different mechanisms have been suggested. It is understood that the appearance of negative longitudinal resistivity or conductivity in a uniform model suffices to explain the observed vanishing resistance. The possibility of absolute negative photoconductance in a 2DEG subject to a perpendicular magnetic field was first explored 30 years ago by Ryzhii. Recent works indicated that the periodical structure of the density of states (DOS) of the 2DEG in a magnetic field and the photon-excited electron scatterings are the origin of the magnetoresistance oscillations. Durst et al. presented a microscopic analysis for the conductivity assuming a $\delta$-correlated disorder and a simple form of the 2D electron self-energy oscillating with the magnetic field, obtaining the correct period, phase and the possible negative resistivity. Shi and Xie reported a similar result using the Tien and Gorden current formula for photon-assisted coherent tunneling. In these studies, however, the magnetic field is to provide an oscillatory DOS only and the high frequency (HF) field enters as if there is no magnetic field or with a magnetic field in Voigt configuration. The experimental setup requires to deal with the magnetic field $B$ perpendicular to the HF electric field. In this Faraday configuration, the electron moving due to HF field, experiences a Lorentz force which gives rise to additional electron motion in the perpendicular direction. In the range of $\omega \sim \omega_c$, the electron velocities in both directions are of the same order of magnitude and are resonantly enhanced. This cyclotron resonance (CR) of the HF current response will certainly change the way the photons assist the electron scattering.

In this Letter, we construct a microscopic model for the interaction of electrons with electromagnetic fields in Faraday geometry. The basic idea is that, under the influence of a spatially uniform HF electric field, the center-of-mass (CM) of the whole 2DEG in a magnetic field performs a cyclotron motion modulated by the HF field of frequency $\omega$. In an electron gas having impurity and/or phonon scatterings, there exist couplings between this CM motion and the relative motion of the 2DEG. It is through these couplings that a spatially uniform HF electric field affects the relative motion of electrons by opening additional channels for electron transition between different states. Based on the theory for photon-assisted magnetotransport developed from this physical idea, we show that the main experimental results of the radiation-induced magnetoresistance oscillations can be well reproduced. We also obtain the secondary peaks and additional maxima and minima observed in the experiments.

For a general treatment, we consider $N_e$ electrons in a unit area of a quasi-2D system in the $x$-$y$ plane with a confining potential $V(z)$ in the $z$-direction. These electrons, besides interacting with each other, are scattered by random impurities/disorders and by phonons in the lattice. To include possible elliptically polarized MW illumination we assume that a uniform dc electric field $E_0$ and ac field $E = E_s \sin(\omega t) + E_c \cos(\omega t)$ of frequency $\omega$ are applied in the $x$-$y$ plane, together with a mag-
the vector potential of the 2D CM momentum and coordinate of the electron system,\textsuperscript{17,18,19} which are defined as $P = \sum_j p_j$ and $R = N_{e-1} \sum_j r_j$ with $p_j = (p_{jx}, p_{jy})$ and $r_j = (x_j, y_j)$ being the momentum and coordinate of the $j$th electron in the 2D plane, and the relative electron momentum and coordinate $\mathbf{p}'_{j} = \mathbf{p}_{j} - \mathbf{P}/N_{e}$ and $\mathbf{r}'_{j} = \mathbf{r}_{j} - \mathbf{R}$, the Hamiltonian of the system can be written as the sum of the 2D CM momentum and coordinate of the electron and phonons, together with couplings of electrons to impurities and the Coulomb interaction. Note that although in the time-dependent part of the Hamiltonian, the effect of higher harmonic current is safely negligible for the determination of the CM coordinate and velocity, consists of a dc part $v_0$ and a stationary time-dependent part that

$$\mathbf{V}(t) = v_0 + v_1 \cos(\omega t) + v_2 \sin(\omega t).$$

This time-dependent CM velocity enters all the operator equations having couplings to impurities and/or phonons in the form of the following exponential factor, which can be expanded in terms of Bessel functions $J_n(x)$:

$$e^{-i\mathbf{q}\cdot\mathbf{V}(s)\mathbf{ds}} = \sum_{n=-\infty}^{\infty} J_n^2(\xi)e^{i(\mathbf{q} \cdot \mathbf{v}_0 - n\omega)(t-t')} + \sum_{m \neq 0} e^{im(\omega t - \varphi)} \sum_{n=-\infty}^{\infty} J_n(\xi)J_{n-m}(\xi)e^{i(\mathbf{q} \cdot \mathbf{v}_0 - n\omega)(t-t')} \cdot$$

Here $\xi \equiv \sqrt{(|\mathbf{q}|^2 \cdot \mathbf{v}_3)^2 + (\mathbf{q} \cdot \mathbf{v}_2)^2}/\omega$ and $\tan \varphi = (\mathbf{q} \cdot \mathbf{v}_2)/(\mathbf{q} \cdot \mathbf{v}_1)$. On the other hand, for 2D systems having electron sheet density of order of $10^{15}$ m$^{-2}$, the intra-band and inter-band Coulomb interactions are sufficiently strong that it is adequate to describe the relative-electron transport state using a single electron temperature $T_e$. Except this, the electron-electron interaction is treated only in a mean-field level under random phase approximation (RPA).\textsuperscript{17,18,19} For the determination of unknown parameters $v_0$, $v_1$, $v_2$, and $T_e$, it suffices to know the damping force up to the base frequency oscillating term $\mathbf{F}(t) = \mathbf{F}_0 + \mathbf{F}_s \sin(\omega t) + \mathbf{F}_e \cos(\omega t)$, and the energy-related quantities up to the time-average term. We finally obtain the following force and energy balance equations:

$$0 = N_e e \mathbf{E}_0 + N_e e (\mathbf{v}_0 \times \mathbf{B}) + \mathbf{F}_0,$$

$$\mathbf{v}_1 = \frac{\varepsilon \mathbf{E}_s}{m \omega} + \frac{\varepsilon \mathbf{F}_s}{N_e m \omega} - \frac{e}{m \omega}(\mathbf{v}_2 \times \mathbf{B}),$$

$$-\mathbf{v}_2 = \frac{\varepsilon \mathbf{E}_e}{m \omega} + \frac{\varepsilon \mathbf{F}_e}{N_e m \omega} - \frac{e}{m \omega}(\mathbf{v}_1 \times \mathbf{B}),$$

$$N_e e \mathbf{E}_0 \cdot \mathbf{v}_0 + S_p - W = 0.$$

Here

$$\mathbf{F}_0 = \sum_{\mathbf{q}_i} |U(\mathbf{q}_i)|^2 \sum_{n=-\infty}^{\infty} \sum_{\mathbf{q}_j} J_n^2(\epsilon \mathbf{q}_i)\Pi_2(\mathbf{q}_i, \omega_0 - n\omega),$$

$$+ \sum_{\mathbf{q}} |M(\mathbf{q})|^2 \sum_{n=-\infty}^{\infty} \sum_{\mathbf{q}_j} J_n^2(\epsilon \mathbf{q}_j)\Lambda_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}} - n\omega)$$

is the time-averaged damping force, $S_p$ is the time-averaged rate of the electron energy-gain from the HF field, $\frac{1}{2}N_e e (\mathbf{E}_s \cdot \mathbf{v}_2 + \mathbf{E}_e \cdot \mathbf{v}_1)$, which can be written in a form obtained from the right hand side of Eq. by replacing the $\mathbf{q}_i$ factor with $n\omega$, and $W$ is the time-averaged rate of the electron energy-loss due to coupling with phonons, whose expression can be obtained from the second term on the right hand side of Eq. by replacing the $\mathbf{q}_i$ factor with $\Omega_{\mathbf{q}}$, the energy of a wavevector-$\mathbf{q}$ phonon. The oscillating frictional force amplitudes $\mathbf{F}_s = \mathbf{F}_{22} - \mathbf{F}_{11}$ and $\mathbf{F}_e = \mathbf{F}_{21} + \mathbf{F}_{12}$ are given by ($\mu = 1, 2$)

$$\mathbf{F}_{1\mu} = -\sum_{\mathbf{q}_i} \eta_{\mu}|U(\mathbf{q}_i)|^2 \sum_{n=-\infty}^{\infty} [J_n^2(\epsilon \mathbf{q}_i)]^\mu \Pi_1(\mathbf{q}_i, \omega_0 - n\omega),$$

$$-\sum_{\mathbf{q}} \eta_{\mu}|M(\mathbf{q})|^2 \sum_{n=-\infty}^{\infty} [J_n^2(\epsilon \mathbf{q})]^\mu \Lambda_1(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}} - n\omega),$$

$$\mathbf{F}_{2\mu} = \sum_{\mathbf{q}_i} \eta_{\mu}|U(\mathbf{q}_i)|^2 \sum_{n=-\infty}^{\infty} 2nJ_n^2(\epsilon \mathbf{q}_i)\Pi_2(\mathbf{q}_i, \omega_0 - n\omega),$$

$$+ \sum_{\mathbf{q}} \eta_{\mu}|M(\mathbf{q})|^2 \sum_{n=-\infty}^{\infty} 2nJ_n^2(\epsilon \mathbf{q})\Lambda_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}} - n\omega).$$

In these expressions, $\eta_{\mu} = q_{\mu} / \omega \xi; \omega_0 = q_{\mu} \cdot v_0; U(\mathbf{q}_i)$ and $M(\mathbf{q})$ stand for effective impurity and
phonon scattering potentials, $\Pi_2(q_\parallel, \Omega)$ and $\Lambda_2(q, \Omega) = 2\Pi_2(q_\parallel, \Omega)[n(\Omega q/T) - n(\Omega/T_0)]$ (with $n(x) = 1/(e^{x} - 1)$) are the imaginary parts of the electron density correlation function and electron-phonon correlation function in the presence of the magnetic field. $\Pi_1(q_\parallel, \Omega)$ and $\Lambda_1(q, \Omega)$ are the real parts of these two correlation functions. The effect of interparticle Coulomb interactions are included in them to the degree of level broadening and RPA screening.

The HF field enters through the argument $\xi$ of the Bessel functions in $F_0$, $F_{\mu \nu}$, $W$ and $S_p$. Compared with that without the HF field ($n = 0$ term only) we see that in an electron gas having impurity and/or phonon scattering (otherwise homogeneous), a HF field of frequency $\omega$ opens additional channels for electron transition: an electron in a state can absorb or emit one or several photons and scattered to a different state with the help of impurities and/or phonons. The sum over $|n| \geq 1$ represents contributions of single and multiple photon processes of frequency-$\omega$ photons. These photon-assisted scatterings help to transfer energy from the HF field to the electron system ($S_p$) and give rise to additional damping force on the moving electrons. Note that $v_1$ and $v_2$ always exhibit CR in the range $\omega \sim \omega_c$, as can be seen from Eqs. A3 and A4 rewritten in the form

$$v_1 = (1 - \omega_c^2/\omega^2)^{-1} \left\{ \frac{e}{m\omega} \left[ E_x + \frac{e}{m\omega} (E_x \times B) \right] \right\}, \quad (9)$$

$$v_2 = (\omega_c^2/\omega^2 - 1)^{-1} \left\{ \frac{e}{m\omega} \left[ E_x - \frac{e}{m\omega} (E_x \times B) \right] \right\} + \frac{1}{N_e m\omega} \left\{ F_x - \frac{e}{m\omega} (F_x \times B) \right\}. \quad (10)$$

Therefore, $\xi$ may be significantly different from the argument of the corresponding Bessel functions in the case with no magnetic field or with a magnetic field in Voigt configuration $\alpha$.

Eqs. A3-A7 can be used to describe the transport and optical properties of magnetically-biased quasi-2D semiconductors subject to a dc field and a HF field. Taking $v_0 = (v_{0x}, 0, 0)$ in the $x$ direction, Eq. A4 yields transverse resistivity $R_{xy} = E_{0y}/N_e e v_{0x} = B/N_e$, and longitudinal resistivity $R_{xx} = E_{0x}/N_e e v_{0x} = -F_0/N_e^2 e^2 v_{0x}$. The linear magnetoresistivity is then

$$R_{xx} = - \sum_{q_\parallel} q_\parallel^2 |U(q_\parallel)|^2 \sum_{n = -\infty}^{\infty} J_0^2(\xi) \frac{\partial \Pi_2}{\partial \Omega} \bigg|_{\Omega = m\omega}$$

$$- \sum_{q_\parallel} q_\parallel^2 |M(q_\parallel)|^2 \sum_{n = -\infty}^{\infty} J_0^2(\xi) \frac{\partial \Lambda_2}{\partial \Omega} \bigg|_{\Omega = \Omega_0 + m\omega}. \quad (11)$$

The parameters $v_1$, $v_2$ and $T_e$ in (11) should be determined by solving equations A3, A6 and A7 with vanishing $v_0$. We see that although the transverse resistivity $R_{xy}$ remains the classical form, the longitudinal resistivity $R_{xx}$ can be strongly affected by the irradiation.

We calculate the unscreened $\Pi_2(q_\parallel, \Omega)$ function of the 2D system in a magnetic field by means of Landau representation:

$$\Pi_2(q_\parallel, \Omega) = \frac{1}{2\pi l_B^2} \sum_{n,n' \parallel} C_{n,n'}(l_B^2 q_\parallel^2/2)\Pi_2(n,n',\Omega), \quad (12)$$

$$\Pi_2(n,n',\Omega) = - \frac{2}{\pi} \int d\varepsilon [f(\varepsilon) - f(\varepsilon + \Omega)] \times \text{Im} G_n(\varepsilon + \Omega) \text{Im} G_{n'}(\varepsilon), \quad (13)$$

where $l_B = \sqrt{1/|eB|}$ is the magnetic length, $C_{n,n'+1}(Y) \equiv n![n+(l+1)]^{-1} Y_{l}^1 e^{-Y}[E_{n}(Y)]^{-1}$ with $E_n(Y)$ the associate Laguerre polynomial, $f(\varepsilon) = \{\exp[\varepsilon - \mu/T_0] + 1\}^{-1}$ the Fermi distribution function, and $\text{Im} G_n(\varepsilon)$ is the imaginary part of the Green’s function, or the DOS, of the Landau level $n$. The real part functions $\Pi_1(q_\parallel, \Omega)$ and $\Lambda_1(q, \Omega)$ can be obtained from their imaginary parts via the Kramers-Kronig relation.

In principle, to obtain the Green’s function $G_n(\varepsilon)$, a self-consistent calculation has to be carried out with all the scattering mechanisms included. In this Letter we do not attempt a self-consistent calculation of $G_n(\varepsilon)$ but choose a Gaussian-type function for the purpose of demonstrating the $R_{xx}$ oscillations ($\varepsilon_n$ is the energy of the $n$-th Landau level) with a broadening parameter $\Gamma = (2e\omega_c/\pi m\mu_0)^{1/2}$, where $\mu_0$ is the linear mobility at temperature $T$ in the absence of the magnetic field and $\alpha > 1$ is a semiempirical parameter to take account the difference of the transport scattering time determining the mobility $\mu_0$, from the single particle lifetime.

The moderate microwave intensity for the $R_{xx}$ oscillation in these high-mobility samples yield only slight electron heating, which is unimportant as far as the main phenomenological is concerned and is neglected for simplicity. We consider scatterings from remote impurities as well as from acoustic phonons. After solving $v_1$ and $v_2$ from Eqs. (9) and (10) the magnetoresistivity $R_{xx}$ can be obtained directly from Eq. (11). At lattice temperature $T = 1K$, the contribution from photon-assisted phonon scattering is minor. The role of acoustic phonons, however, becomes essential at elevated lattice temperatures. Calculations were carried out for linearly polarized MW fields with multiphoton processes included.

Fig.1 shows the calculated longitudinal resistivity $R_{xx}$ versus $\omega/\omega_c \equiv \gamma_c$ subject to a linearly polarized MW radiation of frequency $\omega/2\pi = 100$ GHz at four values of amplitude: $E_x = 20, 45, 65$ and $80$ V/cm. Shubnikov-de Haas (SdH) oscillations show up strongly at high $\omega_c$ side, and gradually decay away as $1/\omega_c$ increases. All resistivity curves exhibit pronounced oscillation having main oscillation period $\gamma_c = 1$ (they are crossing at integer points $\gamma_c = 2, 3, 4, 5$). The resistivity maxima locate around $\gamma_c = j - \delta_x$ and minima around $\gamma_c = j + \delta_x$ with $\delta_x \sim 0.23 - 0.25$ for $j = 3, 4, 5$, $\delta_x \sim 0.17 - 0.21$ for...
resistivity behavior at $\gamma_c < 1$ is shown more clearly in the $\omega/2\pi = 60\,\text{GHz}$ case. As seen in Fig. 1c, a shoulder around $\gamma_c = 0.4-0.6$ with a minimum at $\gamma_c = 0.6$ can be identified from the SdH oscillation background for all three curves, which is related to two-photon process. With increasing MW strength there appears a clear peak around $\gamma_c = 0.68$ and a valley around $\gamma_c = 0.76$. This peak-valley is mainly due to three-photon ($|n| = 3$) process. In the case of 40 GHz, similar peak and valley also show up (Fig. 1d).

Qualitatively, the main oscillation features come from the symmetrical property of the DOS function in a magnetic field. Since $G_n(\epsilon - j\omega c) = G_{n-j}(\epsilon)$ for any integer $j$, the impurity contribution to $R_{xx}$ from the $n$-photon process, which is related to the weighted summation of the derivative $P_\omega$ function over all the Landau levels at frequency $n\omega$ [Eq. (11)], has an intrinsic periodicity characterized by $n\omega = j\omega_c$. The main oscillation of $R_{xx}$ shown in Fig. 1a relates to single-photon process and characterized by $\omega = j\omega_c$. We have also performed calculation using a Lorentz-type DOS function and find that, although the oscillating amplitude and the exact peak and valley positions are somewhat different, the basic periodic feature of the radiation-induced magnetoresistivity oscillation remains.

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