Lateral response formation process between an elastic tire and a supporting surface when a slip angle wheel crosses a single irregularity

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Abstract. The article presents the results of an analytical and experimental study of $R_Y$ lateral response formation between the elastic tire and the support surface when the wheel with the slip angle $\delta$ moves over a single irregularity. In this paper bench methods of experimental study of $R_Y$ lateral response formation process between the elastic tire and the support surface, when the wheel with the slip angle $\delta$ moves over a single irregularity, are used. The experiment results have been obtained in bench conditions on a special test bench developed by the scientists of the Department of Automobile Transport of Irkutsk National Research Technical University. The results of experiments and calculations are presented in the form of graphs. The results obtained allow us to assert that in the research of $R_Y$ lateral response formation between the elastic tire and the support surface, when the wheel with the slip angle $\delta$ crosses a single irregularity, it is necessary to consider the unsteady mode of wheel rolling and take into account the fluctuations of individual tire parts.

1. Introduction

Modern road transport occupies an important place in our lives, primarily, due to the door-to-door delivery of passengers and cargo. With all the obvious advantages, a car has been and remains one of the most dangerous modes of transport. In the last 3 years, in Russia, about 527 thousand road accidents, in which 62510 people are dead and 667711 are injured, have been registered by the General Administration for Traffic Safety [1].

Most of the accidents occur owing to the stability loss and handling at high speeds. The controllability and stability of the vehicle, when driving under the influence of lateral forces, depend on the stability of its tire grip on the road [2-4]. The action of lateral forces on the vehicle is accompanied with the lateral slip angles of wheel rolling.

The technical condition of shock absorbers has a significant impact on the stability of the lateral adhesion of tires to an uneven road while rolling with slip angles [5-6].
The lateral tire grip depends on the processes of \( R_Y \) lateral response formation and normal reactions \( R_Z \) in the contact spot with the supporting surface.

Typically, the rolling of an elastic tire with slip angles is considered in one of two conditions: stationary (steady) and non-stationary (unsteady) [7]. In this case, the lateral response \( R_Y \) is represented either as a function of changing the angle of the tire slip \( R_Y = f(\delta) \) or as a function of the slip change in the contact spot of the tire \( R_Y = f(S) \) (S) [8].

Steady-state rolling of the wheels with elastic tires is considered to obtain the parameters describing the process of the slip function behavior \( f(S) \). These parameters are the coefficient of lateral adhesion \( \phi \), the coefficient of adhesion reduction \( f_b \) and the coefficient of slip resistance \( \eta_S \).

An attempt has been made to study the process of \( R_Y \) lateral response formation in the contact spot of the elastic tire with the support surface, when the wheel with a slip angle \( \delta \) moves over a single irregularity under vehicle operating conditions. It has been a complicated process. At the same time, the motion dynamics of the vehicle sprung and unsprung masses, as well as the forces acting on the wheel from the support surface, was taken into account.

2. Methods

In the course of the research, the dynamics of the tire parts in the vicinity of the contact spot, which had a non-stationary condition, as well as the power and kinematic tire parameter changes, were taken into account. The unsteadiness of the wheel operation mode is associated with an unsteady mode of tire deflection. The deflection of a non-leaning wheel tire is usually determined by three parameters: longitudinal \( x_k \) and lateral \( y_k \) linear displacements of the tire contact spot relative to the wheel disc and the rotation angle \( \varphi_z \) of the contact spot relative to the vertical axis (Figure 1) [7].

![Figure 1. Scheme of the tire contact spot displacements with the support surface relative to the wheel disc.](image)

One of the important conditions of the study was the lack of braking torque action on the rolling process of the wheel with the slip angles. The tire was in a driven mode. This made it possible to identify its greatest potential ability to create lateral responses. The transient nature of the tire deflection characterized the velocities of its lateral \( \dot{y} \) and longitudinal \( \dot{x} \) displacement. The velocity of longitudinal tire deflection was neglected because of being small. The delay in the increase in the turning angle \( \varphi_z \) of the tire contact spot around the vertical axis was also neglected, since the action of the stabilizing moment was not taken into account.

The most convenient model for the analytical study of this process, according to the authors, is the mathematical model developed by A. B. Dik, describing the unsteady rolling mode of a wheel with an elastic tire with a slip [7].

In the work of A. B. Dik, the description of the lateral response forming process in the contact spot of an elastic tire of an automobile wheel during its rolling in non-stationary modes is as follows [7]:

\[
R_Y = R_Z \cdot \varphi_Y \cdot \sin \left( a \cdot \arctg \left( b \cdot \sin \delta - b \cdot \frac{\dot{y}}{V_X} \right) \right),
\]

where \( R_Z \) – a normal reaction from the support surface to the wheel, \( \varphi_Y \) – a tire lateral adhesion coefficient.
to the support surface; $\delta$ – a slip angle, $\dot{y}_S$ – a velocity of the contact spot displacement in the lateral direction, $V_S$ – a velocity of the wheel center, $a$ and $b$ – coefficients that determine the nature of the slip function $f(S)$ and depend on the coefficient of reduction of road grip properties $f_s$ as well as on the specific slip resistance coefficient $\eta_S$ [7].

The velocity $\dot{y}$ of the contact spot center displacement in the lateral direction was determined from the equation describing the oscillation dynamics of a separate tire part. In the course of the study, it was found that, in addition to vibrations of a separate tire part, it is equally important to consider the vibrations of the wheel in the lateral direction relative to the vehicle sprung mass. They occur as a result of elastic and inelastic deformations of the silent blocks, the guide elements of the suspension (levers).

To formulate the equation of dynamic equilibrium of the system, a calculation scheme was made (Figure 2). The scheme takes into account the elastic and damping characteristics of suspension elements ($C_{YD}$ and $K_{YD}$) and tires ($C_Y$ and $K_Y$), the wheel mass $m_k$ and a part of the tire mass $m_s$ in the vicinity of the contact spot, as well as lateral responses $R_Y$ and $R_{YD}$ acting from the support surface to the wheel, and from the wheel to the sprung mass.

**Figure 2.** The design scheme of the oscillatory system of the wheel with an elastic tire under the action of lateral forces.

Based on the scheme (Figure 2), the equation of dynamic equilibrium of the system under consideration, set up on the d'Alembert principle [9], is written as follows:

$$
\begin{align*}
R_Y - C_Y \cdot (y_S - y_K) - K_{SY} \cdot (\dot{y}_S - \dot{y}_K) - m_s \cdot \dot{y}_S &= 0, \\
C_Y \cdot (y_S - y_K) + K_{SY} \cdot (\dot{y}_S - \dot{y}_K) - C_{YD} \cdot y_K - K_{YD} \cdot \dot{y}_K - m_k \cdot \ddot{y}_K &= 0
\end{align*}
$$

where $C_Y$ – the dynamic lateral stiffness of the tire; $K_{SY}$ – a tire damping coefficient; $C_{YD}$ – the suspension guide elements stiffness; $K_{YD}$ – a suspension guide elements damping coefficient; $m_k$ – the weight of the wheel with an elastic tire; $m_s$ – the weight of a separate oscillating part of the tire at the contact spot failure.

The solution of the system equations (2), relative to the higher derivatives (by the Euler numerical method) [10], allows one to obtain the velocities and displacements of the tire contact spot and the wheel rim in the lateral direction at the $i$-th time moment. So the rate of the tire contact spot center displacement in the lateral direction is:

$$
\dot{y}_S = \dot{y}_{S,i \rightarrow i+1} + \dot{y}_{S,i} \cdot dt.
$$

The shift of the tire contact spot in the lateral direction is:

$$
y_S = y_{S,i \rightarrow i+1} + \dot{y}_{S,i} \cdot dt.
$$

The rate of the wheel center displacement in the lateral direction is:

$$
\dot{y}_K = \dot{y}_{K,i \rightarrow i+1} + \dot{y}_{K,i} \cdot dt.
$$

The lateral displacement of the wheel centre is:

$$
y_K = y_{K,i \rightarrow i+1} + \dot{y}_{K,i} \cdot dt.
$$

The lateral response from the wheel to the sprung mass is found by the formula:

$$
R_{YD} = C_{YD} \cdot y_K - K_{YD} \cdot \dot{y}_K.
$$
The normal reaction $R_Z$ from the support surface to the wheel, when rolling and moving over a single irregularity, can vary over a wide range. It depends on the vertical load on the wheel $F_K$, elastic and inelastic characteristics of the tire and geometric parameters of the single irregularity.

For the mathematical description of the change process of the normal reaction $R_Z$ to a wheel at disturbing influence of the road at single irregularity crossing, the design scheme is presented in Figure 3.

**Figure 3.** Calculation scheme for normal reaction determining, when a wheel with an elastic tire crosses a unit irregularity.

The scheme takes into account the elastic and damping characteristics of the suspension elements ($C_{PZ}$ and $K_{PZ}$) and tires ($C_Z$ and $K_{SZ}$). The oscillations of unsprung and sprung masses ($m_{Н}$ and $m_{П}$) and the changes in normal reactions $R_Z$ and $R_{ZD}$, acting from the support surface to the unsprung mass, and from the unsprung to the sprung mass respectively.

Using the calculation scheme (Figure 3), the normal reaction $R_Z$ is determined by the formula:

$$R_Z = C_Z \cdot (\ddot{\xi} - q_i + \Delta_{\xi}) + K_{SZ} \cdot (\ddot{\xi}_{Zi} - \dot{q}_i),$$

where $\Delta_{\xi}$ – a static tire deflection; $\ddot{\xi}$ – an unsprung masses vertical movement; $q_i$ – an ordinate of the cross-section of road with a single asperity; $\ddot{\xi}_{Zi}$ - a velocity of the unsprung mass vertical movement; $\dot{q}_i$ – the first derivative of function, describing the cross-section of road with a single asperity; $K_{SZ}$ – a tire damping coefficient in the radial direction.

The normal reaction $R_{ZD}$ is defined by the formula:

$$R_{ZD} = C_{PZ} \cdot (h_i - \ddot{\xi} + \Delta_{P}) + F_{ш},$$

where $\Delta_P$ – a suspension static deflection; $h_i$ – a sprung masses vertical movement; $F_{ш}$ – a shock absorber resistance force.

The mathematical description of the interaction of a tire with a single irregularity is presented in the form of a smoothing function [11-12]:

$$q = \frac{q_o}{2} \cdot \left(1 - \cos\left(\frac{2 \cdot \pi \cdot t_q}{T}\right)\right),$$

where $q_o$ – a peak value of the function ordinate describing the cross-section of road with a single asperity; $t_q$ – a time value from the beginning of the tire interaction with a single irregularity; $T$ – the
period of the harmonic function describing the cross-section of road with a single irregularity:

$$ T = \frac{l_0}{V_x} = \frac{t_{max}}{V_x}, $$  \hspace{1cm} (11)

where $l_0$ – the harmonic function wavelength.

To determine the kinematic parameters of the oscillatory process of the sprung $m_P$ and unsprung $m_{NP}$ masses according to the scheme (Figure 3), the system including the equations of dynamic equilibrium for the sprung and unsprung masses, in accordance with the d’Alembert principle [9], was planned:

$$ \begin{cases} m_{P} \ddot{h} = m_{P} \cdot g - C_{pz} \cdot (h - \dot{\xi} + \Delta_P) - F_D \\ m_{NP} \ddot{\xi} = m_{NP} \cdot g + C_{pz} \cdot (h - \dot{\xi} + \Delta_P) + F_D - p_{OK} \cdot (C_{D} \cdot (\dot{\xi} - q + \Delta_P) + K_{DZ} \cdot (\dot{\xi} - \dot{q})) \end{cases} $$  \hspace{1cm} (12)

where $m_P$ – a sprung mass; $\dot{h}$ – the sprung mass acceleration along the OZ axis; $g$ – gravitational acceleration; $m_{NP}$ – an unsprung mass; $\ddot{\xi}$ - the unsprung mass acceleration.

The calculation of the resistance forces of the shock absorber $F_D$ (Figure 4) will be performed basing on a piecewise linear function of the following form [13-14]:

$$ F_D = \begin{cases} F_D(V_A), & V_A \leq V_1 \\ F_D(V_A), & V_1 < V_A \leq 0 \\ F_D(V_A), & 0 < V_A \leq V_2 \\ F_D(V_A), & V_A \geq V_2 \end{cases} $$  \hspace{1cm} (13)

where $V_1$ – the velocity of the shock absorber piston relative to the walls of its cylinder, at which the opening / closing of the bypass valve takes place during the rebound stroke; $V_2$ – the velocity of the shock absorber piston relative to the walls of its cylinder, at which the opening / closing of the bypass valve takes place during compression; $V_A$ – the velocity of the shock absorber piston movement relative to the walls of its cylinder.

Let us estimate the velocity of movement of the piston shock absorber $V_A$ as the first derivative of the suspension deformation $\Delta_{PD}$:

$$ V_A = \Delta_{PD} = \dot{h} - \dot{\xi}, $$  \hspace{1cm} (14)

where $\dot{\xi}$ – a vertical movement velocity of the unsprung mass; $\dot{h}$ – a vertical movement velocity of the sprung mass.

**Figure 4.** Graph of piecewise linear function.
The solution of differential equations is performed by the Euler numerical method [11]. The movement velocity of the sprung mass in the vertical direction at the $i$-th time station is determined by the formula:

$$\dot{h}_i = \dot{h}_{i-1} + \ddot{h}_i \cdot dt.$$  \hfill (15)

Then the displacement of the sprung mass is found by the formula:

$$h_i = h_{i-1} + \dot{h}_i \cdot dt.$$  \hfill (16)

Similarly, the velocity of the unsprung mass along the axis at the $i$-th time station is determined by the formula:

$$\dot{\xi}_i = \dot{\xi}_{i-1} + \ddot{\xi}_i \cdot dt.$$  \hfill (17)

The unsprung mass movement is calculated by the following formula:

$$\xi_i = \xi_{i-1} + \dot{\xi}_i \cdot dt.$$  \hfill (18)

To check the adequacy of the mathematical description of the lateral response formation process, the experimental studies were conducted.

The experimental researches, when the wheel with an elastic tire crosses a single irregularity, were performed on a special test bench, a treadmill, designed by the authors (Figure 5). It is manufactured on the basis of a tire tester with a running drum. Its design description and the principles of operation are described in the works [5-15] in more detail.

![Figure 5](image.png)

**Figure 5.** The treadmill for testing the wheel with an elastic tire moving over a single irregularity.

Kinematics and power balance of the developed stand (Figure 5) are equivalent to the kinematics and power balance of the independent suspension of the car. The treadmill has unsprung and sprung masses, interconnected by a guide vane, shock absorber and elastic element. The weight of the sprung mass is $G_{spr} = 3090$ N, the spring stiffness is 26,000 N/m.

On the treadmill drum 14 (Figure 5) the wheel rolls with an elastic tire 12 and the irregularity (obstacle) 13 are fixed. In the process of the pilot study a summer tire by MICHELIN 195/65 R15 and an individual asperity of the rectangular profile of $50 \times 25$ mm were used.
3. Results and discussion

Normal $R_Z$ and lateral $R_Y$ reactions, acting on the wheel in pure form, are very difficult to determine. Therefore, the normal reaction from the unsprung mass to the sprung one of $R_{ZD}$ and the lateral response from the wheel to the sprung mass $R_{YD}$ were measured.

Figure 6 presents the results of the calculation and experiment obtained in the study of the process of the wheel with an elastic tire MICHELIN 195/65 R15 moving, with a slip angle $\delta$ of 2 degrees, through a single irregularity of a rectangular profile with a serviceable Ford Focus shock absorber of the front suspension. The mathematical description of the working characteristics of a new technically serviceable shock absorber is presented in the form:

$$F_D = \begin{cases} 
1226.8 \cdot V_\delta - 376.32, & V_\delta < -0.1 \\
5000 \cdot V_\delta, & -0.1 \leq V_\delta < 0 \\
3000 \cdot V_\delta, & 0 \leq V_\delta < 0.2 \\
781.25 \cdot V_\delta - 443.75, & 0.2 \leq V_\delta 
\end{cases}$$

(19)

**Figure 6.** Graphs of the process of the wheel moving with an elastic tire MICHELIN 195/65, R15 with a slip angle of 2 degrees over a single irregularity of a rectangular profile with a serviceable Ford Focus front suspension shock absorber: a) the normal reaction schedule from the unsprung mass to the sprung one of $R_{ZD}$; b) the graph of the lateral reaction from the wheel to the sprung mass $R_{YD}$.

Shown in Figure 7, the graphs convincingly show that the results of reactions $R_{YD}$ and $R_{ZD}$ calculation are adequate to the experimental results both qualitatively and quantitatively. Therefore, the results of the lateral $R_Y$ and normal $R_Z$ reactions calculation (Figure 7), acting from the support surface to the wheel, will be considered trustworthy.

4. Conclusions

The dynamic processes of the sprung and unsprung masses vibrations, individual parts of the tire, as well as the technical condition of the shock absorber and the adhesion ability of the tire-road pair have a significant effect on the formation of lateral reactions $R_Y$ between the elastic tire and the support surface during the wheel movement with the slip angle $\delta$, over a single irregularity under the operating conditions.

The developed mathematical description allows us to conduct an analytical study of the dynamic processes of the lateral $R_Y$ and normal $R_Z$ reactions formation between the elastic tire and the support surface when the wheel moves with the slip angle $\delta$ over a single irregularity. It takes into account the influence of the tire grip characteristics and the technical condition of the shock absorbers on the process under study.

The relative errors of mathematical model calculation of the kinematic and power parameters of the process do not exceed 1.5-3.2%.
Figure 7. Graphs of the process of the wheel with an elastic tire MICHELIN 195/65, R15 moving with a slip angle of 2 degrees over a single irregularity of a rectangular profile with a serviceable Ford Focus front suspension shock absorber: a) a graph of the normal reaction $R_Z$ in the contact spot of the tire with the support surface; b) a graph of the lateral reaction $R_T$ in the contact spot of the tire with the support surface.

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