Reactor Searches for Neutrino Magnetic Moment as a Probe of Extra Dimensions

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Abstract

We present calculations of the magnetic moment contribution to neutrino electron scattering in large extra dimension brane-bulk models (LED) with three bulk neutrinos. We discuss the cases with two and three large extra dimensions of sizes $R$. The calculations are done using reactor flux from Uranium, $^{235}U$ as the neutrino source. We find that if the electron neutrino mass is chosen to be in the range of one eV, the differential cross section for $\bar{\nu} - e$ scattering for low electron recoil energy can be of the same order as the presently explored values in reactor experiments. Furthermore the spectral shape for the LED models is different from the four dimensional case. Future higher precision reactor experiments can therefore be used to provide new constraints on a class of large extra dimension theories.
I. INTRODUCTION

The possibility that there may be extra hidden small-sized space dimensions has been the subject of a great deal of attention in recent years, mostly motivated by string theories as well as phenomenological considerations having to do with understanding the gauge hierarchy problem. A specially interesting subclass of these theories are the ones where the extra space dimensions have sizes ranging from several micrometers to a millimeter [1]. They have the attractive phenomenological property that they can be tested in laboratory experiments searching for deviations from inverse square law of gravity. In this note we discuss one way to test these models using neutrino experiments.

One of the ways to explain the observed neutrino masses and mixings in large extra dimension (LED) models is to postulate the existence of a singlet neutrino in the bulk [2] and couple it to the known neutrinos on the brane. Since such a particle from a four dimensional point of view is equivalent to a tower of Kaluza-Klein (KK) states, it tends to manifest itself in many experiments even though it has no conventional weak interaction. One such dramatic manifestation is in the effective magnetic moment interaction in $\bar{\nu}e$ and $\bar{\nu}N$ elastic scattering [3]. The reason for this is that the magnetic moment couples the left handed neutrino to the full tower of states. This means that in the magnetic part of $\bar{\nu}e$ scattering one can produce all the states in the KK tower $m_{\nu_R,KK} \leq E_\nu$. All these final states will add incoherently to the magnetic moment part of the cross section and will in general lead to considerable enhancement. One can therefore use reactor neutrino experiments searching for magnetic moment [4,5] to constrain the size of the extra dimensions in the LED models.

In this brief note, we present a calculation of the magnetic moment contribution to $\bar{\nu} - e$ scattering by reactor neutrinos. We calculate the flux averaged $\frac{d\sigma}{dT}$ for a $^{235}U$ neutrino source where $T$ is the electron recoil energy. We find that for the case of two large extra dimensions, the $\frac{d\sigma}{dT}$ can be comparable to the corresponding one for the case where there are no extra dimensions and the $\mu_\nu \sim 10^{-10} \mu_B$, which was calculated in [6]. The value for $\mu_\nu$ chosen is the present upper bound from the previous experiments [4]. For the case of three extra dimensions, the value is lower. We further point out that for the LED models, the spectral shape of $d\sigma/dT$ is different from the case with a single right handed neutrino with an appropriate magnetic moment. This can therefore be a distinguishing signature of the LED models.

II. NEUTRINO MAGNETIC MOMENT IN $\delta$ LARGE EXTRA DIMENSIONS

For the case of $D$ large extra dimensions and $\delta$ small dimensions, the size and the fundamental scale are related by the formula:

$$ M_P^2 = M_{*D}^{2+D/\delta} (\pi R)^D (\pi r)^\delta $$

(1)

where we have assumed all large dimensions to have the same size $R$ and all small ones to have the same size $r$. If we take the $M_{*D}$ to be a TeV, we find that for $D = 2$ and $M_{*D} \pi r = 1$, $R \sim 2$ mm and for $D = 3$ and $M_{*D} \pi r = 1$, $R = 0.02 \mu$m. Below, we will present our calculations for both these cases.

Let us now briefly discuss the minimal neutrino mass scenario for these models where one introduces three singlet neutrinos in the bulk [2,7]. The standard model particles in this
picture reside on the brane whereas the singlet neutrinos $N$ are in the bulk. One introduces a brane bulk coupling in the form:

$$
\mathcal{L}_N = \frac{h_{\alpha\beta}}{(2\pi R)^{D/2}} \int d^D y \delta^D(y) \bar{\psi}_{L,\alpha} H N_\beta + h.c.
$$

(2)

where $R$ is the size of each of the extra space dimensions on which the theory is compactified. The extra dimensions are compactified on an $(S^1/Z_2)^D$ space. We can expand the bulk neutrinos $N_\beta$ into their Fourier modes as

$$
N_\beta(x) = \sum_n N^n_\beta \frac{e^{i n_R x}}{\sqrt{(2\pi R)^D}}
$$

to get the neutrino mass of the form

$$
\mathcal{M}_{\alpha\beta} = \sum_n h_{\alpha\beta} v_{w,\alpha} M_P \bar{\nu}_{L,\alpha} N^{(n)}_{R,\beta}
$$

(3)

This matrix can be diagonalized by a common $3 \times 3$ rotation for all the KK modes for the bulk modes i.e. $N^n_\beta = V^n_\beta M^n_\beta$ and a rotation on the lefthanded brane neutrinos $\nu_\alpha = U_{i\alpha} \nu_i$. The latter leads to the PMNS matrix measured in neutrino oscillation experiments. In this diagonal basis, the mixing of the brane neutrinos to the $n$th KK mode of the bulk singlets are given by a common factor $\xi_i = \sqrt{2m_i R/n}$. The neutrinos in this model are Dirac neutrinos.

Let us now discuss the magnetic moment predictions of this model. As already noted, the magnetic moment will connect the brane neutrino to all the KK excitations of the bulk singlets equally. For example for the one neutrino case, the magnetic moment interaction will be given by [8]:

$$
\mathcal{L}_{\mu E} = \mu_t \mu_B \sum_n \bar{\nu}_{L,\sigma} \sigma^{\mu\nu} N^{(n)} F_{\mu\nu}
$$

(4)

where $\mu_t = \frac{3G_F m_e m_\nu}{4\sqrt{2\pi}} = 3.2 \times 10^{-19} \left[ \frac{m_\nu}{1 \text{ eV}} \right]^2$; $\mu_B$ is the Bohr magneton. The symbol $n$ goes over a square or cubic lattice depending on whether $D = 2$ or $D = 3$. Because of this, when we calculate $\bar{\nu} e$ scattering using this formula, the phase space as well as parameters such as the size $R$ will be very different for the two cases leading to different final results.

### III. MAGNETIC MOMENT CONTRIBUTION TO $\bar{\nu} E$ SCATTERING CROSS SECTION

In this section we calculate the differential cross section as a function of the electron recoil energy $T \equiv E' - m_e$ ($E'$ is the final state electron energy). The techniques used are standard. The only point that need careful treatment is the phase space and implementation of the energy momentum conservation. For instance, a naive estimate of the $\frac{d\sigma}{dT}$ for the case one extra D is to take the four dimensional result and multiply it by $(E_\nu R)$, where $E_\nu$ is the incident neutrino energy. That this is not correct can be seen by noting that the above estimate assumes that all KK states up to $E_\nu$ contribute; however energy momentum conservation allows only modes $\ll E_\nu$ to contribute. Furthermore, energy momentum conservation imposes other constraints.

The calculated cross-section for $\bar{\nu}_e + e \rightarrow e + \bar{N}$ where $N$ is a right handed neutrino of mass $M$ is,
\[
\frac{d\sigma}{dT} = \frac{\pi \alpha^2 \mu_t^2}{8 m_e^2 E^2 T^2} \left[ M^4 T - 2 m_e^2 T (M^2 + 4 E T - 4 E^2) - m_e M^2 (M^2 + 4 E T - 2 T^2) \right]
\]  

(5)

where \(\mu_t\) is the neutrino magnetic moment in units of \(\mu_B\) (see equation (4)). It can easily be checked that in the limit of \(M \to 0\), we get the equation for a massless right handed neutrino as found in [6].

To obtain the differential cross section for LED models, one needs to take into account the tower of KK final states allowed by kinematic considerations. Furthermore to compare experiments, we need to take into account the flux of incoming neutrinos with different energies. For this purpose, we choose the \(^{235}\text{U}\) reactor flux in the parameterization given in [6]. Folding all these effects in, we get

\[
\left\langle \frac{d\sigma}{dT} \right\rangle = \sum_{\text{all } M} \int_{E_{\nu}^{\text{min}}(T,M)}^{\infty} \frac{dN_{\nu}}{dE_{\nu}} d\sigma(E_{\nu}, M, T) dE_{\nu}
\]

(6)

where \(\frac{dN_{\nu}}{dE_{\nu}}\) is the reactor spectrum which can be approximated by

\[
\frac{dN_{\nu}}{dE_{\nu}} = \exp(a_0 + a_1 E_{\nu} + a_2 E_{\nu}^2).
\]

(7)

We will be considering the case of the \(^{235}\text{U}\) reactor, which has the parameters, \(a_0 = 0.870\), \(a_1 = -0.160\) and \(a_2 = -0.0910\).

A few comments are in order here. Although we have a spectrum of all possible incoming neutrino energies, not all of them will contribute to the scattering at recoil energy, \(T\), and KK mass, \(M\). This is because energy momentum has to be conserved. For this reason we do not integrate \(E_{\nu}\) from 0 to \(\infty\). The lower limit on our integration is

\[
E_{\nu}^{\text{min}}(T,M) = \frac{M^2 + 2 m_e T}{2(\sqrt{T^2 + 2 m_e T} - T)}.
\]

(8)

As for the actual increments in \(M\), we took

\[
M = \sqrt{\sum_{i=1}^{D} n_i^2} / R, \quad n_i = 0, 1, 2, \ldots
\]

(9)

where \(D\) is the number of extra dimensions and the radius of the extra dimension(s) \(R\) is obtained from

\[
R = \frac{1}{\pi} \left( \frac{M_{\text{Planck}}}{M_{sD}^{D+2}} \right)^{1/D}
\]

(10)

and \(M_{sD}\) is the \(D + 2\) dimensional Planck mass and is taken to be 1 TeV.

The cross section is sensitive to the mass of the neutrinos since that determines the value of the magnetic moment \(\mu_t\) (see Eq. (4)). In our calculation presented below, we have chosen the electron neutrino mass to be 0.6 eV so as to be compatible with the oscillation data and the current cosmological limits from WMAP and SDSS observations. This corresponds to the case where all the neutrinos are quasi-degenerate in mass. If instead we chose the
normal or the inverted hierarchy, $m_\nu$ would be much smaller and as a consequence our final results will need to be appropriately scaled down.

Figures 1, 2 and 3 show our results for the case of $D = 2$ and $D = 3$ as compared to the phenomenological 4D case as well as the purely weak interaction.

We see from the above figures that the spectral shape of the $d\sigma/dT$ for the case of large extra dimensions is different from the single right handed neutrino case. If the spectral shape was same as the single RH neutrino case, it would be hard to search for extra dimensions using the magnetic moment contribution to $\bar{\nu}e$ scattering. Our results can be used to put limits on the size of extra dimensions in LED type theories. This requires a detailed fitting analysis of the theory versus the observations. It is probably a bit premature to do that since for our choice of parameters, the predictions are roughly at the same level as the current experiments. However the proposed reactor experiments [5] will improve the precision of the data and can help to constrain the size of the extra dimensions further. We also point out that even though we have described our calculation for a model with three bulk neutrinos and Dirac neutrino masses, it is also valid for LED models which have Majorana masses for neutrinos [9].

In conclusion, we have presented calculations of the magnetic moment contribution to the $\bar{\nu}e$ scattering cross section for realistic reactor fluxes for models with large extra dimensions. We find that for the case of two large extra dimensions, the predictions of these models is at the level of current experimental limit on the neutrino magnetic moment. Furthermore, the spectral shape for the case of LED models is different from the case of a single right handed
FIG. 2. The figure shows the folded differential cross section, $\frac{d\sigma}{dT}$, for the magnetic moment contribution to $\bar{\nu}e$ scattering for the case of $D = 3$ and the standard weak contribution. The 7D Planck mass is taken to be 1 TeV.

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FIG. 3. The figure presents the folded differential cross section $\frac{d\sigma}{dT}$ for the magnetic moment contribution to $\bar{\nu}e$ scattering for the cases of $D = 0$, $D = 2$, $D = 3$ and the standard weak contribution. The case $D = 0$ corresponds to the standard model extended by the inclusion of one right handed neutrino per family.

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