Smooth thick braneworlds and the Gibbons-Kallosh-Linde no-go theorem

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Abstract: After working out the so called braneworld sum rules formalism in order to encompass Gauss-Bonnet terms, the generation of thick branes is proposed, even with a periodic extra dimension, what circumvents the Gibbons-Kallosh-Linde no-go theorem in this context.
1 Introduction

Braneworld models are well known to have given rise to several interesting aspects of high energy physics, since the seminal works considering warped geometry [1–5]. To restrict ourselves to particle physics, the warped structure allows, in some models, the solution for the hierarchy problem. Concerning this quite important application, the canonical model is given in [2], whose setup is given by two infinitely thin branes placed at the singular points of a $S^1/Z_2$ (orbifold) extra dimension. Soon after the emergence of such models, important generalisations leading to smooth branes were proposed [6]. The general feeling was that smooth or thick branes were more natural in the sense of precluding the necessity of singular terms (Dirac delta terms necessary to place the brane in the thin brane context) into the action.

A systematic study of the mathematical and the physical constraints arising in different braneworld models [7] has culminated in a comprehensive formalism, correlating physical inputs of a given model with necessary mathematical conditions to be fulfilled, in order to assure a viable braneworld framework [8]. For the use of the so called braneworlds sum rules, obtained in [8], it is mandatory the existence of a compact internal space without boundary (the orbifold extra dimension in the five-dimensional case). Building on such a geometry, the authors in [8] have shown the impossibility of smooth generalizations of the flat periodic two-brane Randall-Sundrum and Kogan-Mouslopoulos-Papazoglou-Ross-Santiago (KMPRS) solutions [9], without singular sources. This is the so called Gibbons-Kalosh-Linde no-go theorem. In practice, the aforementioned theorem statement is transliterated into the fact that the integral along the extra dimension of a quadratic quantity equals zero. Since the constraint cannot be accomplished, the theorem holds.

The aim of this paper is to show that, as far as we use another gravitational theory in the bulk, it is possible to circumvent the Gibbons-Kalosh-Linde theorem hypothesis. In spite of the fact that the general framework be more involved, as we shall see in the next sections, the idea can be straightforwardly depicted: the additional terms coming from the extended gravitational environment can, in principle, alleviate the strong constraint imposed by the sum rules. The necessity of a generalized gravitational theory, indeed, rests
on the robustness of the theorem itself. In fact, its assertion is independent on the assumed scalar field potential, being valid for every scalar field model. In this vein, we shall go one step further, considering the next order Lovelock series term \([10]\), i.e., the Gauss-Bonnet term. By accomplishing so, it is important to stress, the (to be) analyzed framework shall not perform a complete model. As it is going to be clear, some important ingredients in the sum rule formalism (the so called partial traces) do not provide closed relations. Even so, this is a worthwhile exercise. In fact, by assuming that the contribution coming from the Gauss-Bonnet term is weak in the bulk, we are able to show the possibility of a smooth generalization of the Randall-Sundrum-like setup. Besides, as shall be seen, this formalism can also be used in order to avoid the necessity of a negative brane tension.

This paper is organized as follows: in the next section we accommodate the sum rules formalism, in order to encompass the additional terms coming from the Gauss-Bonnet geometry in a given context. The application to the Gibbons-Kallosh-Linde theorem is performed in Section III. In Section IV we point out the conclusion and final remarks.

2 The formalism

In a broader sense, the so called braneworld sum rules were developed in such a way that the gravitational dynamical equation, without being solved, can be used to extract necessary mathematical conditions whose fulfilment enables a physical setup. The formalism, or part of it, is constructed upon purely geometrical considerations. Its impact is much more expressive in dealing with compact internal spaces, just as the one of an orbifold extra dimension, within an warped spacetime.

The general formalism is endowed with the following notation: the bulk is \(D\)-dimensional, whilst the \(p\)-brane has \((p+1)\) dimensions. The internal space, thus, is left with \((D-p-1)\) compact dimensions. The metric is given by

\[
ds^2 = G_{MN} dx^M dx^N = W^2(y)g_{\mu\nu} dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n.
\]

As it can be seen from a number of papers (see, for instance \([11]\)) there is a quite interesting rearrangement fulfilled by the warp factor, the brane and internal space scalars of curvature \((p+1)R\) and \((D-p-1)R\), respectively, and the partial traces given by

\[
\nabla \cdot (W^\alpha \nabla W) = \frac{W^{\alpha+1}}{p(p+1)} \left\{ \alpha \left[ (p+1)RW^{-2} + R_{\mu}^\mu \right] + (p-\alpha) \left[ (D-p-1)R - R_m^m \right] \right\}, \tag{2.1}
\]

where \(R_{\mu}^\mu = W^{-2}g^{\mu\nu}(D)R_{\mu\nu}\) and \(R_m^m = g^{mn}(D)R_{mn}\) are the partial traces, whose sum gives \((D)R = R_{\mu}^\mu + R_m^m\), the bulk scalar of curvature. In Eq. (2.1), \(\alpha\) is just a parameter, and each fixed \(\alpha\) provides a mathematical condition to be respected by the model. From an integration over the internal space it is possible to find a one parameter (the \(\alpha\) parameter, indeed) family of consistency conditions. Before to pursue this program, however, let us implement the generalization concerning the Gauss-Bonnet term. As the partial traces are obtained from the brane and internal space Ricci tensors, the connection between the
geometrical approach and the physical setup is performed by the gravitational equation. In considering the Gauss-Bonnet term, it reads

\[
\begin{align*}
&\frac{(D)R_{MN}}{2} - \frac{1}{2} G_N^{MN} (D)R + 2\alpha_2 \left( (D)R_{SKP}^{MN} (D)R_{SKP}^{NP} - 2 (D)R_{SNP}^{MN} (D)R_{SP}^{MN} - 2 (D)R_{S}^{MN} (D)R_{N}^{S} \right) \\
&+ (D)R^{(D)R_{MN}} - \frac{\alpha_2}{2} G_N^{MN} \left( (D)R_{MNAB}^{M} (D)R_{MNAB}^{N} - 4 (D)R_{AB}^{(D)R_{AB}} + (D)R^2 \right) \\
&= 8\pi G_D T_{MN}^{M}.
\end{align*}
\]  

(2.2)

where \( G_D \) stands for the bulk gravitational constant and \( \alpha_2 \) gives the strength of the Gauss-Bonnet term contribution. In the computation of the partial traces, it is interesting to express the final result in terms of as much stress tensor terms as possible. Notice, hence, that from Eq. (2.2) we have

\[
(D)R = \frac{16\pi G_D}{(2-D)} T + \frac{(D-4)(2-D)}{(2-D)} \alpha_2 GB,
\]

(2.3)

where \( GB \) denotes the scalar

\[
GB = (D)R_{MNAB}^{(D)R_{MNAB}} - 4 (D)R_{AB}^{(D)R_{AB}} + (D)R^2.
\]

Now, by isolating \( \alpha_2 GB \) from Eq. (2.3) and reinserting it back into Eq. (2.2) we obtain

\[
(D)R_{MN} - \frac{1}{2} \left( \frac{2}{4-D} \right) G_N^{MN} (D)R + 2\alpha_2 \left( (D)R_{MABC} (D)R_{N}^{ABC} - 2 (D)R_{MANC} (D)R_{N}^{AC} - 2 (D)R_{MA} (D)R_{N}^{A} + (D)R_{MN} \right)
\]

\[
- 2 (D)R_{MN} - (D)R_{MN} = 8\pi G_D \left( T_{MN} + \frac{G_N^{MN}}{4-D} T \right).
\]

(2.4)

Notice that in the limit \( \alpha_2 \to 0 \) we have \( (D)R = \frac{16\pi G_D}{2-D} T \), and Eq. (2.4) reduces to the \( D \)-dimensional Einstein equation, as expected.

From the general dynamic equation (2.4) we can read the necessary partial traces. Starting from \( R_{\mu}^{\mu} \) we have, after a bit of algebra

\[
R_{\mu}^{\mu} = \frac{8\pi G_D}{(D-4)(1+2\alpha_2 (D)R)} \left( (D-5-p)T_{\mu}^{\mu} - (p+1)T_{m}^{m} \right) - \frac{(p+1)}{(D-4)} (D)R
\]

\[
- 2\alpha_2 W^{-2} \left[ (D)R_{\mu ABC} (D)R_{\mu ABC} - 2 (D)R_{\mu A} (D)R_{A}^{A} - 2 (D)R_{\mu A} (D)R_{A}^{A} \right],
\]

(2.5)

whilst

\[
R_{m}^{m} = \frac{8\pi G_D}{(D-4)(1+2\alpha_2 (D)R)} \left( (p-3)T_{m}^{m} - (D-5-p)T_{m}^{m} \right) - \frac{(D-p-1)}{(D-4)} (D)R
\]

\[
- 2\alpha_2 \left[ (D)R_{m ABC} (D)R_{m ABC} - 2 (D)R_{m A} (D)R_{A}^{A} - 2 (D)R_{m A} (D)R_{A}^{A} \right].
\]

(2.6)
Let us point out some relevant points related to the partial traces. First of all, we note that the used notation yields \( T = T^\mu_\mu + T^m_m \). Therefore, the relation \( R^\mu_\mu + R^m_m = (D)R \) can be seen to be indeed fulfilled. The appreciation of Eqs. (2.5) and (2.6) can also explain what we mean by relations that are not closed, in the Introduction. In fact, since \( (D)R \) is the sum of the partial traces, both relations (for \( R^\mu_\mu \) and \( R^m_m \)) are written, among several quantities, in terms of \( R^\mu_\mu \) and \( R^m_m \). Obviously, this behavior indicates a limitation of the usual formalism. On the another hand, the generalization of Eq. (2.1) in order to fully encompass the Gauss-Bonnet geometry is a extremely hard task. The important point, nevertheless, is that it is possible to extract physical information about the braneworld system, even with this limited approach. The reason is that the bulk scalar of curvature can be faced as an input of the model by hand [7, 8, 11]. Therefore, it is reasonable to express the relevant quantities, performed by the partial traces here, in terms of it.

Before using the partial traces into Eq. (2.1), however, we shall write them in a more explicit way, keeping our presentation in a clear basis. Hence, working out the sum terms of Eq. (2.5) it yields

\[
R^\mu_\mu = \frac{1}{(1 + 2\alpha_2 (D)R)} \left\{ \frac{8\pi G_D}{(D - 4)} \left( (D - p - 5)T^\mu_\mu - (p + 1)T^m_m \right) - \frac{(p + 1)}{(D - 4)} (D)R \right\} - 2\alpha_2 W^{-2} \left[ (p+1)R_{\mu\alpha\beta\gamma} R^\mu\alpha\beta\gamma - 4 (p+1)R_{\mu\alpha} R^\mu\alpha + 3 (D)R_{\mu\alpha\beta\gamma} (D)R^\mu\alpha\beta\gamma \right.
\]

\[
+ 3 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} + (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 4 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 2 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 2 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} \right] ,
\]

(2.7)

and similarly

\[
R^m_m = \frac{1}{(1 + 2\alpha_2 (D)R)} \left\{ \frac{8\pi G_D}{(D - 4)} \left( (p - 3)T^m_m - (D - p - 1)T^\mu_\mu \right) - \frac{(D - p - 1)}{(D - 4)} (D)R \right\} - 2\alpha_2 \left[ (D-\mu^{(p-1)}R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 4 (D-\mu^{(p-1)}R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} \right.
\]

\[
+ 3 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} + (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 4 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 2 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} - 2 (D)R_{\mu\alpha\beta\gamma} (D)R_{\mu\alpha\beta\gamma} \right] ,
\]

(2.8)

It is important to stress that in the broader scope of string theory, the Gauss-Bonnet term is the next leading order term of the string tension inverse expansion [12, 13]. Thus, it is possible expand the \( \alpha_2 \) factors disregarding the non linear contributions, i. e., \( (1 + 2\alpha_2 (D)R)^{-1} \simeq 1 - 2\alpha_2 (D)R \). By performing thus, it is possible to realize that Eq. (2.1)
along with the $\alpha_2$-linear version of (2.7) and (2.8) gives

$$
\nabla \cdot (W^a \nabla W) = \frac{W^{a+1}}{p(p+1)} \left\{ \alpha (p+1)RW^{-2} + (p - \alpha) (D-p-1)R + \gamma_3 (D)R \right. \\
\left. + 8\pi G_D \left( \bar{\gamma}_1 T^m_\mu + \bar{\gamma}_2 T^m_\tau \right) + 2\alpha_2 \left[ \alpha W^{-2} A_{GB} - \gamma_3 (D)R + (p - \alpha) B_{GB} \right] \right\} (2.9)
$$

where $\bar{\gamma}_1 = \frac{p(D-p-1) - 2\alpha(D-p-3)}{(D-4)}$, $\bar{\gamma}_2 = \frac{2\alpha(p-1) - p(p-3)}{(D-4)}$, and $\gamma_3 = \frac{\alpha(p+1) - (p-\alpha)(D-p-1)}{(D-4)}$ are simply coefficients depending on the dimensions of the bulk and of the $p$-brane, while $A_{GB}$ and $B_{GB}$ are purely Gauss-Bonnet corrections provided by

$$
A_{GB} := (p+1) R_{\mu a \beta \gamma} (p+1) R^{\mu a \beta \gamma} - 4 (p+1) R_{\mu \nu} (p+1) R^{\mu \nu} - 4 (D) R_{a \mu \nu} (D) R^{a \mu \nu} - 2 (D) R_{a \mu \nu} (D) R^{a \mu \nu} + (D) R_{a b c \mu} (D) R^{a b c \mu} + 3 (D) R_{a b \mu} (D) R^{a b \mu} + 3 (D) R_{a \alpha \beta \gamma} (D) R^{a \alpha \beta \gamma} - 2 (D) R_{a \mu} (D) R^{a \mu} (2.10)
$$

and

$$
B_{GB} := (D-p-1) R_{a b m n} (D-p-1) R^{a b m n} - 4 (D-p-1) R_{a b} (D-p-1) R^{a b} - 4 (D) R_{a m \gamma} (D) R^{a m \gamma} - 2 (D) R_{a m \gamma} (D) R^{a m \gamma} + (D) R_{a \alpha \beta \gamma} (D) R^{a \alpha \beta \gamma} + 3 (D) R_{a b \mu} (D) R^{a b \mu} + 3 (D) R_{a b c \mu} (D) R^{a b c \mu} - 2 (D) R_{a \mu} (D) R^{a \mu} (2.11)
$$

Note that, apart from the $A_{GB}$ and $B_{GB}$ contributions, Eq. (2.9) is given in terms of source factors (stress-tensor terms) and in terms of the brane, the internal space, and the bulk scalar of curvature as well. This is indeed desirable. Within this expression we can study several physical possibilities by looking at these inputs, or setting its behavior for a given model.

After constructing the functional form of Eq. (2.9), the next step is to write down the stress-tensor, explaining the physical content of the model. In order to prepare the formalism for our applications, let us work with

$$
T^a_\mu = -(p+1) \left[ \frac{\Lambda}{8\pi G_D} + \sum_i T_{q}^{(i)} \Delta(D-q-1)(y - y_i) \right] + \tau^a_\mu, \quad (2.12)
$$

and

$$
T^a_m = -(D-p-1) \frac{\Lambda}{8\pi G_D} - \sum_i (q-p) T_{q}^{(i)} \Delta(D-q-1)(y - y_i) + \tau^a_m. \quad (2.13)
$$

In the above equations, $\Lambda$ stands for the bulk cosmological constant, $T_{q}^{(i)}$ is the tension of the $i^{th}$ $q$-brane placed at $y_i$. Besides, $\Delta(D-q-1)$ is the generalized delta term, necessary to fix the brane position (see [11]) and the $\tau$ terms encompass the possibility of additional fields in the bulk, possibly related to the scalar field generating the brane, in the smooth brane framework. By substituting Eqs. (2.12) and (2.14) back into (2.9) and rearranging
the terms it follows that
\[ \nabla \cdot (W^{\alpha+1} \nabla W) = \frac{W^{(\alpha+1)}}{p(p+1)} \left\{ \alpha^{(p+1)} RW^{-2} + (p - \alpha)^{(D-p-1)}R + \gamma_3^{(D)}R + 8\pi G_D(1 - 2\alpha_2) \right. \]
\[ \times \left. \left[ - \frac{\Lambda}{8\pi G_D} \gamma_1 - \gamma_2 \sum_i T_q^{(i)} \Delta^{(D-p-1)}(y - y_i) + \gamma_2 \tau_m^\mu + \gamma_2 \tau_m^m \right] \right. \]
\[ + 2\alpha_2 \left[ - \gamma_3^{(D)}R + \alpha W^{-2} A_{GB} + (p - \alpha) B_{GB} \right] \right\} \right\}, \] (2.14)
where
\[ \gamma_1 = \frac{4}{D-4}[(D-p)(p-\alpha) + \alpha(p + 2) - p], \] (2.15)
and
\[ \gamma_2 = \frac{1}{D-4} \left\{ (p+1)[(D-p)(p-2\alpha) + 6\alpha - p] + (q-p)[2\alpha(p-1) - p(p-1) + 2p] \right\}. \] (2.16)

It is worthwhile to remark that in the limit \( \alpha_2 \to 0 \) Eq. (2.14) recover the usual case of General Relativity studied in Ref. [11].

Finally, since the internal space is assumed to be compact, the integration of the left-hand side of Eq. (2.9) vanishes. Therefore, the one-parameter family of consistency conditions is given by
\[ \oint W^{\alpha+1} \left\{ \alpha^{(p+1)} RW^{-2} + (p - \alpha)^{(D-p-1)}R + \gamma_3^{(D)}R + 8\pi G_D(1 - 2\alpha_2) \right. \]
\[ \times \left. \left[ - \frac{\Lambda}{8\pi G_D} \gamma_1 - \gamma_2 \sum_i T_q^{(i)} \Delta^{(D-p-1)}(y - y_i) \right] \right. \]
\[ + 2\alpha_2 \left[ - \gamma_3^{(D)}R + \alpha W^{-2} A_{GB} + (p - \alpha) B_{GB} \right] \right\} = 0. \] (2.17)

In the next section we shall investigate the physical outputs of this general formalism.

### 3 Applications

In order to apply the previous formalism to our purposes, let us particularize to the \( D = 5 \), \( p = q = 3 \) case. By means of these specifications we have \( \tilde{\gamma}_1 = 3 + 2\alpha, \tilde{\gamma}_2 = 4\alpha, \gamma_3 = 3(\alpha+1), \gamma_2 = 4\gamma_1, \) and \( \gamma_1 = 4\gamma_3 \). Obviously, the 1-dimensional internal space has a null scalar of curvature \( ^{(1)}R = 0 \). Moreover, by taking into account the cosmological data for our Universe it is fairly safe to set \( ^{(1)}R \) down to zero.

Within the above considerations, Eq. (2.17) reduces to
\[ \oint W^{\alpha+1} \left\{ 8\pi G_5(1 - 2\alpha_2) \left[ - \frac{12\Lambda(\alpha + 1)}{8\pi G_5} + (3 + 2\alpha)\tau_\mu^\mu + 4\alpha\tau_m^m - 4(2\alpha + 3) \sum_i T_q^{(i)} \delta(y - y_i) \right] \right. \]
\[ + 2\alpha_2 \left[ - 3(\alpha + 1)^{(5)}R + \alpha W^{-2} A_{GB} + (3 - \alpha) B_{GB} \right] \right\} = 0. \] (3.1)
Of course, after the dimensional particularization, $A_{GB}$ and $B_{GB}$ assume a simpler form compared to (2.10) and (2.11). For instance, every $D - p - 1 (= 1)$ previous contribution vanishes and so on. The remaining form is, however, still far from trivial. Clearly in the limit $\alpha_2 \to 0$, therefore without the contribution of $A_{GB}$ and $B_{GB}$, and by considering $\tau^\mu_\mu = 0 = \tau^m_m$ it yields

$$\oint W^{\alpha+1} \left\{ - 12(\alpha + 1)\Lambda - 32\pi G_5 \sum_i T_3^{(i)} \delta(y - y_i) \right\} = 0.$$  \hspace{1cm} (3.2)

Therefore it leads, for the case $\alpha = -1$ with two branes, to $T_3^{(1)} + T_3^{(2)} = 0$. It reveals the necessity of a negative brane tension in the standard Randall-Sundrum model, as expected.

We note parenthetically that disregarding the contribution of $\tau^\mu_\mu$ and $\tau^m_m$, but in the two brane Gauss-Bonnet framework, the $\alpha = -1$ case is again insightful. The condition to be fulfilled in this case reads

$$\alpha_2 \oint \left( - \frac{A_{GB}}{W^2} + 4B_{GB} \right) = 16\pi G_5 (1 - 2\alpha_2) \left( T_3^{(1)} + T_3^{(2)} \right),$$  \hspace{1cm} (3.3)

demonstrating the possibility of a consistent model without the necessity of a negative brane tension.

Now, the adaptation from thin to thick (or smooth) branes is accomplished by replacing the delta source terms by a bulk scalar field, whose profile (generally assumed as a kink, or many kinks in the multiple brane case) along the extra dimension generates the brane placed at the region of its maximum slope. Therefore, apart from removing the tension terms from Eq. (3.1) it is necessary to particularize $\tau^\mu_\mu$ and $\tau^m_m$ as \cite{8}

$$\tau^\mu_\mu = -4 \left( \frac{1}{2} \Phi' \cdot \Phi' + V(\Phi) \right)$$ \hspace{1cm} (3.4)

and

$$\tau^m_m = \tau^5_5 = \frac{1}{2} \Phi' \cdot \Phi' - V(\Phi),$$ \hspace{1cm} (3.5)

where the prime denotes derivative with respect to the extra dimension. By taking into account the smooth brane scenario, the $\alpha = -1$ case is once again useful. In fact, the consistency condition is provided by

$$\alpha_2 \oint \left( - \frac{A_{GB}}{W^2} + 4B_{GB} \right) = 16\pi G_5 (1 - 2\alpha_2) \oint \Phi' \cdot \Phi'.$$ \hspace{1cm} (3.6)

Thus, as far as the left-hand side of (3.6) is positive, it is possible to generate a smooth brane in a framework endowed with a periodic extra dimension, circumventing the Gibbons-Kallosh-Linde no-go theorem. Note that without the Gauss-Bonnet contribution the previous statement is no longer valid. Obviously, the theorem still holds for the usual case, regarding General Relativity.
4 Final Remarks

The impossibility of generating smooth branes in a bulk containing a compact internal space performs a strong constraint for braneworld model builders. It is, however, no longer true in a more comprehensive context as the one involving the Gauss-Bonnet geometry. We have shown that in this last case, even in the approximation encoded in Eqs. (2.5) and (2.6), smooth branes are, indeed, allowed.

Keeping in mind the net effect of the Gauss-Bonnet terms, we may wonder whether the same effect is obtained in different relativistic theories, as \( f(R) \) for instance. These matters are under current investigation [14]. Finally, we stress again that in the previous Gauss-Bonnet geometric context, singular sources are also precluded from the 3-branes solutions given in Ref. [9].

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