Effective Lagrangian of Thermal QED with External Magnetic Field and the Static Limit of the Polarization Operator

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Abstract

A low-temperature expansion of QED one-loop effective Lagrangian valid for a wide range of parameters is presented in a form of finite sums of elementary functions. Starting from the effective action components of the one-loop polarization operator responsible for Hall conductivity and Debye screening are obtained. It is shown that in a strong background magnetic field the Debye radius depends on spatial direction.

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1 Introduction

In the vicinity of some stellar objects the magnetic field strength and the fermion density may be very high [1, 2] and the corresponding quantum corrections become important [3, 4]. Many essential properties of the electron-positron plasma with a constant and uniform magnetic field may be obtained in the framework of (3+1)-dimensional quantum electrodynamics (QED) with chemical potential $\mu$, temperature $T$ and constant and uniform magnetic field $B$. This theory have been intensively studied in the last 25 years [5-14] using different approaches and important results were obtained. At the same time, the resulting expressions are usually very cumbersome [9-12] (see discussion in Ref. [4]) and it is quite difficult even to match specific calculations made in different approaches. For that reason an approach providing meaningful results in a simple and easily reproducible form would be useful.

The expression for the one-loop effective Lagrangian $L_{\text{eff}}(B, \mu, T)$ in the finite temperature and density QED is well-known [8, 13, 14]. Nevertheless, at specific values $\mu, T$ and $B$ this expression may be calculated only numerically: one can get an analytical answer for plasma- (i.e. $\mu$- and $T$-dependent) part of the effective action $\tilde{L}_{\text{eff}}(B, \mu, T)$ in the $\mu$ or $B \to \infty$ limits [13] or the $T \to 0$ limit [15], while calculation of $\tilde{L}_{\text{eff}}(B, \mu, T)$ at intermediate values of parameters is difficult. In our previous paper [16] we have shown that it is possible to obtain a low-temperature expansion of the one-loop effective Lagrangian keeping $\mu$ and $B$ finite. Low-temperature corrections (as well as $\mu$-dependent part at $T = 0$) may be expressed in terms of elementary functions as a finite sum over excited Landau levels.

The situation with the one-loop polarization operator $\Pi_{\mu \nu}(p)$ is similar to that with the effective action. Indeed, a general expression for the one-loop polarization operator at $B, \mu, T \neq 0$ was obtained in Refs. [10, 11]. It was presented as a decomposition over six covariant transversal tensor structures, but the scalar coefficients had a form much more complicated than $\tilde{L}_{\text{eff}}(B, \mu, T)$ itself. On the other hand, for some applications one needs only few components of the polarization operator in the static limit ($p_0 = 0, \mathbf{p} \to 0$), not the whole answer. Nevertheless, even the calculation of the static limit from the general expression possess serious problems. At the same time, some components of the polarization operator may be obtained from the fermion density as derivatives with respect to $\mu$ and $B$ and as long as the fermion density may be presented in the simple form, one may obtain a simple expression for the above components, too. Such calculations were performed in Ref. [16] to get the components $\Pi_{00}, \Pi_{01} = \Pi^*_{10}$ and $\Pi_{02} = \Pi^*_{20}$ at a low (and zero) temperature in the static limit, which have been obtained as finite sums of the elementary functions (these calculation were confirmed in Ref. [17] where the above components had been calculated at
$T = 0$ as corresponding one-loop integrals at zero external four-momentum).

In this paper we are continuing the study of the finite density QED with a uniform magnetic field at low temperature started in Ref. [16]. We shall demonstrate that in a finite temperature and density QED in a strong uniform magnetic field a low-temperature expansion may be performed for a wider range of parameters $\mu$ and $B$. We shall pay special attention to show how the finite temperature improves analytical properties of the fermion density and related quantities, which are piecewise continuous at $T = 0$ [15, 16]. We shall also demonstrate that starting from the effective Lagrangian and using the general properties of the polarization operator one can obtain a static screening in the finite temperature and density QED with a strong magnetic field. As an example we shall calculate the Debye screening for some simple charge configurations to demonstrate that magnetic field splits Debye radius in a transversal and a longitudinal parts.

The paper is organized as follows: in Section 2 a low-temperature expansion of the one-loop effective Lagrangian is obtained for the partially filled highest excited Landau level. In Section 3 the components of the polarization operator in the static limit are obtained. In Section 4 the approach used for a partially filled highest Landau level is extended for the Landau level edge crossing the Fermi surface. We calculate the Hall conductivity and show how finite temperature smoothen down the inverse square-root singularities. In Section 5 the Debye screening in the QED plasma with external magnetic field is studied.

2 Low-temperature expansion in the QED with a magnetic field

We shall consider a finite density QED with a uniform magnetic field. At nonzero chemical potential the corresponding Lagrangian is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial - eA - \gamma_0\mu - m)\psi \quad .$$

The one-loop effective Lagrangian at $T, \mu, B \neq 0$, $\mathcal{L}_{\text{eff}}(B, \mu, T)$ may be written as follows [8, 13, 14]:

$$\mathcal{L}_{\text{eff}}(B, \mu, T) = \mathcal{L}_{\text{eff}}(B) + \tilde{\mathcal{L}}_{\text{eff}}(B, \mu, T) \quad ,$$

\footnote{In this paper we shall keep the same notations as in Refs. [15, 16]. We take the external magnetic field to be parallel to the $z$-axis, $F_{12} = -F_{21} = B; \mu, eB > 0.$}
where $\tilde{\mathcal{L}}^{\text{eff}}(B, \mu, T)$,

$$
\tilde{\mathcal{L}}^{\text{eff}}(B, \mu, T) = \frac{1}{\beta (2\pi)^2} \sum_{k=0}^{\infty} b_k \int_{-\infty}^{\infty} dp_\parallel \left\{ \ln[1 + e^{-\beta(\varepsilon_k(p_\parallel) - \mu)}] + \ln[1 + e^{-\beta(\varepsilon_k(p_\parallel) + \mu)}] \right\}
$$

(3)

is the contribution due to the finite temperature and density ($\beta = 1/T$, $p_\parallel$ is the modulus of the momentum parallel to the magnetic field, $\varepsilon_k(p_\parallel) = \sqrt{m^2 + 2eBk + p_\parallel^2}$, $b_k \equiv 2 - \delta_{n,0}$) and $\mathcal{L}^{\text{eff}}(B)$,

$$
\mathcal{L}^{\text{eff}}(B) = -\frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} \left[ eBs \coth(eBs) - 1 - \frac{1}{3}(eBs)^2 \right] \exp(-m^2s)
$$

(4)

is the Euler-Heisenberg effective Lagrangian in the purely magnetic case [18].

Integrating Eq.(3) by parts one has:

$$
\tilde{\mathcal{L}}^{\text{eff}}(B, \mu, T) = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} b_n \int_{-\infty}^{\infty} dp_\parallel \frac{p_\parallel^2}{\varepsilon_n(p_\parallel)} \left( f_+(T) + f_-(T) \right)
$$

(5)

$f_\pm(T)$ denotes the Fermi distribution,

$$
f_\pm(T) = \frac{1}{1 + e^{\beta(\varepsilon_\pm - \mu)}}.
$$

(6)

Using representation (3) one can easily obtain a zero temperature limit: at $T \to 0$ the Fermi distribution approaches the step-function, $\lim_{T \to 0} f_\pm = \theta(\pm\mu - \varepsilon)$ and Eq.(3) reads [15]:

$$
\tilde{\mathcal{L}}^{\text{eff}}(T = 0, B, \mu) = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} b_n \left\{ \mu \sqrt{\mu^2 - m^2 - 2eBn - (m^2 + 2eBn)} \ln \left( \frac{\mu + \sqrt{\mu^2 - m^2 - 2eBn}}{\sqrt{m^2 + 2eBn}} \right) \right\},
$$

(7)

where $[\ldots]$ denotes the integral part. To evaluate a low temperature expansion we make change a of variables and integrate Eq.(5) by parts again:

$$
\tilde{\mathcal{L}}^{\text{eff}}(T, B, \mu) = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} b_n \left\{ \varepsilon \sqrt{\varepsilon^2 - m^2 - 2eBn - (m^2 + 2eBn)} \log(\varepsilon + \sqrt{\varepsilon^2 - m^2 - 2eBn}) \right\} (f_+ + f_-) \bigg|_{\varepsilon = \mu}^{\infty} - \frac{m^2 + 2eBn}{\sqrt{m^2 + 2eBn}} - \frac{m^2 + 2eBn}{\sqrt{m^2 + 2eBn}}
$$

(8)
\[
\int_{\sqrt{m^2+2eBn}}^{\infty} d\varepsilon \left[ \varepsilon \sqrt{\varepsilon^2 - m^2 - 2eBn} - (m^2 + 2eBn) \log(\varepsilon + \sqrt{\varepsilon^2 - m^2 - 2eBn}) \right] \left( \frac{\partial f_+}{\partial \varepsilon} + \frac{\partial f_-}{\partial \varepsilon} \right) .
\]

At low temperature the contributions from \( \frac{\partial f_+}{\partial \varepsilon} \), \( f_- \) and from the upper limit in the first term are exponentially small \((\mu > 0)\). Making change of variables and rewriting the derivative of the Fermi distribution as \( \frac{\partial f_+}{\partial \varepsilon} + \frac{\partial f_-}{\partial \varepsilon} = -\frac{1}{4T} \cosh^{-2} \left( \frac{\varepsilon - \mu}{2T} \right) \) we may present Eq.(8) as follows:

\[
\tilde{\mathcal{L}}^\text{eff}(T, B, \mu) = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} b_n \left\{ - (m^2 + 2eBn) \log(m^2 + 2eBn) \left( \mu - \sqrt{m^2 + 2eBn} \right) + \right.
\]

\[
\int_{\sqrt{m^2+2eBn}}^{\infty} dq \left[ \left( \mu + q \right) \sqrt{\left( \mu + q \right)^2 - m^2 - 2eBn} - (m^2 + 2eBn) \log \left( \left( \mu + q \right) + \sqrt{\left( \mu + q \right)^2 - m^2 - 2eBn} \right) \right] \cosh^{-2} \left( \frac{q}{2T} \right) \right\} .
\]

At low temperature the derivative of the Fermi distribution approaches the \( \delta \)-function and in the \( T \to 0 \) limit we arrive at Eq.(4). To get low-temperature corrections one may extend lower boundary of the integration in Eq.(8) to \(-\infty\) (the function \( \frac{1}{4T} \cosh^{-2} \left( \frac{q}{2T} \right) \) decreases sharply as one moves off the point \( q = 0 \)). Then one should expand the expression in the brackets in a Taylor series at \( q = 0 \) to obtain in the leading order the following low-temperature correction to the zero-temperature Lagrangian [16]:

\[
\Delta \tilde{\mathcal{L}}^\text{eff}(T, B, \mu) = \frac{eBT^2}{6} \sum_{n=0}^{\infty} b_n \frac{\mu}{\left( \mu^2 - m^2 - 2eBn \right)^{1/2}} + O(T^4) .
\]

Hence, only those Landau levels which have the edge laying below the Fermi surface give rise to the effective Lagrangian at a low temperature.

The above expansion is valid as long as

\[
\frac{T}{\mu - \sqrt{m^2 + 2eBn}} \ll 1 ,
\]

which means that the distance from the edge of any Landau level \( \varepsilon_k(p\parallel) = \sqrt{m^2 + 2eBk} \) to the Fermi surface \( \mu \) is much greater than the temperature (this means also that the Landau level with \( n = \left[ \frac{\mu^2 - m^2}{2eB} \right] \) (highest excited) should be partially filled, i.e. its edge cannot coincide with the Fermi surface).
3 Fermion density and polarization operator

Having the expressions for the effective Lagrangian we may move forward to calculate the fermion density \( \rho = \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \mu} \), the magnetization \( M = \frac{\partial \mathcal{L}_{\text{eff}}}{\partial B} \), the Hall conductivity and some components of the polarization operator in the static limit \( p_0 = 0, \ p \to 0 \).

Using Eqs. (7), (9) and definitions of the fermion density \( \rho = \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \mu} \) one has:

\[
\rho(B, \mu, T) = \left( \frac{eB}{2\pi^2} \right)^2 \sum_{n=0}^{\infty} b_n \sqrt{\mu^2 - m^2 - 2eBn} \left\{ 1 - \frac{T^2\pi^2}{6} \frac{m^2 + 2eBn}{(\mu^2 - m^2 - 2eBn)^2} \right\} + O(T^4).
\]

Then, we may calculate five components of the polarization operator in the static limit \( p_0 = 0, \ p \to 0 \). As it was shown in Ref. [19], the \( \Pi_{00} \) component of the polarization operator may be written in the static limit as a derivative of the fermion density with respect to the chemical potential, \( \Pi_{00}(p_0 = 0, \ p \to 0) = e^2 \frac{\partial \rho}{\partial \mu} \),

\[
\Pi_{00}(p_0 = 0, \ p \to 0) = e^2 \frac{eB\mu}{2\pi^2} \sum_{n=0}^{\infty} b_n \left\{ (\mu^2 - m^2 - 2eBn)^{-1/2} + T^2\pi^2 \frac{m^2 + 2eBn}{(\mu^2 - m^2 - 2eBn)^{5/2}} \right\}.
\]

At \( B = 0 \) \( \Pi_{00}(p_0 = 0, \ p \to 0) \) defines the Debye screening radius, \( r_D^2 = \Pi_{00}(p_0 = 0, \ p \to 0) \) [13] but this relation does not hold for \( \mu, B \neq 0 \) as the tensor structure of the polarization operator is more complicated now (modification of the Debye screening will be discussed in Section 5. See also an instructive QED_{2+1} example [20]).

The components \( \Pi_{01} = \Pi_{10} \) and \( \Pi_{02} = \Pi_{20} \) in the static limit may be expressed via derivatives of the fermion density with respect to magnetic field:

\[
\Pi_{0j}(p \to 0) = i e \varepsilon_{ij} p_i \frac{\partial \rho}{\partial B} \quad i, j = 1, 2 \quad ,
\]

which follows from the definition of the polarization operator,

\[
\Pi_{\mu\nu}(x, x') = i \frac{\delta < j_\mu(x) >}{\delta A_\nu(x')} \quad .
\]
Analyzing the general structure and symmetry properties of the polarization operator we may readily define one of the scalar coefficients of the polarization operator. The polarization tensor in QED$_{3+1}$ with a uniform magnetic field at $T, \mu \neq 0$ may be decomposed over six tensor structures [10] (we are using a slightly modified with respect to Refs. [10, 16] expression):

$$\Pi_{\mu \nu}(p|T, \mu, B) =$$

$$\left( g_{\mu \nu} - \frac{p_{\mu}p_{\nu}}{p^2} \right) A + \left( \frac{p_{\mu}p_{\nu}}{p^2} - \frac{p_{\mu}u_{\nu} + u_{\mu}p_{\nu}}{(pu)^2} + \frac{u_{\mu}u_{\nu}}{(pu)^2} \right) B +$$

$$F_{\alpha \beta} F^{\alpha \beta} F_{\nu \phi} F^{\nu \phi} p_{\rho} C + F_{\mu \lambda \rho} p^\lambda F_{\nu \phi} p^\phi D +$$

$$i \left( p_{\mu} F_{\nu \lambda} p^\lambda - p_{\nu} F_{\mu \lambda} p^\lambda + p^2 F_{\nu \mu} \right) E + i \left( u_{\mu} F_{\nu \lambda} p^\lambda - u_{\nu} F_{\mu \lambda} p^\lambda + (pu) F_{\nu \mu} \right) F .$$

(16)

The scalars $A, B, C, D, E$ and $F$ are the functions of $p_0^2, p_3^2 = p_{||}^2, p_1^2 + p_2^2 = p_{\perp}^2$ and $B$ ($u^\mu$ is the 4-velocity of the medium [19], $u^\mu = (1, 0, 0, 0)$), therefore only the last tensor structure may contribute to $\Pi_{0j}$ and we may define the coefficient $F$ in the static limit ($F \neq 0$ at $B, \mu \neq 0$ only [4]):

$$F(p_0 = 0, \ p \to 0) = \frac{e}{B} \frac{\partial \rho}{\partial B} .$$

(17)

It is easy to see that the components $\Pi_{0j}$ describe a conductivity in the plane orthogonal to the magnetic field which is Hall-like [16, 21]:

$$\sigma_{ij} = \left. \frac{\partial j_i}{\partial E_j} \right|_{E \to 0} = \left. \frac{\partial \Pi_{0b}(p)}{\partial p_j} \right|_{p \to 0} = e \varepsilon_{ij} \frac{\partial \rho}{\partial B} \ i, j = 1, 2 .$$

(18)

Substituting into Eq.(18) the expression for the fermion density, Eq.(12) one has

$$\Pi_{0j}(p_0, \ p \to 0) = \frac{e \varepsilon_{ij} p_i}{2\pi^2} \left[ \frac{1}{2} \frac{m^2 - m^2 - 3eBn}{2\pi^2} \right] \sum_{n=0}^{\infty} b_n \times$$

$$\left\{ \frac{\mu^2 - m^2 - 3eBn}{(\mu^2 - m^2 - 2eBn)^{1/2}} - \frac{T^2}{3} \right\} .$$

(19)
It follows from the above expression that the Hall conductivity in the QED$_{3+1}$ is an oscillating function of the chemical potential and the magnetic field and in the $T \to 0$ limit it has an inverse square-root singularity (we would like to remind that polarization operator in QED$_{3+1}$ with a uniform magnetic field has just the same kind of singularities [1]). These oscillations are close to “giant oscillations”, well-known in condensed matter physics [22] and resonant effects in QED [3, 6, 7] and semiconductors [23, 24]. In the next Section we shall calculate finite-temperature corrections at the points $\mu \to \sqrt{m^2 + 2eBk}$ to show how the temperature cures the singularities.

4 Low-temperature expansion at $\mu = (m^2 + 2eBk)^{1/2}$

Now we shall show how a low-temperature expansion at $\mu \to \sqrt{m^2 + 2eBk}$ may be derived. At $T = 0$ the functions, calculated in the previous Section are not smooth and components of the polarization operator even do not possess a continuous limit. As an example we shall consider the Hall conductivity, $e \frac{\partial \rho}{\partial B}$.

Differentiating effective Lagrangian Eq.(3) with respect to $\mu$ to obtain the fermion density at arbitrary $T$ and taking derivative with respect to $B$ (we assume below $T$ to be small, thus $f_-$-dependent part may be omitted) one has:

$$
\sigma = \sigma^{(1)} + \sigma^{(2)} = \frac{e}{(2\pi)^2} \sum_{k=0}^{\infty} b_k \int dp_{\|} f_{\|} \frac{eB}{(2\pi)^2} + \sum_{k=0}^{\infty} b_k \int dp_{\|} \frac{\partial f_{\parallel}}{\partial \varepsilon_k} \frac{\partial \varepsilon_k}{\partial B} . \tag{20}
$$

Let us consider $\sigma^{(2)}$ first. Making change of variables one gets:

$$
\sigma^{(2)} = \frac{eB}{\pi^2} \sum_{k=1}^{\infty} b_k \int_{\sqrt{m^2 + 2eBk} - \mu}^{\infty} dz \frac{1}{\sqrt{z + \mu}^2 - m^2 - 2eBk} (\frac{1}{4T}) \cosh^{-2} \left( \frac{z}{2T} \right) . \tag{21}
$$

Assuming $\mu \to \sqrt{m^2 + 2eBk_0}$ and extracting the $k_0$-term from the sum (21) (the remaining contribution to $\sigma^{(2)}$ may be derived along the same lines as in Section 2) one has:

$$
\lim_{\mu \to \sqrt{m^2 + 2eBk_0}} \sigma^{(2)}_{k_0} = - \frac{eB}{4\pi^2 T} \int_0^{\infty} dz \frac{z^{-1/2}}{\sqrt{z + 2\mu}} \cosh^{-2} \left( \frac{z}{2T} \right) \tag{22}
$$

Expanding the inverse square root in Eq.(22) at $z = 0$ we finally obtain:

$$
\sigma^{(2)}_{k_0} = - \frac{e(2^{3/2} - 1)\zeta(\frac{3}{2})}{8\pi^{3/2}} \frac{\mu^2 - m^2}{\sqrt{2T\mu}} . \tag{23}
$$
Making a similar calculation for $\sigma_{(1)}$ one has $\sigma_{(1)} \sim \sqrt{T}$. Therefore, at a finite temperature the inverse square-root singularity in the expression for the Hall conductivity (as well as for above-mentioned components of the polarization operator) is smoothened down. The leading contribution in the low-temperature limit is finite and proportional to $T^{-1/2}$. Making the similar expansions for the fermion density and magnetization one may see that the finite-temperature correction levels corresponding zero-temperature expressions (cf. \[14, 15\]).

5 Debye screening via effective action

The most interesting application of the above analysis is the possibility to study the static screening properties of the magnetized nondissipative relativistic electron-positron gas. To study the screening the gauge field Green function $D_{\mu\nu}$ with appropriate quantum corrections is required,

$$D_{\mu\nu} = \left(D_{\mu\nu}^{-1} - \Pi_{\mu\nu}\right)^{-1},$$

$D_{\mu\nu}$ is the tree-level propagator.

Formally, there are no obstacles to obtain an expression for $D_{\mu\nu}$ by inverting the corresponding matrix, but with $\Pi_{\mu\nu}$ as complicated as in Eq.(16) it requires unwieldy algebra. Another possibility to calculate $D_{\mu\nu}$ is to solve the eigenvalue equation, $\Pi_{\mu\nu}(p) b^\nu = \kappa b_\mu$ and to rewrite $\Pi_{\mu\nu}(p)$ in the diagonal form \[25\],

$$\Pi_{\mu\nu}(p) = \sum_{i=1}^{3} \frac{\kappa_i}{b_\mu^{(i)} b_\nu^{(i)}} b_\mu^{(i)} b_\nu^{(i)}.$$  

Then, one readily obtains

$$D_{\mu\nu}(p) = \sum_{i=1}^{3} \frac{1}{p^2 - \kappa_i b_\mu^{(i)} b_\nu^{(i)}} ,$$

but this representation either does not simplify the calculation of a specific component of $D_{\mu\nu}$.

To investigate the Debye screening one needs $D_{00}(p_0 = 0, \mathbf{p})$. For this particular case we have made the relevant calculations in the Feynman gauge keeping all structures contained in the expression \[16\]. We have no possibility to present this cumbersome result and show the static limit of the expression only. The coefficients $\mathcal{A}, \mathcal{C}, \mathcal{D}$ and $\mathcal{E}$ happen to be subleading in $p^2$ (see also Ref. \[17\]) and may lead to some finite corrections (of order $e^2$) which do not affect the qualitative picture. The final expression may be written as follows ($\tilde{F} = B\mathcal{F}$):
\[ D_{00} = -\frac{p^2}{(\Pi_{00} + p^2)\mathbf{p}^2 + \mathcal{F}^2 p_\perp^2} \]  

(27)

The Coulomb potential,

\[ A_0(x) = \int d\mathbf{p} e^{-ip_0x} D_{00}(p_0 = 0, \mathbf{p}) j_0(p \mathbf{x}) \]  

(28)

in the case of a point-like test charge may be calculated only numerically so we shall study examples of simpler charge configurations (i.e. these reducing one of momentum integrations in Eq.(28)). For instance, let us consider the field of an infinitely long charged thin rod parallel to \( z \)-axis.

\[ A_0(r) = \rho_c \int_0^\infty p_\perp dp_\perp \int_0^{2\pi} d\phi e^{-ip_\perp r \cos \phi} \frac{1}{\Pi_{00} + \mathcal{F}^2 + p_\perp^2} = \rho_c K_0(r \sqrt{\Pi_{00} + \mathcal{F}^2}) \ , \]  

(29)

\( K_0 \) is the modified Bessel function, \( \lim_{r \to \infty} K_0(r) \sim \sqrt{\frac{\pi}{2r}} e^{-r} \) (cf. screening in QED\(_{2+1} \), Ref. [26]).

Therefore, in the presence of a strong background magnetic field the Debye screening is described by at least two scalar functions and the suggestion that \( \Pi_{00} \) only is responsible for screening at \( B, \mu \neq 0 \) made in Ref. [17] is incorrect.

Then, let us consider potentials of charged planes parallel and orthogonal to magnetic field:

\[ A_0(x_1) = \rho_c \int dp_1 \frac{e^{-ip_1 x_1}}{\Pi_{00} + \mathcal{F}^2 + p_1^2} = \frac{\pi \rho_c}{\sqrt{\Pi_{00} + \mathcal{F}^2}} e^{-x_1 \sqrt{\Pi_{00} + \mathcal{F}^2}} \ , \]  

(30)

\[ A_0(z) = \rho_c \int dp_\parallel \frac{e^{-ip_\parallel z}}{\Pi_{00} + p_\parallel^2} = \frac{\pi \rho_c}{\sqrt{\Pi_{00}}} e^{-z \sqrt{\Pi_{00}}} \ , \]  

(31)

We may see that the Debye radius is splitted in plasma with external magnetic field, \( r^\parallel_D \neq r^\perp_D \); therefore, screening is anisotropic (in Ref. [27] it was shown that the small-distance screening depends on direction at \( \mu = 0 \) and high \( T \)). We would like to stress that at a realistic values of parameters the long distance anisotropy is quantitatively small (in the vicinity of Landau level edge \( \Pi_{00} \) and \( \mathcal{F}^2 \) may be of the same order, but in that case the screening radius will be much less than an average distance between the particles and the Debye approximation fails).

In the above-described examples the anisotropy in the leading order is due to additional tensor structure only (cf. resonant effects described in Refs. [3, 4, 23, 24]).
Supposing that the coefficients $A, C, D$ and $E$ will not affect qualitatively the other components of $D_{\mu\nu}$ we may write the latter in the static limit as follows (we are using the Feynman gauge):

$$D_{\mu\nu}^{-1} = \begin{pmatrix}
-(\Pi_{00} + p^2) & i\tilde{F}_{p2} & -i\tilde{F}_{p1} & 0 \\
-i\tilde{F}_{p2} & p^2 & 0 & 0 \\
i\tilde{F}_{p1} & 0 & p^2 & 0 \\
0 & 0 & 0 & p^2
\end{pmatrix}$$

(33)

It is easy to check that antysymmetric $p$-linear structure in the polarization operator, unlike Chern-Simons term in QED$_{2+1}$ [28], does not lead to magnetic screening (we would like to stress that with the polarization operator Eq.(16) condition $\Pi_{ii} = 0$ [29] does not guarantee the absence of the magnetic mass).

We have demonstrated that it is possible to perform a low-temperature expansion in QED$_{3+1}$ with a uniform magnetic field. In the framework of this expansion both effective Lagrangian and derivative functions have the same structure as in the $T = 0$ limit, i.e. they are finite sums over excited Landau levels. We have calculated the components of the polarization operator responsible for the Hall conductivity and Debye screening in a finite fermion density QED$_{3+1}$ with a uniform magnetic field. We have shown how finite temperature cures zero-temperature singularities. Static potential of some charge configurations was calculated and anisotropy of the Debye screening was demonstrated.

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