Of Matter Less Repulsive than a Cosmological Constant

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The case grows ever stronger that the average density of matter, ordinary and dark, is less than the critical density required for a flat universe. However, most of determinations of the mass density have been dynamical, hence sensitive only to matter which is clustered at or below the scale of the observed dynamical systems. The density may still be critical if there is a dark matter component which is relatively smooth on the scales of galaxies or clusters. Thoughts on this matter have focused on the possibility of an effective cosmological constant or vacuum energy. In this letter we examine an alternative possibility - that there is a second component to the dark matter which has a repulsive self-interaction. We show that given even very weak self-repulsion, this dark matter would remain unclustered. While this repulsive alternative is perhaps aptly named, it is arguably at least as palatable as a cosmological constant.

For many years theoretical cosmologists have favoured flat models with critical total energy density $\Omega = 1$, where $\Omega$ is dominated by non-relativistic matter. This theoretical prejudice is based on the Dick coincidence argument and standard models of inflation. The simplest, most generic models of inflation predict $|\Omega - 1| \ll 1$. Indeed, until the observational evidence favouring low $\Omega$ in clustered matter became overwhelming, $|\Omega - 1| \ll 1$ was widely viewed as the unalterable, defining prediction of inflation (and still is by some).

Confronted by the growing evidence that clustered matter does not add up to more than about 30% of the critical density, two alternative interpretations have been widely circulated. The first view is that the total energy density of the universe is indeed subcritical, so the spatial geometry is hyperbolic, not Euclidean. Advocates of this view relinquish the standard prediction, and standard models of inflation. The alternative view is that the universe is indeed flat, but that the 70% of the critical density is in the form of a cosmological constant, also known as vacuum energy. Since vacuum energy does not cluster, such energy density or its absence, would not have been detected by observations of the dynamics of clustered systems.

The main theoretical argument for a cosmological constant solution is that the universe remains flat, ($\Omega = 1$), so one can continue to accept the standard inflationary models and their predictions. Recent results on the Hubble parameter as a function of redshift from observations of type Ia Supernovae, may favour a cosmological-constant-dominated universe over either an open universe or a flat matter-dominated universe. There is also some preliminary evidence for a doppler peak in the cosmic microwave background power spectrum at multipole number $\ell \sim 200$. This would rule out an open universe with $\Omega = 0.3$, and make a strong case for a flat universe. But an $\Omega = 1$ universe with $\Omega_\nu = 0.3$ in clustered matter does not prove the case for cosmological constant. We need to consider every alternative possibility, and put each of them to the test. Then, "...when you have eliminated the impossible, whatever remains, however improbable, must be the truth."  

One interesting alternative is that there is an additional form of unclustered dark matter, call "quintessence" or $Q$-matter with equation of state $\rho = \omega p$ where $-1 < \omega < 0$. Since $\omega < 0$ this $Q$-matter exerts a repulsive tension that inhibits clustering. The special case $\omega = -1/3$ was first considered by Kolb (Kolb dark matter), who showed that one could then have a closed universe with $\Omega > 1$ even if the energy density in ordinary matter is subcritical, $\Omega_\nu < 1$. However, even this wide class of $Q$-matter models does not exhaust the possibilities. Here we suggest another.

It is possible that the universe is flat, but that 70% of the critical density is composed not of vacuum energy but of unclustered non-relativistic matter. The standard objection to this scenario is that any ordinary matter will fall into the gravitational potential wells defined by galaxies, clusters and large scale structure, and become ipso facto clustered matter. Our solution to this conundrum is to imbue the new type of matter, which we term X-matter, with a repulsive self-interaction strong enough to prevent clustering in existing non-linear structures.

Let us be the first to admit that such a repulsive alternative is neither elegant nor natural. But, as well shall demonstrate, current observational and theoretical constraints do not rule out the possibility. So for now we must add X-matter to our list of possible improbables.

A simple way to accommodate repulsive matter is to postulate a new broken gauge interaction under which ordinary matter, including ordinary dark matter, is neutral (or more generally is a singlet), but under which the X particle carries a charge. We will consider only the Abelian case and take the charge of the X to be unity. The strength of the $U(1)_X$ gauge interaction is characterised by the analogue $\alpha_X$ of the fine structure constant, with an implicit condition that $\alpha_X < 1$. The range of the interaction is determined by the mass $m_X$ acquired by the $U(1)$ gauge boson in the symmetry breaking process. Because the gauge interaction is broken, there is no constraint on the total X-charge of the universe. It has been
suggested to us that the $U(1)$ gauge interaction might somehow be connected with a gauging of lepton and/or baryon number. We have not carefully considered the complications and constraints that implies, given the fact that baryon and lepton number are carried by ordinary matter.

In the neighbourhood of a galaxy or other gravitational potential well, a smooth background of $X$ particles of density $n_X$ will develop an overdensity $\delta n_X/n_X$. This overdensity will grow until the Coulomb repulsion of the $X$’s overwhelms their gravitational attraction to the ordinary matter. The gravitational force on an $X$ particle due to the galaxy or cluster is

$$ F_g = \frac{GMm_X}{r^2} $$

(1)

where $M$ and $r$ are the mass and radius of the galaxy or cluster. The repulsive force on the $X$ particle trying to fall into the galaxy once there is an $X$ overdensity $\delta n_X$ is approximately:

$$ F_c = \frac{4\pi\alpha_X\hbar^2\delta n_X}{3m_B} $$

(2)

If $F_c > F_g$, then the $X$ particle will not fall into the galaxy, and so the overdensity of $X$’s will build until

$$ \frac{\alpha_X\delta n_X}{m_Xm_B} \approx \frac{3GM}{4\pi\hbar^2r^2}. $$

(3)

Taking the astronomical system to be approximately spherical, and expressing its mass in terms of its radius and average density $\rho_{av}$, we can find the $X$ overdensity:

$$ \delta n_X = \frac{Gr\rho_{av}m_Xm_B}{\alpha_X\hbar^2}. $$

(4)

Since the whole point of this exercise is to insist that the $X$ matter not be clustered, we require that

$$ m_X\delta n_x \ll \rho_{av} $$

(5)

This implies that

$$ \frac{m_X^2m_B}{\alpha_X} \ll \frac{\hbar^2}{Gr} = \frac{M_{Pl}^2}{hc} $$

(6)

For a cluster, a rather generous $r \approx 10\text{Mpc}$, gives

$$ \frac{m_X^2m_B}{\alpha_X} \ll 10^4\text{GeV}^3/c^6 $$

(7)

We would also probably prefer $m_X, m_B \gg 1\text{eV}$ in order that the $X$’s not be free streaming during the growth of large scale structure. These limits imply that $X$ and $B$ masses in the eV-GeV range are realistic.

Our third constraint comes from Standard Big Bang Nucleosynthesis (SBBN). The success of the SBBN model in reproducing the light element abundances, informs us that we cannot add too many light degrees of freedom. In this instance, “light” means $m \lesssim 1 - 10\text{MeV}$. Given our constraint (7) above, there would be no problem taking both $m_X$ and $m_B$ heavy enough to evade this bound, however, we might also be interested in having them lighter. The maximum allowed is generally quoted to be approximately one-third of a light neutrino family equivalent. This would be impossible if the light $X$ or $B$ was at the same temperature as the ambient ordinary matter. However, even ordinary neutrinos are expected to be colder than the photons during SBBN.

Neutrinos are thermally coupled to the photons only via weak interactions. Since the strength of the weak interactions falls rapidly with the energy of the constituents, the neutrinos thermally decouple from the ambient plasma at a few times $10^{10}$°K. When the electrons and positrons annihilate soon after, their entropy is injected nearly entirely into the electro-magnetically coupled photon-electron-nucleon plasma, leaving the neutrinos slightly colder: $T_\nu = (4/11)^{1/3}T_{\gamma}$. Which is the cube root of the ratio of the effective number of relativistic degrees of freedom after $e^+e^-$ annihilation, to the effective number of relativistic degrees of freedom before $e^+e^-$ annihilation.

An analogous calculation of the temperature of the $B$ finds that

$$ \frac{T_B}{T_\nu} = \left(\frac{g_\nu}{g_B}\right)^{1/3}. $$

(8)

Here $g_\nu = \frac{43}{7}$ is the effective number of relativistic degrees of freedom before neutrinos thermally decouple. (photons contribute 2, electrons contribute $4 \times \frac{7}{8}$, neutrinos contribute $2 \times \frac{7}{8}$ per family.) Since

$$ \frac{B}{\rho_\nu} = \frac{T_B^4}{\frac{7}{8}(T_\nu)^4} $$

(9)

the requirement that the $B$ contribute less than 0.3 effective neutrino families is $(g_\nu/g_B)^{4/3} < 0.3 \times \frac{7}{8}$. This implies

$$ g_B > 30 $$

(10)

This is easily accomplished; in the $SU(3) \times SU(2) \times U(1)$ standard model $g \gtrsim 60$ before the QCD phase transition at $T \approx 150\text{MeV}$, and $g > 100$ at $T \gtrsim 100\text{GeV}$. Since, the $B$ boson couples only to $X$ charge, and since ordinary matter is $X$-neutral, the $B$ will thermally decouple from the ordinary plasma as soon as the $X$ does.

This also indicates that within the standard model it would also not be particularly difficult to accommodate a light $X$ ($m_X \lesssim 10\text{MeV}$) in addition to a light $B$. The limit would be $g_B > 47$, if the $X$ were a Weyl fermion (like a standard left handed neutrino) $g_B > 63$, if the $X$ were a Dirac fermion (like an electron) and $g_B > 49$. if the $X$ is a boson. Thus so long as the $X$ and $B$ are
$SU(3)_c \times SU(2)_L \times U(1)_Y$ singlets, they would have thermally decoupled early enough to evade the nucleosynthesis constraints on light particles. Since any particle dark matter candidate must have a small interaction cross-section with ordinary matter, the assumption that it carries no $3 - 2 - 1$ quantum numbers is the simplest one.

We also want the $X$ particles to contribute $\Omega_X \simeq 0.7 - 0.8$ to the mass density of the universe. This means that

$$n_X = \frac{\Omega_X \rho_{\text{crit}}}{m_X},$$

where $\rho_{\text{crit}} = 1.88 \times 10^{-26} h^2 \text{kg m}^{-3}$ is the critical density, and $h$ is the Hubble constant in units of $100 \text{km sec}^{-1} \text{Mpc}^{-1}$. The number of $X$’s must be negligible compared to the number of $X$’s (in order for Coulomb repulsion to keep $X$’s from collecting in galaxies and clusters). Thus there must be a nearly perfect $X - \bar{X}$ asymmetry. Large asymmetries are, in general, difficult to estimate for the age of the oldest globular cluster is [9], the magnitude of the asymmetry can best be characterised by the number density $n_X$ of $X$’s, divided by the number density of bosons in thermodinamic equilibrium at temperature $T_B$. We thus require

$$n_X \ll \frac{1}{4} T_B^3.$$  

Expressing $n_X$ in terms of $m_X$, $\Omega_X$ and $\rho_{\text{crit}}$; and $T_B$ in terms of $g_B/g_\gamma$ and the measured value of $T_\gamma = 2.7^\circ \text{K}$, we find

$$m_X \gg 4.6 \times 10^{-7} \text{GeV} \frac{\Omega_X}{0.7} \frac{g_B}{100}$$

Indicating that this is not a particularly restrictive constraint. For an $X$ mass of 1GeV, we see that an $X - \bar{X}$ asymmetry of only $\sim 3 \times 10^{-7}$ is required.

What are the difficulties with this scenario? First there is the age problem. The age of an $\Omega = 1$ matter dominated universe is $t_\circ = 6.52 \times 10^9 h^{-1} \text{yrs}$. The current best estimate for the age of the oldest globular cluster is [3], 11.5 $\pm$ 1.3 $\times 10^9 \text{yrs}$, with a one-sided 95% confidence level lower limit of 9.5Gyr. For $\Omega = 1$, this implies $h \leq 0.67$, with no allowances for any protracted time interval between matter domination and globular cluster formation. An expansion rate of $H_0 = 67 \text{ km sec}^{-1} \text{Mpc}^{-1}$ is consistent with the lower end of most recent determinations of the Hubble parameter.

The second concern is the limits placed by observations of the light curves of type Ia supernovae [10] on $\Omega_{nr}$ in non-relativistic matter, which exclude $\Omega_{nr} = 1$ at better than 95% confidence level. Nevertheless, there are still concerns over both the small sample size in these ongoing studies, and over the systematic effects of reddening.

Finally, there are certainly questions of the “naturalness” of this model. The fact that one seems to require yet another form of dark matter with a considerable asymmetry is not particularly appealing. On the other hand, since the X-matter is non-relativistic, the ratio $\Omega_X/\Omega_B$ (and by implication $\Omega_X/\Omega_{\text{dm}}$) would be time-independent at temperatures below the mass of the lightest of the proton, the $X$ particle and the ordinary dark matter particle. Thus $\Omega_{nr} = 0.3$ would not define a special epoch. This is a philosophical motivation not shared by either a hyperbolic universe or by a universe dominated by a cosmological-constant.

The possibility of preserving the standard prediction of inflation that $\Omega = 1$, without resorting to a cosmological constant holds some appeal. The classic argument forbidding this is that non-relativistic matter would clump in existing gravitationally bound structures such as galaxies and clusters to an extent not consistent with observations. We have pointed out that this argument relies implicitly on the argument that all forms of dark matter are not self-interacting, and that a relatively generic form of interaction could lead to non-clustering dark matter. We make no attempt to extol the great beauty of this model, nor to identify the X-matter with any particular well-motivated, technically or philosophically natural particle physics candidate, leaving this rather to the reader’s own imagination.

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