Quantum vacuum, dark matter, dark energy and spontaneous supersymmetry breaking

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We study the vacuum condensate characterizing many physical phenomena. We show that such a condensate may lead to non-trivial components of the dark energy and of the dark matter and may induce the spontaneous supersymmetry breaking, in a supersymmetric context. In particular, we consider the condensate induced by thermal states, fields in curved space-time and mixed particles.

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I. INTRODUCTION

According to recent experimental data, the universe consists of 68% of dark energy [1]-[3] and about of 27% of dark matter. The rest is ordinary matter. The dark energy and the dark matter problem have been analyzed in different ways [32]-[37], however, their solution represents still a very big challenge.

Another object of study which has had a huge impact on contemporary physics is the Supersymmetry (SUSY) [32]-[37]. SUSY is a symmetry of nature that relates any boson to a fermion (called superpartner) with the same mass and internal quantum numbers, and vice-versa. However, there is no evidence for the existence of the superpartners. Therefore, SUSY must be a broken symmetry, allowing for superparticles to be heavier than the corresponding Standard Model particles, or it must be ruled out as a fundamental symmetry. Intensive study has been devoted to the analysis of the possibility of SUSY breaking.

Here we report on recent results [52] according to which the vacuum condensate characterizing many phenomena [38]-[51], such as Hawking effect and fields in curved space, can explain the origin of dark energy and matter components [52] and can describe the spontaneous SUSY breaking [53]-[57]. In fact, all the phenomena inducing condensates have a non-zero vacuum energy which cannot be removed by use of the normal ordering procedure. The origin of the non trivial vacuum energy is due to the fact that the physical vacuum of such systems is a condensate of couples of particles and antiparticles which generate a positive value of the zero point energy. Such an energy can contribute to the dark sector of the universe and, in a supersymmetric context, induces the spontaneous SUSY breaking [53].

In particular, we analyze thermal states, fields in curved space and mixed particles and we show that dark matter components can be originated by the thermal vacuum of the hot plasma present at the center of a galaxy cluster (intracluster medium), by vacuum fluctuations of fields in curved space [57] and by the flavor neutrino vacuum [52]. Moreover, we show that dark energy contributions are given by vacuum condensates induced by the axion-photon mixing and by superpartners of mixed neutrinos [52].

We then consider the free Wees-Zumino model as a supersymmetric field theory, and we show that the presence of nonvanishing vacuum energy at the Lagrangian level implies that SUSY is spontaneously broken by the condensates. Next experiments using atomic systems characterized by vacuum condensate, could test our conjecture.

In Sec.II, we introduce the Bogoliubov transformations in QFT. In Sec.III, we compute the energy density and pressure of vacuum condensates induced by a generic Bogoliubov transformation for boson and fermion fields. In Secs.IV, V and VI, we present the contribution given to the energy of the universe by thermal states, with reference to the Hawking and Unruh effects, by fields in curved space and by particle mixing phenomena, respectively. The SUSY breaking induced by vacuum condensate is presented in Sec.VII and Sec.VIII is devoted to the conclusions.

II. BOGOLIUBOV TRANSFORMATION AND VACUUM CONDENSATE

In the context of QFT, the Bogoliubov transformations [45], [48] describe phenomena such as the Hawking-Unruh effect [38, 39], the Schwinger effect [40], the BCS theory of superconductivity [41], the Thermo Field Dynamics [43]-[49], the QFT in curved spacetimes [48] and the particle mixing phenomena [50]-[63].

A Bogoliubov transformation for bosons (similar discussion hold for fermions) assumes the form

\begin{align*}
    a^\dagger_k(\xi,t) &= U_k B \ a_k(t) - V^B_{-k} a^\dagger_{-k}(t), \\
    a_{-k}(\xi,t) &= U^{-B}_{-k} a^\dagger_{-k}(t) - V^{-B}_{k} a_k(t),
\end{align*}

with \( a_k(t) = a_k e^{-i\omega_k t} \), annihilators, such that \( a_k |0\>_B = 0 \) and \( \omega_k = \sqrt{k^2 + m^2} \). The coefficients satisfy the condi-
tions $U_k^B = U_{-k}^B$, $V_k^B = V_{-k}^B$, $|U_k^B|^2 - |V_k^B|^2 = 1$, and similar for fermions. The parameter $\xi$ depends on the system one considers.

The transformations $\mathbf{\Delta}$ can be rewritten in terms of a generator $J(\xi,t)$ as $a_k(\xi,t) = J^{-1}(\xi,t) a_k(t) J(\xi,t)$, where $J(\xi,t)$ has the property, $J^{-1}(\xi) = J(-\xi)$. The vacua annihilated by $a_k(\xi,t)$, denoted with $|0(\xi,t)\rangle$, are related to the original vacua $|0\rangle$ by $|0(\xi,t)\rangle = J^{-1}(\xi,t)|0\rangle$. Such a relation is a unitary operation in quantum mechanics, where $\mathbf{k}$ assumes a discrete range of values and there is a finite or countable number of canonical commutation relations. On the contrary, in QFT, $\mathbf{k}$ assumes a continuous infinity of values and the relation $|0(\xi,t)\rangle = J^{-1}(\xi,t)|0\rangle$ is not a unitary transformation any more. In this case, $|0(\xi,t)\rangle_B$ and $|0\rangle_B$ are unitarily inequivalent and the physical vacua of the systems described by Bogoliubov transformations $[38]-[48]$, are the $|0(\xi,t)\rangle$ ones $[44]-[46]$. Notice that $|0(\xi,t)\rangle$ has a condensate structure, i.e.

$$
|0(\xi,t)\rangle a_k^\dagger a_k|0(\xi,t)\rangle = |V_k|^2 ~, \quad (2)
$$

which induces an energy momentum tensor different from zero for $|0(\xi,t)\rangle_\lambda$.

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### III. ENERGY-MOMENTUM TENSOR OF VACUUM CONDENSATE

In order to derive the state equation of the vacuum condensates $|0(\xi,t)\rangle_\lambda, (\lambda = B, F)$ and their contributions to the energy density, one computes the expectation value of the free energy momentum tensor densities $T^\mu_\nu(x)$ for real scalar fields and for Majorana fields on $|0(\xi,t)\rangle_\lambda$.

$$
\Xi^\lambda_{\mu\nu}(x) = \lambda \langle 0(\xi,t) | T^\lambda_{\mu\nu}(x) | 0(\xi,t) \rangle_\lambda
$$

$$
= \lambda \langle 0(\xi,t) | T^\lambda_{\mu\nu}(x) | 0(\xi,t) \rangle_\lambda - \lambda \langle 0 | T^\lambda_{\mu\nu}(x) | 0 \rangle_\lambda
$$

where, : ... : denotes the normal ordering with respect to the original vacuum $|0\rangle_\lambda$. Since the off-diagonal components of $\Xi^\lambda_{\mu\nu}(x)$ are zero, $\Xi^\lambda_{ij}(x) = 0$, for $i \neq j$, the condensates behave as a perfect fluid and one can define their energy density and pressure as $[52]$

$$
\rho^\lambda = \langle 0(\xi,t) | T^\lambda_{00}(x) | 0(\xi,t) \rangle_\lambda \quad (4)
$$

$$
p^\lambda = \langle 0(\xi,t) | T^\lambda_{ij}(x) | 0(\xi,t) \rangle_\lambda \quad (5)
$$

respectively. For bosons, one has $[52]$

$$
\rho_B = \frac{1}{2} \langle 0(\xi,t) | [\pi^2(x) + (\nabla \phi(x))^2 + m^2 \phi^2(x)] | 0(\xi,t) \rangle_\lambda \quad (6)
$$

$$
p_B = \langle 0(\xi,t) | \left( \left( \partial \phi(x)^2 + \frac{1}{2} \left[ \pi^2(x) - (\nabla \phi(x))^2 - m^2 \phi^2(x) \right] \right) | 0(\xi,t) \rangle_\lambda \quad (7)
$$

and, in the particular case of the isotropy of the momenta, $k_1 = k_2 = k_3$, such that, $[\partial \phi(x)]^2 = \frac{1}{3} \left[ \nabla \phi(x) \right]^2$, the energy density, the pressure and the state equation are

$$
\rho_B = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} k^2 \omega_k |V^B_k|^2, \quad (8)
$$

$$
p_B = \frac{1}{6\pi^2} \int_0^\infty \frac{dk}{k} k^2 \left( \frac{k_2^2}{\omega_k} \right) |V^B_k|^2
$$

$$
- \left( \frac{k^2}{\omega_k} + \frac{3m^2}{2\omega_k} \right) |U^B_k||V^B_k| \cos(\omega_k t), \quad (9)
$$

$$
w_B = \frac{1}{3} \int \frac{d^3k}{\omega_k} \frac{k^2}{|V^B_k|^2} |V^B_k|^2
$$

$$
- \frac{1}{3} \int \frac{d^3k}{\omega_k} \left( \frac{k^2}{\omega_k} + \frac{3m^2}{2\omega_k} \right) U^B_k V^B_k \cos(\omega_k t)
$$

$$
\int \frac{d^3k}{\omega_k} |V^B_k|^2, \quad (10)
$$

respectively.

For fermions, the energy density and the pressure are $[52]$

$$
\rho_F = \frac{1}{2} \langle 0(\xi,t) | [ - i \bar{\psi} \gamma_j \partial^j \psi + m \bar{\psi} \psi ] | 0(\xi,t) \rangle_\lambda \quad (11)
$$

$$
p_F = \langle 0(\xi,t) | \left( \frac{i}{2} \bar{\psi} \gamma_j \partial^j \psi \right) | 0(\xi,t) \rangle_\lambda. \quad (12)
$$

and explicitly one has

$$
\rho_F = \frac{1}{\pi^2} \int_0^\infty \frac{dk}{k} k^2 \omega_k |V^F_k|^2, \quad (13)
$$

$$
p_F = \frac{1}{3\pi^2} \int_0^\infty \frac{dk}{k} k^4 \omega_k |V^F_k|^2, \quad (14)
$$

$$
w_F = \frac{1}{3} \int \frac{d^3k}{\omega_k} \left( \frac{k^2}{\omega_k} + \frac{3m^2}{2\omega_k} \right) U^F_k V^F_k \cos(\omega_k t)
$$

$$
\int \frac{d^3k}{\omega_k} |V^F_k|^2. \quad (15)
$$

We also note that, being $J^{-1}(\xi,t) = J^1(\xi,t) = J(-\xi,t)$, one can write

$$
\chi(0(\xi,t) : T^\lambda_{\mu\nu}(x) : |0(\xi,t)\rangle_\lambda =
$$

$$
= \chi(0 | J^\lambda_{\mu\nu}(x) : T^\lambda_{\mu\nu}(x) : J^\lambda_{\mu\nu}(x) |0\rangle_\lambda. \quad (16)
$$
In the following, we denote with \( \Theta(-\xi, x) = J^{-1}(-\xi, t) \Theta(x) J(-\xi, t) \) the operators transformed by \( J(-\xi, t) \). All the equations above presented hold for many systems. The explicit form of the Bogoliubov coefficients specifies the particular system.

IV. VACUUM CONTRIBUTIONS OF THERMAL STATES, HAWKING AND UNRUH EFFECTS

In the context of the Thermo Field Dynamics (TFD) \([43–45]\), the physical vacuum of systems at non-zero temperature is the thermal vacuum state \( \vert 0(\xi(\beta)) \rangle_\lambda \), where \( \beta \equiv 1/(k_B T) \), \( k_B \) is the Boltzmann constant and \( \lambda = B, F \). The state \( \vert 0(\xi(\beta)) \rangle_\lambda \) is obtained by means of a Bogoliubov transformation similar to the ones presented in Section I, (for details see \([45]\)) and the thermal statistical average \( N_{xk}(\xi) \) is given by \( N_{xk}(\xi) = \lambda \langle 0(\xi(\beta))\vert N_{xk} \vert 0(\xi(\beta)) \rangle_\lambda \), where \( N_{xk} = \chi_k \lambda_k \) (\( \chi = a \) for bosons and \( \chi = a \) for fermions) is the number operator \([44]\).

The thermal Bogoliubov coefficients are given by \( U_k^T = \sqrt{\frac{\beta \omega_k}{\beta \omega_k + 1}} \) and \( V_k^T = \sqrt{\frac{1}{\beta \omega_k + 1}} \), with \(- \) for bosons and \( + \) for fermions, and \( \omega_k = \sqrt{k^2 + m^2} \). Such coefficients, used in Eqs. (8, 10) and (13, 14) give the contributions of the thermal vacuum states to the energy and pressure. In particular, for temperatures of order of the cosmic microwave radiation, i.e. \( T = 2.72K \), one find that photons and particles with masses of order of \((10^{-3} - 10^{-4}) eV \) contribute to the energy radiation with \( \rho \sim 10^{-51} GeV^4 \) and state equations, \( w = 1/3 \) \([64]\). On the other hand, non-relativistic particles give negligible contributions. Moreover, the thermal vacuum of the hot plasma filling the center of galaxy clusters, which has temperatures of order of \((10 \div 100) \times 10^6K \), has an energy density of \( (10^{-48} - 10^{-45}) GeV^4 \) and a state equation \( w = 0.01 \). Such values of \( \rho \) and \( w \) are in agreement with the ones of the dark matter.

The thermal states can describe also the Unruh and of the Hawking effects; however, both of the phenomena do not contribute to the energy of the universe, since the temperatures are very low \([64]\).

V. VACUUM CONTRIBUTION OF FIELDS IN CURVED BACKGROUND

Fields in curved background are also characterized by condensed vacuum and Bogoliubov transformations \([48]\). For such fields, the energy density and pressure depend on the particular metric considered. Here one considers the spatially flat Friedmann Robertson-Walker metric \( ds^2 = dt^2 - a^2(t) dx^2 = a^2(t) (dt^2 - dx^2) \), where \( a \) is the scale factor, \( t \) is the comoving time, \( \eta \) is the conformal time, \( \eta(t) = \int_{t_0}^t \frac{dt'}{a(t')} \), with \( t_0 \) arbitrary constant.

The energy density and pressure are expressed as \([65]\)

\[
\rho_{\text{curv}} = \frac{2\pi}{a^2} \int_0^K dkk^2 \left( |\phi_k'|^2 + k^2|\phi_k|^2 + m^2|\phi_k|^2 \right),
\]

\[
p_{\text{curv}} = \frac{2\pi}{a^2} \int_0^K dkk^2 \left( |\phi_k'|^2 - \frac{k^2}{3}|\phi_k|^2 - m^2|\phi_k|^2 \right).
\]

where \( K \) is the cut-off on the momenta, \( \phi_k \) are mode functions and \( \phi_k' \) denotes the derivative of \( \phi_k \) with respect to the conformal time \( \eta \). Assuming at late time the cutoff on the momenta much smaller than the comoving mass of the field, \( K \ll ma \) and setting \( m \gg H \), for an arbitrary Robertson-Walker metric in infrared regime, one has \([57]\)

\[
\rho_{\text{curv}} = \frac{1}{8\pi^2} \int_0^K dkk^2 \left( \frac{2m}{a^3} + \frac{9H^2}{4ma^3} + \frac{k^2}{ma^5} \right),
\]

\[
p_{\text{curv}} = \frac{1}{8\pi^2} \int_0^K dkk^2 \left( \frac{9H^2}{4ma^3} - \frac{k^2}{3ma^5} \right).
\]

The state equation is \( w_{\text{curv}} \simeq 0 \), which coincides with the one of the dark matter. Numerical values compatible with the ones of dark matter are found when \( \frac{ma^3}{K} \simeq 10^{-45} GeV^4 \).

VI. VACUUM CONTRIBUTIONS OF PARTICLE MIXING

The particle mixing concerns neutrinos and quarks in fermion sector, axions, kaons, \( B^0 \), \( D^0 \), and \( \eta - \eta' \) systems, in boson sector. In the case of mixing between two fields, it is expressed as

\[
\varphi_1(\theta, x) = \varphi_1(x) \cos(\theta) + \varphi_2(x) \sin(\theta),
\]

\[
\varphi_2(\theta, x) = -\varphi_1(x) \sin(\theta) + \varphi_2(x) \cos(\theta),
\]

where, \( \theta \) is the mixing angle, \( \varphi_i(x) \) are the mixed fields and \( \varphi_i(x) \) are the free fields, with \( i = 1, 2 \).

The mixing transformations \([21]\) and the mixed annihilation operators can be expressed by means of the generator \( J(\theta, t) \) as \( \varphi_i(\theta, x) \equiv J^{-1}(\theta, t) \varphi_i(x) J(\theta, t) \), and \( \chi_{k, i}(\theta, t) \equiv J^{-1}(\theta, t) \chi_{k, i}(x) J(\theta, t) \), respectively, with \( \chi_{k, i} = a_{k, i}, a_{k, i}^* \), for bosons and fermion, respectively, and \( i = 1, 2 \) \([49, 50]\).

The physical vacuum where particle oscillations appears is \( \vert 0(\theta, t) \rangle \equiv J^{-1}(\theta, t) \vert 0 \rangle_{1, 2} \), where \( \vert 0 \rangle_{1, 2} \) is the vacuum annihilated by \( \chi_{k, i} \). One has

\[
\langle 0(\theta, t) \vert \chi_{k, i}^\dagger \chi_{k, i} \vert 0(\theta, t) \rangle = \sin^2 \theta \vert \Upsilon_{k, i} \vert^2,
\]

where \( \Upsilon_{k, i} \) is the vacuum expectation value of \( \chi_{k, i}^\dagger \chi_{k, i} \).
where \( \lambda = B, F \), \( i = 1, 2 \) and the reference frame \( k = (0, 0, |k|) \) has been adopted. The Bogoliubov coefficient entering the mixing transformation \( \Upsilon_k^i \) assumes the following form for boson and fermion

\[
\begin{align*}
|\Upsilon_k^B| &= \frac{1}{2} \left( \sqrt{\frac{\Omega_{k,1}}{\Omega_{k,2}}} - \sqrt{\frac{\Omega_{k,2}}{\Omega_{k,1}}} \right), \\
|\Upsilon_k^F| &= \frac{(\Omega_{k,1} + m_1) - (\Omega_{k,2} + m_2)}{2\sqrt{\Omega_{k,1}\Omega_{k,2}(\Omega_{k,1} + m_1)(\Omega_{k,2} + m_2)}} |k|.
\end{align*}
\]

(23) - (24)

respectively. One can see that the kinetic and gradient terms of the mixed vacuum are equal to zero \([52]\).

Then, Eqs. (25) and (26) reduce to

\[
\begin{align*}
\rho_{mix}^B &= \frac{1}{2} \langle 0 | : \sum_i \left[ \pi_i^2(-\theta, x) + \left( \nabla \phi_i(-\theta, x) \right)^2 \right] : |0 \rangle; \\
p_{mix}^B &= \langle 0 | : \sum_i \left( \partial_j \phi_i(-\theta, x) \right)^2 + \frac{1}{2} \left[ \pi_i^2(-\theta, x) - \left( \nabla \phi_i(-\theta, x) \right)^2 - m_i^2 \phi_i^2(-\theta, x) \right] : |0 \rangle,
\end{align*}
\]

(25) - (26)

where \( K \) is the cut-off on the momenta.

- In the case of the vacuum condensate induced by mixed bosons are

\[
\rho^{\text{mix}}_{\text{B}} = 2.3 \times 10^{-47} \text{GeV}^4,
\]

which is of the same order of the estimated upper bound on the dark energy.

- In the case of superpartners of the neutrinos, considering masses \( m_1 = 10^{-3} \text{eV} \) and \( m_2 = 9 \times 10^{-3} \text{eV} \), such that \( \Delta m^2 = 8 \times 10^{-5} \text{eV}^2 \) and assuming \( \sin^2 \theta = 0.3 \), one obtains, \( \rho_{mix}^B = 7 \times 10^{-47} \text{GeV}^4 \) for a cut-off on the momenta \( K = 10 \text{eV} \). Smaller values of the mixing angle lead to values of \( \rho_{mix}^B \) which are compatible with the estimated value of the dark energy also in the case in which the cut-off is \( K = 10^{19} \text{GeV} \), indeed \( \rho_{mix}^B \) depends linearly by \( \sin^2 \theta \) \([52]\).

- Fermion mixing - The energy density and pressure of the mixed vacuum condensate induced by mixed bosons are

\[
\rho^{\text{mix}}_{\text{F}} = -\langle 0 | : \sum_i \left[ \psi_i^\dagger(-\theta, x) \gamma_0 \gamma_j \partial_j \psi_i(-\theta, x) + m \psi_i^\dagger(-\theta, x) \gamma_0 \psi_i(-\theta, x) \right] : |0 \rangle;
\]

(30)

\[
p^{\text{mix}}_{\text{F}} = i \langle 0 | : \sum_i \left[ \psi_i^\dagger(-\theta, x) \gamma_0 \gamma_j \partial_j \psi_i(-\theta, x) \right] : |0 \rangle,
\]

(31)

where \( \psi_i(-\theta, x) \) are the flavor neutrino fields or the quark fields. Being

\[
\begin{align*}
0 \langle 0 | : \sum_i \psi_i(-\theta, x) \gamma_j \partial_j \psi_i(-\theta, x) : |0 \rangle = 0,
\end{align*}
\]
one has
\[
\rho_{mix}^F = -\langle 0 | : \sum_i \left[ m_i \bar{\psi}_i^\dagger (\theta, x) \gamma_0 \psi_i (\theta, x) \right] : | 0 \rangle \tag{32}
\]
\[
\rho_{mix}^F = 0 . \tag{33}
\]

The state equation is then \( u_{mix}^F = 0 \), which is the one of the dark matter. The energy density is
\[
\rho_{mix}^F = \frac{\Delta m^2 \sin^2 \theta}{2 \pi^2} \int_0^K dk k^2 \left( \frac{m_2}{\omega_{k,2}} - \frac{m_1}{\omega_{k,1}} \right) .
\]

By considering masses of order of \( 10^{-3}eV \), such that \( \Delta m^2 \approx 8 \times 10^{-5}eV^2 \) and a cutoff on the momenta \( K = m_1 + m_2 \), one obtains \( \rho_{mix}^F = 4 \times 10^{-47}GeV^4 \), which is in agreement with the estimated upper bound of the dark matter. For \( K \) of order of the Plank scale one has \( \rho_{mix}^F \sim \times 10^{-46}GeV^4 \). Notice that the quark confinement inside the hadrons should inhibit the gravitational interaction of the quark vacuum condensate. Thus the quark condensate should not affect the dark matter of the universe.

VII. SUSY BREAKING AND VACUUM CONDENSATE

We show that vacuum condensate provides a new mechanism of spontaneous SUSY breaking. We start by a situation in which SUSY is preserved at the lagrangian level and study the effects of vacuum condensation. We consider a Bogoliubov transformation acting simultaneously and with the same parameters on the bosonic and on the fermionic degrees of freedom in order not to break SUSY explicitly. Since, in any field theory which has manifest supersymmetry at the lagrangian level, a nonzero vacuum energy implies the spontaneous SUSY breaking [34], then the vacuum condensate (which is characterized by nontrivial energy) breaks SUSY spontaneously.

The effects of a Bogoliubov transformation are analyzed in the Wess–Zumino model described by the Lagrangian [67]
\[
\mathcal{L} = \frac{i}{2} \bar{\psi} \gamma_\mu \partial^\mu \psi + \frac{1}{2} \partial_\mu S \partial^{\mu} S + \frac{1}{2} \partial_\mu P \partial^{\mu} P - \frac{m}{2} \bar{\psi} \gamma_\mu \partial^\mu \psi - \frac{m^2}{2} (S^2 + P^2), \tag{34}
\]
where \( \psi \) is a Majorana spinor field, \( S \) is a scalar field and \( P \) is a pseudoscalar field. This Lagrangian is invariant under supersymmetry transformations [67].

The fields are quantized by expanding them in modes:
\[
\psi(x) = \sum_{r=1}^2 \int \frac{d^3 k}{(2\pi)^3} e^{ikx} \left[ u^r_k \alpha_k^r(t) + v^r_k \alpha^r_k(t) \right], \tag{35}
\]
\[
S(x) = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} e^{ikx} \left[ b_k(t) + b_k^\dagger(t) \right], \tag{36}
\]
\[
P(x) = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} e^{ikx} \left[ c_k(t) + c_k^\dagger(t) \right]. \tag{37}
\]
where \( \alpha_k^r(t) = \alpha_k e^{-i\omega_k t} \), \( b_k(t) = b_k e^{-i\omega_k t} \), \( c_k(t) = c_k e^{-i\omega_k t} \) and \( \omega_k = \sqrt{k^2 + m^2} \), and since \( \psi \) is a Majorana spinor, one has \( \bar{\psi}^r_k = i \gamma_0 \bar{\psi}_k \) and \( \bar{\psi}^r_k = i \gamma_0 \bar{\psi}_k \).

The vacuum annihilated by \( \alpha_k^r \), \( b_k \) and \( c_k \) is defined as \( | 0 \rangle = | 0 \rangle^\psi \otimes | 0 \rangle^S \otimes | 0 \rangle^P \).

We perform simultaneous Bogoliubov transformations on fermion and boson annihilators,
\[
\begin{align*}
\alpha_k^r(t) &= U_k^\psi \alpha_k^r(t) + V_k^\psi \alpha_k^r(t) , \\
\alpha_{-k}^r(t) &= U_k^S \alpha_k^r(t) + V_k^S \alpha_k^r(t) , \\
b_k(\eta, t) &= U_k^S b_k(t) - V_k^S b_k(t) , \\
b_{-k}^\dagger(\eta, t) &= U_k^S b_{-k}^\dagger(t) - V_k^S b_{-k}^\dagger(t) , \\
c_k(\eta, t) &= U_k^P c_k(t) - V_k^P c_k(t) , \\
c_{-k}^\dagger(\eta, t) &= U_k^P c_{-k}^\dagger(t) - V_k^P c_{-k}^\dagger(t)
\end{align*}
\]
One has, \( U_k^\psi = U_k^P \) and \( V_k^S = V_k^P \). We denote such quantities as \( U_k^B \) and \( V_k^B \), respectively.

Eqs. [33–10] can be written as
\[
\chi_k^r(\xi, \eta, t) = J^{-1}(\xi, \eta, t) \chi_k(t) J(\xi, \eta, t), \tag{41}
\]
with \( \chi_k = \alpha_k, b_k, c_k \) and \( J(\xi, \eta, t) = J_\psi(\xi, t) J_S(\eta, t) J_P(\eta, t) \), where
\[
\begin{align*}
J_\psi &= \exp \left[ \frac{1}{2} \int d^3 k \chi_k(\xi) \left( \alpha_k^\dagger(t) \alpha_{-k}^r(t) - \alpha_{-k}^r(t) \alpha_k^\dagger(t) \right) \right] , \\
J_S &= \exp \left[ -i \int d^3 k \eta_k(\xi) \left( b_k(t) b_{-k}^\dagger(t) - b_{-k}^\dagger(t) b_k(t) \right) \right] , \\
J_P &= \exp \left[ -i \int d^3 k \eta_k(\xi) \left( c_k(t) c_{-k}^\dagger(t) - c_{-k}^\dagger(t) c_k(t) \right) \right] .
\end{align*}
\]

The transformed vacuum is \( | 0(\xi, \eta, t) \rangle = | 0(\xi, \eta, t) \rangle_\psi \otimes | 0(\eta, t) \rangle_S \otimes | 0(\eta, t) \rangle_P \), where \( | 0(\xi, \eta, t) \rangle_\psi = J^{-1}_\psi(\xi, t) | 0 \rangle_\psi \), \( | 0(\eta, t) \rangle_S = J^{-1}_S(\eta, t) | 0 \rangle_S \), \( | 0(\eta, t) \rangle_P = J^{-1}_P(\eta, t) | 0 \rangle_P \), respectively, and then
\[
| 0(\xi, \eta, t) \rangle = J^{-1}(\xi, \eta, t) | 0 \rangle .
\]

In a supersymmetric context, \( | 0(\xi, \eta, t) \rangle \) is the physical vacuum for the systems listed above. The nonzero energy of \( | 0(\xi, \eta, t) \rangle_\psi \), \( | 0(\eta, t) \rangle_S \) and \( | 0(\eta, t) \rangle_P \) (see above) leads to an energy densities different from zero for \( | 0(\xi, \eta, t) \rangle \).
Indeed, considering the free Hamiltonian $H$ corresponding to the Lagrangian in Eq. (44), $H = H_\psi + H_B$ where $H_B = H_S + H_P$, the expectation values of the fermion and boson Hamiltonians on $|0(\xi, \eta, t)|$ are
\[
\langle 0(\xi, \eta, t)|H_\psi|0(\xi, \eta, t)\rangle = -\int d^3k \omega_k (1 - 2|V_k^\psi|^2),
\]
and
\[
\langle 0(\xi, \eta, t)|H_B|0(\xi, \eta, t)\rangle = \int d^3k \omega_k (1 + 2|V_k^B|^2), \tag{44}
\]
respectively. Then we have the final result
\[
\langle \hat{\theta}(t)|H|\hat{\theta}(t)\rangle = 2\int d^3k \omega_k (|V_k^\psi|^2 + |V_k^B|^2), \tag{45}
\]
which is different from zero and positive.

Notice that, Eq. (45), holds for disparate physical phenomena. As remarked above, the explicit form of the Bogoliubov coefficients $V_k^\psi$ and $V_k^B$ specifies the particular system.

Laser cooling experiments could allow to test the mechanism of SUSY breaking here presented. Indeed, the Wess-Zumino model in $2 + 1$ dimensions can be obtained by a mixture of cold atoms-molecules trapped in two dimensional optical lattices \cite{68}. In this case SUSY is preserved at zero temperature and is broken at $T \neq 0$. Then, a signature of SUSY breaking in such a system can be probed by the detection of the constant background noise due to the nonzero energy of the thermal vacuum, given by $\langle H \rangle = 14\pi \zeta(3) T^3$.

VIII. CONCLUSIONS

The vacuum condensates characterizing many systems can contribute to the dark matter and to the dark energy. Dark matter contributes derive by thermal vacuum of intercluster medium, by the vacuum of fields in curved space and by the neutrino flavor vacuum. Dark energy contribute are given by axion-photon mixing. We have also shown that, in a suprasymmetric field theory, vacuum condensates may lead to spontaneous SUSY breaking.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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