MAJOR 1.5 – A Monte Carlo Generator for Heavy Majorana Neutrinos in ep Collisions

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Abstract: The Monte Carlo generator MAJOR 1.5 simulates the production and decay of heavy Majorana neutrinos via lepton mixing or exchange of ‘light’ right-handed W-bosons in deep inelastic scattering, i.e. \( e^\pm p \to NX \to e^\pm W^\mp X \) or \( \nu_e ZX \). Physics and programming aspects are described in this manual.

1 Physics of included processes

The standard model of electroweak and strong interactions has been remarkably successful in describing the experimental data. Nevertheless, it cannot be the ultimate theory due to its theoretical shortcomings, \textit{e.g.} for the understanding of the mass and family structure of quarks and leptons and the many free parameters. Attempts to solve the theoretical problems of the standard model have been made based on various theoretical grounds and several extended theories have been suggested and studied in detail. These theories are normally based on some larger symmetry group which unifies the interactions and is spontaneously broken down to the standard model gauge group. Particular attention has been given to models with an additional \( U(1) \) symmetry or with left-right symmetry in the form \( SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \), which can both be subgroups of the unification groups \( SO(10) \) and \( E_6 \). The questions of neutrino masses and lepton number violation, which are of general importance (for a review see \cite{1}), enter explicitly in this context and, in particular, heavy Majorana neutrinos may be present \cite{2}.

The simplest process, in terms of minimal new physics assumptions, to produce heavy Majorana neutrinos in \( ep \) collisions is through normal charged current interactions as illustrated in Fig. 1. The cross-section is suppressed, not only by limited phase space due to the large neutrino mass \( m_N \), but also by the required mixing \( \xi \) between light and heavy Majorana neutrinos \cite{2, 3}. The light Majorana neutrinos can be identified with the normal neutrinos \( \nu_e, \nu_\mu \) and \( \nu_\tau \), whose small masses is naturally explained through the see-saw mechanism. The mixing in the leptonic part of the charged current interaction is given by a unitary Kobayashi-Maskawa type matrix \( V \) and the matrix \( \xi \). A power series in \( \xi \) gives the connection between the weak eigenstates \( \nu_L, \nu_R \) and the Majorana mass eigenstates \( \nu, N \) \cite{4}, \textit{i.e.}

\[
\nu_L = \frac{1 - \gamma_5}{2} (\nu + \xi N + ...) , \quad \nu_R = \frac{1 + \gamma_5}{2} (N - \xi^T \nu + ...).
\] (1)
The heavy Majorana neutrinos will decay into either $\nu Z^0$, $\ell^+W^-$ or $\ell^-W^+$ where the weak bosons can be taken as being on-shell since we only consider Majorana neutrino masses above the boson masses ($m_N > m_B$) due to mass limits [4]. Using the narrow width approximation for the heavy Majorana neutrino propagator in Fig. 1, the cross-section for the process $e^-p \rightarrow NX$ where $N \rightarrow \ell^\pm W^\mp$ or $N \rightarrow \nu Z^0$ is [2]

$$\frac{d\sigma}{dx\,dy\,dp_{\ell\perp}\,dy_\ell} = \frac{G_F^2 \mu_N^4 |(V_\xi)e_N|^2}{32\sqrt{2}\pi^3 \hat{s}m_N \Gamma_N} \left( y\hat{s} + m_N^2 \right)^2 \frac{1}{m_{N\perp}}$$

$$\times \left[ \cosh(y_N - y_\ell) - \frac{m_N^2 - m_B^2 - 2p_{\ell\perp}p_{N\perp}}{2m_{N\perp}p_{\ell\perp}} \right]^{-1/2}$$

$$\times \left[ \frac{m_N^2 - m_B^2 + 2p_{\ell\perp}p_{N\perp}}{2m_{N\perp}p_{\ell\perp}} - \cosh(y_N - y_\ell) \right]^{-1/2}$$

$$\times \left\{ A \cdot \left[ d(x, \mu^2) + c(x, \mu^2) \right] + B \cdot \left[ \bar{d}(x, \mu^2) + \bar{c}(x, \mu^2) \right] \right\}$$

For incoming positron the last row in Eq.(2) is replaced by

$$\times \left\{ C \cdot \left[ d(x, \mu^2) + s(x, \mu^2) \right] + D \cdot \left[ \bar{d}(x, \mu^2) + \bar{s}(x, \mu^2) \right] \right\}$$

Another possibility to produce right-handed neutrinos is through the exchange of right-handed $W$-bosons, $W_R$, see e.g. [5]. If it is assumed that the $W_R$ is lighter than the heavy neutrino and that there is no mixing with standard model gauge bosons then the same cross-section formula can be used except that $G_F^2 \mu_N^4 |(V_\xi)e_N|^2 = g_{L_R}^2 |(V_\xi)e_N|^2$ is replaced by the right handed gauge coupling, $g_{R_R}^2$, and everywhere else $m_W$ is replaced by $m_{W_R}$. To simplify the picture one may also assume that the heavy neutrino only decays according to the charged decay modes, $N \rightarrow \ell^\pm W_R^\mp$, thus avoiding any extra assumptions on the $Z'$-mass. Finally, if only two-body decays are considered, then $W_R$ only decays into quark-antiquark pairs due to the larger mass of the right-handed neutrino. It is difficult to obtain model-independent limits on $m_{W_R}$ and according to [6] the $W_R$ can be as light as the ordinary $W_L$ if extreme fine-tuning is allowed.

The independent variables are chosen as the normal deep inelastic scaling variables $x$ and $y$ together with the transverse momentum $p_{\ell\perp}$ and rapidity $y_\ell$ of the final state lepton $\ell$ from the
Majorana neutrino decay. With four-momenta as in Fig. 1, \( s = (P+k)^2 \), \( \hat{s} = xs \), \( W^2 = (P+q)^2 \) and \( Q^2 = -q^2 \). As usual, \( x = Q^2/2P \cdot q \) and \( y = P \cdot q/P \cdot k \), but they are globally constrained to \( \frac{m_N^2}{s} \leq x \leq 1 \) and \( 0 \leq y \leq 1 - \frac{m_N^2}{s} \). The phase space is given by

\[
\frac{1}{s} (p_{N\perp}^\text{min} + m_{N\perp}^\text{min})^2 \leq x \leq 1
\]

\[
\frac{\hat{s} - m_N^2 - \sqrt{(\hat{s} - m_N^2)^2 - 4 \hat{s} p_{N\perp}^\text{min}^2}}{2 \hat{s}} \leq \frac{\hat{s} - m_N^2 + \sqrt{(\hat{s} - m_N^2)^2 - 4 \hat{s} p_{N\perp}^\text{min}^2}}{2 \hat{s}}
\]

\[
0 \leq p_{\ell\perp} \leq \frac{m_N^2 - m_B^2}{2m_N^2} \sqrt{s}
\]

\[
\frac{m_N^2 - m_B^2 - 2p_{\ell\perp} p_{N\perp}}{2m_N^2 p_{\ell\perp}} \leq \cosh(y_N - y_\ell) \leq \frac{m_N^2 - m_B^2 + 2p_{\ell\perp} p_{N\perp}}{2m_N^2 p_{\ell\perp}}
\]

The indices \( N \) and \( W \) denote the Majorana neutrino and the \( W \) boson exchanged in the \( t \)-channel, whereas \( B \) and \( \ell \) denote the weak boson and lepton from the Majorana neutrino decay, respectively. The symbols \( y_\ell, y_N, m, m_\perp = \sqrt{m^2 + p_\perp^2} \) and \( p_\perp \) refer to rapidity, mass, transverse mass and transverse momentum, respectively. All frame dependent quantities are understood to be in the lab frame. \( \Gamma_N \) is the total width of the Majorana neutrino and the functions \( u, c, d \) and \( s \) are the parton density functions in the proton evaluated at a suitable scale \( \mu \).

The coefficient functions \( A, B \) depend on the decay products of the Majorana neutrino and are for the charged decays given by [2]

\[
A(\ell^- W^+) = |(V\xi)_\ell N|^2 \frac{4\hat{s}}{m_W^2} \left[(m_N^2 - m_W^2)(2m_W^2\hat{s} - m_N^4) - 2m_N^2(2m_W^2 - m_N^2)p_{\ell\perp}(xe^{-y_\ell} + e^{y_\ell})\right]
\]

\[
B(\ell^- W^+) = |(V\xi)_\ell N|^2 \frac{8(\hat{s}(1 - y) - m_N^2)}{m_W^2} \left[\hat{s}(1-y)m_W^2(m_N^2 - m_W^2) - m_N^2(2m_W^2 - m_N^2)p_{\ell\perp}xe^{-y_\ell}\right]
\]

\[
A(\ell^+ W^-) = |(V\xi)_\ell N|^2 \frac{4\hat{s}m_N^2}{m_W^2} \left[(m_N^2 - m_W^2)(\hat{s} - 2m_W^2) + 2(2m_W^2 - m_N^2)p_{\ell\perp}(xe^{-y_\ell} + e^{y_\ell})\right]
\]

\[
B(\ell^+ W^-) = |(V\xi)_\ell N|^2 \frac{4m_N^2(\hat{s}(1 - y) - m_N^2)}{m_W^2} \left[\hat{s}(1-y)(m_N^2 - m_W^2) + 2(2m_W^2 - m_N^2)p_{\ell\perp}xe^{-y_\ell}\right]
\]

and for the neutral decay \( N \rightarrow \nu_\ell Z \) by [4]

\[
A(\nu_\ell Z) = |\xi_{\nu_\ell N}|^2 \frac{2\hat{s}(\hat{s} - m_N^2)(m_N^2 - m_Z^2)(m_N^2 + 2m_Z^2)}{\cos^2\theta_W m_Z^2}
\]

\[
B(\nu_\ell Z) = |\xi_{\nu_\ell N}|^2 \frac{2\hat{s}(1 - y)(\hat{s}(1-y) - m_N^2)(m_N^2 - m_Z^2)(m_N^2 + 2m_Z^2)}{\cos^2\theta_W m_Z^2}
\]
For incoming positron the coefficient functions $C, D$ are related to $A, B$ in the following way,
\begin{align*}
D(\ell^+ W^-) &= A(\ell^- W^+) \\
C(\ell^+ W^-) &= B(\ell^- W^+) \\
D(\ell^- W^+) &= A(\ell^+ W^-) \\
C(\ell^- W^+) &= B(\ell^+ W^-) \\
D(\nu_\ell Z) &= A(\nu_\ell Z) \\
C(\nu_\ell Z) &= B(\nu_\ell Z)
\end{align*}

The width of the heavy Majorana neutrino is given by
\begin{align*}
\Gamma_N &= \sum_\ell \left[ 2\Gamma(N \to \ell^\pm W^{\mp}) + \Gamma(N \to \nu_\ell Z) \right] \\
&= \sum_\ell \frac{G_F}{8\sqrt{2}\pi m_N^3} \left[ 2|V_\xi\ell N|^2 (m_N^2 - m_W^2)^2 (m_N^2 + 2m_W^2) \\
&\quad + |\xi_{\nu_\ell N}|^2 (m_N^2 - m_Z^2)^2 (m_N^2 + 2m_Z^2) \right].
\end{align*}

Assuming the Kobayashi-Maskawa type matrix $V$ for the lepton sector to be diagonal, so that $|V_\ell N| = |\xi_{\nu_\ell N}|$, we obtain the branching ratios shown in Fig. 2. Although all three are $1/3$ at large Majorana neutrino masses, for masses not far above the $W$ and $Z$ masses there is a substantial phase space suppression of the decay into the $Z$.

For a detailed study on present limits on Majorana neutrino masses and mixings, as well as their phenomenology and suitable search strategies at $ep$ colliders we refer to [4].

## 2 The Monte Carlo implementation

The importance sampling method is used to generate phase space points according to the complete differential cross-section formula given by Eq. (20). In short, the importance sampling
method can be described as a variable transformation,

$$(x, y, p_{\ell\perp}, y_{\ell}) \rightarrow (H_x(x), H_y(y), H_{p_{\ell\perp}}(p_{\ell\perp}), H_{y_{\ell}}(y_{\ell}))$$

where $H_x(x) = \int x h_x(x') dx'$, etc. and $h$ are the so-called weighting functions. A phase-space point, $(x, y, p_{\ell\perp}, y_{\ell})$ is first chosen from the inverse of the integrals, e.g.

$$x = H_x^{-1}[H_{x,\text{min}} + R(H_{x,\text{max}} - H_{x,\text{min}})]$$

where $H_{x,\text{min}}$ is short for $H_x(x_{\text{min}})$, etc. and $R$ is a random number, $R \in [0, 1]$. The probability for keeping the point is then given by the ‘$g$-function’,

$$g(x, y, p_{\ell\perp}, y_{\ell}) = \frac{(H_{x,\text{max}} - H_{x,\text{min}})(H_{y,\text{max}} - H_{y,\text{min}})(H_{p_{\ell\perp,\text{max}} - H_{p_{\ell\perp,\text{min}}}})(H_{y_{\ell,\text{max}} - H_{y_{\ell,\text{min}}}})}{h_x(x)h_y(y)h_{p_{\ell\perp}}(p_{\ell\perp})h_{y_{\ell}}(y_{\ell})}
\times \frac{d\sigma}{dx dy dp_{\ell\perp} dy_{\ell}}$$

divided by its maximum. The weighting functions $h$ should be chosen to mimic the characteristic behaviour of the differential cross-section. This makes the function $g$ smooth which is important to make the rejection technique efficient. For details on the Monte Carlo methods used, see [7].

From the phase space point $(x, y, p_{\ell\perp}, y_{\ell})$, the four-momenta of the particles (partons) in the Feynman diagram, Fig. 1, are calculated. The Lund Monte Carlo programs LEPTO 6.5 [8] and JETSET 7.4 [9] are then used to produce a complete final state of observable particles. The on-shell $W/\gamma$ (from the heavy neutrino decay) is decayed with the proper branching ratios into a lepton pair or a $q\bar{q}'$ pair, where the polarisation of the $W/\gamma$ is not taken into account. (In the case of a right-handed $W$, it is assumed to decay only hadronically since the right-handed neutrino is assumed to be heavier than the right-handed $W$.) For $W/\gamma \rightarrow q\bar{q}'$, parton showers are included to account for QCD radiation of additional partons, and this parton system is then hadronised using the Lund string model [10, 9]. Similarly, the quark coming into and leaving the deep inelastic scattering may radiate partons through initial and final state parton showers. Together with the proton remnant spectator, this parton system is hadronised with the Lund model. Thus, the complete ‘history’ of the event is generated resulting in a complete final state. For a more complete description of the implementation, see [7].

## 3 Description of program components

The program is written in FORTRAN 77 and consists of a set of subroutines that must be activated by the users main steering program, which should call the subroutine MAINIT to initialize the generator and then call the subroutine MAJOR for each new event to be generated. All subroutine and common-block names start with MA to indicate origin and avoid name clashes. The only exception is the real function AMGFUN which starts with AM to follow the FORTRAN name convention.

### 3.1 Subroutines and functions

The following subroutines should be called by the user:
SUBROUTINE MAINIT

Purpose: Initiate constants and starting values. Calculate the maxima of the \( g \)-function, Eq. (23), if not given by the user. **Has to be called once before MAJOR is called.**

Called by: User

Calls to: LINIT, MADEFM, MADMAX

SUBROUTINE MAJOR

Purpose: Administer the generation of one event, calculate the four momenta and fill the event record. **Has to be called once for every event to be generated.**

Called by: User

Calls to: MAGENE, MAFLAV, MAERRM, LSHOWR, LUSHOW, LUPREP, LUEDITOR, LUDBRB, LUREBO, LUEXEC

Functions used: ULMASS, RLU, ULANGL, LUCOMP, PLU

The following subroutines and functions are called internally:

SUBROUTINE MADEFM

Purpose: Define the Majorana neutrino in Jetset 7.4 code (KF=79).

Called by: MAINIT

Calls to: -

SUBROUTINE MADMAX

Purpose: Set appropriate starting values for MINUIT \cite{1} and call MINUIT for the calculation of the maxima of the \( g \)-function.

Called by: MAINIT

Calls to: MAINEW

SUBROUTINE MASIGX(NPAR,DERIV,DIFSIG,XF,NFLAG)

Purpose: Is called by MINUIT and is used as an interface to the \( g \)-function AMGFUN. Gives minus the \( g \)-function if within range and otherwise it gives the square of the distance to the limit. The search for the maximum is done along the singularity of the \( g \)-function (only \( x \) and \( y \) varies freely while \( p_{\ell\perp} \) and \( y_{\ell} \) are calculated from \( x \) and \( y \)). Stores the maxima found by MINUIT in the array GMAX. For details, see \cite{7}.

Called by: MINUIT-subroutines

Calls to: MAERRRM

Functions used: AMGFUN

SUBROUTINE MAGENE

Purpose: Generate a phase space point according to the differential cross-section.

Called by: MAJOR

Calls to: -

Functions used: RLU, AMGFUN

REAL FUNCTION AMGFUN(C)

Purpose: Calculate the value of the \( g \)-function, Eq. (23), in a given phase space point.

Called by: MASIGX, MAGENE

Calls to: PYSTFU
SUBROUTINE MAFLAV(NINITQ,NSCATQ)

Purpose: Choose flavour of initial quark and scattered quark at the boson vertex according to the relative cross-sections and the CKM matrix.

Called by: MAJOR

Calls to: -

Functions used: RLU, LUCOMP

SUBROUTINE MAERRM(CHERRM,ERRVAL)

Purpose: Handle errors and write error messages.

Called by: MAJOR, MASIGX, MAGENE, AMGFUN

Calls to: -

The following subroutines are modified versions of MINUIT subroutines from LEPTO: MACMND, MAIDAT, MAINEW, MAINTO, MAPRIN, MARAZZ and MASIMP. In addition the following subroutines (S) and functions (F) in LEPTO 6.5 (L) and JETSET 7.4 (J) are used:

| Routine | Purpose |
|---------|---------|
| LINIT (S/L) | Initialize LEPTO |
| LSHOWR (S/L) | Include parton cascades |
| LYSTFU (S/L) | Give the parton distribution functions |
| LUDBRB (S/J) | Perform rotation and boost in double precision |
| LUROBO (S/J) | Perform rotation and boost in single precision |
| LUSHOW (S/J) | Generate timelike parton showers |
| LUPREP (S/J) | Rearrange parton shower end products |
| LUEDIT (S/J) | Exclude unstable or undetectable jets/particles from the event record |
| LUEXEC (S/J) | Administrate the fragmentation and decay chain |
| ULMASS (F/J) | Give the mass of a parton/particle |
| RLU (F/J) | Generate a (pseudo)random number uniformly in 0 < RLU < 1 |
| ULANGL (F/J) | Calculate the angle from the x and y coordinates |
| LUCOMP (F/J) | Give the compressed parton/particle code |
| PLU (F/J) | Provide various real-valued event data |

3.2 Common blocks

The common-block mainly intended for communication with the program is MAUSER, which sets switches, parameters and cuts. In addition, all kinematic variables for a given event can be found in MAKINE and LUJETS in JETSET 7.4 is used to store the event record. All variables are given sensible default values in the block data MADATA and all variable names obey the following name convention: integers start with I-N, single precision reals start with A-C,E-H and O-Z and double precision reals start with D.

COMMON /MAUSER/ MAFLAG(20),CUTM(12),PARM(30)

Switches for controlling the program:

MAFLAG(1) (D=9) choice of parameterisation for the parton densities in the proton. Is transferred to LST(15) in LEPTO 6 which in version 6.5 gives the following choices of relevance. (For information on how to use PDFLIB [12], see LST(16) in LEPTO. The parton densities are obtained from subroutine PYSTFU in PYTHIA 5.7 [9].)
=0: parton density choice and parameters are controlled directly through parameters in Pythia 5.7.
=1: Eichten-Hinchliffe-Lane-Quigg set 1 [13].
=2: Eichten-Hinchliffe-Lane-Quigg set 2 [13].
=3: Duke-Owens set 1 [14].
=4: Duke-Owens set 2 [14].
=5: CTEQ2M (best $\overline{MS}$ fit) [15].
=6: CTEQ2MS (singular at small-$x$) [15].
=7: CTEQ2MF (flat at small-$x$) [15].
=8: CTEQ2ML (large $\Lambda$) [15].
=9: CTEQ2L (best leading order fit) [15].
=10: CTEQ2D (best DIS fit) [15].

MAFLAG(2) (D=0) choice of factorisation scale $\mu^2$ used in the structure-functions

=0: $Q^2$
=1: $p_{N\perp}^2$, the squared transverse momentum of the neutrino
=2: the constant given in PARM(4)

MAFLAG(3) (D=0) regulates the final-state lepton status in the event record, K(I,1)

=0: active final-state lepton, i.e. K(I,1)=1
=1: inactive final-state lepton, i.e. K(I,1)=21

MAFLAG(4) (D=1) regulates the direction of the $x$-axis ($z$-axis in proton direction)

=0: the $x$-axis along $p_{N\perp}$
=1: event rotated randomly in azimuthal angle between 0 and $2\pi$

MAFLAG(5) (D=1) regulates the hadronisation

=0: hadronisation off
=1: hadronisation on

MAFLAG(6) (D=0) regulates the amount of written output.

=0: all output except MINUIT-output
=5: all output

MAFLAG(7) (D=0) regulates the choice of the process number (NPROC)

=0: 1, 2 and 3 mixed according to their respective branching ratios
=1: $e^-p \rightarrow NX \rightarrow e^+W^-X$
=2: $e^-p \rightarrow NX \rightarrow e^-W+X$
=3: $e^-p \rightarrow NX \rightarrow \nu ZX$
=10: 11, 12 and 13 mixed according to their respective branching ratios
=11: $e^+p \rightarrow NX \rightarrow e^-W+X$
=12: $e^+p \rightarrow NX \rightarrow e^+W^-X$
=13: $e^+p \rightarrow NX \rightarrow \nu ZX$

MAFLAG(8) (D=2) simulation of QCD effects in the scattered quark and proton remnant system (for 2-5 cf. LST(8) in Lepto [8])

=1: final state radiation from the scattered quark only, using a simplified treatment (LUSHOW is used on the quark and diquark system). Only valence quarks, i.e. $u$-quarks, are considered in the proton.
=2: initial and final state radiation
=3: initial state radiation
=4: final state radiation
=5: QCD switched off but target remnant as 2, 3 and 4

MAFLAG(9) error flag
=1: maximum of \(g\)-function violated. This is serious if the violation is large and in that case the user has to either set the maxima by hand (see MAFLAG(19)) or change the \(h\)-functions to make the \(g\)-function smoother.

**MAFLAG(10)**  
(D=1) Error handling  
=0: no warnings and execution not stopped on error  
=1: warnings printed but execution not stopped  
=2: warnings printed and execution stopped

**MAFLAG(11)**  
(D=2) choice of \(h_x\)-function (see Section 2 for definition and Table I for suitable choices)  
=−2: \(h_x(x) = 1/x^2\)  
=−1: \(h_x(x) = 1/x\)  
=0: \(h_x(x) = \text{const}\).  
=1: \(h_x(x) = \exp(-A_x x)\)  
=2: \(h_x(x) = x \exp(-A_x x^2)\)

**MAFLAG(12)**  
(D=0) regulates calculation of \(A_x\)  
=0: \(A_x\) is calculated by the program  
=1: \(A_x\) is given in PARM(8)

**MAFLAG(13)**  
(D=1) choice of \(h_y\)-function (see Section 2 for definition and Table I for suitable choices)  
=−2: \(h_y(y) = 1/y^2\)  
=−1: \(h_y(y) = 1/y\)  
=0: \(h_y(y) = \text{const}\).  
=1: \(h_y(y) = \exp(-A_y y)\)  
=2: \(h_y(y) = y \exp(-A_y y^2)\)

**MAFLAG(14)**  
(D=0) regulates calculation of \(A_y\)  
=0: \(A_y\) is calculated by the program  
=1: \(A_y\) is given in PARM(9)

**MAFLAG(15)**  
(D=2) choice of \(h_{p_{\perp\perp}}\)-function (see Section 2 for definition and Table I for suitable choices)  
=−2: \(h_{p_{\perp\perp}}(p_{\perp\perp}) = 1/p_{\perp\perp}^2\)  
=−1: \(h_{p_{\perp\perp}}(p_{\perp\perp}) = 1/p_{\perp\perp}\)  
=0: \(h_{p_{\perp\perp}}(p_{\perp\perp}) = \text{const}\).  
=1: \(h_{p_{\perp\perp}}(p_{\perp\perp}) = \exp(-A_{p_{\perp\perp}} p_{\perp\perp})\)  
=2: gives \(h_{p_{\perp\perp}}(p_{\perp\perp}) = p_{\perp\perp} \exp(-A_{p_{\perp\perp}} p_{\perp\perp}^2)\)

**MAFLAG(16)**  
(D=0) regulates calculation of \(A_{p_{\perp\perp}}\) for \(N \to e^\pm W^\mp\)  
=0: \(A_{p_{\perp\perp}}\) is calculated by the program  
=1: \(A_{p_{\perp\perp}}\) is given in PARM(10)

**MAFLAG(17)**  
(D=0) regulates calculation of \(A_{p_{\perp\perp}}\) for \(N \to \nu Z\)  
=0: \(A_{p_{\perp\perp}}\) is calculated by the program  
=1: \(A_{p_{\perp\perp}}\) is given in PARM(11)

**MAFLAG(18)**  
(D=1) choice between fixed and varying (according to a Breit-Wigner distribution) boson-masses in the neutrino decay  
=0: fixed boson-masses  
=1: varying boson-masses

**MAFLAG(19)**  
(D=0) regulates calculation of \(g\)-function (see section 2) maxima  
=0: maxima of \(g\)-functions are calculated  
=1: maxima of \(g\)-functions given in PARM(12) to PARM(17)
MAFLAG(20) \hspace{1cm} (D=0) \text{choice between left and right handed } W \text{ exchange}

-0: left handed } W \text{ exchange}

=1: right handed } W \text{ exchange}

\textit{Cuts on kinematic variables defined in Section 1:}

CUTM(1): \hspace{1cm} x_{\text{min}} (D=0.)

CUTM(2): \hspace{1cm} x_{\text{max}} (D=1.)

CUTM(3): \hspace{1cm} y_{\text{min}} (D=0.)

CUTM(4): \hspace{1cm} y_{\text{max}} (D=1.)

CUTM(5): \hspace{1cm} Q_{\text{min}}^2 (D=4. \text{ GeV}^2)

CUTM(6): \hspace{1cm} Q_{\text{max}}^2 (D=10^8 \text{ GeV}^2)

CUTM(7): \hspace{1cm} W_{\text{min}}^2 (D=9. \text{ GeV}^2)

CUTM(8): \hspace{1cm} W_{\text{max}}^2 (D=10^8 \text{ GeV}^2)

CUTM(9): \hspace{1cm} p_{\ell \perp \text{min}} (D=0. \text{ GeV})

CUTM(10): \hspace{1cm} p_{\ell \perp \text{max}} (D=1000 \text{ GeV})

CUTM(11): \hspace{1cm} y_{\ell \text{min}} (D=−100)

CUTM(12): \hspace{1cm} y_{\ell \text{max}} (D=100)

\textit{Parameters for input (1-20) and output (21-30):}

PARM(1): \hspace{1cm} (D=100 \text{ GeV}) \text{ mass of heavy Majorana neutrino}

PARM(2): \hspace{1cm} (D=820 \text{ GeV}) \text{ proton-momentum}

PARM(3): \hspace{1cm} (D=30 \text{ GeV}) \text{ electron-momentum}

PARM(4): \hspace{1cm} (D=1000 \text{ GeV}^2) \text{ fixed factorisation scale } \mu^2 \text{ used if MAFLAG(2)=2}

PARM(5): \hspace{1cm} (D=0.01) \text{ degree of mixing between light and heavy Majorana neutrinos, } |(V\xi)_{eN}|^2 = |\xi_{\nu e,N}|^2. \text{ In the case of right handed } W \text{ exchange it is interpreted as the ratio between the right and left handed gauge couplings, } (g_R/g_L)^2 \text{ which normally is assumed to be 1.}

PARM(6): \hspace{1cm} (D=0.001) \text{ cut used around the singularity of the } g\text{-function which divides the } g\text{-function into two parts (see ALSCUT in [7] for details). A larger cut will speed up the simulation but at the same time increase the risk of violating the maximum of the } g\text{-function and cut away a larger part of the phase-space.}

PARM(7): \hspace{1cm} (D=1.1) \text{ safety factor, multiplies the maxima of the } g\text{-function.}

PARM(8): \hspace{1cm} A_x \text{ (see also MAFLAG(12))}

PARM(9): \hspace{1cm} A_p \text{ (see also MAFLAG(14))}

PARM(10): \hspace{1cm} A_{p_{\ell \perp}} \text{ for } N \rightarrow e^\pm W^\mp \text{ (see also MAFLAG(16))}

PARM(11): \hspace{1cm} A_{p_{\ell \perp}} \text{ for } N \rightarrow \nu Z \text{ (see also MAFLAG(17))}

PARM(12): \hspace{1cm} \text{Max of first part of } g\text{-function for process nr 1 or 11 (see also MAFLAG(19))}

PARM(13): \hspace{1cm} \text{Max of second part of } g\text{-function for process nr 1 or 11 (see also MAFLAG(19))}

PARM(14): \hspace{1cm} \text{Max of first part of } g\text{-function for process nr 2 or 12 (see also MAFLAG(19))}

PARM(15): \hspace{1cm} \text{Max of second part of } g\text{-function for process nr 2 or 12 (see also MAFLAG(19))}

PARM(16): \hspace{1cm} \text{Max of first part of } g\text{-function for process nr 3 or 13 (see also MAFLAG(19))}

PARM(17): \hspace{1cm} \text{Max of second part of } g\text{-function for process nr 3 or 13 (see also MAFLAG(19))}

PARM(18): \hspace{1cm} \text{Mass of right-handed } W\text{-boson in GeV}

PARM(19): \hspace{1cm} \text{Width of right-handed } W\text{-boson in GeV}

PARM(20): \hspace{1cm} \text{not used}

PARM(21): \hspace{1cm} \text{Estimate}^1 \text{ of the cross-section in } \text{pb} \text{ (including cuts) for process nr 1 or 11.}

PARM(22): \hspace{1cm} \text{Estimate}^1 \text{ of the standard deviation in PARM(21)}

PARM(23): \hspace{1cm} \text{Estimate}^1 \text{ of the cross-section in } \text{pb} \text{ (including cuts) for process nr 2 or 12.}
PARM(24): Estimate\(^1\) of the standard deviation in PARM(23)
PARM(25): Estimate\(^1\) of the cross-section in \(pb\) (including cuts) for process nr 3 or 13.
PARM(26): Estimate\(^1\) of the standard deviation in PARM(25)
PARM(27): Efficiency\(^2\) in the generation for process nr 1 or 11
PARM(28): Efficiency\(^2\) in the generation for process nr 2 or 12
PARM(29): Efficiency\(^2\) in the generation for process nr 3 or 13
PARM(30): not used

COMMON /MAKINE/ EP,EE,S,SHAT,Q2,W2,X,Y,PNT,PNL,YN,EN,PLT,YL

Kinematical variables in a given event of use for the user:

EP: Proton energy in GeV
EE: Electron energy in GeV
S: CMS-energy squared (Mandelstam s) in GeV\(^2\)
SHAT: \(\hat{s} = x s\)
Q2: Momentum transfer squared, \(Q^2 = -q^2\) in GeV\(^2\)
W2: Hadronic CMS-energy squared, \(W^2 = (p_p + q)^2\) in GeV\(^2\)
X: Bjorken-\(x\), \(x = Q^2/2p_p \cdot q\)
Y: Standard \(y\)-variable, \(y = p_p \cdot q/p_p \cdot p_e\)
PNT: Transverse momentum of heavy \(p_{N\perp}\) Majorana neutrino in GeV
PNL: Longitudinal momentum of heavy Majorana neutrino in GeV
YN: Rapidity \(y_N\) of heavy Majorana neutrino
EN: Energy of heavy Majorana neutrino in GeV
PLT: Transverse momentum \(p_{\ell\perp}\) of final-state lepton from \(N\) decay in GeV
YL: Rapidity \(y_{\ell}\) of final-state lepton

COMMON /LUJETS/ N,K(4000,5),P(4000,5),V(4000,5)
The variables in the common-block LUJETS are described in the Jetset 7.4 manual. The first seven entries in the event record are as follows (in the lab frame with the \(z\)-axis in the proton direction):

1. Incoming electron
2. Incoming proton
3. Exchanged \(W\)-boson
4. Heavy Majorana neutrino
5. Incoming parton before initial shower
6. Incoming quark at boson vertex
7. Scattered quark at boson vertex before final shower

The decay products from the \(N\)-decay are in line K(4,4) (final state lepton) and K(4,5) (on shell boson) respectively.

Common-blocks for internal use: MACROS, MAMAMI, MAMASS, MACONS, MAGSPE.
Lepto 6.5 common-blocks used: LEPTOU, LBOOST, LFLMIX, LPFLAG, LYPARA.
Jetset 7.4 common-blocks used: LUDAT1, LUDAT2, LUDAT3, LUDAT4.

\(^1\) The estimate is updated for each event so it should not be used until all events have been generated.
\(^2\) The efficiency is given by the number of accepted events divided by the number of tries.
Table 1: Examples of suitable choices of $h$-functions at different $cms$ energies and for different heavy Majorana neutrino masses.

| $\sqrt{s}$ [GeV] | $m_N$ [GeV] | HERA 300 | LEP\(\otimes\)LHC 1200 |
|------------------|-------------|---------|------------------|
| $\otimes$        | 100         | 300     | 700              |
| MAFLAG(11)       | 2           | -1      | -2               |
| MAFLAG(13)       | 1           | -1      | -1               |
| MAFLAG(15)       | 2           | 2       | 2                |

3.3 Update history

Updates from version 1.1 [16] to 1.3 [17]:

- inclusion of the neutral current decay $N \rightarrow \nu Z$
- extension and redefinition of the common-block MAUSER and some internal common-blocks
- inclusion of a subroutine for error handling
- correction of a small error in the differential cross-section

from version 1.3 [17] to 1.5:

- inclusion of possibility to have incoming positron (see MAFLAG(7))
- inclusion of possibility to have exchange of right-handed W's which are lighter than the Heavy Neutrino (see MAFLAG(20), PARM(5), PARM(18) and PARM(19))
- update to run with LEPTO 6.5, JETSET 7.4 and PYTHIA 5.7 (see MAFLAG(1))
- inclusion of MINUIT [11] subroutines from LEPTO

4 Usage and availability

MAJOR 1.5 should be loaded together with LEPTO 6.5 [8], JETSET 7.4 and PYTHIA 5.7 [9]. The program is a slave system, which the user must call from his own steering program. Information about the program, the source code and a demonstration job can be found via the WWW on the MAJOR homepage [http://www3.tsl.uu.se/hep/major] or from one of the authors via email. It is not recommended to split the program and making a library of it since this can give problems with the use of LEPTO routines. Examples of suitable choices of $h$-functions are given in Table 1.
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