HYDROMAGNETIC NATURAL CONVECTION FROM A HORIZONTAL POROUS ANNULUS WITH HEAT GENERATION OR ABSORPTION

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This paper deals with a numerical study of free convection in a horizontal cylindrical annulus filled with a fluid-saturated porous medium in the presence of a transverse magnetic field and the heat generation or absorption effect. It is assumed that the inner and outer walls of the cylindrical annulus are maintained at constant temperatures $T_i$ and $T_o$, respectively, as $T_i > T_o$. In addition to the heat equation, the model consists of the equation of motion under the Darcy law and Boussinesq approximation. The system of equations is solved numerically by the alternating-direction implicit finite difference method. This investigation concerns the effects of the magnetic field inclination angle, Hartmann number, and the heat generation or absorption coefficient on heat transfer and the flow pattern. The results demonstrate that the heat transfer rate and flow regime depend mainly on the characteristics mentioned. The obtained data are presented graphically in terms of the streamlines and isotherms.

Keywords: free convection, inclined magnetic field, heat generation/absorption, heat transfer, porous medium.

Introduction. Convective phenomena induced by the thermal buoyancy force in enclosures filled with fluid-saturated porous media have attracted considerable attention during the last decades. This interest is due to a wide range of their numerous applications in geophysics and energy-related engineering problems. Such applications include solar collectors, building heating and cooling systems, heat exchangers, thermal storage systems, and underground transmission lines (see [1, 2] and the references therein).

Natural convection in cavities subjected to heat generation is of crucial interest in various technological applications. Examples are provided by post-accident heat removal in nuclear reactors and geophysical problems associated with underground storage of nuclear waste (as examples, see [3–9]).

The effect of a magnetic field on convective flows is of great interest in numerous industrial applications, such as crystal growth, metal casting, processes in liquid metals, electrolytes, and ionized gases, as well as in MHD generators. There has been a noticeable enlargement in studying the impact of a magnetic field on the performance of many processes and systems with the use of electrically conducting fluids [10–12]. The impact of heat generation/absorption and thermal radiation on convective heat transfer generated by a bidirectional stretching sheet was evaluated in [13]. It was found that an increase in the Hartmann number leads to a significant decrease in the fluid velocity. In [14], MHD natural convection with thermal radiation and heat absorption or generation over a cone was studied. It was shown that the temperature increases with the heat generation coefficient and the velocity decreases when the magnetic field strength increases. In [15], the control volume finite element method was used to analyze heat transfer in the presence of a nanofluid with consideration for the Lorentz effect. It was concluded that the convective heat transfer rate decreases with strengthening the magnetic field.

The present study aims to analyze the influence of the magnetic field inclination angle, Hartmann number, and the heat generation or absorption coefficient on the convective heat transfer rate within horizontal concentric cylinders filled with a saturated porous medium.

Basic Equations. The model considered here includes a porous layer saturated with an incompressible Newtonian fluid and located between two horizontal concentric cylinders of radii $r_i$ and $r_o$, as shown in Fig. 1. Both the inner and outer cylinders are kept at constant temperatures $T_i$ and $T_o$, respectively, where $T_i > T_o$. The fluid is exposed to a uniform external magnetic field of the strength $\mathbf{B}$ at its constant magnitude $B_0$. We assume that radiation, viscous dissipation, and Joule heating are taken to be small enough to be neglected. The fluid is assumed to be in thermal equilibrium with the
The polar coordinates system \( r' - n \) is used. Under these assumptions along with the Boussinesq and Darcy approximations, the governing equations of heat and electric transfer can be written as follows:

\[
\nabla' \cdot \mathbf{V}' = 0 ,
\]

\[
\mathbf{V}' = \frac{K}{\mu} \left( -\nabla' p' + \rho_0 \beta (T' - T_0) \right) g \mathbf{k} + \mathbf{I} \times \mathbf{B} ,
\]

\[
(pC_p)_m \frac{\partial T'}{\partial t'} + (pC_p)_f (\mathbf{V}' \cdot \nabla') T' = k_m \nabla'^2 T' + Q_0 ,
\]

\[
\nabla' \cdot \mathbf{I} = 0 ,
\]

\[
\mathbf{I} = \sigma (-\nabla' \phi + \mathbf{V}' \times \mathbf{B}) .
\]

Here the gradient and Laplace operator in the used polar coordinates are defined in the following manner:

\[
\nabla' F = \left( \frac{\partial F}{\partial r'} + \frac{1}{r'} \frac{\partial F}{\partial \phi} \right) ,
\]

\[
\nabla'^2 F = \frac{1}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial F}{\partial r'} \right) + \frac{1}{r' \phi^2} \frac{\partial^2 F}{\partial \phi^2} ,
\]

where \( K \) is the permeability, \( \mathbf{k} \) is the unit vector in the vertical direction \( (\mathbf{k} = g / |g|) \), \( Q_0 \) is the constant generated or absorbed heat from the source per unit volume, and \( \phi \) is the electric potential. The heat capacity of a saturated porous medium is equal to \( (\rho C_p)_m = \epsilon (\rho C_p)_f + (1 - \epsilon) (\rho C_p)_s \), and the equivalent thermal conductivity is calculated as the weighted average of the solid and fluid conductivities, i.e., \( k_m = \epsilon k_f + (1 - \epsilon) k_s \).

As was shown in [16, 17], Eqs. (4) and (5) reduce to \( \nabla'^2 \phi = 0 \). Unique solution of this equation is \( \phi = 0 \) due to an electrically insulating boundary around the cavity. It follows that an electric field vanishes everywhere (see [18]).

It is more suitable to deal with dimensionless equations. Thus, using the following scales: \( r_1 \) for the radial coordinate, \( \eta_f^2 (pC_p)_m / k_m \) for time, \( k_m / \eta_f (pC_p)_f \) for the velocity, \( \Delta T = T_i - T_o \) for the temperature, and \( k_m \mu / (pC_p)_f \) for the pressure, we obtain the partial differential equations in the dimensionless form as

\[
\nabla \cdot \mathbf{V} = 0 ,
\]

\[
\mathbf{V} = -\nabla p - \text{Ra} T \mathbf{k} - \text{Ha} \mathbf{F} ,
\]

\[
\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T = \nabla^2 T + Q .
\]
Here \( \text{Ra} = \frac{g\beta K \Delta T_1 (\rho C_p)}{k_m \nu} \), \( \text{Ha} = \frac{\sigma KB^2}{\mu} \), \( Q = \frac{Q_0 r^2}{k_m \Delta T} \), \( F = (u \cos^2 \eta + v \sin \eta \cos \eta) e_x + (u \sin \eta \cos \eta + v \sin^2 \eta) \times e_y \), \( \eta = \theta - \varphi \), where \( \theta \) is the inclination angle of the magnetic field. In Eqs. (6)–(8) \( \text{Ra} \), \( \text{Ha} \), and \( Q \) are respectively the thermal Rayleigh number, Hartmann number, and the dimensionless heat generation or absorption parameter. The governing equations for the studied problem in terms of the stream function and temperature are

\[
\nabla^2 \psi = -\text{Ra} \left( \sin \varphi \frac{\partial T}{\partial r} + \frac{\cos \varphi}{r} \frac{\partial T}{\partial \varphi} \right) - \text{Ha} \left( \frac{2 \sin \eta \cos \eta}{r^2} \left( \frac{\partial \psi}{\partial \varphi} - r \frac{\partial^2 \psi}{\partial r \partial \varphi} \right) + \sin^2 \eta \frac{\partial^2 \psi}{\partial r^2} + \frac{\cos^2 \eta}{r^2} \left( \frac{\partial^2 \psi}{\partial \varphi^2} + r \frac{\partial \psi}{\partial r} \right) \right),
\]

\[
\frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = -\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial \psi}{\partial \varphi} \right) = \nabla^2 T + Q,
\]

where \( \psi \) is the stream function such that \( u = \frac{1}{r} \frac{\partial \psi}{\partial \varphi} \) and \( v = -\frac{\partial \psi}{\partial r} \).

The problem is assumed to be symmetric about the vertical line passing through the center of the system, therefore, only the half of the flow domain will be considered. Thus, the boundary conditions can be written as

\[
\frac{\partial \psi}{\partial \varphi} = 0, \quad T = 1 \quad \text{at} \quad r = 1,
\]

\[
\frac{\partial \psi}{\partial \varphi} = 0, \quad T = 0 \quad \text{at} \quad r = R,
\]

\[
\frac{\partial \psi}{\partial r} = \frac{\partial T}{\partial \varphi} = 0 \quad \text{at} \quad \varphi = 0, \pi,
\]

where the radius ratio \( R = r_o/r_i \) represents an another dimensionless key parameter of the problem. Heat transfer across the whole annulus is presented in terms of the average Nusselt number along the hotter (inner) cylinder evaluated as follows:

\[
\overline{\text{Nu}(i)} = \frac{1}{\pi} \int_0^\pi \text{Nu}_i d\varphi,
\]

where \( \text{Nu}_i \) is the local Nusselt number along the hotter cylinder calculated as the ratio between convective and conductive heat transfer:

\[
\text{Nu}_i(\varphi)|_{r=1} = -\ln \left( \frac{r \frac{\partial T}{\partial r}}{R \frac{\partial T}{\partial r}} \right)_{r=1}.
\]

**Numerical Methods.** The dimensionless governing equations are discretized by the use of the finite difference method coupled with the alternating-direction implicit scheme. To solve these discretized equations, the Thomas tridiagonal matrix algorithm is used in conjunction with iterations. As the study concerns steady-state regimes, the iterative procedure is stopped when the residuals become below \( 10^{-8} \).

In order to assess the accuracy of the numerical code developed, the convection problem was solved in the absence of a magnetic field and the heat generation or absorption effects, since solutions for such a problem are available. The results obtained with the use of the present code are in good agreement with the experimental results of [19] and the numerical data reported earlier [19–24] (see Table 1). Moreover, Fig. 2 is indicative of good agreement between the obtained streamlines and
temperature contour plots and the results of [25–27] for \( R = 2 \) and \( Ra = 200 \). In this comparison, we have used a motionless fluid and the temperature distribution in the case of pure conduction at the initial conditions of the problem considered.

For obtaining the best compromise between the accuracy and minimized calculation time, a grid sensitivity analysis was performed. The present code was assessed for grid independence by evaluating the average Nusselt number along the hotter cylinder. Therefore, various numerical experiments were tested for \( R = 2, Ra = 100, Ha = 0.5, Q = 0.01, \) and \( \theta = 0 \), as illustrated in Table 2 (where the reference value \( \bar{Nu}_r \) corresponds to the \( 301 \times 301 \) grid). It was found from the comparison performed that the \( 131 \times 131 \) grid yields the solutions that are reasonably grid-independent.

**TABLE 1. Average Nusselt Number at \( R = 2 \) for Pure Free Convection**

| Work | Grid size | Ra = 50 | Ra = 100 |
|------|-----------|---------|----------|
| [19] | 49 \times 49 | 1.3278  | 1.8286   |
| [20] | 30 \times 44 | 1.335   | 1.844    |
| [21] | 10 \times 10 | 1.341   | 1.861    |
| [22] | 161 \times 101 | 1.338  | 1.861    |
| [23] | 100 \times 240 | 1.343  | 1.868    |
| [24] | 50 \times 50 | 1.324   | 1.830    |
| This study | 49 \times 49 | 1.343  | 1.852    |

**TABLE 2. Average Nusselt Number at \( R = 2, Ra = 100, Ha = 0.5, Q = 0.01, \) and \( \theta = 0 \)**

| Grid size | \( \bar{Nu} \) | \( \frac{\Delta\bar{Nu}}{\bar{Nu}_r}, \% \) |
|-----------|----------------|-------------------------------|
| 31 \times 31 | 1.6239 | 0.0431 |
| 101 \times 101 | 1.6229 | 0.0184 |
| 131 \times 131 | 1.6232 | 0.0002 |
| 151 \times 151 | 1.6232 | 0 |
| 251 \times 251 | 1.6232 | 0 |
Results and Discussion. A numerical investigation of the boundary value problem presented by Eqs. (9)–(11) has been carried out at the following values of the key parameters: $50 \leq Ra \leq 200$, $0 \leq Ha \leq 20$, $-0.5 \leq Q \leq 0.5$, $0 \leq \theta \leq \pi/2$, and at the fixed radius ratio $R = 2$.

Particular efforts have been focused on the impact of these key parameters on the fluid flow and convective heat transfer. Figure 3a illustrates the effect of the inclination angle on the average Nusselt number for $Ra = 50$. It is seen that the average Nusselt number decreases when the Hartman number increases. It is interesting to note that the maximum effect of a magnetic field is observed when the inclination angle is equal to zero. Moreover, convective heat transfer is more pronounced for the values of the inclination angle close to $\pi/2$.

Figure 3b and c illustrates the influence of the Hartmann number on the average Nusselt number for $Ra = 100$ and different values of the heat generation or absorption coefficient. It is seen that, regardless of the value of $Q$, the average Nusselt number decreases when $Ha$ increases. It should also be noted that the effect of the Hartman number on convective heat transfer is less significant when $Ha$ is greater than 15. Besides, it may be concluded that the convective heat transfer rate is a decreasing function of the heat generation or absorption coefficient $Q$. This conclusion is confirmed by Fig. 4 which shows the average Nusselt number as function of $Q$ for $Ra = 100$ and different values of the Hartman number.

It is well known that several types of convective flow regimes may develop depending on the initial conditions of the problem and the values of the key parameters, such as the radius ratio and Rayleigh number [21, 25, 28, 29]. Thus unicellular, bicellular or multicellular flow structures may appear for relatively high values of the Rayleigh number at
The transition from a unicellular flow regime to a bicellular one is connected with the onset of thermoconvective instabilities, as was already demonstrated by earlier investigations [19, 23, 30].

Figure 5a and b shows the streamlines and isotherms for $Ra = 80$, $Q = 0$, and two values of the Hartmann number ($Ha = 0$ and 5). It is clearly seen that the flow structure transfers from a bicellular flow regime for $Ha = 0$ to a unicellular structure at $Ha = 5$. It should be noted that the number of convective counterrotating cells inside the annulus top decreases as the Hartman number increases. In other words, thermoconvective instability is less pronounced when a magnetic field is relatively strong. However, the performed tests demonstrate that the flow structure changes when the heat generation or absorption coefficient exceeds a certain given threshold. Thus, the thermoconvective instabilities may develop for relatively high values of $Q$. Therefore, as shown in Fig. 5c and d, a small increase in $Q$ leads to the development of a new convective cell in the top part of the annulus.

Conclusions. In this paper, we have studied the impact of the magnetic force and heat generation or absorption on convective heat transfer and the flow pattern inside horizontal concentric cylinders filled with a saturated porous medium. Upon introducing the proper similarity variable, the resulting set of differential equations under the Boussinesq and Darcy approximations was represented in the dimensionless form.

The most important conclusions of the investigation can be summarized as follows:

1. In general, the convective heat transfer rate is highly impacted by the magnetic force inclination angle, Hartmann number, Rayleigh number, and the heat generation or absorption coefficient.
2. The impact of the magnetic force becomes more effective when the inclination angle is close to zero.
3. Different flow regimes may appear for various combinations of the Rayleigh number, Hartmann number, and the heat generation or absorption coefficient.
4. The magnetic force has a significant effect on thermoconvective instabilities. A small increase in the Hartmann number influences the flow structure.

5. The thermoconvective instabilities are more pronounced when the heat generation or absorption coefficient increases.

**NOTATION**

\( B_0 \), magnetic field strength, \( \text{Wb-m}^{-2} \); \( C_p \), specific heat, \( 	ext{J/(kg-K)} \); \( g \), gravitational acceleration, \( \text{m-s}^{-2} \); Ha, Hartmann number; \( \eta \), electric current, \( \text{A} \); \( \kappa \), thermal conductivity, \( \text{W-m}^{-1}\text{K}^{-1} \); \( K \), permeability, \( \text{m}^2 \); \( \nu \), kinematic viscosity, \( \text{m}^{-1} \); \( \phi \), electric potential, \( \text{V} \); \( \mu \), dynamic viscosity, \( \text{kg/(m-s)} \); \( \rho \), density, \( \text{kg/m}^3 \); \( \rho_0 \), reference fluid density, \( \text{kg/m}^3 \); \( \sigma \), electrical conductivity of the fluid, \( \Omega^{-1}\text{m}^{-1} \); \( \varphi \), polar angle, rad; \( \beta \), coefficient of thermal expansion, \( \text{K}^{-1} \); \( \varepsilon \), porosity; \( \theta \), inclination angle of the magnetic field, rad; \( \mu \), dynamic viscosity, \( \text{kg/(m-s)} \); \( \nu \), kinematic viscosity, \( \text{m}^{-2}\text{s}^{-1} \); \( \rho \), density, \( \text{kg/m}^3 \); \( \sigma \), electrical conductivity of the fluid, \( \Omega^{-1}\text{m}^{-1} \); \( \varphi \), polar angle, rad; \( \phi \), electric potential, \( \text{V} \); \( \psi \), dimensionless stream function. Indices: i, inner; f, fluid; m, fluid–solid mixture; o, outer; s, solid matrix of the porous medium.

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