A new theory is proposed that offers a consistent conceptual basis for nonrelativistic quantum mechanics. The violation of Bell’s inequality is explained by maintaining realism, inductive inference and Einstein separability.

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Despite the successes of quantum mechanics its basic concepts, especially the measurement and the collapse of the wave function [1] has remained unclear and controversial up to these days. From the theoretical point of view, the most serious problem is probably the violation of Bell’s inequality [2], as it is believed to be the proof that Einstein separability is violated in Nature. Einstein separability is an obvious physical requirement stating that separated systems (i.e., which are prevented from any interaction with each other) cannot influence each other.

As a brief reminder, let us think of two spin-half particles in an entangled state, which is due to some previous interaction between them. Suppose that the particles are already separated so much that they can no longer interact with each other. Imagine that we perform spin measurements in different directions on each of the two separated particles. It is natural to assume that any correlation between the results of the measurements performed on the different particles can come only from the previous interaction which created the entangled state. One may also assume that there are some stable properties attached to each system, so that these properties ‘store’ the correlation after the systems have become separated. Using these assumptions one may derive that the measuring device has no influence on them. The usual conclusion is that Einstein separability is violated.

Nevertheless, we maintain that such a conclusion is physically unacceptable. The principle of Einstein separability has served us well in every branch of physics, even in quantum physics, including the most sophisticated quantum field theories. The only way out can be if there is some further, independent and hidden assumption, which seems to us obvious, but which is not valid in quantum mechanics.

In the present letter it will be shown that it is indeed the case. One may reinterpret the meaning and the interrelations of the quantum states such a manner that the violation of Bell’s inequality gains a natural explanation without giving up realism, inductive inference or Einstein separability. The hidden, not allowed assumption mentioned above is connected to the fact that in the new theory it may happen that different states, although individually exist, cannot be compared. In case of the violation of Bell’s inequality it turns out that the states of the measuring devices and those which ‘store’ the correlations are not comparable as any attempt for a comparison changes the correlations themselves. Therefore, the usual picture about stable properties which ‘store’ the correlations and are comparable in principle at any time with anything does not apply, although the correlations may be attributed exclusively to the ‘common past’ (previous interaction) of the particles.

To begin with, let us consider a simple example, namely, an idealized measurement of an $\hat{S}_z$ spin component of some spin-$\frac{1}{2}$ particle. Be the particle $P$ initially in the state $|\alpha\rangle \rightarrow |\uparrow\rangle + |\beta\rangle \downarrow\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$ and the states $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of $\hat{S}_z$ corresponding to the eigenvalues $\frac{h}{2}$ and $-\frac{h}{2}$, respectively. The dynamics of the measurement is given by the relations $|\uparrow\rangle |m_0\rangle \rightarrow |\uparrow\rangle |m\uparrow\rangle$ and $|\downarrow\rangle |m_0\rangle \rightarrow |\downarrow\rangle |m\downarrow\rangle$, where $|m\uparrow\rangle$, $|m\uparrow\rangle$ and $|m\downarrow\rangle$ stand for states of the measuring device $M$. The linearity of the Schrödinger equation implies that the measurement process can be written as

$$\langle\alpha| \uparrow\rangle + \beta |\downarrow\rangle |m_0\rangle \rightarrow |\Psi\rangle = \alpha |\uparrow\rangle |m\uparrow\rangle + \beta |\downarrow\rangle |m\downarrow\rangle \ . \tag{1}$$

Let us consider now the state of the measuring device $M$ after the measurement. As the combined system $P + M$ is in an entangled state, the measuring device has no own wave function and may be described by the reduced density matrix [3].
choosing a quantum reference system \( R \) means that we imagine what we would experience if we did a measurement on \( R \) that does not disturb \( \hat{\rho}_R(\mathcal{R}) = |\psi_R><\psi_R| \). In order to see that such a measurement exists, consider an operator \( \hat{A} \) (which acts on the Hilbert space of \( R \)) whose eigenstates include \( |\psi_R> \). The measurement of \( \hat{A} \) will not disturb \( |\psi_R> \). Let us emphasize that the possibility of nondisturbing measurements is an expression of realism: the state \( \hat{\rho}_R(\mathcal{R}) \) exists independently whether we measure it or not.

As the dependence of \( \hat{\rho}_S(\mathcal{R}) \) on \( R \) is a fundamental property now, one has to specify the relation of the different states in terms of suitable postulates:

**Postulate A.** If the reference system \( R = \mathcal{I} \) is an isolated one \( ^1 \) then the state \( \hat{\rho}_S(\mathcal{I}) \) commutes with the internal state \( \hat{\rho}_S(S) \).

This means that the internal state of \( S \) coincides with one of the eigenstates of \( \hat{\rho}_S(\mathcal{I}) \). Therefore, we shall call the eigenstates of \( \hat{\rho}_S(\mathcal{I}) \) the possible internal states of \( S \) provided that the reference system \( I \) is an isolated one.

**Postulate B.** The result of a measurement is contained unambiguously in the internal state of the measuring device.

**Postulate C.** If there are \( n \) \( (n = 1, 2, 3, ...) \) disjointed physical systems, denoted by \( S_1, S_2, ... S_n \), all contained in the isolated reference system \( \mathcal{I} \) and having the possible internal states \( |\phi_{S_1,j}> \), \( ..., |\phi_{S_n,j}> \), respectively, then the joint probability that \( |\phi_{S_i,j}> \) coincides with the internal state of \( S_i \) \((i = 1, ... n)\) is given by

\[
P(S_{1,j_1}, ..., S_{n,j_n}) = Tr_{S_1+...+S_n}[\hat{\pi}_{S_1,j_1} ... \hat{\pi}_{S_n,j_n} \hat{\rho}_{S_1+...+S_n}(\mathcal{I})],
\]

where \( \hat{\pi}_{S,j} = |\phi_{S,j}> <\phi_{S,j}| \).

Furthermore, the time dependent Schrödinger equation remains valid for closed systems. Note that in the present theory there is no collapse or reduction of the wave function, and every conclusion should be drawn by using the above rules.

Let us consider now a two-particle system \( P_1 + P_2 \) consisting of the separated spin-half particles \( P_1 \) and \( P_2 \). The initial internal state of the two-particle system be

\[
\sum_j c_j |\phi_{P_1,j}> |\phi_{P_2,j}>
\]

where \( c_1 = a \), \( c_2 = -b \) (certainly \(|a|^2 + |b|^2 = 1\), \( |\phi_{P_1,1}> = |1,\uparrow> \), \( |\phi_{P_2,2}> = |1,\downarrow> \), \( |\phi_{P_1,1}> = |2,\downarrow> \), \( |\phi_{P_2,2}> = |2,\downarrow> \), \( |\phi_{P_1,1}> = |2,\downarrow> \), \( |\phi_{P_2,2}> = |2,\downarrow> \).

\(^1\) An isolated system is such a system that has not been interacting with the outside world. A closed system is such a system that is not interacting with any other system at the given instant of time (but might have interacted in the past).
|φ_{P_2,i} >= |2, ↑>. When the two-particle system is in the state $|\vartheta\rangle$, there are strong correlations between the states $\hat{\rho}_{P_1}(P_1)$, $\hat{\rho}_{P_2}(P_2)$ = $|\psi_{P_1} > < \psi_{P_1}|$ and $|\psi_{P_2} > < \psi_{P_2}|$. Provided that the system $P_1 + P_2$ is initially isolated, applying Postulate C we obtain that the probability that $|\psi_{P_1} > |\phi_{P_1,i} >$ and $|\psi_{P_2} > |\phi_{P_2,j} >$ is $P(P_1, j, P_2, k) = |c_1|^2 \delta_{j,k}$.

Let us consider now a typical experimental situation, when measurements on both particles are performed. Before the measurements the internal state of the isolated system $P_1 + M_1 + P_2 + M_2$ ($P_1, P_2$ standing for the particles and $M_1, M_2$ for the measuring devices, respectively) is given by
\[
\sum_j c_j \hat{U}_t(P_1 + M_1) \left(|\phi_{P_1,j} > |m_0^{(1)} >\right) \times \hat{U}_t(P_2 + M_2) \left(|\phi_{P_2,j} > |m_0^{(2)} >\right),
\]
with a time $t$ later, i.e., during and after the measurements. Here $\hat{U}_t(P_1 + M_1)$ ($i = 1, 2$) stands for the unitary time evolution operator of the closed system $P_1 + M_1$.

Eq. (5) implies (according to Postulate A) that the internal states of the closed systems $P_1 + M_1$ and $P_2 + M_2$ evolve unitarily and do not influence each other. This time evolution can be given explicitly through the relations
\[
|\xi(P_1, j) > |m_0^{(i)} > \rightarrow |\xi(P_1, j) > |m_1^{(i)} > ,
\]
where $i, j = 1, 2$ and $|\xi(P_1, j) >$ is the $j$-th eigenstate of the spin measured on the $i$-th particle along an axis $z^{(i)}$ which closes an angle $\vartheta_i$ with the original $z$ direction. The time evolution of the internal state of the closed systems $P_1 + M_1$ is given explicitly by $|\psi_{P_1} > |m_1^{(i)} > \rightarrow \sum_j < \xi(P_1, j)|\psi_{P_1} > |\xi(P_1, j) > |m_1^{(j)} >$. As we see, the $i$-th measurement process is completely determined by the initial internal state of the particle $P_i$. Therefore, any correlation between the measurements may only stem from the initial correlation of the internal states of the particles.

The calculation of the state $\hat{\rho}_{M_1}(M_1)$ (which corresponds to the measured value, cf. Postulate B) needs the state of the whole isolated system $P_1 + P_2 + M_1 + M_2$. Using Eq. (5) the final state $|\vartheta\rangle$ may be written as
\[
\sum_{j,k} \sum_l c_l < \xi(P_1, j)|\phi_{P_1,l} > < \xi(P_2, k)|\phi_{P_2,l} > |m_0^{(1)} > |m_0^{(2)} > < \xi(P_1, j)|\xi(P_2, k) > |m_1^{(1)} > |m_1^{(2)} > ,
\]
Direct calculation shows that $\hat{\rho}_{M_1}(P_1 + P_2 + M_1 + M_2) = \sum_l \sum_i (|c_l|^2 < \xi(P_1, j)|\phi_{P_1,l} > |^2 |m_0^{(1)} > < m_1^{(1)} > |^2 < \xi(P_2, k)|\phi_{P_2,l} > |^2 |m_0^{(2)} > < m_1^{(2)} > |^2$ and it is independent of the second measurement. According to Postulate A $|\psi_{M_1} >$ is one of the $|m_1^{(1)} >$ states. The probability to observe the $j$-th result (up or down spin in a chosen direction) is $P(M_1, j) = \sum_l |c_l|^2$ .

This may be interpreted in conventional terms: $|c_l|^2$ is the probability that $|\psi_{P_1} > |\phi_{P_1,l} >$, and $| < \xi(P_1, j)|\phi_{P_1,l} > |^2$ is the conditional probability that one gets the $j$-th result if $|\psi_{P_1} > |\phi_{P_1,l} >$. Thus we see that the initial internal state of $P_1$ determines the outcome of the first measurement in the usual probabilistic sense. But doesn’t it mean that the internal states of $P_1$ and $P_2$ play the role of local hidden variables? No, because hidden variables are thought to be comparable with the results of the measurements so that their joint probability may be defined, while in our theory there is no way to define the joint probability $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$, i.e., the probability that initially $|\psi_{P_1} > |\phi_{P_1,l_1} >$ and $|\psi_{P_2} > |\phi_{P_2,l_2} >$ and finally $|\psi_{M_1} > |m_1^{(1)} >$ and $|\psi_{M_2} > |m_2^{(1)} >$. Intuitively we would write
\[
P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t)) = |c_l|^2 \delta_{l_1,l_2} < \xi(P_1, j)|\phi_{P_1,l_1} > |^2 \times < \xi(P_2, k)|\phi_{P_2,l_2} > |^2 ,
\]
as $|c_l|^2 \delta_{l_1,l_2}$ is the joint probability that $|\psi_{P_1} > |\phi_{P_1,l_1} >$ and $|\psi_{P_2} > |\phi_{P_2,l_2} >$ and $| < \xi(P_1, j)|\phi_{P_1,l_1} > |^2$ is the conditional probability that one gets the $j$-th result in the $i$-th measurement if initially $|\psi_{P_1} > |\phi_{P_1,l_1} >$ ($i = 1, 2$). Certainly the existence of such a joint probability would immediately imply the validity of Bell’s inequality, thus it is absolutely important to understand why this probability does not exist.

Let us mention, first of all, that using Postulate C for $n = 2$, we may calculate the correlation between the measurements, i.e., the joint probability that $|\psi_{M_1} > |m_1^{(1)} >$ and $|\psi_{M_2} > |m_2^{(1)} >$. We obtain
\[
P(M_1, j, M_2, k) = \sum_l |c_l|^2 < \xi(P_1, j)|\phi_{P_1,l} > |^2 < \xi(P_2, k)|\phi_{P_2,l} > |^2 .
\]
This is the usual quantum mechanical expression which violates Bell’s inequality and whose correctness is experimentally proven. Thus our theory gives the correct expression for the correlation. Nevertheless, if the joint probability $<\xi(P_1, j)|\phi_{P_1,l} > |^2$ exists, it leads to
\[
P(M_1, j, M_2, k) = \sum_l |c_l|^2 < \xi(P_1, j)|\phi_{P_1,l} > |^2 < \xi(P_2, k)|\phi_{P_2,l} > |^2 \]
which satisfies Bell’s inequality and contradicts Eq. (5). Let us demonstrate that no such contradiction appears. Evidently, the joint probability $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$ can be physically meaningful only if one can compare the initial internal states of $P_1$ and $P_2$ with the final internal states of $M_1$ and $M_2$ by suitable nondisturbing measurements. If we try to compare the initial internal states of
$P_1$ and of $P_2$ with the final internal states of $M_1$ and $M_2$, the first difficulty appears because we want to compare states given at different times. Nevertheless, as the initial internal state of $P_1$ is uniquely related to the final internal state of the system $P_1 + M_1$, the joint probability $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$ (if exists) coincides with $P(P_1 + M_1, l_1, P_2 + M_2, l_2, M_1, j, M_2, k)$, where all the occurring states are given after the measurements. As the systems $P_1 + M_1$, $P_2 + M_2$, $M_1$, $M_2$ are not disjointed, our **Postulates** do not provide us with an expression for the joint probability we are seeking for. If we check $|\psi_{M_1}>$ and $|\psi_{M_2}>$ by suitable nondisturbing measurements, we destroy $|\psi_{P_1+M_1}>$ and $|\psi_{P_2+M_2}>$, inhibiting any comparison. On the other hand, if we check first $|\psi_{P_1+M_1}>$ and $|\psi_{P_2+M_2}>$, then $P(M_1, j, M_2, k)$ changes. In fact, after suitable measurements performed on $P_1 + M_1$ (which do not change the internal states of $P_1 + M_1$) by further measuring devices $M_i$ we get for the internal state of the whole system

$$
\sum_i c_i \left( \sum_j <\xi(P_1,j)|\phi_{P_1,i}> |\xi(P_1,j)> |m_j^{(1)}> \right)
$$

$$
\times \left( \sum_k <\xi(P_2,k)|\phi_{P_2,i}> |\xi(P_2,k)> |m_k^{(2)}> \right)
$$

$$
\times |\tilde{m}_i^{(1)}> |\tilde{m}_i^{(2)}> .
$$

As the systems $M_1$, $M_2$, $\tilde{M}_1$, $\tilde{M}_2$ are disjointed, we may apply **Postulate C** for $n = 4$ and we indeed get for $P(\tilde{M}_1, l_1, \tilde{M}_2, l_2, M_1, j, M_2, k)$ the expression [8]. Do we get then a contradiction with Eq.[6]? No, because applying **Postulate C** directly for $n = 2$, we get in this case Eq.[6] instead of Eq.[8]. Thus we see that the extra measurements have changed the correlations and our theory gives account of this effect consistently.

Summing up, we have seen that the initial internal state of $P_1$ ($P_2$) determines the first (second) measurement process, therefore, these states ‘carry’ the initial correlations and ‘transfer’ them to the measuring devices. As the measurement processes do not influence each other, the observed correlations may stem only from the ‘common past’ of the particles. On the other hand, any attempt to compare the initial internal states of $P_1$ and $P_2$ with the results of both measurements changes the correlations, thus a joint probability for the simultaneous existence of these states cannot be defined. This means that the reason for the violation of Bell’s inequality is that the usual derivations always assume that the states (or ‘stable properties’) which carry the initial correlations can be freely compared with the results of the measurements. This comparability is usually thought to be a consequence of realism. According to the present theory, the above assumption goes beyond the requirements of realism and proves to be wrong, because each of the states $|\psi_{P_1+M_1}>$, $|\psi_{P_2+M_2}>$, $|\psi_{M_1}>$ and $|\psi_{M_2}>$ exists individually, but they cannot be compared without changing the correlations.

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