Adaptive Sliding Mode Control Method for Z-Axis Vibrating Gyroscope Using Prescribed Performance Approach

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Featured Application: The adaptive sliding mode control method using the prescribed performance approach was proposed for Z-axis vibrating gyroscopes, which plays a crucial role in detecting angular velocities. Compared with traditional angular velocity meters, Micro-Electro-Mechanical-System (MEMS) gyroscope has outstanding features such as low cost, easy installation and others. As a result, MEMS gyroscopes have good application prospects in navigation, customer digital devices, and other areas.

Abstract: This paper investigates one kind of high performance control methods for Micro-Electro-Mechanical-System (MEMS) gyroscopes using adaptive sliding mode control (ASMC) scheme with prescribed performance. Prescribed performance control (PPC) method is combined with conventional ASMC method to provide quantitative analysis of gyroscope tracking error performances in terms of specified tracking error bound and specified error convergence rate. The new derived adaptive prescribed performance sliding mode control (APPSMC) can maintain a satisfactory control performance which guarantees system tracking error, at any time, to be within a predefined error bound and the error convergences faster than the error bound. Besides, adaptive control (AC) technique is integrated with PPC to online tune controller parameters, which will converge to their true values at last. The stability of the control system is proved in the Lyapunov stability framework and simulation results on a Z-axis MEMS gyroscope is conducted to validate the effectiveness of the proposed control approach.

Keywords: adaptive control; prescribed performance; sliding mode control; MEMS gyroscope

1. Introduction

As one kind of inertial sensors, gyroscopes are used in application areas where there exist particular requirements for angular velocities measurement and MEMS gyroscopes have attracted increasing attention for the advantages of low cost, easy installation and so on. Due to above mentioned prominent features, MEMS gyroscopes have been widely installed in cell phones, navigation devices, self-balance platforms and other application aspects. Generally speaking, the principle of vibrating gyroscopes is based on the Coriolis phenomenon where a detecting mass is driven into a stable oscillatory motion along the driving axis, Coriolis force occurs along the sense axis when some angular velocity is applied to the gyroscope. However, gyroscope performances are often degraded by manufacturing errors (mainly contribute to aspects of parameter deviation and asymmetric structure) and external
disturbances, which exist in manufacturing and measuring environments. As a result, aiming at improving gyroscope oscillation performance as well as robustness to disturbances, advanced control methods such as sliding mode control (SMC), adaptive control (AC) are practical and feasible choices in the control of gyroscopes.

It is known that there are many factors that may hinder the angular velocity measurement such as asymmetrical structure, temperature [1–4], humidity in packing environment, and other disturbances. Great efforts have been devoted to improve system performances and robustness by suppressing parameter uncertainties and external disturbances [5–9]. Various control methods are adopted to stabilize gyroscope systems and improve system performances. In [5], Mehran used an extended state observer and presented a disturbance rejection-based controller for MEMS gyroscopes. As one kind of advanced control scheme with extraordinary strong robustness [10–13], SMC can also be used to control gyroscope dynamics and the method has been adopted in various implementation occasions for its outstanding features such as easy implementation, insensitive to parameter variation, strong robustness and so on. In [12], Utkin introduced the concept of sliding mode control method where design and analysis for sliding mode systems are investigated. Although SMC is a good tool for control problems in the presence of disturbance and uncertain parameters, SMC can achieve excellent control performance on the basis that there exist a switching term varying with error dynamics which may cause high frequency chattering in control forces. To overcome the drawback of SMC, great efforts have been devoted to the investigation of SMC method, ASMC, dynamic sliding mode control, super-twisting algorithm are gradually introduced and adopted in many areas.

AC technique is one effective method to solve system parameter variation problems and it is often integrated with SMC to alleviate chattering in the face of different kinds of disturbances. The basic principle of AC strategy is that controller parameters can vary automatically according to different system parameters and different system dynamics. As a result, it is natural to combine AC with SMC method for system performance improvement in the presence of parameter deviation. In [14], Fei and Batur proposed a robust sliding mode controller for gyroscopes where all parameters can be estimated by adaptive laws. Fei and Yan [15,16] further proposed terminal sliding mode controller for gyroscopes where finite time error convergence can be achieved.

Although the control methods [14–16] mentioned above can accomplish trajectory tracking under parameter variation and other disturbances, transient performance indexes especially maximum overshoot and undershot [15] and error convergence rate are not fully studied. However, for the fact that MEMS gyroscopes are very small while its structure is quite complex and precise, control of dynamics of gyroscope oscillation are of great importance and deserve more focus on the design of corresponding control strategies. For example, big overshoot or undershoot may cause detecting capacity combs to collide resulting in damage to measure chips. At the same time, error convergence rates are also very important in measurement meters, if it takes a very long time for the tracking errors to converge (in other words a very long settling time), it may lead to a very long measuring time and it is not likely to choose measuring meters which need a long time for measure values to stabilize in practical situations. However, very little attention is paid to these important performances.

To improve the control performance of the systems, terminal sliding mode control, non-quadratic Lyapunov function [17,18] can be utilized in the design of control systems. Meanwhile, prescribed performance control (PPC) method [19–21] is an advanced control method which ensures that system tracking error will not go beyond a predefined error bound and the error dynamics converge faster than the error bound. In [20], Bechlioulis and Rovithakis introduced a kind of PPC method for nonlinear MIMO systems where the “constrained” error dynamic system is stabilized using the proposed control method with the “unconstrained” error signals. For the characteristic of PPC that the method can impose some specified error bound on system error dynamics to achieve a favorable performance. PPC technique has been applied in diverse occasions including space vehicle altitude control [19,22,23] and so on [24,25]. In [26], Lu and Fei adopted PPC method for MEMS gyroscope control, and the method can successfully drive the detecting mass into a desired oscillation. However, the predefined
error bound is treated as a slow time varying function and it shall be further investigated how to accomplish the trajectory tracking objective without the assumption [27].

Motivated by the above mentioned work, this paper aims to systematically study the MEMS gyroscope trajectory tracking via adaptive PPC approach. The main contribution of this paper can be concluded as follows:

The proposed adaptive prescribed performance sliding mode control (APPSMC) utilizes a transformed error in the design of sliding mode control (SMC) surface where a performance bound (PB) function regarding system tracking error is proposed together with an error transformation function to connect the origin system tracking error with the transformed equivalent error.

The MEMS gyroscope tracking error can converge to the 0 and remaining in a predefined PB (both transient state and steady state) under the proposed APPSMC. It is worth pointing out that above mentioned gyroscope control papers cannot guarantee tracking error to be within a specific bound while the gyroscope tracking error in this paper will not go beyond the proposed error bound.

The propose control method can accomplish trajectory tracking with information of the nominal values of the system where some adaptive laws are derived in the Lyapunov stability framework where gains for adaptive laws can be calculated according to the calculation method introduced in the paper.

This paper is organized as follows: the gyroscope dynamics are introduced in Section 2. In Section 3, design and stability proof for the Z-axis gyroscope using APPSMC is investigated. Section 4 provides simulation results and Section 5 gives the conclusions.

2. Dynamics of MEMS Gyroscopes

The schematic structure of a gyroscope model is described in Figure 1. As shown in Figure 1, the vibrating gyroscope contains a proof mass, spring beams, and dampers. The proof mass is driven by electrostatic actuators and it is constrained only to move in the X–O–Y plane where states including position and velocity information of the proof mass can be acquired detecting capacity combs. Some fixed angular velocity is applied to the gyroscope along the Z axis.

![Figure 1. Schematic model for a gyroscope.](image)

Gyroscope dynamics [28,29] can be described in the following form:

\[
\begin{align*}
    m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y &= u_x + 2m\Omega_z y \\
    m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y &= u_y - 2m\Omega_z \dot{x}
\end{align*}
\]

(1)

\(m\) is the proof mass weight; 
\(x\) and \(y\) are the displacements of the mass; and 
\(k_{xx}, k_{yy}\) and \(d_{xx}, d_{yy}\) are the coefficients for springs and dampers in the X and Y directions. 
\(k_{xy}\) and \(d_{xy}\) are coefficients of the coupling terms which are mainly caused by the asymmetric structure due to limits of manufacture technology.
\( \Omega \) is the angular velocity and \( u_x \) and \( u_y \) represent the control forces in the X and Y direction.

We can get the motion dynamics in non-dimensional form through dividing both sides of (1) by the resonance frequency \( \omega_0^2 \) reference length \( q_0 \) and reference mass \( m \).

\[
\begin{cases}
\dot{q}_x + D_{xx}q_x + D_{xy}q_y + K_x^2q_x + K_{xy}q_y = U_x + 2\Theta q_y \\
\dot{q}_y + D_{xy}q_x + D_{yy}q_y + K_y^2q_x + K_{xy}q_y = U_y - 2\Theta q_x
\end{cases}
\]

(2)

where \( q_x = \frac{x}{q_0}, q_y = \frac{y}{q_0}, \dot{q}_x = \frac{\dot{x}}{q_0}, \dot{q}_y = \frac{\dot{y}}{q_0}, D_{xx} = \frac{d_{xx}}{mq_0}, D_{xy} = \frac{d_{xy}}{mq_0}, D_{yy} = \frac{d_{yy}}{mq_0}, \)

\( \Theta = \Omega \omega_0, K_x = \sqrt{\frac{k_x}{m_0q_0}}, K_y = \sqrt{\frac{k_y}{m_0q_0}}, K_{xy} = \frac{k_{xy}}{m_0q_0}, U_x = \frac{u_x}{m_0q_0}, \) and \( U_y = \frac{u_y}{m_0q_0} \).

(2) is rewritten in vector form as:

\[
\dot{q} + Dq + Kq = U - 2\Theta q
\]

(3)

where \( q = \begin{bmatrix} q_x \\ q_y \end{bmatrix} \), \( U = \begin{bmatrix} U_x \\ U_y \end{bmatrix} \), \( D = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix} \), \( K = \begin{bmatrix} K_x^2 & K_{xy} \\ K_{xy} & K_y^2 \end{bmatrix} \), and \( \Theta = \begin{bmatrix} 0 & -\Theta \\ \Theta & 0 \end{bmatrix} \).

Due to manufacture errors and environment changes, actual values for springs, dampers, and their nominal values are not exactly the same. Considering these effects, we modified system dynamics by adding extra terms concerning parameter deviation and external disturbance and the new motion equation in the described in the following form:

\[
\dot{q} = U - (D + 2\Theta)q - Kq + F
\]

(4)

where \( F \) is the lumped unknown disturbances vector including unknown system dynamics (caused by parametric deviation) and external disturbance. The lumped disturbance is further described such that \( F = -(\Delta D + 2\Delta \Omega)q - \Delta Kq + d \) where \( \Delta D, \Delta \Omega, \Delta K \) denote parameter uncertainties of \( D, \Omega, \) and \( K \), respectively, and we allow 10% parameter variations for the spring and damping coefficients with respect to their nominal values such that \( \Delta D = 0.1D, \Delta \Omega = 0.1\Omega, \) and \( \Delta K = 0.1K. \) \( d \) is external disturbance vector caused by temperature change, humidity change, shock, and so on.

State space form of gyroscope dynamics is:

\[
\begin{cases}
\dot{q}_1 = q_2 \\
\dot{q}_2 = -(D + 2\Theta)q_1 - Kq_1 + F + U
\end{cases}
\]

(5)

where system states \( q_1 \) and \( q_2 \) are the displacement and velocity vectors of the detect mass and they are denoted as \( q_1 = q = \begin{bmatrix} q_x \\ q_y \end{bmatrix} \) and \( q_2 = \dot{q} = \begin{bmatrix} \dot{q}_x \\ \dot{q}_y \end{bmatrix} \). \( F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \) represents the lumped disturbance vector.

Assumption 1 regarding \( F \) is made for further discussion.

**Assumption 1.** The lumped disturbance is bounded \([12]\) such that \( |F_i| \leq F_{di}, i = 1, 2. \) \( F_1, F_2 \) are lumped disturbances in the X and Y axis, respectively, and \( F_{d1} \) and \( F_{d2} \) are disturbance bounds in each direction.

The control objective for MEMS gyroscopes is: 1) to propose an adaptive controller so that the proof mass can be driven into oscillation motion at given amplitudes and frequencies in the driving and sensing axis, or in other words, to reach the target that the gyroscope state \( q \) can track a reference trajectory \( q_d \) and the tracking error satisfies specific prescribed performance requirement and 2) controller parameters can be tuned by adaptive laws.

### 3. APPSMC for MEMS Gyroscopes

An APPSMC is introduced to accomplish trajectory tracking where several adaptive laws are derived to adjust controller parameters. The stability of the control system is guaranteed according to Lyapunov stability theory. In the design procedure of the PPC method in this section, an error
transformation function is introduced to transform the origin tracking error into an equivalent unconstrained one and the transformed error is used in the design of adaptive sliding mode controller with prescribe performance.

According to the theory of PPC scheme in [20,21], error dynamics shall always stay in some specified error bound described by a PB function and the error convergence rate is no less than the performance function. Generally speaking, any smooth decreasing function can be chosen as an error bound function. As can be seen from [20,21] that the performance bound function is chosen as a decreasing function. Prescribed performance can be achieved by transforming the original tracking error into an unconstrained equivalent error with appropriate original error bound and steady state error bound. In the paper, we adopted the prescribed performance control method in the control of gyroscopes to provide transient tracking performance of the gyroscopes. Relative articles have been cited in the reference.

In this section, a decreasing function described by Equation (6) is adopted as an error bound function.

$$\rho_i(t) = (\rho_{0i} - \rho_{\text{out}}) e^{-lt} + \rho_{\text{out}}, i = 1, 2$$  (6)

According to the theory of prescribed performance control (PPC) [20,21], $\rho_{0i}$ is a non-zero positive constant and choices for $\rho_{0i}$ shall comply with the principle that the initial tracking error is less than $\rho_{0i}$. $\rho_{\text{out}}$ is also a positive constant and it is the value of the PB imposed on error dynamics at the steady state.

$e$ is the Euler number and the term $e^{-lt}$ determines the convergence speed of PB function $\rho_i(t)$. It can be found from $e^{-lt}$ that $l_i$ is the dominant parameter which is chosen as a positive constant. Different choices of $l_i$ can result in different PB function with different convergence speed.

If gyroscope tracking errors satisfy $-\rho_i(t) < \epsilon_i(t) < \rho_i(t), i = 1, 2$, prescribed performance is guaranteed. A new performance index is introduced construct connection between the origin tracking error and the PB, given in the form:

$$\theta_i(\epsilon_i) = \frac{\epsilon_i(t)}{\rho_i(t)}, i = 1, 2$$  (7)

We can easily get the range of (7) such that $-1 < \theta_i(\epsilon_i) < 1$ if prescribed performance is achieved. A hyperbolic tangent function is chosen to transform the constrained error index $\frac{\epsilon_i(t)}{\rho_i(t)}$ into an equivalent unconstrained one. The error transformation equation is expressed as:

$$\theta_i(\epsilon_i) = \frac{\epsilon_i(t)}{\rho_i(t)} = \frac{e^{\delta_i \epsilon_i} - e^{-\delta_i \epsilon_i}}{e^{\delta_i \epsilon_i} + e^{-\delta_i \epsilon_i}}, i = 1, 2$$  (8)

where $\delta_i$ is the adjust parameter for the transformation function chosen to be a positive constant. $\epsilon_i$ is the derived “equivalent” error which is used to compose the sliding manifold afterwards.

Figure 2 depicts the hyperbolic tangent function.
Lemma 1. If the equivalent error $\varepsilon_i$ in (8) converges 0, the origin error in (8) will also converge to 0.

Proof. From the definition of the hyperbolic tangent function defined in (8) and the graphic depicting the hyperbolic tangent function in (8), we can easily get the conclusion that: $\theta_i(\varepsilon_i)$ converges to 0 with $\varepsilon_i$ converging to 0.

Equivalent error $\varepsilon_i$ can be derived by solving the inverse function of $\theta_i(\varepsilon_i)$, given in the form:

$$\varepsilon_i = \frac{1}{\delta_i} \theta_i^{-1}(\frac{\varepsilon_i}{\rho_i}), i = 1, 2 \quad (9)$$

where $\theta_i^{-1}(*)$ represents the inverse function of $\theta_i(*)$.

In the following parts, $\theta_i(\varepsilon_i)$, $\rho_i(t)$, and $\varepsilon_i(t)$ are denoted as $\theta_i$, $\rho_i$, and $\varepsilon_i$ for simplicity.

Consider (5) as the system dynamics, system tracking error is defined as:

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = q_d - q. \quad (10)$$

where $q_d$ is the reference trajectory, given in the form $q_d = \begin{bmatrix} q_{dx} \\ q_{dy} \end{bmatrix}$.

The derivative of the tracking error is:

$$\dot{e} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \dot{q}_d - \dot{q}. \quad (11)$$

Conventionally, sliding mode control method uses system error and its derivative to form a sliding mode manifold. In APPSMC, the new derived error will be used in the design of the prescribed performance sliding mode manifold.

Consider a linear sliding manifold given in the form:

$$S = \dot{e} + \lambda e \quad (12)$$

where $\lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ is an adjustable parameter of the manifold chosen to be a diagonal matrix with its main diagonal elements being positive constants.

Differentiating (12) leads to:

$$\dot{S} = \dot{\varepsilon} + \lambda \dot{e} \quad (13)$$

$\varepsilon_i$ can be derived according to (9) and $\varepsilon_i$ takes the form:

$$\varepsilon_i = \frac{1}{2\delta_i} \ln \frac{1+\theta_i}{1-\theta_i} = \frac{1}{2\delta_i} (\ln((1+\theta_i) - \ln(1-\theta_i)), i = 1, 2 \quad (14)$$

Differentiating (14) gives:

$$\dot{\varepsilon}_i = \frac{d(\theta_i^{-1}(\frac{\varepsilon_i}{\rho_i}))}{d(\frac{\varepsilon_i}{\rho_i})} \cdot \frac{\varepsilon_i}{\rho_i} = \frac{\delta_i \dot{\theta}_i}{1-\theta_i^2}, i = 1, 2 \quad (15)$$

Differentiating (15) leads to:

$$\ddot{\varepsilon}_i = \frac{(1-\theta_i^2) \dot{\theta}_i + 2\theta_i \dot{\varepsilon}_i}{(1-\theta_i^2)^2}$$

$$\ddot{\varepsilon}_i = \frac{\delta_i \dot{\theta}_i}{1-\theta_i^2} + \frac{2\delta_i \theta_i \dot{\varepsilon}_i}{(1-\theta_i^2)^2}$$

$$\ddot{\varepsilon}_i = k_1 \dot{\theta}(\varepsilon_i) + k_2 \varepsilon_i, i = 1, 2 \quad (16)$$
where $k_{1i} = \frac{\delta_i}{1 - \theta_i^2}$, $k_{2i} = \frac{2\delta_i \theta_i \dot{\theta}_i^2}{(1 - \theta_i^2)^2}$, $i = 1, 2$

It can be proved that $k_{1i}$ and $k_{2i}$ are positive diagonal matrices for the fact that $\delta_i$ and $\theta_i$ are positive constants and $-1 < \theta_i < 1$, $i = 1, 2$.

For simplicity, equation (16) is rewritten in vector form:

$$\ddot{\epsilon} = k_1 \dot{\theta}(\epsilon) + k_2$$

(17)

where $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$, $k_1 = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{12} \end{bmatrix}$, $\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}$, and $k_2 = \begin{bmatrix} k_{21} \\ k_{22} \end{bmatrix}$.

Differentiating (8) with respect to time gives first and second order derivative of $\theta_i$.

$$\dot{\theta}_i = \frac{\epsilon_i \rho_i + \epsilon_t \rho_t}{\rho_t^2}, i = 1, 2$$

(18)

$$\ddot{\theta}_i = \frac{\rho_t^2 \dot{\epsilon}_i - \rho_t \dot{\epsilon}_t \dot{\rho}_t - 2 \rho_t \dot{\epsilon}_t \dot{\rho}_t + 2 \rho_t \dot{\epsilon}_t \dot{\rho}_t^2}{\rho_t^4}$$

(19)

where:

$$r_{1i} = \frac{1}{\rho_t^2}, r_{2i} = -\rho_t \dot{\rho}_t - 2 \rho_t \dot{\rho}_t \dot{\rho}_t + 2 \rho_t \dot{\rho}_t^2, i = 1, 2$$

Equation (19) is rewritten in vector form as:

$$\ddot{\theta} = r_1 \ddot{\epsilon} + r_2$$

(20)

where $\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$, $r_1 = \begin{bmatrix} r_{11} & 0 \\ 0 & r_{12} \end{bmatrix}$, $\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}$, and $r_2 = \begin{bmatrix} r_{21} \\ r_{22} \end{bmatrix}$.

Substitute (20) into (17) gives:

$$\ddot{\epsilon} = k_1 r_1 \ddot{\epsilon} + k_1 r_2 + k_2$$

(21)

Substitute (21) into (13) leads to:

$$\dot{S} = \ddot{\epsilon} + \lambda \dot{\epsilon}$$

$$= k_1 r_1 \ddot{\epsilon} + k_1 r_2 + k_2 + \lambda \dot{\epsilon}$$

(22)

Substitute system dynamics in (4) into (22), it reveals that:

$$\dot{S} = k_1 r_1 (\ddot{q}_d - (U - (D + 2\Omega)\dot{q} + Kq) 1 - Kq_1 + F))$$

$$+ k_1 r_2 + k_2 + \lambda \dot{\epsilon}$$

(23)

Equivalent control force can be derived by ignoring the lumped error $F$ and setting $\dot{S} = 0$:

$$u_{eq} = \ddot{q}_d + (D + 2\Omega)\dot{q} + Kq$$

$$+(k_1 r_1)^{-1}(k_1 r_2 + k_2 + \lambda \dot{\epsilon})$$

(24)

The actual sliding mode control force is designed in the form:

$$U = u_{eq} + u_{sw}$$

(25)

where the switching control force $u_{sw}$ is designed in the form:

$$u_{sw} = \gamma \text{sign}(S)$$

(26)
where $\gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$ is the gain of the switching term. □

**Theorem 1.** If the control force in (25) is applied to the MEMS system with robust gain in (26) larger than the lumped disturbance bound, system tracking error will asymptotically converge to 0.

**Proof.** Choose a Lyapunov candidate in the form:

$$V = \frac{1}{2} S^T S$$  \hspace{1cm} (27)

Differentiating (27) with respect to time leads to:

$$\dot{V} = S^T \dot{S} = S^T (k_1 r_1 (a_d - (U - (D + 2\Omega)\dot{q} - Kq + F))) + k_1 k_2 + k_2 + \lambda \varepsilon$$  \hspace{1cm} (28)

Substitute the control force in (25) into (28) leads to:

$$\dot{V} \leq S^T (k_1 r_1 (-\gamma \text{sign}(S) + F_d)) \leq S^T (k_1 r_1 (-\gamma \text{sign}(S) + F_d)) \leq |S|^2 |(k_1 r_1 (-\gamma + F_d))|$$  \hspace{1cm} (29)

It can be inferred from Assumption 1 that if the robust gain $\gamma$ is set such that $F_d \leq \gamma, i = 1, 2$, together with the fact that $k_1$ and $r_1$ are positive diagonal matrices, the derivative of the Lyapunov function is non-positive definite in the following form.

$$\dot{V} \leq 0$$  \hspace{1cm} (30)

It can be seen from the designed control force (24) that the control force contains three different system parameter matrices which are actually unknown. Thus, we actually cannot carried out the control force directly. AC can be used in the paper to tune control force parameters and the control force in (25) is modified in the following form where three adaptive laws are given in (32)–(34).

$$U = \hat{u}_{eq} + u_{sw} = \hat{a}_d + (\hat{D} + 2\hat{\Omega})\dot{q} + \hat{K}q + (k_1 r_1)^{-1}(k_1 r_2 + k_2 + \lambda \varepsilon) + u_{sw}$$  \hspace{1cm} (31)

where $\hat{D}$, $\hat{\Omega}$, and $\hat{K}$ are set to replace the corresponding parameters and they are the estimates for $D$, $K$, and $\Omega$, respectively.

Adaptive laws for $\hat{D}$, $\hat{\Omega}$, and $\hat{K}$ are set in the following form. □

$$\hat{D}^T = \frac{\dot{\hat{q}} S^T k_1 r_1}{\eta_1}$$  \hspace{1cm} (32)

$$\hat{\Omega}^T = 2 \frac{\dot{\hat{q}} S^T k_1 r_1}{\eta_2}$$  \hspace{1cm} (33)

$$\hat{K}^T = \frac{\dot{\hat{q}} S^T k_1 r_1}{\eta_3}$$  \hspace{1cm} (34)

**Theorem 2.** If the control force in (31) in applied to the MEMS system, adaptive laws (32)–(34) are incorporated to online tune control force parameters, system tracking error will asymptotically converge to 0. Besides, according
to parameter identification theory [30], all system parameters can be correctly estimated if the persistent excitation condition is satisfied.

Figure 3 shows the block diagram of the entire APPSMC system.

![Block diagram of the entire APPSMC system](image)

**Figure 3.** Structure of the adaptive prescribed performance sliding mode control (APPSMC) system.

**Proof.** Select a Lyapunov candidate in the form:

\[ V = \frac{1}{2} S^T S + \frac{1}{2} \text{tr}(\eta_1 \bar{D}^T \bar{D}) + \frac{1}{2} \text{tr}(\eta_2 \bar{K}^T \bar{K}) + \frac{1}{2} \text{tr}(\eta_3 \bar{\Omega}^T \bar{\Omega}) \]  

(35)

where the function contains sliding surface and parameter estimation error.

Parameter estimation errors are more specifically described in the following form:

\[ \bar{D} = D - \bar{D}, \bar{K} = K - \bar{K}, \bar{\Omega} = \Omega - \bar{\Omega} \]

\( \eta_1, \eta_2, \) and \( \eta_3 \) are positive constants which can be viewed as fix gains for adaptive laws.

Differentiating right hand and left hand of (35) and substituting system dynamic in (4) into it yields:

\[ \dot{V} = S^T \dot{S} + \text{tr}(\eta_1 \bar{D}^T \dot{\bar{D}}) + \text{tr}(\eta_2 \bar{K}^T \dot{\bar{K}}) + \text{tr}(\eta_3 \bar{\Omega}^T \dot{\bar{\Omega}}) \]

\[ = S^T (-k_1 \dot{r}_1 (\dot{q}_d - (U + (D + 2\Omega)\dot{q} - Kq + F))) \]

\[ + k_1 k_2 + k_2 + \lambda \dot{e} + \text{tr}(\eta_1 \bar{D}^T \bar{D}) + \text{tr}(\eta_2 \bar{K}^T \bar{K}) \]

\[ + \text{tr}(\eta_3 \bar{\Omega}^T \bar{\Omega}) \]  

(36)

The last three terms in (36) are denoted as \( \text{tr}(*) \) for clarity.

Substituting control force in (31) into (36) yields:

\[ \dot{V} = S^T (k_1 \dot{r}_1 ((D + 2\bar{\Omega})\dot{q} + \bar{K}q + F) \]

\[- k_1 \dot{r}_1 \gamma \text{sign}(S)) + \text{tr}(*) \]

(37)
Recover $tr(*)$ back to its original form, (37) can be rewritten as:

$$
\dot{V} = S^T k_1 r_1 \tilde{D} q + 2 S^T k_1 r_1 \tilde{D} \eta + S^T k_1 r_1 \bar{K} q \\
+ S^T k_1 r_1 F - S^T k_1 r_1 \gamma \text{sign}(S) + tr(\eta_1^T \bar{D} \bar{D}) \\
+ tr(\eta_2^T \bar{K} \bar{K}) + tr(\eta_3^T \Omega \Omega)
$$

(38)

According to the property of matrix trace, (38) can be rewritten in the form:

$$
\dot{V} = tr(q S^T R \bar{D}) + tr(2 q S^T R \bar{D}) + tr(q S^T R \bar{K}) \\
+ S^T k_1 r_1 F - S^T k_1 r_1 \gamma \text{sign}(S) + tr(\eta_1^T \bar{D} \bar{D}) \\
+ tr(\eta_2^T \bar{K} \bar{K}) + tr(\eta_3^T \Omega \Omega)
$$

(39)

Substituting adaptive laws (32), (33), (34) into (39) leads to:

$$
\dot{V} = S^T k_1 r_1 F - S^T k_1 r_1 \gamma \text{sign}(S) \\
= S^T k_1 r_1 (-\gamma \text{sign}(S) + F) \\
\leq S^T k_1 r_1 (-\gamma \text{sign}(S) + |F|) \\
\leq S^T k_1 r_1 (-\gamma \text{sign}(S) + F_d) \\
\leq |S^T| k_1 r_1 (-\gamma + F_d)
$$

(40)

According to Assumption 1 together with the fact that $k_1$, $r_1$ are positive diagonal matrices, $\dot{V}$ can be proved to be semi positive definite with $F_d \leq \gamma \varepsilon_i$, $i = 1, 2$.

$\dot{V}$ is negative definite i.e., $V(t) \leq V(0)$ implies that $S$, $\tilde{D}$, $\bar{K}$, and $\tilde{\Omega}$ are all bounded. Integrating both sides of (40) leads to

$$
\int_0^t |S^T| dt \leq (k_1 r_1 (-\gamma + F_d))^{-1} (V(0) - V(t)).
$$

For the fact that $V(0)$ is bounded, $V(t)$ is bounded and non-increasing, we can obtain $\lim_{t \to \infty} \int_0^t |S^T| dt < \infty$. Besides, since $\dot{S}$ is also bounded, therefore, we can prove that $S$ will asymptotically converge to zero, i.e., $\lim_{t \to \infty} S = 0$ by Barbalart lemma [14]. If the sliding manifold $S = 0$, the equivalent error $\varepsilon_i$ will converge to 0 with the sliding surface (12) being a Hurwitz polynomial. Furthermore, we can infer from Lemma 1 that the origin tracking error will converge to 0 with the equivalent error converging to 0. □

**Remark 1.** In this paper, a prescribed performance is proposed for the control of MEMS gyroscopes. For further improvement of gyroscope oscillation motion performance, design of the control of gyroscopes can be investigated using non-quadratic Lyapunov function [17,18].

4. Simulation Study

Numerical simulations are conducted on a Z-axis gyroscope model using Matlab/Simulink environment to evaluate the effect of the proposed APPSMC method. $D$, $K$, and $\Omega$ are system parameters matrices and actual gyroscope parameters (elements of $D$, $K$, and $\Omega$) are listed in Table 1.
Table 1. Nominal Parameters of the Z-axis gyroscope.

| System Parameters           | Symbol | Value  |
|-----------------------------|--------|--------|
| X axis spring coefficient   | $K_x^2$| 355.3  |
| Y axis spring coefficient   | $K_y^2$| 532.5  |
| Coupling spring coefficient | $K_{xy}$| 70.99  |
| X axis damping coefficient  | $D_{xx}$| 0.01   |
| Y axis damping coefficient  | $D_{yy}$| 0.002  |
| Coupling damping coefficient| $D_{xy}$| 0.01   |
| Angular velocity            | $\Theta$| 0.1    |
| External disturbance        | $d$    | $\text{Rand}(1)$ |
| X axis command trajectory   | $q_{dx}$| $\sin(4.17t)$ |
| Y axis command trajectory   | $q_{dy}$| $1.2\sin(5.67t)$ |

And in the simulation study, we compared control performances of adaptive sliding mode control, sliding mode control, and adaptive prescribed performance control. Table of used acronyms and corresponding full names are provided in Table 2.

Table 2. Table of acronyms.

| Acronym   | Full Name                                      |
|-----------|-----------------------------------------------|
| PPB       | Prescribed Performance Bound                  |
| APPSMC    | Adaptive Prescribed Performance Sliding Mode Control |
| ASMC1     | Adaptive Sliding Mode Control 1               |
| ASMC2     | Adaptive Sliding Mode Control 2               |
| SMC1      | Sliding Mode Control 1                        |
| SMC2      | Sliding Mode Control 2                        |

To verify the robustness of the APPSMC, parameter deviations and random signal disturbance are set in the control system where there exist 10% parameter deviation between actual gyroscope parameters and the origin values (nominal values) in APPSMC such that $D(0) = 0.9D$, $K(0) = 0.9K$, and $\Omega(0) = 0.9\Omega$. Parameters for the PPB and parameters for the proposed controller including robust gain and gains for adaptive laws are listed in Table 3.

Table 3. Parameters of the proposed APPSMC controller.

| Controller Parameters. | Symbol | APPSMC | ASMC1 | ASMC2 | SMC1 | SMC2 |
|------------------------|--------|--------|-------|-------|------|------|
| Initial error bound    | $\rho_{0,i} = 1, 2$| 1      | N/A   | N/A   | N/A  | N/A  |
| Steady state error bound| $\rho_{0,i} = 1, 2$| 0.01   | N/A   | N/A   | N/A  | N/A  |
| Convergence rate coefficient | $l_{i, i} = 1, 2$| 0.5    | N/A   | N/A   | N/A  | N/A  |
| Error transformation coefficient | $b_{i, i} = 1, 2$| 25     | N/A   | N/A   | N/A  | N/A  |
| Parameter of the sliding surface | $\lambda$| $\begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0.5 & 0 \\ 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}$| $\begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}$| $\begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}$| $\begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}$| $\begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix}$|
| Robust gain            | $\gamma$| $\begin{bmatrix} 5 & 0 \\ 0 & 5 \\ 5 & 0 \\ 0 & 5 \end{bmatrix}$| $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$| $\begin{bmatrix} 5 & 0 \end{bmatrix}$| $\begin{bmatrix} 5 & 0 \end{bmatrix}$| $\begin{bmatrix} 80 & 0 \end{bmatrix}$|
| Gain of adaptive law (32) | $\eta_1$| 0.01   | 0.01  | 10    | N/A  | N/A  |
| Gain of adaptive law (33) | $\eta_2$| 0.01   | 0.01  | 10    | N/A  | N/A  |
| Gain of adaptive law (34) | $\eta_3$| 0.01   | 0.01  | 10    | N/A  | N/A  |

Figures 4 and 5 depict trajectory tracking performances using SMC1, SMC2, ASMC1, and ASMC2. Parameters of the controllers above mentioned are listed in Table 3. In Figures 4 and 5, red and green dotted line depict actual MEMS trajectory using SMC2 and SMC1, while yellow and purple dotted lines depict system trajectory using ASMC1 and ASMC2. Reference trajectory are depicted by a blue solid line. It is obvious in Figures 4 and 5 that system trajectory using SMC1 (green) and ASMC1 (yellow) can hardly track the reference trajectory while system trajectory using SMC2 and ASMC2 can
achieve trajectory tracking. Furthermore, blue, red and purple line almost overlap with each other after time = 15 s.

![Figure 4. X axis trajectory tracking comparisons of Sliding Mode Control 1, SMC2, Adaptive Sliding Mode Control 1, and ASMC2.](image1)

![Figure 5. Y axis trajectory tracking comparisons of SMC1, SMC2, ASMC1, and ASMC2.](image2)

Figures 6 and 7 depict system tracking error performances in X and Y axis. According to the analysis of system tracking performances in Figures 4 and 5, it can be found from Figures 6 and 7 that system tracking error using SMC2 (red) and ASMC2 (purple) can converge to 0, which means the mass proof is successfully driven into the desired oscillation mode (fixed amplitude and frequency) by SMC2 and ASMC2. Green and yellow line depict system tracking error using SMC1 and ASMC1 where tracking error does not converge to 0 while it does not diverge to infinity. By connecting Table 3 and Figures 4–7 it can be concluded that, due to parameter uncertainty and external disturbance, SMC1 cannot accomplish trajectory tracking with a small robust gain while tracking error using SMC2 can converge to 0 with a big robust gain. Comparisons between ASMC1 and ASMC2 shows that proper gain in adaptive laws can result in good tracking error performance.

![Figure 6. X axis tracking error comparisons of SMC2, SMC1, ASMC2, and ASMC1.](image3)
Choosing SMC2 and ASMC2 for comparison, system tracking performance using APPSMC is provided in Figures 8 and 9. It can be seen that system dynamic can track the command trajectory after several milliseconds. Furthermore, it can be found that blue, red and purple line almost overlap with each other after time = 15 ms.

Figures 10 and 11 depict trajectory tracking performances using APPSMC, SMC2, and ASMC2. Parameters of the controllers are listed in Table 3. In Figures 10 and 11, blue and green solid line are upper and lower bound described by performance bound function. Red dotted line, purple dotted line and black dotted line depict tracking error performance of SMC2, ASMC2, and APPSMC, respectively. It can be seen from Figures 10 and 11 that all three control methods can achieve trajectory tracking target while only APPSMC and SMC1 can maintain tracking error to be within the performance bound. It shall be pointed out that the great tracking error performance of SMC1 is derived on the basis that all system parameters are well known. Besides, the robust gain in SMC2 is chosen to be very big and the big robust gain will result in severe chattering phenomenon. At the same time, APPSMC can still...
achieve desired error performance using adaptive laws to tune controller parameters even if system parameters are not exactly known and there are no severe chattering in control force for the fact that the robust gain in APPSMC is very small.

![Figure 10. X axis tracking error comparisons of SMC2, ASMC2, and APPSMC.](image)

![Figure 11. Y axis tracking error comparisons of SMC2, ASMC2, and APPSMC.](image)

Table 4 compares system mean absolute errors and root mean square errors of APPSMC, ASMC1, ASMC2, SMC1, and SMC2. It can be concluded from Table 4 that the proposed control method can achieve excellent tracking error performance compared with conventional ASMC. SMC2 have a better tracking error performance on the basis that all system parameters are well known. Besides, the robust gain in SMC2 is larger than the robust gain used in APPSMC and the big robust gain will result in severe chattering in control forces.

**Table 4. Tracking performance comparison of APPSM, ASMC1, ASMC2, SMC1, and SMC2.**

| Performance Index          | APPSMC | ASMC1 | ASMC2 | SMC1  | SMC2  |
|----------------------------|--------|-------|-------|-------|-------|
| X axis mean absolute error | 0.015  | 0.692 | 0.043 | 0.792 | 0.003 |
| Y axis mean absolute error | 0.014  | 0.818 | 0.101 | 0.977 | 0.012 |
| X axis root mean square error | 0.079  | 0.915 | 0.102 | 1.134 | 0.002 |
| Y axis root mean square error | 0.068  | 1.066 | 0.196 | 1.331 | 0.003 |

To further validate the effectiveness and robustness of the proposed control method, some low frequency noise is added to the system. It can be seen from Figures 12 and 13 that system tracking errors in both directions remain in the performance bound. The results mean that the control system has a strong robustness in the presence of diverse disturbances.
Figures 14 and 15 depict adaptation performances of system parameters. It can be found from Figures 14 and 15 and nominal values of system parameters in Table 1 that initial values for system parameters and their true values are not the same at the beginning. But the estimation values for system parameters can change with the adaptive laws and parameter adaptation performances in Figures 14 and 15 indicate that although there are some oscillation in the parameter adaptation performance, all system parameters can converge to bounded range and can almost converge to their true values by adaptive laws. As a consequence, the adaptive mechanism can greatly alleviate chattering phenomenon in the control force caused by parameter deviations.
Figure 15. Adaptation performance of system parameters.

Figure 16 depicts control forces in the X and Y direction. It can be seen from above figure that there are some slight chattering in the control forces and the chattering can be further reduced by substituting sign function in the control force with saturated function.

Figure 16. Control forces in the X and Y direction.

5. Conclusions

In this paper, an adaptive sliding mode controller is proposed for Z-axis MEMS gyroscopes using prescribed performance approach. Transient and stable state error performances are very important in controlling MEMS gyroscopes while little attention is paid to corresponding control methods. The designed APPSMC method imposes a desired performance bound on system tracking error and the tracking error will converge to zero without going beyond performance bound with error convergence rate no less than the performance bound. Adaptive laws are designed in the Lyapunov stability framework, as long as the persistent excitation condition is satisfied, all system parameters can be correctly estimated. Comparisons of SMC, ASMC, and APPSMC are presented showing that the proposed APPSMC has better performance and strong robustness in the presence of diverse disturbances. And to further investigate the control problem of MEMS gyroscopes, we plan to adopt non-quadratic Lyapunov function in the stability analysis of the entire control system for performance evaluation.

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