Yet another comment on
“Nonlocal character of quantum theory”

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Abstract
There has been considerable discussion of the claim by Stapp that quantum theory is incompatible with locality. In this note I analyze the meaning of some of the statements used in this discussion.
Stapp has claimed to have proven that quantum theory is incompatible with relativistic causality \[1\]. This claim has been criticized by Unruh \[2\] and by Mermin \[3, 4\], and Stapp has replied to this criticism, in \[5, 6, 7\].

Stapp’s proof involves a statement (to be explained below) which I will denote by \(S(L_2)\). Stapp claims to show that \(S(L_2)\) is true, that an analogous statement \(S(L_1)\) is false, and that this difference in the truth-values of \(S(L_2)\) and \(S(L_1)\) constitutes a violation of a locality condition he calls LOC2. Unruh and Mermin state that they agree that \(S(L_2)\) is true; thus the dispute between Stapp and his two critics would seem to involve the relation between \(S(L_2)\) and \(S(L_1)\), and, in particular, the meaning and applicability of LOC2. However, as I will point out in this note, the agreement on \(S(L_2)\) is illusory; the version of \(S(L_2)\) agreed to by Unruh and Mermin does not have the same meaning as does \(S(L_2)\) as understood by Stapp. In this note I will discuss the meaning of \(S(L_2)\).

In ref. \[1\], Stapp considers two particles in the (entangled) Hardy state \[8\], on which measurements can be performed in two spacelike-separated regions called Right and Left. On the Right, measurement is made of either of two non-compatible properties called \(R_1\) and \(R_2\); similarly, on the Left either \(L_1\) or \(L_2\) is measured. The result of any given measurement is either + or −. The statement I call \(S(L_2)\) is the statement that, if \(L_2\) is measured, a statement called \(S\) is true; in symbols

\[
S(L_2) := [L_2 \Rightarrow S]
\]

(similarly, \(S(L_1) := [L_1 \Rightarrow S]\)), where \(S\) is defined by

\[
S := \text{“If } R_2 \text{ is measured and yields the result +, then if } R_1 \text{ had been measured it would have yielded the result −”}
\]

Now, what exactly does \(S\) mean? In this (counterfactual) statement, one is describing an “actual” world, in which \(R_2\) is measured, and a “hypothetical” world in which, instead, \(R_1\) is measured; \(S\) then is the assertion that, in the hypothetical world, the result of \(R_1\) would necessarily be −. For this to make sense, it is necessary for the hypothetical world to be specified more fully. \textit{Roughly} speaking, the idea is to specify that, except for the replacement of \(R_2\) by \(R_1\), the hypothetical world agrees closely with the actual world; \(S\) then is the assertion that, in every world that fits the specification, the result of \(R_1\) is −. And so to complete the definition of \(S\), it is necessary to specify exactly in which ways the hypothetical world is required to agree with the actual world.
Here is one way to make the specification: to demand that the hypothetical world agree with the actual world on all events which are not in the invariant future of the measurement on the Right (that is, on all events either spacelike-separated from, or else on or within the backward lightcone from, that measurement). Let $F$ denote this set of events (which are behind the Forward lightcone), and then define $S_F$ to be this version of $S$:

$$S_F := \text{"If } R_2 \text{ is measured and yields the result +, then }$$
$$\text{in every world which agrees with the actual world on } F$$
$$\text{(in particular, which agrees with the actual Left result)}$$
$$\text{and in which } R_1 \text{ rather than } R_2 \text{ is measured,}$$
$$\text{the result of } R_1 \text{ is } -."$$  \hspace{1cm} (3)

It is not important whether or not the definition (3) agrees with what most people would mean by the statement $S$; what is important is to have a clearly-defined definition for $S$. Of course, (3) might be motivated by physical principles such as causality, and those principles might be relevant in proving that $S_F$ is true in some particular case; nevertheless, (3) is just a definition, and we are free to chose a different definition if we like. Here is an example of a different definition: we could specify that the hypothetical world must agree with the actual world on all events in the invariant past of the measurement on the Right. Let $B$ denote that set of events (behind the Backward lightcone), and $S_B$ that version of $S$:

$$S_B := \text{"If } R_2 \text{ is measured and yields the result +, then }$$
$$\text{in every world which agrees with the actual world on } B$$
$$\text{and in which } R_1 \text{ rather than } R_2 \text{ is measured,}$$
$$\text{the result of } R_1 \text{ is } -."$$  \hspace{1cm} (4)

$S_F$ and $S_B$ are not identical statements (truth of $S_B$ implies truth of $S_F$, but not the other way around). No physical principle can tell us whether either (3) or (4) give the “correct” meaning for $S$, since definitions cannot be “correct”. I certainly do not mean to assert that $S_B$ captures the usual meaning of $S$, nor do I mean to suggest $S_B$ as a useful alternative to $S_F$. I have introduced $S_B$ merely to emphasize that, to be unambiguous, statement $S$ must include a specification of the ways in which the hypothetical world is required to agree with the actual world, and to suggest that it is a good idea to spell out that specification as completely as possible.
So far, I have discussed the meaning of statement $S$; for that discussion, the quantum state of the two particles was completely irrelevant. To discuss the conditions under which $S$ is true, it will be necessary to remember that the particles are in the Hardy state $[8]$. Let $S_F(L2)$ denote $S(L2)$ in which $S$ is understood as $S_F$; that is,

$$S_F(L2) := [L2 \Rightarrow S_F],$$

with analogous definitions for $S_B(L2)$, $S_F(L1)$, and $S_B(L1)$. It happens that, for particles in the Hardy state, $S_F(L2)$ is true, and $S_B(L2)$, $S_F(L1)$, and $S_B(L1)$ are all false.

- To see that $S_F(L2)$ is true, note that a quantum calculation shows, for the actual world of $S_F(L2)$ (in which $L2$ and $R2$ are measured and the result of $R2$ is $+$), that the result of $L2$ is necessarily $+$. Since $S_F$ constrains the hypothetical world to agree with the actual world on the Left, in the hypothetical world that result is also $+$. Then a quantum calculation for this hypothetical world (in which $L2$ and $R1$ are measured, and the result of $L2$ is $+$) requires the result of $R1$ to be $-$. Note that no locality assumption is needed here; the truth of $S_F(L2)$ follows simply from its definition and the quantum properties of the Hardy state.

- To see that $S_B(L2)$ is false, note that, again, in the actual world the result of $L2$ is $+$. Now, however, we are free to consider a hypothetical world in which the result of $L2$ is $-$, and in such a world ($L2$ and $R1$ measured, result of $L2$ is $-$), a quantum calculation reveals a non-zero probability for the result of $R1$ to be $+$. Thus there is a hypothetical world consistent with the specification of $S_B(L2)$ in which the result of $R1$ is $+$; therefore $S_B(L2)$ is false. Now it may seem strange to allow a hypothetical world in which the result of $L2$ is $-$, while in the actual world that result is $+$; after all, how could the decision to measure $R1$ rather than $R2$ change the result on the Left? It may help to remember that a hypothetical world is, ipso facto, not the same as the actual world. If I choose to talk about a hypothetical world, then I get to choose what world to talk about; if I (perhaps foolishly) were to adopt $S_B$ as representing the meaning of $S$, then I could find an allowed hypothetical world in which the result of $R1$ is $+$.

2The quantum predictions for the measurements we are discussing, for particles in the Hardy state, are presented in refs. [1, 2, 3].
For completeness, we can see that \( S_F(L1) \) is false by noting that in the actual world (\( L1 \) and \( R2 \) measured, result of \( R2 \) is +), it is allowed that the result of \( L1 \) be \(-\). Then in the hypothetical world (\( L1 \) and \( R1 \) measured, result of \( L1 \) is \(-\)), the result of \( R1 \) is allowed to be +. Finally, since \( S_F(L1) \) is false, \( S_B(L1) \) must be false also.

Again, I am certainly not advocating adopting \( S_B \) to represent the meaning of \( S \). I am suggesting that, whatever we wish \( S \) to mean, it is useful to spell out that meaning explicitly, by specifying the ways in which the hypothetical world is required to agree with the actual world.

Mermin, in ref. [3], denotes by (I) the statement here called \( S(L2) \). From his discussion of why he considers this statement to be true, it is clear that Mermin requires the hypothetical world to agree with the actual world on all events which, in some frame, occur earlier than the measurement on the Right. Since for every event in the set we have called \( F \) there is a frame in which that event does precede the measurement on the Right, Mermin’s understanding of \( S(L2) \) coincides with \( S_F(L2) \). Although Unruh [2] does not give general criteria for the interpretation of counterfactuals, in his discussion of statement \( S(L2) \) (which appears in eqs. 12 and 13 of ref. [2]) he strongly emphasizes that this statement must be understood as requiring that the result of \( L2 \) be +, and that is the aspect of \( S_F(L2) \) which is relevant for all the further discussion of locality. Stapp [1, 5, 6, 7], however, evidently interprets \( S(L2) \) differently. For him, the meaning of \( S \) (as opposed to the conditions required for its proof) involves only the Right region; he writes, for example [6] “This ‘meaning’ of statement \( S \) is strictly in terms of a relationship between the possibilities for the outcomes of alternative possible experiments both of which are confined to the region R(ight).” So statement \( S \) for Stapp is not the same as \( S_F \) (and incidentally, not the same as \( S_B \) either); thus statement \( S \) is used to mean different things by Stapp and by his two critics.

Of course, the issue is not which is the “correct” meaning of statement \( S \)—that is, after all, just a matter of definition. The issue is whether quantum mechanics is incompatible with relativistic locality. But it is difficult to discuss that issue without unambiguous and agreed-upon definitions for the statements which are being discussed.

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3 Humpty Dumpty has remarked “When I use a word it means just what I choose it to mean—neither more nor less.”
Acknowledgement: I have benefited from conversations with (but not necessarily agreement from) Henry Stapp. I would also like to acknowledge the hospitality of the Lawrence Berkeley National Laboratory.

References

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[3] N. D. Mermin, “Nonlocal character of quantum theory?,” quant-ph/9711052 (1997).

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[6] H. P. Stapp, quant-ph/9712036 (1997).

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[9] H. Dumpty, quoted in L. Carroll, Through the Looking Glass, and What Alice Found There, Macmillan and Co., London (1872).