We discuss aspects of the statistical hadronization model for the production of mesons with open and hidden charm in ultra-relativistic nuclear collisions. Emphasis is placed on what can be inferred from the dependence of the yield of charmonia on the number of participants in the collisions.

1. Introduction

Experiments with ultra-relativistic nuclei are performed to produce and study the quark-gluon plasma. This new state of matter is predicted to exist at high temperatures and/or high baryon densities. Numerical solutions of QCD using lattice techniques imply that the critical temperature (at zero baryon density) is about 170 MeV [1]. Comprehensive surveys of the various experimental approaches on how to produce such matter in nucleus-nucleus collisions have been given recently [2–5]. Here we focus on charm production and its recent interpretation in terms of a statistical hadronization model [6]. Since the statistical hadronization model for charm is a consequence of the success of thermal model descriptions for hadrons produced in nuclear collisions we first very briefly review the present state there. We then present an update of the extension of this model to charm production with emphasis on what can be inferred from the dependence of the yield of charmonia on the number of participants in the collisions. Remarks on what can be expected at collider energies will conclude the manuscript.

2. Equilibration at the Phase Boundary

The statistical model used to describe hadron multiplicities is presented in detail in [7]. Like its predecessors presented in [8,9] it is based on the use of a grand canonical ensemble to describe the partition function and hence the density of the hadrons under consideration. Without invoking volume information this model can then be used to describe ratios of particle yields. The surprising result of applying this very simple model to data is that ratios of all hadron yields including those involving multi-strange baryons, where enhancement factors of more than 1 order of magnitude over what had been measured in p+Be collisions are observed, can be described consistently [7] if one assumes a fireball with temperature $T=168$ MeV and baryon chemical potential $\mu_B = 266$ MeV. This is
demonstrated in Fig. 1. On the other hand, the observed enhancement, especially for multistrange hadrons, cannot currently be understood within any of the hadronic event generators [10].

The chemical potentials $\mu_B$ and temperatures $T$ resulting from the such thermal analyses [7–9] place the systems at chemical freeze-out very close to where we currently believe is the phase boundary between plasma and hadrons. This is demonstrated in Fig. 2. The freeze-out trajectory (solid curve through the data points) is to guide the eye.

The closeness of the freeze-out parameters ($T, \mu_B$) to the phase boundary might be the clue to the apparent chemical equilibration in the hadronic phase: if the system prior to reaching freeze-out was in the partonic (plasma) phase, then hadron production in general and strangeness production in particular is determined by larger partonic cross sections as well as by hadronization.

Figure 1. Comparison of measured particle ratios with predictions of the thermal model. For details see text and [7].

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3. Statistical Hadronization of Charm

Charm quarks are heavy ($m_c \gg T_c$) and thermal production of charm quarks and charmed hadrons is not likely in ultra-relativistic nuclear collisions. The situation has been recently discussed [6] with the conclusion that, compared to direct hard production, thermal production of charm quarks can be neglected at SPS energies and is small even at LHC energy. However, in the course of these investigations we proposed a new scenario for
Figure 2. Phase diagram of hadronic and partonic matter. The hadrochemical freeze-out points are determined from thermal model analyses of heavy ion collision data at SIS, AGS and SPS energy. The hatched region indicates the current expectation for the phase boundary based on lattice QCD calculations at $\mu_B=0$. The arrow from chemical to thermal freeze-out for the SPS corresponds to isentropic expansion.

quarkonia and charmed hadron production. We assume \[6\] that all $c\bar{c}$ pairs are produced in direct, hard collisions, i.e. in line with the above considerations we neglect thermal production. For a description of the hadronization of the $c$ and $\bar{c}$ quarks, i.e. for the determination of the relative yields of charmonia, and charmed mesons and baryons, we employ the statistical model, with parameters as determined by the analysis of all other hadron yields \[7\]. The picture we have in mind is that all hadrons form within a narrow time range at or close to the phase boundary.

Since the number of directly produced charm quarks deviates from the value determined by chemical equilibration, we introduced a charm enhancement factor $g_c$ by the requirement of charm conservation. This leads to the following relation between hard open charm production and thermal production:

$$N_{c\bar{c}}^{\text{direct}} = \frac{1}{2} g_c \left( \sum_i N_i^{\text{therm}} + N_i^{\Lambda} \right) + g_c^2 \left( \sum_i N_i^{\psi} \right) + \ldots$$

(1)

Consequently, the charm enhancement factor $g_c$ can be determined if $N_{c\bar{c}}^{\text{direct}}$ is known. Since there are no data yet for open charm production in nuclear collisions we rely, as
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discussed in [1], on the investigations and model calculations of [11], where the information available on charm production in hadron-nucleus collisions is extrapolated to Pb-Pb collisions at SPS energy. The number of J/ψ mesons is then enhanced relative to the thermal model prediction by a factors of $g_c^2$, i.e.

$$N_{J/\psi} = g_c^2 N_{therm}^{J/\psi}.$$  (2)

Application of this model to the data for charmonium production is somewhat complicated since the NA50 collaboration has not provided J/ψ yields as a function of centrality or number of participants in Pb+Pb collisions at SPS energy. Gosset et al. [12] have provided such an analysis based on preliminary 1995 data and this was the basis of the discussion in ref. [6]. We will, for the following discussion, mainly focus on the dependence of J/ψ production on the number of participating nucleons. The shape of this dependence can be analyzed from published NA50 data [13–15]. The absolute normalization we obtained by using for $N_{part} = 100$ that $N_{J/\psi}/N_{h^-} = 1.1 \times 10^{-6}$ [16], leading to $N_{J/\psi}/N_{part} = 1.83 \times 10^{-6}$. These results agree well with a recent analysis [17] of NA50 data of 1996 and 1998 within the framework of ref. [12] and are displayed in Fig. 3. We note that the so obtained data agree with the analysis reported in [12] for low centralities but exceed those data by about 30% for $N_{part}=350$.

In the following we will discuss these data within the framework of ref. [6], with the following modifications. We take into account, in the thermal model description, the full set of charmed mesons and baryons. This increases, as mentioned already in [6], the (grand-canonical) thermal charm yield by about a factor 2.5, mostly because of the large statistical factors of the D* mesons. Since, despite this increase, the number of charm pairs is significantly less than 1 per collision, we treat the system, as proposed by Redlich [18], within canonical thermodynamics, following [19]. A similar approach was recently chosen by [20], to investigate limits on open charm production which are imposed within the present model by the data on J/ψ production.

Neglecting the (very small) quadratic terms in eq. (1) the statistical hadronization model then reads:

$$N_{cc}^{direct} = \frac{1}{2} g_c N_{oc}^{therm} \frac{I_1(g_c N_{oc}^{therm})}{I_0(g_c N_{oc}^{therm})}. \quad (3)$$

Here, $N_{oc}^{therm}$ denotes the total thermal yield per collision of open charm mesons and baryons (calculated within the grand-canonical ensemble) and $I_1$ and $I_0$ are modified Bessel functions. From the properties of the Bessel functions one obtains $\lim_{x \to \infty} \frac{I_1(x)}{I_0(x)} = 1$ so that the grand-canonical limit is obtained for $g_c N_{oc}^{therm} \gg 1$.

Since quarkonia are hidden charm mesons, eq. (2) is still valid. To set the stage for the following discussion we now proceed to evaluate the dependence of J/ψ production on $N_{part}$. For all estimates we assume $N_{cc}^{direct} \propto N_{part}^{(4/3)}$. Since $N_{oc}^{therm} \propto N_{part}$ we get that:

$$\frac{N_{J/\psi}}{N_{part}} \propto g_c^2. \quad (4)$$
Figure 3. Comparison of the dependence of the measured \((J/\psi)/N_{\text{part}}\) ratio on the number of participating nucleons with the predictions of the canonical direct/statistical model (solid line). The data are from a reanalysis of [13–15] and are normalized using the \((J/\psi)/h^-\) ratio deduced in [16]. For details, in particular about the normalization, see text. The dashed line is obtained by increasing the direct open charm yield by 50%.

With the above assumed \(N_{\text{part}}\) dependences one can then solve Eq. 3 to determine the \(N_{\text{part}}\) dependence of \(g_c\) with the result that

\[
\frac{N_{J/\psi}}{N_{\text{part}}} \propto N_{\text{part}}^\alpha,
\]

where \(\alpha = 2/3\) in the grand-canonical limit while \(\alpha = -2/3\) in the canonical limit. From the smooth behavior of the Bessel functions it is clear that the coefficient \(\alpha\) develops smoothly and continuously from \(-2/3\) to \(+2/3\) as \(N_{c\bar{c}}^{\text{direct}}\) increases from values \(\ll 1\) to values \(\gg 1\). The numerically obtained dependence of \(\alpha\) on \(N_{c\bar{c}}^{\text{direct}}\) for fixed \(N_{c\bar{c}}^{\text{therm}}\) is shown in Fig. 4. We conclude from this discussion that, in the range of validity of the statistical hadronization model, the centrality dependence of the yield will be directly related to overall magnitude of the open charm cross section relative to the thermal open charm yield.

\(^1\)Numerical solution indicates a slight \(N_{\text{part}}\) dependence of \(\alpha\) for intermediate values of \(N_{c\bar{c}}^{\text{direct}}\).
Figure 4. Dependence of $\alpha$ on the magnitude of the direct open charm yield. For details see text.

Application of this model to the data, displayed in Fig. 3, now proceeds in the following way. We start the calculation at $N_{\text{part}} = 350$ to avoid that part of the data which is determined by fluctuations [21] not contained within this model. Furthermore, predictions of the model should only be trusted from about $N_{\text{part}} > 150$ on, where also the $\psi'/(J/\psi)$ ratio is close to the thermal value for Pb+Pb data (see Fig. 2 of ref. [3]). In this context it should be mentioned that the thermal character of the $\psi'/(J/\psi)$ ratio was first noticed by [22]. In our approach, ratios for all higher charmonia states including the $\chi_c$ should approach the thermal value from $N_{\text{part}} > 150$ on, implying that for those $N_{\text{part}}$ values feeding to $J/\psi$ should be small. In this picture, there should thus not be different “thresholds” for the disappearance of different charmonia.

We also take the value of direct open charm production from [31], appropriately scaled to $N_{\text{part}} = 350$, i.e. $N_{cc}^{\text{direct}} = 0.144$. With $N_{cc}^{\text{therm}} = 0.55$ for $N_{\text{part}} = 350$ we get, solving eq. (3), a value of $g_c = 1.43$. Using eq. (2) and $N_{J/\psi}^{\text{therm}} = 1.84 \cdot 10^{-4}$ we obtain immediately that $\frac{N_{J/\psi}}{N_{\text{part}}} = 1.08 \cdot 10^{-6}$ for $N_{\text{part}} = 350$ and $\alpha = -0.61$. This result is shown as the
solid line in Fig. 3. Considering the uncertainties in the normalization of the \(J/\psi\) and open charm yields the agreement is surprising. We illustrate the importance of accurately knowing the open charm yield by the dashed line in Fig. 3. Here the open charm cross section was increased by 50\% (well within the uncertainty of the pp data), leading to \(g_c = 1.78\) and \(\alpha = -0.60\). Only a direct measurement of open charm production in Pb+Pb collisions can remove this uncertainty of the present analysis.

Since the present analysis involves absolute densities we would like to point out that, in the model of \([7]\), an excluded volume correction is applied. For the parameters chosen in \([7]\) this leads to a density reduction by a factor of \(\beta \approx 0.65\). This should be taken into account when trying to deduce limits on open charm production from a statistical analysis\([23]\). Alternatively, one can determine the effective volume by requiring, as is done here for the SPS data analysis, that the total charged particle multiplicity is correctly reproduced by the thermal model calculations.

4. Summary and Outlook

Hadron production results from central nucleus-nucleus collisions at ultra-relativistic energies can be quantitatively understood by assuming that the fireball formed in the collision freezes out chemically very near to the phase boundary between quark-gluon plasma and hadron gas. Mesons containing heavy (charm) quarks are not thermally produced but their yield can be quantitatively explained, at least for central collisions, in the statistical hadronization model where all charm quarks are directly produced but all hadrons containing charm quarks are formed according to thermal phase space. For small values of \(N_{c\bar{c}}^{direct}\) it is, as was pointed out by \([18]\) and recently used by \([20]\), important to use the canonical approach for calculating the thermal phase space.

Predicting absolute yields for the rapidity density of \(J/\psi\) mesons at collider energies requires knowledge of the yield of open charm pairs as well as of the total charged particle multiplicity (to fix the volume needed for the calculation of thermal yields). The following estimates are for central (\(N_{part} = 350\)) Au+Au (RHIC) or Pb+Pb (LHC) collisions. They should be considered schematic but illustrative of the qualitatively new features expected within the framework presented here. For full RHIC energy \(N_{c\bar{c}}^{direct}\) is expected to be of order 1.5 per unit rapidity. For a charged particle rapidity density of \(dN_{ch}/d\eta = 1000\) we get, using eqs. (2) and (3), that \(g_c = 9.0, \alpha = -0.25\), resulting in a \(J/\psi\) yield of 0.011 per unit rapidity, close to the expected unsuppressed value. For LHC energies, where \(N_{c\bar{c}}^{direct}\) approaches 50 per unit of rapidity, the grand-canonical picture should be appropriate, i.e. \(\alpha\) will be close to 2/3. The actual computation for \(dN_{ch}/d\eta = 6300\) gives \(\alpha = 0.62\) and \(g_c = 41\), implying a \(J/\psi\) yield of about 1.4 per unit rapidity, about a factor of 20 above the direct yield! In the grand-canonical limit the expected \(J/\psi\) yields is proportional to \((N_{c\bar{c}}^{direct})^2/N_{ch}\) and can be trivially scaled to other values for charm and charged particle production. As already discussed in \([1]\), the statistical hadronization picture should, under these circumstances, lead to strongly enhanced quarkonia production even compared to values for direct hard scattering production. A similar result has been obtained recently \([24]\) using a model based on kinetic equations.
5. Acknowledgements

We would like to acknowledge important and stimulating discussions with K. Redlich.

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