Quintessence, Supersymmetry and Inflation

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Recent data point in the direction of a Λ dominated universe. We briefly review “Quintessence” as a model for a dynamical cosmological term and analyse the role of Susy QCD as a possible particle physics candidate. The multiscalar content of the theory is fully taken into account and interaction with other cosmological fields is discussed. Finally, the possibility of constructing a unified scheme for quintessence and inflation is mentioned.

1. INTRODUCTION

The very last years have witnessed growing interest in cosmological models with Ω_m ∼ 1/3 and Ω_Λ ∼ 2/3, following the most recent observational data (see for example [1] and references therein). A very promising candidate for a dynamical cosmological constant Λ(t) is a “quintessential” scalar field presently rolling down its potential [2], for which particle physics models have also been proposed [3–5]. Several ways of constraining these models from observations are under investigation [6].

The study of scalar field cosmologies has shown [7] that for certain potentials there exist attractor solutions that can be of the “scaling” [8] or “tracker” [9] type: that means that for a wide range of initial conditions the scalar field will rapidly join a well defined late time behavior.

On the other hand, the investigation of quintessence models from the particle physics point of view presents two classes of problems: the construction of a field theory with the required scalar potential and the interaction of the quintessence field with the standard model (SM) fields [10]. The former problem was first considered by Binétruy [3], who pointed out that scalar inverse power law potentials (required by the “tracking” condition) appear in supersymmetric QCD theories with N_c colors and N_f < N_c flavors [11]. The second seems the toughest. Indeed the quintessence field today has typically a mass of order H_0 ∼ 10^{-33}eV. Then, in general, it would mediate long range interactions of gravitational strength, which are phenomenologically unacceptable.

Going on with the analysis, another very interesting question is whether it is possible to construct a successful common scheme for the two cosmological mechanisms involving rolling scalar fields, i.e. quintessence and inflation. This perspective has the appealing feature of providing a unified view of the past and recent history of the universe, but can also “cure” some weak points that the two mechanisms taken separately have. Indeed, inflation could provide the initial conditions for quintessence without any need to fix them by hand, and quintessence could hope to give some more hints in constraining the inflaton potential on observational grounds.

2. QUINTESSENCE

2.1. The cosmological attractors

Consider a cosmological scalar field Q, with potential V(Q) evolving according to

$$\ddot{Q} + 3H \dot{Q} + \frac{\partial V}{\partial Q} = 0$$

and whose equation of state is given by

$$w_Q = \frac{\dot{Q}^2/2 - V(Q)}{\dot{Q}^2/2 + V(Q)}$$

with $H^2 = 8\pi/3M_P^2 (\rho_m + \rho_r + \rho_Q)$, where $M_P$ is the Planck mass, $\rho_m(r)$ is the matter (radiation) energy density, and $\rho_Q$ is the quintessence...
field energy. If \( \rho_Q \ll \rho_B \), where \( \rho_B \) is the energy density of the dominant background (radiation or matter), the attractor can be studied analytically.

In the case of an exponential potential, \( V \sim \exp(-Q) \) the solution \( Q \sim \ln t \) is, under very general conditions, a “scaling” attractor in phase space characterized by \( \rho_Q/\rho_B \sim \text{const} \). This could potentially solve the so called “cosmic coincidence” problem, providing a dynamical explanation for the order of magnitude equality between matter and scalar field energy today. Unfortunately, the equation of state for this attractor is \( w_Q = w_B \), which cannot explain the acceleration of the universe neither during RD \((w_{\text{rad}} = 1/3)\) nor during MD \((w_m = 0)\). Moreover, Big Bang nucleosynthesis constrain the field energy density to values much smaller than the required \( \sim 2/3 \).

If instead an inverse power-law potential is considered, \( V = M^{4+n}Q^{-\alpha} \), with \( \alpha > 0 \), the attractor solution is \( Q \sim t^{1-n/m} \), where \( n = 3(w_Q + 1) \) and \( m = 3(w_B + 1) \). The equation of state turns out to be \( \omega_Q = (\alpha w_B - 2)/(\alpha + 2) \), which is always negative during MD. The ratio of the energies is no longer constant but scales as \( \rho_Q/\rho_B \sim a^{m-n} \) thus growing during the cosmological evolution, since \( n < m \). \( \rho_Q \) could then have been safely small during nucleosynthesis and have grown lately up to the phenomenologically interesting values. These solutions are good candidates for quintessence and have been denominated “trackers” in the literature. There are two main qualitative ways through which the attractor can be joined (see \[3\] for details). If the initial conditions for the scalar field \( Q \) are such that \( \rho_Q^c \leq \rho_Q^{in} \leq \rho_{tr}^m \) (undershoot case), it will remain “freezed” until \( \rho_Q \sim \rho_{tr} \) and then start to scale as the tracker. If, instead, initially \( \rho_{tr}^m \leq \rho_Q^{in} \leq \rho_B^m \) (overshoot case) then \( Q \) will pass through a phase of kinetic energy domination before remaining frozen and eventually join the attractor.

The inverse power-law potential does not improve the cosmic coincidence problem with respect to the cosmological constant case. Indeed, the scale \( M \) has to be fixed from the requirement that the scalar energy density today is exactly what is needed. This corresponds to choosing the desired tracker path. An important difference exists in this case though. The initial conditions for the physical variable \( \rho_Q \) can vary over many tens of orders of magnitude (between the present critical energy density and the initial background energy density), depending on the initial time, and will anyway end on the tracker path before the present epoch, due to the presence of an attractor in phase space \[3\]. On the contrary, in the cosmological constant case, the physical variable \( \rho_A \) is fixed once and for all at the beginning. This allows us to say that in the quintessence case the fine-tuning issue, even if still far from solved, is at least weakened.

### 2.2. The tracker solution in Susy QCD

As already noted by Bin\'etruy \[3\], supersymmetric QCD theories with \( N_c \) colors and \( N_f < N_c \) flavors \[1\] may give an explicit realization of a model for quintessence with an inverse power law scalar potential.

The matter content of the theory is given by the chiral superfields \( Q_i \) and \( \overline{Q}_i \) \((i = 1 \ldots N_f)\) transforming according to the \( N_c \) and \( \overline{N}_c \) representations of \( SU(N_c) \), respectively. In the following, the same symbols will be used for the superfields \( Q_i, \overline{Q}_i \), and their scalar components.

Supersymmetry and anomaly-free global symmetries constrain the superpotential to the unique exact form

\[
W = (N_c - N_f) \left( \Lambda^{(3N_c - N_f)} \right) \frac{\overline{Q}_i \dot{Q}_j}{\text{det} T} \]

where the gauge-invariant matrix superfield \( T_{ij} = Q_i \cdot \overline{Q}_j \) appears. \( \Lambda \) is the only mass scale of the theory. It is the supersymmetric analogue of \( \Lambda_{QCD} \), the renormalization group invariant scale at which the gauge coupling of \( SU(N_c) \) becomes non-perturbative. As long as scalar field values \( Q_i, \overline{Q}_i \gg \Lambda \) are considered, the theory is in the weak coupling regime and the canonical form for the K"ahler potential may be assumed.

We consider the general case in which different initial conditions are assigned to the different scalar VEV’s \( \langle Q_i \rangle = \langle \overline{Q}_i \rangle \equiv q_i \), and the system is described by \( N_f \) coupled differential equations. Taking for illustration the case \( N_f = 2 \), the equa-
tions to be solved are (see [4] for details)
\[ \ddot{q}_1 + 3H \dot{q}_1 - \frac{d \cdot q_1}{(q_1 q_2)} \Lambda^2 a [2 + N, \frac{q_2^2}{q_1^2}] = 0, \]
\[ \ddot{q}_2 + 3H \dot{q}_2 - \frac{d \cdot q_2}{(q_1 q_2)} \Lambda^2 a [2 + N, \frac{q_2^2}{q_2^2}] = 0 \]
with \( d = 1/(N_c - N_f) \) and \( a = (3N_c - N_f)/(N_c - N_f) \).

In analogy with the one-scalar case, we look for power-law solutions of the form
\[ q_{tr,i} = C_i \cdot t^{p_i}, \quad i = 1, \cdots, N_f. \]

It is straightforward to verify that for fixed \( N_f \) (and when \( \rho_Q \ll \rho_B \)), a solution exists with \( p_i = p = p(N_c) \) and \( C_i \equiv C = C(N_c, \Lambda) \) and is the same for all the \( N_f \) flavors [4]. The equation of state of the tracker is given by
\[ w_Q = \frac{1 + r}{2} w_B - \frac{1 - r}{2}, \]
where we have defined \( r \equiv N_f/N_c \).

Following the same methods employed in ref. [7], one can show that this solution is the unique stable attractor in the space of solutions of eqs. [4]. Then, even if the \( q_i \)'s start with different initial conditions, there is a region in field configuration space such that the system evolves towards the equal fields solutions [4], and the late-time behavior is indistinguishable from the case considered in ref. [7], where equal initial conditions for the \( N_f \) flavors were chosen. In spite of this, the two-field dynamics introduces some new interesting features. For example, we have found that for any given initial energy density such that – for \( q_1^n / q_2^n = 1 \) – the tracker is joined before today, there exists always a limiting value for the fields’ difference above which the attractor is not reached in time. A more detailed discussion and numerical results about the two-field dynamics can be found in [4].

The scale \( \Lambda \) can be fixed requiring that the scalar fields are starting to dominate the energy density of the universe today and that both have already reached the tracking behavior. The two conditions are realized if
\[ v(q_0) \simeq \rho_{\text{crit}}^0, \quad v''(q_0) \simeq H_0^2, \]
where \( \rho_{\text{crit}}^0 = 3M_{\text{Pl}}^2 H_0^2/8\pi \) and \( q_0 \) is the present scalar fields VEV. Eqs. [4] imply
\[ \frac{\Lambda}{M_P} \simeq \left[ \frac{3(1 + r)(3 + r)}{4\pi(1 - r)^2 r N_c} \right]^{1/2} \left[ \frac{1}{2r N_c M_P^2} \right]^{1/2}, \]
\[ \frac{q_0^2}{M_P^2} \simeq \frac{3}{4\pi} \frac{(1 + r)(3 + r)}{(1 - r)^2} r N_c. \]

2.3. Interaction with the visible sector

The superfields \( Q_i \) and \( \bar{Q}_i \) have been taken as singlets under the SM gauge group. Therefore, they may interact with the visible sector only gravitationally, i.e., via non-renormalizable operators suppressed by inverse powers of the Planck mass, of the form
\[ \int d^4 \theta \ K^j(\phi^j_+, \phi^j) \cdot \beta^{ji} \left[ \frac{Q^j Q^j}{M_P^2} \right], \]
where \( \phi^j_j \) represents a generic standard model superfield. From [4] we know that today the VEV’s \( \langle q_i \rangle \) are typically \( O(M_P) \), so there is no reason to limit ourselves to the contributions of lowest order in \( |Q|^2/M_P^2 \). Rather, we have to consider the full (unknown) functions \( \beta^j \)'s and the analogous \( \bar{\beta}^j \)'s for the \( \bar{Q}_j \)'s. Moreover, the requirement that the scalar fields are on the tracking solution today, eqs. [4], implies that their mass is of order \( \sim H_0 \sim 10^{-35} \text{ eV} \).

The exchange of very light fields gives rise to long-range forces which are constrained by tests on the equivalence principle, whereas the time dependence of the VEV’s induces a time variation of the SM coupling constants [4]. These kind of considerations set stringent bounds on the first derivatives of the \( \beta^j \)'s today,
\[ \alpha^{ji} \equiv \frac{d \log \beta^{ji}}{dx_i} \bigg|_{x_i = x_i^0}, \]
where \( x_i = q_i/M_P \). To give an example, the best bound on the time variation of the fine structure constant comes from the Oklo natural reactor. It implies that \( |\dot{\alpha}/\alpha| < 10^{-15} \text{ yr}^{-1} \) [2], leading to the following constraint on the coupling with the kinetic terms of the electromagnetic vector superfield \( V \),
\[ \alpha^{Vi} < 10^{-6} \frac{H_0}{\langle \dot{q}_i \rangle} M_P, \]
where $\langle \dot{q}_i \rangle$ is the average rate of change of $q_i$ in the past $2 \times 10^{9}$ yr. Therefore, in order to be phenomenologically viable, any quintessence model should postulate that all the unknown couplings $\beta_i$’s and $\bar{\beta}_i$’s have a common minimum close to the actual value of the $q_i$’s.

The simplest way to realize this condition would be via the least coupling principle introduced by Damour and Polyakov for the massless superstring dilaton in ref. [13], where a universal coupling between the dilaton and the SM fields was postulated. In the present context, we will invoke a similar principle, by postulating that $\beta_i = \beta$ for any SM field $\phi_j$ and any flavor $i$.

The decoupling from the visible sector implied by bounds like [14] does not necessarily mean that the interactions between the quintessence sector and the visible one have always been phenomenologically irrelevant. Indeed, during radiation domination the VEVs $q_i$ were typically $\ll M_P$ and then very far from the postulated minimum of the $\beta$’s. For such values of the $q_i$’s the $\beta$’s can be approximated as

$$\beta \frac{Q_i Q}{M_P^2} = \beta_0 + \beta_1 \frac{Q_i Q}{M_P^2} + \ldots$$  \hspace{1cm} (11)

where the constants $\beta_0$ and $\beta_1$ are not directly constrained by [10]. The coupling between the last expression and the SM kinetic terms, as in [11], induces a SUSY breaking mass term for the scalars of the form

$$\Delta L \sim H^2 \beta_1 \sum_i (|Q_i|^2 + |\bar{Q}_i|^2)$$  \hspace{1cm} (12)

as discussed in [14].

If present, this term would have a very interesting impact on the cosmological evolution of the fields. From a phenomenological point of view, the most relevant effect of the presence of mass terms like [12] during radiation domination is the rise of the scalar potential at large fields values, that induces a (time-dependent) minimum. This results in a significative enlargement of the already large region of initial condition phase space leading to late-time tracking behavior. Numerical confirmation of this qualitative discussion can be found in [1].

3. INFLATION

The idea of studying inflaton potentials $V(\phi)$ which go to zero at infinity is not new [13-17] and has most recently referred to as the “quintessential inflation” [18] or “non oscillatory” [17] scheme. All these models have the appealing feature of providing a natural candidate for quintessence in the tail of the inflaton potential.

The main emphasis of these previous works was on the mechanism of reheating which, due to the usual shape of the potential, could not be achieved in the standard oscillatory way. Gravitational particle production was most often invoked [14,15], but Felder, Kofman and Linde have recently shown [17] that the so-called “instant preheating” mechanism is also a workable option. We will instead focus on the issue of the compatibility of the constraints coming from inflation and quintessence [18].

Regarding inflation, there are four main points to be taken into account:

1. The equation of state of the inflaton $\phi$ must satisfy $w_\phi < -1/3$, for the universe to accelerate.
2. A sufficient number of e-foldings should take place, in order to solve the flatness and horizon problems.
3. The amplitude of scalar perturbations in the cosmic microwave background measured by COBE constrains the normalization of the inflaton potential.
4. We must ensure that at the end of inflation sufficient reheating takes place.

For what concerns quintessence, instead, the following requirements should be fulfilled:

1. In order for the scalar field modeling of the cosmological constant to be sufficiently general, we require that the late-time shape of the potential is of the form $V(\phi) \sim M^{4+\alpha} \phi^{-\alpha}$ with $\alpha > 0$.
2. Secondly, we want the field $\phi$ to be already on track today and its present energy density to correspond to what observations report, i.e. $\Omega_\phi \simeq 2/3$, as discussed in Section 2.

While it is straightforward to find potentials with the required early and late-time behavior, the subtle issue resides in successfully matching the exit conditions for the scalar field after inflation with the range of initial conditions allowed...
for the trackers.

An very promising possibility seems to be that of considering first-order inflation. In this case, if the potential does not have an absolute minimum but instead goes to zero at infinity, the exit conditions of the inflaton $\phi$ from the tunneling would set to a very high precision the initial conditions for the subsequent quintessential evolution of the same field $\phi$. In ref. [18], we study the scalar field dynamics in a potential of the form

$$V(\phi) = \frac{\Lambda^{\alpha+6}}{\phi^\alpha \left[ (\phi - v)^2 + \beta^2 \right]}, \quad \text{with} \quad \frac{\beta}{v} \ll 1\, , (13)$$

where $\Lambda$, $\beta$ and $v$ are constants of mass dimension one. This potential has a barrier in $\phi \simeq v$, while for $\phi \gg v$ it behaves like $V(\phi) \sim \Lambda^{\alpha+6} \phi^{-\alpha-2}$ as required by the tracker condition. Reheating can be easily achieved via bubble collisions after nucleation, as it is usually done in first order inflation.

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