Experimental Validation of Various Temperature Modells for Semi-Physical Tyre Model Approaches

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Abstract. With increasing level of complexity and automation in the area of automotive engineering, the simulation of safety relevant Advanced Driver Assistance Systems (ADAS) leads to increasing accuracy demands in the description of tyre contact forces. In recent years, with improvement in tyre simulation, the needs for coping with tyre temperatures and the resulting changes in tyre characteristics are rising significantly. Therefore, experimental validation of three different temperature model approaches is carried out, discussed and compared in the scope of this article. To investigate or rather evaluate the range of application of the presented approaches in combination with respect of further implementation in semi-physical tyre models, the main focus lies on the a physical parameterisation. Aside from good modelling accuracy, focus is held on computational time and complexity of the parameterisation process. To evaluate this process and discuss the results, measurements from a Hoosier racing tyre 6.0 / 18.0 10 LCO C2000 from an industrial flat test bench are used. Finally the simulation results are compared with the measurement data.

1. Introduction
With enhanced integration of simulation and modelling into vehicle development and assistance systems, the requirements for simulation models regarding precision and computational time are increasing rapidly. In this case, not only demands affecting vehicle modelling but also tyre modelling become more important. In recent years, with improvement in tyre simulation, the needs for coping with tyre temperatures and the resulting changes in tyre characteristics are rising significantly. These changes include, amongst other things, the temperature dependent behaviour of friction coefficients and the loss of cornering stiffness with increasing temperature. To account those effects in modern assistance systems, it will become mandatory to reliably estimate tyre temperatures with information gathered from sensors or state estimators the vehicle can provide. This needs to be done in real time and therefore a reasonable compromise between accuracy and computational time has to be achieved. To generate a comparative basis three different models are tested in regard of the fulfilment of the requirements mentioned. To keep the effort needed for parameterisation as low as possible, the models were designed to handle the simulation with little to no information about the car itself and instead focusing on properties derived from tyre measurements.
2. Basic investigation - mechanics of heat generation and distribution

Before describing the three different types of temperature model mentioned above, a basic overview of the mechanics of heat generation and distribution of a tyre is given in this chapter, see also Figure 1.

Figure 1: Different heat phenomena occurring at a tyre during operation and divided in heat generation and heat exchange, based on [1]

**Heat generation** is described by the different effects which increase the inner and/or outer temperature of a tyre. Two different types of heat generation are common used in the literature, see e.g. [2] or [3]. First, heat is generated by different sliding velocities between road and tyre, see \( \dot{Q}_1 \) in Figure 1, and second heat is generated inside due the hysteresis effect, see \( \dot{Q}_2 \). The heat in the sliding region of the contact patch \( \dot{Q}_1 \) is generated by an differential velocity between tyre and road and is mathematically described with

\[
\dot{Q}_{1,\text{Friction(x/y)}} = F_{x/y} \cdot v_{s,x/y} \cdot \left( c_s \cdot \frac{T_{\text{nom}}}{T_{\text{sur}}} \cdot \frac{1}{2} \right),
\]

(1)

where \( F_{x/y} \) describes the longitudinal and lateral forces and \( v_{s,x/y} \) the sliding velocities. Additionally a parameter \( c_s \) is needed which scales the distribution between sliding and non-sliding region of the contact patch depending on the current longitudinal slip and slip angle, see Figure 1. The term \( \frac{T_{\text{nom}}}{T_{\text{sur}}} \) describes the power distribution between tyre and road where \( T_{\text{nom}} \) stands for a temperature equilibrium where the generated heat is distributed equally among the tyre and the roadway.

As mentioned above, the second type of heat is generated inside the tyre due to hysteresis effect \( \dot{Q}_2 \). In the course of the parameterisation process it could be observed that the heat generation of the tested tyre is governed by the impact of the frictional powers, therefore heat generation due to hysteresis is neglected in this article. Through multiple tests it could also be concluded that the temperature at the center area of the tyre is not influenced by inclination angle, therefore inclination angle is used in this article.

**Heat exchange** owing to conduction between the adhesive part of the contact patch and the roadway \( \dot{Q}_4 \) and outside the contact patch due to convection with the surrounding air \( \dot{Q}_3 \). Furthermore convection is happening constantly between the inner layer and the filling medium \( \dot{Q}_5 \). Equation 2 shows the mathematical approach to model this heat exchange

\[
\dot{Q}_{3/4/5,\text{convection/conduction}} = f\left(\frac{\text{Exp}_i, \lambda_i, \rho_i, c_p(i)}{h_i}, A(i) \cdot \Delta T(i)\right).
\]

(2)
The parameter $A$ describes the area and $\Delta T$ the temperature difference between the affected regions, where $i = \{\text{area in contact with ambient air, track, filling medium}\}$. The convection coefficient $h_i$ is strongly dependent on the velocity $v$, the adjacent material properties ($\lambda$, $\rho$, $c_p$) and the way the tyre is mounted in the car ($Exp$). In open wheel racing the airflow is surrounding the tyre at all times whereas passenger car tyres are mostly covered by the wheel case. Because the examined tyre is used to compare measurements done on an industrial flat track test bench with constant tyre speed, this function is treated as a constant value $h_i$ in this article. Investigations especially on the convection function will be done in the future.

The same is true for the conduction coefficient which is highly dependent on the chemical composition and the surface design of the tyre. Although knowing that the slick tyre tested is characterised by a very soft compound, almost no information about a reasonable value range could be obtained through literature research.

2.1. Single-layer temperature model

The single-layer model is characterised by consisting of the simplest structure in this article. The whole tyre consists of one layer with combined thermal properties from rubber and belt materials, [1]. For this single-layer, the first law of thermodynamics is applied using

$$\Delta T = \frac{\Delta t}{m \cdot c_p} \cdot \left( \sum_i \dot{Q}_i \right),$$

(3)

where $\dot{Q}_i$ describes the different phenomena of heat generation and exchange, see Figure 1, $m$ the tyre mass, $c_p$ the combined thermal heat capacity and $\Delta t$ the time difference between two measurement points. This energy balance contains the heat generation due to friction and also the heat emission to the surrounding environment and the filling medium. With these inputs, temperature alternation from an initial starting temperature and hence the temperature of the layer can be calculated.

2.2. Multi-layer temperature model

The multi-layer model is an improvement of the single-layer model, based on [2], where in this article the tyre is split in three layers with individual thermal properties and input sizes. These layers describe the main constituents of a tyre and for each of these layers the energy balance considers different heat mechanisms. The outer layer for example, is in direct contact with the road and the ambient air whereas the inner layer is exchanging heat with the filling medium. Additionally heat flows among the three layers are taken into account, these are modelled using an adapted form of the stationary heat conduction as shown in Figure 2 and given by

$$\dot{Q}_{\text{conduction}} = \frac{1}{\sum_i \delta_i \cdot \lambda_i} \cdot A \cdot \Delta T,$$

(4)

where $A$ describes the area and $\Delta T$ the temperature difference between adjacent point masses. Combined with thickness $\delta_i$ and thermal conduction coefficients $\lambda_i$ the thermal resistors are modelled properly. Thus the temperature for each zone can be calculated depending on the behaviour of the other zones and therefore a temperature distribution over the tyre depth can be shown. As example, Equation 5 shows the energy balance for the surface layer, where $\dot{Q}_{\text{surface}}$ describes the conduction term as described through Equation 4, this can be seen in Figure 3 and applied for all layers.

$$\Delta T_{\text{surf}} = \frac{\Delta t}{m_{\text{surf}} \cdot c_{p,\text{surf}}} \cdot (\dot{Q}_{1,\text{friction}} + \dot{Q}_{3,\text{convection}} + \dot{Q}_{4,\text{conduction}} + \dot{Q}_{\text{surface}})$$

(5)
2.3. Partial temperature model

The partial temperature model describes the problem of an unsteady heat conduction in the most accurate way compared to the other two models. It is described by an one dimensional partial differential Fourier heat equation, as described in [3] by

$$\lambda_i \frac{\partial^2 T}{\partial \delta^2} = \rho_i c_{p,i} \frac{\partial T}{\partial t}. \tag{6}$$

The term $\frac{\partial^2 T}{\partial \delta^2}$ describes the directional derivative of the temperature over the tyre depth $\delta$ and $\rho_i$ the corresponding density. An infinitesimal element as shown in Figure 4 revolves around the wheel axis and traverses all zones relevant for heat exchange $\dot{Q}_{1-5}$. For that reason the position regarding tyre depth and circumference are additional state variables. Heat flows are described as boundary conditions on the outer and inner surface, therefore, $\dot{Q}_{1-5}$ (Eq. 7/8) have to be adapted slightly compared to Equations 1 and 2. Because of the integration over the circumference of the tyre, areas regarding convection $A_j$ do not occur in the respective equations whereas the heat generation has to be divided by the sliding area $A_{cp,sl}$ as seen in Equation 8.

$$\dot{Q}_{3/4/5} = -\lambda_i \frac{\partial T}{\partial \delta} \bigg|_{\delta=0} = h_i \cdot [T_\infty - T(0,t)] \tag{7}$$

$$\dot{Q}_{1,x/y} = -\lambda_i \frac{\partial T}{\partial \delta} \bigg|_{\delta=0} \approx \frac{1}{A_{cp,sl}} \cdot F_{x/y} \cdot v_{s,x/y} \cdot \left( c_s \cdot \frac{T_{nom}}{T_{sur}} \cdot \frac{1}{2} \right) \tag{8}$$

The material properties regarding the heat flux are changing along the tyre depth, thus modeling tyre geometry correctly, see Fig. 4. Because of the partial access, temperatures can be calculated steplessly throughout the tyre depth and circumference. This model is characterised through a distinctive temperature behaviour as shown in Figure 5. At each revolution the element enters the sliding zone where the temperature rises nearly step like. Afterwards, in the convection and conduction zone, the temperature decreases continuously throughout the remaining rotation until the sliding zone is reached again. The angular position to determine the needed boundary condition is calculated by means of the wheel rotational speed and compared to angles $\alpha_{1-3}$ as shown in Figure 1.

3. Parameterisation Process

The tyre tested is a Hoosier slick 6.0 / 18.0 10 LCO C2000 used by the Formula Student racing team of TU Graz. The tyre is characterised by a slim construction and a very soft compound. The testing procedure was carried out on a flat track test bench and consists of transient manoeuvres with changes in slip angle, slip ratio, contact force and inclination angle.
By measuring the geometric properties of the tyre and combining this information with heat transfer coefficients found in literature, [5], the internal heat flux was described. Because of the wide range of values for convection coefficients found in literature, e.g., [5], these parameters are optimised within a physically justifiable range. The same was done for the conduction coefficient in the adhesive part of the contact patch, with the difference that no explicit upper limit for this kind of material combination could be determined in literature. Also a reference temperature \( T_{nom} \) is optimised where the distribution of heat generation due to friction between tyre and road is in a thermal state of equilibrium. Table 1 summarises the parameters used for the optimisation process. Since IR-sensors, used to measure tyre temperature, scan only the outermost elements of the measured area, it is necessary to have a very responsive layer in the surface region. Therefore, the surface layer of the multi-layer model represents the first millimetre of rubber of the tyre. For the same reason a very fine resolution for the partial access of the PDE-tool (Partial Differential Equation Toolbox, Matlab [6]), used to solve the Fourier equation, was chosen. Because of that thin surface layer, for the multi-layer model an additional parameter \( p_4 \) was implemented, this coefficient scales the power generated due to friction between the surface and the bulk layer, where a value of one represents frictional power only occurring in the surface layer and vice versa. Figure 6 shows the temperature penetration of a tyre experiencing flash heating due to frictional powers, it can be seen that heat generated due to friction is not only penetrating the outermost elements of the tyre, but rather affects internal regions too. Therefore this parameter was necessary to control the temperature balance between the three layers of the multi-layer model. Overall, the parameter solving was performed using the least squares method where the error between measured and simulated temperature was minimised. In terms of the three layer model, this temperature was represented by the surface layer, whereas for the partial-model the temperature of the outermost element was used.

4. Results

Table 2 summarises the parameters solved in the course of the fitting process. Overall the parameters identified are approximately the same size and satisfactorily match the expected values. The convection coefficient is situated at the bottom end of the allowed value range. This can be attributed to the fact that the boundaries found in [5] refer to a passenger car tyre where the turbulent boundary air layer is more pronounced compared to a slick tyre, therefore this coefficient is estimated to be higher at those type of tyres. With respect to the vague information regarding the conduction coefficient between road and tyre the solved values seem reasonable. The single-layer model reached the upper limit set for this parameter which was chosen because of the values identified through the other models. This limit was necessary
Table 1: Parameters used in the fitting process

| Description       | $h_{\text{road}}$ | $h_{\text{amb}}$ | $T_{\text{nom}}$ | $F_{\text{fric}}$ |
|-------------------|-------------------|------------------|------------------|-------------------|
| Partial model     | $p_1$             | $p_2$            | $p_3$            |                   |
| Single-layer model| $p_1$             | $p_2$            | $p_3$            |                   |
| Multi-layer model | $p_1$             | $p_2$            | $p_3$            | $p_4$            |

Figure 6: Temperature penetration due to flash heating from frictional power, [4]

To keep the convection coefficient in an physically meaningful value range. The reference temperature found by the optimisation tool is a few degrees above the road temperature and can be interpreted as an averaged contact temperature between road and tyre, this comes in handy because to calculate this temperature much information would be needed about the track properties, which is unavailable in most cases. Figure 9 shows the results of the temperature simulation in lateral-, longitudinal and combined direction. Although all three models have a satisfactory average error, only the multi-layer and the partial model can reproduce the behaviour of the surface region, in terms of flash temperature, realistically. Additionally the partial model is able to picture the cooling condition of the tyre the most accurate way and can be modified to deal with full braking lock scenarios, where only a specific region of the surface is affected. When comparing the inner regions of the partial and the multi-layer model a difference can be observed. This comes from the different approaches on how the power is introduced into the tyre, where at the partial model the frictional power enters the tyre through a boundary condition whereas at the multi-layer model parts of this power are directly entering the bulk region. Another reason is the different heat distribution characteristic of these approaches as seen in Figure 7. Since there is no measurement data of the inner parts of the tyre, no clear statement could be made which model is more accurate. Another interesting feature of the partial model are three dimensional plots over tyre depth, time and temperature, which can be really illuminating, as evidenced by Figure 8. Concerning computational time, all three models are capable of running online, although it should be noted that the chosen resolution of the partial model had to be adjusted carefully to ensure that premise without loosing accuracy. Table 2 summarise the used material specific values used for the simulation.

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Figure 9: Simulation results compared with measurement data. Left: Different slip ratio and slip angle targets. Right: Temperature comparison by using different model approaches.
Table 2: Parameters identified in the fitting process

| Description          | $h_{\text{road}}$ in W/m²K | $h_{\text{amb}}$ in W/m²K | $T_{\text{nom}}$ in °C | $F_{\text{fric}}$ in - |
|----------------------|----------------------------|----------------------------|------------------------|------------------------|
| Partial model        | 1282.1                     | 36.7                       | 30.2                   |                        |
| Single-layer model   | 2000.0                     | 18.8                       | 34.7                   |                        |
| Multi-layer model    | 1496.3                     | 17.9                       | 31.9 0.55              |                        |

Table 3: Material specific values used to simulate the presented temperature models

| Symbol | Description                        | Unit Value |
|--------|------------------------------------|------------|
| $\lambda_{\text{sbr}}$ | Heat conduction coefficient SBR | W/m K 0.25 |
| $\lambda_{\text{steel}}$ | Heat conduction coefficient steel | W/m K 10 |
| $\rho_{\text{sbr}}$ | Density SBR | kg/m³ 1550 |
| $\rho_{\text{steel}}$ | Density steel | kg/m³ 7850 |
| $c_{\text{p,sbr}}$ | Specific heat capacity SBR | J/kg K 1880 |
| $c_{\text{p,steel}}$ | Specific heat capacity steel | J/kg K 450 |
| $h_{\text{fill}}$ | Convection coefficient inner liner/filling medium | W/m²K 1 |

5. Conclusion
The influence of tyre and road temperature on the tyre’s contact force characteristics is a well-known phenomenon. Different model approaches, with respect of possible implementation in semi-physical tyre models, were compared in model accuracy, computational time and complexity of the parameterisation process in this article. To evaluate this process and discuss the results, measurements from a Hoosier racing tyre 6.0 / 18.0 10 LCO C2000 from an industrial flat test bench are used. It is presented that a single-layer model is easy to parametrise but has lack in accuracy especially on dynamic manoeuvres and investigations depending on the tyre depth. A multi-layer model with three layers shows quite good evaluation results but brings the disadvantage of additional model parameters. A partial-model, based on the Fourier heat equations, brings long calculation time but shows the most accurate results and is furthermore able to model the temperature depending on the circumference of the wheel. Summarised, depending on the application, the presented model approaches are able to model the tyre temperature in an accurate way. To evaluate the model including the parameterisation process investigations will be done on different types of passenger cars tyres.

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