Structural optimization design of crane frame for nuclear power waste

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Abstract. The structural stiffness and strength are the basis to ensure the performance of nuclear power waste crane. The structural topology optimization method is an effective method to ensure the structural performance of nuclear power waste crane in the conceptual design stage. In this paper, the topology optimization method is used to establish the finite element topology optimization model of the nuclear waste crane frame, and the structure optimization design is carried out. The optimization results are verified by the finite element method, which proves the effectiveness of the method.

1. Introduction
Due to the particularity of the workplace, nuclear waste crane must ensure strict safety, reliability and positioning accuracy. The structural stiffness and strength are the basis to ensure the performance of nuclear power waste crane. The structural topology optimization method is an effective method to ensure the structural performance of nuclear power waste crane in the conceptual design stage.

Structural topology optimization originated from Michell’s [1] research on topology optimization design of truss structure in 1904. However, it has attracted a lot of attention since Cheng Gengdong and Olhoff et al. [2] studied the optimization design of the solid plate structure with minimum flexibility. In their research, they innovatively introduced the microstructure into the optimization problem. Later, based on the above ideas, Bendsoe and Kikuchi [3] proposed a topology optimization design method combining microstructure and homogenization. This method transforms the problem of structural topology optimization design into the problem of material distribution. This has played a milestone role in the development of structural topology optimization. In the past 30 years, topology optimization has been developed rapidly.

At present, there are many literatures about topology optimization. According to different optimization objects, topology optimization can be divided into discrete structure topology optimization and continuum structure topology optimization. The topology optimization of discrete structure mainly optimizes the cross-sectional area of members in truss, steel frame and other structures to remove the inefficient members in the structure, so as to obtain the structure with better performance [4,5]. The topology optimization of continuum structure is mainly to optimize the material layout in the solid structure, so as to obtain the structural design scheme meeting the design requirements [6]. The classical topological optimization mathematical model for minimizing the flexibility of continuum structure is shown in formula (1). In the formula, $x_i$ is a function representing the material distribution state in the design domain, $x_i = 1$ indicating that the point $i$ is filled by solid material, while $x_i = 0$ indicating the change point has no material. $u$ represents the field variable, which satisfies the linear or nonlinear
state equation, \( T \) and \( f \) represent the surface load and the volume load respectively, \( V_0 \) represents the initial volume of the design area. Because this paper is mainly about topology optimization of continuum structure, if there is no special explanation, the subsequent content of topology optimization refers to topology optimization of continuum structure.

\[
\begin{align*}
\text{Find:} & \quad x = \{x_1, x_2, \ldots\} \\
\text{Minimize:} & \quad C(x) = \int_{S_p} \mathbf{u}^T \mathbf{T} dS + \int_{V} \mathbf{u}^T \mathbf{f} dV \\
\text{Subject to:} & \quad V(x) = \int_{\Omega} \rho(x) d\Omega \leq V_0 \\
& \quad x_i = 0 \text{ or } 1, \quad \forall i \in \Omega
\end{align*}
\]

(1)

In solving the above topological optimization problem (1), a key part is the structural analysis process. Structural analysis mainly refers to the PDE solution corresponding to the physical location of the structure. At present, there are many numerical methods to solve PDE in topology optimization, such as finite element method, meshless method and polynomial mesh method [7]. The theoretical basis of finite element method is very mature, and it is easy to program. Therefore, in the general topology optimization of linear elastic structure, the finite element method is used to solve the PDE. In this paper, the finite element method is also used to solve the PDE in the topology optimization of multiphase materials.

According to the different methods of structural topology description and material interpolation, topology optimization of continuum structure can be divided into homogenization method [3], variable density method [6], level set method [8], progressive structure optimization method, etc. In this paper, the homogenization method is used to optimize the frame structure of nuclear power crane and the finite element method is used to verify.

2. Establishment of topology optimization model for crane frame of nuclear power plant

Using the weighted method, the mathematical model of multi-objective topology optimization for nuclear power crane can be generally expressed as follows:

\[
\begin{align*}
\min \Phi &= \omega_1 \bar{\Phi}_1 + \omega_2 \bar{\Phi}_2 + \ldots + \omega_m \bar{\Phi}_m \\
\text{s.t.} & \quad g_k (\mathbf{a}, \rho) \leq 0 \quad (k = 1, 2, \ldots, p) \\
& \quad h_l (\mathbf{a}, \rho) = 0 \quad (l = 1, 2, \ldots, q) \\
& \quad \rho_{\text{min}} \leq \rho \leq 1
\end{align*}
\]

(2)

Among them, \( \Phi \) is the newly constructed weighted objective function, \( \bar{\Phi}_m \) represents the normalized \( m \)-th objective function, and \( \omega_m \) is its corresponding weight coefficient, which satisfying the equation \( \sum_{i=1}^{m} \omega_i = 1 \).

For the optimization problem (2), the gradient based method such as mathematical programming method can be used to solve it. Therefore, a step degree (sensitivity) of all objective functions and constraint functions with respect to design variables \( \rho \) must be provided. In this paper, we use the adjoint method to derive the first order analytical sensitivity of the objective function. Consider the \( m \)-th objective function:

\[
\Phi_m = \int_{\Omega} \varphi(\mathbf{a}, \rho) dV
\]

(3)
Among them, $\varphi$ is the defined function in the $\Omega$ domain. In order to derive the sensitivity of the objective function $\Phi_m$, we introduce the Lagrange term to construct a new function $\Phi_m^*$.

$$\Phi_m^* = \Phi_m + \lambda^T (Ka - F)$$

(4)

So:

$$\frac{d\Phi_m}{d\rho} = \frac{\partial\Phi_m}{\partial a} \frac{da}{d\rho} + \lambda^T K \frac{da}{d\rho} + \lambda^T \frac{\partial K}{\partial a}$$

(5)

$$\frac{d\Phi_m}{d\rho} = \frac{\partial\Phi_m}{\partial a} \frac{da}{d\rho} + \lambda^T K \frac{da}{d\rho} + \lambda^T \frac{\partial K}{\partial a}$$

In order to eliminate the implicit term $\frac{da_i}{d\rho}$, we introduce the adjoint problem as follows:

$$K\lambda = -\frac{\partial\Phi_m}{\partial a}$$

(6)

Then, bringing it into the formula (6), the sensitivity of the objective function $\Phi_m$ can be obtained:

$$\frac{d\Phi_m}{d\rho} = \frac{\partial\Phi_m}{\partial a} \frac{da}{d\rho} + \lambda^T \frac{\partial K}{\partial a}$$

(7)

Among them, $\frac{\partial K}{\partial a}$ can be directly obtained by the variational method.

In this paper, MMA is used to solve structural topology optimization problems. By introducing the moving asymptote, the implicit optimization problem is transformed into a series of explicit and simpler strictly convex approximation subproblems. The approximation function is determined by the left and right asymptotic points determined in advance and the derivative of the original objective function and constraint function at each point. MMA is the most effective mathematical programming method to solve multi-objective topology optimization.

3. Topology optimization results and verification

Figure 1 is the design domain model and boundary conditions of the frame beam structure of nuclear power crane. It is the most important bearing structure of the frame. The topology optimization method is used to optimize the structure and the finite element method is used to verify the optimization results.

![Figure 1. Topology optimization design domain and boundary conditions of nuclear waste crane](image_url)
Figure 2. Topology optimization results of frame crossbeam

Figure 3. Assembly finite element model of frame beam structure and Z-direction lifting device

Figure 4. Deformation cloud chart of the device under the limit working state

Figure 5. Stress cloud chart of the device under the limit working state
Figure 2 is the topological optimization design result of the frame crossbeam structure. According to the optimization result, the structure model of the frame crossbeam is constructed as shown in Figures 3-5. Under the limit condition, the stress and deformation of the whole machine are analyzed by finite element method. The maximum stress and deformation of the structure are 116MPa and 0.095mm respectively. The strength and rigidity meet the design requirements.

4. Conclusion
In this paper, the topology optimization method is used to optimize the frame beam structure of nuclear waste crane, and on this basis, the beam structure is designed. Through the finite element simulation analysis under the limit condition, the stiffness and strength indexes of the structure are obtained, and it is found that they can meet the design requirements. The validity of the proposed method is verified.

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