Muons $g - 2$, $B \to K^{(*)}\mu^+\mu^-$ anomalies, and leptophilic dark matter in $U(1)_{\mu - \tau}$ gauge symmetry

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ABSTRACT: We propose a new class of $U(1)_{\mu - \tau}$ gauged model that can explain recent flavor anomalies such as muon $g - 2$, $b \to s\mu^+\mu^-$, as well as include a scalar dark matter candidate while satisfying all the phenomenological constraints. For this purpose, we add new vectorlike quarks and leptons as well as two inert singlet scalars, one leptophilic and the other leptophobic. In our model $b \to s\mu^+\mu^-$ anomalies can be explained by loop induced interactions among the SM, exotic quarks, and $Z'$ gauge boson. In our numerical analysis, we show allowed region of our model to be narrow, and it would be tested soon, for example by searching for 4 muons and dimuon + missing transverse momentum from $pp \to \mu^+\mu^- Z'$ and $pp \to \nu_{\mu,\tau} \bar{\nu}_{\mu,\tau} Z'$ followed by $Z' \to \mu^+\mu^- , \nu_{\mu,\tau} \bar{\nu}_{\mu,\tau}$ at the LHC.

KEYWORDS: Beyond Standard Model, Gauge Symmetry

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1 Introduction

Even after the discovery of the standard model (SM) Higgs boson, there are a number of reasons for physics beyond the SM (BSM), such as Baryon Asymmetry of Universe (BAU), neutrino oscillations [1] and nonbaryonic dark matter [2], all of which are well established. Also there are some anomalies in the flavor sector: the ratio between rare $B$ meson decays of $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ e\bar{e}$ ($R_K$) [3–6], and the muon anomalous magnetic moment (muon $g-2$) [7, 8]. These anomalies may call for explanations within some new physics beyond the SM (BSM), although some anomalies may (partly) originate from poorly understood SM predictions involving nonperturbative QCD.

Especially, the muon $g-2$ is known as a longstanding anomaly that was initially discovered by E821 experiment at Brookhaven National Lab (BNL) two decades ago [7], and has recently been confirmed by E989 Run 1 experiment at Fermilab(FNAL) [8]; combining these measurements we obtain

$$\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}, \quad (1.1)$$

whose deviation from SM prediction [9–29] is 4.2 $\sigma$.

Also $R_K$ anomaly has been recently updated by the LHCb collaboration [30]:

$$R_K = 0.846^{+0.042+0.013}_{-0.039-0.012} \; (1.1\text{GeV}^2 < q^2 < 6\text{GeV}^2), \quad (1.2)$$

where $q^2$ is the squared momentum transfer.
where the first (second) uncertainty is statical (systematic) one and $q^2$ is the dilepton invariant mass squared. Its deviation from the SM prediction is about 3.1 $\sigma$. There are also discrepancies in the measurements of the angular observable $P_\ell$ in the $B$ meson decay $(B \to K^* \mu^+ \mu^-)$ [31–35], as well as in $R_{K^*} = BR(B \to K^* \mu^+ \mu^-)/BR(B \to K^* e^+ e^-)$ [36]; we call these as $b \to s\mu^+ \mu^-$ anomalies.

$U(1)_{\mu-\tau}$ gauge symmetry is anomaly free even without additional chiral fermions beyond the SM fermions [37, 38]. It has been frequently applied to models that explains some of these flavor anomalies. For examples, the muon $g-2$ within the $U(1)_{\mu-\tau}$ gauge models was studied right after the BNL announcement [39]. And the PAMELA $e^+$ excess was studied in the $U(1)_{\mu-\tau}$-charged Dirac fermion dark matter with mass $\sim O(1)$ TeV in ref. [40]. Due to the flavor-dependent feature of $U(1)_{\mu-\tau}$ symmetry, we often obtain flavor specific signals that can be more testable through various experiments than the SM. Moreover, since a new gauge boson does not directly interact with electrons up to small kinetic mixing, we can evade stringent constraints on the gauge coupling and gauge boson mass from LEP (or ILC) easier than the other gauged models.\footnote{Notice here that we have to consider a constraint of neutrino trident production in gauged $U(1)_{\mu-\tau}$ models [41], and we will discuss it in the main text.}

In fact, there is a wide variety of recent applications of $U(1)_{\mu-\tau}$ models, which are distinguished by additional particle contents to address flavor anomalies; refs. [42–47] for neutrino oscillations to control lepton flavor structure, refs. [28, 29, 48–59] for the muon $g-2$, refs. [60–69] for $b \to s\mu^+ \mu^-$ anomalies, and (or) DM. Qualitative features in $U(1)_{\mu-\tau}$ models for flavor anomalies are the following. The muon $g-2$ anomaly is explained by light $Z'$ around $O(10)$ to $O(100)$ MeV scale or by Yukawa interactions with exotic fermions controlled by $U(1)_{\mu-\tau}$ symmetry. For explaining $b \to s\mu^+ \mu^-$ anomalies with $U(1)_{\mu-\tau}$, generally we need to add new particles since the corresponding gauge boson $Z'$ does not couple to quarks. Mixing among the SM quarks and new vector-like quarks are considered to generate flavor violating $Z'$-quark coupling in refs [60, 61, 66], while flavor dependent charge assignment in the SM quark sector is considered in ref. [62].

In ref. [63], the present authors proposed a gauged $U(1)_{\mu-\tau}$ model in which interactions among $Z'$ and quarks are radiatively generated via one-loop diagram with vector-like quarks($Q'$) and scalar dark matter(DM) candidate propagating inside loop.\footnote{See also refs. [64, 65, 67, 68] for similar mechanism and analysis. Also scenarios in which vector-like fermions are mixed with the SM fermions are also found e.g. in refs. [60, 74].} In this work, we focused on a loop diagram associated with $Z'Q'\bar{Q}'$ interaction. But there are contributions associated with $ZQ'\bar{Q}'$ and photon-$Q'\bar{Q}'$ interactions, since vector-like quarks interact with $Z$ and photon that should be taken into account to check consistency of the scenario. Note also that we could not explain the muon $g-2$ in this scenario since $Z'$ from $U(1)_{\mu-\tau}$ should be heavier than $O(10)$ GeV. Also in ref. [63] we introduced only vector-like quarks and not vector-like leptons, and the model looks asymmetric between quark and lepton sectors. Thus it is interesting to add vector-like leptons in addition to vector-like quarks in the previous scenario to make the model look more symmetrical, and explain the muon $g-2$ and consider loop diagrams associated with $Z$ and photon interactions to check consistency for $b \to s\mu^+ \mu^-$ anomalies. We will also introduce two inert complex scalars.
and their Hermitian conjugates, so that newly added vector-like fermions and the SM ones

In addition we forbid these operators,

imposing conditions for the charge of the U(1)_{\mu-\tau} symmetry spontaneously by the nonzero VEV; \langle \varphi \rangle = v_{\varphi}/\sqrt{2}.

\( \chi_q \) (leptophobic) and \( \chi_{\ell} \) (leptophilic) that are charged under U(1)_{\mu-\tau} in such a way that Yukawa couplings among the vector-like quarks (leptons), SM quarks (leptons) and the inert scalar \( \chi_q \) (\( \chi_{\ell} \)) are allowed by gauge symmetries.

In the present work, we extend the previous model [63] by introducing vector-like leptons to explain the muon \( g - 2 \), \( R_K \), and DM in a gauged U(1)_{\mu-\tau} symmetry, where DM is assumed to be a weakly interacting massive particle (WIMP). We then estimate one loop diagrams considering \( Z' \), \( Z \) and photon vertices with vector-like quarks that affects \( b \to s\mu^+\mu^- \) process to explain \( R_K \) anomaly and to check consistency of the model. We also estimate the muon \( g - 2 \) and DM relic density to search for parameters explaining them. In addition we take all the relevant constraints such as neutral meson mixings and \( B_{s(d)}^0 \to \mu^+\mu^- \) into consideration and show the allowed region in our numerical analysis.

This letter is organized as follows. In section 2, we present our model and show several constraints from \( B \to K^{(*)}\ell^+\ell^- \), neutral meson mixings, muon \( g - 2 \), and (leptophilic) dark matter. In section 3 we carry out the numerical analysis and demonstrate our allowed region. Finally, section 4 is devoted to the summary of our results and conclusion.

## 2 Model setup and constraints

In this section we define our model. We introduce a vector-like exotic quark \( Q' \equiv [U', D']^T \) and lepton \( L' \equiv [N', E']^T \) (weak isospin doublet), and a singly charged-lepton \( E \) (a weak isospin singlet), two complex scalar bosons \( \chi_q \) and \( \chi_{\ell} \), and a boson \( \varphi \), where \( \chi_q(\chi_{\ell}) \) couples only to quark(lepton) sector respectively. The singlet scalar \( \varphi \) plays a role in breaking the U(1)_{\mu-\tau} symmetry spontaneously developing the nonzero vacuum expectation value (VEV); \( \langle \varphi \rangle = v_{\varphi}/\sqrt{2} \). Here we take the U(1)_{\mu-\tau} charge of \( \varphi \) so as to retain the inert features of \( \chi_q, \ell; \) we forbid terms \( \chi_{\ell,q}'(\varphi^{(*)})^n \) with \( n \) being integer. This can be achieved by imposing conditions for the charge of \( \varphi \) as follows:

\[
q_{\varphi} \neq \pm \frac{1}{n} \left( \frac{q_x}{2} - 1 \right), \pm \frac{1}{n} q_x.
\]

In addition we forbid these operators, \( \bar{Q}_L Q_{H}(\varphi^{(*)})^n, \bar{L}_L L_L(\varphi^{(*)})^n \) and \( \bar{\ell}_R E_L(\varphi^{(*)})^n \) (\( \ell = e, \mu, \tau \)) and their Hermitian conjugates, so that newly added vector-like fermions and the SM ones

| \( Q' \) | \( \chi_q \) | \( L' \) | \( E \) | \( \chi_{\ell} \) | \( \varphi \) |
|---|---|---|---|---|---|
| SU(3)_C | 3 | 1 | 1 | 1 | 1 |
| SU(2)_L | 2 | 1 | 2 | 1 | 1 |
| U(1)_Y | \( \frac{1}{5} \) | 0 | \(-\frac{1}{2}\) | -1 | 0 | 0 |
| U(1)_{\mu-\tau} | \( q_x \) | \( q_x \) | \( \frac{q_x}{2} \) | \( \frac{q_x}{2} - 1 \) | \( q_x \) |

**Table 1.** Charge assignments of the new fields \( Q', L' \equiv [N', E']^T, E, \chi_q \) and \( \chi_{\ell} \) under SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{\mu-\tau}. \) Here \( Q', L' \) and \( E \) are vector-like fermions. \( \chi_q \) and \( \chi_{\ell} \) are complex inert scalar bosons either of which can be considered as a DM candidate.
do not mix after $U(1)_{\mu-\tau}$ gauge symmetry breaking. In fact, $\bar{Q}_L Q'_{R}\varphi^{(s)}$, $\bar{L}_{L,e,L} L'_{R}\varphi^{(s)}$ and $\bar{e}_R (\mu_R) E_{L} \varphi^{(s)}$ are already forbidden by the conditions in eq. (2.1). To forbid remaining operators we require additional conditions,

$$q_\varphi \neq \pm \left( \frac{q_\mu}{2} + 1 \right).$$  \hspace{1cm} (2.2)

The SM Higgs doublet is denoted by $H$ and its VEV is given by $[0, v/H/\sqrt{2}]^T$. $Q'$ as well as $\chi_q$ is requested to induce sizable $B \to K^{(*)}\ell\ell$ anomalies, $E$ and $L'$ as well as $\chi_\ell$ are required to get sizable lepton anomalous magnetic moment.\(^3\) Charge assignments of these new fields are summarized in table 1. Then, the gauge invariant Lagrangian under these symmetries is given by

$$-L_{\text{VLF}+\chi} = M_Q Q' Q' + M_E E' E' + M_{L,L'} L' L' + y_{L,R} E_R H E_R + m_{\chi_q}^2 |\chi_q|^2 + m_{\chi_\ell}^2 |\chi_\ell|^2$$

$$+ (f_i Q'_{R} Q_{L} \chi_q + g_\mu \mu_R E_L \chi_\ell^i + h_\mu L_{L,R} L'_{R} \chi_i + \text{h.c.}),$$  \hspace{1cm} (2.3)

where $i = 1 - 3$ are generation indices, $Q_L$'s are the SM quark doublets and $\mu_R$ is the right-handed muon. We have abbreviated kinetic terms for simplicity.

The scalar potential in our model is given by

$$V = -M_H^2 H^1 H - M_\varphi^{2} \varphi^* \varphi + \lambda_H (H^1 H)^2 + \lambda_\varphi (\varphi^* \varphi)^2 + \lambda_{H_\varphi} (H^1 H)(\varphi^* \varphi),$$  \hspace{1cm} (2.4)

where we have abbreviated scalar potential associated with $\chi_{q,\ell}$ for simplicity. We expand scalar fields around its VEVs as

$$H = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + \hat{h} + iG_Z) \end{pmatrix}, \quad \varphi = \frac{1}{\sqrt{2}}(v_\varphi + \hat{\varphi} + iG_{Z'})$$  \hspace{1cm} (2.5)

\(^3\)With $E$ only, one cannot get sizable lepton anomalous magnetic moment, since the mass of $E$ has to be very small at about 200 GeV, which would be ruled out by LHC. $L'$ plays a role in evading chiral suppression and thus sizable values are obtained.
where \( \hat{h} \) and \( \hat{\phi} \) correspond to CP-even physical scalar boson states while \( w^+ \), \( G_Z \) and \( G_{Z'} \) are the Nambu-Goldstone(NG) bosons which are absorbed by the weak gauge bosons \( Z \), \( W \) and \( Z' \). The mass matrix for physical scalar bosons is obtained as

\[
M^2 = \begin{pmatrix}
\lambda_H v^2 & \lambda_H \phi v v \\
\lambda_H \phi v v & \lambda_\phi v^2
\end{pmatrix},
\]

(2.6)

The matrix is diagonalized by acting an orthogonal matrix \( U \) such that

\[
U^T M^2 U = \text{diag}(m_{\hat{h}}^2, m_{\hat{\phi}}^2),
\]

(2.7)

where

\[
U = \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix},
\]

(2.8)

and the scalar mixing angle \( \alpha \) is given by

\[
\tan 2\alpha = \frac{2\lambda_H \phi v v}{\lambda_H v^2 - \lambda_\phi v^2}.
\]

(2.9)

Then mass eigenstates are given by

\[
\begin{pmatrix}
\hat{h} \\
\hat{\phi}
\end{pmatrix} = U^T \begin{pmatrix}
\hat{h} \\
\hat{\phi}
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
\hat{h} \\
\hat{\phi}
\end{pmatrix},
\]

(2.10)

where \( h \) corresponds to the SM-like Higgs boson.

Exotic singly-charged fermions \( (E, E') \) mix each other after spontaneous electroweak symmetry breaking, and its mass matrix is given by

\[
\mathcal{M}_E = \begin{pmatrix}
M_E & m_E \\
m_E & M_{L'}
\end{pmatrix},
\]

(2.11)

where \( m_E \equiv y v/\sqrt{2} \). It is diagonalized by a unitary matrix \( V_E \) as \( \text{diag}(M_1, M_2) = V_E^T \mathcal{M}_E V_E \), and \( V_E \) is given by

\[
V_E = \begin{pmatrix}
c_c & -s_c \\
 s_c  & c_c
\end{pmatrix}, \quad \tan 2\theta_c = \frac{2m_E}{M_E - M_{L'}},
\]

(2.12)

where \( c_c(s_c) \) is shorthand symbol of \( \cos \theta_c(\sin \theta_c) \). We write mass eigenstates as \( E_1 \) and \( E_2 \) whose masses are \( M_1 \) and \( M_2 \).

3 Phenomenological constraints

3.1 Wilson coefficients for \( B \to K^{(*)} \ell^+ \ell^- \) decay

Here we calculate Wilson coefficients associated with \( B \to K^{(*)} \ell^+ \ell^- \) processes from the Feynman diagrams in figure 1. Anomalies observed in \( B \to K^{(*)} \ell^+ \ell^- \) decays can be
explained by the shift of the Wilson coefficient $C_9$ which corresponds to the operator $(\bar{s}\gamma_\mu P_L b)(\mu\gamma_\mu\mu)$. In our model, the effective coupling for $Z_\mu'\gamma\mu P_L s + h.c.$ is induced at one loop level as shown in figure 1 with the Yukawa coupling in eq. (2.3). Then the effective Hamiltonian $(\bar{s}\gamma_\mu P_L b)(\mu\gamma_\mu\mu)$ arises from $Z'$ mediation and the contribution to Wilson coefficient $\Delta C_{9}^{\mu\mu}(Z')$ is obtained as:

$$\Delta C_{9}^{\mu\mu}(Z') \simeq -\frac{g_{Z}g_{Z'}^{2}}{4\pi^{2}m_{Z}^{2}m_{Z'}^{2}}C_{SM} \left(\frac{1}{2} + 2s_{w}^{2}\right) \left(-\frac{1}{2} + \frac{1}{3}s_{w}^{2}\right),$$

(3.1)

where $V_{tb} \approx 0.999$, $V_{ts} \approx -0.040$ are the 3-3 and 3-2 elements of CKM matrix respectively, $G_F \approx 1.17 \times 10^{-5}$ GeV is the Fermi constant, $\alpha_{em} \approx 1/137$ is the electromagnetic fine-structure constant, $m_b \approx 4.18$ GeV and $m_s \approx 0.095$ GeV are respectively the bottom and the strange quark masses given in the $\overline{MS}$ scheme at a renormalization scale $\mu = 2$ GeV [75]. Notice here that we have ignored $m_b$ and $m_s$ in the formula for $C_9$ in eq. (3.1), since they are much smaller than $M_{Q'}$ and $m_{\chi_s}$ in our scenario. The preferred value of $\Delta C_{9}^{\mu\mu}$ is around $\Delta C_{9}^{\mu\mu} \sim -1$ from the global fit [78], where the $2\sigma$ range depends on other non-zero Wilson coefficients.

We also obtain lepton flavor universal Wilson coefficient $\Delta C_{9}(Z)$ from the diagram where $Z'$ is replaced to $Z$. Another lepton flavor universal Wilson coefficient $\Delta C_{10}(Z)$ associated with the corresponding operator $(\bar{s}\gamma_\mu P_L b)(\mu\gamma_\mu\ell)$ is also arisen from the $Z$ mediation. The formulas of them are given by

$$\Delta C_{9}(Z) \simeq \frac{g_{Z}g_{Z}^{2}}{4\pi^{2}m_{Z}^{2}m_{Z'}^{2}}C_{SM} \left(-\frac{1}{2} + 2s_{w}^{2}\right) \left(-\frac{1}{2} + \frac{1}{3}s_{w}^{2}\right),$$

(3.2)

$$\Delta C_{10}(Z) \simeq \frac{g_{Z}g_{Z}^{2}}{8\pi^{2}m_{Z}^{2}m_{Z'}^{2}}C_{SM} \left(-\frac{1}{2} + \frac{1}{3}s_{w}^{2}\right).$$

(3.3)

The $\Delta C_{10}$ contributes also to $B_{s}^{0} \to \mu^{+}\mu^{-}$ process. Here we consider the measurement of $B_{s}^{0} \to \mu^{+}\mu^{-}$ branching ratio [70, 71],

$$BR(B_{s}^{0} \to \mu^{+}\mu^{-})^{\exp} = (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9},$$

(3.4)

where the first and second uncertainties are statistical and systematic, respectively. We compare the experimental value with the theoretical one estimated by [72]

$$BR(B_{s}^{0} \to \mu^{+}\mu^{-})^{th} = |1 - 0.24\Delta C_{10}^{\mu\mu}|^{2}BR(B_{s}^{0} \to \mu^{+}\mu^{-})^{SM},$$

(3.5)

where $BR(B_{s}^{0} \to \mu^{+}\mu^{-})^{SM} = (3.65 \pm 0.23) \times 10^{-9}$ is the SM prediction [73]. $\Delta C_{10}^{\mu\mu}$ is restricted by the data of the process and $B \to K^{(*)}\ell^{+}\ell^{-}$. For example, the allowed range at $2\sigma$ is given by [78] including data of $B_{s}^{0} \to \mu^{+}\mu^{-}$ decay with assuming $\Delta C_{9}^{\mu\mu} = -\Delta C_{10}^{\mu\mu}$: $0.30 \leq \Delta C_{10}(Z) \leq 0.59$. In fact this range will be modified as we have lepton flavor universal $\Delta C_{9}(Z)$ and $\Delta C_{10}(Z)$ effects. Here we first impose wider range $0 \leq \Delta C_{10}(Z) \leq 0.80$

\footnote{We follow ref. [78] for the definitions of $\Delta B = 1$ operators relevant to $b \to s\ell^{+}\ell^{-}$.}
to satisfy measurement of $BR(B_s^0 \to \mu^+ \mu^-)$; we here impose our assumption $\Delta C_{10}(Z) > 0$ since it is preferred by data. We further discuss range of the Wilson coefficients when we carry out numerical analysis in section 4.

Furthermore the Wilson coefficient $C_7$, for $\bar{s}\sigma_{\mu\nu}bF^{\mu\nu}$ operator, is modified by $\Delta C_7$:

$$
\Delta C_7 = \frac{f_2 f_3}{2} \int [dX]^3 \frac{xy}{x(x-1)m_b^2 + xm_{\chi_q}^2 + (y+z)M_{Q'}^2} \left( \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \right)^{-1}, \tag{3.6}
$$

where $\int [dX]^3 \equiv \int_0^1 dx dy dz (1-x-y-z)$. The Wilson coefficient $C_7$ is required to be small; for example global analysis in 6-dim Wilson coefficient hypothesis indicates $[78]$:

$$
-0.04 \leq \Delta C_7 \leq 0.03. \tag{3.7}
$$

Note that Wilson coefficient $\Delta C_7'$ is also induced but it is suppressed by $m_s/m_b$ factor compared to $C_7$.

### 3.2 $M - \bar{M}$ mixing

The exotic vector-like quarks and the complex scalar DM $\chi$ induce the neutral meson ($M$)-antimeson ($\bar{M}$) mixings such as $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$, $B_s - \bar{B}_s$, and $D^0 - \bar{D}^0$ from the box type one-loop diagrams. We calculate relevant box diagrams to derive effective operators with corresponding coefficients in terms of our model parameters. Then these operators are used to calculate the mass splittings applying the formulas given in ref. [80]:

$$
\Delta m_K \approx \sum_{a,b=1}^3 |f_1 f_2|^2 G_{\text{box}}^K [m_{\chi_q}, M_{Q'}] \lesssim 3.48 \times 10^{-15} \, \text{[GeV]}, \tag{3.8}
$$

$$
\Delta m_{B_d} \approx \sum_{a,b=1}^3 |f_1 f_3|^2 G_{\text{box}}^{B_d} [m_{\chi_q}, M_{Q'}] \lesssim 3.36 \times 10^{-13} \, \text{[GeV]}, \tag{3.9}
$$

$$
\Delta m_{B_s} \approx \sum_{a,b=1}^3 |f_2 f_3|^2 G_{\text{box}}^{B_s} [m_{\chi_q}, M_{Q'}] \lesssim 1.17 \times 10^{-11} \, \text{[GeV]}, \tag{3.10}
$$

$$
\Delta m_D \approx \sum_{a,b=1}^3 |f_1 f_2|^2 G_{\text{box}}^{D} [m_{\chi_q}, M_{Q'}] \lesssim 6.25 \times 10^{-15} \, \text{[GeV]}, \tag{3.11}
$$

$$
G_{\text{box}}^M (m_1, m_2) = \frac{m_M f_M^2}{3(4\pi)^2} \int [dX]^3 \frac{x}{xm_1^2 + (y+z)m_2^2} \\
= \frac{m_M f_M^2}{6(4\pi)^2} \left[ m_1^4 - m_2^4 + 4m_1^2m_2^2 \ln \left( \frac{m_2}{m_1} \right) \right] \left( m_1 \neq m_2 \right), \tag{3.12}
$$

where relevant quarks $(q, q')$ are respectively $(d, s)$ for $K$, $(b, d)$ for $B_d$, $(b, s)$ for $B_s$, and $(u, c)$ for $D$, each of the last inequalities of the above equations represent the upper bound from the experimental values $[75]$, and $f_K \approx 0.156 \, \text{GeV}$, $f_{B_d(B_s)} \approx 0.191(0.230) \, \text{GeV}$, $f_D \approx 0.212 \, \text{GeV}$, $m_K \approx 0.498 \, \text{GeV}$, $m_{B_d(B_s)} \approx 5.280(5.367) \, \text{GeV}$, and $m_D \approx 1.865 \, \text{GeV}$ $[75, 76]$. 


3.3 Muon anomalous magnetic dipole moment

Recently, E989 Collaboration at Fermilab reported the new result on the muon \((g - 2)\) \cite{8}:

\[
a_\mu^{\text{FNAL}} = 116592040(54) \times 10^{-11},
\]

(3.13)

Combining it with the previous BNL result, the new result on the muon \((g - 2)\) shows a deviation from the SM prediction at 4.2 \(\sigma\) level:

\[
\Delta a_\mu^{\text{new}} = (25.1 \pm 5.9) \times 10^{-10},
\]

(3.14)

which may be a herald for new physics beyond the SM.

In our model, the new dominant contribution to the muon \((g - 2)\) arises from the loop diagrams involving the vector-like leptons and leptophilic DM \(\chi_l\). The dominant terms involve a product of \(g_\mu\) and \(h_\mu\), and its form is given by

\[
\Delta a_\mu^{\text{new}} = m_\mu^2 \left( \frac{4\pi}{2} s_c c_c g_\mu h_\mu \right) \left[ F(q_1, r_1) - F(q_2, r_2) \right] \left[ \frac{1}{M_1} - \frac{1}{M_2} \right],
\]

(3.15)

\[
F(q, r) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{1 - 2y}{1 - x + q^2 (x^2 - x) + r^2 x} \approx -1 + \frac{r^4 - 2r^2 \ln(r^2)}{2(-1 + r^2)^3} + \mathcal{O}(q^2),
\]

(3.16)

where \(q_i \equiv m_\mu / M_i (\ll 1)\) and \(r_i \equiv m_\chi / M_i\). And \(M_i (i = 1, 2)\) and \(s_c (c_c)\) are the mass eigenstates for the exotic singly-charged fermions and their mixings given in eq. (2.12), respectively.

3.4 Oblique parameters

The vector-like fermions and scalars contribute to oblique parameters through vacuum polarization diagram for electroweak gauge bosons. Here we consider \(S-\) and \(T\)-parameters which would constrain our parameter space. They are given by \cite{77}

\[
\alpha_{em} S = 4 e^2 \left( \sum_{\text{VLF}} \left[ \frac{d}{dq^2} \Pi_{33}^{\text{VLF}} - \frac{d}{dq^2} \Pi_{33Q}^{\text{VLF}} \right]_{q^2 = 0} + \left[ \frac{d}{dq^2} \Pi_{33}^{h, H} - \frac{d}{dq^2} \Pi_{33Q}^{h, H} \right]_{q^2 = 0} \right),
\]

\[
\alpha_{em} T = \frac{e^2}{3 s_W^2 c_W m_Z^2} \left( \sum_{\text{VLF}} \left[ \Pi_{33}^{\text{VLF}}(q^2) - \Pi_{33}^{\text{VLF}}(q^2) \right]_{q^2 = 0} + \left[ \Pi_{33}^{h, H}(q^2) - \Pi_{33}^{h, H}(q^2) \right]_{q^2 = 0} \right),
\]

(3.17)

(3.18)

where the superscript \(\text{VLF}\) distinguishes contributions from different combination of our vector-like fermions \(\{E_1, E_2, N, U, D\}\), and \(\Pi_{33,3Q,\pm}\) are obtained from vacuum polarization diagrams which are summarized in the appendix; here \(\Pi_{33, h, H, \text{VLF}}(33, 3Q, \pm)\) indicates beyond the standard model contribution. We impose the constraint on new physics contributions to the \(S-\) and \(T\)-parameters given by \cite{75}

\[
\Delta S \leq 0.00 \pm 0.07,
\]

(3.19)

\[
\Delta T \leq 0.05 \pm 0.06,
\]

(3.20)

where we fix \(U\)-parameter to be zero.

\textsuperscript{5}The contributions proportional to \(|g_\mu|^2\) or \(|h_\mu|^2\) cannot be dominant because of the chiral suppression. Thus, we neglect those terms for simplicity.
3.5 Dark matter

In our scenario, $U(1)_{\mu-\tau}$ is a good global symmetry of the model Lagrangian even after the $U(1)_{\mu-\tau}$ gauge symmetry breaking at renormalizable level, and the lightest $U(1)_{\mu-\tau}$ particle with spin-0 would make a good DM candidate in addition to the $\nu_\mu$ and $\nu_\tau$. Then the lighter field of two complex scalars $\chi_q$ (hadro-philic or leptophobic) and $\chi_\ell$ (leptophilic) can be DM candidate.

In fact DM physics in our model is very rich. For generic choices of $q_x$ and $q_\varphi$, $\chi_q$ and $\chi_\ell$ would be separately stable, and we are with two-component DM models. However this would be no longer true for special choices of $q_\varphi$ and $q_x$. For example, the following operators would be gauge invariant while satisfying eqs. (2.1) and (2.2):

\begin{equation}
\chi_\ell \chi_q \varphi^2, \text{ if } q_\varphi = 1 - \frac{3q_x}{2}, \quad (3.21)
\end{equation}

\begin{equation}
\chi_\ell \chi_q \varphi^2, \text{ if } q_\varphi = \frac{1}{2} \left( 1 + \frac{q_x}{2} \right). \quad (3.22)
\end{equation}

Then $\chi_q$ and $\chi_\ell$ will mix with each other, and we would end up with a single component DM model. In fact the mixing can occur without $\varphi$: $\chi_\ell \chi_q$ is allowed if $q_x = 2/3$. Another interesting possibility is an operator $\chi_\ell^2 \chi_q \varphi^4$ that is allowed for $q_\varphi = 2$. Then after $U(1)_{\mu-\tau}$ symmetry breaking, the effective $\chi_\ell^2 \chi_q$ operator is induced, so that leptophobic DM $\chi_q$ will decay into a pair of leptophilic DM $\chi_\ell$.

In this paper, we shall ignore all these interesting possibilities, and focus on the leptophilic $\chi_\ell$ as a DM candidate\footnote{See ref. [63] for the case where $\chi_q$ is DM.} that is simply denoted by $\chi$ hereafter. Then, it annihilates into the SM leptons via $\chi \chi^* \to \ell^+ \ell^-$ with $E_{1,2}$ propagator, and $\chi \chi^* \to Z' \to \{\mu^+ \mu^-, \tau^+ \tau^-, \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau\}$, indicating our DM is leptophilic. Here, the Yukawa contribution gives an $s$-wave dominant annihilation cross section, while the $Z'$ mediated cross section is the $p$-wave dominant. In fact there is also an annihilation mode $\chi \chi^* \to Z'Z'$ if kinematically allowed. In our analysis we shall not consider this mode assuming $m_\chi < m_{Z'}$ to simplify our analysis. The relic density of DM is given by

\begin{equation}
\Omega h^2 \approx \frac{1.07 \times 10^9}{\sqrt{g^*(x_f)} M_{Pl} J(x_f)[GeV]}, \quad (3.23)
\end{equation}

where $g^*(x_f \approx 25) \approx 100$, $M_{Pl} \approx 1.22 \times 10^{19}$, and $J(x_f) := \int_{x_f}^{\infty} dx \frac{(\sigma v_{\text{rel}})}{x}$ is found as

\begin{equation}
J(x_f) = \int_{x_f}^{\infty} dx \left[ \frac{\int_{4m_\chi^2}^{\infty} ds \sqrt{s - 4m_\chi^2} (\sigma v_{\text{rel}}) K_1 \left( \frac{\sqrt{s}}{m_\chi} \right)}{16m_\chi^2 x K_2(x)^2} \right],
\end{equation}

\begin{equation}
(\sigma v_{\text{rel}}) = (\sigma v_{\text{rel}})_{\mu\bar{\mu}} + (\sigma v_{\text{rel}})_{\tau\tau} + (\sigma v_{\text{rel}})_{\nu_\mu \bar{\nu}_\mu + \nu_\tau \bar{\nu}_\tau}, \quad (3.24)
\end{equation}

\begin{equation}
(\sigma v_{\text{rel}})_{\mu\bar{\mu}} \approx \frac{g^4(q_x - 2)^2 s(s - 4m_\chi^2)}{24\pi(s - m_{Z'}^2)^2}
+ \frac{(s_c e_c h_{\mu} g_\mu)^2}{16\pi} \int_0^{\pi} d\theta \sin \theta \left( \frac{M_1}{t - M_1} - \frac{M_2}{t - M_2} \right)^2 s, \quad (3.25)
\end{equation}
Here $s$ and $t$ are Mandelstam variables, and $K_{1,2}$ are the modified Bessel functions of the second kind of order 1 and 2, respectively. We write second term of eq. (3.25) in terms of integration over the polar angle $\theta$, since analytic form after integration is complicated as $t$ depends on $\theta$. Note that integration over $\theta$ can be carried out easily, since corresponding amplitudes only depend on $s$. In addition we ignore the interference term between $Z'$ and $E_{1,2}$ mediated diagrams since it is suppressed by $m_\mu/m_\chi$. In our numerical analysis below, we use a relaxed experimental range for the relic density: $0.11 \leq \Omega h^2 \leq 0.13$ \cite{81}.

**Direct detection.** We have a spin independent scattering cross section that is constrained by several experiments such as XENON1T \cite{82}. The dominant contribution is Higgs portal via interactions in the scalar potential. However we simply assume the Higgs portal coupling $c_{\chi h}$ for $\chi - \chi - h_{\text{SM}}$ to be tiny enough; $c_{\chi h} \lesssim 10^{-3}$ \cite{83}, in order to evade the direct detection via Higgs exchanges.

4 Numerical analysis

In this section, we perform the numerical analysis. Relevant free parameters are scanned over to search for parameter points satisfying phenomenological requirements discussed in the previous section. The ranges of the other input parameters are set to be as follows:

\[
g' \in [0.001, 0.1], \quad |q_x| \in [0.1, \sqrt{4\pi/g'}], \quad \{f_{1,2}, g_{e,\mu}, h_{e,\mu}\} \in [10^{-2}, 1], \quad m_{Z'} \in [m_\chi, 150] \text{ [GeV]}, \]
\[
m_\chi \in [1, 100] \text{ [GeV]}, \quad m_{\chi_q} \in [1.2m_\chi, 10^4] \text{ [GeV]}, \quad \{M_{E,L'}\} \in [10^2, 10^4] \text{ [GeV]}, \quad m_E \in [10, 100] \text{ [GeV]}, \quad M_{Q'} \in [10^3, 10^4] \text{ [GeV]},
\]

where we take perturbation limit $|q_x g'| \leq \sqrt{4\pi}$, and choose $M_1 < M_2$, and take $m_\chi < 1.2 \times (M_1, m_{\chi_q})$ for simplicity so that we can evade contributions from co-annihilation processes. Here we require $1000\text{ GeV} \leq M_{Q'}$ to avoid constraints from direct search of vector-like quarks at the LHC. For vector-like lepton, constraint $m_{VLL} \geq 700\text{ GeV}$ is given for vector-like lepton decaying into the SM lepton with $W/Z$ boson \cite{85}. But here we just consider LEP limit of $m_{VLL} \gtrsim 100\text{ GeV}$ \cite{84} since our vector-like lepton decays into lepton and $\chi$. In addition we impose the constraint of $550\text{ GeV} \leq m_{Z'}/g'$ which arises from the neutrino trident production \cite{41}. Here we take the scalar mixing $\sin \alpha$ to be sufficiently small so that the scalar boson contribution to oblique parameters ($\propto \sin^2 \alpha$) can be neglected. We find that new scalar contribution to $S$ and $T$ parameter is sufficiently small for $\sin \alpha < 0.1$. Moreover, taking $m_E < M_{E,L'}$, we find that constraint from $S$- and $T$-parameters can be easily avoided since $S$ and $T$ are found to be less than $\sim 0.05$ and $0.01$, respectively.

Firstly, we note that products of couplings $|f_i f_j|$s are constrained from meson-antimeson mixings, $\Delta m_K$, $\Delta m_{B_s}$, $\Delta m_{B_d}$ and $\Delta m_{D}$. In figure 2, we show upper limit of $|f_i f_j|$ as functions of $M_{Q'}$ fixing $m_{\chi_q} = 500\text{ GeV}$ as a reference value. We find that
maximal values have relation of $|f_1 f_2|_{\text{max}} < |f_1 f_3|_{\text{max}} < |f_2 f_3|_{\text{max}}$ where $|f_2 f_3|$ can be $O(1)$ for heavy $Q'$ region. Also parameters $\{m_\chi, M_L', m_E, g', q_x\}$ are constrained to reproduce the observed DM relic density. Then allowed values of $\{f_{2,3}, g', q_x\}$ are used to calculate Wilson coefficients $\Delta C_{9,10}$.

In figure 3, we show $C_{9,10}$ values obtained from parameter sets satisfying the constraints of neutral meson-antimeson mixings, correct relic density $0.11 \leq \Omega h^2 \leq 0.13$ and $0 < \Delta C_{9,10}(Z) < 0.79$; here we show region with $\Delta C_{9,10}^\mu(\mu') < 0$ and $|\Delta C_{9,10}(Z)|$ where region with $\Delta C_{9}^\mu(\mu') > 0$ and $\Delta C_{9,10}$ $\leq (>)0$ similarly appear. We find that $|\Delta C_{9}(Z)|$ tends to be small due to suppression factor of $2(\alpha_m^2 - 1/2)$. We also find that $\Delta C_7$ is tiny in our allowed region as $|C_7| < 0.001$ and the corresponding plot is omitted. Thus we have sizable $\Delta C_{9}^{\mu,\mu}(Z')$ and $\Delta C_{10}(Z)$ while $\Delta C_9(Z)$ and $\Delta C_7$ are small. Then we adopt range of Wilson coefficients from global analysis in ref. [78] for $\{\Delta C_{9}^{\mu,\mu}, \Delta C_{10}\}$ plane where $\Delta C_{10}$ is flavor universal one: here $2\sigma$ ranges are $\Delta C_{9}^{\mu,\mu} \in [-1.27, -0.69]$ and $\Delta C_{10} \in [-0.01, 0.56]$ (see also ref. [79]). The $2\sigma$ interval corresponds to region between vertical dashed lines and below horizontal dashed line respectively, and the colored regions satisfy both $2\sigma$ range of $\Delta C_{9}^{\mu,\mu}$ and $\Delta C_{10}$. Note that non-zero $\Delta C_{10}$ helps to fit $B_s \rightarrow \mu\mu$ data.

In figure 4, we show our values of $\{M_1, \Delta a_\mu\}$ for the parameter sets the same as figure 3. We can obtain $\Delta a_\mu^{\text{new}} = (25.1 \pm 5.9) \times 10^{-10}$ at $1\sigma$ $2\sigma$) for $M_1 \lesssim 2000(2700)$ GeV. Here, the dashed line denotes the allowed region within $2\sigma$.

In figure 5, we show the allowed parameter region for DM mass $m_\chi$ and $m_{Z'}$. The relic density is explained by $\chi\chi \rightarrow Z' \rightarrow \{\mu^+\mu^-, \tau^+\tau^-, \nu_\mu, \bar{\nu}_\mu, \tau\}$ process for the points around $m_{Z'} \sim 2m_\chi$ due to enhancement of the annihilation cross section. On the other hand $\chi\chi \rightarrow \mu^+\mu^-$ via Yukawa interaction explains the relic density for the points which do not give resonant enhancement of the $Z'$ exchange process.

Finally, in the left plot of figure 6, we show parameter points in the $\{m_{Z'}, g'\}$ plane satisfying $0.69 \leq -\Delta C_9^{\mu,\mu} \leq 1.27$, $0.00 \leq \Delta C_{10} \leq 0.56$, $\Delta a_\mu^{\text{new}} = (25.1 \pm 5.9) \times 10^{-10}$ and other phenomenological constraints. In this region the most important one is the LHC

---

$^7$See the scenario 10 in table 4 of the reference.
Figure 3. Left: scattering plot on \{\Delta C^\mu\mu_9(Z'), \Delta C_{10}(Z)\} plain where each point satisfies phenomenological constraints except for \Delta C_{9,10}. Right the same plot on \{\Delta C^\mu\mu_9(Z'), |\Delta C_9(Z)|\} plain.

Figure 4. $M_1$ versus $\Delta a_\mu$ for the parameter sets same as the previous plots.

Figure 5. $m_{\chi^\prime}$ versus $m_{Z'}$ in unit of GeV for the parameter sets same as the previous plots. The dashed line corresponds to $m_\chi = m_{Z'}$ region.
constraint from \( pp \to \mu^+\mu^-Z'(\to \mu^+\mu^-) \) process \[86\] . We find that \( m_{Z'} \lesssim m_Z \) region is preferred to explain experimental anomalies, whereas some parameter region is excluded by the LHC constraint for \( m_{Z'} < 60 \text{ GeV} \). The gauge coupling should be small for small \( m_{Z'} \) region to avoid the LHC constraint, and large \( |q_x| \) is required to explain \( B \)-anomalies in the region. In the right plot in figure 6 we show correlation between \( g' \) and \( |q_x| \) for the same parameter sets as the left plot; both negative and positive values of \( q_x \) appear as same amount and we only show absolute values. We then find \( g'|q_x| \sim \mathcal{O}(1) \) is required to obtain sizable \( \Delta C_9^{\mu\mu} \). Thus the region with \( m_{Z'} > 60 \text{ GeV} \) would be more natural to explain \( b \to s\mu^+\mu^- \) anomalies since required \( |q_x| \) is not too large. The remaining allowed region would be further tested in future at the LHC from the search for \( pp \to \mu^+\mu^-Z'(\to \mu^+\mu^-) \) and \( pp \to \nu_{\mu,\tau}\bar{\nu}_{\mu,\tau}Z'(\to \mu\bar{\mu}) \): namely 4 muons or dimuon + missing transverse momentum.

5 Summary and conclusions

In this paper, we have proposed a \( U(1)_{\mu-\tau} \) gauged model to explain recent anomalies; the muon anomalous magnetic moment and \( b \to s\mu^+\mu^- \) at the same time, satisfying several constraints. Also, we have considered a complex scalar boson DM candidate (\( \chi_l \)) that mainly couples to leptons. In our model \( b \to s\mu^+\mu^- \) anomalies can be explained by loop induced interactions among quarks and \( Z' \). We also estimated one-loop diagrams associated with vector-like quark and \( Z(\gamma) \) vertices that contribute to Wilson coefficients \( C_9(10) \) and \( C_7 \) to check consistency of our model. We then find that \( C_9(10) \) from \( Z \) mediation and \( C_7 \) induced by vector-like quark loop can be small, and we can explain \( b \to s\mu^+\mu^- \) anomalies consistently. In our numerical analysis, we have found allowed regions to explain anomalies while satisfying all the relevant phenomenological constraints. This model would be test soon in future measurement such as \( pp \to \mu^+\mu^-Z'(\to \mu^+\mu^-) \) signal search at the LHC. We also have the processes \( pp \to \mu^+\mu^-Z'(\to \nu_{\mu,\tau}\bar{\nu}_{\mu,\tau}) \) and \( pp \to \nu_{\mu,\tau}\bar{\nu}_{\mu,\tau}Z'(\to \mu\bar{\mu}) \) giving

### Figure 6

Left: parameter points on \((m_{Z'}, g')\) plane satisfying \( 0.69 \leq -\Delta C_9^{\mu\mu} \leq 1.27, \ 0.00 \leq \Delta C_{10} \leq 0.56, \Delta a_\mu^{\text{new}} = (25.1 \pm 5.9) \times 10^{-10} \) and other phenomenological constraints. Shaded region is excluded by the LHC constraint from \( pp \to \mu^+\mu^-Z'(\to \mu^+\mu^-) \) process \[86\]. Right: the same points on \((g', |q_x|)\) plane.
signals of $\mu^+\mu^-$ with missing transverse momentum that is also searched for at the LHC. In addition our model can be tested by searching for vector-like fermions at the LHC where vector-like quarks and leptons can be pair produced providing “dijet + missing transverse momentum” and “dilepton + missing transverse momentum” respectively.

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A Vector-like fermion contributions to electroweak vacuum polarization diagrams

We calculate electroweak vacuum polarization diagrams with vector-like fermions and scalar bosons, and summarize analytic form of their contributions to estimate oblique parameters. We can write the vacuum polarizations for $Z$ and $W$ contributing to oblique parameters such that

$$\Pi_{\mu\nu}^Z = g_{\mu\nu} \frac{e^2}{c^2_W s^2_W} (\Pi_{33}(q^2) - 2 s_W^2 \Pi_{3Q}(q^2) - s_W^4 \Pi_{QQ}(q^2)),$$

$$\Pi_{\mu\nu}^W = g_{\mu\nu} \frac{e^2}{s^2_W} \Pi_\pm(q^2),$$

where $q$ is four momentum carried by gauge bosons.

Non-zero contributions to $\Pi_{33}(q^2)$ are summarized as follows;

$$\Pi_{33}^{E_1E_1}(q^2) = -\frac{s^4}{16 \pi^2} F(q^2, M^2_{E_1}, M^2_{E_1}),$$

$$\Pi_{33}^{E_2E_2}(q^2) = -\frac{c^4}{16 \pi^2} F(q^2, M^2_{E_2}, M^2_{E_2}),$$

$$\Pi_{33}^{E_1E_2}(q^2) = -\frac{s^2 c^2}{8 \pi^2} F(q^2, M^2_{E_1}, M^2_{E_2}),$$

$$\Pi_{33}^{N'N'}(q^2) = -\frac{1}{16 \pi^2} F(q^2, M^2_{N'}, M^2_{N'}),$$

$$\Pi_{33}^{U'U'}(q^2) = -\frac{1}{16 \pi^2} F(q^2, M^2_{U'}, M^2_{U'}),$$

$$\Pi_{33}^{D'D'}(q^2) = -\frac{1}{16 \pi^2} F(q^2, M^2_{D'}, M^2_{D'}).$$
where the superscripts in the left sides indicate particles inside vacuum polarization diagrams, and \( F(q^2, m^2, m'^2) \) is the loop function given by

\[
F(q^2, m^2, m'^2) = \int_0^1 dx dy \delta(1 - x - y) \left( \frac{1}{\epsilon_{\text{MS}}} - \ln \left( \frac{\Delta}{\mu^2} \right) \right) 
\times (2x(1 - x)q^2 - xm'^2 - ym'^2 + mm'), \tag{A.9}
\]

\[
\Delta = -x(1 - x)q^2 + xm'^2 + ym'^2, \quad \frac{1}{\epsilon_{\text{MS}}} \equiv \frac{2}{\epsilon} - \gamma - \ln(4\pi), \tag{A.10}
\]

where \( \mu \) is auxiliary parameter having mass dimension. Dependence on \( \mu \) is canceled when we calculate \( S, T \)-parameter from eqs. (3.16) and (3.15) in section 3.3.

Similarly we obtain non-zero contributions to \( \Pi_{3Q,QQ,\pm} \):

\[
\Pi_{3Q}^{E_1E_1}(q^2) = -\frac{\gamma^2}{8\pi^2} F(q^2, M_1^2, M_1^2), \tag{A.11}
\]

\[
\Pi_{3Q}^{E_2E_2}(q^2) = -\frac{\gamma^2}{8\pi^2} F(q^2, M_2^2, M_2^2), \tag{A.12}
\]

\[
\Pi_{3Q}^{U\prime U\prime}(q^2) = -\frac{1}{12\pi^2} F(q^2, M_{U'}^2, M_{U'}^2), \tag{A.13}
\]

\[
\Pi_{3Q}^{D' D'}(q^2) = -\frac{1}{24\pi^2} F(q^2, M_{D'}^2, M_{D'}^2), \tag{A.14}
\]

\[
\Pi_{3Q}^{E_1E_1}(q^2) = -\frac{1}{4\pi^2} F(q^2, M_1^2, M_1^2), \tag{A.15}
\]

\[
\Pi_{3Q}^{E_2E_2}(q^2) = -\frac{1}{4\pi^2} F(q^2, M_2^2, M_2^2), \tag{A.16}
\]

\[
\Pi_{3Q}^{U\prime U\prime}(q^2) = -\frac{1}{9\pi^2} F(q^2, M_{U'}^2, M_{U'}^2), \tag{A.17}
\]

\[
\Pi_{3Q}^{D' D'}(q^2) = -\frac{1}{36\pi^2} F(q^2, M_{D'}^2, M_{D'}^2), \tag{A.18}
\]

\[
\Pi_{\pm}^{N' N'}(q^2) = -\frac{\gamma^2}{8\pi^2} F(q^2, M_N^2, M_N^2), \tag{A.19}
\]

\[
\Pi_{\pm}^{E_1' E_2'}(q^2) = -\frac{\gamma^2}{8\pi^2} F(q^2, M_{E'}^2, M_{E'}^2), \tag{A.20}
\]

\[
\Pi_{\pm}^{U' D'}(q^2) = -\frac{1}{8\pi^2} F(q^2, M_{E'}^2, M_{E'}^2). \tag{A.21}
\]

Finally non-zero contributions from new scalar particles beyond the SM are given by

\[
\Pi_{33}^{h,H} = -\frac{\sin^2 \alpha}{32\pi^2} \int_0^1 dx \left[ 2m_2^2 \ln \left( \frac{\Delta_h}{\Delta_H} \right) + \Delta_h \left( \frac{1}{\epsilon_{\text{MS}}} - \ln \left( \frac{\Delta_h}{\mu^2} \right) \right) - \Delta_H \left( \frac{1}{\epsilon_{\text{MS}}} - \ln \left( \frac{\Delta_H}{\mu^2} \right) \right) \right],
\]

\[
\Pi_{\pm}^{h,H} = -\frac{\sin^2 \alpha}{32\pi^2} \int_0^1 dx \left[ 2m_1^2 \ln \left( \frac{\Delta_h}{\Delta_H} \right) + \Delta_h \left( \frac{1}{\epsilon_{\text{MS}}} - \ln \left( \frac{\Delta_h}{\mu^2} \right) \right) - \Delta_H \left( \frac{1}{\epsilon_{\text{MS}}} - \ln \left( \frac{\Delta_H}{\mu^2} \right) \right) \right],
\]

\[
\Delta_h = x(1 - x)q^2 + xm_1^2(h) + (1 - x)m_2^2,
\]

\[
\Delta_H = x(1 - x)q^2 + xm_1^2(h) + (1 - x)m_2^2. \tag{A.22}
\]

This completes the new contributions to the vacuum polarization tensors of electroweak gauge bosons in the \( U(1)_{\mu - \tau} \) model considered in this paper. Then the \( S \)- and \( T \)-parameters
can be obtained from eqs. (3.16) and (3.15) in section 3.3. divergent part proportional to \( \epsilon_{\text{SM}} \) will be canceled when we calculate oblique parameters by those equations.

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