Improved strategy for optimal control of Civil Engineering structures

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Abstract. Inside the Industrial Revolution 4.0, the concepts as Smart Cities and Smart Society are important parts in protection of humans against natural disasters, as strong earthquakes. Older or newer protection strategies of Civil Engineering structures (bridges, buildings) must be updated with elements of the actual technological revolution as sensors, drones, robots, nanotechnologies, AI, big data, cloud computing and so on. This paper is focused on a optimal structural active control of buildings started by many researchers during the last decades. Now, the target is to improve an optimal strategy developed by authors by proposing a more efficient use of computation. The success of this strategy leads to a faster response of computers that direct the control devices. This provides the ground for a larger range of control devices based on a larger number of sensors and, therefore, a better energy use together with a more precise anti-seismic structural protection.

1. Introduction
New context of revolutionary changes, placed under Industrial Revolution 4.0 slogan, leads to concepts as Smart Cities and Smart Society. They play important roles in protection of humans against natural disasters as strong earthquakes. Therefore, older or newer protection strategies of Civil Engineering structures (bridges, buildings) must be updated with elements of the actual technological revolution as sensors, drones, robots, nanotechnologies, AI, big data, cloud computing and so on. Some early proposals were issued in [1, 2, 3] and later in [4]. Figure 1 shows two device proposals, from [4], that could lead to implementing Structural Robotics.

![Figure 1. Structural Active Hinge, SAH – concept details [4].](image)

During a previous work, [1], a classical optimal control approach has been used and shown as viable using 3D control systems with multiple actuators for strong earthquake effects mitigation. At that time, a pioneering strategy based on energy approach has been adopted. Due to that strategy, a methodology for limiting to only one scalar the parameters for control objective index had been issued [2, 5].
Latter, the methodology was applied to incorporate the practical problems as time delay, control noise and the introduction of estimators in the control global system. Then the methodology had been shown to be effective for decentralized disjoint and overlapping [6] control systems in Civil Engineering structures. The decentralized overlapping control model from [6] is shown in Figure 2. Other parametric studies had been done for better understanding of control devices work [7].

In this paper, an improvement of the above methodology is proposed. Starting from the previously defined matrices for the optimal objective index, a new procedure is proposed. It mainly consists in using a smaller dimension for the matrices involved in Riccati equations to the goal of a faster control computation chain.

2. Use of classical optimal control approach in Civil Engineering

In Structural Dynamics, the linear, \( n \) dimensional, second order differential equation with active control writes [1, 2]

\[
M_s \dddot{z} + C_s \dot{z} + K_s z = f - u
\]  

(1)

where \( M_s \) is the \( n \times n \) mass matrix, \( C_s \) is the \( n \times n \) damping matrix \( K_s \) is the \( n \times n \) stiffness matrix, \( z \) is the \( n \times 1 \) displacement vector, \( f \) is the \( n \times 1 \) external forces vector, and \( u \) is the \( n \times 1 \) control forces vector. \( z, f, \) and \( u \) are all time dependent vectors.

In the case of unidirectional earthquake action, the vector of external actions might be written as

\[
f = h \dddot{x}_g
\]  

(2)

where \( \dddot{x}_g \) is the time dependent seismic acceleration and \( h \) is the \( n \times 1 \) distribution of seismic action vector.

Also, the vector of control forces is detailed as

\[
u = L_i u_i
\]  

(3)

with \( L_i \) being a \( n \times m \) matrix showing the distribution of active forces on structural degrees of freedom, \( u_i \) - the \( m \times 1 \) vector of active forces, and \( m \) is the number of active forces.

Denoting

\[
x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}
\]  

(4)

the classical canonical full states equation used in mitigation of earthquakes effects on Civil Engineering structures is

\[
\begin{align*}
\dot{x} &= Ax + Bu - u \\
y &= Cx + Du
\end{align*}
\]  

(5)

where: \( x \) is the time dependent \( 2n \) dimensional vector of states, \( y \) is the time dependent \( 2n \) dimensional vector of outputs. Matrices: \( A \) \((2n \times 2n)\), \( B \) \((2n \times n)\), \( C \) \((p \times 2n)\) and \( D \) \((p \times 2n)\) are the canonical equation matrices. \( p \) is the number of available measurements.

For \( A \) and \( B \), the next is available

\[
A = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M_i^{-1} \end{bmatrix}
\]  

(6)
and the matrices $C$ and $D$ are defined depending on the practical measurement system the user is manipulating. In what follows the ideal case of full state measurement is employed in order of simplifying the content of presentation.

The linear dependency on states control low is proposed as

$$u_t = Kx$$

where matrix $K$ is the $m \times 2n$ control matrix. Therefore, (3) becomes

$$u = L_kx$$

Using equations (8) and (2), the first equation of the system in (5) becomes

$$\ddot{x} = (A - BL_k)x + Bh$$

or, in another form, denoting $A_e = A - BL_k$ and using again equation (2)

$$\ddot{x} = A_ex + Bf$$

The linear quadratic control theory is imposing an optimal index to be minimized in the form

$$J = \frac{1}{2} \int_0^t \left[ x'Qx + u'Ru \right] dt$$

where $Q$ and $R$ are $2n \times 2n$ and $m \times m$ parametric weighting matrices of control showing the desire of design to minimize displacements, velocities and accelerations together with a low actuators’ energy consumption.

In [2], based on reducing the structural system energy, the weighting matrix $Q$ was shown as

$$Q = \begin{bmatrix} K_S & 0 \\ 0 & M_S \end{bmatrix}$$

and the weighting matrix $R$ as

$$R = \text{diag} \{ r_1, \ldots, r_m \}$$

or, simpler

$$R = rI$$

with $r$ being a scalar parameter to be obtained by trial and error and $I$ having $m \times m$ dimension. That is leading to a very easy way to establish the control parameters.

The first term under the integral in equation (10) can be rewritten using equation (12)

$$x'Qx = \left\{ z' \quad \dot{z} \right\} \begin{bmatrix} K_S & 0 \\ 0 & M_S \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} = z'K_Sz + \dot{z}'M_S\dot{z}$$

and equation (15) is a sum of terms of energies (potential and kinetic). Therefore, the proposed $Q$ matrix in equation (12) is justified and the optimized index in equation (11) is targeting a minimization of the energies in structural response.

Under the above circumstances, the solution of the optimal problem is based on solving the $2n \times 2n$ dimensional Riccati equation with constant matrices [5].
\[ PA - PBL_iR^{-1}L_i'B'P + A'P + Q = 0 \]  
(16)

where \( P \) is the unknown \( 2n \times 2n \) matrix.

Then, knowing \( P \) matrix, the control matrix is computed

\[ K = R^{-1}L_i'B'P \]  
(17)

Further, the above methodology was applied to incorporate the practical problems as time delay, control noise and the introduction of estimators in the control global system. Then the methodology had been shown to be effective for decentralized disjoint and overlapping [6] control systems in Civil Engineering structures.

**Figure 2.** Bridge model and its decomposition structure for overlapping decentralized active control of a cable stayed bridge [6].

3. **Proposed improvement of the strategy**

The now proposed improvement is based on the observation that \( Q \) matrix in equation (12) is containing two zero \( n \times n \) matrices and suggesting that some simplifications could be applied.

A first step is proposing a partitioning of \( P \) matrix into 4, \( n \times n \) matrices

\[
\begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\]  
(18)

and, furthermore, if imposing \( P \) matrix as a symmetrical one

\[
P_{12} = P_{21}
\]  
(19)

Detailing equation (17) based on equation (18), the next is obtain

\[
K = R^{-1}L_i'B'P = R^{-1}\begin{bmatrix} 0 & L_i'M_i^{-1} \\ L_i'M_i^{-1} & S \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\
P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} R^{-1}L_i'M_i^{-1}P_{21} & R^{-1}L_i'M_i^{-1}P_{22} \end{bmatrix}
\]  
(20)
which implies that only $P_{21}$ and $P_{22}$ \(n \times n\) matrices are really needed for calculation of the $m \times 2n$ control matrix, $K$.

For the present proposal, other helpful notations are

$$Q = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$$  \hspace{1cm} \text{(21)}$$

$$A = \begin{bmatrix} 0 & I \\ -M_{21}^T K_{21} & -M_{22}^T C_{21} \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix}, \quad B_{L1} = \begin{bmatrix} 0 \\ M_{21}^T L_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$  \hspace{1cm} \text{(22)}$$

With all notations above, shown by equations (18), (21) and (22), the $2n \times 2n$ Riccati equation (16) becomes

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} R^{-1} \begin{bmatrix} 0 & B_2 \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} 0 & A_{12} \\ I & A_{22} \end{bmatrix} \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} R^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} \text{(23)}$$

further developed in

$$\begin{bmatrix} P_{12} A_{21} & P_{11} + P_{12} A_{22} \\ P_{22} A_{21} & P_{21} + P_{22} A_{22} \end{bmatrix} - \begin{bmatrix} P_{12} B_2 \\ P_{22} B_2 \end{bmatrix} R^{-1} \begin{bmatrix} B_{21} P_{21} & B_{22} P_{22} \end{bmatrix} + \begin{bmatrix} 0 & A_{12} \\ I & A_{22} \end{bmatrix} \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} R^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} \text{(24)}$$

and

$$\begin{bmatrix} P_{12} A_{21} & P_{11} + P_{12} A_{22} \\ P_{22} A_{21} & P_{21} + P_{22} A_{22} \end{bmatrix} - \begin{bmatrix} P_{12} B_2 R^{-1} B_{21} P_{21} & P_{12} B_2 R^{-1} B_{22} P_{22} \\ P_{22} B_2 R^{-1} B_{21} P_{21} & P_{22} B_2 R^{-1} B_{22} P_{22} \end{bmatrix} + \begin{bmatrix} 0 & A_{12} \\ I & A_{22} \end{bmatrix} \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} R^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$  \hspace{1cm} \text{(25)}$$

Finally, the next four, $n \times n$ dimensional Riccati equations, result as follows

- left-up equation
  \[ P_{12} A_{21} - P_{12} B_2 R^{-1} B_{21} P_{21} + A_{12} P_{21} + Q_1 = 0 \]  \hspace{1cm} \text{(26)}$$

- right-up equation
  \[ P_{11} + P_{12} A_{22} - P_{12} B_2 R^{-1} B_{22} P_{22} + A_{12} P_{22} = 0 \]  \hspace{1cm} \text{(27)}$$

- left-down equation
  \[ P_{22} A_{21} - P_{22} B_2 R^{-1} B_{21} P_{21} + P_{11} + A_{21} P_{21} = 0 \]  \hspace{1cm} \text{(28)}$$

- right-down equation
  \[ P_{21} + P_{22} A_{22} - P_{22} B_2 R^{-1} B_{22} P_{22} + P_{12} + A_{22} P_{22} + Q_2 = 0 \]  \hspace{1cm} \text{(29)}$$

From observing equations (26-29), the next order of solving is proposed
• solve equation (26) for $P_{21}$ taking into account that $P_{12} = P_{21}$

• solve equation (29) for $P_{22}$, replacing $P_{12}$ and $P_{21}$ from above.

This way, the solution of the $2n \times 2n$ dimensional Riccati equation with constant matrices in equation (16) is replaced with solving two $n \times n$ dimensional Riccati equations with constant matrices shown in equations (26) and (29).

This should lead to faster control strategies. Further numerical applications, in views for future work, are needed to prove the competitiveness of the above procedure.

4. Conclusions

In the context of smart cities, new strategies for protection of humans and structures against earthquakes are required, based on new technologies and devices.

The proposed improvement is based on reducing the dimension of the problem involved by obtaining the control matrix. The equation used in obtaining the control matrix of an older strategy in linear quadratic linear control is replaced with two half size similar equation.

This should lead to faster control strategies. However, further numerical applications are needed to prove the competitiveness of the above procedure. This is the goal of future work.

5. References

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