Q-factor of microwave resonators: calibrated vs. uncalibrated measurements

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Abstract.
In this article we present a study about using VNA calibrated or uncalibrated measurements for the purpose of measuring the $Q$-factor of a dielectric loaded microwave resonator. We evaluate the uncertainties in $Q$ involved in both methods. We use a complete and robust transmission resonant model for the fit and parameter extraction. The $Q$ values obtained from uncalibrated and calibrated data were compared by taking into account the S-parameter measurement uncertainty.

1. Introduction
A continuously growing interest in the devices operating at microwaves forces the development of measurement techniques to satisfy a wide range of applications [1, 2]. One of the most powerful techniques makes use of resonators, mainly due to their high sensitivity [3, 4].

In this paper, we concentrate our attention on the dielectric resonator (DR) technique, widely used, e.g., for the material characterization and temperature measurements [5, 6]. A wide range of low-cost Vector Network Analyzers (VNA) could make resonant measurements techniques more accessible also for non-laboratory application even for amateur electronics [7, 8].

VNAs are peerless measuring devices due to their high accuracy ($\pm 0.01$ dB). These accuracy levels are achievable thanks to sophisticate calibrations which allows to compensate the errors introduced by the VNA itself and by the whole transmission line used to connect the Device Under Test (DUT) to VNA. A particular situation occurs when part of the microwave line could not be calibrated for certain reasons. For example, in cryogenic measurements a part of the line remains uncalibrated because of both the absence of the standards operating at low-temperatures and the practical impossibility of calibrating the transmission line at every possible thermalization condition [9, 10]. Here we study the effect of using uncalibrated data in deriving the main resonant parameter, i.e. the resonator quality factor $Q$.

2. Experimental conditions
We estimate and compare uncertainties in $Q$ as a parameter derived from the fit of the calibrated and uncalibrated resonant curves. $Q$ is the fundamental experimental parameter for the determination of the surface resistance $Rs$ of conducting samples by means of a resonator.
method, following the relation [11]:

\[ \frac{1}{Q} = \frac{R_s}{G_s} + \text{background} \quad (1) \]

where \( G_s \) is a constant geometrical factor and “background” represents the resonator contribution to \( Q \), typically removed by measuring the resonator response without the sample. Therefore the main contributions to the uncertainty of \( R_s \) come from the uncertainties \( u(Q) \) and \( u(G_s) \) of \( Q \) and \( G_s \) respectively. Since \( u(G_s) \) originates from the simulation, one has \( u(G_s) < 1\% \) in the worst case.

The estimation of the effect of fitting the uncalibrated resonant curve on the values of \( Q \) was performed using an ad-hoc DR cell.

An open DR of the Hakki–Coleman [12] type was designed where antennas for the coupling were inserted directly in the central pin of Female-adapters. This construction allows to perform a calibration of the whole microwave line up to the plane of the DR cell. Flexible coaxial cables were chosen as connecting lines for an easier calibration and for operation in a wide frequency band (d.c. – 40 GHz).

In Fig. 1 the scheme of the DR test cell is shown. A sapphire dielectric cylinder was used due its low-loss tangent \( \tan \delta \) to make the contribution of the dielectric losses negligible and, correspondingly, to yield the highest \( Q \)-factor of the DR cell. The adjustment of the air gap height, \( H_{\text{gap}} \), in the DR configuration allows for the adjustment of \( Q \) in a wide range of values. To operate below 15 GHz, we choose a dielectric cylinder with diameter 8 mm and height 5 mm, yielding a resonant frequency of the TE\(_{011}\) mode at \( f_{\text{res}} = 14.8 \) GHz at \( H_{\text{gap}} = 0 \). We note that a variable gap determines also a change of \( f_{\text{res}} \). In the experiment \( H_{\text{gap}} \) was adjusted to keep \( f_{\text{res}} \) between 13.17 GHz and 13.26 GHz.

As a result of different gaps, we measured a set of resonant curves with different \( Q \)-factors from 2000 to 9500 on the same setup. During the measurements, the antenna positions with respect to the lower base and dielectric were kept fixed, so to ensure approximately the same coupling. The upper base was attached to a movement screw so that its position (and gap) was adjusted by the screw movement. In addition to the variation of the gap height, to adjust finely the values of \( Q \) we changed one base of the resonator with plates of different metals, aluminum, silver and copper, with different \( R_s \) and then we obtained different \( Q \), see Eq.(1)

It is well known that in real resonant structures a resonant curve is far from the ideal Lorentzian shape. Various additional contributions should be taken into account. Previously in [13,14] it was shown that in the general case the contributions due to cross-coupling \( (S_{cR} + iS_{cX}) \) and phase angle correction \( (\phi) \) should be taken into account. The resulting model for the scattering parameter \( S_{21} \) could be written as:

\[ S_{21}(f) = \left( \frac{S_M}{1 + 2iQ\frac{f-f_{\text{res}}}{f_{\text{res}}} + S_{cR} + iS_{cX}} \right) e^{i\phi} \quad (2) \]

where the first term in parenthesis is the complex representation of Fano resonance [15] with \( S_M = S_{21}(f_{\text{res}}) \).

This model describes a general case and allows to fit even strongly asymmetric resonant curves, reducing possible errors in the modelization. We estimated the uncertainty of fit parameters on the basis of the variance-covariance matrix obtained on the output of Levenberg-Marquardt algorithm [16]. If possible (see further on), the uncertainty on \( S_{21} \) data was taken into account.

Experimental measurements were performed using an Anritsu 37269D VNA with a maximum number of acquisition points equal to 1601. In order to be in conditions as near as possible to
the real measurements, we performed the standard procedure of the VNA calibration. A full 12-term SOLT calibration method was used for calibration. The disadvantage of the calibration applied directly by the VNA is the constraint to operate in a fixed frequency window with a fixed number of data points. This fact reduces the range of both $f_{res}$ and $Q$–factors which could be used for the study because of the limited range of frequencies due to the limited number of data points at resonance, respectively.

To provide a complete description of the uncertainty on the extracted $Q$, one should take into account also the uncertainty on the measured $S$-parameters. It is well known that the uncertainty can be obtained only when correct information regarding the VNA, the microwave line, and the calibration procedures are attainable. For this study, $S_{21}$ calibrated data uncertainty was estimated using advanced modeling of the microwave system within the “Exact Uncertainty Calculator” tool by Anritsu [17].

### Figure 1. Sketch of the dielectric resonator cell with microwave line.

### Figure 2. Calibrated (a) and uncalibrated (b) resonant curves with fit residuals.

### 3. Results and discussion

Each measurement in the set was recorded with and without applying the calibration under the same conditions. To make the resonant curve fit operate in same conditions for all $S_{21}$ curves, data were cut to a frequency band 0.04 GHz wide, with a central frequency corresponding to $f_{res}$. This data crop eliminates, also, the effect of the curve tails.

In Fig. 2 examples of the fit on uncalibrated (Fig. 2a) and calibrated (Fig. 2b) curves are shown. For all measured curves and independently on $Q$–factor values, the general model based on Eq. 2 fits very well the experimental curves (see residuals in Fig. 2). The mean level through all measured curves of the cross-coupling term is $S_{CR} + iS_{CX} = 0.008 + i0.003$, with phase angle correction of 130°. For calibrated curve the worst estimation of the uncertainty level on the modulus $|S_{21}|$ was within 1%.

In Fig. 3 we show an estimation of the systematic error on $Q$ originating from the use of uncalibrated resonant curve instead of calibrated one. We take a measure of the systematic error as the relative variation between $Q$–factors, $(Q_{uncal} - Q_{cal})/Q_{cal}$, where $Q_{cal}$ is obtained from the fit of calibrated curve $|S_{21}|(f)$, and $Q_{uncal}$ is the result of the fit of the uncalibrated $|S_{21}|(f)$ (see Fig. 2 for the curve fit examples).

Fits to calibrated curves must include the uncertainty on $S$–parameters. Taking into account this contribution, we obtain $u(Q_{cal})/Q_{cal}$ below 1%, see Fig. 3, blue diamonds. For the relative uncertainty on $Q$-factor based on uncalibrated data, one might be tempted to assume the uncertainty of the fit algorithm. Clearly, in this case $u(|S_{21}|)$ does not play a role and practically cannot be assigned to uncalibrated data (we set $u(|S_{21}|)$=0). The very small value for $u(Q_{uncal})/Q_{uncal}$, see Fig. 3, is not only a clear underestimation but it does not show
the real effect of the use of uncalibrated data. In fact, the calculation of the systematic error (Fig 3, green triangles) shows a 
\( \frac{Q_{\text{uncal}} - Q_{\text{cal}}}{Q_{\text{cal}}} \sim 7\% \) systematic error, much larger than the uncertainties that can be evaluated on the basis of the fits.

It is clear that in a situation where it is impossible to calibrate all or part of the microwave line, the worst possible case of the error should be considered. Thus, the systematic error becomes the leading estimation of the uncertainty on \( Q \) when uncalibrated data are fitted. In the range of \( Q \) values here presented, one should then consider an uncertainty at \( u(Q)=7\% \) if no or partial calibration is applied.

We finally note that it is likely that a sharper resonance curve, that is higher \( Q \) factors, can mitigate this effect. We could not explore higher \( Q \) values due to the characteristics of our resonators. Further work is required to clarify this aspect.

Figure 3. Relative uncertainty on calibrated \( Q \)-factor compared to the systematic error \( \frac{Q_{\text{uncal}} - Q_{\text{cal}}}{Q_{\text{cal}}} \). The uncalibrated \( u(Q_{\text{uncal}})/Q_{\text{uncal}} \) is presented to illustrate the level of the uncertainty underestimation when no calibration is applied. All data are represented as a function of the measured \( Q \)-factor.

4. Conclusions
We evaluated the uncertainty in the \( Q \)-factor of microwave resonators as obtained from microwave transmission measurements, in the case of fits to the resonance curve as obtained from calibrated and uncalibrated data. We performed an experimental study with a dielectric-loaded resonator able to tune the \( Q \)-factor in the range \((2–10) \cdot 10^3\). It was shown that calibrated curves provide an uncertainty \( u(Q)/Q \) below 1\%. The use of uncalibrated data yields an uncertainty that is completely dominated by systematic errors, at the level \( u(Q)/Q=7\% \).

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