Vibration analysis of the simply supported R/C beam

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Abstract. A mechanical vibration can be of different origins – caused by machines, due to a shock impact, by motors, etc. In this paper the behaviour of a reinforced concrete beam with a static scheme of a simply supported beam under the action of vibrations has been studied. The paper presents differences between static and dynamic analysis of the simply supported reinforced concrete beam. Concrete and the reinforcement bars are modelled with their material characteristics by applying linear-elastic material models. The load area is in the middle of the beam. A numerical model by the Finite Element Method was developed. To perform the vibration analysis, a time-history analysis has been employed. Dynamic analysis of the numerical model was performed using a software product based on the Finite Element Method. The dynamic time-history analysis was adopted to determine the dynamic response of the simply supported R/C beam under the action of time variables.

1. Introduction
A mechanical vibration can be of different origins – caused by machines, due to a shock impact, by motors, cranes etc. Transient or time-history dynamic analysis is a technique used to determine the dynamic response of a structure under the action of time-dependent vibrations. This type of analysis may be utilized to determine the time-varying displacements, strains, stresses, and forces in a structure, as it responds to any combination of static, transient and harmonic loads [1].

Harmonic vibrations
The equation describing the harmonic damping vibrations of a system with a final number of degrees of freedom, and taking into account the resistance forces:

\[ [m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{F(t)\}, \]

and without taking into account the resistance forces

\[ [m]\{\ddot{u}\} + [k]\{u\} = \{F(t)\}\]
2. Numerical models

The concrete and reinforcement material properties is according to Eurocode 2 [4]. More about calculating anchoring length can be found in [8].

![Figure 2. a) Scheme and loading area b) Position of reinforced bars](image)

The concrete properties are: C20/25, $f_{cm}=28$ MPa, $E_{cm}=30$ GPa, Poisson’s ratio is 0.2. Reinforcement bars’ diameters and position is shown on Figure 2b).

Cross section of the simply supported beam – b=0,25 m, h=0,40 m, L=3,00 m.

### 2.1. Static analysis. Linear elastic concrete model.

First numerical model (M1) represents modelling by ANSYS. The load stamp size is 0.25/1.00 m. A static load of 132 kN/m$^2$ ($R_q=33$ kN) on the load stamp is applied. The concrete is modelled by SOLID 65 (CONCRET 65) finite elements as linear elastic concrete material model – Table 1. Reinforcement bars are modelled via BEAM188 finite elements – Table 2.

| Table 1. Concrete properties - linear isotropic          |
|---------------------------------------------------------|
| Concrete material model for C20/25                      |
| Young’s modulus $E_c=3\times10^7$ kN/m$^2$              |
| Poisson’s ratio $\nu=0.2$                               |

| Table 2. Reinforcement properties - linear isotropic    |
|--------------------------------------------------------|
| Reinforcement steel B500 model                         |
| Young’s modulus $E_s=2\times10^8$ kN/m$^2$              |
| Poisson’s ratio $\nu=0.3$                               |
2.2. Static analysis. Nonlinear inelastic concrete model.
Second model (M2) is modeling with material properties according to Table 3, are presented. The boundary conditions adopted are restraints applied on the left side of the simply supported beam - displacements by axes X and Y. The load stamp size is 0.25/1.00 m. The applied force starts at 40 kN/m², by a step of 40 kN/m². The beam is modeled by SOLID 65 (CONCRETE 65) finite elements with nonlinear plasticity material model. Different variables for applying the Willam and Warnke’s material model [5] in ANSYS are required (Table 3). Reinforcement bars are modeled with BEAM188 finite elements – Table 2. In Figure 3, the distribution of the normal stresses and displacement uy in the simply supported beam are shown. Figure 4 shows the formed cracks at model M2.

Tensile strength of concrete under bending - the average tensile strength in the bending moment of reinforced concrete elements depends on the average tensile strength and the height of the cross section. [9]

| Table 3. Concrete material nonlinear inelastic model in tension zone |
|---------------------------------------------------------------|
| 1. Shear transfer coefficient for an open crack ($\beta_t$) | 0.2 |
| 2. Shear transfer coefficient for an close crack ($\beta_c$) | 0.9 |
| 3. Uniaxial cracking stress ($f_r$) | 2800 kN/m² |
| 4. Uniaxial crushing stress ($f_c$) | -1 |

2.3. Dynamic analysis. Harmonic vibrations.
The numerical model of a simply supported beam is a process of modeling run in ANSYS. Applied load at the end of the beam on the load stamp (Figure 2a). The load stamp size is 0.25/1.00 m. The stamp load case is presented. The concrete is modeled by SOLID 65 (CONCRETE 65) finite element as linear elastic concrete material model – Table 1. Reinforcement bars are modeled with BEAM188 finite elements – Table 2. The most important point in the study of building structures of dynamic impacts is the choice of a suitable dynamic model. More information about the dynamic study in structural mechanics can be found in [1], [3] [6] and [7].

The behaviour of the simply supported beam subjected to dynamic vibrations, which become changeable under a harmonious law, has been investigated. For the purpose of the study, on the numerical model, excitations defined with the following dependencies were applied:

\[
q(t) = q \cdot \sin(\theta_i \cdot t),
\]

with

\[
\theta_i = 2. \pi \cdot f_i, \quad f_i = 5, 10, 20, 25, 100 \text{ s}^{-1}.
\]

Amplitude decay factor: $\gamma = 0.005$,

and

\[
q(t) = q \cdot e^{-0.05\theta_i t} \cdot \sin(\theta_i \cdot t),
\]

with

\[
\theta_i = 2. \pi \cdot f_i, \quad f_i = 5, 10, 20, 25, 100 \text{ s}^{-1}.
\]

Amplitude decay factor: $\gamma = 0.005$.

3. Numerical results and discussion

3.1. Static analysis
In Figure 3a) displacement $u_y$ is presented and Figure 3b) shows the distribution of $\sigma_x$ under a load of 132 kN/m². The results from static analysis are presented in Table 4.
3.1.1. Linear elastic material models

![Figure 3](image1.png)

**Figure 3.** a) Displacement $u_y$, b) Distribution of normal stress $\sigma_x$ at the simply supported beam, $q=132$ kN/m$^2$

3.1.2. Nonlinear material model

![Figure 4](image2.png)

**Figure 4.** Nonlinear concrete material model. Crack initiation at applied force $q=132$ kN/m$^2$

### Table 4. Results from static analysis max $u_y$, $q=132$ kN/m$^2$.

| Parameter                        | Linear elastic material model (M1) | Plasticity material model $q=132$ kN/m$^2$ (M2) | Plasticity material model $q>132$ kN/m$^2$ (M2) |
|----------------------------------|-----------------------------------|-------------------------------------------------|-------------------------------------------------|
| max $|u_y|$                            | 0.000441525 m                     | 0.00441943 m                                    | 0.00160113                                      |

3.2. Dynamic analysis – Harmonic vibrations.

### Table 5. Results from dynamic analysis – harmonic vibrations displacement max $u_y$.

| Load                     | max $u_y$, m | max $u_y$, m |
|--------------------------|--------------|--------------|
|                           | $q(t)=q\sin(\theta_i t), \gamma=0.005$ | $q(t)=qe^{-0.05 \theta_i t} \sin(\theta_i t), \gamma=0.005$ |
| Static                   | 0.000441525  | 0.000441525  |
| Dynamic-harmonic         |              |              |
| $q(t)=132\sin(2\pi5t)$   | 0.000575430  | 0.000519005  |
| $q(t)=132\sin(2\pi10t)$  | 0.000840289  | 0.000700378  |
| $q(t)=132\sin(2\pi20t)$  | 0.016037000  | 0.003785650  |
| $q(t)=132\sin(2\pi25t)$  | 0.001941520  | 0.001379130  |
| $q(t)=132\sin(2\pi50t)$  | 0.000292689  | 0.000261438  |
In Table 5 are presented the results from the harmonic analysis – max \( u_y \) upon the applied load: 
\[ q(t) = q \sin(\theta_i t), \] 
with the amplitude decay factor \( \gamma = 0.005 \) and 
\[ q(t) = q e^{-0.05 \theta_i t} \sin(\theta_i t), \] 
with the amplitude decay factor \( \gamma = 0.005 \).

In figure 5, the "time-displacement" relation due to harmonic analysis 
\[ q(t) = q \sin(\theta_i t), \] \( \gamma = 0.005 \) is shown.

In figure 6, the "time-displacement" relation due to harmonic analysis 
\[ q(t) = q e^{-0.05 \theta_i t} \sin(\theta_i t), \] \( \gamma = 0.005 \) is shown.

In Figures 7 and 8 “q(t)- \( u_y \)” relations from harmonic vibrations are presented.
Figure 6. Displacements max $u_y$ from harmonic analysis

\[ q(t) = q \cdot e^{-0.05 \cdot \theta_i \cdot t} \cdot \sin(\theta_i \cdot t), \ \gamma = 0.005 \]

a) $f_i = 10 \text{ s}^{-1}$; b) $f_i = 20 \text{ s}^{-1}$; c) $f_i = 25 \text{ s}^{-1}$; d) $f_i = 50 \text{ s}^{-1}$

Figure 7. “$q(t)$-$u_y$” relation from harmonic analysis

\[ q(t) = q \cdot \sin(\theta_i \cdot t), \ \gamma = 0.005, \ f_i = 20 \text{ s}^{-1} \]
4. Conclusions

The FEM provides opportunities for further studying a stress-and-strain state of constructive members subjected to local concentrated static load and dynamic vibrations, taking into account the material and geometric nonlinearity. The results obtained by these numerical models are much closer to the actual behaviour of the investigated systems and their constructive members.

At static analysis, loads are constant in size and location and are applied slowly to the structure. The accelerations and inertial forces that develop during deformation and cause additional load are very small. In most cases, the loads on the structures and elements have a dynamic character. They cause deformations in elastic systems with high accelerations, so that the additional inertial load cannot be ignored. Dynamic analysis clarifies both quantitatively and qualitatively the actual phenomena related to stresses and strains caused from dynamic loads. At harmonic vibrations of increasing frequency likely to reach frequencies close to resonance – Figure 9, the vertical displacements increase, reaching values from 2 to over 40 times higher than the values obtained in static analysis. For small values of the ratio $\eta$, i.e. when the force changes slowly, the amplitude of the vertical displacements is close to the static load displacements. The curves of the dependence $u_y-\eta$ for the different values of the amplitude decay factor are approaching. When the displacements and deformations continue to increase, at some point in time, destruction occurs if it is made of a brittle material, and if it is made of ductile material, yielding will occur. At values of $\eta>1.20$ or $\eta<1.20$ again approaching curves $u_y-\eta$ are observed.
Dynamic response at small values of $\eta$ - when the force is changing slowly, is approximately the same as a static load and is predetermined by the stiffness. Dynamic response at low values - when the force is changing very quickly and controlled by the mass. Increasing the $\eta$ ratio values to reach the resonant frequency $\theta=\omega$ is expressed via a sharp increase in vertical displacements, respectively strains and deformations. After passing the resonant frequency, the vertical displacement values begin to decrease sharply.

In damped systems, when the $\eta\approx1$ system is very sensitive to damping. The displacements are significantly higher than the values of $u_y$ at static loading. It is necessary to protect the structure from frequencies close to the resonant one.

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5. References
[1] Bankov B 2004 Theory of elasticity, stability and dynamic of elastic systems, UACEG, Sofia
[2] ANSYS Academic Reference Manual, www.ansys.com
[3] Chopra A 1995 Dynamic of structures, Prentice Hall Inc. ISBN 0-13-855214-2
[4] European standard Eurocode 2 – BDS EN 1992-1-1
[5] Willam K J and Warnke E P 1975 Constitutive models for the triaxial behavior of concrete. Proc. Int. Assoc. for Bridge and Structural Engineering, vol 19, pp 1-30
[6] Handruleva An, Matuski V, Kazakov K, Bankov B 2011 Practical examples on Theory of elasticity and dynamics of structures, VSU Publishing house, Sofia, 2011, ISBN 978-954-331-033-3, p.121
[7] Handruleva An 2020 Mechanisms of destruction of spatial frame structures, Sofia 2020, ISBN: 978-954-331-114-9
[8] Cvetkov St 2013 Anchoring length of the bearing reinforcement at reinforced concrete beams, In Proc.:XIII International Scientific Conference VSU2013, ISSN 1314-071X, pp. II-108-II-111
[9] Cvetkov St, Constructors for civil engineers - the prefabricated one-storey buildings made of reinforced concrete elements, VSU Publishing house, ISBN: 978-954-331-103-3