OPTIMAL WEAK-LENSING SKEWNESS MEASUREMENTS

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ABSTRACT

Weak-lensing measurements are starting to provide statistical maps of the distribution of matter in the universe that are increasingly precise and complementary to cosmic microwave background maps. The most accurate measurement is the correlation in alignments of background galaxies, which can be used to infer the variance of the projected surface density of matter. This measurement of the fluctuations is insensitive to the total mass content and is analogous to using waves on the ocean to measure its depths. However, when the depth is shallow, as happens near a beach, waves become skewed. Similarly, a measurement of skewness in the projected matter distribution directly measures the total matter content of the universe. While skewness has already been convincingly detected, its constraint on cosmology is still weak. We address optimal analyses for the Canada-France-Hawaii Telescope Legacy Survey in the presence of noise. We show that a compensated Gaussian filter with a width of 2/5 optimizes the cosmological constraint, yielding $\Delta \Omega_m/\Omega_m \sim 10\%$. This is significantly better than other filters that have been considered in the literature. This can be further improved with tomography and other sophisticated analyses.

Subject headings: cosmology: observations — cosmology: theory — dark matter — gravitational lensing — large-scale structure of universe

On-line material: color figure

1. INTRODUCTION

Mapping the mass distribution of matter in the universe has been a major challenge and focus of modern observational cosmology. The only direct procedure to weigh the matter in the universe is by using the deflection of light by gravity. While this effect is very small, a large statistical sample can provide a precise measurement of averaged quantities.

There are very few direct ways to weigh the universe. The most accurate measurement by combining cosmic microwave background (CMB) data with large-scale structure (Spergel et al. 2003; Contaldi, Hoekstra, & Lewis 2003) results in $\Omega_m \sim 0.27$ with zero geometric curvature, implying a cosmological constant $\Omega_A \sim 0.73$. This type of inference requires combining data measured at different times and on different length scales. Blanchard et al. (2003) have shown that the same data can be consistent with $\Omega_m = 1$ if one gives up perfect scale invariance for the primordial perturbations and allows for a neutrino mass of 1 eV. The physical constraint arises since the CMB measures the fluctuations on large scales $L \sim$ gigaparsecs at high redshifts of $z \sim 1100$. The large-scale structure measures scales of $L \sim 1-100$ Mpc and a low redshift of $z \sim 0$. The scales have only a small overlap (Tegmark & Zaldarriaga 2002). If one requires perfect scale invariance of the fluctuations, one is forced into a low matter density with a cosmological constant. It is perhaps an aesthetic choice to trade scale invariance in time for scale invariance in space.

Weak gravitational lensing provides a direct statistical measure of the dark matter distribution regardless of the nature and dynamics of both the dark and luminous matter intervening between the distant sources and observer. Weak lensing by large-scale structure can lead to shear and magnification of the images of distant faint galaxies. On the basis of the theoretical work done by Gunn (1967), Blandford et al. (1991), Miralda-Escudé (1991), and Kaiser (1992) performed the first calculation of weak lensing by large-scale structure, the result of which showed that the expected distortion amplitude of the weak-lensing effect is at a level of roughly a few percent in adiabatic cold dark matter (CDM) models. Kaiser (1992) also made early predictions for the power spectrum of the shear and convergence using linear perturbation theory. Because of the weakness of the effect, all detections have been statistical in nature, primarily in regimes where the signal-to-noise ratio is less than unity. Fortunately, several groups have recently been able to measure this weak gravitational lensing effect (Bacon, Refregier, & Ellis 2000; Refregier, Rhodes, & Groth 2002; Hoekstra et al. 2002; Van Waerbeke et al. 2002; Jarvis et al. 2003; Brown et al. 2003; Hamana et al. 2003).

In the standard model of cosmology, fluctuations start off small, symmetric, and Gaussian. Even in some non-Gaussian models such as topological defects, initial...
fluctuations are still symmetric; positive and negative fluctuations occur with equal probability (Pen, Spergel, & Turok 1994). As fluctuations grow by gravitational instability, this symmetry can no longer be maintained; overdensities can be arbitrarily large, while underdense regions can never have less than zero mass. This leads to a skewness in the distribution of matter fluctuations. While skewness has already been measured at very high statistical significance (Pen et al. 2003), the measurement has not resulted in a strong constraint on the total matter density \( \Omega_m \). The data has so far been limited by sample variance and analysis techniques. Currently ongoing surveys, such as the Canada-France-Hawaii Telescope (CFHT) Legacy Survey, will provide more than an order of magnitude improvement in the statistics. In this paper we address the optimal analysis of the new data sets and examine the likely plausible accuracies on the direct measurements of matter density that they can achieve. The calculations rely only on subhorizon Newtonian physics.

Several studies have addressed the feasibility of the skewness measurements (Jain, Seljak, & White 2000; White & Hu 2000). These pioneering contributions have provided the first estimates of the expected strengths of the skewness \( S_3 \). A real measurement is limited by the sample variance in \( S_3 \) and noise properties. Furthermore, the density field is always smoothed by some filters. Since gravitational lensing can only measure differences in mass, all such filters must have zero area. In this paper we study a range of filters that have been suggested in the literature. Our goal is to find the scale that maximizes the skewness for simulated weak lensing using different kinds of window functions that much of the literature is based.

In second-order perturbation theory, one finds that the skewness scales as the square of the variance and inversely to density. In terms of the dimensionless surface density \( \kappa \), one can express the square of the variance and the skewness as \( \langle \kappa^2 \rangle \propto \sigma_8 \Omega_m^{0.75} \) and \( S_3 \equiv \langle \kappa^3 \rangle / \langle \kappa^2 \rangle \propto \Omega_m^{-0.8} \), respectively. Therefore, one can break the degeneracy between \( \sigma_8 \) and \( \Omega_m \) only if both the variance and the skewness of the convergence are measured. The skewness of the convergence field has been studied in perturbation theory (Bernardeau, Van Waerbeke, & Mellier 1997; Hui 1999), and initial detections have been reported (Bernardeau, Mellier, & Van Waerbeke 2002). Jain et al. (2000) investigated weak lensing by large-scale structure using ray tracing in \( N \)-body simulations. By smoothing the convergence field using a top-hat window function, they compute \( S_3 \) under two conditions: one with noise and one without noise added in the convergence fields by the third moment for all varieties of cosmological models. Moments are linear in the probability density function; one can combine the moments of different maps, which gives the same answer as combining maps first. Nonlinear methods have also been proposed. One can measure \( S_3 \) by using the conditional second moment of the \( \kappa \) field, specifically, the second moment of positive \( \kappa \) and negative \( \kappa \), which is related to \( S_3 \) in perturbation theory (Nusser & Dekel 1993; Juszkiewicz et al. 1995).

Moments are also additive in the presence of noise, such that skewness-free noise (which realistic symmetric noise sources often have) does not bias the measurement of moments.

White & Hu (2000) presented a calculation for the skewness \( S_3 \) and its standard deviation of weak lensing by large-scale structure based on \( N \)-body simulations. By smoothing the \( \kappa \) field using a top-hat filter, they show that the standard deviation of the skewness after adding simulated shot noise to the \( \kappa \) field is only slightly increased, by about 16%, compared with the case of a pure \( \kappa \) field.

In this paper we present the first extended comparison of skewness for simulated weak lensing using different kinds of window functions to isolate the filter that is optimal for distinguishing cosmological models. We highlight some candidate window functions that have been used separately in the literature. The outline of the paper is as follows: In § 2 we introduce the strategy of map construction of weak lensing from simulations. In § 3 we describe the details of the window functions employed and present the results and summarize our conclusions in § 4.

2. SIMULATIONS AND MAP CONSTRUCTION OF WEAK LENSING

2.1. N-Body Simulations

We ran a series of \( N \)-body simulations with different values of \( \Omega_m \) to generate convergence maps and make simulated catalogs to calibrate the observational data and estimate errors in the analysis. The power spectra for given parameters were generated using CMBFAST (Seljak & Zaldarriaga 1996), and these tabulated functions were used to generate initial conditions. The power spectra were normalized to be consistent with the earlier two-point analysis from this data set (Van Waerbeke et al. 2002). We ran all of the simulations using a parallel, particle-mesh \( N \)-body code (Dubinski et al. 2003) at 1024\(^3\) mesh resolution using 512\(^3\) particles on an eight-node quad processor Itanium Beowulf cluster at the Canadian Institute for Theoretical Astrophysics (CITA). Output times were determined by the appropriate tiling of the light-cone volume with joined comoving boxes from \( z \approx 6 \) to 0. We output periodic surface density maps at 2048\(^2\) resolution along the three independent directions of the cube at each output interval. These maps represent the raw output for the run and are used to generate convergence maps in the thin lens and Born approximations by stacking the images with the appropriate weights through the comoving volume contained in the past light cone.

All simulations started at an initial redshift \( z_i = 50 \) and ran for 1000 steps in equal expansion factor ratios with box size \( L = 200 \ h^{-1} \) Mpc comoving. We adopted a Hubble constant \( h = 0.7 \) and a scale-invariant \( n = 1 \) initial power spectrum. A flat cosmological model with \( \Omega_m + \Lambda = 1 \) was used. Four models were run with \( \Omega_m \) of 0.2, 0.3, 0.4, and 1. The power spectrum normalization \( \sigma_8 \) was chosen as 1.16, 0.90, 0.82, and 0.57, respectively.

2.2. Simulated Convergence Maps

The convergence \( \kappa \) is the projection of the matter overdensity \( \delta \) along the line of sight \( \theta \) weighted by the lensing geometry and source-galaxy distribution. It can be
expressed as
\[ \kappa(\theta, \chi) = \int_0^\chi W(\chi) \delta[\chi, r(\chi)\theta] \, d\chi, \]
where \( \chi \) is the comoving distance in units of \( c/H_0 \) and \( H_0 = 100 \, h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \). The weight function
\[ W(\chi) = \frac{1}{2} \Omega_m g(\chi)(1 + z) \]
is determined by the source-galaxy distribution function \( n(z) \) and the lensing geometry
\[ g(\chi) = r(\chi) \int_\chi^\infty d\chi' n(\chi') \frac{r(\chi' - \chi)}{r(\chi')}, \]
where \( r(\chi) \) is the radial coordinate; \( r(\chi) = \sinh(\chi) \) for an open, \( r(\chi) = \chi \) for a flat, and \( r(\chi) = \sin(\chi) \) for a closed geometry of the universe. The function \( n(z) = n(\chi) d\chi/dz \) is normalized such that \( \int_0^\infty n(z)dz = 1 \). For the CFHT Legacy Survey, we adopt \( n(z) = \beta/(z_0 \Gamma[(1 + \alpha)/\beta])(z/z_0)^\alpha \exp[-(z/z_0)^\beta] \), with \( \alpha = 2 \), \( \beta = 1.2 \), and the source redshift parameter \( z_0 = 0.44 \), which peaks at \( z_p = 1.58z_0 \). The mean redshift is \( \bar{z} = 2.1z_0 \), and the median redshift is \( z_b = 1.9z_0 \) (Van Waerbeke et al. 2002). The source redshift distribution \( n(z) \) adopted here is the same as that for VIRMOS.

During each simulation we store two-dimensional projections of \( \delta \) through the three-dimensional box at every light crossing time through the box along all \( x \)-, \( y \)-, and \( z \)-directions. Our two-dimensional surface density sectional maps are stored on 2048\(^2\) grids. After the simulation, we stack sectional maps separated by a width of the simulation box, randomly choosing the center of each section and randomly rotating and flipping each section. The periodic boundary condition guarantees that there are no discontinuities between any two adjacent boxes. We then add these sections with the weights given by \( W(z) \) onto a map of constant angular size, which is generally determined by the maximum projection redshift. To minimize the repetition of the same structures in the projection, we alternatively choose the sectional maps of \( x \)-, \( y \)-, and \( z \)-directions during the stacking. Using different random seeds for the alignments and rotations, we make 40 maps for each cosmological model. Since the galaxy distribution peaks at \( z \sim 1 \), the peak contribution of lensing comes from \( z \sim 0.5 \) because of the lensing geometry term. Thus, the maximum projection redshift \( z \sim 2 \) is sufficient for the lensing analysis, so we project the \( \Omega_0 = 1 \), 0.4, and 0.3 simulations to \( z = 2 \) and obtain 40 maps each with angular width \( \theta \approx 4^\circ.09 \), 3\( ^\circ.18 \), and 3\( ^\circ.02 \), respectively. To make sufficiently large maps, for \( \Omega_0 = 0.2 \) we project up to \( z = 1.8 \) and obtain maps with angular width \( \theta \approx 2^\circ.86 \). One \( \kappa \) map created from a cosmological simulation of \( \Omega_m = 0.3 \) is shown in Figure 1. The skewness is quite apparent at this resolution of the simulation. Decreasing the cosmological density while maintaining the same variance of convergence \( \kappa \) forces structures to be more nonlinear and thus more skewed. Our challenge is to extract this behavior accurately from realistic data.

We then simulate the CFHT Legacy Survey by adding noise to these clean maps. The noise \( \kappa \) maps have a pixel-pixel variance \( \sigma_N^2 = \langle e^2 \rangle/2/N_{\text{pixel}} \). Here \( \langle e^2 \rangle = 0.47^2 \) is the total noise estimated in the VIRMOS-DESCART survey.

![Fig. 1.—Initial noise-free \( \kappa \) map in the \( N \)-body simulation of a \( \Omega_m = 0.3 \, \Lambda \text{CDM} \) cosmology with a map width of 3\( ^\circ.02 \) and 2048\(^2\) pixels, and the scale is in units of \( \kappa \). [See the electronic edition of the Journal for a color version of this figure.]](image-url)
and we take it as what would be expected by the CFHT Legacy Survey. It includes the dispersion of the galaxy intrinsic ellipticity, point-spread function correction noise, and photon shot noise; \( \langle N_{\text{pixel}} \rangle \) is the mean number of galaxies in each pixel. For VIRMOS, the number density of observed galaxies \( n_g \simeq 26 \text{arcmin}^{-2} \), thus, \( \langle N_{\text{pixel}} \rangle = n_g(\theta_{\text{e}}/1')/N^2 \), where \( N = 2048 \) is the number of grids by which we store two-dimensional maps and the field of view \( \theta_{\text{e}} \) is in arcminutes. The factor of 2 arises from the fact that the shear field has 2 degrees of freedom \((\gamma_1, \gamma_2)\), where the definition of \( \langle \chi^2 \rangle \) sums over both. We use this as our best guess for the CFHT Legacy Survey noise. The maps we obtained through the method described above are nonperiodic after the projection. In order to eliminate edge effects, we crop each smoothed map by a factor of 10% in the margins of each \( \kappa \) map for models with \( \Omega_m = 0.2 \). For comparison, the size of the maps for the other models is the same as that for the models with \( \Omega_m = 0.2 \).

3. THE OPTIMAL FILTER

Our goal is to find the optimal filter for constraining \( \Omega_m \) by the non-Gaussianity of weak lensing. The non-Gaussianity of weak lensing for a clean map is quantified by the skewness \( S_3 \),

\[
S_3(\theta_f) = \frac{\langle \kappa^3 \rangle}{\langle \kappa^2 \rangle^{3/2}},
\]

where \( \theta_f \) is the characteristic radius of the filter function. When noise is present, the definition of skewness can be modified to be (White & Hu 2000)

\[
S_3(\theta_f) = \frac{\langle \kappa^3 \rangle_{S+N}}{\langle \kappa^2 \rangle_{S+N}^{3/2}},
\]

The subscript \( S \) indicates a weak lensing signal, while \( N \) denotes random noise. Since \( \langle \kappa^3 \rangle = \langle \kappa^2 \rangle \), \( \langle \kappa^2 \rangle_{S+N} = \langle \kappa^2 \rangle_{S} \), the \( S_3 \) defined by equation (5) is statistically equivalent to the one defined by equation (4), and the presence of noise has only residual effects on the dispersion of \( S_3 \).

The skewness \( S_3 \) is a function of cosmological density parameter \( \Omega_m \) and the filter function. The noise introduces a large dispersion in \( S_3 \) and also smears its intrinsic dependence on cosmological parameters. Filtering on a large scale reduces this noise but also reduces the intrinsic skewness and increases sample variance. Our goal is to find the optimal smoothing scale. Different filters also have different scale dependences. The general form of this filter is difficult to find, so we employ five parameterized classes of filters in this paper. These are the top-hat, Gaussian, aperture, compensated Gaussian, and Wiener filters. The top-hat filter is normalized to have a sum unity in the two-dimensional window function map, and the Gaussian one is defined as \( W(\theta) = (1/2\pi \theta_f^2) \exp(-\theta^2 / 2\theta_f^2) \), which is normalized by the same as the top hat. The aperture filter is defined as \( W(\theta) = (9/\pi)(1/\theta_f)^3 \left[ 1 - (\theta / \theta_f)^2 \right]^{3/2} \) and zero for \( \theta > \theta_f \), which has zero mean. The compensate Gaussian filter is written as \( W(\theta) = (1/2\pi \theta_f^2) [1 - (\theta^2 / 2\theta_f^2)] \exp(-\theta^2 / 2\theta_f^2) + 1 \), which holds zero area, and is normalized to have a peak amplitude of unity in Fourier space. This has the feature that it will only damp modes, never amplify. Many analytic integrals for the compensated Gaussian filter can be evaluated analytically (Crittenden et al. 2002). The Wiener filter is defined in Fourier space by \( W(\ell) = C_{\ell}(l)/[C_{\ell}(l) + C_0] \), where \( C_{\ell}(l) \) is the angular power spectrum of the signal \( \kappa \), while \( C_0 = 4\pi \alpha^2 f_{\text{sky}}/N^2 \) is that for noise power and \( f_{\text{sky}} = \pi(\theta_{\text{e}}/360)^2 \) is the fractional sky coverage of each map.

Given these filters, one can measure \( S_3 \) and its dispersion \( \Delta S_3 \), which are all functions of \( \Omega_m \). The dispersion \( \Delta S_3 \) causes the inferred \( \Omega_m \) to differ from its true value by a change of \( \Delta \Omega_m \). For each class of filter, there exists an optimal filter radius \( \theta_f \) to minimize \( \Delta \Omega_m \). Comparing the minimum \( \Delta \Omega_m \) of each class of filter, one can then find the optimal one.

From simulated maps, we first calculate the skewness \( S_3 \) and its standard deviation \( \Delta S_3 \) with different filter radii for all kinds of cosmological models. The CFHT Legacy Survey will observe 160 deg\(^2\), which is about 24 times larger than the simulated area. We conservatively decreased the error we obtained in the field-to-field variations by a factor of about 4 to estimate the sensitivity for the Legacy Survey. Therefore, the standard deviation of \( S_3 \) throughout this paper is taken to be one-fourth of the original predicted value from simulation. The skewness \( S_3 \) and its standard deviation \( \Delta S_3 \) of the top-hat, Gaussian, aperture, and compensated Gaussian window functions are shown in Figures 2 and 3, while those of Wiener filter are plotted in Figure 4. Figures 2 and 4 show that the expected \( S_3 \) decreases with the cosmological density parameter \( \Omega_m \) for all of filters, which is in consistent with what is predicted by perturbation theory at large \( \theta_f \) (Gaztanaga & Bernardeau 1998) and nonlinear perturbation theory (Hui 1999; Van Waerbeke et al. 2001). A fixed fluctuation amplitude measured by weak lensing is a smaller fractional fluctuation in a higher \( \Omega_m \) universe, and thus less nonlinear and less non-Gaussian. The value of \( S_3 \) also decreases with filter scale \( \theta_f \), as one would expect from the central limit theorem when smoothing over more independent patches to converge to a Gaussian distribution. Our dependence of \( S_3 \) agrees qualitatively with Van Waerbeke et al. (2001), where the detailed normalization depends on details of the redshift distribution. Estimates of \( S_3 \) are possible analytically. To optimize its measurement, we also need its standard deviation, which is related to a six-point function. This is difficult to compute analytically.

The fit for \( S_3 \) in Figure 2 fails badly not only on small scales but also on large scales of more than 10'. This is due to a larger standard deviation of \( S_3 \) as shown in Figure 3 on both of the two scales. It is also apparent from Figure 3 that there exists a corresponding optimal filtering scale \( \theta_f \) that minimizes \( \Delta S_3 \) (except for the Wiener filter, which does not depend on filter radius). This is due to the trade-off between noise on small scales and sample variance on large scales. The skewness \( S_3 \) as a function of filter radius \( \theta_f \) for the top-hat filter in Figure 2 is in rough agreement with that of White & Hu (2000), where the cosmological model is specified to \( \Omega_m = 0.3 \), but it differs from Jain et al. (2000). We do not understand the behavior of the Gaussian window for large \( \Omega \) at small angular scale, where a smoothing scale smaller than our resolution seems to be preferred, but we do not dwell on this since the Gaussian window is not observable on a shear map.

From the above analyses it is clear that \( S_3 \) and \( \Delta S_3 \) depend not only on the smoothing scale \( \theta_f \) but also the cosmological parameter \( \Omega_m \) for all window functions except the
Wiener filter. In real observations of weak lensing, one must evaluate the uncertainty in \( \Omega_m \) for a given observed \( S_3 \) and \( \Delta S_3 \) to discriminate between cosmological models. One needs to invert the relation \( S_3 = S_3(\Omega_m, \theta_f) \) to obtain \( \Omega_m = \Omega_m(S_3, \theta_f) \) and estimate the uncertainty of inferred \( \Omega_m \) by

\[
\Delta \Omega_m [S_3(\Omega_m, \theta_f), \theta_f] = \Omega_m [S_3(\Omega_m, \theta_f), \theta_f] - \Omega_m [S_3(\Omega_m, \theta_f) + \Delta S_3(\Omega_m, \theta_f), \theta_f].
\]

Because of the irregularity of the data points, the inversion is noisy and may introduce unrealistic artifacts. To overcome this problem, we first fit \( S_3 \) and \( \Delta S_3 \) by a combination of power laws of \( \Omega_m \) and \( \theta_f \) in the presence of noise,

\[
S_3(\Omega_m, \theta_f) = A(\Omega_m) \theta_f^{B(\Omega_m)},
\]

\[
\Delta S_3(\Omega_m, \theta_f) = A_S(\Omega_m) \theta_f^{B_S(\Omega_m)} + A_N(\Omega_m) \theta_f^{B_N(\Omega_m)},
\]

where \( A(\Omega_m) = A_1(\Omega_m)^{B_1}, B(\Omega_m) = A_2(\Omega_m)^{B_2}, A_S(\Omega_m) = A_{S1}(\Omega_m)^{B_{S1}}, B_S(\Omega_m) = A_{S2}(\Omega_m)^{B_{S2}}, A_N(\Omega_m) = A_{N1}(\Omega_m)^{B_{N1}}, \) and \( B_N(\Omega_m) = A_{N2}(\Omega_m)^{B_{N2}} \). The two terms of \( \Delta S_3(\Omega_m, \theta_f) \) represent two sources of the dispersion of \( S_3 \): the intrinsic dispersion of the signal and that caused by noise. Noise dominates at small smoothing scales, so we expect \( B_N(\Omega_m) < 0 \), while we expect \( B_S(\Omega_m) > 0 \) because of the large smoothing behavior of \( \Delta S_3 \) from the signal (Fig. 3). For the Wiener function, we can simply parameterize the dependence of skewness \( S_3 \) and its standard deviation \( \Delta S_3 \) on \( \Omega_m \) as \( S_3(\Omega_m) = A_1(\Omega_m)^{B_1} \) and \( \Delta S_3(\Omega_m) = A_2(\Omega_m)^{B_2} \), respectively, because the filter is independent of smoothing radius.

We fit \( S_3 \) and \( \Delta S_3 \) as functions of \( \Omega_m \) and \( \theta_f \) with equations (7) and (8) for all filters except the Wiener filter. The best-fit coefficients \( A_{S1}, B_{S1}, A_{S2}, B_{S2}, A_{N1}, B_{N1}, A_{N2}, \) and \( B_{N2} \) are listed in Table 1; the best-fit relations are plotted with the smoother lines in Figures 2 and 3. For the Wiener filter, the best-fit coefficients \( A_1 = 68.03 \pm 2.37, B_1 = -0.67 \pm 0.03, A_2 = 18.17 \pm 6.13, \) and \( B_2 = -0.35 \pm 0.26 \), and the dashed lines in Figure 4 show the best fits to \( S_3 \).
and \( \Delta S_3 \). In the fit of skewness \( S_3 \), we weighted using the standard deviation of the skewness. Using these best-fit coefficients, we can calculate \( D / C_{10} \) as a function of \( / C_{18} f \), as shown in Figures 5 and 6. As expected, we find in Figure 5 that there does exist an optimal smoothing scale for each class of filter (except for the Wiener filter) that has a minimum error for the inferred \( / C_{10} m \). This optimal smoothing scale has only a weak dependence on cosmology except for

| Coefficient | Top Hat | Gaussian | Aperture | Compensated Gaussian |
|-------------|---------|----------|----------|----------------------|
| \( A_1 \)   | 62.16 ± 1.21 | 65.79 ± 1.11 | 252.74 ± 4.01 | 90.33 ± 0.69 |
| \( B_1 \)   | -0.75 ± 0.02 | -0.64 ± 0.01 | -0.94 ± 0.02 | -0.81 ± 0.01 |
| \( A_2 \)   | -0.16 ± 0.03 | -0.17 ± 0.02 | -0.24 ± 0.01 | -0.18 ± 0.01 |
| \( B_2 \)   | -0.13 ± 0.11 | -0.07 ± 0.09 | -0.42 ± 0.02 | -0.43 ± 0.03 |
| \( A_{S1} \) | 3.47 ± 0.04 | 5.43 ± 0.08 | 2.20 ± 0.17 | 1.47 ± 0.07 |
| \( B_{S1} \) | -0.25 ± 0.02 | -0.07 ± 0.03 | -0.79 ± 0.10 | -0.71 ± 0.05 |
| \( A_{S2} \) | 0.79 ± 0.01 | 0.84 ± 0.01 | 0.73 ± 0.03 | 0.98 ± 0.02 |
| \( B_{S2} \) | 0.10 ± 0.02 | -0.002 ± 0.02 | -0.06 ± 0.04 | 0.23 ± 0.03 |
| \( A_{S3} \) | 12.18 ± 1.75 | 7.02 ± 0.86 | 380.75 ± 108.86 | 8.11 ± 1.60 |
| \( B_{S3} \) | -0.79 ± 0.13 | -0.62 ± 0.12 | -1.05 ± 0.23 | -0.98 ± 0.17 |
| \( A_{S4} \) | -0.42 ± 0.09 | -0.05 ± 0.05 | -1.90 ± 0.15 | -0.87 ± 0.17 |
| \( B_{S4} \) | -0.43 ± 0.18 | -1.78 ± 0.45 | -0.14 ± 0.07 | -0.35 ± 0.14 |
the Gaussian filter. The minimum $\Delta \Omega_m$ decreases toward lower $\Omega_m$. Because of the $S_3 \propto \Omega_m^{-0.8}$ behavior (Table 1), at low $\Omega_m$ a small change in $\Omega_m$ results in a large change in $S_3$. However, $\Delta S_3$ does not have such a strong $\Omega_m$ dependence; thus, the resulting error in $\Omega_m$ decreases toward lower $\Omega_m$. In addition, we show in Figure 6 the relative uncertainty $\Delta \Omega_m/\Omega_m$ as a function of $\Omega_m$ smoothed at the optimal filter radius for all of the filters. We show that the relative uncertainty $\Delta \Omega_m/\Omega_m$ for the compensated Gaussian filter almost stays constant with $\Omega_m = 0.1$ and takes the smallest value in the range of interest from $\Omega_m = 0.2$ to 0.6, compared with that of the other filters. By comparing the minimum of $\Delta \Omega_m$ for each filter class, we conclude that the compensated Gaussian filter is the optimal filter for all cosmologies. The relative uncertainty $\Delta \Omega_m/\Omega_m$ at the optimal filter scale for this filter is nearly independent of $\Omega_m$, and the corresponding optimal filter scale is about $2/5$.

4. CONCLUSIONS

We have studied the power of weak-lensing surveys to measure the matter density of the universe without relying on any exterior data sets. We have found that the CFHT Legacy Survey can measure a fractional accuracy in $\Omega_m$ of 10%, which is competitive with global joint analyses but bypasses a large number of cross-calibration uncertainties.

We have run a series of high-resolution $N$-body simulations to study statistical skewness properties of weak lensing by large-scale structure in the universe with a range of cosmological matter density parameters. We have added noise due to intrinsic ellipticity of background faint galaxies to the simulated $\kappa$ fields and smoothed it using different filters with a range of smoothing radii. We have calculated the skewness $S_3$ of the smoothed $\kappa$ field with added Gaussian noise and predicted the uncertainty $\Delta \Omega_m$ for the cosmological mass density parameter for a given $S_3$ and smoothing radius $\theta_s$. We have examined the relative discriminating power of different window functions for distinguishing cosmological models in the upcoming CFHT Legacy Survey. Except for the Wiener filter, we have found the optimal smoothing radius for all of the window functions that minimizes $\Delta \Omega_m$. This optimal smoothing scale has only a weak dependence on cosmology. The compensated Gaussian function is the optimal filter for measuring $\Omega_m$ from skewness. The relative uncertainty $\Delta \Omega_m/\Omega_m$ smoothed at the optimal filter radius for the compensated Gaussian filter is about 10%.

To overcome the irregularity of the simulated $S_3$ and $\Delta S_3$, we have fitted their smoothing scale and cosmology dependence with some phenomenological power laws. One could derive these relations analytically using perturbation theory following the theoretical work of Bernardeau et al. (1997), but since skewness is intrinsically nonlinear, such a perturbation approach has to be tested against simulations. In fact, in our work based on simulations, the optimal filter radius is a few arcminutes or $\sim 1$ Mpc $h^{-1}$, which lies in the strongly nonlinear regime, where perturbation theory breaks down (Gaztanaga & Bernardeau 1998). In the nonlinear regime, a semianalytical model, hyperextended perturbation theory (HEPT; Scoccimarro & Frieman 1999), which applies at the highly nonlinear regime, and a fitting formula to interpolate between the quasi-linear and highly
nonlinear regimes (Scoccimarro & Couchman 2001) have been applied to predict $S_3$ (Hui 1999; Van Waerbeke et al. 2001). Since these models rely on simulations for calibration, they by no means can produce better results than simulations. Furthermore, to calculate the lensing $D_{S_3}$ analytically, one has to know the $S_6$ of the density field, which can be predicted by HEPT but has not been tested against simulations (Scoccimarro & Frieman 1999). So we would rather use our fitting formula approach instead of adopting these analytical results.

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REFERENCES

Bacon, D. J., Refregier, A. R., & Ellis, R. S. 2000, MNRAS, 318, 625
Bernardeau, F., Mellier, Y., & Van Waerbeke, L. 2002, A&A, 389, L28
Bernardeau, F., Van Waerbeke, L., & Mellier, Y. 1997, A&A, 322, 1
Blanchard, A., Douspis, M., Rowan-Robinson, M., & Sarkar, S. 2003, A&A, in press
Blandford, R. D., Saust, A. B., Brainerd, T. G., & Villumsen, J. V. 1991, MNRAS, 251, 600
Brown, M. L., Taylor, A. N., Bacon, D. J., Gray, M. E., Dye, S., Metzenger, K., & Wolf, C. 2003, MNRAS, 341, 100
Contaldi, C. R., Hoekstra, H., & Lewis, A. 2003, Phys. Rev. Lett., 90, 221303
Crittenden, R. G., Natarajan, P., Pen, U.-L., & Theuns, T. 2002, ApJ, 568, 20
Dubinski, J., Kim, J., Park, C., & Humble, R. J. 2003, NewA, submitted (astro-ph/0304467)
Gaztanaga, E., & Bernardeau, F. 1998, A&A, 331, 829
Gunn, J. E. 1967, ApJ, 147, 61
Hamana, T., et al. 2003, ApJ, 597, 98
Hoekstra, H., Yee, H. K. C., Gladders, M. D., Barrientos, L. F., Hall, P. B., & Infante, L. 2002, ApJ, 572, 55
Hui, L. 1999, ApJ, 519, L9

Jain, B., Seljak, U., & White, S. 2000, ApJ, 530, 547
Jarvis, M., Bernstein, G., Jain, B., Fischer, P., Smith, D., Tyson, J. A., & Wittman, D. 2003, AJ, 125, 1014
Juszkiewicz, R., Weinberg, D. H., Amsterfamski, P., Chodorowski, M., & Bouchet, F. 1995, ApJ, 442, 39
Kaiser, N. 1992, ApJ, 388, 272
Miralda-Escude, J. 1991, ApJ, 380, 1
Nusser, A., & Dekel, A. 1993, ApJ, 405, 437
Pen, U.-L., Spergel, D. N., & Turok, N. 1994, Phys. Rev. D, 49, 692
Pen, U.-L., Zhang, T., van Waerbeke, L., Mellier, Y., Zhang, P., & Dubinski, J. 2003, ApJ, 592, 664
Refregier, A., Rhodes, J., & Groth, E. J. 2002, ApJ, 572, L131
Scoccimarro, R., & Couchman, H. M. P. 2001, MNRAS, 325, 1312
Scoccimarro, R., & Frieman, J. A. 1999, ApJ, 520, 35
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Spergel, D. N., et al. 2003, ApJS, 148, 175
Tegmark, M., & Zaldarriaga, M. 2002, Phys. Rev. D, 66, 103508
Van Waerbeke, L., Hamana, T., Scoccimarro, R., Colombi, S., & Bernardeau, F. 2001, MNRAS, 322, 918
Van Waerbeke, L., Mellier, Y., Pello, R., Pen, U.-L., McCracken, H. J., & Jain, B. 2002, A&A, 393, 369
White, M., & Hu, W. 2000, ApJ, 537, 1