Superluminal Signal Velocity

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July 31, 2021

Abstract

It recently has been demonstrated that signals conveyed by evanescent modes can travel faster than light. In this report some special features of signals are introduced and investigated, for instance the fundamental property that signals are frequency band limited.

Evanescent modes are characterized by extraordinary properties: Their energy is negative, they are not directly measurable, and the evanescent region is not causal since the modes traverse this region instantaneously. The study demonstrates the necessity of quantum mechanical principles in order to interpret the superluminal signal velocity of classical evanescent modes.

1 Introduction

Tunneling represents the wave mechanical analogy to the propagation of evanescent modes [1]. Evanescent modes are observed, e.g. in the case of total reflection, in undersized waveguides, and in periodic dielectric heterostructures [2]. Compared with the wave solutions an evanescent mode is characterized by a purely imaginary wave number, so that i.e. the wave equation yields for the electric field $E(x)$

$$E(x) = e^{i(\omega t - kx)} \Rightarrow E(x) = e^{i\omega t - \kappa x},$$

(1)

where $\omega$ is the angular frequency, $t$ the time, $x$ the distance, $k$ the wave number, and $\kappa = ik$ the imaginary wave number of the evanescent mode.

Thus evanescent modes are characterized by an exponential attenuation and a lack of phase shift. The latter means that the mode has not spent time in the evanescent region, which in turn results in an infinite velocity in the phase time approximation neglecting the phase shift at the boundary [3].

Two examples of electromagnetic structures in which evanescent modes exist are shown in Fig.1 [2]. The dispersion relations of the respective transmission coefficients are displayed in the same figure.
Figure 1: Two examples of electromagnetic structures with evanescent mode solutions: (a) of a waveguide with an undersized central part and (b) a one-dimensional periodic hetero-structure. $n_1$ and $n_2$ are the refractive index and $c$ and $d$ the thickness of the dielectric materials. In (c) the graphs show the dispersion relations for both structures. The transmission dispersion of the periodic heterostructure displays a forbidden gap which corresponds to a tunneling regime, for details see Ref.[2]. The evanescent regime is characterized by a strong attenuation due to the exponential decay.

2 Signals

Quite often a signal is said to be defined by switching on or off light. It is assumed that the front of the light beam informs my neighbour of my arrival home with the speed of light. Such a signal is sketched in Fig.2. The inevitable inertia of the light source causes an inclination of the signal’s front and tail. Due to the detector’s sensitivity, level $D_s$, the information about the neighbours arrival (switching on) and departure (switching off) becomes dependent on intensity. In this example the departure time is detected earlier with the attenuated weak signal.

This special signal does not transmit reliable information on arrival and departure time. In addition to the dependence on intensity, a light front may be generated by any spontaneous emission process or accidentally by another neighbour. Obviously, a detector needs more than a signal’s front to response properly. The detector needs information about carrier frequency and modulation of the signal in order to obtain reliable information about the cause.

In general, a signal is detected some time after the arrival of the light’s front. Due to the dynamics of detecting systems there are several signal oscillations needed in order to produce an effect \[\square\]. An
Figure 2: Sketch of a light train. The two detected fronts of the same attenuated and not attenuated light beam are $F_1$, $F_2$ and the tails are $T_1$, $T_2$. They are measured at different places and times in consequence of the detector’s responsivity level $D_s$.

*effect* is detected with the energy velocity. In vacuum or in a medium with normal dispersion the signal velocity equals both, the energy and the group velocities.

For example, a classical signal can be transmitted by the Morse alphabet, in which each letter corresponds to a certain number of dots and dashes. In general signals are either frequency (FM) or amplitude modulated (AM) and they have in common that the signal does not depend on its magnitude. A modern signal transmission, where the halfwidth corresponds to the number of digits, is displayed in Fig.3. This AM signal has an infra-red carrier with $1.5\mu m$ wave length and is glass fiber guided from transmitter to receiver. As mentioned above, the signals are independent of magnitude as the halfwidth does not depend on the signal’s magnitude.

The front or very beginning of a signal is only well defined in the theoretical case of an infinite frequency spectrum. However, physical generators only produce signals of finite spectra. This is due to their inherent inertia and due to a signal’s finite energy content. These properties result in a real front which is defined by the measurable beginning of the signal. For example the signals of Fig.3 have a detectable frequency band width of $\Delta\nu = \pm 10^{-4} \cdot \nu_C$, where $\nu_C$ is the carrier frequency.

Frequency band limitation in consequence of a finite signal energy reveals one of the fundamental deficiencies of classical physics. A classical detector can detect a deliberately small amount of energy, whereas every physical detector needs at least one quantum of the energy $\hbar \omega$ in order to respond.
\textbf{3 An Experimental Result}

Superluminal signal velocities have been measured by Enders and Nimtz \cite{4,5,6}. The experiments were carried out with AM microwaves in undersized waveguides and in periodic dielectric heterostructures. The measured propagation time of a pulse is shown in Fig.\ref{fig:figure4}. The microwave pulse has travelled either through air or it has crossed an evanescent barrier \cite{6}. The linewidth of the pulse represents the signal. The experimental result is, that the tunneled signal has passed the airborne signal at a superluminal velocity of \(4.7 \cdot c\). The measurements of the traversal time are carried out under vacuum-like conditions at the exit of the evanescent region, the reason for this will be discussed later.

\textbf{4 Some Implications of superluminal signal velocity}

Measured microwave signals are shown in Fig.\ref{fig:figure4}. The halfwidth (information) of the tunneled signal has traversed the evanescent region at a velocity of \(4.7 \cdot c\). As explained above, signals have a limited frequency spectrum since their energy content \(W\) is always finite and detectable frequency components with \(\omega \geq W/h\) can not exist.

In this experiment all frequency components of the signal are evanescent and move at a velocity faster than \(c\). The beginning of the evanescent signal overtakes that of the airborne signal as seen in Fig.\ref{fig:figure4}. The superluminal velocity of evanescent modes has some interesting features differing fundamentally from luminal or subluminal propagation of waves with real wave numbers. This will be discussed in the following subsections.

\textbf{4.1 Change of chronological order}

The existence of a superluminal signal velocity ensures the possibility of an interchange of chronological order. Let us assume an inertial system \(\Sigma_{II}\) moves away from system \(\Sigma_{I}\) with a velocity \(v_r\). The
Figure 4: Measured barrier traversal time of a microwave packet through a multilayer structure inside a waveguide (barrier length 114.2 mm). The center frequency has been 8.7 GHz with a frequency width of ± 0.5 GHz. The pulse’s magnitudes are normalized. The slow pulse (1) traversed the empty waveguide, whereas the fast one (2) has tunneled the forbidden gap of the same length. The maximum corresponds to the center of mass and equals the group velocity. The group velocity of the tunneled signal was $4.7 \cdot c$ [6]. The transmission dispersion of the barrier is shown in Fig.1(c) curve b). The tunneled signal (i.e. the halfwidth of the pulse) traversed the 114.2 mm long barrier in 81 ps, whereas the signal spent 380 ps to cross the same air distance. The time resolution in the experiment has been better than ± 1 ps [4, 6].
Special Relativity (SR) gives the following relationship for the travelling time $\Delta t$ and for the distance $\Delta x$ of a signal in the system $\Sigma_I$ which is watched in $\Sigma_{II}$

$$\Delta t_{II} = \Delta t_I - v_r \frac{\Delta x_I}{c^2} \left(1 - \frac{v^2}{c^2}\right)^{1/2} \frac{v_S}{c} \left(1 - \frac{v^2}{c^2}\right)^{1/2} \right).$$  

(2)

$v_r \geq c^2/v_S$ is the condition for the change of chronological order, i.e. $\Delta t_{II} \leq 0$, between the systems $\Sigma_I$ and $\Sigma_{II}$. For example, at a signal velocity $v_S \geq 10 c$ the chronological order changes at $v_r \leq 0.1 c$. This result does not violate the SR. The common constraint $v_S \leq c$ is posed by the SR on electromagnetic wave propagation in a dispersive medium and not on the propagation of evanescent modes.

### 4.2 Negative electromagnetic energy

The Schrödinger equation yields a negative kinetic energy in the tunneling case, where the potential $U$ is larger than the particle’s total energy $W$:

$$\frac{d^2 \Psi}{dx^2} + \frac{2m}{\hbar^2} (W - U) \Psi = 0 \quad (3)$$

The same happens to evanescent modes. Within the mathematical analogy their kinetic electromagnetic energy is negative too. The Helmholtz equation for the electric field $E$ in a waveguide is given by the relationship

$$\frac{d^2 E}{dx^2} + (k^2 - k_c^2) E = 0 \quad (4)$$

where $k_c$ is the cut-off wave number of the evanescent regime. The quantity $(k^2 - k_c^2)$ plays a role analogous to the energy eigenvalue and is negative in the case of evanescent modes.

The dielectric function $\epsilon$ of evanescent modes is negative and thus the refractive index is imaginary.

For the basic mode a rectangular waveguide has the following dispersion of its dielectric function, where $k_c = (\pi/b)^2$ holds and $b$ is the waveguide width,

$$\epsilon(\lambda_0) = \left(1 - \frac{\lambda_0}{2b}\right) \quad (5)$$

$\lambda_0$ is the free space wavelength of the electromagnetic wave.

In the case of tunneling it is argued that a particle can only be measured in the barrier with a photon having an energy $\hbar \omega \geq (U - W) \Box$. This means that the total energy of the system is positive. According to Eq.(5) the evanescent mode’s electric energy density $\rho$ is given by the relationship
\[
\rho = \frac{1}{2} \epsilon \epsilon_0 E^2 < 0, \tag{6}
\]

where \(\epsilon_0\) is the electric permeability.

The analogy between the Schrödinger equation and the Helmholtz equation holds again and it is not possible to measure an evanescent mode. Achim Enders and I tried hard to measure evanescent modes with probes put into the evanescent region but failed [8]. Obviously evanescent modes are not directly measurable in analogy to a particle in a tunnel. We might also say this problem is due to impedance mismatch between the evanescent mode and a probe. The impedance \(Z\) of the basic mode in a rectangular waveguide is given by the relationship

\[
Z = Z_0 \epsilon^{-1/2} \tag{7}
\]

where \(Z_0\) is the free space impedance. In the evanescent regime \(k < k_c\) the impedance is imaginary.

### 4.3 The not-causal evanescent region

Evanescent modes do not experience a phase shift inside the evanescent region [2, 3]. They cross this region without consuming time. The predicted [3] and the measured [2] time delay happens at the boundary between the wave and the evanescent mode regime. For opaque barriers (i.e. \(\kappa \cdot x \geq 1\), where \(\kappa\) is the imaginary wave number and \(x\) the length of the evanescent barrier) the phase shift becomes constant with \(\approx 2\pi\) which corresponds to one oscillation time of the mode. In fact, the measured barrier traversal time was roughly equal to the reciprocal frequency in the microwave as well as in the optical experiments, i.e. either in the 100 ps or in the 2 fs time range independent of the barrier length [3]. The latter behaviour is called Hartman effect: the tunneling time is independent of barrier length and has indeed been measured with microwave pulses thirty years after its prediction [5].

### 5 Summing up

Evanescent modes show some amazing properties with which we are not familiar. For instance the evanescent region is not causal since evanescent modes do not spend time there. This is an experimental result due to the fact that the traversal time is independent of barrier length.

Another strange experience in classical physics is that evanescent fields cannot be measured. This is due to their negative energy or to the impedance mismatch. Amazingly enough this is in analogy with the wave mechanical tunneling.

The energy of signals is always finite resulting in a limited frequency spectrum both according to Planck’s energy quantum \(\hbar \omega\). This is a fundamental deficiency of classical physics, which assumes the
measurability of any small amount of energy. A physical signal never has an ideal front. The latter needs infinite high frequency components with a correspondingly high energy.

Another consequence of the frequency band limitation of signals is, if they have only evanescent mode components, they are not Lorentz-invariant. The signal may travel faster than light.

Front, group, signal, and energy velocities all have the same value in vacuum. Bearing in mind the narrow frequency band of signals, the former statement holds also for the velocities of evanescent modes. In first order approximation the dispersion relation of a stop band is constant and a significant pulse reshaping does not take place. This result demonstrate that signals and effects may transmitted with superluminal velocities provided that they are carried by evanescent modes.

6 Acknowledgments

Stimulating discussions with V. Grunow, D. Kreimer, P. Mittelstaedt, R. Pelster, and H. Toyatt are gratefully acknowledged.

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