Dynamic response analysis of polyoxymethylene hydrofoils using the hybrid pitch mode FSI method

W-S Jang¹, W-S Choi¹, S-Y Hong¹, J-H Song² and H-W Kwon³

¹Department of Naval Architecture and Ocean Engineering, Seoul National University, Seoul, South Korea
²Department of Naval Architecture and Ocean Engineering, Chonnam National University, Yeosu, South Korea
³Department of Naval Architecture and Ocean Engineering, Koje College, Koje, South Korea

E-mail: syh@snu.ac.kr

Abstract. In this paper, the objective is to analyze the dynamic responses of 2D pitching polyoxymethylene (POM) hydrofoils in the uniform flow using the hybrid coupling Fluid-Structure Interaction (FSI) method, which takes account of added mass effects and show its robustness by comparing with the experiments. First, FSI analysis using the empirical formula of hydrodynamic forces around hydrofoils was performed and validated by comparing the analysis results of the weak coupling FSI method and the hybrid coupling FSI method with the analytic solution. Next, The FSI method with Computational Fluid Dynamics (CFD) was implemented in the framework of OpenFOAM, the open-source CFD code. Flow analysis and structural analysis were performed for every time step in the calculations. Here, the flow analysis was performed using Large Eddy Simulation (LES) scheme and the structural analysis was performed by solving the equation which considered the pitch mode of deformable hydrofoils. Dynamic responses of 2D NACA0015 POM hydrofoils using the hybrid coupling FSI method well matched with the experimental results qualitatively, where dynamic responses without the hybrid coupling FSI method showed numerical instability. It is shown that the hybrid coupling FSI method considering added mass effects should be considered for analysis of deformable hydrofoils.

1. Introduction
Most ships and submarines generally use rigid propellers made of Nickel-Aluminum-Bronze (NAB) alloy or stainless steel. However, composite propellers and hydrofoils have been a very active area of research recently. There are many advantages of composite materials for propellers and hydrofoils, such as high strength to weight, high stiffness to weight ratios, high Cavitation-Inception-Speed (CIS) and low subcavitating noises [1,2].

Computational Fluid Dynamics (CFD) methods are generally used for predicting performances of rigid hydrofoils. However, it is unable to predict composite hydrofoils with CFD analysis because they suffer from large displacements which leads to instantaneous variations of the flow field. Hence, Fluid-Structure Interaction (FSI) analysis is needed to predict dynamic responses of composite hydrofoils taking account of their large displacements. Fluid-structure interactions represent interactions among hydrodynamic forces, elastic forces, and inertia forces.

Numeric procedures to solve these FSI problems are classified into two approaches, the monolithic
and the partitioned approach [3]. The monolithic approach regards coupled fluid-structure system as a whole, solving them simultaneously. Although it has the advantage of ensuring stability and accelerating convergence, order differences in fluid and structural properties may result in the ill-conditioned matrix, and most of all, it is a great challenge to implement completely new analyzing code [4]. On the other hand, the partitioned approach, also known as the staggered approach, solves fluid domain and structure domain sequentially and each result transfers to the other domain as boundary conditions [5]. This approach is not mathematically rigorous or stable compared to the monolithic approach. However, the partitioned approach is still actively used as FSI analysis approach because it has a great advantage of being able to utilize ‘legacy’ codes used in each area [3].

The partitioned approach is also classified into two coupling methods, the weak (loose) coupling method and the strong (tight) coupling method. The difference between the weak coupling method and the strong coupling method is whether or not there are iterations in the coupling process. The weak coupling method, a coupling method without iterations, takes less computing time while it does not accurately solve situations with large displacements and large added mass. The strong coupling method, on the contrary, a coupling method with iterations, predicts responses more accurately than the weak coupling method while it takes a lot of computation time.

Various attempts are being researched to take large displacements and large added mass effects into account with relatively less computation time. Astorino et al, Fernandez et al developed a semi-implicit coupling method implementing robin boundary conditions [6,7]. Young and Chae et al predicted responses of flexible propellers and hydrofoils using the hybrid coupling method [1,8,9].

In this paper, dynamic responses of 2D pitching polyoxymethylene (POM) hydrofoils in water, which is the case where substantial added mass effects are expected, were predicted using the hybrid coupling FSI method. The simple hybrid coupling FSI analysis was performed using the empirical formula of hydrodynamic forces to validate the hybrid coupling FSI analysis. Next, the hybrid coupling FSI analysis using CFD was performed to predict dynamic responses of pitching POM hydrofoils.

2. Methods
Two-dimensional symmetric hydrofoils (see figure 1) were used for analyses where $c$ denotes the chord length of the hydrofoil and $b (= c/2)$ denotes the mid-chord length. The elastic axis, defined as the locus of shear centers, is positioned at distance $ab (= ac/2)$ from the mid-chord. Moment and pitch angle $\theta$ have clockwise positive direction.

![Figure 1. Two-dimensional symmetric hydrofoil undergoing pitch motion.](image)

2.1. Flow analysis
Consider a 2D hydrofoil undergoing pitch motion with frequency $\omega$ in the uniform flow. The solution for the moment about the elastic axis is written as in equation (1) [10].

$$M_T = \pi \rho b^2 [-V b (0.5 - a) \dot{\theta} - b^2 (0.125 + a^2) \ddot{\theta}] + 2 \pi \rho V b^2 (a + 0.5) C(k) [V \theta + b (0.5 - a) \dot{\theta}]$$

(1)
where \( \rho \) and \( V \) denote fluid density and uniform flow velocity each and \( k (= \omega b/V) \) denotes reduced frequency. \( C(k) \) is Theodorsen function and it is written as equation (2). Theodorsen function represents a relation between quasi-steady and unsteady hydrodynamic forces of hydrofoils undergoing pitch motion at reduced frequency \( k \). Length ratio \( a \), related with elastic axis, contributes to the elasticity because hydrodynamic elastic force \( 2\pi \rho Vb^2(a + 0.5)C(k)V\theta \) is function of \( a \).

\[
C(k) = \frac{H_1'(k)}{H_1'^2(k) + iH_0'^2(k)}
\]

(2)

\( H_n^{(2)}(k) \) denote Hankel functions of the second kind.

Equation (1) is obtained by the potential flow assumption. According to potential flow theory, flow is incompressible, inviscid and irrotational. However, there are viscous effects like boundary layers in practice and they should be considered to predict dynamic responses accurately. Thus, hydrodynamic forces should be obtained by using CFD because the forces using CFD include forces obtained by equation (1) and also viscous forces.

Without CFD, equation (3) represents an empirical formula for the hydrodynamic force of the forced oscillating NACA 0009 hydrofoils in the uniform flow [11].

\[
M_m = -(J_m\ddot{\theta} + \mu_m\dot{\theta} + k_m\theta)
\]

(3)

Where \( J_m = 0.125\rho \pi b^4s \), \( \mu_m = 0.84\rho Vb^3sk^{-0.4} \) for \( k \leq 4 \), \( \mu_m = 0.08\rho Vb^3sk^{0.6} \) for \( k \geq 12 \), \( k_m = 2\rho V^2b^2s(0.18k - 1.6) \) for \( k \leq 4 \), \( k_m = 2\rho V^2b^2s(0.13k - 1.8) \) for \( k \geq 12 \). Here, \( s \) denotes span.

2.2. Structure analysis

Structure analysis considered only pitch motion of hydrofoils. Thus, structure analysis can be performed by equation (3) which is a simple 1-degree-of-freedom equation of motion about pitch angle \( \theta \) of hydrofoils. \( I \), \( C \) and \( K \) denote moment of inertia, structural damping parameter and structural stiffness respectively.

\[
I\ddot{\theta} + C\dot{\theta} + K\theta = F
\]

(4)

2.3. Partitioned hybrid coupling FSI method

In the partitioned approach, there are two boundary conditions for FSI analysis. First, fluid mesh velocity should be equal to the time derivative of structural displacement on the fluid-structure interface. The second boundary condition is that stress at each domain should be equal on the fluid-structure interface. Using the second boundary condition, equation (5) is obtained by coupling equation (4) and the hydrodynamic forces of hydrofoils.

\[
I\ddot{\theta} + C\dot{\theta} + K\theta = M_{\text{CFD}}
\]

(5)

\( M_{\text{CFD}} \) can be replaced by \( M_M \) which is the empirical formula of hydrodynamic forces without CFD analysis [11]. If FSI analysis is performed at each time step in this condition, this is the weak coupling approach.

\( M_{\text{CFD}} \) includes the potential hydrodynamic force in the form of equation (1), which is functions of \( \ddot{\theta}, \dot{\theta}, \theta \). If their effects are reflected in the structure analysis matrix, the dynamic responses of hydrofoils between old time step and new time step can be more accurately predicted during the FSI analysis process. The following equation (6) considers the effects of equation (1) as added mass, added damping and added stiffness and this FSI analysis is called the hybrid coupling FSI method.

\[
(I + I_a)\ddot{\theta} + (C + C_a)\dot{\theta} + (K + K_a)\theta = M_{\text{CFD}} - M_T
\]

(6)

Where \( I_a = \pi \rho b^4(0.125 + a^2) \), \( C_a = \pi \rho b^3V(0.5 - a)(1 - (2a + 1)C(k)) \), \( K_a = -2\pi \rho V^2b^2(a + 0.5)C(k) \).
If $M_{\text{CFD}}$ is replaced by $M_M$, all terms of equation (6) can be described as a function of $\dot{\theta}, \dot{\theta}, \theta$. Therefore, it is possible to obtain an exact solution without numerical analysis.

3. Result and discussion

3.1. Non-CFD FSI analysis

FSI analysis predicting dynamic responses of NACA 0009 hydrofoils of the pitching frequency $\omega$ were performed and validated. The parameters of the analysis are given in table 1. Runge-Kutta 4th order scheme was used for the time derivative scheme and flow analysis was performed by the empirical formula [11].

| Parameter | Rigid hydrofoil | Deformable hydrofoil |
|-----------|----------------|----------------------|
| $I$ (kg$\cdot$m) | $1.0 \times 10^{-2}$ | $1.428 \times 10^{-3}$ |
| $C$ (kg$\cdot$m/s) | 0.253 | 0.096 |
| $K$ (kg$\cdot$m/s$^2$) | 1000 | 1000 |
| $c$ (m) | 0.1 | 0.1 |
| $b$ (m) | 0.05 | 0.05 |
| $a$ (-) | 0 | 0 |
| $V$ (m/s) | 5 | 5 |
| $k$ (-) | $2.63 \times 10^{-2}$ | $4.84 \times 10^{-2}$ |
| $\omega$ (rad/s) | 2.63 | 4.84 |
| $\rho$ (kg/m$^3$) | 1000 | 1000 |
| $M_{\text{forced}}$ (Nm) | 34.9 | 34.9 |
| Time step (s) | $5.0 \times 10^{-3}$ | $2.70 \times 10^{-3}$ |

This analysis was aimed at forced pitching hydrofoils. Thus, there was an additional moment term $M_{\text{forced}} \times \sin(\omega t)$ at the right hand side of equations (5) and (6). The weak coupling FSI method and the hybrid coupling FSI method were used for predicting dynamic responses of two different materials and compared with the analytic solution.

The results of non-CFD FSI analysis were given as figure 2. For rigid hydrofoils which have high structural density, both coupling methods showed the same results with the analytic solution. On the other hand, dynamic responses of deformable hydrofoils varied depending on the coupling method. While the response of the weak coupling FSI method diverged, the response of the hybrid coupling FSI method converged and was shown to be the same as the analytic solution.

In the case of deformable hydrofoils, the structural density is relatively low and fluid around them has relatively high density making mass ratio $\rho_{\text{fluid}}/\rho_{\text{structure}}$ close to 1. Thus, the effects of added mass, added damping, added stiffness by the flow field around the hydrofoil become relatively significant. Hence, the hybrid coupling FSI method that considered those added effects can predict dynamic responses of deformable hydrofoils. It is difficult to predict dynamic responses of deformable hydrofoils with the weak coupling FSI method that ignores the added effects. If the weak coupling FSI method is used to predict dynamic responses of deformable hydrofoils, relatively small time step is required, which is inefficient in the respect of computation time.

The results showed the same results with Young’s results [12].
3.2. FSI analysis

Dynamic responses of POM NACA0015 hydrofoils undergoing only pitch motion in the uniform flow were obtained through FSI analysis using CFD. The FSI analysis was implemented in the framework of OpenFOAM, an open source CFD software. Flow analysis was performed by OpenFOAM’s own solver and structural analysis was performed by the same method as in subsection 3.1. The hybrid coupling FSI method was used to take the added effects into account and POM which has low density was used for hydrofoils’ material to observe dynamic responses of POM hydrofoils and to compare the responses with the results of Chae [8]. Fluid mesh for the FSI analysis was given in Figure 3. To precisely obtain the transient characteristics of the hydrodynamic forces, the Large Eddy Simulation(LES) method was selected for the turbulence modeling method and the y+ value of the fluid mesh around the hydrofoil was set to 1 or less. Detailed parameters were given in Table 2.

The results of the FSI analysis compared to the experimental result by Chae was given by Figure 4 [8]. Results of the FSI analysis using the weak coupling FSI method diverged. Thus, it is difficult to obtain dynamic responses of POM hydrofoils using the weak coupling FSI method and Figure 4 contained only the analysis result by the hybrid coupling FSI method. The result by the hybrid coupling FSI method takes approximately 3 days for parallel calculation using 5 CPUs. The result showed that the order of responses’ amplitude and peak frequency of the FSI analysis is equal to the experimental result. Contrary to FSI results, the experimental results had two frequency peaks because the experiments considered motions of all modes. Therefore, the FSI analysis should consider the other modes to obtain more accurate dynamic response predictions of POM hydrofoils.
Table 2. Parameters for the FSI analysis.

| Parameter | POM hydrofoil |
|-----------|---------------|
| $I$ $(kg\cdot m)$ | $1.28 \times 10^{-4}$ |
| $C$ $(kg\cdot m/s)$ | $1.26 \times 10^{-2}$ |
| $K$ $(kg\cdot m/s^2)$ | 769.88 |
| $c$ $(m)$ | 0.1 |
| $a$ (-) | -0.28 |
| $AOA$ $(\degree)$ | 8 |
| $\rho$ $(kg/m^3)$ | 1000 |
| $Time \ step$ $(s)$ | $1.0 \times 10^{-5}$ |

Figure 4. Dynamic responses of POM hydrofoils obtained by the FSI analysis and experiments.

4. Conclusion
To predict dynamic responses of composite hydrofoils, Fluid-structure interaction (FSI) analysis should be performed and in particular, various FSI methods taking account of added mass effects are being researched. In this paper, dynamic responses of 2D pitching POM hydrofoils in water, which is the case where substantial added mass effects are expected, were predicted using the hybrid coupling FSI method.

Non-CFD FSI analysis based on an empirical formula to predict dynamic responses of NACA 0009 hydrofoils of the pitching frequency $\omega$ in the uniform flow were performed and validated. Dynamic responses of rigid hydrofoils can be obtained by both the weak coupling FSI method and the hybrid coupling FSI method. However, dynamic responses of deformable hydrofoils can be obtained only by the hybrid coupling FSI method because added mass, added damping, added stiffness have significant effects on the dynamic responses.

Dynamic pitching responses of POM NACA0015 hydrofoils in the uniform flow were obtained through FSI analysis using the hybrid coupling FSI method and compared to experimental results. The order of response amplitude and the peak frequency of FSI analysis are equal to the response of experiments. On the contrary to the weak coupling FSI method, FSI analysis using the hybrid coupling FSI method can predict the tendency of dynamic responses of POM hydrofoils. However, there is a limit that FSI analysis considering only the pitch motion has difficulty in predicting accurate dynamic responses.
responses of POM hydrofoils in practice.

For future work, the other modes should be considered in order to obtain accurate dynamic responses of POM hydrofoils. Furthermore, other natural modes can be considered to obtain more accurate dynamic responses.

Acknowledgments
This research was funded by the Education and Research Center for Creative Offshore Plant Engineers. Also, supported by Future Submarine Low Noise Laboratory.

References
[1] Young Y L 2008 Fluid-structure interaction analysis of flexible composite marine propellers J. Fluid and Struc. 24 799-818
[2] Lee H S 2017 A study on the BEM-FEM based hydroelastic analysis of composite marine propellers (Seoul National University)
[3] Hou G, Wang J and Layton A 2012 Numerical methods for fluid-structure interaction: A review Community of Comp. Phys 12 337-77
[4] Hubner B, Walhorn E and Dinkler D 2004 A monolithic approach to fluid-structure interaction using space-time finite elements Comp. Methods Appl. Mech. Eng. 193 2087-104
[5] Campbell R L 2010 Fluid-structure interaction and inverse design simulations for flexible turbomachinery (Pennsylvania State University)
[6] AStrorino M, Chouly F and Fernandez M A 2009 Robin based semi-implicit coupling in fluid-structure interaction: stability analysis and numerics Society for Industrial and Appl. Math. 31 4041-65
[7] Fernandez M A, Mullaert J and Vidrascu M 2013 Explicit robin-neumann schemes for the coupling of incompressible fluids with thin walled structures Comp. Methods of Appl. Mech. Eng. 267 566-593
[8] Chae E J 2015 Dynamic response and stability of flexible hydrofoils in incompressible and viscous flow (University Michigan)
[9] Chae E J, Akcabay D T, Lelong A, Astolfi J A and Young Y L 2016 Numerical and experimental investigation of natural flow-induced vibrations of flexible hydrofoils J. Phys. Fluids. 28 075102
[10] Theodorsen T 1949 General theory of aerodynamic instability and the mechanism of flutter: laboratory report 496 (Washington D C: National Advisory Committee for Aeronautics )
[11] Munch C, Ausoni P, Braun O, Farhat M and Avellan F 2010 Fluid-structure coupling for an oscillating hydrofoil J. Fluids and Struc. 26 1018-33
[12] Young Y L, Chae E J and Akcabay D T 2012 Hybrid algorithm for modeling of fluid-structure interaction in incompressible, viscous flows Acta Mechanica Sinica 28 1030-41