B_K from quenched QCD with exact chiral symmetry

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We present a calculation of the standard model ΔS = 2 matrix element relevant to indirect CP violation in K → ππ decays which uses Neuberger’s chiral formulation of lattice fermions. The computation is performed in the quenched approximation on a 16³ × 32 lattice that has a lattice spacing a ∼ 0.1 fm. The resulting bare matrix element is renormalized non-perturbatively. Our main result is B_K^RGI = 0.87(8)1±14, where the first error is statistical, the second is systematic and the third is an estimate of the uncertainty associated with the quenched approximation and with the fact that our kaons are composed of degenerate s and d quarks with masses m_s/2.

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Introduction. — Ginsparg-Wilson (GW) fermions [1–4], with their exact SU(N_f)_L × SU(N_f)_R chiral–flavor symmetry at finite lattice spacing [5], provide unique opportunities for exploring the physics of light quarks through numerical simulations of lattice QCD (e.g. reviews in [6, 7]). In this regularization the ΔS = 1 effective weak Hamiltonian renormalizes with the same pattern as in the continuum. As a consequence, in the presence of an active charm quark, the ΔI = 1/2 rule in K → ππ decays can be studied from the simpler K → π amplitudes without having to perform difficult power subtractions [8]. In addition, this exact chiral–flavor symmetry implies full O(a) improvement. In this letter we present the results of a quenched calculation of the matrix element of the ΔS = 2 effective weak Hamiltonian relevant to K^0–K^0 mixing, performed using Neuberger’s implementation of GW fermions [3, 4]. The bare matrix element is renormalized non-perturbatively in the RI/MOM scheme à la [9]. This calculation is the first study of a matrix element of a four-quark operator using a lattice formulation which has exact chiral–flavor symmetry at finite lattice spacing. It calls upon many of the ingredients required for studying the ΔI = 1/2 rule or ε′ (three-point functions, non-perturbative renormalization of four-quark operators, etc.), but avoids the numerically challenging computation of eye diagrams. Furthermore, because this ΔS = 2 matrix element has been extensively studied with other fermion formulations (e.g. reviews in [10, 11]), it provides a good test of the reliability of our approach.

In addition to using Neuberger fermions, we make a number of other improvements on the methods traditionally used to compute ΔS = 2 matrix elements. In order to eliminate sizeable unphysical finite-volume contributions from topology-induced fermion zero-modes, we use the fact that the relevant ΔS = 2 operator is purely left-handed and consider correlation functions constructed from left-left quark propagators only. These propagators are free from zero-mode contributions, as the latter are chiral. Thus, to create and destroy kaons at rest, we use the time component of left-handed, quark-bilinear currents. The use of left-handed currents was proposed in [12, 13] in the context of the ε′-regime of Gasser and Leutwyler [14]. We also use chiral sources in the RI/MOM non-perturbative renormalization (NPR) of our operators to avoid zero-mode contributions, which would otherwise appear. This has the added advantage of eliminating the leading chiral-symmetry-breaking contributions to the NPR Green functions.

K^0–K^0 mixing induces indirect CP violation in K → ππ decays, quantified by the parameter ε for which the theoretical expression [15] and experimental value [16] are:

ε e^{-i2} ≃ C_ε C_K(μ)B_K(μ)A^4λ^{10}[(1−ρ)η_2S_0(x_t)} (1)

+P(x_t, x_c, . . .)] = (2.282 ± 0.017) × 10^{-3},

with B_K defined through (F_π = 93 MeV)

⟨K^0|O_{ΔS=2}(μ)|K^0⟩ = \frac{16}{3}M_K^2F_K^2 × B_K(μ) (2)

and

O_{ΔS=2} = [\bar{S}_γμ(1−γ_5)d][\bar{S}_γμ(1−γ_5)d]. (3)

Here, C_ε is obtained from well measured quantities, A, λ, ρ and η are Wolfenstein parameters [17] and η_2, C_K, S_0 and P incorporate perturbative, short-distance physics (P also contains CKM factors). Combined with a determination of B_K, the measurement of ε provides an important hyperbolic constraint on the summit (ρ, η) of the unitarity triangle, as indicated by Eq. (1). It is advantageous to compute B_K instead of the ΔS = 2 matrix element, because many lattice systematic errors cancel in the ratio that defines B_K. A preliminary version of the present calculation was reported in [18]. Preliminary results obtained by the MILC collaboration with a different
implementation of GW fermions were also published in the same volume [19].

**Computational details.**— The simulation is performed in quenched QCD with $\beta = 6.0$ and $V = 16^3 \times 32$. We use a sample of 80 gauge configurations, generated with the standard Wilson gluonic action, retrieved from the repository at the "Gauge Connection" (cf. http://qcd.nersc.gov). Fermion propagators are obtained from a local source, using Neuberger’s action [3, 4] with bare quark masses $am = 0.040, 0.055, 0.070, 0.085, 0.100$. The sign function of the Hermitian Wilson-Dirac operator, $X$, is obtained by making an optimal rational approximation [20–22], after explicit evaluation of the contributions from the lowest eigenvectors of $X^\dagger X$. The computation of the propagators thus uses nested multi-conjugate gradient inversions (for more details see [23, 24]). The lattice spacing is determined from $r_0 = 0.5$ fm, where $r_0$ is the Sommer scale [25]. This gives $a^{-1} = 2.12$ GeV.

**Meson correlation functions and fits.**— From the quark propagators we compute the two-point function

$$C_{J J}(x_0) = \sum_x \langle J_0(x) \bar{J}_0(0) \rangle , \quad (4)$$

and the three-point function

$$C_{J J J}(x_0, y_0) = \sum_{x, y} \langle J_0(x) \mathcal{O}_{\Delta S = 2}^{\text{bare}}(0) J_0(y) \rangle , \quad (5)$$

where $J_\mu = \bar{q} \gamma_\mu (1 - \gamma_5) q$, $\bar{q} = (1 - \gamma^\mu D) q$, $\rho = 1.4$ (see [23, 24]). $\bar{J}_0$ is obtained from $J_0$ with $s \leftrightarrow d$ and $\mathcal{O}_{\Delta S = 2}^{\text{bare}}$ is obtained from Eq. (2) with $d \to \bar{d}$. Statistical errors are estimated with the jackknife method.

The ratio

$$R(x_0, y_0) = \frac{3}{8} \frac{C_{J J J}(x_0, y_0)}{C_{J J}(x_0) C_{J J}(y_0)} \quad (6)$$

is fitted to the asymptotic form ($a \ll x_0 \ll y_0 \ll T$):

$$R(x_0, y_0) \to B_{K}^{\text{bare}} \quad (7)$$

where the fit parameter $B_{K}^{\text{bare}}$ is the bare bag parameter. The kaon mass $M_K$, on the other hand, is determined from a fit of the time-symmetrized, two-point function to the standard asymptotic form ($a \ll x_0$)

$$C_{J J}(x_0) \to \frac{Z^2}{M_K} e^{-M_K T^2 / 2} \cosh [M_K (T / 2 - x_0)] , \quad (8)$$

with $Z = |\langle K^0 | J_0 | 0 \rangle |$. We find that the two-point function is asymptotic from $ax_0 = 5$ on and that the statistically optimal fitting range for $aM_K$ is $5 \leq ax_0 \leq 12, 13, 14, 15$ and $16$ when $am = 0.040, 0.055, 0.070, 0.085$ and $0.100$, respectively.

For the ratio of Eq. (6), we find that the asymptotic behavior sets in for $ax_0 \geq 5$ and $ay_0 \leq 27$. Because our lattice is rather short in the time direction, we have to worry about time-reversed contributions. Assuming that the time-reversed matrix element is approximately the same as the forward one and that the two-kaon energy is approximately $2M_K$ (i.e. that finite-volume effects are not significant), then for $ax_0 \leq 6, ay_0 \geq 26$ the time-reversed contributions never exceed 1.5% of the forward signal. We also checked this explicitly by a fit which includes time-reversed contributions and allows for finite-volume shifts on the time-reversed matrix element and the two-kaon energy. We thus use the range $5 \leq ax_0 \leq 6$ and $26 \leq ay_0 \leq 27$ to calculate our observables. All fits are excellent and our results for $aM_K$ and $B_{K}^{\text{bare}}$ are summarized in Table I.

| $am$    | $aM_K$ | $B_{K}^{\text{bare}}(a)$ |
|---------|--------|-------------------------|
| 0.040   | 0.253(5) | 0.70(1)                |
| 0.055   | 0.288(4) | 0.71(6)                |
| 0.070   | 0.321(3) | 0.73(5)                |
| 0.085   | 0.352(3) | 0.74(4)                |
| 0.100   | 0.382(2) | 0.75(4)                |

**TABLE I:** Mesons masses and $B_{K}^{\text{bare}}(a)$ vs quark mass.

**Non-perturbative renormalization.**— We perform all renormalizations non-perturbatively in the RI/MOM scheme à la [9]. Thus, we fix gluon configurations to Landau gauge and numerically compute appropriate, amputated forward quark Green functions with legs of momentum $p \equiv \sqrt{p^2}$. We define the ratio

$$\mathcal{R}_{\text{RI}}(m, p^2, y_0) = \frac{\Gamma_{J}(m, p^2, y_0)}{\Gamma_{O}(m, p^2)} , \quad (9)$$

where $\Gamma_{O}$ is the value of the non-perturbative, amputated Green function of operator $O$ computed with left-handed quark sources and projected onto the spin-color structure of $O$. We compute this ratio for all five quark masses and perform a linear chiral extrapolation in $m^2$ as prescribed by a next-to-leading order expansion of the Green functions in quark mass. We find that the mass dependence is very mild and is well described by this linear form. We must then isolate the “perturbative part” of this ratio to get the renormalization constant appropriate for renormalizing $B_{K}^{\text{bare}}(a)$. A straightforward operator product expansion (OPE) in $1/p^2$ yields the following $p^2$ behavior for $\mathcal{R}_{\text{RI}}$:

$$\mathcal{R}_{\text{RI}}(0, p^2, y_0) = \cdots + \frac{A}{p^2} + Z_{B_{K}}^{\text{RGI}}(y_0) \times U_{\text{RI}}(p^2) + B \times (ap)^2 + \cdots , \quad (10)$$

where $Z_{B_{K}}^{\text{RGI}}(y_0)$ is the renormalization constant that takes $B_{K}^{\text{bare}}(a)$ to the renormalization-group invariant parameter $B_{K}^{\text{RGI}}$, and $U_{\text{RI}}(p^2)$ describes the running in $p$ of the corresponding RI-scheme renormalization constant $Z_{B_{K}}^{\text{RI}}(p^2, y_0)$. The ellipses in Eq. (10) correspond
to higher-order terms in the OPE and higher-order discretization errors.

To describe the running of $Z_{B_K}^R(p^2, g_0)$, we use the 2-loop expression obtained by combining the $\overline{MS}$–NDR result of [26] and the $\overline{MS}$–NDR $\rightarrow$ RI matching result of [27], with $N_f = 6$. Thus,

$$U^{R\text{I}}(p^2) = \alpha_s(p^2) \frac{\gamma_0}{\beta_0} \left[ 1 + \frac{\beta_1}{\beta_0} \frac{\alpha_s(p^2) + \gamma_1}{4\pi} \right] \frac{\gamma_1^{R\text{I}}}{\gamma_1^{N\text{D}}} \frac{m_0^2}{C_0^2},$$

with $\beta_0 = 11$, $\beta_1 = 102$, $\gamma_0 = 4$ and $\gamma_1^{R\text{I}} = 287/3 - 176 \ln 2$. The strong coupling constant that we use is taken from [28] and corresponds to $\alpha_s(\mu^2_{\text{ref}}) = 0.0926(12)$ in the $\overline{MS}$ scheme with $\mu_{\text{ref}} = 94.1(3.6)/r_0$. To obtain the coupling at other scales, we integrate the 2-loop running equation exactly and solve it numerically.

Our results for $R^{R\text{I}}(0, p^2, g_0)$ are plotted in Fig. 1 as a function of $p^2 \equiv \frac{(a/r_0)^2}{\sigma} \sum_{\mu=0}^3 (\sin a p_{\mu})^2$. Using the lattice definition of momentum significantly reduces discretization errors. Also shown is the fit of these results to the functional form given by Eqs. (10) and (11) with $Z_{B_K}^{R\text{I}}(g_0)$, $A$ and $B$ as parameters, in the range of 2 GeV$^2 \leq p^2 \leq 10$ GeV$^2$. We find that the OPE and discretization error terms kept in Eq. (10) are sufficient to describe the data in this range. The fit actually describes the data in a much larger range, indicating that the retained terms dominate. The results of the fits are summarized in Table II, where we also display a fit to the extended momentum range 0.65 GeV$^2 \leq p^2 \leq 15.9$ GeV$^2$ with additional 1/p$^4$ and (ap)$^4$ terms, and a fit using the continuum $p^2$ in the range 2 GeV$^2 \leq p^2 \leq 10$ GeV$^2$. All fits are excellent and produce compatible results. The renormalization constants at 4 GeV$^2$ in the RI and $\overline{MS}$–NDR schemes are obtained by multiplying $Z_{B_K}^{R\text{I}}(g_0)$ by the appropriate two-loop running expressions $U^{R\text{I}}(4$ GeV$^2)$ and $U^{N\text{D}}(4$ GeV$^2)$, respectively, with $N_f$ and $\alpha_s$ chosen as above. $U^{N\text{D}}(p^2)$ is given by Eq. (11) with $\gamma_1^{R\text{I}} - \gamma_1^{N\text{D}} = -7$ for $N_f = 6$.

**Physical results.**—The central values for $P_{B_K}^\text{bare}(a)$, $Z_{B_K}^{R\text{I}}(g_0)$, $Z_{B_K}^{N\text{D}}(4$ GeV$^2, g_0)$ and $Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0)$ are obtained as described above. While statistical and systematic errors in the $B$-parameters and renormalization constants will be correlated in our final results, we also wish to give results for the renormalization constants themselves. The systematic errors on these constants take into account the following sources of uncertainty: the errors on $\alpha_s(\mu^2_{\text{ref}})$ and $\mu_{\text{ref}}$ given above; the variation due to a $\pm 10\%$ uncertainty on the lattice spacing, which is typical in quenched calculations [29]; the difference between the renormalization constants obtained using the lattice and continuum definitions of quark momentum. The latter yields an estimate of discretization errors which turns out to be the dominant uncertainty. We take it to be a symmetric error. Adding all of these variations in quadrature, we obtain:

$$Z_{B_K}^{R\text{I}}(g_0) = 1.261(9)_{-10}^{+11},$$
$$Z_{B_K}^{N\text{D}}(4$ GeV$^2, g_0) = 0.908(6)_{-7}^{+7},$$
$$Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0) = 0.897(6)_{-7}^{+7}. \quad (12)$$

For comparison, we compute the renormalization constant $Z_{B_K}^{R\text{I}}$ at one-loop in perturbation theory, matching the results of [30] with the standard RI/MOM scheme used here. We find $Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0) = 0.95(5)$ with the value of $\alpha_s$ given above. The central value is obtained by expanding the ratio of $Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0)$ to $Z_{A}^{R\text{I}}(g_0)$ in $\alpha_s$. The uncertainty is determined from the errors on $\alpha_s(\mu^2_{\text{ref}})$ and $\mu_{\text{ref}}$ and from an estimate of $O(\alpha_s^2)$ corrections obtained by considering the unexpanded ratio. While the one-loop correction to $Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0)$ is very large (of order 70\% and the perturbative result for $Z_{A}^{R\text{I}}(g_0)^2$ is approximately 15\% below the non-perturbative value found in [23], the agreement of $Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0)$ with the non-perturbative value given in Eq. (12) is surprisingly good. This is due to the large cancellations in $Z_{B_K}^{R\text{I}}(4$ GeV$^2, g_0)/Z_{A}^{R\text{I}}(g_0)^2$.

We now turn to $B_K$. Our lightest pseudoscalar meson is very close to having the mass of the kaon. Since our results for $B_K$ are linear in $M_{B_K}^2$, we extrapolate them linearly to the physical point, as shown in Fig. 2. To extrapolate to lighter quarks, chiral logarithms should be taken into account. Thus, we also perform a fit to the functional dependence predicted by one-loop quenched chiral perturbation theory (Q$\chi$PT) [31], supplemented by an $(M/4\pi F)^4$ term to parametrize higher-order contributions, which are expected for our pseudoscalar meson masses. Here, $F$ is the leading-order leptonic decay constant, which we take to be $F_\tau$, and $M$ is the leading-order pseudoscalar meson mass, which we set equal to meson

![FIG. 1: $\mathcal{R}^{R\text{I}}(0, p^2, g_0)$ vs $p^2$. The solid triangles correspond to the points used in the fit to Eq. (10) while the open ones are not included. The curve is the result of the fit.](image-url)
TABLE II: Renormalization constants and OPE coefficients obtained from fits of \( R_{\chi}^{}(0,p^2,g_0) \) to the OPE form of Eq. (10) for different momentum ranges and for the lattice and continuum definitions of \( p^2 \).

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
p^2\text{-range} [\text{GeV}^2] & (p^2 \text{ def.}) & Z_{\chi}^{\text{RGI}}(g_0) & 1/p^2 [\text{GeV}^2] & (ap)^2 & 1/p^2 [\text{GeV}^4] & (ap)^4 \\
\hline
2, 10 & (lattice) & 1.261(9) & -0.042(16) & 0.028(2) & 0 & 0 \hline
0.65, 15.9 & (lattice) & 1.264(13) & -0.035(23) & 0.024(5) & -0.013(13) & 0.0010(9) \hline
2, 10 & (continuum) & 1.269(8) & -0.051(16) & 0.022(2) & 0 & 0 \hline
\end{array}
\]

Our results for \( B_K \) in various schemes are summarized in Table III. They are in excellent agreement with world averages for this quantity (e.g. [10, 11]), which are based on quenched, staggered results [34-36]. However, our precision is not sufficient to exclude the rather low values found with domain-wall fermions [37, 38].

In addition to the systematic errors accounted for in the results of Table III, one must also consider the error associated with the fact that our kaon is composed of degenerate quarks with masses \( m_s = m_d \) instead of an \( s \) and a \( d \) quark. This error is thought to be roughly 5\% on the basis of \( \chi \)PT estimates reviewed in [39]. In this same review, Sharpe suggests adding a 15\% quenching uncertainty. We have accounted for part of this error in considering the variations due to the uncertainties in the determination of the lattice spacing and of the strange quark mass. We find these variations to be very much smaller than 15\%, which might suggest that the quenching error estimate given in [39] is rather conservative.

TABLE III: Final results for \( B_K \) at the physical point and in the chiral limit in different schemes. The first error is statistical and the second is systematic (see text for a discussion of additional \( m_s = m_d \) and quenching systematics).

| Scheme | RI (4 GeV^2) | NDR (4 GeV^2) | RGI (4 GeV^2) |
|--------|--------------|---------------|---------------|
| \( B_K \) | 0.87(8)_{-0.1}^{+0.1} | 0.63(6)_{-0.1}^{+0.1} | 0.62(6)_{-0.1}^{+0.1} |

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