A Survey on Location-Driven Influence Maximization

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Abstract

Influence Maximization (IM), which aims to select a set of users from a social network to maximize the expected number of influenced users, is an evergreen hot research topic. Its research outcomes significantly impact the real-world applications such as business marketing. The booming location-based network platforms of the last decade appeal to the researchers embedding the location information into the traditional IM research. In this survey, we provide a comprehensive review of the existing location-driven IM studies from the perspective of the following key aspects: (1) a review of the application scenarios of these works, (2) the diffusion models to evaluate the influence propagation, and (3) a comprehensive study of the approaches to deal with the location-driven IM problems together with a particular focus on the accelerating techniques. In the end, we draw prospects into the research directions in future IM research.

1 Introduction

In the past two decades, the rise of social media platforms (e.g., Twitter) has brought dramatic changes to human society, especially in the area of mass communication and commercial marketing. To better utilize the power of social networks, many researchers turn to the study of influence maximization (IM) [Kempe et al., 2003; Golovin and Krause, 2011; Yuan and Tang, 2017], which aims to select a set of influential users with a given budget to achieve the maximum information spread in a social network. To fit with different practical application scenarios, several variants of the IM problem have been investigated recently, including Topic-aware IM [Guo et al., 2013; Li et al., 2015], Time-aware IM [Feng et al., 2014; Xie et al., 2015; Huang et al., 2019], Community-aware IM [Tsang et al., 2019; Li et al., 2020a; Yadav et al., 2018], Competitive IM [Ou et al., 2016; Tsaras et al., 2021; Becker et al., 2020], Multi-strategies IM [Kempe et al., 2015; Chen et al., 2020a], and IM with semi-bandit [Wen et al., 2017; Lin et al., 2015; Sun et al., 2018; Li et al., 2020b].

With the popularity of GPS-equipped smart devices (e.g., smart phones) and the rise of many location-based social network platforms (e.g., Gowalla) in recent years, large-scale digital data with location information are generated and recorded from online social communications, offline facility information (e.g., electronic billboards) and moving objects (e.g., pedestrian and taxi trajectories). The availability of location and communication datasets makes the study of location-driven IM feasible.

Research along the direction of location-driven IM can be categorized as the following two lines. The first line of research focuses on equipping the traditional IM study with location information [Li et al., 2014; Wang et al., 2018; Güney et al., 2021], while the other line of research concentrates on finding a set of optimal offline objects (e.g., billboards) to maximize the influence spread with a given budget [Zhang et al., 2019b; Zhang et al., 2021].

The motivation of this survey. Several existing survey papers [Tejaswi et al., 2016; Arora et al., 2017; Sumith et al., 2018; Li et al., 2018b; Banerjee et al., 2020] have provided a detailed introduction and analysis of the IM problem. However, some critical emerging trends of IM research haven’t been thoroughly discussed in these surveys, i.e., IM query with location information. This has motivated us to comprehensively summarize the research progress of location-driven IM through a fine-grained classification and a systematic analysis of existing location-driven IM methods.

The structure of this survey. This paper focuses on providing a comprehensive survey on existing location-driven IM studies. As illustrated in Figure 1, we summarize the existing location-driven IM research efforts in our survey paper through the following three critical perspectives, including: (1) the problem description, (2) the influence diffusion model, and (3) the methodology:

- **Problem description.** To better summarize the existing location-driven IM works, we categorize them into two research lines based on their application scenarios, in-
2 Background and Problem Description

This section first introduces the influence maximization (IM) problem. Then, we divide the existing location-driven IM studies into two directions: online location-aware influence maximization (OLIM) and offline billboards influence maximization (OBIM), and formally define the research problems on OLIM and OBIM, respectively.

2.1 The Influence Maximization (IM) Problem

Given a graph $G = (V, E)$ representing a social network, where $V$ is the set of nodes in $G$ (i.e., users) and $E$ is the set of edges in $G$, and each edge $(u, v) \in E$ in $G$ is associated with a propagation probability $p(u, v) \in [0, 1]$.

Let $I(S)$ be the number of vertices that are activated by $S$ in graph $G$ on the influence propagation process. The IM problem aims to find a size-$k$ seed set $S$ with the maximum expected influence spread $E(I(S))$. We formally define the IM problem as follows:

**Definition 1 (IM Problem).** Given a graph $G = (V, E)$, and an integer $k$, the IM problem aims to find an optimal seed set $S^*$ satisfying:

$$S^* = \arg \max_{S \subseteq V, |S| = k} E(I(S))$$  \hspace{1cm} (1)

2.2 Online Location-aware IM

The online location-aware influence maximization (OLIM) problem is a variant of the traditional IM problem which considers the effect of given location information on the influence diffusion to the target users in online social networks. For instance, [Su et al., 2018] focuses on finding a set of users to maximize the influence spread over the target users who have both topic and geographical preferences on influence diffusion.

In brief, we can conclude that the traditional IM problem and the OLIM problem share a common goal with two different settings. The common goal is that both aim to find a set of seed users to maximize the influenced target users in the online social networks. Differently, the IM and OLIM problems differ in the definition of target users and the influence diffusion process. Specifically, the definition of target users in the IM problem can be any users in a social network. In contrast, the target users in OLIM may need to satisfy the location constraint, e.g., the distance between seed users and target users does not exceed a given threshold. Besides, compared with the IM problem, evaluating the influence diffusion process for given seed users in the OLIM problem requires further consideration of the effect of the users’ locations.

2.3 Offline Billboards IM

With the availability of trajectory data of moving objects and offline facilities’ information e.g., billboards’ locations and pedestrian trajectories, recent studies [Zhang et al., 2019b; Zhang et al., 2021] initiate the influence maximization problem in Out-of-Home advertising on billboards. We call it offline billboards influence maximization (OBIM). The goal of the OBIM problem is to maximize the influence for the advertiser (e.g., billboards). We provide a generic definition of the OBIM problem as follows:

**Definition 2 (OBIM Problem).** Given a trajectory database $T$, a set of billboards $U$ to place ads and the budget $L$, the OBIM problem aims to find a subset of billboards $S \subseteq U$, which maximizes the expected number of influenced trajectories.

The critical difference between the OLIM and the OBIM problems is the influence diffusion process. In the OLIM...
problem, user influences can be spread from one user to others in online social networks. On the contrary, in the OBIM problem, a user can be influenced by an outdoor billboard only if this user passes by the billboard within a certain range.

3 The Influence Diffusion Models

In this section, we review the influence diffusion models that are used to capture the information diffusion process in the existing OLIM and OBIM works, respectively.

3.1 Diffusion Models in OLIM

In the OLIM problem, three well-known diffusion models are widely used in modeling the influence diffusion process, which are the independent cascade model, the Linear threshold model [Kempe et al., 2003], and the maximum influence arborescence model [Chen et al., 2010]. We associate each user in the following diffusion models with a status of either inactive or active.

Independent cascade (IC) model: the IC model is one of the widely adopted stochastic models which is used for modeling the influence propagation in social networks. In this model, we represent the social network as a graph $G$, $S_0$ as a set of initial activated users, and generate the activated sets $S_t$ for all $t \geq 1$ according to the following randomized rule. At every time step $t \geq 1$, we first set $S_t$ to be $S_{t-1}$; Each user $u$ activated in time step $t$ has one chance to activate his or her neighbours $v$ with propagation probability $p(u,v)$. If successful, we then add $v$ into $S_t$ and change the status of $v$ to activated. This process continues until no more possible activations. Finally, $S_t$ is returned as the activated user set of $S_0$.

Linear threshold (LT) model: The key idea of the LT model is that a user may change his or her status from inactive to activated if the user has sufficient number of activated neighbours. Specifically, let $G = (V, E)$ represent the social network where each edge $e = (u,v) \in E$ is associated with a weight $w_e$ and each user $u$ is associated with a threshold $\lambda_u$. $S_0$ is a set of initial activated users, and generates the active sets $S_t$ for all $t \geq 1$. At each time step $t \geq 1$, all the activated users in step $t-1$ remain activated in step $t$, and any user $v$ that is inactive in step $t-1$ will be changed to activated in step $t$ if the total edge weights between $v$ and its activated neighbours is no less than $\lambda_v$. The LT influence diffusion terminates when there are no more new activated users.

Maximum influence arborescence (MIA) model: The MIA model is the most known as a reduced model of the IC model. The key idea of the MIA model is to restrict the influence diffusion of each user $u$ to a local tree structure with $u$ as the root. In concise, the MIA model performs the reductions from two aspects: (1) For any user pair $(u,v)$, it considers that $u$ can only influence $v$ through the maximum influence paths (MIPs), while MIP is the path with the maximum influence probability among any paths between users $u$ and $v$, i.e., $\text{MIP}(u,v)$; (2) Given threshold $\theta$, all MIPs with the propagation probabilities less than $\theta$ will be ignored. Therefore, the influence diffusion of a given user $v$ is a set of users $u \in V$ with $\text{MIP}(u,v) \geq \theta$ under the MIA model.

3.2 Diffusion Models in OBIM

Several diffusion models have been widely used in the OBIM problem to evaluate the influence propagation effects for a given offline object (e.g., billboards), including the one-time impression model [Liu et al., 2016], the multi-time impression model [Zhang et al., 2020], and the logistic influence model [Zhang et al., 2019b].

One-time impression (OI) model: Given a trajectory database $T$ with each trajectory $t = \{p_1,p_2,\ldots,p_{|t|}\}$, a billboard $b$ with location information $b.loc$, and a given threshold $\lambda$. For each trajectory $t$, if $\exists p_i \in t$ such that the distance between $p_i$ to $b.loc$ is no less than $\lambda$, we define that trajectory $t$ is influenced by the billboard $b$, i.e., $\text{Distance}(p_i, b.loc) \leq \lambda$.

Multi-time impression (MI) model: Given a trajectory $t = \{p_1,p_2,\ldots,p_{|t|}\}$, a billboard set $B = \{b_1,b_2,\ldots,b_i\}$, we set $pr(b_i, t)$ as a uniform value if a billboard $b_i \in B$ can influence $t$. The value of $pr(b_i, t)$ is correlated with the billboard set size and the billboards exposure frequency, i.e., $pr(b_i, t) = \text{size}(b_i)/A$ where $\text{size}(b_i)$ is the panel size of $b_i$ and $A = \max_{b_i \in B} \text{size}(b_i)$. Then, we use the following equation to compute the influence of a billboard set $S$ to $t$:

$$pr(S,t) = 1 - \prod_{b_i \in S} (1 - pr(b_i,t))$$

Logistic influence (LI) model: Given a trajectory $t = \{p_1,p_2,\ldots,p_{|t|}\}$, a billboard set $B = \{b_1,b_2,\ldots,b_i\}$, and a Bernoulli random variable $I(b_i,t), i \in [1,l]$ to denote the states whether $b$ impresses $t$. When $I(b,t) = 1$, we say $b$ impresses $t$; otherwise, no impression is delivered. The logistic influence model evaluates the effective influence of a set of billboards $S \subseteq B$ to a trajectory $t$, $P(S,t)$, by using the logistic function based equation as follow:

$$P(S,t) = \begin{cases} 1 & \text{if} \exists b_i \in S, I(b_i,t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

In Equation 3, $\alpha$ and $\beta$ are the parameters to control $t$’s turning point for being influenced.

Other models: There also exist several other uncommon diffusion models for evaluating the influence propagation process of the given billboard, such as the quantitative model in [Wang et al., 2019], the application-driven mining model in [Liu et al., 2016], and the voter model in [Wu et al., 2018].

4 Methodology

In this section, we systematically analyze the methodologies of the existing location-driven IM studies.

4.1 Approaches of Existing OLIM Studies

We first introduce the methodologies of the existing OLIM works. An analysis of these works is summarized in Table 1. Specifically, we categorize the framework algorithms of existing OLIM works into three approaches based on how an approach overcomes the #P-hardness of evaluating the influence spread $I(S)$ of given users $S$, including (1) the simulation-based approach, (2) the sketch-based approach, and (3) the proxy-based approach.
Simulation-based Approach

The core idea of this approach is to estimate the influence spread $I(S)$ of a given user set $S$ by using the Monte Carlo (MC) simulations of the diffusion process [Kempe et al., 2003; Leskovec et al., 2007; Zhou et al., 2015a]. Specifically, for a given user set $S$, we simulate the randomized diffusion process with $S$ for $R$ times. Each time we count the number of active users after the diffusion ends, and then take the average of these counts over the $R$ times.

[Chen et al., 2020b] mentions that the existing works try to study influence maximization by only considering the geographic distance, while ignoring the influence of users’ spatio-temporal behaviours on information diffusion or location promotion. Motivated by this drawback, they propose a Similarity-aware Influence Maximization (SIM) model to evaluate the influence diffusion, which is an extension of the IC model by taking the effect of users’ spatio-temporal behaviors into account. Moreover, two greedy algorithms, including influence propagation trees index-based and cutting tails pruning algorithms, are proposed to find optimal seed users to maximize the influence spread under the SIM model based on the simulation-based approach.

Proxy-based Approach

Instead of running heavy MC simulation, the proxy-based approach estimates the influence spread of given users via the proxy models. Precisely, this approach is aligned with a specific diffusion model (e.g., IC model), and the influence spread estimation process of given users is highly accelerated by taking advantage of the properties of the corresponding models. Intuitively, there are two branches of the proxy-based approaches, including (1) Estimate the influence spread of given users by transforming it to easier problems (e.g., Degree and PageRank) [Chen et al., 2010; Galhotra et al., 2016]; and (2) Simplify the typical diffusion model (e.g., IC model) to a deterministic model (e.g., MIA model) [Chen et al., 2010] or restrict the influence propagation range of given users under the typical diffusion model to the local subgraph [Goyal et al., 2011] to precisely compute the influence spread of given users. Compared with the simulation-based approach, the proxy-based approach offers higher performance, but lacks theoretical guarantees. The proxy-based approach has been used in [Li et al., 2014; Zhou et al., 2015b; Wang et al., 2016a; Wang et al., 2016b; Wang et al., 2018; Zhu et al., 2019]. We introduce the details of these efforts as follows.

[Li et al., 2014] adopts the proxy-based approach under the MIA model. To accelerate their algorithm, the authors focus on developing bound estimation techniques for effective pruning. Besides, they use a classic spatial index QuadTree to fast locate the users having influence on the “influences” in the given region. [Zhou et al., 2015b] points out the previous work tends to concentrate on a single aspect of online social networks, treating location as a simple property and ignoring the presence of offline consumption behaviour. To conquer this drawback, they propose an improved influence diffusion model, namely the TP model, which builds on the IC model to further evaluate influence diffusion weighted by the user’s distance from an offline location. Under the TP model, the proxy-based approach is utilized to find the optimal users to maximize the influence spread.

[Wang et al., 2016a; Wang et al., 2016b] focus on maximizing the influence of potential customers in a given promotion location. They adopt the MIA diffusion model for influence approximation and utilize the proxy-based approach as the framework algorithm. To this end, they devise the pruning method to accelerate the framework algorithm by reducing the number of candidate users to be evaluated. [Song et al., 2016] also studies the similar research topic as [Wang et al., 2016a]. Differently, the authors adopt a sketch-based approach to deal with their research problem.

[Wang et al., 2018] points out that most existing IM algorithms are static and location-unaware, which rarely reflect the evolving and location-aware nature of social networks. To track the influential users over social streams and further consider the users’ location information, the authors propose two IM queries, Stream Influence Maximization (SIM) and Location-aware SIM (LSIM). SIM adopts the sliding window model and maintains a seed set with the maximum influence value collectively over the most recent social actions, while LSIM considers the social actions associated with geo-tags and identifies a seed set that maximizes the influence value in a query region over a location-aware social stream.

| OLIM          | Diffusion Model | Algorithms        | Accelerating          | Application Scenarios                     |
|---------------|-----------------|-------------------|-----------------------|-------------------------------------------|
| [Li et al., 2014] | MIA model       | Proxy-based approach | Index & Pruning | Maximizing the influenced users within the given Region. |
| [Zhou et al., 2015b] | TP model        | Proxy-based approach | None              | Online promoting to drive the offline consuming. |
| [Wang et al., 2016a; Wang et al., 2016b] | MIA model       | Proxy-based approach | Index & Pruning | Maximizing the influence of potential customers of a given promoted location. |
| [Song et al., 2016] | IC model        | Sketch-based approach | Heuristic     | Targeted influence maximization. |
| [Wang et al., 2018] | SIC model       | Proxy-based approach | Index              | Retrieving a seed set with the maximum influence value over location-aware social streams. |
| [Li et al., 2018a; Cai et al., 2020] | HIM model       | Sketch-based approach | Index & Pruning | Evaluating the influence diffusion by considering the social connection, spatial connection and preference-based connection, rather than only take social connection into account. |
| [Zhu et al., 2019] | MIA model       | Proxy-based approach | None              | Retrieving $k$ positive users to block the diffusion of influence of negative seed users to the targeted users of a given region. |
| [Chen et al., 2020b] | SIM model       | Simulation-based approach | Index & Pruning | Designing a new IC model based influence diffusion that further take the users’ spatial-temporal behavior into account. |

Table 1: Summary of the existing OLIM research works
[Zhu et al., 2019] generalizes the problem by considering the location-aware targeted influence blocking maximization, which aims to find a positive seed user set to block the influence propagation of negative seed users over the targeted users located in a given region. They propose a problem-solving algorithm by adopting the proxy-based approach together with a QT-tree index that is used to store topics and locations information of users.

**Sketch-based Approach**

To avoid running heavy MC simulations and reserve the theoretical guarantee, the sketch-based approach [Borgs et al., 2014] pre-computes a number of Reverse Reachable (RR) sketches based on a specific diffusion model and then evaluates the influence spread of given users set $S$ by exploiting the portion of RR-sketches which $S$ can reach. Here, the RR-sketch is constructed by randomly selecting a node $u \in V$, and conducting a reverse MC sampling starting from $u$. By performing the above operation iteratively $\theta$ times, the RR-sketches can be obtained. Compared with the simulation-based approach, the sketch-based approach has a lower time complexity under a theoretical guarantee.

[Cai et al., 2020; Li et al., 2018a] study the holistic influence maximization (HIM) query problem, which aims to find a minimum set of users whose holistic influence can cover all target users in the network. They evaluate the influence diffusion by considering the social connection, spatial connection, and preference-based connection, rather than only taking social connection into account. Specifically, they propose a holistic influence diffusion model, named HIM model, under the basis of the IC model that takes into account both cyber and physical user interactions. To answer the HIM query, they propose a basic algorithm that uses the reverse $k$ nearest neighbors technique to evaluate the offline influence spread and adopt the sketch-based approach to calculate the influence of users under the HIM model. In addition, they develop an index-based method to deal with large networks.

### 4.2 Approaches of Existing OBIM Studies

We then analyze the methodologies of the existing OBIM studies. A summary of this analysis is illustrated in Table 2.

**Cluster-based Approach for Single Advertiser**

As observed by [Zhang et al., 2018], the billboards in different areas should have small overlaps in their influenced trajectories. The cluster-based approach exploits the locality property of billboards’ influence to avoid the overlap of influence. Specifically, this approach firstly partitions the billboards into a set of small clusters with low influence overlap, and then finds the local solutions of each cluster by using the enumeration method. After that, a dynamic programming approach is executed to construct the global solution based on the local solution of each cluster.

Some studies [Zhang et al., 2018; Zhang et al., 2019b; Wang et al., 2019; Zhang et al., 2020] focus on helping the single advertisers to achieve the largest influence under their budget constraints by selecting the optimal billboards. Specifically, [Zhang et al., 2018; Zhang et al., 2020] study the problem of trajectory-driven influence billboard placement (TIP), which aims to find a set of billboards within budget such that the placed ads on the selected billboards has the maximum influence under the budget. The authors address the TIP problem under the MI model and adopt a cluster-based approach to deal with the TIP problem. Moreover, they boost the time efficiency of the algorithm by pruning billboards with a low benefit or cost ratio.

Similar with [Zhang et al., 2018], [Zhang et al., 2019b] proposes the problem of optimizing impression counts for outdoor advertising (ICOA), which aims to find a set of billboards that have the maximum influence under the budget. Differently, [Zhang et al., 2019b] uses the LI model to evaluate the influence spread of billboards. Since the objective function of ICOA problem is non-submodularity under the LI model, which means no efficient algorithms with constant approximation ratio to ICOA. To conquer this challenge, they propose an estimation method to compute the upper bound by using a dynamic tangent line to tightly bound the real influence. Based on the upper bound function, the authors propose a branch-and-bound framework to answer the ICOA query. Moreover, they optimize this framework by designing a progressive upper-bound estimation based pruning method with $\frac{\theta}{2(1 - 1/e - \varepsilon)}$-approximation ratio.

**Demos of OBIM**

There exist two demo works of OBIM. [Liu et al., 2016] systematically studies the problem of identifying the optimal billboard locations using the trajectory data. Specifically, they present an interactive visual analytics system, named SmartAdP, to deal with the optimal billboard locations through a greedy heuristic algorithm, called $k$-location query under the OI model, together with several visualizations and interaction techniques. Moreover, [Zhang et al., 2019a] aims to assist users in finding an optimal strategy for outdoor advertising deployment. Specifically, they implement an interactive visual demonstration system, namely ITAA, to find the optimal billboard deployment strategy, while the optimal billboards query techniques are from [Zhang et al., 2018; Zhang et al., 2019b].

**Other OBIM Works**

[Guo et al., 2016] introduces an IM problem over trajectory databases, which aims to find $k$ best trajectories to be attached with a given advertisement and maximizes the expected influence among a large group of users. To deal with this problem, they propose a cluster-based algorithm that partitions the trajectory database into clusters and accesses the clusters in an order. Besides, a trajectory index is proposed to reduce the influence calculation cost. Different from any other IM related works, this work does not consider the influence diffusion.

The latest OBIM work [Zhang et al., 2021] studies the OBIM problem from the host’s perspective, which is responsible for assigning billboards to all advertisers, and formulates this as the *Minimizing Regret for the Out-of-Home Advertising Market Problem* (MROAM). The MROAM problem aims to minimize the regret of the influence providers when dealing with numerous influence purchasers. To deal with this problem, the authors first use the OI model to evaluate the influence diffusion between billboards and users. Then, they propose two greedy heuristic methods as baselines to answer the MROSAM query, where the billboards are assigned
5 Challenges and Opportunities

Some critical issues in location-driven IM still remain for further investigation. In this section, we discuss four challenges and their potential research opportunities.

5.1 Challenge I: Predicting OLIM /OBIM

The existing OLIM/OBIM researches focus on finding a set of seed users (or billboards) under the current social network or trajectory databases. However, commercial companies want their product information to be disseminated to the majority of their customers over a period of time after they have spent their budget. Therefore, the seed users selected by using existing OLIM/OBIM methods may not have a good performance for influence spread in the following period, as the network topology often evolves in the real world [Leskovec et al., 2008].

Opportunities: The temporal link prediction (TLP) technique [de Bruin et al., 2021; Yu et al., 2017] can help predict the topology evolution of social networks and trajectories. Developing effective algorithms to find a set of influential users (or billboards) in the future period based on the TLP method would be an exciting research topic.

5.2 Challenge II: Network Evolution

Reverse Reachable Sketch (RRS) [Borgs et al., 2014] is the state-of-the-art algorithm to deal with the IM problem. However, the RRS method cannot be directly used to deal with the dynamic IM problem in which the graph structure dynamically changes. This is because the structure information of each graph sketch does not remain the same during the process of RRS.

Opportunities: One possible solution is to introduce a new index to dynamically maintain the structural information of each graph sketch generated from the RRS method. It would be a breakthrough in the IM research if we successfully develop a new index-based variant of the RRS method which can efficiently deal with the dynamic IM problem.

5.3 Challenge III: Non-submodularity

As mentioned in this survey, the OBIM problem is NP-hard, and most of the existing solutions are based on greedy algorithms. However, some influence diffusion models of the OBIM problem have a non-submodular diffusion function [Zhang et al., 2019b]. In this situation, greedy algorithms are no longer effective.

Opportunities: There are two possible research directions. The first direction is to design a new influence diffusion model instead of the models with non-submodular diffusion function. The other direction is to loosen the non-submodular diffusion model to satisfy the weakly submodular function.

5.4 Challenge IV: Seed Users vs. Budget

All existing research efforts dedicated to the influence maximization problem depend on a strong assumption – the selected seed users will spread the given information. However, some of the chosen individual seed users may be unwilling to promote the product information for various reasons in the real business market. Motivated by this scenario, a new type of IM problem description is required.

Opportunities: To this point, we suggest studying the optimal influential edges query rather than the traditional seed users selection of IM problem. Specifically, an interesting problem description would be researching for boosting influence spread by arousing the estranged relationships emerged in the past but have drifted apart.

6 Conclusions

This paper conducts a comprehensive survey on the existing location-driven influence maximization (IM) research and classifies the efforts into OLIM and OBIM based on their application scenarios. We systematically review the influence diffusion models in existing OLIM and OBIM research works, respectively. Furthermore, we study the problem-solving methods of all existing OLIM and OBIM works, including the basic framework algorithms and the accelerating techniques to boost the efficiency of the basic framework algorithms. Finally, we provide several exciting research problems and opportunities under the topic of location-driven IM.
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