Scheduling and Dynamic Pilot Allocation For Massive MIMO with Varying Traffic

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Abstract—In this paper, we consider the problem of joint user scheduling and dynamic pilot allocation in a Time-Division Duplex (TDD) based Massive MIMO network under varying traffic condition. One of the main problems with Massive MIMO systems is that the number of available orthogonal pilot signals is limited, and the dynamic allocation of these signals to different users is crucially needed to utilize the full benefit of these systems. In addition, pilot signals are radio resource control (RRC) configured in practice, and hence the frequent reconfiguration causes high signaling overhead and is costly. Using Lyapunov optimization framework, we develop an optimal algorithm that first assigns pilots dynamically based on queue sizes and the channel conditions of users as well as the reconfiguration cost at large time-scale. Then, it schedules users on a small-time scale. Numerical results show the efficacy of our algorithm and demonstrate that pilots do not need to be configured frequently at the expense of increased queue delay.

Index Terms—Massive MIMO, Pilot Allocation, Lyapunov Optimization, random traffic, queue stability.

I. INTRODUCTION

The Massive MIMO is one of the core technologies in future 5G wireless systems in which base stations (BSs) operate with a large number of antennas. Using many antennas at the BS provides very high beamforming gain and the capability of serving multiple users simultaneously via spatial multiplexing at the same time and frequency resources. Consequently, enormous spectral efficiency can be achieved [1, 2].

One of the main challenges for a TDD based Massive MIMO system is the accurate estimation of the channel of users. The estimation is realized through special uplink signals called as pilot signal or Sounding Reference Signal (SRS) allocated to users. Multiplexing and beamforming gain can be achieved only by the users with pilot signals, and the gain and the system performance degrade when those signals are not carefully assigned to the users. For instance, the users with high amount of traffic may be preferred to have pilot signals to maximize the network throughput. On the other hand, the practical problems that obstruct the pilot allocation must also be taken into account. Specifically, pilot signals are configured through RRC signaling [3] and the frequent reconfiguration of the pilots can cause an intolerable signaling overhead in practice. Therefore, in addition to a smart SRS allocation, the minimization of the frequency of SRS configuration needs attention.

Although the impact of reusing the same pilot signals on different cells (i.e., pilot contamination) has been well investigated in the literature, the allocation of these signals with the consideration of varying user’s traffic and channel conditions has received little attention. In [4], [5], [6] and [7] the pilot allocation and scheduling are considered without the impact of the network traffic. In [8], the pilot allocation is done for only special messages and the results cannot be generalized.

Our contributions are summarized as follows: i-) We formulate the problem of the scheduling and pilot allocation for Massive MIMO networks with the associated signaling cost as a stochastic optimization problem under Lyapunov optimization framework; ii-) We develop an optimal algorithm that operates at two different time-scale and does not need the future knowledge of the system. We also derive analytical bounds on the performance of the optimal algorithm in terms of average queue size and the signaling cost; iii-) We implement a realistic network setting, and demonstrate the performance of our algorithm and depict the tradeoff between the signaling cost and the average queue size.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a Massive MIMO capable cellular network where there is a BS with $M$ antennas serving $N$ users. Let $N$ be the set of users with single antenna. The BS can transmit simultaneously up to $K$ users via its Multi-User (MU)-MIMO capability and $K < M$. The system schedules users on timeslot fashion at each $\tau$ (e.g., regular scheduling decision that LTE performs at every 1 ms) and let $k(\tau)$ be the number of users served simultaneously at time $\tau$, $1 \leq k(\tau) \leq K$.

In this case, the transmit power for a scheduled user $n$ is $p_n(\tau) = \frac{P_{tot}}{k(\tau)}$, and $P_{tot}$ is the total transmit power. The transmission rate for a user $n$ at time slot $\tau$, $R_n(\tau) \leq R_{max}$ for all $n$ and $\tau$, is given by,

$$R_n(\tau) = B \gamma \log_2 \left( 1 + \frac{p_n(\tau) | h_n(\tau) w_n(\tau) |^2}{\sum_{k=1}^{k(\tau)} p_k(\tau) | h_n(\tau) w_k(\tau) |^2 + \sigma_n^2} \right),$$

where $h_n(\tau) \in \mathbb{C}^M$ is channel vector of user $n$ in the downlink direction, and $s_n(\tau)$ represents the complex transmit symbol for user $n$. Also, $w_n(\tau) \in \mathbb{C}^N$ is the normalized precoding vector of user $n$ and $\sigma_n^2(\tau)$ is zero mean complex Gaussian additive noise with power $\sigma^2$. Furthermore, $B$ is the system bandwidth and $\gamma$ is the fraction of the total time/frequency resource used for data transmission, and the $(1 - \gamma)$ fraction is donated for obtaining SRS. We note that $\gamma$ depends on the length of the coherence block and pilot signals [1].
Let $I_n(\tau)$ be the scheduling decision given for user $n \in N$ at time slot $\tau$. If user $n$ is scheduled then $I_n(\tau) = 1$, else $I_n(\tau) = 0$. We assume there are $P$ number of pilot signals that can be allocated among $N$ users, where $K < P < N$. The decision for SRS allocation is taken at a larger time-scale denoted as $T > \tau$. In every time slot of the form $t = lT$ where $l = 1, 2, \ldots$, the decision denoted as $I_n(\tau)$ is taken to decide whether user $n, n \in N$, should have SRS or not. If $I_n(\tau) = 1$, the system allocates SRS to user $n$ and it can benefit from the multiplexing and beam forming gain between $[t, t + T - 1]$. If $I_n(\tau) = 0$, user $n$ cannot have a SRS till the next time $t = (k+1)T$. After deciding on $I_n(\tau)$ for every user, a SRS configuration flag is set. If $I_n(\tau) = 1$, for all users (i.e. the same SRS allocation as previous time), then the decision on the reconfiguration of SRS denoted as $I^*(t)$ is set to 0, otherwise $I^*(t) = 1$. That is to say when $I_n(\tau)$ is given for all users, $I^*(t)$ is completely determined.

At each time slot $\tau$, data randomly arrives to the queue of each user. Let $A_n(\tau)$ be the amount of data (bits or packets) arriving into the queue of user $n$ at time slot $\tau$. We assume that $A_n(\tau)$ is a stationary process and it is independent across users and time slots, and $A_n(\tau) \leq A_{max}$ for all $n$ and $\tau$. We denote the arrival rate vector as $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)$, where $\lambda_n = E[A_n(\tau)].$ Let $Q_n(\tau) = (Q_1(\tau), Q_2(\tau), \ldots, Q_N(\tau))$ denote the vector of queue sizes, where $Q_n(\tau)$ is the queue length of user $n$ at time slot $\tau$. A queue is strongly stable if $\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[Q_n(\tau)] < \infty$. Moreover, if every queue in the network is stable then the network is called stable. The dynamics of the queue of user $n$ is

$$Q_n(\tau + 1) = \max\{Q_n(\tau) - I_n(\tau)R_n(\tau), 0\} + A_n(\tau).$$

(1)

Let $\Lambda$ denote the capacity region of the system, which is the largest possible set of rates $\Lambda$ that can be supported by a joint scheduling and SRS allocation algorithm with ensuring the network stability.

We recall that configuring SRS allocation frequently (i.e., at every $T$) causes signaling overhead and costly but the SRS allocation should be also sufficiently adaptive and dynamic. Let $C$ be the cost when SRS allocation is reconfigured, i.e., $I^*(t) = 1$. Then, the average cost is given $C_{avg} = \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E[C I^*(t)].$ The control decisions of the system are $c_n(t, \tau) = [I_n(\tau), I_n(\tau)]$ for user $n$. Let $C(t, \tau)$ be the set of all possible control decisions. We consider the following optimization problem:

$$\min \ C_{avg}$$

s.t.: 1) Netwrok stability

$$2) c_n(t, \tau) \in C(t, \tau), \ \forall n, t, \tau$$

(3) (4)

The problem (2)-(3)-(4) aims at minimizing the average cost by taking the scheduling and SRS allocation decisions optimally. The problem constitutes a stochastic optimization problem and we next propose a solution based on Lyapunov optimization technique.

III. JOINT SRS CONFIGURATION, ALLOCATION AND SCHEDULING

In our work, we use Lyapunov drift and optimization tools [9]. The advantage of this tool is the ability to deal with performance optimization and queue stability problems simultaneously in a unified framework. We first define quadratic Lyapunov function as $L(Q(t)) = \frac{1}{2} \sum_{n=1}^{N} Q_n^2(t)$ measuring the total queue size in the system. We then define the conditional $T$-slot Lyapunov drift that is the expected variation in the Lyapunov function over $T$ slots as follows:

$$\Delta_T(t) \triangleq E[L(Q(t + T)) - L(Q(t))|Q(t)].$$

(5)

The Lyapunov optimization tool allows us to minimize the drift and optimize a given objective simultaneously [9]. We add our system cost $V E[I^*(t)|Q(t)]$ (i.e., $T$ slot drift-plus-penalty) to (5).

$$\Delta_T(t) + V E[I^*(t)|Q(t)],$$

(6)

where $V$ is system parameter that characterizes a tradeoff between performance optimization and delay in the queues.

According to the Lyapunov optimization theory, the problem (2)-(3)-(4) can be reinterpreted as the minimization of (6) which can be done by first deriving an upper bound for (6) in the following Lemma.

**Lemma 1:** Given $V > 1$, and at time $t = lT$, for any feasible decision, we have

$$\Delta_T(t) + V E[I^*(t)|Q(t)]$$

$$\leq B_1 + E \left[ \sum_{\tau=0}^{t+T-1} \sum_{n=1}^{N} Q_n(\tau)A_n(\tau)|Q(t) \right]$$

$$- E \left[ \sum_{\tau=0}^{t+T-1} \sum_{n=1}^{N} Q_n(\tau)R_n(\tau) - CV I^*(t)|Q(t) \right].$$

(7)

where $B_1 = \frac{NT(R_{max}^2 + A_{max}^2)}{2}$

**Proof:** The proof is given in Appendix A.

Now, our aim is to find a method that minimizes the right hand side of (7), and this is realized by maximizing the following term in (8) given in the following problem.

**Opt 1:** Maximize over $c_n(t, \tau) \in C(t, \tau), \forall n, t, \tau$

$$E \left[ \sum_{\tau=0}^{t+T-1} \sum_{n=1}^{N} Q_n(\tau)(R_n(\tau) - A_n(\tau)) - CV I^*(t)|Q(t) \right].$$

(8)

It is clear that the solution of Opt 1 requires the prior knowledge of the future queue sizes and the data rates which depends on the future channel conditions over $[t, t + T - 1]$, and this knowledge cannot be obtained in practice. In order to overcome this issue, first we follow the idea in [10] and approximate the future queue sizes as the current observation, i.e., $Q_n(\tau) = Q_n(t)$ for all $\tau \in [t, t + T - 1]$ and $n \in N$. Then, we obtain a looser but more relaxed bound as follows.

**Lemma 2:** Given $V > 1$, and at time $t = lT$, for any feasible decision, we have the following bound,

$$\Delta_T(t) + V E[I^*(t)|Q(t)]$$

$$\leq B_2 + E \left[ \sum_{\tau=0}^{t+T-1} \sum_{n=1}^{N} Q_n(\tau)A_n(\tau)|Q(t) \right]$$

$$- E \left[ \sum_{\tau=0}^{t+T-1} \sum_{n=1}^{N} Q_n(\tau)R_n(\tau) - CV I^*(t)|Q(t) \right].$$

(9)

**Third Term**
derive the following algorithm that solves Opt 2 optimally.

$t$ as the set of users with SRS at the beginning of time

due to small scale fading can be neglected and it

large, which is the case for Massive MIMO systems. Hence,

Rayleigh fading channel) the fluctuation over the transmission

and uses these upper and lower bounds for $Q_n(t)$ on the R.H.S

of \([7]\) and the rest of the proof is similar to the proof of Lemma

1 and omitted here. ■

Lemma 2 reveals that now the optimal control actions can be
taken by maximizing the third term in the R.H.S of \([9]\), which

yields the optimal solution but with a higher average queue

delay. However, we still need the future channel information
(i.e., transmission rates ) to maximize the third term in the

R.H.S of \([9]\) optimally. Here, we exploit one of key benefits

of a Massive MIMO system: under certain condition (i.e.,

Rayleigh fading channel) the fluctuation over the transmission

rates due to small scale fading can be neglected and it

only depends on the large-scale fading such as path-loss and

shadowing. This is known as channel hardening effect \([11]\) that

occurs when the number of antennas at the BS is sufficiently

large, which is the case for Massive MIMO systems. Hence, the

user rates become nearly deterministic and simplifies the

scheduling and resource allocation problem. This channel

characteristic and the result in Lemma 2 reduce Opt 1 to the

following simple optimization problem:

**Opt 2:**

$$
\max_{I_n^*(t), I_s^*(t)} \left\{ \sum_{n=1}^{N} Q_n(t) R_n(t) - \frac{CV I^s(t)}{T} \right\}
$$

In Opt 2, $Q_n(t)$ and $R_n(t)$ are replaced by $Q_n(t)$ and $R_n(t)$
due to the Lemma 2 and the hardening effect, respectively, and

Opt 2 becomes a deterministic problem. We define $N^s(t)$ as the

set of users with SRS at the beginning of time $t$ and derive the

following algorithm that solves Opt 2 optimally.

**Joint Scheduling and SRS Allocation (JSSA):**

- **Input:** $V, C, T, P, N^s(t)$
- **Step 1.1 (SRS Allocation):** At every $t = lT$, among all
  users, for each set with size $k$, $1 \leq k \leq K$ do:
  - Set the transmit power to $p_n(t) = \frac{P}{k}$
  - Find
    $$
    S^*_k(t) = \arg\max -k \{ Q_n(t) R_n(t) \} , n \in N
    $$
    where $\arg\max -k$ choose the first $k$ elements of a
given set of numbers sorted in decreasing order.
    $S^*_k(t)$ is called as the best set with size $k$.
  - Find the weight $W^*_k(t) = \sum_{n \in S^*_k(t)} Q_n(t) R_n(t)$
- **Step 1.2: Find**
  $$
  k^*(t) = \arg\max_{1 \leq k \leq K} W^*_k(t)
  $$
- **Step 1.3:** Determine the set $S^*_k(t)$ and $W^*_k(t) =
  \sum_{n \in S^*_k(t)} Q_n(t) R_n(t)$. Set $S_1(t) = S^*_k(t)$ and the
  maximum weight $W_1(t) = W^*_k(t)$.

**Proof:** The proof uses the fact that for every $\tau \in [t, t +
T - 1]$

$$
Q_n(t) - (\tau - t) R_{max} \leq Q_n(\tau) \leq Q_n(t) + (\tau - t) A_{max}
$$

and uses these upper and lower bounds for $Q_n(\tau)$ on the R.H.S

of \([7]\) and the rest of the proof is similar to the proof of Lemma

1 and omitted here. ■

Theorem 1: (Lyapunov Optimization) Suppose $\lambda$ is an

interior point in $\Lambda$, and there exits $\epsilon > 0$ such that $\lambda + \epsilon \in \Lambda$.

Then, under JSSA, we have the following bounds:

$$
\limsup_{L \to \infty} \frac{1}{L} \sum_{t=0}^{L-1} E[C^s(t)IT(t)] \leq C^{*}_{avg} + \frac{B_2}{\epsilon} \tag{10}
$$

$$
\limsup_{L \to \infty} \frac{1}{L} \sum_{t=0}^{L-1} \sum_{n=1}^{N} E[Q_n(t)IT(t)] \leq \frac{B_2 + VC}{\epsilon} \tag{11}
$$

Where $C^{*}_{avg}$ is the optimal solution of problem (2)-(3)-(4).

**Proof:** To avoid redundancy with existing literature, we

omit the details here. The sketch of the proof is as follows: it

follows similar steps in Theorem 5.4 of \([9]\) by first showing the

existence of a stationary randomized algorithm that is optimal

and achieves the minimum time average cost by choosing the

control actions independently from $Q(t)$ but according to a

fixed probability distribution known to the system. Then, it is

shown that JSSA is better than the randomized algorithm in

minimizing the R.H.S of (9) and thus it is also optimal. ■

Theorem 1 implies that the average cost under JSSA

approaches to the optimal cost $C^{*}_{avg}$ as $V$ increase, while the

average queue sizes also increase.

**IV. Numerical Analysis**

In our simulations, there is a single cell covering a square of

250 m x 250 m area with a Massive MIMO capable BS.

We set $M = 64$ and there are $N = 300$ users uniformly and

independently distributed in the cell. We adapt the same

channel models and take the related parameters given in \([12]\)

for large-scale and NLOS fading. We apply MMSE precoding,

and set $B = 20$ MHz, $K = 10$ and $P_{tot} = 1$ Watts. We assume

that $\tau = 1$ millisecond, and at each time slot $P = 60$ users

send their SRS to the BS, and $\gamma = 0.8$. 3GPP FTP Model 3 is

considered, where user traffic follows Poisson arrival process

with a payload size of 0.2 MB and different mean arrival rate

varying between 0.5 and 2 seconds.

We first show the performance of JSSA when $T = 20$

ms. Figure \([1]\) depicts the time average (every 100 ms) total

network throughput achieved by JSSA with different values of $V$.

Modified JSSA (M-JSSA) configures SRS without any cost

at every $T$ and it constitutes a benchmark to the performance

of JSSA. When $V = 200$, JSSA achieves almost the same
throughput as that of M-JSSA and the average number of SRS configuration is approximately 0.94, which implies that more than 90% of the time JSSA attempts to reconfigure SRS and hence the cost is high. We observe that when $V = 200$ the average total queue size approaches to a fixed point as shown in Figure 2 which means the network is stabilized.

As $V$ increases to $V = 20000$, the throughput achieved by JSSA and the benchmark approach to the same value, which reveals that the network is still stable. However, the average number of SRS reconfiguration is reduced to 0.3, so the cost decreases. We also observe from Figure 2 that the average total queue size is higher when $V = 20000$ compared to the case with $V = 200$, that is aligned with the theoretical result found in Theorem 1.

V. Conclusion

We have investigated the problem of SRS allocation and scheduling problem in a single cell Massive MIMO network. By applying Lyapunov optimization tool, we have developed a joint scheduling and SRS allocation algorithm that can perform well under random traffic and channel conditions. In simulation results, we show that the average signaling cost can be reduced at the expense of an increase in the average queue delay. The problems of the SRS allocation with different objectives (i.e., reducing delay) with fairness can be other research directions. Also, SRS allocation in a multi-cell setup with pilot contamination would be an interesting research problem.

APPENDIX A

Proof of Lemma 1

The proof starts with finding an upper bound for the Lyapunov drift given in (5) by using the following fact: for user $n$, the following inequality holds.

$$Q_n^2(\tau + 1) - Q_n^2(\tau) \leq R_n^2(\tau) + A_n^2(\tau)$$

By summing (12) over $[t, t+T-1]$ and knowing that $R_n(\tau) \leq R_{\text{max}}$ and $A_n(\tau) \leq A_{\text{max}}$ for all $n$ and $\tau$ we obtain,

$$Q_n^2(t + T) - Q_n^2(t) \leq T R_{\text{max}}^2 + T A_{\text{max}}^2$$

Then, by taking the conditional expectation of (13) with respect to $Q(t)$ and summing over all users, and dividing by 1/2 we have,

$$\Delta_T(t) \leq B_1 - E \left( \sum_{\tau = t}^{t+T-1} \sum_{n=1}^{N} Q_n(\tau) | R_n(\tau) - A_n(\tau) | \right)$$

Finally, we add the penalty term $\sum \mathbb{E} [I^s(t)] |Q(t)|$ to both sides of above inequality and rearranging the resulting terms, we have the bound in Lemma 1. This completes the proof.

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