Geometric model of generation of family of contour-parallel trajectories (equidistant family) of a machine tool

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Abstract. In the present paper the analytic solution to the geometric model of formation of equidistant curves (contour-parallel curves) is considered for the case of flat boundary contour and an island within it. The geometric model is spatial and based cyclographic representation. It differs from the known models of the considered formation task and their respective solutions in that on the stages of computer visualization it allows us to achieve a more complete and vivid spatial representation of interconnection and interrelation of all the geometric objects of the model. An example proving the validity of the proposed geometric model of the considered formation problem is provided. The model can be applied in computer-aided machine tool trajectory design in pocket machining on NC units.

1. Introduction

Contour-parallel trajectories of machine tool, geometrically representing a family of equidistant curves, are widespread as trajectories for milling operations, e.g. in pocket machining [1, 2]. Solution to the task of contour-parallel trajectory generation with application of Voronoi diagram is considered in papers [3, 4]. It is based on finding bisectrix of curves and arcs of circles. In NC machining with the use of software environment of CAD/CAM systems, generation of contour-parallel trajectories of tool motion is based on medial axis transform (MAT) of an area with a boundary contour [5, 6, 7, 8, 9].

The vast majority of applied models of trajectory generation are geometrically planar. In these models, upon generation of contour-parallel trajectories, families of curves are formed, one equidistant with respect to contour of pocket and the other – to contour of island within it. These families are treated as independent geometric objects, which leads to a problem as internal (pocket) and external (island) equidistant curves intersect [10, 11], which requires additional analysis and further development of an algorithm for trimming of the non-working fragments of intersecting curves. The proposed spatial geometric model treats both families as a uniform geometric object. This allows us to achieve a more complete and vivid spatial representation of interconnection and interrelation of all the geometric objects taking part in generation of a family of equidistant curves on the stage of computer-aided visualization and therefore minimize computing costs and errors.

An equidistant curve, depending on its position with respect to the initial contour, can have loops of self-intersection. Therefore, the task of self-intersecting equidistant curve trimming arises. This task is solved by means of (MAT). By “medial axis transform” of an area we mean a multitude of all circles of maximum radius inscribed in the area, i.e. tangent to its boundary contour. Center coordinates (x, y) and radius R of such circles are represented by three numbers, the multitude of which for all the inscribed maximum radius circles is defined by MAT [5, 6, 7, 8]. Centers of all these circles generate medial axis (MA). Self-intersection of equidistant curves takes place in points of MA and is determined on the basis of information on radiuses of inscribed circles. Therefore, MAT, as a multitude of pairs “point, radius”, allows us to solve the task of equidistant curve trimming in order to perform
subsequent transition to automated generation of contour-parallel curves constituting machine tool trajectories.

There is, however, a different approach to generating a family of equidistant curves, further referred to as a multitude of offset curves (OC). This approach is based on representing MAT as a certain spatial image, bijectively correspondent to a given prototype – a flat area with a boundary contour. In this new approach MAT is represented by a spatial curve, restored by spatial cyclographic representation [9,14] on the basis of geometric information of the area and its boundary contour. MAT is generated upon intersection of various linear α-surfaces featuring generatrixes inclined to the plane of the contour on angle \( \alpha = 45^\circ \) [15]. These α-surfaces are formed on the basis of geometric information of the area and its boundary contour. This results in formation of a certain α-shell covering the given area with its boundary contour. The α-shell and the flat area with its boundary contour are bijectively correspondent two-parameter geometric objects. Further section of the acquired α-shell by means of a multitude of horizontal planes along the \( z \) axis with step \( \Delta z = \delta = \text{const} \) results in generation of a family of level curves, orthogonal projections of which onto the plane of the given area constitutes a multitude of OC inside the given area, which is required to perform further automated pocket machining tool trajectory calculation and NC unit programming.

2. Problem definition

Upon comparison of the two approaches to OC formation – the known one and the one suggested in the present paper – it is possible to notice the difference in formation technology. In the first case MA is formed on the basis of compacting interpolation – compaction of a multitude of tangent circles filling a flat area - with further construction of OC multitude and, when necessary, MAT. In the second case, initially, MAT is formed, and then the α-shell is constructed and sectioned by a bundle of horizontal planes with equal step along the \( z \) axis. Then, by means of orthogonal projection onto a flat plane, MA and a multitude of OC are acquired.

In the suggested approach the necessity to perform complex analytic operations over a multitude of tangent circles in order to acquire MA, MAT and construct OC vanishes. At the same time, it is possible to acquire a more complete and vivid spatial representation of interconnection and interrelation of all the geometric objects taking part in OC multitude generation on the stage of computer visualization. On the basis of the suggested approach to OC generation the following problem is set: to develop a geometric model featuring an analytic description, that would realize this approach, and to verify it experimentally.

3. Theory of MAT formation

MAT constitutes a spatial curve bounding an α-shell in height (along \( z \)-axis). An α-shell constitutes a composite linear α-surface compartment. α-shell is resting over boundary contour of the given area on plane \((xy)\). A bijective correspondence is established between two-dimensional multitudes of points of the α-shell and the area. MAT and α-shell are defined entirely by the given area and its boundary contour.

Consider certain closed polygonal contours \( a(a_1,\ldots,a_n) \) and \( b(b_1,\ldots,b_k) \) given on plane \((xy)\). The external contour \( a(a_1,\ldots,a_n) \) is represented by a sequence of segments \( a_i \) connected in their endpoints, and the internal (island) contour \( b(b_1,\ldots,b_k) \) is represented by a sequence of segments \( b_j \) connected in their endpoints. The segments of contours \( a \) and \( b \) are defined by the following equations:

\[
\begin{align*}
    a_i &: \overline{\mathbf{r}}_{ai} = (x_{ai}(t_i), y_{ai}(t_i)) ; \\
    b_j &: \overline{\mathbf{r}}_{bj} = (x_{bj}(t_j), y_{bj}(t_j)) ; \\
    t_i, t_j &\in \mathbb{R} .
\end{align*}
\]

A plane is constructed through each segment: for segments of the external contour \( a \) inclined on angle \( 45^\circ \) with respect to plane \((xy)\); for segments of the internal contour \( b \) inclined on angle \(-45^\circ \) with respect to plane \((xy)\). In order to model the planes an equidistant curve is constructed for each segment of the contours. For segments \( a_i \) of the external contour equidistant curves \( e_{ai(xy)} \) are constructed inside the closed contour \( a \), for segments \( b_j \) of the internal contour equidistant curves \( e_{bj(xy)} \) are constructed outside the closed contour \( b \). Then each of the equidistant curves is displaced along the \( z \) axis in certain direction on the value of its location parameter with respect to the initial contour segment. As a
result, lines \( e_{ai} \) and \( e_{bj} \) are generated. Lines \( a_i \), \( e_{ai} \) and \( b_j \), \( e_{bj} \) pairwise form bounded linear \( \alpha \)-surfaces (\( \alpha \)-planes) \( P_i \) and \( P_j \), for which these lines serve as guide lines:

\[
P_i : \overrightarrow{P_0(t_i, l_i)} = \overrightarrow{P_0(t_i)} + l(\overrightarrow{P_0(t_i)} - \overrightarrow{P_0(t_i)}) ,
\]

\[
P_j : \overrightarrow{P_0(t_j, l_j)} = \overrightarrow{P_0(t_j)} + j(\overrightarrow{P_0(t_j)} - \overrightarrow{P_0(t_j)}) .
\]

In vertices of concave angles of the external contour and in vertices of convex angles of the internal contour conical surfaces of revolution with angle at the vertex equal to 90° are constructed:

\[
\text{Con}_i : x_{vi} = R_i \cos(t_{vi}), y_{vi} = R_i \sin(t_{vi}), z_{vi} = -R_i t_{vi},
\]

\[
\text{Con}_j : x_{vj} = R_j \cos(t_{vj}), y_{vj} = R_j \sin(t_{vj}), z_{vj} = -R_j t_{vj},
\]

where \((x_{vi}, y_{vi})\) represent coordinates of vertices of concave angles of the external contour, \(R_i\) represents radiiuses of external contour cone bases, \((x_{vj}, y_{vj})\) represent coordinates of vertices of convex angles of the internal contour, \(R_j\) represents radiiuses of external contour cone bases. Herewith the vertices of the cones match the vertices of the angles, the axes of the cones are perpendicular to the plane of the contours.

\( \alpha \)-surfaces generate curves of intersection, segments of which represent segments of MAT. Let us designate these line of intersection as follows:

\[
s_1 = P_1 \cap P_2 ; 

s_2 = \text{Con}_1 \cap P_2 ; 

s_3 = \text{Con}_1 \cap P_3 ; 

s_4 = \text{Con}_j \cap P_1 ; 

s_5 = \text{Con}_j \cap P_6 ; 

s_6 = \text{Con}_j \cap \text{Con}_j .
\]

The MAT curve is generated segmentally as a result of combination of segments \( s_i \), \( \alpha \)-projections of each of the segments \( s_i(s_1, \ldots, s_6) \) constitute segments \( a_i \) and \( b_j \). An \( \alpha \)-projection of each of the segments \( s_i(s_1, \ldots, s_6) \) belonging to \( \alpha \)-surfaces is understood to be a multitude of points of intersection of generatrices of the \( \alpha \)-surface and plane \((xy)\). Herewith the generatrices are inclined to plane \((xy)\) on angle equal to 45° and pass through segment \( s_i \). Therefore, segments \( a_i \) and \( b_j \) constitute branches of the curve of intersection of plane \((xy)\) and an envelope of one-parameter multitude of \( \alpha \)-cones with vertices belonging to \( s_i \) and axes perpendicular to plane \((xy)\). The base of each cone on plane \((xy)\) constitutes a circle of radius \( R=\pm \) tangent to segments \( a_i \) and \( b_j \). Therefore, a continuous multitude of triples of numbers \((x,y,R=\pm)\) is generated on plane \((xy)\). This multitude, in accordance with the abovementioned algebraic definition, represents MAT curve.

4. Results of experiments

4.1. Construction of MAT, MA, and OC of polygonal contours

Let us consider construction of geometric objects MAT, MA, and OC given an area bounded by a closed polygonal contour and an island within it. The initial data includes the external contour – line \( ABCDEA \) and the internal contour - line \( KLMK \). The external contour is defined by points \( A(60, 20), B(45, 75), C(110, 84), D(95, 55), E(130, 43) \). The internal contour is defined by points \( K(70, 35), L(72, 62), M(85, 42) \) (fig.1). The external and the internal contours are described by parametric equations of their segments:

\[
\overrightarrow{P_i} = (x_i(t_i), y_i(t_i), z_i(t_i)) ; \quad 0 \leq t_i \leq 1 .
\]

On the basis of cyclographic representation, the \( \alpha \)-planes and \( \alpha \)-surfaces of the cones are formed (ref. section III). The planes pass through the sides of the given polygons. Planes passing through segments of the external contour \( a \) are inclined on angle 45° with respect to plane \((xy)\); planes passing through segments of the internal contour \( b \) are inclined on angle -45° with respect to plane \((xy)\). At the vertex of the concave angle of the external contour and at the vertices of convex angles of the internal contour cones with angle 90° at the vertex are formed (fig.2).

Subsequently, the lines of intersection between the planes, and between the planes and the cones are acquired. As a result, we have 15 equations of lines of intersection \( IL_i \) \((i=1, \ldots, 15)\):

\[
\overrightarrow{P_i} = (x_{IL_i}(t_{IL_i}), y_{IL_i}(t_{IL_i}), z_{IL_i}(t_{IL_i})) , \quad 0 \leq t_{IL_i} \leq 1 .
\]

Then, the points of intersection \( T_1, \ldots, T_{11} \) of the acquired spatial lines \( IL_1, \ldots, IL_{15} \) are found (fig.3). Then the equations of elementary segments \( ES_i \) \( ES_i = T_i T_{i+1} \) are acquired. Hereby \( T_i \) represents the first point of a segment, \( T_{i+1} \) represents the last point of a segment. As a result, the elementary segments \( ES_i \) are formed, their equations are found as follows:
Figure 1. The initial contours: the external ABCDEA and the internal (island) KLMK.

Figure 2. $\alpha$-images of vertices and segments of the external and the internal contours.

$$ES_1 : T_Ei = (x_Ei(t_i), y_Ei(t_i), z_Ei(t_i)), \quad 0 \leq t_i \leq 1;$$

$$ES_1 = T_1T_2, \; ES_2 = T_2T_3, \; ES_3 = T_3T_4, \; ES_4 = T_4T_5, \; ES_5 = T_5T_6, \; ES_6 = T_6T_7, \; ES_7 = T_7T_8, \; ES_8 = T_8T_9, \; ES_9 = T_9T_{10}, \; ES_{10} = T_{10}T_{11}, \; ES_{11} = T_{11}T_1.$$ Combination of the elementary segments $ES_i, i=1, \ldots, 11$ and lines $IL_{12}, IL_{13}, IL_{14}, IL_{15}$ results in MAT line formation.

Figure 3. Formation of composite line $(T_1, \ldots, T_{11})$ of intersection of $\alpha$-surfaces.

4.2. Combination of neighboring elementary segments $ES_i$ of a single surface into a composite segment $CS_i$ of the constructed MAT.

Lines and segments of pairwise intersection of all of the constructed $\alpha$-surfaces constitute elementary segments $ES_i$ of the constructed MAT. In order to form the solid $\alpha$-shell, it is required to combine all the $ES_i$ so that they would all be consistent with the correspondent segments of the external ABCDEA and the internal KLMK contours. Specifically, for the external and the internal contours the combination of $ES_i$ is carried out in such way that the composite segments $CS_i$ and the elementary segments $ES_i$ (not included into any $CS_i$) belong to a common $\alpha$-surface. With respect to the internal (“in”) contour KLMK the following composite segments $CS_{in}$ and elementary segments $ES_{in}$ can be acquired:
Let us replace the parameter with a single equation, a certain method of mathematical formulation of compound curves [16] is applied in the present paper. Elementary segments $ES_i$ included in composite segments $CS_i$ are described by equations

$$\tau_{ES_i}(t_i) = (x_{ES_i}(t_i), y_{ES_i}(t_i), z_{ES_i}(t_i)), \quad t_i \in [t_{i0}^i, t_{i1}^i].$$

Vector equation $\vec{R}(\tau)$ of a composite segment can be constructed in a certain way [16] and represented in the following form:

$$\vec{R}(\tau) = \vec{R}_i(t_i^0) + \sum_{k=1}^n (\vec{R}_k(t_k^0 + (t_k^1 - t_k^0)) P(\tau, \tau_{k-j}, \tau_k - \tau_{k-1})) - \vec{R}_k(t_k^1),$$

where $n$ represents the number of elementary segments $ES_i$, $\vec{R}_k(t_k^0)$ represents the initial point of a composite segment, $\vec{R}_k(t_k^1)$ represents the terminal point of an elementary segment. Scalar function $P(\tau, \tau_{k-j}, \tau_k - \tau_{k-1})$ corresponds to a certain function $P(\tau, a, w)$ [16]:

$$P(\tau, a, w) = \frac{1}{2w} \left( |\tau - a| - |\tau - a - w| \right), \quad (w \neq 0).$$

The function $P(\tau, a, w)$ is continuous with respect to parameter $\tau$. In case $\tau < a$, the function $P(\tau, a, w) = 0$, in case $\tau < a + w$, the function $P(\tau, a, w) = 1$, within interval $a \leq \tau \leq a + w$ the function $P(\tau, a, w)$ matches the linear function $\frac{1}{w} (\tau - a)$ for the case of $w > 0$.

Let us consider the internal contour $KLMK$. Of the compound segment $CS_{inl}(T_{11} - T_{12}) = (ES_{11} \cup ES_1) \subset Con(K)$ we have: the initial point $T_{11} = R_1(t_1^0)$, the point $T_1 = R_2(t_2^0)$ of connection of two elementary segments $ES_{11}$ and $ES_1$ and the terminal point of the elementary segment $T_{2} = R_3(t_3^0)$. In accordance with (2), in the equation for $ES_{11}$

$$\tau_{ES_{11}}(t_{11}) = (x_{ES_{11}}(t_{11}), y_{ES_{11}}(t_{11}), z_{ES_{11}}(t_{11}))$$

let us replace the parameter $t_{11}$ with $P(\tau, 0, 1)$; in the equation for $ES_1$

$$\tau_{ES_{1}}(t_{1}) = (x_{ES_{1}}(t_{1}), y_{ES_{1}}(t_{1}), z_{ES_{1}}(t_{1}))$$

let us replace the parameter $t_1$ with $P(\tau, 1, 2)$. The resulting equation of composite segment $CS_{inl}$ will be of the following form:

$$x_{inl}(\tau) = x(T_{11}) - (x(T_{1}) + x(T_{2})) + x_{ES_1}(P(\tau, 0, 1)) + x_{ES_1}(P(\tau, 1, 2)) - x(T_{11}) - x(T_{1}) \quad (0 \leq \tau \leq 2).$$

The equations of all the other composite segments are constructed likewise.

4.3. Formation of composite $a$-shell from compartments of linear $a$-surfaces
α-compartments form as parts of linear α-surfaces, the directrices of which are the lines of the external contour ABCDEA and the corresponding MAT segments, as well as the lines of the internal contour KLMK and the corresponding MAT segments.

The following holds for the external contour ABCDEA in case the MAT segments belong to a plane, lines $\text{CS}_{\text{ex1}}$ and $AB$, $\text{CS}_{\text{ex2}}$ and $BC$, $\text{ES}_{\text{ex3}} = \text{ES}_{\text{ex0}}$ and $CD$, $\text{ES}_{\text{ex5}}$ and $D \text{E}$, $\text{CS}_{\text{ex6}}$ and $EA$ pairwise generate compartments of linear α-surfaces constituting flat areas $Q_{\text{ext}(j)}$: 

$$
Q_{\text{ext}} : \tau_1(l_1) = \tau_{\text{ext}}(l_1) + l_1(\tau_{AB}(l_1) - \tau_{\text{ext}}(l_1)), \quad 0 \leq \tau_1 \leq 3, \quad 0 \leq l_1 \leq 1;
$$

$$
Q_{\text{ext}} : \tau_2(l_2) = \tau_{\text{ext}}(l_2) + l_2(\tau_{BC}(l_2) - \tau_{\text{ext}}(l_2)), \quad 0 \leq \tau_2 \leq 2, \quad 0 \leq l_2 \leq 1;
$$

$$
Q_{\text{ext}} : \tau_3(l_3) = \tau_{\text{ext}}(l_3) + l_3(\tau_{CD}(l_3) - \tau_{\text{ext}}(l_3)), \quad 0 \leq \tau_3 \leq 1, \quad 0 \leq l_3 \leq 1;
$$

$$
Q_{\text{ext}} : \tau_5(l_5) = \tau_{\text{ext}}(l_5) + l_5(\tau_{DE}(l_5) - \tau_{\text{ext}}(l_5)), \quad 0 \leq \tau_5 \leq 1, \quad 0 \leq l_5 \leq 1;
$$

$$
Q_{\text{ext}} : \tau_6(l_6) = \tau_{\text{ext}}(l_6) + l_6(\tau_{EA}(l_6) - \tau_{\text{ext}}(l_6)), \quad 0 \leq \tau_6 \leq 1, \quad 0 \leq l_6 \leq 1,
$$

where $l_i$ represents a parameter that determines position of a point on generatrix of a surface.

In case MAT segment belongs to surface of a cone, it corresponds to angle vertex of respected contour. For the external contour ABCDEA the segment $\text{ES}_{\text{ex4}}$ corresponds to vertex of angle $D$. Compartment $Q_{\text{ext}}$ of the corresponding linear α-surface is determined the following way (fig.4):

$$
Q_{\text{ext}} : \tau_4(l_4) = \tau_{\text{ext}}(l_4) + l_4(\tau_{D}(l_4) - \tau_{\text{ext}}(l_4)), \quad 0 \leq \tau_4 \leq 1, \quad 0 \leq l_4 \leq 1.
$$

The equations of compartments of linear α-surfaces of the internal contour KLMK are constructed likewise. For the internal contour KLMK the segments belonging to a plane and the line of the internal contour, namely $\text{ES}_{\text{in2}}$ and $KL$, $\text{CS}_{\text{in3}}$ and $LM$, $\text{ES}_{\text{in6}}$ and $MK$ pairwise generate compartments of linear α-surfaces constituting flat areas $G_{\text{in}(j)}$: $G_{\text{in2}}$, $G_{\text{in4}}$, $G_{\text{in6}}$.

MAT segments belonging to surfaces of cones of the internal contour KLMK and corresponding with vertices, namely $\text{CS}_{\text{in1}}$ and $K$, $\text{CS}_{\text{in3}}$ and $L$, $\text{CS}_{\text{in5}}$ and $M$ form compartments of linear α-surfaces $G_{\text{in}(j)}$: $G_{\text{in1}}$, $G_{\text{in3}}$, $G_{\text{in5}}$ (fig.5) in accordance with equations (1-5).

The acquired compartments of α-surfaces form an α-shell. Further section of the acquired α-shell with planes $P_i (\Delta z_i = \delta = \text{const})$ of a horizontal bundle of planes results in generation of a discrete family of level lines belonging to the α-shell (fig. 6) and a discrete family of their orthogonal projections $OC$ of plane vertex (fig.7).

![Figure 4](image1.png)  
**Figure 4.** Compartment $Q_{\text{ext}}$ of α-cone surface.

![Figure 5](image2.png)  
**Figure 5.** Generation of the compound α-shell.
The planes cutting the composite α-shell are described with the equation \( z_i = h \), where \( h = i \delta \); \( i = 0, 1, 2, \ldots, n \); \( n \) represents the number of cutting planes, which is specified depending on design and technological conditions; \( \delta = \frac{z_{\text{max}}(\text{MAT})}{n-1} \), where \( z_{\text{max}}(\text{MAT}) \) represents maximum applicable value of the \( \text{MAT} \) line. Intersection of the \( \alpha \)-planes \( P_i \) and compartments \( G_{in(j)} \) of linear \( \alpha \)-surfaces occurs along the segments of the curves \( L_{in(j)} = P_i \cap G_{in(j)} \) that form composite curves \( OC_{in(j)} = L_{in(1)} \cup L_{in(2)} \cup \ldots \cup L_{in(j)} \) within the internal with respect to \( \text{MAT} \) part of the \( \alpha \)-shell and along segments \( L_{ext(k)} = P_i \cap G_{ext(k)} \), that form composite curves \( OC_{ext(j)} = L_{ext(1)} \cup L_{ext(2)} \cup \ldots \cup L_{ext(k)} \) within the external with respect to \( \text{MAT} \) part of \( \alpha \)-shell. The family of level line segments \( L_{in(j)} \) and \( L_{ext(k)} \) generated through the parameter \( h_i \) are represented with the following parametric equations:

\[
L_{in(j)}: \tau_{in(i,j)} = (x_{in(i,j)}(h_i, \tau_j), y_{in(i,j)}(h_i, \tau_j), z_{in(i,j)}(h_i, \tau_j)),
\]

\[
L_{ext(k)}: \tau_{ext(i,k)} = (x_{ext(i,k)}(h_i, \tau_k), y_{ext(i,k)}(h_i, \tau_k), z_{ext(i,k)}(h_i, \tau_k)),
\]

where \( i \) represents the index of the cutting plane \( P_i \), \( \tau_j \) and \( \tau_k \) represent parameters of shape of segments \( L_{in(j)} \) and \( L_{ext(k)} \) correspondingly, \( j \) represents index of segment of the composite curve \( OC_{in(j)} \), \( k \) represents index of segment of the composite curve \( OC_{ext(j)} \).

5. Consideration of the results

The result of the computational experiment of \( OC \) family generation for the case of biconnected area with closed contour have revealed the particular qualities of the proposed geometric model of generation and have raised the necessity of formulation of a new problem directed at optimization and enhancement of the model for the case of the area with higher connectivity. Formulation of the problem is determined by the following condition: the \( \alpha \)-shell \( \alpha_g \) in the considered example is formed by combination of two components: \( \alpha_x = \alpha_{s(in)} \cup \alpha_{s(ext)} \). Each of these parts, in turn, is formed by combination of compartments of the corresponding \( \alpha \)-planes and linear \( \alpha \)-surfaces:

\[
\alpha_{s(in)} = G_{in(1)} \cup G_{in(2)} \cup \ldots \cup G_{in(j)} \quad \alpha_{s(ext)} = G_{ext(1)} \cup G_{ext(2)} \cup \ldots \cup G_{ext(k)}.\]

For this reason, each of the level curves \( OC \) on the \( \alpha \)-shell is formed as a combination of pairs of compound curves \( OC_i = OC_{in(i)} \cup OC_{ext(i)} \), where \( OC_{in(i)} = L_{in(1)} \cup L_{in(2)} \cup \ldots \cup L_{in(j)} \) and \( OC_{ext(j)} = L_{ext(1)} \cup L_{ext(2)} \cup \ldots \cup L_{ext(k)} \). It is obvious that the principle of sequential generation of separate geometric objects and their further combination is characteristic for the considered model of \( OC \) family generation.

The mentioned principle still requires significant computational and time costs, regardless of the capability to acquire analytical solution to the problem of generation. Thereupon the solution of the accompanying problem of representation of an \( \alpha \)-shell as a solid geometric object with uniform analytic representation and preserving the analytic solution to the task of generation is considered urgent. The solution to the accompanying problem would allow us to minimize the mentioned costs.
and widen the capabilities of the proposed geometric model in relation to the problem of OC family generation of multiply connected areas.

6. Conclusion

A geometric model of OC family generation for multiply connected areas bounded by closed contours and constituting the basis of pocket surface modeling is proposed on the basis of cyclographic representation of space $R^3$. Each stage of analytic solution is accompanied with visualization of geometric objects and their relations in virtual computer space of the geometric model. The proposed algorithm of OC family generation for the case of biconnected polygonal area can serve as the basis of OC family generation for the case of multiply connected areas. The existence of an analytic solution to the problem of OC family generation significantly simplifies automated tool trajectory calculation and control programming for pocket machining on NC units.

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