Role of Modified Chaplygin Gas as a Dark Energy Model in Collapsing Spherically Symmetric Cloud

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In this work gravitational collapse of a spherical cloud, consists of both dark matter and dark energy in the form of modified Chaplygin gas is studied. It is found that dark energy alone in the form of modified Chaplygin gas forms black hole. Also when both components of the fluid are present then the collapse favors the formation of black hole in cases the dark energy dominates over dark matter. The conclusion is totally opposite to the usually known results.

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I. INTRODUCTION

Recent observational data \cite{1, 2} shows the consistency of the inflationary scenario with the power spectrum of the microwave background radiation for cosmic fluid having equation of state in the range $-1 \leq \gamma (= p/\rho) \leq -1/3$. to match with these observational results usually two (dark) components of matter are invoked: the pressureless cold dark matter (or simply dark matter (DM)) and the dark energy (DE) having negative pressure components. The DM contribution ($\Omega_{DM} \sim 0.3$) is mainly motivated by the theoretical study of galactic rotation curves and large scale structure formation, while for dark energy $\Omega_{DE} \sim 0.7$ and is responsible for the acceleration of the distant type Ia supernovae (for recent reviews see \cite{3} and \cite{4}). Though there are no direct laboratory observational or experimental evidence for both of them, yet a unified dark matter-dark energy scenario i.e., they are two different manifestations of a single fluid \cite{5} would be interesting. Recently, unified model has been proposed which is known as modified Chaplygin gas \cite{6, 7} having exotic equation of state

$$p = \gamma \rho - \frac{B}{\rho^\alpha}, \quad B > 0, \quad 0 < \alpha < 1 \quad (1)$$

In this paper, gravitational collapse of a spherically symmetric cloud consists of both dark matter and dark energy (having equation of state given by equation (1)) is considered with energy-momentum tensor

$$T^j_i = (\rho_{DM} + \rho + p)u_iu^j - p\delta^j_i \quad (2)$$

The Einstein equations for spherical space-time with line-element

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2d\Omega^2\right) \quad (3)$$

are given by

$$3\frac{\dot{a}^2}{a^2} = \kappa(\rho_{DM} + \rho) \quad (4)$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \kappa\rho \quad (5)$$

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Now, if $Q(t)$ denotes the interaction between dark matter and dark energy then from the conservation law $T_{\mu\nu} = 0$ one gets

$$\dot{\rho}_{DM} + 3\frac{\dot{a}}{a}\rho_{DM} = Q \quad (6)$$

and

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -Q \quad (7)$$

If $\Sigma : r = r_\Sigma$ denotes the boundary of the spherical collapsing cloud then on $\Sigma$

$$ds_\Sigma^2 = dT^2 - R^2(T)d\Omega^2 \quad (8)$$

where $T = t$ and $R(T) = r_\Sigma a(T)$ is called the area radius. Thus the total mass of the collapsing cloud is

$$M(T) = \frac{1}{2}R(T)\dot{R}(T) \quad (9)$$

If $T_{AH}$ be the time instant at which the whole cloud starts to be trapped then

$$R_\alpha R_\beta g^{\alpha\beta}|_{T=T_{AH}} = 0 \quad i.e., \quad \dot{R}^2(T_{AH}) = 1 \quad (10)$$

As it is natural to assume the cloud to be untrapped initially ($t = t_i$) so one should have

$$\dot{R}(T = T_i) > -1 \quad (11)$$

Note that if the condition (11) holds throughout the collapsing process then the collapse will not produce black holes. in the following two sections, collapsing process will be studied when there is only Chaplygin gas as the collapsing fluid and then a combination of dark matter and Chaplygin gas both with and without interaction. the paper ends with some conclusive remarks.

II. GRAVITATIONAL COLLAPSE OF DARK ENERGY AS CHAPLYGIN GAS MODEL

This section deals with gravitational collapse of dark energy in the form of Chaplygin gas. From the conservation equation (7), integrating once one gets

$$\rho = \left[ \frac{B}{1+\gamma} + C \frac{a^{3(1+\gamma)(1+\alpha)}}{a^{3(1+\gamma)(1+\alpha)}} \right]^{\frac{1}{1+\gamma}}, \quad (\gamma \neq -1) \quad (12)$$

with $C$ is the constant of integration.

Now substituting this expression for $\rho$ into the Friedman equation (4) and integrating the scale factor can be obtained as

$$a^{\frac{3(1+\gamma)}{2}}\frac{\sqrt{3K}}{2} \frac{(1+\gamma)C^x(t_0 - t)}{C(1+\gamma)} = 2F_1[x,x,1+x,-\frac{B}{C(1+\gamma)}a^{\frac{3(1+\gamma)}{2x}}] \quad (13)$$

where $x = \frac{1}{2(1+\alpha)}$ and $2F_1$ is the hypergeometric function.

The expressions for the related physical parameters are
\[ \dot{R}(\tau) = -R_0 a^{-\frac{3(1+\gamma)}{2}} \left[ C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)} \right]^{\frac{1}{2(1+\alpha)}} \] (14)

\[ M(\tau) = \frac{1}{2} R_0^2 r^\Sigma a^{-3\gamma} \left[ C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)} \right]^{\frac{1}{2(1+\alpha)}} \] (15)

One may note that as \( t \to t_0 \)

\[ a^{\frac{3(1+\gamma)}{2}} \simeq \frac{\sqrt{3\kappa}}{2} (1+\gamma) C^{\frac{1}{2(1+\alpha)}} (t_0 - t) \sim 0 \]

Also using the relation [8]

\[ 2F_1[a, b; c; z] = \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} (-z)^{-a} 2F_1[a, 1-c+a, 1-b+a; \frac{1}{z}] + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} (-z)^{-b} 2F_1[b, 1-c+b, 1-a+b; \frac{1}{z}] \]

one gets the limiting value of

\[ a^{\frac{3(1+\gamma)}{2}} 2F_1[\frac{1}{2(1+\alpha)}, \frac{1}{2(1+\alpha)}, 1+\frac{1}{2(1+\alpha)}, -\frac{B}{C(1+\gamma)} a^{3(1+\alpha)(1+\gamma)}] \]

as

\[ \frac{1}{1+\alpha} \left[ \frac{C(1+\gamma)}{B} \right]^{\frac{1}{2(1+\alpha)}} \]

when \( a \) is very large. thus if \( t \to t_s \) as \( a \to \infty \) then from equation (13)

\[ t_s = t_0 - \frac{2}{\sqrt{3\kappa} (1+\alpha)(1+\gamma)} \left( \frac{1+\gamma}{B} \right)^{\frac{1}{2(1+\alpha)}} \] (17)

The limiting value of the physical parameters are

\[ \tau \to \tau_s : \rho \to \left[ \frac{B}{1+\gamma} \right]^{\frac{1}{1+\alpha}}, \quad \dot{R} \to \begin{cases} -\infty & \text{for } \gamma > -5/3 \\ 0 & \text{for } \gamma \leq -5/3 \end{cases}, \quad M(\tau) \to \infty \]

\[ \tau \to \tau_0 : \rho \to \infty, \quad \dot{R} \to -\infty, \quad M(\tau) \to \infty \]

Thus if the collapse starts at an instant close to \( \tau_s \) then for \( \gamma > -5/3 \), initially the collapsing system is trapped and in course of the collapsing process it gets untrapped (provided the maximum value of \( \dot{R} \) is greater than \(-1\)) and then again it is trapped and black hole forms. However, for \( \gamma \leq -5/3 \), the system is initially untrapped and as it approaches to the singularity at \( \tau = \tau_0 \), it gets trapped and leads to the formation of a black hole. Thus dark energy alone in the form of Chaplygin gas favours formation of black hole.
III. COLLAPSING PROCESS UNDER THE JOINT INFLUENCE OF DARK MATTER AND DARK ENERGY

This section is divided into two parts. In the first case, the interaction $Q(t)$ is neglected while in the second case, the influence of $Q(t)$ is considered.

**CASE I : Interaction is neglected :** $Q(t) = 0$

Here the conservation equation for $\rho_{DM}$ gives

$$\rho_{DM} = \frac{\rho_0}{a^3}, \quad \rho_0 > 0 \quad \text{a constant.} \quad (18)$$

The expressions for $\dot{R}(\tau)$ and $M(\tau)$ are

$$\dot{R}(\tau) = -R_0 \ a^{\frac{1}{2}} \left[ \rho_0 + a^{-3\gamma} \left\{ C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)} \right\} \right]^{\frac{1+\gamma}{1+\alpha}} \quad (19)$$

and

$$M(\tau) = \frac{1}{2} R_0^2 \ r_{\Sigma} \ a^2 \left[ \rho_0 + a^{-3\gamma} \left\{ C + \frac{B}{1+\gamma} a^{3(1+\alpha)(1+\gamma)} \right\} \right]^{\frac{1+\gamma}{1+\alpha}} \quad (20)$$

with $R_0 = r_{\Sigma} \sqrt{\frac{2}{3}}$.

As the integral in equation (19) can not be evaluated in general, so only the approximate forms for ‘a’ may be obtained for small and large ‘a’. However, one can determine the behaviour of the physical parameters in these two limits (namely, $a \to 0$ and $a \to \infty$) as follows:

$$a \to 0 : \quad \rho_{DM} \to \infty, \quad \rho \to \begin{cases} \infty \text{ if } 1+\gamma > 0 \\ \text{a constant} \text{ if } 1+\gamma < 0 \end{cases}, \quad \dot{R} \to \begin{cases} 0, \text{ if } \gamma < \frac{1}{3} \\ -\infty, \text{ if } \gamma > \frac{1}{3} \end{cases}, \quad M \to \begin{cases} 0, \text{ if } \gamma < \frac{2}{3} \quad , \quad \mu = R_0 \ C^{\frac{1}{2(1+\alpha)}} \end{cases} \quad (21)$$

$$a \to \infty : \quad \rho_{DM} \to 0, \quad \rho \to \begin{cases} a \text{ constant}, \text{ if } 1+\gamma > 0 \quad , \quad \hat{\rho} \to -\infty, \quad M \to \infty. \end{cases} \quad (22)$$

Thus $a = 0$ is always a singularity of the space-time and it is covered by an apparent horizon for $\gamma \geq 1/3$ (provided $\mu > 1$) while the singularity is naked for $\gamma < 1/3$.

**CASE II : Gravitational Collapse with Interaction:**

Recently, Cai and Wang [9, 10] have assumed the ratio of dark energy density and dark matter density as

$$\frac{\rho}{\rho_{DM}} = A \ a^{3n} \quad (23)$$
with $A > 0$ and $n$ as arbitrary constants. then solving the conservation equations (6) and (7) one obtains

$$\rho_i^{a+1} = \frac{(\alpha + 1)B}{[\alpha(n - 1) - 1]} \frac{(A a^{3n})^{\frac{\alpha}{n}(a+1-na) - (\alpha+1)}}{(A a^{3n+1})^{\frac{\alpha}{n}(a+1-na) + \gamma (\alpha+1)}} \times$$

$$2F_1\left[\frac{1 + a - n\alpha}{n}, \frac{1 + n + \alpha + \gamma + \alpha\gamma}{n}, \frac{1 + n + a - n\alpha}{n}, \frac{A a^{3n}}{1 + A a^{3n}}\right] + z_o \left[A a^{3n} (1 + A a^{3n})^\gamma\right]^{-(\alpha+1)}$$

(24)

where $\rho_i = \rho + \rho_{DM}$ and using (23) one gets

$$\rho = \frac{A a^{3n} \rho_i}{1 + A a^{3n}}, \quad \rho_{DM} = \frac{\rho_i}{1 + A a^{3n}}$$

(25)

Hence from the conservation equation (6) and the Friedman equation (4) the expression for the interaction is

$$Q(t) = -\frac{3(\gamma + n)A a^{3n} \rho_t}{(1 + A a^{3n})^2} \frac{\dot{a}}{a} + \frac{3B(1 + A a^{3n})^{\alpha - 1}}{\rho_i^\alpha A a^{3n\alpha}} \frac{\dot{a}}{a}$$

(26)

where

$$\frac{\dot{a}}{a} = -\frac{\rho_0}{a^{\frac{\alpha}{n}} (1 + A a^{3n})^\frac{\alpha}{2}} \left[\frac{\rho^6(\alpha-1-na) A_6^{\frac{\alpha}{n}(a+1-na)}}{(1 + A a^{3n})^{\frac{\alpha}{2}(a+1-na)}} \timesight.$$

$$2F_1\left[\frac{1 + a - n\alpha}{n}, \frac{1 + n + \alpha + \gamma + \alpha\gamma}{n}, \frac{1 + n + a - n\alpha}{n}, \frac{A a^{3n}}{1 + A a^{3n}}\right] + z_1\left[\frac{1}{2a^{\alpha+1}}\right]$$

(27)

with

$$\rho_0 = \sqrt{\frac{\kappa}{3A}} \left[\frac{(\alpha + 1)B}{\alpha(n - 1) - 1}\right]^{\frac{1}{2a^{a+1}}}, \quad z_1 = z_0 \left[\frac{(\alpha + 1)B}{\alpha(n - 1) - 1}\right]^{\frac{1}{2a^{a+1}}}$$

The equation (27) can be written in the integral form as

$$\int\left[\frac{(1 + A y^{2})^{\frac{\gamma}{2}}}{(1 + A a^{3n})^{\frac{\alpha}{n}(a+1-na)}} \right]^{2F_1\left[\frac{1 + a - n\alpha}{n}, \frac{1 + n + \alpha + \gamma + \alpha\gamma}{n}, \frac{1 + n + a - n\alpha}{n}, \frac{A a^{3n}}{1 + A a^{3n}}\right] + z_1\left[\frac{1}{2a^{\alpha+1}}\right]} dy = -y_0(t - t_0)$$

(28)

with $y = a^{3n/2}$ and $y_0 = \frac{3n}{2} \rho_0$.

The expression for mass function and $\dot{R}$ are

$$M(\tau) = \frac{r_{\Sigma}^3 \rho_0}{2a^{3n-1} (1 + A a^{3n})} \left[\frac{a^{6(\alpha+1-na)} A_6^{\frac{\alpha}{n}(a+1-na)}}{(1 + A a^{3n})^{\frac{\alpha}{n}(a+1-na)}} \timesight.$$

$$2F_1\left[\frac{1 + a - n\alpha}{n}, \frac{1 + n + \alpha + \gamma + \alpha\gamma}{n}, \frac{1 + n + a - n\alpha}{n}, \frac{A a^{3n}}{1 + A a^{3n}}\right] + z_1\left[\frac{1}{2a^{a+1}}\right]$$

(29)
and

\[ \dot{R}(\tau) = -\frac{\rho_0 r\Sigma}{a^{\frac{4n}{3}} (1 + A a^3)^\frac{2}{3}} \left[ \frac{a^{6(\alpha+1-n\alpha)} A^{\frac{2}{3}(\alpha+1-n\alpha)}}{(1 + A a^3)^\frac{2}{3}(\alpha+1-n\alpha)} \times \right. \]

\[ \left. _2F_1\left[ \frac{1 + \alpha - n\alpha}{n}, \frac{1 + n + \alpha + \gamma + \alpha\gamma}{n}, \frac{1 + n + \alpha - n\alpha}{n}, \frac{A a^3}{1 + A a^3} \right] + z \right] \]

(30)

The above expressions for the physical parameters show the following limiting behaviour

\[ \rho_t \sim \begin{cases} a^{-3n}, & a \to 0 \\ a^{-3n(\alpha+1)(\gamma+1)}, & a \to \infty \end{cases} \]

\[ \rho \sim \begin{cases} a \text{ constant}, & a \to 0 \\ a^{-3n(\alpha+1)(\gamma+1)}, & a \to \infty \end{cases} \]

\[ \rho_{DM} \sim \begin{cases} a^{-3n}, & a \to 0 \\ a^{-3n(\alpha+1)(\gamma+1)}, & a \to \infty \end{cases} \]

\[ \dot{R}(\tau) \sim \begin{cases} -a^{1-\frac{n\gamma}{2}}, & a \to 0 \\ -a^{1-\frac{n\gamma}{2}(1+\gamma)}, & a \to \infty \end{cases} \]

\[ M(\tau) \sim \begin{cases} a^{1-3n}, & a \to 0 \\ a^{-3n(\gamma+1)}, & a \to \infty \end{cases} \]

It is to be noted that \( a = 0 \) is always a singularity of the space-time but \( a = \infty \) is singularity if \( 1 + \gamma < 0 \).

The above integral in equation (28) is solvable for the (choice \( \alpha = 1, n = 2 \) and one gets) restriction \( 1 + \alpha = n\alpha \) and one gets

\[ \frac{y}{(1 + z_1)^{\frac{2}{2}} \Gamma(\frac{1}{2} - \frac{\gamma}{2}) A y^{2}} = -y_0(t - t_0) \]

(31)

Thus in the limit as \( a \to 0 \), one finds

\[ a \sim a_0 (t_0 - t)^{\frac{2}{3}}, \quad a_0 = y_0^{\frac{2}{3}} (1 + z_1)^{\frac{2}{3}} \text{ i.e., } a \sim 0 \text{ as } t \to t_0. \]

Further, using the property of the hypergeometric function (see equation (16)), for large \( a \), the solution (31) approximates to

\[ a^{1+\gamma} \sim a_1 (t_0 - t), \quad a_1 = \frac{4a_0}{(1 + \gamma)A^{\frac{2}{2}}} \text{ for } \gamma > -1 \]

and

\[ \frac{\sqrt{\pi}}{2\sqrt{A} (1 + z_1)^{\frac{2}{2}} \Gamma(\frac{1}{2})} = -y_0(t_s - t_0) \]

\[ \text{i.e., } t_s = t_0 - \frac{\sqrt{\pi}}{2\sqrt{A} a_0^{\frac{2}{2}} \Gamma(\frac{1}{2})}, \text{ for } \gamma \leq -1 \]

The limiting value of the physical parameters show that if \( n < 2/3 \) then the space-time collapses to a naked singularity while black hole will form for \( n > 2/3 \). However, the singularity at \( a = \infty \) always corresponds to a black hole solution for \( 1 + \gamma < 0 \).
IV. CONCLUSION

The paper deals with gravitational collapse of a spherically symmetric homogeneous and isotropic fluid having finite radius. The fluid has two component — one component is dark matter in the form of dust and the other, the dark energy component is the modified Chaplygin gas model.

When the collapsing fluid is only in the form of modified Chaplygin gas (the dark energy) then the collapse always leads to the formation of a black hole. But there is some peculiarity for $\gamma > -5/3$. Initially, the space-time is trapped and during the evolution it gets untrapped and again it is covered by an apparent horizon. This feature is interpreted by Cai and Wang [10] (see also [11]) as the evaporation of a white hole by ejecting matter which again re-collapse to form a black hole. Note that the collapsing dark energy in the form of Chaplygin gas can alone form black holes unlike the dark energy model of Cai and Wang [10] is not in favour of black hole.

Section III deals with collapsing fluid having both components with or without interaction. In both cases it is also found that when the dark energy density dominates over the dark matter energy density then the collapse favours formation of black hole. Further, the expression for the interaction parameter has two terms of which the first one is identical to that of Cai and Wang [10] while the second term, due to the Chaplygin gas having negative sign reduces the interaction parameter. Therefore, from the above study, one may conclude that the dark energy is not always against the formation of black holes, it favours the formation of apparent horizon in some cases.

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