The 9th grade students’ mathematical understanding in problem solving based on Pirie-Kieren theory

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Abstract. The purpose of this study is to analyse the mathematical understanding in 9th grade students’ problem solving based on Pirie-Kieren theory. The Pirie-Kieren theory provides a framework to evaluate the students’ mathematical understanding in solving mathematical problem. The layers of mathematical understanding in Pirie-Kieren theory consist at primitive knowing, image making, image having, property noticing, formalising, observing, structuring, and inventizing. This study designed according to case study toward two students who are studying at one of primary school in Bandung district. The participants in this study were selected among voluntary students by considering their mathematics interesting and recommendation from the mathematics teacher. The data was collected by testing and interviewing. One problem solving item was posed for the mathematics test. The interview of this study was collected by through a semi-structured interview. The obtain data was descriptive analysis with qualitative approach based on Pirie-Kieren theory. The result show that two of participant was started working at image having layer and they have placed at formalising layer. The participants have lacked at primitive knowing and image making according to Pirie-Kieren theory.

1. Introduction

The curriculum of 2013 mandates a learning process that involves interaction among learners, between learners with educators and learning resources in the certain learning environment. The Pirie-Kieren theory attempts to elaborate the constructivist definition of understanding as a continuing process of organizing one’s knowledge structure in detail. This theory sees student understanding in the framework of a whole dynamic, leveled but non-linear, and recursive process [1]. We can conclude that the Pirie-Kieren theory shows about how student’s thinking activity. Pirie-Kieren theory identifies eight layers of mathematics understanding, that are: primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventizing [2].

The layer of Pirie-Kieren theory started at primitive knowing, this layer does not imply low level mathematics, but does rather the starting place for the growth of any particular mathematics understand. For the growth of initial understanding of Pythagoras theorem, observer assumed that the students already knew mathematics operation, the direction of wind, and the two-dimensional figure. At the second layer is Image making the learner is able to make distinctions based on previous abilities and knowledge. At the next layer is image having that single-activity associated images are replaced by a mental picture. The image making and image having are student’s activities for making a new image of
revising an existing image, and then for manipulating that image in the mind. Then property noticing layer is an action to identifying properties of the constructed image. In formalizing layer, the learner is able to cognize about the properties to abstract common qualities from classes of images. Descriptions of these class-like mental objects result in the production of full mathematical definitions. The following layer of understanding is observing, entails the ability to consider and reference one’s own formal thinking. The learner is able to produce verbalizations concerning cognitions about the formalized concept. After gaining such awareness the learner can explain the interrelationships of the observation by an axiomatic system at structuring layer. At structuring layer, the learner would able to conceive proofs of properties associated with the concept. The outermost layer in Pirie-Kieren theory is inventizing. The use of inventizing does not imply that one cannot invent at other layers, but rather is used to indicate the ability to break free of a structured knowledge which will result in the development of a new concept. At inventizing layer the learner’s mathematical understanding is unbounded, imaginative and reaches beyond the current structure to contemplate the question “what if?” [3-5].

Even though the term of ‘understanding’ is freely used in literature, a search for a substantial definition of understanding is still on progress [6]. Not all students will arrive at the same level of understanding at the same time, so each student will arrive at their understanding differently. Students who have low achievement can reach formalizing level in Pirie-Kieren theory, based on result of the research the student can construct the difficult concepts by interpreting activities [7]. The other research Pirie-Kieren theory used as a framework to students’ conceptual and procedural understanding of fractions, and the result showed that students had difficulty with the question in the structuring and inventising levels [8]. In the understanding process, students use to do folding back when they do problem solving. Students’ folding back frequently in a variety of ways and for range of reason so that it can be said that students’ understanding continues to growing up [9]. Understanding a student’s understanding requires not only a close examination of his/her written work, but also involves a careful analysis of the student’s thought and ideas while performing the computations [10].

This study purposed to analyse the mathematical understanding students’ problem solving based on Pirie-Kieren theory. The main purpose of learning mathematics is to find ways to solve problem [11]. Learning to solve problems is very important in formal educational settings, that because of the understanding of its processes is limited [12]. Mathematics understanding is needed to problem solving. Some related study about problem solving found Problem-solving skills have important place not only in mathematics but also in real life. By knowing how students understanding process to solve the problem, the teacher can found how process of thinking high achievement student and applying that to low achievement student. Then teacher have a alternative way to improve students problem-solving skill. The problem types used in mathematics divided into two groups, that are routine and routine-problems [13]. This study wills examination specifically at routine problem to determine the mathematical understanding student base on Pirie-Kieren theory.

2. Method
The descriptive analysed with qualitative approach was used in this study. This study participant is consisted of two volunteer 9th grade students who had learned Pythagoras’ theorem and favoured mathematics. The collect data was test and interview. One routine problem (essay) was given to participants related with Pythagoras’ theorem. Each participant obtained item and answer sheet and were not allowed to use calculator and to wipe off the wrong answer for assurance their authentic solutions. Duration for the test is 45 minutes. The interview of the study was collected by a semi-structured interview. The purpose of this study is to analyse in-depth of mathematical understanding (specific in mathematics problem solving) of these student according to Pirie-Kieren theory.
3. Result and discussion

3.1. Result

In this study, test items were used to analyze students’ mathematics understanding on mathematics problem solving item represent eight layers in Pirie-Kieren theory perspective that are, Primitive Knowing, Image Making, Image Having, Property Noticing, Formalizing, Observing, Structuring, and Inventizing. The findings are included about at which layer participants are in the process of constructing information according to the Pirie-Kieren theory. Participants (Student 1 and Student 2) were given test about Pythagoras theorem for junior high school level.

We began our analysis by carefully observing the participant while did the test in order to determine the significant thinking process: what they think after read the question and what they do for the next step. Considering this page of work on Figure 1 and Figure 2, we notice that Student 1 (S1) and Student 2 (S2) have correctly determined the Pythagoras theorem. But both of them have difficult in determining the direction of the wind. The two participant equally started working at image having layer. Them did the folding back to found a knowledge that have connection can have used to answer the given question. Two participants placed at formalizing layer.

![Figure 1. Student S1’s worksheet.](image-url)
3.2. Discussion

From the Pirie-Kieren perspective, Student 1 and Student 2 equally started working from the image having layer. They think the appropriate concept, and them knows that the concept is connect to Pythagoras theorem. After that, both of them did folding back to image making. They try to make an appropriate image by the question. Student 2 make a triangle image and moved on property noticing. At property noticing, she tries to found the properties of Pythagoras theorem. After she found the properties, Student 2 is at formalizing layer that she determines the result and make the conclusion about the shortest distance that will take according the question.

At the different way, Student 1folding back again from image making to the primitive knowing, because she had a problem to define the direction of wind. From the primitive knowing, S1 back to image making to make the image that appropriate with the question, then she move on property noticing layer to found the properties, she can analysed the existing mental image about the order of the shortest distance that requested on question. After the S1 had completed the property noticing layer, she moved on formalizing layer. Using Pythagoras theorem and two-dimensional figure S1 found the correct result of the question.

Student 1 (S1) and Student 2 (S2) equally started at image having, S1 did 2 times folding back and make a connection about all the concept and knowledge that she has, then she gives the answer correctly. S2 did 1 time folding back and didn’t make a connection about all concept and knowledge that she has, so S2 give the incorrect answer. This might happen because less of understanding at the primitive knowing. Description about process construction understanding participant can have identified from Figure 3.
Figure 3. Our Mapping of Student 1 and Student 2 mathematical understanding on the Pirie-Kieren model.

In this discussion we conclude that students always doing folding back in problem solving process, this folding back shows their mathematical understanding. As Martin said in his research that “folding back is likely to lead to a thicker and deeper understanding of the mathematical concept, and it an essential stage in the dynamical growth of mathematical understanding” [9].

4. Conclusion
For this study, we have succeeded to know how students’ mathematics understanding of mathematics problem solving item based on Pirie-Kieren theory at one of junior high school in Kabupaten Bandung. We used 1 item tests that are well-defined (routine) problem. The finding of this study confirmed that the participant equally started working the problem at image having layer. Two of student did folding back to found connection and the right ways to completed the question, but they have a different way to think. We also found that two participants still on formalizing layer. This phenomenon needed to analyses furthermore whether the student achieved object or schema conception of not.

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