A Simplified Approach for Predicting the Time of Consolidation for a Multi-Layered Foundation

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Abstract

This paper deals with a simplified approach of predicting the time of consolidation settlement for a multi-layered foundation using the coupling technique for settlement and unsettlement zones in a specific compressible layer and layers. According to MIT’s lecture, every two layers should combine into one having an equivalent degree of consolidation or having an equivalent thickness of consolidation. From this valuable guideline, several procedures are developed to tackle the specific problem of the settlement of a multi-layered foundation. This study also introduces an idea for coupling the layers, concerning the drainage path during combining every two layers, and some practical points are suggested.

Keywords: Consolidation settlement; Equivalent coefficient of consolidation $C_{v,eq}$; Thickness $H_{eq}$; Settlement zones

Introduction

Estimating the consolidation settlement of a layered foundation is a complicated task. In multi-layered foundation, and the rate of consolidation settlement for such a multi-layered foundation is a non-linear problem approximately solved by many research works such as: Taking the varied compressibility into account [1]; considering the variation in permeability [2]. A technique of coupling every two compressive layers into one equivalent is developed by applying MIT’s suggestion [3].

Figure 1: Settlement zones in a compressible layer.

During the time of consolidation, there are the settlement zones 1a, 2a, and unsettled zones 1b, 2b or still zones recognized (Figure 1), and the compressibility of the zone of settlement (i.e., 1a for layer 1 and 2a for layer 2) has definitely varied, both in permeability.
and compressibility. By assuming that the unsettled layer (i.e., 1b or 2b in Figure 1) has no variation in the compressibility so that each compressible layer has two separate sub-layers (e.g., 1a and 2a of layer 1), which the coefficient of consolidation is not the same as the original values. A question is what occurs if the zone of settlement, e.g., 1a or 2a, which both have changed the compressibility and the coefficient of consolidation could take into account.

This paper aims to further some results studied by [3,4] investigate the consolidation settlement of the multi-layered compressible foundation, concerning the settlement zones into account.

**Theoretical Background**

**Assumptions**

a) Every multi-layered foundation has compressible layers and very low compressible layers (e.g., sand, even a thin layer). The sand layer plays the role of a drainage boundary. As a default concept, the other boundary of drainage is the ground surface. In the other words, in most possible cases, there is a pervious layer in the foundation.

b) The coefficient of consolidation $C_v$ is time-variant quantity, for a varied coefficient of permeability $k$ as expressed in the following equation 1:

$$ C_v = \frac{k}{m_v \gamma_w} \quad (1) $$

c) The coefficient of volume compressibility $m_v$ also varies with respect to void ratio, therefore a time-variant variable.

d) Coefficient of consolidation $C_v$ will change its value within a settlement zone while being constant if outside the settlement zone (zone 1b in Figure 1).

e) Each layer has its constant characteristics of compressibility and permeability. The load is fully constructed at the start of loading, and soil is normally consolidated. The condition of equivalence is the two models have the same degree of consolidation.

f) The permeability varies only within the settled zone (i.e., zone 1a or 2a); the coefficient of permeability $k$ would be of 3-order power of the void ratio, i.e. $k \propto e^3$;

g) The Terzaghi’s theory of 1-D consolidation is still valid within a compression layer.

**Equivalent coefficient of consolidation**

The double-layered foundation are converted to a single layer having the equivalent coefficient of consolidation from the equation (2) without changing the thickness of the foundation (Figures 2 & 3).

$$ C_{v,eq} = \frac{\sum_{i=1}^{n} h_i}{\sum_{i=1}^{n} \sqrt{C_v h_i}} \quad (2) $$

**Equivalent thickness of consolidation settlement**

MIT suggested that an alternative keeps the properties as original values but changes the depth into an equivalent one having the same compressibility (the smaller value of $C_v$ is taken into account) [5].

The key assumption for the latter approach is that both the boundaries are either pervious or impervious, so that the drainage length would be $d = H_{eq}/2$ in which $H_{eq}$ is the equivalent thickness of the converted two-layer foundation.
where \( h_1 \) are the thickness having the smaller value of the coefficient of consolidation, or \( C_{v1} \), \( H_{eq} \) then is the equivalent thickness of the foundation that has the greater value of the coefficient of consolidation \( C_{v2} \).

### A simplified model for estimating the degree of consolidation

Fine-grain soil as clay would be assumed to be time-dependently compressible, while non-cohesive soil like sand will not be compressible. The multi-layered foundation with strata are described in Figure 4:

In each layer, the settlement, denoted \( s_1 \) or \( s_2 \), is easily predictable as the accumulated vertical deformation over the total thickness of the layer, and then there is an unsettlement zone beneath the settlement zone in this layer; \((H_1 - s_1)\) or \((H_2 - s_2)\) as in Figure 4. \( C_{v,eq1} \) is the equivalent coefficient of consolidation combining the compressibility of sub-layer 1, and the unsettlement zone \((H_1 - s_1)\) having original \( C_v \); \( C_{v,i-j} \) is of the combined lower compressible layers. These equivalent converted layers with sub-layers take into account the boundaries of drainage. There are two main patterns of combination: lower and upper equivalent layers which are open layers in the upper part and half-open in the lower part of the foundation. Pattern 1 has the coupled lower conversion, whilst pattern 2 has the upper conversion [6].

The time of consolidation for the two possible cases is mainly different due to the drainage condition, i.e. the length of drainage \( d \) and the effective normal stress in layers. This concern points out that formula (2) using the coefficient of consolidation \( C_v \) in prediction is more practicable and reasonable than formula (3) with \( H_{eq} \).

### A model for validation

The equivalent coefficient \( C_{v,eq} \) and the equivalent thickness \( H_{eq} \) are calculated by the formula (2) and (3). The following model will apply it in calculating the consolidation, concerning the compressibility of settlement zones.

### Table 1: Soil layers in this study.

| No | Depth of Sampling | Void Ratio \( e_0 \) | Coefficient of Consolidation \( C_{v90} \) (\( \text{cm}^2/\text{s} \)) \( \times 10^3 \) | Compression Index \( C_c \) | Preconsolidation Pressure \( \text{KPa} \) | Description |
|----|-------------------|---------------------|--------------------------------|-----------------|-----------------|-------------|
| 1  | 2                 | 3                   | 4                              | 5               | 6               | 7           |
| 1  | 1.8-22            | 2.686               | 0.328                          | 1.494           | 67.3            | Clay mud, liquid state. |
Real soft soil properties at a real site in District 2 Ho Chi Minh City (a typically soft soil foundation) are reviewed as in Table 1. As per the Terzaghi’s assumption for the case in which the loading width extends to infinity or very large as compared to the thickness of compressible layer, the 1D consolidation is applicable. The site subjected a 1D consolidation with total stress 32 kPa over a very wide area (nearly 2 meters backfill height over very soft soil foundation, spreading over a very wide area of backfilling) (Figures 5 & 6).

Data are input into the Plaxis 2D model using Mohr Coulomb model, with drained mode for consolidation analysis.

### Consolidation settlement

| Layer | Level          | Thickness | γsat | C  | φ  | E  | Settlement (mm) |
|-------|----------------|-----------|------|----|----|----|-----------------|
| 1+2   | Ele. -1.8 to Ele. -22 | 2.2       | 13.8 | 6.4 | 3  | 1500 | 139.34          |
| 4     | Ele. -29.8 to Ele. -32 | 2.2       | 20.8 | 41.2 | 15 | 2800 | 11.2            |
| 5     | Ele. -33.8 to -40.0 | 6.2       | 21.6 | 16.7 | 13 | 5200 | 22.8            |
| Total |                |           |      |     |    |     | 173.34 mm       |

Under the overburden pressure $p_o$ (e.g. layer 1, $p_o = 43.4$ kPa) and additional pressure $\Delta p = 32$ kPa, with properties and soil stiffness, the total settlement calculated and described in Table 2.

### Equivalent coefficient $C_{eq}$ and thickness $H_{eq}$

**Case 1:** Taking settlement zone 1a, 2a (Figure 1) into account, calculating the equivalent coefficient of consolidation $C_{eq}$. 

![Figure 5: Plaxis model.](image1)

![Figure 6: Input data for 1D Consolidation Analysis using Plaxis 2D for comparison purpose.](image2)
Case 2: In each layer, the settlement zone and the rest part (unsettlement zone) will be converted to a new layer and so on.

Comparative Results of Consolidation Settlement

According to the results of the abovementioned convert in Table 3, the time-dependent settlement could be computed by the Terzaghi's theory of 1D consolidation, results are described in Table 4. A finite element model is created with Plaxis 2D V.8.5 as in Figure 2, with Mohr-Coulomb model of analysis for comparison purpose. The Plaxis output of the vertical displacement is shown in Figure 4a and the time-dependent settlement of consolidation is plotted as in Figure 4b.

Table 3: Equivalent values of coefficient of consolidation $C_{v,eq}$ and thickness $H_{eq}$.

| Layer | Parameter       | Case 1: Equivalent Coefficient $C_{v,eq}$ | Case 2: Equivalent Thickness $H_{eq}$ |
|-------|-----------------|------------------------------------------|-------------------------------------|
|       | $C_v$ (cm$^2$/s) | $C_{v,eq}$ | Thickness (m) | $H_{eq}$ | $Cv$ (cm$^2$/s) | $H_{eq}$ | $Cv$ (cm$^2$/s) | $H_{eq}$ | $Cv$ (cm$^2$/s) | $H_{eq}$ |
| 1     | Thickness (m)   | 22.4 | 26.59 |
|       | $C_v$ (cm$^2$/s) | 3.28E-04 | 3.26E-04 | 24.59 | 26.86 |
| 4     | Thickness (m)   | 2.2 | 3.27E-04 | 24.54 |
|       | $C_v$ (cm$^2$/s) | 3.27E-04 | 3.32E-04 | 30.8 | 3.28E-04 |
| 5     | Thickness (m)   | 6.2 | 6.2 |
|       | $C_v$ (cm$^2$/s) | 4.77E-04 | 4.77E-04 |

Table 4: Consolidation settlement up to 500 days (settlement in cm).

| Time (Days) | By $C_{v,eq}$ | This Study (Adjusted $C_{v,eq}$) | By $H_{eq}$ | This Study (Adjusted $H_{eq}$) | By Numerical Model (Plaxis) |
|-------------|---------------|---------------------------------|-------------|--------------------------------|------------------------------|
| 0           | 0             | 0                               | 0           | 0                              | 0                            |
| 100         | 4.218         | 6.8                             | 4.972       | 7.8                            | 4.5                          |
| 200         | 7.607         | 9.6                             | 8.109       | 11                             | 7.5                          |
| 300         | 9.013         | 11.8                            | 9.766       | 13.4                           | 9.7                          |
| 400         | 10.419        | 13.6                            | 11.424      | 15.5                           | 11.5                         |
| 500         | 11.825        | 15.2                            | 13.081      | 17.3                           | 13.6                         |

Discussion

a) Results obtained by numerical model Plaxis 2D used the update meshing for large strain indicate a nearly perfect 1-dimension consolidation (cf Figure 7a & 7b).

b) It is tentatively recognized that coupling the layers concerning the settlement zones might result in a greater value of consolidation settlement; the converted foundation which uses the equivalent $C_{v,eq}$ agrees with the results given by the numerical model, i.e Plaxis©, and predicts a longer time of consolidation. As such it is on the safer side for warning purposes.

c) The degree of consolidation might be different between the two approaches of equivalent foundation, especially when the soil layers are highly compressible, the soft soil layers are thick and the technique of equivalent coefficient of consolidation should be studied with care.
Conclusion

Consolidation settlement could be approximately estimated by using an equivalent coefficient of consolidation or equivalent thickness of compressible layers according to MIT’s approximation. This study suggests every two layers will be coupled to convert them into one single and the settled zone may be viewed as a layer having varied compressibility. An equivalent foundation should use the equivalent coefficient of consolidation $C_{v,eq}$ instead of the equivalent thickness $H_{eq}$ for a safer prediction. This idea of this study which takes the settlement zone into account and the equivalent coefficient of consolidation should be studied further for predicting the time-dependent settlement in the long term.

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Figure 7: 1D consolidation with vertical deformation and results of time-dependent settlement with/without concerning settlement zones.