Profile of students' errors in trigonometry equations

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Abstract. Learning activities during lessons do not only teach a material, but also need evaluation of those learning. One way to evaluate students’ learning is by knowing the students' mistakes related to the material being taught. This study aims to determine errors and their causes in solving trigonometric equation problems. Data collection is done through tests, interviews, and observations. The research test instrument consists of 3 trigonometric equation questions. From 203 students of SMA Batik 1 Surakarta, there were 34% of data errors, 32.35% of concept errors 45.15% of strategy errors, 34.81% of calculation error and 30.38% of conclusion error. The common cause of these errors is that students do not understand the concept of trigonometric periods or angles in various quadrants, students forget the trigonometric values at special angles and students are not careful enough in doing or solving the problem.

1. Introduction
The development of science and technology to face the 4.0 industrial revolution demands an increase in the quality of human resources. One effort to increase human resources is through education. Especially in mathematics education has an important role in everyday life. Through mathematics learning students are expected to foster the ability to think logically, critically, systematically, effectively, efficiently in solving problems.

One of the successes of students in understanding mathematics can be assessed from the implementation or not of mathematics learning. Therefore, it is necessary to evaluate learning by giving tests to students. The test can be in the form of daily tests, midterm tests, end of semester tests and national exams. When viewed from the results of national examinations from several years, material that is still considered low is trigonometry. The average absorption capacity of the National Examination on indicators containing trigonometric problems at the national level in the 2015/2016 academic year was 48.75% [1], and in the 2016/2017 school year the absorption rate was 39.68% [2]. These indicate that understanding trigonometric material for students is still low.

Trigonometry is an important part of mathematics subjects taught at the high school level. Knowledge of trigonometry is a prerequisite for understanding topics in physics, architecture, and many other branches of engineering [3]. Students assume that trigonometric material is difficult. Students when working on Trigonometry questions still have many difficulties. The difficulty of students in working on a problem can be seen from the errors in students’ answers. Errors in doing the questions are not only done by the lower groups, but the middle and high groups also make mistakes. The mistakes made by students can be known by doing an error analysis of students in solving trigonometric equation problems. Analysis is an investigation of an event to find out the actual situation [4].
The error comes from the wrong word, which means it is incorrect or deviates from what is supposed to be [4]. The mistakes that students often make according to Newman are errors in reading, understanding, transformation, completion, and conclusion [5]. There are also types of errors according to John Watson (2006), the data error, the procedure is incorrect, data is lost, conclusions are lost, conflict level response, indirect manipulation, problem of hierarchy of skills, and others than those seven errors [6]. In addition, in Rafi's research found errors that were often made by students were technical errors, theorems and definition errors, missing data errors, and checking answer errors [7].

There are 5 types of errors used in this study, namely data errors, misconceptions, strategy errors, calculation errors and conclusion errors. The indicators of each type of error are as follows.

**Table 1. Error type indicator.**

| Error Types       | Indicators                                                                 |
|-------------------|-----------------------------------------------------------------------------|
| Data Error        | Students are writing down existing problem data wrongly                      |
|                   | Students are translating existing problems wrongly                          |
| Concept Error     | Students are determining the formula or theorem or definition to answer the problem incorrectly |
|                   | Students do not write formulas or theorems or definitions to answer problems |
| Strategy Error    | Students try to operate at the right level on a problem, but use procedures or methods that are not appropriate |
|                   | Students try to operate at the right level, but choose inappropriate data information |
| Calculation Error | Students are giving or writing signs of mathematical operation incorrectly  |
|                   | Students are counting maths operations such as adding, subtracting, multiplying and dividing wrongly. |
| Conclusion Error  | Students are determining conclusions incorrectly                             |
|                   | Students do not write conclusions                                             |

Based on students' problems about understanding trigonometric material, the researcher wants to examine students' errors in solving trigonometric equation problems. Thus, information about errors made by students can be used as a consideration for planning new learning strategies to reduce errors that occur when working on trigonometric equation problems.

2. Method

This research is qualitative and quantitative research. Qualitative research is research to understand the phenomena experienced by research subjects (behavior, perceptions, actions, etc.) holistically and by describing (in words and languages) [8]. This is in the form of student error analysis and the causative factor in the trigonometric equation material. Quantitative research is in the form of figures from processed data. This is in the form of a percentage of each type of students' errors. This research was conducted at Surakarta Batik 1 High School. The subject of this study was the XI grade students of MIPA with a total of 203 students. The data in this study were the types, underlying factors and the percentage of students' errors. Data was collected through tests and interviews. The test results of students are corrected one by one and classified as any mistakes made. Then the subject of each type of error is selected to be analyzed about the error and the underlying factors. Data analysis techniques are based on Milles and Huberman's steps, namely data reduction, data presentation, data conclusions [9]. From the mistakes made by students, the percentage of each type of error is obtained, to find out what type of error is often done by students. The error percentage is obtained from:

\[ p\% = \frac{n}{203} \times 100\% , \text{ where } n \text{ is the number of errors.} \]
The trigonometric equation used is three items. There are 4 types of questions, but only differ in numbers and types of trigonometry. Examples of these questions are as follows: Determine the set of resolutions from the following trigonometric equation (type A):

\[
\sin(2x - 10) = \frac{1}{2}\sqrt{3}, \quad \text{for} \quad \{0^\circ \leq x \leq 360^\circ\}
\]

\[
5\sqrt{2}\sin x - 5\sqrt{2}\cos x = 5, \quad \text{for} \quad \{0^\circ \leq x \leq 360^\circ\}
\]

\[
\cos 2x - 3 \cos x + 2 = 0, \quad \text{for} \quad 0 \leq x \leq 360^\circ
\]

3. Result and discussion

3.1. Data Errors

Data errors occur when students are writing data from existing problems incorrectly, or students are wrongly transforming the data into the desired form.

In Figure 1, the student was determining the trigonometric value incorrectly that is \(\sin 30^\circ = \frac{1}{2}\sqrt{3}\). It should be \(\sin 30^\circ = \frac{1}{2}\). It made the next answer also experienced an error. From the results of interviews with students, students forget the trigonometric values at special angles. In Figure 2, the students were determining the value of A and B wrongly, besides the formula \(\tan \alpha = \frac{B}{A}\). Based on interviews with students, students assume that the coefficient that lies ahead are A, and the back coefficient is B. It can be analyzed that students have not understood the concept of trigonometric equations form of \(A \sin x + B \cos x = C\). Supposedly if students use formulas \(\tan \alpha = \frac{B}{A}\) then the value of B is the coefficient of Sinus and the value of A is the coefficient of Cosinus, or formula \(\tan \alpha = \frac{A}{B}\) then the value of A is the coefficient of Sinus and the value of B is the coefficient of Cosinus.

3.2. Concept Errors

Misconceptions occur if students are wrong or do not write formulas, theorems, definitions to answer existing problems.
In Figure 3 the student was wrongly determining the formula of the sine. The sine formula should be $x = \alpha + k \cdot 360$ and $x = (180 - \alpha) + k \cdot 360$. Based on interviews with students, students often flipped back and forth with sine, cosine and tangent formulas. It is possible that students do not understand the concept of angles of various quadrants and the nature of trigonometric angular periods. Students' understanding of the basic concepts of trigonometry is still low. As Weber said that students' mistakes about trigonometry are caused by poor initial understanding of basic concepts such as angles, right triangles and circles [3]. In Kamber's research, many students experience difficulties with the nature of periodicity and trigonometric functions [10].

In Figure 4 the student was writing trigonometric identity incorrectly. It should be $\cos 2x = 2\cos^2 x - 1$. Based on interviews with students, students still forget the trigonometric identity.

3.3. Strategy Errors
A strategic error happens if students try to operate at the right level of a problem, but use procedures or methods that are not appropriate. In addition, students try to operate at the right level but choose inappropriate data information.

In Figure 5 the students were wrong in performing the procedure in finding the value of the form trigonometric equation $\sin(ax + b) = c$. Students immediately look for the $x$ value at the beginning without paying attention to period changes. Based on interviews with students, they do not understand the concept of trigonometric functions, especially the period. They haven’t understood that form $\sin(2x - 10)$ will change the period. The students still answer $360^\circ$ as the period.

In Figure 6 the students were wrong in choosing the procedure in working on the form questions $A \cos x + B \sin x = C$. Students didn’t change the $k \cos(x - \alpha) = C$ form, but did it using $A \cos x = C$ and $B \sin x = C$. Based on interviews with students, students assume that such forms can be changed into two parts, and then they can be solved one by one. It means students haven’t understood about the concept of form trigonometry equations $A \cos x + B \sin x = C$.

3.4. Calculation Errors
Miscalculation occurs if the student is wrong in giving or writing a mathematical operation mark. In addition students are wrong in math counting operations such as adding, subtracting, multiplying and dividing.
In Figure 7 the student made a mistake in the division operation. The 360 value is not divided by 2, it should be 180. Based on interviews with students, students assume that such a division is only divided into 150. It means that the basic ability of students' calculations is still lacking.  
In Figure 8 the student is wrong in giving a sign that should be -20 instead of 20, so that the next calculation is also wrong. Based on interviews with students, students were not careful in doing the calculations.

3.5. Conclusion Errors  
Conclusion errors occur if students are wrong or do not determine conclusions (settlement set) because they are not in accordance with the final calculation results.  
In figure 9 the students are still incomplete in writing the settlement set, because the $x = 60^\circ$ value is not written in the settlement set. Based on interviews with students, students were not careful in writing the settlement set.
In Figure 10 the students incorrectly wrote the set of solutions from the desired one on the problem. In the question the value should be $0 \leq x \leq 360^\circ$, but the value $x = 395^\circ$ is included in the member of the settlement set. Based on interviews with students, students were not careful in writing the settlement set. In addition there are other subjects who do not know the usefulness of the boundary given in questions such as $0 \leq x \leq 360^\circ$. Students just count, regardless of the limits given in the problem.

The results of the 203 students' correction are classified into each type of error. From each type of error calculated the percentage of student errors. The results of the percentage of students' errors are obtained as follows.

| Question | Number of Students | Percentage | Data Error | Concept Error | Strategy Error | Calculation Error | Conclusion Error |
|----------|--------------------|------------|------------|---------------|----------------|------------------|------------------|
| 1        | 11                 | 5.41 %     | 21 %       | 59 %          | 69 %           | 16 %             |
|          | 2                  | 119        | 10.34 %    | 91 %          | 120 %          | 74 %             | 86 %             |
| 2        | 58.62 %            | 44.83 %    | 59.11 %    | 36.45 %       | 42.36 %        |
|          | 3                  | 77         | 85 %       | 96 %          | 69 %           | 83 %             |
|          | 37.93 %            | 41.87 %    | 47.29 %    | 34 %          | 40.89 %        |
|          | Percentage Average | 34 %       | 32.35 %    | 45.15 %       | 34.81 %        | 30.38 %          |

From the table on the percentage of student errors, viewed from each item, the most number of errors made by students was in item 2. In item 1, the most error was the 34% calculation error. In item 2, the most error was the 59.11% strategy error. In item 3, the most error was a 47.29% strategy error. Overall, this study found the percentage of errors from the three items, data error 34%, misconception 32.35%, strategy error 45.15%, miscalculation 34.81% and conclusion error 30.38%. Like research from Parmjit Singh dkk, conclusion errors are classified as low percentages [5]. In addition, research from Rafi dkk, states the largest percentage of technical errors, namely one indicator is an error in determining the calculation algorithm or procedure [7]. This is the same as the strategy error in this study. In this study the biggest percentage of errors was in strategy errors.

**Suggested Solution to Reduce Trigonometric Equation Errors**

Based on the errors profile and analysis of the causes of errors, the researcher has several solutions to reduce these errors in solving trigonometric equation problems.

**Using Geogebra Software in mathematics learning**

If trigonometric material is only taught using the lecture method, students will feel that trigonometric material is very abstract. The lecture method develops very limited in understanding trigonometric material [3]. So it is necessary to visualize the trigonometric material, to make it look more concrete. If trigonometry is taught concretely, the concept of trigonometric periods or angles in various quadrants will be easier for students to understand. These contextual factors can influence the abstraction process in understanding a material [11]. To visualize trigonometric material, teachers can use GeoGebra. GeoGebra can describe a trigonometric function to understand trigonometric periods. Visualization or drawing concepts can encourage students' understanding of the concepts being studied [12]. Like Muhamad Hafizhudin's research the use of GeoGebra software in trigonometric learning can improve student achievement [13]. Geogebra can help low students in understanding trigonometric material in a simple and interesting way. The use of GeoGebra Software can also increase students' motivation in understanding trigonometric material [14]. From the advantages of
using GeoGebra, it can and should be used as an alternative to promote the use of technology in teaching and learning mathematics [15].

**Use of the Discovery Learning model in mathematics learning**

The cause of the error found in the research is that students often forget the material that has been learned, so it is needed a way of learning so that the material can be stored long in the memory of students. One solution from researchers is the need for learning models that require students to play an active role in finding a concept. One such learning model is Discovery Learning. Discovery Learning is a learning model that presents material not in the form of fina, but students are given the opportunity to find and discover the concept themselves [16]. According to prince, learning models through discovery or investigation activities result in long-term results in memory, not easily forgotten by students [17]. The results of research on the application of Discovery Learning provide a positive view of student learning achievement [18][19].

**Review the answers or steps that have been taken**

Reviewing the Polya stage is an important final stage [20]. Students can correct their incorrect steps, check count operations, check final answers, and be able to solve the problem in other ways and compare the results in the previous way. This can reduce students’ errors in the form of students’ inaccuracy in doing or solving the questions.

**Basic trigonometry learning with experimental instruction.**

Students need to build basic angular concepts strongly to be able to understand the problems of trigonometric functions and trigonometric equations [12]. Weber found that experimental instruction can develop an in-depth understanding of trigonometry [3]. Experimental instructions need three stages, namely the procedure as a completion step algorithm, the process as an opportunity to reflect on it, Procept as an anticipation of the results of the process. Students who received experimental instruction were able to demonstrate a strong understanding of trigonometric functions.

**4. Conclusion**

The research can be concluded that Student’ errors are as follows, (1) data errors that students are wrong in determining trigonometric values and students are wrong in determining the elements of the trigonometric equation form \( A \sin x + B \cos x = C \), (2) concept error that the students wrongly determine the formula of the equation of the sine, cosine and tangent forms, and the students are wrong in writing trigonometric identities, (3) strategy errors occur when students are incorrectly using the right procedure, (4) calculation error happen when students are wrongly doing addition operations, division, (5) conclusion error is that the students are wrong in determining the settlement set. Common causes of these errors are students do not understand the concept of trigonometric periods or angles in various quadrants, students forget the value of the angles of exeptional, and students are not careful enough in working on the problem. The number of the data error is 34%, misconception 32.35%, strategy error 45.15%, miscalculation 34.81% and conclusion error 30.38%.

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