Value at risk in the black litterman portfolio with stock selection through cluster analysis

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Abstract. The aim of this paper is to support the new strategy in the previous research where Black Litterman procedure is assisted by clustering for the process of given views. We provide the empirical result and focus on how to compute the Value at Risk (VaR) to illustrate the risk measure on Black Litterman as a reference portfolio.

1. Introduction
Portfolio is a set of investments that involve the stages of identifying securities to be selected and some proportion of funds to be invested in each of these securities. The aim of securities choices is to reduce the risks to be faced. According to Jogiyanto [1] a portfolio is a collection of financial assets in a unit held or made by an investor, investment company, or financial institution. Optimal portfolio is a portfolio that provides the highest combination of returns with the lowest risk.

One way to reduce portfolio risk is diversification. The principles of diversification are divided into two, namely random diversification and Markowitz diversification. Random diversification is investment randomly by investors on different types of stocks or on different types of assets and expect that the variance as a measure of the risk of the portfolio will be reduced. Markowitz diversification is a more efficient diversification of random diversification. This diversification model was introduced by Henry Markowitz around the 1950s which stated that in investing funds not only on one asset, because if the asset fails, then all invested funds will disappear [2].

In the 90s a model emerged known as the Black-Litterman (BL) introduced by Robert Litterman and Fisher Black. The Black-Litterman model is one of the measuring instruments which able to optimize the portfolio construction process. The Black Litterman model appears with a formula that does not ignore the views of an investor. It is expected that the formed portfolio is more profitable because it is obtained not only from the historical data but also accommodates the feeling of the investor which is considered as a result of external factors [3].

Black Litterman's portfolio weighting uses the Markowitz Mean Variance model with a standard deviation risk measure or portfolio variance [4]. Generally, calculating the value of portfolio variance is the result of the sum of all variances and covariances in the variance-covariance matrix multiplied by the weight
of each security included in the Black Litterman portfolio [1]. In addition to use a variance in determining the risk of a portfolio, Value at Risk (VaR) can also be used.

The method which is often used in measuring VaR is a parametric method approach (often referred as the method of variance-covariance). This method is a method that assumes that returns from a single investment or portfolio are normally distributed. These two factors cause the estimation of potential changes in asset prices (volatility) in the future is not too large. In this case the volatility can be expressed as how far the change in asset prices deviates from the expected results, so the spread size is used to measure risk. VaR measurement through the variance-covariance method approach is based on the volatility of an investment portfolio return that can be calculated using the normal distribution of investment portfolio variance [5].

In this study, the VaR calculation for portfolios using the cluster strategy in the selection of shares is given a view on the portfolio, especially Black Litterman, to show the results of the estimated risks that investment managers will be handle. For this reason, we are still using data from previous research [6]. The VaR measurement aim is to estimate the maximum amount of loss that probably experienced by investors at a certain level of confidence because VaR is an estimation of the maximum loss that will be obtained during a certain period of time with a certain level of confidence [7]. From the results of measuring the value of this risk, investors can determine the anticipatory steps of their investment.

2. Literature Review
2.1 Value at Risk (VaR)
Measuring a risk in investment can be calculated from variance as a deviation from data, while VaR is one of method that considering sampling distribution data with a certain level of confidence. The illustration of VaR can be described in figure 1.

![Figure 1. VaR with a certain alpha](image)

The three main approaches of calculating VaR are Variance-Covariance approach, Historical simulation and Monte Carlo simulation and there are two parameters could be considered from the definition, namely holding period and a confidence level [8]. For confidence level, 95 %, 97.5 %, and 99 % are the values that are often used with the example interpretation of 95 % is for about 5 % of time probably to lose more than the number given by the VaR. In other words, if daily VaR of 5 million rupiah means that in normal market condition, we could state that in 95 out of 100 days loss will not exceed 5 million rupiah.

VaR at \((1 - \alpha)\) level of confidence can be written as [9]

\[
    VaR = W_0 Z_{1-\alpha} \sigma(R_p)
\]

(1)
$W_0$ is an initial capital, $Z$ score can be found from normal standard table, and $\sigma(R_p)$ as a standard deviation of portfolio return.

2.2 Black Litterman (BL)
The explanation of this model followed [3],[4] and the previous work [6] where Capital Asset Pricing Model as an equilibrium model is the benchmark of the market and in this case, the view is estimated from time series method. The optimization problem of BL portfolio can be written as follows:

$$E(R_p) = W_{BL}' \mu_{BL}$$
subject to $\text{Var}(R_p) = W_{BL}' \Sigma W_{BL}$

(2)

where $W_{BL}$ is weight/allocation of BL and $\Sigma$ is var-covariance return. $\mu_{BL}$ is BL return which has formula as follows

$$E\mu_{BL} = E(r_{BL}) = \pi + \tau \Sigma P' (\Omega + \tau \Sigma P')^{-1} (q - P\pi)$$

(3)

where $\pi$ is a vector $k \times 1$ for return equilibrium, $\tau$ is a value to represent the confidence of views (range from 0 to 1), $\Omega$ is a diagonal matrix of covariance of views, $P$ is a pick matrix $k \times n$ for return views and $q$ is a vector $k \times 1$ of return views [6].

2.3 Cluster in BL
The cluster in implementation of BL is a proposed strategy that combine clustering method in selection assets and it could be as an alternative way to put the matrix of views in Black Litterman [6]. The goal of this strategy is to maximize the performance index of portfolio.

How to cluster the assets is vary, we can perform it not only through hierarchy or non-hierarchy method but also ant colony algorithm or meta heuristic approach. The previous work shows that in selection of assets could be in same result [10] and [11]. In this research, we describe the flowchart of the process cluster involved in BL portfolio.

![Flowchart Cluster Process in Black Litterman Portfolio](image)

2.4 Backtesting
Backtesting is a method used to test the validity of the VaR model that is built on the basis of the market so a large VaR model as discussed in actual market historical data can be compared to the upper and lower thresholds. There are two kinds of methods used in backtesting calculations, namely methods based on the number of frequencies of tail loss and methods based on the size of losses that occur in the tail of the distribution (size of tail loss). The Backtesting method used in this study is the Kupiec test.

$H_0 : P(X > VaR) \leq P^*$ or $P(t) \leq P^*$ vs $H_1 : P(X > VaR) < P^*$ or $P(t) > P^*$
Ho is rejected if $\alpha^*$ is less than the significance level $\alpha$. If Ho is rejected, it can be said that the formed VaR model is not reliable [13].

3. Result and Discussion

Calculation of Value at Risk (VaR) is done to estimate the maximum loss that might be experienced by a portfolio. A portfolio is prepared to reduce the risk of loss of investment. The size of the portfolio risk will depend on the weight and return of each of the stocks that make up the portfolio. In this study, the method used for VaR is the covariance variance method assuming the assumption that a portfolio return varies and is normally distributed.

The portfolio model used in this study is the Black Litterman portfolio model. In the formation of the Black Litterman portfolio, the shares used were seven selected shares included in the LQ45 stock index, namely Wijaya Karya (Persero) Tbk. (WIKA), Unilever Indonesia Tbk. (UNVR), United Tractors Tbk. (UNTR), Semen Indonesia (Persero) Tbk. (SMGR), Telekomunikasi Indonesia (Persero) Tbk. (TLKM), Sawit Sumbermas Sarana Tbk. (SSMS), and Summarecon Agung Tbk. (SMRA). The Hypothesis testing of normal assumption for the seven stock returns before the calculation of VaR using the covariance variance method is carried out. The results of normality testing for the seven stocks are as follows.

| Stock       | P-Value | Conclusion         |
|-------------|---------|--------------------|
| WIKA        | 0.093778| normal distributed |
| UNVR        | 0.23644 | normal distributed |
| UNTR        | 0.5     | normal distributed |
| SMGR        | 0.2418  | normal distributed |
| TLKM        | 0.41578 | normal distributed |
| SSMS        | 0.047   | normal distributed |
| SMRA        | 0.5     | normal distributed |

Based on the results of the normality test in Table 1, it can be concluded that the seven stock returns fulfill the normality assumption. So that the calculation of VaR with the variance covariance method can be performed.

Before calculating the VaR value on a portfolio, it is necessary to calculate the variance and covariance of each stock return. The diagonal entry of the covariance variance matrix is the variance value of each stock return, while the other entry is the value of covariance between shares. The BL portfolio covariance matrix is given as follows.

$$
\Sigma = \begin{bmatrix}
0.00169 & 0.00020 & 0.00069 & 0.00026 & 0.00023 & -0.0001 & 0.00049 \\
0.00020 & 0.00012 & 0.00054 & 0.00039 & 0.00056 & 0.00077 & 0.00051 \\
0.00069 & 0.00054 & 0.00238 & 0.00054 & 0.00045 & 0.00058 & 0.00054 \\
0.00026 & 0.00039 & 0.00054 & 0.00165 & 0.00024 & 0.00023 & 0.00102 \\
0.00023 & 0.00056 & 0.00045 & 0.00024 & 0.00092 & -0.0002 & 0.00044 \\
-0.0001 & 0.00077 & 0.00058 & 0.00023 & -0.0002 & 0.00023 & 0.00019 \\
0.00049 & 0.00051 & 0.00054 & 0.00102 & 0.00044 & 0.00019 & 0.00210 \\
\end{bmatrix}
$$

In addition to variance and covariance, calculation of weights for each stock is also needed. In this study, the weight used in the BL portfolio is the allocation result as same as the previous research [6]. We still use the Mean variance [14] method as the basic model to calculate the optimal weight by replacing the expected return with the new Black Litterman return [15]:
Tabel 2. Weight of Portfolio Black Litterman

| Stock | Weight |
|-------|--------|
| WIKA  | 0.21   |
| UNVR  | 0.14   |
| UNTR  | 0.002  |
| SMGR  | 0.13   |
| TLKM  | 0.32   |
| SSMS  | 0.20   |
| SMRA  | 0.005  |

The results of the calculation of Black Litterman portfolio stock weight are 21%, 14%, 0.2%, 13%, 32%, 20% and 0.5% respectively for WIKA, UNVR, UNTR, SMGR, TLKM, SSMS and SMRA. The calculation of portfolio VaR begins by computing the standard deviation value of the portfolio calculated based on stock weight values and the covariance variance matrix.

\[
\sigma^2 = w^T \Sigma w
\]

where \( \Sigma \) is the covariance matrix of stock returns. The calculation of VaR is performed using the variance-covariance method at three levels of confidence: 90%, 95%, and 99%.

For each confidence level, the calculated VaR is as follows:

- For 90% confidence (\( \alpha = 0.1 \)):
  \[
  \text{VaR} = Z_{1-\alpha} \times \sqrt{w^T \Sigma w} = 1.281552 \times 0.02130468 = 0.02730304
  \]
- For 95% confidence (\( \alpha = 0.05 \)):
  \[
  \text{VaR} = Z_{1-\alpha} \times \sigma = 1.644854 \times 0.02130468 = 0.03504308
  \]
- For 99% confidence (\( \alpha = 0.01 \)):
  \[
  \text{VaR} = Z_{1-\alpha} \times \sigma = 2.326348 \times 0.02130468 = 0.04956209
  \]

The calculated VaR values for the confidence levels of 90%, 95%, and 99% are 2.73%, 3.5% and 4.96% respectively. This means that at the 90%, 95%, and 99% confidence level, the losses that investors may receive will be informed in a range of 2.73%, 3.5% and 4.96% of the invested funds.

The following step is to check the validation of the VaR calculation using the Kupiec test. Backtesting is a procedure to compare actual profits or losses with a Value at Risk estimate. This hypothesis testing is needed to validate whether the estimated VaR is accurate. If VaR estimates are not accurate, it means the model should be reviewed and many possible factors such as incorrect assumptions, or the wrong parameters or the chosen model [13].

Table 3 Backtesting for estimated VaR
Method & $VaR_{90\%}$ (p-value) & $VaR_{95\%}$ (p-value) & $VaR_{99\%}$ (p-value) & Decision
\hline
Variance covariance & 0.02730304 & 0.03504304 & 0.04556209 & accepted \\
(0.4011483) & (0.4661255) & (0.5051613) & \\
\hline

From the Table 3 we show that the performance for each level of confidence indicates to accept Ho. In other words the VaR estimation using the variance covariance method is reliable.

4. Conclusion
Even though Variance-Covariance approach is the simplest way to be applied but it is based on normal distribution theoretically. Actually, return does not always in normal distribution, meanwhile in this study, Black Litterman portfolio has an assumption normal distributed for its return so that method is fitted in this research. In implementation of the calculation of VaR from the proposed model which is Black Litterman through cluster aid in the view construction shows that at the 90%, 95%, and 99% confidence level, the losses that investors may receive will be informed in a range 2.73%, 3.5% and 4.96% of the invested funds.

The result show that for three level of confidence 90%, 95%, and 99% the model are reliable. If the result of backtesting for VaR estimates are not accurate, the model must be reviewed. There are many methods for confirming the accuracy of model in calculating VaR beside the variance-covariance method based on the assumption. For further research, we will develop how to cope calculating risk measure if data return is not normally distributed in Black Litterman model.

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