Interval valued fuzzy ideals of near algebras

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Abstract
In this paper, we introduced the concept of interval valued fuzzy ideal of a near-algebra over interval valued fuzzy field. Using this notion we have described some results with suitable example.

Keywords
Interval valued fuzzy set, Near-algebra, interval valued fuzzy near-algebra,Fuzzy field, Interval valued fuzzy field.

AMS Subject Classification
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1. Introduction

Zadeh [9] introduced the concept of a fuzzy subset of a non-empty set X as a function from X to [0, 1] in 1965. He also introduced the notion of interval valued fuzzy subset [10] in 1975, in which the values of membership functions are intervals of numbers instead of a single number as in fuzzy set. Brown [2] introduced the concept of near-algebra. Nanda [6] studied the notion of fuzzy algebras over fuzzy fields. Srinivas and Narasimha Swamy [7] have introduced the concept of fuzzy near-algebra over a fuzzy field and extended the results of fuzzy algebras over fuzzy fields obtained by Gu and Lu [4]. In [5] gives some basic information about the interval valued fuzzy set and biswas[1] defined interval-valued fuzzy subgroups of the same nature of Rosenfeld fuzzy subgroups. Davvaz [3] introduced fuzzy ideals of near-rings with interval-valued membership functions. Thillaigovindan et al. [8] have studied interval valued fuzzy ideals and anti fuzzy ideals of near-rings. In this article we introduced an interval valued fuzzy ideal of near-algebra over interval valued fuzzy field and obtained fundamental results to this notion.

2. Preliminaries

In this section, we recall some definitions and basic results of interval valued fuzzy near algebra over interval valued fuzzy field which will be used throughout the paper.

Definition 2.1. An interval valued number $\tilde{a}$ on $[0, 1]$ is a closed subinterval of $[0, 1]$, that is $\tilde{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where $a^-$ and $a^+$ are lower and upper limits of $\tilde{a}$ respectively. The set of all closed sub intervals of $[0, 1]$ is denoted by $D[0, 1]$. In this notation $\tilde{0} = [0^-, 0^+]$ and $\tilde{1} = [1^-, 1^+]$. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any two interval numbers $\tilde{a} = [a^-, a^+]$ and $\tilde{b} = [b^-, b^+]$ on $[0, 1]$, we define

(i) $\tilde{a} \leq \tilde{b} \iff a^- \leq b^- \text{ and } a^+ \leq b^+$
(ii) $\tilde{a} = \tilde{b} \iff a^- = b^- \text{ and } a^+ = b^+$
(iii) $\tilde{a} < \tilde{b} \iff a^- \leq b^- \text{ and } a^+ < b^+$
(iv) $k\tilde{a} = [ka^-, ka^+]$, for $0 \leq k \leq 1$.

For any interval valued numbers $\tilde{a}_i = [a_i^-, a_i^+]$, $\tilde{b}_i = [b_i^-, b_i^+] \in D[0, 1]$, $i \in I$ an index set we define

$\max \{\tilde{a}_i, \tilde{b}_i\} = [\max^i \{a_i^-, a_i^+\}, \max^i \{b_i^-, b_i^+\}]$, $\min \{\tilde{a}_i, \tilde{b}_i\} = [\min^i \{a_i^-, a_i^+\}, \min^i \{b_i^-, b_i^+\}]$.

Definition 2.2. Let $X$ be a non-empty set. A mapping $\tilde{\mu} : X \rightarrow D[0, 1]$ is called an interval valued fuzzy subset of $X$. For all $x \in X, \tilde{\mu} = [\mu^-, \mu^+]$, $\mu^-$ and $\mu^+$ are fuzzy subsets of $X$ such that $\mu^- \leq \mu^+$. Thus $\tilde{\mu}(x)$ is an interval( a closed subset of

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[0, 1]) not a number from the interval [0, 1] as in the case of fuzzy set.

Let μ, ν be IVS of X. Then the following are holds:

1. μ ≤ ν ⇔ μ(x) ≤ ν(x),
2. μ = ν ⇔ μ(x) = ν(x),
3. (μ ∪ ν)(x) = max i μ(x), ν(x),
4. (μ ∩ ν)(x) = min i μ(x), ν(x),
5. (μi ∪ ν)(x) = inf i μi(x) : i ∈ I,
6. (μi ∩ ν)(x) = sup i μi(x) : i ∈ I.

where inf i (μi(x) : i ∈ A) = [inf i μi(x), inf i μi+(x)] is called interval valued infimum norm and sup i (μi(x) : i ∈ A) = [sup i μi(x), sup i μi+(x)] is called interval valued supremum norm.

Definition 2.3. A mapping min : D[0, 1] × D[0, 1] → D[0, 1] defined by min(a, b) = [min(a−, b−), min(a+, b+)] ∀a, b ∈ [0, 1] is called an interval min-norm.

A mapping max : D[0, 1] × D[0, 1] → D[0, 1] defined by max(a, b) = [max(a−, b−), max(a+, b+)] for all a, b ∈ [0, 1] is called interval max-norm. Let min and max be the interval valued min-norm and interval valued max-norm on [0, 1] respectively. Then the following are true.

1. min {a, b} = a ∧ b and max i {a, b} = a ∨ b ∈ D[0, 1].
2. min i {b, a} = min {b, a} and max i {b, a} = max i {b, a}
   ∀ a, b ∈ D[0, 1].
3. If a ≥ b, then
   min i {a, c} ≥ min i {b, c} and max i {a, c} ≤ max i {b, c}.

Definition 2.4. An interval valued fuzzy subset F of a field X is called a interval valued fuzzy field of X if it satisfies the following four conditions for every x, y ∈ X:

(i) F(x + y) ≥ F(x) ∧ F(y),
(ii) F(0) = F(x) ∧ F(y),
(iii) F(xy) ≥ F(x) ∧ F(y),
(iv) F(x−1) ≥ F(x) for every x ≠ 0 ∈ X.

An interval valued fuzzy field F of X is denoted by (F, X).

Definition 2.5. Let Y be a near-algebra over a field X. Let (F, X) be an interval valued fuzzy field. An interval valued fuzzy subset A of Y is called a Interval valued fuzzy near-algebra of Y over a fuzzy field (F, X) if it satisfies the following four conditions:

(i) A(x + y) ≥ min(A(x), A(y)),
(ii) A(λx) ≥ min(F(λ), A(x)),
(iii) A(xy) ≥ min(A(x), A(y)),
(iv) F(1) ≥ A(x) for all x, y ∈ Y and λ ∈ X.

A fuzzy near-algebra A of Y is denoted by (A, Y).

3. Interval valued fuzzy ideal of near algebra

Definition 3.1. Let (A, Y) be an interval valued fuzzy near-algebra over an interval valued fuzzy field (F, X). Then A is called an interval valued fuzzy ideal of Y, if A(xy) ≥ A(x) and A(y(x+i) − xx) ≥ A(i) for every x, y, i ∈ Y.

A is an interval valued fuzzy right ideal of Y if A(xy) ≥ A(x) for every x, y ∈ Y.

A is an interval valued fuzzy left ideal of Y if A(y(x+i) − xx) ≥ A(i) for every x, y, i ∈ Y.

In other words, an interval valued fuzzy subset A of a near-algebra Y over an interval valued fuzzy field (F, X) is said to be interval valued fuzzy ideal if it satisfies the following conditions:

(i) A(x + y) ≥ min(A(x), A(y)), (ii) A(λx) ≥ min(F(λ), A(x)),
(iii) F(1) ≥ A(x), (iv) A(xy) ≥ A(x),
(v) A(y(x+i) − xx) ≥ A(i) (or equivalently A(xy − yx) ≥ A(z − x))

for every x, y, z, i ∈ Y, where 1 is the unity in X.

If A satisfies (i), (ii), (iii) and (iv), then A is called a interval valued fuzzy right ideal of Y.

If A satisfies (i), (ii), (iii) and (v), then A is called a interval valued fuzzy left ideal of Y.

4. Example

Example 4.1. Let X = Z2 = {0, 1} ⊕ Z2. Define an interval valued fuzzy subset F : X → D[0, 1] by F(0) = [0.8, 0.9] and F(1) = [0.7, 0.8]. For any x, y ∈ X, we have x + y ∈ X and for x ≠ 0, xy−1 ∈ X. This implies that X is a field. Also we get F(x−y) ≥ F(x) ∧ F(y) and F(xy−1) ≥ F(x) ∧ F(y). Thus (F, X) is an interval valued fuzzy field. Let Y = {a, b, c} be a set with two binary operations “+” and “·” as follows:

|   | 0 | a | b | c |
|---|---|---|---|---|
| + | 0 | a | b | c |
| 0 | 0 | a | b | c |
| a | a | 0 | b | c |
| b | b | c | 0 | a |
| c | c | b | a | 0 |

Let a scalar multiplication on Y is defined by 0 · x = 0, 1 · x = x for each x ∈ Y; 0, 1 ∈ X. X is a field. A direct verification shows that Y is a near-algebra over a field X. Let A : Y → D[0, 1] be an interval valued fuzzy subset of Y defined by A(0) = [0.5, 0.6], A(1) = [0.3, 0.4].

We have that x + y, xy, λx, y(x+i) − xx ∈ Y for every λ ∈ X and x, y, i ∈ Y. Then A(x + y) ≥ min(A(x), A(y)), A(λx) ≥ min(F(λ), A(x)), A(xy) ≥ A(x) and A(y(x+i) − xx) ≥ A(i). Hence A is an interval valued fuzzy ideal of a near-algebra Y.

5. Main Results

Theorem 5.1. Let A be an interval valued fuzzy ideal of a near-algebra Y over an interval valued fuzzy field F of Y.
Then each level subset $\tilde{A}_t = \{x \in Y : \tilde{A}(x) \geq t, t \in [0, 1]\}$ is an ideal of $Y$, where $F(\lambda) \geq t$ for any $\lambda \in X$.

**Proof.** Let $x, y \in \tilde{A}_t$ and $\lambda \in X$. Then $x, y \in Y$ and $\tilde{A}(x) \geq t, \tilde{A}(y) \geq t$. Since $\tilde{A}$ is an interval valued fuzzy ideal, we get $\tilde{A}(x - y) \geq \min\{\tilde{A}(x), \tilde{A}(y)\} \geq \min(t, t) = t$. Therefore $x - y \in \tilde{A}_t$. Now $\tilde{A}(\lambda x) \geq F(\lambda) \wedge \tilde{A}(x) \geq F(\lambda) \wedge t \geq t \wedge t = t$. Therefore $\lambda x \in \tilde{A}_t$. Thus $\tilde{A}_t$ is a left subspace of $Y$.

Let $x, y \in Y$ and $i \in \tilde{A}_t$. Then $y(x + i) - yx \in Y$, and so $\tilde{A}(y(x + i) - yx) \geq \tilde{A}(i) \geq t$. Therefore $y(x + i) - yx \in \tilde{A}_t$. Thus $\tilde{A}_t$ is a left ideal of $Y$. Let $x, y \in Y, i \in \tilde{A}_t$. Then $\tilde{A}(ix) \geq \tilde{A}(i) \geq t$. Therefore $ix \in \tilde{A}_t$. Thus $\tilde{A}_t$ is a right ideal of $Y$.

Hence $\tilde{A}_t$ is an ideal of $Y$. \qed

**Theorem 5.2.** Intersection of a family of an interval valued fuzzy ideals of a near-algebra $Y$ is an interval valued fuzzy ideal of $Y$.

**Proof.** Let $\{\tilde{A}_i\}_{i \in A}$ be a family of an interval valued fuzzy ideals of near-algebra $Y$ over an interval valued fuzzy field $(\tilde{F}, X)$. Let $\tilde{A}(x) = \cap_{i \in A} \tilde{A}_i(x) = \inf_{i \in A} \tilde{A}_i(x)$. For every $x, y \in Y$ and $\lambda, \mu \in X$ we have

$$\tilde{A}(\lambda x + \mu y) = \inf_{i \in A} (\lambda \tilde{A}(x) + \mu \tilde{A}(y)) \geq \inf_{i \in A} \{\min(\min(\tilde{F}(\lambda), \tilde{A}_i(x)), \min(\tilde{F}(\mu), \tilde{A}_i(y)))\} \geq \min(\min(\tilde{F}(\lambda), \tilde{A}(x)), \min(\tilde{F}(\mu), \tilde{A}(y))) = \min(\min(\tilde{F}(\lambda), \tilde{A}(x)), \min(\tilde{F}(\mu), \tilde{A}(y)))$$

Since each $\tilde{A}_i$ is an interval valued fuzzy ideal, we get $\tilde{F}(1) \geq \tilde{A}_i(x) \geq \inf_{i \in A} \tilde{A}_i(x) = \tilde{A}(x)$ for every $x \in Y$ and $i \in A$. Now $\tilde{A}(xy) = \inf_{i \in A} \tilde{A}_i(xy) \geq \inf_{i \in A} \tilde{A}_i(x) = \tilde{A}(x)$. Thus $\tilde{A}$ is an interval valued fuzzy right ideal of $Y$. Let $x, y, j \in Y$. Then $\tilde{A}(y(x + j) - yx) = \inf_{i \in A} \tilde{A}_i(y(x + j) - yx) \geq \inf_{i \in A} \tilde{A}_i(j) = \tilde{A}(j)$. Thus $\tilde{A}$ is an interval valued fuzzy left ideal of $Y$.

Hence $\tilde{A}$ is an interval valued fuzzy ideal of a near-algebra $Y$. \qed

**Theorem 5.3.** Let $Y$ and $Y'$ be two near-algebras over a field $X$. Let $\tilde{A}$ and $\tilde{B}$ be two an interval valued fuzzy ideals of $Y$ and $Y'$ respectively over an interval valued fuzzy field $(\tilde{F}, X)$. Then $\tilde{A} \times \tilde{B}$ is an interval valued fuzzy ideal of a near-algebra $Y \times Y'$.

**Proof.** Let $\tilde{A}$ and $\tilde{B}$ be two an interval valued fuzzy ideals of near-algebras $Y$ and $Y'$ respectively over an interval valued fuzzy field $(\tilde{F}, X)$. We have that $\tilde{A} \times \tilde{B}((x, x')) = \min(\tilde{A}(x), \tilde{B}(x'))$ where $(x, x') \in Y \times Y'$. Also know that $Y \times Y' = \{(y, y') : y \in Y, y' \in Y'\}$. Let $(x, x'), (y, y') \in Y \times Y'$ and $\lambda \in X$.

$$\tilde{A} \times \tilde{B}((x, x') + (y, y')) = \min(\tilde{A}(x), \tilde{A}(y)) \times \min(\tilde{B}(x'), \tilde{B}(y'))$$

Since $\tilde{A}$ is an interval valued fuzzy ideal of $Y$, we get $\tilde{F}(1) \geq \tilde{A}(x)$ for every $x \in Y$, $1$ is the unity in $X$. And since $\tilde{B}$ is an interval valued fuzzy ideal of $Y'$, then $\tilde{F}(1) \geq \tilde{B}(x')$ for every $x' \in Y'$. Then $\tilde{F}(1) \geq \min(\tilde{A}(x), \tilde{B}(x')) = (\tilde{A} \times \tilde{B})(x, x')$. Now

$$\tilde{A} \times \tilde{B}((x, x')(y, y')) = \min(\tilde{A}(x), \tilde{B}(x')) \times \min(\tilde{A}(y), \tilde{B}(y')) \geq \min(\tilde{A}(x), \tilde{B}(x')) = (\tilde{A} \times \tilde{B})(x', x)$$

Thus $\tilde{A} \times \tilde{B}$ is an interval valued fuzzy right ideal of $Y \times Y'$. Let $(x, x')(y, y'), (i, i') \in Y \times Y'$. Then

$$\tilde{A} \times \tilde{B}((x, x')(y, y')(i, i')) = \min(\tilde{A}(x), \tilde{B}(x')) \times \min(\tilde{A}(y), \tilde{B}(y')) \geq (\tilde{A} \times \tilde{B})(i, i')$$

Thus $\tilde{A} \times \tilde{B}$ is an interval valued fuzzy left ideal of $Y \times Y'$. Hence $\tilde{A} \times \tilde{B}$ is an interval valued fuzzy ideal of $Y \times Y'$. \qed

**Definition 5.4.** Let $\tilde{A}$ and $\tilde{B}$ be two an interval valued fuzzy ideals of a zero symmetric near-algebra $Y$. Let $x \in Y$. Then their sum is denoted by $\tilde{A} + \tilde{B}$ is defined by $(\tilde{A} + \tilde{B})(x) = \sup \{\min(\tilde{A}(y), \tilde{B}(z))\}$, where $y, z \in Y$. Note that, if $x = y + z$ then we can write $x = z + (-z + y + z) = z + y'$ where $y' = -z + y + z$, and then $\tilde{B}(z) = \tilde{B}(y)$. That is $\tilde{B}(y') = \tilde{B}(y)$. From this it is clear that $\tilde{A} + \tilde{B}(x) = (\tilde{A} + \tilde{B})(x)$.

**Theorem 5.5.** If $\tilde{A}$ and $\tilde{B}$ are two an interval valued fuzzy ideals of a zero symmetric near-algebra $Y$, then $\tilde{A} + \tilde{B}$ is also an interval valued fuzzy ideal of $Y$.

**Proof.** Let $\tilde{A}$ and $\tilde{B}$ be two an interval valued fuzzy ideals of a zero symmetric near-algebra $Y$. Now

(i) let $x = x_1 + x_2, y = y_1 + y_2; x, y, x_1, y_1, x_2, y_2 \in Y$. 

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Then \( x + y = x_1 + x_2 + y_1 + y_2 = x_1 + y_1 - y_2 + x_2 + y_1 + y_2 \).
This implies
\[
(\tilde{A} + \tilde{B})(x + y) = \sup \{ \min (\tilde{A}(x_1 + y_1), \\tilde{B}(-y_1 + x_2 + y_1 + y_2)) \} \\
\geq \sup \{ \min (\tilde{A}(x_1), \tilde{A}(y_1), \tilde{B}(y_2)) \} \\
\geq \sup \{ \min \tilde{A}(y_1), \tilde{B}(y_2) \} \\
= \min \{ \tilde{A}(y_1), \tilde{B}(y_2) \} \\
= \min \{ (\tilde{A} + \tilde{B})(x), (\tilde{A} + \tilde{B})(y) \}.
\]

(ii) Let \( x = x_1 + x_2, \lambda x = \lambda x_1 + \lambda x_2; x_1, x_2 \in Y, \lambda \in X \). Then
\[
(\tilde{A} + \tilde{B})(\lambda x) = \sup \{ \min (\tilde{A}(\lambda x_1), \tilde{B}(\lambda x_2)) \} \\
\geq \sup \{ \min (\tilde{F}(\lambda), \tilde{A}(x_1)), \min (\tilde{F}(\lambda), \tilde{B}(x_2)) \} \\
= \min (\tilde{F}(\lambda), \sup (\tilde{A}(x_1), \tilde{B}(x_2))) \\
= \min (\tilde{F}(\lambda), (\tilde{A} + \tilde{B})(x)).
\]

(iii) Since \( \tilde{A}, \tilde{B} \) are an interval valued fuzzy near-algebras, then we have \( \tilde{F}(1) \geq x(x) \) and \( \tilde{F}(1) \geq x(x) \). Let \( x = x_1 + x_2, x_1, x_2 \in Y \). Then \( (\tilde{A} + \tilde{B})(x) = \sup \{ \min (\tilde{A}(x_1), \tilde{B}(x_2)) \} \geq \min (\tilde{F}(1), \tilde{F}(1)) = \tilde{F}(1) \).

(iv) Let \( x = x_1 + x_2; x_1, x_2 \in Y \). Which implies \( xy = x_1 y + x_2 y \in Y \). Then
\[
(\tilde{A} + \tilde{B})(xy) = \sup \{ \min (\tilde{A}(x_1 y), \tilde{B}(x_2 y)) \} \\
\geq \sup \{ \min (\tilde{A}(x_1), \tilde{B}(x_2)) \} \\
= \min (\tilde{A} + \tilde{B})(x).
\]

(v) Let \( z = x_1 + x_2; x_1, x_2 \in Y \). Which implies \( z = x_1 + x_2, x_1, x_2 \in Y \). And so \( yz = x_1 y + x_2 y \), and so \( yz = xy = y(t_1 + t_2 + x) - y(t_2 + x) + y(t_2 + x) - y(t_2 + x) + y(t_2 + x) - y(t_2 + x) \). Then
\[
(\tilde{A} + \tilde{B})(yz) = (\tilde{A} + \tilde{B})(y(t_1 + t_2 + x) - y(t_2 + x) + y(t_2 + x) - y(t_2 + x)) \\
\geq \sup \{ \min (\tilde{A}(y(t_1 + t_2 + x) - y(t_2 + x)), \tilde{B}(y(t_2 + x) - y(t_2 + x))) \} \\
\geq \sup \{ \min (\tilde{A}(t_1 + t_2 + x - t_2), \tilde{B}(t_2 + x - y)) \} \\
= \sup \{ \min (\tilde{A}(t_1), \tilde{B}(t_2)) \} \\
= \sup \{ \min (\tilde{A}(t_1), \tilde{B}(t_2)) \} \\
= \sup \{ \min (\tilde{A}(t_1), \tilde{B}(t_2)) \} \\
= \tilde{A}(z).
\]

Hence \( \tilde{A} + \tilde{B} \) is an interval valued fuzzy ideal of \( Y \).

Theorem 5.6. Let \( Y \) and \( Z \) be two near-algebras over a field \( X \). Let \( f: Y \to Z \) be an onto near-algebra homomorphism. If \( \tilde{A} \) is an interval valued fuzzy ideal in \( Y \), then \( f(\tilde{A}) \) is an interval valued fuzzy ideal in \( Z \).

Proof. Let \( u, v \in Z \) and \( \lambda, \mu \in X \). Now
(i) for all \( u, v \in Z \) there exists \( x, y \in Y \) such that \( u = f(x), v = f(y) \).

Hence \( f(\tilde{A}) \) is an interval valued fuzzy ideal in \( Z \).

Theorem 5.7. Let \( Y \) and \( Z \) be two near-algebras over a field \( X \). Let \( f: Y \to Z \) be an onto near-algebra homomorphism. If \( \tilde{A} \) is an interval valued fuzzy ideal in \( Y \), then \( f^{-1}(\tilde{A}) \) is an interval valued fuzzy ideal in \( Y \).

Proof. For all \( x, y \in Y \) and \( \lambda, \mu \in X \), consider
\[
f^{-1}(\tilde{A})(\lambda x + \mu y) = \tilde{A}(f(\lambda x + \mu y)) = \tilde{A}(\lambda f(x) + \mu f(y)) \\
\geq \min (\tilde{A}(\lambda f(x)), \min(\tilde{A}(\mu f(y))) \\
\geq \min (\min(\tilde{A}(f(x)), \tilde{A}(f(y))), \min(\tilde{A}(f(y)), \tilde{A}(f(\mu y)))) \\
= \min (\min(f(\lambda), f^{-1}(\tilde{A})(x)), \min(f(\mu), f^{-1}(\tilde{A})(y))).
\]

We have \( \tilde{F}(1) \geq \tilde{A}(z) \) for each \( z \in Z \). This implies for every...
Let $y = x_1 + y_1$. Then $\tilde{A}(y) = \tilde{A}(x_1 + y_1) = \tilde{A}(x_1) + \tilde{A}(y_1)$, which is an interval valued fuzzy ideal in $y = x_1 + y_1$. Hence $f^{-1}(\tilde{A}(x))$ is an interval valued fuzzy ideal in $y = x_1 + y_1$.

**Theorem 5.5.** Let $Y$ be a near-algebra over a field $X$. If $\tilde{A}$ is an interval valued fuzzy right ideal of $Y$, then $\tilde{A} \cap B \subseteq \tilde{A} \cap B$.

**Proof.** For any interval valued fuzzy subset $\tilde{A}$ of $Y$, we have

$$\tilde{A}(\tilde{A}(y)) = \sup\{\min(\tilde{A}(y), B(z))\}$$

Let $\tilde{A}$ be an interval valued fuzzy right ideal and $B$ be an interval valued fuzzy left ideal of $Y$. Let $x \in Y$. If $\tilde{A}(x) = 0$, then it is obvious. Otherwise $\tilde{A}(x) = \sup\{\min(\tilde{A}(y), B(z))\}$, where $y, z \in Y$.

**Theorem 5.11.** Let $\tilde{A}$ be an interval valued fuzzy ideal of a near-algebra $Y$. If $\tilde{A}(x) = A(0)$, then $\tilde{A}(x) = \tilde{A}(y)$, where $0$ is the additive identity in $Y$.

**Proof.** Suppose that $\tilde{A}(x) = \tilde{A}(y)$ for every $x, y \in Y$. Then $\tilde{A}(x) = \tilde{A}(x + y + y) = \min(\tilde{A}(x), \tilde{A}(y)) = \tilde{A}(y)$ and $\tilde{A}(y) = \tilde{A}(x + y + y) = \min(\tilde{A}(y), \tilde{A}(x)) = \min(\tilde{A}(x), \tilde{A}(x)) = \tilde{A}(x)$. Hence $\tilde{A}(x) = \tilde{A}(y)$.

**Theorem 5.12.** Let $I$ be a non-empty subset of a near-algebra $Y$ over a field $X$. Let $\mathcal{F}$ be an interval valued fuzzy subset in $Y$ such that $\mathcal{F}$ is onto $\{0, 1\}$ so that $\mathcal{F}$ is the characteristic function of $I$. Then $\mathcal{F}(\mathcal{F})$ is an interval valued fuzzy ideal of $Y$ if and only if $I$ is an ideal of $Y$, where $\mathcal{F}(\mathcal{F}) = 1$ for every $\mathcal{F} \in X$.

**Proof.** Proof is straightforward.

**Definition 5.13.** Let $\tilde{A}$ and $\tilde{B}$ be two interval valued fuzzy ideals of a near-algebra $Y$. Let $x \in Y$. Then

$$\mathcal{F}(\tilde{A}(x)) = \left\{ \begin{array}{ll} \sup\{\min(\tilde{A}(a), \tilde{B}(b))\} & \text{if } x = ab, \text{ where } a, b \in Y, \\
0 & \text{if } x \neq ab \text{ for each } a, b \in Y. \end{array} \right.$$
Now

\[
((\tilde{A} + \tilde{B})\tilde{C})(x) = \sup_{x=uz} \{\min((\tilde{A} + \tilde{B})(y), \tilde{C}(z))\}; x, y, z \in Y \]
\[
= \sup_{x=uz} \{\min(\sup_{y=au+v} (\tilde{A}(u), B(v)), \tilde{C}(z))\}; u, v \in Y
\]
\[
\leq \sup_{x=au+b} \{\min(\min(\tilde{A}(u), \tilde{C}(z)), min(\tilde{B}(v), \tilde{C}(z)))\}
\]
\[
\leq \sup_{x=a+b} \{\min((\tilde{A}\tilde{C})(a), (\tilde{B}\tilde{C})(b)) : a, b \in Y, a+b=x\}
\]
\[
= (\tilde{A}\tilde{C} + \tilde{B}\tilde{C})(x).
\]

Thus \((\tilde{A} + \tilde{B})\tilde{C} \subseteq \tilde{A}\tilde{C} + \tilde{B}\tilde{C}\). Hence \((\tilde{A} + \tilde{B})\tilde{C} = \tilde{A}\tilde{C} + \tilde{B}\tilde{C}\). \(\square\)

Definition 5.15. Let \(\tilde{A}\) be an interval valued fuzzy ideal of a near-algebra \(Y\). Let
\(<\tilde{A}> = \bigcap \{\tilde{D} : \tilde{A} \subseteq \tilde{D}, \tilde{D} \text{ is an interval valued fuzzy ideal of } Y\}.\)
Then \(<\tilde{A}>, \tilde{A}\) is called an interval valued fuzzy ideal of \(Y\) generated by \(\tilde{A}\).
Clearly, \(<\tilde{A}>, \tilde{A}\) is the smallest interval valued fuzzy ideal of \(Y\) containing \(\tilde{A}\).

Definition 5.16. Let \(\tilde{A}\) and \(\tilde{B}\) be two interval valued fuzzy subsets of a non-empty set \(L\). Define \(\tilde{A} \cup \tilde{B}\) is an interval valued fuzzy subset of \(L\) by \((\tilde{A} \cup \tilde{B})(x) = \max\{\tilde{A}(x), \tilde{B}(x)\} = \tilde{A}(x) \vee \tilde{B}(x)\) for every \(x \in Y\).

Theorem 5.17. Let \(\tilde{A}\) and \(\tilde{B}\) be two interval valued fuzzy ideals of a near-algebra \(Y\). Then \(\tilde{A} \cup \tilde{B} = <\tilde{A} \cup \tilde{B}>\).

Proof. Given that \(\tilde{A}\) and \(\tilde{B}\) be two interval valued fuzzy ideals of a near-algebra \(Y\). Let \(\tilde{D}\) be an interval valued fuzzy ideal of \(Y\) such that \(\tilde{A} \cup \tilde{B} \subseteq \tilde{D}\). Then
\[
(\tilde{A} + \tilde{B})(x) = \sup_{x=au+v} \{\min(\tilde{A}(u), \tilde{B}(v)) : x, u, v \in Y\}
\]
\[
\leq \sup_{x=au+v} \{\min(\tilde{D}(u), \tilde{D}(v))\}
\]
\[
= \tilde{D}(u+v)
\]
\[
= \tilde{D}(x).
\]

Therefore \(\tilde{A} + \tilde{B} \subseteq \tilde{D}\). Thus \(\tilde{A} + \tilde{B}\) is the smallest interval valued fuzzy ideal of \(Y\) such that \(\tilde{A} \cup \tilde{B} \subseteq \tilde{A} + \tilde{B}\). Hence \(\tilde{A} + \tilde{B} = <\tilde{A} \cup \tilde{B}>\). \(\square\)

6. Conclusion

In this paper, we introduced and studied the concept of interval valued fuzzy ideal of near algebra over an interval valued fuzzy field. In this notion we improve some results and the results are illustrated with a well-analyzed example in Section 4.

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