Quark-Hadron Duality for Hybrid Mesons at Large-$N_c$

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ABSTRACT

We investigate implications of quark-hadron duality for hybrid mesons in the large-$N_c$ limit. A simple formalism is developed which implements duality for QCD two-point functions of currents of quark bilinears, with any number of gluons. We argue that the large-$N_c$ meson masses share a common parameter, which is related to the QCD string tension. This parameter is fixed from correlators of conserved vector and axial-vector currents, and using lattice QCD determinations of the string tension. Our results predict towers of hybrid mesons which, within expected $1/N_c$ corrections, naturally accommodate the $1^{-+}$ experimental hybrid candidates.
1. Introduction

The possible experimental observation of mesons with quantum numbers that cannot arise from a simple quark-antiquark state, has generated a great deal of interest [1,2]. In effect, search for these hybrid meson states is a primary mission of the proposed Hall D at Jefferson Laboratory. There are currently two experimental $J^{PC} = 1^{-+}$ hybrid candidates, at $\sim 1.4$ GeV [1] and $\sim 1.6$ GeV [2]. One reason for excitement among theorists is the possibility that these states offer a direct experimental probe of the gluonic nature of QCD. Of course, a priori there is no reason why these meson states should not couple strongly to four-quark currents, rather than to currents involving gluons. In this respect, the large-$N_c$ limit comes into its own as an extremely useful tool [3,4].

In the large-$N_c$ limit, mesons are narrow and couple to QCD currents with one quark, one antiquark and any number, $n_g$, of gluons [3,4]. We define a hybrid to be a meson whose space-time quantum numbers require $n_g \geq 1$. As with conventional mesons [3], the QCD correlators of currents which couple to hybrid mesons can be expressed as sums of meson tree graphs, which must be infinite in order to be dual to perturbative QCD at large momentum transfer [4]. By contrast, quenched lattice QCD calculations [5] suggest that the lightest hybrid has $1^{-+}$ and a mass $\sim 2$ GeV. (A survey of results from hadronic models is available in Ref. 6.) The coupling of the hybrid mesons to currents with four-quark states is suppressed in the quenched lattice calculations. One might expect that the discrepancy between the lattice and experiment is due to this suppression [7,5]. The natural forum to test this hypothesis is the large-$N_c$ limit, as the coupling of hybrid mesons to four-quark currents is subleading in the $1/N_c$ expansion [3,4].

In this paper we investigate duality in the large-$N_c$ limit for QCD correlators with hybrid quantum numbers. We develop a simple formalism which implements duality for arbitrary two-point functions of currents of quark bilinears. The Wilson coefficients in the operator product expansion (OPE) of correlators with $n_g = 1$ also appear in the OPE of correlators with $n_g = 0$. We also argue that the large-$N_c$ meson masses depend on a universal parameter, $\Lambda$, which is related to the QCD string tension. Recent work has investigated quark-meson duality in the large-$N_c$ limit for conserved vector and axial-vector QCD currents [8,9] (see also Ref. 10). We use this phenomenology of ordinary mesons as input into our investigation of hybrids.

In Section 2, we develop a simple formalism for investigating quark-meson duality in the large-$N_c$ limit. In Section 3 we apply this formalism to conserved vector and axial-vector QCD currents and their well-known phenomenology. This section is a necessary review of work that has appeared elsewhere [9]. We discuss the issue of setting the scale at which to evaluate the QCD coupling constant in Section 4. We also introduce a universality
conjecture in this section. In Section 5 we investigate vector and axial-vector currents which couple to hybrid mesons. We discuss our results in Section 6.

2. The General Statement of Duality

The basic object we will consider in this paper is the two-point function,

$$\Pi(Q^2) = i \int d^4x e^{iqx} \langle 0 | T[J^\dagger(x) J(0)] | 0 \rangle,$$  \hspace{1cm} (1)

where $J(x)$ is a color-singlet QCD current of the form $\bar{q} \Gamma(n_g) q$, where $\Gamma(n_g)$ is an arbitrary Dirac-Lorentz structure coupled to $n_g$ gluons, and $Q^2 = -q^2$. Here for illustration we have assumed a scalar current with respect to all quantum numbers. The general form of the spectral function of mass-dimension $2m$ in the large-$N_c$ limit is

$$\frac{1}{\pi} \text{Im} \Pi(t) = 2 \sum_{n=0}^{N} F^2(n) M^{2m}(n) \delta(t - M^2(n)),$$  \hspace{1cm} (2)

where $F(n)$ is the decay constant of the $n$th meson, $M(n)$ is its mass, and $N$ regulates the number of mesons, which is strictly speaking, infinite. This spectral function expresses the statement that in the large-$N_c$ limit, $\Pi(Q^2)$ is given by an infinite sum of zero-width mesons. We can now construct $\Pi(Q^2)$ using dispersion theory. At large space-like momenta we obtain the general duality relation

$$2 (-1)^m Q^{2m} \sum_{n=0}^{N} \frac{F^2(n)}{Q^2 + M^2(n)} + \sum_{k=0}^{m} a(N; \mu)_{k} Q^{2k} \rightarrow \mathcal{C}(\mu) Q^{2m} \log \frac{Q^2}{\mu^2} + \sum_{\ell=0}^{\infty} \frac{\langle O(Q^2, \mu) \rangle}{Q^{2\ell}} + \ldots$$  \hspace{1cm} (3)

where $\mathcal{C}$ is an analytic function of $\alpha_s$ which is computed in QCD perturbation theory and $\langle O(Q^2, \mu) \rangle$ are Wilson coefficients of mass-dimension $d = 2(\ell + m)$. The $a(N; \mu)_{k}$ are hadronic counterterms which render the hadronic sums over states independent of $N$, and $\mu$ is a renormalization scale defined in dimensional regularization with $\overline{MS}$. The dots represent higher order perturbative contributions. (For a diagrammatic statement of this duality relation, see Fig. 1 and Fig. 2.) Eq. (3) implies

$$F^2(n)/M^2(n) \rightarrow \frac{1}{n}.$$  \hspace{1cm} (4)

In this paper we will assume that the squared masses are linear in $n$. This is the case in two-dimensional QCD in the large-$N_c$ limit [11]. And, too, linearity in $n$ is expected if large-$N_c$ QCD is in some sense equivalent to a string theory. There is a caveat to this assumption relating to chiral symmetry breaking, which will be addressed below. It follows
from Eq. (4) that \( F^2(n) = F^2 + O(n^{-\epsilon}) \) where \( F \) is a hadronic parameter and \( \epsilon \) is a positive number. We will keep the leading correction to \( F^2 \). We choose the parameterization

\[
M^2(n) = M^2 + \Lambda^2 n \quad (5a)
\]

\[
F^2(n) = F^2 + \frac{E^{2+2\epsilon}}{M^{2\epsilon}(n)} \quad (5b)
\]

Hence, \textit{a priori}, each QCD two-point function is parameterized by four hadronic parameters: \( M, \Lambda, F \) and \( E \). It is straightforward to convince oneself that in order that the leading Wilson coefficient \( (d = 2m) \) not vanish, we must have \( \epsilon = m \). On the hadronic side of Eq. (3) we then have

\[
2 (-1)^m Q^{2m} \sum_{n=0}^{N} \frac{F^2}{Q^2 + M^2(n)} + 2 \sum_{n=0}^{N} \frac{E^{2+2m}}{Q^2 + M^2(n)} + \sum_{k=0}^{m} \bar{a}(N;\mu)_k Q^{2k} =
\]

\[
-2 \left( (-1)^m Q^{2m} \frac{F^2}{\Lambda^2} + \frac{E^{2+2m}}{\Lambda^2} \right) \left( \Psi \left( \frac{Q^2 + M^2}{\Lambda^2} \right) - \Psi(N) \right) + \sum_{k=0}^{m} \bar{a}(N;\mu)_k Q^{2k} \quad (6)
\]

where \( \Psi \) is Euler’s function, defined as the logarithmic derivative of the Gamma function. The counterterms here are barred, indicating that they differ from those in Eq. (3). From Eq. (3) we immediately find

\[
\mathcal{C}(\mu) = (-1)^{m+1} \frac{2F^2}{\Lambda^2} \quad (7a)
\]

\[
\langle \mathcal{O}(Q^2,\mu) \rangle_{d=2m} = \mathcal{W}(\mu) \log \frac{Q^2}{\mu^2} \quad \mathcal{W}(\mu) = -\frac{2E^{2+2m}}{\Lambda^2} \quad (7b)
\]

Using Eq. (3) and Eq. (6), we can determine the counterterms, \( \bar{a}(N;\mu)_k \), and match to the OPE separately for each \( m \). Working to leading order in \( \alpha_s \) and keeping Wilson coefficients up to dimension six, we find

\[
\bar{a}(N;\mu)_0 = \mathcal{C}(\mu) \log \frac{N \Lambda^2}{\mu^2} \quad (8a)
\]

\[
\langle \mathcal{O}(\mu) \rangle_{d=4}^{d=2} = \mathcal{C}(\mu) \left( M^2 - \frac{1}{2} \Lambda^2 \right) \quad (8b)
\]

\[
\langle \mathcal{O}(\mu) \rangle_{d=4}^{d=4} = -\frac{1}{2} \mathcal{C}(\mu) \left( M^4 - M^2 \Lambda^2 + \frac{1}{6} \Lambda^4 \right) \quad (8c)
\]

\[
\langle \mathcal{O}(\mu) \rangle_{d=6}^{d=6} = \frac{1}{3} \mathcal{C}(\mu) \left( M^2 - \frac{1}{2} \Lambda^2 \right) (M^2 - \Lambda^2) M^2 \quad (8d)
\]
for $m = 0$, and

\begin{align}
\bar{a}(N; \mu)_0 &= \mathcal{W}(\mu) \log \frac{NA^2}{\mu^2} + \frac{1}{2} \mathcal{C}(\mu) \left( M^4 - M^2A^2 + \frac{1}{6}A^4 \right), \quad (9a) \\
\bar{a}(N; \mu)_1 &= -\mathcal{C}(\mu) \left( M^2 - \frac{1}{2}A^2 \right), \quad (9b) \\
\bar{a}(N; \mu)_2 &= \mathcal{C}(\mu) \log \frac{NA^2}{\mu^2}, \quad (9c)
\end{align}

\begin{align}
\langle \mathcal{O}(\mu) \rangle^{d=6} &= \left( \frac{1}{3} \mathcal{C}(\mu) \left( M^2 - A^2 \right) M^2 + \mathcal{W}(\mu) \right) \left( M^2 - \frac{1}{2}A^2 \right), \quad (9d)
\end{align}

for $m = 2$. We will consider these two cases in this paper.

One may wonder about the effects of higher-order contributions to $F^2(n)$. An $O(n^{-m-2})$ correction, parameterized by $D^4(m)/M^{2m+2}(n)$, generates a log $Q^2$ correction to the Wilson coefficient with $d = 2(1 + m)$. This correction is higher order in $\alpha_s$ than the order to which we are working.

### 3. Conserved Vector and Axial-Vector Currents

We consider QCD with two massless flavors. This theory has an $SU(2) \times SU(2)$ chiral symmetry. In the large-$N_c$ limit, assuming confinement, this symmetry breaks spontaneously to the $SU(2)_V$ isospin subgroup \[12\]. (Strictly speaking, large-$N_c$ QCD has a $U(2) \times U(2)$ chiral symmetry. In this paper the additional Goldstone mode and its associated $U(1)$ charge are ignored.) Consider the conserved QCD vector and axial-vector currents

\begin{align}
V^a_\mu &= \bar{q}{\gamma_\mu}^1\tau^a q, \quad (10a) \\
A^a_\mu &= \bar{q}{\gamma_\mu}^5\tau^a q. \quad (10b)
\end{align}

These currents fill out a six-dimensional chiral multiplet; they transform as $(3, 1) \oplus (1, 3)$ with respect to $SU(2) \times SU(2)$. We are interested in the correlators

\begin{align}
2i \int d^4x e^{iqx} \langle 0 | T[V^\mu_a(x)V^\nu_b(0)] | 0 \rangle &= \delta_{ab}(q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_V(Q^2), \quad (11a) \\
2i \int d^4x e^{iqx} \langle 0 | T[A^\mu_a(x)A^\nu_b(0)] | 0 \rangle &= \delta_{ab}(q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_A(Q^2). \quad (12a)
\end{align}

Since $\Pi_{V,A}(Q^2)$ are dimensionless, we are here interested in the case $m = 0$.

Chiral symmetry constrains duality. Since chiral symmetry is not broken in perturbation theory, $\Pi_V(Q^2) = \Pi_A(Q^2)$ to all orders in $\alpha_s$. Moreover, because the pion couples to
the axial-vector current, consistency with chiral symmetry requires that the pion, together with its chiral partners, be left out of the tower of states [8,9]. The minimal realistic ansatz for the spectral functions consistent with chiral symmetry is

\[
\frac{1}{\pi} \text{Im} \Pi_V(t) = \frac{2}{\pi} F^2_\rho \delta(t - M^2_\rho) + 2 \sum_{n=0}^{N_V} F^2_V(n) \delta(t - M^2_V(n)), \\
\frac{1}{\pi} \text{Im} \Pi_A(t) = \frac{2}{\pi} F^2_\pi \delta(t) + 2 F^2_{a_1} \delta(t - M^2_{a_1}) + 2 \sum_{n=0}^{N_A} F^2_A(n) \delta(t - M^2_A(n)).
\]

Here we have extracted the lowest-lying vector and axial-vector mesons, \(\rho\) and \(a_1\), respectively. These states, together with the pion, satisfy spectral function sum rules. This is equivalent to the statement that \(\pi, \rho\) and \(a_1\), together with an isoscalar \(S_0\), fill out a reducible \((1, 3) \oplus (3, 1) \oplus (2, 2)\) representation of the chiral group [9]. The parameters are then related by a single mixing angle, \(\phi\), via \(F_\pi = F_\rho \sin \phi, F_{a_1} = F_\pi \cot \phi\) and \(M_\rho = M_{a_1} \cos \phi\). One can further show that each vector meson in the infinite sum must be paired with a degenerate axial-vector chiral partner so that pair-by-pair they fill out irreducible \((1, 3) \oplus (3, 1)\) representations of the chiral group. It follows that \(F_V = F_A \equiv F, \Lambda_V = \Lambda_A \equiv \Lambda\) and \(M_V = M_A \equiv M\) [9]. It is intriguing that only the states outside of the tower exhibit mass splittings.

Reading directly from Eq. (7a) and Eq. (8) we then find the system of equations

\[
\mathcal{C}_{V,A} = -\frac{N_c}{12\pi^2}, \\
\langle O \rangle_{V,A}^{d=2} = 0 = 2 F^2_\pi \csc^2 \phi + \mathcal{C}_{V,A} \left( M^2 - \frac{1}{2} \Lambda^2 \right), \\
\langle O \rangle_{V,A}^{d=4} = \frac{\alpha_s}{12\pi} \langle G_{\mu\nu} G_{\mu\nu} \rangle = -2 F^2_\pi M^2_\rho \csc^2 \phi - \frac{1}{2} \mathcal{C}_{V,A} \left( M^4 - M^2 \Lambda^2 + \frac{1}{6} \Lambda^4 \right),
\]
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & \textbf{TH} & \textbf{EXP} & \textbf{TH} & \textbf{EXP} \\
\hline
$M_{\rho'}$ & INPUT & 1465 ± 25 & $\phi$ & 44.1 ± 0.6 \\
\hline
$M_{a_1'}$ & 1465 ± 25 & 1640 ± 12 ± 30 & $F_{a_1}$ & 96 ± 2 \\
\hline
$M_{\rho''}$ & 1903 ± 45 & 1700 ± 20 & $M_{a_1}$ & 1073 ± 7 \\
\hline
$M_{a_1''}$ & 1903 ± 45 & NO DATA & $F$ & 137 ± 5 \\
\hline
$M_{\rho'''}$ & 2258 ± 56 & 2149 ± 17 & $\pi\alpha_s\langle\bar{q}q\rangle^2$ & 14.8 ± 0.02 \\
\hline
$M_{a_1'''}$ & 2258 ± 56 & NO DATA & $\alpha_s(G_{\mu\nu}G_{\mu\nu})$ & 0.063 ± 0.006 \\
\hline
$L_{10}(10^{-3})$ & -5.5 ± 0.1 & -5.5 ± 0.7 & $\Lambda$ & 1172 ± 43 MeV \\
\hline
\end{tabular}
\caption{Predictions compared to data using $F_\pi = 93$ MeV, $M_\rho = 770$ MeV and $M_{\rho'} = 1465 ± 25$ MeV as input [14]. The theoretical errors are from $M_{\rho'}$. All masses and decay constants are in MeV, $L_{10}$ is dimensionless, $\phi$ is in degrees, the gluon condensate is in units of GeV$^4$ and the quark condensate (squared) is in units of $10^{-4}$ GeV$^6$.}
\end{table}

\begin{align}
\langle O \rangle_{d=6}^{V} &= -\frac{28}{9}\pi\alpha_s\langle\bar{q}q\rangle^2 = 2F_\pi^2M_\rho^4\csc^2\phi + \frac{1}{3}C_{V,A} \left( M^2 - \frac{1}{2}\Lambda^2 \right) \left( M^2 - \Lambda^2 \right) M^2, \quad (14d) \\
\langle O \rangle_{d=6}^{A} &= \frac{44}{9}\pi\alpha_s\langle\bar{q}q\rangle^2 = 2F_\pi^2M_\rho^4\csc^2\phi\cos^2\phi + \frac{1}{3}C_{V,A} \left( M^2 - \frac{1}{2}\Lambda^2 \right) \left( M^2 - \Lambda^2 \right) M^2. \quad (14e)
\end{align}

The Wilson coefficients and $C_{V,A}$ have been computed from the diagrams in Fig. 1 by Ref. 13. We input $F_\pi = 93$ MeV, $M_\rho = 770$ MeV and the mass of the lowest-lying vector excited state, $\rho'(1450)$: 1465 ± 25 MeV [14]. The solution to Eq. (14) with these inputs is given in Table 1. We have included predictions for the first few vector and axial-vector states in the towers of states. Of course the entire spectrum is predicted once $M$ and $\Lambda$ are determined. We have also included a prediction for the chiral perturbation theory parameter $L_{10}$, which is determined by the mixing angle $\phi$ [15]. The error quoted in the predicted quantities is from $\rho'(1450)$. There are, of course, many additional sources of error, including $1/N_c$ corrections, which conservatively generate an error of ±3$F_\pi$ (the delta-nucleon mass splitting). The predicted values of the quark and gluon condensates are somewhat large compared to the independent determinations of Ref. 16 and Ref. 17, respectively.

4. Scale Setting and Universality

A crucial issue of principle and practice is that of setting the scale at which the strong coupling constant and Wilson coefficients are evaluated. Intuitively one would expect that this scale, which can be taken to represent the onset of perturbative QCD, is a universal parameter, common to correlators with any given set of quantum numbers. Proving that
such a universal scale exists is highly non-trivial and we do not purport to do so here. Nevertheless, we will present a plausibility argument that such a universal parameter is present in large-$N_c$ QCD. Consider the basic relation between the hadronic description and the perturbative logarithm in large-$N_c$ QCD. From Eq. (3) and Eq. (5), neglecting the counterterms, we have

$$\sum_{n=0}^{N_i} \frac{F_i^2}{Q^2 + M_i^2 + n\Lambda_i^2} \Rightarrow -\frac{F_i^2}{\Lambda_i^2} \log \frac{Q^2}{N_i\Lambda_i^2} + \ldots.$$  

(15)

Here the subscript $i$ is a collective meson quantum number and the dots represent terms that are suppressed by inverse powers of $Q^2$ and $N_i$. The perturbative logarithm does not depend on the hadronic scale $M_i$. The natural scale at which to evaluate $\alpha_s$, which we interpret as the onset of perturbation theory, is $\sim \Lambda_i$. Naively, the $\Lambda_i$ can be widely disparate. This renders choosing the physical scale at which to evaluate $\alpha_s$ channel dependent. Now consider the following argument. Regulating QCD with an ultraviolet cutoff $\Lambda_\infty$ implies

$$\Lambda_\infty^2 = N_1\Lambda_1^2 = N_2\Lambda_2^2 = N_3\Lambda_3^2 = N_4\Lambda_4^2 = \ldots.$$  

(16)

This is simply the statement that in perturbative QCD there is one scale. While physics is independent of the cutoff $\Lambda_\infty$, seemingly physics depends on the $N_i$. Say we choose to regulate the sums with $N_1 = N_2$. Eq. (16) then implies a new relation between physical quantities, $\Lambda_1 = \Lambda_2$, which is not present if instead we choose $N_1 \neq N_2$. This ambiguity is removed if

$$\Lambda_1 = \Lambda_2 = \Lambda_3 = \Lambda_4 = \ldots \equiv \Lambda.$$  

(17)

There is then a universal scale, $\Lambda$, which is shared by all large-$N_c$ mesons. This common physical scale marks the onset of perturbation theory. In what follows we will treat Eq. (17) as a conjecture. Using the two-loop beta function in the large-$N_c$ limit in $\overline{MS}$ with $\Lambda_{\overline{MS}} = 380$ MeV, we find $\alpha_s(\Lambda) = 0.11$, where we have taken $\Lambda = 1172 \pm 43$ MeV from the
Table 2: Predictions of $1^{-+}$ hybrids using experimental candidates as input. On the left (right), we assume that the $\sim 1.4$ GeV [1] ($\sim 1.6$ GeV [2]) candidate is the ground state. Theoretical numbers are from Eq. (22) using (A) predicted condensates from Table 1 and (B) quark and gluon condensates from Ref. 16 and Ref. 17, respectively (see Table 1).

|               | TH (A,B) | EXP | TH (A,B) | EXP |
|---------------|----------|-----|----------|-----|
| $M_{\psi}$    | INPUT    | $1392 \pm 25$ [1] | INPUT | $1593 \pm 8$ [2] |
| $\Lambda_{\psi}$ | 1421,1543 | -- | 1787,1884 | -- |
| $M_{\psi^{0}}$ | 1990,2078 | $1593 \pm 8$ [2] | 2394,2467 | NO DATA |
| $M_{\psi^{0}}$ | 2445,2588 | NO DATA | 2987,3104 | NO DATA |

Consider the vector and axial-vector currents

$$V_{\mu}^a = g_s q \bar{q} \lambda^\alpha G_{\mu\nu}^{\alpha} \gamma^\nu \frac{1}{2} \tau^a q, \quad (18a)$$

$$A_{\mu}^a = g_s q \bar{q} \lambda^\alpha G_{\mu\nu}^{\alpha} \gamma^\nu \gamma_5 \frac{1}{2} \tau^a q, \quad (18b)$$

where $g_s$ is the QCD coupling constant, $G_{\mu\nu}$ is the gluon field-strength tensor and $\lambda^\alpha$ is an $SU(N_c)$ generator. These currents transform as $(3,1) \oplus (1,3)$ with respect to $SU(2) \times SU(2)$. Since these currents are not conserved, $V_{\mu}^a$ has nonvanishing matrix elements between $1^{-+}$ and $0^{++}$ states, and $A_{\mu}^a$ has nonvanishing matrix elements between $1^{+-}$ and $0^{-+}$ states. At leading order in the $1/N_c$ expansion, the $0^{++}$ and $1^{+-}$ states also couple to currents with $n_g = 0$. Therefore we will have nothing to say about these states here. There are no obvious chiral constraints between the $1^{-+}$ and $0^{-+}$ hybrid states. The nonvanishing matrix elements between the hybrid states and the vacuum are
Figure 3: The left (right) panel gives $M_V$ ($M_P$) as a function of $\Lambda_V$ ($\Lambda_P$) as given by Eq. (22). The shaded horizontal bars give the experimental $1^{-+}$ events at $\sim 1.4$ GeV [1] and $\sim 1.6$ GeV [2]. The solid curves use the predicted condensates from Table 1 and the dashed curves use the quark and gluon condensates from Ref. 16 and Ref. 17, respectively (see Table 1). The blocks over the solid curves are the universality predictions where $\Lambda$ is taken from Table 1. They correspond to $\sqrt{\sigma} = 468 \pm 12$ MeV. The dotted curve in the right panel is evaluated with $\alpha_s = 0.13$.

$$\langle 0| V^a_\mu | \hat{V}^{b\lambda} \rangle = \delta^{ab} F_V M_V^3 \epsilon^{\lambda}_\mu,$$

$$\langle 0| A^a_\mu | \hat{P}^{b\lambda} \rangle = \delta^{ab} F_P M_P^2 p_\mu.$$

Here $\hat{V}$ denotes a $1^{-+}$ hybrid and $\hat{P}$ denotes a $0^{--}$ hybrid. The $0^{++}$ and $1^{+-}$ states are denoted $\hat{S}$ and $\hat{A}$, respectively. The relevant spectral functions are defined through

$$8i \int d^4 x e^{i q x} \langle 0 | T[V^\mu_a(x) V^\nu_b(0)] | 0 \rangle = \delta_{ab} (q^\mu q^\nu - g^{\mu\nu} q^2) \Pi_V(Q^2) + \delta_{ab} g^{\mu\nu} q^2 \Pi_S(Q^2),$$

$$8i \int d^4 x e^{i q x} \langle 0 | T[A^\mu_a(x) A^\nu_b(0)] | 0 \rangle = \delta_{ab} (q^\mu q^\nu - g^{\mu\nu} q^2) \Pi_A(Q^2) + \delta_{ab} g^{\mu\nu} q^2 \Pi_P(Q^2).$$

Since $\Pi_{V,P}(Q^2)$ are dimension four, we are here interested in the case $m = 2$. The spectral functions in the large-$N_c$ limit are

$$\frac{1}{\pi} Im \Pi_V(t) = 2 \sum_{n=0}^{N_V} \frac{F^2_V(n)}{M_V^4(n)} \delta(t - M_V^2(n)),$$

$$\frac{1}{\pi} Im \Pi_P(t) = 2 \sum_{n=0}^{N_P} \frac{F^2_P(n)}{M_P^4(n)} \delta(t - M_P^2(n)).$$

Reading directly from Eq. (7a) and Eq. (9) we find the system of equations
Table 3: Predictions for $1^{++}$ and $0^{--}$ hybrids using universality. As input we use $\Lambda_V = \Lambda_\rho = \Lambda = 1172 \pm 43$ MeV (see Table 1), which corresponds to a string tension of $\sqrt{\sigma} = 468 \pm 12$ MeV.

|       | TH        | EXP   |       | TH   | EXP   |
|-------|-----------|-------|-------|------|-------|
| $M_V$ | 1190 ± 20 | 1392 ± 25 | $M_\rho$ | 4180 ± 5 | NO DATA |
| $\Lambda_V$ | INPUT | -- | $\Lambda_\rho$ | INPUT | -- |
| $M_{V'}$ | 1670 ± 40 | 1593 ± 8 | $M_{\rho'}$ | 4340 ± 16 | NO DATA |
| $M_{V''}$ | 2050 ± 50 | NO DATA | $M_{\rho''}$ | 4495 ± 26 | NO DATA |

The Wilson coefficients and $C_V, \rho$ have been computed from the diagrams in Fig. 2 by Ref. 21. The masses of the lowest-lying hybrids, $M_V, \rho$, require knowledge of $\Lambda_V, \rho$. The purest predictions we can make use the $1^{++}$ experimental candidates as input to determine $\Lambda_V$. These results are given in Table 2. We give predictions using the condensates extracted in the analysis of Section 3, and from independent determinations [16,17] (see Table 1). The spread in values gives a sense of the theoretical error due to truncating the OPE. In Fig. 3 we plot $M_V (M_\rho)$ as a function of $\Lambda_V (\Lambda_\rho)$ as given by Eq. (22). We give curves for both sets of condensate parameters. The sensitivity to $\alpha_s$ is insignificant for $M_V$. In Fig. 3 we show a curve for $M_\rho$ evaluated with $\alpha_s = 0.13$.

6. Discussion

If we treat the $1^{++}$ experimental candidate at $\sim 1.4$ GeV [1] as the lowest-lying state, then we predict the first excited state at $\sim 2$ GeV, as compared to $\sim 1.6$ GeV [2] (see Table 2). This discrepancy is larger than one would expect from a $1/N_c$ correction. If we treat the $1^{++}$ experimental candidate at $\sim 1.6$ GeV [2] as the lowest-lying state, then we predict the first excited state at $\sim 2.4$ GeV. One must then understand the role of the $\sim 1.4$ GeV [1] experimental candidate. We reiterate that in the large-$N_c$ limit, mesons do not couple to QCD currents with four quarks. If such mixing is relevant, it corresponds...
to a large subleading effect in the $1/N_c$ expansion. Without assuming universality, there is no prediction for the lowest-lying hybrid states. With $\Lambda_\Psi$ and $\Lambda_\rho$ between $1 - 2$ GeV, $M_\Psi$ ranges between $1.1 - 1.7$ GeV, while $M_\rho$ ranges between $4.1 - 4.7$ GeV (See Fig. 3). Assuming universality, and taking $\Lambda_\Psi = \Lambda_\rho = \Lambda = 1172 \pm 43$ MeV from the analysis of Section 3, we find a lowest-lying hybrid $\sim 1.2$ GeV, which underestimates the mass of the low-lying experimental candidate, and a first excited state $\sim 1.7$ GeV, which overestimates the mass of the heavier experimental candidate (see Table 3). However, the discrepancies are consistent with expected $1/N_c$ corrections.

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