Optimizing Information Freshness in Two-Way Relay Networks

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Abstract—In this paper, we investigate an amplify-and-forward (AF) based two-way cooperative status update system, where two sources aim to exchange status updates with each other as timely as possible with the help of a relay. Specifically, the relay receives the sum signal from the two sources in one time slot, and then amplifies and forwards the received signal to both the sources in the next time slot. We adopt a recently proposed concept, the age of information (AoI), to characterize the timeliness of the status updates. Assuming that the two sources are able to generate status updates at the beginning of each time slot (i.e., generate-at-will model), we derive a closed-form expression of the expected weighted sum AoI of the considered system. We further minimize the expected weighted sum AoI by optimizing the transmission power at each node under the peak power constraints. Simulation results corroborate the correctness of our theoretical analysis.

I. INTRODUCTION

With the abundance of inexpensive Internet of Things (IoT) devices, there has been a rapid development in real-time monitoring applications, such as autonomous vehicles, wireless industrial automation, and health care monitoring [1], [2]. In these applications, timely delivery of status updates is crucial. To quantify the timeliness of status updates, a new performance metric, termed the age of information (AoI), was recently introduced in [3]. The AoI is defined as the time elapsed since the generation of the latest received status update at the destination. According to the definition, unlike any conventional metrics, the AoI is jointly determined by the transmission interval and the transmission delay [4]. As a result, the AoI has attracted increasing attention as a more comprehensive evaluation criterion for information freshness [1]–[17].

Since the AoI concept was first proposed to characterize the information freshness in a vehicular status update system [5], AoI-oriented problems have been well studied in single-hop networks [6]–[13]. However, only a handful of work considered multi-hop networks [14]–[17]. In [14], both the offline and online policies were proposed to minimize the AoI of a two-hop energy harvesting network. The authors in [15] proved that the preemptive Last Generated First Served (PLGFS) queuing policy is age-optimal in a multi-hop network, where a single source disseminates status updates through a gateway to the network. Both [16] and [17] considered the AoI in multi-source, multi-monitor, multi-hop networks, while [17] was from a global perspective in the sense that every node in the network is both a source and a monitor. All the above work assumed that only one node can transmit in each time slot such that the transmission is always one-way. However, in the applications where two nodes sometimes need to exchange status updates with each other through a single relay (e.g., autonomous vehicles), one-way transmission means that one node must wait while the other node is transmitting. In such multi-hop networks, the two-way relaying scheme, in which the nodes simultaneously transmits status updates to each other, can significantly decrease the AoI in the networks.

To the best of the authors’ knowledge, although the two-way relaying scheme can potentially enhance the information freshness in the multi-hop network, there is no existing work that analyzes and optimizes the AoI of a two-way cooperative status update system. Such an analysis and optimization is non-trivial because the AoI of the two source nodes are tangled together. Specifically, transmitting with higher power at one source node reduces the outage probability and the AoI at the corresponding destination. However, higher transmission power at this source node also means that the interference in its received signal will be stronger, which results in a larger local AoI.

Motivated by this gap, in this paper we investigate an amplify-and-forward (AF) based two-way cooperative status update system, in which two source nodes timely exchange status updates with each other with the help of a relay. In order to keep the information at each destination as fresh as possible, we adopt the two time slot physical-layer network coding (PNC) scheme and the generate-at-will model in the system. We concentrate on analyzing and optimizing the AoI of the considered system. We first define some necessary time intervals to mathematically express the average AoI at each destination node. By representing these time-interval definitions in terms of the transmission success probabilities, we then attain a closed-form expression of the expected weighted sum AoI for the considered system. As the transmission success probabilities are functions of the transmission power at each node, we further minimize the expected weighted sum AoI by optimizing the transmission power at each node under the peak power constraints. Simulation results are then provided to validate the theoretical analysis.

II. SYSTEM MODEL AND SCHEME DESCRIPTION

A. System Model

We consider a two-way cooperative status update system where two source nodes, source A \((S_A)\) and source B \((S_B)\), want to exchange status updates with each other as timely as possible with the help of a single relay \((R)\). \(R\) adopts the
amplify-and-forward (AF) relaying protocol. Specifically, $S_A$ and $S_B$ transmits status updates $x_A$ and $x_B$, which can be generated at the beginning of any time slot (i.e. generate-at-will model [13]), to $R$ with power $P_A$ and $P_B$, respectively. Since the AF protocol is adopted, the received signals at $R$ from $S_A$ and $S_B$ are multiplied by a gain $G$. The amplified signal is then forwarded to the source nodes with the power $P_r$. To quantify the timeliness of the status updates, we adopt a recently proposed AoI metric, which is defined as the time elapsed since the generation of the last successfully received status update. The considered system is time slotted, and the transmission of each status update by any node takes exactly one time slot. The length of each time slot is normalized to one without loss of generality.

We denote by $h_A$ and $h_B$ the channel coefficients between $S_A$ and $R$, and $S_B$ and $R$, respectively. All channels in the system are assumed to be subject to Rayleigh fading. We assume that the mean and the variance associated with the Rayleigh distribution are $0$ and $1$, respectively, such that $h_A, h_B \sim CN(0, 1)$. In addition, we denote $n_A \sim CN(0, \sigma^2_A)$, $n_B \sim CN(0, \sigma^2_B)$ and $n_r \sim CN(0, \sigma^2)$ as the additive white Gaussian noise (AWGN) at $S_A$, $S_B$ and $R$, respectively. All the channels are assumed to be constant during one round of status update exchange between $S_A$ and $S_B$ [19]. With the aid of the instantaneous channel state information (CSI) and the noise statistics, the gain $G$ can be properly chosen such that the instantaneous transmission power at $R$ is constrained [20]. Furthermore, $G\sqrt{T_r}$ is assumed to be known at both source nodes. For notation simplification, we define $\gamma_A = \frac{P_r}{\sigma^2_A}$, $\gamma_B = \frac{P_r}{\sigma^2_B}$, $\gamma_r,A = \frac{P_r}{\sigma^2}$ and $\gamma_r,B = \frac{P_r}{\sigma^2}$.

### B. Two Time Slot PNC Scheme

In this paper, we consider the two time slot PNC scheme, in which $S_A$ and $S_B$ completes one round of status update exchange over two time slots, as shown in Fig. 1. In the first time slot, the two source nodes simultaneously transmit to $R$. In the second time slot, $R$ amplifies and forwards the received signals to the two sources nodes to complete the status update exchange.

Based on [19 Eqs. (8) and (9)], we can express the received signal-to-noise ratio (SNR) at $S_\zeta$, given by

$$\Gamma_{\zeta,TS} = \frac{\gamma_r,\zeta,|h| \cdot |h_{\zeta}|^2}{(\gamma_r,\zeta + \gamma_r,\zeta,|h| \cdot |h_{\zeta}|^2 + \gamma_r,\zeta,|h_r|^2 + 1},$$

where $\zeta, \zeta \in \{A, B\}$ and $\zeta \neq \zeta$. We further show the expression of the transmission success probability, which will be used to optimize the average AoI. The transmission success probability is defined as the probability that the received SNR is higher than an acceptable SNR threshold $\gamma_{th}$. The exact transmission success probability expression for the two slot PNC scheme is complex for the AoI optimization. We thus adopt the asymptotic transmission success probability expression at high SNR. Based on [19 Eq. (33)], the asymptotic transmission success probability through $S_\zeta \sim R - S_\zeta$ link for the two time slot PNC scheme is given by

$$F_{\zeta,TS} = 1 - \frac{\gamma_{th}(\gamma_{r,\zeta} + \gamma_\zeta + \gamma_{r,\zeta})}{\gamma_{r,\zeta} \cdot \gamma_\zeta} + o(\gamma_{th}^2).$$

In the next section, we will use the above expressions to analyze and optimize the average AoI of the two time slot PNC scheme for the considered system.

### III. Analysis and Optimization of Average AoI

In this section, we first derive the closed-form expression of the average AoI of the considered system. Furthermore, we minimize the average AoI by optimizing the transmission power at each node under the peak power constraints.

#### A. Analysis of Average AoI

Since we adopt generate-at-will model in the considered system, we assume that $S_A$ and $S_B$ generate new status updates every two time slots to keep the information at the corresponding destinations as fresh as possible. Let $U_\zeta(t)$ denote the generation time of the most recently received status update at $S_\zeta$ until time slot $t$. The AoI at time slot $t$ at $S_\zeta$ can then be defined as

$$\Delta_{\zeta,TS}(t) = t - U_\zeta(t).$$

Let $S_{\zeta,k}$ and $T_{\zeta,k}$ denote the service time of the $k$th successfully received update at $S_\zeta$, and the interdeparture time between two consecutive successfully received status updates at $S_\zeta$, respectively. Following similar analysis to that in [21], the average AoI at $S_\zeta$ can be expressed as

$$\Delta_{\zeta,TS} = \mathbb{E}[S_{\zeta,k-1}] + \frac{\mathbb{E}[T_{\zeta,k}^2]}{2\mathbb{E}[T_{\zeta,k}]} - \frac{1}{2}.$$

It is obvious that, in the considered system, the service time of each successfully received update is always a constant, which is given by $\mathbb{E}[S_{\zeta,k-1}] = 2$. As mentioned before, $S_A$ and $S_B$ are assumed to generate new status updates every time slots. Therefore, the interdeparture time $T_{\zeta,k}$ is a geometric random variable with mean $\mathbb{E}[T_{\zeta,k}] = 2/F_{\zeta,2TS}$ and second moment $\mathbb{E}[T_{\zeta,k}^2] = 4(2 - F_{\zeta,2TS})/F_{\zeta,2TS}$. By substituting $\mathbb{E}[S_{\zeta,k-1}]$, $\mathbb{E}[T_{\zeta,k}]$ and $\mathbb{E}[T_{\zeta,k}^2]$ into (4), we obtain the closed-from expression of the average AoI at $S_\zeta$ for the two time slot PNC scheme, given by

$$\Delta_{\zeta,TS} = 1 + \frac{2}{F_{\zeta,2TS}}.$$

The expected weighted sum AoI of the considered system can thus be expressed as

$$\tilde{\Delta}_{2TS} = \omega_A \cdot \Delta_{A,2TS} + \omega_B \cdot \Delta_{B,2TS}$$

where $\omega_A$ and $\omega_B$ are weighting coefficients at $S_A$ and $S_B$, respectively, such that $\omega_A + \omega_B = 1$.

In the next subsection, we will manage to minimize the expected weighted sum AoI by optimizing the transmission power at each node in the considered system.
B. Optimization of Expected Weighted Sum AoI

In this subsection, we optimize the expected weighted sum AoI of the considered system under the peak power constraints. By substituting (2) into (6), we can rewrite the expected weighted sum AoI of the two time slot PNC scheme as

\[
\Delta_{2TS} = \frac{1}{2}(\omega_A + \omega_B) + 2 \left[ \frac{\omega_A}{1 - \gamma_{th}(\sigma_A + \gamma_A + \gamma_B)} + \frac{\omega_B}{1 - \gamma_{th}(\sigma_B + \gamma_A + \gamma_B)} \right]
\]

\[
= \frac{1}{2}(\omega_A + \omega_B) + 2 \left[ \frac{\omega_A}{1 - \gamma_{th}(\sigma_A^2 + \frac{\sigma_A^2}{P_A} (1 + \frac{P_A}{P_B}))} + \frac{\omega_B}{1 - \gamma_{th}(\sigma_B^2 + \frac{\sigma_B^2}{P_B} (1 + \frac{P_B}{P_A}))} \right].
\]

(7)

It is obvious that the first term on the RHS of (7) is a constant with a value of 0.5. Therefore, in order to minimize the expected weighted sum AoI of the two time slot PNC scheme, we only need to optimize the second term on the RHS of (7). We are at a point to formally formulate the following optimization problem:

\[
\begin{align*}
\min_{P_A, P_B, P_r} & \left\{ \frac{\omega_A}{F_{A,2TS}} + \frac{\omega_B}{F_{B,2TS}} \right\} \\
\text{s.t.} & \quad 0 < P_A \leq P_A^k, 0 < P_B \leq P_B^k, 0 < P_r \leq P_r^k, \\
& \quad 0 < F_{A,2TS} < 1, 0 < F_{B,2TS} < 1,
\end{align*}
\]

(8)

where \( F_{A,2TS} = 1 - \gamma_{th} \left( \frac{\sigma_A^2}{P_A} + \frac{\sigma_A^2}{P_A} (1 + \frac{P_A}{P_B}) \right) \) and \( F_{B,2TS} = 1 - \gamma_{th} \left( \frac{\sigma_B^2}{P_B} + \frac{\sigma_B^2}{P_B} (1 + \frac{P_B}{P_A}) \right) \). It is worth mentioning that the asymptotic transmission success probabilities are obtained at high SNR, which makes their values inaccurate or even less than 0 in the case of low SNR. Therefore, in order to guarantee the physical meaning of probability, we limit 0 < \( F_{A,2TS} < 1 \) and 0 < \( F_{B,2TS} < 1 \) in problem (8). Note that although \( F_{A,2TS} \) and \( F_{B,2TS} \) are supposed to be high probabilities, we can follow the same procedure as below to solve the optimization problem under this constraint.

As the objective function is a monotonically decreasing function of \( P_r \), the optimal transmission power at \( R \) in problem (8) is given by

\[ P_r^* = P_r^k. \]

Thus, problem (8) is equivalent to the following problem:

\[
\begin{align*}
\min_{P_A, P_B} & \left\{ \frac{\omega_A}{F_{A,2TS}} + \frac{\omega_B}{F_{B,2TS}} \right\} \\
\text{s.t.} & \quad 0 < P_A \leq P_A^k, 0 < P_B \leq P_B^k, \\
& \quad 0 < F_{A,2TS}^r < 1, 0 < F_{B,2TS}^r < 1,
\end{align*}
\]

(10)

where \( F_{A,2TS}^r = 1 - \gamma_{th} \left( \frac{\sigma_A^2}{P_A} + \frac{\sigma_A^2}{P_A} (1 + \frac{P_A}{P_B}) \right) \) and \( F_{B,2TS}^r = 1 - \gamma_{th} \left( \frac{\sigma_B^2}{P_B} + \frac{\sigma_B^2}{P_B} (1 + \frac{P_B}{P_A}) \right) \). Problem (10) is non-convex due to the non-convexity of the object function, and is thus difficult to solve via standard convex optimization techniques \[22\]. In order to solve the non-convex problem, we first fix \( P_A \) or \( P_B \), by which we have the following lemmas:

**Lemma 1:** When \( P_A \) is fixed, problem (10) is converted into a convex problem, in which the optimal \( P_B \) is given by

\[
P_B^* = \min \left\{ P_B^k, P_B^k \right\}, \quad \text{where} \quad P_B^k = \frac{\beta_A \theta_A + \gamma_A}{\kappa_A}, \quad \beta_A = \frac{P_B^k}{2} \frac{P_B^k \omega_A \omega_B (P_A \sigma_A^2 + P_B^k \sigma_B^2)}{\theta_B = P_A \gamma_A} (\sigma_A^2 + \gamma_A^2) + \frac{\gamma_A^2}{\gamma_A^2} (\sigma_B^2 - \sigma_A^2) + \frac{P_B^k \gamma_A \sigma_B^2 \gamma_A}{\gamma_A^2} (\sigma_B^2 - \gamma_A^2) (\sigma_A^2 + \gamma_A^2) + \frac{P_B^k \gamma_A \sigma_B^2 \gamma_A}{\gamma_A^2} (\sigma_B^2 - \gamma_A^2) (\sigma_A^2 + \gamma_A^2),
\]

\[
\gamma_A^2 \frac{\sigma_A^2 \sigma_B^2}{\sigma_A^2} (\gamma_B^2 - \gamma_A^2) (\sigma_A^2 + \gamma_A^2),
\]

\[
\begin{align*}
\text{Proof:} & \quad \text{See Appendix A.}
\end{align*}
\]

**Lemma 2:** When \( P_B \) is fixed, problem (10) is converted into a convex problem, in which the optimal \( P_A \) is given by

\[
P_A^* = \min \left\{ P_A^k, P_A^k \right\}, \quad \text{where} \quad P_A^k = \frac{\beta_A \theta_A + \gamma_A}{\kappa_A}, \quad \beta_A = \frac{P_A^k}{2} \frac{P_A^k \omega_A \omega_B (P_B^k \sigma_B^2 + P_A^k \sigma_A^2)}{\theta_B = P_B^k \gamma_A} (\sigma_B^2 + \gamma_B^2) + \frac{\gamma_B^2}{\gamma_B^2} (\sigma_A^2 - \sigma_B^2) + \frac{P_A^k \gamma_B \sigma_A^2 \gamma_B}{\gamma_B^2} (\sigma_A^2 - \gamma_B^2) (\sigma_B^2 + \gamma_B^2) + \frac{P_A^k \gamma_B \sigma_A^2 \gamma_B}{\gamma_B^2} (\sigma_A^2 - \gamma_B^2) (\sigma_B^2 + \gamma_B^2),
\]

\[
\gamma_B^2 \frac{\sigma_A^2 \sigma_B^2}{\sigma_B^2} (\gamma_A^2 - \gamma_B^2) (\sigma_A^2 + \gamma_B^2),
\]

\[
\begin{align*}
\text{Proof:} & \quad \text{See Appendix B.}
\end{align*}
\]

By applying the results obtained in Lemma 1 and Lemma 2, we can acquire the optimal solution to problem (10) given by the following theorem.

**Theorem 1:** The minimum expected weighted sum AoI is obtained when at least one source node transmits with the peak power. Mathematically, the optimal transmission power at the source nodes, i.e., \( P_A \) and \( P_B \), and the minimum expected weighted sum AoI of the two time slot PNC scheme are given as

\[
(P_A^*, P_B^*) = \arg \min \left\{ (P_A^k, P_B^k); (P_A^k, P_B^k) \right\} \{ f(P_A, P_B) \}, \quad \text{where} \quad f(P_A, P_B) = \frac{\omega_A \omega_B (P_A \sigma_A^2 + P_B \sigma_B^2)}{\theta_B = P_A \gamma_A} (\sigma_A^2 + \gamma_A^2) + \frac{\gamma_A^2}{\gamma_A^2} (\sigma_B^2 - \sigma_A^2) + \frac{P_B \gamma_A \sigma_B^2 \gamma_A}{\gamma_A^2} (\sigma_B^2 - \gamma_A^2) (\sigma_A^2 + \gamma_A^2),
\]

\[
\begin{align*}
\Delta_{2TS}^* = & \frac{1}{2} + f(P_A^*, P_B^*), \quad \text{and} \quad \gamma_{th} = 20dB. \quad \text{For the exact analytical results shown in Fig. 2, we plot them by substituting the exact transmission success probability expression given in [19] Eq. (29) into our AoI analysis. Each simulation curve presented in the section is averaged over 10^7 time slots.}
\end{align*}
\]

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present simulation results to validate our analytical results. Without loss of generality, we consider that the two source nodes are of the same importance in the system, i.e., \( \omega_A = \omega_B = 0.5 \). In the following simulations, it is also assumed that \( P_r^k = 0.75 P_B^k \), \( \sigma_A^2 = \sigma_B^2 = 0.001 \) and \( \gamma_{th} = 20dB \). For the exact analytical results shown in Fig. 2, we plot them by substituting the exact transmission success probability expression given in [19] Eq. (29) into our AoI analysis. Each simulation curve presented in the section is averaged over 10^7 time slots.
We first plot the expected weighted sum AoI of the two time slot PNC scheme against the transmission power at $S_B$ (i.e., $P_B$) by fixing the transmission power at $S_A$ (i.e., $P_A$). As shown in Fig. 2, we consider two cases with $P_A$ being 1 and 1.5, respectively. In both cases, it is apparent that the expected weighted sum AoI first decreases and then increases as $P_B$ increases. This is because when $S_B$ transmits with higher power, although $S_A$ will receive better signals, the received SNR at $S_B$ will be lower. The resulting transmission success probabilities through the $S_B - R - S_A$ link and the $S_A - R - S_B$ link will thus increase and decrease, respectively. However, no matter how high $F'_{A,2TS}$ is, the minimum average AoI at $S_A$ can only be reduced to 2, while the average AoI at $S_B$ will grow rapidly when $F'_{B,2TS}$ is low. Therefore, to high transmission power at $S_B$ in turn increases the expected weighted sum AoI of the considered system. It is also worth mentioning that the optimal transmission power at $S_B$ to minimize the expected weighted sum AoI in the case of $P_A = 1$ and $P_A = 1.5$ are $P_B = 1.196$ and $P_B = 1.718$, respectively, which are consistent with the results calculated by using the formula in Lemma 2. We can also find that, in both cases, the asymptotic analytical result is close to the simulation result and the exact analytical result when $P_B \in (0.75, 2)$. Specifically, in this region, when $P_A = 1$ and $P_A = 1.5$, we can have $F_{A,2TS} > 0.56$, $F'_{B,2TS} > 0.50$ and $F_{A,2TS} > 0.60$, $F'_{B,2TS} > 0.72$, respectively. This observation validates the rationality of using the asymptotic result at high SNR, or high transmission success probability in our analysis.

After verifying the tightness between the asymptotic expected weighted sum AoI and the exact values at high transmission success probabilities, we now investigate the optimal expected weighted sum AoI and the exact values at high transmission success probabilities through the two cases, the asymptotic analytical result is close to the simulation result when $P_B \in (0.75, 2)$. Specifically, in this region, when $P_A = 1$ and $P_A = 1.5$, we can have $F_{A,2TS} > 0.56$, $F'_{B,2TS} > 0.50$ and $F_{A,2TS} > 0.60$, $F'_{B,2TS} > 0.72$, respectively. This observation validates the rationality of using the asymptotic result at high SNR, or high transmission success probability in our analysis.

V. Conclusions

In this paper, we analyzed and optimized the average age of information (AoI) of a two-way cooperative status update system, where two source nodes timely exchange status updates with each other with the help of a relay. By analyzing the evolution of the AoI, we derived a closed-form expression of the expected weighted sum AoI for the considered system as a function of the signal-to-noise ratio (SNR) at each node. Under the peak power constraints, we further figured out the optimal transmission power at each node that minimizes the expected weighted sum AoI. We found that when the AoI of the considered system is minimum, at least one source node transmits with its peak power, while the optimal transmission power of the other source node may be lower than the peak value. Simulation results validated our theoretical analysis.

APPENDIX

A. Proof of Lemma 1

When $P_A$ is fixed, the objective function in problem (10) is a function only of $P_B$, i.e. $f(P_B)$. To verify its convexity, we derive the second-order derivative of $f(P_B)$ with respect to (w.r.t.) $P_B$. After some algebra manipulations, we have

$$
\frac{\partial^2 f(P_B)}{\partial P_B^2} = 2\omega_A \gamma_{th} \left( P_A \sigma_A^2 + P_{th} \sigma_B^2 \right) \left( P_{th}^b - \gamma_{th} \sigma_A^2 \right) \left( P_{B,2TS} \right)^3
\frac{P_B^3 \left( P_{th}^b \right)^2 \left( F'_{A,2TS} \right)^3}{2\omega_A \gamma_{th} \sigma_A^2}.
$$

As $0 < F'_{A,2TS} < 1$ and $0 < F'_{B,2TS} < 1$, the denominators of the first and second terms on the RHS of (15) are always larger than zero. Based on $F'_{A,2TS} < 1$, after some algebra manipulations, we have that $P_{th}^b - \gamma_{th} \sigma_A^2 > 0$. Therefore, we always have $\frac{\partial^2 f(P_B)}{\partial P_B^2} > 0$, which indicates that when $P_A$ is
After some algebraic manipulation on (19), we can readily find \( P_B \) in order to investigate whether \( f(P_B) \) is convex in the hand side (LHS) of (19). From the above two cases, it is obvious that \( g_{\omega P} \) when \( g_{\omega P} \) is the only valid solution to minimize \( f(P_A) \). Recall that \( f(P_A) \) is a convex function when \( P_B \) is fixed. This means that, on the LHS of (19), \( f(P_A) \) is a monotonically decreasing function. Therefore, if \( P_{A1} > P_{pk} \), the minimum value of \( f(P_A) \) is obtained at \( P_{pk} \). This completes the proof.

### B. Proof of Lemma 2

When \( P_B \) is fixed, the objective function in problem (10) is a function only of \( P_A \), i.e., \( f(P_A) \). To verify its convexity, we derive the second-order derivative of \( f(P_A) \) w.r.t. \( P_A \), given by

\[
\frac{\partial^2 f(P_A)}{\partial P_A^2} = \frac{2\omega_B \gamma_{th}(P_B \sigma_B^2 + P_pk \sigma_B^2)(P_pk - \gamma_{th} \sigma_B^2)}{P_A^3 (P_pk)^2 (P_B + 2TS)^2}.
\]

(20)

Based on \( P_{B,2TS} < 1 \), after some algebraic manipulations, we have that \( P_pk - \gamma_{th} \sigma_B^2 > 0 \). According to the analysis of (15) in Appendix A, we can have \( \frac{\partial^2 f(P_A)}{\partial P_A^2} > 0 \), which indicates that when \( P_B \) is fixed, the objective function in problem (10) is convex in the constraint region. In addition, the optimal \( P_A \) to minimize \( f(P_A) \) are given by

\[
P_{A1} = \frac{\beta_A \theta_A + \varphi_A}{\kappa_A} \quad \text{and} \quad P_{A2} = \frac{\beta_A \theta_A + \varphi_A}{\kappa_A},
\]

(21)

where \( \beta_A = \beta_A = P_pk \sqrt{\omega_B} \) and \( \kappa_A = P_pk \). Proof of Lemma 2

Recall that the objective function in problem (10) is given by

\[
f(P_A, P_B) = \frac{\omega_A}{1 - \gamma_{th}} \left[ \frac{\sigma_B^2}{P_B + \sigma_B^2} + \frac{\sigma_B^2}{P_A + \sigma_B^2} \right] + \frac{\omega_B}{1 - \gamma_{th}} \left[ \frac{\sigma_A^2}{P_B + \sigma_A^2} + \frac{\sigma_A^2}{P_A + \sigma_A^2} \right].
\]

(22)

Suppose when \( P_B = P_{pk} \), \( \tilde{P}_A < P_{pk} \) and \( \tilde{P}_B < P_{pk} \), the corresponding \( f(P_A, \tilde{P}_B) \) is minimized. We can always have (a) \( \frac{P_{pk}}{P_B} = \rho \), where \( P_{pk} > P_A \) and \( \tilde{P}_B > P_B \); (b) \( \frac{P_{pk}}{P_B} = \rho \), where \( P_{pk} > \tilde{P}_A \) and \( P_{pk} > \tilde{P}_B \). In the following, we further compare \( f(P_{pk}, \tilde{P}_B) \) and \( f(P_{pk}, \tilde{P}_B) \) with \( f(P_A, \tilde{P}_B) \), respectively.

To investigate \( f(P_{pk}, \tilde{P}_B) \), we now consider \( \tilde{P}_B \) for the following two cases:

1) If \( \tilde{P}_B < P_{pk} \), \( \tilde{P}_B \) is in the feasible region. Based on (22), when \( \frac{\tilde{P}_B}{P_B} = \rho \), \( P_{pk} > \tilde{P}_A \) and \( \tilde{P}_B > \tilde{P}_B \), it is obvious that \( f(P_{pk}, \tilde{P}_B) < f(\tilde{P}_A, \tilde{P}_B) \).

### C. Proof of Theorem 1

Recall that the objective function in problem (10) is given by

\[
f(P_A, P_B) = \frac{\omega_A}{1 - \gamma_{th}} \left[ \frac{\sigma_B^2}{P_B + \sigma_B^2} + \frac{\sigma_B^2}{P_A + \sigma_B^2} \right] + \frac{\omega_B}{1 - \gamma_{th}} \left[ \frac{\sigma_A^2}{P_B + \sigma_A^2} + \frac{\sigma_A^2}{P_A + \sigma_A^2} \right].
\]

(22)
2) If $\tilde{P}_A' > P_{pk}^B$, $\tilde{P}_B$ is not in the feasible region. However, we now have $\tilde{P}_A' > P_{pk}^B$. Therefore, we can always find $\tilde{P}_A'$, which follows $\tilde{P}_A' < P_A^k$, so that $\frac{\tilde{P}_A'}{\tilde{P}_A} = \rho$. As $\frac{\tilde{P}_B}{\tilde{P}_B} = \frac{\tilde{P}_A'}{\tilde{P}_A} = \rho$ and $P_{pk}^B > \tilde{P}_A$, it is obvious that $\tilde{P}_A' > \tilde{P}_A$.

Therefore, we can readily have that $f(\tilde{P}_A', P_{pk}^B) < f(\tilde{P}_A, \tilde{P}_B)$.

Similarly, when investigating $f(P_A^k, P_{pk}^B)$, we have the following two cases:

1) If $\tilde{P}_A > P_{pk}^k$, $f(\tilde{P}_A, P_{pk}^B) = \rho$. Therefore, we can readily have that $\rho > P_{pk}^k$ and $\rho < P_{pk}^B$.

Based on the above cases, we can always find feasible $\tilde{P}_A$ and $\tilde{P}_B$ to ensure that $f(P_A^k, P_{pk}^B) < f(\tilde{P}_A, \tilde{P}_B)$ and $f(P_A^k, P_{pk}^B) < f(\tilde{P}_A, \tilde{P}_B)$, which contradicts our hypothesis. It indicates that at least one of $P_A$ and $P_B$ should be chosen as the corresponding peak value to minimize the objective function $f(P_A, P_B)$. By applying the results obtained in Lemma 1 and Lemma 2, we complete the proof of the theorem.

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