I. G. Aznauryan · V. D. Burkert

Electroexcitation of Nucleon Resonances in a Light-Front Relativistic Quark Model

Received: 10 December 2017 / Accepted: 24 May 2018 / Published online: 8 June 2018
© Springer-Verlag GmbH Austria, part of Springer Nature 2018

Abstract We report the predictions for the 3q core contributions to the electroexcitation of the resonances \( \Delta(1232)_{3/2}^+, N(1440)_{1/2}^+, N(1520)_{3/2}^-, N(1535)_{1/2}^-, \) and \( N(1675)_{5/2}^- \) on the proton obtained in the light-front relativistic quark model (LF RQM). For these states, experimental data on the electroexcitation transition amplitudes allow us to make comparison between the experiment and LF RQM predictions in wide range of \( Q^2 \) and also to quantify the expected meson-baryon contributions as a function of \( Q^2 \).

1 Introduction

Recently extensive investigation of the electroexcitation of nucleon resonances has been made in the light-front dynamics, which is known as most suitable framework for describing the transitions between relativistic bound systems. In wide range of \( Q^2 \), electroexcitation on the proton and neutron is investigated in LF RQM \([1,2]\) for the resonances of the multiplets \([56, 0^+]\), \([56, 0^+]\), \([70, 1^-]\) \([3–7]\). In Refs. \([4–7]\), the parameters of the LF RQM have been specified via description of the nucleon electromagnetic form factors up to \( Q^2 = 16 \text{ GeV}^2 \) by combining the 3q and pion-cloud contributions in the LF dynamics and by employing running quark mass as a function of \( Q^2 \) \([4]\).

In this talk we present the results for the transitions \( \gamma^*p \rightarrow \Delta(1232)_{3/2}^+, N(1440)_{1/2}^+, N(1520)_{3/2}^-, N(1535)_{1/2}^-, \) and \( N(1675)_{5/2}^- \). For these processes experimental data on the transition amplitudes are available in wide range of \( Q^2 \) and, therefore, allow us to make detail comparison between the experiment and LF RQM predictions and also to make conclusions on the inferred meson-baryon contributions.

2 Results and Discussion

The predictions for the \( \gamma^*N \rightarrow N \) form factors and for the amplitudes of resonance electroexcitation on nucleons have been made in Refs. \([4–7]\) under the assumption that in addition to the three-quark (3q) contribution, these transitions contain contributions, which are produced by meson-baryon interaction. The nucleon electromagnetic form factors were described by combining 3q and pion-nucleon loops contributions. With
the pion-nucleon loops evaluated according to the LF approach of Ref. [8], the following form of the nucleon wave function has been found:

\[ |N \rangle = 0.95 |3q \rangle + 0.313 |\pi N \rangle. \]  

(1)

For the resonances \( \Delta (1232) \frac{3}{2}^+, N (1440) \frac{1}{2}^+, N (1520) \frac{3}{2}^-, \) and \( N (1535) \frac{3}{2}^- \), the weights of the \(|3q \rangle\) component in the expansion

\[ |N^* \rangle = c_{N^*} |3q \rangle + \cdots \]  

(2)

were found from experimental data on the \( \gamma^* p \to \Delta (1232) \frac{3}{2}^+, N (1440) \frac{1}{2}^+, N (1520) \frac{3}{2}^-, \) and \( N (1535) \frac{3}{2}^- \) assuming that at \( Q^2 > 2 - 3 \text{ GeV}^2 \) these transitions are determined only by the first term in Eq. (2). The obtained values of the coefficients \( c_{N^*} \) are 0.88 \pm 0.04, 0.93 \pm 0.05, 0.93 \pm 0.06, and 0.91 \pm 0.03, respectively, for the resonances \( \Delta (1232) \frac{3}{2}^+, N (1440) \frac{1}{2}^+, N (1520) \frac{3}{2}^-, \) and \( N (1535) \frac{3}{2}^- \). For the \( N (1675) \frac{5}{2}^- \), we have good description of the data at \( Q^2 > 3.5 \text{ GeV}^2 \) with \( c_{N^*} \approx 1 \); the presented results for the \( 3q \) core contribution to the \( \gamma^* p \to N (1675) \frac{5}{2}^- \) amplitudes correspond to \( c_{N^*} = 1 \).

To study sensitivity to the form of the quark wave function, two forms of the spatial wave function have been employed in Ref. [4]:

\[ \Phi_1 \sim \exp(-M_0^2/6\alpha_i^2), \quad \Phi_2 \sim \exp\left[-(q_0^2 + q_0^2 + q_i^2)/2\alpha_2^2\right], \]  

(3)

where \( M_0 \) is invariant mass of the system of constituent quarks, and \( q_i \) are three momenta of these quarks in their c.m.s. A good description of the nucleon electromagnetic form factors up to \( Q^2 = 16 \text{ GeV}^2 \) has been obtained with the wave functions (3) using the following parameterizations for the running quark mass as a function of \( Q^2 \):

\[ m_q^{(1)}(Q^2) = \frac{0.22 \text{ GeV}}{1 + Q^2/56 \text{ GeV}^2}, \quad m_q^{(2)}(Q^2) = \frac{0.22 \text{ GeV}}{1 + Q^2/18 \text{ GeV}^2}. \]  

(4)

For the resonances, the results for the transition amplitudes obtained with the wave functions (3) and corresponding masses (4) are very close to each other. The role of the running quark mass becomes visible above \( 3 \text{ GeV}^2 \). At \( Q^2 = 5(12) \text{ GeV}^2 \), it increases the transition helicity amplitudes by 25–35% (100–140%) and 10–15% (55–65%) for the wave functions \( \Phi_1 \) and \( \Phi_2 \), respectively.

We note that while in QCD lattice calculations and Dyson–Schwinger equations we deal with the off-shell quarks, and the quark virtuality is determined by their four-momentum square, in constituent quark models, including LF approaches, the quarks are on-mass-shell objects, and their virtuality is characterized...
Electroexcitation of Nucleon Resonances in a Light-Front...

Fig. 2 The $\gamma^* p \rightarrow \Delta(1232)\frac{3}{2}^+$ transition helicity amplitudes (in units of $10^{-3}$ GeV$^{-1/2}$). The solid curves correspond to the LF RQM predictions, the dashed curves present inferred meson-baryon contributions. The legend for the data is as for Fig. 1.

Fig. 3 The $\gamma^* p \rightarrow N(1440)\frac{1}{2}^+$ transition helicity amplitudes (in units of $10^{-3}$ GeV$^{-1/2}$). The solid curves are the LF RQM predictions, the dotted curves present the results obtained within Dyson-Schwinger equations [17,18]. The solid circles and triangles are the amplitudes extracted from the CLAS data on $\pi N$ [10] and $\pi^+\pi^- p$ [19,20] electrooduction off the proton. The solid box at $Q^2 = 0$ is the RPP estimate [21].

We present the results for the electroexcitation of the resonances in terms of transition helicity amplitudes (Figs. 2, 3, 4, 6, 7). The exception is made for the $\Delta(1232)\frac{3}{2}^+$ form factor. For this resonance in Fig. 1 we present also the predictions for the $\gamma^* N \rightarrow \Delta(1232)\frac{3}{2}^+$ magnetic-dipole form factor and the ratios $R_{EM} \equiv ImE_{1^+}/ImM_{1^+}$, $R_{SM} \equiv ImS_{1^+}/ImM_{1^+}$, as these observables are commonly used to present the results on the $\Delta(1232)\frac{3}{2}^+$ extracted from experimental data on the electroproduction of pions on nucleons. From Fig. 1 it is seen that the predictions for the $3q$ contribution to the $\gamma^* N \rightarrow \Delta(1232)\frac{3}{2}^+$ magnetic-dipole form factor are within limits obtained in the dynamical reaction model [26,28], where the bare contribution, that can be associated with the $3q$ contribution, gives at $Q^2 = 0$ about 40-70% of the total magnetic-dipole form factor. For the ratios $R_{EM}$ and $R_{SM}$ one conclude that $3q$ core contribution to $R_{EM}$ will stay small up to $Q^2 = 12$ GeV$^2$, while for $R_{SM}$ it grows in agreement with experimental data and will continue to grow up $Q^2 = 12$ GeV$^2$.

The CLAS measurements made possible, for the first time, the determination of the electroexcitation amplitudes of the Roper resonance $N(1440)\frac{1}{2}^+$ on the proton in wide range of $Q^2$ [9,10]. Comparison of the data with the LF RQM predictions [3,29] provided strong evidence for the identification of the $N(1440)\frac{1}{2}^+$ as

by the invariant mass of the quarks, which is increasing with increasing $Q^2$. The correspondence between $Q^2$ and mean value of $M_0^2$ is following: $Q^2 = 0$, 5, 10, 20 GeV$^2$ correspond, respectively, to $< M_0^2 >= 1.35, 2.66, 3.1, 3.5$ GeV$^2$.

We present the results for the electroexcitation of the resonances in terms of transition helicity amplitudes (Figs. 2, 3, 4, 6, 7). The exception is made for the $\Delta(1232)\frac{3}{2}^+$. For this resonance in Fig. 1 we present also the predictions for the $\gamma^* N \rightarrow \Delta(1232)\frac{3}{2}^+$ magnetic-dipole transition form factor and the ratios $R_{EM} \equiv ImE_{1^+}/ImM_{1^+}$, $R_{SM} \equiv ImS_{1^+}/ImM_{1^+}$, as these observables are commonly used to present the results on the $\Delta(1232)\frac{3}{2}^+$ extracted from experimental data on the electroproduction of pions on nucleons. From Fig. 1 it is seen that the predictions for the $3q$ contribution to the $\gamma^* N \rightarrow \Delta(1232)\frac{3}{2}^+$ magnetic-dipole form factor are within limits obtained in the dynamical reaction model [26,28], where the bare contribution, that can be associated with the $3q$ contribution, gives at $Q^2 = 0$ about 40-70% of the total magnetic-dipole form factor. For the ratios $R_{EM}$ and $R_{SM}$ one conclude that $3q$ core contribution to $R_{EM}$ will stay small up to $Q^2 = 12$ GeV$^2$, while for $R_{SM}$ it grows in agreement with experimental data and will continue to grow up $Q^2 = 12$ GeV$^2$.

The CLAS measurements made possible, for the first time, the determination of the electroexcitation amplitudes of the Roper resonance $N(1440)\frac{1}{2}^+$ on the proton in wide range of $Q^2$ [9,10]. Comparison of the data with the LF RQM predictions [3,29] provided strong evidence for the identification of the $N(1440)\frac{1}{2}^+$ as...
a predominantly first radial excitation of the nucleon [9, 10, 30]. This conclusion was based on the description of the following specific features in the extracted $\gamma^* p \rightarrow N(1520)^{3/2-}$ amplitudes: (1) the specific behavior of the transverse amplitude $A_{1/2}$, which being large and negative at $Q^2 = 0$, becomes large and positive at $Q^2 \approx 2$ GeV$^2$, and then drops slowly with $Q^2$; (2) the positive relative sign between the longitudinal $S_{1/2}$ and transverse $A_{1/2}$ amplitudes above $Q^2 \approx 1$ GeV$^2$; (3) the common sign of the amplitudes $A_{1/2}, S_{1/2}$ extracted from the data on $\gamma^* p \rightarrow \pi N$, that includes the signs from the $\gamma^* p \rightarrow N(1440)^{1/2+}$ and $N(1440)^{1/2+} \rightarrow \pi N$ vertices. All these features are described by the LF RQM [3, 4, 29] assuming that the $N(1440)^{1/2+}$ is the first radial excitation of a three-quark (3q) ground state.

In Fig. 5, we present the comparison of the LF RQM predictions with experimental data for the helicity asymmetry $A_{hel} \equiv (A_{1/2}^2 - A_{3/2}^2)/(A_{1/2}^2 + A_{3/2}^2)$ for the $\gamma^* p \rightarrow N(1520)^{3/2-}$ transition. The data, as well LF RQM predictions, show the rapid helicity switch from the dominance of the $A_{3/2}$ amplitude at the photon point to the dominance of $A_{1/2}$ at $Q^2 > 1$ GeV$^2$. Such behavior was predicted by the nonrelativistic quark models with harmonic oscillator potential [31, 32]. It is reproduced also by the LF RQM.

The approximation of the single quark transition model [33–36] leads to selection rules, which for the resonance $N(1675)^{5/2-}$ result in the suppression of the amplitudes $A_{1/2}(Q^2)$ and $A_{3/2}(Q^2)$ on the proton. According to our results, relativistic effects violate this suppression weakly, and we expect that experimental...
Fig. 6 The $\gamma^*p \to N(1535)\frac{1}{2}^-$ transition helicity amplitudes (in units of $10^{-3}$ GeV$^{-1/2}$). The solid curves are the LF RQM predictions. The solid circles are the amplitudes extracted from CLAS pion electroproduction data [10], the open circles are the amplitudes extracted from CLAS $\eta$ electroproduction data [22,23], the solid triangles are the amplitudes extracted from JLab/Hall C $\eta$ electroproduction data [24,25]. The solid box at $Q^2 = 0$ is the RPP estimate [21]

Fig. 7 The $\gamma^*p \to N(1675)\frac{5}{2}^-$ transition helicity amplitudes (in units of $10^{-3}$ GeV$^{-1/2}$). The solid curves are the LF RQM predictions, the dashed curves present inferred meson-baryon contributions, the dashed-dotted curves are absolute values of the predicted meson-baryon contributions from the dynamical coupled-channel approach of Ref. [26], and the dotted curves correspond to quark model predictions of Ref. [27]. The solid circles are the amplitudes extracted from CLAS pion electroproduction data [10]. The solid box at $Q^2 = 0$ is the RPP estimate [21]

values of these amplitudes should be dominated by the meson-baryon contributions. In contrast with the proton, the quark core contributions to the electroexcitation amplitudes on the neutron for the $N(1675)\frac{5}{2}^-$ are not suppressed and are predicted to be large. In both cases, for the proton and neutron, similar predictions have been obtained in the quark model of Ref. [27].

The meson-baryon contributions presented in Figs. 2, 3, 4, 6, 7 are inferred from the difference of the LF RQM predictions and the data. Most of these contributions have a clear peak at $Q^2 = 0$, except for the $A_{1/2}(Q^2)$ amplitude of $N(1520)\frac{3}{2}^-$ and for the $S_{1/2}(Q^2)$ amplitude of $N(1535)\frac{1}{2}^-$. Such pronounced peaks are also characteristic for the meson cloud contributions in the coupled-channels analyses [26,37]. Concerning the $A_{1/2}(Q^2)$ amplitude for the $N(1520)\frac{3}{2}^-$, we mention that in all coupled-channels analyses the results for the meson cloud contribution are by order of magnitude and $Q^2$ dependence very similar to our result.

At the photon point $Q^2 = 0$, the inferred meson-baryon contributions for the $N(1400)\frac{1}{2}^+$, $N(1520)\frac{3}{2}^-$, $N(1535)\frac{1}{2}^-$, and $N(1675)\frac{5}{2}^-$ can be found on the proton and neutron using the RPP (Review of Particle Physics) estimates [21]. According to our results presented in Table 1, for these resonances the inferred meson-baryon contributions at the photon point are dominated by the isovector component.
Table 1 Transverse transition helicity amplitudes at $Q^2 = 0$ for several states for proton and neutron (in units of $10^{-3} \text{ GeV}^{-1/2}$). The first two columns show the RPP estimates \cite{21}. Columns 3 and 4 show the inferred meson-baryon contributions obtained by subtracting the values obtained in the LF RQM and those from experimental data. The quoted uncertainties are from the experimental estimates.

| Resonance | $A_{1/2}$ | $A_{3/2}$ | $A_{1/2}$ | $A_{3/2}$ |
|-----------|-----------|-----------|-----------|-----------|
|           | exp. [21] | exp - LF RQM |           |           |
| Proton    |           |           |           |           |
| $N(1440)_{\frac{1}{2}^+}$ | $-60 \pm 4$ | $-31 \pm 4$ |           |           |
| $N(1520)_{\frac{3}{2}^-}$ | $-20 \pm 5$ | $140 \pm 10$ | $-17 \pm 5$ | $-174 \pm 10$ |
| $N(1535)_{\frac{1}{2}^-}$ | $115 \pm 15$ |           | $-54 \pm 15$ |           |
| $N(1675)_{\frac{3}{2}^-}$ | $19 \pm 8$ | $20 \pm 5$ | $16 \pm 8$ | $15 \pm 5$ |
| Neutron   |           |           |           |           |
| $N(1440)_{\frac{1}{2}^+}$ | $40 \pm 10$ | $12 \pm 10$ |           |           |
| $N(1520)_{\frac{3}{2}^-}$ | $-50 \pm 10$ | $-115 \pm 10$ | $20 \pm 10$ | $131 \pm 10$ |
| $N(1535)_{\frac{1}{2}^-}$ | $-75 \pm 20$ | $87 \pm 20$ |           |           |
| $N(1675)_{\frac{3}{2}^-}$ | $-60 \pm 5$ | $-85 \pm 10$ | $-13 \pm 5$ | $-23 \pm 10$ |

Acknowledgements This work was supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Contract No. DE-AC05-06OR23177, and the National Science Foundation, State Committee of Science of the Republic of Armenia, Grant No. 15T-1C223.

References

1. I.G. Aznauryan, A.S. Bagdasaryan, N.L. Ter-Isaakyan, Relativistic quark model in infinite momentum frame and static properties of hadrons. Phys. Lett. B 112, 393 (1982)
2. I.G. Aznauryan, A.S. Bagdasaryan, N.L. Ter-Isaakyan, Relativistic quark model in infinite momentum frame and static properties of hadrons. Yad. Fiz. 36, 1278 (1982)
3. I.G. Aznauryan, Electroexcitation of the Roper resonance in relativistic quark models. Phys. Rev. C 76, 025212 (2007)
4. I.G. Aznauryan, V.D. Burkert, Nucleon electromagnetic form factors and electroexcitation of low lying nucleon resonances in a light-front relativistic quark model. Phys. Rev. C 85, 055202 (2012)
5. I.G. Aznauryan, V.D. Burkert, Electroexcitation of the $\Delta(1232)_{\frac{3}{2}^+}$ and $\Delta(1600)_{\frac{1}{2}^+}$ in a light-front relativistic quark model. Phys. Rev. C 92, 035211 (2015)
6. I. G. Aznauryan, V. D. Burkert, Configuration mixings and light-front relativistic quark model predictions for $\Delta(1232)_{\frac{3}{2}^+}$, $N(1440)_{\frac{1}{2}^+}$, and $\Delta(1600)_{\frac{1}{2}^+}$, arXiv:1603.06692, (2016)
7. I.G. Aznauryan, V.D. Burkert, Electroexcitation of nucleon resonances of the $[70, 1^-]$ multiplet in a light-front relativistic quark model. Phys. Rev. C 95, 065207 (2017)
8. G.A. Miller, Light front cloudy bag model: nucleon electromagnetic form factors. Phys. Rev. C 66, 032201 (2002)
9. I.G. Aznauryan et al., CLAS Collaboration, Electroexcitation of the Roper resonance for $1.7 < Q^2 < 4.5 \text{ GeV}^2$ in $ep \rightarrow en\pi^+$. Phys. Rev. C 78, 045209 (2008)
10. I.G. Aznauryan et al., CLAS Collaboration, Electroexcitation of Nucleon resonances from CLAS data on single pion electroproduction. Phys. Rev. C 80, 055203 (2009)
11. V.V. Frolov et al., Electroproduction of the Delta resonance at high momentum transfer. Phys. Rev. Lett. 82, 45 (1999)
12. A.N. Villano et al., Neutral pion electroproduction in the resonance region at high $Q^2$. Phys. Rev. C 80, 035203 (2009)
13. J.J. Kelly et al., Recoil polarization for Delta excitation in pion electroproduction. Phys. Rev. Lett. 95, 102001 (2005)
14. S. Stave et al., Lowest $Q^2$ measurement of the $\gamma^* p \rightarrow \Delta$ reaction: probing the pionic contribution. Eur. Phys. J. A 30, 471 (2006)
15. N.F. Sparveris et al., Determination of quadrupole strengths in the $\gamma^* p \rightarrow \Delta(1232)$ transition at $Q^2 = 0.20(\text{GeV}/c)^2$. Phys. Lett. B 651, 102 (2007)
16. S. Stave et al., Measurements of the $\gamma^* p \rightarrow \Delta$ reaction at low $Q^2$: probing the mesonic contribution. Phys. Rev. C 78, 025209 (2008)
17. J. Segovia, B. El-Bennich, E. Rojas et al., Completing the picture of the Roper resonance. Phys. Rev. Lett. 115, 171801 (2015)
18. C. D. Roberts, J. Segovia, Baryons and the Bromoreno, arXiv:1603.02722 [nucl-th]
19. V.I. Mokeev et al., CLAS Collaboration, a study of the $P_{11}(1440)$ and $D_{13}(1520)$ resonances from CLAS data on $ep \rightarrow e^+\pi^+\pi^- p$. Phys. Rev. C 86, 035203 (2012)
20. V.I. Mokeev et al., CLAS Collaboration, Measurements of $ep \rightarrow e\pi^+\pi^- p$ cross sections with CLAS at 1.40 GeV. Phys. Rev. C 96, 025209 (2017)
21. C. Patrignani et al., (Particle Data Group), 2016 Review of particle physics. Chinese Physics C 40, 100001 (2016)
22. R. Thompson et al., CLAS Collaboration, The $ep \rightarrow e' p \eta$ reaction at and above the S11(1535) baryon resonance. Phys. Rev. Lett. 86, 1702 (2001)
23. H. Denizli et al., CLAS Collaboration, $Q^2$ Dependence of the S11(1535) photocoupling and evidence for a P-wave resonance in eta electroproduction. Phys. Rev. C 76, 015204 (2007)
24. C.S. Armstrong et al., Electroproduction of the S11(1535) resonance at high momentum transfer. Phys. Rev. D 60, 052004 (1999)
25. M.M. Dalton et al., Electroproduction of eta mesons in the S11(1535) resonance region at high momentum transfer. Phys. Rev. C 80, 015205 (2009)
26. B. Julia-Diaz, T.-S.H. Lee, A. Matsuyama, T. Sato, L.C. Smith, Dynamical coupled-channels effects in pion photoproduction. Phys. Rev. C 77, 045205 (2008)
27. E. Santopinto, M.M. Giannini, Systematic study of longitudinal and transverse helicity amplitudes in the hypercentral constituent quark model. Phys. Rev. C 86, 065202 (2012)
28. T. Sato, T.-S.H. Lee, Dynamical study of the $\Delta$ excitation in $N(e,e'\pi)$ reactions. Phys. Rev. C 63, 055201 (2001)
29. S. Capstick, B.D. Keister, Baryon current matrix elements in a light-front framework. Phys. Rev. D 51, 3598 (1995)
30. I.G. Aznauryan, V.D. Burkert, Electroexcitation of nucleon resonances. Prog. Part. Nucl. Phys. 67, 1 (2012)
31. F.E. Close, F.J. Gilman, Helicity structure of nucleon resonance electroproduction and the symmetric quark model. Phys. Lett. B 38, 541 (1972)
32. R. Koniuk, N. Isgur, Baryon decays in a quark model with chromodynamics. Phys. Rev. D 21, 1868 (1980)
33. A.J.G. Hey, J. Weyers, Quarks and the helicity structure of photoproduction amplitudes. Phys. Lett. B 48, 69 (1974)
34. J. Babcock, J.L. Rosner, Single-quark transition analysis of resonance photocouplings. Ann. Phys. (N.Y.) 96, 191 (1976)
35. W.N. Cottingham, I.H. Dunbar, Baryon multipole moments in the single quark transition model. Z. Phys. C 2, 41 (1979)
36. V.D. Burkert et al., Single quark transition model analysis of electromagnetic nucleon resonance transitions in the [70, 1^-] supermultiplet. Phys. Rev. C 67, 035204 (2003)
37. B. Julia-Diaz, H. Kamano, T.-S.H. Lee, A. Matsuyama, T. Sato, N. Suzuki, Dynamical coupled-channels analysis of $p(e,e'\pi)N$ reactions. Phys. Rev. C 80, 025207 (2009)