Methods of quasiclassical Green’s functions in the theory of transport phenomena in superconducting mesoscopic structures

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Abstract. A short introduction to the theory of matrix quasiclassical Green’s functions is given and possible applications of this theory to transport properties of mesoscopic superconducting-normal metal (S/N) structures are considered. We discuss a simplified version of these equations in the diffusive regime and in the case of a weak proximity effect. These equations are used for the calculation of the conductance of different S/N structures and for analysis of kinetic phenomena in these structures. We discuss the subgap conductance measured in SIN tunnel junctions and the mechanism of a nonmonotonic dependence of the conductance of a N wire on temperature $T$ and voltage $V$, observed in an S/N structure.

Long-range, phase-coherent effects are studied in a 4-terminal S/N/S structure under conditions when the Josephson critical current is negligible (the distance between superconductors is much larger than the coherence length in the normal wire). It is shown that the Josephson effects may be observed in this system if a current $I$, in addition to a current $I_1$ in the S/N/S circuit, flows through the N electrode.

1. Introduction

The progress of nanotechnology over the last few years has made it possible to produce conducting nanostructures in which new physical phenomena have been observed. Specifically, hybrid structures consisting of superconductors (S) and normal conductors (N) have been created using metal films [2–5] or semiconductor layers [1,6–8] as the normal conductors. The transport properties of these S/N structures have turned out to be quite unusual. First, the subgap conductance (zero-bias anomaly) has been observed in SIN tunnel junctions at low temperatures ($T < 100$ mK) [1](see also [7,8]). Second, conductivity oscillations have been observed in these mesoscopic structures in a magnetic field $H$ (i.e., in structures with dimensions less than the phase-breaking length $L_\phi$). Oscillations of the con-
ductivity of the N channel appeared if the structure contained superconducting or normal loops [2–6]. Moreover, for an N channel in contact with superconductors a nonmonotonic dependence of the conductivity on the temperature $T$ and voltage $V$ has been observed at $T \ll T_c$ [5]. The main experimental facts have been explained in recent theoretical works (see review articles [23,24]). It was established that the proximity effect plays the main role in the transport properties. For example, the conductivity of an N channel in the structure shown in Fig.1 changes as a result of the contribution of the condensate induced by the proximity effect. Since the condensate is induced by both superconductors in a nonlocal manner, interference appears and a term $-\delta R \cos \varphi$, which depends on the phase difference $\varphi$ between the superconductors, arises in the resistance of the N channel [9–11]. The phase difference increases with the magnetic field $H$, and this results in oscillations of the conductivity of the N channel in a magnetic field. The nonmonotonic dependence of the resistance $R$ of an N channel on $T$ and $V$ has also been explained [12–15] (see also the theoretical works in the Conference Proceedings in Ref. [14]). The nonmonotonic dependence of the resistance $R(T,V)$ of a point contact ScN (c is a constriction) was first obtained theoretically in Ref. [16].

New effects have also been predicted in theoretical work devoted to S/N structures. For example, in Refs. [14,17] it was shown that the critical Josephson current $I_c$ in a structure of the type displayed in Fig.1 depends on the voltage $V_S$ between the S and N conductors, changing sign ($\pi$-contact) if $V_S$ exceeds a certain value. In addition, it has been shown that the Josephson effect also arises in the case when current flows only through one S/N boundary. Several different configurations of S/N structures were studied in Ref. [18], determining that under certain conditions the current-voltage characteristics of the S/N structures have descending segments ($dI/dV < 0$).

An important circumstance was noted in Ref. [19] (see also the works in Ref. [14]). It was shown that the local conductivity of an N channel changes over distances from the S/N boundary which can be much greater than the coherence length $\xi_N = \sqrt{D/2\pi T}$ ($D$ is the diffusion coefficient). Important consequences follow from this fact. For example, phase coherence effects in the conductivity of an N channel remain even if the distance $2L_1$ between the superconductors is much greater than $\xi_N$. This means that the conductivity oscillations in the structure shown in Fig.1b will also be observed in the case of a negligibly low critical current $I_c$. The oscillation conservation effect is due to fact that as $T$ increases, $I_c$ decreases exponentially ($I_c \sim \exp(-2L_1/\xi_N(T))$, and $\delta R$ decreases slowly ($\delta R \sim T^{-1}$) [20,35].

In these lectures we discuss briefly the method of quasiclassical Green’s functions and apply this method to the study of transport phenomena in mesoscopic S/N structures. We restrict ourselves to the dirty limit where the mean free path $l$ is essentially less than geometric dimensions of the system and the coherence length, but exceeds significantly the Fermi wave length $k_F$ (quasiclassic approximation). We will consider mostly a weak proximity effect, when the amplitude of the condensate induced in the normal metal is small compared to the condensate amplitude
in the superconductors S. This case occurs if the S/N interface resistance is larger than the resistance of the normal conductor N. Results obtained for this case remain qualitatively valid in case when these resistances are comparable.

In the next Section we present the main necessary equations for the Green’s functions and a general expression for the current in the N channel in which a condensate is present due to proximity effect. In Section 3 we will give formulas describing the subgap conductance of tunnel S/I/N junctions and discuss a possible physical interpretation of this conductance. In Section 4 we will consider the conductance of a N channel attached to two superconductors and obtain a formula describing, in particular, the oscillatory behaviour of the conductance in an applied magnetic field \( H \). Also a nonmonotonic dependence of the conductance on temperature \( T \) and on bias voltage \( V \) will be analysed. The possibility of observing Josephson-like effects in a S/N/S mesoscopic structure (see Fig.1b) will be considered in Section 5. We will show that zero voltage between superconductors may exist in some interval of the current through the S/N interfaces and Shapiro-steps may be observed even in absence of the real Josephson coupling between superconductors when the distance figure 1.

**FIGURE 1.** Schematic diagram of the system considered
separating superconductors exceeds essentially the coherence length
\[ 2L_1 \gg \xi_N(T) = \sqrt{D/2\pi T}. \] (1)

It is important that these effects arise only in the case when a current \( I \) flows along the N channel and the dissipation takes place [21,35].

### 2. Basic equations for quasiclassical Green’s functions

The Green’s function technique is a powerful tool for the theoretical study of different phenomena in superconductors and superconducting structures. In the case of superconducting systems, we need to introduce condensate Green’s functions of the type \( <\psi^\uparrow(1)\psi^\downarrow(2)> \), therefore all the Green’s functions have a matrix form. For example, the retarded (advanced) Green’s functions are defined as follows [22,23]

\[ \hat{G}^{R(A)} = \pm \theta(t_1(2) - t_2(1)) \left( \hat{G}^> (1, 2) - \hat{G}^< (1, 2) \right) \] (2)

Here \( \hat{G}^< \) and \( \hat{G}^> \) are

\[ \hat{G}_{\alpha\beta}^> = -i <\psi^\alpha(1)\psi^\downarrow(2) > (-1)^{\alpha+1}, \]
\[ \hat{G}_{\alpha\beta}^< = i <\psi^\uparrow(2)\psi^\alpha(1) > (-1)^{\alpha+1} \] (3)

We introduced here spin indices in the Nambu-space: \( \psi^\uparrow(1) = \psi^\uparrow(1), \psi^\downarrow(1) = \psi^\uparrow(1) \). As is well known, the functions \( \hat{G}^{R(A)} \) describe the excitation spectrum of the system. In order to describe nonequilibrium processes, one needs to know the distribution functions which are related to the Green’s function \( \hat{G} \) introduced by Keldysh. It is convenient to define a supermatrix \( \hat{G}(1, 2) \) elements of which are the matrices \( \hat{G}^{R(A)} = \left( \hat{G}(1, 2) \right)_{11,(22)} \) and \( \hat{G} = \left( \hat{G} \right)_{12} \). The element \( (\hat{G})_{21} \) is the zero matrix.

In the quasiclassical approximation all components of the Green’s functions \( \hat{G}_{ex}(1, 2) \) are integrated over the variable \( \xi_p = (p - p_F) v_F \) and in the corresponding equation for \( \hat{G}_{ex}(1, 2) \) an expansion is carried out in the parameters \( (p_Fd)^{-1}, (p_FL)^{-1} \) or \( (p_F\xi_N)^{-1} \), where \( d \) is the thickness of the S or N films, \( \xi_N \) is the coherence length in the N conductor and \( l \) is the mean free path. The quasiclassical Green’s functions are defined by the relation [22,23]

\[ \check{G}(\vec{p}/p, \vec{r}; t_1, t_2) = (i/\pi) \int d\xi_p \check{G}_{ex}(\vec{p}, \vec{r}; t_1, t_2) \] (4)

The subscript ”ex” means exact (nonquasiclassical) Green’s functions. Therefore the quasiclassical Green’s functions \( \check{G}(1, 2) \) depends on the angle of momentum on the Fermi surface, on the coordinate \( \vec{r} \), and on two times.

In what follows we need an equation for the supermatrix \( \check{G}(1, 2) \) only in the N conductor, having the form

\[ D \nabla \left( \check{G} \nabla \check{G} \right) + i\epsilon [\hat{\sigma}_z, \check{G}] = 0. \] (5)
where $\hat{\sigma}_z$ is a diagonal supermatrix elements of which are the Pauli matrix $\hat{\sigma}_z$. Eq.(5) may be averaged over the thickness of the N film $d$. Performing the averaging, we obtain
\[
D \partial_x \left( \hat{G} \partial_x \hat{G} \right) + i \epsilon \left[ \hat{\sigma}_z, \hat{G} \right] = \epsilon_b \theta(x_S) \left[ \hat{G}_S, \hat{G} \right].
\]
(6)

where the coefficient $\epsilon_b$ is a characteristic energy which is proportional to the transmission of the S/N boundary: $\epsilon_b = \rho D/2 R_b \Delta$, $R_b \Delta$ is the resistance of a unit area of the S/N boundary; $\rho$ and $d$ are the resistivity and thickness of the N film.

When deriving Eq. (6), the boundary condition
\[
D \left( \hat{G} \partial_z \hat{G} \right) = (\epsilon_b d_N) \left[ \hat{G}, \hat{G}_S \right]
\]
(7)

was used. Here the $z$-axis is normal to the plane of the S/N interface. The boundary conditions for the quasiclassical Green’s functions $\hat{G}$ have been derived in the general case by Zaitsev [25] and have been reduced to the simple form (7) by Kupriyanov and Lukichev [26] in the dirty case. In the case of a good S/N contact, condition (7) is reduced to the continuity of the Green’s functions at the S/N interface: $\hat{G} = \hat{G}_S$. In the case of a poor contact ($\epsilon_b \to 0$), condition (7) gives the same result for the current through the S/N interface as obtained with the aid of the tunneling Hamiltonian method. However, for a S/N contact with an arbitrary barrier transparency condition (7) is not applicable. The point is that when deriving Eq. (7) Kupriyanov and Lukichev [26] restricted themselves to the Legendre polynomials of the zeroth and first orders in the expansion of the angle-dependent Green’s function $\hat{G}$. Meanwhile, one can easily show that all the Legendre harmonics are excited near the S/N (or N/N') interface. They decay to zero (except the Legendre polynomials of the zeroth and first order) over the mean free path away from the interface. In order to obtain a correction of the next order in $\epsilon_b$ to condition (7), one has to solve an integral equation (see Ref. [27]). In the case of the S/N interface with an arbitrary barrier transparency, the problem of boundary conditions for the quasiclassical Green’s functions becomes complicate (boundary conditions and their applicability are discussed in detail in the Raimondi’s Lecture Notes).

Eq.(6) must be solved in the normal conductor for a particular geometry (see, for example, Fig.1) with boundary conditions at $x = \pm L$. In the case of normal reservoirs the condensate functions $\hat{F}^{R(A)}(\pm L)$ are equal to zero and $\hat{G}^{R(A)}(\pm L) = \pm \hat{\sigma}_z$. In the case of superconducting reservoirs the boundary conditions for the retarded (advanced) Green’s functions are
\[
\hat{G}^{R(A)}(\pm L) = G^{R(A)}(\pm L) \hat{\sigma}_z + \hat{F}^{R(A)}_{S \pm},
\]
(8)

where $G^{R(A)} = \epsilon / \xi^{R(A)}$, $\xi^{R(A)} = (\pm i \hat{\sigma}_x \sin \phi + i \hat{\sigma}_y \cos \phi)$, $\xi^{R(A)} = \sqrt{(\epsilon \pm i \Gamma)^2 - \Delta^2}$, $\Gamma$ is a damping in the excitation spectrum in superconductor, $2\phi$ is the phase difference between superconductors. Eq.(8) is valid if the voltage
between superconductors $2V$ is much less than $\Delta/e$. We assume that there is no barrier between the N conductor and reservoirs.

The Keldysh function $\hat{G}$ describes the kinetic properties of the system and is related to distribution functions

$$\hat{G} = \hat{G}^R \hat{f} - \hat{f} \hat{G}^A$$

where $\hat{f} = f_o \hat{1} + f \hat{\sigma}_z$ is a matrix distribution function. The function $f_o$ enters an expression for the supercurrent (in a superconductor it determines the energy gap), and the function $f$ determines the quasiparticle current (in a superconductor it describes the charge-imbalance and the electric field; see, for example [28]). In reservoirs the functions $f_o$ and $f$ are supposed to have equilibrium form

$$f_o(\pm L) = \left[\tanh((\epsilon + eV)\beta) + \tanh((\epsilon - eV)\beta)\right]/2$$

$$f(\pm L) \equiv \pm F_N(\epsilon) = \pm \left[\tanh((\epsilon + eV)\beta) - \tanh((\epsilon - eV)\beta)\right]/2,$$

where $\beta = (2T)^{-1}$.

If the functions $\hat{G}^R$ and $\hat{G}$ are known, one can easily find a relation between the applied voltage $2V$ and the current $I$ in the N conductor. The expression for the current is [22,23]

$$I = (\sigma d/8) Tr \hat{\sigma}_z \int d\epsilon (\hat{G}^R \partial_{\epsilon} \hat{G} + \hat{G} \partial_{\epsilon} \hat{G}^A)$$

Eqs.(6) and (12) can be simplified significantly in the case of a weak proximity effect when the amplitudes of the condensate functions in the N conductor $\hat{F}_R$ are small. Then the retarded (advanced) Green’s functions in the N conductor have the form

$$\hat{G}^R = G^R \hat{\sigma}_z + \hat{F}_R$$

where $G^R \approx \pm [1 + (F^R)^2/2]$ and $|F^R| \ll 1$. When obtaining the relation between $\hat{G}^R$ and $\hat{F}_R$, we employed the normalization condition [22,23]

$$(G^R)^2 \hat{1} + (F^R)^2 = \hat{1},$$

where $\hat{1}$ is a unit matrix.

The equation for the condensate functions $\hat{F}_R$ follows from Eq.(6) and the expression (14)

$$\partial_{xx} \hat{F}_R - (k^R)^2 \hat{F}_R = -k_b^2 w \left[ \hat{F}_{S+} \delta(x - L_1) + \hat{F}_{S-} \delta(x + L_1) \right]$$

where $k_b^2 = 2\epsilon_b/D$, $(k^R)^2 = (\pm 2i\epsilon + \gamma)/D$, $w$ is the width of the S/N interface in the x-direction and is supposed to be much less than $L$ and $\xi_N(T)$. We introduce here the depairing rate $\gamma$ in the N conductor which determines the phase-breaking
length \( L_\phi = \sqrt{D/\gamma} \). Once the functions \( \tilde{F}^{R(A)} \) are known from a solution of the linear equation (15), we can find the conductance of the N film.

Let us consider for example the system shown in Fig.1. Writing down Eq.(6) for the matrix element (12), we arrive at the equation outside the S/N interface

\[
D \partial_x \left[ \hat{G}^R \partial_x \hat{G} + \hat{G} \partial_x \hat{G}^A \right] + i \epsilon \left[ \hat{\sigma}_z, \hat{G} \right] = 0.
\] (16)

Multiplying Eq.(16) by \( \hat{\sigma}_z \) and calculating the trace, we obtain after the first integration

\[
(\partial_x f) \left[ 1 - m_- \right] = J(\epsilon).
\] (17)

Here \( J(\epsilon) \) is an \( x \)-independent constant and

\[
m_- = (1/8) Tr \left( \hat{F}^R - \hat{F}^A \right)^2
\] (18)

is a function which describes the condensate contribution to the N film conductance. The left side of Eq.(17) stems from the first term in the square brackets in Eq.(16) provided that the condensate functions are small. Therefore, according to Eq.(12), the current \( I \) is an integral from the ”partial current” \( J \)

\[
I = (\sigma d/2) \int d\epsilon J(\epsilon)
\] (19)

Solving Eq.(17) with boundary conditions (11), we can find a relationship between the current and voltage \( I(V) \). In the next Sections we will analyse the conductance of S/N mesoscopic systems.

3. Subgap conductance in SIN junctions

In this Section the subgap conductance in superconductor/insulator/normal metal (S/I/N) tunnel junctions will be discussed. As is well known from conventional theory for S/I/N junctions, the subgap conductance should exponentially decrease with decreasing temperature \( T \) (see, for example, Ref. [29]). However, experiments on Nb/n+InGaAs contacts have established that a peak in conductance appears at zero-bias if the temperature becomes low enough \( (T \ll \Delta) \), and the magnitude of this peak at low temperatures \( (T \approx 50 \text{ mK}) \) is comparable with the conductance in the normal state [1]. This contact can be considered as a tunnel S/I/N junction. A Schottky barrier at the interface plays the role of the insulating layer I. An explanation for anomalous transparency of the SIN junction at low voltages and temperatures \( (T, eV \ll \Delta) \) was suggested in Refs. [9,30–34]. According to the interpretation proposed in Ref. [32], the subgap conductance is due to a component of the current which, in the case of a SIS Josephson junction, gives the so-called interference current. This component can be presented in the form

\[
I_{int} = - (8R_b)^{-1} \int d\epsilon \cdot F_N (\epsilon, V) \left( F^R + F^A \right) \left( F_S^R + F_S^A \right),
\] (20)
where $F_N(\varepsilon, V)$ is defined in Eq. (11); $F_S^{R(A)} = \Delta / \left(\left(\varepsilon \pm i\Gamma\right)^2 - \Delta^2\right)^{1/2}$ are the condensate, retarded (advanced) Green’s functions in the superconductor. This formula can be obtained from the general expression for the current (12) and from Eq.(6) in which we have to put $\theta(x_S) = 1$. If the energy $\varepsilon$ is small, $F^R = F^A \approx -i$. In the case of a S/I/N junction, $F^{R(A)}$ are the condensate functions in the N electrode. To zero order in the barrier transmittance (i.e., in $R^{-1}$), they are equal to zero. If the proximity effect is taken into account, they differ from zero and in the case of a planar S/I/N junction they have the form (see, for example, Ref. [32])

$$F^{R(A)} = \begin{cases} \pm \varepsilon_b / (\varepsilon \pm i\gamma), & \gamma > \varepsilon_b \\ \varepsilon_b / \left[(\varepsilon \pm i\gamma)^2 - \varepsilon_b^2\right]^{1/2}, & \gamma < \varepsilon_b \end{cases}$$

(21)

This formula can be obtained from Eq.(6) in the case of weak and strong proximity effect. It is seen from Eq.(21) that $F^{R(A)}$ are small if $\varepsilon_b \ll \varepsilon, \gamma$, where $\varepsilon$ is determined either by temperature $T$ or voltage $V$. In the opposite limit when $\varepsilon$ and $\gamma$ are small compared with $\varepsilon_b$, $F^{R(A)}$ are not small, and the differential conductance normalized by $R^{-1}$, $S = R_b dI/dV$, calculated from Eq.(20) for $T=0$ and $V=0$ is not small either. The integrand in Eq.(20) is not zero if $|\varepsilon| < \Delta$ because $F^R = F^A$ at $|\varepsilon| < \Delta$ and $F^R = -F^A$ at $|\varepsilon| > \Delta$. This means that the current given by Eq.(1) is caused by the charge-transfer mechanism of the same type as Andreev reflection processes. The second important circumstance leading to the subgap conductance is related to an anomalous proximity effect when the amplitude of the condensate functions $F^{R(A)}$ at small energies $\varepsilon$ is not small.

The density-of-states (DOS) in the N electrode is changed drastically in the case of the strong proximity effect: the DOS is zero at $|\varepsilon| < \varepsilon_b$ and the DOS = $\varepsilon / \sqrt{\varepsilon^2 - \gamma^2}$ in the interval $\Delta \gg \varepsilon \gg \varepsilon_b$. In a one-dimensional S/I/N junction the DOS has a quasigap at $|\varepsilon| < \varepsilon_b$. In both cases of a planar or one-dimensional S/I/N junctions the zero-bias, zero-temperature conductance coincides with it’s value in the normal state [32].

4. Conductance of the Andreev interferometer

Consider the system shown in Fig.1 (the Andreev interferometer). In order to calculate the normalized differential conductance of the N channel $S = R_N dI/dV$ in the presence of a phase difference between superconductors, we must solve Eq.(17) taking into account the boundary condition (11). The function $m_-$ is small by assumption. Therefore the expression for $J$ may be presented in the form

$$J(\varepsilon) = F_N(\varepsilon, V) \left[1 - \langle m_- \rangle\right] / L$$

(22)

where $\langle m_- \rangle = (1/L) \int_0^L dx \cdot m_-^2$. Substituting (22) into Eq.(19), we can obtain an expression for $S$. Here we present the formula for a deviation of the normalized differential conductance from it’s value in the normal state $R_N$: $\delta S = (2L/\sigma d) dI/dV - 1$. We obtain
\[
\delta S = -\frac{1}{2} \int d\epsilon \cdot \beta \cdot F_N' (\epsilon, V) \langle m_- \rangle ,
\]
(23)

where \( F_N' (\epsilon, V) = \left[ \cosh^{-2} (\epsilon + eV) \beta + \cosh^{-2} (\epsilon - eV) \beta \right] / 2 \). By virtue of the definition of \( \langle m_- \rangle \) (see Eqs.(18)), we can write \( \langle m_- \rangle \) in the form

\[
\langle m_- \rangle = Tr \left\{ \left( \hat{F}^R \right)^2 + \left( \hat{F}^A \right)^2 - 2 \hat{F}^R \hat{F}^A \right\} / 8 \]
(24)

The first two terms in (24) determine a change in the DOS of the N channel due to the condensate (this term reduces the conductance), and the last, so-called anomalous, term leads to an increase of the conductance.

As it is seen from Eq.(23), in order to find the conductance, we need to solve Eq.(15) for the condensate functions \( \hat{F}^R (A)(x) \). In this Section we present here the solution for the geometry shown in Fig.1a.

\[
\hat{F}^R (A)(x) = i \hat{\sigma}_y F^R_S (r \left[ \theta^R (A) \cosh \theta^R (A) \right]^{-1} \sinh \left[ k^R (A) (L - |x|) \right] \cdot \cos \varphi
\]
(25)

Here \( r = k_0^2 Lw \) is the ratio of the N channel resistance to the S/N resistance, \( \theta^R (A) = k^R (A) L \). Calculating \( \langle m_- \rangle \) we find

\[
\langle m_- \rangle = (r^2 / 8) \left\{ \text{Re} \left[ \sinh (2\theta) / (2\theta - 1) \right] \left[ \theta \cosh \theta \right]^2 \right. - \left. \sin \left[ (2\theta_1) / 2\theta_1 \right] / \left| \theta \coth \theta \right|^2 \right\} (1 + \cos 2\varphi)
\]
(26)

where \( \theta = \theta_1 + i\theta_2 \). The first term in (26) determines a contribution to the conductance due to a change in the DOS, and the second term is related to the anomalous term \( (\hat{F}^A \hat{F}^R) \). In Fig.2 we show the dependence \( \delta S_{DOS} (V) \) (first term contribution) and the \( \delta S_{an} (V) \) dependence (anomalous term contribution) at \( T = 0 \). It is seen that \( \delta S_{DOS} (V) \) is negative and \( \delta S_{an} (V) \) is positive. The total change in the conductance \( \delta S (V) = \delta S_{DOS} (V) + \delta S_{an} (V) \) is shown by the solid line. This quantity increases with increasing \( V \) from zero, reaches a maximum at \( V_m \approx \epsilon_L / e \), and decays to zero with further increase of \( V \) (here \( \epsilon_L = D / L^2 \) is the so-called Thouless energy).

As follows from (23) and (26), the conductance \( \delta S \) oscillates with increasing the phase difference. In Fig.2 we also plot the temperature dependence of the zero-bias conductance. Both curves, \( \delta S (V) \) at \( T = 0 \) and \( \delta S (T) \) at \( V = 0 \) are similar. Note that nonmonotonous temperature dependence of the conductance was obtained earlier in Ref. [16] where a short ScN contact was analyzed (here \( c \) means a constriction).

5. Dissipative Josephson-like effects in S/N/S structures

In this Section we discuss a possibility to observe Josephson-like effects in mesoscopic S/N/S structures (see Fig.1b) with negligible Josephson coupling between superconductors, i.e., when the inequality (1) is fulfilled [21,35]. Following the same steps as in Section 2, we obtain instead of Eq.(17)
\( (1 - m_-(x)) \partial_x f = \begin{cases} J + J_1 - J_S, & 0 < x < L_1 \\ J, & L_1 < x < L \end{cases} \) (27)

In what follows the function \( m_- \) plays the most important role.

The current on the segment \((0, L_1)\) is determined by Eq. (19) if \( J \) is replaced by \( J + J_1 \). The quantity \( J_S \), the superconducting “current”, is constant over the segment \((L_1, L)\) and \((0, L_1)\) and is equal to

\[
J_S = (1/4) \text{Tr} \sigma_z \left( \hat{F}_R \partial_x \hat{F}_R - \hat{F}_A \partial_x \hat{F}_A \right) \tag{28}
\]

The integral of \( J_S \) (28) over the energy is exponentially small if the condition (1) is satisfied. As follows from Eq. (6), the constant \( J_1 \), is related with the Green’s function and distribution function in the superconductor. It can be written in the form [11]

\[
J_1 = J_q + \tilde{J}_S, \quad J_q = \left( \rho/d \nu_N \nu_S \right) \left[ F_S(\epsilon) - f(L_1) \right] \tag{29}
\]

Here \( R_b = R_{b\Omega}/w \left[ \nu_N \nu_S + (1/8) \text{Tr} \left( \hat{F}_R + \hat{F}_A \right) \left( \hat{F}_S^R + \hat{F}_S^A \right) \right]^{-1} \) is the resistance of the S/N boundary per unit length in the \( y \) direction and \( \nu_N, \nu_S \) are the density of states in the N and S conductors. It can be shown that for \( V_{N,S} \) which are small compared with \( T/e \), the “supercurrent” \( \tilde{J}_S \), flowing throw the S/N boundary equals \( J_S \). The distribution function \( F_S \) is the equilibrium function, i.e., it is identical to the function in Eq. (11), if \( V_N \) is replaced by \( V_S \) (we measure voltages from the point 0, where the voltage is equal to zero). Using the fact that \( m_- \) is small, we can integrate Eq. (27) and find the relation of \( J \) and \( J_q \) with \( F_N \) and \( F_S \) (see the boundary condition (11)). We obtain the normal currents

\[
(d_N/\rho) J = \frac{R_b F_N + R_1 (F_N - F_S)}{R_b R + R_1 R_2},
\]

\[
(d_N/\rho) J_1 \approx J_q (d_N/\rho) = \frac{R_2 F_S + R_1 (F_S - F_N)}{R_b R + R_1 R_2} \tag{30}
\]

**FIGURE 2.** Normalized conductance \( \delta S \) vs. normalized voltage \( eV/\epsilon_L \) at \( T = 0 \) and vs. normalized temperature \( T/\epsilon_L \) at \( V = 0 \) for the structure shown in Fig.1a.
Here $\Re_b$ is determined in Eq. (29); the quantity $\Re = \Re_1 + \Re_2$, $\Re_{1,2} = R_{1,2} (1 + \langle m_- \rangle)$ can be termed the partial resistance. The spatial average $\langle m_- \rangle_{1,2}$ on the segments $(0, L_1)$ and $(L_1, L)$ gives a decrease in the resistances on account of proximity effect ($\langle m_- \rangle$ is negative). All resistances in Eq. (30) depend on the difference of the phases $\varphi$ and on the energy; they can be represented in the form $\Re_b = R_b - \delta \Re_b \cos \varphi$ and $\Re_{1,2} = R_{1,2} - \delta \Re_{1,2} \cos \varphi$. The corrections to the resistances $\delta \Re_b$ and $\delta \Re_{1,2}$ are small in the case of a weak proximity effect. The quantities $\Re_b$ and $R_{1,2}$ depend, generally speaking, on the energy $\epsilon$ (for example, $\nu_S$ depends on $\epsilon$). We assume, for simplicity, that these quantities do not depend on the energy. This is valid if it is assumed that the superconductors are gapless (the results remain qualitatively the same in the case of superconductors with a gap). Then, integrating Eq. (30) over energies, we obtain on the left-hand side the currents $I$ and $I_1$ (see Eq. (19)).

Eliminating $V_N$ from the two equations obtained, we find for $V_S$

$$ V_S = \frac{\hbar \partial t \varphi}{2 e} = I_1 \left[ R_b + R_1 - (\delta R_b + \delta R_1) \cos 2 \varphi \right] + I \left( R_1 - \delta R_1 \cos 2 \varphi \right) $$

(31)

Here we employed the Josephson relation; $R_b$ is the resistance of the S/N boundary, which in the case of zero-gap superconductors is approximately equal to its value in the normal state. The resistance $R_1$ is also approximately equal to $\rho L_1/d_N$ (the $\varphi$-independent correction arising from $\langle m_- \rangle$ is small and unimportant). Integrating Eq. (31), we obtain a relation between the average voltage $V_S$ and the fixed currents $I$ and $I_1$.

$$ V_S = \sqrt{[(I + I_1) R_1 + I_1 R_b]^2 - [(I + I_1) \delta R_1 + I_1 \delta R_b]^2} $$

(32)

The function $V_S(I_1)$ is displayed in Fig.3 for different currents $I$. One can see that for $I \neq 0$ this dependence is identical to the current-voltage characteristic of a standard Josephson contact. In this case the critical current is

$$ I_c = I \frac{\delta R_1 R_b - \delta R_b R_1}{(R_b + R_1)^2} $$

(33)

Therefore $I_c$ increases in proportion to the current $I$. We shall show below that the correction $\delta R_1$ decreases slowly with increasing temperature ($\delta R_1 \sim T^{-1}$), and the correction $\delta R_b$ is small if the condition (1) is satisfied. Therefore, for $R_b \gg R_1$, we obtain $I_c \approx I \delta R_1 / R_b$. The maximum current $I$ is limited by the condition that Joule heating be small and by the condition $eV_N \simeq eIR \ll T$. In the opposite case $\delta R_1$ decreases as $V_N$ increases. If the condition (1) is not satisfied and a finite Josephson coupling exists between the superconductors, then it is easy to show that the critical current of the structure equals $I_c^* = \sqrt{I_c^2 + I_{cJ}^2}$, where $I_{cJ}$ is the critical Josephson current. An expression for $I_{cJ}$ can be easily obtained with the aid of Eq. (28). This expression is presented in Ref. [20]. The equilibrium phase difference $\varphi_0$ for $I_1 + IR_1/(R_b + R_1) = 0$ equals $2\varphi_0 = - \arcsin(I_c/I_c^*)$. 
To determine $\delta R_1$ and $\delta R_b$, it is necessary to find the condensate functions $\hat{F}^{R(A)}$.

For $|x| < L_1$ the solution of Eq. (15) has the form

$$\hat{F}^{R(A)}(x) = F^{R(A)}_S [i\hat{\sigma}_y \cos(\varphi) P_y \cosh(kx) + i\hat{\sigma}_x \sin(\varphi) P_x \sinh(kx)]^{R(A)}$$  \hspace{1cm} (34)

Here $F^{R(A)}_S$ is the amplitude of the condensate functions in the superconductors.

In the zero-gap case

$$F^{R(A)}_S = \pm \frac{\Delta}{\epsilon \pm i\gamma_S},$$

where $\gamma_S$ is the frequency of spin-flip collisions with impurities. The functions $P_x,y$ equal:

$$P_x = b \sinh \theta_2 / (\sinh \theta + b \sinh \theta_1 \sinh \theta_2),$$

$$P_y = b \sinh \theta_2 / (\cosh \theta + b \cosh \theta_1 \sinh \theta_2),$$

where $b = \rho w / (R_b \xi d N_k) k, k^{R(A)} = \sqrt{\pm 2\epsilon/D}, \theta = \theta_1 + \theta_2, \theta_{1,2} \equiv \theta_1' + i\theta_2'' = kL_{1,2}$.

Once the functions $\hat{F}^{R(A)}$ are known, the interference correction $\delta R_1$ to the resistance can be calculated:

$$\delta R_1 = -R_1 \int_0^\infty d\epsilon \beta \cdot \cosh^{-2}(\epsilon \beta) \langle m_-(x,\varphi) - m_-(x,\pi/2) \rangle_1$$  \hspace{1cm} (35)

With the aid of the expressions for $\langle m_-(x,\varphi) \rangle_1$ and for $\hat{F}^{R(A)}$ (see Eq. (34)), we find

$$\delta R_1/R_1 = \int_0^\infty d\epsilon \beta \cdot \cosh^{-2}(\epsilon \beta) M(\epsilon),$$  \hspace{1cm} (36)

where

$$M(\epsilon) = \frac{1}{8} \left\{ |F_S|^2 \left[ |P_y|^2 \left[ \sinh(2\theta_1')/2\theta_1' + \sin(2\theta_1'')/2\theta_1'' \right] - |P_x|^2 \left[ \sinh(2\theta_1')/2\theta_1' - \sin(2\theta_1'')/2\theta_1'' \right] \right] + \text{Re} F_S^2 \left[ P_y^2 \left[ \sinh(2\theta_1')/2\theta_1' + 1 \right] \right] \right\}.$$

---

**FIGURE 3.** $V_S$ versus the current $I_1$ for the following values of the current: $1 - 0$, $2 - 250 \mu A$, $3 - 500 \mu A$, $4 - 750 \mu A$, $5 - 1 mA$. Here $\delta R_1 = 0.1 R_1, R_b = 5 R_1, R_1 = 1 \Omega$. 
$-P_x^2 \left( \frac{\sinh (2\theta_1)}{2\theta_1} - 1 \right)$}. The temperature dependence of $\delta R_1$ is displayed in Fig.4. One can see that for $T > \epsilon_{L_1} = D/(2L_1)^2$ the quantity $\delta R_1$ decreases as $T^{-1}$ with increasing temperature. As noted in Refs. [15,18], the slow decrease of $\delta R_1 (T)$ is due to the so-called anomalous term $F_R F_A$ in $\langle m \rangle_1$. The special role of this term, which is nonanalytic both in the upper and lower planes of $\epsilon$, was noted in Ref. [36].

The Josephson current $I_S$ is determined by the integral of $J_S$ (28), over all energies, i.e., the integral of products of either advanced or retarded Green’s functions. It can be calculated by closing the integration contour in the upper (lower) half plane of $\epsilon$ and switching to summation over the Matsubara frequencies $\omega_n = \pi T (2n + 1)$. For such energies the functions $F_R^{(A)}$ decay exponentially over distances $k^{-1}(\omega_n) \leq \xi_n (T)$ away from the S/N boundary. Therefore the current $I_S$ will be exponentially small ($I_S \sim \exp (-2L_1/\xi_N (T))$. The function $I_S (T)$ for the structure shown in Fig.1b is presented in Ref. [20]. Similar arguments are also applicable to the calculation of $\delta R_{b1}$, since for $T < \gamma_S$ the functions $F_R^b$ and $F_A^b$ can be assumed to be equal and independent of the energy. At the same time, the function $F_R^{(A)}$, appearing in the expression for $\delta R_1$, decreases over a small (compared with $T$) energy $\epsilon_{L_1} = D/(2L_1)^2$ and makes a nonzero contribution. For such energies the characteristic decay length of $F_R^{(A)}(x)$ is of the order $L_1$, i.e., of the order of the distance between the superconductors.

In order to observe long-range Josephson effects, the critical current $I_c$ must exceed the fluctuation current $Te/h$: $I_c \gg Te/h$. On the other hand, the ordinary Josephson effect is negligible if the condition $\epsilon_{L_1} \ll T$ is fulfilled. Combining these inequalities, we obtain the condition

$$TR_{b1}R_1/ (\delta R_{b1}R_Q) \ll \epsilon_{L_1} \ll T \tag{37}$$

\[ \begin{align*}
\text{FIGURE 4. Interference correction } \delta R_1 \text{ to the resistance as a function of temperature in the case } L_1 &= 0.5L, R/R_0 = 0.4, \gamma/\epsilon_L = 100, \Delta/\epsilon_L = 30. \\
\end{align*} \]
which should be satisfied for observation of the effects under consideration. Here $R_Q = h/e^2 \approx 3k\Omega$, and we took into account that a maximal value of $I$ is determined by the relation $eIR \leq \epsilon_{L_1}$. Otherwise $\delta R_1$ decreases with increasing $I$. The first inequality of (37) means that the zeroth Shapiro step on the $I_1(V_S)$ curve is absent at $I = 0$. If the second inequality of (37) is not fulfilled, then the critical current is not zero at $I = 0$ (ordinary Josephson effect). In this case the effective critical current $I_c^*$ should first increase with increasing $I$ and then decrease when $I$ exceeds $\epsilon_{L_1}/eR$.

6. Conclusion
In conclusion we note that, as one can see from Fig.4, the correction $\delta R_1$ to the resistance of the normal channel caused by the proximity effect depends on the temperature $T$ in a nonmonotonic way: it is equal to zero at $T = 0$ (the bias voltage is zero as well), reaches a maximum at $T \approx \epsilon_{L_1}$ and decays to zero at higher $T$. Such behavior of $\delta R_1(T)$ is related, as noted in [15], to different dependencies of two contributions to $\delta R_1$ on the energy $\epsilon$. One contribution which increases the N channel resistance is connected with a decrease of the density-of-states in the normal channel, which is described by the last term in $M(\epsilon)$ (see Eq. (36)). Another contribution (anomalous) which diminishes the resistance of the normal channel is described by the first two terms in $M(\epsilon)$. This contribution exactly compensates a contribution due to a change in the density-of-states of the normal channel at $\epsilon = 0$ and dominates at $\epsilon \neq 0$. At $T > T_c$ it leads to the Maki-Thompson contribution to the paraconductivity. Mathematically, compensation of the two contributions at $\epsilon = 0$ arises because at $\epsilon = 0$ $F^R = F^A$ and $m_-$ in Eq. (35) tends to zero. The nonmonotonic behavior of $\delta R$ has been observed in an experiment [5]. It would be interesting to observe the long-range Josephson effect experimentally.

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