COSMIC-RAY PARALLEL AND PERPENDICULAR TRANSPORT IN TURBULENT MAGNETIC FIELDS

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Received 2013 July 4; accepted 2013 October 17; published 2013 December 3

ABSTRACT

A correct description of cosmic-ray (CR) diffusion in turbulent plasma is essential for many astrophysical and heliospheric problems. This paper aims to present the physical diffusion behavior of CRs in actual turbulent magnetic fields, a model of which has been numerically tested. We perform test particle simulations in compressible magnetohydrodynamic turbulence. We obtain scattering and spatial diffusion coefficients by tracing particle trajectories. We find no resonance gap for pitch-angle scattering at 90°. Our result confirms the dominance of mirror interaction with compressible modes for most pitch angles, as revealed by the nonlinear theory. For cross-field transport, our results are consistent with normal diffusion predicted earlier for large scales. The diffusion behavior strongly depends on the Alfvénic Mach number and the particle’s parallel mean free path. We, for the first time, numerically derive the dependence of $M_A^4$ for the perpendicular diffusion coefficient with respect to the mean magnetic field. We conclude that CR diffusion coefficients are spatially correlated to the local turbulence properties. On scales smaller than the injection scale, we find that CRs are superdiffusive. We emphasize the importance of our results in a wide range of astrophysical processes, including magnetic reconnection.

Key words: cosmic rays – diffusion – magnetohydrodynamics (MHD) – turbulence

1. INTRODUCTION

Astrophysical plasma is generally turbulent due to the large spatial scales involved. Propagation of cosmic rays (CRs) in turbulent magnetic fields plays a key role in understanding many important issues both in space and astrophysics, e.g., solar modulation of CRs, CR acceleration, positron transport, CR anisotropy, and diffuse $\gamma$-ray emission (see Jokipii & Parker 1969; Yan et al. 2012). However, CR diffusion in a turbulent medium is still not fully understood. Current models of CR propagation are often developed by fitting observational data. The conventional assumption is that CR diffusion is isotropic and spatially homogeneous, but this oversimplified assumption faces major problems in interpreting observations. In addition to the conventional problems, such as the ratio of boron to carbon, mounting observational evidence challenges the traditional models of propagation. Examples include inconsistency between the EGRET data and locally measured spectra of CRs (Strong et al. 2004), and diffuse $\gamma$-ray excess in the inner Galaxy (Ackermann et al. 2012). All of these observations imply that a spatially dependent diffusion may hold the key. Additional effects of turbulence on CR transport are discussed in some recent works based on phenomenological arguments (Evoli et al. 2012; Tomassetti 2012).

CR diffusion depends on the turbulent magnetic fields adopted. Recent advances in turbulence studies necessitate corresponding revisions in CR transport theory. As revealed earlier, CR transport in tested models of turbulence is very different from the earlier paradigm and is indeed inhomogeneous and can be anisotropic (Yan & Lazarian 2002, 2004, 2008, hereafter YL02, YL04, and YL08, respectively; see also the book by Yan & Lazarian 2012). In this paper, we shall study numerically the transport of CRs in a tested model of MHD turbulence (Goldreich & Sridhar 1995, henceforth GS95; Lazarian & Vishniac 1999; Cho & Vishniac 2000; Cho & Lazarian 2003; see review by Lazarian et al. 2009 and references therein). We employ realistic turbulent magnetic fields, directly produced by three-dimensional MHD simulations, to provide a reliable description of the diffusion process of CRs. In particular, we shall use compressible MHD turbulence as our input for the following reasons. First, turbulence in nature is compressible with finite plasma $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$. The magnetic pressure $P_{\text{mag}}$ cannot be neglected compared to the gas pressure $P_{\text{gas}}$ for most of the medium in which CRs propagate. Second, the compressible modes, in particular the fast magnetosonic modes, have been identified as the most important for CR scattering by both quasilinear theory (QLT; YL02; YL04) and nonlinear theory (NLT; YL08). Indeed, pseudo-Alfvén modes (the incompressible limit of slow modes) can contribute through the mirror interactions. This process, however, does not function for particles with small pitch angles (YL08).

Perpendicular transport is another issue that we shall concentrate on in this paper. Many astrophysical environments, including the heliosphere and our Galaxy, have a well-defined mean magnetic field. In spite of its fundamental importance, cross-field transport remains an open question. A popular concept of CR cross-field transport is subdiffusion (Kóta & Jokipii 2000; Getman & Zirin 1963; Mace et al. 2000; Qin et al. 2002; Webb et al. 2006), but it fails to reproduce the diffusion process of solar energetic particles observed in the heliosphere (Perri & Zimbardo 2009). The solar energetic particle fluxes measured at different heliocentric distances indicate a faster diffusion process perpendicular to the solar magnetic field than subdiffusion (Maclennan et al. 2001). Moreover, recent studies based on the tested GS95 model of turbulence have shown that subdiffusion does not apply, and instead CR cross-field transport is diffusive on large scales and superdiffusive on small scales (YL08; see review by Yan 2013). On the contrary, superdiffusive behavior in the direction perpendicular to magnetic field, with displacement squared proportional to the third power of the distance along the magnetic field, follows from the GS95 theory (Lazarian & Vishniac 1999; Narayan & Medvedev 2001; Lazarian et al. 2004; Maron et al. 2004; Lazarian 2006; YL08).

$^3$ Otherwise, without the magnetic field the CRs’ propagation would be ballistic, which is against what we know from observations.
This superdiffusion is important for, e.g., particle acceleration (Lazarian & Yan 2013).

In this work, we will focus on investigating the diffusion process of CRs based on the tested model of turbulence. The structure of the paper is as follows. In Section 2, we describe the turbulent magnetic fields we use. In Section 3, we perform test particle simulations in the generated magnetic fields. We investigate particle scattering and parallel diffusion processes in Section 4. In Sections 5 and 6, we present the results on particle perpendicular transport, followed by discussions and a summary in Sections 7 and 8.

2. GENERATION OF TURBULENT MAGNETIC FIELDS

We use the Cho & Lazarian (2002) code to generate isothermal compressible MHD turbulence at $512^3$ resolution. We drive the turbulence solenoidally in Fourier space with the energy injection scale $L$ equal to 0.4 cube size. The turbulence evolves on a Cartesian grid with a mean magnetic field along the $x$ direction. We set initial density and velocity fields to unity, and adopt the same initial gas pressure value for all our simulations. The total magnetic field is a sum of a uniform background component and a fluctuating component, $B = B_{\text{ext}} + b$. Initially, we have $b = 0$, and $B_{\text{ext}}$ is the only controlling parameter in our MHD simulations. By varying the external magnetic field values $B_{\text{ext}}$, we derive a data set of MHD turbulence with different Alfvénic Mach numbers. The Alfvénic Mach number is

$$M_A = \langle |v|/v_A \rangle, \quad (1)$$

where $v$ is the local velocity, $v_A = |B|/\sqrt{\rho}$ is the Alfvénic velocity, $B$ is the local magnetic field, and $\rho$ is the density. Here, $\langle \ldots \rangle$ means a spatially averaged value over all grid points.

$M_A$ describes the perturbation strength of the turbulence with respect to the mean field, and is the single parameter that characterizes the magnetic fields we use. Figures 1(a)–(c) display examples of resulting magnetic fields with the same input parameters except for different $B_{\text{ext}}$ values. These magnetic fields clearly have different structures and $M_A$ values. We divide our data into sub-Alfvénic ($M_A < 1$) and super-Alfvénic ($M_A > 1$) turbulence for the following test particle simulations.

Table 1 lists the $M_A$ values for the magnetic fields we use in this work.

3. TEST PARTICLE SIMULATIONS

After the MHD turbulence is fully developed, we use snapshots of turbulence separated by $\gtrsim$ the turnover time of the largest eddy as different magnetic field realizations. We trace the trajectories of CRs in the test particle simulations. Since the relativistic particles have speed much higher than the Alfvén speed, the magnetic field can be treated as stationary and the electric field in the turbulent plasma can be safely neglected for the study of the transport of CRs. We use the Bulirsh–Stoer method (Press et al. 1986) to trace the trajectories of test particles. The algorithm uses an adaptive time-step method and the particle energy is conserved to a high degree during the simulation.

In each time step, the magnetic fields defined on grid points are interpolated to the position of a test particle using a cubic spline routine. Given the local magnetic field $B$, the trajectory can be computed by integrating the Lorentz force on each particle,

$$\frac{du}{dt} = \frac{q}{mc} u \times B, \quad (2)$$

where $u$ is the particle’s velocity and the remaining symbols have their standard meanings. We also use periodic box boundary conditions to keep the number of test particles unchanged.

In each simulation, we release 1000 test particles with random initial positions and pitch angles through the simulation cube. The particle energy is represented by its Larmor radius, expressed as

$$r_L = \frac{u}{\Omega}. \quad (3)$$

Here, $\Omega$ is the frequency of a particle’s gyromotion,

$$\Omega = \frac{eB}{\gamma mc}, \quad (4)$$

where $\gamma$ is the particle’s gamma factor.

To examine the effects of sample size on statistics, we perform test particle simulations with different numbers of particles.
To examine the variations in numerical results arising from different magnetic field realizations, we perform test particle simulations in different magnetic field realizations with a constant $M_A$. Figure 3(a) shows the $D_\perp$ results derived from four snapshots of the magnetic field with an average $M_A$ value equal to 0.54 (black circles) along with the results from other magnetic field data using a single snapshot (gray circles). The black circles are overlapped due to the marginal difference in $D_\perp$ values. The best fit to the data (dashed line) has a slope of $4.11 \pm 0.66$ with a 95% confidence level. In Figure 3(b), we show $D_\perp$ averaged from the four values using different magnetic field realizations and the error bar is calculated from the standard deviation. Since the error bar has a height comparable to the symbol size, we use a small black dot to exhibit the mean $D_\perp$ value. Other symbols are the same as those in Figure 3(a). The slope of the best fit changes slightly, to $4.21 \pm 0.75$ with a 95% confidence level. We can clearly see that different realizations of magnetic fields only induce marginal differences in the results. So we can safely neglect this effect in our statistical analysis.

Note that it is not possible to generate turbulence data with exactly the same $M_A$ because of statistical fluctuations.

We show the results for the perpendicular diffusion coefficient $D_\perp$ (in units of $\Omega^{-1}$) in Table 2 as an example. We will discuss the measurement for $D_\perp$ in detail below. For magnetic fields with different $M_A$, $D_\perp$ will always become stable when the sample size reaches $\sim 1000$. Our tests show that the statistics do not depend on the sample size when test particle numbers are equal to (or larger than) 1000. Thus, we use 1000 as our sample size in the following test particle simulations.

Figures 2(a) and (b) show sample particle trajectories in three-dimensional turbulent magnetic fields with different $M_A$. Obviously, particle diffusion strongly depends on the properties of the turbulence.

Table 2

| Sample Size | $M_A = 0.30$ | $M_A = 0.54$ | $M_A = 0.73$ |
|-------------|--------------|--------------|--------------|
| 100         | $1.25e-6$    | $1.80e-5$    | $1.22e-4$    |
| 500         | $1.56e-6$    | $1.70e-5$    | $1.31e-4$    |
| 1000        | $1.59e-6$    | $1.76e-5$    | $1.31e-4$    |
| 1500        | $1.62e-6$    | $1.77e-5$    | $1.28e-4$    |
| 2000        | $1.61e-6$    | $1.76e-5$    | $1.29e-4$    |

Figure 2. Particle trajectories (thick black lines) in (a) sub-Alfvénic turbulence with $M_A = 0.30$ and (b) super-Alfvénic turbulence with $M_A = 1.5$. The thin gray lines display the magnetic field stream lines.

Figure 3. $3D_\perp/\Omega$ as a function of $M_A$. The dashed line shows the best fit to the numerical results (filled circles). (a) Black circles are the results for different snapshots of one magnetic field data set, and gray circles are the results using a single snapshot of the magnetic field. (b) Same as (a), except that the small dot at $M_A = 0.54$ is the average value of the four data points (black circles in (a)). The error bar indicates the standard deviation of the four values.
4. PITCH-ANGLE SCATTERING AND PARALLEL DIFFUSION

We perform the scattering experiments using an ensemble of particles with a specific pitch angle at the starting point. During the scattering process, the pitch angle, i.e., the angle between the particle’s velocity vector and the local magnetic field direction, changes with time. We trace the change of the pitch-angle cosine $(\mu - \mu_0)$ in a short time interval to keep the deviation of $\mu$ small (see Beresnyak et al. 2011), and obtain the pitch-angle diffusion coefficient $D_{\mu\mu}$ using the definition

$$D_{\mu\mu} = \frac{\langle (\mu - \mu_0)^2 \rangle}{2t} \quad (5)$$

averaged over the ensemble of particles. Here, $\mu_0$ and $\mu$ are the initial and final pitch-angle cosine, respectively, and $t$ is the integration time. Figure 4 displays the measured $D_{\mu\mu}$ for particles with the same energy, $r_L = 0.03$ cube size, and different $\mu_0$ in the turbulence with $M_A = 0.54$. Error bars in Figure 4 and the following figures are associated with the variance of the Monte Carlo simulations. The fitted $D_{\mu\mu}$ curve smoothly extends from $\mu_0 = 0$ to $\mu_0 \sim 1$. Particles with a wide range of pitch angles, including 90°, are scattered due to the resonance broadening, in contrast to the QLT results. In QLT, the mirror resonance has a sharp peak at large pitch angles, but it is zero at 90° because of the discrete resonant Landau resonance condition $k_z u_L = kv_A$, where $k_z$ is the component of the wavevector $k$ parallel to the mean magnetic field, and $u_L$ is the parallel velocity component of a particle. Gyroresonance also does not function at 90°, according to its resonance condition. In NLT, nonetheless, the small gap around 90° disappears because of resonance broadening. Figure 4 also displays the pitch-angle scattering arising from QLT calculations of gyroresonance (Gyro) and transient time damping (TTD) interactions, calculated with NLT separately, and their total contribution. These results are from the analytical work predicted in YL08. It is clear from Figure 4 that our result agrees well with the prediction of NLT in YL08. Their analytical calculations show that mirror interaction dominates for large pitch angles until 90°.

The pitch-angle scattering determines the diffusion of CRs parallel to the magnetic field. By substituting $D_{\mu\mu}$ into the equation (Earl 1974)

$$\frac{\lambda_{||}}{L} = \frac{3}{4} \int_0^1 d\mu_0 \frac{u(1 - \mu_0^2)^2}{D_{\mu\mu} L} \quad (6)$$

where $u$ is particles’ velocity, we can obtain the corresponding parallel mean free path of the particles. For instance, the corresponding mean free path is $\lambda_{||} \approx 1.3$ in cube size units for the case considered in Figure 4.

To measure the parallel diffusion coefficient, we trace the particles over a long distance until we find that they enter the normal diffusion regime, i.e.,

$$\langle (\bar{x} - \bar{x_0})^2 \rangle \propto t, \quad (7)$$

where $(\bar{x} - \bar{x_0})$ is the distance measured parallel to the local magnetic field, and then we take the averaged square distance over all particles. The diffusion coefficient is calculated following the definition (Giacalone & Jokipii 1999)

$$D_{||} = \frac{\langle (\bar{x} - \bar{x_0})^2 \rangle}{2t} \quad (8)$$

Given the parallel diffusion coefficient, we compute the parallel mean free path $\lambda_{||}$ of particles directly from $D_{||}$ using the relation

$$\lambda_{||} = \frac{3D_{||}}{u} \quad (9)$$

Figure 5 displays the resulting $\lambda_{||}$ for particles with $r_L = 0.01$ cube size versus $M_A$. We find that $\lambda_{||}$ decreases with $M_A$, showing that the increased $M_A$ leads to an enhanced efficiency in particle scattering.

5 The study in YL08 shows that the difference between QLT and NLT is marginal for gyroresonance, which operates only with small-scale fluctuations, unlike TTD.
than the correlation length of turbulence \( L \), we consider space diffusion on large and small scales separately.

On large scales, similar to the parallel diffusion we described above, we observe that

\[
\langle (y - y_0)^2 \rangle \propto t,
\]

where \( y - y_0 \) represents the perpendicular distance, and

\[
D_\perp = \frac{\langle (y - y_0)^2 \rangle}{2 t}.
\]

Note that the perpendicular diffusion coefficient \( D_\perp \) is calculated across the average magnetic field in the global frame of reference.

For the super-Alfvénic turbulence, it is straightforward to see that the transport is isotropic with a uniform diffusion coefficient since there is no mean magnetic field. Thus, we focus on the sub-Alfvénic turbulence \( (M_A < 1) \).

In the sub-Alfvénic turbulence, the mean free paths of the test particles are large because of the low scattering rate (see Figure 5). Due to the limited inertial range of the current MHD simulations, \( \lambda_l \) is larger than the injection scale \( L \), even for particles with the lowest attainable energies. Thus, we consider the case of \( \lambda_l > L \), for instance, the cases of ultra high energy CRs, and the transport of high-energy Galactic CRs in small-scale interplanetary turbulence. We measure the perpendicular diffusion coefficients of particles propagating in sub-Alfvénic turbulence with different \( M_A \). Figure 6 presents \( D_\perp \) of particles with \( r_L = 0.01 \) cube size as a function of \( M_A \). The results can be fitted by a line with a slope of \( \approx 4.21 \), indicating

\[
D_\perp \propto M_A^{4.21 \pm 0.75},
\]

with a 95% confidence level. This relation is consistent with Equation (26) in YL08, and confirms the dependence of \( M_A^{4.21} \) instead of \( M_A^{2} \) scaling in, e.g., Jokipii (1966). This is exactly due to the anisotropy of the Alfvénic turbulence. In the case of sub-Alfvénic turbulence, the eddies become elongated along the magnetic field from the injection scale of the turbulence (Lazarian 2006; YL08). The result indicates that CR perpendicular diffusion depends strongly on the \( M_A \) of the turbulence, especially in magnetically dominated environments, e.g., the solar corona.

6 The relation between the concept of magnetic field lines and the particle trajectories that trace magnetic field lines is discussed in detail in Eyink et al. (2011).

Figure 6. \( 3D_\perp/Lu \) as a function of \( M_A \). The dashed line shows the best fit to the numerical results (filled circles).

Figure 7. \( \langle (\delta z)^2 \rangle^{1/2}/L \) vs. \( \delta t \cdot \Omega \) for \( M_A = 0.41 \). The vertical line denotes the time for particles to travel \( L \) along the direction of magnetic field.

6. PERPENDICULAR TRANSPORT ON SMALL SCALES

We next consider the perpendicular transport specifically at scales smaller than the injection scale \( L \). We simultaneously inject 40 beams of test particles randomly in the simulation cube. There are 20 particles in each beam. The spatial separations between their initial positions are equal to several grids, and their initial pitch angles are equal to \( 0^\circ \). All the particles have the same energy with \( r_L = 0.01 \) cube size. At each time step of the particle simulation, we measure the distances between particle trajectories as \( \delta z \) and we take the rms of this value \( \langle (\delta z)^2 \rangle \) as the perpendicular displacement in the following discussions. We use the same method described in Lazarian et al. (2004), except that here we deal with particle trajectories instead of magnetic field lines. Figure 7 shows how \( \langle (\delta z)^2 \rangle \) evolves with time. Since we focus on the diffusion behavior of particles on small scales, we trace the particles before \( \langle (\delta z)^2 \rangle \) reaches \( L \).

Since the particles in the sub-Alfvénic turbulence usually have \( \lambda_l \) larger than the injection scale in our simulations (see Section 5), we assume that particles move strictly along magnetic field lines during the simulation. We then determine the distance traveled along magnetic field lines by

\[
\delta x = u \delta t,
\]

where \( u \) is a constant velocity derived from the initial Larmor radius, and \( \delta t \) is the corresponding time. Figures 8(a) and (b) display \( \langle (\delta z)^2 \rangle^{1/2}/L \) as a function of the displacement of particles moving along the field \( |\delta z|/L \) for super-Alfvénic and sub-Alfvénic cases. The separation grows among the field lines to the 1.5 power after passing the minimum perpendicular scale of eddies \( \delta_{l_{\perp \min}} \), up to the injection scale of the strong MHD turbulence \( (l_A = L/M_A^2 \) for \( M_A > 1 \) and \( l_{tr} \sim L M_A^2 \) for \( M_A < 1 \); Lazarian 2006; YL08). Our result is also consistent with earlier studies on the separation of field lines in Lazarian et al. (2004) and Maron et al. (2004). This consistency verifies that particle superdiffusion on small scales is determined by the
Notably, in the real astrophysical world, since CRs have much smaller than the injection scale of the turbulence with sufficient amplitude. This addition will not affect the statistical properties of particle transport across the field for the following reasons. First, the small-scale resonant slab modes are uncorrelated with the original turbulence modes. Moreover, the contribution of slab modes to particle cross-field transport is sub-diffusive (see Kótá & Jokipii 2000) and therefore can be neglected.

\[
\langle \delta t \rangle \propto \frac{1}{\Omega u \delta},
\]

consistent with YL08’s predictions (see Equations (30) and (31) in YL08).

Figure 9 displays \((\delta z)^2 / |\delta x|^3\) as a function of \(M_A\). Filled circles are for previous results with constant velocity, and asterisks are for results after the correction for \(u\). The dashed lines are the best fits to the data.

\[
\frac{\langle (\delta z)^2 \rangle}{|\delta x|^3} \propto M_A^{2.58 \pm 0.64}
\]

Figure 9 displays \((\delta z)^2 / |\delta x|^3\) as a function of time in this case. For the scales \(L > |\delta x| > \lambda_{||}\), our result suggests

\[
\langle (\delta z)^2 \rangle^{1/2} \propto \delta t^{0.75},
\]

at 95% confidence. Actually, we notice that even for particles with \(\lambda_{||} \geq L\), the pitch angles change significantly, especially in cases with higher \(M_A\) values. The assumption of no pitch-angle scattering may lead to an overestimate of the parallel distances. Thus, we replace \(u\) with the parallel particle velocity

\[
\delta \cdot t \cdot \Omega
\]

and only consider a specific case with \(\lambda_{||} < L\). To study the regime \(\lambda_{||} < L\), applicable to most of the CRs, we add resonant slab fluctuations to the initial turbulent magnetic fields obtained through MHD turbulence simulations. Since the slab component is very efficient in pitch–angle scattering through gyroresonance, \(\lambda_1\) can be effectively reduced to values smaller than the injection scale of the turbulence with sufficient amplitude. This addition will not affect the statistical properties of particle transport across the field for the following reasons. First, the small-scale resonant slab modes are uncorrelated with the original turbulence modes. Moreover, the contribution of slab modes to particle cross-field transport is sub-diffusive (see Kótá & Jokipii 2000) and therefore can be neglected.

\[
\langle \delta t \rangle \propto \frac{1}{\Omega u \delta},
\]

consistent with YL08’s predictions (see Equations (30) and (31) in YL08).

Figure 10 presents the ratio \((\delta z)^2 / |\delta x|^3\) as a function of \(M_A\). The best fit to the numerical data shows

\[
\frac{\langle (\delta z)^2 \rangle}{|\delta x|^3} \propto M_A^{2.58 \pm 0.64}
\]
correction (also see Figures 8(b) and 10):

$$\langle \delta \vec{x}^2 \rangle / \langle \delta \hat{x} \rangle^2 \propto M_A^{3.84 \pm 0.78},$$  \hspace{1cm} (16)

with 95\% confidence, in good agreement with the theoretical predictions (Lazarian & Vishniac 1999; YL08).

7. DISCUSSION

The spatially dependent CR diffusion we obtain with the physically motivated model of turbulence should help resolve the current observational puzzles in relation to CR propagation in various astrophysical environments. Our conclusions on CR diffusion will contribute to a fundamental understanding of the underlying processes of non-local observables, like CR anisotropy and galactic γ-ray diffuse emission.

Instead of simulations of CR transport that employ a slab/2D composite model and synthetic turbulence data (e.g., Giacalone \\& Jokipii 1999; Qin et al. 2002; Tautz \\& Dosch 2013), we use direct 3D MHD numerical simulations to produce the turbulence data cube. The reasons are as follows. First, slab/2D composite approximation of turbulence is not supported by numerical simulations. Also, generating the numerically confirmed scale-dependent anisotropy with respect to the local magnetic field in synthetic turbulence has not yet been realized (Lazarian \\& Vishniac 1999; Cho \\& Vishniac 2000). Performing test particle simulations in turbulence that is missing essential physics does not lead to reliable results on CR diffusion.

Perpendicular diffusion of CRs across the mean magnetic field has been considered a difficult problem of particle astrophysics for a long time. We, for the first time, demonstrated numerically that perpendicular transport of CRs depends on the $M_A^4$ of the turbulence. Our numerical results can be used for a wide range of applications. On large scales, our results on perpendicular diffusion can be applied to depict the normal diffusion processes we studied in this work have important implications for other issues. Similar diffusion properties and the dependence on $M_A$ can also be applied to thermal particles and our numerical results are consistent with the analytical descriptions in Lazarian (2006). The thermal diffusion has a profound impact on problems such as cooling flows in clusters of galaxies.

As a fundamental astrophysical process, magnetic reconnection is controlled by the turbulent wandering of magnetic fields (Lazarian \\& Vishniac 1999). The diffusion behavior of field lines is essential for determining the reconnection rate in the turbulent medium. The $M_A$ dependence that we numerically confirmed in this paper for the first time can help quantitatively determine the extension degree of the outflow region and the resulting magnetic reconnection rate.

8. SUMMARY

We provide a realistic description of particle transport with test particle simulations in a tested model of compressible turbulence. Our results are, in general, consistent with the nonlinear transport theory developed in YL08 and can be summarized as below.

1. Pitch-angle scattering experiments are consistent with the NLT, showing the dominance of mirror interaction for most of the pitch angles, except for the small ones.

2. The nonlinear effect for pitch angles close to 90° has been confirmed by our simulations.

3. We have demonstrated numerically that CRs are diffusive on large scales. We show that the perpendicular diffusion coefficient depends on $M_A^4$ in the case of $\lambda_{||} > L$ in sub-Alfvénic turbulence.

4. On small scales, CRs experience superdiffusion.

S.X. and H.Y. are supported by NSFC grant AST-11073004. We acknowledge the computing support from FSC-PKU. We have benefited from valuable discussions with Blakesley Burkhart, Alex Lazarian, and Shangfei Liu. We also thank the anonymous referee for their helpful suggestions.

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7
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