Study on POT model of vehicle load under toll-by-weight mode

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Abstract. The overload and overweight operations of vehicles in expressway are particularly serious in China recently. Research on the extreme maximum value of the vehicle loads is an urgent problem in the bridge design. On the basis of statistical analysis of the WIM data from Yu-Zhan Expressway under toll-by-weight mode in Guangdong Province, the POT probability model of gross weight of the vehicle was established by the extreme value theory. It can predict the probability distribution of operating vehicle loads in any recurrence period. Results showed that the overweight vehicles were not accidental, and there may be heavier vehicles in the future. It is necessary to improve and modify the current toll-by-weight charging policy.

1. Introduction
With the continuous development of economy and society and the automobile industry in China, the growing overload and overweight transportation of operating vehicle produce significant variation. Related research and study had become one of the hot topics. In order to curb the increasingly overload situation and guarantee the traffic safety, toll-by-weight model must be implement. Nowadays, the weight of vehicle got by the Weight in Motion (WIM) technology has been successfully applied in the expressway in our country. Under this weight charging mode and the current Toll-by-Weight policy, whether the overloaded or overweight of vehicle has been effectively controlled? What are the development trends of operating vehicle loads? How can make a more scientific and rational prediction? On these issues, some researchers had made some studies and researches, but in general, the conclusions were very different. In this view, adopted the WIM data of a toll station in Yu-Zhan Expressway, the paper used the extreme value theory to establish the vehicle load model and forecasted the development trend.

2. Vehicle load statistics
Yu-Zhan Expressway is an artery highway connecting Chongqing to Zhanjiang with a length of 73 kilometers in Guangdong province, and was opened to traffic in 2005. 307172 WIM data from one vehicle lane at a toll station in Guangdong in 2013 were collected. Through statistical analysis, the numerical characteristics of each wheel vehicle were obtained, as shown in table 1 and figure 1.

Statistical analysis displayed the following. Firstly, 2-wheel vehicles accounted for up to 33.62%, followed by the 4-wheel and the 6-wheel vehicles, accounting for more than 33.57% and 27.01%; 3-wheel and 5-wheel vehicles accounted for less, with less than 5% of the total. This proportion of vehicles reflected the characteristics of the traffic in the post industrial areas. Secondly, the average value of 3–6 wheel vehicles were close to the policy provision limit value of each type, and the
maximum value were 2–3 times of the limit value, which reflected the total weight of the operation traffic load of this Expressway were so heavy.

Table 1. The numerical characteristics of each type vehicle.

| Type   | Number | Proportion (%) | Maximum Value(t) | Average Value(t) | Limit Value(t) | Standard Deviation | Coefficient of Variation |
|--------|--------|----------------|------------------|------------------|----------------|--------------------|-------------------------|
| 2-wheel| 103275 | 33.62          | 55.5             | 9.71             | 17             | 5.31               | 0.55                    |
| 3-wheel| 12874  | 4.19           | 57.9             | 22.96            | 25             | 6.69               | 0.29                    |
| 4-wheel| 103117 | 33.57          | 86.5             | 33.99            | 35             | 8.21               | 0.24                    |
| 5-wheel| 4931   | 1.61           | 96.8             | 34.47            | 43             | 14.42              | 0.42                    |
| 6-wheel| 82975  | 27.01          | 153.4            | 46.66            | 49             | 16.11              | 0.35                    |

Figure 1. Proportion of each vehicle type.

3. Principle of POT model

At present, most researchers usually use the methods of hypothesis test, analysis of the parameters of the technical route to simulate the weight distribution of the vehicle load. These trial and error analysis methods have strong subjectivity with exactly simulate the central distribution, but failing to accurately describe the tail distribution. This paper intends to use the POT model based on the extreme value analysis theory to establish the vehicle load model.

F(x) is an arbitrary distribution function of random variable sequence\{Xi\}, which is assumed to have independent identically distributed random variables, F(x) supports at the top of the point x*, and there is a value of Xi <μ < x*, called Threshold. Xi is a sequence of random variables, and \{Xi-μ\} is for out of sequence. Fμ(y) is defined as the conditional distribution function of the random variable, which can be expressed as in equation (1).

\[
F_\mu(y) = P(X - \mu \leq y | X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)} = \frac{F(x) - F(\mu)}{1 - F(\mu)}
\]

(1)

So

\[
F(x) = F_\mu(y) (1 - F(\mu)) + F(\mu)
\]

(2)

\[
G(x; \mu, \sigma, \xi) = 1 - (1 + \frac{x - \mu}{\sigma})^{-\frac{1}{\xi}}, x \geq \mu, 1 + \frac{\xi(x - \mu)}{\sigma} > 0
\]

(3)

The X is said to obey the General Pareto distribution (GPD distribution). μ ∈ R is a function of the position, σ > 0 is the scale parameter, and ξ ∈ R is the shape parameter.

If there are constants like \(a_n\) and \(b_n\), when reaching \(F(x)\) at the upper end of the point, \(F_\mu(a_n + b_n)\) have continuous limit distribution, the theorem is established.
\[
\lim \sup_{\mu \rightarrow \infty, y \in \mathbb{Y}_t} \left| F_\mu(y) - G(y; \mu, \sigma, \xi) \right| = 0
\]  
(4)

The theorem: for sufficiently large threshold \( \mu \), the majority of unknown distribution function \( F(x) \) beyond the volume distribution function \( F_\mu(y) \) available GPD distribution. \( G(y; \mu, \sigma, \xi) \) approximation is \( F_\mu(y) = G(y; \mu, \sigma, \xi) \), and the type of substitution as formula (5).

\[
F(x) = G(y; \mu, \sigma, \xi)(1 - F(\mu)) + F(\mu)
\]  
(5)

After the determination of the \( \mu \), you can get a large number of \( N_\mu \) than the threshold \( \mu \) in \( \{X_i\} \), according to the formula (5) with the frequency \((1 - N_\mu/n)\) instead of the \( F(\mu) \), you can get the expression of \( F(x) \).

\[
F(x) = F_\mu(y)(1 - F(\mu)) + F(\mu) = 1 - \frac{N_\mu}{n} \left( 1 + \frac{\xi}{\sigma}(x - \mu) \right)^{-\frac{1}{\xi}}
\]  
(6)

Through parameter estimation, \( \hat{\sigma} \) and \( \hat{\xi} \) can be estimated, then equation (7) can be deduced by equation (6).

\[
F(x) = 1 - \frac{N_\mu}{n} \left( 1 + \frac{\xi}{\sigma}(x - \mu) \right)^{-\frac{1}{\xi}}
\]  
(7)

The threshold model was used to estimate the tail distribution of the vehicle load, and the key is the choice of threshold \( (\mu) \).

### 4. Establishment of a POT model

Using 307172 WIM data of 2013 collected from one lane at a toll station of Yu-Zhan Expressway, according to the weight distribution density frequency diagram of the vehicle load, based on GPD theory, a POT model of gross weight of the vehicle was established with properly fitting the tail probability distribution.

First of all, the threshold was selected by kurtosis method, when the threshold value was 70.5t; the amount of sample and the distribution curve were moderate, which come to the best fitting effect. Then, according to the method of maximum likelihood parameter estimation, a Mat lab program was developed to estimate the GPD distribution parameter by using the GDP distribution to fit the data distribution, and the estimated results are shown in table 2. Finally, the distribution probability function of vehicle weight was obtained as equation (8).

| Table 2. Results of parameter estimate. |
|-------------------------------|---|---|---|
| threshold \( (\mu) \) | \( \hat{\sigma} \) | \( \hat{\xi} \) | \( N_\mu \) |
| 70.5 | 5.60754 | 0.028286 | 0.019754 |

\[
F(x) = 1 - 0.019752 \left[ 1 + \frac{0.028286}{5.60754}(x - 70.5) \right]^{-\frac{1}{0.028286}}
\]  
(8)

### 5. The maximum weight of a vehicle prediction

After the probability distribution model of vehicle gross weight is established, if the number of vehicle with a certain weight in a reference period is known, the return level of U(T) over a period of time and the possible maximum value W95 (guaranteed rate is 95%) can be predicted. The return level U(T) means that the weight of more than U (T) of the vehicle will appear 1 times in T years.

Based on above and the vehicle data of Yu-Zhan Expressway, the predicted value of U (T) and the W0.95 (T) in different return period were calculated, and the results were summarized in table 3.

| Table 3. The predicted return level and maximum gross weight. |
|---------------------|-------|---------------------|-------|
| Return Period (year) | Predicted Value (t) | Return Period (year) | Predicted Value (t) |
|                     | U(T)  | W0.95               | U(T)  | W0.95               |
| 5                   | 129.6 | 152.1               | 30    | 143.0               | 166.7 |
From the view of predicted results, more than 129.6 tons of heavy vehicles will appear at least once every 5 years, and a maximum of more than 152.1 tons of heavy vehicles will appear once every 5 years, the maximum weight of vehicle will reach 157.7 tons after 10 years. This was far more than the gross weight limit (49 tons) of vehicle on current highway bridge.

6. Conclusions
Based on the study of the measured data of the WIM with toll-by-weight model, some conclusions can be obtained.

(1) The average value of 3–6 wheel vehicles were close to the policy provision limit value of each type with the maximum value being 2–3 times of the limit value, which reflected the weight of the operation traffic load in the Yu Zhan Expressway were so heavy.

(2) The POT model of operating vehicle load was established by using the extreme value theory, which can properly describe the gross vehicle weight of the tail distribution characteristics. The predicted results showed that the current toll-by-weight policy had a certain inhibitory effect on overweight vehicles, but can not effectively control the variability of overweight vehicles. With the increase in the return period, there is sufficient reason to believe that more overweight vehicles would appear. Therefore, it is necessary to continue to improve and improve the policy of weighing charges. It is necessary to improve and modify the current weight charging policy in Guangdong province.

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