Misconceptions of prospective Mathematics teacher on graphing function

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Abstract. Good learning outcomes supported by a good understanding of concepts. A good understanding of the concept is an understanding that do not contain misconceptions. This study aims to identify the misconceptions of prospective mathematics teacher on graphing function. This research was conducted on 8 undergraduate students. This study used a descriptive research design. Descriptive research is used to analyse conceptual understanding and student misconceptions based on test results using diagnostic test instruments and interview results. The questions used were 5 questions that had been validated by 2 mathematics lecturers with valid results. The results showed that most of the students were still doing plotting without identify of the domain dan characteristics of the function. Some of them were also still stuck with using integers as the reference.

1. Introduction
Misconception is one of the topics in the world of education that has not been completed until now. Misconception is a condition where a concept is not properly understood by students, which is not in accordance with the understanding generally accepted by the scientific community [1]. Misconception is different from error [2]. Misconceptions can be identified as systematic errors due to misconceptions in students [3]. The errors are caused by carelessness, inaccuracy, misinterpretation of the questions, lack of experience in solving questions related to the given topic, or due to the inability to check the answers obtained [4]. Misconceptions are difficult to change, persistent and can prevent students from understanding further learning. The nature of these misconceptions shows that overcoming the misconceptions that occur is not something easy. Moreover, learning in Indonesia tends to be teacher-centered where teachers are the source of authority for knowledge.

The significant impact of misconceptions on both students and undergraduate students are decrease their learning outcomes. Based on this, these misconceptions must be identified and then searched for their roots and then propose the solution so that similar misconceptions do not occur again. Thus the major aim of the research on misconceptions in mathematics should beyond just determining them [5]. Among the several studies in mathematics, most studies were conducted on students at the first or second level, such as in Lin and Almog [6,7]. Other few of them did it at higher levels such as undergraduate students or prospective educators such as in Dzulfiqar, Green, and Baştürk [2,8,9].

Students' prior learning sometimes arises misconceptions either in the classroom or from their interaction with the social and physical world [10]. According to Taber [11], the misconceptions experienced by students come from three sources, namely the books they read, other knowledge from
their environment that they carry before entering class and also from their teachers. Therefore, it’s much more important to firstly make no misconception on the prospective teacher since if prospective teachers did not have adequate knowledge regarding the concept, some misconceptions will also emerged in their teaching process [12] and at the end the students they taught will also inherit the misconception.

Function is a topic in mathematics that has important role. This topic needs to be mastered, especially prospective mathematics teachers because this material is used in many areas of mathematics [13]. So far, to explain about functions, teachers or lecturers tend to use software applications such as Autograph Software and GeoGebra Software. The use of this technology in explaining function graphs to students really helps them understand because the software provides a detailed description. However, on the other hand it also creates problems. Students tend to memorize visible pictures so that when they are asked to describe on their worksheets, the hand-drawn drawings do not match with the existing concept.

It is important to determine and then eliminate misconceptions in mathematics since learning mathematical concepts highly depends on learning previous concepts [14]. Study of misconceptions about graph that conducted on first or second grader mostly in describing about how the students’ understanding or interpreting the graph, namely by Garcia [15]. This research is aimed to investigate the misconceptions that emerge on prospective mathematics teacher while graphing function. Although there are many researches on misconceptions in graphing function, the number of studies that conducted on prospective mathematics teachers is limited. Therefore, we tried to find an answer to the question “what kind of misconception that emerge to prospective mathematics teacher on graphing function”.

2. Methods
This study is carried out to determine prospective mathematics teachers’ misconceptions when they sketch the graph of function. Qualitative research methods are used to reach this aim. Descriptive research aims to measure systematically, factually, accurately about the facts and characteristics of the population and the sample. The subjects in this study were 8 undergraduate students on mathematics education department (prospective mathematics teacher). Those subjects were selected using purposive sampling technique that is the students had already learned differential calculus. Diagnostic tests and interviews were used in data collection. The test is given in order to find out the mathematical misconceptions that occur in prospective teacher students. This test consists of five items that have been validated by 2 experts with all valid items. Meanwhile, interviews were conducted to find out more information about the misconceptions that occurred and their causes. The data obtained in this study were analysed using an interactive model, which includes data reduction, data display, and verifying.

3. Results and findings
Prospective mathematics teacher’s misconception on graphing function is viewed from how they sketch the graph of several function. The study shows that the prospective mathematics teachers have various misconceptions on how to sketch the function graph. Some of them can be summarized as follows: 1) most of the subject were still just doing plotting without identify the characteristic of the function, and 2) some of them were also still stuck with using integers as the reference point.

To identify the misconception about graphing function, we give a diagnostic test and interview to 8 undergraduate students. The result of the diagnostic test is given in Table 1 as follow.

| Graph of          | Correct | Correct with notes | Incorrect | No Answer |
|-------------------|---------|--------------------|-----------|-----------|
| $f(x) = x + 2$    | 3 (37.5%) | 5 (62.5%)           | 5 (62.5%) | 1 (12.5%) |
| $f(x) = 1 - x^2$  | 2 (25%)  | 6 (75%)            | 5 (62.5%) | 1 (12.5%) |
| $f(x) = \frac{1}{x}$ | 2 (25%) | 2 (25%)            | 1 (12.5%) | 1 (12.5%) |
| $f(x) = \frac{x^2 + x - 2}{x - 1}$ | 2 (25%) | 2 (25%)            | 4 (50%)   |           |
| $f(x) = \sqrt{x - 1}$ | 1 (12.5%) | 1 (12.5%)     | 1 (12.5%) |           |

Table 1. Student’s results on graphing function test.
Based on Table 1 above we can observe that less than 50% students give correct answer. Especially for graph of \( f(x) = \frac{1}{x} \), there was no one of student give correct answer. This result shows that their comprehension about graphing function still low. For more analysis of each graph was described on the paragraph below.

3.1. Graph of Linear function \( f(x) = x + 2 \)
Since the function of \( f(x) = x + 2 \) is a linear function, it easily can be sketched by finding the intersection point with the axis and then draw the line through those point. However, all the students’ answer is correct. But, it’s only 3 of 8 (37.5%) as in Table 1 students who are using this method (finding intersection point with the axis). The other 5 are using plotting method with choose some integers as the input. This is not absolutely wrong method, since a line (linear function) can be drawn by at least 2 known points lay on it. But if the students use more than 2 points, its actually ineffective way. Thus, we can say that 5 of 8 students didn’t use the characteristic of the linear function.

![Figure 1](image-url)

**Figure 1.** Student’s answer on linear function.

As seen in the Figure 1, the student sketch the graph by choosing several numbers as inputs then plot them on the coordinate plane. The problem is in the choosing of those numbers as inputs or reference. The following transcripts of the interview with the students make sure that the students didn’t concern to the characteristic of linear function. But they just plotting.

| Interviewer | Why did you choose -3 to 3 as inputs? |
|------------|--------------------------------------|
| Students   | Actually, it’s arbitrary. There is no must in choosing -3 to 3. I just chose them because I think it’s well-matched negative and positive numbers and also 0 as mid number. |

3.2. Graph of quadratic function \( f(x) = 1 - x^2 \)
The quadratic function has some characteristic that can be used to sketch the graph. The graph of a quadratic function is called a parabola and has a curved shape. One of the main points of a parabola is its vertex. It is the highest or the lowest point on its graph. Quadratic function also certainly has an intersection point with \( y \)-axis and in some case with \( x \)-axis. Most of the students answer (6 of 8) on graphing quadratic function is correct but with some notes. Figure 2 and transcript below indicate that the graph is again all about plotting. However, students are aware that a quadratic function has a graph called a parabola which is a curved shape.
Figure 2. Student’s answer on quadratic function.

Interviewer: How did you do to get the graph?
Students: Same with function no.1, I chose some number as inputs then find the value of \( y \) to get the point.
Interviewer: Why the shape is like this?
Students: Yes, we all know that this is quadratic equation so the graph is parabola.

3.3. Graph of rational function \( f(x) = \frac{1}{x} \)

The function \( f(x) = \frac{1}{x} \) have several characteristics that must be emerge on the graph. To sketch the graph of this function must be noticed that the domain is not all of real number and it has asymptote.

As seen in Figure 3, students only sketch the graph with positive value of \( x \). Actually, he is already aware on the domain but he didn’t think that negative value of \( x \) is also available. About the asymptote, the students could explain that the value of \( f(x) \) tends to 0 for \( x \) tends to \( \infty \) which means the graph is getting close to x-axis but will never touch it. While in Figure 4, the student sticked on choosing integers as the inputs. The consequence is, he could only draw graph as two lines. He also can’t explain how this graph should be drawn.

3.4. Graph of rational function \( f(x) = \frac{x^2+x-2}{x-1} \)

Function \( f(x) = \frac{x^2+x-2}{x-1} \) is kindly similar to the function \( f(x) = \frac{1}{x} \) but has a bit different characteristic. The domain is restricted for a number \( x \) that is 1. The graph has no asymptote and it is like a linear
function but discontinue at \( x = 1 \). In this graphing function, 2 of 8 students have already answered correctly, but 5 students still incorrect. Some of incorrect answer given in the following figure.

![Figure 5](image1.png) ![Figure 6](image2.png)

**Figure 5.** Student’s answer indicating misconception on domain. **Figure 6.** Student’s answer indicating that its still stick on integers as inputs.

As seen in Figure 5, the student has already aware that the function is algebraically can be simplify. Therefore, the function now is like linear function. But this student forgot that simplifying the function doesn’t mean change the domain. Otherwise in Figure 6, the student is already aware that for \( x = 1 \) the function is undefined. Therefore, he drew nothing for \( x = 1 \). But the problem is nothing drawn for \( 0 < x < 2 \). From the following transcripts of the interview, it is again indicating that students still stick on the integers.

**Interviewer:** Why did you draw nothing for \( x \) from 0 to 2?

**Students:** Yes, because for \( x = 1 \) the value of the function is \( \frac{0}{0} \), and it’s undefined. So, I don’t know how to draw it.

3.5. Graph of square root function \( f(x) = \sqrt{x - 1} \)

It is easy to find that not all real numbers are available as the input. So, to sketch the graph of this function, firstly we need to find the domain. And it should be known that square root of any non-negative real number must be non-negative. There is 1 of 8 student that have answered correctly, 2 answered correctly with notes, 4 answered incorrectly and 1 give no answer.

![Figure 7](image3.png) ![Figure 8](image4.png)

**Figure 7.** Student’s answer indicating misconception on square root. **Figure 8.** Student’s answer indicating misconception on the curve and stick on the integers as reference.

From Figure 7 and Figure 8, it can be seen that both students were aware about the domain. But, Figure 7 also show us that the student forgot about the concept of square root. Other side, student’s answer on Figure 8 show that the graph is from plotting and stick to the integers as a reference.
4. Conclusion
Based on this research, we can conclude that mathematical misconceptions in prospective mathematics teachers on graphing function were still found. We found four misconception. Misconception on domain, function, sketching method and plotting procedure. The most often founded misconception is about domain part. In the procedure of plotting, it is still stick on choosing integers as reference. In the future, it needs to apply such learning model that can prevent this misconception.

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