Distinguishing between evidence and its explanations in the steering of atomic clocks

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Abstract
Quantum theory reflects within itself a separation of evidence from explanations. This separation leads to a known proof that: (1) no wave function can be determined uniquely by evidence, and (2) any chosen wave function requires a guess reaching beyond logic to things unforeseeable. Chosen wave functions are encoded into computer-mediated feedback essential to atomic clocks, including clocks that step computers through their phases of computation and clocks in space vehicles that supply evidence of signal propagation explained by hypotheses of spacetimes with metric tensor fields.

The propagation of logical symbols from one computer to another requires a shared rhythm—like a bucket brigade. Here we show how hypothesized metric tensors, dependent on guesswork, take part in the logical synchronization by which clocks are steered in rate and position toward aiming points that satisfy phase constraints, thereby linking the physics of signal propagation with the sharing of logical symbols among computers.

Recognizing the dependence of the phasing of symbol arrivals on guesses about signal propagation transports logical synchronization from the engineering of digital communications to a discipline essential to physics. Within this discipline we begin to explore questions invisible under any concept of time that fails to acknowledge unforeseeable events. In particular, variation of spacetime curvature is shown to limit the bit rate of logical communication.

Keywords: evidence, atomic clock, unpredictability, Turing machine, wave function, spacetime curvature.

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1. Introduction

While outcomes are subject to quantum uncertainty, uncertainty is only the tip of an iceberg: how can one “know” that a wave function describes an experimental situation? The distinction within quantum theory between linear operators and probabilities implies a gap between any explanation and the evidence explained\textsuperscript{1, 2, 3, 4}:

Proposition 1. \emph{To choose a wave function to explain experimental evidence requires reaching beyond logic based on that evidence, and evidence acquired after the choice is made can call for a revision of the chosen wave function.}

Because no wave function can be unconditionally known, not even probabilities of future evidence can be unconditionally foreseen. Here we show implications of the unknowability of wave functions for the second as a unit of measurement in the International System (SI), implications that carry over to both digital communications and to the use of a spacetime with a metric tensor to explain clock readings at the transmission and reception of logical symbols.

For reasons including quantum uncertainty, not even the best atomic clocks tick quite alike; they drift in frequency and position. Here we develop implications of the necessity of continually adjusting clocks in response to evidence of deviations from an aiming point, where the aiming point depends on provisional hypotheses—i.e., guesswork subject to revision as prompted by accumulated evidence. Although frequency instabilities approaching \(10^{-18}\) shrink the leeway...

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within which clock adjustments are made [5], adjustments within whatever leeway persists remain indispensable. Clocks that generate Universal Coordinated Time (UTC) are steered toward aiming points that depend on both a chosen wave function and an hypothesized metric tensor field of a curved spacetime. Like the chosen wave function, the hypothesis of a metric tensor, while constrained, cannot be determined by measured data.

Examining how guesses enter the operations of atomic clocks, we noticed ubiquitous computational machinery, operating in a rhythmic cycle. Within this machinery, hypotheses are coded into computational processes that interact in a feedback loop that responds to evidence, leading to the generation of more evidence. The machinery updates records that determine an aiming point, and so involves the writing and reading of records. The writing must take place at a phase of a cycle distinct from a phase of reading, with a separation between the writing and the reading needed to avoid a logical short circuit.

To illustrate how physical clocks depend on computational machinery, Sec. 2 sketches the operation of an atomic clock in which computer-mediated feedback steers an active oscillator in frequency. First, off line, an hypothesis about how to steer the oscillator in response to evidence of scattering of the oscillator’s radiation by one or more passive resonant atoms is developed. Then that hypothesis, though developed off line on the blackboard, so to speak, is encoded into a program in the computer memory that adjusts the oscillator on the workbench.

In Sec. 3, we picture an explanation used in the operation of a clock as a string of characters written on a tape divided into squares, one symbol per square. The tape is part of a Turing machine modified to be stepped by a clock and to communicate with other such machines and with keyboards and displays. We call this modified Turing machine an open machine. The computations performed by an open machine are open to an inflow numbers and formulas incalculable prior to their entry.

Because an open machine (or indeed any digital computer) cycles through distinct phases of memory use, the most direct propagation of symbols from one computer to another requires a symbol from one computer to arrive during a suitable phase of the receiving computer’s cycle. In Sec. 4, we elevate this phase constraint to a principle that defines the logical synchronization necessary to a channel that connects clock readings at transmission of symbols to clock readings at their reception.

Provisional hypotheses, involving guesswork about the atoms of a clock and about signal propagation, are essential to symbol-bearing channels between computers. The recognition that unforeseeable evidence can prompt revision of these
hypotheses raises several types of questions as topics for a discipline of *logical synchronization* within physics, outlined in Sec. 5. The first type of question concerns patterns of channels that are possible aiming points, as determined in a blackboard calculation that assumes a theory of signal propagation. Sec. 6 addresses examples of constraints on patterns of channels under various hypotheses of spacetime curvature, leading to “phase stripes” in spacetime that constrain the channels possible between one open machine and another. An example of a freedom to guess an explanation within a constraint of given channels is characterized by a subgroup of a group of clock adjustments.

Sec. 7 briefly addresses the two other types of questions, pertaining not to hypothesizing possible aiming points ‘on the blackboard’, but to using hypothesized aiming points, copied into feedback-mediating computers, for the steering of drifting clocks. To model drift, we draw on quantum uncertainty relations to express the looseness of the relation between clocks as general-relativistic world-lines and any evidence obtainable from physical clocks. After discussing steering toward aiming points copied from the blackboard, we note occasions that invite revision of a hypothesized metric tensor and of patterns of channels chosen as aiming points.

Sec. 8 suggests giving up ‘global time’ with its predictability, in favor of attention to logical synchronization. A few topics attractive for future investigation are noted.

### 2. Computer-mediated feedback within a single atomic clock

The fact is that time as we now generate it is dependent upon defined origins, a defined resonance in the cesium atom, interrogating electronics, induced biases, timescale algorithms, and random perturbations from the ideal. Hence, at a significant level, time—as man generates it by the best means available to him—is an artifact. Corollaries to this are that every clock disagrees with every other clock essentially always, and no clock keeps ideal or “true” time in an abstract sense except as we may choose to define it [6].

Within any atomic clock computer-mediated feedback is essential. As is well known, in both cesium clocks that realize the SI second and in the most stable optical atomic clocks, the atom or atoms in the atomic clock are passive—they do not “tick”—so the clock needs an active oscillator and other components in addition to the atom(s). An atomic clock operates a feedback loop in which a
guessed hypothesis steers the rate of ticking of its oscillator. An atomic clock’s components include 7, 8, 9, 10, 11, 12:

1. an active oscillator radiating at microwave or optical frequencies, cycling through phases adjustable over a narrow range of frequency (picture a driven pendulum).
2. a controllable “gear box,” called a frequency synthesizer, that produces an output frequency at a variable ratio to that of the oscillator,
3. one or more passive resonant atoms illuminated by radiation from the oscillator;
4. a real-time computer that controls the oscillator and the synthesizer;
5. detectors of the unforeseeable outcomes of interaction of the atom(s) with the oscillator’s radiation that write records into the computer memory;
6. a formula encoded in the computer memory that defines the how the computer steers the oscillator frequency and the synthesizer in response to deviations of accumulating recorded evidence from an hypothesized aiming point.

In designing an atomic clock to realize the SI second, one encounters, among others, the following two problems. (a) The resonance exhibited by the atom or atoms of the clock varies with the details of the clock’s construction and the circumstances of its operation; in particular the resonance shifts depending on the intensity of the radiation of the atoms by the oscillator. (b) The oscillator, controlled by, in effect, a knob, drifts in relation to the knob setting.

Problem (a) is dealt with by introducing a wave function parametrized by radiation intensity and whatever other factors one deems relevant. The second is then “defined” in terms of the resonance the “would be found” at zero temperature (implying zero radiation) 13, 14, 15. For a clock using cesium 133 atoms, this imagined resonance is declared by the General Conference of Weights and Measures to be 9 192 631 770 Hz, so that the SI second is that number of cycles of the radiation corresponding to that imagined resonance 16.

Problem (b) is dealt with by feedback that adjusts a “knob” that controls the oscillator, in response to detections of scattering of the oscillator’s radiation by the atom or atoms of the clock, so that the oscillator is steered toward an aiming point at which the detection rate is sensitive to small displacement of the oscillation from the aiming point.

As illustrated in Fig. 1, the drift of the oscillator is indicated by variation in the detection rate (which, apart from some statistical variation, varies with changes
in the oscillator frequency relative to the resonance of the radiated atoms). The aiming point is set at a fixed detection rate, chosen to be sensitive to variation in the oscillator frequency relative to the resonance of the atoms as radiated by the oscillator. The function illustrated by the bell-shaped curve, which would be obtained from experimental data, is coded into the computer to express detection rate vs. deviation of the oscillation from the resonance of the radiated atom(s). The aiming point for the oscillator frequency differs from the imagined frequency at 0 K for two reasons: (a) it differs from the resonant peak for the atom(s) radiated by the oscillator to get a more sensitive response, and (b) that resonant peak differs by an amount depending on a chosen wave function from the defining imagined resonance at 0 K. The synthesizer gears the output frequency of the clock to the oscillator so as to account for the ratio of the resonance hypothesized at 0 K to the aiming point for the oscillator.
3. Open Turing machine as a model of a computer in a feedback loop

Computer-mediated feedback used in an atomic clock requires logic open to an inflow of inputs beyond the reach of calculation. To model the logic of a computer that communicates with the other devices in a feedback loop, we modify a Turing machine to communicate with external devices, including other such machines. One thinks of a Turing machine, modified or not, as making a record on a tape marked into squares, each square holding one character of an alphabet. Operating in a cyclic sequence of ‘moments’ interspersed by ‘moves’, at any moment the machine scans one square of the tape, on which it can read or write a single character. A move as defined in the mathematics of Turing machines consists (only) of the logical relation between the machine at one moment and the machine at the next moment [17], thus expressing the logic of a computation, detached from its speed, so that that two computations executing at different speeds can be represented in their logic by the same sequence of moves. In a feedback loop, however, computational speed matters, and so we let the moves of the modified Turing machine be stepped by ticks of a clock. A step occurs once per period of revolution of the clock hand. This period is adjustable, on the fly. A cycle of the modified Turing machine corresponds to a unit interval of the readings of its clock.

To express communication between Turing machines, we postulate that the modified Turing machine can receive externally supplied signals and can transmit signals, with both the reception and the transmission geared to the cycle of the machine. In addition, the modified Turing machine registers a count of moments at which signals are received and moments at which signals are transmitted. At a finer scale, the machine records a phase quantity in the cycle of its clock, relative to the center of the moment at which a signal carrying a character arrives. We call such a machine an open machine. An open machine can receive detections and can command action, for instance the action of increasing or decreasing the frequency of the variable oscillator of an atomic clock.

In contradistinction to an open machine, one might speak of the usual Turing machine (which Turing called an automatic machine [17]) as closed. Calculations performed on a closed machine proceed from start to halt by a succession of moves made according to a pre-programmed rule, closed to outside influences, neither receiving anything nor commanding any action. Such “closed” calculations correspond to logic in which atomic propositions never change their truth values. In contrast, calculations performed on an open machine communicating with detectors and actuators proceed by moves made according to a rule that can be modified from outside the machine in the course of its operation. These cal-
Calculations can respond to received influences, such as occurrences of outcomes undervariable from the contents of the machine memory, so that the open machine writes commands on a tape read by an external actuator. The wider physical world shows up in an open machine as both (1) unforeseeable messages from external devices and (2) commands to external devices.

We picture the records made by a real-time computer in a feedback loop as written on the tape of an open machine. The segmentation into moments interspersed by moves is found not just in Turing machines but in any digital computer, which implies

**Proposition 2.** *The logical result of any computation is oblivious to variations in speed at which the clock steps the computer.*

**Corollary 2.1.** *No computer can sense directly any variation in its clock frequency.*

Although it cannot directly sense variation in the tick rate of its clock, the logic of open machine, thought of as stepped by an atomic clock, can still control the adjustment of the clock’s oscillator by responding to variations in the detection rate written moment by moment onto its Turing tape. A flow of unforeseeable detections feeds successive computations of results, each of which, promptly acted on, impacts probabilities of subsequent occurrences of outcomes, even though those subsequent outcomes remain unforeseeable. The computation that steers the oscillator depends not just on unforeseeable inputs, but also on a steering formula coded into a program.

**Remarks:**

1. To appreciate feedback, one needs to distinguish any formula as as written from what it expresses. For example a formula written along a stretch of a Turing tape as a string of characters can name a wave function $\psi$ that depends on a time variable $t$. Like a formula chalked on a blackboard, the formula containing $\psi$, once written, “sits motionless,” in contrast to the time variation that the formula expresses.

2. Although unchanged over some cycles of a feedback loop, a steering formula does not stay put for ever. A feedback loop operates in a larger context, in which steering formulas are subject to evolution. Sooner or later, the string of characters that expresses the steering formula is apt to be over-written by a characters expressing a new formula. Occasions for rewriting steering formulas are routine in clock networks, including those employed in geodesy and astronomy.
3. Noticing feedback raises an opportunity to improve the short-term stability of an atomic clock. Recall that the oscillator is steered in frequency by responding to deviations in detection rate from an assumed operating point. If the clock keeps a record of these deviations, the record could be used to make corrections to the clock’s readings that takes the deviations into account, thereby improving the corrected stability of the clock.

4. Communication channels and logical synchronization

Because open machines (and computers) are stepped through phases by clocks, signals communicating logical symbols from one open machine to another must arrive at a computer during a certain phase and not during other phases. A reading $\zeta_A$ of the clock of an open machine $A$—an $A$-reading—has the form $m.\phi_m$ where an integer $m$ indicates the count of cycles and $\phi_m$ is the phase within the cycle. Thus the clock reading of an open machine passes through an integer value as the phase of the clock hand passes through zero. We adopt the convention that $-1/2 < \phi_m \leq 1/2$. We define a channel from $A$ to $B$, denoted $\overrightarrow{AB}$, as a set of pairs, each pair of the form $(m.\phi_m, n.\phi_n)$. The first member $m.\phi_m$ is an $A$-reading at which machine $A$ can transmit a signal and $n.\phi_n$ is a $B$-reading at which the clock of machine $B$ can register the reception of the signal. A repeating channel is defined to be a channel $\overrightarrow{AB}$ such that

$$\forall \ell \in [\ell_1, \ell_2] \exists m, n, j, k (m + j.\phi_A, n + k.\phi_B, \ell) \in \overrightarrow{AB},$$

For theoretical purposes, it is convenient to define an endlessly repeating channel for which $\ell$ ranges over all integers. Again for theoretical purposes, we sometimes consider channels for which the phases are all zero, in which case one may omit writing the phases.

When two-way repeating channels $\overrightarrow{AB}$ and $\overrightarrow{BA}$ link open machines $A$ and $B$, a lower bound on the clock reading at which $A$ can receive an acknowledgment from $B$ comes from the following

**Definition of echo count:** Suppose that at its reading $m.0$ an open machine $A$ transmits a signal at to an open machine $B$, and the first signal that $B$ can transmit back to $A$ after receiving $A$’s signal reaches $A$ at $m'.\phi'$; then the quantity $m'.\phi' - m$ will be called the echo count $\Delta_{ABA}$ at $m$. 

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Thus $\Delta_{ABA}(m) + m$ is a lower bound on the cycle count at which $A$ can receive an acknowledgement of a transmission sent at $A$-reading $m$ to $B$. In the theoretical case in which receptions occur at null phases, echo counts are integers. Echo count is defined relative to the variably geared output of the adjustable clock of an open machine.

| A’s cycle count | Event | Other: Phase | Other: Rate | Cycle sent |
|-----------------|-------|--------------|-------------|------------|
| :              | :     | :            | :           | :          |
| 17              | send  | B            |             |            |
|                 | rate  | 3.14         |             |            |
| 18              | send  | D            |             |            |
|                 | rate  | 3.14         |             |            |
| 19              | rec’d | B            | 0.17        | 24         |
|                 | rate  | 3.07         |             |            |
|                 | send  | B            |             |            |

Table 1: History recorded in the memory of open machine A, indicating clock rates relative to oscillator and phases at receptions

Because they are defined by local clocks without reference to any metric tensor, channels invoke no assumption about a metric or even a spacetime manifold. For this reason evidence from the operation of channels is independent of any explanatory assumptions involving a manifold with metric and hence is independent of any global time coordinate or any “reference system” [18]. Thus clock readings at the transmission and the reception of signals can prompt revisions of hypotheses about a metric tensor field. Table 1 illustrates evidence in the form of clock readings associated with channels acquired by an open machine $A$. Histories recorded or imagined in the form of Table 1 can be expressed as occurrence graphs [19], specialized to exhibit a distinct trail for each open machine, with the trails linked by edges for signals. Fig. 2 illustrates open machines $A$ and $B$ separately, along with their combined evidence. When “analog” measurements of phases with their idiosyncrasies are forgotten, the occurrence graph for a network of endlessly repeating channels can be “wrapped around” to form a marked graph [20, 28]. Occurrence graphs, marked graphs, and more general Petri nets [21] form categories with interesting graph morphisms.
From the beating of a heart to the bucket brigade, life moves in phased rhythms. It is well known that for a symbol-carrying signal transmitted from one computer to be written into the memory of another computer, the signal must arrive during a phase in which writing can take place, and the cycle must offer room for a distinct other phase. We elevate this engineering commonplace to a principle pertaining to open machines:

**Proposition 3.** A logical symbol can propagate from one open machine to another only if the symbol arrives within the writing phase of the receiving machine; in particular, respect for phasing requires that for some positive $\eta$ any arrival
phase $\phi_n$ satisfy the inequality

$$|\phi_n| < \frac{(1 - \eta)}{2}. \quad (2)$$

Prop. 3 serves as a fixed point to hold onto while hypotheses about signal propagation in relation to channels are subject to revision. We call the phase constraint on a channel asserted by (2) logical synchronization.

In this report we consider only channels that preserve the order of signals in the sense that when two successive signals propagate from any machine $A$ to any other machine $B$, whichever signal is sent later must also arrive later:

**Order preservation:** A channel $\vec{AB}$ preserves order if for any $(m.\phi_m, n.\phi_n)$ and $(m'.\phi_{m'}, n'.\phi_{n'})$, $m'.\phi_{m'} > m.\phi \Rightarrow n'.\phi_{n'} > n.\phi_n$.

**Remarks:**

1. Note that $\phi_n$ in (2) is a phase of a cycle of a variable-rate clock *not* assumed to be in any fixed relation to a proper clock as conceived in general relativity. Indeed, satisfying (2) usually requires the operation of clocks at variable rates.

2. Computers are commonly designed with buffering that detaches the timing of message reception from the stepping of the computer. Buffering, while convenient, inserts delay between transmission and the arrival of symbols in the computer memory [22]. In analyzing open machines we focus on the most direct communication possible, which cannot be buffered, and so employs the character-by-character phase meshing as asserted in Prop. 3.

3. Logical synchronization differs from Einstein synchronization [23], among other ways, by allowing leeway in the arrival of a signal.

4. Designs for logical synchronization that arise in engineering contexts are an extensive subject [22].

5. **A discipline of logical synchronization within physics**

Given the definition of a channel and the condition (2) essential to the communication of logical symbols, three types of questions arise:

**Type I:** What patterns of interrelated channels and echo counts can one try for as aiming points?

**Type II:** How can the steering of open machines be arranged to approach given aiming points within acceptable phase tolerances?

**Type III:** How to respond to deviations from aiming points beyond tolerances?
Such questions point the way to what might be called a discipline of logical synchronization transported from the engineering of digital communications into physics. So far we notice two promising areas of application within this discipline:

1. Provide a theoretical basis for networks of logically synchronized repeating channels, highlighting
   (a) possibilities for channels with null receptive phases as a limiting case of desirable behavior, and
   (b) circumstances that force non-null phases.

2. Explore constraints on receptive phases imposed by gravitation, as a path to exploring and measuring gravitational curvature, including slower changes in curvature than those searched for by the Laser Gravitational Wave Observatory [24].

5.1. Geometry of signal propagation.

Answers to questions of the above Types require hypotheses, if only provisional, about signal propagation. For this section we assume that propagation is described by null geodesics in a Lorentzian 4-manifold $M$ with one or another metric tensor field $g$, as in general relativity. Following Perlick [23] we represent an open machine as a timelike worldline, meaning a smooth embedding $\gamma: \zeta \mapsto \gamma(\zeta)$ from a real interval into $M$, such that the tangent vector $\dot{\gamma}(\zeta)$ is everywhere timelike with respect to $g$ and future-pointing. We limit our attention to worldlines of open machines that allow for signal propagation between them to be expressed by null geodesics. To say this more carefully, we distinguish the image of a worldline as a submanifold of $M$ from the worldline as a mapping. Consider an open region $V$ of $M$ containing a smaller open region $U$, with $V$ containing the images of two open machines $A$ and $B$, with the property that every point $a$ of the image of $A$ restricted to $U$ is reached uniquely by one future-pointing null geodesic from the image of $B$ in $V$ and by one past-pointing null geodesic from the image of $B$ in $V$. And suppose that this works the other way around for every point $b$ of the image of $B$ restricted to $U$. We then say $A$ and $B$ are radar linkable in $U$. We limit our attention to open machines that are radar linkable in some spacetime region $U$. In addition we assume that the channels preserve order. Indeed, we mostly deal with open machines in a gently curved spacetime region, adequately described by Fermi normal coordinates around a timelike geodesic.

For simplicity and to allow comparing conditions for phasing with conditions for Einstein synchronization, we take the liberty of allowing transmission to occur at the same phase as reception, so that both occur during a phase interval satisfying
The perhaps more realistic alternative of demanding reception near values of \( \phi = 1/2 \) can be carried out with little difficulty.

To develop the physics of channels, we need to introduce three concepts.

1. We define a group of clock adjustments as transformations of the readings of the clock of an open machine. As it pertains to endlessly repeating channels, a group \( H \) of clock adjustments consists of functions on the real numbers having continuous, positive first derivatives. Group multiplication is the composition of such functions, which, being invertible, have inverses. To define the action of \( H \) on clock readings, we speak ‘original clock readings’ as distinct from ‘adjusted readings’. An adjustment \( f_A \in H \) acts by changing every original reading \( \zeta_A \) of a clock \( A \) to an adjusted reading \( f_A(\zeta_A) \). As we shall see, clock adjustments can affect echo counts.

2. To hypothesize a relation between the \( A \)-clock and an accompanying proper clock, one has to assume one or another metric tensor field \( g \), relative to which to define proper time increments along \( A \)’s worldline; then one can posit an adjustment \( f_A \) such that \( f_A(\zeta_A) = \tau_A \) where \( \tau_A \) is the reading imagined for the accompanying proper clock when \( A \) reads \( \zeta_A \).

3. We need to speak of positional relations between open machines. For this section we assume that when an open machine \( B \) receives a signal from any other machine \( A \) then \( B \) echoes back a signal to \( A \) right away, so the echo count \( \Delta_{ABA} \) defined in Sec. 4 involves no delay at \( B \). In this case, evidence in the form of an echo count becomes explained, under the assumption of a metric tensor field \( g \), as being just twice the radar distance \( [23] \) from \( A \) to the event of reception by \( B \).

6. Type-I questions: mathematical expression of possible patterns of channels

Questions of Type I concern constraints on channels imposed by the physics of signal propagation. Here we specialize to constraints on channels imposed by spacetime metrics, constraints obtained from mathematical models that, while worked out so to speak on the blackboard, can be copied onto Turing tapes as aiming points toward which to steer the behavior of the clocks of open machines. Questions of Types II and III are deferred to the Sec. 7.
6.1. Channels with null phases as aiming points: two open machines linked by a two-way channel.

We begin by considering just two machines. Assuming an hypothetical space-time \((M, g)\), suppose that machine \(A\) is given as a worldline parametrized by its clock readings: what are the possibilities and constraints for an additional machine \(B\) with two-way repeating channels \(\overrightarrow{AB}\) and \(\overrightarrow{BA}\) linking \(B\) to \(A\) at constant echo count? We assume the idealized case of channels with null phases, which implies integer echo counts. For each \(A\)-tick there is a future light cone and a past light cone. The future light cone from an \(A\)-reading \(\zeta_A = m\) has an intersection with the past light cone for the returned echo received at \(\zeta_A = m + \Delta_{ABA}\). Fig. 3 illustrates the toy case of a single space dimension in a flat spacetime by showing the two possibilities for a machine \(B\) linked to \(A\) by two-way channels at a given constant echo count. In each solution, the clock rate of \(B\) is adjusted so that a tick of \(B\) occurs at each of a sequence of intersections of outgoing and incoming light cones from and to ticks of \(A\). Note that the image of \(B\), and not just its clock rate, depends on the clock rate of \(A\).

![Diagram of a single space dimension in a flat spacetime illustrating the two possibilities for a machine B linked to A by two-way channels at a given constant echo count. In each solution, the clock rate of B is adjusted so that a tick of B occurs at each of a sequence of intersections of outgoing and incoming light cones from and to ticks of A. Note that the image of B, and not just its clock rate, depends on the clock rate of A.](image-url)
Determination of the tick events for $B$ leaves undetermined the $B$ trajectory between ticks, so there is a freedom of choice. One can exercise this freedom by requiring the image of $B$ to be consistent with additional channels of larger echo counts. A clock adjustment of $A$ of the form $\zeta_A \rightarrow \zeta'_A = N\zeta_A$ for $N$ a positive integer increases the density of the two-way channel by $N$ and inserts $N - 1$ events between successive $B$-ticks, thus multiplying the echo count by $N$. As $N$ increases without limit, $B$ becomes fully specified.

Turning to two space dimensions, the image of $B$ must lie in a tube around the image of $A$, as viewed in a three-dimensional space (vertical is time). So the image of any timelike worldline within the tube will do for the image of $B$. For a full spacetime of 3+1 dimensions, the solutions for the image of $B$ fall in the corresponding “hypertube.” The argument does not depend on flatness and so works for a generic, gently curved spacetime in which the channels have the property of order preservation.

Figure 4: (a) Images of worldlines for open machines $A$ and $B$ freely chosen; (b) A lacing of light signals that defines tick events; (c) Interpolated lacings of light signals added to make $\Delta_{ABA} = \Delta_{BAB} = 3$.

A different situation for two machines arises in case only the image of $A$’s worldline is specified while its clocking left to be determined. In this case the
image of any $B$ radar linkable to $A$ can be freely chosen, after which the clocking of both $A$ and $B$ is constrained, as illustrated in Fig. 4 for the toy case of flat spacetime with 1 space dimension. To illustrate the constraint on clocking, we define a “lacing” of light signals to be a pattern of light signals echoing back and forth between two open machines as illustrated in Fig. 4 (b). For any event $a_0$ chosen in the image of $A$, there is a lacing that touches it. In addition to the choice of $a_0$, one can choose any positive integer $N$ to be $\Delta_{ABA}$, and choose $N - 1$ events in the image of $A$ located after $a_0$ and before the next $A$-event touched by the lacing of light signals. The addition of lacings that touch each of the $N - 1$ intermediating events corresponds to a repeating channel $\overrightarrow{AB}$ with echo count $\Delta_{ABA} = N$, along with a repeating channel $\overrightarrow{BA}$ with the same echo count $\Delta_{BAB} = N$. This construction does not depend on the dimension of the spacetime nor on its flatness, and so works also for a curved spacetime having the property of order preservation.

6.2. Example of free choice characterized by a transformation group.

Evidence of channels as patterns of clock readings leaves open a choice of worldlines for its explanation. In the preceding example of laced channels between open machines $A$ and $B$, part of this openness can be reflected within analysis by the invariance of the channels under a subgroup of the group of clock adjustments that “slides the lacings,” as follows. Suppose that transmissions of an open machine $A$ occur at given values of $A$-readings. We ask about clock adjustments that can change the events of a worldline that correspond to a given $A$-reading. If a clock adjustment $f_A$ takes original $A$-readings $\zeta_A$ to a revised $A$-readings $f_A(\zeta_A)$, transmission events triggered by the original clock readings become triggered when the re-adjusted clock exhibits the same readings. As registered by original readings, the adjusted transmission occurs at $\zeta'_A = f_A^{-1}(\zeta_A)$. Based on this relation we inquire into the action of subgroups of $H \times H$ on the readings of the clocks of two open machines $A$ and $B$. In particular, there is a subgroup $K(A,B) \subset H \times H$ that expresses possible revisions of explanations that leave invariant the repeating channels with constant echo count $N$. An element $f_A \times f_B \in K(A,B)$ is a pair of clock adjustments that leaves the channels invariant, and such a pair can be chosen within a certain freedom. For the adjustment $f_A$ one is free to: (a) assign an arbitrary value to $f_A^{-1}(0)$; and (b), if $N > 1$, then for $j,k = 1, \ldots, N - 1$, choose the value of $f_A^{-1}(j)$ at will, subject to the constraints that $k > j \Rightarrow f^{-1}(k) > f^{-1}(j)$ and $f^{-1}(N - 1)$ is less than the original clock reading for the re-adjusted first echo from $f^{-1}(0)$. With these choices, $f_B$ is then
constrained so that each lacing maps to another lacing. The condition (a) slides a lacing along the pair of machines; the condition (b) nudges additional lacings that show up in the interval between a transmission and the receipt of its echo. In this way a freedom to guess within a constraint imposed by evidence becomes expressed by $K(A, B)$.

6.3. Channels among more than two open machines.

Moving to more than two machines, we invoke the

**Definition:** an arrangement of open machines consists of open machines with the specification of some or all of the channels from one to another, augmented by proper periods of the clock of at least one of the machines.

(Without specifying some proper periods, the scale of separations of one machine from another is open, allowing the arrangement to shrink without limit, thus obscuring the effect of spacetime curvature.)

Although gentle spacetime curvature has no effect on the possible channels linking two open machines, spacetime curvature does affect the possible channels and their echo counts in some arrangements of five or more machines, so that the channels that can be implemented are a measure spacetime curvature. The way that spacetime curvature affects the possible arrangements of channels is analogous to the way surface curvature in Euclidean geometry affects the ratios of the lengths of the edges of embedded graphs. The effect on ratios of edge lengths shows up in mappings from embeddings of graphs in a plane to their images on a sphere. For example, a triangle can be mapped from a plane to a generic sphere, in such a way that each edge of the triangle is mapped to an arc of the same length along a great circle on the sphere. The same holds for two triangles that share an edge, as illustrated in Fig. 5 panel (a); however, the Gauss curvature of the sphere implies that the complete graph on 4 vertices generically embedded in the plane, shown in panel (b), cannot be mapped so as to preserve all edge lengths. The property that blocks the preservation of edge ratios is the presence of an edge in the plane figure that cannot be slightly changed without changing the length of at least one other edge; we speak of such an edge as “frozen.”

In a static spacetime, which is all we have so far investigated, a generic arrangement of 4 open machines is analogous to the triangle on the plane in that it can be mapped to any gently curved spacetime in such a way as to preserve all the echo counts.
Proposition 4. Assume four open machines in a static spacetime, with one machine stepped with a proper-time period \( p_r \), and let \( N \) be any positive integer. Then, independent of any gentle Riemann curvature of the spacetime, the four open machines can be arranged, like vertices of a regular tetrahedron, to have six two-way channels with null phases, with all echo counts being \( 2N \).

Proof: Assuming a static spacetime, choose a coordinate system with all the metric tensor components independent of the time coordinate, in such a way that it makes sense to speak of a time coordinate distinct from space coordinates (for example, in a suitable region of a Schwarzschild geometry). Let \( V_1 \) denote the machine with specified proper period \( p_r \), and let \( V_2, V_3, \) and \( V_4 \) denote the other three machines. For \( i, j \in \{1, 2, 3, 4\}, i \neq j \), we prove the possibility, independent of

Figure 5: Plane figures, one of which maps to a sphere while preserving edge lengths
curvature of the channels

\[ \overrightarrow{V_iV_j} = \{(k, k + N.0) | k \text{ any integer}\}. \]  \tag{3}

Let each of four machines be located at some fixed spatial coordinate. Because the spacetime is static, the coordinate time difference between a transmission at \( V_1 \) and a reception at any other vertex \( V_j \) (a) is independent of the value of the time coordinate at transmission and (b) is the same as the coordinate time difference between a transmission at \( V_j \) and a reception at \( V_1 \). For this reason any one-way repeating channel of the form \( (3) \) can be turned around to make a channel in the opposite direction, so that establishing a channel in one direction suffices. For transmissions from any vertex to any other vertex, the coordinate-time difference between events of transmission equals the coordinate-time difference between receptions. A signal from a transmission event on \( V_1 \) propagates on an expanding light cone, while an echo propagates on a light cone contracting toward an event of reception on \( V_1 \). Under the constraint that the echo count is \( 2N \), (so the proper duration from the transmission event to the reception event for the echo is \( 2Np_r \)), the echo event must be on a 2-dimensional submanifold—a sphere, defined by constant radar distance \( Np_r \) of its points from \( V_1 \) with transmission at a particular (but arbitrary) tick of \( V_1 \). In coordinates adapted to a static spacetime, this sphere may appear as a “potatoid” in the space coordinates, with different points on the potatoid possibly varying in their time coordinate. The potatoid shape corresponding to an echo count of \( 2N \) remains constant under evolution of the time coordinate. Channels from \( V_1 \) to the other three vertices involve putting the three vertices on the potatoid. Put \( V_2 \) anywhere on the potatoid. Put \( V_3 \) anywhere on the ring that is intersection of potatoid of echo count \( 2N \) radiated from \( V_2 \) and that radiated from \( V_1 \). Put \( V_4 \) on an intersection of the potatoids radiating from the other three vertices.

Q.E.D.

According to Prop \( \ref{prop:4} \) the channels, and in particular the echo counts possible for a complete graph of four open machines in flat spacetime are also possible for a spacetime of gentle static curvature, provided that three of the machines are allowed to set their periods not to a fixed proper duration but in such a way that all four machines have periods that are identical in coordinate time. The same holds if fewer channels among the four machines are specified.

But for five machines, the number of channels connecting them matters. Five open machines fixed to space coordinates in a static spacetime are analogous to the 4 vertices of a plane figure, in that an arrangement corresponding to an incom-
plete graph on five vertices can have echo counts independent of curvature, while a generic arrangement corresponding to a complete graph must have curvature-dependent relations among its echo counts.

**Proposition 5.** Assuming a static spacetime, consider an arrangement of five open machines obtained by starting with a tetrahedral arrangement of four open machines with all echo counts of \(2N\) as in Prop. 4 and then adding a fifth machine: independent of curvature, a fifth open machine can be located with two-way channels having echo counts of \(2N\) linking it to any three of the four machines of tetrahedral arrangement, resulting in nine two-way channels altogether.

**Proof:** The fifth machine can be located as was the machine \(V_4\), but on the side opposite to the cluster \(V_1, V_2, V_3\).

Q.E.D.

Figure 6: (a) 5 open machines with 9 two-way channels; (b) Five open machines with all 10 two-way channels

In contrast to an arrangement of 5 open machines having 9 two-way channels, illustrated in Fig. 6(a) consider an arrangement analogous to a complete graph on five vertices, having ten two-way channels, as illustrated in Fig. 6(b). For five open machines in a generic spacetime, not all of the ten two-way channels can have the same echo counts. Instead, channels in a flat spacetime as specified below can exist with about the simplest possible ratios of echo counts. Label five open machines, \(A_1, A_2, A_3, B_1,\) and \(B_2\). Take \(B_1\) to be stepped by a clock ticking at a fixed proper period \(p_r\), letting the other machines tick at variable rates to be determined. Let \(X\) be any machine other than \(B_1\). For a flat spacetime
it is consistent for the proper periods of all 5 machines to be \( p_\tau \), for the echo counts \( \Delta_{B_1X,B_1} \) to be \( 4N \) and for the echo counts \( \Delta_{A_iA_j,A_i} \) to be \( 6N \), leading to the following twenty channels, conveniently viewed as in Fig. 6(b) as consisting of ten two-way channels.

\[
\begin{align*}
\overrightarrow{A_nB_j} &= \{(k.0,k+2N.0)|k = 0,1,2\ldots\} \quad (4) \\
\overrightarrow{B_jA_n} &= \{(k.0,k+2N.0)|k = 0,1,2\ldots\} \quad (5) \\
\overrightarrow{B_1B_2} &= \{(k.0,k+2N.0)|k = 0,1,2\ldots\} \quad (6) \\
\overrightarrow{B_2B_1} &= \{(k.0,k+2N.0)|k = 0,1,2\ldots\}, \quad (7) \\
\overrightarrow{A_nA_{n+1}} &= \{(k.0,k+3N.0)|k = 0,1,2\ldots\} \quad (8) \\
\overrightarrow{A_{n+1}A_n} &= \{(k.0,k+3N)|k = 0,1,2\ldots\} \quad (9)
\end{align*}
\]

**Proposition 6.** Consider 5 open machines each fixed to space coordinates in a static curved spacetime in which the machines are all pairwise radar linkable, with 10 two-way channels connecting each machine to all the others; then:

1. Allowing for the periods of the machines other than \( B_1 \) to vary, it is consistent with the curvature for all but one of the ten two-way channels to have null phases and echo counts as in a flat spacetime, but at least one two-way channel must have a different echo count that depends on the spacetime curvature.

2. Suppose \( m \) of the 10 two-way links are allowed to have non-zero phases. If the spacetime does not admit all phases to be null, in generic cases the least possible maximum amplitude of a phase decreases as \( m \) increases from 1 up to 10.

3. The periods of the clocks of the open machines can be taken to be the coordinate-time interval corresponding to the proper period \( p_\tau \) of \( B_1 \).

**Proof:** Reasoning as in the proof of Prop. 4 with its reference to a static spacetime shows that the same echo counts are possible as for flat spacetime with the exception that at least one of the two-way channels must be free to have a different echo count. For \( m < 10 \), similar reasoning shows that allowing \( m+1 \) machines to vary...
in echo count allows reduction in the maximum variation from the echo counts in a flat spacetime, compared to the case in which only \( m \) machines are allowed to vary in echo count.

Q.E.D.

Adding the tenth two-way channel to an arrangement of five open machines “freezes” all the echo counts. To define freezing as applied to echo counts, first note an asymmetry in the dependence of echo counts on clock rates. Consider any two machines \( A \) and \( B \). While \( B \) can change the echo count \( \Delta_BAB \) by changing its clock rate, the echo count \( \Delta_ABA \) is insensitive to \( B \)’s clock rate. An echo count \( \Delta_ABA \) will be said to be to \( B \) and from \( A \).

**Definition:** An arrangement of open machines is frozen if it has an echo count to a machine \( B \) that cannot be changed slightly without changing another echo count to \( B \).

The property of being frozen is important because of the following.

**Proposition 7.** Whether or not a frozen arrangement of open machines is consistent with an hypothesized spacetime depends on the Weyl curvature of the spacetime.

### 6.4. Five open machines near Earth.

Here is a quantitative example in which we think of the 5 open machines linked as in a complete graph by ten two-way channels. We picture the 5 machines as carried by 5 space vehicles following closely a radial geodesic in a Schwarzschild geometry corresponding to the Earth as a central mass. In this example the variation of echo counts necessary to accommodate curvature is small enough to be expressed by non-null phases of reception, without changing the integer part of any echo count. In Fermi normal coordinates centered midway between the radially moving open machines \( B_1 \) and \( B_2 \) one has the metric

\[
\begin{align*}
\frac{ds^2}{c^2} &= -c^2[1 + \mu(y^2 + z^2) - 2x^2] dt^2 \\
&\quad - \frac{2\mu}{3} (xz \, dx \, dz + xy \, dx \, dy - 2yz \, dy \, dz) \\
&\quad + \left( 1 + \frac{\mu}{3} (y^2 + z^2) \right) dx^2 + \left( 1 + \frac{\mu}{3} (x^2 - 2z^2) \right) dy^2 \\
&\quad + \left( 1 + \frac{\mu}{3} (x^2 - 2y^2) \right) dz^2, 
\end{align*}
\]

where \( \mu := GM/(c^2r^3) \), \( r \) is the Schwarzschild radial coordinate to the origin of the Fermi normal coordinates, \( x \) is the radial distance coordinate from from the
center point between $B_1$ and $B_2$, and $y$ and $z$ are transverse to the radial geodesic \cite{25}. To work in SI units rather than the geometrized units of \cite{25}, we write speed of light, $c$, explicitly.

We ignore the temporal variation of $r$ in comparison with the dynamics of light signals between space vehicles, thus treating the Fermi normal coordinates as pertaining to a static spacetime. Locate each of the 5 open machines at fixed values of $x, y, z$, as follows. The metric (10) is symmetric under rotation about the $x$-axis. Let $B_1$ and $B_2$ be located symmetrically at positive and negative values, respectively, of the $x$-axis, and let $A_0, A_1, A_2$ be located on a circle in the plane $x = 0$. With the five machines so located, the coordinate-time difference between transmissions is then the same as the coordinate-time difference between receptions, and the coordinate-time delay in one direction equals that in the opposite direction (as stated in the proof of Prop. 4). We construct seven two-way channels as in (4–7) with null phases and show that the remaining 3 two-way channels can have the equal phases, but that this phase $\phi$ must be non-null and dependent on curvature, as in

$$\vec{A}_nA_{n+1} = \{(k,0,k + 3N.\phi)|k = 0, 1, 2\ldots\}\quad (11)$$

$$\vec{A}_{n+1}A_n = \{(k,0,k + 3N.\phi)|k = 0, 1, 2\ldots\}\quad (12)$$

**Proposition 8.** Under the stated conditions, if the effect of curvature is small enough so that $27GMN^3p_r^2/(4r^3) < 1$ then

$$\phi = -\frac{27GMN^3p_r^2}{8r^3}.\quad (13)$$

**Proof:** (by calculation of $\phi$):

1. Given $p_r$, determine coordinate-time period, denoted $p_t$, to first order in curvature. This is the interval of coordinate time over a proper period $p_r$ of $B_1$, which comes from evaluating $g_{00}$ at $x(B_1)$, with $x(B_1)$ evaluated at order zero:

$$p_t = (1 + \mu N^2 p_r^2 c^2) p_r\quad (14)$$

2. Determine $x(B_1)$ to first order in curvature as the value such that the coordinate time difference for a null geodesic from $B_2$ to $B_1$ is $2Np_t$. This leads to

$$2Np_t = 2 \int_{0}^{x(B_1)} \frac{dt}{dx} dx,\quad (15)$$

25
where \( dt/dx \) is obtained from (10) evaluated at \( y = z = dy = dz = 0 \). Along the \( x \)-axis one finds \( cdt/dx = 1 + \mu x^2 \). Substituting this into (15), integrating, and solving to first order in curvature yields

\[
x(B_1) = Np_t c \left( 1 - \frac{\mu}{3} N^2 p_t^2 c^2 \right).
\]  

(16)

3. Determine the radius of the circle on which \( A_n \) lies, \( n = 0, 1, 2 \). The coordinates are symmetric under rotation about the \( x \)-axis, so one can locate \( A_0 \) on the line \( x = z = 0 \). Then the radius is just the coordinate \( y(0) \). The difference in coordinate time between a null geodesic traversing from \( B_1 \) to \( A_0 \) and the coordinate time for a null trajectory that is linear in the coordinates is zero to first order in curvature. Thus, to first order, the coordinate time difference is that of a null curve following \( z = 0, y = (x_1 - x)y_0/x_1, dy = -(y_0/x_1) dx, \) where we write \( y_0 \) for \( y(A_0) \) and \( x_1 \) for \( x(B_1) \). Eq. (10) implies for this null curve:

\[
c^2 \left[ 1 + \mu \left( (x_1 - x)^2 y_0^2 x_1 - 2x^2 \right) \right] dt^2 = \frac{x_1^2 + y_0^2}{x_1^2} \left( 1 + \frac{\mu x_1^2 y_0^2}{3(x_1^2 + y_0^2)} \right) dx^2,
\]

leading to the relation to first order in curvature:

\[
c \frac{dt}{dx} = \sqrt{x_1^2 + y_0^2} \left( 1 + \frac{\mu x_1^2 y_0^2}{6(x_1^2 + y_0^2)} \right) \left[ 1 - \frac{\mu}{2} \left( (x_1 - x)^2 y_0^2 x_1^2 - 2x^2 \right) \right].
\]  

(18)

Integration gives

\[
2Np_t c = \sqrt{x_1^2 + y_0^2} \left[ 1 + \frac{\mu}{6} \left( \frac{x_1^2 y_0^2}{x_1^2 + y_0^2} + 2x_1^2 - y_0^2 \right) \right].
\]  

(19)

Substituting (14) and (16) into (19) and solving to first order in \( \mu \) for \( y_0 \) yields

\[
y_0 \equiv y(A_0) = \sqrt{3} Np_t c \left( 1 + \frac{\mu}{8} N^2 p_t^2 c^2 \right) + O[(\mu N^2 p_t^2 c^2)^2].
\]  

(20)

4. Determine the coordinate-time delay for transmission from \( A_0 \) to \( A_1 \), which is the same coordinate-time delay for all the transmissions between \( A_m \) and \( A_n, n \neq m, \) for \( n, m = 0, 1, 2 \). The metric is symmetric under rotation about the \( x \)-axis, which allows us to rotate \( A_0 \) to the position \( x = 0, z = y_0/2, y = -(\sqrt{3}/2)y_0, \) and to locate \( A_1 \) at \( x = 0, z = y_0/2, \)
\( y = (\sqrt{3}/2)y_0 \). Again, deviations of null geodesic from the linear relation between coordinates make zero first-order contribution, so we compute the coordinate-time delay for a null-curve from \( A_0 \) to \( A_1 \) along the line \( x = 0, z = y_0/2 \), which to first order in curvature is:

\[
t(A_1 \text{rec}) - t(A_0 \text{x-mit}) = 3Np_t(1 - 9\mu N^2 p_t^2 c^2 / 8) \tag{21}
\]

5. Converting from coordinate-time delay to the coordinate-independent echo count, under the hypothesis that \( 27\mu N^3 p_t^2 c^2 / 8 < 1/2 \), we arrive at the fractional part of \( t(A_1 \text{rec}) - t(A_0 \text{x-mit}) \) being \( \phi \) as stated in the Prop. 8.

Q.E.D.

Note that the proper periods of both \( B_1 \) and \( B_2 \) are \( p_\tau \), while, to first order in curvature, that of the \( A_n \) is \( p_\tau (1 - \mu N^2 p_t^2 c^2) \).

6.5. Changing curvature limits bit rate.

The dependence of echo counts on curvature has an interesting implication. When channels are to be maintained in the face of varying curvature, or in cases where there is uncertainty about what curvature describes their situation, the variability in curvature imposes a lower bound on clock rates and hence an upper bound on the rate at which information can be transmitted from one open machine to another. For the situation of the preceding example, this limit is readily obtained, as follows. For simplicity, assume that the positions and clock rates are continually adjusted to maintain null phases for all but the three channels \( A_n A_{n\pm 1} \). From (6) we have that \( L \approx 2Np_\tau c \), which with Prop. 8 and the fact that \( p_t \approx p_\tau \) implies \( \phi \approx -27ML^3/(32r^3 c^3 p_\tau) \), which with (2) implies

\[
p_\tau > \frac{27GML^3}{32r^3 c^3} \tag{22}
\]

Suppose the cluster of 5 open machines is arranged to have the proper radar distance \( L \) from \( B_1 \) to \( B_2 \) be 6,000 km, and suppose the cluster descends from a great distance down to a radius of \( r = 30,000 \text{ km} \) from an Earth-sized mass \( M_\oplus = 5.98 \times 10^{24} \text{ kg} \). With these values of the parameters, for the phases for the channels \( A_n A_{n\pm 1} \) to satisfy (2), it is necessary that

\[
p_\tau > 1.0 \times 10^{-13} \text{ s}. \tag{23}
\]

In case an alphabet conveys \( b \) bits/character, the maximum bit rate for all the channels in the 5-machine cluster is \( b/p_\tau < 10^{13}b \text{ bits/s} \).
7. Steering while listening to the unforeseeable

The preceding section displays “blackboard models” of clocks, expressed in the mathematical language of differential geometry. Turning from Type-I questions to questions of Type II, we now look at how such models get put to work when they are encoded into programs of computers that steer open machines in clock rate and in position toward aiming points. For questions of Types II and III, besides the models that explain or predict evidence, the evidence itself comes into play. Models taking part in the steering of physical clocks contribute to the generation of echo counts as evidence that, one acquired, can stimulate the guessing of new models that come closer to the aiming point.

7.1. Importing quantum uncertainty into general relativity.

To express the effect of quantum uncertainty on deviations from aiming points, one has to introduce quantum uncertainty into the representation of clocks by general-relativistic worldlines. This introduction hinges on the ever-crucial distinction between evidence and its explanations. Timelike worldlines and null geodesics in explanations, being mathematical, can have no mathematical connection to physical atomic clocks and physical signals. To make any (non-mathematical) connection, one has to invoke the logical freedom to make a guess. Within this freedom, without logical conflict, one can interpret events of signal reception as corresponding to expectation values in the sense of quantum theory. This intermediating layer of modeling explains some of the deviations of an atomic clock from an imagined proper clock, represented as a worldline. This is no “unification” of quantum theory and the theory of general relativity, merely a recognition that both theories are blackboard systems of explanation, distinct from evidence, and that pieces from one can, under certain circumstances, be joined to the other.

7.2. Need for prediction in steering toward an aiming point.

For reasons that include quantum uncertainty, coming close to an aiming point stated in terms of channels and a proper frequency scale requires steering. In steering, evidence of deviations from the aiming point combine with hypotheses concerning how to steer \[26\] \[27\]. For example, consider a case of an aiming for two open machines \(A\) and \(B\), as in the first example of Sec. 6. Recall that the open machine \(A\) is modeled by a given worldline with given clock readings \(\zeta_A\). Machine \(B\) aims to maintain a two-way, null-phase channel of given \(\Delta_{ABA} = \Delta_{BAB}\). To this end \(B\) registers arriving phases of reception and adjusts its clock rate more or less continually to keep those phases small. But \(B\) also needs to steer in position.
Deviations in $B$’s position show up as phases of echoes registered by $A$, so the steering of machine $B$ requires information about receptive phases measured by $A$. The knowledge of the deviation in position of $B$ at $\zeta_B$ cannot arrive at $B$ until its effect has shown up at $A$ and been echoed back as a report to $B$, entailing a delay of at least $\Delta_{BAB}$, hence requiring that machine $B$ predict the error to which its steering responds. Machine $B$ must predict ahead by at least $\Delta_{BAB}$. That is, steering deviations by one open machine are measured in part by their effect on receptive phases of other open machines, so that steering of one machine requires information about receptive phases measured by other machines, and the deviations from an aiming point must increase with increasing propagation delays that demand predicting further ahead.

As is clear from the cluster of five machines discussed in Sec. 6, the aiming-point phases cannot in general all be taken to be zero. For any particular aiming-point phase $\phi_0$ there will be a deviation of a measured phase quantity $\phi$ given by

$$\delta := \phi - \phi_0$$

(24)

Whatever the value of $\phi_0$, adjustments to contain phases within tolerable bounds depends on phase changes happening only gradually, so that trends can be detected and responded to on the basis of adequate prediction.

**Remarks:**

1. While it is often convenient to assume that cycle counts of open machines are free of uncertainty, recognizing uncertainty in measured phases and their deviations from aiming point has an immediate and interesting implication. For logic to work in a network, transmission of logical symbols must preserve sharp distinctions among them; yet the maintenance of sharp distinctions among transmitted symbols requires responses to fuzzy measurements.

2. The acquisition of logical synchrony in digital communications involves an unforeseeable waiting time, like the time for a coin on edge to fall one way or the other [22, 28].

7.3. Adjusting the aiming point.

Here we touch on questions of Type III. Up to this point we have looked at one or another manifold with metric $(M, g)$ as some given hypothesis, whether explored on the blackboard or coded into an open machine to serve in steering toward an aiming point. For it use in steering we think of $(M, g)$ as “given”—one
might say for use “ballistically,” without any “piloting.” But Type-III questions recognize that unforeseen deviations of phases outside of tolerances can happen, and an aiming point based on a hypothesized metric tensor can be found to be unreachable. Recognizing that an hypothesis of a metric tensor can reach the end of its useful life calls for “piloted” hypothesis making, recognizing that each hypothesis of a metric tensor field is provisional, to be revised as prompted by deviations outside allowed tolerances assigned for steering toward an aiming point that incorporates that metric tensor field.

Drawing on measured phases as evidence in order to adjust a hypothesis of a metric tensor is one way to view the operation of the Laser Interferometer Gravitational-Wave Observatory (LIGO) [24]. While LIGO sensitivity drops off severely below 45 Hz, the arrangement of five open machines of Prop. 6 has no low-frequency cutoff, and so has the potential to detect arbitrarily slow changes in curvature.

8. Discussion

In the physics of Newton, “time” as a concept offers a future that flows downstream to the present and into the past. In special relativity Einstein grounds a concept of time on ‘time local to a clock’, spread out by the synchronization of separated clocks (assuming no drift). But except locally, Einstein synchronization is unavailable to the curved spacetimes of general relativity [23]. Still, general relativity holds fast to the image of a predictable future, seemingly unwelcoming of surprise, as if what I do not foresee within my (relativistic) future is the fault of my ignorance, which I can hope to remedy. But if explanations in terms of wave functions are undetermined by any amount of evidence, and therefore are subject to surprises that prompt their revision, how do we float the future on any ‘river of time’?

By accepting unforeseeable events, and by reflecting within itself a range of guesses that respect given evidence, logical synchronization as a discipline within physics gives up the hope of using prediction to evade the need for adjustment, and instead employs predictions, dependent on guesswork, to define the departures that call for adjustments that respond to unforeseeable events, welcoming surprises that lead to the revision of predictions. By giving up predictability in favor of adjustment as fundamental to physics, logical synchronization opens avenues of inquiry invisible under any concept of ‘time’ that fails to acknowledge the foreseeable. Here are three examples:
1. How freedom of choice of wave function for atomic clocks implies freedom of choice in the smoothing of a metric tensor as an explanation of clock readings. Both for arrangements of clocks near and on Earth and for cosmological endeavors, one has occasion to choose a metric tensor field consistent with one or another body of experimental evidence of clock readings in the form of Fig. 2. Clock readings come with error bars on phases contribute evidence used in choosing a metric tensor field. Think of two error bars separated by a variable distance; as the distance shrinks the range of slopes of lines consistent with error bars increases. The resulting indeterminacy in the metric tensor thus involves a scale over which smoothing is applied, and at short scales of time and distance the room for choice of both spacetime curvature and clock-rate variation grows while maintaining consistency with experimental data.

2. How chosen wave functions for atomic clocks affect spacetime curvature. Additional freedom of metric tensor fields chosen to explain clock readings arises because, as sketched in 2, a chosen wave function references the SI second to an imagined resonance at absolute zero temperature. One might hypothesize a fine-scale variation in the 0 K offset of atomic clocks from from moment to moment and clock to clock as what we have called a clock-adjustment field [29], or one might smooth this adjustment field. This difference between the smoothed and unsmoothed clock-adjustment fields implies a difference in the corresponding metric tensor fields chosen to explain clock readings. If the smoothed readings underlie an hypothesized metric tensor field $g$, then the unsmoothed readings would correspond to a metric tensor field as expressed by a conformal transformation $	ilde{g} = (1 + \epsilon(x))g$, where $|\epsilon| \ll 1$ and $x$ is the 4-dimensional spacetime coordinate of some assumed chart. The two metric tensor fields, even if differing only slightly, can imply markedly different spacetime curvatures because two powers of derivatives—e.g. a term $(\nabla_a \epsilon)(\nabla^a \epsilon)$—that appears in relating the curvature of one spacetime to that of the other [30]. Thus the smallness of the clock shift $\epsilon$ is multiplied in its effect on curvature by the rapidity of its variation as expressed in these derivatives.

3. Further development of group representations of freedom of choice in explanations of given evidence. As we have seen above, unforeseeability brings open choices of explanations that respect given evidence, such as the choice of a wave function, resolved by guesswork. Guesses, by definition, cannot be decided by analysis, but in Sec. 6.2 one choice open to guesswork was
characterized by a transformation group. What else is there to do along these lines?

Appendix A. Guesswork in atomic clocks stemming from the gap between evidence and explanation as reflected in quantum theory

The operation of an atomic clock depends on a wave function chosen to explain evidence attributed to the atom or atoms of the clock as these are radiated by the oscillator. How, though, is this wave function chosen? One learns quantum mechanics starting ‘the other way’: given a wave function and a positive operator-valued measure (POVM) expressing a measurement procedure, one learns to calculate probabilities of outcomes. But an atomic clock poses an ‘inverse problem’ of arriving at a quantum model from given probabilities. If one knew the Hilbert space and a sufficient set of measurement operators, the density operator (or the wave function) could be determined, provided that the quantum state under study could be prepared repeatedly [31, 32]. Assuming knowledge of measurement operators, etc., this determination, called “quantum tomography,” is effective in some cases [33]; however, no experiment can determine a Hilbert space, and to determine the measurement operators one has to determine the effects of the measuring procedures that they express on laboratory preparations for which the density operators on the assumed Hilbert space are known. Thus the logic of quantum tomography is circular, which raises the question: given probabilities abstracted from laboratory work, how is one to arrive at a wave function without assuming knowledge of measurement operators that itself presupposes knowledge of some density operators?

Quantum theory reflects within the grammar of its mathematical language a distinction between experimental results and their explanations. This distinction makes it possible to picture an experimental set-up without presupposing any of its possible explanations in terms of quantum states and measurement operators. Picture an experimenter transmitting commands to an experimental set-up and receiving reports of detections via a process-control computer expressed by an open machine [1]. The commands act set values of device parameters which we think of as knobs controlling devices. We call a list of knobs with their possible settings a knob domain, and a family of related experiments, some involving more or different knobs than others corresponds to a lattice of knob domains ; similarly one defines a lattice of detector domains [3]. Given a knob domain $K$ and a detector domain $\Omega$, evidence to be explained quantum mechanically consists of a
parametrized probability measure (PPM) which is a function \( \alpha : K \times \Omega \to [0, 1] \), with the property that

\[
(\forall k \in K) \ \alpha(k, -) : \Omega \to [0, 1] \text{ is a probability measure on } \Omega, \tag{A.1}
\]

so that the PPM can be viewed as a function from a knob domain to probability measures on \( \Omega \). (Any of several metrics defined on spaces of probability distributions induce a topology on a knob domain \([3]\). Lifts from probability distributions to PPMs allow the definition of “metric deviation” as a quantitative difference between two PPMs that have the same knob domain but can differ in their detector domains \([3]\).)

Turning from evidence to its explanation, quantum language is used in various formulations to explain or to predict a PPM with domains \( K \) and \( \Omega \). For example, assume that the knob domain splits into two pieces, \( K = K_{\text{prep}} \times K_{\text{meas}} \), the first for the knobs by which a density operator is selected, the second for the knobs by which a Positive Operator-Valued Measure (POVM) is selected. A quantum-theoretic model of the PPM can be formulated as a structure \((K, \Omega, H, \rho, M)\) with:

1. a Hilbert space \( H \),
2. a function \( \rho : K_{\text{prep}} \to \{\text{density operators on } H\} \),
3. a function \( M : K_{\text{meas}} \to \{\text{POVMs on } \Omega \text{ assigning positive operators on } H\} \)

(Other formulations include unitary time evolution explicitly \([2]\).) Models of a PPM \( \alpha \) express \( \alpha \) through the trace as a functor:

\[
\alpha(k, \omega) = \text{tr}[\rho(k) M(k, \omega)] \tag{A.2}
\]

Metric deviations are defined not only for PPMs but also for density operators belonging to different models which can differ in their Hilbert spaces and for POVMs that can differ in their detector domains. When two models exhibit a positive metric deviation, we say the models are metrically distinct. (In contrast, unitarily equivalent models are not metrically distinct.)

With respect to the ‘inverse problem’ of choosing a model to explain given probabilities, quantum theory enables the proof of the following:

**Proposition A.1**: For any given PPM there is an infinite set of models, all metrically distinct \([2, 3, 4]\).
Proposition A.2: Any two metrically distinct models of a PPM can be extended in their detector domains or their knob domains to imply metrically distinct extended PPMs [2, 3].

Thus many quantum models, involving distinct wave functions and POVMs, map via the trace to any given PPM, but conflict with one another in their implied probabilities for experiments that extend the given PPM. So that choosing a wave function or a linear operator to explain an experiment requires reaching beyond the confines of logic based on evidence, and there is provably always the possibility that newly acquired evidence will call for a change of mind. With the recognition of unforeseeable events implicit in the use of wave functions encoded into clocks, we see quantum theory as a language in which one can think, speak, and trace the logic of conclusions to assumptions, revising the assumptions when that is called for.

Remark: The narrow view of quantum language as expressing evidence by probabilities precludes the application of quantum theory to feedback that reacts promptly to individual occurrences of outcomes. Following common practice, we adopt a wider view by admitting occurrences of outcomes, not just their probabilities, into the language of quantum theory, enabling the discussion of feedback that responds to outcomes of individual occurrences of measurements.

Quantum cryptography offers two related examples of ambiguity in the choice of wave functions, in which the issue is the security of quantum key distribution against undetected eavesdropping. In both examples the wave function that naive tomography produces seriously misleads; in effect Occam’s razor fails. An analysis of security that assumes a low-dimensional Hilbert space overlooks vulnerabilities from physical effects exploited by expanding the detector domain in such a way that a larger Hilbert space along with a larger detector domain is required to express these effects, as has been reported in an example [34]; a more physical example involves the propagation of pulses of polarized light generated by a set of four lasers. A popular model that expresses a density operator only with respect to light polarization asserts security, but an enveloping model in which the density operator also expresses variations in wave lengths emitted by the lasers displays security vulnerabilities [4].

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