Modeling of hydraulic fracture in a medium containing a hollow inclusion

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Abstract. In this work, the problem of the development of a hydraulic fracture crack near a circular cavity in a three-dimensional homogeneous elastic medium is considered. The results of numerical calculations performed using the extended finite element method (XFEM) implemented in the ABAQUS software package are presented. Various variants of the position of the initial disk crack relative to the cavity are considered. Some regularities of the development of a three-dimensional hydraulic fracture near the cavity have been found.

1. Introduction
One of the technologies used in the extraction of natural resources is hydraulic fracturing (HF). The following tasks can be solved using HF: intensification of oil and gas production [1], degassing of coal seams [2], softening of rocks [3, 4], creation of anti-seepage layers around mine workings [5], determination of the stress state of the environment [6, 7]. The hydraulic fracturing method is based on the ability to create cracks inside a rock mass by pumping fluid into it. Prediction of fracture parameters is an important problem, since its solution allows increasing the efficiency of this technology.

It is known that the geometry of the fracture (trajectory of development, length, opening) depends on the properties of the rock, the injected fluid, the structure of the medium and its stress state, the mode of fluid injection, etc. The case when cracks develop near cavities/voids can be separately distinguished. Such conditions can arise, for example, when HF is performed near mine workings.

Some problems of the development of hydraulic fractures near cavities/voids were considered in [8–14]. In [8], a model of hydraulic fracturing in the roof of a mine working, which had a rectangular shape, was proposed. In [9–12], the behavior of HF crack in the vicinity of circular cavities in a 2d elastic medium was investigated. In [13], the problem of modeling HF near a free surface in an elastic and poroelastic model of a medium was considered, where it was shown that the trajectories of cracks in these two cases are different. In [14], the influence of the poroelastic parameters of the medium on HF in the vicinity of a circular cavity was investigated.

In this paper, the problem of the development of a three-dimensional hydraulic fracture crack in an elastic homogeneous medium, near a cylindrical cavity under conditions of hydrostatic stress, is considered. In the first part of the work, the basic equations used to simulate hydraulic fracturing are presented. In the second part, the results of numerical studies performed using the extended finite element method implemented in the ABAQUS software package are presented.
2. Governing equation of the model
In this work, we assume that the hydraulic fracture propagates in a homogeneous elastic medium, the deformations of which are determined by the following equations:

\[ \sigma_{ij} = 2\mu \varepsilon_{ij} + \lambda \varepsilon_{ii} \delta_{ij}, \]  

(1)

where \( \sigma_{ij} \) – stress tensor; \( \varepsilon_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2 \) – strain tensor; \( u_i \) – medium displacement; \( \lambda, \mu \) – Lame parameters; \( \varepsilon_{ii} \) – volume strain.

The fluid flow inside the crack and opening of crack are related by the continuity equation:

\[ \frac{\partial d}{\partial t} + \nabla \cdot \vec{q} = 0, \]  

(2)

where \( d \) – crack opening; \( \vec{q} = (q_1, q_2, q_3) \); \( q_i = d \bar{k}(\partial p / \partial x_i) \); \( \bar{k} \) – effective fracture permeability in the \( i \)-th direction. We assume that the fluid is Newtonian, and the flow inside the fracture is the Poiseuille flow. Then the effective fracture permeability is defined as

\[ \bar{k} = -\frac{d^2}{12\eta_s}, \]  

(3)

where \( \eta_s \) – dynamic viscosity of fracturing fluid. Fluid leak-off from the crack into the medium are not taken into account.

The cohesive fracture model (CZM) will be used to simulate crack growth [15, 16]. This model makes it possible to avoid the singularity at the tip of a crack that arises in the linear elastic fracture mechanics (LEFM). When using CZM, it is assumed that there is an area of material softening in front of the crack tip. The behavior of the material in this area is described by a damage law that links stresses and displacements. The softening of the material begins when a predetermined critical value \( \sigma_c \) (material strength) is reached. Next, the calculation of the new values \( \bar{\sigma}_c \) and modulus of elasticity \( \bar{E} \) is carried out, according to a given damage evolution law, which determines the fracture variable \( D \). New parameters of materials are determined by the following relationships:

\[ \bar{\sigma}_c = (1 - D) \sigma_c, \]
\[ \bar{E} = (1 - D) E, \]  

(4)

and this procedure continues while \( \bar{\sigma}_c, \bar{E} \) will not become zero, which means complete failure of material and crack formation. A more detailed description of this fracture process is given in [13].

The equations described above, which determine the behavior of a material that contains a crack, were solved numerically using an extended finite element method XFEM (eXtended Finite Element Method) [16–19]. The main advantage of this method is the ability to simulate the crack growth process without re-meshing. In this work, the XFEM implementation in the ABAQUS software package was used.

3. Numerical experiments and results
Let us consider the problem of interaction between a hydraulic fracture and a cylindrical cavity. For numerical studies, we use a 3d cube with a side of 10 meters, which contains a through circular cavity with a radius of 0.5 m (Figure 1a). An initial disc crack with a radius of 0.2 m is created in the model.
The distance from the initial crack to the cavity was varied in the course of numerical experiments. In the calculations, a homogeneous elastic medium with the following parameters was used: elastic modulus \(E\) – 3.0 GPa; Poisson's ratio \(\nu\) – 0.3; critical energy of damage \(G_c\) – 120 N/m; critical tensile stress \(\sigma_c\) – 0.75 MPa. The dynamic viscosity of the fluid is 0.001 Pa·s. To achieve the required stress state inside the region, the following boundary conditions were chosen: on the \(y = 0\) plane, displacements along \(y\) are equal to zero; on the plane \(z = 0\) displacements along \(z\) are equal to zero; on the plane \(x = 0\), displacements along \(x\) are equal to zero; pressure \(S_{yy}\) is applied on the plane \(y = 10\) m; pressure \(S_{zz}\) is applied on the plane \(z = 10\) m; pressure \(S_{xx}\) is applied on the plane \(x = 10\) m. A schematic representation of the boundary conditions in the \(xy\) plane is shown in Figure 1b. On the surface of the crack, which is free, the fluid pressure calculated from the solution of the problem (1)–(2) is applied. Also, in Figure 1b, for the convenience of further reasoning, the designations of the cavity radius \(R\) and the distance from the initial crack to the cavity \(s\) are introduced.

Figure 1. (a) 3d model with a cylindrical cavity and an initial disk crack; (b) a schematic representation of the problem statement in the \(xy\) plane.

Figure 2 illustrates the discretization of the model. A mesh formed by hexagons was used in the calculations. The average side size of discrete elements in the crack growth domain was 0.055 meters. Figure 2a shows the mesh in the \(xy\) plane, figure 2b shows the mesh in the slice \(x = 5\) m. near the cavity. The existing elongated elements (the size in one of the directions is much larger than the other) are due to the construction of discretization of the computational domain containing the minimum number of elements. Such elements are located at the edges of the cube and do not affect the results of numerical calculations.

Figure 2. Discretization of the model. (a) mesh in the \(xy\) plane; (b) mesh in the slice \(x = 5\) meters.
As can be seen from the figures, with decreasing distance \(s\), the crack propagates along the axis of the cylindrical cavity, assuming an elongated shape. This behavior is due to the fact that near the boundaries of the cavity, the radial stresses are less than at a distance from it. Due to this, favorable conditions for the growth of the tensile crack are achieved. At the same time, as the distance to the cavity decreases, the circumferential stresses increase, preventing the crack from reaching its boundary even at small values of the distance \(s\) (Figure 3e). If the distance \(s\) is increased, the effect of the cavity will decrease, and the shape of the crack becomes closer to the disk (Figure 3a). It can be noted that for the given parameters of the model, the crack does not significantly propagate around the cavity (in the radial direction), as is well observed when considering the 2d plane problem [11, 12, 14].

Figure 3. 3d hydraulic fracture near a cylindrical cavity at different distances between the initial crack and the cavity under conditions of hydrostatic stress in the medium equal to 3e6 Pa. a), c), e), g) a three-dimensional image of a crack. b), d), f), h) projection of the crack onto the \(xy\) plane. a), b) – the distance between the initial crack and the cavity is equal to two cavity radii; c), d) – the distance is a cavity radius; e), f) – the distance is 1/2 of the cavity radius; g), h) – the distance is 1/4 of the cavity radius.

Table 1 shows the projection areas of the fractures on the \(xy\), \(xz\), and \(yz\) plane, obtained by pumping the same volume of fluid equal to 4.05e-4 m³ and different distances between the initial fracture and the cavity. As above, these results were obtained at a hydrostatic stress in the medium equal to 3 MPa.

| Distance between the initial crack and the cavity (in the radii of the cavity) | Crack projection area on the \(xy\) plane (m²) | Crack projection area on the \(xz\) (m²) | Crack projection area on the \(yz\) (m²) |
|---|---|---|---|
| 2\(R\) | 0.0577 | 0.0611 | 0.8162 |
| \(R\) | 0.1508 | 0.1952 | 1.7462 |
| \(R/2\) | 0.095 | 0.1513 | 1.3112 |
| \(R/4\) | 0.0627 | 0.1671 | 1.2374 |
It can be seen from the table that at $s = 2R$ the crack sizes are smaller than in other cases due to the larger opening. At $s = R$, the maximum crack size was obtained, which then decreased along with a decrease in the distance $s$ due to an increase in crack opening. The increase in crack opening is apparently due to a weakening of the elastic properties of the medium in the region between the crack and the cavity (the thickness of the material between the crack and the surface of the cavity becomes small).

4. Conclusions
In this work, numerical studies of the propagation of a three-dimensional hydraulic fracture crack near a cylindrical cavity in an elastic homogeneous medium are carried out. It is shown that under conditions of a hydrostatic stress state in a medium, the distance between the initial crack and the cavity has a significant effect on crack propagation. With a decrease in this distance, the crack tends to propagate along the axis of the cylindrical cavity, assuming an elongated shape. With increasing distance, the shape of the crack becomes closer to the disk one. In addition, the stresses around the cavity arising from the hydrostatic pressure in the medium prevent the crack from reaching the boundary of the cavity.

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