ARE MASS AND LENGTH QUANTIZED?

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Abstract

We suggest that there are time-varying quanta of mass (gomidia) and of length (somia), thus pointing to a quantization of geometry and gravitation. The present numerical value of the gomidium and somium, are, $10^{-65}$ grams, and $10^{-91}$ centimeters. Gomidia may be responsible for dark matter in the Universe; Heisenberg’s principle, confirms the numerical estimates for gomidia and somia, either for the present Universe, or for Planck’s time.
I. Introduction

We introduce the definitions of micromass and macromass, as well as those of microlength and macrolength, in the spirit of Wesson’s suggestions (Wesson, 2006). We show that by obtaining such quantities for Planck’s time, and the present Universe, both ”micros” coincide with Planck’s mass and length, while for the present Universe, macrolength stands as the radius of the causal Universe, while macromass represents the mass of the Universe.

We find a quantum of mass (”gomidium”) (Berman, 2007; 2007a), and a quantum of length (”somium”), to which we suggest interpretations. In the end of this paper, we discuss the novelties which appear here, in comparison with what has been already published (for instance, by Wesson, 2006). The definitions of macromass, micromass, macrolength, and microlength, given in this paper, are related with gauge parametrizations in penta-dimensional physics (Wesson, 2006).

Dark matter in the Universe, responds for 27% of the total energy density, which is to be represented by the critical one, as far as we accept inflationary scenario (Güth, 1981). So we call the dark matter energy density, \( \rho_\nu \), and we write,

\[
\rho_\nu = 0.27 \rho_{\text{crit}}.
\]  

Berman (2006 b) along with others (see Sabbata and Sivaram, 1994) have estimated that the Universe possess a magnetic field which, for Planck’s Universe, was as huge as \( 10^{55} \) Gauss. The relic magnetic field of the present Universe is estimated in \( 10^{-6} \) Gauss. We can then, suppose that some hypothetical particles with elementary spin, have been aligned with the magnetic field. On the other hand, the spin of the Universe is believed to have increased in accordance with a Machian relation. If we call \( n \) the number of gomidia in the present Universe, and \( n_{Pl} \) its value for Planck’s Universe, we may write, along with Berman (2007; 2007a),

\[
\frac{n}{n_{Pl}} = \frac{L}{L_{Pl}} = 10^{122}.
\]
Then,

\[ n = n_{\text{Pl}} \left[ \frac{R}{R_{\text{Pl}}} \right]^2. \]  

(3)

Thus, \( n \) grows with \( R^2 \).

Now, we write the energy density of \textit{gomidia},

\[ \rho_{\nu} \approx \frac{nm_{\nu}}{3\pi R^3}. \]  

(4)

where \( m_{\nu} \) is the rest mass of the individual \textit{gomidium}.

Berman’s suggestion, in the above citations, imply that all energy densities in the Universe decrease with \( R^{-2} \). Consider for instance, the inertial mass content. Its energy density is given by,

\[ \rho_i = \frac{M}{4\pi R^3}. \]  

(5)

For a Machian Universe, then, \( M \propto R \), from where the \( R^{-2} \) inertial energy density appears in (5).

If, then, \( \rho_{\nu} \propto R^{-2} \), we find:

1\textsuperscript{st.} \( \rho_{\nu} = 0.27 \rho_{\text{Pl}} \left[ \frac{R}{R_{\text{Pl}}} \right]^2 \).  

(6)

2\textsuperscript{nd.} \( m_{\nu} = \frac{\rho_{\text{Pl}} R_{\text{Pl}}^4}{R} \).  

(7)

We see now that while the number of \textit{gomidia} in the Universe increases with \( R^2 \), the rest mass decreases with \( R^{-1} \); we may obtain, with \( R \approx 10^{28} \) cm, that the rest mass of \textit{gomidia} should be, in the present Universe:

\[ m_{\nu} \approx 10^{-65} \text{ g}. \]  

(8)

A law of variation for the number of \textit{gomidia} in the Universe has been found. A law of variation for the rest mass of \textit{gomidia} was also found.

We remind the reader that Kaluza-Klein’s cosmology (Wesson, 1999; 2006; Berman and Som, 1993), considers time varying rest masses, in a penta-dimensional (“induced mass”)
space-time-matter, of which the fifth coordinate is rest mass. The above results can not be rejected, for the time being, by any known data. We point out, that some of the features of the present calculation, resemble some points in a paper by Sabbata and Gasperini (1979).

II. Quantization of inertia and geometry

Wesson (2006), by citing Desloge(1984), comments that by means of the four fundamental ”constants”, Planck’s ($h$), Newton’s ($G$), speed of light ($c$), and cosmological ($\Lambda$), one can obtain two different kind of mass, the micromass ($m$), and the macromass ($M$), given by:

\[ m = \left( \frac{h}{c} \right) \Lambda^{1/2}, \quad (9) \]

and,

\[ M = \frac{c^2}{G} \Lambda^{-1/2}. \quad (10) \]

Micromass involves Planck’s constant, hence its denomination; macromass is defined by means of $G$, so its ”macro” denomination.

Notice that the above constant tetrad, is, of course, overlapping. With the present values for the cosmological ”constant”, $\Lambda = \Lambda_U \approx 10^{-56}$ cm$^{-2}$, it is found,

\[ m(U) \approx 10^{-65} \text{ g}, \quad (11) \]

and,

\[ M(U) \approx 10^{56} \text{ g}. \quad (12) \]

The present Universe’s micromass ($m(U)$), represents a present mass-quantum, i.e., the minimum mass in the present Universe. On the other hand, the present Universe’s macromass ($M(U)$), is approximately the mass of the present Universe ($M_U$).

What Wesson overlooked, is that, when we apply the definitions (9) and (10), by plugging, $\Lambda = \Lambda_{PL} \approx L_{PL}^2 \approx 10^{-66}$ cm$^{-2}$, which stand for Planck’s time values, we find that micromass and macromass coincide approximately with Planck’s mass, $M_{PL}$.
\[ M_{(PL)} = m_{(PL)} = M_{PL} \approx 10^{-5} \text{ g} \] \hspace{1cm} (13)

We are led to consider that, macromass, is always associated to the mass of the Universe \((M_U)\), either in the very early Universe or in the present one.

As to the micromass, we baptize this mass as the quantum mass value \((gomidium, \text{ after F.M.Gomide})\): it is a time-varying mass, because \(\Lambda\) is also so, because we expect its energy density \(\frac{\Lambda}{\kappa}\) depend on \(R^{-2}\) altogether.

We now show, that associated with micromass and macromass, we have two distinct length values, which come associated to the present and Planck’s Universe.

For each mass, we associate two kinds of lengths, namely, the macrolength, \((\lambda_\nu)\), and the microlength \((l_\nu)\); the first one, is Compton’s wavelength, given by,

\[ \lambda_\nu = \frac{\hbar}{m \nu} \] \hspace{1cm} (14)

It is a ”macro”, because it is inversely proportional to micromass.

The second length, that we call ”microlength”, is a gravitationally associated length with mass, which we term the quantum of length, or \(somium\),

\[ l_\nu = \frac{G m}{c^2} \] \hspace{1cm} (15)

It is a ”micro”, because it is proportional to micromass.

We find, a microlength, \(l_\nu\), for the present Universe, with \(m = m_\nu = m_{(U)}\),

\[ l_{\nu(U)} \approx 10^{-91} \text{ cm} \] \hspace{1cm} (16)

while, the macrolength is then found,

\[ \lambda_{\nu(U)} \approx 10^{28} \text{ cm} \] \hspace{1cm} (17)

On the other hand, for Planck’s Universe, the macrolength is found to be,

\[ \lambda_{\nu(PL)} \approx 10^{-33} \text{ cm} \] \hspace{1cm} (18)

and, the microlength has the same numerical value,
\[ l_{\nu(PL)} \approx 10^{-33} \text{ cm} \quad .\] (19)

One can check, that the microlength represents a quantum, the \textit{somium}. Macrolength is represented by the radius of the Universe.

For Planck’s Universe, the \textit{somium}, the macrolength, and the microlength then coincide with the Planck’s radius, \( L_{PL} \).

\section*{III. Heisenberg uncertainty and minimum mass and length}

According to Heisenberg’s uncertainty principle, any two conjugate quantities, in the sense of Hamilton’s canonical ones, carry uncertainties, \( \Delta Q \) and \( \Delta P \), which obey the condition (Leighton, 1959),

\[ \Delta Q \Delta P \approx h \quad .\] (20)

If we consider maxima \( \Delta P \), we obtain minima \( \Delta Q \).

If \( \Delta P \) stands for the uncertainty in linear momentum, given, say, by the product of mass and speed, then, its maximum value must be the product of the largest mass in the Universe by the largest speed,

\[ \Delta P = M_U c \quad .\] (21)

We thus, obtain a minimum length value,

\[ \Delta Q \approx \frac{h}{c M_U} \quad .\] (22)

Now, let us think of the largest time numerical value in the Universe,

\[ t_U \approx 10^{10} \text{ years}. \] (23)

Its conjugate variable, will point out to a minimum inertial energy and a minimum inertial mass (\( \Delta m \)),

\[ \Delta E = c^2 \Delta m \approx \frac{h}{t_U} \quad .\] (24)
It turns out, that we have retrieved, from $\Delta m$ and $\Delta Q$, the minimum mass and length for the present Universe, with the same approximate values attached to $gomidia$ and $somia$, in last Section, also for the present Universe.

Analogously, we could repeat the calculation for Planck’s time and Planck’s mass, and we then would obtain numerically the same values attained by $gomidia$ and $somia$ in Planck’s Universe.

We have, then, support for the quantization of mass and length, in a time-varying fashion, coinciding with the calculation in the last section.

IV. Conclusions

Wesson (2006), dealt classically with $D = 5$ WEP (weak equivalence principle), as a symmetry in the ”induced mass” pentadimensional Kaluza-Klein theory, or even the brane one. The new fifth forces and coordinate are then present. The geodesic equation adds and extra-acceleration.

Microlength and macrolength were found by Wesson, to be good gauge parametrizations for mass, which allow a mass geometry consistent with the rest of Physics. The known laws of Mechanics and conservation of linear momentum, in limiting cases, were also preserved when $m$ is a representation of rest-mass. Because of the standard structure of $D = 5$ Physics as an extension of the $D = 4$ chapter, while introducing an extra coordinate, opens the possibility of quantization in the lower dimension Physics. Wesson even advanced that the Quantum domain would extend to the Cosmos, in the form of a broken symmetry for the angular momenta tied to the gravitational field.

We have therefore, found in our present paper, that the micromass and microlength represent quanta of mass and length. We call them, respectively, $gomidium$ and $somium$, but their numerical values are time-varying: present day’s $gomidium$ is $10^{-65}$ g, while $somium$ is about $10^{-91}$ cm. Planck’s values for $gomidium$ and $somium$, coincide respectively with Planck’s mass and Planck’s length. We have thus hinted that mass is quantized, but geometry is altogether. As gravitation is associated with geometry, quantization of the latter, implies on the former: it seems that quantum gravity has been found. Much of what we
have calculated here, like the Machian derivation, which led to time-varying quanta of mass and length, and also the interpretation, under which macromass and macrolength describe the Universe’s mass and radius, throughout its lifespan, (in particular, Planck’s and present times) are novelties in the literature. Though some of the topics dealt in our paper, were sparsely dealt in Wesson’s books (Wesson, 1999; 2006), and by other authors, we have here given a rational interpretation of otherwise disconnected elements. The numerical value for the present Universe’s microlength (somium), seems to have never appeared in the literature, in any context; our quantization ideas, as far as I know, are also novel; dark matter has been associated with gomidia. All the above, supported by Heisenberg’s uncertainty principle.

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