An interactive web-based tool for fatigue analysis and life prediction

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Abstract

This paper presents an interactive web-based tool for fatigue analysis and life prediction methodology of smooth and notched components. Users around the globe may access it via Internet by means of multiple platforms such as desktop and laptop computers, tablets and/or smart-phones. In particular, the users with a limited fatigue background would benefit from “on-the-fly” fatigue learning experience. It makes a self-explanatory and frustration free web-based education software, which allows users to learn the fatigue fundamentals while expanding their knowledge on modern fatigue analysis methods. The tool is aimed to assist a designer in fatigue analysis and file predictions of parts subjected to constant amplitude, block loading, and variable loading histories. For variable amplitude loading a dedicated spectrum software package is provided, which is essential for a potential clean-up and/or desired modifications of a raw spectrum data. Subsequently, a rainflow method is utilized and the corresponding hysteresis loops at the notch-root or critical location are determined and plotted. In analyses, three convenient and innovative “master curves” techniques are used, namely for Neuber’s rule and also for stress- and strain-based fatigue life approaches.

Keywords: Web-based software, interactive, fatigue life, notch analysis, master curve.

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1. Introduction

In general, fatigue damage occurs in materials when subjected to variable amplitude stresses that are below the ultimate tensile strength, or even the yield strength of the material [1]. To avoid fatigue cracking, components are designed by allowing a sufficient safety factor. Too high safety factor would result in bulky and expensive components. Hence, R&D departments employ highly trained engineers to perform fatigue analysis on components before and during a prototype development. This involves extensive time and resources to create new designs. Due to cut off in R&D budgets many new designs are based on old and proven designs.

In the global and highly competitive market, the R&D departments are mostly afforded by relatively large organizations. These R&D departments routinely acquire dedicated fatigue software, which are accompanied often with relatively expensive annual license fees. On the other hand, smaller organizations rather seldom have R&D departments but they may have a periodic need for fatigue analysis. Therefore, there is an overall demand for a straightforward web-based fatigue analysis tool accessible via Internet, which can be used occasionally for a fee on as needed basis. Specifically, in an age of technological advancement, which is growing exponentially, there is an increasing trend that students in colleges shift more towards an ‘e-learning’ platform to grasp knowledge outside of the classroom environment. In order to carry out a meaningful fatigue analysis it requires some fundamental

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**Nomenclature**

| Symbol | Description |
|--------|-------------|
| $E$ | modulus of elasticity |
| $b$ | fatigue strength exponent |
| $c$ | fatigue ductility exponent |
| $k_i$ | elastic stress concentration factor |
| $k_f$ | fatigue notch factor |
| $2N_f$ | number of reversals to failure |
| $S$ | nominal stress |
| SWT | Smith-Watson-Topper parameter |
| $\varepsilon_a$ | applied strain amplitude |
| $\varepsilon_{a e}$ | elastic strain amplitude, $\sigma_a/E$ |
| $\varepsilon_{a p}$ | plastic strain amplitude |
| $\varepsilon_f$ | fatigue ductility exponent |
| $\gamma$ | exponent in Walker’s equation |
| $\tau_{x y}$ | shear stress amplitude |
| $\sigma_a$ | applied stress amplitude |
| $\sigma_{ar}$ | fully reversed stress amplitude |
| $\sigma_{ar} G$ | fully reversed stress amplitude, Goodman |
| $\sigma_{ar} M$ | fully reversed stress amplitude, Morrow |
| $\sigma_{ar} SWT$ | fully reversed stress amplitude, SWT |
| $\sigma_{ar} W$ | fully reversed stress amplitude, Walker |
| $\sigma_{ax}$ | stress amplitude, x-component |
| $\sigma_{ay}$ | stress amplitude, y-component |
| $\sigma_{a1}$ | stress amplitude, principal component (1) |
| $\sigma_{a2}$ | stress amplitude, principal component (2) |
| $\sigma^*_a$ | effective stress amplitude |
| $\sigma_m$ | uniaxial mean stress |
| $\sigma_{max}$ | uniaxial maximum stress |
| $\sigma_{mx}$ | mean stress, x-component |
| $\sigma_{my}$ | mean stress, y-component |
| $\sigma_{m1}$ | mean stress, principal component (1) |
| $\sigma^*_m$ | effective mean stress |
| $\sigma_u$ | ultimate stress |

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Figure 1 Illustration of the web-based access.
knowledge, which most students would not have learned during their undergraduate studies. Also, most practices in small companies are limited when it comes to fatigue analysis and life prediction know-how experiences. For more than a decade, there are available fatigue analyses sites on-line [e.g. 2, 3]. At present, these sites provide the most comprehensive material databases and elaborated fatigue analyses that include also fatigue crack propagation and multi axial fatigue analyses. Hence, this paper is focused on a simple, self-explanatory and interactive web-based fatigue analyses learning tool dedicated to the first time or unexperienced users.

The aim of this paper is twofold. Firstly, it offers (in Appendix) a brief overview, which discusses fundamental similarities and differences among the most popular fatigue life prediction approaches. Secondly, it presents an interactive web-based tool for learning a modern fatigue analysis and life prediction methodology of smooth and notched components. The web-based tool may be easily access by means of multiple platforms such as desktop and laptop computers, tablets and/or smart-phones as it is illustrated in Fig. 1. A user with limited fatigue background would benefit from the proposed interactive “on-the-fly” learning experience.

At each step, concise fundamental fatigue information is provided without the need of a tutorial handbook. It constitutes interactive, self-explanatory web-based educational tool, which allows the users to learn the fatigue fundamentals as well as to expand and master their knowledge on modern fatigue analysis methods. For variable amplitude loading histories a rainflow method is used to count the cycles and to determine the corresponding hysteresis loops at the critical locations. In addition, a dedicated spectrum analysis tool is provided which enables clean-up and desired modifications of a raw spectrum data. During analysis the relevant interactive graphs and calculated values are displayed. The following sections provide a summary of the proposed web-based interactive fatigue analysis learning tool.

2. Structure/flowchart of the web-based software

The core structure of the proposed web-based software shown in Fig. 2 provides multiple options for users to select.

![Diagram of the web-based software structure](image-url)
Figure 2 General structure of the educational version of the web-based fatigue analysis tool. Unexperienced or first time users could be confused given so many options pertaining in particular to life prediction models. In order to assist such users, a brief overview of the most popular fatigue approaches, i.e. stress-, strain-, and stress/strain-based are provided in Appendix. The objective of this overview on fatigue life predictions approaches is to map-up common similarities and differences among them.

3. Web-based software

The website is structured to be simple and self-explanatory, flowing from Step 1 to Step 7, as it is shown in Fig. 3. The current website features an educational version of the software along with video examples of using the software.
The website also provides additional spectrum analysis software to prime the raw spectrum data before fatigue life prediction analysis can be carried out using the educational version. In this section, the analysis procedure is discussed with different options available on the website at www.fatiguenet.com.

As the user enters the website, the home page is presented. Under ‘Educational Version’, the ‘software’ tab is selected, which presents the life predication software. Below are the steps involved with explanation and screenshot of the steps depicted in Fig. 3. In this educational version, the user would be presented with helpful tips and information at each step in seven languages with English being default.

**Step 1: Select Material**

In this step, the desired material and units are selected. The educational version only provides five materials with an additional feature to input user defined material properties. The full version would provide a large database of materials.

**Step 2: Specimen Type**

In this step, the users are provided with an option to choose a smooth or a notched specimen/component. Selecting a smooth specimen would result in a $k_t$ value of 1 to be used in the analysis, which the users cannot change. While for a notched specimen the users would be able to input the desired $k_t$ (or $k_f$) value ($k_t = 2$ is default) in the proceeding step. In general, the smooth specimen/components represents a situation when the actual elastic-plastic stresses and/or strains are known. On the other hand, the notched specimen/component represents a situation when only linear elastic stresses are known.

**Step 3: Input type**

If smooth specimen is selected in Step 2, users would have the option to select either stress or strain as the input data type. In case notched specimen is selected, users have the option to select nominal stress or the elastic notch-root stress obtained from finite element analysis (FEA) together with $k_t = 1$ as the input data type.

**Step 4: Loading Type**

The user can select three loading type to be analyzed. The software is designed to analyze the following loading types:

1. Constant Amplitude
2. Block Loading
3. Spectrum Loading

**Step 5: Input Data**

In this step $k_t$ or $k_f$ value is specified. By selecting a smooth specimen the value is displayed as 1 and cannot be altered. In case if notched specimen, the default value is 2 but may be changed according to the users need. For a constant amplitude loading the user shall specify the loading levels as $S_{level\,1}$ and $S_{level\,2}$. Loading always starts from zero and continue towards $Level\,1$, which can be the maximum or the minimum stress. Proper information is displayed if the user chooses Block Loading or Spectrum Loading options.

**Step 6: Neuber’s Analysis/Hysteresis Loops**

Selecting “Analyze” in Step 5 displays an interactive Neuber’s “master” curve depicted in Fig. 4, which plays a pivotal role in the proposed stress/strain analysis software. The Neuber’s interactive master curve is unique for a given material. Any point on this curve is related to the apparent elastic notch-root stress, which can be calculated as
a product of the nominal stress S and $k_t$ or can be obtained from linear FEA (see Step 3). Then, selecting the hysteresis button displays the hysteresis loop or loops for the given cyclic loading type.

![Figure 4: Screenshot of the Ramberg-Osgood and Neuber's interactive master curves.](image)

**Figure 4** Screenshot of the Ramberg-Osgood and Neuber’s interactive master curves.

### Step 7: Life Prediction Criteria

In this step the life prediction method based on stress, strain or stress-strain is selected. The stress-based methods include four well known approaches for mean stress effects, i.e. Goodman [5], Morrow [6], SWT [7] and Walker [8]. Strain-based approaches consist of the strain versions of Goodman, Morrow [6] and Kujawski-Ellyin [11] models. The SWT parameter is available as the stress/strain-based method. All three methods are reviewed and discussed in Appendix. A critical plane method of Fatemi-Socie [12] and the deviatoric formulation of the SWT parameter proposed by Kujawski [13] are not active in this educational version.

The output window provides a consolidated numerical values for maximum, minimum, mean and amplitude of stresses and strains at the notch tip along with reversals or repetitions to failure. The “master” life predictions graph (explained in Appendix) is depicted in Fig. 5 and the corresponding fatigue lives and endurance limit stresses are provided in Table 1. After completion of the analysis, three printing options are available such as: print this page, print this table, and print explanations only.

### 4. Conclusions

An interactive web-based learning tool for a modern fatigue analysis and life prediction methodology of smooth and notched components has been developed. Users from around the globe may access it via Internet by means of multiple platforms such as desktop and laptop computers, tablets and/or smart-phones. In particular, the users with a limited fatigue background would benefit from possibility to expand their fatigue learning experience. The tool can be used to analyze constant amplitude, block loading and spectrum loading histories. Dedicated spectrum analysis software is also included. Life predictive capabilities include stress-based, strain-based and stress-strain based methods. In analyses, three convenient and novel “master curves” techniques are used, namely for Neuber’s rule and also for stress- and strain-based fatigue life approaches.
Figure 5 Screenshot of stress-based life prediction curve for various approaches.

Table 1 Screenshot of comparison table showing numerical values of life prediction.

| Material name:2024 T351- AL | Goodman | Morrow | SWT | Walker |
|------------------------------|---------|--------|-----|--------|
| Life(Reversals)              | 794     | 1036   | 1016| 1016   |
| $\sigma_{ar}$                | 436     | 423    | 424 | 424    |
| $\sigma_{a,endurance}$       | 147     | 143    | 141 | 143    |

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Appendix

A1. Stress-based fatigue life prediction models
A1.1 Uniaxial loading

The classical stress-based fatigue life prediction model proposed by Basquin [4] has the following form when is written in terms of reversals for fully-reversed loading

$$\sigma_{ar} = \sigma' \left(2N_f\right)^b$$  \hspace{1cm} (1)

For cycles with mean stress, there are three popular approaches used to estimate the corresponding effective fully-reversed stress amplitude, i.e.

- Goodman approach [5]

$$\sigma_{arG} = \sigma_{a} \left(\frac{1}{1 - \sigma_m / \sigma_u}\right)$$  \hspace{1cm} (2)
For $\sigma_m < 0$ Goodman’s relation is typically set to $\sigma_{arG} = \sigma_a$.

- Morrow approach [6]

$$\sigma_{arM} = \sigma_a \left( \frac{1}{1 - \sigma_m / \sigma_f} \right)$$

(3)

- The stress version of the SWT parameter [7]

$$\sigma_{arSWT} = (\sigma_a \sigma_{max})^{0.5} = \sigma_a \left( 1 + \sigma_m / \sigma_a \right)^{0.5}$$

(4)

- Walker model [8]

$$\sigma_{arW} = \sigma_a^{1-\gamma} \sigma_{max}^{1-\gamma} = \sigma_a \left( 1 + \sigma_m / \sigma_a \right)^{1-\gamma}$$

(5)

for $\gamma = 0.5$ Walker’s model is equivalent to the SWT parameter.

Substituting Eqs. (2), (3), (4) and (5) into Eq. (1), the corresponding fatigue life can be estimated using a “master” Basquin’s S-N curve, Eq. (1), for fully-reversed amplitude:

$$\frac{\sigma_a}{1 - \sigma_m / \sigma_f} = \sigma_f^{1/2} (2N_f)^b$$

(Goodman)

(6)

$$\frac{\sigma_a}{1 - \sigma_m / \sigma_f} = \sigma_f^{1/2} (2N_f)^b$$

(Morrow)

(7)

$$\sigma_a \left( 1 + \sigma_m / \sigma_a \right)^{0.5} = \sigma_f^{1/2} (2N_f)^b$$

(SWT)

(8)

$$\sigma_a \left( 1 + \sigma_m / \sigma_a \right)^{1-\gamma} = \sigma_f^{1/2} (2N_f)^b$$

(Walker)

(9)

In all these approaches, the equivalent fully-reversed stress amplitudes (given by the left hand side of Eqs. (6), (7), (8) and (9)) are calculated by modifying accordingly the applied stress amplitude $\sigma_a$ for a given mean stress $\sigma_m$.

Existing data indicate that Goodman approach may provide the most conservative life predictions for tensile mean stress whereas too optimistic or not conservative predictions for compressive mean stress. The common notion is that stress-based approaches are best suited for high-cycle fatigue (HCF). In general, for smooth specimens a tensile mean stress reduces the initiation and propagation life when compared to fully-reversed (zero mean stress) loading, while a compressive mean stress prolongs fatigue life. This can be rationalized by means of positive and negative cyclic creep (ratcheting), which may promote opening or closing of a fatigue crack, respectively. On the other hand, under stress control loading a mean stress relaxation is restricted.

### A1.2. Biaxial loading

Usually, fatigue cracks start from the free surface where biaxial in-pane state of stress ($\sigma_a$, $\sigma_y$ and $\tau_{xy}$) prevails. For such biaxial loading effective stress amplitude can be defined as

$$\sigma_a^* = \left( \sigma_{ax}^2 - \sigma_{ax} \sigma_{ay} + \sigma_{ay}^2 + 3\tau_{axy}^2 \right)^{0.5}$$

(10a)
In terms of principal stress components (assuming $\sigma_{a3} = 0$) one can write

$$\sigma^*_a = (\sigma^2_{a1} - \sigma_{a1}\sigma_{a2} + \sigma^2_{a2})^{0.5}$$  \hspace{1cm} (10b)

Also, an effective mean stress can be calculated as

$$\sigma^*_m = \sigma_{mx} + \sigma_{my} = \sigma_{m1} + \sigma_{m2}$$  \hspace{1cm} (11)

After the effective amplitude and mean stress are determined, then they can be treated as uniaxial values.

### A2. Stress-based fatigue life prediction models

For cyclic loading when considerable plastic strain is present, such as in plastic zone at a notch, a strain-based approach is commonly utilized. For such situations called low-cycle fatigue (LCF), plastic strain-based relationship proposed by Coffin [8] and Manson [9] is usually adopted

$$\varepsilon_{ap} = \varepsilon_f (2N_f)^c$$  \hspace{1cm} (12)

Dividing both sides of Eq. (7) by $E$ and after arranging, one gets

$$\frac{\varepsilon_{ae}}{1 - \sigma_m / \sigma_f} = \frac{\sigma_f}{E} (2N_f)^b$$  \hspace{1cm} (13)

where $\varepsilon_{ae} = \sigma/E$ in an elastic strain amplitude.

Adding the left hand sides and the right hand sides of Eqs. (12) and (13) results in Morrow’s approach

$$\frac{\varepsilon_{ae}}{1 - \sigma_m / \sigma_f} + \varepsilon_{ap} = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon_f (2N_f)^c$$  \hspace{1cm} (14)

The above Morrow’s approach is usually known in the following form [6]

$$\varepsilon_a = \frac{\sigma_f - \sigma_m}{E} (2N_f)^b + \varepsilon_f (2N_f)^c$$  \hspace{1cm} (15)

which is obtained by modifying Eq. (13) to the following form

$$\varepsilon_{ae} = \frac{\sigma_f - \sigma_m}{E} (2N_f)^b$$  \hspace{1cm} (16)

and replacing the sum of elastic and plastic strain amplitudes $\varepsilon_{ae} + \varepsilon_{ap}$ by the total strain amplitude $\varepsilon_a$.

Also, dividing both sides of Eq. (6) by $E$ and combining it with Eq. (12) one can obtain a strain version of Goodman’s approach

$$\frac{\varepsilon_{ae}}{1 - \sigma_m / \sigma_u} + \varepsilon_{ap} = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon_f (2N_f)^c$$  \hspace{1cm} (17)
Kujawski and Ellyin [11] proposed the following formula to account for mean stress effect

$$\frac{\varepsilon_{ae}}{\sqrt{\frac{\sigma_m}{2\sigma_a}} + \sqrt{(\frac{\sigma_m}{2\sigma_a})^2 + 1}} = \frac{\sigma_f}{E} (2N_f)^b$$

(18)

Combining Eqs. (12) and (18) results in the following strain version of Kujawski and Ellyin’s model

$$\sqrt{\frac{\sigma_m}{2\sigma_a}} + \sqrt{(\frac{\sigma_m}{2\sigma_a})^2 + 1}$$

$$\frac{\varepsilon_{ae}}{\sigma_{max}} = \varepsilon_{ap} = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon_f (2N_f)^c$$

(19)

It can be noted that the right hand sides of Eqs. (14), (17) and (19) represents a strain-based “master curve” which corresponds to fully-reversed strain controlled tests with zero mean stress. For each approach only the elastic strain amplitude is altered accordingly for a given mean stress whereas the plastic strain amplitude is not modified.

A2. Stress/strain-based fatigue life prediction models

The SWT parameter [7] is one of the most widely used stress/strain approaches to account for a mean stress effect in fatigue life analysis. In this approach, a product of $\varepsilon_a\sigma_{max}$ is linked to fatigue life by the following relationship

$$\varepsilon_a\sigma_{max} = \frac{\sigma_f^2}{E} (2N_f)^{2b} + \sigma_f e_f (2N_f)^{b+c}$$

(20)

It can be noted that using the relation $\sigma_{max} = \sigma_d(1+\sigma_m/\sigma_a)$ and Eq. (8) the above SWT parameter can be written in the following strain version as

$$\varepsilon_a \sqrt{1+\sigma_m/\sigma_a} = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon_f (2N_f)^c$$

(21)

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