Analysis and Simulation of the Impact of Inertial Force on the Impact Point of Missile Weapons

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Abstract. In order to explore the impact of inertial force on missile impact point, the law of the missile impact point error with missile range, aiming azimuth and launch point latitude under the influence of inertial force is analyzed. In this paper, based on the basic principles of missile ballistics and theoretical mechanics, the trajectory and the distribution of impact point under the influence of inertial force are simulated and analyzed. The results show that the impact point error affected by inertial force is positive correlation to the range of missile when the aiming azimuth is fixed. In the case of a certain range, with the change of the aiming azimuth, the impact point error will also change according to a certain rule. When the range and aiming azimuth are fixed, the impact point error decreases with the increase of latitude.

1. Introduction
The missile is subjected to five main forces during its flight: engine thrust, aerodynamic force, gravity, Coriolis inertia force and implicated inertia force, of which the coriolis inertia force and implicated inertia force are collectively called the inertial force. The inertial force is the force generated when the missile moves with the rotating earth. Since the inertial force follows the whole flight process of the missile, its influence on the landing point of the missile cannot be ignored. Therefore, it is very important to study the influence of inertial force on missile hitting accuracy.

The causes of missile landing point error are divided into two categories: guidance error and unguidance error. At present, there are many kinds of research on the influencing factors of missile landing point deviation. The research on the impact of the unguided error on the landing point deviation mainly focuses on the influence of the disturbance gravity and vertical deviation on the landing point deviation. There are few researches on the effect of inertial force on missile landing point. The missile landing point under the influence of inertia force is simulated and analyzed in this paper.

2. Analysis of generating mechanism of inertia force
Inertial force is: when the object has acceleration, the inertia of the object itself will cause the object itself to maintain the original motion. [1] Newton's laws of motion do not apply in non-inertial systems, where inertial forces are assumed for ideological convenience. The inertial forces that affect missile flight are divided into implicated inertial forces and Coriolis inertial forces [2].
2.1. Implicated inertia force

The resultant motion of points can be divided into three types: absolute motion, relative motion and implicated motion [3]. At a certain point, absolute motion equals the sum of relative motion and implicated motion.

If we call $\bar{a}_a$ the absolute acceleration and $\bar{a}_r$ the relative acceleration. Then the involved acceleration $\bar{a}_e$ is:

$$\bar{a}_e = \bar{a}_a - \bar{a}_r$$ (1)

Where $\bar{a}_e$ is:

$$\bar{a}_e = \omega^2 \cdot \hat{r} - \hat{\omega} \cdot (\hat{\omega} \cdot \hat{r})$$ (2)

$\hat{\omega}$ is the angular velocity of the earth rotation, and $\hat{r}$ is the geocentric vector diameter of the missile. Suppose $\omega_x$, $\omega_y$, and $\omega_z$ are respectively the components of the earth's rotation angular velocity under the emission system, then

$$\begin{cases}
\omega_x = \omega \cos B_r \cos A_r \\
\omega_y = \omega \sin B_r \\
\omega_z = -\omega \cos B_r \sin A_r
\end{cases}$$ (3)

Where $B_r$ and $A_r$ are the astronomical latitude and astronomical aiming azimuth of the launching point respectively. Make

$$\begin{cases}
a_{11} = \omega_x^2 - \omega_z^2 \\
a_{12} = a_{21} = -\omega_x \omega_y \\
a_{13} = a_{31} = -\omega_x \omega_z \\
a_{22} = \omega_y^2 - \omega_z^2 \\
a_{23} = a_{32} = -\omega_y \omega_z \\
a_{33} = \omega_z^2 - \omega_x^2
\end{cases}$$ (4)

The three components of the missile's involved acceleration under the launch system are

$$\begin{cases}
a_{ex} = a_{11}(R_{ox} + x) + a_{12}(R_{oy} + y) + a_{13}(R_{oz} + z) \\
a_{ey} = a_{21}(R_{ox} + x) + a_{22}(R_{oy} + y) + a_{23}(R_{oz} + z) \\
a_{ez} = a_{31}(R_{ox} + x) + a_{32}(R_{oy} + y) + a_{33}(R_{oz} + z)
\end{cases}$$ (5)

2.2. Coriolis inertia force

Formula (1) is no longer applicable when the implicated motion is a fixed axis rotation. And then there will be an extra acc in the formula for the resultant acceleration called the Coriolis acceleration, and $\bar{a}_c$, is called the Coriolis acceleration, and
The three components of the missile’s Coriolis acceleration under the launch system are

\[
\ddot{a}_c = \begin{bmatrix} \dot{e}_x & \dot{e}_y & \dot{e}_z \\ \omega_x & \omega_y & \omega_z \\ v_x & v_y & v_z \end{bmatrix}
\]  

(6)

make

\[
\begin{align*}
b_{12} &= -b_{21} = 2\omega_z \\
b_{31} &= -b_{13} = 2\omega_y \\
b_{23} &= -b_{32} = 2\omega_x
\end{align*}
\]  

(7)

The three components of the missile’s Coriolis acceleration under the launch system are

\[
\begin{align*}
a_{1x} &= b_{12}v_y + b_{13}v_z \\
a_{1y} &= b_{21}v_x + b_{23}v_z \\
a_{1z} &= b_{31}v_x + b_{32}v_y
\end{align*}
\]  

(8)

3. Drop point deviation calculation

For standard trajectories, range angles and azimuths from launch point to landing point are \( \beta_c \) and \( \alpha_c \). When inertia force is not considered, the range Angle and azimuth Angle from the launching point to the landing point are \( \beta'_c \) and \( \alpha'_c \), then there is

\[
\begin{align*}
\Delta \beta_c &= \beta_c - \beta'_c \\
\Delta \alpha_c &= \alpha_c - \alpha'_c
\end{align*}
\]  

(9)

Then the longitudinal deviation \( \Delta L \) and the transverse deviation \( \Delta H \) are

\[
\begin{align*}
\Delta L &= N_c \Delta \beta_c \\
\Delta H &= N_c \sin \beta'_c \Delta \alpha_c
\end{align*}
\]  

(10)

Where \( N_c \) is the unitary radius of the point. The drop point deviation \( \Delta R \) is

\[
\Delta R = \sqrt{\Delta L^2 + \Delta H^2}
\]  

(11)

4. Simulated analysis

According to the above analysis, the influence of inertial force on the accuracy of missile landing point is related to the range, aiming azimuth and launching point latitude.

4.1. Impact of range

Fixed transmitting point longitude and latitude are 120° and 0° respectively, aiming azimuth is 90°. The range is changed by varying the shutdown time, which decreases from 66 seconds to 46 seconds, recording a set of data every 2 seconds. The corresponding ranges of different shutdown times are shown in Table 1.
Table 1. Range for different shutdown times.

| Shutdown time/s | Range/m  | Shutdown time/s | Range/m  |
|----------------|----------|----------------|----------|
| 66             | 654200.70| 54             | 358071.77|
| 64             | 544655.43| 52             | 298003.63|
| 62             | 535932.56| 50             | 275392.76|
| 60             | 451362.47| 48             | 243310.71|
| 58             | 431020.84| 46             | 231420.61|
| 56             | 360085.41|                |          |

When the launching point latitude and aiming azimuth are fixed, the variation relationship between range and drop point deviation under the influence of inertial force is shown in Fig. 1.

As can be seen from fig. 1, when the latitude of launching point and aiming azimuth are fixed, with the increase of the range, the drop point deviation also increases, and the relationship between the two changes is approximately linear. The range is about 100 times the drop point deviation.

Figure 1. Variation relationship between range and drop point deviation.

4.2. Impact of aiming azimuth

For a flight trajectory with a range of 600km, the longitude and latitude of the fixed launching point are 120° and 0° respectively, then the azimuth Angle of the aiming is within [-180°, 180°], and a set of data is recorded every 30° from -180°. The fall point deviation under the influence of inertial force is shown in Fig. 2.
4.3. Impact of launching point latitude
For a flight trajectory with a range of about 600km, the longitude of the fixed launching point is 120°, and the latitude of the launching point is (0°, 15°, 30°, 60°). The aiming azimuth Angle is within [-180°, 180°]. Starting from -180°, a set of data is recorded every 30°.
As can be seen from Fig. 3, when the launching point is at different latitudes, the variation law of the falling point deviation caused by inertial force is the same with the aiming azimuth, but the variation degree is different. As the latitude of the transmitting point increases, the drop point deviation caused by inertial force becomes smaller.

4.4. Impact of coriolis inertial force and implicated inertial force

The longitude and latitude of the fixed launching point are 120° and 0°, respectively, and the aiming azimuth is 90°. The variation relationship between the range and the missile drop point deviation under the influence of the coriolis inertia force and the implicated inertia force is shown in Fig. 4.

The following conclusions can be drawn from Fig. 4:

1. The drop point deviation under the influence of coriolis inertia force and implicated inertia force increases with the increase of the range;
2. The influence of coriolis inertia force is obviously greater than that of implicated inertia force;
3. The increase of the influence of coriolis inertia force is also greater than that of the implicated inertia force.

![Figure 4. The influence of coriolis inertial force and implicated inertial force on drop point deviation.](image)

5. Conclusion

Aiming at the influence of inertial force on missile landing point, this paper systematically analyzes the mechanism of inertial force during missile flight, and realizes the conversion between launching coordinate system and geocentric coordinate system, and the conversion between geocentric coordinate system and geocentric coordinate system as well as the calculation of landing point deviation. By establishing a ballistic model, the results of missile landing point deviation under the influence of inertial force at different range, aiming azimuth and launching point latitude are obtained, and the variation law of missile landing point deviation with range, aiming azimuth and launching point latitude under the influence of inertial force is analyzed. The research in this paper has important reference significance for improving the accuracy of missile landing point.
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