A symmetry for heavy nuclei: Proxy-SU(3)

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Abstract. The SU(3) symmetry realized by J. P. Elliott in the sd nuclear shell is destroyed in heavier shells by the strong spin-orbit interaction. However, the SU(3) symmetry has been used for the description of heavy nuclei in terms of bosons in the framework of the Interacting Boson Approximation, as well as in terms of fermions using the pseudo-SU(3) approximation. We introduce a new fermionic approximation, called the proxy-SU(3), and we comment on its similarities and differences with the other approaches.

PACS. 21.60.Fw Models based on group theory – 21.60.Ev Collective models

1 Introduction

The SU(3) symmetry has been introduced in nuclear structure by J. P. Elliott [1,2], who considered the sd shell nuclei and showed the microscopic origins of the connection between the nuclear quadrupole deformation and SU(3). A generalization of the Elliott SU(3) scheme to more than one nuclear shell has been obtained in the framework of the microscopic symplectic model [3]. Since then the SU(3) symmetry has been used in the framework of various algebraic models, especially for the study of medium-mass and heavy deformed nuclei, where the LS coupling scheme of the Elliott model breaks down [4], while microscopic calculations are still out of reach. Descriptions in terms of bosons have been given in the framework of the Interacting Boson Model (IBM) [5] and of the Interacting Vector Boson Model (IVBM) [6], while fermionic descriptions have been provided by the Fermion Dynamical Symmetry Model (FDSM) [7]. It underlies also the pseudo-SU(3) scheme [8,9,10,11,12,13,14], which we will discuss below, as well as the quasi-SU(3) symmetry [15,16], in which an approximate restoration of LS coupling in heavy nuclei is obtained, based on the smallness of certain \( \Delta j = 1 \) matrix elements.

On the other hand, many properties of heavy deformed nuclei have been successfully described in detail in terms of the Nilsson model [17,18,19]. Nilsson states are labelled by \( K[Nn_l \Lambda] \), where \( N \) is the number of oscillator quanta, \( n_l \) is the number of quanta along the cylindrical symmetry axis, \( \Lambda \) is the projection of the orbital angular momentum along the symmetry axis, and \( K \) is the the projection of the total angular momentum along the symmetry axis, connected to \( \Lambda \) by \( K = \Lambda + \Sigma \), where \( \Sigma \) is the projection of the spin along the symmetry axis. For large deformations, the Nilsson wave functions reach the asymptotic limit, in which these quantum numbers become good quantum numbers, and they remain rather good even at intermediate deformation values [18].

Ben Mottelson has remarked [20] that the asymptotic quantum numbers of the Nilsson model can be seen as a generalization of Elliott’s SU(3), applicable to heavy deformed nuclei. Working in this direction, we have shown [21,22] that a proxy-SU(3) symmetry of the Elliott type can be developed in heavy deformed nuclei. The development of the proxy-SU(3) scheme is based on the so-called 0[110] pairs of Nilsson orbits related by \( \Delta K[\Delta N_\Lambda n_z \Delta \Lambda] = 0[110] \) [23]. These pairs, which are characterized by high overlaps [24], have been shown to play a key role in the onset and development of nuclear deformation in the rare earth region [23,24].

In the proxy-SU(3) scheme we also focus attention on Nilsson 0[110] pairs, but in a different way. Instead of taking advantage of proton-neutron pairs, we use proton-proton and neutron-neutron pairs. In this way we reveal an approximate SU(3) symmetry in heavy deformed nuclei, which can be used for predicting nuclear properties within the SU(3) symmetry using algebraic methods, as we shall see in Refs. [25,26].
One can thus think of replacing all of the invading $1h_{11/2}$ orbitals (the upper group of dashed lines in Fig. 1), except the $11/2[505]$ orbital (the dotted line in Fig. 1) in the 50-82 shell by their deserting $1g_9/2$ counterparts (the lower group of dashed lines in Fig. 1), expecting nuclear properties related to angular momentum to be little affected, since angular momentum projections remain intact. However, one should carefully take into account that during this replacement the $N$ and $n_1$ quantum numbers have been changed by one unit each, thus changing the sign of the parity. These changes will obviously affect the selection rules of various relevant matrix elements, as well as the avoided crossings in the Nilsson diagrams. Detailed calculations to be shown in Ref. [28] will demonstrate that the changes inflicted in the Nilsson diagrams by these modifications are indeed minimal.

The $1h_{11/2}$ $11/2[505]$ orbit has no $0[110]$ partner in the $1g_9/2$ shell, thus it has been excluded from this replacement. However, this orbit lies at the top of the 50-82 shell in the Nilsson diagrams [17,18], where it is unlikely to find nuclei with large deformations. The same remark applies to similar orbits in other shells such as the 13/2[606] orbit in the 82-126 shell.

After these two approximations have been performed, one is left with a collection of orbitals which form exactly the full $sdg$ shell, which is known to possess a $U(15)$ symmetry, having an $SU(3)$ subalgebra. However, in axially symmetric deformed nuclei the relevant symmetry is not spherical, but cylindrical [30]. As a consequence, the relevant algebras are not $U(N)$ Lie algebras, but more complicated versions of deformed algebras [31,32,33,34,35,36]. Nevertheless, one can expect that some of the $SU(3)$ features would appear within the approximate scheme.

Since the present approximation scheme is based on the replacement of the invading from above abnormal parity orbitals (except the one with highest angular momentum) by their $0[110]$ counterparts deserting to the lower shell, with the latter being used as proxies of the former in subsequent considerations, we are going to call this approximation the proxy-$SU(3)$ model.

The same approximation can be made in the 28-50, 82-126, 126-184 shells, which thus become approximate pf, sd, $sdg$ shells, respectively. These shells are known to correspond to $U(10)$, $U(21)$, $U(28)$ algebras having $SU(3)$ subalgebras (see [29] and references therein).
4) Superdeformed bands \[53-54\].
5) Double-beta decay \[57-58,59\], neutrinoless double-beta decay \[60\], and double-electron capture \[61\].
6) It should be pointed out that in the case of pseudo-SU(3) a unitary transformation connecting the normal parity orbitals to the pseudo-SU(3) space is known \[62\].

It is expected that using a large number of results regarding the study of fermionic systems by algebraic techniques, already developed and used in the pseudo-SU(3) framework, one would be able to perform further complementary studies using the proxy-SU(3) model.

4 Conclusions

A new approximate SU(3) symmetry applicable in heavy deformed nuclei has been suggested \[21,22\], called the proxy-SU(3) model. In Ref. \[28\] a detailed numerical study of deformed nuclei has been suggested \[21,22\], called the proxy-SU(3) scheme. In Ref. \[28\] a detailed numerical study of deformed nuclei has been suggested \[21,22\], called the proxy-SU(3) scheme. In Ref. \[28\] a detailed numerical study of deformed nuclei has been suggested \[21,22\], called the proxy-SU(3) scheme. In Ref. \[28\] a detailed numerical study of deformed nuclei has been suggested \[21,22\], called the proxy-SU(3) scheme.

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