Constraints on $\Xi^-$ nuclear interactions from capture events in emulsion

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Abstract

Five $\Xi^-p \rightarrow \Lambda\Lambda$ two-body capture events in $^{12}C$ and $^{14}N$ emulsion nuclei, in which a pair of single-$\Lambda$ hypernuclei is formed and identified by their weak decay, have been observed in $(K^-, K^+)$ emulsion exposures at KEK and J-PARC. Applying a $\Xi^-$-nucleus optical potential methodology to study atomic and nuclear transitions, we confirm that these capture events occur from Coulomb assisted 1$\rho\Xi^-$ nuclear states. Long-range $\Xi N$ shell-model correlations are found essential to achieve consistency between the $^{12}C$ and $^{14}N$ events. The resulting $\Xi$-nuclear interaction is strongly attractive, with $\Xi$ potential depth in nuclear matter $V_\Xi \gtrsim 20$ MeV. Implications to multi-strangeness features of dense matter are outlined.

Keywords: hyperon strong interactions; $\Xi^-$ atoms and hypernuclei.

1. Introduction and background

Recent two-particle correlation studies of $p\Lambda$, $\Lambda\Lambda$ and $\Xi^-p$ pairs measured by ALICE [1, 2, 3, 4] in $pp$ and $p$-Pb ultra-relativistic collisions at TeV energies have triggered renewed interest in Strangeness $S \neq 0$ baryon-baryon interactions and consequences thereof to strange hadronic matter. In particular, the $\Xi^-p$ interaction was shown to be attractive [3], in good agreement with the recent HAL-QCD lattice calculations reaching $m_\pi = 146$ MeV [5]. Understanding the strength of the $S = -2 \Xi N$ interaction, particularly when embedded in nuclear media, is vital for resolving the Hyperon Puzzle which addresses the fate of hyperons in dense neutron-star matter [6].

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Little is known from experiment on the nuclear interaction of Ξ hyperons \[7, 8\]. A standard reaction production is the nuclear \((K^-, K^+)\) reaction, driven by \(K^-p \rightarrow K^+\Xi^-\) strangeness exchange on protons. Owing to its large momentum transfer, the produced \(\Xi^-\) hyperons populate dominantly the quasi-free continuum region, with less than 1% expected to populate \(\Xi^-\)-nuclear bound states that decay subsequently by the \(\Xi^-p \rightarrow \Lambda\Lambda\) strong-interaction capture reaction. Analysis of old emulsion events attributed to the formation of \(\Xi\) hypernuclei suggested attractive Woods-Saxon \(\Xi\) nuclear potential depth \(V_{\Xi} = 21\text{–}24\text{ MeV} \[9\]. While this range of values is considered sufficient for \(\Xi\) hyperons to play an active role in strange hadronic matter \[10\] and in neutron stars \[11\], somewhat smaller values follow from studies of dedicated \((K^-, K^+)\) counter experiments: \(V_{\Xi} \lesssim 14\text{ MeV} \) in KEK PS-E224 \[12\], \(V_{\Xi} \approx 17 \pm 6\text{ MeV} \) on \(^9\text{Be} \[13\] from BNL AGS-E906 \[15\]. New results from the J-PARC E05 and E70 \((K^-, K^+)\) experiments on \(^{12}\text{C}\) are forthcoming \[16\]. However, no \(\Xi^-\) or \(\Lambda\Lambda\) hypernuclear bound states have ever been observed unambiguously in these experiments. Powerful future experiments by the PANDA Collaboration at FAIR \[17\], using \(\bar{p}p \rightarrow \Xi^-\bar{\Xi}^+\) or \(\bar{p}n \rightarrow \Xi^-\bar{\Xi}_0\) production modes, and also at BESIII \[18\] focusing on the \(J/\psi \rightarrow \Xi^-\bar{\Xi}^+ O(10^{-3})\) decay branch, are likely to change this state of the art.

The situation is different in exposures of light-emulsion CNO nuclei to the \((K^-, K^+)\) reaction, in which a tiny fraction of the produced high-energy \(\Xi^-\) hyperons slow down in the emulsion, undergoing an Auger process to form high-\(n\) atomic levels, and cascade down radiatively. Cascade essentially terminates, when strong-interaction capture takes over, in a 3D atomic state bound by 126, 175, 231 keV in C, N, O, respectively. The 3D strong-interaction shift is less than 1 keV \[19\]. Capture events are recorded by observing \(\Lambda\) hyperon or hypernuclear decay products. Interestingly, whereas the few observed double-\(\Lambda\) hypernucleus production events are consistent with \(\Xi^-\) capture from 3D states \[8\], formation of pairs of single-\(\Lambda\) hypernuclei requires capture from a lower \(\Xi^-\) orbit. Expecting the final two \(\Lambda\) hyperons in \(\Xi^-p \rightarrow \Lambda\Lambda\) capture to be formed in a \(S = 0, 1s^2_\Lambda\) configuration, the initial \(\Xi^-\) hyperon and the proton on which it is captured must satisfy \(l_{\Xi^-} = l_p \[20\] which for \(p\)-shell nuclear targets favors the choice \(l_{\Xi^-} = 1\). Indeed, all two-body \(\Xi^-\) capture events, \(\Xi^- + ^AZ \rightarrow ^A\Lambda^Z + ^A\Lambda^Z\), to twin single-\(\Lambda\) hypernuclei reported from KEK and J-PARC light-emulsion \(K^-\) exposures \[21, 22, 23\], as listed here in Table 1, are consistent with \(\Xi^-\) capture from Coulomb-assisted \(1p_{\Xi^-}\) nuclear states bound by \(\sim 1\text{ MeV}\).
Table 1: Reported two-body \( \Xi^- \) capture events \( \Xi^- + ^A Z \rightarrow ^{A'}_\Lambda Z' + ^{A''}_\Lambda Z'' \) in light-emulsion nuclei to a pair of single-\( \Lambda \) hypernuclei, some in ground states, some in specific excited states marked by asterisk. Only the first and last events are uniquely assigned. Fitted \( \Xi^- \) binding energies \( B_{\Xi^-} \) are listed.

| Experiment | Event | \( ^A Z \) | \( ^{A'}_\Lambda Z' + ^{A''}_\Lambda Z'' \) | \( B_{\Xi^-} \) (MeV) |
|------------|-------|-------------|---------------------------------|--------------------|
| KEK E176 [21] | 10-09-06 12C | \( ^1_\Lambda H + ^9_\Lambda Be \) | 0.82±0.17 |
| KEK E176 [21] | 13-11-14 12C | \( ^4_\Lambda H + ^9_\Lambda Be^* \) | 0.82±0.14 |
| KEK E176 [21] | 14-03-35 14N | \( ^3_\Lambda H + ^{12}_\Lambda B \) | 1.18±0.22 |
| KEK E373 [22] | KISO 14N | \( ^5_\Lambda He + ^{10}_\Lambda Be^* \) | 1.03±0.18 |
| J-PARC E07 [23] | IBUKI 14N | \( ^5_\Lambda He + ^{10}_\Lambda Be \) | 1.27±0.21 |

In Table 1 only the first and last listed events are uniquely assigned in terms of initial emulsion nucleus \( ^A Z \) and final single-\( \Lambda \) hypernuclei \( ^{A'}_\Lambda Z' + ^{A''}_\Lambda Z'' \) ground states, providing thereby a unique value of \( B_{\Xi^-} \) per each event. The events fitted by assuming specific excited states \( ^{A''}_\Lambda Z''^* \) are equally well fitted each by g.s. assignments \( ^{A''}_\Lambda Z'' \), with values of \( B_{\Xi^-} \) as high as \( \sim 4 \) MeV, and the middle event is equally well fitted as a capture event in \( ^{16}_0 O \), to \( ^3_\Lambda H + ^{14}_0 C \) with \( B_{\Xi^-} = 0.46 \pm 0.39 \) MeV or to \( ^4_\Lambda H + ^{12}_0 C \) with \( B_{\Xi^-} = 0.40 \pm 0.27 \) MeV, both consistent with \( \Xi^- \) capture from an atomic 3D state. We note that the listed \( \Xi^- \) binding energy \( B_{\Xi^-} \) values are all around 1 MeV, significantly higher than the purely-Coulomb atomic 2P binding energy values which are approximately 0.3, 0.4, 0.5 MeV in C,N,O atoms, respectively. These \( \sim 1 \) MeV \( B_{\Xi^-} \) values correspond to \( 1p_{\Xi^-} \) nuclear states that evolve from 2P atomic states upon adding a strong-interaction \( \Xi \) nuclear potential\(^1\). This interpretation is the only one common to all five events.

Not listed in the table are multi-body capture events that require for their interpretation undetected capture products, usually neutrons, on top of a pair of single-\( \Lambda \) hypernuclei. Most of these new J-PARC E07 events \[^24\] imply \( \Xi^- \) capture from \( 1s_{\Xi^-} \) nuclear states, with capture rates \( \mathcal{O}(10^{-2}) \) of capture rates from the \( 1p_{\Xi^-} \) nuclear states considered here \[^20\,25\].

In the present work we consider \( \Xi^- \) atomic and nuclear transitions in light emulsion atoms, first with a \( tp \) optical potential, to substantiate that

\[^1\] Nuclear single-particle (s.p.) states are denoted by lower-case letters: \( 1s, 1p, 1d,... \) for the lowest \( l \) values, in distinction from atomic s.p. states denoted by capitals: \( 1S,2P,3D,... \) for the lowest \( L \) values.
Ξ⁻p → ΛΛ capture indeed occurs from a Coulomb-assisted nuclear 1pΞ⁻ state in light emulsion nuclei. The strength of this Ξ-nuclear potential is determined by requiring that it reproduces a 1pΞ⁻ state in ¹²C(0⁺1s.s.) bound by 0.82±0.15 MeV from Table I. Disregarding temporarily the sΞ⁻ = 1/2 Pauli-spin degree of freedom, we proceed to discuss the implications of identifying the value \(B_{\Xi^-} \approx 1.15\pm 0.20\) MeV for ¹⁴N from Table I with the binding energy of \(\mathcal{L}^π = (0^-,1^-,2^-)\) triplet of 1pΞ⁻ nuclear states built on \(J^π(¹⁴N_g.s.)=1^+\), thereby linking the capture process to properties of the Ξ-nucleus residual interaction. This provides the only self consistent deduction of the Ξ-nuclear interaction strength from analysis of the five Ξ⁻ capture events in light nuclear emulsion, fitted to two-body formation of specific Λ hypernuclei, as listed in Table I. The resulting \(tρ\) potential depth at nuclear-matter density \(\rho_0=0.17\) fm\(^{-3}\) is \(V_{\Xi} \approx 24\) MeV. We then improve upon the \(tρ\) leading term of the optical potential by introducing the next, Pauli correlation term in the optical potential density expansion ²⁰. This leads to \(\approx 10\%\) reduction of \(V_{\Xi}\), down to \(V_{\Xi} \approx 22\) MeV, keeping it within the optical potential approach well above 20 MeV. Our results suggest that 1sΞ⁻ nuclear bound states exist all the way down to ⁴He, with potentially far-reaching implications for the role of Ξ hyperons in multi-strange dense matter.

A value \(V_{\Xi} \gtrsim 20\) MeV implies a substantially stronger in-medium ΞN attraction than reported by some recent model evaluations (HAL-QCD ²⁷, EFT@NLO ²⁸ ²⁹, RMF ³⁰) all of which satisfy \(V_{\Xi} \lesssim 10\) MeV. A notable exception is provided by versions ESC16*(A,B) of the latest Nijmegen extended-soft-core ΞN interaction model ³¹, in which values of \(V_{\Xi}\) higher than 20 MeV are derived. However, these large values are reduced substantially by ΞNN three-body contributions within the same ESC16* model.

2. Methodology

The starting point in optical-potential analyses of hadronic atoms ³² is the in-medium hadron self-energy \(\Pi(E,\vec{p},\rho)\) that enters the in-medium hadron (here Ξ hyperon) dispersion relation

\[
E^2 - \vec{p}^2 - m_\Xi^2 - \Pi(E,\vec{p},\rho) = 0,
\]

where \(\vec{p}\) and \(E\) are the Ξ momentum and energy, respectively, in nuclear matter of density \(\rho\). The resulting Ξ-nuclear optical potential \(V_{\text{opt}}\), defined by \(\Pi(E,\vec{p},\rho) = 2EV_{\text{opt}}\), enters the near-threshold Ξ⁻ wave equation

\[
\left[\nabla^2 - 2\mu(B + V_{\text{opt}} + V_c) + (V_c + B)^2\right] \psi = 0,
\]
where $\hbar = c = 1$. Here $\mu$ is the $\Xi^-$-nucleus reduced mass, $B$ is the complex binding energy, $V_c$ is the finite-size Coulomb potential of the $\Xi^-$ hyperon with the nucleus, including vacuum-polarization terms, all added according to the minimal substitution principle $E \rightarrow E - V_c$. Strong-interaction optical-potential $V_{\text{opt}}$ terms other than $2\mu V_{\text{opt}}$ are negligible and omitted here. The use of a Klein-Gordon wave equation (2) for the $\Xi^-$ fermion rather than Dirac equation provides an excellent approximation when $Z\alpha \ll 1$ and fine-structure effects are averaged on, as for the X-ray transitions considered here. $\Xi^-$ nuclear spin-orbit effects are briefly mentioned below.

For $V_{\text{opt}}$ in Eq. (2) we first use a standard $t\rho$ form [32]

$$V_{\text{opt}}(r) = -\frac{2\pi}{\mu} (1 + \frac{A - 1}{A} \frac{\mu}{m_N})[b_0\rho(r) + b_1\rho_{\text{exc}}(r)],$$

where the complex strength parameters $b_0$ and $b_1$ are effective, generally density dependent $\Xi N$ isoscalar and isovector c.m. scattering amplitudes respectively. The density $\rho = \rho_n + \rho_p$ is a nuclear density distribution normalized to the number of nucleons $A$ and $\rho_{\text{exc}} = \rho_n - \rho_p$ is a neutron-excess density with $\rho_n = (N/Z)\rho_p$, implying that $\rho_{\text{exc}} = 0$ for the $N = Z$ emulsion nuclei $^{12}\text{C}$ and $^{14}\text{N}$ analyzed in the next section. Here we used mostly nuclear density distributions of harmonic-oscillator (HO) type [33] where the r.m.s. radius of $\rho_p$ was set equal to that of the known nuclear charge density [34]. Folding reasonably chosen $\Xi N$ interaction ranges other than corresponding to the proton charge radius, or using Modified Harmonic Oscillator (MHO) densities, or replacing HO densities by realistic three-parameter Fermi (3pF) density distributions [35, 36] made little difference: all the calculated binding energies changed by a small fraction, about 0.03 MeV of the uncertainty imposed by the ±0.15 MeV experimental uncertainty of the 0.82 MeV 1$\Xi^- p$ binding energy in $^{12}\text{C}$ listed in Table 1. We note that the central density $\rho(0)$ in all density versions used here is within acceptable values for nuclear matter, i.e., between roughly 0.15 and 0.20 fm$^{-3}$.

The form of $V_{\text{opt}}$ given by Eq. (3) corresponds to a central-field approximation of the $\Xi$-nuclear interaction. Spin and isospin degrees of freedom induced by the most general two-body $s$-wave $\Xi N$ interaction $V_{\Xi N}$,

$$V_{\Xi N} = V_0 + V_o\sigma_\Xi \cdot \sigma_N + V_\tau\tau_\Xi \cdot \tau_N + V_{\sigma\tau}\sigma_\Xi \cdot \sigma_N \tau_\Xi \cdot \tau_N$$

with $V$s functions of $r_{\Xi N}$, are suppressed in this approach. Choosing $^{12}\text{C}_{g.s.}$ with isospin $I = 0$ and spin-parity $J^\pi = 0^+$ for a nuclear medium offers
the advantage that only $V_0$ is operative in leading order since the nuclear expectation value of each of the other three terms in Eq. (4) vanishes. But for $^{14}$N$_{g.s.}$, with $I(J)=0(1^+)$, $V_\sigma$ is operative as well, adding unavoidable model dependence of order $\mathcal{O}(1/A)$ to $\Xi^-$-nuclear potential depth values derived from capture events assigned to this emulsion nucleus. For this reason we start the present analysis with the two $\Xi^-^{12}$C emulsion events of Table [1].

Figure 1: Energy levels (in keV) of the lowest $\Xi^-$ atomic states for $L=0$ (1S,2S) and $L=1$ (2P,3P) in $^{12}$C as a function of the strength parameter $\Re b_0$ (in fm) of the $\Xi^-$ optical potential (3). Spin-orbit splittings of $L=1$ states are suppressed in this figure. The dashed and dotted horizontal lines mark the value $B_{\Xi^-} = 0.82 \pm 0.15$ MeV from Table [1].

3. $\Xi^-$ capture in $^{12}$C

Figure [1] shows a portion of the combined atomic plus nuclear spectrum of $\Xi^-$ in $^{12}$C, $B_{\Xi^-} \leq 2$ MeV, as a function of $\Re b_0$, Eq. (3), for a fixed $\Im b_0 = 0.01$ fm corresponding to a nuclear-matter $\Xi^-$ capture width $\Gamma_{\Xi^-} \approx 1.5$ MeV, compatible with the HAL-QCD weak $\Xi^-p \rightarrow \Lambda\Lambda$ transition.
potential \[5\]. Plotted are the energies of the two lowest states of each orbital angular momentum \(l = 0, 1\), starting at \(Re b_0 = 0\) with almost purely atomic states \(1S, 2P, 2S, 3P\) from bottom up. Of these states the \(1S\) state with Bohr radius about 3.8 fm is indistinguishable from a nuclear \(1s\) state, and indeed it dives down in energy as soon as \(Re b_0\) is made nonzero. It takes considerable strength, \(Re b_0 \gtrsim 0.25\) fm, before the next atomic state, \(2P\) with Bohr radius about 15 fm, overlaps appreciably with the \(^{12}\text{C}_{g.s.}\) nuclear core, diving down in energy to become a nuclear \(1p\) state. The higher two states that start as atomic \(2S, 3P\) remain largely atomic as \(Re b_0\) is varied in Fig. 1. Their slowly decreasing energies indicate a rearrangement of the atomic spectrum \[37\]: \(2S \rightarrow 1S\) and \(3P \rightarrow 2P\). Judging by the marked band of values \(B_{\Xi^-} = 0.82 \pm 0.15\) MeV for the two KEK E176 events listed in Table 1, the figure suggests that they are compatible with a \(1p\) nuclear state corresponding to a \(\Xi^-\) nuclear potential strength of \(Re b_0 = 0.32 \pm 0.01\) fm. The sensitivity to variations of \(Im b_0\) is minimal: choosing \(Im b_0 = 0.04\) fm \[19\] instead of 0.01 fm increases \(Re b_0\) by 0.01 fm to 0.33 \(\pm 0.01\) fm.

Radiative rates for E1 transitions from the \(\Xi^-\) atomic \(3D\) state to the \(\Xi^-\) atomic \(3P\) state, and to the \(1p\) nuclear state that started as atomic \(2P\) state are found comparable to each other, accounting together for 7.6\% of the total \(3D\) width \(\Gamma_{3D} = 3.93\) eV as obtained using the optical potential \[3\]. However, the subsequent \(\Xi^-p \rightarrow \Lambda\Lambda\) capture will proceed preferentially from the \(1p\) nuclear state that offers good overlap between the \(1p\) and \(1p_p\) valence-proton orbits. Since the final \(1s^2\Lambda\) configuration has \(J_f = 0\), and the \(p\)-shell protons in \(^{12}\text{C}\) are mostly in \(j = \frac{3}{2}\) orbits, the requirement \(J_i = J_f = 0\) imposes \(j_{\Xi^-} = \frac{3}{2}\) on the spin-orbit doublet members of the \(1p\) state. The shift of this \(1p_{\Xi^-}(\frac{3}{2})\) sub level from the \(1p_{\Xi^-}(2j + 1)\)-average is estimated, based on the 152 keV \(1p_\Lambda\) spin-orbit splitting observed in \(^{13}\Lambda\text{C}\) \[38\] to be less than 100 keV upward \[39\] and, hence, within the 0.15 MeV listed uncertainty introduced here for the position of the \((2j + 1)\)-averaged \(1p_{\Xi^-}\) state.

4. Spectroscopy of \(^{14}\text{N}_{g.s.} + 1p_{\Xi^-}\) states

Having derived the strength parameter \(Re b_0 = 0.32 \pm 0.01\) fm of \(V_{opt}\) by fitting it to \(B_{\Xi^-}^{1p}\) \(\text{MeV}\), we apply this optical potential to \(^{14}\text{N}\) where it yields \(B_{\Xi^-}^{1p}(^{14}\text{N}) = 2.08 \pm 0.28\) MeV, considerably higher than the value \(B_{\Xi^-} = 1.15 \pm 0.20\) MeV obtained from the three events assigned in Table 1 to \(\Xi^-\) capture in \(^{14}\text{N}\). However, this calculated \(B_{\Xi^-}^{1p}(^{14}\text{N})\) corresponds to a \((2L + 1)\)-average of binding energies for a triplet of states \(L^\pi = (0^-, 1^-, 2^-)\).
obtained by coupling a $1p_{\Xi^{-}}$ state to $J^{\pi}(^{14}N_{g.s.})=1^{+}$, as shown in Fig. 2. We now discuss the splitting of these triplet states. Effects of $\Xi^{-}$ Pauli spin, $s_{\Xi^{-}} = \frac{1}{2}$, are introduced at a later stage.

Figure 2: Energies (in MeV) of $L^{\pi} = (0^{−}, 1^{−}, 2^{−})$ triplet of $^{14}N_{g.s.} + 1p_{\Xi^{-}}$ states, split by a $Q_{N} \cdot Q_{\Xi}$ residual interaction [5]. The $(2L + 1)$-averaged energy $−2.08 \pm 0.28$ MeV was calculated using the same optical potential parameter $b_{0}$ that yields a $^{12}C_{g.s.} + 1p_{\Xi^{-}}$ state at $−0.82 \pm 0.15$ MeV, corresponding to the $\Xi^{-}$ capture events in $^{12}C$ listed in Table 1.

The construction of the $^{14}N_{g.s.} + 1p_{\Xi^{-}}$ spectrum in Fig. 2 follows a similar $^{12}C(2^{+}; 4.44$ MeV) + $1p_{\Lambda}$ spectrum construction in $^{13}_{\Lambda}C$ [10]. The energy splittings marked in the figure are obtained from a two-body spin-independent interaction $V_{0}(r_{\Xi N})$, Eq. (4), between a $p$-shell $\Xi$ hyperon and $p$-shell nucleons, expressed in terms of its shell-model (SM) quadrupole-quadrupole residual interaction $V_{\Xi N}$,

$$ V_{\Xi N} = F_{\Xi N}^{(2)} Q_{N} \cdot Q_{\Xi}, \quad Q_{B} = \sqrt{\frac{4\pi}{5}} Y_{2}(\hat{r}_{B}), $$

where $F_{\Xi N}^{(2)}$ is the corresponding Slater integral [11]. A representative value of $F_{\Xi N}^{(2)} = −3$ MeV is used here, smaller than the value $F_{\Lambda N}^{(2)} = −3.7$ MeV.
established empirically for \(p\)-shell \(\Lambda\) hypernuclei \[12\], in accordance with a \(\Xi N\) strong interaction somewhat weaker than the \(\Lambda N\) strong interaction (see next section). A single \(3D_1\, ^{14}N_{g.s.}\) SM component providing a good approximation to the full SM intermediate-coupling g.s. wavefunction \[13\] was assumed in the present evaluation.

Fig. 2 exhibits a triplet of \(^{14}N_{g.s.} + 1p_{\Xi^-}\) levels, spread over more than 1 MeV. The least bound triplet state, with \(L^\pi = 0^+\), is shifted upward by 0.84 MeV from the \((2L + 1)\) averaged position at \(-2.08 \pm 0.28\) MeV to \(E(0^-) = -1.24 \pm 0.28\) MeV. This is consistent with the averaged position \(\bar{E} = -1.15 \pm 0.20\) MeV of the three \(\Xi^-\)^{14}N_{g.s.} capture events listed in Table 1. We are not aware of any good reason why capture has not been seen from the other two states with \(L^\pi = 1^-, 2^-\). This may change when more events are collected at the next stage of the ongoing J-PARC E07 emulsion experiment.

Table 2: Quadrupole-quadrupole contributions to the energies \(E(L^\pi)\) of the \(^{14}N_{g.s.} + 1p_{\Xi^-}\) triplet of states shown in Fig. 2 with respect to \(E(^{14}N_{g.s.})\), using \(F_{\Xi N}^{(2)} = -3\) MeV, and spin contributions to the splittings \(\Delta E(L) = E(J = L + \frac{1}{2}) - E(J = L - \frac{1}{2})\) of the \(L \neq 0\) states. \(A_{ls}\) and \(A_{ss}\) are spin-orbit \((l_\Xi = 1, s_\Xi = \frac{1}{2})\) and spin-spin \((s_N = s_\Xi = \frac{1}{2})\) energy splittings, respectively, see text.

| Interaction | \(E(0^-)\) | \(E(1^-)\) | \(E(2^-)\) | \(\Delta E(1^-)\) | \(\Delta E(2^-)\) |
|-------------|-----------|-----------|-----------|----------------|----------------|
| \(Q_N \cdot Q_\Xi\) | \(-\frac{7}{25}\, F_{\Xi N}^{(2)}\) | \(\frac{7}{50}\, F_{\Xi N}^{(2)}\) | \(-\frac{7}{250}\, F_{\Xi N}^{(2)}\) | \(-\frac{1}{2}\, A_{ls}\) | \(\frac{5}{6}\, A_{ls}\) |
| \(l_\Xi \cdot s_\Xi\) | \(-\) | \(-\) | \(-\) | \(\frac{1}{2}\, A_{ss}\) | \(\frac{5}{6}\, A_{ss}\) |
| \(s_N \cdot s_\Xi\) | \(-\) | \(-\) | \(-\) | \(\frac{1}{2}\, A_{ss}\) | \(\frac{5}{6}\, A_{ss}\) |

Introducing Pauli spin, \(s_{\Xi^-} = \frac{1}{2}\), the total angular momentum of the uppermost level marked \(0^-\) in Fig. 2 becomes \(J^\pi = \frac{1}{2}^-\), but its position is unaffected by spin-orbit and spin-spin interactions. Each of the other two levels in Fig. 2 splits into a doublet \(J = L \pm \frac{1}{2}\) whose \((2J + 1)\)-average remains in the unsplit position. Estimated splittings are listed in Table 2 in terms of two constituent spin matrix elements: \(A_{ls} \lesssim 300\) keV \[39\] for the \(l_\Xi \cdot s_\Xi\) spin-orbit splitting \(E_{1p_{\Xi^-}^{3/2}}(\frac{3}{2}^-) - E_{1p_{\Xi^-}^{1/2}}(\frac{1}{2}^-)\), and \(A_{ss} \approx 400 \pm 80\) keV for the \(s_N \cdot s_\Xi\) spin-spin splitting \(E(S_{\Xi N} = 0) - E(S_{\Xi N} = 1)\) for \(p\)-shell nucleon and \(\Xi\) hyperon. For estimating \(A_{ss}\) we used the HAL-QCD \[5\] volume integral of \(V_\sigma\), Eq. 4, relative to that of \(V_0\), thereby generating about 20% systematic uncertainty. Incorporating these spin splittings into the \(L^\pi = (0^-, 1^-, 2^-)\) triplet in Fig. 2 keeps the \(L^\pi = 0^-\) state of interest, which has become \(J^\pi = \frac{1}{2}^-\), well separated by at least 0.5 MeV from the rest of the split states.
5. Density dependence, $\Xi$ nuclear potential depth and $1s_{\Xi^-}$ states

So far we have discussed a density independent $t$-matrix element $b_0$ in $V_{\text{opt}}$, Eq. (3), to fit the $\Xi^-$ capture events in $^{12}\text{C}$ from Table 1. The resulting value \( \Re b_0 = 0.32 \pm 0.01 \text{ fm} \) implies, in the limit $A \to \infty$ and $\rho(r) \to \rho_0 = 0.17 \text{ fm}^{-3}$, a value $V_\Xi = 24.3 \pm 0.8 \text{ MeV}$ in nuclear matter, in accordance with the extraction of $V_\Xi$ from old emulsion events [9] but exceeding considerably other values reviewed in the Introduction. To explore how robust this conclusion is, we introduce the next to leading-order density dependence of $V_{\text{opt}}$, replacing $\Re b_k (k=0,1)$ in Eq. (3) by

$$\Re b_k (\rho) = \frac{\Re b_k}{1 + \frac{3k_F}{2\pi} \Re b_{0ab}}, \quad k_F = \left(\frac{3\pi^2 \rho}{2}\right)^{\frac{1}{3}},$$

(6)

where $k_F$ is the Fermi momentum corresponding to nuclear density $\rho$ and $b_{0ab} = (1 + \frac{m_{\Xi^-}}{m_N})b_0$ is the lab transformed form of the c.m. scattering amplitude $b_0$. Eq. (6) accounts for Pauli exclusion correlations in $\Xi N$ in-medium multiple scatterings [26, 44]. Variants of the form (6) have been used in kaonic atoms [45] and mesic nuclei [46, 47, 48] calculations. Shorter-range correlations, disregarded here, were shown in Ref. [46] to contribute less than $\sim 30\%$ of the long-range Pauli correlation term. Applying Eq. (6) in the present context, $B_{1p}^{1s} (^{12}\text{C}) = 0.82 \text{ MeV}$ is refitted by $\Re b_0 = 0.527 \text{ fm}$, lowering the former value $B_{1p}^{1s} (^{14}\text{N}) = 2.08 \text{ MeV}$ to $1.95 \text{ MeV}$ without any substantive change in the conclusions drawn above regarding the five two-body capture events deciphered here. The nuclear-matter $\Xi$-nuclear potential depth $V_\Xi$ decreases from $24.3 \pm 0.8$ to $21.9 \pm 0.7 \text{ MeV}$, a decrease of merely $10\%$, with additional systematic uncertainty of less than $1 \text{ MeV}$. This value of $V_\Xi$ is sufficient to bind $1s_{\Xi^-}$ states in $p$-shell nuclei, with systematic uncertainty of less than $0.5 \text{ MeV}$, as demonstrated in Table 3 which shows a steady decrease of $B_{1s}^{1s}$ and $\Gamma_{1s}^{1s}$ down to $^4\text{He}$. The increased $\Gamma_{1s}^{1s} (^4\text{He})$ reflects a denser $^4\text{He}$ medium. However, expecting corrections of order $\mathcal{O}(1/A)$ to the optical potential methodology, our $^4\text{He}$ result should be taken with a grain of salt. It is worth noting that all listed $1s_{\Xi^-}$ g.s. levels remain bound also when the attractive finite-size Coulomb interaction $V_c$ is switched off. None of such $1s_{\Xi^-}$ states have been observed conclusively in dedicated experiments.

The $T = \frac{1}{2}$ $^{11}\text{B}$ nucleus, the only $T \neq 0$ nucleus listed in Table 3, requires in addition to the isoscalar parameter $b_0 = 0.527 + i0.010 \text{ fm}$ also a knowledge of the isovector parameter $b_1$. Here we used the HAL-QCD [5] volume integral of $V_\tau (r_{\Xi N})$ relative to that of $V_0 (r_{\Xi N})$, Eq. (4), to estimate $b_1$ relative to
Table 3: Binding energies $B^{1s}_{\Xi^{-}}$ and widths $\Gamma^{1s}_{\Xi^{-}}$ (in MeV) in core nuclei $^A Z(J_c)$, g.s. spin $J_c$, obtained by solving Eq. (2) with $b_0 = 0.527 + 10.010 \text{ fm}$ and $b_1 = -0.225 \text{ fm}$ in $V_{opt}$, Eqs. (3), (6). A finite-size Coulomb interaction $V_c$ is included.

| Nucleus | $B^{1s}_{\Xi^{-}}$ | $\Gamma^{1s}_{\Xi^{-}}$ |
|---------|---------------------|------------------------|
| $^{11}\text{N}(1)$ | 11.5 | 1.02 |
| $^{12}\text{C}(0)$ | 9.8 | 0.93 |
| $^{11}\text{B}(\frac{3}{2})$ | 8.4 | 0.89 |
| $^{10}\text{B}(3)$ | 7.6 | 0.77 |
| $^6\text{Li}(1)$ | 2.1 | 0.26 |
| $^4\text{He}(0)$ | 2.0 | 0.45 |

$b_0$, thereby deriving a value $b_1 = -0.225 \text{ fm}$. The resulting value $B^{1s}_{\Xi^{-}}(11\text{B})$ listed in the table is lower by 530 keV than obtained disregarding $b_1$. Next, we introduce $s_{\Xi^{-}} = \frac{1}{2}$ Pauli spin, splitting each of the listed $1s_{\Xi^{-}}$ levels in $J_c \neq 0$ core nuclei into two sub levels $J = J_c \pm \frac{1}{2}$. Using HAL-QCD [5] ratios of volume integrals of $V_\sigma(r_{\Xi N})$ and $V_{\sigma\tau}(r_{\Xi N})$ to that of $V_0(r_{\Xi N})$, Eq. (4), as done above for $V_{\tau}$, we estimate the $1s_{\Xi^{-}}$ spin splittings to be well below 1 MeV. Other potential sources of $\Xi^{-}$ spin splittings that are relevant in $\Lambda$ hypernuclei, such as tensor or induced nuclear spin-orbit terms, are likely to be considerably weaker than evaluated in Ref. [49] and are disregarded here. Of particular interest is the $^{11}\text{B}_{g.s.} + 1s_{\Xi^{-}}$ $J^e = 1^-$ doublet member expected to be formed in the $^{12}\text{C}(K^-, K^+)$ production reaction when the outgoing $K^+$ meson is detected in the forward direction. Our estimates place it about 0.5 MeV deeper than the listed $(2J+1)$-averaged $B^{1s}_{\Xi^{-}}(11\text{B}) = 8.4 \text{ MeV}$, contrasting statements, e.g. [8], that adopt $B^{1s}_{\Xi^{-}}(11\text{B}) \sim 5 \text{ MeV}$ from the BNL AGS-E885 $^{12}\text{C}(K^-, K^+)$ experiment [13]. In fact, the E885 poor resolution prevents making any such conclusive statement.

6. Conclusion

We have shown that all five light nuclear emulsion events identified in KEK and J-PARC $K^-$ exposure experiments as two-body $\Xi^{-}$ capture in $^{12}\text{C}$ and $^{14}\text{N}$ into twin $\Lambda$ hypernuclei correspond to capture from $1p_{\Xi^{-}}$ Coulomb-assisted bound states. This involved using just one common strength parameter of a density dependent optical potential. Long-range $\Xi N$ shell-model correlations were essential in making the $^{14}\text{N}$ events consistent with the $^{12}\text{C}$ events. Earlier attempts to explain these data overlooked this point, therefore

\footnote{Note that Im $b_1 = 0$ because the charge exchange $\Xi^{-} + ^{11}\text{B} \rightarrow ^{\Xi^0} + ^{11}\text{Be}$ is kinematically blocked.}
reaching quite different conclusions [50, 51, 52, 53, 54]. Predicted then are
1sΞ− bound states with \( B_{1s}^{\Xi-} \approx 10 \) MeV in \(^{12}\)C and somewhat larger in \(^{14}\)N, deeper by 4–5 MeV than the 1sΞ− states claimed by a recent J-PARC E07 report of multibody capture events [24]. The Ξ nuclear-matter potential depth derived here within an optical potential methodology, \( V_\Xi = 21.9 \pm 0.7 \) MeV, is considerably larger than \( G \)-matrix values below 10 MeV derived from recent LQCD and EFT ΞN potentials [27, 29]. A systematic optical-potential model uncertainty of less than 1 MeV as discussed in Sect. 5 is short of bridging the gap noted above. Substantial ΞNN three-body attractive contributions to the Ξ nuclear potential depth would be required to bridge this gap. Intuitively one expects repulsive BNN three-body contributions for octet baryons \( B \), e.g. Ref. [31], but in chiral EFT studies, focusing on decuplet-\( B^* \) intermediate \( B^*NN \) and \( BN\Delta \) configurations, this has been proven so far only for \( B = \Lambda \) [55, 56].

To check the procedure practised in Sect. 5 for fitting \( V_\Xi \) to just one \( \Xi^-{12}\)C bound state datum, we apply it to the 1sΛ binding energy in \(^{12}\)C, \( B_{1s}^{\Lambda} = 11.69 \pm 0.12 \) MeV [57]. The fitted strength \( b_0 = 0.866 \pm 0.010 \) fm amounts to a nuclear-matter Λ potential depth \( V_\Lambda = 31.7 \pm 0.2 \) MeV, in good agreement with the accepted value \( V_\Lambda \approx 30 \) MeV [7]. One may slightly improve the derived value of \( V_\Lambda \) by subtracting from \( B_{1s}^{\Lambda} \) a nuclear induced spin-orbit contribution that vanishes in the limit \( A \to \infty \), thereby reducing our input \( B_{1s}^{\Lambda} \) to 10.85 MeV [39]. This gives \( b_0 = 0.798 \pm 0.010 \) fm and \( V_\Lambda = 30.5 \pm 0.2 \) MeV. Here too it is not possible to separate the contribution of \( \Lambda NN \) three-body potential terms from that of the main \( \Lambda N \) two-body potential term.

A strong \( \Xi^- \)-nuclear interaction, such as derived here, may have far-reaching implications to \((N,\Lambda,\Xi)\) strange hadronic matter [10] and particularly to dense neutron star matter [11]. In the latter case a strong \( \Xi^- \)-nuclear interaction might cause a faster depletion of Λ hyperons by \( \Lambda\Lambda \to \Xi^- p \), a process inverse to the \( \Xi^- \) capture reaction considered in the present work. More work is necessary in this direction.

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