Parameter determination of hereditary models of deformation of composite materials based on identification method

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Abstract. In this paper, based on the analysis of some experimental data, a study and selection of hereditary models of deformation of reinforced polymeric composite materials, such as organic plastic, carbon plastic and a matrix of film-fabric composite, was pursued. On the basis of an analysis of a series of experiments it has been established that organo-plastic samples behave like viscoelastic bodies. It is shown that for sufficiently large load levels, the behavior of the material in question should be described by the relations of the nonlinear theory of heredity. An attempt to describe the process of deformation by means of linear relations of the theory of heredity leads to large discrepancies between the experimental and calculated deformation values. The use of the theory of accumulation of micro-damages leads to much better description of the experimental results. With the help of the hierarchical approach, a good approximation of the experimental values was successful only in the first three sections of loading.

1. Introduction

Polymeric and composite materials belong to the class of materials with a long memory. This means that the stresses at this point in time depend not only on the current values of deformations, temperature and other determining parameters, but also on the values of these parameters at all preceding points in time – from the history of the deformation process. Dependence on the history of the process is particularly shown in the fact that in the simplest experiments regarding pure tension such phenomena as creep and relaxation take place [1]-[4].

Defining equations of hereditary type, written in integral form, take into account the influence of load history, temperature history and other certain factors. In connection with this circumstance, they can be used to describe the mechanical behavior under various loading regimes, in particular, creep, relaxation, and other regimes of time-variant loading. The material described by such equations can be considered as a dynamic system determined by a number of interrelated material functions, such as long-term moduli, long-term compliances, creep and relaxation functions, complex moduli, creep and relaxation spectrum [5]-[8].

In this paper, based on the analysis of some experimental data, a study and selection of hereditary models of deformation of reinforced polymeric composite materials, such as organic plastic, carbon plastic and a matrix of film-fabric composite, was pursued [9].
2. Solutions
When constructing the model of creeping of polymeric composite materials under constant loading, they are usually based on the linear theory of heredity:

\[ e = e + e^c, \quad \text{where} \quad e = \sigma / E, \quad e^c = \int_0^t H(t-\tau)\sigma(\tau)d\tau. \quad (1) \]

Here \( E \) is an elastic module of the studied material, \( H \) is a creep kernel, \( e \) is a total deformation, \( e^c \) is a creep deformation, \( e \) is an elastic part of deformation.

One of the simplest cores is Abel core, which well describes creep of the composite materials, at least the initial sections. It has the form:

\[ H(t-\tau) = \frac{C}{(t-\tau)^\alpha}, \quad C \geq 0, \quad 0 < \alpha < 1. \quad (2) \]

Substituting (2) into relation (1), when \( \sigma = \text{const} \) we can obtain:

\[ e^c = e^c(t) = \sigma \frac{C}{1-\alpha} t^{1-\alpha}. \quad (3) \]

First, the experimental data of the organic plastic tests were processed in one-stage and two-stage loading with unloading.

To find the constants \( C, \alpha \) we used the experimental values \( e^c_j \) and \( \sigma_j \) at the different points in time \( t_j \) \((j=1,\ldots,m)\). Substituting them into the expression (3) gives a system of nonlinear algebraic equations with respect to \( C, \alpha \) which can be written in the following form:

\[ \{e_{\exp}\} = f(C,\alpha), \quad \text{where} \quad \{e_{\exp}\} = \{e_1^c, e_2^c, \ldots, e_m^c\}^T. \quad (4) \]

Here an index “\( T \)” means a transposition operation, an index “\( \exp \)” means a experimental data, \( \{\ldots\} \) is a sign of vector.

To ensure the conditions (2), the required values were replaced:

\[ C = s^2, \quad 1-\alpha = \frac{1}{1+c^2}. \]

To determine the unknowns \( s, \theta \) a quadratic discrepancy of the equation system was composed (4):

\[ \rho^2 = (\{f(s,\theta)\} - \{e_{\exp}\})^T (\{f(s,\theta)\} - \{e_{\exp}\}), \quad (5) \]

and its minimization by standard gradient methods in the application package Mathematica was done.

Figure 1 shows a plot of deformation \( e \) versus time \( t \), obtained from the formula (1), the parameters of which were found by the above method in the case of one-step loading. The received values are: \( C = 0.049 \ (\text{MPa} \cdot \text{s}^{1-\alpha})^{-1}, \ \alpha = 0.69 \). The dots on the plot represent the experimental data.

The results of the calculations showed that for different load levels under a single loading the constants \( C, \alpha \) differed greatly. This suggests that it is necessary to use the nonlinear theory of heredity. It can be represented in the following form:

\[ e = \frac{1}{E}[\sigma + \int_0^t H(t-\tau,\sigma(\tau))\sigma(\tau)d\tau]. \quad (6) \]
As a non-linear kernel, a modification of the Abel kernel was used, in which it was assumed that $C = C(\sigma), \alpha = \alpha(\sigma)$.

![Image of Figure 1](image1.png)

**Figure 1.** A plot of deformation $\varepsilon$ versus time $t$.

To ensure the relations (2) we used the replacement of the unknown values like this:

\[
\begin{align*}
C &= (s_0 + s_1 \sigma)^2, \\
1 - \alpha &= \frac{1}{\sqrt{1 + (\alpha_0 + \alpha_1 \sigma + \alpha_2 \sigma^2)^2}}.
\end{align*}
\]  

(7)

Using the forth going procedure a system of equations similar to (4) is constructed. By minimizing the quadratic discrepancy (5), the unknowns were sought $s_0, s_1, \alpha_0, \alpha_1, \alpha_2$.

![Image of Figure 2](image2.png)

**Figure 2.** The deformation $\varepsilon$ dependence on time $t$ for the case of two-stage loading.

Figure 2 shows the deformation dependence $\varepsilon$ on time $t$ for the case of two-stage loading. The dots represent the experimental data, and the curves are constructed on the basis of the mathematical model (6), (2), (7) with the help of the values obtained as a result of minimization $\rho^2$. 

Furthermore, a mathematical model of behavior of carbon plastic was constructed with due account for the process of accumulation of microdamages since an attempt to describe its deformation with the help of (1) used to lead to large discrepancies between the calculated and experimental values of deformations in some experiments [10]. That was observed when the shell was brought to ruin.

In the experiment, a five-step loading with the same tension was considered for all five sections with unloadings between them [9]. Two approaches were considered.

\( a) \) Hierarchical approach

In this case, when writing the creep equations, the possibility is considered that a damage degree of the material (an accumulation degree of microcracks) affects the process of creep. The modernized Abel kernel was used in the form:

\[
H(t - \tau) = \frac{C(\omega)}{(t - \tau)^{\alpha(\omega)}}, \quad C(\omega) \geq 0, \quad 0 < \alpha(\omega) < 1.
\]  

(8)

where \( \omega \) is a damage parameter of the material. The kinetic equation for the damage parameter was taken in the form:

\[
\dot{\omega} = f(\sigma, \omega), \quad f(\sigma, \omega) = \frac{B\sigma^m}{(1 - \omega)^n}, \quad \omega(0) = 0,
\]  

(9)

where \( B, m, n \) are some constants.

The simplified strength condition according to the works of Rabotnova Yu.N. was accepted in the form:

\[
\omega = 1, \quad t = t^* = t_{\text{max}}.
\]  

(10)

When \( \sigma = \text{const} \) after integration (9) and an exception \( B\sigma^m \) the expression for the damage parameter \( \omega \) takes the following form:

\[
\omega = (1 - [1 - \frac{t}{t^*}]^{n+1}), \quad \text{where} \quad t^* = t_{\text{max}}.
\]  

(11)

Just as before, to ensure the relations (2) we used the replacement of the required values, but with \( \omega^* \):

\[
\begin{align*}
C &= (s_0 + s_1 \omega)^2, \\
1 - \alpha &= \frac{1}{1 + (\alpha_0 + \alpha_1 \omega)}. 
\end{align*}
\]  

(12)

To find out \( C, \alpha \), which are already functions \( C = C(\omega), \quad \alpha = \alpha(\omega) \), we used the minimization of the quadratic discrepancy \( \rho^2 \) according to the procedure mentioned above. As a result, the required constants are obtained:

\[
\begin{align*}
s_0 &= -0.00206(\sqrt{\text{MPa} \cdot \text{s}^{1-\alpha}})^{-1}, \\
s_1 &= 0.00267(\sqrt{\text{MPa} \cdot \text{s}^{1-\alpha}})^{-1}, \\
\alpha_0 &= 2.253, \\
\alpha_1 &= -2.6825, \\
n &= 0.0000339.
\end{align*}
\]
Figure 3 shows a graph where the dots represent the experimental data, and the lines represent the calculated curves.

![Graph](image)

**Figure 3.** A graph where the dots represent the experimental data, and the lines represent the calculated curves with using the Hierarchical approach.

**b) Approach to Kachanov’s hypothesis**

In this approach, the assumption is made that the damage parameter is not included in the creep equation, that is, the mechanisms of creep and fracture processes are generally different. This hypothesis for our case can be written in the following form:

\[ \varepsilon = \varepsilon + \varepsilon^* \quad \text{where} \quad \varepsilon^* = \varepsilon^c + \varepsilon^\omega, \]

where \( \varepsilon^c \) is a deformation, responsible for creep, \( \varepsilon^\omega \) is a deformation, responsible for accumulation of microdamages. It is assumed that the simplest expression of the dependence \( \varepsilon^\omega \) on \( \omega \) has this form:

\[ \varepsilon^\omega = s\omega^k. \]

For \( \varepsilon^c \), as before, the expression (1) is accepted. Following the procedure mentioned above, by means of minimization \( \rho^2 \) the required constants are obtained \( s_1, \alpha_1, n, s, k \). On Figure 4 the dots also represent the experimental data, and the lines represent the calculated curves.

Furthermore, the results of processing the experimental data on the creep of a film coating of PVC (polyvinyl chloride) were processed. To describe the creep process, the Abel kernel was used in the form (2), where \( C, \alpha \) are functions of stresses: \( C = C(\sigma), \alpha = \alpha(\sigma) \).

To ensure the conditions (2), the following replacement of the required values was made:

\[
\begin{align*}
C &= (s_0 + s_1\sigma)^2, \\
\alpha &= 1 - \frac{1}{\sqrt{1 + (\alpha_0 + \alpha_1\sigma)^2}}.
\end{align*}
\]
Figure 4. A graph where the dots represent the experimental data, and the lines represent the calculated curves with using the Kachanov’s hypothesis.

Figure 5. The dependence of creep deformation $\varepsilon^c$ on time for PVC at different load levels, where the points represent the experimental data, and the solid line represents the calculated curves.

By minimizing the quadratic discrepancy $\rho^2$ according to the process mentioned above, the required values were obtained:

$$ s_0 = 0.09(\sqrt{MPa \cdot s^{1-\alpha}})^{-1}, $$

$$ s_1 = 0.03(\sqrt{MPa \cdot s^{1-\alpha}})^{-1}, $$

$$ \alpha_0 = 9.14, \alpha_1 = 2.4 / MPa. \quad (16) $$

Figure 5 shows the dependence of creep deformation $\varepsilon^c$ on time for PVC at different load levels, where the points represent the experimental data, and the solid line represents the calculated curves.
3. Results and discussion

On the basis of an analysis of a series of experiments it has been established that organo-plastic samples behave like viscoelastic bodies. It is shown that for sufficiently large load levels, the behavior of the material in question should be described by the relations of the nonlinear theory of heredity. An attempt to describe the process of deformation by means of linear relations of the theory of heredity leads to large discrepancies between the experimental and calculated deformation values. The use of the theory of accumulation of micro-damages leads to much better description of the experimental results. With the help of the hierarchical approach, a good approximation of the experimental values was successful only in the first three sections of loading. According to Kachanov's hypothesis, the calculated curves are closer to the experimental points in all five sections of loading.

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