STATISTICAL PHYSICS OF DARK AND NORMAL MATTER DISTRIBUTION IN GALAXY FORMATION : DARK MATTER LUMPS AND BLACK HOLES IN CORE AND HALO OF GALAXIES.

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ABSTRACT
In unified field theory the cosmological model of the universe has supersymmetric fields. Supersymmetric particles as dark and normal matter in galaxy clusters have a phase separation.

Dark matter in halos have a statistical physics equation of state. Neutralino particle gas with gravitation can have a collapse of dark matter lumps.

A condensate phase due to boson creation by annihilation and exchange can occur at high densities. The collapse of the boson condensate, including neutralinos, into the Schwarzschild radius creates dark matter black holes.

Microscopic dark matter black holes can evaporate with Hawking effect giving gamma ray bursts and create a spectrum of normal particles.

The phase separation of normal and dark matter in galaxy clusters and inside galaxies is given by statistical physics.

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INTRODUCTION
The current understanding of cosmology has dark energy, dark matter and normal matter including electromagnetic radiation as contents of the universe; all arising from an inflationary phase.

A unified field theory [Refs 9] can have scalar fields coupled to others and create spacetime geometry and all forms of particles.

The dark energy gets associated with vacuum and Casimir energy. It is associated with the inflationary scalar field and with the cosmological constant of spacetime geometry in various epochs of the universe.

It can also be a fluid such as Chaplygin gas or phantom or tachyonic particles. Cosmological observational evidence gives a’best estimate’equation of state and models give some properties.

Dark energy has a phase separation from the dark and normal matter around 10 million years from big bang; with voids full of repulsive dark energy and collapsing 3,2,1 dimensional structures of normal and dark matter that appear as filaments with point like galactic nodes at the scale of giga parsecs down to 100 megaparsecs.

The inhomogeneity structure formation from WMAP time of < 1 million years to large scale structure formation of 100s of million years includes this epoch.

The dark matter and normal matter particles are interspersed in the pre-galactic formation epoch. In the process of formation of galaxies, the interspersed voids contain dark matter excess over normal matter.
Supersymmetric particles of dark matter decay and the neutralino is a lowest mass remnant particle. A relativistic gas at low temperatures is formed as can be understood as follows.

The SUSY particles do not thermalise with the background electromagnetic radiation and they do not interact with photons once the electric, colour and flavour charge free states are produced. They do not interact with the leptonic and baryonic normal matter and hence do not loose energy by collisions.

They remain relativistic at the energy when the last decay to neutralino occurred. As neutralinos have very small interaction crosssections they will have negligible thermal energy or temperature.

The dark matter is interspersed and phase separated from normal matter. And there is excess dark matter over normal matter of 3 to 8 times.

Normal matter goes through the well known stages of baryon, nuclei, atom ,molecule, dust formation which interacts with electromagnetic radiation of appropriate frequencies in each epoch of the expanding universe.

This matter also interacts among itself and a thermal distribution is attained. This leads to a non relativistic gas of normal matter at low temperatures.

The neutralino gas with particle rest energy 300 to 1400 GeV and the baryonic/ leptonic gas with 1GeV to 1 MeV rest energy have two different intrinsic temperatures or average thermal energies and they phase separate.

The core of galaxy is formed with a AGN or supermassive black hole and a spherical halo is formed with dark matter; with the normal matter residing in the potential well in between.

This becomes visible as the luminous part of the galaxy in some wavelengths. Star formation and other processes eventually give radiation at all wavelengths from the galaxy.

Throughout this process that lasts from 100 million years to \( \approx \) 10 billion years from origin; the dark matter neutralino gas is a relativistic fermionic gas going from ultrarelativistic to non relativistic, in a self created gravitational potential.

It can create an effective gravitational potential in the galaxy as well as local fluctuations that grow in density and collapse to form dark matter lumps with a wide range of mass spectrum.

Some of these become dark matter black holes; as at short range and high densities the interaction crosssection of neutralinos increases due to exchange bosons including Higgs boson.

The Bose condensation of Higgs and exchange W,Z bosons along with the gravitational collapse of the fermionic neutralino lumps can take all the dark matter mass into the Schwarzschild radius, creating a dark matter black hole.

A mass spectrum of such dark matter black holes gives the possibility that microscopic black holes would be able to quantum evaporate by Hawking effect. This may lead to a flux of gamma rays and normal matter particles. Such events have been reported from galaxy halos, but the data is still insufficient and not conclusive.

The entire process depends on the statistical physics of the dark and normal matter gas including gravitational effects at long range and particle interactions.
at short range. For black hole formation general relativistic effects have to be included.

In an earlier work [Ref 9] the formation and quantum evaporation of primordial black holes was considered.

In this paper the statistical physics of dark matter is worked out and the various conditions of galaxy formation and galaxy halos with dark matter black holes is obtained.

The observational tests for the interspersed dark matter and the evaporating dark matter black holes are mentioned. This work should complement the growing tests for dark matter in galaxy cores, in galaxy halos and galaxy clusters.

As a comparison with other work on this problem consider the results of the papers [Refs 1 to 6] (a) density functions in halos ex. [1] (b) equations of state ex. [2] (c) annihilation crosssections ex. [3] topics.

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STATISTICAL PHYSICS OF NORMAL AND DARK MATTER IN GALAXIES:

The statistical physics of Fermionic and Bosonic particles is developed for normal and dark matter. For the dark matter a neutralino $\chi^0_1$ particle is taken as the remnant of SUSY decays.

The relativistic case of fermionic neutralino gas with limits of non relativistic and ultrarelativistic gas, the low and high temperature limits and low and high density limits is considered in the presence of self gravitation effects and interactions with exchange bosonic particles.

We begin with the standard expressions for free non interacting identical particles.

\[
\langle N \rangle = \sum g(E) \frac{z^{-\exp(\beta E) + a}}{z^{-\exp(\beta E) + a}}
\]

\[
\langle E \rangle = \sum \frac{g(E)E}{z^{-\exp(\beta E) + a}}
\]

\[
\frac{PV}{K T} = \sum a^{-1} g(E) \ln(1 + az \exp(-\beta E))
\]

where $a$ is $+1$ for Fermions, and $-1$ for Bosons,

$z$ is fugacity, $z = \exp(\mu/K T)$

The supersymmetric matter of Fermionic gas of very weakly interacting neutralinos exists in a self created gravitational potential at low densities.

Neutralinos interact with exchange gauge and Higgs bosons at short range when gravitational collapse causes high density.

The gravitational potential that was for low densities now becomes the effective potential in the Schwarzschild geometry at high densities.

The statistical physics during collapse can be described in a comoving coordinate system in this geometry. A Bose condensation can be expected for the bosons. This accelerates the collapse.

If the entire mass of fermions and bosons resides inside the horizon then a dark matter black hole is formed. Otherwise a dark matter lump occurs.
Next the mixture of normal matter and dark matter distribution is considered. At length scales of 100 megaparsecs this is nearly uniform with a $1 : 5$ ratio.

The phase separation into galaxy clusters and intergalactic space happens from 10 million years onwards within the filaments of normal and dark matter formed with voids filled with dark energy.

The intergalactic spaces have a low density dark matter and negligible normal matter. With galaxy formation around 100 million years onwards the phase separation creates galaxy halos with dark matter and interiors with normal matter.

The question of interspersed dark matter within the galaxy and the ratio of dark and normal matter in the galaxy cores that have collapsed into active galactic nuclei as supermassive black holes is an open question.

The statistical physics of the model developed in this paper gives some insight into the phase separations.

With $E = mc^2[(1 + \frac{p}{mc})^2]^{1/2} - 1]$, and $(p/mc) = \sinh(x)$,

$E = mc^2(\cosh(x) - 1)$,

$\frac{dE}{dp} = ctanh(x)$

$g(E)dE = g(p)dp$, $g(p)dp = \frac{4\pi^3}{h^3}p^2dp$

substituted in equations above gives the relativistic quantities.

Then pressure is $P = \frac{4\pi^3}{h^3}\int_0^\frac{\pi}{2} \sinh^4(x)dx$

And total internal energy is $U = 4\pi^3V^4h^3\int_0^\frac{\pi}{2} (\cosh(x)-1)\sinh^2(x)cosh(x)dx$

Consider the relevant approximations:

Ultrarelativistic normal and dark matter, bosonic and fermionic with $T = 10^{12}$ eV and below. Then $z = \exp(0) = 1$ and $\exp(\beta E) = 1$.

The $z^{-1}\exp(\beta E) = 1$.

$q(V, T) = \frac{\mu}{K}\frac{4\pi^3V^4h^3\zeta(4)}{\beta^3}$

with $g_4(1) = \zeta(4) = \pi^4/90$

and $P = 1/3U/V$ for the bosonic particles.

For the fermionic ones it is $q(V, T) = \frac{\mu}{K}\frac{4\pi^3V^4h^3\zeta(3)}{\beta^3}$ and higher order terms in $(\mu/KT)^2$

The expressions for number density $N/V$ are also obtained as

$\frac{4\pi^3V^4h^3\mu}{3(\beta^3)^2} \frac{\mu}{KT}$

for Fermionic and $\frac{8\pi^3V^4g(3)}{\beta^3} \frac{\mu}{KT}$ for Bosonic case.

This is the early epoch of uniformly interspersed dark and normal matter.

The normal matter particles have masses $1MeV$ and $1GeV$ for leptons and baryons.
As the temperatures go below TeV, they go from being ultrarelativistic to relativistic to nonrelativistic after those of SUSY matter whose lightest particles of dark matter, neutralinos are mass $10^3 GeV$.

Hence in the epoch of 100 million years we could take the dark matter as nonrelativistic and the normal matter as relativistic. However the normal matter couples to radiation and equilibrates thermally.

In the expanding universe as the radiation spectrum shifts to longer wavelengths the normal matter also becomes nonrelativistic. In the in between phase due to different approximate equations of state and number densities, energy densities arising, there is a phase separation.

The dark matter interacts negligibly with normal matter and not at all with radiation. It has few energy loss mechanisms.

If the temperature becomes much less than eV compared to the rest and kinetic energies of particles then we could take it to be zero.

Then $z^{-1} \exp(\beta E) = 0$ for the fermionic gas.

Then the relativistic gas has

$$P = \frac{\pi m c^2 A(\sinh(p/mc))}{6n^3}$$

and

$$U = \frac{\pi V m c^2 B(\sinh(p/mc))}{6n^3}$$

where $A$ and $B$ are Airy functions.

Two useful limits are for $\sinh(p/mc) << 1$ and $>> 1$ respectively nonrelativistic and ultrarelativistic case.

For the Pressure and total internal energy given above The Airy functions in these limits give :

$$P = \frac{4\pi p^5}{15nmc} = 2/3U/V$$

and

$$P = \frac{\pi cp^4}{3m^3} = 1/3U/V.$$ 

For dark matter the mass $m$ is 1000 times more than for normal matter leading to much less pressure and energy density. Also the Fermi momentum at which these are evaluated is

$$p = \left(\frac{4\pi^2 N}{15nmc}\right)^{1/3}.$$ 

This is lower for dark matter because the 1 : 5 ratio by total mass of normal and dark matter, and 1 : 1000 ratio of particle masses implies a 200 : 1 ratio of numbers of normal and dark particles.

Hence a smaller $N/V$ for dark matter and a lower $p$ Fermi momentum too. Hence we expect with $200^{1/3} = 6$ and a factor $10^3 6^5$ less pressure for dark matter than normal matter.

It may be expected then that any accumulation of a mixture of normal and dark matter can have a thermodynamic non equilibrium in which the dark matter would accumulate more rapidly due to gravitational effects as it has less thermodynamic pressure and collapses the widely dispersed mixture into pregalactic lumps and intergalactic spaces.

Further the dark matter would collapse to the core of the pregalactic lump leading to a largely dark matter supermassive black hole.

The phase separation of dark matter halos and normal matter in the potential well between core and halo is one important model of galaxies.
However the model of dark matter interspersed in the galaxy with normal matter is a possibility too. If the neutralinos do not thermalise by scattering or radiative methods then they can continue to move at relativistic speeds as they were when they were formed. Virialisation is not a definite or significant process for them.

Hence they will reside for negligible time in the filled spaces by normal matter, such as planets and stars. But they will fill mostly the much more voluminous void spaces in between in the solar system, star clusters as well as in the galaxy. Using crosssections for interaction with baryonic matter while passing through filled regions, a flux limit for detection can be set at Earth.

Dark matter is therefore a relativistic Fermi gas at very low temperatures at the time of pregalactic lump formation, which becomes non relativistic over the life time of a galaxy residing mostly in the halo.

For the collapsing galactic material, at any time there is a volume in the equations above that decreases in time. The gravitational potential energy is added to the thermodynamic energy as well as the gravitational pressure to thermodynamic pressure. Considering a spherically symmetric model for the halos and lumps in halos, the gravitational potential energy inside the halo sphere is:

\[ \phi(r) = a - bR^2 \]

For flat velocity rotation curves a constant potential model is taken. The central cusp model gives various density functions. Keeping density as a \(1/R^2\) function gives a mass function linear in \(R\).

The gravitational pressure is:

\[ P_G = \frac{GM^2}{4\pi R^2} \]

The equilibrium condition then becomes:

\[ U + \phi \text{ minimised and } P + P_G < 0 \]

Actually this condition is dynamical and leads to collapse as the neutralino gas component phase separates.

In the halo density fluctuations around average \(\rho(R)\) create gravitational accretion.

Using limits of low number density and low Fermi momentum for the Airy functions, the relativistic low temperature neutralino 1000\(GeV\) rest mass gas can form a mass spectrum of dark matter lumps

\[ R = \text{const}M^{-1/3}(1000GeV)^{-5/3}. \]

These can range from 100 stellar masses to 1/10 stellar masses. If the other limit of high density and large fermi momentum is taken then

\[ R = \text{const}M^{1/3}(1000GeV)^{-1/3}. \]

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COLLAPSE OF DARK MATTER LUMPS IN GALAXY HALOS AND CORE: FORMATION OF BLACK HOLES

As the collapse proceeds these dark matter lumps go from stellar radii to pebble size. The neutralino annihilation cross section increases and bosons, real and virtual are produced.
The neutralino $\chi_1^0$ particles have very low cross section for interactions, including annihilations, that produce normal particles – particularly Z, W and H bosons. This effect of weakly interacting massive particles (WIMPS) is expected to become significant at high densities.

During the gravitational collapse of dark matter, due to very low thermodynamic pressure, nearly $10^{-7}$ times that of normal matter, there is no mechanism to halt collapse except Fermi degeneracy pressure.

This occurs over length scales of Compton or de Broglie wavelengths, which are very small $10^{-19}$m as neutralino mass is large. If the interparticle separation is of this order then the mass is inside a horizon.

The degenerate Fermi gas collapses by self gravitation and this process is accelerated by excess boson creation as densities increase and interparticle separation decreases. It is shown that the Bose condensation of these Bosons accelerates the collapse in spite of neutralino degeneracy pressure, even for low mass dark matter lumps, thus driving the matter inside the Schwarzschild radius. Hence a mass spectrum of dark matter black holes is expected.

From the studies of primordial black holes and their mass spectrum it is expected that up to $10^{15}$ kg black holes would be unstable due to Hawking effect of quantum evaporation.

Larger mass black holes are classical and will survive and grow. These will acquire discs of dark matter and perturb their local regions in the galaxy halos.

The larger mass black holes will have life time of decay up to life time of galaxies. The last stage of evaporation is rapid as most energy output occurs then. Luminosity is $1/M^2$ and life time is $M^{1/3}$.

This will result in normal matter creation and gamma ray emissions. Some evidence for this in the dark matter halos of galaxies is being found; but better observations are expected in a decade.

Bosons created by SUSY fermionic Neutralino annihilation as well as the exchange bosons in their interaction create a relativistic Bose gas, in the high density stage. This Bose gas is mixed with the fermionic neutralino gas.

As the large hadron collider coming up will likely generate data on the SUSY particles, the physical parameters will become known and will refine the astrophysical observations.

The statistical physics in the high density stage of dark matter collapse requires an effective potential in Schwarzschild geometry. This can be inserted in place of the Newtonian gravitational potential.

Alternately using co falling coordinates the $(1 - 2GM/rc^2)$ factor can be included in rescaled lengths along radial directions.

The conditions for the Bose condensate and the relativistic Fermi gas to collapse into and form a dark matter black hole are given as follows.

The non relativistic Bose condensation condition is

$$N = \frac{\sqrt[3]{3/2}}{\lambda}$$

For ultra relativistic gas $\mu = 0, z = 1$ and no condensation.

This becomes $N = \frac{\sqrt{3}}{\lambda}$ for relativistic case.
Solving for the volume of the condensate and equating it to interior volume of the black hole we obtain a condition for formation of the black hole.

If this number of particles resides in a sphere of radius
\[ R_s = \frac{2GM}{c^2} \]
where \( M = M_b + M_f \),
\[ M_b = N_{100} GeV \] and 
\[ M_f = N_{1000} GeV \] and 
\[ V = \frac{4}{3}\pi R_s^3 \],
then the dark matter black hole condition is satisfied.
The \( 10^2 M_s < M < 10^8 M_s \) clumps form due to annihilation processes at the pregalactic stage and become the galaxy core supermassive black holes. The Fermi degeneracy pressure of neutralino gas is overcome if the condensate volume is inside the Schwarzschild volume \( 4/3\pi(2GM/c^2)^3 \).

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CONCLUSION

There is growing research on the details of galaxy halos, formation of lumps and black holes, as well as the nature of the neutralino gas as dark matter. This paper attempted a simplified model of the formation of galaxies and halos and dark matter black holes.

The statistical physics of normal matter (baryons and leptons) and dark matter (neutralinos) was developed. This provides a basic explanation of phase separation in galaxy halos and interior of galaxies of fermionic gases with two distinct mass species of particles, neutralinos and baryons.

The gravitational collapse of low density and pressure neutralino gas also gave rise to lumps of dark matter in the halos of galaxies.

Further gravitational collapse into black holes was described using Bose condensation of exchange and annihilation bosons along with the neutralino gas overcoming Fermi degeneracy pressure of neutralinos.

Detailed models of density functions and non spherical shapes as well as equations of state using theories of interacting neutralinos and partition functions are being made that will be refined with more observations.

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