Optimal Sensor Arrangement for Cable-Stayed Bridge Based on Multiple Algorithms

Jianjun Liang¹, Kai Peng² and Kai Li¹

¹China First Highway Engineering Company Ltd, Beijing 100024, China
²School of Highway, Chang’an University, Xi’an, Shaanxi Province, 710064, China

Abstract. The arrangement of sensors is one of the key points in health monitoring of long-span bridge structures. In this paper, eight algorithms are applied to the optimal sensor arrangement of a long-span cable-stayed bridge and compared. Firstly, eight algorithms and four criteria are introduced, then eight algorithms are applied to optimize the layout scheme, finally, four criteria are used to evaluate the algorithms comprehensively according to the same weight. The results show that in the optimal arrangement of sensor measuring points, it is necessary to combine the engineering practice, comprehensively consider the weight ratio of different criteria, and choose the best optimization scheme.

1. Introduction

Due to the influence of the external environment, vehicle loads and human factors, as well as the delay in maintenance during the long-term operation of the bridge structure, cumulative damage and fatigue damage will inevitably occur, so it is necessary to establish an efficient health monitoring system. The arrangement of sensor measurement points determines the integrity of dynamic information acquisition of bridge structure and the cost of health monitoring system to a large extent. Therefore, it is necessary to optimize the arrangement of sensor measurement points. At present, the algorithms mainly applied to the optimal arrangement of sensor points include the following: effective independence method¹, Sequence method including Stepwise subtraction method and stepwise accumulation method², EI and MAC hybrid algorithm³, Genetic algorithm⁴ and monkey group algorithm⁵, etc. Although some scholars have compared these algorithms, most of them are based on a single criterion, and the comprehensive evaluation combined with multiple criteria is less.

Taking a long-span cable-stayed bridge as an engineering background, this paper adopts eight algorithms, namely effective independence method, Stepwise subtraction method, stepwise accumulation method, EI and MAC hybrid algorithm, EI-based Stepwise subtraction method, EI-based stepwise accumulation method, genetic algorithm and monkey group algorithm (algorithm for 1-8), to optimize the sensor measurement points of bridge deck layout. Then, the results of the eight algorithms are compared and analyzed by using 4 criteria: minimum mean square deviation, maximum non-diagonal element of MAC matrix, determinant value of Fisher information matrix and condition number of modal matrix⁶. Finally, the advantages and disadvantages of each method are evaluated by comprehensive evaluation system.
2. Introduction of Algorithms and Evaluation Criteria

2.1. Algorithm Introduction

The effective independence method was proposed by Kammer in 1991 based on the minimum criterion of recognition error. The basic idea is to use the idempotency of the complex modal matrix to calculate the effective independence vectors from all possible points and sort them according to the independence of the target modal matrix, removing the degree of freedom that contributes the least to its rank, thereby optimizing the Fisher information matrix, making the modal vector of interest as linear as possible. Both Stepwise subtraction method and Stepwise accumulation method belong to sequence method, and their calculation ideas are opposite. The stepwise subtraction method calculates the structural modal matrix by the finite element method, and rearranges the matrix by QR decomposition, eliminating the degree of freedom that cannot be used as the measuring point in the model. Contrary to the Stepwise subtraction method, the Stepwise accumulation method continuously chooses an optimum addition from the remaining optional positions to the optimum configuration until the optimum number is reached; EI and MAC hybrid algorithms use column principal component QR decomposition technology to obtain the initial measurement points of sensor position, and then use effective independence method to determine the range of candidate added measurement points, that is to select the minimum non-diagonal elements of MAC matrix from the candidate measurement points, add them to the initial measurement points, and continue to cycle until the requirements are met. The EI-based stepwise subtraction method first uses the EI method to determine the position of candidate points, which is equivalent to optimizing the position of candidate points by effective independence method. The EI-based stepwise accumulation method does not screen candidate points outside the initial measurement point, but only uses the effective independence method instead of the QR decomposition method to determine the initial measurement point position and the candidate measurement point. The genetic algorithm is a random method proposed by some scientists represented by Professor Holland in the 1960s to imitate the survival of the fittest in the evolution of nature. The monkey group algorithm is a group intelligent optimization algorithm proposed by Zhao and Tang, its search method is similar to the process of monkey climbing, mainly relying on the three steps of “crawling”, “looking” and “jumping”.

2.2. Introduction of Evaluation Criteria

2.2.1. Minimum Mean Square Variance Criterion. The purpose of the sensor point optimization arrangement is to use the response of finite degree of freedom to construct the response on all degrees of freedom of the structure. Generally, the spline interpolation of the output effect value of the known measuring point is performed to approximate the effect value of other points on the structure. Different optimization schemes obtained by different algorithms will result in different expansion effect values. The minimum mean square variance criterion is to evaluate the advantages and disadvantages of various optimal placement methods by calculating the sum of the mean square error of the displacement values obtained by finite element analysis of modal displacement and expansion effect. The formula for calculating the total mean square error[7] is shown in Equation 1:

$$\sigma_{TMSD} = \sum_{j=1}^{N} \frac{1}{n} \sum_{i=1}^{n} \left( \phi_{ij}^{CS} - \phi_{ij}^{FE} \right)^2$$

$$= \frac{1}{N} \sum_{j=1}^{N} \left[ \sum_{i=1}^{n} \left( \phi_{ij}^{CS} - \phi_{ij}^{FE} \right)^2 \right]$$

Where $N$ is the modal order, $n$ is the modal error test point, $\sigma_j$ is the standard deviation of mode $j$, $\phi_{ij}^{CS}$ is the extended modal value of the $i$th error test point of the $j$th order, $\phi_{ij}^{FE}$ is the extended modal value of the $j$th error test point of the $i$th order.
2.2.2. Modal Assurance Criterion. Since the degree of freedom of measurement is far less than the degree of freedom of the structure itself, coupled with the influence of noise and accuracy, the measured modal vector is difficult to maintain orthogonality, and sometimes important modals are lost due to the small space intersection between vectors. The Modal Assurance Criterion (MAC) is a good tool for evaluating the intersection angle of modal vectors. The elements on the diagonal of the MAC matrix are equal to 1, and the other elements are between 0 and 1. The closer the value of non-diagonal element is to 1, the worse the orthogonality of the two modes corresponding to this location is, on the contrary, the better the orthogonality of the two modal vectors is. The modal confidence MAC matrix formula is shown in Equation (2):

\[
MAC_{ij} = \frac{\left(\phi_i^T \times \phi_j\right)^2}{\left(\phi_i^T \times \phi_i\right)\left(\phi_j^T \times \phi_j\right)}
\]  

(2)

2.2.3. Fisher Information Matrix Criterion. The Fisher information matrix reflects the sensitivity of the location of the sensor to the modal response of the structure. The index adopted is the determinant logarithm of Fisher information matrix of each optimized measurement point, the larger the value, the better the unbiased estimation of the modal, that is the better the optimal layout scheme.

2.2.4. Conditional Number of Mode Matrix. The larger the condition number of the mode matrix, the more unstable the solution process is. Therefore, it can be used as a standard to evaluate the advantages and disadvantages of the point arrangement. The closer the condition number is to 1, the better the stability of the measurement point arrangement, on the contrary, the worse the stability is.

3. Instance verification

3.1. Project Overview and Theoretical Modal
The main bridge of a bridge is a (245+565+245)m three-span double pylons double cable planes steel-concrete composite beams cable-stayed bridge with a total length of 1055m and a semi-floating system in the longitudinal direction. The finite element model of the bridge structure is established by Midas. The full bridge model is shown in Fig. 1:

![Finite Element Model of Full Bridge](image1)

Considering that in the actual bridge monitoring, only the first few modes of the bridge can be accurately monitored, so the first four modes of the bridge deck are considered, and the first four-order vertical modes of the bridge deck are extracted by the Midas finite element model, as shown in Fig. 2:
(a) Vertical 1st mode

(b) Vertical 2nd mode

(c) Vertical 3rd mode

(d) Vertical 4th mode

Fig 2 First four vertical modes
3.2. **optimal arrangement of sensor measurement points**

Fig. 3 compares the optimal placement schemes of sensor points obtained by different algorithms when 9 points are arranged.

(a) Effective Independence Method

(b) Stepwise subtraction method

(c) Stepwise accumulation method

(d) EI and MAC hybrid method

(e) EI-based Stepwise subtraction method

(f) EI-based stepwise accumulation method
From Fig. 3, it can be seen that the effective independence method is mainly concerned with capturing the kinetic energy of the structure, so the optimal measurement points are mostly concentrated in the large displacement of the mid-span structure; The stepwise subtraction method and the stepwise accumulation method mainly start from acquiring the linearly independent characteristics of the modes, and the distribution of the measured points is relatively uniform than that obtained by the effective independence method; The EI-based stepwise subtraction method and the EI-based stepwise accumulation method also have the above characteristics; The EI and MAC hybrid algorithm also has obvious measurement point accumulation phenomenon. The results obtained by the above 6 algorithms all have different degrees of measurement point accumulation phenomenon. Genetic algorithm and monkey group algorithm are random algorithms, and the results are not unique. It is necessary to perform multiple calculations and select the optimal scheme, and the distribution of measurement points is much more uniform than the previous 6 algorithms.

3.3. **Comparative Analysis and Comprehensive Evaluation of Optimization Schemes**

Four criteria, including mean square deviation, maximum value of non-diagonal elements of MAC matrix, determinant value of Fisher information matrix and condition number of mode matrix, are used to evaluate the advantages and disadvantages of eight optimal sensor placement schemes, as shown in Fig. 4.
Figure 4 Evaluation and comparison of different evaluation indicators

Figure 4(a) shows that the results obtained by the eight algorithms are generally unsatisfactory in the case of fewer measuring points, which is in good agreement with the actual situation. That is, it is difficult to accurately fit the whole bridge information when the number of measuring points is small. With the increase of the number of measurement points, the mean square deviation index of the eight algorithms decreases in varying degrees. Except the Stepwise subtraction method, the mean square
deviation criteria of the results of other algorithms are mostly about 0.001, which shows that the results of these algorithms are better.

Figure 4(b) shows that the stepwise subtraction method, the stepwise accumulation method, and the EI-based stepwise accumulation method have a large decrease in the MAC non-diagonal maximum as the number of sensor optimization increases, and the effective independence method, EI-based Stepwise subtraction method. The genetic algorithm, and monkey group algorithm have little change with the number of sensors, and the EI and MAC hybrid algorithms have basically no change.

Figure 4(c) shows that the determinants of stepwise accumulation method are larger and more stable, and do not increase with the increase of the number of measurement points, while the values of the other algorithms increase significantly with the increase of the number of measurement points.

Figure 4(d) shows that the conditional number of the EI-based stepwise accumulation method, genetic algorithm, and monkey group algorithm modal matrix fluctuates greatly, and the other algorithms are relatively stable.

In this paper, the linear dimensionless technology is used. According to the optimal result of each algorithm is 100 points, the worst is 0 points, and the intermediate result is scored by the principle of linear interpolation. The same weight of each evaluation index is specified at 0.25. The comprehensive score of 8 algorithms according to 4 different evaluation indexes is obtained, as shown in Fig. 5.

As can be seen from Fig. 5, with the increase of the number of measurement points, the comprehensive scores of all sensor optimization methods have been improved to varying degrees, and finally tend to be stable. The comprehensive score of the stepwise accumulation method after stabilization is about 95, mainly because the determinant of Fisher information matrix is generally larger, while the other algorithms after stabilization is between 60 and 70.

4. Conclusion
(1) By extracting the location of measurement points, it can be seen that when the first six algorithms increase the number of measurement points, the optimal solution obtained is mainly encrypted at the original location of measurement points, and the solution of the algorithm is basically unique. The solutions of algorithms 7 and 8 are stochastic, so it is necessary to calculate them several times and select the most stable results.

(2) The location of a reasonable measurement point determined under a certain criterion may not be applicable under another criterion. In this paper, the weights of the four evaluation criteria are respectively specified as 0.25. In the actual project, the appropriate weights of 4 different criteria should be specified according to the actual situation for comprehensive evaluation, and the optimal solution is obtained.

(3) Although increasing the number of measuring points can improve the score, when the number of measuring points exceeds a certain value, the increase will no longer increase significantly. At this
time, it is necessary to combine the sensitivity of different algorithms to the number of measuring points to comprehensively consider and select the optimal solution.

Acknowledgments
The authors are grateful for the financial supports from the 2017 transportation research project of Shaanxi Provincial Department of transportation (17-15k).

Reference
[1] Kammer D C. Sensor Placement for On-Orbit Modal Identification and Correlation of Large Space Structures[J]. Journal of Guidance, Control, and Dynamics. 1991: 251-259.
[2] HUANG M S, ZHU H P, SONG J Q. Application of Optimal Sensor Placement in Modal Parameters Test of Bridge Structure[J]. Journal of Highway and Transportation Research and Development. 2008(02): 85-88.
[3] YUAN A M, DAI H, SUN D S. Optimal Sensor Placement of Cable-Stayed Bridge Using Mixed Algorithm Based on Effective Independence and Modal Assurance Criterion Methods[J]. Journal of Vibration, Measurement & Diagnosis. 2009, 29(01): 55-59.
[4] LIU Y, BI D, LI Z X. Optimal placement of accelerometers in long cable-stayed bridges based on genetic algorithm[J]. JOURNAL OF SOUTHEAST UNIVERSITY (Natural Science Edition). 2009, 39(04): 825-829.
[5] ZHANG X D. Optimal sensor placement based on monkey algorithm[D]. Dalian University of Technology, 2013.
[6] Shah P C, Udwadia F E. METHODOLOGY FOR OPTIMAL SENSOR LOCATIONS FOR IDENTIFICATION OF DYNAMIC SYSTEMS.[J]. Journal of Applied Mechanics, Transactions ASME. 1978(1): 188-196.
[7] WU Z Y, DAI F J, SONG J et al. A More Efficient Optimal Sensor Placement Method for Structure Damage Detection[J]. Journal of Northwestern Polytechnical University. 2007(04): 503-507.