Investigation on Dynamic Pressure Characteristics Induced by an Internal Solitary Wave in a Two-layer Fluid of Finite Depth

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ABSTRACT In this paper, the dynamic pressure characteristics induced by an internal solitary wave (ISW) in a two-layer fluid of finite depth are studied. A method for detecting ISWs by dynamic pressure is proposed. First, an accurate controllable ISW numerical flume is established based on the applicability conditions of three kinds of ISW theories. Then the relationship between the maximum dynamic pressure and the amplitude of the ISW under four density stratifications is studied. The results show that the dynamic pressure variation induced by the ISW is synchronous with that of interface displacement. Under the same density stratification, the maximum dynamic pressure has a strong linear relationship with the corresponding amplitude. Furthermore, the nonlinear error is less than 7\%, and the relationship is not affected by the ocean structure. According to the above characteristics of dynamic pressure, a series of dynamic pressure measurement experiments were carried out in a large internal wave flume (IWF) using a very low frequency (VLF) piezoelectric sensor. The experimental results show that the dynamic pressure measurement results are in good agreement with the numerical simulation results. Last, the Miyata-Choi-Camassa (MCC) theory is used to invert the measured maximum dynamic pressure to the amplitude of ISW, and compared with the measured amplitude, the maximum error is less than 6\%, which verifies the feasibility of the inversion method of ISW amplitude by using dynamic pressure measurement proposed in this paper, and provides a new method and idea for detecting ISW at sea in the future.

INDEX TERMS Internal solitary wave, dynamic pressure, numerical flume, detection method.

I. INTRODUCTION

Ocean internal wave is a kind of fluctuation occurring in the density stratified seawater, which widely exists near the offshore continental shelf and island chain, and is a mesoscale ocean phenomenon \cite{1}. The formation of internal waves has obvious seasonal characteristics, especially in summer, due to the uneven density distribution of seawater along the gravity direction, a certain density stratification structure will be formed in seawater. When the seawater is subjected to a small disturbance, the seawater particles will deviate from their equilibrium position to form marine internal waves. Internal solitary wave (ISW) is a special kind of ocean internal wave. The balance between nonlinearity and dispersion makes the ISWs keep the velocity and waveform unchanged in the propagation process. ISWs have large amplitude, strong nonlinearity and large carrying energy, which pose a serious threat to marine structures, submarine activities and underwater weapon launch \cite{2}. For example, in early 2014, a Chinese Navy 372 submarine operating in the South China Sea (SCS) sank sharply after encountering an ISW that triggered a burst of internal pipelines. Therefore, the study of ISW observation methods has important scientific values and military needs.

In recent years, people have used various instruments to observe and study ISW and achieved fruitful results. Xu et al. \cite{3} used the Acoustic Doppler Current Profiler (ADCP) in the SCS to capture the largest velocity ISW since records, with a horizontal velocity of 2.94 m/s. Ma et al. \cite{4} observed the ISWs in the northern SCS by using the conductivity-temperature-depth (CTD) recorders, and they inversed the period and amplitude of the ISWs according to the measured fluctuations of temperature and salt. Huang et al. \cite{5} observed the strongest ISW in the northern SCS using ADCP and CTD, with the amplitude up to 240 m. According to the convergence and divergence effect of sea surface caused by the propagation of ISW, Alpers et al. \cite{6} used synthetic aperture radar (SAR) to study the spatial distribution of ISWs in Andaman Sea according to the convergence and divergence of sea surface caused by the propagation of ISWs. Chonnaniyah et al. \cite{7} used SAR and optical sensors to observe the ISWs in the northern Lombok Strait and estimate its propagation velocity. The occurrence of ISWs in the ocean is highly random, and the observation of ISWs requires high experimental costs. Therefore, the laboratory
internal wave flume (IWF) experiment has become the main way to study the propagation and evolution of ISWs and the interaction between ISWs and marine structures. However, the IWF in the laboratory has the problems of high construction cost and complicated experimental process [8]. With the development of computational fluid dynamics (CFD) technology, the emergence of numerical flume not only solves the pain point of traditional physical flume, but also under the premise of reasonable numerical flume model, the numerical simulation results are usually consistent with the experimental results. Therefore, more and more scholars carry out research on ISWs based on numerical flume model. According to the applicability conditions of ISW theories, Wang et al. [9] proposed a mass source method for numerically generating ISWs in two-layer fluid and realized in the numerical flume. Zhi et al. [10] established the theory of ISW propagation and evolution over slope-shelf topography based on the two-layer fluid numerical flume model of finite water depth and verified it by numerical method. Zhang et al. and Wang et al. [11]-[12] proposed an accurate and controllable numerical flume to calculate the interaction between ISWs and structures.

At present, the observation or the study of ISWs mainly focuses on either the changes of velocity, temperature, density and salinity in seawater induced by the ISWs, or the characteristics of sea surface convergence and divergence caused by the ISW propagation. However, the sudden strong flow induced by the ISW propagation will cause the change of dynamic pressure in the flow field, and there are few studies on the characteristics of dynamic pressure induced by the propagation of ISW. In the present paper, the dynamic pressure induced by ISW is studied by numerical simulation and experiment. Firstly, the CFD software Fluent is used to model the numerical flume based on the actual size of the IWF at Shanghai Jiao Tong University (SJTU) in China. Secondly, the relationship between the maximum dynamic pressure induced by an ISW and the amplitude during the propagation of ISW is studied by using the accurate and controllable numerical flume. Finally, the very low frequency (VLF) piezoelectric sensor is used to carry out the dynamic pressure measurement experiment in the laboratory physical flume to verify the simulation results.

This paper is organized as follows. Section II establishes the numerical flume model, and the dynamic pressure induced by ISW is analyzed theoretically. Section III introduces experimental equipment and procedures. In Section IV, the numerical and experimental results are given, including the dynamic pressure characteristics induced by ISW and the relationship between maximum dynamic pressure and the amplitude of ISW. Section V concludes the paper.

II. NUMERICAL METHOD

In this paper, the numerical flume model uses the Navier-Stokes equation as the governing equation to carry out the numerical study of the ISW. On the premise of ensuring that the results are not affected, the model is simplified to a two-dimensional numerical flume. In addition, the velocity inlet method is used to generate ISWs, which considerably improves the computational efficiency of the model.

A. GOVERNING EQUATION

For an incompressible fluid with a density of $\rho$, the velocity components $u_i$, $w_i$ and pressure $p_i$ in Cartesian coordinates $\Omega_\z$ (Fig. 1) satisfy the continuity equation and the Navier-Stokes equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + w_j \frac{\partial u_i}{\partial z_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \left( \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial z^2} \right)$$  

$$\frac{\partial w_i}{\partial t} + u_j \frac{\partial w_i}{\partial x_j} + w_j \frac{\partial w_i}{\partial z_j} = -\frac{1}{\rho} \frac{\partial p}{\partial z_i} + \nu \left( \frac{\partial^2 w_i}{\partial x^2} + \frac{\partial^2 w_i}{\partial z^2} \right) - g$$

where $x$ and $z$ represent the coordinate components, $g$ is the acceleration due to gravity, $u_i$ and $w_i$ are the velocity vectors in $x$ and $z$ directions, respectively, $\nu$ is the kinematic viscosity, $p_i$ is the pressure, and $i = 1$ and 2 represent the upper and lower fluid, respectively.

In this paper, the velocity inlet method is used for wave generation. This method is superior to the double push-pedals wave maker method and the mass source method in terms of wave generation accuracy and efficiency [9]. If the amplitude of the ISW is $a$ and the phase speed is $c$, the depth-averaged velocities of the upper and lower layers induced by the ISW respectively are given as:

$$\bar{u}_1 = -c \frac{\zeta}{h_1 - \zeta}, \bar{u}_2 = c \frac{\zeta}{h_2 - \zeta}$$

where $\zeta(\xi,t)$ is the interface displacement of two layers of fluid and is a function of time $t$. By giving the upper and lower fluid at the inlet of the numerical flume at speeds $\bar{u}_1$ and $\bar{u}_2$, the wave generation by the velocity inlet method is realized, as shown in Fig. 1.

The volume of fluid (VOF) method is used to capture the change of two-layer fluid interface during the propagation of ISW. The VOF equation (5) is shown as follows [13]:

$$\frac{\partial a_q}{\partial t} + u_i \frac{\partial (a_q)}{\partial x_i} + w_i \frac{\partial (a_q)}{\partial z_i} = 0$$

In VOF equation, $a_q$ represents the volume fraction of the phase fluid in the unit, where $q = 1$ represents the upper fluid and $q = 2$ represents the lower fluid. The sum of the volume fraction of each phase in each unit is 1, that is, the value of $a_q$ satisfies the following conditions: (1) When $a_q = 1$, the unit is completely $q$ fluid; (2) When $a_q = 0$, it means that the unit does not contain $q$ phase fluid; (3) When $0 < a_q < 1$, it means that the unit is located at the interface of the two-layer fluid.

B. BOUNDARY AND INITIAL CONDITIONS

At the interface of the two-layer fluid, the normal velocity and pressure should meet the following continuity conditions:

$$\zeta_i + u_i \zeta_i + u_i \zeta_i = w_i, \zeta_i + u_i \zeta_i = w_i, p_i = p_2$$

As the vertical velocity induced by ISWs on the free surface is very small, the upper boundary can be assumed as a rigid lid [14]. Under the condition that the bottom is an impermeable flat wall, the top and bottom of the numerical flume satisfy the following boundary conditions:

$$w_1(x,h) = 0, w_2(x,-h_2) = 0$$
The boundary conditions of numerical flume in Fluent software are set as shown in Table I.

| Boundary      | Boundary condition type |
|---------------|-------------------------|
| Left boundary | Velocity-inlet          |
| Top boundary  | Wall                    |
| Bottom boundary | Wall                  |
| Right boundary | Pressure-outlet         |

### C. NUMERICAL CONDITIONS

It is necessary to analyze the velocity field induced by the ISW in order to study the dynamic pressure characteristics of ISW propagation. There are two kinds of ISWs: a wave of elevation or depression, respectively. The latter is the most common in the ocean. Figure 2 shows the velocity field and streamline diagram of the depression ISW. It can be observed that with the pycnocline as the boundary, the horizontal velocities of the upper and lower fluids, and the vertical velocities on both sides of the trough are in the opposite direction, which shows the effect of mutual shear.

According to the approximate relationship between the average velocity of the layer and the instantaneous horizontal velocity, the horizontal and vertical velocities induced by ISWs in the upper and lower fluids are as follows [15]:

\[
u_i(X,t) = c \left[ 1 - \frac{h_i}{\eta_i} \left( \frac{\eta_i}{\eta_2} \frac{h_i^2}{h_i} \left( \frac{h_i}{h_i} \frac{2h_i}{\eta_i^2} \right) \right) \right]
\]

\[
u_i(X,t) = (-1)^i \frac{c \chi \eta_i \left( h_i + (-1)^i \zeta \right)}{\eta_i}
\]

where \( X = x - ct \), \( \eta_i = h_i + (-1)^i \zeta \), \( \eta_i^{\prime} = \eta_{i-1} \), \( \eta_i^\prime = \eta_{i+1} \) and \( i = 1, 2 \) denotes the upper (lower) layer fluid.

The dynamic pressure at any position in a two-layer fluid based on the Bernoulli equation is:

\[
\rho_i \frac{1}{2} \rho_i \left( u_i^2 + w_i^2 \right)
\]

near the trough position of wave (namely \( X = X_{o} \)), where \( \eta_i > 0 \) and \( \eta_i^{\prime} = 0 \). It can be seen from equation (9) that the vertical velocity can be ignored [15], so the dynamic pressure at the trough position is:

\[
\rho_i \frac{1}{2} \rho_i u_i^2
\]

As the layer-averaged velocity is very close to the instantaneous horizontal velocity [16], the layer-averaged velocity \( \overline{u_i} \) is substituted into (11) in order to simplify the calculation and obtain the approximate theoretical dynamic pressure value of the trough position. This value is given by the following expression:

\[
\rho_i \frac{1}{2} \rho_i \overline{u_i}^2 \approx \frac{1}{2} \rho_i \left( \frac{c \zeta}{h_i + (-1)^i \zeta} \right)^2
\]

where \c, \zeta and \a denote the phase speed, interface displacement and amplitude of an ISW, respectively.

### III. EXPERIMENTAL FACILITY AND PROCEDURES

A series of experiments were carried out in the large IWF (length: 30 m, width: 0.6 m, height: 1.2 m) at SJTU as shown in Fig. 3. The density stratified fluid was prepared by the injection method. Fresh water with density of 998 kg/m$^3$ and depth of \( h_1 \) was first injected into the flume, and then salt water with density of 1025 kg/m$^3$ was slowly injected into the flume by two mushroom-type water injection devices at the bottom of the flume, so that the upper fresh water was continuously lifted up until the total depth reached 1 m.

The whole IWF system consists of wave generated zone, propagation zone and dissipation zone. The total length of wave generated zone and propagation zone is 24 m, and the length of wave dissipation zone is 6 m. A wave-making device is installed in the wave generated area, and the ISW is generated by the double push-pedals wave maker. When the upper and lower push plates move to the right and left respectively controlled by the motor, the depression ISW will be generated at the interface of the two layers of fluid. In the wave dissipation area at the right end of the flume, a porous medium slope wave dissipation device is set to dissipate wave energy and reduce the reflection of ISW.

The conductivity probe array is installed at \( x = 10 \) m in the IWF to obtain the waveform and amplitude of the ISW. The array
is composed of 13 equidistant probes with a spacing of 3 cm. Due to the linear relationship between conductivity and density, the density change can be obtained by post-processing the conductivity signal, so as to obtain the waveform and amplitude information of the ISW.

**FIGURE 3.** IWF structure schematic.

**FIGURE 4.** Structure diagram of VLF piezoelectric sensor.

**FIGURE 5.** Dynamic pressure measurement experimental facility.

In the experiment, a VLF piezoelectric sensor composed of a piezoelectric ceramic tube and a high impedance low noise preamplifier is used to measure the low frequency dynamic pressure signal induced by ISW [17]. As shown in Fig. 4 shows the schematic diagram of VLF piezoelectric sensor structure. The whole structure adopts air backing structure, which is mainly composed of piezoelectric ceramic tube, interference support rod, axial stress release structure, electronic storehouse, water-tight coating layer, underwater electrical connector and longitudinal watertight cables. Piezoelectric ceramic tube is a high impedance sensing element, which usually needs to be used in conjunction with the pre-amplifier with high input impedance to complete the measurement work. Therefore, the pre-amplifier is installed in the electronic warehouse. At the same time, considering that the pre-amplifier has high input impedance and is vulnerable to environmental electrical noise, the piezoelectric tube element and its positive and negative output lines are wrapped by electromagnetic shielding wire mesh and connected to the input end of the pre-amplifier through SubMiniature version A (SMA) joint. In addition, the water vapor infiltration will lead to the decrease of the sensor insulation resistance, resulting in the decrease of low-frequency sensitivity and charge leakage. Therefore, a layer of polyurethane rubber is poured around the main body of the sensor to play a role of water tightness.

The piezoelectric element belongs to a high impedance signal source and the output signal is weak. The lower the frequency is, the greater the output impedance is, and the higher the input impedance requirement of the receiving end is. If the input
impedance of the pre-amplifier circuit is insufficient, the sensitivity of the piezoelectric sensor in the low frequency band will decrease. Therefore, it is usually necessary to configure the pre-amplifier for impedance transformation and amplification of low-frequency weak signals. In addition to considering the input impedance factor of the pre-amplifier, since the piezoelectric sensor works at VLF (< 0.1 Hz), the equivalent input noise of the device in the preamplifier at this frequency band, especially the 1/f noise, will increase significantly.

The active devices commonly used in amplifier circuits include transistor (BJT), junction field effect transistor (JFET) and metal oxide semiconductor field effect transistor (MOSFET). For high impedance signal sources such as piezoelectric elements (>10^5 Ω), JFET and MOSFET with higher input impedance are generally used to design preamplifier circuits, in which JFET has lower 1/f noise and higher transconductance gm than MOSFET in the low frequency band. Therefore, according to the design requirements of VLF piezoelectric sensors with high input impedance, low frequency and low noise, discrete JFET devices are used in the input stage of the pre-amplifier. As shown in Fig. 4, the high input impedance low noise pre-amplifier consists of two-stage amplifier circuit, the first stage consists of discrete JFET and some passive devices, and the second stage consists of integrated operational amplifier. According to Friis formula [18], the noise of the cascade amplifier circuit mainly depends on the noise of the first stage amplifier circuit. Therefore, the key device Q1 of the first stage amplifier circuit selects IF9030, which is one of the JFET with the best noise performance [19]. Its equivalent input noise at 0.5 Hz is as low as 2.7 nV/√Hz. The noise of the second stage amplifier circuit has little effect on the finishing noise of the preamplifier, so the ultra-low noise precision operational amplifier OPA27 (manufactured by Texas Instruments) is adopted, and its equivalent input noise is only 3.8 nV/√Hz at 10 Hz. The addition of the second stage amplifier circuit can effectively reduce the input capacitance of the previous stage JFET, which is conducive to reducing the noise of the entire circuit. Moreover, the negative feedback circuit composed of the operational amplifier U1 and the previous stage is integrated, so that the gain of the entire preamplifier is stabilized at 20 dB.

The performance of the sensor is tested before the experiment. The frequency response range is 0.06 Hz ~ 10 Hz, and the sensitivity is -166 dB [20]. During the experiment, according to the relationship between the output voltage of the VLF piezoelectric sensor and the dynamic pressure, the dynamic pressure induced by ISW can be obtained by real-time conversion of the data acquisition system.

Since the piezoelectric ceramic element of the sensor adopts the radial polarization mode, namely its side has uniform sensitivity, the axial direction of the sensor is perpendicular to the propagation direction of the ISW. As shown in Fig. 5(a), the sensor is fixed with the 1-shaped fixture at the top of the flume by an annular fixture to place it in the upper fluid and place the entire device at x = 10 m in the IWF. In addition, an appropriate amount of red fuel was added to the lower fluid to facilitate the observation of the propagation of ISW at the interface, as shown in Fig. 5(b).

### IV. NUMERICAL AND EXPERIMENTAL RESULTS

#### A. NUMERICAL IMPLEMENTATION

The numerical flume in this paper is modeled according to the actual size of the large IWF at the SJTU as shown in Fig. 1. In the numerical flume, the length of generation and propagation zone of ISW is 24 m, the length of wave dissipation zone is 6 m, and the total water depth is 1 m. The fluid density of the upper and lower layers is $\rho_1 = 998$ kg/m$^3$ and $\rho_2 = 1025$ kg/m$^3$ respectively. Through the redevelopment tools UDF (User Defined Function) of Fluent software, the velocity of the upper and lower fluid at the inlet of the flume is specified to produce ISWs. The numerical model works as follows. First, the appropriate theoretical solution $\zeta(t)$ of ISW is selected and substituted into (4) to calculate the depth-averaged velocities $u_1$ and $u_2$ of the upper and lower fluids induced by ISW, respectively, as the boundary conditions of the velocity inlet. Then, the inlet boundary condition is written into codes based on the UDF tool to drive the velocity inlet boundary of the numerical flume model, thereby controlling the velocity distribution of the inlet boundary and the volume fraction of the upper and lower fluids.

| Case ID | $h_1$ / m | $h_2$ / m | $a$ / m | ISW theory |
|---------|-----------|-----------|----------|------------|
| A       | 0.2       | 0.8       | 0.0474   | KdV        |
| A2      | 0.2       | 0.8       | 0.0913   | eKdV       |
| A3      | 0.2       | 0.8       | 0.1304   | eKdV       |
| A4      | 0.2       | 0.8       | 0.2033   | eKdV       |
| A5      | 0.2       | 0.8       | 0.2433   | MCC        |
| B1      | 0.25      | 0.75      | 0.0489   | eKdV       |
| B2      | 0.25      | 0.75      | 0.0970   | eKdV       |
| B3      | 0.25      | 0.75      | 0.1140   | eKdV       |
| B4      | 0.25      | 0.75      | 0.1350   | eKdV       |
| B5      | 0.25      | 0.75      | 0.1540   | eKdV       |
| C1      | 0.3       | 0.7       | 0.0340   | eKdV       |
| C2      | 0.3       | 0.7       | 0.0890   | eKdV       |
| C3      | 0.3       | 0.7       | 0.1190   | eKdV       |
| C4      | 0.3       | 0.7       | 0.1506   | eKdV       |
| C5      | 0.3       | 0.7       | 0.1620   | eKdV       |
| D1      | 0.1       | 0.9       | 0.0330   | MCC        |
| D2      | 0.1       | 0.9       | 0.0830   | MCC        |
| D3      | 0.1       | 0.9       | 0.1373   | MCC        |
| D4      | 0.1       | 0.9       | 0.2023   | MCC        |
| D5      | 0.1       | 0.9       | 0.2515   | MCC        |

At present, the commonly used theories to describe the propagation and evolution of ISWs are Korteweg-de Vries theory (KdV) [21], extended KdV (eKdV) theory [22] and Miyata-Choi-Camassa (MCC) theory [14]. The three theories are applicable to describe weak nonlinear, medium nonlinear and strong nonlinear ISWs in turn.

In this paper, a total of 20 numerical conditions are designed as shown in TABLE II. The conditions can be divided into four groups A, B, C and D based on density stratification. Column 5 of TABLE II shows the most suitable ISW theory for this working condition according to the applicability conditions of the three types of ISW theories [23]. In addition, eight probes are set in the numerical flume to monitor the interface
displacement and dynamic pressure changes caused by the propagation of ISWs in real time.

Due to the shape rules of the computational domain, the domain is discretized by structured grids. The $x$-direction grid spacing for the propagation zone is $\lambda / 80$, where $\lambda$ is defined as follows [15]:

$$\lambda = \frac{1}{a} \int_{-\infty}^{\infty} \zeta(X) dX$$  \hspace{1cm} (13)

In (13), $a$ and $X_w$ are the amplitude and position of the maximum displacement of the ISW, respectively. The grid for the propagation region in the $z$-direction is divided into three parts: $-h_0 \leq z \leq -a$, $-a \leq z \leq 0$ and $0 \leq z \leq h$. In the interface region of wave propagation ($-a \leq z \leq 0$), the grid spacing in the $z$-direction is $\lambda / 500$. In the regions above and below this region ($-h_0 \leq z \leq -a$, $0 \leq z \leq h$), the first layer grid is $\lambda / 500$, and the subsequent grid spacing is gradually widened in the ratio of 1.03. The $z$-direction grid in the wave dissipation zone is consistent with the propagation zone. The first layer of the $x$-direction grid is $\lambda / 80$, and the subsequent grid spacing is gradually widened in the ratio of 1.06. Figure 6 shows the grid division effect.

The governing equation of the numerical flume is discretized using the finite volume method (FVM). The convection term is discretized using the second-order upwind scheme. The pressure-velocity coupling is calculated using the pressure implicit with splitting of operators (PISO) algorithm, and a geometric reconstruction method is used for the VOF volume fraction. The time-term is discretized using the first-order upwind scheme. In all simulation cases, the courant number $C = u \Delta t / \Delta x$ is set to 0.2, and the selection of time step size needs to meet $\Delta t \leq \Delta x / u$. Among them, $u$ takes the average velocity of the layer depth of the upper fluid, and $\Delta x$ is the grid size. It can be seen that the time step size is an adjustable range. Therefore, $\Delta t = 0.05$ s, 0.01 s and 0.005 s are selected to verify the time step independence. The test results show that when $\Delta t = 0.01$ s, the numerical results have converged. Finally, we choose the time step $\Delta t = 0.01$ s, which takes into account the accuracy and efficiency of numerical calculation.

![FIGURE 6. 2D numerical flume model of ISW in CFD.](image)

**B. VALIDATION OF NUMERICAL FLUME**

In order to verify the validity of the numerical flume, theoretical and experimental (experimental data derived from reference [23], measured by conductivity probe array) interface displacement values monitored at $x = 8$ m are compared in Fig. 7. The three subfigures represent the weak nonlinear, medium nonlinear and strong nonlinear ISWs in turn. It can be noted that the simulation results agree closely with the experimental and theoretical results, as the maximum relative error is less than 2%. This indicates that under the current numerical settings, the current numerical flume model can accurately generate the waveform whether it is weak nonlinear, medium nonlinear or strong nonlinear ISWs.

[Table: Comparison of theoretical, experimental and CFD simulation waveforms of ISWs for Case A1, A3 and A5.]

**C. DYNAMIC PRESSURE CHARACTERISTICS OF ISW**

Figure 8 shows the density field cloud images of Case A5 at different times. Limited by the large aspect ratio of the flume, the cloud image shows only the top 20 m of the numerical flume, where the blue represents the upper fluid and the red represents the lower fluid. It can be seen from the figure that the interface of ISWs propagates stably and clearly without breaking.
FIGURE 8. Cloud images of density field at different times.

Corresponding to Fig. 8, Fig. 9 is the cloud picture of dynamic pressure field during the propagation of ISWs. It can be noted that as the ISW moves to the right, the dynamic pressure energy is concentrated near the interface of the ISW, while the dynamic pressure at other positions is zero. Further analysis of the dynamic pressure magnitude near the wave interface shows that with the interface as the boundary, the dynamic pressure above the interface is higher than that below the interface, and that near the trough is the largest.

FIGURE 9. Cloud images of dynamic pressure field induced by ISW at different time instants.

In order to further quantitatively study the variation law of dynamic pressure induced by ISWs, eight probes are set up in the whole wave propagation zone as shown in Fig. 1. This setup is equivalent to the arrangement of eight wave altimeters and pressure sensors to monitor the interface displacement and dynamic pressure changes with time at different positions in the numerical flume. Figure 10 shows the temporal changes in dynamic pressure collected by the probes in the upper and lower fluids of Case A5, where Figs. 10(a) and (b) represent the temporal dynamic pressure changes of the upper and lower fluids along the z direction of the flume (x = 8 m), and Fig. 10 (c) represents the changes of the upper fluid along the x direction of the flume.

Figures 10(a) and (b) show that the dynamic pressure in the upper fluid has no noticeable attenuation along the depth, while the dynamic pressure in the lower fluid decreases with the increase of depth. Therefore, as the dynamic pressure of the upper fluid attenuates very little along the depth, the middle point of the upper fluid without disturbance is considered to represent the variation trend of the dynamic pressure of the upper fluid in subsequent studies.

Figure 10(c) shows that in the upper fluid, the induced dynamic pressure has no obvious attenuation during the propagation of the ISW along the x direction of the flume. For the convenience of discussion and comparison, unless stated otherwise, x = 8 m of the numerical flume is considered for the waveform and dynamic pressure monitoring positions in all cases in the follow-up study.

FIGURE 10. Dynamic pressure changes of the upper and lower fluids with time (Case A5).

Figure 11 shows that in Cases A5, the interface displacement and upper fluid’s dynamic pressure collected by the probes at x = 8 m change with time. It can be observed that under the same density stratification, the dynamic pressure and the interface displacement over time are consistent. The dynamic pressure becomes maximum when the trough reaches the probe. Similarly, other cases have the same rules.
The phase speed of the ISW is obtained by measuring the time difference of the wave trough passing through the two probes, as shown in (14) [23]. It can be obtained according to the synchronous change of dynamic pressure and interface displacement by measuring the time difference of the maximum dynamic pressure induced by the ISW passing through the two probes. For example, two dynamic pressure monitoring probes (x = 8 m and x = 12 m) were set up in the numerical flume of Case A5, with the two probes spaced at Δx = 4 m, and the measured maximum dynamic pressure time difference is Δt = 15.86 s. By using these values in (14), the phase speed c is obtained as 0.2522 m/s. The theoretical phase speed of Case A5 is 0.2544 m/s, and the error between the two is only 0.86 %, which indicates the feasibility of indirectly measuring the phase speed of the ISW by using dynamic pressure.

\[ c = \frac{\Delta x}{\Delta t} \]  

The maximum dynamic pressure induced by ISW in the upper fluid under the same density stratification is an important characteristic parameter. Figure 12 shows the relationship between the maximum dynamic pressure induced by ISW in the upper fluid and the corresponding amplitude under four density stratifications. The small circles in the figure represent the CFD simulation results. It can be observed that under the same density stratification, the maximum dynamic pressure increases as the amplitude of the ISW increases. The relationship between the two is approximately linear, and the nonlinear error γa is less than 6 %.

When the amplitude is less than 0.1 m in the four density stratifications, the KdV theory is consistent with the CFD simulation results. When the amplitude is more than 0.1 m, the KdV theory and the CFD simulation results show a large deviation. This behavior occurs because the KdV theory is only suitable for describing the ISW with an amplitude-depth ratio less than 0.1 [23]. Compared with the KdV theory, the eKdV theory is consistent with the CFD simulation results in the range of amplitude limit. Once this limit is approached or exceeded, there is a significant deviation between the eKdV theory and the CFD results. Compared with the other two theories, the MCC theory is always the most consistent with the CFD simulation results in the four density stratifications. This indicates that the MCC theory based on strong nonlinearity and weak dispersion has a wider application range within the range of amplitude limit. TABLE III provides the limit amplitudes of eKdV and MCC theories for each density stratification obtained using (15) [15].

\[ a_{m,eKdV} = \frac{4h_1h_2(h_1 - h_2)}{h_1^2 + h_2^2 + 6h_1h_2}, \quad a_{m,MCC} = \frac{h_1\sqrt{\rho_2 / \rho_1 - h_2}}{\sqrt{\rho_2 / \rho_1} + 1} \]  

The above analysis shows that the MCC theory can effectively describe the relationship between maximum dynamic pressure and amplitude under each density stratification. This means that when the thickness and densities of the upper and lower fluid are known, the maximum dynamic pressure induced by the ISW in the upper fluid is measured. Subsequently, the amplitude of the ISW at the current position can be estimated according to (14) based on the MCC theory.

### TABLE III

| Density Stratification | \( a_{m,eKdV} \) (m) | \( a_{m,MCC} \) (m) |
|------------------------|----------------------|----------------------|
| 20:80                  | 0.2341               | 0.2967               |
| 25:75                  | 0.2143               | 0.2466               |
| 30:70                  | 0.1826               | 0.1967               |
| 10:90                  | 0.2118               | 0.3967               |

### D. DYNAMIC PRESSURE CHARACTERISTICS OF ISW INDUCED IN NUMERICAL FLUME WITH STRUCTURE

In order to develop marine resources, many marine engineering structures have been built. These structures include semi-submersible platform, tension leg platform (TLP) and Spar platform, and are usually made up of complex cylindrical structures [24]. The ISW may interact with the structures during the propagation process. In the following, the variation law of dynamic pressure after the interaction between the ISW and circular structures is numerically studied.

Figure 13 shows a numerical flume model for placement of horizontal cylindrical structures. The velocity of the upper fluid is greater than that of the lower fluid for the depression ISW. In order to attenuate the energy of the ISW as much as possible, the cylindrical structure is fixed in the upper fluid, the central axis is 10 m away from the inlet of the flume, and the diameter D is 0.7 times the depth of the upper fluid. In addition, probes are arranged before and after the numerical flume structure for real-time monitoring of the changes of the interface displacement and dynamic pressure in the flume.

Figure 14 shows the surface changes of ISWs for Case A5 before and after passing through the structure at different times. When \( t = 50 \text{ s} \), as shown in Fig. 14(a), the entire wave surface of the ISW is smooth and symmetric before interacting with the structure. After the interaction, as shown in Figs. 14(b) and (c), the wave surface of the ISW breaks. Figure 14(d) shows that the wave surface of the ISW regain smooth and symmetrical characteristics until the ISW leaves the structure, indicating that the nonlinearity and dispersion of the ISW reach the equilibrium again.

Figure 15 shows the cloud images of the dynamic pressure fields before and after the ISW passes through the cylindrical structure. It can be observed that before the ISW passes through the structure, the dynamic pressure distribution near the wave surface is symmetrical. When the ISW reaches the structure, the dynamic pressure near the wave surface is disordered. When the whole wave surface leaves the structure, the dynamic pressure near the wave surface regains a symmetrical distribution.
FIGURE 12. Variation of maximum dynamic pressure induced by ISWs in upper fluid versus amplitude.

FIGURE 13. Numerical flume model with a cylindrical structure.

FIGURE 14. Waveform changes of ISWs before and after passing through cylindrical structures.

Figure 16 shows the time-varying dynamic pressure obtained from the probe behind the structure in Case A5, where (12,0.1) is the nearest probe to the structure. It can be noted that the dynamic pressure field near this position is relatively disordered. When the ISW propagates to (14,0.1), the dynamic pressure begins to stabilize, indicating that the nonlinearity and dispersion of the ISW are rebalanced at this time.

FIGURE 15. Cloud images of ISW dynamic pressure fields before and after passing through cylindrical structures.
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FIGURE 16. Dynamic pressure changes over time (Case A5).

Figure 17 shows the variation of the maximum dynamic pressure of the ISW versus the corresponding amplitude before and after passing through the structure under different densities. Figure 16 shows that the dynamic pressure field of the probe close to the structure is relatively dispersed. Therefore, the probes at \( x = 14 \) m and after \( x = 14 \) m are selected for studying the maximum dynamic pressure law in each case. In Fig. 17, the probe coordinates in the lower right corner not marked with ‘*’ refer to the absence of structure and that marked with ‘*’ refer to the presence of structure. The first column in Fig. 17 shows that when the ISW does not pass through the structure, the amplitude and maximum dynamic pressure are consistent with those of the corresponding cases of non-structure, and the variation law is consistent with the theoretical solution.

The second column in Fig. 17 shows the variation of the maximum dynamic pressure versus the amplitude under each density stratification after the ISW passes through the obstacle. Compared with the condition without structure, the amplitude and maximum dynamic pressure of the corresponding condition with structure are significantly lower, but the change trend of the two still matches the theoretical curve. It can be observed from the third column in Fig. 17 that when the nonlinearity and dispersion effect of ISW reach the equilibrium again, the maximum dynamic pressure and the corresponding amplitude show minor decay along the propagation direction of ISWs.

The above results show that under the same density stratification, the relationship between the maximum dynamic pressure and the amplitude still satisfies the corresponding ISW theory irrespective of whether the ISW passes through the structure, and the nonlinear error \( \gamma_L \) is less than 7%. This conclusion also shows that although the ISW dissipates a part of energy after interacting with the structure, the ISW characteristic parameters after dissipation can still be estimated by measuring the maximum dynamic pressure in the upper fluid after dissipation and combining with the MCC theory.

E. INTERNAL SOLITARY FLUME EXPERIMENT

To reduce the effect of the sensor structure size on the flow field, the upper fluid depth cannot be too shallow, so the density stratification ratio of the experiment is \( h_1:h_2 = 30:70 \), i.e., 0.3 m for the upper freshwater layer and 0.7 m for the lower brine layer. As accurate density stratification is key to generating ISWs with the expected amplitude, an array of conductivity probes is used to measure the density stratification state of the two fluid layers after we configure the stratified fluid.
Figure 18 shows the measurement results of stratified fluid density profile and the corresponding buoyancy frequency profile when depth ratio $h_1:h_2 = 30:70$. The buoyancy frequency is calculated by $N(z) = \frac{g}{\rho_1}(\frac{\partial \rho}{\partial z})$, where $\rho$ is the vertical density distribution [2]. The diagram shows that there are three layers of fluid system in the flume, including the upper freshwater layer, the density transition layer near the interface and the lower saltwater layer. If the maximum buoyancy frequency is seen as boundary, the upper fluid depth is just 0.3 m, and the lower fluid depth is just 0.7 m. Therefore, the current flume can be regarded as a two-layer fluid system. At the same time, it shows that the prepared stratified fluid meets the experimental requirements. A total of six groups of tests with different amplitudes were carried out in this experiment, as shown in TABLE IV.

Figure 19 shows the change of the interface before and after the ISW passes through the VLF piezoelectric sensor (Case E6).

The four sub-images in Fig. 19 clearly show the changes of the interface before and after the ISW passes through the VLF piezoelectric sensor. Reference arrows are marked before and after the sensor in each sub-image, where the length of the arrow is the same as the depth of the upper fluid when undisturbed and its direction points to the interface. By comparing the distance between the arrowhead and the interface, the characteristics of ISW wave front and wave back can be clearly distinguished. As can be seen from the four sub-images, the wave surface is clear and unbroken before and after the ISW passes through the sensor, indicating that the sensor size structure has a small effect on the flow field of the ISW.

A comparison of the dynamic pressure induced during the propagation of ISWs measured using piezoelectric sensors in Case E1, E3 and E6 with the CFD simulation results is shown in Fig. 20. It can be seen from the figure that the numerical simulation results are in good agreement with the experimental results, indicating that the dynamic pressure induced by ISWs obtained by numerical calculation in the current numerical...
flume is reasonable and credible. It can be seen that the measured maximum dynamic pressure increases with the constant amplitude of the ISW.

As can be seen from the above TABLE V, the maximum error between the inversion values of the ISW amplitude and the measured values under the six working conditions does not exceed 6%, which indicates that the method of inversion of the ISW amplitude is feasible by measuring the dynamic pressure induced by the ISW and combining it with the MCC theory.

V. CONCLUSIONS

In this paper, based on the actual size of large IWF in SJTU, the numerical flume model of ISW is established. The dynamic pressure characteristics induced by ISW propagation are studied by using this model, and the relationship between the maximum dynamic pressure induced by ISW and the amplitude of ISW is clarified. Considering the existence of engineering structures in the actual ocean, the influence of structures on the dynamic pressure induced by ISW is discussed. Finally, the piezoelectric sensor is used to measure the dynamic pressure induced by ISW in the IWF in the laboratory, and the amplitude of ISW is inverted by combining it with the MCC theory. In summary, the following conclusions can be drawn:

1. For the two-layer fluid of finite water depth, the dynamic pressure in the upper fluid had no obvious attenuation along the depth, while the dynamic pressure in the lower fluid decreased with the increase of depth. The dynamic pressure induced by the ISW in the upper fluid was studied. It was found that the change of dynamic pressure induced by the ISW was synchronized with the change of interface displacement. Therefore, the real-time monitoring of ISW could be realized by measuring the dynamic pressure of upper fluid.

2. The relationship between the maximum dynamic pressure induced by the ISWs in the upper fluid and the corresponding amplitude under four density stratifications was studied. It was found that under the same stratification, the maximum dynamic pressure had a strong linear relationship with the amplitude, the nonlinear error was less than 7%, and the relationship could be described by the MCC theory. When the density profile of the stratified fluid was known, the amplitude, phase velocity of the current ISW and the maximum horizontal velocity of the upper fluid could be retrieved by measuring the maximum dynamic pressure of the upper fluid combined with the MCC theory. In addition, when the ISW interacted with a structure, it dissipated a part of energy after passing through the structure. However, the characteristic parameters of the ISW after dissipation could still be estimated by measuring the maximum dynamic pressure in the upper fluid and combining with the MCC theory.

3. In the laboratory IWF, the dynamic pressure induced by ISWs with different amplitudes was measured by a piezoelectric sensor that can sense the dynamic pressure change. The results show that the dynamic pressure measurement results are in good agreement with the measured values under the six working conditions does not exceed 6%, which indicates that the method of inversion of the ISW amplitude is feasible by measuring the dynamic pressure induced by the ISW and combining it with the MCC theory.

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agreement with the numerical simulation results, and the maximum error between the measured amplitude of ISWs and the measured amplitude is less than 6% by using the MCC theory, which verifies the feasibility of the proposed inversion method for the amplitude parameters of ISWs by using the dynamic pressure measurement. The conclusions obtained in this paper are only verified in the laboratory flume scale. For the actual ocean situation, the applicability of the theory and method proposed in this paper needs to be further studied.

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