Degree of Belief Analysis and Control Method of Reliability Assessment Result of Existing Structures

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Abstract. Reliability assessment result of existing structures is analyzed on the basis of a few samples, which often has strong subjective uncertainty and will affect reliability assessment result and structural safety. In this paper, belief function is introduced to quantify the subjective uncertainty and analyze influence of different degree of belief on structural failure assessment. The calculation method of degree of belief of reliability evaluation result of existing structures and optimization model under complex multiple degree of belief environment is proposed. Furthermore, reliability degree of belief control method is proposed to remedy the defect that subjective uncertainty has not been taken into account in past control methods. It is found that calculation results of different failure probability depend on degree of belief, which affects safety assessment of existing structures. The degree of belief calculation method of reliability structure proposed in this paper quantitatively reduces the influence of subjective uncertainty and makes assessment result more accurate and scientific.

1. Introduction

Structural reliability is a probability measure of a structure's completion of a predetermined function within a specified time, under specified conditions, and the reliability control[1]. Among factors affecting reliability of existing structures, the probabilistic characteristics and current values of random things such as action, material properties, geometric parameters should be objective[2]. If limit state (failure criterion) to determine whether structure fails are clear, then the reliability of the structure should have an objective value[3-4]. However, it is difficult to completely ascertained the probabilistic characteristics of random things in engineering practice. Even for current value of existing things, people's understanding may be uncertain, incomplete and vague[5-8].

The evaluation of the performance and state of existing structures is the basis of judging their remaining life, and is also the precondition for choosing a reasonable reinforcement mode[9-10]. In actual engineering, there are great differences between existing structure and proposed structure. Due to the limitation of conditions or the influence of environment, the existing structure can not provide enough samples in accordance with the proposed structure[11-13]; therefore, in few samples environment, the reliability assessment results inferred from only a small number of observation samples will inevitably be affected by uncertainties.

In order to objectively describe the influence of uncertain factors, it is necessary to assess the degree of belief of reliability assessment results in uncertain environments. On the basis of the concept of belief function in evidence theory[14-16], this paper deeply discusses the degree of belief of reliability assessment results of existing structures and puts forward specific degree of belief analysis and control methods of existing structures.
2. Degree of belief analysis in uncertainty problems Introduction

2.1. Basic concepts of belief function
Let $Bel$ be a real-valued set function defined on the proposition field $\Gamma$, that is, according to the set function, any proposition in the proposition field $\Gamma$ corresponds to a specific real number. If this condition is satisfied, $Bel$ is called the trust measure on proposition field $\Gamma$, and $Bel(A)$ is the degree of belief of proposition $A$, which represents the measure of affirmative proposition $A$.

The belief functions have the following attributes[16-18]:

1. Non-negativity. For any proposition $A \in \Gamma$, the $Bel(A)$ is expressed as:

   $$0 \leq Bel(A) \leq 1$$

2. Necessity. On the proposition of necessity $\Omega$, the $Bel(\Omega)$ is expressed as:

   $$Bel(\Omega) = 1$$

3. Additivity. For any proposition $A_i \in \Gamma (i = 1, 2, ..., n)$, the calculation method can be obtained as follows:

   $$Bel \left( \bigcup_{i=1}^{n} A_i \right) = \sum_{i=1}^{n} Bel(A_i) - \sum_{j<i} Bel(A_i \cap A_j) + \sum_{k<j<i} Bel(A_i \cap A_j \cap A_k) - ...$$

Similar to the definition of random variables, Let $\xi = \xi(\omega)$ be a single-valued real function defined on the domain $\Omega$. For any real number $x$, and the existence relation $\omega : \xi(\omega) \leq x \in \Gamma$, that is, the proposition consisting of all basic propositions satisfying $\xi(\omega) \leq x$ conditions belongs to the propositional domain $\Gamma$, then $\xi = \xi(\omega)$ is called uncertainty variable, and obtained the function is expressed as:

$$Bel_{\xi}(x) = Bel\{\omega : \xi(\omega) \leq x\}$$

The belief function of the uncertain variable $\xi$ can also be recorded as $Bel\{\xi \leq x\}$, which indicates the degree of belief in the proposition $\{\xi \leq x\}$. When the belief function $Bel_{\xi}(x)$ is absolutely continuous, the belief density function $bel_{\xi}(x)$ can be introduced, which the function as follows:

$$Bel_{\xi}(x) = \int_{-\infty}^{x} bel_{\xi}(t)dt$$

2.2. Analysis Modeling for the Effect of degree of belief on Reliability
Let the function $Z'$ denote the difference between the standard value and the estimated value of the performance function of the existing structure.

$$Z' = Z(X) - Z(x) = g(X_1, X_2, ..., X_n) - g(x_1, x_2, ..., x_n)$$

Where $X$ is the standard value of the random variable and $x$ is the estimate of the degree of belief $C$ of the random variable.

Considering the influence of degree of belief of resistance random variable on reliability evaluation of existing structures, then, with equation (6), the performance function of the existing structure is expressed as:

$$Z' = Z\{[R(X_1, X_2, ..., X_n), S]\} - Z\{[R(x_1, x_2, ..., x_n), S]\}$$

Assuming that the normal distribution of random variables are $X \sim N(\mu, \sigma^2)$, the random variables with both structural resistance are also subject to normal distribution. However, due to the damage caused by material defects or construction defects and external loads, the average value of random variables remains unchanged and the variance changes. so, the probability density functions(PDF) are shown as figure 1.
According to figure 1, this phenomenon makes the degree of belief of random variables estimates unable to match the degree of belief conditions of design requirements, and \( P(X \leq x_C) \) has the follows obviously:

\[
P(X \leq x_C) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_1}} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right) dx \geq P(X' \leq x_C) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_2}} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right) dx
\]

Where \( P(X \leq x_C) \), \( P(X' \leq x_C) \) denotes that the original random variables and the actual random variables are less than the probability values of the same estimated value, respectively. In order to ensure that the estimated value of the original random variable has the same probability as that of the actual random variable, it is necessary to increase the range of the actual random variable in the existing structure and reduce value of the degree of belief, and the relation is expressed as:

\[
P(X \leq x_C) = P(X' \leq x_{(AC)})
\]

Where \( k \) is adjustment coefficient, and \( k \leq 1 \) or \( C \leq kC \).

### 2.3. Multi-dimensional degree of belief modeling

Based on probability theory and took two random variables as examples, the joint normal PDF of two random variables are considered, the following holds[19-20]:

\[
f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2(1-\rho^2)^{1/2}} \exp\left[-\frac{1}{2(1-\rho^2)} \left\{ \frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} \right\} \right]
\]

Where \( x_1 \) and \( x_2 \) are random variables, \( \mu_1, \sigma_1 \) are mean value and variance of random variable \( x_1 \). \( \mu_2, \sigma_2 \) are mean value and variance of random variable \( x_2 \). \( \rho \) is the correlation coefficient of random variables \( x_1 \) and \( x_2 \). Similar to figure 1, specific values can be supposed, and the joint PDF of the original random variable and the actual random variable can be compared and analyzed with MATLAB, which is shown as figure 2.

According to equation (8), the calculation method of two-dimensional random variables is obtained as follows:

\[
P(X, Y \leq x_C, y_C) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x, y) dxdy \leq P(X', Y' \leq x_C, y_C) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x, y) dxdy
\]

Where \( f(x, y) \) followed equation (10), \( (X, Y) \) are original random variables, \( (X', Y') \) are actual random variables, \( (x_C, y_C) \) are the assessment of random variables with degree of belief \( C \).
3. Calculating model of degree of belief

3.1. Degree of belief calculation of uncertain variables

According to the method of analyzing and calculating random variables in probability theory, the subjective uncertain variables are analyzed and calculated by unascertained measure belief function\[5,21,22\]. For example, for the current resistance \( R \) of existing structures, it should be objectively determined, but it is generally difficult to straightforwardly capture its real value, which is uncertain and should be regarded as unascertained\[5,7\]. Meanwhile, the degree of belief of proposition \( \{ R \leq x \} \) can be written as \( \{ \mu_R(x) \leq \} \) or \( \{ \mu_{Bel_R}(x) \} \), which is called the degree of belief function of resistance \( R \) and its range is [0,1]. Because of the definition domain of resistance \( R \) is continuous, so the degree of belief density function \( \mu_{Bel_R}(x) \) can be set, and it follows:

\[
\int_0^x \mu_{Bel_R}(t)dt = \mu_R(x)
\]  

(12)

and degree of belief mean value of resistance \( R \) follows:

\[
E_{\mu}(R) = \int_0^\infty \mu_{Bel_R}(t)dt
\]  

(13)

At present, membership function \( \mu_{A}(x) \) is usually used to analyze and calculate the fuzzy failure criteria in structural applicability analysis\[23-25\]. It denotes the degree to which \( x \) belongs to the fuzzy set \( A \), while the fuzzy set \( A \) denotes the failure domain of the structure\[26-28\]. In fact, the fuzziness or subjective uncertainty of failure criterion is mainly reflected on the boundary of the fuzzy set \( A \). If the limit values \( W \) of deflection and crack width are taken into account, they should be regarded \( W \) as unascertained variables, and their belief function can be expressed as follows:

\[
Bel_{\mu}(x) = Bel\{W \leq x\}
\]  

(14)

\( Bel_{\mu}(x) \) denotes the degree of belief that \( W \) is not greater than \( x \), and follows:

\[
Bel_{\mu}(x) = \mu_{A}(x)
\]  

(15)

3.2. Degree of belief calculation

3.2.1. Degree of belief calculation of structural reliability.

If there are unascertained variables in structural safety analysis, the performance function of structures
follows as:

$$Z = g(X, Y)$$  \hspace{1cm} (16)

Similarly, the applicability performance function follows as:

$$Z = W - S(X, Y)$$  \hspace{1cm} (17)

Where $g(\cdot)$ is performance function, $S(\cdot)$ is load effect function. $X = \{X_1, X_2, \ldots, X_n\}$ are random vectors, which PDF is $f_X(x; \theta)$, and $\theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ are random vectors. $Y = \{Y_1, Y_2, \ldots, Y_m\}$ are unknown non-random vectors, $W$ is uncertain fuzzy boundary. Due to the fact that all factors affecting structural reliability, i.e. all the basic variables, should be identified as far as possible in engineering practice, the subjective uncertainty, which is composed of basic variables, is not considered here[29-30].

So the failure probability $P_f$ of the structure is a function of the unascertained quantity $\theta, Y$ and $W, \theta, Y$, which can be expressed as follows:

$$P_f = P_f(\theta, Y) = P\{g(X, Y) \leq 0\} = \int f_X(x; \theta) dx$$  \hspace{1cm} (18)

$$P_f = P_f(W, \theta, Y) = P\{\beta - S(X, Y) \leq 0\} = \int f_X(x; \theta) dx$$  \hspace{1cm} (19)

For any given failure probability, the belief functions be obtained as follows[5]:

$$Bel_{P_f}(p_f) = Bel\{P_f(\theta, Y) \leq p_f\} = \int \int bel_\theta(t)bel_y(y) dt dy$$  \hspace{1cm} (20)

$$Bel_{P_f}(p_f) = Bel\{P_f(W, \theta, Y) \leq p_f\} = \int \int \int bel_w(w)bel_\theta(t)bel_y(y) dw dt dy$$  \hspace{1cm} (21)

Then, mean value of degree of belief of failure probability $P_f$ of structures is expressed as follows:

$$E_b(P_f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_f(t, y)bel_\theta(t)bel_y(y) dt dy$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{g(x,y) \leq 0} f_X(x; t) dx bel_\theta(t)bel_y(y) dt dy$$

$$= \int \int \int_{g(x,y) \leq 0} f_X(x; t) bel_\theta(t) dt bel_y(y) dx dy = \int f_X(x; \theta) dx$$  \hspace{1cm} (22)

$$E_b(P_f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_f(w, t, y)bel_w(w)bel_\theta(t)bel_y(y) dw dt dy$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{W-S(X, Y) \leq 0} f_X(x; t) dx bel_w(w)bel_\theta(t)bel_y(y) dw dt dy$$

$$= \int \int \int_{W-S(X, Y) \leq 0} f_X(x; t) bel_\theta(t) dt bel_y(y) \int_{0}^{S(x,y)} bel_w(w) dw dx dy$$

$$= \int \int \int_{W-S(X, Y) \leq 0} f_X(x; t) bel_\theta(t) \mu_x[S(x, y)] dx dy$$

and

$$f_X(x) = \int_{-\infty}^{+\infty} f_X(x; t) bel_\theta(t) dt$$  \hspace{1cm} (24)

Equation (18)–equation (24) shows that:

(1) The larger limit value of failure probability $P_f$, the larger integral domain and greater degree of
belief of proposition \( \{ P_f \leq p_f \} \). When \( p_f = 1 \) integral region is the domain of definitions of independent variables, there should be \( Bel\{ P_f \leq p_f \} = 1 \), that is, the proposition of complete proposition \( \{ P_f \leq 1 \} \), when \( Bel\{ P_f \leq p_f \} = 0 \) integral domain is an empty set and should be \( Bel\{ P_f \leq p_f \} = 0 \), that is, completely negative proposition \( \{ p_f \leq 0 \} \).

(2) The greater variability of unascertained quantity \( W, \theta, Y \), the smaller degree of belief of proposition \( \{ P_f \leq p_f \} \), which actually means that the lower cognitive level of unascertained quantity \( W, \theta, Y \).

(3) If unascertained variables \( W, \theta, Y \) are regarded as a random variables, degree of belief of density function is regarded as a PDF, and the equation (24) is used to determine the PDF of the random variable \( X \), then degree of belief mean value of the failure probability \( E_b(P_f) \) is the same as the failure probability calculated by probability method or fuzzy probability method at present. And if the same substitution method is used and reliability index is calculated according to the current calculation method, the calculated result is actually degree of belief mean value of the reliability index.

(4) When the unascertained quantity \( W, \theta, Y \) are transformed into \( w, t, y \), that is, there is no unascertained quantity in the analysis of structural failure probability \( P_f \), equation (20)–equation (24) can be expressed as follows:

\[
Bel_{p_f}(p_f) = Bel\{ P_f(t,y) \leq p_f \} = Bel\{ \int f_X(x;t)dx \leq p_f \}
\]

\[
Bel_{p_f}(p_f) = Bel\{ P_f(w,t,y) \leq p_f \} = Bel\{ \int f_X(x;t)dx \leq p_f \}
\]

\[
E_b(P_f) = \int_{g(x,y) \leq 0} f_X(x)dx
\]

\[
E_b(P_f) = \int_{W-S(x,y) \leq 0} f_X(x)dx
\]

\[
f_X(x) = f_Y(x;t)
\]

The inequalities in equation (25) and equation (26) are general inequalities without unascertained quantities, which should be judged by general events: when inequalities are valid, \( Bel_{p_f}(p_f) = 1 \), otherwise, \( Bel_{p_f}(p_f) = 0 \). Degree of belief mean value of failure probability in equation (27) and equation (28) are completely equal to the failure probability calculated by the present probability method.

The above conclusions are consistent with the actual engineering judgment. Section (2) quantitatively reveals the relationship between people's cognitive level and the results of structural reliability analysis, and provides a mathematical method for reasonably considering the influence of subjective uncertainty in the analysis and control of structural reliability. Section (3) reveals an important conclusion, that is, when calculating the mean value \( E_b(P_f) \) of degree of belief of structural failure probability, the unascertained quantity can be regarded as a random variable, and the present probability or fuzzy probability method can be used completely, which provides a realistic way to calculate the mean value of degree of belief.

3.2.2. Example analysis.
There is a example to illustrate degree of belief analysis process of structural failure probability \( P_f \). If
the current resistance $R$ of existing structural is an unascertained quantity and its degree of belief distribution is a normal distribution $N(\mu_R, \sigma_R^2)$, then degree of belief density function of resistance $R$ be expressed follows:

$$ \text{bel}_R(r) = \frac{1}{\sigma_R} \phi\left(\frac{r - \mu_R}{\sigma_R}\right) $$

(30)

Where $\phi(\cdot)$ is standard normal PDF, the range of $r$ is $-\infty < r < \infty$, assuming that the resistance $R$ remains unchanged during the service life of the target, the load effect $S$ is a random variable with normal distribution $N(\mu_S, \sigma_S^2)$. Performance function of component holds:

$$ Z = R - S $$

(31)

Failure probability of components can be expressed as:

$$ P_f(x) = P_f(R) = \int_{-\infty}^{\infty} \phi\left(\frac{S - \mu_R}{\sigma_S}\right) ds = 1 - \Phi\left(\frac{R - \mu_R}{\sigma_S}\right) = \Phi\left(-\frac{R - \mu_R}{\sigma_S}\right) $$

(32)

and belief function can be expressed as:

$$ \text{Bel}_{P_f}(p_f) = \int_{-\infty}^{\infty} \text{bel}_R(r) dr = \int_{-\infty}^{\infty} \frac{1}{\sigma_R} \phi\left(\frac{r - \mu_R}{\sigma_R}\right) dr = \Phi\left(\frac{\mu_R - \mu_S + \sigma_S \Phi^{-1}(p_f)}{\sigma_R}\right) $$

(33)

To calculate degree of belief mean value in the following form:

$$ E_{\text{Bel}}(P_f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sigma_R} \phi\left(\frac{r - \mu_R}{\sigma_R}\right) dr dR = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sigma_R} \phi\left(\frac{s - \mu_L}{\sigma_S}\right) ds dR $$

$$ = \Phi\left(-\frac{\mu_R - \mu_S}{(\sigma_R^2 + \sigma_S^2)^{1/2}}\right) $$

(34)

3.3. Computing method of complex multiple degree of belief

Assuming that the estimated values of the standard values of basic variables $X_{1k}, X_{2k}, \ldots, X_{nk}$ are $x_{1k}, x_{2k}, \ldots, x_{nk}$ and corresponding degree of belief are $C_1, C_2, \ldots, C_n$, the failure probability of existing structures determined by the inferred results $x_{1k}, x_{2k}, \ldots, x_{nk}$ are $p_f(x_{1k}, x_{2k}, \ldots, x_{nk})$, which corresponds to the results of reliability evaluation of existing structures and can be regarded as the upper limit estimate of the lower failure probability $P_f(T)$ of a certain degree of belief $C$.

If only $X_{ik}$ ($i = 1, \ldots, n$) is unascertained in the standard value of the basic variable, it should be expressed follows:

$$ \text{Bel}\{p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk}) \leq p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk})\} = C_i $$

$$ \text{Bel}\{p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk}) \geq p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk})\} = 1 - C_i $$

(35)

(36)

It is confirmed that degree of belief of the actual failure probability $P_f(T)$ is not greater than that of the evaluation result $p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk})$ is $C_i$, and not less than that of the evaluation result $p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk})$ is $1 - C_i$. If degree of belief $C$ of the standard value $P_f(T)$ estimator $x_{ik}$ ($i = 1, \ldots, n$) of the basic variable is 0.75, it is known that the failure probability of the existing structure $P_f(T)$ is not greater than degree of belief interval of the evaluation result $p(x_{i1}, \ldots, x_{ik}, \ldots, x_{nk})$ is (0.25, 0.75). When the standard values of basic variables $X_{1k}, X_{2k}, \ldots, X_{nk}$ are unknown, similarly, according to equation (35) and equation (36), there should be given by:
\[ \prod_{i=1}^{n} C_i \leq \text{Bel}\{p(X_{1k}, ..., X_{ik}, ..., X_{nk}) \leq p(x_{1k}, ..., x_{ik}, ..., x_{nk})\} \leq 1 - \prod_{i=1}^{n} (1 - C_i) \]  

(37)

It can be seen that the failure probability \( P_f(T) \) of the existing structure is not greater than the reliability of the evaluation result \( p(x_{1k}, ..., x_{ik}, ..., x_{nk}) \) between \( \prod_{i=1}^{n} C_i \), \( 1 - \prod_{i=1}^{n} (1 - C_i) \).

If \( n = 2 \), \( C_1 = 0.6 \), \( C_2 = 0.75 \). So, the interval of degree of belief is [0.45, 0.9], but this interval is relatively broad. Next, a more optimal estimation interval is proposed for general cases.

Let \( p(X_{1k}, ..., X_{ik}, ..., X_{nk}) \leq p(x_{1k}, ..., x_{ik}, ..., x_{nk}) \) be proposition \( A \) and \( p(x_{1k}, ..., x_{ik}, ..., x_{nk}) \) be proposition \( B \). Their inverse propositions are \( A^C \) and \( B^C \). When proposition \( A \) is established, proposition \( B^C \) may be established, and proposition \( B \) may also be established when proposition \( A^C \) is established. Assuming that degree of belief \( C_{\min} \) of proposition \( B \) is the minimum in \( C_1, C_2, ..., C_n \), the first case is more likely to occur in general than the second case.

\[ \text{Bel}\{A \cap B^C\} \geq \text{Bel}\{A^C \cap B\} \]  

add both sides \( \text{Bel}\{A \cap B\} \), and

\[ \text{Bel}\{A\} \geq \text{Bel}\{B\} \]  

(38)

and

\[ \text{Bel}\{p(X_{1k}, ..., X_{ik}, ..., X_{nk}) \leq p(x_{1k}, ..., x_{ik}, ..., x_{nk})\} \geq C_{\min} \]  

(39)

If degree of belief of proposition \( B \) is \( C_{\max} \), the second case is more likely to occur in general than the first case. And have a relationship:

\[ \text{Bel}\{A \cap B^C\} \leq \text{Bel}\{A^C \cap B\} \]  

(40)

add both sides \( \text{Bel}\{A \cap B\} \), and

\[ \text{Bel}\{p(X_{1k}, ..., X_{ik}, ..., X_{nk}) \leq p(x_{1k}, ..., x_{ik}, ..., x_{nk})\} \leq C_{\max} \]  

(41)

and

\[ \text{Bel}\{A \cap B^C\} \leq \text{Bel}\{A^C \cap B\} \]  

(42)
\[ C_{\text{min}} \leq Bel\{ p(X_{i_1}, \ldots, X_{i_k}, \ldots, X_{i_n}) \} \leq p(x_{i_1}, \ldots, x_{i_k}, \ldots, x_{i_n}) \leq C_{\text{max}} \]  

(43)

Therefore, degree of belief of the existing structure whose failure probability \( P_f(T) \) is not greater than the evaluation result \( p(x_{i_1}, \ldots, x_{i_k}, \ldots, x_{i_n}) \) should generally be between \( C_{\text{min}} \), \( C_{\text{max}} \), and the width of this interval is smaller than that of the previous calculation. If \( n = 2 \), \( C_1 = 0.6 \), \( C_2 = 0.75 \). So, the interval of degree of belief is \([0.60, 0.75]\).

4. Degree of belief control of structural reliability

4.1. Modeling of control

Because the analysis of structural reliability in practical engineering is usually influenced by subjective uncertainty, Degree of belief of reliability analysis results must be considered when using the analysis results to control structural reliability. This is a practical method for people's actual cognitive level.

The control criteria of structural reliability are divided into following three levels[17]:

1. Limit state \( \Lambda \)
2. Maximum failure probability \( [p_f] \) or target reliability \( [\beta] \)
3. Maximal degree of belief \( [c] \)

Limit state \( \Lambda \) is the physical criterion of structural reliability control, which is directly used to determine whether the structure is invalid or not. Maximum failure \( [p_f] \) probability and objective reliability index \( [\beta] \) are probability criteria of structural reliability control, which are used to control the possibility of the structure completing its intended function. Both of them are requirements for the structure's own ability. The lowest degree of belief \( [c] \) is the reliability standard of structural reliability control, which is used to control the reliability of structural reliability analysis results. It is a requirement for understanding and analysis results.

By synthesizing these three levels of control standards, the control of structural reliability in engineering practice should meet the requirements as follows:

\[ Bel\{ P_f \leq [p_f] \} = Bel\{ P\{ Z \leq 0 \} \leq [p_f] \} \geq [c] \]  

(44)

Equation (44) shows that the reliability of structural failure probability \( P\{ Z \leq 0 \} \) is not greater than that of maximum failure probability \( [p_f] \), and that of minimum failure probability \( [c] \), where \( Z \) is performance function of the structure and it contains possible uncertainties. There is basic expression of structural reliability control and considering the influence of randomness and subjective uncertainty, it can be called degree of belief control method of structural reliability.

The basic expression of structural reliability control in present research is as follows:

\[ P\{ Z \leq 0 \} \leq [p_f] \]  

(45)

The commonly used control method only considers the objective structural reliability, but does not consider the influence of subjective uncertainty on the reliability analysis results. It is an ideal probability control method. However, degree of belief control method of structural reliability shown in equation (45) considers the influence of subjective uncertainty in a realistic manner, which makes the analysis results of structural reliability closer to people's actual cognitive level, and has more practical significance for the control of structural reliability.

4.2. Expression of degree of belief control

Generally, belief function of structural failure probability \( Bel_{p_f}(p_f) \) is difficult to express as the analytic expression shown in equation (33). It is usually a complex integral expression in the integral domain, which brings great mathematical difficulties to the reliability control of structural reliability.
Therefore, a feasible control method is proposed here. If degree of belief control of structural safety is considered and the unascertained quantity $Y$, $\theta$ is given a specific value $y^*$, the failure probability of the structure is its limit $[p_f]$, and its performance function can be expressed as follows:

$$Z^* = g(X, y^*)$$  \hspace{1cm} (46)

Where PDF of random vector $X$ is $f_x(x; t^*)$, and $[p_f]$ can be given by:

$$[p_f] = P_f(t^*, y^*) = P\{g(X, y^*) \leq 0\} = \int_{g(x, y) \leq 0} f_x(x; t^*)dx$$  \hspace{1cm} (47)

and degree of belief of the failure probability $P_f$ of the structure is not greater than its limit value $[p_f]$ is expressed as follows:

$$Bel\{P_f \leq [p_f]\} = Bel\{P_f(\theta, Y) \leq P_f(t^*, y^*)\} = \int_{P_f(\theta, Y) \leq P_f(t^*, y^*)} bel_\theta(t)bel_Y(y)dtdy$$  \hspace{1cm} (48)

and equation (47) can be written in the following form:

$$\int_{g(x, y) \leq 0} f_x(x; \theta)dx \leq \int_{g(x, y') \leq 0} f_x(x; t^*)dx$$  \hspace{1cm} (49)

Equation (49) defines the range of unascertained quantity $\theta, Y$. $Z'$ can be determined form:

$$Z' = Z - Z^* = g(X, Y) - g(X, y^*)$$  \hspace{1cm} (50)

Equation (50) is a function of the unascertained quantity $\theta, Y$, and is composed of two isomorphic functions in a linear relationship. Therefore, equation (48) can be expressed equivalently as follows:

$$\int f_x(x; \theta)dx \geq 0$$  \hspace{1cm} (51)

Find

$$P\{Z' \geq 0\} \geq 0$$  \hspace{1cm} (52)

The range of the unascertained quantity $\theta, Y$ defined by equation (52) is the same as that defined by equation (49). And have relationship is given by:

$$Bel\{P_f \leq [p_f]\} = Bel\{P_f\{Z' \geq 0\} \geq 0\}$$  \hspace{1cm} (53)

The value of unascertained quantity $\theta, Y$ should be guaranteed $\{Z' \geq 0\}$ to be valid. Since $Z'$ is composed of two isomorphic functions in a linear relationship, if $\{Z' \geq 0\}$ is valid, the equation can be expressed as follows:

$$P\{Z' \geq 0\} = 1$$  \hspace{1cm} (54)

Therefore

$$Bel\{P_f \leq [p_f]\} = \int_{\infty}^0 \int_{\infty}^0 P\{Z' \geq 0\}bel_\theta(t)bel_Y(y)dtdy$$  \hspace{1cm} (55)

Equation (55) shows that if unascertained quantity $\theta, Y$ is regarded as a random variable, degree of belief $Bel\{P_f \leq [p_f]\}$ of the probability of structural failure $P_f$ is not greater than its limit $[p_f]$, which is equivalent to the reliability probability determined by the performance function $Z'$, and the specific value $t^*, y^*$ of the unascertained quantity $\theta, Y$ should satisfy the requirement of equation (46). The same conclusion can be obtained for degree of belief control of structural applicability, which can
expressed as follows:

\[
Z^* = w^* - S(X, y^*)
\]  

\[
[p_f] = P_f(w^*, t^*, y^*) = P[w^* - S(X, y^*) \leq 0] = \int f_Y(x; t^*) dx
\]

\[
Z^* = Z - Z^* = [W - S(X, Y)] - [w^* - S(X, y^*)]
\]  

According to the calculation method of reliability index at present, the reliability index \( \beta' \) corresponding to performance function is determined, which degree of belief can be given by:

\[
\text{Bel} \{ P_f \leq [p_f] \} = \Phi(\beta')
\]

and

\[
\Phi(\beta') \geq [c]
\]

If

\[
[\beta'] = \Phi^{-1}([c])
\]

The control of structural reliability should satisfy the following relationship:

\[
\beta \geq [\beta']
\]

Where \([\beta']\) corresponds to the target reliability index reflecting degree of belief requirement. This method needs to calculate the limit value of failure probability according to performance function shown in equation (46) or equation (56) to determine the specific value of the unascertained quantity. Although it can not give a simple analytical expression, it can avoid the complex integral operation in the integral domain. It is a feasible control method at present.

4.3. Actual practice

Failure probability and reliability index mean \( E_B(P_f) \) and \( E_B(\beta) \) describe main characteristics of failure probability \( P_f \) and reliability index \( \beta \). They are basic indexes reflecting effects of randomness and subjective uncertainty. Therefore, they can also be considered as control reliability of structures, which can be expressed as follows:

\[
E_B(P_f) \leq [p_f]
\]

\[
E_B(\beta) \geq [\beta]
\]

Compared with the previous control methods, the control indices \( E_B(P_f) \) and \( E_B(\beta) \) in this paper can be determined by the current method of calculating structural reliability, and the application is relatively simple. In fact, in the present structural reliability analysis, besides the fuzziness of failure criteria, other uncertainties, including subjective uncertainties, are described and analyzed indiscriminately as the randomness of objective things. The control method proposed in this paper is different from the previous method of structural reliability control. It defines the concept and theoretical basis of structural reliability analysis and can be called a practical method of structural reliability degree of belief control.

According to the method in this paper, performance function can be given by:

\[
Z = Z - Z^* = [R - S] - [r^* - S] = R - r^*
\]

Where the specific value of resistance \( R \) is \( r^* \), and failure probability can be expressed as follows:

\[
P[r^* - S \leq 0] = \Phi(\frac{r^* - \mu_S}{\sigma_S}) = [p_f]
\]

\[
r^* = \mu_S - \sigma_S \Phi^{-1}([p_f])
\]
\[ P\{Z \geq 0\} = P\{R - r^* \geq 0\} = \Phi\left(\frac{\mu_R - \mu_S + \sigma_S \Phi^{-1}(I[p_f])}{\sigma_R}\right) \]  

Degree of belief control of structural reliability should be calculated as:
\[ \Phi\left(\frac{\mu_R - \mu_S + \sigma_S \Phi^{-1}(I[p_f])}{\sigma_R}\right) \geq [c] \]  

or
\[ \beta = \frac{\mu_R - \mu_S + \sigma_S \Phi^{-1}(I[p_f])}{\sigma_R} \geq [\beta'] \]

5. Conclusions
(1) Degree of belief analysis method of structural reliability presented in this paper reveals the influence of subjective uncertainty on the results of structural reliability analysis.

(2) A detailed degree of belief calculation method is proposed to overcome the mathematical difficulties in reliability analysis of structures, and a new degree of belief optimization model is proposed in the face of complex multiple degree of belief problems. It provides a systematic calculation and analysis model for degree of belief calculation of structures.

(3) In this paper, a minimum degree of belief control method is proposed, in which the structural failure probability is not greater than the maximum failure probability. This method fully considers the influence of subjective uncertainties and makes the structural reliability analysis more scientific and accurate.

(4) The practical control method proposed in this paper clarifies the concept and theoretical basis of structural reliability analysis.

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