Chaos-based cryptograph incorporated with S-box algebraic operation

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Abstract

Error function analysis is an effective attack against chaotic cryptograph [PRE 66, 065202(R) (2002)]. The basin structure of the error function is crucial for determining the security of chaotic cryptosystems. In the present paper the basin behavior of the system used in [21] is analyzed in relation with the estimation of its practical security. A S-box algebraic operation is included in the chaotic cryptosystem, which considerably shrinks the basin of the error function and thus greatly enhances the practical security of the system with a little computational expense.

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I. INTRODUCTION

In the last decade, secure communication based on chaos synchronization has attracted continual attention [1-9]. In early stage, scientists in this field did much effort in inventing methods to hide private message in chaotic signals and in presenting a variety of designs for chaos synchronization of encryption and decryption systems. Recently, the study has gone further into practical aspects. The standard evaluations popularly used in the conventional secure communication on some essential cryptographic properties, such as security, performance and robustness against channel noise and so on, have been applied to chaotic cryptosystems. With these evaluations some advantages and disadvantages of chaos-based cryptosystems have been explored [10-18]. In this paper, we will propose a new chaotic cryptosystem and focus on evaluating its security.

Chaotic cryptograph is based on analytical floating point computation [18], and it has a property of aperiodicity, which is very much desirable for secure communication. Though any trajectory of autonomous chaos must be periodic in computer realizations with finite precision, the actual period of a system with a number of chaotic coupled oscillators (e.g., ten oscillators) may be so large that realistic communications can never reach this period, and thus aperiodicity is practically kept in most of spatiotemporally chaotic systems [19]. Moreover, chaotic systems have positive Lyapunov exponents leading to the sensitivity of trajectories to initial conditions and system parameters, and these features characterize very good properties of bit diffusion and confusion [17,20]. In particular, in spatiotemporal chaos with large numbers of positive Lyapunov exponents, these bit confusion and diffusion are conducted in multiple directions and high dimensions of variable spaces, and may thus become rather strong. This is of crucial importance for the high security of cryptosystems. However, chaotic cryptograph has some serious weakness in its security. First, since any chaotic output is based on floating-point analytical computation, the underlying dynamics, including its parameters of secret key, may be pursued (some times very easily [10-17]). Second, positive Lyapunov exponents determine certain finite bit confusion rate, this finite-
ness may not fast enough to confuse small parameter mismatches and may present certain basin structures around the secret key (see Fig.1 in [21] and Fig.2 in [22]), which can be used by intruders for effective attacks. On the other hand, conventional cryptograph uses algebraic operations on integers, some methods such as S-box transformation can greatly increase the difficulty of attacks of analytical computations, and can effectively distort and even eliminate basin around the secret key. Nevertheless, a single or few S-box operations cannot make sufficiently strong bit diffusion and confusion, and many rounds of S-box or other algebraic operations are needed for reaching high security (e.g., for Advanced Encryption Standard, AES, one needs 10, 12 and 14 rounds for 128bit, 192bits and 256bit keys, respectively). Therefore, it is interesting to combine approaches of both chaotic and conventional cryptographic methods, and to enhance the security of cryptosystems in some convenient ways. This has been done before by a number of authors[17,18,23]. In this paper, we will apply the idea of S-box in our chaotic cryptograph which mainly uses floating point analytical computation.

In [21] we suggested to apply spatiotemporal chaos produced by a one-way coupled one-dimensional map lattice for achieving high practical security. In [22] a two-dimensional map lattice is used for obtaining efficient performance, i.e., fast encryption and decryption speeds, by applying parallel encryption (decryption) operations from different chaotic sites. In this paper we will go further from [21] by incorporating the S-box algebraic operation in the chaotic cryptograph and reach high security with low computation expense. The paper will be organized as follows. In Sec.II we specify our system and present some cryptographic results without S-box, which will be evaluated by the error function. And the basin structure and its influence on the practical security are analyzed in detail. In Sec.III, we include a S-box operation, and show that with the combinatorial applications of spatiotemporal chaos and S-box the key basin of the error function is effectively shrunk to a single point well in the computation precision. This leads to great enhancement of the security of the chaotic cryptosystem. In the last section, brief discussion and conclusion of the practical security of the system against other attacks currently used in chaotic and conventional cryptanalyses.
are presented.

II. CHAOTIC CRYPTOSYSTEM AND ITS SECURITY EVALUATION

We start from the cryptosystem of one-way coupled map lattice used in [21], with a slight modification by using the nonlinear parameters rather than coupling parameters as the secret key. The encryption transformation reads

\begin{equation}
    x_{n+1}(j) = (1 - \varepsilon)f_j[x_n(j)] + \varepsilon f_j[x_n(j - 1)],
\end{equation}

\begin{equation}
    f_j(x) = (3.75 + a_j/4)x(1 - x), \quad j = 1, \ldots, m
\end{equation}

\begin{equation}
    x_{n+1}(j) = (1 - \varepsilon)f[x_n(j)] + \varepsilon f[x_n(j - 1)],
\end{equation}

\begin{equation}
    f(x) = 4x(1 - x), \quad j = m + 1, \ldots, N
\end{equation}

\begin{equation}
    S_n = (K_n + I_n) \mod 2^\nu, \quad K_n = [\text{int}(x_n(N) \times 2^\nu)] \mod 2^\nu
\end{equation}

\begin{equation}
    x_n(0) = S_n/2^\nu
\end{equation}

and the decryption transformation is

\begin{equation}
    y_{n+1}(j) = (1 - \varepsilon)f_j[y_n(j)] + \varepsilon f_j[y_n(j - 1)],
\end{equation}

\begin{equation}
    f_j(x) = (3.75 + b_j/4)x(1 - x), \quad j = 1, \ldots, m
\end{equation}

\begin{equation}
    y_{n+1}(j) = (1 - \varepsilon)f[y_n(j)] + \varepsilon f[y_n(j - 1)], \quad j = m + 1, \ldots, N
\end{equation}

\begin{equation}
    K'_n = [\text{int}(y_n(N) \times 2^\nu)] \mod 2^\nu, \quad I'_n = (S_n - K'_n) \mod 2^\nu
\end{equation}

\begin{equation}
    y_n(0) = S_n/2^\nu = x_n(0)
\end{equation}
In Eqs.(1) and (2), \( a = (a_1, a_2, \ldots, a_m) \) and \( b = (b_1, b_2, \ldots, b_m) \) serve as the secret keys of encryption and decryption, respectively, which can be freely chosen in the interval \((0, 1)\).

The transmitter sends the ciphertext \( S_n \) in the public channel, and the receiver can achieve chaos synchronization with the transmitter by applying the decryption key \( b = a \) and the ciphertext driving \( y_n(0) = x_n(0) = S_n/2^\nu \), and then correctly receive the plaintext as

\[
y_n(N) = x_n(N), \quad K'_n = K_n, \quad I'_n = I_n
\]

We assume that the intruder has the same communication machine, and thus he knows the structure of Eqs.(1) and (2), and knows also the values of all the system parameters \((N, m, \mu, \nu, \varepsilon)\) except the secret key \( a \). Moreover, the intruder has full knowledge (past and present) of ciphertexts \( S_n \) because \( S_n \) is transmitted in the open channel, and a part of past plaintexts \( I_n \) of length \( T \) (say, from \( n = 1 \) to \( n = T \)) due to some chances. With the above messages accessible to the intruder we have to consider resistance against the public-structure and plaintext-known attacks. There exist many attacks of this type, among which we will consider the error function attack (EFA) by computing the following error function

\[
e(b) = \frac{1}{T} \sum_{n=1}^{T} \left| i'_n - i_n \right|, \quad i_n = \frac{I_n}{2^\nu}, \quad i'_n = \frac{I'_n}{2^\nu}
\]

from which the intruder may try to extract the secret key \( a \) and then unmask all future plaintext. Since EFA fully uses all information available for the legal receiver except the secret key only, it can be regarded as a very effective attack.

Now we start our numerical simulation with the following parameters fixed throughout the paper

\[
m = 4, \quad \mu = 52, \quad \nu = 32, \quad \varepsilon = 0.99, \quad a_j = 0.5, \quad j = 1, 2, 3, 4
\]

In Figs.1(a)-(d) we fix \( N = 25, \quad T = 10^5 \), and \( b_2 = b_3 = b_4 = 0.5 \), and run the cryptosystem by varying a single test key parameter \( b_1 \), and plot \( e(b_1) \) vs \( b_1 \) for different test resolutions. The initial values of both transmitter and receiver are chosen arbitrarily, and they vary for
different runs. In computing (4) 100 transient iterations are discarded. In Figs.1(c) and (d) the error function \( e(b_1) \) shows a clear basin around the actual key \( b_1 = a_1 \), and in Figs.1(a) and (b) far away from the basin we have \( e(b_1) \approx \frac{1}{3} \).

Suppose two data sequences \( A_n \) and \( B_n \) are completely random, and uniformly distributed in \([0, 1]\), and they are completely uncorrelated from each other, then the average error between two data sequences is

\[
\langle e(A, B) \rangle = \lim_{T \to \infty} \frac{1}{T} \sum_{n=1}^{T} |A_n - B_n|
\]

(6)

\[
= \frac{1}{3}
\]

The square root of variance of this error for the sequences of length \( T \) reads

\[
\sigma = \left( \frac{1}{T} \sum_{n=1}^{T} (|A_n - B_n| - \langle e \rangle)^2 \right)^{\frac{1}{2}} \approx \frac{1}{3\sqrt{2T}}
\]

(7)

It is confirmed that the numerical results of \( e(b_1) \) plotted in Fig.1 satisfy both relations Eqs.(6) and (7) in very high precision when \( b_1 \) is chosen far from the key basin. This indicates that the output keystream \( K_n \) and \( K_n' \) of Eqs.(1) and (2) have rather good statistical properties of randomness and uniform distributions in \((0, 1)\), and both \( K_n \) and \( K_n' \) are practically independent from each other if \(|b_1 - a_1|\) is considerably larger than the basin width.

In [21], similar error function basin was observed (Fig.1 in [21]). Now we will go further to study the functional behavior of this basin with numerical data, which will be important for predicting the security level of the system against EFA. We find that \( e(b_1) \) can be presented by a united empirical formula

\[
e(b_1) = \frac{1}{3} \frac{1}{1 + \left( \frac{c_1}{|b_1 - a_1|} \right)^{\alpha(b_1)}} \alpha(b_1) = \frac{1 + \frac{5}{3} \frac{|b_1 - a_1|}{c_2}}{1 + \frac{|b_1 - a_1|}{c_2}},
\]

(8)

\[
c_1 = 2.8 \times 10^{-12}, \quad c_2 = 4.0 \times 10^{-9}
\]

which is plotted by the solid curves in Figs.2(a) and (b). The formula prediction of Eq.(8) perfectly coincides with the numerical results of \( e(b_1) \) [circles, \( T = 10^9 \) for (a) and \( 10^7 \) for
(b)] for all ranges from \(|b_1 - a_1| < c_2\) to \(|b_1 - a_1| > c_2\). In both limits of large and small \(|b_1 - a_1|\) the error function has interesting scalings of

\[
e(b_1) \propto |b_1 - a_1|^{\alpha_1}, \quad \alpha_1 = 1, \quad |b_1 - a_1| < c_2
\]

\[
\frac{1}{3} - e(b_1) \propto |b_1 - a_1|^{-\alpha_2}, \quad \alpha_2 = \frac{5}{3}, \quad |b_1 - a_1| > c_2
\]

With the basin structure of Figs. 1 and 2, the intruder can make EFA as follows. Suppose the error function has a key basin of width \(W\). Away from the key basin, the error function is flat \(e(b_1) \approx \frac{1}{3}\) with certain fluctuation, the intruder cannot find any tendency toward the position of the key. Therefore, he can make only brute force attack to find the key basin with average cost of \(L/W\), where \(L\) is the space length available for setting the key (in our case \(L = 1\)). If the intruder finds the key basin, he can find the tendency in the basin towards the key, and can easily expose the key position by certain optimal searching methods, such as adaptive searching. Therefore, the width of the key basin \(W\), is an extremely important quantity for the security of our system against EFA. This width can be conveniently defined from Eq.(8). According to Eq.(8) the error function \(e(b_1)\) is monotonously decreasing towards the key. Nevertheless, as \(\frac{1}{3} - e(b_1)\) \(\ll 1\) (i.e., \(|b_1 - a_1| \gg \frac{1}{3}\)) and \(T\) is finite, the inevitable fluctuation of \(e(b_1)\) characterized by Eq.(7) may be considerably larger than \(\frac{1}{3} - e(b_1)\), and thus can definitely mask the monotonous tendency of Eq.(8). According to this argument the basin width \(W\) may be defined as being proportional to the distance between the right and left boundaries \(b_r\) and \(b_l\)

\[
W = \beta(b_r - b_l)
\]

\[
\frac{1}{3} - e(b_{l,r}) \approx \frac{1}{3\sqrt{2T}}
\]

where the multiple factor \(\beta\) in Eq.(10a) can be roughly chosen such that for \(|b_1 - a_1| > W\) no any tendency towards the key location of \(e(b_1)\) can be observed. Equations (10) and (9b), together, lead to
\[ W \propto T^{\frac{1}{20.2}} = T^{0.3} \] (11)

For larger \( T \) we can certainly have larger width \( W \) and need less tests \( \left( \frac{1}{W(T)} \right) \) for finding the key basin, but meantime we need more computation time for each test \( (T \text{ iterations for each test}) \), and need more cost for collecting longer known plaintext data. Therefore, the total computation cost for finding the key basin in the space of four parameters \((b_1, b_2, b_3, b_4)\) reads

\[ Cost \approx \left( \frac{1}{W(T)} \right)^4 T \] (12)

For numerical simulations we define \( b_l \) and \( b_r \) by the first left and right crossings of \( e(b_1) \) over \( \frac{1}{3} \), respectively, when \( b_1 \) varies from \( b_1 = a_1 \) to large \( |b_1 - a_1| \), and \( W \) is defined as \( W = 10(b_r - b_l) \) according to Eq.(10a). In Figs.3(a)-(d) we plot \( e(b_1) \) for different \( T \)'s by keeping other parameters the same as Fig.1, where \( b_l, b_r \) and \( W \) are clearly indicated. The crossing points \( b_l \) and \( b_r \) may fluctuate in different runs, but this fluctuation does not affect the general tendency between \( W \) and \( T \). In Fig.3(e) we plot \( W \) against \( T \) where the solid line denotes the scaling line

\[
\log W = -10.53 + 0.30 \log T, \quad W = 2.95 \times 10^{-11} T^{0.30}
\] (13)

while dots respect numerical data. These numerical results fully confirm the prediction of Eq.(11) which is deduced from the empirical formula Eq.(8) and the scaling relation Eq.(9b). Inserting Eq.(13) into Eq.(12) we can specify the cost to break the security of the system. The minimal computation cost for breaking our system by EFA can be achieved at \( T \approx 10^{36} \) and \( W(T) \approx 1 \) that produces

\[ Cost \approx 10^{36} \text{ Iterations} \] (14)

III. CHAOTIC CRYPTOGRAPH WITH A S-BOX OPERATION

Though the security given by Eq.(14) is rather high (equivalent to the effective key of 120 bits), the basin of Eq.(8) restricts the security level. This basin structure is due
to the insensitivity of chaos synchronization to very small mismatch of the corresponding parameters in analytical computation of chaos.

With smaller system size $N$, the output keystream $K'_n$ has even lower sensitivity to the variable of $b$ (i.e., has larger basin width $W$), and accordingly the system has even lower security. In order to shrink the basin width and further increase the sensitivity of $K'_n$ to small change of the key $b$ with a given smaller system size (i.e., with less computational expenses), one has to perform some additional (nonanalytical) operation to directly shift the bits corresponding to small key variation to the position corresponding to larger key variation. Specifically, we modify Eqs.(1) and (2) as following.

First, we change (1d) and (2d) to

$$x_n(0) = \begin{cases} 
S_n/2^\nu, & \text{for } \frac{1}{8} \leq S_n/2^\nu \leq \frac{7}{8} \\
S_n/2^\nu + \frac{1}{8}, & \text{for } S_n/2^\nu < \frac{1}{8} \\
S_n/2^\nu - \frac{1}{8}, & \text{for } S_n/2^\nu > \frac{7}{8}
\end{cases} \quad (15)$$

$$y_n(0) = x_n(0)$$

respectively. These changes effectively increase the sensitivity of the keystreams $K_n$ and $K'_n$ to the secret key $a$ and $b$, respectively, for small (large) and even zero $S_n$ (or $S_n/2^\nu = 1$). In Eq.(2) such sensitivity is very low for $0 < S_n/2^\nu \ll 1$ and $0 < 1 - S_n/2^\nu \ll 1$, and this weakness can be intelligently used by the intruder with the chosen-ciphertext attack [24].

Second, we add a S-box in the $(m+1)$th map in both transmitter and receiver, namely, we modify the $(m+1)$th map of Eq.(1) as

$$x'_{n+1}(m+1) = (1 - \varepsilon)f[x_n(m+1)] + \varepsilon f[x_n(m)],$$

$$Q'_n = \left\lfloor x'_{n}(m+1) \times 2^\mu \right\rfloor \mod 2^\nu \quad (16a)$$

$$Q_n = Sbox(Q'_n)$$

$$x_{n+1}(m+1) = Q_n/2^\nu \quad (16b)$$
where the S-box is defined as follows:

\[
A_1 = [(Q_n' \gg 24) \& 255],\quad A_2 = [(Q_n' \gg 16) \& 255],
A_3 = [(Q_n' \gg 8) \& 255],\quad A_4 = Q_n' \& 255,
\]

\[
A_0 = A_1 \oplus A_2 \oplus A_3 \oplus A_4
\]

\[
Q_n = [A_0 \ll 24] + [A_4 \ll 16] + [A_3 \ll 8] + A_2
\]

In Fig.4 we show the operations of Eq.(17) schematically. The operation \(x \gg y\) denotes a right shift of \(x\) by \(y\) bits and the \& operator is bitwise AND and \(\oplus\) means bitwise XOR [25]. After the S-box transformation of (17) all the 32 bits in \(Q_n'\) take place in the highest bits in \(Q_n\) through \(A_0\). And this operation greatly enhances the sensitivity of the output \(S_n\) to the mismatches of the secret key (these mismatches are naturally reflected in the low bits of \(Q_n'\)). The decryption transformation (2) has exactly the same S-box operation as

\[
y_{n+1}(m+1) = (1 - \varepsilon)f[y_n(m+1)] + \varepsilon f[y_n(m)],
\]

\[
\hat{Q}_n = \text{int}(y_n'(m+1) \times 2\mu) \mod 2^\nu
\]

\[
\hat{Q}_n = \text{Sbox}(\hat{Q}_n')
\]

\[
y_{n+1}(m+1) = \hat{Q}_n/2^\nu
\]

The modified chaotic cryptosystem basically performs analytical floating-point computations on continuous variables. However, its performance is incorporating with a very simple conventional operation based on algebraic transformations supported by integer variables. The following operations in the whole arithmetic of the system Eqs.(1), (2) and (15)-(18) are of crucial importance for achieving optimal cryptographic properties:

(i) The transmitted signal \(S_n\) is defined as integer sequences by the int operation in Eqs.(1d), (2d), and this operation effectively enhances the robustness of the communication against noise disturbances in the transmission channel, and against the round-off errors of both transmitter and receiver computers. This improvement is extremely important when
the system has very high security (i.e., very high sensitivity to small changes of system parameters, variables and driving signal).

(ii) We apply modulo operations on some integer variables [Eqs. (1c), (2c), (16), and (18)]. These operations greatly enhance the sensitivity of chaos synchronization to the secret key parameters ($a$ and $b$), and considerably increase the difficulty of any attacks based on analytical computations.

(iii) The S-box operations of Eqs. (16)-(18) dramatically change the basin structure of Figs. 1 and 2, and effectively improve the security of the chaotic cryptograph. Particular attention will be paid on this point in the following discussion.

In Figs. 5(a)-(d) we do the same as Figs. 1(a)-(d), respectively, with modifications (15)-(18) taken into account. It is remarkable that with the S-box operation, the continuous basin in Figs. 1 and 2 shrinks now to a singular needle-like structure with zero width. In Fig. 5 various refinements of detection precision up to $2^{-45}$ precision do not help to amplify the basin width, this is in sharp contrast with Figs. 1 where finer detection precision can show larger and clearer basins [Fig. 1(d)].

Another significant point is that the singular needle-like basin structure cannot be changed by collecting and applying more known past plaintexts, i.e., by increasing $T$. In Figs. 6(a)-(d) we do the same as Figs. 3(a)-(d), respectively, with Eqs. (15)-(18) applied. It is clear that $W = 2^{-45}$ is kept for different $T$’s of which the largest one is up to $T = 10^8$ (3.2 × $10^9$ bits plaintext). For collecting so many data of plaintext one has to illegally hear a given secure talk for about five days without any break (in normal voice communication, the transmission speed is about 8k bits per second). Thus, the practical security of our modified chaotic cryptosystem against EFA is determined by the key number $2^{180}$ (four key parameters, each 45 bits) with the system size $N = 10$, which is much higher than that without the S-box operation [$2^{120}$ given by Eq. (14)] with considerably larger size $N = 25$.

It is emphasized that the S-box of Eq. (17) is very simple and needs little computation cost. This operation can thus hardly produce effective bit confusion and diffusion by itself in the conventional cryptograph. Nevertheless, incorporating with spatiotemporal chaos...
synchronization this algebraic transformation can greatly enhance the practical security of chaotic cryptograph. This is the most significant result of the present paper that a suitable combinatorial applications of both chaotic and conventional cryptographies may yield surprisingly high benefit.

IV. CONCLUSION AND DISCUSSION

In conclusion we would like to remark that the modified chaotic system [Eqs.(1) and (2) with modifications of (15)-(18)] is highly secure against EFA, which is a very strong attack because with this attack the intruder effectively uses all information accessible to the legal receiver, except the secret key only. We assume that the security against EFA may be probably the actual practical security of the system. Nevertheless, in this conclusion part we would like to briefly mention the difficulties and the results (without giving much reasonings) of some other possible attacks in evaluating our system. The detailed analysis on all these attacks is in preparation and will be presented elsewhere [27].

First, the keystream $K_n$ and, accordingly, the transmitted signal $S_n$ have very good statistical properties. They satisfactorily pass the randomness checking of all the 16 types of NIST Standard evaluations of [26] for arbitrary plaintexts. The keystream ($K_n/2^n$) has uniform distributions in the interval (0,1) up to $q$-order ($q$’s from 1 to 4 have been carefully checked, and even higher orders up to $q = 10$ have been roughly checked). These uniform distributions remain unchanged by widely changing the secret key and the plaintexts. The probability distribution of differences between any ciphertexts is practically independent of the difference of the corresponding plaintexts. Thus, many attacks in conventional cryptanalysis such as linear attack and differential attack can hardly work for our system. And all attacks based on statistics analysis may not work effectively.

Second, most of attacks used in chaotic cryptography, such as nonlinear dynamics forecasting and dynamics reconstructs [10-16] are definitely not efficient for evaluating our system, because these approaches apply ciphertext–only and private-structure attacks. With-
out enjoying the information of system dynamics and known plaintext these attacks are incomparably weaker. These attacks often reconstruct the system dynamics by constructing time-delay return maps. Due to the good statistical properties mentioned above these return maps all have uniform distributions from which one can hardly find any trace of the system dynamics as well as the secret key.

Usually, chaotic cryptograph may be easily attacked by analytical computations in the case of one-round encryption. However, in our case nonlinearity together with multiplicity of coupled maps makes analytical attacks difficult. The truncations and S-box operations break simple analytical relations between the key and the output signals. A simple analysis shows that attacks based on analytical solution need much more computational cost than EFA for breaking the security of the system.

Recently, Hu et al proposed a chosen-ciphertext attack to evaluate chaotic cryptosystems[24], which is very strong. It can be shown that this chosen-ciphertext attack can break the security of Eqs.(1) and (2) without S-box operation with much less cost than Eq.(14), i.e., this attack is much stronger than EFA. However, with the S-box operation and the associated modifications((15)-(18)), the chosen-ciphertext attack can be made completely ineffective by including the bits of the secret key (a for encryption and b for decryption). An analysis similar to [24] shows that the computation cost of the chosen-ciphertext attack of [24] is again much more expensive than EFA. This matter will be discussed in detail in our future work. It is well known that chosen-ciphertext attacks are among the strongest attacks if we classify the intensities of various evaluations[18,24]. Thus, it is reasonable to estimate the practical security of our system by Eq.(4) and Fig.5.

With respect to the problem of security, our system has a good advantage over AES. Our system is practically one-time pad cryptograph if its secret key can be well hidden and the encryption time is much shorter than the period of computer realization of chaos. On the contrary, AES is definitely not one-time pad even if its key is kept secret because for AES same inputs always produce same outputs. Our system has fairly fast encryption speed. With the C code running a Pentium III (700MHz) computer it can encrypt 50Mbit
ciphers (with the key length of 180 bits) which is comparable to that of AES having a speed of 83Mbit per second with a key of 192 bits by using the same computer. The encryption speed of our system can be further greatly enhanced by parallel encryption operations of multiple space units [22].

Finally, we would like to emphasize that our system will not suffer from the periodicity of finite precision computer realization of chaos. By increasing the number of coupled maps those period increases rapidly. We certainly cannot directly detect the period of our system because it is too long. An estimation, based on small number of coupled maps shows that the period of our system with $N = 10$ and double precision computation is about $10^{70}$ [19]. The currently best computer in the world needs to work more than $10^{45}$ years to reach this period, and thus spatiotemporal chaos can be surely observed in our system for practically unlimited uses.
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Captions of Figures

Fig.1. Error function $e(b_1)$ defined in Eq.(4) vs the decryption key parameter $b_1$ with different detection resolutions. $T = 10^5$. The basins of the error function around the key position $b_1 = a_1$ can be clearly observed in (c) and (d).

Fig.2 (a) and (b) Comparison of the prediction of Eq.(8) (solid lines) with numerical measurements (circles) of the error function Eq.(4) for $T = 10^3$ [(a)] and $T = 10^7$ [(b)], respectively. Agreements of the analytical predictions with numerical results are satisfactory. The agreement shown in (b) is particularly desirable since there fluctuation of $e(b_1)$ is considerably reduced by taking rather long $T = 10^7$ (3.2 × $10^8$ bits plaintext).

Fig.3 Widths of the basins of error function $e(b_1)$ for different available lengths of plaintexts (a) $T = 10^3$, (b) $T = 10^4$, (c) $T = 10^6$, (d) $T = 10^7$, respectively. The boundaries $b_l$ and $b_r$ are defined by the first crossings of $e(b_1)$ over $e(b_1) = \frac{1}{3}$ when $b_1$ varies from $b_1 = a_1 = 0.5$ to left and right, respectively. And the basin width $W$ is taken as $W = 10(b_r - b_l)$. (e) $W = 10(b_r - b_l)$ plotted vs $T$. The solid line is the prediction of Eq.(11) while the circles represent numerical data. Both coincide with each other satisfactorily.

Fig.4 The schematic figure of the S-box operation of Eq.(17). (a) Four bytes $A_1$, $A_2$, $A_3$ and $A_4$ divided from $Q_n'$ of 32 bits, and $A_0$ produced from $A_i$ via XOR operations. (b) Produce $Q_n$ of 32 bits from the four bytes $A_0$, $A_4$, $A_3$, and $A_2$.

Fig.5 The same as Fig.1 with the modifications (15)-(18) taken into account. $T = 10^5$. The error function basin shrinks now to a needle-like single point, irregarding the detection resolution, this is in sharp contrast with the behavior of Fig.1. In (d) the resolution is up to $2^{-45}$, indicating that the key number of the four parameters $b_1$ to $b_4$ can be up to $2^{180}$.

Fig.6 The same as Figs.3(a)-(d), respectively, with Eqs.(15)-(18) applied. Increasing $T$ can effectively reduce the fluctuation of $e(b_1)$, but cannot change the needle-like basin structure.
Fig. 1

(a) $e(b_1)$

(b) $e(b_1)$

(c) $e(b_1)$

(d) $e(b_1)$
Fig. 2

(a) $T = 10^3$

(b) $T = 10^7$
Fig. 3
Fig. 4

(a) $Q_r(32 \text{ bits})$

\[
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
\downarrow 8 \text{ bits} & \downarrow 8 \text{ bits} & \downarrow 8 \text{ bits} & \downarrow 8 \text{ bits} \\
\end{array}
\]

\[
\begin{array}{cccc}
A_1 \oplus A_2 \oplus A_3 \oplus A_4 & \rightarrow A_0 \\
\end{array}
\]

(b) $Q_r(32 \text{ bits})$

\[
\begin{array}{cccc}
A_0 & A_4 & A_3 & A_2 \\
\downarrow & \downarrow & \downarrow & \\
A_0 & A_4 & A_3 & A_2 \\
\end{array}
\]
Fig. 5

(a) (b)

(c) (d)

\[ e(b_1) \]

\[ b_1 - a_1 \]

\[ \times 2^{-36} \]

\[ \times 2^{-45} \]
Fig. 6

(a) $T=10^5$

(b) $T=10^6$

(c) $T=10^7$

(d) $T=10^8$