D-flation

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Abstract. In a recent paper we showed how Starobinsky-like inflation could emerge from dilaton dynamics in brane cosmology scenarios based on string theory, in which our universe is represented as a three-brane and the effective potential acquires a constant term from a density of effectively point-like non-perturbative defects on the brane: “D-particles”. Here we explore how quantum fluctuations of the ensemble of D-particles during the inflationary period may modify the effective inflationary potential due to the dilaton. We then discuss two specific ways in which an enhanced ratio of tensor to scalar perturbations may arise: via either a condensate of vector fields with a Born-Infeld action that may be due to such recoil fluctuations, or graviton production in the D-particle vacuum.

Keywords: cosmology of theories beyond the SM, modified gravity, alternatives to inflation

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1 Introduction and motivation

In a recent article [1] we showed how Starobinsky-like [2] inflation could emerge from dilaton dynamics in brane cosmology scenarios based on string theory. In such models, our universe is represented as a three-brane, and the effective potential for the dilaton/inflaton acquires an asymptotically-constant term from a density of effectively point-like non-perturbative defects on the brane, termed “D-particles”. This could arise from a flux of such defects from the bulk to the brane, in such a way that the number density of the D-particles on the brane remains approximately constant during the inflationary period. Higher-genus corrections generate corrections to the effective potential that are exponentially damped at large field values, as in the Starobinsky model, but at a faster rate.

Such a scenario leads to a prediction for the tensor-to scalar perturbation ratio $r$ that is smaller than the original Starobinsky model [2]. However, as we also commented, this reduction may be compensated partially by logarithmic deformations on the world-sheet generated by recoil of the defects due to scattering by string matter on the brane, which tend to enhance the tensor-to-scalar ratio [1]. However, the basic features of the Starobinsky model, namely the flatness of the effective potential for large dilaton/inflaton field values, relative to the reduced Planck scale are retained in the scenario of [1], making such models highly consistent with the first installment of data from the Planck satellite [7, 8].

However, the BICEP2 Collaboration [9] has announced strong evidence for B-mode polarization in the cosmic microwave background radiation, which it has interpreted as evidence for gravitational waves at the time of the last scattering, with a tensor-to-scalar ratio $r = 0.16^{+0.06}_{-0.05}$ after dust subtraction [9]. The BICEP2 data are consistent with a scalar spectral index $n_s \simeq 0.96$ and no appreciable running, in agreement with the Planck data [7, 8]. However, the value of the Hubble parameter during slow-roll inflation $H_*$ indicated by the BICEP2 data is much larger than that suggested by Planck. The estimated inflationary energy scale $E_* = V^{1/4}$, where $V_*$ is the inflation potential during inflation (which is assumed to be approximately constant) [10]:

$$E_* = \left(3 H_*^2 M_{Pl}^2 \right)^{1/4} \simeq 2.1 \times 10^{16} \times \left(\frac{r}{0.20}\right)^{1/4}\text{GeV},$$

yielding, for $r = 0.16$, $H_* \sim 1.0 \times 10^{14}\text{GeV}$.

\footnote{For some other recent Starobinsky-like models, see [3–6].}
If confirmed by subsequent experiments, such a large value of \( r \) would exclude Starobinsky-type inflationary potentials. However, it is premature to abandon Starobinsky-like models, as there is currently an active debate whether the BICEP2 signal truly represents primordial gravitational waves, or is polluted by Galactic foregrounds and gravitationally-lensed E-modes. For instance, in [11] it was argued that magnetized dust associated with radio loops due to supernova remnants might contribute to the BICEP2 signal, while [12] and [13] have demonstrated that the BICEP2 signal could be compatible with a cosmology with \( r \ll 0.1 \) if there is a dust polarization effect as large as presently allowed by Planck [7, 8] and other data. These remarks have recently been reinforced by data on the foreground dust in the BICEP2 region released by the Planck collaboration [14], which point to significant foreground pollution that would affect the interpretation of the BICEP2 B-mode polarization data. However, one needs to wait for the results of the planned joint analysis by the Planck and BICEP2 teams, before any definite conclusions are drawn on this. Until this issue is resolved, it is interesting to explore the possible range of \( r \) values that models of inflation could yield.

In [1], we presented D-motivated models with Starobinsky-like potentials for the inflaton \( \varphi \) of the form

\[
\tilde{V} \equiv \frac{1}{2\kappa^2} V = \frac{A}{2\kappa^2} \left( 1 - \delta e^{-B\varphi} + \ldots \right),
\]

where \( \kappa^2 \equiv M_{Pl}^{-2} \) where \( M_{Pl} = M_P/\sqrt{8\pi} \) is the reduced Planck scale, \( A \ll 1 \) to obtain the correct magnitude of scalar perturbations, and \( \delta \) and \( B \) are treated as free parameters that are allowed to vary from the original values \( \delta = 2, B = \sqrt{2}/3 \) in the Einstein-frame Starobinsky model, and the dots represent possible higher-order terms that are \( \mathcal{O}(e^{-2B\varphi}) \). Using the following standard expressions for inflationary observables in the slow-roll approximation [10]

\[
\epsilon = \frac{1}{2} M_{Pl}^2 \left( \frac{V'}{V} \right)^2, \quad \eta = M_{Pl}^2 \left( \frac{V''}{V} \right), \quad n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon,
\]

\[
N_* = -M_{Pl}^{-2} \int_{\varphi_i}^{\varphi_e} \frac{V}{V'} d\varphi,
\]

where \( \varphi_i(e) \) denotes the value of the inflaton at the beginning (end) of the inflationary era, we obtain to leading order in the small quantity \( e^{-B\varphi} \) [6]:

\[
n_s = 1 - 2B^2 \delta e^{-B\varphi}, \quad r = 8B^2 \delta^2 e^{-2B\varphi}, \quad N_* = \frac{1}{B^2 \delta} e^{B\varphi}, \quad n_s = 1 - \frac{2}{N_*}, \quad r = \frac{8}{B^2 N_*^2},
\]

where \( N_* \) is the number of e-foldings during the inflationary phase. Requiring \( N_* = 54 \pm 6 \) yields the characteristic prediction \( n_s = 0.964 \pm 0.004 \), in agreement with both Planck [7, 8] and BICEP2 [9]. However, the Starobinsky-model choice \( B = \sqrt{2}/3 \) yields \( r = 0.0041^{+0.0014}_{-0.0008} \) that, whilst highly consistent with the Planck data [7, 8], is inconsistent with the BICEP2 data [9]. We note that larger values of \( r \) compatible with the BICEP2 data can in principle be obtained if the exponent \( B \) is much smaller than the conventional Starobinsky value \( B_{Star} = \sqrt{2}/3 \). For instance, the BICEP2 central value \( r \sim 0.16 \) is obtained for \( N_* \sim 50 \) and \( B \sim 0.14 \) in (1.2).
However, the string theory considerations we presented in [1], where the dilaton was identified as the inflaton field, pointed towards values of $B$ that are larger than in the conventional Starobinsky model, specifically $B = \sqrt{4/(D-2)}$ for $D$ large uncompactified dimensions, giving $B = \sqrt{2}$ for the physical case of $D = 4$, leading to a value of $r$ that is three times smaller than the Starobinsky value. Indeed, even if the dilaton lives in the maximum number $D = 26(10)$ of uncompactified space-time dimensions in the bosonic (super)string, the resulting value of $B = 1/\sqrt{6} (1/\sqrt{2})$ is much larger than is required to yield $r = 0.16$.

It is the purpose of this note to explore the possible effects of quantum fluctuations of the D-particle defects during dilaton-driven inflation. We assumed in [1] that the dominant role of such fluctuations was simply to provide a cosmological constant term in the potential (1.2). However, such D-particle fluctuations yield, in general, effective vector field degrees of freedom associated with stochastic fluctuations in the corresponding recoil velocities of the D-particles as they interact with the (closed) string degrees of freedom representing bulk space-time gravitons. The coupled dynamics of such fluctuating defects with the space-time metric is described in [15], where it was argued there that there are growing modes in such systems that may result in the formation of large-scale structures on the D3 brane universe at late epochs. Here we discuss the possibility that these stochastic recoil fluctuations (or other vector fields with a Born-Infeld action) may condense, and consider the form of effective potential that they would then generate. Under suitable conditions on the potential parameters, the condensate may suppress the coefficient $B$ of the effective exponential in the potential (1.2), resulting in an enhancement of the tensor-to-scalar ratio $r$ (1.4), possibly into the range suggested by BICEP2 [9]. Another avenue for enhancing $r$ is via graviton production in the ‘out’ state originating from the D-particle vacuum.

The structure of this article is the following. In section 2 we discuss the effective Lagrangian that describes the coupling of the quantum fluctuations of D-particles to graviton degrees of freedom. In section 3 we discuss the formation of condensates of the recoil-velocity field strength and show that they are compatible with the inflationary phase being driven by the dilaton. If the condensate and the dilaton are treated as independent scalar degrees of freedom, then Starobinsky-like inflation with a small value of the tensor-to-scalar ratio $r$ is obtained, incompatible with the BICEP2 result. On the other hand, if the recoil condensate is assumed proportional to the dilaton, then the possibility arises of an enhanced $r$ ratio. In section 4 we discuss briefly the alternative way in which D-particles may enhance the tensor-to-scalar-perturbation ratio, via the mixing of thermal modes of D-particles with de Sitter modes in the ‘out’ state originating from the D-particle vacuum. Finally, our conclusions are presented in section 5.

2 D-particle quantum fluctuations

As suggested in [1], populations of D-particles may induce, together with the dilaton field and/or other moduli fields in string theory, an inflationary phase of Starobinsky type. The D-particles may be truly point-like defects, in which case the string theory is of type I, and the supergravity action in the bulk is of type IIA as in the model of [16–18], involving 8-branes that can eventually be compactified to give 3-brane worlds. Alternatively, they may be “effectively” point-like (i.e., as perceived by a low-energy brane observer), as in the models of [19], which utilize type IIB string theory branes, in which case the role of D-particles is played by 3-branes wrapped around three-cycles, which are embedded in 7-branes that are in turn wrapped around four-cycles. The D-foam is then realized by constructing appropriate
lattices of such defects on the brane Universe. The details and stability of the various compactifications are beyond the scope of the effective four-dimensional field theory approach used in this paper. We assume that stable compactifications (e.g., through fluxes penetrating topologically non-trivial compact manifolds such as tori, etc., free from dilaton tadpoles and other instabilities do exist, leading to a well-defined four-dimensional theory.

We are aware that important issues in the compactification of string theory in the presence of branes are raised by the divergences arising from tadpoles in one-loop string amplitudes, due to the presence of flux forms of Ramond-Ramond (RR) fields, for which D-branes and orientifold planes play the roles of electric and magnetic sources. In the presence of such non-trivial charges, corresponding to current forms $J_m$ of appropriate rank $m$, the associated flux forms satisfy the generic equations \[ [20, 21] \]

\[
dH_n = \star J_{9-n} + \ldots , \quad d \star H_n = \star J_{n-1} + \ldots ,
\]

where the $\ldots$ on the right-hand-side denote corrections due to the Chern-Simons couplings of the RR fields to gauge field strengths and space-time curvature. If one wishes to maintain supersymmetry, consistent low-energy four-dimensional theories exist provided certain conditions for tadpole cancellations exist, which can be represented generically by

\[
\int_{C_k} \star J_{10-k} = 0 ,
\]

for all closed curves $C_k$. Such conditions translate to appropriate constraints on one-loop string amplitudes that restrict the appropriate types of allowed brane backgrounds and/or target-space gauge groups for which a consistent four-dimensional supersymmetric theory can be defined. The dangerous (from the point of view of divergences) tadpoles in the one-loop amplitudes of such theories come from the propagation of massless RR states that are $n$-forms in the $n$ non-compact dimensions of the theory.

Clearly such tadpoles exist generically in the type of theories in which we wish to embed our D-particle foam. In supersymmetric contexts, such tadpoles can be cancelled by the inclusion of orientifolds, which constitute the appropriate “sinks” for the corresponding brane charges that give rise to divergent tadpoles. A lot of work on this issue has been devoted to models associated with type IIB string theories, which contain D3/D7 brane configurations, as in the models of [20–22], in which one may embed our D-foam configurations [19]. The strategy is that one first considers appropriate compactifications of the D3/D7-IIB system, adding appropriate orientifolds so that the tadpole cancellation conditions are obeyed and the four-dimensional theory makes sense. One may then add to the system the D-particle foam lattice as discussed in [19]. Similar considerations apply for the eight-branes in the models of [16–18]. The important point is that the recoil fluctuations of the D-foam break target-space supersymmetry, as a result of brane defects that move with a non-trivial velocity relative to the branes. Thus extra conditions for tadpole cancellation are not expected to arise from these defects. The formal study of a system of many D-particles in type-IIB strings with D3-branes wrapped around appropriate 3-cycles lies beyond the scope of this paper, where we are concerned with the phenomenology of inflationary scenarios that may characterize the low-energy limit of such systems. We therefore leave a detailed mathematical analysis of tadpole cancellations in such a system to future work dedicated to this issue.

We assume that the dominant sources for vacuum energy contributions on the brane worlds are the gravitational masses of the bound-state D-particles on the brane world during

\[ \text{In the context of F-theory and the associated low-energy type IIB string, see also [22].} \]
the inflationary phase and their recoil fluctuations. In general, there exist other interactions among D-particles and the branes, as discussed in detail in [16–18]. In particular, D-particles may have extra $U_D(1)$ vector interactions with a given D-charge, described by stretched open strings between the defects or between the bulk defects and the brane worlds. For the latter, the analysis of [16–18] shows that only the relative motions of the bulk D-particles relative to the brane world in directions perpendicular to the brane contributes to the vacuum energy, via terms of mixed sign depending on the distance of the bulk defect from the brane. In our approach here, we assume that such contributions average out or at most are subdominant compared to the gravitational contribution of the D-particles bound to the brane worlds, which constitute the main source of brane dark energy together with the condensates of the vector fields representing the recoil fluctuations of these brane-bound defects.

We assume here that the D-particles are either neutral under these extra $U_D(1)$ interactions, or that there is (almost) a ‘no force’ condition among the charged D-particle defects. This occurs for certain non-supersymmetric non-BPS D-particles, e.g., those formed by appropriate superpositions of BPS states as discussed in [23–28], where extra vector interactions stabilize the configurations. The existence of charged D-particles would raise the issue of possible “charge polarized” regions among the populations of brane D-particles and their anti-particles. We assume here that during the inflationary period, if such charged branes are present, the brane Universe has an overall D-particle neutrality [29].

Finally, we assume without proof the stabilization of other string moduli, which is an open problem in string theory and model-building that lies beyond the scope of this paper. As in the brane cosmologies of [30, 31], we may envisage that the moduli configurations relax asymptotically in time to constant values. In general, there are very few concrete frameworks currently available, for instance in M-theory [32], in which moduli stabilization has been considered and where our D-foam can be embedded without altering much the associated cosmology of these models. Detailed consideration of such constructions merit a dedicated future publication.

Under the above assumptions and considerations, the effective four-dimensional action that describes the coupled dynamics of gravitons with the vector degrees of freedom $A^\mu$ (with field strength $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$) that describe the recoil of ensembles of D-particles on the brane Universe reads [15].

$$S_{\text{eff 4dim}} = \int d^4x \left[ -\frac{T_3}{g_\text{d}} e^{-\phi} \sqrt{-\det (g + 2\pi \alpha' F)} \left( 1 - \alpha R(g) \right) \right. $$
$$- \sqrt{-g} e^{-2\phi} \frac{1}{\kappa_0} \hat{\Lambda} + \frac{1}{\kappa_0} \sqrt{-g} e^{-2\phi} R(g) + O ((\partial\phi)^2) \left] \right. $$
$$\approx \int d^4x \sqrt{-g} \left[ -\frac{T_3}{g_\text{d}} e^{-\phi} - e^{-2\phi} \frac{1}{\kappa_0} \hat{\Lambda} 
- \frac{T_3}{4g_\text{d}} 2\pi \alpha' e^{-\phi} F_{\mu\nu} F_{\mu\nu} + \alpha \frac{T_3}{4g_\text{d}} 2\pi \alpha' e^{-\phi} F_{\mu\nu} F_{\mu\nu} R(g) 
+ \left( \frac{\alpha}{4g_\text{d}} \frac{T_3}{g_\text{d}} e^{-\phi} + \frac{1}{\kappa_0} e^{-2\phi} \right) R(g) + O ((\partial\phi)^2) \right] + \ldots , \quad (2.1)$$

where $g \equiv \det(g)$ is the determinant of the four-dimensional gravitational field $g_{\mu\nu}$, $T_3 > 0$ is the D3-brane tension, $\phi$ is the dilaton field and the string coupling is given by $g_s = g_\text{d} e^{\phi}$.

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3We use the following conventions: for the metric we use the signature $(-,+,+,+)$ and the Riemann curvature tensor is defined as $R^\gamma_{\delta\gamma\delta} = \partial_\delta \Gamma^\gamma_{\delta\gamma} + \Gamma^\lambda_{\delta\gamma} \Gamma^\gamma_{\lambda\delta} - \gamma \leftrightarrow \delta$. 

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with $g_{s0} < 1$ a (perturbatively) weak string coupling constant. In the second part of the approximate equality we have expanded the action up to second order in the vector recoil fields, which are assumed to be weak.

The reader should note that the Born-Infeld determinant structure for the vector field in (2.1) arises because of the open strings attached on the brane world, representing matter/radiation excitations, while the curvature term inside the parenthesis (with coefficient $\alpha$) represents the gravity induced in the open sector. The quantity $\alpha = \alpha' \zeta(2) = \alpha' \pi^2 / 6$ in the example of [33]. However, we wish to keep the discussion general, so from now on we treat the positive constant $\alpha > 0$ as a parameter of our model; it has dimensions of length squared.

The $O((\partial \phi)^2)$ terms are the kinetic terms of the dilaton, which shall not play an important role in our analysis in this work, for reasons that will be clarified below. The gravitational constant $\kappa_0$ is defined as

$$\frac{1}{\kappa_0} = \frac{V^{(6)}}{g_{s0}^2} M_s^2,$$  \hspace{1cm} (2.2)

where $M_s = 1/\sqrt{\alpha'}$ is the string scale, $\alpha'$ the string tension, and $V^{(6)}$ is the six-dimensional volume factor.

The quantity $\tilde{\Lambda}$ represents a generic vacuum energy term that arises in non-critical string theories. For strings of subcritical dimension, $D < D_{\text{critical}}$, where $D_{\text{critical}} = 10$ for superstrings, such as the case $D = 4$ considered in [1] and here,

$$\tilde{\Lambda} \propto \frac{1}{\kappa_0^2 \alpha'} (D - D_{\text{critical}}) < 0.$$  \hspace{1cm} (2.3)

The vector field $A_\mu$ represents the recoil-velocity degrees of freedom of ensembles of D-particles on the D3-brane [34–36]. Here the term “recoil” velocity is not associated with the interactions of D-particles with individual particles of ordinary (string) matter, but expresses the quantum fluctuations of the D-particle that are represented by open strings stretched between the D-particles and the D3-brane world. One has the freedom [15, 34–36] to covariantize the field $A_\mu$ and express it in terms of the covariant component of the recoil velocity $u_\mu$

$$A_\mu \propto -a^2(t) u_\mu = -a(t) u^{\text{phys}}_\mu,$$  \hspace{1cm} (2.4)

where the proportionality constants are such to give $A_\mu$ dimension of [mass]. They arise from the fact that, when we expand the Born-Infeld determinant in (2.1) in powers of derivatives, the canonical normalization of the kinetic terms of the vector fields, $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ in (2.1), is obtained by absorbing the factor $(2\pi \alpha' e^{-\phi_0} T_3 / g_{s0})^{1/2}$ into a redefinition of the gauge potential $A_\mu$, in the case of (almost) constant dilaton as discussed in [15] and here. In a Robertson-Walker background, the physical (“local”) four velocity $u^{\text{phys}}_\mu \equiv a(t) u_\mu$ obeys the Minkowski-flat constraint: $u^{\text{(phys)}}_\mu u^{\text{(phys)}}_\nu \eta^{\mu\nu} = -1 < 0$, from which it follows that the canonically-normalized vector field has the following time-like constraint in our Robertson-Walker background:

$$A_\mu A_\nu g^{\mu\nu} = -\frac{|T_3|}{g_{s0}} 2\pi \alpha' e^{-\phi_0} < 0.$$  \hspace{1cm} (2.5)
where $\ll \cdots \gg$ indicate appropriate averages over the D-particle population in the foam. The parameter $\sigma_0^2$ is free in our model, though small, and is to be constrained by the data. In general, $\sigma_0^2$ depends on the cosmic time, since it is a function of the (bulk space) density of the foam, which may vary between different cosmological eras \cite{15–18, 37}.

A quantum treatment of the constraint (2.5) has been made in \cite{15} by implementing it in a path integral by means of a Lagrange multiplier field $\rho(x)$. We consider here the phase in which the constraint is irrelevant, i.e., $\langle \rho(x) \rangle = 0$, in which case the vector Dirac-Born-Infeld field is massless. This is the case for an inflationary era \cite{1} in which a sufficiently dense population of D-particles crosses our brane universe and they condense due to quantum effects.\footnote{The phase in which $\langle \lambda(x) \rangle \equiv M^2/2 > 0$, in which case the vector field becomes massive was considered in \cite{15}, where it was assumed to characterize the era of structure formation.}

The condensates may be formed by the higher-order (higher than two derivatives) interaction terms among the vector field strengths in the Dirac-Born-Infeld Lagrangian in (2.1), by analogy with the gluon condensates of Quantum Chromodynamics. Such an assumption was made in \cite{38} for generic Abelian flux fields with Dirac-Born-Infeld world volume actions that characterize D-brane excitations. That analysis indicated that, under certain plausible assumptions on the dominance of the quantum effects on the formation of condensates over classical, statistical ones due to ensemble properties (2.6), it is possible to obtain an equation of state for the vector Dirac-Born-Infeld system that resembles that of a de Sitter phase, with an equation of state $w \simeq -1$. A detailed analysis has been considered in \cite{15}, and will not be repeated here. Instead, we review briefly relevant aspects of those results.

We consider a generic Dirac-Born-Infeld field with Lagrangian (2.1) on a D3-brane world volume. In general, there are two contributions to the vector field strength $F_{\mu\nu}$ condensates that may come from quantum vacuum effects:

\begin{align}
\langle F_{\mu\nu} F^{\mu\nu} \rangle_{\text{vac}} &= \hat{\alpha}(t), \\
\langle F_{\mu\nu} F^{\star\mu\nu} \rangle_{\text{vac}} &= \tilde{\beta}(t),
\end{align}

where $F^\star$ indicates the dual tensor, and the condensates may in general depend on time, but are constant on spatial hyper-surfaces. The isometry structure of the spatial hyper-surfaces led the authors of \cite{38} further to assume the following, which we also adopt here:

\begin{align}
\langle F_{0\nu} F_{0\nu} \rangle_{\text{vac}} &= \frac{\alpha_s(t)}{4} g_{00}, \\
\langle F_{i0} F_{0j} \rangle_{\text{vac}} &= \frac{\alpha_s(t)}{4} g_{ij},
\end{align}

where $i,j$ are spatial indices on the three-dimensional volume of the D3-brane, and we have the relations $\alpha_t + 3\alpha_s = 4\hat{\alpha}$. In addition to the quantum effects, one also has classical thermodynamical effects on the energy density and pressure of the Dirac-Born-Infeld fluid, which are obtained by averaging over the spatial volume. If one decomposes the field strength into “electric”, $F_{0i} \equiv E_i$, and “magnetic” $B_i = \frac{1}{2} \epsilon_{ijk} F^{jk}$ components, one may specify the contributions of these classical effects as follows [38]:

\begin{align}
\langle F_{0\nu} F_{0\nu} \rangle_{\text{class}} &= \left( \sum_{i=1}^{3} E_i^2 \right)_{\text{class}}, \\
\langle F_{i0} F_{0j} \rangle_{\text{class}} &= -\langle E_i E_j \rangle_{\text{class}} + 2\langle B_i B_j \rangle_{\text{class}}.
\end{align}
On making the further (natural) assumption \[38\]

\[
\langle E_i E_j \rangle_{\text{class}} = \langle B_i B_j \rangle_{\text{class}} = C g_{ij}/3,
\]  

(2.10)

we observe that the total contribution of both classical and quantum effects to the vacuum condensates can then be expressed by the relations:

\[
\alpha_t = \tilde{\alpha} - 4C, \quad \alpha_s = \tilde{\alpha} + \frac{4}{3}C.
\]  

(2.11)

Computing the total stress tensor of the Dirac-Born-Infeld fluid in the presence of condensates one arrives at the following expressions for energy density \(\rho^\text{DBI}\) and pressure \(p^\text{DBI}\) \[38\]:

\[
\rho^\text{DBI} = \frac{\lambda}{2} \left( \frac{1 + \tilde{\alpha} - \frac{\alpha_t}{4}}{\sqrt{1 + \tilde{\alpha} - \frac{\alpha_s}{4}}} \right),
\]

\[
p^\text{DBI} = -\frac{\lambda}{2} \left( \frac{1 + \tilde{\alpha} - \frac{3\alpha_s}{4}}{\sqrt{1 + \tilde{\alpha} - \frac{\beta^2}{16}}} \right); \quad \lambda \equiv \frac{T_3}{g_s}.
\]  

(2.12)

The quantum condensates \(\tilde{\alpha}\) and \(\tilde{\beta}\) have not been specified. The only restrictions come from the positivity of the corresponding quantities inside the square root in the Dirac-Born-Infeld action, e.g. in the denominators of (2.12), which imply the following relation between the various quantum condensates:

\[
\frac{\tilde{\alpha}}{2} > -1 + \frac{\tilde{\beta}^2}{16}.
\]  

(2.13)

It is immediately apparent from (2.11) that, on the one hand, if quantum effects are the dominant ones, with \(\tilde{\alpha} \gg C\), then \(\alpha_t \simeq \alpha_s \simeq \tilde{\alpha}\), and hence from (2.12) we obtain an equation of state of cosmological constant (de Sitter vacuum) type, namely \(w^\text{DBI} \simeq -1\) independently of the exact form of the quantum condensate (assuming of course it exists, which is a question that probably cannot be addressed in a generic manner, as it requires specific properties of the brane action). On the other hand, as demonstrated in \[38\], when classical effects dominate, with \(C \gg \tilde{\alpha}\), then eq. (2.11) implies \(\alpha_t = -3\alpha_s \simeq -4C\), so that (2.12) leads to an ordinary relativistic fluid with positive energy density and pressure, \(\rho^\text{DBI} \simeq (\lambda/2)C > 0\) and \(p^\text{DBI} = (1/3)\rho^\text{DBI} \simeq (\lambda/2)C > 0\).

In the context of our study here, the Dirac-Born-Infeld vector field has a microscopic origin, as it describes the dynamics of the recoil degrees of freedom of the D-particles in interaction with electrically-neutral string matter and gravitational fields. In contrast to the case of \[38\], here we have the additional couplings of the space-time curvature to the Dirac-Born-Infeld action in (2.1), whose effects have not been considered in \[38\]. Nevertheless, as we shall argue, for sufficiently small values of the condensate, one can still argue for a late de Sitter era, due to the dominance of quantum effects of the D-particle vacuum.

To this end, we concentrate on the case where the condensate \(\tilde{\alpha}\) is constant in time and small in magnitude, while the condensate \(\tilde{\beta} = 0\), so that an expansion up to order \(\tilde{\alpha}\) in the effective action is sufficient. Since the dilaton drives inflation in our scenario \[1\], we assume that the condensate forms in a de Sitter phase of the Universe, during which the dilaton field is rolling slowly. We denote the nominal value of the dilaton in this slow-roll phase by \(\phi_0\)
which is large and negative. We shall not write explicitly kinetic terms of the dilaton, which are suppressed. As shown in [15], the scalar curvature of the space-time reads

$$R = \frac{1}{\alpha \left( \frac{2 T_3}{g s_0} e^{3\phi_0} - \tilde{\alpha} \right)} \left[ \frac{6}{g s_0} T_3 e^{5\phi_0} + \frac{4\tilde{\Lambda}}{\kappa_0} e^{4\phi_0} \right].$$

(2.14)

This can be a positive constant, as appropriate for a de Sitter space-time, provided

$$\frac{2 T_3}{g s_0} e^{3\phi_0} - \tilde{\alpha} > 0. \quad (2.15)$$

If we assume a maximally symmetric de Sitter form for the Riemann curvature tensor, corresponding to a Hubble constant, \(H_I = \text{const} > 0\), namely,

$$R_{\mu\nu\rho\sigma} = H_I^2 (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}),$$

we obtain from (2.14) the following relation:

$$H_I^2 = \frac{R}{12} = \frac{1}{\alpha \left( \frac{2 T_3}{g s_0} e^{3\phi_0} - \tilde{\alpha} \right)} \left[ \frac{T_3}{2g s_0} e^{5\phi_0} + \frac{\tilde{\Lambda}}{3\kappa_0} e^{4\phi_0} \right] > 0. \quad (2.16)$$

On the other hand, from the graviton equation for the action (2.1), ignoring the matter contributions \(T^m_{\mu\nu}\), which are negligible in the inflationary phase, we obtain [15]

$$H_I^2 = \frac{1}{6} \left[ \frac{T_3}{g s_0} e^{3\phi_0} + \frac{\tilde{\Lambda}}{3\kappa_0} e^{2\phi_0} \right] \left( \frac{1}{\kappa_0} + \alpha \left( \frac{T_3}{g s_0} e^{\phi_0} - \frac{\tilde{\alpha}}{4} e^{-2\phi_0} \right) \right). \quad (2.17)$$

From (2.16) and (2.17) we obtain finally the value of the condensate that characterizes the de Sitter geometry in this phase, namely

$$\tilde{\alpha} = \frac{4 e^{2\phi_0}}{\alpha \kappa_0} \left[ 1 + \alpha \frac{\kappa_0}{g s_0} e^{\phi_0} \left( 1 - \frac{1}{3} \mathcal{G} \right) \right],$$

$$\mathcal{G} \equiv \frac{T_3}{g s_0} e^{\phi_0} + \frac{\tilde{\Lambda}}{\kappa_0}.$$ 

(2.18)

Thus, the condensate is proportional to the factor

$$\frac{e^{2\phi_0}}{\alpha \kappa_0} = \frac{6V^{(6)}}{\pi^2 (g s_0 e^{-\phi_0})^2}, \quad (2.19)$$

using (2.2) and adopting the value of \(\alpha\) that appears in the example of [33]. We note that this factor depends on the size of the compactification volume \(V^{(6)}\).

The mathematical and physical/phenomenological consistency of our solution require the condensate \(\tilde{\alpha}\) and the curvature, i.e., the Hubble constant \(H_I\) (2.16) to be sufficiently small compared to Planck scale, which is easily guaranteed from (2.18) for sufficiently large negative values of the dilaton field \(\phi_0\) that characterizes inflation in the scenario of [1].

Given that in our string scenarios the bulk cosmological constant \(\tilde{\Lambda}\) is negative [1], (2.3), it is
possible that the bulk density of D-particles during the de Sitter phase is such that in order of magnitude
\[ G \gg -1, \quad \text{e.g.} \quad \frac{T_3}{g_{s0}} e^{\phi_0} \sim \frac{2|\tilde{\Lambda}|}{3\kappa_0}, \quad (2.20) \]
in which case eq. (2.18) implies:
\[ \tilde{\alpha} \simeq 2 e^{2\phi_0} \frac{T_3}{g_{s0}} e^{\phi_0} \sim e^{2\phi_0} \frac{4|\tilde{\Lambda}|}{3\kappa_0}, \quad (2.21) \]
and the condensate \( \tilde{\alpha} \ll 1 \) (as required for consistency of the approach) for \(|\tilde{\Lambda}|/\kappa_0 \ll 1 \) and finite, but large negative values of \( \phi_0 \).

In such a case, the formation of the condensate may be understood as stabilizing the brane vacuum. Indeed, as follows from (2.1), upon the formation of constant quantum vacuum condensates \( \tilde{\alpha} \), the dark energy terms in the brane effective action (in the Einstein frame) assume the form
\[ D_3 - \text{Brane Dark Energy} \sim \tilde{\alpha} + e^{2\phi_0} \left( \frac{T_3}{g_{s0}} e^{\phi_0} - \frac{|\tilde{\Lambda}|}{\kappa_0} \right). \quad (2.22) \]

In the absence of a condensate, the brane vacuum energy would be negative, of order \(-|\tilde{\Lambda}|/\kappa_0\), which would indicate an instability of the vacuum. The true stable vacuum of the theory would then be the one in which the condensate forms, with the value (2.21), which implies a positive (de Sitter-type) vacuum energy (2.22) of order
\[ e^{2\phi_0} \frac{|\tilde{\Lambda}|}{\kappa_0} > 0, \quad (2.23) \]
which is small for large negative values of the dilaton \( \phi_0 \ll 0 \).

We remind the reader that we are interested in the slow-roll phase of the dilaton, in which \( \phi_0 \) is almost constant over appropriate time scales considered here.

This bypasses the “no-go” theorems of [39–44] forbidding a de Sitter phase in simple type IIA or IIB effective supergravity theories coming from string/brane models in the absence of D-particle populations on the brane Universe. When they are present, (quantum) recoil fluctuations of the D-particles on the brane, whose dynamics satisfies the highly non-linear Born-Infeld action, can lead to condensate formation and positive vacuum energy. This is consistent with the observation in [39–44] that quantum effects can lead to positive vacuum energy in such theories and hence inflation.

3 D-particle-vacuum condensates and perturbations during Dilaton inflation

We now discuss the possible effects of quantum fluctuations of the condensate field \( \tilde{\alpha}(t) \) on the curvature as well as on the tensor perturbations during the dilaton-driven de Sitter phase. As mentioned above, one may assume, without loss of generality, that in this case the dilaton assumes some large, negative and almost constant value, \( \phi_0 \), which characterizes its slow-roll
phase. Hence, in the following it suffices qualitatively to concentrate only on the effective action obtained from (2.1) by truncating the Born-Infeld expansion to second order in the recoil vector field $A_\mu$ and replacing $F_{\mu\nu}F^{\mu\nu}$ by the condensate quantum fluctuating field $(\frac{1}{2}F_{\mu\nu}F^{\mu\nu}) \sim \dot{\alpha}(t)$.

According to the discussion in [1], we can also add to the effective action in the Einstein frame (w.r.t. to the dilaton factors) a dilaton-independent positive contribution $\mathcal{A} > 0$ to the brane vacuum energy due to classical effects of ensembles of D-particles, which, according to the discussion in [1], is essential for ensuring a de Sitter phase driven by the dilaton. The quantity $\mathcal{A}$ is proportional to the density of D-particles on the brane during the inflationary period.

In this work, we start with the effective action in the string frame [15], and then pass to the Einstein frame. The reason, as we shall see, is that in our case the passage to the Einstein frame requires a conformal rescaling by a combination of scalar fields, the dilaton and a condensate emerging from the recoil degrees of freedom of the D-particle ensemble during their interaction with the string matter. In the string frame, the vacuum energy due to the D-particle ensemble depends on the dilaton as $\mathcal{A}_s \equiv M_s^2 e^{-\phi_0} n_s$, with $n_s$ the proper space density of D-particles in the string frame. In the Einstein frame (w.r.t. dilaton factors) $\mathcal{A}_s$ yields the aforementioned dilaton-independent cosmological constant $\mathcal{A}$. Thus, the string-frame effective action we consider, in the dilaton-induced inflationary period with near-constant $\phi_0$, is [15]:

$$S^{(4)}_{\text{eff}} \simeq \int d^4x \sqrt{-g} \left[ -\left( \frac{T_3}{g_{s0}} e^{-\phi_0} + e^{-2\phi_0} \frac{1}{\kappa_0} \dot{\Lambda} + \mathcal{A}_s \right) - \frac{1}{4} \dot{\alpha}(x) + \alpha \frac{1}{g_{s0}} e^{-\phi_0} + e^{-2\phi_0} \frac{\alpha}{4} \ddot{\alpha}(x) \right] R(g) + \cdots \equiv \int d^4x \sqrt{-g} \frac{e^{-2\phi_0}}{2\kappa^2_{\text{eff}}} \left[ \left( 1 + 2\sigma(x) \right) R - 2\Lambda \sigma(x) - 2\bar{\mathcal{B}} \right] + \cdots,$$

(3.1)

where $2\kappa^2_{\text{eff}} \equiv \left( \alpha \frac{T_3 e^{\phi_0}}{g_{s0}} + \frac{1}{\kappa_0} \right)^{-1} \simeq \kappa_0$ for the large negative values of $\phi_0$ characterizing the inflationary regime [1], is the four-dimensional reduced Planck constant, $\kappa^2_{\text{eff}} = M^2_{\text{Pl}}$, to be used in phenomenological studies, $0 < \Lambda \equiv \frac{\kappa^2_{\text{eff}}}{\alpha} \propto M^2_{\text{Pl}}$ is a positive constant to be determined by comparison with data,

$$2\bar{\mathcal{B}} = \frac{T_3}{g_{s0}} e^{\phi_0} - \frac{1}{\kappa_0} |\dot{\Lambda}| + e^{2\phi_0} \mathcal{A}_s,$$

(3.2)

is an effective cosmological constant, and

$$\sigma(x) \equiv \frac{1}{4} \alpha \kappa^2_{\text{eff}} e^{2\phi_0} \ddot{\alpha}(x)$$

(3.3)

is a dimensionless condensate field. We remind the reader that the parameter $\alpha \propto \alpha' > 0$, e.g., in the example of [33] $\alpha = \frac{\pi^2}{3} \alpha'$.

The action (3.1) describes the dynamics of two scalar fields interacting with gravity, namely the slowly-rolling dilaton $\phi_0$ and the condensate $\sigma$. In what follows we restrict ourselves to the effects of the fluctuations of the D-particle-induced condensate field $\sigma(x)$ on the perturbations during the inflationary phase induced by the dilaton, considering the dilaton field in (3.1) as approximately constant. We shall work in units of the reduced Planck constant, i.e., we set $\kappa^2_{\text{eff}} \simeq \kappa_0 = 1$. 

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**Note:** The above text is a transcription of a mathematical physics document, focusing on the description of a particular model involving scalar fields, dilaton, and condensate, and the derivation of an effective action for an inflationary period. The document discusses the role of dilaton factors, effective actions, and the implications of these factors on cosmological constants and perturbations, providing a detailed analysis of the theoretical framework. The text is dense with mathematical expressions and theoretical physics concepts, typical of advanced research in the field.
We pass into the Einstein frame, denoted by a superscript $E$, by redefining the metric \[ g_{\mu\nu} \rightarrow g^E_{\mu\nu} = (1 + 2\sigma(x)) e^{-2\phi_0} g_{\mu\nu} , \] in which case the field $\sigma(x)$ becomes a dynamical scalar degree of freedom. We define a canonically-normalized scalar field $\varphi(x)$

\[ \varphi(x) \equiv \sqrt{\frac{3}{2}} \ln (1 + 2\sigma(x)) , \]

so that the action (3.1) becomes

\[ S_{\text{eff}}^{(4)} = \frac{1}{2} \int d^4x \sqrt{-g^E} \left( R^E + g^E_{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right) + \mathcal{O} \left( \partial_\mu \varphi \partial_\nu \phi_0 g^{E\mu\nu}, \partial_\mu \phi_0 \partial_\nu \phi_0 g^{E\mu\nu} \right) , \]

with the effective potential $V(\varphi)$ in the inflationary regime of large negative values of $\phi_0$ [1] given approximately by:

\[ V(\varphi) \simeq \left( e^{\sqrt{\frac{2}{3}}\varphi} - 1 \right) \Lambda e^{-2\phi_0} + 2B e^{2\phi_0 - 2\sqrt{\frac{2}{3}}\varphi} , \]

where

\[ 2B \simeq -\frac{1}{\kappa_0} |\tilde{\Lambda}| + e^{-2\phi_0} A . \]

It is important to note that, in order to arrive at (3.7), we took into account the conformal nature of the condensate $\tilde{\alpha}(x) = \frac{1}{4} \langle F_{\mu\nu} F_{\rho\sigma} \rangle g^{\mu\rho} g^{\sigma\nu}$ and have ignored terms that are more than quadratic in the vector potential. Moreover, as already emphasized, for our purposes here we concentrate on the slow-roll phase of the dilaton field $\phi_0$, so any potential-like terms with dilaton time-derivative factors will be ignored. In this approximation we need not worry about the cross-kinetic-terms $\partial_\mu \phi_0 \partial_\nu \varphi$, which can in any case be eliminated by a further redefinition (mixing) of the fields $\varphi$ and $\phi_0$.

There is one more ingredient in the effective action (3.6) that is important for inflationary phenomenology, as we now discuss. This arises because in brane worlds there are other flux gauge fields, in addition to the recoil vector field, that can also be described by appropriate Born-Infeld actions on the D3-brane Universe, and may condense [38] during the dilaton-induced inflationary period. Such condensates are also conformally invariant, just like the recoil vector fields above, leading to additional vacuum energy terms in the four-dimensional effective action of the following form in the Einstein frame:

\[ \int d^4x \sqrt{-g^E} \frac{1}{4} \langle G_{\mu\nu} G^{\mu\nu} \rangle^E = \int d^4x \sqrt{-g^E} D , \]

where $D$ is a constant, independent of the D-particle recoil condensate field and its fluctuations $\varphi(x)$. This would therefore lead to an extra (positive) cosmological constant contribution to the effective potential:

\[ V(\varphi) = \left( e^{\sqrt{\frac{2}{3}}\varphi} - 1 \right) e^{-2\phi_0} \Lambda - \frac{|\tilde{\Lambda}|}{\kappa_0} e^{2\phi_0 - 2\sqrt{\frac{2}{3}}\varphi} + A e^{-2\sqrt{\frac{2}{3}}\varphi} + D . \]

From the discussion in [1], an extra factor $\sqrt{2}$ needs to be absorbed into the dilaton normalization in order to obtain a canonical kinetic term, yielding finally

\[ V(\varphi) = \left( e^{\sqrt{\frac{2}{3}}\varphi} - 1 \right) e^{-\sqrt{2}\phi_0} \Lambda - \frac{|\tilde{\Lambda}|}{\kappa_0} e^{\sqrt{2}\phi_0 - 2\sqrt{\frac{2}{3}}\varphi} + A e^{-2\sqrt{\frac{2}{3}}\varphi} + D . \]
Figure 1. The effective potential (3.9) as a function of the collective scalar field $\varphi$ representing quantum fluctuations of D-particle condensates and the dilaton $\phi_0$ during its slow-roll inflationary phase, as obtained by choosing some representative values, $\Lambda = 10^{-6}$, $|\tilde{\Lambda}| = 10^{-2}$, $A = 10^{-4}$ and $D = 15$ in units of an overall unspecified scale.

For weak condensates $\varphi \ll 1$, where the approximations in this article hold, and large negative values of the dilaton $\phi_0 \ll 0$, as appropriate for the slow-roll inflation of [1], the reader will recognize in (3.10) the Starobinsky-like form (1.2) of the effective action for the dilaton-driven inflation of ref. [1], provided $A + D > 0$. This can fit the Planck data [7, 8] — though not the BICEP2 data [9] — due to the very small value of the tensor-to-scalar ratio $r$ predicted by this class of theories.

The form of the effective potential as a function of both the dilaton $\phi_0$ and the condensate fields field $\varphi$ is illustrated in figure 1, where we see that a weak condensate: $\varphi \ll 1$ is consistent with the minimization at large negative $\phi_0$, which is a consistency check of our approach. It should be understood that a full analysis of the two-field effective action (3.1), together with a study of non-Gaussianities, along the lines of [48, 49], is required before a conclusive comparison with data can be made. However, it is clear from the above analysis that the effects of D-particle recoil through the formation of appropriate condensates, although compatible with an inflationary phase driven by the dilaton, cannot lead to significant enhancement of the tensor to scalar ratio if the scalar degrees of freedom associated with the condensate and the dilaton are treated as independent fields.

On the other hand, we expect the condensate field $\varphi$ to mix with the dilaton via kinetic terms, so that

$$\varphi = C\sqrt{2}\phi_0(t) + \alpha(t)$$

(3.11)

where $C$ is some undetermined constant and $\alpha(t)$ is independent of the dilaton. Looking at the exponents in the action (3.10), we observe that the dilaton effects in the second term of the effective potential (3.10) may then be screened, so that the coefficient of $\phi_0$ in the exponential factor becomes significantly smaller than $\sqrt{2}$ for a suitable value of $C = O(1) > 0$. Specifically, one may obtain an enhanced tensor-to-scalar ratio $r$ that is more compatible with BICEP2 [9], if one assumes suitable values of the model parameters $A$ and $\Lambda$ in (3.10) so that the inflationary potential for $\phi_0$ values of interest is dominated by an effective constant that is approached exponentially with a small coefficient $B$ in the notation of (1.2). According to (1.4), this would then lead to an enhanced value of $r$, and a BICEP2-friendly value is possible if $C \approx 0.55$. 

\[ -13 - \]
4 Hawking radiation effects on tensor perturbations

In this section we mention another way in which tensor perturbations may be enhanced in our D-particle foam scenario [1]. At the end of the dilaton-induced inflation, this scenario yields a new ‘out’ vacuum state, with a significant population of D-particles. Such populations also existed during the inflationary phase, and were responsible for inducing a non-zero cosmological constant in the string effective action $A$ (1.2). Moreover, it has been demonstrated [50, 51] that when the D-particle vacuum is viewed as an ‘out’ vacuum in an expanding universe, there is particle production, and the D-particle vacuum, with stochastic recoil-velocity fluctuations, constitutes a system with an equation of state similar to that of a de Sitter space-time (see also [1]).

This particle production has a thermal spectrum à la Gibbons-Hawking, since it is known via non-perturbative string theory [52] (using duality to a matrix model) that the thermally excited collection of D0-branes in a D3-brane space-time constitute a stringy version of a black hole. Using this description, the “scrambling” time $t_\star$ for this black hole (i.e., the time needed for thermalization of the system of $N$ D0-particles, as seen by an observer who is asymptotically far from the horizon of the black hole) was estimated in [53, 54] in the large $N$ limit to be:

$$t_\star \sim \frac{1}{2\pi} T^{-1} \sqrt{-g_{00}(r)} \log \left( \frac{R_S}{\ell_s} \right),$$

(4.1)

where $R_S$ is the Schwarzschild radius of the black hole, and $g_{00}$ is the temporal component of the metric:

$$ds^2 = -(1 - R_S r^{-1}) dt^2 + \left(1 - r^{-1} R_S\right)^{-1} dr^2 + r^2 d\Omega^2,$$

(4.2)

where $R_S = 2M/M_P$ is the Schwarzschild radius (with $M_P$ the four-dimensional Planck mass).

In (4.1), $T \propto N^{-1/2}$ becomes the Hawking evaporation temperature $T_H = \frac{M_P^2}{2\pi M} = \frac{1}{4\pi R_S}$ for an observer at a large distance $r \to \infty$ from the horizon, and the $\log(R/\ell_s)$ factor is related to $\log S$, where $S \propto \log N$ is the entropy of the black hole. This logarithmic behaviour of the scrambling time is associated in [53, 54] with the fact that in the microscopic description of the black hole the relevant degrees of freedom are delocalized as in the matrix model.

If we assume that D0-branes on the D3-brane Universe thermalize, a cosmological brane observer would find him/her self inside the horizon of the black hole formed by the thermal ensemble of D0-particles [53, 54]. There is a duality between a de Sitter space-time with radius $R = 1/H$ and Gibbons-Hawking temperature $T_{\text{GH}} = H/2\pi$, where $H$ is the Hubble parameter, parametrized by the metric

$$ds^2 = -(1 - r^2 H^2) dt^2 + \left(1 - r^2 H^2\right)^{-1} dr^2 + r^2 d\Omega^2,$$

(4.3)

and a Schwarzschild Black hole of mass $M$ with metric (4.2).

This duality extends the scrambling time [53, 54], with the Hawking temperature of the black hole corresponding to the Gibbons-Hawking temperature. The scrambling time of de Sitter space-time has also been estimated using matrix theory in [53, 54], and found to be analogous to that of the black hole (4.1):

$$t_\star^{\text{dS}} \sim \hbar T_{\text{GH}}^{-1} \log (1/(H\ell_s)) \propto \frac{1}{H} \log (1/(H\ell_s)),$$

(4.4)

where $\ell_s$ is the string length.
If the Hubble parameter during inflation is of order $10^{-5} M_P$, the maximum allowed by the BICEP2 data \((1.1)\), and assuming that the string length is of order the four-dimensional Planck length, $\ell_s \sim \ell_P = M_P^{-1}$, the thermalization time is of order $t_* \sim 10^5 \log \left( 10^5 \frac{M_*}{M_P} \right) M_P^{-1}$, which is plausibly less than the duration of inflation in realistic theories, $\sim 10^9 M_P^{-1}$. Thus, the D-particles and the out vacuum are thermalized at the end of the de Sitter phase.

It was suggested in \([55]\) that, at the exit from a generic inflationary de-Sitter vacuum, the vacuum structure changes to a new ‘out’ vacuum in which there is mixing between the de Sitter modes and the eigenmodes of the ‘out’ vacuum. Assuming that the Bogolyubov coefficients of this mixing have a thermal distribution with temperature $T = H_* / 2\pi$, it was shown in \([55]\) that the two-point quantum correlation functions of (transverse, traceless) graviton tensor modes in the ‘out’ vacuum state would have the spectrum

$$P_T \simeq \frac{2 H_*^2}{\pi^2 M_{Pl}^2} \left( \frac{k}{a H_*} \right)^{-2\epsilon} \left( \frac{\pi}{2 \ a H_*} \right)^2$$

for the scales $k \ll aH_*$ of relevance to observations, providing a contribution to the tensor perturbation spectrum with spectral index

$$n_T = dP_T / d\ln(k) \sim 1 - 2\epsilon.$$ 

On the other hand, dilaton-induced inflation proceeds in such a way that the $\eta$ parameter is negative, corresponding to tachyonic scalar curvature perturbations. These are not thermalized, so the scalar perturbation spectrum is not affected, and assumes the standard form. Hence the scalar-to-tensor ratio for this case has a scale-dependent form that may be different from the standard result.

5 Conclusions

In this article we have explored how D0-brane defects in the stringy cosmological scenario of \([1]\) may affect tensor perturbations so as to yield a larger tensor-to-scalar ratio.

One possibility is the formation of condensates of the vector field representing quantum fluctuations in the recoil velocities of D0-branes on the D3-brane Universe (or some other fields with a Born-Infeld action). These condensates may contain terms proportional to the dilaton field, due to the presence of mixed kinetic terms in the effective lagrangian, which in turn leads to effective potentials of the type \((1.2)\) but with exponents $B$ much smaller than the Starobinsky value. According then to \((1.4)\), an enhanced ratio $r$ of tensor to scalar perturbations may be obtained.

Another possibility is that D-particle defects may generate a non-trivial ‘out’ vacuum, characterized by thermal spectrum particle production, with modes that mix with the de Sitter modes to enhance the tensor-to-scalar ratio and modify the spectrum of tensor perturbations. The blue-tilted spectrum $n_T \sim 1$ of tensor perturbations suggested in this second scenario may be in tension with other constraints, as pointed out in \([56]\), but we leave for future work their analysis in the context of a detailed brane cosmology.

We close by remarking that, although the above ideas are certainly speculative, they may provide sufficient motivation to study such stringy-inflation scenarios more deeply, with a view to accommodating current astrophysical data as well as Planck and BICEP2 data.
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