Scale generation via dynamically induced multiple seesaw mechanisms

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(Dated: January 3, 2017)

We propose a model which accounts for the dynamical origin of the electroweak symmetry breaking (EWSB), directly linking to the mass generation of dark matter (DM) candidate and active neutrinos. The standard model (SM) is weakly charged under the $U(1)_{B-L}$ gauge symmetry, in conjunction with newly introduced three right-handed Majorana neutrinos and the $U(1)_{B-L}$ Higgs.

The model is built on the classical scale invariance, that is dynamically broken by a new strongly coupled sector, what is called the hypercolor (HC) sector, which is also weakly coupled to the $U(1)_{B-L}$ gauge. At the HC strong scale, the simultaneous breaking of the EW and $U(1)_{B-L}$ gauge symmetries is triggered by dynamically induced multiple seesaw mechanisms, namely bosonic seesaw mechanisms. Thus, all the origins of masses are provided singly by the HC dynamics: that is what we call the dynamical scaleogenesis. We also find that an HC baryon, with mass on the order of a few TeV, can be stabilized by the HC baryon number and the $U(1)_{B-L}$ charge, so identified as a DM candidate. The relic abundance of the HC-baryon DM can be produced dominantly via the bosonic-seesaw portal process, and the HC-baryon DM can be measured through the large magnetic moment coupling generated from the HC dynamics, or the $U(1)_{B-L}$-gauge boson portal in direct detection experiments.

I. INTRODUCTION

The standard model (SM) of particle physics has achieved great success and been excellently consistent with experiments so far. In the SM, an elementary scalar field, a Higgs field, plays a role in spontaneously breaking electroweak symmetry and generating masses, and the signals predicted by the SM Higgs boson have been discovered at the LHC [1, 2]. However, the source to trigger the electroweak symmetry breaking (EWSB) seems quite ad hoc and mysterious: one needs to assume square of the Higgs mass parameter to be negative without any dynamical reason. In that sense, the mechanism of the EWSB in the SM is still unsatisfactory, so one is urged to go beyond the SM, including new physics, where the low-energy physics looks much like that of the SM.

Once going beyond the SM, to reveal the origin of the EWSB, triggered by the negative mass squared for the Higgs, one necessarily encounters a problem: cancellation of quantum corrections to the Higgs mass which is proportional to the new physics scale. One way to avoid this problem is to invoke the classical scale invariance, which can forbid all dimensional parameters including the Higgs mass in the theory, hence one is to be free from quantum corrections to the Higgs mass #1. To retrieve the EWSB, one thus needs to generate the nonvanishing and negative squared Higgs mass term somehow.

One idea to generate the Higgs mass in the scale-invariant models is to introduce $U(1)_{B-L}$ gauge symmetry, which would be inspired by the possible existence of grand unified theory. In this scenario, the $U(1)_{B-L}$ symmetry is broken by the newly introduced the vacuum expectation value of the $U(1)_{B-L}$ Higgs boson generated by radiative corrections, so-called Coleman-Weinberg mechanism [6]. Then the mass term of Higgs is induced via the mixing term between the SM Higgs and the $U(1)_{B-L}$ Higgs bosons [6].

Another benefit to introduce the $U(1)_{B-L}$ gauge symmetry involves physics related to neutrinos and their mass generation mechanism. When the $U(1)_{B-L}$ gauge symmetry is encoded into the classical scale invariant scenario, the

#1 Note that the scale symmetry is anomalous to be explicitly broken by quantum corrections, yielding the trace anomaly. The gravitational effects may, however, cancel the trace anomaly and make the theory asymptotically safety [3, 4]. Therefore, we assume that the classical scale invariance is held below the Planck scale, as long as all the couplings in the theory do not reach the Landau pole up to the Planck scale, as argued in [4].
neutrino mass generation is achieved by nonzero vacuum expectation value of $U(1)_{B-L}$ Higgs, where the neutrinos possess the right-handed (RH) Majorana nature. Thus, the extension by the $U(1)_{B-L}$ gauge symmetry can explain the origin of both the EWSB andMajorana mass for RH neutrinos simultaneously.

In the scenario of this class, however, the mixing coupling between the EW Higgs and $U(1)_{B-L}$ Higgs fields is needed to be tuned to be negative to make the square of Higgs mass negative, i.e., to realize the EWSB, so this idea seems to be still unsatisfactory.

Another proposal built on the classical scale invariance has been published in the framework of namely the bosonic seesaw mechanism [8,9], which is triggered by a new strong dynamics [11–13], what we call the hypercolor (HC). In models of this class, the scale-invariance is dynamically broken by the strong scale intrinsic to the HC dynamics, and the negative-mass squared of the Higgs is then dynamically generated by the seesaw mechanism operative between the elementary Higgs field and a composite Higgs field generated from the HC dynamics. (Since the sign is never absorbed by phase rotations in the case of boson fields, the negative sign induced by the seesaw mechanism is manifestly physical to be a trigger of the EWSB.)

In this paper, we develop the bosonic seesaw model including the $U(1)_{B-L}$ gauge symmetry: all the masses, for the SM particles and $U(1)_{B-L}$ Higgs, gauge boson, and right-handed Majorana neutrinos (RHMνs), are generated singly by the new strong dynamics, the HC, via a sequence of bosonic seesaws (multiple seesaws) involving the HC composite Higgs bosons: that is the dynamical scalegenesis. The scale of active neutrino masses is generated via the neutrino seesaw of ordinary type-I form [15], which is induced from the bosonic seesaw term of the elementary and composite $U(1)_{B-L}$ Higgs bosons.

We also find that the lightest $U(1)_{B-L}$-charged HC baryon can be a dark matter (DM) candidate, and that the relic abundance can nonthermally be produced via the bosonic-seesaw portal process to explain the observed amount. The HC-baryon DM possesses the sensitivity enough to be accessible in direct detection experiments, due to the large magnetic-moment $g$ factor generated by the strong HC dynamics, or the sizable $U(1)_{B-L}$ gauge boson portal coupling.

This paper is organized as follows: In Sec. II we introduce our model and show how the dynamical scalegenesis works in Sec. III. We discuss our dark matter candidate in Sec. IV, and finally our conclusion is given in Sec. V. The Appendix A compensates the potential analysis to realize the EWSB and $U(1)_{B-L}$ breaking.

II. MODEL

Our model consists of the HC sector having the $SU(3)_{HC}$ gauge symmetry and the SM sector. The key assumption in this model is presence of the classical scale invariance, so that the Higgs field ($H$) in the SM sector does not have the mass term. The HC sector includes eight HC gluons ($G$) of the $SU(3)_{HC}$ as well as four HC fermions ($F_{i=1,2,3,4}$) forming the the fundamental representation of $SU(4)$, $F_{L/R} = (χ, ψ_1, ψ_2)^T_{L/R}$. The HC dynamical feature is assumed to be a complete analogue of QCD.

In addition to the SM gauge symmetry, we introduce the $B-L$ gauge symmetry, $U(1)_{B-L}$, by which the $U(1)_{B-L}$ gauge boson ($X$) and a new complex scalar ($φ$) weakly couple involving the HC sector and the SM particles. The HC fermions are vector-like charged under the SM and $U(1)_{B-L}$ gauges (see Table I). To make the $U(1)_{B-L}$-gauge anomaly-free, we also introduce right-handed Majorana neutrinos (RHMνs) $N^R_{1,2,3}$.

Regarding the matter contents of the model, the charge assignment for these gauges is summarized in Tables I and II. Reflecting the gauge symmetries read off from Table I and II and the classical scale-invariance, one can uniquely

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#2 When the EW Higgs sector is extended from the minimal structure, the negative mass term can be generated without assuming ad hoc negative quartic coupling mixing.

#3 The idea of the scale generation by dimensional transmutation from a hidden strong dynamics, or the existing QCD, has been discussed in the literature [14] in a context different from the present model based on the bosonic seesaw mechanism.
TABLE II: The charge assignment for the SM quarks ($q^u_L$, $q^u_R$, $d^R_R$), leptons ($l^e_L$, $e^e_R$), Higgs ($H$) and three Majorana neutrinos ($N^\alpha_R$) and a $U(1)_{B-L}$ complex scalar ($\phi$). The upper script $\alpha$ attaching on fermion fields denote the generation index, $\alpha = 1, 2, 3$. All the fields listed here do not carry the $SU(3)_C$ charge.

| Constituent | SU(2)w | U(1)Y | U(1)_{B-L} |
|-------------|--------|--------|------------|
| $q^u_L$     | 3      | 2      | 1/6        | $-1/3$   |
| $q^u_R$     | 3      | 1      | 2/3        | $-1/3$   |
| $d^R_R$     | 3      | 1      | $-1/3$     | $-1/3$   |
| $l^e_L$     | 1      | 2      | $-1/2$     | 1        |
| $e^e_R$     | 1      | 1      | $-1$       | 1        |
| $H$         | 1      | 2      | 1/2        | 0        |
| $N^\alpha_R$| 1      | 1      | 0          | 1        |
| $\phi$      | 1      | 1      | 0          | $-2$     |

TABLE III: The list of the composite scalars and pseudoscalars embedded in the “chiral” $SU(4)_F$-16 plet. In the second row the isosinglet pNGB (like $\eta'$ in QCD) component has been discarded.

write down the model Lagrangian:

$$\mathcal{L} = \mathcal{L}^{\text{gauge-kin.}} + \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{4} F_{\mu\nu}^S F^{S\mu\nu} + \frac{1}{4} F_{\mu\nu}^F F^{F\mu\nu} + \frac{1}{4} F_{\mu\nu}^{\text{P}} F^{\text{P}\mu\nu} + |D_\mu H|^2 + |D_\mu \phi|^2 + \lambda_1 H^\dagger H^2 + \lambda_2 (\phi H^\dagger H)^2 + \lambda_3 (H H^\dagger)^2 + \lambda_4 (\phi H^\dagger H)^2$$

where $\mathcal{L}^{\text{gauge-kin.}}$ stands for the kinetic terms of all gauge fields forming the gauge field strengths; the covariant derivatives ($D_\mu$) can be read off from Tables I and II. The sums over repeated flavor indices $\alpha$ and $\beta$ have been taken into account; we have chosen basis for the $N_R$-flavor structure to be diagonal in the $\nu_N$-Yukawa coupling. The Yukawa couplings $y_H$ and $y_\phi$ are assumed to be much smaller than $O(1)$, $y_H \ll 1$ and $y_\phi \ll 1$, which will be consistent with realization of the EWSB and $U(1)_{B-L}$ breaking, as will be seen later on.

It is the HC dynamics that generates all the mass scales for the model particles: as in QCD, the HC gauge coupling gets strong to dynamically break the scale-invariance by the intrinsic scale $\Lambda_{HC}$, say, $O(5-10)\ TeV$, as the consequence of the dimensional transmutation. As will turn out, the HC dynamics triggers a sequence of seesaw mechanism (multiple seesaws) so that the dynamically generated scale $\Lambda_{HC}$ drives the EWSB as well as the mass generation of the active neutrinos.

### III. BOSONIC SEESAWs: DYNAMICAL SCAlegenesis

The HC sector possesses the (approximate) global “chiral” $SU(4)_F \times SU(4)_F$ symmetry, which is explicitly broken by the gauges as seen from Table II and a couple of Yukawa terms as displayed in Eq. (1). At the strong scale $\Lambda_{HC}$, this approximate “chiral” symmetry is spontaneously broken down to the vectorial $SU(4)_F$, by developing the nonzero “chiral” condensate ($F^\dagger F^2)$ ~ $\Lambda_{HC}^{-2}$, to give rise to the HC fermion dynamical mass of $O(\Lambda_{HC})$.

At the scale $\Lambda_{HC}$, the HC sector dynamics can be described as the “hadron” physics (HC hadron). As in the case of QCD, the lightest HC hadron spectra are then expected to be constructed from the composite scalars ($S$) and pseudoscalars ($P$), pseudo Nambu-Goldsone bosons (pNGBs) associated with the spontaneous “chiral” symmetry.
The mass generation of the pNGBs will be subject to the presence of an extra elementary pseudoscalar as discussed in the literature [12], where the size of masses can be fixed by other couplings irrespectively to those presented in Eq. 3.

TABLE IV: The list of the HC baryons with spin 1/2 forming the $SU(4)_F$-20plet classified by the weak isospin and hypercharge as well as the $U(1)_{B-L}$ charge. The upper script on the HC baryons denotes the electromagnetic charge ($Q_{em} = I_3 + Y$). In the list the isospin doublet $\Xi_{13}$ includes two degenerate states, analogously to the $\Xi_c, \Xi'_c$ baryons predicted in the quark model applied to QCD.

breaking #4. The composite scalars acquire the masses of $O(\Lambda_{HC})$ due to the “chiral” symmetry breaking. The light HC hadrons form the chiral $SU(4)_F \times SU(4)_{FR}$ 16-plet, $M = S + iP$, which can be classified with respect to the weak isospin ($SU(2)_W$), $U(1)_Y$ and $U(1)_{B-L}$ charges. The complete list for the lightest composite scalars and pseudoscalars is provided in Table [11]

Besides the HC scalars and pseudoscalars, the HC baryons formed by HC fermions like $\sim FFF$ are expected to be generated to have the masses of $O(\Lambda_{HC})$ in a way analogously to QCD. According to the QCD baryon spectroscopy, the spin 1/2 baryons form the $SU(4)_F$-20plet, classified by the weak isospin ($SU(2)_W$) and $U(1)_Y$ charges. In Table [11] the spin 1/2 HC baryons are listed.

Among those HC hadrons, we note composite scalars,

$$\Theta_1 \sim \bar{\psi}_1\chi, \quad \Phi \sim \bar{\psi}_2\psi_2,$$

in which the $\Theta_1$ has the same quantum numbers as those of the elementary Higgs doublet $H$, and the $\Phi$ carries the same charges as those the elementary $U(1)_{B-L}$ scalar $\phi$ does. (See Table [11]). Of interest is to note that at the $\Lambda_{HC}$ scale, the Yukawa terms with the couplings $y_H$ and $y_\phi$ in Eq. 3 induce the mixing between $\Theta_1-H$ and $\Phi-\phi$, such as $y_H\Lambda_{HC}^2(\Theta_1 H + h.c.)$ and $y_\phi\Lambda_{HC}^2(\Phi^*\phi + h.c.)$. Thus, the mass matrices of the seesaw form are generated:

$$\begin{pmatrix}
0 & y_H/\phi \Lambda_{HC}^2 \\
y_H/\phi \Lambda_{HC}^2 & \Lambda_{HC}^2
\end{pmatrix}.$$

These matrices yield the negative mass-squared for $H$ and $\phi$, $m_H^2 \simeq -y_H^2\Lambda_{HC}^2$, and $m_\phi^2 \simeq -y_\phi^2\Lambda_{HC}^2$, for small Yukawa couplings. Combined with the quartic potential ($\lambda_H$ and $\lambda_\phi$) terms for $H$ and $\phi$ in Eq. 3, the EWSB and $U(1)_{B-L}$ breaking are thus triggered to develop the vacuum expectation values (VEVs) $v_{EW} \simeq 246$ GeV and $v_{\phi_1} = O(\Lambda_{HC}) = $
Here the decays of the various (off-shell) neutral HC pions listed in Table III, which finally decay to diphoton: $\Lambda_{\text{HC}} \rightarrow \phi_{\pm} \gamma \gamma$.

A. Case I with $q = -1/3$

First, in this case, the electromagnetically charged HC baryons in the isospin multiplets decay to the neutral-isospin partners along with the $W$ boson emission, such as

\begin{eqnarray}
\rho_{\text{HC}}^+ & \rightarrow & n_{\text{HC}}^0 + W^+(\ast), \\
\Sigma_{(1),(2)}^\pm & \rightarrow & \Sigma_{(1),(2)}^0 + W^{\pm}(\ast), \\
\Xi_{(11),(12),(22)}^- & \rightarrow & \Xi_{(11),(12),(22)}^0 + W^{-}(\ast),
\end{eqnarray}

where the charged HC baryons have masses larger than the neutral ones by the size of $O(\alpha_{\text{em}} \Lambda_{\text{HC}}) (= O(100\text{ GeV})$ for $\Lambda_{\text{HC}} = O(5-10\text{ TeV})$). Then, these neutral HC baryons decay to the SM-singlet $\Lambda_{(1)}^0$ or $\Lambda_{(2)}^0$ by emitting the various (off-shell) neutral HC pions listed in Table III, which finally decay to diphoton:

\begin{eqnarray}
n_{\text{HC}}^0 & \rightarrow & P_{\phi_2}^0 + \Lambda_{(1)}^0, \\
\Sigma_{(1),(2)}^0 & \rightarrow & P_{\phi_{0}}^0 + \Lambda_{(1),(2)}^0, \\
\Xi_{(11),(12),(22)}^0 & \rightarrow & \tilde{P}_{\phi_1,\phi_2}^0 + \Lambda_{(1),(2)}^0,
\end{eqnarray}

where the masses of the parent HC baryons are larger than those of the daughters by amount of $O(\alpha_{\text{em}} \Lambda_{\text{HC}})$ due to the weak interaction corrections.

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#5 The value of $q$ would be sensitive to realization of the asymptotic safety condition (i.e. no Landau pole up to the Planck scale) as noted in footnote #1. We find that the gauge coupling of $U(1)_Y$ does not diverge below the Planck scale at the one-loop level, as far as $-1 < q < 1/2$ is satisfied. Including the two-loop corrections, the bound could be relaxed because of the corrections from the Yukawa couplings.

#6 The charge $q$ could take arbitrary fractional numbers (satisfying the asymptotic safety condition in footnote #5) so that some HC baryons other than those in Cases I and II could be stable. In the present study we will disregard this possibility for simplicity.

#7 Here the decays of $P_{\phi_1}$ involve the $\psi_1 \leftrightarrow \chi_{1,2}$ conversion via the $y_H$-Yukawa coupling in Eq. (1) with the $H$-Higgs VEV $v_{\text{EW}}$, while those of $P_{\phi_2}$ do the $\phi_1 \leftrightarrow \phi_2$ conversion, as well as the $\phi_1 \leftrightarrow \chi_{1,2}$ conversion arising from the $y_{\phi}$-Yukawa coupling in Eq. (1) with the $\phi$-Higgs VEV $v_{\phi_1}$. 

IV. DARK MATTER

The HC baryons possess the HC baryon number associated with the global $U(1)_Y$ symmetry, so can be stabilized to be DM candidates. Looking at Table IV, one may expect the isosinglets are favored to be the candidates, i.e., $\Lambda_{(1),(2)}^{1+3\phi}$ or $\Omega_{(12),(22)}^{3\phi}$. Since the DM has to be electromagnetically neutral, below we shall employ the possible two cases with (I) $q = -1/3$ and (II) $q = 0$ #5, and discuss the stability of the DM candidates, the thermal history and the discovery sensitivity in direct detection experiments #6.


Second, the electromagnetically-charged isosinglet $\Omega^{-(12)}$ decays like

$$\Omega^{-(12)} \rightarrow \Lambda^{0}_{(1)} + \tilde{P}_{\Theta_{1}} + \tilde{P}_{\Theta_{1}}^0,$$

where the $\Omega^{-(12)}$ has the mass larger than the $\Lambda^{0}_{(2)}$ mass due to the hypercharge-gauge boson-exchange contribution. The stability of the other charged isosinglet $\Omega^{-(22)}$ is dependent on the choice for the $q'$ value. Since the sufficiently large abundance of such a stable charged particle has already been excluded by astrophysical observations, we may choose the $q'$ value to be $q' < 1$ #9, in such a way that the $\Omega^{-(22)}$ can have the mass larger than the mass of $\Omega^{-(12)}$, which arises from the $U(1)_{B-L}$ gauge boson exchange, and hence is allowed to decay like

$$\Omega^{-(22)} \rightarrow \Omega^{-(12)} + \tilde{P}_{\Phi},$$

and finally decays to $\Lambda^{0}_{(2)}$ as aforementioned above #10.

Finally, consider the mass difference between $\Lambda^{0}_{(1)}$ and $\Lambda^{0}_{(2)}$, arising from the $U(1)_{B-L}$ gauge boson exchanges. It goes like $\sim \Lambda_{HC} \cdot (1 - 3q')$ up to some loop factor. Hence the scenario will be split up to the value of $q'$: i) when $q' < 1/3$ or $1/3 < q' < 1$, either $\Lambda^{0}_{(1)}$ or $\Lambda^{0}_{(2)}$ decays to each of the rest, $\Lambda^{0}_{(2)}$ or $\Lambda^{0}_{(1)}$, along with the $\tilde{P}_{\Phi}$ (with the $B-L$ charge $-2$); ii) when $q' = 1/3$, both of $\Lambda^{0}_{(1),(2)}$ are the lightest HC baryons, hence cannot decay. Thus, the lightest SM-singlet baryon $\Lambda^{0}_{(1)}$ ($\Lambda^{0}_{(2)}$ or both) is most stable to be a dark matter candidate.

In the thermal history, the production of the $\Lambda^{0}$ has taken place in two ways; i) the $\Lambda^{0}$ can annihilate into other light HC hadrons such as HC pions, so the relic abundance would be accumulated by this process at around the temperature, $T = \Lambda_{HC} = \mathcal{O}(5 - 10 \text{ TeV})$, through the thermal freeze-out scenario. However, it would not be a dominant process: by scaling the typical size of QCD hadron annihilating cross section, one gets $\langle \sigma v \rangle \sim 1/m_{\Lambda^{0}}^2$. One thus immediately finds that the freeze-out relic is negligibly small, $\Omega h^2 = \mathcal{O}(10^{-3})$, for $m_{\Lambda^{0}} = \mathcal{O}(\Lambda_{HC}) = \mathcal{O}(5 - 10 \text{ TeV})$; ii) the other possibility would be at hand, thanks to the bosonic seesaw mechanism as pointed out in Ref. [13], that is called the bosonic-seesaw portal process. In the present model, a source of the bosonic seesaw portal coupling can be generated at the $\Lambda_{HC}$ scale like

$$a_{\Lambda_{HC}} \overline{\Lambda^{0}}(\Theta_{1} \Theta_{1}) \Lambda^{0},$$

with $\mathcal{O}(1)$ coupling $a$. The bosonic seesaw, between the elementary Higgs doublet $H$ and the composite Higgs doublet $\Theta_{1}$, yields the mixing such as $\Theta_{1} = y_{H} H_{1} + \cdots \approx y_{H} v_{EW} h_{1} + \cdots$ for $y_{H} \ll 1$, where $H_{1}$ and $h_{1}$ respectively denote the lightest Higgs field and the physical Higgs boson field identified as the SM Higgs boson with the mass $m_{h_{1}} \simeq 125 \text{ GeV}$. Thus the bosonic seesaw generates a Higgs portal coupling for the $\Lambda^{0}$ baryon:

$$a \cdot y_{H} v_{EW} \overline{\Lambda_{HC}} \left( h_{1} \overline{\Lambda^{0}} \Lambda^{0} \right).$$

As noted in [13], this coupling is still operative even after the particles having the mass of $\mathcal{O}(\Lambda_{HC})$ decouple from the thermal equilibrium at around $T = \Lambda_{HC}$, so that the $\Lambda^{0}$ baryon can unilaterally and nonthermally be produced from the SM particles through the induced-Higgs portal coupling. Thus this process is thought to have been dominant for the production in the thermal history. In a way done in Ref. [13], one can estimate the relic abundance to find that the $\Lambda^{0}$ baryon having the mass of $\mathcal{O}(5 - 10 \text{ TeV})$ can explain the presently observed abundance of dark matter for $y_{H} \lesssim \mathcal{O}(10^{-5})(< 1)$, provided the single component scenario. Note also that this smallness of the $y_{H}$ coupling constant is consistent with the bosonic seesaw formalism in Eq.[3].

The $\Lambda^{0}$ dark matter would show up in the direct detection experiments such as the LUX and PandaX-II [16, 17]. Note that in the present Case I with $q = -1/3$, the constituent (valence) HC fermions of the spin $1/2 \Lambda^{0} \sim \chi_{1,2} \psi_{1,2}$ carry the electromagnetic charges, so the $\Lambda^{0}$ can have the electromagnetic form factors even though the composite state is neutral, as in the case of the QCD neutron. Among the form factors, the most stringent coupling to the

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#9 The size of $q'$ would be constrained by the asymptotic safety condition as well as the $q$ as noted in footnote #5. To avoid the Landau pole up to the Planck scale in the one-loop running of the $U(1)_{B-L}$ coupling $g_{X}$, one needs to have $g_{X} \lesssim 0.6$ for $0 < q' \leq 1$, which would be reduced to the constraint on the $U(1)_{B-L}$ gauge boson mass, $m_{X} = g_{X} v_{\phi_{1}} / 2 \lesssim \mathcal{O}(10^{-1}) \cdot v_{\phi_{1}} = \mathcal{O}(5)$ TeV for $v_{\phi_{1}} = 50$ TeV.

#10 The HC pion $P_{\Theta_{1}}^{0}$ decays to diphoton, involving the $\phi_{1} \leftrightarrow \phi_{2}$ conversion twice, as well as the $\phi_{1} \leftrightarrow \chi_{1,2}$ conversion, arising from the $y_{\phi}$-Yukawa coupling in Eq.(11) with the $\phi$-Higgs VEV $v_{\phi_{1}}$. 
photon arises from the magnetic moment interaction due to the sizable $g$ factor of $\mathcal{O}(1)$ generated by the strong dynamics. Such a sizable magnetic-moment portal coupling associated with a new strong (HC) dynamics has been severely constrained by direct detection experiments, as discussed in a context of some strong dynamics\cite{18,20}. The currently most stringent limit, derived from the LUX2016 data\cite{16}, has been placed on the composite baryon-DM mass, $m_{\text{DM}} \gg \mathcal{O}(10)$ TeV with the $g$ factor of $\mathcal{O}(1)$\footnote{This limit can be read off from the fourth reference in \cite{20} with a rough scaling of the upper bound of cross sections by a factor of 1/10 between the 2013 and 2016 data.}.

In the region satisfying $m_{\text{DM}} \sim \mathcal{O}(10)$ TeV, we need the detailed analysis for the LZ and XENON1T experiments\cite{21}, including the $U(1)_{B-L}$ interaction, which is to be pursued in the future.

### B. Case II with $q = 0$

The HC-baryon decay chain in this case is constructed in a way similar to the Case I. First, the charged HC baryons with the higher isospin numbers weakly decay to the isospin partners with the lower isospin numbers:

$$
\begin{align*}
\rho_{\text{HC}} &\rightarrow n_{\text{HC}} + W^+(s), \\
\Sigma^{0,+,-(2)} &\rightarrow \Sigma^{0,+,-(1),(2)} + W^+(s) \rightarrow \Sigma^{0,+,-(1),(2)} + W^+(s) + W^+(s), \\
\Xi^{+,0}_{(1),(2),(22)} &\rightarrow \Xi^{0,+,0}_{(1),(2),(22)} + W^+(s).
\end{align*}
$$

(11)

Then the daughter HC baryons, except $\Xi^{0}_{(22)}$, subsequently decay to the electromagnetically-charged isosinglet baryons $\Lambda^{\pm}_{(1),(2)}$ (having the same $B-L$ charge as those of daughters), plus the (off-shell) HC pions:

$$
\begin{align*}
n_{\text{HC}} &\rightarrow \Lambda^{+}_{(2)} + P_{\Theta_3}, \\
\Sigma^{0,+,0}_{(1),(2)} &\rightarrow \Lambda^{0,+,0}_{(1),(2)} + P_{a_{\text{HC}}}^{0,+,0} + P_{a_{\text{HC}}}^{0,+,0}, \\
\Xi^{0}_{(1),(2),(22)} &\rightarrow \Lambda^{+,0}_{(1),(2)} + \rho_{\Theta_3}. \\
\end{align*}
$$

(12)

The rest, $\Xi^{0}_{(22)}$, decays to the electromagnetically-neutral-isosinglet $\Omega^{0}_{(22)}$ along with the isospin-doublet HC pion $P^{0}_{\Theta_1}$.

The stability of the singly-charged $\Lambda^{+}_{(1),(2)}$ depends on the $B-L$ charge value $q'$. To avoid a stable charged baryon, as done in the Case I, we may take $q' \geq 1/3$, so as to allow the decay channel of the $\Lambda^{+}_{(1)}$ to the $\Lambda^{+}_{(2)}$:

$$
\Lambda^{+}_{(1)} \rightarrow \Lambda^{+}_{(2)} + \rho_{\Phi}. \\
$$

(13)

Note that we have the mass difference between $\Lambda^{0}_{(1)}$ and $\Lambda^{0}_{(2)}$, $\Delta m_{(1)-(2)} \propto (3q' - 1) > 0$, according to the $B-L$ charge. $\Lambda^{+}_{(2)}$ can decay to the neutral $\Omega^{0}_{(12)}$, by emitting the (off-shell) two isospin-doublet HC pions $P^{+}_{\Theta_3}$ and $P^{0}_{\Theta_1}$.

For the remaining $\Omega^{0}_{(12),(22)}$ baryons, the stability again depends on the value of the $B-L$ charge, $q'$: when $q' \neq 1$ is satisfied, either $\Omega^{0}_{(12)}$ or $\Omega^{0}_{(22)}$ can decay to either of the rest, along with the HC pion $P^{0}_{\Phi}$ or $P^{0}_{\Phi}$. In the case of $q' = 1$, these two baryons are degenerate so that both two can be DM candidates. Thus, the lightest $\Omega^{0}$ ($\Omega^{0}_{(12)}$ or $\Omega^{0}_{(22)}$, or both) becomes stable when the $B-L$ charge is taken as $q' \geq 1/3$.

The thermal history of the $\Omega^{0}$ is the same as the $\Lambda^{0}$ in the Case I: the relic abundance has dominantly been produced by the bosonic-seesaw portal process with the portal coupling as in Eq.\cite{10} replacing $\Lambda^{0}$ with $\Omega^{0}$. The desired amount of the abundance can thus be accumulated consistently with the bosonic seesaw mechanism with the small coupling $y_{H} \lesssim \mathcal{O}(10^{-3})$\cite{13}.

As to the discovery sensitivity in direct detection experiments, it is drastically different from the Case I: since the constituent HC fermions of the spin 1/2 $\Omega^{0}$ baryon do not have the electromagnetic charges, the magnetic moment cannot be generated, so the $\Omega^{0}$ DM is free from the severe constraint on the photon portal process in direct detection experiments as discussed in Refs.\cite{18,20}. The most dominant source then turns out to be the $U(1)_{B-L}$-gauge boson portal process. (As noted in Ref.\cite{13}, the bosonic-seesaw portal coupling yields tiny spin-independent nucleon-dark matter scattering cross section, to be negligible compared to the $B-L$ portal contribution.)

Indeed, the $\Omega^{0}-\Omega^{0}$-$U(1)_{B-L}$ gauge boson interaction would be sizable enough to get sensitive at the direct detection experiment. This feature is in contrast to the literature\cite{13}, in which the $U(1)_{B-L}$ gauge has not been introduced.
As done in the effective operator analysis, say, in Ref. 22, the four-fermion coupling \( g_X^2 / m_X^2 = 1 / v^2 \), induced from the \( U(1)_{B-L} \) gauge-boson \((X)\) exchange between the \( \Omega^0 \) and nucleon currents, is constrained by the exclusion limits provided by the detection experiments. The currently most stringent limit from the LUX2016 12 thus constrains the \( \phi \)-Higgs VEV \( v_{\phi_1} \). When \( q' = 1 \) is taken for a benchmark value, we find the lower bound, \( v_{\phi_1} > 5.1(4.3) \) TeV for the \( \Omega^0 \) DM mass \( m_{\Omega^0} = 5(10) \) TeV. The prospected future detection experiments such as the XENON1T and LZ 21 will give more severe limits, to constrain the parameter space in the model.

V. CONCLUSION

In the model presented here, the dynamical scalegenesis has successfully generated masses of the standard-model particle and active neutrino as well as explained the dark matter, by the multiple seesaw mechanisms induced from the new strong dynamics of the hypercolor. We have predicted a number of hypercolor hadrons, the \( B-L \) Higgs, gauge bosons and three heavy right-handed neutrinos, around the order of a few or tens of TeV scale.

Some of hypercolor baryons, with the mass on the order of the hypercolor scale \( \Lambda_{HC} \), say, \( O(5 - 10) \) TeV, can be stabilized due to the hypercolor-baryon number and the \( B-L \) charge. Two classes of dark matter candidates have been discussed by splitting the model in terms of the hypercharge parameter \((q)\). Note that the success of the dynamical scalegenesis is irrespective to the \( q \) value. Those two classes are shown to have different sensitivities to dark-matter detection experiments: one scenario implies that the dominant source to measure the dark matter is provided by the \( L \)-gauge-boson portal coupling (Case I), while the other by the \( B-L \) gauge-boson portal coupling (Case II). Future-planned detection experiments such as the XENON1T and LZ would make it possible to clearly verify which scenario would be favored.

The model can also be tested by the collider signatures of those new particles, as well as the searches for the dark matter. In particular, the lightest hypercolor hadrons, say, the hypercolor pions would show up at the LHC with distinct signals, as addressed in the literature 12, so would be a smoking gun of this model.

More details on the phenomenological analyses, including collider study of the hypercolor hadrons in correlation with the \( U(1)_{B-L} \) gauge boson and flavor physics induced by the couplings to the heavy Higgs \((H_2)\), will be pursued in the future.

Acknowledgments

We are grateful to David Schaich for useful comments on direct detection experiments for baryonic composite dark matters. This work was supported in part by the JSPS Grant-in-Aid for Young Scientists (B) #15K17645 (S.M.).

Appendix A: The realization of EW and \( U(1)_{B-L} \) symmetry breaking

In this Appendix we analyze the potential terms regarding the realization of the EWSB and the \( U(1)_{B-L} \) gauge symmetry breaking.

We employ an effective potential at the \( \Lambda_{HC} \) scale including terms in Eq.(1) and the HC-induced terms,

\[
V = \lambda_H (H^\dagger H)^2 + \lambda_\phi (|\phi|^2)^2 + \kappa_\phi |\phi|^2 (H^\dagger H) + y_H A_{HC}^\dagger \left( \Theta^\dagger H + \text{h.c.} \right) + y_\phi A_{HC}^2 (\Phi^* \phi + \text{h.c.}) \\
+ m_\phi^2 \text{tr}[M^\dagger M] + \lambda_1 \text{tr}[(M^\dagger M)^2] + \lambda_2 (\text{tr}[M^\dagger M])^2,
\]

(A1)

where \( M = S + iP \) denotes the composite scalar field of the \( 4 \times 4 \) matrix form, transforming bifundamentally under the \( U(4)_{FL} \times U(4)_{FR} \) symmetry, \( M \rightarrow g_L \cdot M \cdot g_R^\dagger \) with \( g_{L/R} \in U(4)_{FL/R} \). The \( M \) is expanded with respect to the \( U(4) \) generators \( T^a \) \((a = 0, \ldots, 15)\), normalized by \( \text{tr}[T^a T^b] = 6 \delta^{ab}/2 \) with \( T^0 = 1/2 \sqrt{2} \cdot 4_{1\times 4} \), as \( M = \sum_a M^a T^a \). In terms of the \( S^a \), the composite Higgs doublet \( \Theta_1 \) and the composite \( B-L \) Higgs \( \Phi \) are parametrized as

\[
\Theta_1 = \begin{pmatrix} \Theta_1^+ \\ \Theta_1^- \end{pmatrix} = \begin{pmatrix} \Theta_1^- \\ \Theta_1^+ \end{pmatrix} = \begin{pmatrix} S^1 \pm iS^8 \\ S^1 \mp iS^8 \end{pmatrix},
\]

\[
\Phi = \frac{S^{13} - iS^{14}}{\sqrt{2}}.
\]

(A2)
Since the $y_H$ and $y_{\phi}$ couplings are assumed to be small ($y_{H,\phi} \ll 1$), the potential in Eq. (A1) possesses the approximate “chiral” $U(4)_{F_L} \times U(4)_{F_R}$ symmetry reflected in the $M$ sector. Matching with the underlying vectorlike dynamics of the HC, we choose the VEV of $\mathcal{S}$, $\langle \mathcal{S} \rangle = S^0/2\sqrt{2} \cdot 1_{4\times4} = v/2\sqrt{2} \cdot 1_{4\times4}$, to realize the spontaneous breaking pattern $U(4)_{F_L} \times U(4)_{F_R} \to SU(4)_{F_L} \times U(1)_{F_R}$, with the $U(1)_{F_R}$ anomaly in the underlying HC dynamics taken into account. (The state $S^0$ corresponds to a linear combination of $f_0^{HC}$, the third-adjoint component of $a_0^{HC}$, $\phi_1$ and $\phi_2$ in Table III) The VEV $v$ is equivalent to the HC pion decay constant $f_\pi$, which can be related to the $\Lambda_{HC}$ scale as $f_\pi \simeq \Lambda_{HC}/(4\pi) = O(1)$ TeV for $\Lambda_{HC} = O(5-10\text{ TeV})$. The stationary condition for the $v$ is then derived from Eq. (A1) to be
\[
v \left( m^2_M + \left( \frac{\lambda_1}{4} + \lambda_2 \right) v^2 \right) = 0, \tag{A3}
\]
so that we have the VEV $v^2 = -m^2_M/(\lambda_1/4 + \lambda_2)$.

The physical $S^0$ scalar mass arises by expanding the potential around the VEV $v$, to be
\[
m_{S^0} = \sqrt{2(\lambda_1/4 + \lambda_2)v}. \tag{A4}
\]

As clearly seen from the potential form in Eq. (A1), one can always choose the vacuum where the VEVs of composite scalars are zero (i.e. trivial solutions for the stationary conditions), except for the $\Theta_1$ and $\Phi$ having the quadratic mixing terms with the elementary $H$ and $\phi$. Therefore, we can extract only the $\Theta_1$ and $\Phi$ scalars from the $M$ matrix in the potential Eq. (A1), and derive the effective potential terms relevant to discussion on the EWSB and the $U(1)_{B-L}$ breaking:
\[
V_{\text{eff}} = \lambda_H (H^\dagger H)^2 + \lambda_{\phi} |\phi|^2 + \kappa_{\phi} |\phi|^2 (H^\dagger H) + y_H \Lambda^2_{HC} \left( \Theta^\dagger_1 H \right. + \text{h.c.} \left.) + y_{\phi} \Lambda^2_{HC} (\Phi^* \phi + \text{h.c.}) \right.
- \left. + m^2_S \left( |\Theta|^2_1 |\Theta_1|^2 + |\Phi|^2 \right), \tag{A5}
\]
where $m^2_S = (3\lambda_1/8)v^2 (\simeq 3\Lambda^2_{HC}/16)$ and $\lambda_S = \lambda_1/2 + \lambda_2$. Note the degenerate mass and quartic coupling terms for $\Theta_1$ and $\Phi$, reflecting the approximate “chiral” $SU(4)_{F_L} \times SU(4)_{F_R}$ symmetry.

Solving the quadratic mixing terms for $\Theta_1 H$ and $\Phi\phi$ of the seesaw form in Eq. (A5), to the leading order of expansion in $y_H$ and $y_{\phi}$, one finds the mass eigenstate fields $(H_1, H_2)$ and $(\phi_1, \phi_2)$ related to the original fields $(H, \Theta)$ and $(\phi, \Phi)$ as
\[
\begin{pmatrix} H_1 \\
H_2 \end{pmatrix} \approx \begin{pmatrix} 1 & -y_H \\
y_H & 1 \end{pmatrix} \begin{pmatrix} H \\
\Theta_1 \end{pmatrix}, \tag{A6}
\begin{pmatrix} \phi_1 \\
\phi_2 \end{pmatrix} \approx \begin{pmatrix} 1 & -y_{\phi} \\
y_{\phi} & 1 \end{pmatrix} \begin{pmatrix} \phi \\
\Phi \end{pmatrix},
\]
with the mass eigenvalues
\[
m^2_{H_1} \simeq -y_H^2 \Lambda^2_{HC} m^2_S \left( \simeq -\frac{16}{3} y^2_H \lambda^2_{HC} \right),
m^2_{H_2} \simeq m^2_S \left( \simeq \frac{3}{16} \Lambda^2_{HC} \right),
m^2_{\phi_1} \simeq -y_{\phi}^2 \Lambda^2_{HC} m^2_S \left( \simeq -\frac{16}{3} y^2_{\phi} \lambda^2_{HC} \right),
m^2_{\phi_2} \simeq m^2_S \left( \simeq \frac{3}{16} \Lambda^2_{HC} \right). \tag{A7}
\]
Plugging these expressions into the effective potential and rewriting the terms in terms of the mass eigenstate fields, one finds the stationary conditions under the assumption that the $H_2$ and $\phi_2$ do not develop the VEVs:
\[
-m^2_{H_1} \simeq \frac{1}{2} \lambda_S \left( y_H^2 v^2_1 + y_{\phi}^2 v^2_{\phi_1} \right),
-m^2_{\phi_1} \simeq 4 \lambda_{\phi} v^2_{\phi_1},
-\kappa_{\phi} v^2_{\phi_1} \simeq \lambda_H v^4_1, \tag{A8}
\]
\[
\begin{pmatrix} H_1 \\
H_2 \end{pmatrix} \approx \begin{pmatrix} 1 & -y_H \\
y_H & 1 \end{pmatrix} \begin{pmatrix} H \\
\Theta_1 \end{pmatrix}, \tag{A6}
\begin{pmatrix} \phi_1 \\
\phi_2 \end{pmatrix} \approx \begin{pmatrix} 1 & -y_{\phi} \\
y_{\phi} & 1 \end{pmatrix} \begin{pmatrix} \phi \\
\Phi \end{pmatrix},
\]
where $v_1$ and $v_{\phi_1}$ stand for the VEVs of $H_1$ and $\phi_1$, respectively, and the last condition has come from the vacuum assumption. Thus, we realize the EWSB ($v_1 \neq 0 \simeq 246$ GeV) and $U(1)_{B-L}$ gauge symmetry breaking ($v_{\phi_1} \neq 0$).

Taking into account the stationary conditions in Eq. (A8) and expanding the $H_1$ and $H_2$ around those VEVs as $H_1 = \frac{1}{\sqrt{2}}(0, v_1 + h_1)^T$, $H_2 = \frac{1}{\sqrt{2}}(0, h_2)^T$, and redefining as $\phi_1 \rightarrow \frac{1}{\sqrt{2}}(v_1 + \phi_1)$ and $\phi_2 \rightarrow \frac{1}{\sqrt{2}}\phi_2$, one can find the physical masses in the effective potential,

$$m_{h_1} \simeq \sqrt{2\lambda_H}v_1 (\simeq 125 \text{ GeV}),$$

$$m_{h_2} \simeq m_{\phi_2} \simeq m_S \left( \simeq \frac{\sqrt{3}}{4}\Lambda_{\text{HC}} \right),$$

$$m_{\phi_1} \simeq 2\sqrt{2\lambda_{\phi_1}}v_{\phi_1}.$$  \hspace{1cm} (A9)

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