Fitting of Resistance Model of Underwater Electrodes

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Abstract. This paper measures and analyzes the resistance of a circuit formed by underwater metal electrodes under a low-frequency condition. The conditions for ignoring displacement current and the existence of Ohm’s law are discussed. On this basis, three expressions of resistance are proposed based on the measured data, and the three expressions are analyzed according to their different application emphases. The results show that under partial constraints, the three resistance expressions proposed can accurately calculate the resistance of the circuit formed by underwater metal electrodes.

1. Introduction
In general, the resistance of good conductors such as metal electrodes and metal wires can be obtained through simple measurements. At the same time, since the resistance of the metal conductor used to conduct current is extremely small, its resistance is often ignored in most applications. For example: the resistance of a 500kV AC transmission line is about 0.03Ω/km, which is often ignored in the calculation. However, the resistance of metal conductors is extremely important in some special environments and applications. TAGG proposed a method for measuring metal grounding resistance in soil covering a large area [1]. Bhang studied a design method of ground electrode array for photovoltaic panels floating on the sea [2]. Approximate expressions for the resistance of large-scale underwater grounding networks are given in [3]. However, the frequency of above research is only for the power frequency of the power system: 50Hz or 60Hz, and the metal electrode is huge in size or a large-scale network. In other underwater applications, the volume of the metal electrode may be relatively small, and the electrical signal carried by it will not only stay at 50Hz or 60Hz [4] [5]. For special application environments such as underwater current communication, this paper measures and analyzes the resistance of underwater metal electrodes. The conditions for ignoring displacement current and the existence of Ohm’s law are discussed. On this basis, three expressions of resistance are proposed based on the measured data, and the three expressions are analyzed according to their different application emphases. The results show that under partial constraints, the three resistance expressions proposed can accurately calculate the resistance of the circuit formed by underwater metal electrodes. The methods used in this article can be used in the design process of related fields.

2. Conditions for ignoring displacement current
2.1. Characteristics of current in water
Generally speaking, if there is an alternating current or an alternating electric field, both conduction current and displacement current must be considered. When the displacement current is so small that it can be ignored, an electric field can be regarded as a quasi-stable field. The following will discuss the
conditions under which the above assumptions hold in sea water. Given that the charge density in a conductive medium is \( \rho \), then:

\[
\frac{d\rho}{dt} = -\frac{\sigma}{\varepsilon} \tag{1}
\]

Where \( \varepsilon \) is dielectric constant, \( \sigma \) is conductivity. Therefore:

\[
\rho = \rho_0 \exp \left( -\frac{\sigma}{\varepsilon} t \right) = \rho_0 \exp \left( -\frac{t}{\tau} \right) \tag{2}
\]

Where \( \tau = \frac{\varepsilon}{\sigma}, \rho_0 \) is the charge density at \( t=0 \), \( \tau \) is the relaxation time. Generally speaking, the relaxation time of seawater is \( \tau = 1.77 \times 10^{-10} \) s, \( \sigma = 4 \text{ mhos/m}, \varepsilon = 80 \varepsilon_0 = 7.083 \times 10^{-10} \text{ F/m} \). The relaxation time \( \tau \) of metallic materials is generally of the order of \( 10^{-18} \) s. Therefore, the relaxation time of seawater is small enough to be regarded as a good conductor. From another perspective, the displacement current density is:

\[
j_d = \sigma \varepsilon \frac{\partial D}{\partial t} \tag{3}
\]

Where \( D \) is the electric displacement. The conduction current density is:

\[
j_c = \sigma E \tag{4}
\]

Then the ratio of displacement current density to conduction current density is:

\[
\frac{j_d}{j_c} = \frac{\varepsilon}{\sigma} \frac{\partial E}{\partial t} \tag{5}
\]

Assuming that the electric field is an alternating electric field: \( E = E_0 \exp(j\omega t) \). Where \( E_0 \) is the amplitude and \( \omega \) is the angular frequency. Then the ratio is

\[
\frac{j_d}{j_c} = \frac{\varepsilon}{\sigma} \omega = \tau \omega \tag{6}
\]

Therefore, when \( \tau \omega \ll 1 \), the displacement current can be ignored. To make \( \tau \omega = 1 \), the order of magnitude of the frequency is \( 10^{10} \)Hz. Therefore, if only low-frequency scenarios are considered, the displacement current in seawater can be ignored, and the current in seawater is dominated by conduction current. The electric field formed by electrodes underwater is equivalent to a static electric field in a conductor.

3. Electrodes and measuring method

According to the above conclusions, the current is mainly conducted in seawater, and Ohm’s law holds. Therefore, the resistance of the loop formed by two electrodes in seawater can be observed by measuring current and voltage. As shown in Figure 1, current is injected into the two ends of the metal electrode pair in the water, and the two metal electrode pairs form a current loop through water. When the distance between two electrodes is greater than a certain value, the loop resistance tends to a fixed value. At this time, the loop resistance is only related to the size of the electrodes and the conductivity of sea water.
4. Measurement results and analysis

4.1. Measurement result
Pair and measure according to different electrode sizes, a total of 90 sets of measurement results are shown in Figure 2:
4.2. Fitting of different size parameter models
For cylindrical electrodes, the following size parameters were considered: 1. volume and volume-related parameters 2. surface area and surface area-related parameters 3. the radius and height of the cylinder.

4.2.1. Volume and volume-related parameters
Assuming that the electrodes loop resistance is related to electrode’s volume:

\[ R = A \cdot Volume_A + B \cdot Volume_B + C \]  

(7)

Where \( R \) is the loop resistance, \( Volume_A \) and \( Volume_B \) are the volumes of electrodes A and B respectively, and \( A, B, C \) are three constants. The fitting results are shown in Figure 3, and the fitting indicators of this model are shown in Table 1.

![Figure 3. Fitting result of volume parameters](image)

| Coefficient of determination R-Square | Root mean square error |
|--------------------------------------|-----------------------|
| 0.7942                               | 14.57                 |

It can be seen from the fitting result index that the model composed of the volume of the two electrodes and other parameters cannot accurately express the loop resistance formed by the electrodes. According to the skin effect, the current will concentrate on the outer surface of the conductor when there is alternating current in the conductor. Therefore, for a cylindrical electrode, it is not appropriate to use its volume to form a resistance model.

4.2.2. Surface area and surface area related parameters
Assuming that the electrode loop is related to the surface area of the electrode:
\[ R = A \times \text{Area}_a + B \times \text{Area}_b + C \]  \hspace{1cm} (B)

Where \( R \) is the loop resistance, \( \text{Area}_a \) and \( \text{Area}_b \) are the surface area of electrodes A and B respectively, and \( A, B, C \) are three constants. The fitting results are shown in Figure 4, and the fitting indicators of this model are shown in Table 2.

![Figure 4. Fitting result of surface area parameters](image)

| Coefficient of determination R-Square | Root mean square error |
|--------------------------------------|-----------------------|
| 0.8793                               | 11.15                 |

It can be seen from the above results that this model has improved in terms of indicators. Then consider such a model:

\[ R = A \times \text{Area}_a^{1/2} + B \times \text{Area}_b^{1/2} + C \]  \hspace{1cm} (9)

The fitting results are shown in Figure 5, and the fitting indicators of this model are shown in Table 3.
Figure 5. Fitting result of surface area parameters

Table 3. Surface area fitting results related indicators

| Coefficient of determination R-Square | Root mean square error |
|--------------------------------------|------------------------|
| 0.9104                               | 9.613                  |

It can be seen that the fitting results of the model formed after the square root of the surface area has been further improved.

4.2.3. The radius and height of the cylinder

The relationship between the electrode loop resistance and its volume and surface area has been established above. Let's use a more direct way: the relationship between the electrode loop resistance and the radius and height. Assuming that the loop resistance is a second-order polynomial expression of the radius $r$ and height $l$ of electrodes A and B, after fitting and eliminating irrelevant terms, the following model is obtained:

$$R = a + b \cdot r_a + c \cdot r_b + d \cdot l_a + e \cdot l_b + f \cdot (r_a l_a) + g \cdot (r_a l_b) + h \cdot (l_a l_b) + i \cdot (r_b l_b) + f \cdot l_a^2 + k \cdot l_b^2 + m \cdot r_b^2$$ (10)

The fitting results are shown in Figure 6, and the fitting indicators of this model are shown in Table 4.
Table 4. Radius and height fitting results related indicators

| Coefficient of determination R-Square | Root mean square error |
|----------------------------------------|-----------------------|
| 0.992                                  | 3.08                  |

Compared with the other models above, the polynomial model with radius and height fits better. However, the model expression is relatively complicated and not convenient for practical application. In addition, the parameters in the above three models, except for volume, area, radius, and height, are the results of linear regression and have no actual physical meaning. This creates a situation: once the environment of the electrode pair is different from the environment at the time of measurement, the loop resistance prediction made by this model will produce errors. For such problems, an important physical parameter is the resistivity $\rho_s$ of sea water. The resistance of a rod-shaped metal object in a medium with a resistivity $\rho_s$ can be approximately expressed as $R = \frac{\rho_s}{2\pi l \frac{ln}{r}}$. Try to use the following model to fit the measured values, where A is a constant:

$$R = \frac{\rho_s}{2\pi l \ln} \frac{2l}{r} + A$$

The fitting results are shown in Figure 7, and the fitting indicators of this model are shown in Table 5.
5. Conclusion
By comparing all the models discussed in this article, the model represented by equation (11) not only includes the size of the electrode, but also the resistivity of the environment where the electrode is located, and the fitting result is good. The model can be used to predict the resistance of the electrode in different environments. If the resistivity of the environment where the electrode is located is a fixed value, then the model represented by equation (9) and equation (10) can also be used to predict the electrode resistance after testing.

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