Nonlinear optical rectification of tuned quantum dots under the action of a vertical magnetic field

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Abstract
The changes of optical rectification (OR) coefficient in tuned quantum dots under the action of the external magnetic field, hydrostatic pressure, temperature and quantum dot radius are theoretically studied in detail. The tuned quantum dots are subjected to a uniform magnetic field perpendicular to the structural plane. Within the framework of effective mass approximation and parabolic band approximation, the energy level and wave function are derived, and the nonlinear OR coefficient is calculated by compact density matrix theory and iterative method. Numerical results show that the resonance peak of the OR coefficient moves in the direction of high energy or low energy under different constraint parameters, that is, red shift or blue shift. At the same time, the peak value of the OR coefficient will increase or decrease with the change of the parameters.

Keywords Tuned quantum dots · Nonlinear optical rectification · Density matrix theory · Vertical magnetic field

1 Introduction
Early experiments show that the constraint potential in quantum contact is mainly parabolic. Parabolic constraints are more appropriate when quantum dots are prepared by etching, ion implantation, or electrostatic gates (Baskoutas et al. 2006; Sari et al. 2017; Hashemi et al. 2021; Ozturk et al. 2019; Karimi and Rezaei 2011; Chen et al. 2016). The parabolic confinement potential can accommodate various resonances, and the nonlinear polarization is enhanced due to the constant spacing of the energy levels, which is conducive to light absorption. Subsequently, people have conducted various optical studies on parabolic and semi-parabolic quantum dots (Yu 2013; Hassanabadi et al. 2012).

In recent years, many researchers have conducted many theoretical studies on the nonlinear optical properties of different quantum dots. Li et al. (2007) studied the nonlinear optical rectification of parabolic quantum dots in the presence of electric and magnetic fields. Zhang et al. (2010) analyzed the light absorption coefficient and refractive index...
changes of parabolic quantum dots under the action of electric and magnetic fields. Bouzaïene et al. (2013) studied the effects of hydrostatic pressure and temperature on nonlinear optical rectification of crystal mirror quantum dots. Ganguly et al. (2016) investigated under the interaction of hydrostatic pressure, temperature and noise, the second and third harmonics of doped quantum dots are generated by modulating optical rectification. Khor-dad (2013) considered the effect of magnetic field on the linearity of parabolic quantum dots. Dahiya et al. (2020) studied the effects of temperature and hydrostatic pressure on the optical rectification of the exciton system in semi-parabolic quantum dots. Zhang et al. (2021) discussed the influence of second-harmonic generation under the external electric field and magnetic field of parabolic quantum dots. However, the nonlinear optical rectification in tuned quantum dots under the action of a vertical magnetic field, hydrostatic pressure and temperature have not been studied. Therefore, in this article, we will theoretically study the influence of factors such as the vertical magnetic field and hydrostatic pressure in the tuned quantum dots on the optical rectification coefficient.

In this paper, the OR coefficient of tuned quantum dots under magnetic field is studied in theory. The first section of chapter 2 numerically solves the time-independent Schrödinger equation based on the effective mass approximation and parabolic band approximation, and we obtained the wave function and energy level. In the second section, the compact density matrix method and iterative method are used to derive the simple analytical formula of OR coefficient. In the third chapter, the numerical results are drawn and analyzed. The results show that vertical magnetic field, hydrostatic pressure and temperature have significant effects on OR coefficient. Chapter 4 gives a brief summary.

2 Theory and model

2.1 Wave function and energy eigenvalue

This study is based on a theoretical model of a parabolic inverse square bound potential electron with Gaussian correction and a magnetic field perpendicular to its plane. The binding potential $V_1(r)$ in this model is composed of parabolic and inverse square potential functions, which can be expressed as (Karabulut et al. 2011; Hien et al. 2019; Restrepo et al. 2017)

$$V_1(r) = \frac{1}{2} m(P, T)^* \omega_0^2 r^2 + \frac{\hbar^2}{2m(P, T)^*} \xi r^2,$$

(1)

where $\xi$ is a dimensionless parameter, $\omega_0$ is the harmonic constraint frequency, $\hbar$ is the reduced Planck constant, $m(P, T)^*$ is the effective mass related to hydrostatic pressure $P$ and temperature $T$. The expression is as follows

$$\frac{m_0}{m(P, T)^*} = 1 + \frac{E^\Gamma}{2 E_g(P, T)} + \frac{1}{E_g(P, T) + \Delta_0},$$

(2)

where $m_0$ is the free electron mass, $E^\Gamma_P$ is the energy associated with the momentum matrix element and equal to 7.51 eV, and $\Delta_0$ is the energy of the spin–orbit splitting. For GaAs materials, $E_g(P, T)$ is the energy gap function related to static pressure $P$ (kbar) and temperature $T$ (K), and the analytical form is as follows (Yuan et al. 2016; You et al. 2019)
$E_g^f(P, T) = E_g^f(0, T) + 1.26 \times 10^{-2} \times P - 3.77 \times 10^{-5} \times P^2,$  

(3)

with $E_g^f(0, T) = [1.519 - (5.405 \times 10^{-4} T^2)/(T + 204)]$.

The modified Gaussian potential (MGP) is similar to Gaussian potential (GP) and has continuity at the boundary of quantum dots. This property of MGP makes the approximation of MGP in a finite range of depth and potential. Moreover, the modified Gaussian potential is very flexible and can be used to calculate spectral energy and wave function, which can be expressed by the following expression (Gharaati and Khordad 2010; Khordad and Bahramiyan 2014)

\[ V_2(r) = -V_0 \exp\left[-\left(\frac{r}{R}\right)^q\right]. \]  

(4)

In the Eq. (4), $V_0$ and $R$ are the depth and stretching range of the constrained potential respectively. Where $R$ can be considered as the radius of the quantum dot, and suppose $q = 2$, and $r/R \ll 1$. Therefore, the Gaussian potential can be written as the anisotropic harmonic oscillator potential as (Rezaei et al. 2010)

\[ V_2(r) = -V_0 \exp\left[-\left(\frac{r}{R_0}\right)^2\right]. \]  

(5)

So the total effective potential is the sum of two parts

\[ V(r) = V_1(r) + V_2(r). \]  

(6)

With the effective mass approximation, the eigenstate of the Hamiltonian of the stationary electron in the system is

\[ H = \frac{1}{2m(P, T)} \left[-i\hbar \nabla + \frac{e}{c} \vec{A}\right]^2 + V(r). \]  

(7)

where $c$ is the speed of light, and $\vec{A}$ represents the vector potential, which can be expressed as $\vec{A} = (0, Br/2, 0)$ using the Coulomb criterion, so the above Hamiltonian can also be expressed as

\[ H = \frac{1}{2m(P, T)} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}\right) + \frac{1}{2} m(P, T) \left(\omega_0^2 + \frac{\alpha^2}{4}\right) r^2 + \frac{V_0 r^2}{R_0^2} - V_0 - i\hbar \omega_c \frac{\partial}{\partial \theta} + \frac{\hbar^2}{2m(P, T) r^2} \frac{\partial^2}{\partial \phi^2}. \]  

(8)

where $\Omega^2 = \left(\omega_0^2 + \frac{\alpha^2}{4} + \frac{2V_0}{m \omega_c R_0^2}\right)$ is the effective frequency, and $\omega_c = eB/m^*c$. This Hamiltonian model fully expresses two-dimensional quantum dots with single-carrier electrons, and its confinement potential conforms to the transverse electrostatic confinement of electrons on the x–y plane. The time-independent Schrödinger equation can be expressed as

\[ H\Psi = E\Psi, \]  

(9)

where $\Psi$ is the two-dimensional eigenstate of a pure quantum dot, $E$ is the corresponding eigenvalue, and the above formula is often written as

\[ \Psi(r, \theta) = R(r) e^{im\theta}, \]  

(10)
where \( m \) is the magnetic quantum number and is an integer. Substituting the above equation and the Hamiltonian expression into Eq. (9), and eliminating the same terms on the left and right sides of the equation, we get (Mo et al. 2020; Kumar et al. 2014)

\[
\left[ -\frac{\hbar^2}{2m^2} \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{m^2}{r^2} \right) + \frac{1}{2} \frac{m^* \Omega^2 r^2}{\hbar^2} - V_0 + \frac{\hbar \omega_c}{2} + \frac{\hbar^2}{2m^* r^2} \right] R(r) = ER(r). 
\]

(11)

For the convenience of calculation, further substitutions are made, so that

\[
R(r) = \frac{\chi(r)}{\sqrt{r}}.
\]

(12)

At this point, Eq. (11) becomes

\[
\frac{d^2 \chi(r)}{dr^2} + \left[ \frac{2m^* \lambda}{\hbar^2} - \frac{m^* \Omega^2 r^2}{\hbar^2} - \frac{l(l+1)}{r^2} \right] \chi(r) = 0,
\]

(13)

where \( \lambda = E + V_0 - m \hbar \omega_c / 2, l(l+1) = m^2 + \xi - 1/4 \), \( l \) is always a positive number, you can define \( l = \sqrt{m^2 + \xi - 1/4} \). For further solving, let \( r^2 = \Omega_1^2 z, l_m = \sqrt{m^2 + \xi + 1/4}, \Omega_1 = \sqrt{\hbar/m^* \Omega} \) and \( \Omega_2 = \lambda/2h\Omega \). After substituting Eq. (13), we get

\[
\frac{d^2 \chi(z)}{dz^2} + \frac{1}{2z} \frac{d \chi(z)}{dz} + \left[ \frac{l(m - \frac{1}{2})}{z^2} - \frac{\Omega_2}{z} \right] \chi(z) = 0.
\]

(14)

Since the wave function has asymptotic properties at the origin and infinity, it can be assumed

\[
\chi(z) = z^m e^{-\frac{\xi}{2}} W(z).
\]

(15)

Substitute Eq. (15) into Eq. (14), we can get

\[
z \frac{d^2 W(z)}{dz^2} + \left( 2l_m + \frac{1}{2} \right) - z \frac{dW(z)}{dz} - \left( l_m + \frac{1}{4} - \Omega_2 \right) W(z) = 0.
\]

(16)

Let \( \left( 2l_m + \frac{1}{2} \right) = b \) and \( \left( l_m + \frac{1}{4} - \Omega_2 \right) = a \) further simplify, the original equation becomes

\[
z \frac{d^2 W(z)}{dz^2} + (b - z) \frac{dW(z)}{dz} - a(z) = 0.
\]

(17)

The form of Eq. (17) conforms to Kummer differential equation, and the solution of this equation is a confluence hypergeometric function. In order to make \( \chi(z) \) finite, let \( a = -n \), and we can get

\[
W(-n, b; z) = \frac{\Gamma(1 + n)}{\Gamma(b + n)} \frac{\Gamma(b)}{\Gamma(b - n)} L_n^{(b-1)}(z),
\]

(18)

where \( \Gamma(x) \) is the Euler gamma function, and \( L_n^{(b-1)}(z) \) is the related Laguerre polynomial, so
\[ \chi(z) = z^{ln} e^{-\frac{z^2}{2\Gamma(b+n)} \frac{\Gamma(1+n)}{\Gamma(b+n)} I_n^{(b-1)}(z),} \] (19)

Simultaneous Eqs. (12) and (19) can be obtained

\[ R(r) = \frac{r^{2l_m - \frac{1}{2}}}{\Omega_1^2} e^{\frac{z^2}{2\Omega_1^2}} \frac{\Gamma(1+n)}{\Gamma(b+n)} L_n^{(2l_m-\frac{1}{2})} \left( \frac{r^2}{\Omega_1^2} \right), \] (20)

Therefore, the normalized intrinsic wave function of the quantum dot system can be obtained from Eq. (10) as

\[ \Psi_{mn}(r, \theta) = \sqrt{n! \pi \Gamma(2l_m + n + \frac{1}{2}) \Omega_1^{4l_m+1}} r^{(2l_m-\frac{1}{2})} e^{\frac{z^2}{2\Omega_1^2}} L_n^{(2l_m-\frac{1}{2})} \left( \frac{r^2}{\Omega_1^2} \right) e^{im\theta}, \] (21)

with

\[ a = -n, \]
\[ n = -l_m - \frac{1}{4} + \Omega_2, \]
\[ n = -l_m - \frac{1}{4} + \frac{E + V_0 - \frac{\hbar \omega_0}{2}}{2\hbar \omega} E = \left( 2n + 2l_m + \frac{1}{2} \right) \hbar \omega + \frac{\hbar \omega_0}{2} - V_0. \]

The energy eigenvalue of the quantum dot system is

\[ E_{mn} = \left( 2n + \sqrt{m^2 + \xi} + 1 \right) \hbar \omega + \frac{\hbar \omega_0}{2} - V_0. \] (22)

### 2.2 Optical rectification coefficient

Assuming that the system is excited by an electromagnetic field, the electromagnetic field can be expressed as (Li et al. 2008, 2020; Pal et al. 2019)

\[ E(t) = E_0 \cos(\omega t) = \tilde{E} e^{i\omega t} + \tilde{E} e^{-i\omega t}. \] (23)

The evolution of the density matrix operator \( \rho \) follows the following Schrödinger equation that varies with time

\[ \frac{\partial \rho_{ij}}{\partial t} = \frac{1}{i\hbar} \left[ H_0 - qzE(t), \rho \right]_{ij} - \Gamma_{ij} (\rho - \rho^{(0)})_{ij}, \] (24)

where \( \rho^{(0)} \) is the undisturbed density matrix operator, \( \Gamma_{ij} \) is the relaxation rate, and \( H_0 \) is the Hamiltonian of the system without electromagnetic field. The above formula can be calculated using an iterative method (Stevanović et al. 2019)

\[ \rho(t) = \sum_n \rho^{(n)}(t), \] (25)

with
\[
\frac{\partial \rho_{ij}^{(n+1)}}{\partial t} = \frac{1}{i\hbar} \left\{ [H_0, \rho^{(n+1)}]_{ij} - i\hbar \Gamma_{ij} (\rho - \rho^{(n+1)})_{ij} \right\} - \frac{1}{i\hbar} \left[ q_{ij}, \rho^{(n)} \right]_{ij} E(t). \tag{26}
\]

The electric polarization of the quantum dot due to the electromagnetic field \(E(t)\) can be expressed as
\[
P^{(n)}(t) = \left( \varepsilon_0 \chi^{(1)}_0 e^{i\omega_0 t} + \varepsilon_0 \chi^{(2)}_0 \left| \vec{E} \right|^2 + \varepsilon_0 \chi^{(2)}_{2\omega} e^{i2\omega_0 t} \right) + c.c., \tag{27}
\]
where \(\chi^{(1)}_0, \chi^{(2)}_0, \chi^{(2)}_{2\omega}\) are linear susceptibility, optical rectification, and second harmonic generation, respectively. \(\varepsilon_0\) is the vacuum dielectric constant. The n-th order electron polarization is
\[
P^{(n)}(t) = \frac{1}{V} Tr(\rho^n e z), \tag{28}
\]
where \(Tr\) represents the trace of the matrix, that is, the sum of the diagonal elements of the matrix \(\rho^n e z\), and \(V\) represents the volume of the system. Using the density matrix form and iterative method, the optical rectification coefficient of the system is obtained (Sedehi and Khordad 2021; Ungan 2017)
\[
\chi^2_0 = \frac{4\varepsilon^3 s_v M_{22}^2 \delta_{01}}{\varepsilon_0 \hbar^2} \frac{\omega_{10}^2 (1 + \Gamma_s \Gamma_1)}{\left( \omega_{10} - \omega \right)^2 + \Gamma_s^2} \frac{\Gamma_s}{\Gamma_1}, \tag{29}
\]
where \(s_v\) represents the electron density in the relative quantum dot, \(M_{ij} = \left| \langle \psi_i | z | \psi_j \rangle \right|\) is a non-diagonal matrix, and it reflects the coherence of the system between different states. \(\delta_{01} = |M_{22} - M_{11}|\), \(\omega_{ij} = \frac{E_i - E_j}{\hbar}\) is the transition frequency, \(\Gamma_k = \frac{1}{\tau_k}\) is the damping term related to the life of electrons in the transition.

3 Results and discussion

In this section, the following systematic analysis will be made on the optical rectification coefficients in the quantum dots tuned under the action of a vertical magnetic field. Mainly discuss the influence of vertical magnetic field \(B\), hydrostatic pressure \(P\), quantum dot radius \(R\), temperature \(T\), harmonic restraint frequency \(\omega_0\), limit potential depth \(V_0\) and dimensionless parameter \(\xi\) on the OR coefficient. The parameters used in the calculation are as follows: \(s_v = 5 \times 10^{24} \text{ m}^{-3}\), \(\varepsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}\), \(m_0 = 9.1 \times 10^{-31} \text{ kg}\), \(e = 1.602 \times 10^{-19} \text{ C}\), \(\Gamma = 5 \times 10^{12} \text{ Hz}\) (Yesilgul et al. 2014; Yu et al. 2018; Zhai 2014).

Table 1 shows the geometric factors and values for the various excited states. These numerical results are used to explain the peak amplitude and formant displacement of the nonlinear optical rectification coefficient (Duan et al. 2022). As shown in Fig. 1, the influence of the vertical magnetic field on the optical rectification coefficient of the tuned quantum dot is discussed. Take \(P = 12 \text{ kbar, } T = 400 \text{ K, } \xi = 1, \omega_0 = 30 \times 10^{12} \text{ THz, } R = 12 \text{ nm, } V_0 = 300 \text{ meV}\). The relationship between the optical rectification coefficient and the energy of the incident photon under the action of different vertical magnetic fields is made. The vertical magnetic field strength is respectively taken as \(B = 4, 6, 8 \text{ T}\). It can be seen from
Table 1  Geometric factors and excited states, first excited states, second excited states

| Parameter | Value | $M_{01} \times M_{01} \times \delta_{01}$ ($\times 10^{-26} \text{ m}^3$) | $E_0$ ($\times 10^{-20} \text{ J}$) | $E_1$ ($\times 10^{-20} \text{ J}$) | $E_2$ ($\times 10^{-20} \text{ J}$) |
|-----------|-------|-------------------------------------------------|----------------|----------------|----------------|
| $R$ (nm)  | 10    | 0.1943                                          | 0.648          | 3.372          | 6.096          |
|           | 12    | 0.4030                                          | −0.203         | 2.096          | 4.395          |
|           | 14    | 0.7466                                          | −0.802         | 1.197          | 3.196          |
| $B$ (T)   | 4     | 0.4235                                          | −0.261         | 2.008          | 4.277          |
|           | 6     | 0.4030                                          | −0.203         | 2.096          | 4.395          |
|           | 8     | 0.3768                                          | −0.122         | 2.217          | 4.557          |
| $P$ (kbar)| 0     | 0.4448                                          | −0.116         | 2.227          | 4.569          |
|           | 12    | 0.4030                                          | −0.203         | 2.096          | 4.395          |
|           | 24    | 0.3642                                          | −0.255         | 2.017          | 4.289          |
| $\xi$     | 0.5   | 0.5120                                          | −0.539         | 1.760          | 4.058          |
|           | 1.0   | 0.403                                          | −0.203         | 2.096          | 4.395          |
|           | 1.5   | 0.3372                                          | 0.056          | 2.354          | 4.653          |
| $\omega_0$ (THz) | 25 | 0.4376                                          | −0.343         | 1.885          | 4.113          |
|           | 30    | 0.4030                                          | −0.203         | 2.096          | 4.395          |
|           | 35    | 0.3704                                          | −0.051         | 2.323          | 4.697          |
| $V_0$ (meV) | 250 | 0.3519                                          | 0.363          | 2.544          | 4.726          |
|           | 300   | 0.4030                                          | −0.203         | 2.096          | 4.395          |
|           | 350   | 0.4521                                          | −0.793         | 1.611          | 4.014          |
| $T$ (K)   | 300   | 0.3768                                          | −0.012         | 2.222          | 4.456          |
|           | 400   | 0.4030                                          | −0.203         | 2.096          | 4.395          |
|           | 500   | 0.4305                                          | −0.347         | 2.000          | 4.347          |

Fig. 1  When $B=4, 6, 8$ T, the variation curve of optical rectification coefficient with incident photon energy
the figure that as the intensity of the magnetic field increases, the resonance peak of the OR coefficient moves toward the high-energy region, that is, "blue shift". The main reason is that with the increase of the magnetic field, the quantum confinement effect weakens, and the energy level interval $E_{ij}$ increases accordingly, which leads to the blue shift behavior. At the same time, it can also be found that as the photon energy increases, the peak value of the OR coefficient gradually decreases. This phenomenon is closely related to the density matrix. It can be concluded from Eq. (29) that the decrease of the density matrix $M_{ij}$ will affect the change of the OR coefficient.

As shown in Fig. 2, the influence of hydrostatic pressure $P$ on the optical rectification coefficient in the tuned quantum dot is discussed. Take $B=6$ T, $T=400$ K, $\xi=1$, $\omega_0=30 \times 10^{12}$ THz, $R=12$ nm, $V_0=300$ meV. The relationship between the optical rectification coefficient and the energy of the incident photon under the action of different hydrostatic pressures is made, and the hydrostatic pressures are respectively taken as $P=0$, 12 and 24 kbar. It can be seen from the figure that as the hydrostatic pressure $P$ increases, the resonance peak of the OR coefficient moves toward the low-energy region, that is, a "red shift" occurs. The physical reason is that the hydrostatic pressure has an effect similar to reducing the constraint, which increases the effective mass of electrons, which in turn leads to a decrease in the strength constraint. The physical origin of this "redshift" behavior is that the energy difference between the ground state and the first excited state changes by changing the hydrostatic pressure. The energy difference $E_{10}$ between the ground state and the first excited state decreases with the increase of hydrostatic pressure. This can be seen from Table 1. Therefore, by increasing the hydrostatic pressure, the resonance peak of the OR coefficient moves to the low-energy region and "redshifts". At the same time, it can be observed from the graph of the change of the OR coefficient with the incident photon energy that the peak value of the OR coefficient will decrease with the increase of the hydrostatic pressure. These effects can be easily used by the separation energy $E_{21}$ between the first and second states. According to Eqs. (2)–(3), this is because the effective mass of
electrons increases with the increase of external pressure. Therefore, the "red shift" of photon energy with increasing pressure can be used as an adjustment parameter to obtain the required transition energy.

As shown in Fig. 3, the influence of the radius of the quantum dot on the optical rectification coefficient in the tuned quantum dot is discussed. Take $B = 6 \, \text{T}$, $T = 400 \, \text{K}$, $\xi = 1$, $\omega_0 = 30 \times 10^{12} \, \text{THz}$, $P = 12 \, \text{kbar}$, $V_0 = 300 \, \text{meV}$. A graph of the relationship between the OR coefficient and the energy of the incident photon under the action of different quantum dot radii is made. The quantum dot radii are respectively taken as $R = 10$, $12$ and $14$ nm. It can be seen from the figure that as the radius of the quantum dot increases, the resonance peak of the OR coefficient moves toward the low-energy region, that is, a "red shift" occurs. It can be seen from Eq. (4) that an increase in the radius of the quantum dot will cause the modified Gaussian potential to decrease, resulting in a smaller quantum confinement effect, and the energy level spacing of the confined electrons in the quantum dot will decrease, so the resonance peak of the OR coefficient moves to the lower energy direction, that is, a "red shift" occurs. At the same time, it can be seen from the Figure that the peak value of or coefficient increases with the increase of radius, because the matrix element increases with the increase of quantum dot radius $R$, which can be calculated from the Table 1. From the point of view of physical origin, the limit potential energy will be greatly reduced. In fact, what is reduced is the energy required for conversion between subband levels, and the state overlap will increase. Therefore, a larger OR coefficient can be obtained.

As shown in Fig. 4, the influence of temperature on the optical rectification coefficient in the tuned quantum dot is discussed. Take $B = 6 \, \text{T}$, $R = 12 \, \text{nm}$, $\xi = 1$, $\omega_0 = 30 \times 10^{12} \, \text{THz}$, $P = 12 \, \text{kbar}$, $V_0 = 300 \, \text{meV}$. A plot of the change of OR coefficients with the incident photon energy at different temperatures is made, and the temperatures are respectively taken as: $T = 300$, $400$, $500 \, \text{K}$. It can be seen from the figure that as the temperature increases, the resonance peak of the OR coefficient moves toward the high-energy region, that is, "blue shift". From Eqs. (2)–(3), it can be seen that as the temperature increases, the effective

![Graph showing OR coefficient variation with photon energy](image)

**Fig. 3** When $R = 10$, $12$, $14$ nm, the variation curve of optical rectification coefficient with incident photon energy
mass of the electron decreases, and the energy difference between the ground state and the first excited state becomes larger. Therefore, the photon energy moves toward the high-energy region as the temperature increases. It can be seen from the figure that the peak value of the OR coefficient increases with the increase of temperature. From a numerical point of view, the increase of temperature leads to the decrease of electron effective mass. At the same time, the size of the wave propagation function of the barrier element is reduced, which will reduce the peak value of the wave propagation function of the barrier element and reduce the size of the barrier element. From the perspective of physical origin, the limiting potential energy will be reduced for large. In fact, what is reduced is the energy required for the transition between subband levels, and the overlap of states will increase. Thus, larger OR coefficient can be obtained.

As shown in Fig. 5, the influence of limiting the potential depth $V_0$ on the optical rectification coefficient in the tuning quantum dot is discussed. Take $B = 6$ T, $R = 12$ nm, $\xi = 1$, $\omega_0 = 30 \times 10^{12}$ THz, $P = 12$ kbar, $T = 400$ K. The relationship between the OR coefficient and the incident photon energy under the action of different limiting potential depths is made, and the limiting potential depths are respectively taken as: $V_0 = 250, 300, 350$ meV. It can be seen from the figure that as the limiting potential depth $V_0$ increases, the resonance peak of the OR coefficient moves toward the high-energy region, that is, "blue shift". As the limiting potential depth $V_0$ increases, the energy level interval increases, which leads to blue shift behavior. It can be seen from the figure that the peak value of the OR coefficient increases with the increase of the limit potential depth $V_0$. According to Eqs. (1), (11) and (29), the energy level interval $E_{ij}$ increases as $V_0$ increases, so that the peak value of the OR coefficient shows an increasing trend. From Eqs. (4)–(5), it can be seen that MGP in the limited potential energy is only directly affected by $V_0$ and $R_0$. From Figs. 3 and 5, it can be seen that $V_0$ and $R_0$ have opposite effects on the OR coefficient. This effect includes the peak and resonance peak of the coefficient. Therefore, MGP has good control ability for this type of quantum dots. In addition, this potential model has an
accurate representation of the energy level of zero angular momentum and can be used to simulate the slowly changing limit potential. In short, because MGP is flexible enough to be applied to the realistic limit potential of quantum dots, it has a certain reference value for the design and synthesis experiments of low dimensional semiconductor nano materials.

Fig. 5 Variation curve of optical rectification coefficient with incident photon energy when $V_0 = 250, 300, 350$ meV

Fig. 6 When $\omega_0 = 25, 30, 35$ THz, the variation curve of optical rectification coefficient with incident photon energy
As shown in Fig. 6, the influence of the confinement wave frequency on the optical rectification coefficient in the tuned quantum dot is discussed. Take $B = 6$ T, $R = 12$ nm, $\xi = 1$, $V_0 = 300$ meV, $P = 12$ kbar, $T = 400$ K. The relationship between the optical rectification coefficient and the incident photon energy under the action of different confinement wave frequencies is made. The confinement wave frequency $\omega_0$ is taken as follows: $\omega_0 = 25, 30, 35$ THz. It can be seen from the figure that as the frequency of the confinement wave increases, the resonance peak of the OR coefficient moves toward the high-energy region, that is, "blue shift". The physical source of this displacement is the quantum confinement effect in these nanostructures, which causes energy level separation, and the weaker the confinement effect, the more obvious the separation phenomenon. It can be seen from Eq. (1) that with the increase of $\omega_0$, the binding potential $V_1(r)$ will also increase, and the quantum confinement effect will increase, and the energy level spacing of the confined electrons in the quantum dot will increase, so the resonance peak of the OR coefficient moves to the high-energy direction, that is, "blue shift" occurs.

As shown in Fig. 7, the influence of the dimensionless parameter $\xi$ on the coefficient of light rectification in the tuned quantum dot is discussed. Take $B = 6$ T, $R = 12$ nm, $\xi = 1$, $\omega_0 = 30 \times 10^{12}$ THz, $V_0 = 300$ meV, $T = 400$ K. The relationship between the OR coefficient and the energy of the incident photon under the action of different parameters is made, and the dimensionless parameter $\xi$ is respectively taken as: $\xi = 0.5, 1, 1.5$. It can be seen from the figure that the peak value of OR coefficient decreases gradually with the increase of parameter value, because the increase of parameters will lead to the decrease of matrix elements. At the same time, it can be concluded from Eq. (29) that the decrease of matrix elements leads to the decrease of OR coefficient peak.

![Fig. 7](image-url)  
**Fig. 7** When $\xi = 0.5, 1, 1.5$, the variation curve of optical rectification coefficient with incident photon energy
4 Conclusion

The energy level and wave function are derived by effective mass approximation and parabola approximation, and then the analytical expression of optical rectification coefficient is obtained by density matrix theory and iterative method. Numerical results show that the peak value of OR coefficient increases with the increase of quantum dot radius and decreases with the increase of dimensionless parameters and vertical magnetic field. In addition, the increase of limiting potential depth, temperature and constrained wave frequency will move the formant of OR coefficient towards high energy. With the increase of hydrostatic pressure, the resonance peak of OR coefficient moves to the direction of low energy. Finally, we hope that our research can have some impact on the improvement of practical devices and the practical exploration of quantum size effect in devices.

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Author contributions ZZ: Software, validation, data processing, writing-original. XL: Data curation, methodology, supervision, reviewing draft, financial support. YD: Discussing the results, validation, data processing. CC: Revising the manuscript. LZ: Discussing the results.

Data availability All data generated or analysed during this study are included in this published article.

Declarations

Conflict of interest We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product.

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