Vanishing of black hole tidal Love numbers from scattering amplitudes

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We extract the black hole (BH) static tidal deformability coefficients (Love numbers) and their spin-0 and spin-1 analogs by comparing on-shell amplitudes for fields to scatter off a spinning BH in the worldline effective field theory (EFT) and in general relativity (GR). We point out that the GR amplitudes due to tidal effects originate entirely from the BH potential region. Thus, they can be separated from gravitational non-linearities in the wave region, whose proper treatment requires higher order EFT loop calculations. In particular, the elastic scattering in the near field approximation is produced exclusively by tidal effects. We find this contribution to vanish identically, which implies that the static Love numbers of Kerr BHs are zero for all types of perturbations. We also reproduce the known behavior of scalar Love numbers for higher dimensional BHs. Our results are manifestly gauge-invariant and coordinate-independent, thereby providing a valuable consistency check for the commonly used off-shell methods.

Introduction. — The worldline point-particle effective field theory for gravitational wave sources is a modern toolbox for precision waveform calculations [1--4]. The increasing interest to these calculations is fueled by recent discoveries of gravitational waves from black hole (BH) binaries [5]. In the EFT each compact object in a binary is approximated as a point particle at leading order, while the finite-size effects are captured by the higher-derivative operators on the worldline. These operators generate multipolar corrections to the point mass potential. The most general worldline theory for a perturbed spherically symmetric body at leading order in derivatives of the long-wavelength metric field is given by

\begin{equation}
S_{pp} = -m \int d\tau + c_E \int d\tau E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau B_{\mu\nu}B^{\mu\nu},
\end{equation}

where \( m \) is the mass of a compact body, \( E_{\mu\nu} \) and \( B_{\mu\nu} \) are the electric (parity even) and magnetic (parity odd) parts of the Weyl tensor.\textsuperscript{1} The dimensionful Wilson coefficients \( c_{E,B} \) measure the gravitational response of the body to external tidal fields in the quadrupolar sector. A simple response calculation shows that the action (1) generates the following Newtonian potential \( \Phi_N \) [6--9] (schematically),

\begin{equation}
\Phi_N = \frac{m}{r} - \sum_{m=-2}^{2} \mathcal{E}_{2m} r^2 \left( 1 - \frac{c_E}{r_s^5} \right),
\end{equation}

where \( \mathcal{E}_{2m} \) are tidal harmonic coefficients, we assumed a long distance limit and ignored the magnetic part for simplicity. This expression coincides with a classic Newtonian definition of the static tidal response, which is controlled by the static deformability coefficients, “Love numbers” [10, 11]. Thus, \( c_{E,B} \) provide us with a gauge-invariant definition of Love numbers in general relativity (GR). In order to extract the Love numbers for a particular object, one needs to match some quantity calculated in the full theory (GR) and the EFT.

The Love numbers capture details of body’s internal structure, e.g. they depend on the equation of state for neutron stars [12]. For BHs, however, the application of the EFT leads to a surprising result. Matching the EFT and GR calculations for BH static perturbations implies that their Love numbers vanish identically in GR in four dimensions [6, 8, 9, 11, 13, 14]. This is in sharp contrast with the dimensional analysis (Wilsonian naturalness), suggesting that \( c_{E,B} \sim r_s^5 \), where \( r_s \) is the Schwarzschild radius. The vanishing of the EFT Wilson coefficients for BHs has been a major puzzle in BH physics for many years [15], see [16--19] for proposals addressing this problem.

The vanishing of black hole Love numbers is however, still a controversial topic. The problem is that this result is obtained by comparing certain static field pro-
files in GR with the corresponding EFT off-shell one-point functions that are calculated in a particular coordinate system. In theory, this should not be a problem as one is free to choose any quantity for matching and the result should not depend on whether this quantity is gauge-invariant or not. In practice, however, the matching in specific gauges may be difficult, and often leads to conflicting conclusions, see e.g. a recent debate in the literature on whether the Love numbers vanish for Kerr black holes [9, 14, 20, 21], and discussions on interference between the tidal effects and Post-Newtonian corrections [6, 9, 22–24]. This leaves room for doubt about results of the off-shell Love number extraction, see e.g. [25, 26].

To avoid any confusion, it is desirable to match the EFT and GR by comparing manifestly gauge invariant quantities, such as amplitudes to elastically scatter off a BH geometry in GR versus an S-matrix element in the EFT. One can easily estimate that the Love operator contributes to the graviton-BH cross-section at order \( \sim \omega^8 r_s^2 \sim \omega^8 r_s^{10} \). Since the EFT is valid when \( r_s \omega \ll 1 \), this term is much smaller than the leading long-range (Newtonian) contribution \( \sim r_s^2 \), and also than relativistic corrections to that contribution. The total cross-section should be of the form, schematically

\[
\sigma_{\text{GR}}(\omega) = r_s^2 \left( 1 + (r_s \omega)^2 + ... \right) + r_s^{10} \omega^8 .
\]

The smallness of the finite-size contribution makes it hard to extract. At face value, one needs to compute the full GR cross-section to the fifth Post-Minkowski (PM) order in black hole perturbation theory (BHPT). In addition, one needs to calculate the EFT amplitude at the same 5PM, which is a four loop calculation in the EFT nomenclature, see [27–31] for recent progress in these calculations, including tidal effects.

In this Letter we show that in fact, there is a consistent approximation to BHPT, where the total cross-section is given exclusively by the finite-size operators. Using this approximation, the matching with the EFT can be performed entirely at the tree level.

**General EFT for Kerr black holes.**—A general astrophysical relevant BH is described by the Kerr metric [32]. The Kerr black hole has two horizon, \( r_{\pm} = M \pm \sqrt{M^2 - a^2} \), where \( M \) and \( a \) denote BH mass and spin. Within the EFT, the worldline action inherits symmetries of the underlying gravitational background, i.e. the EFT for spinning black holes must have axial symmetry. In the context of static tides it is sufficient to incorporate this fact by promoting the Love numbers in (1) to tensors \([9, 33]\), i.e. considering the following EFT for metric perturbations,

\[
S_{\text{finite size}} = \int \! d\tau \, \lambda^{ij}_r E^{i'j'} E^{i'j'} + \text{magnetic} , \quad \text{(4)}
\]

where we have switched to the BH rest frame, and \( \lambda^{ij}_r \) is a symmetric real matrix. The above local EFT coupling only captures conservative effects. In order to account for dissipation we introduce the coupling between the long-wavelength tidal field and the composite mass quadrupole \( Q_{ij}(X) \) [2, 24, 33],

\[
S_{\text{diss}} = \int \! d\tau \, Q_{ij}(X) E^{ij} + \text{magnetic} , \quad \text{(5)}
\]

where \( X \) are gapless degrees of freedom on the horizon. Actions (4) and (5) can be used to compute, e.g. a potential contribution to the Weyl curvature scalar for \( \ell = 2 \) \([9]\) (schematically),

\[
\psi^{(\ell=2)}_0 \sim 2 Y_{\ell m}(\theta, \phi) E_{2m} \left( 1 - \frac{k_{2m}^{(2)} + ik_{2m}^{(2)}}{r^5} \right) ,
\]

where \( 2 Y_{\ell m} \) is the spin-weighted spherical harmonic, \( k_{2m}^{(2)} \) is a real coefficient related to \( \lambda_{ij,i'j'} \), whist \( \psi^{(2)} \) is a real number generated by the time-reversal odd (non-conservative) part of the retarded correlator \( \langle Q_{ij} Q_{i'j'} \rangle_{\text{ret}} \) \([9]\). The factor in front of the \( r^{-5} \) term in Eq. (6) is called the response coefficient. Note that its real part captures the conservative effects (Love numbers), while the imaginary part is responsible for BH absorption (dissipation numbers) \([9, 14]\).

The local action (4) can be generalized to the case of a generic test field with an angular multipole \( \ell \) and a positive integer spin \( s \). The most general leading order EFT action needed to reproduce the effect of static Love
numbers and their spin-0 and spin-1 analogs is given by
\[
\sum_{\ell=s} \frac{1}{2\ell!} \int d\tau \left[ \lambda^{(0)}_{L}(\ell) \partial(\ell) \phi^{(L')} \phi \\
+ \lambda^{(1)}_{L}(\ell) \partial(\ell) E_{i_1} \partial(\ell') E_{i_1}^r \\
+ \lambda^{(2)}_{L}(\ell) \partial(\ell) E_{i_1} \partial(\ell') E_{i_1}^r \right],
\]

(7)

where we have omitted the magnetic contributions for brevity, \(L = i_1 \ldots i_\ell\) is the multi-index, \(\partial_{\ell} \equiv \partial_{i_1} \ldots \partial_{i_\ell}\), \((L)\) denotes the trace-free part, \(E^s\) and \(\phi\) are test electric and scalar fields.

The Wilson tensors \(\lambda^{(s)}_{L'}\) have previously been extracted by comparing the static EFT off-shell one-point functions like (6) with BHPT calculations, which implied that all such tensors vanish identically [9, 14]. In contrast, the dissipation numbers \(\nu^{(2)}_{\ell m}\) we found to be non-zero [21]. These results, however, were obtained in a coordinate-dependent fashion. Now we show how the Love numbers and dissipation numbers can be extracted from the on-shell scattering and absorption cross-sections in a gauge-invariant manner.

**Scattering off a Kerr black hole in general relativity.** — The scattering by rotating black holes is an old and well studied subject [34–36]. We present only few essential elements in this Letter and leave other details for future work [37]. Consider a wave of a spin-\(s\) test field impinging along the axis of a Kerr BH. The scattering cross-section is given by
\[
\frac{d\sigma}{d\Omega} = |f_+(\theta)|^2 + |g_+(\theta)|^2,
\]

(8)

where \(\theta\) is the scattering angle, \(f_+(\theta)\) and \(g_+(\theta)\) are helicity-conserving and helicity-reversing amplitudes, respectively. Their partial wave expressions are given by

\[
\begin{align*}
\left\{ f_{\ell s}(\theta) \right\} &= \frac{\pi}{i\omega} \sum_{P=\pm 1} \sum_{\ell=s} \left\{ -sS^P_{\ell s}(\theta, a\omega) \right. \\
&\quad \left. -sS^P_{\ell s}(\pi - \theta, a\omega) \right\} \delta P(-1)^{\ell+s} \\
&-sS^P_{\ell s}(0, a\omega)[\eta^P_{\ell s}(2i\delta^P_{\ell s}) - 1],
\end{align*}
\]

(9)

where \(B^{(inc)}_{-\ell m}\) and \(B^{(refl)}_{-\ell m}\) are complex constants. The phase shifts are given by
\[
\eta^P_{\ell s} e^{2i\delta^P_{\ell s}} = (-1)^{\ell+1} \frac{A_s}{(2\omega)^{2s}} B^{(refl)}_{-\ell m} B^{(inc)}_{-\ell m},
\]

(11)

where \(A_s\) are normalization factors related to the Starobinsky-Teukolsky constants [37].

The constants \(B^{(inc)}_{-\ell m}\) and \(B^{(refl)}_{-\ell m}\) should be extracted from the solution of the appropriate Tekolsky equations. Mano, Suzuki, and Takasugi (MST) have constructed such solutions in a systematic low-frequency expansion [40, 41]. The solution takes the form of an infinite series over hypergeometric functions in the near zone (potential) region, and of an infinite series of Coulomb wavefunctions in the far zone (wave) region. \(B^{(inc)}_{-\ell m}\) and \(B^{(refl)}_{-\ell m}\) are determined by matching these two series. The MST solution is a linear combination of modes labeled by the “renormalized” angular momentum \(\nu = \ell + O(\epsilon^2)\), where \(\epsilon \equiv 2M\omega\) is the PM expansion parameter, and \(\ell \geq s\) is the usual integer angular momentum number.

The partial wave amplitude in the sector \(\nu\) and \(s\), for-
The logarithms then indicate the mixing between loops and finite-size operators in the EFT, which complicates the matching with the GR results. Strictly speaking, a full calculation is needed in this case. However, since logarithms should be present both in the EFT and UV calculations, one may identify the scheme-independent (running) part of the Love numbers from the GR solution. In what follows we will see that for four dimensional Kerr BH scattering there are no logs and thus no mixing, and hence the matching between the EFT and GR is unambiguous.

Scattering in the near zone approximation.— Let us neglect the PM terms completely and compute the cross-section entirely from the near zone term. This amounts to using an approximate BHPT solution obtained through the leading order matching of the potential and radiation regions [14, 44–46]. At this order one matches the first order near zone solution with the zeroth order far zone solution (describing a free motion with an angular momentum $\ell$) in an overlapping region where both solutions are valid. Note that this approximation is unacceptable from the practical point of view as it misses the leading order contributions in (3). However, it is perfectly suitable for our goal to extract the finite size effects.

In the leading near zone approximation the parity-even and parity-odd phase shifts take the same expressions and hence the helicity reversing amplitude vanishes.

\[
\frac{B^{(\text{ref})}_{s_m}}{B^{(\text{inc})}_{s_m}} = \frac{1}{\omega^2 s} \left( 1 + ie^{i\pi \nu} K_{\nu-1} K_{\nu} + \frac{A_{\nu} e^{i\epsilon (2 \ln \epsilon - 1 - \kappa)}}{A_{\nu}} \right),
\]

(12)

where $\kappa = \sqrt{1 - a^2/M^2}$, and $A_+, A_-, K_\nu, K_{-\nu-1}$ are some $\epsilon$-dependent coefficients. Crucially, the above expression factorizes into two distinctive contributions. The first term (in blue) comes from the matching between the potential and the wave region expansions. It is sourced by the perturbed gravitational potential in the near zone and hence contains information about finite-size effects encoded in the multipole expansion of BH perturbations. The second term (in red) stems from the solution of the Teukolsky equation in the wave zone. By construction, it contains PM terms due to non-linearity of gravity but no information about the BH finite size structure. Crucially, this separation holds only in linear BHPT.

The interpretation of Eq. (12) within the EFT is not straightforward. We may associate the two terms in Eq. (12) with two sets of EFT diagrams shown in Fig. 1. The wave zone term captures the scattering of on-shell gravitons off the BH geometry. Naively, it is this part that maps onto the EFT PM loop expansion. Indeed, if we ignore the potential zone contribution and retain the 1PM corrections from the radiation zone, we recover the Newtonian answer, e.g.

\[
\frac{d\sigma^\text{GR}}{d\Omega} \bigg|_{1\text{PM}} = \frac{M^2}{\sin^4\left(\frac{\theta}{2}\right)} \left( \cos^8\left(\frac{\theta}{2}\right) + \sin^8\left(\frac{\theta}{2}\right) \right),
\]

(13)

for a Schwarzschild BH and for the spin-2 field. This result can be reproduced with the tree-level EFT diagrams analogous to the one shown in Fig. 1 [37, 42, 43]. In contrast, the near field contribution in Eq. (12), physically, incorporates finite-size effects. Specifically, the first order near zone expansion should capture the static Love number contribution. The relativistic $\epsilon$- corrections to this result correspond to the graviton-dressed Love number diagrams and frequency-dependent local worldline operators $\sim \lambda_{t(\omega z)} E^2_{ij}$ (“dynamical Love numbers”) [9, 24], shown in Fig. 1. These appear starting at second order in the near zone expansion.

The above arguments suggest that the separation between the near zone and far zone contributions should hold in the EFT. This may not be true if EFT loop diagrams produce logarithmic divergences that should be renormalized by the Love number. The logarithms then indicate the mixing between loops and finite-size operators in the EFT, which complicates the matching with the GR results. Strictly speaking, a full calculation is needed in this case. However, since logarithms should be present both in the EFT and UV calculations, one may identify the scheme-independent (running) part of the Love numbers from the GR solution. In what follows we will see that for four dimensional Kerr BH scattering there are no logs and thus no mixing, and hence the matching between the EFT and GR is unambiguous.

3 Here we employ dimensional regularization, where non-logarithmic divergence vanish identically.

4 For general renormalized angular momentum $\nu$ there are no logarithms in the near zone part of GR amplitudes, and hence there is no ambiguity. The logarithmic corrections appear only in the limit $\nu \to \ell \in \mathbb{N}$. This is akin to dimensional regularization.

5 This solution can be recovered as the $\epsilon \to 0$ limit of Eq. (12).
\( \delta_{\ell s} \sim (\text{near zone part}) + (\text{far zone part}) \)

\[
\begin{align*}
\delta_{\ell s} &= \lambda_{\ell}^s + \lambda_{\ell}^s + \lambda_{\ell}(\omega^2) + \cdots \\
&+ (\text{near zone part}) + (\text{far zone part})
\end{align*}
\]

FIG. 1. Diagrammatic interpretation of the elastic scattering amplitude in the EFT and its representation in terms of the near zone and far zone parts in GR. Curly lines stand for off-shell gravitons. The vertical straight lines depict the worldline. External legs correspond to external spin-\( s \) on-shell fields. The upper diagrams stem from the EFT finite-size action. The lower diagrams describe the scattering of the on-shell fields off background potential modes in the far region.

Assuming that the scattering is perturbative, the total cross-section in the partial wave approach can be decomposed into the elastic and absorptive contributions:

\[
\sigma_{\text{elastic}} = \frac{4\pi}{\omega^2} \sum_{\ell=s}^{\infty} (2\ell + 1) \sin^2 \delta_{\ell s},
\]

\[
\sigma_{\text{abs}} = \frac{\pi}{\omega^2} \sum_{\ell=s}^{\infty} (2\ell + 1) \left(1 - \eta_{\ell s}^2\right).
\]

Performing a matching calculation, we get

\[
\eta_{\ell s} e^{2i\delta_{\ell s}} = 1 + i(-1)^s \frac{(\ell + s)![(\ell - s)!]}{(2\ell)![(2\ell + 1)!]} \left(2\omega(r_+ - r_-)\right)^{2\ell + 1} \Im I_{s\ell s},
\]

where \( I_{s\ell m} \) is the harmonic near zone response function \([9, 14]\),

\[
I_{s\ell m} = i(-1)^{s+1} P_+ \frac{(\ell + s)![(\ell - s)!]}{(2\ell)![(2\ell + 1)!]} \prod_{j=1}^{\ell} (j^2 + 4P_+^2),
\]

and \( P_+ = \frac{am - 2r_+\omega}{r_+ - r_-} \). With Eq. (15), we get

\[
\eta_{\ell s} = 1 - (-1)^s \frac{(\ell + s)![(\ell - s)!]}{(2\ell)![(2\ell + 1)!]} \left(2\omega(r_+ - r_-)\right)^{2\ell + 1} \Im I_{s\ell s},
\]

\[
\delta_{\ell s} = \frac{1}{2}(-1)^s \frac{(\ell + s)![(\ell - s)!]}{(2\ell)![(2\ell + 1)!]} \left(2\omega(r_+ - r_-)\right)^{2\ell + 1} \Re I_{s\ell s}.
\]

Since \( \Im I_{s\ell s} = 0 \) for Kerr BHs, we conclude that the scattering phase shift is zero for all spins \( s \) and multipoles \( \ell \), to all orders of the BH spin,

\[
\sigma_{\text{elastic},\ell}^{\text{GR}} = 0.
\]

Comparing this with the EFT result from the local action (7) we conclude that all static Kerr Love numbers vanish identically, in agreement with previous off-shell calculations \([9, 14]\).

As far as absorption is concerned, it is straightforward to see from Eq. (14) that

\[
\sigma_{\text{abs},\ell} = \frac{2(-1)^s \pi}{\omega^2} \frac{(\ell + s)![(\ell - s)!]}{(2\ell)![(2\ell + 1)!]} \left(2\omega(r_+ - r_-)\right)^{2\ell + 1} \Im I_{s\ell s}.
\]

This generalizes the result of \([46]\) for the \( \ell = s \) case. Note that the imaginary part of the response coefficient generates the absorption cross-section and is directly linked with the EFT dissipation numbers \((6)\) \([9, 14, 24, 33]\).

**Love numbers of higher dimensional BHs.**—Love numbers do not vanish in general for BHs in spacetimes with a number of dimensions \( d \) greater than four \([6, 8]\). Let us perform their explicit matching from near zone scattering amplitudes. For simplicity, we focus on scalar fluctuations of higher dimensional static BHs. The corresponding phase shift is given by

\[
\eta_{\ell} e^{2i\delta_{\ell}} = 1 + \frac{2^{1-d-2\ell}(r_+ \omega)^{d+2\ell}}{\Gamma(\frac{d}{2} + \ell)} \Gamma(1 + \frac{d}{2} + \ell) I_{\ell},
\]

where \( \hat{d} \equiv d - 3, \hat{\ell} \equiv \ell/\hat{d}, \) and the response function

\[
I_{\ell} = \frac{\Gamma(-2\hat{\ell} - 1)\Gamma(1 + \hat{\ell})\Gamma(1 + \hat{\ell} - \frac{2ir_+ \omega}{\hat{d}})}{\Gamma(-\hat{\ell})\Gamma(2\hat{\ell} + 1)\Gamma(-\hat{\ell} - \frac{2ir_+ \omega}{\hat{d}})}.
\]
From this equation, we get
\[ \eta_\ell = 1 - \frac{2^{1-d-2\ell}\pi(r_+\omega)^{d+2\ell}}{\Gamma\left(\frac{d}{2} + \ell\right)\Gamma\left(1 + \frac{d}{2} + \ell\right)}\text{Im}I_\ell, \]
\[ \delta_\ell = \frac{2^{1-d-2\ell}\pi(r_+\omega)^{d+2\ell}}{\Gamma\left(\frac{d}{2} + \ell\right)\Gamma\left(1 + \frac{d}{2} + \ell\right)}\text{Re}I_\ell. \]
(22)

On the EFT side, the worldline action
\[ S_{\text{finite size}} = \int d\tau \sum_{\ell=0}^{\infty} \frac{\lambda_{\ell=0}}{2\ell!}(\partial_{(\ell)}\phi)^2 \]
yields the following tree-level scattering amplitude in the orbital sector \( \ell \),
\[ i T = i\lambda_{\ell=0} \omega^{2\ell} \frac{\ell!(d-2)!}{(2\ell + d - 2)!} P_{\ell}^{(d)}(\cos \theta), \]
(24)

where \( P_{\ell}^{(d)} \) are generalized Legendre polynomials. Comparing this with the phase shift from the UV theory (22), we obtain
\[ \lambda_{\ell=0} = (-1)^\ell \frac{\pi}{2^{d-2}} \frac{\Gamma\left(\frac{5-d}{2} - \ell\right)}{\Gamma\left(\frac{2}{2} - \ell\right) \Gamma\left(\frac{d}{2} - \ell\right)} r^{2\ell+d-3} \text{Re}I_{\ell}, \]
(25)

which identically coincides with the result of off-shell matching from [8]. Note that according to Eq. (21) the Love numbers vanish if \( \ell \) is an integer, exhibit the classical renormalization group running if \( \ell \) is half integer, and are constant numbers \( O(\pi^{2+d-3}) \) otherwise, in full agreement with the off-shell results [6, 8].

Conclusions. — We have matched on-shell amplitudes of massless fields scattering off BHs in the worldline EFT and the near zone approximation of BHPT. We have found that the static tidal Love numbers vanish for four dimensional spinning (Kerr) black holes to all orders of the black hole spin. Our results are gauge-invariant, and hence they remove any uncertainty as to the validity of the previous results based on the coordinate-dependent off-shell matching.

We have also reproduced the known behavior of the spin-0 Love numbers of BHs in a general number of dimensions. This is an important consistency check of our approach and of the former off-shell calculations. We stress that the mapping between the EFT and GR potential and wave regions does not work if Love numbers run logarithmically. Nevertheless, we can still extract the scheme-independent part from the full GR solution in this case [37].

Two facts are key to the matching of elastic cross-sections: the realization that finite-size effects originate from the BH near region, and the absence of logarithms. These have allowed us to bypass both a full calculation of four loop corrections in the EFT, and the construction of the Teukolsky equation solution at the 5PM order. It would be interesting to extend our results to the time-dependent local worldline couplings (“dynamical Love numbers”), which are non-zero for four dimensional Kerr black holes [9]. These correspond to a second order near zone approximation. It is also important to understand the implications of the Love symmetry for on-shell observables. Indeed, the Love symmetry explains the vanishing of Love numbers in the off-shell calculations as a result of an algebraic constraint [16]. It would be curious to see if this constraint manifests itself at the level of on-shell scattering amplitudes.

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