Universality and the sparticle spectrum

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We calculate the particle spectrum of the SSM which follows from the assumption that the commonly assumed universal form of the soft supersymmetry-breaking terms is invariant under renormalisation. It is argued that this “strong” universality might be approached as an infra–red fixed point for the unified theory above the unification scale.
1. Introduction

The unification of the gauge couplings at $M_G \approx 10^{16}$ GeV has been responsible for a much increased interest in the supersymmetric standard model (SSM). It is commonly assumed that the supersymmetry-breaking terms unify likewise, and so are determined ultimately by only four parameters: $m_0$, $A$, $B$ and $M$, which we will define presently. There have been many attempts to justify this universal form for the soft breaking in terms of $N = 1$ supergravity, with or without an underlying string theory. In some scenarios the parameters turn out to be related, so that the soft terms may be characterised by as few as one or two parameters.

At what scale does unification of the soft breakings take place? In view of their gravitational origin a first guess would place this scale at the Planck mass ($10^{19}$ GeV); most analyses, however, assume that soft unification holds, at least to a good approximation, at the gauge unification scale. In fact, in explicit models the soft unification may occur at some intermediate scale, but it seems not unreasonable to explore the consequences of locating it near or at $M_P$. One may then expect model–dependent deviations from universality at $M_G$, and the question is whether these deviations will significantly impact low energy predictions. This program has been pursued recently in Ref. [1], with the conclusion that there can indeed be a quite substantial effect on the sparticle spectrum due to the evolution between $M_P$ and $M_G$.

In a recent paper[2], two of us approached this issue from a different point of view. We asked whether there existed any theories such that universality (in the sense described above) is a renormalisation group invariant property of the theory. Were the unified effective field theory valid for scales between $M_G$ and $M_P$ to have this property, then clearly universality at $M_P$ would imply universality at $M_G$. We found, remarkably, that this strong universality is a property of a class of softly broken theories which satisfy one simple relation among the dimensionless coupling constants. Moreover, the soft breakings are all determined by the gaugino mass, $M$, and their relationships to each other bear intriguing similarity to analogous relations in certain string–based theories. Even if the relations implied by our constraints were not exact properties of the fundamental theory, they might still arise to a good approximation in the infra–red limit at $M_G$, from a more general class of theories at higher scales, and thus still be relevant at low energies. In this paper we pursue the phenomenological consequences of this idea, for the SSM.
2. Universality

The essential results from Ref. [2] are as follows. We start with a supersymmetric theory whose Lagrangian $L_{\text{SUSY}}(W)$ is defined by the superpotential

$$ W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j. \quad (2.1) $$

$L_{\text{SUSY}}$ is the Lagrangian for the $N = 1$ supersymmetric gauge theory, containing the gauge multiplet $\{A_\mu, \lambda\}$ ($\lambda$ being the gaugino) and a chiral superfield $\Phi_i$ with component fields $\{\phi_i, \psi_i\}$ transforming as a (in general reducible) representation $R$ of the gauge group $G$. We assume for simplicity that there are no gauge-singlet fields and that $G$ is simple. (The generalisation to a semi-simple group is trivial.)

The soft breaking is incorporated in $L_{\text{SB}}$, given by

$$ L_{\text{SB}} = (m^2)^i_j \Phi^i \Phi^j + \left( \frac{1}{6} h^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} b^{ij} \Phi_i \Phi_j + \frac{1}{2} M \lambda \lambda + \text{h.c.} \right) \quad (2.2) $$

(Here and elsewhere, quantities with superscripts are complex conjugates of those with subscripts; thus $\Phi^i \equiv (\Phi_i)^\ast$.)

Our fundamental hypothesis is that the dimensionless couplings of the unified theory satisfy the constraint

$$ P^i_j = \frac{1}{3} g^2 Q \delta^i_j. \quad (2.3) $$

where

$$ Q = T(R) - 3C(G), \quad \text{and} \quad P^i_j = \frac{1}{2} Y^{ikl} Y_{jkl} - 2g^2 C(R)^i_j. \quad (2.4) $$

Here

$$ T(R) \delta_{AB} = \text{Tr}(R_AR_B), \quad C(G) \delta_{AB} = f_{ACD} f_{BCD} \quad \text{and} \quad C(R)^i_j = (R_AR_A)^i_j, \quad (2.5) $$

where the $f_{ABC}$ are the structure constants of $G$.

If we impose Eq. (2.3), then the following relations among the soft breakings are renormalisation group invariant through at least two-loops:

$$ h^{ijk} = -MY^{ijk}, \quad (2.6a) $$

$$ (m^2)^i_j = \frac{1}{3} (1 - \frac{1}{16\pi^2} \frac{2}{3} g^2 Q) M M^\ast \delta^i_j, \quad (2.6b) $$

$$ b^{ij} = -\frac{2}{3} M \mu^{ij}. \quad (2.6c) $$
The fact that these relations are preserved under renormalisaton subject only to the simple constraint of Eq. (2.3) requires a miraculous sequence of cancellations among contributions from the various β–functions; for a discussion, see Ref. [2].

In the usual SSM notation, Eq. (2.6) corresponds to a universal scalar mass $m_0$ and universal $A$ and $B$ parameters related (to lowest order in $g^2$) to the gaugino mass $M$ as follows:

$$m_0 = \frac{1}{\sqrt{3}} M, \quad (2.7a)$$

$$A = -M, \quad (2.7b)$$

$$B = -\frac{2}{3} M. \quad (2.7c)$$

Remarkably, relations of this form can arise in effective supergravity theories motivated by superstring theory, where supersymmetry breaking is assumed to occur purely via vacuum expectation values for dilaton and moduli fields [3] [4]. Ignoring string loop corrections and possible phases for the auxiliary fields $F^S$ and $F^T$, where $S$ is the dilaton and $T$ is the overall modulus, then according to Ref. [4],

$$m_0^2 = \frac{1}{3} M^2 + \frac{2}{3} \frac{M^2}{C^2 \sin^2 \theta} (C^2 - 1) \quad (2.8a)$$

$$A = -M \quad (2.8b)$$

and

$$B = -\frac{M(1 + \sqrt{3} \sin \theta + C \cos \theta)}{\sqrt{3} C \sin \theta} \quad (2.9)$$

or

$$B = \frac{2M}{\sqrt{3} C \sin \theta}, \quad (2.10)$$

where Eqs. (2.9) and (2.10) apply according to whether the μ–term is generated by an explicit μ–term in the supergravity superpotential, or by a special term in the Kähler potential.

Here $C$ is related to the vacuum expectation value of the scalar potential and a vanishing cosmological constant corresponds to $C = 1$. $\theta$ is called the goldstino mixing angle, and the values $\theta = 0$ and $\theta = \pi/2$ correspond to modulus–dominated and dilaton–dominated cases respectively. It is easy to see that with $C = 1$, Eqs. (2.8a) and (2.8b) reproduce Eqs. (2.7a) and (2.7b), and for $\theta = 4\pi/3$, Eq. (2.9) gives Eq. (2.7d).
Another particular case that has been subject to some phenomenological investigation \cite{5,6} is that of $C = 1$ and $\theta = \pi/2$ in Eq. (2.10). We will refer to this case as the $DD$ (dilaton-dominated) scenario. It again corresponds to Eq. (2.7) except that now $B = 2M/\sqrt{3}$. We shall see that this difference has considerable impact. The similarity between the conditions on the soft-breaking terms which arise from our strong universality hypothesis and those that emerge from string theory is certainly intriguing.

There is, however, an alternative interpretation of the above results. Consider a unified theory where it would be possible to find Yukawa couplings satisfying Eq. (2.3). The fact that Eqs. (2.3) and (2.7) are renormalisation group invariant is of course equivalent to saying that they are fixed points of the evolution equations; fixed points, moreover, that are approached in the infra-red, at least in simple examples. At first sight it might appear that the difference between $M_P$ and $M_G$ is insufficient to allow significant evolution, but it has recently been argued\cite{7} that in the case of the Yukawa couplings the evolution towards the fixed point occurs more rapidly in the unified theory than in the low energy theory. If we believe that this conclusion holds also for the soft terms, then it is possible to argue that for a wide range of input parameters the boundary conditions of Eq. (2.7) might hold at $M_G$.

Our philosophy now is as follows. We assume that the SSM is valid below gauge unification, and that the unified theory satisfies Eq. (2.3). We then proceed to impose Eq. (2.7a−c) as boundary conditions at the gauge unification scale. These boundary conditions are so restrictive that it is not a priori obvious that a phenomenologically viable solution will exist.

3. The supersymmetric standard model

We start with the superpotential:

$$W = \mu_s H_1 H_2 + \lambda_t H_2 Q\bar{t} + \lambda_b H_1 Q\bar{b} + \lambda_\tau H_1 L\bar{\tau}$$  \hspace{1cm} (3.1)$$

where we neglect Yukawa couplings except for those of the third generation.

The Lagrangian for the SSM is defined by the superpotential of Eq. (3.1) augmented with soft breaking terms as follows:

$$L_{SSM} = L_{SUSY}(W) + L_{SOFT}$$  \hspace{1cm} (3.2)$$
where

\[ L_{\text{SOFT}} = -m_1^2 H_1^* H_1 - m_2^2 H_2^* H_2 + [m_3^2 H_1 H_2 + \text{h.c.}] \]

\[ - \sum_i \left( m_Q^2 |Q_i|^2 + m_L^2 |L_i|^2 + m_T^2 |T_i|^2 + m_H^2 |H_i|^2 + m_T^2 |T_i|^2 \right) \]

\[ + [A_t \lambda_t H_2 Q + A_b \lambda_b H_1 Q + A_\tau \lambda_\tau H_1 L + \text{h.c.}] \]

\[ - \frac{1}{2} [M_1 \lambda_1 \lambda_1 + M_2 \lambda_2 \lambda_2 + M_3 \lambda_3 \lambda_3 + \text{h.c.}] \]

and the sum over \( i \) for the \( m^2 \) terms is a sum over the three generations.

The running coupling and mass analysis of the above theory has been performed many times. The novel feature here is the very restricted set of boundary conditions at gauge unification:

\[ m_1 = m_2 = m_Q = m_L = m_T = m_\tau = m_\bar{\tau} = \frac{1}{\sqrt{3}} M, \]

\[ A_\tau = A_b = A_t = -M, \]

\[ M_1 = M_2 = M_3 = M, \]

\[ m_3^2 = -\frac{2}{3} \mu_s M, \]

where Eq. (3.4d) includes the squarks and sleptons of all three generations. Notice that these boundary conditions satisfy the constraint

\[ \Delta = m_1^2 + m_2^2 + 2\mu_s^2 - 2|m_3^2| > 0 \]

for any \( \mu_s \). We require this (at \( M_G \)) to keep the potential bounded from below, in other words so that \( SU_2 \otimes U_1 \) breaking does not occur with characteristic scale \( M_G \). (See Eq. [A.2].) It is interesting that in the \( DD \) scenario, one obtains

\[ \Delta = 2 \left( \frac{M}{\sqrt{3}} - \mu_s \right)^2. \]

With Eq. (2.10) and a value of the goldstino angle \( \theta \) other than \( \frac{\pi}{2} \), one would need to check that \( \mu_s(M_G) \) indeed gave \( \Delta > 0 \.)

Our procedure is as follows. We input \( \alpha_1, \alpha_2, \alpha_3, m_t \) and \( \tan \beta \) at \( M_Z \), and calculate the unification scale \( M_G \) (defined as the meeting point of \( \alpha_1 \) and \( \alpha_2 \)) by running the dimensionless couplings. Then we input the gaugino mass \( M \) at \( M_G \), and run the dimensionful parameters (apart from \( m_3^2 \) and \( \mu_s \)) down to \( M_Z \). We can then determine \( m_3^2 \) and \( \mu_s^2 \) as usual at \( M_Z \) by minimising the (one–loop corrected) Higgs potential. Then
we run $m_3^2$ and $\mu_s$ back up to $M_G$ (for the two possibilities of sign $\mu_s$) and calculate $B' = B/M = (m_3^2)/(M\mu_s)$. By plotting $B'$ against the input value of $\tan \beta$ we can then determine whether (for a given input $M$) there exists a value of $\tan \beta$ such that Eq. (3.14) is satisfied. Given a set $M, \tan \beta$ satisfying our boundary conditions we can calculate the sparticle spectrum in the usual way and plot the resulting masses against $M$. See the appendix for some comments about the $\beta$–functions and mass matrices.

At this stage we are chiefly interested in demonstrating that phenomenologically viable solutions are possible with our highly restricted boundary conditions. Consequently we ignore threshold corrections to the mass predictions (for a recent discussion of threshold corrections see, for example, Ref. [8]). Nor do we address here the recent concerns [8][9] relating to the apparent incompatibility (in a precision calculation) of the experimental value of $\alpha_3(M_Z)$ and the value required for gauge unification (note that the solution proposed in Ref. [9], to wit non–unified gaugino masses, is not available to us).

We do, however, incorporate the one–loop corrections into the minimisation of the Higgs potential.† In general we have done this by solving the Higgs tadpole equations, but we also checked our results by numerically minimising the Higgs potential in some cases. (Our results for the Higgs tadpoles agree with Ref. [11], apart from one or two minor typos.) We also do include one loop corrections to the mass ($m_h$) of the lighter CP–even Higgs boson, since, as is well known, the radiative corrections are important in this case[12]. More precisely, our results for $m_h$ and $m_H$ are based on the appropriate second derivative of the one–loop corrected effective potential evaluated with the scale $\mu$ set equal to the gaugino mass $M$. While this is crude compared to existing calculations, it incorporates the most important logarithmic effects. Our results for other masses are based on the tree mass matrices but again with all running parameters evaluated at the scale $M$.

Since the two–loop corrections to the $\beta$–functions are now available [13]–[16], we incorporate these. In general their effect is very small, being most noticeable in the Higgs sector; although the mass of the lightest Higgs is essentially unchanged, the other Higgs masses are increased by up to 10% by the two loop corrections. Of course for precise predictions, we should also include threshold corrections, as indicated above.

We use input parameters at $M_Z$ as follows:

$$\alpha_1 = 0.0169, \quad \alpha_2 = 0.0337, \quad \alpha_3 = 0.11$$

$$m_\tau(M_Z) = 1.75 \text{ GeV}, \quad m_b(M_Z) = 3.2 \text{ GeV}, \quad m_t(M_Z) = 170 \rightarrow 200 \text{ GeV}. \quad (3.7)$$

† The necessity for doing this was first pointed out in Ref. [10]
The appropriate input \( m_b(M_Z) \) depends itself on the sparticle spectrum in general, as emphasised recently \cite{17}. Varying \( m_b(M_Z) \) does not, however, affect the qualitative nature of our results.

4. Discussion of the results: \( m_t = 175 \) GeV

In this section we consider in detail the case \( m_t(M_Z) = 173 \) GeV which corresponds to a pole mass \( m_t \approx 175 \) GeV.

Fig. (1) plots \( B' \) against \( \tan \beta \) for \( M = 200 \) GeV and a pole top mass of 175 GeV.

![Fig.1: The \( B' \)-parameter vs. \( \tan \beta \) for input gaugino mass \( M = 200 \) GeV and \( m_t = 175 \) GeV. The solid and dashed lines correspond to \( \mu_s > 0 \) and \( \mu_s < 0 \) respectively. The required value of \( B' \) is obtained for \( \tan \beta \approx 18 \).](image)

Now since we want \( B' = -\frac{2}{3} \), we might expect (with our conventions) to find a solution with \( \mu_s < 0 \) rather than \( \mu_s > 0 \). (This is because for a tree minimum at the weak scale
we necessarily would have $m_3^2 > 0$.) We see, in fact, that with $\mu_s < 0$ we do indeed get $B' < 0$ throughout; but unfortunately, $B' = -\frac{2}{3}$ is not possible for any $\tan \beta$. Surprisingly, the situation is better with $\mu_s > 0$, and we have the desired result for $\tan \beta \approx 18$. For the $DD$ scenario, when the desired result is $B' = \frac{2}{\sqrt{3}}$, notice that the solution (had it existed) would have been for $\mu_s > 0$ and in the small $\tan \beta$ region. This solution is vulnerable to the existence of the well–known Landau pole in the top mass Yukawa, at $m_t \approx 195 \sin \beta$.

Thus Fig. (1) is consistent with the conclusions of Ref. [6], which quotes an upper limit on $m_t$ of 155 GeV for the $DD$ case. (Note that Ref [3] has the opposite sign for $\mu_s$.) This means that the strict $DD$ scenario is ruled out by the recent measurements of $m_t$ [18], though of course the general string–based framework for the origin of the soft terms, in which $B'$ is a free parameter, is not compromised.

In Fig. (2) we plot $\tan \beta$ against the input gaugino mass, $M$, having satisfied Eq. (3.4d). ($M$ is related to the gluino mass $M_g(M_Z)$, by the relation $M_g(M_Z) \approx 2.4M$, but note that, especially for large $M$, the gluino pole mass can differ considerably from $M_g(M_Z)$).

![Fig. 2: $\tan \beta$ vs. $M$ for $m_t = 175$ GeV.](image)
As already mentioned, we find comparatively large values of $\tan \beta$, except for small $M$. As is well known, successful bottom–tau Yukawa unification favours a large top Yukawa coupling \cite{13} \cite{20}, and so we do not obtain it within our approach, at least within our approximation. This conclusion is sensitive to the input $m_b(M_Z)$, which is in turn affected by radiative corrections (especially that involving the gluino) which will not be negligible for $\tan \beta \approx 18$. At first sight, however, these corrections take us further from $b$–$\tau$ unification. This point deserves further investigation, but we are in any case not too concerned, however, since $b$–$\tau$ unification is a model dependent phenomenon.

Fig. (3) plots the Higgs masses against the gaugino mass.*

* In Fig. 3 (and Figs. 4, 5) $\tan \beta$ changes with the gaugino mass in accordance with Fig. 2.
The one–loop radiative corrections, which we have included, raise $m_h$ above the tree bound $m_h < |M_Z \cos 2\beta|$. Our result for $m_h$ is dependent only weakly on the input gaugino mass, with $m_h \approx 115$ GeV. This is consistent with the generally accepted bound $m_h \leq 135$ GeV (or $m_h \leq 146$ GeV in more general models[21]).

For $M = 150$ GeV we have $M_{h,H} = 116,257$ GeV and $m_{A,H\pm} = 246,259$ GeV. It is interesting to compare these results with those obtained if one–loop $\beta$–functions are used throughout, which are $M_{h,H} = 116,242$ GeV and $m_{A,H\pm} = 225,239$ GeV; so the corrections to $m_{A,H\pm}$ are $O(10\%)$. The masses of the sparticles are in general less affected by using two–loop rather than one–loop $\beta$–functions; typically a sparticle mass changes by 5% or so.

Fig. (4) plots the neutralino masses against the gaugino mass.

![Neutralino masses vs input gaugino mass](image)

*Fig.4: The neutralino masses vs. $M$ for $m_t = 175$ GeV. The solid and dashed lines correspond to the Higgsino–dominated neutralinos, and the dotted and dot–dashed lines to the gaugino–dominated neutralinos.*
Except for small gaugino masses, the lightest neutralino is the lightest superpartner. For $M = 150$ GeV, for example, we have $m_{\chi_4}^0 \approx 55$ GeV which is potentially interesting as cold dark matter. Of course the precise $\chi$ relic density is controlled by the $\chi\chi$ annihilation cross–section, so we need to investigate this to test this hypothesis. (For a review of particle physics dark matter candidates, see, for example, Ref.\cite{22}). Fig. (5) plots the $\tau$–slepton masses against the gaugino mass.

![Graph](image)

*Fig.5: The $\tau$-slepton masses vs. $M$ for $m_t = 175$ GeV. The solid and dashed lines correspond to $\tilde{\tau}_{1,2}$ and the dotted line is the $\tilde{\nu}_\tau$. At $M = 150$ GeV we have $M_{\tilde{\tau}_{1,2}} \approx 156, 80$ GeV.*

It will be apparent that the plots presented thus far exhibit linear behaviour for a wide range of input gaugino masses. Rather than give more figures, we therefore summarise our results in Table 1, which gives a good approximation (within a few GeV) for $100$ GeV $< M < 500$ GeV.
With $m_t = 185$ GeV, the dependence of $B'$ on $\tan \beta$ and the resulting sparticle spectrum are very similar. For $m_t \geq 190$ GeV, there is a change, which we discuss in the next section; but we give results for $m_t = 190$ GeV here as well, for simplicity.

| $m_t$ | 175 | 185 | 190 |
|-------|-----|-----|-----|
| $m = aM + b$ | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ |
| $m_h$ | 0.048 | 108 | 0.059 | 108 | 0.070 | 106 |
| $m_{H^0}$ | 1.613 | 15 | 1.800 | 7 | 1.870 | 5 |
| $m_A$ | 1.585 | 8 | 1.782 | 4 | 1.855 | 2 |
| $m_{H^\pm}$ | 1.555 | 25 | 1.755 | 20 | 1.829 | 17 |
| $m_{\tilde{e}_1}$ | 0.872 | 12 | 0.873 | 12 | 0.874 | 11 |
| $m_{\tilde{e}_2}$ | 0.666 | 12 | 0.667 | 12 | 0.668 | 12 |
| $m_{\tilde{\nu}_e}$ | 0.930 | -22 | 0.930 | -21 | 0.930 | -21 |
| $m_{\tilde{\nu}_1}$ | 0.830 | 31 | 0.852 | 22 | 0.861 | 18 |
| $m_{\tilde{\nu}_2}$ | 0.615 | -11 | 0.644 | 1 | 0.657 | 5 |
| $m_{\tilde{\nu}_e}$ | 0.903 | -21 | 0.917 | -21 | 0.923 | -20 |
| $m_{\tilde{\tau}_1}$ | 1.527 | 48 | 1.580 | 46 | 1.601 | 45 |
| $m_{\tilde{\tau}_2}$ | 0.793 | -21 | 0.799 | -23 | 0.805 | -25 |
| $m_{\tilde{\nu}_e}$ | 1.532 | 44 | 1.583 | 44 | 1.603 | 45 |
| $m_{\chi^0_1}$ | 1.566 | 22 | 1.622 | 20 | 1.645 | 18 |
| $m_{\chi^0_2}$ | 0.789 | -19 | 0.793 | -20 | 0.797 | -21 |
| $m_{\chi^0_3}$ | 0.410 | -7 | 0.413 | -8 | 0.417 | -9 |
| $m_{\tilde{u}_1}$ | 2.264 | 26 | 2.266 | 26 | 2.269 | 26 |
| $m_{\tilde{u}_2}$ | 2.189 | 29 | 2.191 | 30 | 2.194 | 30 |
| $m_{\tilde{d}_1}$ | 2.245 | 37 | 2.247 | 37 | 2.251 | 37 |
| $m_{\tilde{d}_2}$ | 2.175 | 33 | 2.177 | 33 | 2.180 | 33 |
| $m_{\tilde{t}_1}$ | 1.829 | 143 | 1.849 | 143 | 1.861 | 142 |
| $m_{\tilde{t}_2}$ | 1.645 | 0 | 1.615 | 18 | 1.609 | 27 |
| $m_{\tilde{b}_1}$ | 2.040 | 56 | 2.113 | 46 | 2.142 | 42 |
| $m_{\tilde{b}_2}$ | 1.963 | 20 | 1.992 | 28 | 2.009 | 30 |

*Table 1: Linear approximations of the form $m = aM + b$ to the mass spectrum for $m_t = 175$ GeV, $m_t = 185$ GeV and $m_t = 190$ GeV.*
We will not perform a detailed analysis of our predictions vis-a-vis current experimental limits; in more general cases many treatments exist (for a recent example, see Ref. [23]). It is clear enough that these will impose a lower bound on $M$ of around 100 GeV, and that for say, $M \approx 150$ GeV we have acceptable phenomenology, with a stable neutralino at 55 GeV, a $\tau$-slepton at 80 GeV, and the light Higgs at 115 GeV.

5. The large $m_t$ region

For large top masses (in the region $m_t \geq 190$ GeV) the nature of the solutions we find to our universality constraints changes in an interesting way. We find that for $\mu_s < 0$ the dependence of $B'$ on $\tan \beta$ ceases to be monotonic and that for a given input gaugino mass there may be three possible values of $\tan \beta$ that give $B' = -\frac{2}{3}$. This behaviour is shown in Fig. 6, for $M = 150$ GeV.

![B' vs tan beta](image)

Fig. 6: The $B'$-parameter vs. $\tan \beta$ for input gaugino mass $M = 200$ GeV and $m_t = 190$ GeV. The solid and dashed lines correspond to $\mu_s > 0$ and $\mu_s < 0$ respectively.

In fact, however, the existence of the two solutions at $\tan \beta \approx 3.6$ and 4.0 depends on our use of the two-loop $\beta$-functions for the dimensionless couplings; if we use the
corresponding one loop ones they do not exist because of the Landau pole in the top Yukawa coupling. They are therefore unreliable, and we ignore them. For the solution at \( \tan \beta \approx 8 \), the spectrum is similar to that described in the last section, and is shown in Table 1, in the previous section. For \( m_t > 195 \text{ GeV} \) we are unable to satisfy Eq. (2.7c) and retain perturbative unification.

6. Conclusions

We have shown that the restrictions imposed by the conjecture of strong universality at \( M_G \) leave a viable and well determined supersymmetric phenomenology. The main new feature of the resulting spectrum is the determination (for given input gaugino mass \( M \)) of \( \tan \beta \). Although this occurs also in the \( DD \) approach, the results for the two cases are readily distinguished. Since (given \( m_t \)) the mass spectrum depends on a single parameter, \( M \), it is clear that the discovery of supersymmetric particles would swiftly decide whether our marriage of universality with the minimal SSM corresponds to reality.

It would obviously be nice if we could construct a unified theory that satisfied (or approached in the infra–red limit at \( M_G \)) our strong universality hypothesis as encapsulated in Eq. (2.3) and (2.6). In this connection, it is worth observing that Eq. (2.3) permits gauge groups with \( U_1 \) factors (in contrast to the finite case, \( P = Q = 0 \)). Then the conditions Eq. (2.6) still suffice for a universal theory as long as we have also that

\[
\text{Tr } (R_A m^2) = m^2 \text{ Tr } R_A = 0 \quad (6.1)
\]

which is the condition that the theory be free of gravitational anomalies \[24\]. In the light of this remark, a theory based on “flipped” \( SU_5 \) (\( SU_5 \otimes U_1 \) – see for example Ref. \[25\]) might be worth a try, though the direct product nature of this case may also pose problems.

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Appendix A. The beta–functions and mass matrices

In this appendix we make a few comments about the $\beta$–functions and sparticle mass matrices for the SSM, as defined in Eq. (3.2).

The one–loop $\beta$–functions and the mass matrices appear in many papers, and the two–loop $\beta$–functions are readily deduced from the results of Ref. [14] (or somewhat less readily from those of Ref [13] and [15]). These $\beta$–functions are calculated in a “hybrid” regularisation scheme, intermediate, in a sense, between dimensional regularisation and dimensional reduction. The raison d’être of the scheme is to remove annoying dependence on $\epsilon$–scalar masses. The nature of the scheme must be taken into account in the calculation of threshold corrections, as explained in Ref [16].

Although, as stated above, the one–loop results have been often reproduced, we feel it worthwhile emphasising the following point. There are various possible conventions for signs, in particular of $\mu_s$ and $M$, and it is important, of course, that the choices made in the $\beta$–functions are consistent with those made in the mass matrices. We have verified all “sensitive” sign choices by means of the identity

$$\text{STr} \ M^4 = 32\pi^2 \left[ \sum_\lambda \beta_\lambda^{(1)} \frac{\partial}{\partial \lambda} - \sum_{i=1,2} \gamma_{H_i}^{(1)} \frac{\partial}{\partial v_i} \right] V_0(v_1, v_2)$$  \hspace{1cm} (A.1)

where $V_0$ is the effective potential in the tree approximation, i.e.

$$V_0 = \frac{1}{2}(m_1^2 + \mu_s^2)v_1^2 + \frac{1}{2}(m_2^2 + \mu_s^2)v_2^2 - m_3^2v_1v_2 + \frac{1}{32}(g^2 + g'^2)(v_1^2 - v_2^2)^2.$$  \hspace{1cm} (A.2)

Eq. (A.1) follows from the renormalisation group equation for the effective potential. The set $\{\lambda\}$ consists of $\{m_1^2, m_2^2, m_3^2, \mu_s, g, g'\}$. The two anomalous dimensions $\gamma_{H_i}^{(1)}$ are the one–loop anomalous dimensions for the background scalar fields $H_{1,2}$ in the (quantum field) Landau gauge, given in a general covariant gauge by

$$16\pi^2 \gamma_{H_1}^{(1)} = \lambda_7^2 + 3\lambda_b^2 - \frac{1}{4}(1 + \alpha)(3g^2 + g'^2)$$

$$16\pi^2 \gamma_{H_2}^{(1)} = 3\lambda_t^2 - \frac{1}{4}(1 + \alpha)(3g^2 + g'^2).$$  \hspace{1cm} (A.3)

As usual $\alpha = 0$ is the Landau gauge. Note that for $\alpha = 1$ (the Feynman gauge), $\gamma_{H_{1,2}}^{(1)}$ are identical to the corresponding anomalous dimensions for the chiral superfields, in a supersymmetric gauge. For completeness, we note that the corresponding anomalous dimensions for the quantum scalar fields, $\tilde{\gamma}_{H_{1,2}}^{(1)}$, are given by

$$16\pi^2 \tilde{\gamma}_{H_1}^{(1)} = \lambda_7^2 + 3\lambda_b^2 - \frac{1}{4}(1 - \alpha)(3g^2 + g'^2)$$

$$16\pi^2 \tilde{\gamma}_{H_2}^{(1)} = 3\lambda_t^2 - \frac{1}{4}(1 - \alpha)(3g^2 + g'^2),$$  \hspace{1cm} (A.4)

differing only in the sign of the gauge parameter term.
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