Are Domain Walls ruled out ?

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Recent analysis of the combined data of cosmic microwave background, galaxy clustering and supernovae type Ia observations have set strong constraints on the equation of state parameter $w_X$. The upper bound $w_X < -0.82$ at 95% c.l. rules out an important class of models, the domain walls ($-2/3 < w_X < -1/3$). Here we revisit the issue of domain walls as a possible alternative to the standard Λ-CDM model by questioning the assumptions made in the choice of priors of the data analysis. The results of our investigation show that domain walls can provide a good fit to the WMAP data for a different choice of priors with “lower” values of the Hubble parameter ($h < 0.65$), (as indicated by Sunyaev-Zeldovich and time delays for gravitational lensing observations), and “higher” values of the matter density ($\Omega_m > 0.35$), (in agreement with recent measurements of the temperature-luminosity relation of distant clusters observed with the XMM-Newton satellite). In this new perspective, their existence would lead to important implications for the CMB constraints on cosmological and inflationary parameters.

1. INTRODUCTION.

The recent results of precision cosmology and the measurements of Cosmic Microwave Background Anisotropies have been extremely important since they provide an excellent agreement of our theoretical picture of the cosmos, incorporating the standard model of structure formation, the inflationary prediction of flatness, the presence of cold dark matter and an amount of baryonic matter consistent with Big Bang Nucleosynthesis constraints (see e.g. [1], [2]). The price-tag of this success story of the combined observations of CMB with complementary cosmological data concerns a very puzzling consequence: the evolution of the universe is dominated by a mysterious form of energy, $X$, coined dark energy, (an unclustered negative pressure component of the mass-energy density), with a present-day energy density fraction $\Omega_X \sim 2/3$ and equation of state $w_X \sim -1$ (see e.g. [3], [4], [5]). This discovery may turn out to be one of the most important contribution to physics in our generation. Hence it is especially important to consider all possible scheme for dark energy.

A true cosmological constant $\Lambda$ may be at works here. Hence it is entirely possible that a dynamic mechanism is giving rise to the observed acceleration of the present Universe. Some of the popular proposed candidates to explain the observations are a slowly-rolling scalar field, “quintessence” [6], [7], or a “k-essence” scalar field with non-canonical kinetic terms in the Lagrangian [8]-[11], and string-inspired models such as the contribution of nonlinear short distance physics to vacuum energy [8], and modified Friedman equations at late time [9] or large distances [10].

Dark energy can also receive contributions from topological defects produced at phase transitions in the early universe (see e.g. [12]).

However, despite a well established theoretical framework, topological defects have not been thoroughly explored due to technical difficulties in the numerical simulations. Moreover, cosmic fluids with $w_X < 0$ have an imaginary sound speed $c_s$ which causes diverging instabilities on small scales incompatible with structure formation.

More recently, a plausible version of dark energy made of a frustrated network of domain walls was proposed by (27, 28). In these “solid dark matter” models (see also 26), a negative equation of state can avoid the short-length instabilities by an elastic resistance to pure shear deformations. Structure formation is therefore preserved and CMB anisotropies are affected only on very large angular scales ($\ell \lesssim 20$).

These models have several appealing features: Firstly, domain walls are ubiquitous in field theory and unavoidable in models with spontaneously broken symmetries. Second, the scale of spontaneous symmetry breaking responsible for the walls is expected to lie in the $10 - 100 \text{ KeV}$ range and can arise naturally in supersymmetric theories (27). In this respect, the domain wall models of dark energy seem much more natural than the quintessence models which assume the existence of a scalar field with a mass of order...
10^{-33}eV. Finally, two firm phenomenological predictions can be made for domain walls models: an equation of state strictly $-1/3 \geq w_X \geq -2/3$ (23) and a sound speed which can be a fraction of the speed of light i.e. $c_s \leq 1$ (27). These models are therefore predictive in the value of the equation of state parameter and distinguishable from a cosmological constant even at zero order on $w_X$, (while, for example, scalar field models can also produce $w_X \sim -1$ although they differ from a cosmological constant which in the first order variation has $\dot{w}_X = 0$).

However, recent combined analyses of CMB, galaxy clustering and SN-Ia luminosity distances data, have constrained $w_X < -0.82$ at 95% C.L. (1,29,30) and therefore seem to rule out domain walls. It is important to notice that the upper bounds on $w_X$ were obtained under the assumption of a specific choice of priors namely the popular values for the cosmological parameters in agreement with the concordance standard model. Therefore the following questions are fully justified: how model independent are the results of our data analysis and, are we yet ready to abandon domain walls? In this brief report we investigate the impact of the priors on the upper bound of $w_X$ by choosing a different data set. Then we argue that a different choice of the priors can bring domain walls models in reasonable agreement with observations. While the final value of the Hubble constant from the HST Key Project is $h = 0.72 \pm 0.02 \pm 0.07$ (18), where the first error is statistical and the second is systematic, other groups using similar techniques (see e.g. 14, 16, 17, 18) find a lower value $h \sim 0.60$. Measurements based on Sunyaev-Zeldovich method (see e.g. 24 but see also 21) and on time delays for gravitational lenses (22,23) are also suggesting a lower value $h \sim 0.5$, at least globally. It is therefore plausible that the true value of $h$ lies in the lower range allowed by the HST Key project. This is in contrast with the WMAP constraint $h = 0.73 \pm 0.03$ (1), derived under the assumption of $\Lambda$CDM. As we will see, a value of the Hubble parameter $h \leq 0.65$ combined with the WMAP data allows a case to be made for domain walls models.

Moreover, in the past years, the abundance of high redshift X-ray selected clusters has been argued to lead to high values of the matter density parameter $\Omega_m$ (see e.g. 19). In particular, analyses of the recent measurements of the temperature-luminosity relation of distant clusters observed with XMM-Newton and Chandra satellites, seem to be consistent with higher values of $\Omega_m \sim 0.8$ (24). Although such high values for a $\Omega_m \sim 1$ are definitely extreme and need to be considered in combination with other data, it is conceivable that the true value of $\Omega_m$ may lie in a range $\Omega_m \sim 0.35 - 0.45$. Again, this is in tension with the WMAP constraint $\Omega_m = 0.27 \pm 0.04$ (1) derived under the assumption of $\Lambda$CDM. As we will see in the next section, domains walls models are clearly accommodated within the WMAP data when a prior $\Omega_m \geq 0.35$ is assumed.

Finally, $c_s < 1$, offers the advantage of reducing the amplitude of the large-scale CMB anisotropies, as reported by the WMAP recent CMB data, while input from the otherwise unknown new physics of the initial conditions of the Universe is required to bring the standard $\Lambda-CDM$ model in agreement with the WMAP findings, (15-39).

II. ANALYSIS

As is well known (see e.g. 48 and 29) a geometrical degeneracy makes virtually impossible any determination of $\Omega_X$ and $w_X$ from the position of the acoustic peaks in the CMB anisotropy spectrum. However, if one restrict the analysis to flat models, a change in $\Omega_X$ must be necessarily compensated by a change in the matter density $\Omega_m = 1 - \Omega_X$. Since for a perfect degeneracy between the CMB peaks one has also to preserve the physical densities in cold dark matter $\Omega_{cdm} h^2$ and baryons $\Omega_b h^2$, the Hubble parameter needs also to vary.

![FIG. 1: Top- Comparison of the $\Lambda$-CDM and Domain Walls best fit models with 1-st year WMAP CMB data. Bottom- Comparison of the $\Lambda$-CDM and Domain Walls best fit models with SLOAN and 2dF galaxy surveys data.](image-url)
that, on linear scales, this model provides a reasonable fit to the present data, and one as good as Λ-CDM.

We study the $w_X - h$ degeneracy more quantitatively in Fig.2 where we plot the WMAP likelihood contours on those 2 parameters. The likelihood contours have been computed as in [29] and include also the ACBAR and CBI datasets. As we can see, there is a clear degeneracy along the $w_X + h = constant$ direction. Moreover, models with $h \leq 0.65$ are excluded at about 2-$\sigma$ from the WMAP+ACBAR+CBI data in the case of Λ-CDM ($w_X = -1$) while models with $h > 0.7$ are excluded at 2-$\sigma$ in the case of domain walls ($-2/3 \leq w_X \leq -1/3$). If one takes at face value the constraint $h = 0.57 \pm 0.03$ from [10] this yields $w_X \geq -0.78$ at 1-$\sigma$. We can therefore conclude that while the HST determination is consistent with $h \sim 0.65$, this is not the case for the WMAP constraint under the assumption of Λ-CDM. An higher value for $w_X$ can solve the discrepancy.

Since the CMB spectrum provides an independent constraint on $\Omega_m h^2$ we can expect a degeneracy between the equation of state parameter $w_X$ and $\Omega_m$. We show this in Fig.3 where we plot the WMAP+ACBAR+CBI likelihood contours on the $w_X - \Omega_m$ plane. Also plotted in the figure is a region of values compatible with results from high redshift X-ray clusters. The abundance of high redshift X-ray selected clusters has been used to constrain the value of $\Omega_m$ in several works. The values obtained range from $\Omega_m \sim 1 - 0.85$ [24], $\Omega_m \sim 0.85 \pm 0.2$ [33], $\Omega_m \sim 0.96 \pm 0.3$ [57]. These results have been obtained under the assumption of Λ-CDM. A variation in $w_X$ would affect the growth factor for these results. However the effect is small and of a few percent amplitude (see e.g. [54]).

We can therefore state conservatively that from those high redshift cluster analyses $\Omega_m \geq 0.35$. Other high redshift cluster analysis suggest a lower value $\Omega_m \sim 0.35 \pm 0.12$ ([55], [40]) but are still compatible with the $\Omega_m \sim 0.35 - 0.45$ range.

Again, as we can see, this range is incompatible at 95% c.l. with the WMAP constraint obtained under the assumption of Λ-CDM. However, higher values of $\Omega_m$ are compatible with higher values of $w_X$. In particular, $\Omega_m \geq 0.35$, indicates $w_X > -0.9$ at 2-$\sigma$.

The above results have been obtained under the assumption of a flat universe, i.e. $\Omega_{Tot} = \Omega_m + \Omega_X = 1$. While this is certainly one of the most general prediction of inflation, is possible to build inflationary models with $\Omega_{Tot} < 1$ (see e.g. [32]). By relaxing the flatness condition, is possible to obtain degenerate CMB power spectrum by just decreasing $w_X$ and $\Omega_X$ without modifying the other parameters (see e.g. [32]). We show this in Fig.4 where a Λ-CDM flat model is compared with a degenerate open ($\Omega_{Tot} = 0.97$) domain walls model.

Another important aspect is the sound speed of the dark energy component [33].

While for quintessence, the sound speed must be in general equal to the speed of light, for domain walls
FIG. 4: CMB flat and open degenerate models. The parameters have been fixed to \( h = 0.73, \Omega_m h^2 = 0.13, \Omega_b h^2 = 0.024 \).

FIG. 5: Likelihood analysis for the dark energy sound speed \( c_s \) from the WMAP data. The low quadrupole prefers \( c_s \sim 0 \).

models is possible to have \( c_s \leq 1 \). The main effect of a lower \( c_s \) is to reduce the power on large scales (see e.g. [27]), thus yielding a better agreement with the low quadrupole as observed by WMAP (see Fig.1 and Fig.4). Even for quintessence, no perturbations are expected when \( w_X = -2/3 \) (see e.g. [31]).

We study the effect more quantitatively in Fig.5 where a likelihood analysis of the WMAP data is performed by varying \( c_s \) and other cosmological parameters as in [20] but keeping \( w_X = -2/3 \). In order to simplify the problem, the perturbations in the “solid” dark energy component are treated as in a non-interacting fluid ([31]). As we can see, a value of \( c_s \sim 0 \) is preferred by the data, yielding a weak constraint \( c_s \leq 0.7 \) at 1 \( \sigma \) level.

The low CMB quadrupole has raised much interest in recent work and several physical mechanism have been proposed to explain this tension (see e.g. [28],[39],[42]) favouring a non-zero running or scale-dependence \( d\eta_S/d\ln k \sim -0.04 \) of the scalar perturbations spectral index \( n_S \). Simple inflationary models predict a running which is an order of magnitude lower. It is therefore important to address the question whether a different modelling of the dark energy component can explain the low quadrupole. We do this in Fig.6, where we plot the WMAP likelihood curves for the running for the cases \( c_s = 1 \) and \( c_s = 0 \). All other parameters are fixed as in the case \( w = -2/3 \) above, except the spectral index \( n_S = 0.93 \) as in the reported WMAP running index best fit [1]. As we can see, lowering \( c_s \) shifts the likelihood to higher values of \( d\eta_S/d\ln k \), yielding models with zero running in better agreement with the data.

FIG. 6: Likelihood analysis for the running of the spectral index \( d\eta_S/d\ln k \) from the WMAP data. \( c_s \sim 0 \) shifts the likelihood towards \( d\eta_S/d\ln k \sim 0 \).

Recent combined analysis of cosmic microwave background, galaxy clustering and supernovae type Ia data have set strong constraints on the equation of state parameter \( w_X \). The bound \( w_X < -0.82 \) at 95\% c.l. rules out an important class of models as those based on domain walls \(( -1/3 > w > -2/3 \) ). Here we have investigated the stability of this result under a different choice of datasets and theoretical modelling. Our conclusion is that domain walls models are not ruled out by the data and in agreement with the WMAP findings when priors for a “low” hubble parameter \( h \leq 0.65 \), or for a “high” matter density \( \Omega_m \geq 0.35 \) are assumed. Those priors are compatible with most of current cosmological observations and motivated by several others. Moreover, if one relaxes the flatness condition, it is possible to construct CMB spectra for domain walls degenerate with \( \Lambda \)-CDM models that keep the same values of the physical parameters \( h, \Omega_m, \Omega_b \). The current CMB evidence

III. CONCLUSIONS
for a flat universe therefore relies on the assumption of the cosmological constant as the dark energy component. A different value of the sound speed $c_s$ can also lead to a biased determination of inflationary parameters such as the running $d\ln\xi/d\ln k$. When compared with CMB data, the domain walls are compatible but not favoured by the HST constraint on the Hubble parameter. A value of $\Omega_m = 0.35$ and $w_X = -2/3$ is not preferred by present SN-Ia data (see e.g. [45]). When compared with the fiducial WMAP Λ-CDM best fit the disagreement is $\chi^2_{wX} = -2/3 - \chi^2_{\Lambda} \sim 4.1$ i.e. $\sim 2.1\sigma$ disagreement. The SN-Ia dataset has therefore the biggest weight in ruling out domain walls 

models in recent combined analysis. The latest SN-Ia results seems also to favour models with $w_X < -1$ or Chaplygin gases (see e.g. [45]). Density profiles of dark matter halos seem also to prefer $w_X < -1$ [18] while statistics of giant arcs in galaxy clusters prefer $w_X \sim -2/3$ [19]. A value of $\Omega_m \sim 0.35$ is strongly ruled out in the the cluster analysis of [20].

While systematics in SN-Ia might be present (see e.g. [49]) future datasets will be able to clearly test “solid” dark energy. As final remark we mention that detection of non gaussianities in CMB maps (as recently claimed by several groups [52, 53]) is a generic prediction of the models considered here.

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