ZUPT-Aided INS Bypassing Stance Phase Detection by Using Foot-Instability-Based Adaptive Covariance

Chi-Shih Jao*, Student Member, IEEE, and Andrei M. Shkel*, Fellow, IEEE

Abstract—In this paper, we propose a Foot-Instability-Based Adaptive (FIBA) covariance to dynamically adjust the covariance matrix for the pseudo-zero-velocity measurements in the Zero velocity UPdAtE (ZUPT)-aided Inertial Navigation Systems (INS). The proposed ZUPT-aided INS using the FIBA covariance is implemented in an Extended Kalman Filter (EKF) framework, which utilizes a time-varying measurement covariance matrix that is updated in each iteration according to the FIBA covariance. The FIBA covariance is designed to have a very high value during the swing phases in a gait cycle, and a value significantly decreases during the stance phases. As a result, the proposed method eliminates a need to use a binary stance phase detector in implementation of the ZUPT-aided INS. Properties of EKF innovation sequences in the algorithm were studied, and two series of indoor pedestrian navigation experiments were conducted to demonstrate the navigation performance of the proposed system. In the first series of experiments, which included cases of walking and running, localization solutions produced by the system using the FIBA covariance demonstrated 36% and 64% improvements in navigation accuracy along the horizontal and vertical directions, respectively. In the second series of experiments, which included a pedestrian walking on different indoor terrains, such as flat planes, stairs, and ramps, the navigation accuracy of the system using the FIBA covariance reduced horizontal and vertical position errors by 12% and 45%, respectively, as compared to the conventional ZUPT-aided INS.

Index Terms— Indoor pedestrian navigation, inertial measurement unit, zero velocity update, extended Kalman filter, inertial navigation, foot-instability-based adaptive covariance, foot-mounted IMU.

I. INTRODUCTION

Accurate and reliable self-contained indoor localization systems are essential for firefighters, first responders, and soldiers as the personnel often operate in environments where signals of Global Navigation Satellite System (GNSS) are degraded or unavailable, visibility is poor due to smoke, and information about surrounding Radio-Frequency (RF) infrastructures might not be accessible [1]. The latter two conditions limit the performance of many existing alternative localization systems, including 1) radio-based navigation systems utilizing Wireless Local Area Network (WLAN), Bluetooth, Long-Term Evolution (LTE), and Ultra-Wide Band (UWB) [2] and 2) vision-based navigation systems that determine position of a camera based on environmental visual features [3]. In such environments, pedestrian Inertial Navigation Systems (INS) are preferable as positioning approaches, because the systems operate in a self-contained manner, require short installation time, and provide consistently available measurements [4].

Pedestrian INS utilizes Inertial Measurement Units (IMUs) mounted or attached on different parts of a human body, including on a head [5], in a pocket [6], and on a foot [7]–[10], and uses a combination of IMU measurements and local bio-mechanical information for positioning. Among these localization solutions, the systems using foot-mounted IMUs have drawn attention for their ability to significantly enhance a strapdown INS using a Zero velocity UPdAtE (ZUPT) algorithm [11]. The strapdown INS performs localization by dead reckoning based on IMU measurements [12]. Due to noise and stochastic time-varying biases of IMUs, small navigation errors build up in each dead reckoning step, and an accumulated position error of inertial navigation can exceed one meter within just a few seconds of navigation using consumer-grade IMUs [13]. The ZUPT algorithm enhances inertial navigation...
based on an observation that velocities of a person’s foot during walking are nearly equal to zero during the stance phase of a gait cycle. Using this navigation approach, the algorithm periodically resets velocity errors of the INS when the stance phase is detected. The ZUPT-aided INS has been theoretically predicted and experimentally demonstrated to achieve an error of less than 1% of the traveling distance with an industrial-grade IMU [14], [15].

A conventional implementation of ZUPT-aided INS uses a stance phase detector to determine when to feedback the pseudo-zero-velocity measurements to compensate for residual velocity of the INS in an Extended Kalman Filter (EKF) framework [16]. In an ideal scenario of the ZUPT algorithm, the velocity of a foot should only be reset to zero when the foot in the stance phase is completely stationary to avoid unmodeled errors. However, the completely stationary scenario almost never happens in practice when a pedestrian is performing daily activities, such as walking, running, climbing stairs, and crawling. If the ZUPT-aided INS does not calibrate the velocity state with the zero-velocity measurements in a long period of time, the velocity error can grow unbounded, and the position estimates would quickly drift to an unacceptable range. Therefore, most existing stance phase detectors, including the conventional detectors, such as the Acceleration-Moving Variance (AMV) detector, Acceleration-Magnitude (AM) detector, Angular Rate Energy (ARE) detector, and the Stance Hypothesis Optimal dEtection (SHOE) detector, as well as detectors utilizing adaptive mechanisms [17], [18], machine learning [19], [20], and sensor fusion [21]–[23], indicate the stance phase even when the foot is not completely still. Although implementations using the referenced approaches have been effective in reducing the errors of standalone INS, they introduced additional modeling errors to the navigation solutions due to a frequent violation of the assumption that the velocities are zero during the stance period [24]. The violation of the assumption misled the ZUPT algorithm to over-confidently correct velocity states with the measurement covariance matrix that had low-value entrees. The modeling error can accumulate at each step, decreasing the navigation accuracy of the ZUPT-aided INS.

In this paper, we propose a novel mechanism that implements a ZUPT-aided INS with a Foot-Instability-Based Adaptive (FIBA) covariance in an EKF framework. The proposed mechanism, which is shown in Fig. 1(b), aims to minimize one of the modeling errors in the ZUPT algorithm by using an adaptive measurement covariance matrix to avoid feeding back the zero-velocity measurements with high confidence when an inertial sensor mounted on a foot is not completely stationary. Compared to the conventional ZUPT-aided INS, shown in Fig. 1(a), the proposed algorithm feedbacks the pseudo measurements of zero velocity at every time instance in the EKF with the measurement covariance matrix updated by the FIBA covariance that varies according to an instability level of a pedestrian’s foot. The foot instability level is quantified in this paper with a log-likelihood ratio statistics calculated based on readings from the foot-mounted IMU. The FIBA covariance is designed to have a significantly higher value when an IMU mounted on the foot is unstable, which are the cases when the IMU experiences forces other than the gravity of the Earth. The higher value of the FIBA covariance in these cases causes the feedback of zero-velocity measurements to have a numerically minimal effect on other navigation states. When the sensor unit is stable, the FIBA covariance automatically declines to low values that are sufficient to compensate for residual velocities of a standalone INS. This property of the FIBA covariance not only reduces the modeling error of the ZUPT algorithm but also enables an alternative implementation of the ZUPT-aided INS without using a stance phase detector.

This paper makes the following contributions:

1) provides a log-likelihood ratio to quantify instability level of a pedestrian’s foot, based on readings from inertial sensors;
2) develops a FIBA covariance that describes the uncertainty of the zero-velocity measurements in different scenarios for the ZUPT-aided INS;
3) implements the ZUPT-aided INS using the developed FIBA covariance in an EKF framework without a stance phase detector;
4) investigates properties of EKF innovation sequences in the proposed ZUPT-aided INS using the FIBA covariance;
5) verifies the proposed ZUPT-aided INS using the FIBA covariance with real-world indoor pedestrian navigation experiments.

The rest of the paper is organized as follows. Section II presents the proposed FIBA covariance for the ZUPT-aided INS. Section II also includes the derivation of the log-likelihood ratio used to quantify the foot instability level and studies the properties of the zero-velocity measurement model used in the proposed algorithm. Experimental results are presented in Section III. Finally, Section IV concludes the paper with a highlight of our main results and gives an outlook for future research directions.
II. THE PROPOSED APPROACH

The proposed ZUPT-aided INS using the FIBA covariance is implemented in an EKF framework, which is illustrated by the block diagram presented in Fig. 1(b). The propagation step and the update step of the EKF used in this paper are similar to the system discussed in [25]. However, in implementation presented in this paper, the zero-velocity measurement covariance matrix $R_z$ is time-varying and adjusted according to the proposed FIBA covariance in each update step of the EKF.

In this section, we discuss the derivation of the log-likelihood ratio that quantifies the foot instability level, the design of the proposed FIBA covariance, parameters selection for the FIBA covariance, and properties of the EKF innovation sequences in the proposed ZUPT-aided INS.

A. Modeling Instability of Foot Dynamics

The FIBA covariance for zero-velocity measurements aims to provide a low uncertainty when a sensor unit mounted on a foot is stable, which is the case of the sensors being completely stationary, and a high uncertainty when the sensor is unstable, which is the case of the sensors experiencing a motion. The moving case and stationary case are denoted as $H_0$ and $H_1$, respectively. To model the instability level of the sensor with a metrics, this paper derives a Likelihood Ratio (LR) statistics that describes the probabilities of occurrence of any of the two cases based on IMU measurements $z_n$. In this paper, the logarithm of the derived LR statistics, which is the log-likelihood ratio, is considered as a metrics that describes the instability level of a pedestrian’s foot.

The derivation of the LR statistics is similar to the derivation of the SHOE detector presented in [26]. The inertial sensor measurements $y_k$ are modeled using the following notations:

$$y_k = \begin{bmatrix} y^a_k \\ y^\omega_k \end{bmatrix} = \begin{bmatrix} s^a_k \\ s^\omega_k \end{bmatrix} = s_k + v_k.$$  

Here, $s^a_k \in \mathbb{R}^3$ and $s^\omega_k \in \mathbb{R}^3$ denote the IMU-experienced acceleration and angular rate, respectively. Vectors $v^a_k \in \mathbb{R}^3$ and $v^\omega_k \in \mathbb{R}^3$ represent the measurement noise of the accelerometer and gyroscope, respectively. In our derivation, we assumed that the measurement noise of accelerometers and gyroscopes has two components. One of the components is described by independent and identically-distributed white Gaussian noises. The other component is stochastic biases. We considered that the stochastic biases could be minimized by subtracting IMU measurements with bias states of the EKF used in ZUPT-aided INS. Therefore, in this derivation, the measurement noise of accelerometers and gyroscopes were modeled as white Gaussian noises with respective variances $\sigma^2_a$ and $\sigma^2_\omega$.

In the case $H_0$, signal patterns of foot-mounted IMU measurements are unknown. In the case $H_1$, we hypothesize that the accelerometer experiences only the gravitational acceleration, and the angular rate experienced by the gyroscope is zero. More formally, for the two cases, we assume the sensor measurements should satisfy the following conditions:

$$H_0 : \exists k \in \Omega_0, s^a_k \neq g u_a, s^\omega_k \neq 0_{3 \times 1},$$

$$H_1 : \forall k \in \Omega_0, s^a_k = g u_a, s^\omega_k = 0_{3 \times 1},$$

where $u_a$ is a $3 \times 1$ unit vector, $g$ is the gravitational constant, and $\Omega_0 = \{l \in \mathbb{N}, n \leq l < N - 1\}$ is a collection of the sensor measurement indexes at time $n$ with a window of length $N$.

Following the derivation described in [26], the probability density function (pdf) of collected IMU measurements $z_n$ under $H_0$ is expressed as

$$p(z_n; H_0) = \frac{1}{(2\pi \sigma^2_a)^{3N/2}(2\pi \sigma^2_\omega)^{3N/2}}.$$  

where $z_n = \{y_{k,n}\}$. The pdf under $H_1$ is given by

$$p(z_n; H_1) = \frac{1}{(2\pi \sigma^2_a)^{3N/2}} \exp\left( -\frac{1}{2\sigma^2_a} \sum_{k \in \Omega_n} \| y^a_k - g \bar{y}^a_k \| ^2 \right) \times \frac{1}{(2\pi \sigma^2_\omega)^{3N/2}} \exp\left( -\frac{1}{2\sigma^2_\omega} \sum_{k \in \Omega_n} \| y^\omega_k \| ^2 \right),$$  

where

$$\bar{y}^a_k = \frac{1}{N} \sum_{k \in \Omega_n} y^a_k.$$  

The LR statistics $L(z_n)$ is derived from (1) and (2), having the following form

$$L(z_n) = \frac{p(z_n; H_0)}{p(z_n; H_1)} = \exp\left( \frac{1}{2\sigma^2_a} \sum_{k \in \Omega_n} \| y^a_k \| ^2 \right) \left( g \bar{y}^a_k \| \bar{y}^a_k \| ^2 + \frac{1}{2\sigma^2_\omega} \| y^\omega_k \| ^2 \right).$$  

An example of the LR statistics profile collected in an indoor walking-and-running experiment is demonstrated in Fig. 2(a). It should be noted that the LR statistics is always positive.

This paper quantifies the foot instability level with a log-likelihood ratio. The log-likelihood ratio, denoted as $S(z_n)$, takes a scaled version of the logarithm of (3) to increase the numerical discrepancy of small values, which corresponds to the case when a foot is relatively stable. The log-likelihood ratio $S(z_n)$ is expressed as follows:

$$S(z_n) = \frac{2}{N} \log(L(z_n)) = \frac{1}{N} \left( \sum_{k \in \Omega_n} \left( \frac{1}{\sigma^2_a} \| y^a_k \| ^2 - g \bar{y}^a_k \| \bar{y}^a_k \| ^2 \right) + \frac{1}{2\sigma^2_\omega} \| y^\omega_k \| ^2 \right).$$  

Fig. 2(b) gives an example of the log-likelihood ratio profile collected in the same experiment involving indoor walking and running. We can see that (4) gives a lower value in the case of the sensor being stationary than in the case of the moving sensor. Two phenomena can be noticed in Fig. 2(b). First, in the case where an IMU is traveling at a constant velocity without any rotation, the value of the log-likelihood ratio would be on the same level as in the case of the IMU being completely stationary. However, based on our observations in pedestrian navigation experiments, the constant velocity scenarios are unlikely to occur when the IMU is mounted.
on a shoe. Second, the log-likelihood ratio has an identical expression to the statistics metrics used by the SHOE detector. However, the statistics metrics and the log-likelihood ratio play two different roles. In the SHOE detector, the statistics metrics is utilized to compare with a threshold for zero-velocity event detection. In this paper, the log-likelihood ratio is considered as an approach to quantify the instability level of a pedestrian’s foot based on inertial sensor readings, enabling the covariance matrix of zero-velocity measurements to dynamically adjust its values in different scenarios.

B. The Foot-Instability-Based Adaptive (FIBA) Covariance

The proposed FIBA covariance is designed to have values that are varying based on the derived log-likelihood ratio in Eq. (3). The derived log-likelihood ratio demonstrates a desired property where the metrics decreases in the stable case and increases in the unstable case. However, the values might not be suitable for selection as an uncertainty of the zero-velocity measurements. In this paper, we hypothesize that the appropriate uncertainty for zero-velocity measurements can be achieved by scaling the log-likelihood ratio. The FIBA covariance, \( \sigma_{\text{FIBA}}(z_n) \), is expressed as follows:

\[
\sigma_{\text{FIBA}}(z_n) = \beta S(z_n)^\gamma, \quad \beta \in \mathbb{R}^+, \quad \gamma \in \mathbb{R}
\]

In (5), \( \beta \) and \( \gamma \) are hyper-parameters of the FIBA covariance that need to be selected. Note that

\[
\sigma_{\text{FIBA}}(z_n) > 0, \quad \forall z_n.
\]

The proposed FIBA covariance \( \sigma_{\text{FIBA}}(z_n) \) is used to update the covariance matrix \( R_{\text{ZUPT}}(z_n) \) for the zero-velocity measurements in each iteration in the EKF framework. We assume that the zero-velocity measurements are uncorrelated and that uncertainties for the measurements along the 3-axis are identical. \( R_{\text{ZUPT}}(z_n) \) is presented as follows:

\[
R_{\text{ZUPT}}(z_n) = \begin{bmatrix}
\sigma^2_{\text{FIBA}}(z_n) & 0 & 0 \\
0 & \sigma^2_{\text{FIBA}}(z_n) & 0 \\
0 & 0 & \sigma^2_{\text{FIBA}}(z_n)
\end{bmatrix}
\]

The diagonal structure of \( R_{\text{ZUPT}}(z_n) \) and the positivity property of the term \( \sigma^2_{\text{FIBA}}(z_n) \) guarantee \( R_{\text{ZUPT}}(z_n) \) to be a proper covariance matrix.

C. Hyper-Parameter Selection

This paper uses a data-driven approach to estimate values of the hyper-parameters \( \beta \) and \( \gamma \). We conducted pedestrian navigation experiments in the Engineering Gateway Building at the University of California, Irvine. The experimental setup and scenarios are shown in Fig. 3. The IMU VectorNav VN-200 was mounted on the toe side of the right shoe. In the experiments, a pedestrian first walked at a speed of approximately 60 step/sec on a straight line for 20 m and then ran at a speed of approximately 100 step/sec along the line for 22.6 m. An example of the trajectory inside the building is illustrated in Fig. 4. The pedestrian repeated the same experiments 10 times. The relative ground truth location of the destination was determined by a ruler.

The selection of hyper-parameters aims to minimize navigation errors. Based on IMU measurements collected in the experiments, we implemented the ZUPT-aided INS with the
proposed FIBA covariance and swept the values of $\beta$ with values $\left[ e^{-15}, e^{-14.5}, e^{-14.0}, \ldots, e^{15.0} \right]$ and $\gamma$ with values $[-3, -2.5, -2, \ldots, 3]$. For each pair of $\beta$ and $\gamma$ values, we calculated the Root Mean Square Errors (RMSEs) based on the loop-closure errors of the ten experiments. Fig. 5 presents the RMSEs when different values of $\beta$ and $\gamma$ were used. The hyper-parameters that corresponded to the minimum error are summarized in TABLE I.

### D. Discussion

With $\beta = e^{-4.5}$ and $\gamma = 1.8$, an example of profile of the FIBA covariance is illustrated in Fig. 2(c), and two zoomed-in versions are shown in Fig. 2(d) and Fig. 2(e), respectively. The blue curves represent the value of the FIBA covariance, varying based on the IMU measurements collected in the experiments described in Section II-C, and the red horizontal line indicates a value of the variance of the zero-velocity measurements that are commonly used in other pedestrian navigation systems using foot-mounted IMUs. The dark and the light gray areas in 2(c) illustrate the stance phases detected by the SHOE detector with a threshold represented by the green and the yellow horizontal lines in 2(b), respectively. The following three features can be observed in Fig. 2(c), Fig. 2(d), and Fig. 2(e).

- In Fig. 2(c), the FIBA covariance during the stance phase decreases to a comparable level defined by the red line, which marked a value commonly used for the variance of zero-velocity measurements in implementation of conventional ZUPT-aided INS. The decrease indicates that the zero-velocity measurements have low uncertainty, and the velocity error would be reduced during this period. During the swing phase, the FIBA covariance increases sharply, leading to a result that the zero-velocity measurements have high uncertainty, and the velocity state in the navigation solutions would not be numerically affected by the zero-velocity feedback. This property of the proposed FIBA covariance eliminates the need to use a stance phase detector for the ZUPT-aided INS.

- In the period pointed by the black arrow in Fig. 2(d), a stance phase highlighted in light gray is detected, but the instability of the shoe is higher than in other regions of the stance phase. In our opinion, the instability was due to the fact that the foot of a pedestrian could still move slightly during the stance phase, and the
movement generated acceleration and angular velocity that contributed to higher values of the log-likelihood ratio. In conventional ZUPT-aided INS using a stance phase and a constant measurement variance, the filter could be over-confident in the zero-velocity measurements, resulting in a modeling error. With the FIBA covariance, the uncertainty was automatically tuned to a higher number, reducing the impact of the modeling error on navigation accuracy.

- Fig. 2(c) shows a segment of the FIBA covariance collected when the pedestrian was running. In this period, the SHOE detector with a threshold indicated by the green line in Fig. 2(b) was not able to detect the stance phase. The SHOE detector with the threshold indicated by the yellow line in Fig. 2(b) could identify a stance phase, but the usage of the constant covariance could introduce a modeling error because instability of the foot when running should be higher than in the case of walking. In this example, the value of the FIBA covariance in the running case was higher than in the walking case, indicating the modeling error is reduced when the FIBA covariance was used for the ZUPT-aided INS.

E. The Zero-Velocity Measurement Model

A ZUPT-aided INS uses the pseudo-zero-velocity measurements to correct the velocity states in the update step of the EKF. In a traditional implementation of the system, illustrated in Fig. 1(a), the zero-velocity measurements are applied only when a stance phase is detected. The implementation of our proposed ZUPT-aided INS using the FIBA covariance, which is described in Fig. 1(b), feedbacks the zero-velocity information in every iteration of the EKF, regardless of the stance phase or the swing phase. The different implementations lead to distinct properties of the zero-velocity measurement model, the quality of which can be evaluated by reviewing innovation sequences of the EKF [27]. In this paper, we investigate the properties of the measurement model by studying the innovation sequences, \( \tilde{y}_{\text{ZUPT}} \), innovation covariances, \( \sigma^2_{\tilde{y}_{\text{ZUPT}}} \), auto-correlation of the innovation sequences, \( \tilde{y}_{\text{ZUPT}} \), presented in (a). (c) amount of velocity corrected in each iteration of the EKF update step. It can be observed that, even though the ZUPT-aided INS using the FIBA covariance feedbacks the zero-velocity measurements regardless of the stance phase and the swing phase, the corrections applied to the velocity states when the foot was very unstable were minimal, which was \( 6.28 \times 10^{-10} \). (d) the estimated velocities. All profiles shown in this figure corresponded to the IMU measurements collected during one complete gait cycle in the walking part of the experiments discussed in Section II-C.

Fig. 6 compares the properties of zero-velocity measurement models in the proposed ZUPT-aided INS using FIBA covariance, the conventional ZUPT-aided INS with the SHOE...
detector using a low threshold indicated by the green line in Fig. 2(b), and the conventional ZUPT-aided INS with the SHOE detector using a high threshold described by the yellow line in 2(b). It is worth noting that, in this dataset, the low threshold was a preferred value for walking, and the high value was favorable in the case of running. All profiles shown in Fig. 6 corresponded to the IMU measurements collected during one complete gait cycle in the walking part of the experiments discussed in Section II-C. Interpretations of the plots shown in Fig. 6 are discussed in the following paragraphs.

1) The Zero-Velocity Innovation Sequences: A few observations can be made based on the innovation sequences shown in Fig. 6(a). First, it can be seen that the innovation sequences have different profiles in different implementations of the ZUPT-aided INS. In the cases of the conventional ZUPT-aided INS using both the low and high thresholds, the innovation sequences were not continuous because the update step of the EKF was performed only when a stance phase was detected. Second, in all the cases, the innovation sequences along the z-axis had larger values than those along the x- and the y-axis. We considered this phenomenon a result of accelerometer’s insufficient bandwidth of the VN-200 IMU, which was 260 Hz, being insufficient to fully reconstruct the forces experienced by the foot-mounted IMU during heel striking phases. The insufficient bandwidth of an IMU in foot-mounted INS has been reported in [28]. Third, the innovation sequences along the x- and the y-axis in the cases of FIBA covariance and the low threshold were bounded by the dotted $\sigma_{ZUPT}$ curves in Fig. 6(a), while the innovation sequence in the case using the high threshold frequently exceeded the $\sigma_{ZUPT}$ curve. We considered that the occurrence of exceeding innovation sequences is a sign of inappropriate setting of the threshold in the stance phase detection, which could introduce unmodeled errors into the navigation systems.

2) Correlation of the Zero-Velocity Measurements: Fig. 6(b) compares statistics of auto-correlations of the innovation sequences $\tilde{ZUPT}$. Five observations can be made in Fig. 6(b):

- The auto-correlations in the three cases show the innovation sequences are correlated within lags of approximately 0.3 seconds. Since stride periods during walking in this experiment were around 0.8 seconds, this phenomenon suggests that the innovation sequences were correlated within a step, which has been reported and referred to as the within-step correlation in [27].

- The auto-correlations in the cases of the low threshold and the high threshold have peaks at lags of approximately every one second. The presence of the peaks indicates that the zero-velocity measurements between two steps were correlated. In [27], this type of correlation was referred to as the between-step correlation. Although the within-step and the between-step correlations are in contrast to the assumptions of EKF and can lead to navigation errors, it is typically assumed that the correlations would die out within a short period of time.

- The correlation of the innovation sequence in the case of using the high threshold was increased, as compared to the low threshold case. In our opinion, the increase can be interpreted as a presence of zero-velocity measurements biases, which was introduced by an inappropriate setting of thresholds in the stance phase detection. In implementation of the conventional ZUPT-aided INS, the additional navigation errors brought by the high correlation can be reduced by improving performance of stance phase detection [22].

- The auto-correlation in the case of the FIBA covariance had increased within-step and between-step correlations, as compared to those in the other two cases. Moreover, the auto-correlation along the x-axis shows negative values at lags of around every 0.5 seconds. In this dataset, 0.5 seconds was approximately the period between the middle points of a stance phase and a swing phase that were adjacent to each other. Our explanation for the increase in correlations was that in the proposed ZUPT-aided INS using the FIBA covariance, the zero-velocity measurements during the entire experiment were utilized in the EKF update step, and the innovation sequences during the swing phases had large biases. While developing the proposed ZUPT-aided INS using the FIBA covariance, we took this phenomenon into account, and our FIBA covariance handles the highly correlated zero-velocity measurements during the swing phases by increasing the measurement covariance matrix to significantly larger values.

3) Velocity Correction During the Swing Phase: In Fig. 6(c), we can see that, in cases of the low threshold and the high threshold, velocity corrections had the maximum value at the beginning of each detected step, and the magnitude of the corrections in the latter case was larger than the former case. Since the high threshold was an inappropriate setting for walking, the large correction in this case can lead to reduced accuracy in velocity estimation. The inaccuracy velocity estimation can be seen in Fig. 6(d), where the difference in the peaks of velocity states, in the case of the low and the high thresholds, was 0.041 m/s. In the case of the FIBA covariance, the maximum correction did not occur at the beginning of a stance phase but at the moment when the foot had the highest stability, which is illustrated in Fig. 6(c). In Fig. 6(c), we could also observe that, even though the ZUPT-aided INS using the FIBA covariance feeds back the zero-velocity measurements regardless of the stance phase and the swing phase, the corrections applied to the velocity states when the foot was very unstable were minimal, which was $6.28 \times 10^{-10}$ in the case shown in Fig. 6(c). This number can be considered to have an insignificant impact to the estimation accuracy of the velocity states during the swing phases in a short- to mid-term navigation mission. As presented in Fig. 6(d), the differences in the peaks of velocity states in the case of the low threshold and the FIBA covariance was less than 0.003 m/s. In this dataset, we were unclear about whether the case using FIBA covariance had a better velocity estimation accuracy or the one with the low threshold. Alternative localization systems, for example, the camera system described in [29], are needed to verify this information.

Based on properties of the innovation sequences demonstrated in Fig. 6, we concluded that even though the
ZUPT-aided INS using the FIBA applied the zero-velocity measurements, which were highly correlated, regardless of the stance phase and the swing phase, the proposed system only gave an influential update to the velocity state when the foot was stable. During the swing phases when the foot was unstable, the velocity correction could be considered insignificant and would not impact the velocity estimation accuracy in this experiment.

### III. Experimental Validation

To investigate the navigation performance of the proposed ZUPT-aided INS using the FIBA covariance, two series of experiments were conducted in the Engineering Gateway building at the University of California, Irvine. The first series investigated the navigation performance in the case of traveling at two different speeds. The second series evaluated the performance in the case of traveling on different terrains. The experimental setup used for the two series of experiments is shown in Fig. 3. A VectorNav IMU VN-200 was mounted on a customized fixture which was firmly attached to the toe side of the pedestrian’s right shoe. The IMU was connected to a laptop held by the pedestrian for data recording. The sampling rate of the IMU was set to 800 Hz. The EKF noise parameters, including Velocity Random Walk (VRW) $\sigma_{VRW}$, Angular Random Walk (ARW) $\sigma_{ARW}$, Rate Random Walk (RRW) $\sigma_{RRW}$, and Acceleration Random Walk (AcRW) $\sigma_{AcRW}$, had values listed in 

| EKF parameter | Value |
|---------------|-------|
| $\sigma_{VRW}$ | $2.1597 \times 10^{-5}$ |
| $\sigma_{ARW}$ | $4.8557 \times 10^{-4}$ |
| $\sigma_{RRW}$ | $1.7141 \times 10^{-6}$ |
| $\sigma_{AcRW}$ | $1.3873 \times 10^{-6}$ |

A. Different Traveling Speeds

The first series of experiments was the same as the experiments conducted for parameter selection of the FIBA covariance, which is described in Section II-C. A reference trajectory is shown in Fig. 4. In this experiments, we evaluated the navigation accuracy of the ZUPT-aided INS using the SHOE detector with a low threshold specified by the green line in 2(b), the SHOE detector with a high threshold illustrated by the yellow line in 2(b), and the FIBA covariance. The zero-velocity variance used for the conventional ZUPT-aided INS was set to 0.02 m/s. For the horizontal displacement...
error, we used Circular Error Probable (CEP), which is a circle centered at the ground truth location with a radius enclosing 50% of the data. For the vertical displacement error, we calculated the RMSE based on the estimated destination.

The navigation solutions are presented in Fig. 7. It can be observed that the horizontal displacement errors of destinations estimated by solutions that used the low threshold and the FIBA covariance had similar values, which were 0.62 m and 0.64 m, respectively. The horizontal CEP in the case of using the high threshold was increased to 1 m. The vertical RMSEs in the case of the FIBA covariance was 0.34, which was smaller than the other two localization solutions. Ratios of position errors and trajectory lengths in this series of experiments are summarized in TABLE III.

Two observations can be made in this series of experiments. First, the ZUPT-aided INS that was using a high threshold had larger displacement errors because the high threshold led to feeding the zero velocity measurements to the EKF when the foot was not stable, resulting in additional modeling errors. The proposed ZUPT-aided INS using the FIBA covariance also feedbacked zero-velocity measurements during this period, but the modeling error was reduced because the uncertainty of the zero-velocity measurements automatically adjusted to a higher value. Second, although the displacement errors at the destinations estimated by the ZUPT-aided INS using the low threshold had errors less than one meter, it can be seen that the estimated trajectory in Fig. 7(a) drifted away when the pedestrian started to run and the maximum error was 4 m along the east direction. The drift was observed because the low threshold was not able to detect the stance phase, and hence no ZUPT algorithm was triggered during this period. In this series of experiments, the proposed ZUPT-aided INS using the FIBA covariance demonstrated an improvement in the localization accuracy, as compared to the conventional ZUPT-aided INS using the SHOE detector with a low threshold and a high threshold.

**B. Different Terrains**

The second series of experiments investigated the navigation performance of the proposed ZUPT-aided INS using FIBA covariance when operating on different terrains. The experiments were conducted in an indoor environment illustrated in Fig. 8. In this series of experiments, a pedestrian walked a closed-loop trajectory for a length of 50 m in 40 s. The path included a flat surface, a ramp, and stairs. The pedestrian repeated the same experiments 10 times.

We compared the navigation performance of the proposed ZUPT-aided INS using the FIBA covariance and the conventional ZUPT-aided INS using the SHOE detector with a constant threshold. The experimental results of the two systems are shown in Fig. 9. The solution with FIBA covariance improved horizontal CEP and the vertical RMSE by 12% and 45%, respectively, as compared to the conventional ZUPT-aided INS. TABLE IV summarizes percentage of position errors in trajectory lengths in this series of experiments. We concluded the improvement in the navigation accuracy to be a direct result of using the FIBA covariance, which
TABLE IV
PERCENTAGE OF POSITION ERROR IN TRAJECTORY LENGTH FOR THE 2ND SERIES OF EXPERIMENTS

|     | FIBA covariance | Constant threshold |
|-----|-----------------|--------------------|
| Horizontal | 0.58%          | 0.66%              |
| Vertical   | 0.52%          | 0.96%              |

reduced the modeling error in the ZUPT-aided INS. This series of experiments demonstrated that the navigation performance of the ZUPT-aided INS using the proposed FIBA covariance outperformed the case of conventional ZUPT-aided INS when traveling on terrains of flat planes, stairs, and slopes.

IV. CONCLUSION AND FUTURE WORK

In this paper, we introduced the FIBA covariance to dynamically adjust uncertainties of the zero-velocity measurements in the ZUPT-aided INS, allowing to reduce the modeling error of the ZUPT algorithm. The proposed ZUPT-aided INS with the FIBA covariance was implemented in the EKF framework, where the measurements covariance matrix for the zero-velocity measurements was updated in each iteration according to the FIBA covariance, which varied based on the instability metrics derived from foot-mounted IMU measurements. The developed FIBA covariance was demonstrated to exhibit a property that it gives a value sufficiently lower when the foot was stable, such that the zero-velocity measurements can effectively reset the velocity states. When the foot was experiencing a motion, the statistics of the FIBA covariance increased sharply, and as such the zero-velocity measurements would not have a significant numerical impact on the velocity state. This property of the FIBA covariance allows to reduce the ZUPT modeling error and eliminates the requirement to have a stance phase detector in the ZUPT-aided INS. Two series of indoor pedestrian navigation experiments were conducted in this paper to evaluate the ZUPT-aided INS using the proposed FIBA covariance. In the first series, including both walking and running activities, the solution using the FIBA covariance showed a maximum improvement in navigation accuracy of 36% horizontally and 64% vertically, as compared to the conventional ZUPT-aided INS using the SHOE detector with a constant threshold. In the second series of experiments, which included walking on different terrains of flat planes, stairs, and slopes, the ZUPT-aided INS using the FIBA covariance reduced horizontal CEP by 12% and vertical RMSE by 45%, as compared to the conventional ZUPT-aided INS. We concluded that using the FIBA covariance in the ZUPT-aided INS can eliminate the need to use a stance phase detector and could be beneficial for navigation accuracy.

Although this paper has demonstrated with real-world pedestrian navigation experiments that the proposed ZUPT-aided INS using the FIBA covariance provided a higher navigation accuracy than the traditional systems using the SHOE detector with constant thresholds, we would like to discuss possible limitations of the FIBA covariance and suggest corresponding future research directions.

1) In this paper, the hyper-parameter selection for the proposed FIBA covariance was conducted based on a series of walking-and-running experiments of 42.5 meters using a single subject. It is needed for future research to investigate the validity of the hyper-parameter selection in different cases, including performing other common pedestrian activities, such as sprinting, jumping, crawling, and side-stepping, on different terrains like stairs, sand, and grass. Moreover, in our experiments, we could see that the innovation sequences along the horizontal and the vertical directions had different behaviors. The different behaviors could be an indication that appropriate choices of the hyper-parameters, $\beta$ and $\gamma$, might not be the same for two different directions.

2) The proposed ZUPT-aided INS using the FIBA covariance feedbacks the zero-velocity measurements regardless of the stance phase or the swing phase. As discussed in Section II-E.2, the zero-velocity measurements during the swing phases are highly correlated. The correlation violates the fundamental assumption of EKF on uncorrelated measurements. In our proposed FIBA covariance, the measurement covariance matrix is increased to a significantly large value to minimize the impact of the correlated measurements. A potential approach to further reduce the impact of violation of the EKF assumption on correlation of measurements is to develop an hybrid approach combining the conventional ZUPT-aided INS with the proposed FIBA covariance. The hybrid approach can work in a way that the zero-velocity measurements with the FIBA covariance are applied in the update step only when the likelihood of stance phase detection is below a high threshold and no measurement updates are performed when the foot is certainly moving.

3) As discussed in Section II, the proposed FIBA covariance aims to avoid applying the zero-velocity measurements with a high confidence when the foot is not completely stationary, and the covariance has low values when the foot is stable and significantly high values when the foot is unstable. In some common pedestrian activities, for example sprinting, the foot of a pedestrian is frequently unstable, even during the ground contacting phases. In these situations, the FIBA covariance would give a low confidence to zero-velocity measurements and the accuracy of estimated velocities in the ZUPT-aided INS would start to decrease with time due to unavoidable sensor noise. This problem could be potentially mitigated through cooperative localization or sensor fusion solutions with other non-inertial sensing modalities.

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Chi-Shih Jiao (Student Member, IEEE) received the B.S. degree in electrical engineering from the National Tsing Hua University, Hsinchu, Taiwan, in 2015, and the M.S. degree in electrical engineering from Pennsylvania State University, University Park, in 2018. He is currently pursuing the Ph.D. degree with the Microsystems Laboratory, Department of Mechanical and Aerospace Engineering, University of California at Irvine, Irvine, CA, USA. His research interests include aided inertial navigation by sensor fusion approach and vision-based inertial navigation. He was a recipient of the 2019–2020 Holmes Fellowship.