Emissivity of neutrinos in supernova in a left-right symmetric model

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(Dated: October 10, 2018)

Abstract

We calculate the emissivity due to neutrino-pair production in $e^+e^-$ annihilation in the context of a left-right symmetric model in a way that can be used in supernova calculations. We also present some simple estimates which show that such process can act as an efficient energy-loss mechanism in the shocked supernova core. We find that the emissivity is dependent of the mixing angle $\phi$ of the model in the allowed range for this parameter.

PACS numbers: 14.60.Lm,12.15.Mm, 12.60.-i

Keywords: Ordinary neutrinos, neutral currents, models beyond the standard model.
I. INTRODUCTION

The existence of neutrinos was postulated by Pauli in 1932 in order to explain the observed continuous electron spectrum accompanying nuclear beta decay. Based on the idea of Pauli, Fermi [1, 2] proposed the beta decay theory, while Bethe and Peierls [3, 5] predicted an extremely small cross-section for the interaction of neutrino with matter, and Gamow [6, 7] and Pontecorvo [8] were the first to recognize the important role played by neutrinos in the evolution of stars. The neutrino emission processes may affect the properties of matter at high temperatures, and hence affect stellar evolution.

Neutrino emission is known to play an important role in stellar evolution, especially in the late stages when the rate of evolution is almost fully dependent on energy loss via neutrinos. This refers to the stage of steady burning prior to the implosion of the stellar core, to the process of catastrophic core-collapse, and to the cooling of the neutron star which is formed.

The explosion energy of the core-collapse is typically $10^{53} \text{erg}$ which makes it one of the most impressive violent events in the universe. This energy comes from the explosion of the progenitor star, and only partly manifests itself in the shock wave that is launched somewhere at the boundary between the iron core of mass $M_{Fe} = (1.2 - 2)M_\odot$ and the innermost regions, collapsing into a neutron star. Even when the mechanism of the core-collapse is not yet understood in great detail, the most distinctive feature is the enormous energy of $(3 - 5) \times 10^{53} \text{erg} = (10 - 15)\% M_{Fe} c^2$ radiated in the form of neutrinos and antineutrinos of all flavors ($\nu_e, \nu_\mu, \nu_\tau$) during a burst of about 10 seconds. While such neutrinos were first observed in the supernova SN1987A, various observatories running or under design, like next generation large-size detectors, could provide us with the luminosity curve from a future (extra)galactic explosion and/or the observation of relic neutrinos from past supernovae. To disentangle the information from such neutrino signals represents a challenging task, since the crucial information from the explosion phenomenon and neutrino properties such as the neutrino hierarchy and the third neutrino mixing angle are intertwined.

The detection of neutrinos from SN1987A by the Kamiokande II [9] and Irvine-Michigan-Brookhaven [10] detectors confirmed the standard model of core-collapse (type II) supernovae [11, 12] and provided a laboratory to study the properties of neutrinos [13, 18] and exotic particles such as axions [19]. The collapse of stellar iron-core into a neutron star is preceded by a high-power pulse of neutrino emission. In general, a bolometric neutrino light
curve that includes all the neutrino and antineutrino flavors consists of two parts. In the first part \((t < 5 \text{ s})\) the non-thermal neutrino emission is dominated by the electron neutrinos \(\nu_e\) produced by the non-thermal neutronization. For \(t \approx 0.5 \text{ s}\), the core is transparent to \(\nu_e\) emitted due to electrons captures by nuclei and free protons. By this time, the mean individual \(\nu_e\) energy becomes \(10^2 - 20 \text{ MeV}\). Thus, the non-thermal neutrinos carry away approximately a small fraction of the total available energy \(E_{\nu_{\text{tot}}} = (3 - 5) \times 10^{53} \text{ erg}\). Nearly 90% of this energy is emitted in the regime of thermal emission once the innermost region of the core becomes opaque to all the neutrino flavors, which get decoupled from the stellar mass at a surface so-called neutrinosphere.

Therefore, one of the crucial parameters which strongly affect the stellar evolution is the cooling rate. During their lifetime, stars can emit energy in the form of electromagnetic or gravitational waves, and a flux of neutrinos. However, in late stages a star mainly looses energy through neutrinos, and this is quite independent of the star mass. In fact, white dwarfs and supernovae, which are the evolution end points of stars formed from very different masses, have cooling rates largely dominated by neutrino production. An accurate determination of neutrino emission rates is therefore mandatory in order to perform a careful study of the final branches of star evolutionary tracks. In particular, a change in the cooling rates at the very last stage of massive star evolution could perceptibly affect the evolutionary time scale and the iron core configuration at the onset of the supernova explosion whose triggering mechanism still waits a full theoretical understanding [20].

The energy loss rate due to neutrino emission receives contributions from both weak nuclear reactions and purely leptonic processes. However, for the large values of density and temperature which characterize the final stage of stellar evolution, the latter are largely dominant, and are mainly produced by four possible interaction mechanisms: \(e^+e^- \rightarrow \nu\bar{\nu}\) (pair annihilation), \(\gamma e^\pm \rightarrow e^\pm \nu\bar{\nu}\) (\(\nu\)-photoproduction), \(\gamma^* \rightarrow \nu\bar{\nu}\) (plasmon decay), \(e^\pm Z \rightarrow e^\pm Z\nu\bar{\nu}\) (bremsstrahlung on nuclei). These mechanisms play an important role in astrophysics and cosmology and have been considered by many authors in various theories of weak interactions.

Actually these processes are the dominant cause of the energy loss rate in different regions in a density-temperature plane. For very large core temperature, \(T \gtrsim 10^9 \text{ oK}\), and not excessively high values of density, pair annihilations are most efficient, while \(\nu\) photoproduction gives the leading contribution for \(10^8 \text{ oK} \lesssim T \lesssim 10^9 \text{ oK}\) and relatively low density,
\[ \rho \lesssim 10^5 g \, cm^{-3} \]. These are the typical ranges for very massive stars in their late evolution.

Our main objective in this paper is to provide suitable expressions for the emissivity of pair production of neutrinos via the process \( e^+e^- \rightarrow \nu\bar{\nu} \) in the context of a Left-Right Symmetric Model (LRSM) \[21–28\] and in a form which can be easily incorporated into realistic supernova models to evaluate the energy lost in the form of neutrinos.

The amplitude of transition \( \mathcal{M} \) in the context of the LRSM can be written as a function of the mixing angle \( \phi \) between \( W^3_L, W^3_R \) and \( B \) bosons of the model to give the physical \( Z_1 \) and \( Z_2 \) and the photon, being \( \phi \) the only extra parameter besides the standard model parameters. For which in this paper we choice the Left-Right symmetric model \[26–28\] to calculate the emissivity of neutrinos in supernova.

This paper is organized as follows: In Sect. II we present the calculation of the transition amplitude of the process \( e^+e^- \rightarrow \nu\bar{\nu} \) in the context of a left-right symmetric model. In Sect. III we calculate the emissivity and, finally, we give our results and conclusions in Sec. IV.

**II. CROSS SECTION OF THE PROCESS** \( e^+e^- \rightarrow \nu\bar{\nu} \)

In this section we obtain the cross section for the \( Z \) exchange process

\[
e^+(p_1) + e^-(p_2) \rightarrow \bar{\nu}(k_1, \lambda_1) + \nu(k_2, \lambda_2),
\]

i.e., in the limit of a four-fermion electroweak interaction no electromagnetic radiative corrections. Here the \( k_i \) and \( p_i \) are the particle momenta and \( \lambda \) is the helicity of the neutrino. We recall that within the context of the standard theory, a neutrino interaction eigenstate \( (\nu_L \text{ or } \nu_R) \) is a superposition of helicity \( (\lambda) \) eigenstates \( \nu_\pm \), where \( \lambda = \sigma \cdot p = \pm 1 \). For a relativistic particle, this translates into the statement that a \( \nu_L \) is predominantly in the \( \lambda = -1 \) state and a \( \nu_R \) is predominantly in the \( \lambda = +1 \) state, with small admixtures of the opposite helicity of the order \( m/E_\nu \).

The amplitude of transition for the process (1) is given by

\[
\mathcal{M} = \frac{g_Z^2}{2M_Z^2} \left\{ \bar{u}(k_2, \lambda_2) \gamma^\mu \frac{1}{2} \left( ag_\nu^V - bg_\nu^A \gamma_5 \right) v(k_1, \lambda_1) \left[ \bar{v}(p_1) \gamma_\mu \frac{1}{2} \left( ag_\nu^V - bg_\nu^A \gamma_5 \right) u(p_2) \right] \right\}, \quad (2)
\]

where the constant \( a \) and \( b \) depend only on the parameters of the LRSM model \[26–28\].
\[ a = \cos \phi - \frac{\sin \phi}{\sqrt{\cos 2\theta_W}} \quad \text{and} \quad b = \cos \phi + \sqrt{\cos 2\theta_W} \sin \phi, \]  

(3)

where \( \phi \) is the mixing parameter of the LRSM [26–28], \( u \) and \( v \) are the usual Dirac spinors, and the electron and positron helicity indexes have been suppressed since they will be averaged over. We then write

\[
\frac{1}{2} \times \frac{1}{2} |M|^2 = \frac{G_F^2}{8} N^{\mu\nu} E_{\mu\nu},
\]

(4)

where

\[
N^{\mu\nu} = \frac{1}{4} Tr[(\vec{k} + m_\nu)(1 + \gamma^5 \gamma_5)(a - b\gamma_5)(\vec{k} - m_\nu)(1 + \gamma^5 \gamma_5)] \gamma^{\mu}(a - b\gamma_5),
\]

(5)

\[
E_{\mu\nu} = \frac{1}{4} Tr[(\vec{p} + m_e)\gamma_\mu(\gamma_5)(a g_V - b g_A)(\vec{p} - m_e)\gamma_\nu(\gamma_5)(a g_V - b g_A)].
\]

(6)

Here \( s_1 \) and \( s_2 \) are the spin four-vectors associated with the antineutrino and neutrino respectively, while \( m_\nu \) and \( m_e \) are the neutrino and electron mass. These spin vectors satisfy the Lorentz invariant conditions

\[
s_i \cdot s_i = -1; \quad s_i \cdot k_i = 0;
\]

(7)

and for a relativistic neutrino, the additional constraint

\[
s_i \parallel \lambda_i k_i \quad \text{for} \quad i = 1, 2
\]

(8)

holds, where

\[
k^\mu = (E_\nu, \hat{k}),
\]

(9)

with \( \hat{k} \) being a unit vector along the three-momentum of the neutrino.

We now introduce two four-vectors associated with the neutrino pair

\[
K_1^{\mu} = k_1^{\mu} + m_\nu s_1^{\mu}; \quad K_2^{\mu} = k_2^{\mu} - m_\nu s_2^{\mu}.
\]

(10)

In conjunction with the properties given in Eqs. (7) and (8), these will allow us to write the amplitude squared for the process under consideration in a compact and physically
revealing form. As a first step towards this, we note that the spin vector may be expressed as

$$s^\mu = \frac{\lambda}{m_\nu}(k, E_\nu \mathbf{k}).$$

(11)

Using this and Eq. (10), we write

$$K_1 = \eta_1(1, \mathbf{k}); \quad K_2 = \eta_2(1, -\mathbf{k});$$

(12)

with

$$\eta_1 = E_\nu + (E_\nu^2 - m_\nu^2)^{1/2}; \quad \eta_2 = E_\nu - (E_\nu^2 - m_\nu^2)^{1/2}.$$ \hspace{1cm} (13)

Note that for $m_\nu << E_\nu$ we have

$$\eta_1 \approx 2E_\nu; \quad \eta_2 \approx \frac{m_\nu^2}{2E_\nu}.$$ \hspace{1cm} (14)

We now evaluate the traces given in Eqs. (5) and (6) and the contraction $N^{\mu\nu}E_{\mu\nu}$ is given by

$$N^{\mu\nu}E_{\mu\nu} = 8 \left( a^2 + b^2 \right) \left\{ \left[ a^2 (g_{V}^e)^2 + b^2 (g_{A}^e) + 4 \frac{a^2 b^2}{(a^2 + b^2)} g_{V}^e g_{A}^e \right] (p_1 \cdot K_1)(p_2 \cdot K_2) 
+ \left[ a^2 (g_{V}^e)^2 + b^2 (g_{A}^e) - 4 \frac{a^2 b^2}{(a^2 + b^2)} g_{V}^e g_{A}^e \right] (p_1 \cdot K_2)(p_2 \cdot K_1) 
+ \left[ a^2 (g_{V}^e)^2 - b^2 (g_{A}^e) \right] m_\nu^2 (K_1 \cdot K_2) \right\},$$

(15)

where $g_{V}^e = -\frac{1}{2} + 2 \sin^2 \theta_W$ and $g_{A}^e = -\frac{1}{2}$. From this expression and Eqs. (12) and (14) above, we see that the amplitude of transition vanishes for massless neutrino, as expected. Furthermore, Eq. (15) is taken as the usual weak pair production amplitude with the replacement $K_i \rightarrow k_i$.

From Eqs. (4) and (15) the explicit form for the squared transition amplitude is

$$\frac{1}{2} \times \frac{1}{2} |\mathcal{M}|^2 = G_F^2 \left( a^2 + b^2 \right) \left\{ \left[ a^2 (g_{V}^e)^2 + b^2 (g_{A}^e) + 4 \frac{a^2 b^2}{(a^2 + b^2)} g_{V}^e g_{A}^e \right] (p_1 \cdot k_1)(p_2 \cdot k_2) 
+ \left[ a^2 (g_{V}^e)^2 + b^2 (g_{A}^e) - 4 \frac{a^2 b^2}{(a^2 + b^2)} g_{V}^e g_{A}^e \right] (p_1 \cdot k_2)(p_2 \cdot k_1) 
+ \left[ a^2 (g_{V}^e)^2 - b^2 (g_{A}^e) \right] m_\nu^2 (k_1 \cdot k_2) \right\},$$

(15)
\[ a^2(g_V^e)^2 + b^2(g_A^e) - 4\frac{a^2b^2}{a^2 + b^2}g_V^e g_A^e \] 
\[ (p_1 \cdot k_2)(p_2 \cdot k_1) \]
\[ + a^2(g_V^e)^2 - b^2(g_A^e) \]
\[ m_e^2(k_1 \cdot k_2) \}, \quad (16) \]

where the contribution of the parameters of the LRSM is contained in the constants \( a \) and \( b \). Upon evaluating the limit when the mixing angle \( \phi = 0 \) and \( a = b = 1 \), Eq. (16) is thus reduced to the expression to the amplitude given in Refs. [29–35].

### III. CALCULATION OF EMISSIVITY

In this section, we calculate the emissivity associated with neutrino pair production by using Eq. (16). The formula of the emissivity is given by [29–31, 33, 36]

\[
Q_{\nu\bar{\nu}} = \frac{4}{(2\pi)^8} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3k_1}{2\epsilon_1} \frac{d^3k_2}{2\epsilon_2} (E_1 + E_2) F_1 F_2 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) |\mathcal{M}|^2, \quad (17)
\]

where the quantities \( F_{1,2} = [1 + \exp(E_{e-} \pm \mu_e)/T]^{-1} \) are the Fermi-Dirac distribution functions for \( e^\pm \), \( \mu_e \) is the chemical potential for the electrons and \( T \) is the temperature (we take \( K_B = 1 \) for the Boltzmann constant).

From the transition amplitude Eq. (16) and the formula of the emissivity Eq. (17) we obtain

\[
Q_{\nu\bar{\nu}}^{[1]} = G_F^2 (a^2 + b^2) \left[ a^2 (g_V^e)^2 + b^2 (g_A^e)^2 + 4\frac{a^2b^2}{a^2 + b^2} g_V^e g_A^e \right] I_1, \quad (18)
\]

where \( I_1 \) is explicitly given by

\[
I_1 = \frac{4}{(2\pi)^8} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3k_1}{2\epsilon_1} \frac{d^3k_2}{2\epsilon_2} (E_1 + E_2) F_1 F_2 \delta^{(4)}(p_1 + p_2 - k_1 - k_2)(p_1 \cdot k_1)(p_2 \cdot k_2). \quad (19)
\]

The integration can be performed by using the Lenard formula, namely [36]

\[
\int \frac{d^3k_1}{2\epsilon_1} \frac{d^3k_2}{2\epsilon_2} k_1^\alpha k_2^\beta \delta^{(4)}(p_1 + p_2 - k_1 - k_2) = \frac{\pi}{24} \left[ g_{\alpha\beta}(p_1 + p_2)^2 + 2(p_1^\alpha + p_2^\alpha)(p_1^\beta + p_2^\beta) \right] \Theta \left[ (p_1 + p_2)^2 \right], \quad (20)
\]

thus Eq. (19) takes the form
\[ I_1 = \frac{1}{24(2\pi)^7} \int \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} (E_1 + E_2) F_1 F_2 \left[ 3m_e^2(p_1 \cdot p_2) + 2(p_1 \cdot p_2)^2 + m_e^4 \right]. \]  

(21)

In a similar way for the second and third term of Eq. (16), we obtain

\[
Q^{[2]}_{\nu\bar{\nu}} = G^2_F \left( a^2 + b^2 \right) \left[ a^2 \left( g_V^e \right)^2 + b^2 \left( g_A^e \right)^2 - \frac{4a^2b^2}{a^2 + b^2} g_V^e g_A^e \right] I_2, \tag{22}
\]

\[
Q^{[3]}_{\nu\bar{\nu}} = G^2_F \left( a^2 + b^2 \right) \left[ a^2 \left( g_V^e \right)^2 - b^2 \left( g_A^e \right)^2 \right] m_e^2 I_3, \tag{23}
\]

where

\[
I_2 = I_1 = \frac{1}{24(2\pi)^7} \int \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} (E_1 + E_2) F_1 F_2 \left[ 3m_e^2(p_1 \cdot p_2) + 2(p_1 \cdot p_2)^2 + m_e^4 \right], \tag{24}
\]

\[
I_3 = \frac{1}{(2\pi)^7} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} (E_1 + E_2) F_1 F_2 \left[ m_e^2 + (p_1 \cdot p_2) \right]. \tag{25}
\]

The calculation of the emissivity can be more easily performed by expressing the latest integrals in terms of the Fermi integral, which is defined as

\[
G_{\pm}^s = \frac{1}{\alpha^{3+2s}} \int_{\alpha}^{\infty} x^{2s+1} \frac{\sqrt{x^2 - \alpha^2}}{1 + e^{x \pm \beta}} dx, \tag{26}
\]

where \( \alpha = \frac{m_e}{kT}, \ \beta = \frac{\mu_e}{kT} \) and \( x = \frac{E}{kT} \).

With these definitions, Eq. (26) becomes

\[
G_{\pm}^s = \frac{1}{m_e^{3+2s}} \int_{m_e/kT}^{\infty} E^{2s+1} \frac{\sqrt{E^2 - m_e^2}}{1 + e^{(E \pm \mu_e)/kT}} dE, \tag{27}
\]

therefore

\[
\int_{m_e/kT}^{\infty} E^n \frac{\sqrt{E^2 - m_e^2}}{1 + e^{(E \pm \mu_e)/kT}} dE = m_e^{n+2} G_{n+1}^{\pm}, \tag{28}
\]

\[
\int_{m_e/kT}^{\infty} E^{n+1} \frac{\sqrt{E^2 - m_e^2}}{1 + e^{(E \pm \mu_e)/kT}} dE = m_e^{n+3} G_{n+2}^{\pm}, \tag{29}
\]

\[
\int_{m_e/kT}^{\infty} E^{n+2} \frac{\sqrt{E^2 - m_e^2}}{1 + e^{(E \pm \mu_e)/kT}} dE = m_e^{n+4} G_{n+3}^{\pm}. \tag{30}
\]

From (28), (29) and (30), Eqs. (24) and (25) are expressed as
\[ I_{1}^{nm} = \frac{m_{n}^{m+n+8}}{6(2\pi)^{5}} \left[ 3G_{\frac{n}{2}}^{-}G_{\frac{m}{2}}^{+} + 2G_{\frac{n+1}{2}}^{-}G_{\frac{m+1}{2}}^{+} + G_{\frac{n-1}{2}}^{-}G_{\frac{m-1}{2}}^{+} + \frac{4}{9} \left( G_{\frac{n+1}{2}}^{-} - G_{\frac{n-1}{2}}^{-} \right) \left( G_{\frac{m+1}{2}}^{+} - G_{\frac{m-1}{2}}^{+} \right) \right], \]  
\[ I_{3}^{nm} = \frac{m_{e}^{n+m+6}}{(2\pi)^{4}} \left[ G_{\frac{n+1}{2}}^{-}G_{\frac{m+1}{2}}^{+} + G_{\frac{n-1}{2}}^{-}G_{\frac{m-1}{2}}^{+} \right]. \]

Therefore, Eqs. (18), (22) and (23) are explicitly

\[ Q_{\nu\bar{\nu}}^{[1]} = G_{F}^{2}(a^{2} + b^{2}) \left[ a^{2} (g_{V}^{e})^{2} + b^{2} (g_{A}^{e})^{2} + \frac{4a^{2}b^{2}}{a^{2} + b^{2}} g_{V}^{e}g_{A}^{e} \right] \left[ I_{1}^{10} + I_{1}^{01} \right], \]  
\[ Q_{\nu\bar{\nu}}^{[2]} = G_{F}^{2}(a^{2} + b^{2}) \left[ a^{2} (g_{V}^{e})^{2} + b^{2} (g_{A}^{e})^{2} - \frac{4a^{2}b^{2}}{a^{2} + b^{2}} g_{V}^{e}g_{A}^{e} \right] \left[ I_{2}^{10} + I_{2}^{01} \right], \]  
\[ Q_{\nu\bar{\nu}}^{[3]} = G_{F}^{2}(a^{2} + b^{2}) \left[ a^{2} (g_{V}^{e})^{2} - b^{2} (g_{A}^{e})^{2} \right] m_{e}^{2} \left[ I_{3}^{10} + I_{3}^{01} \right]. \]

Finally, the expression for the emissivity of neutrino pair production via the process \( e^{+}e^{-} \rightarrow \nu\bar{\nu} \) in the context of a left-right symmetric model is given by

\[ Q_{\nu\bar{\nu}}^{LRSM} (\phi, \beta) = Q_{\nu\bar{\nu}}^{[1]} (\phi, \beta) + Q_{\nu\bar{\nu}}^{[2]} (\phi, \beta) + Q_{\nu\bar{\nu}}^{[3]} (\phi, \beta), \]  
where the dependence of the \( \phi \) mixing parameter of the LRSM is contained in the constants \( a \) and \( b \), while the dependence of the \( \beta \) degeneration parameter is contained in the Fermi integrals \( G_{s}^{\pm} \).

**IV. RESULTS AND CONCLUSIONS**

The numerical result on the emissivity of neutrino pair production via the process \( e^{+}e^{-} \rightarrow \nu\bar{\nu} \) as a function of the mixing angle \( \phi \) and the generation parameter \( \beta \) are present in this section. For our analysis we consider the following data: the Fermi constant \( G_{F} = 1.166 \times 10^{-5} \text{GeV}^{-2} \), angle of Weinberg \( \sin^{2} \theta_{W} = 0.223 \) and the electron mass \( m_{e} = 0.51 \text{MeV} \), thereby obtaining the emissivity of the neutrinos \( Q_{\nu\bar{\nu}}^{LRSM} = Q_{\nu\bar{\nu}}^{LRSM} (\phi, \beta) \).

For the mixing angle \( \phi \) of the left-right symmetric model, we use the reported data of A. Gutiérrez-Rodríguez, et al. [37]:

\[ -1.6 \times 10^{-3} \leq \phi \leq 1.1 \times 10^{-3}, \]  

(37)
with a 90% C.L. Other limits on the mixing angle $\phi$ reported in the literature are given in Refs. [26, 28, 38, 39].

In Figure 1 we show the emissivity as a function of the degeneration parameter $\beta$ for several values representative of the mixing angle $\phi = -0.0016, 0, 0.0011$. We observe that emissivity decreases when $\beta$ increases, which is due to the reduction in the number of positrons available necessary to cause the collision. Also we see that the emissivity is affected by the $\phi$ parameter. There are other effects which may change the emissivity, for example, the radiative corrections at one-loop level.

To analyze the effects of the $\phi$ parameter of the left-right symmetric model on the emissivity $Q_{\nu\bar{\nu}}^{LRSM}(\phi, \beta)$ of the neutrinos, in Fig. 2 we show the ratio $\frac{Q_{\nu\bar{\nu}}^{LRSM}(\phi, \beta)}{Q_{\nu\bar{\nu}}^{SM}(\beta)}$ (ratio of emissivity calculated in the LRSM to the emissivity calculated in the SM [34]), as a function of the $\phi$ parameter for several values representative of degeneration parameter $\beta = 0, 5, 12$.

According to collapse theories, the full energy loss in a stellar collapse is $L_{\nu\bar{\nu}} = Q_{\nu\bar{\nu}} \cdot V_{star} \cdot t_{col} \approx 10^{53}$ erg, where $V_{star}$ is the volume of a star and $t_{col}$ the collapse time. Therefore, if we assume that $t_{col} \sim 1 - 10$ s, the stellar collapse temperature $T_{col} \sim (1.1 - 1.4) \times 10^{11}$ oK will be obtained. On the contrary, if we know the exact stellar collapse temperature, the collapse time can be obtained. For instance, the value $T_{col} \sim 10^{12}$ oK corresponds to $t_{col} \sim 10^{-7}$ s.

In summary, we have analyzed the effects of the mixing angle $\phi$ of a left-right symmetric model on the emissivity of the neutrinos via the process $e^+ e^- \rightarrow \nu \bar{\nu}$. We find that the emissivity is dependent of the mixing angle $\phi$ of the model in the allowed range for this parameter. As expected, in the limit of vanishing $\phi$ we recover the expression for the emissivity $Q_{\nu\bar{\nu}}^{SM}(\beta)$ for the SM previously obtained in the literature [29, 33]. In addition, the analytical and numerical results for the emissivity have never been reported in the literature before and could be of relevance for the scientific community.

Acknowledgments

This work was supported by CONACyT, SNI and PROMEP (México).
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FIG. 1: The emissivity for $e^+e^- \rightarrow \nu \bar{\nu}$ as a function of degeneration parameter $\beta$ for $\phi = -0.0016, 0, 0.0011$.

FIG. 2: Plot of ratio $Q(\phi, \beta) / Q(\beta)$, as a function of mixing angle $\phi$ for $\beta = 0, 5, 12$. 