Integer superspin supercurrents of matter supermultiplets

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ABSTRACT

In recent papers [\textsuperscript{18,21}] we demonstrated that consistent and non-trivial linear transformations of matter supermultiplets generate half-integer superspin supercurrents and the cubic interactions between matter and half-integer superspin supermultiplets. In this work we show that consistent and non-trivial antilinear transformations of matter superfields lead to the construction of integer superspin supercurrents and the cubic interactions between matter and integer superspin supermultiplets. Applying Noether’s method to these transformations, we derive the integer superspin supercurrents and supertraces for the cases of a massless, massive and with a linear superpotential chiral superfield.

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1 Introduction

For non-supersymmetric theories there is a plethora of well-known results on the topic of higher spin conserved currents [1–10] and higher spin cubic interactions [11–16]. Recently some of these results have been extended to supersymmetric theories. In a series of papers [17–24] a variety of supersymmetric, higher spin, currents have been constructed for miscellaneous matter and higher spin supermultiplets while the corresponding cubic interactions between matter and higher spin supermultiplets or between higher spin and higher spin supermultiplets have been discussed.

In most of these considerations, the multiplet of supercurrents were found by solving the appropriate conservation equations. However, for [18, 21] the foundation of the construction was the discovery of a linear non-trivial consistent, first order, higher spin transformation of matter superfields. Specifically, it was shown that the most general linear transformation of matter superfields, which is non-trivial and consistent with the various constraints of matter supermultiplets (chiral, or complex linear) is parametrized by terms that match the gauge symmetry of free, massless, half-integer superspin \(Y = s + 1/2\) supermultiplets \((s + 1, s + 1/2)\). The application of Noether’s method to this kind of deformation lead us to the construction of higher spin supercurrents and higher spin supertraces which generate the cubic interactions of the various matter supermultiplets with the half-integer superspin supermultiplets. The construction is reminiscent of the way that linearized superdiffeorphisms lead to the construction of the supergravity supercurrent and supertrace of matter supermultiplets. Nevertheless, the absence of integer superspin \(Y = s\) supermultiplets \((s + 1/2, s)\) from the above consideration was intriguing.

The purpose of this work is to find appropriate higher spin deformations of the matter superfields that lead via Noether’s method to the construction of integer superspin supercurrents and generate the cubic interactions with free, massless integer superspin supermultiplets. We find that there exist non-trivial, antilinear transformations \(^4\) of the matter superfields that will generate these interactions. Specifically, we write the most general antilinear transformation for a chiral superfield, demand it to be non-trivial and compatible with the chiral constraint. The result is that parameters of the transformation have the same structure with the gauge symmetry of free, massless integer superspin supermultiplets. That means by performing Noether’s method, we can construct the integer superspin supercurrent multiplet (includes the supercurrent \(J_{\alpha(s)} \dot{\alpha}(s-1)\) and the supertrace \(T_{\alpha(s-1)} \dot{\alpha}(s-1)\)) and generate the cubic interactions between the free, massless chiral superfield and the integer superspin supermultiplets \((s + 1/2, s)\). The results are extended to the case of a free, massive chiral and a free chiral with linear superpotential.

It is known that any \(\mathcal{N} = 1\) supersymmetric matter theory can be consistently coupled to supergravity with the help of the gravitational superfield. For that case the calculation of the conserved supercurrent is straightforward. One has to take the functional derivative of the interacting action with respect the gravitational superfield (see e.g. [25, 26]). However, this procedure is not applicable for higher spin theory because we do not know the fully interacting theory at present. The only alternative option we have is to follow Noether’s method in order to construct directly the higher spin supercurrent multiplet of the theory.

The paper is organized as follows. Section 2 reviews the philosophy and the details of Noether’s method as well as it provides to the non-expert reader the essentials for the description of 4D, \(\mathcal{N} = 1\) arbitrary integer superspin supermultiplets for both the Poincaré and conformal cases. In section 3, we consider first order transformations (in the spirit of Noether’s method) of the chiral superfield which are antilinear and

\(^4\)A map \(f: V \rightarrow W\) from one complex vector space \(V\) to another \(W\) is called antilinear if \(f(au + bv) = a^*f(u) + b^*f(v)\) where \(a, b\) are complex numbers and \(u, v\) are elements of \(V\). This is equivalent to a linear map from \(V\) to the complex conjugate vector space \(\bar{W}\). The transformations we consider have this property, they are linear in the complex conjugate of the superfield.
demonstrate the fixing of their parameters by requiring them to be consistent with the chiral constraint of
the superfield and non-trivial. Sections 4 and 5 consider the case of a single, free, massless chiral superfield
and derive the conformal and Poincaré supercurrents respectively using the deformations of section 3. In
section 6, we extend this results for the two case of a free, massive chiral superfield and a free chiral with
linear superpotential. In the last section 7, we discuss and summarize our results.

2 Gauge invariant interacting theories of matter with gauge fields

It is a fact of physics that a manifestly Lorentz invariant and local description of massless degrees of
freedom with spin bigger than \(1/2\) requires the identification of various field configurations (gauge symme-
tries). As we transition perturbatively from free theories to interacting ones, the notion of this identification
has to be re-examined in every step. This can be done systematically by expanding the action
\(S[\phi, h]\) and
the transformation of all fields in a power series of a coupling constant \(g\).

\[
S[\phi, h] = S_0[\phi] + gS_1[\phi, h] + g^2S_2[\phi, h] + \ldots ,
\]

\[
\delta \phi = 0 + g\delta_1 \phi + g^2\delta_2 \phi + \ldots ,
\]

\[
\delta h = \delta_0 h + g\delta_1 h + g^2\delta_2 h + \ldots
\]

In the above expressions we consider the interaction of a set of matter fields represented by \(\phi\) with a set
of gauge fields represented by \(h\). Matter fields do not have a zeroth order gauge transformation \((\delta_0 \phi = 0)\),
whereas gauge fields do \((\delta_0 h \neq 0)\). The terms \(S_i[\phi, h]\) correspond to interaction terms of order \(i + 2\) in
the number of fields and \(\delta_i\) is the part of transformation with terms of order \(i\) in the number of fields. The
invariance of the theory under these transformations can be studied iteratively, order by order. For cubic
order terms \(S_1[\phi, h]\) we get

\[
\int \left\{ \frac{\delta S_0}{\delta \phi} \delta_1 \phi + \frac{\delta S_1}{\delta h} \delta_0 h \right\} = 0
\]

The above expression is a symbolic one. There are a number of hidden summations over ‘repeated fields’,\(^5\) which are compressed down to the integral sign. This invariance condition (up to cubic terms) makes
very clear the importance of the first order correction in the transformation of the matter fields, \(\delta_1 \phi\). The
starting action \(S_0\) is known and the zeroth order transformations of gauge fields are also known. Hence, in
order to find a consistent set of non-trivial cubic interactions \(S_1[\phi, h]\) we must find a non-trivial \(\delta_1 \phi\). In
this consideration, trivial interactions and trivial transformations are the ones that can be absorbed by an
appropriate redefinition of the fields or in other words they vanish under the consideration of the equations
of motion.

Cubic interactions of a matter theory with gauge fields can be written in the form \(j h\) where \(j\) is a current
constructed out of the matter fields which plays the role of the source. For these types of interactions,
condition (4) takes the form

\[
\int \left\{ \frac{\delta S_0}{\delta \phi} \delta_1 \phi + j \delta_0 h \right\} = 0
\]

from which one can recover the conservation law of the current \(j\) by using the equations of motion (up
to the appropriate order, for this case it is \(\frac{\delta S_0}{\delta \phi} = 0\)) and the structure of the gauge transformation of \(h\)
\((\delta_0 h = \partial \lambda)\).

\(^5\)There is a summation over hidden external indices that count the number of matter and gauge fields, there is a
summation over the hidden spacetime indices of the gauge fields and an integration over the spacetime coordinates.
In recent papers [18, 21], this approach has been used in order to construct conserved, higher spin supercurrents for the chiral \((\Phi, \bar{D}_A \Phi = 0)\) and complex linear \((\Sigma, \bar{D}^2 \Sigma = 0)\) supermultiplets. In these papers we considered the most general, non-trivial, first order transformations \(\delta \Phi, \delta_1 \Sigma\) which depend linearly on \(\Phi\) and \(\Sigma\) respectively. These transformations are a higher spin extension of linearized superdiffeomorphism and like superdiffeomorphism generate interactions to supergravity supermultiplet \((2,3/2)\), they generate interactions to arbitrary higher spin supermultiplets of type \((s + 1, s + 1/2)\) (called half-integer superspin supermultiplets) for any non-negative integer \(s\).

In this work, we explore the possibility of non-trivial, first order transformations that depend antilinearly on the matter superfield. In the following sections we will find that such transformations do exist and generate interactions to arbitrary higher spin supermultiplets of type \((s + 1/2, s)\) (called integer superspin supermultiplets). We briefly remind the non-expert reader that the superspace lagrangian description of free, massless, super-Poincaré, arbitrary high \((s \geq 2)\), integer superspin supermultiplet involves a fermionic superfield \(\Psi_{\alpha(s)\dot{\alpha}(s-1)}\) and a real bosonic superfield \(V_{\alpha(s-1)\dot{\alpha}(s-1)}\) with the following zeroth order gauge transformations

\[
\delta_0 \Psi_{\alpha(s)\dot{\alpha}(s-1)} = -\mathcal{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{D}(\dot{\alpha}_{s-1}) \Lambda_{\alpha(s)\dot{\alpha}(s-2)} ,
\]

\[
\delta_0 V_{\alpha(s-1)\dot{\alpha}(s-1)} = \bar{D}^{\dot{\alpha}s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)} .
\]

(6a) (6b)

Off-shell, this supermultiplet carries \(8s^2 + 8s + 4\) bosonic and equal number of fermionic degrees of freedom\(^6\). The physical\(^7\) (propagating) degrees of freedom are described by a field strength\(^8\) superfield \(\mathcal{W}_{\alpha(2s)}\)

\[
\mathcal{W}_{\alpha(2s)} \sim \bar{D}^2 D_{(\alpha_2} \partial_{\alpha_{2s-1}} \dot{\alpha}_{s-1} \partial_{\alpha_{2s-2}} \dot{\alpha}_{s-2} \ldots \partial_{\alpha_{s+1}} \dot{\alpha}_1 \Psi_{\alpha(s)\dot{\alpha}(s-1)}
\]

(7)

The super-field strength is chiral \((\bar{D}_\beta \mathcal{W}_{\alpha(2s)} = 0)\) and on-shell satisfies the following equation of motion:

\[
D^\beta \mathcal{W}_{\beta \alpha(2s-1)} = 0
\]

(8)

There is also a super-conformal integer superspin supermultiplet. It’s lagrangian description is given in terms of the super-field strength \(\mathcal{W}_{\alpha(2s)}\). Similarly with the super-Poincaré case, the super-field strength can be expressed in terms of a prepotential \(\Psi_{\alpha(s)\dot{\alpha}(s-1)}\) (as in (7)) whose gauge transformation saturates the maximum symmetry group of \(\mathcal{W}_{\alpha(2s)}\)

\[
\delta_0 \Psi_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{2!} D(\alpha_s \Xi_{\alpha(s-1)} \dot{\alpha}(s-1)) + \frac{1}{(s-1)!} \bar{D}(\dot{\alpha}_{s-1}) \Lambda_{\alpha(s)\dot{\alpha}(s-2)}
\]

(9)

The interested reader can find the details of 4D, \(\mathcal{N} = 1\) integer and half-integer superspin, irreducible, representations in the following original references [27] (component description with on-shell supersymmetry) [28–31] (superspace description) and later studies [32] (superspace description with use of prepotentials) [33] (BRST component description with on-shell supersymmetry) [34] (off-shell component completion of on-shell supersymmetry).

3 Non-trivial first order transformation

Let’s consider a chiral superfield \(\Phi\). The most general antilinear transformation one can write is \(^9\):

\(^6\)A detailed counting of the off-shell degrees of freedom can be found in [32, 34].
\(^7\)The on-shell degrees of freedom are the 2 helicities of spin \(j = s + 1/2\) and the two helicities of spin \(j = s\)
\(^8\)This is the simplest gauge invariant object that does not vanish on-shell.
\(^9\)We are following the Superspace [25] conventions.
\[
\delta_1 \Phi = \sum_{k=0}^{\infty} \left\{ A^{\alpha(k)\dot{\alpha}(k+1)} D_{\dot{\alpha}_{k+1}} D_{\alpha_k} D_{\dot{\alpha}_k} \ldots D_{\alpha_1} D_{\dot{\alpha}_1} \bar{\Phi} + \Delta^{\alpha(k)\dot{\alpha}(k)} D_{\alpha_k} D_{\dot{\alpha}_k} \ldots D_{\dot{\alpha}_1} \bar{\Phi} + \Gamma^{\alpha(k+1)\dot{\alpha}(k)} D_{\alpha_{k+1}} D^2_{\alpha_k} D_{\dot{\alpha}_k} \ldots D_{\dot{\alpha}_1} \bar{\Phi} + E^{\alpha(k)\dot{\alpha}(k)} D^2_{\alpha_k} D_{\dot{\alpha}_k} \ldots D_{\dot{\alpha}_1} \Phi \right\} .
\]

The consistency of this transformation with the chiral condition of \( \Phi \), \( \bar{D}_\alpha \Phi = 0 \) constraints the parameters of the transformation in the following way:

\[
\bar{D}_\beta A^{\alpha(k)\dot{\alpha}(k+1)} + \frac{1}{(k+1)!} \Delta^{\alpha(k)\dot{\alpha}(k)} \delta^{\dot{\alpha}(k+1)}_\beta = 0 ,
\]

\[
\bar{D}_\beta \Delta^{\alpha(k)\dot{\alpha}(k)} = 0 ,
\]

\[
\frac{k+1}{k+2} \Delta^{\alpha(k+1)\dot{\alpha}(k+1)} C^\beta_{\dot{\alpha}_{k+1}} + \bar{D}_\beta \Gamma^{\alpha(k+1)\dot{\alpha}(k)} = 0 ,
\]

\[
A^{\alpha(k+1)\dot{\alpha}(k+2)} C^\beta_{\dot{\alpha}_{k+2}} + \bar{D}_\beta E^{\alpha(k+1)\dot{\alpha}(k+1)} - \frac{1}{(k+1)!} \Gamma^{\alpha(k+1)\dot{\alpha}(k)} \delta^{\dot{\alpha}(k+1)}_\beta = 0 ,
\]

\[
A^{\alpha} C^\beta_{\dot{\alpha}} + \bar{D}_\beta E = 0 .
\]

The solution of the above set of constraints is:

\[
A^{\alpha(k)\dot{\alpha}(k+1)} = \frac{1}{(k+1)!} \bar{D}_{\dot{\alpha}_{k+1}} \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} - \bar{D}^2 \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} ,
\]

\[
\Delta^{\alpha(k)\dot{\alpha}(k)} = \bar{D}^2 \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} ,
\]

\[
\Gamma^{\alpha(k+1)\dot{\alpha}(k)} = \frac{k+1}{k+2} \bar{D}^2 \bar{\xi}^{\alpha(k+1)\dot{\alpha}(k+1)} ,
\]

\[
E^{\alpha(k)\dot{\alpha}(k)} = -\bar{\ell}^{\alpha(k)\dot{\alpha}(k)} - \bar{D}^{k+1} \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} .
\]

where \( \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} \) and \( \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} \) are arbitrary, unconstrained superfields. The \( \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} \) terms can be ignored because they correspond to a field redefinition of \( \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} \). However, because the \( \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} \) and the \( \bar{\ell}^{\alpha(k+1)\dot{\alpha}(k+1)} \) parts of (12a) match the terms that appear in the complex conjugate versions of the conformal (9) and Poincaré (6a) transformations respectively, it will be convenient to consider separately the \( \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} \) and \( \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} \) parts of (12). Additionally, the redefinition that connects the \( \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} \) with the \( \bar{\ell}^{\alpha(k+1)\dot{\alpha}(k+1)} \) terms in (12) corresponds to the redefinition that relates the \( \Xi^{\alpha(s)\dot{\alpha}(s-1)} \) term of (9) for the conformal case transformation with the \( L^{\alpha(s)\dot{\alpha}(s-1)} \) term of (6a) for the Poincaré case transformation.

This difference between the gauge symmetries of the conformal and Poincaré supermultiplets appears only for integer superspin, whereas for half-integer superspin the corresponding superfield (a real bosonic \( H_{\alpha(s)\dot{\alpha}(s)} \)) has the same gauge transformation in both conformal and Poincaré supermultiplets. In [35] it was demonstrated that for the matter gravitino supermultiplet \( Y = 1 \) \((3/2, 1)\) one can relax the Poincaré gauge transformation to match the conformal one, by adding another compensating superfield with an algebraic (no derivatives) transformation law. Recently [20] this mechanism was applied to higher integer superspin supermultiplets. However, this description is non-economical (requires more superfields than it is necessary) and one can always use the algebraic nature of the transformation of the additional compensator in order to remove it. Hence, we will work using the (6) description where the transformations of the conformal and Poincaré supermultiplets differ.

\[^{10}\text{Notice that the redefinition } \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} \rightarrow \bar{\xi}^{\alpha(k)\dot{\alpha}(k)} - \bar{D}^{\alpha+1} \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} \text{ will absorb all the } \bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)} \text{ terms.}\]
Furthermore, it is straightforward to check that the $\Gamma^{\alpha(k+1)\alpha(k)}$ and $E^{\alpha(k)\bar{\alpha}(k)}$ terms of (10) correspond to trivial redefinition of $\Phi$ (see [18]), hence they can be dropped independently of the values (12c,12d) they take. Therefore, from this point forward we will consider the following two transformations:

1. $\delta_1 \Phi = \sum_{k=0}^{\infty} \left\{ \frac{1}{(k+1)!} \bar{D}^{(\bar{\alpha}_k+1)} \bar{\zeta}(\alpha(k)) \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \partial \bar{\Phi} + \bar{D}^2 \bar{\zeta}(\alpha(k)) \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \bar{\Phi} \right\}$ (13a)

2. $\delta_1 \Phi = \sum_{k=0}^{\infty} \left\{ \bar{D}^2 \bar{\zeta}(\alpha(k+1)) \bar{D}_{\bar{\alpha}_k+1} \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \bar{\Phi} \right\}$ (13b)

We will demonstrate that the first will generate the conformal integer superspin supercurrents and the corresponding interactions with the conformal integer superspin supermultiplets, whereas the second will lead to the Poincaré integer superspin supercurrents and the interactions of matter with Poincaré integer superspin supermultiplets.

### 4 Conformal integer superspin supercurrents for free massless chiral

Let’s consider the case of a single, free, massless chiral superfield

$$S_0 = \int d^8 z \ (\bar{\Phi} \Phi) \ .$$

Using (13a) we calculate the change of the above action to be:

$$\delta_S S_0 = g \sum_{k=0}^{\infty} \int \left[ \frac{1}{(k+1)!} \bar{D}^{(\bar{\alpha}_k+1)} \bar{\zeta}(\alpha(k)) \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \partial \bar{\Phi} \right] + \bar{D}^2 \bar{\zeta}(\alpha(k)) \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \bar{\Phi} + c.c. \ (15)$$

The quantities inside the curly brackets are not uniquely defined because one can consider improvement terms $A_{\alpha(k+1)\alpha(k)}$ and $B_{\alpha(k)\alpha(k)}$ that satisfy $11$:

$$D^{\alpha(k+1)} A_{\alpha(k+1)\alpha(k)} = D^2 B_{\alpha(k)\alpha(k)} \ .$$

A general expression for the improvement terms is

$$A_{\alpha(k+1)\alpha(k)} = \frac{k+1}{(k+2)!} D(\alpha_{k+1}) \bar{\zeta}(\alpha(k)) \bar{\alpha}(k) + \frac{1}{k!} \bar{D}_{\bar{\alpha}_k} D^2 \bar{\alpha}_{\alpha(k+1)\alpha(k+1)} + X_{\alpha(k+1)\alpha(k)} \ ,$$

$$B_{\alpha(k)\alpha(k)} = \bar{\zeta}(\alpha(k)) \bar{\alpha}(k) + \frac{1}{k!} \bar{D}_{\bar{\alpha}_k} D^{\alpha(k+1)} \bar{\alpha}_{\alpha(k+1)\alpha(k+1)} + Y_{\alpha(k)\alpha(k)} \ ,$$

where $D^{\alpha(k+1)} X_{\alpha(k+1)\alpha(k)} = 0$ and $D^2 Y_{\alpha(k)\alpha(k)} = 0$ up to trivial redefinition terms, such as terms that depend on the on-shell equations of motion. The superfield $X_{\alpha(k+1)\alpha(k)}$ may include terms like $D^{\alpha(k+2)} P_{\alpha(k+1)\alpha(k)}^{(1)}$ or $D^2 P_{\alpha(k+1)\alpha(k)}^{(2)}$ which identically satisfy $X$’s constraint due to the algebra of the covariant spinorial derivatives. However, it is important to state that there can be non-trivial solutions which do not fit into this form. An example of this has been demonstrated in [22]. A similar statement holds true for superfield $Y_{\alpha(k)\alpha(k)}$. Therefore, equation (15) can be written in the following way

$$\delta_S S_0 = g \sum_{k=0}^{\infty} \int \left[ \frac{1}{(k+1)!} \bar{D}^{(\bar{\alpha}_k+1)} \bar{\zeta}(\alpha(k)) \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \partial \bar{\Phi} \right] + \bar{D}^2 \bar{\zeta}(\alpha(k)) \bar{D}_{\bar{\alpha}_k} \bar{D}_{\bar{\alpha}_k} \ldots \bar{D}_{\bar{\alpha}_1} \bar{D}_{\bar{\alpha}_1} \bar{\Phi} + c.c. \ (18)$$

where

$^{11}$ $T_P$ to trivial redefinition terms, such as terms that depend on the zeroth order equations of motion. See [18] for examples.
\[ \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} = (-i)^k \Phi \partial^{(k)} \mathcal{D} \Phi + \sum_{k=0}^{\infty} \frac{1}{(k+2)!} \mathcal{D}^{(\alpha(k+1)\dot{\alpha}(k))} + \frac{1}{k!} \tilde{\mathcal{D}}^{(\alpha(k+1)\dot{\alpha}(k))} + X_{\alpha(k+1)\dot{\alpha}(k)}, \quad (19a) \]

\[ \mathcal{T}_{\alpha(k)\dot{\alpha}(k)} = (-i)^k \Phi \partial^{(k)} \Phi + \zeta_{\alpha(k)\dot{\alpha}(k)} + \frac{1}{k!} \tilde{\mathcal{D}}^{(\alpha(k)\dot{\alpha}(k))} + Y_{\alpha(k)\dot{\alpha}(k)}. \quad (19b) \]

Exploiting the freedom of the unconstrained \( \zeta_{\alpha(k)\dot{\alpha}(k)} \) improvement term we can select it appropriately such that \( \mathcal{T}_{\alpha(k)\dot{\alpha}(k)} = 0 \). With this choice, the variation of \( S_0 \) reduces to:

\[ \delta_g S_0 = g \sum_{k=0}^{\infty} \int \left[ \frac{1}{(k+1)!} \mathcal{D}^{(\alpha(k)\dot{\alpha}(k))} \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} \right] + c.c. \quad (20) \]

with

\[ \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} = \frac{(-i)^k}{k+2} \Phi \partial^{(k)} \mathcal{D} \Phi - \frac{k+1}{k+2} (-i)^k \mathcal{D} \Phi \partial^{(k)} \Phi + X_{\alpha(k+1)\dot{\alpha}(k)} - \frac{k+1}{(k+2)!} \mathcal{D}^{(\alpha(k+1)\dot{\alpha}(k))} + \frac{1}{k!} \tilde{\mathcal{D}}^{(\alpha(k+1)\dot{\alpha}(k))} + \frac{1}{(k+2)!} \mathcal{D}^{(\alpha(k+1)\dot{\alpha}(k))} + X_{\alpha(k+1)\dot{\alpha}(k)}, \quad (21) \]

The \( Y_{\alpha(k)\dot{\alpha}(k)} \) and \( \kappa_{\alpha(k+1)\dot{\alpha}(k)} \) terms of (21) can be absorbed by appropriate redefinition of \( X_{\alpha(k+1)\dot{\alpha}(k)} \) (they are consistent with \( \mathcal{D}^{\alpha(k+1)} X_{\alpha(k+1)\dot{\alpha}(k)} = 0 \)), hence we can simplify the expression for \( \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} \):

\[ \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} = \frac{(-i)^k}{k+2} \Phi \partial^{(k)} \mathcal{D} \Phi - \frac{k+1}{k+2} (-i)^k \mathcal{D} \Phi \partial^{(k)} \Phi + X_{\alpha(k+1)\dot{\alpha}(k)} \quad (22) \]

In order to get consistent interactions with conformal integer superspin supermultiplets \((\Phi_\alpha s\dot{\alpha}(s-1))\) we have to consider the full transformation (9) and not just a part of it as it appears in (20). However we can write\(^{12}\)

\[ \delta_g S_0 = g \sum_{k=1}^{\infty} \int \left[ \left\{ \frac{1}{(k+1)!} \mathcal{D}^{(\alpha(k)\dot{\alpha}(k))} + \frac{1}{k!} \tilde{\mathcal{D}}^{(\alpha(k)\dot{\alpha}(k))} \right\} \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} \right] + c.c. \quad (23) \]

if and only if \( \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} \) has the property \( \tilde{\mathcal{D}}^{\dot{\alpha}k} \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)} = 0 \), identically. An encouraging observation towards this direction is that the first term in (22) has this property. However, the last two terms of (22) do not comply for generic \( X_{\alpha(k+1)\dot{\alpha}(k)} \). This is reasonable because both these terms originated from the improvement terms consideration and include a lot of freedom. Hence, we must choose the improvement term \( X_{\alpha(k+1)\dot{\alpha}(k)} \) appropriately such that

\[ \mathcal{D}^{\alpha(k+1)} X_{\alpha(k+1)\dot{\alpha}(k)} = 0 \quad (up \ to \ equations \ of \ motion), \quad (24a) \]

\[ \tilde{\mathcal{D}}^{\dot{\alpha}k} \left[ X_{\alpha(k+1)\dot{\alpha}(k)} - \frac{k+1}{k+2} (-i)^k \mathcal{D} \Phi \partial^{(k)} \Phi \right] = 0 \quad (identically). \quad (24b) \]

These two conditions will uniquely fix the improvement term \( X_{\alpha(k+1)\dot{\alpha}(k)} \). To find the explicit expression of \( X_{\alpha(k+1)\dot{\alpha}(k)} \), let’s consider the ansatz

\[ X_{\alpha(k+1)\dot{\alpha}(k)} = \sum_{p=0}^{k} c_p \partial^{(p)} \mathcal{D} \Phi \partial^{(k-p)} \Phi. \]

Constraints (24) are equivalent to:

\(^{12}\)Notice that we ignored the \( k = 0 \) term, because it does not correspond to higher spin supermultiplets \( k \geq 1 \) but to the matter gravitino multiplet. Although the analysis will go through even in that case, for simplicity reasons we will not include it.
\[ c_{k-p} = -\frac{p+1}{k-p+1} c_p, \quad p = 0, 1, \ldots, k \]  \hspace{1cm} (24c)
\[ c_{k-p-1} = \frac{k-p}{p+1} c_p, \quad p = 1, 2, \ldots, k-2 \]  \hspace{1cm} (24d)
\[ c_{k-1} = k c_0 - \frac{k(k+1)}{k+2} (-i)^k \]  \hspace{1cm} (24e)

It is straightforward to prove that this system of recursive equations has a solution only for odd values of \( k \) (\( k = 2l + 1, \ l = 0, 1, 2, \ldots \))
\[ c_k = c_0 = 0, \]  \hspace{1cm} (25a)
\[ c_p = -\frac{(-i)^k}{k+2} (-1)^p \left( \frac{k}{p} \right) \left( \frac{k+1}{p+1} \right), \quad p = 1, 2, \ldots, k-1, \ k = 2l + 1. \]  \hspace{1cm} (25b)

Therefore, we conclude that a single, free, massless, chiral supermultiplet can have cubic interactions only with even integer superspin conformal supermultiplets (\( Y = 2l + 2, \ l = 0, 1, \ldots \))
\[ S_I = -g \sum_{l=0}^{\infty} \int \Psi^\alpha(2l+2) \hat{\alpha}(2l+1) \ J_{\alpha(2l+2)\hat{\alpha}(2l+1)} + c.c. \]  \hspace{1cm} (26)

which are generated by the integer superspin supercurrent \( J_{\alpha(2l+2)\hat{\alpha}(2l+1)} \):
\[ J_{\alpha(2l+2)\hat{\alpha}(2l+1)} = \frac{i}{2l+3} \sum_{p=0}^{2l+1} (-1)^p \left( \frac{2l+1}{p} \right) \left( \frac{2l+2}{p+1} \right) \tilde{\partial}^{(p)} D\Phi \tilde{\partial}^{(2l+1-p)} \Phi. \]  \hspace{1cm} (27)

Furthermore, one can check that \( J_{\alpha(2l+2)\hat{\alpha}(2l+1)} \) satisfies the following conservation equations:
\[ D^{\alpha(2l+2)} J_{\alpha(2l+2)\hat{\alpha}(2l+1)} = 0, \ \tilde{D}^{\hat{\alpha}(2l+1)} J_{\alpha(2l+2)\hat{\alpha}(2l+1)} = 0. \]  \hspace{1cm} (28)

Expression (27) matches the Minkowski superspace limit of the AdS integer superspin supercurrents constructed in [23].

5 Poincaré integer superspin supercurrents for free massless chiral

Now, let’s consider the effects of (13b) on the single, free, massless chiral action (14). We get:
\[ \delta_g S_0 = g \sum_{k=0}^{\infty} \int \left[ D^2 \ell^{\alpha(k+1)\hat{\alpha}(k)} \left\{ (-i)^k \Phi \tilde{\partial}^{(k)} D\Phi \right\} \right] + c.c. \]  \hspace{1cm} (29)

As mentioned previously, the term inside the curly bracket has the property \( \tilde{D}^{\hat{\alpha}k} \left\{ (-i)^k \Phi \tilde{\partial}^{(k)} D\Phi \right\} = 0 \), hence we can rewrite (29)
\[ \delta_g S_0 = g \sum_{k=1}^{\infty} \int \left[ \left\{ D^2 \ell^{\alpha(k+1)\hat{\alpha}(k)} + \frac{1}{N} \tilde{D}^{\hat{\alpha}k} \chi^{\alpha(k+1)\hat{\alpha}(k-1)} \right\} J_{\alpha(k+1)\hat{\alpha}(k)} \right] + c.c. \]  \hspace{1cm} (30)

where
\[ J_{\alpha(k+1)\hat{\alpha}(k)} = (-i)^k \Phi \tilde{\partial}^{(k)} D\Phi \]  \hspace{1cm} (31)

Following Noether’s method we find that this supercurrent generates the following cubic interactions between free massless chiral supermultiplet and the Poincaré integer superspin supermultiplet
\[ S_I = -g \sum_{s=2}^{\infty} \int \Psi^{\alpha(s)\hat{\alpha}(s-1)} J_{\alpha(s)\hat{\alpha}(s-1)} + c.c. \]  \hspace{1cm} (32)
In contrast with the previous conformal case, the supercurrent (and the cubic interaction) is defined for every positive integer $s$ and not just for the even values. Furthermore, one can prove that $J_{\alpha(s)\bar{\alpha}(s-1)}$ satisfy the following conservation equations:

$$D^2 J_{\alpha(s)\bar{\alpha}(s-1)} = 0 , \quad \bar{D}^{\dot{a}_{s-1}} J_{\alpha(s)\bar{\alpha}(s-1)} = 0$$

(33)

and crucially $D^{a_s} J_{\alpha(s)\bar{\alpha}(s-1)} \neq 0$, thus can not be related to the conformal supercurrent.

6 Integer superspin supercurrent multiplet beyond free, massless, chiral

In [22] we investigated the construction of half-integer superspin supercurrent multiplet for a general class of non-linear sigma models of a single chiral superfields, parametrized by an arbitrary Kähler potential $K(\Phi, \bar{\Phi})$ and a chiral superpotential $\mathcal{W}(\Phi)$. The result was that besides the free, massless case, the arbitrary half-integer superspin supercurrent multiplets exist only for $K(\Phi, \bar{\Phi}) = \Phi \bar{\Phi}$ with $\mathcal{W}(\Phi) = f \Phi$ or $\mathcal{W}(\Phi) = m \Phi^2$, which is consistent with the expectations coming from [36–38]. Therefore, it will be interesting to investigate the existence of arbitrary integer superspin supercurrents for the cases of a free chiral with a linear superpotential or a free massive chiral superfield.

In both cases there is a dimension-full parameter, therefore only the super-Poincaré discussion is relevant. The general cubic interaction of the chiral with the Poincaré integer superspin supermultiplet $Y = s$ (6) has the form

$$S_I = \int d^8 z \left[ \Psi^{\alpha(s)\bar{\alpha}(s-1)} J_{\alpha(s)\bar{\alpha}(s-1)} + \frac{i}{2} V^{\alpha(s-1)\bar{\alpha}(s-1)} \mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)} \right] + \text{c.c.}$$

(34)

where $J_{\alpha(s)\bar{\alpha}(s-1)}$ is the arbitrary integer superspin supercurrent and $\mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)}$ is the arbitrary integer superspin supertrace. Due to the gauge symmetries (6) the supercurrent and the supertrace have to respect the following:

$$D^2 J_{\alpha(s)\bar{\alpha}(s-1)} = \frac{1}{s!} D(\alpha_s \mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)}) ,$$

$$\bar{D}^{\dot{a}_{s-1}} J_{\alpha(s)\bar{\alpha}(s-1)} = 0$$

(35a)

(35b)

where $\mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)}$ is real ($\mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)} = \mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)}$). In previous section, we demonstrated that for the free, massless chiral case the supertrace vanishes. However if we go beyond that, it is reasonable to expect corrections proportional to the dimension-full parameter that controls the added terms.

6.1 Free chiral with linear superpotential

Let’s consider the addition of a linear superpotential term in in (14) controlled by a complex parameter $f$:

$$S_0 = \int d^8 z \bar{\Phi} \Phi + f \int d^6 z \bar{\Phi} + f^* \int d^6 z \bar{\Phi}$$

(36)

It is straightforward to show that in this case the supercurrent and supertrace are:

$$J_{\alpha(s)\bar{\alpha}(s-1)} = (-i)^{s-1} \Phi \partial^{(s-1)} \Phi ,$$

$$\mathcal{T}_{\alpha(s-1)\bar{\alpha}(s-1)} = (-i)^{s-1} f^* \partial^{(s-1)} \Phi + (i)^{s-1} f \partial^{(s-1)} \bar{\Phi}$$

(37a)

(37b)

The term supertrace originates from the cubic interaction of matter supermultiplets with the compensator of Poincaré supergravity (see [26]). We have been using the same terminology for the higher spin version of this type of interactions, meaning the interactions with the compensator of the Poincaré half-integer superspin supermultiplet. For the case of integer superspin supermultiplets, we will continue to use it in the same spirit. The supertrace generates the cubic interactions with the compensator of the Poincaré integer superspin supermultiplet.
The supercurrent is the same as in the free, massless case (31) and both the supercurrent and supertrace can be defined for any \( s \geq 2 \) value of the integer \( s \).

6.2 Free massive chiral

Let’s consider the addition of a mass term in (14)

\[
S_0 = \int d^8 z \bar{\Phi} \Phi + m \int d^6 z \bar{\Phi}^2 + m \int d^6 z \bar{\Phi}^2
\]

with a real mass parameter \( m \). In this case one can show that the integer superspin supercurrent and supertrace exist only for even values of \( s \) \((s = 2, 4, 6, \ldots)\) and they are:

\[
\mathcal{J}_{\alpha(2l+2)\dot{\alpha}(2l+1)} = (-1)^{(l+1)} i \bar{\Phi} \partial^{(2l+1)} \Phi ,
\]

\[
\mathcal{T}_{\alpha(2l+1)\dot{\alpha}(2l+1)} = (-1)^{(l+1)} i m \bar{\Phi} \partial^{(2l+1)} \Phi - (-1)^{(l+1)} i m \bar{\Phi} \partial^{(2l+1)} \bar{\Phi} .
\]

7 Summary and discussion

In recent work [18, 21] it has been demonstrated the existence of a linear, higher spin transformation, of matter superfields, such as the chiral. Noether’s procedure for this type of transformation lead to the cubic interactions between various matter supermultiplets and half-integer superspin supermultiplets by the construction of arbitrary high, half integer superspin supercurrent multiplets that generate these interactions. In this work, we prove the existence of an antilinear, higher spin transformation of the chiral superfield which via Noether’s method leads to cubic interactions between the chiral and the integer superspin supermultiplets and the construction of the corresponding integer superspin supercurrents. This is a very interesting feature because antilinear transformations do not appear frequently in physics, unlike linear transformations (e.g. (super)diffeomorphisms).

We considered the most general, non-trivial, antilinear transformation of a chiral superfield which is consistent with the chiral constraint. In this case non-trivial means that we can not absorb the transformation by trivial redefinitions. For example, we can ignore terms that depend on equations of motion and thus vanish on-shell. We found two classes [(13a) and (13b)] of transformations of this type and surprisingly the parameters of these transformations match the gauge transformations of free, massless integer superspin supermultiplets for the conformal and the Poincaré case respectively.

Moreover, we studied the effect of these transformations by using them to perform Noether’s method on a matter action \( S_0 \) that depends on a single chiral superfield. Considering the simplest possible starting theory, the free, massless, chiral we find that:

\( i \). Transformation (13a) leads to the construction of cubic interactions (26) with the conformal integer superspin supermultiplet \( Y = s \) but only for even values of \( s \) \((s = 2, 4, \ldots = 2l + 2)\). The interactions are generated by the integer superspin supercurrent \( \mathcal{J}_{\alpha(2l+2)\dot{\alpha}(2l+1)} \) given by (27), which satisfies conservation equations (28). This result is consistent with the flat spacetime limit of the results in [23] where the AdS conformal integer superspin supercurrent was constructed by solving the conservation equation.

\( ii \). Transformation (13b) leads to the construction of cubic interactions (32) with the Poincaré integer superspin supermultiplet \( Y = s \) for all values of \( s \). The interactions are generated by the supercurrent \( \mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} \) given by (31) which satisfies the Poincaré conservation equations (33) but not the conformal conservation equations.
Next, we considered matter theories beyond the simple free massless theory. However based on \[36-38,22\] the most general theory we can consider for the construction of higher spin supercurrents is the free, massless theory with the addition of a superpotential with linear and quadratic terms. Due to the presence of dimension-full parameters that control these additional terms, only the Poincaré results can be extended and we found the following:

iii. For a free chiral superfield with a linear superpotential, we can construct cubic interactions with the Poincaré integer superspin supermultiplet \( Y = s \) for all values of \( s \). The interactions are generated by a supercurrent \( J_{\alpha(s)\dot{\alpha}(s-1)} \) and a supertrace \( T_{\alpha(s-1)\dot{\alpha}(s-1)} \) given by (37) with conservation equations (35).

iv. For a free, massive chiral we find cubic interactions with the Poincaré integer superspin supermultiplet \( Y = s \), but only for even values of \( s \) \( (s = 2, 4, \cdots = 2l + 2) \). The supercurrent and supertrace that generate the interaction are given by (39).

In this work, we considered chiral superfields to represent the matter supermultiplets. However, similar constructions can be done for complex linear superfields as is demonstrated in [21]. The fastest and easiest method to extract the corresponding supercurrent multiplets for a complex linear superfield is to use the chiral - complex linear duality. Starting from the supercurrents for the chiral and performing the duality one can get the supercurrents for the complex linear as well as the relative coupling constant which relates the charge of these two matter multiplets for the interaction with higher superspins (see [21]). Additionally, it will be useful to comment that for any of the above higher spin supercurrent multiplets, one can project the corresponding superspace conservation equations to components in order to find the usual spacetime conservation equations and the corresponding higher spin current multiplets. This has been illustrated in detail in [18, 21].

Recently, the results of [18, 21] have been criticized in [23]. For this reason, we want to emphasize once more the context of our work and the findings presented here and in [18, 21]. The aim is to construct consistent cubic interactions between matter (chiral or complex linear) and higher superspin supermultiplets that are generated by supercurrent multiplets. In [18, 21] transformations of the matter superfields which depend linearly in the matter superfield itself were considered. It was demonstrated that Noether’s method for such type of transformations gives interactions only with half-integer superspin multiplets. Additionally, various selection rules where found for the various matter models (massive, massless, linear superpotential). In [23], using a different method (solving the superspace conservation equations) various supercurrents on AdS space have been constructed for both integer and half-integer superspins. The authors of [23] interpreted this as a reason to criticize our results [23].

The general summary of the results presented here and in [18, 21] is that (a.) linear, nontrivial transformations of matter superfields, consistent with their constraints (chiral or linear) generate interactions with the half-integer superspin supermultiplets, (b.) antilinear, nontrivial transformations of the matter superfields, consistent with the corresponding constraints generate interactions with integer superspin supermultiplets and (c.) depending on the characteristics of the matter theory (massive, massless, linear superpotential) there exist various selection rules.

\[14\] In communications of one of the authors (K. K. ) with the authors of [23], we explained the context of our work by finding interactions using Noether’s method to an appropriate set of linear, higher spin transformations. Furthermore, we informed them that the integer superspin supercurrents they constructed will be part of the supercurrents obtained by appropriate antilinear transformations, exactly as demonstrated by the results of this work. In the second preprint version of [23], this argument was used and there was an attempt to describe symbolically such antilinear transformations however the results were trivial because correspond to superfield redefinitions, as explained in [18].
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References

[1] F. A. Berends, G. J. H. Burgers and H. van Dam, “Explicit Construction of Conserved Currents for Massless Fields of Arbitrary Spin”, Nucl. Phys. B 271 (1986) 429.

[2] D. Anselmi, “Higher spin current multiplets in operator product expansions”, Class. Quant. Grav. 17 (2000) 1383 [hep-th/9906167].

[3] M. A. Vasiliev, “Higher spin gauge theories: Star-product and AdS space” in M. Shifman ed., The many faces of the superworld (World Scientific, 2000) [arXiv:hep-th/9910096].

[4] S. E. Konstein, M. A. Vasiliev and V. N. Zaikin, “Conformal higher spin currents in any dimension and AdS/CFT correspondence”, JHEP 0012 (2000) 018 [arXiv:hep-th/0010239].

[5] O. A. Gelfond, E. D. Skvortsov and M. A. Vasiliev, “Higher spin conformal currents in Minkowski space”, Theor. Math. Phys. 154 (2008) 294 [arXiv:hep-th/0601106].

[6] X. Bekaert, E. Joung and J. Mourad, “On higher spin interactions with matter”, JHEP 0905 (2009) 126 [arXiv:0903.3338 [hep-th]].

[7] X. Bekaert, E. Meunier, “Higher spin interactions with scalar matter on a constant curvature space-times: conserved current and cubic coupling generating functions”, JHEP 1011 (2010) 116 [arXiv:1007.3338].

[8] X. Bekaert, E. Joung and J. Mourad, “Effective action in a higher spin background”, JHEP 1102 (2011) 048 [arXiv:1012.2103].

[9] C. Sleight and M. Taronna, “Higher Spin Interactions from Conformal Field Theory: The Complete Cubic Couplings,” Phys. Rev. Lett. 116 (2016) no.18, 181602 [arXiv:1603.00022 [hep-th]].

[10] O. A. Gelfond and M. A. Vasiliev, “Current Interactions from the One-Form Sector of Nonlinear Higher-Spin Equations,” Nucl. Phys. B 931 (2018) 383 [arXiv:1706.03718 [hep-th]].

[11] E. S. Fradkin and R. R. Metsaev, “A Cubic interaction of totally symmetric massless representations of the Lorentz group in arbitrary dimensions,” Class. Quant. Grav. 8 (1991) L89–L94.

[12] R. R. Metsaev, “Note on the cubic interaction of massless representations of the Poincare group in D = 5 space-time,” Class. Quant. Grav. 10 (1993) L39–L42.

[13] R. R. Metsaev, “Cubic interaction vertices of totally symmetric and mixed symmetry massless representations of the Poincare group in D = 6 space-time,” Phys. Lett. B309 (1993) 39–44.
[14] R. R. Metsaev, “Generating function for cubic interaction vertices of higher spin fields in any dimension,” *Mod. Phys. Lett.* **A8** (1993) 2413–2426.

[15] R. R. Metsaev, “Cubic interaction vertices for massive and massless higher spin fields,” *Nucl. Phys.* **B759** (2006) 147–201, arXiv:hep-th/0512342.

[16] R. R. Metsaev, “Cubic interaction vertices for fermionic and bosonic arbitrary spin fields,” *Nucl. Phys. B* **859** (2012) 13 [arXiv:0712.3526 [hep-th]].

[17] S. M. Kuzenko, R. Manvelyan and S. Theisen, “Off-shell superconformal higher spin multiplets in four dimensions,” *JHEP* **1707** (2017) 034 [arXiv:1701.00682 [hep-th]].

[18] I. L. Buchbinder, S. J. Gates, Jr. and K. Koutrolikos, “Higher Spin Superfield interactions with the Chiral Supermultiplet: Conserved Supercurrents and Cubic Vertices,” *Universe* **4** (2018) no.1, 6 [arXiv:1708.06262 [hep-th]].

[19] J. Hutomo and S. M. Kuzenko, “Non-conformal higher spin supercurrents,” *Phys. Lett. B* **778** (2018) 242 [arXiv:1710.10837 [hep-th]].

[20] J. Hutomo and S. M. Kuzenko, “The massless integer superspin multiplets revisited,” *JHEP* **1802** (2018) 137 [arXiv:1711.11364 [hep-th]].

[21] K. Koutrolikos, P. Koči and R. von Unge, “Higher Spin Superfield interactions with Complex linear Supermultiplet: Conserved Supercurrents and Cubic Vertices,” *JHEP* **1803** (2018) 119 [arXiv:1712.05150 [hep-th]].

[22] I. L. Buchbinder, S. J. Gates, Jr. and K. Koutrolikos, “Interaction of supersymmetric nonlinear sigma models with external higher spin superfields via higher spin supercurrents,” *JHEP* **1805** (2018) 204 [arXiv:1804.08539 [hep-th]].

[23] E. I. Buchbinder, J. Hutomo and S. M. Kuzenko, “Higher spin supercurrents in anti-de Sitter space,” *JHEP* **1809** (2018) 027 [arXiv:1805.08055 [hep-th]].

[24] I. L. Buchbinder, S. J. Gates, Jr. and K. Koutrolikos, “Conserved higher spin supercurrents for arbitrary spin massless supermultiplets and higher spin superfield cubic interactions,” *JHEP* **1808** (2018) 055 [arXiv:1805.04413 [hep-th]].

[25] S. J. Gates, Jr., M. T. Grisaru, M. Rocek and W. Siegel, “Superspace Or One Thousand and One Lessons in Supersymmetry,” *Front. Phys.* **58** (1983) 1 [hep-th/0108200].

[26] I. L. Buchbinder and S. M. Kuzenko, “Ideas and methods of supersymmetry and supergravity: Or a walk through superspace,” Bristol, UK: IOP (1998) 656 p

[27] T. Curtright, “Massless Field Supermultiplets With Arbitrary Spin,” *Phys. Lett.* **85B** (1979) 219.

[28] S. M. Kuzenko, A. G. Sibiryakov and V. V. Postnikov, “Massless gauge superfields of higher half integer superspins,” JETP Lett. **57** (1993) 534 [Pisma Zh. Eksp. Teor. Fiz. **57** (1993) 521].

[29] S. M. Kuzenko and A. G. Sibiryakov, “Massless gauge superfields of higher integer superspins,” JETP Lett. **57** (1993) 539 [Pisma Zh. Eksp. Teor. Fiz. **57** (1993) 526]

[30] S. M. Kuzenko and A. G. Sibiryakov, “Free massless higher superspin superfields on the anti-de Sitter superspace,” Phys. Atom. Nucl. **57** (1994) 1257 [Yad. Fiz. **57** (1994) 1326] [arXiv:1112.4612 [hep-th]].
[31] S. J. Gates, Jr. and S. M. Kuzenko, “4D, \( N = 1 \) higher spin gauge superfields and quantized twistors,” *JHEP* **0510** (2005) 008 [hep-th/0506255].

[32] S. J. Gates, Jr. and K. Koutrolikos, “On 4D, \( \mathcal{N} = 1 \) massless gauge superfields of arbitrary superhelicity”, *JHEP* **1406** (2014) 098,
S. J. Gates, Jr. and K. Koutrolikos; “On 4D, \( \mathcal{N} = 1 \) Massless Gauge Superfields of Higher Superspin: Half-Odd- Integer Case”, arXiv:1310.7386 [hep-th];
“On 4D, \( \mathcal{N} = 1 \) Massless Gauge Superfields of Higher Superspin: Integer Case”, arXiv:1310.7385[hep-th].

[33] I. L. Buchbinder and K. Koutrolikos, “BRST Analysis of the Supersymmetric Higher Spin Field Models”, *JHEP* **1512** (2015) 106 [arXiv:1510.06569 [hep-th]].

[34] S. J. Gates, Jr. and K. Koutrolikos, “From Diophantus to Supergravity and massless higher spin multiplets”, *JHEP* **1711** (2017) 063 [arXiv:1707.00194 [hep-th]].

[35] S. J. Gates, Jr. and W. Siegel, “\((3/2, 1)\) Superfield of O(2) Supergravity,” *Nucl. Phys. B* **164** (1980) 484.

[36] S. R. Coleman and J. Mandula, “All possible symmetries of the S-matrix”, *Phys. Rev.* **159** (1967) 1251.

[37] R. Haag, J. T. Lopuszanski and M. Sohnius, “All possible generators of supersymmetries of the S-matrix”, *Nucl. Phys. B* **88** (1975) 257.

[38] J. Maldacena and A. Zhiboedov, “Constraining Conformal Field Theories with A Higher Spin Symmetry,” *J. Phys. A* **46** (2013) 214011 [arXiv:1112.1016 [hep-th]].