Free vibration and sound transmission properties of beetle elytron plate: structural parametric analysis

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Keywords: Aluminum beetle elytron plate, Structural parameters, Free vibration, Sound transmission, Engineering applications

Honeycomb plate (HP), which is a high-strength and lightweight structure, has good vibration characteristics, while beetle elytron plate (BEP) has better mechanical properties. To promote the engineering application of BEPs, the vibration and sound transmission characteristics of aluminium BEPs were investigated in this paper with HP as the comparison object. This paper investigated the effects of the number of trabeculae, the ratio of length and width, skin thickness, core height and core thickness on the first 4 frequencies using finite element method. The results show that (1) the vibration characteristic of BEP is optimal when the number of trabeculae is 6, and its 3rd and 4th modes show mixed mode, i.e., torsion-bending or bending-torsion mode. (2) The frequencies of BEPs are generally lower than those of HPs. Compared with HPs, the ratio of length and width and core thickness have a smaller influence on the mode shapes of BEPs, and the core height has a smaller influence on BEPs' frequencies. When the skin thickness is small, increasing the thickness can effectively change the natural frequencies of BEPs and HPs. (3) Considering the common frequencies of four applications (aircrafts, unmanned aerial vehicles, high-speed trains and automobiles) of sandwich plates, the effects of the abovementioned parameters including the ratio of length and width, skin thickness, core height and core thickness are analysed. (4) Combined with the theoretical calculation formula, the effect of the above structural parameters on the sound transmission characteristic is explored using the index of sound transmission loss, and targeted recommendations are given. This paper progresses the application in engineering.

1. Introduction

Sandwich plates, which are high-strength and lightweight structures with good thermal insulation, vibration and sound transmission properties, have been employed in the aerospace (e.g., aircraft, Figure 1a) [1, 2], transportation (e.g., high-speed trains and automobiles, Figure 1b, c) [3] and other fields [4, 5]. Among them, honeycomb plate (HP), as a typical sandwich structure, has the largest volume and the best stiffness when the same amount of material is used [6]. However, based on the 3D structure of the Trpoxylus dichotomus (Figure 1d), a bionic sandwich plate (now named beetle elytron plate: BEP; Figure 1g) with a "trabeculae - honeycomb" core (Figure 1e, f) was proposed [7]. Its compressive [8], flexural and shear properties are significantly better than those of HP. It can fully realize the performance upgrade of the existing HPs.

Modern industrial technology and practical engineering proposed higher demands for the mechanical properties of structures, including static or dynamic properties and sound insulation. With the successful rolling of aluminum alloy and large-width aluminum foil, the application field of aluminum alloy honeycomb plates has been further broadened [6]. There have been many studies about experiments on vibration and sound transmission properties [15, 16], finite element analysis [17] and equivalent theory [18, 19, 20] of aluminum honeycomb plates. For example, Thamburaj et al. found that the transmission loss of anisotropic sandwich beams increased significantly compared to isotropic materials [21]. The vibration performance of sandwich beams characterised by hybrid cores (honeycomb-corrugation) was investigated by Zhang et al. [22]. In terms of theory, the theories of vibration characteristics mainly include classical plate theory, Reddy's higher-order shear deformation theory and Mindlin's improved plate theory, etc [23]. Among them, Yu et al. [16] investigated the free vibrations of honeycomb plates using above theories. Zeki et al. [24] proposes many theories for the solution of aluminum honeycomb sandwich structures, such as honeycomb panel theory, sandwich panel theory, equivalent panel theory, Kirchhoff theory, and Reissner theory. The theories of sound transmission properties include transmission theory of

https://doi.org/10.1016/j.heliyon.2022.e11683
Received 2 June 2022; Received in revised form 10 October 2022; Accepted 10 November 2022
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single plates, 3D elasticity theory and third order shear deformation theory. For example, Daneshjou et al. studied acoustic transmission through thick functionally graded materials cylindrical shells and found that more accurate results could be got using the 3rd order shear deformation theory (higher order model) [25]. Shen et al. proposed an analytical model to study acoustic transmission loss characteristic of adhesively bonded sandwich plates using 3D elasticity theory [26].

Of course, structural parameters also significantly affect the vibration and sound transmission performances. For example, Griese et al. investigated the effect of different geometrical parameters of honeycomb core on the acoustic transmission and vibration performance under in-plane loading using finite element analysis (FEA) [27]. Oliazadeh et al. studied the effects of the skin thickness and core thickness and internal loss factor on the acoustic transmission [28]. Kumar et al. [29] and Wang

![Figure 1](image1.png)

**Figure 1.** Applications of sandwich plates and the bionic origin and structure of beetle plates [9]. (a) Luggage rack [10] and wing [11] of an aircraft. (b) Train floor [12, 13]. (c) Automobile [14]. (d) The fresh elytron of *T. dichotomus*. (e, f) A trabecula and a three-dimensional structural model, where dash-dotted lines represent honeycomb walls. (g) Beetle elytron plate.

![Figure 2](image2.png)

**Figure 2.** Structure and dimensions. (a) Dimensions of the sandwich plates. (b) Structural dimensions of a complete structure of a hexagonal cell: (b1) HP, (b2) BEP₁, (b3) BEP₂, (b4) BEP₃, (b5) BEP₄, (b6) BEP₅, (b7) BEP₆, (b8) TP, where the subscript of BEP represents the number of trabeculae.
et al. [30] found the thickness of skin and honeycomb walls affects the natural frequencies and mode shapes.

These studies not only affirm the value of the investigation of the vibration and sound transmission characteristics but also illustrate the necessity of the parametric studies. Thus, based on the verification of the validity of the FEA models, the influence of the number of trabeculae per honeycomb cell on the vibration characteristics of BEP will be first investigated, and the optimal trabecular number will then be determined. Subsequently, the effects of single structural parameters, including the ratio of length and width, skin thickness, core height and core thickness, on the vibration characteristics of BEPs are investigated and compared with those of HPs. Meanwhile, combined with the common frequencies of four applications (aircrafts, unmanned aerial vehicles (UAVs), high-speed trains and automobiles), the effects of each structural parameter are analyzed, and targeted recommendations are given. Finally, based on the vibration frequency obtained by numerical simulation, the effects of the above geometric parameters on the sound transmission properties are studied through theoretical formulas, and the design guidance is given pertinently. This paper provides a reference for the design of the vibration and sound transmission properties, enriches the research on the mechanical and acoustic properties of BEPs and accelerates the engineering application of BEPs.

2. Models and methods

2.1. Model design

Considering that the vibration characteristics of sandwich plates are studied by changing a single structural parameter in this paper, the structural dimensions of the standard model BEP and the HP which is used as a comparison object, are first specified here. Taking the standard model BEP as an example, the length \( a \) is 300 mm, width \( b \) is 60 mm (the ratio of length and width \( a/b \) is 5), skin thickness \( t_f \) is 1.4 mm, core height \( h_c \) is 11.4 mm, core thickness \( t \) is 0.06 mm, length of the basic cell \( l \) is 5 mm, and centerline diameter of hollow trabeculae \( d \) is 2 mm.

Based on the standard BEP and HP models and considering the rationality of engineering applications and the realizability of production processing, the first 4 natural frequencies of BEPs are analyzed by changing a single structural parameter, and investigated the effects. The structural parameters include the number of trabeculae \( N \), the ratio of length and width \( a/b \) (\( b \) is a constant value of 60 mm), the skin thickness \( t_f \), the core height \( h_c \) and the core thickness \( t \). Specifically, the number of trabeculae is set as 0 (HP), 1, 2, 3, 4, 5 and 6, corresponding to Figure 2b1–b7, respectively. Meanwhile, to highlight the superiority of BEPs’ vibration performance, the vibration characteristics of the trabecular plate (TP, \( N = 6 \), Figure 2b8) are also studied. The ratio of length and width is set to 4, 5, 6 and 7, the skin thickness is set to 0.4, 1.4, 2.4 and 3.4 mm, the core height is set to 6.4, 11.4, 16.4 and 21.4 mm, and the core thicknesses are set to 0.06, 0.12, 0.18 and 0.24 mm. There is a total of 32 models.

2.2. Finite element method

The core models with different structural parameters are established in SolidWorks (3D modelling software, version 2018, Dassault Systems). Then, they were imported into the ABAQUS/Standard program (version 2016), and the core and skins were assembled. A linear perturbation analysis step is created, and the frequency extraction procedure is carried out.
out with the Lanczos solver. The first 4 frequencies of 32 models are obtained. The interaction between the core and the skin is Tie. One edge of the short side is constrained (displacement and rotation of $x$, $y$ and $z$ are zero). To obtain higher simulation accuracy, liner quadrilateral shell element with induced integration (S4R) is used for the core structure. Eight-node linear brick elements with reduced integration and hourglass control (C3D8R) are used for the skins [31]. After verification of mesh irrelevance, the mesh grid is 1.4 mm, which is one over ten of the height of the standard model (14.2 mm). The total elements employed in the finite element models range from 35,000 to 111,000. The models are made of Al-3003-H18, and the material properties are $E = 70.0$ GPa, $v = 0.3$, and $\rho = 2700.0$ kgm$^{-3}$ [14].

2.3. Validation of finite element model

To ensure the reliability and validity of the FEA results, the models are established in the same way as the geometric parameters of Zhang et al.’s [14] experimental samples (Figure 3a). Then, the numerical and experimental results are compared and analyzed. The results show that the shapes of the first 3 modes (bending, lateral and bending mode, successively) and frequencies of the numerical results agree well with those of the experimental results (Figure 3b, c). The errors of the 1st, 2nd and 3rd natural frequencies with the corresponding experimental results are 3.7%, 2.9% and 8.6%, respectively (within 10%). In addition, to further ensure the validity of the FEA method, a honeycomb sandwich plate that is made of aluminum with dimensions of 100 mm $\times$ 100 mm $\times$ 11 mm is also modelled and analyzed in this paper (Figure A1) [18]. Good agreement is also achieved. The detailed results are shown in Appendix A, which will not be repeated here. Therefore, the verification of the above two types of models confirms the validity of the FEA method.

3. Results and discussion

In this section, the effect of the number of trabeculae $N$ on the vibration characteristics of BEPs will be first investigated to determine the optimal number. Then, the effect of the ratio of length and width $a/b$, skin thickness $t_0$, core height $h_c$ and core thickness $t$ on the vibration characteristics of BEPs will be studied. Reasonable suggestions will be given combined with the common frequencies of different applications of sandwich plates (including aircrafts, UAVs, high-speed trains and automobiles, etc.). Finally, based on the vibration frequencies obtained by numerical simulation, the effect of the above geometric parameters on the sound transmission properties will be studied, and the design guidance will be given. This paper provides a design and application reference.

3.1. Effect of the number of trabeculae

Figure 4 represents the vibration mode shapes of HP ($N = 0$), BEP$_6$ ($N = 6$) and TP ($N = 6$). In engineering application, the lower the natural frequency, the easier it is to be stimulated by the outside environment. Therefore, although the structure has infinite natural frequencies, in many cases, only some lower natural frequencies of the structure are concerned. This paper mainly focuses on the first 4 frequencies. As shown in Figure 4a, the 1st and 4th mode shapes of HP are the 1st and 2nd bending modes, respectively, the 2nd mode shape is the lateral mode, and the 3rd mode is the torsional mode. For the BEP, when the number of trabeculae $N$ is 6, the 3rd and 4th mode shapes of the BEP show a mixture of two modes. Specifically, the 3rd mode shape mainly shows torsion and is supplemented by bending, which is called the torsion-bending mode (Figure 4b, ③), and the 4th mode mainly shows bending and is supplemented by torsion (bending-torsion mode, Figure 4b, ④). The results of the mixed modes show that the maximum displacement appears only at one corner of the free end (Figure 4b, ③ and ④), rather than uniformly in a strip along the width direction as the HP (Figure 4a, ③ and ④). This will be further analyzed in the following section combined with the frequency and torsional angle. Finally, for the TP, the 1st and 3rd mode shapes are the
The 1st and 2nd bending modes, the 2nd mode shape is the torsional mode, and the 4th mode shape is the lateral mode. Thus, compared to HP and BEP, TP more easily undergoes torsion and bending (Figure 4c). This indicates that the torsion and bending resistance of the trabecular structure is poor, and adding honeycomb walls can effectively improve the stability of the trabecular structure.

The first four natural frequencies of HP, TP and BEPs with different numbers of trabeculae \( N \) are shown in Table 1. Figure 5a shows the effect of the number of trabeculae on the frequencies of BEPs. Overall, the frequencies of HP are the largest, followed by those of BEPs, and the frequencies of TP are the smallest (Figure 5a). Specifically, the first four frequencies of TP are approximately 43%–75% of those of HP and BEPs. The frequency corresponding to the lateral modes (4th mode) of TP is basically the same as the frequencies of the lateral modes (2nd mode) of HP and BEPs (Figure 5a). Combined with the aforementioned result that the 2nd bending and torsional modes of TP occur before the lateral mode, it is proven that the vibration characteristics of TP are poor. This also shows that adding honeycomb walls to the trabecular structure can substantially increase the bending and torsional stiffness, while the contribution to lateral stiffness is negligible.

For BEPs, the increase in the number of trabeculae mainly affects the 3rd and 4th frequencies and has little effect on the 1st and 2nd frequencies. In particular, the 2nd frequency remains basically unchanged (Figure 5a). Regarding the changing trend, the 1st, 3rd and 4th frequencies of BEPs gradually decrease (the maximum magnitudes are 7%, 22% and 18%, respectively) when \( N \) increases from 1 to 5. However, when \( N \) is 6, the frequencies increase obviously. The natural frequency is proportional to the structural stiffness and inversely proportional to the mass. This indicates that when \( N \) is 1–5, the mass dominates the frequencies of the sandwich plates rather than the structural stiffness. When \( N \) increases to 6, the structural stiffness is significantly increased compared with the increase in mass. Of course, it also benefits from the fact that the 3rd and 4th mode shapes of the BEP are torsion-bending or bending-torsion modes when \( N \) is 6, as mentioned above. Thus, the concept of the torsional angle is proposed for the first time in this paper to evaluate the difference and superiority among the structures regarding the torsional mode or torsion-bending mode. As shown in Figure 5b, the torsional angle of BEP6 is significantly smaller than that of the other sandwich plates, which directly proves the structural superiority of BEP6. Furthermore, this is consistent with the conclusion that the BEP has the best mechanical properties when \( N = 6 \) in our previous study [7]. Therefore, considering the poor vibration performance (torsion and bending resistance) of the aforementioned TP, the TP will not be examined in the following section. The parametric study will be conducted only for the BEP6 with the optimal performance and the HP used for comparison.

**Table 2.** Common frequencies of different applications of sandwich plates (means of transport or delivery).

| Type                        | Part             | Frequency (Hz) | Reference |
|-----------------------------|------------------|----------------|-----------|
| Aircraft                    | Y-12 propeller  | 31.25 (93.75)  | [32]      |
|                             | Y-12 floor       | 93.75          |           |
|                             | Cessna208 propeller | 27.91 (83.75) |           |
|                             | Cessna208 floor  | 83.75          |           |
| Unmanned Aerial Vehicle     | wheelset         | 338 (381)      | [34]      |
|                             | gearbox           | 405 (578)      |           |
|                             | wheel–rail system| 750            |           |
| Automobile                  | body             | 250            | [35]      |
|                             | chair            | 16.67 (19.65)  | [36]      |
|                             | floor            | 128.5 (139.3)  | [37]      |
|                             | door             | 38.92 (40.45)  | [38]      |

Note: The numbers without brackets or before brackets indicate the 1st frequency. The number in brackets indicates the frequency corresponding to three times the speed of the aircraft propeller for the aircraft and indicates the second frequency for the other applications.

### 3.2. Effect of geometric parameters on the vibration characteristics of BEPs

In this section, the effects of the ratio of length and width \((a/b)\), skin thickness \((t_s)\), core height \((h_c)\) and core thickness \((t)\) on the vibration characteristics of BEPs (hereafter referred to as BEP) will be investigated compared to those of HP. To successfully examine the differences of each sandwich plate and the superiority of BEPs, the vibration characteristics of BEPs and HPs are evaluated using indicators including the difference between the 4th and 1st frequencies and the torsional angle of the torsional mode in this paper. In addition, to prove the mass efficiency of sandwich plates relative to conventional solid plates, the dimensionless structural efficiency parameter is also used as a supplementary evaluation indicator in this paper, which is represented in detail in Appendix B. Finally, combined with the common frequencies of several applications of sandwich plates, the targeted recommendations for different structural parameters are given in Table 2, which provides a design and application reference.

Overall, the first 4 frequencies of BEPs are generally smaller than those of HPs. Among them, the differences between the 1st and 2nd frequencies are slight, but there are significant differences between the 3rd and 4th frequencies (Figure 6a, Figure 7a, Figures 8a and Figure 9a). Meanwhile, the difference between the 4th and 1st frequencies and the torsional angles of the torsional modes of BEPs are smaller. This is mainly...
due to the superiority of the aforementioned BEP structure, where the 3rd and 4th mode shapes are torsion-bending or bending-torsion ones. The effect of each parameter will be analyzed in the following subsection.

### 3.2.1. Effect of the ratio of length and width

With the increase in the ratio of length and width \(a/b\), the first 4 natural frequencies of BEPs and HPs decrease significantly (the total decrease is in the range of 50%–66%, Figure 6a, the upper table). This is because the increase in the ratio of length and width will decrease the structural stiffness and increase the structural mass. In addition, when the ratio of length and width is small, the sandwich plate is a plate system vibration. The range of fixed support boundary conditions imposed on the short side is larger, and the influence of the boundary constraint stiffness is larger, so the natural frequency will be larger [39]. With the increase in the ratio of length and width, the sandwich plate gradually presents the unidirectional force characteristics, and the vibration mode gradually transitions from the plate system to the slab system with unidirectional force, which weakens the influence of the boundary restraint stiffness and decreases the natural frequency of the structure. Regarding the difference between the 4th and 1st frequencies of each sandwich plate, the larger the ratio of length and width is, the smaller the difference (Figure 6b), i.e., the frequencies of the two sandwich plates are more concentrated (Figure 6a) when the length is larger, which is called the “pinching” phenomenon in this paper. Compared with HPs, the first 4 frequencies of BEPs are more pinching. Meanwhile, the increase in the ratio of length and width will lead to an increase in the torsional angle of the torsional mode (Figure 6b). In practical engineering, the torsional angle should be strictly controlled [40], so the ratio of length and width should not be too large.

In addition, regarding the mode shapes of each sandwich plate, BEPs are consistent with those shown in Figure 4b, i.e., the 1st bending mode, lateral mode, torsional mode and 2nd bending mode, successively. For the HP, when the ratio of length and width is 6 and 7, the 3rd mode is the 2nd bending mode, and the 4th mode is the torsional mode (Figure 6c), that is, the 2nd bending mode occurs earlier than the torsional mode. The reason is that the increase in length greatly reduces the bending stiffness. This also shows that the mode shapes of HPs are more sensitive to changes in length, while the mode shapes of BEPs are less sensitive to length because the core structures of BEPs are more stable due to their trabecular structure.

Finally, when the sandwich plates of Figure 6a are applied to UAVs, it will not resonate because the first 4 natural frequencies of sandwich plates are far away from UAVs’ natural frequency. When applied to aircrafts, resonance will occur when the ratio of length and width of HP and BEP is 7 (Table 3) because the frequencies of aircraft propeller and floor are approximately 90 Hz (Table 2). When applied to high-speed trains and the ratio of length and width of HP and BEP is 6 and 7 (Table 3), there are several frequencies close to the train frequency, and resonance will occur easily. When applied to an automobile floor, resonance will occur when the ratio of length and width of HP and BEP is 6. When applied to an automobile body, resonance will occur when the ratio of length and width of BEP is 4. From a comprehensive point of view, the ratio of length and width should not be too large, preferably not more than 5, regardless of the mode shape sensitivity, torsional angle or application fields.

### 3.2.2. Effect of skin thickness

With the increase in skin thickness \(t_s\), the 1st and 2nd natural frequencies of BEPs and HPs increase (7%–19%), while the 3rd and 4th natural frequencies decrease (approximately 20%, Figure 7a, the upper table). The frequencies are pinching as the ratio of length and width. When the skin thickness increases, the mass and stiffness of the sandwich plate increase. For the 1st and 2nd natural frequencies, the stiffness plays a dominant role in the frequencies compared to the structural mass. In contrast, the effect of mass is more significant for the 3rd and 4th natural frequencies.

It is obvious that when the skin thickness increases from 0.4 mm to 1.4 mm, the variation change of the first 4 frequencies is large (9%–16%). However, after the skin thickness reaches 1.4 mm, the trend is stable (Figure 7a). This also shows a consistent trend in the difference between the 4th and 1st frequencies (Figure 7b). Regarding the mode shapes, when the skin thickness is small (0.4 mm), the 2nd bending mode (3rd mode) of HP and BEP occurs earlier than the torsional mode (4th mode). This is mainly caused by the fact that the bending stiffness of sandwich plates is approximately proportional to the third power of the skin thickness, and the decrease in the skin thickness will sharply weaken the bending stiffness, making the 2nd bending mode more easily occur. After the skin thickness reaches 1.4 mm, the first 4 mode shapes of both HP and BEP are consistent with those shown in Figure 4a and b, respectively. This also explains the aforementioned large change in four frequencies when the skin thickness increases from 0.4 mm to 1.4 mm. Furthermore, it can also be inferred that when the skin is thin, increasing the skin thickness can effectively change the natural frequency of the sandwich plate, which can guide the application of the sandwich plate. In addition, the skin thickness has little effect on the torsional angle of the torsional mode (Figure 7b), so the effect of the torsional angle can be disregarded in the design process.

For the skin thickness, when the sandwich plates of Figure 7a are applied to UAVs, aircrafts and automobiles, their natural frequencies (Table 2) are far from the frequencies of the sandwich plates, which will

![Figure 6](image-url)

**Figure 6.** The effect of the ratio of length and width \(a/b\) on the vibration characteristics of BEPs and HPs. (a) The first 4 natural frequencies. (b) The difference between the 4th and 1st frequencies and the torsional angles of the torsional mode. Where, the dashed and solid lines in figure indicate HP and BEP, respectively. The boxed area is the aforementioned standard models HP and BEP. The numbers 1–5 indicate the 1st–4th frequencies. The numbers in the upper table indicate the increase/decrease of the first 4 frequencies of the maximum value of each parameter relative to the minimum value. The above notes are also applicable in the subsequent Figures 7–9, and they will not be repeated in the following subsections. (c) The 3rd and 4th mode shapes of HP with the ratio of length and width \(a/b = 6\).
avoid resonance. However, when applied to the wheel-track system of high-speed trains (750 Hz), the 3rd frequency of HP with skin thicknesses of 2.4 and 3.4 mm (Table 3) and the 4th frequency of BEP with skin thickness of 2.4 are close to 750 Hz, and resonance will occur very easily. Therefore, for high-speed trains, a skin thickness of 1.4 mm can meet the requirements of stiffness and also effectively avoid resonance.

3.2.3. Effect of core height

With the increase in the core height $h_c$, except for the 2nd frequency, which slightly decreases (within 5%), the other three frequencies gradually increase (Figure 8a, the upper table). The reason is that the 2nd mode is the lateral mode, and the increase in core height has little effect on the lateral stiffness but increases the mass, resulting in a gradual decrease in frequency. The 1st, 3rd and 4th modes are mainly bending and torsional modes. The core bending stiffness is proportional to the third power of the core height [41], and the core shear stiffness is proportional to the first power of the core height [42]. Therefore, the increase in the core height can increase the 1st, 3rd and 4th frequencies.

Regarding the increase in the 1st, 3rd and 4th frequencies, the HP shows a nearly linear increase. The increase when the core height is 21.4 mm relative to 6.4 mm is significant (158%, 70% and 101%, respectively Figure 8a, the upper table). The difference between the 4th and 1st frequencies also shows a linear increase (Figure 8b). In general, contrary to the aforementioned influence rules of ratio of length and width and skin thickness, trend of the effect of core height shows a divergence (anti-pinching) phenomenon. For BEPs, the trend is stable after the core height reaches 11.4 mm, which is consistent with the difference between the 4th and 1st frequencies and the torsional angle of the torsional mode (Figure 8b). Furthermore, regarding the mode shapes, only when the core height of BEP is small (6.4 mm) does its 2nd bending mode occur earlier than the torsional mode. This is mainly because a core height that is too small will sharply weaken the bending stiffness of the sandwich plate, making the 2nd bending mode more easily occur. However, once an appropriate height (e.g., 11.4 mm) is reached, continuing to increase the core height will have little effect on the frequencies of BEPs. For the HP, the increase in core height can significantly improve its stiffness without changing its mass and the torsional angle (Figure 8b), so the frequency of the HP can be effectively controlled by changing the core height.

For the core height, similar to the aforementioned skin thickness, each sandwich plate of Figure 8a can be directly applied to UAVs and aircrafts without considering resonance. However, when applied to high-speed trains, if the core height of HP is 6.4 mm and 21.4 mm and after the core height of BEP reaches 11.4 mm, the frequencies of the sandwich plates are close to the frequency of the train components, and resonance occurs easily. When applied to automobile bodies, resonance will occur if the core height of HP is 16.4 mm. Therefore, when applied to high-speed trains and automobiles, the core height should be strictly controlled.

3.2.4. Effect of core thickness

When the core thickness $t$ increases, the 1st and 2nd frequencies decrease slightly (no more than 10%, Figure 9a, the upper table). This is because the increase in the core thickness will increase the mass. For the 3rd and 4th frequencies, as the core thickness increases, although the skin parameters remain unchanged, the structural bending stiffness is enhanced due to the core moving it away from the midplane. The density of the core is much smaller than that of the skin, then the change of the
core thickness has little effect on the plate quality, so the resonance frequency increases with the increase of the core thickness (approximately 30%, Figure 9a, the upper table). The trend of core thickness is consistent with the aforementioned core height, which shows a divergence (anti-pinching) phenomenon. It is also due to the above phenomenon that the difference between the 4th and 1st frequencies gradually increases (Figure 9b). In addition, the change in core thickness has a non-consistent phenomenon with seismic requirements. Moreover, by replacing traditional HPs with BEPs, not only the advantages of BEPs in terms of vibration performance, but also its excellent compressive and bending properties can be more fully exploited [7, 37].

The effect of each structural parameter on the first 4 natural frequencies of BEPs and HPs is introduced systematically, and the influence mechanism of each parameter, then summarize the effect of structural parameters on the sound transmission characteristics of BEPs.

### 3.3 Effect of geometric parameters on the sound transmission characteristics of BEPs

As a lightweight structure, when honeycomb plates are applied to the aforementioned structures such as automobiles, high-speed trains, and aircrafts, these parts often have good noise reduction performance in addition to the vibration properties. In this section, the sound transmission characteristics of BEPs and HPs will be studied using the theoretical calculation formulas. The influence of structural parameters on their sound transmission characteristics will be analyzed quantitatively.

Sound transmission loss (STL) is commonly used to describe the sound insulation effect of sound insulation materials, which is defined as the difference between the incident sound power stage on one side of the sound insulation material and the transmitted sound power stage on the other side. As shown in formula (1), this parameter establishes a quantitative link between the design parameters of the sandwich plate and its STL, which can facilitate the study of the influence mechanism of each parameter, then summarize the design law, and provide guidance to the design of honeycomb sandwich plates with high STL.

$$STL = 10 \log \left\{ \left[ 1 + \frac{\rho_c f}{\rho_f} \left( \frac{f_m}{f} \right)^2 \right]^2 + \left( \frac{\rho_c f}{\rho_f} \right)^2 \left[ 1 - \left( \frac{f_m}{f} \right)^2 \right]^2 \right\} \quad (1)$$

#### Table 3. The effect of structural parameters on the first 4 frequencies of HPs and BEPs.

| Effect of the ratio of length/width | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) |
|------------------------------------|---------------|---------------|---------------|---------------|
| HP                                 | 4 277.0       | 799.9         | 1011.4        | 1176.3        |
|                                   | 5 184.0       | 520.0         | 804.4         | 853.7         |
|                                   | 6 130.5       | 364.2         | 646.2         | 667.4         |
|                                   | 7 97.2        | 269.0         | 504.8         | 570.5         |
| BEP                                | 4 250.2       | 789.6         | 840.3         | 949.6         |
|                                   | 5 174.1       | 513.7         | 695.7         | 766.3         |
|                                   | 6 123.8       | 359.7         | 532.7         | 549.1         |
|                                   | 7 91.8        | 265.5         | 413.9         | 439.1         |

| Effect of the skin thickness (mm) | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) |
|-----------------------------------|---------------|---------------|---------------|---------------|
| HP                                | 0.4 168.7     | 492.0         | 935.8         | 1019.4        |
|                                   | 2.4 193.0     | 525.5         | 747.2         | 823.7         |
|                                   | 3.4 200.7     | 527.8         | 728.5         | 812.6         |
| BEP                               | 0.4 159.9     | 472.9         | 821.5         | 912.8         |
|                                   | 2.4 179.4     | 521.7         | 655.2         | 720.7         |
|                                   | 3.4 184.4     | 525.1         | 655.3         | 709.8         |

| Effect of the core height (mm)    | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) |
|-----------------------------------|---------------|---------------|---------------|---------------|
| HP                                | 6.4 117.1     | 525.9         | 598.6         | 615.9         |
|                                   | 16.4 245.6    | 514.1         | 933.5         | 1060.9        |
|                                   | 21.4 302.4    | 508.4         | 1019.6        | 1236.9        |
| BEP                               | 6.4 117.0     | 521.9         | 612.9         | 642.4         |
|                                   | 16.4 286.7    | 504.9         | 693.9         | 757.3         |
|                                   | 21.4 225.7    | 497.4         | 713.6         | 764.3         |

| Effect of the core thickness (mm) | 1st Mode (Hz) | 2nd Mode (Hz) | 3rd Mode (Hz) | 4th Mode (Hz) |
|-----------------------------------|---------------|---------------|---------------|---------------|
| HP                                | 0.12 186.0    | 508.0         | 964.5         | 971.8         |
|                                   | 0.18 184.0    | 496.9         | 1002.1        | 1059.5        |
|                                   | 0.24 181.1    | 486.6         | 1014.3        | 1111.0        |
| BEP                               | 0.12 177.7    | 496.6         | 825.5         | 901.9         |
|                                   | 0.18 175.5    | 481.4         | 881.6         | 969.5         |
|                                   | 0.24 172.1    | 467.8         | 904.4         | 1009.7        |
where, $\eta$ is the damping loss factor (this paper does not consider damping loss and takes it as 0), $\rho_c$ is the dielectric impedance, $\rho = 1.125 \text{ kg/m}^3$, $c = 340 \text{ m/s}$, $\rho$ is the mass per unit area, $f$ is the excitation frequency (in view of the aforementioned sandwich plate frequency and the frequency of high-speed trains are more likely to resonate, so $f$ is taken as the frequency of the aforementioned high-speed train wheel-track system, 750 Hz), $f_{mn}$ is the structure natural frequency, which is mainly related to the structural parameters.

First, there is no obvious rule for the STL of the structure by the change of the ratio of length and width (Figure 10a), but the general principle is to avoid the resonance frequency. As shown in Figure 10a, when the stimulation frequency (750 Hz) is close to the natural frequency of the structure, the structure will resonate, which will lead to a sharp decrease in the STL, and the curve has an obvious valley point (Figure 10a, pentagram). It’s also suitable for the obvious valley points in the curves of the skin thickness and core height parameters (Figure 10b and c, pentagram).

Secondly, with the increase of skin thickness, the STL basically shows an increasing trend in the whole frequency range (Figure 10b). This is because when the core thickness remains unchanged, the increase of the skin thickness increases the bending stiffness and areal density, so the STL is improved. Therefore, the sound insulation can be improved by increasing the skin thickness in practical application.

Regarding the core height, the STL in the whole frequency range basically shows a decreasing trend, and only the 3rd and 4th orders of HP show an increasing trend (Figure 10c), which is consistent with Figure 8a. This is mainly because the increase of core height can significantly improve the stiffness of the HP without substantially changing the mass of the HP as aforementioned above, thereby increasing its STL.

Finally, increasing the core thickness can effectively improve the STL, which is because the increase in the core thickness can increase the structural frequency (Figure 10d). However, if the core thickness continues to increase (from 0.12 mm to 0.24 mm), the increase in STL is significantly reduced. Therefore, when the stiffness is guaranteed, the STL can be effectively improved by appropriately increasing the core thickness.

4. Conclusions

In this paper, the validity of the finite element analysis method was first verified, and then the effect of the number of trabeculae $N$ on the vibration performance of BEPs was investigated to determine the optimal number of trabeculae. Then, the effect of parameters including the ratio of length and width, skin thickness, core height and core thickness on the first 4 natural frequencies of BEPs and HPs were investigated by changing a single geometric parameter. Then, combined with the common frequencies of different application fields, targeted suggestions are given, which provides significant guidance for engineering. The following conclusions were mainly obtained:

1) It is clarified that the vibration characteristics of BEPs are optimal when the number of trabeculae $N$ is 6. Its 3rd and 4th mode shapes show torsion-bending or bending-torsion modes, and the torsional angle is significantly smaller than those of other sandwich plates.

2) The natural frequencies of HPs are generally higher than those of BEPs. Among them, the differences between the 1st and 2nd frequencies are slight, but there is a significant difference between the 3rd and 4th frequencies. Regarding mode shapes, the first 4 mode shapes of most BEPs and HPs successively show the bending mode, lateral mode, torsional mode and 2nd bending mode.

3) The effect of the ratio of length and width and skin thickness on the first 4 frequencies shows a pinching phenomenon, while the core height and core thickness show a divergence (anti-pinching)
phenomenon. Compared with HPs, the ratio of length and width and core thickness have a smaller influence on the mode shapes of BEPs. When the skin is thin, increasing the skin thickness can effectively change the natural frequencies of BEPs and HPs. When the core height reaches a suitable value, continuing to increase the value has little effect on the frequencies of BEPs but has a significant effect on HPs.

4) The frequencies of the sandwich plate are much higher than the frequencies of UAVs, which can avoid the resonance phenomenon. When applied in aircrafts, it is necessary to focus on controlling the ratio of length and width, preferably not more than 5. When applied in high-speed trains, it must consider the effect of multiple factors, such as the ratio of length and width, skin thickness and core height. When applied in automobiles, it is necessary to focus on the influence of the ratio of length and width and core height. However, in the above four fields, the core thickness is not a key parameter and can be disregarded.

5) For the sound transmission properties, the design principle of the ratio of length and width is to keep the frequency away from the frequencies in actual application conditions as far as possible. The STL can be improved by increasing the skin thickness and the core thickness, but it has limitations. The increase of the core height is not conducive to improving the sound transmission properties of BEPs, but it has a significant improvement on the specific frequency of HP.

Declarations

Author contribution statement

Ning Hao: Conceived and designed the experiments; Performed the experiments; Wrote the paper.

Ziying Wang: Analyzed and interpreted the data; Wrote the paper.

Yiheng Song: Conceived and designed the experiments; Performed the experiments; Wrote the paper.

Sihan Ruan; Chaochao He; Zeyu Dong: Contributed reagents, materials, analysis tools or data.

Funding statement

Ning Hao was supported by National Natural Science Foundation of China [51875102].

Data availability statement

Data included in article/supp. material/referenced in article.

Declaration of interest’s statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

Acknowledgements

The authors would like to express their sincere thanks to Prof. Jinxiang Chen (Southeast University, China) for his valuable guidance and advice.

Appendix A

To further verify the validity of the finite element analysis in this paper, the vibration characteristics of a honeycomb plate made of aluminum are also analyzed by finite element method [18]. The dimension is 100 mm × 100 mm × 11 mm (Figure A1a). The first 3 frequencies and mode shapes are obtained using free boundary conditions. The core height is 10 mm, the skin thickness is 0.5 mm, the length of the honeycomb cell is 6 mm, and the core thickness is 0.2 mm (Figure A1a). The numerical results and the theoretical solution based on the equivalent plate theory (first 3 frequencies are 3970, 5776, and 7090 Hz, successively) also meet good agreement (Figure A1b). Among them, the errors of the 1st, 2nd and 3rd frequencies are 0.9%, 6.3% and 1.2%, respectively, which fully illustrates the reliability of the finite element method in this paper.

Figure A1. The structure of aluminum honeycomb plates and vibration results of finite element analysis. (a) Structural parameters of the aluminum honeycomb plate [16]. (b) The first 3 frequencies and their mode shapes. The deformation scale factor is 0.03.
Appendix B

To reflect the difference and mass efficiency in vibration characteristics between the sandwich plate and the traditional solid plate, a dimensionless structural efficiency parameter is used as an evaluation index in this paper. The parameter is the ratio of the 1st frequency $\omega$ and the 1st frequency $\varpi$ of the corresponding homogeneous solid plate with the same length, width, weight and boundary condition [14, 44]. Here, $\varpi$ is obtained from Equation (B1) [22].

Figure B1 gives the effects of geometric parameters, including the ratio of length and width $a/b$, skin thickness $t_f$, core height $h_c$ and core thickness $t$, on the parameter $\omega/\varpi$. As shown in Figure B1, $\omega/\varpi$ increases with increasing ratios of length and width, core height and core thickness but decreases with increasing skin thickness. This indicates that the mass efficiency of BEPs and HPs relative to solid plates decreases only as the skin thickness increases. Moreover, the $\omega/\varpi$ of BEPs and HPs is closest only for the parameter of skin thickness. While for other geometric parameters, the $\omega/\varpi$ of HPs is always higher than that of BEPs.

$$\omega = \frac{1.01}{2\pi} \sqrt{\frac{Eh}{\rho a^4}}$$  \hspace{1cm} (B1)

where $E$ is the elastic modulus of the solid plate, $h$ is the height, $a$ is the length, and $\rho$ is the density.

Figure B1. Effect of geometric parameters on $\omega/\varpi$. (a) The ratio of length and width $a/b$. (b) Skin thickness $t_f$. (c) Core height $h_c$. (d) Core thickness $t$. 
