Magnetic moments of the pentaquarks

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Abstract

We present in this talk a recent analysis for the magnetic moments of the baryon antidecuplet within the framework of the chiral quark-soliton model with linear $m_s$ corrections considered. We take into account the mixing of higher representations to the collective magnetic moment operator, which comes from the SU(3) symmetry breaking. Dynamical parameters of the model are fixed by experimental data for the magnetic moments of the baryon octet as well as by the masses of the octet, decuplet and of $\Theta^+$. The magnetic moment of $\Theta^+$ is rather sensitive to the pion-nucleon sigma term and ranges from $-1.19$ n.m. to $-0.33$ n.m. as the sigma term is varied from $\Sigma_{\pi N} = 45$ to $75$ MeV, respectively. On top of them, we obtained that the strange magnetic moment of the nucleon has the value of $\mu_N^{(s)} = +0.39$ n.m. within this scheme and turns out to be almost independent of the sigma term.
I. INTRODUCTION

Exotic pentaquark baryons has been a hot issue, since the LEPS collaboration [1] announced the new finding of the \( S = +1 \) baryon \( \Theta^+ \) which was soon confirmed by a number of other experiments [2], together with an observation of exotic \( \Xi_{10} \) states by the NA49 experiment at CERN [3], though it is still under debate. Those experiments searching for the pentaquark states was stimulated by Diakonov et al. [4]: Masses and decay widths of exotic baryon antidecuplet were predicted within the chiral quark-soliton model.

The discoveries of the pentaquark baryon \( \Theta^+ \) and possibly of \( \Xi_{10} \) have triggered intensive theoretical investigations (see, for example, Refs. [5, 6, 7]). The production mechanism of the \( \Theta^+ \) has been discussed in Refs. [8, 9, 10]. In particular, it is of great interest to understand the photoproduction of the \( \Theta^+ \) theoretically, since the LEPS and CLAS collaborations used photons as a probe to measure the \( \Theta^+ \). In order to describe the mechanism of the pentaquark photoproduction, we have to know the magnetic moment of the \( \Theta^+ \) and its strong coupling constants. However, information on the static properties such as antidecuplet magnetic moments and their strong coupling constants is absent to date, so we need to estimate them theoretically. Recently, two of the present authors calculated the magnetic moments of the exotic pentaquarks, within the framework of the chiral quark-soliton model [11] in the chiral limit. Since we were not able to fix all the parameters for the magnetic moments in the chiral limit, we had to rely on the explicit model calculations [12, 13].

A very recent work [14] extended the analysis for the magnetic moments of the baryons, taking into account the effect of SU(3) symmetry breaking so that the necessary parameters are fixed by the magnetic moments of the baryon octet. In the present talk, we would like to present the main results of Ref. [14].

II. CONSTRAINTS ON PARAMETERS

The collective Hamiltonian describing baryons in the SU(3) chiral quark-soliton model takes the following form [15]:

\[
\hat{H} = M_{\text{sol}} + J(J+1) \frac{2I_1}{2I_2} + C_2(\text{SU}(3)) - J(J+1) - \frac{N_c^2}{12} + \hat{H}',
\]

with the symmetry breaking term given by:

\[
\hat{H}' = \alpha D^{(8)}_{88} + \beta Y + \gamma \sqrt{3} D^{(8)}_{8i} \hat{J}_i,
\]

where parameters \( \alpha, \beta, \) and \( \gamma \) are of order \( \mathcal{O}(m_s) \) and are given as functions of the \( \pi N \) sigma term [16]. \( D^{(R)}_{ab}(R) \) denote SU(3) Wigner rotation matrices and \( \hat{J} \) is a collective spin operator. The Hamiltonian given in Eq. (2) acts on the space of baryon wave functions:

\[
|\mathcal{R}, J, B, J_3\rangle = \sqrt{\dim(\mathcal{R})}(-1)^{J_3-Y'/2}D^{(R)\ast}_{Y,T,T_3,Y',J,-J_3}(R).
\]

Here, \( \mathcal{R} \) stands for the allowed irreducible representations of the SU(3) flavor group, i.e. \( \mathcal{R} = 8, 10, 10', \cdots \) and \( Y, T, T_3 \) are the corresponding hypercharge, isospin, and its third component, respectively. Right hypercharge \( Y' = 1 \) is constrained to be unity for the physical spin states for which \( J \) and \( J_3 \) are spin and its third component. The model-independent approach consists now in using Eqs. (1) and (2) (and/or possibly analogous
equations for other observables) and determining model parameters such as $I_1, I_2, \alpha, \beta, \gamma$ from experimental data.

The symmetry-breaking term \(\mu_t\) of the collective Hamiltonian mixes different SU(3) representations as follows: \[13\]

\[
|B_{8}\rangle = |8_{1/2}, B\rangle + c_{10}^B |\bar{10}_{1/2}, B\rangle + c_{27}^B |27_{1/2}, B\rangle,
|B_{10}\rangle = |10_{3/2}, B\rangle + a_{7}^B |27_{3/2}, B\rangle + a_{35}^B |35_{3/2}, B\rangle,
|B_{35}\rangle = |\bar{10}_{1/2}, B\rangle + d_{8}^B |8_{1/2}, B\rangle + d_{27}^B |27_{1/2}, B\rangle + d_{35}^B |35_{1/2}, B\rangle,
\] (4)

where \(|B_{\mathcal{R}}\rangle\) denotes the state which reduces to the SU(3) representation \(\mathcal{R}\) in the formal limit \(m_s \to 0\) and the spin index \(J_3\) has been suppressed. All relevant expressions for the mixing coefficients \(c_{\mathcal{R}}^B, a_{\mathcal{R}}^B,\) and \(d_{\mathcal{R}}^B\) can be found in Ref. [14].

### III. MAGNETIC MOMENTS IN THE CHIRAL QUARK-SOLITON MODEL

The collective operator for the magnetic moments can be parameterized by six constants. By definition in the model-independent approach they are treated as free \[12, 13\] :

\[
\hat{\mu}^{(0)} = w_1 D_{Q3}^{(8)} + w_2 d_{pq3} D_{Qp}^{(8)} \hat{J}_q + w_3 D_{Q8}^{(8)} \hat{J}_3,
\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D_{Qp}^{(8)} D_{Qq}^{(8)} + w_5 \left( D_{Q3}^{(8)} D_{88}^{(8)} + D_{Q8}^{(8)} D_{88}^{(8)} \right) + w_6 \left( D_{Q3}^{(8)} D_{88}^{(8)} - D_{Q8}^{(8)} D_{88}^{(8)} \right).
\] (5)

The parameters \(w_{1,2,3}\) are of order \(\mathcal{O}(m_s^0)\), while \(w_{4,5,6}\) are of order \(\mathcal{O}(m_s)\), \(m_s\) being regarded as a small parameter.

The full expression for the magnetic moments can be decomposed as follows:

\[
\mu_B = \mu_B^{(0)} + \mu_B^{(op)} + \mu_B^{(wf)},
\] (6)

where the \(\mu_B^{(0)}\) is given by the matrix element of the \(\hat{\mu}^{(0)}\) between the purely symmetric states \(|\mathcal{R}, J, B, J_3\rangle\), and the \(\mu_B^{(op)}\) is given as the matrix element of the \(\hat{\mu}^{(1)}\) between the symmetry states as well. The wave function correction \(\mu_B^{(wf)}\) is given as a sum of the interference matrix elements of the \(\mu_B^{(0)}\) between purely symmetric states and admixtures displayed in Eq. (4). These matrix elements were calculated for octet and decuplet baryons in Ref. [13].

The measurement of the \(\Theta^+\) mass constrains the parameter space of the model. Recent phenomenological analyzes indicate that our previous assumption on \(\gamma\), i.e. \(\gamma = 0\), has to be most likely abandoned. Therefore, our previous results for the magnetic moments of 8, 10 and \(\bar{10}\) have to be reanalyzed. Now, we show that a model-independent analysis with this new phenomenological input yields \(w_2\) much larger than initially assumed, which causes \(\mu_{\Theta^+}^{(0)}\) for realistic values of \(\Sigma_{\pi N}\) to be negative and rather small. Our previous results for the decuplet magnetic moments turn out to hold within the accuracy of the model.

The octet and decuplet magnetic moments were calculated in Refs. [12] [13]. For the antidecuplet \(\mu_{\bar{10}}^{(0)}\) can be found in Ref. [11]. In order to calculate the \(\mu_{\bar{10}}^{(wf)}\), several off-diagonal matrix elements of the \(\hat{\mu}^{(0)}\) are required. These have been calculated in Ref. [16] in the context of the hadronic decay widths of the baryon antidecuplet.
Denoting the set of the model parameters by
\[ \vec{w} = (w_1, \ldots, w_6) \] (7)
the model formulae for the set of the magnetic moments in representation \( \mathcal{R} \) (of dimension \( R \))
\[ \vec{\mu}_\mathcal{R} = (\mu_{B_1}, \ldots, \mu_{B_R}) \] (8)
can be conveniently cast into the form of the matrix equations:
\[ \vec{\mu}_\mathcal{R} = A^\mathcal{R}[\Sigma_{\pi N}] \cdot \vec{w}, \] (9)
where rectangular matrices \( A^8, A^{10} \), and \( A^{10} \) can be found in Refs. [12, 13, 14]. Note their
dependence on the pion-nucleon \( \Sigma_{\pi N} \) term.

IV. RESULTS AND DISCUSSION

In order to find the set of parameters \( w_i[\Sigma_{\pi N}] \), we minimize the mean square deviation
for the octet magnetic moments:
\[ \Delta \mu^8 = \frac{1}{7} \sqrt{\sum_B \left( \mu^8_{B,\text{th}[\Sigma_{\pi N}]} - \mu^8_{B,\text{exp}} \right)^2}, \] (10)
where the sum extends over all octet magnetic moments, but the \( \Sigma^0 \). The value \( \Delta \mu^8 \approx 0.01 \)
is in practice independent of the \( \Sigma_{\pi N} \) in the physically interesting range \( 45 - 75 \text{ MeV} \). The values of the \( \mu^8_{B,\text{th}[\Sigma_{\pi N}]} \) are independent of \( \Sigma_{\pi N} \).

Similarly, the value of the nucleon strange magnetic moment is independent of \( \Sigma_{\pi N} \) and
reads \( \mu_N^{(s)} = 0.39 \text{n.m.} \) in fair agreement with our previous analysis of Ref. [13]. Parameters
\( w_i \), however, do depend on \( \Sigma_{\pi N} \). This is shown in Table I. Note that parameters \( w_{2,3} \) are
formally \( \mathcal{O}(1/N_c) \) with respect to \( w_1 \). For smaller \( \Sigma_{\pi N} \), this \( N_c \) counting is not borne by
explicit fits. The \( \mu_B^{(0)} \) can be parametrized by the following two parameters \( v \) and \( w \):
\[
\begin{align*}
v &= (2\mu_n - \mu_p + 3\mu_{\Xi^0} + \mu_{\Xi^-} - 2\mu_{\Sigma^-} - 3\mu_{\Sigma^+})/60 = -0.268, \\
w &= (3\mu_p + 4\mu_n + \mu_{\Xi^0} - 3\mu_{\Xi^-} - 4\mu_{\Sigma^-} - \mu_{\Sigma^+})/60 = 0.060.
\end{align*}
\] (11)
which are free of linear \( m_s \) corrections [13]. This is a remarkable feature of the present fit,
since when the \( m_s \) corrections are included, the \( m_s \)-independent parameters need not be
refitted. This property will be used in the following when we restore the linear dependence
of the \( \mu_B^{10} \) on \( m_s \).

| \( \Sigma_{\pi N} \) [MeV] | \( w_1 \) | \( w_2 \) | \( w_3 \) | \( w_4 \) | \( w_5 \) | \( w_6 \) |
|----------------|---------|---------|---------|---------|---------|---------|
| 45             | -8.564  | 14.983  | 7.574   | -10.024 | -3.742  | -2.443  |
| 60             | -10.174 | 11.764  | 7.574   | -9.359  | -3.742  | -2.443  |
| 75             | -11.783 | 8.545   | 7.574   | -6.440  | -3.742  | -2.443  |

TABLE I: Dependence of the parameters \( w_i \) on \( \Sigma_{\pi N} \).
The magnetic moments of the baryon decuplet and antidecuplet depend on the $\Sigma_{\pi N}$. However, the dependence of the decuplet is very weak, which as summarized in Table II where we also display the theoretical predictions from Ref. [12] for $p = 0.25$. Let us note that the $m_s$ corrections are not large for the decuplet and the approximate proportionality of the $\mu_{B_{10}}$ to the baryon charge $Q_B$ still holds. Finally, for antidecuplet we have a strong dependence on $\Sigma_{\pi N}$, yielding the numbers of Table III. The results listed in Table III are further depicted in Fig. 1.

| $\Sigma_{\pi N}$ [MeV] | $\Delta^{++}$ | $\Delta^+$ | $\Delta^0$ | $\Delta^-$ | $\Sigma_{\pi N}$ [MeV] | $\Sigma^{*+}$ | $\Sigma^{*0}$ | $\Sigma^{*-}$ | $\Xi^{*0}$ | $\Xi^{*-}$ | $\Omega^-$ |
|-------------------------|---------------|------------|------------|------------|-------------------------|---------------|------------|------------|------------|------------|--------------|
| 45                      | 5.40 2.65 2.65 2.65 2.65 2.65 2.65 2.65 2.65 2.65 2.65 2.65 | 60                      | 5.39 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 | 75                      | 5.39 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 2.66 | Ref. [12]   | 3.10 3.10 3.10 3.10 3.10 3.10 3.10 3.10 3.10 3.10 3.10 3.10 |

TABLE II: Magnetic moments of the baryon decuplet.

| $\Sigma_{\pi N}$ [MeV] | $\Theta^+$ | $p^+$ | $n^+$ | $\Sigma^+_{10}$ | $\Sigma^0_{10}$ | $\Sigma^-_{10}$ | $\Xi^+_{10}$ | $\Xi^0_{10}$ | $\Xi^-_{10}$ | $\Xi^{*-}_{10}$ |
|-------------------------|------------|--------|--------|-----------------|-----------------|-----------------|--------------|--------------|--------------|-----------------|
| 45                      | -1.19 -0.97 -0.34 -0.75 -0.02 0.71 | 60                      | -0.78 -0.36 -0.41 0.06 0.15 0.23 | 75                      | -0.33 0.28 -0.43 0.90 0.36 -0.19 | 45                      | -0.53 0.30 1.13 1.95 | 60                      | -0.70 0.93 1.15 | 75                      | -1.51 1.14 0.77 0.39 |

TABLE III: Magnetic moments of the baryon antidecuplet.

FIG. 1: Magnetic moments of antidecuplet as functions of $\Sigma_{\pi N}$.

The wave function corrections cancel for the non-exotic baryons and add constructively for the baryon antidecuplet. In particular, for $\Sigma_{\pi N} = 75$ MeV we have large admixture coef-
icient of 27-plet: $d_{27}^B$ tends to dominate otherwise small magnetic moments of antidecuplet. At this point, the reliability of the perturbative expansion for the antidecuplet magnetic moments may be questioned. On the other hand, as remarked above, the $N_c$ counting for the $u_i$ coefficients works much better for large $\Sigma_{\pi N}$. One notices for reasonable values of $\Sigma_{\pi N}$ some interesting facts, which were partially reported already in Ref.\cite{11}: The magnetic moments of the antidecuplet baryons are rather small in absolute value. For $\Theta^+$ and $p^*$ one obtains negative values although the charges are positive. For $\Xi_0$ and $\Xi^{-}$ one obtains positive values although the signs of the charges are negative.

V. CONCLUSION AND SUMMARY

Our present analysis shows that $\mu_{\Theta^+} < 0$, although the magnitude depends strongly on the model parameters. The measurement of $\mu_{\Theta^+}$ could therefore discriminate between different models. This also may add to reduce the ambiguities in the pion-nucleon sigma term $\Sigma_{\pi N}$.

In the present work, we determined the magnetic moments of the baryon antidecuplet in the *model-independent* analysis within the chiral quark-soliton model, i.e. using the rigid-rotor quantization with the linear $m_s$ corrections included. Starting from the collective operators with dynamical parameters fixed by experimental data, we obtained the magnetic moments of the baryon antidecuplet. The expression for the magnetic moments of the baryon antidecuplet is different from those of the baryon decuplet. We found that the magnetic moment $\mu_{\Theta^+}$ is negative and rather strongly dependent on the value of the $\Sigma_{\pi N}$. Indeed, the $\mu_{\Theta^+}$ ranges from $-1.19 \text{ n.m.}$ to $-0.33 \text{ n.m.}$ for $\Sigma_{\pi N} = 45$ and 75 MeV, respectively.

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