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Non-standard finite difference scheme and analysis of smoking model with reversion class

Anwar Zeb a, *, Abdullah Alzahrani b

a Department of Mathematics, COMSATS University Islamabad, Abbottabad Campus, Abbottabad 22060, Khyber Pakhtunkhwa, Pakistan
b Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Keywords:
Mathematical smoking model
Non-standard finite difference scheme
Numerical results

ABSTRACT

Smokers are at more risk to COVID-19 as the entertainment of smoking because their fingers are in touch with lips regularly during smoking that increases the probability of transmission of virus from hand to mouth. On other hand the smokers may have lung disease (or reduced lung capacity) which would greatly increase risk of serious illness especially COVID-19. For this esteem, in this research work, we first formulate a mathematical model contains the reversion class. Then, using different techniques for finding the local and global stability of the presented model related to equilibrium points that are free smoking and positive smoking equilibrium points. As the model consisting on the nonlinear equations, so we use the non-standard finite difference (NSFD) scheme, ODE45 and RK4 methods to find the numerical results. Finally, we show the graphs numerically through MATLAB.

Introduction

Smoking is that process, which not remains bounds to only the smokers but it affects the other society and economy of the country. Smoking sources such as water pipes often involve the sharing of mouth pieces and hoses, which could facilitate the transmission of COVID-19 in communal and social settings. Similarly, smokers are at more risk to COVID-19 as the entertainment of smoking because their fingers are in touch with lips that increases the probability of transmission of virus from hand to mouth. On other hand the smokers may have lung disease (or reduced lung capacity) which would greatly increase risk of serious illness especially COVID-19. It seems that the most of young smokers became regular smokers and then due to nicotine presence it is difficult to remove from this habit. Social habit of smoking was first discover by Christopher Columbus [1], Jean Nicot was the first man who introduced the tobacco in 1560 and from his name nicotine is deduced. In 1920s, it is discovered that the smoking having a strong relation with cancers and from that time the first campaign against smoking take start. In start, people used the simple form of cigarettes as in the form of pipes while the modern cigarettes take start between 1960 and 1970 from Indonesia. Procedure of tobacco enlarged during the world battles and the cause was, delivering free cigarette to allied groups for their moral boosting exercise, however in 20th century as a result of knowledge about health smoking became less popular. In same sense, due to COVID-19 the peoples became jobless and fall in great tension due to which the smoking became enlarge. Mathematicians are working on infectious diseases to describe the dynamics mathematically for the world. Many mathematical models were presented by many authors for different types of diseases take start from the first mathematical model by Ker-mack and McKendrick [2], see therein [2–5,8,7]. In recent days, smoking become a social habit, which is more dangerous for health because a cigarette contains 40 hundreds of chemicals consisting Ben- zene, Tar, Ammonia, Acetone etc., causes many cancers. Millions of smokers lost their lives due to the smoking. For these reasons, the mathematicians played an important role to present the smoking models for the reason that the people will aware about the dynamics of smoking. For the dynamics of smoking, the first model was presented by Castillo Garsow et al. [8] in 1997, in which they divided the whole population in three categories first one is potential smokers P, second one is smokers S and third one is permanently quit smokers Q. Then Sharomi and Gumel [9] extended the model by announcing the new class of temporarily quit smokers Q_. Ham [10], described the different phases and processes of smoking. Zaman [11] made his contribution in the form of introducing the class of occasional smoker and presented dynamical interaction in an integer order. Then, square root dynamics of smoking model were presented by Zeb et al. [12] for the purpose that the model go to extinction.

* Corresponding author.
E-mail address: anwar@cuiatd.edu.pk (A. Zeb).

https://doi.org/10.1016/j.rinp.2020.103785
Received 9 October 2020; Received in revised form 23 December 2020; Accepted 24 December 2020
Available online 6 January 2021
2211-3797/© 2021 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
in finite time. Many researchers tried on different aspects to developed smoking models and control on the epidemic models [8,7,10–17]. In this work, first we frame a mathematical model contains the reversion class then study the equilibrium points that are free smoking and positive smoking equilibrium points. Then using different techniques for finding the local as well as the global stability of the presented model. As the model consisting on the nonlinear equations, so we use the non-standard finite difference scheme, ODE45 and RK4 methods to find the numerical results. Finally show the graphs sketch through MATLAB.

This paper is organized as follow: In Section “Model formulation”, the formulation of model is presented in which the special case occurs. The stability of the proposed model is presented in Section “Stability and equilibria of the proposed model” and numerical solution is presented in Section “Numerical method and results”. Finally, we give conclusion in Section “Conclusion”.

Model formulation

For the model formulation different classes are considered, which detail as follow:

The compartment \( S(t) \) represents the potential smokers (non-smokers) people who have not smoked yet but may be started in future. This compartment is increased at rate \( \lambda \), which is the recruitment rate. The compartment \( C(t) \) stands for chain smokers, which is increased when potential smokers start to smoke with an incidence rate or contact rate between potential smokers and smokers \( \beta S(t)C(t) \) and the rate \( \rho_1 \) \( R(t) \) of the relapse smokers who revert back to smoking. Some other people will leave or vacate the compartment with rates \( \gamma C(t), \rho_2 C(t) \) and \( \mu C(t) \). The compartment \( R(t) \) shows reversion (relapse) class is increased with rate \( \gamma C(t) \). This compartment is decreased at rate \( \mu R(t), \rho_2 R(t) \) and \( \alpha C(t)R(t) \), where \( \alpha \) is relapse rate. On the similar way \( Q(t) \) stands for permanent quitters.

The following five assumptions are assured for model formulation:

- \( A_1 \): The total population is divided into two habitual (smokers) and two non-habitual (non-smokers) compartments.
- \( A_2 \): The natural death rate \( \mu \) is approximately equal to recruitment rate \( \lambda \).
- \( A_3 \): All the recently new born are assumed to join only potential smokers compartment.
- \( A_4 \): In all compartment’s the natural death rate is same \( \mu \).
- \( A_5 \): No person, is more popular than another, so that interaction between any two compartments are equally likely.

To describe an equation of difference for rate at which population of each compartment changes over discrete time, the entry of people to a compartment is represented by addition and taking off people from a compartment is shown by subtraction. Using assumptions \( A_1 – A_5 \), the Ice smoking mathematical model is described by the following differential equations:

\[
\begin{align*}
\frac{dS(t)}{dt} &= \lambda - \beta S(t)C(t) - \mu S(t), \\
\frac{dC(t)}{dt} &= \beta S(t)C(t) - (\gamma + \mu + \rho_1)C(t) + \alpha C(t)R(t), \\
\frac{dR(t)}{dt} &= \gamma C(t) - (\mu + \rho_2)R(t) - \alpha C(t)R(t), \\
\frac{dQ(t)}{dt} &= \rho_1 C(t) + \rho_2 R(t) - \mu Q(t),
\end{align*}
\]

(1)

where the parameters used for this model is described in the following table. Now (see Table 1)

\[
\frac{dS}{dt} + \frac{dC}{dt} + \frac{dR}{dt} + \frac{dQ}{dt} = \lambda - \mu(S + C + R + Q).
\]

(2)

| Parameter | Definition |
|-----------|------------|
| \( \lambda \) | Birth or migration rate to the host population |
| \( \beta \) | The incidence rate for susceptible population to regular smokers class |
| \( \rho_1 \) | Rate by which the regular smokers go to quit smokers |
| \( \rho_2 \) | Represents the rate at which the reversion individual joins quit individual |
| \( \mu \) | Natural death rate for all classes |
| \( \gamma \) | Rate at which regular smokers go to reversion class |
| \( \alpha \) | Rate at which the relapse class people go to regular class |

Table 1

Parameters and variables of smoking model.

Suppose the total population is \( N = S + C + R + Q \); then Eq. (2) leads the following solution

\[
\lim_{t \to \infty} N(t) = \frac{\lambda}{\mu}.
\]

Thus, population of the feasible solution for model (1) is restricted to the region:

\[
\Gamma = \{ (S, C, R, Q) : S + C + R + Q = \frac{\lambda}{\mu}, S > 0, C, R, Q \geq 0 \}.
\]

(3)

The \( \Gamma \) region is positively invariant because all the solutions of system (1) are bounded and enter the \( \Gamma \). That shows, every solution of model (1), with initial conditions in \( \Gamma \), remains there for all \( t > 0 \).

Stability and equilibria of the proposed model

Without loss of generality first three equations of model (1) are independent of the variable \( Q(t) \), so we omit the forth equation for simplicity and discuss the dynamics of system consists on first three equations. So the free smoking equilibrium point \( E_0 \) of model (1) is a steady state solution, for the entire population is non-smokers. Which is

\[
E_0 = \left( S^0, C^0, 0, 0 \right).
\]

Use the next generation matrix method [18] for smoking generation number \( N_0 \) as follow. Let \( P = (C, R, S) \); then the reduced form of model (1) can be written as

\[
P' = \Psi(P) - Y(P)
\]

such that

\[
\Psi(P) = \begin{bmatrix} \beta SC \\ 0 \\ 0 \end{bmatrix},
\]

(4)

\[
Y(P) = \begin{bmatrix} (\gamma + \mu + \rho_1)C - \alpha CR \\ -\gamma C + (\mu + \rho_2)R + \alpha CR \\ -\lambda + \beta SC + \mu S \end{bmatrix}.
\]

(5)

Now the Jacobian of \( \Psi(P) \) is

\[
J(\Psi(P)) = \begin{bmatrix} \beta S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

at \( E_0 \),
Theorem 1. System (1) has free smoking equilibrium point \( E_0 = (\frac{\lambda}{\gamma}, 0, 0) \) and for a positive smoking equilibrium point \( E' = (S', C', R') \), one has three cases:

(i) If \( R_0 < 1 \) and \( \gamma(\mu + \rho_1) \gg \lambda \alpha \), one has no positive equilibrium point.

(ii) If \( R_0 > 1 \), one has one positive equilibrium point \( E' \).

(iii) If \( R_0 = 1 \), one has no positive equilibrium point.

Proof. To find the positive smoking equilibrium point \( E' \) for system (1) by setting \( \frac{dS}{d\tau} = \frac{dC}{d\tau} = \frac{dR}{d\tau} = 0 \), as

\[
0 = \lambda - \beta S C' - \mu S',
\]

\[
0 = \beta S C' - (\gamma + \mu + \rho_1) C' + aC R',
\]

\[
0 = \gamma C' - (\mu + \rho_2) R' - aC R'.
\]

So Eqs. (10) and (12) implies that

\[
S' = \frac{\lambda}{\mu + \beta C'},
\]

\[
R' = \frac{\gamma C'}{\mu + \rho_2 + aC'}.
\]

and from Eq. (11), we have

\[
\beta S' - (\gamma + \mu + \rho_1) + aR' = 0,
\]

\[
\Rightarrow \frac{\lambda \beta}{\mu + \beta C'} - (\gamma + \mu + \rho_1) + \frac{a\gamma C'}{\mu + \rho_2 + aC'} = 0.
\]

\[
C' = \left( (\mu + \rho_1)(\mu + \rho_1) + \beta(\mu + \rho_1)(\mu + \rho_2) - \lambda \alpha \beta \right) C' + (\mu + \rho_1) \left( \frac{\rho \mu(\mu + \rho_1) - \lambda \beta}{a\beta(\mu + \rho_1)} \right) = 0.
\]

The Eq. (15) has solutions of the form

\[
C'_{1,2} = -\frac{A_1}{2} \pm \frac{1}{2} \sqrt{A_1^2 - 4A_2},
\]

where \( A_1 \) and \( A_2 \) are

\[
A_1 = \left( \beta \rho(\mu + \rho_1) + \mu(\mu + \rho_1) + \beta(\mu + \rho_1)(\mu + \rho_2) - \lambda \alpha \beta \right),
\]

\[
A_2 = (\mu + \rho_1) \left( \frac{\rho \mu(\mu + \rho_1) - \lambda \beta}{a\beta(\mu + \rho_1)} \right).
\]

From here, we conclude that \( A_2 < 0 \) and \( \sqrt{A_1^2 - 4A_2} > A_1 \) for \( R_0 > 1 \). Hence, we have only one positive solution:

\[
C' = \frac{1}{2} \left( -A_1 + \sqrt{A_1^2 - 4A_2} \right).
\]

While \( A_2 = 0 \) and there is no-positive solutions for \( R_0 = 1 \). Further, if \( R_0 < 1 \) and \( \gamma(\mu + \rho_1) \gg \lambda \alpha \) then \( A_1, A_2 > 0 \) implies that there are no-positive solutions. \( \square \)

Theorem 2. If \( R_0 < 1 \), then point \( E_0 \) of model (1) is locally asymptotically stable, and if \( R_0 > 1 \), then \( E_0 \) is unstable.

Proof. Evaluating the Jacobian matrix of model (1) at \( E_0 \) by linearizing the model provides

\[
J = \begin{bmatrix}
-\beta C - \mu & -\beta S & 0 \\
\beta S - (\gamma + \mu + \rho_1) + aR & aC & \gamma - aR \\
0 & -\mu - \rho_2 - aC & -\mu - \rho_2 - aC
\end{bmatrix}.
\]

so
\[
J(E_0) = \begin{bmatrix}
-\mu & -\beta\lambda \\
0 & \frac{\beta\lambda}{\mu} - \left(\gamma + \mu + \rho_1\right) & 0 \\
0 & \gamma & -\mu - \rho_2
\end{bmatrix}.
\]

For finding the eigenvalues solve the equation
\[
det[J(E_0) - \delta I] = 0,
\]
\[
\begin{vmatrix}
-\mu - \delta & -\beta\lambda \\
0 & \frac{\beta\lambda}{\mu} - \left(\gamma + \mu + \rho_1\right) - \delta & 0 \\
0 & \gamma & -\mu - \rho_2 - \delta
\end{vmatrix} = 0,
\]
which follows that
\[
\delta_1 = -\mu, \\
\delta_2 = -\mu - \rho_2,
\]
and
\[
\delta_3 = \left(\gamma + \mu + \rho_1\right)[R_0 - 1].
\]

Hence, the eigenvalues of \(J(E_0)\) are \(\delta_1 < 0, \delta_2 < 0\) and \(\delta_3 = \left(\gamma + \mu + \rho_1\right)[R_0 - 1]\). If \(R_0 < 1\), then \(\delta_2 < 0\). So, for \(R_0 < 1\) all eigenvalues are negative and hence \(E_0\) is locally asymptotically stable and \(\delta_2 > 0\) for \(R_0 > 1\), which follows that there exists a positive eigenvalues, so \(E_0\) is unstable.

Further, we explore the local stability of smoking positive equilibrium \(E'\) by using the following lemma.

**Lemma 3.** (see [21]). Let \(M\) be a \(3 \times 3\) real matrix. If \(\text{tr}(M), \det(M)\) and \(\det(M^2)\) are all negative, then all of the eigenvalues of \(M\) have negative real part.

**Proof.** Let \(M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\) be a matrix which is piecewise smooth on \(\Gamma\) which satisfies the conditions \((g) = 0, \langle \nabla g \rangle < 0 \text{ in the interior of } \Gamma\), where \(\nabla g\) is the normal vector to \(\Gamma\) and \(g = f_1 + f_2 + f_3\) is the Lipschitz continuous field in the interior of \(\Gamma\) and

\[
\langle \nabla g \rangle = \left(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y} \right) - \left(\frac{\partial g_1}{\partial y} + \frac{\partial g_2}{\partial x} \right) + \left(\frac{\partial g_3}{\partial x} - \frac{\partial g_3}{\partial y} \right) \kappa.
\]

Then the differential equations \(\frac{dx}{dt} = f_1, \frac{dy}{dt} = f_2\) and \(\frac{dz}{dt} = f_3\) has no periodic solutions, homoclinic loops, and oriented phase polygons inside \(\Gamma\).

Let \(\Gamma = \{(S, C, R) : \frac{\partial g_1}{\partial x} = f_1, \frac{\partial g_2}{\partial y} = f_2\} \text{ and } \frac{\partial g_3}{\partial x} = f_3\) has no periodic solutions, homoclinic loops, and oriented phase polygons inside the invariant region \(\Gamma\).

**Theorem 5.** The positive smoking equilibrium \(E'\) of model (1) is locally asymptotically stable if \(a_1 < \beta\).

**Proof.** Linearizing model (1) at the equilibrium \((S', C', R')\) provides
\[
J(E') = \begin{bmatrix}
-\beta C' - \mu & -\beta S' & 0 \\
\beta C' & \beta S' - \left(\gamma + \mu + \rho_1\right) + \alpha R' & a C' \\
0 & \gamma - a R' & -\mu - \rho_2 - a C'
\end{bmatrix},
\]
here the term
\[
a_{22} = \beta S' - \left(\gamma + \mu + \rho_1\right) + \alpha R',
\]
now from Eq. (12), it follows that
\[
a_{22} = 0.
\]
So \(J(E')\) becomes
\[
J(E') = \begin{bmatrix}
-\beta C' - \mu & -\beta S' & 0 \\
\beta C' & 0 & a C' \\
0 & \gamma - a R' & -\mu - \rho_2 - a C'
\end{bmatrix},
\]
The second additive compound matrix of \(J^{(2)}(E')\) is
\[
J^{(2)}(E') = \begin{bmatrix}
-\beta C' - \mu & -\beta S' & 0 \\
\beta C' & 0 & a C' \\
0 & \gamma - a R' & -\mu - \rho_2 - a C'
\end{bmatrix},
\]
and \(\text{tr}(J(E'))\) is
\[
\text{tr}(J(E')) = -2\mu - \rho_2 - a C' - \beta C' < 0.
\]
Now the \(\det(J(E'))\) is
\[
\det(J(E')) = a_1 a_2^2 - \left(\gamma + \mu + \rho_1\right) a_2 - \beta a_1^2 - a_2 a_3 - a_1 a_3 - 1 < 0, \quad \text{if } a_1 < \beta.
\]
and in similar way the \(\det(J^{(2)}(E')) < 0\), if \(a_1 < \beta\). Therefore, through Lemma 3, \(E'\) is locally asymptotically stable if \(a_1 < \beta\).

**Theorem 6.** If \(R_0 < 1\), then system (1) is globally stable.

**Proof.** For proof, first we construct the Lyapunov function \(L\);
\[
L = (S - S_0) + ln \left(\frac{C}{C_0}\right) + (R - R_0),
\]
after differentiating and algebraic simplification along \(E_0\), we have
\[
\frac{dL}{dt} = \left(\frac{\beta\lambda}{\mu} - (\gamma + \mu + \rho_1)\right).
\]

Theorem 7. (see [21]) Let \(g(S, C, R) = \left|g_0(S, C, R), g_2(S, C, R), g_3(S, C, R)\right|\) be a vector field which is piecewise smooth on \(\Gamma\) and which satisfies the conditions \((g) = 0, \langle \nabla g \rangle \leq 0 \text{ in the interior of } \Gamma\), where \(\nabla g\) is the normal vector to \(\Gamma\) and \(g = f_1 + f_2 + f_3\) is the Lipschitz continuous field in the interior of \(\Gamma\) and

\[
\langle \nabla g \rangle = \left(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y} \right) - \left(\frac{\partial g_1}{\partial y} + \frac{\partial g_2}{\partial x} \right) + \left(\frac{\partial g_3}{\partial x} - \frac{\partial g_3}{\partial y} \right) \kappa.
\]

Then the differential equations \(\frac{dx}{dt} = f_1, \frac{dy}{dt} = f_2\) and \(\frac{dz}{dt} = f_3\) has no periodic solutions, homoclinic loops, and oriented phase polygons inside \(\Gamma\).

Let \(\Gamma = \{(S, C, R) : \frac{\partial g_1}{\partial x} = f_1, \frac{\partial g_2}{\partial y} = f_2\} \text{ and } \frac{\partial g_3}{\partial x} = f_3\) has no periodic solutions, homoclinic loops, and oriented phase polygons inside the invariant region \(\Gamma\).

**Theorem 8.** Model (1) has no periodic solutions, homoclinic loops, and oriented phase polygons inside the invariant region \(\Gamma\).

**Proof.** Let \(f_1, f_2, f_3\) denote the right hand side of equations in model (1) respectively. Using the equation \(\frac{\partial g_1}{\partial x} = f_1, \frac{\partial g_2}{\partial y} = f_2\) and \(f_3\) in the equivalent forms, as
\[ f_1(S, C) = \lambda - \beta SC - \mu S, \]
\[ f_1(S, R) = \lambda - \mu S - \beta S \left( \frac{R}{S} - R \right), \]
\[ f_2(S, C) = -(\gamma + \mu + \rho_1)C + \beta SC + \alpha C \left( \frac{S}{C} - S - C \right), \]
\[ f_2(C, R) = -(\gamma + \mu + \rho_1)C + \beta \left( \frac{R}{C} - C - R \right)C + \alpha CR, \]
\[ f_3(S, R) = -(\mu + \rho_2)R - (\alpha R - \gamma) \left( \frac{R}{S} - S - R \right), \]
\[ f_4(S, R) = -(\mu + \rho_2)R - \alpha CR + \gamma C. \]

Suppose \( g = (g_1, g_2, g_3) \) be a vector field, where:
\[ g_1 = \frac{f_1(S, R)}{SR} - \frac{f_1(C, R)}{SC}, \]
\[ g_2 = \frac{f_1(S, C)}{SC} - \frac{f_1(C, R)}{CR}, \]
\[ g_3 = \frac{f_2(S, C)}{SR} - \frac{f_1(S, R)}{CR} + \frac{f_2(C, R)}{CR}, \]
\[ g_4 = \frac{f_2(S, C)}{SC} - \frac{f_2(C, R)}{CR} + \frac{f_3(S, R)}{SR}, \]
\[ g_5 = \frac{f_3(S, R)}{SR} - \frac{f_3(S, C)}{SC} - \frac{f_3(C, R)}{CR} + \frac{f_4(S, R)}{SR}. \]

Then \( gf \equiv \nabla g \equiv 0 \) on \( \Gamma \). As
\[ \text{(curl) } \nabla = \left( \begin{array}{c} \frac{\beta \gamma}{R} \frac{\gamma}{R} \frac{\gamma}{S} \frac{\lambda}{S} \frac{\alpha}{S} \frac{\lambda}{S} \frac{\alpha}{S} \frac{\lambda}{S} \\ \frac{\beta \gamma}{R} \frac{\gamma}{R} \frac{\gamma}{S} \frac{\lambda}{S} \frac{\alpha}{S} \frac{\lambda}{S} \frac{\alpha}{S} \frac{\lambda}{S} \end{array} \right) \]

Using the normal vector \( \vec{n} = (\frac{\gamma}{S}, \frac{\gamma}{S}) \) to \( \Gamma \), we have

\[ \implies (\text{curl} \vec{n}) = -\frac{\mu}{S} \frac{\mu}{S} - \frac{\mu}{S} \frac{\gamma}{S} < 0, \]
\[ \implies (\text{curl} \vec{n}) = -\frac{\mu}{S} \frac{\mu}{S} - \frac{\mu}{S} \frac{\gamma}{S} < 0. \]

Hence, model (1) has no periodic solutions, homoclinic loops, and oriented phase polygons inside the invariant region \( \Gamma \). This completes the proof.

Consequently, we have the following result.

**Theorem 9.** If \( R_0 > 1 \), then the positive smoking equilibrium point \( E^* \) of system (1) is globally asymptotically stable.

**Proof.** We know that, if \( R_0 > 1 \) in \( \Gamma \), then \( E_0 \) is unstable. Also \( \Gamma \) is a positively invariant subset of \( \Gamma \) and the \( \alpha \) limit set of each solution of system (1) is a single point in \( \Gamma \) since there is no periodic solutions, homoclinic loops, and oriented phase polygon inside \( \Gamma \). Therefore model (1) at \( E^* \) is globally asymptotically stable.

**Numerical method and results**

NSFD method is used for numerical solution of proposed model (1). Basically NSFD scheme [19–22] is an iterative method in which we get near to solution through iteration. Let non-standard ODEs is given below

\[ x'_k = f(x, x_1, x_2, \ldots, x_n) \]

where \( k = 1, 2, \ldots, n \), then by NSFD method

\[ x'_k = \frac{x_{k+1} - x_k}{h}, \]
\[ x'_k = \frac{x_{k+1} - x_k}{h}, \]
\[ \vdots \]
\[ x'_k = \frac{x_{k+1} - x_k}{h}. \]

Here, using NSFD method for numerical solution of system (1) will leads

\[ S_{k+1} = \frac{\gamma h + S_k}{1 + \beta h C_k + \mu h}, \]

\[ C_{k+1} = \frac{C_k}{(1 - \beta h S_{k+1} + h(\gamma + \mu + \rho_1) - \alpha h R_k)}, \]

\[ R_{k+1} = \frac{\gamma h C_{k+1} + R_k}{1 + (\mu + \rho_1)h + \alpha h C_{k+1}}, \]

\[ Q_{k+1} = \frac{\rho_1 C_{k+1} + \rho_2 R_{k+1} + Q_k}{1 + \mu + h}, \]

\[ N_{k+1} = \frac{\lambda h + N_k}{1 + \lambda h}. \]

In Figs. 1–4, the approximate solution of smoking model of potential smokers, regular (chain) smokers, relapse smokers and quit smokers (quitters) are represented respectively. From figures, it is clear that results of NSFD and RK4 are matching very well for different values of parameters. The parameters values used for the numerical simulations
are $\mu = 0.0001, \beta = 0.0005, \alpha = 0.0002, \rho_1 = 0.004, \rho_2 = 0.002, \gamma = 0.007, \lambda = 0.9$, with initial values $S_0 = 100, C_0 = 79, R_0 = 55, Q_0 = 10$.

Conclusion

In this paper, a smoking mathematical model is presented in the light of some assumptions. First, we formulated the smoking model then discussed the dynamics of proposed model. It is observed that the population of the feasible solution for model (1) is restricted to the region;

$$\Gamma = \left\{ (S, C, R, Q): S + C + R + Q = \frac{\lambda}{\mu}, S > 0, C, R, Q \geq 0 \right\}.$$

The following results are obtained.

* If $R_0 < 1$, the free smoking equilibrium point $E_0$ of model (1) is locally asymptotically stable, and if $R_0 > 1$, then $E_0$ is unstable.
* The positive smoking equilibrium $E^*$ of model (1) is locally asymptotically stable if $\alpha \leq \beta$.
* If $R_0 < 1$, then the system (1) is globally stable.

Finally, a discrete-time, finite difference scheme was constructed using the nonstandard finite difference (NSFD) method. Numerical results of NSFD were compared with RK4 and ODE45, which showed a good agreement.

Availability of data and materials

The authors confirm that the data supporting the findings of this study are available within the article cited there in.

Authors Contribution

Authors are equally contributed in preparing this manuscript.

Funding

This article is supported by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under Grant No. D 1439-158-130.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This project was funded by the Deanship of Scientific Research (DSR) at King Abdulaziz University, Jeddah, under Grant No. D 1439-158-130. The authors, therefore, gratefully acknowledge DSR for technical and financial support.

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