Analysis of stresses and deformations of elements of axle boxes and improvement of their shape

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Abstract. The development of constructive recommendations aimed at increasing the service life of roller bearings of axle boxes of railway cars is described. The elastic volumetric stress-strain state of the bearing elements of the bearings under consideration is analyzed. Numerical analysis of the corresponding mathematical models was carried out using the finite element method in the NX NASTRAN software environment. It is shown that near the edge of the roller with chamfer-rounding, a stress concentration arises, caused by insufficient smoothness of the contour line of the axial section of the roller. It is noted that this concentration is caused by the fact that the line of the contour under consideration has a finite specified curvature in the section corresponding to the chamfer and almost zero curvature in the adjacent straight section corresponding to the cylindrical surface of the roller. It is shown that the use of smoother contour lines for the marginal zone of a roller more than doubles the maximum value of the stress intensity in the boundary zone under consideration and accordingly increases the cyclic life of the bearing. The proposed smoothing of the line of the considered circuit allows one not only to unload the edge of the roller but also to reduce the contact pressure in its middle part. At the same time the stress-strain state of the roller becomes more uniform by increasing the smoothness of the contour line in the area of the roller’s edge and simultaneously reducing the degree of bombing (barrel-shaped) its cylindrical surface.

1. Introduction
Currently, there are significant changes in the operating conditions of wheel sets of railway cars. The greatest contribution to these changes is made by an increase in the speeds of movement of the compositions and an increase in their weight [1–3]. At the same time, the decisive contribution to ensuring the reliability of wheelset operation is made by increasing the operational reliability of the wheels and elements of axle boxes – rollers and bearing rings [4–9]. One of the effective ways to improve them is to search for rational design options based on the use of mathematical models of their stress-strain state (SSS) and experimentally verified criteria for assessing their durability [10–12]. Under the conditions of cyclic operation of high-loaded bearing elements of structures, such a criterion, as a rule, is the intensity of stresses [13–22].

2. Results and Discussion
Let us consider for definiteness a constructive scheme for coupling a bearing roller and its rings by the example of one of the typical axlebox nodes (figure 1) [23].

The durability of the elements of the axle assembly in many cases is determined by the moment the contact fatigue shells [24] appear on the rolling contact surfaces of the bearings and their inner and outer rings (figure 2). The determining factor in the occurrence of fatigue damage is an increased level of
stress intensity in a possible focus of damage. To calculate this factor, a mathematical model of elastic deformation of the elements of the axle box can be used. In the case of a homogeneous isotropic material of the body in question, this model is determined by the differential equilibrium equations of the theory of elasticity:

\[ Lu = (\lambda + \mu) \text{grad div } u + \Delta u = 0, \]  

(1)

where \( L \) is the differential operator of the theory of elasticity [25];

\( u \) is the desired displacement vector at points in the domain \( D \) occupied by the deformable elements of the structure under consideration;

\( \lambda, \mu \) – coefficients that determine the mechanical properties of a homogeneous isotropic material of structural elements (Lame parameters).

**Figure 1.** Constructive scheme of the coupling roller bearing and its rings: (a) - constructive diagram of a typical axle box of a freight car: 1 - box housing; 2 - labyrinth ring; 3 - rear bearing; 4 - front bearing; 5 - cover fastening; 6 - tag; 7 - viewing cover; 8 - disc washer; 9 - lock washer; 10 - M20 bolt of the disc washer; 11 - M12 cover bolt; 12 - sealing ring; (b) - axial section of a fragment of the rear bearing of an axle box corresponding to one roller

**Figure 2.** Contact fatigue shells on the surface of the inner ring of the rear bearing in the zone of contact of the edge of the roller and ring
Equations (1) are considered in the domain \( D \) bounded by the surface \( S = S_u + S_\sigma + S_{u\sigma} \) under boundary conditions

\[
\begin{align*}
\mathbf{u} \big|_{S_u} &= \varphi(M) \text{ with } M \in S_u; \quad (\mathbf{v} \cdot \mathbf{T}) \big|_{S_\sigma} = F(M) \text{ with } M \in S_\sigma \\
(\mathbf{u} \cdot \mathbf{v}) \big|_{S_{u\sigma}} &= \varphi_v(M), (\mathbf{v} \cdot \mathbf{T} \cdot \mathbf{\nu}) \big|_{S_{u\sigma}} = 0 \text{ with } M \in S_{u\sigma},
\end{align*}
\]

where \( S_u, S_\sigma, S_{u\sigma} \) are the surfaces of the considered parts, on which surface displacements, surface forces and mixed (contact) boundary conditions are given, respectively;

\( T \) is the stress tensor;

\( \varphi(M), \varphi_v(M) \) are given vector and scalar functions of point \( M \), respectively;

\( F(M) \) is the vector of surface forces specified on the surface \( S_\sigma \);

\( \mathbf{\nu}, \mathbf{\tau} \) are unit vectors defining the normal and tangential directions to the surface \( S \) at the considered point \( M \).

Numerical solution of problem (1) - (3) was carried out using the finite element method (FEM), based on the preliminary discretization of the studied deformable system [26, 27]. In this case, the FEM ratios were considered from the standpoint of the variational approach, according to which the sought nodal displacements of the discrete analogue of the deformable parts of the axle box were found from the condition of minimizing its potential energy:

\[
\Pi = \frac{1}{2} \int_V \mathbf{\epsilon}^T \sigma dV - \mathbf{U}^T \mathbf{R},
\]

where \( \mathbf{\epsilon} \) is the strain vector at the points of the deformed system under consideration (\( T \) is the transposition symbol) formed from the components of the strain tensor;

\( \sigma \) is a similar stress vector;

\( \mathbf{U} \) is the vector of displacements of the discretization nodes of the deformable structure of the axle box, \( V \) is its volume;

\( \mathbf{R} \) is the vector of external nodal effects on the considered discretized deformable system of elements.

Within a finite element, the displacement vector \( \mathbf{u} \) can be represented using the matrix of interpolating functions \( N \):

\[
\mathbf{u} = N \mathbf{d},
\]

where \( \mathbf{d} \) is the vector of nodal displacements in the considered finite element (FE).

The displacement vector \( \mathbf{u} = [u(x, y, z), v(x, y, z), w(x, y, z)]^T \) is considered in the Cartesian coordinate system, which is global for the deformed system under consideration. When constructing a discrete deformation model, linear CEs were used, the displacements within which were interpolated by functions depending on the \( x, y, z \) coordinates. The deformation vector is determined within the CE through the vector \( \mathbf{d} \) by the relation:

\[
\mathbf{\epsilon} = B \mathbf{d},
\]

in which \( B \) is the corresponding matrix of differential operators (strain matrix), defined by the equality:

\[
B = A N^T,
\]

and the matrix of operators \( A \) determines the transition from displacements to deformations:

\[
\mathbf{\epsilon} = A \mathbf{\mathbf{\epsilon}}^u.
\]

Using the given relations (5) – (8), we represent the potential energy under consideration through the vector \( \mathbf{U} \) – the global vector of the nodal displacements of the discretized deformed system:

\[
\Pi = \frac{1}{2} \int_V \mathbf{U}^T B^T DB U dV - \mathbf{U}^T \mathbf{R},
\]

where \( D \) is the matrix of elastic constants of the material: \( \sigma = D \mathbf{\epsilon} \).

The conditions for minimizing the magnitude \( \Pi \):

\[
\frac{\partial \Pi}{\partial \mathbf{U}} = \begin{bmatrix} \frac{\partial \Pi}{\partial u_1}, \frac{\partial \Pi}{\partial u_2}, ..., \frac{\partial \Pi}{\partial u_{N_u}} \end{bmatrix}^T = 0
\]

(10)
and lead to a system of linear algebraic equations:

\[ KU = R, \]  

(11)
in which \( K \) is the global stiffness matrix of the considered discretized construction of the axle box:

\[ K = \int_V B^TDBdV. \]  

(12)
The matrix \( K \) is built in the process of implementing the FEM by assembling the stiffness matrices of individual finite elements determined by the equality:

\[ \frac{1}{2} d^e_e F_e = \frac{1}{2} \int_{V_e} e^e_e \sigma dV_e \]  

(13)
where \( V_e \) is the volume of the CE number \( e \) in the general array of the CE numbers of the discretized construction of the considered constructive node as a whole (the global EC number, then the index \( e \) in the given relations relating to the individual CE is omitted).

The finite element stiffness matrix \( k \) is determined by the ratio:

\[ F = kd, \]  

(14)
in which \( d \) is the vector of nodal displacements of the considered FE; \( F \) is the corresponding vector of nodal forces.

Using relations (13), (14) for the stiffness matrix of an individual CE, we obtain:

\[ k = \int_{V_e} B^TDBdV_e. \]  

(15)
The given relations (4) – (15) completely determine the construction of system (11) – a system of linear algebraic equations (SLAE), which determines an approximate solution of problem (1) – (3).

Taking into account the specified boundary displacements specified in the nodes of the axle box design under consideration allows replacing the general system (11) with the reduced SLAE:

\[ K_{red}U = R_{red}, \]  

(16)
the determinant of which is non-zero, and present the desired approximate solution of the problem in the form:

\[ U = (K_{red})^{-1}R_{red}. \]  

(17)
The FEM implementation as applied to the solution of the problem of determining the VAT of the elements of the axle box of the wheel set was carried out in the NX NASTRAN software environment. In the same system, the necessary geometric models were built, the axial section of which is shown in Fig. 3-5.

\[ \text{Figure 3. Geometrical models of a solid-rolled wheel (a), axles of a set of wheels (b) and inner rings of a bearing (c).} \]
Figure 4. Geometric models of outer rings (a), roller bearings (b) and axle box housing (c).

Figure 5. The location of the rollers in the front bearing box.

Figure 6. The pattern of stress intensity distribution that occurred when the rollers interact with the bearing rings when a load is applied.
In the process of modeling, it was assumed that a vertical uniformly distributed calculated load of $10^7\text{H}$ acts on the thrust surfaces of the housing of the axle box. The general picture of the stress intensity distribution arising in the roller of the axle box under this load is shown in figure 6. From the figure it can be seen that in the rollers of the rear bearing the stress level is higher than in the rollers of the front one. At the same time, the third roller (with respect to the vertical axial section of the axle box), the rear bearing roller, experiences the greatest load.

When considering the features of contact and rings, a preliminary breakdown of the rollers and bearing rings into fragments was used to achieve the required modeling accuracy (Figure 7a). In this case, each of the fragments was split on the FE independently with increasing concentration as it approaches the contact surface and uniformly within the fragment (Figure 7b).

![Fragmentation scheme](image1)

**Figure 7.** Fragmentation of a discrete model of contact interaction between a roller and bearing rings: (a) – fragmentation scheme; (b) is a finite element breakdown with step condensation.

VAT within the structural elements of the axle box is substantially uneven. So, the size of the stress intensity concentration zone in a roller is determined by the depth of the location of the maximum stress intensity point $\sigma_{\text{max}}$ below the roller contact surface. This size determines the depth of the contact fatigue pits (Figure 2) that appear on the contact surfaces during the operation of the bearing, and is about 0.25 mm (Figure 8).

![Stress intensity distribution](image2)

**Figure 8.** The stress intensity distribution below the contact surface of the roller and bearing ring.
The fragments were considered as separate deformable bodies, the solutions in which were mated as follows. On the inner (non-contact surfaces of the rollers and rings) surfaces of the fragments, the solutions were mated corresponding to the full adhesion (gluing) of the fragments. On the contact surfaces of the rollers and rings, mating conditions were specified, corresponding to the ideal contact of the mating surfaces (contact without friction). The described technique allowed the use of fairly small finite elements in fragments adjacent to the contact surfaces of the rollers and rings, the size of which does not exceed 0.04 mm, and achieve the required modeling accuracy on the used computing equipment (Intel Xeon E5 central processor, 64 GB RAM).

Of considerable interest is the distribution of stress intensity near the edge of the roller. The results of the corresponding modeling for a roller with a rectilinear generatrix (without bombing) and facet rounding are shown in figure 9a. It can be seen from the figure that there is a significant stress concentration near the edge of the examined roller. The reason for the stress concentration near the edge of the roller in this case is the lack of smoothness of the contour of the profile of the roller (the contour of the profile of the axial section of the roller). So in the interface zone of the chamfer-rounding contour and the rectilinear generatrix of the cylindrical surface of the roller, the curvature of the contour line changes abruptly: within the chamfer, this line has a predetermined non-zero curvature, and the curvature of the rectilinear generatrix is zero. An even more significant stress concentration near the edge of the roller is caused by a conical chamfer, which creates a corner point at the edge of the contact surface of the roller and bearing ring. Let us note that the curvature of the curve is proportional to the second derivative of the function describing the contour line. From a mathematical point of view, the profile contour line with chamfer-rounding has a discontinuity of the second derivative of this function, which reduces the smoothness of the solutions of problem (1) – (3), found in the area bounded by the surface with the contour line under consideration [28]. From this circumstance, it follows that the use of smoother lines of the profile of the roller profile will lead to a significant decrease in the level of stress intensity near the edge of the roller.

To check the formulated position, the VAT simulation was carried out near the edge of the roller, the chamfer of which had in the axial section instead of rounding with a constant radius of curvature (Figure 10a) an ellipse shape with a ratio of 1:10 semiaxes (Figure 10b).

The results of stress intensity modeling for a roller with an elliptical shape of a chamfer profile are shown in Figure 9b.

![Figure 9](image-url)  

**Figure 9.** The stress intensity distribution $\sigma_i$ in the axial section of the bearing near the edge of the most loaded roller: (a) – roller with a typical shape of a chamfer profile; (b) – a roller with an elliptical shape of a chamfer profile
3. Conclusions
From the obtained results it follows that the use of rolling elements with a smoothly varying curvature of the line of the profile of their axial section allows to significantly reduce the maximum value of stress intensity near the edge of the roller and, accordingly, increase the fatigue life of its work. The results obtained can be used to improve the profile shape of the bombed rollers. It is significant that the proposed smoothing of the contour line profile of the axial section of the roller allows not only relieving the roller edge, but also reducing the contact pressure in its middle part, making the stress-strain state of the roller more uniform by increasing the smoothness of the contour line in the area of the roller edge and simultaneously reducing the degree of bombing (barrel-shaped) of its cylindrical surface.

4. References
[1] Development strategy of the transport engineering of the Russian Federation for the period up to 2030 Approved by the order of the Government of the Russian Federation of August 17, 2017 № 1756-r
[2] Bayasgalan D 2019 Justification of the necessity of constructing the third railway line between Mongolia and Russia Modern Technologies. System Analysis. Modeling 61:1, 97–101 DOI: 10.26731/1813-9108.2019.1(61).97–101
[3] Bayasgalan D 2018 The necessity of building a second railway line between Mongolia and China Modern Technologies. System Analysis. Modeling 60:4 56–63 DOI: 10.26731/1813-9108.2018.4(60).56-63
[4] Borodin A V, Ivanova Yu A and Gric D B 2014 Influence of the radial load distribution on the life of the axle bearing of a freight car The Journal Omsk Scientific Bulletin 2 35–37
[5] Nevmerzhiczkaya G V, Shlyushenkov A P 2002 Estimated reliability assessment of the axlebox bearing of the car according to the fatigue chipping criterion Dynamics, strength and reliability of transport vehicles (Bryansk: BSTU) 113-117
[6] Tao L, Wei S, Zhichao M, Junzhou H, Jianghui D, Liping W 2019 Dynamic investigation on railway vehicle considering the dynamic effect from the axle box bearings Advances in mechanical engineering 11: 4 DOI: 10.1177/1687814019840503
[7] Hassan M, Bruni S 2019 Experimental and numerical investigation of the possibilities for the structural health monitoring of railway axles based on acceleration measurements Structural health monitoring—an international journal 18:3 902-919 DOI: 10.1177/147592171878642
[8] Tsvik L B, Zapolskij D V, Zen’kov E V and Eremeev V K 2015 Design profile railway wheels on a straight search in space radiuses hollow chamfer Transport of the Urals 3 67–70
[9] Martua L, Ng AK, Sun G 2018 Prediction of Rail Rolling Contact Fatigue Crack Initiation Life via Three-Dimensional Finite Element Analysis Int. Conf. on Intelligent Rail Transportation (ICIRT, Singapore)
[10] Borodin A V, Kulinich E N and Ivanova Ju A 2010 Improvement of the roller box of the freight car Journal of Transsib Railway Studies 2 15–20
[11] Borodin A V, Ivanova Ju A, Gric D B 2016 The axle box of a freight car with increased rigidity of the axle The Journal Omsk Scientific Bulletin 4 5–8
[12] Petrov G and Tarmaev A 2018 Modeling of railway vehicles movement having deviations in the content of running parts Proceedings of the International Conference: Aviamechanical Engineering and Transport (AVENT 2018) (Atlantis Press: Series Advances in Engineering Research) 158 410–415 DOI:10.2991/avent-18.2018.79.
[13] Mahutov N A 2005 Structural strength, life and man-made safety Part 1 (Novosibirsk: Science)
[14] Nevmerzhickaja G V 2006 Computer simulation of contact pressure distribution in a freight car bearing and assessment of its durability Bulletin of Computer and Information Technologies 2 18-21
[15] Wang Q, Zhao Y and Wang H 2014 Experiments and characterization of the probabilistic fatigue lives and strength of D1 railway wheel steel Journal of mechanical engineering 50:14 51-55 DOI: 10.3901/JME.201414.050
[16] Tomashevskij S B 2011 Wheelset – axle box: stress models World of Transport and Transportation] 1 38–42
[17] Tomashevskij S B 2011 Clarification of the solution of contact problems on the example of calculating the details of railway transport Transport of the Urals 2 66-70
[18] Tomashevskij S B 2011 The influence of elastic-plastic deformations on the results of solving contact problems of railway transport Bulletin of Bryansk state technical university 3 17-
[19] Tomashevskij S B 2011 Numerical simulation of machine hardening by surface plastic deformation Science and Technology in Transport 2 60-68
[20] Yao Ch, Zhiwei W, Bingyan Ch, Weihua Zh and Guanhua H 2019 An improved complementary ensemble empirical mode decomposition with adaptive noise and its application to rolling element bearing fault diagnosis ISA transactions DOI: 10.1016/j.isatra.2019.01.038
[21] Yang Z, Deng Xy, Li Zl 2018 Numerical modeling of dynamic frictional rolling contact with an explicit finite element method Tribology international 11 214-231 DOI: 10.1016/j.triboint.2018.08.028
[22] Molodova M., Li Z., Núñez A., Dollevoet R. 2014 Validation of a finite element model for axle box acceleration at squats in the high frequency range Comput Struct 141 84–93.
[23] State standard 15872-2014 2015 Rolling bearings. Axle box cylindrical roller bearings for rolling stock. Specification (Moscow: Standartinform)
[24] Classifier for defects and damage to rolling bearings 2007 (Moscow: Russian Railways)
[25] Parton V Z, Perlin P I 1981 Methods of the mathematical theory of elasticity (Moscow: Science)
[26] Bathe K-J and Wilson E L 1976 Numerical Methods in Finite Element Analysis (Prentice Hall)
[27] Duong Van Lam, A. A. Pykhalov 2018 Interpolation of geometry and inhomogeneity of material of deformable solids when constructing their 3d models with the finite elements method based on the computer tomograph scanning Modern Technologies. System Analysis. Modeling 60:4 8 – 15, DOI: 10.26731/1813-9108.2017.3(55).8-16
[28] Vladimirov V S 1981 Equations of mathematical physics (Moscow: Science)