Designed defects in 2D antidot lattices for quantum information processing

Jesper Pedersen\textsuperscript{a,1}, Christian Flindt\textsuperscript{a}, Niels Asger Mortensen\textsuperscript{a}, and Antti-Pekka Jauho\textsuperscript{a,b}

\textsuperscript{a}MIC – Department of Micro and Nanotechnology, NanoDTU, Technical University of Denmark, Building 345east, DK-2800 Kongens Lyngby, Denmark

\textsuperscript{b}Laboratory of Physics, Helsinki University of Technology, P. O. Box 1100, FI-02015 HUT, Finland

Abstract

We propose a new physical implementation of spin qubits for quantum information processing, namely defect states in antidot lattices defined in the two-dimensional electron gas at a semiconductor heterostructure. Calculations of the band structure of a periodic antidot lattice are presented. A point defect is created by removing a single antidot, and calculations show that localized states form within the defect, with an energy structure which is robust against thermal dephasing. The exchange coupling between two electrons residing in two tunnel-coupled defect states is calculated numerically. We find results reminiscent of double quantum dot structures, indicating that the suggested structure is a feasible physical implementation of spin qubits.

Key words: EP2DS-17, quantum computing, exchange coupling, antidot lattices

PACS: 03.67.Lx, 73.21.Cd, 75.30.Et

The possibility of utilizing the spins of electrons confined in quantum dot systems as the fundamental building blocks for large-scale quantum computing was first introduced by Loss and DiVincenzo in 1998 \cite{1} and has since led to numerous theoretical and experimental studies within this field. In the proposal by Loss and DiVincenzo the exchange coupling between the spins of the electrons serves as the mechanism for coherent manipulation of and interaction between the spin qubits. Inspired by these ideas we have recently proposed to use bound states which form at the location of point defects in periodic antidot lattices as an alternative way of realizing spin qubits \cite{8}. The available fabrication methods suggest that such structures may offer high scalability more readily than conventional gate-defined quantum dots \cite{9}.

We consider a two-dimensional electron gas superimposed with a triangular lattice of antidots with lattice constant $\Lambda$. In the effective-mass approximation the two-dimensional single-electron Hamiltonian is

$$H = -\frac{\hbar^2}{2m^*} \nabla^2 \rho + \sum_i V(\mathbf{r} - \mathbf{R}_i), \quad \mathbf{r} = (x, y), \quad (1)$$

where $m^*$ is the effective mass of the electron and $V(\mathbf{r} - \mathbf{R}_i)$ is the potential of the $i$'th antidot positioned at $\mathbf{R}_i$. We use parameter values typical of GaAs, for which $\hbar^2/2m^* \simeq 0.6$ eVnm$^2$, and assume a lattice constant of $\Lambda = 45$ nm. Modeling each antidot as an infinite circular potential barrier of diameter $d$ allows us to solve the problem using finite-element methods with the Dirichlet boundary condition that the eigenfunctions are zero in the antidots \cite{8}.

The calculated band structure of the periodic antidot lattice is shown in Fig. 1 for two different val-
Fig. 1. Band structure of the periodic antidot lattice with lattice constant $\Lambda = 45$ nm and two different values of the relative antidot diameter $d/\Lambda$. On the left graph the gap $\vartheta_{\text{eff}}$ is indicated, below which no states exist for the periodic structure.

Fig. 2. Energy spectrum for a single defect, showing the three lowest energy eigenvalues as a function of the relative antidot diameter $d/\Lambda$. The full line indicates the height $\vartheta_{\text{eff}}$ of the effective potential in which the localized states reside. The inset shows the absolute square of the localized eigenfunction corresponding to the eigenvalue indicated with a circle.

The existence and location of all band gaps have been verified by density of states calculations (not shown) \cite{11}. The general increase in energies with the antidot diameter is due to the increased confinement of the Bloch states.

We next consider the case where a point defect has been introduced in the lattice by leaving out a single antidot. The gap $\vartheta_{\text{eff}}$ defined in Fig. 1 may be considered as the height of an effective two-dimensional circular step potential surrounding the defect, and thus gives an upper limit to the existence of bound states localized in the defect. Similar states are expected to form in the band gaps of the periodic structure. We refer to these localized states as defect states. These decay to zero far from the location of the defect, allowing us to solve the problem on a domain of finite size, imposing once again Dirichlet boundary conditions on the antidots and on the edge of the domain. The discrete spectrum of a single defect is shown in Fig. 2 for states residing below $\vartheta_{\text{eff}}$. The inset shows the eigenfunction corresponding to the lowest eigenvalue. As expected a defect leads to the formation of a number of localized states at the location of the missing antidot. Calculations have confirmed the existence of similar states in the band gap regions \cite{11}. The results indicate that the number of localized states can be tuned via the relative antidot diameter $d/\Lambda$, allowing for $n = 1, 2, 3, \ldots$ levels in the defect. For $d/\Lambda = 0.5$ the energy splitting between the two lowest states is approximately 3.6 meV, which is much larger than $k_B T$ at subkelvin temperatures, and the energy structure is thus robust against thermal dephasing.

Together with single-qubit operations, the exchange coupling between the spins of electrons confined in double quantum dot structures has been shown to be a sufficient mechanism for implementing a universal set of quantum gates for quantum information processing \cite{10}. The exchange coupling arises as a consequence of the Pauli principle, which couples the symmetries of the orbital and spin degrees of freedom. The splitting of the lowest eigenvalue $E_S$ corresponding to a symmetric orbital wavefunction and the lowest eigenvalue $E_A$ corresponding to an anti-symmetric orbital wavefunction may thereby be mapped onto an effective Heisenberg spin Hamiltonian $H = J \mathbf{S}_1 \cdot \mathbf{S}_2$, where $J = E_A - E_S$ is the exchange coupling.

Analogous to a double quantum dot system we now consider an antidot lattice in which a single antidot and one of its next-nearest neighbors have been left out of the lattice. In the following we tune the coupling between the defects via a metallic split gate defined on top of the 2DEG in order to control the opening between the two defects. By increasing the applied voltage one squeezes the opening. As the exchange coupling depends on the overlap of the defect states, we may thereby control the exchange coupling electrostatically. The split gate is modeled as an infinite potential barrier shaped as shown in
Fig. 3. Exchange coupling $J$ as a function of the relative split gate constriction width $w/\Lambda$ for three different values of the relative antidot diameter $d/\Lambda$. The dashed line indicate the results obtained in the Hubbard approximation for $d/\Lambda = 0.7$. The inset shows the calculated two-electron charge density of the singlet ground state for $d/\Lambda = 0.5$ and $w/\Lambda = 2$. The split gate is shown in grey, while antidots are white.

Using recently developed numerically exact methods [11], we have calculated the exchange coupling for such a double defect geometry. In Fig. 3 we show the calculated exchange coupling as a function of the relative split gate constriction width $w/\Lambda$ for three different values of the antidot diameter. We also show the calculated two-electron charge density of the singlet ground state for $d/\Lambda = 0.5$ and $w/\Lambda = 2$. The exchange coupling varies several orders of magnitude as the split gate constriction width is increased. These results are similar to those obtained for double quantum dot structures, where the exchange coupling has been calculated as a function of interdot distance [2]. The figure also shows the exchange coupling calculated in the Hubbard approximation, $J_H = 4t^2/U$, where $t$ is the tunnel coupling between the defect states while $U$ is the on-site Coulomb repulsion [2]. We note that while this approximation yields qualitatively correct results for the entire range of parameters, the approximation clearly has no quantitative predictive power. We also note that in general the validity of the approximative schemes used to evaluate the exchange coupling in low-dimensional nanostructures is highly dependent on both the form of the potential under consideration as well as the choice of parameter values [12].

In conclusion, we have shown that defect states in antidot lattices may serve as a physical implementation of spin qubits for large-scale quantum information processing. We find that introducing a point defect in an antidot lattice leads to the formation of localized states within the defect, with a level structure which is robust against thermal dephasing. Calculations of the exchange coupling show results similar to those obtained for double quantum dot structures, allowing for electrostatic tuning of the exchange coupling over several orders of magnitude.

References

[1] D. Loss, D. P. DiVincenzo, Phys. Rev. A 57 (1998) 120.
[2] G. Burkard, D. Loss, D. P. DiVincenzo, Phys. Rev. B 59 (1999) 2070.
[3] J. Gorman, D. Hasko, D. Williams, Phys. Rev. Lett. 95 (2005) 90502.
[4] M. Helle, A. Harju, R. M. Nieminen, Phys. Rev. B 72 (2005) 205329.
[5] F. Koppens et al., Science 309 (2005) 1346.
[6] J. R. Petta et al., Science 309 (2005) 2180.
[7] L. X. Zhang, D. V. Melnikov, J. P. Leburton, Phys. Rev. B 74 (2006) 205306.
[8] C. Flindt, N. A. Mortensen, A.-P. Jauho, Nano Lett. 5 (2005) 2515.
[9] A. Fuhrer et al., Superlattices and Microstructures 31 (2002) 19.
[10] D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, K. B. Whaley, Nature 408 (2000) 339.
[11] J. Pedersen, C. Flindt, N. A. Mortensen, A.-P. Jauho, Phys. Rev. B 76 (2007) 125323, arXiv:0706.1859v1 [cond-mat.mes-hall].