Giant Molecular Cloud Formation through the Parker Instability
in a Skewed Magnetic Field

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Abstract

The effect of the magnetic skew on the Parker instability is investigated by means of the linear stability analysis for a gravitationally stratified gas layer permeated by a horizontal magnetic field. When the magnetic field is skewed (i.e., the field line direction is a function of the height), the wavelength of the most unstable mode is $\lambda \sim 10H$ where $H$ is the pressure scale height. The growth rate of the short wavelength modes is greatly reduced when the gradient in the magnetic field direction exceeds 0.5 radian per scale height. Our results indicate that the Parker
instability in a skewed magnetic field preferentially forms large scale structures like giant molecular clouds.

Subject Headings: hydromagnetics — instabilities — interstellar:magnetic fields.

1. INTRODUCTION

The Parker instability has been thought to play a major role on the formation of giant molecular clouds in galaxies (Parker 1966; Shu 1974; Mouschovias 1974; Mouschovias, Shu, and Woodward 1974). Although the Parker instability has many characteristics favorable for the formation of giant molecular clouds, it has one unfavorable characteristic in this regard: the fastest growing mode of the Parker instability has an infinite wave number perpendicular to the magnetic field line and may produce many small gas condensations instead of large scale gas condensations (Asséo et al. 1978). This argument has been used to suggest that the pure Parker instability (without self-gravity) leads to chaotic structures and turbulence rather than large scale cloud complexes (Asséo et al. 1978; Elmegreen 1982). This question, however, is based on a linear stability analysis which assumes that the magnetic field is parallel in the unperturbed state. The Galactic magnetic field, however, is not perfectly parallel (Heiles 1987; Sawa & Fujimoto 1986; Shibata & Matsumoto 1991), but rather disturbed and skewed. When the magnetic field is skewed, the mode having a large wave number perpendicular to the magnetic field is likely to be suppressed, since such modes force the adjacent field lines to intersect. Thus, the fastest growing mode is expected to have a finite wave number and to produce large
scale condensations. This Letter studies the effect of magnetic skew on the Parker instability by means of linear stability analysis and supports the above expectation.

The stability analysis is carried out in §2 and its application to giant molecular cloud formation is discussed in §3.

2. LINEAR STABILITY ANALYSIS

We consider an idealized model of a skewed magnetic field to evaluate the effect of the magnetic skew on the Parker instability. The model consists of an isothermal, stratified gas layer permeated by a horizontal magnetic field whose direction changes as a function of height at a constant pitch, \( \varphi(z) \equiv \tan^{-1}(B_y/B_x) \propto z \). The density distribution and the magnetic field of the model are

\[
\rho = \rho_0 \exp \left(-z/H\right),
\]

\[
B = (B_x, B_y, B_z)
\]

\[
= [B_0 \cos (\varepsilon z/H) \exp (-z/2H), B_0 \sin (\varepsilon z/H) \exp (-z/2H), 0]
\]

where \( H \) is the scale height and \( \varepsilon \) denotes the strength of the magnetic skew. (The magnetic skew is not necessary to be constant to suppress the growth of short wave length modes. We imposed this assumption in order to minimize the number of model parameters and to make our model as simple as possible.) This model atmosphere is in hydrostatic equilibrium under gravity in the \( z \)-direction. From the equation of motion, the scale height is expressed as

\[
gH = c_s^2 + \frac{B_0^2}{8\pi \rho_0},
\]
where \( g \) is the gravitational acceleration and assumed to be constant for simplicity. The plasma beta, \( \beta \equiv \frac{8\pi \rho c_s^2}{B^2} \), is constant in this model atmosphere, and is assumed to be unity, \( \beta = 1 \), in the following except when otherwise is noted. Figure 1 shows a schematic view of the model magnetic field. (Although this configuration of the magnetic field is called “magnetic shear” in the solar physics, we use the word “skew” in order to avoid the confusion with the magnetic field sheared with the Galactic differential rotation.)

We consider a small perturbation superimposed on the equilibrium described above. The perturbation is assumed to be isothermal and to be described by the ideal MHD equations. Since the perturbation equations are standard, their detailed form is omitted here (see, e.g., Horiuchi et al. 1988 for perturbation equations). A normal mode solution of the perturbation equations has the form of

\[
\rho_1(x, y, z) = \rho_1(z) \exp (-i\omega t + ik_xx + ik_yy). \tag{4}
\]

According to the variational principle (see, e.g., Bernstein 1983; Nakamura, Hanawa, and Nakano 1991) the square of the eigenfrequency, \( \omega^2 \), is real. The boundary condition is set so that the energy density of the eigenfunction is periodic,

\[
\rho_1(z + 2\pi H/\varepsilon) = e^{i\theta - \pi/\varepsilon} \rho_1(z). \tag{5}
\]

See Shu (1974) and Nakamura, Hanawa, and Nakano (1991) for the reason why the energy density should be constant in an eigenfunction. The ratio of the phase shift, \( \theta \), to the vertical period of the magnetic skew, \( 2\pi H/\varepsilon \), is an effective wavenumber in the vertical direction (\( k_{z, eff} = \theta \varepsilon /2\pi H \)). Since our model has the helical
symmetry, the eigenfrequency is a function of the total horizontal wave number, $\omega = \omega(\sqrt{k_x^2 + k_y^2})$. [ Our model is invariant under the transformation of $(x, y, z) \rightarrow (x \cos \delta + y \sin \delta, -x \sin \delta + y \cos \delta, z + \delta H/\varepsilon)$, where $\delta$ is an arbitrary number. ]

Figure 2 shows the stability diagram of our model atmosphere for $\varepsilon = 1$ and $\beta = 1$. The ordinate and the abscissa are the growth rate, $-\omega^2 H/g$, and the wave number, $k^2 H^2$, respectively. The growth rate and the wave number are normalized by $\sqrt{g/H}$ and $1/H$, respectively. The stability diagram has a band structure similar to the energy level diagram of a crystal (cf. Kittel 1976). The darkly painted regions denote the allowed bands and the blank regions show the inhibited bands. The most unstable mode has a finite wave number of $kH = 0.70$.

On the top of the uppermost band $v_{1z}$ of the eigenmode has no node in the vertical direction ($\theta = 0$). The value of $\theta$ changes from $2n\pi$ to $2(n + 1)\pi$ in each band. As the number of nodes in the eigenfunction increases ($\theta$ increases), the growth rate decreases continuously but with some gaps. This is a general property of the Parker instability. When the initial magnetic field is parallel, the growth rate decreases continuously as the wavenumber, $|k_z|$, increases (see, e.g., Parker 1979). When the Parker unstable layer has a finite thickness, the growth rate is discrete and decreases with the increase in the number of nodes in the eigenfunction (Horiuchi et al. 1988). In our model, the Parker unstable layers appear periodically in the vertical direction for a given $k$. In the band structure of figure 2 the growth rate is continuous in each band and discrete between bands. This is because our model atmosphere has a periodical structure where each Parker unstable layer has a finite thickness but the number of the Parker unstable layers is infinite.
Figure 3 shows the maximum growth at a given wave number for $\varepsilon = 0.2$, 0.5, and 1.0. The growth rate decreases as $\varepsilon$ increases for a given $k$. The decrease is larger for a larger wave number. When $\varepsilon \geq 0.2$, the mode of $k = \infty$ is not the fastest growing mode. The most unstable mode has a wave length of $\lambda \sim 10H$ for $0.2 \leq \varepsilon \leq 1.0$.

Figure 4 shows the maximum growth at $k = \infty$ as a function of $\varepsilon$. As $\varepsilon$ increases, the growth rate at $k = \infty$ decreases. The growth of the mode having $k$ is suppressed by magnetic tension when $k \cdot B$ is large. Whenever the magnetic field is not parallel and $k$ is sufficiently large, most magnetic field lines are bent with a short wavelength and the magnetic tension suppresses the growth of the mode. From figures 3 and 4 we find that the magnetic skew suppresses the Parker instability of short wavelength modes and that the effect is appreciable for $\varepsilon \geq 0.5$. Note that $\varepsilon = 0.5$ corresponds to the case when the direction of the magnetic field changes $29^\circ$ per scale height.

3. APPLICATION TO GIANT MOLECULAR CLOUD FORMATION

The analysis of the previous section demonstrated that the fastest growing Parker instability has a wavelength of $\lambda = 10H$ when the magnetic field is skewed appreciably. It implies that large scale mass condensations are formed by the Parker instability if the Galactic magnetic field is skewed significantly, i.e., more than several tens degrees per scale height. The Galactic magnetic field is not uniform in the Solar neighborhood (see, e.g., Heiles 1987 and the references therein) and has a substantial nonuniform component. Galactic dynamo theory (see, e.g., Sawa and Fujimoto 1986;
Fujimoto and Sawa (1987) also predicts that the Galactic magnetic field changes its direction as a function of the height; in their model the direction of the magnetic field in the halo (at the height of \( z = 1 \) kpc \( \sim 6H \)) differs \( 180^\circ \) from that in the disk (\( \varepsilon \sim 0.5 \)). Also the nonlinear development of the Parker instability produces the skewed magnetic field owing to the Coriolis force, even when the initial field is not skewed (Shibata and Matsumoto 1991). Consequently, although a firm observational evidence of the magnetic skew has not been obtained, it is very likely that the Galactic magnetic field has a skew component which is significantly strong for the formation of giant molecular clouds through the Parker instability.

Our results (Fig. 3) indicate that the growth time of the Parker instability is

\[
\tau \approx 4.9 \times 10^7 \left( \frac{\omega^2 H/g}{5.8 \times 10^{-2}} \right)^{-1/2} \left( \frac{H}{160 \text{ pc}} \right)^{1/2} \left( \frac{g}{3.5 \times 10^{-9} \text{ cm s}^{-2}} \right)^{-1/2} \text{ years,} \quad (6)
\]

in a skewed field with \( \varepsilon \sim 0.2 - 1.0 \) and \( \beta = P_g/P_m = 1 \) and the mass of the cloud formed by the Parker instability is

\[
M \sim 2\rho \lambda^2 H \sim 2 \times 10^7 \left( \frac{n}{1 \text{ cm}^{-3}} \right) \left( \frac{\lambda}{10H} \right)^2 \left( \frac{H}{160 \text{ pc}} \right)^3 \text{ M}_\odot. \quad (7)
\]

We now briefly comment on the effect of the self-gravity on the Parker instability. Hanawa, Nakamura, and Nakano (1991) has analyzed the linear stability of the Galactic gaseous disk taking account of the magnetic field, the Galactic rotation, and the self-gravity of the gas. The magnetic field was assumed to be parallel in the equilibrium. According to them, the growth rate is larger when the wavenumber perpendicular to the field line is larger. The growth of short wavelength modes are
not suppressed by the self-gravity. “Turbulence” rather than a cloud is likely to be formed as far as the magnetic field is parallel in equilibrium even if the self-gravity is taken into account. Thus, the skewed magnetic field is an important factor to produce a large scale structure like clouds through the Parker instability.

The skewed magnetic field may affect not only the linear growth rate of the Parker instability but also the nonlinear evolution of the Parker unstable modes. The intersection of the magnetic field lines is strictly inhibited in the framework of the ideal magnetohydrodynamics. Thus, the growth of an unstable mode will be saturated at latest at the stage when simple extrapolation of the linear stability analysis predicts the intersection of the magnetic field lines (see Matsumoto et al 1988, 1990 for justification of this conjecture). Figure 5 shows the eigenfunction of the Parker unstable mode in the skewed magnetic field ($\varepsilon = 0.5$ and $\beta = 1$). The ordinate is the velocity perturbation, $v_{1z}$, at the origin $(x, y) = (0, 0)$ and the abscissa is the height $z$. The thick curve is for the mode of $(k_x, k_y) = (0.7/H, 0)$ and the thin curve for that of $(k_x, k_y) = (0, 0.7/H)$. As shown in the figure the velocity, $v_{1,z}$, is not a monotonically increasing function of $z$. At some height, the rising magnetic field line catches up with the foregoing field line as the perturbation grows. The growth of the perturbation is likely to be saturated as the field lines becomes closer to each other. If the two eigenmodes shown in figure 5 are excited simultaneously, however, the field lines are less likely to catch up with the neighboring field lines. The linear combination of the eigenmodes is also an eigenmode since the growth rates are the same $[\omega(k) = \omega(|k|)]$. It implies that the perturbation is less easy to be saturated when more than 2 plane waves are simultaneously excited.
If all the mode having the same wave number, $|k|$, are excited, the perturbation velocity increases with height monotonically $v_{1z} \propto \rho_0^{-1/2} \propto \exp(z/2H)$ at $(x, y) = (0, 0)$. Then, the perturbation grows up to have a large amplitude without suffering serious nonlinear effects. The linear combination of many plane waves leads to a cylindrical wave having a symmetry axis at $(x, y) = (0, 0)$ (see Fig. 5). Thus, we expect that a circular structure would be formed in a skewed magnetic field rather than a stripe.

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Figure Legends

**Fig. 1.** Schematic view of the skewed magnetic field.

**Fig. 2.** The stability diagram for $\varepsilon = 0.5$ and $\beta = 1$. The ordinate is the square of the growth rate, $-(H/g)\omega^2$, and the abscissa is the square of the wave number, $H^2k^2$. The eigenmode has a band structure. The allowed bands are denoted with black and the inhibited bands are denoted with blank.

**Fig. 3.** The maximum growth rate at a given wave number for $\varepsilon = 0.2, 0.5, \text{and } 1.0$.

**Fig. 4.** The maximum growth rate at $k = \infty$ as a function of $\varepsilon$.

**Fig. 5.** The velocity perturbation, $v_{1z}$, of the unstable eigenmode at $(x, y) = (0, 0)$ as a function of the height, $z$ for $\varepsilon = 0.5, \beta = 1.0, |k|H = 0.7, \text{and } \omega^2H/g = -5.82 \times 10^{-2}$. The thick solid curve is for the mode of $(k_xH, k_yH) = (0.7, 0.0)$ and the thin solid curve for that of $(k_xH, k_yH) = (0.0, 0.7)$. The dashed curve denotes the linear combination of many plane wave eigenmodes, where the velocity perturbation is proportional to $v_{1z} \propto \rho^{-1/2}$ at $(x, y) = (0, 0)$.