Formulas for direct determination of real speed in Galactic superluminal sources

Yi-Ping Qin¹,²,³,⁴

¹ Yunnan Observatory, Chinese Academy of Sciences, Kunming, Yunnan 650011, P. R. China; E-mail: ypqin@public.km.yn.cn
² National Astronomical Observatories, Chinese Academy of Sciences
³ Chinese Academy of Science-Peking University joint Beijing Astrophysical Center
⁴ Yunnan Astrophysics Center

In this paper we show explicit and direct relations between the expected quantities and the observed quantities for the components of Galactic superluminal sources. Basic formulas for calculating the real speed and the angle to the line of sight of the components from the data of proper motions and the distance of the source are presented. We point out that the real speed and the angle to the line of sight of components can be uniquely and directly determined from the observed values of the proper motions and the distance of the source. It is not necessary to calculate an intermediate quantity first and then using the resulted value to calculate the two required quantities. The process of the calculation is simple, and in this way, some extra uncertainties are avoided.

Key Words: galaxies: jets — radio continuum: stars

1 Introduction

Superluminal motions found in distant quasars and active galactic nuclei (AGNs) are an interesting astrophysical phenomenon that arouses a worldwide attention. The motions have been inferred for radio emitting components in these objects (e.g., Blandford et al., 1977). The components move away from the central sources at apparent speeds greater than that of light c. A generally accepted explanation is that clouds of plasma are ejected in opposite directions from the central source at real speeds close to c, and the relativistic effects lead to the apparent superluminal motion (Rees, 1966). But the extreme distance of the sources introduces many uncertainties into this interpretation (Kellermann and Owen, 1988). Fortunately, there is now direct evidence for superluminal motions in the radio images of two strong Galactic X-ray transient sources, GRS
1915+105 and GRO J1655-40 (Mirabel and Rodriguez, 1994; Tingay et al., 1995; Hjellming and Rupen, 1995). As pointed out by Mirabel and Rodriguez (1994), the optical, infrared and X-ray properties of GRS 1915+105 (Mirabel et al., 1984; Harmon et al., 1994) suggest that the source is either a neutron star or a black hole that is ejecting matter in a process similar to, but on a smaller scale than that seen in quasars. Because of their relative proximity, these Galactic superluminal sources may offer the best opportunity to gain a general understanding of relativistic ejections seen elsewhere in the Universe.

To understand the power of the sources that are ejecting material in such a violent manner, the measure of the real speed of the components is desired. The difficulty of measuring the real speed of components of quasars and AGNs is that distances to these sources are still uncertain since they depend on the cosmological parameters and on the cosmology itself. Since the distance of Galactic sources is free of cosmological models and can be directly measured, the discovery of superluminal motions within our own Galaxy makes it possible to determine the real speed of the components at a high level of accuracy.

In the following we illustrate how to use the measure of the proper motions of components to determine their real speeds as well as the angles to the line of sight.

2 Determination of the real speed of components of Galactic sources

For Galactic sources, the apparent proper motions of the approaching and receding components moving in opposite directions away from a common origin of a source along an axis at an angle $\theta$ to the line of sight at the real speed $\beta c$ can be expressed as (Pearson and Zensus, 1987; Mirabel and Rodriguez, 1994)

$$\mu_a = \frac{\beta \sin \theta}{1 - \beta \cos \theta} \frac{c}{D},$$

$$\mu_r = \frac{\beta \sin \theta}{1 + \beta \cos \theta} \frac{c}{D},$$

where $D$ is the distance from the observer to the source, and $\mu_a$ and $\mu_r$ are the proper motions of the approaching and receding components, respectively. These two equations lead to

$$\beta = \sqrt{\left(\frac{2\mu_a \mu_r D}{c}\right)^2 + (\mu_a - \mu_r)^2},$$

$$\mu_a + \mu_r,$$
\[ \theta = \arccos \frac{\mu_a - \mu_r}{\sqrt{\left(\frac{2\mu_a\mu_r D}{c}\right)^2 + (\mu_a - \mu_r)^2}}. \]  

(4)

It is clear that, when the proper motions \( \mu_a \) and \( \mu_r \) as well as the distance \( D \) are measured, the real speed \( \beta c \) and the angle to the line of sight \( \theta \) can then be uniquely determined.

In practice, the observed parameters \( \mu_a, \mu_r \) and \( D \) are always presented in the form \( \mu_a \pm \Delta \mu_a \), \( \mu_r \pm \Delta \mu_r \) and \( D \pm \Delta D \). If the errors are relatively small, the errors of \( \beta \) and \( \theta \) can be determined by means of:

\[ \Delta \beta = \frac{\beta (\mu_a + \mu_r)}{\sqrt{\left[\frac{4D^2\mu_a^2}{c^2} + 2\mu_r (\mu_a - \mu_r)\right] \Delta D^2 + \left[\frac{4D^2\mu_a^2}{c^2} + 2\mu_r (\mu_a - \mu_r)\right] \Delta \mu_a^2 + \left[\frac{4D^2\mu_a^2}{c^2} - 2\mu_a (\mu_a - \mu_r)\right] \Delta \mu_r^2}}. \]  

(5)

\[ \Delta \theta = \frac{2D \mu_a \mu_r}{c} \sqrt{\frac{(\mu_a - \mu_r)^2 (\Delta D)^2 + \mu_r^2 (\Delta \mu_a)^2 + \mu_a^2 (\Delta \mu_r)^2}{\left[\frac{4D^2\mu_a^2}{c^2} + (\mu_a - \mu_r)^2\right]^2}}. \]  

(6)

Recently, two Galactic sources were found to have bidirectional relativistic proper motions of radio components and the proper motions as well as the distances of the sources were well measured. They are: (1) GRS 1915+105, \( D = (12.5 \pm 1.5)kpc, \mu_a = (17.6 \pm 0.4)masd^{-1} \), \( \mu_r = (9.0 \pm 0.1)masd^{-1} \) (Mirabel and Rodriguez, 1994); (2) GRO J1655-40, \( D = 3.2kpc, \mu_a = 54masd^{-1} \), \( \mu_r = 45masd^{-1} \) (Hjellming and Rupen, 1995). With these data, the values of the real speed and the angle to the line of sight for these sources can then be uniquely determined:

(1) GRS 1915+105, \( \beta c = (0.917 \pm 0.098)c, \theta = (69.36 \pm 2.35)^\circ \); (2) GRO J1655-40, \( \beta c = 0.910c, \theta = 84.27^\circ \). The errors of \( \beta \) and \( \theta \) for GRO J1655-40 are not given since \( \Delta \mu_a, \Delta \mu_r \) and \( \Delta D \) for the source are not available.

The above method can be applied to extragalactic sources when \( D \) is replaced by \( D_L/(1 + z) \), where \( D_L \) and \( z \) are the luminosity distance and the redshift of the source, respectively. In the Friedmann cosmology, \( D_L \) is determined by

\[ D_L = \frac{cz(1 + z + \sqrt{1 + 2q_0 z})}{H_0(1 + q_0 z + \sqrt{1 + 2q_0 z})}. \]  

(7)

where \( H_0 \) and \( q_0 \) are the Hubble constant and the deceleration parameter of the universe, respectively. In this case,

\[ D = \frac{cz(1 + z + \sqrt{1 + 2q_0 z})}{H_0(1 + z)(1 + q_0 z + \sqrt{1 + 2q_0 z})}. \]  

(8)

To apply Equations (5) and (6), one just replaces the observed \( \Delta D \) with a calculated one, derived
from Equation (8). That is

$$\Delta D = \sqrt{\left(\frac{\partial D}{\partial H_0}\right)^2 \Delta H_0^2 + \left(\frac{\partial D}{\partial q_0}\right)^2 \Delta q_0^2 + \left(\frac{\partial D}{\partial z}\right)^2 \Delta z^2},$$

(9)

with

$$\frac{\partial D}{\partial H_0} = -\frac{D}{H_0},$$

(10)

$$\frac{\partial D}{\partial q_0} = \frac{Dz[1 + z + q_0 z + (1 + z)\sqrt{1 + 2q_0 z}]}{(1 + 2q_0 z)(2 + z + q_0 z) + (2 + z + z \sqrt{1 + 2q_0 z})^2},$$

(11)

$$\frac{\partial D}{\partial z} = \frac{D[2 + 2z + z^2 + 4q_0 z + 2q_0^2 z^2 + q_0^2 z^2 + q_0 z^3 - \frac{1}{2} q_0 z^3 + (2 + 2z + z^2 + 2q_0 z)\sqrt{1 + 2q_0 z}]}{z(1+z)(1+2q_0 z)(2 + z + q_0 z) + (2 + z + z + q_0 z + q_0 z^2)\sqrt{1 + 2q_0 z}},$$

(12)

where \(\Delta H_0, \Delta q_0\) and \(\Delta z\) are the errors of \(H_0, q_0\) and \(z\), respectively.

### 3 Discussion and conclusions

In this paper we show that the real speed and the angle to the line of sight of components of Galactic sources can be uniquely and directly determined from the measured values of the proper motions and the distance of the sources. It is not necessary to calculate an intermediate quantity, e.g. \(\beta \cos \theta\), first and then using the resulted value to calculate the two required quantities. If so, not only the process of calculation is more complicated but also an extra uncertainty, coming from the intermediate step of calculation, might be brought into the final calculation and hence might affect the final result. In the paper of Mirabel and Rodriguez (1994), they used the same data to calculate the quantity \(\beta \cos \theta\) first and then used this value to calculate \(\beta\) and \(\theta\). Their calculation gave \(\beta \cos \theta = 0.323 \pm 0.016, \beta = 0.92 \pm 0.08, \text{and } \theta = (70 \pm 2)^o\). There are slight differences between theirs and ours. The differences might come from the extra uncertainty of \(\beta \cos \theta\) resulted from the intermediate step of calculation, or might come from other aspects. The value of \(\Delta \beta\) derived in this paper is about 20% larger than that from Mirabel and Rodriguez (1994). We notice that \(\Delta D\) is slightly larger than 10\% of \(D\) for the source (12\%). This might be the cause for the difference. In this kind of case, the method of Mirabel and Rodriguez (1994) might probably be better.

This paper presents explicit and direct relations between the desired quantities \(\beta, \theta\) and the measurements of \(\mu_a, \mu_r\) and \(D\). In this way, not only the process of calculation is simple, but also some extra uncertainties are avoided.
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