$m_t^{\text{exp}} = 174 \pm 16 \text{ GeV}$

$M_H = 100 \text{ GeV}$

Figure 1:
$m_t^{\exp} = 174 \pm 16 \text{ GeV}$

Figure 2:
A Remark on the $Z^0 \rightarrow b\bar{b}$ Width

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Abstract

The $Z^0 \rightarrow b\bar{b}$ width, $\Gamma_b$, is analysed in conjunction with the total and hadronic $Z^0$ widths, $\Gamma_T$ and $\Gamma_h$. Assuming, tentatively, that the present $2\sigma$ discrepancy in $\Gamma_b$ will substantiate as time goes on, for large values of $m_H$ it will be sufficient to modify the $Z^0b\bar{b}$ vertex only. In contrast, for small values of $m_H$, the theoretical predictions for both the $Z^0$ width into light quarks and leptons as well as the $Z^0 \rightarrow b\bar{b}$ vertex will have to be modified.
The precise agreement (e.g. ref. [1]) between the predictions of the $SU(2)_L \times U(1)_Y$ electroweak theory [2] and the experimental data [3] is remarkable indeed. The only evidence for a possible discrepancy between theory and experiment was found in the value of the $Z^0 \to b\bar{b}$ width, which deviates from the theoretical prediction by approximately two standard deviations. The data are consistent with the width predicted for $Z^0 \to d\bar{d}$, and accordingly, they do not show the effect expected from the presence of the mass of the heavy top quark in the $Z^0b\bar{b}$ vertex. As the discrepancy amounts to two standard deviations only, it may be wise to wait for further analysis of forthcoming data before reflecting too much on a possible theoretical explanation of it.

In the present note, nevertheless, we deal with the $Z^0 \to b\bar{b}$ width, restricting ourselves, however, to a few general comments on how the $Z^0 \to b\bar{b}$ “anomaly” could be accommodated in case it will substantiate and stand the test of time. We will briefly analyse the data on $\Gamma_b$ in conjunction with the data on the total and hadronic $Z^0$ widths, $\Gamma_T$ and $\Gamma_h$, respectively, in comparison with standard predictions. Our essential point consists of the observation that low and high values of the Higgs mass $m_H$, require different dominant modifications of the theory in order to accommodate the experimental value of $\Gamma_b$ in conjunction with the experimental data for $\Gamma_T$ and $\Gamma_h$.

Our analysis will be based on the experimental data presented at the Glasgow Conference [3],

\[
M_Z = 91.1888 \pm 0.0044 GeV, \\
\Gamma_T = 2497.4 \pm 3.8 MeV, \\
R = \Gamma_h/\Gamma_l = 20.795 \pm 0.040, \\
\sigma_h = \frac{12\pi\Gamma_l\Gamma_h}{M_Z^2\Gamma_T^2} = 41.49 \pm 0.12 nb
\]

(1)

From the values of $R$ and $\sigma_h$ one derives [1] *

\[
\Gamma_l = 83.96 \pm 0.18 \ MeV, \\
\Gamma_h = 1746 \pm 4 \ MeV
\]

(2)

* The correlation matrix between $\Gamma_T, R$ and $\sigma_h$ was taken into account.
and from the measured value of \( R_{bh} \)

\[
R_{bh} = \frac{\Gamma_b}{\Gamma_h} = 0.2192 \pm 0.0018, \tag{3}
\]

one then obtains

\[
\Gamma_b = 382.7 \pm 3.3 \text{ MeV}, \tag{4}
\]

In what follows, we will compare the data for \( \Gamma_b \) in conjunction with the ones for \( \Gamma_T \) and \( \Gamma_h \) with standard theoretical predictions. All three of these quantities can be simultaneously analysed in a unified manner by first of all extracting the \( Z^0 \to b\bar{b} \) width from the experimental total and hadronic widths, \( \Gamma_{T\exp} \) and \( \Gamma_{h\exp} \), respectively, via

\[
\Gamma_b(T) \equiv \Gamma_{T\exp} - 2 \left( \Gamma_{u\exp}^{th} + \Gamma_{d\exp}^{th} \right) - 3 \left( \Gamma_{e\exp}^{th} + \Gamma_{\nu\exp}^{th} \right) \tag{5}
\]

and

\[
\Gamma_b(h) \equiv \Gamma_{h\exp} - 2 \left( \Gamma_{u\exp}^{th} + \Gamma_{d\exp}^{th} \right). \tag{6}
\]

In these formulae, \( \Gamma_{u\exp}^{th}, \Gamma_{d\exp}^{th} \), etc. denote the (radiatively corrected) theoretical partial \( Z^0 \) widths for the \( Z^0 \to u\bar{u}, Z^0 \to d\bar{d}, \) etc. decays, while \( \Gamma_b(T) \) and \( \Gamma_b(h) \) refer to the partial widths for the \( Z^0 \to b\bar{b} \) decay extracted from the total and hadronic \( Z^0 \) widths, \( \Gamma_T \) and \( \Gamma_h \), respectively. It is evident that \( \Gamma_b(T) \) and \( \Gamma_b(h) \) in (5), (6), are “semi-experimental” quantities. They depend on the experimental data on the total and hadronic \( Z^0 \) widths, \( \Gamma_{T\exp} \) and \( \Gamma_{h\exp} \), as well as the theoretical predictions for the other partial \( Z^0 \) widths which are subtracted on the right-hand-sides in (5), (6). Due to the strong dependence on the mass of the top quark, \( m_t \) (via the leading \( m_t^2 \) dependence), also \( \Gamma_b(T) \) and \( \Gamma_b(h) \) will be decreasing functions of \( m_t \). In addition, \( \Gamma_b(T) \) and \( \Gamma_b(h) \) will depend on the Higgs mass, \( m_H \), via \( \ln m_H \).

Upon inserting the necessary theoretical partial widths into (5) and (6), we will compare \( \Gamma_b(T) \) and \( \Gamma_b(h) \) with the theoretical prediction for the \( Z^0 \to b\bar{b} \) width, \( \Gamma_{b\exp}^{th} \), and with the experimental one, \( \Gamma_{b\exp}^{th} \), and draw our conclusions.

\*\* This value of \( R_{bh} \) is obtained \([3]\) upon fixing \( R_c \equiv \Gamma_c/\Gamma_h \) to its Standard Model value of \( R_c = 0.171 \).
The theoretical values for partial decay widths of the $Z^0$ into leptons and quarks are taken from our recent analysis of the electroweak precision data [1], based on

$$\alpha (M_Z^2)^{-1} = 128.87 \pm 0.12,$$

$$G_\mu = 1.16639(2) \cdot 10^{-5} \text{GeV}$$

as well as $M_Z$ from (1) and

$$\alpha_s = 0.118 \pm 0.007,$$

$$m_b = 4.5 \text{GeV}$$

as input parameters.

The results of the present analysis are presented in figs. 1, 2 for the two cases of a low value of $m_H = 100 \text{GeV}$ and a high value of $m_H = 1000 \text{GeV}$, respectively.

We first of all consider the case of $m_H = 100 \text{GeV}$ shown in fig. 1. From this figure one finds rough agreement of the $Z^0 \rightarrow b\bar{b}$ width extracted from the total and hadronic widths with the theoretical prediction, $\Gamma_b^{th}$, i.e.

$$\Gamma_b (T) \approx \Gamma_b (h) \approx \Gamma_b^{th}$$

for

$$m_t \approx 175 \ \text{GeV},$$

$$m_H \approx 100 \ \text{GeV}.$$

Obviously, the result (9), (10) is nothing else but the (known) consistency between theory and experiment in the total $Z^0$ width and in the hadronic $Z^0$ width, expressed, however, in terms of the $Z^0 \rightarrow b\bar{b}$ partial width. This consistency holds for values of $m_t \approx 175 \ \text{GeV}$, the value favored by the results of the direct searches for the top quark [4.]. To remove the (indication of a small) discrepancy with $\Gamma_b^{exp}$ in fig. 1, both, the theoretical prediction for $Z^0 \rightarrow b\bar{b}$ decay, $\Gamma_b^{th}$, as well as $\Gamma_b (T)$ and $\Gamma_b (h)$ will have to be modified, in order to keep the validity of (9). According to (5) and (6), this implies that the theoretical predictions for the $Z^0$ widths into light leptons and quarks will have to decrease. In summary, for small values of $m_H$, the data — always assuming that the minor discrepancy between theory and experiment visible at present will substantiate — require a modification of the theory which enlarges $\Gamma_b^{th}$ and diminishes $\Gamma_u^{th}, \Gamma_d^{th}$, etc.

The situation (for $m_t \approx 175 \ \text{GeV}$) is different in the case of the other extreme, a large mass of the Higgs boson of e.g. $m_H = 1000 \ \text{GeV}$, as shown in fig. 2. In contrast to (9)
we now have
\[ \Gamma_b(T) \approx \Gamma_b(h) \approx \Gamma_b^{exp} \]  
(11)
for
\[ m_t \approx 175 \text{ GeV}, \]
\[ m_H \approx 1000 \text{ GeV}. \]
(12)
For large values of \( m_H \) the (theoretical) values for the \( Z^0 \) widths into light quarks and leptons in (5), (6) are sufficiently suppressed to accommodate the present enhanced experimental value of \( \Gamma_b^{exp} \) within the total and hadronic widths, \( \Gamma_T^{exp} \) and \( \Gamma_h^{exp} \). Accordingly, in this case, it will be sufficient to modify the \( Z^0 \to b \bar{b} \) vertex to obtain consistency with the data for \( \Gamma_b^{exp} \) as well as \( \Gamma_T^{exp} \) and \( \Gamma_h^{exp} \).

In conclusion, the presentation of the data given in figs. 1, 2 clearly illustrates the delicate interplay of the different experimental results and the parameters \( m_t \) and \( m_H \). If the 2\( \sigma \) effect in \( \Gamma_b \) will stand the test of time, its theoretical explanation will have to discriminate between the low-\( m_H \) and the high-\( m_H \) options (always assuming \( m_t \approx 175 \text{ GeV} \)). For low values of \( m_H \) the theoretical predictions for the \( Z^0 \) widths into the light quarks and leptons as well as the \( Z^0 \to b \bar{b} \) width will have to be modified. On the other hand, in the limit of large values of \( m_H \), it will dominantly only be the theoretical prediction for the \( Z^0 \to b \bar{b} \) vertex which must be changed.

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Fig. 1:

In addition to \( \Gamma_b^{exp} \), the figure shows \( \Gamma_b^{th} \) as a function of the mass of the top quark, \( m_t \), as well as the “semi-experimental” quantities \( \Gamma_b(T) \) and \( \Gamma_b(h) \) obtained from the total and hadronic Z\(^0\) widths, \( \Gamma_T \) and \( \Gamma_h \), by subtracting the theoretical predictions for the Z\(^0\) decay widths into light quarks and leptons. The value of \( m_t = 174 \pm 16 \ GeV \) preferred by the CDF searches is also indicated. For the theoretical prediction for \( \Gamma_b^{th} \) and for \( \Gamma_b(T) \) and \( \Gamma_b(h) \) a Higgs-boson of mass of \( m_H = 100 \ GeV \) was adopted. The error in \( \Gamma_b^{th} \) is due to the experimental error in \( \alpha_s \). This error is also taken into account in \( \Gamma_b(T) \) and \( \Gamma_b(h) \).

Fig 2.:

As fig 1, but for \( m_H = 1000 \ GeV \).