A Hyperspectral Anomaly Detection Algorithm Based on Morphological Profile and Attribute Filter with Band Selection and Automatic Determination of Maximum Area

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Abstract: Anomaly detection is one of the most challenging topics in hyperspectral imaging due to the high spectral resolution of the images and the lack of spatial and spectral information about the anomaly. In this paper, a novel hyperspectral anomaly detection method called morphological profile and attribute filter (MPAF) algorithm is proposed. Aiming to increase the detection accuracy and reduce computing time, it consists of three steps. First, select a band containing rich information for anomaly detection using a novel band selection algorithm based on entropy and histogram counts. Second, remove the background of the selected band with morphological profile. Third, filter the false anomalous pixels with attribute filter. A novel algorithm is also proposed in this paper to define the maximum area of anomalous objects. Experiments were run on real hyperspectral datasets to evaluate the performance, and analysis was also conducted to verify the contribution of each step of MPAF. The results show that the performance of MPAF yields competitive results in terms of average area under the curve (AUC) for receiver operating characteristic (ROC), precision-recall, and computing time, i.e., 0.9916, 0.7055, and 0.25 s, respectively. Compared with four other anomaly detection algorithms, MPAF yielded the highest average AUC for ROC and precision-recall in eight out of thirteen and nine out of thirteen datasets, respectively. Further analysis also proved that each step of MPAF has its effectiveness in the detection performance.

Keywords: anomaly detection; hyperspectral; morphological profile; attribute filter; image histogram; entropy; band selection

1. Introduction

A hyperspectral image (HSI) is an image cube that consists of hundreds of spatial images of a particular space with different spectral information. It has better spectral information than a multispectral image because of its large number of narrow spectral bands [1]. Its rich information opens the possibility to differentiate several objects of interest based on their spectral signatures. Therefore, it is widely used in many remote-sensing research fields, such as spectral unmixing [2–4], target detection [5–7], and classification [8–10].

Target detection methods in HSI are mainly classified into supervised and unsupervised categories, based on the utilization of prior information. Anomaly detection is an unsupervised target detection method that aims to identify interesting objects that are different from their surroundings, in terms of spatial and spectral domain, without any prior information about the objects [11–13]. These objects...
Anomaly detection is widely used in many applications, such as urban detection [14], agriculture [15], maritime target detection, fire monitoring, and geology [16]. Hyperspectral anomaly detection has played a crucial role in both military [13,16,17] and civilian projects [13,14,18,19].

One of the important milestones in anomaly detection, developed by Reed and Yu, is the Reed–Xiaoli (RX) method [20]. It uses the multivariate Gaussian distribution to model the background. It then uses the Mahalanobis distance between the test pixel and the background to detect the anomaly. This method became well-known because of its simplicity and satisfying performance. Several extensions based on the RX were also proposed [21–23]. Recent algorithms based on RX and Fractional Fourier Entropy RX (FrFE-RX) [24] used the fractional Fourier transform as a preprocess to remove noise and to enhance the difference between the anomaly and the background. Even though this had a better performance than RX, it required a high computational time to do the preprocessing.

Another well-known approach in anomaly detection is collaborative-representation-based detector (CRD) [25]. This approach assumes that each pixel in the HSI can be approximately represented by its neighborhoods. The neighborhoods are defined by the area between the outer and the inner windows (dual windows) centered at each test pixel. This method, however, is not flexible. The performance of this method depends on the size of the dual windows. There is also a possibility that the anomalous pixels are inside the neighborhoods, which will decrease the performance of CRD. In order to address this issue, variants of CRD were proposed [26–28]. A recent algorithm, called spatial density background purification (SDBP) [29], also aimed to solve this problem. It used a density peak (DP) clustering algorithm to calculate the local density of the neighborhoods and remove some of the neighborhoods that have a low density to acquire pure background pixels in the neighborhoods. Even though the performance of the SDBP was better than that of CRD, this also depended on the size of dual windows. Another problem of CRD and its variants was the long computing time in detecting the anomalous pixels.

Another type of anomaly detection is based on the attribute filter that exploits HSI’s spatial information [30–33]. One of these methods, introduced by Kang et al., was called attribute and edge-preserving filters detector (AED) [33]. This assumed that anomalous pixels usually appear as small objects. AED used a principal component to reduce the HSI’s spectral resolution, the attribute filter to extract the anomalous pixels, and the edge-preserving filter for postprocessing to smooth the output detection and decrease the false alarm rate. This method had a good performance and fast computing time; however, the proposed filtering function introduced in [33] had a risk. This function was a filter that would remove an object if its area is greater than the filter threshold. If the location of anomalous objects was very close, the dilation process would combine those objects into one object. If the area of the combined object was greater than the filter threshold, this function would remove it. This would cause a significant decrease in AED’s performance. The other limitation of using AED was that the maximum area of the anomaly parameter needs to be set manually. Since AED used the attribute filter to extract the anomalous pixels and the area of the anomaly can differ in each HSI, this parameter was considered a crucial one due to its influence on the risk of false-positive and false-negative occurrences. Since the attribute filter was used to filter connected pixels based on the size area, false positive may occur if the size area is too wide so that there is one or more background pixels included in the anomalies. On the contrary, if the size area was too small, one or more anomaly pixels were excluded from the anomalies and considered as the background.

In this paper, we propose an algorithm for anomaly detection called morphological profile and attribute filter (MPAF). Different from previous works, our proposed method used a morphological profile [34] and an attribute filter [35] to detect anomalies in HSI. This study aims to improve the detection accuracy and computing time. This method is proposed based on the facts: (1) only some bands can give clear information about a certain object in HSI and some bands have a high correlation, and (2) anomalous objects are smaller than the background. Based on the first fact, entropy and histogram counts were used to select a band. This step was not only to select the effective band for
anomaly detection but also to reduce the computing time of the algorithm. For the second fact, a morphological profile was used to extract the anomalous pixels and an attribute filter was used to filter the false anomalous pixels. Even though we can use an attribute filter to extract the anomalous pixels, it relies on the gray-level thresholds to label pixels in the image before extracting the anomalous pixels. The more complex the image is, the more ineffective it becomes. Meanwhile, the morphological profile extracted anomalous pixels in the image based on their neighborhood, but it was ineffective for removing large objects. Combining the morphological profile and the attribute filter for anomaly detection will improve the detection accuracy because they complement each other. Moreover, in order to address the issue of previous methods also based on the attribute filter, we propose an algorithm to automatically determine the maximum area of the anomaly.

Instead of using spectral signatures from all bands, several anomaly detection methods ([30,36–38]) applied band selection algorithm to reduce the spectral dimension, remove redundant and interference bands, and as a result, decrease false alarm rate. The selected bands had more discriminative features for anomaly than the discarded bands. This selection gave a positive effect on the performance of the subsequent detection algorithm and reduced the computational complexity. In this work, instead of spectral signatures, we used HSI data’s entropy as a means to observe the information quantity of the HSI bands. Entropy is a quantitative metric of the information contained in an image. This gave us a subset of prospective bands to represent the anomalies using a particular threshold. Since anomalies were much smaller than the background, we further selected one band from this subset using an algorithm based on the histogram that ensured the spatial characteristic between anomalies and the background. In addition, the subsequent step dealt with the algorithm to determine the maximum area of anomalies due to this spatial characteristic. Hence, even if there was a false anomalous pixel due to its same signature as the anomaly’s in the selected band, this algorithm can detect and remove it based on this spatial requirement. This paper shows that our proposed approach yielded superior results on thirteen datasets that are often used as a benchmark in HSI anomaly detection.

The contribution of this work is developing a new approach for HSI anomaly detection that combines two conventional techniques in image processing, added with new algorithms for band selection and determining the maximum area of anomalies. In the first step, the band selection contributes to less computing time while preserving the most representative band based on the entropy and histogram. Even though we applied conventional techniques for the next steps, we found that the results could be improved by suppressing the false-positive cases. Hence, in the filtering step, we propose the algorithm to determine the maximum area of anomalies automatically, that differs from previous works [30–33] in terms of automated fashion. This paper shows that the proposed algorithm increased detection performance by avoiding false-positive results.

The rest of this paper is organized as follows. Section 2 reviews the morphological profile and attribute filter. Section 3 describes the detail of the proposed method in this paper. Section 4 shows the parameter settings, results, and our respective analyses. Finally, Section 5 is the conclusion of this study.

2. Materials and Methods

2.1. Morphological Profile

A morphological profile consists of an opening profile and a closing profile [39], which are created based on mathematical morphology. A mathematical morphology [40] is a set of operations that processes objects in an image based on their shapes. Each pixel in the image is processed based on the value of other pixels in its neighborhood. The neighborhood is defined by the shape and size of the structuring element SE. There are two basic operations in mathematical morphology, i.e., erosion and dilation.
Erosion (⊖) is defined as a local minimum operator. It selects the minimum value between the manipulated pixel and its neighborhood. On the contrary, dilation (⊕) is defined as a local maximum operator. It selects the maximum value between the manipulated pixel and its neighborhood:

\[ f(x, y) \ominus SE = \min \{ f(x + x', y + y') | (x', y') \in D_{SE} \} \] (1)

\[ f(x, y) \oplus SE = \max \{ f(x + x', y + y') | (x', y') \in D_{SE} \} \] (2)

where \( D_{SE} \) is the domain of \( SE \) and \( f \) is a grayscale image with indexes of spatial dimensions \( x \) and \( y \). Erosion tends to remove bright pixels while dilation tends to remove dark pixels.

Other operations of mathematical morphology are called opening and closing, which are derived from erosion and dilation. Opening (\( ImO \)) and closing (\( ImC \)) are defined as follows:

\[ ImO(f, SE) = (f \ominus SE) \oplus SE \] (3)

\[ ImC(f, SE) = (f \oplus SE) \ominus SE \] (4)

Opening tends to remove bright pixels but is less destructive than erosion. Meanwhile, closing tends to remove dark pixels but is less destructive than dilation.

2.2. Attribute Filter

An attribute filter is an adaptive filter introduced by Breen and Jones [41]. It can preserve or remove connected components in an image based on a predefined attribute such as area, standard deviation, the moment of inertia, etc. Here, we briefly describe how the attribute filter worked. Refer to [35] for a detailed explanation.

Binary attribute thinning \( \Gamma^C \) is defined as a filter that removes connected components of true pixels, called a binary connected opening, that does not fulfill criterion \( C \). For example, if given \( C \) is the area that contains more than 25 pixels, \( \Gamma^C \) preserves only the connected components that have more than 25 pixels and removes the others. Whereas, binary attribute thickening \( \Phi^C \) is defined as a filter that removes connected components of false pixels, called binary connected closing, that does not fulfill criterion \( C \).

A grayscale image can be represented as a stack of binary images obtained by thresholding the grayscale image at each of its gray levels. The connected components of a grayscale image are built based on this set of gray levels. If given a set of gray levels \( g \), ranging on the gray levels of \( f \), grayscale attribute thinning \( \gamma^C \) and thickening \( \phi^C \) can be defined as follows:

\[ \gamma^C(f) = \{ \Gamma^C[\text{Thresh}(f, G)] | G = \max(g) \} \] (5)

\[ \phi^C(f) = \{ \Phi^C[\text{Thresh}(f, G)] | G = \min(g) \} \] (6)

where \( \text{Thresh}(f, G) \) is the binary image obtained by thresholding \( f \) at gray level \( G \). In other words, grayscale attribute thinning removes connected components of bright pixels that do not fulfill the criterion \( C \) and grayscale attribute thickening removes connected components of dark pixels that do not fulfill the criterion \( C \).

2.3. Proposed Algorithm

Figure 1 shows that the proposed method, i.e., morphological profile and attribute filter (MPAF), consisted of three steps: (1) selecting a band based on entropy and histogram counts; (2) background removal with the morphological profile (MP); and (3) filtering the image with the attribute filter (AF).
where \( C_i \) is pixel \( i \) of \( X_k \), \( \bar{X}_k \) is the normalized image, \( \overline{X}_k \) is the mean of pixels in the image \( X_k \), and \( \sigma_{X_k} \) is the standard deviation of pixels on the image \( X_k \).

The normalized image made a clear differentiation between background pixels and anomalous pixels. Figure 2 shows the differentiation between the data before and after normalization. Anomalous pixels are shown as small objects in the image, and the other pixels are background pixels, either in images before or after normalization. The histograms after normalization, however, are shifted and expanded so that it is more reliable to take a threshold to define whether the images belong to the band with BA (Figure 2a,b) or DA (Figure 2c,d). Let \( \alpha \) denote the threshold to define extreme-bright pixels (whose values \( v \geq 1 - \alpha \)), and extreme-dark pixels (whose value \( v \leq \alpha \)). Then, each sample image was then classified as follows:

\[
B_{BA} = \begin{cases} 1 & \text{if } \frac{1}{v=0} q_v(\bar{X}_k) < \frac{1}{v=1-\alpha} q_v(\bar{X}_k) \\ 0 & \text{otherwise} \end{cases}
\]

(10)

\[
B_{DA} = \sim B_{BA}
\]

(11)

where \( B_{BA} \) is the bright-anomaly band flag, \( B_{DA} \) is the dark-anomaly band flag, \( \alpha \) is the classification threshold, and \( q_v \) is the normalized histogram counts at pixel value \( v \).
Figure 2. Image and image histogram of (a) ABU-Airport-1 band 5 before normalization and (b) after normalization; and (c) band 165 before normalization and (d) after normalization. The histograms show the grayscale values (in the range of 0–1) in the x-axis, and the number of pixels (counts) in the y-axis.
The sample images were then filtered as follows:

\[
XC = \begin{cases} 
X_{BA} & \text{if } \sum B_{BA} > \sum B_{DA} \\
X_{DA} & \text{otherwise}
\end{cases}
\]  

(12)

where \(XC\) is a matrix that contains filtered images, \(X_{BA}\) = \(\{X_k | B_{BA} = 1\}\) and \(X_{DA}\) = \(\{X_k | B_{DA} = 1\}\). \(X_{BA}\) is a matrix that contains bright-anomaly images, and \(X_{DA}\) is a matrix that contains dark-anomaly images.

In order to avoid using bands that contain noise, the samples were filtered using entropy (\(H\)). Entropy was used to measure the information contained in the image. The filtered images (\(X_H\)) were calculated as follows:

\[
H_X = -\sum q(X) \log_2 q(X)
\]

(13)

\[
X_H = \left\{ XC_j | H_{XC_j} \rangle H_I - 2\sigma_{H_I} \right\}
\]

(14)

where \(q\) is the normalized histogram counts and \(XC_j\) is the \(j\)th element of \(XC\). \(H_I\) is the mean of entropy of all bands in \(I\), and \(\sigma_{H_I}\) is the standard deviation of entropy of all bands in \(I\). Figure 3 shows that bands with very low entropy, i.e., bands 148, 156, and 204, cannot be used for anomaly detection. Meanwhile, the other band with average entropy, i.e., band 6, can be used for anomaly detection.

The band was then selected as follows:

\[
XS = \begin{cases} 
\arg \min_{X_H} \left( \sum_{v=0}^{0.5-\beta} q_v(XH_I) \right) & \text{if } \sum B_{BA} > \sum B_{DA} \\
\arg \min_{X_H} \left( \sum_{v=0.5+\beta} q_v(XH_I) \right) & \text{otherwise}
\end{cases}
\]

(15)

where \(XS\) is the selected band and \(\beta\) is a selection coefficient. The image with the above condition will have more background pixels. Thus, this condition will improve the detection rate when using a morphological profile to detect anomalies.

Figure 3. (a) Entropy curve of all bands. Image of band (b) 6, (c) 148, (d) 156, and (e) 204. (f) Reference map.
2.3.2. Background Removal

Anomalies are shown as small dark or bright objects in the image. MP was used to detect these objects. Anomalies can be detected by the following equations:

\[
R = \begin{cases} 
(XS - ImO(XS, se_1)) \oplus se_2 & \text{if } \sum B_{BA} > \sum B_{DA} \\
(ImC(XS, se_1) - XS) \oplus se_2 & \text{otherwise}
\end{cases}
\] (16)

where \( R \) is the resulting image and \( se_1 \), which represents the width of the anomaly, and \( se_2 \) is the width of square \( SEs \). Opening removes small objects from bright pixels and closing removes small objects from dark pixels. Therefore, Equation (16) results in anomalous objects.

2.3.3. Area Filtering

Even though we can detect objects with MP, the shape of \( SEs \) is fixed [39]. They are ineffective on non-compact shapes, especially on large objects. Therefore, AF was used to filter these objects:

\[
D = \begin{cases} 
XS - \gamma area(\kappa)(XS) & \text{if } \sum B_{BA} > \sum B_{DA} \\
\phi area(\kappa)(XS) - XS & \text{otherwise}
\end{cases}
\] (17)

\[
O = R \odot (D \oplus se_3)
\] (18)

where \( D \) is the differential map, \( \kappa \) is the maximum area of anomalous objects, \( O \) is the output of the filter, \( \odot \) is element-wise multiplication, and \( se_3 \) is the width of square \( SE \). The criteria used in both attribute filters (\( \gamma \) and \( \phi \)) is \( area > \kappa \), which means they will keep the connected components whose area is greater than \( \kappa \). Therefore, the differential map obtained by Equation (17) will remove the objects whose area is greater than \( \kappa \). Since \( D \) is used for filtering, dilating it with \( se_3 \) will expand each area on \( D \) to avoid losing anomalous pixels during the AF process.

2.3.4. Area of Anomaly

In previous works [30–33], the limitation of using AF was that the methods need to set the \( \kappa \) manually. Therefore, in this paper, we proposed a novel method to automatically set not only the \( \kappa \) but also the \( se_1 \).

Anomalous objects in an image usually appear as small objects and have a similar area. We can differentiate the anomalous objects from the background even though they have a similar spectral reflectance. Based on these assumptions, if given a set of areas of suspected anomalous objects, we can model this set as a normal distribution. Thus, we can calculate the maximum area of anomalous objects statistically. Furthermore, to optimize the detection using MP with square \( SE \), the width of \( SE \) should be equal to the width of the smallest square containing the anomalous object. It is equivalent to the maximum value between the width and height of the smallest box containing the anomalous object (bounding box).

To extract the suspected anomalous objects from the image, we proposed using differential map \( D \) like in Equation (17). In [33], the optimal value for the maximum area of anomalous objects was set to be \( N/100 \) pixels, where \( N \) is the number of pixels in HSI. Based on this, \( D \) was first calculated with \( \kappa \) equal to \( N/100 \). The Boolean map \( BW \) was then obtained by thresholding the \( D \) as follows:

\[
BW = \text{Thresh}(D, \theta)
\] (19)

where \( \theta \) is the threshold of the image. The value of \( \theta \) is set with Otsu’s method [42] due to its robust performance. The connected components in \( BW \) represent the suspected anomalous objects. Each connected component in \( BW \) can be labeled. Then, we can get the area of each connected component and the width and height of each connected component’s bounding box.
We let \( A \) be the areas of connected components in \( BW \) and \( W \) be a set of widths and heights of connected components’ bounding boxes. \( \kappa \) and \( se_1 \) are calculated as follows:

\[
A_k = \begin{cases} 
\min(A') & \text{if } \max(A) > \hat{A} + 2\sigma_A \\
\max(A) & \text{otherwise}
\end{cases} \tag{20}
\]

\[
\kappa = \begin{cases} 
\sqrt{\frac{A_k}{\max(A)}}^2 & \text{if } A_k = \max(A) \\
\sqrt{\frac{A_k}{\max(A)}}^2 & \text{otherwise}
\end{cases} \tag{21}
\]

\[
se_1 = \min\left\{ \max(W'), \left[ \sqrt{\frac{N}{100}} \right] \right\} \tag{22}
\]

where \( A' = \{A | A \geq \hat{A} + 2\sigma_A\} \), \( \hat{A} \) is the mean of \( A \), \( \sigma_A \) is the standard deviation of \( A \), and \( W' = \{W | A = A_k\} \).

3. Experimental Results and Analysis

3.1. Data

In this paper, the proposed method was evaluated on real datasets that have been made available online [43], i.e., Airport–Beach–Urban (ABU) datasets. The data were manually extracted from images of the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS), except for ABU-Beach-4, which was manually extracted from the image of the Reflective Optical System Imaging Spectrometer (ROSIS-03). Each dataset had a reference map that has been manually labeled. Table 1 describes some features of these images.

| Airport Scene | Captured Place | Resolution (pixel) | Band |
|---------------|----------------|--------------------|------|
| 1             | Los Angeles    | 100 × 100          | 205  |
| 2             | Los Angeles    | 100 × 100          | 205  |
| 3             | Los Angeles    | 100 × 100          | 205  |
| 4             | Gulfport       | 100 × 100          | 191  |

| Beach Scene   | Captured Place | Resolution (pixel) | Band |
|---------------|----------------|--------------------|------|
| 1             | Cat Island     | 150 × 150          | 188  |
| 2             | San Diego      | 100 × 100          | 193  |
| 3             | Bay Champagne  | 100 × 100          | 188  |
| 4             | Pavia          | 150 × 150          | 102  |

| Urban Scene   | Captured Place | Resolution (pixel) | Band |
|---------------|----------------|--------------------|------|
| 1             | Texas Coast    | 100 × 100          | 204  |
| 2             | Texas Coast    | 100 × 100          | 207  |
| 3             | Gainesville    | 100 × 100          | 191  |
| 4             | Los Angeles    | 100 × 100          | 205  |
| 5             | Los Angeles    | 100 × 100          | 205  |

3.2. Results

The performance of MPAF was evaluated with the most widely used metrics, area under the curve (AUC), and the receiver operating characteristic (ROC) [44]. To verify the performance of the proposed method, other methods, i.e., RX [20], FrFE-RX [24], AED [33], and SDBP-D [29] (SDBP with dual window) detectors, were used for comparison. These methods were either frequently cited in the literature or recent algorithms for HSI anomaly-detection applications.

To generate the results in Table 2, optimal parameters of FrFE-RX, AED, SDBP-D methods were selected for each image to get the best AUC. For FrFE-RX, the fractional-order (\( p \)) was selected by varying it from 0–1. Since the same datasets were used in AED, we set the maximum area of the
anomaly ($\kappa$) for each dataset as the same as the values recommended in [33]. The other parameters of AED were kept at AED’s default parameter setting, i.e., $\lambda = N/100$, $\alpha = 5$, $\beta = 0.5$, and $M = 3$. Similarly, for SDBP-D, we used the values recommended in [29] to set the window size ($\omega_{in}$ and $\omega_{out}$) and the percentage of the highest selected density value ($P$).

For the proposed method, a default parameter setting ($t = 10$, $u = 5$, $\alpha = 0.15$, $\beta = 0.04$, $se_2 = 1$, and $se_3 = 3$) was used for most of the datasets, which will be discussed in the next subsection. This setting could achieve acceptable performances for most of the datasets. If only a small number of bands or no band can represent all anomalous pixels in the reference map, the $se_2$ needed to be adjusted. Specifically, for the ABU-Airport-1 and the ABU-Beach-4 datasets, the $se_2$ was set as $se_2 = 3$.

Table 2. Area under the curve (AUC) values of receiver operating characteristic (ROC) curves for the compared methods, i.e., morphological profile and attribute filter (MPAF), Reed–Xiaoli (RX), fractional fourier entropy RX (FrFE-RX), attribute and edge-preserving filters detector (AED), and spatial density background purification with dual window (SDBP-D) on the ABU datasets.

| Hyperspectral Image (HSI) | MPAF    | RX       | FrFE-RX  | AED     | SDBP-D   |
|---------------------------|---------|----------|----------|---------|----------|
| Airport–1                 | 0.9718  | 0.8221   | 0.9081   | 0.9923  | 0.9615   |
| Airport–2                 | 0.9765  | 0.8404   | 0.969    | 0.9936  | 0.9842   |
| Airport–3                 | 0.9785  | 0.9287   | 0.9424   | 0.9756  | 0.9662   |
| Airport–4                 | 0.9997  | 0.9526   | 0.9854   | 0.9953  | 0.9930   |
| Average                   | 0.9816  | 0.8859   | 0.9512   | 0.9892  | 0.9762   |

| Beach Scene               | MPAF    | RX       | FrFE-RX  | AED     | SDBP-D   |
|---------------------------|---------|----------|----------|---------|----------|
| Beach–1                   | 0.9992  | 0.9807   | 0.9862   | 0.9974  | 0.9959   |
| Beach–2                   | 0.9910  | 0.9106   | 0.9161   | 0.9550  | 0.9876   |
| Beach–3                   | 0.9998  | 0.9998   | 0.9997   | 0.9997  | 0.9999   |
| Beach–4                   | 0.9985  | 0.9538   | 0.9541   | 0.9916  | 0.9763   |
| Average                   | 0.9971  | 0.9612   | 0.9640   | 0.9859  | 0.9899   |

| Urban Scene               | MPAF    | RX       | FrFE-RX  | AED     | SDBP-D   |
|---------------------------|---------|----------|----------|---------|----------|
| Urban–1                   | 0.9986  | 0.9907   | 0.9918   | 0.9981  | 0.9989   |
| Urban–2                   | 0.9990  | 0.9946   | 0.9962   | 0.789   | 0.9984   |
| Urban–3                   | 0.9977  | 0.9513   | 0.9684   | 0.9976  | 0.9946   |
| Urban–4                   | 0.9959  | 0.9887   | 0.9809   | 0.9912  | 0.9972   |
| Urban–5                   | 0.9900  | 0.9692   | 0.9719   | 0.9845  | 0.9877   |
| Average (all scenes)      | 0.9916  | 0.9420   | 0.9657   | 0.9757  | 0.9872   |

As shown in Table 2, the MPAF method achieved the best scores on 61.5% of the datasets (8 out of 13 datasets). The average AUCs of the MPAF method on each ABU dataset were 0.9816, 0.9971, and 0.9962, respectively, and the average of all datasets reached 0.9916. Even though the MPAF method ranked second on the ABU-Airport datasets, the difference was only 0.0076, which is very low. The proposed method also achieved the best score on 50% of the ABU-Airport datasets. The result of the proposed method was also stable, as the minimum AUC score was 0.9718 (ABU-Airport-1). The FrFE-RX method showed better performances than the RX method in most cases, which was consistent with the results reported in [24]. Despite those results, the FrFE-RX’s AUCs still could not match the MPAF’s AUCs in all cases. Both the AED and the SDBP-D method performed well, but still less effectively than the proposed method.

Figures 4–6 show the results obtained by the compared methods for the three scenes. As shown in those figures, anomalous objects can be easily recognized on the MPAF and the AED detection maps visually. On the contrary, it is harder to recognize anomalous objects on the RX and the FrFE-RX detection maps. Most of the MPAF results had fewer false anomalous pixels compared to the AED or the other methods. This means that the attribute filter with the determination of anomaly area proposed...
in this method was effective for removing false anomalous pixels. Moreover, the MPAF method can detect all anomalous objects even though it cannot wholly detect all of the pixels in the objects.

Figure 4. ABU-Airport detection map of the compared methods. (a) Reference map, (b) MPAF, (c) RX, (d) FrFE-RX, (e) AED.

Figure 5. Cont.
Although the performance of AED was relatively stable, its AUC score was very low, i.e., 0.789, on the ABU-Urban-2 dataset. It was the result of performing image dilation before performing its filtering function. As shown in Figure 6, the anomalous objects were very close to each other. Two or more anomalous objects merged into one after the image dilation step. As a result, the area of the merged objects was greater than the threshold of the filtering function. Meanwhile, our proposed algorithm avoided this drawback by the step to determine the area of anomaly.
Figures 7–9 show the ROC curves of different methods, i.e., the MPAF, RX, FrFE-RX, and AED methods, for anomaly detection using the ABU datasets. The probability of the detection of MPAF is always greater than 0.9 before the false-alarm rate reaches 0.1. It means that for a low false-alarm rate, the result of the proposed method has promising results.

**Figure 7.** ROC curves of the compared methods on the ABU-Airport Scene.

**Figure 8.** ROC curves of the compared methods on the ABU-Beach Scene.
In observation, since imbalanced datasets may occur between anomaly and background classes, we also evaluated the performance using precision-recall curve. Instead of using the F1-score that summarizes a method’s performance for a specific probability threshold, a precision-recall curve provided the performance across different thresholds by its AUC. Figures 10–12 depict the precision-recall curves of the compared methods for Airport, Beach, and Urban datasets, respectively. Table 3 shows the AUC values of each curve. One can see that MPAF outperformed the other methods in most cases, i.e., nine out of thirteen datasets with the average value reached 0.7055, which is 0.1080 higher than the second-best method, i.e., AED. Based on these results and the definition of precision and recall, it implies that MPAF was the best approach among others in predicting positive anomalies while reducing false ones and capturing the actual anomalies on the scenes.
Table 3. AUC values of precision-recall curves for the compared methods on the ABU datasets.

|                  | HSI | MPAF  | RX    | FrFE-RX | AED  |
|------------------|-----|-------|-------|---------|------|
| **Airport Scene**|     |       |       |         |      |
| Airport–1        | 0.4284 | 0.0980 | 0.0966 | 0.7247  |
| Airport–2        | 0.3995 | 0.1876 | 0.2720 | 0.3266  |
| Airport–3        | 0.4746 | 0.2391 | 0.2486 | 0.5828  |
| Airport–4        | 0.9399 | 0.2000 | 0.2980 | 0.5601  |
| **Average**      | 0.5606 | 0.1812 | 0.2288 | 0.5601  |
| **Beach Scene**  |     |       |       |         |      |
| Beach–1          | 0.7204 | 0.3780 | 0.3756 | 0.3438  |
| Beach–2          | 0.6454 | 0.1127 | 0.1105 | 0.3362  |
| Beach–3          | 0.8272 | 0.8373 | 0.7763 | 0.7630  |
| Beach–4          | 0.5915 | 0.3118 | 0.2608 | 0.5850  |
| **Average**      | 0.6961 | 0.4100 | 0.3808 | 0.5070  |
| **Urban Scene**  |     |       |       |         |      |
| Urban–1          | 0.8448 | 0.4030 | 0.3877 | 0.8435  |
| Urban–2          | 0.9454 | 0.7464 | 0.7942 | 0.5764  |
| Urban–3          | 0.7545 | 0.0609 | 0.0731 | 0.8796  |
| Urban–4          | 0.8989 | 0.6510 | 0.5998 | 0.7580  |
| Urban–5          | 0.8552 | 0.3388 | 0.3199 | 0.5699  |
| **Average**      | 0.8598 | 0.4400 | 0.4269 | 0.7255  |
| **Average (all scenes)** | 0.7055 | 0.3437 | 0.3455 | 0.5975  |

Bold numbers are used to highlight which score is the best among others in a row.

Figure 11. Precision-recall curves of the compared methods on the ABU-Beach Scene.
4. Discussion

4.1. Computing Time

In this paper, the computing time of the MPAF is also compared with the other methods. All methods were executed using MATLAB on a computer with Intel(R) Core(TM) i5–4300U CPU and 16 GB RAM. The results are shown in Table 4. It was shown that the proposed method’s time is quite fast, ranking second after the RX method. The differences were not too significant (0.03–0.11 s). Moreover, even though the RX method was fast, the AUC results of this method were far less accurate than the AUC results of the proposed method. As expected from the FrFE-RX method, even though it is a modification of the RX method, it required a lot of time to process the datasets and to select the optimum fractional-order, since this process is pixel-based and is done for each band. The computing time of the proposed method should be acceptable since it had promising results.

Table 4. The average computing time (in seconds) of the compared methods on the ABU dataset.

| Scene   | MPAF | RX  | FrFE-RX | AED  |
|---------|------|-----|---------|------|
| Airport | 0.26 | 0.15| 31.08   | 0.55 |
| Beach   | 0.32 | 0.22| 40.13   | 0.69 |
| Urban   | 0.17 | 0.14| 22.89   | 0.41 |
| Average | 0.25 | 0.17| 31.37   | 0.55 |

Bold numbers are used to highlight which score is the best among others in a row.

4.2. Component Analysis

This subsection analyzes the influence of different parameters on the performance of the MPAF method, the effectiveness of its band-selection method, and the effect of the MP and AF combination on the detection method. These parameters were the maximum area of the anomalous objects $\kappa$ and the width-of-square SEs, i.e., $se_1$, $se_2$, and $se_3$. AUC was used to evaluate the performance of the MPAF method under different parameter settings.

4.2.1. Parameter Sensitivity Analysis

Figure 13 shows the average AUCs of the MPAF on each dataset under the influence of different parameter settings. When analyzing the influence of one parameter, the other parameter was set with
the default parameter settings \( t = 10, u = 5, \alpha = 0.15, \beta = 0.04, se2 = 1, \text{and } se3 = 3 \). Both \( \kappa \) and \( se1 \) were set automatically when analyzing \( se2 \) and \( se3 \), since they did not depend on other parameters but images. Either \( \kappa \) or \( se1 \) was set automatically when analyzing one of these parameters.

As shown at the top of Figure 13, the AUCs depended on the area of the anomaly in the MPAF method, i.e., \( \kappa \) and \( se1 \). The average of AUCs increased from zero before they saturated on a particular value. If either \( \kappa \) was less than the area of the anomaly or \( se1 \) was less than the width of the anomaly, the accuracy of detection was low. It means that the proposed algorithm for setting \( \kappa \) and \( se1 \) was proved to be effective since the AUCs in Table 2 were above 0.97. Furthermore, in a particular case, e.g., ABU-Urban-3, some objects had the same reflectance value as the anomaly. Therefore, if either \( \kappa \) was greater than the area of the anomaly or \( se1 \) was greater than the width of the anomaly, some false anomalous pixels were detected and reduced the AUC of MPAF, as shown in Figure 14.

The bottom left of Figure 13 shows the influence of \( se2 \) over the detection performance. In the ABU-Urban and ABU-Beach cases, increasing \( se2 \) decreased detection performance. This meant that in most cases, the dilation step with \( se2 \) was not needed, since \( se2 = 1 \) means there was no dilation. In the ABU-Airport case, Figure 15a shows the influence of \( se2 \) over AUCs for all the ABU-Airport datasets. It clearly shows that only ABU-Airport-1 needed the dilation step. Furthermore, at \( se2 = 1 \), Figure 15b shows that the maximum of AUCs was 0.8919. This means that the MPAF detected most of the anomaly locations, but not the entire area of each detected anomaly. After the dilation step, the detected anomalous objects were expanded, thus increasing the detection accuracy.
with the best AUCs of the MPAF without the band selection. The best AUCs were obtained by running the AF process. The performance went down as the attribute filter expanded the objects in the image. Thus, the objects covered the lost pixels, which were caused by the AF process. The performance went down as these anomalous pixels were detected and reduce the AUC. In most cases, the dilation step with\( \kappa \) was greater than 1 needed the dilation step. Furthermore, at \( s e_2 = 1 \), the objects to expand too much. Some false anomalous pixels were detected and reduced the AUC. The average difference of the AUC on the ABU datasets were 0.005125, 0.000615, and 0.0008, respectively. Even though the band selection process cannot obtain the best result, its performance is still acceptable since the differences were very low while promoting an automatic process. Furthermore, the time consumed was also low because of the spectral resolution reduction due to this band-selection process.

The bottom right of Figure 13 shows the influence of parameters \( \kappa \) and \( s e_1 \) over the MPAF performance on the ABU-Airport-1 dataset. It clearly shows that only ABU-Airport-1 needed the dilation step. Furthermore, at \( s e_2 = 1 \), the AUCs depend on the ABU datasets were 0.005125, 0.000615, and 0.0008, respectively. Even though the band selection process cannot obtain the best result, its performance is still acceptable since the differences were very low while promoting an automatic process. Furthermore, the time consumed was also low because of the spectral resolution reduction due to this band-selection process.

The bottom right of Figure 13 shows the influence of \( s e_3 \) over the detection performance. As expected, the AUCs increased when \( s e_3 \) was increased to a certain value, i.e., \( s e_3 = 3 \). The dilation of the attribute filter expanded the objects in the image. Thus, the objects covered the lost pixels, which were caused by the AF process. The performance went down as \( s e_3 \) was increased further, causing the objects to expand too much. Some false anomalous pixels were detected and reduced the AUC of MPAF.

4.2.2. Band Selection Effectiveness

To analyze the effectiveness of the proposed band selection, we compared the AUCs in Table 2 with the best AUCs of the MPAF without the band selection. The best AUCs were obtained by running the MPAF on each band of each dataset with the same parameter setting as the one that produced the AUCs in Table 2. Then we selected the maximum AUCs from each dataset. This comparison is shown in Table 5. The average differences of the AUC on the ABU datasets were 0.005125, 0.000615, and 0.0008, respectively. Even though the band selection process cannot obtain the best result, its performance is still acceptable since the differences were very low while promoting an automatic process. Furthermore, the time consumed was also low because of the spectral resolution reduction due to this band-selection process.
Table 5. Comparison between AUC of the MPAF’s best band and AUC of the MPAF’s selected band on the ABU datasets.

|                      | Best Band | Band Selection | Difference |
|----------------------|-----------|----------------|------------|
| Airport Scene        |           |                |            |
| Airport-1            | 0.9788    | 0.9718         | 0.007      |
| Airport-2            | 0.9857    | 0.9765         | 0.0092     |
| Airport-3            | 0.9827    | 0.9785         | 0.0042     |
| Airport-4            | 0.9998    | 0.9977         | 0.0001     |
| Beach Scene          |           |                |            |
| Beach-1              | 0.9996    | 0.9992         | 0.0004     |
| Beach-2              | 0.9921    | 0.991          | 0.0011     |
| Beach-3              | 0.99996   | 0.9998         | 0.00016    |
| Beach-4              | 0.9993    | 0.9985         | 0.0008     |
| Urban Scene          |           |                |            |
| Urban-1              | 0.9994    | 0.9986         | 0.0008     |
| Urban-2              | 0.9993    | 0.999          | 0.0003     |
| Urban-3              | 0.9983    | 0.9977         | 0.0006     |
| Urban-4              | 0.9974    | 0.9959         | 0.0015     |
| Urban-5              | 0.9902    | 0.99           | 0.0002     |

4.2.3. MP and AP Effectiveness

To analyze the effect of MP and AF combination on the detection method, we compared the average AUCs of the MPAF in Table 2 with the average AUCs of the MPAF without MP and the MPAF without AF. As shown in Figure 16, the average AUCs of MPAF were always the highest, while the other two AUCs were inconsistent, depending on the datasets. In addition, we evaluated the statistics using a box plot, as shown in Figure 17. The first three boxes from the left correspond to Airport scenes that are performed by MPAF, MPAF without AF, and MPAF without MP, respectively. The following three boxes are for Beach, and the last three are for Urban scenes. One can see that compared with MP without AF and MP without AF, MPAF always showed the highest median values, least dispersed, and the highest minimum AUC values. It proved that combining MP and AF for anomaly detection improves the detection performance.

![Figure 16](image-url)

Figure 16. Average AUCs of ROC for MPAF, MPAF without attribute filter (AF) step, and MPAF without morphological profile (MP) step on the ABU datasets.
4.2.3. MP and AP Effectiveness

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5. Conclusions

In this paper, we proposed a new hyperspectral anomaly detection algorithm called MPAF, based on the conventional morphological profile and attribute filter that improved with novel band selection and maximum area determination algorithms. This approach uses entropy and histogram counts to reduce the spectral dimension of the hyperspectral image, applies the morphological profile to discard the background pixels, and the attribute filter with a proposed area determination algorithm to filter the false anomalous pixels. A sensitivity analysis was conducted and proved the effectiveness of each step of the method. The experiments on real hyperspectral images showed that MPAF achieved high detection performances, reaching 0.9916 on average and superiority in 61.5% of thirteen datasets, with only slight differences in 38.5% of cases. In addition, it involved less computing time, i.e., 0.25 s on average. The performance of MPAF in reducing false anomalous pixels was also proven by reaching the highest score in the area under precision-recall curves, which was 0.1080 points higher than that of the state-of-the-art method. This proves that the proposed method is suitable for high-resolution HSIs and offers higher performance than previous methods.

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