A comparison of Clover and Wilson spectroscopy in the presence of dynamical quarks

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We present preliminary results of light hadron spectroscopy using valence, tadpole-improved, Clover fermions on an ensemble of gauge configurations generated with 2 flavors of staggered fermions at $\beta = 5.6$. We compare the slope and intercept of the curve $M_V$ vs. $M_{PS}^2$ for Clover and Wilson fermions. We show that a higher order chiral perturbation theory ansatz works very well for chiral extrapolations.

1. INTRODUCTION

In this talk, we present preliminary results from a detailed comparison of Clover and Wilson light spectroscopy at a moderate inverse lattice spacing (about 2 GeV). We discuss two analytic methods; 1) (tadpole-improved) Symanzik improvement of the fermion action and 2) higher order chiral extrapolations. In addition, we use correlated fits to multiple smearing functions and an automated plateau finder with different selection criteria in an attempt to reduce systematic fitting errors.

2. THE SIMULATION

The gauge configurations were generated by the HEMCGC collaboration with $\beta = 5.6$ and 2 flavors of staggered fermions at a mass of $am = .01$. Inversions were performed on 100 configurations separated by 20 trajectories, fixed to Coulomb gauge. We use tadpole improvement in our calculations; $u_0 = .867$ is obtained from the plaquette.

Quark propagators were inverted for each $\kappa$ using a gaussian source; at some $\kappa$’s a wall source was also used. Mixed $\kappa$ hadrons were formed between propagators generated in the same run. For the V and PS mesons, each spatial (mesonic) smearing yields two meson interpolating fields; one with spin structure $\Gamma$ and the other with spin structure $\Gamma \ast \gamma_0$. Similarly, we measure two different $\Delta^+ \gamma$ states; $s_z = 3/2$ and $s_z = 1/2$.

3. FITTING

A technique we find very useful is to perform a correlated multi-state fit to multiple correlation functions which have the same quantum numbers. Two types of fit are to a “vector” of smeared source and local sink and to a “matrix” of smeared source and sink correlations. Even if only a single energy is used in the fit ansatz, multiple smearings can help eliminate the problem of spurious plateaus because the multiple propagators will have different overlaps with excited states. A notation we find useful is to call a single state fit involving two smearings a “$1c2s$ fit.”

In choosing our fits, we have tried to avoid human intervention by using automated plateau finders. I discuss two of our criteria here; “C”, which tends to pick very short plateaus (conservative) and ”D”, which prefers long plateaus (aggressive). Because correlation functions are noisier at later times, the trade-off between different criteria is between statistical uncertainties and systematic fitting errors; an aggressive criterion will give smaller error bars but may have more excited state contamination in the ground state mass. The goal is to use the most aggressive criterion for which the statistical uncertainty dominates fitting errors. The definition of C is $\min(T_{\text{min}} + \chi^2/N_{\text{prop}})$; for D it is $\max(Q \ast N_{\text{dof}}/\delta M)$, where $N_{\text{prop}}$ is the number of propagators being fit, Q is the confidence level of the fit, and $\delta M$ is the fit uncertainty in the mass.

To perform chiral fits, we generated 201 boot-
strap ensembles of gauge configurations, with corresponding sets of propagators. After choosing a plateau for a particular fit, we repeated the fit on each bootstrap ensemble, resulting in an ensemble of 201 mass and amplitude values. Note that, unlike [3], the plateau was kept fixed over all bootstrap ensembles. We would like to implement the method of [3], which allows the plateau to vary between bootstrap ensembles, as this will move systematic fitting errors into bootstrap fluctuations where they can be statistically quantified.

Note that the mass ensembles depend both on the type of fit (i.e. 1c1s vs. 1c2s) and on the fit criterion used (i.e. C vs. D). A too aggressive criterion results in large $\chi^2$ in the chiral fit; this is because excited state effects (systematic errors) are about the same size as the statistical errors. This allows investigation of systematic fitting errors; our results should be consistent between different criteria and types of fits.

4. $\kappa_c$

Our first example of a higher order chiral extrapolation is in the determination of $\kappa_c$. In Fig. 1 we show $M_{PS}^2$ vs. $1/\kappa$ for the clover action, along with quadratic and linear fits. The quadratic fit includes all six points shown, while a linear fit is possible only for the first three points. The values of $\kappa_c$ extracted from these fits are 0.140809(16) and 0.140782(16), respectively. A typical problem when determining $\kappa_c$ is that it “runs away”; linear extrapolations to $\kappa_c$ change as lighter $\kappa$’s are included. This does not seem to be the case for the quadratic fit; omitting the lightest two $\kappa$’s from the fit results in a $\kappa_c$ of 0.14079(2), which is consistent with the full quadratic fit. Because of this we trust the quadratic fit; even our lightest $\kappa$ values are probably too heavy to be in the linear regime.

5. MESONS

In Fig. 2 we present our results for $M_V$ vs. $M_{PS}^2$ for the Wilson and clover actions; both degenerate and non-degenerate $\kappa$ combinations are included. The vertical dotted lines represent the $\rho$, $K^*$ and $\phi$ masses. The $\rho$ and $K^*$ were chosen by requiring that the ratio $M_V/M_{PS}$ match experiment. The pseudoscalar mass corresponding to the $\phi$ was chosen using the formula $M_{PS}^2 = 2m_{K^*}^2 - m_{\pi}^2$. The mass values for both V and PS were obtained from 1c2s vector fits using criterion D; the two smearings were the same spatially and differed by $\gamma_0$ in spin structure.
where the first number indicates the order of the
polynomial and the second is the number of co-
efficients set to zero. The 2-1 (quadratic) ansatz
is just lowest order $\chi PT$; the 4-2 (quartic) ansatz
introduces a higher order term that goes like $m^2$.
The cubic term in the 3-1 (cubic) and 4-1 (full
quartic) ansätze were motivated by the one loop
$\chi PT$ correction to baryon masses; at this confer-
ence we learned of $\chi PT$, in which one loop
$m^2$ corrections to the vector meson mass are derived.

The results of chiral fits were qualitatively similar
for Wilson and clover. In both cases, a
quadratic fit breaks down after the sixth point
is included, while both the cubic and quartic (4-
2) fits were able to include many more points
(typically 11-18 points). The cubic fit, however,
seemed to do a better job of describing data at
even higher masses which could not be fit; and
the quadratic fits are shown in Fig. 3. The full
quartic (4-1) ansatz only allowed one or two more
points to be fit than either a cubic or quartic
ansatz; this is a sign that there is no signal to
resolve the additional parameter. The numerical
results of our fits are presented in Tab. 1.

| Fit  | Wilson | Clover |
|------|--------|--------|
|      | $C_0$  | $C_2$  |
| 2-1  | .337(6) | .395(6) |
| 3-1  | .336(4) | 1.23(2) |
| 4-2  | .337(6) | 1.13(5) |
| 3-1* | .334(4) | 1.25(2) |

Note that the cubic fits are consistent with and
have smaller error bars than the quadratic fits.

This indicates that the higher mass points, which
have smaller fluctuations, are controlling the fit,
i.e. the lightest (and most expensive) $\kappa$'s are too
noisy to contribute useful information. To test
this, we omitted all $\kappa$ combinations lighter than
the third degenerate $\kappa$ (6 points for Wilson and 4
for clover) and repeated the 3-1 and 2-1 fits. The
quadratic ansatz only fit one degree of freedom
(3 points) for clover and none for Wilson. The
cubic fits, however, succeeded to the same $\kappa_{max}$
and gave results with the same error bars. This
indicates that it was unnecessary to simulate bel-
low the strange quark mass in order to calculate
the physical $\rho$ mass.

6. BARYONS

For baryons, we have not yet analyzed our Wil-
son results. In addition, we have results with wall
smearings at only two of our six lightest $\kappa$'s. We
will correct these deficiencies in the near future.

Figure 3. Comparison between fit criteria and
between single and multi-propagator fits for the
$\kappa = .1375$ nucleon. Vertical lines indicate $T_{min}$
chosen by criteria D and C (D is always leftmost).
Upper right plot is superposition of effective mass
for shell and wall sources; upper left plot is mass
values for 1c1s fit as function of $T_{min}$.

One interesting result for the nucleon was that
we were unable to obtain good chiral fits when
criterion D was used for 1c1s propagator fits. We
interpret this as an example of too aggressive a fit criterion leading to statistically significant excited state contamination of the ground state energy. This can be corrected by either using a less aggressive fit criterion (i.e. C), or by including a second smearing (i.e. wall). This can be seen in Fig. 3; the 1c1s criterion D fit uses $T_{min} = 1!$ This is changed to 5 and 7 for the C criterion and 1c2s fit, respectively. In Fig. 4 we show that using criterion C results in successful chiral fits, but that C is probably too conservative. The $\Delta$ behaves similarly; criterion D gives mass values which cannot be fit, while criterion C works well. As with the vector, a cubic ansatz allows much heavier quark masses to be included.

Our most striking result is that a cubic ansatz for chiral extrapolations of the vector mesons works so well, and over such a large mass range. Our results seem to indicate that the two $\kappa$ values lighter than $\kappa_{\text{strange}}$ were unnecessary, since their omission from fits caused negligible changes. This is especially important given that the light $\kappa$'s are both the most expensive and the most sensitive to finite volume effects. A potential drawback to this approach was our inability to clearly distinguish between cubic and quartic ansätze, and the lack of a strong signal to justify including both terms at the same time.

7. CONCLUSIONS

Our comparison between Wilson and clover actions has only been completed for mesons; the results were reasonable. For both the intercept and slope of $M_V$ vs. $M_{PS}^2$, we saw 10-20% discrepancy. This is consistent with a naive estimate of 500 MeV $\sim a \approx .25$ for Wilson discretization errors, and suggests that the O($a^2$) clover discretization errors are roughly 5% at this lattice spacing. A comparison between clover results and experiment is less encouraging; as can be seen from Tab. 2, the ratio $J_d = (M_{K^*} - M_{\Omega})/(M^2_{K^*} - M^2_{\pi})$ is 10% too small and the ratio $(M_{\Omega} - M_{\Delta})/M_{K^*}$ is 20% too small. We hope that this effect is due to either finite volume or incorrect dynamical fermion content; if this is not the case then either clover is not correctly removing O($a$) errors or there is some source of error which we do not understand.

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REFERENCES

1. K. Bitar et al., Nucl. Phys. B (Proc. Suppl.) 26 (1992) 259; Phys. Rev. D46 (1992) 2169.
2. F. Butler et al., Nucl. Phys. B421 (1994) 217; B430 (1994) 179.
3. E. Jenkins et al., UCSD-PTH-95-08 (1995); hep-ph 9506356