Higgs inflation and cosmological electroweak phase transition with N scalars in the post-Higgs era

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ABSTRACT: We study the Higgs inflation and the cosmological electroweak phase transition with the N-scalars extended standard model of particle physics (SM). The number of N is strictly bounded by the unitarity, the Higgs precisions and electroweak precision observables (EWPO). When the scalars respects an extra $O(N)$ symmetry, the masses of the N scalars are bounded to TeV scale in the inflation viable parameter regions, and therefore impossible to saturate the DM candidates. The EWPO bound the N to be smaller than 4 and therefore make the cosmological electroweak phase transition never being strongly first order. The $O(N)$ symmetry preserved by the N-scalar can be spontaneous broken to be $O(N-1)$ with remnant $N-1$ Goldstones, which can fake the neutrinos or gain masses through non-perturbative gravity effects and therefore contribute to dark radiations. The cosmological phase transition has been investigated in the Higgs inflation feasible parameter spaces, where the phase transition can be strongly first order through one-step or two-step types. The mixing of extra heavy Higgs with the SM Higgs confronts with severely constraints from LHC, CEPC, ILC, and FCC-ee. The bounds of the mixing angle might preclude the possibility to obtain the slow-roll Higgs inflations and a strong first order Electroweak phase transition (SFOEWPT).
1 Introduction

To our knowledge, the cosmic inflation [1–3] solves the horizon, flatness and monopole problems of the universe successfully. The primordial density fluctuations generated during the inflation can explain the formation of large scale structure of the universe observed by CMB [4]. The inflation scenario is more attractive when the inflaton field can play some important role in the particle physics, one fascinating scenario is the Higgs inflation [5, 6], where the inflaton is the SM Higgs being observed by LHC [7, 8], that can leads to spontaneously breaking of the electroweak symmetry $SU(2)_L \times U(1)_Y$. The original Higgs inflation suffers from the unitarity problem at high scale around $\sim O(10^{13})$ GeV [9–19]. Therefore, the cosmologists and particle physicists turn to investigate the singlet scalars assistant case utilizing Higgs-portal [20, 21] interactions.

In the post-Higgs era, how does the electroweak symmetry breaking occurs comes to be the hot research topic of particle physicists [22]. One outstanding mechanism is the electroweak phase transition. A strong first order electroweak phase transition does not happen in the Standard Model of particle physics [23, 24], to obtain a strongly first order one, the new physics is always necessary where
the cubic Higgs couplings deviation might be able to be probed at high energy colliders [22, 23].

The most simplest way to realize the SFOEWPT can be extending the SM with an additional real singlet scalar [25, 26] or complex singlet scalar [27, 28].

The existence of the dark matter is supported by overwhelming astrophysical and cosmological observations. For the most simplest DM case with the hidden sector coupled with the SM through only the Higgs-portal interaction of $|H|^2 S^2$ [29–31], the one-step type SFOEWPT can be realized with a large quartic coupling that would directly leads to the under-abundant situation [32]. The straightforward approach to ameliorate the situation of requirement of large quartic coupling can be adding to the SM $N$-scalars with $O(N)$ symmetry, therefore the one-step SFOEWPT can be realized with a lower magnitude of Higgs-portal interaction of $|H|^2 S_i S_i (i = 1, ..., N)$ [33–36]. The previous studies of Ref. [34] indicates that the one-step SFOEWPT cannot be addressed together with a correct relic density unless the quartic couplings of $|H|^2 S_i S_i$ and masses of $S_i$ are non-universal, rather one can expect a rather large $N$ (e.g., $N > 50$) to overcome the problem. Here, the additional $N$ hidden scalars might also alleviate the hierarchy problem through positive contributions to radiative corrections of the Higgs boson mass and therefore satisfy the Veltman conditions [39].

For the Higgs inflation in the Higgs-portal scenarios, the typical quartic scalar couplings are required to be around $\sim O(10^{-1})$ [20, 21, 38]. For that magnitude of couplings, the previous studies of N-scalars with an exact $O(N)$ symmetry suggest the one-step SFOEWPT can be realized for a larger $N$, with typical frequency of $\sim O(10^{-3} - 10^{-1})$ Hz gravitational wave signals can be probed [34–36] together with a substantial triple Higgs couplings deviation to be probed by the future colliders [35, 36]. The WIMP DM situation in the classical scale invariant N-scalars model with $O(N)$ symmetry [39] are ruled out for $N > 4$ even for rather large quartic coupling [40]. Here, we stress that the largeness of the quartic couplings can not accommodate inflations though prefer one-step SFOEWPT.

With the increasing pressure from the direct detection experiments, the relic abundance might be partially rather than fully saturated by WIMP DM. In the one-step SFOEWPT scenario, the dimensional six operators can be used to improve the vacuum barrier. While, in the Higgs-portal scenario with the hidden sector obey $Z_2$ symmetry, the dimensional six operator lose the ability to lift up the vacuum barrier at finite temperature due to the cancellation of the different contributions given the $Z_2$ symmetry is spontaneously broken [41]. Nevertheless, in the simplest scenario with real singlet scalar serves as WIMPs DM candidate, one can expect the SFOEWPT through two-step types [32, 42].

In this work, we study the inflation and EWPT in the Higgs-portal N-scalars model. When the N-scalar fields respect the $O(N)$ symmetry, then all the N-scalars can serve as DM candidates. On the other hand, when the symmetry is broken to $O(N - 1)$ spontaneously, there will be $N - 1$ Goldstones that can fake the effective neutrinos from the viewpoint of cosmology. In Section 2 we revisit the $O(N)$ scalar extended SM, and introduce the possible case with the $O(N)$ being spontaneously broken, where we present the relevant theoretical and experimental constraints. The effects of the N scalars on the inflation are explored in Section 3.1 after imposing theoretical constraints. The cosmological electroweak phase transitions are studied around the Higgs inflation valid parameter regions in Section 3.2. In Section 3.3, the dark matter situation in the $O(N)$ model has been explored after taking into account the bounds from future colliders. We explore the EWPO constraints on $N$ and the ability
of the future lepton colliders to probe or exclude these parameter regions in Section. 4. The effective neutrino constraints are used to set bounds on the number of Goldstones, the ability of Goldstones to contribute to dark radiations are estimated also provided they gain masses from non-renormalizable gravity effects. We conclude with the Section. 5.

2 The Models

In this work, we study two scenario of $N$ singlet scalar extended standard models. In the case of the $N$ singlet scalar with $O(N)$ symmetry, one can anticipate the $O(N)$ symmetry broken at finite temperature and restored at zero temperature. The zero temperature tree-level Lagrangian is given by

$$V_0(H,S) = -\mu_h^2 H^+ H + \lambda_h |H^+ H|^2 + \frac{\mu_s^2}{2} S_i S_i + \frac{\lambda_s}{4} (S_i S_i)^2 + \frac{1}{2} \lambda_{hs} |H|^2 S_i S_i,$$

with $H^T = (v + h + iG^0)/\sqrt{2}$. After the spontaneously symmetry breaking of the Electroweak symmetry, the mass term of $S_i$ is given as $m_{S_i}^2 = \mu_h^2 + \lambda_{hs} v^2 / 2$. The $N$ singlet scalar $S_i$ with $O(N)$ being spontaneously broken to $O(N-1)$ symmetry, which is another model setup, in this case we use “s” instead of “S” to differentiate it from the $O(N)$ scenario. The minimization conditions of the potential can be obtained when EW symmetry is broken and the $O(N)$ being broken along the direction of $s_i$ with other directions being $s_i (i = 1, ... , N - 1)^{1}$.

$$\frac{dV_0(h,s,A)}{dh} \bigg|_{h=v} = 0, \quad \frac{dV_0(h,s,A)}{ds} \bigg|_{s=v_s} = 0,$$

which give rise to $\mu_h^2 = \lambda_{hs} v^2 + \lambda_{hs} v_s^2 / 2, \mu_s^2 = - (\lambda_{hs} v^2 / 2 + \lambda_{hs} v_s^2)$. The mass matrix is given by

$$M^2 = \begin{pmatrix} 2v^2 \lambda_h & v v_s \lambda_{hs} \\ v v_s \lambda_{hs} & 2v_s^2 \lambda_s \end{pmatrix},$$

In order to diagonalize the mass matrix, we introduce the rotation matrix $R = ((\cos \theta, \sin \theta), (-\sin \theta, \cos \theta))$ with $\tan 2\theta = -\lambda_{hs} v_s / (\lambda_h v^2 - \lambda_{hs} v_s^2)$ to relate the mass basis and field basis,

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

The mass squared eigenvalues are

$$m_{h_1,h_2}^2 = \lambda_h v^2 + \lambda_{hs} v_s^2 + \frac{\lambda_{hs} v_s^2 - \lambda_h v^2}{\cos 2\theta},$$

Identify the $h_1$ being the 126 GeV SM-like Higgs boson, and requiring the $h_2$ is dominated by $s$ set $\cos \theta > 1/\sqrt{2}$. The quartic couplings can be expressed as functions of the Higgs masses, $v, v_s$ and the

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1The breaking of $O(N)$ can happen in any direction, means we can have any of $s_i$ with $i = 1, 2, ..., N$ obtain VEV, here we assume $O(N)$ breaks in $s_N$ direction.
mixing angle $\theta$,\[ \lambda_h = \frac{m_h^2 \sin^2 \theta + m_{h_1}^2 \cos^2 \theta}{2 v^2}, \quad \lambda_s = \frac{m_h^2 \cos^2 \theta + m_{h_1}^2 \sin^2 \theta}{2 v_s^2}, \quad \lambda_{hs} = \frac{(m_h^2 - m_{h_1}^2) \sin 2\theta}{2 v v_s}. \] (2.6) (2.7) (2.8)

### 2.1 Theoretical constraints

Firstly, due to the additive property of the scalar contributions to the beta functions of quartic couplings, especially when the $N$ is larger, one need to aware the possible perturbativity problem of the model at high scale when one perform the inflation analysis. We impose the following conditions to preserve the perturbativity,

\[ |\lambda_h| < 1, \quad |\lambda_s| < \sqrt{4\pi}, \quad |\lambda_{sh}| < \sqrt{4\pi}. \] (2.9)

Since the perturbative unitarity rely on the quartic coupling of the tree-level potential, one can expect the bound is the same for $O(N)$ and $O(N \to N - 1)$ scenarios, being given by [36],

\[ \frac{1}{32\pi} \left( 3\lambda + (N + 2)\lambda_s + \sqrt{(3\lambda - (N + 2)\lambda_s)^2 + 4N\lambda_{hs}^2} \right) < \frac{1}{2}, \] (2.10)

with the couplings are related with masses and VEVs through Eq. 2.8 for $O(N \to N - 1)$ case. To prevent the unbounded from bellow of the scalar potential, the vacuum stability conditions should be satisfied,

\[ \lambda_h > 0, \quad \lambda_s > 0, \quad \lambda_{sh} > 0 \quad \text{or} \quad \lambda_{sh} > -2\sqrt{\lambda_h \lambda_s}. \] (2.11)

A simple analysis of these theoretical limits on the quartic couplings at the Electroweak scale are given in Fig. 1 and Fig. 2 for $O(N)$ and $O(N \to N - 1)$ models. The perturbativity roughly bounds the upper limit of $\lambda_{hs,s}$, the sharp of the boundary is set by the unitarity bounds, the lower limit of the quartic couplings $\lambda_{hs,s}$ is given by the stability conditions where more parameter spaces are allowed by $\lambda_{sh} > -2\sqrt{\lambda_h \lambda_s}$ in comparison with $\lambda_{sh} > 0$. In the Fig. 2, it’s converted to the bounds on the $m_{h_s}$ and $v_s$ correspondingly. In the following sections, we require the perturbativity, unitarity, and the stability from the electroweak scale to Planck scale for the safe of inflation and EWPT studies, that have been evaluated with the renormalization group equations list in Sec.5.

### 2.2 Higgs precisions

Integrating out the heavy scalar fields results in the dimension-six operators,

\[ \mathcal{L} \subset \frac{c^H}{\Lambda^2} O_H + \frac{c_6}{\Lambda^2} O_6, \] (2.12)

with $O_H \equiv \frac{1}{2} |\partial |H^\dagger H| |^2$ and $O_6 \equiv |H^\dagger H|^3$. Here, the operator $O_H$ can leads to the universal shift of Higgs couplings by the Higgs field redefinition or Higgs wavefunction renormalization. The operator
\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Parameters regions allowed by Perturbativity+Unitarity+Stability in the $O(N)$ scalar model.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Parameters regions allowed by Perturbativity+Unitarity+Stability in the $O(N \rightarrow N - 1)$ scalar model.}
\end{figure}

$O_6$ can alleviate the triple Higgs coupling and is crucial for the strong first order electroweak phase transition [43]. For the Wilson coefficients generated at tree-level, we have [44, 45],

\begin{align}
    c^N_H &= N \frac{\lambda_{hs}^2}{2\lambda_s}, & c^N_6 &= 0, \\
    c^{N \rightarrow N-1}_H &= \frac{\lambda_{hs}^2}{2\lambda_s}, & c^{N \rightarrow N-1}_6 &= 0.
\end{align}

with the $\Lambda \approx \mu_s$ for the two scenario, for $O(N)$ case one have $\mu_s \approx m_{S_i}$ and in the $O(N \rightarrow N - 1)$ case $\mu_s \approx m_{h_2}$ for the small mixing angle limit. The loop-level induced dim-6 operator Wilson coefficients
are

\[ c_H^N = \frac{N\lambda_{hs}}{48\pi^2}, \quad c_S^N = \frac{N\lambda_{hs}^3}{48\pi^2}, \quad (2.15) \]

for the \( O(N) \) case. While, for the \( O(N \to N - 1) \) case which reduces to

\[ c_H^{N \to N-1} = \frac{\lambda_{hs}}{48\pi^2}, \quad c_S^{N \to N-1} = \frac{\lambda_{hs}^3}{48\pi^2}. \quad (2.16) \]

Here, we note that the same as the tree-level induced dim-6 operator, the Wilson coefficients do not have the factor of “N” in \( O(N \to N - 1) \) case because one can only suppose s being heavy sectors and other \( s_{N-1} \) scalars are massless Goldstones. For the quartic coupling \( \lambda_{hs} \sim O(10^{-1}) \) being required by the inflation, we can safely ignore the loop-level induced operator effects on the Higgs precision.

Here, we match the Eq. 2.1 after the s get VEV with the Ref. [44, 45] to obtain the \( c_6^{N \to N-1} \). Due to no symmetry breaking in strict symmetry of \( O(N) \), we have \( c_6^N = 0 \) at tree level. For the \( O(N \to N - 1) \) case, one can map the \( c_H^{N \to N-1} \) with the mixing angle as: \( 1 - \cos \theta \approx c_H^{N \to N-1}v^2/(2m_h^2) \approx c_H^{N \to N-1}v^2/2\mu^2 \) in the small mixing angle limit. The \( O_H \) leads to the modification of the wavefunction of the Higgs,

\[ L_{\text{eff}} \supset (1 + \delta Z_h)\frac{1}{2}(\partial_\mu h)^2, \quad (2.17) \]

with \( \delta Z_h = 2v^2 c_H/m_{S(h_2)}^2 \) for \( O(N) \) (\( O(N \to N - 1) \)) scenarios. Thus one obtain a universal shift of all Higgs couplings. Which therefore induce the correction to the \( e^+e^- \to hZ \) associated production cross section [47],

\[ \delta \sigma_{Zh} = -2\frac{v^2 c_H}{m_{S(h_2)}^2}, \quad (2.18) \]

which has been defined as the fractional change in the associated production cross section relative to the SM case.

For the \( O(N) \) symmetry with \( m_S > m_h \), the Higgs wavefunction renormalization shift the SM-like Higgs couplings to other SM particles by \( c_H^{N \to N-1}v^2/(2m_h^2) \sim N\lambda_{hs}^3 v^2/(2\lambda_4 m_S^2) \). Which results in the constraint on \( c_H \) and therefore \( N, \lambda_{hs}, \) from the LHC[41] as well as ILC, CEPC, and FCC-ee[46].

On the other hand, the operator \( O_H \) induces the operator combinations \( O_W + O_B \) and \( O_T \) operators through RGE [44, 47], which results in the S and T parameters

\[ \Delta S = \frac{1}{12} c_H \frac{v^2}{m_{S(h_2)}^2} \log\left(\frac{m_{S(h_2)}^2}{m_W^2}\right), \quad (2.19) \]

\[ \Delta T = -\frac{3}{16\pi c_W} c_H \frac{v^2}{m_{S(h_2)}^2} \log\left(\frac{m_{S(h_2)}^2}{m_W^2}\right). \quad (2.20) \]

We set bounds on \( m_{S(h_2)} \) and the “N” using the electroweak fit in Ref. [48],

\[ S = 0.06 \pm 0.09, \quad T = 0.10 \pm 0.07, \quad (2.21) \]
with the correlation coefficient between the S and T parameters being +0.91. Here, we point out that in the case of $O(N \rightarrow N - 1)$, the $c_H = \lambda_{h_1 h_2}^2/(2 \lambda_h)$ due to the other $N - 1$ scalars are Goldstones. In the case of $O(N \rightarrow N - 1)$, the parameter spaces is more strictly constrained by T parameter rather than S parameter, which set stringent bounds on the mixing angle and the masses of the heavy Higgs. When the heavy Higgs is not highly decoupled, i.e., $m_{h_2} \sim m_h$, one can obtain the oblique parameter T with the formula of,

$$T = -\left(\frac{3}{16\pi^3 c_W^2}\right)\left\{ \cos^2 \theta \left[ \frac{1}{c_W^2} \left( \frac{m_{h_1}^2}{m_{h_1}^2 - M_Z^2} \right) \ln \frac{m_{h_1}^2}{M_Z^2} - \left( \frac{m_{h_1}^2}{m_{h_1}^2 - M_W^2} \right) \ln \frac{m_{h_1}^2}{M_W^2} \right] + \sin^2 \theta \left[ \frac{1}{c_W^2} \left( \frac{m_{h_2}^2}{m_{h_2}^2 - M_Z^2} \right) \right] \right\},$$

(2.22)

using the feynman diagram method in Ref. [49]. One can obtain severely constraints on the $\theta$ with increasing of $m_{h_2}$.

For the $O(N \rightarrow N - 1)$ scenario, the mixing angle and the heavy Higgs masses are subjective to the bounds coming from the LHC Higgs data, which force the mixing angle $\theta$ to be $|\cos \theta| \geq 0.84$ [26]. After including of the current LHC and High-luminosity LHC Higgs production rates together with the EWPO, a moderate of $\theta \sim \sqrt{\lambda_{h_2}^2 v^2/(4\lambda_h m_{h_2}^2)} = 0.2$ can be safety [41]. Here, we stress that: though for the $O(N \rightarrow N - 1)$ scenario, when fit the Higgs data we need to take into account the invisible decay of the Higgs to the $N - 1$ Goldstone bosons, one can firstly perform the Higgs fit without including the effects of changing of SM Higgs decay width and then constrain the number of Goldstone bosons $N - 1$ with the invisible decay fit results from [50]: $B_{BSM} < 0.34$ at 95% CL. For the case of $O(N)$ symmetry being broken to $O(N - 1)$ at zero temperature, we have the following lagrangian to describe the triple scalar interactions,

$$L \supset \lambda_{h_1 h_2 h_j} h_1 h_2 h_j + \lambda_{h_i s_{N-1} s_{N-1}} h_i s_{N-1} s_{N-1},$$

(2.23)

with $h_{i,j}$ denotes $h_{1,2}$ and $N = 2, ..., N$, the relevant triple scalar couplings are given bellow,

$$\lambda_{h_1 h_2 h_1} = -\frac{m_{h_1}^2}{2 v v_s} \sin(2\theta)(v_s \cos \theta + v \sin \theta)(1 + m_{h_2}^2/2m_{h_1}^2),$$

(2.24)

$$\lambda_{h_2 s_{N-1} s_{N-1}} = m_{h_2}^2 \cos \theta/(2v_s),$$

(2.25)

$$\lambda_{h_1 s_{N-1} s_{N-1}} = -m_{h_1}^2 \sin \theta/(2v_s),$$

(2.26)

$$\lambda_{h_1 h_2 h_2} = \lambda_{h_1 s_{N-1} s_{N-1}}.$$  

(2.27)

From which, when the $m_{h_2} > m_{h_1}$, the decay widths of the SM-like Higgs and the second Higgs are

\footnote{Indeed, if one want the heavy Higgs take part in the EWPT process, it cannot be highly decoupled.}
given by

\[
\Gamma_{h_2}^{\text{tot}} = \Gamma_{h_2}(h_2 \to h_1h_1) + \sin^2 \theta \Gamma_{h} \bigg|_{m_h \to m_{h_2}} + (N - 1) \Gamma_{h_2}(h_2 \to s_{N-1}s_{N-1}) \\
= \Gamma_{h_2}(h_2 \to h_1h_1) + \sin^2 \theta \Gamma_{h_2}^{\text{SM}} \bigg|_{m_h \to m_{h_2}} + (N - 1) \frac{\lambda_{h_2}^2 s_{N-1}s_{N-1}}{32\pi m_{h_2}} 
\]

(2.28)

\[
\Gamma_{h_1}^{\text{tot}} = \cos^2 \theta \Gamma_{h_2}^{\text{SM}} + (N - 1) \Gamma_{h}(h \to s_{N-1}s_{N-1}) \\
= \cos^2 \theta \Gamma_{h_1}^{\text{SM}} + (N - 1) \frac{\lambda_{h_1}^2 s_{N-1}s_{N-1}}{32\pi m_{h_1}}, 
\]

(2.29)

with

\[
\Gamma(h_2 \to h_1h_1) = \frac{\lambda_{h_2}^2 s_{h_1}}{32\pi m_{h_2}} \sqrt{1 - \frac{4m_{h_1}^2}{m_{h_2}^2}}, 
\]

(2.30)

For the case in which \(m_{h_1} > m_{h_2}\) one need take into account the decay of \(h_1 \to 2h_2\), with decay width being given by

\[
\Gamma(h_1 \to h_2h_2) = \frac{\lambda_{h_1}^2 s_{h_2}}{32\pi m_{h_1}} \sqrt{1 - \frac{4m_{h_2}^2}{m_{h_1}^2}}. 
\]

(2.31)

The invisible decay of SM Higgs can be used to set upper bounds to the number of the Goldstones and the mixing angle \(\theta\). At 95% CL, the LHC (ATLAS+CMS) set \(B_{\text{inv}} < 34\%\) [50], see Fig. 3 for the constraints. With the increase of \(v_s\), more parameter space of \(\theta\) is allowed.

\[\text{Figure 3:} \ \text{Invisible decay bounds on} \ N \ \text{and mixing angle} \ \theta \ \text{in} \ O(N \to N-1) \ \text{model coming from} \ LHC, \ B_{\text{inv}} < 34\% \ [50].\]
3 High scale and low scale phenomenologies

We first study the cosmic inflation with large scale fields. With the temperatures of the universe cooling down the low scale fields physics come to us: the possibility to implement the cosmological strong first order phase transitions, and the dark matter physics.

3.1 The Higgs inflation with N singlet scalars

The action in the Jordan frame is

$$S_J = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{2} R - \xi_h (H^+ H) R - \xi_s S^2 R + \partial_\mu H^+ \partial^\mu H + (\partial_\mu S)^2 - V(H,S) \right],$$  \hspace{1cm} (3.1)

where $M_p$ is the reduced Planck mass, $R$ is the Ricci scalar, $\xi_h, \xi_s$ define the non-minimal coupling of the $h, S$-field. The quantum corrected effective Jordan frame Higgs potential at large field values ($h$) can be written as

$$V(h) = \frac{1}{4} \lambda h(\mu) h^4,$$  \hspace{1cm} (3.2)

which is evaluated along the higgs axis, where the scale is $\mu \sim O(h) \approx h$. We impose quantum corrections to the potential following Ref. \cite{51, 52}. After the conformal transformation,

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 \equiv 1 + \frac{\xi_h h^2}{M_p^2} + \frac{\xi_s S^2}{M_p^2}.$$  \hspace{1cm} (3.3)

and a field redefinition

$$\frac{d\chi_h}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2 / M_p^2}{\Omega^4}}, \quad \frac{d\chi_S}{ds} = \sqrt{\frac{\Omega^2 + 6\xi_s^2 S^2 / M_p^2}{\Omega^4}},$$  \hspace{1cm} (3.4)

we obtain

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} M_p^2 R + \frac{1}{2} \partial_\mu \chi_h \partial^\mu \chi_h + \frac{1}{2} \partial_\mu \chi_S \partial^\mu \chi_S + A(\chi_S, \chi_h) \partial_\mu \chi_h \partial^\mu \chi_S - U(\chi_S, \chi_h) \right],$$  \hspace{1cm} (3.5)

where $U(\chi_S, \chi_h) = \Omega^{-4} V(s(\chi_S), h(\chi_h))$ and

$$A(\chi_S, \chi_h) = \frac{6\xi_h \xi_s}{M_p^2 \Omega^4} \frac{dS}{d\chi_S} \frac{dh}{d\chi_h} h S.$$  \hspace{1cm} (3.6)

In this work we consider only $h$ serves as inflaton for the $O(N)$ model and the $O(N \rightarrow N-1)$ model \cite{53–55}. The metric in this case is given by $\Omega^2 = 1 + (\xi_h h^2 + \xi_s S^2) / M_p^2 \approx 1 + \xi_h h^2 / M_p^2$ with $S \sim 0$. The inflationary action in terms of the canonically normalized field $\chi$ is therefore given as

$$S_{inf} = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \frac{1}{2} (\partial \chi)^2 - U(\chi) \right],$$  \hspace{1cm} (3.7)

\(^3\text{Here, we drop the subscript to simplify the notation as in Ref. [53].}\)
with the potential in terms of the canonically normalized field $\chi$ as

$$U(\chi) = \frac{\lambda_h (h(\chi))^4}{4\Omega^4},$$  \hspace{1cm} (3.8)

where the new field $\chi$ are defined by

$$\frac{d\chi}{dh} \approx \left(1 + \xi_h h^2 / M_p^2 + 6\xi_h^2 h^2 / (1 + \xi_h h^2 / M_p^2)^2\right)^{1/2}$$  \hspace{1cm} (3.9)

for $h-$ inflations [53]. Note that $\lambda_h$ and $\xi_h$ have a scale $(h)$ dependence.

The slow-roll parameters are then given by

$$\varepsilon(\chi) = \frac{M_p^2}{2} \left(\frac{dU/d\chi}{U(\chi)}\right)^2, \quad \eta(\chi) = M_p^2 \left(\frac{d^2 U/d\chi^2}{U(\chi)}\right).$$  \hspace{1cm} (3.10)

The field value at the end of inflation $\chi_{\text{end}}$ is obtained when $\varepsilon = 1$, and the horizon exit value $\chi_{\text{in}}$ can be calculated by assuming e-folding numbers being 60 between the two periods,

$$N_{\text{e-folds}} = \int_{\chi_{\text{end}}}^{\chi_{\text{in}}} \frac{1}{M_p \sqrt{2\varepsilon}} \, d\chi.$$  \hspace{1cm} (3.11)

Then, one can relate the inflationary observables of spectrum index $n_s$ and the tensor to scalar ratio of $r$ with the slow-roll parameters at the $\chi_{\text{in}}$, so that

$$n_s = 1 + 2\eta - 6\varepsilon, \quad r = 16\varepsilon.$$  \hspace{1cm} (3.12)

Meanwhile, one can determine the non-minimal gravity couplings using the constraint coming from CMB observations [4], with the amplitude of scalar spectrum fluctuations $\Delta^2_{\mathcal{R}}$ being calculated as

$$\Delta^2_{\mathcal{R}} = \frac{1}{24\pi^2 M_p^4} \frac{U(\chi)}{\varepsilon} = 2.2 \times 10^{-9}.$$  \hspace{1cm} (3.13)

We note that though the above formula can describe the slow-roll inflation of $O(N)$ and $O(N \to N - 1)$ scenarios, the inflation dynamics for the two scenarios are different due to the mass spectrum and the relation between scalar masses and quartic couplings which will be explored in detail in the following sections.

### 3.1.1 $O(N)$ scenario

First, in Fig. 4, we show the Higgs inflation feasible parameter regions in the plane of $(\lambda_s, \lambda_{hs})$ for different N after imposing the theoretical constraints up to planck scale as aforementioned in Sec. 2.1. Where, the upper and lower bounds of $\lambda_{hs}$ are mostly coming from perturbativity and unitarity, and stability conditions. The inflation feasible $(\lambda_s, \lambda_{hs})$ ranges are the largest for $N = 1$ compared with other $N$ cases. The feasible ranges are reduced with the increased $N$ and are overlap for the two neighbour $N$ expect for $N = 1$ and $N = 2$. The decreasing of the inflation valid area with the increase of $N$ is due to the fact that: a larger $N$ leads to more contributions of $\lambda_{hs}$ to $\lambda_h$ at the inflation scale through RG running, and therefore the stability, perturbativity and unitarity set the lower and upper bounds of $\lambda_{hs}$. We plot the RG running of the quartic scalar couplings for the case of $N = 7, 10, 13$ in Fig. 5. The perturbativity of quartic coupling and unitarity can be violated due to RG running of couplings as shown in the right panel of Fig. 5.
Figure 4: Inflation feasible $(\lambda_s, \lambda_{hs})$ plane for different $N$ with in $O(N)$ scalar model, the larger $N$ is shown by the deeper color, the corresponding $N$ are $1 \rightarrow 5$, $6 \rightarrow 10$ and $11 \rightarrow 15$ for left, middle and right panel, respectively.

Figure 5: RG running of couplings for $\lambda_s$, $\lambda_h$ and $\lambda_{hs}$. Left: the quartic couplings where the inflation is valid. Right: the quartic couplings lives in the parameter region where the inflation is invalid.

3.1.2 $O(N \rightarrow N - 1)$ scenario

We show the Higgs inflation valid parameter spaces in Fig. 6, the feasible ranges are reduced with the increased $N$ and are overlap for the two contiguous $N$. The left panel indicates that the $\lambda_{hs}$ increases with the increase of $\lambda_s$ for the $O(N \rightarrow N - 1)$ case, which is different from the $O(N)$ scalar model cases. A interesting triangular shape shows up due to the bounds on $m_{h_2}$ and $v_s$ from perturbativity, unitarity, and stability, together with the relations among quartic couplings and the Higgses masses as well as VEVs. Here, it should be note that the lower value of $m_{h_2}$ is set by the stability bounds, the quartic Higgs coupling will be negative for $m_{h_2} < 330$ GeV. That set the lower boundary of inflation. And the upper boundary is set by the magnitude of $v_s$ assisted by $m_{h_2}$, and the upper bound on $m_{h_2} < 600$ GeV is required to fulfilled the perturbatity and unitarity conditions at high scale. For explicitly, we explain the property in the Fig.7 by taking $N = 1$ case as an example. In the upper plot, yellow region stands for the feasible inflation region for $N = 1$ in the $(\lambda_s, \lambda_{hs})$ plane within the $O(N \rightarrow N - 1)$ scalar model with the number of Goldstone being zero, the values of $m_{h_2}$ and $v_s$ for
the numbers and alphabets can be obtained form the right hand of the upper plot and the middle and bottom panels, respectively. The middle and bottom panels show the RG running of coupling from electroweak scale to plank scale, the corresponded initial scale is the points A, B, C and D in the upper panel, respectively. Note that there is a absolute value for the coupling $\lambda_{hs}$ in the last figure, which is responsible for the downward tip. Which means the stability is violated in the point of D. That indicates that the stability is the lower bound for $(\lambda_s, \lambda_{hs})$ plane in the $O(N \rightarrow N - 1)$ model. The perturbativity and unitarity are violated for the RG running of couplings at B and C points, which indicates that the perturbativity and the unitarity set the upper bound for that case. With the increasing of N, the perturbativity and unitarity conditions, together with stability requirement results in a smaller parameter regions of $\lambda_{hs}$ and $\lambda_s$ as shown in Fig. 6.

### 3.2 Electroweak phase transitions

With the temperature cooling down, the universe can evolve from symmetric phase to the symmetry broken phase. The behavior can be studied with the finite temperature effective potential with particle physics models [56]. Through which one can obtain the critical classical field value and temperature being $v_C$ and $T_C$ to characterize the critical phases. Roughly speaking, a SFOEWPT can be obtained when $v_C/T_C > 1$, then the electroweak sphaleron process is quenched inside the bubble and therefore one can obtain the net number of baryon over anti-baryon in the framework of EWBG. For the uncertainty of the value and possible gauge dependent issues we refer to Ref. [57].

The finite temperature effective potential include the tree level scalar potential, the Coleman-Weinberg potential, and the finite temperature corrections [58]. The tree level scalar potential for $O(N)$ is obtained directly from Eq. 2.1,

$$V_0(h, S) = -\frac{\mu^2 h^2}{2} + \frac{\lambda h^4}{4} + \frac{\mu_s^2 S^2}{2} + \frac{\lambda_s S^4}{4} + \frac{\lambda_{hs} h^2 S^2}{4},$$

(3.14)
Figure 7: Top panel: the feasible of Higgs inflation in the $(\lambda_s, \lambda_{hs})$ plane. Middle and bottom panels: the running coupling $\lambda_{h, hs, s}$ with the corresponding initial values at EW scale can be get from the top panel.

Here, we drop the subscript since all N directions are the same and we assume only one direction get VEV during the EWPT process. The “S” should be the “s” for the $O(N \rightarrow N - 1)$ scenario to indicate the possible symmetry breaking direction with other directions $s_i$ ($i = 1, ..., N - 1$) do not get VEV during the EWPT process. For the $O(N)$ scalar model, the one-loop Coleman-Weinberg potential for
the scalar parts is given by

\[
V_{CW}(h,S) = \frac{1}{64\pi^2} \left[ m^4_{h}(h,S) \left( \log \frac{m^2_{h}(h,S)}{Q^2} - c_i \right) + 2m^4_{G^+}(h,S) \left( \log \frac{m^2_{G^+}(h,S)}{Q^2} - \frac{3}{2} \right) \right. \\
+ \left. m^4_{G^0}(h,S) \left( \log \frac{m^2_{G^0}(h,S)}{Q^2} - 3/2 \right) + Nm^4_{S}(h,S) \left( \log \frac{m^2_{S}(h,S)}{Q^2} - 3/2 \right) \right].
\] (3.15)

If the \( O(N) \) is broken to \( O(N-1) \) we have,

\[
V_{CW}(h,s,N) = \frac{1}{64\pi^2} \left[ m^4_{h_1}(h,s) \left( \log \frac{m^2_{h_1}(h,s)}{Q^2} - \frac{3}{2} \right) + m^4_{h_2}(h,s) \left( \log \frac{m^2_{h_2}(h,s)}{Q^2} - \frac{3}{2} \right) \right. \\
+ 2m^4_{G^+}(h,s) \left( \log \frac{m^2_{G^+}(h,s)}{Q^2} - \frac{3}{2} \right) + m^4_{G^0}(h,s) \left( \log \frac{m^2_{G^0}(h,s)}{Q^2} - \frac{3}{2} \right) \\
+ (N-1)m^4_{S}(h,s) \left( \log \frac{m^2_{S}(h,s)}{Q^2} - \frac{3}{2} \right) \right],
\] (3.16)

For other gauge bosons contributions and fermions contributions we refer to Ref. [59]. The running scale \( Q \) is chosen to be \( Q = 246.22 \text{ GeV} \) in the numerical analysis process. The field dependent masses are given as follows for both \( O(N-1) \) and \( O(N) \) cases (in this case one need do “s”→“S”)

\[
m_{hs}(h,s) = \lambda_{hs}hs,
\]
(3.17)
\[
m_{h_1}^2(h,s) = 3\lambda h^2 - \mu^2 + \frac{\lambda_{hs}s^2}{2},
\]
(3.18)
\[
m_{h_2}^2(h,s) = \frac{\lambda_{hs}h^2}{2} + \frac{\lambda s^2}{2},
\]
(3.19)
\[
m_{G^+}^2(h,s) = \lambda h^2 - \mu^2 + \frac{\lambda_{hs}s^2}{2},
\]
(3.20)
\[
m_{G^0}^2(h,s) = \lambda h^2 - \mu^2 + \frac{\lambda_{hs}s^2}{2},
\]
(3.21)

and when \( O(N) \) is broken to \( O(N-1) \), we need to diagonalization the field dependent mass matrix of \( M = ((m_{h_1}^2, m_{hs}), (m_{hs}, m_{S}^2)) \) to obtain the mass eigenvalue, i.e., \( (m_{h_1}, m_{h_2}) \). The finite temperature corrections to the effective potential at one-loop are given by [56],

\[
V_T(h,S,N,T) = \frac{T^4}{2\pi^2} \left[ J_B \left( \frac{m_{h_1}^2(h,S,N,T)}{T^2} \right) + J_B \left( \frac{m_{G^0}^2(h,S,N,T)}{T^2} \right) + 2J_B \left( \frac{m_{G^+}^2(h,S,N,T)}{T^2} \right) \right. \\
+ NJ_B \left( \frac{m_S^2(h,S,N,T)}{T^2} \right) \right],
\] (3.22)

and

\[
V_T(h,s,N,T) = \frac{T^4}{2\pi^2} \left[ J_B \left( \frac{m_{h_1}^2(h,s,N,T)}{T^2} \right) + J_B \left( \frac{m_{G}^2(h,s,N,T)}{T^2} \right) + 2J_B \left( \frac{m_{G^+}^2(h,s,N,T)}{T^2} \right) \right. \\
+ J_B \left( \frac{m_{h_2}^2(h,s,N,T)}{T^2} \right) + (N-1)J_B \left( \frac{m_S^2(h,s,N,T)}{T^2} \right) \right],
\] (3.23)
for $O(N)$ and $O(N-1)$ scenarios respectively. The functions $J_{B,F}(y)$ are

$$J_{B,F}(y) = \pm \int_0^\infty dx x^2 \ln \left[ 1 + \exp \left( -\sqrt{x^2 + y} \right) \right],$$

(3.24)

with the upper (lower) sign corresponds to bosonic (fermionic) contributions. Here, the above integral $J_{B,F}$ can be expressed as a sum of them second kind modified Bessel functions $K_2(x)$ [60],

$$J_{B,F}(y) = \lim_{N\to\infty} \sum_{l=1}^N \frac{(-1)^l y}{l^2} K_2(\sqrt{y} l).$$

(3.25)

The thermal masses/corrections are given by,

$$m_5^2(h,S,N,T) = m_{h_2}^2 + \frac{1}{16} T^2 (g_1^2 + 3g_2^2 + 4g_3^2) + T^2 \left( \frac{\lambda}{2} + \frac{N\lambda_{hs}}{12} \right),$$

(3.26)

$$m_{G^+}^2(h,S,N,T) = m_{G^+}^2 + \frac{1}{16} T^2 (g_1^2 + 3g_2^2 + 4g_3^2) + T^2 \left( \frac{\lambda}{2} + \frac{N\lambda_{hs}}{12} \right),$$

(3.27)

$$m_{G^0}^2(h,S,N,T) = m_{G^0}^2 + \frac{1}{16} T^2 (g_1^2 + 3g_2^2 + 4g_3^2) + T^2 \left( \frac{\lambda}{2} + \frac{N\lambda_{hs}}{12} \right),$$

(3.28)

$$m_S^2(h,S,N,T) = m_S^2 + T^2 \left( \frac{(N+2)\lambda_s}{4} + \frac{\lambda_{hs}}{3} \right),$$

(3.29)

for the $O(N)$ case, and for the $O(N \to N-1)$ case one needs to replace the “$S$” by “$s$” and replace the thermal mass of the Higgs fields by

$$m_{h_1}^2(h,N,T) = m_{h_1}^2 + \frac{1}{16} T^2 (g_1^2 + 3g_2^2 + 4g_3^2) + T^2 \left( \frac{\lambda}{2} + \frac{N\lambda_{hs}}{12} \right),$$

(3.30)

$$m_{h_2}^2(h,N,T) = m_{h_2}^2 + \frac{1}{16} T^2 (g_1^2 + 3g_2^2 + 4g_3^2) + T^2 \left( \frac{(N+2)\lambda_s}{4} + \frac{\lambda_{hs}}{3} \right),$$

(3.31)

Last but not least, the resummation of ring (or daisy) diagrams are also crucial for the evaluation of $v_C$ and $T_C$ with the finite temperature effective potential [61], which is given by

$$V_{ring}(h,S,N,T) = \frac{T}{12} \left[ (m_h^3 - m_{h_1}^3(h,S,N,T)) + (m_{G^0}^3 - m_{G^0}^3(h,S,N,T)) + 2(m_{G^+}^3 - m_{G^+}^3(h,S,N,T)) \right]$$

$$+ N(m_S^3 - m_S^3(h,S,N,T)),$$

(3.32)

and

$$V_{ring}(h,s,N,T) = \frac{T}{12} \left[ (m_{h_1}^3 - m_{h_1}^3(h,s,N,T)) + 3(m_{G^0}^3 - m_{G^0}^3(h,s,N,T)) + (m_{h_2}^3 - m_{h_2}^3(h,s,N,T)) \right]$$

$$+ (N-1)(m_S^3 - m_S^3(h,s,N,T)),$$

(3.33)

for $O(N)$ and $O(N \to N-1)$ cases. Here, again, we list only the contributions of scalar contributions for $V_T$ and $V_{ring}$, the others particle fields contributions are the same as the SM, see Ref. [59, 61]. It should be note that, the counter terms can keep the VEVs of the potential from shift caused by the $V_{CW}$. 

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we add that parts follows Ref.[59]. Then the critical parameters of EWPT can be calculated when there are two degenerate vacuums with a potential barrier. Due to rich vacuum structures of the potential at finite temperatures, there can be one-step or multi-step phase transitions. A SFOEWPT can be realized in the first or the second step in the two-step scenario. We will investigate two scenarios: one-step and two-step phase transition in O(N) and the O(N \to N - 1) scalar models.

Figure 8: EWPT with dim-6 operators.

Before the detailed study, we firstly recall the one-step strong first order phase transition conditions on Wilson coefficients of the dimensional-six operator [43, 62, 63],

\[ \frac{m_h^2}{3v^4} < c_6 \frac{\Lambda^2}{v^4} < \frac{m_h^2}{v^4}. \]  

(3.34)

In the Fig. 8 we show the physics picture for the energy barrier with different dimensional six operator Wilson coefficients. A suitable \( c_6 = c_6^{cr} \) is needed to obtain a proper vacuum barrier to separate two degenerate vacuum at critical temperature \( T_c \), therefore make the SFOEWPT feasible. It should be note that, with the spontaneous symmetry breaking of \( O(N \to N - 1) \), the two contributions of the \( c_6 \) from \( sh^2 \) and \( s^3 \) terms cancels each other when one following the method of Ref.[45] and therefore the tree-level induced dimensional six operator disappears. Which is the same as in the SM+1 singlet case being studied in Ref. [64]. And in this case the dimensional six operator shows up at loop level which is too small to affect the EWPT dynamics. This property can explain why the SM+1 singlet scalar with \( Z_2 \) does not prefer one-step strong first order EWPT, and here one may needs to pursue the two-step types where the DM can assistant to achieve the SFOEWPT [32, 42]. In this work, we confirm the same property in the \( O(N) \) symmetry and \( O(N \to N - 1) \) cases. The inflation occurs for small quartic scalar couplings, that motivate us mostly focusing on the two-step types.
Figure 9: One- and two-step EWPT types in O(N) for left and right panels, respectively.

3.2.1 O(N) scenario

The corresponding critical temperature and critical field value for one- and two-step EWPT types in O(N) (see the Fig.9) can be evaluated through the following degeneracy conditions,

\[
V(0,0,N,T_C) = V(h^B_C,0,N,T_C),
\]

\[
\frac{dV(h,0,N,T_C)}{dh}|_{h=h^B_C} = 0,
\]

for one step and two step EWPTs. Here the \( s_c \) is the O(N) broken direction, which is analogy to the O(N → N – 1) scenarios. The survey of the one-step EWPT in the O(N) scenario shows negligible effects of the quartic couplings \( \lambda_s \), and the quartic coupling between the SM Higgs and the O(N) scalars \( S_i (\lambda_{hs}) \) should be large in order to make the SFOEWPT occurs, which is not favored by the slow-roll inflation.

Generally, the parameter spaces with a larger \( \lambda_{hs} \) one can only have a SFOEWPT where the inflation is invalid, the perturbativity of quartic coupling and unitarity are violated due to RG running of couplings as explored in Sec. 3.1.1. The numerical survey of the two step EWPT shows that a strong first order EWPT requires \( N \geq 7 \) in the inflation favored parameter regions. Unfortunately, as can be seen from Fig.20, the inflation valid \( N \) should be smaller than 4 after imposing the constraints from EWPO. Which therefore shout down the window to realize the SFOEWPT.

3.2.2 O(N → N – 1) scenario

We demonstrate the one-step and two-step phase transition in Fig.10. The one-step EWPT types in O(N → N – 1), occurs along the \( \overrightarrow{OB} \) line, and the two-step EWPT occurs through the process of \( O \rightarrow A \rightarrow B \). With two degenerate vacuums being separated by a potential barrier structures at the critical temperature, the degeneracy conditions can be expressed as Eqs.3.37 and Eqs.3.38.
Figure 10: One- and two-step phase transition types in $O(N \rightarrow N - 1)$ for left and right panels, respectively.

\[ V(0,0,N,T_C) = V(v_C^B,s_C^B,N,T_C), \]

\[ \frac{dV(h,s,N,T_C)}{dh} \bigg|_{h=h_C^B,s=s_C^B} = 0, \quad \frac{dV(h,s,N,T_C)}{ds} \bigg|_{h=h_C^B,s=s_C^B} = 0. \] (3.37)

\[ V(0,s_A^N,N,T_C) = V(h_C^B,s_C^B,N,T_C), \]

\[ \frac{dV(h,s,N,T_C)}{dh} \bigg|_{h=h_C^B,s=s_C^B} = 0, \quad \frac{dV(h,s,N,T_C)}{ds} \bigg|_{h=h_C^B,s=s_C^B} = 0. \] (3.38)

For the one-step case, one can realize strong first order EWPT with a smaller $\lambda_{hs}$ with increasing of $N$, as shown in Fig.11. Which means that the Goldstones contributions to the EWPT is notable at finite temperature. While this property disappear in the two-step scenario as can be seen in the bottom-right panel. The two-step phase transition can be strongly first order with a relatively lower magnitude of $\lambda_{hs}$ in comparision with the one-step scenario. The analysis also shows that different from one-step scenario, the strong first order EWPT occurs more easy at relatively small $\lambda_{hs}$ for two-step case, that provide the possibility to obtain the SFOEWPT in the inflation favored regions. Moreover, our study demonstrates that the rate of $s_B/s_A$ can be larger or smaller than 1 for different $N$, as shown in Fig.12.

As shown in Fig. 6 previously, for the $O(N \rightarrow N - 1)$ scalar model, the slow-roll inflation are almost excluded for $N \geq 5$ in the $(\lambda_s, \lambda_{sh})$ plane. We show the parameter regions that can valid Higgs inflation and SFOEWPT together with Higgs cubic couplings and Higgs decay widths in $O(N \rightarrow N - 1)$ model in the Fig.13 after taking into account perturbativity, unitarity, and stability conditions from Electroweak scale to Planck scale. The inflation and one(two)-step SFOEWPT are allowed by the Higgs invisible decay bounds from LHC [50] for different $N$, which is marked by cyan. The two-step SFOEWPT shows no obvious relation with $N$ due to $s_A$ can be higher or lower than $s_B$ as shown in Fig.12. In the SFOEWPT allowed parameter spaces with relatively larger quartic couplings, the slow-roll inflation is not allowed, this property is caused by the bound on $m_{m_2}$ and $v_s$ set by perturbative unitarity and stability from weak scale to Planck scale imposed by us when perform the inflation analysis with the RGEs. There are larger parameter spaces of $(\lambda_s, \lambda_{hs})$ allowed by two-step SFOEWPT.
in comparison with the one-step SFOEWPT. In the regions, we find that the ratio of the triple Higgs couplings ($r_{3h_1} = \lambda_{h_1 h_2 h_1}/\lambda_{SM}^{hhh}$ and $r_{h_2 h_1} = \lambda_{h_2 h_1 h_1}/\lambda_{SM}^{hhh}$) increases with the increase(decrease) of $\lambda_{hs}(\lambda_s)$. With the SM Higgs resonance searched by SM Higgs pairs production, one can estimated the cross section with respect to the SM case being $\sigma_{h_1 h_1}/\sigma_{SM}^{hh} \sim \cos^2 \theta \times r_{3h_1}^2 \times \Gamma_{h_1}^{tot}/\Gamma_{SM}^{h} \sim 0.98^2 \times 2^2 \sim 3.8$, and therefore a large enhancement of the cross section can be expected. With the increasing of N, one can expect the ratio $\sigma_{h_1 h_1}/\sigma_{SM}^{hh}$ decrease due to the decrease of the ratio of $r_{3h_1}$. The triple Higgs coupling of $\lambda_{h_2 h_1 h_1}$ varies from $0.3-1.4$ in units of $\lambda_{SM}^{hhh}$ with increasing of $\lambda_{hs}$. For the heavy Higgs search of $h_2$, the cross section of the signal $\sigma_{h_1 h_1}^{h_2} \sim (\sqrt{2}m_t/v)^2 \sin^2 \theta \times \lambda_{h_2 h_1 h_1}^2/(m_{h_2} \Gamma_{h_2}^{tot})$. Due to the

**Figure 11:** Various parameters plane of the one step (top panel) and two step (bottom panel) strong first order EWPT within $O(N \rightarrow N-1)$ model.

**Figure 12:** The two-step EWPT in $O(N \rightarrow N-1)$ model.
**Figure 13:** The $O(N \to N - 1)$ scenario. The $\lambda_{h_1 h_1 h_1}$, $\lambda_{h_2 h_1 h_1}$ (both in units of $\lambda_{h h h}^{SM}$), and $\Gamma_{h_2}^{\text{tot}}$ are shown by red, blue, and brown contours, respectively. Dashed, solid and dotted lines stand for $N = 1, 2, 3$, respectively. Cyan regions present the allowed regions by the invisible Higgs decay bounds from $B_{\text{inv}} < 0.34$ [50], in which the light and deep colors correspond to $N = 2, 3$, respectively. The feasible regions of inflation are shown by green color regions, and the orange regions represent one- and two-step SFOEWPT for the left and right panels, respectively. For those colors, a deeper color corresponds to a larger $N$ for $N = 1, 2, 3$.

$\Gamma_{h_2}^{\text{tot}} \ll m_{h_2}$, the resonance interference explored in Ref. [41] can be safely neglect here. We postpone the detailed collider search of the parameter spaces to a separate publication.

### 3.3 Dark matter/radiations

With the assumption of instantaneous reheating, one can estimate the reheating temperature when the decay of the inflaton starts competing with expansion $H \sim \Gamma_{h_1 h_2}$ for h/s inflation [65].

$$\rho = 3H^2 M_p^2 = 3 \Gamma_{h_1 h_2}^2 M_p^2 \equiv \frac{\pi^2 g_*}{30} T_R^4,$$

(3.39)

where $g_* \approx 100$ is the number of relativistic degrees of freedom in the Universe during the reheating epoch. Freeze out of cold dark matter requires $x_f \equiv m_{DM}/T_{f_s} \approx 20$, here the thermal history can occurs as $T_R > T_C > T_{f_s} > T_{BBN}$ to account for the EWPT and reheating as well as the successful Big Bang Nucleosynthesis (BBN) (with typical temperature of a few MeV). \footnote{Here, we note that the number of Goldstones will affect the reheating temperature during the range of $10^{13} - 10^{14}$ GeV for the Higgs inflation case.} The freeze-out temperature $T_{f_s}$ being smaller than the strong first order electroweak phase transition temperature $T_C$, set the $m_{DM} < 20T_C \sim 2$ TeV with $T_C$ being around $\sim O(10^2)$ GeV.
### 3.3.1 $O(N)$ scenario: dark matter

When the $O(N)$ is kept at zero temperature, the N-singlet scalars can all serve as dark matter candidates. With thermal averaged annihilation cross sections being the same as in Ref. [53] for each $S_i$ and we drop the subscript here. For SM Higgs pair final states one have

\[
\left< \sigma_{v_{rel}} \right>_{hh} = \frac{\lambda_{hs}^2}{64\pi m_s^2} \left[ 1 + \frac{3 m_h^2}{(4 m_s^2 - m_h^2)} + \frac{2 \lambda_{hs} v^2}{(m_h^2 - 2 m_s^2)} \right]^2 \times \left( 1 - \frac{m_h^2}{m_s^2} \right)^{1/2},
\]

the cross section for gauge boson final states are,

\[
\left< \sigma_{v_{rel}} \right>_WW = 2 \left[ 1 + \frac{1}{2} \left( 1 - \frac{2 \lambda_{hs}^2}{m_s^2} \right) \right] \left( 1 - \frac{m_W^2}{m_s^2} \right)^{1/2} \times \frac{\lambda_{hs}^2 m_W^4}{8 \pi m_s^2 \left( (4 m_s^2 - m_h^2)^2 + m_h^2 \Gamma_h^2 \right)},
\]

\[
\left< \sigma_{v_{rel}} \right>_ZZ = 2 \left[ 1 + \frac{1}{2} \left( 1 - \frac{2 \lambda_{hs}^2}{m_s^2} \right) \right] \left( 1 - \frac{m_Z^2}{m_s^2} \right)^{1/2} \times \frac{\lambda_{hs}^2 m_Z^4}{16 \pi m_s^2 \left( (4 m_s^2 - m_h^2)^2 + m_h^2 \Gamma_h^2 \right)}.
\]

and the fermion pair final states cross section is given by,

\[
\left< \sigma_{v_{rel}} \right>_{f f} = \frac{m_W^2}{\pi g^2} \left( \frac{\lambda_{hs}^2}{(4m_s^2 - m_h^2)^2 + m_h^2 \Gamma_h^2} \right) \left( 1 - \frac{m_f^2}{m_s^2} \right)^{3/2}.
\]

The formula of spin independent cross section is given by [30]

\[
\sigma_{SI}^S = \lambda_{hs}^2 \frac{f_N^2}{4\pi} \left( \frac{m_N m_s}{m_N + m_S} \right) \left( \frac{m_N^2}{m_h^2 m_S^2} \right),
\]

where $m_N = 0.946$ GeV is the neutron mass and $m_H = 126$ GeV is the SM-Higgs mass. The strengths of the hadronic matrix elements, $f_N = 0.35$.

The dark matter direct detection constrains the dark matter masses and the quartic Higgs-DM couplings after taking into account the rescale effects supposing the evaluated dark matter relic density will not oversaturated the DM relic abundance,

\[
\sigma_{SI} = \sigma_{SI}^S \times \sum_{i=1}^{N} \frac{\Omega^{S_i} h^2}{\Omega_{DM} h^2}.
\]

With Lee-Weinberg method [66], we can expect $\Omega^{S_i} h^2 \sim 1/\sigma_{v_{rel}} \sim m_S^2 / \lambda_{hs}^2$ for large dark matter masses. Previous studies show that the mass region of Higgs-portal real 1-singlet scalar DM case is excluded up to $\sim$TeV scale by Xenon1T to obtain a correct relic density [30, 38].

It should be note that the future Linear collider constraints would set the dark matter mass in TeV mass regions ($\sim O(1 - 10)$ TeV) in the inflation feasible region as shown in Fig. ??.
Eq. 3.40 or Eq. 3.46 would oversaturate the relic abundance for the inflation feasible $\lambda_{hs}$ though the highly suppress of $\sigma^{SI}$ by large $m_S$ make the mass region safe from Xenon 1T.

For $m_{DM} > 10$ TeV, one obtains $T_{fs} \sim m_{DM}/\chi_f > 500$ GeV, therefore the dark matter freeze out happens earlier than the EWPT, and thus only the seagull diagram process can happen with the Higgs finite states have effectively zero masses. Then, the annihilation cross sections of Eq. 3.40 reduces to

$$\langle \sigma v_{rel} \rangle_{hh} = \frac{\lambda_{hs}^2}{64\pi m_s^2}. \quad (3.46)$$

Here we point out that $m_s \sim \mu_s$ due to the Electroweak symmetry is still kept at temperature higher than $T_c$ within the framework of EWBG. Then, if the relic abundance is partially saturated by $S_i$, one need a larger H-S quartic coupling $\lambda_{hs}$. Though this case could be good for the SFOEWPT, the large $m_S$ in fact would decouple from the EWPT as been studied in Ref. [34]. Furthermore, the largeness of the $\lambda_{hs}$ may results in the perturbativity unitarity problem of quartic couplings of $\lambda_{h,h}$ as shown in Fig. 5 after taking into account the RG running effects and therefore shut down the possibility to realize inflation.

### 3.3.2 $O(N \to N - 1)$ scenario: Goldstone and dark radiation

Suppose gravity violates global symmetries, then the Goldstone boson may acquire a mass through nonpertubative gravitational effects [67, 68]. The non-perturbative gravity effects can break the $O(N)$ symmetry at $M_P$ scale through the lowest high dimension operators, i.e., dim-5 operators, and induce mass terms to Goldstone bosons and make $N - 1$ majoron like particles,

$$\frac{C_1 (H^\dagger H)^2 s_i}{M_P} + \frac{C_2 (H^\dagger H) s_i^3}{M_P} + \frac{C_3 s_i^5}{M_P}, \quad (3.47)$$

For the wilson coefficients $C_i \sim O(1)$ and the VEV of scalar singlet $v_s \sim O(10^3)$ GeV, one can expect the masses of majoron like particle,

$$m_{s_{1...N-1}} = \frac{16C_1 v^4 + 12C_2 v^2 v_s^2 + 5C_3 v_s^4}{2M_P v_s} \sim O(1)\text{eV}. \quad (3.48)$$
In this mass region, we can expect the majoron here decay to diphoton through the non-minimal gravity couple term to break the $O(N-1)$ symmetry as in the [69]. Follow which we find the Goldstone bosons here is long-lived, with $\tau \sim \Gamma^{-1} \sim 10^{46}$s, they can survive until the recombination era and may contribute to the Universe radiation density at the time of recombination or BBN. Now, we consider the possibility of the Goldstone contributing to the dark radiation following Ref. [70].

The effective neutrino number can be expressed in terms of the Goldstone decoupling temperature as[71, 72],

$$N_{\text{eff}} = 3 \left(1 + \frac{\Delta N_{N-1}}{3} \left(\frac{g_*(T^d_v)}{g_*(T^d_{N-1})}\right)^{4/3}\right), \quad (3.49)$$

with $\Delta N_{N-1} = 4(N-1)/7$ due to there is $(N-1)$ Goldstone bosons decoupled at $T > T^d_{N-1}$ and present before the recombination era, the effective number of relativistic degrees of freedoms $g_*(T^d_v) = 43/4$ and $g_*(T^d_{N-1}) = 57/4$ supposing that the Goldstone bosons decouple just before muon annihilation. One can constrain the number of Goldstone as in Fig. 15 using the recent $1\sigma$ experimental data $N_{\text{eff}} = 3.36 \pm 0.34$ [73].

![Figure 15: The Goldstones faked effective neutrino number.](image)

To study how the Goldstone decouples from the thermal bath, we follow Ref. [70]. For the heavy Higgs contributions are typically small, one need to focus on the light Higgs case alternatively, c.f., $m_{h_2} < 2m_{h_1}$. When the decay width $\Gamma_{h_2} \ll m_{h_2}$ and in the small mass region the cross-section of thermal annihilation to $\mu^+\mu^-$, as shown in Fig. 16, is given by,

$$\langle \sigma v \rangle_{SN-1\rightarrow N-1\rightarrow \mu^+\mu^-} = \frac{\lambda^2_{hN}}{128\pi} \frac{m_h^2 T^4}{m_{h_2}^4} \int_{2m_{\mu}/T}^{\infty} w^8 K_1(w) \, dw, \quad (3.50)$$

which leads to the constraints on $\nu_4$ and $m_{h_2}$, as seen in Fig. 17. The invisible decay of $h_1$ requires a smaller $\theta$ or a lower magnitude of $N$. In the resonance enhanced region, $2m_\mu < m_{h_2} < 4$ GeV, using
Figure 16: Goldstone annihilation process.

Figure 17: Decouple conditions satisfied parameter regions in parameter spaces of $m_{h_2}$ and $v_s$, regions in the dashed square frame are allowed by the $B_{inv} < 0.34$ [50]. The decay width of $h_2$ labeled on the blue and orange contours are shown in units of MeV.

the narrow resonance conditions of $\Gamma_{h_2} \ll m_{h_2}$, one obtain,

$$
\langle \sigma v \rangle_{s_{N-1}s_{N-1} \to \mu^+\mu^-} = \frac{\lambda_{h_2}^2}{256} \frac{m_{\mu}^2 m_{h_2}^6}{T^5 m_h^4 \Gamma_{h_2}} \left( 1 - \frac{4m_{\mu}^2}{m_{h_2}^2} \right)^{3/2} K_1(m_{h_2}/T),
$$

which set lower bounds on the mixing angle of $\theta$, see Fig. 18. For a smaller $v_s$, the invisible decay of the $h_1$ set the upper limits on $\theta$ depends on the number of $N$ as shown in the left panel of Fig. 18. For the case of $m_{\mu} < m_{h_2} < 2m_{\mu}$, we have,

$$
\langle \sigma v \rangle_{s_{N-1}s_{N-1} \to \mu^+\mu^-} = \frac{\lambda_{h_2}^2}{128\pi} \frac{m_{\mu}^2}{m_h^4} \int_{2m_{\mu}/T}^{\infty} w^4 K_1(w) dw,
$$
Figure 18: Decouple conditions valid regions for $2m_{\mu} < m_{h_2} < 4$ GeV, regions in the dashed square frame are allowed by the $B_{inv} < 0.34$ [50]. The decay width of $h_2$ ($\Gamma_{h_2}^{\text{tot}}$) labeled on the contours is shown in units of KeV.

Figure 19: Decouple conditions valid parameter regions for $m_{h_2} = 3m_{\mu}/2$, regions in the dashed square frame are allowed by the $B_{inv} < 0.34$ [50]. The decay width of $h_2$ ($\Gamma_{h_2}^{\text{tot}}$) labeled on the contours is shown in units of KeV.

Requiring the Goldstone bosons annihilation process contribute to the equivalent neutrino numbers, we obtain the bounds on mixing angle and the $v_s$, see Fig. 19. A larger $N$ allows a smaller $\theta$ to meet the decoupling conditions due to more Goldstone contributions.
4 The future lepton colliders constraints

The study of Ref. [46] shows that the CEPC, ILC, and FCC-ee can probe the new physics parameter spaces (through the $e^+e^- \rightarrow hZ$ process) much better than that from LHC. The $c_H^{N/m_{S_i}^2}$ is bounded to be smaller than 0.0038, 0.0034, and 0.0028 by CEPC with luminosity of $5 \text{ ab}^{-1}$, ILC with all center of mass energies, and FCC-ee with luminosity of $10 \text{ ab}^{-1}$. With which, we can obtain the bounds on the hidden scalar masses of $m_{S_i} \sim O(1 - 10) \text{ TeV}$, as shown in Fig. 20. As been explored in the Sec. 3.3.1, in this mass regions the N scalars cannot explain the correct relic density.

And in $N = 1$ of the $O(N \rightarrow N - 1)$ case, which is the Higgs-portal 1-singlet scalar case, the corresponding mixing angle $|\sin \theta|$ was constrained to be 0.062, 0.058 and 0.052 at 95% C.L. at CEPC with luminosity of $5 \text{ ab}^{-1}$, ILC with all center of mass energies, and FCC-ee with luminosity of $10 \text{ ab}^{-1}$ [46]. We map that values to constraints our model parameter spaces. Here, we point out that considering the high sensitivity of CEPC, ILC, and FCC-ee, a much smaller mixing angle $\theta$ can be probed, see Fig. 21. At that moment, one obtains a smaller $\lambda_{hs}$, and therefore the stability problem can easily preclude the possibility of slow-roll Higgs inflation. For the case of $N \geq 2$ of the $O(N \rightarrow N - 1)$ model, the ILC from the Higgs Strahlung process set $B_{\text{inv}} < 1\%$ [74], the FCC-ee set $B_{\text{inv}} < 0.5\%$ [75, 76], and CEPC set 0.14% [77]. Fig. 22 imply a larger mixing angle $\theta$ is allowed for a larger $v_s$, which corresponds to the heavy Higgs decouple cases. Generally, to make the Higgs inflation feasible, a relatively larger mixing angle $\theta$ is required to enlarge the value of $\lambda_{hs}$ and therefore to save the Higgs quartic coupling from the vacuum unstability problem. Firstly, the increasing of $\theta$ can leads to the perturbativity problem of $\lambda_{hs}$, and thus we need a lower magnitude of $m_{S_i}^2$. secondly, a larger $v_s$ leads to a smaller $\lambda_{hs}$, therefore to avoid the stability problem we need a larger N. In the narrowed down region of mixing angle $\theta$ coming from the future $e^+e^-$ colliders constraints, the

Figure 20: Colliders, EWPO parameter, and inflation constraints on $m_{S_i}$ in the $(\lambda_{si}, \lambda_{hs})$ plane for different $N$. The magenta line is the allowed magnitude of $m_{S_i} \text{[TeV]}$ by colliders in the feasible inflation regions, the corresponded $N$ is same with feasible inflation regions, and collides constraints is 0.0028, 0.0034 and 0.0038 from left to right panel, respectively. Blue regions is for the feasible inflation within $O(N)$ model, orange regions represent the allowed regions by EWPO parameters confine. For both two colors, a deeper color corresponds to a larger $N$ for $N = 1, 2, 3, 4, 5$. 

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Figure 21: The projected sensitivity of the mixing angle for $O(N \rightarrow N - 1)$ model from lepton colliders.

Figure 22: Various parameters plane of the invisible decay bounds on $N$ in $O(N \rightarrow N - 1)$ model coming from ILC, FCC-ee, and CEPC respectively.

mixing between the heavy Higgs and the SM Higgs is negligible. In this super-weak couple scenario one have a far smaller of $\lambda_{hs}$ to obtain a SFOEWPT, thus one need a much larger $N$ to amplified the effects of the $N - 1$ Goldstone to the potential in order to obtain a SFOEWPT. On the other hand, the Goldstone fake effective neutrino situation would be changed a lot due to the decouple conditions allowed parameter spaces can be covered by the $B_{inv}$ from ILC, FCC-ee and CEPC as aforementioned.

5 Conclusions and discussions

In this work, we study inflation and cosmological electroweak phase transitions utilizing the SM augmented N scalars respecting $O(N)$ symmetry or $O(N \rightarrow N - 1)$ symmetry. With the assistant of N scalars that couple to the SM Higgs, the stability problem can easily been remedied up to the
inflation scale. Therefore, the interaction of the Higgs-portal quartic scalar couplings shouldn’t be too small depending on the number of scalars. The quartic scalar coupling perturbativity and the unitarity constraints set the upper bounds on the quartic scalar couplings. The Higgs inflation valid parameter regions reduces with the increases of the number of N mostly due to the perturbativity and the unitarity bounds.

The Electroweak precision observables set severe bounds on the parameters spaces of both $O(N)$ and $O(N \rightarrow N - 1)$ scenarios. For the $O(N)$ case, the largest number of the scalars that can valid inflation is bounded to $N \leq 3$, which make the strong first order EWPT unreachable, no matter one-step or two-step paths. The future $e^+e^-$ colliders, such as ILC, FCC-ee, and CEPC can set the masses of N-scalars being $\sim O(1 - 10)$ TeV scale. When the $O(N)$ symmetry is an exact symmetry, in principle, all of the N scalars can serve as WIMP DM candidates. While, no way to expect the N-scalar WIMP DM can saturate the correct relic density here. And, in that dark matter mass regions, the freeze out happens earlier that the EWPT process, thus the only relevant DM annihilation process is $S_iS_i \rightarrow hh$ with $m_h(T \geq T_{fs}) \sim 0$.

When the $O(N)$ symmetry is spontaneously broken, one obtains $N - 1$ Goldstones, and the rest one scalar mixed with the SM Higgs. Therefore, the invisible Higgs decay is very powerful to set the bound on the scalar numbers $N$ and the mixing angle $\theta$. With one moderate $\theta = 0.2$ allowed by EWPO and Higgs precisions as well as invisible Higgs decay bounds set by LHC run-I, we explore the possibility to realize Higgs inflation and EWPT through one-step and two-step types. In the parameter regions where one can obtain successful slow-roll Higgs inflation and SFOEWPT, the triple Higgs couplings $\lambda_{h_2h_1h_1}$ and $\lambda_{h_1h_1h_1}$ increase with the increasing of $\lambda_{hs}$, and the decay width of the two Higgses are not larger enough to introduce large interference effects in the resonant mass regions of Higgs pair productions. The future high precision $e^+e^-$ colliders, such as ILC, FCC-ee, and CEPC, set the mixing angle and the number of Goldstones $N - 1$ far more stringent than that from the LHC, which could probe the inflation and SFOEWPT valid parameter regions. The $N - 1$ Goldstones can fake the effective number of neutrinos and contribute to dark radiations supposing they obtained $\sim O(1)$ eV masses induced by non-renormalizable gravity effects. The roughly estimations independent of the mixing angle indicates the N is bounded to be smaller than five at 3$\sigma$. The dark radiations calculations indicates that Goldstones decouples from the thermal bath at different mass range of a small $m_{h_2}$. The parameter spaces of this scenario can be covered by the projected ILC, FCC-ee, and CEPC in the future.

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Appendix: One-loop renormalization group equations

RGE equations are listed bellow,

\[ \beta_{g'} = \frac{g'^3}{(4\pi)^2} \left( \frac{39-x_h}{12} \right) + \frac{g'^3}{(4\pi)^4} \left( \frac{3}{2}g'^2 + \frac{35}{6}g^2 + 12g_s^2 - \frac{3}{2}x_hy_t^2 \right), \]

\[ \beta_{\lambda_s} = \frac{\lambda_s}{(4\pi)^2} \left( 12x_h\lambda_s + 4x_hx_s\lambda_{sh} + 6N\lambda_s^2 \lambda_s + 6y_t^2 - \frac{9}{2}g_s^2 - \frac{3}{2}g^2 \right), \]

\[ \beta_{\lambda_h} = \frac{\lambda_h}{(4\pi)^2} \left( 6(1 + 3x_h^2)\lambda_h^2 - 6y_t^2 + \frac{3}{8}(2g^4 + (g^2 + g'^2)^2) + \lambda_hy_h + \frac{N\lambda_s^2}{2}\lambda_{sh} \right), \]

\[ \beta_{y_t} = \frac{y_t}{(4\pi)^2} \left[ -\frac{9}{4}g^2 - \frac{17}{12}g'^2 - 8g_s^2 + \frac{23 + 4g^2}{6}y_t^2 \right]. \]

with \( y_h = (-9g^2 - 3g'^2 + 12y_t^2), g, g' \) and \( y_t \) are the standard model \( SU(2), U(1) \) and top-quark Yukawa couplings, and

\[ x_h = \frac{1 + \xi_hh^2/M_p^2}{1 + \xi_hh^2/M_p^2 + 6\xi_s^2h^2/M_p^2}, \]

\[ x_s = \frac{1 + \xi_s^2s^2/M_p^2}{1 + \xi_s^2s^2/M_p^2 + 6\xi_s^2s^2/M_p^2}. \]

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