Techniques for the construction of an elliptical-cylindrical model using circular rotating tools in non CNC machines

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Abstract. This paper describes the construction of an elliptical-cylindrical model without spherical aberration using vertical rotating tools. The engine of the circular tool is placed on one arm so that the tool fits on the surface and this in turn is moved by an X-Y table. The test method and computer algorithms that predict the desired wear are described.

1. Introduction

The standard lens manufacturing processes generate a spherical surface with great accuracy, but attempts to broaden the designer’s freedom by permitting the use of aspheric surfaces lead to extremely difficult manufacturing problems.

However, molded aspheric surfaces are very practical and can be used wherever the production rate is sufficiently high to justify the cost of the mold; this applies particularly to plastic lenses made by injection molding. Fairly accurate parabolic surfaces can be generated on glass by special machines and it’s for this reason that in this work we propose a new method of fabrication of this kind of surfaces and in addition with cylindrical symmetry.

2. Design

Before a lens can be constructed it must be designed, that is to say, the radius of curvature of the surfaces, the thicknesses and the types of glass to be used must all be determined and specified.

2.1 Fermat’s Principle

We will focus in the case when a plane wavefront is arrives from the air into the first surface of the lens, is refracted and concentrated in a point that we named F.

In the example shown in Fig. 1, by equating optical paths between any finite ray and the axis, and if \( n \) is the refractive index, using Fermat’s Principle which states that a optical path taken by a ray has to be equal in any path that it takes\(^{[1]}\), we obtain

\[
Bn = x + n[(B - x)^2 + y^2]^{1/2}
\]

(2.1)

where the first term to the left side is the optical path of the ray at the vertex, and the second one is the optical path of the finite ray.
From Eq. 2.1 we have to obtain a relationship between $x$ and $y$ that allows us to find the form of the surface.

Returning to Eq. 2.1, this becomes

$$y^2 = 2Bx \left( \frac{n-1}{n} \right) - x^2 \left( \frac{n^2-1}{n^2} \right) \quad (2.2)$$

and finally

$$\frac{y^2}{B^2(\frac{n-1}{n+1})} + \frac{\left( x - \frac{yn}{(n+1)} \right)^2}{\left( \frac{B_n}{n+1} \right)^2} = 1 \quad (2.3)$$

This expression is an ellipse with semimajor axis equal to $B_n \frac{(n-1)}{(n+1)}$ and semimnior axis $B \left[ \frac{(n-1)}{(n+1)} \right]^{1/2}$, as we have seen with the use of Fermat’s Principle, it is possible to design a lens where the focus of the marginal ray coincides with the paraxial image point without any spherical aberration, Descartes$^{[2]}$.

Now it is necessary to find the form of the second surface that can allow us to preserve this spherical aberration.

### 2.2 Zero Spherical Aberration

We found an elliptical surface that focuses all rays in a point without spherical aberration and this is the first surface of our lens, now we’re going to demonstrate that we can use a spherical surface like second surface and it will be useful to preserve this aberration.

There are four cases in which we can eliminate spherical aberration using a spherical surface: a) the object and the image are both at the vertex of the surface, b) the marginal ray suffers no refraction at the surface Fig.2; this could occur because the object is at the center of curvature of the surface, as also in case c), but it could occur trivially if the refractive index is the same on both sides of the surface. Case d) occurs if $I’=U$ or if $I=U’$ where $I$ is the refracted angle and $U$ is the incidence angle$^{[1]}$. 

![Figure 1. Lens corrected from spherical aberration.](image-url)
It was necessary to choose which case is the one that we really need: a) this option is not the most convenient because in our design we need a focal length of 600mm and that is a lens with a thickness of 600mm which is not practical; c) it is not convenient either because the refractive index is not the same on both sides of the surface; d) this is used in many types of lens, particularly high-power microscope objectives so we have only one case that can be used in our lens b) when the object is at the center of curvature of the surface.

Fig.3 shows how the lens focuses all parallel rays in a point and the cylindrical symmetry that it is going to focus all energy in a straight line instead of a point.

The parameters like radius of curvature, conic constants and focal length of the lens are shown in Tab.1.

| Table 1. Design parameters. |
|-----------------------------|
| **Radius** | **Curvature** | **Conic constant** | **Focal length** |
| ELLIPSE | 200 mm | 0.5 | -1 | 600 mm |
| SPHERE | 590 mm | 0.001 | 0 |

3. Grinding step
In the grinding step of the surface with cylindrical symmetry the main idea is to translate the surface or the grinder both in direction with the optical axis of the cylinder.

We use an Edge Grinder and Saw Model 10J that use a combination saw and plane grinder that does all types of work in an optical workshop, Fig.4. It can do nearly any straight sided glass shape imaginable. If it is equipped with a tilting table, it can grind any degree of bevel, make prisms, as well
as any rectangular shape. This machine was specially designed for glass cutting and we use it for making the generated of our surface.

This machine let us to translate the surface with respect of wheel in direction of the optical axis of cylinder, for this, it was necessary to make the characterization of the machine

3.3 Horizontal displacement
The horizontal axis has a manual handwheel graduated in .001 of an inch increment. The experiment for verifying if the grinder saw was working just like the specifications is shown in Tab2 and Fig.5 shows the experimental graph of this experiment.

![Grinder saw](image)

**Figure 4.** Grinder saw.

| Unities (ua) | Displacement (mm) |
|-------------|-------------------|
| 100         | 1.87              |
| 200         | 4.34              |
| 300         | 7.21              |
| 400         | 9.47              |
| 500         | 12.07             |

**Table 2.** Horizontal Displacement.

![Experimental results of the horizontal displacement.](image)

**Figure 5.** Experimental results of the horizontal displacement.

3.4 Vertical displacement
For the characterization of z axis we did the same process but the change was the reference, the displacements were measured with a digital vernier and this process is shown in Fig.6, some of these results that were obtained for this axis are shown in Tab.3. The Fig.7 shows the experimental results for this axis.
Figure 6. Measurement of the vertical displacement.

Table 3. Vertical displacement

| Unities (ua) | Displacement (mm) |
|--------------|-------------------|
| 100          | 2.4               |
| 200          | 4.92              |
| 300          | 7.45              |
| 400          | 9.99              |
| 500          | 12.53             |

Figure 7. Experimental results of the vertical displacement.

3.5 Making the surface with cylindrical symmetry.

We start with a typically window glass with refractive index of 1.5 and with dimensions 200x100x19 milimeters, this glass is shown in Fig. 8.
The calculus for the coordinates and deepness were made considering the wheel thickness of 2.32 millimetres, Fig. 9 shows the surface obtained in this step.

3.6 Measurement of surface profile
The measurement of surface profile was made with a digital vernier that was positioned in each step of the generated surface; the results obtained in this step are shown in Fig.10.
Fig. 10 shows the graphic of the deepness with respect to the position for the generated surface, there appear two curves the first one with steps correspond to the experimental measurement of the generated surface, and the second one belongs to calculated deepness for the expected surface, the errors shown in Fig. 10 were within of the tolerance that was one millimeter for this step.

Fig. 11 shows the obtained error with an $rms$ equal to 0.760 millimeters and a peak-valley of 1.0 millimeter.

With these results we pass now to the lapping step where the objective was to eliminate the steps and the error obtained in the grinding step.

4. Lapping step
In this step we used a x-y table that has one arm with a freedom grad so when the engine of the circular tool is placed on it, the tool fits on the surface, Fig. 12.

The x-y table is controlled by a PLC’s that sends the instruction to the engine that moves the x and y axis. Fig. 13 shows the lapping process and Fig. 14 shows the lapping surface.

Figure 11. Error of generated surface.

Figure 12. X-Y Table.

Figure 13. Surface lapping.
The profile surface in this step was obtained measuring the deepness with a digital vernier which was placed in different transversal positions of the lapping surface; these experimental results are shown in Fig. 15.

![Figure 14. Surface obtained in the lapping process.](image)

![Figure 15. Lapping surface.](image)

![Figure 16. Error of lapping surface.](image)

Note in Fig. 15 that the steps that were obtained in the generated did not appear in this lapping step making a smooth surface; Fig. 16 shows the graphic of the error obtained in this step with an error of 0.174 millimeters and a peak-valley of 0.500 millimeters, this error was greatly reduced but it did not disappear completely, to eliminate this error it was necessary to make another experiment to find the dependence of the wear as a function of the time.
4.1 Dynamic shapes
The basis of this analysis is to place the vertical tool over the surface and to vary the times of translation; we made a group of fifteen different experiments, in which we varied the time translation with a rate turn of 60 rpm; these experiments are shown in Fig.17 and the measurement of them are shown in Fig.18.

![Figure 17. Dynamic shapes.]

For this experiments the width and deepness were measured, Figs.19-20 shown these experimental results.

![Figure 18. Measuring of dynamic shapes.]

![Figure 19. Widths of dynamic shapes.]

These experimental results were analyzed in OriginPro 8 and the polynomial that fitted better to these results was

\[ P = .00178t - 1.31939e^{-06}t^2 \]  \hspace{1cm} (4.4)
Fig.20 shows the graphic for this expression and how it fits to the experimental measuring. The Tab.4 shows the values of the polynomial fit.

| Model | Polynomial | Value    | Error     |
|-------|------------|----------|-----------|
|       | Intercept  | 0        | --        |
| B1    |            | 0.00178  | 3.326E-5  |
| B2    |            | -1.31939E-6 | 2.634E-7 |

**Table 4. Values of the polynomial fit.**

With these results we made two computational algorithms in Fortran for the wear produced by the vertical tool of glass as a function of time, these algorithms work in Linux and Windows. The Fig.21 shows the error obtained in the last experiment of lapping in which we obtained an error of 0.0132 millimeters and a peak-valley of 0.0509 millimeters.

**Figure 20. Polynomial fit.**

Finally Fig.22 shows the comparison between the final surface obtained in the second lapping step, the expected surface and the initial surface.

**Figure 21. Final error.**
5. Conclusions
We demonstrate that it is possible to make conic-cylindrical lenses with vertical rotating tools, this method reduces the costs of construction of conic-cylindrical lenses because it is not necessary to use CNC machines.
We made an algorithm that predicts the wear as a function of time for the vertical rotating tool. There were obtained the parameters of the conical cylindrical lens. The surface was constructed in the grinder step with a value in the error of $r_{ms}$ = 0.760 millimeters and a peak-valley value of 0.635 millimeters. It was obtained in the first step of lapping an error of 0.174 millimeters and a peak-valley of 0.539 millimeters. In the second step of lapping the error was corrected to 0.0132 millimeters and a peak-valley of 0.0509 millimeters.

References
[1] Kingslake R 1978 *Lens Design Fundamentals* (New York: Academic Press)
[2] Hecht E 2002 *Optics* (San Francisco: Addison Wesley)
[3] Marinescu I and Uhlmann E et al 2007 *Handbook of Lapping and Polishing* (New York: CRC Press)