Measurement-device-independent quantum key distribution with source state errors and statistical fluctuation

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We show how to calculate the secure final key rate in the four-intensity decoy-state MDI-QKD protocol with both source errors and statistical fluctuations with a certain failure probability. Our results rely only on the range of only a few parameters in the source state. All imperfections in this protocol have been taken into consideration without any unverifiable error patterns.

I. INTRODUCTION

Quantum key distribution (QKD) can provide unconditional security based on the laws of quantum physics [1, 2]. But most existing experiment setups such as imperfect single-photon sources and finite-efficiency detectors are not up to the standard of the theoretical models assumed in the security proof and cause security problems [3, 4]. Fortunately, the decoy-state method [6–13] can help us assure the security with imperfect single-photon sources [3, 4]. Theories including device-independent QKD (DI-QKD) [16] and measurement-device-independent QKD (MDI-QKD) [17, 18] have been proposed to overcome the finite-efficiency detector problem.

MDI-QKD is based on the idea of entanglement swapping [17, 18]. There, both Alice and Bob send out quantum signals to the untrusted third party (UTP), but neither of them performs any measurement. The UTP would perform a Bell state measurement for each received pulse pair and announce whether it was a successful event as well as his measurement outcome in the public channel. Those bits corresponding to successful events will be post selected and further processed for the final key. By using the decoy-state method, Alice and Bob can use imperfect single-photon sources [17, 19] securely in the MDI-QKD. The decoy-state MDI-QKD has been studied both experimentally [20, 24] and theoretically [19, 27, 39].

The existing decoy-state MDI-QKD theory assumes perfect control of the source states. This is an impossible task for any real setup in practice. Refs. [13, 15] have shown how to calculate the secure final key rate with intensity fluctuation, or more generally, the source errors in the Bennett-Brassard 1984 (BB84) decoy-state QKD protocol. In our last work [39], we get the formulas of secure final key rate with source errors in asymptotic case in decoy-state MDI-QKD protocol without any presumed conditions. (In most cases, we have no idea about the details of source errors, so we cannot make any assumptions to the source error model.) But in real experiments, the total pulse pairs sent out by Alice and Bob are finite and we have to take the statistical fluctuation into consideration. The ideas proposed in Refs. [36–38] such as global optimization and combined constraints greatly enhance the secure final key rate in nonasymptotic case in decoy-state MDI-QKD protocol. In this paper, we will show how to extend the formulas in Ref. [39] to nonasymptotic case with those brilliant ideas. In our method, we have assumed the worst case that Eve knows exactly the error of each pulse. Our result immediately applies to all existing experimental results. Before going further, we emphasize that our results here are unconditionally correct because we have not assumed any unproven conditions. Although there is another approach reported for the issue of intensity error by using the model of attenuation to pulses from an untrusted source, however, there exists counter examples to the elementary equation in that approach, as was shown in the appendix of Ref. [13].

This paper is arranged as follows. In Sec. II we present the protocol of our method. In Sec. III we show the details of how we extend the formulas in our last work and get the secure final key rate in asymptotic case. In Sec. IV we show the relationship between the asymptotic values and nonasymptotic values with a fixed failure probability $\xi$. And in Sec. V we present the numerical simulations. The article ends with some concluding remarks.

II. PROTOCOL

In this paper, we consider the four-intensity decoy-state MDI-QKD protocol. In this protocol, Alice (Bob) has three sources $v_A, x_A$, and $y_A$ ($v_B, x_B$, and $y_B$) in $X$ basis and one source $z_A$ ($z_B$) in $Z$ basis, and they send pulse pairs (one pulse from Alice, one pulse from Bob)
to the UTP one by one. And if the UTP announces that it’s a successful event, then we say that the ith pulse pair has caused a count. Each pulse sent out by Alice (Bob) is randomly chosen from one of the four sources \(l_A (r_B)\) with constant probability \(p_{l_A} (p_{r_B})\) for \(l, r = v, x, y, z\). Sources \(v_A\) and \(v_B\) are the unstable vacuum sources in \(X\) basis. Generally, these pulses from the unstable vacuum sources are not the exact zero photon-number state. \(x_A, y_A (x_B, y_B)\) are the decoy sources which are used to estimate the lower bound of yield and the upper bound of phase-flip error rate of single-photon pulse pairs. \(z_A \) (\(z_B\)) is the signal source which is used to extract the final key.

We shall use notation \(l_A r_B\) to indicate the two-pulse source when Alice use source \(l_A\) and Bob use source \(r_B\) to generate a pulse pair. For simplicity, we omit the subscripts of any \(l\) and \(r\) for a two-pulse source, e.g., source \(xy\) is the source in which Alice uses source \(x_A\) and Bob uses source \(y_B\). We also denote the number of counts and error counts caused by the two-pulse source \(lr\) as \(N_{lr}\) and \(M_{lr}\) respectively. And we use \(\langle N_{lr} \rangle\) and \(\langle M_{lr} \rangle\) as the asymptotic values of corresponding observed values \(N_{lr}\) and \(M_{lr}\). \(N_{lr}\) and \(M_{lr}\) are observables and will be regarded as known values.

Suppose Alice and Bob send \(N_l\) pulse pairs to the UTP in the whole protocol. In photon-number space, the state of the ith pulse pair from source \(l_A (r_B)\) is

\[
ρ_{l_A}^{i} = \sum_k a^{l,i}_k |k\rangle \langle k|, \quad ρ_{r_B}^{i} = \sum_k b^{r,i}_k |k\rangle \langle k|,
\]

for \(l, r = v, x, y, z\).

We use superscripts \(U, L\) for the upper bound and lower bound of a certain parameter. In particular, given any \(k \geq 0\) in Eq. (9), we denote \(a^{l,U}_k, a^{l,L}_k, b^{r,U}_k, b^{r,L}_k\) for the maximum value and minimum value of \(a^{l,i}_k (b^{r,i}_k)\) for any \(i \in N_l\) and \(l, r = v, x, y, z\). We assume these bound values are known in the protocol.

### III. ANALYSIS FOR FINAL KEY RATE IN ASYMPTOTIC CASE

In this section, we discuss how to get the secure final key rate in asymptotic case. We denote the pulse pair in which Alice send out a \(|j\rangle\)-photon pulse and Bob send out a \(|k\rangle\)-photon pulse as \(|jk\rangle\)-photon pulse pair and we denote the number of counts caused by \(|jk\rangle\)-pulse pairs of source \(lr\) as \(n^{jk}_{lr}\). Therefore we can formulate the total pulse pair counts caused by source \(lr\) by

\[
\langle N_{lr} \rangle = \sum_{j,k \geq 0} n^{jk}_{lr}.
\]

We also need to introduce the following notation

\[
\langle \tilde{N}_{lr} \rangle = \sum_{j \geq 1} n^{j,k}_{lr}, \quad \langle n^{j}_{lr} \rangle = \sum_{k \geq 1} n^{j,k}_{lr} + n^{j,0}_{lr} + n^{j,00}_{lr}.
\]

It is easy to know \(\langle N_{lr} \rangle = \langle \tilde{N}_{lr} \rangle + \langle n_{lr} \rangle\).

As presented in Ref. [39], if

\[
a^{y,L}_{x,U} \geq \frac{a^{y,L}_{x,U}}{\sigma^2_{A} - \sigma^2_{B}}, \quad b^{y,L}_{x,U} \geq \frac{b^{y,L}_{x,U}}{\sigma^2_{A} - \sigma^2_{B}} \geq \frac{b^{y,L}_{1,0}}{\sigma^2_{A} - \sigma^2_{B}},
\]

and

\[
a^{l,i}_{k,i} \geq \frac{a^{l,i}_{k,i}}{\sigma^2_{A} - \sigma^2_{B}}, \quad b^{r,i}_{k,i} \geq \frac{b^{r,i}_{k,i}}{\sigma^2_{A} - \sigma^2_{B}}, \quad \langle l, r = x, y, z \rangle,
\]

hold for all \(k \geq 2\), we could formulate the lower bound of single-photon pulse pairs as follow:

\[
n^{j,k}_{lr} \geq p_up_{r} a^{l,U} b^{r,U} D_{11} - \frac{a^{y,L}_{x,U}}{\sigma^2_{A} - \sigma^2_{B}} \langle \tilde{N}_{yy} \rangle
\]

where

\[
D_{11} = \frac{a^{y,L}_{x,U} b^{r,U}_{1,0}}{\sigma^2_{A} - \sigma^2_{B}} (N_{xx} - \frac{\sigma^2_{A} - \sigma^2_{B}}{p_{r} - p_{x}} \langle \tilde{N}_{yy} \rangle - \frac{\sigma^2_{A} - \sigma^2_{B}}{p_{r} - p_{x}} \langle \tilde{N}_{yy} \rangle)
\]

Our next job is to formulate the upper bound of phase flip error rate. If we denote the error counts caused by \(|jk\rangle\)-photon pulse pair of source \(xx\) as \(m^{xx}_{jk}\), we have

\[
\langle M_{xx} \rangle = \sum_{j,k \geq 0} m^{xx}_{jk} = \langle \tilde{M}_{xx} \rangle + \langle m^{o}_{xx} \rangle \geq \langle m^{o}_{xx} \rangle + m^{xx}_{1,1},
\]

where

\[
\langle \tilde{M}_{xx} \rangle = \sum_{j,k \geq 1} m^{xx}_{jk}, \quad \langle m^{o}_{xx} \rangle = \sum_{j \geq 1} m^{xx}_{j0} + \sum_{k \geq 1} m^{xx}_{0k} + m^{xx}_{00}.
\]

Thus we have

\[
e^{ph}_{11} = \frac{m^{xx}_{1,1}}{n^{xx}_{1,1}} \leq \frac{\langle M_{xx} \rangle - \langle m^{o}_{xx} \rangle}{N_t p_u a^{l,U} b^{r,U} / b^{r,U}_{1,0} / \langle \tilde{N}_{yy} \rangle},
\]

where \(e^{ph}_{11}\) is the phase flip error rate of signal source \(zz\) and \(D_{11} = D_{11}/N_t\) is the lower bound of the yield of single-photon pulse pairs.

The most important conclusion we get in Ref. [39] is that we could formulate the lower and upper bounds of \(\langle \tilde{N}_{xx} \rangle, \langle \tilde{M}_{xx} \rangle, \) and \(\langle \tilde{N}_{yy} \rangle\) without perfect vacuum source. Actually, we only need the upper bounds of \(\langle \tilde{M}_{xx} \rangle\) and \(\langle \tilde{N}_{yy} \rangle\) in this paper. Explicitly, we have

\[
\langle \tilde{M}_{xx} \rangle = \frac{1}{1 - \sigma^2_{A} - \sigma^2_{B}} \left( \langle M_{xx} \rangle - \frac{p_u b^{y,L}_{x,U}}{p_r b^{y,L}_{0,0}} \langle M_{vy} \rangle \right), \quad \langle \tilde{N}_{yy} \rangle = \frac{1}{1 - \sigma^2_{A} - \sigma^2_{B}} \left( \langle N_{yy} \rangle - \frac{p_u b^{y,L}_{x,U}}{p_r b^{y,L}_{0,0}} \langle N_{vy} \rangle \right),
\]

where \(e^{ph}_{11}\) is the phase flip error rate of signal source \(zz\) and \(\langle \tilde{M}_{xx} \rangle\) is the lower bound of the yield of single-photon pulse pairs.

\[
\langle \tilde{N}_{yy} \rangle = \frac{1}{1 - \sigma^2_{A} - \sigma^2_{B}} \left( \langle N_{yy} \rangle - \frac{p_u b^{y,L}_{x,U}}{p_r b^{y,L}_{0,0}} \langle N_{vy} \rangle \right), \quad \langle \tilde{M}_{xx} \rangle = \frac{1}{1 - \sigma^2_{A} - \sigma^2_{B}} \left( \langle M_{xx} \rangle - \frac{p_u b^{y,L}_{x,U}}{p_r b^{y,L}_{0,0}} \langle M_{vy} \rangle \right),
\]
where

\[ \sigma_A^x = \frac{a_0^x L a_1^U}{a_0^x L a_1^U}, \quad \sigma_B^x = \frac{b_0^x L b_1^U}{b_0^x L b_1^U}, \]
\[ \sigma_A^y = \frac{a_0^y U a_1^L}{a_0^y U a_1^L}, \quad \sigma_B^y = \frac{b_0^y U b_1^L}{b_0^y U b_1^L}. \]  

(14)

Given the obvious fact that the bit flip error rate must be 50% if the bit is caused by a \( |j0\rangle\)-photon or \( |0k\rangle\)-photon pulse pair, we have

\[ \langle n_{xx}^2 \rangle = 2 \langle m_{xx}^2 \rangle, \]  

(15)

and we define \( \mathcal{H} = \frac{2 \langle m_{xy}^2 \rangle}{p_x^2 N_t} \) to simplify the following calculation. With Eqs. (3) and (12), it is a pretty easy work to get the lower and upper bounds of \( \mathcal{H} \), which is

\[ \frac{2 \langle M_{xx} \rangle - \langle M_{xx}^U \rangle}{p_x^2 N_t} \leq \mathcal{H} \leq \frac{2 \langle M_{xx} \rangle}{p_x^2 N_t}. \]  

(16)

To formulate the secure final key rate with the observed counting rates, we need to introduce the following notation

\[ \langle S_{lr} \rangle = \frac{\langle N_{lr} \rangle}{p_{lr} N_t}, \quad S_{lr} = \frac{N_{lr}}{p_{lr} N_t}, \]
\[ \langle T_{lr} \rangle = \frac{\langle M_{lr} \rangle}{p_{lr} N_t}, \quad T_{lr} = \frac{M_{lr}}{p_{lr} N_t}, \]  

(17)

where \( S_{lr} \) and \( T_{lr} \) are the observed values of the counting rate and error counting rate of source \( lr \), and \( \langle S_{lr} \rangle \) and \( \langle T_{lr} \rangle \) are their corresponding asymptotic values. Then we could formulate the final key rate with those asymptotic counting rate values. With Eqs. (12) and (16), we could get the lower and upper bounds values of \( \mathcal{H} \)

\[ \mathcal{H}^U = 2 \langle T_{xx} \rangle, \]  

(18)

\[ \mathcal{H}^L = \frac{-2}{1 - \sigma_A^x - \sigma_B^x} \left[ a_0^x L a_1^U (T_{vx}) + b_0^x L b_1^U (T_{xv}) - a_0^y U b_0^x L b_1^L (T_{vx}) - a_0^x U b_1^L (T_{xv}) \right], \]  

(19)

and the upper bound of phase flip error rate is

\[ e_{11}^{ph,U}(H) = \frac{\langle T_{xx} \rangle - \frac{1}{2} \mathcal{H}}{a_1^x L b_1^x L s_{11}(H)}, \]  

(23)

and thus the final key rate is

\[ R = \min[\mathcal{R}(\mathcal{H})], \quad \mathcal{H} \in [\mathcal{H}^L, \mathcal{H}^U]. \]  

(25)

**IV. ANALYSIS FOR STATISTICAL FLUCTUATION**

The formulas of secure final key rate in Sec. III are denoted by expected values, but the direct values we get in experiments are observed values. The relationship between expected values and observed values is discussed as follows. Let \( X_1, X_2, \ldots, X_n \) be \( n \) random samples, detected with the value 1 or 0, and let \( X \) denote their sum satisfying \( X = \sum_{i=1}^{n} X_i \). \( \mu \) is the expected value of \( X \). From the Chernoff bound \[25, 31, 40\] with a fixed failure probability \( \xi \) which is required in experiments, we have

\[ \mu^L(X) = \frac{X}{1 + \delta_1(X)}, \]  

(26)

\[ \mu^U(X) = \frac{X}{1 - \delta_2(X)}, \]  

(27)

where we can obtain the value of \( \delta_1(X) \) and \( \delta_2(X) \) by solving the following equations

\[ \left( \frac{e^{\delta_1}}{(1 + \delta_1)^{1 + \delta_1}} \right)^\frac{X}{\xi} = \frac{\xi}{2}, \]  

(28)

\[ \left( \frac{e^{-\delta_2}}{(1 - \delta_2)^{1 - \delta_2}} \right)^\frac{X}{\xi} = \frac{\xi}{2}. \]  

(29)
With Eqs. (26) and (27), we could get the following useful constraints

\[
\mu^L(L_{ir}S_{ir}) \leq L_{ir}(S_{ir}) \leq \mu^U(L_{ir}S_{ir}),
\]

\[
L_{xx}(T_{xx}) \leq \mu^U(L_{xx}T_{xx}),
\]

\[
\begin{cases}
L_{xx}(S_{xx}) + \mathcal{L}_{vy}(S_{vy}) \geq \mu^L(L_{xx}S_{xx} + \mathcal{L}_{vy}S_{vy}), \\
L_{xx}(S_{xx}) + \mathcal{L}_{vy}(S_{vy}) \geq \mu^L(L_{xx}S_{xx} + \mathcal{L}_{vy}S_{vy}), \\
L_{vy}(S_{vy}) + \mathcal{L}_{vy}(S_{vy}) \geq \mu^L(L_{vy}S_{vy} + \mathcal{L}_{vy}S_{vy}), \\
L_{xx}(S_{xx}) + \mathcal{L}_{vy}(S_{vy}) + \mathcal{L}_{vy}(S_{vy}) \geq \mu^L(L_{xx}S_{xx} + \mathcal{L}_{vy}S_{vy}), \\
\end{cases}
\]

\[
L_{vy}(S_{vy}) + \mathcal{L}_{vv}(S_{vv}) \leq \mu^U(L_{vy}S_{vy} + \mathcal{L}_{vv}S_{vv}),
\]

\[
\begin{cases}
L_{vx}(T_{xx}) + L_{xx}(T_{xx}) \geq \mu^L(L_{vx}T_{xx} + L_{xx}T_{xx}), \\
L_{xx}(T_{xx}) + L_{xx}(T_{xx}) \leq \mu^U(L_{xx}T_{xx} + L_{xx}T_{xx}),
\end{cases}
\]  

where \(L_{ir} = p_{pr}N_t\) is the total pulse pairs sent out by source \(ir\).

With those preparations, we can now calculate the secure final key rate. We first calculate the lower bound of \(S^+\) with Eqs. (21), (30), and (32) and the upper bound of \(S^-\) with Eqs. (22), (33), and (34). And then we calculate the upper and lower bounds of \(H\) with Eqs. (15), (19), (40), (41), and (44). For each certain \(H\) \((H \in [H^L, H^U])\), we could get one corresponding \(R(H)\) with Eqs. (20), (23), and (24). Finally, we get the secure final key rate \(R = \min[\mathcal{R}(H)]\).

V. NUMERICAL SIMULATION

In this section, we present some numerical simulations to show the efforts of intensity fluctuations. Firstly, we shall estimate what values would be probably observed for the yields and error yields in the normal cases by the linear models. We focus on the symmetric case where the two channel transmissions from Alice to UTP and from Bob to UTP are equal. We also assume that the UTP’s detectors are identical, i.e., they have the same dark count rates and detection efficiencies, and their detection efficiencies do not depend on the incoming signals. As discussed before, we know that the methods presented in this paper apply to any sources that satisfy the condition given by Eqs. (14) and (15). For simplicity, we suppose that Alice and Bob use weak coherent states (WCS). The density matrix of the WCS with intensity \(\mu\) can be written into \(\rho = \sum_{k=0}^{\infty} \frac{e^{-\mu} \mu^k}{k!} |k\rangle \langle k|\). The actual intensity of the \(i\)th pulse for source \(l\) out of Alice’s (or Bob’s) laboratory is

\[
\mu_i^L = \mu_1(1 + \delta_i^L), \quad (l = x, y, z),
\]

\[
\mu_i^V \leq \delta_1 \text{ with the boundary conditions } |\delta_i^L| \leq \delta_2 \text{ for } l = x, y, z.
\]

The values of parameters used in the numerical simulations are listed in Table 1. Figures 1 and 2 show the key rates versus transmission distance with different intensity fluctuations. We set the total pulse pairs \(N_t = 10^{11}\) in Figure 1 and \(N_t = 10^{13}\) and the detection efficiency \(\eta_d = 40.0\%\) in Figure 2. Figures 1 and 2 show that the effect of the intensity fluctuation is worse with worse experimental conditions (including smaller data size and worse detectors).

VI. CONCLUSION

In summary, we have shown how to calculate the secure final key rate in the decoy-state MDI-QKD protocol with both source errors and statistical fluctuations with a certain failure probability, provided that the parameters in the diagonal state of the source satisfy Eqs. (14) and (15) and bound values of each parameter in the state are known. By our method, all imperfections in a finite pulse pair size decoy-state MDI-QKD protocol with an unstable source have been taken into consideration. Our
of pulses at each side with different intensity fluctuations under the experimental parameters listed in Table I. Here we set the total number of pulses at each side $N_1 = 10^{13}$ and the detection efficiency $\eta_d = 40\%$.

![Graph](image_url)

FIG. 2: (color online) The optimal key rates (per pulse) versus transmission distance (the distance between Alice and Bob) with different intensity fluctuations under the experimental results.

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