Efficient Gradual Typing

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Abstract
Gradual typing combines static and dynamic typing in the same program. One would hope that the performance in a gradually typed language would range between that of a dynamically typed language and a statically typed language. Existing implementations of gradually typed languages have not achieved this goal due to overheads associated with runtime casts. Takikawa et al. (2016) report up to 100× slowdowns for partially typed programs. In this paper we present a compiler, named Grift, for evaluating implementation techniques for gradual typing. We take a straightforward but surprisingly unexplored implementation approach for gradual typing, that is, ahead-of-time compilation to native assembly code with carefully chosen runtime representations and space-efficient coercions.

Our experiments show that this approach achieves performance on par with OCaml on statically typed programs and performance between that of Gambit and Racket on untyped programs. On partially typed code, the geometric mean ranges from 0.42× to 2.36× that of (untyped) Racket across the benchmarks. We implement casts using the coercions of Siek, Thiemann, and Wadler (2015). This technique eliminates all catastrophic slowdowns without introducing significant overhead. Across the benchmarks, coercions range from 15% slower (fft) to almost 2× faster (matmult) than regular casts. We also implement the monotonic references of Siek et al. (2015). Monotonic references eliminate all overhead in statically typed code, and for partially typed code, they are faster than proxied references, sometimes up to 1.48×.

1 Introduction
Gradual typing combines static and dynamic type checking in the same program, giving the programmer control over which typing discipline is used for each region of code [5, 24, 32, 39, 47]. We would like gradually typed languages to be efficient, sound, and provide interoperability. Regarding efficiency, we would like the performance of gradual typing to range from being similar to that of a dynamically typed language to that of a statically typed language. Regarding soundness, programmers (and compilers) would like to trust type annotations and know that runtime values respect their compile-time types. Third, regarding interoperability, static and dynamic regions of code should interoperate seamlessly.

To date, implementations of gradual typing have only delivered two of these three properties. For example, Typed Racket [48] provides soundness and interoperability but suffers from slowdowns of up to 100× [45, 46] on a partially typed program. Thorn [10, 50] and Safe TypeScript [34] provide better performance but limit interoperability. TypeScript [9, 27] and Gradualtalk [2–4] do not provide soundness and their performance is on par with dynamic languages but not static ones, but they provide seamless interoperability.

Several papers at OOPSLA 2017 begin to address the efficiency concerns for gradually typed languages that are committed to soundness and interoperability. Bauman et al. [7] demonstrate that a tracing JIT can eliminate 90% of the overheads in Typed Racket due to gradual typing. Richards et al. [35] augment the Higgs JIT compiler and virtual machine (VM) [13] for JavaScript, re-purposing the VM’s notion of shape to implement monotonic references [42]. Richards et al. [35] reports that this reduces the worst slowdowns to 45%, with an average slowdown of just 7%. Meanwhile, Muehlboeck and Tate [33] show that for nominally-typed object-oriented languages, efficiency is less of a problem.

In this paper we demonstrate that efficient gradual typing can be achieved in structurally-typed languages by relatively straightforward means. We build and evaluate an ahead-of-time compiler that uses carefully chosen runtime representations and implements two important ideas from the theory of gradual typing. It uses space efficient coercions [20, 30, 40, 43] to implement casts and it reduces overhead in statically typed code by using monotonic references [42].

Contributions This paper makes these contributions.

- A space-efficient semantics for monotonic references and lazy-D coercions (Section 3).
- The first ahead-of-time compiler, named Grift, for a gradually typed language that targets native assembly code. The compiler is the first to implement space-efficient coercions (Section 4).
- Experiments (Section 5.2) showing
  - performance on statically typed code that is on par with OCaml,
  - performance on dynamically typed code that is between Gambit and Racket, and
  - performance on partially typed code ranging from 0.42× to 2.36× that of Racket.
- Experiments showing that coercions eliminate catastrophic slowdowns without adding significant overhead (Section 5.3).
Experiments showing that monotonic references eliminate overhead in statically typed code (Section 5.4).

Section 2 provides background on gradual typing, focusing on runtime casts and the tension between efficiency, soundness, and interoperability.

2 Tensions in Gradual Typing

From a language design perspective, gradual typing touches both the type system and the operational semantics. The key to the type system is the consistency relation on types, which enables implicit casts to and from the unknown type, here written Dyn, while still catching static type errors [5, 24, 39]. The dynamic semantics for gradual typing is based on the semantics of contracts [18, 21], coercions [28], and interlanguage migration [32, 47]. Because of the shared mechanisms with these other lines of research, much of the ongoing research in those areas benefits the theory of gradual typing, and vice versa [14–16, 22, 23, 25, 31, 44]. In the following we give a brief introduction to gradual typing by way of an example that emphasizes the three main goals of gradual typing: supporting interoperability, soundness, and efficiency.

Interoperability and Evolution Consider the example program in Figure 1, written in a variant of Typed Racket that we have extended to support fine-grained gradual typing. On the left side of the figure we have an untyped function for the extended greatest common divisor. With gradual typing, unannotated parameters are dynamically typed and therefore assigned the type Dyn. On the right side of the figure is the same function at a later point in time in which the parameter types have been specified (Int) but not the return type. With gradual typing, both programs are well typed because implicit casts are allowed to and from Dyn. For example, on the left we have the expression (modulo b a), so b and a are implicitly cast from Dyn to Int. On the right, there is an implicit cast around (list b 0 1) from (List Int) to Dyn. The reason that gradual typing allows implicit casts both to and from Dyn is to enable evolution. As a programmer adds or removes type annotations, the program continues to type check and also exhibits the same behavior up to cast errors, a property called the gradual guarantee [41].

Soundness Next consider the function modinv defined below that computes the modular inverse using the second version of egcd. What happens when (read) to an integer before passing it to modinv?

\[
\begin{align*}
\text{(define (modinv a m)} & \quad \text{(let ([r (egcd a m)])}} \\
& \quad \text{(let ([input (read)])}} \\
& \quad \text{(modinv 42 input))} \\
& \quad \text{(if (not (= (car r) 1))}} \\
& \quad \quad \text{(error ...}} \\
& \quad \quad \text{(modulo (cdr r) m))))
\end{align*}
\]

Parameter \(m\) of \text{modinv} has type Dyn, but \(b\) of \text{egcd} has type Int, so there is an implicit cast from Dyn to Int. With gradual typing, this implicit cast comes with a runtime cast that will trigger an error if the input to this program is a string. This runtime cast is required to ensure soundness: without it a string could flow into \text{egcd} and masquerade as an Int. Soundness is not only important for software engineering reasons but it also impacts efficiency both positively and negatively.

Ensuring soundness in the presence of first-class functions and mutable references is nontrivial. When a function is cast from Dyn to a type such as \text{Int \rightarrow Int}, it is not possible for the cast to know whether the function will return an integer on all inputs. Instead, the standard approach is to wrap the function in a proxy that checks the return value each time the function is called [18]. Similarly, when a mutable reference is cast, e.g., from \text{Ref Int} to \text{Ref Dyn}, the reference is wrapped in a proxy that casts from \text{Int} to Dyn on every read and from Dyn to Int on every write [29, 30].

Efficiency Ideally, statically typed code within a gradually typed program should execute without overhead. Likewise, partially typed or untyped code should execute with no more overhead than is typical of dynamically typed languages.

Consider the \text{egcd} on the right side of Figure 1. Inside this \text{egcd}, the expression (modulo b a) should simply compile to an i\text{div} instruction on x86. However, if the language did not ensure soundness as discussed above, then this efficient compilation strategy would result in undefined behavior (segmentation faults at best, hacked systems at worst). It is soundness that enables type-based specialization. However, soundness comes at the cost of the runtime casts at the boundaries of static and dynamic code.

3 Semantics of a Gradual Language

The type system of Grift’s input language is the standard one for the gradually typed lambda calculus [30, 36, 39]. The operational semantics, as usual, is expressed by a translation to an intermediate language with explicit casts.

Consider the source program in Figure 2 which calculates the value 42 by applying the \text{add1} function, by way of variable \(f\), to the integer value 41. The type of \text{add1} does not exactly match the type annotation on \(f\) (which is Dyn \rightarrow Dyn) so the compiler inserts the cast:

\[
\text{(cast add1 (Int \rightarrow Int)) (Dyn \rightarrow Dyn) 12}
\]

The application of \(f\) to 42 requires a cast on 42 from \text{Int} to Dyn. Also, the return type of \(f\) is Dyn, so the compiler inserts a cast to convert the returned value to Dyn to satisfy the type ascription.

In this paper we consider two approaches to the implementation of runtime casts: traditional casts, which we refer to as type-based casts, and coercions. Type-based casts provide the most straightforward implementation, but the proxies they generate can accumulate and consume an unbounded
were originally described using coercions by Siek et al. [38]. The distinction between lazy-D and the more commonly used values of a ground to lazy-D, which requires some changes to the normal forms that compress coercions. Here we adapt that approach define a normal form for coercions and a composition operation for proxied references, which we adapt from the introduction rules for proxied references, and projection operation and the relation

\[\begin{align*}
&\text{let} (\text{add1} : (\text{Int} \rightarrow \text{Int}) \\
&\quad (\lambda ([x : \text{Int}] (+ x 1)))) \\
&\quad (\text{let} ([f : (\text{Dyn} \rightarrow \text{Dyn}) \text{add1}]) \\
&\quad (\lambda ([f (\text{cast add1} (\lambda (x) (+ x 1)))]) \\
&\quad (\text{cast} (f (\text{cast 41 Int Dyn L2}) \text{Dyn Int L3})))
\end{align*}\]

Figure 2. An example of the Grift compiler inserting casts. The L1, L2, etc. are blame labels that identify source code location.

Figure 1. Two gradually typed versions of extended GCD.

The coercions of Henglein [28] solve the space problem with a representation that enables the compression of higher-order casts [30]. For type-based casts, the dynamic semantics that we use is almost covered in the literature. We use the lazy-D cast semantics which is described by Siek and Garcia [37]. (They were originally described using coercions by Siek et al. [38].) The distinction between lazy-D and the more commonly used lazy-UD semantics [49] is not well-known, so to summarize the difference: in lazy-D, arbitrary types of values may be directly injected into type Dyn, whereas in lazy-UD, only values of a ground type may be directly injected into Dyn. For example, Int and Dyn → Dyn are ground types, but Int → Int is not.

The one missing piece for our purposes are the reduction rules for proxied references, which we adapt from the coercion-based version by Herman et al. [30]. In this setting, proxied references are values of the form \((v : \text{Ref}_p T_1 \Rightarrow^\ell \text{Ref}_p T_2)\). The following are the reduction rules for reading and writing to a proxied reference.

\[\begin{align*}
&! (v : \text{Ref}_p T_1 \Rightarrow^\ell \text{Ref}_p T_2) \rightarrow ! v : T_1 \Rightarrow^\ell T_2 \\
& (v_1 : \text{Ref}_p T_1 \Rightarrow^\ell \text{Ref}_p T_2) : v_2 \rightarrow v_1 := (v_2 : T_2 \Rightarrow^\ell T_1)
\end{align*}\]

For monotonically references with type-based casts, the dynamic semantics for lazy-D is given by Siek et al. [42].

Regarding coercions, the dynamic semantics that we used is less well-covered in the literature. Again, we use the lazy-D semantics of Siek et al. [38], but that work, despite using coercions, did not define a space-efficient semantics. On the other hand, Siek et al. [40] give a space-efficient semantics with coercions, but for the lazy-UD semantics. To this end, they define a normal form for coercions and a composition operator that compresses coercions. Here we adapt that approach to lazy-D, which requires some changes to the normal forms and to the composition operator. Also, that work did not consider mutable references, so here we add support for both proxied and monotonically references. Regarding monotonically references, Siek et al. [42] define the lazy-D semantics, but again, they did not define a space-efficient semantics. Here we make it space-efficient by defining the normal forms for reference coercions and the composition operation on them.

Figure 3 defines a representative subset of the types and coercions used in Grift’s intermediate language. The figure also defines the meet operation and the consistency relation on types and the composition operator on coercions. Instead of defining two different languages, one with proxied references and the other with monotonically references, we instead present a single language with both kinds of references. The type \(\text{Ref}_p T\) is for proxied references and \(\text{Ref}_m T\) is for monotonically references. Likewise, \(\text{Ref}_p c d\) is the coercion for proxied references and \(\text{Ref}_m T\) is the coercion for monotonically references.

Space-efficient coercions are defined by a grammar consisting of three rules that enable coercion composition by the composition operator defined in Figure 3. Let \(c, d\) range over space-efficient coercions, \(i\) range over final coercions, and \(g\) range over middle coercions. Space-efficient coercions are either the identity coercion \(I\), a projection followed by a final coercion \((? ; i)\), or just a final coercion. A final coercion is either the failure coercion \(I^p\), a middle coercion followed by an injection \((g ; !)\), or just an intermediate coercion. An intermediate coercion is either the identity coercion \(I\), the function coercion \(c \rightarrow d\), the tuple coercion \(c \times d\), the proxied reference coercion \(\text{Ref}_p c d\), where \(c\) is applied when writing and \(d\) is applied when reading, or the monotonically reference coercion \(\text{Ref}_m T\). The main difference between the lazy-D coercions shown here and those of Siek et al. [40] is in the injection \(!\) and projection \(?\), which take any injectable type (anything but Dyn) instead of only ground
Types and coercions

Base Types

\[ B ::= \text{Int} \mid \text{Bool} \mid \ldots \]

Injectables

\[ I, J ::= B \mid T \rightarrow T \mid T \times T \mid \text{Ref}_p T \mid \text{Ref}_m T \]

Types

\[ T ::= \text{Dyn} \mid I \]

Coercions

\[ c, d ::= i \mid (I^p : i) \mid i \]

Final Crnns

\[ i ::= \bot^{lpj} \mid (g ; !l) \mid g \]

Mid. Crnns

\[ g ::= i \mid c \rightarrow d \mid c \times d \mid \text{Ref}_p c \mid d \mid \text{Ref}_m T \]

Id-free Crnns

\[ f ::= (I^p : i) \mid (g ; !l) \mid c \rightarrow d \mid c \times d \]

\[ \text{Ref}_p c \mid d \mid \text{Ref}_m T \mid [\bot^{lpj}] \]

Consistency

\[
\begin{align*}
\text{Dyn} \sim T & \quad T \sim \text{Dyn} & \quad B \sim B \\
\text{Ref}_p T_1 \sim \text{Ref}_p T_2 & \quad \text{Ref}_m T_1 \sim \text{Ref}_m T_2 \\
T_1 \sim T_2 & \quad T_3 \sim T_4 & \quad T_1 \times T_2 \sim T_3 \times T_4
\end{align*}
\]

Meet operation (greatest lower bound)

\[ T \sqcap T \]

Coercion creation

\[ (B \Rightarrow^l B) = (\text{Dyn} \Rightarrow^l \text{Dyn}) = i \]

\[ (I \Rightarrow^l \text{Dyn}) = !l \]

\[ (\text{Dyn} \Rightarrow^l I) = !p \]

\[ (T_1 \rightarrow T_2) \Rightarrow^l (T'_1 \rightarrow T'_2) = (T'_1 \Rightarrow^l T_1) \rightarrow (T'_2 \Rightarrow^l T_2) \]

\[ (T_1 \times T_2) \Rightarrow^l (T'_1 \times T'_2) = (T'_1 \Rightarrow^l T_1) \times (T'_2 \Rightarrow^l T_2) \]

\[ (\text{Ref}_p f \Rightarrow^l \text{Ref}_p f') = \text{Ref}_p (f' \Rightarrow^l f) \]

\[ (\text{Ref}_m f \Rightarrow^l \text{Ref}_m f') = \text{Ref}_m f' \]

Coercion composition

\[ c \downarrow d = r \]

\[ (g ; !l) \downarrow (I^p ; i) = g \downarrow ((I \Rightarrow^p !) \downarrow i) \]

\[ c \rightarrow d \downarrow c' \rightarrow d' = (c \downarrow c) \rightarrow (d \downarrow d') \]

\[ c \times d \downarrow c' \times d' = (c \downarrow c') \times (d \downarrow d') \]

\[ \text{Ref}_p c \downarrow d \downarrow \text{Ref}_p c' \downarrow d' = \text{Ref}_p (c \downarrow c') \downarrow (d \downarrow d') \]

\[ \text{Ref}_m T \downarrow \text{Ref}_m T' = \text{Ref}_m (T \downarrow T') \]

\[ (I^p ; i) \downarrow c = I^p \downarrow (i \downarrow c) \]

\[ g_1 \downarrow (g_2 ; !l) = (g_2 \downarrow g_2) ; !l \]

\[ i \downarrow c = c \downarrow i \]

\[ g \downarrow \bot^{lpj} = \bot^{lpj} \downarrow c = \bot^{lpj} \]

Figure 3. Types, coercions, and their operations.

This change impacts the coercion composition operation, in the case where an injection I meets a projection J^p we make a new coercions whose source is I and target is J with the coercion creation operation (I \Rightarrow^p J) (Figure 3).

The following is the syntax of Grift’s intermediate language, including both proxied and monotonic references.

\[ u ::= k \mid a \mid \Lambda x.M \mid (u, u) \]

\[ v ::= u \mid (u, v) \mid u(g ; !l) \mid u(c \rightarrow d) \mid u(\text{Ref}_p c d) \]

\[ b ::= \text{blame} p \mid \text{error} \]

\[ M, N ::= b \mid v \mid x \mid M \cdot N \mid (M, M) \mid (\text{fst } M) \mid (\text{snd } M) \mid \text{Ref}_p M \mid \text{Ref}_m M \mid M = : p N \mid \text{Ref}_m M \cdot T \mid \text{Ref}_p M \cdot T \mid M = : M N \mid M = : N @ T \]

Figure 4 defines the dynamic semantics. The language forms \text{Ref}_p M, \text{Ref}_m M, and M = : p N are for allocating, dereferencing, and updating a proxied pointer, respectively. The language form \text{Ref}_m M \cdot T is for allocating a monotonic reference. The form \text{Ref}_m M is for dereferencing a monotonic reference with a fully static type whereas \text{Ref}_m M \cdot T dereferences a monotonic reference that is partially or fully dynamic. Similarly, \text{Ref}_m M \cdot T is for updating a monotonic reference with a fully static type and \text{Ref}_m M \cdot N \cdot T is for updating a partially or fully-dynamic monotonic reference. Regarding the definition of values, the value form u(\text{Ref}_p c d) represents a proxied reference whereas an address a is a regular reference.

The dynamic semantics is given by three reduction relations: cast reductions, program reductions, and state reductions. This organization streamlines the treatment of monotonic references. The heap \mu maps an address to a value (for proxied references) or to a value and a type (for monotonic references). We refer to this type as the run-time type information (RTTI) of the value and write \mu(a)_{\text{val}} to access the value and \mu(a)_{\text{typ}} to access the RTTI.

The cast reduction rules define the semantics of applying a cast to a value. Space efficiency is achieved with the reduction that takes a coerced value u(i) and a coercion c and compresses the two coercions to produce the coerced value u(i \downarrow c). There need not be any cast reduction rules for proxied references, as a cast applied to a reference is a value, similar to the function case. In contrast, when a coercion is applied to a monotonic reference, we cast the underlying value on the heap. This cast is only allowed to make the RTTI more precise. Any attempt to cast to an inconsistent type triggers a runtime error. Also, casting the underlying value causes the heap to become an “evolving heap” which propagates the cast via subsequent state transitions.

Regarding the program reduction rules, we have a different set of reductions for operations on proxied versus monotonic references. For dereferencing a proxied reference, there are two rules, one for raw addresses and the other for a proxy. Thus, an implementation must dispatch on the kind of reference. If it’s an address, the value is loaded from the heap. If it’s a proxy, the underlying reference is dereferenced.
Runtime Structures:

\[ cv ::= v \mid v(c) \mid (cv,cv) \]
\[ \mu ::= \emptyset \mid \mu(a \mapsto v) \mid \mu(a \mapsto v : T) \]
\[ \nu ::= \mu \mid \nu(a \mapsto cv : T) \]
\[ \mathcal{E} ::= \mathcal{F} \mid \mathcal{F}[\square(f)] \]
\[ \mathcal{F} ::= \square \mid \mathcal{E}(\square ; M) \mid \mathcal{E}[v \square] \mid \mathcal{E}(\square \square ; M) \mid \mathcal{E}(v,\square) \mid\]
\[ \mathcal{E}(\text{fst}(\square \square)) \mid \mathcal{E}(\text{snd}(\square \square)) \mid \mathcal{E}(\text{ref}\_p,\square) \mid \mathcal{E}(\text{ref}\_n,\square @ T) \mid \mathcal{E}(y,\square) \mid\]
\[ \mathcal{E}(\square @ T) \mid \mathcal{E}[\square = M \_ M] \mid \mathcal{E}[v = M \_ M] \]
\[ \mathcal{E}[\square = M @ T] \mid \mathcal{E}[v = M @ T] \]

Cast reduction rules:

\[ M, \mu \rightarrow_c N, v \]

Program reduction rules:

\[ M, \mu \rightarrow_e N, v \]

State reduction rules:

\[ M, \mu \rightarrow X N, v \]
\[ X \in \{c, e\} \]
\[ M, \mu \rightarrow N, v \]
\[ \nu(a) = cv : T \quad \nu'(a)_{\text{htu}} \neq T \]
\[ \text{cv}, v \rightarrow_c \text{cv}', v' \]
\[ M, v \rightarrow M, v' \]
\[ M, v \rightarrow M, v'(a \mapsto \text{cv}' : T) \]
\[ \nu(a) = cv : T \]
\[ \text{cv}, v \rightarrow_c \text{cv}', v' \]
\[ M, v \rightarrow E, v' \]

Figure 4. Semantics of the intermediate language of Grift, with proxied and monotonic references.

and then the proxy’s read coercion \( c \) is applied. The story is similar for writing to a proxied reference. For monotonic references, there are two dereference operators. For the fully static dereference, we simply load the value from the heap. For the partially dynamic dereference, we load the value from the heap and cast it from its RTTI to the expected type \( T \). The story is similar for writing to a monotonic reference.

Regarding the state reduction rules, the first rule simply enables transitions according to the cast or program reduction rules. The next three rules propagate casts within an evolving heap. The first of them commits the result of casting a value in the heap whereas the second throws away the result of a cast in the case when the RTTI has changed (due to cycles). The last rule handles the case when a cast fails.

4 The Grift Compiler

The Grift compiler takes programs in an extended version of the gradually typed lambda calculus and compiles them to C, using the Clang compiler to generate x86 executables. The Clang compiler provides low level optimizations. The first step in the Grift compiler is to translate to an intermediate language with explicit casts. This process is standard [30, 36, 39] except for optimizations to avoid unnecessary casts in dynamically typed code, which we describe in Section 4.6.

The next step in the compiler is exposing the runtime functions that implement casts. We describe the representation of values in 4.1. We describe the implementation of type-based casts in Section 4.2, coercions in Section 4.3, and monotonic references in Section 4.4. After lowering casts, Grift performs closure conversion using a flat representation [6, 11, 12], and translates all constructors and destructors to memory allocations, reads, writes, and numeric manipulation.

4.1 Value Representation

Values are represented according to their type. An Int value is a 64-bit integer. A Bool value is also a 64-bit integer, using the C encoding of 1 for true and 0 for false. For values of function type, the representation depends on whether Grift uses type-based casts or coercions. In the former case, a function value is a pointer to a closure. A closure consists of 1) a function pointer, 2) a pointer to a function for casting the closure, and 3) the values of the free variables. In the later case (for coercions), a function value is a pointer to one of two different kinds of closures and the lowest bit of the pointer indicates which kind. The first kind, for regular functions, is the same as the above. The second kind which we call a proxy closure, is for functions that have been cast. It consists of a function pointer (to a “wrapper” function), a pointer to the underlying closure, and a pointer to a coercion.

A value of proxied reference type is a pointer to the data or to a proxy. The lowest bit of the pointer indicates which. The representation of the proxy depends on whether Grift uses type-based casts or coercions. In the former case, the
proxy consists of a reference, the source and target types, and a blame label. In the later case, the proxy consists of a reference and a pointer to a coercion (that is, a reference coercion). A value of monotonic reference type is an address.

A value of type Dyn is a 64-bit integer, with the 3 lowest bits indicating the type of the injected value. For atomic types (e.g. Int and Bool), the injected value is stored in the other 61 bits. For non-atomic types, the other 61 bits are a pointer to a pair of the injected value and its type.

In the following, the macros for manipulating values have all uppercase names to distinguish them from C functions. The macro definitions are listed in Appendix A.

### 4.2 Implementation of Type-based Casts

Type-based casts require a runtime representation of types. Grift allocates all types known at compile time at the start of the program. Each type is a 64 bit value, the lower 3 bits categorize whether it is an atomic, function, proxied reference or monotonic reference type. For atomic types, the other 61 bits indicate which atomic type. For function and reference types, the other 61 bits point to a larger structure. The structure for a function type stores the arity, return type, and types for the parameters. The structure for reference types consists of the referred-to-type.

**Casting Values** Grift implements type-based casts with the C function named cast (Figure 5) that takes a value, two types (source and target), and a blame label, and returns a value or signals an error. If the cast is between two identical types, then cast returns the value unaltered. If the source type is Dyn, then the underlying value and the type are extracted and used to recursively cast to the target type of the original cast. Conversely, if the target type is Dyn, then the value is injected into the representation for Dyn.

In case the source and target of the cast are function types, cast wraps the value in a function that casts the arguments and the return value appropriately. For example, the cast:

```c
obj cast(obj val, type src, type tgt, blame lbl) {
    if (src == tgt) { return val; }
    else if (src == DYN_TYPE) {
        return cast(UNTAG(val), TYPE(val), tgt, lbl);
    } else if (tgt == DYN_TYPE) {
        return INJECT(val, src);
    } else if (TAG(src)==FUN_TAG && TAG(tgt)==FUN_TAG && ARITY(src)==ARITY(tgt)) {
        return UNTAG_FUN(val)->caster(val,src,tgt,lbl);
    } else if (TAG(src)==REF_TAG && TAG(tgt)==REF_TAG) {
        type s = REF_TYPE(src), t = REF_TYPE(tgt);
        return MK_REF_PROXY(val, s, t, lbl);
    } else { raise_blame(lbl); }
}
```

**Figure 5.** The cast function.

### Applying Functions

For type-based casts, Grift doesn’t need to distinguish between the closures created by casting functions and closures created by defining functions. As a result, the calling convention is simple and direct. The generated code at each call site accesses the function pointer at the beginning of the closure and performs an indirect call, passing the closure to the function as an additional argument.

### Reading and Writing to Proxied References

The C functions for reading and writing references are listed in Figure 6. The code for reading dispatches on whether the reference is proxied or not, and if proxied, recurses on the underlying reference (because it could be another proxy). When the

```c
obj ref_read(obj ref) {
    if (TAG(ref) == REF_PROXY_TAG) {
        ref_proxy p = UNTAG_PREF(ref);
        obj v = ref_read(p->ref);
        return cast(v, p->src, p->tgt, p->lbl);
    } else { return *UNTAG_REF(ref); }
}
```

**Figure 6.** Code for reading and writing to references.
whether the coercion is a projection, injection, sequence, or failure. The coercion is implemented by a C function named \texttt{coerce}. This function has previously been coerced. If so, Grift builds a value of type \texttt{T} and then the second coercion. Injection coercions return the value unchanged. Sequence coercions apply the first coercion and then the second coercion. Identity coercions return the value unchanged. The coercion in accordance with the semantics.

**Coercion Representation.** Similar to types, coercions are represented as 64-bit values where the lowest 3 bits indicate whether the coercion is a projection, injection, sequence, failure, or identity. The number of pointer tags is limited, so the rest of the coercions are identified by a tag stored in the first word of their structures. For an identity coercion, the remaining 61 bits are not used. For the other coercions, the remaining 61 bits stores a pointer to heap-allocated structure that depends on the kind of coercion:

- **Projection coercion** ($T\rightarrow^n$) is represented in $2\times64$ bits: the first word is a pointer to the type $T$ of the projection and the second is a pointer to the blame label $p$.
- **Injection coercion** ($T!$) is represented in 64 bits, holding a pointer to the type $T$ of the injected value.
- **Function coercion** ($c_1,\ldots,c_n \rightarrow c_r$) with $n$ parameters is represented in $64(n + 2)$ bits, where the first word stores the secondary tag and arity, the second store a coercion on the return, and the remaining words store $n$ coercions for the arguments.
- **Proxy coercions** ($\text{Ref}_p c_1 c_2$) is represented in $3\times64$ bits, including the secondary tag, a coercion $c_1$ for writing, and another coercion $c_2$ for reading.
- **Monotonic reference coercion** ($\text{Ref}_m T$) is represented in $2\times64$ bits, including the secondary tag and a type.
- **Sequences** ($c_1 ; c_2$) store two coercions in $2\times64$ bits.
- **Failure coercion** ($\bot$) is represented in 64 bits to store a pointer to the blame label.

**Applying a Coercion** The application of a coercion to a value is implemented by a C function named \texttt{coerce}, shown in Figure 7, that takes a value and a coercion and either returns a value or signals an error. The \texttt{coerce} function dispatches on the coercion’s tag. Identity coercions return the value unchanged. Sequence coercions apply the first coercion and then the second coercion. Injection coercions build a value of type \texttt{T}. Projection coercions take a value of type \texttt{T} and build a new coercion from the runtime type to the target of the projection, which it applies to the underlying value.

When coercing a function, \texttt{coerce} checks whether the function has previously been coerced. If so, Grift builds a new proxy closure by copying over the wrapper function and the underlying closure, but its coercion is the result of composing the proxy’s coercion with the coercion being applied via \texttt{compose} (Appendix A Figure 14). If the function has not been previously coerced, then we access its function pointer for casting and apply that to the function and the coercion that needs to be applied. This “caster” function allocates and initializes a proxy closure. Coercing a proxied reference builds a proxy that stores two coercions, one for reading and one for writing, and the pointer to the underlying reference. In case the reference has already been coerced, then we access its function pointer for casting and apply that to the function and the coercion that needs to be applied. This “caster” function allocates and initializes a proxy closure. Coercing a proxied reference builds a proxy that stores two coercions, one for reading and one for writing, and the pointer to the underlying reference.

**Applying Functions** Because the coercions implementation distinguishes between regular closures and proxy closures, one might expect closure call sites to branch on the type of closure being applied. However, this is not the case.
because Grift ensures that the memory layout of a proxy closure is compatible with the regular closure calling convention. The only change from the type-based implementation’s calling convention (described in Section 4.2) is that we have to clear the lowest bit of the pointer to the closure which distinguishes proxy closures from regular closures. This representation is inspired by a technique used in Siek and Garcia [37] which itself is inspired by Findler and Felleisen [19].

Reading and Writing to Proxied References For the coercion implementation, Grift generates code for proxied reference reads and writes that is similar to the code in the type-based cast implementation. However, there are two slight differences: since coercions are space-efficient, we know that there will be at most one proxy, and therefore there is no need to recurse on the proxied reference and the coercion function is used with the coercion contained in a proxy if present.

4.4 Implementation of Monotonic References

A monotonic heap cell has two parts; the first stores runtime type information (RTTI), and the second stores the value. Grift generates pointer dereference and write instructions for reading and writing a fully statically-typed monotonic reference. Otherwise, the value being read or written has to be cast. The details of the latter process and that of casting an address is described below.

Type-based Casts To cast a monotonic reference, we cast the underlying value on the heap. First, the RTTI is read from the first word pointed to by the address. The address is returned if the RTTI equals the target type of the cast. However, the equality check can be expensive if implemented naively because the structures of both types will be traversed. Instead, we hashcons [1] all types to reduce structural equality to pointer equality. If the check fails, we call tglb, which computes the greatest lower bound of two types, and then overwrite the RTTI with the result. Next we call cast on the value to cast it from the old RTTI to the new RTTI. After cast returns, we check if the current RTTI is the same as the one we wrote earlier to the heap and write the new value to the heap only if they are indeed equal. Otherwise, a value with a more precise type has already been written to the heap so we discard the current value and return the address.

Reading from a non-static reference proceeds as follows: the value is read from the second word pointed to by the address, the RTTI is read from the first word, then cast is called on the value, the RTTI, and the statically recorded type. For writing, the RTTI will be read first from the heap and then cast will be called on the new value, the statically recorded type and the RTTI. Again, we check if the RTTI has changed during the casting process, if yes, we drop the casted value, otherwise, we write the new value to the heap.

Coercions The story for coercions is similar. The only difference is that the generated code builds coercions out of the RTTI and the other input type with a call to mk_crcn and calls coerce instead of cast.

Coercions and casts together? An astute reader would notice that the implementation details of coercing is very similar to casting in many scenarios with the difference being creating a coercion out of the source and target types and calling coerce instead of cast. Reading and writing partially typed monotonic references is an obvious example of such scenario where a coercion is created only to be immediately consumed. Grift is clever in optimizing such cases by deferring coercion creation until it is actually needed to be stored or composed and uses the types to simulate coercions in other cases. We refer to this as lazy coercion creation. Our experiments show that this optimization results in performance gains on the quicksort and n-body benchmarks and no performance differences on the others.

4.5 Specializing Casts and Coercions

Typically, types and coercions are inspected at runtime when values get casted. However, for many casts, some or all of the types/coercions involved are known at compile time. Grift can recognize such casts and partially evaluate them, generating more efficient code. Consider the following expression in the intermediate language where casts are explicit:

\[ \text{cast } n \text{ Dyn } \text{Int } 1 \]

a straightforward compilation is a cast call which dispatches on types at runtime. But with specialization, it will get compiled to the following efficient code:

\[ \text{TAG(n) == INT\_TYPE) ? UNTAG\_INT(n) : raise\_blame(l) } \]

which is basically the body of the branch inside cast where the type arguments are Dyn and Int.

While lazy coercion creation reduces the number of coercions that get allocated dynamically in certain cases, specialization simplifies the coercions that are known at compile time. Our experiments show that specializing coercions significantly improves some benchmarks, such as Matrix Multiplication, but only slightly improves the rest (Appendix ??).

4.6 Optimizing Dynamically Typed Code

The straightforward approach to inserting casts [41] can cause unnecessary overhead in dynamically typed regions of code. Consider the function \( \lambda (f) (f 42) \) which applies a function \( f \) injected into Dyn. The straightforward cast insertion would compile it to:

\[ \text{(cast } f \text{ Dyn } (\text{Dyn } \rightarrow \text{Dyn) } l) \text{ (cast 42 Int Dyn l)}) \]

The cast on \( f \) will allocate a function proxy if the source type of \( f \) is not \( \text{Dyn } \rightarrow \text{Dyn) } \). Although allocating a proxy is important to maintain the semantics, the allocation is unnecessary in this case because it will be consumed right away. Instead, Grift specializes these cases by generating
5 Performance Evaluation

In this performance evaluation, we seek to answer a number of research questions regarding the runtime overheads associated with gradual typing.

1. What is the overhead of gradual typing? (Sec. 5.2)
   We subdivide this question into the overheads on programs that are (a) statically typed, (b) dynamically typed, and (c) partially typed.

2. What is the cost of achieving space efficiency with coercions? (Section 5.3)

3. How do monotonic references compare with proxied references? (Section 5.4)

5.1 Experimental Methodology

In these experiments we use benchmarks from a number of sources: the well-known Scheme benchmark suite (R6RS) used to evaluate the Larceny [26] and Gambit [17] compilers, the PARSEC benchmarks [8], and the Computer Language Benchmarks Game (CLBG). We do not use all of the benchmarks from these suites due to the limited number of language features currently supported by the Grift compiler. We continue to add benchmarks as Grift grows to support more language features.

For our experiments on partially typed programs, we randomly sample 90 configurations from across the spectrum of type annotations for each benchmark. Our sampling approach starts from a statically-typed program, and at each type annotation in the program, we generate the array of all types that are less precise than that type. We then sample a program from the lattice at a certain percentage of type annotation by generating random indices into these arrays, use them to choose less precise types, and finally, insert the chosen types into the locations of the original type annotations. We use the Grift compiler to compile these programs and measure their runtime performance.

The situation for languages with fine-grained gradual typing, as is the case for Grift, is considerably more difficult because any type annotation, and even any node within a type annotation, may be changed to Dyn, so there are $2^m$ configurations of the $m$ modules. The random indices are generated in a controlled way to make sure the percentage of type annotations in the resultant sample will fall within the range we are aiming for.

This algorithm ensures that when sampling to produce programs with a particular percentage of type annotations, all programs that satisfy the type percentage constraint are equally likely to be chosen. However, the algorithm suffers from large memory consumption. At this time we are unable to run it on the ray tracing benchmark because of the size of its types. As such, we have omitted ray from the partially typed comparisons.

5.2 Gradual Typing Overhead and Comparison

The purpose of this section is to answer research question 1, i.e., what is the overhead of gradual typing? Of course, to ultimately answer this question one would need to consider all benchmarks that we use are:

- **tak** (R6RS) This benchmark, originally a Gabriel benchmark, is a triply recursive integer function related to the Takeuchi function. It performs the call `(tak 4020 12)`. A test of non-tail calls and arithmetic.
- **ray** (R6RS) Ray tracing a simple scene, 20 iterations. A test of floating point arithmetic. Adapted from Example 9.8 of Paul Graham’s book on ANSI Common Lisp.
- **blackscholes** (PARSEC) This benchmark, originally an Intel RMS benchmark, calculates the prices for a portfolio of European options analytically with the Black-Scholes partial differential equation. There is no closed-form expression for the Black-Scholes equation and as such it must be computed numerically.
- **matmult** (textbook) A triply-nested loop for matrix multiplication, with integer elements. The matrix size is 400×400 in the comparisons to other languages (Sec. 5.2.1 and 5.2.2) and 200×200 for the evaluation of partially typed programs (Sec. 5.3 and 5.4).
- **quicksort** (textbook) The standard quicksort algorithm on already-sorted (worst-case) input, with integer arrays of size 10,000 in the comparison to other languages and 1,000 for the partially typed programs.
- **fft** (R6RS) Fast Fourier Transform on 65,536 real-valued points. A test of floating point.
- **n-body** (CLBG) Models the orbits of Jovian planets, using a simple symplectic-integrator.

**Experimental Setup** All experiments were conducted on an unloaded machine with a 4-core Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz processor with 8192 MB of cache and 16 GB of RAM running Red Hat 4.8.5-16. The C compiler we used is Clang 5.0.0, the Gambit compiler is version 4.8.8, Racket is version 6.10.1, and Chez Scheme is version 9.5.1. All time measurements use real time, and 30 samples were collected of each time measurement and the mean reported.
possible implementations. So the actual question we answer is: what is the overhead of gradual typing in an ahead-of-
time compiler that implements space-efficient coercions and
monotonic references? To answer this question, it is im-
portant to isolate the overheads of gradual typing from other
sources of overhead. Thus, we have implemented a variant
of the Grift compiler, named Static Grift, that requires the
input program to be statically typed and does not generate
any code in support of gradual typing. We then compare
(gradually typed) Grift with Static Grift.

Of course, one threat to validity is the possibility that the
performance of Static Grift could be so poor that the over-
heads due to gradual typing are drowned out. To allay this
fear, we include external comparisons to other programming
language implementations, including statically typed lan-
guages such as OCaml, and dynamically typed languages
such as Scheme and Racket. The upshot of this comparison is
that the performance of Grift is in the same ballpark as these
other implementations. It is tempting to interpret these com-
parisons in a competitive light, but we encourage the reader
to abstain from such thoughts. The point is not that we would
convince the reader to start compiling programs with Grift
(you shouldn’t because it does not support a full language),
but to convince implementors of gradually-typed languages
that coercions and monotonic references are worthwhile.

Finally, we note that the semantics of monotonic refer-
ences is different from proxied references, so the Grift com-
piler implements two different gradually typed languages.
We refer to them as Proxied Grift and Monotonic Grift. Both
versions use coercions (not type-based casts), specialize casts,
and perform lazy coercion creation, as described in Section 4.
In this section we report on the performance of both Proxied
Grift and Monotonic Grift.

5.2.1 Evaluation on Statically Typed Programs

Figure 8 shows the results of evaluating the speedup of Prox-
ied and Monotonic Grift with respect to Static Grift on stat-
ically typed versions of the benchmarks. We see that the
performance of Monotonic Grift is no lower than 0.99× of
Static Grift on all the benchmarks whereas the performance
of Proxied Grift sometimes dips to 0.65× that of Static Grift.
To put the performance of Grift in context, it is comparable
to OCaml and better than fully static Typed Racket.

Answer to research question (1 a): the over-
head of gradual typing for statically typed code
is consistently low with monotonic references
but sometimes high with proxied references.

To mitigate the differences between runtime initialization
we use internal timing. For Type Racket we make sure to
use the floating point specialized math operations, but since
there is no safe and well-performing equivalent operation for
fixed width integers we are forced to use the polymorphic

5.2.2 Evaluation on Dynamically Typed Programs

Figure 9 shows the results of evaluating the speedup of Prox-
ied and Monotonic Grift with respect to Racket on dynami-
cally typed versions of the benchmarks. Note that we have
not implemented a “Dynamic Grift” (analogous to Static
Grift, but for dynamic languages), but instead we compare to
math operators for integers. We also make no attempt to
account for differences in garbage collection.
Racket. The figure also includes results for Gambit and Chez Scheme. We see that the performance of Grift is generally higher than Gambit but lower than Racket and Chez Scheme on these benchmarks. This experiment does not tease apart which of these performance differences are due to gradual typing per se and which of them are due to orthogonal differences in implementation, e.g., ahead-of-time versus JIT compilation, quality of general-purpose optimizations, etc. Thus we draw the following conservative conclusion.

**Answer to research question (1 b)** the overhead of gradual typing for dynamically typed code is currently reasonable but there is still some improvement to be made.

5.2.3 Evaluation on Partially Typed Programs

Figure 10 shows the speedup of Proxied and Monotonic Grift with respect to Racket on a large number of partially typed configurations of each benchmark (recall that Section 5.1 discusses the selection of configurations). The x-axis varies the amount of type annotations in the program, from 0% on the left to 100% on the right. The performance of Racket is the horizontal line at 1. We also include a horizontal line for Static Grift, which represents the best performance one could hope for. In Figure 10 we show the results for three benchmarks. We select matmult, blackscholes, and n-body as representatives of best, middle, and worst case comparisons to Racket.

In matmult, the performance of Monotonic Grift is slightly below Racket for the untyped configurations but then climbs to nearly 10× speedup for configurations that are 80% typed or more. The performance of Proxied Grift trails that of Monotonic, but the trend is similar. Note that the mean for Proxied Grift (purple horizontal line) and for Monotonic Grift (green horizontal line) are both well above Racket.

In blackscholes, Grift starts around 0.3× the speed of Racket and then gradually climbs to match Racket at 60% typed and then exceeds Racket by about 2× at 80% typed. The mean for Grift is similar to Racket on this benchmark.

In n-body, Grift again starts out slower than Racket but becomes faster once the configurations are 80% typed. However, in this benchmark the mean for Grift is significantly lower, around 0.6× the speed of Racket.

**Answer to research question (1 c)** the overhead of gradual typing on partially typed code is currently reasonable but there is still some improvement to be made.

5.3 The Runtime Cost of Space Efficiency

In Figure 11 we compare the performance of Grift with type-based casts, to Grift with coercions, which are space-efficient. We compare these two approaches on partially typed configurations of the benchmarks. The quicksort benchmark is the...
We compare Grift with coercions to Grift with type-based casts (both use proxied references) across partially typed programs to evaluate the cost of space-efficiency. The y-axis is speedup over Racket on a log scale. The x-axis is the percentage of types annotations, from 0% to 100%. Note that some configurations of quicksort exhibit catastrophic slowdowns with type-based casts.

Only one that elicits space efficiency problems. In that benchmark, type-based casts exhibit catastrophic performance on some configuration (the circles at the bottom, between 0.001 and 0.002). In fact, in some configurations, the time complexity of quicksort changes from $O(n^2)$ to $O(n^3)!$ This occurs when the array is cast on each recursive call, and each cast adds another proxy that induces overhead on subsequent reads and writes to the array.

On the other hand, the coercion-based approach successfully eliminates these catastrophic slowdowns. In benchmarks that do not elicit space efficiency problems, we see a general trend of the coercions being roughly equal to the performance type-based casts. Across all benchmarks the speedup of coercions over type-based cast is between 0.82x (the fft benchmark) and 1.93x (the matmult benchmark).

**Answer to research question** (2): Space-efficient coercions offer a “pay as you go” cost model. On benchmarks without space-efficiency issues, we sometimes see a mild speedup and sometimes a mild slowdown. However, where space efficiency is needed, coercions eliminate the catastrophic slowdowns.

**5.4 Monotonic versus Proxied References**

We return our attention to Figure 10, but this time with an eye towards evaluating whether monotonic references perform better than proxied references. Indeed, monotonic references are faster by 1.48x in matmult and by 1.08x in n-body. Additionally, the experiment shows that monotonic references match the performance of Static Grift in cases where the benchmark source code is closer to be fully typed.

**Answer to research question** (3): Monotonic references are more efficient than proxied ones on partially typed programs and enable especially low overhead in statically typed code.

**6 Conclusion**

We have presented Grift, a compiler for exploring implementations of gradually typed languages. In particular, we implement several technologies that enable gradual typing: type-base cast, space-efficient coercions, traditional proxied references, and monotonic references. Our experiments show that Grift with monotonic references is competitive with OCaml on statically typed code. For dynamically typed code, Grift is on par with Scheme implementations using both proxied and monotonic implementations. On partially typed code, our experiments show that coercions eliminate the catastrophic slowdowns caused by space-inefficient casts. Furthermore, we see significant speedups (10x) as 60% or more of a program is annotated with types. Future work remains to improve the efficiency of coercions.
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A Values, Macros, and Compose

Figure 12 lists the C structs use to represent values. Figure 13 lists the macros for manipulating values. Figure 14 show the code for compose runtime function which directly follows the equations for compose in Figure 4.

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/* Types */
typedef type* ref_type;
typedef struct {int64_t arity; type ret; type args[]} fun_type;
typedef union {int64_t atm; ref_type* ref; fun_type* fun;} type;
/* Coercions */
typedef struct {type to; blame info;} project_crcn;
typedef type* inject_crcn;
typedef struct {crcn fst; crcn snd;} seq_crcn;
typedef struct {snd_tag second_tag; int32_t arity; crcn ret; crcn args[]} fun_crcn;
typedef struct {snd_tag second_tag; crcn write; crcn read} pref_crcn;
typedef struct {snd_tag second_tag; crcn rtti} mref_crcn;
typedef struct {char* lbl} fail_crcn;
#define ID NULL
/* Values */
#ifdef TYPE_BASED_CASTS
  typedef struct {obj* ref; type source; type target blame info;} ref_proxy;
  typedef struct {void* code; (obj)(*caster)(obj, type, type, blame); obj[];} closure;
#elseif COERCIONS
  typedef struct {obj* ref; crcn cast;} ref_proxy;
  typedef struct {void* code; (obj)(*caster)(obj, type, type, blame); union {crcn coerce; obj[] fvars;} } closure;
#endif
typedef struct {obj value; type source} nonatomic_dyn;
typedef union {int64_t atomic; nonatomic_dyn*} dynamic;
typedef union {int64_t fixnum; double flonum; closure* clos; dynamic dyn} obj;

Figure 12. Value representations
`/* All allocated values have 3 bits that can be used for tagging */
#define TAG(value) (((int64_t)value)&0b111)
#define UNTAG_INT(value) (((int64_t)value)&~0b111)
#define TAG_INT(value, tag) (((int64_t)value)|tag)

/* Macros that manipulate types */
#define HEAD(type) (TAG(type))
#define ARITY(type) (((fun_type)UNTAG_INT(type))\->arity)
#define REF_TYPE(type) (*((ref_type)UNTAG_INT(type)))

/* Macros that manipulate values in the obj union */
#define UNTAG_REF(ref) ((obj*)UNTAG_INT(ref))

#ifdef TYPE_BASED_CASTS
#define UNTAG_FUN(fun) ((closure*)(fun))
#define MK_REF_PROXY(v, s, t, l) (tmp_rp = (ref_proxy*)GC_MALLOC(RP_SIZE), tmp_rp=\value=v, \tmp_rp=\source=s,tmp_rp=\target=t,tmp_rp=\info=l, (obj)TAG_INT(tmp_rp, REF_PROXY_TAG)
#elseif COERCIONS
#define UNTAG_FUN(fun) ((closure*)UNTAG_INT(fun))
#define MK_REF_PROXY(v, c) (tmp_rp = (ref_proxy*)GC_MALLOC(RP_SIZE), tmp_rp=value=v, \tmp_rp=coerce=c, (obj)TAG_INT(tmp_rp, REF_PROXY_TAG)
#endif

/* Macros that manipulate values in the dynamic union */
#define UNTAG_NONATOMIC(value) ((nonatomic_dyn)UNTAG_INT(value))
#define UNTAG(v) ((TAG(v) == INT_TAG) ? (obj)(UNTAG_INT(v)>>3) : \(TAG(v) == UNIT_TAG) ? (obj)UNIT_CONSTANT : ... (obj)UNTAG_NONATOMIC(v).value)
#define TYPE(v) ((TAG(v) == INT_TAG) ? (type)INT_TYPE : (TAG(v) == UNIT_TAG) ? (type)UNIT_TYPE : ... \UNTAG_NONATOMIC(v)\->source)
#define INJECT(v, s) ((s==INT_TYPE) ? TAG_INT(v<<3, INT_TAG) : (source==UNIT_TYPE) ? DYN_UNIT_CONSTANT : ... \(tmp_na = (nonatomic_dyn*)GC_MALLOC(NA_DYN_SIZE), tmp_na=\value=value, tmp_na=\source=s, (obj)tmp_na)

/* Macros that manipulate types in the crcn union */
#define UNTAG_2ND(c) (\(struct \{snd_tag second_tag;\}*UNTAG_INT(c))
#define UNTAG_PRJ, UNTAG_FAIL, UNTAG_SEQ are similar to UNTAG_INJ */
#define UNTAG_INJ(inj) ((inject_crcn)UNTAG_INT(inj))
*/ MK_SEQ, MK_PROJECTION, MK_INJECTION are similar */
#define MK_REF_COERCION(r, w) (tmp_rc = (ref_crcn*)GC_MALLOC(RC_SIZE), tmp_rc=\second_tag=REF_COERCION_TAG,\tmp_rc=\read=r, tmp_rc=\write=w, (crcn)(TAG_INT(tmp_rc, HAS_2ND_TAG)))

Figure 13. Macros for manipulating values
crcn compose(crcn fst, crcn snd) {
    if (fst == ID) { return snd; }
    else if (snd == ID) { return fst; }
    else if (TAG(fst) == SEQUENCE_TAG) {
        sequence s1 = UNTAG_SEQ(fst);
        if (TAG(s1->fst) == PROJECT_TAG) {
            return MK_SEQ(s1->fst, compose(s1->snd, snd));
        } else if (TAG(snd) == FAIL_TAG) {
            return snd;
        } else {
            type src = UNTAG_INJ(s1->snd)->type;
            type tgt = UNTAG_PRJ(s2->fst)->type;
            blame lbl = UNTAG_PRJ(s2->fst)->lbl;
            crcn c = mk_crcn(src, tgt, lbl);
            return compose(compose(seq->fst, c), s2->snd);
        }
    } else if (TAG(snd) == SEQUENCE_TAG) {
        if (TAG(fst) == FAIL) {
            return fst;
        } else {
            crcn c = compose(fst, s2->fst);
            return MK_SEQ(c, UNTAG_SEQ(seq)->snd);)
        } else if (TAG(snd) == FAIL) {
            return TAG(fst) == FAIL ? fst : snd;
        } else if (TAG(fst) == HAS_2ND_TAG) {
            snd_tag tag = UNTAG_2ND(fst)->second_tag;
            if (tag == FUN_COERCION_TAG) {
                return compose_fun(fst, snd);
            } else if (tag == REF_COERCION_TAG) {
                ref_crcn r1 = UNTAG_REF(fst);
                ref_crcn r2 = UNTAG_REF(snd);
                if (read == ID && write == ID) return ID;
                else {
                    crcn c1 = compose(r1->read, r2->read);
                    crcn c2 = compose(r2->write, r1->write);
                    return MK_REF_COERCION(c1, c2);
                }
            } else {
                raise_blame(UNTAG_FAIL(fst)->lbl);
            }
        }
    } else { raise_blame(UNTAG_FAIL(fst)->lbl); }
}

Figure 14. The compose function for normalizing coercions.