Testing Time Reversal Invariance in Exclusive Semileptonic $B$

Meson Decays

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Abstract

We demonstrate that polarization measurements in exclusive semileptonic $B$ decays are powerful tools for unraveling non-standard model sources of $T$-violation. Measurements of the transverse polarization of the $\tau$ lepton in the $B \to D\tau\bar{\nu}$ and $B \to D^*\tau\bar{\nu}$ decays probe separately effective scalar and pseudoscalar $CP$-violating four-Fermi interactions, whereas the $D^*$ polarization in the $B \to D^*\ell\bar{\nu} (\ell = e, \mu)$ decay is sensitive to an effective right-handed quark current interaction. Two $T$-odd polarization structures exist involving the $D^*$ polarization and they can be isolated and studied separately. An estimate of these $T$-odd effects is also given in the context of supersymmetric theories.

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1. Introduction. Testing the standard model (SM) Cabibbo-Kobayashi-Maskawa (CKM) paradigm of CP violation in hadronic B meson decays is a central theme at future B factories. The semileptonic decay modes are of particular interest both because they are relatively clean theoretically and because they have rather large branching ratios. They thus provide a means of accurately measuring the CKM angles, for example. In this letter we examine some of the exclusive semileptonic decay modes of the B meson and argue that polarization measurements in these modes act as sensitive probes of non-standard model sources of time reversal invariance violation. We start by classifying the various T-violating operators in terms of effective four-Fermi interactions. We then calculate several T-odd polarization observables (TOPO’s) in terms of these effective interactions. As an example, we provide estimates of these observables arising from squark family mixings in supersymmetry (SUSY). CPT symmetry will be assumed throughout this work.

It is well known that the SM CKM phase has a negligible effect on CP-violating observables in meson semileptonic decays. By way of contrast, non-SM sources of CP violation could lead to large observable effects in these decays. The first experimental attempt to measure the transverse muon polarization in kaon semileptonic decays has triggered many theoretical discussions of the muon polarization in both the \( K^+ \rightarrow \pi^0 \mu^+ \nu \) (\( K^+_{\mu 3} \)) and \( K^+ \rightarrow \mu^+ \nu \gamma \) (\( K^+_{\mu 2\gamma} \)) decays, both of which will be measured to a high accuracy at the on-going KEK E246 experiment and at a recently proposed BNL experiment. Similar analyses of the lepton polarization in D and B meson semileptonic decays have also been discussed in multi-Higgs models and in supersymmetric theories.

The B mesons have more semileptonic decay channels than do the kaons, which leads to the interesting possibility that different sources of non-SM CP violation in the B system could be disentangled by performing polarization measurements in several different decay modes. Three types of polarization measurements will be discussed in this letter. The first two involve the lepton transverse polarization in the \( B \rightarrow Dl\bar{\nu} \) and \( B \rightarrow D^*l\nu \) decays, which receive contributions from effective scalar and pseudoscalar interactions, respectively. Since the lepton polarization involves a chirality flip, we concentrate on the case \( l=\tau \) in order to...
take advantage of the large $\tau$ mass. The third type of measurement involves the polarization of the vector meson in the $B \to D^* l \nu$ decay. The TOPO’s in this case can in general receive contributions from both an effective pseudoscalar interaction and from an effective right-handed quark current interaction. We find the $l = e, \mu$ modes to be ideal in searching for a right-handed current effect – besides their large branching ratios relative to the $l = \tau$ mode, they have no pseudoscalar contributions, due to the small Yukawa coupling of the lepton. Furthermore, the average $D^*$ polarization for the $l = e, \mu$ decay can be a factor of three bigger than for the $l = \tau$ decay. It is noted that two TOPO’s can be constructed for the $D^*$ polarization, and that they can be separately measured.

We note as an aside that the electromagnetic final state interactions (FSI) in these decays could also lead to $T$-odd correlation effects [5]. Such effects can be minimized by using charged $B$ decays. More efficiently, FSI effects can be eliminated by taking the difference of the TOPO between two $CP$-conjugate processes, either for charged or for neutral $B$ mesons.

2. Form factors. We consider the semileptonic decays $B \to D l \nu$ and $B \to D^* l \nu$, where $l = e, \mu, \tau$. The contributing hadronic matrix elements of the vector and axial-vector currents are parameterized by the form factors $f_{\pm}$ and $F_{V,A}^{\pm}$ as follows:

$$\langle D(p') | \bar{c} \gamma_\mu b | B(p) \rangle = f_+ (p + p')_\mu + f_- (p - p')_\mu$$  \hspace{1cm} (1a)

$$\langle D^*(p', \epsilon) | \bar{c} \gamma^\mu b | B(p) \rangle = i \frac{F_V}{m_B} \epsilon^{\mu \alpha \beta} \epsilon_\alpha^* (p + p')_\beta q$$  \hspace{1cm} (1b)

$$\langle D^*(p', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(p) \rangle = -F_{A0} m_B \epsilon^*_\mu - \frac{F_{A+}}{m_B} (p + p')_\mu \epsilon^* \cdot q - \frac{F_{A-}}{m_B} q_\mu \epsilon^* \cdot q ,$$  \hspace{1cm} (1c)

where $p$ and $p'$ are the four-momenta of the $B$ and $D$ ($D^*$) respectively, $\epsilon$ is the polarization vector of the $D^*$ vector meson, $q = p - p'$, and the form factors are functions of $q^2$. The convention $\epsilon_{0123} = 1$ will be used. The Dirac equation may be applied to the above expressions in order to obtain the corresponding hadronic matrix elements for the scalar and pseudoscalar currents.

The above form factors are relatively real to a good approximation in the SM, and they are expected to be measured more accurately at the $B$ factories. At present we must rely on theoretical models to evaluate them. For numerical estimates, we adopt the results of
heavy quark effective theory (HQET), in which all of the form factors may be expressed in terms of one unknown Isgur-Wise function $\xi(w)$ \[20\], with $w = \frac{m_b^2 + m_c^2 - q^2}{2m_Bm_{D(*)}}$ and $\xi(1) = 1$.

It is most convenient to parametrize the physics of semileptonic meson decays by general effective four-Fermi interactions of the form,

\[
\mathcal{L} = -\frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\alpha (1 - \gamma_5) b \bar{\ell} \gamma^\alpha (1 - \gamma_5) \nu + G_S \bar{c} \ell (1 - \gamma_5) \nu + G_P \bar{c} \gamma_5 b \bar{\ell} (1 - \gamma_5) \nu + G_R \bar{c} \gamma_\alpha (1 + \gamma_5) \bar{\ell} \gamma^\alpha (1 - \gamma_5) \nu + h.c.,
\]  \[2\]

where $G_F$ is the Fermi constant, $V_{cb}$ is the relevant CKM matrix element, and $G_S$, $G_P$ and $G_R$ denote the strengths of the non-SM interactions due to scalar, pseudoscalar and right-handed current exchange respectively. In many models, tensor effects are small compared to the effects due to the effective interactions of Eq. (2). For simplicity, we neglect tensor interactions in this letter. Note that only left-handed neutrinos need to be considered, as $T$ violation involves the interference between the SM amplitude, which contains a left-handed neutrino, and the non-SM amplitude.

The effects of new physics on the decay amplitude are succinctly encapsulated in three dimensionless parameters \[13\].

\[
\Delta_S = \frac{\sqrt{2} G_S}{G_F V_{cb}} \frac{m_B^2}{(m_b - m_c)m_l}, \\ 
\Delta_P = \frac{\sqrt{2} G_P}{G_F V_{cb}} \frac{m_B^2}{(m_b + m_c)m_l}, \\ 
\Delta_R = \frac{\sqrt{2} G_R}{G_F V_{cb}},
\]  \[3\]

where $m_b$, $m_c$, and $m_l$ denote respectively the masses of the bottom and charm quarks and the mass of the lepton $l$. Note that $\Delta_S$ and $\Delta_P$ are typically independent of the lepton mass because of the leptonic Yukawa coupling contained in $G_S$ and $G_P$ \[21\]. These $\Delta$ parameters are in general complex and could give rise to observable $CP$-violating effects.

**3. $\tau$ polarization.** The transverse polarization of the $\tau$ lepton in the $B \rightarrow D^{(*)}\tau\bar{\nu}$ decays is defined as

\[
P_{\tau}^\perp = \frac{d\Gamma(\vec{n}) - d\Gamma(-\vec{n})}{d\Gamma_{\text{total}}},
\]  \[4\]
where \( \vec{n} = \frac{p_D^{(*)} \times p_\tau}{|p_D^{(*)} \times p_\tau|} \) is a unit vector perpendicular to the decay plane in the \( B \) rest frame, \( d\Gamma(\pm \vec{n}) \) is the partial differential width with the \( \tau \) polarization along \( \pm \vec{n} \), and \( d\Gamma_{\text{total}} \) denotes the partial width summed over the polarizations.

In the \( B \) rest frame, it is convenient to introduce the variables \( x = 2p \cdot p'/p^2 = 2E_D^{(*)}/m_B \) and \( y = 2p \cdot p_l/p^2 = 2E_l/m_B \) as a measure of the \( D^{(*)} \) and lepton energies, as well as two dimensionless quantities \( r_D = m_{D^{(*)}}^2/m_B^2 \) and \( r_l = m_l^2/m_B^2 \). The average tau polarization over a region of phase space \( S \) may then be defined as

\[
\bar{P}_\tau \equiv \frac{\int_S dx dy \rho(x, y) P_{\tau}^\perp(x, y)}{\int_S dx dy \rho(x, y)},
\]

in which \( \rho(x, y) \) is proportional to the partial width of the decay in question, \( \frac{d^2\Gamma(B \to D^{(*)} \tau \nu)}{dx dy} = \frac{G_F^2 |V_{cb}|^2}{128\pi^3} \rho_D^{(*)}(x, y) \). This average is a measure of the difference between the number of \( \tau \) leptons with their spins pointing above and below the decay plane divided by the total number of \( \tau \) leptons in the same region of phase space \( S \).

The analysis of the \( B \to D\tau\nu \) decay proceeds in complete analogy with that of the \( K^{+}_{\mu3} \) decay \[7\]. As the TOPO in this case arises from the interference between the vector and scalar hadronic matrix elements, its effect will be directly proportional to the strength of the induced scalar interaction. In the \( B \) rest frame, the transverse polarization of the \( \tau \) lepton is given by

\[
P_{\tau}^\perp(D)(x, y) = -\sigma_D(x, y) Im\Delta_S
\]

\[
\sigma_D(x, y) = h_D(x) \frac{\sqrt{r_\tau}}{\rho_D(x, y)} \sqrt{(x^2 - 4r_D)(y^2 - 4r_\tau) - 4(1 - x - y + \frac{1}{2}xy + r_D + r_\tau)^2}
\]

\[
h_D(x) = 2f_+^2(1 - r_D) + 2f_- f_+(1 - x + r_D).
\]

In HQET, \( h_D(x) \to (1 - r_D)(1 + \frac{x}{2\sqrt{r_D}})\xi^2 \). Various parameterizations of \( \xi(w) \) exist and may be obtained from the literature \[22\]. Notice that whereas \( \rho_D(x, y) \) has a quadratic dependence on \( \xi \), the polarization function \( \sigma_D(x, y) \) is independent of it. The average polarization as defined in Eq. (5) varies slightly for the various forms suggested for \( \xi \). These comments apply generally to polarization observables, including those relating to the \( D^* \) polarization to be discussed below. For numerical estimates we use the form \( \xi(w) = \exp(1 - w) \), which is
consistent with the experimental data [23]. The average $\tau$ polarization over the whole phase space is then given by

$$\overline{P_\tau^{(D)}} = -\overline{\sigma_D} \text{Im}\Delta S = -0.22 \times \text{Im}\Delta S,$$

which depends only on the non-SM effective scalar interactions.

The transverse polarization of the $\tau$ in the decay $B \to D^*\tau\bar{\nu}$ can be similarly calculated,

$$P_{\tau}^{\perp(D^*)} = -\sigma_{D^*}(x, y) \text{Im}\Delta p,$$

where $\sigma_{D^*}(x, y)$ is similar in form to $\sigma_D(x, y)$, with the replacement in Eq. (8) of $\rho_D(x, y)$ by $\rho_{D^*}(x, y)$ and $h_D(x)$ by a function $h_{D^*}(x)$ which depends on the three axial form factors. In HQET, $h_{D^*}(x)$ coincides with $h_D(x)$ to leading order in the $1/m_{b,c}$ expansion (neglecting QCD corrections).

The $\tau$ polarization in the $B \to D^*\tau\bar{\nu}$ decay is sensitive only to an effective pseudoscalar interaction. This can be understood by examining the polarization components of the $D^*$ involved in the interferences between the vector, axial-vector, and pseudoscalar hadronic matrix elements. It can be seen from Eq. (1) that the vector-axial-vector interference involves only the two transverse polarizations of the $D^*$, and that their effects cancel against each other [17]. Therefore the transverse $\tau$ polarization due to this interference vanishes after summing over the $D^*$ polarizations. The $\tau$ polarization receives a non-zero contribution solely from the axial-vector-pseudoscalar interference, which involves only the longitudinal polarization of the $D^*$. This leads to its dependence on the effective pseudoscalar interaction.

Averaging over the whole phase space in this case gives

$$\overline{P_\tau^{(D^*)}} = -\overline{\sigma_{D^*}} \text{Im}\Delta p = -0.067 \times \text{Im}\Delta p.$$  

Comparing Eqs. (7) and (9), we see that the coefficient of the latter is smaller by about a factor of three. This may be understood roughly by looking at the definition of the average polarization given in Eq. (5) and noting that, for the $B \to D^*\tau\bar{\nu}$ decay, effectively only one of the three polarization states of the $D^*$ contributes to the numerator whereas all three contribute to the denominator.
4. \(D^*\) polarization. We now consider the \(D^*\) polarization in the \(B \to D^*\ell\nu\) decay. The polarization of the \(D^*\) can be measured by studying the angular distribution of its decay products in the decays \(D^* \to D\pi\) and \(D^* \to D\gamma\). \(CP\)-violating effects can therefore also be studied by considering \(T\)-odd momentum correlations in a specific four-body final state of the \(B\) decay \([3,24]\). To be completely general, we present our calculation in terms of the \(D^*\) polarization vector. The resulting expressions are then generically applicable to pseudoscalar decays of the type \(P \to V\ell\nu\), in which the vector meson \(V\) may decay differently than the \(D^*\).

Working in the \(B\) rest frame, we denote the three-momenta of the \(D^*\) and \(\ell\) by \(p_{D^*}\) and \(p_\ell\), and define three orthogonal vectors \(\vec{n}_1 \equiv \frac{(p_{D^*} \times p_\ell) \times p_{D^*}}{|(p_{D^*} \times p_\ell) \times p_{D^*}|}\), \(\vec{n}_2 \equiv \frac{p_{D^*} \times p_\ell}{|p_{D^*} \times p_\ell|}\), and \(\vec{n}_3 = \frac{p_{D^*}}{|p_{D^*}|} m_{D^*} E_{D^*}\).

The vector \(\vec{n}_3\) has been chosen such that the constraint \(\epsilon^2 = -1\) becomes symmetric in the \(\vec{n}\)’s; i.e. \((\vec{\epsilon} \cdot \vec{n}_1)^2 + (\vec{\epsilon} \cdot \vec{n}_2)^2 + (\vec{\epsilon} \cdot \vec{n}_3)^2 = 1\). The polarization vectors of the \(D^*\) can be taken to be real, corresponding to plane polarizations. In this linear basis, the TOPO’s have a clear physical interpretation and can be easily constructed. Note that the \(D^*\) polarization projection transverse to the decay plane, \(\vec{\epsilon} \cdot \vec{n}_2\), is \(T\)-odd, while those inside the decay plane, \(\vec{\epsilon} \cdot \vec{n}_1\) and \(\vec{\epsilon} \cdot \vec{n}_3\), are \(T\)-even. TOPO’s arising from interference between different amplitudes should therefore involve the product \((\vec{\epsilon} \cdot \vec{n}_2)(\vec{\epsilon} \cdot \vec{n}_1)\) or \((\vec{\epsilon} \cdot \vec{n}_2)(\vec{\epsilon} \cdot \vec{n}_3)\).

A measure of the \(T\)-odd correlation involving the \(D^*\) polarization can be defined as

\[
P_{D^*}(x, y) = \frac{d\Gamma - d\Gamma'}{d\Gamma_{total}} = \frac{2d\Gamma_{T-odd}}{d\Gamma_{total}},
\]

where \(d\Gamma'\) is obtained by performing a \(T\) transformation on \(d\Gamma\), \(d\Gamma_{T-odd}\) is the \(T\)-odd piece in the partial width, and \(d\Gamma_{total}\) is the partial width summed over \(D^*\) polarizations.

It can be shown \([2]\) that both the \(G_R\) and \(G_P\) interactions of Eq. \((2)\) contribute to \(P_{D^*}\) of Eq. \((10)\), and that the \(G_P\) effect is Yukawa-suppressed by \(r_1 = m_\ell^2/m_B^2\). Thus by using the \(e\) or \(\mu\) mode of the \(B\) decay, the \(G_R\) effect can be isolated and measured. To a good approximation, we may neglect the masses of the electron and muon by setting \(r_1 = m_{\ell}^2/m_B^2 = 0\). The \(D^*\) polarization for the \(B \to D^*\ell\nu\) \((\ell = e, \mu)\) decay is then simply given by
\[ P_{D^*}(x, y) = (\sigma_1(x, y)(\vec{\epsilon} \cdot \vec{n}_1) + \sigma_2(x, y)(\vec{\epsilon} \cdot \vec{n}_3))(\vec{\epsilon} \cdot \vec{n}_2)Im\Delta_R, \]  

with the two polarization functions defined as

\[ \sigma_1(x, y) = -\frac{8(x^2 - 4r_D)y^2 - 32(1 - x - y + \frac{1}{2}xy + r_D)^2}{\rho_{D^*}(x, y)\sqrt{x^2 - 4r_D}} F_{A0}F_V \]  

\[ \sigma_2(x, y) = \frac{8(x + 2y - 2)\sqrt{(x^2 - 4r_D)y^2 - 4(1 - x - y + \frac{1}{2}xy + r_D)^2}}{\rho_{D^*}(x, y)\sqrt{4r_D(x^2 - 4r_D)}} \times F_V \left( F_{A+}(x^2 - 4r_D) + F_{A0}(x - 2r_D) \right). \]

The two terms in the \( D^* \) polarization of Eq. (11) have quite different polarization structures. Note that the term proportional to \( \sigma_1 \) involves transverse components of the polarization vector only, while the term proportional to \( \sigma_2 \) contains both transverse and longitudinal polarization components and would be absent for on-shell massless vector bosons such as the photon. Using symmetry properties of \( \sigma_1 \) and \( \sigma_2 \), two TOPO’s can be constructed which correspond separately to the two polarization structures of Eq. (11).

In order to isolate the \( \sigma_1 \) term of Eq. (11), we observe that \( \rho_{D^*}(x, y)\sigma_2(x, y) \) is antisymmetric under the exchange of lepton and anti-neutrino energies and that the allowed phase space is symmetric under the same exchange. Thus, performing the average in Eq. (5) over the whole phase space (or any region \( S \) of the phase space which is symmetric in the lepton and neutrino energies) eliminates the \( \sigma_2 \) term and leaves only the first polarization structure:

\[ \left. P_{D^*}^{(1)} \right|_{alt} \simeq 0.51 \times (\vec{\epsilon} \cdot \vec{n}_1)(\vec{\epsilon} \cdot \vec{n}_2)Im\Delta_R. \]

This simple form is a consequence of the near masslessness of the lepton (i.e. \( r_l \approx 0 \)) and the symmetry of the integration region. It is valid independent of the functional form of the form factors, which depend only on \( x \).

The second polarization structure may be separated out by making use of the reflection symmetry of \( \rho_{D^*}(x, y)\sigma_1(x, y) \) under \( y \rightarrow 2 - x - y \) and \( x \rightarrow x \). This symmetry amounts to reflecting the lepton energy with respect to its mid-point value \( y_{mid} = (y_{min} + y_{max})/2 = 1 - x/2 \) for a given \( x \). We can thus define the following asymmetric average over the whole phase space to eliminate the \( \sigma_1 \) term,
\[ \bar{P}^{(2)}_{D^*} \equiv \frac{\int dx \left( \int_{y_{\text{mid}}}^{y_{\text{max}}} dy - \int_{y_{\text{min}}}^{y_{\text{mid}}} dy \right) \rho_{D^*}(x, y) P_{D^*}(x, y)}{\int dxdy \rho_{D^*}(x, y)} \approx 0.39 \times (\vec{e} \cdot \vec{n}_2)(\vec{e} \cdot \vec{n}_3) \text{Im} \Delta R. \]  

Unlike the TOPO of Eq. (13), this method of isolating the transverse-longitudinal polarization structure works independent of the lepton mass. In fact, this procedure also eliminates the pseudoscalar contribution which is generally present for non-zero lepton masses. We find, however, that the average \( D^* \) polarization for the \( \tau \) mode using this prescription is about a factor of three smaller than that for the electron or muon mode.

The two observables of Eqs. (13) and (14) can be separately related to two \( T \)-odd momentum correlation observables in the four-body final state \( B \to D^*(D\pi)l\bar{\nu} \) [25]. Complementary measurements of both observables may then be used to provide a consistency check regarding the possible existence of a right-handed current effect.

5. SUSY effects. As an example, we now estimate the size of the TOPO’s from squark family mixings in SUSY. In the supersymmetric standard model with \( R \)-parity conservation, mass matrices of the quarks and squarks are generally expected to be diagonalized by different unitary transformations. The relative rotations in generation space between the \( \tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \) and \( \tilde{d}_R \) squarks and their corresponding quark partners are denoted by \( V_{U_L}, V_{U_R}, V_{D_L}, \) and \( V_{D_R} \) respectively. These mixing matrices appear in the quark-squark-gluino couplings, and the products of different \( V_U \)'s \( (V_D \)'s) are constrained by flavor-changing-neutral-current (FCNC) processes in the up (down) quark sector [26]. Charged current processes, on the other hand, involve products of \( V_U \) and \( V_D \) which may be of order unity without violating the experimental FCNC bound [3][4]. We consider only top and bottom squark loop diagrams in our calculation, as they tend to dominate in the presence of large squark-generational mixings. The relevant mixing matrix elements involved in \( B \) meson semileptonic decays are \( V_{32}^U \) and \( V_{33}^D \). For simplicity, we consider only phases in the mixing matrices and ignore the phases in other SUSY parameters. This is sufficient for an estimate of the maximal \( T \)-odd polarization effects.

The leading contribution to \( \Delta_S \) and \( \Delta_P \) comes from the charged Higgs exchange diagram.
at one loop [13]. The $m_t$-enhanced effect is given by

$$
\Delta_S = -\frac{\alpha_s}{3\pi} I_H \tan \beta \frac{m_B m_t}{m_H^2} \times \frac{\mu + A_t \cot \beta}{m_\tilde{g}} \times \frac{[V_{33} V_{33}^D V_{32}^U]^{*}}{V_{cb}},
$$

(15)

$$
\Delta_P = \frac{\alpha_s}{3\pi} I_H \tan \beta \frac{m_B m_t}{m_H^2} \times \frac{\mu + A_t \cot \beta}{m_\tilde{g}} \times \frac{[V_{33} V_{33}^D V_{32}^U]^{*}}{V_{cb}},
$$

(16)

where $\alpha_s \approx 0.1$ is the QCD coupling evaluated at the mass scale of the sparticles in the loop, $A_t$ is the soft SUSY breaking $A$ term for the top squark, $\mu$ denotes the two Higgs superfields mixing parameter, $\tan \beta$ is the ratio of the two Higgs VEVs, $m_\tilde{g}$ is the mass of the gluino and $V_{ij}^H$ is the mixing matrix in the charged-Higgs-squark coupling $H^\pm u_i^* \tilde{d}_j^L$. $I_H$ is an integral function of order one, defined previously in Refs. [9,13].

To estimate the maximal allowed SUSY effect, we assume $|V_{33}^{DR}| = |V_{33}^{H}| \sim 1$, and take $m_H = 100$ GeV and $\tan \beta = 50$ to saturate the bound $\tan \beta / m_H < 0.5$ GeV$^{-1}$ from the $b \to c \tau \nu$ decay [27]. With maximal squark mixings, $|V_{32}^{UR}| = \frac{1}{\sqrt{2}}$ [28]. Setting $|\mu| = A_t = m_\tilde{g}$ and taking $m_b = 4.5$ GeV, $m_c = 1.5$ GeV, and $I_H = 1$, we find $|\Delta_S| \leq 1.6$ and $|\Delta_P| \leq 0.78$. The $\tau$ polarization in the $B \to D \tau \nu$ decay is then $|\bar{P}_\tau(D)\rangle \leq 0.35$, and for $B \to D^* \tau \nu$ decay $|\bar{P}_\tau(D^*)\rangle \leq 0.05$. Both limits scale as $(\frac{100 \text{ GeV}}{m_H})^{2 \tan \beta} \frac{\text{Im}[V_{33}^{H} V_{33}^{D L} V_{33}^{U R}^*]}{\sqrt{2}/2}$. An effective $G_R$ interaction can be induced at one loop by the $W$-boson exchange with left-right mass insertions in both the top and bottom squark propagators. In this insertion approximation, the $\Delta_R$ parameter is computed to be [13]

$$
\Delta_R = -\frac{\alpha_s}{36\pi} I_0 \frac{m_t m_b(A_t - \mu \cot \beta)(A_b - \mu \tan \beta)}{m_\tilde{g}^4} \frac{V_{33}^{SKM} V_{32}^{UR} V_{33}^{DR} V_{cb}}{V_{cb}},
$$

(17)

where $A_b$ is the soft SUSY breaking $A$ term for the bottom squark, and $V_{ij}^{SKM}$ is the super CKM matrix associated with the $W$-squark coupling $W^+ u_i^* \tilde{d}_j^L$. The integral function $I_0 = 1$ for $m_\tilde{t}_{L,R} = m_\tilde{b}_{L,R} = m_\tilde{g}$, but can be of order 10 for reasonable squark-gluino mass splittings [13]. To estimate the maximal size of $\Delta_R$, we take $I_0 = 5$, $\tan \beta = 50$, $A_t = A_b = |\mu| = m_\tilde{g} = 200$ GeV, $|V_{33}^{DR}| = |V_{33}^{SKM}| = 1$, and $|V_{32}^{UR}| = \frac{1}{\sqrt{2}}$ for maximal squark family mixings. For $|V_{cb}| = 0.04$, we have $|\Delta_R| \leq 0.08$. Therefore the average polarizations of the $D^*$ over all phase space are simply given by $|\bar{P}_D^{(1)}|_{all} < 0.02$ and $|\bar{P}_D^{(2)}|_{all} < 0.016$. 

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where the optimal case occurs when $(\vec{e} \cdot \vec{n}_i)(\vec{e} \cdot \vec{n}_j) = 1/2 \ (i \neq j)$. These limits scale as
\[
\frac{1}{2} \left( \frac{200 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \left( \frac{1}{50} \left( \tan \beta \right) \right) \left( \frac{\text{Im}[V_{33}^{SKM} V_{12}^{UR} V_{33}^{DR}]}{\sqrt{2}/2} \right),
\]
where $M_{\text{SUSY}}$ is the SUSY breaking scale.

6. Conclusion. The $T$-odd polarization observables constructed and estimated in this letter can be separately measured to identify the sources of non-SM $T$ violation in semileptonic $B$ decays, or to put bounds on them. A model-independent comparison can be made among the various TOPO’s by combining the averages given in Eqs. (7,9,13,14) with the branching ratios for the various decay modes. It is particularly interesting to note that the decay $B \rightarrow D^* l \nu \ (l = e, \mu)$ has a branching ratio which is about 10 times that of the $B \rightarrow D \tau \nu$ mode, so that by including both the electron and muon modes in the $D^*$ polarization measurement, one can expect an event rate which is about 20 times higher than that for the $\tau$ polarization measurement in $B \rightarrow D \tau \nu$ decay. It thus appears that polarization measurements in $B$ decays may generically have a better sensitivity to a $CP$-violating right-handed current effect ($\Delta_R$) than to a scalar ($\Delta_S$) or pseudoscalar ($\Delta_P$) interaction.

We have also estimated the maximal size of the various polarization observables from squark family mixings in SUSY. Although these effects arise at the one-loop level, they are not necessarily small if a large mixing between the right-handed charm and top squarks ($V_{32}^{UR}$) exists. The induced scalar effect seems to have a better chance for detection at the planned $B$ factories than do the induced pseudoscalar or right-handed current effects.

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