Data-driven Low-energy Generic Generator for CMD-3

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Abstract. A Monte Carlo generator to simulate events of single-photon annihilation to hadrons at center-of-mass energies below 2 GeV is described. The generator is based on existing data on cross sections of various exclusive channels of $e^+e^-$ annihilation obtained in scan and ISR experiments. It is extensively used in the software packages for analysis of experiments at the Novosibirsk $e^+e^-$ colliders VEPP-2000 and VEPP-4M aimed at high-precision measurements of hadronic cross sections with different applications, e.g. to calculations of the hadronic vacuum polarization for the muon anomalous magnetic moment.

1. Introduction
Since 2010 the VEPP-2000 electron-positron collider in Budker Institute of Nuclear Physics has been operated in the center-of-mass (c.m.) energy range $\sqrt{s} = 320 – 2000$ MeV [1]. The VEPP-2000 has two interaction points in which the Cryogenic Magnetic Detector (CMD-3) [2] and Spherical Neutral Detector (SND) [3] are located. The physics program of experiments includes high-precision measurements of the total and exclusive cross sections of $e^+e^-$ → hadrons ($R$ measurement), studies of excitations of $\rho$, $\omega$, $\phi$ vector mesons, tests of conservation of vector current in $e^+e^-$ annihilation and $\tau$ decays, production of nucleon-antinucleon pairs near threshold, two-photon physics. Cross sections of $e^+e^- \rightarrow$ hadrons have numerous applications, e.g. to evaluation of the hadronic contribution to the muon anomalous magnetic moment $a_\mu$ [4].

Monte Carlo (MC) simulation of signal and background is an important part of data analysis allowing determination of the detection efficiency and comparison of data with expectations to be performed. In this work a new generic MC generator of multihadron production in $e^+e^-$ annihilation, MHG2000, is discussed, in which all hadronic final states kinematically allowed at $\sqrt{s} < 2$ GeV can be generated. Since perturbative Quantum Chromodynamic fails at these energies, standard generators of multihadron production like LUND [5] or PYTHIA [6] cannot be used. For this reason, simulation of background, the main task of MHG2000, is performed by constructing a generator based on experimental data.

The structure of the work is the following. In Section 2, we highlight some details of MHG2000 related to experimental data. In Section 3, all the necessary information for a user is listed on how to run the MHG2000 code. Section 4 describes the generator output and gives some examples of its application. In Section 5, some concluding remarks are made.
2. Use of experimental data

In the MHG2000 generator all experimentally measured processes of $e^+e^-$ annihilation into hadrons up to 2 GeV are used to calculate a total cross section at a given c.m. energy and events of each final state are sampled with a probability proportional to the fraction of its measured cross section in the total one. To simplify the calculations, all the data are approximated by the functions, which are motivated by underlying physics. Information about the final state and corresponding references for the data used in the generator are listed in the table 1 below:

| Process         | Experimental data | Process         | Experimental data       |
|-----------------|-------------------|-----------------|-------------------------|
| $\pi^+\pi^-$    | BaBar[7]          | $\pi^+\pi^0\pi^0$ | DM2[26], CMD2[27]       |
| $\pi^0\gamma$   | SND[8]            | $\pi^+\pi^-\pi^0\eta$ | CMD3[28]               |
| $\eta\gamma$    | SND[9], CMD2[10]  | $K^+K^-\pi^+\pi^-$ | BaBar[29], CMD3[30]    |
| $\rho\rho$      | BaBar[11], CMD3[12]| $K^+K^-\pi^0\pi^0$ | BaBar[29]               |
| $n\pi$          | SND[13]           | $K^-\pi^0\pi^0$  | BaBar[31]               |
| $K^+K^-$         | BaBar[14] CMD2[15]| $K_S^0K_L^0\pi^+\pi^-$ | BaBar[16]              |
| $K_S^0K_L^0$    | BaBar[16], CMD2[17], SND[18] | $K_S^0K_L^0\pi^0\pi^0$ | BaBar[32]              |
| $\pi^+\pi^-\pi^0$ | SND[19], CMD2[20], BaBar[21] | $K^0\pi^-\pi^0$  | BaBar[31]               |
| $\pi^+\pi^-\eta$ | SND[22]          | $K^+\pi^0\pi^-\pi^0$ | BaBar[31]              |
| $K^+K^-\pi^0$   | BaBar[23]         | $2\pi^+2\pi^-\pi^0$ | BaBar[33]              |
| $K^+K^-\pi^-\eta$ | BaBar[23] | $\pi^+\pi^-3\pi^0$ | BaBar[34]              |
| $K^+K^0\pi^+$   | BaBar[23]         | $3\pi^+3\pi^-$   | BaBar[35], CMD3[36]    |
| $K^+K^0\pi^-$   | BaBar[23]         | $2\pi^+2\pi^-2\pi^0$ | BaBar[35]              |
| $\pi^+\pi^-\pi^+\pi^-$ | BaBar[24], CMD3[25] | $\pi^+\pi^-4\pi^0$ | IR                     |

IR in the Table means that at the present time data for that final state are not yet available, so that we use the isospin relations to evaluate the corresponding cross section.

The procedure of selecting a final state to generate consists of three main parts: approximation of all the data at a given energy, calculation of the probabilities of various processes and sampling of the number of the process that will be generated.

An example of the approximation is shown in Fig. 1.

The data shown in Fig. 1 are taken from the BaBar work [7]. These data are approximated by the following function:

$$A(s) = \frac{BW^G_{\rho}(s, m_\rho, \Gamma_\rho)}{1 + c_\rho + c_{\rho^0} + c_{\rho'^0}} + \frac{c_\rho' BW^G_{\rho'}(s, m_{\rho'}, \Gamma_{\rho'})}{1 + c_\rho + c_{\rho^0} + c_{\rho'^0}} + \frac{c_{\rho^0} BW^{KS}_{\rho}(s, m_{\rho^*}, \Gamma_{\rho^*}) + c_{\rho'^0} BW^{KS}_{\rho'}(s, m_{\rho'^*}, \Gamma_{\rho'^*})}{1 + c_\rho + c_{\rho^0} + c_{\rho'^0}}$$

(1)

where

$$A(s) = \frac{\text{scale}}{(3s)^3/2},$$

$$BW^{KS}(s, m, \Gamma) = \frac{m^2 s}{s - m^2 - i m s}$$

and $c_\omega = |c_\omega| e^{i \phi_\omega}$ and $s = x^2$. Parameters of this function obtained from the approximation are shown in Fig. 1. The Gounaris-Sakurai parametrization $BW^{GS}(s, m, \Gamma)$ is described in detail in the paper [7].
3. Input of MHG2000

The following input parameters should be set to use MHG2000. First of all, it is necessary to set the collision c.m. energy in GeV. Then the generation mode is selected: event generation without an ISR photon (the corresponding flag equals zero), event generation with an ISR photon (flag equals one). Then we can create the list of final states that will be generated. For example, only one process $e^+e^-\rightarrow \pi^+\pi^-$ is generated or all the rest 30 processes are generated. The default value corresponds to generation of all the processes currently included. Such a possibility is useful when it is necessary to simulate all background events without the signal.

4. Output of MHG2000

The output of the MHG2000 generator is an event, which is a random set of the four-momenta of all final particles (values of energy and momentum in GeV) and the process number. In the programming language an event from MHG2000 is a structure, which is an element of GenEvent class from HepMC lib. This structure consists of the vertices. The vertex is the element of GenVertex subclass. The vertex is added to the event by GenEvent class method.

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**Figure 1.** Energy dependence of the cross section of the process $e^+e^-\rightarrow \pi^+\pi^-$ of the $\pi^+\pi^-$ invariant mass

For each final state such functions are required to evaluate the total hadron cross section at any c.m. energy up to 2 GeV. The probability of any process at a given c.m. energy is determined as a ratio of its cross section to the total hadron cross section. The number of the final state (process) $n$ is a discrete random variable with a probability density given by the calculated probabilities of all the processes. After selecting a process by sampling this discrete variable an event of this process is generated.

MHG2000 can also generate the final state with an additional ISR photon. In this case the selection of process is changed. For each process its cross section is corrected for radiation by convolving the hadron cross section with the radiator function. The radiator function $D(x, s)$ defined in Ref. [37] is given by the following formula:

$$D(x, s) = \frac{1}{2} \beta(1 - x)^{\beta/2 - 1}(1 + \frac{2}{\beta} - \frac{1}{16} \beta^2(\frac{1}{4} L + \pi^2 - \frac{47}{8})) - \frac{1}{4} \beta(1 + x) + \frac{1}{32} \beta^2(4(1 + x)\ln\frac{1}{1-x} + \frac{1+3x^2}{1-x} \ln\frac{1}{1-x} - 5 - x)$$

with $\beta = \frac{2\alpha}{\pi}(L - 1)$; $L = \ln\frac{s}{m^2}$

Here $x$ is a variable and $s$ is the square of c. m. energy.
Figure 2. Two-photon invariant mass in the $\pi^+\pi^-\pi^0\eta$ ($\eta \rightarrow 2\gamma$) study [28]: MHG2000 simulation without a signal, sum of signal MC and background, data are shown by the full and empty histograms and dots, respectively.

add_vertex. Several vertices are used to simulate events produced at hadron accelerators but only one vertex is used to simulate events produced at $e^+e^-$ machines. The vertex structure consists of the particles. The particle is the element of the GenParticle subclass. The particle is added to the vertex by GenVertex subclass method add_particle_out. The particle includes its code (Particle ID) and its four-momentum. The correspondence between the particle code (Particle ID) and its PDG name is described in more detail in Ref. [38]. The classes and methods of HepMC library are described in Ref. [39].

5. First applications of MHG2000
The main application of MHG2000 is estimation of the hadronic background during data analysis. The generator was applied for estimation of the background in various CMD-3 studies, e.g. those described in Refs. [30, 28]. Fig. 2 shows the invariant mass of two photons for the final state $\pi^+\pi^-\pi^0\eta$ ($\eta \rightarrow 2\gamma$) [28]. It can be seen that energy deposition due to neutral final states is currently somewhat underestimated. Another example is the difference between the collision energy and the sum of four particle energies in the study of the $\pi^+\pi^-\pi^+\pi^-$ final state, see Fig. 3.

The MHG2000 generator was also applied to the measurement of $R$ between 1.84 and 3.05 GeV at the KEDR detector [40], where it was used to estimate systematic uncertainties. The multiplicity distribution produced by various MC generators is presented in Fig. 4. It is clear that the MC distribution produced by MHG2000 agrees reasonably well with the experimental one.

6. Conclusion
The Monte Carlo code MHG2000 provides generation of hadron production in electron positron annihilation at the c.m. energy below 2 GeV. This generator has been successfully applied in several CMD-3 data analyses as an instrument for estimation of the background contribution. Further applications of the code include its usage at other $e^+e^-$ colliders. Work is in process to
Figure 3. Difference between the collision energy and the sum of four particle energies, assuming a pion mass in the $\pi^+\pi^-\pi^+\pi^-$ study: MHG2000 simulation without a signal, signal MC, data are shown by the vertical, horizontal shaded histograms and dots, respectively.

Figure 4. Properties of hadronic events produced in the $uds$ continuum at 1.84 GeV. Here $N$ is the number of events, $N_{trk}$ is the number of tracks in the event. The experimental distribution and MC simulation based on LUARLW, JETSET and MHG2000 are plotted. All distributions are normalized to unity.

extend the list of available hadronic states and expand the c.m. energy range up to 2.5 GeV.

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