Single-photon Mach-Zehnder interferometry for High Schools

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Abstract. The superposition states, allowed by quantum theory, are dramatically diverse with respect to classical states and represent the true quantum states. By using a Mach-Zehnder interferometer we experimentally show how a single photon in a polarization superposition state can produce interference. The experimental setup includes a strongly attenuated laser source, polarizing beam splitters, half-waveplates, polarizers and single-photon detectors. The experimental activity has been performed with high-school students and teachers in several different contexts.

1. Introduction
Since the school year 2014-2015, Quantum Mechanics has been included in the Italian physics curriculum for the final year of scientifically-oriented High Schools (Liceo Scientifico). Most High-school textbooks present the principles of Quantum Mechanics with a historical approach, only discussing the basic experiments and results obtained in the first two decades of XX century. What is usually missing is the presentation of the properties of quantum states approached in a more general framework, for instance by introducing the wavefunction, solution of the Schrödinger equation or by introducing a simplified Dirac representation. As a consequence, the superposition principle and the quantum-mechanical concept of probability, which are at the origin of the weirdness of Quantum Mechanics, remain unexplained. Nevertheless, from the cultural point of view, these concepts are absolutely relevant, also in view of the forthcoming quantum technologies applied to quantum computing and quantum information [1].

To illustrate the existence of superposition states, textbooks often describe single-electron interference occurring when electrons impinge on a double slit one at a time. This is always nothing more than a gedanken experiment, since, unfortunately, this experiment cannot be easily reproduced [2]. The analogous experiment performed with single photons in a superposition of polarization states has been studied in several papers since late ‘90 [3] and extensively discussed in the book “Sneaking a Look at God’s Cards” by Giancarlo Ghirardi [4].

Here we present an experimental realization of a single-photon interference, in which the approximation of the single-photon state is obtained by attenuating a laser beam down to the so-called “single-photon” level, and the light is detected by single-photon avalanche detectors. The experiment can be performed by exploiting the properties of a Mach-Zehnder (MZ) interferometer containing polarizing beam splitters (PBS). With this apparatus, the interference at the output of the second PBS can be observed with any kind of single-mode macroscopic light that propagates in the interferometer by following the two possible internal paths. What is interesting for the discussion of superposition states is that, if diagonally-polarized single photons are injected in the interferometer, an interference pattern can be observed even when the single-photon approximation is fulfilled and the detectors click very rarely. The approach to the physical problem is based on a simplified formalism requiring a reasonable amount of mathematics accessible to students of scientifically oriented High Schools.
We used the experimental apparatus to support training sessions for in-service teachers and advanced courses for senior High-School students.

![Portable experimental apparatus](image1)

**Figure 1.** Portable experimental apparatus: optical setup (left panel) and detectors (right panel) are mounted on two separate breadboards and the light is delivered to detectors through optical fibers.

2. **The approach**

Our path follows the *reductio-ad-absurdum* reasoning described in [4], to demonstrate the wave-particle duality of light using a *gedanken experiment* based on polarization states and birefringent crystals. This research line is one of the best-known achievements of the group of prof. Michelini in Udine [2]. In our approach we use a MZ interferometric setup based on polarizing beam-splitters (PBS), which represents a versatile version of the Young double-slit experiment. We aim at introducing a simple mathematical notation to represent quantum states in a superposition of some quantum properties. The double nature of photons can be demonstrated by measuring the building-up of interference fringes down to the single-photon level. The experimental apparatus also allows for further experimental expansions, for instance towards the discussion of the statistics of photons or the generation of random numbers.

In figure 1 we show the portable setup we built to implement the experiments in High Schools.

2.1. **The experimental setup**

A diagram of the experimental apparatus is drawn in figure 2. A spatially filtered and enlarged laser beam is strongly attenuated, polarized by a sheet polarizer (P) and injected into a MZ interferometer realized with two PBS. Each PBS transmits the horizontal polarization component (H) and reflects the vertical one (V). A half-wave plate is inserted between the polarizer and the first beam splitter to prepare the polarization state entering the interferometer by rotating the polarization angle to 45° with respect to the vertical direction. A second sheet polarizer is placed at the output of the interferometer, also oriented at 45°, and the light passing through it is finally collected by a lens (an achromatic doublet) and focused into an optical fiber that delivers it to a single-photon avalanche detector (SPAD, MPD, Bolzano, Italy, detection efficiency larger than 50%). The output signal of the detector is finally processed by a counter and stored in a computer.

The final result of the measurement is the intensity profile of the interference pattern generated at the output of the interferometer (figure 3).

During the measurements, the apparatus is kept in complete dark by enclosing the setup and the detectors in sealed wooden boxes.
Figure 2. Scheme of the experimental setup. PBS: polarizing beam splitter; HWP: half-wave plate; M: mirror; F: attenuating filter; P: polarizer; L: converging lens system; MOF: multimode optical fiber; SPD: single-photon detector. A slight misalignment of the mirrors is introduced to generate interference fringes and modelled as a phase shift. The moving slit samples the interference pattern.

2.2. The logic path
The experiment follows the step-by-step the reasoning described in Ghirardi’s book [4]. It is based on a reductio-ad-absurdum argument that drives the reader through a series of apparently solid facts, to an unexpected logical contradiction. The full path, as implemented with our apparatus, is described in [5]. Here we summarize the steps of the argument as follows.

With reference to figure 2, a single photon polarized at 45° enters the interferometer at port $a$. From repeated measurements of the number of detected single photons (counts) on the single arms beyond the first PBS ($b$ and $c$), we observe that the photon can be measured only in one of the two arms of the interferometer, so that we can say that half of the photons exit in $b$ and half in $c$.

Thus, we are led to attribute a defined polarization to the photons entering the first PBS, which selects the output port of the PBS, even when we do not measure it.

The consequent description of the light entering the interferometer is a mixture of two populations of photons with orthogonal polarizations: each photon has a defined horizontal or vertical polarization and the collection of many photons displays the properties of a 45°-polarized light.

This model correctly accounts for the number of counts at the output of the second PBS ($e$ and $f$), but it gives the wrong prediction of counts beyond the linear polarizer at the output of the MZ ($g$). Actually, the number of measured counts exhibits a modulation that resembles interference.

The adopted model for the polarization of the photon cannot account for a coherent superposition of paths inside the interferometer and hence cannot predict interference.

The correct result is obtained by assuming that each photon is in a superposition of polarization states.
2.3. Simplified mathematical description

The state entering the interferometer in port $a$ is made by a single photon polarized at 45° that is described as a superposition of vertical and horizontal polarizations, while the state entering in $b$ is in the vacuum state

$$|\psi\rangle_a = |1\rangle_{45°} = \frac{1}{\sqrt{2}} (|1\rangle_H + |1\rangle_V) \quad |\psi\rangle_b = |0\rangle$$ (1)

By virtue of the PBS, which perfectly transmits the horizontal polarization and perfectly reflects the vertical one, at the two outputs of the first PBS we have

$$|\psi\rangle_c = \frac{1}{\sqrt{2}} (|1\rangle_H + |0\rangle_V) \quad |\psi\rangle_d = \frac{1}{\sqrt{2}} (|0\rangle_H - |1\rangle_V)$$ (2)

The two states $|\psi\rangle_c$ and $|\psi\rangle_d$ propagate inside the interferometer and along two different paths having slightly different lengths. The path difference is described by introducing a (position dependent) phase shift on path $d$ so that

$$|\psi\rangle_d = \frac{e^{i\phi}}{\sqrt{2}} (|0\rangle_H - |1\rangle_V)$$ (3)

The states $|\psi\rangle_c$ and $|\psi\rangle_d$ combine in the second PBS and become

$$|\psi\rangle_c = |0\rangle \quad |\psi\rangle_f = \frac{1}{\sqrt{2}} (|1\rangle_H + e^{i\phi} |1\rangle_V)$$ (4)

at its outputs. Note, that the only non-zero light exits at port $f$, being it essentially a rotated version of the input state in $a$. Finally, after passing through the polarizer on port path $f$, the state becomes

$$|\psi\rangle_e = \frac{1}{2\sqrt{2}} (1 + e^{i\phi}) (|1\rangle_H + |1\rangle_V)$$ (5)

so that the probability of detecting a photon irrespective of the polarization reads
\[ p = \frac{1}{\sqrt{2}} \left( 1 + e^{\delta \phi} \right) \]

Since, in general, \( \phi \) is position dependent, equation (6) describes spatial interference fringes (see experimental results in figure 3).

The results in equations (1)-(6) can be analytically evaluated by representing the polarization states in a vector form and applying the Jones calculus to the matrix representation of the optical elements in the apparatus.

3. Discussion

To explore the paradox of interference with single quantum objects, the light used in the apparatus should be a single-photon state, which is extremely difficult to achieve, mainly in a didactic context. Didactic experiments with true single photons have been realized in the past in some universities [6-8], with very expensive equipment. Presently more simplified and less expensive schemes are becoming achievable [9] and also some portable apparatus can be bought [10]. Anyway, almost the totality of the experimental systems include strongly attenuated laser beams, and some wrong statements such as “the single photons emerge when light is attenuated” are frequent in the didactic literature.

In general a light state can be described as a superposition of states at defined number of photons, the so-called Fock states. So, in general, the expression “number of photons entering the interferometer” is meaningless. For instance, coherent superpositions of Fock states describe the laser light and incoherent superpositions describe the thermal light. These states by no means can be approximated by a single-photon state. The complete description of a light state can be achieved with the methods of Quantum Optics, which describe light as a quantized electromagnetic field, where the photons are the field excitations. Of course these methods are beyond the scope of the High School physics programs. The calculations outlined in section 2.3 show that the behavior of the light inside a MZ interferometer with PBS only depends on the properties of the optical elements and on the polarization state of light, but not on the particular light state. In some sense, we can describe the MZ interferometer in an abstract “geometrical” way as a set of boundary conditions set on the field modes, which are in the vacuum state till they are excited in a particular state by shining some light from the outside. The differences among the various states of light (coherent, thermal, Fock) reflect on the photon number statistics measured at the output. Moreover, also the interpretation of the measurement results depends on the light and on the light detector: in the case of classical light, the observed interference pattern maps light intensity, while, for single-photon states, the recorded detection events reconstruct probabilities. Actually, the single-photon state is a particular case of quantum state for which the final result is far from being obvious.

4. Didactic implementation

The experimental activity was initially set-up in 2006 as a laboratory course for Insubria University students and then proposed to High School classes in the framework of the “Progetto Lauree Scientifiche” (PLS) of the Italian Ministry of Education. Students had to come to University to perform the experiment. In 2009 we realized a portable version of the apparatus, reasonably stable and easy to align, to bring the experiment directly in the High Schools.

While performing the experiment, the students are guided to follow the path described in [4]. Time is necessary to proceed very slowly to avoid logical errors. The topic is rather difficult and the reasoning unusual, closer to philosophy than to experimental physics, but part of the interest of the subject is in its oddity. Most of the didactic efforts are concentrated in discussing the differences between classical and quantum description of physical states (state superposition) and of probability. At the end of the activity, the students are able to discuss the various steps of the logical path and to correctly make previsions on the output of the interferometer. Moreover they can understand the experimental implementation of the theoretical path. In some implementations of the activities, the students took data and analyzed them.
As a didactic strategy, we proposed the experiment either to entire class groups, together with their physics teachers, so that the activity could be inserted in the physics curriculum, or to self-selected groups of interested students, as a university career counselling [11]. In the first case, the entire teaching/learning sequence required about 10 hour work: 4 hours with teachers at the university, 4 hours with students at school (activity leaded by teachers), 2 hours for the experimental activity with students and teachers. This choice involves students having different levels of mathematical and physics skills and different levels of interest in the subject. For this reason, the satisfaction of the students was variable, strongly dependent on the personal involvement of the class teachers. In fact, not surprisingly, the best results and appreciation came from students whose teachers took active part in the preparation of the activity, by attending the introductory lectures at University and by developing suitable lectures for their students. The second choice brings together much more interested and skilled students and opens the possibility of deeper discussions based on the questions put forth by the students themselves. In this case, the satisfaction of the students was very high. The time allocated for this activity is 4-6 hours. The collaboration with students and their teachers led to the production of certain number of works presented at the High School final exam and to the participation to some national workshops. Moreover some teachers participating in the activity wrote educational papers describing the topic and the development of the activity [12].

5. Conclusion

Even if the topic of the superposition states is hard to handle even for specialists, we have demonstrated that the subject can be made understandable to High-School students, provided that enough time is allocated for the discussion of all the questions coming from the students.

We are presently working on the development of a mathematical formulation of the transformations inside the MZ interferometer by using simplified Dirac vectors and transformation matrices, accessible to 4th year high-school students. Each optical element in the apparatus (mirror, polarizer, PBS, phase shift, wave-plates...) can be described by matrices operating on polarization vectors belonging to a bi-dimensional space.

From the didactical point of view, in the framework of the PLS we have activated a collaboration with a group of in-service physics teachers to develop a didactic module based on the discussion of superposition quantum states and supported by experiments performed with our MZ interferometer. Teachers will participate in a comprehensive training (November-January), then will implement the path in the classes (February-April). The experiments will be performed in university laboratories.

The point is not to teach Quantum Optics to High School students, but to teach them a critical approach to the concepts and to be careful about the hypotheses we want to test: the idea of photon as a light bullet is abused ever since Einstein paper.

As a final observation, we note that the experimental apparatus can be also used to perform experiments with coherent light and introduce the idea of statistics and granularity of light just talking about detection. We can also address other aspects of the quantum nature of light, such as the statistical nature of the measured number of photons, and to show possible applications of the quantum properties, such as the conditional generation of random numbers.

The final goal of the activity with the teachers is to introduce some topics of quantum information and quantum computing.

6. References

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