Probing the largest cosmological scales with the CMB-Velocity correlation

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Cross-correlation between the CMB and large-scale structure is a powerful probe of dark-energy and gravity on the largest physical scales. We introduce a novel estimator, the CMB-velocity correlation, that has most of his power on large scales and that, at low redshift, delivers up to factor of two higher signal-to-noise ratio than the recently detected CMB-dark matter density correlation expected from the Integrated Sachs-Wolfe effect. We propose to use a combination of peculiar velocities measured from supernovae type Ia and kinetic Sunyaev-Zeldovich cluster surveys to reveal this signal and forecast dark-energy constraints that can be achieved with future surveys. We stress that low redshift peculiar velocity measurements should be exploited with complementary deeper large-scale structure surveys for precision cosmology.

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INTRODUCTION

The Integrated Sachs-Wolfe (ISW) effect carries information on the evolution of the universe at low and moderate redshifts ($z < 3$) and thus is a powerful probe of dark-energy (DE) properties [1–3]. Since the ISW effect is most sensitive to gravity on the largest scales (hundreds of Mpc), it also offers a novel way to distinguish dark-energy from modified gravity theories [4–6]. Measuring the ISW signal that contributes to the total CMB temperature power spectrum is mainly limited by sampling variance errors on large scales, that has traditionally challenged its detection. An alternative path to extracting the ISW effect is by cross-correlating CMB maps with tracers of the large-scale structure, such as galaxies.

Recently, first detections of the ISW effect from cross-correlation analyses of WMAP data with different galaxy surveys, and x-ray maps have been obtained [7–13]. Reported detections are at the $2-4\,\sigma$ level and favor a DE dominated universe, independent of Supernovae type Ia (SNe hereafter). Future galaxy surveys (such as DES, Pan-STARRS, WFMOS) hold the promise of raising the significance of these detections by going deeper and wider to optimally sample the ISW signal. Primary anisotropies of the CMB are the dominant source of noise in such correlation analyses that seriously limits the ability with which this probe can constrain cosmology [14, 15]. In addition, CMB-galaxy correlations are affected by systematics such as galaxy bias and shot noise. Moreover reconstructing the gravitational potential (and its time evolution) from the density field is a noisy process involving second order spatial derivatives.

Alternatively, velocity flow measurements from estimates of the luminosity distance and redshift are unbiased tracers of the gravitational potential, and the latter can be simply reconstructed through first spatial derivatives of the previous. Also, these measurements are not affected by shot-noise unlike galaxy densities (specially in deep samples). Peculiar velocities have more power on the largest (linear) scales than the dark matter density, what follows from the continuity equation (see below), and thus provides a natural counterpart to the density field in reconstructing the gravitational potential. However probing large-scale velocity flows is not an easy endeavor. Although galaxy surveys now sample large volumes, the intrinsic inaccuracy plaguing distance estimations (20-25%) limits the volume available for such measurements [16–18]. Nonetheless better distance indicators and other probes of the velocity fields are already used in cosmology but their power to reveal the ISW effect and the physics behind it has been neglected so far: SNe currently yield distance measurements with a 5%-10% error [19, 20] and kinetic Sunyaev-Zeldovich (kSZ, [21]) could in principle deliver redshift independent velocity errors around $100\,\text{km.s}^{-1}$. These surveys already allow to map the large-scale velocity field at low $z$ with a good agreement with other probes [19, 22, 23].

In this paper we propose to use the large-scale velocity flows from low-redshift SNe and kSZ cluster surveys to measure the ISW effect from the CMB-velocity correlation. We propose to include velocity measurements at low redshift in combination with deeper large-scale structure probes in tomographic analyses to better probe DE and gravity on the largest scales. The complementarity of velocities with respect to densities comes from the larger signal-to-noise of the CMB-velocity correlation estimator at low redshift (up to a factor of 2 gain depending on cosmology), as we will show in \Sbelow. Unless otherwise stated, we will focus on flat $\Lambda$CDM models with $\Omega_{DE} = 0.75$, $\Omega_b = 0.05,n_s = 1,h = 0.7,\sigma_8 = 0.9$, and assume large-scale structure sources follow a density distribution $dn/dz \propto z^2 \exp[-(z/\sqrt{2\sigma_m})^{1.5}]$, with a width, $\sigma_z \approx z_m/2$, where $z_m$ is the survey median redshift.


**MODELLING THE OBSERVABLES**

Dynamics of the fluctuations of the cosmological fields on the largest scales can be accurately described by linear perturbation theory. In this regime, the Poisson and continuity equation in Fourier space are given by (Peebles 1980, §2) \(-k^2/a^2\Phi(k,t) = 3/2H^2\Omega D\delta(k,t_0)\), where we have used that \(H^2 = (8\pi G/3)\bar{\rho}/\Omega\), with \(\bar{\rho}\) being the mean matter density, \(a(t)\) is the FRW scale factor, \(k\) is the comoving wavenumber and \(D(t)\) accounts for the linear growth of dark-matter density perturbations. The linear continuity equation reads, \(\mathbf{v}(k,t) = -i\bar{D}a \delta(k,t_0)/k\mathbf{k}\) where \(k = k/k\) and \(\delta\) denotes derivative with respect to comoving time \(t\). Combining both Fourier equations we get the linear evolution of velocity modes as a function of the present day gravitational potential \(\Phi(k,t_0)\). The line evolution of gravitational potential modes and \(g(t) = 2\bar{a}^2/3\Omega_0\sqrt{\Omega_m/\Omega_\Lambda^3 + \Omega_\Lambda} \, dD/da\).

Angular cross-power spectra between the CMB temperature-velocity, \(TV\) (top), and the temperature-density, \(TD\) (bottom), for different mean survey depths, \(z_m\). Most of the signal in the \(TV\) correlation comes from larger scales (lower multipoles) than the \(TD\). We amplify the velocity correlation signal by 100 to match the ISW amplitude from the density correlation.

**OBSERVATIONAL PROSPECTS**

Current probes of the velocity flow on large scales include galaxy peculiar velocities, SNe galaxy host peculiar velocities and the kinetic Sunyaev-Zel’dovich effect (kSZ). The radial component of peculiar velocities of galaxies and SNe, \(v\) is determined by inferring their distance, \(d\), and then subtracting off the Hubble flow contribution to the measured redshift, \(z\), as \(v = cz - H_0d\) [34]. The empirical correlation – between light-curve shape and luminosity, and between color-curve shape and extinction – used to infer the SNe luminosity distance currently yield a dispersion in apparent magnitude, \(m\), of \(m_0 = 0.1 - 0.15\). Since \(m\) is related to the luminosity distance in Mpc, \(d_L\), and the absolute magnitude, \(M\), as \(m = 5\log_{10} d_L + M\), a dispersion of \(\sigma(m) = 0.1\) entails a redshift independent distance measurements errors of 5% when ignoring the effect of the marginalization over the constant \(M\). Therefore measuring the velocity of the host galaxy of one SNe can be as accurate as the velocity obtained from 25 galaxies using the \(D - \sigma\) relation.
In turn, the redshift for nearby galaxies (SNe host or not) is currently measured using narrow lines from the host galaxies with an uncertainty $\sigma(cz) = 30\text{km}\cdot\text{s}^{-1}$ that is dominated by statistical errors. As such for a Hubble constant of $h = 0.72$, errors in redshift measurements will be subdominant as compared to errors in $H_0d$ for $z > 0.02$ (0.004) if we use SNe (galaxy) based distance estimators, respectively. Since galaxy peculiar radial velocities typically have a rms of $300\text{ km}\cdot\text{s}^{-1}$ we reach a signal-to-noise ratio $S/N \approx 1$ per source at $z \approx 0.02 (0.004)$. If we were to volume-average the distance measurements coming from SNe observations over samples at fixed $z$, the systematic error limit would be $\sigma(m_i) = 0.02 [27]$. Assuming that we had enough SNe to reach this limit within boxes of depth $\sigma(cz) = 30\text{km}\cdot\text{s}^{-1}$, then one could get a 1% distance measurement and the $S/N$ per source would reach $1$ at $z \approx 0.1$.

On the other hand, the kSZ determined peculiar velocities of galaxy clusters do not rely on a distance determination, and associated $z$-independent errors of $\sim 100\text{ km}\cdot\text{s}^{-1}$ may be achievable [28, 29]. However the signal is limited to $z \gtrsim 0.1$ in order for the primordial CMB not to be a dominant source of confusion. Its power as a tracer of the large-scale gravitational potential has already been studied in [24] and as a DE probe in conjunction with other density tracers in [30, 31].

The nature of the signal we are investigating demands wide but rather shallow surveys in comparison to other probes. To illustrate the detectability of this new correlation, we will consider two full-sky surveys, with a distribution of sources with median redshift $z_m = 0.15$ or $z_m = 0.3$, and study their performance in ISW detection and DE constraints as compared to the $T\delta$ correlation. In particular, we propose to combine full-sky peculiar velocity surveys from SNe at $z \lesssim 0.1$ and, complementarily, kSZ cluster surveys for sources at $z \gtrsim 0.1$. We define the cross-correlation signal-to-noise ratio $S/N$ as

$$
(S/N)^2 = \sum_\ell (2\ell + 1) f_{\text{sky}} \frac{(C_\ell^X)^2}{(C_\ell^X)^2 + C_\ell^{vv} C_\ell^{\delta\delta}}
$$

where $X = v$ or $\delta$. Fig. 3 shows the contribution of different multipoles to the $S/N$ depending on survey depth, for a $\Lambda$CDM model with $\Omega_{DE} = 0.75$ that will be our baseline. The area under the curves give the total $(S/N)^2$. We find that for low $z$, $T\nu$ peaks at similar (albeit slightly larger) $\ell$’s than $T\delta$, i.e., $\ell \sim 10 - 20$. For our baseline cosmology one gets, for the $z_m = 0.3$ survey, $(S/N)_{T\nu} = 5$, that is 25% larger than the corresponding significance for the ISW detection from the $T\delta$ correlation. The shallower survey, $z_m = 0.15$, leads to more moderate significance, $(S/N)_{T\nu} = 3.2$. However, this is about factor of 2 larger than that for $T\delta$. This comes from the fact that the lowest redshift sources give the dominant contribution to the overall $S/N$. In other words, as shown by Fig. 2, progressively deeper velocity surveys are less optimal for ISW measurements since the $T\nu$ signal quickly drops with redshift unlike $T\delta$. In general, the relative gain in $S/N$ when using velocities as compared to densities can be understood by computing the ratio of $S/N$ for $T\nu$ with respect to $T\delta$ for a given multipole $\ell$, that scales as $(C_\ell^{TV}/C_\ell^{T\delta}) \sqrt{C_\ell^{\delta\delta}/C_\ell^{vv}}$. Although at low multipoles

**FIG. 2:** Contribution to the ISW effect power spectra from different 3D wavenumbers, $k$, and thin redshift $z$ shells, for various multipoles $\ell$. As in Fig. 1, top panels display the $T\nu$ kernel, amplified by a factor of 100, whereas the bottom panels show the $T\delta$ kernel. Kernels are given by $\ell(\ell+1)/2\pi$ times the integrand in eq. (1).

**FIG. 3:** Contribution of multipoles to the signal-to-noise ratio $S/N$ for $T\nu$ (solid) and $T\delta$ (dashed) for full-sky surveys with different depths, for our baseline $\Lambda$CDM model. Total $(S/N)^2$ is given by the area under the curves. Velocities give a $S/N$ in cross-correlation with the temperature that is systematically larger than that for densities, $T\delta$, the gain factor depending on survey depth and cosmology. Note that $S/N$ scales as $\sqrt{f_{\text{sky}}}$, where $f_{\text{sky}}$ is the fraction of sky covered by the survey.
the signal is about two orders of magnitude smaller for velocities, \( (C_{\ell}^{T_v}/C_{\ell}^{T_\delta}) \sim 0.01 \) (see Fig. 1), the corresponding ratio of noise terms arising from the auto-correlations \( C_{\ell}^{\delta\delta}/C_{\ell}^{uu} \gtrsim 10^4 \), what makes the relative S/N exceed unity. We note that this \( T_v \) significance gain over \( T_\delta \) increases for more strongly DE dominated cosmologies. In particular, for a shallow survey \( (z_m = 0.15) \), velocities can measure the cosmic low multipoles with a significance \( \gtrsim 2 \) times larger than dark-matter density tracers, such as galaxies, in cross-correlation with the CMB.

**COSMOLOGICAL IMPACT AND CONCLUSIONS**

In the context of flat CDM models, an increased significance in the ISW measurement from the \( T_v \) correlation directly translates into tighter DE constraints. Fig. 4 displays the expected parameter constraints that can be achieved for two shallow full-sky surveys. For comparison purposes, we show constraints from \( T_v \) (solid lines) and the usual \( T_\delta \) (dashed) correlations. In general, the \( T_v \) estimator yields better DE constraints than the \( T_\delta \) irrespective of the baseline cosmology used or survey depth, provided \( z_m \lesssim 0.3 \), beyond which the usual \( T_\delta \) cross-correlation outperforms \( T_v \). A shallow velocity survey with \( z_m = 0.15 \) can deliver a (symmetrized) error \( \sigma(\Omega_{DE}) = 0.065 \) for our baseline \( \Lambda \)CDM cosmology, that is almost a factor of 2 better accuracy than what densities can deliver (see e.g. lower left panel in Fig 4). These constraints are comparable to what is expected for \( \sigma(\Omega_{DE}) \) from PLANCK [35] (see comparison between thin and thick solid lines in the left panels), although such an outstanding CMB dataset will largely help breaking the degeneracy with \( w \). For the deeper survey, \( z_m = 0.3 \), velocities yield a factor of 2 better measurement of the DE density \( \sigma(\Omega_{DE}) = 0.03 \) than the shallower survey (solid contours in right panels as compared to same lines in left panels), but this is comparable to what a density tracer would deliver (dashed contours in corresponding panels). Velocities can also help breaking the degeneracy with \( w \), but, as expected from a low redshift survey, to a much lower extent than PLANCK. Note however how the DE errors depend on the baseline cosmology used: \( T_v \) measurements can provide significantly sharper constraints on quintessence models than on \( \Lambda \)CDM and yield better measurements of \( \Omega_{DE} \) than PLANCK. This is shown in the lower panels of Fig 4 where errors from the \( T_v \) correlation alone (thin solid lines) do not improve when adding PLANCK priors (thick solid lines).

In practice several factors could degrade these optimal expectations. In cross-correlations \( S/N \propto \sigma_8 \) and thus current estimates of \( \sigma_8 \approx 0.8 \) would imply a 10% weaker detection than we estimated from our baseline model. Lensing could introduce noise in \( T_v \) as we go deeper in redshift although its large scale contribution is smaller. More importantly, we neglected so far the fact that we deal with a discrete sampling of the underlying velocity field. Note that for velocities no shot noise affects their measurement, unlike the case for densities. However, if we consider the case for a \( z_m = 0.3 \) survey, 90% of sources lie below \( z = 0.8 \). Since there are around \( 10^5 \) clusters with masses greater than \( 10^{14} h^{-1} M_\odot \) in this volume, a full-sky cluster kSZ survey up to \( z = 0.8 \) would provide a good sampling of the field for modes \( k \lesssim 0.1 \) Mpc\(^{-1} \) which is sufficient for the CMB-velocity signal according to Fig. 2.

In conclusion, the use of wide and shallow velocity surveys can substantially increase our ability to extract the ISW signal from a cross-correlation with the CMB and thus help other cosmological probes to distinguish gravity from alternatives. Peculiar velocities are highly correlated with other large-scale structure tracers such as the galaxy density distribution but probe larger physical scales. In addition, the newly proposed correlation is a priori less affected by systematics that plague the temperature-galaxy correlation such as galaxy bias and shot noise. In particular, combining shallow velocity surveys with complementary data from planned deep surveys (such as LSST, DES, Pan-STARRS, WFMOS) can fully exploit the ability of tomographic analyses of future surveys to reveal the nature of dark-energy and gravity on the largest physical scales [32, 33]. Furthermore,
the temperature-velocity correlation is a more powerful probe of the CMB lowest multipoles than the standard correlation with dark-matter density tracers, such as galaxies, and hence it could shed new light on inflation and the physics of the primordial universe.

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[34] we ignored here the velocity of the observer since it is well measured through the CMB dipole
[35] we use Fisher matrix priors from PLANCK temperature and polarization data, as given by [32], i.e, we assume $\sigma(\Omega_{DE}) = 0.1$, $\sigma(w) = 0.32$ for $\Lambda$CDM with $\Omega_{DE} = 0.75$, and $\sigma(\Omega_{DE}) = 0.2$, $\sigma(w) = 0.5$ for a quintessence model with $w = -2/3$ and $\Omega_{DE} = 0.75$. 

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