We explore the use of policy approximation for reducing the computational cost of learning Nash equilibria in multi-agent reinforcement learning scenarios. We propose a new algorithm for zero-sum stochastic games in which each agent simultaneously learns a Nash policy and an entropy-regularized policy. The two policies help each other towards convergence: the former guides the latter to the desired Nash equilibrium, while the latter serves as an efficient approximation of the former. We demonstrate the possibility of using the proposed algorithm to transfer previous training experiences to different environments, enabling the agents to adapt quickly to new tasks. We also provide a dynamic hyper-parameter scheduling scheme for further expedited convergence. Empirical results applied to a number of stochastic games show that the proposed algorithm converges to the Nash equilibrium while exhibiting a major speed-up over existing algorithms.

Keywords: Reinforcement Learning, Nash Equilibrium, Game Theory

1 Introduction

Both biological and artificial intelligent agents tend to follow goal-directed behaviors in order to survive and achieve desired outcomes. Learning to choose actions that maximize rewards is the foundation behind reinforcement learning (RL) [1]. While the success of RL has been demonstrated in many single agent domains [2, 3], more work is needed to extend the same results to multi-agent scenarios. The difficulty of extending single-agent RL methods to multi-agent scenarios stems from the fact that the interactions between the agents make learning difficult, since changes in the policy of one agent will affect that of the others, and vice versa [4].

One widely adopted framework to address multi-agent systems is via Stochastic Games (SG). The resulting multi-agent reinforcement learning (MARL) framework assumes a group of autonomous agents that share a common environment in which the agents choose actions independently and interact with each other [5] to reach an equilibrium. When all agents are rational, and under the assumption of common knowledge, the most natural solution concept is the one of a Nash Equilibrium (NE) [6–8].

The blanket assumptions of perfect rationality makes computing NE challenging for many scenarios, especially those involving more than a handful of alternative strategies. There is a need for algorithms that generate rational, yet computationally efficient policies that can be used in a wide variety of multi-agent situations. Several approaches have been proposed to find alternative ways to compute policies in multi-agent scenarios. One popular approach is to simplify the learning problem [9][12] and thus allow the agents to compute “fast” but less rational policies. Such algorithms are unlikely to converge to a rational (Nash) equilibrium, however. Other algorithms learn directly a NE using computationally demanding operators [13][16], such as Minimax-Q [13] and Nash-Q [14]. Under certain conditions, agents using these algorithms are more logical/rational (in the sense that they try to compute NE) but in doing so, they tend to take more time to compute the corresponding policies.

Intelligent agents also tend to efficiently utilize prior knowledge to learn new tasks and develop important competencies [17]. Entropy-regularized soft Q-learning [18], originally introduced to address the maximization-bias, has the potential of transferring previous experiences to new environments by incorporating a prior (a reference policy or a previously learnt policy) thus restricting exploration only to policies close to the prior. The soft Q-learning idea was recently extended to two-agent zero-sum and team games [19]. This two-agent Soft-Q algorithm avoids the use of the expensive Nash operator to update the game value at each learning step: the two agents, instead, compute closed-form soft-optimal policies under entropy regularization that explore the policy space close to the priors. Since none of the agents computes Nash policy, the output policies of the two-agent Soft-Q may be far from optimal.
To achieve convergence to NE while maintaining computational efficiency, we propose a new entropy-regularized Q-learning algorithm for zero-sum games, called Soft Nash Q-learning (SNQ2). The proposed algorithm learns two different Q-values: the standard Q-value and the soft Q-value. The two values are used asynchronously in a “feedback” learning mechanism: the soft-Q policies act as an efficient approximation of the Nash policies to update the standard Q-value; the Nash policies from the standard Q-value are periodically used to update the priors guiding the soft-Q policies. Consequently, SNQ2 significantly reduces the frequency of using the expensive Nash operator (Minimax in the zero-sum case), and thus expedites convergence to the NE. Furthermore, the priors used in the soft-Q policy updates provide the opportunity to transfer knowledge from previous training experience or from human experts to a new environment, and thus “warm-start” the learning process. This is demonstrated in the numerical examples section of the paper.

Since the balance between the learning of the two Q-values plays a critical role in the performance of the SNQ2 algorithm, we also introduce a dynamic scheduling scheme that actively changes the frequency of the updates of the priors with the new Nash policies. We empirically show the convergence of SNQ2 to a Nash equilibrium for several stochastic games demonstrating major speed-up over existing algorithms.

2 Related Work

Games, first explored in the economics community [20, 21], offer a natural framework to generalize single-agent Markov Decision Processes [6] to multi-agent settings. The simplest approach to extend learning in multi-agent settings is to use independently learning agents. This was the approach with the use of Q-learning in [22], but such approach fails in many cases due to the nonstationarity of the environment [4]. Some previous works addressed nonstationarity by introducing an extra mechanism, such as centralized critic [10, 23], conjectures on other agents’ policies [9], variable learning rates [12], and neural fictitious play [24].

Many of these works ensure convergence to a NE only in simple versions of stochastic games, such as repeated games [25]. Other algorithms are designed specifically for cooperative settings, such as, optimistic and hysteretic Q-updates [26–28], policy parameter sharing [29] and applications to Markov team games [16]. Deep Learning techniques, such as deep Q-learning, have also been investigated for multi-agent scenarios; see, for example [30]. While these approaches may be computationally tractable, they are either not applicable in competitive settings or lack guarantees of convergence to a NE, a critical concept in competitive games [7]. Approaches that focus on solving for NE policies on the other hand [13–15], although principled, are generally computationally demanding due to the complexity of computing a NE [31]. Different methods have been investigated to alleviate this problem. The algorithm in [32] applies a mean field approximation to simplify the interactions within a population of agents to that of a single agent and the average effect from the rest of the population. The algorithm in [33] uses approximate dynamic programming to reduce complexity. M3DDPG [34] uses a minimax formulation to extend the popular MADDPG [10] algorithm to competitive settings. These algorithms still rely on performing rather expensive minimax operations, and they could still potentially benefit from the ideas utilized in the proposed SNQ2 algorithm.

3 Background

Two-agent Zero-sum Stochastic Games. In two-agent stochastic games two agents, henceforth referred to as the Player (pl) and the Opponent (op), interact in the same stochastic environment. In this paper we are interested, in particular, in zero-sum games where one of the agent’s gain is the other agent’s loss [7]. A zero-sum stochastic game is formalized by the tuple $$(S, A^p, A^o, T, \mathcal{R}, \gamma)$$, where $S$ denotes a finite state space, and $A^p$ and $A^o$ are the finite action spaces of the two agents. To choose actions, the Player uses a stochastic policy $π^p : S × A^p → [0, 1]$ and the Opponent uses $π^o : S × A^o → [0, 1]$, which together produce the next state according to the state transition function $T : S × S × A^p × A^o → [0, 1]$. As a consequence of simultaneously taking these actions, the agents receive a reward $R : S × A^p × A^o → [R_{\min}, R_{\max}]$. The constant $γ ∈ (0, 1)$ represents the reward discount factor across time. For zero-sum games, the Player seeks to maximize the total expected reward, whereas the Opponent seeks to minimize it. We denote the value at each state induced by the policy pair $π = (π^p, π^o)$ as $V^π(s) = E_{π}^s \left[ \sum_{t=0}^{∞} γ^t R(s_t, a^p_t, a^o_t) \right]$.

Nash Equilibrium. The classical solution to a zero-sum game is the Nash equilibrium (NE) [21]. Operating at a NE, no agent can gain by unilaterally deviating from the equilibrium. Even though multiple Nash equilibria could exist in a zero-sum SG, the optimal value at a NE is unique and can be computed [7] via $V^*(s) = \max_{π^p} \min_{π^o} V^{π^pπ^o}(s)$. The resulting optimal Q-value at a NE is then computed via $Q^*(s, a^p, a^o) = R(s, a^p, a^o) + γE_{s'} \left[ V^*(s') \right]$.

Minimax Q-Learning. The unique value property of zero-sum games enables the development of algorithms that learn the NE of such games similarly to Q-learning [35]. The Minimax-Q algorithm [13] learns the optimal Q-value for a
zero-sum game via the following learning rule:

\[
Q_{t+1}(s_t, a^\theta_t, a^\phi_t) \leftarrow (1 - \alpha_t)Q_t(s_t, a^\theta_t, a^\phi_t) + \alpha_t \left[ R(s_t, a^\theta_t, a^\phi_t) 
+ \gamma \max_{\pi^\theta} \min_{a^\phi} \sum_{a^\theta \in \Delta^\theta} Q_t(s_{t+1}, a^\theta, a^\phi)\pi^\theta(a^\theta | s_{t+1}) \right],
\]

(1)

where the max-min optimization problem is normally solved as a linear program [30]. Performing these optimizations at each learning step is inefficient especially at the beginning of the learning process, when the Q-values are inaccurate.

**Soft Q-Learning.** Soft Q-learning was originally introduced to reduce the maximization bias of standard Q-learning [18] and for constructing flexible entropy-regulated policies [37]. Two-agent Soft Q [19] introduces two entropy-regulation terms to the reward structure in (2) and thus restricts the policy exploration to a neighborhood of the given priors. Given the policies \( \pi^\theta \) and \( \pi^\phi \) of the two agents, the soft-value function is defined as in (2) with \( \beta^\theta > 0 \) and \( \beta^\phi < 0 \):

\[
V_{KL}^{\pi^\theta, \pi^\phi}(s) = E_{\pi^\theta, \pi^\phi} \left[ \sum_{t=0}^{\infty} \gamma^t \left( R(s_t, a^\theta_t, a^\phi_t) - \frac{1}{\beta^\theta} \log \pi^\theta(a^\theta_t | s_t) - \frac{1}{\beta^\phi} \log \pi^\phi(a^\phi_t | s_t) \right) \right],
\]

(2)

where \( \beta^\theta(a^\theta_t | s_t) \) and \( \beta^\phi(a^\phi_t | s_t) \) are some prior policies for the Player and the Opponent, respectively.

Similar to unregulated zero-sum stochastic games, the *unique* soft-optimal value of entropy-regulated games is defined via \( V_{KL}^*(s) = \max_{\pi^\theta, \pi^\phi} V_{KL}^{\pi^\theta, \pi^\phi}(s) \), and the soft-optimal Q-value is defined as \( Q_{KL}^*(s, a^\theta, a^\phi) := R(s, a^\theta, a^\phi) + \gamma E_{\pi^\theta, \pi^\phi} [V_{KL}^*(s')] \). It has been shown in [19] that the optimal strategies of an entropy-regulated game can be computed as follows. The Player first marginalizes \( Q_{KL}^* \) via

\[
Q_{KL}^{\pi^\theta, \pi^\phi}(s, a^\theta) = \frac{1}{Z^\theta(s)} \beta^\theta(a^\theta | s) \exp \left( \beta^\theta Q_{KL}^*(s, a^\theta, a^\phi) \right),
\]

(3)

and then she obtains her soft-optimal policy from

\[
\pi_{KL}^{\pi^\theta, \pi^\phi}(a^\theta | s) = \frac{1}{Z^\theta(s)} \beta^\theta(a^\theta | s) \exp \left( \beta^\theta Q_{KL}^{\pi^\theta, \pi^\phi}(s, a^\theta) \right),
\]

(4)

where \( Z^\theta(s) \) is the normalizing factor. The Opponent’s marginalization and her soft-optimal policy are defined symmetrically. The policies in (4) take the form of a Boltzmann distribution, where \( \beta^\theta \) is the inverse temperature. As \( \beta^\theta \) goes to zero, the entropy-regularization term in (2) dominate the reward structure and the soft-optimal policy of the Player collapses to its prior. On the other hand, as the magnitude of \( \beta^\phi \) tends to infinity, the Player chooses the action(s) that maximizes the marginalized Q-value. Essentially, for large values of \( \beta^\theta \) and \( \beta^\phi \) the algorithm trusts the Q-values more than the priors, and vice versa for small values of \( \beta^\theta \) and \( \beta^\phi \).

Based on the Player’s marginalized soft-optimal Q-values in (3), the game’s soft-optimal Q-value \( Q_{KL}^*(s, a^\theta, a^\phi) \) can be learnt via the recursive rule [19]

\[
Q_{KL,i+1}(s_t, a^\theta_t, a^\phi_t) \leftarrow (1 - \eta_i)Q_{KL,i}(s_t, a^\theta_t, a^\phi_t) + \eta_i \left[ R(s_t, a^\theta_t, a^\phi_t) 
+ \gamma \beta^\theta \log \sum_{a^\phi \in A^\phi} \beta^\phi(a^\phi | s) \exp \left( \beta^\phi Q_{KL}^*(s, a^\phi) \right) \right].
\]

(5)

### 4 Two-Agent Soft Nash Q²-Learning

In this section, we present the two-agent Soft Nash Q²-Learning algorithm (SNQ2). As the name suggests, the algorithm relies on two different Q-values: one is the standard Q-value and the other one is the soft Q-value. A schematic of the algorithm is presented in Figure 1.

**Learning Rule for** \( Q_{KL} \). The learning rule of \( Q_{KL} \) is presented in (5). The priors are updated periodically using the Nash policies as indicated by the red arrow in Figure 1.

**Learning Rule for** \( Q \). To learn the standard Q-values, we apply the following update rule recursively [13, 14]

\[
Q_{t+1}(s_t, a^\theta_t, a^\phi_t) \leftarrow (1 - \alpha_t)Q_t(s_t, a^\theta_t, a^\phi_t) + \alpha_t \left[ R(s_t, a^\theta_t, a^\phi_t) + \gamma V_t(s_{t+1}) \right],
\]

(6)
where $V_t(s_{t+1})$ is the estimated optimal value based on the current Q-values at learning step $t$. We employ two methods to compute $V_t$. The soft update (7) uses the entropy-regulated soft-optimal policies as an approximation to update the value. The Nash update (8) resembles Minimax-Q and computes the Nash value via optimization.

\[
V_t(s) = \sum_{a^l} \sum_{a^{op}} Q_t(s, a^l, a^{op}) \pi^{pl}_{KL,t}(a^l|s) \pi^{op}_{KL,t}(a^{op}|s),
\]

(7)

Nash Update: $V_t(s) = \max_{a^l} \min_{a^{op}} \sum_{a^l} \sum_{a^{op}} Q_t(s, a^l, a^{op})\pi^{pl}(a^l|s)$.

(8)

The soft update is used most frequently and is indicated by the blue arrow in Figure 1. The Nash update is applied only periodically and is indicated by the black dashed arrow.

**Nash Prior Updates.** In addition to updating the value function using the soft update and the Nash update as in (7) and (8), the proposed SNQ2 algorithm also updates the priors required by the $Q_{KL}$ as in (3) periodically. Given the current Q-value, one can find the Nash policies at each state by solving the following two Linear Programming problems (7)

\[
\max v, \quad \min u,
\]

subject to $v \mathbb{1}^T - \pi^pl(s)^T Q_t(s) \leq 0$, subject to $u \mathbb{1} - Q_t(s)\pi^{op}(s) \geq 0,$

(9)

where $\pi(s)$ is the policy cast in vector form and $Q(s)$ is the reward matrix at state $s$. We denote the solutions to (9) as $\pi^{pl}_{Nash,t}(s)$ and $\pi^{op}_{Nash,t}(s)$, respectively, which are used to update the priors. The periodic Nash prior update is represented by the red dashed arrow in Figure 1. By default SNQ2 is initialized with uniform priors, according to the principle of maximum entropy (38).

We summarize the Soft Nash Q²-Learning algorithm in Algorithm 1.

**Algorithm 1:** Soft Nash Q²-Learning Algorithm

1. **Inputs:** Priors $\rho; Learning rates $\alpha$ and $\eta$; initial prior update episode $M = \Delta M_0$; Nash update frequency $T$;
2. Set $Q(s, a^l, a^{op}) = Q_{KL}(s, a^l, a^{op}) = 0$ for all states $s$ and actions $a^l, a^{op}$;
3. Set $\beta^l$ and $\beta^{op}$ to some large values;
4. while $Q$ not converged do
5. while episode $i$ not end do
6. Collect transition $(s_t, a^l_t, a^{op}_t, r_t, s_{t+1})$ where $a^l_t \sim \pi^{pl}_{KL}(s_t), a^{op}_t \sim \pi^{op}_{KL}(s_t)$;
7. if $t \mod T = 0$ then
8. Compute $V(s_{t+1}) = \max_{a^l} \min_{a^{op}} \sum_{a^l} \sum_{a^{op}} Q(s_{t+1}, a^l, a^{op})\pi^{pl}(a^l|s_{t+1})$;
9. else
10. Compute $V(s_{t+1}) = \pi^{pl}_{KL}(s_{t+1})^T Q(s_{t+1}) \pi^{op}_{KL}(s_{t+1})$;
11. end
12. Update $Q(s_t, a^l_t, a^{op}_t)$ via (6);
13. Update $Q_{KL}(s_t, a^l_t, a^{op}_t)$ via (5);
14. if $i == M$ then
15. Compute $\pi^{pl}_{Nash}(s)$ and $\pi^{op}_{Nash}(s)$ using Nash solver (6) based on current $Q$ for all states $s$;
16. $\rho^{pl}_{law} \leftarrow \pi^{pl}_{Nash}, \rho^{op}_{law} \leftarrow \pi^{op}_{Nash}$;
17. $\Delta M, \beta_{law} = DynamicSchedule(\rho^{pl}_{law}, \rho^{op}_{law}, \beta^l, \beta^{op}, \Delta M, Q)$ as in Algorithm 2;
18. M += $\Delta M$;
19. Update $\rho^l \leftarrow \rho^{pl}_{law}, \rho^{op} \leftarrow \rho^{op}_{law}, \beta^l \leftarrow \beta_{law}, \beta^{op} \leftarrow \beta^{op}_{law}$;
20. Decrease learning rates $\alpha$ and $\eta$;
21. end
22. end
23. return $Q(s, a^l, a^{op})$.

4.1 Scheduling of the Hyper-parameters $M$ and $\beta$

As presented in Algorithm 1, the trade-off between computation efficiency and approximation accuracy is embedded in the prior update frequency and the Nash update frequency. Intuitively, when the new Nash priors are close to the old
ones, the algorithm is close to convergence. In this situation, the prior update frequency should be decreased (increase \(\Delta M\)) and the algorithm should trust and exploit the priors (decrease \(\beta^p\) and \(\beta^o\)).

The default number of episodes between two Nash prior policy updates \(\Delta M_0\) and the default decay rate of the inverse temperature \(\lambda \in (0, 1)\) are first computed as

\[
\Delta M_0 = \frac{N_{\text{states}} \times N_{\text{action pairs}}}{\alpha_0 \times T_{\text{max}}}, \quad \lambda = \left(\frac{\beta_{\text{end}}}{\beta_0}\right)^{1/N_{\text{updates}}},
\]

where \(\alpha_0\) is the initial learning rate and \(T_{\text{max}}\) is the maximum length of a learning episode; \(\beta_0\) and \(\beta_{\text{end}}\) are the initial and estimated final inverse temperatures for both \(\beta^p\) and \(\beta^o\), and \(N_{\text{updates}}\) is the estimated number of prior updates.

This value of \(\Delta M_0\) allows the algorithm to properly explore the state and action spaces so that the first prior update is performed with an informed Q-function. In our numerical experiments we found that \(\beta_0 = 20\), \(\beta_{\text{end}} = 0.1\) and \(N_{\text{updates}} = 10\) are good set of values.

Algorithm 2 summarizes the dynamic scheduling scheme, where the parameter \(\sigma \in (0, 1)\) is a decrease factor and \(\text{RelativeDifference}\) captures the performance difference between old and new priors.

### Algorithm 2: Dynamic Schedule for \(M\) and \(\beta\)

1. **Inputs:** old and new priors \(p, \rho_{\text{new}}\); old prior update length \(\Delta M\); old inverse temperatures \(\beta\); current Q;
2. Compute \(\nu_{\text{old}}(s) = [p^o(s)]^T Q(s)p^o(s)\) and \(\nu_{\text{new}}(s) = [\rho_{\text{new}}(s)]^T Q(s)\rho_{\text{new}}(s)\), for all \(s\);
3. Compute \(\text{RelativeDifference}(s) = |\nu_{\text{new}}(s) - \nu_{\text{old}}(s)|/|\nu_{\text{old}}(s)|\);  
4. Count the number of states \(n\), where \(\text{RelativeDifference}(s) < \delta\);
5. **if** \(n/|S| \geq \text{Threshold}\) **then**
6. \(\Delta M = \min\{\frac{1}{2} \Delta M, \Delta M_{\text{max}}\}\), \(\beta_{\text{new}} = \max\{\lambda \beta_{\text{old}}, \beta_{\text{min}}\}\);
7. **else**
8. \(\Delta M = \max\{\sigma \Delta M, \Delta M_{\text{min}}\}\), \(\beta_{\text{new}} = \beta_{\text{old}}\);
9. **end**
10. **return** \(\Delta M, \beta_{\text{new}}\).

### 4.2 Warm-Starting

One can warm-start the proposed SNQ2 algorithm by first initializing the priors \(p^o\) and \(p^p\) based on some previously learnt policies or on expert demonstrations, and then also postponing the first prior update to exploit these priors. Using a prior policy instead of the value to warm-start the algorithm has two major advantages: first, human demonstrations can be converted into prior policies in a more streamlined manner than into value information; second, prior policies provide more consistent guidance than the values, as the priors are not updated till the first prior policy update. We demonstrate the effect of transferring previous training experience in Section 5.

### 5 Numerical Experiments

To evaluate the performance of the proposed algorithm, we tested and compared SNQ2 with several existing algorithms (Minimax-Q [13], WoLF-PHC [12], Single-Q [22]) for three zero-sum game environments: a two-agent Pursuit-Evasion Game (PEG) [39], in which the Pursuer aims to capture the Evader; a sequential Rock-Paper-Scissor game (sRPS), in which the two agents play Rock-Paper-Scissor repeatedly till one gets four consecutive wins; and a Soccer game as in [13].

#### 5.1 Evaluation Criteria

Two metrics were used to evaluate the performance of the algorithms: the number of states achieving a NE and the running time. For each game, we computed four different value functions and compare them to determine whether a state has achieved a NE: (1) the ground truth Nash value \(\nu_{\text{Nash}}\) solved exactly via Shapley’s method [7]; (2) the learnt value \(\nu_{\text{Learn}}\); (3) the one-sided MDP value \(\nu_{\text{Learn}}^p\) computed by fixing the Opponent to her learnt policy and letting the Player maximize; (4) the one-sided MDP value \(\nu_{\text{Learn}}^o\). We assume that the learnt policies achieve a NE at state \(s\), if

\[
\frac{|\nu_{\text{Nash}}(s) - \nu_{\text{Learn}}(s)|}{|\nu_{\text{Nash}}(s)|} < \epsilon \quad \land \quad \frac{|\nu_{\text{Nash}}(s) - \nu_{\text{Learn}}^p(s)|}{|\nu_{\text{Nash}}(s)|} < \epsilon \quad \land \quad \frac{|\nu_{\text{Nash}}(s) - \nu_{\text{Learn}}^o(s)|}{|\nu_{\text{Nash}}(s)|} < \epsilon.
\]

The single-threaded experiments were run in a Python environment with AMD Ryzen 1920x and 32G 2666MHz RAM. Scipy’s optimization `linprog` (a simplex method) is used to solve the matrix games at each state.
Here, we pick a tolerance of $\epsilon = 0.03$. Notice that the evaluation criterion in (10) is stricter than simply collecting empirical win rates of different agents competing in the environment as in [13, 40], since any deviation from the NE could be exploited by either agent in our criteria.

5.2 The Game Environments

Pursuit-Evasion. The pursuit-evasion game (PEG) is played on $4 \times 4$, $6 \times 6$ and $8 \times 8$ grids, as depicted in Figure 2. Both agents seek to avoid collision with the obstacles (marked in black). The Pursuer strives to capture the Evader by being in the same cell as the Evader. The goal of the Evader is to reach one of the evasion cells (marked in red) without being captured. Two agents simultaneously choose one of four actions on each turn: $N$, $S$, $E$, and $W$. Each executed action results in a stochastic transition of the agent’s position. In the default setting, an agent has a 60% chance of successfully moving to its intended cell and a 40% chance of landing in the cell to the left of the intended direction. The $4 \times 4$ and $8 \times 8$ PEGs use the default transitions and the $6 \times 6$ PEG uses deterministic transitions. In such setting, a deterministic policy cannot be a Nash policy.

Soccer. The soccer game [13] is played on a $4 \times 5$ grid as in Figure 3. Two agents, A and B, can choose one of five actions on each turn: $N$, $S$, $E$, $W$, and Stand, and the two actions selected are executed in random order. The circle represents the ball. When the agent with the ball steps in to the goal (left for A and right for B), that player scores and the game restarts at a random state. When an agent executes an action that would take it to the cell occupied by the other agent, possession of the ball goes to the stationary agent. Littman [13] argued that the Nash policies must be stochastic.

Sequential Rock-Paper-Scissor. In a sequential Rock-Paper-Scissor game (sRPS), two agents (pl and op) play Rock-Paper-Scissor repeatedly. One episode ends when one of the two agents wins four consecutive games. The states and transitions are shown in Figure 5. State $s_0$ corresponds to the initial state where no one has won a single RPS game, and states $s_4$ and $s_8$ are the winning (terminal) states of pl and op respectively. The Nash policies of sPRS at each state are uniform for both agents.

5.3 Comparison to Existing Methods

We implemented and evaluated SNQ2 on the game environments presented in Section 5.2 using the evaluation criteria in Section 5.1. The SNQ2 algorithm was initialized with two types of priors, a uniform prior (SNQ2-U) and a previous experience (SNQ2-PE). Previous experience for PEGs and Soccer is learnt in a training session of the same game but with a different dynamics. For sRPS, the previous experience is a perturbed uniform strategy. The algorithm also has the option of a fixed schedule (SNQ2-FS) and a dynamic schedule (SNQ2-DS). Unless otherwise specified, SNQ2 uses a dynamic schedule. SNQ2 was compared with three popular alternative algorithms, namely, Minimax-Q, Soft-Q, WoLF-PHC and Single-Q. We fine-tuned these algorithms to get the best performance so as to demonstrate their actual capabilities. We summarize the performance in terms of convergence of these algorithms on the five game environments in Figure 4.

For sRPS, the default prior is randomly generated, as uniform policies are the Nash policy for this game.
Figure 4: Comparison of performance at convergence (left) and computation time (right) for all algorithms. All results are averaged over ten runs. The computation time on the right is normalized by that of Minimax-Q. We cut off the computation at 600,000 episodes for $8 \times 8$ PEG, due to the large state space. In most of the experiments, SNQ2 achieves a slightly better convergence to NE than Minimax-Q, while exhibiting a significant reduction in computation time. Soft-Q has relatively good performance in four of the games but fails to converge in sRPS. Single-Q and WoLF-PHC are fast but fail to converge to NE in all five games hence are not shown in the right plot.

The sRPS game with a uniform Nash equilibrium shows that SNQ2 is capable of converging to a fully mixed strategy. In this game, Soft-Q learning fails to converge to the uniform NE, as its reward structure is regularized as in (2). Single-Q learning tries to learn a pure policy but does not converge; WoLF-PHC converges to NE at state $s_3$ and $s_7$ but with large value deviation, which propagates to other states and results in poor policies.

We then examine the $4 \times 4$ PEG game. Despite the relatively small state space, the stochastic transition at all non-terminal states requires extensive exploration of the environment. We plot the convergence trends over 300,000 episodes and over 12,000 seconds for Minimax-Q and SNQ2 in Figure 5.

Figure 5: Convergence trends of different algorithms in $4 \times 4$ PEG.

The time to convergence in Figure 5(b) demonstrates the expedited convergence of SNQ2 in terms of computation time. The episode-wise trend is depicted in Figure 5(a), where it is shown that SNQ2 maintains the same level of convergence performance as Minimax-Q, albeit with significantly reduced computation time. This shows that our entropy-regulated policy approximation approach is both accurate and computationally efficient. The performance oscillation of SNQ2 at the beginning is the result of the first few prior updates. As the Q-values are inaccurate at the beginning, the priors generated by these Q-values are also not accurate, which has long-lasting effects on the subsequent value updates as seen in Figure 5(a). Thus, updating the priors based solely on the current Q-values may be detrimental to performance. We have conducted experiments of using Polyak-averaging on the Q-values and the results showed that averaging does indeed alleviate the initial oscillations.

### 5.4 Effect of Warm-Starting

In PEGs, the agents learn their previous experience in the same grid but with a 75% success rate. To reduce overfitting to the previous environment, the prior policy fed to the agent for a new training session is the average of the previously learnt policy and a uniform policy. As seen in Figure 5(b), previous experience does not shorten the time till convergence but instead significantly reduces the time to reach a reasonable performance. We plot the cutoff time performance of the $4 \times 4$ and $8 \times 8$ PEGs in Figure 6. In the $4 \times 4$ example the time to reach over 90% Nash convergence is halved from 1,200 seconds with uniform

Figure 6: The convergence performance at cutoff times for $4 \times 4$ (left) and $8 \times 8$ (right) PEGs.
prior down to 600 seconds with previous experience. In the 8×8 example the time to reach 80% convergence was halved from 7,200 seconds to 3,600 seconds. One also observes from Figure 5(a) that the policies generated by SNQ2 with previous experience converge, episode-wise, slightly faster than Minimax-Q. This “warm-start” feature has appealing real-world applications, where the number of episodes interacting with the environment is the main constraint instead of computation time. In this case, one can first train prior policies using a simulator and use these as priors to the agents. With SNQ2 one can train the agents to a reasonable performance in fewer episodes, while maintaining a relatively low computation overhead.

5.5 Effect of Dynamic Scheduling

The effectiveness of dynamic scheduling is demonstrated in Figure 5(a) for the case of the 4×4 PEG where it is shown that dynamic scheduling reduced the number of episodes till convergence. For 6×6 PEG and Soccer, which have less stochasticity, dynamic scheduling improves the performance of the algorithm at convergence, as seen in Figure 7. Dynamic scheduling also reduces the length that SNQ2 exploits a potentially bad prior when the Q-values are far from convergence, especially at the beginning. As the initial learning rate is large, a Q-value based on a bad prior policy could be difficult to correct later on. The proposed dynamic scheduling also reduces the prior policy update frequency when the algorithm is close to convergence and thus reduces oscillations, as demonstrated in Figure 5.

![Figure 7: Convergence trends of algorithms in different environments.](image)

6 Conclusions and Future Work

We have proposed a new algorithm for solving zero-sum games where the agents use entropy-regularized policies to approximate the Nash policies and thus reduce computation time till convergence. Empirically, the proposed algorithm converges to a Nash equilibrium as demonstrated using several stochastic games of different sizes and levels of stochasticity. Our numerical experiments showed that the algorithm significantly reduces computation time while maintaining a good performance, when compared with other algorithms, such as Minimax-Q, WoLF-PHC, and independent Q-learning. The performance of the algorithm can be improved by applying a dynamic scheduling scheme of the prior update frequency and the annealing of inverse temperature. We also demonstrated the effectiveness of warm-starting in a new environment using previous experience.

For future work, we wish to combine SNQ2 with deep learning in order to tackle more complicated games and extend this idea to continuous state and action spaces. We would also like to extend the current algorithm to generalized-sum games and games with more than two agents.
References

[1] E. O. Neftci and B. B. Averbeck, “Reinforcement learning in artificial and biological systems,” Nature Machine Intelligence, vol. 1, no. 3, pp. 133–143, 2019.

[2] V. Mnih et al., “Human-level control through deep reinforcement learning,” Nature, vol. 518, no. 7540, pp. 529–533, 2015.

[3] T. P. Lillicrap et al., “Continuous control with deep reinforcement learning,” Preprint arXiv:1509.02971, 2015.

[4] L. Matignon, G. Laurent, and N. Fort-Plat, “Independent reinforcement learners in cooperative Markov games: A survey regarding coordination problems,” The Knowledge Engineering Review, vol. 27, pp. 1–31, 2012.

[5] L. Busoniu, R. Babuska, and B. De Schutter, “A comprehensive survey of multiagent reinforcement learning,” IEEE Transactions on Systems, Man, and Cybernetics, vol. 38, no. 2, pp. 156–172, 2008.

[6] D. P. Bertsekas and J. N. Tsitsiklis, Neuro-Dynamic Programming. Athena Scientific, 1996.

[7] J. Filar and K. Vrieze, Competitive Markov Decision Processes. Springer, 2012.

[8] G. Owen, Game Theory. Academic Press, 1982.

[9] J. Foerster et al., “Learning with opponent-learning awareness,” in Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems, pp. 122–130, 2018.

[10] R. Lowe, Y. Wu, A. Tamar, J. Harb, P. Abbeel, and I. Mordatch, “Multi-agent actor-critic for mixed cooperative-competitive environments,” in Proceedings of the 31st International Conference on Neural Information Processing Systems, pp. 6382–6393, 2017.

[11] L. Panait and S. Luke, “Cooperative multi-agent learning: The state of the art,” Autonomous Agents and Multi-Agent Systems, vol. 11, pp. 387–434, 2005.

[12] M. Bowling and M. Veloso, “Rational and convergent learning in stochastic games,” in Proceedings of the 17th International Joint Conference on Artificial Intelligence, (Seattle, WA), pp. 1021–1026, Aug. 4–10, 2001.

[13] M. L. Littman, “Markov games as a framework for multi-agent reinforcement learning,” in Proceedings of the 11th International Conference on Machine Learning, pp. 157–163, 1994.

[14] J. Hu and M. P. Wellman, “Nash Q-learning for general-sum stochastic games,” Journal of Machine Learning Research, vol. 4, pp. 1039–1069, 2003.

[15] M. L. Littman, “Friend-or-foe Q-learning in general-sum games,” in Proceedings of the Eighteenth International Conference on Machine Learning, pp. 322–328, 2001.

[16] X. Wang and T. Sandholm, “Reinforcement learning to play an optimal Nash equilibrium in team markov games,” in Proceedings of the 15th International Conference on Neural Information Processing Systems, pp. 1603–1610, 2002.

[17] J. D. Bransford et al., How People Learn, vol. 11. National Academy Press, 2000.

[18] R. Fox, A. Pakman, and N. Tishby, “Taming the noise in reinforcement learning via soft updates,” in Proceedings of the 32nd Conference on Uncertainty in Artificial Intelligence, pp. 202–211, 2016.

[19] J. Grau-Moya, F. Leibfried, and H. Bou-Ammar, “Balancing two-player stochastic games with soft q-learning,” in Proceedings of the 27th International Joint Conference on Artificial Intelligence, pp. 268–274, 2018.

[20] O. Morgenstern and J. Von Neumann, Theory of Games and Economic Behavior. Princeton University Press, 1953.

[21] J. Nash, “Non-cooperative games,” Annals of Mathematics, vol. 54, no. 2, pp. 286–295, 1951.

[22] M. Tan, “Multi-agent reinforcement learning: Independent vs. cooperative agents,” in Proceedings of the tenth international conference on machine learning, pp. 330–337, 1993.

[23] J. Foerster, G. Farquhar, T. Afouras, N. Nardelli, and S. Whiteson, “Counterfactual multi-agent policy gradients,” AAAI Conference on Artificial Intelligence, 2018.

[24] K. Kawamura, N. Mizukami, and Y. Tsuruoka, “Neural fictitious self-play in imperfect information games with many players,” in Computer Games, pp. 61–74, Springer, 2018.

[25] M. L. Littman and P. Stone, “A polynomial-time Nash equilibrium algorithm for repeated games,” Decision Support Systems, vol. 39, no. 1, pp. 55–66, 2005.

[26] M. Lauer and M. A. Riedmiller, “An algorithm for distributed reinforcement learning in cooperative multi-agent systems,” in Proceedings of the 17th International Conference on Machine Learning, pp. 535–542, 2000.
[27] L. Matignon, G. J. Laurent, and N. Le Fort-Piat, “Hysteretic Q-learning: an algorithm for decentralized reinforcement learning in cooperative multi-agent teams,” in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 64–69, 2007.

[28] S. Omidshafiei, J. Pazis, C. Amato, J. P. How, and J. Vian, “Deep decentralized multi-task multi-agent reinforcement learning under partial observability,” in *Proceedings of the 34th International Conference on Machine Learning*, pp. 2681–2690, 2017.

[29] J. K. Gupta, M. Egorov, and M. Kochenderfer, “Cooperative multi-agent control using deep reinforcement learning,” in *Autonomous Agents and Multiagent Systems*, pp. 66–83, 2017.

[30] A. Tampuu et al., “Multiagent cooperation and competition with deep reinforcement learning,” *PLOS ONE*, vol. 12, pp. 1–15, 2017.

[31] C. Daskalakis, P. W. Goldberg, and C. H. Papadimitriou, “The complexity of computing a Nash equilibrium,” in *Proceedings of the 38th Annual ACM Symposium on Theory of Computing*, pp. 71–78, 2006.

[32] Y. Yang, R. Luo, M. Li, M. Zhou, W. Zhang, and J. Wang, “Mean field multi-agent reinforcement learning,” in *Proceedings of the 35th International Conference on Machine Learning*, vol. 80, pp. 5571–5580, 2018.

[33] J. Perolat, B. Scherrer, B. Piot, and O. Pietquin, “Approximate dynamic programming for two-player zero-sum Markov games,” in *Proceedings of the 32nd International Conference on Machine Learning*, vol. 37, pp. 1321–1329, 2015.

[34] S. Li, Y. Wu, X. Cui, H. Dong, F. Fang, and S. Russell, “Robust multi-agent reinforcement learning via minimax deep deterministic policy gradient,” *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 33, pp. 4213–4220, 2019.

[35] C. J. C. H. Watkins and P. Dayan, “Q-learning,” in *Machine Learning*, pp. 279–292, 1992.

[36] D. Bertsimas and J. Tsitsiklis, *Introduction to Linear Optimization*. Athena Scientific, 1997.

[37] T. Haarnoja, H. Tang, P. Abbeel, and S. Levine, “Reinforcement learning with deep energy-based policies,” in *Proceedings of the 34th International Conference on Machine Learning - Volume 70*, pp. 1352–1361, 2017.

[38] E. T. Jaynes, “Information theory and statistical mechanics,” *Physical Review*, vol. 106, pp. 620–630, 1957.

[39] Y. Guan, D. Maity, C. M. Kroninger, and P. Tsiotras, “Bounded-rational pursuit-evasion games,” *Preprint arXiv:2003.06954*, 2020.

[40] M. G. Lagoudakis and R. Parr, “Value function approximation in zero-sum markov games,” in *Proceedings of the 18th Conference on Uncertainty in Artificial Intelligence*, pp. 283–292, 2002.