Hadron spin-flip amplitude: 
an analysis of the new $A_N$ data from RHIC$^1$

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Abstract

Through a direct analysis of the scattering amplitude, we show that the preliminary measurement of $A_N$ obtained by the E950 Collaboration at different energies are mainly sensitive to the spin-flip part of the amplitude in which the proton scatters with the $^{12}C$ nucleus as a whole. The imaginary part of this amplitude is negative, and the real part is positive and has a large slope. We give predictions for $p_L = 600$ GeV/$c$, which depend mainly on the size and energy dependence of the real part of the amplitude.

The new RHIC fixed-target data, from E950, consists in measurements of the analysing power

$$A_N(t) = \frac{\sigma^{(1)} - \sigma^{(4)}}{\sigma^{(1)} + \sigma^{(2)}}$$

for momentum transfer $0 \leq |t| \leq 0.05$ GeV$^2$, for a polarised $p$ beam hitting a (spin-0) $^{12}C$. In this region of $t$, the electromagnetic amplitude is of the same order of magnitude as the hadronic amplitude, and the interference of the imaginary part of $A_{nf}$ with the spin-flip part of the electromagnetic amplitude $A_{em}^{sf}$ leads to a peak in the analysing power $A_N$, usually referred to as the Coulomb-Nuclear Interference (CNI) effect$^1$. The first RHIC measurements at $p_L = 22$ GeV/$c$ in $p^{12}C$ scattering indicated however that $A_N$ may change sign already at very small momentum transfer. Such a behaviour cannot be described by the CNI effect alone. Indeed, it also requires some contribution of the hadron spin-flip amplitude. The first analysis of the preliminary data$^3$ gives for the ratios of the real and imaginary parts of the reduce spin-flip amplitude to the imaginary part of the spin non-flip amplitude $R = 0.088 \pm 0.058$ and $I = -0.161 \pm 0.226$, with the reduced spin-flip amplitude being defined as $A_{sf}(s,t) = 2 m_p A_{sf}(s,t)/\sqrt{|t|}$. The large error on $I$ unfortunately leads to a high uncertainty on the size of the hadronic spin-flip amplitude.

Isoscalar targets such as $^{12}C$ simplify the calculation as they suppress the contribution of the isovector reggeons $\rho$ and $a_2$ by some power of the atomic number. Also, as $^{12}C$ is spin 0, there are only two independent helicity amplitudes: proton spin flip and proton spin non flip. However, nuclear targets lead to large theoretical uncertainties because of the difficulties linked to nuclear structure, and because of the lack of high-energy proton-nucleus scattering experiments. Given these problems, we shall not rely on theoretical models (such as the Glauber formalism) but rather parametrise the scattering amplitude directly from data, and take the interference terms fully into account in the form of the analysing power.

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Figure 1: \( \frac{d\sigma}{dt} \) calculated by Glauber model \[3\] (formula (37), the previous plus the maximal possible inelastic \( \frac{d\sigma}{dt} \), which fall with \( t \to 0 \), first plus inelastic without the decreasing coefficient, and by our formula(9) (long-dashed, dashed, dotted, solid curves correspondingly). The experimental data: circles - \[11\], triangles - \[13\].

The elastic and total cross sections and the analysing power \( A_N \) for \( p^{12}C \) scattering are given by

\[
\frac{d\sigma}{dt} = 2\pi \left( |A_{nf}|^2 + |A_{sf}|^2 \right),
\]

\[
\sigma_{\text{tot}} = 4\pi \text{Im}(A_{nf}),
\]

\[
A_N \frac{d\sigma}{dt} = -2\pi \text{Im}(A_{nf}A_{sf}^*]).
\]

Each term includes a hadronic and an electromagnetic contribution: \( A_i(s, t) = A_{h}^i(s, t) + \alpha_{em} \delta, (i = nf, sf) \), where \( A_{h}^i(s, t) \) describes the strong interaction of \( p^{12}C \), and \( A_{em}^i(t) \) the electromagnetic interaction. \( \alpha_{em} \) is the electromagnetic fine structure constant, and the Coulomb-hadron phase \( \delta \) is given by \( \delta = Z\alpha_{em}\varphi_{CN} \) with \( Z \) the charge of the nucleus, and \( \varphi_{CN} \) the Coulomb-nuclear phase \[5\]. The electromagnetic part of the scattering amplitude can be written as

\[
A_{nf}^{em} = \frac{2\alpha_{em} Z}{t} F_{em1}^{12C} F_{em}^p,
\]

\[
A_{sf}^{em} = -\frac{\alpha_{em} Z}{m_p \sqrt{|t|}} F_{em1}^{12C} F_{em2}^p,
\]

where \( F_{em1}^p \) and \( F_{em2}^p \) are the electromagnetic form factors of the proton, and \( F_{em}^{12C} \) that of \( ^{12}C \). We obtain \( F_{em}^{12C} \) from the electromagnetic density of the nucleus.

The parts of the scattering amplitude due to strong interaction are assumed to be approximated by falling exponentials in the small-\( t \) region. The slope parameter \( B(s, t)/2 \) is then the derivative of the logarithm of the amplitude with respect to \( t \). If one considers only one contribution to the amplitude, this coincides with the slope of the differential cross section. In a more complicated case, there is no direct correspondence with the cross section because of interference terms.

The quick growth of the differential cross sections in elastic hadron-nucleus scattering, which is reflected by the large slope at small momentum transfer, does not permit to make a hard theory input. Pure Glauber theory gives in this region a behaviour for the slope which is very close to that obtained in the black disk limit:
the slope increases with $|t|$ in accordance with low-energy data. However, at high energies, the slope slightly increases as $t \to 0$. For example, in the data of \cite{13}, the slope of the proton-Carbon elastic cross section slightly increases when $|t| \to 0$ and at $|t| \approx 0.01$ GeV$^2$ is equal to $B = 74$ GeV$^{-2}$ at $p_L = 170$ GeV/c. More recent data \cite{5} give $B = 60$ GeV$^{-2}$ at $p_L = 600$ GeV/c in the region $0.01 \leq |t| \leq 0.03$ GeV$^2$.

The description of part of the experimental data by different approaches is shown in Fig.1. The Glauber model gives a sharp minimum at $|t| \approx 0.1$ GeV$^2$. An additional contribution to the differential cross section, coming from quasi-elastic scattering, cannot remove the effect of this sharp minimum in this region. The data of \cite{8}, as the old data \cite{13}, show a smooth change of the slope at small momentum transfer. Hence, if we want to obtain a good description of the diffraction peak in proton-Carbon scattering, we need to take a more complicated form of the scattering amplitude than given by the Glauber model or the simple exponential behaviour. Thus, for our phenomenological analysis of the preliminary data, we use the representation of the hadron spin non-flip amplitude in the form of two exponentials \cite{9,10}. It gives us a large slope at very small momentum transfer which smoothly changes at higher $|t|$. We shall not discuss here the origin of these exponentials. Note that they can be connected with different interaction mechanisms at small and large distances.

\begin{align}
\mathcal{A}_{nf}^h(s, t) &= \mathcal{A}_{nf}^{h1}(s, t) + \mathcal{A}_{nf}^{h2}(s, t), \\
\mathcal{A}_{nf}^{h1}(s, t) &= (1 + \rho^{h1}) \frac{\sigma_{tot}^{h1}(s)}{4\pi} \exp\left(\frac{B^{h1}}{2} t\right), \\
\mathcal{A}_{nf}^{h2}(s, t) &= (1 + \rho^{h2}) \frac{\sigma_{tot}^{h2}(s)}{4\pi} \exp\left(\frac{B^{h2}}{2} t\right),
\end{align}

where $B^{h1} > B^{h2}$ and the amplitude $\mathcal{A}_{nf}^{h1}(s, t)$ gives the main contribution to the differential cross sections when $t \to 0$. For simplicity, we take the additional amplitude $\mathcal{A}_{nf}^{h2}(s, t)$ with small slope as proportional to the nucleon-nucleon total cross section. The normalisation is determined by comparison of our calculation with the experimental data at $-t = 0.1$ GeV$^2$, where the Glauber model has the first minimum in the differential cross sections. We neglect the possible contribution of inelastic effects, as we do not know its size and as at small $t$ it must decrease, see for example \cite{12}. Our $d\sigma/dt$ is a little lower than the data of \cite{11}, indicating where possible inelastic contributions might set in. The coincidence of the hard and electromagnetic form factors at small $t$ supports our representation.

The experiment \cite{8} on $pC$ scattering at $p_L = 600$ GeV/c gives us $\sigma_{tot}^{pC} = 341$ mb; $Br^C(t \approx 0.02$ GeV$^2) = 62$ GeV$^{-2}$. To obtain the values of these parameters for other energies, we make the following assumptions on their energy dependence: some analysis \cite{14} and the data \cite{15} show that the ratio $R_{C/p}$ of $\sigma_{tot}(p^{12}C)$ to $\sigma_{tot}(pp)$ decreases very slowly in the region $5 \leq p_L \leq 600$ GeV/c. We take its energy dependence, according to the data \cite{14}, as $R_{C/p} = 9.5 (1 - 0.015 \ln s)$. From this we obtain $\sigma_{tot}^{h1}(s) \equiv \sigma_{tot}^{pC}(s) - \sigma_{tot}^{h2}(s)$. We assume that the slope slowly rises with $\ln s$ in a way similar to the $pp$ case, and normalise it so that the full amplitude \cite{4} has a slope of 62 GeV$^{-2}$ at $p_L = 600$ GeV/c and $|t| = 0.02$ GeV$^2$. This gives $B^{h1} = 70 (1 + 0.05 \ln s)$.

We do not know the energy dependence of $\rho^{h1}$, but because the $\rho$ and $a_2$ trajectories are suppressed, and because they contribute negatively, it must be higher than in the $pp$ case, where it is about $-0.1$ in this energy region. In fact, the data from \cite{8,13} indicate that $\rho^{h1}$ can be positive. We also know that at very high energy, $\rho^{h1}$ should be of the order of $\rho^{pp}$, which is about 0.1. We thus assume that, at RHIC energies, it is of the order of 0.05, and that it changes logarithmically with
The analysing power $A_N$ (in %) at $p_L = 24$ GeV/c (a) and 100 GeV/c (b), compared with the data \[2, 4\] (only statistical errors are shown). The two scenarios \(7\) and \(8\) lead respectively to the upper and lower curves (these are indistinguishable at 24 GeV/c). The dot-dashed curves correspond to the addition of a small spin-flip contribution from \(9\).

We parametrise the spin-flip part of $p^{12}C$ scattering also by two exponents

$$
\mathcal{A}_{nf}^{hf}(s,t) = (k_2 \rho^{h1} + i k_1) \frac{\sqrt{|t|} \sigma_{4h1}^{h1}(s)}{4\pi} \exp \left( \frac{B^{h1}}{2} t \right),
$$

$$
\mathcal{A}_{nf}^{h2}(s,t) = A_{nf}^{h2}(s,t)/10.
$$

We have assumed here that the spin-flip and the spin-non-flip amplitude have the same slope. One could of course allow for more freedom and take different slopes, but the data are not yet precise enough to test this.

From the full scattering amplitude, the analysing power is given by

$$
A_N \frac{d\sigma}{dt} = -4\pi [Im(A_{nf})Re(A_{nf}) - Re(A_{nf})Im(A_{nf})],
$$

each term having electromagnetic and hadronic contributions. We can now calculate the form of the analysing power $A_N$ at small momentum transfer with different coefficients $k_1$ and $k_2$ chosen to obtain the best description of $A_N$ at $p_L = 24$ GeV/c.
Figure 3: The predictions for $A_N$ (in %) at $p_L = 600$ GeV/c. The curves are as in Fig. 2.

and $p_L = 100$ GeV/c. Of course, we only aim at a qualitative description as the data are only preliminary and as they are normalised to those at $p_L = 22$ GeV/c [2].

The preliminary data show that $A_N$ decreases very fast after its maximum and is almost zero in a large region of momentum transfer. This behaviour can be explained only if one assumes a negative contribution of the interference between different parts of the hadron amplitude, that changes slowly with energy.

The data at $p_L = 100$ GeV/c decrease faster than those at $p_L = 24$ GeV/c, and the zero of $A_N$ moves to lower values of $|t|$. This change of sign is independent from the normalisation of the data. It would be very interesting to obtain new data with higher accuracy and at higher energies in order to distinguish between the two scenarios (7) and (8) (see Fig. 3). Both variants give the same size and the same negative sign for the imaginary part of the spin-flip amplitude. As mentioned above, such an amplitude gives an additional positive contribution to the CNI-effect at the maximum. Its size is mostly determined by the magnitude of $A_N$ at small $|t|$. The fast change of sign of $A_N$ is explained by the interference of different parts of the hadronic amplitude.

Hence, the shape and energy dependence of the analysing power depend mostly on the size and energy dependence of $p_{pC}$. If we choose another size and energy dependence, we can obtain a different shape for $A_N$ and different magnitudes for $k_1$ and $k_2$. However all conclusions will stand and $I = \text{Im}(r_5)$ will remain negative. Note that a positive $I$ with our choice of $p$ would lead to an increase with energy of the value of $p_L$ at which $A_N$ has a zero.

The ratio of the reduced spin-flip amplitude to the spin-non-flip amplitude is approximately 15%. Note that the description of the experimental data and of their energy dependence is heavily correlated with the size of the slope of the spin-flip amplitude. We obtain a slope equal to 85.5 GeV$^{-2}$ for $p_L = 24$ GeV/c and 94.6 GeV$^{-2}$ for $p_L = 600$ GeV/c.

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References

[1] J. Schwinger, Phys. Rev. D (1948) 407.
[2] J. Tojo et al., [BNL-AGS E250 Collaboration], Phys. Rev. Lett. 89 (2002) 052302, arXiv:hep-ex/0206057.

[3] B. Z. Kopeliovich and T. L. Trueman, Phys. Rev. D 64 (2001) 034004, arXiv:hep-ph/0012091.

[4] D. Svirida et al., in Proc. ASI “Symmetry and Spin”, Prague, Czech. Rep., June 15 - 28, 2002.

[5] O. V. Selyugin, Phys. Rev. D 60 (1999) 074028.

[6] L. A. Jansen et al., Nucl. Phys. A 188 (1972) 342.

[7] Atomic Data and Nuclei Data Tables 36 (1987) 495.

[8] U. Dersch et al., [SELEX Collaboration], Nucl. Phys. B 579 (2000) 277, arXiv:hep-ex/9910052.

[9] Y. Akimov, et al., Phys. Rev. D 12 (1975) 3399.

[10] C. Garsia-Recio et al., Phys. Rev. C 51 (1995) 237.

[11] G. Bellettini et al., Nucl. Phys. B 79 (1966) 609.

[12] R.J. Glauber, G. Mattiae, Nucl. Phys. B 21 (1970) 135.

[13] A. Schiz et al., Phys. Rev. D 21 (1980) 3010.

[14] P. J. Karol, Phys. Rev. C 46 (1992) 1988.

[15] P. V. Ramana Murthy, C. A. Ayre, H. R. Gustafson, L. W. Jones, M. J. Longo, Nucl. Phys. B 92 (1975) 269.