Decomposing the stock market intraday dynamics

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Abstract

The correlation matrix formalism is used to study temporal aspects of the stock market evolution. This formalism allows to decompose the financial dynamics into noise as well as into some coherent repeatable intraday structures. The present study is based on the high-frequency Deutsche Aktienindex (DAX) data over the time period between November 1997 and September 1999, and makes use of both, the corresponding returns as well as volatility variations. One principal conclusion is that a bulk of the stock market dynamics is governed by the uncorrelated noise-like processes. There exists however a small number of components of coherent short term repeatable structures in fluctuations that may generate some memory effects seen in the standard autocorrelation function analysis. Laws that govern fluctuations associated with those various components are different, which indicates an extremely complex character of the financial fluctuations.

1 Introduction

One of the great challenges of econophysics is to properly quantify and, following this, to explain the nature of financial correlations and fluctuations. The efficient market hypothesis [1] implies that they are dominated by noise. Indeed, the spectrum of the correlation matrix accounting for correlations among the stock market companies agrees very well [2, 3, 4] with the universal predictions of random matrix theory [5, 6]. Locations of extreme eigenvalues differ however from these predictions and thus identify certain system-specific, non-random properties such as collectivity. In addition, these former properties turn out [⁵, ⁶] to depend on time reflecting a competitive character of the financial dynamics.
The character of the financial time correlations is however very complex, still poorly understood and many related issues remain puzzling. The auto-correlation function of the financial time series, for instance, drops down to zero within few minutes which is interpreted as a time horizon of the market inefficiency [8]. At the same time, however, the correlations in volatility are significantly positive over the time intervals longer by many orders of magnitude. The fat-tailed return distributions seem to be not Lévy stable [9] on short time scales, but on longer time scales it appears difficult to identify their convergence to a Gaussian as expected from the central limit theorem. In addressing this sort of issues below we use the concept of the correlation matrix whose entries are constructed from the time series of price changes representing the consecutive trading days. The method focuses then entirely on the time correlations and their potential existence can parallelly be detected on various time scales. Analogous methodology has already been successfully applied [10] to extract from noise some repeatable structures in the brain sensory response, and its somewhat similar variant, the correlation matrix of the delay matrix, to study the business cycles of economics [11]. The present study is an extension of our recent work [12] and is based on an example of high-frequency (15 s) recordings [13]. As it can be seen from Fig. 1, this is an interesting period which comprises the whole richness of the stock market
dynamics like strong increases and decreases, and even a clearly identifiable hierarchy of the log-periodic structures \[14\].

2 Definition of correlations

In the present application the entries of the correlation matrix are constructed from the time-series \(g_\alpha(t_i)\) of normalized price returns representing the consecutive trading days labelled by \(\alpha\). Starting from the original price time-series \(x_\alpha(t)\) these are defined as

\[
g_\alpha(t_i) = G_\alpha(t_i) - \langle G_\alpha(t_i) \rangle_t, \quad \sigma(G_\alpha) = \sqrt{\langle G_\alpha^2(t) \rangle_t - \langle G_\alpha(t) \rangle_t^2},
\]

with

\[
G_\alpha(t_i) = \ln x_\alpha(t_i + \tau) - \ln x_\alpha(t_i) \simeq \frac{x_\alpha(t_i + \tau) - x_\alpha(t_i)}{x_\alpha(t_i)} ,
\]

where \(\tau\) is the time-lag and \(\langle \ldots \rangle_t\) denotes averaging over time.

The result is \(N\) time series \(g_\alpha(t_i)\) of length \(T\) (the number of records during the day) i.e. an \(N \times T\) matrix \(M\). The correlation matrix can then be defined as

\[
C = \left(\frac{1}{T}\right) MM^T.
\]

Its entries \(C_{\alpha,\alpha'}\) are thus labelled by the pairs of different days. By diagonalizing \(C\)

\[
Cv^k = \lambda_k v^k,
\]

one obtains the eigenvalues \(\lambda_k\) \((k = 1, ..., N)\) and the corresponding eigenvectors \(v^k = \{v^k_\alpha\}\).

A useful null hypothesis is provided by the limiting case of entirely random correlations. In this case the density of eigenvalues \(\rho_C(\lambda)\) defined as

\[
\rho_C(\lambda) = \frac{1}{N} \frac{dn(\lambda)}{d\lambda},
\]

where \(n(\lambda)\) is the number of eigenvalues of \(C\) less than \(\lambda\), is known analytically \([15]\), and reads

\[
\rho_C(\lambda) = \frac{Q}{2\pi\sigma^2} \sqrt{(\lambda_{\text{max}} - \lambda)(\lambda - \lambda_{\text{min}})},
\]

with \(\lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}}, \ Q = T/N \geq 1,\) and where \(\sigma^2\) is equal to the variance of the time series which in our case equals unity.
As mentioned above our related study is based on the DAX recordings with the frequency of 15 s during the period between November 28th, 1997 and September 17th, 1999. After this last date the DAX was traded significantly longer during the trading day. By taking the DAX intraday 15 s variation between the trading time 9:03 and 17:10 which corresponds to $T = 1948$, and rejecting several days with incomplete recordings, one then obtains $N = 451$ complete and equivalent time series representing different trading days during this calendar period. Using this set of data we then construct the $451 \times 451$ matrix $C$.

One characteristic of interest is the structure of eigenspectrum. The resulting probability density of eigenvalues, shown in Fig. 2, displays a very interesting structure. There exist two almost degenerate eigenvalues visibly repelled from the bulk of the spectrum, i.e., well above $\lambda_{\text{max}}$ (for $Q = 1948/451$, $\lambda_{\text{max}} \approx 2.19$) which indicates that the dynamics develops certain time specific repeatable structures in the intraday trading. The bulk of the spectrum, how-
ever, agrees remarkably well with the bounds prescribed by purely random correlations. This indicates that the statistical neighbouring recordings in our time series of 15 s DAX returns share essentially no common information.

A significance of this result can be evaluated by the following numerical experiment. From a Gaussian distribution we draw $N = 451$ series $x_n(i) \ (n = 1, \ldots, N)$ of random numbers of length $T = 1948 \ (i = 1, \ldots, T)$ and determine the spectrum of the resulting correlation matrix. The result (histogram) versus the corresponding theoretical result expressed by the Eq. (6) is shown in the upper part of Fig. 3. As expected, the agreement is unquestionable. In the second step, in each previous series we retain only every third number, e.g., $x_n(1), x_n(4), \ldots$ This omission is compensated by insertion between every two remaining original numbers, say $x_n(i)$ and $x_n(i + 3)$, the two new $x_n(i + 1), x_n(i + 2)$ numbers such that they are functionally (here linearly) dependent on $x_n(i)$ and $x_n(i + 3)$. The net result is the same number $N$ of series of the same length $T$ as before, thus $Q = T/N$ formally remains unchanged. The structure of eigenspectrum of the corresponding correlation matrix, which is shown in the lower part of Fig. 3, changes however completely. In fact, it now perfectly agrees with the theoretical formula of Eq. (6) but for the three times shorter ($T/3$) series, i.e., it nicely reflects a real information content. From this we can conclude that a common information shared by neighbouring events in our DAX time series is basically null and that a whole nonrandomness can be associated with the two largest eigenvalues.

It is also interesting to perform an analogous study of the volatility correlations. This corresponds to replacing $G_\alpha(t_i)$ in Eq. (6) by $|G_\alpha(t_i)|$, for instance. The structure of eigenspectrum of the resulting correlation matrix is shown in Fig. 4. Surprisingly, even in this case the bulk of the spectrum is consistent with purely random correlations. As compared to Fig. 2, one can now identify however three outlying eigenvalues and the largest of them is repelled significantly higher, as far up as 13.3.

The structure of eigenspectrum of a matrix is expected to be related \cite{16, 17} to the distribution of its elements. For this reason in Fig. 5 we show the distributions of such elements of $C$ corresponding to the above specified procedure for our two cases under consideration. The upper part of this figure corresponds to the returns time series and the lower part to the volatility time series. In the first case this distribution is symmetric with respect to zero, a Gaussian like (dashed line) on the level of small matrix elements, but sizably thicker than a Gaussian on the level of large matrix elements, where
Figure 3: (a) The probability density (histogram) of eigenvalues of the correlation matrix $C$ calculated from $N = 451$ series of length $T = 1948$ of the Gaussian distributed uncorrelated random numbers versus the corresponding null hypothesis (dashed line) formulated in terms of Eq. (6). (b) The same as (a) but here only every third original random number in each series is retained. The removed pairs of numbers are replaced by the new ones which are functionally dependent on the neighbouring two original numbers. The dashed line indicates the result of Eq. (6) for $N$ series of length $T/3$. 

\[ \rho_c(\lambda) \]
a power law with the index of about 5.5 - 5.7 (which however is far beyond the Lévy stable regime as consistent with the distribution of returns) provides a reasonable representation. It is these tails which generate the two largest eigenvalues seen in Fig. 2. The volatility correlation matrix, on the other hand, reveals a somewhat different distribution. First of all, the center of this distribution is shifted towards positive values and this is responsible for the largest eigenvalue. Secondly, this distribution is asymmetric. This originates from the fact that the volatility fluctuations are strongly asymmetric relative to their average value. The slope on the right hand side cannot be here reliably measured in terms of a single power law, but its even smaller value as compared to the previous case (of returns) is evident. On the other hand, on the negative side the distribution drops down faster than a Gaussian and, therefore, the separation between the two remaining large eigenvalues is significantly more pronounced than of their returns counterparts (two largest ones) from Fig. 2.

In quantifying the differences among the eigenvectors it is instructive to look at the superposed time series of normalized returns. One possibility adopted here reads:

\[
g_{\lambda k}(t_i) = \sum_{\alpha=1}^{N} \text{sign}(v_{\alpha}^k) |v_{\alpha}^k|^2 g_{\alpha}(t_i). \tag{7}\]
Figure 5: Distribution of matrix elements $C_{\alpha,\alpha'}$ of the $N \times N$ ($N = 451$) correlation matrix $C$ calculated from the 15 s frequency DAX variation during the intraday trading time 9:03–17:10. $\alpha$ labels the different trading days during the calendar period December 28, 1997 – September 17, 1999. The upper (a) part corresponds to the time series of returns and the lower (b) part to the time series of volatilities. The solid lines in (a) indicate the power law fits to the tails of the distribution, while the dashed one represents a Gaussian best fit. The numbers in (a) reflect the corresponding scaling indices.
Figure 6: The superposed time series of normalized returns calculated according to eq. (7) for $k = 1$ (a), $k = 2$ (b) and $k = 200$ (c).

Figure 7: The superposed time series of normalized volatilities calculated according to eq. (8) for $k = 1$ (a), $k = 2$ (b) and $k = 3$ (c).
In this definition $|v^k_\alpha|^2$ is used instead of $v^k_\alpha$ for the reason of preserving normalization and the sign of $v^k_\alpha$ in order not to destroy any possible coherence among the original signals. A collection of such superposed time series of returns for $k = 1, 2$ and $200$ is shown in Fig. 6. As it is illustrated by an example of $k = 200$, for a statistical value of $k$ such a superposed signal develops no coherent structures and $g_{\lambda_k}(t_i)$ basically does not differ from a simple average. The first two differ however significantly and indicate the existence of the very pronounced, up to almost ten times of the mean standard deviations of the original time series, repeatable structures at the well defined instants of time through many days. As it is clearly seen, the two collective signals correspond to two disconnected and well determined periods of an enhanced synchronous market activity. The first ($k = 1$) of them corresponds to the period just before closing in Frankfurt during the time interval considered here, and the second one ($k = 2$) to the period immediately after 14:30, which reflects the DAX response to the North-American financial news release exactly at this time. It is also very interesting to see that in the first case (before closing) the coherent burst of activity expressed by $g_{\lambda_1}(t)$ is oriented to the negative values while in the second case (just after 14:30) ($g_{\lambda_2}(t)$) it points predominantly to the positive values. Surprisingly, the DAX response to the Wall Street opening at 15:30 develops no visible synchronous structure in neither of the eigenstates.

The first three ($k = 1, 2, 3$) analogously superposed volatility signals

$$v_{\lambda_k}(t_i) = \sum_{\alpha=1}^{N} \text{sign}(v^k_\alpha)|v^k_\alpha|^2|g_\alpha(t_i)|.$$  \hspace{1cm} (8)

are shown in Fig 7. The first of them is associated with the largest eigenvalue of the volatility correlation matrix and reflects the magnitude of average volatility as a function of time. The next two ($k = 2, 3$) constitute counterparts of the first two superposed return signals. Interestingly, this correspondence holds in reversed order, however.

Another characteristics which carries some information about the stock market dynamics is the probability distribution of the eigenvector components $v^k_\alpha$. Several relevant histograms, either for single eigenstates or for a collection of them, are shown in Fig. 8. In both cases, of the returns as well as of the volatility correlation matrices, the distributions of eigenvector components from the bulk of the spectrum agree very well with a Gaussian. In the transition region, illustrated here by the eigenstates 3-7, some deviations can already be observed. Significantly different are the distributions for the
Figure 8: Distributions of eigenvector components $v^k_\alpha$ for $k = 1$, $k = 2$ and for the two sets $k = 3 - 7$ and $k = 8 - 451$ of eigenvectors. The upper (lower) part corresponds to the returns (volatility) correlation matrix.
outliers. In the case of the returns correlation matrix it is the second ($k = 2$) eigenvector whose distribution deviates more from a Gaussian; its components are concentrated more at zero but, at the same time, the tails of the distribution are thicker. This indicates that fewer days ($\alpha$’s), but with a larger weight, contribute to the signal seen at 14:30 than to the one just after 17:00. The $k = 1$ and $k = 2$ eigenvector components of the volatility correlation matrix are distributed asymmetrically relative to zero, as consistent with the distribution of the entries of this matrix. In this second case ($k = 2$), a long tail of the negative eigenvector components develops, which makes the corresponding superposed volatility $\nu_{\lambda_2}(t_i)$ signal negative.

The above decomposition of the stock market dynamics allows also to shed some more light on the issue of the probability distribution of price changes [9]. That the nature of such changes is very complex can be concluded by looking for instance at the probability distributions of fluctuations associated with different returns ‘eigensignals’ $g_{\lambda_k}(t_i)$. Some examples are shown in Fig. 9. To quantify such characteristics like Lévy stability or nonstability, based on this analysis, would definitely be premature for many reasons. One is the statistics which is here too poor [13]. What one however can clearly see is that the probability density of fluctuations connected with the bulk of the spectrum drops down much faster than the ones connected with the more collective ($k = 1, 2$) eigenvectors. In the first case a power law index, of the order of 5.8-5.9, can even be assigned, similar on both positive and negative sides. The $k = 1$ and $k = 2$ signals trace however a completely different, much thicker tailed, distribution [14]. Their extreme events, which can be qualified as outliers [20], carry essentially the same features as the ones identified [21] in the Dow Jones draw downs on much longer time scales.

4 Summary

The present study quantifies several characteristics relevant for understanding the dynamics of the stock market time evolution. One principal related issue is a question of how the market inefficiency manifests itself. For the Deutsche Aktienindex our study thus shows that on the time scales of up to one trading day there exist two well defined short periods of the spectacularly synchronous repeatable bursts of activity during the intraday trading between 9:03 and 17:10, a phenomenon somewhat in the spirit of an idea of marginally efficient markets [22]. On the other hand it turns out that generically the con-
Figure 9: Probability density functions of fluctuations of the superposed returns as expressed by the Eq. (7). Squares correspond to $k = 1$, triangles to $k = 2$ and the circles to the average of $k = 11 - 451$. Both, positive (upper part) and negative (lower part) sides of those distributions are shown.
secutie returns carry essentially no common information even when probed with the frequency of 15 s. The fluctuations associated with the so identified distinct components are governed by the different laws which reflects an extreme complexity of the stock market dynamics.

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[19] These observations remain largely unchanged even if the sign$(v^k_\alpha)$ in Eq. (7) is omitted. Then, only the extreme events at 14:30, dominating $g_{\lambda_2}(t_i)$, are somewhat reduced, but still remain outliers.

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