Comments on $|V_{ub}/V_{cb}|$ and $|V_{ub}|$ from Non-leptonic B Decays within the Perturbative QCD Approach

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Abstract

We revisited the extracting $|V_{ub}/V_{cb}|$ and $|V_{ub}|$ through calculating the ratios $\hat{B}(B^0 \to D^{(*)}_s + \pi^-)/\hat{B}(B^0 \to D^{(*)}_s + D^{(*)}_s)$ in the perturbative QCD approach, which is regarded as an effective theory in dealing with non-leptonic charmed B decays. Utilizing this approach, we could calculate the form factors effectively as well as non-factorizable and annihilation contributions. Within the updated distribution amplitudes and the latest experimental data, we get $|V_{ub}/V_{cb}| = 0.083 \pm 0.007$, which favors a bit smaller $|V_{ub}|$ compared with the averaged PDG value, but agrees well with the exclusively measured values. Furthermore, we predict the branching ratio of $\bar{B}_s^0 \to D^{(*)}_s \rho^+ \sim (2.7 \pm 1.2) \times 10^{-5}$, which could be measured in $B$ factories near future. In our calculation, the major uncertainty is due to our poor knowledge of heavy meson wave functions. We also comment that it is not trivial to generalize this approach to $B_s$ system, primarily because both $\bar{B}_s^0$ and $B_s^0$ can decay into the same final states $D^{(*)}_s K^-$ and $D^{(*)}_s D_s^-$.

PACS numbers: 13.25.Hw, 11.30.Er, 13.25.-k

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The precise measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the three related angles, namely $\alpha$, $\beta$ and $\gamma$, is one of the most important topics in flavor physics [1]. Through measurements of those observables we can test the standard model (SM) stringently and seek for a possible new physics beyond the SM. For instance, nonzero $|V_{ub}|$ plays critical role in explaining the CP violation in $B$ and $K$ decays. Furthermore, the ratio between $|V_{ub}|$ and $|V_{cb}|$ is very important as it is proportional the length of the side of the unitary triangle opposite to the well-measured angle $\beta$. Since the end of last century, the obvious example of the progress was made in measuring the ratio $|V_{ub}/V_{cb}|$ and $|V_{ub}|$ as acutely as we can [2].

In both experimental and theoretical sides, the determinations of $|V_{ub}|$ are mostly on semi-leptonic $B$ decays, involving inclusive $B \to X_s\ell\bar{\nu}$ decay and exclusive $B \to \pi\ell\bar{\nu}$ decay. However, both of them suffer from large uncertainties due to the unknown non-perturbative dynamics. For inclusive $B \to X_s\ell\bar{\nu}$ decay, in principle, it is the most appreciated way to extract $|V_{ub}|$. In practice, however, since $|V_{cb}| \gg |V_{ub}|$, the total width of $B \to X_s\ell\bar{\nu}$ cannot be measured due to the large charm background from $B \to X_s\ell\bar{\nu}$ decays. In most regions of phase space, where the charm background is kinematically forbidden, the hadronic physics enters via unknown non-perturbative functions, so-called shape functions. Although there is only one shape function at leading order in $\Lambda_{QCD}/m_b$, which can be extracted from the photon energy spectrum in $B \to X_s\gamma$, the subleading shape functions are modeled in the current calculations [3, 4]. For the exclusive decay, the determination of $|V_{ub}|$ requires a theoretical calculation of the form factors using non-perturbative methods such as Lattice QCD [5, 6], QCD sum rules [7] or light front approach [8], all of which have systematic uncertainties that are hard to be quantified. The $B \to \pi\ell\bar{\nu}$ branching ratio is now well known to 6%. Unquenched lattice QCD calculations of the $B \to \pi\ell\bar{\nu}$ form factor for $q^2 > 16\text{GeV}^2$ are available and yield [5, 6]

$$|V_{ub}| = (3.4 \pm 0.2^{+0.6}_{-0.4}) \times 10^{-3}. \quad (1)$$

Light-cone QCD sum rules are applicable for $q^2 < 14\text{GeV}^2$ and yield similar results [7]. The theoretical uncertainties in extracting $|V_{ub}|$ from inclusive and exclusive decays are very much different. In PDG [9], the values obtained from inclusive and exclusive determinations are

$$|V_{ub}| = (4.12 \pm 0.43) \times 10^{-3}, \quad \text{(inclusive)} \quad (2)$$

$$= (3.5^{+0.6}_{-0.5}) \times 10^{-3}, \quad \text{(exclusive)}. \quad (3)$$

Above two determinations are independent each other, and the dominant uncertainties are on multiplicative factors. If we weight the two values by their relative errors and treat the uncertainties as Gaussian, we find [9]

$$|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}, \quad \text{(combined)} \quad (4)$$

whose determination is dominated by the inclusive measurement. Given the theoretical and experimental progress, it will be interesting to see how the inclusive and exclusive determinations develop.

Although the complexity of QCD and weak dynamics had prevented us from measuring CKM elements through non-leptonic decays, many methods have been explored in determining $|V_{ub}|$ though analyzing decays of $B$ to exclusive non-leptonic two meson final states. In Refs. [10, 11], the authors had pointed out that we could extract the $|V_{ub}|/|V_{cb}|$ and $|V_{ub}|$ by studying the ratio of the branching fractions

$$\mathcal{B}(B^0 \to D_s^+\pi^-)/\mathcal{B}(B^0 \to D_s^+D^-).$$

For decays $B^0 \to D_s^+\pi^-$ and $B^0 \to D_s^+D^-$, either of decay modes involves a single isospin component $I = 1/2$, so the final state interaction is not important. Furthermore, the $B^0 \to D_s^+\pi^-$ is only induced by tree operators; for $B^0 \to D_s^+D^-$, although
penguin type diagrams will pollute this decay, the tree operators are still dominant. In Ref. [11], the penguin contribution has been considered in the generalized factorization scheme, and the decay modes with vector meson have been also considered. Within contemporary data, the authors estimated the ratios and gave the theoretical upper limit of $|V_{ub}/V_{cb}|$. However, in the generalized factorization approach, the effective color number $N_c^{eff}$, a non-physical constant, should be introduced in order to make up the non-factorizable diagrams’ contributions. In the past ten years, many efforts have been promoted to estimate the non-factorizable contributions quantitatively, such as the QCD factorization approach (QCDF) [12], the perturbative QCD approach (PQCD) [13, 14] and the soft collinear effective theory (SCET) [15]. Due to charmed meson involved, QCDF and SCET cannot be used here. In this letter, we reinvestigate the way proposed in [11] within the PQCD approach. In Refs. [16–19], the decay modes with $D$ or $D_s$ have been also calculated, but their results cannot be used here directly because different parameters have been used in different modes. Here we will adopt the consistent parameter space and the updated distribution amplitudes. Moreover, through this study we can cross-check the applicability of the PQCD with charmed meson, which is another starting point of this work.

For the decay modes here we discuss, the related Hamiltonian and the corresponding Wilson coefficients are given in Ref. [20]. In the PQCD approach, after the integration over momenta of positive and negative direction, the decay amplitude for $B \to M_2M_3$ can be conceptually written as

$$\mathcal{A} \sim \int dx_1dx_2dx_3b_1b_2b_3d\bar{b}_3 \text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_i(x_i) e^{-S(t)} \right], \quad (5)$$

where $b_i$ is the conjugate space coordinate of intrinsical transverse momentum of light quark, and $x_i$ is momenta fraction of light quark in each meson. $Tr$ denotes the trace over Dirac and color indices, $C(t)$ is the Wilson coefficient evaluated at scale $t$, and $H(x_i, b_i, t)$ is the hard QCD part, which can be calculated perturbatively. The function $\Phi_M$ is the wave function of a meson $M$, and the function $S_i(x_i)$ describes the threshold re-summation which smears the end-point singularities on $x_i$. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively. More detailed information on the PQCD can be refereed to Ref. [14]. In the PQCD picture, the decay amplitudes of related decay modes are written as

$$\mathcal{A}(\bar{B}^0 \to D_s^-D^+) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{cb}^* \left[ F_e^{LL}(C_2 + \frac{1}{3}C_4) + F_e^{LR}(C_4 + \frac{1}{3}C_3 + C_{10} + \frac{1}{3}C_9) \right] - V_{ub}V_{cd}^* \left[ F_e^{LL}(C_4 + \frac{1}{3}C_3 + C_{10} + \frac{1}{3}C_9) \right] + F_e^{LL}(C_3 + C_9) \right\},$$

$$\mathcal{A}(\bar{B}^0 \to D_s^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{ub}V_{cs}^* \left\{ F_e^{LL}(C_2 + \frac{1}{3}C_4) + F_e^{LL}(C_4) \right\}. \quad (6)$$

In above formulae, $F_e^{LL(R)}$ is the amplitude from (non)-factorizable emission-diagrams with $(V-A)(V-A)$ (or $(V-A)(V+A)$ operators. $SP$ stands for the operators $-2(S-P)(S+P)$, which comes from the Fierz translation of $(V-A)(V+A)$ of different color structure. Also, $a(n)$ means the contribution from (non)-factorizable annihilation diagrams. The detailed expression of each $F$ can be found in Refs. [17–19], and we would not list them explicitly here. Note that the $F_i$’s are different for different decay modes, as they are mode-dependent. In the calculation, the parameters we input are the non-perturbative parts, namely the distribution amplitudes and decay constants. For the light mesons $\pi$ and $\rho$, the distribution amplitudes are well studied within the light-cone QCD sum rules, and we will use the latest versions [21]. However, for heavy mesons $B$ and $D$, their wave functions and distribution amplitudes are still less known, even though much efforts have been made [22]. For the $B$ meson distribution
TABLE I: The numerical results of $T$ and $P$ for each decay mode. Theoretical Results are the predictions of the branching ratios in the PQCD approach. For comparison, we also list the experimental data.

| Mode          | $T$     | $P$     | Theoretical Results                  | Experimental Data [9] |
|---------------|---------|---------|--------------------------------------|-----------------------|
| $B^0 \to D^-\pi^+$ | 1.56 - 0.01 i | 0       | $(2.6^{+0.9+1.2}_{-0.6-0.8}) \times 10^{-5}$ | $(2.4 \pm 0.5) \times 10^{-5}$ |
| $B^0 \to D_s^0\rho^+$ | 1.64 - 0.04 i | 0       | $(3.1^{+1.0+1.3}_{-0.8-1.0}) \times 10^{-5}$ | $< 2.4 \times 10^{-4}$ |
| $B^0 \to D_s^-\pi^+$ | 1.47 - 0.02 i | 0       | $(2.7^{+0.9+1.2}_{-0.6-0.8}) \times 10^{-5}$ | $(2.6^{+0.5}_{-0.4}) \times 10^{-5}$ |
| $B^0 \to D_s^-D^+$ | 2.67 - 0.17 i | 0.23 - 0.01 i | $(7.7^{+1.9+4.3}_{-1.4-2.7}) \times 10^{-3}$ | $(7.4 \pm 0.7) \times 10^{-3}$ |
| $B^0 \to D_s^-D^{*+}$ | 2.68 - 0.17 i | 0.23 - 0.02 i | $(7.6^{+1.9+4.3}_{-1.4-2.7}) \times 10^{-3}$ | $(8.2 \pm 1.1) \times 10^{-3}$ |
| $B^0 \to D_s^-\pi^+$ | 2.72 - 0.21 i | 0.25 - 0.02 i | $(7.7^{+1.9+4.3}_{-1.4-2.7}) \times 10^{-3}$ | $(7.6 \pm 1.6) \times 10^{-3}$ |

amplitude, we adopt the model [13]:

$$
\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[- \frac{1}{2} \frac{x M_B}{\omega_B} \right] - \frac{\omega_B^2 b^2}{2},
$$

with the shape parameter $\omega_B = 0.40 \pm 0.05$ GeV, which has been tested in many channels such as $B \to \pi\pi, K\pi$ [14]. The normalization constant $N_B$ is related to the decay constant $f_B = 190$ MeV [13]. As for $D$ or $D_s$ meson, the distribution amplitude, determined in Ref. [17] by fitting, is

$$
\phi_D = \frac{1}{2 \sqrt{6}} f_D 6x(1 - x) \left[1 + C_D(1 - 2x)\right] \exp \left[- \frac{\omega^2 b^2}{2} \right],
$$

where $C_D = 0.5, \omega = 0.1$ for $D$ meson and $C_D = 0.4, \omega = 0.2$ for $D_s$.

Because of the unitarity of the CKM matrix, we have a relation:

$$
V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0.
$$

Within the parametrization proposed by Wolfenstein, $|V_{ub}V_{us}^*|$ is about $\lambda^4$, while $|V_{tb}V_{ts}^*|$ and $|V_{cb}V_{cs}^*|$ are about $\lambda^2$, where $\lambda \approx 0.22$. Therefore, $|V_{ub}V_{us}^*|$ can be ignored safely, which leads to $V_{ub}V_{us}^* = -V_{cb}V_{cs}^*$. For simplicity, Eqs. (6–7) could be re-written as:

$$
\mathcal{A}(B^0 \to D^-\pi^+) = V_{cb}V_{cs}^*(T_{DD_s} + P_{DD_s}),
$$

$$
\mathcal{A}(B^0 \to D^-\pi^+) = V_{ub}V_{us}^* T_{D_s\pi}.
$$

$T_{DD_s}, P_{DD_s}$ and $T_{D_s\pi}$ can be calculated directly in the PQCD approach, whose central values are listed in Table. 1. In Table 1, we also list the branching ratios calculated within the PQCD approach with the parameter values [12, 9, 23], i.e. $|V_{ub}| = 0.00359, |V_{cb}| = 0.041$ and $|V_{cs}| = 0.973$. The first error is from the threshold re-summation, and the second one comes from the distribution amplitude of heavy meson. For uncertainties of heavy meson, please look at Refs. [17, 19]. Comparing our theoretical predictions with experimental data, we find that our predictions are quite close to the data after combining all kinds of uncertainties, even though the central values are a bit larger than those of experimental data, which may hint a smaller $|V_{ub}|$. It should be also noted that our result of $B^0 \to D^-\pi^+$ is smaller than that from Ref. [13], mainly because the different light meson distribution amplitudes are used.
Following Ref. [11], we define the ratios and estimate the values as,

\[ R_{\pi/D} = \frac{\mathcal{B}(B^0 \to D_{s}^{+} \pi^{+})}{\mathcal{B}(B^0 \to D_{s}^{0} D^{+})} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{T_{D_{s} \pi}}{T_{D_{s} D} + P_{D_{s} D}} \left( \frac{p_{\pi}^{T}}{p_{\pi}^{D}} \right) = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \times 0.44 \pm 0.07, \]  

(13)

\[ R_{\rho/D} = \frac{\mathcal{B}(B^0 \to D_{s}^{+} \rho^{+})}{\mathcal{B}(B^0 \to D_{s}^{0} D^{+})} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{T_{D_{s} \rho}}{T_{D_{s} D} + P_{D_{s} D}} \left( \frac{p_{\rho}^{T}}{p_{\rho}^{D}} \right) = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \times 0.53 \pm 0.09, \]  

(14)

\[ R_{\pi/D'} = \frac{\mathcal{B}(B^0 \to D_{s}^{0} \pi^{+})}{\mathcal{B}(B^0 \to D_{s}^{0} D^{+})} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{T_{D_{s} \pi}}{T_{D_{s} D'} + P_{D_{s} D'}} \left( \frac{p_{\pi}^{T}}{p_{\pi}^{D'}} \right) = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \times 0.46 \pm 0.08, \]  

(15)

\[ R_{\rho/D'} = \frac{\mathcal{B}(B^0 \to D_{s}^{0} \rho^{+})}{\mathcal{B}(B^0 \to D_{s}^{0} D^{+})} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{T_{D_{s} \rho}}{T_{D_{s} D'} + P_{D_{s} D'}} \left( \frac{p_{\rho}^{T}}{p_{\rho}^{D'}} \right) = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \times 0.55 \pm 0.09, \]  

(16)

\[ \bar{R}_{\pi/D} = \frac{\mathcal{B}(B^0 \to D_{s}^{-} \pi^{+})}{\mathcal{B}(B^0 \to D_{s}^{-} D^{+})} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{T_{D_{s} \pi}}{T_{D_{s} D} + P_{D_{s} D}} \left( \frac{p_{\pi}^{T}}{p_{\pi}^{D}} \right) = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \times 0.45 \pm 0.07, \]  

(17)

where \( p_{X}^{T} \) is the c.m. momentum of the decay particle \( X \). Combining the experimental data of Table 1, we arrive at the following ratios:

\[ R_{\pi/D} = (0.32 \pm 0.07) \times 10^{-2}; \]  

(18)

\[ R_{\pi/D'} = (0.29 \pm 0.07) \times 10^{-2}; \]  

(19)

\[ \bar{R}_{\pi/D} = (0.34 \pm 0.10) \times 10^{-2}. \]  

(20)

After compared with the predictions of the PQCD approach [13,17], we can get:

\[
\left| \frac{V_{ub}}{V_{cb}} \right| = \begin{cases} 
0.085 \pm 0.011 & \text{from } R_{\pi/D}, \\
0.079 \pm 0.011 & \text{from } R_{\pi/D'}, \\
0.087 \pm 0.014 & \text{from } \bar{R}_{\pi/D},
\end{cases}
\]  

(21)

and the averaged value is

\[
\left| \frac{V_{ub}}{V_{cb}} \right| = 0.083 \pm 0.007.
\]  

(22)

If we set \( |V_{cb}| = 0.0415 \pm 0.0011 \) which is the global fit PDG result, we can get

\[
|V_{ub}| = (3.44 \pm 0.30) \times 10^{-3}.
\]  

(23)

Compared with the PDG values Eqs. (2,4), our central value favors a bit smaller \( |V_{ub}| \), but agrees well with the exclusive data. We also notice that the uncertainty is still large, which is because there are larger uncertainties in both theoretical calculations and experimental data. With much more knowledge in QCD and precisely measured experimental data in future, the errors in our prediction will be reduced.

With the averaged ratio of \( |V_{ub}/V_{cb}| \) Eq. (22), by using both a relation [14] and experimental data for \( B^0 \to D_{s}^{+} D^{+} \), we can predict the branching ratio of \( B^0 \to D_{s}^{-} \rho^{+} \) as:

\[ \mathcal{B}(B^0 \to D_{s}^{-} \rho^{+}) = (2.7 \pm 1.2) \times 10^{-5}, \]  

(24)

which would be measured easily in future B factories. Please also note that our theoretical estimate within the PQCD approach for the branching ratio, as shown in Table 1, is

\[ \mathcal{B}(B^0 \to D_{s}^{-} \rho^{+}) = (3.1^{+1.0+1.3}_{-0.8-1.0}) \times 10^{-5}. \]  

(25)
To our best knowledge about this channel, still there are only upper limits from CLEO [24] and ARGUS [25]. We hope future B factories could measure this channel to test our prediction.

Finally we add one comment on $B_s$ decay: With the help from CDF, DØ and LHC, the measurement of branching ratios of $B^0_s$ decay modes becomes much more precise than before. In principle, we can generalize the strategy of $B_d$ decays to $B_s$ by using SU(3) symmetry. We can define the ratio similarly:

$$R^{B^0_s}_{K/D} = \frac{\mathcal{B}(B^0_s \to D^-_s K^+)}{\mathcal{B}(B^0_s \to D^-_s D^+_s)}.$$  (26)

However, both decay modes $B^0_s \to D^-_s K^+$ and $B^0_s \to D^-_s D^+_s$ are much more complicate than decay modes of $B^0_d$. Primarily because both $B^0_s$ and $\bar{B}^0_s$ can decay to the same final state $D^-_s K^+$, we cannot discriminate whether the final states come from $B^0_s$ or $\bar{B}^0_s$. The same situation occurs in $B^0_s \to D^-_s D^+_s$ decay mode. Furthermore, the $B^0_s \to D^-_s K^+$ mode also has the annihilation diagrams, which will lead to larger uncertainty in calculating the $R^{B^0_s}_{K/D}$. In practice, in order to measure the CKM angle $\gamma$, the experimentalists often analyze the time dependent CP violation of these kinds of decay modes.

As summary, we revisited extracting $|V_{ub}/V_{cb}|$ and $|V_{ub}|$ through calculating the ratios $\mathcal{B}(B^0 \to D^{(*)+}_s (\pi^-, \rho^-))/\mathcal{B}(B^0 \to D^{(*)+}_s D^{(*)-}_s)$ in the PQCD approach. Using the updated distribution amplitudes and the latest experimental data, we got $|V_{ub}/V_{cb}| = 0.083 \pm 0.007$, which favors a bit smaller $|V_{ub}|$ compared to the value of PDG. Moreover, we predicted the branching ratio of $\bar{B}^0 \to D^-_s \rho^+ \sim (2.7 \pm 1.2) \times 10^{-5}$, which can be measured in future B factories. Furthermore, we could test the applicability of the PQCD with charmed mesons through comparing the future data and our calculation.

Acknowledgement

We would like to thank Y.J. Kwon for his valuable comments. The work of C.S.K. is supported in part by Basic Science Research Program through the NRF of Korea funded by MOEST (2009-0088395), in part by KOSEF through the Joint Research Program (F01-2009-000-10031-0), and in part by WPI Initiative, MEXT, Japan. The work of Y.L. was supported by the Brain Korea 21 Project and by the National Science Foundation under contract Nos.10805037 and 10625525.
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