Damage simulation of fiber reinforced composites using mean-field homogenization methods

Peter Lenz and Rolf Mahnken

Our work presents a two-scale mean-field homogenization framework for constitutive multiscale (meso-macro) modeling of fiber reinforced composites (FRC) including damage. The FRC is decomposed into three distinct linear thermo-chemo-elastic material phases. The three phases are unidirectional fibers surrounded by an interface, which is surrounded by a matrix material. Different mean-field homogenization methods can be used to determine the effective properties. For the damage a distinction is made between four different damage modes namely matrix damage, fiber damage, interface damage and a simultaneous damage of the matrix and the fiber called matrix-fiber damage. An element deletion algorithm is used to simulate a crack propagation. A representative example demonstrates the different types of damage and a resulting crack propagation in an FRC.

1 Introduction

Effective properties of polymer based fiber reinforced composites (FRC) depend on a degree of cure \( z \) of the matrix material. Starting with the initial uncured state of the matrix, a mixture of resin and curing agent, undergoes a polymerization process which cures the matrix to a solid material. The curing process is highly temperature dependent and the matrix shrinks, which introduces thermal and chemical eigenstresses into the fibers. Eigenstresses can lead to an initial damage state inside an FRC, which gains with an increasing load state. So far, only a final curing state of the matrix can be considered and not the process itself.

A distinction is made between four different damage modes: matrix damage, fiber damage, interface damage and a simultaneous damage of the matrix and the fiber called matrix-fiber damage. For matrix damage a critical yield stress is decisive within the matrix. The fiber damage is caused by a high normal stress in fiber direction. The shear stress inside the interface is responsible for an interface damage. An element deletion algorithm is used to simulate a crack propagation. If the matrix-fiber damage occurs in all integration points of a finite-element, it gets marked for the element deletion algorithm. A representative example demonstrates the different types of damage and a resulting crack propagation in an FRC.

2 fibre reinforced composites for linear thermo-chemo-elasticity including damage

In Fig 1.a) a two scale model for an FRC is illustrated. On the macroscopic scale the material is homogeneous. The mesoscopic scale is represented by a representative volume element (RVE). The meso-RVE is decomposed into a fiber \((F)\), an interface \((I)\), and a matrix \((M)\). Furthermore, at the micro-RVE it is assumed that the strain tensor of a geometrically linear theory \( \mathbf{e}_i \) is additively decomposed into elastic, thermal, and chemical parts, respectively, \( \mathbf{e}_i = \mathbf{e}_i^i + \mathbf{e}_i^h + \mathbf{e}_i^{cu} \) for \( i = I, F, M \), where \( \mathbf{u}_i \) is the displacement vector defined at any point \( P_i \) inside the meso-RVE. The equilibrium condition is written at \( P_i \) as \( \nabla \cdot \mathbf{\sigma}_i = 0 \). Furthermore, the following constitutive equations are employed for the individual strain contribution

\[
1. \quad \mathbf{e}_i^i = \frac{1}{(1 - D_i)} \mathbf{C}_i^{-1} : \mathbf{\sigma}_i, \quad 2. \quad \mathbf{e}_i^h = \mathbf{\alpha}_i \Delta \theta, \quad 3. \quad \mathbf{e}_i^{cu} = \mathbf{\beta}_i z_i, \quad i = I, F, M, \tag{1}
\]

with the fourth order elasticity tensor \( \mathbf{C}_i \), the damage variable \( D_i \), the second order thermal-dilatation tensor \( \mathbf{\alpha}_i \), the temperature change \( \Delta \theta = \Delta \theta(x) \), the second order curing-dilatation tensor \( \mathbf{\beta}_i \) and the degree of cure \( z_i \) of phase \( i \), respectively. Moreover, in Eq. (1) the damage variable \( D_i = 1 - \exp\{-b_i (\gamma_i - g_i^0)\} \) is given explicit by an exponential function, where \( b_i \) is a material parameter, \( g_i^0 \) is an initial damage threshold value and \( \gamma_i \) represents the maximum of an equivalent effective stress \( g_i \) of phase \( i \).

The effective macroscopic properties are given by

\[
1. \quad \mathbf{C}^* = \sum_{i=F,I,M} c_i \mathbf{A}_i^T : \mathbf{C}_i, \quad 2. \quad \mathbf{\alpha}^* = \sum_{i=F,I,M} c_i \mathbf{B}_i^T : \mathbf{\alpha}_i, \quad 3. \quad \mathbf{\beta}^* \mathbf{z} = \sum_{i=F,I,M} c_i \mathbf{B}_i^T : \mathbf{\beta}_i z_i, \tag{2}
\]

where \( \mathbf{C}^* \) is the effective elasticity tensor, \( \mathbf{\alpha}^* \) is the effective thermal-dilatation tensor, \( \mathbf{\beta}^* \) is the effective curing-dilatation tensor, \( \mathbf{z} \) is the volume average degree of cure and \( c_i \) is the volume fraction of phase \( i \). Moreover, in Eq. (2) \( \mathbf{A}_i \) and \( \mathbf{B}_i \) are the local strain and the stress concentration tensors [2], respectively. In this case \( \mathbf{A}_i = \mathbf{A}_i[F, I, M] \) and \( \mathbf{B}_i = \mathbf{B}_i[F, I, M] \) are...
functions in terms of all three phases including the properties \( C_i \) and the geometry of phase \( i \), see [1] for more details. Based on the effective properties in Eq. (2) the stress in phase \( i \) is given by

\[
\sigma_i = B_i : C^* : (\varepsilon - \alpha^* \Delta \theta - \beta^* \varepsilon), \quad i = F, I, M,
\]

where \( \varepsilon \) is the macroscopic strain tensor. Furthermore, the equivalent effective stress \( g_i \) is given in terms of \( \sigma_i \) as

\[
1. g_F = |m : \sigma_F : m|, \quad 2. g_M = \sqrt{\frac{3}{2}} \sigma_M^{\text{dev}} : \sigma_M^{\text{dev}}, \quad 3. g_I = |m : \sigma_I : n|
\]

where \( m \) and \( n \) are the longitudinal direction and the surface normal vector of a fiber and \( \sigma^{\text{dev}} \) is the deviatoric part of \( \sigma \).

### 3 Numerical results

As a numerical example, we consider a plate with a hole as shown in Fig 1.a). The material is an FRC with fibers aligned in \( x \) direction. The parameters for the individual phases are given in Eq. (5). The effective properties in Eq. (2) are obtained with the Mori-Tanaka homogenization method [2]. In Fig. 1.b)-g) numerical results including a crack propagation are illustrated. Due to the isotopic damage \( D_i \) in Eq. (1), the crack propagation follows the maximum stress, which occurs at the crack tip, see Fig 1.b). Moreover, only elements where the fiber-matrix damage occurs are deleted, see Fig 1.c)-e). The damage inside the interface has a high influence on thermal and chemical stresses inside the fiber, see Fig 1.d),f) and g).

\[
E_F = 236316 \text{ MPa} \quad \nu_F = 0.1 \quad \alpha_F = -1e^{-7} \frac{1}{K} \quad c_F = 0.65 \quad b_F = 1e^{-5} \quad z_I = 1 \quad g_F^0 = 4600 \text{ MPa}
\]

\[
E_M = 3671 \text{ MPa} \quad \nu_M = 0.43 \quad \alpha_M = 65e^{-6} \frac{1}{K} \quad c_M = 0.3 \quad \beta_M = 4e^{-5} \quad z_M = 0.8 \quad g_M^0 = 85 \text{ MPa}
\]

\[
E_I = \frac{E_F + E_M}{2} \quad \nu_I = \frac{\nu_F + \nu_M}{2} \quad \alpha_I = 65e^{-6} \frac{1}{K} \quad c_I = 0.3 \quad \beta_I = 4e^{-5} \quad z_I = 0.8 \quad g_I^0 = 100 \text{ MPa}
\]

Fig. 1: A two-scale model overview is given in a). Numerical results including a crack propagation: b) macro von Mises stress, c) fiber damage variable \( D_F \), d) interface damage variable \( D_I \), e) matrix damage variable \( D_M \), f) thermal von Mises stress and in g) curing von Mises stress inside the fiber, respectively.

**Acknowledgements** This work is based on investigations of the “SPP 1712 - Intrinsische Hybridverbunde für Leichtbautragstrukturen”, which is kindly supported by the Deutsche Forschungsgemeinschaft (DFG).

**References**

[1] C. Friebel, I. Doghri, and V. Legat, Int J Solids Struct **43**, 2513 - 2541 (2006).

[2] G. Dvorak, Micromechanics of Composite Materials (Springer Netherlands, 2013).