Cellular automaton traffic model considering driver’s reaction to velocity and headway distance with variable possibility of randomization

Guoqiang Zhang\textsuperscript{1,2,3,*} and Weiling Xu\textsuperscript{1,2,3}

1Jiangsu Key Laboratory of Urban ITS, Southeast University, Nanjing 210096, China; 2Jiangsu Province Collaborative Innovation Center of Modern Urban Traffic Technologies, Southeast University, Nanjing 210096, China; 3School of Transportation, Southeast University, Nanjing 210096, China

*Corresponding author e-mail: guoqiang.zhang@163.com

Abstract. Because of its simplicity and flexibility, cellular automaton model is widely used to simulate traffic flows. Based upon the NS (Nagel and Schreckenberg) Model, the paper proposes a cellular automaton model to simulate drivers’ reaction to velocity and headway distance in single lane traffic flow by modification of deceleration and randomization of the rules. Analysis of average velocity is performed to study the state and stability of traffic flow in various situations. For low densities, traffic is in free flow state, with average velocity fluctuating around the mathematical expectation stably. With the increasing of density, traffic becomes unstable and goes into jammed flow state. The behaviours of the model are very similar to those of the real traffic flows observed in the field. It is potential that the model can be used to predict traffic state for the engineering practice of traffic control and management.

1. Introduction
With the rapid development of computer capacity, many kinds of microscopic traffic models have been set up to simulate behavior details of each car to get evolution rules. Among the various models, cellular automaton model (CA Model) has received tremendous attention due to its highly practical importance. Because of properties such as the discreteness of space and time, the simplicity and flexibility of algorithm and the easiness to be simulated on computer, it is widely used to simulate vehicular movements in traffic researches. Compared with continuum models, CA Model is much simpler and more convenient. Besides, CA model can model the complexities of nonlinear characters of traffic flows and produce intuitive physical images, which are extraordinarily helpful for comprehension of the complicated process of traffic flow evolution.

In 1992, Nagel and Schreckenberg proposed the famous NS Model, which is the basis of many CA Models [1]. The model simulates single-lane traffic flow of cars by a one-dimensional cellular chain under periodic boundary conditions. In the model, all of the vehicles are arranged in a discrete cellular chain, whose length is L, according to some distribution requirement. Each cell may either be empty or be occupied by one vehicle. All vehicles are assumed to move from the left to the right and the velocity of each vehicle is an integer between zero and \( v_{\text{max}} \). The \( n \)th vehicle in the time-step \( t \) is located at the position \( x_n(t) \), moving with an integral velocity \( v_n(t) \).
The headway distance between consecutive vehicles is $d_n(t) = x_{n+1}(t) - x_n(t) - 1$, which is in fact the empty cells in front of the $n$th vehicle. With these notations, the system evolves according to the synchronous rules as follows:

(i) Rule 1: acceleration, $v_n(t + 1) = \min(v_n(t) + 1, v_{\max})$;
(ii) Rule 2: deceleration, $v_n(t + 1) = \min(v_n(t + 1), d_n)$;
(iii) Rule 3: randomization, $v_n(t + 1) = \max(v_n(t + 1) - 1, 0)$, with probability $p$;
(iv) Rule 4: update of location, $x_n(t + 1) = x_n(t) + v_n(t + 1)$.

In spite of its simplicity, the NS Model can reproduce such phenomena as phase transition, phase separation, traffic congestion and so on, and these stand for characteristics of real traffic flows. Applying only four simple rules, the NS Model is able to simulate single-lane traffic flows quite successfully. However, just because of the simplicity, it cannot simulate some details of vehicular movements. As a result, the NS Model cannot reproduce some important aspects of real traffic flows, such as metastability and hysteresis.

In order to simulate real traffic more accurately, various CA Models are proposed, based upon the NS Model. In these models, some characteristics of vehicular movements have been modelled by modification of one or more rules of the NS Model. It is observed that acceleration of stopped or slow vehicles is delayed compared to that of moving or faster cars and such slow-to-start actions are simulated by some slow-to-start rules [2]. When one car moves along a lane, drivers will pay attention to not only the position of the front car but also its velocity, which is absent in Rule 2 of the NS Model. To make up the shortage, velocity effect of front cars has been explored by modification of the deceleration rule [3]. Similarly, impact of headway distance has been modeled by modification of Rule 2 or Rule 3 [4]. The heterogeneity of vehicular traffic (e.g., vehicles with different maximal velocity, deceleration rule, probability of randomization, length and so on) is also an important feature observed in real traffic, and therefore CA Models are proposed to simulate some aspects of the phenomenon [5]. Besides, the NS Model has been expanded and modified to simulate more complicated traffic systems [6].

In real traffic, vehicular movements are influenced by drivers’ reaction to all sorts of situations. Of the various impacting factors, velocity and headway distance are two most important ones which affect drivers’ reaction and influence how vehicles accelerate or decelerate under some specific circumstances. On the basis of analysis of driver’s behavior, a CA Model is proposed, modeling drivers’ reaction to velocity and headway distance more thoroughly.

2. Model

When a vehicle is moving along, its driver controls the vehicular velocity and decides whether to accelerate or decelerate according to not only the velocity and headway distance of his or her own car but those of the front car as well. Such reaction to velocity and headway distance can be modeled by a modification of Rule 2, where decision of acceleration or deceleration for the next time-step is impacted by present velocities and headway distances of both the vehicle under consideration and the front vehicle, as is proposed by the Velocity Effect Model (VE Model) [6]. By applying a “virtual velocity”, which reflects driver’s assumption as to how front vehicle will move in the next time-step, the VE Model is able to simulate higher maximum flux than the NS Model. However, there is a drawback in the model, for it implies that drivers behave boldly without consideration of possible errors or accidents.

In reality, with increasing of velocity and shortening of headway distance, drivers tend to become more cautious, which means that vehicles are more likely to slow down in order to avoid potential conflicts with front cars. Such reaction can be modeled by variable possibility of randomization for Rule 3. That is to say, a vehicle is assigned with bigger possibility to slow down when its velocity is equal to or bigger than its headway distance.

To simulate driver’s reaction as has been proposed above, our model applies the rules as follows:

(i) Rule 1: acceleration, $v_n(t + 1) = \min(v_n(t) + 1, v_{\max})$;
(ii) Rule 2: deceleration, $v_n(t + 1) = \min(v_n(t + 1), d_n(t) + v'_{n+1}(t + 1))$,
where \( v_{n+1}(t+1) \) is an estimate of velocity of the front car in the time-step \( t+1 \), which is determined by both velocity and headway distance of the front car and is calculated as

\[
v_{n+1}(t+1) = \min(v_{\text{max}} - 1, v_{n+1}(t), \max(0, d_n(t) - 1))\]

(iii) Rule 3: randomization, \( v_n(t+1) = \max(v_n(t+1) - 1, 0) \), with probability \( p_s \) when \( v_n(t+1) \) is smaller than \( d_n(t) \), and with probability \( p_b \) otherwise , where \( p_s \) is smaller than \( p_b \);

(iv) Rule 4: update of location, \( x_n(t+1) = x_n(t) + v_n(t+1) \).

The modified model, which we call the Reaction to Velocity and Headway Distance Model (RVHD Model), has taken into consideration the most important factors that impact a driver’s reaction while he or she is following a front car. It can be seen that, for Rule 2, the next time-step velocity of a vehicle under consideration is decided by present velocity and headway distance of both the vehicle itself and its front vehicle. For Rule 3, when future (the next time-step) velocity of a vehicle is bigger than or equal to present headway distance, it is more likely to be slowed down than otherwise, which simulates the fact that a driver becomes more cautious, when compared with headway distance, the car is running at a rather faster speed. When \( p_s \) is equal to \( p_b \), the RVHD Model becomes the VE Model.

### 3. Simulations and Discussion

#### 3.1. Simulations

In the simulations, the length of each cell corresponds to 7.5 m on a real road, an automaton time-step is 1 s and a velocity unit corresponds to about 27 km/h. It is assumed that \( v_{\text{max}} \) is 5, which corresponds to 135 km/h, the normal free-flow speed in the real traffic. The total number of cells, denoted by \( L \), is assumed to be 2000 and the total number of vehicles is denoted by \( N \). The global density of traffic flow is \( \rho = N / L \), the average velocity is \( \bar{v}(t) = (\sum v_n(t) / n) \) and the average flux is \( \bar{f}(t) = \rho \times \bar{v}(t) \). In each time-step, velocities of all the vehicles are renewed according to the updating rules that are proposed above and then the vehicles move forwards simultaneously. The periodic boundary condition is applied when necessary. Two different kinds of initial conditions have been used in the simulation: homogeneous and jammed. In a homogeneous initial condition, vehicles are distributed as evenly as possible and the initial velocity is \( v_{\text{max}} \) for each vehicle. In a jammed initial condition, vehicles are arranged in the cells contiguously. In this case, except the leading vehicle, whose velocity is \( v_{\text{max}} \), velocities of all the other vehicles are zero.

#### 3.2. Average Velocity

Broadly speaking, average velocity of vehicles reflects the general situation of traffic flow. Therefore, by analyzing average velocity over a period of time, we can explore how traffic flow performs and evolves. For example, if average velocity stays near the maximum velocity \( v_{\text{max}} \) steadily, it means that most vehicles are running at the maximum speed and therefore, the traffic is in a free flow state. And otherwise, it shows that the traffic is in a jammed state. If average velocity fluctuates dramatically over a period of time, it shows that the traffic flow is not stable during this period, and more simulation steps should be run before traffic flow can reach a stable state.

In the simulations for analyzing average velocity, \( p_s = 0.2 \) and \( p_b = 0.9 \). In addition, the initial conditions are homogeneous. Theoretically, if traffic is in a free flow state, headway distance is usually fairly large, which means that \( v_n(t+1) \) is smaller than \( d_n(t) \), except that about eighty percent of vehicles are running at the maximum velocity 5, approximately twenty percent of vehicles are running at a slower velocity 4, due to randomization. And the mathematical expectation of average velocity can be calculated as 4.8.

Figure 1 shows the change of average velocity of traffic flow with \( \rho = 0.05 \), over a period of 2000 time-steps. Average velocities of consecutive time-steps fluctuate around the mathematical expectation 4.8, indicating that traffic flow is in free flow state. Figure 2 is the graph for average velocity of traffic flow with \( \rho = 0.10 \), which shows similar characteristics as Figure 1. This indicates that traffic flow is still in free flow state when density increases to 0.10.
Figure 3 depicts the inconsistent evolution pattern of average velocity of traffic flow with $\rho = 0.15$. At first, the curve for average velocity fluctuates between 4.4 and 4.6, showing a seemingly stable state. However, after 700 time-steps, it goes down steadily, and then fluctuates between 3.4 and 3.2. This indicates that traffic flow with such a density is unstable at the beginning and data concerning traffic flow should not be collected until it has reached a more stable state.

Figure 4 depicts the dramatic change for average velocity of traffic flow with $\rho = 0.20$. At the beginning of the simulation, average velocities fluctuate above 4. After 28 simulation time-steps, average velocities decrease steadily until they reaches an area between 2.5 and 2, where they fluctuate for quite a long time, with no tendency to decline. This indicates that with a higher density, vehicles are more likely to be involved in slow-moving jammed state very quickly, since more densely distributed vehicles impact each other more severely, giving less opportunity for drivers to choose a faster speed.

![Figure 1. The average velocity of free traffic flow when $\rho = 0.05$, $p_s =0.2$ and $p_b =0.9$.](image1)

![Figure 2. The average velocity of free traffic flow when $\rho = 0.10$, $p_s =0.2$ and $p_b =0.9$.](image2)
4. Conclusion
A cellular automaton model is proposed to simulate drivers’ reaction to velocity and headway distance in single lane traffic flow. Both deceleration and randomization of the model have been modified to take drivers’ reaction into consideration. Analysis of average velocity indicates that for low densities, traffic is in free flow state, with average velocity fluctuating around the mathematical expectation stably. With the increasing of density, traffic becomes unstable and goes into jammed flow state. Results of the model are in agreement with traffic phenomena observed in the field. This indicates that the model might be used to forecast traffic situations for the control and management of traffic flows.

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