A Radio Frequency Helical Deflector for keV Electrons

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Abstract
This paper describes a helical deflector to perform circular sweeps of keV electrons by means of radio frequency fields in a frequency range 500-1000 MHz. By converting the time dependence of incident electrons to a hit position dependence on a circle, this device can potentially achieve extremely precise timing. The system can be adjusted to the velocity of the electrons to exclude the reduction of deflection sensitivity due to finite transit time effects. The deflection electrodes form a resonant circuit, with quality factor Q in excess of 100, and at resonance the sensitivity of the deflection system is around 1 mm per V of applied RF input.

Keywords: Radio Frequency Electron Deflector

1. Introduction
Circular scanning is commonly used in high-resolution timing devices such as streak cameras and the first experiments in this technique were started in the 1950’s. Much effort has gone into the development of deflection systems, which would give a reasonable deflection of a beam of electrons using moderate RF voltages applied to the deflectors. Different types of deflectors have been proposed, developed and applied. Rudenberg showed that loss in dynamic sensitivity, due to transit time effects in parallel plate deflectors, could be reduced by substituting parallel wire deflectors. The analysing system consisted of two crossed pairs of shorted Lecher-wires, resonant at the frequency (3000 MHz) of the beam modulating system under investigation. It was used to measure ps electron bunches, where the time dispersion of secondary electron emission process was determined to be less than 6 ps.

However analysing systems based on parallel plates have also been developed and applied. In these circular-scan systems deflection frequencies up
to 300 MHz were employed. Later, circular-scan streak tubes operating at frequencies of 200 MHz \[8\] and 320 MHz \[9\] have been developed for use in a laser ranging systems. By applying a 320 MHz, 13 W RF signal, which had a phase shift applied between the deflection plates, a 6 mm circular sweep was obtained \[9\]. A circular sweep speed (which essentially defines the achievable time resolution of the device) of \(6.44 \times 10^8\) cm/s was achieved, providing a temporal resolution of 30-35 ps and a range difference jitter of less then 6 ps.

With further development, parallel-plate deflection systems have achieved even faster circular sweeps. A speed of \(2.85 \times 10^9\) cm/sec was achieved by Sibbet et al. by applying 300 MHz, 15 W RF power to a Photochron IIC streak tube \[10\]. A 30 mm diameter scan was obtained, enabling an instrumental time resolution better then 6 ps. \(6.25 \times 10^9\) cm/s was achieved by Bryukhnevitch et al. by applying 500 MHz, 17 W RF power to a PV006S streak tube \[11\]. A 40 mm diameter circular scan and time resolution of less than 5 ps were achieved.

At higher frequencies other types of deflection systems have been investigated in the 1970’s. A travelling wave RF analysing system was used by Kalibjian et al. \[12\], giving a time resolution of about 0.7 ps when a 3.6 GHz RF field was applied. Alternatively a 10 GHz slit-aperture-resonator deflection system was employed by Butslov et al., giving a time resolution of 0.5 ps \[13\] and a similar RF deflector has been investigated by the Yerevan group for 3 GHz frequency applications \[14\]. More recently a cylindrical RF cavity was used to perform circular sweeps, using a transverse magnetic field in TM110 mode, with circular polarization and a frequency of 2.856 GHz \[15\]. The time resolution was about 0.6 ps.

In this article we present a sensitive and compact circular sweep deflection system for keV electrons, capable of operating in the 500-1000 MHz frequency range. The theory of the deflector and calculations based on simulation studies are described in Sec. 2. Results of experimental studies of the deflector are presented in Sec. 3.

2. The Circular Scanning Deflector For keV Electrons

When an electron beam passes through a parallel-plate deflection system that has an axial length \(\Lambda\), a separation \(d\) and a potential difference \(U_d\) between the plates (Fig. 1), it is deflected vertically by an angle \(\tan \theta = \frac{eU_d\Lambda}{mdv_z^2}\), where \(e, m, v_z\) are respectively the charge, mass and velocity of the electron. The accelerating potential \(U_a\) of the electron “gun” of the system produces an energy \(eU_a = \frac{mv_z^2}{2}\) and hence \(\tan \theta = \frac{U_d\Lambda}{2U_ad}\). This produces a deflection \(\Delta Y\) at a distance \(D\) from the plates:

\[
\Delta Y = D\tan \theta = \frac{DU_d\Lambda}{2U_ad}
\]

The static sensitivity of the \(Y\) deflection, i.e. the sensitivity of the deflector at low frequencies, is defined by \(\varepsilon_s = \Delta Y/U_d\). The main problem for transverse
deflection of keV electrons at high frequencies is connected to the finite time that electrons take to travel through the deflector. If the RF field changes according to \( \sin \omega t \), where \( \omega = 2\pi/T \) and \( T \) is the period of the RF Voltage, the dynamic sensitivity of the parallel plate RF deflector \( \varepsilon_d \), can be expressed as

\[
\varepsilon_d = \frac{\varepsilon_s \sin \left(\frac{\omega T_0}{2}\right)}{\omega T_0/2}
\]

where \( T_0 = \frac{\Lambda}{v_z} \) is the transit time of the electron through the deflector and \( \omega T_0 \) is the so called transit angle. The dynamic sensitivity is a function of \( \omega, \Lambda, v_z \) and at very high frequencies the transit time of the electrons through the deflecting field reduces the magnitude of the deflection angle, i.e. \( \varepsilon_d < \varepsilon_s \) unless the transit time is smaller than a half-period of the highest frequency to be used (see Ref.[1, 16]).

To avoid transit time effects at high frequencies a dedicated RF deflector was proposed by Shamaev\(^1\) in the 1960’s (the theory is described in Ref.[16]). The Shamaev deflector is based on a pair of helical electrodes (Fig. 2) of periodic length \( \Lambda \) and separation \( d \). Electrons move in the direction of the \( Z \) axis, which coincides with the axis of the deflection system, with a constant velocity \( v_z \). An applied sinusoidal RF Voltage across the deflection plates produces a transverse \((E_z = 0)\) electric field so that the equation of motion of electrons in this field can be written as:

\[
\frac{dv_\perp}{dt} = -\frac{eE_\perp}{m}, \quad \frac{dv_z}{dt} = 0
\]

where \( v_\perp = v_x + iv_y \) and \( E_\perp = E_x + iE_y \) are complex variables.

When the wavelength of the applied RF field \( \lambda \gg \Lambda \), one can to first approximation ignore the variation of the field along the length of the electrode, in which case \( E_\perp \approx E(t) \exp(i2\pi z/\Lambda) \), where \( E(t) = E_d \sin(\omega t + \phi) \), \( E_d = U_d/d \) is the amplitude of the electric field in the deflector for an applied RF voltage of \( U_d \) and \( \phi \) is the RF phase at entry of an electron into the deflector. The oscillating field can be represented by a sum of two fields which sweep back and forth, by means of the Euler formula:

\(^1\)The Ph.D. thesis of Y.M. Shamaev is referred to in Ref.[17], but we were unable to access this unpublished document.
Assuming that electrons with constant velocity \(v_z\) enter the RF deflector at \(t = 0\), the equation of motion in the \(X - Y\) plane is given by:

\[
\frac{dv_y}{dt} = \frac{eE_d}{2m} \{ \exp [i(\omega_c - \omega)t + i\phi] - \exp [i(\omega_c + \omega)t - i\phi] \}
\]

where \(\omega_c = \frac{2\pi z}{\Lambda} \quad \frac{2\pi v_z}{\Lambda} = \frac{2\pi}{T_c}\). The transverse velocity of the electron after the deflector \(v_y\) can be determined by integration of Eq.(4) over the range \(t = 0 - \tau\), where \(\tau = l/v_z\) and \(l\) is the length of the deflector. \(T_c\) is the transit time over one periodic length of the deflector (\(l = \Lambda\)).

\[
v_y = \frac{eE_d \tau}{2m} \left\{ \sin x_2 \frac{\sin (x_2 + \phi)}{x_1} - \sin x_1 \frac{\sin (x_1 - \phi)}{x_1} \right\}
\]

where \(x_1 = \frac{\omega_c - \omega}{2v_z} \tau = \frac{\omega_c - \omega}{2v_z} l\) and \(x_2 = \frac{\omega_c + \omega}{2v_z} \tau = \frac{\omega_c + \omega}{2v_z} l\). The \(x, y\) components of the transverse velocity are:

\[
v_x = -\frac{eE_d \tau}{2m} \left\{ \frac{\sin x_2}{x_2} \cos (x_2 + \phi) + \frac{\sin x_1}{x_1} \cos (x_1 - \phi) \right\}
\]

Simulations were carried out to investigate the behaviour of the helical RF deflector at different values of \(k = \omega/\omega_c\) and \(l = n\Lambda/4\ (n = 1, 2, 3, 4)\). Plots of \(v_x\) vs. \(v_y\) in units of \(v_0 = eE_d\Lambda/8mv_z\), for different values of \(k\) and \(n\), are presented in Fig.3. These studies show that the \(x - y\) locus is circular only for the \(k = 1\) and \(n = 2, 4\) cases. In the following we concentrate on the case \(k = 1\), \(n = 4\) where the transit time of electrons through the deflector with \(l = \Lambda\) is equal to the period of the applied high frequency sinusoidal voltage (\(\omega = \omega_c\)).
I. $k=0.5$; a) $n=1$; b) $n=2$; c) $n=3$; d) $n=4$

II. $k=1.0$; a) $n=1$; b) $n=2$; c) $n=3$; d) $n=4$

III. $k=1.5$; a) $n=1$; b) $n=2$; c) $n=3$; d) $n=4$

IV. $k=2$; a) $n=1$; b) $n=2$; c) $n=3$; d) $n=4$

V. $n=4$; a) $k=0.75$; b) $k=0.825$; c) $k=1.5$; d) $k=1.65$

Figure 3: Transverse velocity components $v_x$ vs. $v_y$ in units of $v_0$. 
We refer to this as the Shamaev resonance condition, where one has maximum transverse velocity \( v_\perp = 4v_0 \), and Eq. 6 takes a simple form. In the case of \( k = 1, n = 4 \) then \( x_1 = 0, x_2 = 2\pi \), leading to \( \sin x_1/x_1 = 1, \sin x_2 = 0 \) and thus \( v_x = -4v_0 \sin \phi, v_y = -4v_0 \cos \phi \). The deflection angle is given by:

\[
\tan \theta = \frac{v_\perp}{v_z} = \frac{eE_d\tau}{2mv_z} \sqrt{\left( \frac{\sin x_1 - \sin x_2}{x_1 - x_2} \right)^2 + \frac{4\sin x_1 \sin x_2}{x_1 x_2} \sin^2 \left( \frac{\omega \tau}{2} + \phi \right) }
\]

which for \( k = 1 \) and \( n = 4 \) reduces to \( \tan \theta = \frac{4v_0}{v_z} = \frac{eE_d\Lambda}{2mv_z^2} = \frac{U_d\Lambda}{4dU_a} \). When the Shamaev resonance condition is applied to a helical deflector, transit-time effects are avoided and \( \varepsilon_d = \varepsilon_s/2 \) (the factor 2 in the denominator is related to the helical shape of the deflector electrodes). Electron trajectories can be analysed in two parts: those inside and outside the deflector. The electron’s transverse coordinates at the end of the RF deflector, \( r_x = \int v_x dt \) and \( r_y = \int v_y dt \), are obtained by integrating over the range \( t = 0 \rightarrow 1/v_z \), resulting in:

\[
\begin{align*}
r_x &= \frac{eE_d^2}{4mv_z^2} \left\{ \cos \phi \left[ \frac{1}{x_1} \left( 1 - \frac{\sin 2x_1}{2x_1} \right) + \frac{1}{x_2} \left( 1 - \frac{\sin 2x_2}{x_2} \right) \right] + \sin \phi \left[ \frac{\sin^2 x_1}{x_1^2} + \frac{\sin^2 x_2}{x_2^2} \right] \right\} \\
r_y &= \frac{eE_d^2}{4mv_z^2} \left\{ -\sin \phi \left[ \frac{1}{x_1} \left( 1 - \frac{\sin 2x_1}{2x_1} \right) - \frac{1}{x_2} \left( 1 - \frac{\sin 2x_2}{x_2} \right) \right] + \cos \phi \left[ \frac{\sin^2 x_1}{x_1^2} + \frac{\sin^2 x_2}{x_2^2} \right] \right\}
\end{align*}
\]

For \( k = 1 \) and \( n = 4 \) these formulae simplify to \( r_x = R_0 [\sin \phi + \cos \phi/2\pi] \) and \( r_y = R_0 [\cos \phi + \sin \phi/2\pi] \), where \( R_0 = \frac{eE_d\Lambda^2}{4mv_z^2} = \frac{U_d\Lambda^2}{4dU_a^2} \). At the end of the deflector the electrons will scan through an ellipse of half axes \( a = R_0 [1 + 1/2\pi] \) and \( b = R_0 [1 - 1/2\pi] \). If the electrons are detected on a screen at a distance \( D \) from the deflector, the transverse coordinates will be \( R_x = r_x + D\tan \theta_x \) and \( R_y = r_y + D\tan \theta_y \), where \( \tan \theta_x = -4v_0 \sin \phi/v_z \) and \( \tan \theta_y = -4v_0 \cos \phi/v_z \), leading to:

\[
\begin{align*}
R_x &= r_x - D\frac{4v_0 \sin \phi}{v_z} \tag{9} \\
R_y &= r_y - D\frac{4v_0 \cos \phi}{v_z}
\end{align*}
\]

For \( U_a = 2.5 \text{ kV} \) and \( \Lambda = 6 \) (3 cm), corresponding to Shamaev resonance conditions at 500 (1000) MHz, one obtains, for \( U_d = 20 \text{ V} \) and \( d = 1 \) cm, \( R_0 = 0.036 \) (0.009) cm and \( \tan \theta = 0.012 \) (0.006). At distances \( D = 0, 12, 24 \) cm, circles of radius 1, 5, 9 (1, 9, 17) \( R_0 \) respectively are produced. In Sec. 3 it is demonstrated that the deflector forms a resonant circuit with a Q-factor in excess of 100 at these frequencies. This results in an observed deflection more than an order of magnitude larger than has been calculated here.
3. Experimental studies

The experimental set up for investigation of the characteristics of the RF deflector is presented schematically in Fig. 4. It consists of a pumped vacuum vessel with various electrical and mechanical feed throughs. A thermionic cathode (1) emits electrons, which are accelerated by a voltage applied between the cathode and an electron transparent electrode (2). An electrostatic lens (3) then focuses the accelerated electrons on to the phosphor screen (5) at the far end of the tube. The RF deflection system, consists of electrodes (4), a RF source and tuning circuit. The experimental commissioning followed the procedure: i. mechanical and vacuum testing; ii. electron beam optics tuning; iii. RF deflector tuning.

The size of the focused thermionic electron beam spot on the phosphor screen, mounted 12 cm from the end of the deflector, was less than 1 mm diameter. The RF deflection system consisted of one pair of helical electrodes (Fig. 2), which served as a capacitive element, containing the applied RF power within the deflector. For a given set of electrodes we observed that at some frequencies the deflection angles, or radii of the scanned circles, were about an order magnitude higher than anticipated from theory. Indeed for $U_d = 20 \text{ V}$, $\omega = 1000 \text{ MHz}$, $\Lambda = 3 \text{ cm}$, $d = 1 \text{ cm}$ and $U_a = 2.5 \text{ kV}$, the radius of the scanning circle at a distance $D = 12 \text{ cm}$ was predicted to be 0.81 mm, while it was measured at 20 mm (Fig. 5a). For a tuned deflector at $\omega = 750 \text{ MHz}$ frequency, electrons scan on to an ellipse (Fig. 5b). These two cases correspond to the theoretical predictions presented in Fig. 3Id ($n = 4$, $k = 1$) and Fig. 3Va ($n = 4$, $k = 0.75$). The observed factor $\sim 25$ larger deflection than predicted by Eq. 9 can be explained if the deflection electrodes form a resonant circuit at these frequencies with a high Q-factor. The resonant frequency can be fixed at a desired value, i.e. near the Shamaev resonant frequency, by tuning the
Figure 5: Images of the deflected 2.5 keV continuous electron beam at the phosphor screen.

The parameters of the deflector and test set up are: \( \Lambda = 3 \) cm, \( d = 1 \) cm, \( U_d = 20 \) V, \( U_a = 2.5 \) kV, \( D = 12 \) cm. Applied RF frequencies are: a) 1000 MHz; b) 750 MHz. The radius of the scanned circle is 2 cm. The length of the major axis of the ellipse is also around 2 cm.

Figure 6: An equivalent circuit of the RF deflector and resonant tuning system. \( U_d \): applied RF Voltage; \( C_v \): variable capacitance; \( C_d \): deflector capacitance; \( R_d \): deflector resistance; \( L_d \): deflector inductance.

capacitance of the deflection system. This may be realized by suitable design of the electrodes or, for given electrodes, by tuning of the deflector capacitance using a coaxial cavity or an additional variable capacitor.

The equivalent circuit of the helical deflector and tuning system is presented in Fig.6. The Q-factor of the resonant circuit was determined experimentally, by measuring the diameter of the scanning circle as a function of the applied RF frequency. An example of the resonant behaviour at frequencies around 500 MHz for a deflector with parameters \( \Lambda = 6 \) cm, \( d = 1 \) cm, \( U_d = 10 \) V, \( U_a = 2.5 \) kV, \( D = 12 \) cm. is presented in Fig.7. From this it follows that the Q-factor of the formed resonance is about 130. Thanks to this resonance, about 1 W (into 50\( \Omega \)) of RF power is enough to scan 2.5 keV electrons circularly at a radius of 2 cm. The sensitivity of this new and compact RF deflector is about 1 mm/V or 0.1 rad/W\(^{1/2}\) which is an order of magnitude higher than the sensitivities of other RF deflectors used previously. Potentially it has a
number of applications in fixed-frequency cathode-ray-tube based instruments, for example opto-electronic devices such as RF Streak Cameras or the RF Photomultiplier Tube [18, 19]. The latter has been proposed for ps resolution timing measurements in nuclear physics experiments.

4. Summary

This paper describes a RF deflector of helical geometry to perform circular sweeps of keV energy electrons using high frequency electromagnetic fields. The sweep can be used convert the temporal dependence of a pulse of electrons passing through the deflector to a position dependence on a circle. By employing a position sensitive electron detector very precise timing information at the ps level may then be obtained. A major advantage of the helical electrodes is that the system can be optimised to the velocity of the transiting electrons, so that loss of deflection sensitivity due to finite transit time effects is avoided.

The theory of operation of the deflector is derived and optimum deflector dimensions calculated for operational frequencies of 500 and 1000 MHz. After fine tuning the effective capacitance of the system, the deflection electrodes form a resonant circuit, with a quality factor Q in excess of 100. On resonance, the sensitivity of the deflection system is about 1 mm/V or 0.1 rad/W^{1/2} and around 1 W (into 50Ω) of RF power is sufficient to scan 2.5 keV electrons circularly at a radius of 2 cm radius. This makes the deflector suitable for ultra-high resolution timing devices such as the Radio Frequency Photomultiplier Tube [18, 19], where
development continues. Future developments include a dual helical deflection system to produce a spiral rather than circular scanning pattern.

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