An Equivalent Circuit Model for Separate Absorption Grading Charge Multiplication Avalanche Photodiode

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Abstract. With the scale of microwave photonics system getting larger and the structure of it getting more complex, the CAD tools and accurate circuit models have become essential for designers, since difficulty for system design increases greatly. In this work, an equivalent circuit model for separate absorption grading charge multiplication avalanche photodiode (SAGCM APD) has been proposed. The model is applied to simulate the device performance and the simulating results show a well reasonable agreement with the physical model calculation and experimental data. A detailed comparison has been made for InP/InGaAs, InAlAs/InGaAs and Si/Ge APD with different structures. Our results show that Si/Ge APD is the most promising candidate for high performance optical communication system.

1. Introduction

SAGCM-APD has drawn a lot of attention in optical communication systems because of its internal gain, low noise and high bandwidth compared with APDs made in other structures\textsuperscript{[1-3]}. For material used in APDs, ionization coefficient ratio \( k \) for InP is about 0.4-0.5\textsuperscript{[4]}, while for InAlAs that is about 0.2-0.3, which leads to a higher gain-bandwidth product (GBP) for InAlAs APD\textsuperscript{[5]}. In recent years, Silicon has been considered to be the most promising multiplication material due to its favorable ionization coefficient ratio (\( k<0.1 \))\textsuperscript{[6]}. In addition, compared with other semiconductors, Si/Ge APD has overwhelming advantages, such as high GBP (>200GHz), low excess noise, temperature independence, CMOS-compatible and feasible monolithic integration with Transimpedance amplifier (TIA).

As optical communication systems push toward multigigabit per-second transmission rates, the frequency response of these APDs has become a critical issue. The circuit model of APD is helpful to the optimization of APD\textsuperscript{[5-7]}. In this work, an equivalent circuit model for separate absorption grading charge multiplication avalanche photodiode (SAGCM APD) has been proposed. The model is applied to simulate the device performance and the simulating results show a well reasonable agreement with the physical model calculation and experimental data. A detailed comparison has been made for InP/InGaAs, InAlAs/InGaAs and Si/Ge APD with different structures. Our results show that Si/Ge APD is the most promising candidate for high performance optical communication system.

2. Physical model
The two most important characteristics of a photodetector are its quantum efficiency and bandwidth. Increasing the thickness of absorption layer results in the collection of much more photons, and hence improves the quantum efficiency of the photodetector. However, the increase of the thickness of absorption layer leads to an increase of the transit time, and the 3-dB bandwidth decreases. According to the previous reports, WG-APDs or RCE-APDs can make a trade off between the bandwidth and the quantum efficiency of the photodetector. Based on Deen’s theory, the frequency response of primary, second electrons and holes of RCE-SACM-APD are given in (1)–(4), respectively:

\[
N(\omega) = \frac{P}{hv} \left[ (\eta_N + \eta_p) \left[ 1 - \exp(-\alpha W_c) \right] \times \left[ 1 - \exp(-j\omega(x_i + x_n) / v_r) \right] + \frac{1}{j\omega} \exp(-j\omega(x_i + x_n) / v_r) \times \left[ \eta_p - \eta_p \exp(-\alpha W_c) \right] \right. \\
+ \exp(-j\omega(x_i + x_n) / v_r) \times \left[ \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-\alpha W_c) - \exp(-j\omega W_c / v_r) \right] \right] \right. \\
+ \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-\alpha W_c) - \exp(-j\omega W_c / v_r) \right] \right]
\]

\[
P(\omega) = \frac{P}{hv} \left[ (\eta_N + \eta_p) \left[ 1 - \exp(-\alpha W_c) \right] \times \left[ 1 - \exp(-j\omega x_i / v_r) \right] + \frac{1}{j\omega} \exp(-j\omega x_i / v_r) \times \left[ \eta_p - \eta_p \exp(-\alpha W_c) \right] \right. \\
+ \exp(-j\omega x_i / v_r) \times \left[ \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-\alpha W_c) - \exp(-j\omega W_c / v_r) \right] \right] \right. \\
+ \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-\alpha W_c) - \exp(-j\omega W_c / v_r) \right] \right]
\]

\[
P_r(\omega) = \frac{P(M-1)}{hv} \exp(-j\omega x_i + \Delta u_n / v_r) \times \left[ 1 - \exp(-j\omega(x_i + W_n + x_i + \Delta u_n) / v_r) \right] \\
\times \left[ \frac{1}{1 + j\omega(1 + M-1)r_n} \times \left[ \eta_p \left[ \frac{1}{j\omega - \alpha v_r} - \frac{1}{j\omega} \right] \times \left[ \exp(-W_n) - \exp(-j\omega W_c / v_r) \right] \right. \\
- \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-W_n) - \exp(-j\omega W_c / v_r) \right] \right. \\
- \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-W_n) - \exp(-j\omega W_c / v_r) \right] \right. \\
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- \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-W_n) - \exp(-j\omega W_c / v_r) \right] \right. \\
- \frac{\eta_p}{\alpha v_r - j\omega} \times \left[ \left[ \exp(-W_n) - \exp(-j\omega W_c / v_r) \right]
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thickness \( x \) and the relative permittivity \( \varepsilon \) in the depletion region, and \( A \) is the active area of the devices.

3. Circuit model

In signal and system theory, for a linear time-invariant system, the output signal equals the convolution of input signal and the impulse response of the system in time domain. On the other hand, the output signal is equal to the product of signal and the frequency response of the system in frequency domain. In time domain, the overall impulse response of the cascade of multiple linear time-invariant systems is the convolution of the impulse responses of individual systems. In frequency domain, the total frequency response of the cascade of multiple systems is the product of the individual frequency responses \(^{[18]}\). Based on this theory, \( H(\omega)H_{\text{sys}}(\omega) \) is divided into small portions which are treated as frequency responses of sub-systems. From (1)-(4), five typical relationships of sub-system and the output signal are presented as follows (8)-(12):

\[
I_{\text{out}}(\omega) = z \cdot I_{\text{in}}(\omega) \quad (8)
\]

\[
I_{\text{out}}(\omega) = I_{\text{in}}(\omega) \cdot \frac{1}{j\omega \cdot z} \quad (9)
\]

\[
I_{\text{out}}(\omega) = I_{\text{in}}(\omega) \cdot \frac{1}{j\omega} \quad (10)
\]

\[
I_{\text{out}}(\omega) = I_{\text{in}}(\omega) \cdot \frac{1}{z + j\omega} \quad (11)
\]

\[
I_{\text{out}}(\omega) = I_{\text{in}}(\omega) \cdot \frac{1}{z - j\omega} \quad (12)
\]

Based on JiaYin Wu’s work \(^{[17]}\), the equivalent circuit model of a RCE-SACM-APD is presented in Fig.1

4. Results and discussion

On basis of the former theory \(^{[2]}\), the bandwidth of APD is mainly affected by transit time, RC time and avalanche buildup time. How these factors limit the bandwidth of APD will be discussed as follows.
Fig. 1 The equivalent circuit model of RCE-SAGCM-APD
In Fig.2, the gain-bandwidth (G-BW) characteristics of RCE-SACM-APD is simulated. The simulating results show that the circuit model simulation, physical model calculation and experimental data [19] are consistent with each other very well, which confirm the validity and application of circuit model for different structures.

Fig. 2 Circuit model simulated gain-bandwidth characteristics of RCE-SACM-APD compared with a physical model and experimental data from reference [19].

Fig. 3 illustrates G-BW characteristics for APD with different absorption layer’s thickness and structures, where APD1, APD2 and APD3 represent REC-SAGCM-APD, WG-SAGCM-APD and normal incidence SAGCM-APD using InAlAs for multiplication material, respectively. As shown in Fig.3, at low gain, an increase of the thickness of absorption layer leads to a decrease of bandwidth of APDs. The increase of absorption layer’s thickness results in an increase of transit time due to an increase of the distance which the photogenerated carriers in absorption layer have to travel, and hence the bandwidth decreases. At high gain, however, the bandwidth does not change obviously with the varying of the absorption layer’s thickness, which proves that at high gain, the change of the thickness of absorption layer does not result in an obvious change in the bandwidth of APDs, which may be ascribed to the avalanche buildup time determined APD’s bandwidth. The avalanche buildup time is dependent on the multiplication layer’s thickness, and the devices with the same thickness of multiplication layer have the same gain-bandwidth product. The simulated results show that the bandwidth of RCE-SAGCM-APD or WG-SAGCM-APD is higher than that normal incidence of the APD.

Fig.3 Circuit model simulated gain-bandwidth characteristics for APDs with different thickness of absorption layer and different structures (solid: \( x = 1 \) um, empty: \( x = 3 \) um)

According to (7), increasing the area of photosensitive region of APD leads to an increase of capacitance, and hence a decrease of the bandwidth of APD due to an increase of RC time. The G-BW characteristics for APDs with different the area of photosensitive region and different multiplication materials are simulated as shown in Fig.4. In this figure, APD4, APD5 and APD6 are Si/Ge, InAlAs/InGaAs and InP/InGaAs APD, respectively. The simulating results show that at low gain, the bandwidth of APD decreases with the increase of the area of photosensitive region. However, the
effect of the area of photosensitive region for the bandwidth of APD is not significant at high gain, because the bandwidth is dominated by avalanche buildup time not RC time. In addition, APD using Si for multiplication region has a higher bandwidth compared with other materials due to its preferable ionization coefficient rate. It demonstrates that Si/Ge APD is the most promising candidate for high performance optical communication system.

**Fig.4** Circuit model simulated gain-bandwidth characteristics for APDs with different areas of photosensitive region and different multiplication materials (solid: diameter is 10um; empty: diameter is 20um).

In an APD’s packaging module, the APD chip is connected usually to other device (like TIA) by bonding wires which can be equivalent to an inductance. The effect of varying of parasitic inductance are shown in Fig.5, where the frequency response of Si/Ge APD is simulated for different parasitic inductance at M=10. As shown in this figure, the bandwidth increases when the parasitic inductance increases from 0nHz to 1nHz. However, the bandwidth of APD decreases with further increase of inductance, while the resonant peak moves to lower frequency region, and the maximum $S_{21}$ value increases. The resonant effect of parasitic capacitance and inductance demonstrates that the bandwidth can be compensated to some extent using the packaging technology according to the resonant effect.

**Fig.5** Circuit model simulated frequency response of Si/Ge APD chip for different inductance of bonding wires in the packaging state.

5. Conclusion
In this paper, an equivalent circuit model for Resonant Cavity Enhanced-Separated Absorption Graded Charge Multiplication-Avalanche Photodetector (RCE-SAGCM-APD) has been proposed, and the simulating results show a reasonable agreement with the physical model calculation and experimental data. A detailed comparison has been made for InP/InGaAs, InAlAs/InGaAs and Si/Ge APD with different structures. Our results show that Si/Ge APDs is the most promising candidate for high
performance optical communication system. In addition, our circuit model can be applied for circuit simulation software, compared with simple circuit model of APDs, the circuit model is closed to the real APD’s performance, which is helpful to the optimization of APDs.

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Appendix

NOMENCLATURE
Frequency of the incident light.

Quantum efficiency for APDs with forward and backward incidence, respectively.

Planck’s constant.

Thickness of the absorption, the grading, the charge and the multiplication layers, total thickness of the grading and the charge layers, and thickness of the two grading layers surrounding the absorption layer, respectively.

Width of the whole depletion region.

Absorption coefficient of the absorption region.

Multiplication gain.

Avalanche buildup time.

Saturation velocities for the electrons and holes, respectively.

Power of Incident light

Photogenerated primary and secondary electrons, respectively.

Photogenerated primary and secondary holes, respectively.

Output extrinsic current and dc photocurrent of APDs, respectively.

Imaginary unit and angular frequency, respectively.