Are explanations of the Poynting-Robertson effect correct?

J. Klačka, J. Petržala, P. Pástor, L. Kómar

Faculty of Mathematics, Physics and Informatics, Comenius University
Mlynská dolina, 842 48 Bratislava, Slovak Republic
e-mail: klacka@fmph.uniba.sk, petrzala@fmph.uniba.sk
Pavol.Pastor@fmph.uniba.sk, komar@fmph.uniba.sk

Abstract. Physics of the Poynting-Robertson (P-R) effect is discussed and compared with the statements published in the past thirty years. Relativistically covariant formulation reveals the essence of the P-R effect and points out to nonphysical explanations in scientific papers and monographs. Although the final equation of motion
\[ m \frac{dv}{dt} = \left( S A' \bar{Q}'_{pr} / c \right) \{(1 - v \cdot e/c) e - v/c\} \]

has been usually correctly presented and used, its derivation and explanation of its essence is frequently incorrect.

The relativistically covariant form of the equation of motion yields the P-R effect as an action of the radiation pressure force on a moving spherical body. No "P-R drag", as a particular relativistically covariant equation of motion, exists. Omission of the nonphysical term "P-R drag" excludes any confusion in the published definitions.

The difference between the effects of solar electromagnetic and corpuscular (solar wind) radiation is stressed. The force acting on the particle due to the solar wind (the simple case of radial solar wind velocity is considered) is
\[ F_{sw} = F_{sw} \left[ (1 - v \cdot e/v_{sw}) e - x' v/v_{sw} \right], \]
where \( F_{sw} \) is the force on the stationary particle, \( v_{sw} \) is the heliocentric solar-wind speed, and, the value of \( x' \) depends on material properties of the particle (1 < \( x' < 3 \)).

Secular evolution of orbital elements is presented. Initial conditions are included.

Key words. cosmic dust – radiation pressure – equation of motion – relativity theory – Poynting-Robertson effect – solar/stellar wind effect – celestial mechanics
1. Introduction

Motto:
What is popular is not always right;
what is right is not always popular.

The action of electromagnetic radiation on a moving spherical body is called the Poynting-Robertson (P-R) effect. It is used in astrophysical modeling of orbital evolution of dust grains for many decades (e.g., Poynting 1903, Robertson 1937, Wyatt and Whipple 1950, Robertson and Noonan 1968, Dohnanyi 1978, Burns et al. 1979, Kapišinský 1984, Jackson and Zook 1989, Leinert and Grün 1990, Gustafson 1994, Dermott et al. 1994, Reach et al. 1995, Murray and Dermott 1999, Woolfson 2000, Danby 2003, Quinn 2005, Harwit 2006, Grün 2007, Sykes 2007, Krügel 2008, Wyatt 2009). The P-R effect holds when material of the particle is distributed in a spherically symmetric way. It is assumed that the spherical particle can be used as an approximation to real, arbitrarily shaped, particle. Relativistically covariant equation of motion for arbitrarily shaped dust particle can be found in Klačka (2004, 2008b), where also thermal emission of the particle is taken into account (Mishchenko 2001: the radiation pressure on arbitrarily shaped particles arises also from an anisotropy of thermal emission). Krauss and Wurm (2004) have published the first experimental evidence that nonspherical dust grains behave in a different way than the spherical ones.

We will deal with the P-R effect in this paper. It has elapsed thirty years since the time of Dohnanyi’s statement “Much confusion in the literature has existed on this subject and to some extent still does, unfortunately.” (Dohnanyi 1978, p. 563). The situation has not changed and many ”explanations” of the P-R effect have been offered during the past thirty years. It is important to have correct explanations of the effect, in disposal.

The paper presents many statements on the Poynting-Robertson effect, which exist in the literature. We have tried to show physically incorrect points of the statements and to put them into a correct way. The enormous amount of incorrect presentations probably confirms the ideas of Weisskopf (1973): ”The pressure of fast publication is so great that people rush into print with hurriedly written papers and books that show little concern for careful formulation of ideas. .... Essential ideas of physics are often lost ...”

We have already tried to explain the physics of the P-R effect (see, e.g., Klačka 1992a, 1993b, 2000b, 2001b). However, our comments and explanations have not met, in general, with a positive reaction and the incorrect statements and derivations are repeatedly cited and reproduced in many other scientific papers and textbooks. This avalanche propagation is the reason why we have decided
to write a paper presenting comments on various derivations and explanations. This will lead to a deeper and more complete understanding of the effect.

Our paper is based on the results of Klačka (2008a, 2008b). At first, we summarize the most important results on the P-R effect. Then, we discuss many "explanations" of the P-R effect, which have been presented in the literature during the past thirty years. The scientists should concentrate on Sec. 6.1, where comments on the most frequently cited paper by Burns et al. (1979) are given: not only the important parts of Klačka (1992a – Sec. 2.5) are repeated, but the relativistic results of Klačka (2008a, 2008b) are used. It is clear that the statement "the interpretation of the low-order velocity terms in classical (nonrelativistic) physics are still debated" (Gustafson 1994, p. 565) is already not relevant, now. Really, even the simplest case of Poynting (1903) and Robertson (1937) is correctly understood on the basis of relativistic approach. Moreover, the dimensionless efficiency factor $\bar{Q}_{pr}$, $\bar{Q}_{ext}$ are obtained from Mie's solution of Maxwell's equations and it cannot be said that Maxwell's equations belong to classical (nonrelativistic) physics. These relativistic results are used also in other parts of the paper; e.g., the case of the aberration of light is discussed in detail in Sec. 6.2.

To be sure that the physical results are correct, one has to consider relativistically covariant equation of motion. Klačka (2008a, 2008b) explicitly showed the advantage of relativistically covariant formulations: the application of this approach enabled to found that even the statement of Poynting (1903) and Robertson (1937) (and other explanations up to the year 2008) is incorrect: as for the outgoing radiation, the particle's "own radiation outwards being equal in all directions has zero resultant pressure" (Poynting 1903) in the particle's own frame of reference, or, "the process of absorption and re-emission produces no net force on a particle when one chooses to work with a stationary frame referred to the particle" (Wyatt and Whipple 1950).

The importance of relativistically covariant equations of motion is stressed, e. g., by Feynman and Mould. Feynman states: "The laws of physics must be such that after a Lorentz transformation, the new form of the laws looks just like the old form." (Feynman et al. 2006, volume II, p. 25-1), or, "There is, however, another reason for writing our equations this way. It has been discovered – after Einstein guessed that it might be so – that all of the laws of physics are invariant under the Lorentz transformation. That is the principle of relativity. Therefore, if we invent a notation which shows immediately when a law is written down whether it is invariant or not, we can be sure that in trying to make new theories we will write only equations which are consistent with the principle of relativity.

... the interesting physical thing is that every law of physics must have this same invariance under the same transformation. ... It is because the principle of relativity is a fact of nature that in the notation of four-dimensional vectors the equations of the world will look simple." (Feynman et al. 2006, volume
Feynman has motivated a reader to the importance of using tensors (or, as a special case, four-vectors) in formulation of equation of motion. In order to be the reader more familiar with the relevance of this access, we reproduce the corresponding explanation from Mould (2002, p. 123): "It is a goal of relativity to formulate the fundamental laws of nature in covariant form; that is, in a form that is independent of coordinates. A four-vector equation, for instance, is covariant in this sense. The particular values of a four-vector may very well depend on coordinates, but a four-vector equation will have the same form in all coordinate systems. Tensor equations have this same property. They have the same form in all coordinate systems even though their specific component values differ from one coordinate system to another. Scalar functions have a form invariance as well as value invariance."

Sections 2-5 present the most relevant results on the physics of the P-R effect. Sec. 6 discusses various "explanations" of the P-R effect, published during the past 30 years. Sec. 7 offers two simple correct explanations of the P-R effect. Finally, Sec. 8 summarizes the relevant equations for the secular evolution of orbital elements of a spherical body under the action of central star’s gravity and electromagnetic radiation. Discussions presented in the paper can help in understanding the P-R effect. The paper should ensure that only correct explanations will be published in future.

2. Covariant formulations

Relativistically covariant formulations are useful in understanding of the interaction between a particle and an incident electromagnetic radiation. Spherically symmetric mass distribution within the particle is tacitly assumed.

Interaction of the spherical particle with the incident electromagnetic radiation is known as the Poynting-Robertson effect. The emission component of the radiation force, due to thermal emission, is zero, i.e. $F'_e = 0$. The conditions for the incoming and outgoing radiation are (Klačka 2008b):

$$\begin{align*}
\frac{dp_{\text{in}}^\mu}{d\tau} &= \frac{w^2 S A' \bar{Q}'_{\text{ext}}}{c} b^\mu , \\
\frac{dp_{\text{out}}^\mu}{d\tau} &= \frac{w^2 S A' \bar{Q}'_{\text{ext}}}{c} \left\{ \bar{Q}' \frac{w^\mu}{c} + (1 - \bar{Q}') \ b^\mu \right\} , \\
\bar{Q}' &= \frac{\bar{Q}'_{\text{pr}}}{\bar{Q}'_{\text{ext}}} ,
\end{align*}$$

(1)

where the left-hand sides are four-momenta, e.g. $p_{\text{out}}^\mu = (E_{\text{out}}/c ; p_{\text{out}})$, $c$ is the speed of light in vacuum, $S$ is the flux density of radiation energy (energy flow through unit area perpendicular to the ray per unit time), $A'$ is geometrical cross-section of the spherical particle, $\bar{Q}'_{\text{ext}}$ is dimensionless extinction efficiency factor given by optical properties of the particle and averaged over the stellar spectrum, $\bar{Q}'_{\text{pr}}$ is dimensionless efficiency factor for radiation pressure integrated over stellar spectrum.
and calculated for radial direction (as for the dimensionless factors of effectiveness of radiation pressure and extinction, see Mie 1908, or, e.g., Klačka and Kocifaj 2007, or, Sec. 4.5 in Bohren and Huffman 1983) and 4-vectors \(u^\mu\) (four-velocity), \(b^\mu\) and the quantity \(w\) are given by the following equations:

\[
\begin{align*}
\left.\begin{array}{l}
u^\mu = (\gamma c; \gamma v) , \\
b^\mu = \left(\frac{1}{w}; \frac{e}{w}\right) , \\
w = \gamma \left(1 - \frac{v \cdot e}{c}\right),
\end{array}\right\} \quad \text{(2)}
\end{align*}
\]

where \(v\) is particle’s velocity in the reference frame of the star and \(e\) is unit position vector of the particle with respect to the star; \(\gamma\) is the Lorentz factor. As was pointed out by Klačka (2008b), Eq. (1) differs from the geometrical approach presented in the literature since the time of Poynting and Robertson.

Equations for \(dp^\mu_{\text{out}}/d\tau\) and \(dp^\mu_{\text{in}}/d\tau\) yield the following covariant relation between the outgoing and the incoming radiation:

\[
\frac{dp^\mu_{\text{out}}}{d\tau} = (1 - \bar{Q}') \frac{dp^\mu_{\text{in}}}{d\tau} + \frac{w^2 S A' \bar{Q}'_{\text{ext}}}{c} \bar{Q}' u^\mu.
\]

\[
\bar{Q}' = \frac{Q'_{\text{pr}}}{Q'_{\text{ext}}}, \quad \text{(3)}
\]

Eq. (3) can be considered as a fundamental equation for the validity of the P-R effect.

### 2.1. Incoming radiation and particle’s acceleration

The action of the incoming radiation influences particle’s motion according to the following equation of motion:

\[
\frac{dp^\mu}{d\tau} = \frac{dp^\mu_{\text{in}}}{d\tau}, \quad \text{(4)}
\]

where the right-hand side is given by Eq. (1) and

\[
p^\mu = mu^\mu \quad \text{(5)}
\]

is a four-momentum of the particle. Inserting Eqs. (1) and (5) into Eq. (4), we obtain

\[
m \frac{du^\mu}{d\tau} + \frac{dm}{d\tau} u^\mu = \frac{w^2 S A' \bar{Q}'_{\text{ext}}}{c} b^\mu.
\]

\[
\left(\frac{dm}{d\tau}\right)_{\text{in}} = \frac{w^2 S A' \bar{Q}'_{\text{ext}}}{c^2}. \quad \text{(6)}
\]

Multiplication of Eq. (6) by \(u_\mu\) yields, on the basis of \(u_\mu \, du^\mu/d\tau = 0\), \(u_\mu \, u^\mu = c^2\), \(u_\mu \, b^\mu = c\):

\[
\left(\frac{dm}{d\tau}\right)_{\text{in}} = \frac{w^2 S A' \bar{Q}'_{\text{ext}}}{c^2}. \quad \text{(7)}
\]
Inserting Eq. (7) into Eq. (6), we obtain four-acceleration
\[
\left( \frac{du^\mu}{d\tau} \right)_{in} = \frac{w^2 S A' \bar{Q}'_{ext}}{mc} \left( b^\mu - \frac{u^\mu}{c} \right). \tag{8}
\]

The result represented by Eq. (7) can be understood in the sense that \( \frac{dE_{in}}{d\tau} = \frac{w^2 S A' \bar{Q}'_{ext}}{mc} \) corresponds to the energy of the incoming radiation interacting with the particle \( \frac{dE_{in}}{d\tau} = c^2 \frac{dm}{d\tau}_{in} \). The real system consists of radiation and the particle and the system cannot be divided into the individual components. Thus, the result presented by Eq. (7) does not correspond to the real change of the particle mass, but only to a virtual change. No direct real experiment can verify the validity of Eq. (7). Indirect evidence of Eq. (7) is the validity of the result \( 0 < \bar{Q}'_{pr} / \bar{Q}'_{ext} < 2 \) and experimental confirmation of Maxwell’s equations and relativity theory.

It may seem surprising that \( \frac{dm}{d\tau}_{in} \) depends on the particle velocity and \( A' \bar{Q}'_{ext} \), only. E.g., perfectly reflecting and absorbing spherical surfaces, within geometrical optics approximation, yield the same result for \( \frac{dm}{d\tau}_{in} \).

### 2.2. Outgoing radiation and particle’s acceleration

The action of the outgoing radiation influences the motion of the particle according to the following equation of motion:
\[
\frac{dp^\mu}{d\tau} = - \frac{dp^\mu_{out}}{d\tau}, \tag{9}
\]

where the right-hand side is given by Eq. (1). Inserting Eqs. (1) and (5) into Eq. (9), we obtain
\[
m \frac{dw^\mu}{d\tau} + \frac{dm}{d\tau} u^\mu = - \frac{w^2 S A' \bar{Q}'_{ext}}{c} \left\{ \bar{Q}' \frac{u^\mu}{c} + (1 - \bar{Q}') b^\mu \right\}, \tag{10}
\]

\[
\bar{Q}' = \frac{\bar{Q}'_{pr}}{\bar{Q}'_{ext}}.
\]

Multiplication of Eq. (10) by \( u_\mu \) yields, on the basis of \( u_\mu u^\mu = c^2 \), \( u_\mu \frac{du^\mu}{d\tau} = 0 \), and, \( u_\mu b^\mu = c \):
\[
\frac{\frac{dm}{d\tau}}{out} = - \frac{w^2 S A' \bar{Q}'_{ext}}{c^2} \tag{11}
\]

Inserting of Eq. (11) into Eq. (10), we obtain four-acceleration
\[
\left( \frac{du^\mu}{d\tau} \right)_{out} = \frac{w^2 S A' \bar{Q}'_{ext}}{mc} \left( b^\mu - \frac{u^\mu}{c} \right), \tag{12}
\]

\[
\bar{Q}' = \frac{\bar{Q}'_{pr}}{\bar{Q}'_{ext}}.
\]
2.3. Equation of motion

Equation of motion of the particle is given by the condition

\[
\frac{dp^\mu}{d\tau} = \frac{dp^\mu_{\text{in}}}{d\tau} - \frac{dp^\mu_{\text{out}}}{d\tau}.
\]  

(Eq. 13)

Eqs. (1) and (13) yield

\[
\frac{dp^\mu}{d\tau} = \frac{w^2 S A' Q'_{\text{ext}}}{c} \bar{Q}' \left( y^\mu - \frac{u^\mu}{c} \right),
\]

\[
\bar{Q}' \equiv \frac{Q'_{\text{pr}}}{Q'_{\text{ext}}}. \tag{14}
\]

Insertion of Eq. (5) into the left-hand side of Eq. (14) yields

\[
\frac{dp^\mu}{d\tau} = m \left( \frac{du^\mu}{d\tau} \right) + \left( \frac{dm}{d\tau} \right) u^\mu.
\]

Using this relation and subsequent multiplication of Eq. (14) by \(u^\mu\), one obtains \(\frac{dm}{d\tau} = 0\), if also \(u^\mu \left( \frac{du^\mu}{d\tau} \right) = 0\) and \(u^\mu \cdot u^\mu = c^2\), \(u^\mu \cdot b^\mu = c\) are used. The relation \(\frac{dm}{d\tau} = 0\) states that the mass of the particle is conserved. Thus, Eq. (14) may be rewritten to the form

\[
\frac{du^\mu}{d\tau} = \frac{w^2 S A' Q'_{\text{ext}}}{m c} \bar{Q}' \left( y^\mu - \frac{u^\mu}{c} \right),
\]

\[
\bar{Q}' \equiv \frac{Q'_{\text{pr}}}{Q'_{\text{ext}}}. \tag{15}
\]

3. The fundamental condition

In order to be able to correctly understand physics of the P-R effect, we have to realize that the fundamental condition for the validity of the P-R effect exists (Eq. 3). The essence of the P-R effect is that nonradial components of the particle’s radiation pressure and components of the thermal force are zero (in particle’s frame of reference). Thus, the P-R effect holds for spherical particles.

The fundamental condition states: the total momentum (per unit time) of the outgoing radiation is nonzero, in general, and

\[
E'_{\text{outgoing}} = E'_{\text{incoming}},
\]

\[
p'_{\text{outgoing}} = (1 - \bar{Q}') p'_{\text{incoming}},
\]

\[
\bar{Q}' \equiv \frac{Q'_{\text{pr}}}{Q'_{\text{ext}}},
\]

\[
F'_c = 0, \tag{16}
\]

in the proper frame of reference of the particle. The condition is obtained from its covariant form presented by Eq. (3). The equation states that the P-R effect holds if the total momentum (per unit time) of the outgoing radiation, integrated over the whole space angle, is in the direction of the incident radiation, in the particle’s frame of reference.
It is generally believed that the special case $\bar{Q}' = 1$ corresponds to the perfectly absorbing sphere, in geometrical optics approximation. The equation of motion for this special case was discussed by Poynting (1903) and finally written by Robertson (1937). It is believed that this special case yields

i) $p'_\text{outgoing} \equiv d\bar{p}'\text{out}/d\tau = 0$ (see Eqs. 1 or 3) and this corresponds to the hypothesis of "isotropic reemission of the radiation", and, ii), the case $\bar{Q}' = 1$ is also produced by specular reflection on the spherical particle (within the geometrical optics approximation). However, the physical condition for the two cases, perfect absorption and specular reflection, is $\bar{Q}' = 1/2$, in reality (Klačka 2008a, 2008b).

4. Approximation of the equation of motion

It is sufficient to approximate space component of the covariant equation of motion (and also acceleration of the particle) to the first order in $v/c$. Eq. (14) yields the following relation between its time and space components:

$$\frac{dp^0}{d\tau} = \frac{v}{c} \cdot \frac{dp}{d\tau}. \quad (17)$$

For this reason, we have to do an approximation of the time component to the second order in $v/c$.

At first we will present approximations for the incident and outgoing radiation. Eqs. (1) yield the following approximation (the results are exact, accurate to all orders in $v/c$):

$$\frac{dE_{\text{in}}}{dt} = S A' Q'_\text{ext} \left(1 - \frac{v \cdot e}{c}\right),$$

$$\frac{dp_{\text{in}}}{dt} = \frac{SA' \bar{Q}'_\text{ext}}{c} \left(1 - \frac{v \cdot e}{c}\right) e. \quad (18)$$

The results for the outgoing radiation (to the second order in $v/c$ in energy) are:

$$\frac{dE_{\text{out}}}{dt} = S A' Q'_\text{ext} \left\{1 - \frac{v \cdot e}{c} - \bar{Q}' \left[\frac{v \cdot e}{c} - \left(\frac{v \cdot e}{c}\right)^2 - \left(\frac{v}{c}\right)^2\right]\right\},$$

$$\frac{dp_{\text{out}}}{dt} = \frac{SA' \bar{Q}'_\text{ext}}{c} \left\{\bar{Q}' \frac{v}{c} + (1 - \bar{Q}') \left(1 - \frac{v \cdot e}{c}\right) e\right\},$$

$$\bar{Q}' \equiv \frac{\bar{Q}'_{\text{pr}}}{\bar{Q}'_\text{ext}}, \quad (19)$$

where $t$ is time measured in the stationary reference frame, i.e. the reference frame in which the particle moves with the velocity $v$. Acceleration of the particle, due to the incoming and outgoing radiation, we get from Eqs. (8) and (12)

$$\left(\frac{d v}{d t}\right)_{\text{in}} = \frac{SA' \bar{Q}'_\text{ext}}{mc} \left[(1 - \frac{v \cdot e}{c}) e - \frac{v}{c}\right],$$

$$\left(\frac{d v}{d t}\right)_{\text{out}} = - \frac{SA' \bar{Q}'_\text{ext}}{mc} (1 - \bar{Q}') \left[(1 - \frac{v \cdot e}{c}) e - \frac{v}{c}\right],$$

$$\bar{Q}' \equiv \frac{\bar{Q}'_{\text{pr}}}{\bar{Q}'_\text{ext}}. \quad (20)$$
Covariant equation of motion of the particle yields
\[
\frac{dE}{dt} = S A' \bar{Q}'_{ext} \bar{Q}' \left\{ \frac{v \cdot e}{c} - \left( \frac{v \cdot e}{c} \right)^2 - \left( \frac{v}{c} \right)^2 \right\},
\]
\[
\frac{dp}{dt} = \frac{SA' \bar{Q}'_{ext}}{c} \bar{Q}' \left\{ \left(1 - \frac{v \cdot e}{c}\right) e - \frac{v}{c} \right\},
\]
\[
\bar{Q}' = \frac{\bar{Q}'_{pr}}{\bar{Q}'_{ext}}
\]
and the acceleration of the particle is
\[
\frac{dv}{dt} = \frac{SA' \bar{Q}'_{ext}}{mc} \bar{Q}' \left\{ \left(1 - \frac{v \cdot e}{c}\right) e - \frac{v}{c} \right\},
\]
\[
\bar{Q}' = \frac{\bar{Q}'_{pr}}{\bar{Q}'_{ext}}
\]

5. Radiation pressure and the Poynting-Robertson drag

The P-R effect is characterized by the following equation of motion to the first order in \(v/c\):
\[
\frac{dv}{dt} = \beta \frac{GM}{r^2} \left\{ \left(1 - \frac{v \cdot e}{c}\right) e - \frac{v}{c} \right\},
\]
\[
\beta \equiv \frac{\pi R^2 L}{4\pi GMmc} \bar{Q}'_{pr},
\]
where \(G\) is a gravitational constant, \(L\) and \(M\) are luminosity and mass of the central star around which the particle moves, \(r\) is a distance of the particle from the star and \(R\) is a radius of the spherical particle.

Gustafson (1994, p. 566) writes: "The velocity-independent radial term representing force [acceleration] due to radiation pressure... – although not a pressure, it is often referred to simply as 'radiation pressure' ":
\[
\left( \frac{dv}{dt} \right)_{\text{pressure}} = \beta \frac{GM}{r^2} e.
\]

According to a definition (see, e.g. Gustafson 1994, p. 566, or, Dohnanyi 1978, pp. 562-565), the velocity-dependent part of Eq. (23) is the P-R drag:
\[
\left( \frac{dv}{dt} \right)_{\text{P-R drag}} = -\beta \frac{GM}{r^2} \left\{ \left( \frac{v \cdot e}{c} \right) e + \frac{v}{c} \right\}.
\]

We can decompose the velocity vector \(v\) of the particle into the radial and transversal parts
\[
v = v_R e + v_T e_T \equiv \dot{r} e + r \dot{\theta} e_T,
\]
where the unit vector \(e_T\) is normal to \(e\) in the orbital plane. Substituting Eq. (26) into Eq. (23) yields
\[
\frac{dv}{dt} = \beta \frac{GM}{r^2} \left\{ \left(1 - 2 \frac{\dot{r}}{c}\right) e - \frac{r \dot{\theta}}{c} e_T \right\}.
\]
On the basis of Eqs. (25) and (26) we can write for the P-R drag

\[
\left( \frac{dv}{dt} \right)_{P-R \ drag} = - \beta \frac{GM}{r^2} \left( 2 \frac{\dot{r}}{c} e + \frac{r \dot{\theta}}{c} e_T \right).
\]

(28)

We have to stress that any decomposition to the "radiation pressure" (Eq. 24) and the "P-R drag" (Eqs. 25 and 28) is nonphysical. As follows from covariant formulation, the physical equation is Eq. (23).

Moreover, avoiding the usage of the "P-R drag" not only represents correct physics, but any confusion about the "P-R drag" is removed. This is also important since, besides definition represented by Eq. (25), another "P-R drag" is used. It contains only the term proportional to \(-v/c\) in the equation of motion – the first term in brackets on the right-hand side of Eq. (25) is omitted (see, e.g., Wyatt 2009). One must bear in mind that the two different definitions of the P-R drag lead to two different results for secular changes of orbital elements \(\frac{da_\beta}{dt} = - \beta \frac{(GM/c)}{A_{P-R \ drag}} (a_\beta (1 - e_\beta)^{3/2}), \frac{de_\beta}{dt} = - \beta \frac{(GM/c)}{E_{P-R \ drag}} (a_\beta^2 (1 - e_\beta)^{1/2})\), where \(A_{P-R \ drag} = 2 + 3 e_\beta^2, E_{P-R \ drag} = (5/2) e_\beta\) for the first type of the P-R drag and \(A_{P-R \ drag} = 2 + 2 e_\beta^2, E_{P-R \ drag} = 2 e_\beta\) for the second type of the P-R drag; \(a_\beta\) and \(e_\beta\) denote semimajor axis and eccentricity of the particle’s orbit around the central star, see Eqs. 50–67 in Klačka 2004]. Unfortunately, another definition of the P-R drag exists:

\[
\left( \frac{dv}{dt} \right)_{P-R \ drag} = - \beta \frac{(GM/r^2)}{(r \dot{\theta}/c)} e_T,
\]

see, e.g. Minato et al. (2004). Really, confusion in the literature exists. And physics is ignored.

The P-R effect is the effect of radiation pressure on a moving spherical (nonrotating) body. Thus, any separation between the radiation pressure and the P-R effect is also nonphysical (see, e.g., Leinert and Grün 1990; Mann 2009; see also Minato et al. 2004 – Sec. 3.2).

6. P-R effect and statements presented in the literature

As we have already mentioned at the beginning of the paper, much confusion exists in explanations of the P-R effect. As for the papers by Poynting (1903), Robertson (1937), Wyatt and Whipple (1950) and Burns et al. (1979), the physics was partially commented in Klačka (2008a, 2008b) and also in Klačka 1992a (Sec. 2.5). We will deal mainly with newer explanations presented in the literature, in this section.

We have to bear in mind that the results obtained by Klačka (2008a, 2008b) are consistent with Maxwell’s equations and relativity theory. Since relativity theory was motivated by better understanding of Maxwell’s equations, we consider Maxwell’s equations (and their consequences) as an indispensable part of the relativity theory. Classical physics considers that interactions spread with
an infinite speed. Thus, all the results presented in Kláčka (2008a, 2008b) have to be considered as a consequence of the relativity theory. Any statement of the type "radiation forces – including the Poynting-Robertson drag – are fundamentally classical forces and are not produced by relativistic effects as commonly believed" (Burns et al. 1979, p. 8) is not physically correct. As an example, we can mention the calculation of $\bar{Q}'_{pr}$. It is based on Mie’s solution of Maxwell’s equations and it cannot be based on classical physics inconsistent with relativity theory. Similarly, the flux density of radiation energy transforms according to the relation $S' = S (1 - 2 \mathbf{v} \cdot \mathbf{e}/c)$ and not according to $S' = S (1 - \mathbf{v} \cdot \mathbf{e}/c)$ which corresponds to "fundamentally classical" result of Burns et al. (1979, p. 5). Similarly, the fundamental condition is $p'_{\text{outgoing}} = (1 - \bar{Q}'_{pr} / \bar{Q}'_{ext}) p'_{\text{incoming}}$ (see Eq. 16) and not the "fundamentally classical" condition $p'_{\text{outgoing}} = (1 - \bar{Q}'_{pr}) p'_{\text{incoming}}$ corresponding to ideas of Poynting (1903), Robertson (1937), Wyatt and Whipple (1950), Burns et al. (1979, pp. 10-11) and others. Similarly, Eqs. (7) and (11) hold. Similarly, if only thermal emission is considered, then the decrease of particle’s mass is $dm/d\tau = -E'_o/c^2$, where $E'_o$ is the outgoing energy per unit time (see Eqs. 40-41 in Kláčka 2008b). Similarly, light does not obey the law of addition of velocities, according to Einstein’s second postulate ["Principle of the invariance (constancy) of the speed of light: The speed of light in a vacuum has the same value in each inertial reference frame, irrespective of the velocities of the light source or the light receiver." (Freund 2008, p. 9)] Simply, the processes participating in the P-R effect are correctly explained using ideas of relativity theory and not ideas of classical nonrelativistic physics.

6.1. Burns et al. (1979)

Since the paper by Burns et al. (1979) is generally considered to be the most relevant contribution on the P-R effect, we will present the most important comments on the paper. Some information can be found also in papers by Kláčka (1992a – Sec. 2.5, 2008a, 2008b).

1.

Burns et al. (1979, p. 5) write:

"The force on a perfectly absorbing particle due to solar (photon) radiation can be viewed as composed of two parts: (a) a radiation pressure term, which is due to the initial interception by the particle of the incident momentum in the beam, and (b) a mass-loading drag, which is due to the effective rate of mass loss from the moving particle as it continuously reradiates the incident energy."
Physics:

We have $\dot{Q}_{\text{ext}} = 2$, $\dot{Q}_{\text{pr}} = 1$ for the case (see Secs. 7.3.3 and 7.3.4 in Klačka 2008b). As for the incoming radiation, we can write, on the basis of Eq. (1) (see also Eq. 75 in Klačka 2008b),

$$
\frac{dp_{\text{in}}}{d\tau} = (2 w^2 SA'/c) b^\mu ,
$$

$$
\frac{dm}{d\tau}_{\text{in}} = 2 w^2 SA'/c^2 ,
$$
or, to the first order in $v/c$,

$$
\frac{dp_{\text{in}}}{dt} = (2 SA'/c) (1 - v \cdot e/c) e .
$$

The outgoing radiation effects on the particle by the following force (Eq. 76 in Klačka 2008b):

$$
\frac{dp_{\text{out}}}{d\tau}_{\text{particle}} = - (w^2 SA'/c) (b^\mu + u^\mu/c) ,
$$

$$
\frac{dm}{d\tau}_{\text{out}} = - 2 w^2 SA'/c^2 ,
$$
or, to the first order in $v/c$,

$$
\frac{dp_{\text{out}}}{dt}_{\text{particle}} = - (SA'/c) \left[ (1 - v \cdot e/c) e + v/c \right] .
$$

If $(2 SA'/c) (1 - v \cdot e/c) e$ would be the "radiation pressure term", then $- (SA'/c) \left[ (1 - v \cdot e/c) e + v/c \right]$ should be the "P-R drag" ("mass loading drag")! The readers should be utterly confused by the "physics" (see also Sec. 6.5; probably, the authors Burns et al. had not in mind these equations, when they formulated the above presented text).

2.

Burns et al. (1979, p. 5) write:

"$S' = S (1 - \dot{r}/c)$ , \hspace{1cm} (1-B)"

... The momentum removed per second from the incident beam as seen by the particle is then $(S'A/c)$ $\hat{S}$.

Physics:

The momentum removed per second from the incident beam as seen by the particle is $(S'A/c)$ $\hat{S}'$, where $S' = S (1 - 2 \dot{r}/c)$ ,

if the accuracy to the first order in $\dot{r}/c$ is used (see also Sec. 2.5 in Klačka 1992a, or, Eqs. 20-25 in Klačka 2004, or, Klačka 2008b).

3.

Burns et al. (1979, p. 5) write about the perfectly absorbing particle:
"The absorbed energy flux $S' A$ is continuously reradiated from the particle. Since the reradiation is nearly isotropic (small particles being effectively isothermal), there is no net force exerted thereby on the particle in its own frame. However, the reradiation is equivalent to a mass loss rate of $S' A/c^2$ from the moving particle – which has velocity $v$ as seen from the inertial frame of the Sun – and, as measured in the solar frame, this gives rise to a momentum flux from the particle $- (S' A/c^2) v$, schematically shown in Fig. 2."

Physics:

The force exerted on the particle, in its own frame, due to the outgoing radiation, is:

\[
\frac{dp_{\text{out}}}{d\tau}_{\text{particle}} = - \left( w^2 S A' \bar{Q}'_{\text{ext}} / c \right) \left\{ \bar{Q}' \, u^\mu / c + (1 - \bar{Q}') \, b^\mu \right\},
\]

\[
\bar{Q}' \equiv \frac{Q'_{\text{pr}}}{Q'_{\text{ext}}},
\]

or, putting $v = 0$ and $\bar{Q}'_{\text{pr}} = 1$, $\bar{Q}'_{\text{ext}} = 2$,

\[
\frac{dp_{\text{out}}}{d\tau}_{\text{particle}} = - (S' A'/c) \, e',
\]

according to Eq. (1). Thus, the statement of Burns et al. (1979, p. 5) that "there is no net force exerted thereby on the particle in its own frame" is not true.

The following statement of Burns et al. (1979, p. 5) is: "However, the reradiation is equivalent to a mass loss rate of $S' A/c^2$ from the moving particle". We have, in reality, according to Eq. (11),

\[
\frac{dm}{d\tau}_{\text{out}} = - 2 \, w^2 S A'/c^2 .
\]

since $\bar{Q}'_{\text{ext}} = 2$ for the case discussed by the authors.

Again, the authors write: "this gives rise to a momentum flux from the particle $- (S' A/c^2) \, v$, schematically shown in Fig. 2." In reality, the force exerted on the particle due to the outgoing radiation is, according to Eq. (1):

\[
\frac{dp_{\text{out}}}{d\tau}_{\text{particle}} = - \left( w^2 S A' \bar{Q}'_{\text{ext}} / c \right) \left\{ \bar{Q}' \, u^\mu / c + (1 - \bar{Q}') \, b^\mu \right\},
\]

\[
\bar{Q}' \equiv \frac{Q'_{\text{pr}}}{Q'_{\text{ext}}},
\]

or, for the case treated by the authors,

\[
\frac{dp_{\text{out}}}{d\tau}_{\text{particle}} = - \left( w^2 S A'/c \right) \left( u^\mu / c + b^\mu \right) .
\]

Approximation to terms of order $v/c$ yields:

\[
\frac{dp_{\text{out}}}{dt}_{\text{particle}} = - \left( w^2 S A'/c \right) \left( v/c + e/w \right) .
\]

4.

On the basis of the correct results summarized and discussed in the previous three points (1., 2. and 3.) the reader immediately see that also Fig. 2 is physically incorrect. The outgoing radiation in the
particle’s frame is not isotropic. Also the results $S' = (1 - 2 \dot{r}/c) S$ are not taken into account. Also the result
dp_{out}/dt)_{particle} = - (w^2 S A' / c) (v/c + e/w)
is not considered.

5.
Burns et al. (1979, p. 6) write:
"The net force on the particle is then the sum of the forces due to the impulse exerted by the incident beam and the momentum density loss from the particle or, for a particle of mass $m$,
m \dot{v} = (S' A/c) \vec{S} - (S' A/c^2) v \quad \ldots \quad (2)"

Physics:

The equation does not correspond to correct physics.

At first, since $S'$ is the integrated flux density measured by the moving particle (Burns et al. 1979, p. 5), the quantity $(S' A/c)$ cannot be used as the relevant quantity for the equation of motion of the particle in the rest frame of reference of the source of light (Sun). The second Newton’s law does not admit such kind of mixing between two different reference frames (simultaneous considering of momenta measured in two different reference frames).

Secondly, the statement of Burns et al. (1979 – Eq. 2) that
dp_{in}/dt = (S' A/c) \vec{S}
(dp_{out}/dt)_{particle} = - (S' A/c^2) v
is incorrect. The forces acting on the particle are given by Eqs. (1), or, to the first order in $v/c$:
dp_{in}/dt = (2 w^2 S A' / c) e/w ,
(dp_{out}/dt)_{particle} = - (w^2 S A'/c) (v/c + e/w) ,
w = 1 - v \cdot e /c .

6.
Burns et al. (1979, p. 8) write:
" ... we have that
$F_{rad} = m \dot{v} = Q_{pr} M' (c - v) ,$
where $M'$ is the "mass" of the photons which strike the particle or $(S' A/c^2)$. In the above form, the radiation force can be viewed as a drag force caused by the relative velocity $c - v$, of the particle
through a beam of photons. Finally, to terms of order $v/c$, we can write the net force on the particle as

$$m \, \dot{v} = \left( SA/c \right) Q_{pr} \left[ (1 - \dot{r}/c) \hat{S} - v/c \right]. \quad (5)$$

For a totally absorbing particle this reduces to (2) since then $Q_{pr} = 1$. Equation (5) is identical to Robertson’s expression except for being more general by the inclusion of the important factor $Q_{pr}$. We note, however, from the above derivation, that radiation forces – including the Poynting-Robertson drag” are *fundamentally classical* forces and are not produced by relativistic effects as commonly believed.”

Physics:

Our Eq. (8) yields that

$$M' \equiv (dm/d\tau)_{in} = w^2 S A' \bar{Q}_{ext}/c^2.$$  

Even if we take into account that $S' = w^2 S$ (although we have already discussed that Burns *et al.* use incorrect result given by their Eq. 1), the correct result for $M'$ differs from the result presented by the authors. Substituting the correct result for $M'$ into the equation presented above Eq. (5) of Burns *et al.*, the authors would not obtain Eq. (5).

We must admit that we have received the correct result for $M'$ on the basis of relativity theory (Klačka 2008a, 2008b) and not on the basis of *fundamentally classical* physics, as was the access of the Burns *et al.*

There is another problem with the access of the authors. They use ”the relative velocity $c - v$, of the particle through a beam of photons”. There is a problem. We know that we can write $S' = n' h \nu' c$ and, similarly, $S = n h \nu c$, in the case of relativity theory (Klačka 2008b – Eqs. 20-23). Using transformations between $n$ and $n'$, $\nu$ and $\nu'$, we obtain the correct relation $S' = w^2 S$. One can say that the transformations between $n$ and $n'$, $\nu$ and $\nu'$ can be obtained also on the basis of *fundamentally classical* physics (see Sec. 2.4 in Klačka 1992a and Doppler effect). And the result is consistent with the result known from electromagnetism (see Sec 2.5 in Klačka 1992a). But the access still assumes that $c' = c$, so there is no *fundamentally classical* physics. How is it possible? The explanation is evident: the result of electromagnetism is consistent with Maxwell’s equations and they are consistent with relativity (Maxwell’s equations motivated the creation of relativity). If one would like to use also the transformation suggested by the authors, ”the relative velocity $c - v$”, one would obtain, on the basis of *fundamentally classical* physics, that $S' = w^3 S$. And this result would be different from the correct $S' = w^2 S$ and also from the result used by Burns *et al.* (1979 – Eq. 1) $S' = w S$. 

It seems to us that the statement "radiation forces are fundamentally classical forces and are not produced by relativistic effects as commonly believed" (Burns et al. 1979, p. 8) is irrelevant. The complete correct results were obtained on the basis of relativistic approach (Klačka 2008a, 2008b) and the results are, at least in some partial steps, different from those presented before the two papers (Klačka 2008a, 2008b).

7. As for the relativistic approach presented by Burns et al. (1979, Sec. IV), the authors consider
\[ p_o = (1 - Q_{pr}) E_i/c \ldots \]
(see Eqs. 10, 11, 14 in Burns et al. 1979). Although the motivation for the value
\[ E_i = (1 - \dot{r}/c) S \]
is not clear (see also Klačka 1992a – Sec. 2.5), the important result is that the usage of \( p_o = (1 - Q_{pr}) \ldots \) would produce inconsistencies between relativity theory and Mie’s solution of Maxwell’s equations (Klačka 2008a, 2008b). The approach of Burns et al. (1979) leads to the inequality \( 0 < Q_{pr} < 2 \), which is not consistent with the Mie’s solution of Maxwell’s equations. Thus, even the relativistic approach by Burns et al. (1979, Sec. IV) is not physically correct. It is not consistent with relativity theory. The correct equation is given by Eq. (16). Moreover, the correct equation for \( E_i \) is
\[ E_i = wSA'Q_{ext}' \]
(Klačka 2008b – Eqs. 11, 19, and 23) and this is different from the result presented by Burns et al. (1979 – below Eq. 16).

8. We have discussed the result for \( (dm/d\tau)_\text{in} \) in Sec. 2.1. The approach of Burns et al. (1979) would yield \( (dm/d\tau)_\text{in} = wSA'/c^2 \) (but the authors did not treat the problem in a relativistically covariant form, so they could not present the result for \( (dm/d\tau)_\text{in} \)). Thus, the velocity dependence would be given by \( w^1 \) and not by \( w^2 \). Moreover, optical properties of the particle would be represented only by the particle geometrical cross-section \( A' \) and not by the extinction cross-section \( A'Q_{ext}' \).

9. Burns et al. (1979, Sec. V) discuss "solar wind corpuscular forces”. We want to say that the right-hand side of their Eq. (17*) must be multiplied by a factor \( x' \) depending on material properties of dust particle (\( 1 < x' < 3 \), approximately, see Eq. 29 in Klačka and Saniga 1993). The force for the solar wind (if only radial velocity of the solar wind is considered) is:
\[ F_{sw} = F_{sw} \left[ (1 - \mathbf{v} \cdot \mathbf{e}/v_{sw}) \mathbf{e} - x' \mathbf{v}/v_{sw} \right], \]

where \( F_{sw} \) is the force on the particle and \( v_{sw} \) is the heliocentric solar-wind speed.

### 6.2. Shive and Weber (1982), Dohnanyi (1978)

Shive and Weber (1982, p. 221) present: "The particle absorbs light and heat from the Sun and re-radiates photons uniformly in all directions. However, as seen by an observer in a frame of reference stationary with respect to the Sun, the photons emitted by the particle in the forward direction of its orbital motion will be Doppler-shifted towards higher frequencies than those emitted in the backward direction. The energy density ahead of the particle will accordingly be greater than that behind it, and the particle experiences a net tangential radiation pressure which acts as a drag on its orbital motion. So it spirals inward and is eventually engulfed by the Sun. Most of the interplanetary dust originally present at the creation of the solar system has long ago been swept out by this process."

Similar explanation can be found in Dohnanyi (1978, pp. 562-563): "In the system of coordinates in which the particle is at rest, the Sun’s rays (and the force of radiation pressure as well) will not appear to come from the centre of the Sun, but at an angle and in such a direction as to oppose the particle’s transverse component of motion. Analysis of this effect, known as the aberration of light ... The opposing transverse force is then [equal to the P-R effect]. An alternative explanation of this effect is obtained when we view the particle’s motion from the system of coordinates in which the Sun is at rest. The direction of the Sun’s rays and that of the force of radiation pressure on the particle are now exactly in the radial direction. Because of the particle transverse motion, the radiation reflected, diffracted, or emitted by the particle is shifted in its wavelengths asymmetrically: radiation leaving the particle in the direction of motion is blue-shifted and in the other direction it is red-shifted. The net result is a drag force of the same magnitude as equation [for the P-R effect]."

Physics of the phenomenon:

1. If the case treated by Poynting (1903) and Robertson (1937) is considered, then \( \dot{Q}'_{pr}/\dot{Q}'_{ext} = 1/2 \) and Eq. (12), or Eq. (20), yields that the outgoing radiation causes acceleration of the particle and not its deceleration \( (d\mathbf{v}/dt \propto \mathbf{v}/c). \) This is different from the statement done by Shive and Weber (1982) and Dohnanyi (1982). If \( \dot{Q}'_{pr} / \dot{Q}'_{ext} = 1 \), then acceleration of the particle equals to zero in both reference frames, i. e., in the frame of reference of the particle and in the frame of reference stationary with respect to the Sun (see Eqs. 12 or 20). The important phenomenon is that mass of the particle
decreases in the process of reradiation and this leads to an increase of particle’s acceleration: $dp/dt = \ldots$, $dv/dt = (1/m) [ - (dm/dt) v + \ldots]$, and the value $dm/dt$ is given by Eq. (11) in the case of the P-R effect.

Maybe, there exists more simple illustration of the correct physics. If the particle is not irradiated and only outgoing radiation due to the thermal emission exists, then Eqs. (40)-(41) in Kláčka (2008b) hold. As for the spherical particle, one obtains

$$\frac{dp^\mu}{d\tau}_e = - \frac{E'_o}{c} \frac{u^\mu}{c},$$

$$\frac{du^\mu}{d\tau}_e = 0,$$

$$\frac{dm}{d\tau}_e = - \frac{E'_o}{c^2}. \quad (29)$$

where $E'_o$ is thermal emission energy per unit time. The first of Eqs. (29) really shows that a force acting against the motion of the particle exists, but the second of Eqs. (29) states that no acceleration of the particle exists and this holds in all reference frames. The particle is decelerated if the following asymmetry exists, in the proper frame of the particle: the total energy per unit time emitted in the forward direction of particle’s motion (with respect to the surroundings) is greater than the corresponding energy emitted in the backward direction (see Eqs. 40-41 in Kláčka 2008b).

The flux density of radiation energy is influenced not only by the Doppler effect, but also by the change of concentration of photons: $S' = w^2 S$. Moreover, the term $-u^\mu/c$ in Eq. (14) is generated by the conservation of particle’s mass (see Sec. 7.4.5 in Kláčka 2008b) and not by the shifts in wavelengths or frequencies. Similarly, the last two equations of Eqs. (29) immediately lead to the first of Eqs. (29), if one uses the fact that $p^\mu = m u^\mu$.

2.

As for the force acting on the particle due to the outgoing radiation for the P-R effect, one has to use Eqs. (1) and (9):

$$dp^\mu/d\tau = - ( w^2 SA' \bar{Q}'_{ext}/c ) \left[ \left( \frac{Q'_{pr}}{Q'_o} / \bar{Q}'_{ext} \right) \frac{w^\mu}{c} + (1 - \frac{Q'_{pr}}{Q'_o} / \bar{Q}'_{ext}) \frac{b^\mu}{c} \right].$$

It is easily seen that this force is not of the type represented by Eq. (29), if $\bar{Q}'_{pr} / \bar{Q}'_{ext} \neq 1$ ($\bar{Q}'_{pr} / \bar{Q}'_{ext} = 1/2$ for the case treated by Poynting, Robertson, Shive and Weber, Dohnanyi). Thus, the result does not correspond to the idea of Shive and Weber (1982) and Dohnanyi (1978).

3.

Let us consider the following simple thought experiment. Photons are emitted from the two opposite rectangular faces of a cuboid (rectangular prism). The total outgoing energy per unit time is $E'_o = 2$
S′A′, where A′ is geometrical cross-section of the rectangular faces. Then
\( (dp^\mu /dT)_{c} = -2 \left( S′A′/c \right) u^\mu /c \),
according to Eq. (29). This result can be obtained using the idea presented by Dohnanyi (1978) and Shive and Weber (1982). However, also the change of concentration of photons has to be taken into account, not only the Doppler effect:
\( S′ = w_f^2 S_f = w_b^2 S_b \), where the subscripts "f", "b" denote abbreviations of "forward" and "backward", \( w_f = \gamma(1 - v/c) \), \( w_b = \gamma(1 + v/c) \). According to Eqs. (18)-(20) and (24) in Klačka (2008b), we can write
\( (dp^\mu /dT)_{c} = - \left[ \left( w_f^2 S_f A′/c \right) b_f^\mu + \left( w_b^2 S_b A′/c \right) b_b^\mu \right] \),
where \( b_f^\mu = (1/w_f; \hat{v}/w_f) \), \( b_b^\mu = (1/w_b; -\hat{v}/w_b) \); \( \hat{v} \) is unit vector in the forward direction and orientation of the emitted photons. As a consequence, deceleration of the cuboid equals to zero:
\( du^\mu /dT = 0 \).

Comment:
The four-vectors \( b_f^\mu \), \( b_b^\mu \) are important in the previous equation. We can easily show it as follows. The forward and backward directions would yield (to the first order in \( v/c \))
\( S_f = S′ (1 + 2 v/c) \), \( S_b = S′ (1 - 2 v/c) \). But the form \( dp/dt = -(S_f A′/c - S_b A′/c) = -4 (S′A′/c) v/c \) would not correspond to the physical result. Surprisingly, the idea of Burns et al. (1979) \( S_f(B) = S′ (1 + v/c) \), \( S_b(B) = S′ (1 - v/c) \) – results of classical mechanics – would yield the correct final result! Perhaps, Shive and Weber (1982) were influenced by the statement of Burns et al. (1979). Maybe, classical access would be of the form \( dp/dt = - \left[ S_f A′/(c + v) - S_b A′/(c - v) \right] = -2 (S′A′/c) v/c \). However, understanding nature requires much more effort than simple collection and combination of some ad hoc formulae.

4. Aberration of light?
As for the heuristic explanation of the term containing \(-v/c\) in the P-R effect as a consequence of the aberration of light, we refer the reader to Sec. 7.4.5 in Klačka (2008b). Moreover, this can be easily seen from Eqs. (1)-(2), or from the form to the first order in \( v/c \):
\( F_{in} = (w_f^2 S A′ Q′_{ext}/c) (1 + v \cdot e/c) e \).
The form of the force \( F_{in} \) does not contain the aberrational term
\( (1 + v \cdot e/c) e - v/c \).
Perhaps, the following idea can appear: the aberrational term exists in the acceleration \( a_{in} \equiv \dot{v}_{in} \). Really, something like this is produced by the covariant equation Eq. (8):
\( dv_{in}/dt = [w_f^2 S A′ Q′_{ext}/(mc)] \left[ (1 + v \cdot e/c) e - v/c \right] \)
and this term is even almost consistent with the P-R effect! However, there is the term \( Q′_{ext} \) instead
of the term $Q'_{pr}$, and, the relevant four-vector $b^\mu - u^\mu/c$ is not aberration of light. And, for the special case treated by Poynting (1903), Robertson (1937) and others, $\bar{Q}'_{ext} = 2, \bar{Q}'_{pr} = 1$ (Klačka 2008a, 2008b). Thus, aberration of light does not explain the term $-v/c$ in the P-R effect.

6.3. Mignard (1982)

Mignard (1982, p. 349) writes: "$Q \ [\bar{Q}'_{pr}]$ is a global coefficient which expresses the particle surface properties related to the incoming light."

Physics:

The dimensionless efficiency factor for radiation pressure (integrated over stellar spectrum and calculated for radial direction) considers volume properties of the particle, in general. It results from the solution of Maxwell’s equations (Mie 1908).

6.4. Kapišinský (1984)

Kapišinský (1984) discusses disturbing and dissipative (destructive and disruptive) effects acting on interplanetary dust particles. The author explains the action of the solar electromagnetic radiation (Kapišinský 1984, disturbing forces – p. 104):

"Effect of solar electromagnetic radiation (direct light pressure):

This effect is closely related to the Poynting-Robertson effect. Among the non-gravitational effects, these two are of the greatest importance for studying the dynamics of small particles in the Solar System. The outdirected solar pressure always acts on the particle against the gravitational force of the Sun."

Continuation on the same page is:

"Poynting-Robertson effect: Apart from the outdirected solar pressure acting on the particle, the absorption of solar energy by the particle and its isotropic emission causes a small force to be generated along the tangent to the trajectory which decelerates the particle in the orbit. This tangential force decreases the kinetic energy and the orbital angular momentum of the particle. Consequently, the particle is forced to move to an orbit closer to the Sun."
Physics:

The "effect of solar electromagnetic radiation" on spherical dust particle is equivalent to the Poynting-Robertson effect, not to the "direct light pressure". Direct light pressure is a velocity independent part of the P-R effect. Thus, it is not possible to classify "direct light pressure" and the P-R effect as two different disturbing effects influencing motion of the particle.

Similarly, we have the Poynting-Robertson effect and not three effects: "direct light pressure", "Poynting-Robertson drag" and the "Poynting-Robertson effect". We have only the "Poynting-Robertson effect" and the other two "effects" are indispensable parts of the "Poynting-Robertson effect". The two effects cannot be physically separated from the P-R effect. Relativistically covariant equation of motion, represented by Eq. (14), describes the P-R effect and it contains both the "direct light pressure" and the "Poynting-Robertson drag".

Let us consider perfectly absorbing spherical particle, within geometrical optics approximation. The "isotropic emission" is, in reality, described by the value (see Eq. 16)

\[ \bar{Q}' = \frac{\bar{Q}'_{pr}}{\bar{Q}'_{ext}} = \frac{1}{2}, \]

and by the condition

\[ p'_{\text{outgoing}} = (1/2) p'_{\text{incoming}}. \]

Subsequently, the force corresponding to the incoming and outgoing radiation contains also a component tangent to the trajectory which decelerates the particle in the orbit. This tangential component of the force decreases the total (kinetic plus potential) energy of the particle. The tangential component of the force increases kinetic energy of the particle.

6.5. Leinert and Grün (1990)

Leinert and Grün (1990, p. 226) write:

"The impact of solar wind ions also exerts an outward pressure on orbiting dust particles. It is more than three orders of magnitude weaker than radiation pressure. But the aberration angle \( v_{\text{tan}}/v_{\text{sw}} \), where \( v_{\text{sw}} \) is the velocity of the solar wind, is much larger, making the resulting "ion impact" drag
comparable to the Poynting-Robertson effect.” (Leinert and Grün 1990, pp. 226-227).

Physics:

1. At first, Leinert and Grün (1990, Secs. 5.4.2 and 5.4.3) discuss "radiation pressure" and the "Poynting-Robertson effect" as two different phenomena. However, the P-R effect is the effect of radiation pressure on a moving body.

2. As for the forces, the mathematical forms for the P-R effect seem to be consistent with the statements of the authors. However, the force for the solar wind (under the assumption that only radial velocity of the solar wind is considered) is:

\[ F_{sw} = F_{sw} \left[ (1 - v \cdot e/v_{sw}) e - x' \frac{v}{v_{sw}} \right], \]

where \( F_{sw} \) is the force on the particle, \( v_{sw} \) is the heliocentric solar-wind speed, and, the value of \( x' \), \( 1 < x' < 3 \) (approximately), depends on material properties of the particle. This force does not correspond to the statement of the authors. The presence of the term \( x' \) shows that any idea about importance of the aberration effect is not acceptable (see also Sec. 7.4.5 in Kláčka 2008b, as for light pressure).

Moreover, one has to take into account that Eqs. (1) hold. Even if the authors want to take into account the simplest case treated by Poynting (1903) and Robertson (1937), the explanations of the authors will not produce the consistent results for the incoming and outgoing radiation: it is sufficient to consider that \( \bar{Q}'_{ext} = 2, \bar{Q}'_{pr} = 1 \) (see Kláčka 2008a, 2008b). The tacitly assumed values \( \bar{Q}'_{ext} = 1, \bar{Q}'_{pr} = 1 \), considered by Poynting (1903), Robertson (1937), Burns et al. (1979) and others, are not correct.

3. Let us look if the substitution "force" → "acceleration" would make the statements of the authors correct.

Eqs. (8) and (20) (see also Eqs. 106-107 of Kláčka 2008b) yield

\[ \frac{dv}{dt}_{in} = a_{ph} \left[ (1 - v \cdot e_R/c) e_R - v/c \right], \]

and Eqs. (12) and (20) (see also Eqs. 109-110 of Kláčka 2008b) yield

\[ \frac{dv}{dt}_{out} = -a_{ph} \left( 1 - \frac{Q'_{pr}/Q'_{ext}}{Q'_{ext}} \right) \left[ (1 - v \cdot e_R/c) e_R - v/c \right]. \]
$a_{ph} = S A Q_{ext}' / (m c)$. If $Q_{pr}' / Q_{ext}' < 1$, then "scattering and thermal emission" cause acceleration of the particle [the case treated by Poynting (1903), Robertson (1937), and others, corresponds to $Q_{pr}' / Q_{ext}' = 1/2$], $(dv/dt)_{out} \propto + v/c$. This result differs from the statement of Leinert and Grün. The incoming radiation causes deceleration of the particle, $(dv/dt)_{in} \propto - v/c$.

6.6. Banaszkiewicz et al. (1994)

The authors do not try to explain the P-R effect, but they use the electromagnetic Poynting-Robertson force $F_e$ in the form (Banaszkiewicz et al. 1994 – Eq. 10)

$F_e = [S_0 A r_0^2 / (c r^2)] Q_{pr} [((1 - 2 \dot{r} / c) \mathbf{r} - (r/c) \mathbf{v})]$, where ...

The same relevant terms are used in Banaszkiewicz (et al. 1994 – Eq. 22).

Physics:

Besides misprinting in the exponent of $r$ in the denominator, the numerical factor 2 at the term $\dot{r} / c$ is not correct and the correct factor equals 1.

Maybe, Banaszkiewicz et al. (1994) were aware that the flux of the incoming radiation, as seen by the particle, is $S' = S (1 - 2 \dot{r} / c)$ and not by the relation used by Burns et al. (1979, p. 5 – Eq. 1). However, correct equation of motion is

$F_e = [S_0 A r_0^2 / (c r^3)] Q_{pr} [(1 - \dot{r} / c) \mathbf{r} - (r/c) \mathbf{v})]$, see also Eq. (22) in this paper.

6.7. Gustafson (1994)

Gustafson (1994, p. 566) writes: "Poynting-Robertson drag can be thought of as arising from an aberration of the sunlight as seen from the particle and a Doppler-shift induced change in momentum. To the first order in $v/c$, the radiation force acting on a spherical particle is

$|F_g| \beta \left[ (1 - 2 \dot{r} / c) \mathbf{r} - \left( r \Theta / c \right) \hat{\Theta} \right]$, (6-G)

where the unit vector $\hat{\Theta}$ is normal to $\hat{r}$ in the orbital plane..." ($\hat{r}$ is the unit heliocentric radius vector).

The author continues: "The second term along $\hat{r}$ is due to Doppler shift. With the transverse last term, this velocity-dependent part of Equation (6) [Eq. 6-G in our notation] is the Poynting-Robertson (PR) drag. Doppler shift enters twice: once due to the rate at which energy is received and a second
time due to the reradiated and scattered radiation.

Physics:

As it was written in Sec. 7.4.5 in Klačka (2008b), the third term does not originate from aberration of light. The second term is a result of combination of the change of concentration of photons, the Doppler shift of incident radiation, the term $1/w$ present in $b^\mu$ (see Eqs. 2 and 14) and the conservation of particle’s mass. The Gustafson’s statement, that this term is due to the Doppler shift of the absorbed and re-radiated radiation, is not correct.

6.8. Considine (1995)

Considine (1995, p. 2534) writes:

"Poynting-Robertson effect. The effect upon the motion of a micrometeoroid or other very small particle due to the absorption and emission of radiation. The particle absorbs radiation from the sun only in one direction, but reradiates energy in all directions. This effect produces a drag upon the particle that is directly tangential to the orbit of the particle about the sun, and thus decreases the orbital angular momentum. Since this angular momentum varies as the square root of the orbital radius, so that the particle follows a spiral path directing steadily closer to the sun. Although the solar radiation pressure upon the particle opposes this effect, it is not sufficient to offset it completely."

Physics:

The effect is characterized by an equation of motion and it holds for (non-rotating) spherical body, irrespective of its size. In general, it is not relevant that the process of absorption occurs, since the effect holds also for total specular reflection (see Sec. 7.3.3 in Klačka 2008b). May be, the explanation presented above concentrates to the case treated by Poynting and Robertson, when perfectly absorbing (within geometrical optics approximation) spherical particle is considered. However, even in this special case the radiation is absorbed only partially and the effect of diffraction plays equally important role. In general, the fact that the particle ”reradiates energy in all directions” does not determine the behavior corresponding to the Poynting-Robertson effect (see experimental results of Krauss and Wurm 2004 and Eqs. 9-11, 38-39 in Klačka 2008b). Eq. (16) must be fulfilled for the Poynting-Robertson effect.

The effect produces a drag upon the particle that consists both from radial term and the term "directly tangential to the orbit of the particle about the sun".
The solar radiation pressure upon the particle does not oppose the effect, since it is an inevitable part of the Poynting-Robertson effect. More correctly, the Poynting-Robertson effect is the effect of radiation pressure force for a moving spherical body.

6.9. Murray and Dermott (1999)

Frequent idea about the essence of the P-R effect is that "the term $-\nu/c$ in the equation of motion exists under the hypothesis of isotropic reemission of radiation". However, this idea is in contradiction with the following statement: "P-R drag is caused by the nonuniform reemission of the sunlight that a particle absorbs." (Murray and Dermott 1999, p. 121).

Physics:

In order to be able to correctly understand physics of the P-R effect, we have to realize that the fundamental condition for the validity of the P-R effect exists: see Sec. 3 and Eq. (16).

The essence of the P-R effect is that nonradial components of the particle’s radiation pressure and components of the thermal force are zero (in particle’s frame of reference). Eq. (16) states that the P-R effect holds if the total momentum (per unit time) of the outgoing radiation, integrated over the whole space angle, is in the direction of the incident radiation, in the particle’s frame of reference.

If the spherical particle does not absorb, e.g., in the case of specular reflection, no thermal emission exists and particle orbital evolution corresponds to the P-R effect – Eq. (16) is fulfilled. This result also illustrates that the formulation "isotropic reemission of radiation is essential for the P-R effect" is not correct. If at least part of the incident radiation is absorbed, then, according to Eq. (16), thermal emission force equals zero and reemission of the absorbed radiation is isotropic. Thus, the statement "P-R drag is caused by the nonuniform reemission of the sunlight that a particle absorbs" is not correct (in the reference frame of the particle).

In general, the P-R effect admits both isotropic condition for the outgoing radiation

$$p'_{\text{outgoing}} = 0,$$

$$\bar{Q}' \equiv \bar{Q}'_{\text{pr}}/\bar{Q}'_{\text{ext}} = 1,$$  \hspace{1cm} (30)

and anisotropic

$$p'_{\text{outgoing}} = (1 - \bar{Q}') \cdot p'_{\text{incoming}},$$

$$\bar{Q}' \equiv \bar{Q}'_{\text{pr}}/\bar{Q}'_{\text{ext}} \neq 1,$$  \hspace{1cm} (31)
also. The form of anisotropy is exactly given by Eq. (31).

Conclusion:
Any statement on thermal emission is not sufficient for the existence of the P-R effect, in general. Real conditions for the validity of the P-R effect are presented by Eq. (16).

6.10. Srikanth (1999)

1. Srikanth (1999) introduces "the two parameter-model". However, this model does not respect the physics followed from Maxwell’s equations. Correct physics is given by Eq. (1) (Eqs. 75-76 in Klačka 2008b).

2. Srikanth (1999, p. 231) states:
"The principal conclusion is that the motion of a dust in circumsolar orbit is governed only by solar radiation absorption and not by the asymmetry of reemission, even when viewed in the rest-frame of the Sun."

Physics:

Motion of a body is governed by an equation of motion. As for the force generated by the incoming radiation, we have, on the basis of Eq. (1) (see also Eq. 75 in Klačka 2008b)
\[
\frac{dp_{in}}{dr} = \left( w^2 S A' \bar{Q}_{ext}' / c \right) b^\mu ,
\]
or, to the first order in \(\nu/c\),
\[
\frac{dp_{in}}{dt} = \left( w^2 S A' \bar{Q}_{ext}' / c \right) \left( 1 + \nu \cdot e/c \right) e ,
\]
where \(\bar{Q}_{ext}' = 2\) for the case treated by Poynting (1903), Robertson (1937), or, Srikanth (1999). The above presented equations do not generate the P-R effect. Thus, the outgoing radiation plays equally important role in the P-R effect:
\[
\left( \frac{dp_{out}}{d\tau} \right)_{\text{particle}} = - \left( w^2 S A' \bar{Q}_{ext}' / c \right) \left\{ \bar{Q}' \nu / c + (1 - \bar{Q}') b^\nu \right\} ,
\]
\[
\bar{Q}' = \bar{Q}_{pr}' / \bar{Q}_{ext}',
\]
or, to the first order in \(\nu/c\),
\[
\left( \frac{dp_{out}}{dt} \right)_{\text{particle}} = \left( S A' \bar{Q}_{ext}' / c \right) \left\{ \bar{Q}' \nu / c + (1 - \bar{Q}') \left( 1 - \nu \cdot e / c \right) e \right\} ,
\]
\[
\bar{Q}' = \bar{Q}_{pr}' / \bar{Q}_{ext}'
\]
(see Eqs. 1, 2, 19). The accelerations are given by Eqs. (8), (12), (20) (see also Eqs. 106-111 in Klačka
3. Srikhant (1999, p. 233) discusses the origin of the term \(1 - \mathbf{v} \cdot \mathbf{e} / c \equiv 1 - \dot{r} / c\), in the P-R effect. He states that the term "is a flux-modification factor and not red-shift factor".

Physics:

Flux-modification factor is \(1 - 2 \dot{r} / c\), as it follows from the relation

\[ S' = w^2 S, \quad w \doteq 1 - \mathbf{v} \cdot \mathbf{e} / c. \]

This is discussed in Klačka (1992a – Sec. 2.5; 1993b; 2000a; 2004 – Sec. 3.1, Eqs. 20-23; 2008b – Sec. 3.1, Eqs. 20-23). The change of concentration of photons and red-shift (Doppler effect) are equally important. The origin of the term \(1 - \mathbf{v} \cdot \mathbf{e} / c\) is evident from Eqs. (1) and (2) (see also Sec. 10 in Klačka 2008b).

6.11. Woolfson (2000)

Woolfson (2000, p. 401) considers perfectly absorbing spherical particle (within geometrical optics approximation) of radius \(a\) and density \(\rho\) at a distance \(R\) from the Sun. This particle absorbs energy. Woolfson writes: "The total energy absorbed per unit time is

\[ P = L_\odot a^2 / (4R^2) . \quad (\text{V.1-W}) \]

The energy comes to the particle radially but then is re-emitted by the particle moving in its orbit at speed \(v\). The radiated energy thus possesses momentum which must be taken from the particle. Considering the mass equivalent of the radiated energy the rate of change of momentum of the particle, or the tangential force on it, is

\[ F = L_\odot a^2 v / (4R^2 c^2) . \quad (\text{V.2-W}) \]

This force exerts a torque that changes the angular momentum of the particle at the rate (the sign in front of \(L_\odot\) is corrected)

\[ dh/dt = - FR = - L_\odot a^2 v / (4Rc^2) . \quad (\text{V.3-W}) \]

If the mass of the particle is \(m\) then its angular momentum is \(h = m \sqrt{GM_\odot R}\) so that

\[ dh/dt = (1/2) m \sqrt{GM_\odot / R} \ dR/dt = (1/2) m v \ dR/dt . \quad (\text{V.4-W}) \]

From (V.3-W) and (V.4-W) and expressing \(m\) in terms of \(a\) and \(\rho\) we find

\[ dR/dt = - 3L_\odot / (8\pi Rac^2 \rho) . \quad (\text{V.5-W}) " \]
Woolfson’s argument that re-radiated energy possesses momentum which must be taken from the particle is irrelevant because the incident (absorbed) energy also possesses momentum and it is given to the particle. The author, maybe, tacitly assumes that the radially incoming radiation does not change the particle’s orbital angular momentum and he does not take it into account.

However, according to Woolfson, the decrease of the particle’s angular momentum due to the re-emitted radiation would be responsible for spiralling of the particle onto the Sun. This idea is wrong. The correct rate of change of momentum of the particle, due to the outgoing radiation, is given by Eq. (19) and not by Eq. (V.2-W).

Let us look at the rate of change of the particle angular momentum due to the incoming and the outgoing radiation. For the outgoing radiation we get to the first order in $v/c$

\[
\left( \frac{dh}{dt} \right)_\text{out} = \frac{d}{dt} (mr \times v) = \left( \frac{dm}{dt} \right)_\text{out} r \times v + m r \times \left( \frac{dv}{dt} \right)_\text{out} = - \frac{SA'Q'_\text{ext}}{c^2} r \times v + \frac{SA'Q'_\text{ext}}{c^2} (1 - \bar{Q}') r \times v = - \frac{SA'Q'_\text{ext}}{c^2} \bar{Q}' r \times v ,
\]

where we used Eqs. (11) and (20). Similarly, for the incoming radiation we have (see Eqs. 7 and 20)

\[
\left( \frac{dh}{dt} \right)_\text{in} = \frac{dm}{dt} r \times v + m r \times \left( \frac{dv}{dt} \right)_\text{in} = \frac{SA'Q'_\text{ext}}{c^2} r \times v - \frac{SA'Q'_\text{ext}}{c^2} r \times v = 0 .
\]

We see that the outgoing radiation really decreases the angular momentum of the particle. But acceleration is the quantity determining the particle’s motion. Eq. (20) yields that $\left( \frac{dv}{dt} \right)_\text{out} \propto v$, for $\bar{Q}' = 1/2$ corresponding to perfectly absorbing spherical particle within geometrical optics approximation. Thus, the particle would accelerate and its heliocentric distance would increase if the outgoing radiation would be the only relevant process. Deceleration of the particle is caused by the incident radiation, see Eq. (20). We can say, from this viewpoint, that the incident radiation is significant in spiralling of the particle into the Sun. Eq. (32) shows that the decrease of the particle angular momentum, in the case of the outgoing radiation, is caused only by the decrease of the particle’s mass (re-emission of radiation). Thus, Woolfson’s calculation (Eqs. V.3-W, V.4-W, V.5-W) is not correct, since he assumes that the change of angular momentum is connected only with the change of orbital radius. On the other side, the incident radiation does not change particle’s angular momentum (see Eq. 33).
Formulae to the first order in $v/c$ will be considered.

The relation between Eqs. (V.1-W) and (V.2-W) is based on the relativistic relation

$$p = \frac{E}{c^2} v .$$  \hspace{1cm} (34)

This relation yields

$$\frac{dp}{dt} = \frac{\dot{E}}{c^2} v + \frac{E}{c^2} \frac{dv}{dt} .$$  \hspace{1cm} (35)

Woolfson access corresponds to $\dot{E}_{in} = \dot{E}_{out}$ and he uses $-\dot{E}_{out}$ instead of $\dot{E}$. Woolfson neglects the second term on the right-hand side of Eq. (35). He obtains

$$\dot{p}_W = -(\dot{E}_{out}/c^2) v .$$

If we would use the correct result represented by Eq. (19), then

$$\dot{p}_W = -\frac{S A'}{c^2} \dot{Q'}_{ext} v .$$  \hspace{1cm} (36)

This equation should correspond to Eq. (V.2-W).

However, one can immediately see that even the result in Eq. (36) does not correspond to the result presented in Eq. (V.2-W). The difference can be easily explained: Woolfson’s considerations are consistent with those of Poynting (1903), Robertson (1937), Wyatt and Whipple (1950), Burns et al. (1979) and others – all of them state that $\dot{Q'}_{ext} = 1$, but the correct result is $\dot{Q'}_{ext} = 2$.

Moreover, Eq. (36) is not correct, in reality. It differs from Eq. (21). The reason is that the second term in Eq. (35) cannot be neglected! The first term in Eq. (35) is negligible, in reality! We will show it, now.

We have, on the basis of Eqs. (18)-(19),

$$\frac{dE}{dt} = \frac{dE_{in}}{dt} - \frac{dE_{out}}{dt} = S A' \dot{Q'}_{ext} \dot{Q'} \frac{v \cdot e}{c} .$$  \hspace{1cm} (37)

Using $E = m c^2$, Eqs. (35) and (37) yield

$$\frac{dp}{dt} = m \frac{dv}{dt} .$$  \hspace{1cm} (38)

Using $\dot{v} = (dv/dt)_{in} + (dv/dt)_{out}$, Eqs. (20) yield the result consistent with Eq. (21). But Eq. (21) differs from Eq. (36)!

Surprisingly, vector products $r \times (dp/dt)$ and $r \times \dot{p}_W$, using Eqs. (21) and (36), yield the same results if $\dot{Q'}_{ext} = 1$ (incorrect) is assumed!
Let us continue. Woolfson writes \( h = m \sqrt{G M_\odot R} \). If we consider total energy \( E \), as we have just discussed, then \( m \) is a constant. But, at first, the form of the relation for angular momentum \( h \), corresponding to Woolfson’s idea, is \( m \sqrt{G M_\odot (1 - \beta) R} \) (see Secs. 6.1 and 6.2 in Klačka 2004).

Secondly, the relation for \( h \) assumes a circular orbit. But, in this case, Eq. (37) yields \( dE/dt = 0 \) and the Woolfson’s access yields \( \dot{p} = 0 \) and, thus, \( dR/dt = 0 \). But it is O.K., since \( R \) does not change when a particle moves in a circular orbit. Of course, this is a trivial result and one does not need the complicated ”physics” presented by Woolfson.

Eq. (21), or Eq. (22), is the correct equation of motion (gravity of the Sun is also added, this is tacitly assumed). The size of the angular momentum \( h \) is \( m \sqrt{G M_\odot (1 - \beta) p_\beta} \), where \( p_\beta \) is semi-latus rectum. Eqs. (21)-(22) yield

\[
\frac{dh}{dt} = - \frac{S A'}{m c^2} \bar{Q}'_{ext} + \beta \frac{G M_\odot}{c R^2} h ,
\]

\( h = m \sqrt{G M_\odot (1 - \beta) p_\beta} \). \hspace{1cm} (39)

Averaging over one orbital period (see Sec. 6.1 in Klačka 2004) yields

\[
\frac{dh}{dt} = - \beta \frac{G M_\odot}{c a_\beta^2 \sqrt{1 - e_\beta^2}} h ,
\]

where \( a_\beta \) is semimajor axis and \( e_\beta \) is eccentricity of the elliptical orbit. We can use \( p_\beta = a_\beta (1 - e_\beta^2) \) and Eq. (40) yields

\[
\frac{dp_\beta}{dt} = - 2 \beta \frac{G M_\odot}{c a_\beta^2 \sqrt{1 - e_\beta^2}} p_\beta \hspace{1cm} (41)
\]

Eq. (41) is correct equation for secular evolution of semi-latus rectum. If \( e_\beta^2 \) can be neglected in comparison with 1, then (but see also Klačka and Kaufmannová 1992, 1993, Breiter and Jackson 1998, Klačka 2001a) \( p_\beta \) equals approximately \( a_\beta \) and Eq. (41) yields

\[
a_\beta = \sqrt{a_\beta^2 (t - t_0) - 4 \beta \frac{G M_\odot}{c} (t - t_0)} \hspace{1cm} (42)
\]

if \( a_\beta (t_0) \) is the value of semimajor axis at the time \( t_0 \); this results is consistent with the Eq. (V.5-W).

6.12. McDonnell et al. (2001)

McDonnell et al. (2001, p. 164) write that ”small particulates are subject to forces which significantly alter or dominate their dynamics” and also the P-R effect is included:

”the Poynting-Robertson effect (non-radial radiation pressure due to aberration of sunlight) resulting in a gradual decrease in eccentricity and semimajor axis”.
Physics:

If we would consider arbitrarily shaped dust particle, then also non-radial components of radiation pressure may exist. Spherical particle exhibits only radial component of the radiation pressure. This is exact physics and it is defined in the proper frame of reference of the particle. In the reference frame of the Sun, there is also a non-radial term coming from $-v/c$. But this term does not come from the aberration of light, see Sec. 7.4.5 in Klačka (2008b), see also Sec. 6.2, and it is a consequence of the constancy of particle’s mass. Moreover, other physical effects are important in the P-R effect (Doppler effect, change of concentration of the incident photons) and the P-R effect cannot be reduced only to the term $-v/c$. This term would not yield the correct result for the secular decrease of eccentricity and semimajor axis. One has to bear in mind that the P-R effect is the effect of radiation pressure on moving spherical particles. Moreover, if the particles are small, then they can be expelled from the Solar System.

6.13. Grün et al. (2001a)

Grün et al. (2001a, p. 335) write: "Besides the direct effect of radiation pressure on trajectories of small dust grains there is also a more subtle effect: the Poynting-Robertson effect. This is caused by radiation pressure force, that is not perfectly radial on moving dust particle but has small component opposite to the particle motion. The strength is order of $v_t/c$ ($v_t$ is the tangential particle velocity) and it leads to loss of angular momentum and orbital energy of orbiting particle. The effect is strongest when the particle speed is highest, i.e. close to the Sun at its perihelion. Therefore particle orbits slowly circularized while they spiral toward the Sun."

Physics:

The Poynting-Robertson effect is the effect of radiation pressure on moving spherical particles. The "direct effect of radiation pressure" is an indispensable part of the P-R effect. There are not two different effects, "the direct effect of radiation pressure" and "a more subtle effect: the Poynting-Robertson effect", as the authors state.

The P-R effect is caused by radiation pressure force which is perfectly radial. And this holds in the proper frame of reference of the particle. Yes, the effect contains also a velocity component $v_t/c$, which leads to the loss of angular momentum and orbital energy of the orbiting particle. But $v_t$ is
the transversal and not the tangential particle velocity. The velocity vector is always tangential to the orbit of the particle. The velocity vector can be decomposed into radial and transversal components.

6.14. Dermott et al. (2001)

Dermott et al. (2001, pp. 574-575) write: "The component of the radiation force tangential to a particle’s orbit is called the P-R drag force. This force is also proportional to \( \beta \). It results in an evolutionary decrease in both the the semi-major axis and the osculating eccentricity of the particle’s orbit."

Physics:

According to the conventional definition (see Sec. 5), the P-R drag is the part of equation of motion which contains velocity terms. So, not only the term containing \(-v/c\), which is tangential to the particle’s orbit. However, relativistic covariant formulation does not admit to decompose the P-R effect into various parts.

The osculating eccentricity does not exhibit only decrease: \( de/dt \) can be also positive (see Eqs. 55 and 94, 96 in Klačka 2004), but secular value of \( de/dt \) is negative (if analytical time averaging over one period is admitted). An interesting result is presented in Figs. 5-6 in Klačka et al. (2007).

6.15. Grün et al. (2001b)

Grün et al. (2001b, p. 791 - glossary) explain: "Poynting-Robertson drag: Drag on a moving particle that is due to the asymmetrical momentum distribution of scattered and absorbed light; the drag primarily affects small grains, causing them to spiral toward the central object that they orbit, whether the Sun or planet."

Physics:

There is a preferred axis. It is given by the unit vector of the incoming radiation \( \hat{p'}_{\text{incoming}} \). Total outgoing radiation preserves this symmetry, since the condition given by Eqs. (16) is fulfilled:

\[
p'_{\text{outgoing}} = (1 - \hat{Q}'_{pr}/\hat{Q}'_{ext}) \hat{p'}_{\text{incoming}} .
\]

As a consequence, the equation of motion of the spherical particle

\[
dp'/d\tau = (\hat{Q}'_{pr}/\hat{Q}'_{ext}) \hat{p'}_{\text{incoming}}
\]

also fulfills the symmetry.
The drag $-v/c$ is generated by the first condition in Eqs. (16):
\[ E'_{\text{outgoing}} = E'_{\text{incoming}}. \]
The condition corresponds to the conservation of particle’s mass.

Motion of spherical totally reflecting (within geometrical optics approximation) body is also described by the P-R effect. Thus, absorption of the incoming light is not an important condition for the P-R effect.

6.16. Matzner (2001)

In the book of Matzner (2001, p. 524) we can find the following explanation of the Poynting-Robertson effect. It is "a drag force arising on particles orbiting the Sun, because solar radiation striking the leading surface is blueshifted compared to that striking the following surface. Thus, the particles receive a component of momentum from the radiation pressure which is opposite to the direction of motion."

Physics:

If we consider a homogeneous flow of radiation from star at particle’s location (it is a good approximation, in practice), then this radiation impacts on the particle in one direction. There is no difference between Doppler shifts of photons striking different parts of particle’s surface. The origin of drag force component is in the conservation of particle’s mass. The role of the Doppler effect is in the relation $S' w^2 S$, where $S$ is the flux of radiation energy (see Eqs. 1 and 2).

6.17. Murdin (2001)

Murdin (2001) presents that Poynting-Robertson effect is "a net force acting on small particles which causes their orbits to decay, also known as Poynting-Robertson drag". His explanation of this effect is following:

"When photons of light strike a dust particle in orbit around the Sun, they impinge not directly 'side-on' but slightly on its leading side (the 'forward' side in terms of its orbital motion). Their energy is absorbed, and subsequently re-radiated, but it is re-radiated isotropically - in all directions. The transfer of energy from photon to particle can be thought of as the application of a force to the particle. There is a component of this force that acts in the opposite direction to the tangential component of the particle’s orbital motion which is not compensated for when the absorbed energy
is re-radiated. As a result the particle loses kinetic energy, which reduces its velocity, so it spirals inward, toward the Sun. ... The effect can be counteracted by radiation pressure, more so with decreasing particle size below about 1 µm.”

Physics:

The first mistake is that Murdin distinguishes radiation pressure and P-R drag which denotes as the P-R effect. In reality, only the P-R effect exists. The P-R effect is the effect of radiation pressure on a moving spherically symmetric body (see also Sec. 6.2).

It is evident that Murdin considers perfectly absorbing spherical particle. He writes that the incident radiation causes force with component tangential to the particle’s orbit due to the aberration. This is not true in the stationary reference frame connected with the Sun. As we can see from Eqs. (18), the force originating from incident radiation has radial direction. Mentioned tangential force originates just from the outgoing radiation. Moreover, this tangential force decelerates the particle in the orbit and decreases the total (kinetic plus potential) energy of the particle. But increases the kinetic energy.

6.18. Bottke et al. (2002a), Lauretta and McSween (2006)

Bottke et al. (2002a, p. 769), Lauretta and McSween (2006, p. 914) write: ”Poynting-Robertson effect – an effect of radiation on a small particle orbiting the Sun that causes it to spiral slowly toward the Sun. It occurs because the orbiting particle absorbs energy and momentum streaming radially outward from the Sun, but reradiates energy isotropically in its own frame of reference.”

Physics:

The P-R effect holds for any spherically symmetric dust particle, not only for ”perfectly absorbing” particle which ”isotropically reemits/reradiates the absorbed radiation, in the reference frame of the particle”. E. g., also for totally reflecting sphere (within the geometrical optics approximation), although no absorption of the incoming radiation occurs. Moreover, even within the geometrical optics approximation, one must be careful in stating that the radiation is perfectly absorbed and isotropically reradiated in the particle’s own frame of reference. Although these ideas come back to Poynting (1903), and Robertson (1937), and although they are conventionally used (e. g., Wyatt and Whipple 1950, Burns et al. 1979), they do not contain the whole physics. In reality, the particle accepts momentum
per unit time given by Eq. (1), or,

\[ dp'_\text{in}/d\tau = (S'_A'\bar{Q}'_{\text{ext}}/c) e', \]

in the frame of reference of the particle, and, the outgoing radiation is described by Eq. (16). The case treated by Poynting, Robertson and others corresponds to \( \bar{Q}'_{\text{pr}} = 1, \bar{Q}'_{\text{ext}} = 2 \), in reality:

\[ p'_\text{outgoing} = (1/2) p'_\text{incoming} \]

and not \( p'_\text{outgoing} = 0 \) (Poynting 1903, Robertson 1937, Wyatt and Whipple 1950, Burns et al. 1979).

The P-R effect is the effect of radiation pressure. The pressure can cause escape of small dust particles from the Solar System. Only larger particles (of radius greater than 0.1 micron, approximately – this depends on optical properties of the particles) spiral slowly towards the Sun, due to the P-R effect.

6.19. Bottke et al. (2002b), Bottke et al. (2002c)

Bottke et al. (2002b, p. 11), Bottke et al. (2002c, p. 396) write: "Poynting-Robertson drag – a radiation effect that causes small objects to spiral inward as they absorb energy and momentum streaming radially outward from the Sun and then reradiate this energy isotropically in their own reference frame."

Physics:

The P-R drag is not a physical effect, which can be treated as an independent phenomenon. It is an indispensable part of the P-R effect. It holds for any spherically symmetric dust particle. The "perfect absorption" is not the essence of the P-R effect, as it was discussed in the previous Sec. 6.18.

6.20. Danby (2003)

Danby (2003, p. 366) writes: "We shall examine the Poynting-Robertson effect. A small particle moves in the solar system, subject to the gravitational attraction of the Sun and to radiation pressure from the Sun."

Danby continues: "... aberration of light ... leads to a transverse drag component of the radiation force, proportional to the transverse component of velocity, divided by the speed of light." And then: "Next, consider radial motion away from the Sun with speed \( \dot{r} \). Because of this, the particle will absorb less radiative energy in unit time than it would if it were at rest. Also, the radiation that it receives is diluted by the Doppler effect. So, relative to the 'unperturbed' model, we have a radial
drag force proportional to $2 \dot{r}/r^2$.

Physical comments:

1. It must be said that spherical particle with spherically distributed mass is considered. Moreover, equations presented by the author assume that optical properties of the particle do not change. It may be wise to be stressed, too.

2. As for the explanation of the term containing $-\frac{\dot{v}}{c}$ in the P-R effect as a consequence of the aberration of light, we refer the reader to Sec. 7.4.5 in Klačka (2008b), and, also to Sec. 6.2. The term is not a consequence of the aberration of light.

3. Danby states, that the term $-2 \dot{r}/c$ comes from a decrease of radiative energy and Doppler effect. This does not correspond to physics. The access of Danby corresponds to the procedure presented in Burns et al. (1979) which was commented in Sec. 2.5 in Klačka (1992a), see also Sec. 6.1: this would require violation of the second Newton’s law, if one would like to obtain the correct formula for the Poynting-Robertson effect.

4. According to physics, the energy flux density measured by the particle generates the term $-2 \dot{r}/c$ (see Eqs. 113-114 in Klačka 1992a, Eq. 23 in Klačka 2004) and the term is generated by the decrease of concentration of photons and frequency, i.e. Doppler effect (see Eqs. 32 and 36 in Klačka 1992a, Eqs. 7-8 in Klačka 1993b, Eqs. 17 and 22 in Klačka 2004, Eqs. 17, 20, 22 and 23 in Klačka 2008b).

5. The radial drag force proportional to $2 \dot{r}/r^2$ comes from the transformation (if the first order in $v/c$ is taken into account)

$S' = w^2 S = (1 - 2 \dot{r}/c) S$ ,

then from the term

$b = e/w = (1 + \dot{r}/c) e$ ,
and, finally, from the radial part of the term
\[-(v/c)_{rad} = -\dot{r}/c.\]
The term $1/r^2$ comes from the radiation energy flux density.

6.21. Parker (2003), Moore (2002), Köhler and Mann (2002)

Dictionary of Astronomy (Parker 2003, p. 99) explains the Poynting-Robertson effect in the following way: "The gradual decrease in orbital velocity of a small particle such as a micrometeorite in orbit about the sun due to the absorption and reemission of radiant energy by the particle."

Similar explanation is presented in Moore (2002):
"Poynting-Robertson effect. Non-gravitational force produced by the action of solar radiation on small particles in the Solar system; it causes the particles to spiral inwards towards the Sun. When particles in orbit around the Sun absorb energy from the Sun and re-radiate it, they lose kinetic energy and their orbital radius shrinks slightly. The effect is most marked for particles of a few micrometers in size."

Similarly, Köhler and Mann (2002) explain:
"The Poynting-Robertson effect reduces the kinetic energy of the particles. The semimajor axis and the eccentricity of the orbits decrease so that finally the particles fall into the stars."

Physical explanation is based on the two-body problem:
At first, we immediately formulate the correct statement: the P-R effect reduces the total (kinetic plus potential) energy of the particles.

The Keplerian orbital velocity of a body orbiting the Sun is characterized by magnitude
\[
v = \sqrt{G(M_{\text{sun}} + m) \left(\frac{2}{r} - \frac{1}{a}\right)}, \tag{43}
\]
where $a$ and $r$ are semi-major axis and heliocentric distance of the body of mass $m$. Time averaging over one period yields
\[
\langle v^2 \rangle = \frac{G(M_{\text{sun}} + m)}{a}. \tag{44}
\]

Let us consider the P-R effect and let us assume that $\beta$ is a constant. Then, $M_{\text{sun}} + m \rightarrow M_{\text{sun}}$ for a micrometeoroid. One possibility is to use as a central Keplerian acceleration $-GM_{\text{sun}} r/r^3$ and
the corresponding secular value of semi-major axis is $a$. If the non-velocity radiation term is included into the central acceleration, then the central Keplerian acceleration is $-GM_{\text{sun}}(1 - \beta) \frac{r}{r^3}$ and the corresponding secular value of semi-major axis is $a_\beta$. We can write

$$ \langle v^2 \rangle = \frac{GM_{\text{sun}}(1 - \beta)}{a_\beta} $$

for a micrometeoroid. Since semi-major axis is a decreasing function of time for the P-R effect, also

$$ \frac{d}{dt} \langle v^2 \rangle > 0. \tag{46} $$

Qualitative physical explanation:

The above cited Parker’s statement would be correct under the assumption that potential energy is constant. If a particle is decelerated, then its total energy is a decreasing function of time. The time averaged values of the total, potential and the kinetic energies (per unit mass) of the particle are

$$ \langle E \rangle = -\frac{GM_{\text{sun}}(1 - \beta)}{2a_\beta}, $$

$$ \langle U \rangle = 2 \langle E \rangle, $$

$$ \langle K \rangle = -\langle E \rangle. \tag{47} $$

We want to stress that Eqs. (47) are results corresponding to secular evolution. The last two equations of Eqs. (47) are similar to the virial theorem, where energy $E$ is constant. Eqs. (47) immediately yield

$$ \frac{d}{dt} \langle E \rangle = \frac{GM_{\text{sun}}(1 - \beta)}{2a_\beta^2} \frac{da_\beta}{dt} < 0, $$

$$ \frac{d}{dt} \langle U \rangle = \frac{GM_{\text{sun}}(1 - \beta)}{a_\beta^2} \frac{da_\beta}{dt} < 0, $$

$$ \frac{d}{dt} \langle K \rangle = -\frac{GM_{\text{sun}}(1 - \beta)}{2a_\beta^2} \frac{da_\beta}{dt} > 0. \tag{48} $$

The particle’s potential energy decreases more rapidly than the total energy, and, thus, the particle’s kinetic energy must increase in order to reduce the decrease of potential energy. The increase of $\langle K \rangle = \langle v^2 \rangle / 2$ occurs. So, the particle’s velocity is an increasing function of time.

Quantitative physical explanation:

Squared orbital velocity, averaged over one period, is

$$ \langle v^2 \rangle = \frac{GM_{\text{sun}}(1 - \beta)}{a_\beta \text{ in}} \frac{e_{\beta \text{ in}}^{4/5}}{1 - e_{\beta \text{ in}}^2} \frac{1 - e_{\beta}^2}{e_{\beta}^{4/5}} \tag{49} $$
where $a_{\beta \text{ in}}$ and $e_{\beta \text{ in}}$ are initial semi-major axis and eccentricity of the particle. The value of the eccentricity for a time $t$ is given by the following differential equation:

$$
\frac{de_{\beta}}{dx} = -\left(1 - e_{\beta \text{ in}}^2\right)^{3/2} \int_{e_{\beta \text{ in}}^3}^{e_{\beta \text{ in}}^5} \frac{z^{3/5}}{(1 - z^2)^{3/2}} \, dz,
$$

$$
t = x \, T, \quad 0 \leq x \leq 1 \quad (50)
$$

and $T$ is the time of spiralling of the particle into the Sun:

$$
T = \frac{2}{5} \left(\beta \frac{GM_{\text{sun}}}{c}\right)^{-1} a_{\beta \text{ in}}^2 \left(1 - e_{\beta \text{ in}}^2\right) \int_{e_{\beta \text{ in}}^3}^{e_{\beta \text{ in}}^5} \frac{z^{3/5}}{(1 - z^2)^{3/2}} \, dz. \quad (51)
$$

Eq. (50) shows that eccentricity is a decreasing function of time. As a consequence, $\langle v^2 \rangle$ is an increasing function of time, according to Eq. (49).

Finally, we have to stress that the P-R effect holds also for nonabsorbing particles (totally reflecting sphere). Although the case treated by Poynting (1903) and Robertson (1937) holds for perfectly absorbing nonrotating spherical particle, the condition for absorption is not relevant (and, moreover, Eq. 16 is valid).

\textbf{6.22. Encrenaz \textit{et al.} (2004)}

Encrenaz \textit{et al.} (2004, pp. 189-190) write:

The particles in the rings are subject to two effects:

- solar radiation pressure, which pushes the particles towards the outer regions of the Solar System. Because it is proportional to the surface area of a grain, while gravitational attraction is proportional to the volume, radiation pressure is particularly effective on small-sized particles, about one micrometre across;

- the Poynting-Robertson effect results from the fact that a particle orbiting the Sun receives solar radiation proportional to its cross-section, but radiates it away isotropically. The result is preferentially radiation in the forward direction (the frequency of the photons is increased because of the velocity of the particle) and this translates to a loss energy by the particle. This form of braking, which continuously decreases the eccentricity of the grain’s elliptical path, tends to turn the latter into a circle, and finally into a spiral. It is calculated that a grain one micrometre across, subject to the Poynting-Robertson effect alone would fall into the Sun in a few thousand years. In practice, grains of micrometre size are more sensitive to the radiation pressure that tends to push them towards the outer edge of the Solar System. The Poynting-Robertson effect is thus particularly
important for particles in the centimetre size range.

Physics:

Radiation pressure is an indispensable part of the P-R effect.

The incoming radiation generates the force acting on the spherical particle. The force is proportional to the extinction cross-section of the particle (see Eq. 1). If the particle is perfectly absorbing, within geometrical optics approximation, then $p_{\text{outgoing}}' = (1/2) p_{\text{incoming}}'$ and not $p_{\text{outgoing}}' = 0$ (isotropic outgoing radiation), as it was pointed out by Klačka (2008a, 2008b).

Momentum (per unit time) of the outgoing radiation is characterized by Eq. (1), or, Eq. (19), where $\bar{Q}' = 1/2$ for the case treated by Encrenaz et al. (2004):

$$d\mathbf{p}_{\text{out}}/dt = (SA'/c) \left[ \mathbf{v}/c + (1 - \mathbf{v} \cdot \mathbf{e}/c) \mathbf{e} \right].$$

A loss of energy by the particle, due to the outgoing radiation, follows from Eq. (19):

$$dE/dt = -2 S A' \left[ 1 - (3/2) \mathbf{v} \cdot \mathbf{e}/c \right].$$

6.23. Encrenaz et al. (2004)

Encrenaz et al. (2004, pp. 446) write:

"The Poynting-Robertson effect must be taken into account. As explained earlier ... this effect results from the fact that the – radial – force produced by the photons, which propagate with the velocity of light $c$, is exerted on a body moving at a velocity $v$, which is not co-linear with $c$. The transfer of momentum creates a braking force, which leads to a path that spirals in towards the Sun."

Physics:

The Poynting-Robertson effect results from the fact that a particle moves with respect to the source of radiation. The final equation of motion – represented by Eqs. (14), (21), or, (22) – holds for any relative direction and orientation of $\mathbf{v}$ and $\mathbf{e}$, including their co-linearity.

6.24. Quinn (2005)

Quinn (2005, pp. 194-195) distinguishes the radiation pressure and the Poynting-Robertson drag. He writes: "... as radiation is absorbed and re-radiated by the particle, the re-radiated radiation is
anisotropic in the rest frame of the sun. This results in a drag force opposite the direction of motion of the particle referred to as Poynting-Robertson Drag. The total radiation force including this effect to the first order in $v/c$ is:

$$F_{rad} = \left( \frac{L_{\odot} Q_{pr} A}{4\pi cr^2} \right) \left[ (1 - 2v_r/c) \hat{r} - (v_\theta/c) \hat{\theta} \right]$$

where $L_{\odot}$ is the solar luminosity, $r$ is the radius of the particle, and $v_r$ and $v_\theta$ are the radial and tangential component respectively of the particle velocity in the frame of the Sun. The first term in this expression is the radiation pressure, and the second and third terms are the Poynting-Robertson drag. There is a factor of 2 in the second radial term because of the combination of Doppler shift in absorbing radiation and Doppler shift in emission.”

Physics:

The factor 2 in the second radial term is a result of combination of the change of concentration of photons, the Doppler shift of the incident radiation, the term $1/w$ present in $b^\mu$ (see Eqs. 2 and 14) and the conservation of particle’s mass. The Quinn’s statement that the second radial term is due to the Doppler shift of the absorbed and re-radiated radiation, is not correct.

Quinn’s statements are in practice the same as Gustafson’s ones (Gustafson 1994). Thus, the correct explanation of the origin of individual terms in expression for drag force is also the same. See also Sec. 6.7.

6.25. Harwit (2006)

Harwit (2006, p. 174-175; see also Harwit 1988, p. 176-177, Harwit 1973, p. 176-177) belongs to the authors who try to explain the Poynting-Robertson drag through a decrease of the angular momentum.

Harwit writes:

"Consider a grain of dust in interplanetary space. As it orbits the Sun it absorbs sunlight, and re-emits this energy isotropically. We can view this process from two different viewpoints.

(a) Seen from the Sun, a grain with mass $m$ absorbs light coming radially from the Sun and re-emits it isotropically in its own rest frame. A re-emitted photon carries off angular momentum proportional: (i) to its equivalent mass $h\nu/c^2$, (ii) to the velocity of the grain $R\dot{\theta}$; and (iii) to the grain’s distance from the Sun $R$. Considering only terms linear in $V/c$, and neglecting any higher terms we see that the grain loses orbital angular momentum $L$ about the Sun at a rate

$$dL = (h\nu/c^2) \dot{\theta} R^2, \quad (1/L) \frac{dL}{dt} = (h\nu)/(mc^2), \quad (5 - 45 - H)$$
for each photon whose energy is absorbed and re-emitted, or isotropically scattered in grain’s rest frame.

(b) Seen from the grain, radiation from the Sun arrives at an aberrated angle \( \theta' \) from the direction of motion, instead of at \( \theta = 270^\circ \). Hence,

\[
\cos \theta = \frac{\cos \theta' + (V/c)}{1 + (V/c) \cos \theta'} = 0, \quad \cos \theta' = -\frac{V}{c}.
\]

Here \( V \) is \( \dot{\theta}R \), the grain’s orbital velocity, and the photon imparts an angular momentum \( pR \cos \theta' = -(h\nu/c^2)R^2 \dot{\theta} \) to the grain.

For a grain with cross-section \( \sigma_g \)

\[
\frac{dL}{dt} = -\frac{(L_{\text{sun}} \sigma_g)}{(4\pi R^2 mc^2)} L, \quad (5 - 47 - H)
\]

where \( L_{\text{sun}} \) is the solar luminosity.

Either way, the grain’s velocity decreases on just absorbing sunlight. From the first viewpoint, this happens because the grain gains mass, which it then loses on re-emission; from the second, it is because the grain is slowed down by the transfer of angular momentum.³

Physics:

We have already explained in Sec. 6.11 that although the orbital angular momentum of the perfectly absorbing particle decreases due to the outgoing radiation (Eqs. 32, 39), the process increases acceleration of the particle (see Eq. 12, \( \vec{Q}' = 1/2 \)). The outgoing radiation acts on the particle by the force containing also the term \( -v/c \) (see Eqs. 19, where \( \vec{Q}' = 1/2 \)). Thus, the outgoing radiation participates on the Poynting-Robertson drag force, but the corresponding acceleration is positive (see Eqs. 20)!

Harwit’s considerations are strictly based on the idea of Poynting (1903), Robertson (1937) and others, when the authors assume that the outgoing radiation corresponds to the isotropically re-emitted radiation. We know that this is not true.

Moreover, even if the outgoing radiation would be equivalent to the re-emitted radiation, the considerations of Harwit would be incorrect. The Harwit’s statement that each re-emitted photon carries off the same angular momentum is not true, in a stationary reference frame. One photon, emitted in a direction of unit vector \( \vec{n}' \), carries a momentum \( dp' = (h\nu'/c)\vec{n}' \), in the grain’s proper reference frame. Each photon emitted in an arbitrary direction is characterized by the frequency \( \nu' \). Thus, in the grain’s reference frame, the photons are emitted isotropically and the total momentum
carried off by the photons per unit time is zero \( (F'_e = 0) \). The momentum \( dp' \) transforms to the stationary reference frame as

\[
dp = \frac{h\nu}{c} n = \frac{h\nu'}{c} \left\{ n' + \left[ (\gamma - 1) \frac{v \cdot n'}{v^2} + \frac{\gamma}{c} \right] v \right\},
\]

(52)
due to the Doppler effect and the aberration of light. Thus, the angular momentum carried off by the given photon is

\[
dL = R \times dp = \frac{h\nu'}{c} R \left( e \times n' + e \times \frac{v}{c} \right),
\]

(53)
if the first order in \( v/c \) is considered.

In his second explanation (from the view of the grain), Harwit mixes terms from stationary and proper reference frames. The incoming radiation really hits the grain in an aberrated direction and the radiation acts on the grain by a force (see Eqs. 18 and 20). But particle’s velocity equals zero at every moment, in the particle’s proper frame of reference. Thus, the particle has no orbital angular momentum which could be decreased in the proper frame.

6.26. de Pater and Lissauer (2006)

de Pater and Lissauer (2006, pp. 35) state:

"Sections ... above describe the gravitational interactions between the Sun, planets and moons. Solar radiation, which provides an important force for small (\( \leq 1 \) m) particles in the Solar System, has been ignored. Three effects can be distinguished:

– **Radiation pressure**, which pushes particles (primarily micrometer-sized dust) outwards from the Sun.
– **Poynting-Robertson drag**, which causes centimeter-sized particles to spiral inward towards the Sun.
– The **Yarkovski effect**, which changes the orbits of meter to kilometer-sized objects due to uneven temperature distributions at their surfaces.

The solar wind produces a corpuscular drag similar in form to the Poynting-Robertson drag; corpuscular drag is more important for submicrometer particles."

The authors continue with a subsection "Radiation Force (micrometer-sized particles)"

"The Sun’s radiation exerts a repulsive force, \( F_{rad} \), on all bodies in our Solar System. This force is given by:

\[
F_{rad} \approx \left[ \frac{L_\odot A}{(4\pi c r^2_\odot)} \right] Q_{pr} \hat{r},
\]

(2.45-P-L)
where \( A \) is the particle’s geometrical cross-section, \( L_\odot \) the solar luminosity, \( r_\odot \) the heliocentric distance, \( c \) is the speed of light, and \( Q_{pr} \) the radiation pressure coefficient. The radiation pressure coefficient accounts for both absorption and scattering and is equal to unity for a perfectly absorbing particle. Relativistic effects produced by the Doppler shift between the frame of the Sun and that of the particle are generally small, and have been omitted from equation (2.45-P-L), but they will be considered in Sec. 2.7.2-P-L. The parameter \( \beta \) is defined as the ratio between the forces due to the radiation pressure and the Sun’s gravity:

\[
\beta \equiv \left| \frac{F_{rad}}{F_g} \right| = 5.7 \times 10^{-5} \frac{Q_{pr}}{(\rho R)} , \tag{2.46-P-L}
\]

with the particle’s radius, \( R \), in cm and its density, \( \rho \), in \( g \text{ cm}^{-3} \). Note that \( \beta \) is independent of heliocentric distance and that the solar radiation force is only important for micrometer and submicrometer-sized particles. Extremely small particles are not strongly affected by radiation pressure, because \( Q_{pr} \) decreases as the particle radius drops below the (visible wavelength) peak in the solar spectrum (Fig. 2.12-P-L). The magnitude of the Sun’s effective gravitational attraction is given by:

\[
F_{g,eff} = -(1-\beta) \frac{G m M_\odot}{r_\odot^2} . \tag{2.47-P-L}
\]

It is clear that small particles with \( \beta > 1 \) are repelled by the Sun’s radiation, and thus quickly escape the Solar System, unless they are gravitationally bound to one of the planets. Dust released at a keplerian velocity from bodies on circular orbits is ejected from the Solar System if \( \beta > 0.5 \); critical values of \( \beta \) for dust released from bodies on eccentric orbits are calculated in Problem 2.32-P-L.

The importance of solar radiation pressure can, for example, be seen in comets (Sec. 10.4.1.-P-L): Cometary tails always point in the anti-solar direction due to the Sun’s radiation pressure. The dust tails are curved rather than straight as a result of the continuous ejection of dust grains from the comet, which itself is on an elliptical orbit around the Sun.”

de Pater and Lissauer (2006, pp. 36-37) continue with a new subsection "Poynting-Robertson Drag (centimeter-sized grains)":

"A particle in orbit around the Sun absorbs solar radiation and reradiates the energy isotropically in its own frame. The particle thereby preferentially radiates (and loses momentum) in the forward direction in the inertial frame of the Sun (Fig. 2.13-P-L). This leads to a decrease in the particle’s energy and angular momentum and causes dust in bound orbits to spiral sunward. This effect is called Poynting-Robertson drag.

Let us consider a perfectly absorbing, rapidly rotating, dust grain. The flux of solar radiation absorbed by the grain is equal to

\[
\left[ \frac{L_\odot A}{(4 \pi r_\odot^3)} \right] \left( 1 - \frac{v_r}{c} \right) , \tag{2.48a-P-L}
\]
where \( v_r = v \cdot \hat{r} \) is the radial component of the particle’s velocity (i.e., the component which is parallel to the incident beam of light). The second term in expression (2.48a-P-L) accounts for the Doppler shift between the Sun’s rest frame and that of the particle; the transverse Doppler shift is of order \( (v_\theta/c)^2 \ll 1 \) and will be ignored here. The absorbed flux is reradiated isotropically and can be written as a mass loss rate in the particle’s frame of motion (using \( E = mc^2 \)):
\[
\left[ \frac{L_\odot}{4 \pi c^2 r_\odot^2} \right] (1 - v_r / c) \ . \quad (2.48b-P-L)
\]
As the particle moves relative to the Sun with velocity \( v \), there is a momentum flux from the particle as seen in the rest frame of the Sun, since the particle emits more momentum in the forward direction than in the backward direction (Fig. 2.13-P-L). The momentum flux is equal to
\[
\left[ - \frac{L_\odot}{4 \pi c^2 r_\odot^2} \right] (1 - v_r / c) \ v \ , \quad (2.48c-P-L)
\]
which can be generalized to the case in which the particle reflects and/or scatters some of the radiation impinging upon it via multiplication by \( Q_{pr} \). The net force on the particle in this more general case is given by:
\[
\mathbf{F}_{\text{rad}} = \left[ \frac{L_\odot Q_{pr}}{4 \pi c^2 r_\odot^2} \right] \left( 1 - v_r / c \right) \hat{r} - \left[ \frac{L_\odot Q_{pr}}{4 \pi c^2 r_\odot^2} \right] \left( 1 - 2 v_r / c \right) \hat{v} - \left[ \frac{L_\odot Q_{pr}}{4 \pi c^2 r_\odot^2} \right] \left( v_\theta / c \right) \hat{\theta} \ . \quad (2.49a-P-L)
\]
\[
\approx \left[ \frac{L_\odot Q_{pr}}{4 \pi c^2 r_\odot^2} \right] \left[ (1 - 2 v_r / c) \hat{r} - (v_\theta / c) \hat{\theta} \right] . \quad (2.49b-P-L)
\]
The first term in equation (2.49b-P-L) is that due to radiation pressure and the second and third terms (those involving the velocity of the particle) represent the Poynting-Robertson drag.

From the above discussion, it is clear that small dust grains in the interplanetary medium disappear, with (sub-)micrometer-sized grains being blown out of the Solar System, while centimeter-sized particles spiral inward towards the Sun.”

Physics:

The authors distinguish the two effects: the radiation pressure and the Poynting-Robertson drag. However, physics states that the effect of radiation pressure on a moving spherical particle is equivalent to the Poynting-Robertson effect. The Poynting-Robertson effect is the relevant physical effect. The mentioned two ”effects”, the radiation pressure and the Poynting-Robertson drag cannot be separated from the P-R effect, as it follows from the relativistic formulation of the equation of motion. Similarly, the two ”effects” are relevant for the same particles: it is not correct to say that one part of the P-R effect is relevant for the micrometer-sized dust, while the rest part is relevant for the centimeter-sized particles. Moreover, the P-R effect is not the dominant effect for centimeter-sized interplanetary particles – collisions among particles, impact erosion are important. Moreover, the P-R
As for the solar wind effect ("corpuscular drag"), it is important for the same sizes of the particles as the P-R effect. As for submicrometer-sized particles, the Lorentz force, due to the presence of interplanetary magnetic field, is relevant. Small particles can be blown out from the Solar System, due to the (electromagnetic) radiation pressure.

As for the Sec. 2.7.2-P-L: "A particle in orbit around the Sun absorbs solar radiation and reradiates the energy isotropically in its own frame. The particle thereby preferentially radiates (and loses momentum) in the forward direction in the inertial frame of the Sun (Fig. 2.13-P-L). This leads to a decrease in the particle’s energy and angular momentum and causes dust in bound orbits to spiral sunward. This effect is called Poynting-Robertson drag.”

As the authors state, the derivations presented in the textbook is based on the derivation presented by Burns et al. (1979). Unfortunately, the derivation is physically incorrect (Klącka 1992a - see mainly Sec. 2.5). We will present the relevant comments, now.

1. "Let us consider a perfectly absorbing, rapidly rotating, dust grain. The flux of solar radiation absorbed by the grain is equal to
\[ \frac{L_\odot A}{4 \pi r_\odot^2} \left(1 - \frac{v_r}{c}\right) , \quad (2.48a-P-L) \]

Comment:
If one would like to consider "rapidly rotating" particle, the result would be more complicated. So, we will consider that the particle does not rotate. The correct result for the flux of the incoming solar radiation interacting with the grain equals to
\[ \tilde{Q}'_{ext} \left[ \frac{L_\odot A}{4 \pi r_\odot^2} \right] (1 - 2 \frac{v_r}{c}) , \quad (2.48a-P-L-correct) \]
\[ \tilde{Q}'_{ext} = 2 \] for the special case treated by the authors. The second term in expression (2.48a-P-L-correct) accounts for the Doppler shift and the change of concentration of photons between the Sun’s rest frame and that of the particle.

The difference between Eqs. (2.48a-P-L) and (2.48a-P-L-correct) is evident. Incorrect form of Eq.
(2.48a-P-L) can be found also in, e. g., Mukai and Yamamoto (1982).

2. "The absorbed flux is reradiated isotropically and can be written as a mass loss rate in the particle’s frame of motion (using \( E = mc^2 \)):
\[
\left[ \frac{L \odot \ A}{(4 \pi \ c^2 \ r_{\odot}^2)} \right] (1 - v_r / c). \quad \text{(2.48b-P-L)}
\]

Comment:
In general, the absorbed flux can be reradiated in an arbitrary way, not only isotropically. The mass loss rate, corresponding to the outgoing radiation in the particle’s frame of motion, is:
\[
\bar{Q}_{\text{ext}} \left[ \frac{L \odot \ A}{(4 \pi \ c^2 \ r_{\odot}^2)} \right] (1 - \frac{2}{\hat{r}} v_r / c). \quad \text{(2.48b-P-L-correct)}
\]

3. "As the particle moves relative to the Sun with velocity \( \mathbf{v} \), there is a momentum flux from the particle as seen in the rest frame of the Sun, since the particle emits more momentum in the forward direction than in the backward direction (Fig. 2.13-P-L). The momentum flux is equal to
\[
\left[ - \frac{L \odot \ A}{(4 \pi \ c^2 \ r_{\odot}^2)} \right] (1 - \frac{2}{\hat{r}} v_r / c) \ \mathbf{v}, \quad \text{(2.48c-P-L)}
\]
which can be generalized to the case in which the particle reflects and/or scatters some of the radiation impinging upon it via multiplication by \( Q_{\text{pr}} \).

Comment:
As the particle moves relative to the Sun with velocity \( \mathbf{v} \), there is a momentum flux from the particle as seen in the rest frame of the Sun, since the particle emits more momentum in the forward direction than in the backward direction (Fig. 2.13-P-L). The momentum flux is equal to, for the case treated by Poynting (1903) and Robertson (1937),
\[
\left[ - 2 \frac{L \odot \ A}{(4 \pi \ c^2 \ r_{\odot}^2)} \right] \left[ (1 - \frac{v_r}{c}) \ \hat{r} - (1/2) \times \left( (1 - \frac{v_r}{c}) \ \hat{r} - (1 - 2 \frac{v_r}{c}) \ \mathbf{v} / c \right) \right], \quad \text{(2.48c-P-L-correct)}
\]
see Eq. (76) in Klačka (2008b). The last result can be generalized to the case in which the particle reflects and/or scatters some of the radiation impinging upon it. However, this cannot be done in the simple way as stated by the authors (via multiplication by \( Q_{\text{pr}} \)). The correct result is given by Eq. (76) in Klačka (2008b).

4.
The net force on the particle in this more general case is given by:

\[ F_{\text{rad}} = \left[ \frac{L_{\odot} Q_{\text{pr}} A}{4 \pi c r_{\odot}^2} \right] (1 - v_r / c) \hat{r} \]
\[ - \left[ \frac{L_{\odot} Q_{\text{pr}} A v}{4 \pi c^2 r_{\odot}^2} \right] (1 - v_r / c) \hat{v} \]  
\[ \approx \left[ \frac{L_{\odot} Q_{\text{pr}} A}{4 \pi c r_{\odot}^2} \right] \left[ (1 - 2 v_r / c) \hat{r} - (v_\theta / c) \hat{\theta} \right]. \]  

(2.49a-P-L)

(2.49b-P-L)

The first term in equation (2.49b-P-L) is that due to radiation pressure and the second and third terms (those involving the velocity of the particle) represent the Poynting-Robertson drag.

Comment:

The net force on the particle in this more general case is given by:

\[ F_{\text{rad}} = \left[ \frac{L_{\odot} Q_{\text{ext}} A}{4 \pi c r_{\odot}^2} \right] (1 - v_r / c) \hat{r} \]
\[ - \left[ \frac{L_{\odot} Q_{\text{ext}} A v}{4 \pi c^2 r_{\odot}^2} \right] (1 - v_r / c) \hat{v} - (Q_{\text{pr}} / Q_{\text{ext}}) \times \]
\[ ((1 - v_r / c) \hat{r} - (1 - 2 v_r / c) \hat{v}) \]
\[ \approx \left[ \frac{L_{\odot} Q_{\text{pr}} A}{4 \pi c r_{\odot}^2} \right] \left[ (1 - 2 v_r / c) \hat{r} - (v_\theta / c) \hat{\theta} \right]. \]  

The last force is the force due to the radiation pressure.

6.27. Carroll and Ostlie (2007)

We find the following definition in Carroll and Ostlie (2007, p. 806):

"The Poynting-Robertson effect (a consequence of the headlight effect discussed in example 4.3.3) can cause ring particles to spiral in toward the planet. When particles in the rings absorb sunlight, they must re-radiate that energy again if they are to remain in thermal equilibrium. The original light was emitted from the Sun isotropically, but in the Sun’s rest frame the re-radiated light is concentrated in the direction of motion of the particle. Since the re-radiated light carries away momentum as well as energy, the particle slows down and its orbit decays."

Physics:

The statement introduced above is one of the wide-spread ideas about the essence of P-R effect. Let us consider spherical particle. Re-emitted thermal radiation carries off energy and momentum per unit time (in the proper frame of particle)

\[ E_T' = C_{\text{abs}}' S', \]
\[ p_T' = - F_e' = 0, \]  

(54)
where $C'_{abs}$ is an absorption cross-section of the particle. I.e., the four-momentum of thermal radiation per unit time is

$$p^\mu_T = \frac{1}{c} C'_{abs} S' (1; 0). \quad (55)$$

Due to the emission of the energy, the rest mass of the particle will decrease:

$$\left( \frac{dm}{d\tau} \right)_T = - \frac{E'_T}{c^2} = - \frac{C'_{abs} S'}{c^2}. \quad (56)$$

In the stationary frame, the four-momentum of thermal radiation per unit time will be

$$p^\mu_T = \frac{1}{c} C'_{abs} S' \frac{u^\mu}{c}, \quad (57)$$

where $u^\mu$ is the particle’s four-velocity. Thus, thermal radiation acts on the perfectly absorbing particle by four-force

$$\left( \frac{dp^\mu}{d\tau} \right)_T = - p^\mu_T = - \frac{1}{c} C'_{abs} S' \frac{u^\mu}{c}. \quad (58)$$

Using the fact that

$$\left( \frac{dp^\mu}{d\tau} \right)_T = \left( \frac{dm}{d\tau} \right)_T u^\mu + m \left( \frac{du^\mu}{d\tau} \right)_T \quad (59)$$

and also Eqs. (56) and (58), the four-acceleration of the particle is

$$\left( \frac{du^\mu}{d\tau} \right)_T = 0, \quad (60)$$

due to the thermal emission. Thus, the re-emitted thermal radiation produces no acceleration of the spherical particle. The particle is accelerated by incoming and by outgoing scattered radiation. But although the acceleration has the component in opposite direction to the particle’s velocity $v$, the particle does not slow down, but its orbital velocity increases (see Sec. 6.21).

### 6.28. Meyer et al. (2007)

Meyer et al. (2007, p. 580) write:

"The radial component of the radiation force is known as radiation pressure." ... "The tangential component of the radiation force is known as the Poynting-Robertson (P-R) drag. This acts on all grains and causes their orbits to decay into the star (where the grains evaporate) at a rate $\dot{a} = -2 \alpha / a$, where ..."

Physics:

In general, the radiation pressure force has three components, for arbitrarily shaped body. One radial and two nonradial. Equation of motion does not correspond to the P-R effect, in this general...
case. The difference in motion of spherical and nonspherical dust grains was confirmed experimentally (Krauss and Wurm 2004). Only spherical grains behave in accordance with the P-R effect.

The Poynting-Robertson effect is the effect of radiation pressure upon the motion of spherical particle. Nonradial components of the radiation pressure force equal zero, in this case. As for the P-R drag definition, much confusion exists in the literature, see Sec. 5. In any case, relativistically covariant equation of motion yields the P-R effect and its decomposition into several parts (classified as "effects") is nonphysical.

6.29. Sykes (2007)

Sykes (2007, p. 685) writes: "Comets were long thought to be the origin of the zodiacal cloud. However, estimates of dust production by short-period comets fell far short of that needed to maintain the cloud in steady state against losses from particles spiralling into the Sun. This mechanism, where the absorption and reemission of solar radiation continually decrease particle velocity, is called Poynting-Robertson drag."

Physics:

What does physics say, in reality? Particle speed of the spiralling toward the Sun is an increasing function of time, according to Eqs. (46)-(48). The same states Eq. (45), since particle’s semi-major axis \( a \) is a decreasing function of time and \( \beta \) is considered to be a constant. The same result follows from Eqs. (49)-(50): Eq. (50) yields decrease of eccentricity \( e \) and Eq. (49) produces an increase of the speed of motion for a decreasing eccentricity. Thus, the statement of the Sykes that "the absorption and reemission of solar radiation continually decreases particle velocity" does not correspond to reality. Moreover, totally reflecting particle is also described by the P-R effect – no absorption/reemission exists in this case.

6.30. Krügel (2008)

Krügel (2008, p. 173-174) offers very similar explanation as Woolson (2000), see also Sec. 6.9. He considers "a grain of mass \( m \), radius \( a \) and geometrical cross-section \( \sigma_{geo} = \pi a^2 \) circling a star at distance \( r \) with frequency \( \omega \) and velocity \( v = \omega r \). The angular momentum of the grain is \( l = m r^2 \omega = m rv \). Further he states:" Let \( M_* \) and \( L_* \) be the mass and luminosity of the star. Because there is mostly optical radiation, the absorption coefficient of the grain is \( C_{abs} \approx \sigma_{geo} \) and the particle absorbs
per unit time the energy
\[ \Delta E = \frac{L \sigma_{geo}}{(4\pi r^2)}. \]
In thermal balance, the same amount is reemitted. Seen from a non-rotating rest frame, the stellar photons that are absorbed travel in radial direction and carry no angular momentum, whereas the emitted photons do because they partake in the circular motion of the grain around the star. If we associate with the absorbed energy \( \Delta E \) a mass \( m_{\text{phot}} = \frac{\Delta E}{c^2} \), the angular momentum of the grain decreases per unit time through emission by
\[ \frac{dl}{dt} = - m_{\text{phot}} rv = - \left( \frac{\Delta E}{c^2} \right) rv = - \frac{L \sigma_{geo}}{(4\pi c^2 r^2 m)}. \]

Next procedure is in principle the same as the Woolfson’s one. Krügel calculates a decrease of the particle’s distance from the star based on the decrease of the orbital angular momentum of the grain.

Krügel also tries to explain the P-R effect from the view of the observer in a frame corotating with the particle (i.e. particle’s proper frame). He states: "In such a reference frame, the stellar photons approach the grain not exactly along the radius vector from the star, but hit it slightly head-on in view of the aberration of light.... The photons thus decrease the angular momentum of the grain and force it to spiral into the star."

Physics:

From the view of the Sun.
Calculation in Sec. 6.27 shows that the re-emitted thermal radiation from spherical grain acts on it by nonzero force. Thus, the re-emitted radiation changes the grain’s orbital angular momentum. But this action is caused only by the decrease of the grain’s mass due to the re-emission of the radiation and the corresponding acceleration equals zero. Thus, this phenomenon is not an essence of the Poynting-Robertson effect in the sense that electromagnetic radiation gives some acceleration to the particle. Moreover, thermal radiation does not represent the whole outgoing radiation which decreases particle’s angular momentum. See Secs. 6.11 and 6.25 for more detail explanation.

From the view of the particle.
Krügel’s statement that the force originating from the incident radiation decreases (orbital) angular momentum of the particle is, similarly to Harwit’s one (see Sec. 6.25), senseless. The velocity of the particle equals zero in its proper reference frame and thus the particle has no orbital angular momentum in this frame.
6.31. Liou and Kaufmann (2008)

1. Liou and Kaufmann (2008, p. 426) write about "forces due to solar radiation and solar wind, including solar radiation pressure, PR drag, and solar wind drag."

Comment:
There are two physical forces: solar radiation force and solar wind force. These forces cannot be physically decomposed into various parts, as it is presented by the authors. This holds also for explanations presented in Liou and Kaufmann (2008, p. 426-427). Similarly, physically correct solar wind force should automatically contain possibility of nonradial velocity of the solar wind and time dependent properties of the solar wind, but no separation of these "effects" is physically acceptable.

2. Liou and Kaufmann (2008, p. 426) write about the drag forces (PR drag and solar wind drag): "Although these drag forces are weak compared to the velocity-independent radial pressure force component, they dissipate energy and momentum and thereby cause the particles to eventually spiral into the Sun."

Comment:
The velocity-dependent terms of the radiation pressure and solar wind pressure forces dissipate total energy. But the momentum is not dissipated, since the speed of the particle increases during the process of spiralling toward the Sun. The magnitude of the momentum increases. The magnitude of the angular momentum decreases. Moreover, nonradial velocity component of the solar wind can increase particle's total energy and angular momentum (Klaćka et al. 2008).

3. Liou and Kaufmann (2008, pp. 426-427) write: "The PR drag can be thought of as arising from an aberration of the sunlight as seen from the particle and a Doppler-shift induced change in momentum."

Comment:
As it is discussed in Sec. 7.4.5 in Klaćka (2008b), the velocity-dependent term \(-\frac{v}{c}(v - \text{heliocentric velocity of the particle, } c \text{ is the speed of light in vacuum})\) in the P-R effect is not produced by the aberration of light. The same holds for the effect of solar wind. Moreover, the force for the solar wind
effect does not contain the term \(- v/u\) where \(u\) is the speed of solar wind particles (see Eq. 29 in Klačka and Saniga 1993). The effect of aberration is discussed also in Sec. 6.2.

In reality, not only the Doppler effect causes the P-R effect. Also change of concentration of photons plays an equally important role (see also Klačka 1992a: Eqs. 31, 32, 36, 37, or, Secs. 2.4 and 2.5; see also Klačka 1993b: Eqs. 2-5).

4.
Liou and Kaufmann (2008, p. 427) write: "Thus, forces due to collisions with solar wind particles are analogous to forces due to radiation."

Comment:
Yes, we can say that the force for the solar wind and solar electromagnetic radiation effects are analogous. However, one has to bear in mind that there are important differences. The force for the solar wind effect on dust particle contains decrease of mass of the dust particle (Klačka and Saniga 1993, Klačka 1993a, Kocifaj and Klačka 2008 – this physical difference is often not considered, although the change of mass of the particle is admitted; see, e.g., Dohnanyi 1978 – Secs. 2.1.3 and 2.2.6, Mukai and Yamamoto 1982, Mann 2009 – Eqs. 7-10, 7-11), nonradial velocity of the solar wind (Klačka 1994, Klačka et al. 2008), time dependence of the solar wind (Svalgaard 1977).

5.
Liou and Kaufmann (2008, p. 427) write: "The contribution of the solar wind to the drag forces on dust particles is typically parametrized as a constant fraction \(sw\) of the PR drag."

Comment:
The statement of the authors is used in their equations: Eqs. (5-LK), (7-LK), (10-LK), (11-LK), (17-LK), (18-LK), (21-LK), (23-LK), ..., i.e., the equations contain the term \(\beta(1 + sw)\). The same access can be found, e.g., in Liou and Zook (1997), Holmes et al. (2003). However, this is not correct. The correct form is (except for the forces) \(\beta(1 + sw/Q_{pr}')\) as it is presented in Klačka (1994, Eq. 24 – although the value of \(sw\) is different from those used by Liou and Kaufmann), Klačka (2004: Eqs. 176, 177, 182, 184, 186, ...), Klačka et al. (2008: Eqs. 4, 5, 15, 16). Moreover, the correct form for the force is even more complicated:

\[
F_{\text{drag}} = - \left[ S_0 A Q_{pr}/(r_0^2 c) \right] \left[ (1 + sw/Q_{pr}) \left( \mathbf{v} \cdot \hat{r}_0/c \right) \hat{r}_0 + (1 + x'sw/Q_{pr}) \mathbf{v}/c \right],
\]
where the value of $x'$ (> 1) depends on material properties of dust particle. The last equation is the correct equation, not Eq. (5) in Liou and Kaufmann (2008, p. 427).

6.32. http://en.wikipedia.org/wiki/Poynting-Robertson_effect (last modification August 23, 2008)

In this very popular webpage we can find the following definition of Poynting-Robertson effect:

"The Poynting-Robertson effect, also known as Poynting-Robertson drag, ..., is a process by which solar radiation causes a dust grain in the solar system to slowly spiral inward. The drag is essentially a component of radiation pressure tangential to the grain’s motion."

There is also an explanation of the effect:

"From the perspective of the grain of dust circling the Sun ..., the Sun’s radiation appears to be coming from a slightly forward direction (aberration of light). Therefore the absorption of this radiation leads to a force with a component against the direction of movement. ... From the perspective of the solar system as a whole (...), the dust grain absorbs sunlight entirely in a radial direction, thus the grain’s angular momentum remains unchanged. However, in absorbing photons, the dust acquires added mass via mass-energy equivalence. In order to conserve angular momentum (which is proportional to mass), the dust grain must drop into a lower orbit.

Note that the re-emission of photons, which is isotropic in the frame of the grain, does not affect the dust particle’s orbital motion. However, in the frame of the solar system, the emission is beamed anisotropically, and hence the photons carry away angular momentum from the dust grain. It is somewhat counter-intuitive that angular momentum is lost while the orbital motion of the grain is unchanged, but this is an immediate consequence of the dust grain shedding mass during emission and that angular momentum is proportional to mass.

The Poynting-Robertson drag can be understood as an effective force opposite the direction of the dust grain’s orbital motion, leading to a drop in the grain’s angular momentum. It should be mentioned that while the dust grain thus spirals slowly into the Sun, its orbital speed increases continuously.

The Poynting-Robertson force is equal to:

$$F_{PR} = \frac{Wv}{c^2} = \left[\frac{r^2}{(4c^2)}\right]\sqrt{\frac{GM_sL_s^2}{R^5}}$$

where $W$ is the power of the incoming radiation, $v$ is the grain’s velocity, $c$ is the speed of light, $r$ the object’s radius, $G$ is the universal gravitational constant, $M_s$ the Sun’s mass, $L_s$ is the solar
luminosity and $R$ the object's orbital radius."

Comments:

At first, we must point out that the Poynting-Robertson drag is not the same as the P-R effect, according to the standard approach used in literature (see also Sec. 5). The radial radiative pressure is also included into this effect. Therefore also the introduced equation for the P-R force is not correct, it corresponds only to the component proportional to $-\nu/c$, moreover not precisely (see Eqs. 21).

We have to stress that the P-R effect is the effect describing motion of a spherical body (with spherically symmetric mass distribution) under the action of radiation pressure. It has no sense to give a special names to individual parts of the total force. At first, the confusion exists in scientific literature, and, moreover, the readers are utterly confused by such nonphysical steps.

It has been already mentioned, for several times, that the aberration of the incident light does not lead to a force with component against the direction of particle’s movement. For an explanation see Sec. 7.4.5 in Klačka (2008b) and also Sec. 6.2.

From the author’s explanation it follows that dynamics of the particle is influenced (its orbit is changed) only by the incident (absorbed; he does not consider complete interaction including also scattering) radiation because of conservation of particle’s orbital angular momentum at absorption of this radiation. The outgoing (thermal re-emitted) radiation does not influence the particle’s dynamics.

The incident radiation really does not change particle’s angular momentum (see Eqs. 8 and 18 and also Sec. 6.11) because the action of the increase of particle’s mass is compensated by the action of deceleration of the particle. Thus, the author is partially correct in this sense. But his argumentation implies that the total acceleration of the particle should be proportional to $\tilde{Q}_{abs}'$ (particle’s mass increase is proportional to this one) and not to $\tilde{Q}_{pr}'$, as it actually is (see Eq. 22). Moreover, the statement of the author is not consistent with the fundamental condition represented by Eq. (16). Decisive role plays incoming and outgoing radiation. In reality, the incoming radiation generates particle’s acceleration proportional to $\tilde{Q}_{ext}' = 2$, for the case treated by the author (see Eq. 20).

Equation of motion is the relevant physical result. The equation of motion contains (linear) momentum vector of the particle, not its angular momentum vector. If we have equation of motion in disposal, it is not problem to obtain exact equations for the angular momentum of the particle. Really,
we can consider angular momentum of the particle
\[ \mathbf{L}_{in} = \mathbf{r} \times \mathbf{p}_{in} \equiv m \mathbf{H}_{in}, \]
where \[ \mathbf{H}_{in} \equiv \mathbf{r} \times \mathbf{v}_{in} \]
is angular momentum per unit mass of the particle, and, similarly,
\[ \mathbf{L}_{out} = \mathbf{r} \times \mathbf{p}_{out} \equiv m \mathbf{H}_{out}, \]
where \[ \mathbf{H}_{out} \equiv \mathbf{r} \times \mathbf{v}_{out}. \]
The results for these quantities can be summarized as follows:
\[ \frac{d\mathbf{L}_{in}}{dt} = 0, \]
\[ \frac{d\mathbf{H}_{in}}{dt} = -\left(\frac{|dm/dt|}{m}\right) \mathbf{H}_{in}, \]
\[ \frac{d\mathbf{L}_{out}}{dt} = -\left(\frac{|dm/dt|}{m}\right) \left(\frac{Q'_\text{pr}}{Q'_\text{ext}}\right) \mathbf{L}_{out}, \]
\[ \frac{d\mathbf{H}_{out}}{dt} = +\left(\frac{|dm/dt|}{m}\right) \left(1 - \frac{Q'_\text{pr}}{Q'_\text{ext}}\right) \mathbf{H}_{out}, \]
where \[ |dm/dt| = w^2 S A' Q'_{ext}/c^2 \] (of course, \( \mathbf{v}_{in} = \mathbf{v}_{out} = \mathbf{v} \)). These results immediately yield, e.g.,
that the magnitude of the vector \( \mathbf{H}_{in} \) decreases (particle spirals toward the central star) and the
magnitude of the vector \( \mathbf{H}_{out} \) increases for the cases \( Q'_\text{pr}/Q'_\text{ext} < 1 \) (the particle would spiral outward
from the central star, if only "out" process would be important) – everything is consistent with the
results obtained from linear momenta vectors (compare with Sec. 7.4.1 in \( \) Klačka 2008b).

6.33. Zimmermann and Gürtler (2008)

Zimmermann and Gürtler (2008, p. 307) write: "P-R-Effekt, das Phänomen, womach kleine sich um
die Sonne bewegende Teilchen infolge der Absorption von Sonnenstrahlung abgebremst werden, was
zur allmählichen Annäherung an die Sonne führt. Die von der Sonne emittierten Lichtquanten trager
den einen zu ihre. Energie proportionalen Impuls, der auf ein aborbierendes Teilchen übertragen wird.
Infolge der Bahngeschwindigkeit der umlaufenden Partikeln und der endlichen Lichtgeschwindigkeit
ergibt sich bei der Absorption ein Aberrationseffekt. Der auf ein Teilchen übertragene Impuls hat
außer einer von der Sonne wegge richteten radialen Komponente eine der Bewegung des Teilchens
entgegenge richtete tangentielle, abbrem deswirkende Komponente. Je kleiner und mas seärmer eine
Partikel ist, umso größer ist des Verhältnis des tangentialen Impulses gegenüber dem Umlaufimpuls
des Teilchens, umso schneller erfolgt die Abbremsung. Andererseits wächst mit abnehmendem
Teilchenradius das Verhältnis des radialen Strahlungsdrucks zur gravitativen Anziehung durch die
Sonne. Bei Teilchen größer als etwa 0.1 \( \mu \)m überwiegt die Abbremsung, die Teilchen spieralen
in die Sonne. Bei kleineren Partiklen überwiegt der Strahlungsdruck, sie werden von der Sonne
The P-R effect holds for spherically symmetric bodies (see also laboratory measurements obtained by Krauss and Wurm, 2004). Solar electromagnetic radiation and gravity cause secular spiralling of spherical grains toward the Sun (if their radii are larger than $\approx 0.1 \mu m$).

The essence of the P-R effect is not an absorption of the incoming light. Even motion of the particle characterized with mirror-like spherical surface (specular reflection) is characterized by the P-R effect (see Sec. 7.3.3 in Klačka 2008b).

The aberration of the incoming light does not lead to a force with component against the direction of particle’s movement. For an explanation see Sec. 6.2 (or Sec. 7.4.5 in Klačka 2008b).

The P-R effect is the effect of electromagnetic radiation pressure on moving spherically symmetric bodies.

6.34. Mann (2009)

Mann discusses “radiation pressure force” (Mann 2009, Sec. 7.3.1). Moreover, she discusses the “Poynting-Robertson effect”, independently (Mann 2009, Sec. 7.3.3). Finally, she writes: "Particles in bound orbit about the star for which radiation pressure force is smaller than the stellar gravity force migrate toward the star due to the azimuthal component of the radiation pressure force and this is called the Poynting Robertson effect.” (Mann 2009, Sec. 7.3.3).

Mann (2009) makes difference between the radiation pressure force and the P-R effect. As we have already mentioned in Sec. 5, this access does not correspond to physics. The P-R effect is the radiation pressure force acting on moving (nonrotating) body with spherically symmetric mass
distribution. Moreover, the P-R effect contains not only the transversal velocity term, but also other terms, including radial velocity term. In reality, nonradial components of radiation pressure force are zero for spherical bodies. The nonradial components of radiation pressure force may exist for nonspherical particles (Klačka 2004, 2008b) – proper frame of reference of the particle is the relevant system for defining radial and nonradial components.

2. Mann (2009, p. 201) states:

"The stellar wind force in the frame of the moving particles depends on the dust velocity \( \mathbf{v} \) as:

\[
\mathbf{F}_{sw} = \mathbf{F}_{sw} \left[ (1 - \frac{\mathbf{v} \cdot \mathbf{r}}{(rv_{sw})}) \frac{r}{r} - \frac{\mathbf{v}}{v_{sw}} \right],
\]

(7.10-M)

where \( \mathbf{F}_{sw} \) is the force on the dust for \( \mathbf{v} = 0 \) and \( v_{sw} \) is the bulk velocity of the wind. The non-radial term in (7.10) is referred to as plasma or pseudo Poynting-Robertson drag force." (We have corrected the evident erroneous term in the original equation, Eq. (7-10-M) is correct.)

Physics:

i) The solar-wind force acting on a body is not of the form presented by Mann (2009 – see Eq. 7.10-M). The correct form is given by Klačka and Saniga (1993 – Eq. 29):

\[
\mathbf{F}_{sw} = \mathbf{F}_{sw} \left[ (1 - \frac{\mathbf{v} \cdot \mathbf{r}}{(rv_{sw})}) \frac{r}{r} - x' \frac{\mathbf{v}}{v_{sw}} \right],
\]

(29-K-S)

where \( x' \) denotes the part of the incident energy which is lost from the grain during the interaction with the solar wind (measured in the reference frame of the grain; \( 1 < x' < 3 \), approximately, depending on material properties of the particle). The force is even more complicated if nonradial component of the solar wind velocity is included.

ii) What is the sense of the words "plasma or pseudo Poynting-Robertson drag force"? We have just shown that the last term of the real force \( \mathbf{F}_{SW} \) does not correspond to that presented by Mann (2009 - Eq. 7.10). Thus, the term "plasma or..." is misleading. And what about the rest velocity term?

3. Mann (2009, p. 202) states:

"The ratio of plasma P-R drags force to (photon) P-R drag force can be written as
The factor \( c/v_{sw} \) results from the difference of the aberration angles for photons or solar wind particles.

Physics:

We are aware that this kind of explanation is presented also in Burns et al. (1979, p. 12), or, Leinert and Grün (1990, pp. 226-227) – see also Sec. 6.5. However, the solar-wind force does not produce the term \(-F_{sw} v/v_{sw}\) (see Eq. 29-K-S presented above). Thus, explanation of the term as a consequence of the aberration is incorrect. Even the light-term \(-F_{ph} v/c\) is not produced by the aberration of light, see Sec. 6.2 and also Sec. 7.4.5 in Klačka (2008b).

6.35. Minato et al. (2004)

We have already mentioned an analogy between the solar electromagnetic and corpuscular radiation in Secs. 6.30, 6.34. We will present detail comments on the paper by Minato et al. (2004), now: the incorrect statements on the solar wind effect may enable better understanding of the P-R effect.

1. Minato et al. (2004 – Sec. 2.1) state that the contribution of solar-wind forces to the motion of dust grain is described as

\[
F_{sw} \left[ (1 - 2 \frac{\dot{r}}{v_{sw}}) e_R - (\dot{\theta}/v_{sw}) e_{\theta} \right], \quad (1-M)
\]

where \( v \) is the velocity of the grain \( v = \dot{r} e_R + r \dot{\theta} e_{\theta} \) and \( v_{sw} \) is the heliocentric solar-wind speed.

Physics:

The solar-wind force acting on a body is not of the form presented by Minato et al. (2004 – Sec. 2.1, see Eq. 1-M). The correct form is given by Klačka and Saniga (1993 – Eq. 29):

\[
F_{sw} \left[ (1 - (1 + x') \frac{\dot{r}}{v_{sw}}) e_R - (x' r \dot{\theta}/v_{sw}) e_{\theta} \right], \quad (29-K-S)
\]

where \( x' \) denotes the part of the incident energy which is lost from the grain during the interaction with the solar wind (measured in the reference frame of the grain; \( x' > 1 \)). The force is even more complicated if one uses also the nonradial component of the solar wind velocity.

2.
"The second term on the right-hand side of Eq.(1-M) expresses the pseudo PR drag which causes dust grains to spiral into the Sun." (Minato et al. 2004 – Sec. 3.2)

Physics:

i) There does not exist the term $-r\dot{\theta}/v_{sw} \ e_\theta$ in the solar wind force. The real term is multiplied by the value $x'$. This is one of the differences between the P-R effect and the solar wind effect. Thus, there does not exist a simple analogy to the "P-R drag".

ii) There is another inconsistency in defining the "P-R drag". We have already discussed (see Sec. 5) that various definitions are used. Now, Minato et al. (2004 – Sec. 3.2) present another definition for the "P-R drag". Thus, according to the authors, the "P-R drag" is neither the term $-F_{ph} \ [2 \ (\dot{r}/c) \ e_R + (r\dot{\theta}/c) \ e_\theta ]$, nor the term $-F_{ph} \ [(\dot{r}/c) \ e_R + (r\dot{\theta}/c) \ e_\theta ]$, but the term $-F_{ph} \ (r\dot{\theta}/c) \ e_\theta$. And what about the rest velocity terms in the P-R effect? How are they called in the "physics" of Wyatt (2009), Mann (2009) and Minato et al. (2004)?

Really, the radiation pressure force for a body with spherically distributed mass corresponds to the P-R effect. Relativistically covariant formulation of the equation of motion shows that any separation of the equation into various (ad-hoc) components is of no physical sense and the term "P-R drag" should be omitted in future.

By the way, the term $-F_{sw} \ (r\dot{\theta}/v_{sw}) \ e_\theta$ may not cause that dust grain spirals into the Sun. Also the outspiralling from the Sun may occur if the solar wind bombardment significantly decreases mass of the grain. Moreover, non-radial solar wind velocity may be more important than the term $-F_{sw} \ (r\dot{\theta}/v_{sw}) \ e_\theta$.

3. "The ratio of $\gamma$ of the pseudo P-R drag to the P-R drag is given by $\gamma = (F_{sw}/F_{ph}) \ c/v_{sw}$, where the factor $c/v_{sw}$ comes from the difference of the aberration angles for solar wind and light." (Minato et al. 2004 – Sec. 3.2)
Physics:

The solar-wind force does not produce the term $-F_{sw} r \dot{\theta} / v_{sw} \mathbf{e}_\theta$. Thus, explanation of it as a consequence of the aberration is incorrect. Even the light-term $-F_{ph} (r \dot{\theta} / c) \mathbf{e}_\theta$ is not produced by the aberration of light, see Sec. 6.2 (Sec. 7.4.5 in Klačka 2008b).

7. Simple explanation of the P-R effect

Astronomers often want to present a simple explanation of a physical effect. We have just presented many incorrect explanations of the P-R effect. The correct explanation can be found in Klačka (2008b). However, very simple explanation can be very useful for scientists and students.

7.1. Simple explanation without equations

Physical correct and simple explanation with no equation can be as follows:

**Poynting-Robertson effect** – The effect of electromagnetic radiation pressure on a moving spherical particle. The incident radiation can be represented by a parallel beam of photons and the particle is a nonrotating body of spherically symmetric mass distribution.

Let the Sun (or another star) be the only source of radiative and gravitational forces acting on the particle. The particle of radius smaller than about 0.1 micron (the value depends on the particle’s optical properties and the luminosity of the star) is expelled from the Solar System (the star) while the larger particle slowly spirals toward the Sun. Solar radiation pressure force acting on small dust particles (less than 0.1 micron in radii) is larger than the solar gravitational force. As for larger particles, the conservation of particle’s mass, during particle’s interaction with the incoming solar radiation, is important. The conservation of the mass produces the term proportional to $-\mathbf{v} / c$ in the radiation pressure force acting on the particle; $\mathbf{v}$ is particle’s heliocentric orbital velocity and $c$ is the speed of light. The drag/frictional term proportional to $-\mathbf{v} / c$ causes decrease of particle’s total energy (and angular momentum) and the particle slowly spirals toward the Sun. The magnitude of the momentum (and the speed of the particle) slowly increases.

7.2. Mathematical description

Homogeneous beam of parallel photons interacts with non-rotating spherical body/particle. If the incoming momentum per unit time is $\mathbf{p'}_{\text{incoming}}$ in the proper frame of reference of the particle, then
the total outgoing momentum per unit time is
\[ p_{\text{outgoing}}' = (1 - C'_{pr}/C'_{ext}) p_{\text{incoming}}'. \]

\( C'_{pr} \) and \( C'_{ext} \) are pressure and extinction cross-sections. The cross-sections are calculated from Mie’s solution of Maxwell’s equations (Mie 1908). The following inequalities hold: \( C'_{pr} > 0 \) and \( 0 < C'_{pr}/C'_{ext} < 2 \). The case of "perfectly absorbing spherical particle" (treated by Poynting 1903 and Robertson 1937) is, in reality, characterized by \( C'_{pr} = \pi R^2, C'_{ext} = 2\pi R^2 \), where \( R \) is radius of the particle. The force acting on the particle is
\[ dp'/d\tau = p_{\text{incoming}}' - p_{\text{outgoing}}' = (C'_{pr}/C'_{ext}) p_{\text{incoming}}', \]

where \( \tau \) is proper time measured in the reference frame of the particle. Using the fact that \( p_{\text{incoming}}' = (S'C'_{ext}/c) e' \), where \( S' \) is the flux density of radiation energy (energy flow through unit area perpendicular to the ray per unit time), \( c \) is the speed of light and \( e' \) is unit vector describing direction and orientation of the travelling photons with respect to the particle. The last two equations yield \( dp'/d\tau = (S'C'_{pr}/c) e' \).

This equation represents an electromagnetic (light) pressure acting on the particle. Moreover, the mass of the particle does not change and this corresponds to the condition for the energy of the particle \( E' \)
\[ dE'/d\tau = 0. \]

The left-hand sides of the last two equations are components of the energy-momentum four-vector (four-momentum). This enables us to find the equation of motion of the particle in any frame of reference, if we use Lorentz transformation. If the particle moves with velocity \( \mathbf{v} \) in a frame of reference, then the equation of motion of the particle is
\[ dp'^\mu /d\tau = (w^2SC'_{pr}/c) (b'^\mu - u'^\mu/c), \quad (*) \]

where \( p'^\mu = (E/c ; \mathbf{p}) \) is four-momentum of the particle, \( w = \gamma (1 - \mathbf{v} \cdot \mathbf{e}/c), b'^\mu = (1/w ; \mathbf{e}/w), u'^\mu = (\gamma c ; \gamma \mathbf{v}) \) is four-velocity of the particle, \( \mathbf{e} \) is unit vector describing direction and orientation of the travelling photons with respect to the stationary frame of reference (laboratory frame) in which the particle moves with velocity \( \mathbf{v} \), and, finally, \( \gamma = 1/\sqrt{1 - (\mathbf{v}/c)^2} \) is the Lorentz factor.

The equation of motion \((*)\) is known as the Poynting-Robertson effect. The Poynting-Robertson effect is the effect corresponding to the electromagnetic (light) pressure.

To the first order in \( v/c \), Eq. \((*)\) reduces to
\[ d\mathbf{v}/dt = [SC'_{pr}/(mc)] [ (1 - \mathbf{v} \cdot \mathbf{e}) \mathbf{e} - \mathbf{v}/c ] . \]
Remark:

The process of interaction between the incident radiation and the spherical particle can be treated as a two-step process.

The incoming radiation acts on the particle with a force
\[
\frac{dp^\mu}{d\tau}_{\text{in}} = \left( w^2 S \tilde{C}'_{\text{ext}} / c \right) b^\mu,
\]
and the force causes the following acceleration of the particle
\[
\frac{du^\mu}{d\tau}_{\text{in}} = \left[ w^2 S C'_{\text{ext}} / (mc) \right] (u^\mu - u^\mu / c),
\]
or, to the first order in \( v / c \):
\[
\frac{dv}{dt}_{\text{in}} = \left[ S \tilde{C}'_{\text{ext}} / (mc) \right] \times \left[ (1 - v \cdot e / c) e - v / c \right].
\]

The outgoing radiation acts on the particle with a force
\[
\frac{dp^\mu}{d\tau}_{\text{out}} = - \left( w^2 S \tilde{C}'_{\text{ext}} / c \right) \left[ (C'_{\text{pr}} / \tilde{C}'_{\text{ext}}) u^\mu / c + (1 - C'_{\text{pr}} / \tilde{C}'_{\text{ext}}) b^\mu \right],
\]
and the force causes the following acceleration of the particle
\[
\frac{du^\mu}{d\tau}_{\text{out}} = - \left[ w^2 S C'_{\text{ext}} / (mc) \right] (1 - \tilde{C}'_{\text{pr}} / \tilde{C}'_{\text{ext}}) (b^\mu - u^\mu / c),
\]
or, to the first order in \( v / c \):
\[
\frac{dv}{dt}_{\text{out}} = - \left[ S \tilde{C}'_{\text{ext}} / (mc) \right] \times \left[ (1 - \tilde{C}'_{\text{pr}} / \tilde{C}'_{\text{ext}}) \times \left[ (1 - v \cdot e / c) e - v / c \right] \right].
\]

It can be easily seen that \( \frac{dv}{dt}_{\text{out}} \propto + v \) if \( \tilde{C}'_{\text{pr}} / \tilde{C}'_{\text{ext}} < 1 \). This holds also for the case treated by Poynting and Robertson \( (\tilde{C}'_{\text{pr}} / \tilde{C}'_{\text{ext}} = 1 / 2) \).

Often the dimensionless efficiency factors of extinction and pressure are used: \( \tilde{Q}'_{\text{ext}} = \tilde{C}'_{\text{ext}} / (\pi R^2) \), \( \tilde{Q}'_{\text{pr}} = \tilde{C}'_{\text{pr}} / (\pi R^2) \), where \( R \) is radius of the particle.

Orbital evolution of interplanetary spherical dust particle under the action of solar electromagnetic radiation and the solar gravity is thoroughly understood and explained in Klačka (2004) and Klačka et al. (2007). Orbital evolution of spherical particle under the action of solar gravity and electromagnetic radiation leads to a secular decrease of semi-major axis and eccentricity. Shift of perihelion may exist already in the case when the equation of motion is considered to the first order in \( v / c \), if optical properties of the particle change.
8. Secular evolution of orbital elements

Let us consider orbital evolution of a body under the action of gravity and electromagnetic radiation of a central star (e.g., Sun). The action of electromagnetic radiation is taken to be the P-R effect. Accuracy to the first order in $v/c$ is considered.

We have already discussed that physics does not admit to divide the P-R effect into partial components. Thus, we have to take into account that the P-R effect represents the disturbing force to the Keplerian motion given by the gravity of the central star. We are interested in secular evolution of orbital elements (short-term oscillations in orbital elements are not important for us). We will proceed in accordance with Klačka (1994) and Klačka (2004 – Sec. 6).

If parameter $\beta$ changes, then only numerical calculations can be realized. If equation of motion in the form

$$\dot{v} = -\frac{GM}{r^3} r + \beta \frac{GM}{r^2} \left\{ \left(1 - \frac{v \cdot e}{c}\right) e - \frac{v}{c}\right\},$$

(61)

is numerically solved, then orbital elements can be calculated on the basis of Eqs. (47) in Klačka (2004 – the correct right-hand side of the last equation contains $v \cdot e_T/(e \sqrt{GM_\odot/p}) - 1/e$, if $M = M_\odot$, instead of $... - 1$).

In any case, an approximation of constant $\beta$ can be useful, at least for comparison. This case can be treated in a fully analytical approach, if secular evolution of orbital elements is searched. The results can be summarized as follows, as for semi-major axis $a$ and eccentricity $e$:

$$a = a_\beta \left(1 - e_\beta^2\right)^{3/2} \frac{1}{2\pi} \int_0^{2\pi} \frac{1 + \beta \left(1 + e_\beta^2 + 2e_\beta \cos x\right) \left(1 - e_\beta^2\right)}{(1 + e_\beta \cos x)^2} dx,$$

$$e = \left(1 - e_\beta^2\right)^{3/2} \frac{1}{2\pi} \int_0^{2\pi} \sqrt{\frac{(1 - \beta)^2 e_\beta^2 + \beta^2 - 2\beta (1 - \beta) e_\beta \cos x}{(1 + e_\beta \cos x)^2}} dx,$$

(62)

where

$$\frac{da_\beta}{dt} = -\beta \frac{GM}{c} \frac{2 + 3e_\beta^2}{a_\beta \left(1 - e_\beta^2\right)^{3/2}},$$

$$\frac{de_\beta}{dt} = -\frac{5}{2} \beta \frac{GM}{c} \frac{e_\beta}{a_\beta^2 \left(1 - e_\beta^2\right)^{1/2}}.$$

(63)

Orbital elements with subscript $\beta$ correspond to the central Keplerian acceleration $-GM(1 - \beta)r/r^3$, non-velocity radial term of acceleration caused by radiation force in Eq. (61) is added to the gravitational acceleration of the star (however, it is often stated that Eqs. 63 hold for central acceleration...
\[ \Delta = \Delta v_R \mathbf{e}_{PR} + \Delta v_T \mathbf{e}_{PT} + \Delta v_N \mathbf{e}_{PN}, \]

while

\[
\begin{align*}
\mathbf{v} &= v_R \mathbf{e}_{PR} + v_T \mathbf{e}_{PT}, \\
v_R &= \sqrt{GM \frac{p_P^{-1}}{e_P}} \sin(\Theta_P - \omega_P), \\
v_T &= \sqrt{GM \frac{p_P^{-1}}{e_P}} \left[ 1 + e_P \cos(\Theta_P - \omega_P) \right], \\
p_P &= a_P (1 - e_P^2), p_{\beta} = a_{\beta} (1 - e_{\beta}^2) \text{ and } e_{PN} = e_{PR} \times e_{PT},
\end{align*}
\]

\[ p_{\beta \text{ in}} = \frac{p_{\beta}}{1 - e_{\beta}^2} \left( \frac{v_T + \Delta v_T}{v_T^2} \right)^2 + \left( \frac{\Delta v_N}{v_T} \right)^2, \]

\[ e_{\beta \text{ in}}^2 = \frac{\left( 1 + e_P \cos f_P \frac{v_T^2}{v_T^2} \right)^2}{\left( 1 - \beta \frac{v_T^2}{v_T^2} \right)^2} \left( \frac{v_R + \Delta v_R}{v_T} \right)^2, \]

\[ a_{\beta \text{ in}} = \frac{p_{\beta \text{ in}}}{1 - e_{\beta \text{ in}}^2}, \]

\[ \cos i_{\beta \text{ in}} = \frac{v_T + \Delta v_T}{v_T^2} \cos i_P - \frac{\Delta v_N}{v_T} \cos \Theta_P \sin i_P, \]

\[ \sin i_{\beta \text{ in}} \cos \Omega_{\beta \text{ in}} = \frac{v_T + \Delta v_T}{v_T^2} \sin i_P \cos \Omega_P + \frac{\Delta v_N}{v_T} \sin i_P \cos \Theta_P \cos \Omega_P - \sin i_P \sin \Omega_P, \]

\[ \sin i_{\beta \text{ in}} \sin \Omega_{\beta \text{ in}} = \frac{v_T + \Delta v_T}{v_T^2} \sin i_P \sin \Omega_P + \frac{\Delta v_N}{v_T} \sin i_P \cos \Theta_P \sin \Omega_P + \sin i_P \cos \Omega_P, \]

\[ e_{\beta \text{ in}} \cos f_{\beta \text{ in}} = \frac{p_{\beta \text{ in}}}{p_P} \left( 1 + e_P \cos f_P \right) - 1, \]

\[ e_{\beta \text{ in}} \sin f_{\beta \text{ in}} = \frac{1 + e_P \cos f_P}{1 - \beta} \frac{v_T^2}{v_T^2}, \]

\[ \sin i_{\beta \text{ in}} \cos \Theta_{\beta \text{ in}} = \frac{v_T + \Delta v_T}{v_T^2} \sin i_P \cos \Theta_P + \frac{\Delta v_N}{v_T} \cos i_P, \]

\[ \sin i_{\beta \text{ in}} \sin \Theta_{\beta \text{ in}} = \sin i_P \sin \Theta_P, \]
\[ v_{TS}^2 \equiv (v_T + \Delta v_T)^2 + (\Delta v_N)^2. \] (66)

The set of equations represented by Eqs. (62)-(66) fully corresponds to detailed numerical calculations of vectorial equation of motion, if we are interested in secular evolution of eccentricity and semi-major axis for the case when central acceleration is defined by gravity alone. We have to stress that "\(e_\beta \lessgtr 1\)" and "\(e_\beta\) does not correspond to pseudo-circular orbit" are tacitly assumed.

![Fig. 1. Dependence of \(a/a_\beta\) as a function of secular value of \(e_\beta\). Dashed lines depict functions \((1 - e_\beta) / [1 - \beta e_\beta + (1 + e_\beta)]\) and \((1 + e_\beta) / [1 + \beta (1 - e_\beta)]\). The dashed lines confine the zone where instantaneous ratio of semi-major axis (central acceleration \(- GMr/r^3\)) and \(a_\beta\) can occur.]

It is worth mentioning that instantaneous time derivatives of semi-major axis and eccentricity defined by central acceleration \(- GMr/r^3\) may be both positive and negative, while secular evolution yields that semi-major axis \(a\) and eccentricity \(e\) are decreasing functions of time. As for the central acceleration \(- GM(1-\beta)r/r^3\), the instantaneous time derivative of semi-major axis is always negative, while the instantaneous time derivative of eccentricity may be both negative (near apocentre) and positive (secular evolution yields that semi-major axis \(a_\beta\) and eccentricity \(e_\beta\) are decreasing functions of time – see Eqs. 63).
Fig. 2. Dependence of $e$ as a function of secular value of $e_\beta$. Dashed lines depict functions $| (1 - \beta) e_\beta - \beta |$ and $(1 - \beta) e_\beta + \beta$. The dashed lines confine the zone where instantaneous eccentricity (central acceleration $- G M r / r^3$) can occur.

For the special case $\Delta = 0$ Eqs. (66) reduce to

\[ a_{\beta \, in} = a_P (1 - \beta) \left(1 - 2 \beta \frac{1 + e_P \cos f_P}{1 - e_P^2}\right)^{-1}, \]

(67)

\[ e_{\beta \, in} = \sqrt{1 - \frac{1 - e_P^2 - 2 \beta (1 + e_P \cos f_P)}{(1 - \beta)^2}}, \]

(68)

where $f_P \equiv \Theta_P - \omega_P$, $\omega_{\beta \, in}$ has to be obtained from

\[ e_{\beta \, in} \cos (\Theta_P - \omega_{\beta \, in}) = \frac{\beta + e_P \cos f_P}{1 - \beta}, \]

\[ e_{\beta \, in} \sin (\Theta_P - \omega_{\beta \, in}) = \frac{e_P \sin f_P}{1 - \beta}, \]

(69)

\[ \Omega_{\beta \, in} = \Omega_P, \quad i_{\beta \, in} = i_P, \quad \Theta_{\beta \, in} = \Theta_P. \]

(70)

Physics of Eqs. (67)-(70) is following: meteoroid escapes from the Solar System when the orbital energy becomes positive and this can happen when the energy due to the radial component of the radiation force is included, without it being necessary for this force to exceed the gravitational attraction (Harwit 1963). Some figures may be found in Klačka (1992b, 1993c). As an example we may mention that
Fig. 3. Time evolution of semi-major axes $a$ and $a_\beta$. $T$ is the time of spiralling toward the star. The value of $a_{in}$ is given by the first of Eqs. (62). The following time limits hold: $\lim_{t \to T} a = \lim_{t \to T} a_\beta = 0$.

A particle of $\beta = (1 - e_P)/2$ moves in parabolic orbit if ejected at perihelion of the parent body, and, in an orbit with eccentricity $e_{\beta in} = |1 - 3e_P|/(1 + e_P)$ if released at aphelion; $e_{\beta in} = 0$ for $\beta = e_P = 1/3$ and aphelion ejection.

We may mention that a little more simple procedure is obtained when semi-major axis $a_\beta$ is replaced by $p_\beta = a_\beta(1 - e_\beta^2)$ (a quantity referred to as semi-latus rectum). Then $p_\beta = p_{\beta in}(e_\beta/e_{\beta in})^{4/5}$, and:

$$\frac{dp_\beta}{dt} = -2 \beta \frac{GM}{c} \left[1 - e_{\beta in}^2 \left(\frac{p_{\beta}}{p_{\beta in}}\right)^{5/2}\right]^{3/2}$$

$$\frac{de_\beta}{dt} = -\frac{5}{2} \beta \frac{GM}{c} \frac{e_{\beta in}^{8/5}}{p_{\beta in}^2 \left(1 - e_{\beta}^2\right)^{3/2}}$$

Eqs. (62)-(63) show that secular evolutions of semi-major axes $a$, $a_\beta$ and eccentricities $e$, $e_\beta$ are decreasing functions of time. This means that eccentric orbits become more circular, during the particle’s orbital evolution (constant $\beta$ is assumed). This corresponds to the fact that apocentric distances (aphelion distances) $Q$, $Q_\beta$ decrease more rapidly than the pericentric distances (perihelion distances)
Fig. 4. Time evolution of eccentricities $e$ and $e_\beta$. $T$ is the time of spiralling toward the star. The following time limits hold: $\lim_{t \to T} e = \beta$, $\lim_{t \to T} e_\beta = 0$.

$q_\beta$, $q_\beta$, as it is evident also from the equations

$$q = a_\beta \left(1 - e_\beta^2\right)^{5/2} \frac{1}{2\pi} \int_0^{2\pi} Z_q \, dx,$$

$$Z_q = \frac{1 - \sqrt{(1 - \beta)^2 \epsilon_3^2 + \beta^2 - 2\beta (1 - \beta) e_\beta \cos x}}{\left[1 - e_\beta^2 + \beta \left(1 + e_\beta^2 + 2e_\beta \cos x\right)\right] \left(1 + e_\beta \cos x\right)^2},$$

(73)

$$Q = a_\beta \left(1 - e_\beta^2\right)^{5/2} \frac{1}{2\pi} \int_0^{2\pi} Z_Q \, dx,$$

$$Z_Q = \frac{1 + \sqrt{(1 - \beta)^2 \epsilon_3^2 + \beta^2 - 2\beta (1 - \beta) e_\beta \cos x}}{\left[1 - e_\beta^2 + \beta \left(1 + e_\beta^2 + 2e_\beta \cos x\right)\right] \left(1 + e_\beta \cos x\right)^2},$$

(74)

where Eqs. (63)-(66) can be used (or, Eqs. 71-72, too), or

$$a_\beta = \frac{1}{2} (q_\beta + Q_\beta),$$

$$e_\beta = \frac{1 - q_\beta/Q_\beta}{1 + q_\beta/Q_\beta},$$

(75)

and $q_\beta$, $Q_\beta$ are given by the following equations:

$$\frac{dq_\beta}{dt} = -\beta \frac{GM}{c} q_\beta \frac{1}{2Q_\beta} \frac{5q_\beta + 3Q_\beta}{q_\beta + Q_\beta},$$

$$\frac{dQ_\beta}{dt} = -\beta \frac{GM}{c} q_\beta \frac{1}{Q_\beta} \frac{5q_\beta + 3Q_\beta}{q_\beta + Q_\beta},$$

$$\frac{dq_\beta}{dt} = -\beta \frac{GM}{c} q_\beta \frac{1}{Q_\beta} \frac{5q_\beta + 3Q_\beta}{q_\beta + Q_\beta},$$

$$\frac{dQ_\beta}{dt} = -\beta \frac{GM}{c} q_\beta \frac{1}{2Q_\beta} \frac{5q_\beta + 3Q_\beta}{q_\beta + Q_\beta},$$

(76)
\[
\frac{dQ_{\beta}}{dt} = -\beta \frac{GM}{c} \frac{Q_{\beta}}{2q_{\beta}} \frac{1}{\sqrt{q_{\beta}Q_{\beta}}} \frac{3q_{\beta} + 5Q_{\beta}}{q_{\beta} + Q_{\beta}},
\]
\[
\left( \frac{dq_{\beta}}{dt} \right) / \left( \frac{dQ_{\beta}}{dt} \right) = \left( \frac{q_{\beta}}{Q_{\beta}} \right)^2 \left( 1 - \frac{2}{3q_{\beta} + 5Q_{\beta}} \right) < 1. \] 

For completeness we have to introduce initial conditions
\[
q_{\beta in} = \frac{p_{\beta in}}{1 + e_{\beta in}}, \quad Q_{\beta in} = \frac{p_{\beta in}}{1 - e_{\beta in}},
\]
where \( p_{\beta in} \) and \( e_{\beta in} \) are given by Eqs. (66).

It is worth mentioning that instantaneous time derivatives of pericentric and apocentric distances defined by central acceleration \(-GMr/r^3\) may be both positive and negative, while secular evolution yields that \(q\) and \(Q\) are decreasing functions of time. As for the central acceleration \(-GM(1-\beta)r/r^3\), the instantaneous time derivatives of pericentric and apocentric distances are both less than or equal zero (secular evolution yields that \(q_{\beta}\) and \(Q_{\beta}\) are decreasing functions of time – see Eqs. 76).

The time of spiralling of the particle into the star is
\[
T[\text{yrs}] = \frac{6.4 \times 10^2}{\beta} \frac{M_{\text{sun}}}{M} \frac{(a_{\beta in}[\text{AU}])^2}{e_{\beta in}^{8/5}} \frac{1 - e_{\beta in}^2}{e_{\beta in}^{8/5}} \frac{1}{(1 - z^2)^{3/2}} \int_0^{e_{\beta in}} \frac{z^{3/5}}{dz}. \] 

(see also Eq. 51).

Figs. 1 and 2 depict the behaviour of Eqs.(62). The dashed lines correspond to the dependences \((1 - e_{\beta}) / [1 - e_{\beta} + \beta(1 + e_{\beta})]\) and \((1 + e_{\beta}) / [1 + e_{\beta} + \beta(1 - e_{\beta})]\) for the ratio of instantaneous values of semi-major axes and \([1 - \beta)e_{\beta} - \beta\) and \((1 - \beta)e_{\beta} + \beta\) for instantaneous values of eccentricities corresponding to the central acceleration \(-GMr/r^3\) (see Eqs. 96-97 in Klačka 2004). Time evolution of semi-major axes and eccentricities is presented in Figs. 3 and 4. The time of spiralling toward the central star is given by Eq. (78). While the values of \(a\), \(a_{\beta}\) and \(e_{\beta}\) approach to zero, the value of \(e\) tends to the value \(\beta\) (see Eqs. 62-63).

9. Conclusion

We have presented physical comments to “explanations” of the Poynting-Robertson effect published during the past thirty years by almost hundred of scientists working in the field. We have used mainly scientific papers and monographs. Although we have only restricted access to literature, we hope that physics of other published presentations can be covered by our paper. Moreover, we hope that our paper, together with papers by Klačka (1992a, 1993b, 2004, 2008a, 2008b), can help in better understanding the dynamical interaction of electromagnetic radiation with a moving body.
Moreover, we have added two simple explanations of the P-R effect (see Sec. 7). They are not only simple, but they concentrate the physical essence of the phenomenon. These simple explanations (or their modifications) may be used in scientific literature, encyclopaediae and textbooks.

The confusion in the scientific literature about the term called the "Poynting-Robertson drag" (see, e.g., Dohnanyi 1978, Gustafson 1994, Minato et al. 2004, Wyatt 2009, Mann 2009) can be easily removed if one takes into account that the relativistically covariant form of the P-R effect does not enable any division into partial (covariant) terms. The correct physics yields the P-R effect as a whole effect of the radiation force on a moving body and no "P-R drag" exists as a partial relativistically covariant equation of motion.

The difference between the effects of solar electromagnetic and corpuscular (solar wind) radiation is stressed (see Secs. 6.34, 6.35).

Sec. 8 presents secular evolution of orbital elements of a spherical particle if the action of gravity of a central star and it’s electromagnetic radiation is considered.

The physical understanding of the P-R effect enables correct derivation of the action of stellar winds on evolution of dust grains. This can help in physical explanation of the observed dust distribution at stars with planetary systems.

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References

Banaszkiewicz M., Fahr H. J., Scherer K., 1994. Evolution of dust particle orbits under the influence of solar wind outflow asymmetries and the formation of the zodiacal dust cloud. *Icarus* **107**, 358-374.

Bohren C. F., Huffman D. R., 1983. *Absorption and Scattering of Light by Small Particles*, John Wiley & Sons, Inc., New York.

Bottke W. F. Jr., Cellino A., Paolicchi P., Binzel R. P., 2002a. Asteroids III. In: Asteroids III, W. F. Bottke (Jr.), A. Cellino, P. Paolicchi, R. P. Binzel (eds.), The University of Arizona Press, Tucson, in collaboration with Lunar and Planetary Institute, Houston, 785 pp.

Bottke W. F. Jr., Cellino A., Paolicchi P., Binzel R. P., 2002b. An Overview of the Asteroids: The Asteroids III Perspective. In: Asteroids III, W. F. Bottke (Jr.), A. Cellino, P. Paolicchi, R. P. Binzel (eds.), The University of Arizona Press, Tucson, in collaboration with Lunar and Planetary Institute, Houston, 3-15.

Bottke W. F. Jr., Vokrouhlický D., Rubincam D. P., Brož M., 2002c. The Effect of Yarkovsky Thermal Forces on the Dynamical Evolution of Asteroids and Meteorites. In: Asteroids III, W. F. Bottke (Jr.), A. Cellino,
P. Paolicchi, R. P. Binzel (eds.), The University of Arizona Press, Tucson, in collaboration with Lunar and Planetary Institute, Houston, 395-408.

Breiter S., Jackson A. A., 1998. Unified analytical solutions to two-body problems with drag. *Mon. Not. R. Astron. Soc.* **299**, 237-243.

Burns J. A., Lamy P. L., Soter S., 1979. Radiation forces on small particles in the Solar System. *Icarus* **40**, 1-48.

Carroll B. W., Ostlie D. A., 2007. An Introduction to Modern Astrophysics, Pearson Education, Inc., Publishing as Addison-Wesley, San Francisco, 2nd ed., 1278 pp.

Considine D. M., 1995. Van Nostrand’s Scientific Encyclopedia, Van Nostrand Reinhold – A division of International Thomson Publishing Inc., New York, 8th ed., 3455 pp.

Danby J. M. A., 2003. Fundamentals of Celestial Mechanics. Willmann-Bell, Inc., Richmond, 2nd ed., 483 pp.

Dermott S. F., Jayraman S., Xu Y. L., Gustafson B. A. S., Liou J. C., 1994. A circumsolar ring of asteroidal dust in resonant lock with the Earth. *Nature* **369**, 719-723.

Dermott S. F., Grogan K., Durda D. D., Jayaraman S., Kehoe T. J. J., Kortenkamp S. J., Wyatt M. C., 2001. Orbital Evolution of Interplanetary Dust. In: Interplanetary Dust. E. Grün, B. A. S. Gustafson, S. F. Dermott, and H. Fechtig (eds.), Springer-Verlag, Berlin, 569-639.

Dohnanyi J. S., 1978. Particle dynamics. In: Cosmic Dust, J. A. M. McDonnell (ed.), Wiley-Interscience, Chichester, 527-605.

Encrenaz T., Bibring J.-P., Blanc M., Barucci M.-A., Roques F., Zarka Ph., 2004. The Solar System. Springer-Verlag, Berlin, 512 pp.

Feynman R. P., Leighton R. B., Sands M., 2006. The Feynman Lectures on Physics (The Definitive Edition). Pearson Addison-Wesley, San Francisco.

Freund J., 2008. Special Relativity for Beginners. World Scientific Publishing Co., Pte. Ltd., Singapore, 314 pp.

Gajdošik M., Klačka J., 1999. Cometary dust trails and ejection velocities. In: Evolution and Source Regions of Asteroids and Comets, Proc. IAU Coll. 173, J. Svoreň, E. M. Pittich, and H. Rickman (eds.), Astron. Inst. Slovak Acad. Sci., Tatranská Lomnica, pp. 239-242.

Gustafson, B. A. S., 1994. Physics of Zodiacal Dust. *Annual Review of Earth and Planetary Sciences* **22**, 553-595.

Grün E., 2007. Solar System dust. In: Encyclopedia of the Solar System, L.-A. McFadden, P. R. Weissmann and T. V. Johnson (eds.), Academic Press (Elsevier), San Diego, 2nd ed., 621-636.

Grün E., Baguhl M., Svedhem H., Zook H. A., 2001a. In Situ Measurements of Cosmic Dust. In: Interplanetary Dust. E. Grün, B. A. S. Gustafson, S. F. Dermott, and H. Fechtig (eds.), Springer-Verlag, Berlin, 295-346.

Grün E., Gustafson B. A. S., Dermott S. F., Fechtig H., 2001b. Glossary. In: Interplanetary Dust. E. Grün, B. A. S. Gustafson, S. F. Dermott, and H. Fechtig (eds.), Springer-Verlag, Berlin, 787-792.

Harwit M., 1963. Origins of the Zodiacal cloud. *J. Geophys. Res.* **86**, 2171-2180.

Harwit M., 1973. Astrophysical Concepts. Springer, New York, 1st ed., 561 pp.
Harwit M., 1988. Astrophysical Concepts. Springer-Verlag, New York-Berlin-Heidelberg, 2-nd ed., 631 pp.
Harwit M., 2006. Astrophysical Concepts. Springer Science+Business Media, LLC, New York, 4th ed., 714 pp.
Holmes E. K., Dermott S. F., Gustafson B. A. S., Grogan K., 2003. Resonant structure in the Kuiper disk: An asymmetric Plutino disk. *Astrophys. J.* 597, 1211-1236.
Jackson A. A., Zook H. A., 1989. A Solar System dust ring with the Earth as its shepherd. *Nature* 337, 629-631.
Kapiński I., 1984. Nongravitational effects affecting small meteoroids in interplanetary space. *Contr. Astron. Obs. Skalnaté Pleso* 12, 99-111.
Klačka J., 1992a. Poynting-Robertson effect. I. Equation of motion. *Earth, Moon, and Planets* 59, 41-59.
Klačka J., 1992b. Poynting-Robertson effect. II. Perturbation equations. *Earth, Moon, and Planets* 59, 211-218.
Klačka J., 1993a. Interplanetary dust particles: disintegration and orbital motion. *Earth, Moon, and Planets* 60, 17-21.
Klačka J., 1993b. Misunderstanding of the Poynting-Robertson effect. *Earth, Moon, and Planets* 63, 255-258.
Klačka J., 1993c. Poynting-Robertson effect and orbital motion. *Earth, Moon, and Planets* 61, 57-62.
Klačka J., 1994. Interplanetary dust particles and solar radiation. *Earth, Moon, and Planets* 64, 125-132.
Klačka J., 2000a. Electromagnetic radiation and motion of real particle. 
Klačka J., 2000b. On physical interpretation of the Poynting-Robertson effect. arXiv:astro-ph/0006426
Klačka J., 2001a. On the Poynting-Robertson effect and analytical solutions. In: Dynamics of Natural and Artificial Celestial Bodies, H. Pretka-Ziomek, E. Wnuk, P. K. Seidelmann and D. Richardson (eds.), Kluwer Academic Publishers, Dordrecht, 215-217 (astro-ph/0004181).
Klačka J., 2001b. On physics of the Poynting-Robertson effect. arXiv:astro-ph/0108210
Klačka J., 2004. Electromagnetic radiation and motion of a particle, *Celestial Mech. and Dynam. Astron.* 89, 1-61.
Klačka J., 2008a. Mie, Einstein and the Poynting-Robertson effect. arXiv: astro-ph/0807.2795.
Klačka J., 2008b. Electromagnetic radiation, motion of a particle and energy-mass relation. arXiv: astro-ph/0807.2915.
Klačka J., Kaufmannová J., 1992. Poynting-Robertson effect: ’circular’ orbit. *Earth, Moon, and Planets* 59, 97-102.
Klačka J., Kaufmannová J., 1993. Poynting-Robertson effect and small eccentric orbits. *Earth, Moon, and Planets* 63, 271-274.
Klačka J., Saniga M., 1993. Interplanetary dust particles and solar wind. *Earth, Moon, and Planets* 60, 23-29.
Klačka J., Kocifaj M., 2007. Scattering of electromagnetic waves by charged spheres and some physical consequences. *J. Quant. Spectrosc. Radiat. Transfer* 106, 170-183.
Klačka J., Kocifaj M., Pástor P., Petržala J., 2007. Poynting-Robertson effect and perihelion motion. *Astron. Astrophys.* 464, 127-134.
Klačka J., Kómar L., Pástor P., Petřžala J., 2008. The non-radial component of the solar wind and motion of
dust near mean motion resonances with planets. *Astron. Astrophys.* **489**, 787-793.
Kocifaj M., Klačka J., 2008. Dynamics of dust grains with a vaporable icy mantle. *Mon. Not. R. Astron. Soc.*
**391**, 1771-1777.
Köhler M., Mann I., 2002. Model calculations of dynamical forces and effects on dust in circumstellar debris
disks. In: Proceedings of Asteroids, Comets, Meteors (ACM 2002), European Space Agency (ESA-SP-500),
ESTEC, Noordwijk, 771-774 pp.
Krauss O., Wurm G., 2004. Radiation pressure forces on individual micron-size dust particles: a new experi-
mental approach. *J. Quant. Spectroscopy & Radiative Transfer* **89**, 179-189.
Krügel E., 2008. An introduction to the physics of interstellar dust. Taylor & Francis, Boca Raton, 387 pp.
Lauretta D. S., McSween H. Y. Jr., 2006. Meteorites and the Early Solar System II. In: Meteorites and the
Early Solar System II, D. S. Lauretta and H. Y. McSween (Jr.) (eds.), The University of Arizona Press,
Tucson, in collaboration with Lunar and Planetary Institute, Houston, 785 pp.
Leinert Ch., Grün E., 1990. Interplanetary dust. In: Physics of the Inner Heliosphere I, R. Schwen and E.
Marsch (eds.), Springer-Verlag, Berlin, 207-275.
Liou J.-C., Kaufmann D. E., 2008. Structure of the Kuiper belt dust disk. In: The Solar System Beyond
Neptune. M. A. Barucci, H. Boehnhardt, D. P. Cruikshank, A. Morbidelli (eds.), The University of Arizona
Press, Tucson in collaboration with Lunar and Planetary Institute, Houston, 592 pp.
Liou J.-Ch., Zook H. A., 1997. Evolution of interplanetary dust particles in mean motion resonances with
planets. *Icarus* **128**, 354-367.
Mann I., 2009. Evolution of dust and small bodies: physical processes. In: Small Bodies in Planetary Systems,
I. Mann, A. M. Nakamura and T. Mukai (eds.), Springer-Verlag, Berlin, 189-230.
Matzner R. A., 2001. Dictionary of Geophysics, Astrophysics and Astronomy. CRC Press, Boca Raton, 524 pp.
McDonnell T., McBride N., Green S. F., Ratcliff P. R., Gardner D. J., Griffiths A. D., 2001. Near Earth
Environment. In: Interplanetary Dust. E. Grün, B. A. S. Gustafson, S. F. Dermott, and H. Fechtig (eds.),
Springer-Verlag, Berlin, 163-231.
Meyer M. R., Backman D. E., Weinberger A. J., Wyatt M. C., 2007. Evolution of Circumstellar Disks Around
Normal Stars: Placing Our Solar System in Context. In: Protostars and Planets V, B. Reipurth, D. Jewitt and
K. Keil (eds.), The University of Arizona Press, Tucson, in collaboration with Lunar and Planetary
Institute, Houston, 573-588.
Mie G., 1908. Beiträge zur Optik trüber Medien speziell kolloidaler Metalösungen. *Ann. Phys.* **25**, 377-445.
Mignard F., 1982. Radiation pressure and dust particle dynamics. *Icarus* **49**, 347-366.
Mignard F., 1992. On the radiation forces. In: *Interrelations Between Physics and Dynamics for Minor Bodies
in the Solar System*, D. Benest and C. Froeschlé (eds.), Frontières, pp. 419-451.
Minato T., Köhler M., Kimura H., Mann I., Yamamoto T., 2004. Momentum transfer to interplanetary dust
from the solar wind. *Astron. Astrophys.* **424**, L13-L16.
Mishchenko M., 2001. Radiation force caused by scattering, absorption, and emission of light by nonspherical particles. *J. Quant. Spectrosc. Radiat. Transfer* **70**, 811-816.

Moore P. (ed.), 2002. Astronomy Encyclopedia. Philip’s, London, 465 pp.

Mould R. A., 2002. Basic Relativity. Springer-Verlag, New York.

Mukai T., Yamamoto T., 1982. Solar wind pressure on interplanetary dust. *Astron. Astrophys.* **107**, 97-100.

Murdin P. (ed.), 2001. Encyclopedia of Astronomy and Astrophysics, volume 3: O - Sol, IOP Publishing Ltd and Nature Publishing Group, Bristol, 3670pp.

Murray C. D., Dermott S. F., 1999. Solar System Dynamics. Cambridge University Press, Cambridge, 592 pp.

Parker S. P. (ed.), 2003. Dictionary of Astronomy, McGraw-Hill, New York, 2nd ed., 180pp.

de Pater I., Lissauer J. J.: 2006, Planetary Sciences. Cambridge University Press, Cambridge, 528 pp.

Poynting J. M., 1903. Radiation in the Solar System: its Effect on Temperature and its Pressure on Small Bodies. *Philosophical Transactions of the Royal Society of London Series A* **202**, 525-552.

Quinn, T., 2005. Planet Formation. In: Chaos and Stability in Planetary Systems. R. Dvorak, F. Freistetter, J. Kurths (eds.), Springer-Verlag, Berlin, 187-217.

Reach W. T., Franz B. A., Welland J. L., Hauser M. G., Kelsall T. N., Wright E. L., Rawley G., Stemwedel S. W., Splesman W. J., 1995. Observational confirmation of a circumsolar dust ring by the COBE satellite. *Nature* **374**, 521-523.

Robertson H. P., 1937. Dynamical effects of radiation in the Solar System. *Mon. Not. R. Astron. Soc.* **97**, 423-438.

Robertson H. P., Noonan T. W., 1968. Relativity and Cosmology. Saunders, Philadelphia, 456 pp.

Shive J. N., Weber R. L., 1982. Similarities in Physics. Adam Hilger Ltd., Bristol, 277 pp.

Srikanth R., 1999. Physical interpretation of the Poynting-Robertson effect. *Icarus* **140**, 231-234.

Svalgaard, L., 1977. Solar wind and interplanetary space. In: Illustrated Glossary for Solar and Solar-Terrestrial Physics. A. Bruzek and C. J. Durrant (eds.), Reidel, Dordrecht, 191-201 (Slovak edition).

Sykes M. V., 2007. Infrared Views of the Solar System from Space. In: Encyclopedia of the Solar System, L.-A. McFadden, P. R. Weissmann and T. V. Johnson (eds.), Academic Press (Elsevier), San Diego, 2nd ed., 681-694.

Weisskopf V. F., 1973. Foreword to: Pauli W., Wave Mechanics. MIT Press, Cambridge, Massachusetts.

Woolfson M., 2000. The Origin and Evolution of the Solar System. Institute of Physics Publishing, Bristol, 420 pp.

Wyatt S. P., Whipple F. L., 1950. The Poynting-Robertson effect on meteor orbits. *Astrophys. J.* **111**, 134-141.

Wyatt M. C., 2009. Dynamics of small bodies in planetary systems. In: Small Bodies in Planetary Systems. I. Mann, A. M.Nakamura and T. Mukai (eds.), Springer-Verlag, Berlin, 37-70.

Zimmermann H., Gürtler J.: 2008, ABC Astronomie. Spektrum Akademischer Verlag, Heidelberg, 9. Auflage, 503pp.