Magnetic compensation of the gravity by using superconducting axisymmetric coils: spherical harmonics method

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Abstract. An important work for space research is to study in weightless conditions (microgravity) the behavior of fluids such as liquid oxygen and liquid hydrogen. In addition, some magnetic ground-based stations allow to compensate the gravity and to meet space conditions since 1991. The magnetic devices in order to simulate micro-gravity allow low-cost experiments with unlimited time. The issue of the present techniques is to reach the same or better conditions (residual acceleration of the studied fluid) than the ones during the parabolic flights. In this paper, several specific distributions of the magnetic field are determined. These distributions allow to compensate the gravity by means of axisymmetric coils (solenoids). This paper introduces several distributions of the residual forces useful for different kinds of micro-gravity experiments.

1. Introduction

The magnetic field exerts a force density proportional to \( \vec{V} \vec{B}^2 = \vec{G} \), on weakly magnetic materials (dia- and para-magnetic materials) in vacuum, expressed as:

\[
\frac{df}{dV} = \frac{1}{2\mu_0} \chi_m \vec{V} \vec{B}^2
\]

where \( \frac{df}{dV} \) is the magnetic force density (N/m\(^3\)), \( \mu_0 \) the vacuum permeability (H/m), \( \chi_m \) the magnetic susceptibility (dimensionless), \( \vec{B} \) the magnetic flux density (T).

A magnetic force density cannot be constant in a 3D domain, thus a perfect compensation of gravity is unreachable in a 3D zone of the space only in using magnetic fields. This theorem has been demonstrated in previous works.

The relative error between the perfect compensation \( \vec{G}_i \) and the effective compensation \( \vec{G} \) at the considered point is defined by the inhomogeneity vector \( \vec{\varepsilon} \) :

\[
\vec{\varepsilon} = \frac{\vec{G} - \vec{G}_i}{\vec{G}_i}
\]
\[ g = \frac{2\mu_0 \rho}{\chi_m} \]  

(3)

where \( g = 9.81 \text{ m.s}^{-2} \) the terrestrial acceleration, and \( \rho \) the density (kg/m³).

In this paper, a method for determining the \( \vec{G} \) field is developed, this method is all interesting for the axisymmetric geometry (solenoidal coils). The method has been first used by Garrett⁷,⁸ in order to obtain very uniform fields (NMR coils) within solenoidal systems. This method starts with a harmonic decomposition of the scalar magnetic potential \( V \) in the useful zone assumed without currents (resolution of the Laplace’s equation). This suggests the spherical harmonic decomposition of the magnetic field. In this paper the value of \( \vec{G} \), according to these field harmonics, is calculated.

The configurations of the inhomogeneity vector \( \vec{e} \), depending on the desired conditions of microgravity, can be expressed by the cancellation of some derivatives of the vector \( \vec{G} \). These conditions directly are transcribed on the field harmonics. To conclude the determination of the corresponding field sources is obtained by resolution of the inverse problem of the magneto-static, leading to determine the spatial harmonic of the currents providing the desired fields.

2. Calculating method

2.1. Definition of the geometry

The spherical coordinates can be reduced to the \((r, \theta)\) couple for a axisymmetric system. The symmetry axis is defined by \( \theta = 0 \) (figure 1). The sense of the \( x \)-axis is chosen opposite to the gravity vector \( \vec{g} \):

![Spherical coordinates and geometry of the system](image)

**Figure 1.** Spherical coordinates and geometry of the system

2.2. Vector \( \vec{G} \) at any point in the sphere

The distributions of the field in a \( R_0 \) radius sphere, centred on the \( O \) and named working zone, are studied. The axisymmetric current sources are assumed to be either on the surface or outside of the working zone. An infinity of current distributions can create the same magnetic field distribution within the working zone. In order to calculate the exact solutions of the magnetic field at any point of the space, the current sources are, here, assumed to be made up of a surface current layer on the \( R_0 \) radius sphere. One assumes the norm of the magnetic field at the centre equal to \( B_1 \):

\[ |B(0, \theta)| = B_1 \]

Previous works⁴ demonstrated the more homogeneous the compensation, the higher the magnetic field is. This suggests the use of superconducting coils with very high field. At the centre of the working zone \((r=0)\), the value \( G_1 \) of the norm of \( \vec{G} \) perfectly counterbalances the gravity for the considered fluid⁴:

\[ |\vec{G}(0, \theta)| = G_1 \]

The resolution of the Laplace’s equation for the magnetic scalar potential, in spherical coordinates, leads to the components of the magnetic field. The boundary conditions provide two different
solutions, a first one inner to the sphere and a second one outer to the sphere. The inner and outer magnetic fields can be expressed by the polynomials of Legendre $P_n$. The components of the $n$-th field harmonics, along the vectors $\hat{e}_r$ and $\hat{e}_\theta$, are:

$$H_{\text{int}} = \begin{cases} -n.C_n.r^{-n+1}.P_n(\cos\theta) & r<R_0 \\ -n.C_n.r^{-n+1}.P_n(\cos\theta) & r>R_0 \end{cases}$$

$$H_{\text{ext}} = \begin{cases} (n+1)\frac{C_n}{r^{n+2}}.P_n(\cos\theta) & r>R_0 \\ -\frac{C_n}{r^{n+2}}.P_n(\cos\theta) \end{cases}$$

The $C_n$ coefficients will be calculated from the homogeneity conditions of the inner quantities. The continuity conditions allow to determine $C'_n$ and the surface current density. The magnetic field expression (4) within the working zone can be obtained by an infinity of current distributions, but the two relations (4) and (5) together are true only for surface current density on the $R_0$ radius sphere. The relation (4) involves:

$$B_{\text{int}}^2 = \mu_0^2 \sum_{n=1}^{\infty} n.C_n.r^{-n+1}.P_n(\cos\theta) \sum_{p=1}^{\infty} p.C_p.r^{p-1}.P_p(\cos\theta) + \mu_0^2 \sum_{n=1}^{\infty} C_n.r^{-n+1}.P_n(\cos\theta) \sum_{p=1}^{\infty} C_p.r^{p-1}.P_p(\cos\theta)$$

The vector $\vec{G}$ derives from this quantity:

$$G(r,\theta) = 2\mu_0^2 \begin{bmatrix} \sum_{n=1}^{\infty} n.C_n.r^{-n+1}.P_n(\cos\theta) \sum_{p=1}^{\infty} p.(p-1).C_p.r^{p-2}.P_p(\cos\theta) & \sum_{n=1}^{\infty} n.C_n.r^{-n+1}.P_n(\cos\theta) \sum_{p=1}^{\infty} (p-1).C_p.r^{p-2}.P_p(\cos\theta) \\ \sum_{n=1}^{\infty} n.C_n.r^{-n+1}.P_n(\cos\theta) \sum_{p=1}^{\infty} (p-1).C_p.r^{p-2}.P_p(\cos\theta) & \sum_{n=1}^{\infty} n.C_n.r^{-n+1}.P_n(\cos\theta) \sum_{p=1}^{\infty} p.C_p.r^{p-2}.P_p(\cos\theta) \end{bmatrix}$$

Any distribution of magnetic forces in a spherical free space cavity (without current) can be expressed by the expression (7), namely by the $C_n$ coefficients.

2.3. Magneto-gravitarian potential
The inhomogeneity vector derives from a « magneto-gravitarian » potential $\Sigma_L$ (in meter) defined from the expression (2):

$$\Sigma_L = \frac{B^2}{G_1} - z$$

where $B$ is the magnetic flux density (T), $G_1$ the norm of the gradient allowing the levitation of the considered material ($T^2/m$), and $z$ the height (m). If a static fluid, near its critical point, that is to say with a surface tension close to zero, is only subjected to the gravity and the magnetic force, then its free surface must be given by the equipotentials « iso$\Sigma_L$ ». The new working out of the problem, dealing with the magnetic compensation of gravity, by the potential $\Sigma_L$ provides interesting results whom an example is given thereafter.

2.4. Choice of the homogeneity conditions and residual forces
The conditions on the homogeneities define the values of the field harmonics $(C_i)_{i=N}$ by the relations on the $n$-th derivatives of the vector $\vec{G}$. Three distinct conditions are examined. Each one describes a specific inhomogeneity (resulting acceleration sketched on the figure 2) and leads to interesting
experimental conditions of micro-gravity. The residual acceleration vector is orthoaxial, that is to say
central in a plane perpendicular to the symmetry axis (figure 2a), or orthogonal to the yOz plane
(figure 2b), or central (figure 2c). The $\Sigma_1$ equipotentials are respectively cylinders centred on the Oz
axis, or perpendicular plane to the Oz axis, or spheres centred at O.

figure 2. Distributions of the residual accelerations

- In the orthoaxial case, $\varepsilon(r,0)$ tends towards zero, and involves :

$$\forall n \in \mathbb{N}, \quad \frac{\partial^n G(r,0)}{\partial r^n} \bigg|_{r=0} = 0$$

(9)

- In the orthogonal case, $\varepsilon(r,\frac{\pi}{2})$ and $\varepsilon(r, \frac{\pi}{2})$ tend towards zero :

$$\forall n \in \mathbb{N}, \quad \frac{\partial^n G(r, \frac{\pi}{2})}{\partial r^n} \bigg|_{r=0} = 0$$

(10)

- In the central case, $\varepsilon(r, \frac{\pi}{2})$ and $\varepsilon(r,0)$ must be equal :

$$\forall n \in \mathbb{N}, \quad \frac{\partial^n G(r,0)}{\partial r^n} \bigg|_{r=0} = \frac{\partial^n G(r, \frac{\pi}{2})}{\partial r^n} \bigg|_{r=0}$$

(11)

The equations (8), (9) and (10) allow to calculate the values of the coefficients of the field harmonics
$(C_i)_{i=1}$, these values are given in the table 1.

Table 1. Values of the first six harmonics of the field for the three configurations

|        | $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  | $C_6$  |
|--------|--------|--------|--------|--------|--------|--------|
| Orthoaxial | $\frac{B_i}{\mu_0}$ | $\frac{G_i}{4\mu_0 B_i}$ | $-\frac{G_i^2}{24\mu_0 B_i^3}$ | $\frac{G_i^3}{64\mu_0 B_i^5}$ | $-\frac{G_i^4}{128\mu_0 B_i^7}$ | $\frac{7G_i^5}{1536\mu_0 B_i^9}$ |
| Orthogonal | $\frac{B_i}{\mu_0}$ | $\frac{G_i}{4\mu_0 B_i}$ | $\frac{G_i^2}{48\mu_0 B_i^3}$ | $0$ | $-\frac{G_i^4}{3840\mu_0 B_i^7}$ | $-\frac{G_i^5}{46080\mu_0 B_i^9}$ |
| Central | $\frac{B_i}{\mu_0}$ | $\frac{G_i}{4\mu_0 B_i}$ | $-\frac{G_i^2}{48\mu_0 B_i^3}$ | $\frac{G_i^3}{128\mu_0 B_i^5}$ | $-\frac{4G_i^4}{6400\mu_0 B_i^7}$ | $\frac{23G_i^5}{7680\mu_0 B_i^9}$ |

One notices that the two first harmonics are the same in any case. If the harmonic orders higher than
two are zero, a fourth configuration appears where the magneto-gravitarian equipotentials are
spheroids with an eccentricity equal to two.

2.5. Calculation of the surface current density
The determination of the currents from the established distributions of the field is a inverse problem in
magnetism, with an infinity of solutions. The theoretical easiest solution is the surface current
distribution on a $R_0$ radius sphere. In this case, the coefficients of the relations (4) and (5) are linked as follow:

$$C_n' = -\frac{n}{n+1} R_0^{2n+1} C_n$$  \hspace{1cm} (12)

According to the relations (4), (5), (12) and the equality between the tangential components of the field and the currents, the surface current density harmonics are expressed as:

$$K_n = \frac{2n+1}{n+1} R_0^{n-1} C_n R_0^l (\cos \theta) e_\theta$$  \hspace{1cm} (13)

3. Results of the numerical simulation

The numerical simulation of the different configurations of micro-gravity is possible in choosing the sources of current given by the relation (13), with the values of the coefficients $C_n$ of the table 2. In our simulation, the surface current density is arbitrarily truncated to the sixth harmonic.

The simulations are carried out on liquid oxygen at 90K, with a gradient $G_1=8 \text{T/m}$ and a magnetic field at the origin of $B_1=10 \text{T}$. The surface current density, previously obtained, is spread over a 0.5 meter radius sphere.

The figures of the first line of the table 2 are obtained with a finite elements software. Inside a 0.4 meter sphere, the black arrows represent the vector $\vec{e}$, in fact the residual acceleration. The norm of the inhomogeneity (in %) is provided by the colour bar on the right of each figure. The bluish lines are the iso$\Sigma_l$ (in meter). These iso$\Sigma_l$ also are plotted in the second line of the table 2, according to a analytical calculation taking into account only the first six harmonics. In each figure of the second line is drawn a 0.4 radius circle because the scale are not normed.

**Table 2.** Representation of the three configurations with only the first six harmonics

| Orthoaxial | Orthogonal | Central |
|------------|------------|---------|
| ![Orthoaxial](image) | ![Orthogonal](image) | ![Central](image) |
One notices, according to the tables (1) and (2), that the third harmonic seems to fix the anisotropy of the residual acceleration. The first three harmonics respectively make it possible to fix, the magnetic field and the gradient, at the centre, and the resulting acceleration vector.

4. Conclusion
The method, briefly described in this paper, uses, in part, previous works of our team\(^6\). This method provides precious elements in order to elaborate magnetic levitation device. The practical design of the superconducting coils needed to create the desired magnetic fields is not introduced in this paper. It will be the subject of further works. The design of these devices uses the same method as the one for the superconducting coils of the MNR machines.

This method allows to introduce the useful concept of magneto-gravitarian potential describing the residual forces at the moment of the magnetic compensation of the gravity. The distributions of the resulting forces reachable by this method can be various. A wise choice of the harmonic coefficients previously defined allows to adapt the distributions. The various choice of the coefficients allows to carry out a wide range of experiments in the ground-based simulation station of micro-gravity. The three examples introduced above lead to interesting kind of experiments, for the study of fluids or granular matter in space conditions.

This general method, succinctly developed here, can be adapted for other levitation devices, in cylindrical geometry\(^4\) for example. In this case, these devices could be built from multipoles as for particle accelerators, mainly dipoles, quadrupoles and sextupoles.

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References
[1] Wunenberg R, Chatain D, Garrabos Y and Beysens D 2000 Magnetic compensation of gravity forces in (-p) hydrogen near critical point: Applications to weightless conditions Physical Review E 62, pp 469-476
[2] Chatain D and Nikolayev V S 2002 Using magnetic levitation to produce cryogenic targets for inertial fusion energy: experiment and theory Cryogenics 42, pp 253-261
[3] Beaugnon E and Tournier R 1991 Levitation of organic materials Nature 349, pp 470
[4] Lorin C and Mailfert A 2007 Magnetic compensation of gravity, and centrifugal forces ELGRA 2007 conference, Florencia submitted to Microgravity Science and Technology
[5] Quettier L, Félice H, Mailfert A, Chatain D and Beysens D 2005 Magnetic compensation of gravity forces in liquid/gas mixtures: surpassing intrinsic limitations of a superconducting magnet by using ferromagnetic inserts The European Physical Journal Applied Physics 32, pp 167-175
[6] Quettier L 2003 Contribution méthodologique à la conception de systèmes supraconducteurs de lévitation magnétique Thesis INPL
[7] Garett M W 1967 Thick cylindrical coil systems for strong magnetic fields with field or gradient homogeneities of the 6\(^{th}\) to 20\(^{th}\) order Journal of Applied Physics 38, 6, pp 2563-2586
[8] Garett M W 1951 Axially symmetric systems for generating and measuring magnetic fields Journal of Applied Physics 22, 9, pp 1091-1107