Nernst effect as a probe of superconducting fluctuations in disordered thin films

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New Journal of Physics 11 (2009) 055071 (18pp)
Received 12 February 2009
Published 29 May 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/5/055071

Abstract. In amorphous superconducting thin films of Nb\textsubscript{0.15}Si\textsubscript{0.85} and InO\textsubscript{x}, a finite Nernst coefficient can be detected in a wide range of temperature and magnetic field. Due to the negligible contribution of normal quasi-particles, superconducting fluctuations easily dominate the Nernst response in the entire range of study. In the vicinity of the critical temperature and in the zero-field limit, the magnitude of the signal is in quantitative agreement with what is theoretically expected for the Gaussian fluctuations of the superconducting order parameter. Even at higher temperatures and finite magnetic field, the Nernst coefficient is set by the size of superconducting fluctuations. The Nernst coefficient emerges as a direct probe of the ghost critical field, the normal-state mirror of the upper critical field. Moreover, upon leaving the normal state with fluctuating Cooper pairs, we show that the temperature evolution of the Nernst coefficient is different depending on whether the system enters a vortex solid, a vortex liquid or a phase-fluctuating superconducting regime.

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1. Introduction

In recent years, the Nernst effect has emerged as an important probe of superconducting fluctuations, following the observation of an anomalous Nernst signal in the high temperature phase of underdoped cuprates [1]. Because of the small superfluid stiffness expected in underdoped cuprates [2], and because vortices are a well-known source of a Nernst response [3], these authors proposed the vortex-like excitations of a phase-disordered superconductor as a natural source of this Nernst signal [4].

This discovery motivated numerous experimental and theoretical works on the Nernst effect. On one hand, several studies on correlated metals of various families resolved an unexpectedly large Nernst coefficient (for a review see [5]). Observations of large Nernst responses have been reported for organic superconductors [6, 7], heavy fermion systems [8]–[11], as well as for the CDW superconductor NbSe$_2$ [12]. In some cases, a large Nernst signal could be observed even in total absence of superconductivity. The most illuminating example was bismuth, the semi-metallic element in which Nernst and Ettingshausen discovered in 1886 the effect which bears their name [13]. The Nernst coefficient in bismuth [14] is three orders of magnitude larger than what is typically seen in any type II superconductor. In fact, the large magnitude of the Nernst coefficient in bismuth is in agreement with the implications of the semiclassical transport theory [5, 15, 16] and therefore, a large Nernst signal does not necessarily imply superconducting fluctuations (either of phase or amplitude of the order parameter). In particular, see [17] for a recent interpretation of Nernst effect enhancement in a high-$T_c$ superconductor as the onset of stripe order.

On the other hand, this led to the first theoretical study of the Nernst response of fluctuating Cooper pairs [18] (see also [19, 20]). These fluctuations are usually described in the Gaussian approximation within the Ginzburg–Landau framework [21] and are known to give rise to the phenomenon of paraconductivity [22], i.e. an excess of conductance due to short lived Cooper pairs in the normal state, and to so-called fluctuation diamagnetism [23]. Theoretical calculations by Ussishkin, Sondhi and Huse (USH) [18] have shown that Cooper-pair fluctuations should also produce a sizable Nernst signal. As Cooper-pair fluctuations imply both amplitude and phase fluctuations of the superconducting order parameter (SOP), this work established that a finite Nernst signal is not necessarily restricted to the cases where there are only superconducting phase fluctuations.
This prediction was put to the test through measurements of the Nernst effect in amorphous thin films of low-$T_c$ superconductors. The normal state of these systems is a simple dirty metal with a totally negligible Nernst response. These last studies [24]–[26] demonstrated that the Nernst signal of amorphous superconducting films is exclusively generated by superconducting fluctuations, thus providing a remarkable test bed for theories. In quantitative agreement with USH theory close to $T_c$, these measurements established that conventional Gaussian fluctuations do indeed generate a Nernst signal.

Following this observation, we now need to learn how to distinguish other regimes of superconducting fluctuations from those simple Cooper-pair fluctuations, in particular, regimes with only thermal or quantum fluctuations of the phase of the SOP as expected in the underdoped cuprates, or in the vicinity of quantum superconductor–insulator transitions. Furthermore, in the presence of an applied magnetic field, we want to learn how to distinguish the regime of Cooper-pair fluctuations from the vortex fluid with long-lived vortices that exist in any type-II superconductor. Thus, one major ambition in the field is to identify the characteristic signatures of those different regimes of fluctuations in the Nernst data.

In this paper, we review our observation of the Nernst signal by Cooper-pair fluctuations and our identification of the ghost critical field (GCF) in the amorphous superconducting films Nb$_x$Si$_{1-x}$ [24, 25] and InO$_x$ [26]. Then, we describe the evolution of the Nernst signal within their superconducting phase diagrams, from the regime of Cooper-pair fluctuations to the vortex solid, across the vortex liquid. In finite magnetic field, a large increase in the Nernst signal is observed in the crossover from the regime of Cooper pair fluctuations to the vortex liquid phase, i.e. one non-superconducting dissipative state. In the zero magnetic field limit, where a true second-order transition takes place between the regime of Cooper-pair fluctuations and the dissipationless vortex solid, the Nernst coefficient diverges at the approach of the superconducting transition, i.e. following the diverging correlation length, and becomes zero in the vortex solid region. No abrupt increase of the Nernst signal due to vortices is observed as the temperature range for the existence of the vortex liquid shrinks to zero in the zero magnetic field limit.

The organization of this paper is as follows. Section 2 describes different regimes of superconducting fluctuations, whose existence in amorphous thin films or cuprates has been postulated. Section 3 reviews sample characteristics and the experimental set-up. Section 4 describes the Nernst signal generated by the vortex flow; section 5, the Nernst signal generated by Cooper-pair fluctuations. Section 6 describes the evolution of the Nernst coefficient across the transition from the regime of Cooper-pair fluctuations, i.e. normal state, to the vortex solid. Finally, we discuss the effect of thermal and quantum fluctuations of the SOP on the Nernst response of the vortex fluid.

2. Regimes of superconducting fluctuations

According to Bardeen–Cooper–Schrieffer (BCS) theory, cooling a superconductor below its superconducting transition temperature leads simultaneously to both the formation of Cooper pairs and their Bose condensation into a macroscopically coherent quantum state. However, several subjects of contemporary studies in superconductivity ask us to consider the possibility that Cooper pairs may exist without macroscopic phase coherence, mostly as a consequence of thermal or quantum fluctuations of the SOP [2], [27]–[29]. The magnitude of these fluctuations and their predominance in the phase diagram depends on materials parameters such as the
amount of random impurities, i.e. quenched disorder, dimensionality or correlation length value [27].

One such electronic phase is well known, found in many conventional and non-conventional superconductors, the vortex-liquid phase. This vortex fluid results from the melting of the vortex-solid above some characteristic magnetic field, $B_m$ [27, 28], as a consequence of thermal fluctuations of the phase of the SOP. This vortex fluid is separated from the normal state only by a crossover at the upper critical field $B_{c2}$, as shown on the phase diagram (figure 1(a)).

In high-temperature superconductors, a combination of high temperature, small correlation length, large magnetic penetration depth and quasi-two-dimensionality, conspire to increase the effects of thermal fluctuations and $B_m$ can be significantly smaller than the upper critical field $B_{c2}$.

In contrast, in bulk low-$T_c$ superconductors, $B_m$ almost coincides with $B_{c2}$. However, as the vortex lattice is unstable against the introduction of quenched disorder [30], i.e. random pinning sites, the superconducting phase diagram of amorphous thin films usually displays a large vortex liquid region.

As the effects of thermal fluctuations are enhanced, either by increasing disorder, reducing dimensionality, or reducing superfluid density, a phase-disordered vortex liquid state may survive in the limit of zero magnetic field [27, 28], giving rise to a phase diagram as shown in figure 1(b). In this diagram, in the zero magnetic field limit, a second characteristic temperature emerges for the establishment of superconductivity, where macroscopic coherence sets in.

One similar situation has been intensively studied theoretically in two dimensions by Berezinsky, Kosterlitz and Thouless (BKT) [29, 31]. They found that, in two dimensions and zero magnetic field, there exists a temperature, $T_{BKT}$, that corresponds to a transition between two distinct regimes of superconducting fluctuations where only the phase degree of freedom is altered by the transition. The low temperature state ($T < T_{BKT}$) is quasi-ordered.

Figure 1. Evolution of the phase diagram of a type-II superconductor as the effects of thermal fluctuations increase—panel (a) to panel (b)—and the effects of quantum fluctuations increase—panel (b) to panel (c). A second-order phase transition, i.e. with diverging correlation length, separates the vortex glass from the vortex liquid phase at $B_m$ (thick line). A crossover is only expected between the vortex liquid and the normal state, at $B_{c2}$ (dashed line).
with algebraically decaying correlation functions. The high temperature state \((T > T_{\text{BKT}})\) is phase-disordered due to thermally generated vortex–antivortex pairs that dissociate and populate the ground state. This leads to a phase-incoherent superconducting state with exponentially decaying correlation functions. Strict experimental realizations of this model for a charged superfluid are still lacking; however, some variations of it are being considered to apply in some part of the phase diagram of the cuprates and in the vicinity of the quantum superconductor–insulator transition observed in amorphous and granular superconducting thin films [32].

In cuprates, the observation of a pseudogap above \(T_c\), in the underdoped region of their phase diagram, was interpreted as a possible signature of two characteristic temperatures for superconductivity. The higher temperature, where the pseudogap forms in the electronic spectrum, may correspond to Cooper-pair formation, and the second, lower temperature, akin to \(T_{\text{BKT}}\), would correspond to the transition toward the phase-coherent superconducting state [33]. This regime of phase-only fluctuations is fundamentally different from the order parameter fluctuations as described in the context of Ginzburg–Landau theory [21]. In this last theory only one single critical temperature, \(T_c\), or critical magnetic field, \(B_c\), corresponding to the Cooper pair formation, is required to describe the fluctuations. Remarkably, within the Ginzburg–Landau framework, there is no upper temperature limit for the existence of these fluctuations; they are expected to survive far above \(T_c\) in the normal state. In contrast, the regime of phase-only fluctuations implies two distinct characteristic temperatures or magnetic fields: one higher temperature for Cooper pair formation and one lower temperature for the establishment of the phase coherence. Between these two temperatures, there exists a fluctuation regime characterized by long-lived, phase-incoherent, Cooper pairs and freely moving vortex–antivortex pairs. In the context of cuprate physics, Emery and Kivelson [2] extended the concept of phase-coherence temperature introduced by BKT. They suggested that, for any superconductor in any dimension, vortex–antivortex pairs should appear spontaneously when the thermal energy, \(k_B T\), is larger than the energy cost for their formation; this energy cost results from the kinetic energy associated with superfluid flow around the vortices. This defines a characteristic temperature for phase coherence, \(T_{\text{COH}}\), above which spontaneous nucleation of vortices is possible. In conventional superconductors, this coherence temperature largely exceeds \(T_{\text{BCS}}\), the Cooper pair forming temperature, and superconducting fluctuations exist only as fluctuations of both the amplitude and phase of the SOP. In contrast, for a low density superfluid, such as the underdoped cuprates, \(T_{\text{COH}} < T_{\text{BCS}}\). This implies that the temperature for the superconducting transition is controlled by the superfluid density. In the context of cuprate physics, this provides an explanation of the Uemura plot [34], where \(T_c\) is found to scale with the magnetic penetration depth which is inversely proportional to superfluid density.

Finally, in addition to quenched disorder and thermal fluctuations, quantum fluctuations of the SOP provide another origin for the quantum melting of the vortex solid. This leads to a phase diagram as shown in figure 1(c), where a quantum liquid of vortices is expected in the zero-temperature limit, separated from the superconducting state by a second-order transition whose critical behavior is controlled by quantum fluctuations [35]. Fine-tuning of the transition can be achieved either by applying a perpendicular magnetic field [36]–[45] or by varying the sheet resistance \(R_{\Box}\) of the films—using film thickness [46]–[48] or an electrostatic field [49].

The systems discussed in this paper are amorphous superconducting films for which distinct regimes of superconducting fluctuations are possible. Well above the mean field superconducting transition temperature \(T_c\), we expect the conventional Cooper-pair fluctuations.
Below $T_c$, different regimes may exist according to the amount of thermal or quantum phase fluctuations. One quantum origin is possible as quantum superconductor–insulator transitions have been observed in both systems [44, 45].

3. The compounds studied and the experimental technique used

In this paper, we present the evolution of the Nernst signal across the phase diagram of two different disordered superconductors, Nb$_{0.15}$Si$_{0.85}$ and InO$_x$.

The two amorphous thin films of Nb$_{0.15}$Si$_{0.85}$ used for this study were prepared by Dumoulin’s group. The samples are deposited by co-evaporation of Nb and Si in an ultra-high vacuum chamber, as described elsewhere [50, 51]. On the other hand, the 300 Å-thick amorphous InO$_x$ film was prepared by Z Ovadyahu’s group. The sample is deposited on a glass substrate by e-gun evaporation of In$_2$O$_3$ in an oxygen atmosphere [52]. The as-prepared film has an insulating-like behavior down to the lowest measured temperature of 60 mK. After thermal annealing at 50 °C under vacuum as described elsewhere [53], the room temperature sheet resistance decreases by about 30% and a superconducting state appears. During all measurements, the film has been kept below liquid nitrogen temperature to avoid aging effects.

Several characteristics of InO$_x$ indicate that effects of thermal or quantum fluctuations are stronger in this system than in Nb$_{0.15}$Si$_{0.85}$. While Nb$_{0.15}$Si$_{0.85}$ has a high carrier density $n = 8 \times 10^{22}$ cm$^{-3}$, comparable to any ordinary metal, the carrier density of InO$_x$ is 80 times smaller, $n = 10^{21}$ cm$^{-3}$, comparable to values found for the underdoped cuprates. According to an argument put forward by Emery and Kivelson [2], this low carrier density increases the probability for the spontaneous nucleation of vortices and so the amount of phase fluctuations. A second difference between both systems is the larger sheet resistance of InO$_x$, $R_{\square} \approx 4000 \Omega$, which implies enhanced quantum fluctuations with respect to Nb$_{0.15}$Si$_{0.85}$, $R_{\square} \approx 350 \Omega$. Finally, one last striking difference between both systems is the observation of a large negative magnetoresistance in InO$_x$. This phenomena has been interpreted as a possible indication of the pair-breaking effect of a magnetic field on localized Cooper pairs [42, 54, 55].

The Nernst effect is the transverse thermoelectric response, $N = E_y/\nabla_x T$, of a sample submitted to a thermal gradient and a magnetic field applied perpendicular to the sample plane. One usually defines the Nernst coefficient, $\nu = N/B$, and within linear response theory, one also defines the Peltier conductivity tensor:

$$\left(\begin{array}{c}
\mathbf{j}_e \\
\mathbf{j}_{th}
\end{array}\right) = \left(\begin{array}{cc}
\hat{\sigma} & \hat{\alpha} \\
\hat{\alpha} & \hat{\kappa}
\end{array}\right) \left(\begin{array}{c}
\mathbf{E} \\
\nabla T
\end{array}\right).$$

(1)

From the condition, $\mathbf{j}_e = 0$, one gets:

$$N = \frac{\sigma_{xx}\alpha_{xy} - \sigma_{xy}\alpha_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}.$$  

(2)

For all samples discussed, the Hall angle is small, and so is $\sigma_{xy}$. This leads to a simple relationship between the Nernst coefficient, $\nu$ and the Peltier coefficient, $\alpha_{xy}$.

$$\nu \approx \frac{\alpha_{xy}}{B\sigma_{xx}}.$$  

(3)

In our experimental set-up, the Nernst signal is measured using a one heater–two RuO$_2$ thermometers set-up. It allows measurements of diagonal and off-diagonal thermoelectric and
electric transport coefficients with the same contacts. At low temperature, \( T < 4.2 \text{ K} \), a dc voltage of 1 nV can be resolved and typical relative resolution of \( 10^{-3} \) on the magnitude of the temperature gradient can be achieved.

In superconductors, the two most important contributions expected are, below \( T_c \), the vortex contribution, \( N^S \), and above \( T_c \), the normal electrons contribution, \( N^n \). The measured Nernst signal is the sum of both contributions

\[
N = N^S + N^n. \tag{4}
\]

In the amorphous superconductors studied here, the Nernst signal due to normal quasi-particles is particularly low as this contribution scales with electron mobility \([5]\). This characteristic of amorphous superconductors is of utmost importance as it allows an unambiguous identification of the Nernst signal measured deep into the normal state with the contribution of superconducting fluctuations.

Part of the Nernst data presented here have been discussed previously, where we have shown that, in \( \text{Nb}_{0.15}\text{Si}_{0.85} \), Cooper-pair fluctuations could generate a Nernst signal up to very high temperature (\( 30 \times T_c \)) and high magnetic field (\( 4 \times B_c^2 \)) in the normal state \([24, 25]\).

4. Long-lived vortices and Nernst effect

Previous works on conventional superconductors \([3, 56]\) and cuprates \([4, 57]\) have shown that a large Nernst signal is generated by vortices as they are displaced by an applied heat current. This can be described phenomenologically by considering the forces exerted on the vortices. There is the force exerted by the thermal gradient, \( f = S_\phi (-\nabla T) \), where \( S_\phi \) is the entropy transported per vortex. Moving vortices with speed \( \mathbf{v} \) are also subject to the frictional force \( \mathbf{f}_f = \eta \mathbf{v} \), where the damping viscosity \( \eta \) may be inferred from the flux-flow resistivity, \( \rho = B \phi_0 / \eta \), where \( \phi_0 = h/2e \) is the superconducting flux quantum. In the steady state, when the frictional force balances the thermal force, the Nernst signal is given by:

\[
N = \frac{BS_\phi}{\eta} = \frac{\rho S_\phi}{\phi_0}. \tag{5}
\]

Figure 2 shows the temperature dependence of resistivity and Nernst coefficient data across the superconducting transition of one 35 nm thick film of \( \text{Nb}_{0.15}\text{Si}_{0.85} \), one 30 nm thick film of \( \text{InO}_x \) and the underdoped cuprate \( \text{La}_{1.94}\text{Sr}_{0.06}\text{CuO}_4 \), taken from \([24, 25], [26] \) and \([58]\), respectively. For \( \text{Nb}_{0.15}\text{Si}_{0.85} \), we observe a sharp increase of the Nernst coefficient at the superconducting transition. As we will see later, in the zero magnetic field limit, this large enhancement of the Nernst coefficient reflects the diverging correlation length at the approach of the superconducting transition. We will show that a comparison of the magnetic field dependence of the Nernst signal, figure 3, allows one to establish a fundamental distinction between the data measured above and below \( T_c \). At finite magnetic field, as the only genuine
superconducting phase is the dissipation-less vortex solid, the large enhancement of the Nernst coefficient actually reflects a crossover between two regimes of fluctuations, the regime of Cooper-pair fluctuations and the vortex fluid with frozen amplitude fluctuations of the order parameter.

For InO_x and La_{1.94}Sr_{0.06}CuO_4, figure 2 shows that the Nernst coefficient changes continuously across the transition and does not increase abruptly at the transition. For InO_x, this reflects the absence of a true phase transition, with diverging correlation length, and so the absence of long range superconducting order in this system.

5. Cooper-pair fluctuations and GCF

Figure 3 shows the magnetic field dependence of the Nernst signal for Nb_{0.15}Si_{0.85} and InO_x. In the normal state, for both systems, the Nernst data follow a characteristic tilted tent profile with a maximum at the field value \(B^*\). The magnitude of \(B^*\) increases with increasing temperature.

For Nb_{0.15}Si_{0.85}, below \(T_c\), the vortex-induced Nernst signal increases steeply with magnetic field, when the vortices become mobile following the melting of the vortex solid state. It reaches a maximum and decreases at larger magnetic fields when the excess entropy of the vortex core is reduced. In contrast to the high temperature regime, the position of the maximum \(B^*\) shifts toward higher magnetic fields upon decreasing the temperature. This is not surprising, since in the superconducting state, all characteristic fields associated with superconductivity, such as \(B_{c2}\) and \(B_{an}\), are expected to increase with decreasing temperature. Plotting the position of \(B^*\), above and below \(T_c\), on the phase diagram figure 4 shows that \(B^*\) goes to zero just at \(T_c\). This observation is the most definitive signature that the nature of superconducting fluctuations at the origin of the Nernst signal observed above \(T_c\) is fundamentally distinct from that below \(T_c\).
Figure 3. Nernst signal measured below and above $T_c$: for Nb$_{0.15}$Si$_{0.85}$, panels (a) and (b), respectively, and for InO$_x$, panels (c) and (d), respectively. The maxima occurring at $B^*$ are indicated by arrows. Below $T_c$, $B^*$ increases with decreasing temperature, like $B_{c2}$ and $B_m$. Above $T_c$, the temperature dependence of $B^*$ is reversed, it increases with increasing temperature as expected for the GCF.

Below $T_c$, the Nernst signal is generated by the long-lived vortices of the vortex liquid, above $T_c$, the Nernst signal is generated by Cooper-pair fluctuations.

These fluctuations correspond to spatial and temporal fluctuations of the SOP, $\Psi(x, t)$, and are described by the Ginzburg–Landau theory [21]. The typical size of these superconducting fluctuations is set by the correlation length, $\xi_d$. It sets the characteristic length on which the correlation function $\langle \psi(x_0)\psi(x_0 - x) \rangle$ decreases to zero. Upon cooling, this correlation length increases and diverges at the approach of the superconducting transition as $\xi_d = \xi_0 \epsilon^{-1/2}$, where $\epsilon = \ln(T/T_c)$ is the logarithm of the reduced temperature. At the microscopic level, these fluctuations correspond to short-lived Cooper pairs whose lifetime is controlled by their decay into free electrons:

$$\tau = \frac{\pi \hbar}{8k_B T_c} \epsilon^{-1}. \quad (6)$$

These Cooper pair fluctuations give rise to the phenomena of paraconductivity [22] and fluctuation diamagnetism [23]. As normal quasi-particles contribute significantly to
**Figure 4.** Top panels: resistance curves of Nb$_{0.15}$Si$_{0.85}$, panel (a), and InO$_x$, panel (c). Bottom panels: phase diagram displaying the field value $B^*$ as a function of temperature. For Nb$_{0.15}$Si$_{0.85}$, panel (b), this field value goes to zero at $T_c$. Below $T_c$, this field reflects the field position where the vortex-induced Nernst signal reaches a maximum. Above $T_c$, this reflects the GCF. For InO$_x$, panel (d), only the GCF is clearly identified in the data. It keeps decreasing as the temperature is swept across the superconducting transition. In contrast to Nb$_{0.15}$Si$_{0.85}$, there is no distinct signature of the large Nernst signal due to vortex flow. For both samples the critical field for the superconductor–insulator transition $B_{SIT}$ is also shown, as extracted from the crossing point of the resistance curves plotted as function of magnetic field, insets of top panels.

Conductivity and magnetic susceptibility, the sensitivity of these probes to superconducting fluctuations is limited to a narrow region close to the superconducting transition [59]. In contrast, in these amorphous films, as the elastic mean free path is only a few Ångstrom long, the contribution of free electrons to the Nernst signal is particularly weak, orders of magnitude lower than the measured Nernst signal due to superconducting fluctuations. This explains why the Nernst signal generated by short-lived Cooper pairs can be detected up to very high temperatures ($30 \times T_c$) and high magnetic field ($4 \times B_{c2}$), deep into the normal state [24, 25]. Furthermore, because of this weak contribution of normal quasi-particles excitations, a direct and unambiguous comparison of Nernst data with superconducting fluctuation theories is possible.
Figure 5. Nernst coefficient $\nu$ (bottom panels) and Peltier coefficient $\alpha_{xy}/B$ (top panels) for Nb$_{0.15}$Si$_{0.85}$ (left) and InO$_x$ (right). The similarity between the plots shows that the evolution of the Peltier coefficient is controlled by the variations of the Nernst coefficient. For both systems, we find that at low field $B < B^*$ those coefficients are independent of magnetic field, they are set only by the temperature dependent correlation length. In the opposite limit, $B > B^*$, those coefficients are independent of temperature, they are determined by the magnetic length.

Treating the fluctuations of the SOP in the Gaussian approximation, USH obtained a simple analytical formula, valid close to $T_c$ and in the zero magnetic field limit, relating the off-diagonal Peltier coefficient $\alpha_{xy}$ to fundamental constants and the correlation length [18]

$$\frac{\alpha_{xy}^{SC}}{B} = \frac{1}{6\pi} \frac{k_B e^2}{\hbar^2} \xi^2,$$

(7)

where $\frac{\alpha_{xy}}{B}$ is simply related to the Nernst coefficient and the measured conductivity through the formula $\frac{\alpha_{xy}}{B} \approx \sigma_{xx} \nu$. Above $T_c$, as the conductivity of samples changes only weakly with temperature and magnetic field, the evolution of the Peltier coefficient is mostly controlled by the Nernst coefficient value, as shown in figure 5, where $\nu$ and $\frac{\alpha_{xy}}{B}$ are plotted side by side.

One remarkable characteristic of formula (7) is that the coefficient $\alpha_{xy}^{SC}/B$ is independent of magnetic field. A plot of this coefficient obtained experimentally for Nb$_{0.15}$Si$_{0.85}$ and InO$_x$, figure 5, shows that this is indeed the case at low magnetic field.
Figure 6. Peltier coefficient $\frac{\alpha_{SC}}{B}$ in the zero magnetic field limit plotted as a function of temperature for Nb$_{0.15}$Si$_{0.85}$ and InO$_x$. The data for Nb$_{0.15}$Si$_{0.85}$ are compared with USH theory.

From those plots, the value of $\frac{\alpha_{SC}}{B}$ in the zero magnetic field limit, ($B \rightarrow 0$), is extracted and compared to the USH equation (7), as shown in figure 6.

For Nb$_{0.15}$Si$_{0.85}$, the quantitative agreement with the theoretical prediction is found close to $T_c$. At high temperature, the data deviate from the USH theoretical expression. Recent theoretical works have extended the calculations of the Nernst effect due to Gaussian fluctuations beyond the regime of validity of USH theory, to higher temperature and magnetic field \cite{60, 61} and have found quantitative agreement with those data as well.

Thus, these last experimental and theoretical works have established that well defined vortex-like excitations are not required for superconducting fluctuations to generate a Nernst signal, and that the magnitude of the Nernst coefficient in the regime of Gaussian fluctuations is simply related to the correlation length. Remarkably, these measurements also demonstrated that even at high magnetic field and high temperature, the Nernst coefficient is simply related to that single characteristic length, the size of the superconducting fluctuations \cite{25, 26}. In the zero-field limit, this size is set by the correlation length, $\xi_d$. In the high field limit, the size of the superconducting fluctuations is set by the magnetic length, $\ell_B = (\hbar/2eB)^{1/2}$ when this length becomes shorter than the correlation length at zero magnetic field.

The shrinking effect of the magnetic field on superconducting fluctuations is well known from studies of fluctuation diamagnetism in low temperature superconductors \cite{23} and cuprates \cite{62}. While in the low field limit, the magnetic susceptibility should be independent of the magnetic field, i.e. in the Schmidt limit \cite{63}, the magnetic susceptibility is experimentally observed to decrease with the magnetic field, following Prange’s formula \cite{64}, which is an exact result within the Ginzburg–Landau formalism. At high magnetic field, the superconducting fluctuations are described as evanescent Cooper pairs arising from free electrons with quantized cyclotron orbits \cite{59}.
As a consequence of this phenomenon, at a given temperature \( T > T_c \), the size of superconducting fluctuations decreases from the value \( \xi_d(T) = \xi_0 e^{-1/2} \), at low magnetic field, to the magnetic length value \( \ell_B \), when the magnetic field exceeds \( B^* = \phi_0 / 2\pi \xi_d^2 \). This characteristic field was identified for the first time by Kapitulnik et al [65] in the magnetoresistance data of mixture films of InGe. As it mirrors, above \( T_c \), the upper critical field below \( T_c \), it has been dubbed the ‘GCF’ by these last authors.

As shown in figure 3, above \( T_c \), this crossover is responsible for the observed maximum in the field dependence of the Nernst signal. Upon increasing the magnetic field, the Nernst signal increases linearly with field, reaches a maximum at \( B^* \) and decreases beyond that field value. As extensively discussed in our previous publications [25, 26], we recall here the arguments demonstrating that the Nernst coefficient is set by the size of the superconducting fluctuations and that \( B^* \) is set by the GCF.

- At low magnetic field, the Nernst coefficient depends only on the temperature and is independent of the magnetic field. Indeed, when \( \ell_B > \xi_d \), the size of the superconducting fluctuations is set by the temperature dependent correlation length \( \xi(T) \) (see figure 5).
- Above \( T_c \), the magnitude and the temperature dependence of \( B^* \) follows the field value set by the Ginzburg–Landau correlation length, \( \xi_0 = \frac{\phi_0}{\sqrt{\epsilon}} \) through the relation \( B^* = \frac{\phi_0}{2\pi \xi_0} \) where \( \phi_0 \) is the flux quantum and \( \epsilon = \ln \frac{T}{T_c} \) the logarithm of reduced temperature. See [24] and [26] for details regarding the determination of the correlation length in Nb, Si\( _{1-x} \) and InO\( _x \), respectively. The position of the maximum \( B^* \) is the field where \( \ell_B = \xi_d \). As shown in figure 4(b) for Nb\( _{0.15} \)Si\( _{0.85} \), above \( T_c \) it mirrors the upper critical field below \( T_c \).
- At high magnetic field, \( B > B^*(T) \), the data for the Nernst coefficient converge toward a weakly temperature-dependent curve. Indeed, when \( \ell_B < \xi_d \), the size of the superconducting fluctuations is set by the magnetic length, which is obviously independent of temperature (see figure 5).
- As shown in figure 7 for Nb\( _{0.15} \)Si\( _{0.85} \), when one substitutes temperature and magnetic field by their associated length scales, the zero-field superconducting correlation length, \( \xi_d(T) \) and the magnetic length, \( \ell_B(B) \), we find that the Nernst coefficient is symmetric with respect to the diagonal \( \xi_d(T) = \ell_B \). This shows that the Nernst coefficient depends only on the size of the superconducting fluctuations, no matter what sets it, the magnetic length or the correlation length.

Finally, we noticed previously for Nb\( _{0.15} \)Si\( _{0.85} \) that \( B^* \) goes to zero at \( T_c \). It appears now clearly that this is the consequence of the divergence of the correlation length at the transition, which drives the GCF to zero. This characteristic temperature dependence of \( B^* \) is a remarkable signature of the superconducting transition and is expected in any conventional superconductor with a phase diagram as depicted in figure 2(a).

A quite distinct phenomenon is observed in InO\( _x \). \( B^* \) keeps decreasing in the temperature range where the superconducting transition is expected, according to resistivity measurements. This indicates that the correlation length does not diverge in this sample, implying the absence of a true superconducting transition. Most likely, strong superconducting fluctuations prevent the establishment of the superconducting order in this sample [26]. These fluctuations could also be held responsible for the weak vortex-induced Nernst signal in this system. Indeed, the nature of vortices existing in conventional vortex fluids is quite distinct from the vortex-like excitations expected in the BKT-type fluctuating regime. While vortices are long-lived in the
vortex fluid, they have a short lifetime in the presence of phase fluctuations of the SOP. Most likely, such a reduction of the lifetime of vortices should reduce the Nernst signal.

This situation bears much similarity with the underdoped cuprates, where the weak Nernst signal observed at high temperature has been attributed to short-lived vortex excitations of a regime with phase-only superconducting fluctuations. However, in contrast to our InO$_x$ sample, where the superconducting order is never reached in our measurements, a genuine superconducting transition, with diverging correlation length, occurs in the cuprates. Consequently, as for Nb$_{0.15}$Si$_{0.85}$, it is expected that the GCF should decrease to zero at $T_c$. While this characteristic field has never been discussed and identified in the magnetic field dependence of the Nernst signal in cuprates, it appears clearly in the Nernst data shown in figures 11, 12, 15 and 16 from [4] for Bi$_2$Sr$_{1.6}$La$_{0.4}$CuO$_6$, Bi$_2$Sr$_{1.8}$La$_{0.2}$CuO$_6$, La$_{1.83}$Sr$_{0.17}$CuO$_4$ and Bi$_2$Sr$_{1.6}$La$_{0.4}$CuO$_6$, respectively.

Despite the distinct characteristics of the three families of materials discussed, Nb$_{0.15}$Si$_{0.85}$, InO$_x$ and the cuprates, we find that the GCF is a robust feature of the Nernst signal generated by superconducting fluctuations, regardless of the precise nature of those fluctuations, i.e. Cooper-pair fluctuations or phase-only fluctuations of the SOP. As a measure of the temperature dependence of the correlation length, the GCF provides a remarkable tool for the characterization of superconducting fluctuations.

6. From Cooper-pair fluctuations to the vortex liquid

As discussed earlier, $B_m$, the melting field of the vortex solid, is believed to be the only second-order transition within the temperature–magnetic field phase diagram of disordered type-II superconductors. On the other hand, the upper critical field line $B_{c2}$ is believed to represent only a crossover between the vortex fluid and the regime of Cooper-pair fluctuations. As we established that, in the zero magnetic field limit, the Nernst coefficient diverges at the transition
as the correlation length, this led us to speculate that the evolution of the Nernst coefficient across the superconducting transition should be markedly different in a finite magnetic field. Indeed, while in the zero-field limit, the transition occurs directly between the regime of Cooper-pair fluctuations and the vortex solid, in finite magnetic field, the vortex fluid emerges between those two phases and prevents the divergence of the correlation length within the regime of Cooper-pair fluctuations.

To locate the vortex fluid within the phase diagram of Nb$_{0.15}$Si$_{0.85}$, figure 8(a), shows the Nernst coefficient as a function of magnetic field measured at temperatures above and below $T_c$.

The high-field boundary of the vortex liquid phase is defined as the characteristic field below which the Nernst signal exceeds values expected for Cooper-pair fluctuations. On this figure, we see that the curve at $T_c$ provides an upper envelope for the Nernst curves measured above $T_c$ (the dotted lines) and a separator from the curves measured below $T_c$. All these curves merge with the curve measured at $T_c$ above a field of about 0.9 T. This field value turns out to

**Figure 8.** Panel (a): magnetic field dependence of the Nernst coefficient of Nb$_{0.15}$Si$_{0.85}$ for temperatures above $T_c$ (dotted lines) and below $T_c$ (continuous lines). Panel (b): phase diagram of Nb$_{0.15}$Si$_{0.85}$ on a log scale. See text for the determination of three characteristic fields: the GCF $B^*$, the superconductor–insulator transition critical field $B_{SIT}$ and the melting field $B_m$ of the vortex solid.
be close to the critical field $B_{\text{SIT}}$ for the superconductor–insulator transition. This transition is identified through the observation of a crossing point in the field dependence of the resistivity curves, as shown in the insets of figure 4, and the finite size scaling of the data [45]. Our measurements show that the vortex-induced Nernst signal may be damped by this transition. This is an unexpected observation as the usual understanding of the superconductor–insulator transition implies that the insulating phase should correspond to a quantum fluid of vortices.

The low field boundary of the vortex fluid phase is obtained as the field value where the Nernst coefficient approaches zero. While it should be recognized that this criterion depends on experimental resolution, it provides a reasonable estimate of the melting field $B_m$ of the vortex solid.

Those two fields, $B_m$ and $B_{\text{SIT}}$, are reported on the phase diagram shown on a log scale, figure 8(b), together with the GCF line obtained from the position of the maximum in the field dependence of the Nernst data, measured above $T_c$.

This diagram shows that in the low-field limit, the temperature range for the existence of the vortex liquid is very narrow, and explains why the temperature dependence of the Nernst coefficient shows a sharp peak centered at $T_c$, figure 2(d). This peak is the consequence of the diverging correlation length for Cooper-pair fluctuations and is not due to the vortex fluid motion. Just below $T_c$, the Nernst coefficient decreases as the system enters the vortex solid regime.

At finite magnetic field, see curve at $B = 0.15$ T, figure 2(d), the temperature dependence of the Nernst coefficient shows a peak that becomes broader than that in the zero-field limit as a consequence of the intervening vortex liquid.

7. Conclusion

Superconducting fluctuations are at the center of important contemporary issues in strongly correlated electronic systems. In cuprates, the identification of the nature of superconducting fluctuations in the underdoped–high temperature part of the phase diagram may help to elucidate the origin of the pseudogap observed in the electronic spectrum. If so, this will undoubtedly bring us closer to the solution of the high-$T_c$ problem. In amorphous superconducting thin films, the proper characterization of the superconducting fluctuations on the insulating side of the quantum superconductor–insulator transition would shed light on the nature of this transition and the characteristics of the Bosonic insulator.

This context explains the significant attention devoted to the Nernst effect. While it has been known for a long time to be highly sensitive to the vortices of the vortex fluid, only recently did we discover that it is also highly sensitive to Cooper-pair fluctuations. Theoretically, while the vortex-induced Nernst signal is exceedingly difficult to analyze as it depends on microscopic details such as the vortex pinning, the Nernst signal arising from Cooper-pair fluctuations is simple to analyze as it only depends on the size of the superconducting fluctuations. This leads to a simple relationship between the Nernst coefficient and the superconducting correlation length, as expressed by the USH formula close to $T_c$, and gives rise to a GCF in the field dependence of the Nernst signal. Our description of the evolution of the Nernst coefficient across the superconducting phase diagram of those superconducting films shows that the examination of unconventional superconducting fluctuations should be done by considering the deviations with respect to the Nernst signal generated by Cooper-pair fluctuations, which are expected to exist in any superconductor.
Acknowledgments

We thank C A Marrache-Kikuchi, L Dumoulin and Z Ovadyahu who provided us with amorphous superconducting thin films and C Capan for the La\textsubscript{1.94}Sr\textsubscript{0.06}CuO\textsubscript{4} data. The financial support of the Agence National de la Recherche (ANR-08-BLANC-0121-02) is acknowledged.

References

[1] Xu Z A, Ong N P, Wang Y, Kakeshita T and Uchida S 2000 Nature 406 486
[2] Emery V J and Kivelson S A 1995 Nature 374 434
[3] Vidal F 1973 Phys. Rev. B 8 1982
[4] Wang Y, Li L and Ong N P 2006 Phys. Rev. B 73 024510
[5] Behnia K 2009 J. Phys.: Condens. Matter 21 113101
[6] Nam M S, Ardavan A, Blundell S J and Schlüter J A 2007 Nature 449 584
[7] Wu W, Lee I J and Chaikin P M 2003 Phys. Rev. Lett. 91 056601
[8] Bel R, Jin H, Behnia K, Flouquet J and Lejay P 2004 Phys. Rev. B 70 220501
[9] Bel R, Behnia K, Nakajima Y, Izawa K, Matsuda Y, Shishido H, Settai R and Onuki Y 2004 Phys. Rev. Lett. 92 217002
[10] Sheikin I, Jin H, Bel R, Behnia K, Proust C, Flouquet J, Matsuda Y, Aoki D and Onuki Y 2006 Phys. Rev. Lett. 96 077207
[11] Onose Y, Li L, Petrovic C and Ong N P 2007 Europhys. Lett. 79 17006
[12] Bel R, Behnia K and Berger H 2003 Phys. Rev. Lett. 91 066602
[13] Ettingshausen A V and Nernst W 1886 Wied. Ann. 29 343
[14] Behnia K, Measson M A and Kopelevich Y 2007 Phys. Rev. Lett. 98 076603
[15] Oganesyan V and Sondhi S L 2004 Phys. Rev. B 70 054503
[16] Varlamov A A and Kavokin A V 2008 arXiv:0811.2614 [cond-mat]
[17] Cyér-Choiniere O et al 2009 Nature 458 743
[18] Ussishkin I, Sondhi S L and Huse D A 2002 Phys. Rev. Lett. 89 287001
[19] Ussishkin I 2003 Phys. Rev. B 68 024517
[20] Ussishkin I and Sondhi S L 2004 Int. J. Mod. Phys. B 18 3315
[21] Larkin A I and Varlamov A A 2005 Theory of Fluctuations in Superconductors (Oxford: Oxford University Press)
[22] Glover R E 1967 Phys. Lett. A 25 542
[23] Gollub J P, Beasley M R, Callarot R and Tinkham M 1973 Phys. Rev. B 7 3039
[24] Pourret A, Aubin H, Lesueur J, Marrache-Kikuchi C A, Berge L, Dumoulin L and Behnia K 2006 Nat. Phys. 2 683
[25] Pourret A, Aubin H, Lesueur J, Marrache-Kikuchi C A, Berge L, Dumoulin L and Behnia K 2007 Phys. Rev. B 76 214504
[26] Spathis P, Aubin H, Pourret A and Behnia K 2008 Europhys. Lett. 83 57005
[27] Blatter G, Feigelman M V, Geshkenbein V B, Larkin A I and Vinokur V M 1994 Rev. Mod. Phys. 66 1125
[28] Fisher D S, Fisher M P A and Huse D A 1991 Phys. Rev. B 43 130
[29] Kosterlitz J M and Thouless D J 1973 J. Phys. C: Solid State Phys. 6 1181
[30] Larkin A I and Ovchinnikov Y N 1979 J. Low Temp. Phys. 34 409
[31] Ambegaokar V, Halperin B I, Nelson D R and Sigga E D 1980 Phys. Rev. B 21 1806
[32] Goldman A M and Markovic N 1998 Phys. Today 51 39
[33] Lee P A, Nagaosa N and Wen X G 2006 Rev. Mod. Phys. 78 17
[34] Uemura Y J et al 1989 Phys. Rev. Lett. 62 2317
[35] Sondhi S L, Girvin S M, Carini J P and Shahar D 1997 Rev. Mod. Phys. 69 315
[36] Hebard A F and Paalanen M A 1990 Phys. Rev. Lett. 65 927

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[37] Paalanen M A, Hebard A F and Ruel R R 1992 Phys. Rev. Lett. 69 1604
[38] Yazdani A and Kapitulnik A 1995 Phys. Rev. Lett. 74 3037
[39] Ephron D, Yazdani A, Kapitulnik A and Beasley M R 1996 Phys. Rev. Lett. 76 1529
[40] Markovic N, Christiansen C and Goldman A M 1998 Phys. Rev. Lett. 81 5217
[41] Markovic N, Mack A M, Martinez-Arizala G, Christiansen C and Goldman A M 1998 Phys. Rev. Lett. 81 701
[42] Gantmakher V F, Golubkov M V, Dolgopolov V T, Tsydynzhapov G E and Shashkin A A 2000 JETP Lett. 71 160
[43] Bielejec E and Wu W H 2002 Phys. Rev. Lett. 88 206802
[44] Sambandamurthy G, Engel L W, Johansson A and Shahar D 2004 Phys. Rev. Lett. 92 107005
[45] Aubin H, Marrache-Kikuchi C A, Pourret A, Behnia K, Berge L, Dumoulin L and Lesueur J 2006 Phys. Rev. B 73 094521
[46] Jaeger H M, Haviland D B, Orr B G and Goldman A M 1989 Phys. Rev. B 40 182
[47] Markovic N, Christiansen C, Mack A M, Huber W H and Goldman A M 1999 Phys. Rev. B 60 4320
[48] Marrache-Kikuchi C A, Aubin H, Pourret A,Behnia K, Lesueur J, Berge L and Dumoulin L 2008 Phys. Rev. B 78 144520
[49] Parendo K A, Sarwa K H, Tan B, Bhattacharya A, Eblen-Zayas M, Staley N E and Goldman A M 2005 Phys. Rev. Lett. 94 197004
[50] Dumoulin L, Berge L, Lesueur J, Bernas H and Chapellier M 1993 5th Int. Workshop on Low Temperature Detectors (LTD-5) (Berkeley, CA: Plenum) p 301
[51] Marnieros S, Berge L, Juillard A and Dumoulin L 2000 Phys. Rev. Lett. 84 2469
[52] Ovadyahu Z 1993 Phys. Rev. B 47 6161
[53] Ovadyahu Z 1986 J. Phys. C: Solid State Phys. 19 5187
[54] Gantmakher V F and Golubkov M V 2001 JETP Lett. 73 131
[55] Steiner M A, Boebinger G and Kapitulnik A 2005 Phys. Rev. Lett. 94 107008
[56] Huebener R P and Seher A 1969 Phys. Rev. 181 710
[57] Ri H C, Gross R, Golnik F, Beck A, Huebener R P, Wagner P and Adrian H 1994 Phys. Rev. B 50 3312
[58] Capan C, Behnia K, Li Z Z, Raffy H and Marin C 2003 Phys. Rev. B 67 100507
[59] Skocpol W J and Tinkham M 1975 Rep. Prog. Phys. 38 1049
[60] Serbyn M N, Skvortsov M A, Varlamov A A and Galitski V 2008 arXiv:0806.4427 [cond-mat]
[61] Michaeli K and Finkel’stein A 2008 arXiv:0812.4268 [cond-mat]
[62] Carballeira C, Mosquera J, Revcolevschi A and Vidal F 2000 Phys. Rev. Lett. 84 3157
[63] Schmid A 1969 Phys. Rev. 180 527
[64] Prange R E 1970 Phys. Rev. B 1 2349
[65] Kapitulnik A, Palevski A and Deutscher G 1985 J. Phys. C: Solid State Phys. 18 1305

New Journal of Physics 11 (2009) 055071 (http://www.njp.org/)