Dechirp-receiving radar target detection based on generalized Radon-Fourier transform

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Abstract
The dechirp-receiving radar measures range by the frequency difference between the target echo and the reference signal. It can operate at a low sampling frequency while transmitting a wideband signal, which simplifies the radar hardware. However, when detecting high-speed and highly maneuvering targets, residual video phase (RVP), frequency mismatch, frequency modulation (FM) rate mismatch, and across range-Doppler unit (ARDU) problems occur after dechirp receiving, which cause difficulties in integrating the target echo and result in severe performance loss. To solve these problems, the dechirp generalized Radon-Fourier transform (DGRFT) and an algorithm for its fast implementation are proposed. The DGRFT compensates for the frequency and FM rate mismatch in the fast-time domain and the RVP in the fast-time frequency domain, and then implements joint envelope and phase compensations to overcome the ARDU phenomenon. After these compensations, the target echo can be effectively integrated. Experiments with simulated and measured data show that the performance of the DGRFT is better than that of the conventional moving target detector.

1 | INTRODUCTION

The linear frequency modulated (LFM) signal is a commonly used waveform in radar. In addition to matched filtering, it can be processed by the dechirp-receiving method. In the dechirp-receiving radar, the frequency modulation (FM) rate of the target echo is the same as that of the transmitted signal. Therefore, after mixing the target echo with the reference signal, which has the same FM rate, the FM rate in the target echo is eliminated. However, the propagation delay results in a frequency difference between the target echo and the reference signal. The propagation delay can be obtained by calculating the frequency difference using the Fourier transform (FT). Subsequently, the target range can be determined using a linear transform [1].

The dechirp-receiving method, which is also called stretch processing, uses the same principle as LFM continuous wave (LFMCW) radar [2], Ch. 4.6.5. In practice, the transmitted signal does not have to be continuous and can adopt multiple pulses with a pulse repetition time (PRT) larger than the pulse width. Because the dechirp-receiving radar can work with a sampling frequency significantly lower than the signal bandwidth, it is suitable for applications with a high range resolution and limited range windows, such as in cases with a small range detection extent or rough priori range information. Compared with the pulse radar, dechirp-receiving radar equipment is simple and inexpensive. Therefore, it is widely used, such as detecting unmanned aerial vehicles (UAVs) with X-band LFMCW radars [3, 4], designing an X-band LFMCW radar for automobile anticollision [5], monitoring vehicles with a Ka-band LFMCW inverse synthetic aperture radar (ISAR) [6], tracking a corner reflector held by a walking person with a Ka-band LFMCW interferometric ISAR [7], and imaging simulation of space targets with an X-band LFMCW ISAR [8].

However, three problems occur when a dechirp-receiving radar is used to detect high-speed and highly maneuvering targets:

(1) The residual video phase (RVP), which is a quadratic phase term, exists in the target echo after dechirp receiving.
(2) The motion of the target produces a scale effect on the target echo, causing the frequency and FM rate of the LFM signal in the target echo to deviate from those of the...
reference signal. This results in the dechirp mismatch, that is, frequency mismatch and FM rate mismatch.

(3) The range and velocity of high-speed and highly maneuvering targets easily migrate more than one resolution unit: the across range-Doppler unit (ARDU) phenomenon occurs.

These problems degrade the detection performance and ranging accuracy of common methods.

Research on target detection algorithms for the dechirp-receiving radar has mainly focused on solving the range-velocity coupling phenomenon, which occurs owing to difficulties in distinguishing the frequency corresponding to the target range from the intrapulse Doppler frequency caused by the frequency mismatch. Methods for solving the range-velocity coupling problem can be classified into two types: one is the positive and negative FM class, and the other is the velocity information correction class.

In positive and negative FM methods, two LFM signals with positive and negative FM rates are transmitted in turn [9]. After the two parts are dechirp received, the intrapulse Doppler frequencies are equivalent, but the frequencies corresponding to the target range have opposite signs. Therefore, an equation set is established, and the target range and velocity can be calculated using this equation set. However, in the case with multiple targets, an equation pairing operation is required to find the correct equation set for each target. Unfortunately, incorrect pairings often occur, causing target losses or false alarms. To solve this problem, the algorithm in [10] adds a single frequency signal to measure the intrapulse Doppler frequencies of targets, which can then be used to select the correct pairings. In [11, 12], the single frequency signal is replaced with LFM signals of different FM rates, which improves the pairing results. However, even with an additional signal, incorrect pairings still occur in positive and negative FM methods.

The basic idea behind velocity information correction methods is to compensate for the ranging error according to the velocity information obtained in other ways. In [13], the moving target detector (MTD) [2], which is commonly used in pulse radars, is applied to target detection using a dechirp-receiving radar. After multiple LFM signals are transmitted and dechirp received with a constant PRT, the velocity of the target can be obtained by the interpulse Doppler frequency, which can then be used to correct the ranging error. In this way, the target range and velocity can be measured simultaneously using a two-dimensional fast FT (FFT). In addition, the integration of multiple pulses can improve detection performance. However, the MTD integrates only the echo staying in a single range-Doppler unit, causing a severe loss of performance in the case with ARDU phenomenon. In [14, 15], the Keystone transform (KT), which can also provide velocity information for correcting ranging errors, is used for detecting targets by dechirp-receiving radars. However, the KT is suitable only for targets with uniform velocities and has a loss of performance when the velocity changes.

To improve the detection performance of dechirp-receiving radars for high-speed and highly maneuvering targets, an algorithm is proposed based on the generalized Radon-FT (GRFT) [16, 17] used in pulse radars, namely dechirp GRFT (DGRFT). The DGRFT compensates for frequency and FM rate mismatches in the fast-time domain. Thereafter, the RVP is compensated for after the transformation of the echo into the fast-time frequency domain by FT. Finally, the target echo is focussed into the motion parameter space through joint envelope and phase compensations. The target can be detected using the focussed peak. Moreover, because the DGRFT implemented by an ergodic search induces a huge computational complexity burden, a fast implementation of DGRFT is proposed based on the chirp-Z transform (CZT) [18]. Experiments with simulated and measured data show that the DGRFT achieves better performance than the MTD after overcoming all of the RVP, frequency mismatch, FM rate mismatch and ARDU problems.

The remainder of this work is organised as follows: In Section 2, the signal model in the dechirp-receiving radar is introduced, and the RVP, frequency mismatch, FM rate mismatch and ARDU problems are analysed. In Section 3, the DGRFT and an algorithm for its fast implementation are proposed. In Section 4, experiments with simulated and measured data are presented to verify the effectiveness of the proposed algorithms.

2 | SIGNAL MODEL IN DECHIRP-RECEIVING RADAR

2.1 | Principle of dechirp receiving

The dechirp-receiving principle is shown in Figure 1. Assuming that an LFM signal with an initial frequency \( f_0 \), bandwidth \( B \) and time width \( T_p \) is transmitted, there will be a constant frequency difference \( f \) between the echo and reference signal because they are delays of the transmitted signal with different initial times. After mixing with the reference signal, the frequency \( f \) of the target echo becomes proportional to echo delay \( t_0 \) and target range \( r \), that is,

\[
 f = \frac{2\alpha}{c} r, \\
 r = \frac{c}{2\alpha} f, 
\]

and the range resolution is

\[
 \rho_t = \frac{c}{2\alpha} \frac{1}{T_p} = \frac{c}{2B} \]

where \( c \) is the speed of light, and \( \alpha = B/T_p \) is the FM rate.

When the range detection extent is small, the maximum frequency difference is far less than the signal bandwidth: that is, \( f_{\text{max}} \ll B \). Therefore, a frequency significantly lower than the bandwidth of the transmitted signal can be used for sampling, which can achieve a large bandwidth using a greatly simplified set of hardware equipment.
2.2 | Signal model of a single pulse

For a target with an initial range $r$ and velocity $v$, the range is expressed as $r(t_0) = r + vt_0$. After a two-way delay, the signal transmitted at time $t_0$ is received at time:

$$t_r = t_0 + \frac{2r(t_0)}{c - v} = \frac{t_0}{1 - \frac{v}{c}} + t_d.$$

(3)

Therefore, the target echo is written as $s_R(t) = As_T(t)$, that is,

$$s_R(t_0) = As_T(t_0 - t_d),$$

(4)

where $t_0$ is the fast time, $A$ is the complex amplitude of the echo, $\rho$ is the scale coefficient depending on the velocity, $t_d$ is the echo delay, and

$$\rho = \frac{c - v}{c + v}$$

(5)

$$t_d = \frac{2r}{c - v}.$$

Assuming that the zero time is at the dechirp centre, the transmitted LFM signal is written as

$$s_T(t_0) = \text{rect} \left( \frac{1}{T_p} \left( t_0 - \frac{T_p}{2} \right) \right) \exp \left[ j2\pi f_0 t_0 + j\pi \alpha t_0^2 \right],$$

$$t_0 \in [0, T_p].$$

(6)

Substituting Equation (6) into Equation (4), the echo is expressed as

$$s_R(t_0) = A\text{rect} \left( \frac{1}{T_p} \left( t_0 - \frac{T_p}{2} - t_d \right) \right)$$

$$\times \exp \left[ j2\pi f_0 (t_0 - t_d) + j\pi \alpha \rho^2 (t_0 - t_d)^2 \right],$$

(7)

where $T_p = T_p/\rho$ is the length of the rectangular window after scaling. After dechirp receiving, that is, mixing with the reference signal, the echo can be simplified to

$$s(t_0) = s_R(t_0)s_T^*(t_0)$$

$$= A \text{rect} \left( \frac{1}{T_p} \left( t_0 - \frac{T_p}{2} \right) \right) \text{rect} \left( \frac{1}{T_p} \left( t_0 - \frac{T_p}{2} - t_d \right) \right)$$

$$\times \exp \left[ j\pi \alpha (\rho^2 - 1) t_d^2 \right] \exp \left[ j2\pi f_0 (\rho - 1 - \alpha \rho^2 t_d) t_0 \right]$$

$$\times \exp \left[ j\pi \alpha \rho^2 t_d^2 \right] \exp \left[ - j2\pi f_0 \rho t_d \right].$$

(8)

where $\cdot^*$ denotes the conjugate operation.

For analysis, the four exponential terms in Equation (8) are denoted by $H_2(t_0)$, $H_1(t_0)$, $H_{1a,2}(t_d)$ and $H_{1d,1}(t_d)$, respectively, and two variables are defined as

$$\eta_{t0} = \frac{2r}{cT_p},$$

$$\eta_t = \frac{vT_p}{\rho_t},$$

(9)

where $\eta_{t0}$ is approximately equivalent to the ratio of the echo delay to the pulse width, and $\eta_t$ is the ratio of the target moving range within $T_p$ to the range resolution. To guarantee that most of the echo is received, the echo delay should be much less than the pulse width ($\eta_{t0} \ll 1$). In addition, $\eta_{t0}T_p$ is much less than $\rho_t$, for common targets, so $\eta_t \ll 1$ is generally satisfied. However, for targets with very high speeds, $\eta_t$ will be close to or even greater than 1.

First, it can be demonstrated that $\rho^2 \approx 1 - 4v/c$ using the first-order Taylor expansion, so $H_2(t_0)$ can be approximated as

$$H_2(t_0) \approx \exp \left( - j \frac{4\pi \alpha v}{c} t_0 \right).$$

(10)

Second, because the first-order Taylor expansion shows that $\rho \approx 1 - 2v/c$ and $\rho^2 t_d \approx (1 - 3v/c)2r/c$, the phase of $H_1(t_0)$ is approximately

$$\angle H_1(t_0) \approx -2\pi \left( \frac{2vf_0}{c} + \frac{2ar}{c} \right) t_0 + 2\pi \frac{3v}{c} \frac{2r}{c} t_0.$$

(11)

The second phase term can be ignored because it is not greater than $3\pi \eta_{t0} \eta_t$. Therefore, $H_1(t_0)$ is approximately equal to

$$H_1(t_0) \approx \exp \left( - j \frac{4\pi \left( vT_p + aT \right)}{\rho c} t_0 \right).$$

(12)

Then, for $\rho t_d \approx (1 - v/c)2r/c$, the phase of $H_{1d,1}(t_d)$ is approximately

$$\angle H_{1d,1}(t_d) \approx -2\pi \frac{2vf_0}{c} + 2\pi \frac{2rv}{c\lambda}.$$
Finally, $p^2 t_0^2 \approx (1 - 2v/c)4r^2/c^2$, so

$$2H_{id,2}(t_0) \approx \frac{4\pi ar^2}{c^2} - \frac{2\pi}{c} \frac{2v}{4r^2} c^2. \quad (15)$$

The second phase term is simplified to $\pi t_0^2 H$, and can also be ignored. Therefore,

$$H_{id,2}(t_0) \approx \exp \left( -\frac{4\pi c}{c^2} \right). \quad (16)$$

To summarise, the echo in the time domain is approximately

$$s(t_0) \approx Ay(t_0) \exp \left( -\frac{4\pi c}{c^2} \right) \exp \left( -\frac{4\pi (v t_0 + ar)}{c} t_0 \right)$$

$$\times \exp \left( -\frac{4\pi c}{c^2} \right) \exp \left( \frac{4\pi c}{c^2} \right). \quad (17)$$

where two rectangular windows approximately overlap and are represented by $\gamma(t_0)$.

### 2.3 Signal model of multiple pulses

If the radar transmits multiple pulses, strictly speaking, the target velocity may not be constant in each pulse. However, considering the short duration of the pulses, it can be assumed that the target velocity does not change in each pulse and varies only with different pulses; that is, there is a ‘stop-and-hop’ in the target velocity. Under this assumption, the multiple pulses in the time domain can be approximately expressed as

$$s(t, t_0) \approx Ay(t_0) \exp \left( -\frac{4\pi c}{c^2} \psi(t) t_0^2 \right)$$

$$\times \exp \left( -\frac{4\pi c}{c^2} (f_0 \psi(t) + ar(t)) t_0 \right)$$

$$\times \exp \left( -\frac{4\pi c}{c^2} \right) \exp \left( \frac{4\pi c}{c^2} r^2(t) \right), \quad (18)$$

where $t$ is the slow time, $T_a$ is the total time of the echo, and $r(t)$ and $\psi(t)$ are the target range and velocity at time $t$, respectively. Here, polynomial models are used, where $r(t)$ and $\psi(t)$ are determined using the target initial range $r$, velocity $\psi$ and high-order motion parameters such as the initial acceleration $a$. Therefore, $r(t)$ and $\psi(t)$ are expressed as

$$r(t) = r + vt + \frac{1}{2}at^2 + \cdots, \quad (19)$$

$$\psi(t) = \psi + at + \cdots.$$ 

Here, ‘⋯’ denotes that higher-order motion parameters, such as jerk, can be considered.

In Equation (18), the first exponential term caused by the FM rate mismatch is a quadratic phase term of the fast time, which causes an incomplete integration and reduces the envelope amplitude when measuring the range using FT. The second exponential term is a single frequency signal related to both the range and velocity. The frequency mismatch produces the velocity part, which is called intrapulse Doppler and causes errors when measuring the range. The fourth exponential term is the RVP, which affects the integration efficiency along the slow time when the target range changes significantly. In addition, after transforming into the fast-time frequency domain, the echo envelope and the third exponential term change with the target velocity. That is, the ARDU phenomenon may occur, which also affects the integration efficiency along the slow time.

### 3 DECHIRP GENERALIZED RADON-FOURIER TRANSFORM AND ITS FAST IMPLEMENTATION

For high-speed and highly maneuvering target detection, the effects of RVP, FM rate mismatch, frequency mismatch and ARDU phenomenon cannot be ignored. When the conventional MTD algorithm is used, a significant loss of performance is incurred. To solve these problems, we propose the DGRFT and its fast algorithm.

### 3.1 Principle of the DGRFT

The idea behind the DGRFT is integration after envelope and phase compensations. Because the frequency and FM rate mismatches cause first-order and quadratic phases of fast time $t_0$, respectively, they are compensated for in the fast-time domain. Furthermore, the remaining envelope and phase changes, including the ARDU phenomenon and RVP, are more convenient to compensate for in the fast-time frequency domain.

The final DGRFT output is defined in the motion parameter space. Defining the reference parameters in the motion parameter space as $r_p, \psi_p, \alpha_p$, and so on, the reference range and velocity functions determined by the reference parameters are

$$r_p(t) = r + \psi_p t + \frac{1}{2} \alpha_p t^2 + \cdots, \quad \psi_p(t) = \psi + \alpha_p t + \cdots, \quad (20)$$

where $r_p, \psi_p$ and $\alpha_p$ denote the reference range, velocity and acceleration, respectively. As in Equation (19), higher-order
reference parameters can be considered in this equation. After the frequency and FM rate mismatches are compensated for by multiplying the phase factors with Equation (18), the echo is transformed into the fast-time frequency domain, which is expressed as

$$S_c(t,f) = \text{FT}_t \{ s(t, t_0) H_c(t, t_0) \}, \quad (21)$$

where the phase factors are

$$H_c(t, t_0) = H_{c1}(t, t_0) H_{c2}(t, t_0),$$

$$H_{c1}(t, t_0) = \exp \left( j \frac{4 \pi f_0}{c} v_p(t) t_0 \right),$$

$$H_{c2}(t, t_0) = \exp \left( j \frac{4 \pi \alpha}{c} v_p(t) t_0^2 \right).$$

\text{FT}_t \{ \cdot \} \text{ denotes the FT along } t_0. \text{ Because the target range is proportional to the fast-time frequency, the RVP and ARDU problems can be compensated for according to the change of the target range, that is, } r_p(t). \text{ Therefore, the DGRFT can be expressed as}

$$f_{\text{DGRFT}}(r_p, v_p, a_p, \ldots) = \int_{-T_s/2}^{T_s/2} S_c \left( t, \frac{2 \alpha}{c} r_p(t) \right) \times \exp \left( j \frac{4 \pi f_0}{c} v_p(t) \right) H_{c,RVP}(t) dt,$$

where $$H_{c,RVP}(t)$$ compensates for the RVP and can be expressed as

$$H_{c,RVP}(t) = \exp \left( - j \frac{4 \pi \alpha}{c^2} r_p(t) \right). \quad (24)$$

The remaining parts of Equation (23) represent joint envelope and phase compensations to overcome the ARDU phenomenon, like the normal GRFT [16, 17].

If the target moves with a uniform velocity, \( r(t) = r + vt \) and \( v(t) = v \). In this case, the DGRFT degenerates into the dechirp Radon-FT (DRFT); that is,

$$S_c(t,f) = \text{FT}_t \{ s(t, t_0) H_c(t_0) \}$$

$$f_{\text{DRFT}}(r_p, v_p) = \int_{-T_s/2}^{T_s/2} S_c \left( t, \frac{2 \alpha}{c} \left( r_p + v_p t \right) \right) \times \exp \left( j \frac{4 \pi f_0}{c} \left( r_p + v_p t \right) \right) H_{c,RVP}(t) dt,$$

where

$$H_c(t_0) = \exp \left( j \frac{4 \pi}{c} (f_0 + \alpha t_0) v_p t_0 \right),$$

$$H_{c,RVP}(t) = \exp \left( - j \frac{4 \pi \alpha}{c^2} \left( r_p + v_p t \right)^2 \right). \quad (26)$$

For convenience, we write the motion parameters in vector format and denote the transpose operation by \((\cdot)^T\). In the DGRFT and DRFT, when the reference parameters \([r_p, v_p, a_p, \ldots]^T\) are the same as the target parameters \([r, v, a, \ldots]^T\), \( H_c(t, t_0) \) and \( H_{c,RVP}(t) \) can compensate for frequency mismatch, FM rate mismatch and RVP, respectively. In addition, the ARDU phenomenon can be solved by joint envelope and phase compensations as in the normal GRFT. Therefore, the maximum integration value appears at \([r, v, a, \ldots]^T\). Even if a single parameter of \([r_p, v_p, a_p, \ldots]^T\) is different from the corresponding parameter in \([r, v, a, \ldots]^T\), it can be obtained that \( r_p(t) \neq r(t) \), so the compensations are mismatched and the echo cannot be integrated together. Therefore, a peak is observed in the motion parameter space at the position corresponding to the real motion parameters, and detection can be performed according to the peak.

### 3.2 Fast implementation of the DGRFT based on the CZT

One method of implementing the DGRFT is to calculate \( f_{\text{DGRFT}}(r_p, v_p, a_p, \ldots) \) using Equations (21)–(23) for all possible combinations of \([r_p, v_p, a_p, \ldots]^T\). However, this method performs an ergodic search in the multidimensional motion parameter space, which causes a heavy computational burden and makes it unsuitable for practical applications. To solve this problem, we propose a fast implementation of the DGRFT based on the CZT.

First, for convenience, we consider the fast implementation of the DRFT. After writing the FT in Equation (25) as an integral, the DRFT can be expressed as

$$f_{\text{DRFT}}(r_p, v_p) = \int_{-T_s/2}^{T_s/2} \int_{0}^{T_p} s(t, t_0) H_{c,RVP}(t) H_c(t_0) \times \exp \left( j \frac{4 \pi f_0}{c} \left( f_0 + \alpha t_0 \right) \left( r_p + v_p t \right) \right) dt_0 dt,$$

To achieve a fast implementation, the compensation of RVP (product of the target echo and \( H_{c,RVP}(t) \)) is approximately achieved by multiplying a phase factor in the fast-time frequency domain; that is,

$$S_c(t, t_0) = \text{IFT}_f \{ \text{FT}_t \{ s(t, t_0) \} H_{c,RVP,c}(f) \}, \quad (28)$$
where the phase factor is
\[ H_{c,\text{RVP},r}(f) = \exp \left( -j \frac{f^2}{\alpha} \right) \] (29)
and \( \text{IFT}(\cdot) \) denotes the inverse FT (IFT) along \( f \). According to the relation between \( f \) and \( r \), this phase factor can also be written as
\[ H_{c,\text{RVP},r}(r) = \exp \left( -j \frac{4\pi r^2}{c^2} \right). \] (30)

Note that \( r \) in Equation (30) represents the range axis, which is different from the initial range. The process shown in Equation (28) is not the same as multiplying \( H_{c,\text{RVP}}(t) \) in Equation (26) and can compensate for the RVP only at the envelope peaks of the echo in the fast-time frequency domain. Fortunately, this process is sufficient because the envelope peaks in the pulses contribute to the final integration peak.

After substituting \( s(t,t_0)H_{c,\text{RVP}}(t) \) in Equation (27) with Equation (28) and changing the order of the two integrals, the DRFT can be simplified to
\[ f_{\text{DGRFT}}(r_p, \varphi_p) \approx \int_{t_0}^{T_p} s_c(t, t_0) \int_{-T_f/2}^{T_f/2} \exp \left( \frac{4\pi i r_0}{c} (f_0 + a(t_0)) r_p \right) \, \text{d}t \, \text{d}t_0, \] (31)
which can further be expressed as
\[ f_{\text{DGRFT}}(r_p, \varphi_p) \approx \exp \left( \frac{4\pi f_0 r_p}{c} \right) \int_0^{T_p} F_{\text{DGRFT}}(\varphi_p, t_0) \times H_c(t_0) \exp \left( \frac{4\pi i r_0 a(t_0)}{c} \right) \, \text{d}t_0, \] (32)
where
\[ F_{\text{DGRFT}}(\varphi_p, t_0) = \int_{-T_f/2}^{T_f/2} s_c(t, t_0) \exp \left( \frac{4\pi i f_0}{c} (f_0 + a(t_0)) \varphi_p t \right) \, \text{d}t. \] (33)

The DRFT is approximately equal to the FT of the product of Equation (33) and \( H_c(t_0) \) along \( t_0 \). Because Equation (33) has the exact form as the CZT [18, Ch. 6.3.2], a fast implementation of DRFT can be achieved using a fast CZT; that is, by FFTs and complex multiplications. Considering Equations (28), (32) and (33), a flowchart for fast DRFT is shown in Figure 2. The process along the fast time includes RVP and dechirp mismatch compensations, which correspond to Equations (28) and (32), respectively. The process along the slow time is the fast CZT, that is, the calculation of Equation (33), which includes joint envelope and phase compensations and integration. Phase factor 3 contains two parts for the last phase compensation of CZT and dechirp mismatch compensation, respectively. The two steps can be combined in a single complex multiplication operation.

Now we consider the fast implementation of the DGRFT. After writing the FT in Equation (21) as an integral and substituting it into Equation (23), the DGRFT can be expressed as
\[ f_{\text{DGRFT}}(r_p, \varphi_p, a_p, \ldots) \approx \int_0^{T_p} \int_{-T_f/2}^{T_f/2} s_c(t, t_0) H_{c}(t_0) \exp \left( \frac{4\pi i f_0}{c} (f_0 + a(t_0)) r_p \right) \, \text{d}t_0 \, \text{d}t. \] (34)

Through the operation shown in Equation (28), the effect of RVP is approximately eliminated. Therefore, the DGRFT can be approximately expressed as
\[ f_{\text{DGRFT}}(r_p, \varphi_p, a_p, \ldots) \approx \int_0^{T_p} \int_{-T_f/2}^{T_f/2} s_c(t, t_0) H_{c}(t_0) \times H_{c}(t_0) \exp \left( \frac{4\pi i f_0}{c} (f_0 + a(t_0)) r_p \right) \, \text{d}t_0 \, \text{d}t. \] (35)

Different from \( H_{c}(t_0) \) in Equation (31), \( H_{c}(t_0) \) in Equation (35) relies on both fast time \( t_0 \) and slow time \( t \), and only parts of \( H_{c}(t_0) \) can be moved out of the integration of \( t \). After separating reference range \( r_p \) and velocity \( \varphi_p \) from the motion parameters, we can express the range and velocity changes that depend on the high-order motion parameters as
\[
    r_{p,H}(t) = \frac{1}{2} \alpha t^2 + \ldots, \quad v_{p,H}(t) = \alpha t + \ldots, \quad (36)
\]

where ‘...’ represents that higher-order parameters can be included. Therefore, \( H_c(t, t_0) \) is expressed as a product of two exponential terms, that is,

\[
    H_c(t, t_0) = \exp \left( \frac{j4\pi}{c} (f_0 + \alpha t_0)v_{p,H}(t_0) \right) \times \exp \left( \frac{j4\pi}{c} (f_0 + \alpha t_0) v_{p,H}(t)t_0 \right). \quad (37)
\]

Here, the first exponential term can be moved out of the integration of \( t \), and the second exponential term relies on only the high-order motion parameters. Therefore, the DGRFT can be approximately expressed in the form of fast DRTFs as

\[
    f_{\text{DGRFT}}(r_p, v_p, a_p, \ldots) \approx \int_0^{T_0} \int_{-T_0/2}^{T_0/2} s_c(t, t_0, a_p, \ldots) \times \exp \left( \frac{j4\pi}{c} (f_0 + \alpha t_0) \left( r_p + v_{p,H}(t) \right) \right) dt dt_0, \quad (38)
\]

where \( H_c(t_0) \) is shown as Equation (26), and

\[
    s_c(t, t_0, a_p, \ldots) = \exp \left( \frac{j4\pi}{c} (f_0 + \alpha t_0) \left( r_p + v_{p,H}(t) \right) \right) \quad (39)
\]

The difference between Equations (31) and (38) is the echo. As shown in Equation (39), the echo in Equation (38) is the product of \( s_c(t, t_0) \) and an exponential term related to the high-order motion parameters rather than the range and velocity. For every combination of high-order motion parameters, that is, for each combination of \( [a_p, \ldots]^T \), the DGRFT can be achieved through phase compensation and the flowchart of the fast DRTF shown in Figure 2. Therefore, the total DGRFT can be branched into several fast DRTFs, as shown in Figure 3. The phase factors are used to compensate the high-order motion parameters before the CZT of Figure 2. The compensations shown in Equation (39) and first phase compensation of the CZT can be calculated by a single complex multiplication operation to reduce the computational burden.

### 3.3 Computational complexity analysis

In this section, we consider the case with a second-order motion, that is the uniformly accelerated motion case. Here, the motion parameter space searched by the DGRFT contains range, velocity and acceleration dimensions. It is assumed that these dimensions are divided into \( N_r, N_v, N_a \) grids, respectively.

As shown in Equations (21) and (23), the DGRFT with an ergodic search contains phase compensation, FFT, phase compensations in \( N_r \) range grids and summations in \( N_v \) range grids. If the modular square operation of the DGRFT result is included, the total computational complexity of the DGRFT with an ergodic search can be expressed as

\[
    I_{\text{DGRFT}} = N_rN_v \left( 6MN + 5MN \log_2 N + N(6M + 2M) \right) + 3NN_rN_a
    = NN_rN_v \left( 5M \log_2 N + 14M + 3 \right).
    \quad (40)
\]

where \( M \) is the total number of pulses.

In the fast implementation of DGRFT, a single complex multiplication operation is sufficient to achieve both phase compensation for high-order motion parameters in Equation (39) and first phase compensation of the CZT process in Figure 2. This is because the phase factors can be pre-prepared, multiplied together and stored in the memory. In total, fast DGRFT completes the flowchart steps (in Figure 2) \( N_r \) times, the computational complexities of which are shown in Table 1. Therefore, the computational complexity of fast DGRFT can be expressed as

\[
    I_{\text{DGRFT,f}} = NN_r \left( 5M \log_2 N + 5M \log_2 N + 10 \log_2 J \right) + 18M + 15N_v, \quad (41)
\]

where \( J = M + N_v \).
In the target detections of both pulse radars and dechirp-receiving radars, the ARDU phenomenon disperses the target echo in multiple range-Doppler units and results in a severe loss of performance. The normal GRFT, which implements joint envelope and phase compensations, is an excellent algorithm for solving the ARDU problem in pulse radars. Unfortunately, the normal GRFT cannot be directly applied to a dechirp receiving radar because it uses a different principle for measuring the range and phase terms in the target echo. From those of the GRFT because of the different principles but the phase factors used for compensations are different.

The DGRFT has some similarities with the normal GRFT [16, 17]:

1. The DGRFT and GRFT both realise the integration of the target echo by joint envelope and phase compensations to overcome the ARDU phenomenon.
2. They can be realised in the fast-time domain or fast-time frequency domain of the target echo.

However, they also have some differences:

1. In the normal GRFT, the target range corresponds to the fast time. However, in the DGRFT, the target range is proportional to the fast-time frequency. In other words, the fast-time frequency domain in the DGRFT corresponds to the fast-time domain in the GRFT, whereas the fast time in the DGRFT is similar to the fast-time frequency in the GRFT.
2. The GRFT ignores the effect of dechirp mismatch. However, in dechirp receiving, the effect of dechirp mismatch must be considered. Therefore, the DGRFT compensates both the frequency and FM rate mismatch problems.

In addition, a unique RVP appears in dechirp receiving, which is also compensated for by the DGRFT.

(3) Because that linear phase corresponds to signal shift before and after the FT, the GRFT can realise envelope compensation through phase compensation in the fast-time frequency domain. Furthermore, the GRFT can simultaneously achieve envelope compensation, phase compensation and integration of multiple pulses by the CZT in the fast-time frequency domain. A similar implementation is performed by the fast DGRFT in the fast-time domain, but the phase factors used for compensations are different from those of the GRFT because of the different principles of measuring the range and phase terms in the target echo.

Compared with pulse radars, a dechirp-receiving radar achieves a low sampling frequency by sacrificing the range detection extent, resulting in cost-effective hardware. Therefore, dechirp-receiving radars and the proposed DGRFT have advantages in applications with a high range resolution and limited range windows. One application is in vehicle or UAV monitoring, in which the ARDU problem may occur because of a high range resolution and a variable target velocity. Other problems may have little effect because the target velocity is relatively low. In this case, the DGRFT can be flexibly adjusted, such as by omitting a part of or the entire RVP, frequency mismatch and FM rate mismatch compensations. A complete DGRFT process can also definitely be adopted. Another example is the detection before ISAR imaging of space or celestial targets with ephemerides. The ephemerides contribute to the reduction of range detection extent. However, the target velocity is high, which makes the ARDU, RVP, frequency mismatch and FM rate mismatch problems significant. Therefore, a complete DGRFT process should be considered. In Section 4.4, we demonstrate an experiment with measured data to show the possibility of UAV detection using the DGRFT.

### 4.1 Outputs of DGRFT

In this section, simulations are presented to show the outputs of the DGRFT. The simulation parameters are listed in Table 2.

#### Table 2 Simulation parameters

| Parameter          | Value |
|--------------------|-------|
| Initial frequency  | 3 GHz |
| Sampling frequency | 10 MHz|
| Bandwidth          | 500 MHz|
| Pulse width        | 400 μs|
| Dechirp centre     | 500 μs|
| PRT                | 1 ms  |
| Number of pulses   | 1280  |

Abbreviation: PRT, pulse repetition time.
First, the DGRFT outputs with an ergodic search and fast implementation are shown in Figures 4 and 5, respectively. The initial range of the target is 75.5 km, which is 500 m from the dechirp centre. The initial velocity and acceleration of the target are 312.5 m/s and 20 m/s², respectively. The results show two peaks in the outputs at the real parameter position, illustrating the effectiveness of the two implementation methods. In addition, the DGRFT with an ergodic search has a certain amplitude loss, because it can only obtain integer samples rather than the real peaks when the envelope migrates noninteger range units. Fortunately, the fast DGRFT can realize envelope compensation of noninteger range units by multiplying phase factors in the fast-time domain, so the amplitude has no rounding loss.

Then, simulations presented in Figure 6 show the effects of RVP and dechirp mismatch compensations. For the earlier target with uniform acceleration, the result without RVP or dechirp mismatch compensations is shown in Figure 6a. The defocused peak indicates an ineffective integration. After the RVP compensation, as shown in Figure 6b, the peak becomes taller, but the dechirp mismatch still causes a certain amplitude loss. When the FM rate mismatch is compensated further, the integration result improves again, as shown in Figure 6c. In Figure 6d, the frequency mismatch compensation does not affect the peak value but moves the peak to the real target position. The one-dimensional range slices for Figure 6 simulations are shown in Figure 7. The DGRFT improves the integration efficiency through the RVP and FM rate mismatch compensations, and frequency mismatch compensation further corrects the ranging error. A peak representing the target is finally obtained in the motion parameter space after these steps.

Finally, simulations are performed for four targets with an initial range of 75.5 km. The initial velocities are 311.5, 312, 312.5 and 313 m/s, respectively. The initial accelerations are 0, 0, 20 and 20 m/s², respectively. The MTD output is shown in Figure 8a. The result is seriously defocused because the high velocity and acceleration cause severe ARDU problems in the echo. Conversely, as shown in Figure 8b,c, the DGRFT can overcome the ARDU phenomenon, so four peaks appear in the motion parameter space at the real target positions, which indicates better integration and resolution performance.

**FIGURE 4** Dechirp generalized Radon-Fourier transform output with an ergodic search. (a) Range-velocity slice; (b) Velocity-acceleration slice; (c) Range-acceleration slice

**FIGURE 5** Dechirp generalized Radon-Fourier transform output with fast implementation. (a) Range-velocity slice; (b) Velocity-acceleration slice; (c) Range-acceleration slice
4.2 Comparison of detection performance

In this section, Monte-Carlo simulations are performed to compare the detection performance of the MTD, DRFT and DGRFT. The number of simulations is 1000 and the false alarm probability is $10^{-6}$.

For a target with an initial range of 75.5 km, a velocity of 312.5 m/s and an acceleration of 0 or 20 m/s$^2$, the detection probability curves for the three methods are shown in Figure 9, where the signal to noise ratio (SNR) refers to the ratio of the signal peak power to the noise power in the fast-time frequency domain. The results show that the MTD can effectively integrate only the echo staying in the same range-Doppler unit. In other words, the performance of the MTD is severely affected by the ARDU phenomenon, so it cannot detect the targets even in high-SNR cases. When the target velocity is high but with no acceleration, both the DRFT and DGRFT can achieve excellent performance. However, when the target also has high acceleration, only the DGRFT can compensate for the acceleration and maintain good performance.

In fact, when the resolution is high, the ARDU phenomenon occurs even with a relatively low velocity and acceleration. The detection performance for such a case is shown in Figure 10, where the SNR demand refers to the SNR in the fast-time frequency domain when the detection probability reaches 0.9. The results show that the DGRFT has the lowest SNR demand and the best detection performance in different velocity and acceleration cases. The DRFT, which is a special case of the DGRFT, considers only the compensation of the target velocity rather than the high-order motion parameters. Therefore, the detection performance of DRFT is the same as...
FIGURE 8  Comparison of outputs of moving target detector (MTD) and dechirp generalized Radon-Fourier transform (DGRFT): (a) Result of the MTD; (b) Range-velocity slice of DGRFT with an acceleration of 0 m/s²; (c) Range-velocity slice of DGRFT with an acceleration of 20 m/s².

FIGURE 9  Detection probability curves for high-speed and highly maneuvering targets: (a) Curves for target with a velocity of 312.5 m/s and an acceleration of 0 m/s²; (b) Curves for target with a velocity of 312.5 m/s and an acceleration of 20 m/s²; DGRFT, dechirp generalized Radon-Fourier transform; DRFT, dechirp Radon-Fourier transform; MTD, moving target detector; SNR, signal to noise ratio.

FIGURE 10  Detection performances with different velocities and accelerations: (a) Detection probabilities with a velocity of 1 m/s and an acceleration of 0 m/s²; (b) Signal to noise ratio (SNR) demand with an acceleration of 0 m/s²; (c) Detection probabilities with a velocity of 0 m/s and an acceleration of 0.3 m/s²; (d) SNR demand with a velocity of 0 m/s; DGRFT, dechirp generalized Radon-Fourier transform; DRFT, dechirp Radon-Fourier transform; MTD, moving target detector.
that of the DGRFT for targets with uniform velocities but deteriorates in cases with uniform acceleration. The MTD cannot integrate an echo that crosses more than one range or velocity unit and exhibits poor detection performance once the ARDU phenomenon occurs.

### 4.3 Computational complexity comparison

In this section, the computational complexities of DGRFTs with an ergodic search and fast implementation are compared for a range search extent of 1.2 km and an acceleration search extent of $40 \text{ m/s}^2$. The results are presented in Figure 11. The computational complexity of the ergodic search DGRFT is proportional to the velocity search extent. For the fast DGRFT, to calculate the CZT along the slow time, the number of FFT points should not be less than the sum of the numbers of velocity grids and pulses. When the velocity search extent is small, the number of FFT points is significantly more than the number of result points, indicating greater computational complexity of the fast DGRFT than an ergodic search DGRFT. However, as the velocity search extent increases, the number of velocity grids gradually approaches the number of pulses, so the number of FFT points to calculate the CZT gradually becomes larger. In this way, the advantage of FFT appears, so the computational complexity of the fast DGRFT becomes lower and increases more slowly than that of an ergodic search DGRFT. When the velocity search extent is large, the CZT along the slow time is highly efficient and can greatly reduce the computational complexity.

To highlight the advantages of the fast DGRFT over an ergodic search DGRFT, the platform and computing times are listed in Table 3. The range search extent remains unchanged, but the velocity and acceleration search extents are set to $30 \text{ m/s}$ and $2 \text{ m/s}^2$, respectively, to guarantee an acceptable time cost. Compared with an ergodic search, the fast implementation reduces the time cost to approximately $1/447$ of the original time. The selected platform is not highly efficient but is easily programmable. For practical applications, faster hardware (digital signal processor or graphics processing unit), a more efficient programing language and 32-bit float data type can be used to further reduce the time cost.

### 4.4 Verification with measured data

To verify the performance of the DGRFT in real applications, an experiment was conducted to obtain the measured data, as shown in Figure 12. The target was a UAV with a diameter (including rotor arms) of approximately 2 m. An LFMCW radar was used with parts of the transmitted LFM signals being processed. The parameters are listed in Table 4. A fixed filter was used to eliminate ground clutter.

---

**TABLE 3** Computing time comparison between DGRFTs with an ergodic search and fast implementation

| Implementation | Ergodic | Fast |
|----------------|---------|------|
| Hardware       | Intel Core i7-6700HQ CPU |      |
| Operating system | Windows 10 |      |
| Software       | MATLAB R2017b |      |
| Data type      | 64-bit double |      |
| Computing time | 11650.62 s | 26.04 s |

Abbreviation: DGRFT, dechirp generalized Radon-Fourier transform.

**TABLE 4** Experiment parameters for obtaining the measured data

| Parameter          | Value |
|--------------------|-------|
| Band               | Ku     |
| Extracted bandwidth| 320 MHz|
| Blind speed        | 20.5 m/s |
| UAV height         | 10–20 m |
| UAV velocity       | ≤10 m/s |
| Integration time   | 0.4 s  |

Abbreviation: UAV, unmanned aerial vehicle.
**Figure 13** Measured data and moving target detector (MTD) output: (a) Measured data in the fast-time frequency domain; (b) MTD output

**Figure 14** Dechirp generalized Radon-Fourier transform output for the measured data: (a) Range-velocity slice; (b) Velocity-acceleration slice; (c) Range-acceleration slice

**Figure 15** Moving target detector results for measured data 1: (a) Range result; (b) Velocity result; (c) Acceleration result; (d) Amplitude result
**FIGURE 16** Dechirp generalized Radon-Fourier transform results for measured data 1: (a) Range result; (b) Velocity result; (c) Acceleration result; (d) Amplitude result

**FIGURE 17** Results of moving target detector (MTD) and dechirp generalized Radon-Fourier transform (DGRFT) for measured data 2: (a) MTD result; (b) DGRFT result

**FIGURE 18** Results of moving target detector (MTD) and dechirp generalized Radon-Fourier transform (DGRFT) for measured data 3: (a) MTD result; (b) DGRFT result
The echo in the fast-time frequency domain between 312 and 312.4 s for data 1 is shown in Figure 13a. The outputs of the MTD and DGRFT are shown in Figures 13b and 14, respectively. The peak value in the MTD result is significantly smaller than that in the DGRFT result, which indicates better integration efficiency of the DGRFT than the MTD.

To compare the performance in the low-SNR case, noise was added to the measured data, and tracking was performed for the MTD and DGRFT. The results for data 1 are shown in Figures 15 and 16, respectively. The MTD and DGRFT have similar maximum detection ranges in the first away and back processes. In the second away process, the performance of the MTD deteriorates and no track can be formed. In contrast, the integration efficiency of DGRFT is much higher and there are more detected dots to form a continuous track. The results for data 2–4 are shown in Figures 17, 18 and 19, respectively. The maximum detection ranges are listed in Table 5. The improvement in detection ranges verifies the effectiveness and superiority of the DGRFT.

5 CONCLUSION

To improve the detection performance of dechirp-receiving radars for high-speed and highly maneuvering targets, this work analyses the effects of RVP, frequency mismatch, FM rate mismatch and ARDU phenomenon on the integration of the target echo, and then proposes the DGRFT to solve these problems. In the DGRFT, frequency and FM rate mismatches are compensated for in the fast-time domain, and RVP is compensated for in the fast-time frequency domain. Subsequently, the ARDU phenomenon is overcome by the joint envelope and phase compensations, which effectively focus the echo into the motion parameter space. A CZT-based fast implementation of the DGRFT is also proposed that can significantly reduce computational complexity compared with the ergodic search method. Experiments with simulated and measured data show that the DGRFT performs better than the conventional MTD.

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