The emission positions of kHz QPOs and Kerr spacetime influence

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Abstract

Based the Alfven wave oscillation model (AWOM) and relativistic precession model (RPM) for twin kHz QPOs, we estimate the emission positions of most detected kHz QPOs to be at \(r = 18 \pm 3\,\text{km} / (R/15\,\text{km})\) except Cir \(X - 1\) at \(r \sim 30 \pm 5\,\text{km} / (R/15\,\text{km})\). For the proposed Keplerian frequency as an upper limit to kHz QPO, the spin effects in Kerr Spacetime are discussed, which have about a 5\% (2\%) modification for that of the Schwarzchild case for the spin frequency of 1000 (400) Hz. The application to the four typical QPO sources, Cir \(X - 1\), Sco \(X - 1\), SAX J1808.4-3658 and XTE 1807-294, is mentioned.

Key words: kHz QPO, neutron star, low-mass X-ray binaries

1 Introduction

In thirty more low-mass X-ray binaries (LMXBs), the kiloHertz quasi-periodic oscillations (kHz QPOs) have been found, where 2/3 of them show the twin peak kHz QPOs\[^1\], upper and lower frequencies, in the ranges of \(\sim 100\,\text{Hz} - \)

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1300 Hz for the sources with the different spectrum states, e.g. Atoll and Z[2]. The separations of twin kHz QPOs are not constant[13][15][67], which are inconsistent with the beat model[8][9]. The low frequency QPOs have also been found, which follow the tight correlations with the kHz QPOs[14]. Some kHz QPO models have been proposed, most of which are ascribed to the accretion flow[10], and the Alfven wave mode oscillation[11][12]. To account for the varied kHz QPO separation, the relativistic precession model (RPM) is proposed by Stella and Vietri[15], which ascribes the upper frequency to the Keplerian frequency of orbiting material in an accretion disk and the lower frequency to the periastron precession of the same matter.

However, for the detected twin kHz QPOs of neutron star (NS) in a LMXB, their average ratio value is also 3 : 2, but varies with the accretion, which may indicate some distinctions between BHC and NS[1]. In this short letter, we will investigate the orbital positions of kHz QPO emissions, based on the Alfven Wave Oscillation Model (AWOM)[13][14] and RPM[15]. The Kerr spacetime modification is discussed by considering the spin influence on the Keplerian frequency.

2 AWOM/RPM for kHz QPOs

AWOM ascribes an upper frequency to the Keplerian frequency of orbiting matter at radius r, and a lower frequency to the Alfven wave oscillation frequency at the same radius, as described in the following)[13][14],

$$\nu_2 = \nu_k = 1850AX^{\frac{7}{4}}$$

(1)

with the parameter $X = R/r$ (ratio between star radius R and disk radius r) and $A = (m/R_6^3)^{1/2}$ with $R_6 = R/10^6(cm)$ and $m$ the mass $M$ in the units of solar masses. The ratio of twin kHz QPO frequencies can be obtained as,

$$\nu/\nu = (1 + (1 - x)^{1/2})^{1/2}/X^{\frac{4}{7}}$$

(2)

which only depends on the position parameter $X = R/r$, and has nothing to do with the other parameters. The twin kHz QPO separation is obtained as,

$$\nu_2 - \nu_1 = \nu_2[1 - (1 - (1 - x)^{1/2})^{1/2}]X^{\frac{4}{7}}$$

(3)

In FIG.1, the upper kHz QPO frequency is plotted against the position parameter $X = R/r$ ($Y = 3Rs/r$, $Rs$ is the Schwarzschild radius) for AWOM (RPM). For the detected twin kHz QPOs, the mass density parameter A is...
Fig. 1. Upper kHz QPO frequency vs. the position function \((X = R/r, Y = 3Rs/r, Rs = 2GM/C^2)\). The upper (down) solid curve represents AWOM with the mass density parameters \(A = 0.7 (A = 0.45)\), where the maximum frequency is 1850A (Hz); The upper (down) dashed curve represents RPM with the mass parameters \(m = 2 (m = 3)\) solar masses, where the maximum frequency is 2200/m (Hz).

Fig. 2. Twin kHz QPO separation vs. the position function. Curve 1 and 2 represent AWOM with mass density parameters \(A = 0.7\) and \(A = 0.45\) respectively. Curve 3 (4) represents RPM with mass parameter \(m = 2 (m = 3)\), respectively.
found to be about 0.7 (e.g. Sco $X-1$) \cite{13,14}. In most cases (except Cir $X-1$), the position parameter $X = R/r$ is lies in the range from 0.7 to 0.92, or radius from $r = 1.1R$ to $r = 1.4R$. This implies that the emission positions of most kHz QPOs are close to the surface of the NS $X = 1$ for AWOM (for RPM the emission positions are close to $3Rs$), which means that the maximum kHz QPO frequency occurs at the surface (ISCO of star $r = R$ for AWOM (or $r = 3Rs$ for RPM)). In FIG.2, the twin kHz QPO separation vs. position parameter is plotted, where the maximum separation 375 (200) Hz is achieved for $A = 0.7$ (0.45) at $X = 0.7$ for AWOM. The kHz QPO data of two accretion powered millisecond X-ray pulsars (AMXPs), Sax J 1808.4-3658 and XTE 1807-294, approximately hint at the condition of $A = 0.45$, which presents relatively low kHz QPO separations. For RPM, the maximum kHz QPO separations are 360 Hz (210 Hz) for the different choices of mass parameter $m = 2(3)$ solar masses, which occurs at $Y = 0.76$. For the two AMXPs, Sax J 1808.4-3658 and XTE 1807-294, RPM has to assume their star masses are close to the NS mass upper limit, 3 solar masses, if consistent theoretical curves with the detected data can be fitted. FIG.3 is the diagram of twin kHz QPO ratios vs. position parameter. It can be noticed that the averaged ratio 1.5 of the detected kHz QPOs corresponds to the position $X = 0.83$ for AWOM ($Y = 0.89$ for RPM). The ratios of all sources but Cir $X-1$ lie in the regimes between $ratio = 1$ and $ratio = 2$. The kHz QPO data of Cir $X-1$ implies that its kHz QPO emitting positions are far away from the star, i.e. $0.4 < X < 0.6$ or $2.5R > r > 1.6R$, centered at about $2R$. 

Fig. 3. Twin kHz QPO ratio vs. the position function (Same meaning as shown in FIG.1). The ratio 1.5 (1) is the averaged (minimum limit) value of the detected twin kHz QPOs.
3 Kerr spacetime effect on the kHz QPO

If the influence of Kerr spacetime on the Keplerian frequency is taken into account, then the orbital frequency of a spinning point mass $M$ with angular momentum $J$ is expressed as below\cite{11}

$$\nu_2 = \nu \nu_k \xi; \quad \nu_k = (GM/4\pi r^3)^{\frac{3}{2}}$$

(4)

with the Kerr modification parameter

$$\xi = 1 + j R_g^\frac{2}{3}; \quad R_g = R_s/2$$

(5)

$$j = J c/GM^2; \quad J = 2\pi I \nu_s$$

(6)

where $I$ is the moment of inertia, with the maximum value for the homogeneous sphere $I = (2/5)MR^2$. In the Schwarzschild geometry, $j = 0$, Eq\cite{11} recovers the conventional Keplerian frequency; $0 < j < 1$ represents a pro-grade orbit. To put the NS mass ($m = M/M_\odot$, radius and spin frequency parameters, we have the following simplified expressions,

$$j = 4\pi \nu_2 R^2/R_g C = (0.22/m)R_6^2(\nu_s/400 HZ)$$

(7)

$$\xi = 1 + (0.0013m)R_6(\nu_s/400 hz)$$

(8)

If we set the conventional values $M = 1.4M_\odot$, $R = 15km$ and $s = 400Hz$, then the Kerr modification parameter has about a 2% contribution to the Keplerian frequency, which cannot have too much influence on the kHz QPO model based on the Keplerian frequency. For the maximum spin frequency $1122Hz$, the Kerr modification contributes about 5% to the Schwarzschild spacetime, so this influence should be considered when we estimate the NS parameters.

4 Discussions and results

The kHz QPO emission positions are analyzed by the models (AWOM and RPM), which shows that most kHz QPOs (e.g. Sco X − 1) come from the regimes of several kilometers away from the stellar surface. This may correspond to the condition of a spinning up NS, since the detected NS spin frequencies are averaging $400Hz$\cite{16}, less than the upper frequencies. In RPM, the star mass can be derived by the detected twin kHz QPOs, then it usually
gives a value of 2 solar masses, higher than the typical NS mass of 1.4 solar masses. One reason for RPM’s prediction of high NS mass may be originating from its assumption of the vacuum circumstance around the star in introducing the perihelion precession term[15], but the accretion disk does not satisfy this clean condition. A value of about 3 solar masses for SAX J1808.4-3658 [17] (e.g. XTE 1807-294) is obtained, which seems to suggest that RPM should be modified. AWOM cannot predict a stellar mass by QPO but rather an averaged mass density \( A \sim M^{1/2}/R^{3/2} \), by which one can evaluate the equation of state (EOS) of the star. For the presently known kHz QPO frequencies, AWOM cannot give the prediction of quark matter[18] inside the star unless the QPO frequency over 1500 Hz is detected. In addition, the Kerr spacetime influence is investigated, and a 5% modification factor in Keplerian frequency exists for a high spin frequency of 1000 Hz, which will increase the estimation of the mass density parameter. Though the spectral properties of Cir X – 1 are typical of those of Z sources[19,20], its detected 11 pairs of kHz QPOs are generally low frequencies, 230 Hz to 500 Hz for the upper QPO and 56 Hz to 225 Hz for the lower QPO, increasing with accretion rate, which is contrary to those of the other LMXBs. The peak separation lies at 175-340 Hz, similar to those of other LMXBs [6]. Since the kHz QPO emitting positions of Cir X – 1 are estimated to be beyond the orbit of 25 kilometers, we guess that its rotating frequency is low, e.g. a hundred Hz.

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