ECCENTRICITY EVOLUTION OF MIGRATING PLANETS

N. Murray, 1,2 M. Paskowitz, 1 and M. Holman 3

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ABSTRACT

We examine the eccentricity evolution of a system of two planets locked in a mean motion resonance, in which either the outer or both planets lose energy and angular momentum. The sink of energy and angular momentum could be a gas or planetesimal disk. We analytically calculate the eccentricity damping rate in the case of a single planet migrating through a planetesimal disk. When the planetesimal disk is hot (the average eccentricity is much less than 1), the circularization time \( \tau_c \) is comparable to the inward migration time \( \tau_m \), as previous calculations have found for the case of a gas disk. If the planetesimal disk is cold (the average eccentricity is much less than 1), the circularization time \( \tau_c \) is comparable to the inward migration time \( \tau_m \), as previous calculations have found for the case of a gas disk. If the planetesimal disk is hot, \( \tau_m \) can be an order of magnitude shorter than \( \tau_m \). We show that the eccentricity of both planetary bodies can grow to large values, particularly if the inner body does not directly exchange energy or angular momentum with the disk. We present the results of numerical integrations of two migrating resonant planets showing rapid growth of eccentricity. We also present integrations in which a Jupiter-mass planet is forced to migrate inward through a system of 5–10 roughly Earth-mass planets. The migrating planet can eject or accrete the smaller bodies; roughly 5% of the mass (averaged over all the integrations) accretes onto the central star. The results are discussed in the context of the currently known extrasolar planetary systems.

Subject heading: planetary systems: formation

1. INTRODUCTION

The 60 or so extrasolar planetary systems known to date have revealed three striking features. 4 First, the distribution of orbital semimajor axes of the planets ranges from \( \sim 3 \) AU down to an almost incredible 0.038 AU. Second, most of the objects have high eccentricity by solar-system standards, with a typical value being around \( e = 0.4 \), but ranging up to 0.927. Third, the parent stars are highly metal rich, and some may have accreted iron-rich material after having reached the main sequence (Gonzalez et al. 2001; Santos, Israeli, & Mayor 2000; Laughlin 2000; Murray & Chaboyer 2001).

The simplest interpretation of the small orbits is that Jupiter-mass planets experience large-scale migrations in some cases, but not in others; Jupiter falls into the latter class. There are currently two viable explanations for the migration: tidal interactions between the planet and the gas disk out of which it formed (Goldreich & Tremaine 1980; Lin, Bodenheimer, & Richardson 1996), and gravitational interactions between the planet and a massive (1–5 Jupiter masses) planetesimal disk (Murray et al. 1998).

The most straightforward interpretation of the high eccentricities, that they result from collisions or near collisions of two or more Jupiter-mass planets, is appealing, but requires that most systems be dynamically unstable, in addition to undergoing migration. However, the ratio of the planetary escape velocity \( v_p \) to the orbital escape velocity \( v_{esc} \),

\[
v_{esc} = \sqrt{\frac{G M}{a_p}}
\]

\[
v_p = \sqrt{\frac{G (M + m_p)}{r_p}}
\]

is of order unity for \( a_p \approx 0.5 \) AU; here \( m_p \) and \( r_p \) are the mass and radius of the planet, \( a_p \) is the orbital semimajor axis, and \( M_p \) is the stellar mass. This fact suggests that the outcome of such a dynamical instability will typically be a merger. The body resulting from such a merger will be on a nearly circular orbit. Indeed, a recent exhaustive numerical study of the problem indicates that the fraction of observed systems with low eccentricities is smaller than would be produced by gravitational instability (Ford, Havlickova, & Rasio 2001).

In this paper we investigate another possible mechanism for producing large eccentricities: resonant migration. We suppose that a Jupiter-mass planet is forced to migrate inward, either by tidal torques or by ejection of planetesimals, and that a second (possibly much less massive) object is in a mean motion resonance with the first. We further assume that the migration process does not significantly damp the eccentricity of the inner body. This could occur in migration in a gas disk if the inner disk manages to drain onto the central star while leaving behind the planets and a substantial outer gas disk. It would almost inevitably occur in migration through a massive planetesimal disk, since the planetesimals are likely to accrete into terrestrial mass or larger bodies; we show below by direct numerical integrations that such terrestrial-mass bodies will be trapped into mean motion resonances.

We show that the inward migration of two planets trapped in a mean motion resonance can produce eccentricities \( e_p \approx 0.5 \) or larger. We also show that in the case of migration by planetesimal ejection, the final state may or may not have two resonant planets. Whether the distribution of eccentricity with planetary mass and semimajor axis produced by such resonant migrations is consistent with the observed distribution is a question left for later work.

As a by-product of our numerical simulations, we find that the fraction of planetesimal disk mass that accretes onto the star is likely to be much smaller than found in the work of Quillen & Holman (2000); that work studied the
accretion of massless test particles subject to gravitational perturbations from a migrating Jupiter-mass planet. The authors found that of the order of half the mass in the disk would accrete. Using our more realistic, but less extensive, integrations of massive planetesimals, we find a much smaller fraction (\(\sim 5\%\)) of the disk mass accreting onto the star in the early stages of the migration. Another \(\sim 5\%-10\%\) of the disk mass will fall on the star if the planet approaches within \(\sim 0.1\) AU (Hansen, Murray, & Holman 2001).

The remainder of the paper is organized as follows. Section 2 gives a short derivation of the eccentricity evolution in the case of planetesimal-driven migration. We assume that the planet does not accrete a substantial fraction of the planetesimals. This is appropriate when \(v_s\) is larger than \(v_{esc}\). As with the more commonly studied case of migration by tidal torques (Goldreich & Tremaine 1980), we find that the eccentricity of the planet is damped on a timescale \(\tau_e\) comparable to the migration time \(\tau_m\) when the planetesimal disk is cold, \(e \gg e_p \ll 1\). However, for a hot disk, \(e \gg e_p\), we find that \(\tau_e \approx \tau_m/10\). Section 3 describes the process of capture into resonance and the evolution of the eccentricities of both resonant bodies as the migration proceeds. Section 4 presents the results of numerical integrations of two resonant bodies, with parameters appropriate for planetesimal migrations, as well as integrations involving up to 11 planets. Section 5 gives a discussion of our results, and contrasts the two types of migration, while § 6 presents our conclusion.

2. MIGRATION AND ECCENTRICITY EVOLUTION

We examine the eccentricity evolution of a Jupiter-mass body migrating inward due to the extraction of energy and angular momentum. The energy \(E_p\) and total angular momentum \(L_p\) of the planet are given by

\[
L_p = m_p \sqrt{GM_\ast a_p(1 - e_p^2)} ,
\]

\[
E_p = -\frac{GM_\ast m_p}{2a_p} ,
\]

where \(G\) is Newton’s constant. Taking the time derivative of equations (1) and (2), we find the time variation of \(e_p\) in terms of the time variation of the planetary energy, assuming the planetary mass is fixed:

\[
\frac{e_p}{1 - e_p^2} \frac{de_p}{dt} = -\frac{1}{2} \left( \frac{1}{E_p} \frac{dE_p}{dt} \right) \left[ 1 + 2 \left( \frac{E_p}{L_p} \frac{dL_p}{dE_p} \right) \right] \quad (3)
\]

The quantity

\[
\beta \equiv \left[ 1 + 2 \left( \frac{E_p}{L_p} \frac{dL_p}{dE_p} \right) \right]
\]

is a convenient measure of the rate at which the planetary eccentricity changes. Both \(E_p\) and \(dE_p/dt\) are negative for an inward migration, so \(e_p\) decreases if \(\beta > 0\). Conservation of energy and angular momentum implies that \(dL_p/dE_p = (dL/dE)_T\), where the latter quantity is the ratio of the rates at which angular momentum and energy are removed from the system by whatever process is driving the migration. The planetary eccentricity decreases as long as

\[
\left( \frac{dL}{dE} \right)_T < -\frac{L_p^2}{2E_p} \frac{1 - e_p^2}{n_p} ,
\]

where \(n_p = [G(M_\ast + m_p/a_p^3)]^{1/2} \approx (GM_\ast/a_p^3)^{1/2}\).

We now specialize to the case of planetesimal migration. To find \(dL_p/dE_p\), we calculate the total change in \(E\) and \(L\) for a planetesimal of mass \(m\) (where \(m \ll m_p\)) from its initial orbit, with semimajor axis \(a\) and eccentricity \(e\), to the point at which it is ejected, then use conservation of energy and angular momentum. We note that this is not adequate for cases in which the planet eats the planetesimal, since some orbital energy will be lost in the form of radiation in that case.

Figure 1 illustrates the constraints on the evolution of a planetesimal in the \(E-L\) plane, in the case \(e_p = 0.1\). The solid curved line corresponds to \(e = 0\); planetesimal orbits must lie to the left of this line and cannot transit the region to the right of the line. In analyzing motion involving close encounters with a planet, it is useful to introduce the Jacobi parameter \(J = E - n_a L\); it is useful because in the limit \(e_p \rightarrow 0, J = \text{const}\), and even when \(e_p \neq 0\), the time or ensemble average of \(J\) is constant. The diagonal solid line corresponds to \(J/m = -3/2\) (in units where \(GM_\ast = a_p = 1\)); this corresponds to the Jacobi parameter for \(a = a_p, e = 0\). We note that in order to be ejected from \(a < a_p\), the asteroid must reach \(J/m \geq -3/2\), since it must pass through \(a = a_p\) with \(e \geq 0\) to be ejected.

We assume that the planetesimal is ejected with zero total energy; if it is ejected with a larger energy, the damping rate will be smaller than the estimate we obtain below. The initial energy and momentum of the planetesimal are given by expressions analogous to equations (1) and (2). Rather than calculating the change in \(L\) directly, we calculate the change in the Jacobi parameter. We do so because while \(J\) undergoes a random walk when \(e_p \neq 0\), the average value of...
$J$ is constant during the planet-crossing phase of the asteroid's evolution for those asteroids with initial $a \approx a_p$ (Öpik 1976). To lowest order in $m/m_p$, we have

$$
\frac{dL}{dE} = \frac{(1 - dJ/dE)}{n_p}. \tag{6}
$$

If the planetesimal disk is originally cold ($e \ll 1$) and $e_p \ll 1$, few planetesimals will cross the orbit of the planet. However, planetesimals trapped in resonance with the planet will suffer chaotic perturbations that on average transfer angular momentum, but not energy, from the asteroid to the planet. This causes $J$ to increase, while leaving $E$ fixed. Once enough angular momentum has been removed from the asteroid's orbit, the asteroid can suffer close encounters with the planet. The first close encounter that changes the energy of the asteroid by a significant amount removes the asteroid from the resonance; because the initial value of $e_p$ is small, this encounter leaves $J$ fixed. Subsequent encounters extract or supply energy and angular momentum to the asteroid in such a way as to leave $J$ constant on average, as noted above. Eventually, the planetesimal is ejected with $E \geq 0$; taking $E = 0$, we find

$$
\frac{dJ}{dE} = 2\left(\frac{a}{a_p}\right)^{3/2} \left(\sqrt{1 - e^2} - \sqrt{1 - e_p^2}\right), \tag{7}
$$

where $e_c \equiv a_p(1 - e_p)/a - 1$ is the eccentricity at which the planetesimal just crosses the orbit of the planet. Note that $dJ/dE \geq 0$. In arriving at equation (7), we have assumed that the final Jacobi parameter

$$
\frac{J_f}{m} = \frac{J_e}{m} = -\frac{2a}{2a} - \sqrt{a(1-e_c^2)} > -3/2 \tag{8}
$$

(we again use $GM_\ast = a_p = 1$, so that $n_p = 1$).

If the Jacobi parameter at planet crossing ($J_e$) is not larger than $-3m/2$, the planetesimal must diffuse to higher $J$ in order to be ejected, since it has to get past the solid curve in Figure 1. Setting $J_f/m = -3$, we find

$$
\frac{dL}{dE} = 2\left(\frac{a}{a_p}\right)^{3/2} \left[\frac{1}{2} e^2 + \frac{3}{2} \left(\frac{a}{a_p}\right)^{1/2}\right]. \tag{9}
$$

When $J_e/m > -3/2$, equation (7) should be used, while equation (9) is appropriate if $J_e/m < -3/2$.

Combining equations (3) and (6), the expression for the rate of change of the planet's eccentricity is

$$
e_p \frac{d_e}{1 - e_e^2} \frac{dt}{dE} = -\frac{1}{2} \left(\frac{1}{E_p} \frac{dE_p}{dt}\right) \left[1 - \frac{(1 - dJ/dE)}{\sqrt{1 - e_p^2}} \right] > -\frac{1}{2} \left(\frac{1}{E_p} \frac{dE_p}{dt}\right)^2 \tag{10}
$$

It can be shown using equations (7) and (9) that

$$
\frac{dJ}{dE} \geq 1 - \sqrt{1 - e_p^2}. \tag{11}
$$

Equations (6), (10), and (11) show that inward planetesimal migration as described here always damps the eccentricity of a single planet. We can estimate the value of $dJ/dE$ when $e$ and $e_p$ are small; for example, for $J_e/m < -3/2$ [which requires $e \lesssim (3 + \sqrt{3})e_p + O(e_p^2)$], equation (9) evaluated at the maximum $a/a_p = (1 - e_p)/(1 + e)$ gives

$$
\frac{dJ}{dE} \approx \frac{3}{4} e_p^2 + \frac{3}{2} e_p e - \frac{1}{4} e^2. \tag{12}
$$

For $e \approx e_p = 0.05$, we find $dJ/dE \approx 2e_p^2 \approx 0.005$.

For smaller values of $a$, this increases, as shown in Figure 2. Figure 2 (left panel) shows $\beta$ and $dJ/dE$ for a planet with $e_p = 0.05$ ejecting a planetesimal with initial $e = 0.05$, as a function of the initial $a$ of the planetesimal. The relevant value of $dJ/dE$ depends on the average initial $a$ of the planetesimals that are ejected, i.e., it depends on which resonance is most actively ejecting objects. Early on in the evolution of the system, we expect the planetesimal disk to be truncated inside the chaotic zone produced by the overlap of first-order mean motion resonances (the $\mu^{1/7}$ chaotic region); Wisdom 1980). Under those circumstances, the relevant resonance is the 5:3, at $a/a_p \approx 0.71$ and $dJ/dE \approx 0.066$ for that case. However, as the migration proceeds, material is supplied to the $\mu^{1/7}$ zone, and $dJ/dE$ will be on average smaller, at least while $e_p \ll 1$.

The discussion in the previous paragraph assumes that the average $e$ of planetesimals is small. If the planetesimal disk is hot ($e \gg e_p$), then planetesimals with $a \ll a_p$ can be ejected by the planet. This will lead to much larger eccentricity damping rates (large $\beta$), as shown by the dotted line in Figure 2 (left panel), which illustrates a case with $e_p = 0.05$, but $e = 0.5$. The minimum $\beta$ is about 0.01, already larger than $e_p^2$. However, even high-order resonances are strongly chaotic for such large $e$, so that many resonances at small $a$ [but with $a > 0.5(1 - e_p)$] will be active. As the plot shows, typically $\beta \approx 0.2$, 10–100 times larger than $e_p^2$.

For small $m_1$, where $m_1$ is the mass of the inner resonant planet (see below), the planetesimal disk will not be significantly heated before the Jupiter-mass planet sweeps through. However, if $m_1$ is of the order of a Jupiter mass, then the inner planet will heat the disk before the outer planet has a chance to interact with the planetesimals. In that case, $e_p$ could be limited to rather small values.

As $e$ increases, equations (7) and (9) show that $dJ/dE$ no longer increases as rapidly as $e_p^2$, it effectively saturates near $dJ/dE \approx 0.3–0.4$. Figure 2 (right panel) shows $\beta$ and $dJ/dE$ in the case $e_p = e = 0.5$. Planetesimals never get the chance to reach the 5:3 resonance, since they start crossing the planet orbit at a much smaller semimajor axis. The damping rate is of the order of 0.3.

We can calculate the circularization time for planetesimal migration in terms of the migration time. Following the notation employed in satellite studies (Lissauer, Peale, & Cuzzi 1984),

$$
\frac{dE_p}{dt} = -n_p T + H, \tag{13}
$$

$$
\frac{dL_p}{dt} = -T, \tag{14}
$$

where $T$ is the (average) torque exerted on the planet by the ejection of planetesimals, and $H$ is responsible for removing energy from the radial motion of the planet, i.e., it damps the eccentricity. (Actually, it would be better to use $dE_p/dt = -[n_p T/(1 - e_p^2)^{1/2} + H]$, which becomes appa-
ent below). It follows from these equations that

\[ H = -\beta \frac{dE_p}{dt}, \]

(15)

where we have replaced the missing factor of \((1 - e_p^2)^{1/2}\). Using this in equation (13), we define the migration time

\[ \tau_m \equiv \frac{a}{da/dt} = \frac{GM_* m_p^2}{2a_T h_p T^2} (1 - \beta). \]

(16)

The circularization time \(\tau_c\) is given by equation (3):

\[ \tau_c \equiv \frac{e_p}{de_p/dt} = \frac{2e_p^2}{\beta(1 - e_p^2)}, \]

(17)

For small \(e\) and \(e_p\), \(\beta\) is a few times \(e^2_p\), and this reduces to \(\tau_c \approx \tau_m\); the circularization time is somewhat shorter than the migration time. As noted above, if the planetesimal disk is hot \((e \gg e_p)\), then \(\beta \gg e_p^2\) and \(\tau_c \approx \tau_m\). This might arise if two Jupiter-mass planets migrate together (see § 3.1 below).

Note that in deriving equations (7) and (9), we have ignored the finite extent \(D \equiv (m_p/3M_p)^{1/3}a_p\) of the planet's Hill sphere, the region over which the planet's gravity exceeds the tidal acceleration from the central star. For a Jupiter-mass planet, \(D \approx 0.07a_p\). Including the effect of the Hill sphere in our analysis effectively increases \(e_p\) to \(e_p^* = e_p + (m_p/3M_p)^{1/3}\); this is why we use \(e_p = 0.1\) in Figure 1.

3. RESONANCE CAPTURE AND ECCENTRICITY EVOLUTION

We have seen that a single planet embedded in a planetesimal disk suffers eccentricity damping; it appears that a similar statement applies to a single planet in a gas disk (Papaloizou, Nelson, & Masset 2001). However, a Jupiter-mass planet migrating through a disk of planetesimals will capture bodies into resonance; in the early stages, this is how the migration proceeds. We show in this section that these resonant bodies tend to increase the eccentricity of the Jupiter-mass planet; if \(10 - 20\) Earth masses (denoted \(M_\oplus\)) are trapped into resonance, then this resonant eccentricity driving exceeds the eccentricity damping described in the previous section, and the eccentricity of the planet will increase as the planet migrates inward.

We begin by describing capture into resonance. We consider the gravitational interaction of two planets in orbit around a much more massive central body. For simplicity, we consider only the planar problem. In the absence of dissipative effects, the Hamiltonian describing the motion is

\[ H = -\frac{\mu_1^2 m_1}{2L_1} - \frac{\mu_2^2 m_p}{2L_p} \]

\[-\left[ \frac{Gm_1 m_p}{a_1} \sum_j \Phi_j(a_1, a_p)e_p^{j_1}e_p^{j_2} \right] \times \cos \left( j_1 \lambda - j_p \lambda_p + j_3 \sigma_1 + j_4 \sigma_p \right). \]

(18)

Here \(\mu_1 \equiv G(M_\oplus + m_1)\), where \(L_1 = (\mu_1 a_1)^{1/2}\), with similar definitions for the outer planet (labeled with a subscript \(p\)). The third term in equation (18) represents the mutual perturbations of the two planets. It produces variations in the orbital elements \((a, e, \text{and so forth})\) of the order of the planetary mass \(m_1\).

The coefficient \(\Phi \sim [a_1/(a_p - a_1)]^{1/2} - j_p\) (Holman & Murray 1996). The integers \(j_i\) satisfy the relation \(j_1 - j_p + j_3\)
+ j_k = 0. Each cosine term in the sum is referred to as a resonant term or simply as a resonance. Resonances with $|j_1 - j_p| = q$ are proportional to $q$ powers of eccentricity, and are said to be $q$th-order mean motion resonances. The planets are said to be in resonance if one or more of the arguments of the cosines are bounded. Since $n_1 \equiv \langle \dot{j}_1 \rangle$ (where the angle brackets refer to an average over a single orbit) and $n_p$ are much larger than $n_1$ and $m_p$, the condition for resonance is roughly equivalent to

$$j_1 n_1 - j_p n_p = 0,$$  \hspace{1cm} (19)

or $a_p/a_1 = (j_p/j_1)^{2/3}$. Throughout this section, we ignore nonresonant terms of second order in the planetary masses.

Suppose that $a_p/a_1$ is initially larger than this resonant value, but that some dissipative process acts to reduce $a_p$, while leaving $a_1$ unchanged. Then the torques represented by the resonant cosine term will increase, since $\Delta a \equiv a_p - a_1$ is decreasing, and the torques are proportional to $(a_1/\Delta a)^2$. Another way to say this is that the depth of the potential well represented by the resonant term is increasing, as is the width of the resonance. As $a_p/a_1$ passes through the resonant value $(j_1/j_p)^{2/3}$, the planets may be trapped into resonance. The torques represented by the resonant term in equation (18) will then transfer energy and angular momentum between the two planets in just such a way as to maintain the resonance while both bodies move toward the sun (Goldreich 1965).

Capture is much less likely if the planets are moving away from each other, for example, if $a_1$ is decreasing, or if both semimajor axes are decreasing, but that of the inner planet decreases more rapidly; in that case, the size of the resonance is decreasing, and the planets will usually pass through the resonance without being trapped.

Henceforth, we assume that the outer planet is moving toward the inner planet, resulting in capture into resonance.

Suppose for the moment that the inner planet has a sufficiently small mass that it cannot effectively scatter planetesimals, so that it does not lose energy or angular momentum directly to the planetesimal bath. (In the case of gas migration, we assume that the inner disk is nonexistent.) By virtue of its resonance interaction with the outer planet, it nevertheless does supply energy and angular momentum indirectly to material driving the migration. Another way to say this is that $dE_p/dL_p$ no longer equals $(dE/dL)_T$, the quantity calculated for the case of planetesimal migration in the previous section. Here we calculate the relation between these two quantities that obtains when a second object of mass $m_1$ is in resonance with the Jupiter-mass object.

We relate $dE_p/dL_p$ to $(dE/dL)_T$ using the conservation of energy and angular momentum. Conservation of energy gives

$$\frac{dE}{dE_p} = 1 + \frac{m_1}{m_p} \left( \frac{j_1}{j_p} \right)^{2/3},$$  \hspace{1cm} (20)

while conservation of angular momentum implies

$$\frac{dL_p}{dE_p} = \left( \frac{dL}{dE} \right)_T \left[ 1 + \frac{m_1}{m_p} \left( \frac{j_1}{j_p} \right)^{2/3} \right] - \frac{dL_1}{dE_p}.$$  \hspace{1cm} (21)

We also need

$$\frac{L_1}{L_p} = \frac{m_1}{m_p} \left( \frac{j_1}{j_p} \right)^{1/3} \sqrt{\frac{1 - e_1^2}{1 - e_p^2}}.$$  \hspace{1cm} (22)

Using all these in equation (3), we find

$$\frac{e_p}{1 - e_p^2} \frac{de_p}{dt} = -\frac{1}{2} \left( \frac{dE_p}{dE} \right)_T \times \left[ 1 + 2 \frac{E_p}{L_p} \frac{dL}{dE} \right]$$

\[\times \left( \frac{dE}{dE_p} \right)_T \frac{de_p}{dt},\]  \hspace{1cm} (23)

Note that the $dE/dE_p$ factor multiplying $(dL/dE)_T$ is larger than 1; it effectively increases $(dL/dE)_T$. From equation (5), we see that this tends to increase $e_p$. The term proportional to $dL_1$ tends to decrease $E_p$, but we show below that its effect is smaller than that of the term involving $dE/dE_p$; this is the origin of the increase in eccentricity of two resonant bodies undergoing migration. From the expression for $L_1$, and using the resonance condition, we have

$$\frac{1}{L_1} \frac{dL_1}{dt} = -\frac{1}{2} \frac{1}{E_p} \frac{dE_p}{dt} - \frac{e_1}{1 - e_1^2} \frac{de_1}{dt}.$$  \hspace{1cm} (24)

As just noted, the first term on the right tends to damp the eccentricity of the outer planet; the second term on the right also damps $e_p$, as long as $de_1/dt > 0$. Combining the last two equations, we find

$$\frac{e_p}{1 - e_p^2} \frac{de_p}{dt} = -\frac{1}{2} \left( \frac{dE_p}{dE} \right)_T \left[ \frac{1}{1 - e_1^2} \frac{de_1}{dt} \right]$$

\[\times \left( \frac{dE}{dE_p} \right)_T \frac{dE}{dt} \frac{dE_p}{dt} \]  \hspace{1cm} (26)

where we have added and subtracted $(m_1/m_p)(j_1/j_p)^{2/3}$ inside the curly brackets.

We identify

$$\frac{dE_p}{dt} = \frac{dE}{dt},$$

as the rate at which the expulsion of planetesimals removes energy from the outer planet. With this identification, and recalling equation (3), it becomes clear that the first term in curly brackets in equation (25) is the time rate of change of $e_p$ due to the expulsion of planetesimals. The final result is

$$\frac{e_p}{1 - e_p^2} \frac{de_p}{dt} = -\frac{1}{2} \left( \frac{dE_p}{dE} \right)_T \left[ 1 - \left( \frac{1 - dL/dE}{\sqrt{1 - e_p^2}} \right) \right]$$

\[\times \left( \frac{dE}{dE_p} \right)_T \frac{dE}{dt} \frac{dE_p}{dt} \]  \hspace{1cm} (27)
Note that this expression reduces to equation (10) when $m_1 \to 0$.

Equation (27) has two undetermined quantities, $de_p/dt$ and $de_1/dt$. Bodies that are trapped in a mean motion resonance typically have their apsidal lines locked as well, so that $\sigma_1 = \sigma_2$. Using the equations of motion for $\sigma_1$ and $\sigma_2$, we can find a relation between $e_1$ and $e_p$ that depends on the precession rates of the apsidal lines. The latter are determined both by the mutual perturbations of the two planets and by the distribution of mass in the planetesimal (or gas) disk. When the eccentricities are small, we can obtain an analytic expression. However, the eccentricities of many of the currently known planetary systems are not small, and the analytic expressions are not useful. One will likely have to resort to numerical integrations to make detailed comparisons with such systems.

In the Appendix, we present another derivation in which we allow for the possibility that the inner, less massive planet is comparable to or more massive than the inner body. We expect resonance capture of terrestrial bodies with masses ranging upward from $1M_\oplus$; for migration in a gas disk, the inner body must be massive enough to open a gap (else it migrates inward faster than the outer planet and will not be captured), suggesting a lower limit of $\sim 100M_\oplus$. The plausible range for $\gamma$ is then $5 \times 10^{-3} - 4 \times 10^{-1}$. For migration in a planetsesimal disk, we expect $\beta$ to be in the range 0.01 or less (for $e_p \ll 1$) to 0.3 (for $e_p \geq 0.5$). Taking 1 AU as a representative value for $a_p$ (although some extrasolar planets are in much smaller orbits), with $a_p \sim 5$--10 AU, we find final eccentricities in the range 0.1--0.6, with $e_p \approx 0.45$ being a typical value.

3.1. Two Jupiter-Mass Planets

The situation is likely to be more subtle when both bodies are of roughly Jupiter mass; in that case, the inner body also experiences a tidal force that tends to decrease $e_1$, since it will efficiently scatter bodies from small $a$ to larger $a$. This scattering will tend to damp $e_1$, for the same reason that the eccentricity of the outer planet is damped. The scattering also tends to reduce $a_1$, supplementing the resonant forcing provided by the outer planet's inward migration. From equation (A10) in the Appendix, we see that an inward tidal force on the inner body tends to reduce the rate of eccentricity growth of both bodies.

Finally, there is likely to be an increase in the damping rate $\beta$ experienced by the outer planet when the inner planet is a Jupiter-mass planet. The reason for this increase can be seen in Figure 2 (left panel: dotted line). When the inner planet is massive enough to efficiently perturb and scatter planetesimals, the outer planet has access to planetesimals with small $a$, e.g., $a \sim 0.55$--0.6, if the inner planet is trapped in the 2:1 resonance. This is true even when $e_p$ is small, e.g., $e_p \approx 0.05$, as in the figure, since the inner planet will scatter planetesimals into orbits with large $e$. As long as the outer planet is comparable to or more massive than the inner planet, the outer planet will eject a larger fraction of the planetesimals, since $v_p/v_{esc}$ is larger for the outer planet. From Figure 2 (left panel), we see that $\beta$ is 0.1 for such planetesimals; this is more than an order of magnitude larger than $e_p^2$ for $e_p \approx 0.05$. This will reduce the circularization time for the outer planet by a factor of 10 or more relative to the estimate given above, so that $\tau_c \approx \tau_{esc}/10$ or less. This suggests that the eccentricity of the outer
planet may be smaller than in the case of a large mass ratio $m_p/m_1$, where the inner planet cannot efficiently scatter planetesimals.

The case of two well-separated Jupiter-mass planets migrating inward may be intermediate between the (roughly) equal-mass resonant migration case and the resonant migration of a single Jupiter-mass planet with a much smaller mass object. The passage of the first Jupiter-mass planet will heat the planetesimal disk (as well as reducing its mass), so that the second, distant (nonresonant) planet will encounter planetesimals with a wide range of eccentricities. For that reason, the outer planet is likely to experience stronger eccentricity damping than in the unequal-mass resonant case. However, the damping is likely to be weaker than in the case of two equal-mass bodies migrating in resonance. In a resonant migration, the outer body will interact with the population of high-$e$ planetesimals that the inner planet is in the process of ejecting; in a nonresonant migration, the inner planet is more likely to eject the high-$e$ planetesimals before they get a chance to interact with the outer planet. This leaves fewer high-$e$ planetesimals to damp the eccentricity of the outer planet. Performing a numerical experiment to test this hypothesis will be a daunting task.

4. Numerical Calculations

For most runs, we employ a Bulirsch-Stoer integrator with a variable step size. We require that at each time step, the relative accuracy of the integration (as measured in phase space) be $10^{-12}$. Typical orbital periods are of the order of $1$–$10$ yr, while the integrations can extend up to $10^8$ yr. In test runs in which no energy is removed from the planet, the largest variation in total energy is typically less than a part in $10^9$. In most of the runs reported on here, we remove energy and angular momentum from the largest planet; the variations in energy and angular momentum from the expected amounts are similarly small. We also have employed a Wisdom-Holman integrator (Wisdom & Holman 1991, 1992), modified to account for the removal of energy and angular momentum in the case of only two planets. The results using the two different integrators are essentially the same.

In runs with multiple massive planets, collisions often occur. We assume that the smaller planets are rocky bodies with bulk densities of $3\,\text{g cm}^{-3}$, that a collision occurs when planets are within 2 times the sum of the planetary radii (to allow for capture by tidal disruptions), and that the captures are completely inelastic, with no loss of mass to small fragments. This is reasonable for collisions involving the more massive objects.

We assume that the most massive body (“Jupiter”) migrates inward by ejecting numerous planetesimals from the system (Murray et al. 1998). We simulate this by extracting energy and angular momentum from the orbit of Jupiter. We implement this numerically as follows.

The energy and angular momentum of the planet are given by

$$ E = \frac{1}{2} m_p(v_x^2 + v_y^2) - \frac{GM_p m_2}{r} \quad (34) $$

and

$$ L = m_p(v_x v_y - y v_x) \quad , \quad (35) $$

where $v_x$ and $v_y$ are the velocity of the planet (we suppress the subscript $p$ for ease of reading). Recall that we assume the planet and planetesimal are coplanar. Taking time derivatives, we invert to find

$$ \frac{dv_x}{dt} = \left( \frac{1}{E_p} \frac{dE_p}{dt} \right) \frac{E_p}{m_p r} \cdot \mathbf{v} \left[ x + \frac{v_x}{n_p} \sqrt{1 - e_p^2} (\beta - 1) \right] , $$

$$ \frac{dv_y}{dt} = \left( \frac{1}{E_p} \frac{dE_p}{dt} \right) \frac{E_p}{m_p r} \cdot \mathbf{v} \left[ y - \frac{v_x}{n_p} \sqrt{1 - e_p^2} (\beta - 1) \right] , \quad (36) $$

where we have used equation (4). We assume that the close encounters that lead to changes in the planetesimal’s orbital elements occur on timescales much shorter than the orbital period, so that we can take the position of the planet to be fixed.

Using these equations for numerical work is problematic, since the vector dot product in the denominator vanishes at periapse and apoapse. We regularize the equations by multiplying the right-hand side by $2\sin^2 f$, where $f$ is the true anomaly.

If the terrestrial-mass objects in our simulations do not have high enough escape velocities to efficiently eject smaller bodies, we do not force them to migrate.

We have integrated the equations of motion for the two-body problem (the star and a massive planet), modified to account for the drag imposed by the ejection of planetesimals, as given in equation (36), regularized as noted above. The eccentricity $e_p$ and semimajor axis $a_p$ decay as expected. We discuss the behavior of systems with more than one massive planet in the following subsections.

4.1. Two Massive Bodies

To test the basic idea that the migration of two resonant bodies induces the growth of eccentricity, we have started two planets just outside resonance and have applied the “tides” described in the previous paragraphs. An example is shown in Figure 3. In Figure 3 (left panel), the inner body has a mass of $40M_\oplus$, and the outer body has a mass equal to that of Jupiter ($\approx 318M_\oplus$). The bottom plot shows the semimajor axes of both bodies as a function of time, while the top two plots show the eccentricities. Energy and angular momentum are removed only from the outer body. The resonance interaction forces the inner body to migrate inward as well, as can be seen in the figure. One can also see from the figure that the eccentricity of both bodies increases.

The value of $e_1$ predicted by equation (29) is shown as the smooth line in Figure 3 (left panel). Since this expression ignores the effect of the precession frequency $\omega$ on the evolution, as noted below equation (27), it does not capture the fact that $e_1$ has a maximum value less than unity.

The prediction based on equation (33) for the eccentricity of the outer, more massive body is shown as the smooth line in the top panel. The predicted $e_1$ is too large, which is consistent with the fact that we neglected $de_1/dt$ in deriving equation (33). By the end of the integration equation, the relative error is about 15%.

These results support the notion that the inward migration of a Jupiter-mass object can result in the resonant capture of nonmigrating bodies in orbits inside that of the migrating planet, and that subsequently the eccentricities of both planets will increase. The rate at which $e_1$ and $e_p$
increase are qualitatively consistent with the analytic estimates given above. Comparisons with observed multiplanet systems will require extensive numerical integrations, but even then, uncertainties in such basic parameters as the eccentricity damping rate will make quantitative comparisons difficult.

Figure 3 (right panel) shows the migration of two planets with a mass ratio of 1:3, and a period ratio of 2:1, similar to the properties of the system in GJ 876. The predictions of the analytic theory are shown as the nonoscillatory lines; the relative error for the outer planet is $\leq 50\%$ in this case.

4.2. Multiple Massive Bodies

The planetesimal disk we postulate is very massive, 1–3 Jupiter masses. It is likely that multiple bodies with masses comparable to or larger than that of the Earth will form in such a massive disk. As a first step toward a realistic simulation of migration in such a disk, we have run a number of cases involving 5–10 roughly Earth mass bodies placed in orbits with random semimajor axes and small eccentricities inside the orbit of a Jupiter-mass planet. We then force the Jupiter-mass body to migrate inward toward the Earth-mass planets.

Figure 4 shows the result of one such integration. We started five bodies with masses randomly distributed between $0.3M_\oplus$ and $10M_\oplus$, with semimajor axes between 0.5 and 4 AU. As Jupiter migrated inward, three of the small planets merged to form a $6.3M_\oplus$ planet, one small planet crashed into Jupiter, and one small planet was ejected. Both the latter two events illustrate the migration mechanism we are postulating.

After the three small planets merged to form a $6.3M_\oplus$ body, the resulting planet was captured into the 3:1 mean motion resonance with Jupiter. Subsequently, the eccentricities of both bodies increased (we employed a rather low value of $\beta/\epsilon_p^2$ for this run). Both planets migrated inward until the inner planet struck Jupiter, when Jupiter was at 0.12 AU, with an eccentricity of 0.4.

In this run, most of the mass in the disk actually accreted onto the Jupiter-mass planet. Part of the reason for this was that the smaller planets were started at small radii, where the escape velocity from the system exceeded the escape velocity from the Jupiter-mass planet; in that case, we expect that most planetesimals will accrete onto Jupiter rather than be ejected from the system. Only one body was ejected. On the other hand, no bodies hit the star.

The latter result is representative of most of our runs; the fraction of mass accreting onto the star is small, $\sim 5\%$. The fraction ejected varies with the initial semimajor axis and the mass of the Jupiter-mass body; both larger initial $a_p$ and larger $m_p$ produce a larger fraction of ejected bodies (relative to bodies accreted onto the massive planet). These rather low accretion fractions are in stark contrast to those found by Quillen & Holman (2000). The difference appears to be that we employ massive planetesimals, while their simulations employed only test particles, which did not interact with each other. In our simulations, only the second-most massive body (the first being “Jupiter”) remains for long in a resonance; this second-most massive body lords it over his smaller brethren, kicking them out of nearby resonances that they might like to occupy. This tends to prevent the smaller objects from reaching the extremely high eccentricities (greater than 0.9) needed to strike the star.

In other runs, the final state includes two planets in a mean motion resonance. Since we start with such low planetesimal masses and numbers, the mass ratio is always large. However, we expect that if we allow larger terrestrial bodies
FIG. 4.—Evolution of a system of five bodies with masses randomly distributed between 0.3$M_\oplus$ and 10$M_\oplus$, placed inside the orbit of a Jupiter-mass object that is forced to migrate inward. Three of the small planets merged with each other to create a 6.3$M_\oplus$ mass planet. One of the small planets merged with the Jupiter-mass planet, while the fifth small planet was ejected. Left: Semimajor axis of one of the low-mass planets that merged to form the mass body. The mass increases with time because of collisions with other bodies, seen as jumps in $a$. This body was caught into resonance with the Jupiter-mass planet at $t = 9 \times 10^6$ yr. Right: Eccentricity of the Jupiter-mass body as a function of time. It damps slowly up until the time it captures the smaller mass body in the left panel, then rises rapidly. The value of $\beta$ used in this run assumed that $e = 0.05$, which is not appropriate for the final $e_f \approx 0.4$.

to grow, we may well find final states with mass ratios nearer to unity. Finally, we note that recent simulations of planetesimal migration show that two Jupiter-mass objects placed in a planetesimal disk will on some occasions migrate toward each other (Hansen et al. 2001). This could lead to resonance capture followed by inward migration. As noted in §3.1, interactions between the massive bodies and the planetesimal disk would likely tend to damp the eccentricity of both bodies. This would reduce the final eccentricities relative to the case we have studied here.

5. DISCUSSION

The mechanism we have proposed for the growth of eccentricity with inward migration is essentially the same as that used to explain the nonzero eccentricities of the inner Jovian satellites. It is well understood and quite robust. It does not rely on the details of the migration mechanism; in the case of the Jovian satellites, the migration is a result of the tidal bulge raised by Io on Jupiter; the bulge exerts a torque on Io that transfers energy from Jupiter’s spin to the orbit of Io. Io in turn exerts, through a 2:1 mean motion resonance, a torque on Europa. In the satellite case, the eccentricity damping, produced by tidal flexing in both satellites as they oscillate from peri- to apo-Jove, is very strong. This limits the eccentricity to a value that, while small, is sufficient to dissipate enough energy to power the volcanism on Io.

5.1. Resonant Migration by Tidal Torques

In the previous section, we examined resonant eccentricity growth in the context of planetesimal migration, but it can work in the context of migration due to tidal torques imposed by a gas disk as well. (After this paper was submitted, we learned that another group was exploring this possibility: Snellgrove, Papaloizou, & Nelson 2001.) Suppose that two Jupiter-mass bodies embedded in a gas disk are locked in a mean motion resonance. Suppose that the gas between the planets is removed, as numerical integrations indicate (Kley 2000; Bryden et al. 2000). The gas inside the orbit of the inner planet will accrete onto the star, possibly with some fraction being removed by a disk or stellar wind. The planets are likely to follow the inner disk inward; if they do not, the normal viscous spreading of the inner disk would move the outer edge of the disk outward, until it experiences tidal torques from the inner planet. This interaction would produce a back reaction that would tend to damp the eccentricity of the inner planet. Large planetary eccentricities are unlikely to arise in that case.

However, it may be possible that the inner disk drains onto the star, leaving the planets behind. This could occur, for example, if the inner disk had a mass only slightly larger than that of the planets (Nelson et al. 2000). Then only the outer planet would experience significant tidal torques, since the first-order resonances of the inner planet lie in the region between the planets that is depleted of gas. Both planets would then migrate inward, and the eccentricity of both would grow. This is exactly analogous to the planetesimal migration described above, and the expressions we have given describe the growth and equilibrium values of the eccentricity once the appropriate eccentricity damping rate for the outer planet is introduced, namely $\tau_e \approx \tau_m$, see, e.g., Goldreich & Tremaine (1980).

In addition to the lack of an inner gas disk, the tidal-torque scenario also requires that both planets have a mass sufficient to open a gap in the gas disk. If not, then one or
both bodies will undergo type I migration (Ward 1997), making capture or even survival unlikely, as well as leading to eccentricity damping on both bodies. An approximate criterion for gap formation is (Lin & Papaloizou 1986)

$$\frac{m}{M_\ast} \gtrsim 40\pi^2 \left( \frac{c_s/v_k}{a} \right)^2,$$

(37)

where $x$ is the (Shakura & Sunyaev 1973) viscosity parameter, $c_s$ is the sound speed in the gas disk, and $v_k$ is the Keplerian rotation velocity. If the disk is ionized, then the Balbus-Hawley instability (Balbus & Hawley 1991; Hawley & Balbus 1991) is likely to produce a rather larger (effective) $x$ of the order of 0.5. The planet must then have a mass of the order of 200 Jupiter masses ($M_\ast$) in order to open a gap. At small orbital radii $a \lesssim 0.1$ AU, the disk will be ionized (Gammie 1996). This suggests that if the capture into resonance occurs at very small radii, or if the planets migrate to very small radii, the eccentricity of both bodies will be damped.

However, at larger radii, protoplanetary disks are believed to be substantially neutral, so that they are not subject to the Balbus-Hawley instability (Gammie 1996). If so, they will likely have a small effective viscosity, and equation (37) predicts that gap opening will occur for small- (sub–Jupiter-) mass planets. In terms of $x$, currently favored viscosities have $x$ in the range $10^{-5} - 10^{-2}$. The latter value yields a mass for a gap-clearing planet of about 2$M_\ast$. Smaller values of $x$ would yield smaller masses, but it is believed that in that case, a second criterion is relevant, namely $m/M_\ast > 3(c_s/v_k)^2$ (Papaloizou & Lin 1984). At 5 AU, this yields $m \gtrsim M_\ast$, falling to about $m \gtrsim 0.3M_\ast$ at 0.2 AU. Bodies less massive than this will not open gaps in the disk, and will undergo rapid type I migration.

Another constraint is that the migration torque not exceed the resonant torque. If it does, the resonance will be broken, and the eccentricity of the outer body will drop. The outer body may then migrate inward, perhaps to be captured into a stronger resonance. This constraint is likely to be important in the capture phase, particularly in a migration produced by tidal torques. In that situation, both planets are likely to have very small eccentricities. The tidal torque is given by (Ward 1997)

$$T_{\text{disk}} \approx \left( \frac{GM_\ast m_p}{2a} \right) \frac{m_p}{M_\ast} \frac{M_{\text{disk}}}{M_\ast} \left( \frac{v_k}{c_s} \right)^3,$$

(38)

assuming that the outer planet does not open a gap. For a disk of mass $M_{\text{disk}} = 10^{-2}M_\ast$ and a planet at 5 AU, this is about 50($GM_\ast m_p/2a$). If the outer planet is massive enough to open a gap, the torque is set by the viscosity in the gas disk,

$$T_{\text{gap}} \approx \left( \frac{GM_\ast m_p}{2a} \right) \frac{m_p}{M_\ast} \left( \frac{c_s}{v_k} \right)^2.$$

(39)

This is much smaller than the torque in the gapless case, $T_{\text{gap}} \approx 10^{-5}(\alpha/10^{-2})(GM_\ast m_p/2a)$. The resonant torque is

$$T_{\text{res}} \approx \left( \frac{GM_\ast m_p}{2a} \right) \frac{m_1}{m_p} \left( \frac{e a_1}{a_1 - a_p} \right)^4,$$

(40)

where $e$ is the larger of the eccentricities of the two planets. This eccentricity is likely to be small; if we take $e$ to be 0.01, then the resonant torque is $T_{\text{res}} \approx 10^{-2}(m_1/m_p)(GM_\ast m_p/2a)$ for the 2:1 first-order resonance; for a first-order resonance near the inner edge of the gap (where $a/\Delta a \approx 10$), this will rise by about 10.

If both bodies are of roughly Jupiter mass, capture is possible into first- or second-order resonances, that is, resonances with $|j_1 - j_p| = q$, where $q = 1$ or 2, since $T_{\text{gap}}$ is the appropriate torque to use. This case may arise in the scenario mentioned above, in which gas caught between two giant planets can leak out over several hundred orbital periods. The tidal torques from the gas inside the inner planet will then tend to push it outward, while the gas outside the outer planet will tend to push it inward; capture into the 2:1 or possibly the 3:1 mean motion resonance could then occur.

Could a smaller body migrate toward a Jupiter-mass planet and subsequently grow by accretion (first of solid material, later of gas) into a Jupiter-mass object? If the outer body initially has a mass substantially smaller than $M_\ast$, it will not open a gap in the gas disk. If it has a mass comparable to or larger than $1M_\odot$, the hydrodynamic drag it experiences will be much smaller than the tidal torques. It will undergo rapid type I inward migration, easily passing through any mean motion resonances (see eq. [38]). According to Ward (1997), the time for this inward migration will be less than 10$^3$ yr. The inward migration will not halt until the outer body enters the gap produced by the inner Jupiter-mass object. It will then experience a torque similar to that felt by the inner Jupiter-mass body, and both bodies will migrate inward without a substantial change in their eccentricity.

Since the initial inward migration is so rapid, it seems unlikely that an outer planet with initial mass less than about 10$M_\odot$ will be able to accrete sufficient solid material to trigger the accretion of gas before it enters the gap produced by the inner planet. Once it enters the gap, the outer planet could grow by eating other inward-migrating bodies, a la the scenario proposed by Ward (1997) for explaining the very short period Jupiter-mass objects. However, unlike Ward’s case, here the outer planet cannot emerge far enough from the inner edge of the outer disk that its 2:1 resonance can leave the disk, slowing the inward migration; the inner planet is in the way. Given the mismatch between the tidal and resonant torques, it seems likely that the smaller planet will be subsumed by the more massive body.

To summarize, resonant migration in a gas disk will produce large eccentricities in both migrating bodies. Unlike the case of planetesimal migration, both bodies must be of roughly Jupiter mass.

5.2. Planetesimal or Tidal-Torque Migration?

There may be ways to distinguish migration by tidal torques and migration by ejection of planetesimals. Planetesimal migration is likely to produce dynamically isolated (although possibly not unaccompanied) Jupiter-mass bodies in small eccentric orbits; as we have seen, the small-mass inner body responsible for driving the eccentricity up to large values is often ingested into the star or the Jupiter-mass object.

In those cases in which the inner body survives the migration process, it should be possible to detect it with high-precision radial-velocity observations. In some cases, the inner body may have a large mass, since planetesimal migration involving more than one Jupiter-mass object sometimes produces a convergence of the semimajor axes of two bodies (Hansen et al. 2001). This might produce systems
such as those recently discovered around GJ 876 (Marcy et al. 2001). Alternately, such a system could be the outcome of the migration of two Jupiter-mass planets in a gas disk.

We noted in §3.1 that if two Jupiter-mass objects migrate through a planetesimal disk, the circularization time of the outer planet might be much shorter than the migration time. The inner planet excites the eccentricities of the planetesimals before they cross the orbit of the outer planet. As a result, a large fraction of the planetesimals ejected by the outer planet have rather small semimajor axes (and large eccentricities) when they first interact with the outer planet. While this paper was under review, we received a preprint by Lee & Peale (2001) that points out that the small eccentricity $e \approx 0.03$ of the outer planet in GJ 876 is difficult to understand if $\tau_e \approx \tau_m$; their simulations suggest that $\tau_e \lesssim \tau_m/10$ is required to reproduce that system. We are investigating this problem further.

Resonant migration in a gas disk is less likely to produce a single Jupiter-mass planet in a moderately eccentric orbit than is migration in a planetesimal disk. The immediate outcome of a resonant gas migration is two Jupiter-mass planets in eccentric orbits. The subsequent evolution may result in ejection or accretion (onto the star) of one body, but this is essentially the gravitational instability problem discussed in the introduction, in which it was noted that the most likely outcome was a single planet in a low-eccentricity orbit. We say essentially, but there is one difference; the total angular momentum in the resonant migration case is much smaller, relative to the total energy, than in the numerical experiments of Ford et al. (2001). Because the total angular momentum is smaller than in the case of two planets on initially circular orbits of the same energy, the result of a merger would have a higher eccentricity than in their simulations. It would be instructive to generalize their initial conditions and do more simulations.

In a planetesimal migration, the low mass of the inner planet combined with the frequency of close encounters with numerous smaller bodies tends to keep the amplitude of libration large. We have seen several cases in our numerical integrations in which the inner planet collides with the Jupiter-mass body or with the central star. Thus, some systems may harbor low-mass resonant companions to the known Jupiter-mass planets, or the Jupiter-mass planets may have disposed of their resonant partners.

The fact that our simulations show accretion of planetesimals onto the star suggests another way to distinguish the two scenarios. The planetesimal migration is inevitably accompanied by the accretion of $\sim 5\%$ of the planetesimal disk mass onto the star; we see this even in simulations in which we halt the migration at large semimajor axis. Since the mass of the disk is of the order of $300 M_\oplus \sim 600 M_\oplus$, this amounts to $\sim 20 M_\oplus$ of rocky material accreted onto the star. This material will include $5 M_\oplus \sim 7 M_\oplus$ of iron, altering the apparent metallicity of the parent stars dramatically. Note that Jupiter contains only about $2 M_\oplus$ of iron.

Moderate-period (longer than 40 day period) systems produced by gas migration are unlikely to pollute their parent stars so dramatically; there is no reason to expect that such systems have a few hundred Earth masses of rocky material lying around. The parent stars could accrete metal-rich Jupiter-mass bodies after reaching the main sequence, but again, there is no reason to expect that every moderate-period system would do so, when it is known that most stars that lack such companions do not (Murray et al. 2001). We say this because planets with 40 day or longer periods and moderate eccentricities ($\sim 0.5$ or less) are dynamically uncoupled from very short period planets (4 days or so), and so are unlikely to cause them to fall onto the star.

5.3. Comparison with Currently Known Systems

There are several systems that have sub-Jupiter-mass planets in highly eccentric orbits, including HD 108147 ($M \sin i = 0.35 M_J, a = 0.098 AU, e = 0.56,$ and $K = 37 \text{ m s}^{-1}$), HD 83443 ($M \sin i = 0.17 M_J, a = 0.174 AU, e = 0.42,$ and $K = 14 \text{ m s}^{-1}$; the system contains at least two planets, currently not dynamically linked), and HD 16141 ($M \sin i = 0.22 M_J, a = 0.351 AU, e = 0.28,$ and $K = 10.8 \text{ m s}^{-1}$). The first system is particularly interesting as a test of the type of migration involved, assuming the eccentricity is produced by resonant migration. The putative resonant planet must have a mass less than about $1/3$ the mass of the detected planet, in order to escape detection (since the survey is clearly capable of finding planets with $K \approx 10 \text{ m s}^{-1}$; this would give $M \sin i \approx 0.1 M_J$). For a typical inclination, we would expect a mass of about $60 M_\oplus$. Whether such a low-mass object could open a gap in a gas disk is an interesting question. We note that the planet in HD 108147 is near the radius at which the Balbus-Hawley instability is believed to operate. In a gas disk, a planet in this region would be subject to rapid eccentricity damping. As radial-velocity surveys improve below the $10 \text{ m s}^{-1}$ level, the discovery of even lower mass objects in eccentric orbits would indicate that some mechanism other than resonant migration in a gas disk was operating to produce the high eccentricities.

There are also a handful of multiple-planet systems known, including $\gamma$ And, HD 83443, GJ 876, HD 74156, HD 168443, 47 UMa, and of course, the solar system. The diversity of eccentricities in these systems is impressive. 47 UMa appears to be a rough analog of our solar system in that the two most massive planets are in fairly long-period low-eccentricity orbits. The two outer planets in $\gamma$ And have substantial eccentricities, 0.18 for the middle and 0.41 for the outer planet. Similarly, the outer planet in HD 83443 has $e = 0.42$ (the inner planet is likely tidally circularized).

It is tempting to suggest that the planets in 47 UMa are in roughly circular orbits because, at 2.09 and 3.73 AU, they have not migrated very far. However, the single planet in 14 Her at $a \approx 3.2$ AU has $e = 0.45$, while the outer planet in HD 74156 has $e \approx 0.4$ at $a \approx 3.5$ AU. It is possible that the latter planets formed at much larger radii and migrated a significant distance to their current locations, while the planets in 47 UMa formed near where they are now, but this is not a very satisfying explanation.

In contrast, in the one system that is currently believed to be in resonance, GJ 876, the outer planet has a very low eccentricity. Any scenario that explains this system is likely to have difficulties explaining the eccentricities of the other multiple-planet systems, since the outer planets in the other systems (excluding 70 UMa and the solar system) have substantial eccentricities. We indicated one way that such a situation might arise in the planetesimal migration scenario. In both GJ 876 and the other multiple- (Jupiter–mass–) planet systems, the passage of the inner massive planet would have heated the disk, so that is not the source of the difference. The distinction between GJ 876 and the other systems is that in GJ 876, the mass ratio of the resonant
Combining this with equation (A2), we expect equation yields

\[ T \equiv \left( \frac{\ln \mathcal{L}}{\ln \mathcal{L}_0} \right) \],

where the subscript \( T \) represents the tidal-torque scenario. The first body is subject to the constraint that the planets are locked in a resonance (Lissauer et al. 1984; Gomes 1998).

6. CONCLUSIONS

This paper has described how two resonant planets undergoing inward migration can reach large eccentricities. The eccentricity growth is largest when the inner planet is subject to eccentricity damping. Such a situation may arise either in planetesimal migration or in migration driven by tidal torques in which the inner gas disk has been removed by accretion or mass loss in a wind. Migration in a planetesimal disk, in which a Neptune-mass body is trapped at an inner resonance of a Jupiter-mass body, could explain the number of apparently single-planet systems with large eccentricities; migration in a gas disk requires that the second body be nearly as massive as the outer body in order to produce large \( e \), so this scenario does not explain the bulk of the observed systems.

Equations (29) and (33) give approximate expressions for the relation between \( e \) and the initial and final semimajor axes of the resonant planets.

Section 4 presented numerical integrations showing that in some cases, a planetesimal migration will produce a single Jupiter-mass object with a large eccentricity; in other cases, the Jupiter-mass object may be accompanied by a resonant object with a similar mass or by a Neptune-mass companion. In the case of a Neptune-mass body, the inner companion will have a very large eccentricity.

We have also described integrations involving 10 or more roughly Earth-mass bodies, together with a Jupiter-mass planet; the latter is forced to migrate inward, with the migration process tending to damp its eccentricity. It typically captures one or more of the least massive bodies into mean motion resonance. Usually only the second-most massive planet (which may not be the body that was originally second-most massive behind “Jupiter”) survives long in resonance. In fact, this second-most massive body tends to accrete its smaller companions.

We have suggested two possible ways to tell the difference between systems produced by planetesimal or gas migration. First, if there exist a large number of systems having an eccentric planet with either no resonant companion or with only very low mass (non-gap-opening) companions, a finding requiring very high precision radial-velocity measurements, it would strongly suggest that resonant migration by tidal torques was not responsible. Second, the planetesimal migration picture predicts that several Earth masses of iron will be accreted in the migration process, well after the central star has reached the main sequence. The tidal-torque scenario is mute regarding this point; accretion of material after the gas disk vanishes and close correlation with the presence of Jupiter-mass planets are not natural features.

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APPENDIX A

DERIVATION OF ECCENTRICITY VARIATION WITH DISSIPATIVE FORCES

To derive the expression for the variation of the eccentricities of two planets caught in a mean motion resonance when one or both are subject to dissipative forces, we examine the equations describing the evolution of energy and angular momentum, subject to the constraint that the planets are locked in a resonance (Lissauer et al. 1984; Gomes 1998).

We start with the energy, which is given by

\[ E = -\frac{GM_1 m_p}{2a_1} - \frac{GM_2 m_p}{2a_p}. \] (A1)

We assume \( a_1 < a_p \). It will prove useful to employ the variables \( \mathcal{L}_i \equiv (GM_i a_i)^{1/2} \); then the resonance condition in equation (19) implies

\[ \frac{d \ln \mathcal{L}_i}{dt} = \frac{d \ln \mathcal{L}_p}{dt}. \] (A2)

This equation is accurate only in an average sense; on times shorter than the libration time of the resonance, it is violated.

We assume that some dissipative force removes both energy and angular momentum from the orbits:

\[ \frac{dE}{dt} = \left( \frac{dE_i}{dt} \right)_T + \left( \frac{dE_p}{dt} \right)_T, \] (A3)

where the subscript \( T \) (for “tides”) represents the effect of the nonconservative force. After some algebra, the energy evolution equation yields

\[ m_p \frac{d \mathcal{L}_p}{dt} + \frac{j_p}{j_1} m_1 \frac{d \mathcal{L}_1}{dt} = m_p \left( \frac{d \mathcal{L}_p}{dt} \right)_T + \frac{j_p}{j_1} m_1 \left( \frac{d \mathcal{L}_1}{dt} \right)_T. \] (A4)

Combining this with equation (A2), we find

\[ \frac{d \ln \mathcal{L}_1}{dt} = \frac{d \ln \mathcal{L}_p}{dt} = \frac{1}{1 + (m_1/m_p)(j_p/j_1)^{2/3}} \left[ \left( \frac{d \ln \mathcal{L}_p}{dt} \right)_T + \frac{m_1}{m_p} \left( \frac{j_p}{j_1} \right)^{2/3} \left( \frac{d \ln \mathcal{L}_1}{dt} \right)_T \right]. \] (A5)
Next we examine the angular momentum
\[ L = m_1 \sqrt{GM_a a_1(1 - e_1^2)} + m_p \sqrt{GM_a a_p(1 - e_p^2)}, \]  
which evolves according to
\[ \frac{dL}{dt} = \left( \frac{dL_1}{dt} \right)_T + \left( \frac{dL_p}{dt} \right)_T. \]  
Introducing the auxiliary variables \( Y_i = (1 - e_i^2)^{1/2} \), we find
\[ \left[ \frac{dY_1}{dt} - \frac{dY_1}{dt} \right] + \frac{m_p}{m_1} \left( \frac{j_p}{j_1} \right)^{1/3} \left[ \frac{dY_p}{dt} - \frac{dY_p}{dt} \right] = Y_1 \left[ \left( \frac{d \ln \mathcal{L}_1}{dt} \right)_T - \left( \frac{d \ln \mathcal{L}_1}{dt} \right)_T \right] + \frac{m_p}{m_1} \left( \frac{j_p}{j_1} \right)^{1/3} Y_p \left[ \left( \frac{d \ln \mathcal{L}_p}{dt} \right)_T - \left( \frac{d \ln \mathcal{L}_p}{dt} \right)_T \right]. \]  
Combining equations (A5) and (A8) gives (Gomes 1998)
\[ \left[ \frac{dY_1}{dt} - \frac{dY_1}{dt} \right] + \frac{m_p}{m_1} \left( \frac{j_p}{j_1} \right)^{1/3} \left[ \frac{dY_p}{dt} - \frac{dY_p}{dt} \right] = \left[ \left( \frac{d \ln \mathcal{L}_1}{dt} \right)_T - \left( \frac{d \ln \mathcal{L}_1}{dt} \right)_T \right] Y_1 - \frac{(j_p/j_1)^2}{1 + (m_1/m_p)(j_p/j_1)^{2/3}}. \]  
Writing this in terms of \( e_i \) and \( a_i \), we find
\[ \frac{e_1}{\sqrt{1 - e_1^2}} \left[ \frac{de_1}{dt} - \frac{de_1}{dt} \right] + \frac{e_p}{\sqrt{1 - e_p^2}} \left[ \frac{de_p}{dt} - \frac{de_p}{dt} \right] = \left[ \left( \frac{d \ln a_1}{dt} \right)_T - \left( \frac{d \ln a_1}{dt} \right)_T \right] \frac{(j_p/j_1)^2}{2[1 + (m_1/m_p)(j_p/j_1)^{2/3}]} \]  
\[ (A10) \]

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