TRAVEL TIME RELIABILITY IN TRANSPORTATION NETWORKS:

A REVIEW OF METHODOLOGICAL DEVELOPMENTS

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Abstract

The unavoidable travel time variability in transportation networks, resulted from the widespread supply-side and demand-side uncertainties, makes travel time reliability (TTR) be a common and core interest of all the stakeholders in transportation systems, including planners, travelers, service providers, and managers. This common and core interest stimulates extensive studies on modeling TTR. Researchers have developed a range of theories and models of TTR, many of which have been incorporated into transportation models, transport policies, and project appraisals. Adopting the network perspective, this paper aims to provide an integrated framework for summarizing the methodological developments of modeling TTR in transportation networks, including its characterization, evaluation and valuation, and traffic assignment. Specifically, the TTR characterization provides a whole picture of travel time distribution in transportation networks; TTR evaluation and TTR valuation (also known as the value of reliability, VOR) interpret abstract characterized TTR in a simple and intuitive way in order to be well understood by different stakeholders of transportation systems; and lastly TTR-based traffic assignment investigates the effects of TTR on the individual users’ travel behavior and consequently the collective network flow pattern. As the above three topics are mainly separately studied in different disciplines and research areas, the integrated framework allows to better understand their relationships and may contribute to developing more possible combinations of TTR modeling philosophy. Also, the network perspective enables to pay more attention to some common challenges of modeling TTR in transportation networks, especially the uncertainty propagation from the uncertainty sources to the TTR at various spatial levels including link, route, and the entire network. Some potential directions for future research are discussed in the era of new data environment, applications, and emerging technologies.

Keywords: travel time, travel time reliability, travel time distribution, value of reliability, traffic assignment

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1. **INTRODUCTION**

As a critical factor in the efficiency and service quality of transportation networks, travel time reliability (TTR) exerts a strong influence on the stakeholders in transportation networks, including users (travelers), service providers, planners, and managers. To be specific, empirical studies have increasingly concluded that the importance of TTR is equal to or even greater than that of travel time in travelers’ choice behaviors (e.g., Bates *et al.*, 2001; Lam and Small, 2001; Small *et al.*, 2005; Hollander, 2006; Asensio and Matas, 2008; Li *et al.*, 2010, Carrion and Levinson, 2013). To account for TTR, different reliability costs may be added to different projects, and these may significantly affect the results of planers’ project appraisals. Therefore, it is unsurprising that the cost of TTR is recommended or even required to be included in transportation project appraisals (de Jong and Bliemer, 2015; New Zealand Transport Agency, 2016; Organization for Economic Co-operation and Development (OECD), 2016). As for service providers and traffic managers, the reliability of travel time is one of the key performance indicators they used for monitoring or improving the service quality of transportation networks (Lyman and Bertin, 2008; Kim, 2014) and it is also their responsibility or objectives to improve the reliability of transportation systems. This common interest inspires and stimulates extensive studies on modeling TTR since Herman and Lam (1974) and Sterman and Schofer (1976) firstly recognize the need of modeling the TTR, making TTR an active research area. We searched Scopus database for published papers using the phrase "travel time reliability" in the search category "Article title, Abstract, Keywords". There are 3,121 records, and 66.26% are published in the past ten years (i.e., 2012-2021). Figure 1 presents the total publication numbers per year from 2012 to 2021 and the top 10 transportation journals contributing to TTR, including Transportation Research Part A/B/C, Transportation, and IEEE Transactions on Intelligent Transportation Systems, etc. These results clearly show that TTR has already been a hot research topic for decades. In fact, researchers have developed a range of ideas, theories, and models of TTR, many of which have been incorporated into transportation models, transport policies, and project appraisals.

This paper aims to summarize the literature and provide a methodological review of modeling TTR from the network perspective. First, the methodological review makes this paper concentrate on summarizing the methods of modeling TTR, especially the modeling rationales, modeling frameworks, and modeling techniques. Second, the network perspective makes this paper focus on the big picture of modeling TTR in transportation networks (with a large number

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1 Other definitions of reliability in networks include connectivity or terminal reliability (Wakabayashi and Iida, 1992; Bell and Iida, 1997), capacity reliability (Chen *et al.*, 2002b), and travel demand satisfaction reliability (Heydecker *et al.*, 2007), etc.
of diverse links, routes, and travelers), including what is TTR, how to assess TTR, and what is the behavioral and network effect of TTR. To this end, we provide an integrated framework as shown in Figure 2 to review the methodological developments in the literature from the network perspective. Specifically,

- The travel time distribution (TTD) characterization answers what is TTR through accurately capturing the manifestation of TTR. It uses a mathematical way to provide a whole picture of TTR in transportation networks, which lays the foundation of assessing TTR for different stakeholders.

- Assessing TTR can in turn interpret the abstract characterized TTD in a simple and intuitive way in order to be well understood by the different stakeholders of transportation networks. There are two ways to assess TTR, including TTR evaluation and TTR valuation, which have different applications. The former quantifies the reliability performance using various reliability measures, whereas the latter quantifies the VOR in monetary unit to understand users’ behavioral response to TTR.

- The TTR has a direct and indirect effect on the travelers of a transportation network—the individual users’ travel behavior and the collective network flow pattern, which corresponds to the TTR-based traffic assignment problem.

![Number of papers per year about travel time reliability since 2012](image)

![Top 10 transportation journals contributing to travel time reliability](image)

Figure 1. A summary of the results of searching Scopus database for published papers using “travel time reliability” (Journal names are abbreviated under ISO 4 standard).
What’s the TTR?

Travel time dataset
Characterized TTDs
Empirically fitting

Uncertainty source
- Demand-side, supply-side, and both
Theoretically deducing

How to assess the TTR?

Reliability performance
Behavior response

TTR evaluation
Reliability measures
- Probability-based, moment-based, percentile-based, tail-based, bound-based, PDF-based, utility-based

TTR valuation
Valuation measures

Theoretical valuation models
- Mean-variance
- Schedule delay model
- Mean-lateness model
- Utility maximization network model

Various VORs

What’s the effect of TTR?

TTR-based route choice criteria
- Objective quantification
- Subjective perception

Uncertainty propagation

Network flow pattern

Figure 2. An integrated framework for modeling TTR in transportation networks

Although there are already some review papers about TTR, this paper contributes to the literature in several ways. First, the network perspective makes this paper put the above three research questions and topics together, which are mainly separately studied and reviewed by researchers from different disciplines and research areas, e.g., statistics, economics, and network optimization. For example, many previous studies or review papers focus on TTR valuation (e.g., Noland and Polak, 2002; Li et al., 2010; Small, 2012; Taylor, 2013; Wardman and Batley, 2014; Carrion and Levinson, 2012; Shams et al., 2017), while some papers are related to TTR evaluation (e.g., Iida, 1999; Pu, 2011; Gu et al., 2020) or TTR-based traffic assignment (Chen et al., 2011a; Nikolova and Stier-Moses, 2014). In contrast, this paper with an integrated review framework as shown in Figure 2 allows us to better understand their relationships and to develop more possible combinations of modeling philosophy. Second, we endeavor to summarize methodological developments (e.g., modeling rationales, modeling frameworks and modeling techniques) and to provide corresponding general formulas for modeling TTR. However, previous review papers lack introduction about the modeling methods. For example, the early review on TTR by Iida (1999) only summarizes the basic concepts of TTR proposed by Asakura and Kashiwadani (1991) and Asakura (1996), while Taylor (2013) seeks to provide an overview of TTR research from travel behavior and network optimization.
performance appraisal without modeling formulations. Third, the network perspective allows this paper to pay attention to the research progress about some challenges of modeling TTR in transportation networks, while previous review papers usually ignore such progress. For example, (1) the TTDs of different links and/or routes at different time periods in a transportation network are heterogeneous, and thus, the methods for characterizing TTDs in networks must be flexible enough to capture this heterogeneity while guaranteeing modeling accuracy; (2) different stakeholders in a transportation system, e.g., travelers with different preferences, service providers, planners and managers, have different understanding about the impacts of TTR on their behavioral responses, investments and decision-makings; and (3) a transportation network has a large number of links, routes, and origin-destination (O-D) pairs, which requires the modeling of TTR to explicitly capture the uncertainty propagation from the sources to the TTR at various spatial levels.

The remainder of this paper is organized as follows. Section 2 reviews the characterization of TTDs, including modeling frameworks, existing modeling rationales, and TTD models. The methods and measures for TTR evaluation are summarized in Section 3. Section 4 reviews the mathematical models, measures, and dimensions of TTR valuation. Section 5 reviews the methods used in TTR-based traffic assignment, including route choice criteria, mathematical models, and solution algorithms. Section 6 focuses on the methods for modeling uncertainty propagation, and Section 7 concludes the paper with a discussion of potential future research.

2. **Travel Time Distributions Characterization**

In this section, we discuss the meaning and necessity of characterizing TTDs, summarize the current modeling frameworks, and review the modeling rationales and corresponding TTD models\(^2\), followed by some discussions.

Although using empirical distributions (e.g., empirical cumulative distribution function (CDF)) seems to be the most straightforward method, many studies focus on developing fitted or deduced TTD models for several reasons. First, a well-fitted TTD model and its analytical expression are key preliminaries for evaluating TTR, valuing TTR, and incorporating TTR into transportation models in transportation networks. Second, the statistical representativeness of empirical distributions cannot be guaranteed, as empirical datasets may not collect all values, especially “extreme” values that indeed existed in the tail of TTD (Taylor, 2017). Nevertheless,

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\(^2\) The TTD is also a crucial input for other calculations such as the estimation of arrival time, travel time prediction, vehicle routing problem, etc. Although all of the methods for characterizing the TTD can theoretically be used for assessing the TTR, this paper reviews only the methods under the context of TTR in transportation networks.
these “extreme” values are critical for the reliability/risk analysis of transportation networks (Xu et al., 2014; Taylor, 2017; Zang et al., 2018a; Esfeh et al., 2020). Last, route-level or network-level travel time datasets are not readily available for large-scale urban transportation networks, which makes it hard to directly obtain the route or network TTD models. To derive route or network TTD models, current typical method is to aggregate the TTD from link to route (or network) levels, and thus a well-fitted link TTD model is a prerequisite for this process.

There are two modeling approaches for characterizing a TTD: fitting and deducing. Fitting methods use assumed distribution functions to fit travel time datasets, whereas deducing methods derive the distribution functions of travel time from assumed uncertainty sources of TTR. Hence, the inputs for models of characterizing TTD are empirical datasets or the assumed sources of variability in the fitted and deductive approaches, respectively; and the outputs are the fitted or deduced distribution functions of travel time, respectively. The distribution functions by these models include the probability density function (PDF), CDF, and inverse CDF, which is equivalent to the percentile point function (PPF).

2.1 Modeling Frameworks and Challenges

Figure 3 summarizes the general frameworks of the two modeling approaches to characterizing TTD in transportation networks, namely fitting and deducing. Based on Figure 3, the frameworks are described in detail below.

- For the fitting approach, the typical process for fitting TTD in transportation networks is to fit the link TTD and then to derive the route/network TTD model by aggregating the fitted link TTD models, which is marked by the blue solid line in Figure 3. The first step of this typical process is to select or develop an appropriate TTD model. Such selection depends on many factors, such as modeling rationale, the characteristics of the dataset, the application purposes, etc. Besides, it is hard to directly fit the route/network TTD due to the lack of route/network travel time datasets, although in theory, the fitted route/network TTD can be obtained from empirical route/network datasets.

- For the deducing approach, the distribution functions of the link TTD models are firstly deduced based on the assumed uncertainty sources, and then the deduced link TTD models are aggregated to derive the route/network TTD model. Section 2.2.4 will review the sources of uncertainty considered in this process and the corresponding methods for deducing the distribution functions.
Some challenges exist in characterizing TTDs under these two frameworks as follows.

- Because of their heterogeneity, TTDs can have various statistical features, i.e., symmetric or asymmetric, left- or right-skewed, long or short tail, and unimodal or multimodal, which makes characterizing TTDs very challenging. Many empirical studies have verified that TTDs are right-skewed with a long/fat tail (Polus, 1979; Fosgerau and Fukuda, 2012; Susilawati et al., 2013; Srinivasan et al., 2014; Kim and Mahmassani, 2015; Delhomme et al., 2015; Taylor, 2017). However, despite with this typical feature, some TTDs may present quite different statistical features, e.g., TTDs may be left-skewed or symmetric (van Lint and van Zuilen, 2005; van Lint et al., 2008; Yang et al., 2014b; Zang et al., 2018a). van Lint and van Zuilen (2005) and Zang et al. (2018a) identify TTDs with negative skewness, near-zero skewness, positive skewness, and strong positive skewness. Other studies further identify the multimodality of TTDs in urban transportation systems (e.g., Dong and Mahmassani, 2009; Guo et al., 2010; Taylor and Somenahalli, 2010; Kazagli and Koutsopoulos, 2012; Chen et al., 2014b; Yang et al., 2014a).

- The theoretical and empirical applications of TTD models have different requirements. The TTD models used in empirical applications focus on accuracy and computational efficiency, which are non-trivial problems because of the heterogeneity of TTDs. In contrast, theoretical applications use closed-form expressions of TTD models as the basis for conceptualizing and formulating computationally tractable or analytically derived traffic models that incorporate TTR.

- Both the fitting and deducing approaches derive a route/network TTD model from the link TTD model, which involves aggregating uncertainty from the link level to the...
As transportation networks often consist of a large number of links, modeling such uncertainty aggregation is quite challenging, which requires a trade-off between computational efficiency and modeling accuracy. It should be noted that uncertainty aggregation is also an important part of TTR evaluation and TTR-based traffic assignment models.

Table 1 lists the models for characterizing TTDs in transportation networks developed within these two modeling frameworks, indicating their modeling rationales, applications, closed-form formulae, and forms of random variables. Section 2.2 summarizes the four modeling rationales underlying these two modeling frameworks, and presents a general formula for each modeling rationale and TTD models that correspond to each modeling rationale. Section 2.3 discusses how studies overcome the first two challenges in characterizing TTD outlined above, and the methods for addressing the third challenge are reviewed in Section 6.

2.2 Modeling Rationales and Corresponding TTD Models

The four modeling rationales underlying the models for characterizing TTD are the single distribution modeling rationale, the mixture distribution modeling rationale, the moment-based modeling rationale, and the source-based derivation modeling rationale. The first three are the basis for models that fit TTD based on travel time datasets, and the last is the basis for models that deduce TTD from an assumed uncertainty source.

Let a unified notation, i.e., $z$, denote the general formulation for each of the four modeling rationales. Assume that $T$ is a random variable that denotes travel time or its variants, $\rho$ denotes the probability or confidence level, and let $f(T)$, $F(T)$, and $F^{-1}(\rho)$ denote the PDF, CDF, and PPF, respectively. The variants of travel time in TTD models include standardized travel time, total travel time, pace, and (extreme) travel time delay. Standardized travel time can be obtained by $(\text{Travel time} - \text{mean})/\text{standard deviation}$ and is usually used in VOR research (Fosgerau and Fukuda, 2012; Taylor, 2017; Zang et al., 2018a, 2018b; Li, 2019), which corresponds to the general assumption discussed in TTR valuation, i.e., the standardized travel time distribution is independent of departure time. Total travel time is the sum of all users’ travel times in a transportation network, and thus it is used in network-wide reliability models (Chen et al., 2014a; Xu et al., 2013, 2014). Pace equals the random travel time divided by link/path length and is used to exclude the travel time variation that arises from variations in link length (Daganzo, 1997; Mahmassani et al., 2013; Saberi et al., 2014). The difference between (extreme) travel time and the minimum (extreme) travel time is (extreme) travel time delay,
which is used by Kim and Mahmassani (2015) and Esfeh et al. (2020) to explore the direct proportionality between the mean and standard deviation of (extreme) travel time.

2.2.1 Single distribution modeling rationale

The classic single distribution model is the most widely used method to characterize TTR. Its general formula can be written as

\[ z = f(T \mid \theta, l, k) \]

where \( \theta \) is the scaling factor, \( l \) is the location factor, and \( k \) is the shape factor. Under this modeling rationale, two-parameter statistical distribution functions such as the Normal (Bell and Iida, 1997), Lognormal (Emam and Al-Deek, 2006), Weibull (Al-Deek and Emam, 2006), and Gamma distributions (Polus, 1979) are first used to fit the TTDs\(^3\). However, the verification of the heterogeneity of TTDs by many empirical studies has led to the adoption of distribution functions with three or more parameters to accurately capture the TTDs’ multiple statistical properties, e.g., shifted Lognormal (Srinivasan et al., 2014), compound Gamma (Kim and Mahmassani, 2015), Generalized Beta (Castillo et al., 2012), Stable (Fosgerau and Fukuda, 2012), (compound) generalized extreme value (Lei et al., 2014), Burr (Susilawati et al., 2013)\(^4\), and tailored Wakeby-type distribution (Zhang et al., 2018). Note that compound distribution is the probability distribution with the assumption that its random variable follows a distribution type with an unknown parameter that also follows the same distribution type. Therefore, the compound Gamma distribution and compound generalized extreme value are still viewed as the models under the single distribution modeling rationale.

2.2.2 Mixture distribution modeling rationale

It is difficult for single distribution models to characterize the multimodal travel time resulting from interrupted flows at intersections or congested traffic conditions in urban transportation systems (e.g., Dong and Mahmassani, 2009; Guo et al., 2010; Taylor and Somenahalli, 2010; Kazagli and Koutsopoulos, 2012; Chen et al., 2014b; Yang et al., 2014a). However, mixture distribution models can deal with such multimodality of TTDs. The mixture model is usually a weighted combination of several single distribution models of the same type and the component distribution can be any of the unimodal TTD models developed under the single distribution modeling rationale. Its general formula is

\[ z = \sum \lambda_i f_i(T \mid \theta_i, l_i, k_i) \]

where \( \lambda_i \) is the \( i \)th weight associated with the \( i \)th single distribution. Mixture distribution models

\(^3\) For each TTD model, we list only one or two representative references in the text. Table 1 lists more related papers.

\(^4\) The Burr distribution is also referred to as the Singh–Maddala distribution. The latter is used by Guessous et al. (2014) to characterize TTD.
can establish a connection between TTDs and the related traffic states, which leads to a better fitting for multimodal TTDs (Guo et al., 2010; Rahmani et al., 2015; Chen et al., 2017). The models based on this modeling rationale include the Normal mixture model (Guo et al., 2010), Lognormal mixture model (Kazagli and Koutsopoulos, 2012), Gamma mixture model (Yang and Wu, 2016), finite mixture of the regression model (Chen et al., 2014b), and kernel density estimation (Fosgerau and Fukuda, 2012; Li, 2019).

2.2.3 Moment-based modeling rationale
Both the single distribution and mixture distribution modeling rationales need a prior assumption about the distribution type that travel time follows, which is unknown in reality. Furthermore, empirical studies show that although TTDs are typically right-skewed with long/fat tails, they can also be left-skewed or have near-normal distributions (van Lint and van Zuylen, 2005; van Lint et al., 2008; Yang et al., 2014b; Zang et al., 2018a). To circumvent these challenges, some studies directly use statistical information of travel time datasets to fit heterogeneous TTDs, avoiding the need to assume the distribution type. Moments are the most common statistical information used under this modeling rationale, and thus we refer to this as the moment-based modeling rationale. The general formula, which is based on the first $n$ moments, can be expressed as

$$ z = f(T \mid \xi_1, \xi_2, \ldots, \xi_n) $$

where $\xi_i$ is the $i$th moment of travel time. The Cornish–Fisher expansion (Zang et al., 2018a), Gram–Charlier expansion (Hou and Tan, 2009), and Johnson curves (Clark and Watling, 2005) are three representative methods, and they all use the first four moments of travel time, i.e., mean, standard deviation, skewness, and (excess) kurtosis. Ng et al. (2011) uses the probability inequalities based on the first $n$ moments to obtain the upper bounds of the tail probabilities instead of an exact probability distribution.

2.2.4 Source-based derivation modeling rationale
The above three modeling rationales use empirical datasets to fit TTDs. An alternative rationale is the source-based derivation modeling rationale that uses assumptions about the uncertainty sources of TTR to deduce TTD models. The uncertainty sources of demand and/or supply are main assumptions, e.g., the link capacity variations belonging to supply-side uncertainty in Lo and Tung (2003) and Lo et al. (2006), the day-to-day demand fluctuations belonging to demand-side uncertainty in Clark and Watling (2005) and Shao et al. (2006a, 2006b), and the

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5 The kernel density estimation is also a combination of its component kernels, so it is considered as a method of the mixture distribution modeling rationale despite that the component kernel may not be a distribution function.
Table 1. A summary of existing TTD models for TTR in transportation networks

| TTD model       | Modeling Rationale | Formula (PDF) | Random variable | Application | Closed-forms | References                                                                 |
|-----------------|--------------------|---------------|-----------------|-------------|--------------|-----------------------------------------------------------------------------|
| Normal          | Single distribution| $f(T|θ, I)$    | TT              | Theoretical | PDF CDF PPF  | Bell and Iida (1997); Lam and Xu (1999); Yin and Ieda (2001); Lomax et al. (2003); Lo and Tung (2003); Watling (2006); Siu and Lo (2013) |
| Lognormal       | Single distribution| $f(T|θ, k)$    | (T)TT           | Theoretical | PDF CDF      | Kaparias et al. (2008); Pu (2011); Chen et al. (2014b) Herman and Lam (1974); Richardson and Taylor (1978); Emam and Al-Deck (2006); Rakha et al. (2006, 2010); van Lint et al. (2008); Arezoumandi (2011) |
| Shifted Lognormal| Single distribution| $f(T|θ, l, k)$ | TT              | Empirical   | Empirical PDF CDF | Srinivasan et al. (2014) Lee et al. (2019) |
| Weibull         | Single distribution| $f(T|θ, k)$    | TT              | Empirical   | PDF CDF      | Al-Deck and Emam (2006) |
| Gamma           | Single distribution| $f(T|θ, k)$    | TT              | Empirical   | PDF CDF      | Polus (1979) |
| Compound Gamma  | Single distribution| $f(T|θ_1, θ_2, k)$ | TD/dist.       | Empirical   | None         | Kim and Mahmassani (2015) |
| Generalized Beta| Single distribution| $f(T|θ, l, k)$ | TT              | Theoretical | PDF          | Castillo et al. (2012) |
| Stable          | Single distribution| $f(T|θ, l, k_1, k_2)$ | STT             | Empirical   | None         | Fosgerau and Fukuda (2012) |
| Burr            | Single distribution| $f(T|θ, k_1, k_2)$ | (S)TT           | Empirical   | PDF CDF PPF  | Taylor (2012, 2017); Susilawati et al. (2013); Guessous et al. (2014) |
| Tailored Wakeby-type | Single distribution| $f(T|θ, l_1, l_2, k_1, k_2)$ | Bus HAR | Empirical | PPF          | Zhang et al. (2018) |
| Model                                   | Distribution | Formula            | Method                  | PDF  | CDF  | PPF  | References                                                                 |
|----------------------------------------|--------------|--------------------|-------------------------|------|------|------|----------------------------------------------------------------------------|
| Generalized extreme value (GEV)        | Single       | $f(T | \theta, l, k)$ | Pace                    |      |      |      | Lei et al. (2014); Zhang et al. (2019b)                                    |
| Generalized Pareto                     | Single       | $f(T | \theta, l, k)$ | Pace                    |      |      |      | Lei et al. (2014)                                                          |
| Compound GEV                           | Single       | $f(T | l_1, l_2, k_1, k_2, s_\mu)$ | ETD                     | Empirical |      |      | None                         | Esfeh et al. (2020)                                      |
| Mixture model                          | Mixture      | $\sum \lambda_i f(T | \theta_i, l_i, k_i)$ | TT                      | Empirical | PDF  |      | Dong and Mahmassani (2009); Jintanakul et al. (2009); Guo et al. (2010); Taylor and Somenahalli (2010); Guo et al. (2012); Kazagli and Koutsopoulos (2012); Chen et al. (2014b); Yang et al. (2014a); Ma et al. (2016); Yang and Wu (2016) |
| Kernel density estimation              |              |                    | (S)TT                   | Empirical | PDF  |      | Fosgerau and Fukuda (2012); Yang et al. (2014b); Rahmani et al. (2015); Li (2019) |
| Gram–Charlier expansion                | Moment-based | $f(T | \xi_1, \xi_2, \xi_3, \xi_4)$ | TT                      | Empirical | PDF  |      | Hou and Tan (2009)                                                        |
| Johnson curves                         | Moment-based | $f(T | \xi_1, \xi_2, \xi_3, \xi_4)$ | TTT                     | Theoretical | PDF  | CDF  | PPF  | Clark and Watling (2005)                                                   |
| Cornish–Fisher expansion               | Moment-based | $f(T | \xi_1, \xi_2, \xi_3, \xi_4)$ | (T)TT                   | Theoretical | PPF  |      | Lu et al. (2005, 2006); Di et al. (2008); Chen et al. (2011b); Xu et al. (2013, 2014) Zang et al. (2018a, 2018b) |
| N.A.                                   | Sourced-based derivation | N.A. | TT | Empirical | PDF  |      | Kharoufeh and Gautam (2004); Zheng et al. (2017) |
|                                        | Sourced-based derivation and single distribution | N.A. | TTT | Theoretical | N.A. |      | Lo and Tung (2003); Lo et al. (2006); Shao et al. (2006a, 2006b); Lam et al. (2008); Ng and Waller (2010a); Li et al. (2017) |

Note: 1. $\theta$ is the scaling factor; $l$ is the location factor; $k$ is the shape factor; $s_\mu$ is the seasonal mean of the ETD in compound GEV; “None” means that it does not have any closed-form formulae; N.A. means that this column is not applicable.

2. TT: travel time; STT: standardized travel time; TTT: total travel time; (E)TD: (extreme) travel time delay; HAR: headway adherence ratio; dist.: distance.
adverse weather conditions with different rainfall intensities belonging to both demand-side and supply-side uncertainties in Lam et al. (2008) and Li et al. (2017). Interested readers may refer to van Lint et al. (2008) and Chen and Zhou (2010) for a more complete summary about the typical uncertainty factors of supply side or demand side causing travel time variability.

Generally, this modeling rationale is used to derive route or network TTDs for assessing the network reliability. The general formula for these models depends on the assumed uncertainty source and the assumed link travel time function. Suppose that the link travel time $T$ is a function of the link flow $v$ and link capacity $C$; then, we have the following formula for link $a$:

$$T_a = t_a(v_a, C_a), \forall a \in A$$

(4)

where $A$ is the set of links in a transportation network.

Different assumed sources of uncertainty lead to different expressions of the random link travel time. To illustrate this, we use a tilde to highlight the stochasticity of this variable due to the assumed source of uncertainty. Let $t_a(\tilde{v}_a, C_a)$, $t_a(v_a, \tilde{C}_a)$, and $t_a(\tilde{v}_a, \tilde{C}_a)$ represent the deduced link travel time function based on demand-side uncertainty, supply-side uncertainty, and both demand-side and supply-side uncertainties, respectively. Then, the random link travel time is expressed as

$$\tilde{T}_a = t_a(\tilde{v}_a, C_a) \text{ or } t_a(v_a, \tilde{C}_a) \text{ or } t_a(\tilde{v}_a, \tilde{C}_a), \forall a \in A$$

(5)

Taking $t_a(v_a, \tilde{C}_a)$ as an example, the mean and variance of the link travel time, i.e., $\mu_a^T$ and $(\sigma_a^T)^2$, can be computed as follows:

$$\mu_a^T = E[\tilde{T}_a] = E[t_a(v_a, \tilde{C}_a)], \forall a \in A$$

(6)

$$(\sigma_a^T)^2 = Var[\tilde{T}_a] = Var[t_a(v_a, \tilde{C}_a)], \forall a \in A$$

Now, we have expressions of link travel time and corresponding mean travel time and travel time variance. Below, we briefly introduce the general formula of route or network TTDs.

For transportation networks with routes that consist of many links, the travel time of route $j$ between O-D pair $\omega$ can be obtained by summing the corresponding link travel time:

$$\tilde{T}_{\omega j} = \sum_{a \in A} \tilde{T}_a \delta_{aj}, \forall j \in J_\omega, \forall \omega \in \Omega_\omega$$

(7)
where $J_\omega$ and $\Omega_\omega$ are the set of routes between O-D pair $\omega$ and the set of O-D pairs in the network, $\delta_{aj}$ is the link-route incidence indicator, and $\delta_{aj} = 1$ if link $a$ is on route $j$, and 0 otherwise. Similarly, the route travel time is a random variable, so the mean and the variance, denoted as $\mu_T^{\omega_j}$ and $(\sigma_T^{\omega_j})^2$, of the route travel time can be expressed as

$$\mu_T^{\omega_j} = \sum_{a \in A} \mu_a \delta_{aj}, \forall j \in J_\omega, \forall \omega \in \Omega_\omega$$

$$\left(\sigma_T^{\omega_j}\right)^2 = \sum_{a \in A} \delta_{aj} \text{Var}\left[\tilde{T}_a\right] = \sum_{a \in A} \delta_{aj} \left(\sigma_T^a\right)^2, \forall j \in J_\omega, \forall \omega \in \Omega_\omega$$

With calculated mean and variance, many methods have been developed to derive route TTDs including Central Limit Theorem, cupula, convolution, Markov chain, etc., to be reviewed in detail in Section 6.2. Taking the Central Limit Theorem as an example, the route travel time would follow a Normal distribution regardless of the link TTDs (e.g., Lo and Tung, 2003; Lo et al., 2006; Shao et al., 2006a, 2006b). Consequently, the route TTD can be expressed as

$$\tilde{T}_{\omega j} \sim N\left(\mu_T^{\omega j}, \left(\sigma_T^{\omega j}\right)^2\right), \forall j \in J_\omega, \forall \omega \in \Omega_\omega$$

For transportation networks consisting of many links, the total travel time is the product of the link flow and the link travel time of all of the links:

$$\tilde{T} = \sum_{a \in A} \tilde{T}_a \tilde{v}_a$$

Then, we can deduce the formulae of the first $n$ moments of total travel time:

$$\xi_1 = E\left[\tilde{T}\right], \xi_2 = E\left[\tilde{T}^2\right], \ldots, \xi_n = E\left[\tilde{T}^n\right]$$

With the deduced moments of TTT, the methods mentioned in Section 2.2.3 can be directly used to derive the distribution function of total travel time:

$$z = f\left(T | E\left[\tilde{T}\right], E\left[\tilde{T}^2\right], \ldots, E\left[\tilde{T}^n\right]\right)$$

It should be noted that in the literature, the stochastic traffic process is another typical assumption regarding the origin of travel time uncertainty at the operational level in urban transportation systems. For example, Kharoufeh and Gautam (2004) assumes that a vehicle’s speed follows a stochastic speed process, and they use a partial differential equation and Laplace transforms to derive the link TTDs. Zheng and van Zuylen (2010) and Zheng et al. (2017) assume that urban travel times are the result of many stochastic factors (e.g., stochastic traffic flow and arrivals, and departures) and traffic control, and they use shockwave theory to derive the TTDs of urban signalized arterial roads.
| Distribution | Formula (PDF or PPF) | Distribution | Formula (PDF or PPF) | Distribution | Formula (PDF or PPF) |
|--------------|---------------------|--------------|---------------------|--------------|---------------------|
| Normal       | \( f(T \mid \theta, l) = \frac{1}{\sqrt{2\pi \theta}} e^{-\frac{1}{2} \left( \frac{T-l}{\theta} \right)^2} \) | Lognormal    | \( f(T \mid \theta, k) = \frac{1}{Tk\sqrt{2\pi}} e^{-\frac{(T-l)^2}{2Tk^2}} \) | Shifted Lognormal | \( f(T \mid \theta, l, k) = \frac{1}{(T-l)k\sqrt{2\pi}} e^{-\frac{(T-l)^2}{2(T-l)k^2}} \) |
| Weibull      | \( f(T \mid \theta, k) = k \left( \frac{T}{\theta} \right)^{k-1} e^{-\left( \frac{T}{\theta} \right)^k} \) | Gamma        | \( f(T \mid \theta, k) = \frac{1}{\Gamma(k)\theta^k} T^{k-1} e^{-\frac{T}{\theta}} \) | Mixture model     | \( \sum \lambda_i f_i(T \mid \theta, l, k) = \sum \lambda_i \frac{1}{\theta_i\sqrt{2\pi}} e^{-\frac{(T-l)^2}{2\theta_i^2}} \) |
| Kernel density estimation | \( f(T \mid \theta, l, k) = \frac{1}{k^2} \sum_{m=1}^{k} K \left( \frac{T-l}{\theta} \right) \) | Burr         | \( f(T \mid \theta, k_1, k_2) = \frac{k_1 k_2}{\theta^k} \left( \frac{T}{\theta} \right)^{k-1} \left[ 1 + \left( \frac{T}{\theta} \right)^{k+1} \right] \) | Generalized Beta | \( f(T \mid \theta, l, k_1, k_2) = \frac{\Gamma(k_1 + k_2)}{\Gamma(k_1) \Gamma(k_2)} \left( \frac{T-l}{\theta} \right)^{k_1} \left( 1 - \frac{T-l}{\theta} \right)^{k_2} \) |

Tailored Wakeby-type

\[
F^{-1}(\rho \mid l, \theta_1, \theta_2, k_1, k_2) = l + \frac{\theta_1}{k_1} \left( 1 - (1 - \rho)^{k_1} \right) + \frac{\theta_2}{k_2} \left( 1 - (1 - \rho)^{k_2} \right)
\]

Generalized Extreme Value

\[
f(T \mid \theta, l, k) = \frac{1}{\theta} \phi(T)^{k-1} e^{-\rho(T)}, \text{ where } \phi(T) = \left( 1 + k \left( \frac{T-l}{\theta} \right) \right)^{-1/k} \text{ (if } k \neq 0) \text{ or } \phi(T) = e^{-(T-l)^{\theta}} \text{ (if } k = 0)\]

Generalized Pareto

\[
f(T \mid \theta, l, k) = \frac{1}{\theta} \left( 1 + k \left( \frac{T-l}{\theta} \right) \right)^{(k+1)/k}, \text{ for } T > l \text{ when } k > 0 \text{ and } l \leq T \leq l - \theta/k \text{ when } k < 0\]

Gram–Charlier

\[
f(T \mid \xi_1, \xi_2, \xi_3, \xi_4) = \frac{1}{\sqrt{2\pi}} e^{-\frac{T^2}{2\sigma^2}} \left( 1 + \frac{\xi_4}{6} (\phi' - 3\phi) + \frac{\xi_3}{24} (\phi'^2 - 6\phi' + 3) \right), \text{ where } \phi = (T - \xi_2)/\xi_4\]

Cornish–Fisher

\[
F^{-1}(\rho \mid \xi_1, \xi_2, \xi_3, \xi_4) = \xi_1 + \xi_2 \left( U_\rho + \xi_3 (U_\rho^2 - 1)/6 + \xi_4 (U_\rho^3 - 3U_\rho)/24 - \xi_5 (2U_\rho^4 - 3U_\rho^2)/36 \right), \text{ where } U_\rho \text{ is } \rho \text{ quantile of the standard Normal.}\]

Note: 1. The formula of the mixture model depends on the single distribution element; here, we provide the formula of the mixture Normal model as an example.
2. The Compound Gamma distribution does have a closed-from PDF, but cannot be expressed using one formulation. Please refer to Kim and Mahmassani (2015) for details.
2.3 Discussions of the Existing TTD Models

2.3.1 Capturing the heterogeneous TTDs based on different modeling rationales

As verified by Plötz et al. (2017) and Zang et al. (2018a), the performance of the same TTD model in fitting different travel time datasets may be different. Consequently, many different TTD models, based on different modeling rationales, have been developed to fit heterogeneous TTDs. As can be seen from Table 1, the TTD models based on single distribution and mixture modeling rationales have been widely studied for decades. In particular, the single distribution model is suitable for fitting a TTD with a typical right-skewed distribution and a long/fat tail, whereas the mixture distribution model can deal with the multimodality that arises from interrupted flow at intersections or congested traffic conditions in the urban transportation systems. Although relatively less attention has been paid to TTD models based on the moment-based modeling rationale, the performance of TTD models based on this modeling rationale are promising because they do not need assumptions about the distribution of travel time and can adaptively fit heterogeneous TTDs using the actual travel time data. The fourth set of methods to derive TTD models, based on the source-based derivation modeling rationale, is mainly used in network-wide theoretical studies.

Furthermore, as shown in Table 1, some TTD models include multiple modeling rationales. For example, the widely used Normal and Lognormal distributions used in the single distribution modeling rationale are usually combined with an assumed uncertainty source of travel time variability in TTD models (e.g., Lo and Tung, 2003; Lo et al., 2006; Shao et al., 2006a, 2006b; Lam et al., 2008; Li et al., 2017). In addition, some TTD models adopt both source-based derivation and moment-based modeling rationales. For instance, considering day-to-day demand fluctuations, Clark and Watling (2005) applies Johnson curves based on the first four moments of total travel time to characterize their TTD model for evaluating network-wide TTR.

2.3.2 Requirements of TTD models for different applications

TTD models have both theoretical and empirical applications. In theoretical applications, researchers directly assume the distribution type of the travel time and use this to explore further theoretical components of TTR-based models such as travel behavior (Siu and Lo, 2013), reliable route finding (Chen and Ji, 2005; Chen et al., 2014a; Srinivasan et al., 2014; Lee et al., 2019), and network equilibrium (Watling, 2006; Castillo et al., 2012). In empirical applications, researchers develop TTD models to fit real travel time datasets (e.g., Fosgerau and Fukuda, 2012; Susilawati et al., 2013; Zang et al., 2018a, 2018b). Obviously, the criteria for TTD models for these two applications are different. Desirable mathematical properties are crucial for TTD models used in theoretical applications, whereas TTD models for empirical
applications must be accurate and computationally efficient. Therefore, the Normal and Lognormal distributions are the two most widely used TTD models in theoretical applications, but more advanced or complicated TTD models based on the mixture or moment-based methods are more common in empirical applications and seldom used in theoretical applications, with an exception of the Cornish–Fisher expansion (e.g., Lu et al., 2005, 2006; Di et al., 2008, Chen et al., 2011b, Xu et al., 2013, 2014).

The closed-form expressions of TTD models are critical for assessing TTR or including TTR into traffic models. For example, as Zang et al. (2018a) notes (and as to be shown in Table 3), the PPF (i.e., travel time percentile function) is a basic and common element of most TTR measures. Because of the existence of the closed-form PPF, the Burr distribution and Cornish–Fisher expansion are preferred for calculating Fosgearu’s TTR ratio (Taylor, 2017; Zang et al., 2018a). In addition, the closed-form expression of the TTD model is the basis for conceptualizing and formulating computationally tractable or analytically derived traffic models that consider TTR. Thus, the Normal distribution has been widely used in TTR-based traffic models due to its simple and closed-form PDF (e.g., Lam and Xu, 1999; Yin and Ieda, 2001; Lo and Tung, 2003; Lo et al., 2006; Watling, 2006; Lam et al., 2008; Siu and Lo, 2013), although very few studies use it to empirically fit TTD in the context of TTR.

Table 1 documents whether existing TTD models have closed-form expressions of PDF, CDF, and PPF. For reference, Table 2 presents the models with closed-form expressions of the PDF or the inverse CDF. As shown in Table 1, the modeling rationale and closed-form formula of TTD models are closely related. Specifically, most TTD models based on the single distribution modeling rationale have at least one closed-form expression, e.g., Normal, Lognormal, shifted Lognormal, Weibull, Gamma, Generalized Beta, Burr, tailored Wakeby-type, and Generalized extreme value. However, the Stable, Compound Gamma and Compound Generalized extreme value distributions do not have any closed-form expressions, although they are based on the single distribution modeling rationale. Most of the models that adopt the mixture distribution modeling rationale have only closed-form PDF. Similarly, models based on moment-based methods have only closed-form PPFs. As for TTD models based on the source derivation modeling rationale, Kharoufeh and Gautam (2004) derives a closed-form expression of CDF, whereas Zheng et al. (2017) derives a closed-form expression of PDF. For TTD models based on both single distribution and source derivation modeling rationales, the existence of closed-form expressions depends on the assumed distribution type of the unimodal TTD model. For

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6 The earliest adoption of Normal distribution in travel time reliability can only date back to Bell and Iida (1997) in which Normal distribution was introduced to characterize TTD model.
example, the TTD models in Lo and Tung (2003), Lo et al. (2006), Shao et al. (2006a, 2006b), and Lam et al. (2008) have a closed-form PDF due to the assumed Normal distribution, and the TTD model in Li et al. (2017) has a closed-form PDF due to the assumed Lognormal distribution.

3. **Travel Time Reliability Evaluation**

In this section, we review the modeling perspectives and the corresponding methods used in TTR evaluation, and then summarize and analyze the reliability measures.

As characterized TTD is the basis of TTR evaluation in transportation networks, the two common modeling perspectives of evaluating TTR are each associated with one of the modeling perspectives of characterizing TTD: (1) evaluating TTR directly using travel time datasets, and (2) evaluating TTR based on assumed distributions of the sources of uncertainty. Figure 4 summarizes these two modeling perspectives, giving the methods used in each for developing reliability measures to evaluate TTR. Specifically, directly using travel time data is preferred when appropriate datasets are available, as this can eliminate the need to assume the probability distributions of the sources of uncertainty and automatically account for all sources of uncertainty. Evaluations based on assumptions about the sources of uncertainty are useful alternatives, especially for large-scale networks without sufficient travel time data. However, the accuracy of the assumed distributions is not guaranteed when there is no (or insufficient) data available for calibration.

![Figure 4. Frameworks and methods for evaluating TTR](image)

Below is a detailed discussion of the methods used in these two modeling perspectives for TTR evaluation.
• There are two ways to develop reliability measures to evaluate TTR directly using travel time data. The first is to directly develop reliability measures using the moment information of the travel time datasets. Current methods include probability inequalities and generalized moment problems. The second is to characterize the TTD and then develop reliability measures using the characterized TTD. A number of methods use this strategy, including statistical methods, mathematical programming-based methods, and utility-based methods.

• The modeling framework based on assumed distributions of uncertainty sources also provides two ways for developing reliability measures to evaluate TTR. The first way is to directly develop reliability measures using assumed distributions of the uncertainty sources; game theory is a typical method under this approach. The second is to obtain the characterized TTDs and then to develop reliability measures. Current methods of developing reliability measures after obtaining the characterized TTDs is the same as those in the second way of the first modeling perspective (i.e., the above bullet point).

In early studies, TTR is evaluated as the probability that travel times will remain below acceptable levels, and travel times are obtained using extensive simulations designed to solve traffic assignment problems (e.g., Asakura and Kashiwadani, 1991; Du and Nicholson, 1997; Bell et al., 1999). Recently, Sumalee and Watling (2003, 2008) and Ng et al. (2011) propose different methods to obtain upper bounds instead of exact probabilities, which improve the computational efficiency of the methods. In addition to these single scalar performance indices or bounds, some studies construct the whole PDF of the total travel time to obtain a complete picture of TTR (Clark and Watling, 2005; Ng and Waller, 2010a). In summary, the reliability measures developed under these two modeling perspectives can be divided into three classes: (1) point-based measures, including probability-based, moment-based, percentile-based, tail-based, and utility-based measures, (2) bound-based measures, and (3) PDF-based measures.

Reliability measures for TTR evaluation are summarized in Table 3, regarding their types, formulae, methods, assessing level, and representative references. In the “assessing level” column, “all” means that the reliability measures can be used for evaluating link TTR, route TTR, and network TTR; “route” or “network” means that the reliability measures can be only used for evaluating route TTR or network TTR. In general, reliability measures are assumed to be valid at all assessing levels unless specified otherwise. In Sections 3.1 and 3.2, we review the methods associated with each of the above two modeling perspectives, provide their general formulas, and discuss the corresponding reliability measures.
| Type               | Measure | Formula | Method | Assessing level | References                                                                 |
|--------------------|---------|---------|--------|-----------------|--------------------------------------------------------------------------|
| Probability-based  | FR      | $F(T) = \int_0^T f(x) \, dx$ |        | All             | Bell and Iida (1997) and Iida (1999); Lam and Xu (1999); Lei et al. (2014); Chen et al. (2017) |
|                    | FR      | $1 - P(T < (1 + \rho) \cdot F^{-1}(50\%))$ |        | All             | Lomax et al. (2003)                                                     |
|                    | FoC     | $R(T) = \exp \left( - \int_0^T f(x) \, dx \right)$ | Statistic | All             | Al-Deek and Emam (2006); Emam and Al-Deek (2006)                           |
| Moment-based       | SD      | $\sigma$ |        | All             | Chen et al. (2003); Fosgerau et al. (2008); Carrion and Levinson (2013); Mahmassani et al (2013); Yang and Wu (2016) |
|                    | Inverse SD | $1/\sigma$ | Statistic | All             | Polus (1979)                                                             |
|                    | Variance | $\sigma^2$ |        | All             | Rakha et al. (2006, 2010)                                               |
|                    | CoV     | $\sigma/\mu$ |        | All             | Lomax et al. (2003); Chen et al. (2018)                                 |
|                    | $\rho$-PTT | $F^{-1}(90\%)$ or $F^{-1}(95\%)$ |        | All             | FHWA (2009); Arezoumandi (2011)                                          |
|                    | TTI     | $\mu F^{-1}(15\%)$ | Statistic | All             | Nie and Wu (2009); Nie (2011)                                            |
|                    | PTI     | $F^{-1}(95\%) / F^{-1}(15\%)$ |        | All             | NCHRP (2008)                                                            |
|                    | LTI     | $\mu F^{-1}(1-(1-\rho)/2)$ | Link Route | Pu (2011)               | Kaparias et al. (2008)                                                  |
ELI  \( F^{-1}\left( (1 - \rho) / 2 \right) / \mu \)

N.A.  \( F^{-1}(80\%) - F^{-1}(50\%) \)  All  Loon et al. (2011)

Buffer time  \( F^{-1}(95\%) - F^{-1}(50\%) \)  All  FHWA (2009)

BI  \( \frac{F^{-1}(95\%) - F^{-1}(50\%)}{F^{-1}(50\%)} \)  All  Lomax et al. (2003); SHRP (2009)

\( \lambda^{\text{skew}} \)  \( \frac{F^{-1}(90\%) - F^{-1}(50\%)}{F^{-1}(50\%) - F^{-1}(10\%)} \)  All  van Lint et al. (2008)

\( \lambda^{\text{var}} \)  \( \frac{F^{-1}(90\%) - F^{-1}(10\%)}{F^{-1}(50\%)} \)  All  van Lint et al. (2008)

N.A.  \( (F^{-1}(90\%) - F^{-1}(10\%))(1 - \rho_n) + (F^{-1}(90\%) - F^{-1}(10\%))\rho_r \)  Route  Tu et al. (2012)

(T)TTB  \( \min \left\{ T \mid P \left( T \leq T \right) \geq \rho \right\} \)  All  Lo et al. (2006); Shao et al. (2006a, 2016b); Chen et al. (2014a)

MI  \( \int_{\mu_n}^{1} F^{-1}(x) \, dx - \frac{\mu}{\mu} \)  All  Lomax et al. (2003); FHWA (2009)

Tail-based  Statistics

ME(T)TT  \( \frac{1}{1 - \rho} \int_{\rho}^{1} F^{-1}(x) \, dx \)  All  Chen and Zhou (2010); Chen et al. (2011b); Xu et al. (2013, 2014, 2017)
| Metric         | Formula                                                                 | Reference                                      |
|---------------|-------------------------------------------------------------------------|-----------------------------------------------|
| TTRR          | $\frac{\beta + \gamma}{\alpha} \int_{\frac{\gamma}{\gamma}}^{1} F^{-1}(x) \, dx$ | All Fosgerau (2017)                            |
| UA            | $\int_{\rho}^{1} \left( F^{-1}(x) - F^{-1}(\rho) \right) \, dx$          | All Zang et al. (2021)                        |
| Bound-based   | **UBP** N.A. N.A.                                                        | Lempel-Ziv entropy All Li et al. (2019)       |
|               | **UB** $P(T > \bar{T}) \leq UB$                                         | Probability inequalities All Sumalee and Watling (2003, 2008) |
|               | N.A.                                                                     | Generalized moment problem All Ji et al. (2019) |
| PDF-based     | **PDF** $f(T)$                                                           | Moment-based Fourier transform All Clark and Watling (2005) |
|               |                                                                        |                                               | Ng and Waller (2010a) |
| Utility-based | **TDC** $\sum_{i} \sum_{\omega} U_{i}^{\omega} \cdot q_{i}^{\omega}$     | Traffic assignment All Yin and Ieda (2001); Yin et al. (2004) |
|               | **LAP** $\eta E[\max(0, T - \bar{T})]$                                  | Traffic assignment All Watling (2006)          |
|               | **Reliability premium** N.A.                                              | Schedule delay Route Batley (2007); Beaud et al. (2016); Zang et al. (2021) |
|               | **NED** $\mu + \eta E[T - \bar{T}]$                                     | Schedule delay All Lee et al. (2019)          |

Note: 1. FR: failure rate; FoC: frequency of congestion; SD: standard deviation; CoV: coefficient of variation; PTT: percentile of travel time; TTI: travel time index; PTI: planning time index; LTI: lateness index; ELI: earliness index; BI: buffer index; TTB: travel time budget; MI: misery index; METT: mean-excess travel time; TTRR: travel time reliability ratio; UA: unreliability area; UBP: upper bound of predictivity; UB: upper bound; TDC: total disutility of commuter; LAP: late arrival penalty; NED: normalized expected disutility.

2. N.A. means that (1) it is hard to give a specified name of the reliability measures, or (2) it is hard to state the measure in a simple expression; readers can refer to the original references for details.

3. $F_{c}^{-1}$ is the percentile travel time in free flow conditions; $F_{c}^{-1}$ is the percentile travel time in the congested conditions after breakdown; $\rho_{br}$ is the probability of traffic breakdown.
3.1 Evaluating TTR with Travel Time Datasets

3.1.1 Evaluating TTR with only moments information of travel times

Note that probability inequalities and generalized moment problem are proposed for evaluating network-wide TTR, so their corresponding travel time variable is total travel time.

(1) Probability inequalities

Specifically, the upper bound of the reliability engineering function (also called as the survivor function) of total travel time is used as the reliability measure to evaluate network TTR. Let \( R(\vec{T}) \) denote the reliability function of total travel time \( T \) at a specified threshold \( \vec{T} \), i.e., \( R(\vec{T}) = P(T \geq \vec{T}) \).

Then, the upper bound of the reliability function can be written as

\[
\max R(\vec{T}) = \max P(T \geq \vec{T})
\]

(13)

Using common probability inequalities, including the Markov inequality (Ross, 2002) and Madansky inequality (Madansky, 1959), Ng et al. (2011) derives several specific upper bounds of reliability function to assess the network reliability performance, including (1) upper bound based on the first-order moment, (2) upper bound based on both the first and second-order moments, and (3) upper bound based on the first \( n \) moments.

(2) Generalized moment problem

Similarly, to assess network TTR, Ji et al. (2019) proposes a generalized moment problem for obtaining the upper bound of the reliability function \( R(\vec{T}) \). Assume that \( t = (v_1l_1, v_2l_2, \ldots, v_al_a, a \in A) \), which is the vector of total link travel times; \( T = \sum_{a \in A} v_at_a \) is the total travel time, \( \Omega_t \) is the support information about \( t \); \( S = \{T \geq \vec{T}, t \in \Omega_t\} \); and \( 1_S = 1 \) if \( t \) is in the set \( S \), and \( 1_S = 0 \) otherwise. Then, the reliability function can be rewritten as

\[
R(\vec{T}) = P(T \geq \vec{T}) = \int_{\Omega_t} f(s)ds = \int_{\Omega_t} 1_S f(t)dt
\]

(14)

The upper bound of the reliability function can be formulated as a semi-definite program with finite constraints based on the given first \( n \) moments (\( \xi_i \) is the \( i \)th moment of \( t \))

\[
\max \int_{\Omega_t} 1_S f(t)dt
\]

s.t. \( \int_{\Omega_t} t^i f(t)dt = \xi_i, \forall i \in \mathbb{N}^n \)

(15)

\[ f(t) \geq 0 \]
Ji et al. (2019) refers to this semi-definite program as a generalized moment problem and mentions that the upper bounds that they derive are tighter than those in Ng et al. (2011).

3.1.2 Evaluating TTR with characterized travel time distributions

Note that all of the following evaluation methods yield a single scalar performance index.

(1) Statistical methods: probability

According to Asakura and Kashiwadani (1991), Bell and Iida (1997), and Iida (1999), TTR is the probability that a trip can be achieved within a given period. In more recent studies, this given period is interpreted as the upper threshold of travel time accepted by travelers, which consists of two components: expected travel time \( \mu \) and additional time \( \delta \) (Florida Department of Transportation, 2000). This definition of TTR can be formulated as:

\[
R(\bar{T}) = P(T \leq \bar{T}) = P(T \leq \mu + \delta) = \int_{-\infty}^{\mu+\delta} f(x)dx = F(\bar{T})
\]

This definition is used by Lam and Xu (1999), Lei et al. (2014), and Chen et al. (2017), and can be interpreted as the probability of completing a trip successfully. In contrast, some studies (Lomax et al., 2003; Al-Deek and Emam, 2006; Emam and Al-Deek, 2006) highlight the importance of the probability of failure rate in evaluating TTR. Furthermore, the frequency of congestion (FHWA, 2009) quantifies the probability that the travel time will exceed a specified expected threshold. Consequently, an alternative definition of TTR is

\[
R(\bar{T}) = P(T \geq \bar{T}) = 1 - F(\bar{T}) = e^{-\int_{0}^{\bar{T}} g(s)ds} = e^{-\int_{0}^{\bar{T}} \frac{f(s)}{R(s)}ds}
\]

where \( g(x) \) is the failure rate function: \( g(x) = f(x)/R(x) \). The definition of TTR given in Eq. (17) is known as the reliability engineering function. Interested readers can refer to Rausand and Hoyland (2003) for more details.

(2) Statistical methods: moment

The moments of a function are quantitative measures related to the shape of the function’s distribution. According to Carrion and Levinson (2012), TTR is a measure of the spread of TTD, and thus, it is natural to use the moments of travel time to evaluate TTR. Given a continuous TTD, the first moment is the expected travel time \( \mu \) and the \( i \)-th central moment can be obtained by

\[
R(T) = \xi_i = \int_{-\infty}^{\infty} (T - \mu)^i f(T)dT, \quad \text{for} \quad i > 1
\]

Among the first \( n \) moments of travel time, the two most used measures for assessing the reliability performance of transportation networks are the second central moment (variance) (Rakha et al., 2006, 2010) and the associated standard deviation (Chen et al., 2003; Fosgerau...
et al., 2008; Carrion and Levinson, 2013; Mahmassani et al., 2013). Other moment-based measures include the inverse standard deviation (Polus, 1979) and the coefficient of variation (Lomax et al., 2003).

(3) Statistical methods: percentile
As extensive empirical studies have demonstrated the asymmetric and right-skewed features of TTDs, the percentile function has received increasing attention. Generally, the formula of the percentile function is

\[ R(T) = F^{-1}(\rho) \]

where \( \rho \) is the reliability requirement. As shown in Table 3, percentile-based reliability measures include 90th/95th percentile (Chen et al., 2003; FHWA, 2009, Arezoumandi, 2011) or \( \rho \)-percentile travel time (Nie and Wu, 2009; Nie, 2011), travel time index, planning time index (NCHRP, 2008), buffer time and buffer index (Lomax et al., 2003; SHRP, 2009), lateness index and earliness index (Kaparias et al., 2008), and skewness-width \( \lambda_{\text{skew}} \), \( \lambda_{\text{var}} \) (van Lint et al., 2008). Furthermore, the reliability measure by Tu et al. (2012), which combines both the uncertainty and the instability in travel times, can also be viewed as a percentile-based measure.

(4) Statistical methods: distribution tail
Empirical studies (Odgaard et al., 2005; van Lint et al., 2008; Franklin and Karlström, 2009; Sikka and Hanley, 2013) show that the unexpected delays resulting from the distribution tail of travel time can lead to serious consequences. Tail-based reliability measures have been developed to consider the effects of distribution tails in TTR evaluation. Tail-based measures are also referred to as tardy trip indicators, and are used to answer the question “How often will a traveler be unacceptably late?” The key for these measures is modeling the distribution tail, which has the following general formula:

\[ R(\rho) = \int_{\rho}^{1} F^{-1}(x) dx \]

The classic tail-based measures include the misery index (FHWA, 2009), mean-excess travel time (Chen and Zhou, 2010), travel time reliability ratio (Fosgerau, 2017), and unreliability area (Zang et al., 2021). Chen et al. (2022a, b) propose a conservative expected travel time measure for disseminating travel time distribution information with an explicit consideration of distribution tail to the public and service platforms.

(5) Mathematical programming method
Using mathematical programming to define TTR measures is an important alternative method for TTR evaluation. This method considers travelers’ tradeoff between trip efficiency and trip cost and is associated with the behavioral assumption of travelers in considering TTR. In other words, travelers tend to add a safety margin $\delta$ beyond the expected travel time $\mu$ to improve trip TTR (Garver, 1968; Thomson, 1969; Knight, 1974; Hall, 1983; Senna, 1994). The sum of the mean travel time and the specified safety margin is referred to as the effective travel time (Hall, 1983). Mathematically, the optimal $\delta$ in the effective travel time can be determined by the following chance-constrained programming:

$$TTB(\rho) = \min \{\mu + \delta\}$$

s.t. $$\int_{-\infty}^{\mu+\delta} f(t) dt \geq \rho$$

The effective travel time is further conceptualized as the travel time budget in Lo et al. (2006), which replaces the safety margin with the travel time margin, i.e., the product of the route’s standard deviation ($\sigma$) and travelers’ punctuality requirement ($\lambda$): $\delta = \lambda \sigma$. As Wu and Nie (2011) notes, when travel times on all routes of an O-D pair follow the same type of distribution, there is one-to-one correspondence between travel time budget (TTB) and percentile travel time. That is, $TTB(\rho) = \mu + \delta = \mu + \lambda(\rho)\sigma = F^{-1}(\rho)$. As can be seen from Eq. (21), the effective travel time or travel time budget considers the reliability aspect of TTD while ignoring the unreliability aspect. To capture the unreliability aspect of travel time related to excessively late trips, Chen and Zhou (2010) defines the mean-excess travel time (METT) as the conditional expectation of travel times beyond the travel time budget:

$$METT(\rho) = \min \left\{TTB(\rho) + \frac{1}{1-\rho} \int_{\rho}^{1} (F^{-1}(t) - TTB(\rho)) dt \right\}$$

(6) Utility-based method

In utility-based methods, travelers usually have a target travel time or preferred arrival time for their trips. Because of travel time variability, travelers may arrive early or late at the destination, defined as schedule delay early or schedule delay late. To improve trip reliability, travelers attach penalty costs to uncertainty in arrival times. The corresponding utility-based measures mainly include the total disutility of commuters (Yin and Ieda, 2001; Yin et al., 2004), late arrival penalty (Watling, 2006), normalized expected disutility (Lee et al., 2019), and reliability premium (Batley, 2007; Beaud et al., 2016; Zang et al., 2021). The expressions of the above utility-based measures can be found in Table 3. The explicit formula for the reliability premium depends on the linear utility function, travelers’ risk attitudes, etc. Interested readers can refer
to Batley (2007) and Beaud et al. (2016) for the expressions of the reliability premium under different assumptions. Note that the theoretical basis of the utility-based method is the valuation models of TTR to be reviewed in Section 4.1.

It should be noted that many reliability measures such as travel time budget and mean-excess travel time can be regarded as route utilities in the route choice model. However, reliability measures are categorized as utility-based measures in this paper only when they are explicitly based on utility theories.

3.2 Evaluating TTR with Assumed Distribution of Uncertainty Sources

3.2.1 Obtaining the characterized TTDs and then developing reliability measures

We briefly review how to obtain the characterized TTDs based on an assumed distribution of uncertainty from the supply side (Lo and Tung, 2003; Lo et al., 2006), the demand side (Shao et al., 2006a, 2006b), and both the demand and supply sides (Lam et al., 2008).

(1) Demand-side uncertainty

Assume that the stochastic demand \( q_\omega \) of O-D pair \( \omega \) follows an independent theoretical distribution with a mean of \( q_\omega \) and a variance of \( \left( \sigma_q^\omega \right)^2 \). Typically, it is assumed that (1) the route flow denoted by \( f_{o\omega j} \) follows the same type of probability distribution as the O-D demand; (2) the coefficient of variation (CoV) of the route flow is equal to that of the O-D demand; and (3) the route flows are mutually independent. According to the flow conservation constraint, the mean and standard deviation of the stochastic route flow are

\[
\mu_{j}^{o\omega} = E\left[ \tilde{f}_{o\omega j} \right] = \rho_{o\omega j} E\left[ \tilde{q}_{\omega} \right] = \rho_{o\omega j} q_{\omega}, \quad \forall j \in J_\omega, \quad \forall \omega \in \Omega_\omega \quad \text{and}
\]

\[
\sigma_{j}^{o\omega} = \sqrt{\text{Var}\left[ \tilde{f}_{o\omega j} \right]} = f_{o\omega j} \text{CoV}_{\omega} = f_{o\omega j} \frac{\sigma_{q}^{\omega}}{q_{\omega}}, \quad \forall j \in J_\omega, \quad \forall \omega \in \Omega_\omega \quad \tag{23}
\]

where \( \rho_{o\omega j} \) is the choice probability of route \( j \) between O-D pair \( \omega \). Then, based on the definitional constraint, the mean and standard deviation of the stochastic link flow are

\[
\mu_a^{o} = E\left[ \tilde{v}_a \right] = \sum_{o \in \Omega_\omega} \sum_{j \in J_\omega} \delta_{o\omega j} E\left[ \tilde{f}_{o\omega j} \right] = \sum_{o \in \Omega_\omega} \sum_{j \in J_\omega} \delta_{o\omega j} \mu_{j}^{o\omega}, \quad \forall a \in A \quad \text{and}
\]

\[
\sigma_a^{o} = \sqrt{\sum_{o \in \Omega_\omega} \sum_{j \in J_\omega} \delta_{o\omega j} \text{Var}\left[ \tilde{f}_{o\omega j} \right]} = \sqrt{\sum_{o \in \Omega_\omega} \sum_{j \in J_\omega} \delta_{o\omega j} \left( f_{o\omega j} \right)^2 \left( \sigma_{q}^{\omega} / q_{\omega} \right)^2}, \quad \forall a \in A \quad \tag{24}
\]

As we have \( \bar{T}_a = t_a \left( \tilde{v}_a, C_a \right) \), the mean and variance of the link travel time can be computed as:
\[ \mu_r^a = E[\tilde{T}_a] = E[t_a(\bar{v}_a, C_a)], \ \forall a \in A \quad \text{and} \]
\[ (\sigma_r^a)^2 = Var[\tilde{T}_a] = Var[t_a(\bar{v}_a, C_a)], \ \forall a \in A \]

(2) **Supply-side uncertainty**

In this condition, the uncertainty directly propagates from the source to the link cost through a link cost function. Given the link capacity distribution or free-flow TTD, one can derive an explicit expression of the link TTD (i.e., the mean and variance of the link travel time) based on the link travel time function. Assuming that the link capacity is stochastic, we have
\[ \tilde{T}_a = t_a(v_a, \bar{C}_a), \]
and then the mean and variance of the link travel time can be computed as:
\[ \mu_r^a = E[\tilde{T}_a] = E[t_a(v_a, \bar{C}_a)], \ \forall a \in A \quad \text{and} \]
\[ (\sigma_r^a)^2 = Var[\tilde{T}_a] = Var[t_a(v_a, \bar{C}_a)], \ \forall a \in A \]

To have more detailed explicit formulas of mean and variance, one can further assume that the link capacity follows a specific distribution (e.g., the uniform distribution). Interested readers may refer to Lo et al. (2006) for more details.

(3) **Both demand- and supply-side uncertainties**

Assuming that the link capacity and the demand are both stochastic, we have
\[ \tilde{T}_a = t_a(\bar{v}_a, \bar{C}_a), \]
and then the mean and variance of the link travel time can be computed as follows:
\[ \mu_r^a = E[\tilde{T}_a] = E[t_a(\bar{v}_a, \bar{C}_a)], \ \forall a \in A \quad \text{and} \]
\[ (\sigma_r^a)^2 = Var[\tilde{T}_a] = Var[t_a(\bar{v}_a, \bar{C}_a)], \ \forall a \in A \]

With the derived random link travel time and corresponding statistics, the link (or route or network) TTDs can be obtained based on the methods reviewed in Section 2.2.4. With the characterized TTDs, it is straightforward to develop reliability measures, as discussed in Section 3.1.2. Although all of the reliability measures listed in Section 3.1.2 can be computed in theory, the whole PDF (Clark and Watling, 2005; Ng and Waller, 2010a), travel time budget (Lo et al., 2006; Shao et al., 2006a), and mean-excess travel time (Chen and Zhou, 2010) are some common reliability measures under this modeling perspective.
3.2.2 Directly developing reliability measures
Game theory is a representative methodology for directly developing reliability measures based on assumed sources of uncertainty. The source of uncertainty in game theory is the stochastic traffic supplies that are the result of link failures. In particular, compared to the route choice criterion used in traditional user equilibrium or stochastic user equilibrium, the route choice criterion in game theory assumes that the demons select links that will cause the maximum damage to travelers, while travelers accordingly seek the best routes to avoid link failures (e.g., Bell, 2000; Bell and Cassir, 2002; Szeto et al., 2006; Szeto, 2011). The resulting trip cost obtained by the game theory model is used as the measure to evaluate the network performance reliability. Please refer to Section 5.2.1 for the general formula of game theory model.

3.3 Analysis of TTR Evaluation Measures
This section summarizes the existing analysis of reliability measures from different perspectives, including behavioral assumptions, consistency, and accuracy.

(1) Behavioral assumptions
A widely used behavior assumption in the development of reliability measures is that travelers add a safety margin beyond the mean travel time to hedge against travel time variability (Garver, 1968; Thomson, 1969; Knight, 1974; Hall, 1983; Senna, 1994). Specifically, the reliability measures that consider the reliable aspect of travel times use 90th/95th percentile or \(\rho\)-percentile travel time, buffer time and buffer index, and travel time budget. The measures that consider both the reliable and unreliable aspects of travel times include misery index, mean-excess travel time, and travel time reliability ratio. Reliability measures that only consider the consequences of the worst trips include failure rate, frequency of congestion, and unreliability area. Besides, Tan et al. (2014) examines the Pareto efficiency of four widely used reliability measures for risk-taking behavior in terms of mean and standard deviation of travel times, i.e., the percentile travel time, travel time budget, mean-excess travel time, and quadratic disutility function.

(2) Consistency analysis
Most of reliability measures summarized in Table 3 are related to the statistical properties (particularly the shape) of the TTDs. However, this does not mean that these measures behave consistently for the same assessment object. Some studies demonstrate that there are consistencies between some measures. For example, Higatani et al. (2009) finds that buffer time and buffer time index have tendencies similar to those of standard deviation and coefficient of variation, respectively. Dowling et al. (2009) and Fosgerau (2017) find that standard deviation is a good proxy for several other measures, whereas Pu (2011) demonstrates
that coefficient of variation rather than standard deviation is a good proxy for several other measures. In contrast, many studies have verified the inconsistencies between reliability measures. For example, Loon et al. (2011) finds that the correlations between six common measures of the same route give inconsistent outputs. Because of this inconsistency, different studies recommend different TTR evaluation measures. For instance, van Lint et al. (2008) and Tu et al. (2007) argue that percentile-based measures are more robust for capturing the often wide and (left) skewed TTDs; Chen and Zhou (2010) states that the mean-excess travel time performs better than travel time budget for risk-averse travelers, which is further confirmed by Xu et al. (2014) for network-wide reliability assessment and by Zang et al. (2021) for valuing the TTR. Wakabayashi and Matsumoto (2012) suggests that TTR evaluation measures should be selected according to the study’s purpose and the characteristics of the study route. To understand the fundamental causes of the observed consistencies and inconsistencies, Pu (2011) theoretically examines their mathematical relationships and interdependencies with the assumed lognormal distributed travel time, and Xu et al. (2021) theoretically examines the mathematical and behavioral (in)consistency of the schedule delay, travel time budget, and mean-excess travel time models.

(3) Accuracy analysis

Some studies examine the accuracy of TTR measures in different applications. Yang and Cooke (2018) develops a bootstrapping technique-based framework to explore the accuracy of TTR measures and finds that moment-based TTR measures are not sufficiently accurate for evaluating freeway TTR. Considering the computational burden, Rakha et al. (2006, 2010) propose five methods for calculating trip travel time variance based on link travel time variance and conclude that the accuracy of trip travel time variance can be ensured when calculating the coefficient of variation of trip travel time as the conditional expectation over all of links. To improve the estimation of network travel time reliability, Saedi et al. (2020) uses network partitioning to divide the heterogeneous large-scale network into homogeneous regions with well-defined network fundamental diagrams.

4. TRAVEL TIME RELIABILITY VALUATION

In this section, we review theoretical models and valuation measures for quantifying the VOR, and then summarize the existing valuation dimensions of TTR.

According to Li et al. (2010) and Carrion and Levinson (2012), the VOR reflects the value of
the monetary unit that travelers are willing to pay or place to improve the reliability of their travel time. There are four types of theoretical models for quantifying the VOR in transportation networks: mean-variance, schedule delay, mean-lateness, and network utility maximization models. In addition to theoretical models, stated preference and revealed preference surveys can empirically estimate the VOR (see, e.g., Bates et al., 2001; Lam and Small, 2001; Bhat and Sardesai, 2006; Liu et al., 2004, 2007; Batley and Ibáñez, 2009; Carrion and Levison, 2013). As this paper focuses on the theoretical models, interested readers may refer to Li et al. (2010), Carrion and Levison (2012), and Shams et al. (2017) for in-depth reviews of the VOR research based on stated preference and revealed preference surveys. Table 4 briefly summarizes the disutility source, theoretical foundation, utility function, model components, and representative references of these four types of models.

The main difference among the four types of models for valuing TTR is their different assumptions regarding the sources of disutility, and the different assumed sources of disutility further affect their model components considered.

- For the mean-variance model, the source of disutility is the whole TTD. Therefore, the associated valuation metrics generally measure the width of the TTD, including both the right-hand and left-hand sides of the TTD, and the mean-variance model consists of two utilities: the utility of mean travel time and the utility of standard deviation.
- For the schedule delay model, the disutility is incurred when the traveler does not arrive at the destination at the preferred arrival time, and thus the corresponding valuation metrics focus on schedule delay, including both schedule delay early and schedule delay late. Consequently, the schedule delay model contains utilities of travel time, schedule delay early, and schedule delay late.
- As late arrival is much more serious than early arrival, the mean-lateness model assumes that the disutility only results from late arrival. Thus, the mean-lateness model has only two utility components, i.e., the utility of travel time and the utility of the lateness at arrival.
- In the network utility maximization model, the disutility comes from the utility maximization behavior of the drivers in transportation networks. This model is expressed as a mathematical programming subject to several budget constraints.

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7 Schedule delay is also described as the variations to the preferred arrival time in Bates et al. (2001).
| Mathematical model | Disutility source | Theoretical foundation | Utility function | Components | References |
|--------------------|-------------------|------------------------|------------------|------------|------------|
| Mean-variance      | Travel time variability | Expected utility theory | Constant          | Travel time and variability measure | Jackson and Jucker (1982); Pells (1987); Black and Towriss (1993) |
|                    |                   |                        | Polynomial or exponential | Travel time and variability measure | Polak (1987) |
|                    |                   |                        | General form       | Travel time, variability measure, and cost | Senna (1994); Small et al. (1999) |
| Schedule delay     | Not arriving at the destination at the preferred arrival time | Utility maximization | Constant          | Travel time, SDE, and SDL (lateness penalty) | Small (1982); Noland and Small, (1995); Bates et al. (2001); Hollander (2006); Fosgerau and Karlström (2010) |
|                    |                   |                        | Non-constant       | Travel time, SDE, and SDL (lateness penalty) | Tseng and Verhoef (2008); Li et al. (2010); Fosgerau and Engelson (2011); Jenelius et al. (2011) |
| Mean lateness      | Mean lateness at departure and/or arrival | Expected utility theory | Constant          | Travel time and SDL | ATOC (2005); Batley and Ibáñez (2009, 2012) |
| Network utility maximization | Utility maximization behavior of the drivers in the network | Utility maximization behavior in a network | Cobb–Douglas | Objective function, budget constraint, and driver behavior | Uchida (2014); Kato et al. (2020) |

Note: SDE: schedule delay early; SDL: schedule delay late.
In general, the mean-variance model, schedule delay model, and mean-lateness model investigate VOR on the trip level, although some studies extend them to trip chains (e.g., Jenelius et al., 2011; Jenelius, 2012) or bottlenecks (e.g., Siu and Lo, 2009; Coulombel and de Palma, 2014a, 2014b; Zhu et al., 2018). Figure 5 provides an illustration to help readers have a better understanding of these three models, particularly their differences and similarities. In contrast, Uchida’s (2014) network utility maximization model estimates the value of time and VOR in transportation networks using the network structure and drivers’ route choice behavior.

4.1 Theoretical Models for Valuing TTR

This section reviews four theoretical models for valuing TTR. It is important to note that all of the \( \eta \) coefficients in the following equations are parameters to be estimated.

4.1.1 Mean-variance model

The basic idea of the mean-variance model is that both expected travel time and travel time variability are sources of disutility (Jackson and Jucker, 1982; Pells, 1987; Black and Towriss, 1993), and thus the utility in the mean-variance model is defined as a function of expected travel time and travel time variability. The mean-variance model is sometimes referred to as the Bernoulli approach (e.g., Beaud et al., 2016), as it dates back to the expected utility approach originally proposed by Bernoulli (1738) and perfected by von Neuman and Morgenstern (1947).

In the transportation literature, Jackson and Jucker (1982) first proposed the mean-variance framework to study the traveler’s tradeoff between travel time and travel time variability,
where travelers minimize the sum of the disutility of travel time and the disutility of travel time variability as follows:

\[ U(T) = \eta_1 \mu + \eta_2 \text{Var}[T] \]  

(28)

Using the framework in Jackson and Jucker (1982), Polak (1987) considers alternative formulae of utility function to understand the risk behavior of travelers in response to travel time variability, i.e., the polynomial of second degree and exponent form of travel time. By combining the mean-variance model in Jackson and Jucker (1982) and the expected utility approach in Polak (1987), Senna (1994) derives a general algebraic term of degree with respect to the travel time, which makes it possible to measure risk aversion (or proneness). In addition, Senna’s model has another attribute: trip cost \( c \), i.e.,

\[ EU(T) = \eta_1 E[T] + \eta_2 \text{Var}[T] + \eta_3 c \]  

(29)

### 4.1.2 Schedule delay model

Although the mean-variance model assumes a trade-off between the expected travel time and travel time variability for travelers, it ignores the travelers’ scheduling cost. To fill in this gap, Small (1982) extends the seminal work of Gaver (1968) and Vickery (1969) on the schedule delay model to include the scheduling cost. In the schedule delay model, the scheduling cost plays a major role in the travelers’ departure time choice, and travelers’ disutility is due to not arriving at the preferred arrival time, whether early or late. Let \( \text{Arr} \) and \( D \) denote the arrival time and departure time, respectively. The schedule delay model in Small (1982) is expressed as follows:

\[ U(D, T) = \eta_1 T + \eta_2 (SDE) + \eta_3 (SDL) + \eta_4 D_L \]  

(30)

where \( SDE \) is the schedule delay early defined as \((\text{PAT} - \text{Arr})^+\), and \( SDL \) is the schedule delay late defined as \((\text{Arr} - \text{PAT})^+\). \((\cdot)^+\) denotes a function such that \( x^+ = x \) if \( x > 0 \), and 0 otherwise. \( \eta_1, \eta_2, \) and \( \eta_3 \) are the traveler’s preference parameters, representing the marginal utility per unit of mean travel time, marginal utility per unit of SDE, and marginal utility per unit of SDL, respectively. \( D_L \) is a binary term that is equal to 1 if \( SDL \geq 0 \), and 0 otherwise, and \( \eta_4 \) is an additional discrete lateness penalty associated with \( D_L \).

To account for the effect of different levels of congestion, Noland and Small (1995) extends Small’s scheduling model by including the probability distribution of travel time. Therefore, the formula of the expected utility of the scheduling model is

\[ EU(D, T) = \eta_1 E[T] + \eta_2 E[SDE] + \eta_3 E[SDL] + \eta_4 D_L \]  

(31)
For simplicity, the binary lateness penalty can be discarded by assuming that $\eta_4 = 0$, and the corresponding simpler form would be Eq. (32), which is widely used in VOR research (e.g., Fosgerau and Karlström, 2010; Fosgerau and Engelson, 2011; Xiao and Fukuda, 2015; Zang et al., 2018b, 2021). Note that $\eta_1$, $\eta_2$, and $\eta_3$ in Eq. (32) are commonly replaced by $\alpha$, $\beta$, and $\gamma$ in the literature.

$$EU (D, T) = \eta_1 E[T] + \eta_2 E[SDE] + \eta_3 E[SDL]$$

Given the above discussion, Bates et al. (2001) argues that the simple schedule delay model can be approximated by a mean-variance model when travel time follows assumed distributions. Fosgerau and Karlström (2010) theoretically demonstrates that this equivalence holds as long as the distribution of standardized travel time $X$ is independent of the departure time. Specifically, with the standardized travel time $X = (T - \mu)/\sigma$, the mean-variance model for approximating the optimal expected utility $EU^*$ is the weighted sum of the mean and the product of the standard deviation and mean-lateness factor $H$:

$$EU^* = \eta_1 \mu + (\eta_2 + \eta_3) H \left( \Phi, \frac{\eta_2}{\eta_2 + \eta_3} \right) \sigma$$

where $H \left( \Phi, \frac{\eta_2}{\eta_2 + \eta_3} \right) = \int_{\eta_2}^{1} \Phi^{-1} (\rho) d\rho$, and $\Phi^{-1}(\rho)$ is the inverse CDF of standardized travel time $X$.

In Small’s scheduling model, the utility function takes a piece-wise constant form, and thus the corresponding schedule delay model is also referred to as the step model. Based on Vickrey (1973), recent studies (Tseng and Verhoef, 2008; Fosgerau and Engelson, 2011) generalize Small’s scheduling model with time-dependent preference parameters, and thus, the utility function takes a linear form, which is referred to as the slope model. Figure 6 gives the diagrams of the utility functions in the slope model and the step model, where $h(t)$ is the utility of staying at home and $w(t)$ is the utility of staying at work at time $t$. The analytical expression of the slope model with respect to the linear utility function is

$$U = \int_{0}^{D} h(t) dt + \int_{0}^{\Delta t} w(t) dt = \int_{0}^{D} (\eta_1 + \eta_2 t) dt + \int_{\Delta t}^{0} (\eta_3 + \eta_4 t) dt$$

Many studies (e.g., Fosgerau et al., 2008; Fosgerau and Karlström, 2010; Fosgerau and Engelson, 2011; Fosgerau, 2017) demonstrate that the step model is preferable for theoretically

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8 Börjesson et al. (2012) refers to the approximated mean-variance model as the reduced form of the scheduling model and further points out that such an approximation may not capture all of the variability costs. This conclusion has been later verified by Xiao and Fukuda (2015) and Abegaz et al. (2017).
quantifying VOR due to its simple structure, and it is also useful for empirical studies when the preferred arrival time is observable; however, the slope model provides a better explanation of observed scheduling behavior because of its time-varying utility rate, and its related valuation measure (namely travel time variance) is additive over parts of a trip. In addition to the linear utility function shown in Figure 6, non-linear utility functions have been proposed for different purposes, e.g., accounting for risk behavior in Li et al. (2012) and Hensher et al. (2013) and evaluating VOR for trip chains in Jenelius et al. (2011) and Jenelius (2012).

Note that the schedule delay model characterizes congestion as an exogenous phenomenon and thus fails to capture the impact of travelers’ departure time choice on travel time variability/congestion. The trip scheduling equilibrium model based on the bottleneck model addresses this shortcoming (Siu and Lo, 2009; Coulombel and de Palma, 2014a, 2014b; Zhu et al., 2018). Although the trip scheduling equilibrium model uses the same framework of the schedule delay model, it takes into account the equilibrium mechanisms/interactions between individuals’ departure time choice and congestion. Under the trip scheduling equilibrium model, the expected cost \( EC \) for individual \( i \) is

\[
EC = Q(i) / C_b + E(RD) + \eta_1 E[SDE(i)] + \eta_2 E[SDL(i)]
\]

where \( Q(i) \) is the queue length when individual \( i \) entering the bottleneck with the capacity \( C_b \) and \( RD \) is the random delay.

Figure 6. The utility functions of staying at home \( h(t) \) and at work \( w(t) \) at time \( t \) in the (a) step model and (b) slope model (PAT: preferred arrival time)

4.1.3 Mean-lateness model

As another approach to measuring VOR, the mean-lateness model considers only the mean lateness at departure and/or arrival and does not consider mean earliness (i.e., negative lateness).
The original model proposed by the Association of Train Operating Companies (2005) is shown in Eq. (36). It describes the reliability of passenger rail transport in the UK and contains two elements under the expected utility paradigm: scheduled journey time and lateness.

\[ EU = \eta_1 SchedT + \eta_2 L^+ \]  

The scheduled journey time \( SchedT \) is defined as the travel time between the scheduled departure time and scheduled arrival time; lateness, and the lateness \( L^+ \), is the difference between the actual arrival time and scheduled arrival time (i.e., the lateness at destination). Batley and Ibáñez (2009, 2012) extend this model by replacing the original lateness \( L^+ \) with both the lateness at boarding \( LB^+ \) (i.e., the difference between the actual departure time and scheduled departure time) and the lateness at destination \( LD^+ \). For more details about the applications and developments of lateness, please refer to Batley et al. (2011) and Wardman and Batley (2014).

### 4.1.4 Network utility maximization model

To estimate the value of travel time and VOR in transportation networks, Uchida (2014) formulates a utility maximization problem under three budget constraints related to the travel time \( T \), travel cost \( c \), and travel time variance \( \sigma^2 \):

\[
\begin{align*}
\max \quad & U_{cd} = \sum_{o \in \Omega_o} \int_0^{q_o} \frac{\lambda_{wo}}{x+1} \, dx = \sum_{o \in \Omega_o} \lambda_{wo} \ln (q_o + 1) \\
\text{s.t.} \quad & \sum_{a \in A} \int_0^{v_o} t_o(x) \, dx \leq T_{\text{max}} \\
& \sum_{a \in A} \int_0^{v_o} c_a(x) \, dx \leq c_{\text{max}} \\
& \sum_{a \in A} \int_0^{v_o} \sigma^2_a(x) \, dx \leq \sigma^2_{\text{max}} \\
& q_o + e_o = Q_o, \quad \forall o \in \Omega_o
\end{align*}
\]

(37)

where the utility \( U_{cd} \) to be maximized follows the style of the Cobb–Douglas utility function (Cobb and Douglas, 1928), \( \lambda_{wo} (>0) \) is a parameter, and \( \sum_{o} \lambda_{wo} = 1 \). \( e_o \) and \( Q_o \) are the excess demand and assumed maximal traffic demand, respectively. Let \( \varphi_1, \varphi_2, \) and \( \varphi_3 \) denote the optimal Lagrangian multipliers associated with the three budget constraints on \( T, c, \) and \( \sigma^2 \). This utility maximization problem can then be reformulated as a mathematical program that has the same structure as the UE traffic assignment problem with elastic demand:

\[
\min \quad Z = \sum_{a \in A} \int_0^{v_o} x_a(x) \, dx - \sum_{o \in \Omega_o} \int_0^{q_o} \frac{\lambda_{wo}}{x+1} \, dx
\]

(38)
\[ f_{\omega j} = \rho_{\omega j} q_{\omega} \quad \forall j \in J_{\omega}, \quad \forall \omega \in \Omega_{\omega} \]

\[ v_{\omega} = \sum_{\omega \in \Omega_{\omega}, j \in J_{\omega}} \delta_{\omega j} f_{\omega j}, \quad \forall \omega \in A \]

\[ q_{\omega} + e_{\omega} = Q_{\omega}, \quad \forall \omega \in \Omega_{\omega} \]

where \( \chi_{\omega}(v_{\omega}) \) is a monotonic increasing function of \( v_{\omega} \) about link \( a \), and its expression is

\[ \chi_{\omega}(v_{\omega}) = \varphi_{1, a} t_{v_{\omega}} + \varphi_{2, a} c_{\omega}(v_{\omega}) + \varphi_{3, a} \sigma_{\omega}^{2}(v_{\omega}) \]  

(39)

Then, the value of travel time (VOT) and the VOR can be given by

\[ VOT = \frac{\varphi_{1}}{\varphi_{2}} \quad \text{and} \quad VOR = \frac{\varphi_{3}}{\varphi_{2}} \]  

(40)

The above network model is derived under the assumption that link travel times are monotonic and separable; interested readers may refer to Uchida (2014) for the network model in which link travel times are non-monotonic and non-separable. In addition, Kato et al. (2020) extends the model to consider heterogeneous drivers.

### 4.2 Valuation Measures and Reliability Ratio

Generally speaking, all of valuation measures used to quantify VOR belong to the category of reliability measures summarized in Table 3 of TTR evaluation, because valuation measures need to be able to quantitatively evaluate TTR. However, only a small subset of the TTR evaluation measures are appropriate for valuing TTR, and the existing valuation measures used for empirical studies and theoretical studies of VOR can be divided into two types. For the expressions of the valuation measures described below, please refer to Table 3.

The first type of valuation measures assesses the spread of TTD, but different measures capture such spread from different aspects.

- Standard deviation (e.g., Jackson and Jucker, 1982; Senna, 1994; Hollander, 2006; Batley et al., 2008; Fosgerau and Karlström, 2010; Carrion and Levinson, 2013; Coulombel and de Palma, 2014a, 2014b; Zhu et al., 2018)
- Variance (e.g., Fosgerau and Engelson, 2011; Uchida, 2014)
- Coefficient of variation (Abdel-Aty et al., 1995; Small et al., 1995)
- Inter-quantile difference (e.g., Lam and Small, 2001; Brownstone and Small, 2005; Small et al., 2005)
- Width of TTD (Hensher, 2001)
- Unreliability area (Zang et al., 2021)
Standard deviation, variance, and coefficient of variation can be viewed as moment-based measures, which mainly measure the width of the spread of travel times with respect to the mean and are suitable for characterizing a symmetrical distribution. Compared to standard deviation, variance is additive and thus is convenient for network modeling purposes. Inter-quantile difference and width of TTD can be considered as percentile-based measures, which quantify the reliability of the TTD under a specified confidence level. The unreliability area (Zang et al., 2021) focuses on unexpected delays in the tail (i.e., unreliable aspect) of TTD.

The second type of valuation measures uses delays, defined as the difference between the arrival time and a specified value, e.g., preferred arrival time, the most frequently encountered travel time (usual travel time), or timetable. These measures belong to a subset of the utility-based measures summarized in Table 3, including the following.

- Delay early or late (Hollander, 2006, Tilahun and Levinson, 2010)
- Lateness factor (e.g., Franklin and Karlstrom, 2009)
- Reliability premium (e.g., Batley, 2007; Beaud et al., 2016; Zang et al., 2021)

Based on these mathematical models and valuation measures, the outputs of many VOR studies are the estimated VOT and VOR. However, it is meaningless to directly compare the estimated values of VOT and VOR because studies are done in different areas and use different model structures and different valuation measures. To make a fair comparison among different VOR studies, it is necessary to use the so-called reliability ratio, defined as the ratio of the VOR to the VOT (Jackson and Jucker, 1982; Black and Tovriss, 1993). Its expression is

\[ \text{Reliability ratio} = \frac{\Delta U/\Delta \mu_T}{\Delta U/\Delta \sigma_T} = \frac{\text{VOR}}{\text{VOT}} \]  

(41)

where \( \mu_T \) and \( \sigma_T \) are the valuation measures for the VOT and VOR, respectively. The reliability ratio can indirectly reveal the importance of reliability relative to the expected time, and hence many reviews use the estimated reliability ratios in VOR studies to investigate how large the VOR is, such as Chang (2010), Li et al. (2010), Carrion and Levinson (2012), de Jong et al. (2014), and Batley et al. (2019).

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9 Engelson and Fosgerau (2011) shows that the variance is a special case of a more general travel time cost measure, i.e., cumulant generating function measure.
4.3 Dimensions of Valuing Travel Time Reliability

Based on the well-established schedule delay model, Bates et al. (2001) develops a general model of the VOR for personal travel, which brought on huge empirical and theoretical studies of the VOR. Building on this work, Fosgerau and Karlström (2010) derives the simple expression of the optimal expected utility shown in Eq. (33), providing a foundation for subsequent VOR research. In addition to the standard dimension of VOR (e.g., Bates et al., 2001; Fosgerau and Karlström, 2010; Fosgerau and Engelson, 2011), there are a variety of dimensions of VOR. Below, we briefly introduce these valuation dimensions.

(1) The value of service (headway) and service reliability

Scheduled services, e.g., public transportation, train, or air transportation, usually have long service intervals (i.e., headway), which constrain users’ scheduled departure times. To investigate the impact of headway on users’ choice, Fosgerau (2009) defines the value of headway as the marginal increase of users’ cost with the increased headway. Benezech and Coulombel (2013) extends Fosgerau’s work for variable headways and gives two alternative definitions: the value of service and the value of service reliability. These two values quantify the marginal effect of a change in the mean and standard deviation of headway on users’ expected travel cost, respectively. Mathematically, a main difference between deriving the value of service or service reliability and deriving the VOR is that the travel time $T$ in the schedule delay model is replaced by the sum of waiting time $T_w$ and in-vehicle time $T_v$.

(2) The value of information

Empirical evidence shows that disseminating information of TTR to travelers helps them to make better travel choices (Ettema and Timmermans, 2006; Zhang and Levinson, 2008; Zhu and Timmermans, 2010). To quantify the benefits to users, it is straightforward to define the value of information as the difference between trip utility with and without information (e.g., Engelson and Fosgerau, 2020), i.e., $U(D^*, T) - U(D_{info}^*, T)$. $D^*$ and $D_{info}^*$ are the optimal departure time for a trip without and with information, respectively. Except for the value of information derived from a better departure time choice, other studies show that the value of information can be also derived from a better route choice (de Palma and Picard, 2006; Gao et al., 2010; de Palma et al., 2012) and a better departure time and route choice (Soriguera, 2014). In addition, to accurately quantify the value of information, it is necessary to include some critical impact factors in the calculation, such as the cost of obtaining information (e.g., Chorus et al., 2006; Fosgerau and Jiang, 2019; Jiang et al., 2020), the regimes of information release...
(de Palma et al., 2012; Lindsey et al., 2014), and the accuracy of the provided information (Jenelius et al., 2011; Engelson and Fosgerau, 2020).

(3) The cost of misperceived travel time variability

Xiao and Fukuda (2015) derives the misperceived cost of travel time variability by considering how travelers misperceive TTD. To capture the misperceived cost, a general scheduling delay model is formulated based on the rank-dependent utility theory. Under this modeling framework, the misperceived TTD makes the departure time of travelers deviate from the optimal departure time $D^*$, resulting in a suboptimal departure time $D_{misp}$. This deviation allows to quantify the cost of misperceived travel time variability, which is defined as the difference between the expected trip utility under the optimal departure time and the expected trip utility under the suboptimal departure time, i.e., $|EU(D^*, T) – EU(D_{misp}^*, T)|$. Xiao and Fukuda (2015) presents two analytical expressions of the cost of misperceived travel time variability based on the step utility function and slope utility function, respectively.

(4) The value of unreliability

In most VOR studies using the standard scheduling model, the two most widely used measures are the standard deviation and travel time variance (e.g., Hollander, 2006; Fosgerau and Karlström, 2010; Fosgerau and Engelson, 2011; Uchida, 2014; Coulombel and de Palma, 2014a, 2014b; Fosgerau, 2017; Zhu et al., 2018). However, the existence of highly-skewed TTDs with long/fat tails has been verified by considerable empirical evidence and thus unexpected delays could have more serious consequences than expected or modest delays (van Lint and van Zuilen, 2005; Fosgerau and Fukuda, 2012; Susilawati et al., 2013; Delhome et al., 2015; Kim and Mahmassani, 2015; Taylor, 2017; Zang et al., 2018b; Li, 2019). To address this phenomenon, Zang et al. (2021) distinguishes the value of unreliability from the VOR to capture the unreliability cost caused by the long fat tails of real TTDs. Travelers who consider travel time unreliability will depart early to avoid the serious consequences of unexpected delays, leading to a new departure time $D_{adjust}$. The cost of this adjustment, $|EU(D_{adjust}, T)| – |EU(D^*, T)|$, is the unreliability cost that the traveler must pay to allow for travel time unreliability in the departure time choice. Therefore, the value of unreliability can be defined by the unreliability cost with respect to the unreliability measure.

(5) The marginal social cost of travel time reliability
Although the above definitions of VOR reflect how different foci affect the valuation of TTR, their calculations are similar. That is, under all of these extended definitions, VOR can be derived through the difference between utilities under different situations, namely the difference between utilities with the original optimal departure time and the new or adjusted departure time. As discussed above, many factors may affect the departure time, such as service headway, provided information, travelers’ misperception of travel time variability, etc. These adjustments to departure time affect and even improve TTR. However, the feedback effect of this adjustment on the congestion profile has been neglected in the above definitions of VOR. To this end, some studies take the congestion as an endogenous factor rather than an exogenous factor to model the interactions between individuals’ departure time choice and congestion (e.g., Siu and Lo, 2009; Coulombel and de Palma, 2014a, 2014b; Xiao et al., 2017; Zhu et al., 2018). The general formula of this equilibrium trip scheduling is given in Eq. (35). The estimated VOR obtained by this equilibrium trip scheduling model is thus interpreted as the marginal social cost of TTR (Coulombel and de Palma, 2014a).

5. TRAVEL TIME RELIABILITY-BASED TRAFFIC ASSIGNMENT

This section summarizes the methodologies for modeling traffic assignment with consideration of TTR, which will be referred to as TTR-based assignment for short. The general framework of TTR-based assignment is presented in Figure 7.

![Figure 7. Framework for TTR-based traffic assignment models](image)

As TTR affects individual travelers’ route choice behaviors and the collective network flow pattern, TTR-based assignment models have received abundant attention. In these studies, the uncertainty source of travel time variability is typically first specified as coming from the supply side, demand side, or both. Then, based on these assumptions, the uncertainty propagates from its source to the travel time, resulting in travel time variability. A specified route choice criterion for capturing traveler’s risk attitude toward such variability is selected for loading the travel demand onto the network. As the uncertainty propagation from the source to the travel time is discussed in Section 2.2.4, the following discussion only introduces the
route choice criterion with consideration of TTR, the corresponding traffic assignments models, and their solution algorithms.

Before introducing how to capture travelers’ attitudes to TTR in the route choice criterion, it is worthwhile to point out that in the TTR-based traffic assignment, the route choice criterion is used to determine the optimal reliable path for loading the travel demand onto the network. The procedure of determining the optimal reliable path corresponds to the reliable path finding problem in the literature, which is an active research problem conducted by many researchers from different aspects. For example, Miller-Hooks and her colleagues propose several efficient procedures for finding the reliable paths with the least expected time as reliability measure in stochastic and time-varying networks (Miller-Hooks and Mahmassani, 1998, 2000, 2003; Miller-Hooks, 2001; Opasanon and Miller-Hooks, 2006). Shahabi et al. (2013, 2015) discuss the robustness of the reliable path finding problem and design solution algorithms for this problem. The Lagrangian relaxation-based algorithms (Xing and Zhou, 2011; Khani and Boyles, 2015; Yang and Zhou, 2017) and simulation-based method (Zockaie et al., 2014) are developed for handling link travel time correlation as such correlation cannot be neglected in modeling path travel time reliability. As for the computational efficiency, several algorithms are proposed, including the iterative learning approach in Fakhrmoosavi et al. (2019), the Lagrangian substitution algorithm in Zhang and Khani (2019), and the online shortest path algorithm in Khani (2019).

5.1 Route Choice Criterion with Consideration of Travel Time Reliability

There are two dimensions considered in capturing travelers’ attitudes to TTR: objective quantification and subjective perception. Similar to Chen et al. (2002a), Table 5 classifies TTR-based traffic assignment models according to these two dimensions. Objective quantification means transforming the abstract TTR (i.e., characterized TTD) into an intuitive risk measure/utility, as it is difficult to compare different routes using only their TTDs, whereas risk measures in the form of scalars make such comparisons convenient. Obviously, the risk or utility measures used in the TTR-based route choice criterion are the measures for quantitatively assessing TTR and are thus a subset of reliability measures of TTR evaluation summarized in Table 3. Subjective perception reflects how travelers perceive the TTR and can be divided into deterministic perception error and stochastic perception error. Below, we review how these two dimensions are incorporated into the TTR-based route choice criterion.

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10 In the literature, TTR-based route choice criterion may be also called as route choice criterion under uncertainty.
5.1.1 Dimension 1: objective quantification of travel time reliability

To account for TTR in traffic assignment, various risk or utility measures are used to capture the role of travelers’ attitudes toward TTR in route choice criterion. Table 6 summarizes the representative types and the corresponding measure. The first type is based on utility and mainly consists of expected utility-based and non-expected utility-based measures. Expected utility-based measures assume that decision-making under uncertainty aims to maximize the expected utility, e.g., mean-variance (Jackson and Jucker, 1982), late arrival penalty (Watling, 2006), quadratic utility function (Mirchandani and Soroush, 1987), and exponential utility function (Tatineni et al., 1997; Chen et al., 2002a). In the above utility measures, travelers’ different risk attitudes toward TTR can be determined with different parameters. In contrast, the non-expected utility-based measures are based on the idea that decision-making under uncertainty may not be made according to the expected utility maximization, as demonstrated in the cumulative prospect theory (e.g., Avineri, 2006; Connors and Sumalee, 2009; Xu et al., 2011), regret theory (Chorus, 2012; Li and Huang, 2017), and fuzzy set theory (Miralinaghi et al., 2016). Cumulative prospect theory assumes that travelers compare a summary statistic of the perceived utility (instead of the expected utility) of each route before choosing one. Regret theory uses the modified utility function to calculate route utility and assumes that a traveler can anticipate the possibility that the chosen route may be slower or faster than a non-chosen route, causing regret or satisfaction, respectively. Finally, fuzzy set theory uses the membership value function to calculate route utility, where the mode of the random travel time of each link has the highest membership value and, accordingly, the highest route utility.

Another type of route choice criterion is based on safety margin-based risk measures, which assumes that beyond the mean travel time, travelers add a safety margin to improve TTR (Garver, 1968; Thomson, 1969; Knight, 1974; Hall, 1983; Senna, 1994). Such safety margin-based risk measures include travel time budget (Lo et al., 2006; Shao et al., 2006a, 2006b; Siu and Lo, 2008), percentile travel time (Nie, 2011; Ordóñez and Stier-Moses, 2010; Wu and Nie, 2013), and mean-excess travel time (Chen and Zhou, 2010; Chen et al., 2011b; Xu et al., 2017). The former two consider the reliability aspect of travel time variability, and the third one explicitly considers both the reliability and unreliability aspects of travel time variability in the route choice decision process.

| Objective quantification | Subjective perception | No | Yes |
|--------------------------|-----------------------|----|-----|
| User equilibrium         | No                    | TTR-based user equilibrium |
| Stochastic user equilibrium | Yes | TTR-based stochastic user equilibrium |

Table 5. Classification of TTR-based traffic assignment models

| Dimension 1: objective quantification of travel time reliability |
|-------------------------------------------------------------------|
| To account for TTR in traffic assignment, various risk or utility measures are used to capture the role of travelers’ attitudes toward TTR in route choice criterion. Table 6 summarizes the representative types and the corresponding measure. The first type is based on utility and mainly consists of expected utility-based and non-expected utility-based measures. Expected utility-based measures assume that decision-making under uncertainty aims to maximize the expected utility, e.g., mean-variance (Jackson and Jucker, 1982), late arrival penalty (Watling, 2006), quadratic utility function (Mirchandani and Soroush, 1987), and exponential utility function (Tatineni et al., 1997; Chen et al., 2002a). In the above utility measures, travelers’ different risk attitudes toward TTR can be determined with different parameters. In contrast, the non-expected utility-based measures are based on the idea that decision-making under uncertainty may not be made according to the expected utility maximization, as demonstrated in the cumulative prospect theory (e.g., Avineri, 2006; Connors and Sumalee, 2009; Xu et al., 2011), regret theory (Chorus, 2012; Li and Huang, 2017), and fuzzy set theory (Miralinaghi et al., 2016). Cumulative prospect theory assumes that travelers compare a summary statistic of the perceived utility (instead of the expected utility) of each route before choosing one. Regret theory uses the modified utility function to calculate route utility and assumes that a traveler can anticipate the possibility that the chosen route may be slower or faster than a non-chosen route, causing regret or satisfaction, respectively. Finally, fuzzy set theory uses the membership value function to calculate route utility, where the mode of the random travel time of each link has the highest membership value and, accordingly, the highest route utility. Another type of route choice criterion is based on safety margin-based risk measures, which assumes that beyond the mean travel time, travelers add a safety margin to improve TTR (Garver, 1968; Thomson, 1969; Knight, 1974; Hall, 1983; Senna, 1994). Such safety margin-based risk measures include travel time budget (Lo et al., 2006; Shao et al., 2006a, 2006b; Siu and Lo, 2008), percentile travel time (Nie, 2011; Ordóñez and Stier-Moses, 2010; Wu and Nie, 2013), and mean-excess travel time (Chen and Zhou, 2010; Chen et al., 2011b; Xu et al., 2017). The former two consider the reliability aspect of travel time variability, and the third one explicitly considers both the reliability and unreliability aspects of travel time variability in the route choice decision process. |
| Types                     | Risk measures                                                                 | References                                                                 | Basic idea                                                                                                                                 |
|--------------------------|-------------------------------------------------------------------------------|----------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Expected utility-based   | Mean-variance                                                                 | Jackson and Jucker (1982); de Palma and Picard (2005); Di et al. (2008); Sumalee et al. (2011); Nikolova and Stier-Moses (2014); Prakash et al. (2018) | A linear combination of mean and standard deviation of travel time.                                                                    |
|                          | Late arrival penalty                                                          | Watling (2006)                                                             | The route disutility incorporates both the standard generalized travel time and the travel time acceptability in the form of a lateness penalty. |
|                          | Quadratic utility function                                                    | Mirchandani and Sorush (1987); Yin et al. (2004); Zhang et al. (2011)       | A quadratic disutility function that can depict travelers’ avoidance attitude toward risk with different parameters.                     |
|                          | Exponential utility function                                                  | Tatineni et al. (1997); Chen et al. (2002a)                                | An exponential disutility function that can reflect different travelers’ risk attitude with different parameters.                        |
| Non-expected utility-based| Perceived value or prospect                                                   | Avineri (2006); Connors and Sumalee (2009); Sumalee et al. (2009); Xu et al. (2011); Yang and Jiang (2014) | A route’s prospect value is calculated by the reference point, value function, and weight function.                                  |
|                          | Expected modified utility (regret)                                            | Chorus (2012); Li and Huang (2017)                                         | A route’s utility (regret) is determined by its performance difference with the competing route.                                     |
|                          | Fuzzy membership function                                                     | Miralinaghi et al. (2016)                                                   | Travelers’ perceptions of available routes are described as fuzzy sets and their final                                             |
| Safety margin-based                      | Travel time budget | Lo et al. (2006); Shao et al. (2006a, 2006b); Siu and Lo (2006, 2008); Lam et al. (2008); Chen et al. (2011c) | The sum of mean travel time and a safety margin. |
|                                         | Mean-excess travel time | Chen and Zhou (2010); Xu et al. (2014, 2017, 2018) | The conditional expectation of travel times beyond the travel time budget. |
|                                         | Percentile travel time   | Nie (2011); Wu and Nie (2013) | The percentile of travel time. |
| Others                                  | Robust shortest route    | Ordonez and Stier-Moses (2010) | The route that has the best worst-case travel time without being overly conservative by considering a reasonable estimate for the maximum deviation of the route travel time. |
|                                         | Game theory-based measure | Bell (2000); Bell and Cassir (2002); Szeto et al. (2006) | Demons select links to cause the maximum damage to travelers, while travelers seek the best routes to avoid link failures. |
There are also some risk measures in the route choice criteria that do not fall into the above two types. For example, in the game theory model in Bell and Cassir (2002), travelers make route choice decisions based on the expected cost triggered by the uncertain disruption caused by the demon. The expected cost therein is different from that in the Wardrop equilibrium, as the link cost is in a binary state (one value in the normal state and another in the disrupted state) rather than following a distribution. Ordonez and Stier-Moses (2010) assumes that travelers will select the robust shortest route with the minimum worst-case travel time. Wu and Nie (2011) uses stochastic dominance to model heterogeneous risk-taking route choice behavior, and Wang et al. (2014) assumes that travelers seek routes based on an explicit bi-objective criterion, i.e., minimizing the expected travel time and the standard deviation of travel time.

5.1.2 Dimension 2: subjective perception error of travel time reliability
After determining the risk measure, to assign travelers to the network, it is necessary to consider travelers’ subjective perception error in the route choice criterion. There are three types of assumptions: no perception error, deterministic perception error, and stochastic perception error.

These three types of assumptions lead to different route cost structures as shown in Figure 8.

- In the first type, it is assumed that travelers have perfect knowledge about the actual route TTD and can choose the optimal route based on some risk measure imposed on the actual route TTD. The route cost is the actual travel cost, denoted by $c$, and thus is a deterministic value that does not have perception error. This leads to the extension of the user equilibrium traffic assignment model (Wardrop, 1952). Representative studies include the probabilistic user equilibrium in Lo and Tung (2003), the travel time budget-based user equilibrium in Lo et al. (2006), the late arrival penalized user equilibrium in Walting (2006), the mean-excess travel time-based user equilibrium in Chen and Zhou (2010), and the multi-class percentile user equilibrium in Nie (2011).

- In the second type, it is assumed that travelers’ route choice decisions are based on the perceived route TTD, where the perception error is independent of the stochastic travel time. Consequently, the route cost is the perceived cost $c_p$, which is the sum of the actual travel cost $c$ and a deterministic perception error $\varepsilon$ that is independent of $c$, i.e., $c_p = c + \varepsilon$. This assumption is more realistic, as travelers may have imperfect knowledge of the TTD, particularly in congested networks. Accordingly, Siu and Lo (2006) extends the probabilistic user equilibrium model to the stochastic travel time budget equilibrium model. Similarly, Shao et al. (2006b) proposes the travel time budget-based stochastic user equilibrium model that is also used by Lam et al. (2008), Shao et al. (2008), and Siu and
Lo (2008). Note that the above models use the common Gumbel variate adopted in Logit-based stochastic user equilibrium model (Dial, 1971; Fisk, 1980) as the perception error term to construct the perceived travel time budget. Recently, Xu et al. (2021a) develops a Weibit-based mean-excess travel time stochastic user equilibrium model in the context of day-to-day dynamics, where the perception error follows the Weibull distribution.

- In the third type, travelers’ perception error is dependent on the actual travel time distribution, and is thus called “stochastic perception error”. The perceived travel cost $c_p(T_p)$ is a function of the perceived travel time $T_p$, which is the sum of the actual TTD and a stochastic perception error conditional on the actual TTD (i.e., $T_p = T + \varepsilon |T$). In proposing this idea, Mirchandani and Soroush (1987) argues that travelers’ route choice decisions are based on the perceived TTD rather than on the actual TTD. Chen et al. (2011b) proposes a stochastic mean-excess traffic equilibrium model to account for the stochastic perception error. Xu et al. (2013) and Wang and Sun (2016) also assume a stochastic perception error.

In addition to the above three types, Walting (2002) proposes a generalized stochastic user equilibrium model to account for day-to-day uncertainty in traffic flows caused by stochastic variation in both the demand and the route choice proportions, conditional on the demands. Different from the conventional stochastic user equilibrium models, in which the perceived cost is made up of the cost at mean flows and perception error, Walting (2002) adds a third part, called “uncertainty”, to the perceived cost. This uncertainty is generally a random quantity (between days and between drivers) based on the actual variation in traffic conditions. As a result, the route flows are random variables rather than deterministic, which is why Walting (2002)’s model is considered a more general stochastic user equilibrium model.
5.2 Mathematical Models and Solution Algorithms

5.2.1 Mathematical models

Because of the non-additive route cost structure, most TTR-based traffic assignment models are formulated as a variational inequality problem (Shao et al., 2006a; Waltling 2006; Chen and Zhou, 2010; Chen et al., 2011b; Nie, 2011; Wu and Nie, 2013), a non-linear complementarity problem (Ordoñez and Stier-Moses, 2010), or a fixed point problem (Mirchandani and Soroush, 1987; Lam et al., 2008; Sumalee et al., 2011) in terms of the route flow variables. The existence of a solution is guaranteed if the route cost function is continuous and the feasible set is convex. An exception to the above route-based formulations is the link-based mean-excess traffic equilibrium model proposed by Xu et al. (2017). Taking advantage of the sub-additivity of the mean-excess travel time, Xu et al. (2017) proposes a link-based mean-excess traffic equilibrium model in the form of a Beckmann-like transformation. Below we will summarize the general formulations of the above TTR-based traffic assignment models.

(1) Variational inequality

The route flow vector $f^*$ is at equilibrium if the following variational inequality holds:
\[ c(f^*)^\text{Transpose} (f - f^*) \geq 0, \quad \forall f \in \Omega_f, \quad \Omega_f = \left\{ f_{aj} : \sum_{j \in J_\omega} f_{aj} = q_\omega, \quad \forall \omega \in \Omega_\omega \right\} \]  

(42)

where \( \Omega_f \) is the feasible set of the route flow vector. Note that if different forms of route cost functions \( c(\cdot) \) are adopted, as shown in Section 5.1, Eq. (42) would lead to different equilibrium models in the variational inequality form.

(2) Non-linear complementarity problem

The route flow vector \( f \) is at equilibrium if the following non-linear complementarity problem holds:  
\[ 0 \leq f \perp (c(f) - c_{\text{min}}) \geq 0 \]  
\( c_{\text{min}} \) is the vector of the minimum travel cost for each O-D pair. Likewise, different forms of the route cost functions \( c(\cdot) \) would lead to different equilibrium models in the non-linear complementarity problem form.

(3) Fixed point problem

Traffic equilibrium problems under uncertainty, particularly stochastic user equilibrium problems, can also be formulated as fixed point problems. \( f = H(f) \) is the generic formulation of a fixed point problem, where \( H(\cdot) \) is a mapping function; for instance, \( H(f) \) may equal the product of O-D demand and route choice probability.

(4) Game theory model

Bell and Cassir (2002) proposes a game theory model to formulate risk-averse traffic assignment problems. The upper-level problem for the demon maximizes the total expected cost imposed on network users; while the lower-level problem for travelers is a standard deterministic user equilibrium assignment problem, where travelers select the routes with the lowest expected costs in different disruption scenarios caused by the demon. The mathematical formulations of game theory-based equilibrium model with multiple O-D pairs are as follows. For each O-D pair \( \omega \), solve the following simultaneously.

\[
\begin{align*}
\text{Upper level:} & \quad \max_{\rho_{\omega}} \sum_{j \in J_\omega} \sum_{s \in S_\omega} \rho_{as} c_{a_js}(f) f_{aj}, \quad \text{s.t.} \quad \sum_{s \in S_\omega} \rho_{as} = 1, \quad \rho_{\omega} \geq 0 \\
\text{Lower level:} & \quad \max_{v_\omega} \sum_{a \in A} \sum_{s \in S_\omega} \rho_{as} \int_0^{c_{a}(f)} c_{as}(x) dx, \quad \text{s.t.} \quad v_\omega = \sum_{j \in J_\omega} \delta_{aj} f_{aj}, \quad \sum_{j \in J_\omega} f_{aj} = q_\omega
\end{align*}
\]

(43)

where \( c_{a_js}(f) \) and \( c_{as}(x) \) are the cost of route \( j \) and the flow-dependent cost on link \( a \) under scenario \( s \), respectively; \( S_\omega \) is the set of scenarios; \( \rho_{as} \) is the probability of scenario \( s \) (or link damage) for O-D pair \( \omega \), and \( p_{\omega} \) is the corresponding vector of scenario (or link damage) probabilities for O-D pair \( \omega \).
If the route cost is additive, the equilibrium link flow pattern $v^*$ can be obtained by solving the following mathematical programming in the form of a Beckmann’s transformation (Beckmann, 1956).

$$\min \sum_{a \in A} \int_0^{v_{a}(r)} t_a(x)dx$$

s.t.

$$v_a = \sum_{\omega \in \Omega_a} \sum_{j \in J_{\omega}} \delta_{aj} f_{aj}, \quad \forall a \in A$$

$$\sum_{j \in J_{\omega}} f_{aj} = q_{\omega}, \quad \forall \omega \in \Omega_{\omega}$$

$$f_{aj} \geq 0, \quad \forall \omega \in \Omega_{\omega}, \quad j \in J_{\omega}$$

(44)

Due to the non-additive cost structure (e.g., travel time budget and percentile travel time), most TTR-based traffic assignment models cannot have the above form. However, the mean-excess travel time-based equilibrium model can be approximately formulated in this way because of the sub-additivity of the mean-excess travel time (see Xu et al., 2017 for more details).

5.2.2 Solution algorithms

Table 7 summarizes the existing solution algorithms for solving TTR-based traffic assignment problems, which can be classified into two types: route-based algorithms and link-based algorithms. Because of the non-additive structure of most risk measures, the majority of studies adopt a route-based algorithm. The route-based algorithms can be further categorized into four types: the alternating direction method, the gradient projection method, the route-based method of successive averages, and optimization algorithms using gap function reformulations of the variational inequality problem. Some of the proposed approaches require enumerating routes in advance (Shao et al., 2006a; Chen and Zhou, 2010; Chen et al., 2011b), whereas others use the column generation procedures to avoid the need of a pre-specified route set (Wu and Nie, 2013; Chen et al., 2011c). Only a few studies use a link-based algorithm, such as the well-known Frank-Wolfe algorithm, because of additive cost structures in their models. Exceptions include Chen et al. (2002a), where the route utility is the exponential function of route travel time, which can be obtained by summing up the link travel time, and Xu et al. (2017), where the route cost is the sum of the mean-excess travel times of the links making up the route.

Although TTR-based traffic assignment problems adopt similar solution algorithms as conventional traffic assignment problems, the former are generally more computationally demanding than the latter for two reasons. First, the properties of the route cost under uncertainty may be different from the properties without uncertainty. For example, the route
cost under uncertainty may not be monotonic with respect to the route flow, meaning that only a heuristic solution can be found. Second, calculating the risk measures in TTR-based traffic assignment problems requires the consideration of a series of uncertainty propagations (to be reviewed in Section 6), which is much more complex than calculating the deterministic link travel time function as used in conventional traffic assignment problems.

| Type               | Method                                | References                                      |
|--------------------|---------------------------------------|------------------------------------------------|
| Route-based algorithm | Alternating direction | Shao et al. (2006b); Chen et al. (2010, 2011b) |
|                    | Gradient projection                   | Nie (2011); Zhang et al. (2011); Wu and Nie (2013); Ji et al. (2017) |
|                    | Route-based method of successive averages | Waltling (2002); Shao et al. (2006a, 2008) |
|                    | Gap function                          | Lo et al. (2006); Szeto et al. (2006); Siu and Lo (2008) |
| Link-based algorithm | Frank–Wolfe algorithm                | Chen et al. (2002a); Cheu et al. (2007); Xu et al. (2017) |

5.3 Discussions of Travel Time Reliability-Based Traffic Assignment Models

This section briefly reviews the dynamic traffic assignment with the consideration of travel time reliability and the applications of travel time reliability-based traffic assignment model in different research problems, mainly including network design problem, road pricing, and evacuation models.

(1) Dynamic traffic assignment with the consideration of travel time reliability

Compared to the static traffic assignment, dynamic traffic assignment can more realistically capture some important time-dependent phenomena, especially queue formation, link spillover, and temporal bottlenecks formation, and thus has the potential to more effectively support policy evaluation and traffic operation strategy design. Specifically, travel time reliability is considered in the route choice component of the dynamic traffic assignment model. For example, Boyce et al. (1999) proposes a stochastic dynamic user optimal model, where it is assumed that route travel times are variable and perceived by travelers at each time instant and travelers selected routes with the minimum perceived disutilities at each time. Szeto and Sumalee (2009) and Szeto et al. (2011) propose the reliability-based stochastic dynamic user optimal route choice principle, which uses the effective travel time to capture travelers’ attitudes towards the risk of late arrivals owing to travel time variability. Later on, Ng and Waller (2012) considers travel time reliability as the probability that the real travel time
deviates from the expected value in their linear programming cell transmission-based dynamic traffic assignment model. To assess the within-day dynamics of transportation networks and travel time reliability of public railways, Nakayama et al. (2012) establishes a semi-dynamic traffic assignment model under stochastic travel times. Besides, Jiang et al. (2011) and Zockaie et al. (2015) develop the multi-criterion dynamic user equilibrium traffic assignment model, in which the route choice framework explicitly considers heterogeneous users who want to minimize travel time, out-of-pocket cost, and travel time reliability.

(2) The applications of travel time reliability-based traffic assignment model
As network users are concerned with the reliability of reaching their destinations on time, travel time reliability concept is generally embedded into the network design problem. For example, Yang et al. (2000) suggests that combining capacity reliability and travel time reliability together could provide a valuable tool for designing reliable transportation networks. Chootinan et al. (2005) formulates the reliability-based network design problem as a bi-level program of which the lower level sub-program uses the probit-based stochastic user equilibrium to capture travel time reliability. Later on, Chen et al. (2007) proposes alpha reliable network design model to minimize the total travel time budget required to satisfy the total travel time reliability constraint while considering network users’ route choice behavior. Ng and Waller (2009) proposes a mean-variance type of system-optimal network design model with probabilistic guarantees on system-wide travel time, which could yield a confidence interval within which the total system-wide travel time lies when implementing the prescribed capacity expansion decisions. In addition, to make a tradeoff between capacity reliability and travel time reliability, Chen et al. (2011a) further presents a bi-objective reliable network design model to determine the optimal link capacity enhancements under travel demand uncertainty.

Besides the above network design problem, TTR-based traffic assignment has also been have incorporated into evacuation models. For instance, Waller and Ziliaskopoulos (2006) develops a system optimum-dynamic traffic assignment to provide a robust solution with a user specified level of reliability for evacuation modeling. Later on, a model proposed by Ng and Waller (2010b) could ensure a user-specified reliability level on the evacuation plan, which only requires the knowledge of the mean values and upper and lower bounds on the evacuation demand and road capacities. Ng and Lin (2015) extends their work by providing new and complementary probability inequalities for the case when only knowing the means and variances of the uncertain quantities. Lim et al. (2015) further considers capacity uncertainty of road links and proposes a reliability-based evacuation route planning model, which uses the
breakdown minimization principle to find the reliable evacuation routes to load evacuation flow in the network.

Due to the unavoidable travel time variability, researchers also consider travel time reliability in road pricing problem. For example, Jiang et al. (2011) proposes a multi-criterion dynamic user equilibrium model to explicitly consider travel time reliability for designing and analyzing road pricing strategies, while Fakhrmoosavi et al. (2021) presents an equitable pricing scheme with total travel time as reliability measures for heterogeneous users who have different perception values of time and reliability.

6. Uncertainty Propagation in Modeling Travel Time Reliability

In this section, we review the methods for addressing a common difficulty in modeling TTR in transportation networks—uncertainty propagation, i.e., the propagation of uncertainty from its source to TTR at different spatial levels. Specifically, we review the uncertainty propagation from the uncertainty source to the link TTD, and subsequently to the route or network TTR.

6.1 Uncertainty Propagation from Source to Link TTD

In early studies on TTR-based traffic assignment, TTD is assumed to be exogenous (i.e., provided explicitly) and flow-independent. Thus, these studies do not make any assumptions about the source and there is no propagation consideration of uncertainty (Mirchandani and Soroush, 1987; Uchida and Iida, 1993). Recent studies on TTR-based traffic assignment assume that TTDs are flow-dependent and that travel time variability originates from the uncertainty in certain sources, such as the supply side (Lo and Tung, 2003; Lo et al., 2006), the demand side (Shao et al., 2006a, 2006b), or both (Lam et al., 2008). Figure 9 summarizes the three common paradigms for understanding the uncertainty propagation processes in TTR-based traffic assignment problems.

• In Paradigm 1, supply side uncertainty is the sole uncertainty source and includes two cases: link capacity variation (Lo and Tung 2003; Lo et al., 2006) and free-flow travel time variation (Chen et al., 2010). Under this paradigm, the uncertainty would directly propagate from the source to the link travel time through a link cost function.

• In Paradigm 2, demand side uncertainty is the sole uncertainty source (e.g., Clark and Watling, 2005; Shao et al., 2006a, 2006b). Under this paradigm, the uncertainty propagates first from the source to the route flow through an assumed route choice model, and then to the link travel time through the definitional constraint between route flow and link flow and the link cost function.
• In Paradigm 3, uncertainties in both the demand and supply sides are considered to be the uncertainty sources (Lam et al., 2008). As a result, the uncertainty propagation process in this paradigm combines those of the above two paradigms. Interested readers may refer to Lam et al. (2008) for more details.

Note that the general formulae for characterizing the TTR based on the above uncertainty sources have been provided in Sections 2.2.4 and 3.2.1.

![Figure 9. Three paradigms for understanding uncertainty propagation in traffic assignment under travel time uncertainty](image)

### 6.2 Uncertainty Propagation from Link TTD to Route (Network) TTR

Generally, there are two ways of modeling uncertainty propagation from link TTD to route (network) TTR, as summarized in Figure 10. The first way shown by the dashed line in Figure 10 avoids TTD aggregation from link to route/network and directly obtains the route or network TTR from the link TTR. The second way showed by the solid line in Figure 10 considers the aggregation of TTD from the link to route (or network) level, and then the route or network TTR is obtained via the corresponding TTD. The methods for capturing uncertainty propagation under these two ways are summarized in Table 8, indicating their inputs and outputs, spatial and temporal dependence, TTD assumptions, references and notes.
Figure 10. Typical ways of aggregating the uncertainty from link TTD to route/network TTR

6.2.1 From Link TTD to Route (Network) TTD and to Route (Network) TTR

Aggregating TTD from link to route/network requires a trade-off between computational efficiency and modeling accuracy, and appropriately handling the correlations between link TTDs is a key to this process. Generally, studies handle the correlations in three ways: (1) neglecting the correlations between link TTDs; (2) considering the correlations between link TTDs with TTD assumptions; and (3) considering the correlations between link TTDs without TTD assumptions.

(1) Neglecting the correlations between link TTDs

These methods sacrifice modeling flexibility/realism for mathematical tractability. For example, many studies adopt the Central Limit Theorem when aggregating link TTDs to route TTDs (Lo and Tung, 2003; Lo et al., 2006; Shao et al., 2006; Chen and Zhou, 2010), especially in TTR-based traffic assignment problems. If link TTDs are independently and identically distributed (i.i.d.), the Central Limit Theorem would make the route TTDs follow the Normal distribution. Namely, we have

$$T_j = \sum_{a \in A} T_a \delta_{aj} \sim N \left( \sum_{a \in A} \mu_a \delta_{aj}, \sum_{a \in A} \sigma_a^2 \delta_{aj} \right) \quad (45)$$

Some variants of the Central Limit Theorem weaken the strict independently and identically distributed assumption. For example, link TTDs have to be independent but not necessarily identically distributed in the Lyapunov Central Limit Theorem and Lindeberg Central Limit Theorem (Nie, 2011). Although these variants to some extent weaken the independently and identically distributed assumption, empirical datasets may still not satisfy the remaining assumptions. Xu et al. (2017) shows that the route TTD does not always follow a Normal distribution, particularly when the number of links on a route is less than 30. This means that the Normal distribution assumption of route TTD in the Central Limit Theorem may sacrifice too much accuracy for computational efficiency, and thus it fails to capture the empirical
characteristics of TTDs (i.e., positive skew and long upper tail). Furthermore, link TTDs are generally interdependent in reality, while the Central Limit Theorem requires them to be independent. This inconsistency also leads to a loss of accuracy in the aggregation process.

Another intuitive method for aggregating link TTD is the convolution integral (Fan et al., 2005; Ng et al., 2010; Nie, 2011; Ramezani and Geroliminis, 2012; Yang et al., 2013), which assumes that link TTDs are independent. Compared to the Central Limit Theorem, the convolution integral has two advantages: (1) it does not require all of the link TTDs to be identically distributed, and (2) it does not have a requirement on the number of links to ensure its accuracy. Considering that a route $j$ consists of $n$ links, then the PDF of the route TTD is

$$f_j(T) = f_1(T) * f_2(T) * \cdots * f_n(T),$$

where $f_i(T) * f_{i+1}(T) \triangleq \int_{-\infty}^{\infty} f_i(x)f_{i+1}(T-x)dx$ (46)

where $f_i(\cdot) * f_{i+1}(\cdot)$ represents the convolution integral of two link TTDs. It is time-consuming when routes consist of more than two links because of the recursive integral, as revealed in Eq. (46). Therefore, some researchers use approximation algorithms to reduce the computational burden, such as the Laplace transform (Fan et al., 2005) and the fast Fourier transform (Ng et al., 2010). However, link TTDs in real transportation networks are usually interdependent, and the convolution integral ignores the spatiotemporal correlations between link TTDs, which may lead to errors in obtaining route TTDs (Ramezani and Geroliminis, 2012; Chen et al., 2017).

(2) Considering the correlations between link TTDs with TTD assumptions

To alleviate the loss of accuracy caused by neglecting the correlations between link TTDs, some researchers concentrate on modeling the correlations in TTD aggregation with assumed distribution type of link travel time.

Lognormal distribution is a typically assumed distribution type for link TTD for capturing the correlations between link TTDs. Then, the Fenton–Wilkinson approximation is used for obtaining route TTD (Srinivasan et al., 2014; Chen et al., 2018). This approximation method is first proposed by Fenton (1960) to model the transmission loss in communication engineering. Fenton (1960) numerically finds that the sum of several Lognormal random variables is still a Lognormal distribution. That is, the mean of the route TTD equals the sum of the mean of the component link TTDs, and the variance equals the sum of all of the elements of the covariance matrix of the component link TTDs. Based on the Fenton–Wilkinson approximation, the parameters of route TTD can be obtained from the parameters of the link TTDs. Modeling flexibility has been further expanded through the use of 3-parameter
Lognormal (i.e., shifted Lognormal) distributions (Srinivasan et al., 2014). This new shift parameter of route TTD can also be expressed by the shift parameters of the link TTDs.

In a nutshell, the Fenton–Wilkinson approximation uses the approximated additivity of a Lognormal distribution. Similarly, Castillo et al. (2013) uses the property of location-scale family distributions and assumes that the route TTD follows a location-scale family distribution whose parameters can be obtained with the mean and covariance of the link TTDs. The Fenton–Wilkinson approximation is a special case of the moment generating function (MGF) method that is often used to compute a distribution’s moments (Rudin, 1976). The MGF is the expectation of a function of the random variable, which can be defined as follows:

\[
MGF_r(x) = E[e^{\alpha x}]
\]  

(47)

MGF is an alternative expression of a distribution that uniquely determines the distribution. Ma et al. (2017) adopts the MGF method for approximating the sum of the correlated Normal and Lognormal link TTDs. The route TTD can be derived using its MGF, and its MGF can be calculated from the component link TTDs: \( MGFR_{\text{route}} = MGFR\{T_1, T_2, \ldots, T_n\} \).

Note that MGF may not have a general closed form; in such cases, MGF can be approximated by other methods. For example, the MGF of a Lognormal distribution does not have a closed form, so Mehta et al. (2007) uses a Gauss–Hermite expansion to approximate it. However, not all distributions have MGFs.

(3) Considering the correlations between link TTDs without TTD assumptions

As a powerful tool to describe the dependency between random variables, the copula function is a more general method for modeling the aggregation of link TTDs (e.g., Chen et al., 2017; Chen, et al., 2019; Luan et al., 2019; Yun et al., 2019; Prokhorchuk et al., 2019; Qin et al., 2020; Samara et al., 2020; Yu et al., 2020). Before being adopted in transportation research, the copula function has been widely used in finance and insurance (Nelsen, 2006). According to Sklar’s theorem, a copula function can be used to connect multiple univariate marginal distributions to express a corresponding multivariate joint distribution (Nelsen, 2006). Following this rule, the joint TTD of a route consisting of \( n \) links can be expressed by marginal link TTDs with the help of copula function \( C \):
Table 8. Methods for determining uncertainty propagation from the link level to the route (network) level

| Method                        | Input                                         | Output                          | Dependence | TTD assumption                                         | Literature                                                                 | Note                                                                 |
|-------------------------------|-----------------------------------------------|----------------------------------|------------|--------------------------------------------------------|--------------------------------------------------------------------------|----------------------------------------------------------------------|
| Central limit theorem (CLT)   | Mean and variance of link TTD                 | Route/network TTD                | No         | Link TTD i.i.d.; Route TTD follows a normal distribution | Lo and Tung (2003); Lo et al. (2006); Shao et al. (2006a, 2006b); Chen and Zhou (2010) | In some variants of CLT, such as Lyapunov CLT and Lindeberg CLT, link TTDs are not required to be identically distributed |
| Convolution integral          | Link TTD                                      | Route TTD                        | No         | Link TTDs are independent                              | Fan et al. (2005); Ng et al. (2010); Nie (2011); Ramezani and Geroliminis (2012); Yang et al. (2013); Filipovska and Mahmassani (2020); Filipovska et al. (2021) | Different algorithms are used to calculate convolution integral, e.g., the Laplace Transform and the Fast Fourier Transform |
| Moment-Based                  | Moments and covariance matrix of link travel time | Route/network TTD                | Yes        | Link TTD follows a multivariate normal or lognormal distribution | Srinivasan et al. (2014); Ma et al. (2017); Chen et al. (2018); Chen et al. (2020) | Some also assume that route TTD follows a lognormal distribution |
| Copula function               | Link TTD and copula function                  | Route TTD                        | Yes        | No                                                     | Chen et al. (2017); Chen, et al. (2019); Luan et al. (2019); Yun et al. (2019); Prokhorchuk et al. (2019); Qin et al. (2020); Samara et al. (2020); Yu et al. (2020) | /                                                                     |
| Method                        | Traffic states; transition probability; and link TTD | Route TTD | Mean and variance of link TTD | Mean and variance of route TTD | Boolean algebra function | Network reliability | Independent/dependent | Ramezani and Geroliminis (2012); Ma et al. (2017); Yu et al. (2020) | Rakha et al. (2006); Kaparias et al. (2008) | Al-Deek and Emam (2006); Emam and Al-Deek (2006) | Note. i.i.d : independently and identically distributed. |
|-------------------------------|-------------------------------------------------------|-----------|------------------------------|-------------------------------|--------------------------|------------------------|----------------------|---------------------------------------------------------------------|-----------------------------------------------|--------------------------------------------------|-----------------------------------------------------------------|
| Markov Chain framework        | Traffic states; transition probability; and link TTD | Route TTD | Yes                           | Yes                           | No                       | No                     | Yes                  | Yes                                                                  | No    | No                   | No                                                               | Often used in combination with other aggregation methods     | Route coefficient of variation is the mean coefficient of variation over all links | Link reliability function can be expressed by using the well-defined reliability engineering functions |
| Empirical evidence            | Mean and variance of link TTD                         | Yes       | Yes                           | No                            | No                       | No                     | No                   | Rakha et al. (2006); Kaparias et al. (2008)                          | No    | No                   | No                                                               | No    | No                   | No                                                               | Often used in combination with other aggregation methods     | Route coefficient of variation is the mean coefficient of variation over all links | Link reliability function can be expressed by using the well-defined reliability engineering functions |
\[ f(T_1, T_2, \ldots, T_n; \theta) = C(F(T_1, \theta_1), F(T_2, \theta_2), \ldots, F(T_n, \theta_n); \theta) \prod_{i=1}^{n} f(T_i; \theta_i) \]  

where \( f(T_1, T_2, \ldots, T_n; \theta) \) represents the joint route PDF with the parameter \( \theta \); \( f(T_i, \theta_i) \) and \( F(T_i, \theta_i) \) are the marginal link PDF and CDF with the parameter \( \theta_i \), respectively. Many copula families have been developed in the literature and interested readers may refer to Bhat and Eluru (2009) for bivariate copulas and Luan et al. (2019) for multivariate copulas. Generally, modeling the aggregation of link TTDs using copula functions requires three steps: (1) measuring the dependency between link TTDs, (2) selecting optimal copula functions, and (3) modeling the route TTD. The computation used in the copula-based method is challenging in higher dimensions (i.e., routes consisting of more than two links) and therefore is cumbersome for large-scale networks. Yun et al. (2019) proposes a pair-copula to overcome this challenge, and further verification of its performance with a larger number of links is needed.

Another method used in modeling TTD aggregation is the Markov chain (Ramezani and Geroliminis, 2012; Ma et al., 2017; Yu et al., 2020), which is used to describe the stochastic process where one state transitions to another based on corresponding probabilistic rules. The method using Markov chain consists of three stages: (1) defining traffic states, (2) estimating the transition probability of traffic states, and (3) estimating the route TTD. Generally, a Markov chain serves as the framework for the whole TTD aggregation, and it is often combined with other aggregation methods in stage three to estimate Markov route TTDs. For instance, Ramezani and Geroliminis (2012), Ma et al. (2017), and Yu et al. (2020) respectively adopt convolution integral, MGF, and copula in stage three of the Markov chain framework to aggregate link TTDs.

Note that this section only reviews methods that explicitly consider link TTD aggregation. Some studies propose methods to fit route/trip TTD directly based on travel time data, which may implicitly involve link TTD aggregation, e.g., the generative adversarial network method to model trip TTD with GPS data in Zhang et al. (2019a), and the statistical model to characterize route TTD for probe vehicle data in Jenelius and Koutsopoulos (2013).

6.2.2 From Link TTD to Link TTR and to Route (Network) TTR

As shown in Figure 10, directly obtaining route or network TTR from link TTR is another approach to aggregating TTR that can circumvent calculating/fitting route or network TTDs. These methods require that link TTR is well-defined in terms of additivity or other similar properties. However, few studies adopt such methods as it is difficult to satisfy these criteria mathematically.
A typical method is to use the variance as the reliability measure and then it is straightforward to calculate the route TTR from the link TTR because the variance is additive (Sen et al., 2001; Yin and Hitoshi, 2001). To circumvent storing the covariance matrix in estimating the variance of route travel time (Rakha et al., 2006; Kaparias et al., 2008), some studies use empirical approximations which assumes that the coefficient of variation of the route TTD is the mean coefficient of variation of all of the component links.

Another method is to calculate the route or network TTR based on the probability of failure or hazard of links in terms of travel time from the perspective of system reliability engineering. In fact, this method is widely used to assess connectivity reliability (Bell and Iida, 1997). Specifically, the calculation uses Boolean algebra based on the configuration of the networks (i.e., series configuration, parallel configuration, or both) (Al-Deek and Emam, 2006; Emam and Al-Deek, 2006). First, the state of link $a$ can be defined as a binary variable based on link TTR: $R_a = 1$ if link $a$ functions and $R_a = 0$ otherwise. Then, the route TTR $R_j$ with series and parallel configurations can be expressed by the following equation:

$$R_j = \begin{cases} 
\prod_{a \in A} R_a & \text{for series configuration} \\
1 - \prod_{a \in A} (1 - R_a) & \text{for parallel configuration}
\end{cases}$$

The computational burden of Boolean algebra would be high if the network consists of complicated combinations of series and parallel configurations.

7. **Conclusions and Future Research**

This paper reviewed the methodological developments of modeling TTR in transportation networks. Adopting a network perspective, this review concentrated on the methods of modeling TTR from depicting the whole variability picture (i.e., characterizing TTDs) to assessing the TTR (i.e., TTR evaluation and TTR valuation) and lastly to investigating the effects of TTR on individual users’ travel behavior and the collective network flow pattern (i.e., TTR-based traffic assignment). Also, this paper reviewed the methods for addressing a common challenge for modeling TTR in transportation networks: uncertainty propagation from the uncertainty source to link TTR and to route/network TTR. This review, particularly with the integrated review framework and network-wide perspective, is expected to provide a better and deeper understanding about the relationships and common components of four topics mainly studied from different disciplines, and to help developing more possible combinations of TTR modeling philosophy.
Specifically, this paper summarized two common modeling frameworks for characterizing TTD (i.e., fitting and deducing the TTD) and three difficulties in applying these frameworks: (1) characterizing heterogeneous TTDs, 2) satisfying different requirements for theoretical and empirical applications, and (3) deriving route or network TTD based on link TTD. Under these two frameworks, this paper identified four modeling rationales and provided their general formulas and corresponding TTD models. To make the abstract characterizations of TTD understandable and intuitive, TTR evaluation quantitatively assesses the reliability performance using various reliability measures, and TTR valuation focuses on quantifying the VOR in monetary units to understand users’ responses. In particular, TTR evaluation can be directly based on empirical datasets or on assumed uncertainty sources. Therefore, we presented the general formulas of the methods under these two perspectives and summarized the associated behavior assumption, consistency, and accuracy of the reliability measures. In summarizing the valuation of TTR, this paper reviewed the four mathematical models used to measure VOR: the mean-variance model, schedule delay model, mean-lateness model, and network utility maximization model. Then, we summarized the valuation measures and presented the dimensions of the VOR according to the general theoretical frameworks used to derive the valuations. To further investigate the effects of TTR on travelers’ individual route choice behaviors and collective network flow pattern, we reviewed the route choice criteria and corresponding TTR-based assignment models as well as their solution algorithms. Two key dimensions in modeling travelers’ route choice criteria are identified, i.e., objective travel time variability and subjective perception error.

Although this paper reviewed a number of novel ideas and approaches for modeling TTR in transportation networks, many areas remain open to be further investigated. Some potential directions for future research close to the reviewed research topics of this paper are as follows.

- **Unified approach with closed-form expressions for characterizing TTDs in transportation networks.** Although many different TTD models based on different modeling rationales have been developed, it is hard to identify an optimal distribution function for heterogeneous TTDs. In addition, conceptualizing and formulating a computationally tractable or analytically derived TTR model needs closed-form expressions of TTD models. Therefore, it is necessary to develop a unified approach with closed-form expressions to characterize heterogeneous TTDs in transportation networks. To this end, **Zang et al. (2018b)** makes some attempts but the proposed model only has closed-form expressions of PPF. More efforts in this direction are needed for exploring other possible ways and promoting TTR applications in large-scale networks.
• **Simple but useful criteria to select reliability measures.** As discussed in Section 3.3, the existing reliability measures may not behave consistently for the same assessment object. Therefore, it is necessary to establish some criteria to support selecting or developing an optimal reliability measure for practical applications. These criteria should be simple to be well understood by users but useful to identify different optimal reliability measures for different application purposes. To unify and streamline the pool of TTR measures, we need to have a complete and quality dataset to explore the relationship and connection among the existing measures.

• **Reasonable network-wide TTR monitoring method.** In the literature, when analyzing the network-wide TTR, we usually compute some reliability measures with respect to total travel time. Total travel time is the product of link flow and link travel time, which is a simple aggregation from link level to network level. However, this is not suitable for network-wide TTR monitoring. On the one hand, this measure is too aggregated due to the summation and multiplication, making it insensitive to identify network perturbations. On the other hand, it cannot easily tell the source of an identified abnormal network state. Hence, innovative methods are needed to achieve this purpose.

• **Quantifying the VOR at route or network level.** Although the value of travel time (VOT) and VOR are critical to many traffic network models, there are very limited studies to estimate VOT and VOR at network level. Accordingly, the VOT and VOR are generally given and assumed values in network models. Uchida (2014) firstly estimates the VOT and VOR in transportation networks considering the network structure and drivers’ route choice behavior. However, how to quantify the VOR at route, O-D, or network level remains to be open for further investigation. Furthermore, it is important to examine the relationship between classical valuation models at trip level and the new network-level valuation models.

The following identifies some potential future research directions that involve more topics of modeling TTR in transportation networks, especially in the era of new data environment, applications, and emerging technologies.

• **Constructing large-scale baseline travel time datasets at multiple spatial levels.** Due to the lack of large-scale datasets of transportation networks, it is hard to testify the validation of the TTR models developed based on various assumptions and also hard to conduct fair comparisons of TTR models. The advent of big data is making large-scale travel time datasets at the route/network level available, therefore it is necessary to construct the baseline travel time datasets for the development of modeling TTR in transportation
networks. The datasets should be well customized to suit different application purposes, such as TTD characterization at heterogenous links, routes and time periods, and route choice modeling under uncertainty. Based on the datasets, several unique and less studied research questions could be investigated, e.g., (1) the optimal selection of time aggregation interval of modeling TTR for different application purposes, (2) validating the modeling of uncertainty propagation at different spatial levels by “matching” travel time data and traffic flow data; (3) studying the feasibility of still using the deterministic link travel time function like BPR and developing possible adjustment schemes of BPR parameters; and (4) developing data and model integrative driven methodologies for estimating stochastic O-D demand, route flow and travel time. The large sample size of big data is closely related to stochasticity, which should not be simply averaged and then input into the traditional deterministic models.

- **Estimating the values of the reliability/risk parameters through a non-answered way in the era of big data.** This review shows that many TTR models need travelers’ risk parameters as inputs, such as the preference parameters in VOR models, the probability required in reliability measures, and the risk parameters in route choice models. However, it is difficult to set these values in advance and it is impossible to collect each user’s parameters via surveys. Travelers even may not know their own exact values and these parameters may be flow-dependent rather than fixed. The data acquisition techniques, such as GPS, AVI, RFID, Bluetooth, cellular data, can automatically collect long-term massive individual datasets. It will be very promising to study how to mine (e.g., via deep learning) the travelers’ reliability/risk parameters through a non-answered way.

- **Modeling the TTR of multimodal transportation networks.** Most studies of TTR concentrate on a single travel mode, but with the fast development of city scale and urban agglomeration, many trips involve more than one travel mode. Also, emerging trip-oriented services, such as mobility as a services (MaaS) and route guidance, highlight the need to understand the reliability performance of multimodal urban/regional/international transportation systems. Under a multimodal transportation system, travelers may not be able to complete their whole trip if they cannot access one of the necessary travel modes, and thus it would be important to develop methods for modeling reliability on routes involving multiple travel modes, which could be called “door-to-door TTR”. Assessing a door-to-door TTR that consists of different travel modes (e.g., car, bus, metro, bicycle, shared mobility, train, air traffic, etc.) requires redeveloping the methods of assessing TTR with both flexible and scheduled services and modeling the uncertainty propagation reviewed in this paper. From the viewpoint of service providers, it is also interesting to design the insurance packages of
on-time arrival of MaaS and on-time delivery of app-based services, given that the travel time is highly uncertain.

- Investigating the challenges and benefits of emerging technologies on modeling TTR. The popularization of emerging technologies, such as connected and automatic vehicles, can reduce traffic congestion and road crashes (Poczter and Jankovic, 2014; Fagnant and Kockelman, 2015; Papadoulis et al., 2019), leading to less supply-side uncertainty than in systems for conventional vehicles. However, before achieving the highest level of automation (i.e., full self-driving), the road traffic will still be mixed with both conventional vehicles and connected and automatic vehicles. During this long transition period, mixed traffic may generate new driving risk. As traffic incidents contribute significantly to the supply-side uncertainty of transportation networks (van Lint et al., 2008; Chen and Zhou, 2010), it would be worthwhile to explore how the emerging technologies at different transition stages could affect the TTR of transportation networks.

### SUMMARY OF ACRONYMS AND NOTATIONS AND DEFINITIONS

**Table A9. Summary of acronyms.**

| Acronyms | Note                        | Acronyms | Note                        |
|----------|-----------------------------|----------|-----------------------------|
| CDF      | cumulative distribution function | TTR      | Travel time reliability     |
| DTA      | dynamic traffic assignment  | TTD      | Travel time distribution    |
| MGF      | moment generating function  | O-D      | origin-destination         |
| PDF      | probability density function| UE       | user equilibrium            |
| PPF      | percentile point function   | VOR      | Value of travel time reliability |
| SUE      | stochastic user equilibrium | VOT      | Value of travel time        |

**Table A2. Summary of notations and definitions.**

| Notation | Definition                                                                 |
|----------|-----------------------------------------------------------------------------|
| $z$      | The general formula of the PDF of travel time distribution                  |
| $T, \bar{T}$ | Random travel time, and the specified travel time or threshold               |
| $t$      | Time                                                                        |
| $f(T), F(T), F^{-1}(\rho)$ | PDF, CDF and inverse CDF of travel time $T$                                |
| $\phi(X), \Phi(X), \Phi^{-1}(\rho)$ | PDF, CDF and inverse CDF of standardized travel time $X$                   |
| $\mu, \sigma, \sigma^2$ | Mean, standard deviation, and variance                                     |
| $l, \theta, k$ | Location factor, scaling factor, and shape factor                          |
| $\xi$    | Moment of travel time                                                       |
| $\lambda$ | Weight/degree                                                               |
| $\rho$   | Probability or confidence level or reliability requirement                  |
| $a, A$   | Link and the set of links                                                   |
$t_a(\cdot), v_a, C_a, f, q, e, Q$  
Link travel time function, link volume, and link capacity of link $a$  
Path flow and O-D demand  
Excess demand and a constant larger than the maximal traffic demand  
Stochasticity of a variable due to assumed source of uncertainty  
Count number and constant number  
Set  
O-D pair and the set of O-D pairs  
Path and the set of paths of O-D pair $\omega$  
Expectation and variance operators  
Link-path incidence indicator  
Normal distribution  
General formulation of travel time reliability  
Additional travel time  
Scheduling preference parameters in the schedule delay model  
Preference parameter in valuation (utility) models  
TTB and METT for a desired reliability requirement $\rho$  
Utility, expected utility and Cobb–Douglas utility  
travel cost (function)  
Arrival time, departure time and optimal departure time  
Utility of staying at home and staying at work at time $t$  
Mean lateness factor, and lateness at boarding  
Lagrange multiplier  
Valuation measures for VOT and VOR  
perception error  
Mapping function in the fixed point problem  
Convolution operator  
State/scenario  
Moment generating function  
Copula function

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REFERENCES

Abdel-Aty, M., Kitamura, R., Jovanis, P.P., 1995. Travel time variability on route choice using repeated measurement stated preference data. Transportation Research Record 1493, 39–45.

Abegaz, D., Hjorth, K., Rich, J., 2017. Testing the slope model of scheduling preferences on stated preference data. Transportation Research Part B 104, 409-436.

Al-Deek, H., Emam, E., 2006. New methodology for estimating reliability in transportation networks with degraded link capacities. Journal of Intelligent Transportation Systems 10(3), 117–129.

Arezoumandi, M., 2011. Estimation of travel time reliability for freeways using mean and standard deviation of travel time. Journal of Transportation Systems Engineering and Information Technology 11(6), 74–84.

Asakura, Y., 1996. Reliability measures of an origin and destination pair in a deteriorated road network with variable flows, in Bell M.G.H. (Eds.), Transportation Networks: Recent Methodological Advances, Pergamon Press, Oxford, 273-288.

Asakura, Y., Kashiwadani, M., 1991. Road network reliability caused by daily fluctuation of traffic flow. In 19th PTRC Summer Annual Meeting, Briton.

Asensio, J., Matas, A., 2008. Commuters’ valuation of travel time variability. Transportation Research Part E 44, 1074–1085.

Association of Train Operating Companies (ATOC), 2005. Passenger Demand Forecasting Handbook. Association of Train Operating Companies, London.

Avineri, E. 2006. The effect of reference point on stochastic network equilibrium. Transportation Science 40(4), 409-420.

Bates, J., Polak, J., Jones, P., Cook, A., 2001. The valuation of reliability for personal travel. Transport Research Part E 37, 191–229.

Batley, R., 2007. Marginal valuations of travel time and scheduling, and the reliability premium. Transportation Research Part E 43(4), 387-408.

Batley, R., Bates, J., Bliemer, M., Borjesson, M., Bourdon, J., Cabral, M.O., Chintakayala, P.K., Choudhury, C., Daly, A., Dekker, T., Drivyla, E., Fowker, T., Hess, S., Heywood, C., Johnson, D., Laird, J., Mackie, P., Parkin, J., Sanders, S., Sheldon, R., Wardman, M., Worsley, T., 2019. New appraisal values of travel time savings and reliability in Great Britain. Transportation 46 (3), 583–621.

Batley, R., Dargay, J., Wardman, M., 2011. The impact of lateness and reliability on passenger rail demand. Transportation Research Part E 47(1), 61-72.

Batley, R., Grant-Muller,S., Nellthorp, J., de Jong, G., Watling, D., Bates, J., Hess, S., Polak, J., 2008. Multimodal Travel Time Variability. Final Report, Department of Transport, UK.

Batley, R., Ibáñez, J. N., 2012. Randomness in preference orderings, outcomes and attribute tastes: An application to journey time risk. Journal of Choice Modeling 5(3), 157-175.

Batley, R., Ibáñez, J.N., 2009. Randomness in preferences, outcomes and tastes; an application to journey time risk. In: Proceedings of the International Choice Modeling Conference, Harrogate, UK.

Beaud, M., Blayac, T., Stéphan, M., 2016. The impact of travel time variability and travelers’ risk attitudes on the values of time and reliability. Transportation Research Part B 93, 207-224.

Beckmann, M.J., McGuire, C.B., Winsten, C.B., 1956. Studies in Economics of Transportation. Yale University Press, New Haven, Connecticut.

Bell, M.G.H. 2000. A game theory approach to measuring the performance reliability of transport networks. Transportation Research Part B 34(6), 533-545.
Bell, M.G.H., Cassir, C. 2002. Risk-averse user equilibrium traffic assignment: an application of game theory. *Transportation Research Part B* 36(8), 671-681.

Bell, M.G.H., Cassir, C., Iida, Y., Lam, W.H.K., 1999. A sensitivity-based approach to network reliability assessment. In: Proceedings 14th ISTTT, Jerusalem, pp. 283–300.

Bell, M.G.H., Iida, Y., 1997. *Transportation Network Analysis*. Wiley: Chichester.

Benezech, V., Coulombel, N., 2013. The value of service reliability. *Transportation Research Part B* 58, 1-15.

Bernoulli, D., 1738. Exposition of a new theory on the measurement of risk. In: *Econometrica* 22, 1954, translated from Latin into English by Dr. Louise Sommer

Bhat, C., Sardesai, R., 2006. The impact of stop-making and travel time reliability on commute mode choice. *Transportation Research Part B* 40, 709–730.

Bhat, C.R., Eluru, N., 2009. A copula-based approach to accommodate residential self-selection effects in travel behavior modeling. *Transportation Research Part B* 43(7), 749-765.

Black, I.G., Towriss, J.G., 1993. *Demand Effects of Travel Time Reliability*, Centre for Logistics and Transportation. Cranfield Institute of Technology.

Börjesson, M., Eliasson, J., Franklin, J.P., 2012. Valuations of travel time variability in scheduling versus mean–variance models. *Transportation Research Part B* 46, 855–873

Boyce, D.E., Ran, B., Li, I.Y., 1999. Considering travelers’ risk-taking behavior in dynamic traffic assignment, in *Transportation Networks: Recent Methodological Advances*, M.G.H. Bell (ed.), Elsevier, Oxford.

Brownstone, D., Small, K., 2005. Valuing time and reliability: assessing the evidence from road pricing demonstrations. *Transportation Research Part A* 39 (4), 279–293.

Carrion, C., Levinson, D., 2012. Value of travel time reliability: A review of current evidence. *Transportation Research Part A* 46(4), 720-741.

Carrion, C., Levinson, D., 2013. Valuation of travel time reliability from a GPS-based experimental design. *Transportation Research Part C* 35, 305-323.

Castillo, E., Calviño, A., Sánchez-Cambronero, S., Nogal, M., Rivas, A., 2013. A percentile system optimization approach with and without path enumeration. *Computers & Operations Research* 40(11), 2711–2723.

Castillo, E., Nogal. M., Menendez. J., Sanchez-Cambronero. S., Jimenez. P., 2012. Stochastic demand dynamic traffic models using generalized beta-gaussian Bayesian networks. *IEEE Transactions on Intelligent Transportation Systems* 13(2), 565–581.

Chang, J. S., 2010. Assessing travel time reliability in transport appraisal. *Journal of Transport Geography* 18(3), 419-425.

Chen, A., Ji, Z., 2005. Path finding under uncertainty. *Journal of Advanced Transportation* 39(1), 19-37.

Chen, A., Ji, Z.W., Recker, W., 2002a. Travel time reliability with risk-sensitive travelers. *Transportation Research Record* 1783, 27-33.

Chen, A., Kim, J., Zhou, Z., Chootinan, P., 2007. Alpha reliable network design problem. *Transportation Research Record* 2029(1), 49-57.

Chen, A., Yang, H., Lo, H. K., Tang, W. H., 2002b. Capacity reliability of a road network: an assessment methodology and numerical results. *Transportation Research Part B* 36(3), 225-252.

Chen, A., Zhou, Z., 2010. The α-reliable mean-excess traffic equilibrium model with stochastic travel times. *Transportation Research Part B* 44(4), 493-513.

Chen, A., Zhou, Z., Chootinan, P., Ryu, S., Yang, C., Wong, S.C., 2011a. Transport network design problem under uncertainty: a review and new developments. *Transport Reviews* 31(6), 743-768.
Chen, A., Zhou, Z., Lam, W.H.K. 2011b. Modeling stochastic perception error in the mean-excess traffic equilibrium model. *Transportation Research Part B* 45(10), 1619-1640.

Chen, B.Y., Lam, W.H.K., Sumalee, A., Li, Q., Tam, M.L., 2014a. Reliable shortest path problems in stochastic time-dependent networks. *Journal of Intelligent Transportation Systems* 18, 177–189.

Chen, B.Y., Lam, W.H.K., Sumalee, A., Shao, H., 2011c. An efficient solution algorithm for solving multi-class reliability-based traffic assignment problem. *Mathematical and Computer Modeling* 54(5-6), 1428-1439.

Chen, C., Skabardonis, A., Varaiya, P., 2003. Travel-time reliability as a measure of service. *Transportation Research Record* 1855, 74-79.

Chen, M., Yu, G., Chen, P., Wang, Y., 2017. A copula-based approach for estimating the travel time reliability of urban arterial. *Transportation Research Part C* 82, 1-23.

Chen, P., Tong, R., Lu, G., Wang, Y., 2018. The α-reliable path problem in stochastic road networks with link correlations: a moment-matching-based path finding algorithm. *Expert Systems with Applications* 110, 20-32.

Chen, P., Tong, R., Yu, B., Wang, Y., 2020. Reliable shortest path finding in stochastic time-dependent road network with spatial-temporal link correlations: a case study from Beijing. *Expert Systems with Applications* 147, 113192.

Chen, P., Yin, K., Sun, J., 2014b. Application of finite mixture of regression model with varying mixing probabilities to estimation of urban arterial travel times. *Transportation Research Record* 2442, 96–105.

Chen, P., Zeng, W., Chen, M., Yu, G., Wang, Y., 2019. Modeling arterial travel time distribution by accounting for link correlations: a copula-based approach. *Journal of Intelligent Transportation Systems* 23(1), 28-40.

Chen, R., Xu, X., Chen, A., Yang, C., 2022a. A conservative expected travel time approach for traffic information dissemination under uncertainty. *Transportmetrica B*, 10.1080/21680566.2022.2060369.

Chen, R., Xu, X., Chen, A., Zhang, X., 2022b. How to disseminate uncertain waiting time in app-based transportation services considering attractiveness and credibility. *Transportmetrica A*, 10.1080/23249935.2022.2077857.

Choe, R. L., Kreinovich, V., Manduva, S. R., 2007. Traffic assignment for risk averse drivers in a stochastic network. *In the Proceedings of the 87th Annual Meeting of the Transportation Research Board*, Washington, D.C.

Chootinan, P., Wong, S.C., Chen, A., 2005. A reliability-based network design problem. *Journal of Advanced Transportation* 39(3), 247-270.

Chorus, C. G., Arentze, T. A., Molin, E. J., Timmermans, H. J., Van Wee, B., 2006. The value of travel information: Decision strategy-specific conceptualizations and numerical examples. *Transportation Research Part B* 40(6), 504-519.

Chorus, C.G., 2012. Regret theory-based route choices and traffic equilibria. *Transportmetrica* 8(4), 291-305.

Clark, S., Watling, D., 2005. Modeling network travel time reliability under stochastic demand. *Transportation Research Part B* 39, 119–140.

Cobb, C.W., Douglas, P.H., 1928. A theory of production. *American Economic Review* 18(1), 139–165.

Connors, R.D., Sumalee, A. 2009. A network equilibrium model with travellers' perception of stochastic travel times. *Transportation Research Part B* 43(6), 614-624.

Coulombel, N., de Palma, A., 2014a. Variability of travel time, congestion, and the cost of travel. *Mathematical Population Studies* 21(4), 220-242.
Coulombel, N., de Palma, A., 2014b. The marginal social cost of travel time variability. *Transportation Research Part C* 47, 47-60.

Daganzo, C.F., 1997. *Fundamentals of Transportation and Traffic Operations*. Pergamon, Oxford.

de Jong, G. C., Bliemer, M. C., 2015. On including travel time reliability of road traffic in appraisal. *Transportation Research Part A* 73, 80-95.

de Jong, G., Kouwenhoven, M., Bates, J., Koster, P., Verhoef, E., Tavasszy, L., Warffemius, P., 2014. New SP-values of time and reliability for freight transport in the Nwetherlands. *Transportation Research Part E* 64, 71-87.

de Palma, A., Lindsey, R., Picard, N., 2012. Risk aversion, the value of information, and traffic equilibrium. *Transportation Science* 46(1), 1-26.

de Palma, A., Picard, N. 2005. Route choice decision under travel time uncertainty. *Transportation Research Part A* 39(4), 295-324.

de Palma, A., Picard, N., 2006. Equilibria and information provision in risky networks with risk averse drivers. *Transportation Science* 40(4) 393–408.

Delhomme, R., Billot, R., El-Faouzi, N.E., 2015. Moment-ratio diagram for travel time reliability: Empirical study on urban and perirurban links. In *Proceeding of the 6th International Symposium on Transportation Network Reliability*, Nara, Japan.

Di, S., Pan, C., Ran, B., 2008. Stochastic multiclass traffic assignment with consideration of risk-taking behaviors. *Transportation Research Record* 2085, 111–123.

Dial, R.B., 1971. A probabilistic multipath traffic assignment model which obviates path enumeration. *Transportation Research* 5(2), 83-111.

Dong, J., Mahmassani, H. S., 2009. Flow breakdown and travel time reliability. *Transportation Research Record* 2124(1), 203-212.

Dowling, R. G., Skabardonis, A., Margiotta, R. A., Hallenbeck, M. E., 2009. Reliability Breakpoints on Freeways. *Presented at 88th Annual Meeting of the Transportation Research Board*, Washington, D.C.

Du, Z.P., Nicholson A.J., 1997. Degradable transportation systems: sensitivity and reliability analysis. *Transportation Research Part B* 31(3), 225-237.

Emam, E., Al-Deek, H., 2006. Using real-life dual-loop detector data to develop new methodology for estimating freeway travel time reliability. *Transportation Research Record* 1959, 140–150.

Engelson, L., Fosgerau, M., 2011. Additive measures of travel time variability. *Transportation Research Part B* 45, 1560–1571.

Engelson, L., Fosgerau, M., 2020. Scheduling preferences and the value of travel time information. *Transportation Research Part B* 134, 256-265.

Esfeh, M. A., Kattan, L., Lam, W. H. K., Esfe, R. A., Salari, M., 2020. Compound generalized extreme value distribution for modeling the effects of monthly and seasonal variation on the extreme travel delays for vulnerability analysis of road network. *Transportation Research Part C* 120, 102808.

Ettema, D., Timmermans, H., 2006. Costs of travel time uncertainty and benefits of travel time information: Conceptual model and numerical examples. *Transportation Research Part C* 14(5), 335-350.

Fagnant, D. J., Kockelman, K., 2015. Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations. *Transportation Research Part A* 77, 167-181.
Fakhrmoosavi, F., Zockaie, A., Abdelghany, K., 2021. Incorporating travel time reliability in equitable congestion pricing schemes for heterogeneous users and bimodal networks. *Transportation Research Record* 2675(11), 754–768.

Fakhrmoosavi, F., Zockaie, A., Abdelghany, K., Hashemi, H., 2019. An iterative learning approach for network contraction: Path finding problem in stochastic time-varying networks. *Computer -Aided Civil and Infrastructure Engineering* 34(10), 859-876.

Fan, Y.Y., Kalaba, R.E., Moore, J.E., 2005. Arriving on time. *Journal of Optimization Theory and Applications* 127, 497–513.

Fenton, L., 1960. The sum of log-normal probability distributions in scatter transmission systems. *IRE Transactions on Communications Systems* 8 (1), 57-67.

FHWA. 2009. Travel time reliability: making it there on time, all the time. Texas Transportation Institute and Cambridge Systems, Inc. [http://ops.fhwa.dot.gov/publications/tt_reliability/](http://ops.fhwa.dot.gov/publications/tt_reliability/).

Filipovska, M., Mahmassani, H.S., 2020, September. Reliable least-time path estimation and computation in stochastic time-varying networks with spatio-temporal dependencies. In 2020 IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC). IEEE.

Filipovska, M., Mahmassani, H.S., Mittal, A., 2021. Estimation of path travel time distributions in stochastic time-varying networks with correlations. *Transportation Research Record* 2675(11), 498-508.

Fisk, C., 1980. Some developments in equilibrium traffic assignment. *Transportation Research Part B* 14(3), 243-255.

Florida Department of Transportation. 2000. The Florida reliability method: In Florida’s mobility performance measures program. [http://www.dot.state.fl.us/planning/statistics/mobilitymeasures/reliability.pdf](http://www.dot.state.fl.us/planning/statistics/mobilitymeasures/reliability.pdf).

Fosgerau, M., 2009. The marginal social cost of headway for a scheduled service. *Transportation Research Part B* 43(8–9), 813-820.

Fosgerau, M., 2017. The valuation of travel time variability. International Transport Forum Report on Quantifying the Socio-Economic Benefits of Transport, OECD Publishing, Paris. [http://dx.doi.org/10.1787/9789282108093-en](http://dx.doi.org/10.1787/9789282108093-en).

Fosgerau, M., Engelson, L., 2011. The value of travel time variance. *Transportation Research Part B* 45, 1-8.

Fosgerau, M., Fukuda, D., 2012. Valuing travel time variability: Characteristics of the travel time distribution on an urban road. *Transportation Research Part C* 24, 83–101.

Fosgerau, M., Hjorth, K., Brems, C. R., Fukuda, D., 2008. *Travel time variability: Definition and valuation*. DTU Transport, Denmark.

Fosgerau, M., Jiang, G., 2019. Travel time variability and rational inattention. *Transportation Research Part B* 120, 1-14.

Fosgerau, M., Karlström, A., 2010. The value of reliability. *Transportation Research Part B* 44, 38–49.

Franklin, J. P., Karlström, A., 2009. Travel time reliability for Stockholm roadways: modeling mean lateness factor. *Transportation Research Record* 2134(1), 106-113.

Gao, S., Frejinger, E., Ben-Akiva, M., 2010. Adaptive route choices in risky traffic networks: a prospect theory approach. *Transportation Research Part C* 18 (5), 727–740.

Garver, D.P., 1968. Headstart strategies for combating congestion. *Transportation Science* 2(3), 172-181.

Guessous, Y., Aron, M., Bhouri, N., Cohen, S., 2014. Estimating travel time distribution under different traffic conditions. *Transportation Research Procedia* 3, 339-348.
Guo, F., Li, Q., Rakha, H., 2012. Multistate travel time reliability models with skewed component distributions. *Transportation Research Record* 2315, 47-53.

Guo, F., Rakha, H., Park, S., 2010. Multistate model for travel time reliability. *Transportation Research Record* 2188, 46–54.

Hall, R.W. 1983. Travel outcome and performance: The effect of uncertainty on accessibility. *Transportation Research Part B* 17(4), 275-290.

Hensher, D. A., 2001. The valuation of commuter travel time savings for car drivers: evaluating alternative model specification, *Transportation* 28, 101-118.

Hensher, D. A., Li, Z., Rose, J. M., 2013. Accommodating risk in the valuation of expected travel time savings. *Journal of Advanced Transportation* 47(2), 206-224.

Herman, R., Lam, T., 1974. Trip time characteristics of journeys to and from work. In: Buckley, D.J. (Ed.), *Transportation and Traffic Theory*, Sydney.

Heydecker, B.G., Lam, W. H., Zhang, N., 2007. Use of travel demand satisfaction to assess road network reliability. *Transportmetrica* 3(2), 139-171.

Higatani, A., Kitazawa, T., Tanabe, J., Suga, Y., Sekhar, R., Asakura, Y., 2009. Empirical analysis of travel time reliability measures in Hanshin expressway network. *Journal of Intelligent Transportation Systems* 13(1), 28-38.

Hollander, Y., 2006. Direct versus indirect models for the effects of unreliability. *Transportation Research Part A* 40, 699–711.

Hou, L., Tan, J., 2009. Estimating travel time reliability in urban transportation using Gram-Charlier distribution (in Chinese). *Chinese Journal of Management Science* 17(6), 139-146.

Iida, Y., 1999. Basic concepts and future directions of road network reliability analysis. *Journal of Advanced Transportation* 33(2), 125-134.

Jackson, W., Jucker, J., 1982. An empirical study of travel time variability and travel choice behavior. *Transportation Science* 16, 460–475.

Jenelius, E., 2012. The value of travel time variability with trip chains, flexible scheduling and correlated travel times. *Transportation Research Part B* 46, 762–780.

Jenelius, E., Koutsopoulos, H. N., 2013. Travel time estimation for urban road networks using low frequency probe vehicle data. *Transportation Research Part B* 53, 64-81.

Jenelius, E., Mattsson, L.G., Levinson, D., 2011. Traveler delay costs and value of time with trip chains, flexible activity scheduling and information. *Transportation Research Part B* 45, 789–807.

Ji, X., Ban, X., Li, M., Zhang, J., Ran, B. 2017. Non-expected Route Choice Model under Risk on Stochastic Traffic Networks. *Networks & Spatial Economics* 17(3), 777-807.

Ji, X., Ban, X., Zhang, J., Ran, B., 2019. Moment based travel time reliability assessment with Lasserre’s relaxation. *Transportmetrica B: Transport Dynamics* 7(1), 401-422.

Jiang, G., Fosgerau, M., Lo, H.K., 2020. Route choice, travel time variability, and rational inattention. *Transportation Research Part B* 132, 188-207.

Jiang, L., Mahmassani, H.S., Zhang, K., 2011. Congestion pricing, heterogeneous users, and travel time reliability: Multicriterion dynamic user equilibrium model and efficient implementation for large-scale networks. *Transportation Research Record* 2254(1), 58-67.

Jiantanakul, K., Chu, L., Jayakrishnan, R., 2009. Bayesian mixture model for estimating freeway travel time distributions from small probe samples from multiple days. *Transportation Research Record* 2136, 37-44.
Kaparias, I., Bell, M.G.H., Belzner, H., 2008. A new measure of travel time reliability for in-vehicle navigation systems. *Journal of Intelligent Transportation Systems* 12(4), 202–211.

Kato, T., Uchida, K., Lam, W. H., Sumalee, A., 2020. Estimation of the value of travel time and of travel time reliability for heterogeneous drivers in a road network. *Transportation* 48, 1639–1670.

Kazaglis, E., Koutsopoulos, H.N., 2012. Estimation of arterial travel time from automatic number plate recognition data. *Transportation Research Record* 2391, 22–31.

Khani, A., 2019. An online shortest path algorithm for reliable routing in schedule-based transit networks considering transfer failure probability. *Transportation Research Part B* 126, 549-564.

Khani, A., Boyles, S.D., 2015. An exact algorithm for the mean–standard deviation shortest path problem. *Transportation Research Part B* 81, 252-266.

Kharoufeh, J. P., Gautam, N., 2004. Deriving link travel-time distributions via stochastic speed processes. *Transportation Science* 38(1), 97-106.

Kim, J., 2014. Travel Time Reliability of Traffic Networks: Characterization, Modeling and Scenario-based Simulation. PhD dissertation, Northwestern University.

Kim, J., Mahmassani, H.S., 2015. Compound Gamma representation for modeling travel time variability in a traffic network. *Transportation Research Part B* 80, 40–63.

Knight, T.E., 1974. An approach to the evaluation of changes in travel unreliability: a “safety margin” hypothesis. *Transportation 3*(4), 393-408.

Lam, T.C., Small, K., 2001. The value of time and reliability: measurement from a value pricing experiment. *Transportation Research E* 37 (2–3), 231–251.

Lam, W. H. K., Shao, H., Sumalee, A., 2008. Modeling impacts of adverse weather conditions on a road network with uncertainties in demand and supply. *Transportation Research Part B* 42(10), 890-910.

Lam, W. H., Xu, G., 1999. A traffic flow simulator for network reliability assessment. *Journal of Advanced Transportation* 33(2), 159-182.

Lei, F., Wang, Y., Lu, G., Sun, J., 2014. A travel time reliability model of urban expressways with varying levels of service. *Transportation Research Part C* 48, 453-467.

Li, H., He, F., Lin, X., Wang, Y., Li, M., 2019. Travel time reliability measure based on predictability using the Lempel–Ziv algorithm. *Transportation Research Part C* 101, 161-180.

Li, M., Huang, H.J. 2017. A regret theory-based route choice model. *Transportmetrica A: Transport Science* 13(3), 250-272.

Li, M., Zhou, X., Rouphail, N. M., 2017. Quantifying travel time variability at a single bottleneck based on stochastic capacity and demand distributions. *Journal of Intelligent Transportation Systems* 21(2), 79-93.

Li, Z., Hensher, D.A., Rose, J.M., 2010. Willingness to pay for travel time reliability in passenger transport: A review and some new empirical evidence. *Transportation Research Part E* 46, 384–403.

Li, Z., Tirachini, A., Hensher, D.A., 2012. Embedding risk attitudes in a scheduling model: application to the study of commuting departure time. *Transportation science* 46(2), 170-188.
Lim, G.J., Rungta, M., Baharnemati, M.R., 2015. Reliability analysis of evacuation routes under capacity uncertainty of road links. *IIE Transactions* 47(1), 50-63.

Lindsey, R., Daniel, T., Gisches, E., Rapoport, A., 2014. Pre-trip information and route-choice decisions with stochastic travel conditions: Theory. *Transportation Research Part B* 67, 187-207.

Liu, H., He, X., Recker, W., 2007. Estimation of the time-dependency of values of travel time and its reliability from loop detector data. *Transportation Research Part B* 41, 448–461.

Liu, H., Recker, W., Chen, A., 2004. Uncovering the contribution of travel time reliability to dynamic route choice using real-time loop data. *Transportation Research Part A* 38, 435–453.

Lo, H. K., Tung, Y. K., 2003. Network with degradable links: capacity analysis and design. *Transportation Research Part B* 37(4), 345-363.

Lo, H.K., Luo, X.W., Siu, B.W.Y. 2006. Degradable transport network: Travel time budget of travelers with heterogeneous risk aversion. *Transportation Research Part B* 40(9), 792-806.

Lomax, T., Schrank, D., Turner, S., Margiotta, R., 2003. *Selecting travel reliability measures*. Texas Transportation Institute, Texas, USA.

Loon, R. V., Rietveld, P., Brons, M., 2011. Travel-time reliability impacts on railway passenger demand: a revealed preference analysis. *Journal of Transport Geography* 19(4), 917-925.

Lu, J., Ban, X., Qiu, Z., Yang, F., Ran, B., 2005. Robust route guidance model based on advanced traveler information systems. *Transportation Research Record* 1935, 1-7.

Lu, J., Yang, F., Ban, X., Ran, B., 2006. Moments analysis for improving decision reliability based on travel time. *Transportation Research Record* 2046(1), 1-10.

Luan, S., Chen, X., Su, Y., Dong, Z., Ma, X., 2019. Modeling travel time volatility using copula-based Monte Carlo simulation method for probabilistic traffic prediction. *Transportmetrica A: Transport Science* 7, 1-26.

Lyman, K., Bertini, R., 2008. Using travel time reliability measures to improve regional transportation planning and operations. *Transportation Research Record* 2046(1), 1-10.

Ma, Z., Ferreira, L., Mesbah, M., Zhu, S., 2016. Modeling distributions of travel time variability for bus operations. *Journal of Advanced Transportation* 50(1), 6-24

Ma, Z., Koutsopoulos, H. N., Ferreira, L., Mesbah, M., 2017. Estimation of trip travel time distribution using a generalized Markov chain approach. *Transportation Research Part C* 74, 1-21.

Madansky, A., 1959. Bounds on the expectation of a convex function of a multivariate random variable. *The Annals of Mathematical Statistics* 30(3), 743–746.

Mahmassani, H.S., Hou, T., Saberi, M., 2013. Connecting networkwide travel time reliability and the network fundamental diagram of traffic flow. *Transportation Research Record* 2391, 80-91.

Mehta, N.B., Wu, J., Molisch, A.F., Zhang, J., 2007. Approximating a Sum of Random Variables with a Lognormal. *IEEE Transactions on Wireless Communications* 6 (7), 2690-2699.

Miller - Hooks, E., 2001. Adaptive least - expected time paths in stochastic, time-varying transportation and data networks. *Networks: An International Journal* 37(1), pp.35-52.

Miller-Hooks, E., Mahmassani, H., 2003. Path comparisons for a priori and time-adaptive decisions in stochastic, time-varying networks. *European Journal of Operational Research* 146(1), 67-82.

Miller-Hooks, E.D., Mahmassani, H.S., 1998. Least possible time paths in stochastic, time-varying networks. *Computers & Operations Research* 25(12), 1107-1125.

Miller-Hooks, E.D., Mahmassani, H.S., 2000. Least expected time paths in stochastic, time-varying transportation networks. *Transportation Science* 34(2), 198-215.
Miralinaghi, M., Lou, Y., Hsu, Y.T., Shabanpour, R., Shafahi, Y. 2016. Multiclass fuzzy user equilibrium with endogenous membership functions and risk-taking behaviors. *Journal of Advanced Transportation* 50(8), 1716-1734.

Mirchandani, P., Soroush, H. 1987. Generalized Traffic Equilibrium with Probabilistic Travel-Times and Perceptions. *Transportation Science* 21(3), 133-152.

Nakayama, S. I., Takayama, J. I., Nakai, J., Nagao, K., 2012. Semi-dynamic traffic assignment model with mode and route choices under stochastic travel times. *Journal of Advanced Transportation* 46 (3): 269–281.

NCHRP, 2008. *NCHRP Report 618: Cost-Effective Performance Measures for Travel Time Delay, Variation, and Reliability*. Transportation Research Board of the National Academies, Washington, D.C.

Nelsen, R.B., 2006. *An introduction to copulas*, Springer Science & Business Media.

New Zealand Transport Agency, 2016. *Economic Evaluation Manual*. New Zealand Transport Agency, Wellington, New Zealand. [www.nzta.govt.nz](http://www.nzta.govt.nz).

Ng, M., Lin, D.Y., 2015. Sharp probability inequalities for reliable evacuation planning. *Transportation Research Part C* 60, 161-168.

Ng, M., Waller, ST., 2009. Reliable system-optimal network design: convex mean-variance model with implicit chance constraints. *Transportation Research Record* 2090(1), 68-74.

Ng, M., Waller, S.T., 2010a. A computationally efficient methodology to characterize travel time reliability using the fast Fourier transform. *Transportation Research Part B* 44(10), 1202-1219.

Ng, M., Waller, ST., 2010b. Reliable evacuation planning via demand inflation and supply deflation. *Transportation Research Part E* 46(6), 1086–1094.

Ng, M., Waller, ST., 2012. A dynamic route choice model considering uncertain capacities. *Computer-Aided Civil and Infrastructure Engineering* 27(4), 231-243.

Ng, M.W., Szeto, W.Y., Waller, S.T., 2011. Distribution-free travel time reliability assessment with probability inequalities. *Transportation Research Part B* 45(6), 852-866.

Nie, Y. 2011. Multi-class percentile user equilibrium with flow-dependent stochasticity. *Transportation Research Part B* 45(10), 1641-1659.

Nie, Y.M., Wu, X., 2009. Shortest path problem considering on-time arrival probability. *Transportation Research Part B* 43(6), 597-613.

Nikolova, E., Stier-Moses, N.E. 2014. A Mean-Risk Model for the Traffic Assignment Problem with Stochastic Travel Times. *Operations Research* 62(2), 366-382.

Noland, R., Small, K., 1995. Travel-time uncertainty, departure time choice, and the cost of morning commutes. *Transportation Research Record* 1493, 150–158.

Noland, R.B., Polak, J.W., 2002. Travel time variability: a review of theoretical and empirical issues. *Transport Reviews* 22(1), 39-54.

Odgaard, T., Kelly, C., Laird, J., 2005. Current practice in project appraisal in Europe. in: *Proceedings of the European Transport Conference*, 3–5 October, Association for European Transport, Strasbourg.

OECD, 2016. Quantifying the socio-economic benefits of transport roundtable. International Transport Forum. Organization for Economic Cooperation and Development, Paris. [www.itf-oecd.org/quantifying-socio-economic-benefits-transport-roundtable](http://www.itf-oecd.org/quantifying-socio-economic-benefits-transport-roundtable).

Opasanon, S., Miller-Hooks, E., 2006. Multicriteria adaptive paths in stochastic, time-varying networks. *European Journal of Operational Research* 173(1), 72-91.
Ordonez, F., Stier-Moses, N.E. 2010. Wardrop Equilibria with Risk-Averse Users. *Transportation Science* 44(1), 63-86.

Papadoulis, A., Quddus, M., Imprialou, M., 2019. Evaluating the safety impact of connected and autonomous vehicles on motorways. *Accident Analysis & Prevention* 124, 12-22.

Pells, S., 1987. The evaluation of reductions in travel time variability. PhD thesis, University of Leeds, UK.

Plötz, P., Jakobsson, N., Sprei, F., 2017. On the distribution of individual daily driving distances. *Transportation Research Part B* 101, 213–227.

Pocztar, S. L., Jankovic, L. M., 2014. The google car: driving toward a better future?. *Journal of Business Case Studies* 10(1), 7-14.

Polak, J., 1987. A more general model of individual departure time choice. In: *PTRC Summer Annual Meeting, Proceedings of Seminar C*, England.

Polus, A., 1979. A study of travel time and reliability on arterial routes. *Transportation* 8(2), 141–151.

Prakash, A.A., Seshadri, R., Srinivasan, K.K. 2018. A consistent reliability-based user-equilibrium problem with risk-averse users and endogenous travel time correlations: Formulation and solution algorithm. *Transportation Research Part B* 114, 171-198.

Prokhorchuk, A., Dauwels, J., Jaillet, P., 2019. Estimating travel time distributions by bayesian network inference. *IEEE Transactions on Intelligent Transportation Systems* 21(5), 1867-1876.

Pu, W., 2011. Analytic relationships between travel time reliability measures. *Transportation Research Record* 2254, 122-130.

Qin, W., Ji, X., Liang, F., 2020. Estimation of urban arterial travel time distribution considering link correlations. *Transportmetrica A: Transport Science* 16(3), 1429-1458

Rahmani, M., Jenelius, E., Koutsopoulos, H.N., 2015. Non-parametric estimation of route travel time distributions from low-frequency floating car data. *Transportation Research Part C* 58, 343–362

Rakha, H., El-Shawarby, I., Arafeh, M., 2010. Trip travel-time reliability: Issues and proposed solutions. *Journal of Intelligent Transportation Systems* 14, 232–250.

Rakha, H., El-Shawarby, I., Arafeh, M., Dion, F., 2006. Estimating Path Travel-Time Reliability. *2006 IEEE Intelligent Transportation Systems Conference*, Toronto, Canada.

Ramezani, M., Geroliminis, N., 2012. On the estimation of arterial route travel time distribution with Markov chains. *Transportation Research Part B* 46(10), 1576-1590.

Rausand, M., Hoyland, A., 2003. *System Reliability Theory: Models, Statistical Methods, and Applications* (Vol. 396). John Wiley & Sons.

Richardson, A.J., Taylor, M.A.P., 1978. Travel time variability on commuter journeys. *High Speed Ground Transportation Journal* 6, 77–79.

Rudin, W., 1976. *Principles of mathematical analysis*, New York, McGraw-Hill Education.

Saberi, M., Mahmassani, H., Hou, T., Zockaie, A., 2014. Estimating network fundamental diagram using three-dimensional vehicle trajectories: Extending Edie’s definitions of traffic flow variables to networks. *Transportation Research Record* 2422, 12–20.

Saedi, R., Saeedmanesh, M., Zockaie, A., Saberi, M., Geroliminis, N., Mahmassani, H.S., 2020. Estimating network travel time reliability with network partitioning. *Transportation Research Part C* 112, 46-61.

Samara, A., Rempe, F., Göttlich, S., 2020. Modeling arterial travel time distribution using copulas. In *IEEE 23rd International Conference on Intelligent Transportation Systems*, Rhodes, Greece.
Sen, S., Pillai, R., Joshi, S., Rathi, A. K., 2001. A mean-variance model for route guidance in advanced traveler information systems. *Transportation Science* 35(1), 37-49.

Senna, L., 1994. The influence of travel time variability on the value of time. *Transportation* 21, 203–228.

Shahabi, M., Unnikrishnan, A., Boyles, S.D., 2013. An outer approximation algorithm for the robust shortest path problem. *Transportation Research Part E* 58, 52-66.

Shahabi, M., Unnikrishnan, A., Boyles, S.D., 2015. Robust optimization strategy for the shortest path problem under uncertain link travel cost distribution. *Computer-Aided and Infrastructure Engineering* 30(6), 433-448.

Shams, K., Asgari, H., Jin, X., 2017. Valuation of travel time reliability in freight transportation: A review and meta-analysis of stated preference studies. *Transportation Research Part A* 102, 228-243.

Shao, H., Lam, W. H., Meng, Q., Tam, M. L., 2006a. Demand-driven traffic assignment problem based on travel time reliability. *Transportation Research Record* 1985(1), 220-230.

Shao, H., Lam, W.H., Tam, M. L., Yuan, X. M., 2008. Modeling rain effects on risk-taking behaviours of multi-user classes in road networks with uncertainty. *Journal of Advanced Transportation* 42(3), 265-290.

Shao, H., Lam, W.H.K., Tam, M.L. 2006b. A reliability-based stochastic traffic assignment model for network with multiple user classes under uncertainty in demand. *Networks & Spatial Economics* 6(3-4), 173-204.

SHRP, 2009. SHRP 2 Report S2-C02-RR: Performance Measurement Framework for Highway Capacity Decision Making. Transportation Research Board of the National Academies, Washington, D.C.

Sikka, N., Hanley, P., 2013. What do commuters think travel time reliability is worth? Calculating economic value of reducing the frequency and extent of unexpected delays. *Transportation* 40(5), 903-919.

Siu, B., Lo H.K., 2013. Punctuality-based route and departure time choice. *Transportmetrica A: Transport Science* 1(3), 195-225.

Siu, B.W., Lo, H.K., 2009. Equilibrium trip scheduling in congested traffic under uncertainty. In: *Transportation and Traffic Theory 2009: Golden Jubilee*. Springer US, pp. 147–167.

Siu, B.W.Y., Lo, H.K. 2006. Doubly uncertain transport network - Degradable link capacity and perception variations in traffic conditions. *Transportation Research Record* 1964, 59-69.

Siu, B.W.Y., Lo, H.K. 2008. Doubly uncertain transportation network: Degradable capacity and stochastic demand. *European Journal of Operational Research* 191(1), 166-181.

Small, K.A., Noland, R. B., Koskenoja, P., 1995, Socio-economic attributes and impacts of travel reliability: a stated preference approach, University of California, Irvine, California PATH research report.

Small, K.A., 1982. The scheduling of consumer activities: work trips. *American Economic Review* 72 (3), 467-479.

Small, K.A., 2012. Valuation of travel time. *Economics of Transportation* 1, 2–14.

Small, K.A., Noland, R.B., Chu, X., Lewis, D., 1999. Valuation of travel-time savings and predictability in congested conditions for highway user-cost estimation. NCHRP Report 431, Transportation Research Board, National Research Council.

Small, K.A., Winston, C., Yan, J., 2005. Uncovering the distribution of motorists’ preferences for travel time and reliability. *Econometrica* 73(4), 1367–1382.

Soriguera, F., 2014. On the value of highway travel time information systems. *Transportation Research Part A* 70, 294–310.

Srinivasan, K.K., Prakash, A.A., Seshadri, R., 2014. Finding most reliable paths on networks with correlated and shifted log-normal travel times. *Transportation Research Part B* 66, 110–128.
Sterman, B.P., Schofer, J.L., 1976. Factors Affecting Reliability of Urban Bus Services. *ASCE Transportation Engineering Journal of ASCE* 102 (1), 147–159.

Sumalee, A., Connors, R.D., Luathep, P., 2009. Network equilibrium under cumulative prospect theory and endogenous stochastic demand and supply. In *Transportation and Traffic Theory 2009: Golden Jubilee*, pp. 19-38.

Sumalee, A., Uchida, K., Lam, W.H.K. 2011. Stochastic multi-modal transport network under demand uncertainties and adverse weather condition. *Transportation Research Part C* 19(2), 338-350.

Sumalee, A., Watling, D.P., 2003. Travel time reliability in a network with dependent link modes and partial driver response. *Journal of Eastern Asia Society for Transportation Studies* 5, 1687–1701.

Sumalee, A., Watling, D.P., 2008. Partition-based algorithm for estimating transportation network reliability with dependent link failures. *Journal of Advanced Transportation* 42 (3), 213–238.

Susilawati, S., Taylor, M.A.P., Somenhalli, S.V.C., 2013. Distributions of travel time variability on urban roads. *Journal of Advanced Transportation* 47(8), 720–736.

Szeto, W. Y., 2011. Cooperative game approaches to measuring network reliability considering paradoxes. *Transportation Research Part C* 19(2), 229-241.

Szeto, W.Y., Lo, H.K., 2004. A cell-based simultaneous route and departure time choice model with elastic demand. *Transportation Research Part B* 38(7), 593-612.

Szeto, W.Y., O'Brien, L., O'Mahony, M. 2006. Risk-averse traffic assignment with elastic demands: NCP formulation and solution method for assessing performance reliability. *Networks & Spatial Economics* 6(3-4), 313-332.

Szeto, W.Y., Sumalee, A., 2009. Multi-class reliability-based stochastic-dynamic-user-equilibrium assignment problem with random traffic states. In Proceedings of the 88th Annual Meeting of Transportation Research Board.

Tan, Z.J., Yang, H., Guo, R.Y. 2014. Pareto efficiency of reliability-based traffic equilibria and risk-taking behavior of travelers. *Transportation Research Part B* 66, 16-31.

Tatkinen, M., Boyce, D., Mirchandani, P. 1997. Comparisons of Deterministic and Stochastic Traffic Loading Models. *Transportation Research Record* 1607, 16-23.

Taylor, M.A.P., 2012. Modeling travel time reliability with the Burr distribution. *Procedia-Social and Behavioral Sciences* 54, 75-83.

Taylor, M.A.P., 2013. Travel through time: the story of research on travel time reliability. *Transportmetrica B: Transport Dynamics* 1(3), 174-194

Taylor, M.A.P., 2017. Fosgerau’s travel time reliability ratio and the Burr distribution. *Transportation Research Part B* 97, 50–63.

Taylor, M.A.P., Somenhalli, S., 2010. Travel time reliability and the bimodal travel time distribution for an arterial road. *Rout & Transport Research: A Journal of Australian and New Zealand Research and Practice* 19(4), 37-50.

Thomson, J.M., 1968. The value of traffic management. *Journal of Transport Economics and Policy* 1(1), 3-32.

Tilahun, N. Y., Levinson, D. M., 2010. A moment of time: reliability in route choice using stated preference. *Journal of Intelligent Transportation Systems* 14(3), 179-187.

Tseng, Y., Verhoef, E., 2008. Value of time by time of day: a stated-preference study. *Transportation Research Part B* 42, 607–618.
Tu, H., Li, H., van Lint, J.W.C., van Zuylen, H.J. Lint, H. V., Zuylen, H. V., 2012. Modeling travel time reliability of freeways using risk assessment techniques. *Transportation Research Part A* 46(10), 1528-1540.

Tu, H., van Lint, J.W.C., van Zuylen, H.J., 2007. The impact of traffic flow on travel time variability of freeway corridors. *Transportation Research Record* 1993, 59–66.

Uchida, K., 2014. Estimating the value of travel time and of travel time reliability in road networks. *Transportation Research Part B* 66, 129-147.

Uchida, T., Iida, Y. 1993. Risk assignment: a new traffic assignment model considering risk of travel time variation. In *Proceedings of the 12th International Symposium on Transportation and Traffic Theory*, pp. 89-105.

van Lint, J.W.C., van Zuylen, H.J., Tu. H., 2008. Travel time unreliability on freeways: Why measures based on variance tell only half the story. *Transportation Research Part A* 42 (1), 258-277.

Vickrey, W.S., 1969. Congestion theory and transport investment. *American Economic Review* 59, 251–260.

Vickrey, W.S., 1973. Pricing, metering, and efficiently using urban transportation facilities. *Highway Research Record* 476, 36–48.

von Neuman, J., Morgenstern, O., 1947. *Theory of Games and Economic Behaviour 2nd edition*. Princeton University Press, Princeton

Wakabayashi, H., Iida, Y., 1992. Upper and lower bounds of terminal reliability of road networks: an efficient method with Boolean algebra. *Journal of Natural Disaster Science* 14, 29-44.

Wakabayashi, H., Matsumoto, Y., 2012. Comparative study on travel time reliability indexes for highway users and operators. *Journal of Advanced Transportation* 46(4), 318-339.

Wang, J.Y.T., Ehrgott, M., Chen, A. 2014. A bi-objective user equilibrium model of travel time reliability in a road network. *Transportation Research Part B* 66, 4-15.

Wang, W., Sun, H.J. 2016. Cumulative prospect theory-based user equilibrium model with stochastic perception errors. *Journal of Central South University* 23(9), 2465-2474.

Wardman, M., Batley, R., 2014. Travel time reliability: a review of late time valuations, elasticities and demand impacts in the passenger rail market in Great Britain. *Transportation* 41(5), 1041-1069.

Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. *Proceeding of Institute of Civil Engineers, Part II*, 325–378.

Watling, D. 2006. User equilibrium traffic network assignment with stochastic travel times and late arrival penalty. *European Journal of Operational Research* 175(3), 1539-1556.

Watling, D., 2002. Stochastic network equilibrium under stochastic demand. In: Patriksson, M., Labbe, M. (Eds.), Transportation Planning: State of Art. Kluwer Academic, Dordrecht, Netherlands, pp. 33–51.

Wu, X., Nie, Y. 2011. Modeling heterogeneous risk-taking behavior in route choice: A stochastic dominance approach. *Transportation Research Part A* 45(9), 896-915.

Wu, X., Nie, Y. 2013. Solving the multiclass percentile user equilibrium traffic assignment problem: a computational study. *Transportation Research Record* 2334(1), 75-83.

Xiao, Y., Coulombel, N., De Palma, A., 2017. The valuation of travel time reliability: does congestion matter?. *Transportation Research Part B* 97, 113-141.

Xiao, Y., Fukuda, D., 2015. On the cost of misperceived travel time variability. *Transportation Research Part A* 75, 96–112.
Xing, T., Zhou, X., 2011. Finding the most reliable path with and without link travel time correlation: A Lagrangian substitution based approach. *Transportation Research Part B* 45(10), 1660-1679.

Xu, H.L., Lou, Y.Y., Yin, Y.F., Zhou, J. 2011. A prospect-based user equilibrium model with endogenous reference points and its application in congestion pricing. *Transportation Research Part B* 45(2), 311-328.

Xu, X., Chen, A., Cheng, L., 2013. Assessing the effects of stochastic perception error under travel time variability. *Transportation* 40, 525–548.

Xu, X., Chen, A., Cheng, L., Lo, H.K., 2014. Modeling distribution tail in network performance assessment: A mean-excess total travel time risk measure and analytical estimation method. *Transportation Research Part B* 66, 32–49.

Xu, X., Chen, A., Cheng, L., Yang, C., 2017. A link-based mean-excess traffic equilibrium model under uncertainty. *Transportation Research Part B* 95, 53-75.

Xu, X., Chen, A., Lo, H.K., Yang, C. 2018. Modeling the impacts of speed limits on uncertain road networks. *Transportmetrica A* 14(1-2), 66-88.

Xu, X., Qu, K., Chen, A., Yang, C., 2021a. A new day-to-day dynamic network vulnerability analysis approach with Weibit-based route adjustment process. *Transportation Research Part E* 153, 102421.

Xu, X., Zang, Z., Chen, A., Yang, C., 2021b. Mathematical and behavioral consistency in the schedule delay, travel time budget, and mean-excess time models. Working paper.

Yang, F., Yun, M. P., Yang, X. G., 2014a. Travel time distribution under interrupted flow and application to travel time reliability. *Transportation Research Record* 2466(1), 114-124.

Yang, H., Lo, K. K. Tang,W., 2000. Travel time versus capacity reliability of a road network, in: M.G. H. Bell and C. Cassir (Eds) *Reliability of Transport Networks*, 119–138 (Baldock, England: Research Studies Press Ltd).

Yang, J.F., Jiang, G.Y. 2014. Development of an enhanced route choice model based on cumulative prospect theory. *Transportation Research Part C* 47, 168–178.

Yang, L., Zhou, X., 2017. Optimizing on-time arrival probability and percentile travel time for elementary path finding in time-dependent transportation networks: Linear mixed integer programming reformulations. *Transportation Research Part B* 96, 68-91.

Yang, Q., Wu, G., Boriboonsomsin, K., Barth, M., 2013. Arterial roadway travel time distribution estimation and vehicle movement classification using a modified Gaussian Mixture Model. In *16th International IEEE Conference on Intelligent Transportation Systems*, The Hague, Netherlands.

Yang, S., Cooke, P., 2018. How accurate is your travel time reliability?—Measuring accuracy using bootstrapping and lognormal mixture models. *Journal of Intelligent Transportation Systems* 22(6), 463-477.

Yang, S., Malik, A., Wu, Y.J., 2014b. Travel time reliability using the Hasofer-Lind-Rackwitz-Fiessler algorithm and kernel density estimation. *Transportation Research Record* 2442, 85-95.

Yang, S., Wu, Y., 2016. Mixture models for fitting freeway travel time distributions and measuring travel time reliability. *Transportation Research Record* 2594, 95–106.

Yin, Y., Ieda, H., 2001. Assessing performance reliability of road networks under non-recurrent congestion. *Transportation Research Record* 1771, 148-155.

Yin, Y., Lam, W., Ieda, H., 2004. New technology and the modeling of risk-taking behavior in congested road networks. *Transportation Research Part C* 12(3-4), 171-192.

Yu, Y., Chen, M., Qi, H., Wang, D., 2020. Copula-based travel time distribution estimation considering channelization section spillover. *IEEE Access* 8, 32850-32861.
Yun, M., Qin, W., Yang, X., Liang, F., 2019. Estimation of urban route travel time distribution using markov chains and pair-copula construction. *Transportmetrica B: Transport Dynamics* 7(1), 1521-1552.

Zang, Z., Batley, R., Xu, X., Chen, A., Wang, D., 2021. The value of travel time unreliability. Working paper.

Zang, Z., Xu, X., Yang, C., Chen, A. 2018a. A closed-form estimation of the travel time percentile function for characterizing travel time reliability. *Transportation Research Part B* 118, 228-247.

Zang, Z., Xu, X., Yang, C., Chen, A., 2018b. A distribution-fitting-free approach to estimating travel time reliability ratio. *Transportation Research Part C* 89, 83–95.

Zhang, C., Chen, X.J., Sumalee, A. 2011. Robust Wardrop's user equilibrium assignment under stochastic demand and supply: Expected residual minimization approach. *Transportation Research Part B* 45(3), 534-552.

Zhang, K., Jia, N., Zheng, L., Liu, Z., 2019a. A novel generative adversarial network for estimation of trip travel time distribution with trajectory data. *Transportation Research Part C* 108, 223-244.

Zhang, L., Levinson, D., 2008. Determinants of route choice and value of traveler information: a field experiment. *Transportation Research Record* 2086(1), 81–92.

Zhang, M., Meng, Q., Kang, L., Li, W., 2018. Tailored wakeby-type distribution for random bus headway adherence ratio. *Transportation Research Part C* 86, 220-244.

Zhang, Y., Khani, A., 2019. An algorithm for reliable shortest path problem with travel time correlations. *Transportation Research Part B* 121, 92-113.

Zhang, Z., He, Q., Gou, J., Li, X., 2019b. Analyzing travel time reliability and its influential factors of emergency vehicles with generalized extreme value theory, *Journal of Intelligent Transportation Systems* 23(1), 1-11.

Zheng, F., van Zuylen, H., 2010. Uncertainty and predictability of urban link travel time: Delay distribution–based analysis. *Transportation Research Record* 2192(1), 136-146.

Zheng, F., van Zuylen, H., Liu, X., 2017. A methodological framework of travel time distribution estimation for urban signalized arterial roads. *Transportation Science* 51(3), 893-917.

Zhu, S., Jiang, G., Lo, H. K., 2018. Capturing value of reliability through road pricing in congested traffic under uncertainty. *Transportation Research Part C* 94, 236-249.

Zhu, W., Timmermans, H. J. P., 2010. Modeling simplifying information processing strategies in conjoint experiments. *Transportation Research Part B* 44(6), 764–780.

Zockaie, A., Nie, Y.M., Mahmassani, H.S., 2014. Simulation-based method for finding minimum travel time budget paths in stochastic networks with correlated link times. *Transportation Research Record* 2467(1), 140-148.

Zockaie, A., Saberi, M., Mahmassani, H. S., Jiang, L., Frei, A., Hou, T., 2015. Activity-based model with dynamic traffic assignment and consideration of heterogeneous user preferences and reliability valuation: application to toll revenue forecasting in Chicago, Illinois. *Transportation Research Record* 2493(1), 78-87.