Learning self-triggered controllers with Gaussian processes

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Abstract—This paper investigates the design of self-triggered controllers for networked control systems (NCSs), where the dynamics of the plant is unknown apriori. To deal with the unknown transition dynamics, we employ the Gaussian process (GP) regression in order to learn the dynamics of the plant. To design the self-triggered controller, we formulate an optimal control problem, such that the optimal pair of the inter-communication time step and control input can be determined based on the GP dynamics of the plant. Moreover, we provide an overall implementation algorithm that jointly learns the dynamics of the plant and the self-triggered controller based on a reinforcement learning framework. Finally, a numerical simulation illustrates the effectiveness of the proposed approach.

Index Terms—Event-triggered/self-triggered control, Optimal control, Gaussian process regression.

I. INTRODUCTION

In networked control systems (NCSs), sensors, actuators, and controllers reside in multiple areas linked by wired/wireless communication network. Due to the progress in communication technology and many practical advantages such as a low-cost maintenance and flexibility for modifications, NCSs have been developed in a wide variety of applications, including manufacturing plants, autonomous robots/vehicles, traffic networks, to name a few [1]. In recent years, event-triggered and self-triggered control have been attracted much attention and are known to be useful strategies for the NCSs [2]. This is due to the fact that, it leads to the potential saving of resources that are present in NCSs, such as a limited battery capacity or a limited communication bandwidth, by transmitting sensor measurements over the communication network only when it is needed. So far, various event/self-triggered controllers have been proposed in the literature, see, e.g., [3] for survey papers. Early works consider designing event/self-triggered control based on input-to-state stability (ISS) or $L_2$-gain performance [4]–[6]. More recently, event-triggered control has been formulated as the hybrid dynamical systems [7], [8]. In addition, some approaches to combine event/self-triggered control and optimal control have been also provided in recent years [9]–[17].

In the aforesaid event-triggered and self-triggered control framework, it is generally assumed that the transition dynamics, which represents the underlying model of the plant, is known apriori. This implies that, when the event/self-triggered controllers are applied to the real world (actual) control systems, the resulting performance is heavily dependent on how the system model is accurate with respect to the true dynamics. However, it may be the case in practice when an accurate model of the plant is hard to obtain based on the first principles from physics, due to the fact that the dynamics is complex and highly nonlinear. Examples include mechanical systems [18], autonomous vehicles [19], power consumption of multi-story buildings [20], periodic errors in astrophotography systems [21], to name a few.

Motivated by the above, in this paper we investigate the design of a novel self-triggered controller for NCSs, where the dynamics of the plant is assumed to be unknown apriori. To this end, we make use of the Gaussian process (GP) regression [22] in order to learn the dynamics of the plant. The use of GP offers many benefits, such as the ability to incorporate prior knowledge about the model (e.g., smoothness, periodicity) by selecting suitable kernel functions, as well as the ability to provide uncertainty of the model for prediction values. To design the self-triggered controller, we first formulate an infinite horizon optimal control problem, such that both the cost for control performance and the communication cost can be taken into account. We then provide an approach to solving the optimal control problem, from which both the optimal inter-communication time step and the control input for each state can be determined based on the GP dynamics of the plant. In particular, a moment matching technique is employed to approximate predictive distribution of states under constant control inputs. Finally, we provide an overall implementation algorithm that jointly learns the dynamics of the plant, learns the self-triggered controller, and executes the self-triggered controller in a reinforcement learning framework.

Our approach is related to several techniques that have been provided in the literature. Using the GP in control community has been attracted much attention in recent years [18]–[21], [23]–[26]. In particular, our approach is related to the ones based on optimal control framework, see, e.g., [19]–[21], [24]–[26]. For example, in [19], the authors have utilized the GP model to learn the dynamics of the plant, and they have formulated a model predictive control (MPC), in which the optimal control problem is solved for each time step based on the knowledge about the dynamics learned by the GP. In contrast to these previous methods, we provide an approach that jointly learns the dynamics of the plant and the self-triggered controller, aiming at reducing the number of communication time steps for NCSs. As previously mentioned and will be clearer in later sections, this is achieved by formulating a value iteration algorithm, such that the optimal pair of the
control input and the inter-communication time steps can be determined for each state based on the GP dynamics of the plant. With regard to the event/self-triggered control, some model-free or model-based approaches with unknown transition dynamics have been proposed [27]–[33]. For example, in [28], the authors have proposed a deep reinforcement learning framework to design the event-triggered control. In contrast to these previous approaches, we make use of the GP model to learn the dynamics of the plant as well as the self-triggered controller.

Notation. Throughout the paper, we make use of the following notations. Let $\mathbb{N}$, $\mathbb{N}_\geq 0$, $\mathbb{N}_{>0}$, $\mathbb{N}_{\neq 0}$ be the set of integers, non-negative integers, positive integers, and the set of integers in the interval $[a,b]$, respectively. Let $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{R}_{>0}$ be the set of reals, non-negative reals and positive reals, respectively. For a square matrix $Q$, we use $Q > 0$ to denote that $Q$ is positive definite. Let $\text{diag}(a_1,a_2,\ldots,a_N)$ be the diagonal matrix whose (diagonal) elements are given by $a_1,\ldots,a_N \in \mathbb{R}$. Moreover, let $\text{Blockdiag}(A_1,A_2,\ldots,A_N)$ be the block diagonal matrix that consists of a set of matrices $A_1,\ldots,A_N$.

II. PRELIMINARIES OF GAUSSIAN PROCESS REGRESSION

In this section, we provide some basic concepts and useful properties of the Gaussian process (GP) regression. Consider a nonlinear function $h : \mathbb{R}^n \to \mathbb{R}$ expressed as

$$y = h(x) + \varepsilon,$$

where $x \in \mathbb{R}^n$ is the input, $y \in \mathbb{R}$ is the output, and $\varepsilon \sim \mathcal{N}(0,\sigma^2)$ is the Gaussian distributed white noise. In the GP regression, we assume that the function $h$ follows the GP. That is, for every set of a finite (or possibly infinite) number of inputs $x_i \in \mathbb{R}^n, i = 1,\ldots,N$, the joint probability of the corresponding set of outputs $y = [y_1,y_2,\ldots,y_N]^T$ follows the multivariate Gaussian distribution, i.e., $y \sim \mathcal{N}(0,K)$, where $K \in \mathbb{R}^{N \times N}$ is the covariance matrix. In the GP regression, it is assumed that the covariance matrix is characterized by $K_{ij} = k(x_i,x_j)$, where $K_{ij}$ is the $(i,j)$-component of $K$ and $k : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is the positive definite kernel function.

In this paper, we assume that the kernel function $k$ is given by the squared exponential covariance function:

$$k(x_i,x_j) = \alpha^2 \exp \left( -\frac{1}{2} (x_i - x_j)^T \Lambda^{-1} (x_i - x_j) \right),$$

where $\Lambda = \text{diag} (\lambda^2_1,\ldots,\lambda^2_N)$ and $\{\alpha,\lambda_1,\ldots,\lambda_N\}$ are the hyper-parameters. For a given set of input-output training data $D = \{x_n,y_n\}_{n=1}^N$, the predictive distribution of the output for a new test input $x$ follows the Gaussian distribution, i.e., $p(y|x,D) = \mathcal{N}(\mu(x),\sigma^2(x))$. Here the mean and the variance are given by

$$\mu(x) = k^T(x)(K + \sigma^2 I)^{-1}y,$$

$$\sigma(x) = k(x,x) - k^T(x)(K + \sigma^2 I)^{-1}k(x),$$

where $y = [y_1,y_2,\ldots,y_N]^T$ and

$$k(x) = [k(x,x_1),\ldots,k(x,x_N)]^T.$$

Suitable selections of the hyper-parameters $\{\alpha,\lambda_1,\ldots,\lambda_N\}$ are given by evidence maximization, see, e.g., [22]. For simplicity of presentation, we write $h \sim \mathcal{GP}$ if the function $h$ follows the GP.

III. PROBLEM STATEMENT

A. Dynamics

We consider a networked control system (NCS) illustrated in Fig.1. As shown in the figure, the controller and the learning agent are connected to the plant over the communication network. Roughly speaking, the learning agent is responsible for learning the dynamics of the plant as well as the optimal control and communication policies. On the other hand, the controller is responsible for transmitting the control inputs to operate the plant based on the control and communication policies derived by the learning agent. This implementation will be formally given later in this paper. Throughout the paper, we assume that the communication network is ideal; it induces neither packet dropouts nor any network delays.

The dynamics of the plant is given by the following nonlinear systems:

$$x_{k+1} = f(x_k,u_k), \quad u_k \in U,$$

for all $k \in \mathbb{N}_{>0}$, where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^n$ is the control input, $U \subset \mathbb{R}^n$ is the set of control inputs, and $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the transition dynamics that is assumed to be unknown apriori. While the transition dynamics is unknown, it is assumed here that the equilibrium point is known; without loss of generality, we assume that the origin has the equilibrium point, i.e., $0 = f(0,0)$. The control goal is to stabilize the system towards the origin.

Since $f$ is unknown apriori, we consider that each component of the unknown function, i.e., $f_i, i \in \mathbb{N}_{1:n}$ ($f = [f_1,f_2,\ldots,f_{n}]^T$) is modeled by the GP. That is, $f_i$ is learned from the input-output training data $D_i = \{x_i,y_i\}$, where

$$X = \begin{bmatrix} x_0^* & u_0^* \cr x_1^* & u_1^* \cr \vdots & \vdots \cr x_{N-1}^* & u_{N-1}^* \end{bmatrix},$$

$$y_i = [x_{i,1}^* x_{i,2}^* \ldots x_{i,N}^*]^T.$$

In (7) and (8), $N \in \mathbb{N}_{>0}$ denotes the number of training data points, $[x_n^*,u_n^*], n \in \mathbb{N}_{1:N}$ are the training inputs following the dynamics (6) (i.e., $x_{n+1}^* = f(x_n^*,u_n^*), n \in \mathbb{N}_{0:N-1}$), and $x_{i,n}^*$ is the $i$-th element of $x_{n}^*$ as the set of training outputs. We denote by $k_{i,\cdot}, K_i$ and $\{\alpha_i,\lambda_{i,1},\ldots,\lambda_{i,N}\}$ the kernel function, covariance matrix and the hyper-parameters for the GP model of $f_i$, respectively. Moreover, we denote by $\mu_i(x,u), \sigma_i(x,u)$ the mean and the covariance for the
GP model of \( f_i \) with an arbitrary test input \( \hat{x} = [x^T, u^T]^T \), respectively, i.e.,

\[
\mu_i(x, u) = \mathbf{K}_{x,i}(\hat{x}) (K_i + \sigma^2 I)^{-1} y_i,
\]

\[
\sigma_i(x, u) = \mathbf{k}_{x,i}(\hat{x}, \hat{x}) - \mathbf{K}_{x,i}(\hat{x}) (K_i + \sigma^2 I)^{-1} \mathbf{k}_{x,i}(\hat{x}),
\]

where \( \mathbf{k}_{x,i}(\hat{x}) = [k_i(x, x_1), \ldots, k_i(x, x_N)]^T \). That is, letting \( \hat{f}_i \) be the GP model of \( f_i \), we have

\[
\hat{f}_i(x, u) \sim \mathcal{N}(\mu_i(x, u), \sigma_i(x, u)).
\]

Then, the overall GP model for \( f = [f_1, f_2, \ldots, f_{n_x}]^T \) is given by

\[
\hat{f}(x, u) \sim \mathcal{N}(\mu(x, u), \Sigma(x, u)),
\]

where \( \hat{f} = [\hat{f}_1, \hat{f}_2, \ldots, \hat{f}_{n_x}]^T \) and

\[
\mu(x, u) = [\mu_1(x, u), \ldots, \mu_{n_x}(x, u)]^T,
\]

\[
\Sigma(x, u) = \text{diag} (\sigma_1(x, u), \ldots, \sigma_{n_x}(x, u)).
\]

\[\text{Step 1}\] the plant measures the state \( x \in \mathbb{X} \) and transmits \( x \) to the controller;

\[\text{Step 2}\] the controller computes the control input and the inter-communication time steps as \( u_k = \pi_{\text{inp}}(x_k) \) and \( m_k = \pi_{\text{com}}(x_k) \);

\[\text{Step 3}\] the controller transmits \( \{u_k, m_k\} \) to the plant, and the plant applies \( u_k \) constantly until the next communication time, i.e., \( u_k = u_k \), \( \forall k \in \mathbb{N}_{k_i, k_i+1-1} \), where \( k_i+1 = k_i + m_i \).

\[\text{Cost function to be minimized}\]

In this paper, we consider the following infinite-horizon cost function to be minimized:

\[
J^\pi(x_{k_i}) = \sum_{\ell = i+1}^{\infty} \mathbb{E}^\pi_{x_k} \left[ C_1(x_{k_\ell}) + \gamma C_2(m_\ell) \right],
\]

where \( \mathbb{E}_x^\pi[\cdot] \) denotes the expectation with respect to \( x \), \( C_1 : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0} \) represents the stage cost for the state, \( C_2 : \mathbb{N}_{1:M} \to \mathbb{R}_{\geq 0} \) represents the communication cost that aims to penalize the inter-communication time steps, and \( \gamma > 0 \) is the weight associated to the communication cost. We assume that the cost for the state is characterized by polynomials or exponential functions. For example, exponential type of the cost function is given by

\[
C_1(x_{k_\ell}) = 1 - \exp \left\{ \frac{1}{2} x_{k_\ell}^T Q x_{k_\ell} \right\},
\]

where \( Q \) is a positive definite matrix. Moreover, polynomial cost functions include quadratic type:

\[
C_1(x_{k_\ell}) = x_{k_\ell}^T Q x_{k_\ell}.
\]

As will be clearer, the above characterizations will allow us to provide analytical computations of the integrals with respect to the Gaussian probability distribution.

The communication cost is characterized as follows:

\[
C_2(m_\ell) = M - m_\ell.
\]

Recall that \( M \) is the maximum inter-communication time steps, i.e., \( m_\ell \leq M, \forall \ell \in \mathbb{N} \). Hence, the total cost function defined in (15) aims at taking the cost of the control performance and the communication into account, and the parameter \( \gamma \) regulates the trade-off between them.

As will be formalized in later sections, we design the optimal control and communication policies \( \pi = \{\pi_{\text{inp}}, \pi_{\text{com}}\} \), such that (13) is minimized. Note that, since the function \( f \) is unknown apriori and is learned by the GP, we will make use of the GP model \( \hat{f} \) (see (12)) in order to derive the optimal solution; for details, see Section IV.

\section*{IV. APPROXIMATING MULTIPLE-AHEAD PREDICTIONS UNDER CONSTANT CONTROL SIGNALS}

Suppose that, for given GP model \( \hat{f} \) and a pair \( (x_k, u) \in \mathbb{R}^{n_x} \times U \), we aim at computing the predictive distribution of the states with the constant control input \( u \), i.e.,

\[
p(x_{k+1}|x_k, u), p(x_{k+2}|x_k, u), \ldots, \]

where \( x_{k+m} \), \( m \in \mathbb{N}_{0} \) represent the state from \( x_k \) by applying \( u \) constantly for \( m \) time steps. In this paper, we employ a moment matching technique [34] in order to approximate the predictive distributions by the Gaussian distribution. Since the functions \( f_i, i \in \mathbb{N}_{1:n_x} \) are modeled by the GP, the predictive distribution of the state for \( k + 1 \) is given by \( p(x_{k+1}|x_k, u) = \mathcal{N}(\mu_{k+1}, \Sigma_{k+1}) \), where \( \mu_{k+1} = \mu(x_k, u), \Sigma_{k+1} = \Sigma(x_k, u) \) with

\[
\mu(x_k, u) = [\mu_1(x_k, u), \ldots, \mu_{n_x}(x_k, u)]^T
\]

\[
\Sigma(x_k, u) = \text{diag} (\sigma_1(x_k, u), \ldots, \sigma_{n_x}(x_k, u)).
\]
distribution for \( k + \ell + 1 \) can be derived as follows:

\[
p(x_{k+\ell+1}|x_k, u) = \int p(\tilde{x}_{k+\ell}|x_k, u)p(x_{k+\ell+1}|\tilde{x}_{k+\ell}, x_k, u)d\tilde{x}_{k+\ell},
\]

where we let \( \tilde{x}_{k+\ell} = [x_k^T, u_k^T]^T \) and \( u_{k+\ell} \) denotes the control input applied at \( k + \ell \). Since the analytical computation of the integral in (21) cannot be given, we compute the mean and the covariance of the right hand side of (21) and approximate \( p(x_{k+\ell+1}|x_k, u) \) by the Gaussian distribution. The integral in (21) involves the joint distribution \( p(\tilde{x}_{k+\ell}|x_k, u) \), which is further computed as

\[
p(\tilde{x}_{k+\ell}|x_k, u) = p(x_{k+\ell}, u_{k+\ell}|x_k, u) = p(x_{k+\ell}|x_k, u)p(u_{k+\ell}|x_k, u)
\]

Since \( u \) is applied constantly, it follows that \( u_{k+\ell} = u \), i.e.,

\[
p(u_{k+\ell}|x_k, u) = \text{Dirac}(u_{k+\ell} - u),
\]

Dirac(\( \cdot \)) denotes the Dirac delta function. Hence, (21) leads to

\[
p(x_{k+\ell+1}|x_k, u) = \int p(x_{k+\ell}|x_k, u)p(x_{k+\ell+1}|\tilde{x}_{k+\ell}, x_k, u)d\tilde{x}_{k+\ell} + \int p(x_{k+\ell+1}|x_k, u)d\tilde{x}_{k+\ell},
\]

where \( p(x_{k+\ell+1}|x_k, u) \approx N(\mu_{k+\ell+1}, \Sigma_{k+\ell+1}) \). Moreover, using the GP model in (12), we have \( p(f(x_{k+\ell}|x_k, u)) \approx p(f(x_{k+\ell}+u)|x_k, u) = N(\mu(x_{k+\ell}, u), \Sigma(x_{k+\ell}, u)) \), where \( \mu(x_{k+\ell}, u) = [\mu_1(x_{k+\ell}, u), \ldots, \mu_{n_x}(x_{k+\ell}, u)]^T, \Sigma(x_{k+\ell}, u) = \text{diag}(\sigma_1(x_{k+\ell}, u), \ldots, \sigma_{n_x}(x_{k+\ell}, u)) \).

In the above, \( \mu_i(\cdot) \) and \( \sigma_i(\cdot) \) \((i \in \mathbb{N}_{1:n_x})\) are computed according to (9) and (10), respectively.

Based on the above, let us compute the mean and the covariance of the right hand side of (22). From (22), the mean of \( p(x_{k+\ell+1}|x_k, u) \) is given by

\[
\mu_{k+\ell+1} = \mathbb{E}[x_{k+\ell+1} | x_{k+\ell+1}| x_k, u] = \mathbb{E}[x_{k+\ell} | x_k, u] \mu(x_{k+\ell}, u) = \int p(x_{k+\ell}|x_k, u)\mu(x_{k+\ell}, u)d\tilde{x}_{k+\ell},
\]

where \( \mathbb{E}[x_{k+\ell+1}|x_k, u] \approx N(\mu_{k+\ell+1}, \Sigma_{k+\ell+1}) \).

The integral in (23) can be computed analytically and is given by \( \mu_{k+\ell+1} = \beta_1^T \eta_i \), where \( \mu_i_{k+\ell+1} \) denotes the \( i \)-th component of \( \mu_{k+\ell+1} \), \( \beta_i = (K_i + \sigma^2 I)^{-1} \beta_i \) and \( \eta_i = [\eta_{i,1}, \eta_{i,2}, \ldots, \eta_{i,N}] \) with

\[
\eta_{i,n} = \frac{\alpha^2}{2} \left( (\Lambda_i^{-1} - \tilde{\Sigma}_{k+\ell})^T I \right)^{-1/2} \times \exp \left( -\frac{1}{2} (\mu_{k+\ell} - \tilde{x}_n)^T (\Lambda_i^{-1} - \tilde{\Sigma}_{k+\ell})^{-1} (\mu_{k+\ell} - \tilde{x}_n) \right),
\]

for all \( i \in \mathbb{N}_{1:n_x}, n \in \mathbb{N}_{1:N} \). In the above, we let \( \mu_{k+\ell} = [\mu_{k+\ell}^T, u_{k+\ell}^T]^T, \tilde{x}_n = [x_n^T, u_n^T]^T \) (recall that \( x_n, u_n \) are the \( n \)-th training input defined in (7)), and \( \tilde{\Sigma}_{k+\ell} = \text{Bkl} \text{diag}(\Sigma_{k+\ell}) \) with \( \Sigma_{n_x \times n_x} \) being the \( n_x \times n_x \) zero matrix. The covariance matrix \( \Sigma_{k+\ell+1} \) can be obtained by considering diagonal elements \( \sigma_{i,i,k+\ell+1}, i \neq j \) (see, e.g., (34)). The diagonal elements are given by

\[
\sigma_{i,i,k+\ell+1} = \mathbb{E}[x_{k+\ell+1, i} | x_{k+\ell+1, i}] + \mathbb{E}[x_{k+\ell+1, i}]^2 - \mathbb{E}[x_{k+\ell+1, i}] \times \mathbb{E}[x_{k+\ell+1, i}] + \mathbb{E}[x_{k+\ell+1, i}] - \mu_{k+\ell+1, i}^2,
\]

where \( \mathbb{E}_x[\cdot] \) is the variance with respect to \( x \), \( L_i \) is the \( N \times N \) matrix, whose \((p, q)\)-component (denoted as \( L_{i,p,q} \)) is given by

\[
L_{i,p,q} = |R_i|^{-1/2} k_i(\tilde{x}_p, \tilde{\mu}_{k+\ell}) k_j(\tilde{x}_q, \tilde{\mu}_{k+\ell}) \times \exp \left( 2(\Lambda_i^{-1} - \tilde{\Sigma}_{k+\ell})^{-1} \tilde{x}_{pq,i,j} \right),
\]

where \( \tilde{x}_{pq,i,j} = \frac{1}{2} (\tilde{x}_p + \tilde{x}_q) - \tilde{\mu}_{k+\ell}, R_i = 2 \Lambda_i^{-1} \tilde{\Sigma}_{k+\ell} + I \). The off-diagonal elements are given by

\[
\sigma_{i,j,k+\ell+1} = \beta_i^T L_{i,j} + \mu_{k+\ell+1,m} \mu_{k+\ell+1,m} + 1,
\]

where \( L_{i,j} \) is the \( N \times N \) matrix, whose \((p, q)\)-component (denoted as \( L_{i,p,q} \)) is given by

\[
L_{i,p,q} = |R_{ij}|^{-1/2} k_i(\tilde{x}_p, \tilde{\mu}_{k+\ell}) k_j(\tilde{x}_q, \tilde{\mu}_{k+\ell}) \times \exp \left( -\frac{1}{2} (\tilde{x}_{pq,i,j})^T (\Lambda_i^{-1} + \Lambda_j^{-1} + \tilde{\Sigma}_{k+\ell})^{-1} \tilde{x}_{pq,i,j} \right),
\]

where \( \tilde{x}_{pq,i,j} = \Lambda_i^{-1} (\tilde{x}_p - \tilde{\mu}_{k+\ell}) + \Lambda_j^{-1} (\tilde{x}_q - \tilde{\mu}_{k+\ell}) \).

Based on the above, we can approximate \( p(x_{k+\ell+1}|x_k, u) \) by the Gaussian distribution as

\[
p(x_{k+\ell+1}|x_k, u) \approx N(\mu_{k+\ell+1}, \Sigma_{k+\ell+1}) \).
\]

Hence, by recursively applying the above procedure for all \( \ell = 1, \ldots, m - 1 \), we can approximate \( p(x_{k+m}|x_k, u) \) by the Gaussian distribution.

V. APPROXIMATE VALUE ITERATION

In this section, we provide an approach to deriving the optimal self-triggered controller that minimizes (15), provided the GP model of the transition dynamics (12) is given. Let \( J^*(x_k) = \min_{u_{k+\ell+1}} J^*(x_{k+\ell+1}) \). From (15), the corresponding Bellman equation is given by

\[
J^*(x_k) = \min_{u_{k+\ell+1}} \left\{ \mathbb{E}_x[x_{k+\ell+1}] + J^*(x_{k+\ell+1}) \right\},
\]

where \( \mathbb{E}_x[x_{k+\ell+1}] \) and the input space \( U \) for the dynamics in (6) are both infinite, deriving an explicit solution to (28) is in general
intractable. Thus, we derive an approximated solution to (28) by employing a finite number of representative points in the state space and the input space, which are denoted as \(x_{R,1}, x_{R,2}, \ldots, x_{R,N_x} \in \mathbb{R}^{n_x}\) and \(u_{R,1}, u_{R,2}, \ldots, u_{R,N_U} \in U\), respectively, with \(N_x\) and \(N_U\) being the number of representative points. For simplicity of presentation, we let \(X_R = \{x_{R,0}, x_{R,1}, \ldots, x_{R,N_x}\}\), \(U_R = \{u_{R,1}, u_{R,2}, \ldots, u_{R,N_U}\}\). The optimal cost function (denoted as \(J^*\)) and the optimal control policy (denoted as \(\pi^*\)) are then approximated by the exponential Radial Basis Functions (RBFs):

\[
\hat{J}^*(x) = \sum_{n=1}^{N_x} w_{J,n} \exp \left( -\frac{1}{2\sigma_J^2} \|x - x_{R,n}\|^2 \right),
\]

\[
\hat{\pi}^*_m(x) = \sum_{n=1}^{N_x} w_{\pi,n} \exp \left( -\frac{1}{2\sigma_u^2} \|x - x_{R,n}\|^2 \right),
\]

where \([w_{J,n}]_{n=1}^{N_x}\) are the weights and \(\sigma_J, \sigma_u\) are the width of the RBFs for \(\hat{J}^*\) and \(\hat{\pi}^*_m\), respectively, which are the hyper-parameters to be designed and will be updated during the algorithm. Moreover, the optimal communication policy is approximated by \(\hat{\pi}^*_{\text{com}}(x) = \|\hat{\pi}^*_{\text{com}}(x)\|\), where \(\|a\|\) denotes the closest positive integer to \(a\) (i.e., \(\|a\| = \text{arg min}_j\{|a - j| : j \in \mathbb{N}_{>0}\}\)) and

\[
\hat{\pi}^*_{\text{com}}(x) = \sum_{n=1}^{N_x} \exp \left( -\frac{1}{2\sigma_c^2} \|x - x_{R,n}\|^2 \right).
\]

Here, \([w_{c,n}]_{n=1}^{N_x}\) and \(\sigma_c\) are the hyper-parameters to be updated.

The iterative procedure to solve (28) follows the so-called value iteration [35], which is summarized in Algorithm 1. As shown in the algorithm, for each \(x \in X_R\), we compute \(D(x, u, m)\) for all \(u \in U_R\) and \(m \in \mathbb{N}_{1:M}\), which are specifically defined as

\[
D(x, u, m) = \mathbb{E}_{x_m} \left[ C_1(x_m) + \gamma C_2(m') + \hat{J}^*(x_m) \right]
\]

\[= \int p(x_m | x, u) C_1(x_m) dx_m \tag{33} \]

\[+ \gamma \int p(x_m | x, u) C_2(m') dx_m \tag{34} \]

\[+ \int p(x_m | x, u) \hat{J}^*(x_m) dx_m \tag{35} \]

where \(x_m\) is the state that is reached from \(x\) by applying \(u\) constantly for \(m\) time steps, \(m'\) is the inter-communication time steps determined for the state \(x_m\), i.e., \(m' = \hat{\pi}^*_{\text{com}}(x_m)\).

As shown in (33)-(35), it is required to compute the distribution \(p(x_m | x, u)\), as well as the three expected values (integrals) with respect to this distribution. In what follows, we provide a detailed way of computing these three terms.

(Computation of \(p(x_m | x, u)\)): The term \(p(x_m | x, u)\) is the predictive distribution of the state from \(x\) by applying \(u\) constantly for \(m\) time steps, which can be indeed approximated by the moment matching technique as discussed in Section IV. That is, we can approximate the distribution as \(p(x_m | x, u) \approx \mathcal{N}(\mu_m, \Sigma_m)\), where \(\mu_m\) and \(\Sigma_m\) denote the mean and the covariance of \(p(x_m | x, u)\) that are computed by following the technique described in Section IV.

(Computation of (33)): Using the Gaussian approximation of \(p(x_m | x, u)\), the first term (33) is given by

\[
\int p(x_m | x, u) C_1(x_m) dx_m \approx \int \mathcal{N}(\mu_m, \Sigma_m) C_1(x_m) dx_m. \tag{36}
\]

Since we assume that \(C_1\) is characterized by polynomials or exponential, we can analytically compute the integral in (36). For example, if \(C_1\) is given by (16), the integral in (36) further leads to

\[
\int \mathcal{N}(\mu_m, \Sigma_m) C_1(x_m) dx_m = 1 - \int \mathcal{N}(\mu_m, \Sigma_m) \exp \left\{ -\frac{1}{2} x_m^T Q x_m \right\} dx_m
\]

\[= 1 - \delta(\mu_m, \Sigma_m), \]

where \(\delta(\cdot)\) is given by \(\delta(\mu_m, \Sigma_m) = |I + \Sigma_m Q|^{-\frac{1}{2}} \exp \left( -\frac{1}{2} \mu_m Q (I + \Sigma_m Q)^{-1} \mu_m \right).

(Computation of (34)): The second term (34) can be com-
puted as
\[
\int p(x_m|x,u)C_2(m')dx_m
= \int p(x_m|x,u)(M - \tilde{\pi}^*_{com}(x_m))dx_m
= M - \int p(x_m|x,u)\tilde{\pi}^*_{com}(x_m)dx_m. \tag{37}
\]
which requires to compute \( \int p(x_m|x,u)\tilde{\pi}^*_{com}(x_m)dx_m \). Using (31), we approximate this term as follows:
\[
\int p(x_m|x,u)\tilde{\pi}^*_{com}(x_m)dx_m \\
\approx \sum_{n=1}^{N_X} \int p(x_m|x,u)\tilde{\pi}^*_{com}(x_m)dx_m \\
= \sum_{n=1}^{N_X} \int p(x_m|x,u)\tilde{\pi}^*_{com}(x_m)dx_m. \tag{38}
\]
where \( \delta_{e,s}() \) is given by \( \delta_{e,s}(\mu_m, \Sigma_m) = |I + \sigma_c^{-2}\Sigma_m|^{-\frac{1}{2}} \exp \left( -\frac{1}{2\sigma_c^2}(\mu_m - x_{R,n})^T(I + \Sigma_m\sigma_c^{-2})^{-1}(\mu_m - x_{R,n}) \right) \).

(Computation of (35)): The third integral (35) can be approximated in a similar manner to the computation of (34). From (29) and using \( p(x_m|x,u) \approx N(\mu_m, \Sigma_m) \), we have
\[
\int p(x_m|x,u)\tilde{f}^*(x_m)dx_m = \sum_{n=1}^{N_X} \int p(x_m|x,u)\tilde{f}^*(x_m)dx_m. \tag{39}
\]
where \( \delta_{j,n}() \) is given by \( \delta_{j,n}(\mu_m, \Sigma_m) = |I + \sigma_j^{-2}\Sigma_m|^{-\frac{1}{2}} \exp \left( -\frac{1}{2\sigma_j^2}(\mu_m - x_{R,n})^T(I + \Sigma_m\sigma_j^{-2})^{-1}(\mu_m - x_{R,n}) \right) \).

As shown in the algorithm (line 3 – line 10), for each \( x \in X_R \) we pick the smallest value among \( D(x, u, m), u \in U_R, m \in \mathbb{N}_{1:M} \), as well as the corresponding pair of the control input and inter-communication time steps, which we denote by \( D^*(x), u^*(x), \) and \( m^*(x) \), respectively. Consequently, we obtain \( \{D^*(x_{R,n}), u^*(x_{R,n}), m^*(x_{R,n})\}_{n=1}^{N_X} \), and these are used as the new training data to update the hyper-parameters of \( \tilde{f}^*, \tilde{\pi}^*_{inp}, \tilde{\pi}^*_{com} \) in (29), (30), (31). For example, \( \tilde{f}^* \) is updated by using the training data \( \{x_{R,n}, D^*(x_{R,n})\}_{n=1}^{N_X} \), where \( x_{R,n}, n \in \mathbb{N}_{1:N_X} \) are the training inputs and \( D^*(x_{R,n}), n \in \mathbb{N}_{1:N_X} \) are the training outputs.

VI. IMPLEMENTATION

In this section, we provide an overall implementation algorithm that jointly learns the dynamics of the plant, learns the self-triggered controller, and executes the self-triggered controller based on a reinforcement learning framework.

The overall algorithm is summarized in Algorithm 2. Since we assume that the learning agent has no knowledge about the dynamics of the plant, we set the communication policy as \( \tilde{\pi}^*_{com}(x) \leftarrow 1, \forall x \in \mathbb{R}^n_u \) (i.e., communication is given at every time step), so that the learning agent can efficiently collect the training data and learn the dynamics of the plant at the initial phase. For each episode, we implement the following two parts: execution/exploration phase, and learning phase. During the execution/exploration phase, the controller implements the self-triggered controller in an \( \varepsilon \)-greedy fashion. In the algorithm, Uniform(0, 1) (line 2) is a function that generates a random real number from the interval \([0, 1]\) according to the uniform distribution. That is, with the probability \( \varepsilon \), a random control input with the one step inter-communication time step is sampled, and, otherwise, the computed optimal control and communication policies are chosen to be executed. Here, the one step inter-communication time step is chosen (with the probability \( \varepsilon \)) so that the learning agent is able to utilize the consecutive states (i.e., \( x_{k,t}, x_{k,t+1} \) with \( k_{t+1} = k_t + 1 \)) to update the GP model of \( f \). In the learning phase, the learning agent utilizes the new training data \( D \) to update the GP model of the plant, and compute the

Algorithm 2 Overall reinforcement learning algorithm.

Input: \( x_{\text{init}} \) (initial state), \( N_{\text{epi}} \) (number of episodes), \( \varepsilon \in [0,1) \) (threshold for the greedy policy);
Output: \( \hat{\pi}^*_{inp}, \hat{\pi}^*_{com} \) (approximated optimal control and communication policies);
1: Initialize the hyper-parameters to represent \( \hat{\pi}^*_{inp} \), and set \( \pi_{com}(x) \leftarrow 1, \forall x \in \mathbb{R}^n_u \);
2: \( X \leftarrow \{\}, \forall i \in \mathbb{N}_{1:n_x} \);
3: \( y_i \leftarrow \{\}, \forall i \in \mathbb{N}_{1:n_x} \);
4: \( D_i \leftarrow \{X, y_i\}, \forall i \in \mathbb{N}_{1:n_x} \) (initialize the training data);
5: for Episode = 1 : \( N_{\text{epi}} \) do
6: \( \ell \leftarrow 0; \)
7: \( k_t \leftarrow 0; \)
8: \( x_{k_t} = x_{\text{init}}; \)
9: The plant transmits \( x_{k_t} \) to the controller;
10: [Execution/Exploration phase]
11: for \( \ell = 0 : N_{\text{max}} - 1 \) do
12: Sample \( r \sim \text{Uniform}[0, 1]; \)
13: if \( r < \varepsilon \) then
14: \( m_t \leftarrow 1; \)
15: Select \( u_{k,t} \) randomly from \( U; \)
else
16: \( u_{k,t} \leftarrow \pi^*_\text{com}(x_{k_t}); \)
17: \( m_t \leftarrow \pi^*_\text{inp}(x_{k_t}); \)
18: end if
19: end for
20: \( k_{t+1} \leftarrow k_t + m_t; \)
21: The controller transmits \( \{u_{k,t}, m_t\} \) to the plant;
22: The plant applies \( u_{k_t} \) constantly for \( m_t \) time steps and transmit \( x_{k_{t+1}} \) to the controller;
23: if \( m_t = 1 \) then
24: \( X \leftarrow \{X \cup [x_{k_t}^T, u_{k_t}^T]\}; \)
25: \( y_i \leftarrow \{y_i \cup x_{k_{t+1}}\}, \forall i \in \mathbb{N}_{1:n_x}; \)
26: \( D_i \leftarrow \{D_i \cup \{X, y_i\}\}; \)
27: end if
28: [Learning phase]
29: The learning agent learns the GP model of the plant by using the new training data \( D = \{D_i\}_{i=1}^{n_x}; \)
30: The learning agent executes Algorithm 1 to update the (approximated) optimal policies \( \hat{\pi}^*_{inp}, \hat{\pi}^*_{com}; \)
31: end for
(approximated) optimal control and communication policies according to Algorithm \[\text{1}\].

VII. SIMULATION RESULTS

In this section, we illustrate the effectiveness of the proposed approach through a simulation example. The simulation was conducted on Matlab 2016a under Windows 10, Intel(R) Core(TM) i7 4.20 GHz, 32 GB RAM. As a simulation example, we consider a control problem of an inverted pendulum, whose dynamics is governed by

\[ x_{1,k+1} = x_{1,k} + \Delta x_{2,k} \]
\[ x_{2,k+1} = x_{2,k} + \Delta (\sin x_{1,k} - x_{2,k} + u_k) , \]

where \( x_{1,k} \) and \( x_{2,k} \) with \( x_k = [x_{1,k}; x_{2,k}] \) are the states that represent the angular position and the velocity of the mass, \( u_k \in \mathbb{R} \) is the control input, and \( \Delta = 0.2 \) denotes the sampling time interval. It is assumed that \( U = [-1.5, 1.5] \) and the initial state is given by \( x_{\text{init}} = [x_{1,0}; x_{2,0}] = [1.0; 0.2] \). The maximum inter-communication time step is \( M = 10 \), and the representative points for the state space to solve (25) is selected by the grid points in the set \( X = [-1.6, 1.6] \times [-1.6, 1.6] \) with the interval 0.4, i.e., \( X_R = [X]_{0.4} \). The representative points for the input space is given by \( U_R = [U]_{0.3} \). We use the exponential type for the stage cost in (16) with \( Q = I_2 \), and we set \( \gamma = 0 \) for the cost function in (15).

Fig. 2(a) illustrates the trajectories of the states by applying the self-triggered controller obtained by Algorithm \[\text{2}\] with Episode = 1 (red dotted) and 100 (blue solid). The figures illustrate that, while the state diverges at the initial learning phase, it is indeed stabilized towards the origin as the number of episode increases. The computed inter-communication time steps corresponding to the simulation result in Fig. 2(a) (Episode = 100) are illustrated in Fig. 2(b) which shows that the communication is given aperiodically according to the derived self-triggered controller. Note that, even for the case \( \gamma = 0 \), communication reduction can be potentially achieved by minimizing (15). This is due to the fact that the total cost in (15) is defined by summing the stage costs only for the communication time steps, i.e., the cost will be accumulated only when the communication is given. Hence, reducing the number of communication leads to the reduction of the total cost, and, therefore, minimizing (15) leads to the communication reduction even for the case \( \gamma = 0 \).

To indicate the robustness of the derived self-triggered controller, we also illustrate in Fig. 3 several trajectories from different initial states around \( x_{\text{init}} \). The figure illustrates that the states are indeed stabilized to the origin regardless of the deviation of the initial states.

To analyze the effect of \( \gamma \), we illustrate in Fig. 4(a) and Fig. 4(b) the resulting state trajectories under different selections of \( \gamma \) (\( \gamma = 0.01, 0.02, 0.05 \)) and the corresponding inter-communication time steps, respectively. Here, Algorithm \[\text{2}\] has been implemented for each \( \gamma \) with 100 episodes (\( N_{\text{epi}} = 100 \)). From Fig. 4(b), it is shown that larger inter-communication time steps are more likely to be selected as \( \gamma \) is selected larger. This is due to the fact that, by selecting larger \( \gamma \), it will penalize more for the communication cost. Note that, for the case \( \gamma = 0.05 \), the resulting state trajectory converges farther from the origin than for the other cases (while it achieves
larger inter-communication time steps), which may be due to the fact that achieving large inter-communication time steps is too emphasized.

Finally, to analyze the effect of the maximum inter-communication time steps, we illustrate in Table I the number of communication time steps over the time interval $k$ with different selection of $M$, as well as the corresponding total execution time [s] after completing 100 episodes. From the table, we can achieve a more communication reduction by increasing the parameter $M$, while, on the other hand, it requires a heavier computation load for the algorithm to be terminated. The increase of the execution time is due to the derivation of $D(x, u, m)$ in Algorithm 1, the number of evaluations to compute $D(x, u, m)$ increases as $M$ increases. Hence, it is shown that there exists a tradeoff between the communication load for NCSs and the computation load of Algorithm 1, and such tradeoff can be regulated by tuning $M$.

### VIII. Conclusion and Future Work

In this paper, we investigate the self-triggered controller for NCSs with the unknown transition dynamics. To this end, we use the GP to learn the dynamics of the plant. We first formulate an optimal control problem, such that both the cost for the control performance and the communication cost can be taken into account. Then, we illustrate that the optimal control problem can be solved via a value iteration algorithm, in which the optimal pair of the control input and the inter-communication time steps can be determined based on the GP model of the plant. Then, we provide overall reinforcement learning algorithm that jointly learns the dynamics and execution of the self-triggered controller implemented by the controller and the learning agent. Finally, a numerical simulation is given to illustrate the effectiveness of the proposed approach.

When solving the value iteration algorithm (Algorithm 1), for each representative point of the state, input, and the inter-communication time step $(x, u, m) \in \mathcal{X}_R \times \mathcal{U}_R \times \mathbb{N}_{1:M}$, the corresponding value of $D(x, u, m)$ needs to be computed. Since the number of representative points to precisely approximate the optimal solution needs to be larger as the state (input) dimension increases, the computational complexity of Algorithm 1 should be higher as $n_x$ ($n_u$) increases. Hence, future work involves improving the computational complexity of the algorithm in order to accommodate high-order systems. Moreover, future work involves analyzing some theoretical issues (e.g., stability of the closed loop system, convergence property of the value iteration algorithm, etc.) for the GP dynamics of the plant. Finally, extending the proposed framework to the case when network uncertainties, such as random packet dropouts or network delays, should be investigated for our future research.

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**Table I**

| Communication instances | 1   | 5   | 10  |
|-------------------------|-----|-----|-----|
| Execution time [s]      | 1730| 9520| 19440|

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When solving the value iteration algorithm (Algorithm 1), for each representative point of the state, input, and the inter-communication time step $(x, u, m) \in \mathcal{X}_R \times \mathcal{U}_R \times \mathbb{N}_{1:M}$, the corresponding value of $D(x, u, m)$ needs to be computed. Since the number of representative points to precisely approximate the optimal solution needs to be larger as the state (input) dimension increases, the computational complexity of Algorithm 1 should be higher as $n_x$ ($n_u$) increases. Hence, future work involves improving the computational complexity of the algorithm in order to accommodate high-order systems. Moreover, future work involves analyzing some theoretical issues (e.g., stability of the closed loop system, convergence property of the value iteration algorithm, etc.) for the GP dynamics of the plant. Finally, extending the proposed framework to the case when network uncertainties, such as random packet dropouts or network delays, should be investigated for our future research.

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