COMPTONIZATION SIGNATURES IN THE RAPID APERIODIC VARIABILITY OF GALACTIC BLACK HOLE CANDIDATES

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ABSTRACT

We investigate the effect of Compton scattering of flares of soft radiation in different geometries of a hot, Comptonizing region and a colder accretion disk around a solar mass black hole. The photon energy–dependent light curves, their Fourier transforms, power spectra, and Fourier period–dependent time lags of hard photons with respect to softer photons are discussed. On the basis of a comparison with existing data we find arguments against Comptonization of external soft radiation as well as Comptonization in a homogeneous medium as dominant mechanisms for the rapid aperiodic variability in Galactic black hole candidates. Possible further observational tests for the influence of Comptonization on the rapid aperiodic variability of Galactic black hole candidates are suggested.

Subject headings: accretion, accretion disks — black hole physics — radiation mechanisms: thermal — radiative transfer — X-rays: stars

1. INTRODUCTION

The rapid aperiodic and quasiperiodic variability of the X-ray signals from Galactic black hole candidates (GBHCs) and low-mass X-ray binaries (for a review see van der Klis 1995) contains valuable information about the source of high-energy emission in these objects. Its typical timescales, typical repetition frequencies, power spectrum densities (PSDs), autocorrelation functions, time lags between different energy bands, etc., give hints toward important parameters such as the extent of the emitting region, the dominant microscopic timescales, the dominant emission mechanism, and the geometry of the source.

With the new generation of X-ray telescopes such as the PCA and the HEXTE on board the Rossi X-Ray Timing Explorer (RXTE), it is now possible to measure in great detail the above properties of the rapid variability of X-ray binaries with high timing resolution and good spectral resolution. As we will see in this paper, the photon energy–dependence of the rapid aperiodic variability can provide important diagnostics regarding the nature of the X-ray–emitting regions.

Early measurements of the Fourier frequency–dependence of time lags between the signals in different X-ray energy channels from GBHCs (Miyamoto et al. 1988; Miyamoto et al. 1993) have been interpreted as evidence that Comptonization of soft photons in a hot, uniform plasma could not be the dominating mechanism for the production of hard X-rays. This conclusion, however, is strongly geometry-dependent and does not hold for a very extended, inhomogeneous Comptonizing region, as was recently found by Kazanas, Hua, & Titarchuk (1997). In that paper and two subsequent papers (Hua et al. 1997a, 1997b), the effect of radial density gradients in the hot Comptonizing regions in GBHCs on the power-density spectra and the phase and time lags between different energy bands were discussed. From the comparison of the measured Fourier frequency–dependent time lags between two energy channels of the signal from Cyg X-1, they deduced that a radial density–dependence $n(r) \propto r^{-1}$ in the Comptonizing hot plasma is appropriate to account for the observed hard time lags and the hard X-ray spectrum at the same time. However, their work was restricted to a geometry with a central soft photon source surrounded by a spherical corona of hot Comptonizing plasma. Furthermore, the density was the only parameter allowed to vary radially.

In this paper, we extend the investigation of the effects of Comptonization and the rapid variability in X-ray binaries, discussing two fundamentally different source geometries and radial gradients of other physical parameters, such as the electron temperature. This is done primarily with Monte Carlo simulations of instantaneous flares of soft radiation (described as a $\delta$ function in time) being Comptonized by the X-ray–emitting region. The generalization to the case of multiple, randomly distributed flares (shot noise) is straightforward. The light curves resulting from the Monte Carlo simulations are then Fourier transformed, and the PSD, as well as the Fourier frequency–dependent phase and time lags between different energy bands, are calculated.

In § 2 we describe our Monte Carlo simulations leading to the energy-dependent light curves for the different geometrical situations together with analytical approximations for these light curves. Their Fourier transforms and the corresponding time lags between different energy bands are discussed in § 3. In § 4 we investigate specific differences between the PSDs and hard time lags resulting from different geometries and compare the predictions of both scenarios to the power spectra and hard time lags observed for some GBHCs. We suggest further observational tests that could either confirm a preferred geometry or rule out the Comptonization scenario in general.

2. SIMULATED LIGHT CURVES

For our time-dependent Monte Carlo simulations on Compton scattering in a hot plasma we use an improved version of the Comptonization code developed by Canfield, Howard, & Liang (1987) (see also Liang 1993; Wandel & Liang 1991). It can handle spherically symmetric or slab geometry of the Comptonizing region with thermal soft photon sources located in the center of the Comptonizing cloud, distributed throughout the cloud, and/or located outside the Comptonizing region. All parameters (such as density, electron temperature, and radiation temperature of
an internal soft photon field) may have arbitrary radial dependences within the cloud.

Practically, we split the Comptonizing region into 20 radial zones, each zone having the same (radial) Thomson depth, within which the parameters are set to a constant value corresponding to an average of the respective radial dependence over the zone. For several of our simulations, we run tests with a finer radial grid (50–100 zones) in order to check that the step function dependence of the parameters in the Comptonizing region does not introduce artificial structures in the light curves. No significant difference to our simpler (and faster) 20 zone simulations has been found in any of these tests.

In this paper, we focus on two basic situations with respect to the location of the flaring soft photon source: a source located at the center of the cloud and a source located outside the cloud. The former situation is representative of an accretion disk corona model (Liang & Price 1977; Bisnovatyi-Kogan & Blinnikov 1977; Haardt & Maraschi 1993) in which photons resulting from a flare in the cold, optically thick disk (Shakura & Sunyaev 1973) are Compton-uncattered by the hot corona to produce the hard X-ray spectrum. The latter case represents a hot inner-disk model (Shapiro, Lightman, & Eardley 1976) or an advection-dominated accretion flow (Narayan & Yi 1994; Chen et al. 1995), assuming that a soft photon flare occurs in the cold, optically thick outer disk and its radiation impinges on the hot inner disk (torus), producing the hard X-rays via Compton upscattering.

For all simulations shown in this paper, the soft photon spectrum is assumed to be a thermal blackbody of \( kT \) = 0.2 keV.

For each of the light curves resulting from central soft photon injection, a total of \( 9 \times 10^6 \) photons being injected instantaneously at \( t = 0 \) are simulated. The photons leaving the system are collected in five energy channels and in equal time steps appropriate to reveal the detailed structure of the light curves (typically \( \Delta t \sim 10^{-2} R/c \), where \( R \) is the radius of the Comptonizing region). The light curves resulting from external soft photon injection prove to be mathematically more complicated and require a higher numerical signal-to-noise ratio in order to yield reliable Fourier transforms. Therefore, the simulations for this geometry have been carried out with \( 25 \times 10^6 \) photons each.

For both geometries, we investigate the effect of density and temperature gradients within the hot Comptonizing region. Figure 1 shows the energy-dependent light curves for the case of a central photon source inside a homogeneous cloud. We did further simulations for Comptonizing clouds with density profiles \( n(r) \propto r^{-p} \) for \( p = 1 \) and 3/2 and for clouds with temperature profiles of the same form. Generally, the light curves for a central source can be parameterized by a gamma distribution,

\[
f_{\text{inl}}(t) = A t^{\alpha - 1} e^{-\beta t} \Theta(t - \frac{R}{c})
\]

as suggested by Hua et al. (1997a), where

\[
\Theta(t) = \begin{cases} 
0 & \text{if } x < 0 \\
1 & \text{if } x > 0 
\end{cases}
\]

is the Heaviside function and \( A \) is a normalization constant. Inspecting the simulated light curves for the different cases, we find that both \( \alpha \) and \( \beta \) are energy dependent in both cases of a homogeneous cloud and a cloud with parameters spatially varying.

Both the early time slope (\( \alpha - 1 \)), describing the light curve immediately after the onset of the flare (accounting for the light-travel time) and the cutoff time \( \beta \) (which is of the order of the photon escape time \( t_{\text{esc}} \approx \tau_T R/c \)) tend to increase with photon energy. The assumption of a density gradient (conserving the total Thomson depth \( \tau_T \) and extent \( R \) of the region) leads to a considerable reduction of \( \alpha \) and a moderate reduction of \( \beta \). These changes are much more pronounced for the high photon energies than for the lower energy channels, which implies that the energy dependence of \( \alpha \) and \( \beta \) is reduced because of the spatial gradient.

The effect of a temperature gradient in a medium of constant density, which might be a more appropriate choice in the case of an accretion-disk corona model (Haardt & Maraschi 1991; Haardt & Maraschi 1993; Dove et al. 1997), is qualitatively similar to the effect of a density gradient in a medium of uniform temperature, provided the respective gradients are not too steep. In this paper, we concentrate on a detailed discussion of the effects of a density gradient. Temperature gradients, together with more elaborate single-pulse shapes (instead of \( \delta \) function-like flares) will be investigated in future work.

The values of \( \alpha \) and \( \beta \) yielding a good representation of the simulated light curves for central soft photon injection are listed in Table 1. We point out, however, that in the case of radial parameter gradients the light curves of the two higher energy channels show significant deviations from a power law at the onset of the flare of Comptonized radiation.

For the case of radiation impinging from outside onto the Comptonizing region, we assume that the source of soft photons is located close to the outer boundary of the hot plasma region, which implies that the radiation enters the Comptonizing region at angle cosines with respect to the radial direction that are randomly distributed between \(-1\) and 0. In this case, the light curve can generally be parameterized by the sum of a power law resulting from single scattering over the light crossing time through the source, \( 2R/c \) plus a multiple scattering component, described by a
light curves at except in the two highest photon energy channels, where the formally the same as in the case of a central soft photon source, more important and dominates the shape of all light curves initially. With a more pronounced radial extent and Thomson depth. The resulting Comptonization spectra of these two situations are similar to the ones obtained from the situations assumed in the case of central photon injection.

For a homogeneous cloud, clearly the first term in equation (3) is dominant, i.e., the parameterization is basically the same as in the case of a central soft photon source, except in the two highest photon energy channels, where the light curves at $t \ll R/c$ resemble a broken power law with a flat early time slope. With a more pronounced radial density gradient, the second term in equation (3) becomes more important and dominates the shape of all light curves until $t \approx R/c$ for density profiles with $p \gtrsim 1$. However, although the parameterization of the light curves for a uniform cloud scattering radiation impinging from the outside is formally the same as for scattering of radiation coming from a central soft photon source, the energy dependence of the early time slope ($\alpha - 1$) of the light curves for $t \ll R/c$ is significantly weaker in the case of the external source. This becomes obvious when comparing Tables 1 and 2 in which the parameters characterizing the light curves for the two geometrically different situations are listed, and from inspection of Figure 10 in which the energy dependence of the early time slope of the light curves is plotted.

It should be noted that the light curves resulting from our numerical simulations differ significantly from the analytical results of Lightman & Rybicki (1979b) for the problem of inverse Compton reflection off a hot plasma. This discrepancy is due to the fact that Lightman & Rybicki (1979b)
used the photon escape probability corresponding to the case of a semi-infinite, plane-parallel medium with infinite Thomson depth. Lightman & Rybicki (1979) have shown that the inverse Compton spectrum in that case is basically given by a power law of photon index $1$, which is much harder than the observed hard X-ray spectrum of Galactic black hole candidates (typically $\Gamma \sim 1.5-2$). Therefore we must consider a medium of finite Thomson depth, $r_T \ll 10$.

3. FOURIER TRANSFORMS, POWER SPECTRA, AND TIME LAGS

We performed a numerical Fourier Transform for each energy band in our simulated light curves and calculated the resulting power spectrum densities. Figures 4 and 5 show the PSDs corresponding to the simulations for external soft-photon injection for the cases $p = 0$ and $p = 1$, respectively, illustrated in Figures 2 and 3. Knowing the Fourier transform $F_k(\omega) = r_k(\omega)e^{i\phi_k(\omega)}$ of the light curve in energy channel $k$ at Fourier frequency $\omega = 2\pi f$, the phase difference $\Delta\phi_k(\omega)$ between two energy channels $k$ and $l$ can be computed. The resulting time lag between the signals in different energy channels is then given by $\Delta t_{kl} = \Delta\phi_k(\omega)/\phi_k(\omega)$.

Purely from the complex value of $F_k(\omega)$, however, the absolute value of the phase $\phi_k(\omega)$ is not known a priori. Its normalization is given as follows: for a signal extending over a finite time interval, $0 \leq t \leq t_{\text{max}}$, the low-frequency limit of the Fourier transform at $\omega \ll t_{\text{max}}$ is given by

$$F_k(\omega) \bigg|_{\omega \ll t_{\text{max}}^{-1}} \approx \int_{-\infty}^{\infty} dt_{kl}(t) + i\omega \int_{-\infty}^{\infty} dt_{kl}(t) + O[(\omega t_{\text{max}})^2],$$

where $O(x)$ denotes the Landau symbol. Thus, in the low-frequency limit we find

$$\phi_k(\omega) \bigg|_{\omega \ll t_{\text{max}}^{-1}} \approx \arctan (\omega \langle t \rangle_k) \approx \omega \langle t \rangle_k,$$

where $\langle t \rangle_k = \int dt_{kl}(t)/\int dt_{kl}(t).$ This implies that the hard time lags at Fourier periods $P \gg t_{\text{max}}$ are

$$\Delta t_{kl} \bigg|_{P \gg t_{\text{max}}} = \langle t \rangle_k - \langle t \rangle_l.$$  

The time lags as a function of Fourier period $P = 1/\nu$ resulting from our simulations for central soft photon injection into a homogeneous ($p = 0$) cloud and a cloud with $p = 1$ are displayed in Figures 6 and 7, respectively. Figures 8 and 9 show the respective time lag curves for the case of external soft photon injection. The solid curves in Figures 6–9, corresponding to an exactly linear dependence, have been plotted for comparison. In order to suppress numerical noise contained in the simulated light curves, we use smooth analytical curves following the light curves closely for the computation of the hard time lags. In Figures 4 and 5, the PSDs from our analytical representations (thick lines) are shown in comparison to the PSDs of the simulated light curves (thin lines). The figures demonstrate that the simulated signal of the highest energy channel (with the worst photon statistics) is strongly dominated by numerical noise at high frequencies. We therefore do not consider our results concerning the highest energy channel (31–70 keV) in the case of external photon injection as fully reliable.
Generally, for both the central and the external soft photon source, the time lags are increasing with Fourier period for frequencies $\omega \gtrsim \beta_k^{-1} \sim c/(\tau_r R)$ in the case of a central soft photon source and for frequencies $\omega \gtrsim \omega_{cr} \equiv c/(2R)$ for an external soft photon source. For lower frequencies (longer periods) the respective time lags turn into a flat curve where the time lags are given by equation (6).

In all cases investigated here, the light curves in the lowest two energy channels are very similar to each other, being characterized by roughly equal typical escape timescales $\langle \tau \rangle$ (see Figs. 1–3). This implies that the largest hard time lags calculated for these channels (eq. [6]) are significantly smaller than the typical light crossing time.

For an external soft photon source, the $\Delta \lambda(P)$ dependence deviates from linearity in the lowest photon energy channels that are close to the energy of the injected soft photons. The photons in these channels are mainly produced by a single-scattering reflection of soft photons in the outer parts of the scattering region, impinging at large angles with respect to normal to the surface, resulting in relatively small scattering angles and little energy gain.

In the case of a central source the gradual rise of the light curves at the onset of the flare, systematically deviating from a power law, causes a weak flattening of the $\Delta \lambda(P)$ dependence with increasing photon energies. In Figure 6 (central source, $p = 0$), the time lag between the lowest two energy channels depends on the Fourier period to a power greater than unity, which we attribute to the fact that the early time slopes of the two light curves in these energy channels are nearly equal (see below).

The power spectra and hard time lags resulting from our simulations show a significantly different energy dependence for the two different geometries. In the case of the central soft photon source and a density or temperature gradient, the PSD is flat for low-photon energies, while for higher photon energies it becomes gradually steeper to higher Fourier frequencies. For the $p = 1$ case, this can be approximated by a broken power law with break frequency $\omega_{br} \sim \beta^{-1}$. For the solid sphere, the PSD can generally be well described by a broken power law with a flat, low-
frequency slope and the high-frequency slope increasing with photon energy.

For an external soft photon source the PSD can roughly be represented by a broken power law for all energy channels with break frequency $\omega_{br} \sim c/(2R)$ and a flat low-frequency branch. The power-law slope depends only very weakly on photon energy and is always close to 2.

The analytical representations (1) and (3) of the light curves in the two geometrical cases considered here can be Fourier transformed analytically. The Fourier transforms are

$$F_{\text{int}}(\omega) = A \frac{\Gamma(\beta)^{\beta}}{[1 + (\beta \omega)^{2}]^{\beta/2}} e^{i \pi \text{arctan}(\beta \omega)}$$  \hspace{1cm} (7)

for the central soft photon source and

$$F_{\text{ext}}(\omega) = A \frac{\Gamma(\beta)^{\beta}}{[1 + (\beta \omega)^{2}]^{\beta/2}} e^{i \pi \text{arctan}(\beta \omega)}$$

$$+ B \frac{\omega_{\text{cr}}^{\kappa}}{\kappa} e^{-i \omega_{\text{cr}} \omega} M\left(1; \kappa + 1; i \frac{\omega}{\omega_{\text{cr}}} \right),$$  \hspace{1cm} (8)

where $M(a; b; z)$ is Kummer’s function for the external source. In the limiting cases $\omega \ll \beta^{-1}$, $\omega_{\text{cr}}$ and $\omega \gg \beta^{-1}$, $\omega_{\text{cr}}$, these Fourier transforms are

$$F_{\text{int}}(\omega) \approx \begin{cases} \Gamma(\beta)^{\beta} e^{i \pi \beta \omega} & \text{for } \omega \ll \beta^{-1} \\ \Gamma(\beta)^{\beta} e^{-i \pi \beta \omega/2} & \text{for } \omega \gg \beta^{-1} \end{cases}$$  \hspace{1cm} (9)

and

$$F_{\text{ext}}(\omega) \approx \begin{cases} \Gamma(\beta)^{\beta} e^{i \pi \beta \omega} + B \frac{\omega_{\text{cr}}^{-\kappa}}{\kappa} e^{-i \omega_{\text{cr}} \omega} & \text{for } \omega \ll \omega_{\text{cr}} \\ \Gamma(\beta)^{\beta} e^{-i \pi \beta \omega/2} + \Gamma(\kappa) \omega^{-\kappa} e^{-i \beta \omega/2} & \text{for } \omega \gg \omega_{\text{cr}} \end{cases}$$  \hspace{1cm} (10)

For the central soft photon source, this yields a Fourier frequency-independent power spectrum in the low-frequency limit, while for high frequencies $PSD_{\text{int}} \propto \omega^{-2}$. In general, the hard time lags between two energy bands in the high- and low-frequency limits are (Hua et al. 1997a)

$$\Delta t_{\text{int}} = \begin{cases} \Delta(\alpha \beta) & \text{for } \omega \ll \beta^{-1} \\ \Delta(\alpha \beta) P/4 & \text{for } \omega \gg \beta^{-1} \end{cases},$$  \hspace{1cm} (11)

If the values of $\alpha$ for two energy bands are exactly equal, then the phase difference in the high-frequency limit is $\Delta \phi_{\text{int}} \propto \omega^{-1}$ and $\Delta t_{\text{int}} = \Delta \beta P^2/(4\pi^2 \beta_1 \beta_2)$.

From our simulations, we find that the Fourier transforms of the light curves in the case of an external photon source are dominated by the second term in equations (8) and (10) in limits the $\omega \ll \omega_{\text{cr}}$ and $\omega \gg \omega_{\text{cr}}$, respectively, if the scattering medium has a strong radial density gradient ($p \geq 0.5$). The high-frequency limit of the Fourier transform in equation (8) is completely determined by the shape of the light curves at $t \ll R/c$ and can always be obtained by simply Fourier transforming a function $f(t) = t^{p-1}G(t)$ and describing the light curve for $t \ll R/c$ without accounting for the cutoff at $t \approx 2R/c$ or $t = R$. Therefore, for $\omega \gg \omega_{\text{cr}}$ we have PSD $\propto \omega^{-2\mu}$, where $\mu = \alpha$ for a weak density gradient ($p \leq 0.5$) and low-photon energies and $\mu = \kappa$ else.

However, this relation between the early time slope of the light curve and the high-frequency branch of the PSD does not apply to the case of the free-fall density gradient $p = 3/2$ and internal soft photon injection, where the light curves at the onset of the flare significantly deviate from power-law behavior, which leads to a deviation of the PSD from a simple power law at high Fourier frequencies.

For the hard time lags we find in the case of external soft photon injection

$$\Delta t_{\text{ext}} = \begin{cases} \frac{1}{\omega_{\text{cr}}} & \text{for } \omega \ll \omega_{\text{cr}} \\ \pi^2 P \Delta \mu & \text{for } \omega \gg \omega_{\text{cr}} \end{cases}$$  \hspace{1cm} (12)

The qualitative similarity of the asymptotic expressions for the Fourier transforms for both injection geometries is a consequence of the fact that in both cases the rise of the Compton-scattered flare is approximately described by a power law, which results in a power-law PSD at high frequencies, while for both geometries there is a fast decay after one light crossing time $t_c$, which always results in a constant PSD at frequencies $\omega \ll t_c^{-1}$.

Assuming that the real signal we measure from an X-ray binary is composed of multiple shots randomly distributed in time and with a certain distribution of shot amplitudes $A_n$, i.e.,

$$f(t) = \sum_{n=-\infty}^{\infty} A_n f_0(t - t_n)$$  \hspace{1cm} (13)

where $f_0$ is one of the single-shot light curves (1) or (3), yields a Fourier transform

$$F(\omega) = F_0(\omega) \sum_{n=-\infty}^{\infty} A_n e^{-i \omega \omega_{\text{int}}}$$  \hspace{1cm} (14)

where $F_0$ is the respective single-shot Fourier transform (7) or (8). From equation (14) it is obvious that the time lags between different energy bands remain unaffected by the summation of multiple shots, and in the PSD only near the average shot repetition frequency some additional structure is introduced. Therefore the asymptotic expressions (9) and (10) yield a very good description of the expected high-frequency branch of the PSD from a model of Comptonized random shots of soft photons in the two geometries discussed here, and equations (11) and (12) remain valid without modification.

4. COMPARISON TO OBSERVATIONS

The analysis of the frequency-dependent light curves and their Fourier transforms enables us to extract important information from observed or observable properties of the rapid aperiodic variability of Galactic black hole candidates. Observable quantities that we will focus on now are the following: (1) the high-frequency power-law index of the PSD and its photon energy–dependence; (2) the turnover frequency of the PSD; (3) the photon energy–dependence of hard time lags with respect to the soft X-rays; and (4) the turnover frequency in the hard time lag versus Fourier-period relation.

As we have seen in the previous section, the power-law index of the PSD at high-Fourier frequencies is dominated by the short-term behavior of individual flares and is described by the parameter $\alpha$ or $\kappa$ of equations (1) or (3), respectively. This dependence can be written as $PSD \propto \omega^{-2\alpha}$, where $\mu$ is the dominant short-time light curve slope ($\alpha$ or $\kappa$). In Figure 10, we have plotted this slope as a function of the mean photon energy in the energy bins
in which the light curves were sampled for density gradients of \( p = 0, 1, \) and \( 3/2, \) respectively, for both central and external soft-photon injection. As we mentioned earlier, in the case of central soft-photon injection and a density gradient of \( p \leq 1, \) a significant photon energy–dependence results, while for an external soft-photon source the curves are consistent with a constant slope (recalling that we do not attribute high significance to our results concerning the highest energy channel for external soft photon injection).

For the external soft photon source, we find on the basis of our simulations \( \mu_{\text{ext}} \approx 1, \) irrespective of radial parameter gradients. This yields a PSD declining as \( \text{PSD} \propto \omega^{-2}, \) which is much steeper than the observed power spectra that are generally flat below a frequency of \( \nu \sim 0.1 \text{ Hz} \) and turn into a power-law PSD \( \propto \nu^{-1}. \) The slope \( \Gamma \) has been measured for Cyg X-1 to be \( \Gamma \approx 1 \) (Brinkmann et al. 1974; Nolan et al. 1981), while for GX 339-4 Motch et al. (1983) found \( \Gamma = 1.6 \pm 0.2 \) and Maejima et al. (1984) evaluated \( \Gamma = 1.7. \)

The universality of the external source PSD slope \( \Gamma \approx 2 \) is therefore consistent with the value found for GX 339-4 but seems to argue strongly against this case in the case of Cyg X-1.

For the case of the central source and a density gradient \( p = 1, \) our values of \( \alpha, \) which are in agreement with the ones found by Hua et al. (1997a), are consistent with PSD \( \propto \omega^{-1} \) for the energy channel 2.2–4.6 keV. For lower energies, our simulations predict a slightly flatter PSD, while for higher energies a steeper PSD is found.

An important observational test of the model of Comptonization of centrally injected soft photons into an inhomogeneous hot medium would be a reliable determination of the photon energy–dependence of the PSD power-law slope at high Fourier frequencies.

In the case of Cyg X-1, Nolan et al. (1981) did not find any significant difference in the PSD between the energy channels 12–30 keV and 30–140 keV, while Brinkmann et al. (1974) and Miyamoto & Kitamoto (1989) found evidence for a flattening of the PSD with increasing photon energy, in striking contrast to the prediction of the Comptonization model in both geometries investigated here.

For GX 339-4, no photon energy–dependence of the PSD power-law slope was detected (Motch et al. 1983; Maejima et al. 1984), consistent with external photon injection.

In the case of an external soft photon source, the early time slope \( \mu \) of the single-flare light curves also determines the hard time lag (e.g., eq. [12]) at high Fourier frequencies and high photon energies \( E \gg kT. \) The predicted amount of the resulting time lags is comparable for both scenarios. The deviation from a power-law behavior of the light curves in the case of a central soft photon source, however, leads to a weak systematic deviation from linearity in the \( \Delta t(P) \) dependence at high photon energies, while for external photon injection we predict a strong deviation from linearity at low photon energies. With the parameters used in Hua et al. (1997a) and in the energy range considered in Miyamoto et al. (1988), internal photon injection predicts a linear \( \Delta t(P) \) relation, but for higher photon energy channels a slight flattening of this dependence would be observable if Comptonization of centrally injected soft photons is responsible for the rapidly varying hard X-rays.

Recently Crary et al. (1997) found a dependence \( \Delta t \propto P^{0.8} \) for the time lag of the 50–100 keV photons with respect to the 20–50 keV signal from Cyg X-1. This seems to be a hint toward the central soft photon injection scenario, and the observed approximate dependence of \( \Delta t_{\text{FL}} \propto \ln(1/E_2/E_1) \) (Miyamoto et al. 1988) is more consistent with this geometry.

The available data on the phase lags in GX 339-4 and GS 2023+38 (Miyamoto et al. 1993) do not show an obvious trend and definitely need to be supplemented by more sensitive measurements comparing a larger number of energy channels in order to test the Comptonization models investigated in this paper.

In the discussion of § 3 we have seen that the break frequencies defining the transition between a flat PSD and a steep power law, as well as the transition between the linear dependence of the hard time lag on Fourier period to a constant time lag, are both expected around

\[
\nu_b = \begin{cases} 
\frac{c}{2\pi T_1 R} & \text{for central injection} \\
\frac{c}{4\pi R} & \text{or external injection} 
\end{cases}
\]

for \( \tau \approx 1. \) Since for most Galactic black hole candidates a Thomson depth of 1 to a few is appropriate to model the hard X-ray spectrum via Comptonization of soft photons, equation (15) yields roughly the same size estimate for both injection geometries,

\[
R \sim \begin{cases} 
5 \times 10^{10} (\nu_{b,1}^{-1} - \nu_{b,-1}^{-1})^{-1} \text{ cm for central injection} \\
2.5 \times 10^{10} \nu_{b,-1}^{-1} \text{ cm for external injection} 
\end{cases}
\]

where \( \nu_{b,-1} \) is the break frequency in units of 0.1 Hz. Since the observed flattening frequencies of the PSDs of GBHCs are of the order \( \nu_{\text{break}} \sim 0.1 \text{ Hz} \) and the measurements of Miyamoto et al. (1988, 1993) yield a lower limit on the turnover Fourier period of the hard time lag in the same frequency range, equation (16) reveals a major problem of the Comptonization scenario as an explanation for the rapid aperiodic variability of Galactic black hole candidates in general. If indeed the spectral breaks in the PSD and the hard time lags are the result of inverse Compton scattering as analyzed in the previous sections, then the estimated size of the Comptonizing region is much larger than the expected extent of an accretion-disk corona. This problem is obviously even more severe for the case of external photons impinging on a hot inner-disk torus.

The problem resulting from the break in the power spectrum could be overcome assuming a soft radiation flare of much longer intrinsic duration than the light crossing time of the inner disk region. The real signal would then be the convolution of the soft photon injection light curve with the single-shot light curves considered in this paper. In this case, the turnover frequency of the PSD would yield an estimate for the typical intrinsic duration of the flare, \( \nu_{\text{flare}} \sim 1–10 \text{ s}, \) which is of the order of the radial drift timescale in the inner accretion disk. The convolution of the two fundamental light curves would generally produce an additional break in the PSD at the turnover frequency of the single-shot power spectrum. A second break or a smooth steepening of the PSD around \( \sim 10 \text{ Hz} \) has indeed been observed in the low states of several GBHCs.

We expect, however, that the assumption of a broad flare as opposed to a \( \delta \) shot in time does not shift the turnover of the hard time lag versus Fourier-period curves if it is indeed because of inverse Compton scattering. Therefore, the Fourier-period–dependence of the hard time lag is still a
puzzle in the framework of Comptonization models. As one possible solution to this problem we suggest an intrinsic hardening of the soft input photon spectrum during the evolution of an extended flare, which could be caused by a hot spot in the accretion disk heating up as it spirals inward. This idea will be investigated in detail in a forthcoming paper.

5. SUMMARY AND CONCLUSIONS

We presented a systematic study on the predicted Fourier power spectra and hard time lags resulting from two fundamentally different geometries in Comptonization models for the hard X-ray emission of Galactic black hole candidates: a flaring soft photon source located in the center of the Comptonizing region (e.g., a hot corona) and a soft photon source located outside a hot inner disk region. The different scenarios yield different predictions for the Fourier power spectra and fundamental differences in the photon energy–dependence of the Fourier PSD slope and of the slope describing the hard time lag versus Fourier-period relation.

We found that Comptonization of flares of external soft radiation generally leads to a weak photon energy–dependence of the power spectrum and the hard time lag, with the power spectrum being marginally consistent with the PSD measured for GX 339–4, but significantly steeper than for Cyg X-1. Comptonization of centrally injected soft photon flares, in turn, leads to a significant photon energy–dependence in the PSD power-law slope at high Fourier frequencies and weak, photon energy–dependent deviations from linearity of the hard time lags as a function of Fourier period. With a density gradient \( n(r) \propto r^{-1} \), the simulated PSD for medium-energy X-rays is consistent with the slope observed in Cyg X-1.

The turnover in the Fourier frequency–dependent hard time lag curve poses a severe challenge to Comptonization models for the rapid aperiodic variability in GBHCs in general because of the resulting large size estimates.

Detailed measurements of the energy-dependent properties of the rapid aperiodic GBHC X-ray variability are strongly encouraged. Using the diagnostic tools developed in this paper, they can shed light on the question of the geometry of black hole accretion disks and the location of the Comptonizing region and may serve to rule out a specific geometry or even Comptonization models as mechanisms involved in the rapid aperiodic X-ray variability of Galactic black hole candidates in general.

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