Covariant Harmonic Supergraphity for
$N = 2$ Super Yang–Mills Theories

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Mills theories formulated in the N = 2 harmonic superspace. The covariant harmonic
supergraph technique is then applied to rigorously prove the N = 2 non-renormalization
theorem as well as to compute the holomorphic low-energy action for the N = 2 SU(2)
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for N = 4 SU(2) super Yang-Mills theory.

1 Introduction

Manifest covariance is one of the imperative principles in modern theoretical
physics. It means that any physical theory possessing some symmetries must be
formulated and studied in such a form where all the symmetries are manifest
both at the classical and quantum levels.

The present paper is a brief review of recent progress in constructing the
manifestly covariant quantum formulation for the N = 2 supersymmetric Yang-
Mills (SYM) theories (Buchbinder et al. (1998b,c,d)) on the base of the N = 2
harmonic superspace developed by V.I. Ogievetsky and collaborators (Galperin
et al. (1984)). As we understand now, the harmonic superspace approach is an
elegant and universal setting to formulate general N = 2 SYM theories (Galperin
et al. (1984)) and N = 2 supergravity (Galperin et al. (1987a,b)) in a manifestly
supersymmetric way. Its universality follows simply from the fact that all known
D = 4, N = 2 supersymmetric theories can be naturally realized in harmonic
superspace. In particular, the formulations for N = 2 SYM theories in the con-
tentional N = 2 superspace (Howe et al. (1984)) and in the N = 2 projective
superspace (Lindström et al. (1990)) turn out to be gauge fixed and truncated
versions, respectively, of that in harmonic superspace. It is the N = 2 harmonic
superspace which allows us to realize the general N = 2 SYM theories in terms of
unconstrained superfields. Therefore, just the harmonic superspace approach is
an adequate and convenient base for developing N = 2 supersymmetric quantum
field theory.
The manifestly $N = 2$ supersymmetric Feynman rules in harmonic superspace have been developed by Galperin et al. (1985a,b). One of the basic purposes of the present paper is to extend these rules in order to have manifest gauge invariance along with $N = 2$ supersymmetry. As is well known, the most efficient way to realize such a goal is the background field method.

The paper is organized as follows. In section 2 we review the (harmonic) superspace formulation for the $N = 2$ SYM theories. Section 3 is devoted to the presentation of the background field method for such theories. In section 4 we use the background field formulation developed to prove the $D = 4$, $N = 2$ non-renormalization theorem. The structure of the one-loop effective action is discussed in section 5. Finally, in section 6 we compute the low-energy holomorphic action for the pure $N = 2$ $SU(2)$ SYM theory as well as the non-holomorphic action for the $N = 4$ $SU(2)$ SYM theory.

2 \textbf{N = 2 super Yang-Mills theories in superspace}

We start with a brief review of $N = 2$ SYM theories in superspace.

2.1 \textbf{N = 2 SYM in standard superspace}

The constrained geometry of $N = 2$ super Yang-Mills field is formulated in standard $N = 2$ superspace $\mathbb{R}^{4|8}$ with coordinates $z^M \equiv (x^m, \theta^\alpha, \bar{\theta}^\dot{\alpha})$ in terms of the gauge covariant derivatives

$$D_M \equiv (\partial_m, D^i, \bar{D}^\dot{i}) = D_M + iA_M, \quad A_M = A_M^a(z)T^a$$

satisfying the algebra (Grimm et al. (1978))

$$\{D^i_\alpha, \bar{D}^\dot{j}_\dot{\alpha}\} = -2i\delta^i_j D_{\alpha\dot{\alpha}}, \quad \{D^i_\alpha, D^j_\beta\} = 2i\varepsilon_{\alpha\beta}\varepsilon^{ij}W, \quad \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{ij}\bar{W}.$$ \hspace{1cm} (2)

Here $D_M \equiv (\partial_m, D^i, \bar{D}^\dot{i})$ are the flat covariant derivatives, $T^a$ are the generators of the gauge group. The covariantly chiral strength $W$ satisfies the Bianchi identities

$$\bar{D}_{\dot{\alpha}}W = 0, \quad D^a(i\bar{D}^\dot{i}_j)W = \bar{D}_{\dot{\alpha}(i)}D^a(j)\bar{W}.$$ \hspace{1cm} (3)

The covariant derivatives and a matter superfield multiplet $\varphi(z)$ transform as follows

$$D'_M = e^{i\tau}D_M e^{-i\tau}, \quad \varphi' = e^{i\tau}\varphi$$ \hspace{1cm} (4)

under the gauge group. Here $\tau = \tau^a(z)T^a$, and $\bar{\tau} = \bar{\tau}^a$ are unconstrained real parameters. The set of all transformations (4) is said to form the $\tau$-group.

The gauge invariant action of the $N = 2$ pure SYM theory reads (Grimm et al. (1978))

$$S_{SYM} = \frac{1}{2g^2} \text{tr} \int d^4xd^4\theta W^2 = \frac{1}{2g^2} \text{tr} \int d^4xd^4\bar{\theta} \bar{W}^2.$$ \hspace{1cm} (5)
2.2 \( N = 2 \) SYM in harmonic superspace

To realize the \( N = 2 \) pure SYM theory as a theory of unconstrained dynamical superfields, Galperin et al. (1984) extended the original superspace to \( N = 2 \) harmonic superspace \( \mathbb{R}^{4|8} \times S^2 \). A natural global parametrization of \( S^2 = SU(2)/U(1) \) is that in terms of the harmonic variables \((u_i^-, u_i^+) \in SU(2)\) which parametrize the automorphism group of \( N = 2 \) supersymmetry,
\[
u_i^+ = \varepsilon_{ij} u_j^+, \quad u_i^+ u_i^- = 1 .
\]

Tensor fields over \( S^2 \) are in a one-to-one correspondence with functions on \( SU(2) \) possessing definite harmonic \( U(1) \)-charges. A function \( \Psi(p)(u) \) is said to have the harmonic \( U(1) \)-charge \( p \) if
\[
\Psi(p)(e^{i\varphi} u^+, e^{-i\varphi} u^-) = e^{ip\varphi} \Psi(p)(u^+, u^-) , \quad |e^{i\varphi}| = 1 .
\]

A function \( \Psi(p)(z, u) \) on \( \mathbb{R}^{4|8} \times S^2 \) with \( U(1) \)-charge \( p \) is called a harmonic \( N = 2 \) superfield.

Introducing a new basis of covariant derivatives
\[
D_\alpha^+ = D^i_\alpha u_i^+ , \quad \overline{\Phi}_\alpha^+ = \overline{\Phi}_\alpha^+ u_i^+
\]
the covariant derivative algebra (2) implies
\[
\{D_\alpha^+, D_\beta^+\} = \{\overline{\Phi}_\alpha^+, \overline{\Phi}_\beta^+\} = \{D_\alpha^+, \overline{\Phi}_\beta^+\} = 0
\]
and, hence,
\[
D_\alpha^+ = e^{-i\Omega} D^+ e^{i\Omega} , \quad \overline{\Phi}_\alpha^+ = e^{-i\Omega} \overline{\Phi}_\alpha^+ e^{i\Omega}
\]
for some Lie-algebra valued harmonic superfield \( \Omega = \Omega^a(z, u)T^a \) with vanishing \( U(1) \)-charge, which is called the ‘bridge’.

As a consequence of (8), one can define covariantly analytic superfields constrained by
\[
D_\alpha^+ \phi(p) = \overline{\Phi}_\alpha^+ \phi(p) = 0 , \quad (10)
\]
where \( \phi(p)(z, u) \) carries \( U(1) \)-charge \( p \) and can be represented as follows
\[
\phi(p) = e^{-i\Omega} \phi(p) , \quad D_\alpha^+ \phi(p) = \overline{\Phi}_\alpha^+ \phi(p) = 0 .
\]

The superfield \( \phi(p) \) is, in general, an unconstrained function over an analytic subspace of the harmonic superspace (Galperin et al. (1984)) parametrized by
\[
\zeta \equiv \{x^m_A, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u_i^+\} , \quad \phi(p)(z, u) \equiv \phi(p)(\zeta) ,
\]
where (Galperin et al. (1984))
\[
x^m_A = x^m - 2i \theta^i (\sigma^m \bar{\theta}^i) u^+ u^- , \quad \theta^{+\alpha} = \theta^i u_i^+ , \quad \bar{\theta}^{+\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}} u_i^+ .
\]
That is why such superfields are called analytic.
The analytic subspace (12) is closed under $N = 2$ supersymmetry transformations and real with respect to the generalized conjugation $\gamma \equiv ^\ast$ (Galperin et al. (1984)), where the operation $^\ast$ is defined by

$$
(u_i^+)^\ast = u_i^-, \quad (u_i^-)^\ast = -u_i^+ \quad \Rightarrow \quad (u_i^\pm)^{\ast\ast} = -u_i^\mp .
$$

A remarkable property of this generalized conjugation (called below the ‘smile-conjugation’) is that it allows us to consistently define real analytic superfields with even $U(1)$-charge.

Without loss of generality, the bridge $\Omega$ (9) can be chosen to be real with respect to the smile-conjugation, $\tilde{\Omega} = \Omega$. The bridge possesses a richer gauge freedom than the original $\tau$-group. Its transformation law reads

$$
e^{i\Omega'} = e^{i\lambda}e^{i\Omega}e^{-i\tau}
$$

with an unconstrained analytic gauge parameter $\lambda = \lambda^a(\zeta)T^a$ being real with respect to the smile-conjugation, $\tilde{\lambda}^a = \lambda^a$. The set of all $\lambda$-transformations form the so-called $\lambda$-group (Galperin et al. (1984)). The $\tau$-group acts on $\Phi(p)$ and leaves $\phi(p)$ unchanged while the $\lambda$-group acts only on $\phi(p)$ as follows

$$
\phi'(p) = e^{i\lambda}\phi(p) .
$$

The superfields $\Phi(p)$ and $\phi(p)$ are said to correspond to the $\tau$- and $\lambda$-frames respectively.

The $\lambda$-frame is most useful to work with the covariantly analytic superfields. At the same time, it is the $\lambda$-frame in which a single unconstrained prepotential of the $N = 2$ SYM theory naturally emerges. Let us, first of all, introduce the harmonic derivatives (Galperin et al. (1984))

$$
D^{\pm\pm} = u_i^\pm \frac{\partial}{\partial u_i^\mp} , \quad D^{0} = u_i^+ \frac{\partial}{\partial u_i^+} - u_i^- \frac{\partial}{\partial u_i^-} ,
$$

where $D^{\pm\pm}$ are two independent derivatives on $S^2$, and $D^{0}$ is the operator of $U(1)$ charge, $D^{0}\phi(p) = p\phi(p)$. Operators $D_M \equiv (D_M, D^{++}, D^{--}, D^0)$ form a full set of gauge covariant derivatives in the $\tau$-frame. The $\lambda$-frame is defined by the following transform

$$
D_M \rightarrow \nabla_M = e^{i\Omega}D_M e^{-i\Omega} , \quad \Phi(p) \rightarrow \phi(p) = e^{i\Omega}\Phi(p) \quad \text{(18)}
$$

$$
\nabla_\alpha = D_\alpha^+ , \quad \nabla^{\alpha} = D^{\alpha}_+ , \quad \nabla^{\pm\pm} = D^{\pm\pm} + iV^{\pm\pm} . \quad \text{(19)}
$$

Since $[\nabla^{++}, \nabla^{\alpha}_+] = [\nabla^{++}, \nabla^{\alpha}] = 0$, the connection $V^{++} = V^{++0}T^0$ is a real analytic superfield, $\tilde{V}^{++} = V^{++}$, $D_\alpha^+ V^{++} = \overline{D^{\alpha}_+ V^{++}} = 0$, and its transformation law is

$$
V'^{++} = e^{i\lambda}V^{++}e^{-i\lambda} - i e^{i\lambda}D^{++}e^{-i\lambda} .
$$

The analytic superfield $V^{++}$ turns out to be the single unconstrained prepotential of the pure $N = 2$ SYM theory and all other objects are expressed in
terms of it. In particular, action (5) can be rewritten via $V^{++}$ as follows (Zupnik (1987))

$$S_{\text{SYM}} = \frac{1}{g^2} \text{tr} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \int d^{12}z d^n u \frac{V^{++}(z, u_1) \ldots V^{++}(z, u_n)}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}.$$  \hspace{1cm} (21)

The rules of integration over $SU(2)$ as well as the properties of harmonic distributions were given by Galperin et al. (1984) and Galperin et al. (1985a).

### 2.3 Supersymmetric matter

Harmonic superspace provides us with two possibilities to describe $N = 2$ supersymmetric matter in terms of unconstrained analytic superfields (Galperin et al. (1984)). A charge hypermultiplet, transforming in a complex representation $R_q$ of the gauge group, is described by an unconstrained analytic superfield $q^+(\zeta)$ and its conjugate $\bar{q}^+(\zeta)$ ($q$-hypermultiplet). A neutral hypermultiplet, transforming in a real representation $R_\omega$ of the gauge group, is described by an unconstrained analytic real superfield $\omega(\zeta)$, $\bar{\omega} = \omega$ ($\omega$-hypermultiplet). The matter action reads

$$S_{\text{MAT}} = -\int d\zeta (-4) \bar{q}^+ \nabla^{++} q^+ - \frac{1}{2} \int d\zeta (-4) \nabla^{++} \omega^T \nabla^{++} \omega,$$ \hspace{1cm} (22)

where the integration is carried out over the analytic subspace (12).

### 3 Background field quantization

To quantize the pure $N = 2$ SYM theory, we split $V^{++}$ into background $V^{++}$ and quantum $v^{++}$ parts

$$V^{++} \rightarrow V^{++} + g v^{++}. \hspace{1cm} (23)$$

Then, the original infinitesimal gauge transformations (20) can be realized in two different ways:

(i) background transformations

$$\delta_B V^{++} = -D^{++} \lambda - i[V^{++}, \lambda] = -\nabla^{++} \lambda, \hspace{0.5cm} \delta_B v^{++} = i[\lambda, v^{++}] \hspace{1cm} (24)$$

(ii) quantum transformations

$$\delta_Q V^{++} = 0, \hspace{0.5cm} \delta_Q v^{++} = -\frac{1}{g} \nabla^{++} \lambda - i[v^{++}, \lambda]. \hspace{1cm} (25)$$

It is worth pointing out that the form of the background-quantum splitting (23) and the corresponding background and quantum transformations (24), (25) are much more analogous to the conventional Yang-Mills theory than to the $N = 1$ non-abelian SYM model. Our aim now is to construct an effective action as a gauge-invariant functional of the background superfield $V^{++}$. 
Upon the splitting (23), the classical action (21) takes the form

$$S_{\text{SYM}} = S_{\text{SYM}}[V^{++}] + \frac{1}{4g} \text{tr} \int d\zeta (-4) v^{++} (D^{+})^{2} W_{\lambda} + \Delta S,$$

(26)

where $\Delta S[v^{++}, V^{++}]$ reads

$$\Delta S = -\text{tr} \sum_{n=2}^{\infty} \frac{(-i g)^{n-2}}{n} \int d^{12} z d^{n} u \frac{v^{++}_{\tau}(z, u_1) \ldots v^{++}_{\tau}(z, u_n)}{(u_{11} u_{22}) \ldots (u_{nn} u_{11})}. $$

(27)

Here $W_{\lambda}$ and $v^{++}_{\tau}$ denote the $\lambda$- and $\tau$-transforms of $W$ and $v^{++}$, respectively, with the bridge $\Omega$ corresponding to the background covariant derivatives constructed on the base of the background connection $V^{++}$. The quantum action $\Delta S$ given in (27) depends on $V^{++}$ via the dependence of $v^{++}_{\tau}$ on $\Omega$, the latter being a complicated function of $V^{++}$. Each term in the action (26) is manifestly invariant with respect to the background gauge transformations. The linear in $v^{++}_{\tau}$ term in (26) determines the equations of motion. This term should be dropped when considering the effective action.

To construct the effective action, we can use the Faddeev-Popov Ansatz. Within the framework of the background field method, we should fix only the quantum transformations (25). Let us introduce the gauge fixing function in the form

$$F^{(4)} = \nabla^{++} v^{++}, \quad \delta_{Q} F^{(4)} = \frac{1}{g} \{ \nabla^{++} (\nabla^{++} \lambda + ig [v^{++}, \lambda]) \}. $$

(28)

Eq. (28) leads to the Faddeev-Popov determinant

$$\Delta_{FP}[v^{++}, V^{++}] = \text{Det} \{ \nabla^{++} (\nabla^{++} + ig v^{++}) \}. $$

(29)

To get a path-integral representation for $\Delta_{FP}[v^{++}, V^{++}]$, we introduce two analytic fermionic ghosts $b$ and $c$, in the adjoint representation of the gauge group, and the corresponding ghost action

$$S_{FP}[b, c, v^{++}, V^{++}] = \text{tr} \int d\zeta (-4) b \nabla^{++} (\nabla^{++} c + ig [v^{++}, c]).$$

(30)

As a result, we arrive at the effective action $I_{\text{SYM}}[V^{++}]$ in the form

$$e^{i I_{\text{SYM}}[V^{++}]} = e^{i S_{\text{SYM}}[V^{++}]} \int \mathcal{D}v^{++} \mathcal{D}b \mathcal{D}c \times e^{i (\Delta S[v^{++}, V^{++}] + S_{FP}[b, c, v^{++}, V^{++}])} \delta[F^{(4)} - f^{(4)}],$$

(31)

where $f^{(4)}(\zeta)$ is an external Lie-algebra valued analytic real superfield, and $\delta[F^{(4)}]$ is the proper functional analytic delta-function.

To bring the effective action to a form more adapted for calculations, we average (31) with the weight

$$\Xi[V^{++}] \exp \left\{ \frac{1}{2 \alpha} \text{tr} \int d^{12} z d u_1 d u_2 f^{(4)}_{\tau}(z, u_1) \frac{(u_{11} u_{22})}{(u_{11} u_{22})^3} f^{(4)}_{\tau}(z, u_2) \right\}.$$
Here $\alpha$ is an arbitrary (gauge) parameter, and $f^{(4)}_\tau$ is the $\tau$-transform of $f^{(4)}$. The functional $\Xi[V^{++}]$ is represented as follows (Buchbinder et al. (1998b))

$$
\Xi[V^{++}] = \left(\text{Det}_{(4,0)} \square \right) \hat{\int} D\phi e^{iS_{NK}[\phi,V^{++}]}
$$

$$
S_{NK}[\phi,V^{++}] = \frac{-1}{2} \text{tr} \int d\zeta^{(-4)} \nabla^{++} \phi \nabla^{++} \phi
$$

(33)

with the integration variable $\phi$ being a bosonic real analytic superfield, with its values in the Lie algebra of the gauge group, and presenting itself a Nielsen-Kallosh ghost for the theory. The gauge-covariant operator $\square$ defined by

$$
\square = -\frac{1}{2} (\nabla^+)^4 (\nabla^-)^2 = -\frac{1}{2} (D^+)^4 (\nabla^-)^2
$$

(34)

moves every harmonic superfield into an analytic one, and it is equivalent to the second-order differential operator

$$
\hat{\square}_\tau = e^{-i\Omega} \square e^{i\Omega} = D^\alpha D_m + \frac{i}{2} (D^{+\alpha} W) D^-\alpha + \frac{i}{2} (D^\alpha W) D^{-\alpha}

- \frac{i}{4} (D^{+\alpha} D^-_m W) D^{--} + \frac{i}{8} [D^{+\alpha},D^-_m] W + \frac{1}{2} [\mathcal{W},\mathcal{W}]
$$

(35)

when acting on the covariantly analytic superfields. This operator is said to be the analytic d’Alambertian. The functional $\text{Det}_{(4,0)} \square$, which enters the first line of eq. (33), is defined by the following path integral

$$
\left(\text{Det}_{(4,0)} \square\right)^{-1} = \int D\rho^{(+4)} D\sigma \exp \left\{-i \text{tr} \int d\zeta^{(-4)} \rho^{(+4)} \square \sigma \right\}
$$

(36)

over unconstrained bosonic analytic real superfields $\rho^{(+4)}$ and $\sigma$.

Upon averaging the effective action with the weight (32), for $\alpha = -1$ one gets the following path integral representation (Buchbinder et al. (1998b))

$$
e^{i\mathcal{S}_{\text{SYM}}[V^{++}]} = e^{iS_{\text{SYM}}[V^{++}]} \left(\text{Det}_{(4,0)} \square \right)^{-1} \hat{\int}

\times \int Dv^{++} Db Dc D\phi e^{iS_Q[v^{++},b,c,\phi,V^{++}]},
$$

(37)

where action $S_Q$ reads

$$
S_Q[v^{++},b,c,\phi,V^{++}] = S_2 + S_{\text{int}}
$$

(38)

$$
S_2 = \text{tr} \int d\zeta^{(-4)} \left\{-\frac{1}{2} v^{++} \square v^{++} + b (\nabla^{++})^2 c + \frac{1}{2} \phi (\nabla^{++})^2 \phi \right\}
$$

(39)

$$
S_{\text{int}} = -\text{tr} \int d^{12}z d^n u \sum_{n=3}^\infty \frac{(-ig)^{n-2} v^{++}_n(z,u_1) \ldots v^{++}_n(z,u_n)}{(u_1^+ u_2^+ \ldots u_n^+) (u_1^- u_2^- \ldots u_n^-)} = -ig \text{tr} \int d\zeta^{(-4)} \nabla^{++} b [v^{++},c].
$$

(40)

\footnote{We use the notation $(D^+)^4 = \frac{1}{16} (D^+)^2 (D^+)^2$, $(D^\pm)^2 = D^{\pm\alpha} D^\alpha$, $(D^\alpha)^2 = D^\alpha D^{\alpha}$ and similar notation for the gauge-covariant derivatives.}
Eqs. (37–40) completely determine the structure of the perturbation expansion for calculating the effective action $\Gamma_{\text{SYM}}[V++]$ of the pure $N = 2$ SYM theory in a manifestly supersymmetric and gauge invariant form.

So far we have considered the pure $N = 2$ SYM theory only. In the general case, the classical action contains not only the pure SYM part given by (5) (or, what is equivalent, by (21)), but also the matter action (22). Our previous consideration can be easily extended to the case of the general $N = 2$ SYM theory. The only non-trivial new information, however, is the explicit structure of the matter superpropagators associated with the action (22). They read as follows

$$i < q^+(1) \bar{q}^+(2) > = -\frac{1}{\sq(D_1^+)^4} \left\{ \delta^{12} (z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} e^{i\Omega(1)} e^{-i\Omega(2)} \right\} \sq(D_2^+)^4$$

$$i < \omega(1) \bar{\omega}^T(2) > = \frac{1}{\sq(D_1^+)^4} \left\{ \delta^{12} (z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} e^{i\Omega(1)} e^{-i\Omega(2)} \right\} \sq(D_2^+)^4.$$ 

The Green’s functions (41) and (42) are to be used for loop calculations in the background field approach.

The propagators of the gauge and ghost superfields follow from (39). For the gauge superfield one get

$$i < v^+ + (1) v^+ + (2) >= \frac{1}{\sq(D_1^+)^4} \left\{ \delta^{12} (z_1 - z_2) \delta(-2,2)(u_1, u_2) \right\}$$

with $\delta(-2,2)(u_1, u_2)$ being the proper harmonic delta-function (Galperin et al. (1985a)). The propagator of the Faddeev-Popov ghosts $b$ and $c$ is completely analogous to the $\omega$-hypermultiplet propagator (42). The third ghost $\phi$ contributes at the one-loop level only.

4 The $D = 4$, $N = 2$ non-renormalization theorem

Let us apply the covariant harmonic supergraph technique to analyse the divergence structure of the theory. The result is formulated as the $D = 4$, $N = 2$ non-renormalization theorem: there are no ultraviolet divergences beyond the one-loop level (Howe et al. (1984), Buchbinder et al. (1998c)).

Consider the loop expansion of the effective action within the background field formulation. Then, the effective action is given by vacuum diagrams (that is, diagrams without external lines) with background field dependent propagators and vertices. In our case, the corresponding propagators are defined by eqs. (41–43), and the vertices can be read off from eqs. (22) and (40). It is obvious that any such diagram can be expanded in terms of background fields, and leads to a set of conventional diagrams with an arbitrary number of external legs.
As follows from eqs. (22) and (40), the gauge superfield vertices are given by integrals over the full superspace, while the matter vertices and the Faddeev-Popov ghosts vertices are given by integrals over the analytic subspace. Note, however, that propagators (41–43) contain factors of \((D^+)^4\), which can be used to transform integrals over the analytic subspace into integrals over the full superspace if we make use of the identity

\[
\int d\zeta (-4)(D^+)^4 L = \int d^{12} z \, du \, L .
\]

The cost of doing this is, as a rule, the removal of one of the two \((D^+)^4\)-factors entering each matter and ghost propagator (41,42). There is, however, one special case. Let us consider a vertex with two external \(\omega\)-legs, and start to transform the corresponding integral over the analytic subspace into an integral over the full superspace. To do this, we should remove the factor \((D^+)^4\) from one of the two gauge superfield propagators (43) associated with this vertex. As a result of transforming all integrals over the analytic subspace into integrals over the full superspace, each of the remaining propagators will contain, at most, one factor of \((D^+)^4\). Thus, any supergraph contributing to the effective action is given in terms of the integrals over the full \(N = 2\) harmonic superspace. Since this conclusion is true for each conventional supergraph in the expansion of a given background field supergraph, we see that an arbitrary background field supergraph is also given by integrals over the full \(N = 2\) harmonic superspace. This is in complete analogy with \(N = 1\) supersymmetric field theories.

Once we have constructed the supergraphs with all vertices integrated over the full \(N = 2\) harmonic superspace, we can perform all but one of the integrals over the \(\theta\)'s, step by step and loop by loop, due to the spinor delta-functions \(\delta^8(\theta_i - \theta_j)\) contained in the propagators (41–43). To do this, we remove the \((D^+)^4\)-factors acting on the spinor delta-functions in the propagators by making an integration by parts. This allows one to obtain spinor delta-functions without \((D^+)^4\)-factors. One can then perform the integrals over the \(\theta\)'s. We note that in the process of integration by parts, some of the \((D^+)^4\)-factors can act on the external legs of the supergraph. To obtain a non-zero result in the case of an \(L\)-loop supergraph, we should remove \(2L\) factors of \((D^+)^4\) attached to some of the propagators using the identity

\[
\delta^8(\theta_1 - \theta_2)(D^1_1)^4(D^2_2)^4 \delta^8(\theta_1 - \theta_2) = (u^+_1 u^+_2)^4 \delta^8(\theta_1 - \theta_2) .
\]

Thus, any supergraph contributing to the effective action is given by a single integral over \(d^8\theta\).

Now, it is not difficult to calculate the superficial degree of divergence for the theory under consideration. Let us consider an \(L\)-loop supergraph \(G\) with \(P\) propagators, \(N_{\text{MAT}}\) external matter legs and an arbitrary number of gauge superfield external legs. We denote by \(N_D\) the number of spinor covariant derivatives acting on the external legs as a result of integration by parts in the process of
transforming the contributions to a single integral over $d^8\theta$. Then, the super-

dicial degree of divergence $\omega(G)$ of the supergraph $G$ turns out to be (Buchbinder et al. (1998c))

$$\omega(G) = -N_{\text{MAT}} - \frac{1}{2} N_D.$$  

We see immediately that all supergraphs with external matter legs are automatically finite. As to supergraphs with pure gauge superfield legs, they are clearly finite only if some non-zero number of spinor covariant derivatives acts on the external legs. Let us now show that this is always the case beyond one loop.

The Feynman rules for $N = 2$ supersymmetric field theories in the harmonic superspace approach have been formulated in the $\lambda$-frame, where the propagators are given by eqs. (41–43). As we have noticed, all vertices in the background field supergraphs, including the vertices of matter and Faddeev-Popov ghosts superfields, can be given in a form containing integrals over the full $N = 2$ harmonic superspace only. To be more precise, this property is stipulated by the identity in the $\lambda$-frame

$$(D^+)^4 \tilde{\square} = \tilde{\square} (D^+)^4.$$  

This identity allows one to operate with factors $(D^+)^4$ as in case without background field, and use them to transform the integrals over the analytic subspace into integrals over the full superspace directly in background field supergraphs. Let us consider the structure of the propagators in the $\lambda$-frame (41–43). The background field $V^{++}$ enters these propagators via both $\tilde{\square}$ and the background bridge $\Omega$. The form of the propagators (41–43) has one drawback: if we use this form, we can not say how many spinor derivatives act on the external legs since the explicit dependence of $\Omega$ on the background field is rather complicated. To clarify the situation when a number of spinor derivatives act on external legs, we use a completely new step (in comparison with the conventional harmonic supergraph approach developed by Galperin et al. (1985a,b)) and transform the supergraph to the $\tau$-frame (after restoring the full superspace measure at the matter and ghost vertices). The propagators in the $\tau$-frame are given by (Buchbinder et al. (1998c))

$$i < q^\tau_1(1) \bar{q}^\tau_2(2) > = -\frac{1}{\tilde{\square}_{\tau}} (D^+_{\tau})^4 \left\{ \delta^{12}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \right\} (D^+_{\tau})^4$$

$$i < \omega_\tau(1) \omega^T_\tau(2) > = \frac{1}{\tilde{\square}_{\tau}} (D^+_{\tau})^4 \left\{ \delta^{12}(z_1 - z_2) \frac{(u_1 u_2)}{(u_1^+ u_2^+)^3} \right\} (D^+_{\tau})^4$$

$$i < v^{++}_\tau(1) v^{++}_\tau(2) > = \frac{1}{\tilde{\square}_{\tau}} (D^+_{\tau})^4 \left\{ \delta^{12}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2) \right\}.$$  

They contain, at most, one factor of $(D^+)^4$ after restoring the full superspace measure at the matter and ghost vertices. The essential feature of these propagators is that they contain the background field $V^{++}$ only via the $\tilde{\square}_{\tau}$ and $D^+_{\tau}$-factors; that is, only via the $u$-independent connections $A_M$ (1). But all connections $A_M$ contain at least one spinor covariant derivative acting on the
background superfield $V^{++}$ (Galperin et al. (1984)). Therefore, if we expand any background field supergraph in the background superfield $V^{++}$, we see that each external leg must contain at least one spinor covariant derivative. Thus, the number $N_D$ in eq. (46) must be greater than or equal to one. As a consequence, $\omega(G) < 0$ and, hence, all supergraphs are ultravioletly finite beyond the one-loop level. This completes the proof of the non-renormalization theorem.

5 The one-loop effective action

As is clear from the above analysis, the one-loop effective action requires a separate investigation. In what follows, we restrict our attention to the part $\Gamma[V^{++}]$ of effective action, which depends on the gauge superfield only. It is $\Gamma[V^{++}]$ which (i) determines the one-loop ultraviolet divergences; (ii) constitutes the effective dynamics in the Coulomb branch of $N = 2$ SYM theories.

It follows from eqs. (22,37,39) that the one-loop effective action $\Gamma^{(1)}[V^{++}]$ of the general $N = 2$ SYM theory reads

$$\Gamma^{(1)}[V^{++}] = \frac{i}{2} \text{Tr}_{(2,2)} \ln \Phi - \frac{i}{2} \text{Tr}_{(4,0)} \ln \Phi - \frac{i}{2} \text{Tr}_{ad} \ln(\nabla^{++})^2 + i \text{Tr} R_q \ln(\nabla^{++})^2. \quad (49)$$

Here the contribution in the first line comes not only from the overall factor in (37), but also from the gauge superfield,

$$\left(\text{Det}_{(2,2)} \Phi\right)^{-\frac{1}{2}} = \int Dv^{++} \exp \left\{ -\frac{i}{2} \text{tr} \int d\zeta^{(-4)} v^{++} \Phi v^{++} \right\}. \quad (50)$$

The second line in (49) represents the joint contribution from the Faddeev-Popov ghosts $b, c$ and the third ghost $\phi$. Finally, the third line includes the contributions from the matter $q$- and $\omega$-hypermultiplets.

The joint contribution of the Faddeev-Popov ghosts and the third ghost differs only in sign from that of an $\omega$-hypermultiplet in the adjoint representation of the gauge group. In case of the $N = 4$ SYM theory realized in the $N = 2$ harmonic superspace, the matter sector is formed by a single $\omega$–hypermultiplet in the adjoint representation (Galperin et al. (1985b)), and the classical action reads

$$S_{N=4}^{SYM} = \frac{1}{2g^2} \text{tr} \int d^4x d^4\theta \mathcal{W}^2 - \frac{1}{2g^2} \text{tr} \int d\zeta^{(-4)} \nabla^{++} \omega \nabla^{++} \omega. \quad (51)$$

Therefore, the corresponding one-loop effective action is given by the first line of eq. (49),

$$\Gamma^{(1)}_{N=4}[V^{++}] = \frac{i}{2} \text{Tr}_{(2,2)} \ln \Phi - \frac{i}{2} \text{Tr}_{(4,0)} \ln \Phi. \quad (52)$$
It is the contributions in the second and third lines of (49) which (i) are responsible for all the ultraviolet divergences of the theory and (ii) generate the low-energy holomorphic action (see Buchbinder et al. (1998c) for more detail). By now, we have a well elaborated perturbative scheme to compute such quantum hypermultiplet corrections (Buchbinder et al. (1997), Buchbinder et al. (1998a)). As concerns the $N = 4$ SYM effective action (52), it is free of ultraviolet divergences, but its calculation turns out to be a nontrivial technical problem. The point is that the one-loop supergraphs contributing to $\Gamma^{(1)}_{N=4}[V^{++}]$ in the harmonic superspace approach contain coinciding harmonic singularities, that is harmonic distributions at coinciding points. The problem of coinciding harmonic singularities in the framework of harmonic supergraph Feynman rules was first discussed by Galperin et al. (1987c). Such singularities have no physical origin, in contrast to ultraviolet divergences. They can appear only at intermediate stages of calculation and should cancel each other in the final expressions for physical quantities. The origin of this problem is an infinite number of internal degrees of freedom associated with the bosonic internal coordinates.

To get rid of the one-loop coinciding harmonic singularities, Buchbinder et al. (1998b) introduced, as is generally accepted in quantum field theory, some regularization of harmonic distributions. Unfortunately, this regularization proved to be unsuccessful; its use led us to the wrong conclusion $\Gamma^{(1)}_{N=4}[V^{++}] = 0$. In a sense, the situation in hand is similar to that with the well-known supersymmetric regularization via dimensional reduction which leads to obstacles at higher loops. The harmonic regularization we used turned out to be improper already at the one-loop level.

We would like to emphasize that the problem of coinciding harmonic singularities is associated only with perturbative calculations of the effective action and has no direct relation to the $N = 2$ background field method itself. The problem of coinciding harmonic singularities has been solved by Buchbinder et al. (1998d) for a special $N = 2$ SYM background

$$D^{\alpha(i)}\mathcal{D}^{\alpha(j)}\mathcal{W} = 0. \quad (53)$$

In this case the effective action can be equivalently represented in the form

$$\exp \left\{ i \Gamma^{(1)}_{N=4} \right\} = \frac{\int DG^{++} \exp \left\{ -\frac{1}{2} \text{tr} \int d\zeta(-4) G^{++} \Box G^{++} \right\}}{\int DG^{++} \exp \left\{ \frac{1}{2} \text{tr} \int d\zeta(-4) G^{++} G^{++} \right\}} \quad (54)$$

where the analytic integration variable $G^{++}$ is constrained by

$$\nabla^{++} G^{++} = 0. \quad (55)$$

Representation (54) allows us to perturbatively compute $\Gamma^{(1)}_{N=4}$. Moreover, it can to used to prove equivalence of the $N = 2$ covariant supergraph technique to the famous $N = 1$ background field formulation for the $N = 4$ SYM (Grisaru et al. (1979)), when the lowest $N = 1$ superspace component of the $N = 2$ vector multiplet is switched off (Buchbinder et al. (1998d)).
6 Low-energy effective action

In the Coulomb branch of the $N = 2$ SYM theory, the matter hypermultiplets are integrated out and the gauge superfield lies along a flat direction of the $N = 2$ SYM potential. If the gauge group is $SU(2)$, only the $U(1)$ gauge symmetry survives, upon the spontaneous breakdown of $SU(2)$, and the gauge superfield $V^{++} = V^{++a}T^a$ ($T^a = \sqrt{2}\sigma^a$, $a = 1, 2, 3$) takes the form

$$V^{++} = V^{++3}T^3 \equiv V^{++}T^3. \quad (56)$$

Here $V^{++}$ consists of two parts, $V^{++} = V_0^{++} + V_1^{++}$, where $V_0^{++}$ corresponds to a constant strength $W_0 = \text{const}$, and $V_1^{++}$ is an abelian gauge superfield. The presence of $V_0^{++}$ leads to the appearance of mass $|W_0|^2$ for matter multiplets (see Buchbinder et al. (1997)).

Since the effective action $\Gamma[V^{++}]$ is gauge invariant, it presents itself a functional of the chiral strength $W$ and its conjugate $\bar{W}$. Assuming the validity of momentum expansion, one can present the effective action $\Gamma[W, \bar{W}]$ in the form

$$\Gamma[W, \bar{W}] = \left( \int d^4x d^4\theta L^{(c)}_{\text{eff}} + \text{c.c.} \right) + \int d^4x d^8\theta L_{\text{eff}}. \quad (57)$$

Here the chiral effective Lagrangian $L^{(c)}_{\text{eff}}$ is a local function of $W$ and its space-time derivatives, $L^{(c)}_{\text{eff}} = F(W) + \ldots$, and the higher-derivative effective Lagrangian $L_{\text{eff}}$ is a real function of $W$, $\bar{W}$ and their covariant derivatives, $L_{\text{eff}} = H(W, \bar{W}) + \ldots$

At the one-loop level, it is the Faddeev-Popov ghosts, the third ghost and the matter hypermultiplets which contribute to $F(W)$. As concerns the quantum correction in the first line of (49), it contributes to the higher-derivative action $H(W, \bar{W})$. A general analysis of covariant harmonic supergraphs given by Buchbinder et al. (1998c) shows that the holomorphic action $F(W)$ is completely generated by the one-loop contribution. Another consequence of such an analysis is that there is no two-loop contribution to $H(W, \bar{W})$.

The covariant harmonic supergraph technique allows us to easily compute the holomorphic effective action. Let us restrict, for simplicity, our consideration to the case of the pure $N = 2$, $SU(2)$ SYM theory. If we are interested in the low-energy holomorphic action, it is proper to use the following approximation

$$\Gamma^{(1)}_{SU(2)}[V^{++}] \approx -\Gamma_\phi[V^{++}] \quad (58)$$

with $\Gamma_\phi[V^{++}]$ the effective action of a real $\omega$-hypermultiplet in the adjoint representation of $SU(2)$ coupled to the external gauge superfield $V^{++}$:

$$e^{i\Gamma_\phi[V^{++}]} = \int D\phi \exp \left\{ -\frac{i}{2} \text{tr} \int d\zeta (-4) \nabla^{++}\phi \nabla^{++}\phi \right\}$$

$$\phi = \phi^a T^a, \quad \nabla^{++}\phi = D^{++}\phi + i[V^{++}, \phi]. \quad (59)$$
Since the gauge superfield has the form (56), $\phi^3$ completely decouples. Unifying $\phi^1$ and $\phi^2$ into the complex $\omega$-hypermultiplet $\omega = \phi^1 - i\phi^2$, we observe
\[
\nabla^{++} \omega = D^{++} \omega + i\sqrt{2} V^{++} \omega ,
\]
(hence the $U(1)$-charge of $\omega$ is $e = \sqrt{2}$). As was shown by Buchbinder et al. (1997), the effective actions of the charged complex $\omega$-hypermultiplet and the charged $q$-hypermultiplet, interacting with background $U(1)$ gauge superfield $V^{++}$, are related by $I_\omega[V^{++}] = 2 I_q[V^{++}]$ and the leading contribution to $I_q[V^{++}]$ in the massive theory is given by
\[
I_q[V^{++}] = \int d^4x d^4 \theta \ F(W) + c.c. , \quad F(W) = -\frac{e^2}{64\pi^2} W^2 \ln \frac{W^2}{\Lambda^2} . \quad (61)
\]
Here $e$ is the charge of $q^+$ (it coincides with the charge of $\omega$ in the above correspondence), $\Lambda$ is the renormalization scale. Since in our case $e = \sqrt{2}$, from eqs. (58,61) we finally obtain the perturbative holomorphic of the $N = 2$ $SU(2)$ SYM theory
\[
I^{(1)}_{SU(2)}(W) = \frac{1}{16\pi^2} W^2 \ln \frac{W^2}{\Lambda^2} . \quad (62)
\]
This is exactly Seiberg’s low-energy effective action (Seiberg (1988)) found by integrating the $U(1)$ global anomaly and using the component analysis.

Let us finally turn to the $N = 4$ $SU(2)$ SYM theory (51). Here the non-holomorphic action $H(W, \overline{W})$ constitutes the leading low-energy quantum correction. Its calculation is based on the representation (54). Using the technique developed in our paper (Buchbinder et al. (1998d)) and under additional restrictions on the background superfields, one can represent the effective action $I^{(1)}_{N=4}$ by a path integral over an unconstrained $N = 1$ complex superfield $V$ and its conjugate
\[
\exp\{i I^{(1)}_{N=4}\} = \int \mathcal{D}\overline{V} \mathcal{D}V \exp\left\{\frac{i}{2} \text{tr} \int d^8 z \Delta V \right\}
\]
\[
\Delta = D^a D_a - e W^\alpha D_\alpha + e \overline{W}_\alpha D^a + e^2 |\phi|^2 . \quad (63)
\]
Here $\phi$ and $W_\alpha$ are the $N = 1$ projections of $W$: $\phi = W|$, $2iW_\alpha = D^a W|$. Being rewritten in terms of the $N = 1$ projections, the leading non-holomorphic correction to $I^{(1)}_{N=4}$ takes the form
\[
\int d^{12}z \ H(W, \overline{W}) = \int d^8 z W^\alpha W_\alpha \overline{W} \partial^4 H(\phi, \overline{\phi}) + \cdots \quad (64)
\]
To calculate $\partial^4 H(\phi, \overline{\phi})/\partial \phi^2 \partial \overline{\phi}^2$, we use a superfield proper-time introducing the Schwinger kernel for the operator $\Delta$ (63). Then one gets $\partial^4 H(\phi, \overline{\phi})/\partial \phi^2 \partial \overline{\phi}^2 = (4\pi \phi \overline{\phi})^{-2}$. One can easily find a general solution to this equation. Since the effective action of the $N = 4$ SYM theory should be scale and chiral invariant, we finally get
\[
H^{(1)}_{N=4}(W, \overline{W}) = \frac{1}{4(4\pi)^2} \ln \frac{W^2}{\Lambda^2} \ln \frac{\overline{W}^2}{\overline{\Lambda}^2} . \quad (65)
\]
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The details can be found in our work (Buchbinder et al. (1998d)). This action was independently computed by Periwal et al. (1998) and Gonzalez et al. (1998). The possibility of quantum corrections of the form \((65)\) in the effective action for the \(N = 4\) SU(2) SYM theory was first argued by Dine et al. (1997).

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