Energy dependence of pion interferometry scales in ultra-relativistic heavy ion collisions

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Abstract

A study of energy behavior of the pion spectra and interferometry scales is carried out for the top SPS, RHIC and LHC energies within the hydrokinetic approach. The latter allows one to describe evolution of quark-gluon and hadron matter as well as continuous particle emission from the fluid in agreement with the underlying kinetic equations. The main mechanisms that lead to the paradoxical, at first sight, behavior of the interferometry scales, are exposed. In particular, a slow decrease and apparent saturation of $R_{\text{out}}/R_{\text{side}}$ ratio with an energy growth happens due to a strengthening of positive correlations between space and time positions of pions emitted at the radial periphery of the system. Such an effect is a consequence of the two factors: a developing of the pre-thermal collective transverse flows and an increase of the initial energy density in the fireball.

Keywords: A+A collisions, SPS, RHIC, LHC, Interferometry radii, HBT puzzle, Transverse spectra, Pre-thermal flows, Energy density

1. Introduction

The pion interferometry method, which is based on the Bose-Einstein or Fermi-Dirac interference of identical particles, has been proposed first in Refs. \cite{1, 2} for measurements of the geometrical sizes and shapes of the interaction region in hadronic collisions. Then it has been developed in Refs. \cite{3, 4, 5, 6} as a tool for a study of rapidly expanding fireballs formed in ultra-relativistic heavy ion collisions. Despite the extremely small sizes of such systems - the order of the value is around $10^{-14}$ m - they have, nevertheless, a pronounced inhomogeneous structure. The generalized treatment of the interferometry measurements asserts that the measured scales - the interferometry radii - are associated just with the homogeneity lengths in the system \cite{7}. Only in the very particular case of a finite homogeneous system such lengths correspond to the total geometrical sizes, but normally they are smaller than the latter. The interferometry scanning of femto-systems at various total momenta of pion pairs allows one to analyze the homogeneity lengths related to different space-time regions of the expanding fireball \cite{3, 8}. An understanding of interferometry in terms of the homogeneity length, as opposed to the simple-minded geometrical picture, provides explanation to some, at first sight paradoxical, results at RHIC.

Naively it was expected that when the energy of colliding nuclei increases, the pion interferometry volume $V_{\text{int}}$ - product of the interferometry radii in three orthogonal directions - rises at the same maximal centrality for Pb+Pb and Au+Au collisions just proportionally to $dN_{\pi}/dy$. However, when experiments at RHIC starts, an increase of the interferometry volume with energy turns out to be essentially flatter then the proportionality low. This is one of the components of so-called RHIC HBT puzzle \cite{9}.

Another item of the HBT puzzle is an unexpected RHIC result as for the ratio of the two transverse interferometry radii: one that is measured in the direction of the sum of transverse momenta of the
(pion) pair, \( R_{\text{out}} \), and another, \( R_{\text{side}} \), - orthogonally to it and the beam axis. While the latter is associated with transverse homogeneity length, the former includes besides that also additional contributions, in particular, the one which is related to a duration of the pion emission. Since the lifetime of the systems obviously should grow with collision energy, if it is accompanied by an increase of the initial energy density and/or by a softening of the equation of state due to phase transition between hadron matter and quark-gluon plasma (QGP), then the duration of pion emission should grow with energy and \( R_{\text{out}}/R_{\text{side}} \) ratio could increase. A very large value of the ratio was predicted as a signal of the QGP formation. The RHIC experiments brought the result: the ratio \( R_{\text{out}}/R_{\text{side}} \approx 1 \) and similar or even smaller than at SPS.

In this letter we analyze the pion spectra and the interferometry scales as depending on the energy of central Pb+Pb and Au+Au collisions supposing the formation of QGP. The aims of the study are to describe these pion observables at the top SPS and RHIC energies and make predictions for LHC energies, also to understand the physical mechanisms responsible for the peculiarities of energy dependence of the interferometry radii, in particular, the \( R_{\text{out}}/R_{\text{side}} \) ratio. Numerical calculations are based on the HydroKinetic Model (HKM) that allows one to describe the evolution of quark-gluon and hadron matter and continuous particle emission from the fluid in agreement with the underlying kinetic equations.

2. Hydro-kinetic approach to \( A+A \) collisions

Let us briefly describe the main features of the HKM. It incorporates hydrodynamical expansion of the systems formed in \( A+A \) collisions and their dynamical decoupling described by escape probabilities. The method corresponds to a generalized relaxation time approximation for the particle emission functions applied to inhomogeneous systems expanding into vacuum according to the Boltzmann equations. This method also allows one to describe the viscous effects at the hadronic stage of the evolution. The basic hydro-kinetic code, proposed in [2], is modified now to include decays of resonances into expanding hadronic non-chemically equilibrated system and, based on the resulting composition of the hadron-resonance gas at each space-time point, to calculate the equation of state (EoS) in a vicinity of this point. The obtained local EoS allows one to determine the further evolution of the considered fluid elements. The complete picture of the physical processes in central Pb + Pb and Au + Au collisions encoded in calculations is the following.

2.1. Initial conditions

Our results are all related to the central rapidity slice where we use the boost-invariant Bjorken-like initial condition. We consider the proper time of thermalization of quark-gluon matter as the minimal one discussed in the literature, \( \tau_0 = 1 \text{ fm/c} \). The initial energy density in the transverse plane is supposed to be Glauber-like, i.e. proportional to the participant nucleon density for Pb+Pb (SPS) and Au+Au (RHIC, LHC) collisions with zero impact parameter. The height of the distribution - the maximal initial energy density - \( \epsilon(r=0) = \epsilon_0 \) is the fitting parameter. From analysis of pion transverse spectra we choose it for the top SPS energy to be \( \epsilon_0 = 9 \text{ GeV/fm}^3 \), for the top RHIC energy \( \epsilon_0 = 16.5 \text{ GeV/fm}^3 \). The brackets \( < ... > \) correspond to mean value over the distribution associated with the Glauber transverse profile. We also demonstrate results at \( \epsilon_0 = 40 \text{ GeV/fm}^3 \) and \( \epsilon_0 = 60 \text{ GeV/fm}^3 \) that probably can correspond to the LHC energies of 4 and 5.6 ATeV. As for the latter, according to [14], at the top LHC energy \( \epsilon_{\text{max}} \) re-calculated to the time 1 fm/c is 0.07 * 700 = 49 GeV/fm³. We suppose that soon after thermalization the matter created in A+A collision at RHIC energies is in the quark-gluon plasma (QGP) state. Also, at time \( \tau_i \), there is peripheral region with relatively small initial energy densities: \( \epsilon(r) < 0.5 \text{ GeV/fm}^3 \). This part of the matter ("corona") does not transform into QGP and have no chance to be involved in thermalization process [15]. By itself the corona gives no essential contribution to the hadron spectra. This part of the matter, probably, is not thermalized. One should consider it separately from the thermal bulk of the matter and should not include in hydrodynamic evolution.

At the time of thermalization, \( \tau_0 = 1 \text{ fm/c} \), the system already has developed collective transverse velocities [16, 17]. The initial transverse rapidity profile is supposed to be linear in radius \( r_T \):

\[
y_T = \frac{\alpha}{R_T} \text{ where } R_T = \sqrt{<r_T^2>},
\]

Here \( \alpha \) is the second fitting parameter. Note that the fitting parameter \( \alpha \) should include also a pos-
rive correction for underestimated resulting transverse flow since in this work we do not take directly into account the viscosity effects [15] neither at QGP stage nor at hadronic one. In formalism of HKM [12] the viscosity effects at hadronic stage are incorporated in the mechanisms of the back-reaction of particle emission on hydrodynamic evolution which we ignore in current calculations. Since the corrections to transverse flows which depend on unknown viscosity coefficients are unknown, we use fitting parameter $\alpha$ to describe the "additional unknown portion" of flows, caused both factors: by a developing of the pre-thermal flows and the viscosity effects in quark-gluon plasma. The best fits of the pion transverse spectra at SPS and RHIC are provided at $\alpha = 0.28$ $(v_T = 0.178)$ for SPS energies and $\alpha = 0.28$ $(v_T = 0.25)$ for RHIC ones. The latter value we use also for LHC energies aiming to analyze just influence of energy density increase. From a fitting of the pion spectra we can conclude that an "additional portion" of flow is bigger at RHIC than at SPS where the parameter $\alpha$ is compatible with that free streaming of partons brings [16, 17]. One can conclude, thus, that longer increase. From a fitting of the pion spectra we can conclude that an "additional portion" of flow is bigger at RHIC than at SPS where the parameter $\alpha$ is compatible with that free streaming of partons brings [16, 17]. One can conclude, thus, that longer viscous QGP evolution at RHIC has a bigger impact for a developing of transverse flows than at the SPS energies.

2.2. Equation of state

Following Ref. [12] we use at high temperatures the EoS [19] adjusted to the QCD lattice data with the baryonic chemical potential $\mu_B = 0$ and matched with chemically equilibrated multi-component hadron resonance gas at $T = 175$ MeV. Such an EoS could be a good approximation for the RHIC and LHC energies; as for the SPS energies we utilize it just to demonstrate the energy-dependent mechanism of formation of the space-time scales. We suppose the chemical freeze-out for the hadron gas at $T_{ch} = 165$ MeV [20]. It guarantees us the correct particle number ratios for all quasi-stable particles (here we calculate only pion observables) at least for RHIC. Below $T_{ch}$ a composition of the hadron gas is changed only due to resonance decays into expanding fluid. We include 359 hadron states made of u, d, s quarks with masses up to 2.6 GeV. The EoS in this non-chemically equilibrated system depends now on particle number densities $n_i$ of all the 359 particle species $i$: $p = p(\epsilon, \{n_i\})$. Since the energy densities in expanding system do not directly correlate with resonance decays, all the variables in the EoS depend on space-time points and so an evaluation of the EoS is incorporated in the hydrodynamic code. We calculate the EoS below $T_{ch}$ in the Boltzmann approximation of ideal multi-component hadron gas.

2.3. Evolution

At the temperatures higher than $T_{ch}$ the hydrodynamic evolution is related to the quark-gluon and hadron phases which are in chemical equilibrium with zero baryonic chemical potential. The evolution is described by the conservation law for the energy-momentum tensor of perfect fluid:

$$\partial_\mu T^{\mu\nu}(x) = 0$$

(2)

At $T < T_{ch} = 165$ MeV the system evolves as non-chemically equilibrated hadronic gas. The concept of the chemical freeze-out implies that afterwards only elastic collisions and resonance decays take place because of relatively small densities allied with a fast rate of expansion at the last stage. Thus, in addition to (2), the equations accounting the particle number conservation and resonance decays are added. If one neglects the thermal motion of heavy resonances the equations for particle densities $n_i(x)$ take the form:

$$\partial_\mu (n_i(x) u^\mu(x)) = -\Gamma_i n_i(x) + \sum_j b_{ij} \Gamma_j n_j(x)$$

(3)

where $b_{ij} = B_{ij} N_{ij}$ denote the average number of $i$th particles coming from arbitrary decay of $j$th resonance, $B_{ij} = \Gamma_{ij}/\Gamma_{j, tot}$ is branching ratio, $N_{ij}$ is a number of $i$th particles produced in $j \rightarrow i$ decay channel. We also can account for recombination in the processes of resonance decays into expanding medium just by utilizing the effective decay width $\Gamma_{i, eff} = \gamma \Gamma_i$. We use $\gamma = 0.75$ supposing thus that near 30% of resonances are recombinating during the evolution. All equations (2) and 359 equations (3) are solved simultaneously with calculation of the EoS, $p(x) = p(\epsilon(x), \{n_i(x)\})$, at each point $x$.

2.4. System’s decoupling and spectra formation

During the matter evolution, in fact, at $T \leq T_{ch}$, hadrons continuously leave the system. Such a process is described by means of the emission function $S(x, p)$ which is expressed for pions through the gain term, $G_\pi(x, p)$, in Boltzmann equations and the escape probabilities $\mathcal{P}_\pi(x, p) = \exp(-\int ds R_{\pi + h}(s, r + p/\rho(s-t), p))$: $S_\pi(x, p) =$
$G_\pi(x,p)P_\pi(x,p)$ \cite{11, 12}. For pion emission in relaxation time approximation $G_\pi \approx f_\pi R_{\pi+h} + G_{H-\pi}$ where $f_\pi(x,p)$ is the pion Bose-Einstein phase-space distribution, $R_{\pi+h}(x,p)$ is the total collision rate of the pion, carrying momentum $p$, with all the hadrons $h$ in the system in a vicinity of point $x$, the term $G_{H-\pi}$ describes an inflow of the pions into phase-space point $(x,p)$ due to the resonance decays. It is calculated according to the kinematics of decays with simplification that the spectral function of the resonance $H$ is $\delta(p^2 - \langle m_H \rangle^2)$. The cross-sections in the hadronic gas, that determine via the collision rate $R_{\pi+h}$ the escape probabilities $P(x,p)$ and emission function $S(x,p)$, are calculated in accordance with the UrQMD method \cite{21}. The spectra and correlation functions are found from the emission function $S$ in the standard way (see, e.g., \cite{11}).

3. Results and conclusions

The pion emission function per unit (central) rapidity, integrated over azimuthal angular and transverse momenta, is presented in Fig. 1 for the top SPS, RHIC and LHC energies as a function of transverse radius $r$ and proper time $\tau$. The two fitting parameters $\epsilon_0$ and $\langle \tau T \rangle$ are fixed as discussed above and marked in figures. The pion transverse momentum spectrum, its slope as well as the absolute value, and the interferometry radii, including $R_{out}$ to $R_{side}$ ratio, are in a good agreement with the experimental data both for the top SPS and RHIC energies.

As one can see particle emission lasts a total lifetime of the fireballs; in the central part, $r \approx 0$, the duration is half of the lifetime. Nevertheless, according to very recent results \cite{12, 22}, the Landau/Cooper-Frye presentation of sudden freeze-out could be applied in a generalized form accounting for momentum dependence of the freeze-out hypersurface $\sigma_\mu(x)$; now $\sigma_\mu(x)$ corresponds to the maximum of emission function $S(t_\mu(r,p), r, p)$ at fixed momentum $p$ in an appropriate region of $r$. This finding gives one possibility to keep in mind the known results based on the Cooper-Frye formalism, applying them to a surface of the maximal emission for given $p$. Then the typical features of the energy dependence can be understood as follows. The inverse of the spectra slopes, $T_{eff}$, grows with energy, since as one sees from the emission functions, the duration of expansion increases with initial energy density and, therefore, the fluid element gets more transverse collective velocities $v_T$ when reach a decoupling energy densities. Therefore the blue shift of the spectra becomes stronger. A rise of the transverse collective flow with energy leads to some compensation of an increase of $R_{side}$: qualitatively the homogeneity length at decoupling stage is $R_{side} = R_{Geom}/\sqrt{1 + (\tau T/m_H)^2}$, (see, e.g., \cite{8}). So, despite the significant increase of the transverse emission region, $R_{Geom}$, seen in Fig.1, a magnification of collective flow partially compensates this. It leads to only a moderate increase of the $R_{side}$ with energy. Since the temperatures in the regions of the maximal emission decrease very slowly when initial energy density grows (e.g., the temperatures for SPS, RHIC and LHC are correspondingly 0.105, 0.103 and 0.95 MeV for $p_T = 0.3$ GeV/c) the $R_{long} \sim \tau T/m_H$ \cite{5} grows proportionally to an increase of the proper time associated with the hypersurface $\sigma_{p_T}(x)$ of maximal emission. As we see from Fig. 1 this time grows quite moderate with the collision energy.

A non-trivial result concerns the energy behavior of the $R_{out}/R_{side}$ ratio. It slowly drops when energy grows and apparently is saturated at fairly high energies at the value close to unity (Fig.1). To clarify the physical reason of it let us make a simple half-quantitative analysis. As one can see in Fig. 1, the hypersurface of the maximal emission can be approximated as consisting of two parts: the "volume" emission ($V$) at $\tau \approx const$ and "surface" emission ($S$). A similar picture within the Cooper-Frye prescription, which generalizes the blast-wave model \cite{23} by means of including of the surface emission has been considered in Ref. \cite{24}. If the hypersurface of maximal emission $\tilde{\tau}(r)$ is double-valued function, as in our case, then at some transverse momentum $p_T$ the transverse spectra and HBT radii will be formed mostly by the two contributions from the different regions with the homogeneity lengths $\lambda_{i,V} = \sqrt{< (\Delta r_i)^2 >}$ ($i = side$, out) at the $V$-hypersurface and with the homogeneity lengths $\lambda_{i,S}$ at the $S$-hypersurface. Similar to Ref. \cite{8}, one can apply at $m_T/T \gg 1$ the saddle point method when calculate the single and two particle spectra using the boost-invariant measures $\mu_V = d\sigma^V_\mu p^\mu = \tilde{\tau}(r) r dr d\phi d\eta(m_T \cosh(\eta - y) - p_T d\phi d\eta cos(\phi - \alpha))$ and $\mu_S = d\sigma^S_\mu p^\mu = \tilde{\tau}(\tau) r dr d\phi d\eta(m_T \cosh(\eta - y) - p_T d\phi d\eta + p_T \cos(\phi - \alpha))$ for $V$- and $S$- parts of freeze-out hypersurface correspondingly (here $\eta$ and $y$ are space-time and par-
pion transverse momentum of the pion pairs \( \pi \) into account that at \( T \approx 0 \) for pions \( \beta_{out} = p_{out}/p^{th} \approx 1 \). All homogeneity lengths depend on mean transverse momentum of the pion pairs \( p_T \). The slope \( df/d\tau \) in the region of homogeneity expresses the strength of \( r-\tau \) correlations between the space and time points of particle emission at the \( S \)-hypersurface \( \tau(\tau) \). The picture of emission in Fig. 1 shows that when the energy grows the correlations between the time and radial points of the emission become positive, \( df/d\tau > 0 \), and they increase with energy density. The positivity is caused by the initial radial flows \( \dot{u}(\tau_0) \), which are developed at the pre-thermal stage, and the strengthening of the \( r-\tau \) correlations happens because the non-central \( i \)th fluid elements, which produce after their expansion the surface emission, need more time \( \tau_i(\epsilon_0) \) to reach the decoupling density if they initially have higher energy density \( \epsilon_0 \). (Let us characterize this effect by the parameter \( \kappa = \frac{df(\epsilon_0)}{d\epsilon_0} > 0 \). Then the fluid elements before their decays run up to larger radial freeze-out position \( r_i \); if \( a \) is the average Lorentz-invariant acceleration of those fluid elements during the system expansion, then roughly for \( i \)th fluid elements which decays at time \( \tau_i \) we
have at $\sigma \tau \gg 1$: $r_i(\tau_i) \approx r_i(\tau_0) + \tau_i + (u_i'(\tau_0) - 1)/u_i$. Then the level of $r - \tau$ correlations within the homogeneous freeze-out "surface" region, which is formed by the expanding matter that initially at $\tau_0$ occupies the region between the transversal radii $r_1(\tau_0)$ and $r_2(\tau_0) > r_1(\tau_0)$, is

$$\frac{d\tau}{d\tau} \approx \frac{r_1(\tau_1) - r_2(\tau_2)}{\tau_1 - \tau_2} \approx 1 - \frac{R}{\epsilon_0 \kappa}$$

(6)

and, therefore, the strength of $r - \tau$ correlations grows with energy: $\frac{d\tau}{d\tau} \to 1$. Note that here we account for $\tau_2 - \tau_1 \approx \kappa(\epsilon_0(r_2(\tau_0)) - \epsilon_0(r_1(\tau_0)))$ and that $\frac{d\tau}{d\tau} \approx -\frac{\tau}{\tau}$ where $\epsilon_0 \equiv \epsilon_0(r = 0)$ and $R$ is radius of nuclear. As a result the second S-term in Eq. (6) tends to zero at large $\epsilon_0$, therefore, the $R_{out}/R_{side}$ ratio. In particular, if $\lambda^2_{\text{side},V} \gg \lambda^2_{\text{side},S}$ then, accounting for a similarity of the volume emission in our approximation and in the blast wave model, where as known $\lambda_{\text{side},V} \approx \lambda_{\text{out},V}$, one can get: $\frac{R_{out}}{R_{side}} \approx 1 + \text{const} \cdot \frac{R}{\epsilon_0 \kappa} \to 1$ at $\epsilon_0 \to \infty$. It is worth noting that measure $\mu_S$ also tends to zero when $\frac{d\tau}{d\tau} \to 1$ that again reduces the surface contribution to side and out radii at large $p_T$. The presented qualitative, in fact, analysis demonstrates the main mechanisms leading to the non-trivial behavior of $R_{out}$ to $R_{side}$ ratio exposed in detailed HKM calculations, see Fig.1 (bottom, right).

4. Summary

We conclude that the energy behavior of the pion interferometry scales and transverse spectra can be understood if they are analyzed within fairly developed hydro-kinetic models. The latter should be based on EoS which accounts for a crossover transition between quark-gluon and hadron matters at high collision energies and non-chemically equilibrated expansion of the hadron-resonance gas at the later stage. The process of particle emission from expanding fireball, that is not sudden and lasts about system’s lifetime, should be correctly treated. The pre-thermal flow of transverse flow have to be taken into account. Then the main mechanisms that lead to the paradoxical behavior of the interferometry scales find a natural explanation. In particular, a slow decrease and apparent saturation of $R_{out}/R_{side}$ ratio around unity at high energy happens due to a strengthening of positive correlations between space and time positions of pions emitted at the radial periphery of the system. Such an effect is a consequence of the two factors accompanying an increase of collision energy: a developing of the pre-thermal collective transverse flows and an increase of initial energy density in the fireball.

Acknowledgments

The authors thank Peter Braun-Munzinger and Darinsz Miskovic for discussions and careful reading of the manuscript. This work was supported by the Bilateral Award DLR (Germany) - MESU (Ukraine) for the UKR 06/008 Project. The research is carried out within the scope of the EURO: European Ultra Relativistic Energies Agreement (European Research Group: Heavy ions at ultrarelativistic energies) and supported in part by the Fundamental Researches State Fund of Ukraine: Agreement with MESU No F33/461-2009.

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