Cosmic Fluctuations and Dark Matter from Scalar Field Oscillations

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Scale-invariant fluctuations and cold dark matter could originate from two different modes of a single scalar field, fluctuations from massless Goldstone oscillations and matter from massive Higgs modes. Matching the fluctuations and dark matter density observed requires a heavy scale ($\phi_0 \approx 10^{16}$GeV) for the potential minimum and an extremely small self coupling ($\lambda \approx 10^{-83}$). Mode coupling causes the dark matter to form in lumps with nonnegligible velocities, leading to early collapse of dense dark matter “miniclusters” and halos on the scale of compact dwarf galaxies.

I. INTRODUCTION

It is generally acknowledged that new physics is required to explain the origin of cosmic structure, both in the cosmic background radiation and in the galaxy distribution. At some level, all suggestions at present require activity of a new scalar field. The favorite hypothesis is that quantum fluctuations in the fields driving cosmic inflation create scale-invariant fluctuations [1, 2]. If inflation leaves behind a universe which is too smooth, similar structures can be introduced later by large-scale classical motions of scalar fields [3, 4]. New physics is also required to produce the apparent prevalence of nonbaryonic, nonrelativistic dark matter in the universe. Among the many possibilities are scalar and pseudoscalar bosons such as the axion, which condense into nonrelativistic dark matter during vacuum phase transitions [15, 16]. Usually however the fields considered as dark matter candidates have little to do with the sources of cosmic fluctuations.

In this paper I explore the possibility that the two unresolved phenomena—large scale fluctuations and dark matter—might arise from two different modes of oscillation of a single global scalar field. I use a simple Mexican-hat model potential to describe the generic (topology-independent) classical dynamical behavior of multicomponent scalars in an expanding universe, in which classical modes are excited by the Kibble mechanism. I argue that in general the Goldstone modes of the field produce scale-invariant fluctuations and the Higgs modes become cold dark matter. Combining these functions into one field is not merely economical and tidy, but makes a physical difference: during the early stages of oscillation when the Higgs amplitudes are large, the modes couple to each other, leading to additional isocurvature fluctuations and peculiar velocities in the Higgs matter.

The behavior is determined quantitatively by two standard parameters describing the shape of the potential $V(\phi)$. The width of the hat, $\phi_0$, controls the strength of the scale-invariant gravitational perturbations; the self-coupling, $\lambda$, controls the amount of dark matter. Fixing $\phi_0$ to match the COBE/DMR fluctuation amplitude on large scales [6], the present density of dark matter requires a tiny self-coupling parameter, $\lambda \approx 10^{-83}$. With these parameters, mode coupling causes the Higgs matter to condense in lumps of astrophysically interesting sizes and velocities.

There is little new here apart from the particular way a lot of familiar ideas are spliced together from many other scenarios which create cosmic structure with active scalars. The Goldstone-mode part of the model discussed here corresponds to massive global cosmic strings with mass per length $\approx \phi_0^2$—even though the strings are much thicker, and much less dense, than the usual situation considered with potentials of central height $V_0 \approx \phi_0^4$. The self-ordering dynamics create scale-invariant fluctuations even in theories with no topological defects [5]: simulations of this process [7] have however excluded Higgs degrees of freedom. The Higgs-mode condensate is a standard element in inflation theory [8, 9], and resembles the direct-condensation process for forming cosmic axions [10, 11] (although here it is the radial and not the axial degrees of freedom that provide the harmonic potential to condense in). The coupling of Higgs with Goldstone modes resembles radiation of axions from cosmic strings [12, 13]. The formation of the isocurvature fluctuations is by a process similar to that envisioned for “axion miniclusters” [14, 15], except that the scale is now large enough to be more astrophysically interesting. The same condensation process was also considered in a late-phase-transition model [16] with still weaker coupling; that scenario however included no Goldstone modes. Models with weaker coupling still (i.e., Higgs masses $m \equiv 2\lambda\phi_0^2 < H_0 \approx 10^{-32}$eV) and $\phi_0 \approx m_{P}(H_0/m)$ (and no Goldstone modes, which would cause disastrous fluctuations in this situation) can produce a cosmological constant [17, 18]. This work shows that in some supergravity models, exponentially small masses arise naturally from anomalies, so there is at least a plausible context for the parameters proposed here.
II. HIGGS AND GOLDSTONE MODES

For definiteness consider the behavior of a complex classical scalar field described by Lagrangian density \[^2\]
\[
L = \partial_{\mu} \phi \partial^\mu \phi/2 - V(\phi),
\]
with a potential of the familiar form
\[
V(\phi) = (\lambda/4)(\phi^2 - \phi_0^2)^2,
\]
assuming as usual \(c = \hbar = 1\). The potential has the form of a Mexican hat of height at center \(V_0 = \lambda \phi_0^4/4\) and a set of degenerate minima forming a circle at \(|\phi| = \phi_0\), characterized by an internal phase angle \(\theta\). The field obeys the evolution equation
\[
\ddot{\phi} + 3H \dot{\phi} - \nabla^2 \phi + \partial V/\partial \phi = 0,
\]
where \(H = \dot{a}/a\) and \(a\) denotes the cosmic scale factor. As this simple theory has no dissipative couplings, there is no "reheating" in the theory \[^3\,^4\], and energy losses are all adiabatic.

The system supports two kinds of oscillations. The first are "classical Goldstone modes," corresponding to motion within the circle of minima. Quantum mechanically these modes are massless Goldstone bosons; the classical modes are oscillations in which the gradient term \(\nabla^2 \phi\) plays the role of a restoring force which tries to correct misalignments in \(\phi\). In these modes the field is not perfectly aligned but has spatial variations in \(\theta\); the misalignments propagate in space at unit velocity, with a characteristic frequency determined by the wavelength.

The second type of oscillations are "classical Higgs modes," corresponding to harmonic motion in the hat’s radial direction. The characteristic frequency is
\[
\omega^2 = V''(\phi_0) = 2m^2 = 2\lambda\phi_0^2
\]
where \(m\) is the mass of the Higgs particle. The zero momentum modes are spatially uniform and do not propagate.

Both types of modes carry energy with distinctive equations of state. The density and pressure are \[^2\]
\[
p_\phi = \dot{\phi}^2/2 + V(\phi) + (\nabla \phi)^2/2
\]
\[
p_\phi = \dot{\phi}^2/2 - V(\phi) - (\nabla \phi)^2/6
\]
where the gradient term contributes an anisotropic pressure in the direction of \(\nabla \phi\). A static uniform \(\phi\) field yields the familiar inflationary (de Sitter, steady state, cosmological constant) equation of state, \(p = -\rho = -V\); a changing \(\phi\) contributes an ultra-stiff component, \(p = \rho = \dot{\phi}^2\); and a stationary spatial gradient contributes \(p = -1/3\rho = -(\nabla \phi)^2/6\) (Which incidentally in spite of carrying energy, has zero Newtonian gravity; a universe made of such material mimics an open universe even with zero space curvature).

On timescales comparable to the oscillation period, the pressure and density of the modes fluctuates between these various extreme equations of state. On timescales long compared to an oscillation period, the equations of state of the two types of modes are harmonic time averages of these expressions over the oscillation. The Goldstone modes have \(\langle V \rangle = 0\) so their equation of state averages \(\dot{\phi}^2/2\) and \(\langle (\nabla \phi)^2 \rangle\), with the latter multiplied times \(+1/2\) and \(-1/6\) for the density and pressure respectively, to produce simple relativistic matter with \(p = \rho/3\). In the the zero-momentum Higgs modes the gradient vanishes and the average of \(V\) and \(\dot{\phi}^2/2\) produces a cancellation, yielding pressureless matter \(p = 0\); this phenomenon is familiar from the formation of a cosmological axion condensate. In both cases the density and pressure are proportional to the squared amplitude of the oscillations.

III. FLUCTUATIONS AND DARK MATTER

Now consider the evolution of the classical field in an expanding universe. The effective potential at high temperatures is as usual driven to have a minimum at \(\phi = 0\), which is the initial condition of the system apart from small fluctuations. For \(T < 2\phi_0\), this minimum disappears, so that the classical system rolls down from the center of the hat to someplace on the circle of minima. This process in general excites both types of modes, but in different ways and at different times.
The Goldstone modes are excited by the same Kibble mechanism responsible for the formation of cosmic strings. (In fact, for the specific potential used here strings actually form as well; but the Goldstone oscillations are in addition to any such topological defects). Widely separated portions of the universe choose independently the value of $\theta$ they relax to; thus any random initial conditions of the field naturally produce large-amplitude (in $\theta$) spatial gradients in $\phi$ on all scales. These gradients are approximately preserved, frozen in comoving coordinates, on each comoving scale as long as the wavelength exceeds $H^{-1}$; they excite propagating Goldstone modes of unit amplitude on each scale when the wavelength comes within $H^{-1}$.

The expansion rate is related to the cosmic density $\rho$ by the Friedmann equation $H^2 = \rho/m_p^2$, where $m_p \equiv (3h c/8\pi G)^{1/2} = 4.5 \times 10^{18}$ GeV is the Planck mass. The Goldstone modes on each scale contribute density fluctuations when they cross the horizon of the order of their fluctuating density. Since they always move locally at the speed of light, the gradient is determined by the expansion rate $H$,

$$\delta \rho = \rho_0 \equiv (\nabla \phi)^2 \approx \delta \theta^2 \phi_0^2 H^2 = \rho(\phi_0/m_p)^2 \delta \theta^2.$$  \hspace{1cm} (7)

The modes on each scale are initially excited with $\delta \theta \approx 1$. (The amplitude decreases by Hubble damping after they come within the horizon and oscillations begin, leading to a stochastic background of [practically undetectable] relativistic Goldstone waves.) This process thus produces approximately scale-free fluctuations with amplitude $\delta \rho/\rho \approx (\phi_0/m_p)^2$. Matching the COBE/DMR amplitude requires $(\phi_0/m_p)^2 \approx 10^{-5}$, or $\phi_0 \approx 10^{18}$ GeV. Detailed simulations, and exact solutions for the (nearly Gaussian) case of many $\phi$ components, confirm the nearly scale-invariant spectrum, and provide the precise normalization of CBR anisotropy to matter fluctuations \[9,10,12\]. In general, gravitational effects of strings or textures, if any, are up to logarithmic factors comparable to (and additional to) the Goldstone effect.

The radial Higgs mode is also excited in this system, just by starting at the top of the hat (at the origin), rolling off and overshooting the minimum. The natural timescale for initial relaxation to produce the zero-momentum mode is just the oscillation time $\omega^{-1}$. Even if the rolling starts very early ($T \approx \phi_0$), the oscillations as such do not start until $\omega \approx H$. (This is also when the Higgs and Goldstone modes decouple from each other; after this the coupling is weak because the Higgs amplitude is $<< \phi_0$ and the Higgs frequency no longer matches the $\delta \theta = 1$ Goldstone modes.) The temperature $T_{\omega H}$ when this happens can be estimated by setting $\omega^2 = 2\lambda \phi_0^2$ equal to $H^2 = a_s T_{\omega H}^4/m_p^2$, yielding

$$T_{\omega H}/m_p = (2/a_s)^{1/4} \lambda_1^{1/4}(\phi_0/m_p)^{1/2}.$$  \hspace{1cm} (8)

(This has assumed radiation domination $\rho_{rad} = a_s T^4$, where $a_s = \pi^2 N_{eff}/15$ and $N_{eff}$ is the number of effective photon degrees of freedom). At this time, the density in the Higgs modes, like the Goldstone modes at all times as they enter the horizon, is

$$(\rho_\phi/\rho_{rad})_{\omega H} \approx V_0/a_s T_{\omega H}^4 = (\phi_0/m_p)^2 \approx 10^{-5}.$$  \hspace{1cm} (9)

The subsequent behavior is very different however. Although subsequent excitation of Higgs modes is very inefficient, these initial excitations, like those of the Goldstone modes, have their amplitudes reduced only by the expansion. However, as the Higgs correspond to pressureless matter, they do not lose energy as quickly as the relativistic matter: $\rho_\phi \propto \rho_{rad}/T$. They contribute a component of pressureless dark matter today whose ratio to the radiation density at the present temperature $T_0 \approx 5 \times 10^{-32}m_p$ is then

$$\rho_\phi/\rho_{rad} \approx (\phi_0/m_p)^2(T_{\omega H}/T_0) \approx (\phi_0/m_p)^{2.5}(m_p/T_0)^{1/4}.$$  \hspace{1cm} (10)

Let $\Omega_\phi$ denote the fraction of critical density today in the form of cosmic cold dark matter in the Higgs oscillations, and $\Omega_{rad} = 4 \times 10^{-5}h^{-2}$ the fraction in radiation. A reasonable density requires a tiny coupling: $\Omega_\phi h^2 \approx 4 \times 10^{-20}$, $\lambda \lesssim 10^{-68}(\Omega_\phi h^2)^4$, and a tiny Higgs mass, $m = \sqrt{2} \lambda^{1/2} \phi_0 \approx 6 \times 10^{-17}(\Omega_\phi h^2)^2$ eV. The characteristic energy scale of the unbroken vacuum is $V_0^{1/4} = \phi_0 \lambda^{1/4} \approx 20(\Omega_\phi h^2)$ keV. The formation of the Higgs condensate occurs at redshift $z_{\omega H} \approx (\rho_\phi/\rho_{rad})(\phi_0/m_p)^{-3} \approx 2.5 \times 10^{9}(\Omega_\phi h^2)$, at a temperature $T_{\omega H} \approx 7 \times 10^3(\Omega_\phi h^2)$ K, or about 600$(\Omega_\phi h^2)$ keV. The Compton wavelength of the Higgs particles is $2\pi/m \approx 2\pi/H \approx 2 \times 10^{12}(\Omega_\phi h^2)^{-2}$ cm, corresponding to a characteristic oscillation period of about $60(\Omega_\phi h^2)^{-2}$ sec.

The Higgs is not excited in a spatially uniform zero momentum mode. The initial mixing with the Goldstone modes ensures fractional variations of the order of unity in the rolloff time, since the spatial gradients in the field accelerate rolloff in some places and retard it in others, leading to spatial variations in the phases and initial epochs for the oscillations. The amplitude of the Higgs oscillation therefore fluctuates spatially, with about unit fractional amplitude on the scale of the Hubble length at $T_{\omega H}$. This variation leads to dark matter forming in coherent lumps,
with the Higgs matter having coherent peculiar velocity of the order of unity on this scale. The lumps are created with a distribution of dark matter masses roughly in the range [1 to (2\pi)^3] \times V_0/m^3 \approx [2 to 550](\Omega_{\phi}h^2)^{-2} M_\odot, with 100(\Omega_{\phi}h^2)^{-2} M_\odot a typical value. These “compensated isocurvature fluctuations” \[ \Delta \] in dark matter density form in addition to the Goldstone modes considered already. Since the lumps are laid down with no large-scale correlations (according to original hypothesis of the Kibble domain formation), the isocurvature fluctuation spectrum corresponds to white noise on larger scales.

IV. ASTROPHYSICAL CONSEQUENCES

Isocurvature fluctuations grow at a rate of the order of \( (\rho_\phi/m_P^2)^{-1/2} \) — slower than \( H \) until the epoch of equal matter and radiation densities \( t_{eq} \), at the usual rate \( H \) thereafter. Eventually the perturbations grow to be nonlinear and collapse into bound dark matter “miniclusters” \[ [21] \] in virial equilibrium. A spherical model \[ [21] \] estimates the density of the virialized system after collapse, \( \rho = 140\Delta^3(\Delta + 1)\rho_{eq} \), where \( \rho_{eq} = 3 \times 10^{-16}(\Omega_0 h^2) \) g cm\(^{-3}\) is the matter density at \( t_{eq} \), and \( \Delta \) is the initial fractional overdensity. For the first miniclusters, on the scale of the Higgs lumps, \( \Delta \) is of the order of unity. Linear initial fluctuations on scales larger than this have the white noise spectrum of rms fluctuations in spheres of mean mass \( M \),

\[
\Delta_{rms}(M) \approx [M/10^2(\Omega_{\phi}h^2)^{-2} M_\odot]^{-1/2}\Omega_{\phi}/\Omega_0. \tag{11}
\]

These fluctuations cause hierarchical clustering earlier than with standard CDM, starting with the minicluster formation and proceeding to larger scales. Applying the spherical model for each mass scale in the hierarchy of clustering, the dark matter forms into virialized systems which have a virial radius

\[
10^{16} \text{cm}[M/10^2(\Omega_{\phi}h^2)^{-2} M_\odot]^{5/6}[(\Omega_{\phi}h^2)(\Omega_{\phi}h^2)^5]^{-1/3}, \tag{12}
\]

and a virial velocity \( \approx \sqrt{GM/R} \)

\[
\approx 10 \text{ km sec}^{-1}[M/10^2(\Omega_{\phi}h^2)^{-2} M_\odot]^{1/12}(\Omega_0/\Omega_{\phi})^{1/6}, \tag{13}
\]

a characteristic velocity high enough that clusters can accrete baryons at high redshift. The computed velocity dispersion approximately matches that of the densest dark halos yet found, compact dwarf galaxies \[ [23] \] \[ [22] \]. Although halos of this dispersion extend to lower masses, and higher density, than in standard CDM, on larger scales the fluctuations are dominated by the Goldstone modes and the predictions are almost the same.

The Higgs matter is not perfectly cold, since the same coupling that creates the lumps also produces spatial gradients in Higgs mode phase, corresponding to particle peculiar velocities. Although still cold compared to massive neutrinos, the relic particle velocities for the Higgs matter are much larger for the usual CDM candidates such as WIMPs or axions. The velocity dispersion is of the order of \( (\rho_{rad}/\rho_\phi)(\phi_0/m_P)^2 = 3\text{km/sec at } t_{eq} \). Since this is comparable to the virial velocity of the miniclusters, they form with nonsingular cores comparable in size to the virial radius \[ [33] \].

Another constraint is that dynamics cannot increase the classical particle phase space density \[ [14] \]. The initial condensation creates mildly relativistic \( (\gamma \approx 1) \) particles with mass density \( \phi_0^2 m^2 \); thus there are \( \phi_0^2 m \) particles per volume, spread out over a wavenumber volume of order \( m^3 \). Although the fine-grained phase space density of the particles is extremely high, once orbits cross and coherence is lost the effective coarse-grained phase density is \( \approx \phi_0^2/m^2 \), leading to an upper limit on the density of subsequently formed systems of velocity dispersion \( v \): \( \rho < \phi_0^2 m^2 v^3 \), which is about \( 4 \times 10^{-15}(v/10\text{km sec}^{-1})^3(\Omega_{\phi}h^2)^4 \text{ g cm}^{-3} \) for the parameters determined above.

If any of the original miniclusters survive they might be detected by gravitational microlensing of quasars. The Einstein radius at the Hubble distance is \[ [33] \] \[ [36] \]

\[
R_E = 2\sqrt{GM/H} = 8 \times 10^{17} h^{-1/2}(M/10^2 M_\odot)^{1/2} \text{ cm.} \tag{14}
\]

The smallest miniclusters are themselves smaller than this, so individual miniclusters are compact enough to amplify distant objects significantly, allowing a variety of observational probes \[ [35] \] \[ [34] \]. The incidence of such events however depends on the number of surviving miniclusters, which is difficult to compute.
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