Heat and Mass Transfer on unsteady MHD free convective flow through a porous medium over an infinite vertical plate with time dependent permeability and oscillatory suction

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Abstract

We considered the free convective flow over an infinite vertical porous plate under the influence of uniform transverse magnate field with time dependent permeability and oscillatory suction. The equations of the flow are solved by perturbation method for small amplitude of the permeability. The closed form solutions for the velocity, temperature and concentration have been derived analytically and also its behavior is computationally discussed with reference to different flow parameters with the help of profiles. The skin fiction on the Nusselt number and Sherwood number are also attained and their behavior computationally discussed.

Keywords: Heat transfer; mass transfer; oscillatory suction; porous medium; MHD flow; vertical plates.

Subject Classification codes: 80A20, 76Sxx, 76N20, 76A05, 76A10

Nomenclatures

\( C \) Species concentration;
\( \phi \) Non-dimension species concentration;
\( D \) Molecular diffusivity;
\( Gr \) thermal Grashof number
\( Gm \) mass Grashof number
\( g \) Acceleration due to gravity;

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1. Introduction

Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation studied by Chen. Chamkha analyzed MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Hayat and Mehmood analyzed slip effects on MHD flow of third order fluid in a planar channel. Pal and Chatterjee found heat and mass transfer in MHD non-Darcian flow of a micro polar fluid over a stretching sheet embedded in a porous media with non-uniform heat source and thermal radiation. Kim studied unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Chamkha discussed unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Das studied free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. Soundalgekar examined free convection effects on steady MHD flow past a vertical porous plate. Hossain et al. studied the effect of radiation on free convection from a porous vertical plate. Srinivasacharya and Mendu studied free convection in MHD micro polar fluid with radiation and chemical reaction effects. Srinivasacharya and RamReddy studied natural convection heat and mass transfer in a micro polar fluid with thermal and mass stratification.

The fluid under consideration undergoes in some chemical reactions e.g. air and benzene reacts chemically, so also water and sulphuric acid. During such chemical reactions, there is always generation of heat. Makinde
et al.\textsuperscript{17} investigated unsteady convection with chemical reaction and radiation. Kandasamy et al.\textsuperscript{15} have analyzed effects of chemical reaction. Das et al.\textsuperscript{7} analyzed effects of mass transfer on flow. Anjalidevi and Kandasamy\textsuperscript{1} studied effects of chemical reaction. Seddeek et al.\textsuperscript{26} discussed effects of chemical reaction and variable viscosity on hydromagnetic mixed convection. Ibrahim et al.\textsuperscript{11} studied effect of the chemical reaction and radiation absorption. Patil and Kulkarni\textsuperscript{24} investigated effects of chemical reaction on free convective flow. Pal and Mondal\textsuperscript{21} discussed effects of Soret, Dufour, chemical reaction and thermal radiation on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet. Pal and Talukdar\textsuperscript{22} analyzed analytically, unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow. Muthucumaraswamy and Ganesan\textsuperscript{19} studied first-order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Dulal Pal (1999) studied effect of chemical reaction on the dispersion of a solute in a porous medium. Mehta and Tiwari(2011) analyzed dispersion in the presence of slip and chemical reactions in porous wall tube flow. Hayat and Nawaz (2009) studied the Soret and Dufour effects on the mixed convection flow of a second grade fluid subject to Hall and ion-slip currents. Patil et al. (2014) analyzed double diffusive mixed convection flow over a moving vertical plate in the presence of internal heat generation and a chemical reaction. Srinivasacharya and Kaladhar (2014) studied mixed convection flow of chemically reacting couple stress fluid in a vertical channel with Soret and Dufour effects. Srinivasacharya and Upendar\textsuperscript{29} studied Soret and Dufour effects on MHD free convection in a micro polar fluid. Kaladhar and Srinivasacharya\textsuperscript{14} analyzed mixed convection flow of chemically reacting couple stress fluid in an annulus with Soret and Dufour effects. Hsiao (2010 & 2011), Srinivasacharya and Swamy Reddy\textsuperscript{32} & Srinivasacharya and Kaladhar\textsuperscript{31} examined Soret and Dufour effects. Recently, Krishna and Swarnalathamma (2016) and Swarnalathamma and Krishna\textsuperscript{34} discussed the peristaltic MHD flows. Krishna and M.G.Reddy (2016) and Krishna and G.S.Reddy (2016) discussed MHD free convective rotating flows. Motivated by the above studies, in this paper we have considered the free convective flow over an infinite vertical porous plate.

2. \textit{Formulation and solution of the problem}:

We considered the unsteady MHD free convection flow of an incompressible viscous electrically conducting fluid with simultaneous heat and mass transfer over an infinite vertical plate through porous medium with time dependent permeability and oscillatory suction under the influence of uniform transverse magnetic field of strength $H_0$. The $y$-axis is taken along the plate and $x$-axis perpendicular to it and $u$ is the velocity along the $x$-direction. The physical configuration of the problem is as shown in Fig. 2.1.

![Fig. 2.1 Physical configuration of the problem](image)

The basic assumptions are made as following.

1. All fluid proportions are constant.
2. The plate as well as the fluid is assumed to be at the same temperature and the concentration of species is raised or lowered.
3. The magnetic Reynolds number is small so that the induced magnetic field can be neglected in comparison to the applied magnetic field.
4. The permeability of the porous medium is assumed to be
   \[ K(t) = k(1 + \varepsilon e^{i\omega t}) \]  
   (1)
5. The suction velocity is assumed to be
   \[ v(t) = -v_0(1 + \varepsilon e^{i\omega t}) \]  
   (2)
   Where, \( v_0 \) represents the suction or injection velocity at the plate.
6. The pressure is assumed to be constant.
7. If the plate is extended to infinite length, then all the physical variables are functions of \( y \) and \( t \) alone.

The equations for the unsteady MHD free convection flow over an infinite vertical plate through porous medium are given by

Equation of continuity:
\[ \frac{\partial u}{\partial y} = 0 \]  
(3)

Momentum equation:
\[ \frac{\partial u}{\partial t} - v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu H_0^2}{\rho} u - \frac{v}{K(t)} u + g \beta(T - T_\infty) + g \beta^* (C - C_\infty) \]  
(4)

Equation of energy:
\[ \frac{\partial T}{\partial t} - v \frac{\partial T}{\partial y} = \frac{\epsilon}{\epsilon y^*} - S_1(T - T_\infty) \]  
(5)

Equation of concentration
\[ \frac{\partial C}{\partial t} - v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_1(C - C_\infty) \]  
(6)

The boundary conditions are
\[ u(y, t) = T(y, t) = C(y, t) = f(t) \quad \text{at} \quad y = 0 \]
\[ = 0 \quad \text{at} \quad y \rightarrow \infty \]  
(7)

Where, \( f(t) = 1 + \varepsilon e^{i\omega t} \). With foregoing assumptions and taking usual Boussinesq’s approximation into account as well as the following non-dimensional variables:

\[ u^* = \frac{u}{v_0}, \quad y^* = \frac{y}{v_0}, \quad t^* = \frac{t}{v_0^2}, \quad \omega^* = \frac{\omega}{v_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \]

Making use of non-dimensional variables, the governing equations reduces to (Dropping asterisks)

\[ \frac{\partial u}{\partial t} - v_0(1 + \varepsilon e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} \left( M^2 + \frac{1}{K(1 + \varepsilon e^{i\omega t})} \right) u + Gr \theta + GmC \]  
(8)
\[
\frac{\partial \theta}{\partial t} - v_0 (1 + \varepsilon e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} - \text{Pr} S \theta
\]

\[
\frac{\partial \phi}{\partial t} - v_0 (1 + \varepsilon e^{i\omega t}) \frac{\partial \phi}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} - \text{Sc} \text{Sc} \phi
\]

The corresponding boundary conditions are

\[
u(y, t) = \theta(y, t) = \phi(y, t) = 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad y = 0
\]

\[
u(y, t) = \varepsilon \quad \text{at} \quad y \to \infty
\]

Where \( M^2 = \frac{\sigma B_0^2 v}{\rho v_0^2} \) is the Hartmann number (Magnetic field parameter), \( K = \frac{v^2}{k v_0} \) is the permeability parameter (Porosity or Darcy parameter), \( \text{Pr} = \frac{v}{\alpha} \) is the Prandtl number, \( \text{Sc} = \frac{v}{D} \) is the Schmidt number, \( \text{Kc} = \frac{k v}{v_0} \) is the chemical reaction parameter, \( S = \frac{S v}{v_0^3} \) is the Heat Source parameter, \( \text{Gr} = \frac{g \beta v (T_w - T_0)}{v_0^2} \) is the thermal Grashof number and \( \text{Gm} = \frac{g R^* v (C_w - C_\infty)}{v_0^3} \) is the mass Grashof number.

In order to solve the equations, (8) – (10) using boundary conditions (11), we assume the solutions of the following form, because the amplitude \( \varepsilon (< 1) \) of permeability is very small.

\[
u(y, t) = \nu_0(y) + \varepsilon \nu_1(y) e^{i\omega t}
\]

\[
\theta(y, t) = \theta_0(y) + \varepsilon \theta_1(y) e^{i\omega t}
\]

\[
\phi(y, t) = \phi_0(y) + \varepsilon \phi_1(y) e^{i\omega t}
\]

Substituting the Eqs. (12) – (14) into the Eqs. (8) – (10) respectively and equate the harmonic and non-hormonic terms to obtain the zeroth and first orders ordinary differential equations for momentum, temperature and concentration distributions.

**Zeroth order:**

\[
\frac{\partial^2 \phi_0}{\partial y^2} + \text{Sc} v_0 \frac{\partial \phi_0}{\partial y} - \text{Kc} \text{Sc} \phi_0 = 0
\]

\[
\frac{\partial^2 \theta_0}{\partial y^2} + \text{Pr} v_0 \frac{\partial \theta_0}{\partial y} - \text{Pr} S \theta_0 = 0
\]

\[
\frac{\partial^2 u_0}{\partial y^2} + v_0 \frac{\partial u_0}{\partial y} - \left( M^2 + \frac{1}{K} \right) u_0 = -\text{Gr} \theta_0 - \text{Gm} \phi_0
\]

Corresponding boundary conditions are

\[
\phi_0 = 1, \theta_0 = 1, u_0 = 1 \quad \text{at} \quad y = 0
\]

\[
\phi_0 = 0, \theta_0 = 0, u_0 = 0 \quad \text{at} \quad y \to \infty
\]
Solving the equations (15) – (17) with relevant boundary conditions (18) and (19), we obtained the zeroth order concentration, temperature and velocity.

First order:

\[
\frac{\partial^2 \phi_1}{\partial y^2} + S c v_0 \frac{\partial \phi_1}{\partial y} - S c(S c + K c i) \phi_1 = - S c v_0 \frac{\partial \phi_0}{\partial y} \quad (20)
\]

\[
\frac{\partial^2 \theta_1}{\partial y^2} + P r v_0 \frac{\partial \theta_1}{\partial y} - P r(S + i o) \theta_1 = - P r v_0 \frac{\partial \theta_0}{\partial y} \quad (21)
\]

\[
\frac{\partial^2 u_1}{\partial y^2} + v_0 \frac{\partial u_1}{\partial y} - \left( M^2 + \frac{1}{K} \right) u_1 = -v_0 \frac{\partial u_0}{\partial y} - G r \theta_1 - G m \phi_1 \quad (22)
\]

Corresponding boundary conditions are

\[
\phi_1 = 1, \theta_1 = 0, u_1 = 0 \quad \text{at} \quad y \to 0 \quad (23)
\]

\[
\phi_1 = 0, \theta_1 = 0, u_1 = 0 \quad \text{at} \quad y \to \infty \quad (24)
\]

Solving the equations (20) – (22) with relevant boundary conditions (23) and (24), we obtained the first order concentration, temperature and velocity.

\[
\phi_0 = e^{-m_1 y} \quad (25)
\]

\[
\theta_0 = e^{-m_2 y} \quad (26)
\]

\[
u_0 = A_1 e^{-m_1 y} - \frac{G m}{A_3} e^{-m_1 y} \left( 1 - \frac{S c v_0 m_1}{m_1^3 + S c v_0 m_1 - S c(S c + i o)} \right) e^{-m_1 y} + \frac{S c v_0 m_1 e^{-m_1 y}}{m_1^3 + S c v_0 m_1 - S c(S c + i o)} \quad (27)
\]

\[
\theta_1 = \left( 1 - \frac{P r v_0 m_3}{m_3^3 + P r v_0 m_3 - P r(S + i o)} \right) e^{-m_2 y} + \frac{P r v_0 m_3 e^{-m_1 y}}{m_3^3 + P r v_0 m_3 - P r(S + i o)} \quad (28)
\]

\[
u_1 = (1 - B_1 + G B_2 + G m B_3) e^{-m_1 y} + \frac{A_8}{A_1} e^{-m_1 y} + \frac{A_9}{A_2} e^{-m_1 y} + \frac{A_{10}}{A_3} e^{-m_1 y}
\]

\[
- G m \left( \frac{A_4}{A_{14}} e^{-m_1 y} + \frac{A_5}{A_{15}} e^{-m_1 y} \right) - G m \left( \frac{A_6}{A_{15}} e^{-m_1 y} + \frac{A_7}{A_{13}} e^{-m_1 y} \right) \quad (29)
\]

The skin friction at the plate in terms of amplitude and phase is given by

\[
\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} + e \left( \frac{\partial u}{\partial y} \right)_{y=0} = F_1 + e |F_2| \cos(\omega t + \psi) \quad (30)
\]

Where \( F_1 = \left( \frac{\partial u}{\partial y} \right)_{y=0} \); \( F_2 = \left( \frac{\partial u}{\partial y} \right)_{y=0} \), and \( \tan(\psi) = \frac{\text{Re}[F_2]}{\text{Im}[F_2]} \).

The Nusselt number at the plate in terms of amplitude and phase is given by

\[
N u = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} + e \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = F_3 + e |F_4| \cos(\omega t + \gamma) \quad (31)
\]
Where, \( F_3 = \left( \frac{\partial \theta_0}{\partial y} \right)_{y=0} ; F_4 = \left( \frac{\partial \theta_0}{\partial y} \right)_{y=0} \) and \( \tan (\gamma) = \frac{\text{Re}[F_4]}{\text{Im}[F_4]} \).

The Sherwood number at the plate in terms of amplitude and phase is given by

\[
Sh = -\left( \frac{\partial \phi}{\partial y} \right)_{y=0} = \left( \frac{\partial \phi_0}{\partial y} \right)_{y=0} + \varepsilon \left( \frac{\partial \phi_0}{\partial y} \right)_{y=0} = F_5 + \varepsilon |F_6| \cos(\omega t + \zeta) \tag{32}
\]

Where, \( F_5 = \left( \frac{\partial \phi_0}{\partial y} \right)_{y=0} ; F_6 = \left( \frac{\partial \phi_0}{\partial y} \right)_{y=0} \) and \( \tan (\zeta) = \frac{\text{Re}[F_6]}{\text{Im}[F_6]} \).

\[
m_1 = \frac{\text{Sc} v_0 + \sqrt{\text{Sc}^2 v_0^2 + 4 \text{Kc} \text{Sc}}}{2}, \quad m_2 = \frac{\text{Sc} v_0 + \sqrt{\text{Sc}^2 v_0^2 + 4 \text{Sc} (\text{Kc} + i\omega)}}{2},
\]

\[
m_3 = \frac{\text{Pr} v_0 + \sqrt{\text{Pr}^2 v_0^2 + 4 \text{Pr} \text{S}}}{2}, \quad m_4 = \frac{\text{Pr} v_0 + \sqrt{\text{Pr}^2 v_0^2 + 4 \text{Pr} (\text{S} + i\omega)}}{2},
\]

\[
m_5 = \frac{\sqrt{v_0^2 + 4 \left( M^2 + \frac{1}{K} \right)}}{2}, \quad m_6 = \frac{\sqrt{v_0^2 + 4 \left( M^2 + \frac{1}{K} + i\omega \right)}}{2}.
\]

\[
A_1 = 1 - \frac{\text{Gr}}{A_2}, \quad A_2 = m_3^2 - m_3 v_0 - \left( M^2 + \frac{1}{K} \right), \quad A_3 = m_4^2 - m_4 v_0 - \left( M^2 + \frac{1}{K} \right), \quad A_4 = 1 - A_5,
\]

\[
A_5 = \frac{\text{pr} v_0 m_5}{m_5^2 + \text{Sc} v_0 m_5 - \text{Sc} (\text{Kc} + i\omega)}, \quad A_6 = -m_3 A_1, \quad A_7 = \frac{\text{Gr} m_4}{A_2}, \quad A_8 = \frac{\text{Gm} m_1}{A_1},
\]

\[
A_{11} = m_4^2 - v_0 m_4 - \left( M^2 + \frac{1}{K} + i\omega \right), \quad A_{12} = m_3^2 - v_0 m_3 - \left( M^2 + \frac{1}{K} + i\omega \right), \quad A_{13} = m_1^2 - v_0 m_1 - \left( M^2 + \frac{1}{K} + i\omega \right),
\]

\[
A_{14} = m_2^2 - v_0 m_2 - \left( M^2 + \frac{1}{K} + i\omega \right), \quad A_{15} = m_3^2 - v_0 m_3 - \left( M^2 + \frac{1}{K} + i\omega \right),
\]

\[
B_1 = \frac{A_6}{A_{11}} + \frac{A_8}{A_{12}} + \frac{A_7}{A_{13}}, \quad B_2 = \frac{A_1}{A_{14}}, \quad B_3 = \frac{A_2}{A_{15}}, \quad B_4 = \frac{A_3}{A_{16}}.
\]

3. Results and Discussion

Figures (2-5) represent the velocity, the temperature and concentration distributions respectively. The stresses, Nusselt number and Sherwood number at the plate are evaluated numerically and discussed with governing parameters and are tabulated in the tables (1-3). Fixing the parameters \( M=2, K=1, \text{Pr}=0.71, S=1, \text{Sc}=0.22, \text{Kc}=1, \text{Gr}=5, \text{Gm}=10, v_0=0.2 \) and \( \omega = \pi / 6 \), we draw the profiles varying for each parameter while the other parameters being fixed. From the Figures 2(a-f), we noticed that the magnitude of the velocity component...
$u$ reduces with increasing the intensity of the magnetic field $M$, Prandtl number $Pr$ and Heat source parameter $S$. Whereas the velocity component $u$ enhance with increasing permeability parameter $K$, thermal Grashof number $Gr$ or mass Grashof number $Gm$ throughout the fluid region. Lower the permeability of the porous medium lesser the fluid speed in the entire region. The Figures 3 (a-d) depict the velocity component $u$ experiences retardation in the flow field with increasing the chemical reaction parameter $Kc$, Schmidt number $Sc$, the suction velocity $v_0$ or the frequency of oscillation entire the fluid region. Figures 4(a-d) showed the effect of Heat source parameter $S$, the Prandtl number $Pr$, suction velocity $v_0$ and the frequency of oscillation $\omega$ on the temperature of the flow field. We noted that the temperature of the flow field diminishes as the Prandtl number increases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prandtl number. With increasing heat source parameter reduces the temperature of the flow field. This may happen due the elastic property of the fluid. It is observed that temperature of the flow field diminishes as the suction parameter or the frequency of oscillation increases. Figures 5(a-d) depict the effect of the Schmidt number $Sc$ and the frequency of oscillation $\omega$ on concentration distribution. The concentration distribution decreases at all points of the flow field with the increase in the Schmidt number $Sc$ or chemical reaction parameter $Kc$. This shows that the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field. Also, it is observed that presence of the frequency of oscillation $\omega$ or increasing the suction velocity reduces the concentration distribution. The skin friction is significant phenomenon which characterizes the frictional drag force at the solid surface. From Table 1, it is observed that the skin friction increases with the increase in permeability parameter $K$, thermal Grashof number $Gr$, Mass Grashof number $Gm$ and suction velocity $v_0$, but it is interesting to note that the skin friction decreases with the increase in Hartmann number $M$, Prandtl number $Pr$, Schmidt number $Sc$, chemical reaction parameter $Kc$, Heat source parameter $S$ and the frequency of oscillation. Similarly the amplitude augments with increase $K$, $Sc$, $Kc$, $Gr$, $Gm$ and $v_0$ retards with increase $M$, $Pr$, $S$ and $\omega$. The magnitude of the phase angle increase with $Pr$, $S$ and $Gm$ and reduces with $K$, $Sc$, $Kc$, $Gr$ and $\omega$. From Table 2, it is to note that all the entries are positive. It is seen that Heat source parameter $S$, the Prandtl number $Pr$ and the suction velocity increase amplitude and the rate of heat transfer (Nusselt number $Nu$) at the surface of the plate. Reduce the phase angle and increase the amplitude and Nusselt number with increase the frequency of oscillations $\omega$. From Table 3 it is to note that all the entries are positive. It is observed that Schmidt number $Sc$, chemical reaction parameter $Kc$ and the suction velocity increase amplitude and the rate of mass transfer at the surface of the plate. Increase the frequency of oscillations $\omega$, enhance the amplitude, the rate of mass transfer decrease the phase angle. Table 4 represents comparison of the results. Results are very good agreement with the results of Ashraf (2013) when $Sc=0$. 

\[ u \]
Fig 2(a-f): The velocity Profiles against $M$, $K$, $Gr$, $Gm$, $Pr$ and $S$ with $\varepsilon = 0.001$, $t = 1$

Fig 3 (a-d): The velocity Profiles against $Sc$, $Kc$, $v_0$ and $\omega$ with $\varepsilon = 0.001$, $t = 1$
Fig 4 (a-d): The temperature profiles against Pr, S, ω and v₀ with 

\[ \varepsilon = 0.001, \ t = 0.2 \]

Fig 5 (a-d): The Concentration profiles against Sc, Kc, v₀ and ω with 

\[ \varepsilon = 0.001, \ t = 0.2 \]
Table 1. Shear stresses with $\varepsilon = 0.001, t = 0.2$

| M | K | Pr | Sc | Kc | S | Gr | Gm | $v_0$ | $\omega$ | Amplitude $|F_2|$ | Phase Angle ($\psi$) | $\tau$ |
|---|---|----|----|----|---|----|----|------|--------|-----------------|-------------------|-------|
| 0.5 | 2 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 11.6485 | -1.08662 | 9.91165 |
| 2 | 2 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 4.27658 | -1.55299 | 3.45314 |
| 3 | 2 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 1.29628 | -1.41396 | 0.97579 |
| 2 | 3 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 12.9525 | -1.00391 | 10.8927 |
| 2 | 4 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 13.1681 | -0.92668 | 11.5053 |
| 2 | 3 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 10.9926 | -1.28004 | 8.71502 |
| 2 | 2 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 10.8139 | -1.30829 | 8.13573 |
| 2 | 2 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 12.9525 | -1.00391 | 10.8927 |
| 2 | 2 | 0.71 | 0.22 | 1 | 1 | 5 | 10 | 0.2 | $\pi/6$ | 13.1681 | -0.92668 | 11.5053 |

Table 2. Nusselt number (Nu) with $\varepsilon = 0.001, t = 0.2$

| Pr | S | $v_0$ | $\omega$ | Amplitude $|F_4|$ | Phase Angle $\gamma$ | Nu |
|----|---|------|--------|-----------------|-------------------|-----|
| 0.71 | 1 | 0.2 | $\pi/6$ | 1.00365 | 1.39869 | 0.91666 |
| 3 | 1 | 0.2 | $\pi/6$ | 2.20629 | 1.44928 | 2.05788 |
| 7 | 1 | 0.2 | $\pi/6$ | 3.60564 | 1.47239 | 3.43676 |
| 0.71 | 2 | 0.2 | $\pi/6$ | 1.31588 | 1.48355 | 1.26473 |
| 0.71 | 3 | 0.2 | $\pi/6$ | 1.56988 | 1.51423 | 1.53210 |
| 0.71 | 1 | 0.5 | $\pi/6$ | 1.10256 | 1.46078 | 1.03861 |
| 0.71 | 1 | 0.8 | $\pi/6$ | 1.21656 | 1.48335 | 1.17317 |
| 0.71 | 1 | 0.2 | $\pi/4$ | 1.06122 | 1.31218 | 0.91670 |
| 0.71 | 1 | 0.2 | $\pi/3$ | 1.12272 | 1.24229 | 0.91673 |
From Table 3 it is to note that all the entries are positive. It is observed that Schmidt number $Sc$, chemical reaction parameter $Kc$ and the suction velocity increase amplitude and the rate of mass transfer at the surface of the plate. Increase the frequency of oscillations $\omega$, enhance the amplitude, the rate of mass transfer decrease the phase angle. Table 4 represents comparison of the results. Results are very good agreement with the results of Ashraf (2013) when $Sc=0$.

4. Conclusions

We have considered the free convective flow over an infinite vertical porous plate under the influence of uniform transverse magnate field with time dependent permeability and oscillatory suction. The conclusions are made as the following.

1. The velocity reduces with increasing the intensity of the magnetic field or Prandtl number $Pr$ or Heat source parameter $S$. 

| Sc  | Kc | $v_0$ | $\omega$ | Amplitude $|F_4|$ | Phase Angle ($\zeta$) | Sh |
|-----|----|-------|----------|----------------|---------------------|----|
| 0.22 | 1  | 0.2   | $\pi/6$  | 0.536249       | 1.365880            | 0.491611 |
| 0.3  | 1  | 0.2   | $\pi/6$  | 0.632410       | 1.373015            | 0.578602 |
| 0.6  | 1  | 0.2   | $\pi/6$  | 0.916662       | 1.393062            | 0.836984 |
| 0.22 | 1.5| 0.2   | $\pi/6$  | 0.629483       | 1.429490            | 0.596922 |
| 0.22 | 2  | 0.2   | $\pi/6$  | 0.712364       | 1.464392            | 0.685691 |
| 0.22 | 1  | 0.5   | $\pi/6$  | 0.574058       | 1.422405            | 0.527289 |
| 0.22 | 1  | 0.8   | $\pi/6$  | 0.603609       | 1.454160            | 0.565233 |
| 0.22 | 1  | 0.2   | $\pi/4$  | 0.566008       | 1.276890            | 0.491634 |
| 0.22 | 1  | 0.2   | $\pi/3$  | 0.599008       | 1.206869            | 0.491650 |

| Sc  | $v_0$ | $\omega$ | Ashraf et al.(2013) | Present work [$Kc=0$] |
|-----|-------|----------|---------------------|----------------------|
| 0.22 | 0.2   | $\pi/6$  | 0.045262            | 0.044020             |
| 0.3  | 0.2   | $\pi/6$  | 0.061854            | 0.060238             |
| 0.6  | 0.2   | $\pi/6$  | 0.120552            | 0.120325             |
| 0.22 | 0.5   | $\pi/6$  | 0.115245            | 0.110179             |
| 0.22 | 0.8   | $\pi/6$  | 0.176334            | 0.176133             |
| 0.22 | 0.2   | $\pi/4$  | 0.046625            | 0.044236             |
| 0.22 | 0.2   | $\pi/3$  | 0.046246            | 0.044253             |
The velocity enhances with increasing thermal Grashof number $Gr$ or mass Grashof number.

The resultant velocity enhances with increasing the permeability parameter $K$ throughout the fluid region. Lower the permeability of the porous medium lesser the fluid speed in the entire region.

The reversal behaviour is observed with increasing Schmidt number $Sc$, chemical reaction parameter $K_c$, suction parameter $v_0$ or the frequency of oscillation $\omega$.

The magnitude of the temperature of the flow field diminishes as the Prandtl number, Heat source parameter $S$ or suction parameter $v_0$ or the frequency of oscillation.

The concentration reduces at all points of the flow field with the increase in the Schmidt number $Sc$, chemical reaction parameter $K_c$, suction parameter $v_0$ and presence of the frequency of oscillation $\omega$.

The skin friction increases with the increase in $K Gr$, $Gm$ and $v_0$ and decreases with the increase in $M$, $Pr$, $Sc$, $K_c$, $S$ and $\omega$.

Nusselt number (Nu) at the surface of the plate and amplitude increase with increase $S$, $Pr$ and $v_0$. Also enhance the amplitude, the rate of heat transfer decrease the phase angle with increase the frequency of oscillations $\omega$.

Schmidt number $Sc$, chemical reaction parameter $K_c$, the frequency of oscillations $\omega$ and suction parameter $v_0$ increase the amplitude, the phase angle the rate of mass transfer at the surface of the plate.

Increase the frequency of oscillations $\omega$, enhance the amplitude, the rate of mass transfer decrease the phase angle.

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