An Optimal Sinsing Algorithm for Multiband Cognitive Radio Network

Gamal A. F. M. Khalaf
Department of Electronics, Communications and Computer Engineering, Faculty of Engineering, Helwan, Cairo, Egypt

ABSTRACT
In cognitive radio networks, the biggest problem preventing optimum performance is the optimal setting of individual decision thresholds while keeping the interference on the primary network bounded to a reasonably low level. In this paper, the multiband spectrum sensing problem is formulated as a class of optimization problem in which the opportunistic aggregate throughput is maximized subject to practical constraints. In order to achieve this, the detection problem is formulated based on the eigen-decomposition technique. Analytic and simulation results have indicated that the system being proposed achieves optimal opportunistic throughput for the secondary users and guarantee an acceptable level of interference to the primary users.

Keyword: Cognitive Radio Multiband Optimal Sensing

1. INTRODUCTION
With the rapid growth in wireless applications, spectrum resource becomes scarce. Although the current static spectrum management avoids interference effectively, this comes with the price of very low spectrum utilization. While some frequency bands are overcrowded, others are rarely used. Cognitive radio (CR) promises to increase the utilization of frequency bands that are under-utilized by providing opportunistic spectrum access. This is done through authorizing the unlicensed users (secondary users, SUs) to access the band assigned to the licensed user (primary users, PUs) when it is unoccupied. Spectrum sensing plays an essential role in cognitive radio since SUs need to detect PU's signals in order to make decisions about the occupancy of the spectrum bands. When a wide-band spectrum is assigned to a number of PUs, SUs can search for unoccupied channels (spectrum holes) within the wide-band spectrum and communicate in that band. The way to detect holes in a spectrum is channel-by-channel scanning. In order to implement this, an RF front-end with a bank of tunable and narrow band pass filters is needed. The occupancy of each channel can be determined by comparing the signal energy at the output of each filter with a given threshold level. However, in a CR network (CRN), the biggest problem preventing optimum performance is the optimal setting of individual decision thresholds while keeping the interference with the primary network bounded to a reasonably low level.

The remainder of the paper is organized as follows. Section 2, presents some researches that are related to the multiband sensing problem. In section 3, the detection problem is formulated based on eigen-decomposition. An algorithm that searches for detection thresholds to maximize the opportunistic throughput is formulated in section 4. In Section 5, we numerically evaluate the algorithms presented in this paper and finally, conclusions are drawn in Section 6.
context of spectrum sensing, various algorithms have been proposed. Basically, they can be categorized into three general groups, energy detection [2], coherent detection [3] and cyclostationary feature detection [4]. While energy detectors are a popular choice due to their simplicity, but it is not robust to noise-level uncertainty [5]; the induced sensitivity to threshold setting is a characteristic common to many energy detectors [6, 7]. Due to this, accurate estimation of noise variance becomes an important task in spectrum sensing, which is hampered by the fact that it is unknown whether the PU’s signal is present or not at the sensing time.

When it comes to wideband spectrum sensing [8], the authors in [9] have considered a wideband sensing scheme by tuning a narrowband band pass filter on the secondary user's RF front-end in order to sense one channel at a time. There also have been studies on sensing different frequency bands simultaneously [10, 11, 12]. In [13], a wavelet approach was suggested to estimate the power spectral density of the received signal and decompose it into non-overlapping subbands by taking advantage of irregularities in the frequency domain.

In this paper, the multiband spectrum sensing problem is formulated as a class of optimization problem in which the opportunistic aggregate throughput is maximized subject to practical constraints. The objective is to find threshold values that maximize the aggregate throughput. In this respect, signals received at the CR’s antenna are fed into number of filters within the wide-band spectrum. Then, signal energy levels over multiple channels are evaluated and compared with the thresholds in order to detect the spectrum holes. The performance of this approach has been studied through analytic and simulation validations for different number of channels.

3. DETECTION ALGORITHM

Consider a CRN wherein PUs employ Frequency Division Multiplexing with fixed channelization known to the spectrum monitor. Several primary channels are sensed simultaneously by selecting a band containing K such channels, down converting the signals to basebands and sampling each baseband signal at F_s samples/s, thus obtaining N complex-valued samples per channel. Let r(t) be the continuous-time received signal of a given channel at the SU’s receiver after unknown channel with unknown flat fading. The received signal samples are:

\[ r(n) = r(nT_s), \quad n=0, 1, 2, \ldots \]

In order to detect PU’s signals, we have two hypotheses:

\[ H_0: \quad r(n) = w(n), \]
\[ H_1: \quad r(n) = s(n) + w(n) \]

where \( s(n) \) is the (noiseless) samples of the received PU’s signal and is non-white wide-sense stationary (WSS) model which is more appropriate for signal samples, \( w(n) \) is zero-mean white Gaussian noise. Two probabilities are of interest to evaluate sensing performance: detection probability, \( P_d(H_1 | r(n) = s(n) + w(n)) \) and false alarm probability, \( P_f(H_1 | r(n) = w(n)) \). Assume spectrum sensing is performed upon the statistics of the r-th sensing segment, \( \Gamma_r \), consisting of the following observation vector,

\[ r_i = \{ r(i), r(i + 1), \ldots, r(i + N) \}^T, \quad i=1, 2, \ldots K- \text{subband} \]

where \( (.)^T \) denotes transpose. The sample covariance matrix of \( r_i \) is given by,

\[ R_i = E\{r_i r_i^T\} \]

The eigen-decomposition of \( R_i \) is:

\[ R_i = \{ \phi_1 A_i \phi_1^T, \}
\]

where

\[ \phi_i = \{ \phi_{1,i}, \phi_{2,i}, \ldots, \phi_{N,i} \}, \]

and

\[ A_i = \text{diag}\{ \lambda_{1,i}, \lambda_{2,i}, \ldots, \lambda_{N,i} \}, \]

with diag(.) designates diagonal of matrix \( A_i \), \( \lambda_{ij} \) is the jth eigenvalue of \( R_i \) satisfying \( \lambda_{1,i} > \lambda_{2,i} > \ldots > \lambda_{N,i} \) and \( \phi_{ij} \) is the jth eigenvector corresponding to eigenvalue j. Since \( s(n) \) and \( w(n) \) are uncorrelated, the distribution of received signal vector, \( r_i \), under the two hypothesis can be represented as:
where $R_s$ and $\sigma^2$ are, respectively, the signal covariance and noise variance with $I$ denoting the identity matrix. Assuming that the detection is based upon the statistics of the sensing vectors $r_i$ in $\Gamma_r$, and that $\{ r_i \}, i = 1, ..., N$ are i.i.d., then the likelihood function for $\Gamma_r$ under $H_0$ condition can be:

$$P(\Gamma_r | H_0) = \prod_{i=1}^{N} P(r_i | H_0)$$

$$= \prod_{i=1}^{N} \frac{1}{(2\pi \sigma^2)^{N/2}} \exp\left( -\frac{1}{2\sigma^2} r_i^T r_i \right),$$

and, the likelihood function for $\Gamma_r$ under $H_1$ condition is:

$$P(\Gamma_r | H_1) = \prod_{i=1}^{N} P(r_i | H_1)$$

$$= \prod_{i=1}^{N} \frac{1}{(2\pi)^{N/2} \det^{1/2} (R_s+\sigma^2 I)} \exp\left( -\frac{1}{2} r_i^T (R_s+\sigma^2 I)^{-1} r_i \right),$$

In signal detection, it is desired to design an algorithm maximizing the $P_d$ for a given $P_f$. According to Neyman-Pearson theorem, this can be done by the likelihood ratio test (LRT):

$$L(\Gamma_r) = \frac{P(\Gamma_r | H_1)}{P(\Gamma_r | H_0)}$$

Wherein, $H_1$ is true if $L(\Gamma_r)$ is greater than a threshold value $\gamma$, which is determined by desired $P_f$. In this context, we adopt the assumption that the minimum eigenvalue, $\lambda_{1,N}$, actually gives an adequate estimation of the noise power and that, under $H_1$ hypothesis, the ratio $\frac{\lambda_{1,N}}{\lambda_{1,N}}$ provides a good estimator of the SNR. These observations have, actually, been used successfully in system identification [14, 15] as well as in direction of arrival estimation [16]. Moreover, the average of the eigenvalues, $\bar{\lambda}_{1,n}, n = 1,2, ..., N - 1$, is almost the same as the signal energy. Therefore, the following ratio will be used in our hypothesis testing procedures,

$$H_1 \text{ is true if } \frac{\lambda_{1,N}}{\lambda_{1,N}} > \gamma.$$

The difference between the conventional energy detection and the approach we have adopted here is that, not like the energy detection which needs to estimate the noise power for its hypothesis testing procedures, our approach significantly alleviates this difficulty by adopting the ratio in (equ. 14) which can be calculated directly from the covariance matrix of the observations.

### 4. OPTIMAL SENSING ALGORITHM

In CRNs, there are two parameters associated with spectrum sensing: probability of detection and probability of false alarm. The higher the probability of detection, the better the PUs is protected. However, from the SU’s perspective, the lower the probability of false alarm, the more chances the channel can be reused when it is available, thus the higher the achievable throughput for the secondary network. An example on CRN is plotted in Fig. (1). It illustrates the spectrum of a typical cognitive system in the range of (0, 2)GHz with three primary users respectively, a TV channel (470-500) MHz, GSM-900 uplink (824-849) MHz and downlink (869-894) MHz and a DECT system (1880-1900) MHz.

In the following, the wideband spectrum sensing problem is formulated as an optimization problem in which the opportunistic aggregate throughput is maximized subject to practical constraints. More specifically, there are two scenarios for which the SUs can operate at the PU’s frequency bands.

- **Scenario I:** When the PU is not present and no false alarm is generated by the SU, the achievable throughput of the secondary link is denoted by $C_{0}$.
- **Scenario II:** When the PU is active but it is not detected by the SU, the achievable throughput of the secondary link is denoted by $C_{1}$. 

The probabilities for which Scenarios I or II can happen are 

\[ (1 - P_1) P(H_0) \] and 

\[ (1 - P_d) P(H_2). \]

Then the average throughput for the SU is given by,

\[ T = C_0 (1 - P_1) P(H_0) + C_1 (1 - P_d) P(H_2). \] (15)

Assuming that \( C_0 > C_1 \), the first term in the right hand side of (15) dominates the achievable throughput. Therefore, the average throughput can be approximated by,

\[ T = C_0 (1 - P_1) P(H_0) \] (16)

![Fig.(1), Spectrum of Cognitive Radio (example).](image)

Consider now a cognitive radio sensing a given band in order to opportunistically utilize the unused subbands for transmission. Let \( T^K(\gamma^K) \) denotes the throughput achievable over the k-th subband and, let \( (1 - P^K_d(\gamma^K)) \) designates the opportunistic spectrum utilization of subband k as function of its corresponding sensing threshold, \( \gamma^K \). Therefore, the aggregate, opportunistic, throughput is defined as follow,

\[ T(\gamma) = \sum_{k=1}^{K} T^K(\gamma^K) \] (17)

with,

\[ T^K((\gamma^K) \equiv C_0 (1 - P^K_d(\gamma^K)) \]

Consequently, the optimal sensing algorithm can be formulated as the follow constraint optimization problem,

\[
\begin{align*}
\text{Maximize} & \quad T(\gamma) \\
\text{s. t.} & \quad \sum_{k=1}^{K} C_1 \left(1 - P^K_d(\gamma^K)\right) \leq \varepsilon
\end{align*}
\] (18-19)

where the constrain \( \left(1 - P^K_d(\gamma^K)\right) \) signifies the interference induced into the PUs from CR-terminal-k, hence causing harmful effects and, \( \varepsilon \) designates the acceptable upper bound of such effect. As can be seen, the solution to problem (18) results in an optimal set of individual decision thresholds while keeping the interference to the PUs bounded to the predefined level, \( \varepsilon \). Of importance here to note that, the aggregate throughput in (18), is seen to be convex for the range of \( \{\gamma^K\} \) such that, \( \{0.5 > P^K_F\} \) and \( \{0.5 > (1 - P^K_d)\} \) [17].

In solving the constraint optimization problem (18-19), an exhaustive search algorithm is needed in order to find the optimal set of sensing thresholds. Alternatively, we rewrite the problem (18-19), making the inequality constraints implicit in the objective such that the problem is converted into another unconstraint optimization problem that lends itself to simpler numerical search algorithms. This is carried out by first multiplying the objective function (18) by \( t \), and considering the following equivalent form [18],

\[ \text{Maximize} \quad T(\gamma) t \]

\[ \text{s. t.} \quad \sum_{k=1}^{K} C_1 \left(1 - P^K_d(\gamma^K)\right) t \leq \varepsilon \]
Maximize \( t \, T(\gamma) + \phi(\gamma) \) \hspace{1cm} (20)

where \( \phi(\gamma) = -\frac{1}{t} \sum_{k=1}^{K} \log (\varepsilon - C_1(1 - P_t^K(\gamma^k))) \), \hspace{1cm} (21)

which has the same maxima. The function \( \phi(\gamma) \) is known as the logarithmic barrier which is convex and its domain is the set of points that satisfy constraints (19). No matter what value the positive parameter \( t \) has, the logarithmic barrier grows without bound as long as \( \phi(\gamma) \rightarrow 0 \). Of course, the problem (20-21) is only an approximation of the original problem. In the next section, the performances of the proposed algorithms are tested.

5. NUMERICAL VALIDATIONS

In this section, analytic and simulation results are presented in order to evaluate the sensing-throughput tradeoff for the multiband detection problem. Subbands are characterized by their respective statistical parameters including the set of (optimal) thresholds and SNRs, \( \{ \gamma^k, \text{SNR}^k, k = 1, 2, 3, ... \} \). On the other hand, use has been made of the average to minimum eigenvalues ratio of the covariance matrix of the received signals which is seen to serve well as our criteria for hypothesis testing implementation (i.e., equ. 14). In the simulation course, a multipath fading channels and additive complex white Gaussian noise (AWGN) with zero mean and variance: \( N(0, \sigma^2) \) are considered. Moreover, in order to make fair comparisons, all signal (noise) observations are normalized to a unit variance by pre-multiplication by \( d = \frac{\sigma \sqrt{N}}{m} \) where \( m, \sigma^2, N \) are, respectively, the mean, variance and size of the observation samples.

Let us start validations by, first, plotting \( P_f(\gamma) \), for different observation statistics as shown in Fig. (2). In this, "un-optimized thresholding" case, the hypothesis testing is based on the following thresholds [19]

\[
\frac{1}{2} \sum_{k=1}^{K} \frac{\ln \gamma^k}{d^k} + \frac{d^k}{2}
\]

As can be seen, large threshold values result in small false alarm probabilities and hence, larger missing probabilities which is known to harm badly the PUs and reduces their, presumably, guaranteed protections. Next, a channelized band consisting of 7-subbands is simulated. Then, the algorithm which searches for the optimal thresholds that maximize the objective function (equ. 18) is added to the simulation model. In this case, the missing probabilities are assumed to be upper bounded by \( \varepsilon=0.1: \{ (1 - P_m^k(\gamma^k)) < 0.1, k = 1, 2, ..., 7 - \text{subbands} \} \). Fig. (3), depicts the set of optimal thresholds within the same framework of the present simulations. As can be seen, the optimization algorithm compromises the false alarm probabilities for a guaranteed missing probability, \( P_m \), limit over all the 7-subbands. The fundamental tradeoff between \( P_m \) and \( P_f \) has a significant implication in the context of opportunistic spectrum access: a high \( P_m \) results in missing the presence of primary users with high probability, which in turn increases the interference inflicted on the PUs.

![Fig.(2), Un-Optimized Thresholding.](image-url)
On the other hand, a high $P_f$ inevitably results in low spectrum utilization since false-alarms increase the number of missed access opportunities (white spaces).

In the second phase of validations, we extend the previous results by presenting the impact of optimal "thresholding" on the performance of the opportunistic spectrum access by SUs. Assume a cognitive terminal (CT) moving in a given area, observing the 'external world' by sensing a number of subbands, searching for radio resources in order to increase its access opportunities. Each individual subband is characterized by different statistical parameters and the CT searches for optimal thresholds in order to maximize its available throughput within the same framework presented above. In order to evaluate the overall performance of such a scenario, we assume that measurements of different subbands are statistically independent, and that the aggregate false alarms is given by,

$$P_f = \sum_{k=1}^{K} \frac{p^K(y_k)}{p^K(y)} P_T(y)$$  \hspace{1cm} (23)

Once again, assuming that $C_0 > C_1$, and that the first term in the right hand side of (equ.15) dominates the overall achievable throughput. Therefore the average throughput can be approximated by, $(1-P_f)$. Fig. (4), plots the aggregate missed (access) throughput in the secondary network versus the averaged (optimal) threshold. It is evident in the figure that our, proposed, framework achieves a superior performance (minimal missed access) compared to the non optimized case. More importantly, is the observation which is
notable here: Other than the detection-threshold optimization, it is evident that increasing the number of subbands (from 4 to 7 in this case) improves the performance even more.

Another validation result is depicted in Fig. (5), which shows the opportunistic throughput versus averaged sensing thresholds. As can be seen, our algorithm maximizes the available throughput by joint optimization of the detection thresholds in response to variations in channel conditions. This observation reveals that, our framework provides an "optimally adaptive" spectrum sensing scheme for a time-varying environment wherein cognitive radio networks will most likely operate.

6. SUMMARY AND CONCLUSIONS

In the context of multiband opportunistic spectrum access in cognitive radio networks, the probability of missing determines the level of interference-protection provided to the primary users, while the probability of false-alarm is the percentage of white spaces (free bands) falsely declared occupied (i.e. the percentage of missed opportunities). Therefore, a sensible design criterion is to minimize the probability of false alarm while guaranteeing that probability of missing remains below a certain acceptable threshold. This, in effect, presents the baseline for the work carried out in this paper. In order to accomplish this criterion, a detection problem is formulated based on eigen-decomposition technique. Then, an algorithm that searches for thresholds to maximize the opportunistic throughput is formulated by converting an inherently constraint optimization problem into unconstraint one in order to simplify the computations needed to implement such a system in real life. The performance of the proposed system is validated via a simulation model which is built for the environment under consideration. Analytic and simulation results have indicated that the system being proposed achieves optimal opportunistic throughput for the secondary users of cognitive radio network while guarantee an acceptable level of interference to its primary users.

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**BIOGRAPHY OF AUTHORS**

Prof. Gamal A. F. M. Khalaf is working as professor of Computer Communication Networks at Electronics, Communications & Computer Engineering Department, Faculty of Engineering, at Helwan, Cairo, Egypt. He has taught various subjects such as stochastic processes, detection, estimation models, etc. at Postgraduate Level. He is the supervisor of several Ph. D, and M.Sc. theses. He worked as Head of Electronics, Communications & Computer Engineering Department in 2010-2012.