Research Article

Stiffness and Crack Behavior of Unbonded Posttensioned Concrete Beam Strengthened with Aluminum Alloy Plate

Hong Chang and Wei Zhou

1School of Civil Engineering, Harbin Institute of Technology, Harbin, China
2Key Lab of Structures Dynamic Behavior and Control, Ministry of Education, Harbin Institute of Technology, Beijing, China
3Key Lab of Smart Prevention and Mitigation of Civil Engineering Disaster, Ministry of Industry and Information Technology, Harbin Institute of Technology, Beijing, China

Correspondence should be addressed to Wei Zhou; zhouwei-hit@163.com

Received 14 December 2019; Revised 11 July 2020; Accepted 4 September 2020; Published 30 September 2020

Copyright © 2020 Hong Chang and Wei Zhou. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Corrosion resistance of aluminum alloy plates externally bonded by magnesium phosphate cement provides the ability to strengthen inshore infrastructures in harsh environments subject to moisture and humidity. In this study, the aim is to study the stiffness and cracking behavior of concrete beams using this strengthening technique. Six damaged unbonded posttensioned concrete beams were repaired and strengthened and then subjected to monotonic load until failure. This technique improved the stiffness and limited the development of cracks. The formula of elastic-plastic stiffness coefficient related to the comprehensive reinforcement index was established. An influence coefficient $\delta$ considering the effect of aluminum alloy plates and unbonded tendons was introduced, and the crack expansion coefficient under short-term load was obtained by statistical analysis. Finally, some simplified methods were proposed to evaluate the stiffness and cracks of unbonded posttensioned concrete beams strengthened with aluminum alloy plates.

1. Introduction

Numerous studies have verified the mechanical properties of structures strengthened by thin steel plates and fiber reinforced polymer (FRP) sheets, and the research results have contributed to the development of specifications, codes, and standards. Meanwhile, these techniques have been used widely in various infrastructures [1–8]. However, the corrosion of steel in wet or humid conditions can decrease its strengthening effect. In addition, the brittle failure characteristics of FRP does not meet the ductile requirements of concrete structures. As a strengthening material, there are two advantages of aluminum alloy (AA) compared to steel and FRP: (1) it has a similar tensile strength to steel, and lower density and (2) it has a similar corrosion resistance to FRP, and higher ductility. In addition, epoxy used as an adhesive for structural strengthening behaves poorly under high temperature and aging, and has a pungent odor. Magnesium phosphate cement (MPC) has the advantages of high strength, low shrinkage, high temperature resistance, aging resistance, and nontoxicity, and therefore has the potential to replace epoxy as the adhesive for structural strengthening in harsh environments [9, 10].

Previous studies [11–14] have shown experimentally that the technique of strengthening with AA plates can effectively improve the flexural capacity of reinforced concrete (RC) beams, with better ductility than that of beams strengthened with FRP. The RC beams strengthened by this technique have the behavior of debonding under the loading point, which is similar to the behavior shown by FRP sheet and steel plate and causes the premature failure of strengthened beams. The strengthening technique of near-surface mounting with AA plates and external bonding using AA plates with anchoring can effectively avoid early failure of strengthened beams caused by debonding of AA plates. The formula of ACI 440.2R-08 is found to be feasible to predict
the debonding strain of the beams strengthened by AA plates. Epoxy was used as the adhesive for AA plates in most previous studies. Researchers have applied MPC to the interface between AA plates and concrete, studied the behavior of the interface through bilateral shear tests, and proposed the constitutive model of the bond-slip relationship [15]. The feasibility of using MPC as the adhesive bonding AA plates to strengthen unbonded posttensioned concrete (UPC) beams has been proved [16]. The ductility of the strengthening beam decreased while the flexural capacity was effectively improved. Studies on stiffness and cracking of UPC beams strengthened by AA plates bonded with MPC have not yet been carried out.

In this study, an investigation was carried out on the stiffness and cracking of UPC beams strengthened with AA plates bonded by MPC. Six damaged UPC beams were repaired and strengthened, and then loaded to failure under a monotonic load, and attention was specifically paid to the deflection and cracks of the beams. Finally, a calculation method of stiffness and cracking of UPC beams strengthened with AA plates has been proposed based on experimental results and available theory.

2. Specimens and Methods

2.1. Specimens. A total of six damaged UPC beams were repaired and strengthened. Figure 1 shows the dimensions, reinforcement details, cross-section details, and unbonded prestressed details of the specimens. Each beam has a length of 6000 mm and different reinforcement ratios. The properties of specimens are shown in Table 1.

The reinforcement index of the non-prestressed reinforcements $\beta_p = f_y A_s / f_y b h_p$, that of the unbonded tendons $\beta_p = \sigma_p A_s / f_y b h_p$, that of the AA plate $\beta_a = \sigma_a A_a / f_y b h$, that of the compression reinforcement $\beta_c = f_y A_c / f_y b h_p$, and the composite reinforcement index $\beta_0 = \beta_p + \beta_p + \beta_a - \beta_c$, where $b$ is the width of the UPC beams; $h_a$, $h_p$, and $h_b$ are the distance from the top face to the centroid of the non-prestressed reinforcement, unbonded tendons, and AA plate, respectively. $f_y$, $\sigma_p$, $\sigma_a$, and $f_y'$ are the yield stress of non-prestressed reinforcement, unbonded tendons, AA plate, and compression reinforcement, respectively.

All six UPC beams were damaged by static loads, followed by the crushing of the concrete in the compression zone. All the damaged beams exhibited relatively small residual deformation, and most of the tensile cracks were closed. The reinforcement yielded before the concrete in the compression zone crushing. The initial damage of each damaged UPC beam is shown in Table 2.

The diameter of the stirrup of the specimens was 8 mm, which was distributed in the range of 2000 mm at the end of the beams with a spacing of 100 mm (see Figure 1). The stainless-steel bolts with a diameter of 10 mm were anchored into the hole with a depth of 50 mm at the bottom of the beams with MPC. The uses of the stainless-steel bolts are as follows: (1) to bear the interfacial shear force when the bonding fails; (2) to avoid the debonding of the AA plates before the failure of strengthened beams; (3) to apply uniform pressure to the AA plates in the setting process of MPC to improve the adhesion effect. Considering the above factors, it was determined that the stainless-steel bolts would be staggered with a spacing of 330 mm (see Figure 2).

2.2. Properties of Materials. Concrete with grade 40 was used to cast the beams. P.O 42.5 Portland cement and class F fly ash were used as binders, with a water-binder ratio ($w/b$) of 0.3. Sand particles in zone II were used as the fine aggregate, and crushed rocks were used as coarse aggregate. The axially compressive strength of the concrete was 45 MPa. The elastic modulus of the concrete $E_c$ was 33600 MPa.

Steel strands of grade 1860 with a diameter of 17.8 mm were used as unbonded tendons. Their tensile strength was 1915 MPa, and nominal yield strength was 1732 MPa. The mechanical properties of non-prestressed reinforcements are shown in Table 3.

A unidirectional tensile test of 5083 AA plates was carried out on a universal electronic test machine (see Figure 3), which showed that its yielding strength was 112 MPa, ultimate strength was 210 MPa, elastic modulus $E_a$ was 278000 MPa, and ultimate elongation was 27%.

MPC is composed of ammonium dihydrogen phosphate or potassium dihydrogen phosphate, magnesium oxide, and a setting retarder. Borax was used as a retarder agent in this experiment, and the mix ratio of MPC by weight was ammonium dihydrogen phosphate $(\text{NH}_4\text{H}_2\text{PO}_4)$: borax $(\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O})$: magnesium oxide $(\text{MgO})$: water $= 26 : 4 : 51 : 19$. The compressive strength and flexural strength of the MPC were 23.5 MPa and 7.5 MPa, respectively.

2.3. Experimental Designs. Two-point loads were applied at the one-third span points of the beam (see Figure 4). Three displacement sensors were set under the loading points and the midspan of the beam, two sensors were located at the end of the AA plate to measure the relative slip between the AA plate and the beam, and another two were set above the supports to measure the beam end movement. The AL beams were loaded with 5 kN for each level, and the AS beams with 3 kN for each level.

2.4. Failure Modes. Among the six specimens, the failure mode of SS3 was the sudden fracture failure, and the failure of the other five beams can be attributed to concrete crushing in the compression zone. The details are as follows.

3. Stiffness Analysis

3.1. Relationship between Load and Deflection. The typical curve of load-deflection of prestressed concrete beams should have obvious turning at the points of cracking of beams and yielding of non-prestressed reinforcements. In this study, the test beams have been damaged and repaired, which has a certain impact on the stiffness of strengthened beams. As a result, the stiffness modification of strengthened beams was not obvious, as shown in Figure 5. The attached figures show the characteristics of a trilinear model with the inflection points of the crack load,
yield load, and ultimate load of strengthened beams. It can be observed from Figure 5 that the curves still have similar characteristics to those of the prestressed concrete beams.

The three dotted lines were divided into three stages, with the slope of each segment being the stiffness of the corresponding stage.

### Table 1: Properties of specimens.

| No. | Cross-sectional area (mm × mm) | Steel reinforcements | Steel strand |
|-----|-------------------------------|----------------------|--------------|
| SL1 | 200 × 400                     | 2C12                 | 1A³17.8      |
| SL2 | 200 × 400                     | 2C18                 | 1A³17.8      |
| SL3 | 200 × 400                     | 2C25                 | 1A³17.8      |
| SS1 | 200 × 300                     | 2C12                 | 1A³17.8      |
| SS2 | 200 × 300                     | 2C18                 | 1A³17.8      |
| SS3 | 200 × 300                     | 2C22                 | 1A³17.8      |

### Table 2: Initial damage of each damaged beam.

| No. | Cracking load (kN) | Ultimate load (kN) | Maximum crack width (mm) | Number of cracks | Initial prestress (MPa) |
|-----|--------------------|--------------------|--------------------------|------------------|------------------------|
|     |                    |                    |                          | Front | Back      |                          |
| SL1 | 80                 | 204                | 5                        | 13    | 12        | 1570                     |
| SL2 | 60                 | 180                | 1                        | 16    | 16        | 1570                     |
| SL3 | 44                 | 200                | 0.3                      | 18    | 20        | 1570                     |
| SS1 | 40                 | 84                 | 2                        | 17    | 16        | 1361                     |
| SS2 | 30                 | 104                | 0.2                      | 19    | 18        | 1570                     |
| SS3 | 34                 | 118                | 0.4                      | 20    | 20        | 1570                     |
Stage I is the stage before the concrete cracks. The beam behaves mainly elastically. The relationship between stress and strain is basically linear. This stage continued until the tensile stresses exceeded the tensile strength of concrete.

Stage II is the serviceability stage that occurs after the concrete has been cracked and non-prestressed reinforcements take up almost all the tension force, but the non-prestressed reinforcements have not yet yielded. The neutral axis shifts upward with the increase of the applied loads.

Stage III starts with the yielding of non-prestressed reinforcements and ends with the failure of the beam. In this stage, the stiffness of the beam is weakened further. The load-deflection relationship becomes clearly nonlinear. Finally, the excessive developing of cracks and the crushing of concrete in the compression zone lead to the ultimate collapse of the beam.

Figure 5 shows that the stiffness changes of the SL1 and SS1 beams with the lowest reinforcement ratio are obvious at
Figure 5: Load-deflection curves: (a) SL1; (b) SL2; (c) SL3; (d) SS1; (e) SS2; (f) SS3.
the crack load, while those of other beams are not obvious. The behavior at the yield load is opposite to that at the crack load; the stiffness of the SL1 and SS1 beams does not change significantly at the yield load, while the stiffness of other beams changes significantly at the yield load. It can be concluded that (1) the influence of cracking on the stiffness is inversely proportional to the reinforcement ratio and (2) the influence of the yielding of non-prestressed reinforcement on the stiffness is proportional to the reinforcement ratio.

Figure 6 shows the failure modes of the specimens.

3.2. Stiffness Calculation. A formula for calculating the deflection of homogeneous elastic materials is proposed in [17], \( f = \frac{aM^2}{E_iI_0} \), but the concrete is a heterogeneous and inelastic material, so the bending stiffness of the interface changes during bending. The bending stiffness not only decreases with the increase of load but also decreases with the increase of load duration, so the reduction coefficient, namely, the elastic-plastic stiffness coefficient, should be considered. According to the empirical values used in the Chinese design standards, the elastic-plastic stiffness coefficient of concrete beams before cracking is 0.85, and the elastic-plastic stiffness coefficient after cracking \( \beta' \) needs further research.

According to the bilinear stiffness model, the bending moment-deflection curve of prestressed concrete beams can

![Figure 7: Bending moment-deflection curve of the prestressed beam.](image-url)
be regarded as consisting of two straight sections OA and AB. With the cracking point as the turning point, the deflection under the cracking moment \( M_{cr} \) is \( f_{cr} \), and the deflection under the serviceability moment is \( f_s \) as shown in Figure 7.

According to Figure 5, \( f = f_{cr} + f_s = a \frac{l^2}{E_c I_0} ((M_{cr}/0.85) + (M_s - M_{cr}/\beta')) \).

The elastic-plastic stiffness coefficient under the increment of bending moment \((M_s - M_{cr})\) is then obtained as

\[
\beta' = a \frac{l^2}{(f_s - f_{cr})E_c I_0} M_s - M_{cr}
\]  

(1)

where \( a \) is the coefficient of load and support conditions, which according to the material mechanics, in this case, is 0.106.

In order to study the elastic-plastic stiffness coefficient \( \beta' \) after cracking, it is necessary to analyze the bending moment increment, and \( M_s \) is calculated as \(1.67M_{cr} \). It was found by calculation that when the \( M_s \) of the SL1 beam was 150 kN, the non-prestressed reinforcements have yielded, so no research was conducted on the SL1 beam. According to the experimental data of the other five strengthened beams, the elastic-plastic stiffness coefficient was calculated in Table 4.

By analysis of the experimental data, it was found that there was an approximate linear relationship between \(1/\beta' \) and \(1/\beta_0 \) and the fitting equation is as equation (2) with a correlation coefficient of 0.9588. Figure 8 shows the relationship between experimental data and the fit line:

\[
\frac{1}{\beta'} = 2.3 + \frac{0.52}{\beta_0}
\]  

(2)

Thus,

\[
\beta' = \frac{\beta_0}{2.3\beta_0 + 0.52}
\]  

(3)

The stiffness of a UPC beam changes during bending; therefore, in order to accurately calculate the deformation of the beam under short-term load, an appropriate average stiffness \( B_s \) is proposed as the short-term stiffness. Thus, on the basis of \( f = aMl^2/B_s \), the short-term stiffness of UPC beams strengthened by AA plates can be expressed as

\[
B_s = \frac{0.85\beta' M_s E_c I_0}{\beta' M_{cr} + 0.85(M_s - M_{cr})}
\]  

(4)

3.3. Prediction and Analysis of Deflection. A UPC beam can be regarded as equivalent to an RC beam with a pair of prepressure in the anchorage position, which still show the characteristics of an RC beam after cracking. Therefore, the stiffness of UPC beams strengthened with AA plates can be measured using the same research method as the RC beams.

The stiffness under short-term load of the strengthened beam was constant before cracking, so the stage before cracking has not been investigated, and the deflection under short-term load of five strengthened beams, apart from SL1, was analyzed.

### Table 4: Calculation results of elastic-plastic stiffness coefficient.

| No. | \( M_{cr} \) (MPa) | \( f_{cr} \) (mm) | \( M_s \) (MPa) | \( f_s \) (mm) | \( \beta' \) |
|-----|------------------|-----------------|----------------|---------------|---------|
| SL2 | 65               | 16              | 108            | 31            | 0.19    |
| SL3 | 60               | 11              | 100            | 19            | 0.23    |
| SS1 | 45               | 28              | 75             | 62            | 0.18    |
| SS2 | 42               | 20              | 70             | 40            | 0.22    |
| SS3 | 39               | 19              | 65             | 34            | 0.23    |

![Figure 8: Relationship between 1/\beta' and 1/\beta_0.](image)

The value of \( B_s \) is substituted into equation (4) to calculate the deflection of the strengthened beam after cracking, and the calculation results are shown in Table 5. Although most of the deflection of UPC beams in the initial period of loading was restored by the prestress, there were still some residual deflection that was not restored, which has been taken into account.

The calculation errors by the method proposed in this study were all within 10%, so the calculation method is applicable to the stiffness calculation of UPC beams strengthened by AA plates.

4. Crack Analysis

4.1. Mean Crack Spacing. The crack development mechanism of UPC beams is similar to that of RC beams. When the concrete in the tensile zone cracks, the concrete is no longer in tension, but the non-prestressed reinforcement continues to undergo tension. Due to the difference in the elastic modulus of concrete and steel, the tension causes a relative slip between the non-prestressed reinforcements and the concrete bonded with them, which causes the development of cracking. During this process, there is no bond force between the unbonded prestressed tendons and concrete, and the tensile stress of concrete in the tensile zone is offset by the prestress of unbonded prestressed tendons in the initial period of loading. Prestress can delay the occurrence of cracking in the concrete, but it cannot control the distribution of cracks, which are mainly controlled by the relationship of bond-slip between the non-prestressed reinforcement and the concrete.
The experimental results show that the cracks in the tensile concrete of the strengthened beams are all the same as in the initial damage, and no new cracks appeared. The experimental data also show that there is a regular relationship between the mean crack spacing $l_{cr}$ and $d_{eq}/\rho_s$ and $t_a/\rho_a$, with the regression equation as follows:

$$l_{cr} = 0.042 \frac{d_{eq}}{\rho_s} - 0.033 \frac{t_a}{\rho_a} + 74.76. \tag{5}$$

The correlation coefficient of the above equation is 0.9538, where $d_{eq}$ is the equivalent diameter of the reinforcement, $t_a$ is the thickness of the AA plate, $\rho_s$ and $\rho_a$ are the reinforcement ratios of the non-prestressed reinforcement and AA plate, respectively.

Table 6 shows the results calculated by equation (5) and the experimental results. The calculated errors are all less than 5%, indicating that it is suitable for calculating the mean crack spacing of UPC beams strengthened by AA plate.

### 4.2. Mean Crack Width

#### 4.2.1. Stress Increment of Unbonded Prestressed Tendons in Serviceability Stage.

In the initial loading, the stress in the unbonded prestressed tendons grew slowly. After the concrete in the tension zone cracked, the stress in the unbonded prestressed tendons grew quickly. The cracks then gradually developed, the stiffness gradually declined, and the deformation increased more quickly. Due to the different reinforcement ratio, initial prestress, and other parameters of each strengthened beam, the stress increment of the unbonded prestressed tendons could not be compared directly. In order to have a unified comparison standard for the strengthened beams, the relative values of the stress increment and prestress of each strengthened beam were analyzed. Based on the experimental results, it was found that there was a linear relationship between $\Delta\sigma_p/\Delta\sigma_{py}$ and $(M_k-M_{cr})/(M_y-M_{cr})$ in the serviceability stage, described as follows:

$$\frac{\Delta\sigma_p}{\Delta\sigma_{py}} = 0.583 \frac{M_k-M_{cr}}{M_y-M_{cr}} + 0.376. \tag{6}$$

The correlation coefficient of the fit line was 0.94, where $\Delta\sigma_p$ is the stress increment of unbonded tendons in the serviceability stage, $\Delta\sigma_{py}$ is the stress increment of the unbonded tendons when the non-prestressed reinforcements were yielding, $M_k$ is the bending moment in the serviceability stage, and $M_y$ is the bending moment when the non-prestressed reinforcements were yielding. The relationship between the fit line and the experimental data of each test beam is shown in Figure 9.

#### 4.2.2. Stress Increment of Unbonded Tendons When the Non-Prestressed Reinforcement Is Yielding.

In order to calculate $\Delta\sigma_p$, using equation (6), the equation governing $\Delta\sigma_{py}$ needs to be established.

### Table 5: Comparison of calculation results and experimental results of deflection.

| No. | $\beta'$ | Calculation results of deflection (mm) | Initial residual deflection (mm) | Experimental result of deflection (mm) | Error (%) |
|-----|---------|--------------------------------------|----------------------------------|--------------------------------------|-----------|
| SL2 | 0.19    | 79.5                                 | 15                               | 90.0                                 | 6.67      |
| SL3 | 0.23    | 63.5                                 | 5                                | 66.8                                 | 3.29      |
| SS1 | 0.18    | 95.4                                 | 20                               | 106.4                                | -8.08     |
| SS2 | 0.22    | 83.8                                 | 25                               | 119.0                                | 8.4       |
| SS3 | 0.23    | 72.0                                 | 15                               | 89.7                                 | 3.01      |

### Table 6: Calculation results and experimental data of mean crack spacing.

| No. | $\rho_s$ | $\rho_a$ | Experimental data of $l_{cr}$ (mm) | Calculation results of $l_{cr}$ (mm) | Error (%) |
|-----|---------|---------|------------------------------------|--------------------------------------|-----------|
| SL1 | 0.006   | 0.015   | 154                                | 157                                  | 1.9       |
| SL2 | 0.013   | 0.015   | 133                                | 128                                  | -3.9      |
| SL3 | 0.025   | 0.015   | 111                                | 111                                  | 0         |
| SS1 | 0.008   | 0.02    | 142                                | 137                                  | -3.7      |
| SS2 | 0.017   | 0.02    | 114                                | 114                                  | 0         |
| SS3 | 0.025   | 0.02    | 105                                | 106                                  | 0.9       |
Analysis of the experimental data shows that $\Delta \sigma_{py}$ has a regular relationship with $\beta_0$ and $\beta_s$, with the fitting equation as follows:

$$\Delta \sigma_{py} = \frac{11.04\beta_0 - 0.56}{\beta_s}.$$  \hspace{1cm} (7)

The fitting curve has a correlation coefficient of 0.9309. The relationship between experimental data and fit line is shown in Figure 10.

The results of $\Delta \sigma_{py}$ calculated by equation (7) and the experimental data are shown in Table 7. It can be seen that the error was within 10% except for the SS2 beam with large dispersion. Therefore, equation (7) is applicable to the calculation of the stress increment of unbonded tendons when the non-prestressed reinforcement is yielding.

Equation (6) can be translated into

$$\Delta \sigma_p = \left(0.583 \frac{M_k - M_{cr}}{M_y - M_{cr}} + 0.376\right) \Delta \sigma_{py}.$$ \hspace{1cm} (8)

When equation (8) is substituted into equation (7), the formula for $\Delta \sigma_{py}$ can be obtained as follows:

$$\Delta \sigma_p = \left(0.583 \frac{M_k - M_{cr}}{M_y - M_{cr}} + 0.376\right) \frac{11.04\beta_0 - 0.56}{\beta_s}.$$ \hspace{1cm} (9)

4.2.3. Bending Moment in Serviceability Stage. The serviceability stage of a UPC beam usually refers to the period between beam cracking and yielding of non-prestressed reinforcement. The cracking moment can be expressed as

$$M_{cr} = W_0 (\sigma_{pc} + f_{tk}),$$ \hspace{1cm} (10)

where $W_0$ is the bending modulus of the transformed section, $\sigma_{pc}$ is the prestress of the tensile edge of the concrete while deducting all losses, and $f_{tk}$ is the characteristic value of axial compressive strength.

The distribution of stress and strain of the cross section when the non-prestressed reinforcement was yielding is shown in Figure 11.

From Figure 11, the tensile force is equal to the resultant compression force:

$$0.5(A_t f_y + A_p \sigma_p + \sigma_a A_a) = E_c \varepsilon_c by.$$ \hspace{1cm} (11)

In equation (11),

$$\sigma_p = \sigma_{pc} + \Delta \sigma_{py}.$$ \hspace{1cm} (12)

The experimental data show that the yielding in the non-prestressed reinforcement was earlier than that in the AA plate. Therefore, when the non-prestressed reinforcement was yielding, the distribution of stress and strain in the cross section conforms to the plane-section assumption. So,
According to equations (11), (12), (13), and (15), the depth of the compression zone when the non-prestressed reinforcement was yielding can be obtained, and the yielding moment can be expressed as

\[ M_y = \left[ A_p f_y + A_s \left( \sigma_{pu} + \sigma_{ap} \right) \right] (h_0 - 0.5y) + A_s \sigma_a (h - 0.5y). \]  

(16)

4.2.4. Mean Crack Width. The distribution of stress and strain of the cracking section in the serviceability stage is shown in Figure 12.

For equilibrium, the following equation can be established:

\[ M_k - N_{pe} (z - e_p) = \sigma_A A_p \eta_s h_0 + a \Delta \sigma_p A_p \eta_p h_p + \sigma_a A_a \eta_a h, \]

(17)

where \( N_{pe} \) is the prepressure compression, \( \eta_s h_0, \eta_p h_p, \) and \( \eta_a h \) are, respectively, the internal force arms of the non-prestressed reinforcement, unbonded tendons, and AA plate which undergoes the bending moment.

In the process of crack development of UPC beams, the contribution of unbonded prestressed tendons to the crack resistance of the UPC beam is less than that of the same quantity of bonded prestressed tendons. Therefore, a reduction factor \( \alpha \) for the unbonded tendons has been introduced in equation (17), which is the ratio of the stress increment of unbonded prestressed tendons to that of non-prestressed reinforcement at the same location with service load. According to the existing research [18], the recommended value of \( \alpha \) is 0.23.

According to the plane-section assumption, the stress in non-prestressed reinforcement in the serviceability stage is as follows:

\[ \sigma_{sk} = \frac{M_k - N_{pe} (z - e_p) - 0.23 \Delta \sigma_p A_p \eta_p h_p + \sigma_a A_a \eta_a h}{A_s \eta_s h_0}. \]  

(18)

This can be translated to

\[ \sigma_{sk} = \frac{M_k - N_{pe} (z - e_p)}{A_s \eta_s h_0 (1 + \delta)}, \]

(19)

\[ \delta = \frac{(h - y)E_s \eta_s h}{(h_0 - y)E_s \eta_s h_0} + \frac{0.23(h_p - y)E_p A_p \eta_p h_p}{(h_0 - y)E_s \eta_s h_0}. \]

In the above, \( z \) is the distance from the point of the resultant force of non-prestressed reinforcement and unbonded tendons to the centre of compression of the concrete, which can be calculated by the following equations:

\[ z = \left[ 0.87 - 0.12 \left( \frac{h_0}{e} \right)^2 \right] h_0, \]

(20)

and

\[ e = e_p + \frac{M_k}{N_{pe}}. \]

(21)

where \( e \) is the distance between the action point of axial pressure and that of the resultant force of the non-prestressed reinforcement and \( e_p \) is the distance between the action point of prepressure and that of the resultant force of the unbonded tendons, non-prestressed reinforcement, and AA plate.

According to the development mechanism of cracks in reinforced concrete beams, the mean crack width of the beams is approximately the difference between the elongation \( \epsilon_{elt} \) of non-prestressed reinforcement and the
4.3. Maximum Crack Width. Previous studies showed that the crack width of the RC beams has a large dispersion, and the reasonable maximum crack width should be determined by statistical analysis. According to this study, the distribution of the \( i \)-th crack width \( w_i \) to the mean crack width \( w_m \) under short-term loading, it was found that the value of \( w_i/w_m \) was approximately a normal distribution, as shown in Figure 13.

The fitting equation of the normal distribution curve is as follows:

\[
f = 0.014 + \frac{1}{0.258 \sqrt{2\pi}} \exp \left[ -\left( \frac{(w/w_m - 1.05)^2}{(2 \times 0.258^2)} \right) \right],
\]

with a correlation coefficient of 0.9661.

The maximum crack width is determined by the guarantee rate of 95%, and the characteristic value corresponding to \( w_i/w_m \) is the crack expansion coefficient under short-term loading \( \tau_s \):

\[
\tau_s = 1.05 + 1.645 \times 0.258 = 1.474.
\]

Thus, the equation for calculating the maximum crack width under the short-term load is as follows:

\[
w_{\text{max},s} = \tau_s w_m = 1.474 \times 0.85 \\
\times \psi \frac{\sigma_p}{E_s} \left( 0.042 \frac{d_{\text{eq}}}{\rho_s} - 0.033 \frac{t_a}{\rho_a} + 74.76 \right).
\]

Even if the load remains unchanged under a long-term load, due to shrinkage, creep, and slippage of the concrete in the tension zone, the concrete in tension between the cracks will continuously break away, and the strain of the non-prestressed reinforcement near the cracks will gradually increase. The crack width of the beam will thus increase over time. Therefore, the effect of long-term loading should be considered in the calculation of cracks, and the expansion coefficient \( \xi \) should be introduced. According to previous studies, \( \xi = 1.5 \) [19]. Thus, the calculation equation for the maximum crack width of UPC beams strengthened with AA plates under long-term loading is as follows:
\[ w_{\text{max}} = \tau_1 r_s w_m = 1.5 \times 1.474 \times 0.85 \times \psi \frac{d_{\text{eq}}}{\rho_s} - 0.033 \frac{t_s}{\rho_a} + 74.76 \]  

(27)

5. Conclusion

(1) The technique of strengthening UPC beams with AA plates can improve stiffness and limit the development of cracks.

(2) The relationship of the load-deformation curve of damaged beams strengthened with AA plates is different from that of typical reinforced concrete beams, i.e., the characteristic trilinear model with cracking and yielding as inflection points of the strengthened beams was less obvious than that of undamaged reinforced concrete beams.

(3) Based on the double broken line model, an equation for the elastic-plastic stiffness coefficient in the serviceability stage with integrated reinforcement index $\rho_0$ as parameter was proposed, and a calculation method for the stiffness of a UPC beam strengthened with AA plates was proposed.

(4) The relative relationship between average crack spacing $l_{cr}$, $d_{\text{eq}}/\rho_s$, and $t_s/\rho_a$ was analyzed, and a calculation method of mean crack spacing for UPC beams strengthened with AA plates was proposed by regression.

(5) Based on the analysis of the stress increment of unbonded prestressed tendons in the serviceability stage, a calculation method was proposed. An influence coefficient considering the effect of unbonded prestressed tendons and AA plates was introduced, and a calculation method for the crack width in UPC beams strengthened with AA plates was proposed.

The behavior of concrete structures strengthened with aluminum alloy plate in corrosive environments should be studied in future research.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

Disclosure

The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

The authors are grateful to the members of their research group in Civil of HIT, and they are appreciated for their effort in the investigation. This research has been supported by the National Natural Science Foundation of China (NSFC) under grant no. 51778186.

References

[1] M. C. Sundarraja and S. Rajamohan, “Strengthening of RC beams in shear using GFRP inclined strips—an experimental study,” *Construction and Building Materials*, vol. 23, no. 2, pp. 856–864, 2009.

[2] I. F. Kara, A. F. Ashour, and M. A. Koroğlu, “Flexural behavior of hybrid FRP/steel reinforced concrete beams,” *Composite Structures*, vol. 129, pp. 111–121, 2015.

[3] B. H. Oh, J. Y. Cho, and D. G. Park, “Static and fatigue behavior of reinforced concrete beams strengthened with steel plates for flexure,” *Journal of Structural Engineering*, vol. 129, no. 4, pp. 527–535, 2003.

[4] L. Huang, B. Yan, L. Yan, Q. Xu, H. Tan, and B. Kasal, “Reinforced concrete beams strengthened with externally bonded natural flax FRP plates,” *Composites Part B: Engineering*, vol. 91, pp. 569–578, 2016.

[5] S. Aykac, I. Kalkan, B. Aykac, S. Karahan, and S. Kayar, “Strengthening and repair of reinforced concrete beams using external steel plates,” *Journal of Structural Engineering*, vol. 139, no. 6, pp. 929–939, 2013.

[6] S. Choobbor, R. A. Hawileh, A. A. Abu-Obiedah, and J. Abdalla, “Performance of hybrid carbon and basalt FRP sheets in strengthening concrete beams in flexure,” *Composite Structures*, vol. 227, Article ID 111337, 2019.

[7] O. R. Abudodeh, J. A. Abdalla, and R. A. Hawileh, “Prediction of shear strength and behavior of RC beams strengthened with externally bonded FRP sheets using machine learning techniques,” *Composite Structures*, vol. 234, Article ID 111698, 2020.

[8] M. Z. Naser, R. A. Hawileh, and J. A. Abdalla, “Fiber-reinforced polymer composites in strengthening concrete structures: a critical review,” *Engineering Structures*, vol. 198, Article ID 109542, 2019.

[9] A.-J. Wang, N. Song, X.-J. Fan et al., “Characterization of magnesium phosphate cement fabricated using pre-reacted magnesium oxide,” *Journal of Alloys and Compounds*, vol. 696, pp. 560–565, 2017.

[10] H. Ma and B. Xu, “Potential to design magnesium potassium phosphate cement paste based on an optimal magnesium-to-phosphate ratio,” *Materials & Design*, vol. 118, pp. 81–88, 2017.

[11] J. H. Zhu, L. L. Wei, M. C. Zhu, W. W. Li, and F. Xing, “Experimental study of bond behavior on aluminum alloy plate-to-concrete interface,” *Applied Mechanics and Materials*, vol. 501–504, pp. 805–810, 2014.

[12] J. H. Zhu, M. C. Zhu, L. L. Wei, W. W. Li, and F. Xing, “Bond behavior of aluminum laminates in NSM technique,” *Applied Mechanics and Materials*, vol. 501–504, pp. 1053–1060, 2014.

[13] J. H. Zhu, L. L. Wei, W. T. Wu, F. Xing, and R. Feng, “Experimental study of concrete strengthened by stiffened aluminum plate,” *Applied Mechanics and Materials*, vol. 584–586, pp. 997–1000, 2014.

[14] H. A. Rasheed, J. Abdalla, R. Hawileh, and A. K. Al-Tamimi, “Flexural behavior of reinforced concrete beams strengthened...
with externally bonded Aluminum Alloy plates,” *Engineering Structures*, vol. 147, pp. 473–485, 2017.

[15] H. Chang and W. Zhou, “Flexural behaviour of unbonded posttensioned concrete beam strengthened with aluminium alloy plates,” *Mathematical Problems in Engineering*, vol. 2020, Article ID 6535609, 13 pages, 2020.

[16] H. Chang and W. Zhou, “Experiment on bond behavior of aluminum alloy bonded to concrete by inorganic adhesive,” *Journal of Harbin Institute of Technology*, vol. 51, no. 6, pp. 58–63, 2019.

[17] F. P. Beer, E. Russell Johnston, J. T. DeWolf, and D. F. Mazurek, *Mechanics of Materials*, China Machine Press, Beijing, China, 6th edition, 2013.

[18] W. Z. Zheng and H. Y. Xie, "Calculation methods of stiffness and crack width of UPC beam in accordance with bonded pc beam," *Journal of Building Structures*, vol. 3, pp. 65–69, 2005.

[19] Ministry of Construction, *Code for Design of Concrete Structures. GB 50010-2010*, Ministry of Construction, Beijing, China, 2010.