Measuring Lepton Flavor Violation at LHC

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A new process with the lepton flavor violation (LFV) is presented in the setup for LHC. The LFV is induced by the one-loop effect through Higgs bosons in the framework of the type-III two Higgs doublet model. It is demonstrated that the vast of parameter space in the Yukawa sector could be accessed by current and future LHC experiments.

I. INTRODUCTION

Lepton Flavor Violation (LFV) is not only a consequence of the nonzero neutrino masses and oscillations but also a tool to search for various types of theoretical models beyond the Standard Model (SM). For instance, the SM prediction for $\mu \rightarrow e\gamma$ is too small to be observed in the foreseeable experiments [1, 2]. Therefore, any signal of LFV gives some hint on new physics beyond the SM. Indeed, it is well known that supersymmetric (SUSY) models generically give rise to LFV effects through soft SUSY breaking effects in the slepton sector [3, 4]. In conjunction with the nonzero masses of the neutrinos, right-handed neutrinos are highly motivated particle that may explain not only the neutrino masses via the seesaw mechanism [5–10] but also the baryon asymmetry of the Universe [11]. By putting these particles into the SUSY framework, LFV in the slepton sector may be induced by radiative corrections, even when underlying physics behind the SUSY breaking has nothing to do with LFV [12–17].

In testing such LFV models, $\mu \rightarrow e\gamma$ often gives the most stringent constraint. The SUSY model with right-handed neutrinos can be a typical example to get a feeling of the constraint, where the LFV appears at one-loop with sleptons and chargino/neutralino inside the loop, and thus the amplitude is proportional to the soft term $\tilde{m}_{Lij}^2$ with $i \neq j$ and $i,j = 1,2,3$ denoting the lepton-sector generation. By taking $\tilde{m}$ as a typical SUSY particle mass, the branching ratio may roughly be estimated as $\text{Br}(\tilde{l}_i \rightarrow l_j \gamma \sim \alpha^4 |\tilde{m}_{Lij}^2|/\tilde{m}^8 G_F^2)$. The LFV soft term is generated through a self-energy diagram of sleptons, where the right-handed neutrinos come inside the loop with a neutrino Yukawa coupling $y_{\nu_i}$. By neglecting details of the loop such as logarithmic piece and contributions from different type of soft terms etc., $\tilde{m}_{Lij}^2 \sim (16\pi^2)^{-1}\tilde{m}^2(y_{\nu_i}y_{\nu_j})_{ij}$ can be obtained, yielding, for instance, $\text{Br}_{\mu \rightarrow e\gamma} \sim 10^{-7}|(y_{\nu_i}y_{\nu_j})_{ij}|^2(m_W/\tilde{m})^4$ which should be compared with the current limit $\text{Br}_{\mu \rightarrow e\gamma} < 4.2 \times 10^{-13}$ [18]. Therefore, the LFV measurements have been one of the powerful tools to look for physics beyond the SM.

From the current experimental searches for the LFV processes, the most stringent constraint has been given to the LFV effects involving gauge interactions, such as $\mu \rightarrow e\gamma$. On the other hand, it could be that searches for the LFV involving Yukawa interactions give a complementary path to probe new physics. Collider experiments provide such opportunity that the both types of LFV processes can be explored simultaneously.

The LFV processes have been searched through the rare decay of the SM particles of $Z$ and SM Higgs boson [19–22] as well as exotic particles [23, 24] at LHC. Their limits so far are $\text{Br}_{Z \rightarrow \tau\tau} < 8.1 \times 10^{-6}$, $\text{Br}_{Z \rightarrow \mu\tau} < 9.5 \times 10^{-6}$ [19] and $\text{Br}_{h_{SM} \rightarrow \tau\tau} < 2.2 \times 10^{-3}$ and $\text{Br}_{h_{SM} \rightarrow \mu\tau} < 1.5 \times 10^{-3}$ [22], respectively. It should be noted that those studies had been carried by searching or measuring the resonance particles of $Z$ or $h_{SM}$.

In this paper, LFV processes of $W^+W^- \rightarrow l_i l_j$ at LHC is investigated, based on the type-III two Higgs doublet model (THDM) which provides a generic parametrization of LFV couplings in the Yukawa sector. Within the framework, such LFV processes may arise at one loop level mediated by heavy neutral and charged Higgs bosons while the tree level contributions are largely suppressed at the hypothesis with large $\tan\beta$ region in THDM. It will be shown that, although the cross section is loop-suppressed, it is still accessible at future LHC runs, especially, for the parameter spaces where the extra Higgs
bosons are at multi-TeV scales, and thus, complementary parameter spaces in LFV couplings can be covered.

The paper is organized as follows. Our framework and parametrization of the type-III two-Higgs doublet model are explained in sec. II. The one loop calculation and their event generation at LHC condition are described in sec. III and the numerical results follow in sec. IV. Finally, the feasibility study to constrain the relevant parameters on the LFV couplings is given in sec.V, then sec. VI is devoted to the discussion and conclusion.

II. MODEL

Among various possible sources for LFV, the LFV couplings in the Higgs sector with two Higgs doublet fields are considered in the rest of the paper. In the absence of a flavor symmetry, Higgs-mediated flavor changing neutral current (FCNC) often becomes problematic, since it is not always the case where the Yukawa couplings and fermion mass matrices can be simultaneously diagonalized. The problematic FCNC can be avoided if there is a $Z_2$ symmetry under which, for instance, only one of the two

\[
\kappa \equiv \delta_{ij} \left( \begin{array}{c} \delta_{ij} \delta_{ij} H_d \end{array} \right) + \left( \begin{array}{c} \delta_{ij} \delta_{ij} \delta_{ij} H_u \end{array} \right) \left( \begin{array}{c} \delta_{ij} \delta_{ij} \delta_{ij} \delta_{ij} \end{array} \right) + h.c.,
\]

where $i,j = e, \mu, \tau$, $H_u = (H_u^0 H_u^0)^T$, $H_d = (H_d^0 H_d^0)^T$, $\kappa_L$ and $\kappa_R$ parametrize the flavor off-diagonal contributions in the mass eigenstate basis of leptons. After taking the mass eigenstates for the Higgs fields, the effective Yukawa interactions become [26–29]

\[
- \mathcal{L}_{lep} \simeq \bar{L}_i R_i \left[ y_{li} \delta_{ij} H_d^0 \right] + \left( y_{li} \kappa_{Rij} + \kappa_{Lij} y_{lj} \right) H_d^{0T} L_j + h.c.,
\]

where $i,j = e, \mu, \tau$, $H_u = (H_u^0 H_u^0)^T$, $H_d = (H_d^0 H_d^0)^T$, $\kappa_L$ and $\kappa_R$ parametrize the flavor off-diagonal contributions in the mass eigenstate basis of leptons. After taking the mass eigenstates for the Higgs fields, the effective Yukawa interactions become [30].

III. SET UP

At the decoupling limit, where the lighter Higgs boson $h$ is close to the SM Higgs boson ($c_\alpha \approx 0$, $m_{H/A} > 500$ GeV), the heavier Higgs bosons $H$ and $A$ have a sizeable couplings of $H \rightarrow l_i l_j (i \neq j)$ proportional to the $\kappa_{ij}/c_\beta^2$ when the $t_\beta$ is large. However, the coupling with gauge bosons ($W^+ W^- \rightarrow H$) is largely suppressed. The $A$ boson coupling with gauge bosons even does not exist. Thus, the $s$-channel mode at tree level is largely suppressed. The higher order diagrams instead play an important role
the SM interactions in the model. After the code generation, the tree level vertices are replaced with the corresponding LFV effective 1-loop vertices.

The output is checked with LoopTools [35] and our previous study [36]. All relevant vertex formula's are implemented with same manner. The relevant tree-level vertex with LFV interaction, the 1-loop amplitude is expressed as

\[
\Gamma(\mu \rightarrow \ell \nu) = \frac{1}{16\pi^2} \left[ \left( \int_0^1 \int_0^1 y \ln D(x,y) dx dy - \frac{1}{2} C_{UV} \right) \gamma_\mu 
+ \int_0^1 \int_0^1 \frac{y dx dy}{D(x,y)} (m_1 - \bar{p})(-2\bar{p} + 2p_1 + p_2)_\mu \right],
\]

where the coupling constants are omitted and \(C_{UV} \equiv \frac{1}{2} + \ln 4\pi\) is an ultraviolet divergent part. The \(D(x,y)\) is the outcome of the Feynman integral defined as

\[
D(x,y) = -y\bar{p}\{(1-x)p_1 + p_2\} + xy p_1 p_2 + \bar{m}^2,
\]

with

\[
\bar{p} = (1 + xy - y)p_1 + (1 - y)p_2, \quad \bar{m}^2 = y\{(1-x)m_1^2 + x m_2^2\} + (1-y)m_3^2.
\]

The numerical integration is performed inside code. The output is checked with LoopTools [35] and our previous study [36]. All relevant vertex formula's and coupling constants with the LFV interactions are implemented with same manner. The relevant tree-level vertex with \(W \rightarrow \mu \nu\) is now replaced.
with Eq. (6) together with the corresponding coupling constants. The typical order of such loop correction is

$$\Gamma_{\mu} \sim 10^{-2} \gamma_{\mu} + 10^{-4} p_{\mu},$$

(9)

for $m_A = 1$ TeV, $t_\beta = 40$, $\kappa_{23} = \kappa_{13} = 0.1$ at LHC condition. Each coefficient corresponds to the parameters $A(=B)$ and $C(=D)$ in Eq.(5). Those parameters varies to the input momenta used in the vertex calculation. The outgoing leptons ($\mu^+\mu^-$) are also replaced with the relevant lepton flavors, that results in the LFV in the end.

Another type of the LFV process is through the self-energy diagrams. Typical diagram is shown in Fig.2. This diagram is known to have a logarithm mass-dependence ($\sim \log(m_H)$) in the loop structure. Therefore, the amplitude diverges as an increase of the input Higgs boson mass. To avoid such divergence, the renormalization scale $\mu_R$ is set to be $m_A$ ($\sim m_{H^{\pm}}$ at $m_A > 500$ GeV) to cancel the mass-dependence. This is interpreted that the perturbation is only valid at this scale. This choice minimizes the contributions from the flavor-changing self-energy diagram. Thus, predicts minimal production cross sections.

Soft-photon in the loop is a source of a logarithmic divergence and could be canceled by the real photon emission process at tree level. But such diagrams are raised by the $s$-channel process, where $h$ or $H$ bosons are propagated. Since those diagrams have either of the coupling of the $h \to l^+l^-$ or $WW \to H$, those contributions are negligibly small. Therefore, the soft-photon term is neglected in the calculation. For the same reason, the box-type diagrams are also ignored.

Though the $W^+W^- \to l_il_j$ (and $Z/\gamma Z/\gamma \to l_il_j$) is produced through the vector boson scattering process at LHC, the loop corrections are applied to the vertices in $VV \to \mu^+\mu^-$ process. At decoupling limit in THDM ($m_A > 500$ GeV, $t_\beta > 10$), the tree level Higgs decay mode into the LFV is suppressed and found to be less than 1% contribution to the 1-loop diagram calculation, and thus, the calculation is performed with the 1-loop order only. The schematic view is illustrated in the Fig.3. The Matrix Element is based on the $2 \to 4$ body process and the core part of the $VV \to l^+_il^-_j$ interactions is based on 1-loop order calculation.

The production cross section with the lepton $l_i$, $l_j$ ($i \neq j$) is thus expressed as

$$\sigma(pp \to l_il_j + qq + X) = \int x_1 x_2 f_1(x_1)f_2(x_2)\hat{\sigma}(q^2)dx_1dx_2d\Psi,$$

(10)

where $x_1$ and $x_2$ are the momentum fraction of the
PDF $f_1$ and $f_2$ respectively. All combination of the incoming and outgoing quarks is taken into account in the calculation. The BASES/SPRING package [37] handles numerical integration for the full-phase space mapping and the unweighted event. The 4-vector information for the initial and final state particles are stored with common format in the file [38]. Such file is interfaced by hadronization packages in later stage to simulate realistic events at LHC.

IV. RESULT

The production cross section is presented as a function of $m_A$ ($\simeq m_H$ ($m_{H^+}$)) at $t_\beta = 40$ and $\kappa_{13} = \kappa_{23} = 0.1$ with LHC 14 TeV condition in Fig.4 for each LFV mode, $\mu e$, $\mu \tau$ and $e\tau$, respectively. In the calculation, $\overline{\text{MS}}$ scheme is used. The renormalization scale is fixed at $\mu_R = m_A$, while the factorization scale $\mu_F$ is set as (square-root of) the invariant mass of the incoming partons ($\sqrt{s}$) with 50% to 200% systematic variation as uncertainty, where PDF set (NNPDF30_lo_as_0118) is used [34]. The following physics parameters are also used,

EW parameters,

$$m_W = 80.419 \text{ GeV}, m_Z = 91.188 \text{ GeV},$$

$$m_h = 125.0 \text{ GeV}, \alpha_{em} = 1/128.07,$$

and for neutrino mixing parameters,

Normal ordering,

$$\theta_{12} = 33.44^\circ, \theta_{13} = 8.57^\circ, \theta_{23} = 49.20^\circ, \delta_{CP} = 197^\circ,$$

Inverted ordering,

$$\theta_{12} = 33.45^\circ, \theta_{13} = 8.60^\circ, \theta_{23} = 49.30^\circ, \delta_{CP} = 282^\circ,$$

where $m_W$, $m_Z$ and $m_h$ are masses of $W$, $Z$, and the SM Higgs bosons, respectively. The $\alpha_{em}$ is a fine structure constant defined at $m_Z$. The $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and $\delta_{CP}$ are the neutrino mixing parameters with normal (inverted) ordering taken from the latest combined results [39]. The following kinematical cuts are applied in the calculation,

for leptons,

$$p_T > 15 \text{ GeV}, \ |\eta| < 2.5, m_{ll} > 200 \text{ GeV},$$

for outgoing quarks,

$$p_T > 20 \text{ GeV}, \ |\eta| < 4.5, m_{jj} > 300 \text{ GeV},$$

where any leptons and jets should be separated by $\Delta R_{ll(lj)} > 0.2$ and jets must be separated by $\Delta R_{jj} > 0.4$.

The cross sections are stable at high $m_A$ region due to the fixed renormalization scale of $\mu_R = m_A$. This cross section gives the lower limit that minimizes the contribution from the self-energy divergence according to the input Higgs masses. Ignoring the interference between diagrams, the leading diagrams in the production are extracted as presented in Fig.5. In general, any combinations that have couplings with $h_{SM} \rightarrow$ LFV or $WW \rightarrow H/A$ are largely suppressed by the decoupling condition. Thus, the $s$-channel diagrams do not contribute. This is why the tree-level direct production process in the neutral (non-LFV) MSSM Higgs boson searches do not have the VBF contribution. Meanwhile, the $t$-channel diagrams are dominant in LFV process through the loop contribution.

The flavor exchange occurs at the triangle loop vertex through charged Higgs boson (Fig.5 (a)) while it happens at the tree level vertex in the $W$ boson coupled with leptons through PMNS mixing matrix (Fig.5 (b)). The self-energy diagram in the $t$-channel neutrino mixing is not negligible according to an input $\kappa$ and $t_\beta$ parameters (Fig.5 (c)). The neutrino mixing parameter plays an important role in the LFV. Since the flavor exchange at the tree level vertex in the $W$ boson account only at once in the
is opposite at low $t_\beta$. Also, the cross sections decrease as $m_A$ increases for $\mu\tau$ and $e\tau$ final states while it is stable for the $\mu e$ final state due to lack of the $s$-channel contributions with a fermion loop in the $\mu e$ final state because the Yukawa coupling with $e$ or $\mu$ is negligible (Fig.5 (d)). The $s$-channel contributions in $\mu\tau$ and $e\tau$ final states are visible up to $m_A = 2$ TeV at LHC condition. Table.II summarizes the production cross sections with various parameter space for normal and inverted ordering of the neutrino mixing matrix.

The production cross sections depend on the $t_\beta$. The dependence is more pronounced in the $\mu e$ final state that contributes by a factor $1/c_\beta^4$ in the diagram (c). Then, the cross section becomes smaller than those in $\mu\tau$ and $e\tau$ final states at $t_\beta \sim 15$ since the enhancement by the $t_\beta$ is canceled by the $\kappa$ parameters ($c_\beta^4 \lesssim 1$).

The $\kappa$ parameters are also scanned at the fixed $m_A(\approx 2$ TeV$)$ and $t_\beta(=40)$. Focusing on the diagrams (a) and (b), the asymmetric parameterization of the $\kappa_{13}$ and $\kappa_{23}$ gives rise to not only an asymmetric production rate between $\mu\tau$ and $e\tau$ final states, but also asymmetric contributions between diagrams. As summarized in Table II, the $\kappa_{13}(\kappa_{23})$ is less sensitive to the $\mu\tau (e\tau)$ final state. Smaller $\kappa$ relatively enhances the diagram (b) thus the neutrino mixing parameter becomes sensitive. Given the fact that the observed mixing parameters are almost compatible between normal and inverted ordering of the neutrino mass hierarchy while only $\delta_{\text{CP}}$ distinguishes the mass ordering, the difference of the production cross sections indicates the dependence of the $\delta_{\text{CP}}$ parameter. At smaller $\kappa$, for instance, $\kappa_{23} = 0.01$, about $30\%$ difference could be observed between normal and inverted ordering.

V. FEASIBILITY STUDY FOR LHC

The signal events are interfaced by Pythia [40] to adopt a parton shower in the hard-process and

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1 Even number of flavor-exchanges by the $W$-boson undergoes GIM suppression by imposing the unitary condition of $\sum_{k=1,2,3} |U_{PMNS}^{ik}|^2 \cdot |U_{PMNS}^{kj}|^2 = 0$ where $i \neq j$. The off-diagonal elements are canceled out, thus no LFV occurs.
TABLE II. Summary of the production Cross section in different final states (scanning of the $m_A$ has rather sharp falling while a mild slope with Higgs hadronically decaying tau-lepton are only considered. The invariant masses of lepton pair are presented in Fig.6 (a) $\mu e$ and (b) $\mu\tau + e\tau$ final states, respectively. As shown in Fig.5, the $\mu e$ final state has rather sharp falling while a mild slope with Higgs mass peaks in $\mu\tau + e\tau$ final states due to the corresponding $s$-channel diagrams. At large $t_\beta$, although the $\mu e$ final state has larger production cross section.

An experimental feasibility is evaluated under those configurations. For simplicity, the muon and electron are assumed to be identified by 100% efficiency within a fiducial volume of detector $|\eta| < 2.5$. No trigger efficiency is assumed. The jets are reconstructed with $p_T > 25$ GeV within $|\eta| < 5.0$. The hadronically decaying tau-lepton are only considered as the tau object ($\tau_h$) and assume 75 % identification efficiency. The background rejection for quark and gluon jets misidentified is also taken into account as 3 % for 1-prong and 0.4 % for 3-prong. The $b$-jet is identified with 85 % efficiency within the tracking volume of $|\eta| < 2.5$ and a light-flavour jet rejections 3.5 %.

The signal topology is two high energy leptons plus two jets. The flavor of leptons must be different with the opposite charges. Two jets are observed in opposite hemisphere with large invariant mass ($m_{jj} > 500$ GeV) and $\eta$ separation ($|\Delta\eta_{jj}| > 5.0$). There is no missing transverse energy ($E_T^{miss} < 10$ GeV). The background processes are rejected by lepton ($\mu, e$ or $\tau_h$) $p_T > 100$ and 50 GeV, respectively. Since the neutrino is also associated in the $\tau_h$, the direction between $\tau_h$ and $E_T^{miss}$ is used as $|\Delta\phi(\tau_h, E_T^{miss})| < 0.05$ instead of $E_T^{miss}$ cut for $\mu\tau$ and $e\tau$ final state. After $b$-jet veto is applied to suppress the $t\bar{t}$ background, 13 events for $\mu e$, 11 for $\mu\tau$, and 13 for $e\tau$ are expected to be observed at the luminosity of 3000 fb$^{-1}$ against 53 background events for $\mu e$, 15 for $\mu\tau$, and 11 for $e\tau$ in the $m_{tb} > 500$ GeV region.

The excess with 3$\sigma$ significance is evaluated as a function of $m_A$ for $\kappa_{13} = \kappa_{23} = 0.2$ by counting the number of signal and background events in Fig.7, where the limits from three final states are combined. Two different luminosity scenarios with 300 and 3000 fb$^{-1}$ are considered. Current limits

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\[2\] For simplicity, same calorimeter segment is used as the $\eta$ coverage of 4.9 with 0.025 fine cell granularity in $\phi$ and $\eta$ directions with 15% $\sqrt{GeV}$. 

| Parameters | Normal ordering | | Inverted ordering | |
|---|---|---|---|---|
| $m_A, \tan\beta, \kappa_{13}, \kappa_{23}$ | $\mu e$ [fb] | $\mu\tau$ [fb] | $e\tau$ [fb] | $\mu e$ [fb] | $\mu\tau$ [fb] | $e\tau$ [fb] |
| (500GeV, 40, 0.1, 0.1) | $2.258(6) \times 10^{-1}$ | $1.862(3) \times 10^{-1}$ | $2.100(5) \times 10^{-1}$ | $2.30(1) \times 10^{-1}$ | $1.831(9) \times 10^{-1}$ | $2.09(1) \times 10^{-1}$ |
| (800GeV, 40, 0.1, 0.1) | $2.27(8) \times 10^{-1}$ | $1.389(7) \times 10^{-1}$ | $1.612(3) \times 10^{-1}$ | $2.29(1) \times 10^{-1}$ | $1.346(8) \times 10^{-1}$ | $1.62(1) \times 10^{-1}$ |
| (1000GeV, 40, 0.1, 0.1) | $2.26(7) \times 10^{-1}$ | $1.212(4) \times 10^{-1}$ | $1.443(3) \times 10^{-1}$ | $2.29(1) \times 10^{-1}$ | $1.195(5) \times 10^{-1}$ | $1.465(5) \times 10^{-1}$ |
| (2000GeV, 40, 0.1, 0.1) | $2.255(6) \times 10^{-1}$ | $9.70(4) \times 10^{-2}$ | $1.198(3) \times 10^{-1}$ | $2.29(1) \times 10^{-1}$ | $9.45(4) \times 10^{-2}$ | $1.224(4) \times 10^{-1}$ |
| (5000GeV, 40, 0.1, 0.1) | $2.264(7) \times 10^{-1}$ | $9.36(2) \times 10^{-2}$ | $1.144(8) \times 10^{-1}$ | $2.27(1) \times 10^{-1}$ | $9.11(6) \times 10^{-2}$ | $1.189(5) \times 10^{-1}$ |
| (1000GeV, 10, 0.1, 0.1) | $4.34(2) \times 10^{-6}$ | $2.54(1) \times 10^{-4}$ | $2.60(1) \times 10^{-4}$ | $4.22(2) \times 10^{-6}$ | $2.55(1) \times 10^{-4}$ | $2.58(1) \times 10^{-4}$ |
| (1000GeV, 20, 0.1, 0.1) | $9.17(8) \times 10^{-4}$ | $3.92(1) \times 10^{-3}$ | $4.24(5) \times 10^{-3}$ | $9.21(6) \times 10^{-4}$ | $3.95(1) \times 10^{-3}$ | $4.20(2) \times 10^{-3}$ |
| (1000GeV, 30, 0.1, 0.1) | $2.27(1) \times 10^{-2}$ | $2.33(1) \times 10^{-2}$ | $2.64(1) \times 10^{-2}$ | $2.33(2) \times 10^{-2}$ | $2.29(1) \times 10^{-2}$ | $2.70(1) \times 10^{-2}$ |
| (2000GeV, 40, 0.01, 0.1) | $2.40(3) \times 10^{-2}$ | $5.10(3) \times 10^{-2}$ | $3.55(1) \times 10^{-3}$ | $2.69(1) \times 10^{-2}$ | $6.29(2) \times 10^{-2}$ | $4.11(2) \times 10^{-3}$ |
| (2000GeV, 40, 0.02, 0.1) | $2.98(2) \times 10^{-2}$ | $5.44(3) \times 10^{-2}$ | $6.97(2) \times 10^{-3}$ | $3.22(1) \times 10^{-2}$ | $6.56(7) \times 10^{-2}$ | $8.32(3) \times 10^{-3}$ |
| (2000GeV, 40, 0.05, 0.1) | $6.91(4) \times 10^{-2}$ | $6.92(5) \times 10^{-2}$ | $2.85(1) \times 10^{-2}$ | $7.31(3) \times 10^{-2}$ | $7.63(4) \times 10^{-2}$ | $3.23(1) \times 10^{-2}$ |
| (2000GeV, 40, 0.1, 0.01) | $2.54(1) \times 10^{-3}$ | $4.29(4) \times 10^{-4}$ | $4.04(8) \times 10^{-2}$ | $2.69(2) \times 10^{-3}$ | $3.30(4) \times 10^{-4}$ | $2.90(3) \times 10^{-2}$ |
| (2000GeV, 40, 0.1, 0.02) | $9.44(3) \times 10^{-3}$ | $1.77(3) \times 10^{-3}$ | $4.34(3) \times 10^{-2}$ | $9.55(5) \times 10^{-3}$ | $1.41(1) \times 10^{-3}$ | $3.47(2) \times 10^{-2}$ |
| (2000GeV, 40, 0.1, 0.05) | $5.68(4) \times 10^{-2}$ | $1.34(3) \times 10^{-2}$ | $6.73(3) \times 10^{-2}$ | $5.4(1) \times 10^{-2}$ | $1.24(1) \times 10^{-2}$ | $5.73(2) \times 10^{-2}$ |
from the non-LFV MSSM Higgs boson searches \[44\] by the ATLAS experiment are also overlaid as reference, to see the sensitivity does not reach to higher mass region while such degradation is not observed in the LFV $t$-channel searches. With 300 fb$^{-1}$, the region of $t_\beta > 30$ is excluded for entire mass range. The limit of the $\kappa$ parameters are also scanned for given $t_\beta$. Figure 8 presents the contour region of $3\sigma$ exclusion limits in $\kappa_{13}$ and $\kappa_{23}$ plane for $m_A = 1$ TeV and 3000 fb$^{-1}$ luminosity by single experiment. The $\mu\tau$ and $e\tau$ final states constrain the $\kappa_{23}$ and $\kappa_{13}$, respectively. While the $\mu e$ final state constrains both $\kappa_{13}$ and $\kappa_{23}$. With 3000 fb$^{-1}$ of data, the exclusion of $\kappa$ parameters reaches $\approx 0.1$.

The $\kappa_{13}, \kappa_{23} > 0.44$ are already excluded in the tau decay measurements at $m_A = 1$ TeV by the FCNC searches of $Z$ boson \[19\]. The limits from the SM $H$ decaying into LFV processes \[20, 21\] do not contribute in the constraint of the $\kappa_{13}$ and $\kappa_{23}$ due to the large suppression by the $\cos\alpha \approx 0$ at large $m_A$ region. Meanwhile, the non-LFV neutral MSSM $H \rightarrow \tau\tau$ \[45, 46\] could be re-interpreted from the observed cross section limit to constrain the $\kappa$ parameters. Their limits are about $\sigma(H/A \rightarrow \tau\tau) \lesssim 1-2$ fb at $m_A = 1$ TeV, which gives $\kappa_{13}(\kappa_{23}) \approx 0.3$ at $t_\beta = 40$.

VI. DISCUSSION AND CONCLUSION

The LFV measurements at LHC should be compared with the constraints set by the measurements of LFV in the $\tau$ decay. The most stringent constraints come from the rare decay of $\tau \rightarrow l\gamma$ and $\tau \rightarrow \eta$. It is known that in THDM the constraints from the rare processes $\tau \rightarrow l\gamma$ are the strongest
limit for heavier mass of $A$ due to the non-decoupling effect in the Barr-Zee diagrams [47, 48]. On the other hand, the decay width of $\tau \to l\gamma$ is strongly suppressed at $m_A \sim 700$ GeV due to cancellations at two loops, where the constraints from $\tau \to l\eta$ becomes most stringent. The constraints from $\tau \to l\gamma$ for generic Yukawa interaction including LFV have been discussed in Ref. [49], and the constraints on each branching ratios are given as $\text{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$, $\text{Br}(\tau \to e\eta) < 9.2 \times 10^{-8}$, $\text{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$, $\text{Br}(\tau \to \mu\eta) < 6.5 \times 10^{-8}$ [50]. For these channels, Belle II experiment is expected to improve a sensitivity by more than one order of magnitude when assuming an integrated luminosity of 50 ab$^{-1}$ [51]. These experimental bounds, in particular $\tau \to l\gamma$, can be translated into the constraints on $\kappa_{i3}$ [49]. For $m_A \gtrsim m_W$,

$$\kappa_{i3} \lesssim 0.07 \times \left(\frac{10}{l_\beta}\right)^2 \sqrt{\frac{\text{Br}_{\tau \to l\gamma}^{\text{exp}}/\text{Br}_{\tau \to l\gamma}^{\text{exp}}}{2 \times 10^{-7}}}$$

(11)

can be obtained, where $\text{Br}_{\tau \to l\gamma}^{\text{exp}}$ is the experimental upper bound on the $\tau \to l\gamma$ channel, and $\text{Br}_{\tau \to l\gamma}^{\text{exp}}$ is the observed branching fraction. Notice that this bound does not strongly depend on $m_A$ because of the non-decoupling nature of the Barr-Zee diagrams.

The VBF production mode in the heavy Higgs boson search with LFV will be a new physics process ever analyzed at LHC and provide new channels complementary to the LFV measurements in the $\tau$ decay. Especially, unlike a conventional decay mode of the Higgs boson to LFV, the $\mu e$ mode is enhanced by $l_\beta$ at high mass region. The dominant process through the 1-loop diagram is the $t$-channel production, thus the experimental search is accessible even higher mass region, which is not limited by the colliding energy. With 3000 fb$^{-1}$ of data, vast of the parameters space is explored at HL-LHC experiment. This will also serve as an input for the future collider experiments.

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