Non-Newtonian Gravity, Fluctuative Hypothesis and the Sizes of
Astrophysical Structures

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We show that the characteristic sizes of astrophysical and cosmological structures, where gravity is the only overall relevant interaction assembling the system, have a phenomenological relation to the microscopic scales whose order of magnitude is essentially ruled by the Compton wavelength of the proton. This result agrees with the absence of screening mechanisms for the gravitational interaction and could be connected to the presence of Yukawa correcting terms in the Newtonian potential which introduce typical interaction lengths. Furthermore, we are able to justify, in a straightforward way, the Sanders-postulated mass of a vector boson considered in order to obtain the characteristic sizes of galaxies.

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1. INTRODUCTION

Explaining the large scale structures of the Universe is one of the hardest task of modern astrophysics since the growing amount of observations seems to escape any coherent scheme able to connect all the parts of the puzzle.

Essentially, from the fundamental physics point of view, we would like to re-conduct cosmic structures and their evolution to some unifying theory in which all the today observed interactions are treated under the same standard. In this case, what we observe today on cosmological and astrophysical scales would be just a consequence of quantum fluctuations at early epochs. Then, we should seek for some “enlarging” mechanism which, after one (or more than one) symmetry breaking would be capable of yielding structures like clusters of galaxies, galaxies and then stars from primordial quantum spectra of perturbations.

The so called “inflationary paradigm” related to several unifying theories (e.g. superstrings, GUT, SUSY, and so on) should be successful if some “experimentum crucis” would select the right model.

On the other hand, particle physicists need cosmological predictions and observations since the energies for testing unified theories are so high that it is extremely unlikely they will be ever reached on Earth-based laboratories.

As a matter of fact, cosmology needs particle physics and vice versa. The point is that remnants of primordial epochs should be found by cosmological observations and, by them, one should constrain
elementary particle physics models.

This philosophy has been pursued by several researchers; first of all by Sakharov [2] who in 1965 argued that quantum primordial fluctuations should have expanded towards the present epoch leading first to classical energy–density perturbations and, after the decoupling from the cosmological background, to the observed galaxies, clusters, super-clusters of galaxies, and afterwards stars. Shortly, the underlying issue of any modern theory of cosmological perturbation is this: primordial quantum fluctuations should be enlarged by cosmological dynamics to the present large scale structures. Now the problem is not only whether observations agree with this scheme (e.g. COBE and IRAS data or large scale structure surveys) but, mainly, whether the astrophysical and cosmological systems “remember” their quantum origin or not.

Despite of the apparent sharp division of the classical and quantum worlds, macroscopic quantum phenomena exist and some behaviors of classical systems can be explained only in the framework of quantum mechanics. The high $T_c$ superconductivity and several other macroscopic coherent systems (e.g. optical fibres) are famous instances of these peculiar phenomena in which a quantum “memory” persists at the macroscopic scale.

Recently, a new intriguing conjecture has been proposed to find signatures of $h$ at the classical, macroscopic scales: in [4], it has been argued about a possible gravitational origin of quantization emerging thanks to the universal interaction of every particle in the Universe with the gravitational stochastic background field generated by all other particles.

In the framework of this fluctuative Machian scheme, it is possible to show how classical nonlinearity and chaoticity of the gravitational interaction yields a characteristic unit of action per particle that coincides, in order of magnitude, with $h$.

Further studies [5] have generalized this scheme to the other fundamental interactions responsible for macroscopic structures, finding that $h$ is the characteristic action per particle also for macroscopic systems not bound by gravitational interactions but by other forces, e.g. electromagnetic.

In this scheme, classical laws of force describing the interactions among the constituents of $N$–particle systems of mean length scale $R$, lead to a quantum characteristic action per particle. The forces considered can be, for instance, the electromagnetic interactions between charged particles in large macroscopic systems as charged beams in particle accelerators, plasmas, and neutral dipolar crystals, or the strong interactions between quarks in hadronic bound aggregates, and so on.

Having such a procedure at hand, it seems very natural to investigate whether it can be applied to determine the existence of quantum signatures or “memories” for astrophysical structures.

In a recent paper by the authors [6], it was shown that the order of magnitude of characteristic observed radii of typical galaxies can be inferred starting from the microscopic fundamental scales through a phenomenological scaling law. Further scaling relations can be derived, showing that such a connection can equivalently be obtained either by considering the nucleons as the elementary constituents of a galaxy, or, as usual in astrophysics, by considering stars as its granular components. The key issue is that we can define an interaction length $\lambda$ and a typical number of elementary constituents $N$ (e.g. $N_n$, the number of nucleons with $\lambda_c$, the Compton wavelength, or $N_s$ the number of stars with $\lambda_s$, the typical interaction length of a star, for example the "size" of the Solar System) and, by the relation

$$R \simeq \lambda \sqrt{N},$$

(1)

to obtain the observed typical size (in the case of galaxies we obtain $\sim 10$ kpc a typical galactic scale).

In the present paper, we show that such a relation holds for other self-gravitating systems as stellar globular clusters, clusters and super-clusters of galaxies and we show that it can be implemented by taking into account a gravitational interaction explicitly scale-dependent, as predicted by several effective quantum field theories in the low energy limit.

More precisely, several renormalizable quantum theories of gravity require a modification, in the low energy limit, of Newton potential. Furthermore, if we do not require enormous amounts of dark matter as the only mechanism to explain the puzzle of the present day astrophysical observations, a scale–dependent
Gravitational interaction is needed. This point of view is supported by several authors \[7\] which state that recent experiments on cosmic microwave background as BOOMERANG \[8\] or astrophysical structures can be explained by taking into account modified Newtonian dynamics \[9, 10\].

The paper is organized as follows. In Sec. 2, we deal with the weak field limit of extended theories of gravity which, in general, gives rise to Yukawa-like corrections to the Newtonian potential, which, naturally imply characteristic lengths. Sec. 3 is devoted to the discussion of the characteristic sizes of astrophysical structures. In Sec. 4, we discuss how time-statistical fluctuations of granular components of self-gravitating systems are related to their characteristic sizes. Conclusions are drawn in Sec. 5.

II. NEWTONIAN LIMIT OF EXTENDED THEORIES OF GRAVITY AND THE EMERGENCE OF CHARACTERISTIC INTERACTION LENGTHS

Extended theories of gravity have become a sort of paradigm in the study of gravitational interaction since several motivations push for enlarging the traditional scheme of Einstein general relativity. Essentially, they consist to add corrections due to scalar fields or curvature invariants of the form, $\phi^2 R$, $R^2$, $R_{\mu\nu}R^{\mu\nu}$, or $R\Box R$ in the Einstein-Hilbert gravitational action. Such issues come, essentially, from cosmology and quantum field theory.

In the first case, it is well known that higher-derivative theories \[11\] and scalar-tensor theories \[12\] furnish inflationary cosmological solutions capable, in principle, of solving the shortcomings of standard cosmological model \[13\].

In the second case, every unification scheme as superstrings, supergravity or grand unified theories, gives effective actions where nonminimal couplings to the geometry or higher order terms in the curvature invariants come out. Such contributions are due to one-loop or higher-loop corrections in the high curvature regimes near the full (not yet available) quantum gravity regime \[14\]. However, in the weak-limit approximation, all these theories should be expected to reproduce the Einstein general relativity which, in any case, is experimentally tested only in this limit \[15\].

This fact is matter of debate since several relativistic theories do not reproduce Einstein results at the Newtonian approximation but, in some sense, generalize them. In fact, as it was first noticed by Stelle \[16\], a $R^2$–theory gives rise to Yukawa–like corrections to the Newtonian potential which could have interesting physical consequences.

For example, some authors have shown that a conformal theory of gravity is nothing else but a fourth-order theory containing such terms in the Newtonian limit and, by invoking these results, it could be possible to explain the missing matter problem "without" dark matter \[17\].

In general, it can be shown \[16, 19, 20\] that most of the extended theory of gravity have a weak field limit of the form

$$V(r) = -\frac{G_\infty m}{r} \left[ 1 + \sum_{k=1}^{n} \alpha_k e^{-r/\lambda_k} \right], \quad (2)$$

where $G_\infty$ is the value of the gravitational constant as measured at infinity, $\lambda_k$ is the interaction length of the $k$-th component of non-Newtonian corrections. The amplitude $\alpha_k$ of each component is normalized to the standard Newtonian term (see \[17, 21\] for further details). The discussion involves also the variation of the gravitational coupling. As an example, let us take into account only the first term of the series in (2) which is usually considered the leading term (this choice is not sufficient if other corrections are needed). We have

$$V(r) = -\frac{G_\infty m}{r} \left[ 1 + \alpha_1 e^{-r/\lambda_1} \right]. \quad (3)$$

The effect of non-Newtonian term can be parameterized by $(\alpha_1\lambda_1)$. For large distances, at which $r \gg \lambda_1$, the exponential term vanishes and the gravitational coupling is $G_\infty$. If $r \ll \lambda_1$, the exponential becomes unity and, by differentiating, we get

\[
V(r) = \frac{G_\infty m}{r} \left[ 1 + \alpha_1 e^{-r/\lambda_1} \right].
\]
\[ G_{\text{lab}} = G_\infty \left[ 1 + \alpha_1 \left( 1 + \frac{r}{\lambda_1} \right) e^{-r/\lambda_1} \right] \simeq G_\infty (1 + \alpha_1), \]  

(4)

where \( G_{\text{lab}} = 6.67 \times 10^{-8} \text{ } \text{g}^{-1}\text{cm}^3\text{s}^{-2} \) is the usual Newton constant measured by Cavendish-like experiments. Of course, \( G_\infty \) and \( G_{\text{lab}} \) coincide in standard gravity. It is worthwhile to note that, asymptotically, the inverse square law holds but the measured coupling constant differs by a factor \((1 + \alpha_1)\). In general, any exponential correction introduces a characteristic length that acts at a certain scale for the self-gravitating systems.

This approach has been pursued by several authors who tested non-Newtonian corrections by ground-based experiments using totally different techniques \([22], [23], [24]\). The general outcome of these experiments, even retaining only the term \( k = 1 \), is that a "geophysical window" between the laboratory and the astronomical results has to be taken into account. In fact, the range

\[ |\alpha_1| \sim 10^{-2}, \quad \lambda_1 \sim 10^2 \div 10^3 \text{ m}, \]  

(5)

is not excluded at all. The sign of \( \alpha_1 \) tells us if corrections are attractive or repulsive. Another interesting suggestion has been given by Fuji \([25]\), which proposed that the exponential deviation from the Newtonian standard potential (the "fifth force") could arise from the microscopic interaction which couples to nuclear isospin and baryon number.

The astrophysical counterpart of these non-Newtonian corrections seemed ruled out till some years ago due to the fact that experimental tests of general relativity predict "exactly" the Newtonian potential in the weak energy limit, "inside" the Solar System. Recently, as we said above, indications of an anomalous, long–range acceleration revealed from the data analysis of Pioneer 10/11, Galileo, and Ulysses spacecrafts makes these Yukawa–like corrections come into play \([18]\). Besides, Sanders \([9]\) reproduced the flat rotation curves of spiral galaxies by using

\[ \alpha_1 = -0.92, \quad \lambda_1 \sim 40 \text{ kpc}. \]  

(6)

His main hypothesis is that the additional gravitational interaction is carried by an ultra-soft vector boson whose range of mass is \( m_0 \sim 10^{-27} \div 10^{-28} \text{ eV} \). The action of this boson becomes efficient at galactic scales without the request of enormous amounts of dark matter to stabilize the systems.

On the other hand, by asking for a characteristic length emerging from the standard theory of cosmological perturbation, it is possible to explain the observed segregation of hot stellar systems in the so called fundamental plane of galaxies ("ordinary" and "bright" galaxies) \([26]\). In that case, the length is the "Jeans length" of the protogalaxy \( (\lambda \sim 3 \div 10 \text{ kpc}) \) and, due to this characteristic size, a Yukawa correction was found in the gravitational potential with a characteristic interaction "length" of the same order of magnitude of that proposed by Sanders.

In this paper, we discuss the emergence of characteristic lengths by a time-statistical fluctuation of the granular components of self-gravitating systems. Our guess is that such lengths give rise to non-Newtonian corrections in the gravitational potential.

In the next section we discuss what we intend for the characteristic size of a self-gravitating system and then we discuss the fluctuative hypothesis.

### III. CHARACTERISTIC SIZES OF ASTROPHYSICAL SELF-GRAVITATING SYSTEMS

In general, the concept of "size" of a self-gravitating system is not well-based since, in several cases, the boundary cannot be univocally defined. Let us briefly define globular clusters, galaxies, clusters and
super-clusters of galaxies by their typical lengths and masses.

A *globular cluster* is a very compact self-gravitating stellar system whose typical radius is \( R_{gc} \sim 10 \) pc. It contains up to \( 10^6 \) stars \( (M_{gc} \sim 10^6 M_\odot) \) and is assumed completely virialized due to collisional interactions between stars.

A *galaxy* is a collisionless, diffuse gravitating system without an effective boundary. Astronomers define operative characteristic sizes as the *effective radius* \( R_e \) which is the radius of the isophote containing half of the total luminosity, or the *tidal radius* \( R_t \) corresponding to the distance from the center where the density drops to zero \[27\], \[28\]. Other definitions are possible by using photometry or kinematics but, assuming as a typical interaction size a length \( R_g \sim 1 \div 10 \) kpc is quite reasonable from dwarf to giant galaxies \[2\]. Typical masses are \( M_g \sim 10^{10 \div 12} M_\odot \) for giant galaxies and \( M_g \sim 10^{8 \div 9} M_\odot \) for dwarf galaxies.

As a *cluster of galaxies*, following Abell \[29\], we define a self-gravitating system whose granular components are galaxies with a typical radius \( R_{cg} = R_a \simeq 1.5 h^{-1} \) Mpc (the Abell radius) and a typical mass \( M_{cg} \sim 10^{15} h^{-1} M_\odot \) for rich clusters, where \( h \) is the dimensionless Hubble constant whose value is in the range \( 0.5 < h < 1 \) \[32\].

A *super-cluster* is a self-gravitating system of clusters of galaxies whose typical size is \( R_{sc} \sim 10 \div 100 h^{-1} \) Mpc and typical mass is \( M_{sc} \sim 10^{15 \div 17} h^{-1} M_\odot \).

*Groups of galaxies* are systems containing \( 10 \div 20 \) galaxies, as our Local Group, but there are no evidences that they could be considered self-gravitating systems and, in any case, they are always part of more extended cluster of galaxies (in the case of Local Group, it is a part of the Virgo Cluster).

The main difference between a globular cluster and the other systems is that the former is a collisional system while the others are collisionless. This fact implies a completely different dynamical treatment \[27\].

The properties of these self-gravitating systems can be deduced by assuming them to be relaxed and virialized systems where gravity is the only overall interaction \[27\]. This assumption is, some times, not completely justified. In fact, we have to keep in mind that these systems undergo environmental effects, being never completely isolated; they always belong to larger gravitationally bound systems and the observational times are so short that the overall dynamics can only be extrapolated \[27\], \[28\].

Furthermore, as we said, the dynamics of astrophysical systems have to be related to cosmological evolution so that, in today observed dynamics, some quantum signature of primordial quantum perturbations should be present \[3\].

However the main difficulty, is to provide a physical route connecting the sizes of astrophysical structures with the extremely small numbers of quantum mechanics.

As a first step, we can build a model of the above structures composed of self–gravitating microscopic constituents (nucleons) which undergo some statistical fluctuations \[1\].

\[1\]In this paper, we are taking into account only systems where gravity is the only overall interaction acting between the components. In this sense, a star is not a purely self-gravitating system since, inside it, gravity is balanced by the pressure due to electromagnetic and nuclear interactions. However, we can take into account stars as granular constituents of globular clusters and galaxies and define a typical interaction length as the "size" of a planetary system around a star.

\[2\]Several authors assumes that the halo of giant galaxies can extend as far as \( 100 \) kpc from the center taking into account the dark matter component. Here we do not assume any dark matter hypothesis and do not want to enter into details of galactic dynamics and morphology. For example, Milky Way, a typical spiral galaxy, has an observed disk scale length \( R_d \simeq 3.5 \pm 0.5 \) kpc while kinematics of globular clusters and 21 cm-radio observations of neutral hydrogen give a maximal halo extension of \( 20 \div 30 \) kpc. For our purposes, assuming \( 10 \) kpc as a characteristic size, with a possible error of an order of magnitude, is a good number.
In a semi-quantitative analysis of the model, we introduce as the only observational input the number of nucleons contained. The characteristic dimension of such a model, as we shall see below, is a function of the microscopic nucleon scales (the Compton wavelength of a nucleon $\lambda_c \sim 10^{-13} \text{ cm}$) and of the number of microscopic constituents.

The result is that the characteristic radii so deduced, numerically, coincide with those observed considering the usual gravitational constituents (stars for globular clusters and galaxies or galaxies themselves for clusters and super-clusters of galaxies, not nucleons).

Besides, we obtain a scaling relation between the units of length and action of the granular gravitational components, ranging from nucleons up to stars and galaxies.

On the other hand, the characteristic dimensions of astrophysical structures appear to be independent of the scale of the constituents considered. It is only needed that they depend on a minimal scale of length, which is, in order of magnitude, the Compton wavelength of a nucleon.

All these results could suggest a sort of macroscopic quantum coherence for large scale gravitational systems.

Furthermore, the emergence of these characteristic scales could have a dynamical counterpart in the non-Newtonian, Yukawa corrections of gravitational potential as in Eq. (2).

**IV. THE FLUCTUATIVE HYPOTHESIS FOR SELF-GRAVITATING SYSTEMS**

In order to get a general rule to define "sizes" for self-gravitating systems, we start by considering the total action for a bound system with a very large number $N$ of constituents. Let $E$ be the total energy. Let $T$ be the characteristic global time of the system (e.g. the time in which a particle crosses the system, or the time in which the system evolves and becomes relaxed or virialized). By these two quantities, we get

$$A \approx ET,$$

which is the total action of the system. The only hypothesis which we need is that the system may undergo a time-statistical fluctuation, so that the characteristic time $\tau$ for the stochastic motion per particle is $\tau \approx \sqrt{\frac{T}{N}}$.

This fluctuative hypothesis naturally emerges from the fact that, if $N$ is large, dynamics is affected by some kind of statistical (chaotic) fluctuation.

We can then define an energy per granular component

$$\epsilon \equiv \frac{E}{N},$$

so that the characteristic (minimal) unit of action $\alpha = \epsilon \tau$ per granular component is expressed by the scaling relation

$$\alpha = \epsilon \tau \approx \frac{A}{N^{3/2}}.$$  

Let us now consider the observational data for the above self-gravitating systems, in order of magnitude. Since we are taking into account virialized systems, we can assume

$$2E_k + U = 0,$$

where $E_k$ is the kinetic energy and $U$ the gravitational energy.
The total energy \( E \) can then be assumed to be

\[
E \approx E_k \approx NMv^2
\]

where \( N \) is the typical number of granular components (e.g. stars in a galaxy or in a globular cluster, or galaxies in a cluster or super-cluster of galaxies; \( M \) is the typical mass (e.g. \( 1M_\odot \) for a Main Sequence star or \( 10^{10} \div 10^{11}M_\odot \) for a galaxy like Milky Way); \( v \) is a characteristic typical velocity which we choose to be the circular speed of the stars in the disk of galaxies (\( \simeq 10^7 \div 8 \) cm/sec) or the velocity dispersion of the galaxies inside a cluster (\( \simeq 10^8 \div 9 \) cm/sec). All these numbers are quite accurately measured by the methods of stellar kinematics, statistics and photometry \(^3\). All the above quantities entering into the definitions of the energy, time and action scales are quantities coming from observations. In particular, nowhere we introduce the characteristic radius \( R \) of the structures since this is what we wish to predict in the framework of our considerations.

Let us start by taking into account a galaxy. The energy per unit of mass is of the order \( 10^{15} \) (cm/sec)\(^2\), while the period of a galactic rotation, which can be assumed as the characteristic global time, is of the order \( 10^{15} \) sec,

\[
T_{rot} \approx 10^{15} \text{sec},
\]

and finally the total mass of a typical galaxy is of the order \( 27 \)

\[
M_g = N_g^{(g)} M_s \approx 10^{44} \text{gr}.
\]

From Eq.(7), combining these numbers, we get the typical action

\[
A \approx 10^{74} \text{erg sec}.
\]

The typical number of nucleons in a galaxy is \( 27 \)

\[
N \approx 10^{68}.
\]

Inserting these numbers in Eq.(10) we see that, up to an order of magnitude, the characteristic unit of action \( \alpha \) of a galaxy, considered as an aggregate of nucleons, is of the order of the Planck action constant, \( h \sim 10^{-27} \) erg sec. It is worthwhile to stress that also if dark matter is considered, the result does not change dramatically since the mass to luminosity ratio is of the order \( 10 \div 100 \).

As a further step, we note that Eq.(10), together with the numerical result \( \alpha \approx h \), can be re-formulated as a scaling relation for the mean action per microscopic component \( a \equiv A/N \), that is

\[
a \approx h \sqrt{N}.
\]

We can then deduce that the fluctuative factor \( \sqrt{N} \) provides the rescaling coefficient from the microscopic scales to the characteristic macroscopic dimensions.

Let us now take into account the lengths. Given the nucleons as the basic microscopic constituents in our model, the natural quantum unit of length associated to each single constituent is the Compton wavelength \( \lambda_c = \hbar/mc \), with \( c \) the velocity of light, and \( m \approx m_p \approx 10^{-24} \) gr, the proton mass. In analogy with Eq.(17), we have, in general,

\[
a \approx h \sqrt{N}.
\]

\(^3\)Due to the virial theorem and the conservation of energy, we are assuming that we are dealing with almost isolated systems also if they belong to larger gravitationally bound systems. We discard considerations on the potential energy \( U \) which imply statements on rotation curves, the dependence of the mass from the radius, and the introduction of dark matter.
\[ R \cong \lambda_c \sqrt{N}. \]  
(18)

For a galaxy, with \( N \) given by Eq. (16), we obtain
\[ R_g \cong 10^{21} \div 10^{22} \text{cm} \simeq 1 \div 10 \text{kpc} \]  
(19)

which as we said above is a length of a galaxy. In particular, the numerical agreement of Eq. (18) with the observed galactic radii, is interesting, independently of the present derivation, since it links the scale of a large structure like a galaxy to the Compton wavelength of the elementary constituents (the nucleons) and to the total number of such constituents.

It is worth noticing that, for typical galaxies, \( R_g \) is the characteristic dimension where their rotation curve can be assumed flat [27] and where the halo and the disk stabilize each other.

The validity of Eq. (18) is not restricted to the galaxies, but provides the correct order of magnitude of the observed radii also if one considers the other structures which we mentioned above.

In the case of globular clusters, considering the right \( N \) (which we easily deduce by the number of stars which constitute them, i.e. \( 10^6 \)), we get
\[ R_{gc} \cong 10^{18} \div 10^{19} \text{cm} \simeq 1 \div 10 \text{pc}. \]  
(20)

For clusters of galaxies we obtain \( R_{cg} \cong 1 \text{ Mpc} \) and for super-clusters \( R_{sg} \cong 10 \div 100 \text{ Mpc} \).

The discussion can be extended to the whole Universe, and to other astrophysical objects, such as planetary systems (like the Solar System), provided one inserts in Eq. (18) the correct value of the number of nucleons \( N \) contained in such structures.

These findings indicate that the quantum parameter \( \lambda_c \) and the number of nucleonic constituents \( N \), determine the observed astrophysical and cosmological dimensions.

The crucial objection to these results would be that stars or single galaxies, rather than nucleons, are the natural candidates as elementary gravitational constituents of a typical galaxy or a typical cluster of galaxies.

This apparent difficulty can be solved deriving a simple scaling law, which holds true at any scale. Let us take into account, for example, the number of stars \( N_s^{(g)} \) contained in a typical galaxy, and the number \( N_n^{(s)} \) of nucleons in a star. We can then obviously write, for the total number of nucleons in a typical galaxy
\[ N \cong N_s^{(g)} N_n^{(s)}. \]  
(21)

By Eq. (21), we can write Eq. (18) as
\[ R_g \cong \lambda_s \sqrt{N_s^{(g)}}, \]  
(22)

where
\[ \lambda_s \equiv \frac{A_s}{M_sc}, \quad A_s \equiv h[N_n^{(s)}]^{3/2}, \quad M_s \equiv mN_n^{(s)}. \]  
(23)

Here, as above, \( M_s \) is the total mass of a star, while the quantity \( A_s \) is the characteristic unit of action of a star in the framework of our model, taking the stars as the elementary constituents of a typical galaxy. Inserting the numerical values [27] \( N_n^{(s)} \cong 10^{57} \), \( N_s^{(g)} \cong 10^{10} \div 10^{12} \), we obtain
\[ \lambda_s \cong 10^{13} \div 10^{15} \text{cm}, \]  
(24)

which agrees with the typical range of interaction of a star (e.g. that of the Solar System), while for \( R_g \) we obviously obtain again the value (19).

Therefore, Eqs. (18) and (24) show that we can derive the observed galactic radius \( R_g \) either by considering a galaxy as a gas of \( N \) nucleons with the fluctuation \( \delta \) defined with respect to \( N \), or by
considering, as usual, a typical galaxy as a gas of stars and assuming the fluctuative ansatz (8) rescaled with respect to the number of stars \(N_{g}(s)\).

The reason for the validity of this relation (which, in principle, holds on any scale) relies on the existence of a minimal scale of action which is needed for mechanical stability. In fact, the numerical value of the unit of action \(A_{s}\) defined in Eq.(23) is \(\approx 10^{58}\) erg s and thus coincides, in order of magnitude, with the total action for a typical star. Thus the rescaling relations (10) and (18) hold true also for a star, and \(\lambda_{s}\) appears as the effective macroscopic “Compton wavelength” of a star. However \(\lambda_{s}\) is the typical range of interaction also in the case of a globular cluster giving \(R_{gc} \approx \lambda_{s} \sqrt{N_{s}^{(gc)}} \sim 1 \div 10\) pc.

Immediately we derive \(\lambda_{g}\) as the range of interaction for galaxies considered as granular constituents of clusters and super-clusters. Analogously, we have

\[
\lambda_{g} \equiv \frac{A_{g}}{M_{g}c}, \quad A_{g} \equiv h[N_{n}(g)]^{3/2}, \quad M_{g} \equiv mN_{n}^{(g)}. \tag{25}
\]

and then \(\lambda_{g} \approx 10 \div 100\) kpc. In this case, we can hierarchically consider a cluster or a super-cluster of galaxies as a gas of nucleons, stars or galaxies. It is interesting to note that \(\lambda_{g}\) is the observed typical separation length between galaxies in a cluster.

At this point, it is straightforward the connection to the non-Newtonian gravitational potential (2). As we discussed above, the Yukawa corrections have to be reconducted to the emergence of typical scales for self-gravitating systems. For example, as discussed in [9] and in [10], by the interaction ranges of some vector bosons, it is possible to explain the flat rotation curves of spiral galaxies without asking for large amounts of dark matter. Yukawa corrections naturally emerge in relation to these interaction ranges. The main shortcoming of their approach is that, till now, no ultra-light vector boson has been detected and the requested interaction lengths \(\lambda \sim 10\) kpc are very hard to justify.

By our fluctuative hypothesis, as we discussed above, \(\lambda_{g} \sim 10\) kpc naturally emerges by taking into account stochastic fluctuations of the granular components of a galaxy. Using, Eq.(3) where we assume \(\lambda_{1} = \lambda_{g}\), the arguments in [9] and in [10] are easily recovered.

Besides, the anomalous, long-range acceleration reported in [18] immediately outside the Solar System, could be explained considering a Yukawa correction in the Newtonian potential related to a length as \(\lambda_{s}\) which can be considered as the typical range of interaction of a star as the Sun (a system with gravitationally bound planet).

**V. DISCUSSION AND CONCLUSIONS**

In this paper, we have discussed the possibility that the characteristic sizes of astrophysical self-gravitating systems could be deduced by scaling laws relating the observed macroscopic dimensions to the microscopic fundamental scales. These scaling laws emerge taking into account the stochastic behaviour of complex systems whose characteristic sizes come out from statistical fluctuations. The net effects are Yukawa-like corrections to the Newtonian potential which become relevant in the range \(r \sim \lambda\) and, in general, modify the ”strength” of gravitational coupling (e.g. \(G_{lab} = G_{\infty}(1 + \alpha)\)). This fact could be connected to the well-known absence of screening mechanisms for gravity (see, for example [33] and reference therein, for the discussion of quantum gravity effects on large scale structures).

Before drawing the conclusions, we have to discuss the scales of action involved. Another link between the quantum unit of action \(h\) and the cosmological scales is provided by the so-called Eddington–Weinberg relation \(h \approx G^{1/2}m^{3/2}R^{1/2}\), where \(G\) is the Newton gravitational constant, \(m\) is the mass of the nucleon, and \(R\) is the radius of the Universe. If one takes for \(R\) the various definitions of cosmological radius (Hubble radius, causal radius, or last scattering radius) [62] which range from \(R = 10^{26}\) cm to \(R = 10^{30}\) cm, one obtains a value for the unit of action \(\alpha\) ranging from \(10^{-26}\) erg sec to \(10^{-27}\) erg sec which is usually assumed to coincide with the Planck constant \(h\) in order of magnitude [34].
A similar relation can be deduced also for the self-gravitating structures which we have discussed, if one inserts in the equations for \( A \) and \( \alpha \) the gravitational energy \( U(R) \) and a characteristic gravitational time \( T \) needed for the relaxation of the system. In this case, a sort of Eddington–Weinberg relation can be derived and appears to hold also for galaxies, and other large scale structures as clusters and super-clusters of galaxies yielding a microscopic unit of action of the order of \( 10^{-27} \pm 28 \) erg sec. However, in the framework of our model, this quasi–coincidence is of no real significance, because, as explained above, the correct way to compute the characteristic energy, time, action, and length scales for the astrophysical structures must depend only on observational kinetic and photometric quantities, and not on energy and time scales explicitly dependent on the characteristic dimension \( R \) that one is seeking to predict. Therefore, the fundamental relations in our model, are the set of Eqs.\[1\] \( (\ref{E:7}) \) and \( (\ref{E:18}) \) obtained above, and not the Eddington–Weinberg relation. This is more clear if one moves to consider a planetary system or a globular cluster which are not very large astrophysical structures. As discussed above, the scheme presented in this paper \( (R = \lambda_c \sqrt{N}) \) perfectly applies also to these cases, yielding the correct values for the characteristic radii. However, if one tries to reinterpret them in terms of the Eddington–Weinberg relation, one finds a microscopic unit of action of the order of \( 10^{-32} \) erg sec, thus devoid of any physical significance (according to our present knowledge of the microscopic world). Therefore, the numerical coincidence up to two orders of magnitude of the Eddington–Weinberg relations for the Universe and for large scale structures is, in our opinion, purely accidental. Therefore, as we have shown, what is really significant in our model, both on astrophysical and cosmological scales, are the micro/macro scaling relations and connectivity factors, that is Eqs. \( (\ref{E:8}) \), \( (\ref{E:10}) \), and \( (\ref{E:18}) \), which hold true for all systems and are built starting from the basic assumption of nuclear granularity, from the statistical fluctuative hypothesis (which provides the factor \( \sqrt{N} \)), and from purely kinetic, statistical and photometric observational quantities.

In conclusion, the typical hierarchical sizes of astrophysical structures could be explained by taking into account a fluctuative hypothesis which yields typical interaction ranges for the given granular components. Dynamics is implemented by a non-Newtonian gravitational potential where Yukawa corrections effectively act at that typical scale. However, as sketched in Sec.2, the value of the gravitational coupling is different at the various distances depending on the interaction ranges \( \lambda_c, \lambda_s, \lambda_g \). Finally, we want to stress that no vector boson or additional particle have been introduced and, thanks to the fluctuative hypothesis, the standard nucleons can completely account for sizes and stability of astrophysical structures. The results are in agreement with the statistical approach to the structure of spacetime (the so called “statistical geometry” as it is widely discussed in \[5\]).

REFERENCES

[1] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison–Wesley, New York, 1990).
[2] A. Sakharov, Zh. Eksp. Teor. Fiz. 49, 245 (1965).
  V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. 215, 203 (1992).
[3] V. de Lapparent, M. J. Geller and J. P. Huchra, Astrophys. Jour. 302, L1 (1986).
  M. J. Geller and J. P. Huchra, Science 246, 897 (1989).
[4] F. Calogero, Phys. Lett. A 228, 335 (1997).
[5] S. De Martino, S. De Siena and F. Illuminati, Mod. Phys. Lett. B 12, 291 (1998); Physica A 271, 324 (1999).
[6] S. Capozziello, S. De Martino, S. De Siena, and F. Illuminati, Mod. Phys. Lett. A 16, 1063 (2000).
[7] S.S. McGaugh, Ap. J. Lett. 541, L33 (2000).
[8] P. de Bernardis, et al., Nature 404, 955 (2000).
[9] R.H. Sanders, Astron. Astroph. 136, L21 (1984), Astron. Astrophys. Rev. 2, 1 (1990).
[10] D. H. Eckhardt, Phys. Rev. D 48, 3762 (1993).
[11] A.A. Starobinsky, Phys. Lett. B 91, 99 (1980).
[12] D. La and P.J. Steinhardt, Phys. Rev. Lett. 62, 376 (1989).
[13] S. Weinberg, Gravitation and Cosmology, Wiley, 1972 New York N.Y.
[14] I.L. Buchbinder, S.D. Odintsov, and I.L. Shapiro, Effective Action in Quantum Gravity, IOP Publishing (1992) Bristol.
[15] C.M. Will, Theory and Experiments in Gravitational Physics (1993) Cambridge Univ. Press, Cambridge.
[16] K. S. Stelle, Phys. Rev. D 16, 953 (1977); Gen. Rel. Grav. 9, 353 (1978).
[17] P.D. Mannheim and D. Kazanas, Astroph. Jou. 342, 635 (1989).
[18] O.V. Barabash and Yu. V. Shtanov, Phys. Rev. D60, 064008 (1999).
[19] J.D. Anderson et al., Phys. Rev. Lett. 81, 2858 (1998).
[20] I. Quant and H.-J. Schmidt, Astron. Nachr. 312, 97 (1991).
[21] M. Kenmoku, Y. Okamoto, and K. Shigemoto, Phys. Rev. 48 D, 587 (1993).
[22] A.H. Cook in 300 Years of Gravitation, Eds. S. Hawking and W. Israel, Cambridge Univ. Press, Cambridge 1987.
[23] E. Fischbach, D. Sudarsky, A. Szafer, C. Talmadge, and S.H. Aroson, Phys. Rev. Lett. 56, 3 (1986).
[24] C.C. Speake and T.J. Quinn, Phys. Rev. Lett. 61, 1340 (1988).
[25] D.H. Eckhardt, C. Jeheli, A.R. Lazarewicz, A.J. Romaides, Phys. Rev. Lett. 60, 2567 (1988).
[26] Y. Fujii, Phys. Lett. B 202 246, (1988).
[27] M. Capaccioli, S. Capozziello, G. Longo, and R. de Ritis, Astron. Nachr. 318, 1 (1997).
[28] J. Binney and S. Tremaine, Galactic Dynamics (Princeton University Press, Princeton, 1987).
[29] B.A. Vorontsov–Vel’yaminov, Extragalactic Astronomy Harwood Academic Pub. (London) 1987.
[30] G.O. Abell, Astroph. Jou. S. 3, 211 (1958).
[31] A. Camacho Quintana, Mod. Phys. Lett. A 14, 275 (1999).
[32] S. Roy Statistical Geometry and Applications to Microphysics and Cosmology Kluver Academic Pub., Dordrecht 1998.
[33] P. J. E. Peebles, Principles of Physical Cosmology (Princeton University Press, Princeton, 1993).
[34] J.D. Barrow and P.J. Tipler, The Anthropic Cosmological Principle (Oxford Univ. Press, Oxford 1986).