Improving Deep Learning with Differential Privacy using Gradient Encoding and Denoising

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Abstract

Deep learning models leak significant amounts of information about their training datasets. Previous work has investigated training models with differential privacy (DP) guarantees through adding DP noise to the gradients. However, such solutions (specifically, DPSGD), result in large degradations in the accuracy of the trained models. In this paper, we aim at training deep learning models with DP guarantees while preserving model accuracy much better than previous works. Our key technique is to encode gradients to map them to a smaller vector space, therefore enabling us to obtain DP guarantees for different noise distributions. This allows us to investigate and choose noise distributions that best preserve model accuracy for a target privacy budget. We also take advantage of the post-processing property of differential privacy by introducing the idea of denoising, which further improves the utility of the trained models without degrading their DP guarantees. We show that our mechanism outperforms the state-of-the-art DPSGD; for instance, for the same model accuracy of 96.1% on MNIST, our technique results in a privacy bound of $\epsilon = 3.2$ compared to $\epsilon = 6$ of DPSGD, which is a significant improvement.

1 Introduction

Deep neural networks (DNN) are used in a wide range of learning applications. Unfortunately, the high capacities of DNNs make them susceptible to leaking private information about the datasets they use for training. Recent works have demonstrated various types of privacy leakage in DNNs, most notably, through membership inference [19, 17, 5, 14] and model inversion [10] attacks.

A promising approach to alleviate such privacy leakage is to train DNN models with differential privacy (DP) guarantees. In particular, DPSGD by Abadi et al. [2] adds DP noise to clipped gradients, during the training process, to train models with DP privacy protection. Unfortunately, existing DP-based solutions significantly reduce the utility (prediction accuracy) of the trained models [11].
In this work, we present a framework to train DNN models with DP guarantees that offer much better tradeoffs between utility and privacy (i.e., they offer better prediction accuracies for the same privacy budgets). We use two key techniques in building our framework. First, while DPSGD uses Gaussian noise, recent works have shown that other noise distributions can improve utility for the same privacy budgets in different settings. For instance, Bun et al. \[4\] have shown that the Student-t noise distribution can achieve better utility for trimmed mean applications. However, deriving the privacy bounds of DNN for arbitrary noise distributions is not a trivial task; this is because estimating the privacy bound requires one to compute the distance between two probability distributions with infinite points, which is computationally infeasible and theoretically hard. To be able to use arbitrary noise distributions, we encode gradients into a finite vector space, and use numerical techniques to obtain DP privacy bounds for the given noise distribution. Specifically, we derive privacy bounds by calculating the Rényi distances within our finite set of encoded gradients. This allows us to search in the space of various noise distributions to identify those with better utility-privacy tradeoffs.

Our second key technique is a denoising mechanism that leverages the post-processing property of DP. Specifically, our denoising mechanism modifies privatized (i.e., noisy) gradients (by scaling them based on their useful information) to improve their contributions to model’s utility without reducing their DP privacy bounds.

We evaluate our framework on the CIFAR and MNIST datasets, demonstrating that it outperforms DPSGD on the privacy-utility tradeoff. For instance, we show that on MNIST, DPSGD achieves a 96.1% prediction accuracy with a privacy budget $\epsilon = 6$ while our framework achieves similar accuracy with an $\epsilon = 3.2$ (using $\delta = 10^{-5}$). Note that this is a significant improvement given the exponential impact of the privacy budget parameter ($\epsilon$) on privacy leakage.

2 Background

We overview the basics of differential privacy and deep learning. Table 1 lists the notations used across the paper.

2.1 Differential Privacy

Differential privacy \[7 \cite{dwork06}\] is the gold standard for data privacy. It is formally defined as below:

**Definition 1** (Differential Privacy). A randomized mechanism $M$ with domain $\mathcal{D}$ and range $\mathcal{R}$ preserves $(\epsilon, \delta)$-differential privacy iff for any two neighboring datasets $D, D' \in \mathcal{D}$ and for any subset $S \subseteq \mathcal{R}$ we have:

$$
\Pr[M(D) \in S] \leq e^\epsilon \Pr[M(D') \in S] + \delta
$$

where $\epsilon$ is the privacy budget and $\delta$ is the failure probability.

Rényi Differential Privacy (RDP) is a commonly-used relaxed definition for differential privacy.

**Definition 2** (Rényi Differential Privacy (RDP) \[15\]). A randomized mechanism $M$ with domain $\mathcal{D}$ is $(\alpha, \epsilon)$-RDP with order $\alpha \in (1, \infty)$ iff for any two neighboring datasets $D, D' \in \mathcal{D}$:

$$
D_\alpha(M(D)||M(D')) := \frac{1}{\alpha - 1} \log \mathbb{E}_{\lambda \sim M(D')} \left[ \frac{M(D)}{M(D')}^{\alpha} \right] \leq \epsilon
$$
### Table 1: Description of notations

| Notation | Description |
|----------|-------------|
| $D, D'$  | training datasets |
| $\mathcal{M}$ | privacy mechanism |
| $\theta$ | model parameter |
| $\nabla_{\theta}$ | gradient w.r.t model parameters $\theta$ |
| $\nabla_{\theta}^{\prime}$ | gradient w.r.t model parameters $\theta$ for a micro-batch |
| $\nabla_{\theta}^{E}$ | encoded gradient w.r.t model parameters $\theta$ for a micro-batch |
| $\nabla_{\theta}^{G}$ | aggregated vector of encoded gradients for all micro-batch in a minibatch |
| $\tilde{\nabla}_{\theta}^{G}$ | privatized gradient (after adding noise) for a mini-batch |
| $\hat{\nabla}_{\theta}^{G}$ | denoised (rescaled) privatized gradient vector for a mini-batch |
| $\vec{\psi}$ | a preselected gradient vector for encoding |
| $\Psi$ | the set of all preselected gradient vectors |
| $Z_t$ | probability distribution for the private mechanisms |
| $Z$ | the set of all probability distributions used in the learning |
| $\epsilon, \delta$ | privacy bound metrics |

**Lemma 1** (Adaptive Composition of RDP [15]). Consider two randomized mechanisms $\mathcal{M}_1$ and $\mathcal{M}_2$ that provide $(\alpha, \epsilon_1)$-RDP and $(\alpha, \epsilon_2)$-RDP, respectively. Composing $\mathcal{M}_1$ and $\mathcal{M}_2$ results in a mechanism with $(\alpha, \epsilon_1 + \epsilon_2)$-RDP.

**Lemma 2** (RDP to DP conversion [15]). If $\mathcal{M}$ obeys $(\alpha, \epsilon)$-RDP, then $\mathcal{M}$ is $(\epsilon + \log(\frac{1}{\delta})/(\alpha - 1), \delta)$-DP for all $\delta \in (0, 1)$.

**Lemma 3** (Post-processing of RDP [15]). Given a randomized mechanism that is $(\alpha, \delta)$—Rényi differentially private, applying a randomized mapping function on it does not increase its privacy budget, i.e., it will result in another $(\alpha, \delta)$—Rényi differentially private mechanism.

### 2.2 Deep Learning with Differential Privacy

Several works have used differential privacy in traditional machine learning algorithms to protect the privacy of the training data [12, 6, 9, 21, 3]. Many of these works [9, 3, 6] use properties such as convexity or smoothness for their privacy analysis, which is not necessarily true in deep learning, and therefore, one cannot use many of such methods in practice. Recently, Abadi et. al. [2] designed a deep learning training algorithm, DPSGD, where they used gradient clipping to limit the sensitivity of the learning algorithm, and then add noise to a clipped model gradient proportional to its sensitivity. They also introduced the *momentum accountant* technique allowing them to compute much tighter privacy bounds. In particular, they used the momentum accountant method to obtain high model prediction accuracies using single digit privacy budgets. McMahan et al. [13] applied the momentum accountant method on language models. They also showed the feasibility of using differential privacy in federated learning to achieve user-level privacy guarantees with an acceptable loss in model prediction accuracy. Wang et al. [20] showed that Rényi differential privacy can be used instead of accountant momentum to improve the privacy bounds. We use Rényi to compute the privacy bounds in this work. For a fair comparison, we also use Rényi differential privacy to compute the bounds for DPSGD.
Algorithm 1 Differentially Private Discrete SGD

Require: learning rate $\eta$, $\mu$-batchsize $\mu$, batchsize $n$, noise models $Z = \{Z_1, Z_2, \ldots, Z_T\}$, preselected gradient vectors $\Psi$

1: Initiate $\theta$ randomly
2: for $t \in \{T\}$ do
3: $B_t \leftarrow$ Sample $n$ instances from dataset randomly
4: $\nabla^G_{\theta} \leftarrow 0$
5: for $\mu$-batch $b \in B_t$ do
6: $\nabla^\mu_{\theta} \leftarrow$ gradient of micro-batch $b$
7: $\nabla^\mu_{\theta} \leftarrow$ encode gradients by solving equation (3) $\triangleright$ Encoding
8: $\nabla^G_{\theta} \leftarrow \nabla^G_{\theta} + \nabla^\mu_{\theta}$
9: end for
10: $Z \sim Z_t$
11: $\nabla^G_{\theta} \leftarrow \nabla^G_{\theta} + Z \triangleright$ Noise Addition
12: $\nabla^{\hat{G}}_{\theta} \leftarrow$ Error correction on $\nabla^{G}_{\theta}$ $\triangleright$ Error Correction
13: $\theta \leftarrow \theta - \eta \nabla^{\hat{G}}_{\theta}$
14: end for
15: return output $\theta$

3 Our Framework

In this paper, we present a generic framework to learn models with differential privacy guarantees, expanding over prior works \cite{2, 13}. Our work differs from prior works in two ways: First, our framework allows us to use different noise distributions (ones that have closed-form formulas and well-defined mean). Second, we leverage the post processing property of differential privacy to enhance the learning procedure and improve model accuracy without impacting privacy.

Overview: Here we present an overview of our framework. In each iteration of training, we select a random mini-batch from the training dataset. We further divide the mini-batch into micro-batches, where the number of micro-batches is a hyperparameter. For each micro-batch, we compute the gradient of the loss function ($l$) with respect to the model parameters. Then, we apply a particular encoding on the obtained gradients, as will be explained. The purpose of this encoding is to map gradients into a finite vector space, allowing us to apply arbitrary noise distributions on the encoded gradients while being able to compute their DP privacy guarantees. Next, we aggregate the encoded gradients from different micro-batches and apply a calibrated DP noise on them. Our framework allows us to use arbitrary noise distributions (thanks to our encoder); we evaluate the effect of different noise distributions on the performance of training. The final stage of our algorithm is a post-processing phase, called denoising, in which we use an error correction algorithm to improve the utility of the trained model given the adversary’s prior knowledge. The final gradients are then used to update the model.

Below, we describe the two key components of our framework, gradient encoding and denoising. We will then present our privacy analysis.
3.1 Gradient Encoding

Deriving differential privacy guarantees for arbitrary noise distributions is non-trivial; therefore, prior DP approaches for ML privacy [2, 13] mainly stick to specific distributions like Gaussian distribution for which the privacy bounds can be computed. A promising approach to compute the differential privacy bounds of a given noise distribution is to compute the Rényi distance (see (2)) for all possible neighboring datasets. While we can analytically compute the bound for a few mechanisms (e.g. Gaussian [20]), this is not computationally feasible for other distributions unless the input space is finite. To this aim, we use gradient encoding to map gradients into a finite space, enabling us to obtain Rényi DP bounds for arbitrary noise distributions.

**Empirical distribution of gradients:** In order to design our encoder, we first need to obtain the distribution of typical parameter gradients. We derive this distribution empirically by calculating the gradients for different models and datasets, and across different epochs of training. Figure 1 shows the empirical distribution of gradients for CIFAR and MNIST datasets in the first epoch of training, which can be fitted to a Gaussian distribution (the type of fitted distribution is not important in privacy bounds).

**Sampling the distribution for mapping:** We sample the empirical distribution of gradients to obtain a set \( \Psi = \{ \vec{\psi}_1, \vec{\psi}_2, \ldots, \vec{\psi}_n \} \) where each \( \vec{\psi}_i \) is a preselected gradient vector, and \( n \) is the number of preselected vectors. Each preselected gradient vector \( \vec{\psi}_i \) is a vector of size \( k \), where \( k \) is the size of the model. For each \( \vec{\psi}_i \), we draw its elements i.i.d. from the distribution of gradients.

**Encoding gradients using preselected vectors:** To encode a gradient vector \( \nabla \theta \), we replace it with a preselected gradient vector from \( \Psi \) that is closest to \( \nabla \theta \); therefore, we solve the following optimization problem:

\[
\text{Encode}(\nabla \theta) = \arg \min_{\vec{\psi}_i \in \Psi} ||\vec{\psi}_i - \nabla \theta||
\]

where \( ||\cdot|| \) is the cosine distance. Below, we optimize the computation complexity of this mechanism.

**Practical speedup of encoding:** Solving (3) can be time/memory exhausting, since the size of each vector \( \vec{\psi}_i \) equals the size of the DNN model (typically, millions of parameters), and to perform an effective encoding, we need to have thousands of preselected gradient vectors (i.e., \( \vec{\psi}_i \)'s). We use the following adjustments to speed up solving (3).
Algorithm 2 Encoding gradients

Require: Preselected sorted gradient vectors $\Psi$ and a gradient vector $\nabla^\mu_\theta$

1: sorted-inds ← argsort $|\nabla^\mu_\theta|$
2: $\vec{\psi} \leftarrow \arg\min_{\vec{\psi} \in \Psi} \frac{\vec{\psi} \cdot \nabla^\mu_\theta[\text{sorted-inds}]}{|\vec{\psi} \times |\nabla^\mu_\theta||}$
3: $t \leftarrow 0$
4: $\nabla^\mu_\theta \leftarrow \vec{0}$
5: for $i$ in sorted-indexes do
6: $\nabla^\mu_\theta[i] = \min(\vec{\psi}[t], \max(\nabla^\mu_\theta[i], -\vec{\psi}[t]))$
7: $t = t + 1$
8: end for
9: return $\nabla^\mu_\theta$

First, based on the empirical distribution of gradients (Figure 1), we see that many gradient values are close to zero; therefore, gradient vectors can be represented using sparse vectors. This allows us to reduce the size of each preselected gradient vector by a sparse representation, which will reduce both memory and time complexity of solving (3).

Second, using the translation invariance property of RDP [15] we are able to order the elements of each gradient vector. We sort the gradient vectors before encoding, therefore, we only compute the distance between the sorted gradient vectors and sorted preselected gradient vectors, not every possible permutation. Algorithm 2 summarizes our optimized mechanism for gradient encoding.

3.2 Denoising

As overviewed before, encoded gradients from different micro-batches are aggregated and summed with a noise vector; we call the resulted vector a privatized gradient vector. We leverage the post-processing property of RDP (Lemma 3) to modify privatized gradient vectors in order to improve model utility (prediction accuracy) without impacting their RDP privacy bounds.

Our denoising process scales privatized gradients, based on their value to model utility, before using them to update the model. Consider an oracle $O$ that measures the “usefulness” (i.e., information value) of a privatized gradient vector $\tilde{\nabla}^G_\theta$ as $O(\tilde{\nabla}^G_\theta)$. At the end of each iteration, our model is updated as $\hat{\theta} \leftarrow \theta - \eta O(\tilde{\nabla}^G_\theta) \cdot \tilde{\nabla}^G_\theta$. Note that based on the post-processing property of RDP, such denoising does not change RDP’s privacy guarantees as long as $O$ is a randomized process which does not use the private data (Lemma 3).

We formulate the utility usefulness of a privatized gradient vector as the closeness (inverse distance) between that privatized vector and its corresponding original gradient vector (the vector before noise addition). This is because, intuitively, the closer a private gradient vector is to the original gradient, it will contribute more to model accuracy. However, using the original vector to compute the scaling factor (the distance) will effect RDP’s privacy guarantees as it will make use of the private data. Instead, we approximate the usefulness of a privatized gradient with its distance from the noise distribution used by our algorithm (note that we measure the distance with the noise “distribution”, not the specific noise sample used by our training algorithm as doing the latter will violate privacy bounds). Therefore, we denoise a privatized gradient vector $\tilde{\nabla}^G_\theta$ as $\text{KS}(\tilde{\nabla}^G_\theta, \tilde{Z}_t) \cdot \nabla^G_\theta$, where $\tilde{Z}_t$ is the noise distribution and $\text{KS}(.)$ is the Kolmogorov-Smirnov (KS) distance metric (note that we also experimented with other distance metrics, and KS achieved the best results in our experiments). Therefore, our denoising formula is given by $\nabla^G_\theta \leftarrow \eta \text{KS}(\nabla^G_\theta, \tilde{Z}_t) \cdot \nabla^G_\theta$. 

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4 Privacy Analysis

In this section, we analyze the privacy bounds of our private learning framework. We use Rényi differential privacy to evaluate RDP guarantees, which can be converted to equivalent DP bounds using Lemma 2. To compute the RDP privacy parameter, we need to bound the Rényi distance between any two neighboring training datasets \( D, D' \in D \), where \( D \) is the underlying data distribution and \( D' = D \cup \{ n \} \). We start by computing the privacy bound for one iteration, then use the composition Lemma 1 to estimate an upper bound for the overall privacy parameter.

Given a training dataset \( D \), our training process can be formulated as a randomized algorithm given by

\[
\mathcal{M}(D) = \sum_{x \in b_i} \nabla_{\theta} x + Z,
\]

where \( b_i \) is a mini-batch sampled from \( D \), \( Z \) is the noise, and \( \nabla_{\theta} x \) is encoded gradient of the micro-batch \( x \) (we have omitted the denoising function as it does not impact privacy analysis). Now, we need to solve the following problem to bound the Rényi distance:

\[
\sup_{|D \setminus D'| = 1} D_{\alpha}(\mathcal{M}(D)||\mathcal{M}(D'))
\]

(4)

We know that each encoded gradient belongs to a finite set of preselected vectors, say a specific vector \( \vec{\psi} \in \Psi \). Now we find the Rényi differential privacy of one iteration given this specific gradient vector.

**Lemma 4.** Consider one iteration of Algorithm 1, and suppose there is only one preselected gradient vector, i.e., \( \Psi = \{ \vec{\psi} \} \). Algorithm 1 with the sampling rate \( q \), and a probability distribution described by its pdf \( z(\cdot; \mu) \), where \( \mu \) is the mean of the distribution, obeys \( (\alpha, \epsilon) \)-RDP, for a given \( \alpha \in \mathbb{N}/\{1\} \); \( \epsilon \) can be computed as follows:

\[
\epsilon(\alpha; q, \vec{\psi}, z) \leq \frac{1}{1 - \alpha} \log \sum_{k=0}^{\alpha} \binom{\alpha}{k} q^k (1 - q)^{\alpha - k} \prod_{\tau \in \vec{\psi}} \int_{-\infty}^{\infty} \frac{z(x; \tau)}{z(x; 0)} \frac{dx}{z(x; 0)}
\]

(5)

**Proof.** Please refer to the supplementary material document.

Next, we extend Lemma 4 to the case where \( \Psi \) has more than one element. To do this, we compute the bound for each element of \( \Psi \) separately, and then compute the maximum bound to obtain an upper bound on the Rényi distance. Lemma 5 summarizes this.

**Lemma 5.** One iteration of Algorithm 1 with a set of preselected gradient vectors \( \Psi = \{ \vec{\psi}_1, \vec{\psi}_2, \cdots, \vec{\psi}_n \} \), a sampling rate \( q \), and a probability distribution described by its pdf \( z(\cdot; \mu) \), where \( \mu \) is the mean of the distribution, obeys \( (\alpha, \epsilon) \)-RDP for a given \( \alpha \in \mathbb{N}/\{1\} \); \( \epsilon \) can be computed as follow:

\[
\epsilon(\alpha; q, \Psi, z) \leq \max_{\vec{\psi} \in \Psi} \epsilon(\alpha; q, \vec{\psi}, z)
\]

(6)

**Proof.** For any change in the training dataset \( D \), the aggregated gradient will change by one preselected gradient vector, therefore, the distance will not change more than the maximum possible distance.

Now, using Lemma 5 we can compute the privacy bounds for one iteration of our learning framework. Finally, we can use the composition theorem of RDP to bound the final privacy leakage of our learning framework.
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Figure 2: Comparing the privacy bound of our framework with DPSGD [2] for different model accuracies. We use a constant number of iterations and a $\delta = 10^{-5}$.

**Theorem 1.** Algorithm 1 with a set of preselected gradient vectors $\Psi = \{\psi_1, \psi_2, \cdots, \psi_n\}$, sampling rate $q$, number of iterations $T$, and probability distributions $\mathcal{Z} = \{Z_1, Z_2, \cdots, Z_T\}$ described by their pdf $z_t(\cdot; \mu)$, where $\mu$ is the mean of the distribution, obeys $(\alpha, \epsilon)-RDP$, for a given $\alpha \in \mathbb{N}/\{1\}$; $\epsilon$ can be computed as follow:

$$\epsilon(\alpha; q, \Psi, \mathcal{Z}, T) \leq \sum_{t=1}^{T} \epsilon(\alpha; q, \Psi, z_t)$$  \hspace{1cm} (7)

**Proof.** Using Lemma 5 and the composition Lemma 1.

As mentioned earlier, since the encoded gradients belong to a finite set $\Psi$, we can compute the privacy bound even for probability distributions where we do not know the general formulation of their Rényi distances.

## 5 Experimental Results

**Setup.** We have implemented our algorithm in PyTorch [18] and Tensorflow [1]. Unlike SGD, we need to compute the gradients per sample (or per $\mu$-batch), which makes it computationally expensive. We leverage multiple GPUs in parallel to speed up the training process. We train models on MNIST and CIFAR10 datasets, with cross-entropy as the loss function. We choose a training set comparable to that of Abadi et al. [2].

In our experiments, we used random sampling with replacement to create our training mini-batches (which is the correct way of creating the training batches based on the analysis)\footnote{Available at \url{https://github.com/tensorflow/privacy}}.

**Comparing performances for constant epochs.** Figure 2 compares the accuracy-$\epsilon$ trade-off of our system with the state-of-the-art system of DPSGD [2] for CIFAR and MNIST. As mentioned earlier, our framework allows using arbitrary noise distributions; we are only showing our results with Gaussian and Student-t noise.

\footnote{This is why we see some inconsistencies between our results and those at \url{https://github.com/tensorflow/privacy}, which uses shuffling to create mini-batches.}
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Figure 3: Comparing the best privacy bound of each mechanism for different accuracy values (to find the best privacy bound, we change various hyper-parameters such as the number of epochs, sample rates, and noise parameters)

when we use Gaussian and Student-t distributions, which performed best among all the noise distributions we experimented (we also experimented with Laplace, Cauchy, Variance Gamma (with $\lambda = 2$), and Sech distributions). To have fair evaluations, we use the same number of epochs for our framework as well as DPSGD (100 for CIFAR and 60 for MNIST). To achieve different privacy bounds, we varied the parameters of each of the noise distributions; for Gaussian, we used variances from 0.8 to 1.4, and for Student-t, we used 9 degrees of freedom with variances from 0.9 to 1.5. Note that we used a non-standardized form of Student-t distribution to be able to change the variance independent of the freedom degree. We also used the same learning rate, model architecture, and hyper-parameters across different noise distributions and methods. To select the preselected gradient vectors, we sample 1000 vectors with sizes similar to the size of the model gradient from a standard Gaussian distribution, and we replaced the values smaller than $10^{-5}$ with zero to avoid underflow in our privacy bound calculations. Then we clip the norm of each vector to be equal to one (to have a fair comparison with DPSGD). Also note that we use the standard $(\epsilon, \delta)$-DP metric for easier comparison with DPSGD (we convert RDP parameters to DP using Lemma 2). In all experiments we set $\delta = 10^{-5}$ similar to DPSGD.

As shown in Figure 2, our framework outperforms the state-of-the-art DPSGD by large margins. Specifically, using the Student-t distribution we can achieve similar accuracies with better privacy bounds (or equivalently, better accuracy with similar privacy budget). For instance, we see that for CIFAR our framework achieves a 55.5% accuracy with $\epsilon = 3.6$ compared to DPSGD’s $\epsilon = 5$, and on MNIST we achieve a 96 accuracy with $\epsilon = 2.5$ where DPSGD reaches the same accuracy with $\epsilon = 5$. We outperform DPSGD for two main reasons. First, framework allows using arbitrary noise distributions, and as can be seen, using Student-t noise results in a better performance compared to Gaussian (which is used by DPSGD). Second, we see that even our framework with Gaussian noise outperforms DPSGD (which also use Gaussian), which is due to our denoising mechanism.

Comparing the best performances. In our evaluations of Figure 2 we used the same, constant number of iterations across the systems (100 for CIFAR and 60 for MNIST). Alternatively, in Figure 3 we report the best accuracy each system can achieve for a given privacy budget but by varying other settings. Specifically, for each privacy budget, we tried different combinations of noise model parameters ($Z$), number of iterations ($T$), and sampling rates $q$, and reported the best results.
for each system. Note that, in practice searching the hyper-parameters for the best results will impact privacy bounds \cite{2}, however, we do this experiment to compare the best performances of different systems. From Figure 3, we see that our system still outperforms DPSGD. For instance, for CIFAR our framework achieves a 55% accuracy with $\epsilon = 3$ while DPSGD achieves similar accuracy with $\epsilon = 4.2$. Similarly, on MNIST we achieve a 96.1% accuracy with $\epsilon = 3.2$ compared to $\epsilon = 6$ for DPSGD. This shows a significant improvement given the exponential impact of $\epsilon$ on privacy leakage. Also by comparing Figure 2 and Figure 3, we see a similar gap exists between Student-t and DPSGD.

**Other takeaways.** An interesting takeaway of our experiments is the exponential behavior of the privacy budget $\epsilon$ with respect to model accuracy. Therefore, for higher accuracies, we need to spend much larger privacy budgets to further improve model accuracy. For instance, in the MNIST dataset, we can improve the accuracy by one percent from 94% to 95% by increasing the privacy budget by 0.6, however, increasing the accuracy from 96% to 97% requires to increase the privacy budget by 2. We observe a similar behavior for CIFAR. Note that the privacy budget itself has an exponential impact on privacy, which further demonstrates the difficulties of training private models with high accuracies.

**Training trajectory.** In Section 5, we presented results for the final accuracy of our models for different privacy budgets. To demonstrate why our algorithms achieve better results, here we take a look at the accuracy of the models over the training iterations. Figure 4 presents the model prediction accuracy for different iterations. When we look at the first iterations of training, our technique (Algorithm 1) yields noticeably higher accuracies compared to DPSGD for both MNIST and CIFAR; this allows our algorithm to spend most of the privacy budget for fine-tuning and attaining an overall higher model prediction accuracy.

![Figure 4](image-url)

Figure 4: Comparing model prediction accuracy for different iterations of training for constant hyper-parameters (i.e., comparable noise parameters, $\sigma = 1.1$ for Gaussian and $\sigma = 1$ for Student-t)

**Impact of our denoising component.** To demonstrate the impact of our denoising technique on the overall performance of our framework, we train two models using our framework (Algorithm 1) with and without the denoising component. Figure 5 compares model accuracies, showing that the denoising component has a substantial impact on improving our overall accuracy (by scaling up privatized gradients that are less noisy).

**Impact of different noise distributions on model accuracy.** Figure 6 shows the $l_1$ distance between privatized gradient vectors and the corresponding original gradient vectors for different
probability distributions, for different Rényi privacy budgets (with $\alpha = 2$). As we can see Student-T results in the least overall noise compared to other distributions, which leads to models with overall higher prediction accuracies.

![Figure 5: Effect of denoising on the learning trajectory](image1)

**Figure 5:** Effect of denoising on the learning trajectory

**Figure 6:** The $l_1$ distance between privatized gradient vectors and their originals for different Rényi privacy budgets ($D_2$)

**The convergence rate of our algorithm.** The convergence rate of a training algorithm is the number of iterations needed to reach a certain test accuracy; the convergence rate can impact the privacy budget and also the energy consumption. In Figure 7 we compare the convergence rate of our algorithm with DPSGD (for comparable noise parameters, $\sigma = 1.1$ for Gaussian, and $\sigma = 1$ for Student-t). The results show that Algorithm 1 can achieve similar accuracies to DPSGD [2] with in less training iterations, which results in lower training time, privacy budget and energy consumption.

![Figure 7: Comparing the convergence rate for constant hyper-parameters (i.e., comparable noise parameters, $\sigma = 1.1$ for Gaussian, and $\sigma = 1$ for Student-t)](image2)

**Figure 7:** Comparing the convergence rate for constant hyper-parameters (i.e., comparable noise parameters, $\sigma = 1.1$ for Gaussian, and $\sigma = 1$ for Student-t)
6 Conclusions

Despite their increasing adoption in a wide-range of applications, deep learning models are known to leak information about their training datasets. A promising approach to train DNN models with privacy protection is applying differential privacy noise on the gradients during the training process. However, existing approaches based on differential privacy result in large degradations in the utility (prediction accuracy) of the trained models. In this work, we design a framework to train DNN models with differential privacy guarantees while preserving utility significantly better than prior works. We specifically introduce two novel techniques to improve the utility-privacy tradeoff. First, we encode gradients into a finite vector space; this allows us to obtain privacy bounds for arbitrary noise distributions applied on the gradients, therefore enabling us to search among different noise distributions for the best privacy-utility tradeoffs. Second, we post-process obfuscated gradients, a technique we call denoising, to improve the utility of the trained model without impacting its privacy bounds. Our evaluations on two benchmark datasets show that our framework outperforms existing techniques by substantial margins. For instance, for the same model accuracy of 96.1% on MNIST, our technique results in a privacy bound of $\epsilon = 3.2$ while the state-of-the-art DPSGD results in $\epsilon = 6$.

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A Proof of Lemma 4

In this section, we present our proof for Lemma 4. We use Theorem 2 in our proof. The overall approach is similar to the proof of the subsampled Gaussian mechanism [16, 20]. We limit the Rényi differential privacy analysis only to integer \( \alpha \), then we use binomial expansion. Next, we use the addition rule for integrals and reorder them to our final bounds.

Theorem 2 (Mironov et al. [16]). Let \( P \) and \( Q \) be two differentiable distributions on \( \mathcal{X} \) such that there exists a differentiable mapping \( \nu: \mathcal{X} \mapsto \mathcal{X} \) satisfying \( \nu(\nu(x)) = x \) and \( P(x) = Q(P(\nu(x))) \). Then the following holds for all \( \alpha \leq 1 \) and \( q \in [0, 1] \):

\[
D_\alpha((1 - q)P + qQ) \geq D_\alpha(Q)((1 - q)P + qQ)
\]

Lemma 4 Consider one iteration of Algorithm 4, and suppose there is only one preselected gradient vector, i.e., \( \Psi = \{\vec{\psi}\} \). Algorithm 4 with the sampling rate \( q \), and a probability distribution described by its pdf \( z(x; \mu) \), where \( \mu \) is the mean of the distribution, obeys \((\alpha, \epsilon)\)-RDP, for a given \( \alpha \in \mathbb{N}/\{1\} \); \( \epsilon \) can be computed as follows:

\[
\epsilon(\alpha; q, \vec{\psi}, z) \leq \frac{1}{1 - \alpha} \log \sum_{k=0}^{\alpha} \binom{\alpha}{k} q^k (1 - q)^{\alpha-k} \prod_{\tau \in \vec{\psi}} \int_{-\infty}^{\infty} \left( \frac{z(x; \tau)}{z(\vec{x}; 0)} \right)^{k} z(x; 0) dx
\]

Proof. To show our mechanism obeys \( \epsilon, \alpha \)-Rényi differential privacy, we should show for any two neighbor datasets:

\[
D(\mathcal{M}(D) || \mathcal{M}(D')) \leq \epsilon
\]

and

\[
D(\mathcal{M}(D') || \mathcal{M}(D)) \leq \epsilon
\]

We use a probability distribution \( Z(\cdot; \vec{M}) \) in our privacy preserving mechanism, where \( \vec{M} \) is the mean vector of probability distribution and the covariance is \( \Sigma = \sigma I \). Now, for given a subsampling rate \( q \), we can rewrite the above mechanism as:

\[
D_\alpha((1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{M})) \leq \epsilon
\]

and

\[
D_\alpha(Z(\cdot; \vec{0}) || (1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{M})) \leq \epsilon
\]

We should show the validity of the above inequality for every possible mean vector. First, using Theorem 2 by letting \( \nu(\vec{x}) = \vec{M} - \vec{x} \) we get:

\[
D_\alpha((1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{M})) \geq D_\alpha(Z(\cdot; \vec{0}) || (1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{M}))
\]

Now, we only should compute the bound for \( D_\alpha((1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{M})) || Z(\cdot; \vec{0})) \). In lemma 4 we assumed we have only one possible mean vector \( M = \vec{\psi} \), so we can compute:

\[
D_\alpha((1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{M})) || Z(\cdot; \vec{0})) = D_\alpha((1 - q)Z(\cdot; \vec{0}) + qZ(\cdot; \vec{\psi})) || Z(\cdot; \vec{0}))
\]

\[
= \frac{1}{1 - \alpha} \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} Z(x_1, x_2, \ldots, x_n; \vec{0}) \left(1 - q + q \frac{Z(x_1, x_2, \ldots, x_n; \vec{\psi})}{Z(x_1, x_2, \ldots, x_n; 0)} \right)^{\alpha} dx_1 dx_2 \cdots dx_n
\]
Since covariance is diagonal and using the translation invariance of Rényi divergence, we have:

\[
\frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} Z(x_1, x_2, \cdots, x_n; \vec{0}) \left( 1 - q + q \frac{Z(x_1, x_2, \cdots, x_n; \vec{\psi})}{Z(x_1, x_2, \cdots, x_n; \vec{0})} \right)^{\alpha} dx_1 dx_2 \cdots dx_n
\]

\[= \frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} z(x_1; 0) \cdots z(x_n; 0) \left( 1 - q + q \frac{z(x_1; \psi^{(0)}) \cdots z(x_n; \psi^{(n)})}{z(x_1; 0) \cdots z(x_n; 0)} \right)^{\alpha} dx_1 dx_2 \cdots dx_n \]

\(17\)

Given that \(\alpha\) is an integer, we can write:

\[
\frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} z(x_1; 0) \cdots z(x_n; 0) \left( 1 - q + q \frac{z(x_1; \psi^{(0)}) \cdots z(x_n; \psi^{(n)})}{z(x_1; 0) \cdots z(x_n; 0)} \right)^{\alpha} dx_1 dx_2 \cdots dx_n \]

\[= \frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} z(x_1; 0) \cdots z(x_n; 0) \left( \sum_{k=0}^{\alpha} (1 - q)^{\alpha-k} q^k \frac{z(x_1; \psi^{(0)}) \cdots z(x_n; \psi^{(n)})}{z(x_1; 0) \cdots z(x_n; 0)} \right)^{\alpha} dx_1 dx_2 \cdots dx_n \]

\(18\)

Using the addition rule we can rewrite this as:

\[
\frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} z(x_1; 0) \cdots z(x_n; 0) \left( \sum_{k=0}^{\alpha} (1 - q)^{\alpha-k} q^k \frac{z(x_1; \psi^{(0)}) \cdots z(x_n; \psi^{(n)})}{z(x_1; 0) \cdots z(x_n; 0)} \right)^{\alpha} dx_1 dx_2 \cdots dx_n \]

\[= \frac{1}{1-\alpha} \log \sum_{k=0}^{\alpha} (1 - q)^{\alpha-k} q^k \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} z(x_1; 0) \cdots z(x_n; 0) \left( \frac{z(x_1; \psi^{(0)}) \cdots z(x_n; \psi^{(n)})}{z(x_1; 0) \cdots z(x_n; 0)} \right)^{k} dx_1 dx_2 \cdots dx_n \]

\(19\)

\[
= \frac{1}{1-\alpha} \log \sum_{k=0}^{\alpha} (\frac{\alpha}{k}) q^k (1 - q)^{\alpha-k} \prod_{\tau \in \vec{\psi}} \int_{-\infty}^{\infty} \left( \frac{z(x; \tau)}{z(x; 0)} \right)^k z(x; 0) dx \]

\(20\)