Undrained cyclic simple shear simulation of liquefiable granular particles subjected to large shear strains up to 100% by 3D DEM

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ABSTRACT

To clarify the influence of large shear strain on the cyclic behavior on liquefiable granular materials, a series of undrained cyclic simple shear simulations were carried out by utilizing 3D discrete element method (DEM). The amplitudes of the cyclic shear strain loaded on specimens range from 0.3% to 100%. The results show that when the cyclic shear strain loaded exceeds a certain amplitude, both the loose and the dense specimens exhibited high energy absorption properties, positive dilatancy characteristics, and non-liquefaction behaviors. This phenomenon might be caused by the uneven spatial distribution of particles during cyclic shear loadings. Moreover, the liquefiable cyclic shear strain amplitude range is affected by the density and particle size of specimens.

Keywords: cyclic simple shear, large shear strain, liquefaction, 3D discrete element method

1 INTRODUCTION

It has been recognized that the cyclic shear behavior of soils is highly depended on the strain level. As the cyclic shear strain amplitude increases, the behaviors of soil change from elastic to elastoplastic (Vucetic 1994; Ishihara 1996). However, speaking a liquefaction problem, studies are mainly concentrated in the strain range that no more than the order of 10^{-2} so far. The cyclic shear behavior of liquefiable soils at a shear strain level above this order of magnitude remains unknown.

The response of soils under cyclic shear loading is closely related to the changes in its microstructure. While, in traditional laboratory tests, it is hard to make an observation on the micro-behavior of soils directly. Discrete Element Method (DEM) is an effective tool to undertake the qualitative study on the granular material. It was put forward by Cundall and Strack (1979) and had been developed from 2D to 3D. 3D DEM overcomes some defects of traditional laboratory tests. On the one hand, it allows for the interpretation of phenomena through some parameters, such as potential energy (Kazama et al. 2006), which provides a new perspective. On the other hand, it can be used to simulate the complex behavior of soil elements, such as the application in the research of surface waves (Jiang et al. 2019).

Generally, there are two states of shear deformation: pure shear and simple shear. The cyclic simple shear mode is closer to the deformation mode of free-field level ground during seismic events (Kammerer et al. 2001). In order to discuss the effect of large shear strain (up to 100%) on the cyclic shear behavior of liquefiable granular materials, a series of undrained cyclic simple shear simulations were performed on soil elements at different levels of cyclic shear strain through 3D DEM. This condition cannot be reproduced by the actual experimentation due to mechanical limitations in reality. The results show that specimens will exhibit high energy absorption properties, positive dilatancy characteristics, and non-liquefaction behaviors when the cyclic shear strain loaded exceeds certain amplitudes, which are affected by the specimen density and particle size.

2 3D DEM SIMULATIONS

2.1 Specimen model set-up

A 3D DEM specimen model is shown in Fig. 1. At first, mono-sized spherical particles were generated and fall from the inlet to the cubic space that constrained by six frictionless rigid walls. Subsequently, the specimens were K_0-consolidated by moving the upper boundary wall downward at a constant speed. By adjusting the rolling resistance coefficient during the processes above, two types of specimens with different density but almost the same initial mean effective stress were generated. The void ratio of 0.8 represents a loose state, while the void ratio of 0.75 represents a dense state. The generated specimens, with a size of 100 mm (L) \times 100 mm (W) \times 100 mm (H), have an initial mean effective stress of 20 kPa. Finally, before the cyclic shear strain was loaded, the rolling resistance coefficient of particles was uniformly adjusted to 0.35.
2.2 Interaction models and parameters

The interaction models utilized in this study consist of the normal force model, the tangential force model, and the rolling resistance model. The normal force model is linear spring dashpot type; the tangential force model is linear spring Coulomb limit type, which is made up of the series connection of a linear spring and a slider; the rolling resistance model can be simplified as the series connection of a torsion spring and a rotation slider.

In this study, the particle of specimens is mono-sized. Especially, to confirm the particle size effects, tests were conducted at the same condition for each of three kinds of specimens which are different in particle sizes. The friction coefficient and rolling resistance coefficient between particles were determined by the hollow cylinder method to make the angle of repose of specimens around 30 degrees.

2.3 Undrained cyclic simple shear simulation

The constant volume method was utilized in this study. Without coupling a fluid phase, the volume of a specimen was kept constant during cyclic shear loadings to simulate the undrained condition. The deformation of specimens was strain-controlled and the cyclic shear strain was loaded by the pendulum movement of the lateral boundary walls, as shown in Fig. 1. The shear strain loading procedure is similar to the 3D version of that conducted by Yu et al. (2017). The two lateral boundary walls rotate around point A and B separately in a specific frequency. It should be noted that the cyclic shear strain amplitude $\gamma_a$ ranges from 0.3% to 100%. In order to keep the average shear strain rates the same, the pendulum frequency and cyclic shear strain amplitude were inversely proportional at different strain levels. The

| Particles | Diameter (mm) | 4, 8, and 10 |
|-----------|---------------|--------------|
| Density (g/cm³) | | 2.667 |
| Elastic modulus (N/m²) | | $1.0 \times 10^7$ |
| Rolling resistance coefficient | | 0.35 |

| Boundaries | Elastic modulus (N/m²) | $1.0 \times 10^{11}$ |
|------------|------------------------|---------------------|
| Initial confining pressure (kPa) | | 20±0.1 |

| Interactions | Friction coefficient (particle-particle) | 0.7 |
|--------------|----------------------------------------|-----|
| Friction coefficient (particle-boundary) | | 0.0 |
| Coefficient of restitution | | 0.3 |

| Computational parameters | Gravity (m/s²) | 0.0 |
|--------------------------|---------------|-----|
| Time step (s) | | $5.0 \times 10^{-6}$ |

3 RESULTS

3.1 Secant shear modulus ratio

The secant shear modulus ratio, defined as the ratio of the secant shear moduli to the secant shear modulus at the initial loading cycle, is an important index to evaluate the stiffness loss of soils.

The results for the specimens with a particle size of 4 mm are shown in Fig. 2. At a small cyclic shear strain amplitude below 0.3%, the stiffness of specimens almost did not decrease with the number of cycles; as $\gamma_a$ increa-
-ses, the stiffness of specimens starts to decrease with the number of cycles and becomes 0 at last; as $\gamma_a$ continues to increase and exceeds a threshold, the stiffness of specimens will decrease with the number of cycles at the beginning but stabilize at a non-zero region at last. In general, compared with the loose specimen, the stiffness reduction rate of the dense specimen is slower under the same shear strain condition. Particularly, 0.3% is considered as an intermediate-range of strain (Ishihara 1996). In this study, however, there is almost no decrease in stiffness of the specimens at this cyclic shear strain amplitude. It might be attributed to the size effect and mono-sized particles.

### 3.2 Damping ratio

The damping ratio represents the energy absorption property of soils during cyclic shear loadings. The results for the specimens with a particle size of 4 mm are shown in Fig. 3. As $\gamma_a$ increases, both the loose and the dense specimens have an increase in the initial damping ratio. At a small cyclic shear strain amplitude, like 0.3%, there is almost no energy dissipation in specimens. Specimens

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**Fig. 3.** Relation between the damping ratio and the number of cycles in specimens with a particle size of 4 mm.

Fig. 3 shows the damping ratio behavior under different cyclic shear strain amplitudes. As the cyclic shear strain amplitude increases, the damping ratio generally decreases to a value close to 0 when the specimen enters a fluid-like state. However, when $\gamma_a$ exceeds the threshold, the damping ratio of specimens

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**Fig. 4.** Relation between shear stress and shear strain of the specimens with the particle size of 4 mm.

Fig. 4 illustrates the shear stress-strain behavior of specimens under different conditions. The relation is plotted for various cases, including different particle sizes and shear strain amplitudes. The graphs demonstrate the cyclic behavior of the specimens, showing hysteresis loops that indicate the energy dissipation during shear loadings.

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hardly decreases with the number of cycles. This means that the accumulated dissipated energy in specimens will increase linearly with the number of cycles. In Fig. 3, the threshold is between 30% and 45%.

3.3 Shear stress-shear strain relationship

As shown in Fig. 4, the shear stress-shear strain response of specimens exhibited typical hysteresis loops. Their shapes are closely related to the damping ratios. As γ increases from 3% to 100%, the shape of the initial hysteresis loop changed from a spindle shape to a parallelogram-like shape.

There are two typical liquefaction behaviors for the saturated sands under undrained cyclic shear conditions: flow liquefaction and cyclic mobility (Casto 1975). In Fig. 4, phase 1 represents the state before the initial liquefaction, while phase 2 is on the contrary. The initial liquefaction is assumed to occur once the mean effective stress of specimens below 0.01 kPa. It is found that both the loose and the dense specimens experience the flow liquefaction at the cyclic shear strain amplitude of 3%. After initial liquefaction, their stiffness recovery is negligible. However, as γ increases to 30%, both the loose and the dense specimens exhibited cyclic mobility behaviors. Specimens repeat the process of gaining and losing stiffness after the initial liquefaction. The loose specimens finally entered a fluid-like state while the dense one did not. When γ increases to 100%, the specimens almost did not lose their stiffness. It can be learned that the liquefaction behavior is highly correlated to the cyclic shear strain amplitude and affected by the density of specimens.

3.4 Dilatancy behavior

In this study, the excess pore water pressure ratio is defined as below:

\[ \frac{\Delta u(t)}{\sigma_{m,0}} = \frac{\sigma'_{m,0} - \sigma'_m}{\sigma_{m,0}} \]

where, \( \Delta u(t) \) is the excess pore water pressure at time \( t \); \( \sigma_{m,0} \) is the initial mean effective stress, \( \sigma'_m \) is the mean effective stress at time \( t \). Fig. 5 shows the excess pore water pressure ratios of specimens after each cycle of loadings. In a shear strain amplitude larger than 45%, large negative excess pore water pressure ratios, more than -1, were generated in both loose and dense specimens. It indicates that when the cyclic shear strain exceeds a certain amplitude, the specimen will have obvious positive dilatancy characteristics.

3.5 The size effect

Figure 6 shows that the normalized accumulated dissipation energy (NDE) of specimens increases with the number of cycles. The NDE of specimens was calculated from the equation below:

\[ \Delta W = \frac{1}{\sigma_{m,0}} \int_0^t \tau(\gamma_i)\gamma_i(t)dt \]

where \( \Delta W \) is the NDE by time \( t \), \( \tau(\gamma_i) \) is the shear stress corresponding to the shear strain at time \( t \); \( \gamma_i(t) \) is the strain velocity at time \( t \). NDE is a useful index to estimate the liquefaction potential of soils (Kazama et al. 2000). It will increase linearly with the number of cycles if the specimen is non-liquefiable within a specific range of cyclic shear strain. In contrast, the NDE of specimens will remain constant after the specimens enter a fluid-like state after liquefaction. For specimens in the cyclic mobility liquefaction state, the growth of NDE will be slower than that in the pre-liquefaction state. Through the change of NDE under cyclic shear loadings, it is convenient to judge the liquefaction characteristic of specimens.

It can be seen from Fig. 6 that the particle size also has a great influence on the cyclic behavior of specimens. With the same cyclic shear strain amplitude and density, a specimen composed of larger particles will have a larger NDE after the same number of cycles. In other words, specimens with larger particle sizes have higher liquefaction resistance. Especially, a specimen having a particle size of 10 mm and a void ratio of 0.75 could hardly liquefy under any cyclic shear strain amplitude.

There should be two threshold cyclic shear strain amplitudes that separate the liquefaction behavior of specimen into flow liquefaction, cyclic mobility, and non-liquefaction. Yet, larger particle size will make the lower threshold move to a higher cyclic shear strain amplitude region and the higher threshold move to a lower cyclic shear strain amplitude region, which results in a narrower liquefiable cyclic shear strain amplitude range.

3.6 Behavior in micro-scales

Figure 7 shows the spatial distribution of particles at some points during cyclic shear loading by utilizing the volume fraction, which is the ratio of particle volume to the space volume. Due to the boundary condition, the volume fraction near the boundary wall is lower than that in the interior space and is shown in mint green in Fig. 7. As shown in Fig. 7. a and Fig. 7. b, at the cyclic shear strain amplitude of 3%, no matter the shear strain \( \gamma \) is near 0 or 3%, particles are uniformly distributed during

Fig. 5. The excess pore water pressure ratios of specimens with a particle size of 4 mm after each cycle of loadings.
cyclic shear loadings. However, as shown in Fig. 7. c and Fig. 7. d, at the cyclic shear strain amplitude of 100%, particles are unevenly distributed during cyclic shear loadings. The aforementioned characteristics of specimens during large cyclic shear loading might be explained by the uneven distribution of particles: Particles in some regions of the specimen are more densely packed than others. These relatively dense regions would have a stronger tendency to expand. However, due to the constant volume and the restriction of force chains, it cannot expand, which makes it impossible for particles to be transferred freely from the relatively dense regions to relatively loose regions. As a result, the specimens as a whole have a tendency to swell, with the generation of a large negative excess pore water pressure. The size and location of relatively dense regions will change regularly during cyclic shear loadings. It corresponds macroscopically to the plastic deformation, in which samples exhibit highly energy-absorbing properties. Moreover, at large cyclic shear strain amplitude, such as 100%, the above phenomenon does not disappear with the progress of cyclic shear loadings. Correspondingly, the specimen is non-liquefiable at this cyclic shear strain amplitude.

Fig. 6. The NDE of specimens during cyclic shear loadings
4 CONCLUSIONS

Through a series of cyclic simple shear simulations by 3D DEM, the conclusions about the granular material subjecting to cyclic shear loadings with large amplitude were summarized:

1) The stiffness loss and energy absorption of the specimen are highly affected by the cyclic shear strain amplitude. Over a certain large cyclic shear strain amplitude, usually several ten percents, specimens would have a limited stiffness loss and almost no decease in damping ratios;

2) The liquefaction behavior of specimens is also affected by the cyclic shear strain amplitude. When the cyclic shear strain loadings exceed a certain amplitude, specimens exhibit cyclic mobility or non-liquefaction characteristic, and is accompanied by obvious positive dilatancy characteristics;

3) The density and particle size have a significant influence on the cyclic behavior of specimens. The higher density and larger particle size make the specimens have a narrower range of liquefiable cyclic shear strain amplitude;

4) The aforementioned characteristics of specimens during large cyclic shear loading might be explained by the uneven spatial distribution of particles inside the specimen.

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