3D Chern-Simons Term breaks Meissner Effect

Yong Tao†

College of Economics and Management, Southwest University, Chongqing, China

Abstract: It is well-known that (2+1)-dimensional Chern-Simons term may lead to topological Meissner effect. However, we show that 3-dimensional Chern-Simons term, which is different from (2+1)-dimensional Chern-Simons term, will instead destroy traditional Meissner effect. This means that the presence of a 3-dimensional Chern-Simons term naturally guarantees the validity of “pre-formed pairs” picture explaining high-$T_c$ superconductivity, because such a term breaks Meissner effect induced by paired electrons. In particular, if 3-dimensional Chern-Simons gauge fields indeed emerge in the “pre-formed pairs” regime, our theory predicts that around the transition temperature $T_c$ the Hall conductivity $\sigma$ should exhibit a scaling law $\sigma \propto (T - T_c)^D$ with $D$ being the dimension of the superconducting material.

Keywords: Chern-Simons term; Meissner effect; Vortex; Landau-Ginzburg action; Pseudogap regime

PACS numbers: 74.72.Kf; 74.20.De; 74.25.Uv; 64.60.ae; 11.15.Yc

1. Introduction

Although 30 years have passed since the discovery of high-$T_c$ superconductivity, there is still no consensus on its physical origin. This is in large part due to a lack of understanding of pseudogap phase. Nowadays, the elucidation of the pseudogap has been a major challenge in condensed matter physics [1-9]. To describe pseudogap phase, one of the most popular models is based on the existence of pre-formed pairs [10-12]. In such a picture, the pairs form at the characteristic temperature $T_{pair}$, and would condense at the transition temperature $T_c$. It indicates that the pseudogap phase is a precursor to the superconducting state; that is, pairing without long range coherence. However, because there are some scholars who argue [4, 9] that $T_c \leq T_{pair} < T^*$ (where $T^*$ denotes the temperature that pseudogap opens), from here on we will use the notion “pre-formed pairs regime” rather than “pseudogap regime” to describe the temperature interval $[T_c, T_{pair}]$. If we denote the

† Corresponding author.
E-mail address: taoyingyong@yahoo.com
superconducting order parameter by \( \psi(x) = |\psi(x)|e^{i\theta(x)} \), the phase \( \theta(x) \) may fluctuate as a function of the spatial coordinate \( x \), for example driven by thermal fluctuations and quantum fluctuations enhanced in low-dimensional systems. Here the amplitude \( |\psi(x)| \) is closely related to the energy gap in the excitation spectrum. For temperature below the Berezinskii–Kosterlitz-Thouless (BKT) transition temperature \( T_{BKT} = T_c \), the order parameter \( \psi(x) \neq 0 \). For higher temperature \( T > T_c \) phase fluctuations destroy long range order, that is, \( \langle e^{i\theta(x)} \rangle = 0 \); while the amplitude (or modulus) \( |\psi(x)| \) remains finite up to the pairing crossover temperature \( T_{pair} \). The regime between \( T_c \) and \( T_{pair} \) is referred to as the “pre-formed pairs” regime. The powerful support for “pre-formed pairs” picture comes from the finding of “Nernst effect” [1]. In the cuprates, the Nernst effect persists far above \( T_c \) for underdoped compounds [13-14], indicating the presence of vortex-like excitations in the “pre-formed pairs” regime. This means that Landau-Ginzburg action would apply to “pre-formed pairs” regime, and further gives Abrikosov’s vortex picture.

Although the “pre-formed pairs” regime with \( |\psi(x)| > 0 \) is quite fascinating, it may lead to Meissner effect, which is often thought of as a fundamental proof that superconductivity occurs [15]. To realize this point, let us recall that the super-current equation in Landau-Ginzburg picture reads [15]: \( J_s \propto |\psi|^2 \cdot \left( \nabla \theta(x) + \frac{2e}{\hbar} A \right) \), where \( J_s \) denotes current density vector, \( e \) denotes electric charge, \( \hbar \) denotes Planck constant, and \( A \) denotes magnetic vector potential. In “pre-formed pairs” picture, although \( \langle \psi(x) \rangle = |\psi| \langle e^{i\theta(x)} \rangle = 0 \), we still have \( |\psi| \neq 0 \). Therefore, the super-current equation \( J_s \propto |\psi|^2 \cdot \left( \nabla \theta(x) + \frac{2e}{\hbar} A \right) \) holds. Unfortunately, combining Maxwell equation \( \nabla \times B \propto J_s \) and super-current equation yields \( \nabla^2 B \propto |\psi|^2 B \), which leads to Meissner effect with the penetration depth \( l_p \propto |\psi|^{-1} \), where \( B = \nabla \times A \). Here we have used the fact that \( \nabla \times \nabla \theta(x) = 0 \) holds except at vortex cores. Thus, we have to question why one cannot observe Meissner effect in “pre-formed pairs” regime. The main purpose of this paper is to show that if 3-dimensional Chern-Simons (CS) term emerges in “pre-formed pairs” regime, the Meissner effect will not take place. Furthermore, we predict that if 3-dimensional CS term indeed exists in the “pre-formed pairs” regime, the Hall conductivity will obey a scaling law around the transition temperature \( T_c \). At the end of this paper, we also point out the physical meaning of 3-dimensional CS term. In this paper, we set \( c = 1 \) and \( \hbar = 1 \), where \( c \) denotes light velocity.

The presentation is organized as follows: In Section 2 we point out the difference between (2+1)-dimensional and 3-dimensional CS terms. In Section 3 we show that Landau-Ginzburg action combined with a 3-dimensional CS term does not produce Meissner effect. In Section 4 we propose that Landau-Ginzburg action combined with a 3-dimensional CS term can be considered as an alternative model of describing high-\( T_c \) superconductivity. In Section 5 we show that our model can reproduce the results that have been universally accepted. In Section 6 our conclusion follows.

2. Difference between 3D and (2+1)D Chern-Simons terms
It is well known that (2+1)-dimensional CS term may lead to topological Meissner effect [16-20]; therefore, many scholars attempt to use it to describe the mechanism of superconductivity [16, 18, 19, 21-22]. Nevertheless, nobody checks the physical difference between (2+1)-dimensional and 3-dimensional CS terms. We will immediately show that 3-dimensional CS term, which is different from (2+1)-dimensional CS term, does not lead to Meissner effect. To see this, let us consider the following Maxwell-Chern-Simons action:

\[ \mathcal{L}_{MCS} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\sigma}{2} \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho, \]  

where \( A_\mu \) denotes the electromagnetic potential, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and \( \varepsilon_{\mu\nu\rho} = \sqrt{g} e_{\mu\nu\rho} \).

Here \( e_{\mu\nu\rho} \) denotes the totally anti-symmetric pseudo-tensor \( (e_{012} = 1) \), and \( g \) denotes the determinant of metric tensor \( g_{\mu\nu} \). If we denote the contravariant tensor of \( g_{\mu\nu} \) by \( g^{\mu\nu} \), then one has \( F^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \) and \( \varepsilon^{\mu\nu\rho} = g^{\mu\alpha} g^{\nu\beta} g^{\rho\gamma} \varepsilon_{\alpha\beta\gamma} \).

Moreover, the coefficient \( \sigma \) has the physical meaning of Hall conductivity [23-25].

For (2+1)-dimensional space-time, substituting metric (2) into equation (6) yields:

\[ \frac{\partial^2}{\partial t^2} B^\rho - \nabla^2 B^\rho + \sigma^2 B^\rho = 0. \]  

Using polar coordinates \((r, \varphi)\), the solution of equation (7) yields:

\[ B^0 \propto e^{i\lambda t} r^{-\frac{3}{2}} e^{-\frac{r}{\lambda_p}}, \]

where \( \lambda_p = \sqrt{\frac{1}{\sigma^2 - \kappa^2}} \) denotes the penetration depth, \( B^0 \) denotes the magnetic field.
perpendicular to 2-dimensional plane, and $\kappa$ denotes a constant.

The equation (8) clearly indicates a Meissner effect. In fact, many scholars have pointed out that (2+1)-dimensional CS term would lead to Meissner effect [16-20]. However, we immediately show that 3-dimensional CS term, which is different from (2+1)-dimensional CS term, does not lead to Meissner effect.

For 3-dimensional space, substituting metric (3) into equation (6) yields:

$$\nabla^2 B^\rho + \sigma^2 B^\rho = 0.$$  \hspace{1cm} (9)

Using cylindrical polar coordinates $(r, \varphi, z)$, the solution of equation (9) yields:

$$B^0 \propto e^{\pm i\kappa r},$$  \hspace{1cm} (10)

which indicates a magnetic wave with the wave length $\frac{2\pi}{\kappa}$ rather than a Meissner effect.

3. Breaking Meissner effect

The results (8) and (10) demonstrate that 3-dimensional CS term, which is different from (2+1)-dimensional CS term, does not produce Meissner effect. Next we further show that 3-dimensional CS term will destroy traditional Meissner effect. To this end, let us consider the following Lagrangian function [25]:

$$\mathcal{L} = \left[ (\partial_\mu + i q A_\mu) \phi \right]^* \left[ (\partial_\mu + i q A_\mu) \phi \right] + \lambda_2 |\phi|^2 + \lambda_4 |\phi|^4 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\sigma}{2} \varepsilon^{\mu\nu\rho\sigma} A_\mu \partial_\nu A_\rho,$$  \hspace{1cm} (11)

where $\phi = \frac{1}{\sqrt{2m}} \psi$, $m$ denotes the effective mass of the paired electrons, $q$ denotes the effective charge of the paired electrons, and $\psi$ denotes the order parameter.

Applying Euler-Lagrange variational procedure into Lagrangian function (11) one obtains:

$$\partial_\mu F^{\mu\nu} - \frac{\sigma}{2} \varepsilon^{\nu\alpha\beta} F_{\alpha\beta} = J^\nu,$$  \hspace{1cm} (12)

$$J^\nu = -\frac{i q}{2m} (\psi^* \partial^\nu \psi - \psi \partial^\nu \psi^*) + \frac{q^2}{m} |\psi|^2 A^\nu.$$  \hspace{1cm} (13)

Because the Lagrangian function (11) only holds around the transition temperature $T_c$, we assume that the order parameter has a constant magnitude, $|\psi|$, and a phase $\theta(x)$, which varies only slowly with position $x$.

Substituting $\psi(x) = |\psi| e^{i\theta(x)}$ into equation (13) yields:

$$J^\nu = \omega_0 \partial^\nu \theta + \omega A^\nu,$$  \hspace{1cm} (14)

where $\omega_0 = \frac{q}{m} |\psi|^2$ and $\omega = \frac{q^2}{m} |\psi|^2$.

Combining equations (12) and (14) one obtains:

$$\partial_\mu F^{\mu\nu} - \frac{\sigma}{2} \varepsilon^{\nu\alpha\beta} F_{\alpha\beta} - \omega A^\nu - \omega_0 \partial^\nu \theta = 0.$$  \hspace{1cm} (15)

Substituting equation (5) into equation (15) yields:
\[ \partial_\mu \partial_\nu \partial_\rho \partial_\sigma B^\rho + (\sigma^2 - 2\omega) \partial_\nu \partial_\sigma B^\rho + \omega^2 B^\rho = 0. \] \hspace{1cm} (16)

This paper has two main results. The equation (16) is the first one. The detailed derivation of obtaining equation (16) can be found in Appendix B.

To obtain the solution of equation (16), we adopt the method developed by Swain and Andelman [27]. Thus, let us write equation (16) in the form:

\[ (\partial_\mu \partial^\mu - \eta_+^2)(\partial_\nu \partial^\nu - \eta_-^2)B^\rho = 0, \] \hspace{1cm} (17)

where

\[ \eta_\pm^2 = -\frac{\sigma^2 - 2\omega \pm \sqrt{\sigma^4 - 4\omega \sigma^2}}{2}. \] \hspace{1cm} (18)

If we write the equation (17) in the form:

\[ (\partial_\nu \partial^\nu - \eta_+^2)B^\rho = B_+^\rho, \] \hspace{1cm} (19)

\[ (\partial_\mu \partial^\mu - \eta_-^2)B_+^\rho = 0, \] \hspace{1cm} (20)

then the solution of equation (17) can be written as:

\[ B^\rho = c_+ B_+^\rho + c_- B_-^\rho, \] \hspace{1cm} (21)

where \( c_+ \) and \( c_- \) are constants, and

\[ (\partial_\mu \partial^\mu - \eta_-^2)B_-^\rho = 0. \] \hspace{1cm} (22)

The equations (20) and (22) are known as second-order Helmholtz equations. Therefore, using cylindrical polar coordinates \((r, \varphi, z)\), the solution (21) of equation (16) can be rewritten in the form:

\[ B^0 \propto c_0 e^{\eta_+ r} + c_1 e^{-\eta_+ r} + c_2 e^{\eta_- r} + c_3 e^{-\eta_- r}, \] \hspace{1cm} (23)

where \( c_0, c_1, c_2, c_3 \) are undermined parameters.

Using equation (18), it is easy to check that:

\[ \eta_+ = i \frac{\sqrt{\sigma^2 - 4\omega}}{2} + i \frac{\omega}{2}, \] \hspace{1cm} (24)

\[ \eta_- = -i \frac{\sqrt{\sigma^2 - 4\omega}}{2} + i \frac{\omega}{2}, \] \hspace{1cm} (25)

where \( i^2 = -1 \).

Obviously, \( \eta_+ \) and \( \eta_- \) are both pure imaginary numbers if \( \sigma^2 - 4\omega \geq 0 \). This means that Meissner effect will not occur when \( \sigma^2 - 4\omega \geq 0 \). If \( 0 < \sigma^2 < 4\omega \), then \( \eta_+ \) and \( \eta_- \) are both complex numbers. This means that \( 0 < \sigma^2 < 4\omega \) will lead to “skin effect” in traditional conductor. Furthermore, it is easy to verify that \( \eta_+ \) and \( \eta_- \) are both real numbers if and only if \( \sigma = 0 \). In fact, when \( \sigma = 0 \), the equation (16) becomes the famous London equation. Therefore, we conclude that Meissner effect occurs at \( \sigma = 0 \). Because \( \sigma = 0 \) implies that 3-dimensional CS term must vanish, we verify that such a term indeed destroys Meissner effect. Thus, if 3-dimensional CS term emerges in “pre-formed pairs” regime, we can naturally explain why Meissner effect does not occur in such a regime.

In contrast, (2+1)-dimensional CS term will induce Meissner effect. To see this, let us replace \( \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \) in Lagrangian (11) by \( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \); otherwise, Meissner effect
will not occur at $\sigma = 0$ (This contradicts Landau-Ginzburg picture). Then by using Minkowskian metric (2) it is easy to check that $\eta_+ = \frac{-\sqrt{\sigma^2 + 4\omega + \sigma}}{2}$ and $\eta_- = \frac{\sqrt{\sigma^2 + 4\omega + \sigma}}{2}$, which are both real numbers. Therefore, Meissner effect occurs at $\sigma > 0$.

4. Model

The main hypothesis of this paper is to assume that the Lagrangian function (11) holds in the “pre-formed pairs” temperature interval $[T_c, T_{pair}]$. Following such a hypothesis, the Lagrangian function describing high-$T_c$ superconducting picture should have the form as below:

$$
\mathcal{L} = \begin{cases} 
\mathcal{L}_0 + \frac{\sigma}{2} e^{\mu \nu} A_\mu \partial_\nu A_\rho & T \in (T_c - 0^+, T_c) \\
\mathcal{L}_0 & T \in (T_c, T_c + 0^+) 
\end{cases}
$$

(26)

where $\mathcal{L}_0 = \left[ (\partial_\mu + iq A_\mu)\phi \right]^\dagger \left[ (\partial^\mu + iq A^\mu)\phi \right] + \lambda_2 |\phi|^2 + \lambda_4 |\phi|^4 + \frac{1}{4} F_{\mu \nu} F^{\mu \nu}$, $T$ denotes temperature, and $0^+$ denotes a tiny-positive number.

Clearly, to guarantee that high-$T_c$ superconducting picture (26) holds, one must have $\sigma \to 0$ if $T \to T_c$. Next we verify this fact.

Let us consider the path integral:

$$
Z = \int D\phi \int D\phi^* \int DA_\mu e^{-\int d^Dx \sqrt{g} \mathcal{L}}
$$

(27)

for $T \in (T_c, T_c + 0^+)$, where $D$ denotes the dimension of superconducting material.

Applying the standard procedure of renormalization group into path integral (27) Tao obtained [25]:

$$
q' = (q + \Delta q)(1 + \Delta F)^{-\frac{1}{2}} b^{-\frac{D-2}{2}},
$$

(28)

$$
\lambda_2' = (\lambda_2 + \Delta \lambda_2)(1 + \Delta E)^{-1} b^{-2},
$$

(29)

$$
\lambda_4' = (\lambda_4 + \Delta \lambda_4)(1 + \Delta E)^{-2} b^{D-4},
$$

(30)

$$
\sigma' = (\sigma + \Delta \sigma)(1 + \Delta F)^{-1} b^{-1},
$$

(31)

where $\Delta \lambda_2$, $\Delta \lambda_4$, $\Delta E$, $\Delta q$, $\Delta \sigma$, and $\Delta F$ stand for perturbative terms.

If one only considers the simplest case where perturbative terms are ignored, Tao obtained [25]:

$$
\sigma = \sigma_0 \left( \frac{48m^2\pi^2 T_c^2}{7(3N\varepsilon_F \lambda_0)} \right)^{\frac{1}{2-D}},
$$

(32)

where $\zeta(x)$ denotes Riemann’s zeta function, $\varepsilon_F$ denotes Fermi energy, and $N$ denotes number density of the electrons at the normal state. Moreover, $\sigma_0$ and $\lambda_0$ are undermined parameters.

Therefore, the equation (32) indicates a scaling law:

$$
\sigma \propto (T_c m)^{\frac{2}{D-5}}.
$$

(33)
On the other hand, the first-order perturbation of equations (29) and (30) yields the renormalization group equations:

\[
\frac{d\lambda_2}{dl} = -2\lambda_2, \quad (34)
\]

\[
\frac{d\lambda_4}{dl} = (D-4)\lambda_4, \quad (35)
\]

where \( l \) is a length somewhat larger than atomic dimensions, and \( b = 1 + \frac{8l}{l} \).

Combining equations (34) and (35) one obtains:

\[
\lambda_2 \propto \lambda_4^{\frac{2}{4-b}}. \quad (36)
\]

By Bardeen-Cooper-Schrieffer (BCS) Hamiltonian of superconductivity Gorkov has shown [25, 28]:

\[
\lambda_2 = \frac{2m(T-T_c)}{\lambda T_c}, \quad (37)
\]

\[
\lambda_4 = \frac{4m^2}{\lambda N}, \quad (38)
\]

where \( \lambda = \frac{7\zeta(3)\varepsilon_F}{12\pi^2 T_c^2} \).

Substituting equations (37) and (38) into equation (36) yields:

\[
T_c m \propto (T - T_c)^{\frac{4-D}{b}}. \quad (39)
\]

Combining equations (33) and (39) one obtains:

\[
\sigma \propto (T - T_c)^\frac{2}{b}. \quad (40)
\]

The scaling law (40) is the second one of two main results in this paper. It verifies that \( \sigma \rightarrow 0 \) if \( T \rightarrow T_c \). This means that Lagrangian function (11) will naturally lead to the high-\( T_c \) superconducting picture (26). The scaling law (40) can be regarded as a main prediction of the Lagrangian function (11). Using high-\( T_c \) superconducting materials one can test if there exists such a scaling law around the transition temperature \( T_c \).

Although here we only consider the first order perturbation of equations (29) and (30), the result (40) should be accurate enough. This is because combining equations (34), (35), (37) and (38) will produce the famous scaling laws: \( \phi \propto (T_c - T)^\beta \) and \( \xi \propto (T_c - T)^{-\nu} \) with \( \beta = \frac{D-2}{D} \) and \( \nu = \frac{2}{D} \), where \( \xi \) denotes the correlation length.

The detailed calculation refers to Appendix C. From Table 1, we note that theoretical values of \( \beta \) and \( \nu \) agree well with experimental results. The deviation is about an accuracy of 10\(^{-2}\). Therefore, we believe that the critical exponent \( \frac{2}{D} \) in the scaling law (40) is also accurate enough.
5. The validity of model

Next, we further show that Lagrangian function (11) can reproduce the results that have been universally accepted. If we consider the temperature interval \([0, T_c]\), then by equations (26) and (40) one can note that Lagrangian function (11) returns to Landau-Ginzburg action. So here we only check the case that \(T > T_c\). It is well-known that there exists a universal scaling law (i.e., Homes law) above \(T_c\) [29-34]:

\[
\lambda_p^{-2} \propto \sigma_{dc} T_c,
\]

(41)

where \(\sigma_{dc}\) denotes the dc conductivity measured at approximately \(T_c\), and \(\lambda_p\) denotes the penetration depth.

The scaling law (41) applies equally well to conventional and high-\(T_c\) superconductors [35]. Now we show that the Lagrangian function (11) will produce it.

Let us define the Green function:

\[
G(x, x') = -i\langle T(\psi(x)\psi^+(x'))\rangle.
\]

(42)

Using the technique developed by Abrikosov, Gor’kov and Dzyaloshinskii [36], substituting equation (42) into equation (13) yields:

\[
J^\mu(k) = Q(k)A^\mu(k),
\]

(43)

where

\[
Q(k) = \frac{N(q/2)^2}{2\pi T} \Delta(0)^2 \sum_{n=1}^{\infty} \frac{1}{[2(n+1)^2 \pi^2 T^2 + \Delta(0)^2]^{1/2}}.
\]

(44)

Here \(\Delta(0) \propto T_c\) denotes energy gap at zero temperature and \(\tau\) denotes the scattering relaxation time. To obtain equation (43), we have used \(\sigma \approx 0\) around the transition temperature \(T_c\) (refer to equation (40)).

If one orders \(u = (2n + 1)\pi T\), then by transforming the sum into an integral according to \(2\pi T \sum_{n=1}^{\infty} \rightarrow \int_0^\infty du\) one can rewrite equation (44) as:

\[
Q(k) = \frac{N(q/2)^2}{2\pi T} \Delta(0)^2 \int_0^\infty \frac{1}{(u^2 + \Delta(0)^2)^{1/2}} du. \tag{45}
\]

Thus, solving (45) we obtain:

\[
Q(k) = \frac{N\pi \Delta(0) \tau(q/2)^2}{2m} \left(1 - \frac{4}{\pi \sqrt{1 - \varepsilon}} \tan^{-1} \frac{\sqrt{1 - \varepsilon}}{\sqrt{1 + \varepsilon}}\right), \tag{46}
\]

where \(\varepsilon = \frac{1}{2\tau \Delta(0)}\) and we have used the formula [37]:

\[
\int_0^\infty \frac{dx}{(1+x^2)(\sqrt{1+x^2}+y)} = \frac{\pi}{2y} - \frac{2}{y(1-y^2)} \tan^{-1} \frac{1-y}{1+y}, \tag{47}
\]

\(\text{Table 1. The critical exponent for 3-dimensional material (} D = 3 \) [39]

| Critical exponent | Theoretical value | Experimental value |
|-------------------|-------------------|-------------------|
| \(\beta\)         | 1/3               | 0.327 ± 0.002     |
| \(\nu\)           | 2/3               | 0.64 ± 0.01       |
On the other hand, we have [36]:

\[ \lambda_p = \frac{1}{\sqrt{4\pi Q(k)}}, \]  

\[ \sigma_{dc} = \frac{N(q/2)^2}{m}. \]  

Substituting equations (46) and (49) into equation (48) yields:

\[ \lambda_p^{-2} \propto \sigma_{dc} T'_c \left( 1 - \frac{4}{\pi \sqrt{1-\epsilon^2}} \tan^{-1} \frac{1-\epsilon}{\sqrt{1+\epsilon}} \right), \]  

which successfully reproduces the scaling law (41).

6. Discussion and conclusion

Finally, we attempt to explain why 3-dimensional CS term may exist in the “pre-formed pairs” regime. To realize this point, let us recall that vortices emerge in the “pre-formed pairs” regime [13, 14]. Because each vortex carries one unit of magnetic flux, we can regard the “flux” as the “charge” of vortex. In this sense, the vortex, looks like an electron which carries one unit of electric charge, may be thought of as a “particle” which carries one unit of flux “charge”. In fact, a vortex can be regarded as the dual particle of a “Cooper pair”. If we introduce a complex scalar field \( \Phi \) to describe the vortices, the duality representation of Landau-Ginzburg Lagrangian \( \mathcal{L}_0 \) (see equation (26)) can be written in the form (see page 312 in [38]):

\[ \mathcal{L}_{vortex} \propto \left( \partial_\mu + i \ell A_\mu \right) \Phi^* \left( \left( \partial_\mu + i \ell A^\mu \right) \Phi \right) + W(\Phi^* \Phi) + \kappa \epsilon^{\mu \nu \rho} A_\mu \partial_\nu A_\rho. \]  

where, \( \ell \) denotes one unit of magnetic flux, \( \kappa \) denotes a constant, and \( W(\Phi^* \Phi) \) is a polynomial function of \( \Phi^* \Phi \).

The Lagrangian function (51) indicates that CS term will emerge if vortices exist. A single vortex cannot exist below the BKT transition temperature \( T_c \), since vortices \( \Phi \) and anti-vortices \( \Phi^* \) are tightly bound below \( T_c \). However, vortices and anti-vortices are liberated above \( T_c \). Therefore, we conclude that 3-dimensional CS term may emerge in the temperature interval \([T_c, T_{pair}]\).

In conclusion, we have proved that 3-dimensional CS term, which is different from (2+1)-dimensional CS term, does not lead to Meissner effect. Instead, we further show that 3-dimensional CS term will destroy traditional Meissner effect. This means that if 3-dimensional CS term emerges, we can naturally explain why Meissner effect does not occur in “pre-formed pairs” regime. As a result, the presence of a 3-dimensional CS term may guarantee the validity of “pre-formed pairs” picture explaining high-\( T_c \) superconductivity. In particular, if 3-dimensional CS term indeed emerges in the “pre-formed pairs” regime, our theory predicts that around the transition temperature \( T_c \) the Hall conductivity \( \sigma \) should exhibit a scaling law

\[ \sigma \propto (T - T_c)^D \]  

with \( D \) being the dimension of the superconducting material.
Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (Grant No. SWU1409444)

Appendix A

In this appendix we derive equation (6). To obtain the equation (6), we need to introduce a formula:

\[ \epsilon^{\nu \alpha \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = \partial_k \partial^\lambda A_\lambda - \partial^\lambda \partial_k A_\lambda. \]  

\[ \text{(A.1)} \]

**Proof.** Because \( \epsilon_{\nu k \lambda} \) is anti-symmetric, we have:

\[ \epsilon^{\nu \alpha \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = \epsilon^{\nu \alpha \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta + \epsilon^{\nu \alpha \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta. \]  

\[ \text{(A.2)} \]

On the one hand, one has:

\[ \epsilon^{\nu \lambda \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = \epsilon^{\nu \lambda \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = g^{\nu \gamma} g^{\beta \lambda} g^{\epsilon \delta} \epsilon_{\nu \epsilon \delta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta. \]  

\[ \text{(A.3)} \]

On the other hand, one has:

\[ \epsilon^{\nu \alpha \lambda} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = \epsilon^{\nu \alpha \lambda} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = g^{\nu \gamma} g^{\beta \lambda} g^{\epsilon \delta} \epsilon_{\nu \epsilon \delta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta. \]  

\[ \text{(A.4)} \]

Substituting equations (A.3) and (A.4) into equation (A.2) yields:

\[ \epsilon^{\nu \alpha \beta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta = g^{\nu \gamma} g^{\beta \lambda} g^{\epsilon \delta} \epsilon_{\nu \epsilon \delta} \epsilon_{\nu k \lambda} \partial^\lambda \partial_\alpha A_\beta. \]  

\[ \text{(A.5)} \]

where we have used the formula:

\[ g^{\nu \gamma} g^{\beta \lambda} g^{\epsilon \delta} = \frac{1}{g} \]  

\[ \text{(A.6)} \]

for \( \nu \neq \lambda \neq k \).

Substituting \( \epsilon_{\mu \nu \rho} = \sqrt{g} \epsilon_{\mu \nu \rho} \) into equation (A.5) and observe that \( \epsilon_{\mu \nu \rho} \) denotes the totally anti-symmetric pseudo-tensor, one can verify equation (A.1). □

The equation (4) can be rewritten in the form:

\[ \partial_\mu \partial^\mu A_\nu - \partial_\nu \partial^\mu A_\mu - \sigma \epsilon^{\nu \alpha \beta} \partial_\alpha A_\beta = 0, \]  

\[ \text{(A.7)} \]

where we have used

\[ \epsilon^{\nu \alpha \beta} \partial_\alpha A_\beta = \frac{1}{2} \epsilon^{\nu \alpha \beta} F_\alpha \beta. \]  

\[ \text{(A.8)} \]

Substituting Coulomb gauge

\[ \partial_\mu A_\mu = 0 \]  

\[ \text{(A.9)} \]

into equation (A.7) yields:

\[ \partial_\mu \partial^\mu A_\nu - \sigma \epsilon^{\nu \alpha \beta} \partial_\alpha A_\beta = 0. \]  

\[ \text{(A.10)} \]
Acting the operator $\varepsilon_{k\lambda\nu}\partial^{\lambda}$ on the equation (A.10) we have
\[
\partial_\mu \partial^\mu (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) - \sigma (\varepsilon^{\nu\alpha\beta}\varepsilon_{k\lambda\nu}\partial^{\lambda}\partial_\alpha A_\beta) = 0. \tag{A.11}
\]
On the other hand, substituting Coulomb gauge (A.9) into equation (A.1) one obtains:
\[
\varepsilon^{\nu\alpha\beta}\varepsilon_{vk\lambda}\partial^{\lambda}\partial_\alpha A_\beta = -\partial^{\lambda}\partial_\lambda A_k. \tag{A.12}
\]
Substituting equation (A.12) into equation (A.11) yields:
\[
\partial_\mu \partial^\mu (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) + \sigma \partial_\rho \partial^\rho A_k = 0. \tag{A.13}
\]
Substituting equation (A.10) into equation (A.13) one obtains:
\[
\partial_\mu \partial^\mu (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) + \sigma^2 g^\nu k \varepsilon^{\nu\alpha\beta} \partial_\alpha A_\beta = 0. \tag{A.14}
\]
Acting $g^{\rho k}$ on the equation (A.14) yields:
\[
\partial_\mu \partial^\mu (\varepsilon^{\rho\alpha\beta}\partial_\alpha A_\beta) + \sigma^2 \delta^\rho_\nu \varepsilon^{\nu\alpha\beta} \partial_\alpha A_\beta = 0, \tag{A.15}
\]
where we have used $g^{\rho k} \varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu = \varepsilon^{\rho\alpha\beta}\partial_\alpha A_\beta$ and $\delta^\rho_\nu = g^{\rho k} g_{vk}$.

By equation (5) we have:
\[
B^\mu = \varepsilon^{\mu\nu\rho} \partial_\nu A_\rho. \tag{A.16}
\]
Substituting equation (A.16) into equation (A.15) we finally obtain:
\[
\partial_\mu \partial^\mu B^\rho + \sigma^2 B^\rho = 0.
\]

**Appendix B**

In this appendix we derive equation (16). To obtain equation (16), let us rewrite equation (15) in the form:
\[
\partial_\mu \partial^\mu A^\nu - \sigma \varepsilon^{\nu\alpha\beta} \partial_\alpha A_\beta - \omega A^\nu - \omega_0 \partial^\nu \theta = 0, \tag{B.1}
\]
where we have used Coulomb gauge (A.9) and $\varepsilon^{\nu\alpha\beta} \partial_\alpha A_\beta = \frac{1}{2} \varepsilon^{\nu\alpha\beta} F_{\alpha\beta}$.

Acting the operator $\varepsilon_{k\lambda\nu}\partial^{\lambda}$ on the equation (B.1) we have:
\[
\partial_\mu \partial^\mu (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) - \sigma (\varepsilon^{\nu\alpha\beta}\varepsilon_{k\lambda\nu}\partial^{\lambda}\partial_\alpha A_\beta) - \omega (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) = 0, \tag{B.2}
\]
where we have used
\[
\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu = 0. \tag{B.3}
\]
The equation (B.3) indicates that we have ignored the vortex cores. This means that the equation (16) holds everywhere except at vortex cores.

Substituting equation (A.12) into equation (B.2) yields:
\[
\partial_\mu \partial^\mu (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) + \sigma \partial_\mu \partial^\mu A_k - \omega (\varepsilon_{k\lambda\nu}\partial^{\lambda}A^\nu) = 0. \tag{B.4}
\]
Using equation (A.16) one can rewrite equation (B.4) in the form:
\[ \partial_\mu \partial^\mu B_k + \sigma \partial^\lambda \partial_\lambda A_k - \omega B_k = 0. \]  
(B.5)

On the other hand, using equation (A.16) one can obtain:

\[ B^\nu = e^{\nu \alpha \beta} \partial_\alpha A_\beta = \frac{1}{\sigma} \left( \partial_\mu \partial^\mu A^\nu - \omega A^\nu - \omega_0 \partial^\nu \theta \right), \]  
(B.6)

where we have used equation (B.1).

Substituting equation (B.6) into equation (B.5) yields:

\[ \frac{1}{\sigma} \left[ \partial_\lambda \partial^\lambda \left( \partial_\mu \partial^\mu A_k - \omega A_k - \omega_0 \partial_k \theta \right) \right] + \sigma \partial^\lambda \partial_\lambda A_k - \frac{\omega}{\sigma} \left( \partial_\mu \partial^\mu A_k - \omega A_k - \omega_0 \partial_k \theta \right) = 0. \]  
(B.7)

Equation (B.7) can be rewritten as:

\[ \partial_\mu \partial^\mu A_k + (\sigma^2 - 2\omega) \partial_\nu \partial^\nu A_k + \omega A_k \]  

\[ - \omega_0 \partial_\mu \partial^\mu \partial_\nu \partial_\nu A_k + \omega \omega_0 \partial_\nu \partial_\nu A_k = 0. \]  
(B.8)

Acting the operator \( \epsilon^{\nu \rho \lambda} \partial_\rho \) on the equation (B.8) we obtain:

\[ \partial_\mu \partial^\mu \partial_\nu \partial^\nu B^\rho + (\sigma^2 - 2\omega) \partial_\nu \partial^\nu B^\rho + \omega^2 B^\rho = 0, \]

where we have used the equation (B.3).

**Appendix C**

In this appendix we derive the following two scaling laws:

\[ \phi \propto (T_c - T)^\beta \]  
(C.1)

\[ \xi \propto (T_c - T)^{-\nu} \]  
(C.2)

with \( \beta = \frac{D-2}{D} \) and \( \nu = \frac{2}{D} \).

First, Wilson has shown [40]:

\[ \phi \propto \left( -\frac{\lambda_2}{\lambda_4} \right)^{\frac{1}{2}}. \]  
(C.3)

Substituting equations (37) and (38) into equation (C.3) yields:

\[ \phi \propto \left( \frac{T_c - T}{m T_c} \right)^{\frac{1}{2}}. \]  
(C.4)

Substituting equation (39) into equation (C.4) we obtain:

\[ \phi \propto (T_c - T)^{\frac{D-2}{D}}, \]  
(C.5)

which agrees with equation (C.1).

Second, Wilson has shown [40]:

\[ \xi \propto (\lambda_2)^{\frac{1}{2}}. \]  
(C.6)

Substituting equations (37) and (39) into equation (C.6) we obtain:

\[ \xi \propto (T_c - T)^{-\frac{2}{D}}, \]  
(C.7)
which agrees with equation (C.2).

References:

[1]. M.R. Norman, D. Pines and C. Kallin, Advances in Physics 54, 715 (2005)
[2]. B. Fauque et al., Physical Review Letters 96, 197001 (2006)
[3]. S. Hufner et al., Rep. Prog. Phys. 71, 062501 (2008)
[4]. T. Kondo et al., Nature 457, 296 (2009)
[5]. Y. Li et al., Nature 468, 283 (2010)
[6]. R. Daou et al., Nature 463, 283 (2010)
[7]. M. Feld et al., Nature 480, 75 (2011)
[8]. M. Hashimoto et al., Nature Materials 14, 37 (2015)
[9]. A. Kaminski et al., Philosophical Magazine 95, 453 (2015)
[10]. V. J. Emery and S. A. Kivelson, Nature 374, 434 (1995)
[11]. V. J. Emery and S. A. Kivelson, Physical Review Letters 74, 3253 (1995)
[12]. C. Meingast et al., Physical Review Letters 86, 1606 (2001)
[13]. Z. A. Xu et al., Nature 406, 486 (2000)
[14]. Y. Wang et al., Physical Review Letters 95, 247002 (2005)
[15]. J. F. Annett, Superconductivity, Superfluids and Condensates (Oxford University, 2004)
[16]. S. Randjbar-Daemi, A. Salam and J. Strathdee, Nuclear Physics B 340, 403 (1990)
[17]. A. Cabo and D. Oliva, Physics Letters A 146, 75 (1990)
[18]. G. W. Semenoff and N. Weiss, Physics Letters B 250, 117 (1990)
[19]. E. J. Ferrer, R. Hurka, and V. De La Incera, Mod. Phys. Lett. B 11, 1 (1997)
[20]. P. Kotetes and G. Varelgiannis, Physical Review B 78, 220509 (R) (2008)
[21]. M. Asorey, F. Falceto and G. Sierra, Nuclear Physics B 622, 593 (2002)
[22]. N. Banerjee, S. Dutta and D. Roychowdhury, Classical and Quantum Gravity 31, 245005 (2014)
[23]. C. T. Hill, arXiv:0907.1101 (2009)
[24]. Y. Tao, Eur. Phys. J. B 73, 125 (2010)
[25]. Y. Tao, Scientific Reports 6, 23863 (2016)
[26]. G. V. Dunne, arXiv:hep-th/9902115 (1999)
[27]. P. S. Swain and D. Andelman, Langmuir 15, 8902 (1999)
[28]. L. P. Gorkov, Soviet Phys. JETP 9, 1364 (1959)
[29]. C. C. Homes et al., Nature 430, 539 (2004)
[30]. C. C. Homes et al., Physical Review B 72, 134517 (2005)
[31]. J. L. Tallon et al., Physical Review B 73, 180504 (R) (2006)
[32]. A. T. Bollinger et al., Nature 472, 458 (2011)
[33]. Y. Imry, M. Strongin and C. C. Homes, Physical Review Letters 109, 067003 (2012)
[34]. V. G. Kogan, Physical Review B 87, 220507 (R) (2013)
[35]. J. Zaanen, Nature 430, 512 (2004)
[36]. A. A. Abrikosov, L. P. Gor’kov, and I. E. Dzyaloshinskii, Methods of Quantum
Field Theory in Statistical Physics (Prentice-Hall, Englewood Cliffs, NJ, 1963).
[37]. V. G. Kogan et al., Physical Review B 54, 12386 (1996)
[38]. A. Zee, Quantum Field Theory in a Nutshell, Princeton University Press, 2003
[39]. R. Guida and J. Zinn-Justin, J. Phys. A: Math. Gen. 31, 8103 (1998)
[40]. K. G. Wilson, Reviews of Modern Physics 55, 583 (1983)