Mutual adaptation of a Faraday instability pattern with its flexible boundaries in floating fluid drops

G. Pucci, 1 E. Fort, 2 M. Ben Amar, 3 and Y. Couder 1

1 Matière et Systèmes Complexes, Université Paris Diderot, CNRS - UMR 7057, Bâtiment Condorcet, 10 rue Alice Domon et Léonie Duquet, 75013 Paris, France
2 Institut Langevin, ESPCI ParisTech and Université Paris Diderot, CNRS UMR 7587, 10 rue Vauquelin, 75231 Paris Cedex 05, France
3 Laboratoire de Physique Statistique, Ecole Normale Supérieure and Université Pierre et Marie Curie, 24 rue Lhomond, 75231 Paris Cedex 05, France

Hydrodynamic instabilities are usually investigated in confined geometries where the resulting spatio-temporal pattern is constrained by the boundary conditions. Here we study the Faraday instability in domains with flexible boundaries. This is implemented by triggering this instability in a floating fluid drop. An interaction of Faraday waves with the shape of the drop is observed, the radiation stress of the waves exerting a force on the surface tension held boundaries. Two regimes are observed. In the first one there is a co-adaptation of the wave pattern with the shape of the domain so that a steady configuration is reached. In the second one the radiation stress dominates and no steady regime is reached. The drop stretches and ultimately breaks into smaller domains that have a complex dynamics including spontaneous propagation.

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Fluid dynamics instabilities usually appear in two types of situations corresponding to confined or opened geometries respectively. For instance thermal buoyancy, when confined in a box, gives rise to patterns of Rayleigh-Bénard rolls adapted to the boundaries [1]. In contrast in an open medium, an isolated source of heat generates a thermal plume in which the growing turbulent structures define the envelop of the unstable region [2, 3]. We address an intermediate situation in which an instability develops inside a finite domain with flexible boundaries and we study the interplay between the pattern and its borders. We use the Faraday instability in which waves form on the surface of a fluid submitted to vertical oscillations [4] and whose resulting patterns have been widely studied in confined geometries [5, 6]. Here we choose to confine the Faraday instability in a drop of low viscosity floating on a very viscous, stable, immiscible fluid [Fig.1(a)]. We use a classical Faraday experiment set-up. A circular cell (radius 10cm, depth 0.8cm) is placed on a vibration exciter generating a vertical oscillation of acceleration $γ(t) = γ_m \cos(2πf_0t)$. The investigated frequency and amplitude ranges are respectively 50Hz < $f_0$ < 250Hz and $0 < γ_m/g < 10$, where g is the acceleration of gravity. The motion can be observed by a stroboscopic video camera or a high speed video camera. The heavier and more viscous fluid (fluid 1) fills the cell and forms a bath of thickness 5mm. A controlled quantity $V_2$ of the less viscous fluid (fluid 2) is then deposited becoming a floating drop that forms a single circular pancake far from the boundaries. Its radius $r_0$ and thickness $h_2$ at rest result from the equilibrium between buoyancy and capillarity [7, 8]. The size of the bordering meniscus fixed by the capillary length is small compared to the horizontal size.

FIG. 1: (a) Vertical section of an isopropanol drop floating on perfluorated oil in absence of oscillations. (b-d) A drop of volume $V_2 = 1.00 \pm 0.02$ml forced at $f_0 = 130$Hz for three amplitudes of forcing. (b) Faraday waves of weak amplitude, $γ_m/g = 3.28$, (c) steady elongated state, $γ_m/g = 3.93$ and (d) highly perturbed state $γ_m/g = 6.48$. (e) A drop of volume $V_2 = 10.0 \pm 0.1$ml square when forced at $f_0 = 160$Hz with $γ_m/g = 4.90$. The bars are 1cm long.
In all our experiments we find $h_2 \simeq 2\text{mm}$ and $r_0 \simeq 1.3\text{cm}$. There is a range of forcing amplitude for which the Faraday instability forms in the drop only. The resulting waves are initially disordered and generate fluctuations of the drop boundary [Fig.2(b)]. In return the fluctuations of the boundary generate an unsteadiness of the wave pattern. Two possible archetypes of evolution can then be observed [Fig.1] that depend on the fluids. We investigated many pairs of immiscible fluids, having different viscosity contrasts and wetting properties, and found only these two possible behaviours.

In the first archetype, the system self-organizes by a co-evolution of the wave field and the boundaries so that an equilibrium is reached [9]. The example shown in fig.1 was obtained with isopropanol (density $\rho_2 = 785\text{kg/m}^3$, viscosity $\mu_2 = 1.8 \times 10^{-3}\text{Pa} \cdot \text{s}$ and surface tension against vapor $\sigma_2 = 24\text{mN/m}$) floating on perfluorinated oil ($\rho_1 = 1850\text{kg/m}^3$, $\mu_1 = 26 \times 10^{-3}\text{Pa} \cdot \text{s}$ and $\sigma_1 = 15\text{mN/m}$). The measured interfacial tension is $\sigma_{1,2} = 6.3\text{mN/m}$.

In the second type of evolution the displacement of the boundary due to the waves does not lead to equilibrium [10]. The formation of parallel standing waves results in a constantly increasing elongation of the drop into a snake-like structure which breaks into fragments having a large variety of dynamical behaviors. The example shown in fig.2 was obtained with ethanol ($\rho'_2 = 789\text{kg/m}^3$, $\mu'_2 = 0.9 \times 10^{-3}\text{Pa} \cdot \text{s}$, $\sigma'_2 = 23\text{mN/m}$) floating on silicon oil ($\rho'_1 = 965\text{kg/m}^3$, $\mu'_1 = 100 \times 10^{-3}\text{Pa} \cdot \text{s}$ and $\sigma'_1 = 20\text{mN/m}$). The measured interfacial tension is $\sigma'_{1,2} = 0.7\text{mN/m}$.

The difference between the two regimes can be understood by a dimensional analysis evaluating the ratio of the destabilizing factor (the radiation pressure) to the restoring one (capillary pressure) as will be discussed below.

We first characterize experimentally the former archetype as a function of the frequency and the forcing amplitude. A phase diagram depicting the different states of the system is shown in fig.3. When the forcing amplitude is increased, Faraday waves appear in the drop above a first onset ($\gamma_m > \gamma_m^D$). They form complex unsteady patterns which deform the drop [Fig.1(b)], its average shape remaining circular. Above a second thresh-

![FIG. 2: The case of an ethanol drop of volume $V'_2 = 1.00 \pm 0.02\text{ml}$ deposited on silicon oil. (a) Sketch of the vertical section of the floating drop in the absence of oscillations. (b) It is circular at rest with area and thickness $A_2 = 459 \pm 5\text{mm}^2$ and $h_2 = 2.2\text{mm}$ respectively. (c-e) Three successive images showing the temporal evolution of the drop when forced at $f_0 = 130\text{Hz}$ with $\gamma_m/g = 7.00$. The bar is 1cm long.](image)

![FIG. 3: Phase diagram of the different regimes observed in the case of an isopropanol drop of volume $V_2 = 1.00 \pm 0.02\text{ml}$ deposited on perfluorinated oil. The area at rest is $A_2 = 531 \pm 11\text{mm}^2$, its average thickness $h_2 = 1.9\text{mm}$. C = circular, D = deformed, E = elongated, HP = highly perturbed, F = Faraday instability in oil.](image)
capillary waves for the Faraday frequency \( f_0/2 \).

The first type of behavior can be understood as resulting from the effect of the radiation pressure \( P_r \) of the surface waves distorting the boundaries of the pancake. We thus seek stable solutions in which the radiation and hydrostatic pressures are balanced by capillarity. We assume an unidirectional standing wave along the x-axis, with small thickness for the drop and small wavelength for the wave, the initial radius of the drop being the length unit. The small thickness assumption allows a lubrication approximation transforming the 3D boundary value problem in a 2D one while the small wavelength approximation allows an average on larger length-scales.

An asymptotic analysis adapted from [12] shows that the function \( y(x) \) describing the drop shape satisfies a two-dimensional Laplace law modified by the radiation pressure along the normal. It is a Riccati equation:

\[
P_h + P_r \frac{y^2(x)}{1 + y^2(x)} = -\sigma_2 f(\theta) \frac{d}{dx} \frac{y'(x)}{\sqrt{1 + y^2(x)}}
\]

where \( P_h \) is a term of hydrostatic pressure (assumed constant for quasi-boundary), \( f(\theta) \) depends on the wetting angle and \( P_r \) is the radiation pressure

\[
P_r = \frac{P_2^2 \rho_1 - P_2^2}{\rho_1} \omega^2 A^2 F^2
\]

with \( A \) the amplitude of the Faraday waves, \( \omega = \pi f_0 \) the angular frequency and \( F = e^{\gamma_1 - \gamma_2}(1 + B \cosh k h_2) - B \sinh k h_1 \), with \( B = -k/\omega^2[(\rho_2 - \rho_1)/\rho_2 g - \sigma_1 k^2/\rho_1] \), a factor that takes into account thickness effects and waves transmission for a baroclinic mode in fluid 1, \( k \) being the wave vector modulus. Using the volume conservation, the parameters of eq. [4] reduce only to one free parameter responsible for the final equilibrium shape, that is the ratio \( a = P_r/P_h \). In this case we have found explicit analytical solutions of eq. [3]. The drop shape is given by

\[
y(x) = \pm \frac{1}{P_h \sqrt{1 + a}} \log(\sqrt{1 + a}(\cos P_h' \sqrt{ax} + \\
+ \sin^2 \Phi_0 - \sin^2 P_h' \sqrt{ax}))
\]

with \( \sin^2 \Phi_0 = a/(1 + a) \) and \( P_h' = P_h/|\sigma_2 f(\theta)| \). The conservation of the area gives \( P_h' = |\log((1 + a)/a)^{1/2} \) being \( P_h' = 1 \) at rest \( (P_r = 0) \). When \( R > 0.25 \) good fits of the experimental shapes are obtained [Fig.3(b, c)]. The evolution of the aspect ratio \( R \) with forcing can also be obtained analytically

\[
R = \frac{\log \left( \sqrt{a} + \sqrt{1 + a} \right)}{\sqrt{1 + a} \arcsin \sqrt{a}/(1 + a)}
\]

We measured the amplitude \( A \) of the waves in the drops by lateral zoomed-in films and checked that it varies as \((\gamma_m - \gamma_d)/r_0\). Using it to evaluate \( P_r \) and thus \( a \), we obtain fits of \( R(\gamma_m) \) for various frequencies [Fig.4(a)]. The computed shapes become inconsistent with the initial small wavelength and small thickness hypothesis of the model whenever the radius of curvature at the tip of the computed shape becomes small. For this reason the very elongated shapes \((R < 0.25) \) obtained with this model cannot account for reality. Experimentally we observe that the shape of the tip becomes fixed while the drop keeps elongating, the curvature radius of the tip being of the order of the wavelength.

We can now turn to the second archetype. The main characteristic is that ethanol is wetted by silicon oil. At rest an oil film is observed to cover the upper surface of the drop [Fig.2(a)]. The instability in ethanol appears in a subcritical way i.e. large amplitude waves form at threshold. These waves stretch the drop [Fig.2(c)] but there is no convergence towards a final stable shape. There is no theory for this dynamical regime yet.

However its existence can be understood by dimensional analysis. We use \( a_0 = P_r/P_h \), the ratio of the estimated wave radiation pressure [Eq.2] to the 2D pressure due to capillary effects in the circular drop at rest. For drops of centimetric size and Faraday waves of the usually observed amplitude this analysis applied to the

![FIG. 4: (a) Evolution of the aspect ratio of an elongated isopropanol drop on perfluorated oil as a function of the forcing amplitude for different frequencies. The lines are the fits by eq.4 (b, c) Photographs of two drops in an elongated state at \( f_0 = 130 \) Hz for \( \gamma_m/g = 3.97 \) and \( R = 0.42 \) (b) and \( \gamma_m/g = 4.62 \) and \( R = 0.28 \) (c) and the predicted shapes by the corresponding solutions [Eq.3]. The bar is 1cm long.](attachment:fig4.png)
first archetype gives $a_0 \simeq 0.1$. In the second archetype, because of the wetting, the capillary tension is the interfacial one $\sigma_{1,2} = 0.7\text{mN/m}$. As a result we find $a_0 \simeq 2$. While in the first case the pressure exerted by the waves is only a perturbation of the capillary equilibrium of the drop, in the second this radiation pressure exceeds the possible response of capillary forces so that no steady solution can be reached.

The elongation is followed by buckling [Fig.2(d)] then breaking into several fragments [Fig.2(c), Fig.3(a)]. We measured the total area $A'_0$ covered by the pattern and found that it increases with forcing, proving that the drops are stretched by the waves (their thickness decreases). By buckling large fragments can take croissant or a horseshoe shapes [Fig.5(d)]. These shapes are stationary but propagate in the direction of the curvature. This is one more example of self-propagation by a spontaneous symmetry breaking [13]. The velocity is constant and a croissant (or a horseshoe) could move indefinitely in an infinite bath. In practice a bath of finite size is covered with moving and motionless fragments [Fig.5(a)] that keep colliding, merging and splitting. The global aspect is reminiscent of the interplay of structures obtained in cellular automata such as Conway’s game of life [13]. Very small fragments are observed to remain steady with a stationary shape that recalls the elongated one of the first regime [Fig.5(b)]. This possibility of a steady regime for small drops can be understood by the dimensional analysis: $a_0$ becomes smaller when the radius of the drop at rest is very small.

A remarkable feature of the snake-like structure is that they have constant transverse widths and tip radii, both being of the order of the Faraday wavelength. Their stretching and instabilities affect mostly the middle of their length. The buckling could be due to the generation of a streaming flow [15] that has been already observed in Faraday instability [16]. This streaming effect could also be responsible for their motion on the substrate. The investigation of the resulting dynamical regimes is beyond the scope of this letter.

We have found that, by slow dynamics, a mutual adaptation is possible between an instability and its boundaries. This phenomenon is related to the self-tuning of oscillators. In optical cavities the radiation pressure was shown [17] to create a coupling between the mirrors degrees of freedom and the optical field. Similarly, a forced mechanical oscillator with an additional degree of freedom [18] exhibits a slow drift by which it can self-tune. The present phenomenon is more general and should show up in other type of instabilities confined in adaptable boundaries.

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