FROM PRIMORDIAL $^4$He ABUNDANCE TO THE HIGGS FIELD

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ABSTRACT

We constrain the possible time variation of the Higgs vacuum expectation value ($v$) by recent results on the primordial $^4$He abundance ($Y_p$). For that, we improve the analytic models of the key processes in our previous analytic calculation of the primordial $^4$He abundance. Furthermore, the latest results on the neutron decay, the baryon-to-photon ratio based on 5 year WMAP observations, and a new dependence of the deuteron binding energy on $v$ are incorporated. Finally, we approximate the weak freeze-out, the cross section of photodisintegration of the deuteron, the mean lifetime of the free neutron, the mass difference of the neutron and proton, the Fermi coupling constant, the mass of the electron, and the binding energy of the deuteron in terms of $v$, to constrain its possible time variation by recent results on the primordial $^4$He abundance, $|\Delta s/v| \leq 1.5 \times 10^{-4}$.

Subject headings: cosmological parameters — cosmology: theory — early universe

1. INTRODUCTION

The standard model (Griffiths 1987) is a remarkably successful description of fundamental particle interactions. The theory contains parameters—such as particle masses—which are still unknown and which cannot be predicted, but whose values are constrained through their interactions with the conjectured Higgs field. The Higgs field is assumed to have a nonzero value in the ground state of the universe—called its vacuum expectation value $v$—and elementary particles that interact with the Higgs field obtain a mass proportional to this fundamental constant of nature.

Although the question whether the fundamental constants are in fact constant has a long history of study (see Uzan 2003 for a review), comparatively less interest has been directed toward the consequences of a possible variation of $v$ (Gaßner & Lesch 2008; Dent et al. 2007; Chamoun et al. 2007; Landau et al. 2006; Li & Chu 2006; Yoo & Scherrer 2003; Ichikawa & Kawasaki 2002; Kujat & Scherrer 2000; Scherrer & Spergel 1993; Dixit & Sher 1988). A macroscopic probe to determine the allowed variation range is given by the network of nuclear interactions during the big bang nucleosynthesis (BBN; see Yao et al. [2006] for a review of the standard big bang nucleosynthesis [BBBN] model), with its final primordial abundance of $^4$He. The relevant key parameters are the freeze-out concentration of neutrons and protons, the so-called deuterium bottleneck (the effective start of the primordial nucleosynthesis), and the neutron decay. Their dependency on $v$ and the final impact on the resulting primordial abundance of $^4$He can be understood more clearly by an in-depth approach.

Here we present a revised calculation of the primordial $^4$He abundance (Gaßner & Lesch 2008), where the analytic models of all key processes have been improved. The opening of the deuterium bottleneck and the weak freeze-out are determined more accurately, and a new dependence of the deuteron binding energy on $v$ is incorporated, based on different nucleon-nucleon potential models. The analytic approach enables us to take important issues into consideration that have been ignored by previous authors, as there are the $v$-dependence of the relevant cross sections of deuterium production and its photodisintegration. Furthermore, we take a nonequilibrium Ansatz for the freeze-out concentration of neutrons and protons and incorporate the latest results on the neutron decay and the baryon-to-photon ratio.

Finally, we approximate the weak freeze-out, the cross section of photodisintegration of the deuteron, the mean lifetime of the free neutron, the mass difference of the neutron and proton, the Fermi coupling constant, the mass of the electron, and the binding energy of the deuteron in terms of $v$, to constrain its possible time variation by recent results on the primordial $^4$He abundance (Peimbert et al. 2007; Izotov et al. 2007).

We briefly note, that constraints on the spacial variation of $v$ require a measurement of helium abundance anisotropy or inhomogeneity versus the position in the sky and an inhomogeneous theoretical BBN model. The homogeneous formalism used throughout the paper thus assumes a spacial invariance of the Higgs vacuum expectation value.

2. CALCULATIONS

All relevant processes of SBBN took place at a very early epoch, when the energy density was dominated by radiation, leading to a time-temperature relation for a flat universe,

$$ t = \sqrt{\frac{90h^3c^5}{32\pi^2k^4Gg_s}} \frac{1}{T^2} \ [s], \quad (1) $$
where \( c \) is the velocity of light, \( k \) is the Boltzmann constant, \( G \) denotes the gravitational constant, \( \hbar \) is the Planck constant divided by \( 2\pi \), and \( g_s \) counts the total number of effectively massless \( (mc^2 \ll kT) \) degrees of freedom, given by \( g_s = (g_b + \frac{1}{2} g_f) \), in which \( g_b \) represents the bosonic and \( g_f \) the fermionic contributions at the relevant temperature.

At very high temperatures \((T \gg 10^{10} \text{ K})\), the neutrons and protons are kept in thermal and chemical equilibrium by the weak interactions

\[
\begin{align*}
    n + e^+ &\rightarrow p + \bar{\nu}_e, \\
    n + \nu_e &\rightarrow p + e^-, \\
    n &\rightarrow p + e^- + \bar{\nu}_e,
\end{align*}
\]

until the temperature drops to a certain level, at which the inverse reactions become inefficient. This so-called “freeze-out” temperature \( T_f \) and time \( t_f \) denote the start of the effective neutron beta decay.

Assuming chemical and thermal equilibrium, the rate of neutron to proton concentration at freeze-out is commonly calculated, assuming chemical and thermal equilibrium,

\[
\frac{n_n}{n_p}(T_f) = e^{-Q/(kT_f)}, 
\]

where \( Q \) denotes the energy difference of neutron and proton rest masses. However, the deviation from equilibrium at freeze-out is already significant. Hence, we have to derive nonequilibrium concentrations, where we follow the example calculations of Mukhanov (2004).

The four-fermion interaction \( a + b \rightarrow c + d \) can be calculated using the Fermi theory, where the differential cross section is given by

\[
\frac{d\sigma_{ab}}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|M|^2}{(p_a + p_b)^2} \sqrt{\frac{(p_c \cdot p_d)^2 - m_f^2 m_e^2 c^8}{(p_a \cdot p_b)^2 - m_f^2 m_e^2 c^8}}. 
\]

The quantities \( (p_a \cdot p_b) \) and \( (p_c \cdot p_d) \) denote the scalar products of the four-momenta, and the matrix element is given by

\[
|M|^2 = 16 \left( 1 + 3g_A^2 \right) G_F^2 (p_a \cdot p_b)(p_c \cdot p_d),
\]

where \( G_F \simeq 1.166371 \times 10^{-11} \ [\text{MeV}^{-2}] \) (Yao et al. 2006) denotes the Fermi coupling constant and \( g_A = 1.2739 \) (Abele et al. 2002) the axial vector coupling constant.

First, we consider the reaction \( n + \nu_e \rightarrow p + e^- \) at the relevant temperatures around a few MeV and below, where the nucleons are nonrelativistic,

\[
(p_n + p_{\nu_e})^2 \simeq m_n^2 c^4, \quad (p_n \cdot p_{\nu_e}) = m_n c^2 \nu_e, \quad (p_p \cdot p_e) = m_p c^2 \nu_e,
\]

\[
\sqrt{(p_p \cdot p_e)^2 - m_p^2 c^2} \simeq m_p c^2 \nu_e \sqrt{1 - \left(\frac{m_p c^2}{\nu_e}\right)^2} = m_p c \nu_e \nu_e,
\]

where \( m_p, m_n, \) and \( m_e \) denote the mass of the proton, neutron, and electron, respectively, \( \nu_e \) is the velocity of the electron, \( \epsilon_e \) is the energy of the incoming neutrino, and \( \epsilon_e \simeq \nu_e + Q \) is the energy of the outgoing electron. Substituting all terms into equation (3) we obtain

\[
\frac{d\sigma_{n\nu_e}}{d\Omega} = \frac{1}{(8\pi)^2} 16 \left( 1 + 3g_A^2 \right) G_F^2 \frac{m_{\nu_e} c \nu_e}{m_n} \frac{m_p c^2 \epsilon_e}{m_n^2} \frac{(m_n \epsilon_e/c)}{(m_n \epsilon_e/c)^2 - (m_n m_e c^4)^2},
\]

where we neglect the neutrino mass \( m_{\nu_e} \). Integration leads to

\[
\sigma_{n\nu_e} = \frac{1 + 3g_A^2}{\pi} G_F^2 \frac{m_p c^2}{m_n^2} \epsilon_e \frac{2 \nu_e}{c}.
\]

Next, we have to consider that at temperatures \( kT > 2m_e c^2 \) the possible states for the electron are partially occupied by electron-positron pairs. According to the Pauli exclusion principle, this reduces the appropriate cross section to

\[
\sigma_{e^+e^-} = \sigma_{e^+e^-} \frac{1}{1 + e^{\nu_e/kT}}.
\]

This enables us to calculate \( \Delta N_n \), the reduction of neutrons within a time interval \( \Delta t \) in a given volume, containing \( N_n \) neutrons,

\[
\Delta N_n = - \left( \sum_{\epsilon_e} \sigma_{n\nu_e} c \nu_e \Delta \epsilon_e \right) N_n \Delta t.
\]
where

$$n_{\nu} = \frac{1}{1 + e^{\epsilon_{\nu}/(kT)}}$$  \hspace{1cm} (9)

denotes the neutrino occupation number ($\epsilon_{\nu}$ and $T_{\nu}$ are the velocity and the temperature of the neutrinos) and

$$\Delta g_{\nu} = \frac{1}{2\pi} \int_{\epsilon_{\nu}}^{\epsilon_{\nu} + \Delta \epsilon_{\nu}} |p|^2 \, dp \simeq \frac{1}{2\pi} \epsilon_{\nu}^2 \Delta \epsilon_{\nu}$$  \hspace{1cm} (10)

denotes the phase volume element. Introducing the relative concentration of the neutrons

$$X_n = \frac{n_n}{n_n + n_p}$$  \hspace{1cm} (11)

and assuming baryon conservation, we obtain the rate of change of the neutron concentration due to the $n\nu$-process,

$$\left( \frac{dX_n}{dt} \right)_{n\nu} = -\lambda_{n\nu} X_n,$$  \hspace{1cm} (12)

where $\lambda_{n\nu}$ denotes the decay rate. Substituting the cross section from equation (7) into equation (8), we obtain

$$\lambda_{n\nu} = \frac{1 + 3g_3^2}{2\pi^3} G_F^2 \frac{m_p^2}{m_n^2} I(T_{\nu}),$$  \hspace{1cm} (13)

where

$$I(T_{\nu}) = \int_0^\infty \epsilon_{\nu}^2 \sqrt{1 - \left( \frac{m_{\nu}c^2}{\epsilon_{\nu}} \right)^2} \frac{1}{1 + e^{\epsilon_{\nu}/(kT)}} \frac{\epsilon_{\nu}^2}{1 + e^{\epsilon_{\nu}/(kT)}} \, d\epsilon_{\nu}.$$  \hspace{1cm} (14)

Below the temperature $kT \approx 2m_{\nu}c^2$, the Pauli exclusion principle, represented by the term $(1 + e^{-\epsilon_{\nu}/(kT)})$, loses importance, and numerically, we notice a deviation of 1% only, when we set this term to 1. Expanding the square root $(m_{\nu}c^2/\epsilon_{\nu} \ll 1)$, keeping only the first two terms, and introducing the integration variable $x = \epsilon_{\nu}/(kT_{\nu})$, we derive

$$I(T_{\nu}) = (kT_{\nu})^5 \int_0^\infty x^2 \left[ \frac{Q}{(kT_{\nu})} \right]^{1/2} \left[ \frac{45\zeta(5)}{2} \frac{(kT_{\nu})}{Q} \right] \left[ \frac{7\pi^4}{60} \frac{(kT_{\nu})}{Q} + \frac{3\zeta(3)}{2} \left( 1 - \frac{m_{\nu}c^4}{2Q^2} \right) \right] \, dx,$$  \hspace{1cm} (15)

$$= Q^5 \left( \frac{kT_{\nu}}{Q} \right)^3 \left[ \frac{45\zeta(5)}{2} \frac{(kT_{\nu})}{Q} \right] \left[ \frac{7\pi^4}{60} \frac{(kT_{\nu})}{Q} + \frac{3\zeta(3)}{2} \left( 1 - \frac{m_{\nu}c^4}{2Q^2} \right) \right].$$  \hspace{1cm} (16)

where $\zeta$ is the Riemann zeta function. In the last step, we completed the square approximately. Finally, we convert $\lambda_{n\nu}$ from MeV to s$^{-1}$ and derive

$$\lambda_{n\nu} \approx \frac{1 + 3g_3^2}{1.75 \times 10^{-21}} G_F^2 \frac{m_p^2}{m_n^2} Q^5 \left( \frac{kT_{\nu}}{Q} \right)^3 \left( \frac{kT_{\nu}}{Q} + 0.25 \right)^2 \left[ \text{s}^{-1} \right].$$  \hspace{1cm} (18)

Similarly, we find the decay rate of the reaction $n + e^+ \rightarrow p + \bar{\nu}$ (we interchange $\epsilon_{\nu}$ with $\epsilon_{\nu}$ and $m_{\nu}$ with $m_{\nu} = 0$),

$$\lambda_{ne} = \frac{1 + 3g_3^2}{2\pi^3} G_F^2 \frac{m_p^2}{m_n^2} \int_0^\infty \epsilon_{\nu}^2 \sqrt{1 - \left( \frac{m_{\nu}c^2}{\epsilon_{\nu}} \right)^2} \frac{\epsilon_{\nu}^2}{1 + e^{\epsilon_{\nu}/(kT)}} \, d\epsilon_{\nu}.$$  \hspace{1cm} (19)

Assuming $T_{\nu} = T$, the rates of the inverse reactions are related to the rate of the direct reactions as

$$\lambda_{pe} = e^{-Q/(kT)} \lambda_{n\nu},$$  \hspace{1cm} (20)

$$\lambda_{p\nu} = e^{-Q/(kT)} \lambda_{ne}.$$  \hspace{1cm} (21)
Hence, we can write the following balance equation for \( X_n \),
\[
\frac{dX_n}{dt} = - (\dot{\lambda}_{nv} + \dot{\lambda}_{ne})X_n + (\dot{\lambda}_{pe} + \dot{\lambda}_{pu})(1 - X_n) = - (\dot{\lambda}_{nv} + \dot{\lambda}_{ne}) \left( 1 + e^{-Q/(kT)} \right) (X_n - X_n^\text{eq}),
\]
with the equilibrium neutron concentration
\[
X_n^\text{eq} = \frac{1}{1 + e^{Q/(kT)}}.
\]
To solve this linear differential equation (22), we take the initial condition \( X_n(t = 0) = X_n^\text{eq} \) and obtain
\[
X_n(t) = X_n^\text{eq}(t) - \int_0^t \exp \left\{ - \int_0^t (\dot{\lambda}_{nv}(y) + \dot{\lambda}_{ne}(y)) \left( 1 + e^{-Q/(kT)} \right) dy \right\} X_n^\text{eq}(\tilde{t}) d\tilde{t},
\]
where the overdot denotes the derivative with respect to time. Using the auxiliary function \( F(t) \),
\[
F(t) = \int_0^t (\dot{\lambda}_{nv}(t) + \dot{\lambda}_{ne}(t)) \left( 1 + e^{-Q/(kT)} \right) dt,
\]
we express the integral in equation (24) in the form
\[
\int_0^t e^{-F(t)} X_n^\text{eq}(\tilde{t}) d\tilde{t}
\]
and expand in the small parameter \((t - \tilde{t})\), since the integral is dominated by the contribution of \( \tilde{t} \approx t \) if \( F(t) \) is a quickly growing function of \( t \),
\[
X_n(t) = X_n^\text{eq}(t) - \int_0^t \left[ \frac{\dot{X}_n^\text{eq}(t)}{X_n^\text{eq}(t)} (\tilde{t} - t) + \ldots \right] e^{-F(t - \tilde{t})} \left[ 1 + \frac{1}{2} \tilde{F}(t)(t - \tilde{t})^2 + \ldots \right] dt.
\]
We integrate term by term using
\[
\int_0^t e^{-(t - \tilde{t})^n} d\tilde{t} \simeq A^{-n-1} n!,
\]
where we neglect exponentially small terms of order \( e^{-A} \), deriving
\[
X_n(t) = X_n^\text{eq}(t) \left[ 1 - \frac{1}{(\dot{\lambda}_{nv} + \dot{\lambda}_{ne})(1 + e^{-Q/(kT)}) X_n^\text{eq}(t)} + \ldots \right].
\]
Later, when the temperature has dropped significantly, \( X_n^\text{eq} \) goes to zero and the integral in equation (24) approaches the finite limit. As a result, the neutron concentration freezes out at \( X_n(t \to \infty) \). Effectively, this freeze-out occurs when the deviation from equilibrium becomes significant, hence when
\[
\left| \frac{\dot{X}_n^\text{eq}}{X_n^\text{eq}} \right| \approx (\dot{\lambda}_{nv} + \dot{\lambda}_{ne}) \left( 1 + e^{-Q/(kT)} \right).
\]
Assuming this happens before \( e^{\pm} \)-annihilation and after \( kT \) has dropped below \( Q \), we set \( \dot{\lambda}_{nv} + \dot{\lambda}_{ne} \approx 2 \dot{\lambda}_{nv} \) and neglect the term \( \exp(-Q/(kT)) \). Substituting all terms and taking the time-temperature relation from equation (1) into account, we finally obtain an equation for the freeze-out temperature \( T_f \),
\[
\sqrt{\frac{G g_\nu(T_f)}{h^2 c^5}} 3.7 \times 10^{-35} = (1 + 3 g_\nu) G_F^2 Q^3 \left( \frac{kT_f}{Q} \right)^2 \left( \frac{kT_f}{Q} + 0.25 \right)^2.
\]
The quadratic term \( kT_f/Q \) leads to
\[
T_f \simeq 1.16 \times 10^6 \frac{Q}{k} \left\{ -\frac{1}{8} + \sqrt{1 \frac{8}{64} + \sqrt{3.7 \times 10^{-35} (1 + 3 g_\nu) G_F^2 Q^3 h^2 c^5}} \right\} \text{[K]}.
\]
At \( T_f \) the effectively massless species in the cosmic plasma are neutrinos (left-handed only), antineutrinos (right-handed only), electrons, positrons, and photons. For the case of three neutrino families \( (N_\nu = 3) \), we obtain \( g_\nu(T_f) = 2 + \frac{7}{8}(4 + 2N_\nu) = 10.75 \).
To calculate the relevant neutron concentration at $T_f$, we go back to equation (24). Since $X_n^{eq} \rightarrow 0$ as $T \rightarrow 0$, we have to calculate the integral term in equation (24) in the limit $t \rightarrow \infty$. The main contribution to the integral comes at temperatures above the rest mass of the electron, where again $\lambda_{ne} + \lambda_{me} \simeq 2\lambda_{nu}$ ($\lambda_{nu}$ is given by eq. [18]). Furthermore, we use equation (1) to change the integration variable from $dt$ to $dT$,

$$X_n(T_f) = \int_0^\infty Q \exp \left\{ -2.68 \times 10^{34} \sqrt{h^3 c^5 / \left[ G g_e(T_f) \right]} (1 + 3g_2^2) G_1^2 \int_0^T (x + Q/4)^2 (1 + e^{-Q/4}) \, dx \right\} \frac{dT}{2T^2 [1 + \cosh(Q/T)]},$$

with $Q$ and $T$ in units of MeV. For later purposes, we finally state the neutron-to-proton ratio at freeze-out,

$$\frac{n_n}{n_p}(T_f) = \frac{1}{1/X_n(T_f) - 1}. \quad (34)$$

In comparison, the equilibrium rate from equation (2) is 14% higher; thus, as mentioned above the deviation is significant, which justifies the effort.

From now on the loss of free neutrons via $n \rightarrow p + e^- + \bar{\nu}_e$, with a mean lifetime ($\tau_n = 878.5 \text{ s}$), can no longer be compensated. Thus, whereas the neutron density decreases as $n_n(t) = n_n(t_f) e^{(-t-t_f)/\tau_n}$, the proton density increases as $n_p(t) = n_p(t_f) + [n_n(t_f) - n_n(t)]$, and we obtain

$$\frac{n_p}{n_n}(t) = \frac{e^{(t-t_f)/\tau_n}}{X_n(T_f)} - 1. \quad (35)$$

At the relevant densities in the early universe, fusion reactions can only proceed efficiently through sequences of two-body collisions, and the starting product of these collisions is the weakly bound deuteron ($B_d \simeq 2.225 \text{ MeV}$), which is highly affected by photodisintegration. The start of nucleosynthesis, $t_N$, is therefore usually referred to as the “deuterium bottleneck.”

Once the deuteron production dominates the photodisintegration and the expansion of the universe, our calculation in some sense “produces only deuteron,” disregarding that deuteron is also destroyed by the fusion of light elements. In fact, we do not consider the detailed fusion reactions with their intermediate products that finally lead to $^4\text{He}$. We are interested in the point $t_N$, from that onward we can assume neutron conservation, because enough neutrons have reached stable states inside light nuclei (no matter whether inside deuterons or further fusion products of deuteron). Hence, we obtain $t_N$ assuming two constraints. First, the deuteron production must dominate the photodisintegration and the expansion of the universe. Second, to justify neutron conservation, the deuteron density must exceed the density of free neutrons. In fact, it turns out, that we can reproduce the numerical as well as the observational results for $Y_p$, assuming neutron conservation when 52%–66% of the neutrons have reached stable states inside light nuclei.

The interval between $t_f$ and $t_N$ is a substantial fraction of the neutron lifetime and, therefore, plays an essential role for the outcome of the primordial helium production. Thus, we have to calculate the rates of deuteron production $\Gamma_{(np \rightarrow d\gamma)}$, deuteron photodisintegration $\Gamma_{(d \rightarrow np)}$, and the rate of deuteron density reduction by the expansion of the universe $\Gamma_{exp}$, to determine $t_N$ (hence $T_N$), when

$$\Gamma_{(np \rightarrow d\gamma)} > \Gamma_{(d \rightarrow np)} + \Gamma_{exp}. \quad (36)$$

The rates for production and photodisintegration of deuteron are given by the product of the relevant number densities, velocities, and cross sections, whereas the expansion rate of a radiation-dominated, flat universe is given by $1/(2t)$ with $t$ from equation (1), leading to

$$n_n \frac{\eta n_n}{1 + n_n/n_p} \sqrt{\frac{8kT}{\pi m_N}} \sigma_{(np \rightarrow d\gamma)} > n_d n_n^2 \sigma_{(d \rightarrow np)} + \frac{n_d}{2t}, \quad (37)$$

where $\sigma_{(d \rightarrow np)}$ denotes the cross section of deuteron photodisintegration, $\sigma_{(np \rightarrow d\gamma)}$ denotes the cross section of deuteron production, $m_N$ is the nucleon mass, $\eta \simeq 6.226 \times 10^{-10}$ is the baryon-to-photon ratio based on WMAP (Komatsu et al. 2008) and Steigman (2006), $n_d$ and $n_n$ denote the number densities of deuterons and photons, respectively, and $n_d^*$ is the number density of photons which supply enough energy to disintegrate the deuteron and do not lose this energy in much more likely Compton scattering on electrons.

The number density of photons at a certain temperature $T$ is given by

$$n_\gamma = \frac{8\pi}{(hc)^3} \int_0^\infty \frac{E_\gamma^2}{e^{E_\gamma/(kT)} - 1} dE_\gamma = 16\pi \zeta(3) \left( \frac{kT}{hc} \right)^3,$$

where $\zeta$ is the Riemann zeta function. The number density of these photons supplying a minimum energy $E_\gamma > B_d \gg kT$ is

$$n_{(\gamma > B_d)} = \frac{8\pi}{(hc)^3} \int_{B_d}^\infty E_\gamma^2 e^{-E_\gamma/(kT)} dE_\gamma = 8\pi \left( \frac{kT}{hc} \right)^3 \left( \frac{B_d}{kT} + 1 \right)^2 + 1 \ e^{-B_d/(kT)}, \quad (39)$$
but most of them will lose energy in Compton scattering on electrons, leading to

\[ n_\gamma^* = n_{(\gamma B_d)} \frac{n_p \sigma_{(\gamma d - np)}}{n_p \sigma_{(\gamma e - e^+_\gamma)}} , \]  

(40)

where \( \sigma_{(\gamma e - e^+_\gamma)} \) denotes the Klein-Nishina cross section (Rybicki & Lightman 1979) for Compton scattering on electrons,

\[ \sigma_{(\gamma e - e^+_\gamma)} = \frac{1}{8\pi} \left( \frac{e^2}{\epsilon_0 m_e c^2} \right)^2 \left( 1 + \beta \frac{2(1 + \beta)}{\beta^2} \ln \left( 1 + \frac{1 + \beta}{\beta} \right) + \frac{\ln \left( 1 + \frac{1 + \beta}{\beta} \right) - 1 + 3\beta}{(1 + \beta)^2} \right) , \]

(41)

where

\[ \beta = \frac{\langle E_\gamma \rangle}{m_e c^2} , \]

(42)

and the mean incident photon energy \( \langle E_\gamma \rangle \) is given by

\[ \langle E_\gamma \rangle = \frac{1}{n_{(\gamma B_d)}} \frac{8\pi}{(hc)^3} \int_{B_d} E_\gamma^3 e^{-E_\gamma/(kc)} dE_\gamma = kT \left\{ \frac{[B_d/(kT)]^2}{[B_d/(kT) + 1]^2 + 1} \right\} \simeq B_d + kT . \]

(43)

The interaction cross section of deuteron photodisintegration can be well approximated by (Bethe & Longmire 1950)

\[ \sigma_{BL} = E1 + M1 , \]

(44)

where we express the electric dipole contribution as

\[ E1 = \frac{2}{3} \frac{e^2 \hbar}{\epsilon_0 m_N E_\gamma^3} \left( 1 - \frac{r_s}{\hbar} \sqrt{m_N B_d} \right) \]

(45)

and the magnetic dipole contribution as

\[ M1 = \frac{e^2 (\mu_p - \mu_n)^2}{6\epsilon_0 m_N^2 c^5} \left( \frac{B_d}{E_\gamma} - \frac{B_d}{E_\gamma} \right) ^2 \left\{ 1 - \frac{m_N B_d (a_s + a_t + a_s a_t)}{[m_N B_d/(4\hbar^2)]} - a_s (r_s + r_t) \right\} ^2 \]

\[ \left\{ 1 + a_t^2 \left[ m_N (E_\gamma - B_d)/\hbar^2 \right] \right\} \left( 1 - \frac{r_t}{\hbar} \sqrt{m_N B_d} \right) \]  

(46)

in terms of \( B_d \) and \( E_\gamma \), where \( \epsilon_0 \) denotes the electric constant, \( a_s \) and \( a_t \) are the singlet and triplet scattering lengths, respectively, \( r_s \) and \( r_t \) denote the singlet and triplet effective ranges, respectively, and \( \mu_p \) and \( \mu_n \) are the magnetic moments of protons and neutrons, respectively. With this dependence of \( \sigma_{BL} \) on the incident photon energy \( E_\gamma \), we derive \( \sigma_{(\gamma d - np)} \) as the mean cross section of photodisintegration per photon with \( E_\gamma > B_d \),

\[ \sigma_{(\gamma d - np)} = \frac{1}{n_{(\gamma B_d)}} \frac{8\pi}{(hc)^3} \int_{B_d} E_\gamma^3 e^{-E_\gamma/(kc)} \sigma_{BL}(E_\gamma) dE_\gamma . \]

(47)

We go back to equation (36) and divide by \( \Gamma_{(\gamma d - np)} \) in order to receive two terms, which we analyze separately,

\[ \frac{\Gamma_{(\gamma p - d)} / \Gamma_{(\gamma d - np)}}{\Gamma_{(\gamma d - np)}} = 1 + \frac{\Gamma_{exp}}{\Gamma_{(\gamma d - np)}} . \]

(48)

We start with

\[ \frac{\Gamma_{(\gamma p - d)}}{\Gamma_{(\gamma d - np)}} = \frac{n_p \eta \sqrt{kT/(\pi m_N)} \sigma_{(\gamma p - d)}}{n_p (1 + n_p/n_p) \left( n_p/n_p \right) \sigma_{(\gamma d - np)}} = \frac{3.84 \eta \sqrt{kT/(m_N c^2)} \sqrt{B_p/(kT)} (n_p/n_p) \left( \sigma_{(\gamma e - e^+_\gamma)} / \sigma_{(\gamma d - np)} \right) \sigma_{(\gamma p - d)}}{\left( 1 + n_p/n_p \right) \left[ B_p/(kT) + 1 \right]^2 + 1} \]  

(49)
where \( \sigma_{(np-dp)} \) is related to \( \sigma_{(d-np)} \) by the detailed balance

\[
\frac{\sigma_{(np-dp)}}{\sigma_{(d-np)}} \approx \frac{3\langle E_\gamma \rangle^2}{2m_Nc^2(\langle E_\gamma \rangle - B_d)},
\]

and \( \langle E_\gamma \rangle \) is given by equation (43), leading to

\[
\frac{\Gamma_{(np-dp)}}{\Gamma_{(d-np)}} = \frac{5.755\eta}{1 + (n_n/n_p)(T_N)} \left( \frac{kT}{m_Nc^2} \right)^{3/2} e^{B_d/(kT_N)} \frac{\sigma_{(n-n-d)}}{\sigma_{(d-n-n)}} \left( \frac{n_n}{n_p} \right) \left( \frac{n_p}{n_n} \right) = 1.
\]

Next, we analyze the term on the right-hand side of equation (48),

\[
\frac{\Gamma_{n}}{\Gamma_{d-n-n}} = \frac{n_d/2t}{n_d/m_n^2 c \sigma_{(d-n-n)}} = 1.040 \sqrt{\frac{G_{g_4}(T_N)h^3}{c} \frac{kT_t B_d/(kT)}{(B_d + kT)^2} \frac{\sigma_{(n-n-d)}}{\sigma_{(d-n-n)}} \frac{n_n/n_p}{n_p/n_n} (T_N) = 1,
\]

and finally, the corresponding neutron-to-proton ratio,

\[
\frac{n_n}{n_p}(T_N) = \frac{1}{X_n(T_f)} \exp \left[ \frac{1}{T_n} \sqrt{90h^4c^5}{32\pi^2k^4G} \left( \frac{1}{\sqrt{g_4(T_N)T_t}} - \frac{1}{\sqrt{g_4(T_f)T_t}} \right) \right] - 1,
\]

where \( X_n \) is given by equation (33).

The neutrinos have decoupled from equilibrium before the annihilation of the electron-positron pairs. Therefore, the entropy due to this annihilation is transferred exclusively to the photons, i.e., \( g_4(T_N) = 2 + (7/8)2N_A(41/11)^{43} \).

Assuming neutron conservation after \( T_N \), we finally calculate \( Y_p \), the primordial \( ^4\text{He} \) abundance by weight. Since \( ^4\text{He} \) is not further transformed into heavier nuclei, because elements with nucleon mass numbers \( A = 5 \) and 8 are insufficiently stable to function successfully as intermediate products for nucleosynthesis at the available densities, we derive

\[
Y_p = \frac{(1/2)n_n m_{1\text{He}}} {n_n m_{1\text{He}} + (n_p - n_n) m_P} = \frac{1}{1 + 2(m_p/m_{1\text{He}}) [(n_p/n_n)(T_N) - 1]},
\]

where \( m_{1\text{He}} \) denotes the mass of the helium nucleus and \( (n_p/n_n)(T_N) \) is given by equation (56).

This analytic expression for \( Y_p \) reproduces the observation-based results and the numerical results (see § 3 for details), assuming neutron conservation when 52%–66% of the neutrons have reached stable states inside light nuclei. Therefore, equation (57) provides our basis for finding the dependence of \( Y_p \) and the possible deviation of \( v \) from its present value \( v_0 \), in order to finally constrain \( v/v_0 \) by recent results on the primordial \( ^4\text{He} \) abundance.

Expressing all the key parameters of \( Y_p \) in terms of \( v \), we start with the most important one, the deuteron binding energy. Within our narrow range of interest \( |(v - v_0)/v_0| < 0.5\% \), we determine its dependence on \( v \) by varying the pion mass in different nucleon-nucleon potential model calculations, based on Arenhövel & Sanzzone (1991), and derive

\[
B_d(v) = B_d(v_0) \left[ A - (A - 1)\sqrt{v/v_0} \right],
\]

where \( A \) is a model-dependent constant, which is given as follows:

- Bonn-A potential (Machleidt et al. 1987): \( A = 2.3 \).
- Paris potential (Lacombe et al. 1980): \( A = 28.2 \), and
- Argonne V14 potential (Wiringa et al. 1984): \( A = 61.4 \).

For our purpose, we take the mean average \( A = 30.6 \).
As \( B_d \) changes, \( E_s \), and the cross sections \( \sigma_{(d \rightarrow np)} \) and \( \sigma_{(np \rightarrow d\gamma)} \) change accordingly. Furthermore, we have to consider that the mass of the electron varies proportionally,

\[
m_e (v) = m_e (v_0) \frac{v}{v_0},
\]
which enters the Klein-Nishina cross section.

Concerning \( \tau_n \), the mean lifetime of the free neutron, we use the expression (Gaßner \& Lesch 2008, based on Mueller et al. 2004),

\[
\tau_n (v) \approx \left( 1 - 4.88 \frac{v - v_0}{v_0} \right) \tau_n (v_0).
\]

Next, we have to consider the change on \( Q \), the neutron-to-proton mass difference, which influences the freeze-out concentration. We separate the electromagnetic contribution (Gasser \& Leutwyler 1982) and obtain

\[
Q \approx \left( -0.76 + 2.0533317 \frac{v}{v_0} \right) \text{[MeV]}.
\]

The Fermi coupling constant \( G_F \) is related to \( v \) by (Dixit \& Sher 1988)

\[
G_F (v) = \frac{1}{v \sqrt{2}} \text{[GeV}^{-2}].
\]

Finally, we derive a relation between \( Y_P \) and \( v \), to constrain the permitted variation of the Higgs vacuum expectation value by the primordial \(^4\text{He} \) abundance,

\[
\Delta Y_P \approx -38 \left( \frac{\Delta v}{v} \right)^2 - 2.08 \left( \frac{\Delta v}{v} \right) + 0.0355 \left( \frac{N_v - 3}{3} \right) + 0.0874 \left( \frac{\Delta G}{G} \right) + 0.0042 \ln \left( \frac{\Delta \eta}{\eta} + 1 \right).
\]

Varying each parameter separately (assuming the others fixed), we derive

\[
\Delta Y_P \approx 0.106 \left( \frac{\Delta B_d}{B_d} \right) + 0.056 \left( \frac{\Delta \tau_n}{\tau_n} \right) - 0.235 \left( \frac{\Delta G_F}{G_F} \right) - 0.352 \left( \frac{\Delta Q}{Q} \right) - 0.006 \left( \frac{\Delta m_e}{m_e} \right) - 0.195 \left( \frac{\Delta g_A}{g_A} \right) + 0.0355 \left( \frac{\Delta N_v}{3} \right) + 0.0874 \left( \frac{\Delta G}{G} \right) - 0.2602 \left( \frac{\Delta h}{h} \right) + 0.0042 \ln \left( \frac{\Delta \eta}{\eta} + 1 \right).
\]

We briefly note, that using equation (63), one can also obtain constraints on \( N_v \), \( G \), \( h \), and \( \eta \).

3. RESULTS

Constraining the possible time variation of \( v \), we use observation-based results as well as the SBBN code, developed by Wagoner (1973) and Kawano (1992). This standard code still seems adequate for our purpose, although newer nuclear reaction rates have been evaluated (Descouvemont et al. 2004).

The baryon-to-photon ratio is given by \( \eta \approx (273.9 \pm 0.3)10^{-10} \Omega_b h^2 \) (Steigman 2006), where \( \Omega_b \) is the present ratio of the baryon mass density to the critical density and \( h \) is the present value of the Hubble parameter in units of 100 km \( s^{-1} \text{Mpc}^{-1} \). We take \( 100\Omega_b h^2 = 2.273 \pm 0.062 \), the 5 year mean value of \textit{WMAP} (Komatsu et al. 2008) and obtain

\[
6.049 \times 10^{-10} \leq \eta \leq 6.403 \times 10^{-10}.
\]

Implementing \( \eta \) and \( \tau_n = 878.5 \text{ s} \) (Serebrov et al. 2005), the numerical code delivers

\[
Y_1 = 0.2467 \pm 0.0003,
\]
and thus, equation (63) constrains the possible time variation of \( v \),

\[
\left| \frac{\Delta v}{v} \right| \leq 1.5 \times 10^{-4}.
\]

Using the observation-based results of Izotov et al. (2007), we derive

\[
Y_2 = 0.2516 \pm 0.0011 \Rightarrow \left| \frac{\Delta v}{v} \right| \leq 5.6 \times 10^{-4},
\]
and the results of Peimbert et al. (2007) lead to

\[ Y_3 = 0.2477 \pm 0.0029 \Rightarrow \frac{\Delta Y}{Y} \leq 1.4 \times 10^{-3}. \]

Combining the observations of \( H \) ii regions (\( Y_2 \) and \( Y_3 \)) and the numerical simulation based on \( WMAP \) results (\( Y_1 \)), according to

\[ \Delta Y = Y_p(\text{\( H \) ii} \text{regions}) - Y_p(\text{\( WMAP + SBBN \)}), \]

we alternatively derive (using eq. [63]) two more conservative estimates,

\[ Y_2 - Y_1 = 0.0049 \pm 0.0011 \Rightarrow \frac{\Delta Y}{Y} = (2.4 \pm 0.6) \times 10^{-3}, \]

\[ Y_3 - Y_1 = 0.0010 \pm 0.0029 \Rightarrow \frac{\Delta Y}{Y} = (0.5 \pm 1.4) \times 10^{-3}. \]

We avoid the term “observational results” because the cited publications more or less consist of interpretations of the observational \( ^4\text{He} \) abundance plus theoretical input and constraints by the cosmic microwave background. The different interpretations as a result of the badly understood systematics lead to incompatible data. For consistency, we only cite data based on recent He i recombination coefficients by Porter et al. (2005, 2007).

4. CONCLUSIONS

BBN offers the deepest reliable probe of the early universe. Its predictions of the light-element abundances play a major role in constraining cosmological models. The increasing precision of observational results on primordial abundances opens new scientific fields that can be tested by BBN. We present a calculation, how observations on primordial \( ^4\text{He} \) may provide insight into the fundamental property of elementary particles, the Higgs vacuum expectation value. We find constraints on its allowed time variation \( |\Delta v/v| \leq 1.5 \times 10^{-4} \).

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