On the mass correction of heavy hadrons of arbitrary spin in heavy hadron effective theory *

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The mass correction forms of the arbitrary spin heavy hadrons are derived by using the projection operator method. The Bjorken sum rule for finite mass is derived by using the results of here .

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I. INTRODUCTION

The Heavy Quark Effective Theory(HQET) has been used to describe the heavy hadron phenomena successfully in the past few years [1]. The mass correction for HQET is needed to compensate the infinite heavy quark mass limit simplicity [2]. In the quark language, the heavy hadrons may be taken as composed of one heavy quark and light brown muck. The confinement interactions between heavy quark and light brown muck are almost the QCD energy scale $\Lambda_{QCD}$. As the mass $m_Q$ of the heavy quark being much larger than $\Lambda_{QCD}$, one can specify the ratio of the total momentum $P_Q^\mu$ over $m_Q$ to be the velocity $v^\mu$ which characterize the heavy quark states. Since the most momentum of heavy hadron is carried by the heavy quark , thus one can identify heavy hadron total momentum $P_Q^\mu$ to be $Mv^\mu$ if the hadron mass $M$ is equal to $m_Q + \Lambda_H$, where the $\Lambda_H$ is the relevant parameter to denote the contribution of light brown muck to heavy hadron mass. After extracting the velocity part of total momentum , the rest is defined as the residual momentum $k^\mu$ of the heavy quark to denode the momentum fluctation of heavy quark due to interactions with light brown muck. The systematic expansion in HQET is the expansion in terms of $k^\mu$ and $m_Q^{-1}$. Many mass expansion schemes have been derived according to different physical and mathematical reasons [3]. Although full physical green functions are independent of mass expansion scheme, but the results from HQET are really dependent on mass expansion scheme. This dependence comes from that one can not calculate to all orders of mass expansion. Such limitation confronts us to need a good mass expansion scheme which can at least satisfy the properties of a free field theory [4]. Mass expansion scheme can be separated into two types, one is the lagrangian type mass expansion and the other is the field type mass expansion. The former is derived by functional integral method and the latter is to solve the mass expansion equation. The idea is to find the transformation between mass correction field and effective field. Since most physical interests are about the green functions which are composed of relevant fields. Thus the field type mass expansion scheme is appropriate. We developed the projection operator method to meet the requirements of the field type mass expansion [4].

If we set the relevant energy scale as the chiral symmetry energy scale $\Lambda_{\chi}$, then we can combine the heavy quark symmetry and chiral symmetry to write down the heavy hadron effective theory (HHET) [5]. Such an approach opens an new era for applying the heavy quark symmetry to the heavy hadron transitions involving light pions. Although mass correction for HHET is analogous to that of HQET, but the mass expansion scheme of HHET is still not developed plentily as compared to that of HQET [6]. The main obstacle is beacuse now the basic objects are heavy hadrons which are more complicated than heavy quark. To directly apply the mass expansion scheme in HQET for HHET is not so easy and in some cases will lead to wrong [6]. In [6] we used the idea of the projection operator method in HQET to develope the heavy meson effective theory HMET type projection operator method and get the mass correction. Here we will extend such a approach to all kinds of heavy hadron. In the following we will firstly review the projection operator method in HQET and HMET respectively.

A. Review of projection operator method in HQET

For a heavy quark field $Q(x)$ we define the correction filed as

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\[ H(x) = e^{imQv \cdot x} Q(x), \]  
and the effective field as
\[ h_v(x) = \lim_{m_Q \to \infty} H(x), \]  
and their Fourier transformation forms as \( H, h_v \). In momentum space \( H \) and \( h_v \) have their projection operators
\[ \Lambda^+ = \sum_r H_r \mathcal{P}_r \]
\[ = \frac{1 + v^2}{2} + \frac{k}{2m_Q} \]  
(1.3)
\[ \Lambda^+_v = \sum_r h_v,r \mathcal{P}_r \]
\[ = \frac{1 + v^2}{2} \]  
(1.4)

By applying \( \Lambda^+ \) on \( H \) and noting that the \((1 + v^2)/2 \) part of \( H \) is not equal to \( h_v \), one can have the following mass expansion equation
\[ \frac{1 - v^2}{2} H = \frac{k}{2m_Q} H \]
\[ = \frac{k}{2m_Q} \frac{1 + v^2}{2} \]  
(1.5)

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(1.5)

Making an ansarz that \((1 + v^2)/2 \) is proportional to \( h_v \), we can solve Eq. 1.5 and get the relation between \( H \) and \( h_v \) as
\[ H = \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q} h_v}. \]  
(1.6)

\section*{B. Review of projection operator method in HMET}

For heavy mesons with quantum number \( J^P = (0^-, 1^-) \), we can combine them into a compact heavy meson field \( M(x) \)
\[ M(x) = \frac{m_M + i d}{2m_Q} (-P(x)\gamma_5 + P^{\star \mu}(x)\gamma_\mu). \]  
(1.7)

Define the correction compact heavy meson field as
\[ \tilde{M}(x) = e^{imMv \cdot x} M(x) \]
\[ = \left( \frac{1 + v^2}{2} + \frac{i d}{2m_Q} + \frac{\Lambda_M}{m_Q} \right) (-\hat{P}(x)\gamma_5 + \hat{P}^{\star \mu}(x)\gamma_\mu), \]  
(1.8)

and the effective compact heavy meson field as
\[ H_M(x) = \lim_{m_Q \to \infty} \tilde{M}(x). \]  
(1.9)

From Eq. 1.8, we can take pseudoscalar \( \hat{P}(x) \) as example to identify
\[ \left( \frac{1 + v^2}{2} + \frac{i d}{2m_Q} \right) \hat{P}(x) \]
as its heavy quark part \( P_Q(x) \). Thus going to the momentum space and finding the relevant projection operators, we can do the similar procedure in subsection A to get the result.
\[ \hat{P} = \sqrt{\frac{1 + \frac{k}{2m_Q}}{1 - \frac{k}{2m_Q}}} P_h \]  

(1.10)

where \( \hat{P} \) is the correction pseudoscalar and \( P_h \) is the \((1 + \frac{k}{2})/2\) projected effective pseudoscalar \( P_v \). Finally, we combine everything to arrive at the correction compact heavy meson field related to effective heavy meson field with the following form

\[ \hat{M} = \left( 1 + \frac{\Lambda_M}{m_Q} \frac{1 + \frac{k}{2m_Q}}{2} \right) \sqrt{\frac{1 + \frac{k}{2m_Q}}{1 - \frac{k}{2m_Q}}} H_M. \]  

(1.11)

where \( H_M \) is defined as

\[ H_M = \frac{1 + \frac{k}{2}}{2} (-P_v \gamma_5 + P_v^{\pm \mu} \gamma_\mu). \]  

(1.12)

The above review tells us that the heavy quark mass correction for heavy quark and heavy mesons is irrelevant to the properties of the light brown muck excepting for the total wave function mass corrections(see below). Such a superising result can be tracked back to the factorizability of matrix elements of a heavy quark current between hadronic states into heavy and light matrix elements in the heavy quark mass infinite limit as

\[ \langle \Psi(v') | J(q) | \Psi(v) \rangle = \langle Q'(v') | J(q) | Q(v) \rangle \pm \frac{1}{2} \langle \text{light, } v', \ j', \ m' \ | \text{light, } v, \ j, \ m \rangle \]  

(1.13)

Our organization is as following. In section [II] we derive the mass correction for heavy baryons. In section [III] we derive the mass correction for heavy mesons. In section [IV] we want to generealize the Bjorken sum rule to include the mass corrections. In section [V] is the summary.

II. MASS CORRECTION OF HEAVY BARYONS

To discuss the mass correction of heavy baryon, one need to firstly specify which quantum number is good or almost good. Since in the infinite heavy quark mass limit the quantum number of the angular momentum of the light brown muck , \( j \) appears to be good, this comes from the spin interaction between heavy degree of freedom (d.o.f) and light brown muck(l.b.m) is proportional to 1/m_\(Q\). Applying the same argument for finit mass cases , one can still use \( j \) (almost good) to classify the finit mass heavy baryons to undo the mass correction derivation by requiring that the heavy d.o.f is on-shell. The reason can be thought as following. Although now \( j \) is nomore a good one , but since the heavy d.o.f is on-shell the spin interactions between heavy d.o.f and l.b.m will not affect the heavy d.o.f’s equation of motion. This means that the complicate interactions should be realized via the mass expansion series and while one derives the mass correction formalism unnecessary to consider these complicate dynamical effects and should leave them to the lagrangian one gets. So the whole program of the derivation of the mass correction is now easy and can be picture as that the mass correction is a transformation between the finite mass field and infinite mass field ,while both are free. And ,since the on-sell is always true for infinite field , then the finit mass field should be also on-shell. The on-shell requirment of heavy d.o.f is the source of the “velocity reparametrization invariant”. Thus the whole argument is self-consistent. The general heavy baryons can be still classified according to the light brown muck total angular momentum \( j \geq 1 \) with two specified heavy quark spin states. Thus we classify the heavy baryons with total angular momentum \( J = j \pm \frac{1}{2} \) and write them as

\[ J = j + \frac{1}{2} : \ B^{\mu_1 \mu_2 \ldots \mu_j}(x) \]  

(2.1)

\[ J = j - \frac{1}{2} : \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j + 1)}} (\gamma^{\mu_1} + \frac{id}{m_{B_j}}) \gamma_5 B^{\mu_1 \ldots \mu_i \ldots \mu_j}(x) \]  

(2.2)

and the superfield

\[ S^{\mu_1 \mu_2 \ldots \mu_j}(x) = B^{\mu_1 \mu_2 \ldots \mu_j}(x) + \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j + 1)}} (\gamma^{\mu_1} + \frac{id}{m_{B_j}}) \gamma_5 B^{\mu_1 \ldots \mu_i \ldots \mu_j}(x). \]  

(2.3)
where $\tilde{\mu}_j$ means that this index is subtracted. These baryons and superfield satisfy their respective equations of motion and constrains as following

\begin{align}
    i \frac{d}{dx} B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= m_{B_j} B^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.4) \\
    i \frac{d}{dx} B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.5) \\
    B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.6) \\
    \gamma_{\mu_1} B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.7)
\end{align}

\begin{align}
    i \frac{d}{dx} B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= m_{B_j} B^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.8) \\
    i \frac{d}{dx} B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.9) \\
    B^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.10)
\end{align}

and

\begin{align}
    i \frac{d}{dx} S^{\mu_1 \mu_2 \cdots \mu_j}(x) &= m_{B_j} S^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.11) \\
    i \frac{d}{dx} S^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.12) \\
    S^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.13) \\
    \gamma_{\mu_1} S^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0. \quad (2.14)
\end{align}

Now we can define the correction heavy baryons according to the above basis

\begin{align}
    \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= e^{im_{B_j} vx} B^{\mu_1 \mu_2 \cdots \mu_j}(x) \quad (2.15) \\
    \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j+1)}} (\gamma^{\mu_1} + \nu^{\mu_1} + \frac{i d^{\mu_1}}{m_{B_j}}) \gamma_5 \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j+1)}} (\gamma^{\mu_1} + \nu^{\mu_1} + \frac{i d^{\mu_1}}{m_{B_j}}) \gamma_5 e^{im_{B_j} vx} B^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.16)
\end{align}

\begin{align}
    \tilde{S}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= e^{im_{B_j} vx} B^{\mu_1 \mu_2 \cdots \mu_j}(x) \quad (2.17)
\end{align}

The equations of motion and constrains then become

\begin{align}
    (\gamma + i \frac{d}{dx}) \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.18) \\
    (\nu + i \frac{d}{dx}) \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.19) \\
    \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.20) \\
    \gamma_{\mu_1} \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0. \quad (2.21)
\end{align}

\begin{align}
    (\gamma + i \frac{d}{dx}) \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.22) \\
    (\nu + i \frac{d}{dx}) \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0, \quad (2.23) \\
    \tilde{B}^{\mu_1 \mu_2 \cdots \mu_j}(x) &= 0. \quad (2.24)
\end{align}

and
\[(g' + \frac{id}{m_{B_j}}) \hat{S}^{\mu_1 \mu_2 \cdots \mu_j}(x) = \hat{S}^{*\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.25)\]
\[(v_\mu + \frac{id_\mu}{m_{B_j}}) \hat{S}^{\mu_1 \mu_2 \cdots \mu_j}(x) = 0, \quad (2.26)\]
\[\hat{S}^{\mu_1 \mu_1 \cdots \mu_j}(x) = 0, \quad (2.27)\]
\[\gamma_\mu \hat{S}^{\mu_1 \mu_2 \cdots \mu_j}(x) = 0. \quad (2.28)\]

To define the effective heavy baryons, we firstly separate the mass \(m_{B_j}\) into \(m_Q + \Lambda_{B_j}\) and apply the limit \(m_Q \to \infty\) to obtain
\[B_v^{*\mu_1 \mu_2 \cdots \mu_j}(x) = \lim_{m_Q \to \infty} \hat{B}^{*\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.29)\]
\[\sum_{i=1}^{j} \sqrt{\frac{1}{j(2j+1)}} (\gamma^{\mu_i} + v^{\mu_i}) \gamma_5 B_v^{\mu_1 \cdots \mu_i \cdots \mu_j}(x) = \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j+1)}} (\gamma^{\mu_i} + v^{\mu_i}) \gamma_5 \lim_{m_Q \to \infty} \hat{B}^{\mu_1 \cdots \mu_i \cdots \mu_j}(x). \quad (2.30)\]
\[S_v^{\mu_1 \mu_2 \cdots \mu_j}(x) = \lim_{m_Q \to \infty} \hat{S}^{\mu_1 \mu_2 \cdots \mu_j}(x), \quad (2.31)\]
\[= B_v^{*\mu_1 \mu_2 \cdots \mu_j}(x) + \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j+1)}} (\gamma^{\mu_i} + v^{\mu_i}) \gamma_5 B_v^{*\mu_1 \cdots \mu_i \cdots \mu_j}(x). \quad (2.32)\]

One sees that the effective heavy baryons just those defined in \([9]\) and their related equations of motion and constraints also just defined in there.

Let’s concentrate on the derivation of explicit forms of \(\hat{B}^{*\mu_1 \mu_2 \cdots \mu_j}(x)\) and \(\hat{B}^{\mu_1 \cdots \mu_i \cdots \mu_j}(x)\). It is better to the momentum space. Apply the heavy quark projection operator \(\Lambda^+ = (1 + \gamma^5)/2 + \gamma^5/2m_Q\) on \(\hat{B}^{*\mu_1 \mu_2 \cdots \mu_j}\) and \(\hat{B}^{\mu_1 \cdots \mu_i \cdots \mu_j}\) to project out their heavy quark contents
\[\hat{B}_Q^{*\mu_1 \mu_2 \cdots \mu_j} := \frac{1 + \gamma^5}{2} \hat{B}^{*\mu_1 \mu_2 \cdots \mu_j}, \quad (2.33)\]
\[\hat{B}_Q^{\mu_1 \cdots \mu_i \cdots \mu_j} := \frac{1 + \gamma^5}{2} \hat{B}^{*\mu_1 \cdots \mu_i \cdots \mu_j}. \quad (2.34)\]

Following the same procedure in subsection \([11]\) one can finally get
\[\hat{B}_Q^{*\mu_1 \mu_2 \cdots \mu_j} = \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}} B_v^{*\mu_1 \mu_2 \cdots \mu_j}, \quad (2.35)\]
\[\hat{B}_Q^{\mu_1 \cdots \mu_i \cdots \mu_j} = \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}} B_v^{\mu_1 \cdots \mu_i \cdots \mu_j}, \quad (2.36)\]
and use the following relations
\[\left[\frac{1 + \gamma^5}{2}, \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}}\right] = -\frac{\gamma^5}{2m_Q} \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}}, \quad (2.37)\]
\[\sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}} \left\{\frac{1 + \gamma^5/2m_Q}{2} \right\} B_v^{*\mu_1 \mu_2 \cdots \mu_j} B_v^{\mu_1 \cdots \mu_i \cdots \mu_j} = \left\{\frac{1 + \gamma^5}{2}, \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}}\right\} \hat{B}_v^{*\mu_1 \mu_2 \cdots \mu_j} \hat{B}_v^{\mu_1 \cdots \mu_i \cdots \mu_j}. \quad (2.38)\]

So the results we get
\[\hat{B}^{*\mu_1 \mu_2 \cdots \mu_j} = \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}} B_v^{*\mu_1 \mu_2 \cdots \mu_j}, \quad (2.39)\]
\[\hat{B}^{\mu_1 \cdots \mu_i \cdots \mu_j} = \sqrt{\frac{1 + \gamma^5/2m_Q}{1 - \gamma^5/2m_Q}} B_v^{\mu_1 \cdots \mu_i \cdots \mu_j}, \quad (2.40)\]
and the corresponding $J = j \pm \frac{1}{2}$ baryons are

$$
\hat{B}^{*\mu_1\mu_2\cdots\mu_j} = \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} B_w^{\ast\mu_1\mu_2\cdots\mu_j},
$$

(2.41)

$$
\sum_{i=1}^{j} \sqrt{\frac{1}{j(2j + 1)}} (\gamma^{\mu_i} + \nu^{\mu_i} + \frac{k^{\mu_i}}{m_{B_j}}) \gamma_5 \hat{B}_{\mu_1\cdots\mu_i\cdots\mu_j} = \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j + 1)}} (\gamma^{\mu_i} + \nu^{\mu_i} + \frac{k^{\mu_i}}{m_{B_j}}) \gamma_5 \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} B_w^{\ast\mu_1\cdots\mu_i\cdots\mu_j},
$$

(2.42)

and the superfield

$$
\hat{S}^{\mu_1\mu_2\cdots\mu_j} = \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} B_w^{\ast\mu_1\mu_2\cdots\mu_j} + \sum_{i=1}^{j} \sqrt{\frac{1}{j(2j + 1)}} (\gamma^{\mu_i} + \nu^{\mu_i} + \frac{k^{\mu_i}}{m_{B_j}}) \gamma_5 \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} B_w^{\ast\mu_1\cdots\mu_i\cdots\mu_j}.
$$

(2.43)

For the $j = 0$ we just list the result

$$
\hat{B} = \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} B_w
$$

(2.44)

Having arrived at these mass correction heavy baryons, we may see that the whole mass correction factor

$$
\sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}}
$$

is very similar to the multiplying factor for the wave function. The reader may check that this factor can be written as

$$
\exp \left( \frac{1}{2} \ln \frac{1 + k/2m_Q}{1 - k/2m_Q} \right).
$$

It can have its inverse form under Dirac conjugate

$$
\exp \left( -\frac{1}{2} \ln \frac{1 + k/2m_Q}{1 - k/2m_Q} \right).
$$

So, it is easy to see that such factor is unitary. This is an important property to make sure that the unitarity of whole theory will not be broken by mass correction.

III. MASS CORRECTION OF HEAVY MESONS

In this section we want to discuss the mass correction of heavy mesons. The light brown muck for heavy meson is more complicated than that in heavy baryons. The light brown muck in heavy meson may appear in two kinds of states specified according to $j = l \pm \frac{1}{2}, l \geq 1$. We identify these states as

$$
\hat{R}^{\mu_1\cdots\mu_i}(x),
$$

(3.1)

$$
\hat{R}^{\mu_1\cdots\mu_i}(x)\gamma_5 = 0,
$$

(3.3)

$$
\hat{R}^{\mu_1\cdots\mu_i}(x)i\gamma_5 = 0,
$$

(3.4)

$$
\hat{R}^{\mu_1\cdots\mu_i}(x)\gamma_{\mu_1} = 0,
$$

(3.5)

$$
\hat{R}^{\mu_1\cdots\mu_i}(x)\gamma_{\mu_1} = 0,
$$

(3.6)
and for $j = l - \frac{1}{2}$ states are
\begin{align}
\overline{R}^{\mu_1 \cdots \mu_l} (x) i \gamma_5 &= 0, \\
\overline{R}^{\mu_1 \cdots \mu_l} (x) i \partial_\mu \gamma_5 &= 0, \\
\overline{R}^{\mu_1 \cdots \mu_l} (x) \gamma_5 &= 0, \\
\overline{R}^{\mu_1 \cdots \mu_l} (x) \gamma_5 \gamma_{\mu_1} &= 0.
\end{align}

If we redefine them as
\begin{align}
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) &:= \overline{R}^{\mu_1 \cdots \mu_l} (x) e^{i \Lambda_M j} v^x, \\
\overline{R}^{\mu_1 \cdots \mu_l} (x) \gamma_5 &:= \overline{R}^{\mu_1 \cdots \mu_l} (x) \gamma_5 e^{i \Lambda_M j} v^x.
\end{align}

, then their equations of motion and constraints become
\begin{align}
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) (\gamma_5 (j + \frac{i \gamma_5}{\Lambda_M j}) &= 0, \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) (v_{\mu 1} + \frac{i \gamma_5}{\Lambda_M j}) &= 0, \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) &= 0, \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) \gamma_{\mu_1} &= 0,
\end{align}

and
\begin{align}
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) \gamma_5 (j + \frac{i \gamma_5}{\Lambda_M j}) &= 0, \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) \gamma_5 (v_{\mu 1} + \frac{i \gamma_5}{\Lambda_M j}) &= 0, \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) \gamma_5 &= 0, \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l} (x) \gamma_5 \gamma_{\mu_1} &= 0.
\end{align}

The additional limit accompanied with heavy quark mass infinite limit
\begin{equation}
\frac{i \gamma_5}{\Lambda_M j} \to 1
\end{equation}

will show that the results are just those defined in [9]. So, we apply such a limit on the redefined light brown muck
\begin{align}
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l}_v (x) &= \lim_{m_Q \to -\infty} \overline{R}^{\mu_1 \cdots \mu_l} (x), \\
\overline{\mathcal{R}}^{\mu_1 \cdots \mu_l}_v (x) \gamma_5 &= \lim_{m_Q \to -\infty} \overline{R}^{\mu_1 \cdots \mu_l} (x) \gamma_5.
\end{align}

Having these basis, we write our compact heavy mesons according to the above two kinds of light brown muck
\begin{equation}
J^P = (l^-, (l + 1)^-)_j = l+1/2 : \ M^{(-)}_{\mu_1 \cdots \mu_l} (x) =
\end{equation}
\begin{equation}
\left( \frac{m_{M_j} + i \gamma_5}{2 m_Q} \right) \left( \sqrt{\frac{2l+1}{2l+2}} P^{\mu_1 \cdots \mu_l} (x) \gamma_5 \delta_{\nu_1 \cdots \nu_l} - \frac{1}{2j+1} \sum_{i=1}^{l} \gamma_{\nu_i} (\gamma_5 \delta_{\nu_1 \cdots \nu_i} - \frac{i \gamma_5}{m_{M_j} \nu_1 \cdots \nu_i}) \delta_{\nu_1 \cdots \nu_l} + P^{\mu_1 \cdots \mu_{l+1}} (x) \gamma_{\mu_{l+1}} \right),
\end{equation}

(3.23)
and
\[
J^P = ((l-1)^+, (l)^+)_{j=l-1/2} : \quad M^{(+)}_{\mu_1...\mu_l}(x) = \\
\left( \frac{m_{M_j} + i \not{d}}{2m_Q} \right) \left( \sqrt{\frac{2l+1}{2l+2}} S^{\nu_1...\nu_l}(x) \left[ \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} - \frac{1}{2j+1} \sum_{i=1}^{l} \gamma_{\nu_i} (\gamma^{\nu_i} + \frac{\not{d} \not{M}_j}{m_{M_j}}) \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} \right] + S^{*\mu_1...\mu_{l+1}}(x) \gamma_5 \gamma_{\mu_{l+1}} \right).
\]
(3.24)

Redefine these heavy mesons
\[
\tilde{M}^{(-)}_{\mu_1...\mu_l}(x) = e^{im_{M_j} \not{x}} M^{(-)}_{\mu_1...\mu_l}(x) = \\
\left( \frac{1+ \gamma^l}{2} - \frac{1}{2m_Q} \frac{\Lambda_{M_j} + 1}{m_Q} \not{d} \right) \times \\
\left( \sqrt{\frac{2l+1}{2l+2}} \tilde{P}^{\nu_1...\nu_l}(x) \gamma_5 \left[ \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} - \frac{1}{2j+1} \sum_{i=1}^{l} \gamma_{\nu_i} (\gamma^{\nu_i} - \not{d}^{\nu_i} \not{M}_j \gamma_{\nu_i}) \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} \right] + \tilde{P}^{*\mu_1...\mu_{l+1}}(x) \gamma_5 \gamma_{\mu_{l+1}} \right), \quad (3.25)
\]
and
\[
\tilde{M}^{(+)}_{\mu_1...\mu_l}(x) = e^{im_{M_j} \not{x}} M^{(+)}_{\mu_1...\mu_l}(x) = \\
\left( \frac{1+ \gamma^l}{2} - \frac{1}{2m_Q} \frac{\Lambda_{M_j} + 1}{m_Q} \not{d} \right) \times \\
\left( \sqrt{\frac{2l+1}{2l+2}} \tilde{S}^{\nu_1...\nu_l}(x) \left[ \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} - \frac{1}{2j+1} \sum_{i=1}^{l} \gamma_{\nu_i} (\gamma^{\nu_i} + \not{d}^{\nu_i} \not{M}_j \gamma^{\nu_i}) \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} \right] + \tilde{S}^{*\mu_1...\mu_{l+1}}(x) \gamma_5 \gamma_{\mu_{l+1}} \right). \quad (3.26)
\]

Go to the momentum space and follow the similar procedure in subsection [12]. We finally get the mass correction compact heavy mesons related to the effective compact heavy mesons as
\[
J^P = ((l^-,(l+1)^-))_{j=l+1/2} : \\
\tilde{M}^{(-)}_{\mu_1...\mu_l} = \left( 1+ \frac{\Lambda_{M_j} + 1}{m_Q} \frac{\gamma^l}{2} \right) \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} M^{(-)}_{\mu_1...\mu_l},
\]
\[
\left( \frac{1+ \gamma^l}{2} \right) \left( \sqrt{\frac{2l+1}{2l+2}} D^{\nu_1...\nu_l}(x) \gamma_5 \left[ \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} - \frac{1}{2j+1} \sum_{i=1}^{l} \gamma_{\nu_i} (\gamma^{\mu_i} + \not{d}^{\mu_i} \not{M}_j \gamma^{\mu_i}) \delta^{\mu_1...\mu_l}_{\nu_1...\nu_l} \right] + D^{*\mu_1...\mu_{l+1}}(x) \gamma_5 \gamma_{\mu_{l+1}} \right), \quad (3.27)
\]
and
\[
J^P = ((l^-,(l)^+))_{j=l-1/2} : \\
\tilde{M}^{(+)}_{\mu_1...\mu_l} = \left( 1+ \frac{\Lambda_{M_j} + 1}{m_Q} \frac{\gamma^l}{2} \right) \sqrt{\frac{1 + k/2m_Q}{1 - k/2m_Q}} M^{(+)}_{\mu_1...\mu_l},
\]
\[ = \left(1 + \frac{\Lambda_{Mj}}{m_Q} \frac{1 + \gamma}{2} \right) \sqrt{\frac{2l + 1}{2l + 2}} \Gamma = \left(1 + \frac{\Lambda_{Mj}}{m_Q} \frac{1 + \gamma}{2} \right) \sqrt{\frac{2l + 1}{2l + 2}} S^n_{\mu_1 \cdots \mu_l} \left[ \delta^n_{\mu_1 \cdots \mu_1} - \frac{1}{2j + 1} \sum_{i=1}^{l} \gamma_{\nu_i} (\gamma^\mu_i + v^\mu_i + \frac{k^\mu_i}{m_{Mj}}) \delta^n_{\mu_1 \cdots \mu_i \cdots \mu_1} \right] + S^n_{\mu_1 \cdots \mu_l + 1} \gamma \gamma_{\mu_1} \right). \] 

(3.28)

For the \( j = \pm \frac{1}{2}, l = 0 \) heavy mesons can be referred to [13]. The whole derivation of the mass correction heavy mesons is straightforward and repeats same techniques. If our present heavy quark effective theory approach is right, then our projection operator method presented here will reveal itself to be a good one.

One remark should be noted that the normalization convections for baryons and mesons are different. The baryons’ convection is as usual while that of mesons is defined as \( \sqrt{m_Q} \).

**IV. MASS CORRECTION OF BJORKEN SUM RULE**

As an application of our mass correction of heavy hadrons, we choose to use the formalisms derived here to the Bjorken sum rule. Firstly, let’s look at the following situation

\[ \sum_{s,s'} \langle h(v,s) | Q_{\nu} \Gamma Q_{\nu'} | h(v',s') \rangle \langle h(v',s') | Q_{\nu} \Gamma Q_{\nu'} | h(v,s) \rangle = \sum_{n',s'} \langle \Psi(v) | Q_{\nu} \Gamma Q_{\nu'} | X^{n'}(v',s') \rangle \langle X^{n'}(v',s') | Q_{\nu} \Gamma Q_{\nu'} | \Psi(v) \rangle, \]  

(4.1)

where \( Q_{\nu} \Gamma Q_{\nu'} \) are the the correction heavy quark currents, \( \Psi(v) \) is the effective baryon \( l = 0 \) state and \( X^{n'}(v',s') \) are the baryons intermediate states. Since the infinite mass limit \( m_Q, m_{Q^Q} \rightarrow \infty \) of the Eq. [13] will reproduce the Bjorken sum rule, we can use this formalism to study its mass correction formalism. To use the formalisms derived here, one can use the following matching identitites

\[ Q_{\nu} \Gamma Q_{\nu'} \rightarrow C_{B,X(I)} \Gamma \tilde{B}_{v,v'} X^{\nu'}_{\nu}, \]  

(4.2)

if the final and initial states are \( | \Psi(v) \rangle \) and \( | X^{n'}(v',s') \rangle \). We substitute these into Eq. [4.1] and we obtain

\[ \sum_{n',s'} \langle \Psi(v) | C_{B,X(I)} \Gamma \tilde{B}_{v,v'} X^{\nu'}_{\nu} | X^{n'}(v',s') \rangle \langle X^{n'}(v',s') | C_{X'(I),B} X^{n'}_{\nu'} \Gamma \tilde{B}_{v,v'} \Psi(v) \rangle, \]  

(4.3)

where the \( C_{B,X(I)} \) and \( C_{X'(I),B} \) are relevant form factors with corresponding \( \tilde{B}_{v,v'}, \tilde{B}_{v,v'} \) which are relevant velocity variable, \( g_{\mu\nu} \) and \( \gamma \)-matrix resp.. Thus we have derived the mass correction Bjorken sum rule. To further study the above equation, we can substitute the mass expansion forms of the relevant baryon field operators. For the meson case we also have the same results. In general, if we let \( \tilde{H}_v \) as any correction heavy hadrons, the Bjorken sum rule for finite mass should take the following form

\[ \sum_{n',s'} \langle \tilde{H}(v) | C_{H,X(I)} \Gamma \tilde{B}_{v,v'} X^{n'}_{\nu'} | X^{n'}(v',s') \rangle \langle X^{n'}(v',s') | C_{X'(I),B} X^{n'}_{\nu'} \Gamma \tilde{B}_{v,v'} \tilde{H}_v | H(v) \rangle. \]  

(4.4)

This general formalism then can be studied for every order of \( 1/m_Q \). The detail structure will need careful analysis of every related matrix element, so we leave it to our future publication.
V. SUMMARY

We have derived the mass correction of heavy hadrons of arbitrary spin. We list them by their relevant quantum numbers. These correction field operators can be used to derive mass correction lagrangian for heavy hadron effective theory. As an application we use them to derive the Bjorken sum rule for finite mass.

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