Effect of cylinders on the characteristics of a fine-bubble plume

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Received: 22 March 2020; Revised: 28 August 2020; Accepted: 15 September 2020

Abstract
The characteristics of a fine-bubble plume passing through two tandem cylinders are investigated. Fine-bubbles with a mean diameter of 0.055 mm are released by water electrolysis from electrodes placed at the bottom of a tank. They induce an upward water flow around them with rising due to buoyancy. Orthogonally to the axis of such fine–bubble plume, two cylinders with a diameter D of 30 mm are arranged in tandem. The distance between the cylinders, L, ranges between 1.5D and 3D. This study visualizes the bubbles and water flow around the cylinders. It also measures the bubble velocity distribution. The experiments reveal the water and bubble shear layers originating at the sides of the lower cylinder and make clear their behavior around the upper cylinder. This study elucidates the effects of L on the characteristics of the fine-bubble plume, such as the stagnant bubbly flow and the bubbly wake around the cylinders.

Keywords: Fine-bubble plume, Two tandem cylinders, Shear layer, Wake, Experiment

1. Introduction

The flow around an arrangement of cylinders placed within a uniform flow has been attracting considerable attention in relation to the understanding and prediction of the cooling of integrated circuits, vibration of heat exchangers, and vegetation drag. The flow past two cylinders is one of the most fundamental flow and has been investigated experimentally over a wide range of Reynolds numbers (Thomas et al., 1964; Tanida et al., 1973; Zdravkovich, 1987; Sumner et al., 1999; Ataie and aslani, 2013; More et al., 2015). These experiments demonstrated the presence of separated shear layers originating at the surface of the front cylinder and their reattachment to the rear cylinder. The studies also revealed the effect of the vortical flow induced by the front cylinder on the development of vortices at the rear cylinder. Numerical simulations have also been conducted (Jester and Kallinderis, 2003; Meneghini et al., 2001; Vakil and Green, 2013; Mittal et al., 1997; AlQadi et al., 2015; Luc et al., 2016) to explore the effect of the Reynolds number and the distance between cylinders on the flow around the cylinders and the drag acting on them.

When continuously released into a liquid, gas bubbles rise due to buoyancy, thus inducing a flow around them. Such bubble driven flow, known as bubble plume, is used in various engineering processes involving matter and heat transfer. Several studies proposed methods to predict the entrained liquid flow rate (Hussain and Siegel, 1976; Leitch and Baines, 1989) and the characteristics of the bubble plume (Socolofsky and Adams, 2005). The effect of the bubble flow rate on the meandering motion of the rising bubbles was also investigated (Alam and Arakeri, 1993). Moreover, methods to simulate bubble plumes were presented by the authors (Uchiyama and Matsumura, 2010; Wang et al., 2013; Chen et al., 2014), and the flow observed around solid bodies when placed within a bubble plume was studied. An author proposed a finite element method to simulate a bubbly flow across a cylinder (Uchiyama, 1999a; Uchiyama, 1999b) and applied the method to simulate a bubbly flow across tube bundles (Uchiyama, 2000a; Uchiyama, 2000b). The same author also presented a finite element method to simulate a bubbly flow across moving bodies (Uchiyama, 2001) and applied such method to simulate a bubbly flow around an oscillating cylinder (Uchiyama, 2002; Uchiyama, 2003).
Murai et al. (Murai et al., 2005) experimentally visualized bubble plumes across cylinders and triangular prisms, and demonstrated the formation of a large scale wake with no bubbles behind such solid bodies. The bubble diameter ranged from 3 to 5 mm. The size of fine bubbles used for cooling and cleaning semiconductors is in the range of 0.01 mm to 0.1 mm, so the bubble diameter used by Murai et al. is relatively large. Since the characteristics of a bubble plume depend on the bubble terminal velocity or the bubble diameter, a bubble plume around a solid body would be affected by the bubble diameter. Thus, the authors (Uchiyama and Ishiguro; 2016) experimentally investigated the flow around a cylinder placed inside a bubble plume generated by fine-bubbles with a mean diameter of 0.05 mm. The investigation elucidated the bubble behavior and the water flow, such as the separated shear layers originating at the cylinder surface, their roll up, and the bubble entrainment in the resulting large scale eddies. Conversely, if more than one cylinder is arranged within a fine-bubble plume, the flow past those cylinders is expected to be different from the flow past a single cylinder; however, this is something that has never been studied.

In this study, two cylinders with a diameter $D$ of 30 mm at a distance $L$ between them were arranged in tandem orthogonally to the axis of a fine-bubble plume generated by rising gas bubbles with a diameter smaller than 0.055 mm. The bubble motion and the water flow were visualized for $L/D = 1.5$, $2$, and $3$, and the bubble velocity was also measured. The Reynolds number, as calculated based on the bubble velocity just below the lower cylinder and the kinematic viscosity of water, was 411. In the present study, the behavior of water and bubble shear layers from the side of the lower and upper cylinders are investigated. Furthermore, the effects of $L$ on the flow around the cylinders, such as the stagnant bubbly flow and the bubbly wake are discussed.

2. Experimental
2.1. Experimental set-up

Figure 1 (a) shows a schematic diagram of the experimental set-up. The water tank is made of transparent acrylic resin so as to enable flow visualization. Two cylinders with a diameter $D$ of 30 mm are arranged in tandem with their axis in horizontal position inside the tank. The width and depth of the tank are $14.7D$, and the height is $19.7D$. The water level is $16.7D$. The top of the tank is open and in contact with the atmosphere. In addition, five carbon rods with a diameter of $0.16D$ are arranged horizontally at the bottom of the tank. The origin of the coordinates is at the center of the carbon rods. The $x - y$ plane is horizontal, and the $z$-axis is vertical.

Figure 1 (b) shows the carbon rod arrangement and its position relative to the cylinders. The length of each rod is $3.3D$. The rods are arranged at the bottom of the tank along the $x$-axis separated by a distance of $0.7D$. Two rods are used as anodes, and the rest are used as cathodes. A DC voltage is applied between the electrodes, generating small oxygen and hydrogen bubbles at the anodes and cathodes, respectively. The generated bubbles rise in the water due to buoyancy, inducing an upward water flow around them and creating a bubble plume. The cylinders are arranged in tandem within this bubble plume. The length of each cylinder is $6.5D$ and their axes are orthogonal to the carbon rods. The distance between the tank bottom and the lower cylinder is $5D$. The distance between the cylinders, $L$, is variable.
2.2. Experimental methods and conditions

The volume of oxygen bubbles $Q_o$ and that of hydrogen bubbles $Q_h$, generated per unit time at the carbon rods, are determined by the electric current $I$. Figure 2 shows $Q_o$ and $Q_h$ measured by the downward displacement of water. The measurement time is 300 s, the water temperature is 298 K, and the atmospheric pressure is 1012 hPa. The $Q_h$ values agree nearly with the values calculated by the Faraday’s laws of electrolysis. In this study, the experiments are conducted at $I = 0.3$ A, where $Q_o + Q_h = 39$ mm$^3$/s.

The bubbles are visualized using a laser light sheet with a 0.5 W power, 532 nm wavelength, and 1 mm thickness. The images of the bubbles in the central vertical section ($x−z$ section), perpendicular to the cylinders, and the images of the bubbles in the central vertical section ($y−z$ section), parallel to the cylinders, are captured using a video camera.

The diameter of the bubbles in the upper region of the cylinders is measured from the bubble images acquired at 23 positions in the central vertical section ($x−z$ section). A single-lens reflex camera (Canon EOS80D) with 1/8000 s of shutter speed is used to photograph the bubble diameter. The effective pixel count of the camera is 6000 × 4000. At each position, a small rectangular section is arranged, as shown in Figure 3. The side lengths of these sections along the $x$− and $z$−axes are 0.53$D$ and 0.33$D$, respectively. That is, the spatial resolution of the camera is 0.0025 mm. The diameter of the bubbles inside each section is measured and the average bubble diameter is calculated. The bubble diameter is smaller than 0.06 mm in all sections, as discussed below.

The bubble velocity in the central vertical section ($x−z$ section), perpendicular to the cylinders, is measured with a particle image velocimetry (PIV) system by treating the bubbles as tracer particles. The frame rate and shutter speed of the high-speed camera used for PIV measurement are 200 fps and 1/1000 s, respectively. The effective pixel count of the
camera is $1920 \times 1080$. At this time, the shooting area is $12.2D \times 7.2D$, and the spatial resolution is 0.19 mm.

The authors (Uchiyama and Ishiguro, 2016) visualized the water flow past a single cylinder placed within a fine-bubble plume using a fluorescent dye (Rhodamine B). In this study, the visualization method is applied. Firstly, the lower-half surface of the lower cylinder is thinly coated with a transparent glue mixed with the fluorescent dye, as shown in Fig. 4. The length of the coated part is $1.7D$. Once the glue dries, the cylinder is placed inside the water tank. As the water flow dissolves the dye layer from the cylinder surface, the water flow can be visualized. The trajectories of the dye correspond to the water streaklines.

Two cameras equipped with band–pass filters are used to separately acquire the image of the bubbles and that of the dye in the central vertical sections of the tank.

3. Results

3.1. Bubble plume characteristics in cylinder-free conditions

Firstly, the fundamental characteristics of the bubble plume for this particular experimental set-up are investigated. Thus, the electrolysis of water is conducted without cylinders inside the tank. The bubbles generated at the electrodes arranged at the bottom of the tank rise due to buoyancy. A bubble plume forms once the bubbles reach the water surface. Figure 5 shows images of the bubble plume visualized at a height of $2 \leq z/D \leq 15$. Figure 5 (a) shows the image at the central vertical section ($x-z$ section), parallel to the electrodes. A number of bubbles concentrate within the plume central region ($-1.0 \geq x/D \geq 1.0$), while other bubbles form clusters locally, especially at the periphery of the plume ($x/D \sim \pm 1.0$). This is because the bubbles are entrained by large–scale eddies induced by the water velocity shear layers at the periphery. Figure 5 (b) shows the image at the central vertical section ($y-z$ section), perpendicular to the electrodes. The bubbles are distributed in a similar region as compared with the $x-z$ section. At a height of $z/D = 5$, where the lower cylinder would be placed, the bubble distribution along the $y$–axis (cylinder centerline) is almost uniform.

Figure 6 shows the averaged distribution of the bubble diameter $d_b$. The size of the error bar represents the standard
deviation of the bubble diameter variation. The variations of \(d_b\) along the \(x\)- and \(z\)-axes are not substantial. The average value of \(d_b\) is 0.055 mm. The standard deviation of \(d_b\) at each measuring section ranges from 0.003 mm to 0.006 mm. The bubble diameter distribution for the experiment performed with cylinders is similar to that shown in Fig. 6.

According to the measurements performed by Parkinson et al., (2008) the terminal velocity of a bubble with a diameter smaller than 0.1 mm, \(u_t\), can be estimated by the following equation:

\[
 u_t = \frac{d_b^2 \Delta \rho}{12 \mu} g \tag{1}
\]

where \(d_b\) is the bubble diameter, \(\Delta \rho\) is the density difference between the bubble and the ambient fluid, \(\mu\) is the viscosity of the ambient fluid, \(g\) is the gravitational constant. It should be noted that the ultra-clean water used by Parkinson et al. is expected to change the terminal velocity of bubbles, but this effect was not considered in this study due to the small particle size. The \(u_t\) value for this experiment \((d_b = 0.055\) mm\) is 2.8 mm/s. In this study, the terminal velocity \(u_t\) is used as the representative velocity for dimensionless.

For the experiment performed without cylinders, the bubble velocity \(u\) is measured by PIV system in the central vertical section \((x-z\) section\), parallel to the electrodes, and the time–averaged bubble velocity \(\overline{u}\) during 60 s is calculated. Figure 7 shows the distribution of the time-averaged bubble velocity \(\overline{u}\) at a height of \(2.7 \leq z/D \leq 12.3\), where the velocity is expressed in a non-dimensional form using \(u_t\) (= 2.8 mm/s). The vertical component \(\overline{u}_z\) is dominant, indicating that high-velocity upward water flow is induced by the rising bubbles. The \(\overline{u}_z\) value is the maximum at the plume centerline \((x/D = 0)\), approximately seven times as large as \(u_t\). Since the averaged bubble diameter is smaller than 0.06 mm, as shown in Fig. 6, the bubble velocity is supposed to be identical to the water. The plume width becomes larger as \(z\) increases. These are typical characteristics of a bubble plume. It is observed that the flow directed towards the plume centerline is originated at the plume periphery. This is due to plume entrainment.
Fig. 9 (a) Bubbles and (b) water streaklines visualized around a single cylinder on the central vertical section of the tank.

Fig. 10 Distribution of the time-averaged bubble velocity around a single cylinder on the central vertical section of the tank.

The distribution of $\bar{\vec{u}}_z$ along the horizontal ($x$) axis is shown in Fig. 8. It is expressed in a non-dimensional form using the centerline velocity $u_{z0}$ and the half-width $x_{0.5}$. Wang et al. (2019) showed that the velocity and plume width in a weak bubble plume become similarity with a Gaussian distribution. The distributions on four sections at $z/D \geq 4$ are identical, demonstrating the similarity of the velocity field.

Interestingly, when two cylinders are placed inside the bubble plume, the distribution of $\bar{\vec{u}}_x$ along the horizontal axis at $z/D = 4$, just below the lower cylinder, is almost the same as in the experiment conducted without cylinders. The Reynolds number $Re ( = u_{z0}D/\nu )$ is 411, where $\nu$ is the kinematic viscosity of water and $u_{z0}$ is the centerline velocity at $z/D = 4$. When a cylinder is placed in a single-phase uniform flow with such $Re$ value, the flow separates at the surface of the cylinder and the Karman vortices are shed from the cylinder (Zhou and Alam, 2016).

3.2. Flow past a single cylinder placed inside a bubble plume

The bubble motion and the water flow are investigated when a single cylinder is placed across the plume centerline. Figure 9 (a) shows the bubbles in the central vertical section ($x - z$ section), perpendicular to the cylinder. The rising bubbles collide with the lower surface of the cylinder and form a layer along the lower–half surface of the cylinder. This bubble layer separates from both sides of the cylinder ($x/D = \pm 0.5$, $z/D = 5$), and the separated bubbles flow almost vertically. The bubbles are more broadly distributed across the horizontal direction in the upper region of the cylinder. This is due to collision between the rising bubbles and the cylinder. Figure 9 (b) shows the image of the water dyed with Rhodamine B at the same instance as Fig. 9 (a). The separated shear layers originating at both sides of the cylinder convect upwards as they roll up. There are few bubbles inside the region enclosed by the separated shear layers, demonstrating the
presence of a stagnant region. The water and bubbles separate at the cylinder surface and convect together. Large-scale eddies are induced by the roll-up of the separated shear layers in the upper part of the stagnant region, and some bubbles are entrained by such eddies. The close interrelation between the bubbles and the water shear layers is clearly visualized.

The distribution of the time-averaged bubble velocity measured in the central vertical section ($x - z$ section), perpendicular to the cylinder, is shown in Fig. 10. The distribution at a height of $3.1 \leq z/D \leq 11.3$ (same as in Fig. 7) is presented. The velocity decreases along the plume centerline in the direction towards the lower edge of the cylinder. A stagnant region is observed at the lower edge of the cylinder. The separation of the bubbles at both sides of the cylinder is confirmed. The separated shear layers rise almost vertically. The velocity is extremely low in the region just behind the cylinder. Such stagnant region almost disappears at $z/D \geq 7$. However, the velocity still takes its minimum value at $x/D = 0$ downstream of the cylinder, indicating the presence of a wake. The velocity decreases significantly along the plume centerline in the upper region of the cylinder when compared to the experiment performed without the cylinder (Fig. 7). This is because the bubbles disperse greatly in the horizontal direction after colliding with the cylinder, as shown in Fig. 9 (a), and as a result, the velocity of the water flow induced by the bubbles is lower.

3.3. Flow past two cylinders arranged in tandem inside a bubble plume

Figure 11 shows the images of the central vertical section ($x - z$ section) when two cylinders are arranged in tandem along the plume centerline ($x = 0$). The distance between the cylinders, $L$, is $L = 1.5D$. As shown in Fig. 11 (a), the bubbles separate at the sides of the lower cylinder, giving rise to bubble shear layers. The separated shear layers rise almost vertically and reattach to the sides of the upper cylinder. The reattached shear layers separate again and rise vertically. A
few bubbles are observed just behind the upper cylinder, and the bubble shear layers roll up in the upper region of such stagnant region. Figure 11 (b) shows the water streaklines visualized with the Rhodamine B. The separation of the water flow at the sides of the lower cylinder can be clearly seen, as well as the reattachment of the separated shear layers to the sides of the upper cylinder, and the re–separation. The stagnant region and the roll–up of the separated shear layers can also be observed in the upper region of the upper cylinder. No water flow is observed between the cylinders.

Carmo et al. (2010) investigated the flow around two cylinders arranged in tandem within a single–phase uniform flow by numerical simulations. The simulation for $Re = 200$ and $L/D = 1.5$ showed that the separated shear layers originating at the surface of the front cylinder reattach to the rear cylinder and that a stagnant region appears between the cylinders. The present experiments reveal that such separated shear layers and stagnant region also form between the cylinders in a similar way as in a single–phase flow.

Figure 12 shows the distribution of the time–averaged bubble velocity $\bar{u}$ for $L/D = 1.5$. A stagnant region is observed at the lower edge of the lower cylinder. The separation of the bubble layers at the sides of the lower cylinder is also observed. These separated shear layers rise almost vertically. A stagnant region is observed between the two cylinders and just behind the upper cylinder. There is a wake in the upper region of the cylinders. The velocity reaches the minimum at the plume centerline. When compared to the experiment performed with a single cylinder (Fig. 10), the wake is longer and the velocity defect in the wake is more pronounced. In summary, the arranging two cylinders affects a broader region of wake, and as a result, the bubble plume does not fully develop.

Figure 13 shows the images of the bubbles and water streaklines visualized with Rhodamine B for $L/D = 2$. The flow characteristics are almost the same as those for $L/D = 1.5$. The distribution of the time–averaged bubble velocity $\bar{u}$
Figure 15 shows the images for $L/D = 3$. The water and bubble shear layers originating at the sides of the lower cylinder roll up below the upper cylinder, demonstrating the presence of a flow between the cylinders. The effect of the upper cylinder on the flow between the cylinders is less pronounced than when $L/D = 1.5$ and 2. According to the simulations for a single-phase flow performed by Carmo et al. (2010), when $L/D = 5$, eddies are released from the surface of the front cylinder into the region between the cylinders. In the present study, such vortical flow between the cylinders is observed when $L/D = 3$.

Figure 16 shows the distribution of $\bar{\varepsilon}$ for $L/D = 3$. The presence of a water flow between the cylinders due to the roll-up of the water shear layers is confirmed. The wake in the upper region of the upper cylinder is wider than when $L/D = 1.5$ and 2, indicating a larger effect of the two cylinders in tandem on the development of the bubble plume.

The distributions of the vertical component of the time-averaged bubble velocity, $\bar{u}_z$, on six horizontal sections are shown in Fig. 17. The pink region in Fig. 17 (0 $\leq x/D \leq 0.5$) is the area that overlaps when there is a cylinder. The broken lines show the distributions in the absence of cylinders, and the solid lines show the distributions when placing a single cylinder at $z/D = 5$. At $0.5 \leq x/D \leq 1$, the bubble velocity $\bar{u}_z$ with a single cylinder is larger than without the cylinders, as found from the distribution at $z/D = 5$. This is attributable to the fact that the bubbles flowing along the cylinder surface are accelerated due to the curvature of the streamlines. Similar results are observed in the sections at $z/D \geq 8$ due to the rising of the bubble shear layers originating at the sides of the cylinder. When a single cylinder is placed, a stagnant region with $\bar{u}_z \sim 0$ is observed around the centerline just behind the cylinder ($z/D = 6$). The velocity recovers at $z/D \geq 7$. However, a velocity defect is still present even at $z/D = 10$. When two cylinders are arranged in tandem, the velocity...
distribution just below the lower cylinder \((z/D = 4)\), as well as at the sides of the lower cylinder \((z/D = 5)\), is almost the same as when placing a single cylinder. Thus, no marked differences are observed in the bubble distribution and the water streaklines. The upper cylinder has a less effect on the flow on these sections. However, in the upper region of the lower cylinder \((z/D \geq 6)\), the velocity is lower than when placing a single cylinder around the plume axis \((|x/D| \leq 1.4)\). Furthermore, the velocity distribution changes with the distance between the cylinders \(L\). At \(z/D = 6\), a stagnant region exists around the centerline when \(L/D = 1.5\), and a downward flow \((\overline{u}_z < 0)\) is observed just below the upper cylinder. It is worth noting that the velocity distribution when \(L/D = 2\) and 3 is almost similar to that observed when placing a single cylinder. The section at \(z/D = 6\) passes though the top edge of the upper cylinder when \(L/D = 1.5\). A stagnant region \((\overline{u}_z \approx 0)\) is observed around the centerline. When \(L/D = 3\), the section at \(z/D = 7\) is located between the cylinders, and the velocity recovery is higher around the centerline when compared to the velocity distribution at \(z/D = 6\). The section at \(z/D = 8\) corresponds to the top edge of the upper cylinder for \(L/D = 1.5\) and 2. The velocity recovers when \(L/D = 1.5\), but a stagnant region where \(\overline{u}_z \approx 0\) is observed around the centerline when \(L/D = 2\). The section at \(z/D = 10\) is situated on the top edge of the upper cylinder when \(L/D = 1.5\) and 2. The velocity recovers markedly around the centerline, but the velocity defect still remains when \(L/D = 3\).

Using the images of the bubbles in the central vertical section \((x - z\) section), perpendicular to the cylinders, the luminance of the bubble, \(G\), is measured. The measuring position is set at \((0.3D, 6D)\) just behind the lower cylinder placed at \((0, 5D)\). When measuring \(G\), a square region of 11 pixel × 11 pixel is set around \((0.3D, 6D)\) to obtain the luminance at 121 pixels represented by 256 gradations, and the average value is calculated. Figure 18 shows the time variation of \(G\), where the deviation from the time-averaged value \(G_0\), \((G - G_0)/G_0\), is plotted against the nondimensional time \(u_t/D\). Here, the luminance of the bubble \(G\) largely depends on the setting of the initial value, but the luminance value and the particle number density are linearly dependent. Therefore, it should be note that the luminance value can be treated as an objective index by normalizing with the average value \(G_0\) of it. When no cylinders are placed, \(G\) varies irregularly, as shown in the top part of Fig. 18. This is because the water shear layers with the bubbles pass through the position where \(G\) is being measured, as seen in Fig. 5 (a). When a single cylinder is placed within the bubble plume, the amplitude slightly decreases. This is because the position where \(G\) is being measured is located inside the stagnant region just behind the cylinder and, as expected, the frequency of the passage of bubbles is lower. The effect of the cylinder on the oscillating period of \(G\) is not clearly observed. Conversely, when two cylinders are arranged in tandem, amplitudes are different from those observed in the experiment conducted with a single cylinder. When \(L/D = 1.5\) and 2, the amplitude is markedly lower. This is consistent with the fact that a stagnant region appears between the cylinders and as a result,
Fig. 18 Effect of the distance between the two cylinders on the variation with time of the luminance measured just behind the lower cylinder.

the frequency of the passage of bubbles is lower. By contrast, the amplitude is larger when $L/D = 3$. This is because the effect of the upper cylinder is less pronounced and therefore, the frequency of the passage of shear layers and bubbles is slightly higher.

Figure 19 shows the power spectra normalized by the root-mean-square of $G$, $G_{rms}$. The measuring position is set at $(0.3D, 6D)$ just behind the lower cylinder placed at $(0, 5D)$. When no cylinders are placed, the spectrum peaks at $fD/\bar{u}_t = 5$, and monotonically attenuates toward higher frequencies. When a single cylinder is placed within the bubble plume, there is a large energy on the low frequency side, but that energy is about half that of the case without a cylinder. In addition, dominant peaks appeared at $fD/\bar{u}_t = 30$ and 60. This is the effect of the vortex street caused by the separation of the flow through the cylinder. When a two cylinders are placed within the bubble plume, there is no spectrum peak in the low frequency band as observed in cases of no cylinder and single cylinder. For $L/D = 1.5$, some dominant spectra are observed. As $L/D$ increases, the spectrum changes to a continuous shape. This suggests that the presence of the upper cylinder affects the vortex shedding behind the lower cylinder.

Figure 20 shows the relationship between $G_{rms}$ and $L$. The $G_{rms}$ value obtained with two cylinders in tandem is lower than when placing a single cylinder or no cylinders. It is also found that $G_{rms}$ is higher when $L/D = 3$.

4. Conclusion

Two cylinders with a diameter $D$ of 30 mm were arranged in tandem separated by a distance $L$ orthogonal to the axis of a fine-bubble plume induced by rising gas bubbles with a diameter smaller than 0.055 mm. The bubble and the water flow were visualized for $L/D = 1.5, 2,$ and $3$, and the bubble velocity was also measured. The Reynolds number, as calculated based on the bubble velocity just below the lower cylinder and the kinematic viscosity of water, was 411. The following results were obtained:

1. The upper cylinder hardly affects the bubble plume below the lower cylinder.
2. The bubbles colliding with the lower surface of the lower cylinder flow along the lower half surface of the cylinder, and they separate at both sides of the cylinder. The separated bubbles form shear layers that rise almost vertically and reattach to the sides of the upper cylinder when $L/D = 1.5$ and 2. However, these shear layers are directed towards the centerline between the cylinders when $L/D = 3$.
3. The upward water flow induced by the rising bubbles also separates at both sides of the lower cylinder, forming a stagnant region just behind the lower cylinder. For $L/D = 1.5$ and 2, such separated shear layers rise almost vertically and reattach to the sides of the upper cylinder. Due to the roll up of the separated shear layers, the stagnant region between the cylinders is smaller when $L/D = 3$ than when $L/D = 1.5$ and 2.
(4) When \( L/D = 1.5 \), a downward bubble flow develops locally in the vicinity of the lower edge of the upper cylinder.

(5) The separated shear layers reattach to the sides of the upper cylinder and subsequently, separate again, forming a stagnant region just behind the cylinder. A wake is generated in the upper region of the cylinders. The scale of such stagnant region and wake is larger than that observed when placing a single cylinder, and is significantly larger when \( L/D = 3 \).

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