Review

A Review on the Unit Commitment Problem: Approaches, Techniques, and Resolution Methods

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Abstract: Optimizing the schedule of thermal generators is probably the most important task when the operation of power systems is managed. This issue is known as the unit commitment problem in operational research. It has been profoundly studied in the literature, where several techniques have been proposed to address a computationally tractable solution. In turn, the ongoing changes of paradigms in energy markets focus the attention on the unit commitment problem as a powerful tool to handle new trends, such as the high renewable energy sources penetration or widespread use of non-conventional energy-storage technologies. A review on the unit commitment problem is proposed in this paper. The easy understanding of the diverse techniques applied in the literature for new researchers is the main goal of this state-of-art as well as identifying the research gaps that could be susceptible to further developments. Moreover, an overview of the evolution of the Mixed Integer Linear Programming formulation regarding the improvements of commercial solvers is presented, according to its prevailing hegemony when the unit commitment problem is addressed. Finally, an accurate analysis of modeling detail, power system representation, and computational performance of the case studies is presented. This characterization entails a significant development against the conventional reviews, which only offer a broad vision of the modeling scope of their citations at most.

Keywords: unit commitment; optimal thermal generation; numerical optimization; evolutionary optimization; optimization techniques; decomposition techniques; uncertainty management

1. Introduction

The unit commitment problem (UC) is a traditional optimization problem where the best schedule for a group of thermal units is obtained. Optimizing the electrical generation entails many advantages for market players and final customers. However, that is not an easy task according to the big size of the problem and the computational limitations.

For that reason, there are many works in the literature where different approaches are proposed to find an optimal solution to this problem, constituting an essential target for the advances in operational research. This paper presents a new review of the state-of-art of the unit commitment problem, where the distinctions between optimization techniques, problem formulations, and resolution algorithms are exposed in order to facilitate their understanding.

This section provides a brief description of the main issues that are frequently considered in the unit commitment problem. Several mathematical approaches have been proposed over the years. The principal modeling ideas are gathered in this paper and referenced for more detailed explanations. There are multiple techniques that have been applied to solve the unit commitment problem, being the most popular its presentation as a conventional optimization problem:

\[ \min (\text{Production cost} + \text{SU cost} + \text{SD cost} + \text{Emission cost} + \text{Maintenance cost}) \] (1)
subject to:

\[ \text{Technical and economical constraints.} \] (2)

The different terms of the objective function (OF) are described below:

- **Production cost**: This cost is related to the fuel consumption of the thermal units when electricity is generated. Its behavior is usually described through a linear or quadratic function, where there are a fixed term, a linear term concerning the power production, and a quadratic term that multiplies the squared power generation of the unit. The last term can be omitted to work with linear objective functions. Moreover, a piecewise approximation can also be used to linearize the quadratic function. The utilization of integer variables is mandatory for correctly modeling this cost.

- **Start-up (SU) cost**: This cost is related to the fuel consumption of the starting-up process before a thermal unit is totally committed. It has an exponential behavior according to the number of hours that the unit had been offline. Nevertheless, it is commonly linearized through a stairwise function. Integer variables are also employed for achieving an accurate representation.

- **Shut-down (SD) cost**: This cost is applied when a thermal unit is shut-down. It is usually modeled as a fixed cost where integer variables are used to define its treatment. Sometimes, this cost is not considered.

- **Emission cost**: This cost is related to the polluting compounds or the greenhouse gases generated as a consequence of electricity production. It is not linked to the fuel prices, such as those mentioned above, but it is related to fuel consumption and technological efficiency. Its value depends on the local regulation and the emission allowance trading market scheme if it exists. Within the European Union, it relies on CO\(_2\) prices.

- **Maintenance cost**: This cost represents the increase of the maintenance operations when the thermal unit is running for a longer time. It is modeled as a linear function with respect to power generation, and it is often internalized in the production cost for the sake of simplicity. Integer variables are also associated with this cost.

Fuel consumption is the main cost of the unit commitment problem. The importance of a proper representation of these costs is pointed out in [1], where it is mentioned that saving 0.5% of fuel in electricity generation represents a yearly benefit of millions of euros for a utility. Accordingly, research is focused on improving the modeling detail to increase profitability. In past decades, coal plants were generally the thermal basis of power systems. Meanwhile, combined cycles gas turbines were relegated for high-demand periods, and fast-ramping gas turbines were used to cover demand peaks. This operation was steady over time and did not demand great modeling developments to achieve an efficient asset management.

Nowadays, the trend is changing. The dismantling process of coal plants accounted by many power systems according to greenhouse policies and the loss of their competitiveness due to the implementation of emission-allowance trading markets has brought a paradigm shift. Moreover, the higher renewable-energy source penetration has increased the frequency of the start-up and shut-down processes of the thermal units. Hence, the proper representation of the fuel consumption during these operations is gaining force in current formulations. With regard to the technical constraints used in the unit commitment problem, the following considerations are often assumed:

- **Demand constraint**: This is a balance equation to assure that the electricity generation meets the load demand for every represented time period. An energy storage term can be added if more accurate management is desired. In turn, it is also possible to introduce a spillage term in the equation to represent situations of production surpluses. It is a linear equation with continuous variables.

- **Reserve constraint**: This inequality guarantees a technical necessity of power systems, which is the availability of an extra generation capacity reserved for compromising
situations, such as a failure in a committed thermal unit, to keep the security of supply. It is a linear inequality that also employs continuous variables.

- **Capacity limits**: This inequality is used to assure that the electricity generation of each thermal unit respects the minimum and maximum power output according to their technical limits. This inequality is linear and uses integer variables.

- **Ramping limits**: This inequality assures that the difference between the power generation of a thermal unit during the previous and the current time step does not exceed the ramping rates. It is a linear inequality that uses continuous variables.

- **Logic constraint**: It establishes logic in the commitment decisions at every time step, indicating the relationships between start-ups, shut-downs, and commitment status along the whole time span. This equation is linear and utilizes integer variables.

- **Minimum time up (TU) and time down (TD)**: This inequality is used to guarantee that the unit is online for a minimum period of time since it is started-up or that it is offline for a minimum time since it is shut-down, in order to accomplish with technical limitations that reduce the risk of failure. It is linear and employs integer variables.

- **Operating constraints**: This group gathers the constraints that are utilized for a more accurate representation of the operation of the thermal units from a technical point of view. Some examples are situations where some units must run or have a fixed power output, an outage of a unit due to maintenance tasks, or unforeseen problems, etc.

- **Emission constraints**: They are imposed to bound specific emissions along a time span.

- **Network constraints**: These constraints are implemented with the aim of representing technical limitations regarding the consideration of the power grid. They increase the accuracy of the unit commitment problem but also the complexity of its resolution. For that reason, the network is frequently disregarded unless a more secure generation schedule is desired. In that case, the capacity of the optimal schedule to overcome an unexpected failure safely is sought. It is known as the Security Constrained Unit Commitment problem (SCUC).

The unit commitment problem is frequently addressed as a minimization problem, where the sum of the total costs constitutes the objective function, and the demand constraint is an equation where the generation meets the power consumption. Sometimes, a non-served energy term is added to the balance equation. If the associated cost is high, there will only be non-served energy situations if the demand exceeds the generation limit. On the contrary, if it is not high enough, there could appear situations in which the electricity production is not profitable.

This representation was suitable before the liberalization of energy markets accomplished in many power systems. Nowadays, the objective function of the unit commitment problem is also represented through the difference between benefits and costs of producing electricity, which is sought to be maximized. In turn, price forecasts are implemented to evaluate profitability, and the demand constraint is transformed into an inequality or omitted. The production of a market player will be based on profitability, which is known as the Price-Based Unit Commitment problem (PBUC). The representation of the competitiveness behavior of market players has also been studied in this problem. Nevertheless, this research topic is out of the scope of this survey.

The following section describes the techniques that can be applied to solve the unit commitment problem further than the optimization problem presentation. However, according to its importance, this methodology is predominantly studied in this paper. The programming options with relation to the formulation are exposed in Section 3.1. In turn, the consideration of uncertainty is addressed in Section 3.2. The decomposition techniques that can be applied to facilitate its resolution are mentioned in Section 3.3. On the other hand, the resolution algorithms that can be applied to find the optimal solution to the optimization problem are exposed in Section 3.4. Section 4.1 describes the modeling trends when the unit commitment problem is formulated as a Mixed Integer Linear Programming (MILP), which is currently the most widespread approach. Meanwhile, Section 4.2 presents an accurate analysis of modeling detail, power system representation, and computational performance.
of the case studies described in all the references. Finally, Section 5 concludes the article with a brief contribution summary, identifying research gaps for further developments.

2. Optimization Techniques and Unit Commitment

In order to achieve an optimal solution to the unit commitment problem, several optimization techniques have been applied in the literature. These methodologies require a set of input data and return the best thermal schedule obtained after their applications. Figure 1 presents an illustrative diagram where these optimization techniques are classified. In contrast with many other arrangements made in the literature, this layout can help to clarify the differences between optimization techniques, problem formulations, decomposition techniques, and resolution methods, as well as different options of uncertainty management inside the proposed scheme.

Popular optimization methods are described in this section. They constitute different approaches to face the unit commitment problem, whose complexity has often evolved together with the advances in computation. Furthermore, it is also possible to combine them in hybrid methodologies to exploit some of their advantages at once. Regarding the literature, the following classification is made:

- Exhaustive Enumeration (EE): It consists of the evaluation of all the feasible solutions in order to identify the best value as the optimal solution. Exhaustive enumeration is a brute force method that is not computationally affordable. Its scope is very limited [2].
- Expert System (ES): The underlying idea of this method resides in the creation of an algorithm where the good practices and knowledge of proceedings in the resolution of the unit commitment problem are computed. It was employed to save in computational costs [3], but it fell into disuse because of its sub-optimal solutions.
- Priority List (PL): This technique is based on ordering elements of target sets according to their contribution to the objective function. The decisions taken on these target elements during the resolution process will conclude in a better or worse approach to the optimal solution. For that reason, it usually has a mathematical background behind it. Despite returning a sub-optimal solution, it is an attractive optimization technique from a computational perspective due to the obtaining of near-optimal solutions in reasonable run times [4–7].
- Fuzzy Logic (FL): This method allows the application of abstract reasoning into the computational logic to solve a mathematical problem. It utilizes if–then rules in order to generalize some input data. This information is treated according to a background (if conditions) and output data are obtained consequently (then reactions). The utilization of fuzzy logic techniques helps accelerate the resolution of the unit commitment problems but returns a less accurate solution. They are usually applied at a beginning step and combined with other optimization techniques in hybrid proceedings [8–10].
- Neural Networks (NN): This artificial intelligence technique is based on establishing patterns that transform some input data into a near-optimal solution. Its structure consists of a group of interconnected nodes where some mathematical functions are applied to process the information. In order to achieve good results, these processes are trained with a benchmark database. However, it provides sub-optimal solutions, and its implementation and adjustment are quite difficult [11,12]. This machine-learning approach is still being used to solve the unit commitment problem nowadays. For further information about the application of machine learning techniques in the unit commitment, the reader is referred to [13].
- Optimization Problem (OP): The unit commitment problem is frequently addressed as a classical optimization problem, where an objective function is proposed, subject to a set of constraints. This methodology entails the most widespread approach used to solve the unit commitment problem, and it is described in depth in Section 3.
- Hybrid Methodologies (HM): These optimization techniques are sometimes combined in hybrid methodologies in order to improve their performance. Some approaches made in the literature of the unit commitment problem are [14–21].
A comparison of these optimization techniques is presented in Table 1. References are focused on the thermal UC. For representations where the optimization of power systems adds hydro valleys or energy storage facilities, the reader is referred to [22,23], respectively.

![Figure 1](image-url)

**Figure 1.** Optimization methodologies applied to the unit commitment problem. At the first level, the optimization techniques are exposed. Later, the most widely used is visually described, underlying the two choices of addressing the optimization problem and its stages. The formulating option and the resolution algorithm that are needed to solve the problem are presented. In turn, the uncertainty representation and the utilization of decomposition techniques are introduced as possible alternatives to enhance the accuracy of the representation of the problem and the performance of its resolution.
| Technique | Advantages | Disadvantages |
|-----------|------------|---------------|
| EE        | Optimal solution | Computationally intractable |
| ES        | Fast resolution  
Handles a lot of information  
Combines theoretical–practical knowledge | Non-optimal solution  
Difficult implementation  
Problems if schedules are unexpected in database |
| PL        | Fast resolution  
Mathematical background  
Generally easy to implement | Non-optimal solution  
Difficult identification of new cost-saving trends  
Solution can be very far from the optimum value |
| FL        | Qualitative interpretation  
Effective solution to complex problems  
Handles any type of unit characteristic data | Non-optimal solution  
Fuzzy rules are difficult to implement  
Results are difficult to analyze in-depth |
| NN        | Handles complex systems efficiently  
Hidden relationships can be identified  
Flexible utilization of non linear functions  
Flexible treatment of noisy data | Non-optimal solution  
Intricate network structure  
New additions require a retraining  
Exponential computation-time/problem-size rate |
| OP        | Optimal solution  
Generally easy to implement  
Advances in commercial solvers  
Much information in the literature | Exponential computation-time/problem-size rate  
Modeling simplifications are sometimes needed  
Function linearizations are sometimes needed  
Unable to work with noisy data |

3. Unit Commitment as an Optimization Problem

As mentioned, the resolution of the unit commitment problem is frequently addressed as an optimization problem. Two approaches can be differentiated within this technique. Conventional optimization (CO) exposes the objective function and its constraints as is. On the other hand, dynamic programming (DP) utilizes the principle of optimality proposed by Bellman [24], dividing the problem into overlapping subproblems and optimal substructures [25–27]. However, the curse of dimensionality forces the implementation of heuristic rules to deal with real-size problems, providing near-optimal solutions. Although nowadays, DP is not as relevant as it was before, it is sometimes used to manage uncertainty [28–30]. Furthermore, other distinctions in OP can be made according to their formulation, uncertainty consideration, utilization of decomposition techniques, or the optimization algorithm that is employed to determine an optimal solution.

3.1. Formulating Options

There are different classifications depending on how an OP can be formulated. According to the variables, a distinction can be made between discrete or continuous variables, as well as the utilization of a mix of them. In contrast, the nature of the objective function and constraints can be discriminated into linear, quadratic, or non-linear functions. Combinations of these programming techniques lead to the classification made in Appendix A. In the unit commitment problem, the most popular formulating option is Mixed Integer Linear Programming. In turn, it can also be faced by Mixed Integer Quadratic Programming (MIQP), Mixed Integer Quadratically Constrained Prog. (MIQCP), Mixed Integer Second-Order Cone Programming (MISOCP), and Mixed Integer Non-Linear Programming (MINLP).

3.2. Uncertainty Representation

The operation of real power systems is implicitly subject to uncertainty. The demand forecasts are exposed to inaccuracy or sudden changes according to unexpected situations. Furthermore, the penetration of renewable energy sources in current power markets increases the uncertainty. Wind and solar generation entail the risk of altering the thermal demand at any time. For those reasons, the consideration of uncertainty in the unit commitment problem enhances the reliability of the optimal schedule that is obtained. If the
When the election of representing uncertainty is made, there are several alternatives to transform a deterministic problem into a probabilistic one. The most popular techniques applied to the unit commitment problem are briefly described below and compared in Table 2.

- **Stochastic Optimization (SO):** This methodology manages the representation of uncertainty through the utilization of probability distributions that are connected to risk variables. These distributions can be directly included in constraints that require some statistical parameters. Nonetheless, the most common practice is to consider different scenarios. These scenarios are obtained through probability-distribution discretizations. Each scenario has an associated weight according to its frequency of occurrence.
  - **Two-Stage (TS) Stochastic Programming:** This technique is based on dividing the problem into two steps, distributing decision variables and constraints in these stages. When the first step is accomplished, the first-stage choices are made. Later, these decisions are considered fixed, and the second stage is solved. The two-stage stochastic programming utilizes scenarios to consider uncertainty. When all the scenarios are solved, a solution to the problem is calculated according to the weight of each scenario. It has been widely applied in the UC literature [31–39].
  - **Multi-Stage (MS) Stochastic Programming:** This technique uses a combinatorial tree where every combination of scenarios is represented. The tree is divided into successive nodes that are linked. A branch represents the path between the initial node and a final solution. The weights of scenarios are set in the linking connection between nodes. Thus, it is easy to determine the probability of each solution obtained when each branch is solved, which corresponds to a scenario. Some of its applications to the unit commitment problem are [40–42]. Robust solutions are provided, since decisions are taken dynamically. However, the associated computational burden requires an excellent scenario sampling and reduction to reach acceptable run times.
  - **Risk Consideration (RC) Stochastic Programming:** This method is based on the addition of some constraints in order to respect the risk exposure of some decisions when the problem is solved. These equations require statistical information of the probability distributions as input and the significant value desired to respect. Thus, a solution is obtained according to the confidence interval that is introduced in the problem. This technique has been applied to the unit commitment problem in order to represent situations such as the expected non served load, loss of load probability, or the variance of the total profit [43–45].
  - **Chance Constrained (CCO) Stochastic Programming:** It is considered a particularization of two-stage stochastic programming. This technique allows the solution to violate a set of constraints according to a predefined confidence level. As with the risk consideration stochastic programming, it works with probability distributions instead of scenarios. However, the solution has a probabilistic touch. It will be the optimal solution with a confidence interval, not according to a confidence interval associated with risk variables [46–48].

- **Robust Optimization (RO):** The underlying idea of this methodology is to reach an optimal solution avoiding the worst possible combination of circumstances that can happen according to the uncertainty associated with the presence of risk variables. Robust optimization does not work with probability distributions. It employs a bounded range from which a risk variable can take its value. The bounds imposed on the risk variables are applied in two ways. They are directly used as upper and lower bounds in an inequality that is defined for each dimension in which the variables are formulated and indirectly through the establishment of an uncertainty set. This uncertainty budget is linked to the deviation of the forecasted value associated with
each risk variable from their bounds along the evaluated horizon. It can not exceed a predefined value. The more restrictive the uncertainty set is, the more robust the obtained solution. The load demand is used as the target of RO in the unit commitment problem [49–51].

- **Interval Optimization (IO):** This technique handles the uncertainty representation through the creation of bounds according to a predefined confidence interval. Firstly, a forecasted value is provided for each considered risk variable. Later, the confidence interval is used to generate an envelope around the expected value. The higher the value of the confidence interval, the tighter the bounding to the risk variable. Afterwards, the problem is optimized at the expected central value of the interval, being also capable of providing a feasible solution for those deviations from the forecast that are contained inside the interval [52–54].

- **Monte Carlo Simulation (MCS):** The Monte Carlo methodology is frequently employed to achieve an accurate sampling from a set of probabilistic distributions. However, it can be extended to manage a complete uncertainty representation. In that case, the obtained scenarios are optimized as deterministic problems. Later, the output data are processed, and a probabilistic distribution is associated with each result [55–57].

- **Hybrid Uncertainty Implementation (HUI):** These techniques can be combined to improve their performances. Some examples in the unit commitment literature are [58–60].

| Technique  | Advantages                                                                 | Disadvantages                                                                 |
|------------|-----------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| SO-TS      | Less conservative solution                                                   | Static assumption of uncertainty                                               |
|            | Generally easy to implement                                                 | Necessity of scenario-generation techniques                                   |
|            | Working with discretized scenarios                                          | Necessity of scenario-reduction techniques                                     |
|            | Easy weight-assignment to each solution                                      | Information of probability distributions                                        |
| SO-MS      | Dynamic assumption of uncertainty                                           | Curse of dimensionality                                                         |
|            | Accurate decision-making modeling                                           | Necessity of scenario-generation techniques                                   |
|            | Working with discretized scenarios                                          | Necessity of scenario-reduction techniques                                     |
|            | Easy-traceable paths in the scenario tree                                   | Information of probability distributions                                        |
|            | Easy weight-assignment to each solution                                      | Difficult construction of scenario trees                                       |
| SO-RC      | Flexible interval confidence for input data                                | Necessity of statistical information                                           |
|            | Risk variables define robustness                                            | Working with probabilistic distributions                                       |
|            | Generally easy to implement                                                 | Solution according to a interval confidence                                    |
| SO-CCO     | Less conservative solution                                                   | Solution is not valid in some situations                                       |
|            | Solution with a probabilistic touch                                         | Working with probabilistic distributions                                       |
|            | Flexible setting of violable constraints                                     | Modeling demands extra binary variables                                        |
| RO         | Optimization over the whole horizon                                         | Over-conservative solutions                                                    |
|            | Do not need probabilistic distributions                                     | Tight optimality/uncertainty set relation                                      |
|            | Flexible protection from adverse situations                                 | Expertise to construct the uncertainty set                                      |
|            | Useful if forecasting deviations are bounded                                | Difficult to incorporate uncertainty dynamics                                   |
| IO         | Easier to incorporate uncertainty dynamics                                  | Over-conservative solutions                                                    |
|            | Do not need probabilistic distributions                                     | Tight optimality/forecasting bounds relation                                  |
|            | Flexible protection from adverse situations                                 | Expertise to establish forecasting deviations                                 |
|            | Useful if forecasting deviations are bounded                                | Optimization over particular central values                                     |
| MCS        | Computational efficiency                                                    | Implementation can be difficult                                                |
|            | High-efficient sampling process                                             | Necessity of scenario-generation techniques                                   |
|            | Working with discretized scenarios                                          | Necessity of scenario-reduction techniques                                     |
|            | Resolution process can be parallelized                                       | Information of probability distributions                                        |
|            | Results have a probabilistic distribution                                  | Solutions are not valid in some situations                                     |
3.3. Decomposition Techniques

Once the formulation of the unit commitment problem is chosen and the decision of considering uncertainty or not is taken, it is time to evaluate if the application of decomposition techniques to the problem is desirable. These techniques can be a powerful tool to solve the unit commitment problem with reasonable run times. Although they were frequently employed when the performance of numerical optimization solvers was computationally limited, particularly the Lagrangian Relaxation, their use has drastically decreased currently because of the advances in commercial solvers and disadvantages such as its difficult implementation and convergence toward an optimal solution. However, the application of decomposition techniques in the unit commitment problem is sometimes addressed nowadays:

- Dantzig-Wolfe Decomposition (DWD): [33,40,61].
- Benders Decomposition (BD): [35,39,48].
- Lagrangian Relaxation (LR): [31,37,62].

3.4. Optimization Algorithms

Finally, when an optimization problem is completely defined, optimization algorithms are applied to find an optimal or near-optimal solution to the problem. The resolution techniques can be classified into numerical methods or meta-heuristic methods.

Numerical optimization (NO) is preferably used to solve the unit commitment problem. In a first step, the improvements in these iterative algorithms were implemented directly by researchers looking to enhance the resolution processes. Currently, commercial solvers have advanced notoriously, and the research is focused on exploiting their properties as best as possible. In order to solve MILP, MIQP, and MIQCP problems, two of the most renowned commercial solvers are Gurobi [63] and CPLEX [64]. In addition, for MISOCP or MINLP problems, MOSEK [65] and BARON [66] respectively offer one of the best performances. For further information about NO, the reader is referred to [67].

On the other hand, evolutionary optimization (EO) is also applied to solve the unit commitment problem. Although these methodologies can not guarantee finding an optimal solution, they can handle more complex formulations without the necessity of utilizing too many simplifications. These meta-heuristic algorithms are based on simulating behaviors that are observed in nature to map the feasible region of an optimization problem looking for the optimal solution. Many of these approaches have been employed to solve the unit commitment problem:

- Genetic Algorithm (GA): [68].
- Simulated Annealing (SA): [69].
- Tabu Search Algorithm (TSA): [70].
- Evolutionary Programming (EP): [71].
- Particle Swarm Optimization (PSO): [72].
- Ant Colony Optimization (ACO): [73].
- Ant Lion Optimizer (ALO): [74].
- Artificial Bee Colony Algorithm (ABCA): [75].
- Artificial Fish Swarm Algorithm (AFSA): [76].
- Artificial Immune System Algorithm (AISA): [77].
- Artificial Sheep Algorithm (ASA): [78].
- Cuckoo Search Algorithm (CSA): [80].
- Differential Evolution Algorithm (DEA): [81].
- Exchange Market Algorithm (EMA): [82].
- Firefly Algorithm (FFA): [83].
- Fireworks Algorithm (FWA): [84].
- Gravitational Search Algorithm (GSA): [85].
- Grey Wolf Optimizer (GWO): [86].
• Imperialist Competitive Algorithm (ICA): [87].
• Intensify Harris Hawks Optimizer (IHHO): [88].
• Memory Management Algorithm (MMA): [89].
• Quantum-Inspired Evolutionary Algorithm (QIEA): [90].
• Quasi-Oppositional Teaching Learning Based Algorithm (QOTLBA): [91].
• Shuffled Frog Leaping Algorithm (SFLA): [92].
• Sine–Cosine Algorithm (SCA): [93].
• Whale Optimization Algorithm (WOA): [94].

In turn, these algorithms can be combined with each other to achieve a better performance, or even with commercial solvers if the problem is divided into multiple resolution stages. Some examples in the unit commitment literature are in [95–99]. Furthermore, an excellent state-of-art on EO is presented in [100].

4. Trends in Computational Efficiency to Solve the Unit Commitment Problem

4.1. Evolution of Unit Commitment Modeling Trends and Current Situation

In spite of the application of advanced evolutionary algorithms to the resolution of the unit commitment problem, the mainstream research trend is the utilization of numerical optimization algorithms to find the global optimum. As it was mentioned in Section 3, the implicit computational burden of this problem limited the scope of numerical optimization.

In the beginning, research was mainly focused on improving optimization algorithms. In turn, decomposition techniques were also deeply studied in order to accelerate the resolution processes. Despite their computational advantages, the utilization of decomposition techniques introduces complexity in the problem, such as difficulties in the implementation or the oscillations in the iterative convergence to the optimal solution.

For that reason, when commercial solvers gained competitiveness, the research in the UC problem started to change its direction. The advances in computation and the huge developments in numerical optimization promoted an exponential enhancement of MIP solvers [101]. Commercial solvers were able to address the unit commitment problem in reasonable run times even if decomposition techniques were not utilized. Then, the research began to focus on the formulation, instead of the decomposition, in order to exploit the properties of the solvers efficiently.

Suddenly, it was possible to think about improving the detail representation in the unit commitment problem. In turn, these advances needed to be in accordance with the solver trends. The performance of MILPs was more developed than MIQPs or MIQCPs and much more than MINLPs. For that reason, even quadratic formulations that were able to be solved as is were linearized in order to achieve a faster resolution.

The foundations of the UC modeling were laid. The detailed representation of the quadratic production costs was assumed through a piecewise linearization. Moreover, the dependence between start-up costs and the offline time steps accounted by the thermal units was modeled by a stairwise function [102]. The importance of convexity was manifested, both for thermal units and hydro generators [103]. Regarding the reserves [104] and power trajectories, efficient formulations were proposed to deal with them [105]. In these formulations, it was usual to employ three binary variables for each thermal unit and time step: one to define the commitment status and the others to represent a start-up or shut-down process. Despite that, binary variables were also utilized for piecewise linearizations, whose performance has been studied as well [106], and for stairwise representations [107].

Nevertheless, despite the improvements accomplished by the commercial solvers, the UC remains a strongly NP-hard problem [108]. For that reason, if greater horizons, thermal portfolios, or technical representations are desired, a trade-off between the modeling detail and the size of the corresponding problem must be chosen.

Modeling simplifications are mandatory when bidding processes are included in the representation [109], even more, if congestion in transmission lines is also desired to be included [110]. In turn, disaggregation methods are studied as an alternative for extending the scope of the representation. Ref. [111] breaks the horizons and solves shorter time
spans sequentially. Ref. [112] decomposes the problem according to power market stages to increase the detail and links them heuristically later. Ref. [113] links longer-term with day-ahead decisions. Ref. [114] also uses multiple stages to simulate the competitive behavior of the market players, and in [115], a sequential resolution is proposed, too.

Additionally, the efficiency of the formulations is continuously studied. The optimization of the number of constraints and variables, especially binary variables, to model the same operation is addressed in depth in order to gain accuracy without a high computational cost. Ref. [116] points out the importance of reducing the number of binary variables that are employed per thermal unit and time step. Meanwhile, ref. [117] analyzes the response of the same thermal portfolio when the optimization is addressed as a cost minimization problem and as a PBUC. On the other hand, ref. [118] studies how to increase the flexibility in the operation efficiently, especially regarding fuel consumption.

According to the apparent complications introduced by binary variables, ref. [119] proposed to work with them as relaxed variables and penalize the non-integrality in the OF. On the other hand, ref. [120] uses a formulation with only one bin and imposes constraints to achieve their integrality behavior after a more efficient resolution. This approach reduces the number of branches that are created by the resolution algorithm because of the lower binary variables defined. This philosophy is also adopted in [121], where bidding processes are considered. Despite the defense of their utilization, the results of [121] show that the conventional three-bin formulations achieve practically the same performances.

This issue was further studied in [122,123] where the tightness of a UC formulation was defined as the proximity of the relaxed solution and the integer solution. These solutions are desired to be close in order to enhance the performance of the solver. For that reason, some additional constraints are proposed to tighten the feasible region when the problem is relaxed in the iterations of the Branch & Cut. Moreover, ref. [123] concludes that the advances in commercial solvers cause the obsolescence of the formulation proposed in [120], since the solver can manage the branch creation properly and utilizes integer variables to generate cuts as well. If the integrability is not defined, the cuts will ignore these continuous variables that should be integer ones, hindering the performance of the solver. The Tight formulation proposed in [123] is quite renowned in the literature.

Tight formulations improve the performance of the solver by approximating the gap in the resolution process. However, it is also important for the tightened relaxed problem to be easy to solve. In that way, if many constraints are added, the computational cost of this step will be notably increased. Ref. [124] defines the concept of compactness according to the necessity of the lower number of variables, constraints, and non-zero elements in the problem matrix. For that reason, it is mandatory to evaluate a trade-off between Tightness and Compactness. Ref. [124] proposes a Tight and Compact (T&C) formulation where the importance of power trajectories is exposed, as it is reiterated later in [125]. In [126], the T&C formulation is extended to a thermal portfolio and, in [127], it is analyzed in depth and improved. The T&C formulation with power trajectories is exploited to achieve a proper evaluation of the reserve management in real operations of real power systems, Ref. [128] and their properties are deeply studied in [129]. T&C remains a powerful and renowned formulation nowadays.

Since that moment, UC formulations have tried to be as tight and compact as possible. For this reason, the proposition made in [130] to apply perspective functions in the UC problem was retaken. Perspective reformulations can be utilized to replace quadratic constraints by linear or second-order cone programming [131], enhancing the resolution process. The computational advances of [131] are exploited in [132] to develop an efficient tight formulation. Moreover, this technique is evolved in [133–135] towards the proposition of a two-bin formulation. Ref. [135] is a renowned formulation that projects the thermal production in continuous variables that are bounded in [0, 1], and it does not need the utilization of the shut-down binary variables.

On the other hand, the detail representation takes advantage of the computationally advanced formulations, choosing their equations as efficient ways to model technical
operations. In turn, they are complemented with aspects that are desired to be submitted to study. That is the case of emission targets. The inclusion of emission targets in the UC problem supposes its transformation to a multi-objective problem, which has to be solved through heuristics [136–138]. However, it can be addressed as an MIP problem by giving a penalty to the emissions in the OF, such as [139]. Moreover, non-convex functions that represent emissions targets can be linearized, and the problem can rely on some emission parameters that are introduced as input data [140]. In turn, modeling transmission constraints is also gaining importance in detailed models [141,142], as well as considering non-conventional energy-storage options [143].

Meanwhile, important issues for solving the UC problem are also studied, such as the presence of symmetry [144]. This phenomenon occurs when there are several identical generation units. In that case, the resolution process is slower because the solver does not cut branches that offer the same solution with a different combination of scheduled units. Symmetric-breaking constraints are added to avoid these situations [145], and their performances are compared against the developments in commercial solvers to handle with symmetry [146]. There is not a clear consensus. Apparently, current solvers can deal with symmetry properly, although the constraints seem to be useful in small size problems [147]. Furthermore, this symmetry can be exploited; Ref. [148] solves aggregated units and later decomposes the results after the optimization.

In turn, the current trends of renewable-energy-sources penetration in modern power systems point out an increment in the amount of start-up and shut-down processes accounted by thermal generators [149]. These ongoing changes highlight the necessity of extending the horizon of the UC problem [150] for considering medium-term technical constraints, such as maintenance schedule [151]. This could be achieved through clustering techniques such as [152]. The reduction of variables due to the aggregation of identical generators can be harmonized with an extension of the time span, keeping the run times assumed before the clustering. According to the research gap for enhancing the start-up costs [153], a more accurate stairwise function representation is presented in [154], and a new efficient formulation is proposed in [155]. Furthermore, more detailed generation schedules can be obtained by considering the possible turbine configurations of a power plant. With this purpose, a tight and compact formulation to represent this accurate operation is presented in [156].

The more efficient modeling techniques are translated into more accurate models with a higher scope. Ref. [157] is focused on a more detailed representation of thermal units, hydro generators, and pumped storage with a clear differentiation between turbine-production and pumping processes, as well as upper and lower reservoirs with their respective inflows. Ref. [158] models these issues and a simplified transmission network, too. The transmission representation was enhanced in [159] and later complemented with a simplified natural gas network in [160], where fluctuating natural gas flows in the pipelines toward the thermal unit that consume this fuel are considered.

Finally, it is important to highlight that new efficient formulations have been proposed during the last years. Ref. [161] is specially designed for fast-ramping thermal units. In [162], the commitment status binary variables are replaced by transition state variables, achieving great computational results. These renowned formulations [126,135,162] are compared in [163], manifesting the good performance of [126] despite being proposed a few years earlier than the others. Another comparison and new formulations are exposed in [164], pointing out the importance of this research field nowadays. Additionally, a standardized method to test formulations is employed, and an excellent explanation of the unit commitment evolution since a formulating point of view is described.

4.2. Precise Description of the Modeling Detail Adopted in the Literature

As it was manifested in [164], it is essential to establish a standardized benchmark to compare the efficiency of the unit commitment formulations. Despite traditional thermal portfolios [165] and power systems such as the IEEE 118-bus [166,167], or the IEEE RTS-
are frequently employed in the literature, it is also desirable to set other relevant issues. These concerns are different modeling details, algorithm performances, solver options, and computational resources to handle the optimization process.

The unit commitment is a widely studied optimization problem, and, as a consequence, there are several good reviews on this topic. Recent publications are [100,169–175]. There, comparisons about optimization techniques [169,170], uncertainty representation [171,172], and resolution techniques [100,173,174] are presented. However, advantages and disadvantages are usually exposed from a general perspective, as shown in Tables 1 and 2.

The literature reviews proposed in [100,173,174] gathered hardware specifications (if given), case study systems, run times, and a general explanation about the UC constraints that were employed in their references. Nevertheless, a more exhaustive analysis on modeling detail is necessary if a clear vision of the scope of the citations is required. On the other hand, an intensive description of UC modeling is made in [175], but a literature benchmark with the modeling properties of a set of different case studies and methodologies is not presented.

For that reason, bringing out a structured presentation of the modeling detail achieved through the different techniques referenced in this paper satisfies one of the most urgent requirements of the literature reviews. A general description of the technical and economic aspects represented in the unit commitment problem is made in Section 1. Although the theoretical scope of the representation is practically unlimited, reality demands a tight trade-off between modeling detail, system size, and computational resources to find an optimal or near-optimal solution. Considering that, Table 3 exposes a meticulous characterization of these features.

This summary provides the reader with a precise vision of the trade-off decisions assumed by the different authors when presenting their methodologies. An exact idea about the accuracy of the represented power system (detail, size, and horizon) and the corresponding computational tractability (run time and optimality of the solution) is given. In this way, the reader can discern between the scopes of the optimization proceedings presented in this paper, keeping in mind that it is a small collection of the most representative techniques applied to solve the unit commitment problem.

According to the information gathered in Table 3, the following clarifications are done:

- The number of segments in piecewise linearizations is specified when they are reported. If they are not given in the table, it means that quadratic coefficients were presented in the paper, and the author just said that the function was linearized.
- Hourly granularity is considered unless another specification is shown in the time span column. Additionally, time period chronology is also supposed to be respected except if a disaggregation technique is mentioned in that column.
- The symbol (r) means replication. It is shown when the number of units that compose a generation portfolio is repeated to deal with bigger systems in the case study or when the data presented for a shorter time span (typically a day) are imitated to make it longer. It entails computational conditions such as symmetry or identifiable patterns.
- The information presented in the demand constraint column denotes the elements that participate in the balance equation. If it is described as equal or greater, the unit commitment problem is addressed as a cost minimization problem where the load demand has to be matched or exceeded, respectively. In the case of PBUC, it is assumed that the maximization has no generation limits unless a (lower) term is added to show that total generation must not exceed the load demand.
- The information presented in the reserve constraints column is not always consistent. Some systems include spinning reserve in primary, secondary, or tertiary reserves. However, other systems establish different distinctions. The reserve dependence on the regulatory framework is out of the scope of this review. For that reason, reserves are shown as they are specified in each paper.
• The operative constraints column is sometimes used to show miscellaneous information, such as hydro representation, market specifications, etc., for the sake of clarity. Thus, the whole case study information is presented in the same row and table, despite space limitations.

• The optimal column determines the capability of the proposed methodology to assure finding the global optimum. In turn, when the paper reports the value of the relative optimality criterion, which is imposed on MIP solvers, it is also exposed in this column. This optimality gap is the quotient of the difference between primal and dual bound, in absolute value, and the maximum of both. This feature can be specified before solving an MIP problem. It should not be confused with the dual gap (defined when decomposition techniques are applied) or the integrality gap (calculated when the optimization ends). It clarifies the scope of the methodologies, providing a significant idea about their efficiency and the trade-off between modeling detail, run time, and computational performance.

• Executions are made in regular computing machines up to the date of the publications to whom any researcher could have access. If they are run in high efficient clusters, whose affordability is limited to generation companies or universities, it will be specified in the run time column.

• The data used in each case study are supposed to be given or properly cited. If some technical or economic aspect is modeled, but any input data are provided, it is specified with a (*). Moreover, if these data are apparently given, but the link is offline by the date that this review is sent to the publisher or the referenced article does not provide the information, it is specified with a (**).

• The generation limits of the thermal units are always considered to be given, except if the rest of the information is also missing.

• If there is at least one start-up cost represented in the methodology, the logic constraint that establishes commitments, start-ups, and shut-downs is formulated.
Table 3. Technical and economic modeling detail accomplished in the literature to represent power systems and computational performance of the methodologies.

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|---------------|--------------|--------------|-------------------|-------------------|-------------|-------------------|--------------------|----------------------|---------------------|---------------------|----------|---------|
| [2] | EE quadratic | 1 SU | - | - | yes | equal NSOE | OE | - | - | - | - | - | - | near-opt. | 10 units | 1 year through 6 time steps | - |
| [3] | ES linear | 1 SU | - | - | - | base level | mid. level | peak level | - | - | - | - | - | - | near-opt. | 26 units | 24 h | ~20 min |
| [4] | PL quadratic | 2 SU | - | - | - | equal | primary | - | yes | - | - | - | - | near-opt. | 10 units | 0.01 s | 0.06 s | <1 s |
| [5] | PL quadratic | 2 SU | - | - | - | equal | spinning | yes | yes | - | - | - | - | near-opt. | 10 units | 24 h | 2 s | 174 s |
| [6] | PL quadratic | 2 SU | - | - | - | equal | spinning | yes * | yes | - | - | - | - | near-opt. | 10 units | 0.154 s | 0.275 s | - |
| [7] | PL quadratic | 2 SU | - | - | - | equal | ND-RES EVs | spinning | - | yes | - | - | - | battery capacity limits | near-opt. | 10 units | 24 h | - |
| [8] | FL OP(CO) PSO quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | - | near-opt. | 10 units | 18.34 s | 1734.67 s | - |
| [9] | FL OP(CO) heur. alg. quadratic | 1 SU | - | - | - | PBUC | - | yes | yes | - | - | - | - | near-opt. | 5 units | 24 h | - |
| [10] | FL OP(CO) PSO quadratic | 2 SU | - | - | - | equal | dem. response | ND-RES | ND-RES-C EVs | spinning | - | yes | calculated after resolution | demand response—curtailment | near-opt. | 10 units | 24 h | - |
| [11] | NN quadratic | - | - | - | - | equal | - | - | - | - | - | - | - | near-opt. | 26 units | 24 h | 168 h | - |
| [12] | NN quadratic | 2 SU | - | - | - | equal | spinning * | yes * | yes | - | - | prohibited operating zones | - | near-opt. | 10 units | 24 h | <0.02 s | - |
| [13] | HM IS & OP(DP) exponential | - | - | - | - | PBUC lower | - | yes | - | - | - | - | - | near-opt. | 110 units | 24 h | 15 min | - |
| [14] | HM NN & OP(DP) exponential | - | - | - | - | equal | spinning | - | yes | - | - | prohibited operating zones | - | near-opt. | 7 units | 24 h | 4.5 s | - |
| [15] | HM NN & PL & OP(CO) exponential | 1 SU | - | - | - | equal | spinning | yes | yes | - | - | - | - | near-opt. | 26 units | 24 h | 2.17 s | 1.84 s | - |
## Table 3. Cont.

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|----------------|--------------|---------------|------------------|--------------------|-------------|--------------------|---------------------|---------------------|---------------------|---------|-------------|-----------|----------|
| [17] | HM PL & OP(CO) quadratic | 2 SU | - | - | equal | spinning | | | | | | | | | | 10 units | 100 units (r) | 24 h | 10 s |
| [18] | HM FL & OP(DP) SA quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [19] | HM PL & OP(CO) ACO quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | 252 s |
| [20] | HM ES & OP(CO) PSO quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | 143 s |
| [21] | HM PL & OP(CO) PSO quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | 1700 s |
| [22] | OPT(DP) heuristics quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | 906 s |
| [23] | OPT(DP) heuristics quadratic | 1 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [24] | OPT(DP) heuristics quadratic | 1 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [25] | OPT(DP) heuristics quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [26] | OPT(DP) heuristics quadratic | 1 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [27] | OPT(DP) heuristics quadratic | 2 SU | - | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [28] | OPT(DP) heuristics quadratic | 1 SU | 1 SD | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |
| [29] | OPT(DP) heuristics quadratic | 1 SU | 1 SD | - | equal | spinning | yes | yes | | | | | | | | | 10 units | 100 units (r) | 24 h | |

- **Note:** The table continues with similar entries, detailing various optimization methods and their respective parameters for energy production and operational constraints.
| Ref  | Method                  | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Optimal | Problem Size | Time Span  | CPU Time  |
|------|-------------------------|-----------------|---------------|---------------|--------------|---------------|-----------------|-------------------|-------------|-------------------|---------------------|---------|--------------|------------|----------|
| [30] | OP(DP) heuristics SO-MS | quadratic **     | 1 SU **       | 1 SD **       | -            | -             | equal **          | ND-RES **       | spinning ** | yes **           | yes **               | -       | SU/SD capacity limits ** | CCGT configuration modes | capacity limits per mode ramp rates per mode ramp rates per edge TU/TD per mode SU/SD cost per mode mode transition costs gas fuel rates for gens. pipelines—cap. limits limits gas sources—cap. limits compressors—comp. ratios | flow limits ** | shift factor ** | near-opt. | 24 buses | 38 lines | 33 units | 6 CCGTs | 2 wind gen. | 24 h by 30 min time steps | ~28.3 min | 200 scen. |
| [31] | OP(CO) SO-TSM MILP solver | linear **        | 1 SU **       | -             | -            | -             | equal **          | ND-RES TL       | yes **       | yes **            | yes **               | -       | SU/SD capacity limits ** | near-opt. | - | - | 225 buses | 375 lines | 130 units | wind gen. | 24 h | ~5.7 h | 10 scen. | 10 cores | ~7.4 h | 50 scen. | 50 cores | 177 h | 9 scen. |
| [32] | OP(CO) SO-TSM MILP solver | stepwise marginal cost function | 1 SU **       | -             | -            | -             | equal **          | ND-RES ND-RES C | non-spinning | yes *        | yes *               | -       | interval ramp capabilities * SU/SD durations * SU/SD power outputs * | yes ** | yes *       | 0.1% gap | 20 thermal | 17 hydro | 1 wind gen. | 1 year by 15 min time steps and 36 h look ahead horizons | 177 h | 9 scen. |
| [33] | OP(CO) SO-TSM MILP solver | linear **        | 1 SU **       | -             | -            | -             | equal **          | ND-RES ESS      | spinning ** | yes **       | yes **               | -       | reservoir energy limits ** | yes ** | 0.1% gap | 27 lines | 130 units | 16 PSUs | 24 h | ~37.5 min | 50 scen. | 118 bus | 186 lines | 25 coal plants | 24 h | 1.7 h | 30 scen. |
| [34] | OP(CO) SO-TSM MILP solver | marginal cost    | 1 SU **       | -             | -            | -             | equal **          | ND-RES TL       | reserve       | yes             | -                   | hourly max. gen-emiss. (2-block piecewise) | max. reserve per unit | flow limits voltage limits | yes ** | 0.0001% gap | 9 bus | 9 lines | 3 thermal | 2 wind gen. | 118 bus | 179 lines | 54 thermal | 8 hydro | 168 h | 10 min | 5.5 min | 1 scen. |
| [35] | OP(CO) SO-TSM MILP solver | linear **        | 1 SU | 1 SD | -            | -             | equal **          | ND-RES ND-RES C | spinning &2 | yes             | yes             | spinnidng &2 r. cost tertiary spin. r. cost non-spinning r. cost reservoir inflows reservoir spillage reservoir volume limits reservoir discharge limits | flow limits shift factor | yes | 240 bus | 448 lines | 85 thermal | wind gen. | 24 h | ~200 s | 50 scen. | ~800 s | 100 scen. |
| Ref  | Method          | Production | Start-up | Shut-down | Emission | Mainten. | Demand | Reserve | Ramp | TU/TD | Emission | Operative | Network | Optimal   | Problem | Time Span | CPU Time |
|------|----------------|------------|----------|-----------|----------|----------|--------|---------|------|-------|----------|-----------|---------|-----------|---------|-----------|----------|
|      |                | Cost       | Cost     | Cost      | Cost     | Cost     | Cost   | Constraints | Limits | Constraints | Constraints | Constraints | Constraints | Constraints | Size     | Span      | Time      |
| [37] | OP(CO)SO-TSMILP | linear     | 1 SU     | -         | -         | -        | equal   | HS      | TL    | spinning | yes      | yes      | -         | reservoir inflows | reservoir spillage | reservoir final volume | reservoir volume limits | reservoir discharge limits | flow limits | shift factor | yes       | 83 bus | 143 lines | 28 hydro | 24 h |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | 15 min | 4 scen. |
| [38] | OP(CO)SO-TSMILP | linear**   | 1 SU**   | -         | -         | -        | equal   | NSE     | reserve** | yes**    | yes**    | -         | flexible line rating recourse | (off, normal, high) | line min. hours normal ** | line max. hours high ** | flow limits | shift factor | yes       | 364 bus | 594 lines | 26 fast units | 24 h |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | 18 h    | 10 scen. |
| [39] | OP(CO)SO-TSMILP | piecewise** | 1 SU**   | -         | -         | -        | equal   | NSE     | ND-RES  | ND-RES-C | ESS     | ESS      | -         | expected unserved energy | ESS charge ramp rates | ESS discharge ramp rates | flow limits | shift factor | yes       | 39 bus | 46 lines | 10 units | 1 wind gen. 1 ESS | 24 h |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | 551 s   | 4 scen. |
| [40] | OP(DP)SO-MSMIP | quadratic  | 1 SU     | -         | -         | -        | greater | -       | -      | yes      | -        | -        | -         | generation company offers | self-consumption in units | reservoir energy inflows | reservoir energy limits | reservoir energy limits | flow limits | shift factor | yes       | 10 units | 20 units (r) | 20 units (r) | 48 h (r) |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | 24 h    | 2 h scen. |
| [41] | OP(DP)SO-MSMILP | linear**   | -        | -         | -         | -        | yes**   | residual demand and revenue curves 10-20 blocks ** | -       | yes**    | yes**    | -         | -         | generation company offers | self-consumption in units | reservoir energy inflows | reservoir energy limits | reservoir energy limits | flow limits | shift factor | yes       | 168 h | 24 h look ahead capacity | ~1500 s | 8 scen. |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          |         |         |
| [42] | OP(DP)SO-MSMILP | linear     | 1 SU     | -         | -         | -        | equal   | -       | -      | yes      | yes      | yes      | -         | spinning reserve costs | payments for possible being committed for a particular hour | - | yes 5% gap | 32 units | 24 h (discrete) | 24 min data continuous cubic spline | 40 min | 24 scen. |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | 67 h    | 24 scen. |
| [43] | OP(CO)SO-RCMILP | piecewise  (3 blocks) | 1 SU     | -         | -         | -        | equal   | NSE     | TL     | spinning | yes      | yes      | -         | expected energy not served load probability | unit outage probability | flow limits | reactivities | phase angles | yes       | 73 bus | 147 lines | 96 units (r) | 24 |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | <1 h    |         |
| [44] | OP(CO)SO-RCMILP | piecewise  (3 blocks) | 1 SU     | -         | -         | -        | equal   | NSE     | spinning | yes      | yes      | -         | expected energy not served load probability | unit outage scenarios | - | yes 0.5% gap | 96 units (r) | 24 |
|      |                | solver     |          |           |          |          |        |         |       |         |          |          |           |                        |                        |                        |                        |                        |            |          | <1200 s  |         |
| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|--------|----------------|---------------|----------------|---------------|---------------|-------------------|--------------------|--------------|-------------------|---------------------|---------------------|----------------------|--------|-------------|---------|--------|
| [45] | OP(CO) SO-RC MILP solver | linear | 1 SU | - | linear | - | equal NSE, ND-RES, SS-ESS | spinning | yes | yes | - | - | expected energy not served loss of load probability | yes | 0.5% gap | 71 slow units | 35 fast units | 1 PSU | 24 h by 15 min time steps | 5.5 h to 6.9 h |
| [46] | OP(CO) SO-CCO MILP solver | quadratic | 1 SU | - | - | - | equal ND-RES-ctl | spinning | non-spinning | yes | yes | - | - | reserve ramp limits | yes | - | 30 bus 41 lines 6 units 2 wind gen. | 24 h | tens of minutes |
| [47] | OP(CO) heuristics SO-CCO MILP solver | linear | 1 SU | - | - | - | equal ND-RES-ctl | spinning | yes | yes | - | - | reserve costs | yes | - | 118 bus 186 lines 15 fast units 3 wind gen. | 24 h | 432.6 s |
| [48] | OP(CO) SO-CCO MILP solver | linear | 2 SU | 1 SD | - | - | equal ND-RES-ctl | spinning | yes | yes | - | - | spinning reserve costs | yes | 0.5% gap | 118 bus 186 lines 54 units 10 wind gen. | 24 h | 1092 s 500 scen. |
| [49] | OP(CO) RO MILP solver | piecewise (3 blocks) | 2 SU | - | - | - | equal ND-RES-ctl | reserve | yes | yes | - | - | 1 EIE is composed by: Electricity generators - EFs (batch operation) - EAs (always on) | yes | - | 24 bus 40 lines 32 units (r) 1 wind gen. 1 EIE = 4 gen. units 12 EFs 1 EA | 24 h | 175.74 s |
| [50] | OP(CO) RO MILP solver | linear | 2 SU | 1 SD | - | - | equal ND-RES-ctl | spinning | yes | yes | - | - | power trajectories | warm-up duration | yes | 118 bus 186 lines 54 units 3 wind gen. | 24 h | 13.8 s |
| [51] | OP(CO) RO MILP solver | piecewise | 1 SU | - | - | - | greater | yes | yes | - | - | prohibited operating zones | yes | - | 118 bus 186 lines 54 units 3 wind gen. | 24 h | - |
| [52] | OP(CO) RO MILP solver | piecewise | 2 SU | - | - | - | equal ND-RES-ctl | spinning | operative | yes | yes | - | - | horizon max. gen-emiss. ** SU emiss. ** SD emiss. ** | yes | - | 118 bus 186 lines 54 units 3 wind gen. | 24 h | 73 s |
| [53] | OP(CO) RO MILP solver | piecewise | 1 SU | - | - | - | equal ND-RES-ctl | spinning | yes | yes | - | - | max. spin. reserve per unit | yes | - | 118 bus 186 lines 54 units 4 wind gen. | 24 h | 246.39 s |


| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|--------|----------------|---------------|----------------|---------------|---------------|------------------|-------------------|-------------|------------------|---------------------|---------------------|---------------------|---------|-------------|----------|----------|
| [54] | OP(CO) IO MILP solver | piecewise | 1 SU | - | - | - | equal ND-RES ND-RES-C TL | - | yes | yes | - | - | flow limits reactions phase angles | yes | 1% gap | 73 bus 147 lines 96 units (r) 4 wind gen. | 24 h | <2500 s |
| [55] | OP(CO) MCS MIP solver | non specified function | 1 SU | - | - | - | equal ND-RES TL | spinning operative | yes | yes | - | - | flow limits reactions phase angles | yes | 118 bus 186 lines 76 units 1 wind gen. | 24 h | <30 min 10 scen. |
| [56] | OP(CO) MCS rMIP solver | linear | 1 SU | 1 SD | linear | yes | equal NSE * OE * HS * ND-RES * ND-RES-C * ESS * imports exports | spinning * | yes * | yes * | - | - | competitive gen. companies * unplanned unavailabilities * programmed mainten. * hydro condition profiles * reservoir energy inflows * reservoir energy limits * turb. and pump. limits * pumping efficiency * cascade reservoirs * run-of-river generation * wind generation profiles * solar generation profiles * regulatory framework * | flow limits at system intercon. facilities * | yes | Spanish system 800,000 variables 350,000 const. after presolve? | 1–2 month through ≤40 time steps | <2 min (relaxed) per scen. |
| [57] | OP(CO) MCS rMIP solver | linear | 1 SU | 1 SD | linear | yes | equal NSE * OE * HS * ND-RES * ND-RES-C * ESS * imports exports | spinning * | yes * | yes * | - | - | competitive gen. companies * unplanned unavailabilities * programmed mainten. * hydro condition profiles * reservoir energy inflows * reservoir energy limits * turb. and pump. limits * pumping efficiency * cascade reservoirs * run-of-river generation * wind generation profiles * solar generation profiles * grid-stability commitments * regulatory framework * | flow limits at system intercon. facilities * | yes | Spanish system French system Portuguese system 720000 const. after presolve | 3 year through 940 time steps | ~2000 s (relaxed) per scen. |
| [58] | OP(CO) HUI SO-YS & SO-RC MILP solver | piecewise | 1 SU | - | - | - | equal NSE ND-RES ESS TL | spinning * | yes | yes | - | - | electricity price elasticity max. spin. reserve per unit ESS generation capacities ESS storage capacities loss of load probability | flow limits reactions phase angles | yes | 118 bus 186 lines 54 units 1 wind gen. 4 ESSs | 24 h | ~60 min 100 scen. |
Table 3. Cont.

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|----------------|---------------|---------------|-------------------|-------------------|-------------|-------------------|---------------------|----------------------|---------------------|---------|--------------|-----------|---------|
| [59] | OP(CO) HUI SO-TS & IO MILP solver | | piecewise (3 blocks) | 2 SU | - | - | - | equal | NSEND | ND-RES | ND-RES-C TL | - | yes | yes | - | - | - | flow limits | reactances | phase angles | yes | 0.1% gap | 24 bus | 40 lines | 32 units (r) | 1 win gen. | ∼800 s | 10 scen. hour 1 switch SO→IO, ∼15,000 s 10 scen. hour 24 switch SO→IO |
| [60] | OP(CO) HUI SO-TS & RO MILP solver | linear | 1 SU | - | - | - | equal | NSEND | ND-RES | ND-RES-C TL | - | yes | yes | - | - | - | SU/SD capacity limits ** | flow limits | reactances | phase angles | yes | 0.03% gap | 73 bus | 147 lines | 96 units (r) | 15 win gen. | 1322.7 s | 50 scen. |
| [61] | OP(DP) heuristics MILP solver | quadratic | 1 SU | - | - | - | greater | - | - | - | yes | - | - | - | - | - | near-opt. | 20 units (r) | 24 h | 137 s |
| [62] | OP(CO) heur. alg | quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | - | - | near-opt. | 29 units (r) | 24 h | ∼37 s |
| [63] | OP(CO) GA | quadratic | 2 SU | - | - | - | equal | spinning | yes * | yes | - | - | - | - | - | near-opt. | 10 units | 100 units (r) | 24 h | 3.4 s |
| [64] | OP(CO) SA | quadratic | exponential | - | - | - | equal | spinning | - | yes | - | - | - | - | - | near-opt. | 12 units | 24 h | 2 h 25 min |
| [65] | OP(CO) TSA | quadratic | 1 SU | - | - | - | greater | spinning | - | yes | - | - | bounded spinning reserve | - | near-opt. | 40 units | 24 h | 187 s |
| [66] | OP(CO) EP | quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | - | - | near-opt. | 100 units (r) | 24 h | 1 h 42 min |
| [67] | OP(CO) PSO | quadratic | 2 SU | - | - | - | equal | spinning | yes | yes | - | - | - | - | - | near-opt. | 10 units | 24 h | ∼9 s |
| [68] | OP(CO) ACO | quadratic | 2 SU | - | - | - | equal | spinning | yes * | yes | - | - | - | - | - | near-opt. | 10 units | 100 units (r) | 24 h | 39 s |
| [69] | OP(CO) ALO | quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | - | - | near-opt. | 10 units | 24 h | ∼4 min |
| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|---------------|---------------|--------------|------------------|-------------------|-------------|------------------|---------------------|---------------------|---------------------|---------|--------------|-----------|----------|
| [75] | OP(CO) ABCA | quadratic | 2 SU | - | - | - | PBUC lower | - | yes | - | - | - | near-opt. | 10 units | 100 units (r) | 1000 units (r) | 24 h | 135.23 s (1 core) | 1216.08 s (1 core) | 8456.52 s (1 core) | 14.87 s (20 cores) | 109.16 s (20 cores) | 457.24 s (20 cores) | |
| [76] | OP(CO) AFSA | quadratic | 1 SU | - | - | - | PBUC lower | spinning | yes | yes | - | - | - | near-opt. | 10 units | 100 units (r) | 24 h | ∼13.25 s | ∼664.52 s | |
| [77] | OP(CO) AISA | quadratic | 2 SU | - | - | - | PBUC lower | spinning * | yes | yes | - | - | - | near-opt. | 10 units | 24 h | 81 s | |
| [78] | OP(CO) ASA | quadratic | 2 SU | - | - | - | equal ND-RES-E | spinning | yes | - | reservoir energy limits | turb. and pump. limits | - | near-opt. | 10 units | 1 PSU | wind gen. | 1 solar gen. | 24 h | - | |
| [79] | OP(CO) BFA | quadratic | 2 SU | - | - | - | equal | spinning * | yes * | yes | - | - | - | near-opt. | 10 units | 100 units (r) | 24 h | 110 s | 5800 s | |
| [80] | OP(CO) CSA | quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | near-opt. | 4 units | 8 h | 1.91 s | |
| [81] | OP(CO) DEA | quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | near-opt. | 10 units | 100 units (r) | 24 h | 27.4 s | 663.9 s | |
| [82] | OP(CO) EMA | quadratic | - | - | - | equal | - | yes | - | prohibited operating zones | generation losses | - | near-opt. | 15 units | 1 time step | <1 s | |
| [83] | OP(CO) FFA | quadratic | 2 SU | - | - | - | equal | spinning | yes * | yes | - | - | - | near-opt. | 100 units (r) | 24 h | 113 s | |
| [84] | OP(CO) FWA | quadratic | 2 SU | - | - | - | equal PBUC lower | spinning | yes | yes | - | - | - | near-opt. | 10 units | 100 units (r) | 24 h | 61.57 s (equal) | 1039.80 s (equal) | 58.77 s (PBUC) | 3124.87 s (PBUC) | |
| [85] | OP(CO) GSA | quadratic | - | - | - | equal | - | - | - | fuel cost of valve points | (sinusoidal function) | - | near-opt. | 13 units | 1 time step | 150.32 s | |
| [86] | OP(CO) GWO | quadratic | 2 SU | - | - | - | equal ND-RES | ND-RES-C | yes | yes | - | - | - | near-opt. | 10 units | 1 wind gen. | 24 h | - | |
### Table 3. Cont.

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|----------------|---------------|---------------|------------------|-------------------|-------------|-------------------|----------------------|---------------------|---------------------|---------|-------------|-----------|----------|
| [87] | OP(CO)ICA quadratic | 2 SU | - | - | - | equal TL | - | yes | yes | horizon max. gen-emiss. (quadratic) | reactive generation limits | flow limits | voltage limits | near-opt. | 118 bus 186 lines 54 units | 24 h | 71.2 s |
| [88] | OP(CO)IHHO quadratic * | 1 SU * | - | - | - | equal * | spinning * | - | yes * | - | - | - | near-opt. | 10 units | 24 h | - |
| [89] | OP(CO)MMA quadratic | 1 SU | - | - | - | PBUC lower | spinning | - | - | - | - | - | near-opt. | 10 units | 24 h | - |
| [90] | OP(CO)QHIA quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | near-opt. | 10 units | 24 h | ~30 s |
| [91] | OP(CO)QTLBA quadratic | 2 SU | - | - | - | equal | spinning | yes * | yes | - | - | - | near-opt. | 100 units (r) | 24 h | 61.4 s |
| [92] | OP(CO)SFLA quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | near-opt. | 10 units | 24 h | - |
| [93] | OP(CO)SCLIA quadratic | 2 SU | - | - | - | PBUC lower | spinning | yes * | yes | - | - | - | near-opt. | 10 units | 24 h | 34.86 s |
| [94] | OP(CO)WIA quadratic | 2 SU | - | - | - | PBUC lower | spinning | yes * | yes | - | - | - | near-opt. | 10 units | 24 h | 37.86 s |
| [95] | OP(CO)BFA & GA quadratic | 1 SU | 2 SU | 2 SU | - | equal | spinning | yes | yes | - | - | - | near-opt. | 38 units | 459 s | - |
| [96] | OP(CO)EP & PSO quadratic | 2 SU | - | - | - | PBUC | spinning | yes | yes | - | - | - | bilateral contracts | near-opt. | 10 units (r) | 24 h |
| [97] | OP(CO)DE & SM quadratic | 1 SU | - | - | - | PBUC lower | spinning | - | yes | - | - | - | near-opt. | 10 units | 24 h | 116 s |
| [98] | OP(CO)DE & PSO quadratic | 1 SU | 1 SD | - | - | equal TL | - | yes | yes | TSO-DSO coordination 1 DSs bus ~5 generators | flow limits | reactances | phase angles | near-opt. | 118 bus 186 lines 54 units 2 DSS | 24 h | 2.53 s |
| [99] | OP(CO)SA & PSO quadratic | 2 SU | - | - | - | equal | spinning | - | yes | - | - | - | near-opt. | 10 units | 24 h | - |
| [100] | OP(CO)MILP solver piecewise (3 blocks) | 11 SU | 1 SD | - | - | PBUC | spinning | yes | yes | - | - | - | SU/SD capacity limits | - | yes | 1 unit | 24 h | slight |
Table 3. Cont.

| Ref  | Method        | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|------|---------------|-----------------|---------------|----------------|---------------|---------------|------------------|---------------------|-------------|-----------------|----------------------|---------------------|---------------------|---------|-------------|-----------|----------|
| [103] | OP(CO) MIP solver piecewise *(3 blocks) | 1 SU | 1 SD | - | - | equal HS imports exports | spinning * | - | yes | - | spinning reserve bounds | reservoir inflows | cascade reservoirs | reservoir volume limits | reservoir discharge limits | HR function (piecewise) | yes | reservoir SU/SD costs | reservoir TU/TD import/export limits | import loss coefficient | export loss coefficient | system loss coefficient |
| [104] | OP(CO) MILP solver piecewise (10 blocks) | 11 SU | 1 SD | - | - | PBUC | 10 min spinning | 10 min non-spin. | yes | - | - | power trajectories | SU/SD capacity limits | AGC representation | - | yes | 1 unit | 24 h | 51.51 s |
| [105] | OP(CO) MILP solver linear | 1 SU | 1 SD | - | - | PBUC | - | yes | yes | - | power trajectories | SU/SD durations | SU/SD power trajectories | - | yes | 1 unit | 24 h | 0.66 s |
| [107] | OP(DP) SO-MS MILP solver piecewise * stairwise * | - | - | - | greater ESS | spinning * | - | yes * | - | reservoir energy limits | turb. and pump. limits | pumping efficiency | - | yes | 25 thermal | 192 h | 97.61 s |
| [109] | OP(CO) MILP solver piecewise (4 blocks) | 1 SU | - | - | - | residual demand and price-quotacurves | - | yes | yes | - | - | flow limits | reactances | phase angles | near-opt. | 8 bus | 11 lines | 6 units | 24 h | - |
| [110] | OP(CO) heur. alg. quadratic | - | - | - | piecewise supply curve | TL | - | - | - | - | - | - | - | - | near-opt. | 10 units | 24 h | 19 s |
| [112] | OP(CO) heuristics MILP solver linear * | 2 SU * | 1 SD * | - | - | equal * NSE HS ESS | spinning * | yes * | - | - | reservoir inflows | reservoir spillage | reservoir final volume | reservoir volume limits | reservoir discharge limits | HR func. (3-block piecewise) | pumping capacity limits | pumping efficiency | - | near-opt. | 44 thermal | 102 hydro | 6 PSUs | 168 h | 180 s | ~40 min |
Table 3. Cont.

| Ref  | Method          | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|------|-----------------|-----------------|----------------|----------------|---------------|---------------|-------------------|--------------------|-------------|-------------------|---------------------|----------------------|-----------------------|---------|-------------|-----------|----------|
| [113] | OP(CO) MILP solver | linear *          | -              | -              | -              | -              | equal *            | NSE                | -           | -                 | dispatch-deviation penalties | reserve-deviation penalties | non-smooth power profile penalties | AGC representation | -          | yes          | 20 thermal 6 hydro 5 PSUs | 28 h | 20 min time steps | <10 s |
| [114] | OP(CO) heur. alg. | quadratic        | 1 SU           | -              | -              | -              | PBUC lower        | spinning            | -           | -                 | -                   | dispatch-deviation penalties | reserve-deviation penalties | non-smooth power profile penalties | - | yes          | 10 units 100 units (r) | 24 h | 6 s | 27 s |
| [115] | OP(CO) heuristics MILP solver | piecewise (10 blocks) | 1 SU           | -              | -              | -              | equal            | spinning           | yes         | yes               | -                   | -                   | near-opt. | 54 units | 24 h | 6.57 s |
| [116] | OP(CO) MILP solver | piecewise        | 1 SU           | -              | -              | -              | PBUC              | non-spinning       | yes         | yes               | bilateral contracts | CCGT configuration modes | mode transition fuel | - | yes          | 54 thermal 12 CCGTs | 24 h | ~90 s |
| [117] | OP(CO) MILP solver | piecewise *      | 1 SU *         | 1 SD *         | -              | -              | PBUC *            | equal *            | spinning      | yes               | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| [118] | OP(OF) heur. alg. | quadratic        | 1 SU           | -              | -              | -              | ESS TL            | spinning operative | yes         | yes               | horizon max. ho. unit max. gen-emiss. (linear) SU emiss. (1 step) | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| [119] | OP(CO) heuristics rMIP solver | quadratic *   | 1 SU *         | -              | -              | -              | equal HSE ESS imports | operative         | yes         | -                 | -                   | reservoior energy inflows | reservoior energy limits | reservoior energy splits turb. and pump. limits | pumping efficiency import facilities max. LNG consumption | - | near-opt. |
| [120] | OP(CO) MILP solver | piecewise (4 blocks) | 2 SU           | 1               | -              | -              | equal            | spinning           | yes         | yes               | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| [121] | OP(CO) MILP solver | piecewise (3 blocks) | 10 SU          | 1 SD           | -              | -              | PBUC              | yes                | yes         | yes               | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| [122] | OP(CO) MILP solver | piecewise       | -              | -              | -              | -              | PBUC              | yes                | yes         | yes               | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| [123] | OP(CO) MILP solver | piecewise (2 blocks) | 2 SU           | -              | -              | -              | greater           | spinning           | yes         | yes               | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| [124] | OP(CO) MILP solver | linear           | 5 SU           | 1 SD           | -              | -              | PBUC              | yes                | yes         | yes               | reactive generation limits | max. sustained ramp rate | CCGT configuration modes | TU per configuration fuel blending operation fuel switching operation unit fuel consump. limits | horizon fuel consump. limits | flow limits | voltage limits | near-opt. |
| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|---------|-----------------|--------------|---------------|---------------|---------------|-----------------|-------------------|-------------|-------------------|-------------------|---------------------|---------------------|--------|-------------|----------|----------|
| [125] | OP(CO) MILP solver linear | 2 SU | 1 SD | - | - | equal **NSE 118 bus | reserve yes yes | power trajectories SU/SD durations | flow limits shift factor | yes | 0.05% gap | 118 bus | 186 lines | 54 slow units | 10 fast units | 3 wind gen. | 24 h by 5 min time steps | <2 h 20 scen. |
| [126] | OP(CO) MILP solver linear | 2 SU | 1 SD | - | - | equal **NSEND-RES | reserve yes yes | - | SU/SD capacity limits | - | yes | 0.001% gap | 280–1870 units (r) | 28–187 units (r) | 24 h | 1% gap or 10 h |
| [127] | OP(CO) MILP solver linear | 1 SU | - | - | - | equal **NSE-RES-CTL | reserve yes yes | - | SU/SD capacity limits | - | yes | 0.001% gap | 10 units | 24 h | 9.1 s | 267.23 s |
| [128] | OP(CO) MILP solver linear | 2 SU | 1 SD | - | - | equal **NSE-RES-CTL | reserve yes yes | - | SU/SD capacity limits | - | yes | 0.001% gap | 10 units | 24 h | 9.1 s | 267.23 s |
| [130] | OP(CO) MILP solver piecewise | 1 SU | - | - | - | equal **H5 | reserve yes yes | - | reservoir inflows | reservoir spillage | cascade reservoirs | reservoir volume limits | reservoir discharge limits | yes | 0.5% gap | 200 thermal (r) | 100 hydro (r) | 24 h | 314 s |
| [131] | OP(CO) MISOCP solver quadratic | 2 SU ** | - | - | equal **reserve ** | reserve yes yes | - | SU/SD capacity limits | - | yes | 0.01% gap | 200 units (r) | 50 units (r) | 24 h | 1243 s | 4877 s |
| [132] | OP(CO) heuristics rMISOCP solver quadratic | 2 SU | - | - | - | equal **spinning | reserve yes yes | - | SU/SD capacity limits | - | near-opt. | 10 units | 100 units (r) | 24 h | 1.58 s | 29.09 s |
| [133] | OP(CO) MILP solver quadratic | 2 SU ** | - | - | - | equal **spinning | reserve yes yes | - | SU/SD capacity limits | - | yes | 0.1% gap | 10 units | 200 units (r) | 24 h | 4.25 s | 104.87 s |
| [134] | OP(CO) heuristics rMISOCP solver quadratic | 2 SU ** | - | - | - | equal **spinning | reserve yes yes | - | SU/SD capacity limits | - | near-opt. | 10 units | 200 units (r) | 24 h | 1.05 s | 21.33 s |
| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Optimal Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|---------------|--------------|--------------|------------------|-------------------|-------------|-----------------|------------------|-----------|----------|
| [135] | OP(CO) MILP solver | quadratic | 2 SU | - | - | - | equal | spinning | yes | yes | - | SU/SD capacity limits | - | 10 units | 24 h | 0.21 s |
| | | | | | | | | | | | | 0.5% gap | 19.28 s | |
| | | | | | | | | | | | | 0.1% gap | 51.37 s | |
| | | | | | | | | | | | | 0.1% gap | 3775.95 s | |

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Optimal Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|---------------|--------------|--------------|------------------|-------------------|-------------|-----------------|------------------|-----------|----------|
| [136] | OP(CO) Pareto optimz. | quadratic | 1 SU | - | - | - | PBUC lower | - | yes | yes | horizon max. prod-emiss (quadratic & exponential) | trade-off curve (201 runs) | - | near-opt. | 6 units | 168 h | 0.05 s |
| | | | | | | | | | | | | per run | |
| [137] | OP(CO) Pareto optimz. | quadratic | 2 SU | - | - | - | PBUC lower | - | yes | yes | horizon max. prod-emiss (quadratic & exponential) | trade-off curve (201 runs) | - | near-opt. | 15 units | 168 h | 0.12 s |
| | | | | | | | | | | | | per run | |
| [138] | OP(CO) heur. alg. | quadratic | 2 SU | - | quadratic | - | equal TL | spinning | yes | yes | - | flow limits reactions phase angles | near-opt. | 118 bus | 186 lines | 24 h | 208 s |
| | | | | | | | | | | | | bilateral contracts | |
| | | | | | | | | | | | | SU/SD capacity limits | |
| | | | | | | | | | | | | max. sustained ramp rate | |
| | | | | | | | | | | | | reservoir inflows | |
| | | | | | | | | | | | | reservoir spillage | |
| | | | | | | | | | | | | cascade reservoirs | |
| | | | | | | | | | | | | reservoir volume limits | |
| | | | | | | | | | | | | reservoir discharge limits | |
| | | | | | | | | | | | | HR function (3-block piecewise) | |
| | | | | | | | | | | | | reservoir SU/SD costs | |
| | | | | | | | | | | | | reservoir TU/TD | |
| | | | | | | | | | | | | - | near-opt. | 13 thermal | 24 h | reasonab. |
| | | | | | | | | | | | | 5 hydro | |
| [139] | OP(CO) heuristics MILP solver | piecewise (5 blocks) | 1 SU | 1 SD | linear | - | PBUC HS | spinning | yes | yes | - | bilateral contracts | |
| | | | | | | | | | | | | SU/SD capacity limits | |
| | | | | | | | | | | | | max. sustained ramp rate | |
| | | | | | | | | | | | | reservoir inflows | |
| | | | | | | | | | | | | reservoir spillage | |
| | | | | | | | | | | | | cascade reservoirs | |
| | | | | | | | | | | | | reservoir volume limits | |
| | | | | | | | | | | | | reservoir discharge limits | |
| | | | | | | | | | | | | HR function (piecewise) | |
| | | | | | | | | | | | | reservoir SU cost | |
| | | | | | | | | | | | | reservoir TU/TD | |
| | | | | | | | | | | | | flow limits | |
| | | | | | | | | | | | | reactions phase angles | |
| | | | | | | | | | | | | - | yes | 20 units (r) | 24 h | 9.06 s |
| | | | | | | | | | | | | yes | 652.78 s | |
| [140] | OP(CO) MILP solver | piecewise (5 blocks) | 1 SU | - | piecewise | - | PBUC lower | - | yes | yes | - | yes | 20 units (r) | 24 h | - |
| | | | | | | | | | | | | - | yes | 652.78 s | |

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Optimal Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|---------------|--------------|--------------|------------------|-------------------|-------------|-----------------|------------------|-----------|----------|
| [141] | OP(CO) MILP solver | piecewise | 1 SU | - | - | - | equal HS TL | spinning * | yes | yes | - | prohibited operating zones | |
| | | | | | | | | | | | | max. sustained ramp rate | |
| | | | | | | | | | | | | reservoir inflows | |
| | | | | | | | | | | | | reservoir spillage | |
| | | | | | | | | | | | | cascade reservoirs | |
| | | | | | | | | | | | | reservoir volume limits | |
| | | | | | | | | | | | | reservoir discharge limits | |
| | | | | | | | | | | | | HR function (piecewise) | |
| | | | | | | | | | | | | reservoir SU cost | |
| | | | | | | | | | | | | reservoir TU/TD | |
| | | | | | | | | | | | | yes | 12 thermal | 24 h | - |
### Table 3. Cont.

| Ref | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal Problem Size | Time Span | CPU Time |
|-----|--------|-----------------|---------------|---------------|--------------|--------------|------------------|-------------------|-------------|------------------|---------------------|---------------------|---------------------|---------------------|-----------|----------|
| [142] | OP(CO) MIQCP solver | quadratic * | 2 SU * | - | - | - | equal * NSE TL | spinning * | yes * | yes * | - | fuel consumption limits * | gas-consumption limits * | load losses limits * | flow limits * | shift factor * | yes | 194 thermal 9 CCGTs 1 HCP 4 IPPs 2 gas sector Mexican transmission network | 24 h | 15.12 min |
| [143] | OP(CO) MILP solver | piecewise | 1 SU | - | - | - | equal ND-RES ND-RES-C ESS TL | spinning non-spinning | yes | yes | horizon max. gen-emiss. (piecewise) | fuel consumption limits | reactive generation limits | CAES volume limits | CAES inject limits | CAES release limits | inject efficiency | release efficiency | flow limits | voltage limits | yes | 118 bus 186 lines 54 units 1 CAES 3 wind gen. | 24 h | - |
| [144] | OP(CO) MILP solver | quadratic | - | - | - | - | equal | NSE ES-RES | - | - | - | branching symmetry exploiting | - | - | - | yes | 0.2% gap | 118 bus 186 lines (9 parallel) 54 units | 4 h | 875 s |
| [145] | OP(CO) MILP solver | piecewise | 2 SU | - | - | - | equal | spinning | yes | yes | - | symmetry breaking constraints | - | - | - | yes | 0.0% gap | 100 units (r) | 24 h | 115.64 s |
| [146] | OP(CO) MILP solver | piecewise (2 blocks) | 2 SU | - | - | - | equal | spinning | yes | yes | - | SI/SD capacity limits | - | - | - | yes | 46-72 units (r) | 24 h | ~73.09 s |
| [147] | OP(CO) MILP solver | piecewise | 2 SU | - | - | - | equal | spinning | yes * | yes | - | symmetry breaking constraints | SI/SD capacity limits * | - | - | yes | <0.03% gap | 100 units (r) | 24 h | ~7200 s |
| [148] | OP(CO) MILP solver | piecewise (2 blocks) | 2 SU | - | - | - | equal | spinning | yes | yes | - | SI/SD capacity limits | - | - | - | yes | 0.01% gap | ≤8 agg. units (28-187 units r) | 24 h | ~38.03 s |
| [149] | OP(CO) MILP solver | piecewise (3 blocks) | 3 SU | 1 SD | - | - | equal | NSE ND-RES | secondary | tertiary spinning | tertiary non-spin. | yes | yes | calculated after resolution | AGC representation | ramp rates under AGC | - | yes | 45 units (r) 1 wind gen. 1 solar gen. | 336 h daily runs with 38 h look ahead capacity | ~1 h |
| [151] | OP(CO) MILP solver | linear | 1 SU | - | linear | yes | equal | NSE ND-RES ND-RES-C | - | yes | yes | - | must run units | mayor overhaul costs | relation with increases of SU in slow&fast units | - | yes | 0.5% gap | 301 units (r) | 336 h | - |
| [152] | OP(CO) MILP solver | linear | 1 SU | - | - | - | equal | NSE ND-RES ND-RES-C TL | secondary | yes | yes | - | SI/SD capacity limits | flow limits | shift factor | yes | 0.01% gap | 118 bus 186 lines 54 agg. units (540 units r) 3 wind gen. | 24 h | 406 s |
Table 3. Cont.

| Ref | Method          | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Operative Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time |
|-----|----------------|----------------|---------------|----------------|---------------|---------------|------------------|--------------------|-------------|-------------------|----------------------|---------------------|--------------------|---------|--------------|----------|----------|
| [155] | OP(CO) MILP solver piecewise (1 to 10 blocks) | \( \leq 2 \text{ SU} \) | - | - | - | equal ND-RES-ND-RES-C | spinning | yes | yes | - | SU/SD capacity limits | - | yes | 0.1% gap | PJM Inteconnection | 48 h | 136 s | 2% wind |
| | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| [156] | OP(CO) MILP solver linear ** | 1 SU ** | - | - | - | - | spinning ** | yes ** | yes ** | - | - | CCGT configuration modes ** capacity limits per mode ** ramp rates per mode ** transition ramp rates ** mode transition costs ** prod. costs per mode ** SU cost per mode ** TU/TD per mode ** | - | yes | 0.1% gap | 4 units (r) | 10 units (r) | 24 h | 2.4 s | 4.8 s | 124.9 s |
| | | | | | | | | | | | | | | | | |
| [157] | OP(CO) MILP solver piecewise ** | 1 SU ** | - | - | - | equal ESS | - | - | - | - | reservoir inflows cascade reservoirs reservoir volume limits reservoir discharge limits HR function (s-block piecewise) pumping capacity limits pumping efficiency | - | yes | 0.5% gap | 198 thermal | 72 hydro | 168 h | <10 s | 1 HR 3 blocks | 1080.4 s | 3 HR 6 blocks |
| [158] | OP(CO) MILP solver piecewise | 2 SU | - | - | - | greater ESS TL | spinning | yes | yes | - | reservoir inflows cascade reservoirs reservoir volume limits reservoir discharge limits HR function (piecewise) pumping capacity limits pumping efficiency | - | yes | 0.0% gap | 31 bus | 43 lines | 24 h | 116.9 s | |
| [159] | OP(CO) MILP solver linear | 2 SU | - | - | - | greater TL | spinning | yes | yes | - | SU/SD capacity limits * flow limits reactances phase angles | yes | 0.0% gap | 31 bus | 43 lines | 24 h | 104.5 s | 37.2 s |
| Ref  | Method | Production Cost | Start-up Cost | Shut-down Cost | Emission Cost | Mainten. Cost | Demand Constraint | Reserve Constraints | Ramp Limits | TU/TD Constraints | Emission Constraints | Network Constraints | Optimal | Problem Size | Time Span | CPU Time  |
|------|--------|-----------------|---------------|----------------|--------------|--------------|------------------|--------------------|-------------|-------------------|---------------------|---------------------|---------|--------------|-----------|-----------|
| [160] | OP(CO)MILP solver linear | 2 SU | - | - | - | - | greater ESS TL | - | yes | yes | - | reservoir inflows | - | yes | yes | 31 bus | 43 lines | 13 no-gas units | 3 gas units | 7 gas nodes | 5 gas loads | 5 pipelines | 1 compressor | 2 supply nodes | 2 PSUs | 24 h | 480.5 s |
| [161] | OP(CO)MILP solver piecewise (2 blocks) | 1 SU | 1 SD | - | - | equal TL | - | yes | yes | - | flow limits | reactances | phase angles | - | yes | 0.01% gap | 118 bus | 186 lines | 54 units | 24 h | ~296 s |
| [162] | OP(CO)MILP solver piecewise (1 to 10 blocks) | ≤2 SU | - | - | - | - | equal spinning | - | yes | yes | - | - | flow limits | reactances | phase angles | - | yes | 0.01% gap | 280–1870 units (r) | 28–187 units (r) | ∼367.8 s | ∼34.3 s | ~664.7 s |
| [163] | OP(CO)MILP solver piecewise (1 to 10 blocks) | ≤2 SU | - | - | - | - | equal NSE ND-RES ND-RES-C TL | - | yes | yes | - | SU/SD capacity limits | - | yes | 0.1% gap | 800 units (r) | 24 h | ~285 s |

* The constraint is formulated, but the corresponding input data for the case study are not provided. ** The constraint is formulated, but the corresponding input data for the case study are missing due to an offline link or the lack of information in a reference.
5. Conclusions

This article presents a literature review on the different available alternatives to set out and solve the unit commitment problem. One of the main goals is establishing a hierarchical classification of the computational techniques that could be applied to each stage of the problem. This proposal differs from other surveys, which mix identification principles and describe several modeling techniques, resolution algorithms, etc., without any more profound distinction.

The exposed layout allows to quickly acquire a general idea of the different options to represent and optimize the operation of power systems. Furthermore, their advantages and disadvantages are elaborately described, allowing an easy recognition of the research gaps for further developments and introducing a solid basis to implement new improvements.

Some examples of research necessities in this field are enhancing the computational efficiency when handling uncertainty; improving modeling detail, such as a more precise characterization of the start-up processes, or going in-depth with the operational flexibility representation; extending the unit commitment horizon to facilitate coordination with markedly medium-term tasks, such as hydro management, fuel purchases, or financial contracting; representing the real demand variability for thermal generators in current and future electricity markets with high penetration of non-dispatchable energy sources; or studying the most efficient procedure for achieving widespread integration of non-conventional energy-storage technologies in modern power systems.

In order to not provide an immeasurable amount of references, the most popular and recent unit commitment approaches have been cited throughout the different sections. To conclude this review, the historical events and ongoing trends in the most widespread technique applied to the unit commitment problem are introduced. Hence, the current state is presented, focusing on the thermal generation of electricity markets which is optimized through commercial solvers. The reader is referred to recent surveys if interested in obtaining a deeper knowledge of specific problems, such as the hydro unit commitment problem, the operational representation of energy-storage technologies, and evolutionary optimization.

Moreover, the scope of the different methodologies described in the paper is clearly identified through a comparison table, where the modeling detail adopted in the references is analyzed in depth. In turn, the problem sizes are also precisely presented, providing the elements that compound the power systems represented in the case studies. Furthermore, the computational performance of each proceeding can be discerned, too, according to the corresponding run times and optimality of the solutions.

This characterization entails a significant development against the conventional reviews on the unit commitment problem, which only offer a broad vision of the modeling scope of their cites at most. Additionally, this accurate description can also be used as a benchmark to look for different options to model a specific technical or economic aspect, choosing the formulation that apparently offers a greater computational efficiency according to the desired methodology.

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Abbreviations

The following abbreviations are used in this manuscript:

ACO  Ant Colony Optimization
AGC  Automatic Generation Control
ALO  Ant Lion Optimizer
ABCA Artificial Bee Colony Algorithm
AFSA Artificial Fish Swarm Algorithm
AISA Artificial Immune System Algorithm
ASA  Artificial Sheep Algorithm
BD   Benders Decomposition
BFA  Bacterial Foraging Algorithm
CAES Compressed Air Energy Storage
CCGT Combined Cycle Gas Turbine
CO   Conventional Optimization
CCO  Chance Constrained Optimization
CSA  Cuckoo Search Algorithm
DEA  Differential Evolution Algorithm
DS   Distribution System
DSO  Distribution System Operator
DP   Dynamic Programming
DWD  Dantzig-Wolfe Decomposition
EA   Electrolytic Aluminium series
EE   Exhaustive Enumeration
EF   Electrolytic arc Furnace
EIE  Energy Intensive Enterprise
EMA  Exchange Market Algorithm
EO   Evolutionary Optimization
EP   Evolutionary Programming
ES   Expert System
ESS  Energy Storage System
EV   Electric Vehicle
FBU  Fuel Blending Unit
FFA  Firefly Algorithm
FL   Fuzzy Logic
FSU  Fuel Switching Unit
FWA  Fireworks Algorithm
GA   Genetic Algorithm
GSA  Gravitational Search Algorithm
GWO  Grey Wolf Optimizer
HCCP Hybrid Combined Cycle Plants
HM   Hybrid Methodologies
HS   Hydro Spillage
HR   Head Range
HUI  Hybrid Uncertainty Implementation
ICA  Imperialist Competitive Algorithm
IHHO Intensify Harris Hawks Optimizer
IO   Interval Optimization
IP   Integer Programming
IPP  Independent Power Producer
LNG  Liquefied Natural Gas
LP   Linear Programming
LR   Lagrangian Relaxation
MCS  Monte Carlo Simulation
MILP Mixed Integer Linear Programming
MIQP Mixed Integer Quadratic Programming
MIQCP Mixed Integer Quadratically Constrained Programming
MINLP Mixed Integer Non Linear Programming
Appendix A. Formulating Options

- Integer Programming (IP): The formulation employs only discrete variables. A discrete variable, that is commonly called an integer variable, accomplishes an integer value after the problem resolution \((x_I \in \mathbb{Z})\). According to the numerical methods utilized by computers to solve the problem, the integrality of these variables is related to a tolerance. Formulating exclusive-integer problems is not common. The habitual practice is to utilize continuous variables as well. For that reason, the integer programming term is usually employed as a reference to the usage of integer variables in the unit commitment problem and energy models.
• Mixed Integer Programming (MIP): The formulation uses a mix of integer and continuous variables ($x_C \in \mathbb{R}$). The MIP has several subgroups attending to the mathematical formulation of the OF and constraints. If the OF and all the constraints are linear, the problem is called MILP. If the constraints are linear but the OF is quadratic, it is called MIQP. On the other hand, if there are some quadratic constraints and a linear OF, the problem is an MIQCP. This quadratic constraints can be represented as second-order cones in a MISOCP problem. Finally, if there is any non-linear constraint or OF, that will be an MINLP problem.

• Linear Programming (LP): The formulation only employs continuous variables. In turn, both their objective function and constraints are linear. LP can be a result of relaxing the discrete variables of a MIP. When the integer variables of MILP are relaxed (considered as continuous in order to facilitate the resolution of the problem), the new problem is known as an rMILP and there is not any difference with an LP.

• Non-Linear Programming (NLP): The formulation utilizes continuous variables; meanwhile, the objective function or some constraint is a non-linear function. As well as in LP, the relaxation of an MINLP turns out into a problem which it is solved as an NLP, an rMINLP. Quadratic Programming (QP), Quadratically Constrained Programming (QCP), and Second-Order Cone Programming (SOCP) are non-linear techniques. Nevertheless, they are not frequently included in this group in the literature because their convexity properties allow them to be solved easier than NLP used to be. Relaxing MIQP, MIQCP, or MISOCP converts the problem in an rMIQP, rMIQCP, or rMISOCP that is also solved such as a QP, QCP, or SOCP problem.

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