Soliton, breather-like and dark-soliton-breather-like solutions for the coupled long-wave–short-wave system

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Received: 17 December 2021 / Accepted: 4 January 2022 / Published online: 20 January 2022
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Abstract In this paper, we will obtain the exact $N$-soliton solution of the coupled long-wave–short-wave system via the developed Hirota bilinear method. Through manipulating the relevant parameters, we will construct different types of solutions which include breather-like solutions and dark-soliton-breather-like solutions. Moreover, we will demonstrate that the interactions of two-soliton and two-breather-like solutions are all elastic through asymptotic analysis method. Finally, we will display the interactions through illustrations.

Keywords Coupled long-wave–short-wave system · $N$-soliton solution · Breather-like solutions · Dark-soliton-breather-like solutions · Hirota bilinear method · Asymptotic analysis method

1 Introduction

It is well known that Davey–Stewartson system is a classical $(2+1)$-dimensional model that describes weak nonlinear water waves, namely

$$iA_t + \frac{1}{2} \omega''(k) A_{zz} + \frac{c_g}{2k} A_{yy} = \gamma_1 |A|^2 A + \gamma_2 A \Phi_z,$$  \hspace{1cm} (1.1a)

and

$$\left(gh - \frac{c_g^2}{g}\right) \Phi_{zz} + gh \Phi_{yy} = -\gamma_3 |A|^2 \Phi_z,$$  \hspace{1cm} (1.1b)

where $\Phi$ is the amplitude of the long wave, $A$ is the complex amplitude of the short wave, $g$ is the acceleration of gravity, $h$ is the mean depth of the water, $\omega$, $k$ and $c_g$ denote the frequency, wavenumber and group velocity of the weak nonlinear water waves, respectively, $\gamma_1$, $\gamma_2$ and $\gamma_3$ are the variables related to $\omega$, $k$ and $c_g$ [1–4]. Early literature studies have supported the result that the two equations can be converted into a single equation by comparing Eq. (1.1) and the nonlinear Schrödinger equation [3–7]. The forms of Eq. (1.1) are completely different from that of the nonlinear Schrödinger equation, but it can be written in another way which makes the connection obvious [3,8]. So the equivalent forms for Eq. (1.1) can be given as follows:

$$iA_t + \frac{1}{2} \omega''(k) A_{zz} + \frac{c_g}{2k} A_{yy} = \nu |A|^2 A + \nu_1 A Q,$$  \hspace{1cm} (1.2a)

and

$$\left(gh - c_g^2 \right) Q_{zz} + gh Q_{yy} = \kappa \left(|A|^2 \right)_{yy}.$$  \hspace{1cm} (1.2b)

where $Q$ is the amplitude of the long wave, $\nu$, $\nu_1$ and $\kappa$ are the variables related to $\omega$, $k$ and $c_g$, the other variables are the same as those in Eq. (1.1). On the one hand, Eq. (1.2) can be reduced to the nonlinear Schrödinger equation in the case where $A$ is independent of $y$ and $Q = 0$. On the other hand, it should be noted that Eq. (1.2) are invalid for the case ($\nu = 0$) of long-wave resonates with short-wave ones [2,3].
when the more generic nonlinear Schrödinger equation breaks down due to a singularity of the coefficient of the cubic nonlinear term, Ref. [4] derives the scalar long-wave–short-wave resonance equations describing this resonance through a different analysis and scaling, the dispersion of the short wave is balanced by the nonlinear interaction of the short wave and the long wave, while the evolution of the long wave is driven by the self-interaction of the short wave. The scalar long-wave–short-wave resonance equations can be given as follows:

\[ i A_t + \frac{1}{2} \omega'' (k) A_{zz} = BA, \quad (1.3a) \]

\[ B_t = -\alpha \left( |A|^2 \right)_z, \quad (1.3b) \]

where \( B \) denotes the amplitude of the long longitudinal wave, \( A \) denotes the complex amplitude of the short transverse wave, \( \alpha \) is a variable related to \( \omega, k \) and \( c_g \), the definitions of \( \omega, k \) and \( c_g \) are the same as those in Eq. (1.1), and \( t \) and \( z \) are time and space coordinates, respectively [9, 10]. Equation (1.3) cannot describe the resonance interaction in the deep-water limit, and its traveling wave solutions are unstable [4]. A general theory that reveals various phenomena related to physical properties such as resonances, instabilities and steady-state solutions in interactions between short waves and long waves has been analyzed [11]. Hence, we consider the coupled long-wave–short-wave system which generalizes the scalar long-wave–short-wave resonance equations. It has been given as follows:

\[
\begin{align*}
  i p_t - 2 p_{zz} + 2 p (\eta - \omega) &= 0, \\
  i q_t - 2 q_{zz} + 2 q (\eta - \omega) &= 0, \\
  \eta_t - 4 (pq + q\bar{p})_z &= 0.
\end{align*}
\]

(1.4)

This system consists of two short-wave components and one long-wave component, in which the group velocity of the transverse wave represented by two short-wave components resonates with the phase velocity of the longitudinal wave represented by a long wave. Besides, \( p \) and \( q \) are complex functions representing two short-wave components, \( \eta \) is a real function representing the long wave, \( \omega \) is an arbitrary real constant, \( \bar{p} \) and \( \bar{q} \) are complex conjugations of \( p \) and \( q \), respectively [12–14]. System (1.4) is Lax integrable and can be extensively applied in various aspects such as capillary–gravity waves, internal surface waves, and short and long gravity waves on fluids of finite depth [11, 12].

To our knowledge, many different types of solutions of System (1.4) have been obtained. For example, inclined periodic breather solutions, Ma breather solutions, Akhmediev breather solutions, rogue wave solutions and rational homoclinic solutions of System (1.4) have been obtained by Hirota two-soliton method [13, 14]. In particular, the traits of various breather solutions have been discussed which may provide us with useful information on the dynamics of the relevant physical fields. Moreover, a Bäcklund transformation for plane-wave solutions with linear instability of System (1.4) has been obtained using the dressing method [12]. The explicit expression of the periodic orbit on the homoclinic manifold of a torus of plane waves has been constructed by evaluating the Bäcklund transformation.

However, there are still some solutions that have not been discussed for System (1.4). The purpose of this paper is mainly to generate bright soliton, dark soliton, breather-like and dark-soliton-breather-like solutions via the developed Hirota bilinear method [15, 16] and discuss the dynamic behaviors of two-soliton and two-breather-like solutions. Hirota bilinear method is an important tool for solving the exact solutions of nonlinear evolution equations. Once the bilinear equations of System (1.4) are obtained through certain transformation, a series of exact solutions can be obtained regularly by using the truncated parameter expansions at different levels. The developed Hirota bilinear method can be used to obtain new solutions with breathing behaviors [17–22]. Asymptotic analysis method can be used to study the dynamic behaviors of solitons before and after the interactions, and related physical properties of solitons can be derived which may provide us with useful information on the dynamics of the relevant physical fields [23–25]. Moreover, the amplitudes, phases and stability of solitons are important for industrial informatics and many more applications [26–30].

The structure of this paper is designed as follows. We will obtain the exact \( N \)-soliton solution of the coupled long-wave–short-wave system by using the developed Hirota bilinear method in Sect. 2. The breather-like solutions are discussed in Sect. 3, and mixed (dark-soliton-breather-like) solutions are discussed in Sect. 4. The dynamic behaviors of two-soliton and two-breather-like solutions are illustrated in Sect. 5. The conclusions are summarized in Sect. 6.
2 Soliton solutions for the coupled long-wave–short-wave system

2.1 Developed Hirota bilinear method

Firstly, we introduce the relevant variable transformation \([31, 32]\)

\[
p = \frac{G}{F},
q = \frac{H}{F},
\]

\[
\eta = b_0 - 2 (\ln F)_{zz}.
\]

Substituting Eq. (2.1) into System (1.4), the coupled bilinear differential equations of System (1.4) can be obtained as

\[
\begin{aligned}
\left\{ \begin{array}{ll}
[i D_t - 2 D_z^2 + 2 (b_0 - \omega)] G \cdot F = 0, \\
[i D_t - 2 D_z^2 + 2 (b_0 - \omega)] H \cdot F = 0, \\
(D_t D_z - \lambda) F \cdot F + 4 (GH + \bar{G}H) = 0,
\end{array} \right.
\end{aligned}
\]

where \(G\) and \(H\) are complex-valued functions, \(F\) is a real-valued function, \(b_0\) is a constant, \(\lambda\) is an integral constant, and \(\bar{G}\) and \(\bar{H}\) are complex conjugates of \(G\) and \(H\), respectively. The \(D\)-operator is defined by

\[
D^n F (z, t) G (z, t) = \left( \frac{\partial}{\partial t} - \frac{\partial}{\partial z} \right)^n \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} \right)^m F (z, t) G (z, t) \bigg| _{z = z', t = t}.
\]

Next, we expand the functions \(G\), \(H\) and \(F\) with the parameter \(\eta\) as follows:

\[
G = mg_0 + n \varepsilon g_1 + m \varepsilon^2 g_0 g_2 + n \varepsilon^3 g_3
\]

\[
+ m \varepsilon^4 g_0 g_4 + n \varepsilon^5 g_5 + m \varepsilon^6 g_0 g_6 + \cdots,
\]

\[
H = ag_0 + be g_1 + a \varepsilon^2 g_0 g_2 + b \varepsilon^3 g_3
\]

\[
+ a \varepsilon^4 g_0 g_4 + b \varepsilon^5 g_5 + a \varepsilon^6 g_0 g_6 + \cdots,
\]

\[
F = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \varepsilon^6 f_6 + \cdots.
\]

Substituting Eq. (2.4) into (2.2), we can obtain a set of equations by comparing the coefficients of \(\varepsilon^n\):

\[
\begin{align*}
0 &= D_1 g_0 - 1, \\
0 &= D_2 (f_2 + g_0 g_2 - 1), \\
0 &= D_3 (f_2 + 1 + f_2 + 4 \left( (m \mu + a \bar{m}) |g_0|^2 (g_2 + \bar{g}_2) + (n \bar{b} + b n) |g_1|^2 \right), \\
0 &= D_4 (g_3 + g_1 + f_2), \\
0 &= D_5 (g_0 g_4 + g_0 g_2 \cdot f_2 + g_0 \cdot f_4), \\
0 &= D_6 (f_4 \cdot 1 + f_2 \cdot f_2 + 1 \cdot f_4) + 4 \left( (m \mu + a \bar{m}) |g_0|^2 (g_2 + \bar{g}_2) + (n \bar{b} + b n) (g_3 g_1 + g_1 g_3) \right),
\end{align*}
\]

with

\[
D_1 = i D_t - 2 D_z^2 + 2 (b_0 - \omega),
D_2 = D_t D_z - \lambda.
\]

In the above expressions, \(\eta\) is an arbitrary real constant, \(m, n, a\) and \(b\) are arbitrary complex constants, \(\bar{m}, \bar{n}, \bar{a}\) and \(\bar{b}\) are complex conjugates of \(m, n, a\) and \(b\), respectively. In the process of calculation, we find that the parameters satisfy the relations \(m \mu + a \bar{m} \neq 0, n \bar{b} + b n \neq 0, m \bar{b} + a \bar{n} = 0\) and \(n \bar{a} + b \bar{m} = 0\). Because \(G\) and \(H\) are symmetrical, it can be found that \(p\) and \(q\) have similar properties.

2.2 One-soliton solutions

In order to obtain the one-soliton solution of System (1.4), we can take

\[
\begin{align*}
G &= mg_0 + n \varepsilon g_1 + m \varepsilon^2 g_0 g_2, \\
H &= a g_0 + b \varepsilon g_1 + a \varepsilon^2 g_0 g_2, \\
F &= 1 + \varepsilon^2 f_2.
\end{align*}
\]

Substituting Eq. (2.5) into (2.2), we can obtain

\[
\begin{align*}
g_0 &= g_0 e^{i \eta_0}, \\
g_1 &= g_1 e^{i \eta_1}, \\
g_2 &= g_2 e^{i \eta_1 + A}, \\
f_2 &= f_2 e^{i \eta_1 + \bar{\eta}_1 + \mu}, \\
\eta_0 &= k_0 t + P_0 z, \\
\eta_1 &= k_1 t + P_1 z + \varphi_0.
\end{align*}
\]

We assume \(\Pi = m \mu + a \bar{m}, \Theta = n \bar{b} + b n\), then

\[
\begin{align*}
\lambda &= 4 \Pi |\gamma_0|^2, \\
e^A &= \frac{p_0 + i p_1}{p_0 - i p_1}, \\
e^{\mu} &= \frac{\Theta |\alpha|^2}{(p_1 + \bar{p}_1)^2} \left[ i (p_1 - \bar{p}_1) + \frac{\Pi |\gamma_0|^2}{|p_1 - i p_0|^2} \right]^{-1}.
\end{align*}
\]
In this case of wave system; the parameters are as follows: \( \varepsilon = 1 \), \( \gamma_0 = 0 \), \( p_0 = 0 \), \( \alpha_1 = 2 \), \( b_0 = 1 \), \( \omega = 1 \), \( \varphi_0 = 3 \), \( a = 1 \), \( b = 1 \), \( m = 1 \), \( n = -1 \), \( p_1 = 1 + 1.2i \)

Thus, we obtain the one-soliton solutions as follows:

\[
p = \frac{m \gamma_0 e^{i \eta_0} (1 + e^{\eta_1 + \bar{\eta}_1 + A}) + n \alpha_1 e^{\eta_1}}{1 + e^{\eta_1 + \bar{\eta}_1 + \mu}}, \\
q = \frac{a \gamma_0 e^{a \eta_0} (1 + e^{\eta_1 + \bar{\eta}_1 + A}) + b \alpha_1 e^{\eta_1}}{1 + e^{\eta_1 + \bar{\eta}_1 + \mu}}, \\
\eta = b_0 - 2 \left( \ln \left( 1 + e^{\eta_1 + \bar{\eta}_1 + \mu} \right) \right)_{zz}.
\]

In the above expressions, \( p_0 \) is an arbitrary real constant, \( \gamma_0, \varphi_0, \alpha_1 \) and \( p_1 \) are arbitrary complex constants. In this case of \( \gamma_0 = 0 \), the one-soliton solutions can be expressed as follows:

\[
p = \frac{n \alpha_1 e^{\eta_{11}}}{2e^{\frac{\alpha_1}{2}} \cosh \left( \eta_{11} + \frac{\mu}{2} \right)}, \\
q = \frac{a \gamma_0 e^{a \eta_{11}}}{2e^{\frac{\alpha_1}{2}} \cosh \left( \eta_{11} + \frac{\mu}{2} \right)}, \\
\eta = b_0 - 2 \left( \ln \left( 1 + e^{2 \eta_{11} + \mu} \right) \right)_{zz},
\]

where \( \eta_{11} \) represents the real part and imaginary part of \( \eta_1 \), respectively. Particularly, when \( p_1 = 1 + 1.2i \), we can get the bright one-soliton and dark one-soliton solutions of System (1.4) as shown in Fig. 1a, b, respectively.

2.3 Two-soliton solutions

In order to obtain the two-soliton solution of System (1.4), we can take

\[
G = mg_0 + n e g_1 + m e^2 g_0 g_2 + n e^3 g_3 + m e^4 g_0 g_4, \\
H = a g_0 + b e g_1 + a e^2 g_0 g_2 + b e^3 g_3 + a e^4 g_0 g_4, \\
F = 1 + e^2 f_2 + e^4 f_4.
\]

Substituting Eq. (2.8) into (2.2), we can obtain

\[
G = m \gamma_0 e^{i \eta_0} \left( 1 + e^{\eta_1 + \bar{\eta}_1 + A_{11}} + e^{\eta_1 + \bar{\eta}_2 + A_{12}} + e^{\eta_1 + \bar{\eta}_2 + A_{21}} + e^{\eta_1 + \bar{\eta}_2 + A_{22}} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \rho} \right) + n \left( \alpha_1 e^{\eta_1} + \alpha_2 e^{\eta_2} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \chi_1} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \chi_2} \right),
\]

\[
H = a \gamma_0 e^{i \eta_0} \left( 1 + e^{\eta_1 + \bar{\eta}_1 + A_{11}} + e^{\eta_1 + \bar{\eta}_2 + A_{12}} + e^{\eta_1 + \bar{\eta}_2 + A_{21}} + e^{\eta_1 + \bar{\eta}_2 + A_{22}} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \rho} \right) + b \left( \alpha_1 e^{\eta_1} + \alpha_2 e^{\eta_2} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \chi_1} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \chi_2} \right),
\]

\[
F = 1 + e^{\eta_1 + \bar{\eta}_1 + \mu_{11}} + e^{\eta_1 + \bar{\eta}_2 + \mu_{12}} + e^{\eta_2 + \bar{\eta}_1 + \mu_{21}} + e^{\eta_2 + \bar{\eta}_2 + \mu_{22}} + e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \Omega}.
\]

The relevant variables are as follows:

\[
\eta_0 = k_0 t + p_0 z, \quad k_0 = -2(\omega - b_0 - p_0^2), \\
\eta_j = k_j t + p_j z + \varphi_j, \quad k_j = -2i(\omega - b_0 + p_j^2), \\
\lambda = 4\Pi |\gamma_0|^2, \quad e^{\lambda \Omega} = \frac{p_0 + ip_j}{p_0 - i p_j} e^{\rho \Omega}, \\
e^{\lambda_1} = \frac{(p_2 - p_1)^2}{\eta_1} \left[ i (p_2 + p_1) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_1)(p_0 + ip_1)} \right] \\
\times \left( p_0 + ip_1 \right)(p_0 + ip_1), \\
e^{\lambda_2} = \frac{(p_2 - p_1)^2}{\theta^2} \left[ i (p_2 + p_1) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_1)(p_0 + ip_1)} \right] \\
\times e^{i \mu_{12} + \mu_{12}}, \\
e^{\rho \Omega} = \frac{(p_0 + ip_1)(p_0 + ip_2)}{(p_0 - ip_1)(p_0 - ip_2)} e^{2\Omega}, \\
\Omega = \left( p_0 + ip_1 \right)^2 \left( p_0 + ip_2 \right)^2 \left( p_0 - ip_1 \right)^2 \left( p_0 - ip_2 \right)^2.
\]

2.3 Two-soliton solutions

In order to obtain the two-soliton solution of System (1.4), we can take

\[
G = mg_0 + n e g_1 + m e^2 g_0 g_2 + n e^3 g_3 + m e^4 g_0 g_4, \\
H = a g_0 + b e g_1 + a e^2 g_0 g_2 + b e^3 g_3 + a e^4 g_0 g_4(2.8) \\
F = 1 + e^2 f_2 + e^4 f_4.
\]

Substituting Eq. (2.8) into (2.2), we can obtain
In the above expressions, $j, k = 1, 2$, $p_0$ is an arbitrary real constant, $\alpha_j, p_j, \varphi_j$ $(j = 1, 2)$ and $\gamma_0$ are arbitrary complex constants. Moreover, $e^{\mu_{11}}$ and $e^{\mu_{22}}$ are real constants, and $e^{\mu_{12}}$ and $e^{\mu_{21}}$ are complex constants. When the two-soliton solutions take some specific values, the two-soliton solutions of System (1.4) can be obtained. When $p_1 = 1.2 + i$, $p_2 = 1.4 + i$, we can get the parallel two-soliton solution as shown in Fig. 2a. When $p_1 = 1.2 + 0.4i$, $p_2 = 0.4 + 2i$, we can obtain the intersectant two-soliton solution as shown in Fig. 2b, and the shapes and amplitudes of the two solitons remain unchanged before and after collision. Similarly, we can obtain the parallel dark-two-soliton and the intersectant dark-two-soliton solutions as shown in Fig. 2c, d, respectively.

2.4 Three-soliton solutions

In order to obtain the three-soliton solution of System (1.4), we can take

$$G = mg_0 + n e g_1 + m e^2 g_0 g_2 + n e^3 g_3 + m e^4 g_0 g_4 + n e^5 g_5 + m e^6 g_0 g_6,$$

$$H = a g_0 + b e g_1 + a e^2 g_0 g_2 + b e^3 g_3 + a e^4 g_0 g_4 + b e^5 g_5 + a e^6 g_0 g_6,$$

$$F = 1 + e^2 f_2 + e^4 f_4 + e^6 f_6.$$

Substituting Eq. (2.11) into (2.2), we can get

$$g_0 = \gamma_0 e^{i p_0}, \quad g_1 = e^{\eta_1} + e^{\eta_2} + e^{\eta_3},$$

$$g_2 = e^{\eta_1 + \eta_2 + A_{11}} + e^{\eta_1 + \eta_2 + A_{12}} + e^{\eta_1 + \eta_2 + A_{13}} + e^{\eta_1 + \eta_2 + A_{21}} + e^{\eta_1 + \eta_2 + A_{22}} + e^{\eta_2 + \eta_3 + A_{23}},$$

$$f_2 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_3 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_4 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_5 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_6 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$g_3 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$g_4 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$g_5 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$g_6 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_0 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_1 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_2 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_3 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_4 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_5 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

$$f_6 = e^{\eta_1 + \eta_2 + \eta_3 + A_{31}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{32}} + e^{\eta_1 + \eta_2 + \eta_3 + A_{33}},$$

in which

$$\eta_0 = k_0 t + p_0 z, \quad k_0 = -2 \left( \omega - b_0 - p_0^2 \right).$$

$$\eta_j = k_j t + p_j z + \varphi_j, \quad k_j = -2i \left( \omega - b_0 + p_j^2 \right).$$

$$\lambda = 4 \Pi |\gamma_0|^2, \quad e^{A_{jk}} = \frac{p_0 + i p_j}{p_0 - i p_k} e^{\mu_{jk}},$$

$$e^{X_{1/3/3}} = \frac{(p_{j2} - p_{j1})^2}{\Theta} \times \left[ \left( p_{j2} + p_{j1} \right) + \frac{\Pi |\gamma_0|^2}{(p_0 + i p_{j2})(p_0 + i p_{j1})} \right] \times e^{\mu_{1/1} + \mu_{2/2} - \mu_{3} - 3},$$

$$e^{P_{1/2/3/4}} = \frac{(p_0 + i p_{j2})(p_0 + i p_{j1})}{(p_0 - i p_{j1} - 3)(p_0 - i p_{j2} - 3)} e^{\Omega_{1/2/3/4}}.$$
Fig. 2 Two-soliton solutions for the coupled long-wave–short-wave system; the parameters are as follows: $\epsilon = 1$, $\gamma_0 = 0$, $p_0 = 0$, $\alpha_1 = 1$, $\alpha_2 = 1$, $b_0 = 1$, $\omega = 1$, $\phi_1 = 10$, $\phi_2 = 1$, $a = 1$, $b = 1$, $m = 1$, $n = -1$ and $a$

\[ p_1 = 1.2 + i, \quad p_2 = 1.4 + i; \quad b \quad p_1 = 1.2 + 0.4i, \]
\[ p_2 = 0.4 + 2i; \quad c \quad p_1 = 1 + 1.2i, \]
\[ p_2 = 1 + 1.21i; \quad d \quad p_1 = 1 + 1.5i, \]
\[ p_2 = 1 + 0.8i \]

\[ e^{5j_{34}} = \frac{(p_1-p_2)^2(p_1-p_3)^2(p_2-p_3)^2(p_{j_3-3}-p_{j_4-3})^2}{\Theta^4} \]
\[ \times \left[ i (p_1 + p_2) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_2)(p_0 + ip_1)} \right] \]
\[ \times \left[ i (p_1 + p_3) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_3)(p_0 + ip_1)} \right] \]
\[ \times \left[ i (p_2 + p_3) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_2)(p_0 + ip_1)} \right] \]
\[ \times \left[ -i (\tilde{p}_{j_3-3} + \tilde{p}_{j_4-3}) \right] \]
\[ + \frac{\Pi |\gamma_0|^2}{(p_0 - i \tilde{p}_{j_3-3}) (p_0 - i \tilde{p}_{j_4-3})} \]
\[ \times e^{\mu_{j_3-3}\mu_{j_4-3}}e^{\eta_{j_3-3}\eta_{j_4-3}}, \]

\[ e^{\sigma} = \frac{(p_0 + ip_1)(p_0 + ip_2)(p_0 + ip_3)(p_0 - i \tilde{p}_1)(p_0 - i \tilde{p}_2)(p_0 - i \tilde{p}_3)}{e^\Theta}, \]

with

\[ e^{\mu_{jk}} = \frac{\Theta}{(p_j + \tilde{p}_k)^2} \left[ i (p_j - \tilde{p}_k) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_j)(p_0 - i \tilde{p}_k)} \right]^{-1}, \]

\[ e^{\Omega_{j_1j_2j_3j_4}} = \frac{(p_{j_1} - p_{j_2})^2(\tilde{p}_{j_3-3} - \tilde{p}_{j_4-3})^2}{\Theta^2} \]
\[ \times \left[ i (p_{j_1} + p_{j_2}) + \frac{\Pi |\gamma_0|^2}{(p_0 + ip_{j_2})(p_0 + ip_{j_1})} \right] \]
\[ \times \left[ -i (\tilde{p}_{j_3-3} + \tilde{p}_{j_4-3}) \right] \]
\[ + \frac{\Pi |\gamma_0|^2}{(p_0 - i \tilde{p}_{j_3-3})(p_0 - i \tilde{p}_{j_4-3})} \]
\[ \times e^{\mu_{j_1j_3-3}\mu_{j_2j_4-3}}e^{\eta_{j_1j_3-3}\eta_{j_2j_4-3}}, \]

Thus, we obtain the three-soliton solutions as follows:

\[ p = \frac{m \gamma_0 e^{i\eta_0} (G_1) + n (G_2)}{F_1}, \]
\[ q = \frac{a \gamma_0 e^{i\eta_0} (G_1) + b (G_2)}{F_1}, \quad \quad (2.12) \]
\[ \eta = b_0 - 2 (\ln (F_1))_{zz}, \]

with

\[ G_1 = 1 + \sum e^{\eta_{j_1} + \eta_{j_3-3} + \eta_{j_4-3}} + \sum_e^{\eta_{j_1} + \eta_{j_3} + \eta_{j_4-3}} + \sum_e^{\eta_{j_1} + \eta_{j_3} + \eta_{j_4} + \eta_{j_5-3} + \eta_{j_6-3} + \eta_{j_7-3} + \eta_{j_8-3}}, \]
\[ G_2 = e^{\eta_{j_1} + \eta_{j_2} + \eta_{j_3-3}} + \sum e^{\eta_{j_1} + \eta_{j_2} + \eta_{j_3-3} + \eta_{j_4-3}}, \]
\[ F_1 = 1 + \sum e^{\eta_{j_1} + \eta_{j_2} + \eta_{j_3-3} + \eta_{j_4-3} + \Omega_{j_1j_2j_3j_4}}, \]
\[ F_2 = e^{\eta_{j_1} + \eta_{j_2} + \eta_{j_3-3} + \eta_{j_4-3} + \eta_{j_5-3} + \eta_{j_6-3}}, \]

In the above expressions, $j, k, j_1, j_2 = 1, 2, 3, j_3, j_4 = 4, 5, 6$. $p_0$ is an arbitrary real constant,
Soliton, breather-like and dark-soliton-breather-like solutions

When one soliton intersects with the other two solitons. When 

\[ \eta_0 = k_0 t + p_0 z, \quad k_0 = -2 (\omega - b_0 - p_0^2), \]

\[ \eta_j = k_j t + p_j z + \varphi_j, \quad k_j = 2i (\omega - b_0 + p_j^2), \]

\[ \eta_{j+N} = \tilde{\eta}_j, \quad p_{j+N} = \tilde{p}_j, \quad \lambda = 4\Pi |\eta_0|^2, \]

with

\[ \sum_{\beta=0,1} \Gamma_1 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j \mu_{ij} \right), \]

\[ F = \sum_{\beta=0,1} \Gamma_1 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j \mu_{ij} \right), \]

\[ \eta_0 = k_0 t + p_0 z, \]

\[ \eta_j = k_j t + p_j z + \varphi_j, \]

\[ \eta_{j+N} = \tilde{\eta}_j, \]

\[ p_{j+N} = \tilde{p}_j, \quad \lambda = 4\Pi |\eta_0|^2, \]

\[ \Gamma_1 (\beta) = \begin{cases} 1, & \sum_{j=1}^{N} \beta_j + N = \sum_{j=1}^{N} \beta_j, \\ 0, & \sum_{j=1}^{N} \beta_j + N \neq \sum_{j=1}^{N} \beta_j. \end{cases} \]

\[ \Gamma_2 (\beta) = \begin{cases} 1 + \sum_{j=1}^{N} \beta_j + N = \sum_{j=1}^{N} \beta_j, \\ 0, & 1 + \sum_{j=1}^{N} \beta_j + N \neq \sum_{j=1}^{N} \beta_j, \end{cases} \]

and

\[ e^{\mu_{ij}} = \left( \frac{p_0 + ip_j}{p_0 - ip_j} \right) \exp \left( \frac{ip_i}{(p_0 - ip_j) (p_0 - ip_j)} \right), \]

\[ e^{A_{ij}} = \left( \frac{(p_0 - ip_j) (p_0 - ip_j)}{(p_0 + ip_j) (p_0 + ip_j)} \right) \exp \left( \frac{ip_i}{(p_0 - ip_j) (p_0 - ip_j)} \right), \]

\[ \frac{\omega}{(p_i - p_j)^2} \left[ i (p_i - p_j) + \frac{\Pi |\eta_0|^2}{(p_0 + ip_j) (p_0 - ip_j)} \right]^{-1}, \]

\[ \frac{1}{(p_i - p_j)^2} \left[ i (p_i + p_j) + \frac{\Pi |\eta_0|^2}{(p_0 + ip_j) (p_0 - ip_j)} \right], \]

\[ \frac{1}{(p_i - p_j)^2} \left[ i (p_i + p_j) + \frac{\Pi |\eta_0|^2}{(p_0 - ip_j) (p_0 - ip_j)} \right], \]

\[ i = N + 1, \ldots, 2N, \]

\[ i = N + 1, \ldots, 2N, \]

Thus, we obtain the N-soliton solutions as follows:

\[ p = \frac{m_0 \eta_0 e^{i\eta_0}}{\sum_{\beta=0,1} \Gamma_1 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j A_{ij} \right) / \sum_{\beta=0,1} \Gamma_1 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j A_{ij} \right)}, \]

Fig. 3  Three-soliton solutions for the coupled long-wave–short-wave system: the parameters are as follows: \( \epsilon = 1, \gamma_0 = 0, \)

\( p_0 = 0, b_0 = 1, \omega = 1, \varphi_1 = 10, \varphi_2 = 1, \varphi_3 = 6, a = 1, \)

\( b = 1, m = 1, n = -1 \) and \( a \)

\[ p_1 = 1.2 + i, p_2 = 1.3 + i, \]

\( p_3 = 1.4 + i; b \)

\[ p_1 = 1 + 1.9i, p_2 = 1.5 + 0.5i, p_3 = 0.8 + 1.9i; \)

\( c \)

\[ p_1 = 1 + 1.5i, p_2 = 1.5 + 0.5i, p_3 = 1.8 + 0.7i; d \)

\[ p_1 = 1 + 1.2i, p_2 = 1 + 1.22i; e \]

\[ p_1 = 1 + 1.2i, p_2 = 1 + 0.4i, \]

\( p_3 = 1 + 1.22i; f \)

\[ p_1 = 1 + 1.9i, p_2 = 1 + 0.4i, p_3 = 1 + 1.15i. \]

\( p_j, \varphi_j (j = 1, 2, 3) \) and \( \gamma_0 \) are arbitrary complex constants. Besides, \( e^{\mu_{ij}} (j = k) \)

\[ e^{\epsilon_{1/2,1/4}} (j_3 - 3 = j_1, j_4 - 3 = j_2) \) are real constants and \( e^{\mu_{ij}} (j \neq k) \)

and \( e^{\epsilon_{1/2,1/4}} (j_3 - 3 \neq j_1 \) or \( j_4 - 3 \neq j_2) \) are complex constants. When the parameters take some specific values, the three-soliton solutions of System (1.4) can be obtained. When \( p_1 = 1.2 + i, p_2 = 1.3 + i, \)

\( p_3 = 1.4 + i, \) the parallel three-soliton can be obtained as shown in Fig. 3a. When \( p_1 = 1 + 1.9i, \)

\( p_2 = 1.5 + 0.5i, p_3 = 0.8 + 1.9i, \) the three-soliton solution can be obtained as shown in Fig. 3b, in which one soliton intersects with the other two solitons. When \( p_1 = 1 + 1.5i, p_2 = 1.5 + 0.5i, p_3 = 1.8 + 0.7i, \) the intersector three-soliton solution can be obtained as shown in Fig. 3c. Similarly, we can get the corresponding dark three-soliton solutions as shown in Fig. 3d–f.

2.5 N-soliton solutions

Proceeding further, \( G, H \) and \( F \) in the N-soliton solution can be expressed as follows:

\[ G = m_0 \eta_0 e^{i\eta_0} \left[ \sum_{\beta=0,1} \Gamma_1 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j A_{ij} \right) \right] \]

\[ + n \sum_{\beta=0,1} \Gamma_2 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j \mu_{ij} \right), \]

\[ H = a_0 \eta_0 e^{i\eta_0} \left[ \sum_{\beta=0,1} \Gamma_1 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j A_{ij} \right) \right] \]

\[ + b \sum_{\beta=0,1} \Gamma_2 (\beta) \exp \left( \sum_{j=1}^{2N} \beta_j \eta_j + \sum_{1 \leq i < j} \beta_i \beta_j \mu_{ij} \right). \]
3 Breather-like solutions for the coupled long-wave–short-wave system

In this part, we consider the case of \( \gamma_0 \neq 0 \). Because \( p \) and \( q \) have similar properties, we only consider the results obtained by \( p \).

3.1 One-breather-like solutions

In the case of \( \gamma_0 \neq 0 \), Eq. (2.6) can be rewritten as follows:

\[
p = \frac{1}{2} \gamma_0 e^{i \eta_0} \left[ 1 + \kappa - (1 - \kappa) \tanh \left( \eta_{1R} + \frac{\mu}{2} \right) \right] \\
+ \frac{1}{2} m \alpha_1 e^{i \eta_{1l}} e^{i \eta_{1l}l} e^{\frac{\mu}{2} \cos \left( \eta_{1R} + \frac{\mu}{2} \right)},
\]

\[
q = \frac{1}{2} a \gamma_0 e^{i \eta_0} \left[ 1 + \kappa - (1 - \kappa) \tanh \left( \eta_{1R} + \frac{\mu}{2} \right) \right] \\
+ \frac{1}{2} b \alpha_1 e^{i \eta_{1l}} e^{i \eta_{1l}l} e^{\frac{\mu}{2} \cos \left( \eta_{1R} + \frac{\mu}{2} \right)},
\]

\[
\eta = b_0 - 2 \left( \ln \left( 1 + e^{2 \eta_{1R} + \mu} \right) \right)_{zz},
\]

with

\[
\kappa = \frac{p_0 + i p_1}{p_0 - i p_1},
\]

where \( \eta_{1R} \) and \( \eta_{1l} \) represent the real part and imaginary part of \( \eta_1 \), respectively. Therefore, when \( \gamma_0 = 1 \), \( p_1 = 1 + 1.2i \), the one-breather-like solution can be obtained as shown in Fig. 4a. We can see such obvious breather behavior as peaks and valleys. Compared with Fig. 4a, when the imaginary part of \( p_1 \) decreases, the propagative direction of one-breather-like in Fig. 4b changes. When the real part of \( p_1 \) decreases, it can be found that the amplitude of one-breather-like in Fig. 4c diminishes. In Fig. 4d, we can find that the initial position of one-breather-like changes with the changes of \( \varphi_0 \).

3.2 Two-breather-like solutions

Here, we consider the case of \( \gamma_0 = 1 \) in Eq. (2.10). When \( p_1 = 1.2 + i \), \( p_2 = 1.4 + i \), the parallel two-breather-like solution can be obtained as shown in Fig. 5a. When the real part of \( p_2 \) or \( p_1 \) decreases, the amplitude of corresponding breather-like reduces as shown in Fig. 5b–d, respectively.

Next, we consider the case of \( \gamma_0 = 0.3 \) in Eq. (2.10). When \( p_1 = 0.4 + i \), \( p_2 = 1 + 0.1i \), the intersectant two-breather-like solution can be obtained as shown in Fig. 6a. Compared with Fig. 6a, when the real part of \( p_1 \) increases, it can be found that the amplitude of corresponding breather-like increases in Fig. 6b. In Fig. 6c, when the imaginary part of \( p_1 \) increases, the angle between the two breather-like augments. Finally, it can be found that the initial position of one of the two breather-like changes when \( \varphi_1 \) changes by comparing Fig. 6a, d.
3.3 Three-breather-like solutions

Similarly, the three-breather-like solutions can be obtained as shown in Figs. 7. It can be seen that the initial position of one of the three breather-like changes when \( \varphi_3 \) changes by comparing Fig. 7a with 7b. Compared with Fig. 7a, when real parts of \( p_1 \), \( p_2 \) and \( p_3 \) all decrease, it can be found that the amplitudes of three breather-like all decrease in Fig. 7c. What’s more, Fig. 7d shows the three-breather-like solution that one breather-like intersects with the other two breather-like.

4 Dark-soliton-breather-like solutions for the coupled long-wave–short-wave system

In this part, we consider the case of \( \gamma_0 = 1 \) and \( p_0 = -0.7 \). Because \( p \) and \( q \) have similar properties, we only consider the results obtained by \( p \).

4.1 Dark-soliton-one-breather-like solutions

Here, we consider the case of \( \gamma_0 = 1 \), \( p_0 = -0.7 \) in Eq. (2.10). When \( p_1 = 0.6 + i \), \( p_2 = 0.75 + 0.5i \), the dark-soliton-one-breather-like solution can be obtained as shown in Fig. 8a. Compared with Fig. 8a, when the real part of \( p_1 \) reduces, it can be seen that the amplitude of the breather-like decreases in Fig. 8b. In Fig. 8c, when the imaginary part of \( p_1 \) increases, it can be found that the propagative direction of the breather-like changes. Finally, Fig. 8d shows that the dark-soliton-one-breather-like solution can degenerate into one-breather-like solution when the values of \( p_1 \) and \( p_2 \) are both \( 0.6 + i \).

4.2 Dark-soliton-two-breather-like solution

Next, we consider the case of \( \gamma_0 = 1 \) and \( p_0 = -0.7 \) in Eq. (2.12). In this case, we can obtain the dark-soliton-two-breather-like solution as shown in Fig. 9a. When the values of \( p_1 \) and \( p_3 \) are both \( 0.7 + i \),

Fig. 5 Two-breather-like solutions for the coupled long-wave–short-wave system; the parameters are as follows: \( \varepsilon = 1 \), \( \gamma_0 = 1 \), \( p_0 = -1 \), \( \alpha_1 = 2 \), \( \alpha_2 = 1 \), \( b_0 = 1 \), \( \omega = 1 \), \( \varphi_1 = 10 \), \( \varphi_2 = 1 \), \( a = 1 \), \( b = 1 \), \( m = 1 \), \( n = -1 \) and a \( p_1 = 1.2 + i \), \( p_2 = 1.4 + i \); b \( p_1 = 1.2 + i \), \( p_2 = 0.7 + i \); c \( p_1 = 0.6 + i \), \( p_2 = 1.4 + i \); d \( p_1 = 0.6 + i \), \( p_2 = 0.7 + i \)

Fig. 6 Two-breather-like solutions for the coupled long-wave–short-wave system; the parameters are as follows: \( \varepsilon = 1 \), \( \gamma_0 = 0.3 \), \( p_0 = -1 \), \( \alpha_1 = 1 \), \( \alpha_2 = 1 \), \( b_0 = 1 \), \( \omega = 1 \), \( \varphi_2 = 1 \), \( a = 1 \), \( b = 1 \), \( m = 1 \), \( n = -1 \) and a \( \varphi_1 = 1 \), \( p_1 = 0.4 + i \), \( p_2 = 1 + 0.1i \); b \( \varphi_1 = 1 \), \( p_1 = 0.6 + i \), \( p_2 = 1 + 0.1i \); c \( \varphi_1 = 1 \), \( p_1 = 0.4 + 1.5i \), \( p_2 = 1 + 0.1i \); d \( \varphi_1 = 5 \), \( p_1 = 0.4 + i \), \( p_2 = 1 + 0.1i \)

Fig. 7 Three-breacher-like solutions for the coupled long-wave–short-wave system; the parameters are as follows: \( \varepsilon = 1 \), \( \gamma_0 = 1 \), \( p_0 = -1 \), \( \alpha_1 = 2 \), \( b_0 = 1 \), \( \omega = 1 \), \( \varphi_1 = 20 \), \( \varphi_2 = 1 \), \( a = 1 \), \( b = 1 \), \( m = 1 \), \( n = -1 \) and a \( \varphi_3 = 10 \), \( p_1 = 1.2 + i \), \( p_2 = 1.3 + i \), \( p_3 = 1.4 + i \); b \( \varphi_3 = 5 \), \( p_1 = 1.2 + i \), \( p_2 = 1.3 + i \), \( p_3 = 1.4 + i \); c \( \varphi_3 = 10 \), \( p_1 = 0.6 + i \), \( p_2 = 0.65 + i \), \( p_3 = 0.7 + i \); d \( \varphi_3 = 1 \), \( p_1 = 0.5 + 1.9i \), \( p_2 = 0.75 + 0.5i \), \( p_3 = 0.4 + 1.9i \)
the dark-soliton-two-breather-like solution can degrade into dark-soliton-one-breather-like solution as shown in Fig. 9b.

5 Asymptotic analysis

In this part, we consider the cases of \( \alpha_1 = 1 \), \( \alpha_2 = 1 \) in Eq. (2.10).

Case 1: \( \gamma_0 \neq 0 \).

(i) Before collision \( (t \to -\infty) \)

(a) \( (\eta_{1R} \approx 0, \eta_{2R} \to -\infty) \):

\[
p \to S_{1-}^2 = \frac{m_0 e^{i\eta_0} (1 + e^{2i\eta_{1R} + A_{11}})}{1 + e^{2i\eta_{1R} + \mu_{11}}} \left[ 1 + \kappa_1 - (1 - \kappa_1) \right] + n A_1 e^{i\eta_{11}} \sec h \left( \eta_{1R} + \frac{\mu_{11}}{2} \right),
\]

with

\[
\kappa_1 = \frac{p_0 + i p_1}{p_0 - i p_1}, \quad A_1 = \frac{1}{2} \exp \left( -\frac{\mu_{11}}{2} \right).
\]

(b) \( (\eta_{2R} \approx 0, \eta_{1R} \to -\infty) \):

\[
p \to S_{-}^{-} = \frac{m_0 e^{i\eta_0} (1 + e^{2i\eta_{2R} + A_{22}})}{1 + e^{2i\eta_{1R} + \mu_{11}}} \left[ 1 + \kappa_2 - (1 - \kappa_2) \right] \tan \left( \eta_{2R} + \frac{\mu_{22}}{2} \right) \sec h \left( \eta_{2R} + \frac{\mu_{22}}{2} \right),
\]

with

\[
\kappa_2 = \frac{p_0 + i p_2}{p_0 - i p_2}, \quad A_2 = \frac{1}{2} \exp \left( -\frac{\mu_{22}}{2} \right).
\]

(ii) After collision \( (t \to \infty) \)

(a) \( (\eta_{1R} \approx 0, \eta_{2R} \to \infty) \):

\[
p \to S_{1+}^+ = \frac{m_0 e^{i\eta_0} (e^{A_{22} + A_{11}} + e^{2i\eta_{1R} + \rho}) + ne^{\eta_1 + \chi_2}}{e^{2i\eta_{1R} + \Omega} + e^{2i\eta_{1R} + \Omega}} \left[ 1 + \kappa_1 - (1 - \kappa_1) \right] \tan \left( \eta_{1R} + \frac{\Omega - \mu_{22}}{2} \right) \sec h \left( \eta_{1R} + \frac{\Omega - \mu_{22}}{2} \right),
\]

with

\[
A_3 = \frac{1}{2} \exp \left( \chi_2 - \frac{\Omega + \mu_{22}}{2} \right).
\]

(b) \( (\eta_{2R} \approx 0, \eta_{1R} \to \infty) \):

\[
p \to S_{+}^+ = \frac{m_0 e^{i\eta_0} (e^{A_{11} + A_{22}} + e^{2i\eta_{2R} + \rho}) + ne^{\eta_2 + \chi_1}}{e^{\mu_{11} + e^{2i\eta_{2R} + \Omega}} + e^{\mu_{11} + e^{2i\eta_{2R} + \Omega}}} \left[ 1 + \kappa_2 - (1 - \kappa_2) \right] \tan \left( \eta_{2R} + \frac{\Omega - \mu_{11}}{2} \right) \sec h \left( \eta_{2R} + \frac{\Omega - \mu_{11}}{2} \right),
\]
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\[ \Lambda_4 = \frac{1}{2} \exp \left( \chi_1 - \frac{\Omega + \mu_{11}}{2} \right). \]

Here, we notice that \(|\xi_1| = |\xi_2| = 1, |A_1|^2 = |A_3|^2, |A_2|^2 = |A_4|^2\). That is to say, the amplitude of each breather-like in the two-breather-like solutions remains unchanged before and after the collision. The corresponding visual illustrations are shown in Fig. 6.

**Case 2.** \( \gamma_0 = 0 \).

(i) Before collision \((t \to -\infty)\)

(a) \((\eta_1 R \approx 0, \eta_2 R \to -\infty)\):

\[
p \to S_1^- = \frac{n e^{\eta_1} \sec h \left( \eta_1 R + \frac{\mu_{11}}{2} \right)}{1 + e^{2\eta_1 R + \mu_{11}}},
\]

(b) \((\eta_2 R \approx 0, \eta_1 R \to -\infty)\):

\[
p \to S_2^- = \frac{n e^{\eta_2} \sec h \left( \eta_2 R + \frac{\mu_{11}}{2} \right)}{1 + e^{2\eta_2 R + \mu_{11}}},
\]

(ii) After collision \((t \to \infty)\)

(a) \((\eta_1 R \approx 0, \eta_2 R \to \infty)\):

\[
p \to S_1^+ = \frac{n e^{\eta_1 + \xi_2} \sec h \left( \eta_1 R + \frac{\Omega - \mu_{22}}{2} \right)}{e^{\mu_{22}} + e^{2\eta_1 R + \Omega}},
\]

(b) \((\eta_2 R \approx 0, \eta_1 R \to \infty)\):

\[
p \to S_2^+ = \frac{n e^{\eta_2 + \xi_1} \sec h \left( \eta_2 R + \frac{\Omega - \mu_{11}}{2} \right)}{e^{\mu_{11}} + e^{2\eta_2 R + \Omega}}.
\]

The relevant physical quantities for function \(p\) before and after the collision are listed in Table 1. Here, we notice that \(|A_1|^2 = |A_3|^2, |A_2|^2 = |A_4|^2\). Therefore, Table 1 shows that the collision of two solitons in the two-soliton solutions is elastic. The corresponding visual illustration is shown in Fig. 2b.

**6 Conclusions**

In this paper, we have obtained various types of solutions of the coupled long-wave–short-wave system through the developed Hirota bilinear method. Variations of relevant parameters determine different types of solutions. In the case of \( \gamma_0 = 0 \), we have obtained the exact \(N\)-soliton solution. In the case of \( \gamma_0 \neq 0 \), we have derived the breather-like solutions and studied the influences of \(p_j\) and \(\varphi_j\) for the transmission of the breather-like. In particular, we have obtained the dark-soliton-breather-like solutions in the case of \( \gamma_0 = 1, p_0 = -0.7 \). Finally, we have demonstrated that the interactions of two-soliton and two-breather-like solutions are all elastic through asymptotic analysis method.

**Acknowledgements** We express our sincere thanks to each member of our discussion group for their suggestions. This work has been supported by the National Natural Science Foundation of China under Grant No. 11905155 and the Shanxi Province Science Foundation for Youths, China, under Grant Nos. 201801D221023 and 201801D121016.

**Data availability** The datasets generated during or analyzed during the current study are available from the corresponding author on reasonable request.

**Declarations**

**Conflict of interest** The authors declare no conflict of interest.
References

1. Ablowitz, M.J., Segur, H.: On the evolution of packets of water waves. J. Fluid Mech. 92, 691–715 (1979)
2. Babaoglu, C.: Long-wave short-wave resonance case for a generalized Davey–Stewartson system. Chaos Solitons Fractals 38, 48–54 (2008)
3. Davey, A., Stewartson, K.: On three-dimensional packets of surface waves. Proc. R. Soc. Lond. A Math. Phys. Sci. 338, 101–110 (1974)
4. Djordjevic, V.D., Redekopp, L.G.: On two-dimensional packets of capillary-gravity waves. J. Fluid Mech. 79, 703–714 (1977)
5. Song, L.M., Yang, Z.J., Li, X.L., Zhang, S.M.: Interaction theory of mirror-symmetry soliton pairs in nonlinear nonlinear Schrödinger equation. Appl. Math. Lett. 90, 42–48 (2019)
6. Song, L.M., Yang, Z.J., Li, X.L., Zhang, S.M.: Controllable Gaussian-shaped soliton clusters in strongly nonlinear local media. Opt. Express 26, 19182–19198 (2018)
7. Song, L.M., Yang, Z.J., Zhang, S.M., Li, X.L.: Spreading anomalous vortex beam arrays in strongly nonlinear nonlinear media. Phys. Rev. A 99, 063817 (2019)
8. Hasimoto, H., Ono, H.: Nonlinear modulation of gravity waves. J. Phys. Soc. Jpn. 33, 805–811 (1972)
9. Chan, H.N., Ding, E., Kedziora, D.J., Grimshaw, R., Chow, K.W.: Rogue waves for a long-wave short-wave resonance model with multiple short waves. Nonlinear Dyn. 85, 1–15 (2016)
10. Zhai, Y., Geng, X., Xue, B.: Riemann theta function solutions to the coupled long wave short-wave resonance equations. Anal. Math. Phys. 56, 1–26 (2020)
11. Benney, D.J.: A general theory for interactions between short and long waves. Stud. Appl. Math. 56, 81–94 (1977)
12. Wright, O.C., III.: On a homoclinic manifold of a coupled long-wave short-wave system. Commun. Nonlinear Sci. Numer. Simul. 15, 2066–2072 (2010)
13. Wang, C., Dai, Z.: Various breathers and rogue waves for the coupled long-wave short-wave system. Adv. Differ. Equ. 2014, 1–10 (2014)
14. Chen, W., Chen, H., Dai, Z.: Rational homoclinic solution and rogue wave solution for the coupled long-wave short-wave system. Pramana 86, 713–717 (2016)
15. Wang, X.M., Zhang, L.L., Hu, X.X.: Various types of vector solitons for the coupled nonlinear Schrödinger equations in the asymmetric fiber couplers. Optik 219, 164989 (2020)
16. Wang, X.M., Zhang, L.L.: The nonautonomous N-soliton solutions for coupled nonlinear Schrödinger equation with arbitrary time-dependent potential. Nonlinear Dyn. 88, 2291–2302 (2017)
17. Zhang, F.: Intelligent task allocation method based on improved QPSO in multi-agent system. J. Ambient Intell. Humaniz. Comput. 11, 655–662 (2020)
18. Fu, Y., Kumar, J., Gaithia, B.P., Neware, R.: Nonlinear dynamic measurement method of software reliability based on data mining. Int. J. Syst. Assur. Eng. Manag. 2021. https://doi.org/10.1007/s13198-021-01389-0
19. Pei, S., Ye, L., Zhou, W.: Application of convolutional neural network under nonlinear excitation function in the construction of employee incentive and constraint mode. Int. J. Syst. Assur. Eng. Manag. 2021. https://doi.org/10.1007/s13198-021-01389-0

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