Drivability analysis based on the mathematical model of soil flow resistance

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Abstract. When a pile is driven, the soil around the pile reaches a critical condition. This results in a failure condition for the soil, which then moves as a viscous element. The flow is caused by the external driving force exceeding the static resistance of the soil. When the failure condition is reached, the dynamic resistance of the soil will counter the penetrating force of the pile. The rheology model is accurate in representing the soil flow when the critical condition is reached, because the parameters used for the mathematical model of soil flow resistance represent static and dynamic resistance. The dynamic resistance is influenced by the shear viscosity of the soil, which was determined using the Bingham and Casson models. Discrepancy exists between the analytical output used in the mathematical model of soil flow resistance and the actual result. Therefore, a correction factor is needed to calibrate the displacement of the pile.

1. Introduction
In civil engineering, driven pile is one of the foundations used for structural building. However, in construction work, time and efficiency are the most important factors that determining the quality of the work performed by a contractor. The construction processes of the piles are determined by the soil resistance. In this study, rheological approach is used to determine the parameters needed in soil flow resistance analysis. Hence, verification of the analysis results and the actual drivability is necessary.

2. Methods and Basic Concept
Two rheology models, namely, the Bingham and Casson models, were used for the analytical method. The rheology parameters used for the mathematical model of soil flow resistance are the cohesion ($c_u$) and shear viscosity ($\eta$), which was determined by the fall cone test (Mahajan and Budhu, 2008). Table 1 presents the cone parameters and constants used in determining the rheology parameters of the soils.

| Parameters                                      | Constants |
|------------------------------------------------|-----------|
| Cone mass, $m$ (g)                             | 80        |
| Cone weight, $W$ (N)                           | 0.78      |
| Bearing capacity factor of the cone considering the heaving, $N_{ch}$ (dimensionless) | 7.45 |
| Cone angle, $\beta$ (°)                        | 30        |
| Resistance factor of the cone, $F$ (dimensionless) | 1.68 |
| Fall cone factor to determine the soil’s shear strength, $K$ (dimensionless) | 1.33 |

Table 1. Cone parameters and constants for the rheology parameters
The mathematical model of the soil flow resistance used for the analytical method is divided into static and dynamic resistance. The static resistance uses a conventional method of bearing capacity (Budhu 2010), and the dynamic resistance uses a method of calculating the viscous drag force on a rigid body ($f_z$) that moves through the viscous liquid with low Reynold number (Mahajan and Budhu, 2008). The mathematical expression of soil flow resistance is shown in Equation (1), where $\alpha$ is the adhesion factor of soil, $H_c$ is the length of the cone, $H_r$ is the length of the rod, $r_o$ is the radius of the pile, $v_z$ is the penetration velocity of the pile, and $\eta$ is the shear viscosity of the soil.

$$F_z = N_{ct}c_o \pi \tan \left( \frac{\beta}{2} \right) H_c^2 + ac_o \pi \left( H_r^2 \sec \left( \frac{\beta}{2} \right) \tan \left( \frac{\beta}{2} \right) + 2\tau_r H_r \right) + \beta_o 2\pi \eta \pi \left( H_c + H_r \right)$$  (1)

The mathematical model helps define the pile-driving mechanism aside from the wave equation analysis proposed by Smith (1960). The mathematical model output is the settlement per blow, which is similar to the actual pile-driving record on the field. Moreover, the mathematical model result could be used to predict the pile-driving record. The mathematical expressions for the Bingham and Casson models are shown in Equations (2) and (3):

$$\tau = \tau_B + \eta_B \gamma \quad \text{(Bingham model)}$$  (2)

$$\sqrt{\tau} = \sqrt{\tau_B} + \sqrt{\eta_B} \gamma \quad \text{(Casson model)}$$  (3)

The value of a critical state zone ratio ($\lambda_o$) is the ratio between the radius of the soil that reaches the critical condition ($R_o$) and the radius of the pile ($r_o$). In the normally consolidated clay, the value of $\lambda_o$ is 4, resulting in $\beta_o = 1.2478$.

$$\lambda_o = \frac{R_o}{r_o}$$  (4)

$$\beta_o = \frac{\lambda_o^4 + 4\lambda_o^2 + 4\ln \lambda_o + 3}{(1-\lambda_o^2)(1+\ln \lambda_o - 1) - 2\ln \lambda_o} = 1.2478$$  (5)

In the driving pile with drop-hammer mechanism, the total force obtained using the equation of Budhu (2010) is shown in Equation (6). $F_z$ is the energy delivered by the hammer pile driving and the pile load capacity, $\varepsilon$ is the efficiency factor, $M.g$ is the weight of the ram, $L$ is the height of fall, and $\delta_i$ is the penetration per blow. The final mathematical expression of soil flow resistance for drop-hammer mechanism is shown in Equation (6).

$$F_z = F_s + F_v,$$

where

$$F_s = e \frac{m.g.L}{\delta_i}$$

$$F_v = 1.2478 \pi \eta \left( \frac{m \sqrt{2g (L + \Sigma \delta_i)}}{m + M} + \sqrt{2g \delta_i} \right) (\Sigma \delta_i + \delta_i)$$  (6)

$$F_t = F_{iq} + F_{as}$$
The output from Equation (6) is the value of number of blows versus penetration per blow that will be plotted for each variety of water content. The value should be corrected with the correction factor $x$ for the initial penetration and the correction factors $k$ and $n$ for the $i$ blow. Equation (7) is the mathematical expression of the analysis output correction, where $H$ is the corrected penetration of the pile, $\delta_0$ is the initial penetration, and $\delta_i$ is the penetration for blow number $i$.

$$H = x\delta_0 + kd_i^n$$

The materials used in this study are kaolinite and clay collected from Marunda Center, North Jakarta, Indonesia. The purpose of using two different materials is to obtain the general correction factors for each soil’s phase (plastic and viscous-liquid phases). The simulations of pile-driving mechanism for each material are proposed using various values of liquidity index of the material. In this study, two varieties of liquidity index for each material are used, which result in four simulations. The setting of the simulations is presented in Table 2, and the mechanism of the pile-driving simulation is shown in Figure 1.

**Table 2. Setting of the pile-driving simulation**

| Parameters            | Constants |
|-----------------------|-----------|
| Hammer mass, $m_h$ (g) | 99.95     |
| Height of fall, $L$ (m) | 0.5       |
| Rod length, $H_r$ (m)  | 0.634     |
| Cone length, $H_c$ (m) | 0.026     |
| Pile radius, $r_p$ (m) | 0.015     |

As shown in Figure 1, when the pile is on static condition (a), the value of shear stress ($\tau$) is at point $w$ with zero shear rate. When the pile is driven, the value of external forces exceeds the static resistance of the soil (b), and the soil around the pile also reached the critical state, hence the soil will flow similarly to a fluid element (point $x$). The flow of the soil around the critical state zone allow dynamic resistance, thereby preventing the external forces of the hammer. When the cumulative force of the static and dynamic resistance is equal to the external force of the hammer-fall (c), the shear stress will go back to the yield stress of the material (points $z$ and $w$).
3. Results and Discussions

3.1. Rheology Parameters Based on Mahajan and Budhu’s Method
On the basis of the method proposed by Mahajan and Budhu (2008) using a cone penetrometer, two results were obtained that were used as key parameters for the mathematical model of soil flow resistance. The values of rheological parameters for each material are presented in Table 3 and Table 4.

Table 3. Rheological and geotechnical parameters of kaolinite clay

| Parameters                                | Plastic state | Viscous-liquid state |
|-------------------------------------------|---------------|----------------------|
| Water content, \( w (%) \)               | 69            | 78                   |
| Plastic limit, \( PL \)                  | 51            |                      |
| Liquid limit, \( LL \)                   | 77            |                      |
| Specific gravity, \( G_s \)              |               | 2.70                 |
| Liquidity index, \( LI \)                | 0.69          | 1.04                 |
| Dynamic equilibrium position, \( h_{eq} \) (mm) | 9.1           | 13.7                 |
| Final depth of penetration, \( h_f \) (mm) | 16.6          | 21.5                 |
| Cohesion undrained, \( c_u \) (Pa)       | 3786.71       | 2257.36              |
| Shear rate, \( \dot{\gamma} \) (1/s)    | 2.64          | 2.32                 |
| Total shear stress, \( \tau \) (Pa)      | 5632.80       | 2485.23              |
| Shear viscosity of the Bingham model (Pa\cdot s) | 699.57     | 98.27                |
| Shear viscosity of the Casson model (Pa\cdot s) | 69.22      | 2.36                 |

3.2. Pile-Driving Simulation
Four sample variations of water content are observed, in which each sample represents the soil state. The pile-driving simulation in the laboratory is aimed at obtaining the actual blow count versus the depth of penetration. The result is used as a benchmark in determining the correction factors for each soil state. The pile-driving simulation result is shown in Figure 2.

Table 4. Rheological and geotechnical parameters of Maruna Center clay

| Parameters                                | Plastic state | Viscous-liquid state |
|-------------------------------------------|---------------|----------------------|
| Water content, \( w (%) \)               | 67            | 79                   |
| Natural water content, \( w_n \) (%)     | 40            |                      |
| Plastic limit, \( PL \)                  | 20            |                      |
| Liquid limit, \( LL \)                   | 78            |                      |
| Specific Gravity, \( G_s \)              | 2.61          |                      |
| Liquidity index, \( LI \)                | 0.83          | 1.04                 |
| Dynamic equilibrium position, \( h_{eq} \) (mm) | 9.5           | 12.7                 |
| Final depth of penetration, \( h_f \) (mm) | 18.0          | 19.5                 |
| Cohesion undrained, \( c_u \) (Pa)       | 3220.57       | 2744.15              |
| Shear rate, \( \dot{\gamma} \) (1/s)    | 2.53          | 2.43                 |
| Total shear stress, \( \tau \) (Pa)      | 5168.45       | 2892.01              |
| Shear viscosity of the Bingham model (Pa\cdot s) | 768.43      | 60.73                |
| Shear viscosity of the Casson model (Pa\cdot s) | 90.47       | 0.80                 |
Figure 2 indicates that 279 blows are needed to penetrate the pile to the depth of 39.6 cm for the kaolinite clay on the plastic state and 36 blows to penetrate the pile to the depth of 48.8 cm for the kaolinite clay on the viscous-liquid state. Using the Marunda clay requires 486 blows for penetrating the pile to the depth of 30 cm for the plastic state and 453 blows for penetrating the pile to the depth of 47.1 cm for the viscous-liquid state.

3.3. Analysis and Verifications of Soil Flow Resistance

The rheology parameters based on the Bingham and Casson models used for analytical method are quite different for each model, resulting in the difference of the final penetration. On the basis of the analytical method, the final penetration of the Bingham model is smaller than that of the Casson model. The difference is caused by the varying values of the shear viscosity. Shear viscosity of the Bingham model is higher than that of the Casson model. An example of the output for total force of soil flow resistance is shown in Figure 3.

On the basis of the analysis of the soil flow resistance mathematical model, the Bingham model has dominated in terms of the dynamic drag force, in which higher value of the total drag force is obtained. By contrast, the Casson model has dominated in terms of static drag force, in which smaller value of the total drag force is obtained.

3.4. Correction Factors of the Mathematical Model of Soil Flow Resistance

Discrepancies between the analysis curve of the soil flow resistance mathematical model and the curve of the pile-driving simulation are observed. These discrepancies are caused by the initial penetration at blow count 0, which was not considered in the mathematical model. Results of the correction factors used for the soil flow resistance mathematical model for kaolinite clay and Marunda clay are presented in Tables 5 and 6, respectively.
Table 5. Correction factors of the mathematical model of soil flow resistance for kaolinite clay

| Soil phase          | Bingham model | Casson model |
|---------------------|---------------|--------------|
|                     | $x_B$ | $k_B$ | $\eta_B$ | % of error | $x_C$ | $k_C$ | $\eta_C$ | % of error |
| Plastic             | 8.35  | 0.68  | 1.17     | 15.59      | 4.18  | 0.61  | 0.75     | 19.57      |
| Viscous-liquid      | 19.04 | 0.43  | 1.35     | 10         | 15    | 0.39  | 1.3      | 9          |
| Average             | 13.7  | 0.56  | 1.26     | 13         | 9.59  | 0.5   | 1.03     | 14         |

Table 6. Correction factors of the mathematical model of soil flow resistance for Marunda clay

| Soil phase          | Bingham model | Casson model |
|---------------------|---------------|--------------|
|                     | $x_B$ | $k_B$ | $\eta_B$ | % of error | $x_C$ | $k_C$ | $\eta_C$ | % of error |
| Plastic             | 4.82  | 0.68  | 1.17     | 15.59      | 2.28  | 0.61  | 0.75     | 19.6       |
| Viscous-liquid      | 4.59  | 0.43  | 1.35     | 10         | 4     | 0.39  | 1.3      | 9          |
| Average             | 4.71  | 0.56  | 1.26     | 13         | 3.14  | 0.5   | 1.03     | 14         |

The correction factors for the $i$-blow will be used for the drivability analysis on the field. The analysis used the mathematical model of soil flow resistance where rheology parameters are obtained using laboratory test.

3.5. Drivability Analysis Based on the Mathematical Model of Soil Flow Resistance

The key rheology parameters are soil cohesion ($c_u$) and shear viscosity ($\eta$). The soil cohesion parameter was obtained using the unconfined compression test in the laboratory. The shear viscosity parameter was obtained using the correlation between the value of liquidity index and shear viscosity as proposed by Mahajan (2006). Table 7 shows the parameters used in predicting the drivability on the field. Based on the simulation, the Bingham model was valuated to be more accurate; therefore, the Bingham model was chosen to measure the correlation between the liquidity index and the value of shear viscosity. The correction factors used for the mathematical model of soil flow resistance were only based on the plastic state soil.

Table 7. Rheological parameters used for the drivability analysis using the mathematical model of soil flow resistance

| Depth (m) | $c_u$ (kPa) | $\eta_B$ (P•s) |
|-----------|-------------|-----------------|
| 0–14      | 10          | 632.18          |
| 14–21     | 16          | 5.10            |
| 21–32     | 64          | 95.71           |

The output of the drivability analysis is shown in Figure 4, where the values of blow count and depth of penetration are the benchmark in determining the drivability of the pile.
Figure 4. Comparison between the actual and predicted pile-driving records for (a) blow count per 0.5 m depth and (b) cumulative blow

4. Conclusions
The Bingham model is evaluated to be better than the Casson model in drivability analysis based on the mathematical model of soil flow resistance in the pile driving with drop-hammer mechanism.

On the one hand, the Bingham model dominated in terms of the dynamic resistance and has a higher drag force. On the other hand, the Casson model dominated in terms of the static resistance and has a smaller drag force.

Correction factors used for the analysis based on the Bingham model in plastic state soil have an average error level at 15.59%, whereas those in viscous-liquid state soil have an average error level at 10%. In the analysis based on the Casson model, the correction factors in plastic state have an average error level at 19.57%, whereas those in viscous-liquid state have an average error level at 9%.

The level of average error based on the comparison between the actual and predicted the pile-driving records using the mathematical model of soil flow resistance is 16.2%.

References
[1] Mahajan S P and Budhu M 2008 Proc. 12th Int. Conf. of Int. Ass. for Computer Methods and Advances in Geomechanics (Goa: Goa) pp 16-23
[2] Budhu M 2010 Soil Mechanics and Foundations (New York: Wiley)
[3] Smith E A L 1960 J. of the Soil Mech. and Found. Division, 86 35-64
[4] Mahajan S P 2006 Viscous effects on penetrating shafts in clay (University of Arizona)