Shape optimization for stiffness maximization of geometrically nonlinear structure by considering fluid-structure-interaction

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Received: 12 February 2021; Revised: 26 February 2021; Accepted: 8 March 2021

Abstract
This paper presents numerical solution to a shape optimization for stationary fluid structure interactive fields. In the fluid structure interactive analysis, a weak coupled analysis is used to alternately analyze the governing equations of the flow field domain and the structural field considering geometrically nonlinear. A mean compliance minimization problem is formulated in order to achieve stiffness maximization on the structural field. Shape derivative, which means the sensitivity in the shape optimization problem, is derived theoretically by using the Lagrange multiplier method and adjoint variable method, and the formulae of the shape derivative with respect to domain variation of the distribution function. Reshaping is carried out by the $H^1$ gradient method proposed as an approach to solving shape optimization problems. Numerical analysis program for the problem is developed by using FreeFEM, and validity of proposed method is confirmed by numerical results of 2D problems.

Keywords: Shape optimization, Adjoint variable method, Finite element method, Fluid-structure-interactive, $H^1$ gradient method

1. Introduction

Fluid-structure-interactive (FSI) problem is an interesting issue not only in mechanical engineering but also in fields such as marine engineering and bioengineering (Tsugawa and Takizawa, 2015). Moreover, in such an engineering field, the shape optimization problem for improving performance is an important issue, and in the bioengineering field, the problem of nonlinear structure is often dealt with. In this study, we discuss shape optimization of nonlinear structure for the FSI problem.

Shape optimization can be divided into topology shape optimization and shape optimization as one classification. Research on the shape optimization of FSI problem has been carried out by several researchers. Yoon (2010) proposed topology shape optimization using a monolithic formulation. Jenkins and Maute (2015) classified "dry optimization", which optimizes only the internal structure, and "wet optimization", which considers the shape of the fluid-structure interface as the design boundary, for the topology shape optimization of the FSI field, and then they analyzed the "dry optimization" problem. Picelli et al. (2017) proposed an analysis method using an evolutionary topology optimization method. However, in these topology shape optimizations, linear strain was assumed in the structural field. For these, Lund et al. (2003), Jang et al. (2014), and Aghajari et al. (2015) have performed analysis based on shape optimization assuming nonlinear strain on the structural field. In the studies on shape optimization conducted by Lund et al. (2003), Jang et al. (2014), and Aghajari et al. (2015), the design boundary was represented by a B-spline function and the design variables were minimized using a parametric analysis.

On the other hand, the authors have proposed solutions to fundamental shape optimization problems for maximizing the stiffness of the linear thermoelastic bodies (Katamine and Arai, 2017) and for minimizing the dissipative energy in viscous flow fields (Katamine et al. 2005), and demonstrated their validity. These solutions use the $H^1$ gradient method...
2.1. Governing equations for elastic bodies

The governing equations based on the second Piola–Kirchhoff stress $\Sigma(u'(x))$ is expressed as follows (Ciarlet, 1993):

$$\begin{align*}
-\text{div}((I + \nabla u'(x)) \Sigma(u'(x))) &= f'(x), \quad x \in \Omega^f, \\
u'(x) &= 0, \quad x \in \Gamma_0^d, \\
(I + \nabla u'(x)) \Sigma(u'(x)) n &= h(u'(x)), \quad x \in \Gamma^s,
\end{align*}$$

where $f'(x)$ is the body force, $h(u'(x))$ is the surface traction acting from the fluid side. Since the shape optimization problem shown later is formulated only by the state variables of the elastic body, the surface traction $h$ is considered as a function of $u'(x)$. According to the Piola transformation (Ciarlet, 1993), $h(u'(x))$ is expressed as follows:

$$h(u'(x)) = T(x)n = (\det \varphi(x)) T^s(x^s) (\nabla \varphi(x))^{-T} n = (\det(I + \nabla u'(x))) T^s(x^s) (I + \nabla u'(x))^{-T} n,$$

where $T(x)$ denotes the stress tensor (the first Piola–Kirchhoff stress tensor) at the position prior to deformation $x$, and $T^s(x^s)$ denotes one at the position after deformation $x^s$, which is called the Cauchy stress tensor. For simplicity, we will omit $(x)$; for example, displacement $u'(x)$ as $u'$ in subsequent discussions.

The governing equation (1) above is multiplied by test function $u''(w') = 0$ on $\Gamma_0^d$ and integrated over the domain. By considering the symmetry of the second Piola–Kirchhoff stress, $\Sigma(u')$, a weak formulation of the governing equation can be derived as follows:

$$a'(u', u'', w') = l'(w') + d'(u', w') \forall u'' \in \Omega^f.$$  

Note that terms $a'(u', u'', w')$, $l'(w')$, and $d'(u', w')$ can be derived as follows:

$$a'(u', u'', w') = \int_{\Omega^f} \Sigma(s'(u')) : \text{det}(s'(u')) [w''] dx, \quad l'(w') = \int_{\Omega^f} f^s \cdot w' dx, \quad d'(u', w') = \int_{\Gamma^s} h(u') \cdot w' d\Gamma.$$
where $\varepsilon'(u^i)$ is Green–St Venant strain, defined as

$$
\varepsilon'(u^i) = \frac{1}{2}(\nabla u^i)^T + \nabla u^i + (\nabla u^i)^T (\nabla u^i),
$$

and the first variation $de'(u^i)[w^i]$ is calculated as

$$
de'(u^i)[w^i] = \frac{1}{2}((\nabla u^i)^T + \nabla u^i + (\nabla u^i)^T (\nabla (w^i))^T (\nabla (u^i))).
$$

Stress $\Sigma(e'(u^i))$ is expressed as $\Sigma(e'(u^i)) = \lambda(\varepsilon'(u^i))I + 2\mu e'(u^i)$ with Lamé constants $\lambda$ and $\mu$.

Given the surface traction, $h$, acting on the elastic body from the fluid side, elastic displacement $u^i$ can be analyzed by applying the Newton–Raphson method to nonlinear term $a^i(u^i, u^j, w^s)$ of equation (5). The linearized weak form for given $u^i_n$ at the $n$-th iteration and unknown $\delta u^i_n$ is represented as

$$
da^i(u^i_n, \delta u^i_n, w^i) = a^i(u^i_n, u^i, w^i) - l^i(u^i_n) - d^i(u^i_n, w^i) \forall w^i \text{ in } \Omega^i,
$$

where the first derivative, $da^i(u^i, v^i, w^i)$, for $a^i(u^i, v^i, w^i)$ is calculated as

$$
da^i(u^i, v^i, w^i) = \int_{\Omega^i} [\Sigma(\varepsilon'(u^i))]: d\varepsilon'(u^i)[v^i] + d\varepsilon'(u^i)[\varepsilon'(u^i)] dx,
$$

and the second derivative, $d^2\varepsilon'(u^i)[\delta u^i_n, w^i]$, for $\varepsilon'(u^i)$ is calculated as

$$
d^2\varepsilon'(u^i)[\delta u^i_n, w^i] = \frac{1}{2}((\nabla \delta u^i_n)^T \nabla w^i + (\nabla w^i)^T (\nabla \delta u^i_n)),
$$

which is independent of $u^i_n$.

### 2.2. Governing equations for flow fields

The weak forms of the Navier–Stokes and continuity equations with fluid density $\rho^f$ and coefficient $\mu^f$ of viscosity in the flow field domain, $\Omega^f$, are given as (Katamine et al., 2005)

$$
a^f(u^f, w^f) + a^f_1(u^f, u^i, w^f) + b^f(p, w^f) = 0 \text{ in } \Omega^f,
$$

$$
b^f(q, u^f) = 0 \text{ in } \Omega^f.
$$

Note that terms $a^f(u^f, w^f)$, $a^f_1(u^f, u^i, w^f)$, and $b^f(p, w^f)$ are defined as follows:

$$
a^f(u^f, w^f) = \int_{\Omega^f} 2\mu^f \varepsilon^f(u^f) : \varepsilon^f(u^f) dx,
$$

$$
a^f_1(u^f, v^f, w^f) = \int_{\Omega^f} \rho^f (u^f \cdot \nabla v^f) \cdot w^f dx,
$$

$$
b^f(p, w^f) = -\int_{\Omega^f} \nabla \cdot w^f p dx,
$$

where $u^f$ is given on $\Gamma^f_0$ and $p$ are the flow velocity and pressure, respectively, and $w^f(w^f = 0$ on $\Gamma^f_0$) and $q$ are the test functions. In addition, the stress $\sigma^f(p, u^f)$ in flow field domain is defined as follows:

$$
\sigma^f(p, u^f) = -p I + 2\mu^f \varepsilon^f(u^f), \quad \varepsilon^f(u^f) = \frac{1}{2}((\nabla u^f + (\nabla u^f)^T).
$$

### 2.3. Surface traction of elastic fields based on stress in the flow field domain

At the common boundary of $\Gamma^f$ on the deformed domain, the balance of the force is written as

$$
h^s(x^s) + \sigma^f(p(x^s), u^i(x^s))n^s(x^s) = 0, \quad \text{where } h^s(x^s) \text{ is surface traction after deformation } x^s,
$$

with $n^s(x^s) = -n^s(x^s)$, which is the unit normal vector to the fluid domain at point $x^s$. According to the Piola transformation, surface tractions $h(u^i(x))$ and $h^s(x^s)$ and unit normal vectors $n$ and $n^s$ have the following relations (Ciarlet, 1993):

$$
h(u^i(x)) = \text{det} \nabla \varphi(x) \nabla \varphi(x)^{-T} n^s(x^s),
$$

$$
n^s = \frac{\text{Cof}(\nabla \varphi(x)) n}{|\text{Cof}(\nabla \varphi(x))|}, \quad \text{with } h(u^s) = \text{det} \nabla \varphi(x) \nabla \varphi(x)^{-T} n.
$$

By combining these relations using $|\text{Cof}(\nabla \varphi(x))| = \text{det}(\nabla \varphi(x)) \nabla \varphi(x)^{-T} n$, $h(u^s)$ is represented as

$$
h(u^s) = (\text{det} \nabla \varphi(x)) \sigma^f(p(x^s), u^i(x^s)) (\nabla \varphi(x)^{-T} n = (\text{det}(I + \nabla u^s)) \sigma^f(p, u^i) (I + \nabla u^i)^{-T} n.
$$

The final elastic displacement $u^t$, flow velocity $u^t$, and pressure $p$ in the FSI problem are then analyzed based on the iterative calculations of elastic field equation (5) and the flow field equations, (12) and (13).
2.4. Algorithm for FSI analysis

The analysis of FSI is performed by the following weak coupled scheme, and the common boundary \( \Gamma^s \) and the fluid domain \( \Omega^f \) are updated sequentially.

**Step 1.** Define the common boundary \( \Gamma^s \) between the fluid and elastic domain (iteration: \( k = 1 \), and \( u^s_k(x) = 0 \)).

**Step 2.** For the deformed fluid domain \( \Omega^f_k(x^s) \), \( u^s_k \) and \( p_k \) are analyzed using equations (12) and (13), and the stress \( \sigma_k^f(p_k(x), u^s_k(x^s)) \) at the common boundary \( \Gamma^s_k \) is calculated. If \( k < 2 \), the stress \( \sigma_k^f(p_k(x), u^s_k(x)) \) at the \( \Gamma^s_k \) in the reference fluid domain \( \Omega^f_k(x) \) is calculated.

**Step 3.** Calculate the \( b_k(u^s_k(x)) \) using equation (18).

**Step 4.** For the reference elastic domain \( \Omega^s \), \( u^s_k(x) \) is analyzed using equation (5).

**Step 5.** \( \| u^s_k(x) - u^s_{k-1}(x) \| \) is calculated, and if it is converged for iteration \( k \), the analysis ends.

**Step 6.** Based on the \( u^s_k(x) \), the boundary \( \Gamma^s_k(x) \) is moved to \( \Gamma^s_{k+1}(x^s = x + u^s_k(x)) \), and the fluid domain \( \Omega^f_k(x^s) \) is updated to \( \Omega^f_{k+1}(x^s) \) by \( k = k + 1 \), and returns to **Step 2**.

3. Shape optimization of structure in FSI field

3.1. Stiffness maximization problem for structure

This problem is formulated as follows:

Find \( \Omega_{opt}^s \)

that minimize \( I^s(u^s) + d^s(u^s, u^s) \)

subject to \( (5) \) and \( \int_{\Omega^s} dx \leq M \). \hfill (19)

Equation (19) is the objective functional indicating mean compliance considering the influence of the fluid field. \( \int_{\Omega^s} dx \leq M \) is the constraint equation in the structural domain, where \( M \) is the size of the initial domain. For simplification, we ignore the domain variation of the boundary \( \Gamma^s \) for the shape optimization.

3.2. Shape derivative function

This problem can be formulated as a stationary problem without constraints by using the Lagrange multiplier method. In this case, the Lagrange function \( L(\Omega^s, u^s, w^s, \Lambda) \) is given by the following equation:

\[
L(\Omega^s, u^s, w^s, \Lambda) = I^s(u^s) + d^s(u^s, u^s) = -a^s(u^s, u^s, w^s) + I^s(u^s) + d^s(u^s, w^s) + \Lambda \left( \int_{\Omega^s} dx - M \right),
\hfill (21)
\]

where \( w^s \) is the adjoint variables for the governing equation in elastic domain \( \Omega^s \). Also, \( \Lambda \) is the Lagrange multiplier with respect to the domain size constraint.

Calculating for the shape derivative of \( L(\Omega^s, u^s, w^s, \Lambda) \) with respect to domain variation for the elastic domain \( \Omega^s \), the following conditional expressions can be derived with respect to the adjoint displacement \( w^s \), and Lagrange multiplier \( \Lambda \), based on optimality conditions for the shape optimization (Azegami, 1994, 2014):

\[
\int_{\Omega^s} \left[ \Sigma d\varepsilon(u^s)[u^s'] : d\varepsilon(u^s)[u^s'] + \Sigma \varepsilon(u^s) : d^2\varepsilon(u^s)[u^s', u^s''] \right] dx - d^s(u^s, w^s)[u^s'] - I^s(u^s') = 0 \hfill (22)
\]

\[
\Lambda \geq 0, \int_{\Omega^s} dx \leq M, \Lambda \left( \int_{\Omega^s} dx - M \right) = 0. \hfill (23)
\]

Here, \( (\cdot)' \) represents the derivative of the distribution function fixed in spatial coordinates with respect to the domain variation. In equation (22), the shape derivative \( d''(u^s, u^s)[u^s', u^s''] \) for \( d'(u^s, u^s) \) is represented as

\[
d''(u^s, u^s)[u^s', u^s''] = \int_{\Omega^s} \left[ \delta (h'(u^s)) \cdot u^s + h(u^s) \cdot u^s' \right] d\Gamma.
\]

\[
\int_{\Omega^s} \left[ \delta (h'(u^s)) \cdot u^s + h(u^s) \cdot u^s' \right] d\Gamma = \int_{\Omega^s} \left[ \delta (h'(u^s)) \cdot u^s + h(u^s) \cdot u^s' \right] d\Gamma.
\]

To obtain (24), we used the shape derivatives for domain Jacobi determinant and domain Jacobi inverse matrix (Azegami, 2020, section 9.2.1), which is \( \delta (h'(u^s))' [u^s'] = \nabla \cdot u^s' \) and \( \delta (h'(u^s))' [u^s'] = - \nabla u^s'. \) Also, the shape derivative \( d''_w(u^s, w^s)[u^s'] \) for \( d'(u^s, w^s) \) in (22), where \( \delta (\cdot) = \partial (\cdot)/\partial u^s \), is represented as \( d''_w(u^s, w^s)[u^s'] = \int_{\Omega^s} h'(u^s)[u^s'] \cdot w^s d\Gamma. \)
The shape derivative function $\bar{g} = g^n$, which shows the sensitivity on the design boundaries for the shape optimization, can be derived as follows:

$$g^n = -\Sigma(u^n(u^n)) : de(s^n(u^n)[u^n]+f^n \cdot (u^n + w^n) + \Lambda.$$  (25)

As the shape derivative function $\bar{g}$ can be derived for this problem, the $H^1$ gradient method, called the traction method assuming linear strain for the domain variation of elastic domain (Azegami, 2014, section 9.2), can be applied for this shape optimization problem.

### 3.3. Numerical Analysis procedure for shape optimization

The proposed shape optimization analysis can be performed by repeating the following steps (shown as Fig. 2):

**Step 1.** Give initial shape.

**Step 2.** Analyze the elastic displacement, $u^s$, flow velocity $u^f$, and pressure $p$ in the FSI problem by using the weak coupling analysis based on the algorithm of section 2.4.

**Step 3.** Stop when the objective functional has converged.

**Step 4.** Analyze the adjoint elastic displacement, $w^s$, based on the adjoint equation (22).

**Step 5.** Calculate the shape derivative function, $\bar{g}$, based on the equation (25).

**Step 6.** Analyze the domain variation based on the $H^1$ gradient method to update the elastic domain. Return to **Step 2**.

### 4. Numerical examples

In this study, the numerical analysis program was developed using FreeFEM (Ootsuka and Takaishi, 2014). Based on Algorithm 1b in the tutorial (Suzuki, 2018), the FSI field was analyzed using the P2/P1 elements for the flow velocity and pressure of the flow field and the P2 element for the displacement of the elastic field. Moreover, for analyzing the FSI, the fixedborder option of FreeFEM was used to prevent reassignment of the finite element nodes at the common boundary, $\Gamma^s$, of elastic domain $\Omega^s$ and flow field domain $\Omega^f$.

![Fig. 3 For the structure on the cavity flow, the stiffness maximization problem is set with the circular holes as the design boundaries.](image)

**Fig. 3** For the structure on the cavity flow, the stiffness maximization problem is set with the circular holes as the design boundaries.

![Fig. 4 Flow velocity and pressure distribution for flow field on cavity flow, there is almost no difference in the flow field, as compared with the comparison of linear and non-linear in the initial shape.](image)

**Fig. 4** Flow velocity and pressure distribution for flow field on cavity flow, there is almost no difference in the flow field, as compared with the comparison of linear and non-linear in the initial shape.
4.1. Structure on cavity flow

We analyzed the structure on the cavity flow, as shown in Fig. 3. For the boundary conditions in the viscous flow field, a flow velocity was applied on the left boundary and set to zero at the other boundaries. For the boundary conditions in the structure, the left and right boundaries were fixed, while the lower boundary was set as \( \Gamma^s \) between the fluid and structure. With a flow velocity on the left side of \( \dot{u} = 1 \text{ m/s} \), density \( \rho_f \) and coefficient of viscosity \( \mu_f \) were set such that the Reynolds number was \( Re = 100 \). In addition, Poisson’s ratio was set as \( \nu = 0.3 \), and Young’s modulus was set as \( E = 6 \text{ kPa or } 50 \text{ kPa} \). For \( E = 6 \text{ kPa} \), we performed an analysis by assuming linear strain. Then, Lamé constants \( \lambda \) and \( \mu \) were calculated based on Young’s modulus \( E \) and Poisson’s ratio \( \nu \) as \( \lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \) and \( \mu = \frac{E}{2(1 + \nu)} \), respectively. For the model dimensions, \( L = 1 \text{ m} \) and \( w = L/5 \text{ m} \). The holes in the structure were set as the design boundaries, \( \Gamma^s_{\text{design}} \), and body force \( f_s \) was disregarded.

Figures 4 to 7 show the numerical results. Figure 4 shows the flow velocity and pressure distributions for the deformed flow field for the initial shape of the cases of \( E = 6 \text{ kPa} \) (Linear) and \( E = 6 \text{ kPa} \) (Non-linear). Based on Fig. 4 and Fig. 5, it can be confirmed that there is a difference in the deformation of the structure, but the difference in the flow field is slightly, as compared with the comparison of linear and non-linear in the initial shape. As shown in Fig. 5(c), when \( E = 50 \text{ kPa} \), the deformation was so small that nonlinearity could be ignored. The comparison of Figs. 5 (a) and (b) shows that when \( E = 6 \text{ kPa} \), the amount of deformation is smaller in Fig. 5 (b), where nonlinearity is considered. This is because of the deformation restraining phenomenon called the lifting effect (Izui and Sakai, 2010), which is characteristic of the geometrically nonlinear structure of the beams fixed at both ends. Figures 6 shows the deformed state for optimized shapes. The optimized shapes shown in Figs. 7 (a) and (c) are similar because in Fig. 7 (c), the Young’s modulus is large and nonlinearity is not appeared, so the optimum shapes of Case (a) and (c) are similar. The comparison between Cases (a) and (b) shows the results for consideration of geometrically nonlinear.
For the obstacle in channel, the stiffness maximization problem is set with the circular holes as the design boundaries.

Fig. 8 For the obstacle in channel, the stiffness maximization problem is set with the circular holes as the design boundaries.

Fig. 9 Flow velocity and pressure distribution for flow field on obstacle problem in channel, Case of $E = 5$ kPa (Non-linear)

enough that the effect of nonlinearity is not observed. In contrast, the comparison of Figs. 7 (a) and (b) shows that the effect of nonlinearity appears in the optimized shape in Fig. 7 (b). In addition, the objective functional decreased by 26% for $E = 6$ kPa (linear), 23% for $E = 6$ kPa (nonlinear), and 26% for $E = 50$ kPa (nonlinear), and the stiffness improved with constant domain size. In Fig. 6, the stress concentration part can be seen. However, since the purpose of this study is to maximize the stiffness, not the strength, it is unavoidable. In fact, the similar results can be seen in other studies (Azegami, 2020, Katamine and Arai, 2017) on shape optimization for stiffness maximization based on minimizing mean compliance.

4.2. Obstacle in pipe channel flow

The problem is shown in Fig. 8. For the boundary conditions in the flow field, a Poiseuille flow was applied to the left boundary, the right boundary was set as a natural boundary, and the flow velocities at the upper and lower boundaries were set to zero. For the boundary conditions in the structure, the lower boundary was fixed and the other boundaries were set as the common boundaries, $\Gamma_s$, between the fluid and structure. With a maximum flow velocity on the left side of $u^f = 1$ m/s, density $\rho^f$ and coefficient of viscosity $\mu^f$ were set such that the Reynolds number was $Re = 100$. We set Poisson’s ratio $\nu = 0.3$ and Young’s modulus as $E = 5$ kPa (nonlinear) and $E = 40$ kPa (nonlinear). The model dimensions were set as $D = 4$ m, $L = 16$ m, $h = D/4$ m, $w = h/5$ m, and the distance from the inflow boundary, $\Gamma_0^f$, to the center of the obstacle was $L/4$ m. The boundaries with initially round shapes inside the obstacle were set as $\Gamma_{design}$, and body force $f^s$ was disregarded.

Figures 9 to 10 show the numerical results. Figure 9 shows the flow velocity and pressure distributions for the

Fig. 10 Deformed shapes (Initial and optimized) and initial and optimized shapes (Dashed line: Initial, Solid line: Optimized) for obstacle in channel. The optimum shapes of Case (b) are almost symmetric with respect to the left and right side of the obstacle, with a negligible effect of nonlinearity. In contrast, Case (a), the effect of nonlinearity was large, and thus the optimum shapes lost symmetry.
deformed flow field for the case of $E = 5$ kPa. Figure 10 shows the numerical results, with a large deformation in Fig. 10 (a) $E = 5$ kPa such that the deflection exceeds the obstacle width. In contrast, the deformation in Fig. 10 (b) $E = 40$ kPa is very small. For the optimized shapes in Fig. 10 (b), the shapes are almost symmetric with respect to the left and right side of the obstacle, with a negligible effect of nonlinearity. In contrast, in Fig. 10 (a), the effect of nonlinearity was large, and thus the shapes lost symmetry. The objective functional decreased by 13% for $E = 5$ kPa and 15% for $E = 40$ kPa, this showing an improvement in the stiffness.

5. Conclusions

In this paper, we discussed a shape optimization problem for maximizing stiffness of a geometrically nonlinear structure by considering stationary FSI. We formulated the problem, derived the shape derivative function for the problem, and presented examples of numerical analysis using the $H^1$ gradient method to show the validity of the proposed solution.

Acknowledgments

The advice on FSI problem of Dr. Atsushi Suzuki, Osaka University in Japan, was helpful in conducting the present study. The present study was supported in part by The OGAWA Science and Technology Foundation in Japan.

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