Boson-fermion stars: exploring different configurations

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We use the flexibility of the concept of a fermion-boson star to explore different configurations, ranging from objects of atomic size and masses of the order $10^{18}$ g, up to objects of galactic masses and gigantic halos around a smaller core, with possible interesting applications to astrophysics and cosmology, particularly in the context of dark matter.

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I. INTRODUCTION

Scalar fields have been playing an increasingly important role in recent investigations in cosmology and astrophysics. A number of such fields have been considered, ranging from the dilaton in superstrings, to the Higgs field and from the postulated inflaton, to the axion. This is one of the reasons for the interest in the properties of stellar objects made out of boson fields, called boson stars, since it is quite possible that such objects play, or have played, a considerable role as components of the dark matter of the universe. Moreover, such exotic objects provide us with unique insights into very high energy physics which, apart from the imprints left in the Early Universe, is currently almost completely unaccessible to experiment.

Boson stars can be seen as macroscopic boson condensates, formed by the vacuum expectation value of scalar fields, the particles being the excitations over the vacuum expectation value. First studied by Ruffini and Bonnazola, their work was later generalised to include self-interactions, gauge charges, excited solutions of the coupled Einstein and Klein-Gordon equations, as well as non-topological soliton solutions, Q-stars, oscillating soliton stars, and even stars with two bosons with different masses. The stability of these stars was studied in two complete reviews on the subject can be seen in references and . Two complete reviews on the subject can be seen in references and .

However, if these stars had their origin in some primordial gas of both bosons and fermions, one would certainly expect to find objects made out of a mixture of these two types of particles. Even if they were purely bosonic or fermionic at their origin, they would later be susceptible to a considerable amount of contamination by bosons or fermions, respectively. This is why it is important to investigate these mixed objects, now called boson-fermion stars. We shall not study here the important question of how these stars, both pure boson or boson fermion might have been formed, by gravitational condensation, out of a primordial gas. This issue has been addressed in part, for pure scalar fields, in and . We shall assume these mixed stellar objects to exist and study the configurations they can take as stable systems.

Their stability and main properties have been investigated in . Generalising simple heuristic arguments by Thirring, it was then pointed out that many of the general properties could be understood on the basis of an interplay between the Pauli principle and the Heisenberg principle, the first being ultimately responsible for the stability of the fermionic component and the uncertainty principle being responsible for the stability of the bosonic component. The dependence on these two fundamental principles of quantum mechanics is responsible for the very different characteristics shown by these stars, when we have configurations that are either predominantly fermionic or predominantly bosonic.

In most of the former investigations on the subject, the fermions were taken to be neutrons. We now relax this assumption, taking the fermion mass as a free parameter, accepting as reasonable that we may have different varieties of fermions in a primordial gas. As for the boson field, we take it to be a real, massive, self-interacting scalar field, with the mass of its associated particle also taken as a free parameter.

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Now that new ideas, like the one provided by fermion balls\cite{20} and supermassive boson stars\cite{21}, are being put forward to explain the properties of galaxies, particularly the possible existence of supermassive dark objects at galactic centers, or of using boson stars in the role of gravitational lenses\cite{22}, it is appropriate to come back to the question of fermion-boson stars and to investigate if they can provide us with alternative scenarios of interest in astrophysics and cosmology, in particular when coupled with the problem of dark matter. As a result of the properties of bosons and fermions, we are able, for instance, to construct compact fermionic cores, more or less massive, depending on the mass taken by the fermion, surrounded by immense halos of bosons, whose sizes are also dependent on the mass of the corresponding boson. Very different objects can be constructed, by varying the masses of their components.

Modern high-energy physics theories providing us with a large variety of bosons and fermions as serious candidates for dark matter, including massive sterile neutrinos, gravitinos, axinos as well as Higgs fields, axions, heavy scalars resulting from non-thermal decays of inflatons, and still others\cite{23}, make it worth while to pursue these theoretical constructions and explorations.

II. THE MODEL

The equations have been derived before, nevertheless, and for completeness, we repeat them here. The metric we shall use is the one appropriate to a stationary, spherically symmetric distribution of matter:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2;$$ (1)

The Lagrangian for the massive real scalar field is defined by

$$\mathcal{L} = -\frac{1}{2}\gamma^\mu\gamma^\nu \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4}\lambda \varphi^4$$ (2)

and from it we derive the energy-momentum tensor:

$$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - \gamma_{\mu\nu} \left( \frac{1}{2}\gamma^\rho\gamma^\sigma \partial_\rho \varphi \partial_\sigma \varphi + \frac{1}{2}m^2\varphi^2 + \frac{1}{4}\lambda \varphi^4 \right).$$ (3)

We expand the scalar field in terms of creation and annihilation operators in the usual manner\cite{15,16,17}, in such a way that $\varphi^+ = \varphi$, as is appropriate for a real field:

$$\varphi(r, t) = \Sigma_n \frac{1}{(2\omega_n)^{1/2}} (a_n \varphi_n(r)e^{-i\omega_n t} + a_n^+ \varphi_n^+(r)e^{i\omega_n t})$$ (4)

with

$$[a_m, a_n^+] = \delta_{mn}.$$ (5)

We shall assume in our problem all the bosons to be in the discrete quantum state defined by $n = 0$, defining the lowest energy of the star, $\omega_0$. This corresponds to a spherically symmetric and nodeless wave function $\varphi_0(r)$. It also means that the bosons, in number $N_B$, will be in the quantum state

$$|N_B\rangle = (N_B!)^{-1/2}(a_0^+)^N|0\rangle.$$ (6)

Following Breit et al.\cite{24}, we find that the canonical commutation relations require that

$$\int 4\pi r^2 \sqrt{\frac{A}{B}} |\varphi_0|^2 dr = 1.$$ (7)

The Einstein equations being classical equations, in order to write their right hand side we take the following expectation value for the energy-momentum tensor:

$$T_{\mu\nu} = \langle N_B| T_{\mu\nu} |N_B\rangle = \partial_\mu \varphi_c^+ \partial_\nu \varphi_c - \frac{1}{2}\gamma_{\mu\nu} \left( \gamma^\rho\gamma^\sigma \partial_\rho \varphi_c^+ \partial_\sigma \varphi_c + m^2\varphi_c^+ \varphi_c + \frac{1}{2}\lambda |\varphi_c|^4 \right)$$ (8)

where we introduced, with the help of eq. (6), the semi-classical field

$$\varphi_c(r, t) = (\omega_0)^{1/2}(N_B + \frac{1}{2})^{1/2}\varphi_0(r)e^{-i\omega_0 t}.$$ (9)
The normalisation condition now takes the form
\[
\int 4\pi r^2 \sqrt{A/B} \varphi^2 dr = \frac{1}{\omega_0} \left( N_B + \frac{1}{2} \right).
\]  
(10)

It is well known that Fermi statistics leads the energy-momentum tensor to approach the perfect fluid form, when the number of fermions present is large [2, 21]. The equation we shall use for the degenerate fermion gas was first derived by Chandrasekhar and has the following parametric form:
\[
\rho = K (\sinh T - T) 
\]  
(11)
\[
p = \frac{K}{3} (\sinh T - 8 \sinh \frac{T}{2} + 3T) 
\]  
(12)
the constant \( K \) being given by \( K = m_f^2/(32 \pi^2) \) where \( m_f \) is the mass of the fermion, which we shall leave as a free parameter. The parameter \( T \) is defined in terms of the maximum value of the momentum of the Fermi distribution at radius \( r \) by
\[
T(r) = 4 \log \left( \frac{p_0}{m_f} + \sqrt{1 + \frac{p_0^2}{m_f^2}} \right). 
\]  
(13)

Finally, the particle density \( n(r) \) is given by the expression
\[
n(r) = \frac{p_0^3}{3 \pi^2} = \frac{m_f^3}{3 \pi^2} \sinh^3 \left( \frac{r}{r_0} \right). 
\]  
(14)

In the absence of an explicit non-gravitational interaction between fermions and bosons, the Einstein equations are
\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G (T_{\mu\nu}(\varphi_c) + T_{\mu\nu}(\rho, p)) , 
\]  
(15)
with \( T_{\mu\nu}(\varphi_c) \) given by eq. (5) and the fermion part is defined by the perfect fluid expression
\[
T_{\mu\nu}(\rho, p) = (\rho + p) u_\mu u_\nu + g_{\mu\nu} p 
\]  
(16)
u_\mu being the 4-velocity of the fluid. From the Bianchi identities we find that the covariant derivative of the energy-momentum tensor must be zero, from which condition we derive the following two equations, which will then be coupled to the Einstein equations:
\[
d\rho \over dr = -\frac{1}{2} \frac{B'}{B} (\rho + p) 
\]  
(17)
and, for the scalar field,
\[
\Box \varphi_c = m_f^2 \varphi_c + \lambda |\varphi_c|^2 \varphi_c 
\]  
(18)
where \( \Box \) is the d’Alembertian operator associated with our metric. Before we proceed, we must remember that the explicit time dependence that appears in this equation and in eq. (5) disappears, the exponential factor \( \exp(-\omega_0 t) \) being cancelled out; the result of the action of time derivatives is then the term \( \omega_0^2 \varphi_c \).

We introduce some redefinitions of the variables [17], which enable us to write the equations in a very convenient dimensionless form \( (c = \hbar = 1) \):
\[
x = mr; \quad \sigma(x) = (4\pi G)^{1/2} \varphi_c(r, 0); \quad \Omega = \frac{\omega_0}{m}; \quad \Lambda = \frac{\lambda}{4\pi G m^2}; \quad \bar{\rho} = \frac{4\pi G \rho}{m^2}; \quad \bar{\rho} = \frac{4\pi G p}{m^2}. 
\]  
(19)
The equations to be integrated become:
\[
A' = -\frac{A(A-1)}{x} + x A^2 \left( \frac{\Omega^2 \sigma^2}{B} + \frac{\sigma^2}{A} + \sigma^2 + \frac{1}{2} \lambda \sigma^4 + 2\bar{\rho} \right); 
\]  
(20)
\[
B' = \frac{B(A-1)}{x} + x A B \left( \frac{\Omega^2 \sigma^2}{B} + \frac{\sigma^2}{A} - \sigma^2 - \frac{1}{2} \lambda \sigma^4 + 2\bar{\rho} \right); 
\]  
(21)
\[
\sigma'' = -\left( \frac{B'}{2B} + \frac{A'}{2A} + \frac{2}{x} \right) \sigma' - \frac{A}{B} \Omega^2 \sigma + A \sigma + A \lambda \sigma^3; 
\]  
(22)
\[
T' = -\left( \frac{d\bar{\rho}}{dT} \right)^{-1} \frac{B'}{2B} (\bar{\rho} + \bar{p}); 
\]  
(23)
The last equation is derived from eq. 17 and
\[ \bar{\rho} = \alpha (\sinh T - T) , \]  
\[ \bar{p} = \frac{1}{3} \alpha \left( \sinh T - 8 \sinh \frac{T}{2} + \frac{T}{3} \right) \]  
and \( \alpha = m_f^4/(8\pi m^2 m_{Pl}^2) \), \( m_{Pl} = G^{-1/2} \) being the Planck mass.

To calculate the total mass of the star we use

\[ A(x) = \left( 1 - 2 \frac{M(x)}{x} \right)^{-1} . \]  

Integrating the equations up to a point sufficiently far out that we may consider the fermion and boson terms to be negligible, we obtain the mass from \( x \)

\[ M(x) = \frac{x}{2} \left( 1 - \frac{1}{A} \right) . \]  

The number of fermions and the number of bosons in our star can also be obtained from analytical expressions. From eqs. 10 and 14 we find

\[ N_B = \omega \int 4\pi r^2 \sqrt{\frac{A}{B}} \varphi_c^2 dr = \Omega \left( \frac{m_{Pl}}{m} \right)^2 \int \sqrt{\frac{A}{B}} \sigma x^2 dx \]  

\[ N_f = \int 4\pi r^2 \sqrt{A} n dr = \frac{4}{3\pi} \left( \frac{m_f}{m} \right)^3 \int \sinh^2 \frac{T}{4} \sqrt{A} x^2 dx , \]

where we have written both the original expressions and the ones obtained after the redefinitions introduced above. These values enter into the calculation of the binding energy, defined by

\[ E_B = M - (m_b N_b + m_f N_f) . \]

A positive value of \( E_B \) means an unstable configuration. For a more complete discussion of the issue of stability in boson-fermion stars we refer the reader to [16].

Although we cannot separate the mass of the fermionic component of the star, we introduce in addition, as a useful approximation, but only as an approximation, the following quantity for the fermion core:

\[ M_f = \int 4\pi r^2 \rho dr . \]  

From the structure of the equations, we find that the initial conditions are subjected to the following restrictions: \( A(0) = 1 \) and \( \sigma'(0) = 0 \), with \( \sigma(0) \) and \( T(0) \) free parameters. There is no restriction on \( B \), except that \( B = 1 \) at infinity. As the equations are linear in \( B \), this can always be obtained by rescaling \( B \). Apart from \( B \), only \( \Omega \) needs to be rescaled.

Using the redefined variables, once we fix the initial values \( \varphi_c(0) \) and \( T(0) \), and the value of \( \alpha \), our equations and their solutions are mathematically the same, for any combination of the fermion and boson masses giving the same \( \alpha \) in eqs. 21 and 25. Only the scales of the problem will be changed. From eq. 19 we see that, putting \( \sigma(0) = 1 \), \( \varphi_c(0) \) is close to the Planck energy.

In the numerical simulations that follow we put \( \lambda = 0 \). The dynamical influence of this constant on the total mass of the star has been explored in references [3], [15] and [17], but such influence is not particularly relevant for our considerations here. We could also have introduced direct couplings between the fermion and the boson fields, either in the spirit it has been done in [18] and [26] or, in a more phenomenological way, in [27]. They lead to limits in the couplings and masses of the scalar particles. We made sure that, using values within the limits set up in [27], no important alterations would come to our results.

### III. NUMERICAL SIMULATIONS AND DISCUSSION

From the equation for the scalar field, comparing the mass term with the term involving the eigenvalue, corresponding to the second time derivative of \( \varphi \), we would in general expect the eigenvalue \( \omega \) to be approximately of the same
or expected masses. Likewise, we shall explore a large domain of masses for the fermions and the bosons, not restricting ourselves to known values are obviously scaled accordingly.

\( \sigma \) that, by displacing the node of we deal with a combination of masses such that the boson wave function quickly decreases to very small values, well questionable. It is thus important that we do not restrict our attention to a too small domain of values for \( \phi \).

holes, then planckian-like values seem reasonable. When the stars are formed at later stages, this assumption is more such stars to have formed in the early universe, as usually is the case when we deal with objects like primordial black

\( \rho \) appears as a factor in front of eq. (28) for the number of scalars. In most simulations, the mass of the scalar field is order of magnitude as the mass of the scalar field. This is confirmed by our numerical simulations. The eigenvalue \( \omega \) fer- mions and the bosons, not restricting ourselves to known or expected masses.

We shall apply an iterative shooting method to determine the eigenvalue \( \omega \). A technical problem appears when we deal with a combination of masses such that the boson wave function quickly decreases to very small values, well inside the fermion core. We have to make sure this is not due to an overshooting. The way to do it is to make sure that, by displacing the node of \( \sigma(x) \), there are no changes in the total mass of the star and in the number of bosons, which in turn requires that the eigenvalue be fixed with great accuracy [17]. In what follows we shall study two different situations which differ mainly by the relative magnitude of the bosonic and fermionic terms in the Einstein equations.

### A. Fermionic and bosonic terms of the same order of magnitude

To set the problem, we begin by what looks like a natural choice, taking the fermion mass equal to the neutron mass and assuming the different terms in our equations to be of the same order of magnitude. This requires a \( \phi \) not very distant from the Planck scale. We will now analyse four different cases:

**Case 1:** \( T(0) = 1 \); \( \phi(0) = 2.3 \times 10^{17} \text{GeV} \) (\( \sigma(0) = 0.1 \)); \( m_f = m_n \); \( m_b = 2.71 \times 10^{-20} \text{GeV} \)

In this case we find

\[
\begin{align*}
\rho(0) & = 9.96 \times 10^{13} \text{g cm}^{-3} ; \\
M & = 4.76 \times 10^{32} \text{g} \approx M_\odot \ (M_f = 3.44 \times 10^{32} \text{g} \approx 0.1 M_\odot ) ; \\
N_b & = 9.32 \times 10^{76} \ (N_b m_b = 4.50 \times 10^{31} \text{g} \approx M_\odot ) ; \\
N_f & = 2.08 \times 10^{56} \ (N_f m_f = 3.47 \times 10^{32} \text{g} \approx 0.1 M_\odot ) ; \\
E_B / c^2 & = -9.0 \times 10^{31} \text{g} ; \\
r_c & = 15.9 \text{km} ; \\
r_{90} & = 68.9 \text{km} ;
\end{align*}
\]

where \( \rho(0) \) is the central density corresponding to the fermionic component, \( r_c \) is the radius of the fermion core and \( r_{90} \) the radius within which 90% of the total mass of the star is concentrated. This is a value we shall always quote whenever, as is the present case, the tail of the wave function of the scalar field extends beyond the fermion core, forming a halo around it. From the inspection of \( N_b m_b \) and \( N_f m_f \), or \( M \) and \( M_f \), we conclude that the present configuration is dominated by the boson component which accounts for more than 90% of the total mass. This is an example of values we have found in ref. [17]. The binding energy was converted into the mass equivalent.

Keeping the same fermion mass, we go to higher boson masses by giving smaller values to \( \alpha \). For instance, with \( \alpha = 10^{-24} \), we find \( m_b = 2.7 \times 10^{-8} \text{GeV} \) and we have to decide what is a reasonable choice for \( \phi \). One possibility is to assume that, once the gravitational field couples to the energy, the terms in the equations are of the same order of magnitude, which was the philosophy we used above, except that this time this requires a value for \( \phi \approx 10^6 \text{GeV} \). It is instructive to compare this with the case where we give to the scalar field a value close to the Planck scale. First the case where the terms are about equal.

**Case 2:** \( T(0) = 1 \); \( \phi(0) = 2.3 \times 10^{6} \text{GeV} \) (\( \sigma(0) \approx 10^{-12} \)); \( m_f = m_n \); \( m_b = 2.71 \times 10^{-8} \text{GeV} \)

Now we get:

\[
\begin{align*}
\rho(0) & = 9.96 \times 10^{13} \text{g cm}^{-3} ; \\
M & = 6 \times 10^{32} \text{g} \approx 0.1 M_\odot \ (M_f \approx M) ; \\
N_b & = 3.70 \times 10^{47} \ (N_b m_b = 1.79 \times 10^{16} \text{g} \approx 10^{-17} M_\odot ) ;
\end{align*}
\]
\[ N_f = 3.62 \times 10^{56} \ (N_f m_f = 6.04 \times 10^{32} \text{ g} \approx 0.1 M_\odot); \]
\[ E_B/c^2 = -5.44 \times 10^{30} \text{ g}; \]
\[ r_c = 20.8 \text{ km}; \]

where \( r_{90} \) is not quoted, as now the scalar wave function goes practically to zero well inside the fermion radius. The star is entirely fermion dominated and it has the corresponding typical physical quantities, like mass and radius. If we increase \( T(0) \), the number of fermions increases, as expected, but the number of bosons remains practically the same. This is a typical configuration to be considered when dealing with neutron stars contaminated by WIMPS. If the masses of the WIMPS are of the order of \( 10^{-5} \) eV or above, they will concentrate in the core of the star.

In the following example, the scalar field is closer to the Planck scale:

**Case 3:** \( T(0) = 1; \ \varphi_c(0) = 2.3 \times 10^{17} \text{GeV} (\sigma(0) = 0.1); \ m_f = m_n; \ m_b = 2.71 \times 10^{-8} \text{GeV} \)

Now we find

\[ \rho(0) = 9.96 \times 10^{13} \text{ g cm}^{-3}; \]
\[ M = 5.23 \times 10^{21} \text{ g} = 10^{-12} M_\odot \ (M_f = 3.14 \times 10^{-3} \text{ g}); \]
\[ N_b = 1.11 \times 10^{53} \ (N_b m_b = 5.33 \times 10^{21} \text{ g} \approx 10^{-12} M_\odot); \]
\[ N_f = 1.89 \times 10^{21} \ (N_f m_f = 3.17 \times 10^{-3} \text{ g}); \]
\[ E_B/c^2 = -9.99 \times 10^{19} \text{ g}; \]
\[ r_c = 3.13 \times 10^{-6} \text{ cm}; \]
\[ r_{90} = 7.0 \times 10^{-6} \text{ cm}. \]

We have a configuration which is entirely different from the preceding case, although we have kept the same central density \( \rho(0) \). It is now a typical soft boson star, with a microscopic radius, but a still respectable mass. The fermion component has almost completely disappeared. The increase in \( \varphi_c(0) \) has resulted, not only in a substantial increase in \( N_b \), but also in an enormous reduction in the number of fermions. No wonder this has happened as, with the choice made for \( \varphi_c(0) \), the terms in the equations associated with the bosons are very much larger than the terms associated with the fermions. In this sense, it is not a natural choice.

If we increase \( T(0) \) we find that, around \( T(0) = 3 \), the configuration will suddenly become fermion dominated, a property that has been noticed and investigated in the work of reference [17], and we shall have a configuration similar to the one in case 2 above, albeit with a very different value for \( \varphi_c(0) \).

Before going to a different kind of limit, we mention one case corresponding to the fermions and bosons having much higher masses than the ones used so far:

**Case 4:** \( T(0) = 1; \ \varphi_c(0) = 2.3 \times 10^6 \text{GeV} (\sigma(0) \approx 10^{-12}); \ m_f = 10^7 \text{GeV}; \ m_b = 3.08 \times 10^6 \text{GeV} \)

In this case:

\[ \rho(0) = 1.29 \times 10^{42} \text{ g cm}^{-3}; \]
\[ M = 5.28 \times 10^{18} \text{ g} \approx 10^{-15} M_\odot \ (M_f \approx M); \]
\[ N_b = 2.90 \times 10^{19} \ (N_b m_b = 1.59 \times 10^2 \text{ g}); \]
\[ N_f = 2.99 \times 10^{35} \ (N_f m_f = 5.33 \times 10^{18} \text{ g}); \]
\[ E_B/c^2 = -4.79 \times 10^{16} \text{ g}; \]
\[ r_c = 1.83 \times 10^{-8} \text{ cm}. \]

It is really an object of atomic size, with characteristics close to primordial black holes, without being one. The value we took for \( \varphi_c(0) \) is within the same order of magnitude as the masses and is a case where the fermion and boson terms in the equations are of the same order of magnitude.

Again, it is completely fermion dominated. Except for the case 1 above, in all the others, whenever the terms associated with the bosons and the fermions are of about the same order of magnitude, the resulting configuration is dominated by fermions. This happens, for \( m_f = m_n \), when \( m_b \) is larger than \( 10^{-5} \) eV, as in case 2 above.

**B. Fermionic terms dominate**

We shall now investigate what happens when \( m_f \) is equal to or smaller than the nucleon mass and the boson mass is really very small. This means that we have to increase \( \alpha \). The fermion terms will necessarily dominate the equations, whatever value we use for \( \varphi_c(0) \), if we keep it, as we shall do, equal or below the Planck scale. However, due to the
smallness of the boson mass, this does not mean that the star will be fermion dominated. In case of pure boson stars, we know that the total mass of the star tends to change according to $1/m_b^2$. As a first example we take:

**Case 5:** $T(0) = 1$; $\varphi_c(0) = 2.3 \times 10^{37}$ GeV ($\sigma(0) \approx 0.1$); $m_f = m_n$; $m_b = 8.56 \times 10^{-23}$ GeV

Now we have

\[
\begin{align*}
\rho(0) & = 9.96 \times 10^{13} \text{ g cm}^{-3}; \\
M & = 1.65 \times 10^{36} \text{ g} \approx 10^9 M_\odot (M_f = 5.99 \times 10^{32} \text{ g} \approx 0.1 M_\odot); \\
N_b & = 1.10 \times 10^{82} \ (N_b m_b = 1.68 \times 10^{36} \text{ g} \approx 10^3 M_\odot); \\
N_f & = 3.61 \times 10^{56} \ (N_f m_f = 6.04 \times 10^{32} \text{ g} \approx 0.1 M_\odot); \\
E_B/c^2 & = -3.16 \times 10^{34} \text{ g}; \\
r_c & = 20.8 \text{ km}; \\
r_{90} & = 2.18 \times 10^4 \text{ km}.
\end{align*}
\]

We have a complicated configuration, with a core typical of a neutron star, and a very massive boson halo extending well beyond. The bosons dominate the total mass by many orders of magnitude.

Had we used a still smaller boson mass, for instance $m_b = 2.71 \times 10^{-32}$ GeV, we would have got a star with a total mass of $M = 5.26 \times 10^{45} \text{ g} \approx 10^{12} M_\odot$ and a halo extending up to $6.92 \times 10^{13} \text{ km} = 2.24 \text{ pc}$, most of the mass coming from the boson component (the approximate value for the fermion component is $M_f = 5.99 \times 10^{32} \text{ g} \approx 0.5 M_\odot$ and $r_c = 20.8 \text{ km}$, the same as above).

The authors of [21] have tried to show that, choosing a fermion mass of about 16 KeV (for instance, a sterile neutrino), it was possible to explain the main dynamical features of the supermassive compact dark objects, known to exist in the Galactic center and in the center of M87, with the help of what they have dubbed fermion balls, which they propose as an alternative explanation to the more usual black hole model. Better observations [28] have, however, ruled out this model, but still leaving opened the possibility of a supermassive boson star [21].

These ideas suggest that we consider a boson-fermion star with both a small fermion mass and a very small boson mass. When we consider this possibility we get the following configuration:

**Case 6:** $T(0) = 1$; $\varphi_c(0) = 2.3 \times 10^{37}$ GeV ($\sigma(0) \approx 0.1$); $m_f = 10^{-5}$ GeV; $m_b = 3.08 \times 10^{-30}$ GeV

For this last case we have

\[
\begin{align*}
\rho(0) & = 1.29 \times 10^{-6} \text{ g cm}^{-3}; \\
M & = 4.18 \times 10^{43} \text{ g} \approx 10^{10} M_\odot \ (M_f = 3.03 \times 10^{42} \text{ g} \approx 10^9 M_\odot); \\
N_b & = 7.22 \times 10^{96} \ (N_b m_b = 3.96 \times 10^{43} \text{ g} \approx 10^{10} M_\odot); \\
N_f & = 1.71 \times 10^{71} \ (N_f m_f = 3.05 \times 10^{42} \text{ g} \approx 10^9 M_\odot); \\
E_B/c^2 & = -7.98 \times 10^{41} \text{ g}; \\
r_c & = 1.40 \times 10^{11} \text{ km} = 4.53 \times 10^{-3} \text{ pc} \\
r_{90} & = 6.09 \times 10^{11} \text{ km} = 1.97 \times 10^{-2} \text{ pc}
\end{align*}
\]

The fermion core and the boson halo form a supermassive compact object with the halo extending up to $2 \times 10^{-2}$ pc. Keeping the same fermion mass, but taking, for instance, a boson mass in the range of $10^{-32}$ GeV, we would get a total mass of the order of $10^{13}$ solar masses, with a halo extending up to 6 pc, while the fermion component would have practically the same mass and the same radius as in case 6.

Would it in principle be possible to distinguish a boson-fermion system from a fermion ball? Once again this could be done through the careful investigation of the orbits of stars around a candidate object. We must take into account that the mass density profile of a boson-fermion configuration, with an extended boson halo, would be different from the profile of a fermion ball. This should be reflected in the dynamics of the orbits of stars inside such a halo, exactly as there appears a difference between the fermion ball and the black hole models [21]. We are currently investigating this possibility and the results will be reported in a separate publication [29].

We are thus in the presence of a system which can take many different configurations, characterised by very different masses and radii, ranging from objects of atomic size and masses of the order of $10^{18}$ g, to objects having galactic masses and extending up to a few light years. To show this flexibility was the main purpose of the present work.
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