Experimental Validation of Off-design Combustion for Liquid-propellant Rocket Engine High-frequency Instability*

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The mechanism causing high-frequency combustion instability was presumed to be the progression of combustion at an off-design location; that is, in the vicinity of the faceplate. Experiments were carried out to prove this. In the experiments, water was used to simulate the propellant and the increase in pressure caused by off-design combustion was simulated by injecting nitrogen gas. Pressure increase and decay were measured to within approximately 1 ms, and the maximum local pressure was approximately twice the pressure prior to increasing the pressure. To estimate the pressure change in actual engines, a simplified simulation calculation model was constructed. After calibration using the experimental results, the time and amplitude of pressure change were on the same order of magnitude of those of the engines. The present study results show that the off-design combustion model can be a cause of the high-frequency combustion instability experienced when using liquid rocket engines.

Key Words: Rocket Engine, Instability, High Frequency, Combustion, Interaction

Nomenclature

- $a$: acceleration, ambient, width
- $b$: length
- $C$: constant
- $c$: depth
- $D$: diameter
- $d$: slit width
- $F$: force
- $L$: length
- $l$: reference length
- $m$: mass, kg
- $\dot{m}$: mass flow rate
- $p$: pressure
- $R$: gas constant
- $T$: temperature
- $t$: time
- $V$: volume
- $v$: velocity
- $x$: distance
- $\rho$: density
- $\theta$: angle
- $L$: large
- $\dot{m}$: mass flow
- $a$: oxidizer, no gas injection, out
- $p$: propellant
- $p-p$: peak-to-peak
- $S$: small
- $s$: sonic speed, segment
- $wtr$: water
  1: increase in pressure, initial
  2: decrease in pressure, end

1. Introduction

High-frequency combustion instability is one of the traditional problems of liquid-propellant rocket engines. It damages and melts the engines instantaneously. Phenomena of the instability such as injection, mixing, vaporization and combustion have been studied to elucidate the mechanism of instability. It has been revealed that the characteristic time of reaction is significantly shorter than that of the high-frequency instability and that of vaporization is on the same order of magnitude as that of the high-frequency instability. The instability is divided into a transverse mode and a longitudinal mode. The longitudinal mode was analyzed, and it was found that it does not cause serious damage to engines. However, the transverse mode induces damage, caused by a mechanism of energy-transfer from combustion to vibration. As for the mechanism of cyclic motion and amplification, for example, Crocco et al. hypothesized a feedback mechanism and tried to explain the pressure oscillation. However, the relation of each physical phenomenon to the increase in pressure has not been sufficiently clarified. Table 1 shows the injection and instability conditions of some engines. Typical injection styles (i.e., concentric
and impinging injections) and propellants are selected. $\Delta p_{p-p}$ is the peak-to-peak amplitude of pressure change. There are varieties in $\Delta p_{p-p}$ and frequencies. In concentric injectors, the oxidizer injector is inside of the fuel injector and the diameter of the fuel injector, $D_f$, is larger than that, $D_o$, of the oxidizer injector. Pitch or spacing between the injectors is on the order of magnitude of 10 mm.

Most liquid-propellant rocket engines utilize the impinging-jet injector or the coaxial/concentric injector to supply propellants. In both injection methods, base space is formed on the faceplate, as shown in Fig. 1, surrounded by the propellant jets and fans. The base space is filled with vapor and droplets of propellant. However, the combustion chamber is not designed to ignite propellant in this space. If combustion progresses in the base space under a high-pressure condition, the faceplate will melt easily. The author proposed the idea of a mechanism of high-frequency combustion instability using off-design combustion. Herein, off-design combustion is defined as combustion in the base space shown in Fig. 1.

Injected propellant merges and forms a fan or mixing region. This area has large inertia and pressure increases due to the off-design combustion in the base space (Fig. 2(a)). When pressure in the base space is higher than that in the propellant manifold, propellant supply to the combustion chamber stops. The mixing region is moved by the high pressure in the base space, and the off-design combustion gas flows out (Fig. 2(b)). Pressure in the base space decreases as the gas flows out. After pressure in the base space becomes lower than that in the propellant manifold, propellant is once again injected into the combustion chamber. The base space is filled with propellant vapor and droplets again. After sufficient vaporization and mixing, the off-design combustion starts again. For example, during the cycle, flame is anchored to the injectors.

In a previous paper, the characteristic time related to the off-design combustion model was estimated. The time caused by off-design combustion was less than 1 ms and on the same order of magnitude of the times of high-frequency instability listed in Table 1.

Experiments were conducted to verify the increase and decrease in pressure caused by the off-design combustion mechanism. A simplified model for simulating the pressure increase and decrease was constructed. The calculated results were calibrated with the experimental results and the model was applied to the operating conditions of actual engines to determine whether or not the off-design combustion model could be a mechanism causing high-frequency combustion instability. In this study, the experimental and calculated results are presented, and it is discussed off-design combustion can be a cause of instability.

### 2. Experiments

Experiments were conducted to verify that the off-design combustion mechanism causes pressure to increase and decrease. Figure 3 shows schematic diagrams of the plumbing to the combustor model. Water was used in place of propellant and nitrogen gas was used to simulate a pressure increase by combustion.

The velocity of the injected water was approximately $10 \text{ m/s}$, which is on the same order of magnitude of actual engines. Kyowa PGM-3500KP and PGM-100KP pressure sensors were used. Accuracies were within 0.4%. Two kinds of SMC high-speed valves, SX11-KJ and SX11-BJ, were used, the specifications of which are listed in Table 2. Traveling times of the valves is less than 1 ms. As shown later, the period of pressure change in the model was less than those times. In this paper, SX11-KJ is abbreviated as L-valve, and SX11-BJ is S-valve. Nitrogen gas flows out of the valves even when they are closed. Therefore, pressure in the model is measured to be larger than ambient pressure even before the valves are opened.

Figure 4 shows a schematic diagram of the experimental combustor model. Nitrogen gas was injected into the base.

![Table 1. Injection and instability conditions.](image-url)

| Fuel | Oxidizer | $p_{c}, \text{MPa}$ | $\Delta p_{p-p}, \text{MPa}$ | Freq., kHz | Injection style | $D_f, \text{mm}$ | $D_o, \text{mm}$ | Spacing, mm |
|------|---------|-------------------|-----------------|---------|----------------|---------|---------|-----------|
| NASA Lewis | H$_2$ | O$_2$ | 2.07 | 1 | 1.5, 3–3.5, 3.8, 5 | Concentric | 3.6 | 1.3 | 11.6 |
| F-1 | RP-1 | O$_2$ | 7.76 | 12 | 0.44–0.54 | Like impinging | 7.14 | 6.15 | 11 |
| JPL | Aniline/Furfuryl alcohol | Nitric acid | 2.07 | 1.1–10 | 1.8–3.5 | Unlike impinging | 2.5 | 4.4 | 30 |
| DLR | H$_2$ | O$_2$ | 7 | 6 | 10–12 | Concentric | 3.6 | 4 | 10 |

![Fig. 1. Schematic of off-design combustion.](image-url)

![Fig. 2. Schematic of off-design combustion cycle.](image-url)
space to simulate volume increase due to combustion. Two kinds of models, Model 1 and Model 2, were used. Model 2 consisted of Model 1 and a plate to reduce the inside volume of the model. The plate had a hole to measure pressure and a ditch to direct the flow of nitrogen gas injected into the base space. Water was injected at an angle of $60^\circ$ from the injector wall. The slit width of the water injector was 1 mm. The reference length (i.e., imaginary width of the base space) was 12 mm for Model 1, which is twice the reference length for Model 2 (i.e., 6 mm). The reference length was 6 mm for Model 2. They were on the same order of the magnitude of actual engines, as listed in Table 1.

### 3. Calculation Model

To simulate the flow of gas from the base space, a two-dimensional simplified calculation model was constructed.

**Table 2. Valves for $N_2$ injection into the combustor models.**

| High-speed valve | Injection rate, $L_{\text{inj}}$/min | Traveling time, ms |
|------------------|-------------------------------------|-------------------|
| SX11-KJ          | 150 (0.25 MPa)                      | 0.8               |
| SX11-BJ          | 50 (0.25 MPa)                       | 0.55              |

Figure 5 shows a schematic diagram of the calculation model. The propellant jet was divided to $n$ elements, and the length of each jet element was $L_i$. When the velocity of the propellant jet was $v_p$, the traveling time of the element, $\Delta t_i$, for $L_i$ was set to be

$$\Delta t_i = L_i/v_p$$  \hspace{1cm} (1)

Calculation was advanced with this $\Delta t_i$. Under this condition, an element moves to a neighboring position in the flow direction after $\Delta t_i$. Figure 6 shows the movement of the propellant jet elements. The elements moved in the outer direction due to $F$, which was caused by pressure difference. On the other hand, the element neighboring the injector exit was always at the same position. As the exit width increased, the gas flow rate from the base space increased and mass and pressure in the space decreased. When pressure in the base space became equal to the ambient pressure, $p_a$, calculation was terminated.

The velocity of the gas flow was sonic when pressure in the base space is high. When pressure became lower and the sonic condition was not satisfied, the gas flowed out at subsonic speed. The specified propellant flow rate, however, does not change in this simplified calculation model even though pressure in the base space changes.

When the experiments of the present study were simulated, the gas to simulate combustion was supplied even as pressure decreased, as well as during the experiments.
4. Results and Discussion

4.1. Pressure change in experiments

Figure 7 shows typical, measured pressure changes using Model 2 and the L-valve. Table 3 lists the operating conditions of each test. The pressure change with no water injection is also plotted for comparison. The time of pressure increase was approximately 0.3 to 0.4 ms, and that of decrease was approximately 0.4 to 0.5 ms. A quick increase in pressure with large amplitude and decrease was demonstrated. The time pressure increased was less than the valve traveling time. Therefore, gas was still injected into the base space as pressure decreased.

Figure 8 shows the changes using Model 1 and the L-valve. The maximum pressure was lower than those of Model 2, since the inside volume was larger in Model 1 than Model 2. Quick changes in pressure were observed. Figure 9 shows the changes using the S-valve and Model 2. Quick changes in pressure were also observed.

The quick increase and subsequent quick decrease of pressure in the base space were demonstrated through the experiments using high-density, large inertia water injections to simulate combustion.

4.2. Mechanism of pressure increase

Figures 7 to 9 show the tendency of a shorter period of pressure change with higher maximum pressure. Figure 10 shows the relationship between the ratio of maximum pressure to ambient pressure and the time of pressure increase. According to Fig. 10, an inversely proportional relationship was anticipated between them. Hereinafter pressure increase mechanism is discussed.

The propellant jet is pushed outward due to the pressure difference between $p_b$ and $p_c$. When the mean pressure difference is $\Delta \bar{p}$, propellant acceleration is given by

$$ \bar{a}_p = \Delta \bar{p} / (\rho_p \cdot d) $$

(2)

Table 3. Test conditions and results using Model 2 and the L-valve.

| Test No. | $p_{b,\text{max}}$ | Pressure increase time, ms | $N_2$ gas pressure, kPaG | Water pressure, kPaG |
|----------|---------------------|-----------------------------|--------------------------|----------------------|
| 821      | 1.85                | 0.32                        | 494                      | 170                  |
| 806      | 1.66                | 0.40                        | 494                      | 130                  |
| 834      | 1.14                | —                           | 694                      | —                    |

Table 4. Test conditions and results using Model 1 and the L-valve.

| Test No. | $p_{b,\text{max}}$ | Pressure increase time, ms | $N_2$ gas pressure, kPaG | Water pressure, kPaG |
|----------|---------------------|-----------------------------|--------------------------|----------------------|
| 339      | 1.44                | 0.40                        | 559                      | 150                  |
| 324      | 1.33                | 0.43                        | 592                      | 120                  |
| 301      | 1.09                | —                           | 692                      | —                    |

Table 5. Test conditions and results using Model 2 and the S-valve.

| Test No. | $p_{b,\text{max}}$ | Pressure increase time, ms | $N_2$ gas pressure, kPaG | Water pressure, kPaG |
|----------|---------------------|-----------------------------|--------------------------|----------------------|
| 659      | 1.85                | 0.30                        | 690                      | 130                  |
| 643      | 1.58                | 0.34                        | 692                      | 100                  |
| 679      | 1.13                | —                           | 390                      | —                    |
The time required for the propellant with density \( d \), calculated as

\[
d = \Delta \rho \cdot \frac{r^2}{2}
\]

Then

\[
d = \Delta \rho \cdot \frac{r^2}{(2\rho \cdot d)}
\]

Therefore,

\[
\Delta \rho \cdot r^2 = 2\rho \cdot d^2
\]

The pressure increased almost linearly as time increased. Therefore,

\[
\Delta \rho = \Delta p_{\text{max}}/2
\]

and

\[
\Delta p_{\text{max}} \cdot r^2 = 4\rho \cdot d^2
\]

According to Eq. (7), the square of the time required for pressure increase, \( r^2 \), is inversely proportional to \( \Delta p_{\text{max}} \). In Fig. 10, Eq. (7) is plotted for comparison. \( \rho \) is the value of water, 1000 kg m\(^{-3}\), \( d \) is the slit width of the models, 1 mm, and \( p_a \) is the ambient pressure of \( p_a \), 101 kPa. Equation (7) shows a partial relationship with the experimental results. The correlation factor between the experimental results and that calculated is 0.200, being not sufficiently large. The primary reason is the splash caused by the water jet. This was due to using a small experimental model. The size was specified by the ability to supply gas and water. Since some relationship is found between the experimental and calculated results, a sufficiently larger model and the ability supply of gas and water will improve experimental results.

Figure 11 shows the average \( \Delta \rho \cdot r^2 \) to water injection pressures. According to Eq. (7), \( \Delta \rho \cdot r^2 \) has no relationship to the pressure. Even though \( \Delta \rho \cdot r^2 \) showed distribution, it was within 0.001 to 0.003, which is near the 0.002 result calculated using Eq. (5). Figures 10 and 11 show that the presumed pressure increase mechanism can be related to the quick pressure change.

### 4.3. Pressure decrease

Figure 12 shows the time required for the pressure to decrease as a ratio of pressure change. The time approximately depends on the kind of the valve used. When the L-valve was used, the time was approximately 0.5 ms. When the S-valve was used, it was approximately 0.3 to 0.4 ms. In the experiments, nitrogen gas from the high-speed valves still flowed out even during the pressure decrease phase. Therefore, the time was larger when using the L-valve than when using the S-valve.

Hereinafter, time required for the pressure to decrease with no gas injection, \( \Delta t_0 \), is estimated, using the experimental and valve correlation results. The mass in the base space, \( m_0 \), is written for the L-valve as

\[
m_0 = m_{b,i} - (\dot{m}_{\text{out}} - \dot{m}_{i,L}) \cdot \Delta t_L
\]

When \( m_{b,i} \) is the initial mass in the base space, \( \dot{m}_{\text{out}} \) is the outflow rate of gas from the base space, \( \dot{m}_{i,L} \) is the inflow rate to the space, and \( \Delta t_L \) is the flow-out time using the L-valve. For the S-valve,

\[
m_b = m_{b,i} - (\dot{m}_{\text{out}} - \dot{m}_{i,S}) \cdot \Delta t_S
\]

\( \dot{m}_{i,S} \) is the inflow rate of S-valve. When there is no inflow gas, then

\[
m_b = m_{b,i} - \dot{m}_{\text{out}} \cdot \Delta t_0
\]

\( \dot{m}_{\text{out}} \) is presumed to be the same for all three conditions. From Eqs. (8) to (10), \( \Delta t_0 \) is written as

\[
\Delta t_0 = (\dot{m}_{i,L} - \dot{m}_{i,S}) \Delta t_L \Delta t_S / (\dot{m}_{i,L} \Delta t_L - \dot{m}_{i,S} \Delta t_S)
\]

According to calibration of the valves, the volume flow rate is 1640 mm\(^3\)s\(^{-1}\) for the L-valve and 778 mm\(^3\)s\(^{-1}\) for the S-valve at the gas supply pressure of 500 kPaG. With a nitrogen
gas density of 1.14 kg m\(^{-3}\) and the time pressure decreased at the valves, \(\Delta t_0\) is calculated to be 0.22 ms. The standard deviation is 0.13 for Model 1 data and 0.12 for Model 2 data. The data dispersion is caused by the splashing of the water jet, mentioned in the previous section.

Next, the mechanism resulting in pressure decrease is discussed. Figure 13 shows a schematic of the gas flowing out from the base space. The decreasing mass in the base space is written together with the decreasing pressure in the base space as

\[
\frac{\Delta m}{\Delta t} = \frac{\Delta p_b \cdot V_b}{(R \cdot T_b)}
\]

In the experiments, the ratios of maximum base pressure to ambient pressure were 1.6 to 2. The velocity of the gas flowing out from the base space, \(v_s\), is sonic for simplicity here. When the gas flow area is \(C_\text{out}\), then the time for decreasing pressure, \(\Delta t\), is written as

\[
\Delta t = \frac{\Delta m}{(\rho_b v_s C_\text{out})} = \frac{\Delta p_b \cdot V_b}{(\rho_b \sqrt{\gamma RT_b} \cdot C_\text{out})}
\]

Re-writing for \(C_\text{out}\),

\[
\frac{C_\text{out}}{L_\text{utr} \cdot c} = \frac{\Delta p_b \cdot V_b}{\rho_b \sqrt{\gamma RT_b} \cdot \Delta t \cdot \frac{1}{L_\text{utr} \cdot c}}
\]

\(L_\text{utr}\) is the length of the injected water sheet, and \(c\) is depth of the combustor model. \(L_\text{utr} \cdot c\) is the surface area of the water sheet, and \(C_\text{out}/(L_\text{utr} \cdot c)\) is a ratio of the gas flow area to the surface area of the injected water sheet. When \(p_b\) is the average pressure in the base space, then Eq. (14) is re-written as

\[
\frac{C_\text{out}}{L_\text{utr} \cdot c} = \frac{(\Delta p_b/p_a) \cdot V_b}{[1 + (\Delta p_b/p_a)/2] \sqrt{\gamma RT_b} \cdot \Delta t \cdot \frac{1}{L_\text{utr} \cdot c}}
\]

Figure 14 shows that the relationship between \(C_\text{out}/(L_\text{utr} \cdot c)\) and \(\Delta P_{b,\text{max}}/p_a\). \(C_\text{out}/(L_\text{utr} \cdot c)\) was on the order of magnitude of 0.01 to 0.1. The ratio is approximately proportional to the pressure increase ratio. A higher pressure will break the water sheet into a larger number of fragments and nitrogen gas will easily flow out from the base space.

The ratio of the area where the gas flows out of the S-valve is larger than that of the L-valve. This is because the time pressure decreases using the S-valve is shorter, as discussed in this section. This is caused by a smaller injection flow rate from the S-valve during the pressure decrease phase.

4.4. Simplified calculation model

Calculation results using the simplified model are compared with the experimental results for three typical condi-
Initial $p_c$ was 100 kPa prior to the gas injection. The number of propellant elements was 100. The temperature of the gas in the base space was 290 K. Figure 15 shows the calculation results.

As can be seen, the original calculation results did not agree with the experimental ones. This is due to breaks in the injected water sheet created during the process of increasing the pressure. In the calculation model, gas was presumed to flow out from the gap between the end of the water sheet and the wall, as shown in Fig. 6. In the experiments, gas broke the water sheet into fragments and flowed out between the fragments. The gaps caused the effective gas flow rate to decrease. At the same time, the effective force due to the pressure difference acting on the water sheet, $F_{ef}$, decreases. The modification coefficients are incorporated to the calculation model.

\[
C_m = \frac{\dot{m}_{g,ef}}{\dot{m}_g} \quad (16)
\]
\[
C_F = \frac{F_{ef}}{F} \quad (17)
\]

$\dot{m}_{g,ef}$ is the effective gas flow rate from the gap. The results are shown in Fig. 15 with the caption of "(modified)". The modification coefficients are listed in Table 6. With the modification, the calculated results approximately agreed with the experimental ones. According to the coefficients, it is found that most of the gas flowed out through the fragments.

Figure 16 shows the effects of $C_m$ and $C_F$, respectively, in the form of test calculation of No. 821. The effective gas flow rate using $C_m$, that is, effective pressure increase due to combustion, affects the magnitude of pressure increase. The effective force using $C_F$, that is, effective pressure difference acting on the propellant sheet, affects not only the magnitude of pressure increase, but also the time of pressure change.

### 4.5. Liquid rocket engine

A simplified calculation model was applied to the operating conditions of rocket engines. The temperatures prior to and after combustion are $T_1$ and $T_2$, respectively. The pressure after combustion is written as

\[
p_2 = (T_2/T_1) \cdot p_1 \quad (18)
\]

and

\[
\Delta p_{b,\text{max}} = [(T_2/T_1) - 1] \cdot p_1 \quad (19)
\]

Substituting Eq. (19) for Eq. (7), the pressure increase time, $t_1$, then becomes

\[
t_1 = \frac{\rho_p \cdot d^2}{\rho_p \cdot d^2} \sqrt{\frac{(T_1/T_1) - 1}{p_1}} \quad (20)
\]

When $T_1 = 500$ K, $T_2 = 1000$ K, $\rho_p = 1000$ kg·m$^{-3}$ and $p_1 = 5$ MPa, then $t_1$ is 0.1 ms at $d = 10$ mm. When $d = 20$ mm, $t_1 = 0.2$ ms. In actual engines, the pressure in the base space quickly increases due to combustion. The time calculated corresponds to the traveling time from the initiation of pressure increase to when pressure starts to decrease. They are on the same order of magnitude of times of the engines.

Next, the pressure change of the rocket engines is parametrically calculated using the simplified calculation model. The
propellant injection velocity is $30 \text{ m s}^{-1}$. The width of the injection slit (i.e., the width of the injected propellant sheet of $d$) is a parameter. The reference length of $l$ is $10 \text{ mm}$. $C_F = 0.01$ of test calculation No. 821 is used here for calibration. The temperature in the base space is set to increase from $T_1$ to $T_2$ in $0.07 \text{ ms}$. Figure 17 shows the calculation results.

At the propellant sheet width of $d = 0.1 \text{ mm}$, the pressure ratio changes smoothly. On the other hand, when the width is large, the ratio decreases abruptly from the peak ratio. When the propellant sheet is thin, the sheet moves easily, the exit width of the propellant sheet becomes large quickly and the gas flow rate from the base space becomes large prior to the termination of temperature increase in the base space. So the pressure ratio changes smoothly. On the other hand, when the propellant sheet is thick, the sheet does not move easily and the pressure ratio increases until the termination of the temperature increase in the base space. After the termination of the temperature increase, the pressure ratio begins to decrease. Therefore, the change of the ratio is not smooth when the propellant sheet is thick.

The maximum pressure is twice that of the initial pressure due to the temperature change. The temperature increase ended in about $0.07 \text{ ms}$ under the present calculation conditions. The results show a quick change in base pressure. In the instability cycle, the time for vaporization, mixing and reaction is necessary. Sirignano et al. explained that characteristic times of mixing, vaporization and reaction, and time of vaporization are on the same order of magnitude as that of high-frequency instability. The present calculation lacks mixing, vaporization or reaction. If the present model includes the times, the characteristic time based on the present calculation approximately doubles. The calculated characteristic time is on the same order of magnitude as that of the measured characteristic times. Therefore, the total cycle period due to off-design combustion is on the same order of magnitude. This proves that off-design combustion can be a cause of high-frequency combustion instability.

When the width of the propellant sheet is small, pressure begins decreasing before the temperature stops increasing. When the width of the propellant sheet is large, pressure increases to the maximum value and then decreases.

5. Conclusions

The mechanism of high-frequency combustion instability in liquid rocket engines due to off-design combustion was verified through experiments. Estimation of the characteristic time was conducted using actual engines together with a simplified simulation model. In this study, the following were clarified.

(1) Experiments using the injection of water in place of propellant and the injection of nitrogen gas in place of combustion demonstrated a quick change of pressure with amplitudes of up to twice the initial pressure in the base space. A relationship between the time of pressure increase and the maximum pressure was found.

(2) A simplified simulation model of the pressure change was constructed to estimate the changes in actual engines. The calculated results showed a time and ampli-
tude of pressure change on the same order of magnitude as the changes in rocket engines.

(3) Based on the results, it was proven that off-design combustion can be a cause of high-frequency combustion instability.

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