Research on Utilization Risk of Reservoir Flood Resource

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Abstract. Flood resource utilization risk is a kind of speculative risk with hazard but higher rate of return. It includes flood control operation risk, structural risk and ecological environment risk. The research should contain efficiency and associated risk assessment from multiple perspectives. Appropriate risk management decision-making model and evaluation system should be established. In the paper, it takes the flood resources utilization of the referred reservoir as an example. Firstly, the paper has a comprehensive evaluation using a combination of multi-factor risk assessment methods under assumptions. Secondly, the paper calculates and analyses the degree of risk of dynamic control flood water level.

1. Introduction
At present, the research on flood resource risk is mainly about discrete distribution of individual risk’s obtained based on empirical data assessments. But for risk management and decision-making departments concerned, the urgent need is the risk level under the combination factors. The paper taking a reservoir as an example, has a preliminary analysis about the reservoir flood water level control methods and scheduling mode and the consequent risk, which aims at improving the flood resources utilization efficiency.

2. Research on risk analysis method of flood resources utilization of multi-factor combinations
Assuming the flood resources complete works Ω contains several elements. It means that there are several risks in the process of flood resources. Every risk may has variety of uncertain outcomes. Now they are represented by random variables $X_1, X_2, \ldots, X_n$. Due to the need to take both pure risk and speculative risk into consideration, $X_i$ represents the loss or gain of the $i$th kind of risk, so $X_i \in \mathbb{R}$. The distribution function of $X_i$ is $F_i$. Its density function is $f_i$. Now $F^{(i)}$ represents the overall distribution function of $X_1 + X_2 + \ldots + X_n$. So the distribution function of the sum of all risk result can be expressed as following:

$$
F^{(n)}(s) = \Pr\{S \leq s\} = \Pr\left\{\sum_{i=1}^{n} X_i \leq s\right\} \quad (1)
$$

In order to obtain a distribution function of $n$ risks, now we let $n=2$, then the function is as following:

$$
F^{(2)}(s) = \Pr\{S \leq s\} = \Pr\{X_1 + X_2 \leq s\} \quad (2)
$$

In equation (2), $X_2 \leq s - \min\{X_1\}$, supposing $s' = s - \min\{X_1\}$
\[ F^{(2)}(s) = \sum_{x_2 \leq s} \Pr \{ X_1 + X_2 \leq s \mid X_2 = x_2 \} \Pr \{ X_2 = x_2 \} \]

\[ = \sum_{x_2 \leq s} \Pr \{ X_1 \leq s - x_2 \mid X_2 = x_2 \} \Pr \{ X_2 = x_2 \} \quad (3) \]

Supposing \( X_i \) and \( X_j \) \((i \neq j)\) are independent random variables, the above equation can be written as following:

\[ F^{(2)}(s) = \sum_{x_2 \leq s} F_{X_i}(s - x_2) f_{X_2}(x_2) \quad (4) \]

The corresponding density function is as following:

\[ f^{(2)}(s) = \sum_{x_2 \leq s} f_{X_i}(s - x_2) f_{X_2}(x_2) \quad (5) \]

For continuous random variables, the corresponding equation of (3), (4) and (5) are as following:

\[ F^{(2)}(s) = \int_{-\infty}^{s} \Pr \{ X_1 \leq s - x_2 \mid X_2 = x_2 \} f_{X_2}(x_2) dx_2 \quad (6) \]

\[ F^{(2)}(s) = \int_{-\infty}^{s} F_{X_i}(s - x_2) f_{X_2}(x_2) dx_2 \quad (7) \]

\[ f^{(2)}(s) = \int_{-\infty}^{s} f_{X_i}(s - x_2) f_{X_2}(x_2) dx_2 \quad (8) \]

In the above equation, the computing of (5) and (8) is called convolution of \( F^{(2)}(s) \) to \( F_{X_i} \) and \( F_{X_2} \).

For the distribution of the sum of more than two random variables, it can be calculated through successive convolution operation:

\[ F^{(2)} = F_2 * F^{(1)} = F_2 * F_1, \quad F^{(3)} = F_3 * F^{(2)}, \ldots, F^n = F_n * F^{(n-1)} \]

Taking the corresponding relations of the moment generating function and the distribution function, we can use the moment generating function to calculate the overall distribution function of multiple risks. The moment generating function of random variables \( X \) is defined as \( M_X = E(e^{\alpha X}) \). If it is limited for all the \( t \) expectations of the opening interval, \( M_X(t) \) is only the moment generating function of \( X \). According to this uniqueness, \( S = X_1 + X_2 + \ldots + X_n \) is as following:

\[ M_X(t) = E[e^{\alpha X}] = E[e^{\alpha (X_1 + X_2 + \ldots + X_n)}] = E[e^{\alpha X_1} e^{\alpha X_2} \ldots e^{\alpha X_n}] \quad (9) \]

If \( X_1, X_2, \ldots, X_n \) are independent, the product expectation of equation (9) equals to \( E[e^{\alpha X_1}] E[e^{\alpha X_2}] \ldots E[e^{\alpha X_n}] \), so

\[ M_S(t) = E[e^{\alpha X}] = E[e^{\alpha X_1}] E[e^{\alpha X_2}] \ldots E[e^{\alpha X_n}] \quad (10) \]

The corresponding distribution to equation (9) is \( S \) distribution. Then we can calculate various characteristics digits of \( S \), such as Expectations \( \mu \), Variance \( \sigma^2 \), Entropy \( r \), Degree of risk \( d \), etc. Depending on the meaning these figures represent, we can have a comprehensive assessment on risk in flood resource utilization process. These figures can also provide the basis for the management and decision-making department to control risk.

### 3. Project Case Study

#### 3.1 Project Overview

The referred reservoir is a large water hub of comprehensive benefit with an area of 864 km². The average annual precipitation is 387.8 billion m³. It mainly concentrates in the flood season from May to September, accounting for about 66.1 percents. Supplying for the south-north water transfer project, the project plans to transfer 130 billion square water. Considering the demand of flood protection,
water offering and reducing the loss of reservoir inundation, the project heighten the main dam to a normal water level of 170m [1].

3.2. Risk Assessment Methods Analysis of Multivariate Combination

According to the water resources utilization of the flood basin, its risk factors mainly contain: upstream flood uncertainty risk, water discharged hazardous risk, making mistakes risk of reservoir operation. The result distributions of each risk is represented by random variable \( X_1, X_2 \) and \( X_3 \). Its value can be seen in Section 1 to 3, Table 1. The process of risk assessment is as follows:

1) Calculating distribution function \( F_1(x) \) depending on the probability function and formula of \( X_1 \);

2) Calculating the convolution of \( F^{(2)}(x) \) to \( F_{x_1} \) and \( F_{x_2} \). (Section 5, Table 1) For example:

\[
F^{(2)}(3) = \sum_{x_1, x_2} F_{x_1}(3-x_2) f_{x_2}(x_2) = F_1(2) \times f_2(1) + F_1(1) \times f_2(2) + F_1(0) \times f_2(3) = 0;
\]

3) Calculating the convolution of \( F^{(3)}(x) \) to \( F^{(2)}(x) \) and \( F_{x_1} \). (Section 6, Table 1)

4) According to the formula (5) or the difference of section 5 or 6, calculating the sum of two variables and the corresponding probability function of three variables. (Section 7 and 8, Table 1)

5) Calculating the total risk of flood resources utilization and having a analysis according to the distribution function of \( F^{(3)}(x) \) and \( f^{(3)}(x) \).

| \( x \) | \( f_1(x) \) | \( f_2(x) \) | \( f_3(x) \) | \( F_1(x) \) | \( F^{(2)}(x) \) | \( F^{(3)}(x) \) | \( f^{(2)}(x) \) | \( f^{(3)}(x) \) |
|-------|------------|------------|------------|-------------|-------------|-------------|------------|------------|
| 1     | 0.0        | 0.1        | 0.1        | 0.0         | 0.0         | 0.0         | 0.0        | 0.0        |
| 2     | 0.0        | 0.5        | 0.0        | 0.0         | 0.0         | 0.0         | 0.0        | 0.0        |
| 3     | 0.1        | 0.1        | 0.0        | 0.1         | 0.0         | 0.0         | 0.0        | 0.0        |
| 4     | 0.2        | 0.1        | 0.2        | 0.3         | 0.01        | 0.00        | 0.01       | 0.00       |
| 5     | 0.4        | 0.2        | 0.0        | 0.7         | 0.08        | 0.00        | 0.01       | 0.00       |
| 6     | 0.2        | 0.0        | 0.0        | 0.9         | 0.23        | 0.008       | 0.15       | 0.09       |
| 7     | 0.1        | 0.7        | 0.7        | 1.0         | 0.47        | 0.023       | 0.24       | 0.015      |
| 8     | 0.0        | 0.0        | 0.0        | 1.0         | 0.67        | 0.049       | 0.20       | 0.026      |
| 9     | 0.0        | 0.0        | 0.0        | 1.0         | 0.82        | 0.083       | 0.15       | 0.034      |
| 10    | 0.0        | 0.0        | 0.0        | 1.0         | 0.93        | 0.128       | 0.11       | 0.045      |
| 11    | 0.0        | 0.0        | 0.0        | 1.0         | 0.98        | 0.194       | 0.05       | 0.066      |
| 12    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.252       | 0.02       | 0.058      |
| 13    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.425       | 0.00       | 0.173      |
| 14    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.615       | 0.0        | 0.190      |
| 15    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.765       | 0.0        | 0.150      |
| 16    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.874       | 0.0        | 0.109      |
| 17    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.951       | 0.0        | 0.077      |
| 18    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 0.986       | 0.0        | 0.035      |
| 19    | 0.0        | 0.0        | 0.0        | 1.0         | 1.00        | 1.000       | 0.0        | 0.014      |

Then we get the relative value of total loss risk: Expectations \( \mu = 13.652 \), Variance \( \sigma^2 = 6.8913 \), Degree of risk \( d = \sigma / \mu = 0.19229 \), Entropy...
\[ r = - \sum P(S = s) \log P(S = s) = 1.01016 \]. Compared with the warning value set by the regulatory authorities, we can determine the overall risk level of the project.

3.3. Risk Calculations of Flood Water Level Considering Rainfall Forecasts Dynamic Control

In some assumptions and simplifications, the paper analyzes the following three kinds of risk according to the risk assessment methods proposed by Wang Ben-de [2,3]:

1) Result of design flood pre-building, pre-releasing, forecasting and dispatching in different return periods

The dispatched flood water level is 160.5m while the designed flood water level is 160m. The highest flood water level at different frequencies in both cases are presented in table 2. According to analysis result of rainfall prediction accuracy, the corresponding rainfall frequency distribution of “No rain” and “rain” in the next 24 hours is presented in table 3.

Table 2. Flood results under pre-building and pre-releasing

| Flood water level (m) | Design frequency (%) | 0.01 | 0.1 | 1 | 5 | 10 | 20 |
|----------------------|----------------------|------|-----|---|---|----|----|
| 160.5                | Highest flood level (m) | 172.99 | 172.35 | 172.02 | 168.2 | 167.42 | 164.52 |
| 160.0                | Highest flood level (m) | 172.8 | 172.05 | 171.7 | 168.1 | 167.0 | 164.1 |

Table 3. Actual rainfall frequency distribution table of “No rain” and “rain” in the next 24 hours

| Frequency (%) | 0.01 | 0.05 | 0.1 | 0.2 | 0.5 | 1 | 2 | 5 | 10 | 20 | 50 | 75 | 95 |
|---------------|------|------|-----|-----|-----|---|---|---|----|----|----|----|----|
| 24h no rain forecast (mm) | 25.5 | 22.2 | 16.1 | 13.4 | 10.0 | 7.6 | 5.3 | 2.7 | 1.2 | 0.3 | 0 | 0 | 0 |
| 24h rain forecast (mm) | 45.98 | 40.5 | 33.2 | 29.4 | 24.5 | 20.8 | 17.1 | 12.4 | 9.0 | 5.7 | 1.8 | 0.5 | 0.1 |

2) Limit risk rate calculation by the 24 h “no rain” or “rain” forecast information

① Limit risk rates calculation of dam safety

The flood water level remains keep at 160.5m after receiving the 24h “no rain” or “rain” forecast information. The limit risk rates calculating of dam safety is as follows: according to Risk rate PZ forum, now let ZLd=160.5m, Zm0=172.8m(Limit risk control indicators is defined as checked flood level 172.8m), we can obtain PZ(ZLd, Zm0)=0.02% according to table 2.

If the next 24h forecast is “no rain”, from table 3 we can get the actual happened PP(X>2)≈9% considering rain evaporation 2 mm. Then PZ=PP(X>2)×PZ(ZLd, Zm0)=9%×0.02%=0.0018%<0.01%.

If the next 24h forecast is “rain”, from table 3 we can get the actual happened PP(X>8)≈19%, Then PZ=PP(X>2)×PZ(ZLd, Zm0)=19%×0.02%=0.0038%<0.01%.

② Limit risk rate calculations of downstream flood safety

The flood water level remains keep at 160.5m after receiving the 24 h “no rain” or “rain” forecast information. The limit risk rates calculating of downstream flood safety is as follows: For reservoir assumes the downstream multi-level compensation flood control task, according to Risk rate PZ forum, now let Zm0.1%=171.7m. Zm0.5%=168.1m. Zm0.10%=167.0m. Zm0.20%=164.1m, we can obtain PZ (ZLd, Zm0.1%)=0.9%. PZ (ZLd, 168.1) =4.9%. PZ (ZLd, 167) =8%. PZ (ZLd, 164.1) =19% according to table 2.

If the next 24h forecast is “no rain”, from table 3 we can get the actual happened PP(X>2)=9%. Then

\[ PZ = PP(X>0) \times PZ(ZLd, Zm0=171.7) = 9% \times 0.90% = 0.081% < 1% \]
\[ PZ = PP(X>0) \times PZ(ZLd, Zm0=168.1) = 9% \times 4.9% = 0.441% < 5% \]
\[ PZ = PP(X>0) \times PZ(ZLd, Zm0=167.0) = 9% \times 8.0% = 0.72% < 10% \]
The most possible risk rate of solving overall risk is the highest flood level. According to frequency forecast information, designed at frequency 20%, the highest flood level at frequency 20% is 161.4 m from the level of 160.5 m, which is 2.7~2.0 m lower than the designed at frequency 20%. Therefore, while receiving the 24h “no rain” or “rain” forecast information, the flood water level can be controlled at 160.5 m. The most possible risk rate of exceeding the highest flood level at frequency 20% is “0” and the expected risk rate is also “0”.

Table 4. The corresponding various designed resistant rainfall

| Design frequency (%) | 0.01 | 0.1 | 1 | 5 | 10 | 20 |
|----------------------|------|-----|---|---|----|----|
| W7: Seven days magnanimity (10^3m^3) | 234  | 188 | 137 | 103 | 89 | 71.3 |
| Run-off R (mm) | 245.8 | 197.5 | 143.9 | 108.2 | 93.5 | 74.9 |
| P1 rain-fall (mm) | 260.8 | 212.5 | 158.9 | 123.2 | 98.5 | 89.9 |
| Highest flood level (m) | 172.8 | 172.05 | 171.7 | 168.1 | 167.0 | 164.1 |

Remark: Rainfall flood losses is little, so let it equals to 15mm. It means \( P_t=R+15 \)

Table 5. The most possible risk analysis of flood water level 160.5m’s dynamic controlling according to rainfall forecast information

| Absolute error (mm) | Total water (10^3m^3) | Highest flood level (m) |
|---------------------|-----------------------|-------------------------|
| 25.5 | 7.28 | 161.4 |
| 22.2 | 6.34 | 161.3 |
| 16.1 | 4.60 | 161.1 |
| 13.4 | 3.83 | 161.0 |
| 10.0 | 2.86 | 160.9 |
| 7.6 | 2.17 | 160.8 |
| 5.3 | 1.51 | 160.7 |
| 2.7 | 0.77 | 160.6 |
| 1.2 | 0.34 | 160.6 |
| 0.3 | 0.09 | 160.5 |
| 0.0 | 0.00 | 160.5 |

Introduction: Watershed area=952000km^2, Run-off coefficient a =0.3; the designed 20% highest flood level=164.1m

4. Conclusion and Outlook
The solving overall risk method uses convolution formula and moment generating function in the paper. It is suitable for obtaining sum for multiple types of risk distribution. It is also beneficial for assessment on flood resources. The paper verifies the controllability of risk through the research on dam safety risk rate limit under different rainfall conditions; the possible biggest risk rate of
downstream flood safety and expected risk rate. Due to the complexity of flood resources utilization risk affecting factor, the result of assessment only for single risk or finite risk of individual projects can’t be treated as the basis for administration. Therefore it is the urgent need to establish multi-factor portfolio risk assessment system and risk management model. For example, in addition to risk assessment of individual reservoirs or water conservancy projects in theory, we should also establish risk evaluation index system including social economic and ecological indicators.

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