Electromagnetic two-point functions and Casimir densities for a conducting plate in de Sitter spacetime

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Abstract

We evaluate the two-point function for the electromagnetic field tensor in \((D+1)\)-dimensional de Sitter spacetime assuming that the field is prepared in Bunch-Davies vacuum state. This two-point function is used for the investigation of the vacuum expectation values (VEVs) of the field squared and the energy-momentum tensor in the presence of a conducting plate. The VEVs are decomposed into the boundary-free and plate-induced parts. For the latter, closed form analytical expressions are given in terms of the hypergeometric function. For \(3 \leq D \leq 8\) the plate-induced part in the VEV of the electric field squared is positive everywhere, whereas for \(D \geq 9\) it is positive near the plate and negative at large distances. The VEV of the energy-momentum tensor, in addition to the diagonal components, contains an off-diagonal component which corresponds to the energy flux along the direction normal to the plate. Simple asymptotic expressions are provided at small and large distances from the plate compared with the de Sitter curvature scale. For \(D \geq 4\), all the diagonal components of the plate-induced vacuum energy-momentum tensor are negative and the energy flux is directed from the plate.

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1 Introduction

It is well-known that the properties of vacuum state for a quantum field crucially depend on the geometry of background spacetime. Closed expressions for physical characteristics of the vacuum, such as expectation values of various bilinear products of the field operator, can be found for highly symmetric background geometries only. On one hand these analytic results are interesting on their own and on the other, they help understanding the influence of the gravitational field on the quantum vacuum for more complicated geometries. From this perspective, de Sitter (dS) spacetime is among the most interesting backgrounds. There are several reasons for that. dS spacetime is the maximally symmetric solution of Einstein’s equations with a positive cosmological constant and, owing to this symmetry, numerous physical problems are exactly solvable on this background. In accordance with the inflationary cosmology scenario \([\Pi]\), in the early stages of the cosmological expansion our universe passed through a phase in which its local geometry closely resembles that of dS spacetime. During an inflationary epoch, quantum

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fluctuations in the inflaton field introduce inhomogeneities which play a central role in the generation of cosmic structures from inflation. More recently, astronomical observations of high redshift supernovae, galaxy clusters and cosmic microwave background indicated that at the present epoch the universe is accelerating and can be approximated by a world with a positive cosmological constant. If the universe were to accelerate indefinitely, the standard cosmology would lead to an asymptotic dS universe. It is therefore important to investigate physical effects in dS spacetime for understanding both the early universe and its future.

The interaction of a fluctuating quantum field with the background gravitational field leads to vacuum polarization. The boundary conditions imposed on the field operator give rise to another type of vacuum polarization. These conditions can arise because of the presence of boundaries having different physical natures, like macroscopic bodies in QED, extended topological defects, horizons and branes in higher-dimensional models. They modify the zero-point modes of a quantized field and, as a result, forces arise acting on constraining boundaries. This is the familiar Casimir effect (for reviews see [3]). The particular features of the Casimir forces depend on the nature of a quantum field, the type of spacetime manifold, the boundary geometry, and the specific boundary conditions imposed on the field.

An interesting topic in the investigation of the Casimir effect is its explicit dependence on the geometry of the background spacetime. In the present paper, an exactly solvable problem with both types of polarization of the electromagnetic vacuum will be considered. We evaluate the two-point function for the field tensor and the vacuum expectation values (VEVs) of the field squared and the energy-momentum tensor induced by a conducting plate in (D+1)-dimensional dS spacetime. The corresponding problem for a massive scalar field with general curvature coupling parameter has been recently considered in [4] and [5] for flat and spherical boundaries respectively (see also [6, 7] for special cases of conformally and minimally coupled massless fields). It has been shown that the curvature of the background spacetime decisively influences the behavior of VEVs at distances larger than the curvature scale of dS spacetime. In another class of models with boundary conditions, the latter arise because of the nontrivial topology of the space. The periodicity conditions imposed on the field operator along compact dimensions lead to the topological Casimir effect. This effect for scalar and fermionic fields, induced by toroidal compactification of spatial dimensions in dS spacetime has been investigated in [8].

The outline of the paper is as follows. In the next section we present a complete set of mode functions for the electromagnetic field defining the Bunch-Davies vacuum state in (D+1)-dimensional boundary-free dS spacetime. Then, these mode function are used for the evaluation of the two-point function of the electromagnetic field tensor. In Section 3 we consider the geometry with a conducting plate. The corresponding mode functions for the vector potential are presented and they are used for the evaluation of the two-point functions. The latter are expressed in terms of the boundary-free two-point functions. Then, we evaluate the parts in the VEVs of the electromagnetic field squared and the energy-momentum tensor induced by the conducting plate. Closed expressions for these VEVs are derived and their asymptotic behavior is investigated near the plate and at large distances compared with the dS curvatures scale. The main results are summarized in Section 4.

2 Electromagnetic modes and two-point functions

In this section we present a complete set of mode functions for the electromagnetic field in dS spacetime and then the two-point functions for the field tensor are evaluated. The two-point functions \( \langle A_\mu(x)A_\nu(x') \rangle \) for both massive and massless vector fields in dS spacetime are obtained in [9] by using the arguments based on the maximal symmetry. In [10], the Wightman two-point functions for massive and massless vector fields are investigated in 4-dimensional dS spacetime by making use of the construction based on analyticity properties offered by the complexified
pseud-Riemannian manifold in which the dS manifold is embedded. Infrared pathologies in the behavior of the photon two-point functions in dS spacetime have been recently discussed in [11] and it has been shown that these are purely gauge artifacts.

2.1 Mode functions

We consider the electromagnetic field in the background of $(D + 1)$-dimensional dS spacetime, described in the inflationary coordinates:

$$ds^2 = dt^2 - e^{2t/\alpha} \sum_{l=1}^{D} (d_l')^2,$$

(1)

where the parameter $\alpha$ is related to the positive cosmological constant $\Lambda$ by the formula $\alpha^2 = D(D - 1)/(2\Lambda)$. Below, in addition to the comoving synchronous time coordinate $t$ we use the conformal time $\tau$, defined as $\tau = -\alpha e^{-t/\alpha}$, $-\infty < \tau < 0$. In terms of this coordinate the metric tensor takes a conformally flat form: $g_{\mu\nu} = (\alpha/\tau)^2 \text{diag}(1, -1, ..., -1)$. For the action integral of the electromagnetic field one has

$$S = -\frac{1}{16\pi} \int d^{D+1}x \sqrt{|g|} F_{\mu\nu}(x) F^{\mu\nu}(x),$$

(2)

where $F_{\mu\nu}$ is the electromagnetic field tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Imposing the gauge conditions $A_0 = 0$, $\nabla_\mu A^\mu = 0$, we expand the vector potential as a Fourier integral

$$A_l(x) = \int d\mathbf{k} \, A_l(\tau, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{z}}, \quad A_l(\tau, -\mathbf{k}) = A_l^*(\tau, \mathbf{k}),$$

(3)

with, $x = (\tau, \mathbf{z})$, $\mathbf{z} = (z^1, ..., z^D)$, $\mathbf{k} = (k_1, ..., k_D)$ and $\mathbf{k} \cdot \mathbf{z} = \sum_{l=1}^{D} k_l z^l$. From the gauge condition, for the Fourier components of the vector potential one has $\sum_{l=1}^{D} k_l A_l(\tau, \mathbf{k}) = 0$.

In terms of the Fourier components, the action integral (2) is written in the form

$$S = \frac{(2\pi)^D}{4} \sum_{l=1}^{D} \int d\tau \int d\mathbf{k} \left( \frac{\alpha}{\eta} \right)^{D-3} \left[ \partial_\tau A_l(\tau, \mathbf{k}) \partial_\tau A_l^*(\tau, \mathbf{k}) - k^2 A_l(\tau, \mathbf{k}) A_l^*(\tau, \mathbf{k}) \right],$$

(4)

where $\eta = |\tau|$. The variational principle applied to (4) leads to the equation:

$$\partial^2_\tau A_l(\tau, \mathbf{k}) - \frac{D-3}{\tau} \partial_\tau A_l(\tau, \mathbf{k}) + k^2 A_l(\tau, \mathbf{k}) = 0.$$  

(5)

The general solution of this equation is a linear combination of the functions $\eta^{D/2-1} H^{(1)}_{D/2-1}(k\tau)$ and $\eta^{D/2-1} H^{(2)}_{D/2-1}(k\tau)$, with $H^{(1,2)}_{\nu}(x)$ being the Hankel functions. Different choices of the coefficients in the linear combination correspond to different vacuum states. Here we assume that the field is prepared in dS invariant Bunch-Davies vacuum state which is the only dS invariant vacuum state with the same short-distance structure as the Minkowskian vacuum.

For Bunch-Davies vacuum state $A_l(\tau, \mathbf{k}) \propto \eta^{D/2-1} H^{(2)}_{D/2-1}(k\tau)$. By taking into account that

$$H^{(2)}_{\nu}(k\tau) = -e^{i\nu \pi} H^{(1)}_{\nu}(k\eta),$$

(6)

the complete set of mode functions defining this state are given by

$$A_l(\mathbf{k}) = C_{l}(\eta) e^{i\mathbf{k} \cdot \mathbf{z}}, \quad l = 1, ..., D,$$

(7)

where $\sigma = 1, ..., D - 1$ correspond to different polarizations. The polarization vectors obey the relations

$$\sum_{l=1}^{D} \epsilon_{l}(\sigma) k_l = 0, \quad \sum_{l=1}^{D} \epsilon_{l}(\sigma) \epsilon_{l}(\sigma') = \delta_{\sigma \sigma'},$$

(7)
and
\[ \sum_{\sigma=1}^{D-1} \epsilon_{(\sigma)l} \epsilon_{(\sigma)m} = \delta_{lm} - \frac{k_l k_m}{k^2}. \]  
(8)

The normalization factor \( C \) is determined from the condition
\[ \int d\mathbf{z} \sum_{l=1}^{D} [A_{(\sigma k)l}(x) \partial_{\sigma'} A_{(\sigma' k')l}(x) - A_{(\sigma' k')l}(x) \partial_{\sigma} A_{(\sigma k)l}(x)] = -i \frac{4\pi \delta_{\sigma'\sigma}}{(\alpha/\eta)^{D-3}} \delta(k - k'). \]  
(9)

By using the expression (6), one finds
\[ |C|^2 = \frac{\alpha^{3-D}}{4(2\pi)^{D-2}}. \]  
(10)

For \( D = 3 \), by taking into account that \( H_{1/2}^{(1)}(z) = -i \sqrt{2/(\pi x)} e^{iz} \), we see that the mode functions coincide with the corresponding functions in Minkowski spacetime (the \( D = 3 \) mode functions have also been considered in [12]). This is a consequence of the conformal invariance of the electromagnetic field in \( D = 3 \).

### 2.2 Two-point functions

Having a complete set of normalized mode functions for the vector potential we can evaluate the two-point function for the electromagnetic field by using the mode-sum formula (here and below the Latin indices for tensors run over 1, 2, \ldots, \( D \))

\[ \langle A_l(x) A_m(x') \rangle_0 = \sum_{\sigma=1}^{D-1} \int dk A_{(\sigma k)l}(x) A_{(\sigma k)m}(x'), \]  
(11)

where \( \langle \cdots \rangle_0 \) stands for the VEV in the boundary-free dS spacetime. By taking into account the expression (6), one gets the integral representation

\[ \langle A_l(x) A_m(x') \rangle_0 = \frac{\alpha^{3-D}(\eta\eta')^{D/2-1}}{4(2\pi)^{D/2}} \int dk \left( \delta_{lm} - \frac{k_l k_m}{k^2} \right) H_{D/2-1}^{(2)}(k\eta) H_{D/2-1}^{(1)}(k\eta') e^{ik\cdot \Delta \mathbf{z}}, \]  
(12)

with \( \Delta \mathbf{z} = \mathbf{z} - \mathbf{z}' \).

First let us consider the part with \( \delta_{lm} \). By writing the product of the Hankel functions in terms of the Macdonald function, after the integration over the angular part of \( k \), one finds:

\[ I = \int dk H_{D/2-1}^{(2)}(k\eta) H_{D/2-1}^{(1)}(k\eta') e^{ik\cdot \Delta \mathbf{z}} \]
\[ = \frac{4(2\pi)^{D/2}}{\pi^2 |\Delta \mathbf{z}|^{D/2-1}} \int_0^\infty dk k^{D/2} K_{D/2-1}(ik\eta) K_{D/2-1}(-ik\eta') J_{D/2-1}(k|\Delta \mathbf{z}|), \]
(13)

with \( J_\rho(x) \) been the Bessel function. The last integral is evaluated by using the formula from [13] and we get

\[ I = 4(2\pi)^{(D-3)/2} \Gamma(D - 1) \frac{P_{(D/2-1/2)}^{(1-D)/2}(-Z)}{(\eta\eta')^{D/2} (Z^2 - 1)^{(D-1)/2}}, \]  
(14)

where \( P_\rho^\mu(x) \) is the associated Legendre function,

\[ Z = 1 + \frac{(\Delta \eta)^2 - |\Delta \mathbf{z}|^2}{2\eta\eta'}, \]  
(15)
and $\Delta \eta = \eta - \eta'$. The quantity $Z$ is invariant under the action of the isometry group of dS spacetime. One has $Z > 1$ and $Z < 1$ for timelike and spacelike related points $x$ and $x'$, respectively. An alternative form for the integral (14) is obtained by using the relation between the associated Legendre function and the hypergeometric function [14]:

$$I = \frac{2\pi^{(D-3)/2} \Gamma(D-1)}{\Gamma((D+1)/2) (\eta \eta')^{D/2}} F \left( D - 1, 1; \frac{D+1}{2}; z \right),$$

with the notation

$$z = \frac{Z - 1}{2} = 1 + \frac{(\Delta \eta)^2 - |\Delta z|^2}{4 \eta \eta'}. \quad (17)$$

Hence, the two-point function for the vector potential is presented in the form

$$\langle A_i(x) A_m(x') \rangle_0 = \frac{\delta_{lm} \alpha^{3-D} \Gamma(D-1)}{4(2\pi)^{D-2}} \frac{\alpha^{3-D}(\eta \eta')^{D/2-1}}{4(2\pi)^{D-2}} \int d\mathbf{k} \frac{k_i k_m}{k^2} H_{D/2-1}^{(2)}(k\eta) H_{D/2-1}^{(1)}(k\eta') e^{i\mathbf{k} \cdot \Delta \mathbf{z}}. \quad (18)$$

One has no closed form for the second term in the right-hand side of (18). However, this term does not contribute to the two-point functions for the electromagnetic field tensor and it will not be needed in the further consideration.

By using (18), we find the following expressions for the two-point functions of the electromagnetic field tensor:

$$\langle F_{0i}(x) F_{0m}(x') \rangle_0 = \frac{(\eta \eta')^{-2}}{2B_D \alpha^{D-3}} \left[ (\delta_{lp} \delta_{mq} - \delta_{lm} \delta_{pq}) \frac{\Delta z^p \Delta z^q}{2 \eta \eta'} \partial_z + (D - 1) \delta_{lm} \right] G_D(z),$$

$$\langle F_{pl}(x) F_{0m}(x') \rangle_0 = \frac{(\eta \eta')^{-2}}{B_D \alpha^{D-3}} \delta_{[pl} \delta_{q]m} \frac{\Delta z^q}{2 \eta \eta'} \left[ \frac{z + (\eta + \eta')}{2\eta} \partial_z \right] F_D(z), \quad (19)$$

$$\langle F_{pl}(x) F_{qm}(x') \rangle_0 = \frac{(\eta \eta')^{-2}}{B_D \alpha^{D-3}} \delta_{[pl} \delta_{q]m} \frac{\Delta z^p \Delta z^q}{2 \eta \eta'} \partial_z + \delta_{[pq} \delta_{|m]} \right] F_D(z),$$

where

$$B_D = (4\pi)^{(D-1)/2} \Gamma((D+3)/2), \quad (20)$$

and the square brackets enclosing the indices mean the antisymmetrization over these indices: $a_{[i,j,\ldots,k]} = (a_{i,j,\ldots,k} - a_{i,k,\ldots,j})/2$. In (19) we have introduced the functions

$$F_D(z) = \Gamma(D) F \left( D, 2; \frac{D+3}{2}; z \right),$$

$$G_D(z) = 2\Gamma(D-1) F \left( D - 1, 3; \frac{D+3}{2}; z \right). \quad (21)$$

For odd values of $D$ one has simple expressions:

$$F_3(z) = G_3(z) = 2(z - 1)^{-2},$$

$$F_5(z) = 12(z - 2)(z - 1)^{-3}, \quad G_5(z) = -12(z - 1)^{-3}, \quad (22)$$

$$F_7(z) = 144 \frac{2z^2 - 6z + 5}{(z - 1)^4}, \quad G_7(z) = -48 \frac{2z^2 - 5}{(z - 1)^4}. \quad (23)$$

The expression for the two-point function $\langle F_{0m}(x) F_{pl}(x') \rangle$ is obtained from the expression (19) for $\langle F_{pl}(x) F_{0m}(x') \rangle$ by changing the sign and by the interchange $\eta \leftrightarrow \eta'$. 

5
3 Two-point functions and Casimir densities in the geometry with a conducting plate

3.1 Two-point functions

As an application of the formulas for the two-point functions given above here we consider the change in the properties of the electromagnetic vacuum induced by the presence of a perfectly conducting plate placed at $z^D = 0$ (for the electromagnetic Casimir effect in higher-dimensional spacetimes see, for instance, [15, 16]). We consider the region $z^D > 0$. On the plate the field obeys the boundary condition [15] $n^\mu * F_{\mu\nu} = 0$, with the tensor $* F_{\mu\nu}$ dual to $F_{\mu\nu}$, and $n^\mu$ is the normal to the plate. The corresponding mode-functions for the vector potential are given by the expressions

$$A_{(\sigma k)l}(x) = i C_b \epsilon_{(\sigma)} l \eta^{D/2-1} H_{D/2-1}^{(2)}(k \tau) \sin(k D z^D) e^{i k_l z_l},$$

$$A_{(\sigma k)D}(x) = C_b \epsilon_{(\sigma)D} \eta^{D/2-1} H_{D/2-1}^{(2)}(k \tau) \cos(k D z^D) e^{i k_D z_D},$$

(23)

where $l = 1, \ldots, D-1$, $k_l = (k_1, \ldots, k_{D-1})$, $z_l = (z^1, \ldots, z^{D-1})$ and $k = \sqrt{k_l^2 + k_D^2}$. The polarization vectors obey the same relations (17) and (18). The normalization coefficient is determined from the condition (9), with the difference that now the integration goes over the region $z^D > 0$. In this way, we can see that $|C_b|^2 = 4 |C|^2$, where $|C|^2$ is given by the expression (10).

The two-point functions for the vector potential are evaluated by using the mode-sum formula similar to (11). Substituting the mode functions, the two-point functions are presented in the decomposed form

$$\langle A_l(x) A_m(x') \rangle = \langle A_l(x) A_m(x') \rangle_0 + \langle A_l(x) A_m(x') \rangle_b,$$

(24)

where the second term in the right-hand side is induced by the presence of the conducting plate. For the latter one finds

$$\langle A_l(x) A_m(x') \rangle_b = -\langle A_l(x) A_m(x'_-) \rangle_0,$$

$$\langle A_l(x) A_D(x') \rangle_b = \langle A_l(x) A_D(x'_-) \rangle_0,$$

(25)

where $l = 1, \ldots, D$, $m = 1, \ldots, D-1$, and $x'_-$ is the image of $x'$ with respect to the plate: $x'_- = (\tau', z^1, \ldots, z^{D-1}, -z^D)$. For the two-point function of the electromagnetic field tensor in the region $z^D > 0$ we get a similar decomposition

$$\langle F_{pl}(x) F_{qm}(x') \rangle = \langle F_{pl}(x) F_{qm}(x') \rangle_0 + \langle F_{pl}(x) F_{qm}(x') \rangle_b,$$

(26)

with the plate-induced parts given by

$$\langle F_{pl}(x) F_{qm}(x') \rangle_b = -\langle F_{pl}(x) F_{qm}(x'_-) \rangle_0,$$

$$\langle F_{pl}(x) F_{Dm}(x') \rangle_b = \langle F_{pl}(x) F_{Dm}(x'_-) \rangle_0,$$

(27)

with $p, l = 0, 1, \ldots, D$, and $q, m = 0, 1, \ldots, D - 1$.

Here we have considered the perfectly conducting boundary condition. The case of the infinitely permeable boundary condition, $n^\mu F_{\mu\nu} = 0$, is treated in a similar way. This condition is imposed in the bag model for hadrons.

3.2 Casimir densities

We consider a free field theory and the two-point functions given above encode all the properties of the vacuum state. In particular, having these functions we can evaluate the VEVs for various
physical observables characterizing the vacuum state. First let us consider the VEV of the electric field squared. It can be obtained from the two-point functions given above in the coincidence limit of the arguments
\[ \langle E^2 \rangle = -g^{00}g^{lm} \lim_{x' \to x} \langle F_{0l}(x)F_{0m}(x') \rangle. \] (28)

On the base of the decomposition (26) we can write a similar decomposition for the electric field squared: \( \langle E^2 \rangle = \langle E^2 \rangle_0 + \langle E^2 \rangle_b \). For points away from the plate the divergences in the coincidence limit of the two-point functions are contained in the boundary-free part \( \langle E^2 \rangle_0 \) only and, hence, the renormalization is needed for this part only. Because of the maximal symmetry coincidence limit of the two-point functions are contained in the boundary-free part field squared:
\[ \langle E^2 \rangle_0 \]

On the base of the decomposition (26) we can write a similar decomposition for the electric field squared:
\[ \langle E^2 \rangle_b = \frac{D-1}{2B_D\alpha^{D+1}} [2(1-y)\partial_y - D + 2] G_D(y), \] (29)

with the notation
\[ y = 1 - \left( \frac{z^D}{\eta} \right)^2. \] (30)

The plate-induced part (29) depends on \( z^D \) and \( \eta \) in the combination \( z^D/\eta \). The latter is the proper distance from the plate measured in units of \( \alpha \). For \( D = 3 \), from the general result (29) one finds \( \langle E^2 \rangle_b = 3(\alpha z^D/\eta)^{-4}/(4\pi) \). The latter is obtained from the corresponding result in Minkowski spacetime by the standard conformal transformation.

Let us consider the behavior of the plate-induced part in the VEV of the electric field squared in the asymptotic regions of the ratio \( z^D/\eta \). At proper distances from the plate much smaller than the dS curvature scale \( \alpha \) one has \( z^D/\eta \ll 1 \). By using the asymptotic formula
\[ G_D(y) \approx \Gamma \left( \frac{D+3}{2} \right) \Gamma \left( \frac{D+1}{2} \right) \left( \frac{z^D}{\eta} \right)^{-D-1}, \] (31)

we get
\[ \langle E^2 \rangle_b \approx \frac{3(D-1)\Gamma((D+1)/2)}{2(4\pi)^{(D-1)/2}(\alpha z^D/\eta)^{D+1}}. \] (32)

As it is seen, the plate-induced part diverges on the boundary. This type of divergences are well-known in quantum field theory with boundaries. For points near the plate the boundary-induced part (29) dominates in the VEV of the electric field squared.

At distances from the plate much larger than the dS curvature scale, \( z^D/\eta \gg 1 \), one has \( y \ll -1 \). In this case, we use the asymptotic expressions
\[ G_D(y) \approx \frac{2^{D-4}}{\sqrt{\pi}(-y)^3} \Gamma \left( \frac{D+3}{2} \right) \Gamma \left( \frac{D}{2} - 2 \right), \quad D > 4, \]
\[ G_D(y) \approx \Gamma(D-1) \frac{\Gamma((D+3)/2)\Gamma(2-D/2)}{2^{D-3}\sqrt{\pi}(-y)^{D-1}}, \quad D < 4, \] (33)

and
\[ G_4(y) \approx 3(-y)^{-3} [2 \ln (-4y) - 3], \quad D = 4. \] (34)

For the VEV of the field squared we get
\[ \langle E^2 \rangle_b \approx \frac{(D-1)(8-D)\Gamma(D/2-2)}{2^{4-D}(4\pi)^{D/2}\alpha^{D+1} (z^D/\eta)^6}, \quad D > 4, \]
\[ \langle E^2 \rangle_b \approx \frac{2^{3-D}\Gamma(D+1)\Gamma(2-D/2)}{(4\pi)^{D/2}\alpha^{D+1} (z^D/\eta)^{2(D-1)}}, \quad D < 4, \] (35)
and
\[ \langle E^2 \rangle_b \approx 3 \frac{\ln(2z^D/\eta) - 5/8}{\pi^2 \alpha^5 (z^D/\eta)^6}, \quad D = 4. \] (36)

For \( D = 3 \), the expression (35) coincides with the exact result. In \( D = 8 \) the leading term of (35) vanishes. Numerical calculations show that for \( 3 \leq D \leq 8 \) the plate-induced part \( \langle E^2 \rangle_b \) is positive everywhere. For \( D \geq 9 \), \( \langle E^2 \rangle_b \) is positive near the plate and negative at large distances from the plate. In this case, \( \langle E^2 \rangle_b \) has a minimum for some intermediate value of \( z^D/\eta \).

In a similar way, for the plate-induced part in the VEV of the Lagrangian density one finds
\[ - \frac{1}{16\pi} \langle F_{\mu\nu} F^{\mu\nu} \rangle_b = \frac{1}{8\pi} \left\{ \langle E^2 \rangle_b + \frac{D - 1}{2BD\alpha^{D+1}} [2(1 - y)\partial_y + D - 4] F_D(y) \right\}. \] (37)

The second term in the figure braces presents the magnetic contribution to the Lagrangian density. For \( D = 3 \) it coincides with \( \langle E^2 \rangle_b \).

Another important characteristic of the vacuum state is the VEV of the energy-momentum tensor. In addition to describing the local structure of the vacuum state, it acts as the source of gravity in the quasiclassical Einstein equations and plays an important role in modelling self-consistent dynamics involving the gravitational field. Similar to the case of the field squared, the VEV of the energy-momentum tensor is decomposed into the boundary-free and plate-induced parts:
\[ \langle T_{\mu}^{\nu} \rangle = \langle T_{\mu}^{\nu} \rangle_0 + \langle T_{\mu}^{\nu} \rangle_b. \] (38)

Again, for points outside the plate the renormalization is required for the boundary-free part only. Because of the maximal symmetry of the background geometry, the latter is proportional to the metric tensor: \( \langle T_{\mu}^{\nu} \rangle_0 = \text{const} \cdot \delta_{\mu}^{\nu} \).

Here we are interested in the plate-induced part which is directly evaluated by using the formula
\[ \langle T_{\mu}^{\nu} \rangle_b = -\frac{1}{4\pi} \lim_{x' \to x} \langle F_{\mu}^{\beta}(x) F_{\nu}^{\beta}(x') \rangle_b + \frac{\delta^{\nu}_{\mu}}{16\pi} \langle F_{\beta\sigma} F^{\beta\sigma} \rangle_b. \] (39)

By taking into account the expressions (19), (27) and (37), for the VEVs of the diagonal components one finds (no summation over \( l = 1, \ldots, D - 1 \))
\[
\langle T_{0}^{\nu} \rangle_b = \frac{\alpha^{-D-1}}{A_D} \left\{ [2(1 - y)\partial_y - D + 2] G_D(y) - [2(1 - y)\partial_y + D - 4] F_D(y) \right\},
\]
\[
\langle T_{D}^{\nu} \rangle_b = \frac{\alpha^{-D-1}}{A_D} [2(1 - y)\partial_y - D] [F_D(y) - G_D(y)],
\]
\[
\langle T_{l}^{\nu} \rangle_b = -\frac{\alpha^{-D-1}}{A_D} \frac{2}{D - 1} \left\{ [2(1 - y)\partial_y - (D - 4) \frac{D - 1}{D - 3}] G_D(y)
\]
\[+ [2(1 - y)\partial_y + (D - 4) \frac{D - 1}{D - 3} - 4] F_D(y) \right\}, \] (40)

where
\[ A_D = (4\pi)^{(D+1)/2}(D + 1) \Gamma \left( \frac{D - 1}{2} \right). \]

In addition to the diagonal components, one has also a nonzero off-diagonal component of the vacuum energy-momentum tensor:
\[ \langle T_{0}^{D} \rangle_b = \frac{\alpha^{-D-1} 4z^D}{A_D} \eta \left[ (1 - y)\partial_y - 2 \right] F_D(y). \] (41)

The latter corresponds to the energy flux along the direction normal to the plate.
As in the case of the field squared, the VEV of the energy-momentum tensor depends on the coordinates \(z^D\) and \(\eta\) in the combination \(z^D/\eta\). This is a consequence of the maximal symmetry of the background spacetime and of the Bunch-Davies vacuum state. For \(D = 3\), by using the expressions \((22)\) for the functions \(F_D(y)\) and \(G_D(y)\), we can see that \(\langle T_\nu^\nu \rangle_b = 0\). This result is a direct consequence of the conformal invariance of the electromagnetic field in \(D = 3\).

It can be checked that the plate-induced parts of the vacuum energy-momentum tensor obey the covariant conservation equation \(\nabla_\nu \langle T_\nu^\nu \rangle_b = 0\). For the problem under consideration it reduces to the following two equations:

\[
\eta \partial_\nu \langle T_0^0 \rangle_b - D \langle T_0^0 \rangle_b - \eta \partial_D \langle T_0^D \rangle_b + \sum_{k=1}^{D} \langle T_k^k \rangle_b = 0,
\]

\[
\eta \partial_\nu \langle T_0^D \rangle_b - (D + 1) \langle T_0^D \rangle_b + \eta \partial_D \langle T_0^D \rangle_b = 0.
\]

(42)

Let us consider the behavior of the VEV of the energy-momentum tensor at small and large distances from the plate. At small distances, \(z^D/\eta \ll 1\), by using the asymptotic formulas for the hypergeometric function, to the leading order one finds (no summation over \(l = 0, \ldots, D - 1\))

\[
\langle T^l_l \rangle_b \approx -\frac{\eta}{z^D} \langle T^D_0 \rangle_b \approx \frac{D - 1}{(z^D/\eta)^2} \langle T^D_0 \rangle_b \approx -\frac{(D - 3)(D - 1)\Gamma((D + 1)/2)}{2(4\pi)^{(D+1)/2} \alpha D+1(z^D/\eta)^D+1}.
\]

(43)

In this region, all the components are negative for \(D \geq 4\) and one has \(\langle T^0_0 \rangle_b \gg \langle T^D_0 \rangle_b\). At large distances from the plate, by using the asymptotic expression

\[
F_D(y) \approx \frac{2^{D-3}}{\sqrt{\pi} y^2} \Gamma \left(\frac{D + 3}{2}\right) \Gamma \left(\frac{D}{2} - 1\right), \quad D \geq 3,
\]

valid for \(|y| \gg 1\), for \(D > 4\) one gets (no summation over \(l = 1, \ldots, D\))

\[
\langle T^0_0 \rangle_b \approx \frac{D}{D - 4} \langle T^l_l \rangle_b \approx -\frac{2^{D-4}D(D - 1)\Gamma(D/2 - 1)}{(4\pi)^{D/2+1} \alpha D+1(z^D/\eta)^D},
\]

\[
\langle T^D_0 \rangle_b \approx \frac{2^{D-2}(D - 1)\Gamma(D/2 - 1)}{(4\pi)^{D/2+1} \alpha D+1(z^D/\eta)^5}.
\]

(44)

For \(D = 4\) the asymptotic expressions for the components \(\langle T^0_0 \rangle_b\) and \(\langle T^D_0 \rangle_b\) are still given by \((44)\), whereas for the stresses one has (no summation over \(l = 1, \ldots, D\))

\[
\langle T^l_l \rangle_b = -\frac{3\alpha^{-5} \ln(z^D/\eta)}{16\pi^3(z^D/\eta)^6}.
\]

(45)

As it is seen, at large distances the stresses are isotropic.

In figure 1 we have plotted the plate-induced parts in the components of the vacuum energy-momentum tensor as functions of \(z^D/\eta\) for \(D = 4\). The full curves correspond to the diagonal components, \(\alpha D+1 \langle T^l_l \rangle_b\), and the numbers near these curves are the values of the index \(l\). The dashed curve corresponds to the off-diagonal component, \(\alpha D+1 \langle T^D_0 \rangle_b\). As we see, all the diagonal components of the plate-induced energy-momentum tensor are negative, whereas the off-diagonal component corresponding to the energy flux is positive. The numerical calculations have shown that this is the case for other values of \(D\). In particular, the energy flux is directed from the plate.

Formulas \((40)\) and \((41)\) present the components of the energy-momentum tensor in the coordinates \((\tau, z^1, \ldots, z^D)\). For the components in the coordinates \((t, z^1, \ldots, z^D)\), denoted as \(\langle T^\nu_\mu \rangle_b\), one has (no summation over \(l = 0, 1, \ldots, D\)) \(\langle T^l_l \rangle_b = \langle T_l^l \rangle_b\), \(\langle T^D_0 \rangle_b = (\eta/\alpha) \langle T^D_0 \rangle_b\).
(s indicates the components in the synchronous time coordinate). Let $E_{V}^{(b)}$ be the boundary-induced part of the vacuum energy (with respect to the time coordinate $t$) in the spatial volume $V$ with a boundary $\partial V$:

$$E_{V}^{(b)} = \int_{V} d^{D}z \sqrt{|T_{(s)0}|_{b}},$$

(46)

where $\gamma$ is the determinant of the spatial metric tensor $\gamma_{ij} = -g_{ij}$ and the Latin indices run over $1, 2, \ldots, D$. From the asymptotic expression (44) it follows that the plate induced part in the total energy (per unit surface area of the plate) in the region $z_{1}^{D} \leq z^{D} < \infty$ is finite. From the equation $\nabla_{\nu}\langle T_{(s)\mu}\rangle_{b} = 0$ with $\mu = 0$, it follows that

$$\partial_{t} E_{V}^{(b)} = -\int_{\partial V} d^{D-1}z \sqrt{|n_{l}|_{b}} \langle T_{l}^{l}_{(s)0}\rangle_{b} + \frac{1}{\alpha} \int_{V} d^{D}z \sqrt{|n_{l}|_{b}} \langle T_{l}^{l}_{(s)0}\rangle_{b},$$

(47)

where $n_{l}, \gamma^{i}n_{i}n_{i} = 1$, is the external normal to the boundary $\partial V$ and $h$ is the determinant of the induced metric $h_{il} = \gamma_{il} - n_{i}n_{l}$. The first term in the right-hand side of Eq. (47) describes the energy flux through the boundary $\partial V$. As a volume $V$ let us take a cylinder with the axis perpendicular to the plate and with the bases at $z_{1}^{D}$ and $z_{2}^{D}$. If $S$ is the area of the cylinder base, $S = \int dz_{1} \cdots dz_{D-1}$, then the proper area is given by $(\alpha/\eta)^{D-1}S$. With this choice of the volume $V$ we have $n_{l}|_{z^{D} = z_{1}^{D}} = (-1)^{j}\delta_{l}^{D}\alpha/\eta$, and

$$\int_{\partial V} d^{D-1}z \sqrt{|n_{l}|_{b}} \langle T_{l}^{l}_{(s)0}\rangle_{b} = (\alpha/\eta)^{D} S \left[ \langle T_{l}^{D}_{(s)0}\rangle_{b}|_{z^{D} = z_{2}^{D}} - \langle T_{l}^{D}_{(s)0}\rangle_{b}|_{z^{D} = z_{1}^{D}} \right].$$

From here it follows that $\langle T_{0}^{D}\rangle_{b} = (\alpha/\eta)\langle T_{(s)0}\rangle_{b}$ is the energy flux per unit proper surface area. In particular, for the energy in the region $z^{D} \geq z_{1}^{D}$ we get

$$\partial_{t} E_{z^{D} \geq z_{1}^{D}}^{(b)} = (\alpha/\eta)^{D-1}S \left[ \langle T_{0}^{D}\rangle_{b}|_{z^{D} = z_{1}^{D}} + \frac{1}{\eta} \int_{z_{1}^{D}}^{\infty} dz^{D} \langle T_{l}^{l}\rangle_{b} \right].$$

(48)

The first term in the square brackets of (48) is positive whereas the second one is negative.
4 Conclusion

In the present paper we have considered the two-point functions for the electromagnetic field in background of dS spacetime assuming that the field is prepared in the Bunch-Davies vacuum state. To this aim, a complete set of mode functions was constructed. Then we have evaluated the two-point functions in the geometry of a conducting plate. By using these functions the VEVs of the field squared and the energy-momentum tensor are investigated. These VEVs are decomposed into the boundary-free and plate-induced parts. For points outside of the plate the renormalization is needed for the first parts only. Because of the maximal symmetry of the background spacetime and of the Bunch-Davies vacuum state the boundary-free parts do not depend on spacetime coordinates. The plate-induced parts depend on the coordinates \( z^D \) and \( \eta \) in the form of the ratio \( z^D / \eta \). The latter is the proper distance of the observation point from the plate, measured in the units of the dS curvature scale \( \alpha \). The plate-induced part in the VEV of the electric field squared is given by formula (29). For \( 3 \leq D \leq 8 \) this contribution is positive everywhere, whereas for \( D \geq 9 \) it is positive near the plate and negative at large distances. Simple asymptotic expressions, (32), (35), (36), are obtained at small and at large distances from the plate. At large distances, the plate-induced part decays as \((z^D / \eta)^{-6}\) for \( D > 4 \) and as \((z^D / \eta)^{2(1-D)}\) for \( D < 4 \). For \( D = 4 \) one has the asymptotic behavior \((z^D / \eta)^{-6} \ln(2z^D / \eta)\).

The plate-induced parts in the VEVs of the diagonal components of the energy-momentum tensor are given by the expressions (40). In addition to these components we have also nonzero off-diagonal component (11) which describes energy flux along the direction normal to the plate. The plate-induced part in the VEV of the energy-momentum tensor vanishes for \( D = 3 \). In this case the electromagnetic field is conformally invariant and this result is directly obtained from the corresponding result for a perfectly conducting plate in Minkowski spacetime by using the standard conformal transformation. For the components of the vacuum energy-momentum tensor one has asymptotic expressions (43), (44) and (45). Near the plate the stresses along the directions parallel to the plate are equal to the energy density, whereas for the normal stress and the energy flux one has \( \langle T^0_0 \rangle_b \approx (z^D / \eta)^2 \langle T^0_0 \rangle_b / (D - 1) \) and \( \langle T^D_0 \rangle_b \approx -(z^D / \eta) \langle T^0_0 \rangle_b \).

At large distances from the plate the vacuum stresses are isotropic. For \( D > 4 \) the diagonal components of the plate-induced parts decay as \((z^D / \eta)^{-4}\) and the off-diagonal component decays like \((z^D / \eta)^{-5}\). For \( D = 4 \) the asymptotic behavior for the components \( \langle T^0_0 \rangle_b \) and \( \langle T^D_0 \rangle_b \) remain the same, whereas the stresses behave as \((z^D / \eta)^{-6} \ln (z^D / \eta)\). For the numerical examples we have considered, all the diagonal components of the plate-induced vacuum energy-momentum tensor are negative, whereas the off-diagonal component \( \langle T^D_0 \rangle_b \) is positive. In particular, the energy flux is directed from the plate.

We have considered here the electromagnetic field interacting with boundaries and with the background gravitational field only. An interesting development of the problem in question would be the investigation of the role of loop corrections on the Casimir effect in dS spacetime. For a self-interacting scalar field the loop corrections on boundary-free dS spacetime have been attracting a lot of attention recently (see, for example, the recent review [17]). The calculations have shown that loop diagrams typically exhibit large infrared logarithms which are of key importance in discussing quantum field theory on dS background.

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References

[1] A.D. Linde, Particle Physics and Inflationary Cosmology, Harwood Academic Publishers, Chur, Switzerland, 1990.

[2] A.G. Riess et al., Astrophys. J. 659 (2007) 98; D.N. Spergel et al., Astrophys. J. Suppl. Ser. 170 (2007) 377; E. Komatsu et al., Astrophys. J. Suppl. Ser. 180 (2009) 330.

[3] E. Elizalde, S.D. Odintsov, A. Romeo, A.A. Bytsenko, S. Zerbini, Zeta Regularization Techniques with Applications, World Scientific, Singapore, 1994; V.M. Mostepanenko, N.N. Trunov, The Casimir Effect and Its Applications, Clarendon, Oxford, 1997; K.A. Milton, The Casimir Effect: Physical Manifestation of Zero-Point Energy, World Scientific, Singapore, 2002; M. Bordag, G.L. Klimchitskaya, U. Mohideen, V.M. Mostepanenko, Advances in the Casimir Effect, Oxford University Press, Oxford, 2009; Lecture Notes in Physics: Casimir Physics, edited by D. Dalvit, P. Milonni, D. Roberts, F. da Rosa, Springer, Berlin, 2011, Vol. 834.

[4] A.A. Saharian, T.A. Vardanyan, Class. Quantum Grav. 26 (2009) 195004; E. Elizalde, A.A. Saharian, T.A. Vardanyan, Phys. Rev. D 81 (2010) 124003; A.A. Saharian, Int. J. Mod. Phys. A 26 (2011) 3833.

[5] K.A. Milton, A.A. Saharian, Phys. Rev. D 85 (2012) 064005.

[6] M.R. Setare, R. Mansourii, Class. Quantum Grav. 18 (2001) 2331; M.R. Setare, Class. Quantum Grav. 18 (2001) 4823.

[7] P. Burda, JETP Lett. 93 (2011) 632.

[8] A.A. Saharian, M.R. Setare, Phys. Lett. B 659 (2008) 367; S. Bellucci, A.A. Saharian, Phys. Rev. D 77 (2008) 124010; A.A. Saharian, Class. Quantum Grav. 25 (2008) 165012; E.R. Bezerra de Mello, A.A. Saharian, JHEP 12 (2008) 081; S. Bellucci, A.A. Saharian, H.A. Nersisyan, arXiv:1302.

[9] B. Allen, T. Jacobson, Commun. Math. Phys. 103 (1986) 669.

[10] J.-P. Gazeau, M.V. Takook, J. Math. Phys. 41 (2000) 5920; T. Garidi, J.-P. Gazeau, S. Rouhani, M.V. Takook, J. Math. Phys. 49 (2008) 032501.

[11] A. Youssef, Phys. Rev. Lett. 107 (2011) 021101.

[12] I.I. Cotăescu, C. Crucean, Prog. Theor. Phys. 124 (2010) 1051.

[13] A.P. Prudnikov, Yu.A. Brychkov, O.I. Marichev, Integrals and Series, Gordon and Breach, New York, 1986, Vol. 2.

[14] Handbook of Mathematical Functions, edited by M. Abramowitz and I. A. Stegun, Dover, New York, 1972.

[15] J. Ambjørn, S. Wolfram, Ann. Phys. (N.Y.) 147 (1983) 1.

[16] H. Alnes, K. Olaussen, F. Ravndal, I.K. Wehus, J. Phys. A: Math. Theor. 40 (2007) F315; A. Edery, V.N. Marachevsky, JHEP 12 (2008) 035; L.P. Teo, JHEP 10 (2010) 019; L.P. Teo, Phys. Rev. D 83 (2011) 105020.

[17] E.T. Akhmedov, Int. J. Mod. Phys. D 23 (2013) 1430001.