Stability of compressed sheathings of wood composite plate-ribbed structures

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Abstract. Wood composite plate-ribbed structures working under the conditions of lateral bending in coverings and floors in buildings of various purposes are considered. The results of studies of the influence of various parameters on the stability of the upper compressed sheathing of wood composite panels with bonded joints of sheathings and ribs (type of connections, coefficient of stiffness of the bonded joints, span size, width of panels and material of anisotropic sheathing) are presented. To implement the task, the Lekhnitsky theory of calculating the stability of anisotropic plates is used, in particular, the energy method for determining the critical force for a flat anisotropic plate loaded along the edges by tangential efforts. For describing the distribution function of tangential efforts on the edges of the sheathings, the Rzhanitsyn theory of calculating composite rods is applied. As a result of numerical studies, the coefficients of longitudinal bending, which can be used in the engineering calculation of wood composite panels with mechanical and glued joints are obtained. Keywords: wood composite structures, ribbed panel and anisotropic sheathing, adhesive and mechanical joints, bonded joints, sheathing stability.

1. Introduction
Panel structures on a wooden frame have been widely used in both domestic and foreign construction, as bearing and enclosing elements of coverings, floors and wall fencing. Various wood-based sheet materials are used for sheathings. These are structural plywood (SP) [1], OSB [2] and CSB [3,4].

Despite the fact that at the "rib-sheathing" border rigid adhesive bond [5-8] is traditionally used, a series of experimental and theoretical studies [9-12] showed that sheathing when fixed to the ribs using modern bonded mechanical joints should be taken into account. The use of joints with increased resistance to longitudinal shear [13] allows their use as an alternative to adhesive joints [14,15]. A significant theoretical contribution to the improvement of the design of the panels and the calculation method was made by A. B. Gubenko, V.F. Ivanov, K.P. Kashkarov, M.E. Kagan, G.G. Carlsen, M.F. Kovalchuk, I.M. Linkov, et al. The research of such scientists as Abovskiy, P.A. Dmitriev, V.I. Zhadanov, A.A. Zhuravlev, I.S. Inzhutov, S.A. Korzon, V.G. Lenov, B.V. Labudin, B.K. Mikhailov, A.Y. Naichuk, R.B. Orlovich, K.P. Pyatikrestovsky, E.N. Serov, V.Y. Terentyev and others is devoted to the problems of development and improvement of the theory of lamellar structures design.
2. Relevance, scientific significance of the issue
In terms of calculations panel structures are single-span elements, which can be calculated according to the beam scheme (figure 1). Method of calculation of the panels on a wooden frame is presented in national and international standards of design and in recommendations [16, 17, 18, 19]. It regulates the procedure for calculating only panels with plywood sheathing, which is joint to the ribs by glue. However, the use of bonded joints at the "rib-sheathing" border changes the nature of the distribution of tangential tensions, and for materials such as CSB and OSB there is a significant difference in the elastic anisotropic properties compared to plywood. These factors will undoubtedly have an impact on the stability of the compressed upper sheathing, which should also be taken into account while calculating.

The purpose of the work is using the theory of calculating the stability of Lechnitsky flat anisotropic plates [20,21] to investigate the influence of various structural parameters (type of bonds, rigidity of bonded joints, material of the sheathing) on the stability of the compressed sheathing and to obtain coefficients of longitudinal bending for practical engineering calculations depending on the structural solution of the panel.

3. Defining the problem
A floor panel is considered as a spatial system consisting of a thin slab and longitudinal ribs supporting it. In accordance with the requirements [16], the upper compressed sheathing of glued plywood panels (figure 1) is tested for strength (possible loss of stability being taken into account) by the formula (1):

\[
\sigma_{pl.c} = \frac{M}{\phi_{pl} \cdot W_{ap}} \leq R_{pl.c},
\]

where \(M\) is the maximum bending moment; \(W_{ap}\) - the moment of inertia of the approximate cross-section of the panel; \(\phi_{pl}\) - coefficients of longitudinal bending, taking into account the possible loss of stability of the plywood sheathing; \(R_{pl.c}\) - calculated resistance of plywood to compression.

![Figure 1. Structure of ribbed panel with upper sheathing with mechanical and adhesive joints: a - a general view of the structure and a cross-section of the panel; b - scheme of calculation of the panel.](image)

However, as noted in [22], such an approach leads to an unnecessary waste of materials of designed panels and does not reflect the actual work of the structure. A formula (2), for calculating the loss of stability of the sheathing is proposed in the article; the validity of the formula is verified by the authors of the article on large-scale models and natural size structures:
\[
\sigma_{pl,c} = \frac{M}{(n \cdot I_r + \varphi_{pl} \cdot I_{pl})} (h_p - y_0) \leq R_{pl,c},
\]

where \( n \) - the coefficient of approximating wood to plywood, \( n = E_w / E_{pl} \); \( I_r \) - the moment of inertia of the main bearing ribs; \( I_{sh} \) - the moment of inertia of compressed sheathing; \( h_p \) - the height of the panel; \( y_0 \) - the distance from the neutral axis to center of gravity sheathing.

The coefficient of longitudinal bending \( \varphi \) depends on the ratio of the distance between the ribs of the panel in the light to the thickness of the sheathing \( \delta_{pl} \) and is determined by the formulas (3, 4) [16]:

- at \( a/\delta < 50 \):
  \[
  \varphi_{pl} = 1 - \frac{(a/\delta)^2}{5000};
  \]
- at \( a/\delta \geq 50 \):
  \[
  \varphi_{pl} = \frac{1250}{(a/\delta)^2}
  \]

Foreign standards [17,18] suggest the coefficients that take into account the possibility of loss of stability of compressed sheathing, which depend only on the thickness and type of plywood. Data on calculations of stability of compressed sheathings of other wood-based materials do not exist. Stability being tested, ductility of mechanical joints is not taken into account.

4. Theoretical part

To determine the critical force, we use the energy method for calculating stability [20, 21], the equation (5) of which is:

\[
\lambda_{cr} \cdot A = V_{bend},
\]

where \( A \) is the work of forces distributed along the longitudinal edges; \( V_{bend} \) - the potential energy of bending; \( \lambda_{cr} \) - stability of sheathing reserve coefficient.

Tangential forces \( \tau(x) \), distributed along the edges of the plate (figure 2), will do the work:

\[
A = 2\tilde{\lambda} \int_{0}^{h/2} \int_{0}^{l} \tau(x) \cdot \left( \frac{\partial w}{\partial x} \right)^2 dx dy,
\]

where \( \tilde{\lambda} \) is a variable factor for which the critical value \( \lambda_{cr} \) is determined.

![Figure 2. Diagram of the action of tangential forces on the longitudinal edges of the sheathing (plan view).](image)

Potential bending energy \( V_{bend} \) of the plate can be expressed the following way:
\[ 
\int_0^b \int_0^l D_1 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + 2D_2 \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + D_3 \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 4D_4 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \, dx \, dy, \tag{7} 
\]

wherein \( D_1, D_2 \) - cylindrical stiffnesses around the main axes, determined by the formula (8), \( D_t \) - torsional rigidity (8); \( w \) is the function of plate deflections (9); \( l, b \) - the length and width of the sheathing, respectively.

\[ D_1 = \frac{E_{sh,x} \cdot \delta^3}{12(1-\nu_{xy} \cdot \nu_{yx})}; \quad D_2 = \frac{E_{sh,y} \cdot \delta^3}{12(1-\nu_{xy} \cdot \nu_{yx})}; \quad D_t = \frac{G \cdot \delta^3}{12}, \tag{8} \]

where \( E_{sh,x}, E_{sh,y} \) - sheathing elasticity modules along the main axes; \( \nu_{xy}, \nu_{yx} \) - Poisson's coefficients; \( \delta \) is the thickness of the sheathing.

The function of deflections satisfying the conditions of fixing the sides (swivel bearing on the contour) has the form (9):

\[ w(x, y) = A_{m1} \cdot \sin \left( \frac{m\pi x}{l} \right) \sin \left( \frac{n\pi y}{b} \right), \tag{9} \]

where \( A_{m1} \) - indefinite constant coefficient; \( m, n \) - the number of half-waves of sheathing deflections, determining the shape of the loss of stability.

For a panel with bonded joints with evenly distributed loads, shear stress on sheathing edge \( \tau(x) \) according to (23) is described by dependency (10):

\[ \tau(x) = \frac{1}{2} \left( \frac{\xi q \left( \lambda_i^2 l - 2\lambda_i^2 x + 2\lambda_i \text{sh} (\lambda_i l) - \lambda_i \text{ch} (\lambda_i l) \right) + 2 \sum EI \cdot \lambda_i^4}{2 \sum EI \cdot \lambda_i^4 \text{ch} (\lambda_i l)} + \frac{\xi q \cdot \text{ch} (\lambda_i x) \cdot (\lambda_i l \text{sh} (\lambda_i l) - 2 \text{ch} (\lambda_i l) + 2)}{2 \sum EI \cdot \lambda_i^4 \text{sh} (\lambda_i l)} \right), \tag{10} \]

where \( x \) is the coordinate measured from the left support of the panel; \( q \) – evenly distributed load, approximated to the linear on one rib; \( l \) – span of a panel; \( \xi \) — stiffness coefficient of bonded joints; \( \lambda_i \) - coefficient determined by the formula (11):

\[ \lambda_i = \sqrt{\xi \cdot \gamma}; \tag{11} \]

\[ \gamma = \frac{1}{E_{sh,x} \cdot A_{sh}} + \frac{1}{E_{r} \cdot A_{r}} + \frac{e^2}{\sum EI}, \tag{12} \]

where \( E_r \) - module of elasticity of timber of a rib in the direction of span of the panel; \( A_{sh}; A_{r} \) - cross-sectional area of sheathing and rib respectively; \( \sum EI \) - the sum of the bending stiffnesses of sheathing and rib.

The coefficient \( \xi \) is determined by formula (11) and depends on the shear stiffness of the connections used and on the spacing of their placement [14].

\[ \xi = \frac{n \cdot c_{\text{con}}}{S_{\text{con}}}, \tag{13} \]

where \( c_{\text{con}} \) - the stiffness coefficient of one connection; \( S_{\text{con}} \) - the space between connections, \( n \) - the number of ribs.

For a panel with a rigid adhesive joint (\( \xi \to \infty \)) the distribution function of tangential forces at the "rib-sheathing" border (10) takes the form (14):
\[
\tau(x) = \frac{1}{2} \frac{cq(l - 2x)}{2 \sum EI \gamma}
\]

(14)

Taking (6) and (7) into account, we obtain a formula (15) for determining the reserve coefficient of stability \(\lambda_{cr}\).

\[
\lambda_{cr} = \frac{\int_{0}^{b} \int_{0}^{l} D_1 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + 2D_2 \frac{\partial^2 w}{\partial y^2} + D_2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dx dy}{2\int_{0}^{b} \int_{0}^{l} \tau(x) \left( \frac{\partial w}{\partial x} \right)^2 dx dy}
\]

(15)

Coefficient of longitudinal bending \(\varphi\) for sheathing is determined by the formula (16) [24]:

\[
\varphi = \frac{\sigma_{cr}^{\max}}{\sigma_{lim}}
\]

(16)

where \(\sigma_{cr}^{\max}\) - maximum stresses in the sheathing corresponding to the load at which the loss of stability of sheathing occurs; \(\sigma_{lim}\) - the maximum stresses corresponding to the limit of the material’s strength.

It should be noted that the normal stresses directed along the panel axis arising in the sheathing will have maximum value at the ribs and decrease with distance from them. In the middle of the sheathing, the stresses will be minimal. The algorithm for calculating the maximum stresses is presented in our work [25] and is not given here.

5. Practical significance

Let us consider the above presented solution with the following example: wood-composite panel, sizes \(b \times l\) with two longitudinal ribs (redwood (pine), grade II), supported at the ends by the walls with the help of swivels. The cross-section of the ribs is 50×150 mm. The sheathing is connected to the ribs by bonded joints. Let us consider three variants of sheathings: structural plywood and SP; OSB/3; CSB. The span is \(l = 3\) m, which corresponds to the maximum sheet length of most wood composite materials. Let us investigate the dependence of the coefficient of longitudinal bending \(\varphi\) on the material of sheathings (SP, OSB, CSB), ductility of bonds (\(\xi = 5000, 15000, 30000\) and \(\infty\), kN/m²), ratios \(a/\delta\), where \(a\) is the distance between the axes of longitudinal ribs (for panels with mechanical joints) or the distance between the ribs in the light (with adhesive joints of sheathing); \(\delta\) is the thickness of the sheathing. Results are shown in table 1 and figure 3.

Table 1. Values of coefficients of longitudinal bending in sheathing.

| \(a/\delta\) | \(\xi = 5000\) | \(\xi = 15000\) | \(\xi = 30000\) | \(\xi = \infty\) | \(\xi = 5000\) | \(\xi = 15000\) | \(\xi = 30000\) | \(\xi = \infty\) |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Plywood     | SP             | OSB            | CSB            |                |                |                |                |                |
| 200         | 0.061-         | 0.061-         | 0.061-         | 0.06           | 0.061-         | 0.061-         | 0.061-         | 0.061-         |
| 175         | 0.092-         | 0.092-         | 0.092-         | 0.091          | 0.092-         | 0.092-         | 0.092-         | 0.092-         |
| 150         | 0.131-         | 0.131-         | 0.131-         | 0.129          | 0.131-         | 0.131-         | 0.131-         | 0.131-         |
| 125         | 0.190-         | 0.190-         | 0.190-         | 0.195          | 0.190-         | 0.190-         | 0.190-         | 0.190-         |
| 100         | 0.269-         | 0.269-         | 0.269-         | 0.272          | 0.269-         | 0.269-         | 0.269-         | 0.269-         |
| 75          | 0.498-         | 0.498-         | 0.498-         | 0.504          | 0.498-         | 0.498-         | 0.498-         | 0.498-         |
| 50          | -a             | -a             | -a             | 0.500          | -a             | -a             | -a             | -a             |

The sign "-a" means that sheathing material is depleted before it loses its stability.
Figure 3. Dependency of coefficient of longitudinal bending $\varphi$ on the ratio $a/\delta$: a - panel with bonded mechanical joints; b – panel with a rigid adhesive joint; plywood sheathing; OSB sheathing; CSB sheathing.

From the graphs in figure 3, we can see that coefficients of longitudinal bending largely depend on the material of sheathing, as well as on the ratios ($a/\delta$), and can vary over a very wide range of values. At the same time, the rigidity of joints of shear does not have much influence, within the limits of change of stiffness coefficient $\xi = 5000 \ldots 30000$ kN / m$^2$ (the difference does not exceed 1%). However, when using a rigid adhesive joint, there is a slightly larger discrepancy between the values of the coefficient $\varphi$, (up to 2.63% for plywood, 7.6% for OSB and 1.47 % for CSB). When calculating the critical load, shape of loss of stability was also taken into account by varying the values of $m$ and $n$ (the number of half-waves of deflection of sheathing in the longitudinal and cross-sectional directions, respectively) and for all the considered options the smallest values of the coefficient of reserve are typical for $n = 1$. At the same time, with a decrease in the ratio ($a/\delta$) $m$ increased from 1 to 5 ... 7, depending on the material of sheathings. Values of coefficients $\varphi$, given in the norms [16], are greatly underestimated and differ from calculated values by 48 ... 55%. Coefficients of stability presented in foreign standards [17, 18] take into account the thickness, but do not take into account the width of the sheathing, so a comparison of the results obtained is not given here.

6. Conclusion
To bring the paper to a close, we summarize the key points here:

1. The presented technique allows checking the stability of compressed sheathings of wood composite panels with both bonded mechanical joints and rigid adhesive joints. Calculated width of sheathing with mechanical joints should be equal to the distance between the ribs in the axes, and with rigid adhesive joints to the distance between the ribs in light.

2. Material of sheathing, its physical-mechanical properties and the ratio of span $a$ to the thickness $\delta$ significantly affect the coefficient longitudinal bending $\varphi$, the values of which for
materials considered above can be determined by the graphs (figure 3), depending on the type of connections. Coefficient of rigidity of mechanical joints has no significant effect on the stability coefficient, thus, this factor can be ignored while calculating.

3. Coefficients longitudinal bending $\phi$ in domestic and foreign standards are presented only for the plywood sheathings and their values are significantly different from those obtained. The method of determining the coefficient $\phi$ for other wood-based sheet materials does not exist. The obtained results are recommended to be used in calculations of wood composite panels, that allows to clarify the existing method for determining the coefficient $\phi_{pl}$ in regulations.

4. It is necessary to continue research in the field of influence of the operating conditions, humidity and concentrated loads on the stability of the sheathings.

References

[1] 2004 GOST 3916.1-96 General purpose plywood with outer layers of hardwood veneer. Technical conditions (Minsk: Standards Publishing) p 27
[2] 2014 GOST 32567-2013 Wood plates with oriented chips. Technical conditions (Moscow: Standardinform) p 17
[3] 2016 GOST 26816–2016 Cement-particle boards. Technical conditions (Moscow: Standardinform) p 15
[4] CNIISK named by Kucherenko 1981 Recommendations for the use of cement-particle boards in enclosing structures for industrial, rural industrial and housing construction (Moscow) p 14
[5] Tisevich E V 2008 Compressed-flexural glued plywood wall panels with sheathing included in the overall design work: dissertation of the candidate of technical sciences (Orenburg) p 209
[6] Ukrainchenko D A Wooden construction with standardized panel glued board sheathing: dissertation of the candidate of technical sciences (Orenburg) p 188
[7] Zhadanov V I Low-rise buildings and structures of combined edge-based wood-based structures: dissertation of the doctor of technical sciences (Orenburg) p 423
[8] Nikitin V M 2009 Large-size ribbed plates with a combined sheathing for building coatings: dissertation of the candidate of technical sciences (Krasnoyarsk) p 138
[9] Chernyh A S 2015 Improving the design and production technology of wood-framed wall panels: dissertation of the candidate of technical sciences (Arkhangelsk) p 168
[10] Mekhanikov V M 1995 Connection of elements in structures using DSP: dissertation of the candidate of technical sciences (Moscow) p 211
[11] Kavelin A S 2014 Construction-2014: modern problems of industrial and civil construction. Materials of the international scientific-practical conference. Rostov State University of Civil Engineering, Institute of Industrial and Civil Engineering pp 98–100
[12] Labudin B V, Voronkov S A, Gmyrina A P and Ruslanova A P 2015 Construction science – XXI: theory, education, practice, innovation of the North-Arctic region. Proceedings of the International Scientific and Technical Conference 28–30.06.2015 (Saint Petersburg) pp 187–193
[13] Popov E V, Tyurikova T V, Labudin B V and Melekhov V I 2016 Structural Mechanics and Structures Calculation 4 23–28
[14] Popov E V, Filippov V V, Melekhov V I, Labudin B V and Tyurikova T V 2016 The effect of shear bond stiffness when calculating ribbed panels on a wooden frame Forest Journal (Arkhangelsk) 4 123–134
[15] Popov E V, Filippov V V, Zhuravleva T P, Melekhov V I and Labudin B V 2016 Security of the construction site of Russia. Problems and Solutions. International Academic Readings 24–25.11.2016 (Kursk) p 111–116
[16] 2017 SP 64.13330.2017 Wooden structures. Updated edition SNiP II–25–80 (with a change №1) (Techekspert: electronic fund of legal and regulatory and technical documentation) http://docs.cntd.ru/document/456082589, free
[17] 2014 DIN EN 1995–1–1/A2–2014 Eurocode 5: Design of timber structures 1–1: General – Common rules and rules for buildings (German version EN 1995–1–1:2004/A2:2014)
[18] Chernyh A G and Byzov V E 2015 International regulatory framework for design (Eurocodes) (Moscow: Publishing house ACU) p 74
[19] CNIISK named by Kucherenko 1982 Recommendations for the design of panel structures using wood and wood materials for industrial buildings (Moscow) p 120
[20] Lekhnickij S G 1947 Anisotropic plates (Moscow, ASP Gostekhizdat) p 355
[21] Lehnitsky S G 1943 Stability of anisotropic plates. Handbook for aircraft designers (Moscow, ASP Gostekhizdat) p 80
[22] Zhadanov V I, Inzhutov I S, Ukrainchenko D A and Yarichevsky I I 2016 Proceedings of higher educational institutions. Construction 2(686) 15–24
[23] Rzhanitsyn A R 1986 Composite rods and plates: scientific publication (Moscow: Stroiizdat) p 314
[24] Pyatikrestovskij K P 2014 Nonlinear methods of mechanics in the design of modern wooden structures: monograph (Moscow, MSCU) p 320
[25] Stolypin D A, Sopilov V V, Popov E V and Labudin B V 2018 Engineering personnel - the future of the innovation economy of Russia: materials of the IV All-Russian Student Conference 5: Innovations in construction, environmental engineering and technosphere safety (Yoshkar–Ola, Volga State University of Technology) pp 107–115