Physical unitarity in the Lagrangian
$Sp(2)$-symmetric formalism

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Abstract

The structure of state vector space for a general (non-anomalous) gauge theory is studied within the Lagrangian version of the $Sp(2)$-symmetric quantization method. The physical $S$-matrix unitarity conditions are formulated. The general results are illustrated on the basis of simple gauge theory models.

1. Introduction

The majority of advanced field theory models are formulated in terms of gauge theories. The manifestly covariant quantization of gauge theories is carried out in the Lagrangian formalism with the use of functional integration. These methods can be divided into two groups depending on whether gauge theories quantization is based on the principle of invariance under the BRST (Becchi–Rouet–Stora–Tyutin) symmetry [1, 2] or the quantization rules are underlied by a realization of the principle of invariance under the extended BRST symmetry transformations including, along with the BRST transformations, also the so-called antiBRST transformations [3, 4].

For arbitrary gauge theories (general gauge theories), the BRST symmetry principle has first been realized within the BV (Batalin–Vilkovisky) quantization scheme [5, 6] well-known at present. The same principle provides the basis of the method [7] of superfield quantization discovered for general gauge theories quite recently.

The studies of Refs. [8–10] by Batalin, Lavrov and Tyutin have suggested a quantization scheme (the Lagrangian $Sp(2)$-symmetric formalism), in which, for general gauge theories, the extended BRST symmetry has been manifestly realized. Note that the extended BRST symmetry principle also underlies a superfield quantization method recently proposed for general gauge theories in Ref. [11].

Among the number of questions arising in connection with the Lagrangian quantization of gauge theories, two problems are of great importance. This is first of all the unitarity problem of a theory. Next comes the issue of the dependence of a theory upon the gauge. These long-standing problems have been explicitly formulated by Feynman [12] (the unitarity problem) and Jackiw [13] (the issue of gauge dependence).

For the Yang–Mills type theories, the problem of gauge dependence has been thoroughly studied in Refs. [12–19] (for more references, see Ref. [20]), and for general gauge theories, in Refs. [21, 22]. The studies of Refs. [21, 22] have proved general theorems on the gauge dependence of both the non-renormalized and renormalized Green’s functions and the $S$-matrix for general gauge theories in arbitrary gauges. The theorems themselves have been
proved on general assumptions of the absence of anomalies, the use of loop expansions and the existence of regularization respecting the Ward identities. The latest outburst of interest in the gauge dependence within gauge theories has been caused by Ref. [23], in which the author calculated the one-loop effective action for Einstein gravity within a special class of background gauges and found the effective action to depend upon the gauge on-shell which conflicts with the statements of the general theorems on gauge dependence [21, 22]. There have been papers either maintaining the result [23] and giving reasons for possible gauge dependence in arbitrary non-renormalizable gauge theories [24] or expressing a doubt [25] about the applicability of the general statements [21, 22], as formal ones, to specific theories (Einstein gravity, in particular). The study of Ref. [26] carried out the calculation of the one-loop effective action for Einstein gravity within the class of gauges suggested in Ref. [23]; it was shown, firstly, that the general assumptions applied for the proof of the theorems [21, 22] are valid in the particular case, secondly, that the one-loop effective action for Einstein gravity does not depend upon the gauge on-shell in exact conformity with the theorems [21, 22] and, finally, that the gauge dependence asserted in Ref. [23] results from incorrect calculations (this observation is also valid as regards Ref. [24]).

For the non-renormalized Green’s functions and the S-matrix, the gauge dependence within the Lagrangian Sp(2)-symmetric quantization scheme was studied in Ref. [27].

Turning again to the unitarity problem in quantum gauge theories within the Lagrangian formalism, note that for the Yang–Mills type theories it was efficiently analyzed in Ref. [28] by Kugo and Ojima in the framework of a formalism discovered by them and based on the study of the physical subspace \( V_{\text{phys}} \) of the total state vector space \( V \) with indefinite inner product \( \langle | \rangle \) (note that vector spaces having indefinite inner product are also commonly referred to as vector spaces with indefinite metric; see, for instance, Ref. [29]).

The subspace \( V_{\text{phys}} \equiv \{|\text{phys}\rangle\} \) is specified by the operator \( \hat{Q}_{\text{BRST}} \) (\( \hat{Q}^\dagger_{\text{BRST}} = \hat{Q}_{\text{BRST}} \))

\[
\hat{Q}_{\text{BRST}}|\text{phys}\rangle = 0
\] (1.1)

being the generator of the BRST symmetry transformations and possessing an important nilpotency property

\[
\hat{Q}_{\text{BRST}}^2 = 0.
\] (1.2)

In the Yang–Mills type theories, the nilpotency of the operator \( \hat{Q}_{\text{BRST}} \) follows immediately from the nilpotency of the BRST transformations.

Even though in arbitrary gauge theories the algebra of the BRST transformations is generally open (off-shell), one can still prove (on the assumption of the absence of anomalies) that within such theories, for the corresponding operator \( \hat{Q}_{\text{BRST}} \) the nilpotency property holds [30]. Thus, one can assume that the Noether charge operator \( \hat{Q}_{\text{BRST}} \) in the BV quantization scheme satisfies Eq. (1.2) and that the Kugo–Ojima formalism, discovered for the Yang–Mills type theories, applies to the analysis of the unitarity problem for general gauge theories.

In discussing the property (1.2), it is important to bear in mind that the widespread opinion that the nilpotency of the operator \( \hat{Q}_{\text{BRST}} \) guarantees the unitarity of a theory (see, for example, Ref. [31]) proves to be incorrect [32], and that a more accurate examination of physicality conditions fulfilment ensuring the unitarity of a theory is then required. To this end, we shall now recall the main results of analysis of the unitarity problem within the framework of the formalism proposed by Kugo and Ojima.

In Ref. [28] it was shown that if a theory satisfies the following conditions (physicality criteria) for the Hamiltonian \( \hat{H} \) and the physical subspace \( V_{\text{phys}} \) in the total state vector space \( V \) with indefinite inner product \( \langle | \rangle \)

(i) hermiticity of the Hamiltonian \( \hat{H} = \hat{H}^\dagger \) (or (pseudo-)unitarity of the total
(iii) positive semi-definuteness of inner product $<\psi|\psi> \geq 0$),
then the physical $S$-matrix $S_{\text{phys}}$ is consistently defined in a Hilbert space $H_{\text{phys}}$ equipped with positive definite inner product (the probabilistic interpretation of the quantum theory thus secured). Namely, $H_{\text{phys}}$ can be identified with a (completed) quotient space

$$V_{\text{phys}}/V_0 \ni |\tilde{\Phi}>, |\tilde{\Phi}>=|\Phi> + V_0, |\Phi> \in V_{\text{phys}}$$

of $V_{\text{phys}}$ with respect to the zero-norm subspace $V_0$

$$V_0 = \{|\chi> \in V_{\text{phys}} : <\chi|\chi> = 0\}, V_{\text{phys}} \perp V_0,$$

where positive definite inner product in $V_{\text{phys}}/V_0$ is defined by $<\tilde{\Phi}|\tilde{\Psi}> = <\Phi|\Psi>$. Given this, the physical $S$-matrix in $H_{\text{phys}}$

$$H_{\text{phys}} = \overline{V_{\text{phys}}/V_0}, S_{\text{phys}}|\tilde{\Phi}>=S|\Phi>$$

the unitarity property holds

$$S_{\text{phys}}^\dagger S_{\text{phys}} = S_{\text{phys}} S_{\text{phys}}^\dagger = 1.$$ 

In this connection, note first of all that the subsidiary condition (1.1) ensures, on the assumption of hermiticity of the Hamiltonian, the fulfilment of the condition (1.3), (ii) of invariance $V_{\text{phys}}$ under the time development ($V_{\text{phys}}^\text{in} = V_{\text{phys}}^\text{out}$). In Ref. [27], the analysis of the condition (1.3), (iii) for an arbitrary theory (1.2) was based on the study of representation of the algebra of the operator $\hat{Q}_{\text{BRST}}$ and the ghost charge operator $i\hat{Q}_C$ ($[\hat{Q}_C, \hat{H}] = 0$

$$[i\hat{Q}_C, \hat{Q}_{\text{BRST}}] = \hat{Q}_{\text{BRST}}$$

(the other commutators trivially vanish) in the one-particle subspace of the total Fock space $V$.

The one-particle subspace of the theory generally consists of the so-called genuine BRST-singlets, singlet pairs, and quartets [28]. By definition, the BRST-singlets are introduced as state vectors $|k, N > (i\hat{Q}_C |k, N > = N |k, N >)$ from the physical subspace $V_{\text{phys}}$ which cannot be represented in the form $|k, N > = \tilde{Q}_{\text{BRST}} |* >$ for any state $|* >$. Here, $k$ stands for all the quantum numbers (except the ghost one) which specify the state. At that, if $N = 0$, then these BRST-singlets are called genuine ones and identified with physical states having positive norm. Meanwhile, if $N \neq 0$, then state vectors $|k, -N >, |k, N >)$ from the physical subspace (1.1) possess zero norm and form a singlet pair with non-vanishing inner product

$$<k, -N|k, N >= 1.$$ 

Finally, the states $|k, N >, |k, -N >, |k, N + 1 >, |k, -(N + 1) >)$ such that

$$|k, N + 1 >= \tilde{Q}_{\text{BRST}} |k, N >, |k, -N >= \tilde{Q}_{\text{BRST}} |k, -(N + 1) >, <k, -(N + 1)|k, N + 1 >= <k, -N|k, N >= 1$$

form a quartet.

The study of Ref. [28] discovered a general mechanism, called the quartet one, by virtue of which any state that belongs to the physical subspace $V_{\text{phys}}$ of the total Fock space and contains quartet particles has vanishing norm. At the same time, the condition (1.3), (iii) of positive semi-definiteness of inner product $<| >$ in $V_{\text{phys}}$ is taken over by a requirement [28] of the absence of singlet pairs, which thus guarantees the physical $S$-matrix unitarity in the Hilbert space $H_{\text{phys}} = \overline{V_{\text{phys}}/V_0}$. 


quantization. With that, the structure of asymptotic space is analysed on the basis of representation of the algebra of operators $\hat{Q}^a$, $i\hat{Q}_C$. Here, $\hat{Q}^a$ is an $Sp(2)$-doublet of scalar Noether charge operators being the generators of the extended BRST symmetry transformations. The physical unitarity analysis implies such general assumptions [28] as the non-degeneracy of indefinite inner product $< | >$, the absence of spontaneous symmetry breaking ($\hat{Q}^a|0> = \hat{Q}_C|0> = 0$), the asymptotic completeness of state vector space $V$.

The paper is organized as follows. In Section 2 we summarize the key points of the Lagrangian $Sp(2)$-symmetric method and discuss the algebraic properties of the extended BRST symmetry transformations (as well as their generators) for general gauge theories; in this section we use the condensed notations suggested by De Witt [33] and the designations adopted in Refs. [8–10]. Section 3 is devoted to the study of the general structure of state vector space and to the formulation of the physical unitarity conditions. In Section 4 we illustrate the application of the proposed physical unitarity analysis on the basis of the study of state vector spaces in concrete gauge theory models [32, 34] within the Lagrangian $Sp(2)$ quantization method.

2. $Sp(2)$-symmetric Lagrangian quantization

Let us now bring to mind the key points of the Lagrangian $Sp(2)$-symmetric method [8–10]. To this end, note first of all that the quantization of an arbitrary gauge theory within the formalism [8–10] involves introducing a complete set of fields $\phi^A$ and the set of the corresponding antifields $\phi^*_A$ ($a=1, 2$), $\bar{\phi}_A$ (the doublets of antifields $\phi^*_Aa$ play the role of sources of the BRST and antiBRST transformations while the antifields $\phi_A$ are the sources of the mixed BRST and antiBRST transformations) with the following distribution of the Grassmann parity

$$\varepsilon(\phi^A) \equiv \varepsilon_A, \quad \varepsilon(\phi^*_A) = \varepsilon_A + 1, \quad \varepsilon(\bar{\phi}_A) = \varepsilon_A$$

and the ghost number

$$\text{gh}(\phi^*_A) = (-1)^a - \text{gh}(\phi^A), \quad \text{gh}(\bar{\phi}_A) = -\text{gh}(\phi^A).$$

The specific structure of configuration space of the fields $\phi^A$ (including the initial classical fields, the ghosts, the antighosts and the Lagrangian multipliers) is determined by the properties of original classical theory, i.e. by the linear dependence (reducible theories) or independence (irreducible theories) of generators of gauge transformations. Namely, the studies of Refs. [8, 9] have shown that the fields $\phi^A$ form components of irreducible completely symmetric $Sp(2)$-tensors. The basic object of the Lagrangian $Sp(2)$-symmetric scheme is the bosonic functional $S = S(\phi, \phi^*_A, \bar{\phi})$, which enables one to construct the generating functional of Green’s functions and satisfies the equations [8–10]

$$\frac{1}{2}(S, S)^e + V^eS = i\hbar \Delta^eS$$  \hspace{1cm} (2.1)

with the boundary condition

$$S|_{\phi^*_A=\bar{\phi}=\hbar=0} = \mathcal{S},$$  \hspace{1cm} (2.2)

where $\mathcal{S}$ is the initial gauge invariant classical action. In Eq. (2.1), $\hbar$ is the Planck constant; $(\ , \ )^e$ is the extended antibracket introduced for two arbitrary functionals $F = F(\phi, \phi^*_A, \bar{\phi})$ and $G = G(\phi, \phi^*_A, \bar{\phi})$ by the rule

$$(F, G)^a = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi^*_A} - \frac{\delta G}{\delta \bar{\phi}^A} \frac{\delta F}{\delta \bar{\phi}_A} (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}.$$
on the fields of complete configuration space only. It should also be pointed out that the transformations (2.5) provide a basis for the proof of physical equivalence between the quantization of a general gauge theory in the BV formalism and the one in the Lagrangian $Sp(2)$-symmetric quantization scheme, by the rule

$$Z(J) = Z(J, \phi^*_a, \bar{\phi})|_{\phi^*_a = \phi = 0},$$

which enables one, in particular, to establish the compatibility [8] of Eq. (2.1).

The algebra of operators $\Delta^a$, $V^a$ and the properties of the extended antibracket were studied in detail in Ref. [8], and we omit here the corresponding discussion.

The study of Ref. [10] proved the existence theorem for solutions of Eq. (2.1) with the boundary condition (2.2) in the form of expansions in $\hbar$ powers and described the characteristic arbitrariness of solutions.

The generating functional $Z(J)$ of Green’s functions for the fields of complete configuration space is constructed, within the Lagrangian $Sp(2)$-symmetric quantization scheme, by the rule

$$Z(J) = Z(J, \phi^*_a, \bar{\phi})|_{\phi^*_a = \phi = 0},$$

where the extended functional $Z(J, \phi^*_a, \bar{\phi})$ of Green’s functions is defined in the form of a functional integral [8]

$$Z(J, \phi^*_a, \bar{\phi}) = \int d\phi \exp \left\{ \frac{i}{\hbar} \left( S_{\text{ext}}(\phi, \phi^*_a, \bar{\phi}) + J_A \phi^A \right) \right\}.$$  

In Eq. (2.3), (2.4) $J_A$ are the conventional sources to the fields $\phi^A$ ($\varepsilon(J_A) = \varepsilon_A$, $gh(J_A) = -gh(\phi)$), while $S_{\text{ext}} = S_{\text{ext}}(\phi, \phi^*_a, \bar{\phi})$ is a bosonic functional given by

$$\exp \left\{ \frac{i}{\hbar} S_{\text{ext}} \right\} = \exp \left\{ -i\hbar \tilde{T}(F) \right\} \exp \left\{ \frac{i}{\hbar} S \right\},$$

where $S = S(\phi, \phi^*_a, \bar{\phi})$ is a solution of Eq. (2.1) with the boundary condition (2.2), and $\tilde{T}(F)$ is an operator defined as

$$\tilde{T}(F) = \frac{1}{2} \varepsilon_{ab} [\Delta^b, [\Delta^a, F]_-]_+.$$  

Here, $F = F(\phi)$ is a bosonic functional fixing a concrete choice of admissible gauge, i.e. chosen so as the functional $S_{\text{ext}} = S_{\text{ext}}(\phi, \phi^*_a, \bar{\phi})$ be non-degenerate in $\phi$ (examples of such functionals $F$ have been given in Refs. [8, 9]). It follows from the algebraic properties of the operators $\Delta^a$ ($[\Delta^a, \Delta^b]_+ = 0$) that the functional $S_{\text{ext}}$ (2.5) satisfies Eq. (2.1). Note that the gauge fixing (2.5) is in fact a particular case of the transformation generated by $\tilde{T}(F)$, with any bosonic operator chosen for $F$, and describing the arbitrariness of solutions of Eq. (2.1). It should also be pointed out that the transformations (2.5) provide a basis for the proof [10] of physical equivalence between the quantization of a general gauge theory in the BV formalism and the one in the Lagrangian $Sp(2)$-symmetric formalism.

By virtue of the explicit form of the operator $\tilde{T}(F)$ (2.6) with the functional $F$ depending on the fields of complete configuration space only

$$i \int dF \frac{\delta}{\delta \phi} - 1 = \frac{\delta}{\delta \phi} + \frac{\delta^2}{\delta \phi^2} F = \frac{\delta}{\delta \phi}.$$
the generating functional \( Z(J, \phi^*_a, \bar{\phi}) \) of Green’s functions (2.3), (2.4) is representable in the form [8]

\[
Z(J) = \int d\phi d\phi^*_a d\bar{\phi} d\pi^a \exp \left\{ i \left( \frac{\hbar}{\lambda^A} \left( S(\phi, \phi^*_a, \bar{\phi}) + \phi^*_a \pi^{Aa} + \bar{\phi} \lambda^A - \frac{1}{2} \varepsilon_{ab} \pi^{Aa} \pi^{Ab} + J_A \phi^A \right) \right) \right\},
\]

(2.7)

where \( \pi^{Aa}, \lambda^A \)

\[
\varepsilon(\pi^{Aa}) = \varepsilon + 1, \quad gh(\pi^{Aa}) = (-1)^a + gh(\phi^A),
\]

\[
\varepsilon(\lambda^A) = \varepsilon, \quad gh(\lambda^A) = gh(\phi^A)
\]

are auxiliary variables introducing the gauge.

The validity of Eq. (2.1) for the functional \( S = S(\phi, \phi^*_a, \bar{\phi}) \) enables one, firstly, to establish an important fact that the integrand in Eqs. (2.7) is invariant for \( J = 0 \) under the following global supersymmetry transformations

\[
\delta \phi^A = \pi^{Aa} \mu_a, \quad \delta \phi^A_{Aa} = \mu_a \frac{\delta S}{\delta \phi^A}, \quad \delta \bar{\phi} = \varepsilon_{ab} \mu_a \phi^*_b,
\]

(2.8)

\[
\delta \pi^{Aa} = -\varepsilon_{ab} \lambda^A \mu_b, \quad \delta \lambda^A = 0,
\]

where \( \mu_a \) is an \( Sp(2) \)-doublet of constant anticommuting infinitesimal parameters. The transformations (2.8) realize the extended BRST symmetry transformations in terms of the variables \( \phi, \phi^*_a, \bar{\phi}, \pi^a, \lambda \) and permit establishing the independence [8] of the \( S \)-matrix on a choice of the gauge within the Lagrangian \( Sp(2) \)-symmetric quantization scheme. Namely, let us denote the vacuum functional as \( Z(0) \equiv Z_F \) and change the gauge \( F \rightarrow F + \Delta F \). Then, making in the functional integral for \( Z_{F+\Delta F} \) the change of variables (2.8) and choosing for the parameters \( \mu_a \)

\[
\mu_a = \frac{i}{2\hbar} \varepsilon_{ab} \frac{\delta \Delta F}{\delta \phi^A} \pi^{Ab},
\]

we find that \( Z_{F+\Delta F} = Z_F \) and conclude that the \( S \)-matrix is in fact gauge-invariant.

Secondly, by virtue of Eqs. (2.1), (2.5), the extended generating functional \( Z(J, \phi^*_a, \bar{\phi}) \) of Green’s functions satisfies the Ward identities of the form [8]

\[
\left( J_A \frac{\delta}{\delta \phi^A_{Aa}} - \varepsilon_{ab} \phi^*_b \frac{\delta}{\delta \phi^A} \right) Z(J, \phi^*_a, \bar{\phi}) = 0.
\]

(2.9)

The study of Ref. [27] showed, with the help of Eq. (2.9), that the generating functional \( \Gamma = \Gamma(\phi, \phi^*_a, \bar{\phi}) \) of vertex functions (derivatives with respect to the sources \( J \) are understood as the left-hand ones)

\[
\Gamma(\phi, \phi^*_a, \bar{\phi}) = \frac{\hbar}{i} \ln Z(J, \phi^*_a, \bar{\phi}) - J_A \phi^A, \quad \phi^A = \frac{i}{\hbar} \frac{\delta \ln Z(J, \phi^*_a, \bar{\phi})}{\delta J_A},
\]

calculated on its extremals \( \delta \Gamma / \delta \phi^A = 0 \), does not depend upon the gauge on the hypersurface \( \phi^* = 0 \).

At the same time, with allowance for Eq. (2.9), one derives for \( \Gamma = \Gamma(\phi, \phi^*_a, \bar{\phi}) \) the Ward identities

\[
\frac{1}{2} (\Gamma, \Gamma)^a + V^a \Gamma = 0,
\]

(2.10)
In particular, Eq. (2.10), considered at $\phi^*_a = \bar{\phi} = 0$, results in the invariance of the effective action $\tilde{\Gamma} = \Gamma(\phi)$

$$\tilde{\Gamma} = \Gamma|_{\phi^*_a = \bar{\phi} = 0} \tag{2.11}$$

of the fields $\phi^A$ under the following transformations

$$\delta \phi^A = \frac{\delta \Gamma}{\delta \phi^A}|_{\phi^*_a = \bar{\phi} = 0} \mu_a \tag{2.12}$$

(we shall refer to Eq. (2.12) as quantum extended BRST symmetry transformations); namely,

$$\delta \tilde{\Gamma} = \frac{\delta \Gamma}{\delta \phi^A} \frac{\delta \Gamma}{\delta \phi^*_a} \mu_a = -\varepsilon^{ab} \phi^*_a \frac{\delta \Gamma}{\delta \phi^*_A} |_{\phi^*_a = \bar{\phi} = 0} \mu_a = 0. \tag{2.13}$$

By virtue of Eq. (2.10), one readily finds that the algebra of the symmetry transformations (2.12), (2.13) is open off-shell

$$\delta(1)\delta(2)\phi^A \equiv \delta(2)\delta(1)\phi^A = (-1)^{\varepsilon^{AB}} \delta \tilde{\Gamma} \frac{\delta \Gamma}{\delta \phi^B} \frac{\delta \Gamma}{\delta \phi^*_B} \frac{\delta \Gamma}{\delta \phi^*_A} |_{\phi^*_a = \bar{\phi} = 0} \mu(1)\mu(2) \tag{2.14}$$

(here, the symbol { } denotes the symmetrization with respect to the $Sp(2)$ indices: $A^{(ab)} = A^{ab} + A^{ba}$).

In this connection, note that the study of Ref. [30] investigated the properties of the symmetry transformations $\delta_\alpha$ which form an open algebra within the Lagrangian formulation of an arbitrary non-degenerate theory. Here, $q^i$ are configuration space variables, $f^i_{\alpha\beta}$ are some structure coefficients (depending generally on $q^i$) and $\Delta^i_{\alpha\beta}$ are some functions vanishing on-shell. In Ref. [30] it was shown, on the assumption of the absence of anomalies, that within the quantum theory constructed in accordance with the Dirac procedure, the following relations hold

$$[\hat{Q}_\alpha, \hat{H}] = 0, \quad [\hat{Q}_\alpha, \hat{Q}_\beta] = f^i_{\alpha\beta} \hat{Q}_i, \tag{2.16}$$

where $\hat{H}$ is the Hamiltonian operator and $\hat{Q}_\alpha$ are the Noether charge operators generating, on the quantum level, the symmetry transformations $\delta_\alpha$.

The comparison of Eq. (2.14) with Eqs. (2.15), (2.16) yields the algebra of the operators of Hamiltonian $\hat{H}$ and Noether charges $\hat{Q}^{(1)} \equiv \hat{Q}^a \mu^{(1)a}$, $\hat{Q}^{(2)} \equiv \hat{Q}^a \mu^{(2)a}$ corresponding to the transformations $\delta^{(1)}$, $\delta^{(2)}$ (2.12), (2.14)

$$[\hat{Q}^{(1,2)}, \hat{H}] = 0, \quad [\hat{Q}^{(1)}, \hat{Q}^{(2)}] = 0. \tag{2.17}$$

By virtue of the arbitrariness of parameters $\mu^{(1)a}$, $\mu^{(2)a}$, Eq. (2.17) implies the relations

$$[\hat{Q}^a, \hat{H}] = 0, \quad [\hat{Q}^a, \hat{Q}^b]_+ = 0.$$

Hence it follows that within a general gauge theory (the anomalies out of account) there exists a doublet of nilpotent anticommuting operators $\hat{Q}^a$ generating the quantum transformations of the extended BRST symmetry.
3. Representation of the algebra of $Q^a$, $Q_C$

Let us consider the representation of the algebra

$$\left[ Q^a, Q^b \right]_+ = 0, \quad (3.1)$$

$$[iQ_C, Q^a] = (-1)^a Q^a$$

of the operators $\hat{L} = (\hat{Q}^a, i\hat{Q}_C)$ in the one-particle subspace $V^{(1)}$ of the total Fock space $V$ with indefinite inner product $\langle | >$

$$\hat{L}V^{(1)} \subset V^{(1)}, \quad < \Psi | \hat{L}\Phi > = < \hat{L}^\dagger \Psi | \Phi >, \quad |\Psi >, |\Phi > \in V^{(1)}, \quad (3.2)$$

$$(\hat{Q}^a)^\dagger = (-1)^a \hat{Q}^a, \quad (\hat{Q}_C)^\dagger = \hat{Q}_C.$$

We shall demonstrate it here that the space $V^{(1)}$ of representation of the algebra (3.1) is generally a direct sum

$$V^{(1)} = \bigoplus_n V_n^{(1)}, \quad \hat{L}V_n^{(1)} \subset V_n^{(1)}, \quad (3.3)$$

$$V_n^{(1)} \cap V_{n'}^{(1)} = \emptyset, \quad n \neq n',$$

where subspaces $V_n^{(1)}$ include the following one-particle state complexes

(i) genuine BRST–antiBRST-singlets (physical particles),
(ii) pairs of BRST–antiBRST-singlets,
(iii) BRST-quartets,
(iv) antiBRST-quartets,
(v) BRST–antiBRST-quartets,
(vi) BRST–antiBRST-sextets,
(vii) BRST–antiBRST-octets.

In order to construct the basis of representation (3.3), (3.4) explicitly, note that for an arbitrary state $|\Phi >$ one of the following conditions holds

$$\frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b|\Phi > \not= 0, \quad (3.5)$$

$$\frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b|\Phi > = 0. \quad (3.6)$$

If a state $|\phi_{(k,N)} > \in V_n^{(1)} (i\hat{Q}_C|\phi_{(k,N)} > = N|\phi_{(k,N)} >)$ satisfies the condition (3.5), then, by virtue of Eq. (3.1), there exists a set of linearly independent states

$$|\phi_{(k,N)} >, \quad \hat{Q}^a|\phi_{(k,N)} >, \quad \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b|\phi_{(k,N)} >, \quad (3.7)$$

which form a basis of a four-dimensional representation of the algebra (3.1). Given this, owing to the properties (3.1), (3.2), the states

$$\hat{Q}^a|\phi_{(k,N)} >, \quad \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b|\phi_{(k,N)} >$$

have vanishing norm, in particular, $|k, N > \equiv \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b|\phi_{(k,N)} >$

$$< k, N | k, N >= 0. \quad (3.8)$$

In accordance with Ref. [28], for an arbitrary one-particle zero-norm (3.8) state $|k, N >$ there exists some (one-particle) state $|k, -N >$ such that
(by virtue of Eq. (3.2), any states $|k, N >$, $|k', N' >$ can only have a non-vanishing inner product $< k', N' | k, N >$ when $N = -N'$). At the same time, among all the basis states, the vector $|k, -N >$ subject to the normalization (3.9) is without the loss of generality unique [28]. In fact, in the subspace of linearly independent states (|k, -N >, {|l, -N >}) with the properties $< k, -N | k, N > = < l, -N | k, N > = 1$, one can always choose a basis (|k, -N >, \{ |l, -N > = |-k, -N > \}) such that $< l, -N | k, N > = 0$. Note that, owing to Eqs. (3.8), (3.9), the basis in the subspace of states $|\Psi > = \{ |l, N >, l \neq k \}$, $< k, N | \Psi > = 0$ can always be chosen so as $< k, -N | l, N > = 0$. Indeed, in order to go over from the basis states $|k, N >, \{ |l, N > \}$

\[
< k, N | k, N > = 0, \quad < k, -N | k, N > = 1, \\
< k, N | l, N > = 0, \quad < k, -N | l, N > = 1, \quad \forall l
\]

to an equivalent linearly independent set $|k, N >, \{ |l, N > \}$

\[
< k, N | k, N > = 0, \quad < k, -N | k, N > = 1, \\
< k, N | l, N > = 0, \quad < k, -N | l, N > = 0, \quad \forall l
\]

it is sufficient, for example, to identify

\[
|k, N > = |k, N >, \quad |l, N > = |l, N > - |k, N >, \quad \forall l.
\]

From Eqs. (3.8), (3.9) and the hermiticity assignment (3.2) it follows that there exists a set of four states

\[
|\tilde{\phi}(k, -N) >, \quad \hat{Q}^a |\tilde{\phi}(k, -N) >, \quad \frac{1}{2} \varepsilon_{ab} \hat{Q}^a \hat{Q}^b |\tilde{\phi}(k, -N) >,
\]

which are also linearly independent and form a basis of representation of the algebra (3.1). Here, $|\tilde{\phi}(k, -N) >$ is a state (3.5) chosen from the condition

\[
\frac{1}{2} \varepsilon_{ab} < \tilde{\phi}(k, -N) |\hat{Q}^a \hat{Q}^b |\phi(k, N) > = 1.
\]

By virtue of Eq. (3.11), the state vectors $|\tilde{\phi}_a > = \{ |\tilde{\phi}_1 >, |\tilde{\phi}_2 > \}$ satisfying the normalization $< \tilde{\phi}_1 |\hat{Q}^1 \phi > = < \tilde{\phi}_2 |\hat{Q}^2 \phi > = 1$ that corresponds to the zero-norm states $\hat{Q}^a |\phi >$ can be chosen in the form $|\tilde{\phi}_a > = \varepsilon_{ba} (\hat{Q}^b)\dagger |\phi >$. Note that without the loss of generality, one can assume

\[
< \tilde{\phi}(k, -N) |\phi(k, N) > = 0,
\]

since if there does exists such $\alpha \neq 0$ that

\[
< \tilde{\phi}(k, -N) |\phi(k, N) > = \alpha,
\]

then one can choose the basis in the subspace (3.7) so as

\[
|\phi'(k, N) >, \quad \hat{Q}^a |\phi'(k, N) >, \quad \frac{1}{2} \varepsilon_{ab} \hat{Q}^a \hat{Q}^b |\phi'(k, N) >,
\]

\[
\frac{1}{2} \varepsilon_{ab} < \tilde{\phi}(k, -N) |\hat{Q}^a \hat{Q}^b |\phi'(k, N) > = 1, \quad < \tilde{\phi}(k, -N) |\phi'(k, N) > = 0,
\]

where $|\phi'(k, N) > = \alpha^{-1} |\phi(k, N) > - \frac{1}{2} \varepsilon_{ab} \hat{Q}^a \hat{Q}^b |\phi(k, N) >$.

Let us consider the conditions of the linear dependence of the whole set of states (3.7), (3.10). In fact, let there among the numbers ($\beta$, $\beta_a$, $\tilde{\beta}$, $\gamma$, $\gamma_a$, $\tilde{\gamma}$) be a non-zero one and let $|\phi(k, N) > = \gamma^{-1} |\phi >$, $|\tilde{\phi}(k, -N) > = \gamma^{-1} |\tilde{\phi} >$.
Eq. (3.12), in turn, implies, by virtue of Eq. (3.1), that there exists such \( \alpha \neq 0 \) that

\[
\frac{\alpha}{2} \varepsilon_{ab} \hat{Q}^a \hat{Q}^b |\phi_{(k,N)}\rangle \geq \frac{1}{2} \varepsilon_{ab} \hat{Q}^a \hat{Q}^b |\phi_{(k,-N)}\rangle > .
\]

Eq. (3.13)

Namely, if \( \beta = \gamma = \beta_a = \gamma_a = 0 \), then from the condition \( \tilde{\gamma} \neq 0 \) it follows that \( \gamma \neq 0 \) (reversely, \( \gamma \neq 0 \Rightarrow \tilde{\gamma} \neq 0 \)) with \( \alpha = \beta \gamma^{-1} \). In the case \( \exists a : \beta_a \neq 0 \) the condition \( \beta = \gamma = 0 \) implies \( \gamma_a \neq 0 \) (similarly, \( \gamma_a \neq 0 \Rightarrow \beta_a \neq 0 \)), here \( \alpha = \beta_a \gamma_a^{-1} \) (no summation). Finally, if \( \beta \neq 0 \) (or, equivalently, \( \gamma \neq 0 \)), then we have \( \alpha = \beta \gamma^{-1} \). By virtue of Eq. (3.13), the state \( |\phi_{(k,-N)}\rangle > \) can be chosen from the normalization condition (3.11) in the form

\[
|\phi_{(k,-N)}\rangle > = \alpha |\phi_{(k,N)}\rangle > .
\]

Hence, evidently, \( N = 0 \), and the spaces of representations corresponding to the vector sets (3.7), (3.10) coincide. For a set of basis vectors we choose, say, (3.7), i.e. (\( |\phi_{(k,N=0)}\rangle > = k > \))

\[
|k,0> , \hat{Q}^a |k,0> , \frac{1}{2} \varepsilon_{ab} \hat{Q}^a \hat{Q}^b |k,0> > .
\]

Eq. (3.15)

Given this, owing to Eq. (3.14), the relation holds

\[
\frac{\alpha^*}{2} \varepsilon_{ab} < k,0|\hat{Q}^a \hat{Q}^b |k,0> = 1.
\]

Eq. (3.16)

By virtue of Eq. (3.16), the set of states (3.15) can be represented in the form of both a BRST-quartet (\( (\hat{Q}^1)^\dagger = \hat{Q}^1 \))

\[
|k,0> , \hat{Q}^1 |k,0> , |k,1> , |k,1> , \hat{Q}^1 |k,1> ,
\]

Eq. (3.17)

\[
< k,0|k,0> = < k,-1|k,1> = 1
\]

(choosing for \( |k,1> \equiv -\alpha \hat{Q}^2 |k,0> \)), and an antiBRST-quartet (\( (i\hat{Q}^2)^\dagger = i\hat{Q}^2 \))

\[
|k,0> , \hat{Q}^2 |k,0> , |k,-1> , |k,1> , \hat{Q}^2 |k,1> ,
\]

Eq. (3.18)

\[
< k,0|k,0> = < k,-1|k,1> = 1
\]

(choosing for \( |k,1> \equiv -i\alpha^* \hat{Q}^1 |k,0> \)). In what follows, we shall refer to the state complexes of the form (3.15), (3.16) as BRST–antiBRST-quartets (3.4), (vii).

In case of linear independence, the states (3.7), (3.10) form a BRST–antiBRST-octet (3.4), (vii) and can be represented as both a pair of BRST-quartets

\[
(|\phi_{(k,N)}\rangle > , -\hat{Q}^1 \hat{Q}^2 |\phi_{(k,-N)}\rangle > , \hat{Q}^1 |\phi_{(k,N)}\rangle > , -\hat{Q}^2 |\phi_{(k,-N)}\rangle > ),
\]

Eq. (3.19)

\[
(|\phi_{(k,-N)}\rangle > , -\hat{Q}^1 \hat{Q}^2 |\phi_{(k,N)}\rangle > , \hat{Q}^1 |\phi_{(k,-N)}\rangle > , -\hat{Q}^2 |\phi_{(k,N)}\rangle >
\]

and a pair of antiBRST-quartets

\[
(|\phi_{(k,N)}\rangle > , \hat{Q}^2 \hat{Q}^1 |\phi_{(k,-N)}\rangle > , i\hat{Q}^2 |\phi_{(k,N)}\rangle > , -i\hat{Q}^1 |\phi_{(k,-N)}\rangle >
\]

Eq. (3.20)
Further, we shall consider the states (3.7), (3.10), (3.11) (i.e. BRST–antiBRST-quartets and octets), provided they do exist in a specific theory, as components of the basis in subspace $V^{(1)}$. The state vectors (3.7), (3.10), (3.11) evidently exhaust all the states (3.5). At the same time, by construction, linear combinations of the vectors (3.7), (3.10) constitute, by construction, a subspace of states $|\Psi\rangle$, having non-degenerate inner product ($\forall |\Psi\rangle \neq 0, \exists |\Psi'\rangle : <\Psi|\Psi'\rangle \neq 0$), which is invariant under the action of the operators $\hat{L}$.

Consider now the states $|\Phi\rangle \in V^{(1)}$ which cannot be represented as linear combinations of the vectors (3.7), (3.10), (3.11) (i.e. those which do not belong to BRST–antiBRST-quartets or octets); from the previous treatment it follows immediately that the states $|\Phi\rangle$ under consideration satisfy the condition (3.6). Making allowance for the properties of the zero-norm vectors from the set (3.7), (3.10) a basis in the subspace of states $|\Phi\rangle$ can always be chosen so as $<\Psi|\Phi\rangle = 0$ ($|\Psi\rangle$ is an arbitrary linear combination of BRST–antiBRST-quartet or octet vectors), and hence, the states $|\Phi\rangle$ form a space of representation of the algebra of the operators $\hat{L}$. Given this, the following conditions generally hold

$$\exists a : \hat{Q}^a |\Phi\rangle \neq 0,$$

$$\forall a : \hat{Q}^a |\Phi\rangle = 0.$$  \hspace{1cm} (3.21)

Let us first turn ourselves to the states of the form (3.21). For such states the condition is valid ($|\ast\rangle$ implies arbitrary one-particle states)

$$|\Phi\rangle \neq \hat{Q}^a |\ast\rangle,$$  \hspace{1cm} (3.23)

since otherwise the states $|\Phi\rangle$ under consideration would be some linear combinations of the states (3.7). An arbitrary state $|\Phi\rangle$ (3.6), (3.21), (3.23), in its turn, satisfies one of the three conditions

(i) $\hat{Q}^1 |\Phi\rangle \neq 0, \hat{Q}^2 |\Phi\rangle \neq 0$,

(ii) $\hat{Q}^1 |\Phi\rangle \neq 0, \hat{Q}^2 |\Phi\rangle = 0$,

(iii) $\hat{Q}^1 |\Phi\rangle = 0, \hat{Q}^2 |\Phi\rangle \neq 0$.  \hspace{1cm} (3.24)

If a state $|\phi_{(k,N)}\rangle$ satisfies the condition (3.24), (i), then, by virtue of Eq. (3.1), there exist linearly independent states

$$|\phi_{(k,N)}\rangle, \hat{Q}^a |\phi_{(k,N)}\rangle,$$  \hspace{1cm} (3.25)

which form a basis of a three-dimensional representation of the algebra (3.1). At the same time, the states $\hat{Q}^a |\phi\rangle$ (we omit, for the sake of brevity, the notations of the quantum numbers) have vanishing norm

$$<\hat{Q}^1\phi|\hat{Q}^1\phi>=<\hat{Q}^2\phi|\hat{Q}^2\phi>=0.$$  \hspace{1cm} (3.26)

From the above relations it follows, with allowance made for Eqs. (3.2), (3.8), (3.9), that there exist three linearly independent states

$$|\phi_a\rangle, \frac{1}{2} (<\hat{Q}^a|\hat{Q}^a|\phi_a\rangle),$$  \hspace{1cm} (3.26)

where the states $|\phi_a\rangle \neq \hat{Q}^a |\ast\rangle$, chosen without the loss of generality as eigenvectors for the ghost charge operator $i\hat{Q}_C$, satisfy the normalization conditions

$$<\phi_b|\hat{Q}^a\phi>=\delta^a_b.$$

(here, $i\hat{Q}_C|\phi_a>=-(N-(1)^a)|\phi_a>$ and the conditions $<\phi_2|\hat{Q}^1\phi>=<\phi_1|\hat{Q}^2\phi>=0$ hold, therefore, automatically); at the same time, by virtue of Eqs. (3.2), (3.27), we have

$$<\hat{Q}^a|\phi_a\rangle |\phi_a\rangle = 1.$$
\[ <(\hat{Q}^a)^\dagger \phi_b | \hat{Q}^a \phi_a > = 0, \quad <\phi_a | \phi > = 0 \]

(the inequality \(<\phi_a | \phi > \neq 0 \) leads one to the condition \(\exists a : N = N = (-1)^a \) and, therefore, does not hold for any \(N\).) Owing to Eqs. (3.27), (3.28), the bases (3.25), (3.26) \(|\phi >, \hat{Q}^a |\phi >\) \(\equiv |e_i >, (\phi_a >, \frac{1}{2} (\hat{Q}^a)^\dagger |\phi_a >) \equiv |f_i >\) are dual with respect to each other \(<f_i | e_j > = \delta_{ij}\). Hence follows the non-degeneracy of bilinear form \(<| >\) defined on the pair \(X \equiv \{|e_i >\}, Y \equiv \{|f_i >\}\) of state spaces corresponding to the vector sets (3.25), (3.26).

This fact implies that in the space \(Y\) exists the (unique) representation \(\hat{L}|e_i > = (\hat{L})_{ij}|f_j >\) of the algebra (3.1) conjugate to the representation \(\hat{L}|e_i > = (\hat{L})_{ij}|e_j >\) defined in \(X\), i.e. \((\hat{L})_{ij} = (\hat{L})_{ji}^*\). Namely,

\[(\hat{Q}^a)^\dagger |\phi_b > = \frac{1}{2} \delta_b^a (\hat{Q}^c)^\dagger |\phi_c >, \quad (3.29)\]

\[(\hat{Q}^a)^\dagger (\hat{Q}^b)^\dagger |\phi_b > = 0\]

(for the ghost charge operator \(i\hat{Q}_C\), the basis states of the subspace \(Y\) are by construction eigenvectors).

Let us show, with Eqs. (3.1), (3.2) taken into account, that the whole set of states (3.25), (3.26)

\[ |\phi >, \hat{Q}^a |\phi >, |\phi_a >, \frac{1}{2} (\hat{Q}^a)^\dagger |\phi_a > \quad (3.30)\]

is linearly independent. Indeed, assuming the reverse, i.e.

\[ \beta |\phi > + \beta_a \hat{Q}^a |\phi > + \gamma_a |\phi_a > + \frac{\gamma}{2} (\hat{Q}^a)^\dagger |\phi_a > = 0 \]

(the numbers \((\beta, \beta_a, \gamma_a, \gamma)\) not all vanishing), one arrives, by virtue of Eq. (3.6) and the normalization conditions (3.27), at the relation

\[ \exists a : <(\hat{Q}^a)^\dagger |\phi > \equiv \alpha^a \neq 0 \]

representable as

\[ \beta \neq 0 \iff \exists a : \gamma^a \neq 0, \quad \alpha^a = (-1)^a \gamma^a / \beta, \]

\[ \beta = \gamma = 0, \quad \gamma \neq 0 \iff \exists a : \beta_a \neq 0, \quad \alpha^a = -\gamma / \beta_a. \]

If we now suppose, for example, that \(a = 1\), then, owing to Eq. (3.27) \(<\hat{Q}_1^1 |\phi_1 > = 1\), the eigenvalues of the ghost charge operator \(i\hat{Q}_C\) that correspond to the states \(\hat{Q}_1^1 |\phi > \) and \(\hat{Q}_1^1 |\phi_1 >\)

\[ i\hat{Q}_C |\phi > = (N + 1) |\phi >, \]

\[ i\hat{Q}_C |\phi_1 > = -N |\phi_1 > \]

must coincide, i.e. \(N + 1 = -N\). In the case \(a = 2\) we similarly have \(N - 1 = -N\) and find that neither condition can be satisfied for an integer \(N\).

Note that the states (3.30), (3.27), (3.29) are representable in the form of a BRST-quartet

\[ |\phi >, \hat{Q}_1^1 |\phi_1 >, \hat{Q}_1^1 |\phi >, |\phi_1 >, \]

\[ <\hat{Q}_1^1 |\phi_1 > = <\phi_1 |\hat{Q}_1^1 |\phi > = 1 \]

and a pair of BRST-singlets \((\hat{Q}_2^2 |\phi >, |\phi_2 >)\)
\[ \hat{Q}^1|\hat{Q}^2\phi \rangle = \hat{Q}^1|\phi_2 \rangle = 0, \ |\phi_2 \rangle \neq \hat{Q}^1|\ast \rangle, \ |\hat{Q}^2\phi \rangle \neq \hat{Q}^1|\ast \rangle, \]
as well as in the form of an antiBRST-quartet ((\(\hat{Q}^1\))\(\dagger\)|\(\phi_1 \rangle = (\hat{Q}^2)^\dagger|\phi_2 \rangle )
\[ |\phi \rangle, \ -\hat{Q}^2|\phi_2 \rangle, \ i\hat{Q}^2|\phi \rangle, \ i|\phi_2 \rangle, \]
\[ < i\phi_2|i\hat{Q}^2\phi >= - < \hat{Q}^2\phi_2|\phi >= 1 \]
and a pair of antiBRST-singlets (\(\hat{Q}^1|\phi \rangle, \ |\phi_1 \rangle \))
\[ < \phi_1|\hat{Q}^1\phi >= 1, \] \[ (3.32) \]
\[ \hat{Q}^2|\hat{Q}^1\phi >= \hat{Q}^2|\phi_1 >= 0, \ |\phi_1 \rangle \neq \hat{Q}^2|\ast \rangle, \ |\hat{Q}^1\phi \rangle \neq \hat{Q}^2|\ast \rangle. \]
We shall refer to the states (3.30), (3.27) as BRST–antiBRST-sextets (3.4), (vi) and consider them (supposing they generally exist in a theory) as a part of the basis state vectors in subspace \( \mathcal{V}^{(1)} \).

The above considerations imply that the variety of linear combinations of BRST–antiBRST-sextet states contain all the states of the form (3.24), (i); at the same time, the sextet representations (3.30), (3.29) partly include the states (3.24), (ii), (iii), that is to say
\[ (|\phi_1 \rangle, \ |\phi_2 \rangle) \neq \hat{Q}^a|\ast \rangle, \] \[ (3.33) \]
\[ \hat{Q}^1|\phi_1 >= -\hat{Q}^2|\phi_2 \rangle \neq 0, \ \hat{Q}^2|\phi_1 >= \hat{Q}^1|\phi_2 >= 0. \]
Reversely, any states (3.33) belong to a BRST–antiBRST-sextet
\[ |\phi \rangle, \ \hat{Q}^1|\phi \rangle, \ \hat{Q}^2|\phi \rangle, \ |\phi_1 \rangle, \ |\phi_2 \rangle, \ \hat{Q}^1|\phi_1 >= -\hat{Q}^2|\phi_2 \rangle, \] \[ (3.34) \]
where |\phi \rangle is chosen from the relations
\[ < \hat{Q}^1\phi_1|\phi >= - < \hat{Q}^2\phi_2|\phi >= 1. \]
Hence, for the further analysis of representations of the algebra (3.1) that contain the states specified by the conditions (3.6), (3.21), (3.23), it is sufficient for us to confine ourselves to the states of the form (3.24), (ii), (iii) not representable as linear combinations of BRST–antiBRST-sextet states (such states belong, without the loss of generality, to the subspace orthogonal to BRST–antiBRST-sextet states and, therefore, invariant under the action of the operators \( \hat{L} \)). For the states
\[ |\phi \rangle \neq \hat{Q}^a|\ast \rangle, \ \hat{Q}^1|\phi \rangle \neq 0, \ \hat{Q}^2|\phi \rangle = 0, \] \[ (3.35) \]
\[ |\bar{\phi} \rangle \neq \hat{Q}^a|\ast \rangle, \ \hat{Q}^2|\bar{\phi} \rangle \neq 0, \ \hat{Q}^1|\bar{\phi} \rangle = 0 \]
(3.36)
under consideration, the following supplementary conditions hold
\[ \hat{Q}^1|\phi \rangle \neq \hat{Q}^2|\ast \rangle, \]
\[ (3.37) \]
\[ \hat{Q}^2|\bar{\phi} \rangle \neq \hat{Q}^1|\ast \rangle. \]
(3.38)
Let us show that the violation, for instance, of the condition (3.37) leads one to a contradiction. In fact, \(|\ast \rangle\) is not, by definition, representable as a linear combination of BRST–antiBRST-sextet states, and, consequently, the relation \(\hat{Q}^1|\phi >= \hat{Q}^2|\ast \rangle\) is only possible when \(|\ast \rangle\) belongs to the states (3.36), i.e., without the loss of generality, one has
From the above relation it follows, by virtue of Eqs. (3.33)–(3.36), that the states \(|\phi\rangle, |\varphi\rangle, Q^1|\varphi\rangle = -\bar{Q}^2|\varphi\rangle\) belong to some BRST–antiBRST-sextet (3.34). The inequality (3.38) is proved in a similar way. Eqs. (3.37), (3.38) imply, in particular, that the spaces of representations (3.35) and (3.36) respectively cannot be transformed into each another by the action of the operators \(L\).

By repetition of the given above considerations with respect to Eqs. (3.35)–(3.38) we find that the state complexes (3.35), (3.37) constitute some BRST-quartets (3.4), (iii)

\[
|\phi\rangle, |\varphi\rangle, Q^1|\varphi\rangle, \bar{Q}^1|\varphi\rangle,
\]

\[
<\varphi'|\bar{Q}^1\varphi|\varphi'\rangle = <\bar{Q}^1|\varphi|\varphi'> = 1,
\]

\[
|\Phi\rangle \equiv (|\phi\rangle, |\varphi\rangle),
\]

\[
\bar{Q}^1|\Phi\rangle = 0, |\Phi\rangle \neq Q^a|*\rangle, \bar{Q}^1|\Phi\rangle \neq \bar{Q}^2|*\rangle
\]

(|\phi\rangle > (3.39) is orthogonal to all the BRST–antiBRST-sextet states and, in particular, to any state |\psi\rangle: \forall\alpha, Q^2|\psi\rangle \neq 0; hence, |\varphi'\rangle also satisfies Eqs. (3.35), (3.37)), representable as well in the form of two pair of antiBRST-singlets

\[
(|\phi\rangle, \bar{Q}^1|\varphi\rangle), \ (|\varphi'\rangle, \bar{Q}^1|\varphi\rangle).
\]

Similarly, the states (3.36), (3.38) constitute antiBRST-quartets (3.4), (iv)

\[
|\bar{\varphi}\rangle, |\bar{\varphi}'\rangle, i\bar{Q}^2|\bar{\varphi}\rangle, i\bar{Q}^2|\bar{\varphi}'\rangle,
\]

\[
<\bar{\varphi}'|i\bar{Q}^2\bar{\varphi}|\bar{\varphi}\rangle = <i\bar{Q}^2|\bar{\varphi}|\bar{\varphi}'\rangle = 1,
\]

\[
|\bar{\Phi}\rangle \equiv (|\bar{\varphi}\rangle, |\bar{\varphi}'\rangle),
\]

\[
\bar{Q}^1|\bar{\Phi}\rangle = 0, |\bar{\Phi}\rangle \neq \bar{Q}^a|*\rangle, \bar{Q}^2|\bar{\Phi}\rangle \neq \bar{Q}^1|*\rangle
\]

and BRST-singlet pairs

\[
(|\bar{\varphi}\rangle, i\bar{Q}^2|\bar{\varphi}'\rangle), \ (|\bar{\varphi}'\rangle, i\bar{Q}^2|\bar{\varphi}\rangle).
\]

Thus, with allowance for Eqs. (3.25)–(3.32), (3.35)–(3.42), we have described the structure of representations containing the states of the form (3.6), (3.21), (3.23).

Finally, we turn to the states \(|\Phi\rangle > not representable as linear combinations of the above considered BRST–antiBRST-quartets, sextets, octets and states (3.35)–(3.38) (the state vectors just mentioned are without the loss of generality all orthogonal to \(|\Phi\rangle\)). One readily finds that these restrictions can only be met by the states

\[
|\Phi\rangle \equiv \{|\phi_{(k,N)}\rangle\} \neq \hat{Q}^a|*\rangle
\]

of the form (3.32) \(\hat{Q}^a|\Phi\rangle = 0\), which, for \(N = 0\), we shall identify, following Ref. [28], with physical particles \(|\phi_k\rangle \equiv |\phi_{(k,N=0)}\rangle\) (genuine BRST–antiBRST-singlets (3.4), (i))

\[
<\phi_k|\phi_k\rangle = 1, \ \hat{Q}^a|\phi_k\rangle = 0, \ |\phi_k\rangle \neq \hat{Q}^a|*\rangle.
\]

Meanwhile, in the case \(N \neq 0\) (<\( \phi_{(k,N)}|\phi_{(k,N)}\rangle = 0\)) we shall refer to the states \(|\Phi\rangle \equiv (|\phi_{(k,-N)}\rangle, |\phi_{(k,N)}\rangle)\)

\[
<\phi_{(k,-N)}|\phi_{(k,N)}\rangle = 1, \ \hat{Q}^a|\Phi\rangle = 0, \ |\Phi\rangle \neq \hat{Q}^a|*\rangle.
\]

as BRST–antiBRST-singlet pairs (2.4), (ii); with that, the state complexes (3.43) and (3.44) are orthogonal to each other.

Thus, taking Eqs. (3.37)–(3.44) into account, we have in a general case described the structure (3.4) of the one-particle state subspace \(\mathcal{V}^{(1)} \supset \mathcal{V}^{(1)}_n \supset \mathcal{V}^{(1)}_{n'}\), \(n \neq n'\) as a space of representation \(\bar{L} \mathcal{V} \subset \mathcal{V}, \ \bar{L} = (\hat{Q}^a, i\hat{Q}^c)\) of the algebra (3.1) of the generators \(\hat{Q}^a\) of extended BRST symmetry transformations and the ghost charge operator \(i\hat{Q}^c\). By construction, indefinite inner product \(<|\cdot\rangle\rangle\) is non-degenerate in each subspace \(\mathcal{V}^{(1)}_n\) (see the normalization conditions (3.11), (3.27), (3.39), (3.41), (3.43), (3.44) for basis vectors),
4. Physical unitarity conditions

We now consider, with allowance for Eqs. (3.3), (3.4), (3.7)–(3.44), the conditions of the physical S-matrix unitarity in the Hilbert space $H_{phys} = V_{phys}/V_0$, where the physical subspace $V_{phys} \ni |phys>$ is specified by the $Sp(2)$-covariant subsidiary condition

$$\hat{Q}^{a}|phys> = 0 \quad (4.1)$$

(which evidently ensures the invariance of $V_{phys}$ under the time development). By virtue of Eq. (4.1), the structure of $V_{phys}$ has the form

$$V_{phys} = V^1_{phys} \cap V^2_{phys},$$

where

$$V \supset V^1_{phys}, \quad \hat{Q}^1V^1_{phys} = 0,$$

$$V \supset V^2_{phys}, \quad \hat{Q}^2V^2_{phys} = 0.$$ 

In particular, for the zero-norm subspace $V_0 \subset V$ we have

$$V_0 = V^1_0 \cap V^2_0,$$

$$V^1_0 \subset V^1_{phys}, \quad V^2_0 \subset V^2_{phys}. \quad (4.2)$$

The analysis of representations (3.3), (3.4), (3.7)–(3.44) on the basis of the quartet mechanism [28] shows that the state vectors from $V_{phys}$ containing particles of BRST–antiBRST-quartets (3.4), (v) and octets (3.4), (vii) (i.e. state complexes simultaneously representable as BRST- (3.17), (3.19) and antiBRST- (3.18), (3.20) quartets) belong to the zero-norm subspace $V_0$ (4.2). The remaining unphysical particles (3.4), (ii), (iii), (iv), (vi) generally contain BRST- (antiBRST-) singlet pairs (3.31), (3.32), (3.40), (3.42), (3.44) and are present in the physical subspace $V_{phys}$. In this connection, the physical S-matrix conditions within the suggested approach are the requirements of absence of the pointed out unphysical particles, i.e. BRST–antiBRST-singlet pairs (3.4), (ii), BRST-quartets (3.4), (iii) (antiBRST-singlet pairs (3.40)), antiBRST-quartets (3.4), (iv) (BRST-singlet pairs (3.42)) and BRST–antiBRST-sextets (3.4), (vi).

5. State vector spaces in antisymmetric tensor field models

In this section, in order to illustrate the general results of the paper, we shall study the physical unitarity conditions for two simple gauge theory models [32, 34] within the $Sp(2)$-symmetric Lagrangian formalism.

Consider the theory of a non-abelian antisymmetric field $B^p_{\mu\nu}$ suggested by Freedman and Townsend [34] and described by the action

$$S = S(A^p_\mu, B^p_{\mu\nu}) = \int d^4x\{-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}G^p_{\mu\nu}B^p_{\rho\sigma} + \frac{1}{2}A^p_\mu A^p_\nu\}, \quad (5.1)$$

where $A^p_\mu$ is a vector gauge field with the strength $G^p_{\mu\nu} = \partial_\mu A^p_\nu - \partial_\nu A^p_\mu + f^{pqr}A^q_\mu A^r_\nu$ (the coupling constant is absorbed into the structure coefficients $f^{pqr}$), and $\varepsilon^{\mu\nu\rho\sigma}$ is a constant completely antisymmetric four-rank tensor ($\varepsilon^{0123} = 1$).

The action (5.1) is invariant under the gauge transformations
where $\xi_p$ are arbitrary parameters; $D^p_\mu$ is the covariant derivative with potential $A^p_\mu$ ($D^p_\mu = \delta^p_\mu \partial_\mu + f^p_{\mu\nu} A^\nu_\mu$). The algebra of the gauge transformations is abelian, and the generators $R^{pq}_{\mu\alpha}$ have at the extremals of the action (5.1) the zero-eigenvectors $Z^{pq}_{\mu} \equiv D^{pq}_{\mu} R_{pr\mu\nu\alpha} Z^{rq}_{\alpha} = \varepsilon_{\mu\nu\alpha\beta} f^{prq}_{\alpha} \delta B^{\alpha\beta}$, (5.3)

which, in their turn, are linearly independent. According to the generally accepted terminology, the model (5.1)–(5.3) is an abelian gauge theory of first stage reducibility, and its quantization can be carried out, for example, within the BV scheme [6] for the theories with linearly dependent (reducible) generators. The study of Ref. [32] showed that the application of the rules [6] to the model (5.1)–(5.3) leads one to a physically unitary theory, equivalent to the principal chiral field model (non-linear $\sigma$ model for $d = 4$). We should also mention Refs. [35, 36] devoted to various aspects of quantization of the model (5.1)–(5.3) within the standard BRST symmetry.

Next, consider the gauge model [32], in which the set of fields $(A^p_\mu, B^{p\mu}_\nu)$ is extended on account of a scalar gauge field $\omega^p$ with the transformation rule

$$\delta \omega^p = \partial_\mu \xi^p_\mu;$$

and the initial action of the fields $(A^p_\mu, B^{p\mu}_\nu, \omega^p)$ is chosen as

$$S(A^p_\mu, B^{p\mu}_\nu, \omega^p) = S(A^p_\mu, B^{p\mu}_\nu).$$

(5.5)

Here, $S(A^p_\mu, B^{p\mu}_\nu)$ is defined in Eq. (5.1).

The action (5.5) is invariant under the gauge transformations (5.2), (5.4). The generators of these transformations are linearly independent (irreducible), and their algebra is abelian. At the same time, the Lagrangian quantization of the model (5.5), (5.2), (5.4) within the standard BRST symmetry fails [32] to provide physical unitarity.

As mentioned above, the study of Ref. [10] proved the physical equivalence between the Lagrangian quantizations of a general gauge theory within the standard (BV formalism) and extended ($Sp(2)$-covariant formalism) BRST symmetries. In this connection, we shall reveal, with the help of the analysis of asymptotic states of the reducible (5.1)–(5.3) and irreducible (5.5), (5.2), (5.4) models within the Lagrangian $Sp(2)$-symmetric formalism, the reason for the physical unitarity in (5.1)–(5.3) and the origin of unitarity violation in (5.5), (5.2), (5.4) (see the proof of the physical $S$-matrix unitarity of the theory (5.1)–(5.3) within the Lagrangian $Sp(2)$-symmetric formalism in Ref. [37]).

Consider the model (5.5), (5.2), (5.4) in the Lagrangian $Sp(2)$-symmetric quantization of irreducible gauge theories [8]. To this end, note that the manifest structure of complete configuration space $\phi^A$ of the theory has the form

$$\phi^A = (A^{p\mu}, B^{p\mu\nu}, \omega^p, B^{p\mu}, C^{p\mu\alpha}),$$

where $C^{p\mu\alpha}, B^{p\mu}$ are the $Sp(2)$-doublets of the Faddeev–Popov ghosts and the auxiliary fields respectively, introduced according to the number of gauge parameters $\xi^p$ in Eqs. (5.2), (5.4). The set of antifields $\phi^*_A, \overline{\phi}_A$ corresponding to the fields $\phi^A$ reads explicitly

$$\phi^*_{Aa} = (A^*_p, B^{p\mu}_a, \omega^{*}_{pa}, B^{*}_{p\mu}, C^{*}_{p\mu\alpha}),$$

$$\overline{\phi}_A = (\overline{A}_{p\mu}, \overline{B}_{p\mu\nu}, \overline{\omega}_p, \overline{B}_{p\mu}, \overline{C}_{p\mu\alpha}).$$

The Grassmann parities and the ghost numbers of the fields $\phi^A$ take on the values

$$\varepsilon(A^{p\mu}) = \varepsilon(B^{p\mu\nu}) = \varepsilon(\omega^p) = \varepsilon(B^{p\mu}) = 0,$$
\[ \text{gh}(A^\mu) = \text{gh}(B^{\mu\nu}) = \text{gh}(\omega^\mu) = \text{gh}(B^\mu) = 0, \]
\[ \text{gh}(C^{\mu a}) = 3 - 2a. \]

The solution of the generating equations (2.1) with the boundary condition (2.2) for the model under consideration can be found in a closed form. In order to avoid the overloading of the following relations with an abundance of indices, we shall further omit the gauge indices \( p \). Then the bosonic functional \( S = S(\phi, \phi^*_a, \bar{\phi}) \) for the theory (5.5), (5.2), (5.4) can be represented as

\[ S = S + \int d^4x \left\{ B^a_{\mu a}(D^\mu C^a - D^\nu C^{a\mu}) + \omega^a_\mu \partial^\mu C^a - \right. \]
\[ \left. - \varepsilon^{ab} C_{\mu a|b} B^\mu + B_{\mu \nu} (D^\mu B^\nu - D^\nu B^\mu) + \bar{\omega} \partial^\mu B_\mu \right\}, \]

where \( S \) is the initial classical action defined in Eqs. (5.5), (5.1); besides, we have used for the fields \( A^\mu \equiv A, B^\mu \equiv B \) the notations

\[ A^\mu B^\nu \equiv \langle AB \rangle, \]
\[ D_\mu B \equiv \partial_\mu B + A_\mu \wedge B, \quad (A \wedge B)^\nu = f^{\nu pq} A^p B^q. \]

Consider the generating functional \( Z(J) \) of Green's functions represented in the form of the functional integral (2.7) and choose for the gauge fixing Boson \( F = F(\phi) \)

\[ F = \int d^4x \left\{ -\frac{1}{4} B_{\mu \nu} B^{\mu \nu} + \frac{1}{2} \omega^2 - \frac{1}{4} \varepsilon_{ab} C^a_\mu C^{ab}_\mu \right\}. \]

Integrating in Eq. (2.7) over the variables \( \lambda^A, \pi^A_a, \bar{\phi}_A, \phi^*_a \), we obtain, for the theory concerned, the following representation of the generating functional \( Z(J) \)

\[ Z(J) = \int d\phi \exp \left\{ \frac{i}{\hbar} \left( S + S_{\text{FP}}(\phi) + S_{\text{GF}}(\phi) + J_A \phi^A \right) \right\}, \]

where

\[ S_{\text{FP}} = -\frac{1}{2} \int d^4x \left\{ \varepsilon_{ab} \partial^\mu C^a_\mu \partial^\nu + \varepsilon_{ab} D_\mu C^a_\nu (D^\mu C^{ab} - D^\nu C^{ab}) \right\}, \]

\[ S_{\text{GF}} = \int d^4x \left\{ (D^\nu B_{\nu \mu} + \partial_\mu \omega) B^\mu - \frac{1}{2} B_\mu B^\mu \right\}. \]

The application of the Dirac procedure [38] to the quantum action \( S + S_{\text{FP}} + S_{\text{GF}} \) (5.6) enables one to establish the fact that half the constraints of the theory (the constraints are all second-class ones) have the form \( \pi = 0 \). According to the theorem [39], these momenta and the corresponding conjugate coordinates can be eliminated with the help of the constraint equations; the remaining (physical) variables form canonical pairs

\[ (A^i, \pi_{(A)i} = -\frac{1}{2} \varepsilon_{ijkl} B^{jk}), \]
\[ (B^{ai}, \pi_{(B)ai} = B_i), \]
\[ (\omega, \pi_{(\omega)} = B_0), \]
\[ (C^{ia}, \pi_{(C)ia} = \varepsilon_{ab} (D_o C^{ib} - D_i C^{ob})), \]
\[ (C^{oa}, \pi_{(C)oa} = \varepsilon_{ab} \partial^\mu C^{ob}_\mu), \]
The quantum action (5.6) of the theory is invariant under the following transformations of the extended BRST symmetry

$$\delta B^{\alpha\beta} = (\mathcal{D}^{\alpha} C^{\beta\alpha} - \mathcal{D}^{\beta} C^{\alpha\alpha}) \mu_{\alpha}, \quad \delta A^{\alpha} = 0,$$

$$\delta \omega = \partial_{\alpha} C^{\alpha a} \mu_{a}, \quad (5.8)$$

Specifically, the Lagrangian

$$\mathcal{L} = -\frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} G_{\mu \nu} B_{\rho \sigma} + \frac{1}{2} A_{\mu} A^{\mu} - \frac{1}{2} \varepsilon_{ab} \partial^{\mu} C_{\mu}^{a} C_{\nu}^{b} - \frac{1}{2} B_{\mu} B^{\mu} - \frac{1}{2} \varepsilon_{ab} \mathcal{D}_{\mu} C_{\nu}^{a} (\mathcal{D}^{\mu} C_{\nu}^{b} - \mathcal{D}^{\nu} C_{\mu}^{b}) + (\mathcal{D}^{\nu} B_{\nu \mu} + \partial_{\mu} \omega) B^{\mu}$$

corresponding to the quantum action (5.6) changes under the transformations (5.8) by the total derivative $\delta \mathcal{L} = \partial^{\nu} F_{\nu}$

$$F_{\nu} = \left\{ -\frac{1}{2} \varepsilon_{\nu \gamma \rho \sigma} G^{\rho \sigma} C^{\gamma a} + (\mathcal{D}_{\nu} C^{\alpha a} - \mathcal{D}_{\gamma} C^{\alpha a}) B^{\gamma} + B_{\nu} \partial^{\nu} C^{a}\right\} \mu_{a}.$$ 

This implies the conserved Noether currents $J_{\nu}^{a}$

$$J_{\nu}^{a} \equiv J_{\nu}^{a} \mu_{a} = \frac{\partial \mathcal{L}}{\partial (\partial^{\nu} \phi)} \delta \phi - F_{\nu}, \quad \partial^{\nu} J_{\nu} = 0$$

(the variations $\delta \phi$ of fields are given by Eq.(5.8)), and the corresponding Noether charges $Q^{a} = \int d^{3}x J_{0}^{a}$, expressed in terms of the physical variables (5.7), have the form

$$Q^{a} = \int d^{3}x \{ \frac{1}{2} \varepsilon^{abij} C_{ij}^{a} + \varepsilon^{ab} \pi_{(C)}^{i} \pi_{(B)}^{(i)} + \varepsilon^{ab} \pi_{(C)}^{i} \pi_{(a)}^{i} \}.$$ 

(5.9)

The algebra of the charges $Q^{a}$ with respect to the Poisson superbracket $\{ , \}$ constructed by the canonically conjugate variables (5.7) is abelian

$$\{ Q^{a}, Q^{b} \} = 0.$$ 

As is well-known, within the canonical quantization (according to Dirac), classical variables correspond to operators subject to canonical (anti-)commutation relaitons resulting from the replacement of Poisson (super)brackets by (anti-)commutators (with respect to the Grassmann parities of the variables) in accordance with the rule $[ , ]_{\pm} = i\{ , \}$. In particular, the Noether charges $Q^{a}$ (5.9) correspond to the operators $\hat{Q}^{a}$, generating the extended BRST symmetry transformations in terms of the operators of physical variables

$$[\hat{A}^{i}, \hat{Q}^{a}] = 0, \quad [\hat{\pi}_{(A)}^{i}, \hat{Q}^{a}] = -i \varepsilon_{0ijk} (\partial^{j} \hat{C}^{ka} + \hat{A}^{j} \land \hat{C}^{ka}),$$

$$[\hat{B}^{ib}, \hat{Q}^{a}] = i \varepsilon^{ab} \hat{\pi}_{(C)}^{i} \hat{\pi}_{(B)}^{ib}, \quad [\hat{\pi}_{(B)}^{ib}, \hat{Q}^{a}] = 0,$$

$$[\hat{\omega}, \hat{Q}^{a}] = i \varepsilon^{ab} \hat{\pi}_{(C)}^{i} \hat{\pi}_{(B)}^{ib}, \quad [\hat{\pi}_{(\omega)}^{i}, \hat{Q}^{a}] = 0,$$

$$[\hat{C}^{ib}, \hat{Q}^{a}] = i \varepsilon^{ab} \hat{\pi}_{(B)}^{ib}, \quad [\hat{\pi}_{(C)}^{i}, \hat{Q}^{a}] = 0, \quad [\hat{\pi}_{(B)}^{ib}, \hat{Q}^{a}] = i \varepsilon_{0ijk} \delta^{a}_{b} (\partial^{j} \hat{A}^{k} + \frac{1}{2} \hat{A}^{j} \land \hat{A}^{k})$$

$$[\hat{C}^{ib}, \hat{Q}^{a}]_{+} = i \varepsilon^{ab} \hat{\pi}_{(B)}^{ib}, \quad [\hat{\pi}_{(C)}^{i}, \hat{Q}^{a}]_{+} = 0, \quad [\hat{\pi}_{(B)}^{ib}, \hat{Q}^{a}]_{+} = i \varepsilon^{ab} \hat{\pi}_{(\omega)}^{i}, \quad [\hat{\pi}_{(C)}^{i}, \hat{Q}^{a}]_{+} = 0.$$

Given this, a direct verification yields
Consider the asymptotic state space of the model (5.6), (5.8) and study its structure. To this end, we confine ourselves to the analysis of free in-fields (assuming the existence of the corresponding in-limits). The quadratic approximation $S^{(0)}$ of the quantum action (5.6) reads ($F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$)

$$S^{(0)} = \int d^4x \left\{ -\frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} A_\mu A^\mu - \frac{1}{2} \varepsilon_{ab} \partial_\mu C^a_\mu \partial_\mu C^b_\mu - \frac{1}{2} B_\mu B^\mu + \left( \partial^\nu B_{\nu\mu} + \partial_\mu \omega \right) B^\mu \right\}.$$ (5.10)

From Eq. (5.10) follow the equations of motion for the in-fields (the equations for the in-operators have the same form)

$$\Box B_{\mu\nu} = 0, \quad \Box \omega = 0, \quad \Box C^a_\mu = 0,$$ (5.11)

$$A_\mu = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial^\nu B^\rho_\sigma, \quad B_\mu = \partial^\nu B_{\nu\mu} + \partial_\mu \omega$$

(we omit the symbol of in-limit). The solution of Eq. (5.11) for the operator-valued field $\hat{B}_{\mu\nu}(x)$ is representable as $((\hat{B}_{\mu\nu}^{(-)})^\dagger = \hat{B}_{\mu\nu}^{(+)}$)

$$\hat{B}_{\mu\nu}(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3k_0}} \left( \hat{B}_{\mu\nu}^{(-)}(k)e^{-ikx} + \hat{B}_{\mu\nu}^{(+)}(k)e^{ikx} \right).$$ (5.12)

Similar decompositions are valid for the operators $\hat{\omega}(x), \hat{C}^a_\mu(x)$ with allowance made for

$$(\hat{\omega}^{(-)})^\dagger = \hat{\omega}^{(+)}, \quad (\hat{C}^a_\mu^{(-)})^\dagger = (-1)^{a+1}\hat{C}^a_\mu^{(+)}.$$ (5.13)

At the same time, the analysis of equal-time (anti-)commutation relations yields

$$[\hat{B}_{\mu\nu}^{(-)}(k), \hat{B}_{\rho\sigma}^{(+)}(k')] = \eta_{\mu\rho}\eta_{\nu\sigma}\delta(k - k'),$$

$$[\hat{\omega}^{(-)}(k), \hat{\omega}^{(+)}(k')] = \delta(k - k'),$$

$$[\hat{C}^a_\mu^{(-)}(k), \hat{C}^b_\nu^{(+)}(k')] = \varepsilon^{ab}\eta_{\mu\nu}\delta(k - k').$$

The action (5.10) of the in-fields is invariant under the following (non-vanishing) transformations

$$\delta B^{\alpha\beta} = (\partial^\alpha C^\beta_\alpha - \partial^\beta C^\alpha_\alpha)\mu_a,$$

$$\delta \omega = \partial_\alpha C^\alpha_\mu \mu_a,$$

$$\delta C^\alpha_\mu = -\varepsilon^{ab} B^\alpha_\mu \mu_b.$$ 

In terms of the creation operators, the corresponding transformations for $\hat{B}_{\mu\nu}, \hat{\omega}, \hat{C}^a_\mu$ have the form

$$[\hat{B}_{\mu\nu}^{(+)}(k), \hat{Q}^a] = -\left( k_\mu \hat{C}^a_\nu^{(+)}(k) - k_\nu \hat{C}^a_\mu^{(+)}(k) \right),$$

$$[\hat{\omega}^{(+)}(k), \hat{Q}^a] = -k^\mu \hat{C}^a_\mu^{(+)}(k),$$ (5.14)
In Eq. (5.14), $\hat{Q}^a$ is a doublet of generators of the extended BRST symmetry transformations for the operatorial in-fields, its normal form being

$$
\hat{Q}^a = \int d^3k \, k^\mu \left( \hat{\omega}^{(+)}(\vec{k}) \hat{C}^a_{\mu}^{(+)}(\vec{k}) + C^a_{\mu}^{(+)}(\vec{k}) \hat{\omega}^{(-)}(\vec{k}) + \hat{B}^{(+)}_{\mu\nu}(\vec{k}) \hat{C}^a_{\mu\nu}^{(+)}(\vec{k}) + \hat{C}^{a(+)\nu}(\vec{k}) \hat{B}^{(-)}_{\mu\nu}(\vec{k}) \right).
$$

The analysis of asymptotic state space structure is conveniently carried out with the help of the following local basis

$$
e^a_L(\vec{k}) = \frac{1}{2k_0^2} k^\mu = \frac{1}{2k_0^2} (k^0, \vec{k}),
$$

$$
e^a_T(\vec{k}) = \frac{1}{2k_0^2} \tilde{k}^\mu = \frac{1}{2k_0^2} (k^0, -\vec{k}),
$$

$$
e^a_\lambda(\vec{k}) = (0, \bar{\epsilon}_\lambda(\vec{k})), \quad \bar{k} \bar{\epsilon}_\lambda(\vec{k}) = 0, \quad \bar{\epsilon}_\lambda(\vec{k}) \bar{\epsilon}_{\lambda'}(\vec{k}) = \delta_{\lambda\lambda'}, \quad \lambda = 1, 2.
$$

(5.15)

Given this, the decomposition of any vector $a^\mu(\vec{k})$ in the basis vectors (5.15) have the form

$$
a^\mu(\vec{k}) = e^a_L(\vec{k}) a_L(\vec{k}) + e^a_T(\vec{k}) a_T(\vec{k}) + e^a_\lambda(\vec{k}) a_\lambda(\vec{k}),
$$

where

$$
a_L(\vec{k}) = k^\mu a_\mu(\vec{k}), \quad a_T(\vec{k}) = \tilde{k}^\mu a_\mu(\vec{k}), \quad a_\lambda(\vec{k}) = -e^a_\lambda(\vec{k}) a_\mu(\vec{k}).
$$

Note that for the study of one-particle state space it is sufficient to analyze the structure of creation operators. We shall decompose all the vector creation operators in the basis (5.15), representing the operator $\hat{B}^{(+)}_{\mu\nu}$ as

$$
\hat{B}^{(+)}_{\mu\nu} = \left( \hat{B}^{(+)}_{\mu\nu} + 1 \bar{\omega}_{\mu\nu} \right) \hat{B}^{(+)}_{\mu\nu} \equiv \hat{D}^{(+)}_{\mu\nu}.
$$

(5.16)

Namely,

$$
[D^{(+)}_L(\vec{k}) - D^{(+)}_T(\vec{k}), \hat{Q}^a] = 0,
$$

$$
[D^{(+)}_\lambda(\vec{k}), \hat{Q}^a] = -k_0 \bar{\epsilon}_{\lambda\lambda'} \hat{C}^{a(+)\lambda'}(\vec{k}), \quad \bar{\epsilon}_{\lambda\lambda'} = -\bar{\epsilon}_{\lambda'\lambda}, \quad \epsilon_{12} = -1,
$$

$$
[\hat{B}^{(+)}_L(\vec{k}) - \hat{B}^{(+)}_T(\vec{k}), \hat{Q}^a] = 2k_0 \hat{C}^{a(+)\lambda}(\vec{k}),
$$

$$
[\hat{B}^{(+)}_\lambda(\vec{k}), \hat{Q}^a] = -k_0 \hat{C}^{a(+)\lambda}(\vec{k}),
$$

$$
[\hat{\omega}^{(+)}(\vec{k}), \hat{Q}^a] = -\hat{C}^{a(+)}(\vec{k}),
$$

$$
[C^{a(+)}_L(\vec{k}), \hat{Q}^a]_+ = -2\epsilon^{ab} \{ \hat{\omega}^{(+)}(\vec{k}) + \frac{1}{2k_0} (\hat{B}^{(+)}_L(\vec{k}) - \hat{B}^{(+)}_T(\vec{k})) \},
$$

$$
[C^{a(+)}_T(\vec{k}), \hat{Q}^a]_+ = 0,
$$

$$
[C^{a(+)}_\lambda(\vec{k}), \hat{Q}^a]_+ = -\epsilon^{ab} k_0 \hat{B}^{(+)}_\lambda(\vec{k}).
$$

Hence it follows that among the 15 ($p, \vec{k}$ fixed) one-particle states there is only one genuine BRST–antiBRST-singlet: $(\hat{D}^{(+)}_L - \hat{D}^{(+)}_T)|0\rangle$. There are two BRST–antiBRST-quartets (respectively, for $\lambda = 1, 2$)
and a BRST–antiBRST-sextet

\[ \hat{w}^{(+)}|0 >, \hat{Q}^a \hat{w}^{(+)}|0 >, \hat{C}_L^{a(+)}|0 >, \frac{1}{2} \varepsilon_{ab} \hat{Q}^a \hat{C}_L^{b(+)}|0 >. \]

The presence of the BRST–antiBRST-sextet in the state space thus accounts for the physical S-matrix unitarity violation [32] in the theory concerned.

Let us now turn ourselves to the quantizaiton of the reducible model (5.1)–(5.3) within the Lagrangian Sp(2)-symmetric scheme [8–10].

In accordance with the rules [9], we introduce the set of fields \( \phi^A \)

\[ \phi^A = (A^\mu, B^{\mu\nu}, B^\mu, B^a, \phi^\mu, C^{\mu a}, C^{ab}) \]

and the sets of the corresponding antifields \( \phi^{*A} \), \( \bar{\phi}_A \)

\[ \phi^{*A} = (A^{*\mu}, B^{*\mu a}, B^{*\mu}, B_{a[b]}, C^{*\mu a[b]}, C^{*a[b]}), \]

\[ \bar{\phi}_A = (\bar{A}_\mu, \bar{B}_{\mu a}, \bar{B}_{\mu}, \bar{C}_{\mu a}, \bar{C}_{ab}). \]

Note that \( C^{ab}, B^a \) are the ghost fields (symmetric second rank Sp(2)-tensors) and Sp(2)-doublets of first stage respectively, introduced in accordance with the number of gauge parameters \( \xi \equiv \xi^a \) for the generators \( R_{1 \mu a} = R_{\mu a}^\nu Z^{\nu a} \). Given this,

\[ \varepsilon(C^{ab}) = 0, \; \text{gh}(C^{ab}) = 6 - 2(a + b), \]

\[ \varepsilon(B^a) = 1, \; \text{gh}(B^a) = 3 - 2a \]

(the remaining fields \( A^\mu, B^{\mu\nu}, B^\mu, C^{\mu a} \) have been described above).

The solution \( S = S(\phi, \phi^{*A}, \bar{\phi}) \) of the generating equations (2.1) with the boundary condition (2.2) for the model (5.1)–(5.3) can be represented as

\[
S = S + \int d^4x \left\{ B_{\mu a}^* (D^\mu C^{\nu a} - D^\nu C^{\mu a}) - \varepsilon^{ab} C_{\mu a b}^* B^\mu + \bar{B}_{\mu} (D^\mu B^\nu - D^\nu B^\mu) + C_{\mu a b}^* D^{\mu a} C^{ab} - 2\varepsilon^{ab} C_{a b c}^* B^c - B^a D^{\mu a} B^\mu + 2\bar{C}_{\mu a} D^\mu B^a - \varepsilon^{\mu \nu \rho \sigma} B^*_{\mu \nu a} (\bar{B}_{\rho \sigma} \wedge B^a) + \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} (B^*_{\mu \nu a} \wedge B^*_{\rho \sigma b}) C^{ab} \right\},
\]

where \( S \) is the classical action (5.1).

Choosing for the gauge fixing bosonic functional \( F = F(\phi) \)

\[
F = \int d^4x \left\{ -\frac{1}{4} B_{\mu a} B^{\mu a} - \frac{1}{4} \varepsilon_{ab} C_{\mu a} C^{\mu b} - \frac{1}{12} \varepsilon_{ab} \varepsilon_{cd} C^{ac} C^{bd} \right\}
\]

and integrating in Eq. (2.7) over the variables \( \lambda, \pi^a, \bar{\phi}, \phi^{*A} \), one arrives at the generating functional \( Z(J) \) of Green’s functions of the form

\[
Z(J) = \int d\phi \Delta \exp \left\{ \frac{i}{\hbar} \left( S + S_{\text{FP}}(\phi) + S_{\text{GF}}(\phi) + J_A \phi^A \right) \right\},
\]

where

\[
S_{\text{FP}} = \int d^4x \left\{ \frac{1}{4} C_{\mu a}^a M_{ab} K^{c[b\mu]}(\rho) G_{\rho c}^c - \frac{1}{4} \varepsilon_{ab} \varepsilon_{cd} D_{\mu} C^{ac} D^\mu C^{bd} \right\},
\]

\[
S_{\text{GF}} = \int d^4x \left\{ B_{\mu} D_{\nu} B^{\mu \nu} + \varepsilon_{ab} B^a D_{\mu} C^{\mu b} - \frac{1}{2} B_{\mu} B^\mu - \frac{1}{2} \varepsilon_{ab} B^a B^b \right\},
\]

\[
\Delta = \left\{ \varepsilon^{2i} \int d^4x \left\{ \left( \sqrt{D_{\mu} K^c} \right) \right\} \right\}.
\]
In Eq. (5.17), we have used the following notations

\[ K_b^{a}[\mu \nu | \rho \sigma] \equiv \frac{1}{2} \{ \delta_b^a (\eta^{\mu \rho} \eta^{\nu \sigma} - \eta^{\mu \sigma} \eta^{\nu \rho}) + X_b^a \epsilon_{\mu \nu \rho \sigma} \}, \]

\[ G_{\mu \nu}^a \equiv D_\mu C_\nu^a - D_\nu C_\mu^a - \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} B^a \wedge B^{\rho \sigma}. \]

The matrix \( M_{ab} \) is the inverse of \( M^{ab} \)

\[ M^{ab} \equiv \epsilon^{ab} - X_c^a \chi^b_d \epsilon^{cd}, \quad M^{ac} M_{cb} = \delta_b^a, \]

while the action of the matrix \( X_b^a \) on the objects \( E \equiv E^p \) carrying the gauge indices \( p \) is defined by the rule

\[ X_b^a E \equiv \epsilon_{be} (C^{ac} \wedge E). \]

The functional \( \Delta (5.17) \) can be considered as a contribution to the integration measure (in \( \phi \) space), invariant under the extended BRST symmetry transformations

\[ \delta B^{a \beta} = - \epsilon^{ab} M_{bc} K_d^{[a \beta]} G_{d \mu \nu}^{[\gamma \delta]} \eta_{\gamma \delta} A_{\mu}, \quad \delta A^a = 0, \]

\[ \delta C^{a \alpha} = (D^a C^{ab} - \epsilon^{ab} B^a \mu_b) \eta_{\mu}, \quad \delta B^a = D^a B^a \mu_a, \]

\[ \delta C^{ab} = B^{(a \in b) \mu} \mu_{\alpha}, \quad \delta B^a = 0 \]

for the quantum action \( S + S_{FP} + S_{GF} (5.18) \). The corresponding Noether charges \( Q^a \)

\[ Q^a = \int d^3 x \left\{ \frac{1}{2} \epsilon^{ijkl} G_{ijk} C_i^a + \pi_{(C)ij} D^j C^{ab} - \epsilon^{ab} \pi_{(C)0i} D^j B_{0j} + \epsilon^{ab} \pi_{(C)0i} D^j B_{0j} \right\} \]

expressed in terms of physical variables

\[ (A^i, \pi_{(A)i} = - \frac{1}{2} \epsilon_{ijkl} B^{jk}), \]

\[ (B^a, \pi_{(B)a} = B_a), \]

\[ (C^{ai}, \pi_{(C)ia} = - M_{ab} (G_{0b} + \frac{1}{2} \epsilon_{ijkl} X^i G_{jk} C_{ij})), \]

\[ (C^{a \alpha}, \pi_{(C)a \alpha} = - \epsilon_{ab} B^b), \]

\[ (C^{ab}, \pi_{(C)ab} = - \frac{1}{2} \epsilon_{ac} \epsilon_{bd} D_{0c} C^{cd}), \]

have the algebraic properties

\[ \{ Q^a, Q^b \} = 0 \]

with respect to the Poisson bracket in phase space (5.18). The Noether charge operators \( \hat{Q}^a \) ([\( \hat{Q}^a, \hat{Q}^b \]) = 0) generate the transformations

\[ [\hat{A}^i, \hat{Q}^a] = i \{ \epsilon_{ijkl} (\hat{X}^k + \hat{A}^j \wedge \hat{C}^{ka}) + \hat{\pi}_{(C)j} \wedge \hat{C}^{ab} - \epsilon^{ab} \hat{\pi}_{(C)0i} \wedge \hat{B}_{0j} \}, \]

\[ [\hat{B}^0, \hat{Q}^a] = i \epsilon^{ab} \hat{\pi}_{(C)j} \wedge \hat{B}^0, \quad [\hat{\pi}_{(B)0i}, \hat{Q}^a] = - i \epsilon^{ab} (\hat{X}^k + \hat{A}^j \wedge \hat{C}^{ka}), \]

\[ [\hat{C}^{ab}, \hat{Q}^a]_+ = i (\hat{X}^i \hat{C}^{ab} + \hat{A}^i \wedge \hat{C}^{ab} + \epsilon^{ab} \hat{\pi}_{(C)0i}), \]
The equations of motion for the in-factors following from Eq. (5.19) are representable in the form

\[ \Box B_{\mu\nu} = 0, \quad \Box C^{\alpha}_{\mu} = 0, \quad \Box C^{ab} = 0, \]

\[ A_{\mu} = \frac{1}{2} \xi_{\mu\rho\sigma} \partial^\rho B^{\sigma\rho} + B_{\mu} = \partial^\rho B^{\rho\mu}, \quad B^{a} = \partial^\mu C^{a}_{\mu}. \]

The decompositions of \( \hat{B}_{\mu\nu}(x) \), \( \hat{C}^{\alpha}_{\mu}(x) \) in the creation and annihilation operators are given by the relations (5.12), (5.13), while in the corresponding decomposition for \( \hat{C}^{ab}(x) \) one should make allowance for

\[ (\hat{C}^{ab}(-))^\dagger = (-1)^{a+b+1} \hat{C}^{ab}(+). \]

The (anti-)commutation relations for the creation and annihilation operators read

\[ [\hat{B}_{\mu\nu}(\vec{k}), \hat{B}^{\rho\sigma}(\vec{k}')] = \eta_{\mu}[\rho, \eta_{\nu}]_{\sigma}\delta(\vec{k} - \vec{k}'), \]

\[ [\hat{C}^{\alpha}_{\mu}(\vec{k}), \hat{C}^{\nu}_{\nu}(\vec{k}')] = \varepsilon^{\alpha}_{\rho\sigma}\eta_{\mu}\delta(\vec{k} - \vec{k}'). \]

Owing to Eq. (5.20), the doublet of operators \( \hat{Q}^{a} \)

\[ \hat{Q}^{a} = \int d^{3}k \, k^{\mu} \left( \hat{B}^{\mu}_{\nu}(\vec{k})\hat{C}^{a\nu}(\vec{k}) + \hat{C}^{a\nu}(\vec{k})\hat{B}^{\mu}_{\nu}(\vec{k}) \right) + \varepsilon^{ab}(\hat{C}^{ab}(\vec{k})\hat{C}^{\mu}(\vec{k}) + \hat{C}^{\mu}(\vec{k})\hat{C}^{ab}(\vec{k})) \]

generates for the in-operators the extended BRST symmetry transformations

\[ \delta \hat{B}^{\mu\nu} = -\left( \partial^\nu \hat{C}^{\beta}_{\mu} - \partial^\mu \hat{C}^{\beta a}_{\nu} \right) \mu_{a}, \]

\[ \delta \hat{C}^{\alpha}_{\mu} = \left( \partial^\nu \hat{C}^{ab}_{\nu} - \varepsilon^{ab} \hat{B}^{\alpha}_{\mu} \right) \mu_{b}, \quad \delta \hat{B}^{\alpha} = \partial^\mu \hat{B}^{\alpha}_{\mu} \mu_{a}, \]

\[ \delta \hat{C}^{ab} = \hat{B}^{(a}_{\nu} \hat{B}^{b)}_{\mu} \mu_{c}, \]

which, for the creation operators, take on the form

\[ [\hat{B}^{\mu}_{\nu}(\vec{k}), \hat{Q}^{a}] = k_{\mu} \hat{C}^{a\nu}(\vec{k}) - k_{\nu} \hat{C}^{a\mu}(\vec{k}). \]
\[
[\hat{C}^{bc\mu}(\vec{k}), \hat{Q}^a] = -k^\mu \varepsilon^a_{\mu b} \hat{C}^{c\mu}(\vec{k}).
\]

Making use of the decompositions of operators in the local basis (5.15) and taking the notation (5.16) into account, we find, by virtue of Eq. (4.21), that the 17 one-particle states of the theory form a genuine BRST–antiBRST-singlet
\[
(\hat{D}_L^{(+)} - \hat{D}_T^{(+)})|0>,
\]
two BRST–antiBRST-quartets
\[
(\hat{B}_1^{(+)} + \hat{D}_2^{(+)})|0>, \quad \hat{Q}^a(\hat{B}_1^{(+)} + \hat{D}_2^{(+)})|0>, \quad \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b(\hat{B}_1^{(+)} + \hat{D}_2^{(+)})|0>,
\]
\[
(\hat{B}_2^{(+)} - \hat{D}_1^{(+)})|0>, \quad \hat{Q}^a(\hat{B}_2^{(+)} - \hat{D}_1^{(+)})|0>, \quad \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b(\hat{B}_2^{(+)} - \hat{D}_1^{(+)})|0>
\]
and a BRST–antiBRST-octet
\[
\hat{C}^{1L\mu}(\vec{k})|0>, \quad \hat{Q}^a\hat{C}^{1L\mu}(\vec{k})|0>, \quad \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b\hat{C}^{1L\mu}(\vec{k})|0>,
\]
\[
\hat{C}^{2L\mu}(\vec{k})|0>, \quad \hat{Q}^a\hat{C}^{2L\mu}(\vec{k})|0>, \quad \frac{1}{2}\varepsilon_{ab}\hat{Q}^a\hat{Q}^b\hat{C}^{2L\mu}(\vec{k})|0>.
\]

Given this, the absence of BRST–antiBRST-singlet pairs and BRST–antiBRST-sextets as well as BRST-quartets (antiBRST-singlet pairs) and antiBRST-quartets (BRST-singlet pairs) ensures the physical unitarity of the theory (see [37]).

### 6. Conclusion

In this paper, we have studied the unitarity problem for general gauge theories within the Lagrangian $Sp(2)$-symmetric scheme proposed by Batalin, Lavrov and Tyutin in Refs. [8–10] and underlaid by the principle of invariance under the extended BRST transformations. The present study is based on the investigation of asymptotic state space structure with the help of representations of the algebra of generators $\hat{Q}^a$ ($a = 1, 2$) of the extended BRST symmetry transformations and the ghost charge operator $i\hat{Q}_C$. It is shown that the space of representation of the algebra of the operators $\hat{Q}^a$, $i\hat{Q}_C$ can be described by 7 types of one-particle state complexes referred to in the paper as genuine BRST–antiBRST-singlets (physical particles), pairs of BRST–antiBRST-singlets, BRST-quartets, antiBRST-quartets, BRST–antiBRST-sextets and BRST–antiBRST-octets. The conditions of the $S$-matrix unitarity are formulated. To provide the unitarity of a gauge theory in the Lagrangian $Sp(2)$-symmetric scheme, we require that in the theory be no BRST–antiBRST-singlet pairs, BRST-quartets, antiBRST-quartets and BRST–antiBRST-sextets. The general results are exemplified on the basis of the well-known Freedman–Townsend model [34] and the antisymmetric tensor field model with auxiliary gauge fields, proposed in Ref. [32].
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