An improved cutting force model in micro-milling considering the comprehensive effect of tool runout, size effect, and tool wear

Tongshun Liu1 · Yayun Liu1 · Kedong Zhang1

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Abstract
Tool runout, cutting edge radius-size effect, and tool wear have significant impacts on the cutting force of micro-milling. In order to predict the micro-milling force and the related cutting performance, it is necessary to establish a cutting force model including tool runout, cutting edge radius, and tool wear. In this study, an instantaneous uncut thickness (IUCT) model considering tool runout, a nonlinear shear/ploughing coefficient model including cutting-edge radius, and a friction force coefficient model embedded with flank wear width are respectively constructed. By integrating the IUCT, the nonlinear shear/ploughing coefficient and the friction force coefficient, a comprehensive micro-milling force model including tool runout, cutting edge radius, and tool wear is derived. Experiment results show that the proposed comprehensive model is efficient to predict the nonlinear cutting force of micro-milling with variable tool runout, cutting edge radius, and tool wear.

Keywords Micro-milling · Cutting force · Tool runout · Size effect · Tool wear

Nomenclature

- \( f_z \) Feed per tooth (μm)
- \( h \) Instantaneous uncut chip thickness (μm)
- \( h_m \) Minimum uncut chip thickness (μm)
- \( \theta_s \) Stagnant angle (rad)
- \( \beta_\tau \) Friction angle (rad)
- \( \sigma_m \) Ploughing coefficient (N/μm²)
- \( \tau_m \) Friction stress in ploughing region (N/μm²)
- \( \tau_s \) Shear stress (N/μm²)
- \( r_e \) Cutting edge radius (μm)
- \( r_o \) Length of tool runout (μm)
- \( \gamma_o \) Angle of tool runout (rad)
- \( \tau_v \) Tangential friction stress (N/μm²)
- \( \sigma_v \) Radial friction stress (N/μm²)
- \( V_B \) Flank wear width (μm)
- \( V_B^* \) Width of elastic contactregion (μm)
- \( K_{c,vb} \) Tangential friction force coefficient (N/μm)
- \( K_{r,vb} \) Radial friction force coefficient (N/μm)
- \( K_{c,sp} \) Shear-ploughing coefficient in tangential direction (N/μm²)
- \( K_{r,sp} \) Shear-ploughing coefficient in radial direction (N/μm²)

\( \Delta \theta^2 \) The angle between the equivalent radii (rad)

1 School of Mechanical and Electric Engineering, Soochow University, Suzhou 215021, Jiangsu, China

1 Introduction

Micro-milling, as the name suggests, is generally milling carried out at the microscale [1]. With the high machining efficiency and the ability to cut diverse materials, micro-milling is viewed as one of the most promising technology for fabricating the micro-electromechanical devices [2]. The cutting force, which is related to many machining performance such as the vibration [3], deflection [4], stability [5], and energy consumption [6, 7], plays an important role in micro-milling. Constructing an accurate cutting force model is of great importance for the application of micro-milling.

With the development of micro-milling technology, the mechanism of micro-milling force has been intensively investigated. Over the past decade, different factors such as tool runout [8], elastic recovery [9], tool wear [10], dead metal zone [11], edge radius size effect [12], and chip thickness accumulation [13] have been considered into the micro-milling force model. Among those factors, tool runout, edge radius size effect, and tool wear have particularly significant
impacts on the micro-milling force, and attract more attentions in the field of micro-milling research. Due to the manufacturing and clamping errors, the tool runout is inevitable in the micro-milling process. As the feed speed per tooth in micro-milling is usually within a few microns, slight tool runout will significantly affect the uncut chip thickness (UCT) and the cutting force. Bao and Tansel [8] consider the UCT formed by the adjacent cutting teeth and built an analytic UCT model for micro-milling under tool runout. Li et al. [14] built a generic UCT model for flat micro-end milling, in which the UCTs formed by all of the teeth are considered. Zhu and Zhang [15] extended the generic UCT model to ball end milling, and improved the force prediction accuracy in micro-end milling.

Limited by the tool sharpening technology, the cutting edge is not absolutely sharp. The cutting-edge radius of the micro-milling tool is usually comparable to the uncut chip thickness. The comparability of the cutting-edge radius and the uncut chip thickness, viz., the edge radius size effect [16], leads to an obvious minimum uncut chip thickness (MUCT) phenomenon [17] and ploughing effect [18], which greatly changes the micro-milling force. Liu et al. [19] deduced that the ratio of the MUCT to the edge radius is a function of the effective flow stress and the shear strength. By assuming that the shear angle is equal to the stagnant angle, Son et al. [20] expressed the stagnant angle corresponding to the MUCT as a function of the friction angle. Based on the minimum cutting energy principle and the infinite stress principle [21], Malekian et al. [22] concluded that the stagnant angle equals to the friction angle.

Under the high rotation speed, the tiny-size micro-milling tool wears rapidly, resulting in a sharp increase in micro-milling force. Bao and Tansel [23] built an experienced relationship between the tool wear and the amplitude of micro-milling force. Based on the friction stress distribution formular [24], Lu et al. [10] built a flank wear-included force model for Niki micro-milling. Compared to the flank wear, the effect of cutting-edge wear on the cutting force is much less studied. By assuming that both of the edge radius and the flank wear width increase with the deteriorating tool wear condition, Zhou et al. [25] attempted to build a cutting force model considering the edge wear and flank wear.

The works mentioned above have deeply investigated the individual effect of the tool runout, edge radius effect, and tool wear on micro-milling force. Besides, some other studies have explored the influence of the pairwise combination of the three factors on the cutting force of micro-milling. The combined effect of the tool wear and cutting-edge radius on the force was reported in study [25]. Jing et al. [9] considered the edge radius and runout, and built an analytic micro-milling force model. Li et al. [26] adopted the spatial analytic geometry method to analyze the combined effect of the tool runout and wear upon the micro-milling force. Liu et al. [27] built a micro-milling force model including the tool runout and tool wear, and proposed a cutting force model-based tool wear monitoring method under varying tool runout. However, so far, to our best knowledge, no studies have considered the comprehensive effect of tool runout, cutting-edge radius, and tool wear on the micro-milling force. To bridge this gap, this study builds a mechanics micro-milling force model considering the tool runout, cutting-edge radius, and tool wear, and investigates the comprehensive effect of the three factors on the micro-milling force.

The cutting force modeling process in this study is as follows. Firstly, the UCT model including the tool runout is constructed. Then, a nonlinear shear/ploughing cutting coefficient model including the IUCT and the cutting-edge radius is derived, and the friction force coefficient in the flank wear region is represented as a function of the flank wear width. Finally, by integrating the IUCT, the shear/ploughing cutting coefficient, and the friction force coefficient, a comprehensive micro-milling force model including tool runout, cutting edge radius, and tool wear is constructed.

This paper evolves as follows. In Sect. 2, the comprehensive micro-milling force model is constructed. Section 3 proposes a genetic optimization-based model parameters calibration approach. In Sect. 4, micro-milling experiment is conducted to examine the efficiency of the proposed model. The paper is concluded in Sect. 5.

2 The improved cutting force model for micro-milling

The proposed model is shown in Fig. 1. The input layer of the model consists of the tool runout, cutting edge radius, and the flank wear. The second layer is the middle variables including the IUCT, the shear-ploughing coefficient, and the friction coefficient. The output layer is the theoretical micro-milling force.

The comprehensive model in Fig. 1 is

![Flowchart of comprehensive micro-milling force model](image-url)

Fig. 1 The flowchart of constructing the comprehensive micro-milling force model
\[
\begin{align*}
\text{d}F_c &= \int_0^h K_{c_{vp}}(h^* | \lambda_x, r_c, \alpha) \cdot \text{dh}^* + K_{c_{vb}}(VB | \lambda_v) \text{dz} \\
\text{d}F_r &= \int_0^h K_{r_{vp}}(h^* | \lambda_x, r_c, \alpha) \cdot \text{dh}^* + K_{r_{vb}}(VB | \lambda_v) \text{dz}
\end{align*}
\]  

(1)  

(2)

Notation \( \text{d}F_c \) is the tangential force, \( \text{d}F_r \) is radial force, and \( \text{dz} \) is the unit axial cutting depth. Notation \( h \) is the instantaneous uncut chip thickness, which depends on the tool runout. Notation \( VB \) is the flank wear width, \( r_c \) is the cutting-edge radius related to the flank wear width, \( \alpha \) is the ideal rake angle. Notation \( K_{c_{vp}} \) is the shear-ploughing coefficient in tangential direction, \( K_{r_{vp}} \) is the shear-ploughing coefficient in radial direction. The shear-ploughing coefficient varies with the IUCT. If the partial IUCT \( h^* \) is greater than the minimum uncut chip thickness \( h_m \), the shear-ploughing coefficient represents the cutting force resulting from the shear effect (Fig. 2a). Otherwise, the shear-ploughing coefficient corresponds to the ploughing force (Fig. 2b). Notation \( K_{c_{vb}} \) is the tangential friction force coefficient in flank wear region; \( K_{r_{vb}} \) is the radial friction force coefficient. Mechanical parameter set \( \lambda_x = \{ \tau_x, \beta_x, \tau_m, \sigma_m \} \) includes the shear stress \( \tau_x \), friction angle \( \beta_x \), ploughing coefficient \( \sigma_m \), and the friction coefficient \( \tau_m \) in ploughing region. Mechanical parameter set \( \lambda_v = \{ VB^*, \tau_v, \sigma_v \} \) consists of the radial friction stress \( \sigma_v \), tangential friction stress \( \tau_v \), and the width of elastic contact region \( VB^* \).

The construction of the IUCT model considering tool runout, the shear/ploughing coefficient model considering cutting edge radius, and the friction force coefficient model with flank wear width is elaborated in the following three subsections.

### 2.1 Uncut chip thickness model considering tool runout

By changing the equivalent tool radius and the angle between the radii, the axial tool runout changes the trochoidal trajectory and the IUCT of each tooth. Because the axial cutting depth is much smaller than the length of tool, the radial tool runout could be regarded as the translation of the cutting part at the bottom of the micro-milling tool, and thus, the tool runout could be represented by the length \( r_c \) and the angle \( \gamma_e \) of the translation vector. According to studies [14, 27], the IUCT of the \( k \)th equivalent radius at cutting depth \( z \) with reference position angle \( \phi \) could be written as

\[
h_{k,z}(\phi) = \max \left\{ \min_m \left[ R_{k,z} - R_m \cdot z + f_z \sin(\theta_k) \cdot M \cdot \Delta \theta_m \right] \right\} \subseteq 0
\]

(3)

Notation \( R_{k,z} \) is the \( k \)th equivalent radius at cutting depth \( z \), \( \Delta \theta_m \) is the angle that the \( k \)th equivalent radius clockwise leads the \( m \)th equivalent radius at depth \( z \). \( \phi_k \) is the rotation angle of the \( k \)th equivalent radius at depth \( z \); \( f_z \) is the feed per tooth. The detailed calculation process of determining the equivalent radius \( R_{k,z} \) and the equivalent angles \( \Delta \theta_m \) by the tool runout parameters \( (r_c, \gamma_e) \) could refer to study [27].

### 2.2 Shear/ploughing coefficient model considering cutting edge radius

The comparability of the IUCT and cutting-edge radius, viz., the tool edge radius size effect, lead to the minimum uncut chip thickness and nonlinear IUCT-varying shear/ploughing coefficient. In this study, an analytic MUCT model is derived by assuming the normal ploughing force and the normal shear force are equivalent at the MUCT point (Fig. 3). The proposed assumption is mathematically represented as

\[
r_c \sigma_m \text{d} \theta \text{d}z = \frac{\tau_v \cos(\beta_v)}{\sin^2(\theta_v)} \text{d}h \text{d}z
\]

(4)

The left of Eq. (4) is the norm ploughing force \( \text{d}F_{p,n} \) in Fig. 3, and the right is the normal shear force \( \text{d}F_{s,n} \) under the minimum energy principle [21], \( \theta_v \) is the stagnant angle corresponding to MUCT.
The shear/ploughing coefficient in the three different part is

\[
K_{c,sp} (h | \lambda_s, r_e, \alpha) = \begin{cases} 
\frac{\tau_s \sin (\theta_{im} - \beta_s)}{\sin^2 \left( \frac{\theta_{im} + \beta_s}{2} \right)} h \geq h_{lim} \\
\frac{\tau_s \sin (\theta - \beta_s)}{\sin \left( \frac{\theta}{2} \right)} h_m \leq h < h_{lim} \\
\sigma_m + r_m \cot \theta \leq h < h_m 
\end{cases}
\]

(7)

\[
K_{r,sp} (h | \lambda_s, r_e, \alpha) = \begin{cases} 
\frac{\tau_s \cos (\theta_{im} - \beta_s)}{\sin \left( \frac{\theta_{im} + \beta_s}{2} \right)} h \geq h_{lim} \\
\frac{\tau_s \cos (\theta - \beta_s)}{\sin \left( \frac{\theta}{2} \right)} h_m \leq h < h_{lim} \\
\sigma_m \cot \theta - r_m \leq h < h_m 
\end{cases}
\]

(8)

The shear/ploughing force and the normal shear force are equivalent at the MUCT point, as shown in Fig. 3.

By solving Eq. (4), the analytic stagnant angle is derived as

\[
\theta_s = \pi - \arcsin \left( \frac{\sigma_m}{\sqrt{4r_s^2 \cos^4 \beta_s + (r_s \sin 2 \beta_s + \sigma_m)^2}} \right) - \arctan \left( \frac{\tan \beta_s + \sigma_m}{2r_s \cos^2 \beta_s} \right) + \beta_s
\]

(5)

The MUCT value is

\[
h_m = r_e (1 - \cos \theta_s)
\]

(6)

It worth noting that Eq. (4) has different periodic solutions as the right of Eq. (4) is a periodic function of the stagnant angle. In this study, the minimum solution among the solutions greater than the friction angle is set as the final solution. Malekian et al. [22] indicated that the stagnant angle is around the friction angle. Therefore, the final solution should be near the friction angle. From the right of Eq. (4), it could be found that the shear force, viz., the integral of the shear coefficient, will be infinite if the stagnant angle is smaller than the friction angle. Infinite shear force does not match the practical machining process in which the cutting force is always bounded. Therefore, the final solution should be greater than the friction angle. Considering the constraint conditions, the solution in Eq. (5), viz., the minimum solution among the solutions greater than the friction angle, is set as the final solution.

The angle of the intersection point of rake face and round nose is \(\theta_{lim}\), and the corresponding UCT is \(h_{lim} = r_e - r_e \cos \theta_{lim}\). According to the UCT value, the whole ploughing region could be divided into three parts: the ploughing region with \(h < h_m\), the shear region with UCT-varying shear force coefficient \((h_m \leq h < h_{lim})\), and the part with constant shear force coefficient \((h \geq h_{lim})\).

2.3 Friction force coefficient in flank wear region

The relationship between the flank tool wear and the friction force coefficient has been clearly revealed in the existed studies. According to the friction stress distribution formula [10], the relationship between the friction force coefficient and the flank wear could be written as

\[
K_{c,ib} (V B | \lambda_s) = \begin{cases} 
\frac{5}{3} \cdot \frac{V B}{r_e} & VB < VB^* \\
\tau_s \left( VB - \frac{2}{3} VB^* \right) & VB \geq VB^* 
\end{cases}
\]

(9)

\[
K_{r,ib} (V B | \lambda_s) = \begin{cases} 
\frac{2}{3} \cdot \frac{V B}{\sigma_r} & VB < VB^* \\
\sigma_r \left( VB - \frac{2}{3} VB^* \right) & VB \geq VB^* 
\end{cases}
\]

(10)

Many studies showed that the cutting-edge radius varies with the flank wear width. However, due to the uncertain build up edge and micro-chipping that affect the effective cutting-edge radius, it is difficult to build a deterministic analytic model to represent the relationship between the edge radius and the flank tool wear width. To reveal the dependency of the cutting-edge radius on the flank wear width, in the experimental validation part, empirical relationships between the calibrated edge radius and the measured flank wear width is analyzed by statistical correlation analysis method.
3 Calibration of the model parameters

Genetic optimization algorithm is adopted to calibrate the model parameters. Besides the two unknown mechanical parameter sets $\lambda_s$ and $\lambda_v$, the tool runout and the effective cutting-edge radius are also calibrated via the genetic optimization algorithm. Practically, the static tool runout could be directly measured by optic instrument. However, the optic measurement–based method is incompetent to measure the dynamic rotation–speed-dependent runout in cutting process. Due to the unregular buildup edge and microchipping, it is also difficult to directly measure the effective cutting-edge radius by optical microscope or atomic force microscope. Therefore, in this study, the dynamic tool runout and the effective edge radius are also calibrated via the genetic optimization algorithm. The flank wear band is regular, and the flank wear width could be directly measured by the optical microscope. In this study, the flank wear width is measured by Olympus Toolmakers microscope during the recess of tool holder. The flank wear width of each tooth is obtained from the bottom picture of the tool taken by the microscope, as Fig. 4 shows. The average wear width of all tooth is adopted to indicate the tool wear condition.

Including the mechanical parameter sets $\lambda_s$ and $\lambda_v$, tool runout parameters, and effective cutting-edge radius, there are 12 parameters need to be calibrated. The parameters need to be calibrated are listed in Table 1. The forces in X (feed) and Y (normal) directions are utilized to calibrate the parameters of the proposed model. The purpose of the optimization-based calibration is to find out the 12 optimum parameters, such that the gap between the theoretical force and the measured force is smallest. The measured force is acquired by commercial dynamometer. By decomposing the tangential and radial forces in Eqs. (1) and (2) into X and Y directions and integrating the elemental forces, the theoretical forces in X and Y directions are obtained.

In order to reduce the computation cost of the calibration process, instead of jointly optimizing the 12 parameters, the 12 parameters to be calibrated are optimized by three steps. Firstly, the cutting force signal generated by the fresh tool with known edge radius and flank wear width is utilized to calibrate the machinal parameters $\lambda_s$, and the parameter set $\lambda_s$ is set as a shared input of the subsequent calibration processes with worn tool. Then, with the sharing mechanical parameters $\lambda_s$, the friction force coefficient is calibrated under the worn tool. Finally, the parameter set $\lambda_v$ that reflects the relationship between the

![Fig. 4 Measurement of the flank wear. (a) Fresh tool; (b) flank wear on the worn tool](image)

### Table 1 Parameters to be calibrated

| Parameter to be calibrated                      | Notation | Unit      |
|------------------------------------------------|----------|-----------|
| Calibrated with fresh tool                      |          |           |
| Friction angle                                  | $\beta_s$| rad       |
| Normal cutting stress                           | $\sigma_m$| N/μm²     |
| Tangential cutting stress                       | $\tau_m$ | N/μm²     |
| Shear stress                                    | $\tau_s$ | N/μm²     |
| Calibrated with worn tool                       |          |           |
| Cutting edge radius                             | $r_e$    | μm        |
| Length of tool runout                           | $r_o$    | μm        |
| Angle of tool runout                            | $\gamma_o$| rad       |
| Tangential friction force coefficient           | $K_{c,ab}$| Nμm      |
| Radial friction force coefficient               | $K_{r,ab}$| Nμm      |
| Tangential friction stress                      | $\tau_v$ | N/μm²     |
| Radial friction stress                          | $\sigma_v$| N/μm²     |
| Width of elastic contact region                 | $VB^*$   | μm        |
flank wear and the friction force coefficient is calibrated with the calibrated friction force coefficients and the measured flank wear widths. The stepwise calibration process is shown in Fig. 5.

4 Experimental validation

4.1 Experimental setup

Micro-slot milling is conducted to validate the proposed comprehensive micro-milling force model. The machine used in the experiments is MIKRON HSM600U vertical milling machine. The C-CES-2005–0150 tool produced by UNION TOOL is adopted to machine steel AISI4340 (40CrNi2MoA). The tool is a carbide tool of two-edged micro-end milling cutter with UT coating. The cutting force is measured with a Kistler9109AA2 3-channel dynamometer. The sampling rate is 24 kHz. The flank wear width is measured by Olympus Toolmakers. According to the technical manual provided by UNION TOOL, three experiments with different cutting parameters are designed. Each cutting experiment has 10 cutting passes, and there are 30 cutting passes in the experiments. In each cutting pass, a 3-cm-long slot is machined. The experimental setup is shown in Fig. 6. The cutting parameters used in this study are listed in Table 2. The geometric parameters of the fresh tool are listed in Table 3.

4.2 Parameter calibration results

The cutting force signal acquired during the first cutting pass is adopted to calibrate the four mechanical parameters: the shear stress, friction angle, ploughing coefficient, and friction stress in ploughing region. It is assumed that the tool at the first cutting pass is fresh. The tool edge radius at the first cutting pass is the initial cutting-edge radius (2 μm), and the flank wear width at the first cutting pass is 0 μm. By taking the initial cutting-edge radius and the flank wear width into the proposed cutting force model and comparing the theoretical force and the measured cutting force, the four mechanical parameters along with the runout parameters are

Table 2 Cutting parameters used in this study

| Cutting condition | Spindle speed (rpm) | Cutting speed (m/min) | Axial cutting depth (μm) | Feed speed (mm/min) | Feed per tooth (μm) |
|-------------------|--------------------|-----------------------|--------------------------|---------------------|---------------------|
| C1                | 18,000             | 45.24                 | 80                       | 144                 | 4                   |
| C2                | 24,000             | 60.32                 | 100                      | 96                  | 2                   |
| C3                | 30,000             | 75.40                 | 60                       | 360                 | 6                   |

Fig. 6 Experimental setup
Table 3 Geometric parameters of the micro-milling tool

| Tooth number | Tool diameter | Rake angle | Clearance angle | Initial cutting edge radius | Initial flank wear width |
|--------------|---------------|------------|-----------------|-----------------------------|-------------------------|
| 2            | 0.5 mm        | 0°         | 6°              | 2 μm                        | 0 μm                    |

Table 4 Calibration of the mechanical parameters with fresh tool

| Cutting condition | $\beta_i$ | $\sigma_m$ | $\tau_m$ | $\tau_o$ | $r_o$ | $\gamma_o$ |
|-------------------|---------|---------|--------|--------|-------|--------|
| C1                | 0.557  | 0.027  | 0.026  | 0.001  | 1.022 | 2.046  |
| C2                | 0.439  | 0.024  | 0.012  | 0.001  | 0.318 | 0.950  |
| C3                | 0.514  | 0.025  | 0.017  | 0.001  | 0.764 | 2.164  |

The calibration results are listed in Table 4. Taking the calibrated shear stress, friction angle, and the ploughing coefficient into the MUCT model proposed in Sect. 2.2, the ratios of the MUCT value to the cutting-edge radius are calculated as 0.3537, 0.2775, and 0.3363 for cutting condition C1, C2, and C3. This result is consistent to the findings in most of the studies [28–30] which indicate the ratio of the MUCT value to the cutting-edge radius is around 0.3.

The mechanical parameters $\lambda = \{\tau_c, \beta, \tau_m, \sigma_m\}$ calibrated at the first cutting pass are shared with the subsequent cutting passes. At the following cutting passes, the cutting-edge radius and the flank wear width vary with the tool wear condition. The flank wear width is directly measured. The effective cutting-edge radius, the flank wear width-dependent friction force coefficients, and the runout parameters are jointly calibrated via the calibration algorithm proposed in Sect. 3. Table 5 presents the calibrated parameters with worn tool. As the tool is re-clamped at the beginning of each cutting pass, the runout parameter cannot keep constant for different cutting passes. This could be noticed from Table 5.

The varying process of the flank wear width and the effective cutting-edge radius under condition C1 are shown in Fig. 7. It clearly shows that the cutting-edge radius increases as the flank wear width increases. The correlation coefficient of the cutting-edge radius and the flank wear width is calculated as 0.8842, implying that the effective cutting-edge radius is related to the flank wear width. The same conclusion is found in experiments C2 and C3.

The calibrated friction force coefficients and the measured flank wear width are utilized to calibrate the tangential friction stress, radius friction stress, and the constant width of elastic contact region. The calibrated friction force coefficients of C1 are shown in Fig. 8. It shows that the growth curve of the friction force coefficient could be approximately represented by a polyline with the width of elastic contact region as the turning point. To obtain the mechanical parameters, the calibrated friction force coefficients are fitted by the polylines defined in Eqs. (9) and (10). The constant width of elastic contact region is calibrated as $VB^* = 18.678$ μm. The friction force stresses associated with the slope of the fitting polyline are calibrated as $\sigma_o = 0.125 \times 10^{-2}$ N/μm$^2$, $\tau_o = 0.076 \times 10^{-2}$ N/μm$^2$. Comparing the calibrated shear stress, cutting stress in ploughing region, and the friction stress in flank wear region, it could be found that the cutting stress in ploughing is much bigger than the other two
The flank wear and the cutting-edge wear under cutting condition C1: (a) flank wear width and cutting-edge radius VS cutting pass; (b) cutting edge radius VS flank wear width

Fig. 7 Friction force coefficient vs Flank wear width under condition C1

Table 6 The prediction error of the proposed model

| Cutting pass | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Prediction error (%) | C1  | 21.20 | 17.09 | 16.30 | 18.99 | 24.80 | 28.84 | 13.75 | 24.03 | 22.29 | 23.93 |
|               | C2  | 22.93 | 20.14 | 17.35 | 17.64 | 22.12 | 25.07 | 15.40 | 22.76 | 21.95 | 24.47 |
|               | C3  | 19.41 | 15.22 | 14.38 | 17.31 | 20.40 | 22.18 | 13.06 | 19.53 | 19.90 | 20.35 |

Table 7 Comparison between different models

| Model                              | Prediction error (%) |
|------------------------------------|----------------------|
| Tool runout and cutting-edge radius | 26.84                |
| Tool runout and tool wear          | 33.52                |
| Cutting-edge radius and tool wear  | 24.08                |
| Tool runout and cutting-edge radius and tool wear | 18.74 |

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stresses. This implies that the micro-milling force is most concentrated in the ploughing region.

4.3 Micro-milling force prediction results

The cutting forces are predicted via the proposed comprehensive model. The residual force is defined as the predicted force minus to the measured force. The estimated
error is defined as the ratio of the second norm of the residual force to the second norm of the measured force. The predicted error is listed in Table 6. It shows that the proposed comprehensive model is accurate to predict the micro-milling force.

Three conventional cutting force models are also utilized to predict the micro-milling force in this study. Different from the proposed comprehensive model, the three conventional models only consider two factors. The first conventional model includes tool runout and cutting-edge radius. The second model considers tool runout and tool wear. The third one considers the cutting-edge radius and tool wear. Table 7 lists the average prediction error of the 30 cutting passes. It could be found that the proposed comprehensive model is more accurate than the three traditional models. As Table 7 shows, the model without considering the cutting-edge radius has a much higher prediction error than the other three models. This implies the cutting-edge radius has the most significant effect on the micro-milling force. This could also be concluded from Fig. 9 where the predicted cutting forces at cutting pass 2 of C1 are presented.

5 Conclusion

In order to accurately predict the micro-milling force, this study construct a micro-milling force model considering the comprehensive effect of the tool runout, cutting edge radius, and tool wear. The IUCT model inducing tool runout, nonlinear shear/ploughing coefficient model considering tool wear-dependent cutting-edge radius, and the friction force coefficient function embedded with the flank wear are separately constructed. By integrating the IUCT, shear/ploughing coefficient, and the friction force coefficient into the mechanism milling force model, a comprehensive micro-milling force model considering the tool runout, cutting edge radius, and tool wear is derived. A genetic optimization–based stepwise calibration method is proposed to calibrate the parameters of the model. Experimental results show that the proposed model is efficient to predict the micro-milling force under varying tool runout, cutting edge radius, and tool wear condition. Some conclusions are as follows.

1. Including tool runout, cutting edge radius, and tool wear, the proposed comprehensive model is more accurate than the conventional models.
2. The cutting-edge radius increases as the flank wear width increases. The effective cutting-edge radius could be utilized to indicate the tool wear condition of micro-milling.
3. The cutting stress in ploughing region is much higher than the shear stress and the friction stress in the flank wear region. The micro-milling force is most concentrated in the ploughing region.
4. Among the tool runout, cutting edge radius, and tool wear, the cutting-edge radius has the most significant impact on the micro-milling force.
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Author contribution Tongshun Liu constructed the theoretical model and wrote the manuscript. Yayun Liu designed and directed the project. Kedong Zhang analyzed the data.

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Declarations

Ethics approval This paper does not contain any studies with human participants or animals performed by any of the authors.

Consent to participate All of the authors consent to participate.

Consent for publication All of the authors consent for publication.

Conflict of interest The authors declare no competing interests.

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