Analyzing airflow in static ice caves by using the calcFLOW method

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Abstract

In this paper we present a method to detect airflow through ice caves and to quantify the corresponding airflow speeds by the use of temperature loggers. The time series of temperature observations at different loggers are cross-correlated. The time shift of best correlation corresponds to the travel time of the air and is used to derive the airflow speed between the loggers. We apply the method to test data observed inside Schellenberger Eishöhle (ice cave). The successful determination of airflow speeds depends on the existence of distinct temperature variations during the time span of interest. Moreover the airflow speed is assumed to be constant during the period used for the correlation analysis. Both requirements limit the applicability of the correlation analysis to determine instantaneous airflow speeds. Nevertheless the method is very helpful to characterize the general patterns of air movement and their slow temporal variations. The correlation analysis assumes a linear dependency between the correlated data. The good correlation we found for our test data confirms this assumption. We therefore in a second step estimate temperature biases and scale factors for the observed temperature variations by a least squares adjustment. The observed phenomena, a warming and a damping of temperature variations depending on the distance the air traveled inside the cave, are explained by a mixing of the inflowing air with the air inside the cave. Furthermore we test the significance of the determined parameters by a standard $F$ test and study the sensitivity of the procedure to common manipulations of the original observations like smoothing. In the end we will give an outlook on possible applications and further development of this method.

1 Introduction

Ice cave research in its historical dimension has a long history in Europe (Grebe, 2010), which dates back to the 16th century. Theories about the origin of the cave ice are equally old, numerous and contradictory depending on the scientific knowledge
and ability to conduct measurements in the respective century (Grebe, 2010). In the 19th century the first instrumental measurements were conducted (compare Thury, 1861; Fugger, 1888; Lohmann, 1895; Crammer, 1899) before modern ice cave research found its beginning with the works of, e.g. Bock (1913), Racovitza (1927), Saar (1956) and others. Evidently, right from the beginning the main focus was to understand the processes and dynamics of the ice body and specific cave climate elements, among those the course of the air temperature in the specific study sites as well as the airflow regime were a main focus. Until today long-term measurements are rare but do exist in several European commercial caves, e.g. Scarisoara ice cave (Racovitza and Onac, 2000), Dachstein Rieseneinshöhle (Saar, 1956), Dobsinska ice cave (Korzystka et al., 2011), Schellenberger Eishöhle (Meyer et al., 2014). In addition, short-term measurements are conducted by speleological organizations and others at many sites, thus covering numerous ice cave sites worldwide. The full potential of these study sites and recorded data is not exploited yet. Most climate studies in ice caves are concentrating on air, ice and rock temperature, as temperature loggers are available for relatively low prices and thus also affordable for private studies by, e.g. speleological organizations. Depending on the individual questioning this may be sufficient for a basic cave climate analysis. Beside the financial reasons, ice cave studies are facing two other problems in general: the accessibility of the study site and the energy supply for technical devices. The study sites are in many cases in remote places in the high mountains, exposed to avalanches and winter conditions often lasting several months. As a consequence for example airflow measurements using sonic anemometers are not always possible, though an understanding of the airflow regime is indispensable for the understanding of these complex systems. For the development but also degradation of subterranean ice the airflow regime is the main influencing factor beside the time/amount of water and the thermal conditions or the heat transfer between the different media (rock, ice, water, air) (Korzystka et al., 2011). Racovitza states in his paper “La classification topoclimatique des cavites souterraines” that the main factor to characterize a cave in general is the air temperature (Racovitza, 1975), but among the deduced topocli-
matological factors the airflow regime, which is first of all determined by the thermal relation between the exterior atmosphere and the cave atmosphere, is the factor that influences the cave air temperature and the humidity in the cave the most. Moreover the airflow regime is the most important physical factor to describe the topoclimate of a cave (Racovitza, 1975). For this reason Racovitza proposes to classify the different types of cave topoclimate using the diverse types of airflow regimes (Racovitza, 1975). Luetscher and Jeannin (2004a) propose, for the specific case of ice caves in temperate regions, to classify on the basis of two criteria: cave air dynamics and the type of ice. They explain this by the importance of the airflow regime as the “dominating process at the origin of cave ice” in, e.g. static or dynamic ice caves, just to mention the best known ice cave types. Numerous case studies highlight the role of airflow for the development of ice caves, (e.g. Luetscher and Jeannin, 2004b; Pflitsch et al., 2007; Morad et al., 2010). For these reasons we here present a practical attempt to use the given database which is available for the majority of the ice caves – air temperature measurements – for computing air fluxes. Understanding the airflow regime and the thermal behavior of ice caves, is a fundamental step to understand the specific cave climate and the related processes and dynamics. For this reason we worked out the calcFLOW-method with the major goal to track air movement during the open period of static ice caves by using air temperature measurements. In this paper we present the basic principles and the methodology of the calcFLOW-method and apply it to Schellenberger Eishöhle (Germany). The results will be useful to install a refined network of temperature loggers inside the cave. In the last part of this paper applications of the calcFLOW-method will be presented and discussed. All calculations were conducted by using the GNU Octave open source software\textsuperscript{1}.

\textsuperscript{1}https://www.gnu.org/software/octave/
2 The model

Two different stages of a static ice cave have to be distinguished. A closed phase, where the air temperature in the cave is below the temperature outside and no interaction by gravitational air mass transport between the inside and outside atmosphere takes place. In this case only gravitational layering of the air is considered, the densest (coldest) air occupying the deepest ranges of the cave. As long as the slow warming of the cave during the closed phase is ignored, the difference in temperature observed by two loggers at different locations in the cave is constant over time and may be described by a simple bias:

\[ T_B(t) = T_A(t) + b, \]  

(1)

\( T_A \) and \( T_B \) being the temperatures observed at time \( t \) by the loggers at locations \( A \) and \( B \) inside the cave. \( b \) is the temperature bias observed between both loggers and in this simple model considered to be constant over time.

We focus on the open phase respectively the so-called “winter situation” (compare Meyer et al., 2014) in Schellenberger Eishöhle. Due to its morphology and the fact that it has only one entrance, the cave acts like a cold air trap depending on the external air temperature. The winter situation is limited to external temperatures below 0 °C causing inflow of cold air from outside into the cave driven by gravitational flow.

When outside temperatures drop below the current cave air temperature the specific colder air replaces the warm air inside the cave. The cold air enters the cave along the floor of the cave passages while the warm air is pushed out traveling along the ceiling towards the cave entrance. The temperatures observed close to the cave floor and at the ceiling therefore may differ greatly. For this reason care has to be taken in the selection of the positions for the temperature loggers to capture the airflow of interest.

By mixing of cold and warm air flows and by contact of the inflowing cold air with the cave walls and cave ice the inflowing air will gradually warm and on the other hand the cave is cooled down from the entrance towards its inner reaches. As a consequence the
gravitational layering of the cave air is disturbed. It is replaced by a positive correlation between air temperature and distance traveled by the air, normally the distance to the entrance. In case of a constantly dropping cave passage (as it is typical for a static ice cave) this may lead to a complete inversion of the temperature gradient. As soon as the outside temperatures rise above the cave temperature and the inflow of cold air stops the gravitational layering of the air is restored.

During the open phase loggers at different locations in the cave will record a completely different scenario than during the closed phase. We expect a temperature bias, but now with inverted sign, the cave being warmer the further inside the logger is placed. We furthermore expect the variations in air temperature that are driven by the weather and the day/night cycle outside the cave to be measurable also inside the cave, but damped, due to mixing of the inflowing air with the more stagnant air inside the cave. Thirdly we assume, that the cold inflowing air needs some time to travel from logger A to logger B. Our model for the air temperature measurements taken by different loggers during the open phase of a static ice cave includes all three parameters: bias, scale factor (damping of temperature variations), and travel time of the air from logger A to logger B. The model for the open phase therefore reads:

\[
T_B(t) - \bar{T}_B = s \cdot (T_A(t - \Delta t) - \bar{T}_A).
\]

\textit{T}_A, \textit{T}_B \text{ and } \textit{t} \text{ are defined as above. The model is augmented by a scale factor } s \text{ and the travel time } \Delta t \text{ of the air moving from logger } A \text{ to logger } B. \bar{T}_A \text{ and } \bar{T}_B \text{ are the mean temperatures measured by loggers } A \text{ and } B. \text{ The terms } T_B(t) - \bar{T}_B \text{ and } T_A(t - \Delta t) - \bar{T}_A \text{ describe the temperature variations around mean recorded by the two loggers, that are damped by factor } s \text{ at logger } B \text{ due to the mixing of the inflowing air with stagnant air along the way from logger } A \text{ to logger } B. \text{ The bias } b = \bar{T}_B - \bar{T}_A \text{ is hidden in the difference between the mean temperatures at } A \text{ and } B.
We express the temperature modeled for logger \( B \) as a function of the temperature measured by logger \( A \):

\[
T_B(t) = s \cdot (T_A(t - \Delta t) - \overline{T}_A) + b^*, \quad b^* = \overline{T}_B = \overline{T}_A + b. \tag{3}
\]

The parameters \( b^* \) and \( s \) of this simple model may be estimated from the observed temperature data by a standard least squares adjustment process (Koch, 1999). To keep things simple, the single temperature measurements are assumed to be independent from each other and not affected by colored noise (i.e. their errors are assumed to be normally distributed).

To set up the design matrix \( A \) of the adjustment process we have to compute the partial derivatives of the modeled temperatures at logger \( B \) with respect to the unknown parameters \( b^* \) and \( s \):

\[
A = \begin{pmatrix}
\frac{\partial T_B(t_1)}{\partial b^*} & \frac{\partial T_B(t_1)}{\partial s} \\
\vdots & \vdots \\
\frac{\partial T_B(t_n)}{\partial b^*} & \frac{\partial T_B(t_n)}{\partial s}
\end{pmatrix}, \quad \frac{\partial T_B(t)}{\partial b^*} = 1, \quad \frac{\partial T_B(t)}{\partial s} = T_A(t - \Delta t) - \overline{T}_A. \tag{4}
\]

The optimal solutions \( \hat{b}^* \) and \( \hat{s} \) of the sought for parameters are found by solving equation

\[
\begin{pmatrix}
\hat{b}^* \\
\hat{s}
\end{pmatrix} = (A^T PA)^{-1} A^T P T_B, \tag{5}
\]

\( T_B \) is the column vector of temperatures measured at logger \( B \). The weight matrix \( P \) is the identity matrix, as long as all temperatures are observed with comparable quality (otherwise it is a diagonal matrix with the diagonal elements equal to the inverse of the square of the assumed a priori errors). With the estimated parameters \( \hat{b}^* \) and \( \hat{s} \) the disclosure between observed and modeled temperatures at logger \( B \), determined by the sum of squares of the residuals, is minimized.
To determine the third unknown parameter $\Delta t$ in the same way, we would have to compute the partial derivative

$$\frac{\partial T_B}{\partial \Delta t} = \frac{\partial T_B}{\partial T_A} \frac{\partial T_A}{\partial \Delta t} = s \cdot \frac{\partial T_A}{\partial \Delta t}. \quad (6)$$

Neither an a priori value for $s$ nor $\partial T_A/\partial \Delta t$ are known. We therefore propose to determine the time shift $\Delta t$ independently by cross-correlation of the time series of observed temperatures $T_A$ and $T_B$.

Correlation analysis

The idea behind the calcFLOW-method is that a weather induced temperature pattern is visible at all measuring stations inside the cave and that it is sufficiently unique to produce a distinct maximum of correlation when cross-correlating the observed temperature time series of two different loggers. For this purpose one of the time series is shifted in time until maximum correlation is reached. The time shift corresponding to optimal correlation of both time series is equal to the travel time of the air between the two temperature loggers. To determine the airflow speed the length of the passage between the two loggers has to be divided by the travel time of the air. An analogous method is used, e.g. in hydrology to determine the travel time of a flood pulse or, when applied to karst springs, the time delay between rainfall and discharge (see, e.g. Padilla and Pulido-Bosch, 1994; Laroque et al., 1998). In case of hydrology the medium is water, not air, and the observable is the flow rate, not the temperature. Analogous to our case the signal is damped by mixing with stagnant water (in our case stagnant air).

Pearson’s correlation coefficient between two linearly correlated time series $X$ and $Y$ of $n$ samples each is computed by

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}, \quad (7)$$
where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i$ are the mean values of the corresponding time series. The correlation coefficient $r$ will take values between $-1$ and $1$. A value of 1 validates the assumption that $Y = b + s \cdot X$ with bias $b$ and scale $s$. Note that this assumption exactly corresponds to our simple model in Sect. 2 and therefore $r$ may additionally serve to validate the applicability of the model.

Two parameters have to be chosen carefully when actually correlating the temperature data. First we have to define the number $n$ of samples we want to use for correlation. We inherently assume that the airflow speed is constant for the time period covered by the $n$ samples. It is therefore desirable to choose $n$ as small as possible if we are interested in the temporal variability of the airflow speed in the cave. On the other hand the part of the time series under consideration has to be long enough to show a unique temperature pattern for correlation. Due to the smoothness of the observed temperatures they will resemble a linear trend during short stretches of time. Cross-correlating two straight lines will produce constant correlation coefficients of 1 and no distinct maximum will be distinguishable.

To find an adequate $n$ it is helpful to actually take a look at the correlation function of example data observed in Schellenberger Eishöhle. We analyzed temperatures observed by four different loggers during periods of large temperature variations (Fig. 1) or small temperature variations (Fig. 2). The temperatures at logger T1 were taken as a reference, while the temperatures recorded by loggers T2, T3 and T4 were cross-correlated with the temperatures at logger T1 using different numbers of samples (see Sect. 3 for a characterization of the different loggers). During periods with large temperature variations only a small number of samples is needed to produce distinctive maxima in the correlation function (Fig. 1, bottom panels). During periods of little temperature variations on the other hand no distinction of a maximum of correlation is possible, if to few samples $n$ are considered for cross correlation (Fig. 2, middle and bottom panels). Generally we may assume that a time span of one day (corresponding roughly to a correlation length $n$ of 51 samples in Figs. 1 and 2) will most probably suffice in most cases to get a clear correlation peak due to the day/night cycle in outside temper-
nature. Shorter time spans may suffice during periods of pronounced weather patterns. Fine tuning of \( n \) will be worthwhile, whenever time resolution of the determined airflow speeds is in the center of interest.

The second parameter we have to choose is the maximum number of samples we shift time series \( Y \) against time series \( X \). From a computational point of view it is desirable to keep this number small. Moreover periodic temperature patterns like the day/night cycle will lead to secondary maxima in the correlation function, if we shift one time series by a full period of the cycle (i.e. one day). A rough idea of the expected airflow speeds is helpful to adjust this parameter. If the air is expected to move within 10 min from logger \( A \) to logger \( B \), it is in principle not necessary to shift the time series at logger \( B \) by more than 10 min to catch the maximum in the correlation function.

It has to be stressed that the sampling rate of the temperature measurements limits the time resolution of the correlation analysis. The time shift of maximum correlation will always be an integer multiple of the sampling rate. Even if the smooth nature of the temperature measurements suggests to increase the sampling rate by interpolation, this will not introduce new information for the correlation analysis. On the other hand it does not disturb the analysis according to our experience.

The time shifts determined by the correlation analysis are inserted into Eq. (4) to compute the partial derivatives with respect to the scale factors. In a consecutive step biases and scale factors of our simple model can be determined for each pair of data loggers, as illustrated in Sect. 3 on the basis of data recorded in Schellenberger Eishöhle.

3 Application to data

To illustrate the methods introduced above we apply them to temperature measurements recorded in the static ice cave Schellenberger Eishöhle (total length: 3621 m, total depth: +39, −221 m) is situated at 1570 m a.s.l. on the foot of the NE-walls of Untersberg in the South–East of Germany (see Fig. 3). The access to the cave is
marked by a 4 m high and 20 m wide portal, which leads to the largest room in the
cave Josef-Ritter-von-Angermayer-Halle with a length of 70 m and a width of 40 m that
is illuminated by daylight. The floor 17 m below the entrance level of this hall
completely consists of a major ice monolith, which is surrounded by the cave trail. The
two passages Wasserstelle and Mörkdom connect to the deepest part of the ice cave
called Fuggerhalle, 41 m below entrance level. They are also partly covered with ice.

Temperature loggers were placed in Angermayerhalle (T1 and T4), along one of the
passages leading down (Wasserstelle: T2), and in Fuggerhalle (T3, see Fig. 4). The
loggers recorded temperature data with an interval of 10 resp. 15 min. The original
purpose of the temperature measurements was not to determine air speeds and there-
fore it was not emphasized to synchronize the sampling rates of the different loggers.

For our analysis observations at common 30 min intervals were chosen. It turned out
that for the determination of wind speeds a higher sampling rate would be beneficial.
It therefore is planned to synchronize and increase the sampling rate in the future.
Apart from the 500 m long ice cave part there is one major non-ice part, which forks of
Angermayerhalle in a northeasterly direction and leads through several deep shafts to
the deepest point of the cave (−221 m).

In a first step time shifts between one of the loggers in Angermayerhalle (T1) and
all the other loggers (T2, T3, and T4) were determined for an example epoch following
the procedure described in Sect. “Correlation analysis”. The number of samples for
correlation analysis was chosen to be 51 (slightly longer than one day, as proposed
in Sect. “Correlation analysis”). In a second step temperature biases and scale fac-
tors between the corresponding loggers were determined from the same set of data
according to the least squares formalism introduced in Sect. 2.

Figure 5 shows the fit of observations for logger T1 to the observations of either T2,
T3, or T4, Fig. 6 the inverse fit of T2, T3 and T4 to data of logger T1. While the results
shown in Fig. 6 correspond to the procedure used in Sect. “Correlation analysis”, both
definitions are in principle valid and are shown here to prove the robustness of the
method.

Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper |
As described in Meyer et al. (2014) the two loggers in Angermayerhalle show a very different behaviour (Fig. 6a). Our analysis revealed a significant time shift (Fig. 6b) as well as a pronounced positive temperature bias (Fig. 6c) of T4 relative to T1 as well as a pronounced damping of the temperature variations recorded (Fig. 6d) by T4. We therefore assume that logger T1 records the cold inflowing air, while T4 records the relatively warmer air flowing out of the cave close to T4. We further assume that the cold inflowing air passes by logger T2 to the deepest point in Fuggerhalle, where logger T3 is positioned. The sampling rate of 30 min proved to be too long to determine the airflow speed along this way for the example epoch. This means that the air took significantly less than 30 min for the distance of approximately 65 m between T1 and T3.

While T2 shows distinctive variations of rather short duration (and unknown origin) that clearly correspond to the temperature variations recorded by T1, the same variations are very much damped at T3. This may be explained by the distance of Fuggerhalle to the cave entrance and the damping of the temperature signal on this way. Moreover Fuggerhalle acts as a dead end to those passages where the inflowing cold air is probably thoroughly mixed with the stagnant air.

From T3 at the furthest end of Fuggerhalle the warm air takes a significant time before it reaches T4 on its way out of the cave. Not much more damping or warming takes place along this path. Unfortunately at the time period analyzed no logger was positioned in the second passage (Mörkdom) connecting Angermayerhalle and Fuggerhalle, so it cannot be clarified if one of the passages acts as the primary way down for the cold air while the other channels the warm air back to the surface. But based on our data analysis we assume that Mörkdom is the primary way for the outflowing warm air (Meyer et al., 2014). We hope to be able to prove this theory by further investigations.

The parameters determined from the example data are listed in the legends of Figs. 5 and 6. In Fig. 6 the temperatures recorded by loggers T2, T3 and T4 were shifted, biased and scaled to fit to T1. In Sect. 2 we had defined the parameters the other way.
round, so we have to change the sign of the time shifts and biases quoted in Fig. 6 and invert the corresponding scales to compare the results. Applying these modifications the biases are positive and increase with the distance the air has traveled into the cave (as long as we assume that T4 records the out-flowing air), as predicted by our model. The scaling factors are smaller than 1 (damping of signal) and inversely proportional to the distance the air traveled. Both experiments (Figs. 5 and 6) lead to very much comparable results. The validity of our model is further confirmed by the good fit achieved for the example data (Figs. 5d and 6d).

3.1 Temporal variability

In Sect. “Correlation analysis” it was mentioned that the airflow speed is supposed to be constant during the time period considered for correlation. In this section we will estimate airflow speeds (time shifts) for a period of about 6 days to check if this requirement is met. We use either 51 or 101 samples for correlation. We also try the effect of smoothing (by a moving mean of 5 samples) to filter out the short term variations of unknown origin mentioned in Sect. 3.

The time span analyzed corresponds to the period shown in Fig. 7. The determined time shifts and the corresponding maxima of correlation are displayed in Fig. 8. The latter serve to validate the time shifts. Correlation coefficients below 0.97 indicate questionable results.

Only between loggers T1 and T2 the correlation, at least of the smoothed temperature data, is high during the whole period analyzed and the determined airflow speed is quite constant. As already mentioned the sampling rate of 30 min is to coarse to really resolve it, the time shift varies between 0 and −30 min, indicating a true value between both limits. The at first glance puzzling negative time shift may hint at a placement of logger T1 too high above the ground. The cold air entering the cave moves along the floor of the passage below T1 and reaches T2, before it is recorded by T1.

The results of the correlation analysis between loggers T1 and T3 indicate that the airflow speed in fact is not constant. The larger time shifts determined for the begin-
ning of the time period correspond to higher temperatures and consequently a less pronounced gravitational airflow. Near the end of the period the temperatures rise so much that the air movement stops, the open period of the ice cave is interrupted and our model is not longer valid. Consequently the correlation analysis fails. This result is confirmed by the airflow speeds determined between T1 and T4.

The somewhat different values determined from the analysis of either 51 or 101 samples indicate that the slow airflow at the beginning of the period affects the results for a longer time if 101 samples are considered for correlation. In general the correlation of a larger amount of samples leads to smoother results.

In case of the analysis of loggers T1 and T4 we get very variable results for the airflow speed as well as for the value of maximum correlation. A closer look at the correlation function at single epochs would reveal that side maxima distort the analysis. The results achieved for 51 or 101 samples agree best during the middle of the period. The smoothing of the data generally improves correlation, but does not significantly alter the determined airflow speeds.

After applying the determined time shifts to the time series of temperature observations at loggers T2, T3 and T4 optimal biases and scale factors were estimated for each epoch. The results are summarized in Fig. 9 and show a strong dependency on the temperature of the cold inflowing air. The larger the temperature gradient of the cold outside air with respect to the more stagnant cave environment, the stronger the energy flux between cave (air, rock, ice) and inflowing air.

Again the parameters were fitted either from 51 temperature samples or from 101 samples. Because the fit is optimal to all samples used, an averaging takes place and the results obtained from more samples look considerably smoother. In this context a smoothing (moving mean) of the temperature time series prior to the estimation of biases and scales will not alter the results significantly (as confirmed by Fig. 9).
3.2 Validation of model parameters

The time shifts derived from the correlation analysis could most easily be validated by actual airflow measurements. But we do not have airflow measurements available and so we depend on internal validation methods. In Sect. “Correlation analysis” it was already mentioned that the shape of the correlation function is an indicator for the reliability of the determined time shift. A distinct maximum indicates a reliable determination of the corresponding time shift. The maximum value of the correlation coefficient moreover validates the general applicability of the linear model assumed. In our analysis of data collected in Schellenberger Eishöhle correlation was generally high (> 0.9 for most of the time analyzed) and we can safely assume the model to be valid.

A further internal method (without the use of other measurements) for validation of the model and the determined parameters is the study of the formal errors of the parameters derived from the least squares adjustment and the post fit error of the modeled temperatures when compared to the actually observed ones.

The post fit standard deviation $\sigma$ of the modeled temperatures is easily computed from the sum of squares of the residuals

$$v^2 = \sum_n (T_{B,\text{observed}} - T_{B,\text{modelled}})^2$$

$$\sigma = \sqrt{\frac{v^2}{n-u}}$$

with $n$ the number of observations used to fit the model (in our examples so far chosen to be equal to the number $n$ of samples used for the correlation analysis) and $u$ the number of unknown parameters estimated. The time shift is determined independently of bias and scale factor, nevertheless we chose $u = 3$. 

5305
The formal errors of bias $\sigma_b$ and scale factor $\sigma_s$ are taken from the covariance matrix of the least squares adjustment

$$K = \sigma^2 \cdot \begin{pmatrix} \sigma_b^2 & \sigma_{bs} \\ \sigma_{sb} & \sigma_s^2 \end{pmatrix} = \sigma^2 \cdot (A^T PA)^{-1}. \quad (10)$$

$K$ is a symmetric matrix, $\sigma_{bs}$ and $\sigma_{sb}$ are identical. Keep in mind that in Sect. 2 we chose $P$ to be the identity matrix. The formal errors are scaled by the post fit standard deviation $\sigma$. Note that from the covariance matrix one can also compute the correlation coefficient between bias and scale factor

$$r_{bs} = \frac{\sigma_{bs}}{\sigma_b \cdot \sigma_s} \quad (11)$$

that has not been evaluated in this study. In case of the data analyzed from Schellenberger Eishöhle the correlation between bias and scale is small and could also be neglected (corresponding to a separate estimation of both parameters).

The formal errors of bias and scale factor, scaled with the post fit standard deviation, are shown in Fig. 10 for the time period analyzed in Sect. 3.1. The temperature biases are rather well defined, the scale factors profit from a smoothing of the data. Rising errors to the end of the period correspond to the rising outside temperatures that finally lead to ceasing air movements in the cave and an interruption of the open period.

Finally the significance of the estimated parameters may be calculated assuming that their errors are normally distributed (their variances are $\chi^2$-distributed). To test the significance of the parameter under question, two different models are compared: one including all parameters (full model), the other one including all but the parameter under question (reduced model).

The reduced model to test the significance of $\Delta t$ reads

$$T_{B,\Delta t}(t) = s \cdot (T_A(t) - \bar{T}_A) + b^*, \quad (12)$$
the reduced model to test the significance of \( s \) reads

\[
T_{B_b}(t) = T_A(t - \Delta t) + b^*,
\]

and finally the reduced model to test the significance of \( b^* \) reads

\[
T_{B_{b^*}}(t) = s \cdot (T_A(t - \Delta t) - T_A) + T_A.
\]

Note that it is not correct to determine the parameters of the full model once and subsequently insert them into the reduced models. Instead the parameters of each of the reduced models have to be determined in a separate estimation procedure to also take into account the correlations between the different parameters. As mentioned before the correlations may be neglected here for the test of bias and scale, which can be determined quite independently, but for the significance test of the time shift, both parameters of the reduced model (Eq. 12) have to be re-estimated with a time shift of \( \Delta t = 0 \).

We perform a \( F \) test (e.g. Snedecor and Cochrane, 1989) computing the ratio:

\[
\Phi = \frac{(v_f^2 - v_f^2)}{(r_f - r_f)} / \frac{v_f^2}{r_f},
\]

\( v_f^2 \) and \( v_f^2 \) are the sum of squares of the observed temperatures after subtraction of the modeled ones (see Eq. 8), subscript “f” refers to the full model, subscript “r” to the reduced model. \( r_f \) and \( r_f \) are the corresponding degrees of freedom \( n - u \) of the two models, the number of unknowns \( u_f = u_f - 1 \) of the reduced model being smaller than that of the full model \( u_f \) and therefore \( r_f = r_f + 1 \).

\( \Phi \) is \( F \) distributed, its probability density function \( F_{nm}(\Phi) \), with \( n = r_f \) and \( m = r_f - r_f \), is a measure for the probability, that the additional parameter in the full model could have been estimated in the same way from normally distributed random numbers. We evaluate the associated cumulative distribution function and reject all parameters for
which it is smaller than 0.99 (corresponding to a 99 % confidence level). The remaining biases, scales and time shifts are marked in Fig. 11 for a correlation length of 101 samples, and in Fig. 12 for a correlation length of 51 samples. The determination of bias and scale turns out to be very robust, again verifying the linear model. The determination of the time shift is only robust at the 99 % confidence level for the pair of loggers T1 and T4. For the other loggers the time shift cannot be determined reliably during considerable periods of time, especially in case of a short correlation length. This is most probably due to the short time it takes the air to travel from logger T1 to T2 or T3, which cannot be resolved by the coarse sampling rate of temperature observations at 30 min intervals. If the air moves within 15 min from one logger to the next, then a time shift of 30 min is equally wrong as no time shift at all and the parameter is rated not significant. Again, a higher sampling rate could significantly improve the determination of wind speeds.

3.3 Discussion on results

The aim of this paper was to present the principles and the methodology of the calcFLOW-method we developed in order to be able to use air temperature measurements in static ice caves for multiple purposes. The idea of calcFLOW is based on the fact that in many ice caves in remote places airflow measurements are difficult. But in every ice cave, where cave climate related studies are conducted, at least temperature measurements (air, rock, ice) are performed. With this given database we calculated three different parameters to characterize the processes inside the cave better and to understand the temperature differences between the measuring points: airflow speed, increase of the air temperature and damping of the temperature signal. But the primary idea is to calculate airflow speeds inside a static ice cave by cross-correlating air temperature data of different logger sites. The method was applied to temperatures recorded during the open period, when air movement inside the cave is governed by gravitational flow.
We can state that we received realistic results in Sect. 3, e.g. 2–3 m min\(^{-1}\) for inflowing air, 0.5 m min\(^{-1}\) for outflowing air, but the sampling rate limited the time resolution of the correlation analysis. This does not reduce the validity of our model, but clearly shows that the time resolution of the temperature data has to be chosen carefully. However even with rather large observation intervals we were able to characterize the general patterns of air movement and their slow temporal variations. Our suggestion would be to define shorter observation intervals in a future measuring campaign and then process the data with calcFLOW to get refined results.

Care has to be taken also in the selection of the positions for the temperature loggers to capture the airflow of interest. While we conducted numerous calculations to test the calcFLOW-method, we also figured out that – as an interesting side effect – we resolved several phenomena observed in the basic data analysis, which were not well understood before. For example, the different behavior of the temperatures recorded by the two loggers at Angermayerhalle lower part (T1) and Angermayerhalle upper part (T4) could be explained. While the first (T1) is recording the cold inflowing air, the other one (T4) records the warmed outflowing air, like we expected already before (Meyer, 2014). Moreover the former logger was found to be positioned to high above the floor to catch the inflowing air immediately, which the logger at Wasserstelle (T2) does with greater success. While T2 shows distinctive variations of rather short duration that clearly correspond to the temperature variations recorded by T1, the same variations are very much damped at T3 (Fuggerhalle). Here we record significantly warmed-up air with strongly damped variations due to the mixing of stagnant air with the already warmed-up inflowing air from outside.

The airflow regime at Schellenberger Eishöhle seems to be more complex than we expected. Like we described before we assume a major stream of inflowing cold external air from Angermayerhalle through Wasserstelle down to the deepest part and a major stream of outflowing warm air from the deepest part of the cave through Mörkdom passing logger T4 at Angermayerhalle upper part. So far we thought that all cold air is descending first to Fuggerhalle before the relatively warmer air is pushed out
along the ceiling towards Mörkdom. Depending on the airflow speed, with which the cold airstream arrives at the crossing point between Fuggerhalle and Mörkdom, it may be also possible that only a minor side stream descends to Fuggerhalle and the major stream flows directly towards Mörkdom. To prove this theory it would be necessary to install more air temperature loggers in the passage between Fuggerhalle and Mörkdom, but this is part of possible activities for future measurements.

In summary we can state that our simple linear model is able to explain diverse phenomena observed at Schellenberger Eishöhle, but also indicates new questions in the analysis. The additional information we gather with calcFLOW enables us to understand the airflow dynamics and the specific cave climate in more detail. Additionally, temperature biases and scaling factors for the temperature variations observed by the different loggers were determined. The values determined match the simple model of inflowing cold air that mixes with stagnant and relatively warm air inside the cave. Basically one can say that the warming of inflowing external air depends on the travel distance and travel time of the air inside the cave. The damping of the temperature variations due to continuous mixing of external and cave air also shows the dependency on travel length, i.e. stronger damping with longer travel distance and time. Both parameters agree well with the air movements determined by the correlation analysis and may be used to validate those results.

4 Conclusions

The method of cross-correlation we use for calcFLOW in general depends on rather distinctive temperature variations to successfully correlate the observations of different loggers. On the other hand the airflow speed is supposed to be relatively constant during the time span used for correlation. These two requirements contradict each other and it has to be shown by further studies to what extent the temporal variability of the air movements inside the cave may be resolved. Probably the reliability of the analysis will benefit from an increased sampling rate of the temperature observations. Regard-
less of the complexity of the situation at our test site we may state that the presented
method is well suited to uncover the complicated air movements in the cave. The re-
results of the analysis will help to optimize the placement of the loggers. An increased
number of loggers positioned near the floor as well as near the ceiling of the passages
will allow to distinguish the paths of the inflowing and outflowing air with much better
resolution and reliability. Decreased sampling intervals will enable the determination of
the speed of the rather fast inflowing cold air and generally improve the reliability of the
correlation analysis.

We already tested calcFLOW with air temperature data from Fossil Mountain Ice
Cave (USA), but these results will be part of future publications. What we can already
state for the moment is that calcFLOW is applicable to other ice caves, too. And this
is one major outcome of this pilot study and also a reason for us to keep the model
as simple as possible. We want to present a basic tool for cave climate studies, which
allows everyone to use it for their specific site. To summarize the outcome of this study,
we can say that calcFLOW is useful to:

1. characterize the airflow regime inside a static ice cave,
2. compute (interpolate) with one simple model, based on only 3 determined param-
eters, the temperature at any time and any place between two loggers,
3. indicate possible problems in the measuring set-up (e.g. position and height of
loggers), and
4. indicate useful observation intervals.

In a next step we will validate the calculated airflow speeds by comparison to real-
time airflow measurements in order to improve our model. With these results we will be
able to differentiate specific cave parts based on the cave climate and to determine the
influence of the cold air on the cave climate, e.g. for questions of energy exchange in
the cave. Moreover we would like to calculate the energy balance of the cave based on
the air exchange. For this attempt we would need a dense measuring network, which
records all important airstreams with a temporal resolution high enough to compute the airflow speeds. As the open periods of Schellenberger Eishöhle are temporally limited to some days/weeks during wintertime, we will continue our experiments with data from Fossil Mountain Ice Cave, which shows an open period of cold inflowing air of several months. This will give us the opportunity to calculate with a larger amount of data and to determine seasonal aspects in a better way.

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C. Meyer et al.

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C. Meyer et al.

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Figure 1. Observed temperatures (left panels) and correlation functions (right panels) during a period of large temperature variations, well suited for correlation analysis. Data of loggers T2, T3, or T4 are cross-correlated with the data of logger T1 using a correlation length of 101 (a, b), 51 (c, d) and 25 (e, f) samples. Correlation maxima are more distinctive the less samples are used.
Figure 2. Observed temperatures (left panels) and correlation functions (right panels) during a period of small temperature variations, apparently not so well suited for correlation analysis. Data of loggers T2, T3, or T4 are cross-correlated with the data of logger T1 using a correlation length of 101 (a, b), 51 (c, d) and 25 (e, f) samples. Despite only little temperature variations distinctive maxima are visible in (b), but the less samples are used the worse the correlation maxima are defined respectively are not visible anymore. Time shifts determined from (d) or (f) are meaningless or even wrong because of side maxima.
Figure 3. Map of German show caves with the position of Schellenberger Eishöhle (©Bundesamt für Kartographie und Geodäsie, www.bkg.bund.de).
Figure 4. Groundmap with positions of all used measuring points in Schellenberger Eishöhle.

Schellenberger ice cave
ice part- ground view
based on the survey of Fritz Eigert 1959
drawing: F. Seewald
digitization: Christiane Meyer
1: 200

Mörkdom*
Fuggerhalle
Angermayerhalle
Angermayerhalle
lower part
Achieve

* No data for the example epoch of our calculation.

air temperature measurements

T1
T2
T3
T4

Figure 4. Groundmap with positions of all used measuring points in Schellenberger Eishöhle.
Figure 5. A time span of 51 samples was used for correlation analysis and to adjust temperature biases and scale factors. For the correlation analysis raw data (a) of logger T1 (Angermayerhalle, inflowing) were shifted along the extended period displayed for loggers T2 (Wasserstelle), T3 (Fuggerhalle) and T4 (Angermayerhalle, outflowing) for best correlation (b). In a further step (c) temperature biases at logger T1 were applied to fit loggers T2, T3 or T4, before the temperature variations at logger T1 were scaled to fit loggers T2, T3 or T4 (d).
Figure 6. In this example the raw data of loggers T1, T2, T3 and T4 (a) were used. A time span of 51 samples was used for correlation analysis and to adjust temperature biases and scale factors. For the correlation analysis raw data of loggers T2 (Wasserstelle), T3 (Fuggerhalle) and T4 (Angermayerhalle, outflowing) were shifted in time along the extended period displayed for logger T1 (Angermayerhalle, inflowing) to be correlate (b). Furthermore temperature biases at loggers T2, T3 and T4 are applied (c) and temperature variations at loggers T2, T3 and T4 are scaled to fit those at logger T1 (d).
Figure 7. Temperature observations of all 4 loggers during the time period analyzed.
Figure 8. Value of best correlation (left panels) and corresponding time shift (right panels) for the three pairs of loggers T1 : T2 (top panels), T1 : T3 (middle panels) and T1 : T4 (bottom panels); for smoothing the moving mean of 5 samples was computed. Times of bad correlation correspond to periods or little temperature variations (compare to Fig. 7). A negative time shift may indicate a placement of T1 to high above the floor. The sudden drop after 2 February may be caused by a ceasing of the gravitational airflow due to rising outer temperatures. Slow airflow speeds on 27 January correspond to mild outside temperatures. On 2 February ventilation stops completely. Due to the strong damping of signal at T4 a long correlation length results in more plausible correlation. Sudden jumps around 29 January and 2 February are caused by wrong maxima of correlation.
Figure 9. Bias (left panels) and scale factors (right panels) for the three pairs of loggers T1 : T2 (top panels, T1 : T3 (middle panels) and T1 : T4 (bottom panels); for smoothing the moving mean of 5 samples was computed. Times of large biases correspond to periods or low outside temperatures (compare to Fig. 7) and consequently large temperature gradients. Scale factors for the logger pair T1, T2 are rather variable and probably strongly influenced by the short term variations visible in both time series. The jumps after 2 February are caused by a loss of correlation due to ceasing airflow. Strong damping of the signal (small scale factors) generally corresponds to low outside temperatures and consequently large temperature gradients. Jumps indicate problems in correlating both time series due to the strong damping of the variations at T4.
Figure 10. Standard deviations of bias (left panels) and scale factors (right panels) for the three pairs of loggers T1 : T2 (top panels), T1 : T3 (middle panels) and T1 : T4 (bottom panels). The short period variations of T1 and T2 seem to complicate the fit, but smoothing cures this problem. The determination of scale factors generally benefits significantly from a smoothing of the data. The fit is generally better for smaller numbers of samples (as can be expected), smoothing in (c) and (e) does not lead to better results because the short term variations farther in the cave are already much damped. The rise and jump on 2 February (c–f) indicate problems in the model due to the interruption in the open phase. The uncertainty of the fit increases with the longer distance between the loggers T1 and T4.
Figure 11. Significantly determined parameters, correlation length 101 samples.
Figure 12. Significantly determined parameters, correlation length 51 samples.