Study of the all orders multiplicative renormalizability of a local matter confining Gribov-Zwanziger action in the MAG

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Abstract

We address the issue of the all order multiplicative renormalizability of SU(2) Gribov-Zwanziger theories quantized in the maximal Abelian gauge in presence of confined matter fields. The non-linear character of the maximal Abelian gauge requires the introduction of quartic interaction terms in the Faddeev-Popov ghosts as well as in the localizing Zwanziger fields, extended a well known feature of this gauge. We show that, when scalar matter fields are introduced, a second quartic interaction term between scalar fields, Faddeev-Popov ghosts and Zwanziger-like fields naturally arises. A BRST invariant action accounting for those quartic interaction terms is identified and proven to be multiplicative renormalizable to all orders by means of the algebraic renormalization procedure.

1 Introduction

One of most important problems of modern physics is how to explain the colored fields confinement inside hadrons, i.e., the experimental fact that quarks and gluons have not been detected in isolation, but only as constituents of hadrons. Nowadays, the maximal Abelian gauge [65, 39, 38] is widely employed in order to investigate such a phenomena by means of isolation of physical relevant parameters in the infrared sector, namely this gauge turns out to be suitable for the study of the dual superconductivity mechanism for color confinement [43, 49, 64], according to which SU(N) Yang-Mills theories in the low energy region should be described by an effective U(1)\(_{N-1}\) Abelian theory [26, 63, 62, 35, 53] in the presence of monopoles. The condensation of these magnetic charges leads to a dual Meissner effect, where the QCD vacuum behave as a superconductor of chromo-magnetic current, compressing field lines of chromo-electric charge in a flux tube (string), resulting in quark confinement by generating a linear interquark potential, as Abrikosov' string is formed in a Cooper pairs medium. In particular, to avoiding unnecessary complications, when restricted to SU(2) YM theory, the Abelian configuration is identified with the diagonal components \(A_3^\mu\) of the gauge field corresponding to the diagonal generator of the Cartan subgroup of SU(2). The remaining off-diagonal components \(A_\alpha^\mu\), \(\alpha = 1, 2\), corresponding to the off-diagonal generators of SU(2), are expected to acquire a mass through a dynamical mechanism, thus decoupling at low energies. This phenomenon is known as Abelian dominance and is object of intensive
investigation, both from analytic and from numerical lattice simulations. Evidence for the dynamical mass generation for the off-diagonal components of the gauge field from the analytic side can be found in, while are devoted to numerical studies.

Besides being a renormalizable gauge, the maximal Abelian gauge enjoys the important property of exhibiting a lattice formulation, a property which allows to compare analytic and numerical results. In particular, this important feature of the maximal Abelian gauge has made possible the study, from the numerical lattice point of view, of the behaviour of the two-point gluon correlation function in the non-perturbative infrared region, providing evidence for the Abelian dominance as well as for the confining character of the propagator of the Abelian gluon component. This issue has also been addressed through analytical methods by taking into account the existence of the Gribov copies, which, as in any covariant and renormalizable gauge, affect the maximal Abelian gauge. Here, proceeding in a way similar to the Landau gauge, a few properties of the so-called Gribov region have been derived together with the restriction of the domain of integration in the functional integral to the Gribov horizon, see for instance refs. for the details of the Gribov issue on the maximal Abelian gauge. Remarkably, the agreement between the lattice numerical results and the analytic calculations based on the restriction to the Gribov region looks quite good, confirming the expectation that the study of the Gribov problem is of great relevance for gluon confinement.

Nevertheless, so far, the analytical study of the infrared aspects in the maximal Abelian gauge has been done only for the gluon sector, without including matter fields, i.e. spinor and scalar fields. This work focuses on continuing the analytic study of the non-perturbative behaviour of the matter fields in the maximal Abelian gauge starting in, along the lines recently outlined in the case of the Landau gauge, where it has been possible to recover the behaviour of the propagators for scalar and spinor fields observed in lattice simulations from an analytic point of view. This study is relevant in the MAG context because to extend the Abelian dominance affects to matter sector, make prediction for the propagator of scalars and quark fields which might be compared with lattice numerical simulations, study of the confining character of the correlation functions.

The first step in this endeavour was performed in, where the all orders multiplicative renormalizability of the SU(2) Yang-Mills theory fixing in the maximal Abelian gauge in presence of matter fields was established, a topic which, till then, has not yet been addressed. The goal of the present paper is to extend this proof to the case in which the Gribov problem is taking into account and a Gribov-like confinement mechanism for matter fields is implemented. Although the renormalizability of the maximal Abelian gauge in presence of the matter fields is an expected feature, we had noted that it is not a straightforward matter, requiring in fact a nontrivial analysis. This is due to the non-linear character of the maximal Abelian gauge which gives rise to a rather complex Faddeev-Popov operator. It was already pointed out that the structure of this operator requires the introduction of a quartic interaction between ghosts, only at the very end of the whole renormalization process the gauge parameter entering the quartic interaction can be set to zero, thus recovering the genuine maximal Abelian gauge condition. In, we had seen that this feature generalizes to the case of scalar matter fields, i.e. a quartic interaction between scalar fields and Faddeev-Popov ghosts naturally arises due to the non-linearity of the gauge condition. As a consequence, a second gauge parameter associated to this new term has to be introduced. As in the case of the quartic ghost term, this second gauge parameter can be set to zero only at the very end of the renormalization process.

Concerning the Gribov problem in the maximal Abelian gauge, although the situation cannot be compared to that of the Landau gauge, a few results are already available, where the analogous of Zwanziger’s horizon function as well as of the Gribov-Zwanziger action and of its refined version have been constructed. 

A study of the maximal
Abelian gauge within the context of the Schwinger-Dyson equations can be found in [36]. Gribov analysis starts noting that Faddeev-Popov quantization program is incomplete, because equivalent gauge fields configurations survive to gauge-fixing procedure as consequence of the presence of zero modes of the Faddeev-Popov operator [3 34 18], given by (12) for the SU(2) symmetry group. By restricting the integration in functional Feynman integral to the so-called Gribov region, where Faddeev-Popov operator is strictly positive, a large number of copies could be eliminated, as proved in [19]. In complete analogy to Landau case, can be proved that this restriction is equivalent to adding to the original Faddeev-Popov action the called horizon term [14] (see the next section for details), which is proportional to massive parameter $\gamma$, called Gribov parameter $\gamma^2$. Although horizon term is non-local, it can be cast in local form following the Zwanziger method by introducing a quartet of the so-called Zwanziger fields, resulting in a renormalizable Gribov-Zwanziger action [20]. As in Landau gauge [25 24], this auxiliary fields developed a non-trivial dynamics as consequence of infrared non-vanishing value of Gribov parameter [21]. In this scenario, off-diagonal gluons decoupling in low energy by generation of dynamical mass term due to condensation of dimension two gluon operator $A^a_\mu A^a_\mu$ [37 23], i.e., $\langle A^a_\mu A^a_\mu \rangle \sim m^2$, doing his propagators of the Yukawa type. Another two condensates arise for ghost fields and auxiliary fields needed to localize the horizon function. Dynamical mass associated with one of this new condensates, $\langle \bar{\phi}\phi - \bar{\omega}\omega - \bar{c}c \rangle \sim \mu^2$, introduce modifications of the Gribov-Stingl type in the diagonal gluon propagator, which attain non-vanishing value at zero momentum, $k^2 = 0$, and lacks the Källén-Lehmann spectral representation, can not be associated with the propagation of physical particles. This fact means that diagonal gluons are not physical excitations of the theory. Then, refinement GZ framework by taking into account the condensation of two dimensional operators with MAG fixing condition for pure Yang-Mills theories confirm the confining character of diagonal gluon propagators, as predicted by lattice simulations [1 2 45 44 29 30]. In this way, abelian dominance conjecture can be considerer as the analytical confining criterion emerging from fundamental Yang-Mills theory by taking into account the existence of Gribov copies when maximal Abelian gauge fixing is using in the quantization procedure of the theory.

In this work, we use the extension of this statement when matter fields are included, following the proposal of [17] for Landau gauge, according to which Faddeev-Popov operator coupling in universal way to any colored field. Thus, for all generic matter field $F^i$ in a given representation of SU(N) (with Latin indexes) specified by the generators $(T^a)^{ij}$, a non-local term similar to horizon function should be added to full action (vide also [50]), namely:

$$H_{\text{matter}}(G) = G g^2 \int d^4 x d^4 y F^i(x) (T^a)^{ij} (M^{-1})^{ab}(x,y) (T^b)^{jk} F^k(y)$$

(1)

proportional to generalized Gribov parameter, $G$, for matter sector. In MAG case, for adjoint representation, since Faddeev-Popov operator contains off-diagonal color index only, just diagonal matter fields can be coupling to this, as happens for gauge fields. In this sense, propagators should show abelian dominance. In fundamental representation, there is no qualitative distinction with the propagator in Landau case reported in [17]. Moreover, in the same way that Landau case, localizing Zwanziger fields for horizon matter term [21] develop a non-trivial dynamics associated to new Gribov-like parameter for horizon matter term which modifying the infrared behaviour of the propagators.

It is necessary to emphasize that recently in [4 5 13] was established BRST invariant formulation of the Gribov-Zwanziger theory, in particular in the maximal Abelian gauge [13], which is local, albeit nonpolynomial. This construction has allowed for a geometrical resolution of the Gribov problem in the class of the linear covariant gauges, however for the specific cases of Landau gauge and MAG, the invariant Gribov-Zwanziger formulation correspond to rewrite the Nakanishi-Lautrup sector in terms of a

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1 Notations and definitions are given in the Sec. [2].

2 This is a dynamical quantity determined by gap equation as function of coupling constant and invariant scale [19], being suppressed in UV sector. Thus, horizon functions affects the non-perturbative infrared behavior of gluodynamics only.
infinite power series of the divergence of the gauge field, a trivial change of variables in the path integral because the Jacobian is unitary \[4, 5, 13\]. At this sense, both the BRST-exact and no-exact formulations are equivalent via a reparametrization, \textit{ergo} they share the same formal properties. In particular, in \[9\] a BRST-exact horizon-like matter term was implemented following the aforementioned statement.

The aim of this work is to prove the important fact that a new horizon term as (1) in matter sector does not spoil the renormalizability of the theory. In \[21\] an all order multiplicative renormizability proof for SU(2) pure Yang-Mills theories is performed when Gribov issue is taking into account. The inclusion of Zwanziger fields for localizing the horizon term are controlled by existence of a new class of symmetries in this fields, allowing us to define a large set of Ward identities which guaranteed the algebraic renormalization proof. Moreover, in previous work \[13\] it was proven that Yang-Mills theories remain renormalizable when self-interacting scalar matter minimally coupled to gauge field is present. Inclusion of horizon term should not modify UV-sector since just introduce low-energy effects in the theory. In this paper we are going to show that a new symmetry arise too for localizing fields of horizon term for matter, in the same way that gauge sector, guaranteed a large set of Ward identities in analogy to identities involving Zwanziger fields for gauge sector. This fact will be taken as sufficient evidence that the renormalizability is not jeopardized by the inclusion of confining horizon terms, while the BRST-exact and no-exact equivalence via the Nakanishi-Lautrup reparametrization will be taken as sufficient evidence of the renormalization of the BRST-exact formulation given in \[9\].

The present work is organized as follows. In Sect. 2 we briefly discuss the maximal Abelian gauge condition and Gribov problem. In order to fix the YM action in the maximal Abelian gauge (MAG), we start by considering a SU(2)-Lie algebra valued gauge field \(A_\mu = A_\mu^a T^a\), where the algebra generators \(T^a\) \((a = 1, ..., 3)\)

\[
[T^a, T^b] = i\varepsilon^{abc} T^c
\]

are chosen to be anti-Hermitean and to obey the orthonormality condition \(\text{Tr} (T^a T^b) = \delta^{ab}\). Following \[65, 39, 58\], the gauge field can be decomposed into diagonal and off-diagonal components, namely

\[
A_\mu = A_\mu^a T^a + A_\mu T^3
\]

2 Matter confinement model

2.1 Maximal Abelian gauge condition and Gribov problem

To avoid unnecessary complications and with no loss of generality, for the rest of this paper, we restrict ourselves to the case of the gauge group \(SU(2)\). In order to fix the YM action in the maximal Abelian gauge (MAG), we start by considering a \(SU(2)\)-Lie algebra valued gauge field \(A_\mu = A_\mu^a T^a\), where the algebra generators \(T^a\) \((a = 1, ..., 3)\)

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\[
A_\mu = A_\mu^a T^a + A_\mu T^3
\]
where $\alpha = 1, 2$ and $T^3 \equiv T$ is the diagonal generator of the Cartan subgroup of $SU(2)$. Thus, the commutation relations (2) adopt the form
\[
\left[ T^\alpha, T^\beta \right] = i\epsilon^{\alpha\beta\gamma} T^\gamma, \\
\left[ T^\alpha, T \right] = -i\epsilon^{\alpha\beta\gamma} T^\beta, \\
\left[ T, T \right] = 0
\] (4)
where $\epsilon^{\alpha\beta} = \epsilon^{\alpha\beta3}$.

Analogously, we can express the Yang-Mills action as
\[
S_{YM} = \frac{1}{4} \int d^4x \left( F^\alpha_{\mu\nu} F^\alpha_{\mu\nu} + F_{\mu\nu} F^{\mu\nu} \right) \] (5)
by using the following explicit field strength decomposition
\[
F^\alpha_{\mu\nu} = D^{\alpha\beta}_{\mu} A^\beta_{\nu} - D^{\alpha\beta}_{\nu} A^\beta_{\mu} \\
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g\epsilon^{\alpha\beta} A^\alpha_{\mu} A^\beta_{\nu} \] (6)
with $D^{\alpha\beta}_{\mu}$ being the covariant derivative defined with respect to the Abelian component $3$, namely
\[
D^{\alpha\beta}_{\mu} = \delta^{\alpha\beta} \partial_\mu - g\epsilon^{\alpha\beta} A_\mu \] (7)
and which is left invariant under the following infinitesimal gauge transformations,
\[
\delta A^\alpha_{\mu} = -D^{\alpha\gamma}_{\mu} \xi^\gamma - g\epsilon^{\alpha\beta} A^\gamma_{\mu} \xi^\beta \] \hspace{1cm} (8a) \\
\delta A_\mu = -\partial_\mu \xi - g\epsilon^{\alpha\beta} A^\alpha_{\mu} \xi^\beta \] \hspace{1cm} (8b)

The maximal Abelian gauge condition amounts to impose that the off-diagonal components $A^\alpha_{\mu}$ of the gauge field obey the following nonlinear condition
\[
D^{\alpha\beta}_{\mu} A^\beta_{\mu} = 0 \] (9)
which follows by requiring that the auxiliary functional
\[
R[A] = \int d^4x A^\alpha_{\mu} A^\alpha_{\mu} \] (10)
is stationary with respect to the gauge transformations (8). Moreover, as it is apparent from the presence of the covariant derivative $D^{\alpha\beta}_{\mu}$, equation (9) allows for a residual local $U(1)$ invariance corresponding to the diagonal subgroup of $SU(2)$. This additional invariance has to be fixed by means of a further gauge condition on the diagonal component $A_\mu$, which is usually chosen to be of the Landau type, namely
\[
\partial_\mu A_\mu = 0 \] (11)

The Faddeev-Popov operator, $M^{\alpha\beta}$, corresponding to the gauge condition (9) is easily derived by taking the second variation of the auxiliary functional $R[A]$, being given by
\[
M^{\alpha\beta} = -D^{\alpha\gamma}_{\mu} D^{\beta\gamma}_{\mu} - g^2 \epsilon^{\alpha\gamma\epsilon\beta\omega} A^\gamma_{\mu} A^\omega_{\mu} \] (12)
It enjoys the property of being Hermitian and, as pointed out in [3], is the difference of two positive semidefinite operators given, respectively, by $-D^{\alpha\omega}_{\mu} D^{\beta\omega}_{\mu}$ and $g^2 \epsilon^{\alpha\omega\epsilon\beta\rho} A^\omega_{\mu} A^\rho_{\mu}$. It is worth to point out that the operator $M^{\alpha\beta}$ is non-linear in the gauge fields, a feature which has nontrivial consequences in the

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\[3\] From here, we use always the notation without colour super-index, $A_\mu = A^3_\mu$, for Abelian components.
renormalization process both in the case of the gauge and matter sector. The gauge fixed Yang-Mills action in the MAG is written as

$$S_{\text{MAG}}^{\text{FP}} = S_{\text{YM}} + \int d^4x \left\{ b^\alpha D^\mu_\alpha A_\mu^\beta - e^\alpha M_\alpha^\beta c^\beta + g e^{\alpha\beta} c^\alpha (D^\mu_\alpha A_\mu^\delta) c + b \partial_\mu A_\mu + c \partial_\mu (\partial_\mu c + g e^{\alpha\beta} A_\mu^\alpha c^\beta) \right\}$$

(13)

where \((c^\alpha, \bar{c}, e^\alpha, c)\) are the Faddeev-Popov ghosts and \((b^\alpha, b)\) are the Lagrange multipliers implementing the gauge conditions (9) and (11).

As any other covariant gauge, also the maximal Abelian gauge is plagued by the existence of Gribov copies, vide refs. [3, 14, 15] for explicit examples of zero modes of the Faddeev-Popov operator [12]. An analogous of the Gribov region of the Landau gauge can be introduced in the MAG by restricting the functional integral to the region where the Faddeev-Popov operator \(M_{\alpha^\beta}\) is strictly positive, \(i.e. M_{\alpha^\beta} > 0\), then a large number of copies could be eliminated, as proven in [19, 16]. Furthermore, in complete analogy with the case of the Landau gauge, this restriction is implemented by adding to the original Faddeev-Popov action (13) a non-local horizon term which, in the case of the maximal Abelian gauge, turns out to be given by the expression [19, 20, 21, 16]

$$H_{\text{MAG}}(A) = g^2 \int d^4x d^4y A_\mu(x) \varepsilon^{\alpha\beta} (M^{-1})^{\alpha\delta}(x, y) \varepsilon^{\delta\beta} A_\mu(y)$$

(14)

Therefore, for the analogous of the Gribov-Zwanziger action in the maximal Abelian gauge, we have

$$S_{\text{MAG}}^{\text{GZ}} = S_{\text{MAG}}^{\text{FP}} + \gamma^2 H_{\text{MAG}}(A)$$

(15)

where \(\gamma^2\) stands for the Gribov parameter of the maximal Abelian gauge. Proceeding as in the case of the Landau gauge, expression (15) can be cast in local form by introducing a pair of auxiliary bosonic fields, \((\varphi_\alpha^\mu, \varphi_\alpha^\beta)\), and a pair of auxiliary fermionic fields, \((\bar{\varphi}_\mu^\alpha, \bar{\varphi}_\mu^\beta)\), namely

$$S_{\text{MAG}}^{\text{GZ}} = S_{\text{MAG}}^{\text{FP}} + \int d^4x \left\{ \varphi_\alpha^\beta M^{\alpha\beta} \varphi^{\beta\delta} - \bar{\varphi}_\mu^\alpha \bar{\varphi}_\mu^\beta M^{\alpha\beta} \varphi^{\delta\beta} + \bar{\varphi}_\mu^\beta \bar{\varphi}_\mu^\beta + \bar{\varphi}_\mu^\beta \bar{\varphi}_\mu^\beta + g \gamma^2 \varepsilon^{\alpha\beta} (\varphi - \bar{\varphi})_{\mu}^\alpha A_\mu \right\}$$

(17)

As shown in [19, 20, 21, 16], the action \(S_{\text{MAG}}^{\text{GZ}}\) enables us to implement the restriction in the functional integral to the Gribov region \(\Omega_{\text{MAG}}\) of the maximal Abelian gauge, defined as

$$\Omega_{\text{MAG}} = \left\{ A_\mu^\alpha, A_\mu \mid \partial_\mu A_\mu = 0, D_\mu^\alpha A_\mu^\beta = 0, M_{\alpha^\beta}(A) = -\left( D_\mu^\delta D_\mu^\beta + 2 \varepsilon^{\alpha\sigma} \varepsilon^{\beta\delta} A_\mu^\alpha A_\mu^\delta \right) > 0 \right\}$$

(18)

Although the understanding of the Gribov issue in the maximal Abelian gauge cannot yet be compared to that reached in the Landau gauge, a few properties of the region \(\Omega_{\text{MAG}}\) have been already obtained. In particular, in [19], it has been established that \(\Omega_{\text{MAG}}\) is unbounded along the diagonal directions in field space. This feature seems to be consistent with the aforementioned Abelian dominance hypothesis, according to which the diagonal configurations, corresponding to the Abelian Cartan subgroup, should be the dominant configurations in the infrared. Moreover, in [32], it has been shown that when an Abelian configuration is gauge-transformed to the Landau gauge, it is mapped into a point of the boundary of the Gribov region \(\Omega\) of the Landau gauge, eq. (15), \(i.e.\) into a point of the Gribov horizon\(^5\). These features give further support to the restriction of the domain of integration to the region \(\Omega_{\text{MAG}}\).

\(^4\)Note the presence of the term

\[ F^{\alpha\beta} = 2 g \varepsilon^{\gamma\delta} \left( \partial_\mu c + g \varepsilon^{\alpha\beta} A_\mu^\delta c \right) D_\mu^{\gamma\delta} + g \varepsilon^{\alpha\beta} \partial_\mu \left( \partial_\mu c + g \varepsilon^{\gamma\delta} A_\mu^\delta c \right) - g^2 \left( \varepsilon^{\alpha\gamma} \varepsilon^{\beta\delta} - \varepsilon^{\alpha\delta} \varepsilon^{\beta\gamma} \right) A_\mu^\alpha \left( D_\mu^\gamma c + g \varepsilon^{\gamma\sigma} A_\mu^\sigma c \right) \]

(16)

in (17), which come from the trivial shift \(\bar{\varphi}_\mu^\alpha \rightarrow \bar{\varphi}_\mu^\alpha - M^{-1}_{\alpha^\delta} F_\mu^{\gamma\delta} \varphi_\mu^\gamma\), whose corresponding Jacobian is field-independent, and it is necessary to write the local \(\gamma\)-independent horizon term in a BRST-exact form, due to the transformation of the hidden \(\bar{A}_\mu\) field in the covariant derivative including in the Faddeev-Popov operator.

\(^5\)See Sect.V of [32].
We can now address the issue of the existence of a nilpotent non-perturbative BRST symmetry for the action (17). In [11] use has been made of non-local expressions for the gauge field and the infrared modified BRST operator in order to build-up a gauge-invariant horizon function in the MAG by generalizing the construction proposed in [4] for the Landau and linear covariant gauges. As it was turn out, the construction of the transverse gauge-invariant field \( A^h_\mu \) follows from the minimization of the Hilbert norm along the gauge orbit of a given configuration \( A_\mu \), which correspond to the minimal of the functional

\[
f_A[U] \equiv \text{Tr} \int d^4x A^U_\mu A^U_\mu
\]

where we shall require that both \( A^a_\mu \) and the local gauge transformations, \( U \in SU(N) \), are square-integrable, as showed in cf. [69, 40, 6, 4]. Making use of this configurations, we can rewrite expression (14) as

\[
H_{\text{MAG}}(A) = H_{\text{MAG}}(A^h) - F(A) \partial A - F^\alpha(A) \partial A^\alpha
\]

where we are using the short-hand notation \( F(A)(\partial A) = \int d^4xd^4y F(x,y)(\partial A)_y \) and \( F^\alpha(A)(\partial A^\alpha) = \int d^4xd^4y F^\alpha(x,y)(\partial A^\alpha)_y \), and \( F(A) \) stands for an infinite non-local power series of \( A_\mu \). The residual terms in \( F(A) \) can be reabsorbed by a harmless shift of the fields \( (b^a, b^h, b^h, \alpha) \); namely, by introducing the redefined Lagrange multipliers \( b^h, b^{h,\alpha} \) as following

\[
\begin{align*}
b^h &= b - \gamma^4 F(A) + \gamma^4 \int_{-\infty}^{x} dy^\mu \left( g\epsilon^{\alpha\beta} F^\alpha(A) A^{\beta}_{\mu} \right)_y \\
b^{h,\alpha} &= b^\alpha - \gamma^4 F^\alpha(A)
\end{align*}
\]

we can rewrite the action (15) as

\[
S_{\text{MAG}}^{\text{GZ}} = S_{\text{MAG}}^{\text{FP}}(b^h, b^{h,\alpha}) + \gamma^4 H_{\text{MAG}}(A^h)
\]

which enables us to write down an exact nilpotent non-perturbative BRST symmetry [11]. Notice that equations (21) correspond to a linear change of variables in the functional integral in the \( \delta \)-sector of the theory, thus corresponding to a trivial Jacobian.

The aim of the last analysis is that of establishing a equivalence between the Gribov-Zwanziger actions (15) and (22) via the reparametrizations (20) and (21). As mentioned previously, this means that both the actions share the same physical properties, and in particular its renormalizability. The action (15) suffers from a BRST soft symmetry breaking in the conventional context of the GZ framework, but can be embedded into a larger action so that, following the general lines of the procedure proposed in [70], the BRST symmetry can be restored by means of a suitable set of external sources. This process has the advantage of being much simpler than implementing the algebraic renormalization for a nonpolynomial model like (22), where the renormalization factors can be nonlinear, i.e. they can be power series, as in the linear covariant gauge case [10]. Of course, we avoid unnecessary complications by using the polynomial form (15) in the MAG case with no loss of generality.

In the next Subsection, we proceed to add scalar matter following the conjecture in [17] of universal coupling of the Faddeev-Popov operator to any coloured field in order to universalize the confining character of the Gribov-Zwanziger construction.

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6Mathematical features are not relevant at the present discussion, since the change of variables implemented so far has a illustrative character; the reference [11] is remitted for technical details about the \( A^h \) construction.

7Note that at this case we are forced to use the nonpolynomial form in order to implement the Gribov-Zwanziger framework for this class of gauges.
2.2 Confinement action for scalar matter in the adjoint representation

Considering the case which scalar matter SU(2)–valued, $\phi = \phi^a T^a$, is present, then we added a Klein-Gordon action with self-interaction in the adjoint representation minimally coupled to gauge field with the full covariant derivative, i.e., the coupling term in the covariant derivative include diagonal and off-diagonal components of gauge field. For consistency, we perform the Cartan decomposition of matter field too, i.e. we decompose the field into off-diagonal and diagonal components,

$$\phi^a T^a = \phi^a T^a + \phi T^3$$

The corresponding matter action is given by

$$S_{\text{scalar}} = \int d^4 x \left( \frac{1}{2} (D_{\mu} \phi^a)^2 + \frac{m_\phi^2}{2} \phi^a \phi^a + \frac{\lambda}{4!} (\phi^a \phi^a)^2 \right)$$

$$= \int d^4 x \left\{ (\partial_\mu \phi^a)(\partial_\mu \phi^a) + (\partial_\mu \phi)(\partial_\mu \phi) - 2g^2 \varepsilon^{\alpha \beta} \left[ (\partial_\mu \phi^a) \phi^a A^\beta_\mu - (\partial_\mu \phi^a) \phi A^\beta_\mu \right] + g^2 \left[ A_{\mu A}^a A^a_\mu \left( \phi^a \phi^a + \phi \right) + A_\mu A_\lambda \phi^a \phi^a - A_{\mu \beta} A^a_\mu \phi^a \phi^a - 2A_{\alpha \beta} A_{\mu} A_\beta \phi^a \phi^a \right] + \frac{m_\phi^2}{2} \phi^a \phi^a + \frac{m_\phi^2}{2} \phi_{\text{diag}} \phi_{\text{diag}} + \frac{\lambda}{4!} \left[ (\phi^a \phi^a)^2 + 2\phi^2 \phi^a \phi^a + \phi^4 \right] \right\}$$

The classical action ($S_{\text{YM}} + S_{\text{scalar}}$) is left invariant by the gauge transformations

$$\delta A^a_\mu = -D^a_\mu \omega^\beta - g\varepsilon^{a \beta} A^\beta_\mu \omega$$

$$\delta \phi^a = -\partial_\mu \phi^a - g\varepsilon^{a \beta} A^\beta_\mu \omega$$

and

$$\delta \phi = -g\varepsilon^{a \beta} \phi^a \omega^\beta$$

In full analogy with the pure gluon sector of the Gribov-Zwanziger action, a non-local non-perturbative term is included for matter following the general prescription proposed in [17]. By using (1), the confining character for the matter sector is implemented by adding a horizon term in the matter sector with the inverse of Faddeev-Popov operator coupled only to Abelian component of matter field in the adjoint representation

$$S_{\text{matter}} = S_{\text{scalar}} + \sigma^4 H_{\text{matter}}(\phi)$$

where

$$H_{\text{matter}}(\phi) = g^2 \int d^4 x d^4 y \epsilon^{\alpha \beta} \phi(x)(\mathcal{M}^{-1})^{\alpha \gamma}(x,y)\epsilon^{\gamma \beta} \phi(y)$$

In this case, the parameter $\sigma$ would be the Gribov parameter for matter sector, in analogy to gauge sector case. The horizon-type function for matter can be cast in local form by introducing one pairs of bosonic fields, $(\bar{\eta}, \eta)^{a \beta}_\mu$, and other of anti-commutating fields, $(\bar{\theta}, \theta)^{a \beta}_\mu$:

$$H_{\text{matter}}^{\text{local}} = S_{\bar{\eta} \theta} + S_{\sigma}$$

$$= \int d^4 x \left\{ \bar{\eta}^{a \beta} \mathcal{M}^{\alpha \gamma} \eta^{\gamma \beta} - \bar{\theta}^{a \beta} \mathcal{M}^{\alpha \gamma} \theta^{\gamma \beta} + \bar{\theta}^{a \gamma} \mathcal{F}^{\alpha \beta} \eta^{\gamma \beta} + \sigma^2 g\varepsilon^{a \beta} (\eta - \bar{\eta})^{a \beta} \phi \right\}$$
Finally, the complete physical action is given by
\[
S_{\text{phys}} = S_{\text{YM}} + S_{\text{MAG}} + S_{\phi \omega} + S_{\text{matter}} + S_{\bar{\theta} \theta} + S_{\gamma}
\]
\[
= \lim_{\alpha, \xi \to 0} \int d^4x \left\{ \frac{1}{4} (F_{\mu \nu})^2 + \frac{\alpha}{2} (g^\gamma)^2 + \frac{\xi}{2} b^2 + b^\alpha \partial_{\mu} A_\mu - c^\alpha \partial_{\mu} A_\mu + g \varepsilon^{\alpha \beta} \varepsilon^{\gamma \delta} (D_\mu A_\mu)^2 \right\}
\]
+ \bar{c} \partial_{\mu} (\partial_{\nu} c + g \varepsilon^{\alpha \beta} A_\mu c) + \varphi_{\mu} M^{\alpha \beta} \varphi_{\beta} - \omega_{\mu} \omega_{\mu} + g \gamma^{2} \varepsilon^{\alpha \beta} (\varphi - \varphi_{\mu} A_\mu)
\]
+ \left( \partial_{\mu} \varphi_{\beta} \right)^2 + \frac{\lambda}{4!} (\varphi_{\mu} \varphi_{\nu})^2 + \bar{\eta} \omega M^{\alpha \beta} \bar{\varphi} \omega - \bar{\varphi} \omega M^{\alpha \beta} \varphi \omega + g \sigma^{2} \varepsilon^{\alpha \beta} (\eta - \bar{\eta})^{\alpha \beta} \phi \right) \] (30)

The BRST variations of the Faddeev-Popov fields can be logically extended for all the Zwanziger(-type) localizing fields to remains nilpotent \( (s^2 = 0) \)

\[
s A_\mu = -(\partial_{\mu} c + g \varepsilon^{\alpha \beta} A_\mu c) , \quad s A_\mu = -(\partial_{\mu} c + g \varepsilon^{\alpha \beta} A_\mu c)
\]
\[
s c = g \varepsilon^{\alpha \beta} \varepsilon^{\gamma \delta} c , \quad s c = g \varepsilon^{\alpha \beta} \varepsilon^{\gamma \delta} c , \quad s c = b , \quad s c = b , \quad s b = s b = 0
\]
\[
s \phi = g \varepsilon^{\alpha \beta} (\phi c - \phi c) , \quad s \phi = -g \varepsilon^{\alpha \beta} \phi c c
\]
\[
s \bar{\varphi}_{\mu} = \varphi^\alpha \ , \quad s \varphi^\alpha = 0 , \quad s \varphi^\alpha = \omega^\alpha , \quad s \omega^\alpha = 0
\]
\[
s \bar{\eta}^\alpha = \bar{\eta}^\alpha , \quad s \bar{\eta}^\alpha = 0 , \quad s \bar{\eta}^\alpha = 0 , \quad s \bar{\eta}^\alpha = 0 \] (31)

### 3 Extended BRST-exact form of physical action for scalar matter

Here we would like to prove the renormalizability of the of full physical action (30). In order to achieve this aim, the first step is note that (30) exhibits a soft breaking of the BRST symmetry due to the presence of the Gribov (and Gribov-like) parameter(s), namely

\[
s S_{\text{phys}} = -g \gamma^{2} \varepsilon^{\alpha \beta} \int d^4x \left\{ \left[ \varphi^\alpha (\partial_{\mu} c + g \varepsilon^{\gamma \omega} A_\mu c) - \omega^\alpha A_\mu \right] + \left[ \eta^\beta (g \varepsilon^{\omega \gamma} \varphi^\alpha c) - \theta^\beta \phi \right] \right\} \] (32)

where \( s \) denoting the nilpotent BRST transformations (31). In order to restore the BRST symmetry, we embed (30) into a larger BRST-exact action, following the method introduced by Zwanziger in (70). The detailed construction of this general action follows the same steps described in (20, 21, 17, 13, 12) and we need to keep in mind that the physical action is re-obtained from the extended action, denoted as \( \Sigma \), when the set of external sources and parameters attain their physical values, \( i.e.\)

\[
\Sigma |_{\text{phys}} = S_{\text{phys}} \] (33)

Let us introduce the following two BRST quartet of external sources

\[
s N_{\mu \nu}^{\alpha \beta} = -M_{\mu \nu}^{\alpha \beta} , \quad s M_{\mu \nu}^{\alpha \beta} = 0 , \quad s M_{\mu \nu}^{\alpha \beta} = N_{\mu \nu}^{\alpha \beta} , \quad s N_{\mu \nu}^{\alpha \beta} = 0
\]
\[
s U^{\alpha \beta} = -V^{\alpha \beta} , \quad s V^{\alpha \beta} = 0 , \quad s V^{\alpha \beta} = U^{\alpha \beta} , \quad s U^{\alpha \beta} = 0 \] (34)

where \( (N_{\mu \nu}^{\alpha \beta}, N_{\mu \nu}^{\alpha \beta}), (V^{\alpha \beta}, V^{\alpha \beta}) \) are anti-commuting sources and \( (M_{\mu \nu}^{\alpha \beta}, M_{\mu \nu}^{\alpha \beta}), (U^{\alpha \beta}, U^{\alpha \beta}) \) are commutating. The sources in (34) are necessary to restore the BRST invariance of the model which is broken when the parameters \( \gamma^2 \) and \( \alpha^2 \) are introduced. Notice that this symmetry restoration is only possible because the breaking in (32) is soft (70). We can write now an BRST exact action with a following sources
and its related sources can contract with the multi-indices, then, one can write that

The term $S_{\text{inv}}$ when the external sources attain their physical values

We call a particular attention to the new BRST-exact action, given by

which displays two new global symmetries, the U(8) for pure gauge sector and the U(2) for pure matter sector, meaning that we make use of the composite indixes, or multi-indixes, $I, J, K, \cdots \equiv \{\alpha, \mu\} = 1, \ldots, 8$ and the indexes $i, j, k, \cdots \equiv \alpha = 1, 2$, for global U(8) and U(2) symmetries, respectively. The last is not really a composite index, but usual off-diagonal color index, and is done just to standardize the notation. The indexes $I, J, K, \ldots$ provide that contractions like $\bar{\varphi}_\mu^\alpha \varphi_\mu^\beta$ could be written as $\bar{\varphi}_I^\alpha \varphi_I^\beta$ and forbid contractions like $A_\mu^\alpha A_\nu^\beta \delta_{\mu\nu} \bar{\varphi}_\mu^\alpha \varphi_\mu^\beta$. In fact, the U(8) symmetry provides that only the Zwanziger fields and its related sources can contract with the multi-indixes, then, one can write that

The U(8) symmetry is broken when the sources attain their physical values. Analogously, the U(2) symmetry provides special contractions among the localizing matter sector fields and its related sources. For example, contractions like $\bar{\eta}^\alpha \eta^\beta \equiv \bar{\eta}_I^\alpha \eta_I^\beta$ are allowed while contractions like $\bar{\varphi}_I^\alpha A_\mu^\beta \partial_\mu \bar{\varphi}_I^\alpha \varphi^\beta$ or $\bar{\varphi}_I^\alpha \partial_\mu \eta^\alpha \eta^\beta$ are forbidden. Thus one can write that

The U(2) symmetry is also broken in the physical limit. This two global symmetries can be expressed in a functional form by

in such a way that the trace of the operators $Q_{IJ}$ and $Q_{ij}$ define new additional quantum number in the auxiliary localizing Zwanziger(-like) sector, the $Q_5$ and $Q_2$ charge. The corresponding values of this charge for each field and source are summarized in the Tables in the end this section.

Besides, notice that the BRST transformations of the gauge, ghost and matter fields in are non-linear. In implementation of algebraic renormalization procedure, we need to properly take into account

---

\[ Q_{IJ} (\Sigma_0) = 0, \quad Q_{ij} (\Sigma_0) = 0 \]
the corresponding composite operators, a task which is achieved by introducing a set of external sources coupled to the non-
linear BRST transformations

$$\Sigma_{\text{ext}}^{(1)} = s \int d^4x \left\{ -\Omega^\alpha_{\mu} A^\alpha_{\mu} - g \varepsilon^{\alpha\beta} \chi^\alpha \phi^\beta c - \Omega_{\mu} A^\alpha_{\mu} + L^\alpha c^\alpha + Lc - \bar{\Omega}^\alpha \phi^\alpha - g \varepsilon^{\alpha\beta} \bar{\xi}^\alpha \phi^\beta c - \bar{\Omega} \phi \right\}$$

$$= \int d^4x \left\{ -\Omega^\alpha_{\mu} \tau^\alpha_{\mu} c^\beta - g \varepsilon^{\alpha\beta} \tau^\alpha_{\mu} A^\beta_{\mu} c + \xi^\alpha_{\mu} \left[ g \varepsilon^{\alpha\beta} \left( D^\beta_{\mu} c^\alpha \right) c - \frac{g^2}{2} \varepsilon^{\alpha\beta} \varepsilon^{\omega\rho} A^\beta_{\mu} c^\omega c^\rho \right] \right.$$

$$- \Omega_{\mu} \left( \partial^\mu c + g \varepsilon^{\alpha\beta} A^\alpha_{\mu} c^\beta \right) + g \varepsilon^{\alpha\beta} L^\alpha c^\beta c + \frac{g^2}{2} \varepsilon^{\alpha\beta} Lc^\alpha c^\beta + \bar{\Omega}^\alpha \varepsilon^{\alpha\beta} \phi^\beta c^\beta - g \varepsilon^{\alpha\beta} \bar{\tau}^\alpha \phi^\beta c^\beta$$

$$- g^2 \varepsilon^{\alpha\beta} \bar{\xi}^\alpha \left( \varepsilon^{\beta\omega} \phi^\omega c^\alpha + \frac{1}{2} \varepsilon^{\omega\rho} \phi^\beta c^\omega c^\rho \right) - \bar{\Omega} g \varepsilon^{\alpha\beta} \phi^\alpha c^\beta \right\}$$

(41)

where to guarantee the BRST invariance we require that all sources are \(s\)-invariants except for

$$s \xi^\alpha_{\mu} = - (\Omega^\alpha_{\mu} - \tau^\alpha_{\mu}), \quad s \xi^\alpha_{\mu} = - (\bar{\Omega}^\alpha_{\mu} - \bar{\tau}^\alpha_{\mu})$$

(42)

and thus providing the Slavnov-Taylor identity. After the renormalization process they can be taken do

zero. Furthermore, as was shown in [20], the gauge sector action is left invariant by the following set of transformations:

- \(\delta_I\)-symmetry

$$\delta_I c^\alpha = \varphi^\alpha, \quad \delta_I \varphi^\alpha = \delta_I c^\alpha, \quad \delta_I b^\alpha = g \varepsilon^{\alpha\beta} \varphi^\beta c, \quad \delta_I \Omega^\alpha_{\mu} = M^\alpha_{\mu I}$$

(43)

- \(\bar{\delta}_I\)-symmetry

$$\delta_I c^\alpha = \omega^\alpha_I, \quad \delta_I \omega^\alpha_I = - \delta_I c^\alpha, \quad \delta_I b^\alpha = g \varepsilon^{\alpha\beta} \omega^\beta_I c, \quad \delta_I \Omega^\alpha_{\mu} = - N^\alpha_{\mu I}$$

(44)

- \(d_I\)-symmetry

$$d_I c^\alpha = \omega^\alpha_I + g \varepsilon^{\alpha\beta} \varphi^\beta c, \quad d_I \varphi^\alpha = \delta_I g e^{\alpha\beta} c^\beta, \quad d_I b^\alpha = g \varepsilon^{\alpha\beta} \omega^\beta_I c + \frac{g^2}{2} \varepsilon^{\alpha\beta} \varepsilon^{\omega\rho} \varphi^\beta c^\omega c^\rho, \quad d_I \omega^\alpha_I = \delta_I c^\alpha, \quad d_I \Omega^\alpha_{\mu} = N^\alpha_{\mu I}, \quad d_I \xi^\alpha_{\mu} = - M^\alpha_{\mu I}$$

(45)

- \(\bar{d}_I\)-symmetry

$$\bar{d}_I c^\alpha = - \varphi^\alpha + g \varepsilon^{\alpha\beta} \omega^\beta_I, \quad \bar{d}_I \varphi^\alpha = \delta_I g e^{\alpha\beta} c^\beta, \quad \bar{d}_I b^\alpha = - g \varepsilon^{\alpha\beta} \varphi^\beta c + \frac{g^2}{2} \varepsilon^{\alpha\beta} \varepsilon^{\omega\rho} \omega^\beta_I c^\omega c^\rho, \quad \bar{d}_I \omega^\alpha_I = - \delta_I c^\alpha, \quad \bar{d}_I \Omega^\alpha_{\mu} = - M^\alpha_{\mu I}, \quad \bar{d}_I \xi^\alpha_{\mu} = \bar{N}^\alpha_{\mu I}$$

(46)

This symmetries are very important for guaranteed perturbative renormalizability, since allows to
control with respect to Zwanziger auxiliary fields through the existence of a set of associated Ward
identities. A remarkable fact is the existence of a new set of symmetries for auxiliary fields in the matter
sector in perfect analogy to symmetries above, given by:

- \(\delta_I\)-symmetry

$$\delta_I c^\alpha = \eta^\alpha_i, \quad \delta_I \eta^\alpha_i = \delta_{ij} c^\alpha, \quad \delta_I b^\alpha = g \varepsilon^{\alpha\beta} \eta^\beta_i c, \quad \delta_I \Omega^\alpha_{i} = V^\alpha_i$$

(47)

- \(\bar{\delta}_I\)-symmetry

$$\bar{\delta}_I c^\alpha = \bar{\eta}^\alpha_i, \quad \bar{\delta}_I \eta^\alpha_i = - \delta_{ij} c^\alpha, \quad \bar{\delta}_I b^\alpha = g \varepsilon^{\alpha\beta} \bar{\eta}^\beta_i c, \quad \bar{\delta}_I \Omega^\alpha_{i} = - U^\alpha_i$$

(48)
\begin{itemize}
\item \(d_{t}\)-symmetry
\[
d_{t}c^{\alpha} = \theta_{1}^{\alpha} + g\varepsilon^{\alpha\beta}\eta_{1}^{\beta}c, \quad d_{t}\eta_{1}^{\alpha} = \delta_{ij}g\varepsilon^{\alpha\beta}\bar{c}^{\beta}c, \quad d_{t}\bar{b}^{\alpha} = g\varepsilon^{\alpha\beta}\bar{\theta}_{1}^{\beta}c + \frac{g^{2}}{2}\varepsilon^{\alpha\beta\gamma\delta}\eta_{1}^{\beta}\varepsilon^{\omega\rho}\eta_{1}^{\gamma}\varepsilon^{\omega\rho}c, \quad d_{t}\bar{\theta}_{1}^{\alpha} = \delta_{ij}\bar{c}^{\alpha}, \quad d_{t}\bar{\Omega}^{\alpha} = U_{1}^{\alpha}, \quad d_{t}\bar{\xi}^{\alpha} = -V_{1}^{\alpha}
\]
\item \(\bar{d}_{t}\)-symmetry
\[
\bar{d}_{t}c^{\alpha} = -\eta_{1}^{\alpha} + g\varepsilon^{\alpha\beta}\bar{\theta}_{1}^{\beta}c, \quad \bar{d}_{t}\theta_{1}^{\alpha} = \delta_{ij}g\varepsilon^{\alpha\beta}\eta_{1}^{\beta}c, \quad \bar{d}_{t}\bar{b}^{\alpha} = -g\varepsilon^{\alpha\beta}\eta_{1}^{\beta}c + \frac{g^{2}}{2}\varepsilon^{\alpha\beta\gamma\delta}\theta_{1}^{\beta}\varepsilon^{\omega\rho}\bar{c}^{\gamma}c, \quad \\
\bar{d}_{t}\eta_{1}^{\alpha} = -\delta_{ij}c^{\alpha}, \quad \bar{d}_{t}\bar{\Omega}^{\alpha} = -\bar{V}_{1}^{\alpha}, \quad \bar{d}_{t}\bar{\xi}^{\alpha} = \bar{U}_{1}^{\alpha}
\end{itemize}

This will ensure the control on Zwanziger matter field renormalizability, in the same way as for the gauge sector. In particular, permiten controlar a new set of quartic terms permitted by power counting. In [21] was shown that in addition to quartic terms in Nakanishi-Lautrup and ghost fields [47 [27 [31] which arise of nonlinearity of MAG condition, new terms in Zwanziger fields appear. The \(\delta-\) and \(d-\) symmetries require the choice of the same gauge parameter for all. However, the inclusion scalar matter interacting with the gauge field introduce in the action generate a new class of UV-divergent Feynman diagrams, as showed in a previus work [13]. Hence, we need to add another quartic terms in a BRST invariant fashion in order to renormalizes these new divergences, including the localizing fields of the horizon function of matter sector too, such that satisfies all original symmetries of gauge sector and the new \(\delta-\) and \(d-\) symmetries of matter sector. After a simple algebra, we obtain that the most general term necessary to deal with the new divergences which obey the full set Ward identities displays in the next section, is given by

\[
\Sigma_{\text{qua}} = \alpha s \int d^{4}x \left\{ c^{\alpha}b^{\alpha} - g\varepsilon^{\alpha\beta}c^{\alpha}c^{\beta}c - 2g^{2}c^{\alpha}c^{\beta}c^{\gamma} + 2g^{2}c^{\alpha}c^{\beta} \left( \varphi^{\beta}_{1}\omega_{1}^{\alpha} + \eta_{1}^{\alpha}\bar{\theta}_{1}^{\beta} \right) + g\varepsilon^{\alpha\beta\gamma\delta}\eta_{1}^{\beta}\varepsilon^{\omega\rho}\eta_{1}^{\gamma}\varepsilon^{\omega\rho}c \right\} + \\
+\beta s \int d^{4}x \left\{ c^{\alpha}b^{\alpha} + g\varepsilon^{\alpha\beta}c^{\alpha}c^{\beta}c + 2g^{2}c^{\alpha}c^{\beta}c^{\gamma} - 2g^{2}c^{\alpha}c^{\beta}c^{\gamma}c - g^{2}c^{\alpha}c^{\beta}c^{\gamma}c^{\delta} \left( \varphi^{\beta}_{1}\omega_{1}^{\alpha} + \eta_{1}^{\alpha}\bar{\theta}_{1}^{\beta} \right) + g\varepsilon^{\alpha\beta\gamma\delta}\eta_{1}^{\beta}\varepsilon^{\omega\rho}\eta_{1}^{\gamma}\varepsilon^{\omega\rho}c \right\} = \\
\alpha \int d^{4}x \left\{ b^{\alpha}b^{\alpha} - 2g\varepsilon^{\alpha\beta}b^{\alpha}c^{\beta} - g^{2}b^{\alpha}c^{\beta}c^{\gamma} + 2g^{2}b^{\alpha}c^{\beta}c^{\gamma}c^{\delta} \right\} + \\
+\beta \int d^{4}x \left\{ g\varepsilon^{\alpha\beta}c^{\alpha}c^{\beta}c^{\gamma}c^{\delta} + g\varepsilon^{\alpha\beta}c^{\alpha}c^{\beta}c^{\gamma}c^{\delta} + g^{2}c^{\alpha}c^{\beta}c^{\gamma}c^{\delta} \right\} = \left(51\right)
\]

where arise an second gauge-type parameter, \(\beta\). All the terms in \(51\) are proportional to the parameters \(\alpha\) or \(\beta\), and they are arise due to the effect of the nonlinearity of the off-diagonal gauge condition, \(\partial_{\mu}A_{\alpha}^{\mu} = g\varepsilon^{\alpha\beta}A_{\mu}^{\alpha}A_{\mu}^{\beta}\). Such nonlinearity generates extra interaction vertices\(^9\) that give rise to quartic ghost interaction terms. Then, for renormalization purposes, these parameters need to be introduced. A notable fact is that (\(\ldots\)). For a detailed discussion on this subject we refer [13]. In the present case these

\(^9\)When compared to the Landau and linear covariant gauges.
extra interaction terms are generalized to include the Zwanziger ghosts \( \{ \varphi, \bar{\varphi}, \omega, \bar{\omega} \} \) and the localizing auxiliary fields of the scalar matter sector \( \{ \eta, \bar{\eta}, \theta, \bar{\theta} \} \) in such a way to preserve the identities (53–54). Looking in the equation of motion for the off-diagonal Nakanishi-Lautrup field

\[
\frac{\delta S}{\delta b_{\alpha}} = D_{\mu}^{\alpha \beta} A_{\mu}^{\beta} + \alpha \left( b^\alpha - g \varepsilon^{\alpha \beta} \bar{c}^\beta - 2g^2 c^\alpha (\varphi^I_\alpha \bar{c}_I^\beta + \eta^I_\alpha \bar{\theta}_I^\beta) \right) + \frac{\beta}{2} g \varepsilon^{\alpha \beta} \bar{\phi} \phi^{\beta} \tag{52}
\]

we see that the original maximal Abelian gauge condition is recovered in the limit \( \alpha, \beta \to 0 \). New composite operators contained in these symmetries are adding in a renormalizable fashion using two sets of BRST doublets of external sources \( (X^I_\alpha, Y^I_\alpha) \) and \( (\tilde{X}^I_\alpha, \tilde{Y}^I_\alpha) \) for gauge sector, and two BRST doublets for matter sector, \( (\bar{X}^\alpha_\alpha, \bar{Y}^\alpha_\alpha) \) and \( (\tilde{X}^\alpha_\alpha, \tilde{Y}^\alpha_\alpha) \), according to

\[
\Sigma^{(2)}_{\text{ext}} = s \int d^4 x \left\{ X^I_\alpha \varphi^I_\alpha c + Y^I_\alpha \bar{c}^I_\alpha c + \tilde{X}^\alpha_\alpha \bar{c}^\alpha c + \tilde{Y}^\alpha_\alpha \bar{c}^\alpha c \right\} = g \varepsilon^{\alpha \beta} \left( \varphi^I_\alpha \bar{c}^I_\alpha c + \tilde{X}^\alpha_\alpha \bar{c}^\alpha c + \tilde{Y}^\alpha_\alpha \bar{c}^\alpha c - \tilde{Y}^\alpha_\alpha \bar{c}^\alpha c \right) \tag{53}
\]

where the \( s \)-transformations for the new set of external sources for composite operators is

\[
s Y^\alpha_\alpha = X^I_\alpha, \quad X^I_\alpha = 0, \quad s \tilde{X}^\alpha_\alpha = -\tilde{Y}^\alpha_\alpha, \quad \tilde{Y}^\alpha_\alpha = 0; \quad s X^I_\alpha = \tilde{X}^\alpha_\alpha, \quad X^I_\alpha = 0, \quad s \tilde{Y}^\alpha_\alpha = -\tilde{Y}^\alpha_\alpha, \quad \tilde{Y}^\alpha_\alpha = 0 \tag{54}
\]

We emphasize that the sources in (53) have the same role of the BRST external sources, but in this case, they are necessary to take into account nonlinear transformations (43–50). Quantum numbers for all fields and sources are displayed in the Tables 1, 2 and 3. Thus, finally, the full local and BRST invariant action for Yang-Mills theory with scalar matter with horizon function, \( \Sigma \), is given by

\[
\Sigma = \int d^4 x \left\{ \frac{1}{4} \left( F_{\mu \nu} F^{\mu \nu} + F_{\mu \nu} F^{\mu \nu} \right) + b^\alpha D_{\mu}^{\alpha \beta} A_{\mu}^{\beta} - \varepsilon^\alpha M^{\alpha \beta \epsilon} \bar{c}^\beta + g \varepsilon^{\alpha \beta} \bar{c}^\beta c \bar{D}_{\mu}^{\alpha \beta} A_{\mu}^{\beta} + b \theta_{\mu} A_{\mu} \right. \\
+ c \partial_{\mu} \left( \partial^\alpha c + g \varepsilon^{\alpha \beta} A_{\beta}^\alpha c \right) + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta \\
+ M_{\mu}^{\alpha \beta} \partial_{\mu} \phi^\beta + M_{\mu}^{\alpha \beta} \partial_{\mu} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta + \bar{c} \partial_{\mu} M^{\alpha \beta \epsilon} \phi^\beta \\
+ N_{\mu}^{\alpha \beta} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta \\
\left. + (\partial_{\mu} \phi^\alpha \partial_{\nu} \phi^\beta) + (\partial_{\mu} \phi^\alpha \partial_{\nu} \phi^\beta) - 2g^2 \varepsilon^{\alpha \beta} \left\{ (\partial_{\mu} \phi^\alpha) A_{\mu}^\alpha - (\partial_{\mu} \phi^\alpha) A_{\mu}^\alpha + (\partial_{\mu} \phi^\alpha) A_{\mu}^\alpha \right\} \\
+ g^2 A_{\mu}^{\alpha \beta} A_{\mu}^{\alpha \beta} (\phi^\alpha \phi^\beta + \phi^\beta \phi^\alpha) + A_{\mu} A_{\mu} A_{\mu} A_{\mu} (\phi^\alpha \phi^\beta + \phi^\beta \phi^\alpha - 2A_{\mu} A_{\mu} A_{\mu} A_{\mu} \phi^\alpha \phi^\beta) \\
+ \frac{m_0^2}{2} (\phi^\alpha \phi^\alpha + \phi^\beta \phi^\beta) + \lambda \left\{ (\phi^\alpha \phi^\alpha)^2 + 2\phi^\alpha \phi^\beta \phi^\alpha \phi^\beta + \phi^4 \right\} + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta \\
+ g \varepsilon^{\alpha \beta} (U^{\alpha \omega} \phi^\beta + U^{\alpha \omega} \phi^\beta - g \varepsilon^{\alpha \beta} \phi^\beta c \phi^\beta c) + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta + \bar{c} \partial_{\mu} \bar{c}^\beta \\
\right. \]

We emphasize that the sources in (53) have the same role of the BRST external sources, but in this case, they are necessary to take into account nonlinear transformations (43–50). Quantum numbers for all fields and sources are displayed in the Tables 1, 2 and 3. Thus, finally, the full local and BRST invariant action for Yang-Mills theory with scalar matter with horizon function, \( \Sigma \), is given by
\[ + g^2 \phi \phi^\alpha (\varepsilon^{\alpha \rho} \xi^\rho + \varepsilon^{\alpha \omega} \xi^\omega) \left( \eta_i \bar{\theta}_i - \varphi_i \bar{\omega}_i \right) + 2 g^2 \varepsilon^{\alpha \omega} \phi \xi^\omega \left( \eta_i \bar{\theta}_i + \varphi_i \bar{\omega}_i \right) \]

\[ - \Omega_\mu^\alpha D_\mu^\beta \xi^\beta \bar{c} \xi^\alpha - g \varepsilon^{\alpha \beta} \xi^\alpha A_\mu^\beta c + \varepsilon_\mu^\alpha \left[ g e^{\alpha \beta} \left( D_\mu^\beta \xi^\omega \right) c - \frac{g^2}{2} \varepsilon^{\alpha \beta} \varepsilon^{\omega \rho} A_\mu^\beta \xi^\rho \right] \]

\[ - \Omega_\mu \left( \partial_\mu c + g e^{\alpha \beta} A_\mu^\beta c \right) + g e^{\alpha \beta} L_\mu^\beta c^\alpha + \frac{g_2}{2} \varepsilon^{\alpha \beta} L c^\beta + g e^{\alpha \beta} \bar{\gamma}_\mu^\omega \bar{\xi}_\mu^\beta c + g e^{\alpha \beta} \bar{X}_\mu^\alpha \omega \bar{\bar{c}}^\omega c \]

\[ + g e^{\alpha \beta} \bar{X}_\mu^\alpha \left( \omega_\mu^\beta c + \frac{g_2}{2} \varepsilon^{\alpha \beta} \bar{\bar{\xi}}_\mu^\omega \bar{c}^\omega \right) - g e^{\alpha \beta} \bar{Y}_\mu^\alpha \left( \bar{\bar{\xi}}_\mu^\beta c - \frac{g}{2} \varepsilon^{\alpha \beta} \bar{\bar{c}}^\beta \bar{c}^\alpha \right) \]

\[ + \bar{\bar{\xi}}_\alpha^\beta \phi \bar{c}^\beta - g e^{\alpha \beta} \bar{\bar{\xi}}_\alpha^\beta \phi \bar{c}^\beta - \frac{g_2}{2} \varepsilon^{\alpha \beta} \bar{\bar{\xi}}_\alpha^\beta \bar{c}^\beta \left( \varepsilon^{\beta \omega} \phi \bar{c}^\omega c + \frac{1}{2} \varepsilon^{\beta \omega} \phi \bar{c}^\omega \bar{c}^\beta \right) - \Omega_\mu g e^{\alpha \beta} \phi \bar{c}^\beta \]

\[ + g e^{\alpha \beta} \bar{X}_\alpha^\omega \left( \theta_\alpha^\omega c + \frac{g_2}{2} \varepsilon^{\alpha \beta} \eta \bar{\bar{c}}^\beta \bar{c}^\alpha \right) - g e^{\alpha \beta} \bar{X}_\alpha^\omega \theta \bar{\bar{c}}^\beta \]

\[ - g e^{\alpha \beta} \bar{Y}_\alpha^\omega \left( \bar{\bar{\xi}}_\alpha^\beta c - \frac{g}{2} \varepsilon^{\alpha \beta} \bar{\bar{c}}^\beta \bar{c}^\alpha \right) \] \hfill (55)

The physical action \( S_{\text{phys}} \) is reobtained from \( \Sigma \) after the renormalization procedure at the limit case when this large set of external sources and parameters achieve its physical value, namely

\[ \{ \alpha, \beta \} \to 0, \]

\[ \{ \Omega_\mu, \Omega_\mu^\alpha, \xi^\alpha, \bar{\phi}^\alpha, \bar{\bar{\xi}}^\alpha, \bar{\bar{\phi}}^\alpha \} \to 0, \]

\[ \{ X^\alpha, \bar{X}^\alpha, \bar{\bar{Y}}^\alpha, \bar{\bar{\bar{Y}}}^\alpha, \bar{\bar{\bar{Y}}}^\alpha \} \to 0, \]

\[ M_{\mu \nu} \bigg|_{\text{phys}} = - M_{\mu \nu} \bigg|_{\text{phys}} = \gamma^2 \delta_{\mu \nu} \delta_{\alpha \beta}, \quad N_{\mu \nu} \bigg|_{\text{phys}} = N_{\mu \nu} \bigg|_{\text{phys}} = 0, \]

\[ \tilde{V} \alpha \beta \bigg|_{\text{phys}} = - V \alpha \beta \bigg|_{\text{phys}} = \sigma^2 \delta_{\alpha \beta}, \quad \tilde{\bar{\bar{V}}} \alpha \beta \bigg|_{\text{phys}} = \bar{\bar{V}} \alpha \beta \bigg|_{\text{phys}} = 0 \] \hfill (56)

To end this section we display in the following tables the quantum numbers of the field and sources present in the theory.

**Table 1:** Quantum numbers of fields and sources of the gauge sector. The nature is “B” for bosons and “F” for fermions.

| Gauge sector | A | b | c | \( \varphi \) | \( \bar{\varphi} \) | \( \omega \) | \( \bar{\omega} \) | M | \( \bar{M} \) | N | \( \bar{N} \) |
|--------------|---|---|---|----------------|----------------|---|----------------|---|----------------|---|----------------|
| **DIMENSION** | 1 | 2 | 2 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| **GHOST NUMBER** | 0 | 0 | -1 | 1 | 0 | 0 | -1 | 0 | 0 | 1 | -1 |
| **Q_8-CHARGE** | 0 | 0 | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| **NATURE** | B | B | F | F | B | F | F | B | B | F | F |

**Table 2:** Quantum numbers of fields and sources of the matter sector. The nature is “B” for bosons and “F” for fermions.

| Matter sector | \( \phi \) | \( \theta \) | \( \bar{\theta} \) | \( \eta \) | \( \bar{\eta} \) | V | \( \bar{V} \) | U | \( \bar{U} \) |
|---------------|---|---|---|---|---|---|---|---|---|
| **DIMENSION** | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| **GHOST NUMBER** | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 1 | -1 |
| **Q_8-CHARGE** | 0 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| **NATURE** | B | F | F | B | B | B | F | F | F |

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Table 3: Quantum numbers of the external sources coupled to composite operators. The nature is “B” for bosons and “F” for fermions.

| External sources | $\Omega$ | $\tau$ | $\xi$ | $L$ | $X$ | $Y$ | $Y$ | $\bar{X}$ | $\bar{Y}$ | $\bar{Y}$ |
|------------------|---------|-------|-------|-----|-----|-----|-----|--------|--------|--------|
| Dimension        | 3       | 3     | 3     | 4   | 3   | 3   | 3   | 3      | 3      | 3      |
| Ghost number     | -1      | -1    | -2    | -2  | 0   | -2  | -1  | -1     | -1     | 0      |
| $Q_8$-Charge      | 0       | 0     | 0     | 0   | 0   | 0   | 1   | 0      | 0      | 0      |
| $Q_2$-Charge      | 0       | 0     | 0     | 0   | 0   | 0   | 0   | 0      | 0      | 0      |
| Nature           | F       | F     | B     | B   | B   | F   | F   | B      | B      | F      |

4 Ward identities and stability for scalar matter case

In this section we derive the large set of Ward identities fulfilled by the complete action (55). These Ward identities will be the starting point for the analysis of the algebraic characterization of the most general invariant counterterm. It is easily checked that $\Sigma$ obeys the following identities:

- The Slavnov-Taylor identity:
  \begin{align}
  S(\Sigma) = 0 \tag{61}
  \end{align}

  with
  \begin{align}
  S(\Sigma) &= \int d^4x \left\{ \left( \frac{\delta \Sigma}{\delta \Omega_{\mu}} + \frac{\delta \Sigma}{\delta \tau_{\mu}} \right) \frac{\delta \Sigma}{\delta A_{\mu}} + \frac{\delta \Sigma}{\delta \Omega_{\mu}} \frac{\delta \Sigma}{\delta A_{\mu}} \right. \\
  &\quad + \left[ \omega^{\alpha} \Omega_{\mu} + N_{\mu} \Omega_{\alpha} \Omega_{\alpha} - \left( M_{\mu} \Omega_{\alpha} \Omega_{\alpha} + N_{\mu} \Omega_{\alpha} \Omega_{\alpha} \right) \right] - \left( \frac{\delta \Sigma}{\delta \phi_{\alpha}} \right) \frac{\delta \Sigma}{\delta \phi_{\alpha}} \\
  &\quad + Y_{\alpha} \frac{\delta \Sigma}{\delta X_{\alpha}} \left( \frac{\delta \Sigma}{\delta \tau_{\alpha}} + \theta_{\alpha} \frac{\delta \Sigma}{\delta \theta_{\alpha}} + \delta_{\alpha} \frac{\delta \Sigma}{\delta \delta_{\alpha}} - V_{\alpha} \frac{\delta \Sigma}{\delta \phi_{\alpha}} + U_{\alpha} \frac{\delta \Sigma}{\delta \phi_{\alpha}} \\
  &\quad - \left( \tilde{\Omega}_{\alpha} - \tilde{\tau}_{\alpha} \right) \frac{\delta \Sigma}{\delta \xi_{\alpha}} + \tilde{X}_{\alpha} \frac{\delta \Sigma}{\delta \tilde{Y}_{\alpha}} + \tilde{Y}_{\alpha} \frac{\delta \Sigma}{\delta \tilde{X}_{\alpha}} \right\} \tag{62}
  \end{align}

which is nothing but the BRST invariance of the action (55) when expressed in a functional form.

The BRST transformations of fields and sources can extracted from the Slavnov-Taylor identity above. For example, the transformations of the gauge, scalar and ghost fields are given by:

\begin{align}
  sA^\alpha_\mu &= \frac{\delta \Sigma}{\delta \Omega_{\alpha}} + \frac{\delta \Sigma}{\delta \tau_{\alpha}} \mu \\
  s\phi^\alpha &= \frac{\delta \Sigma}{\delta \Omega_{\alpha}} + \frac{\delta \Sigma}{\delta \tau_{\alpha}} \phi \\
  sc^\alpha &= \frac{\delta \Sigma}{\delta L_{\alpha}} \\
  \end{align}

(63)
The remaining transformations are BRST doublets. Let us also introduce, for further use, the so-called linearized Slavnov-Taylor operator $B_{\Sigma}$, defined as

$$B_{\Sigma} = \int d^4x \left\{ \left( \frac{\delta \Sigma}{\delta \Omega_{\mu}^\alpha} + \frac{\delta \Sigma}{\delta \tau_{\mu}^\alpha} \right) \frac{\delta}{\delta A_{\mu}^\alpha} + \frac{\delta \Sigma}{\delta A_{\mu}^\alpha} \left( \frac{\delta}{\delta \Omega_{\mu}^\alpha} + \frac{\delta}{\delta \tau_{\mu}^\alpha} \right) \right\}$$

$$+ \frac{\delta \Sigma}{\delta \bar{c}} \frac{\delta}{\delta \phi} + \frac{\delta \Sigma}{\delta \phi} \frac{\delta}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta \bar{c}} \frac{\delta}{\delta \phi} + \frac{\delta \Sigma}{\delta \phi} \frac{\delta}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta b} \frac{\delta}{\delta \phi} + \frac{\delta \Sigma}{\delta \phi} \frac{\delta}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta b} \frac{\delta}{\delta \phi} + \frac{\delta \Sigma}{\delta \phi} \frac{\delta}{\delta \bar{c}}$$

$$- \bar{M}^\alpha_{\mu} \frac{\delta}{\delta N^a_{\mu}} + N_{\mu I}^a \frac{\delta}{\delta M^a_{\mu I}} - \left( \Omega^\alpha_{I \mu} - \tau^\alpha_{I \mu} \right) \frac{\delta}{\delta \Omega^\alpha_{I \mu}} + \bar{X}^I_\alpha \frac{\delta}{\delta \bar{X}^I_\alpha} + \bar{Y}^I_\alpha \frac{\delta}{\delta \bar{Y}^I_\alpha} + \left( \frac{\delta \Sigma}{\delta \Omega^\alpha_{I \mu}} + \frac{\delta \Sigma}{\delta \tau^\alpha_{I \mu}} \right) \frac{\delta}{\delta \phi^a}$$

$$+ \frac{\delta \Sigma}{\delta \phi^a} \left( \frac{\delta}{\delta \Omega^\alpha_{I \mu}} + \frac{\delta}{\delta \tau^\alpha_{I \mu}} \right) + \frac{\delta \Sigma}{\delta \phi^a} \frac{\delta}{\delta \bar{c}} + \frac{\delta \Sigma}{\delta \bar{c}} \frac{\delta}{\delta \phi^a} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \bar{X}^I_\alpha} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \bar{Y}^I_\alpha}$$

$$- \bar{V}^i_\alpha \frac{\delta}{\delta U^i_\alpha} + U_i^\alpha \frac{\delta}{\delta \bar{V}^i_\alpha} - \left( \Omega^\alpha_i - \tau^\alpha_i \right) \frac{\delta}{\delta \phi^a} + \bar{X}^i_\alpha \frac{\delta}{\delta \bar{X}^i_\alpha} + \bar{Y}^i_\alpha \frac{\delta}{\delta \bar{Y}^i_\alpha} \right\} \tag{64}$$

The operator $B_{\Sigma}$ has the important property of being nilpotent

$$B_{\Sigma} B_{\Sigma} = 0 \, . \tag{65}$$

- The diagonal Nakanishi-Lautrup field equation:

$$\frac{\delta \Sigma}{\delta \phi} = \partial_\mu A^\alpha_{\mu} \, . \tag{66}$$

- The diagonal anti-ghost equation:

$$\frac{\delta \Sigma}{\delta \bar{c}} + \partial_\mu \frac{\delta \Sigma}{\delta \Omega_{\mu}^\alpha} = 0 \, . \tag{67}$$

- The local diagonal ghost equation:

$$G(\Sigma) \equiv \frac{\delta \Sigma}{\delta \phi} + g e^{\alpha \beta} \left( \eta^\alpha_i \frac{\delta}{\delta \phi^a_i} + \omega^\alpha \frac{\delta}{\delta \phi^a} + \phi^\alpha \frac{\delta}{\delta \phi^a} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \phi^a} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \phi^a} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \phi^a} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \phi^a} + \bar{\theta}^a_\alpha \frac{\delta}{\delta \phi^a} \right)$$

$$= -\partial_\mu (\partial_\mu \bar{c} + \Omega_{\mu}) - g e^{\alpha \beta} \left( L^\alpha \bar{c} - \tau^\alpha_{I \mu} A^\alpha_{\mu} - \tau^\alpha \phi^\beta + \bar{Y}^I_\alpha \varphi^I_i + \bar{X}^I_\alpha \omega^I_i + \bar{X}^I_\alpha \omega^I_i - \bar{Y}^I_\alpha \varphi^I_i \right)$$

$$+ \bar{Y}^I_\alpha \theta^I_i + \bar{X}^I_\alpha \theta^I_i - \bar{Y}^I_\alpha \eta^I_i \right\} \tag{68}$$

Notice that the right-hand side of eq. (68) is linear in the quantum fields. As such, it is a linear breaking, not affected by the quantum correction [52].

- The $U(1)$ residual local symmetry:

$$\mathcal{W}^{U(1)} \Sigma = -\partial^2 b \tag{69}$$

where

$$\mathcal{W}^{U(1)} \equiv \partial_\mu \frac{\delta}{\delta \Omega_{\mu}^\alpha} + g e^{\alpha \beta} \sum_\Psi \frac{\Psi^\alpha}{\delta \Psi^\beta} \tag{70}$$

being the summation over $\Psi$ a sum over all off-diagonal fields and sources, namely

$$\Psi^\alpha = \{ A^\alpha_{\mu}, b^\alpha, c^\alpha, \phi, \varphi^I_i, \omega^I_i, \bar{c}, \bar{\phi}, \bar{\varphi}^I_i, \bar{\omega}^I_i, \bar{\theta}^I_i, \bar{\bar{\theta}}^I_i, \bar{\theta}^I_i, M^a_{\mu I}, \bar{M}^a_{\mu I}, N^a_{\mu I}, \bar{N}^a_{\mu I}, V^\alpha, \bar{V}^i_\alpha, U_i^\alpha, \bar{U}^i_\alpha, \bar{\Omega}^I_{\mu i}, \bar{\Lambda}^I_{\mu i}, \bar{\bar{\Omega}}^I_{\mu i}, \bar{\bar{\Lambda}}^I_{\mu i}, \bar{\bar{\bar{\Omega}}}_{\mu i}, \bar{\bar{\bar{\Lambda}}}_{\mu i}, \bar{\bar{\bar{\bar{\Omega}}}}_{\mu i}, \bar{\bar{\bar{\bar{\Lambda}}}}_{\mu i} \} \tag{71}$$
As noticed in [27], the $U(1)$ Ward identity (69) can be obtained by anticommuting the diagonal ghost equation, eq. (68), with the Slavnov-Taylor identity, eq. (61). This identity shows in a very clear way the fact that the diagonal component $A_{\mu}$ of the gauge field behaves like a $U(1)$ Abelian connection, while all off-diagonal components of the gauge and matter fields play the role of a kind of charged $U(1)$ fields, precisely like in a QED-like theory.

- The discrete symmetry

\[ \Psi^1 \rightarrow \Psi^1, \quad \Psi^2 \rightarrow -\Psi^2, \quad \Psi^{\text{diag}} \rightarrow -\Psi^{\text{diag}} \]  

(72)

where $\Psi^\alpha$ and $\Psi^{\text{diag}}$ stand, respectively, for all off-diagonal and diagonal fields and sources, namely with $\Psi^\alpha$ given by (71) and

\[ \Psi^{\text{diag}} = \{ A_\mu, b, c, \bar{c}, \phi, \Omega_\mu, L \} \]  

(73)

As pointed out in [27], this discrete symmetry plays the role of the charge conjugation with respect to the $U(1)$ Cartan subgroup of $SU(2)$.

- The functional $\delta I$'s-symmetries of gauge sector:

\[ \mathcal{W}_I(\Sigma) \equiv \int d^4x \left\{ \varphi_i^\alpha \frac{\delta \Sigma}{\delta \varphi_i^\alpha} + c^\alpha \frac{\delta \Sigma}{\delta \varphi_i^\alpha} + \frac{\delta \Sigma}{\delta Y_i^\alpha} \frac{\delta \Sigma}{\delta b^\alpha} + M_{\mu I}^\alpha \frac{\delta \Sigma}{\delta Y_i^\alpha} - \bar{Y}_i^\alpha \frac{\delta \Sigma}{\delta L^\alpha} \right\} = 0 \]  

(74)

\[ \bar{\mathcal{W}}_I(\Sigma) \equiv \int d^4x \left\{ \tilde{\varphi}_i^\alpha \frac{\delta \Sigma}{\delta \tilde{\varphi}_i^\alpha} - c^\alpha \frac{\delta \Sigma}{\delta \tilde{\varphi}_i^\alpha} + \frac{\delta \Sigma}{\delta X_i^\alpha} \frac{\delta \Sigma}{\delta \tilde{b}^\alpha} - \bar{N}_{\mu I}^\alpha \frac{\delta \Sigma}{\delta X_i^\alpha} + \bar{X}_i^\alpha \frac{\delta \Sigma}{\delta \bar{L}^\alpha} \right\} = 0 \]  

(75)

This Ward identities corresponding to $\delta I$ and $\tilde{\delta} I$ symmetries, eqs. (43) e (44), respectively, and they are responsible for the renormalization of the horizon function of the MAG (14), as proven in [20].

- The rigid $\tilde{\delta} I$'s-symmetries of matter sector:

\[ \tilde{\mathcal{W}}_I(\Sigma) \equiv \int d^4x \left\{ \tilde{\varphi}_i^\alpha \frac{\delta \Sigma}{\delta \tilde{\varphi}_i^\alpha} + c^\alpha \frac{\delta \Sigma}{\delta \tilde{\varphi}_i^\alpha} + \frac{\delta \Sigma}{\delta \tilde{Y}_i^\alpha} \frac{\delta \Sigma}{\delta \tilde{b}^\alpha} + V_{\mu I}^\alpha \frac{\delta \Sigma}{\delta \tilde{Y}_i^\alpha} - \tilde{Y}_i^\alpha \frac{\delta \Sigma}{\delta \tilde{L}^\alpha} \right\} = 0 \]  

(76)

\[ \bar{\tilde{\mathcal{W}}}_I(\Sigma) \equiv \int d^4x \left\{ \tilde{\varphi}_i^\alpha \frac{\delta \Sigma}{\delta \tilde{\varphi}_i^\alpha} - c^\alpha \frac{\delta \Sigma}{\delta \tilde{\varphi}_i^\alpha} + \frac{\delta \Sigma}{\delta \tilde{X}_i^\alpha} \frac{\delta \Sigma}{\delta \tilde{b}^\alpha} - \tilde{U}_i^\alpha \frac{\delta \Sigma}{\delta \tilde{X}_i^\alpha} + \tilde{X}_i^\alpha \frac{\delta \Sigma}{\delta \tilde{L}^\alpha} \right\} = 0 \]  

(77)

This Ward identities corresponding to $\tilde{\delta}_I$ and $\bar{\tilde{\delta}}_I$ symmetries, eqs. (47) e (48), respectively. This identities are completely analogous to the identities (74) – (75). Then, this set of WI will be responsible to guarantee the renormalization of the horizon function of the matter sector (28), or its local version (29).

- The $d I$'s-symmetries of gauge sector:

\[ \mathcal{Q}_I(\Sigma) \equiv \int d^4x \left\{ \left( \omega_i^\alpha \frac{\delta \Sigma}{\delta \omega_i^\alpha} + \frac{\delta \Sigma}{\delta Y_i^\alpha} \frac{\delta \Sigma}{\delta b^\alpha} + \frac{\delta \Sigma}{\delta X_i^\alpha} \frac{\delta \Sigma}{\delta \tilde{b}^\alpha} + \frac{\delta \Sigma}{\delta \bar{L}^\alpha} \right) \right\} = 0 \]  

(78)

\[ \tilde{\mathcal{Q}}_I(\Sigma) \equiv \int d^4x \left\{ \left( \delta \Sigma \frac{\delta \Sigma}{\delta X_i^\alpha} - \varphi_i^\alpha \right) \right\} = 0 \]  

(79)

This Ward identities corresponding to $d I$ and $\tilde{d} I$ symmetries, but also they can be obtained by anticommuting and commuting, respectively, the identities $\mathcal{W}_I$ and $\tilde{\mathcal{W}}_I$ with the Slavnov-Taylor identity (61).
• The \( d_i \)'s-symmetries of matter sector:

\[
\mathcal{Q}_i(\Sigma) \equiv \int d^4x \left\{ \left( \theta_i^a + \frac{\delta \Sigma}{\delta X_i^a} \right) \frac{\delta \Sigma}{\delta \theta_i^a} + \frac{\delta \Sigma}{\delta Y_i^a} \frac{\delta \Sigma}{\delta \theta_i^a} + \frac{\delta \Sigma}{\delta L^a} \left( \frac{\delta \Sigma}{\delta \eta_i^a} - \tilde{X}_i^a \right) \\
+ e^\alpha \frac{\delta \Sigma}{\delta \theta_i^a} - V_i^a \frac{\delta \Sigma}{\delta \xi^a} + U_i^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{80}
\]

\[
\tilde{\mathcal{Q}}_i(\Sigma) \equiv \int d^4x \left\{ \left( \frac{\delta \Sigma}{\delta X_i^a} - \eta_i^a \right) \frac{\delta \Sigma}{\delta \theta_i^a} + \frac{\delta \Sigma}{\delta Y_i^a} \frac{\delta \Sigma}{\delta \theta_i^a} + \frac{\delta \Sigma}{\delta L^a} \left( \frac{\delta \Sigma}{\delta \eta_i^a} - \tilde{X}_i^a \right) \\
- e^\alpha \frac{\delta \Sigma}{\delta \eta_i^a} - \tilde{V}_i^a \frac{\delta \Sigma}{\delta \xi^a} + \tilde{U}_i^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{81}
\]

This Ward identities corresponding to \( d_I \) and \( \tilde{d}_I \) symmetries, but also they can be obtained by anticommuting and commuting, respectively, the identities \( \mathcal{W}_i \) and \( \tilde{\mathcal{W}}_i \) with the Slavnov-Taylor identity \( [61] \).

• The rigid \( \mathcal{R} \)-symmetries of gauge sector:

\[
\mathcal{R}^{(1)}_{I,J}(\Sigma) \equiv \int d^4x \left\{ \frac{\varphi_I^\alpha}{\delta \omega_J^a} - \frac{\varphi_J^\alpha}{\delta \omega_I^a} + M_{\mu I}^a \frac{\delta \Sigma}{\delta \mu J^a} + \tilde{N}_{\mu J}^a \frac{\delta \Sigma}{\delta \mu I^a} + Y_I^a \frac{\delta \Sigma}{\delta \Omega^a} - \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{82}
\]

\[
\mathcal{R}^{(2)}(\Sigma) \equiv \int d^4x \left\{ \frac{\omega_I^a}{\delta \omega_I^a} - \tilde{N}_{\mu I}^a \frac{\delta \Sigma}{\delta \mu I^a} - \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{83}
\]

\[
\mathcal{R}^{(3)}(\Sigma) \equiv \int d^4x \left\{ \frac{\varphi_I^\alpha}{\delta \omega_I^a} - \frac{\varphi_I^\alpha}{\delta \omega_I^a} - M_{\mu I}^a \frac{\delta \Sigma}{\delta \mu J^a} - \tilde{N}_{\mu J}^a \frac{\delta \Sigma}{\delta \mu I^a} - \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} + \tilde{Y}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{84}
\]

• The rigid \( \tilde{\mathcal{R}} \)-symmetries of matter sector:

\[
\tilde{\mathcal{R}}^{(1)}_{I,J}(\Sigma) \equiv \int d^4x \left\{ \eta_I^a \frac{\delta \Sigma}{\delta \eta_J^a} - \tilde{\eta}_J^a \frac{\delta \Sigma}{\delta \eta_I^a} + \tilde{V}_I^a \frac{\delta \Sigma}{\delta \Omega^a} + \tilde{U}_I^a \frac{\delta \Sigma}{\delta \Omega^a} + \tilde{Y}_I^a \frac{\delta \Sigma}{\delta \Omega^a} - \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{85}
\]

\[
\tilde{\mathcal{R}}^{(2)}(\Sigma) \equiv \int d^4x \left\{ \tilde{\theta}_I^a \frac{\delta \Sigma}{\delta \theta_I^a} - \tilde{U}_I^a \frac{\delta \Sigma}{\delta \Omega^a} - \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{86}
\]

\[
\tilde{\mathcal{R}}^{(3)}(\Sigma) \equiv \int d^4x \left\{ \tilde{\theta}_I^a \frac{\delta \Sigma}{\delta \theta_I^a} - \tilde{\eta}_I^a \frac{\delta \Sigma}{\delta \theta_I^a} - \tilde{V}_I^a \frac{\delta \Sigma}{\delta \Omega^a} - \tilde{U}_I^a \frac{\delta \Sigma}{\delta \Omega^a} - \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} + \tilde{Y}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} = 0 \tag{87}
\]

Note that this rigid invariances in both the gauge and matter sectors, egs. \([82, 87]\), as well as the discrete symmetry \([22]\), are “blind” with respect to the off-diagonal indices which are “hidden” in the indices \( \{I, i\} \).

• The global \( U(8) \) symmetry, here written in the multi-index notation:

\[
\mathcal{Q}_{IJ}(\Sigma) \equiv \int d^4x \left\{ \frac{\varphi_I^\alpha}{\delta \omega_J^a} - \frac{\varphi_J^\alpha}{\delta \omega_I^a} + \omega_I^a \frac{\delta \Sigma}{\delta \omega_I^a} - \omega_J^a \frac{\delta \Sigma}{\delta \omega_J^a} + M_{\mu I}^a \frac{\delta \Sigma}{\delta \mu J^a} - \tilde{M}_{\mu J}^a \frac{\delta \Sigma}{\delta \mu I^a} + \tilde{X}_I^a \frac{\delta \Sigma}{\delta \Omega^a} + \tilde{Y}_I^a \frac{\delta \Sigma}{\delta \Omega^a} \right\} \Sigma = 0 \tag{88}
\]

This WI can be immediately viewed as \( \mathcal{Q}_{IJ} \Sigma = 0 \), i.e. as a linear operator \( \mathcal{Q}_{IJ} \) acting on \( \Sigma \). Then, the eigenvalues of the trace of the operator \( \mathcal{Q}_{IJ} \) define the \( Q_8 \)-charge in the Tables 1 and 3.
• The global $U(2)$ symmetry:

$$Q_{ij}(\Sigma) = \int d^4x \left\{ \eta^a_{ij} \frac{\delta \Sigma}{\delta \eta^a_{ji}} - \bar{\eta}^a_{ij} \frac{\delta \Sigma}{\delta \bar{\eta}^a_{ji}} + \frac{\theta^a_{ij}}{2} \frac{\delta \Sigma}{\delta \theta^a_{ij}} - \bar{\theta}^a_{ij} \frac{\delta \Sigma}{\delta \bar{\theta}^a_{ij}} + U^a_{ij} \frac{\delta \Sigma}{\delta U^a_{ji}} + \bar{U}^a_{ij} \frac{\delta \Sigma}{\delta \bar{U}^a_{ji}} \right\} \Sigma = 0 \quad (89)$$

Analogously to the previous WI, the equation above can be written as $Q_{ij} \Sigma = 0$ and the trace of the linear operator $Q_{ij}$ defines the $Q_2$-charge in the Tables 2 and 3.

4.1 Renormalization factors

The corresponding counterterm, \textit{i.e.}, the most general integrated local polynomial in the fields and sources, with dimension four and ghost number zero, compatible with all symmetries of the action, that can be freely added at order $\epsilon$ in the perturbative expansion, is given by

$$\Sigma_{c.t.} = \Sigma_0 + B_\Sigma \Delta^{-1} \quad (90)$$

where $\Sigma_0$ stands for the nontrivial part of the cohomology of the operator $B_\Sigma$, being given by

$$\Sigma_0 = a_0 S_{YM} + \int d^4x \left( \frac{m^2}{2} \phi^a \phi^a + a_2 \frac{\lambda}{4!} (\phi^a \phi^a)^2 \right) \quad (91)$$

and $\Delta^{-1}$ is given by an integrated local polynomial in the fields with dimension 4, ghost number $(-1)$ and with vanishing $Q_8$ and $Q_2$ charges. Taking into account the full set of symmetries of the last section, one can write $\Delta^{-1}$ as

$$\Delta^{-1} = \int d^4x \left\{ \alpha (a_3 + a_4) (\Omega^{(a)} A_{\mu}^{a} + g \varepsilon^{a\beta} \xi_{\alpha} A_{\mu}^{b} c) + (a_4 + a_5) \xi_{\alpha} (D_{\mu} A^{a \beta} c) - a_6 \varepsilon^{a\beta} D^{a \beta} A_{\mu}^{b} + a_7 \epsilon \alpha L^a 

+ (a_3 - a_5 + a_7) (\bar{N}^{a \beta} M_{\mu}^{a \beta} \bar{D}^{a \beta} - M_{\mu}^{a \beta} D^{a \beta} \bar{N}^{a \beta}) + (a_5 + a_7) \bar{N}^{a \beta} M_{\mu}^{a \beta} + a_9 x N_{\mu} M_{\mu}^{a} 

+ (a_{10} + a_{11}) (\bar{N}^{a \beta} \partial \phi^a + g \varepsilon^{a\beta} \xi_{\alpha} \phi^a c - \bar{\Omega} \phi) + (-a_{11} + a_{12}) g \varepsilon^{a\beta} \xi_{\alpha} \phi^a + a_{13} \bar{\chi} U_{\mu} A_{\mu}^{a} 

- \alpha \left[ a_{14} (\epsilon^a \phi^a - g \varepsilon^{a\beta} \xi_{\alpha} \phi^a c) + (a_7 + 2a_{14}) g^2 \epsilon^a \epsilon^a \phi^a (\phi^a \bar{\phi}^a + \eta_i \bar{\eta}_i) \right] 

+ \alpha (a_7 + a_{14}) g^2 \left( 2 \bar{\omega}_{\alpha} \omega^a_{\alpha} + \eta_i \bar{\eta}_i (\bar{\phi}^a \phi^a - \bar{\omega}_{\alpha} \omega^a_{\alpha}) + 2 \bar{\theta}_i \bar{\theta}_i + \bar{\eta}_i \bar{\eta}_i \right) 

+ \beta (a_7 + a_{15}) \left( \varepsilon^{a\beta} \phi^a \bar{\phi}^a + g \phi^a \phi^a (\phi^a \bar{\phi}^a + \eta_i \bar{\eta}_i) - g \phi^a \phi^a (\phi^a \bar{\phi}^a + \eta_i \bar{\eta}_i) + g \phi^2 (\phi^a \bar{\phi}^a - \eta_i \bar{\eta}_i) \right) \right\}$$

where $\{a_k\}_{k=1}^{15}$ are independent arbitrary coefficients. The counterterm $\frac{1}{2} \eta^2$ can be reabsorbed in the classical action by a multiplicative renormalization of the fields, sources and parameters:

$$\Sigma [F_0, J_0] + O(\epsilon^2) = \Sigma [F, J] + \epsilon \Sigma_{CT} \quad (93)$$

where, the label “0” indicates a bare (nonrenormalized) quantity, $\epsilon$ is the expansion parameter, $F$ stands for the fields and $J$ stands for the external sources and parameters. By convention we choose the renormalization factors as

$$F_0 = Z_{F}^{1/2} F = \left( 1 + \frac{\epsilon}{2} z_F \right) F$$

$$J_0 = Z_J J = (1 + \epsilon z_J) J \quad (94)$$
where the coefficients \( \{ z_F, z_J \} \) are certain linear combinations of \( \{ a_k \} \). By direct inspection, one can find that

\[
(Z^{\text{off}}_A)^{1/2} = 1 + \varepsilon \left( \frac{a_0}{2} + a_3 + a_4 \right) \quad (95)
\]
\[
Z_g = 1 - \varepsilon \frac{a_0}{2} \quad (96)
\]
\[
(Z^{\text{off}}_\phi)^{1/2} = 1 + \varepsilon (a_{10} + a_{11}) \quad (97)
\]
\[
Z_{m\phi} = 1 + \varepsilon \frac{a_1}{2} \quad (98)
\]
\[
Z_\lambda = 1 + \varepsilon a_2 \quad (99)
\]
\[
(Z^{\text{off}}_b)^{1/2} = 1 - \varepsilon \left( \frac{a_0}{2} + a_6 \right) \quad (100)
\]
\[
Z_\alpha = 1 + \varepsilon (a_0 + 2a_6 - 2a_{14}) \quad (101)
\]
\[
(Z^{\text{off}}_c)^{1/2} = (Z^{\text{off}}_c)^{1/2} = 1 - \varepsilon \left( \frac{a_6 + a_7}{2} \right) \quad (102)
\]
\[
(Z^{\text{diag}}_c)^{1/2} = 1 + \varepsilon \left( \frac{a_7 - a_6}{2} \right) \quad (103)
\]
\[
Z^{1/2} = Z^{1/2} = 1 + \varepsilon \left( \frac{a_6 - a_7}{2} - a_1 \right) \quad (106)
\]
\[
Z_N = 1 + \varepsilon \left( \frac{a_0}{2} - a_3 + a_5 \right) \quad (107)
\]
\[
Z_N = 1 + \varepsilon \left( \frac{a_0}{2} - a_3 + a_5 + a_6 - a_7 \right) \quad (108)
\]
\[
Z_\chi = 1 + \varepsilon (2a_3 - a_6 + a_7 - a_9) \quad (109)
\]
\[
Z^{1/2} = Z^{1/2} = 1 - \varepsilon \left( \frac{a_7}{2} \right) \quad (110)
\]
\[
Z^{1/2} = Z^{1/2} = 1 + \varepsilon \left( \frac{a_7}{2} - a_7 \right) \quad (111)
\]
\[
Z^{1/2} = Z^{1/2} = 1 - \varepsilon \left( \frac{a_0 - a_6}{2} \right) \quad (112)
\]
\[
Z^{1/2} = Z^{1/2} = 1 + \varepsilon \left( \frac{a_0 + a_7}{2} + a_{10} - a_{12} \right) \quad (113)
\]
\[
Z_U = 1 + \varepsilon \left( \frac{a_0 - a_{10} + a_{12} - \frac{a_6}{2}}{2} \right) \quad (114)
\]
\[
Z_U = 1 + \varepsilon \left( \frac{a_0 - a_{10} + a_7 + \frac{a_6}{2}}{2} \right) \quad (115)
\]
\[
Z^{1/2} = Z^{1/2} = 1 - \varepsilon (a_0 + 2a_{10} - 2a_{12} + a_7 - a_{13}) \quad (116)
\]

and

\[
(Z^{\text{diag}}_A)^{1/2} = (Z^{\text{diag}}_b)^{1/2} = Z_g^{-1} \quad (117)
\]
\[
(Z^{\text{diag}}_\phi)^{1/2} = (Z^{\text{off}}_c)^{1/2} \quad (118)
\]
\[
(Z^{\text{diag}}_c)^{1/2} = (Z^{\text{diag}}_c)^{-1/2} \quad (119)
\]
\[
Z^{1/2} = Z^{1/2} = (Z^{\text{off}}_c)^{1/2} \quad (120)
\]

(121)
Finally, we note that the non-renormalization theorem of the maximal Abelian gauge \cite{27}, namely \(Z_g(Z_A^\text{diag})^{1/2} = 1\) remains true in the presence of the horizon matter function, extending the result finding in \cite{13}. This ends the proof of the multiplicative renormalization of the SU(2) Gribov-Zwanziger action in the MAG with confining scalar matter.

5 Conclusion

In this work we have addressed the issue of the all orders perturbative renormalization of the SU(2) Gribov-Zwanziger model in the maximal Abelian gauge in the presence of confined scalar matter fields in the adjoint representation as well as fermion matter in the fundamental representation. Following the conjecture of universal coupling for Faddeev-Popov operator to any coloured field, proposed in \cite{17}, an additional Gribov-like term in the matter sector is implemented in order to compel the confinement character of the matter field, which shares great similarity with the horizon function introduced in the pure gauge sector, providing that a similar picture can be consistently achieved in the matter case via the Gribov-Zwanziger picture. Due to the non-linearity of the gauge fixing condition a new quartic interaction term between scalar matter fields, off-diagonal Faddeev-Popov ghosts and Zwanziger-like localizing fields are required for renormalizability. This new term is BRST-invariant, as expressed by eq.\(51\), and proportional to a new gauge-like parameter \(\beta\), generalizing the main result reported in \cite{13} to the Gribov context. Moreover, the most remarkable fact is the existence of a new set of symmetries relating the auxiliary localizing Zwanziger-like fields in the matter sector and the Faddeev-Popov sector fields, eqs. \(47-50\), in perfect analogy to the symmetries in the pure gauge sector reported in \cite{20,21}, allow us to control the ultraviolet finiteness of the new horizon-like term in the matter through the existence of a set of associated Ward identities..

The analysis of the all orders perturbative renormalizability of the maximal Abelian gauge in presence of matter fields is the first necessary step towards the investigation of the non-perturbative effects of the Gribov copies, which deeply affect the maximal Abelian gauge \cite{20,21,16,14}. We underline that the requirements of localizability and renormalizability are unavoidable in order to have at our disposal a consistent computational framework. Besides, although the proof of the renormalizability given here refers to the gauge group SU(2), it can be easily generalized to other gauge groups as well as to other representations of the scalar fields.

The resulting local form of the full action is obtained by the introduction of auxiliary Zwanziger-like fields which, as in the case of the localizing Zwanziger fields of the pure gauge sector, develop their own dynamics giving rise to the formation of dimension two condensates, as explicitly checked through one-loop computations in \cite{9}. Moreover, the condensates arising in the matter sector can be taken into account through an effective action which looks much alike the refined Gribov-Zwanziger action which accounts for the existence of similar condensates in the gluon sector. The inclusion of the dimension two operators is straightforward and don’t spoil the renormalization of the model \cite{21}.

Finally, the inclusion of the usual Dirac action for spinors does not pose any additional problem. In the same way as before, the renormalizability is guaranteed by a new set of Ward identities analogous to the above-mentioned, as showed in the eqs. \(129,132\) at the Appendix \(A\). Also, unlike the case of scalar matter fields, BRST invariance and power counting do not allow for additional interaction terms between spinors and Faddeev-Popov ghosts.
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A Renormalization for fermionic matter case

The inclusion of the usual Dirac action for spinors does not pose any additional problem. In fact, as in the Landau gauge \[17, 12\], the extension to the case of fermionic matter in the MAG is immediate and its renormalizability follows by analogy with the renormalization of the scalar matter case. The same arguments presented for the scalar case can be repeat in the spinorial case, providing that an analogous set of Ward identities can be established.

In complete analogy to the Subsection 2.2, considering the case which spinor matter is present, then we added the Dirac action in the fundamental representation (whose indexies are represented by lowercase Latin letters) minimally coupled to gauge field, i.e.

$$S_{\text{spinor}} = \int d^4x \left[ \bar{\psi}^i_\alpha (\gamma_\mu)_\alpha{}^\beta D^\mu_\beta \psi^{j\beta} - m_\psi \bar{\psi}^i_\alpha \psi^i_\alpha \right]$$

where covariant derivative is in fundamental representation. Note that at this case the circumflexed Greek indexies are spinorial, they do not correspond to the off-diagonal components in a Cartan decomposition, as the case of no-circumflexed Greek letters. The confining character of spinorial matter is implemented by the existence of an horizon function in this sector which coupled the inverse of Faddeev-Popov operator \((12)\) to off-diagonal generators in the fundamental representation, according to

$$H_{\text{spinor}}(\psi) = -g^2 \int d^4xd^4y \bar{\psi}^i_\alpha(x) (T^\alpha)_{ij} \psi^{j\alpha}(y)$$

The complete non-local IR matter action in this case is given by

$$S_{\text{matter}} = S_{\text{spinor}} + M^3 H_{\text{spinor}}$$

The parameter \(M\) is analogous to the Gribov parameter for the case of spinorial matter. Evidently, in same way as before, the non-local horizon function \((12)\) can be cast in local form by the usual method, introducing the fields \((\bar{\eta}^{ai}_\alpha, \eta^{ai}_\alpha)\) and \((\bar{\lambda}^{ai}_\alpha, \lambda^{ai}_\alpha)\), anticommuting and commuting, respectively, where we change the notation of the Zwanziger fields in order to keep the harmony with \[17, 12, 9\]. Thus, the local action of quark matter fields coupled with the gauge sector in a non-perturbative way is expressed as

$$H_{\text{spinor}}^{\text{local}} = \int d^4x \left\{ \bar{\lambda}^{ai}_\alpha \lambda^{\alpha\beta} \lambda^{\beta\alpha i} - \bar{\eta}^{ai}_\alpha \eta^{\alpha\beta} \eta^{\beta\alpha i} - gM^3/2 \left[ \bar{\lambda}^{ai}_\alpha (T^\alpha)_{ij} \psi^{j\alpha} - \bar{\psi}^i_\alpha (T^\alpha)^{ij} \lambda^{\alpha\beta j} \right] \right\}$$

After localization, we can see that full matter action \((12)\) exhibits a soft breaking of the nilpotent BRST symmetry

$$s\psi^{j\alpha}_i = -ig(T^a)^{ij} \epsilon^a \psi^{j\alpha}_i , \quad s\bar{\psi}^{j\alpha}_i = -ig \bar{\psi}^{j\alpha}_i (T^a)^{ji} \epsilon^a$$

$$s\bar{\eta}^{ai}_\alpha = \bar{\lambda}^{ai}_\alpha , \quad s\lambda^{ai}_\alpha = 0 , \quad s\bar{\lambda}^{ai}_\alpha = \eta^{ai}_\alpha , \quad s\eta^{ai}_\alpha = 0$$

due to the presence of Gribov-like parameter \(M\). Note the difference between the Latin and Greek indices in the above transformations. As usual, we write the BRST-exact form for this case by introducing a
quartet of sources \( \left( \tilde{U}^{ij \alpha}_{\tilde{\alpha} \tilde{\beta}}, U^{ij \alpha}_{\tilde{\alpha} \tilde{\beta}}, \tilde{V}^{ij \alpha}_{\tilde{\alpha} \tilde{\beta}}, V^{ij \alpha}_{\tilde{\alpha} \tilde{\beta}} \right) \), namely \[10\]

\[
S_{UV} = s \int d^4 x \left\{ \tilde{U}^{ij \alpha}_{\tilde{\alpha} \tilde{\beta}} \tilde{\psi}^{i \alpha} g(T^i)^{\alpha}_{\beta} \lambda^{\alpha \beta} \tilde{\lambda}^{\beta \delta} \tilde{g}(T^j)^{\delta}_{\alpha} \tilde{\psi}^{j \beta} + \tilde{V}^{ij \alpha}_{\tilde{\alpha} \tilde{\beta}} \tilde{\psi}^{i \alpha} g(T^i)^{\alpha}_{\beta} \lambda^{\alpha \beta} \tilde{\lambda}^{\beta \delta} \tilde{g}(T^j)^{\delta}_{\alpha} \tilde{\psi}^{j \beta} + \zeta m_\psi \tilde{V}^{ij}_{\tilde{\alpha} \tilde{\beta}} \right\}
\]

The last term in expression above, proportional to the dimensionless coefficient \( \zeta \), is a vacuum term allowed by power-counting. The term proportional to Gribov parameter \( M \) in \[12\] is recovered from the invariant action \( S_{UV} \) when the external sources attain the so-called physical value, \( i.e. \)

\[
V^{ij}_{\alpha \beta}\big|_{phys} = \tilde{V}^{ij}_{\tilde{\alpha} \tilde{\beta}}\big|_{phys} = M^{3/2} \delta^{ij} \delta_{\alpha \beta}, \quad U^{ij}_{\alpha \beta}\big|_{phys} = \tilde{U}^{ij}_{\tilde{\alpha} \tilde{\beta}}\big|_{phys} = 0 \quad (128)
\]

A composite index \( \hat{l} \equiv \{i, \tilde{\alpha}\} \) (a combination of fundamental representation and spinorial indexes) can be introduced, which relies on an exact \( U(8) \) symmetry. Therefore a new symmetry arise in perfect analogy with \[14\] and \[15\], which relate Zwanziger-like spinorial sector with the Faddeev-Popov sector fields, namely

\[10\]

- \( \hat{d}I \)-symmetry
  \[
  \hat{d}I^{c^\alpha} = \lambda^{\alpha}_{i}, \quad \hat{d}J^{\alpha}_{i} = \hat{d}^{\alpha}_{J^i}, \quad \hat{d}b^{\alpha} = g \varepsilon^{\alpha \beta \gamma} \lambda^{\beta}_{j} c, \quad \hat{d}J^{\alpha}_{\tilde{\alpha}} = V^{\overline{i}}_{\overline{\alpha} l}
  \]

- \( \overline{\hat{d}}I \)-symmetry
  \[
  \hat{d}^{\overline{c^\alpha}} = \eta^{\overline{\alpha}}_{i}, \quad \hat{d}^{\overline{\alpha}_{J^i}} = \hat{d}^{\overline{\alpha}}_{J^i}, \quad \hat{d}^{\overline{\beta}} = g \varepsilon^{\alpha \beta \gamma} \lambda^{\beta}_{j} c, \quad \hat{d}J^{\overline{\beta}}_{\overline{\alpha}} = -\tilde{U}^{\overline{i}}_{\overline{\alpha} l}
  \]

- \( \hat{d}I \)-symmetry
  \[
  \hat{d}^{c^\alpha} = \eta^{\alpha}_{i} + g \varepsilon^{\alpha \beta \gamma} \lambda^{\beta}_{j} c, \quad \hat{d}I^{\overline{c^\alpha}} = \eta^{\overline{\alpha}}_{i}, \quad \hat{d}^{\alpha}_{J^i} = \hat{d}^{\alpha}_{J^i}, \quad \hat{d}^{\beta}_{J^i} = \hat{d}^{\beta}_{J^i}, \quad \hat{d}J^{\overline{\alpha}}_{\overline{\beta}} = U^{\overline{i}}_{\overline{\alpha} l}, \quad \hat{d}J^{\overline{\beta}}_{\overline{\alpha}} = U^{\overline{i}}_{\overline{\alpha} l}
  \]

- \( \overline{\hat{d}}I \)-symmetry
  \[
  \hat{d}^{\overline{c^\alpha}} = \eta^{\overline{\alpha}}_{i} + g \varepsilon^{\alpha \beta \gamma} \lambda^{\beta}_{j} c, \quad \hat{d}^{\overline{\alpha}_{J^i}} = \hat{d}^{\overline{\alpha}}_{J^i}, \quad \hat{d}^{\overline{\beta}} = g \varepsilon^{\alpha \beta \gamma} \lambda^{\beta}_{j} c, \quad \hat{d}J^{\overline{\alpha}}_{\overline{\beta}} = \hat{d}^{\overline{\alpha}}_{J^i}, \quad \hat{d}J^{\overline{\beta}}_{\overline{\alpha}} = \hat{d}^{\overline{\alpha}}_{J^i},
  \]

where, because the non-linearity of \[12\] for spinors, we introduce external \( s \)-invariant sources \( \{\tilde{J}^{\alpha}_{\tilde{\alpha}}, J^{\alpha}_{i}\} \equiv (\tilde{J}_{\tilde{I}}, J_{I}) \) and \( \{\tilde{K}^{\alpha}_{\tilde{\alpha}}, K^{\alpha}_{i}\} \equiv (\tilde{K}_{\tilde{I}}, K_{I}) \) coupled to the nonlinear BRST transformations in the off-diagonal and diagonal Faddeev-Popov ghosts, in such a way that \( s \ell^{\alpha}_{\tilde{\alpha}} = -(J^{\alpha}_{\tilde{\alpha}} - K^{\alpha}_{\tilde{\alpha}}) \) and similarly for a source \( \tilde{\ell}^{\alpha}_{\tilde{\alpha}} \). Also, unlike the case of scalar matter fields, BRST invariance and power counting do not allow for additional interaction terms between spinors and Faddeev-Popov ghosts, as the \( \beta \) term in \[11\] or something of the kind, which is the biggest difference in relation to scalar matter case.

\[10\] Cf. the second line in \[34\] and the eq. \[35\].

\[11\] Cf. the equations \[61\] and \[62\].
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