Confronting SUSY GUT with Dark Matter, Sparticle Spectroscopy and Muon \((g - 2)\)

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ABSTRACT

We explore the implications of LHC and cold dark matter searches for supersymmetric particle mass spectra in two different grand unified models with left-right symmetry, \(SO(10)\) and \(SU(4)_c \times SU(2)_L \times SU(2)_R\) (4-2-2). We identify characteristic differences between the two scenarios, which imply distinct correlations between experimental measurements and the particular structure of the GUT group. The gauge structure of 4-2-2 enhances significantly the allowed parameter space as compared to \(SO(10)\), giving rise to a variety of coannihilation scenarios compatible with the LHC data, LSP dark matter and the ongoing muon g-2 experiment.

Keywords: Grand Unification, Supersymmetry, Dark Matter, LHC, sparticle spectroscopy

1 INTRODUCTION

In recent years, LHC results \([1, 2, 3, 4]\) and dark matter searches \([5, 6, 7, 8, 9, 10, 11]\) severely constrain some of the simplest standard model (SM) extensions. Nevertheless, we know that we have to find a way to go beyond the standard theory, which cannot accommodate massive neutrinos, nor explain the observed baryon asymmetry of the universe and recent cosmological observations \([12, 13, 14, 15]\). These issues can be addressed by imposing further unification, including grand unified theories and supersymmetry, which among others provides a natural candidate for dark matter \([16, 17]\).

Here we consider two supersymmetric models with gauge unification at \(M_{GUT}\), \(SO(10)\) \([18, 19]\) and \(SU(4)_c \times SU(2)_L \times SU(2)_R\) (4-2-2) \([20, 21, 22]\). We assume that at that scale the SUSY soft terms still preserve the group symmetry. This idea has been implemented in previous works for several GUTs \([27, 28, 29, 30, 31, 32]\). Here, we focus on the effects derived from the gauge structure of the two groups, while the sfermion mass terms remain universal, with all matter being contained in a single representation of the gauge group \(^1\). However, the soft higgs masses differ, since this sector is contained in different representations. The main difference between the two groups arises in the gaugino sector: for \(SO(10)\) it is natural to assume gauge mass universality, since this symmetry is broken to SM at the GUT scale. In the

\(^1\) This is the case in SO(10), but also in the LR symmetric 4-2-2. A larger study relaxing the LR symmetry in the 4-2-2 case is presented in \([33]\)
case of 4-2-2, however, the SM gauge couplings arise from combining broken and unbroken symmetries of the group, allowing different GUT relations among the gaugino masses.

Even though the two models differ in just a single relation for the gaugino masses, this results to vastly different phenomenological predictions. Among others, the possibility of small LSP masses in 4-2-2 enables satisfying the relic density predictions due to coannihilations, and gives rise to direct detection cross sections in the range of the current experiments, as well as visible signals at the LHC. It is also interesting to investigate whether in any of the two groups the discrepancy between the experimental value of the anomalous magnetic moment of the muon and the SM prediction \cite{34}, can be explained through supersymmetric contributions.

Within this framework, we study the predictions for sparticle spectroscopy in the two scenarios, also requiring LSP dark matter through coannihilations. We identify distinct differences between the two groups, which result in direct correlations between the experimental measurements and the group structure and symmetries, which can be tested in future searches.

2 GUT MODELS AND INPUT PARAMETER

As in \cite{27}, where SU(5) has been compared with SU(5) and flipped SU(5), we assume that SUSY breaking occurs at a scale $M_X > M_{GUT}$, through a mechanism that generates flavour blind soft-terms. Between the scales $M_X$ and $M_{GUT}$, renormalisation and additional flavour symmetries may induce non-universalities for soft terms that belong to different representations; on the contrary, particles that belong to the same representation have common soft masses.

The soft terms for the fields in an irreducible representation $r$ of the unification group are defined as multiples of a common scale $m_0$ as:

$$m_r = x_r m_0,$$

while the trilinear terms are defined as:

$$A_r = Y_r A_0, \quad A_0 = a_0 m_0.$$

Here, $Y_r$ is the Yukawa coupling associated to the $r$ representation. We use the standard parametrisation, with $a_0$ being a dimensionless factor, which we consider as representation independent. This is justified because the representation dependence is already taken into account in the Yukawa couplings, and including a further factor can be confusing.

Let us see how the above are applicable to the two gauge groups under discussion:

- \textit{SO(10)}

The simplest possibility arises within an \textit{SO(10)} GUT, in which all quarks and leptons are accommodated in the same $16$ representation, leading to left-right symmetric mass matrices. We assume that the up and down higgses are in a pair of $10$ representations. This assignment determines sfermion mass matrices and beta functions, and results in a common mass for all sfermions and two different higgs masses $m_{h_u}$ and $m_{h_d}$ (thus identified with the NUHMSSM).
In addition to the CMSSM, therefore, we introduce two new parameters $x_u$ and $x_d$ defined as:

$$m_{16} = m_0, \quad m_{H_u} = x_u m_{16}, \quad m_{H_d} = x_d m_{16}. \quad (3)$$

Similarly, the $A$-terms are defined as:

$$A_{16} = a_0 \cdot m_0. \quad (4)$$

• 4-2-2

The main features of the 4-2-2 that are relevant for our discussion are summarised below. The 4-2-2 gauge symmetry can be obtained from a spontaneous breaking of SO(10) by utilizing either the 54 dimensional or the 210 dimensional representation, allowing for some freedom of choice. The former case, where $SO(10)$ breaks through a Higgs 54-plet can be naturally combined with a left-right symmetry [34]. By contrast, this left-right symmetry is explicitly broken in the latter case. Here, we will mostly focus on the left-right symmetric 4-2-2 model, which has the minimal number of free parameters and can be more directly compared with $SO(10)$. In this case, the gaugino masses associated with $SU(2)_L$ and $SU(2)_R$ are the same, while the gluino mass, associated with $SU(4)_C$, can differ.

The main relations are therefore the following:

• Gaugino masses: The hypercharge generator from 4-2-2 implies the relation

$$M_1 = \frac{3}{5} M_2 + \frac{2}{5} M_3. \quad (5)$$

• Soft masses: All sfermions are accommodated in a 16 representation, and have a common mass $m_{16} = m_0$. The Higgs fields are in a 10-dimensional representation with D-term contributions that result to $m_{H_u,d}^2 = m_{10}^2 \pm 2 M_D^2$. In our notation, these values are:

$$r_u = \frac{m_{H_u}}{m_{16}}, \quad r_d = \frac{m_{H_d}}{m_{16}}; \quad (6)$$

with $r_u < r_d$.

In our computations we assume a common unification scale $M_{GUT}$ defined as the meeting point of the $g_1$ and $g_2$ gauge couplings. The GUT value for $g_3$ is obtained by requiring $\alpha_s(M_Z) = 0.187$. Above $M_{GUT}$ we assume a unification group that breaks at this scale. SUSY is broken above $M_{GUT}$ by soft terms that are representation-dependent while preserving flavour blindness. We perform a parameter space scan using as a guide the representation pattern at the GUT scale for soft scalar terms. For this purpose, we extend the CMSSM universal scenario through non-unified soft terms, consistent with the representations of SO(10) and 4-2-2. Even in their simplest versions, these scenarios result in soft term correlations that enlarge the size of the parameter space that can is compatible with the neutralino relic density from Planck.

We perform runs with soft terms up to 10 TeV and a parameter range summarised below:
$100\text{GeV} \leq m_0 \leq 10\text{TeV}$

$50\text{GeV} \leq M_1 \leq 4\text{TeV}$

$50\text{GeV} \leq M_2 \leq 4\text{TeV}$

$-10\text{TeV} \leq A_0 \leq 10\text{TeV}$

$2 \leq \tan\beta \leq 65$

$-1.9 \leq x_u \leq 1.5$

$0 \leq x_d \leq 3.4$

$x_u \leq x_d$ \hspace{1cm} (7)

3 EXPERIMENTAL CONSTRAINTS AND PARAMETER SPACE SCAN

For our analysis we use Superbayes \cite{36,37,38}, a package that includes MSSM RGEs, relic density computations, phenomenological bounds and updated LHC bounds on SUSY particles, as well as phenomenological constraints derived from b-physics. We further impose the constraint on the neutralino-nucleon cross section provided by the more recent Xenon-100 and LUX upper limits. SuperBayeS-v2.0 performs a sample algorithm using the code MultiNest v2.18 \cite{39}.

It is well known that, if the required amount of relic dark matter is provided by neutralinos, particular mass relations must be present in the supersymmetric spectrum. In addition to mass relations, we use the neutralino composition to classify the relevant points of the supersymmetric parameter space. The higgsino fraction of the lightest neutralino mass eigenstate is characterized by the quantity

$$h_f \equiv |N_{13}|^2 + |N_{14}|^2,$$ \hspace{1cm} (8)

where the $N_{ij}$ are the elements of the unitary mixing matrix that correspond to the higgsino mass states. Thus, we classify the points that pass the constraints discussed in Section 2 according to the following criteria:

- **Higgsino $\chi_1^0$:**

$$h_f > 0.1, \quad |m_A - 2m_{\chi}| > 0.1m_{\chi}.$$ \hspace{1cm} (9)

In this case, the lightest neutralino is higgsino-like and, as we discuss later, the lightest chargino $\chi_1^\pm$ is almost degenerate in mass with the $\chi_1^0$. The couplings to the SM gauge bosons are not suppressed and $\chi_1^0$ pairs have large cross sections for annihilation into $W^+W^-$ and $ZZ$ pairs, which may reproduce the observed value of the relic abundance. Clearly, coannihilation channels involving $\chi_1^\pm$ and $\chi_2^0$ also contribute.

- **$A/H$ resonances:**

$$|m_A - 2m_{\chi}| \leq 0.1m_{\chi}.$$ \hspace{1cm} (10)
The correct value of the relic abundance is achieved thanks to $s$-channel annihilation, enhanced by the resonant $A$ propagator. The thermal average $\langle \sigma_{ann} v \rangle$ spreads out the peak in the cross section, so that neutralino masses for which $2m_\chi \simeq m_A$ does not exactly hold, can also undergo resonant annihilations.

- $\tilde{\tau}$ coannihilations:

$$ h_f < 0.1, \quad (m_{\tilde{\tau}_1} - m_\chi) \leq 0.1 m_\chi $$

The neutralino is bino-like, annihilation into leptons through $t$-channel slepton exchange is suppressed, and coannihilations involving the nearly-degenerate $\tilde{\tau}_1$ are necessary to enhance the thermal-averaged effective cross section.

In the 4-2-2 model we get three additional types of coannihilation:

- $\tilde{\chi}^+$ coannihilations:

$$ h_f < 0.1, \quad (m_{\tilde{\chi}^+} - m_\chi) \leq 0.1 m_\chi. $$

The lightest chargino is light and nearly degenerate with the bino-like neutralino.

- $\tilde{g}$ coannihilations:

$$ h_f < 0.1, \quad (m_{\tilde{g}} - m_\chi) \leq 0.1 m_\chi. $$

The gluino is light and nearly degenerate with the bino-like neutralino.

- $\tilde{t}_1$ coannihilations:

$$ h_f < 0.1, \quad (m_{\tilde{t}_1} - m_\chi) \leq 0.1 m_\chi. $$

The $\tilde{t}_1$ is light and nearly degenerate with the bino-like neutralino. These coannihilations were found to also be present in the flipped SU(5) model (but not $SO(10)$ or $SU(5)$).

We note that the LR symmetry in the scalar soft masses does not allow scenarios with $\tilde{\nu} - \tilde{\tau} - \chi$ coannihilations, which appear in the SU(5) scenarios of [27] or in the LR asymmetric 4-2-2.

4 PLANCK COMPATIBLE REGIONS, MUON ANOMALOUS MAGNETIC DIPOLAR MOMENT AND DARK MATTER SEARCHES

In $SO(10)$, the DM regions compatible with the Planck neutralino relic density are similar to the CMSSM. However, their parameter space is enlarged due to the two independent higgs mass terms (essentially reproducing the non-universal Higgs CMSSM). Due to the additional freedom, $\tilde{\tau} - \chi$ coannihilations (orange circles in the Figs.) correspond to neutralino masses below 650 GeV. Resonances of the pseudoscalar Higgs mediated neutralino annihilation channels (brown crosses in the Figs.) are present in the entire range of $m_\chi$ under consideration, to be contrasted to the funnel-like area of the CMSSM. Neutralinos with a large higgsino component are localized at values $m_\chi$ around 1 TeV.
Figure 1. Prediction for $\delta a_{\mu}^{\text{SUSY}}$ versus $m_{\chi}$. The red lines denote the 3-\(\sigma\) bounds for the experimental discrepancy of $a_{\mu}$ with respect to the SM prediction. The left plot corresponds to SO(10) and the right to 4-2-2. Different symbols and color codes are assigned to each class of models and this notation is maintained in the rest of the plots: Turquoise dots stand for Higgsino DM, black crosses for $\tilde{\chi}^{\pm} - \chi$ coannihilations, brown crosses for $A/H$ resonances, blue crosses for $t - \chi$ coannihilations, orange dots for $\tilde{\tau} - \chi$ coannihilations, and green triangles for $\tilde{g} - \chi$ coannihilations.

In 4-2-2, the GUT relation among the Higgsino masses results to wino-like charginos and low mass gluinos that can coannihilate with the lightest neutralino. Moreover, even if the sfermion masses are kept universal at the GUT scale, it is possible to find models where light stops can coannihilate with the neutralino. In 4-2-2, therefore, the additional freedom among the GUT values of the gaugino masses introduces three new kind of coannihilations that satisify the Planck neutralino relic density requirements.

4.1 Muon g-2

The discrepancy between the experimental measurement of the muon $g - 2$ and the respective SM prediction, $\delta a_{\mu}^{\text{SUSY}} = (28.7 \pm 8.2) \times 10^{-10}$, can be attributed to additional contributions from SUSY particles. However, for these contributions to be above the 3 – \(\sigma\) level, sparticle masses below \(\sim 500\) GeV are required. By contrast, the experimental values for the Higgs mass [1, 2] can be accommodated in SUSY models with universal soft terms at the GUT scale with a heavy SUSY spectrum. Since in 4-2-2 unification the less restrictive gaugino mass relations allow a lighter SUSY spectrum than in SO(10) (also compatible with DM, especially for the models with chargino coannihilations), we expect that the study of $\delta a_{\mu}^{\text{SUSY}}$ favours this unification group.

In Figure 1 we display the prediction for $\delta a_{\mu}^{\text{SUSY}}$ versus the neutralino mass values. Although this will be discussed later we include, already at this stage, the points excluded by the LHC analysis. We can see that the value of $\delta a_{\mu}^{\text{SUSY}}$ is always lower than the central value. In the case of SO(10), we can see that just a few points can produce a significant contribution to the muon g-2 (however, these points are excluded by the LHC bounds). In the case of 4-2-2 however, several points are compatible with a contribution to the muon g-2; even though the respective region is small, it is compatible with chargino and stau coannihilation regions that are not excluded by the LHC bounds.

4.2 Dark Matter Searches

The relation among different soft-terms will determine the composition of the neutralino, which is important for its detection. In this respect, different GUTs result to different predictions, which can
tested in both direct and indirect detection experiments. Here we compare the spin independent (SI) neutralino-nucleon cross section of the models under consideration, with the experimental bounds and prospects. Although spin dependent [11] and indirect detection bounds also exclude many SUSY models, the current and projected experiment for the SI neutralino-nucleon cross section can test the predictions of the majority of the models considered here.

Fig. 2 is indicative of how a change on the gauge unification conditions enables a significant enhancement of the parameter space when passing from $SO(10)$ to 4-2-2. It is also possible to see how the direct detection bounds can impose important constraints on the parameter space. For instance, we can see that the latest update of the Xenon-1T bound [9] excludes many points with A-resonances and higgsino DM regions, especially in the case of the 4-2-2 model. But further than that, Fig. 2 shows that projected experiments can be sensitive to most of the parameter space presented here. In the case of $SO(10)$, DARWIN will cover the entire parameter space. In the case of 4-2-2, certain areas of $\tilde{\chi}^\pm - \chi$ still remain below detection prospects.

5 LHC SEARCHES

In this section we investigate the constraints imposed by the LHC on the unified SUSY models under consideration. Each model can be associated to a particular set of particle hierarchies and decays, which are then compared with the generic data provided by the ATLAS and CMS collaborations [42, 43]. These comparisons are made with the help of Simplified Model Spectra (SMS) which can be defined by a set of hypothetical SUSY particles and a sequence of products and decay modes that have to be compared with those expected in our specific model. As a result, an individual check has to be done for every model, while, due to mismatches between the theoretical and the experimental results, it is not possible to provide contour plots where one can easily see which mass ranges are excluded. This task is simplified by using public packages like Smodels-v1.1.1 [44], which provides a powerful tool to perform a fast analysis of a large number of models [45, 46]. By using this package, the theoretical models are decomposed in SMS and can be contrasted with the existing LHC bounds if there is a match in the respective topologies.
Figure 3. Chargino masses versus $m_\chi$ for the different models of SO(10) (left) and 4-2-2 (right). The points that can be compared with the LHC bounds follow the same code as in Figure 2, while the ones excluded are inside a purple square. Grey points correspond to models that cannot be decomposed to SMS LHC signals (circles for A-resonances, squares for higgsino DM and diamonds for stop coannihilations). The red line corresponds to the largest masses that LHC can probe, according to the CMS and ATLAS public analysis.

Figure 4. As in Fig. 3 for the $m_{\tilde{g}}$ versus $m_\chi$ scatter plot.

In Figs. 3 – 6 we classify the models as (i) the ones that can be compared with the LHC data (either satisfying the bounds or being excluded) and (ii) those that cannot be tested at the LHC; the latest are points that either predict processes with very low cross sections or result in topologies that are not tested at the LHC. For the points of category (i), we follow the same notation as in previous sections for points that satisfy the LHC bounds, and denote by magenta squares those excluded. The points of category (ii) that cannot be tested, are drawn in grey. For clarity, we display only points corresponding to A-resonant channels (circles), higgsino DM (squares) and stop coannihilations (diamonds) which lie at high mass areas in which the non-tested models dominate; other classes of models lie in the same regions with the tested ones. The red solid boundary is obtained by combining the simplified model bounds from LHC searches. Since these models often do not apply to our particular cases it cannot be considered as an exclusion line (however, excluded points must be included in at least one of these contours). Nevertheless, it is useful to include this line for illustrative purposes, since it gives an idea of the range of masses explored at the LHC for every SUSY particle.
Figure 5. As in Fig. 3 for the $m_{\tilde{t}}$ versus $m_{\chi}$ scatter plot.

As we have already emphasized, despite the fact that the left-right symmetric 4-2-2 unification differs from SO(10) by just one additional gaugino mass, this gives rise to novel possibilities for DM models. Even similar classes of models that satisfy the DM constraints, now correspond to a different range of SUSY masses. This is due to the fact that the relic density constraints are satisfied through coannihilations, resonances or low values of the $\mu$ term, only in models with certain mass conditions (the areas of higgsino DM and A-resonances have larger values of SUSY partners in SO(10)). Consequently, the resulting LHC signals are very different in the two groups.

The SO(10) unified models are highly constrained by the LHC bounds. Most of the points that Smodels can compare with the LHC bounds are in the stau coannihilation area, and a good part of them are already excluded. These include a large majority of the models with a muon g-2 contribution at the $3 - \sigma$ level. No points on the Higgsino DM region can be tested at the LHC according to the Smodels analysis and only a few points on the A-resoance area can be reached, none of them excluded by the current bounds. The excluded points are affected mostly by the gluino and stop bounds. We can see in Fig. 6 that the predicted sbottom masses are outside the LHC accessible area. The same happens with signals involving quarks of the lighter generations.

The 4-2-2 models have a richer structure with respect to experimental signatures at the LHC, than the ones arising from SO(10). In this case we find points with higgsino DM and A-resonances in a wider spectrum of masses; moreover, we find a larger number of points that Smodels can decompose in signals that can be compared with the LHC bounds. In Fig. 3 we can see that models with chargino coannihilations have a compressed spectrum that results to light charginos compatible with the LHC bounds. However, we can see in Fig. 4 that such points are excluded by the gluino bounds. Despite that, some points predicting a relevant SUSY contribution to the muon g-2 are not yet excluded by the LHC. We can see in Fig. 5 that bounds on stop searches affect many of the excluded points, while only a few models predict sbottom masses that can be affected by LHC. Processes involving first and second generation squarks are outside the scope of the current LHC bounds.

6 CONCLUSIONS

In this work we have performed a comparative study of SO(10) and $SU(4)_c \times SU(2)_L \times SU(2)_R$ grand unification with respect to the LHC sparticle mass spectra, cold dark matter and muon g – 2 predictions.
Based on the remarkable complementarity between the LHC and dark matter searches, we show how the different patterns of soft SUSY-breaking terms at the GUT scale can be used to distinguish the two groups in experimental searches.

In particular, the gauge and symmetry breaking structure of 4-2-2 enhances significantly the allowed parameter space as compared to SO(10), and gives rise to three additional coannihilation scenarios for dark matter (chargino, gluino and stop coannihilations). Even similar types of coannihilations that satisfy the DM constraints, now correspond to a different range of SUSY masses. Moreover, areas where the discrepancy between the theoretical and experimental values of muon g-2 can be reduced via a supersymmetric contribution are identified.

Gluino coannihilations are particularly important, since they are a unique feature of 4-2-2 and do not appear in other GUT schemes. They are a direct outcome of the particular gaugino mass relations of the model that results in relatively light gluinos. Chargino coannihilations are also found and, together with higgsino dark matter, are the most frequently encountered scenarios. Stop coannihilations also arise (these can also appear in flipped SU(5) \[27\]). In all cases, we get concrete predictions for the gaugino mass ratios that favour the respected scenarios and can be tested in future searches.

The overall message from the significant phenomenological differences between two groups that share so many common features is clear: although no SUSY signal has been found so far, there are still several alternative possibilities to explore and the 4-2-2 group is one of them.

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