PARAMETRIZATIONS OF THE SEESAW
or
CAN THE SEESAW BE TESTED?

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This proceedings contains a review, followed by a more speculative discussion. I review different coordinate choices on the 21-dimensional parameter space of the seesaw, and which of these 21 quantities are observable. In MSUGRA, there is a 1-1 correspondence between the parameters, and the interactions of light (s)particles. However, not all of the 21 can be extracted from data, so the answer to the title question is “no”. How to parametrise the remaining unknowns is confusing—different choices seem to give contradictory results (for instance, to the question “does the Baryon Asymmetry depend on the CHOOZ angle?”). I speculate on possible resolutions of the puzzle.

1. Introduction
The seesaw mechanism \(^1\) is a theoretically elegant way to get the small neutrino masses we observe. It predicts that the light neutrino masses are majorana, which could be verified in neutrinoless double $\beta$ decay experiments. In the absence of Supersymmetry, it predicts that lepton flavour violation (LFV), and CP violation are suppressed by powers of the neutrino mass, making the rates very low outside the neutrino sector. On the other hand, if partners were discovered, for instance at the LHC, observable CP and flavour violation can be imprinted by the seesaw into the slepton mass matrices. Experimentally verifying these predictions would increase our confidence in the seesaw. Measuring something different—for instance majorana masses, no SUSY, and large neutrino magnetic moments—would indicate that there is other new physics, or more new physics in the lepton sector than just the seesaw (see e.g. Smirnov, in this volume). The aim

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here is to ask if we can test the seesaw, or as discussed below, a particular implementation of the seesaw mechanism.

This proceedings is written from a bottom-up phenomenological perspective. I want to make as few assumptions as possible about the theory at scales above $m_W$, so I assume the particle content is the Standard Model (SM), or the MSSM with universal soft masses, plus three $\nu_R$, and allow all possible renormalisable interactions. This gives the Lagrangian (in the SM case)

$$L = Y_e \bar{e}_RH_d \cdot \ell_L + Y_\nu \bar{\nu}_R H_u \cdot \ell_L + \frac{M}{2} \bar{\nu}_R \nu_R + h.c.$$  

(1)

where $\ell$ are the lepton doublets, $\bar{\nu}_R = (C \nu_R^\dagger)\gamma_0$, and generation indices are suppressed. The index order on the Yukawa matrices is right-left.

To test this implementation of the seesaw mechanism, we need to

1. extract the unknown parameters of eqn (1) from data
2. predict an additional observable calculated from those parameters
3. verify the prediction

These proceedings discuss the first step. If it could be accomplished successfully, we could calculate the baryon asymmetry produced in various leptogenesis mechanisms (see Hambye and Raidal in this volume), which would be a fabulous cross-check of particle physics and cosmology.

There are many other versions of the seesaw (2 $\nu_R$, type II with scalar triplets, with extra singlets...), which are motivated from various theoretical perspectives (see T Hambye in this volume). The model used here contains three $\nu_R$ because there are three generations, and only three $\nu_R$ because it is useful to know how well the simple model works before adding complications.

I want to test the seesaw mechanism, rather than a particular model, so GUT models, textures, and theoretical considerations of “naturalness” are avoided (insofar as possible). The seesaw mechanism can accomodate any neutrino masses and mixing angles (And almost any sneutrino mass matrix). Particular models may prefer certain ranges for observables, so data can provide hints about the theory that gives the Lagrangian of eqn (1). This is discussed elsewhere in this volume (G Ross and P Ramond). However, if these theoretical expectations are not fulfilled, it is difficult to know if the model was wrong, or if there is more new physics in addition to the seesaw mechanism.
2. Parametrisations

Twenty-one parameters are required to fully determine the Lagrangian of eqn (1). Three of the possible ways these can be chosen are discussed here.

The usual “top-down” description of the theory is as follows. At energy scales $\Lambda \gtrsim M$, where the $\nu_R$ are propagating degrees of freedom, one can always choose the $\nu_R$ basis where the mass matrix $M$ is diagonal, with positive and real eigenvalues: $M = D_M$. Similarly, one can choose the $\ell$ basis such that the charged lepton Yukawa $Y_e$ is diagonal on its LH indices: $Y_e^T Y_e = D_2 Y_e$. The remaining neutrino Yukawa matrix $Y_\nu$ is an arbitrary complex matrix, from which three phases can be removed by phase redefinitions on the $\ell_i$. It is therefore described by 9 moduli and 6 phases, giving in total 21 real parameters for the seesaw. See $^5$ for a more elegant counting, in particular of the phases.

To relate various parametrisations of the seesaw, it is useful to diagonalise $Y_\nu$, which can be done with independent unitary transformations on the left and right:

$$ Y_\nu = V_R^\dagger D_Y V_L $$

(2)

So in the top-down approach, the lepton sector can be described by the nine eigenvalues of $D_M$, $D_Y$, and $D_{Y_e}$, and the six angles and six phases of $V_L$ and $V_R$. Notice that in this parametrisation, the inputs are masses and coupling constants of the propagating particles at energies $\Lambda$, so it makes “physical” sense.

The effective mass matrix $m$ of the light neutrinos can be calculated, in the $D_Y$ basis (charged lepton mass eigenstate basis):

$$ m = \kappa \langle H_u \rangle^2 = Y_{\nu}^T D_M^{-1} Y_{\nu} \langle H_u \rangle^2 = V_L^T D_Y V_R^* D_M^{-1} V_R^\dagger D_Y V_L \langle H_u \rangle^2 $$

(3)

$\kappa$ is introduced to avoid the Higgs vev $\langle H_u \rangle$ cluttering up formulae. The leptonic mixing matrix $U$ is extracted by diagonalising $\kappa$:

$$ \kappa = U^* D_\kappa U^\dagger $$

(4)

where $D_\kappa = \text{diag}\{\kappa_1, \kappa_2, \kappa_3\}$, and $U$ is parametrised as

$$ U = \hat{U} \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta}) $$

(5)

$\alpha$ and $\beta$ are “Majorana” phases, and $\hat{U}$ has the form of the CKM matrix

$$ \hat{U} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} $$

(6)
Alternatively, the (type I) seesaw Lagrangian of eqn (1) can be described with inputs from the left-handed sector. This is referred to as a “bottom-up” parametrisation, because the left-handed (SU(2) doublet) particles have masses \( \lesssim \) the weak scale. \( D_Y, U \) and \( D_\kappa \), can be taken as a subset of the inputs. To identify the remainder, imagine sitting in the \( \ell \) basis where \( \kappa \) is diagonal, so as to emphasize the parallel between this parametrisation and the previous one (this is similar to the \( \nu_R \) basis being chosen to diagonalise \( M \)). If one knows \( Y_\nu^\dagger Y_\nu \equiv W_L D_\kappa^2 W_L^\dagger \) in the \( D_\kappa \) basis, then the \( \nu_R \) masses and mixing angles can be calculated:

\[
M^{-1} = D_Y^{-1} W_L^\dagger D_\kappa W_L^\dagger D_Y^{-1} = V_R^* D_M^{-1} V_R^\dagger \tag{7}
\]

In this parametrisation, there are three possible basis choices for the \( \ell \) vector space: the charged lepton mass eigenstate basis (\( D_Y \)), the neutrino mass eigenstate basis (\( D_\kappa \)), and the basis where the \( Y_\nu \) is diagonal. The first two choices are physical, that is, \( U \) rotates between these two bases. \( D_Y, D_\kappa \) and \( U \) contain the 12 possibly measurable parameters of the SM seesaw. The remaining 9 parameters can be taken to be \( D_{\nu R} \) and \( V_L \) (or \( W = V_L U \)). In SUSY one can hope to extract these parameters from the slepton mass matrix.

The Casas-Ibarra\(^7\) parametrisation is very convenient for calculations. It uses \( D_M, D_\kappa \) and \( D_Y \) as inputs, and the transformations \( U \) and \( R \), which go between the bases where these matrices are diagonal. \( U \) is the usual leptonic mixing matrix. The matrix \( R = D_M^{-1/2} Y_\nu D_\kappa^{-1/2} \), is a complex orthogonal matrix, which transforms between the \( D_M \) and \( D_\kappa \) bases. (Since \( M \) and \( \kappa \) are respectively in the RH and LH neutrino vector spaces, it is unsurprising that the transformation matrix is not unitary.) \( R \) can be written as \( R = \text{diag}\{\pm 1, \pm 1, \pm 1\} \hat{R} \) where the \( \pm 1 \) are related to the CP parities of the \( N_i \), and \( \hat{R} \) is an orthogonal matrix with complex angles:

\[
\hat{R} = \begin{bmatrix} c_{13} c_{12} & c_{13} s_{12} & s_{13} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} & c_{23} c_{12} - s_{23} s_{13} s_{12} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} & -s_{23} c_{12} - c_{23} s_{13} s_{12} & c_{23} c_{13} \end{bmatrix} \tag{8}
\]

The aim of this proceedings is to reconstruct the RH seesaw parameters from the LH ones, many of which are accessible at low energy. However, as discussed in the following section, reconstruction is impossible. We can at best try to establish relations between observables, which turns out to be quite confusing. \( R \) will be helpful in discussing these puzzles.

In summary, the lepton sector of the SM + seesaw can be parametrised with \( D_{\nu R} \), the real eigenvalues of two more matrices, and the transforma-
tions among the bases where the matrices are diagonal. The matrices-to-be-diagonalised can be chosen in various ways:

1. “top-down”—input the $\nu_R$ sector: $D_M$, $D_{Y, Y^\dagger}$, and $V_R$ and $V_L$.
2. “bottom-up”—input the $\nu_L$ sector: $D_\kappa$, $D_{Y^\dagger, Y}$, and $V_L$ and $U$.
3. “intermediate”—the Casas-Ibarra parametrization: $D_M$, $D_\kappa$, and $U$ and a complex orthogonal matrix $R$.

3. (Supersymmetric) reconstruction?
If the matrices $D_\kappa$, $D_{Y, Y^\dagger}$, $V_L$ and $U$ were known, it would be possible to reconstruct the masses and mixing angles of the $\nu_R$. Can the elements of these matrices be determined?

We know the masses of the charged leptons, so we know $D_{Y, e}$ (modulo $\tan \beta$ in SUSY models).

We know two mass differences in the neutrino sector. If the light neutrinos are degenerate, measuring the overall scale of their masses is possible and would determine $D_\kappa$. However, if the mass pattern is hierarchical or inverse hierarchical, we would know only $\kappa_3$ and $\kappa_2$. See the contribution of K Heeger, for present and future accuracy on $D_\kappa$, and $U$.

In the mixing matrix $U$, we currently know two angles. We hope to measure the third, and also the “Dirac” phase $\delta$. But the “majorana” phases appear only in slow lepton number changing processes, so at the moment do not seem experimentally accessible.

The remaining parameters to be determined are the eigenvalues of $Y_\nu$, and the matrix $V_L$. In supersymmetric models $Y_\nu$ contributes via loops to the slepton mass matrix. Consider a model, such as gravity- or anomaly-mediated SUSY breaking, where the soft masses are universal at a scale $\Lambda > M_3$. In renormalisation group running between $\Lambda$ and $m_W$, the slepton mass matrix will acquire flavour off-diagonal terms, due to loops involving the $\nu_R$ (see Masiero in this volume). Using the leading log approximation for the RG running, the sneutrino mass matrix, in the $D_{Y, e}$ basis, is:

$$[m_{\tilde{\nu}}^2]_{ij} \simeq (\text{diag part}) - \frac{3m_0^2 + A_0^2}{8\pi^2} (Y_\nu^t)^{ik} (Y_\nu)^{kj} \frac{\Lambda}{M_k}$$

where $m_0$ and $A_0$ are the universal soft parameters at scale $\Lambda$.

It is tantalising that the seesaw contribution to flavour violation in the sleptons is potentially observable, and depends on the heavy neutrino masses in a different way than $\kappa$. If we could determine $[m_{\tilde{\nu}}^2]$ exactly (the three masses, three mixing angles, and three phases), and if we take seriously the assumption of universal soft masses, then we could reconstruct...
the renormalisable interactions of the high-scale seesaw—that is, the $\nu_R$ masses and Yukawa couplings—from the mass matrices and mixing angles of weak-scale particles.

Unfortunately, neither of these conditions is likely to be fulfilled. Firstly, not all the parameters of $[m^2_{\tilde{\nu}}]$ can be measured with the required accuracy. The diagonal elements of the second term of eqn (9) shift $m^2_{\tilde{\nu}}$ by of order $y^2_i$, so a large $Y_{\nu_e}$ eigenvalue $\sim 1$ could have a measurable effect. However, if $Y_{\nu}$ has a hierarchy similar to the quark Yukawas, the effects of the first and second generation $y_i$ are (undetectably) small.

The flavour-changing elements of eqn (9) could be seen at colliders and induce rare decays, such as $\mu \to e\gamma$. A very optimistic experimental sensitivity of order $BR(\tau \to e\gamma) \sim 10^{-9}$ (the current limit is $\sim 10^{-7}$), could probe $|[V_L]_{3\mu}[V_L]_{3e}^\ast y^2_3| \gtrsim 10^{-(1/2)}$. $\mu - e$ flavour violation is more encouraging: there are plans to reach $BR(\mu \to e\gamma) \sim 10^{-13}$, which would be sensitive to $|[V_L]_{3e}[V_L]_{3\mu}^\ast y^2_3| \gtrsim 10^{-(3/4)}$. However, to extract a “measurement” of either of the $[V_L]_{3\ell}$ from rare decays would require knowing all the masses and mixing angles for the other SUSY particles contributing to the decay.

For hierarchical $Y_{\nu}$ eigenvalues, eqn (9) implies that the three off-diagonal elements of $[m^2_{\tilde{\nu}}]$, are determined by two matrix elements of $V_L$. So one angle of $V_L$ is unknown, and there should be some correlation between $[m^2_{\tilde{\nu}}]_{\tau\mu}$, $[m^2_{\tilde{\nu}}]_{\tau\tau}$, and $[m^2_{\tilde{\nu}}]_{\mu\tau}$. Notice, however, that this is a prediction of hierarchical $Y_{\nu}$. In the bottom-up parametrisation, the slepton mass matrix determines $V_L$ and $D_{Y_{\nu}}$, rather than the seesaw making predictions for $[m^2_{\tilde{\nu}}]$.

Now we come to the three phases of $V_L$. To extract all of these is quite hypothetical; it would require three independent measurements of CP violation in the sleptons. Two possibilities at colliders are charged lepton asymmetries in slepton decays and sneutino-anti-sneutrino oscillations. The slepton phases also contribute to CP violating observables in the leptons, in conjunction with phases from other SUSY particles. This is discussed in this volume by Hisano.

The second objection to extracting seesaw parameters from eqn (9), is that we do not know that soft masses are universal. It is a reasonable assumption in top-down analyses, because we know that flavour violation mediated by sparticles must be suppressed. But I know of no way to distinguish contributions to $[m^2_{\tilde{\nu}}]$ that come from the RG running with the seesaw, from those that come from non-universal soft masses, threshold effects, other particles with flavour off-diagonal couplings, etc... So
measuring $[m^2_\nu]$ exactly could be used to set an upper bound on the seesaw contributions (if one makes the reasonable assumption that there are no cancellations among different contributions), but would not determine them.

It is also possible-in-principle to reconstruct the non-SUSY seesaw: Broncano et al.\textsuperscript{15} observed that the 21 parameters can be extracted from the coefficients of dimension 5 and 6 operators in the Standard Model. However, the coefficients of the dimension 6, lepton number conserving operators are suppressed by two powers of the $\nu_R$ mass, so are (unobservably) small.

In summary, the parameters of the type I seesaw cannot be extracted from data. This should hardly be surprising—we do not usually expect to reconstruct high-scale theories (\textit{e.g.} which GUT, and how does it break?) from weak-scale observations. So why do we even ask if it is possible in the seesaw? I am aware of two peculiarities, which make the seesaw “reconstructable in principle”: the $\nu_R$ only have interactions with light particles (via $Y_\nu$), and the effective operators induced at low energy are experimentally accessible (in principle!) for all flavour indices. To see why these features are significant, compare to proton decay— which I assume to be mediated by a “triplet higgsino” dressed with a squark loop. However accurately we measure every available proton decay channel, we cannot determine the mass and couplings of the triplet higgsino, because we must always sum over squark flavours in the loop, and we only measure proton decay with first generation quarks in the initial state (unlike the three generation $\nu$ and $\bar{\nu}$ mass matrices).

4. Independence, orthogonality and relations when we cannot reconstruct

The 21 parameters of the seesaw cannot be determined from observation, but some sort of partial reconstruction, using the available data, could be possible. This turns out to be much more confusing than one would anticipate. To identify the problem, imagine calculating the baryon asymmetry as a function of parameters separated into three categories: those we know now, those we hope to know, and those we will never know. It then seems straightforward to study how the asymmetry depends on, for instance, $\theta_{13}$. But in practise it is anything but transparent (see eqn 11): the asymmetry is independent of $U$ in the Casas-Ibarra parametrisation, but does depend on $U$ in the bottom-up version. That is, the choice of parametrisation for
the unmeasurables, changes the dependence of one observable (the baryon asymmetry) on another ($\theta_{13}$). It would be better to ask “is $\epsilon_1$ sensitive to $\theta_{13}$?”—this has a unique and useful answer, as discussed in the next section.

The aim of this section is to explore how different coordinates on seesaw parameter space depend on each other, and what we mean by “depends on” and “independent”. I start by reviewing some contradictory statements which can be derived using various parametrisations. Then I present a toy model using parametrisations of the plane, where these same contradictions arise, and where the resolution is obvious. Lastly I suggest how the analogy of the plane could be related to the seesaw.

It has been claimed in various papers that $\epsilon_1$, the CP asymmetry of thermal leptogenesis, is independent of the leptonic mixing matrix $U$. This seems intuitively reasonable, because leptogenesis involves the $\nu_R$, and is independent of $Y_e$. In the limit of hierarchical $\nu_R$:

$$\epsilon_1 \simeq -\frac{3M_1}{8\pi|Y_{\nu}Y_{\nu}^T|_{11}} \Im \{Y_{\nu} \kappa^* Y^T\} = -\frac{3M_1}{8\pi} \frac{\Im \{R_{1k}^2 \kappa_k^2\}}{|R_{1k}|^2 \kappa_k}$$  \hspace{1cm} (10)

$$\propto \Im \left\{ V_L U D_{\kappa}^2 U^T V_L^T D_{Y_e}^{-2} V_L^* D_{\kappa} U^* V_L^T D_{Y_e}^{-2} \right\}$$  \hspace{1cm} (11)

where the second equality of (10) is in the Casas-Ibarra parametrisation. To translate eqn (10) into bottom-up coordinates, requires calculating the mass and eigenvector (first column of $V_R$) of $\nu_{R1}$, which gives short analytic formulae in some limits. However, for hierarchical $\nu_R$ (the limit in which eqn (10) is valid), $\epsilon_1$ is proportional to a Jarlskog invariant $^{16}$, which gives eqn (11). We see that $U$ does not appear in the expression for $\epsilon_1$ in Casas-Ibarra, but does appear in the bottom-up parametrisation. So it is unclear whether $\epsilon_1$ depends on $U$—what do we mean by “depend”? If a mathematical definition can be constructed, then there should be a unique answer.

We can draw an analogy between coordinate choices on a manifold, and parametrisation choices for the seesaw. Different coordinate choices on the plane, all used with the same $^a$ metric $\delta_{\alpha\beta}$, give confusing results that resemble the puzzle about whether $\epsilon$ depends on $\theta_{13}$. If we use the appropriate metrics, results are independent of the coordinate system, which can be chosen for calculational convenience. There is no metric given on “seesaw parameter space”, but this analogy suggests that inventing one would resolve the confusion.

$^a$Of course, we know that this is wrong; the metric should change with the coordinate system.
Consider two choices of coordinates on the upper half plane:

1. the cartesian \((y, z)\) with \(y > 0\) and metric \(g_{\alpha\beta} = I\).
2. \(R = \sqrt{y^2 + z^2}\) and \(Z = z\), with \(R > Z\) and metric

\[
g_{AB} = \frac{R^2}{R^2 - Z^2} \begin{bmatrix} 1 & -Z/R \\ -Z/R & 1 \end{bmatrix}
\]

(12)

These are equally good coordinate choices for the same flat 2-d surface. The seesaw analogy we want to address is: does \(R\) “depend” on \(Z\)?

In any coordinate system, the coordinates vary independently. So by definition

\[
\frac{\partial R}{\partial Z} = 0
\]

(13)

which could be taken to mean that “\(R\) is independent of \(Z\)”. A more intuitive quantity is the total derivative, or by analogy with general relativity, the change of \(R\), treated as a scalar function, along the curve of varying \(Z\):

\[
\begin{pmatrix} \frac{\partial R}{\partial y} & \frac{\partial R}{\partial z} \\ \frac{\partial Z}{\partial y} & \frac{\partial Z}{\partial z} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial Z}{\partial y} \\ \frac{\partial Z}{\partial z} \end{pmatrix} = \frac{Z}{R}
\]

(14)

which is the expected answer. Notice that we need to know how to transform to cartesian coordinates (equivalently, the metric on \(R, Z\) space) for this calculation.

To summarise, \(\epsilon\) and \(\theta_{13}\) are functions of seesaw parameter space, and can be defined such that

\[
\frac{\partial \epsilon}{\partial \theta_{13}} = 0
\]

(15)

by a suitable choice of parameters. However, a better measure of whether \(\epsilon\) depends on \(\theta_{13}\) would be something like eqn (14). To evaluate this, we need a metric on seesaw parameter space \(b\).

How to choose this metric? The top-down parametrisation is the most natural, so in two generations, the obvious choice is to take \(\{D_Y, V_L, D_{Y'}, V_R, D_M\}\) as cartesian coordinates. With this metric, it is straightforward to show that \(\epsilon\) does vary with the angle of the matrix \(U\). (This is simple, because in 2 generations it is easy to calculate the angle of \(W_L\) in terms of RH parameters.) However, in three generations, “distance”

\(b\)When doing seesaw parameter space scans, one must choose the distribution of input points in parameter space. This number density (“measure on parameter space”) is motivated by some theoretical model for the origin of seesaw parameters, so is not intrinsic to the seesaw. Therefore it is not related to this “metric”.

on the unitary transformations should be invariant under reparametrizations (e.g. \( V_R = U_{12} U_{13} U_{23} \) or \( = U_{23} U_{13} U_{12} \)), suggesting a metric similar to the one for polar coordinates.

It is clear, from this section, that the “dependance” of one seesaw observable on another is not clear. For example, the coordinates on seesaw parameter space can be chosen such that either \( \epsilon_1 \) is a function of the MNS matrix, or it is not. This confusion can be resolved by inventing a notion of “orthogonality” for coordinates, that is, a metric on parameter space. However, the metric seems an esoteric solution, and how to find the correct one is not obvious.

5. Rethink: what happens in the Standard Model?

In the Standard Model, the Lagrangian parameters can be reconstructed from data—in fact, there are many more measurements than parameters, so the SM is tested at part-per-mil accuracy. But some parameters are better determined than others, so the difference with respect to the seesaw is just the size of the error bars.

In the SM, the key is the sensitivity of data to a parameter. For instance, to determine \( m_t \) from electroweak data, one should choose an observable with large \( m_t^2 \) corrections, and a parametrisation (e.g. definition of \( s_W^2 \)), where these are easy to identify. If the parameters other than \( m_t \) are sufficiently well determined, a range for \( m_t \) can be extracted \(^c\). This is self-evident; the data allows a model to occupy a subset (often a multidimensional ellipse) in parameter space.

We say an observable \( Ob \) is sensitive to a parameter \( P \), if measuring \( Ob \) constrains \( P \) to sit in a certain range. Conversely, \( Ob \) is insensitive to \( P \), if measuring \( Ob \) is consistent with any value of \( P \) (possibly because one adjusts other unknowns to compensate for variations in \( P \)).

So returning to the seesaw, one could conclude that “does \( \epsilon_1 \) depend on \( \theta_{13} \)?” is the wrong question. If instead, one asks “is \( \epsilon_1 \) sensitive to \( \theta_{13} \)?”, then the answer at present is clearly no. It is easy to see, in the parametrisation using \( R \), that any value of \( \theta_{13} \) is consistent with the observed baryon asymmetry.

\(^c\)In reality this is a crude approx to doing a combined fit.
6. Summary

The seesaw generates small neutrino masses, by introducing heavy majorana $\nu_R$s, which share a Yukawa coupling with the lepton doublets of the Standard Model. It is theoretically possible to establish a 1-1 correspondence between observables (in the quantum mechanical sense), and the 21 parameters of the seesaw (type I, 3 generations). This correspondence, and two other parametrisations of the seesaw, are discussed in section 2. Unfortunately, this peculiarity of the seesaw does not mean the parameters can be extracted from data; some of the “observables” are not realistically measurable, and others cannot be determined accurately enough (see section 3). This makes the seesaw mechanism difficult to test, according to the definition of test outlined in the introduction.

This is sad because the dream test of the seesaw would be to extract its parameters from data, calculate the baryon asymmetry produced in leptogenesis—and get the right answer. More realistically, we can ask “is the baryon asymmetry sensitive to any of the seesaw’s measurable parameters?” For instance, does generating the baryon asymmetry by a specific leptogenesis scenario imply that $\theta_{13}$, or the phase $\delta$, should occupy restricted ranges? Again, the answer sadly seems to be “no”. More generally, one could study which observables are sensitive to which parameters, e.g. would $BR(\mu \rightarrow e\gamma) \neq 0$ restrict the majorana phases of MNS? Most studies to date have looked at whether an observable $O$ “depends” on a parameter $P$—which is not such a useful question, because the answer depends on the parametrisation. Section 4 attempts to construct a parametrisation-independent definition of “depend”, not very successfully. So it is better to ask if $O$ is sensitive to $P$, which does have a unique answer, as discussed in section 5.

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\[d\text{In the Casas-Ibarra parametrisation, } BR(\mu \rightarrow e\gamma) \text{ depends on theses phases}^{18}\]
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