Application of Transfer Matrix Method in Calculating Flexible Deployment Mechanisms Modes

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Abstract: Flexible deployment mechanisms are widely used in the aerospace field. During the satellite antenna's attitude adjustment and deployment process, vibration will greatly affect the accuracy of the satellite antenna profile after deployment. The finite element method (FEM) has the disadvantages of high calculation order, slow calculation speed and difficult optimization in the process of establishing the dynamic model of the flexible deployment mechanism. This paper uses the principle of the transfer matrix method to propose a calculation method for the natural frequency and vibration mode of a flexible deployment mechanism, which is used to calculate the vibration characteristics of a radial rib deployment antenna. Finally, its accuracy is verified by the finite element method (FEM).

1. Introduction
With the development of satellite technology, the design requirements for large flexible self-deployed antennas are getting higher and higher. However, due to the large and complex structure of large-size flexible self-deployed antennas, they belong to a multi-body rigid-flexible coupling mechanism [1]. Coupling vibration is very prone to occur during posture and deployment, affecting the accuracy of the antenna's profile and its ability to communicate with the ground. By establishing its accurate damping dynamic model to analyze the natural frequency and mode shape of the deployed antenna, effective vibration control methods can be targeted to ensure the accuracy of the profile of the deployed antenna and increase the stability after deployment.

At present, most scholars usually use the finite element model method to establish the dynamic model of the flexible self-deploying truss structure. Masayoshi M, et al. Conducted a theoretical analysis of the dynamics and structural dynamics of petal-shaped, umbrella-shaped, and ring-column flexible deployment structures [2]. Zhang Chun et al. Used the finite element method to establish a finite element model of the antenna reflector structure in the working state [3]. Li Dongying et al. Solved the dynamic characteristics of a radial stiffener-film umbrella antenna structure by finite element method [4]. However, due to the complex structure of the large flexible self-deployment device and the large number of state variables, the calculation order of the finite element model is very high [5], the calculation speed is slow, and the finite element method has certain restrictions on the optimization after the dynamic model is established. This paper adopts the principle of transfer matrix method to establish the dynamic model of flexible deployment mechanism. The advantage of the transfer matrix method modeling is that
the dimension of the transfer matrix does not increase with the increase of the degree of freedom of the system. When saving the computer, it is more suitable for dynamic modeling of large truss structures such as self-expanding devices, and it can also be more conveniently optimized. The antenna structure parameters and damping structure design parameters improve the design efficiency and reduce the calculation amount. This paper combines the equivalent structure of a radial rib flexible deployment antenna, establishes a transfer matrix to solve the vibration characteristics, and finally verifies the practicability and effectiveness of the method by the finite element method.

2. Establishment and solution of transfer matrix

The overall flexible expansion antenna transfer matrix can be established by establishing transfer matrices for different antenna components and combining them based on continuous conditions. The flexible deployment antenna can be equivalently formed by connecting a plurality of beams of different materials and different cross-sectional shapes, and some components at the node can be equivalent to adding a transfer function for concentrated mass.

This article introduces the calculation method by taking a radial rib-spread antenna as an example. As shown in Figure 1, the radial rib deployment antenna can be equivalent to a cantilever beam containing multiple complex components. The transfer matrix of different components can be discussed in sections, and all components are transferred via the point transfer matrix at the connection. The matrices are merged into a unified transfer matrix, and the overall transfer matrix is solved by boundary conditions.

2.1. Transfer matrix of beam

Let $S = [y, \theta, M, Q]^T$ be the state vector of the section, where $y$ is the deflection, $\theta$ is the angle of rotation, $M$ is the bending moment, and $Q$ is the shear force. Based on the Euler beam theory and the uniformly distributed mass model, the free vibration differential equation of the beam is:

$$EI \frac{d^4y}{dx^4} + \bar{m} \frac{d^2y}{dt^2} = 0 \tag{1}$$

Where, $EI$ is the bending stiffness, and $\bar{m}$ is the linear density.

The general solution of equation (1) can be obtained by using the separated variable method:

$$Y(x) = A_1 \cosh \lambda x + A_2 \sinh \lambda x + A_3 \cos \lambda x + A_4 \sin \lambda x \tag{2}$$

Where, $\lambda = \left(\frac{\bar{m}w^2}{EI}\right)^{-1/4}$, $w$ is the natural circular frequency of the lateral vibration.

From the relationship between deflection, rotation angle, bending moment and shear force in material mechanics:

$$\theta_x = A_1 \lambda \sinh \lambda x + A_2 \lambda \cosh \lambda x - A_3 \lambda \sin \lambda x + A_4 \lambda \cos \lambda x \tag{3}$$

$$M_x = A_1 EI \lambda^2 \cosh \lambda x + A_2 EI \lambda^2 \sinh \lambda x - A_3 EI \lambda^2 \cos \lambda x - A_4 EI \lambda^2 \sin \lambda x \tag{4}$$
\[ Q_y = A_1 EI \lambda^3 \sinh \lambda x + A_2 EI \lambda^3 \cosh \lambda x + A_3 EI \lambda^3 \sin \lambda x - A_4 EI \lambda^3 \cos \lambda x \]  

(5)

Where, \( A_1, A_2, A_3, A_4 \) is the undetermined coefficient determined by the boundary conditions at the left and right ends of the beam.

The above equation can be written by the Krylov function as the following equation:

\[
\begin{align*}
Y(x) &= L_1 S(x) + L_2 T(x) + L_3 U(x) + L_4 V(x) \\
\theta_x &= \lambda (L_1 V(x) + L_2 S(x) + L_3 T(x) + L_4 U(x)) \\
M_x &= -EI\lambda^2 (L_1 U(x) + L_2 V(x) + L_3 S(x) + L_4 T(x)) \\
Q_y &= -EI\lambda^3 (L_1 T(x) + L_2 U(x) + L_3 V(x) + L_4 S(x))
\end{align*}
\]

(6)

(7)

(8)

(9)

Where, the Krylov function is:

\[
\begin{align*}
S(x) &= \frac{1}{2} [\cosh(x) + \cos(x)] \\
T(x) &= \frac{1}{2} [\sinh(x) + \sin(x)] \\
U(x) &= \frac{1}{2} [\cosh(x) - \cos(x)] \\
V(x) &= \frac{1}{2} [\sinh(x) - \sin(x)]
\end{align*}
\]

(10)

\( L_1, L_2, L_3, L_4 \) is the undetermined coefficient determined by the boundary conditions at the left and right ends of the beam.

When \( x = 0 \),

\[
\begin{align*}
Y(x) &= Y_l, \quad \theta_x = \theta_l, \quad M_x = M_l, \quad Q_y = Q_l
\end{align*}
\]

(11)

\( L_1, L_2, L_3, L_4 \) can be obtained:

\[
\begin{align*}
L_1 &= Y_l, \quad L_2 = \frac{\theta_l}{\lambda}, \quad L_3 = -\frac{M_l}{EI\lambda^2}, \quad L_4 = -\frac{Q_l}{EI\lambda^3}
\end{align*}
\]

(12)

The transfer function of the left and right ends of the beam can be obtained as:

\[
\begin{bmatrix}
Y_r \\
\theta_r \\
M_r \\
Q_r
\end{bmatrix} =
\begin{bmatrix}
S(\lambda l) & \frac{T(\lambda l)}{\lambda} & \frac{U(\lambda l)}{EI\lambda^2} & \frac{V(\lambda l)}{EI\lambda^3} \\
\lambda V(\lambda l) & S(\lambda l) & \frac{T(\lambda l)}{EI\lambda} & \frac{U(\lambda l)}{EI\lambda^2} \\
EI\lambda^2 U(\lambda l) & EI\lambda V(\lambda l) & S(\lambda l) & \frac{T(\lambda l)}{\lambda} \\
EI\lambda^3 T(\lambda l) & EI\lambda^2 U(\lambda l) & \lambda V(\lambda l) & S(\lambda l)
\end{bmatrix}
\begin{bmatrix}
Y_l \\
\theta_l \\
M_l \\
Q_l
\end{bmatrix}
\]

(13)

The transfer matrix of the beam is:

\[
Z_1 =
\begin{bmatrix}
S(\lambda l) & \frac{T(\lambda l)}{\lambda} & \frac{U(\lambda l)}{EI\lambda^2} & \frac{V(\lambda l)}{EI\lambda^3} \\
\lambda V(\lambda l) & S(\lambda l) & \frac{T(\lambda l)}{EI\lambda} & \frac{U(\lambda l)}{EI\lambda^2} \\
EI\lambda^2 U(\lambda l) & EI\lambda V(\lambda l) & S(\lambda l) & \frac{T(\lambda l)}{\lambda} \\
EI\lambda^3 T(\lambda l) & EI\lambda^2 U(\lambda l) & \lambda V(\lambda l) & S(\lambda l)
\end{bmatrix}
\]

(14)

2.2. Transfer Matrix of Concentrated Mass

For the transfer matrix of concentrated mass \( m \), the relationship between the deflection, rotation angle, bending moment and shear-force of the left side and the right side at the mass point acting section is:

\[
Y_r(x) = Y_l(x), \quad \theta_r(x) = \theta_l(x), \quad M_r(x) = M_l(x), \quad Q_r(x) = mw^2 Y_l(x) + Q_l(x)
\]

(15)

Written as a matrix:
\[
\begin{bmatrix}
Y_l \\
\theta_l \\
M_l \\
Q_l
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\text{mw}^2 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
Y_l \\
\theta_l \\
M_l \\
Q_l
\end{bmatrix}
\]

(16)

Therefore, the transfer matrix of concentrated mass is:

\[
Z_2 =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\text{mw}^2 & 0 & 0 & 1
\end{bmatrix}
\]

(17)

2.3. Continuous Condition

Because the joint of the structural rod and the connector obeys the continuous condition, that is, the state vectors of the left and right ends of the cross section of the connected parts are equal. So you can get the matrix form:

\[
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_i =
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_r
\]

(18)

2.4. Transfer Matrix of Overall Structure

After the state transfer matrices of each segment of beam and concentrated mass are determined, the total transfer matrix relationship of the state vector transferred from the initial point state to the end position state vector is:

\[
S_n = Z_n Z_{n-1} \cdots Z_1 S_0 = Z \cdot S_0
\]

(19)

Where,

\[
Z =
\begin{bmatrix}
H_{11} & H_{12} & H_{13} & H_{14} \\
H_{21} & H_{22} & H_{23} & H_{24} \\
H_{31} & H_{32} & H_{33} & H_{34} \\
H_{41} & H_{42} & H_{43} & H_{44}
\end{bmatrix}
\]

(20)

2.5. Solution of Transfer Matrix

The boundary conditions of a radial rib-expanded antenna can be equivalent to a fixed end at one end and a free end, the displacement and rotation angle at the fixed end are zero, and the bending moment and shear force at the free end are zero. The initial point state vector \( S_0^L \) and the end point state vector \( S_0^R \) can be obtained as:

\[
S_0^L = [0, 0, M, Q]^T, \quad S_0^R = [y, \theta, 0, 0]^T
\]

(21)

The boundary conditions at both ends of the flexible expanded antenna are brought into equation (18) to obtain the transfer function. To make the transfer function solvable, let \( |Z| = 0 \), and the natural frequency \( w \) can be solved. Bringing the natural frequency back into the transfer function can solve the initial state vector. Through the initial point state vector and the transfer matrix of each beam, the state vectors of all points of the satellite antenna can be solved, and finally the modal shape of the flexible expanded antenna can be obtained.

3. Modal example of radical rib flexible deployment antenna

3.1. Calculation of Modal Features

In this paper, a radial rib antenna with a diameter of 9m is used as an example. It consists of 5 structural members and 5 connecting members. The structural parts and connecting parts are made of M60j carbon.
fiber material, and their dimensions, cross-sectional areas and other physical parameters are shown in Table 1.

**Table 1. Physical parameters of radial rib antenna structure**

| Physical parameters | Structural Members | Connector Members |
|---------------------|--------------------|-------------------|
| Length /m           | 0.5                | 0.07              |
| Diameter /mm        | Φ25×1              | Φ28×1.5           |
| Density (kg/m³)     | 1910               | 1910              |
| Elastic Modulus /Pa | 5.88×10¹¹          | 5.88×10¹¹         |

3.2. Calculation Results

By solving the physical parameters of the flexible deployment antenna using the method described above, the first third-order natural frequency and mode shape of the antenna deployment rib can be calculated. The results are shown in Table 2 and Figure 2.

**Table 2. Calculation result of natural frequency**

| Frequency                      | Radial Rib Antenna Structure /Hz |
|--------------------------------|----------------------------------|
| First-order Natural Frequency  | 2.6                              |
| Second-order Natural Frequency | 16.9                             |
| Third-order Natural Frequency  | 48.1                             |

![Figure 2. First third-order mode of radial rib antenna structure](image)

4. Finite element certification

Based on the external dimensions of the radial rib flexible deployment antenna geometric model, a three-dimensional solid model is established, and the model is analyzed by frequency using the finite element method. The modeling results are shown in the Figure 3, and the analysis results of the finite element method and the calculation results of the method in this paper are shown in Table 3.

![Figure 3. Finite element model of radial rib antenna structure](image)
Table 3. Comparison of calculation results between the finite element method and the method of this paper

| Frequency            | Method of This Article/Hz | Finite Element Method /Hz | Error /% |
|----------------------|---------------------------|---------------------------|----------|
| First-order Frequency| 2.6                       | 2.5                       | 3.84     |
| Second-order Frequency| 16.9                     | 15.6                      | 7.69     |
| Third-order Frequency| 48.1                      | 43.6                      | 9.35     |

As can be seen from Table 3, the method in this paper can accurately calculate the modal parameters of the radial rib flexible deployment antenna. Therefore, the mathematical model established in this paper has good practicability and effectiveness for calculating the vibration characteristics of flexible deployment mechanisms such as satellite antennas.

5. Conclusion

Based on the Euler-Bernoulli quantity theory, the transfer matrix method is used to derive the transfer matrix of the cantilever part of the radial rib flexible deployment mechanism. The state vectors of different positions of the deployment mechanism are solved, and the transfer relationship between different positions of the flexible deployment mechanism is established. The calculation examples show that the results of the transfer matrix in this paper are accurate and have faster calculation speed than the finite element method, which is more convenient to establish the dynamic model of the flexible self-deploying mechanism.

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