The Frobenius Structure of Local Cohomology

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For a local ring \((R, m)\) of prime characteristic \(p > 0\), the local cohomology modules with support in the maximal ideal \(H^i_m(R)\) inherit a natural Frobenius action \(F\). The talk with present results on the lattice of submodules \(N\) of the local cohomology that are invariant under the Frobenius action, that is \(F(N) \subseteq N\). This lattice of submodules contains remarkable information about the structure of the ring \(R\). A result of Karen E. Smith, from a decade or so ago, states that, under mild conditions, a Cohen-Macaulay ring \(R\) is \(F\)-rational (an analogue of rational singularities in positive characteristic) if and only if 0 is the only proper \(F\)-invariant submodule of the highest local cohomology module. We will explore the properties of the lattice of \(F\)-invariant submodules in detail. Part of the work is joint with Mel Hochster.