Constraints on Neutron Star Masses and Radii from Kilohertz QPOs

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Abstract. The frequencies of the highest-frequency kilohertz QPOs recently discovered in some sixteen neutron stars in low-mass X-ray binary systems are most likely orbital frequencies. If so, these QPOs provide tight upper bounds on the masses and radii of these neutron stars and interesting new constraints on the equation of state of neutron star matter. If the frequency of a kilohertz QPO can be established as the orbital frequency of the innermost stable circular orbit, this would confirm one of the key predictions of general relativity in the strong-field regime. If the spin frequency of the neutron star can also be determined, the frequency of the QPO would fix the mass of the neutron star for each assumed equation of state. Here we describe how bounds on the stellar mass and radius can be derived and how these bounds are affected by the stellar spin. We also discuss detection of the innermost stable circular orbit.

INTRODUCTION

Determination of the equation of state of neutron stars has been an important goal of nuclear physics for more than two decades. Progress toward this goal can be made by establishing astrophysical constraints as well as by improving our understanding of nuclear forces. The discovery using the Rossi X-ray Timing Explorer of highly coherent kilohertz QPOs in the persistent emission of some sixteen neutron stars in low-mass X-ray binaries ([1]) is likely to provide important new constraints. In both the sonic-point [2] and magnetospheric [3] beat-frequency interpretations, the highest frequencies observed are the orbital frequency at the inner edge of the Keplerian disk flow (see [4]). Such high orbital frequencies yield interesting bounds on the masses and radii of these neutron stars and interesting constraints on the equation of state of neutron star matter. Here we describe how bounds on masses and radii can be derived from the properties of the kilohertz QPOs and how these bounds are affected by stellar rotation. We also discuss detection of the innermost stable circular orbit (ISCO).
CALCULATIONS

Suppose that $\nu_{QPO2}^*$ is the highest Keplerian frequency observed from a given neutron star and that the star is not rotating (we use a superscript zero to indicate relations that are valid only for a nonrotating star). The radius of the star must be smaller than the radius $R_{\text{orb}}$ of the gas with orbital frequency $\nu_{QPO2}^*$, so the representative point of the star in the $R,M$ plane must lie to the left of the curve $M^0(R_{\text{orb}}, \nu_{QPO2}^*)$ that relates $R_{\text{orb}}, \nu_{QPO2}^*$, and the mass of the star. In addition, in order to produce a wave train with tens of oscillations, the gas producing the QPO must be outside the radius $R_{\text{ms}}$ of the marginally stable orbit (ISCO), so the representative point must also lie below the intersection of $M^0(R_{\text{orb}}, \nu_{QPO2}^*)$ with the curve $M^0(R_{\text{ms}})$ that relates $R_{\text{ms}}$ and the mass of the star. Figure 1a shows the allowed region of the $R,M$ plane for $\nu_{QPO2}^* = 1220$ Hz. The maximum allowed mass and radius are

$$M_{\text{max}}^0 = 2.2 (1000 \text{ Hz} / \nu_{QPO2}^*) M_\odot \text{ and } R_{\text{max}}^0 = 19.5 (1000 \text{ Hz} / \nu_{QPO2}^*) \text{ km.} \quad (1)$$

Figure 1b compares the $M$-$R$ relations given by five equations of state with the allowed regions of the radius-mass plane for three values of $\nu_{QPO2}^*$.

The spin of the star affects both the structure of the star and the spacetime, altering the stellar mass-radius relation, the frequency of an orbit of given radius, and the value of $R_{\text{ms}}$ for a given stellar mass. The parameter that characterizes the importance of these effects is the dimensionless quantity $j \equiv cJ/GM^2$, where $J$ and $M$ are the angular momentum and gravitational mass of the star. For the spin frequencies $\sim 300$ Hz inferred in the kilohertz QPO sources, $j \sim 0.1$–0.3. For such small values of $j$, a first-order treatment is adequate.

To first order in $j$, the frequency of the prograde orbit at $R_{\text{ms}}$ around a star of given mass $M$ and dimensionless angular momentum $j$ is (see [5]) $\nu_{K,\text{ms}} \approx 2210 (1 + 0.75j)(M_\odot/M) \text{ Hz.}$. The corresponding upper bounds on the mass and radius are given implicitly by

$$M_{\text{max}} \approx [1 + 0.75j(\nu_{\text{spin}})]M_{\text{max}}^0 \text{ and } R_{\text{max}} \approx [1 + 0.20j(\nu_{\text{spin}})]R_{\text{max}}^0 , \quad (2)$$

where $j(\nu_{\text{spin}})$ is the value of $j$ for the observed stellar spin rate at the maximum allowed mass for the equation of state being considered. Figure 1c indicates the first-order effects of spin rates $\sim 300$ Hz on the allowed region of the $R$-$M$ plane.

Our calculations [2] show that the mass of the neutron star in 4U 1636–536 must be less than $\sim 2.2 M_\odot$ and its radius must be less than $\sim 17$ km. As just explained, the precise upper bounds depend on the equation of state assumed. For further details, see [2].

If the stellar spin frequency is $\sim 500$ Hz or higher, spin affects the structure of the star as well as the exterior spacetime, which then differs substantially from the Kerr spacetime and must be computed numerically for each assumed equation of state. We have carried out such computations and find [6] that if the neutron star is spinning rapidly, the constraints on the equation of state are dramatically tightened.
FIGURE 1. (a) Radius-mass plane, showing how bounds on the mass and radius of a nonrotating neutron star with $\nu_{\text{QPO2}} = 1220$ Hz can be constructed. The steps in the construction are similar for rotating stars. (b) Comparison of the $M-R$ relations for nonrotating neutron stars given by five representative equations of state (solid curves) with the regions of the radius-mass plane allowed for nonrotating stars with three different circumstellar orbital frequencies. The region bounded by the heavy solid line is the region that would be allowed for 4U 1636−536 if it were not rotating. (c) Regions allowed for rotating neutron stars with various values of $j$ and $\nu_{\text{QPO2}} = 1220$ Hz, when the first-order effects of the stellar spin are included. The gas generating the QPO is assumed to be in a prograde orbit. (d) Characteristic variation of the sonic-point orbital frequency with accretion luminosity from fully general relativistic calculations of the gas dynamics and radiation transport. The sonic-point orbital frequency increases steeply with increasing accretion luminosity until it reaches the frequency of the ISCO, at which point it stops changing. See [2] for further details.
INNERMOST STABLE CIRCULAR ORBIT

Establishing that an observed QPO frequency is the frequency of the ISCO would be an important step forward in our understanding of strong-field gravity and the properties of dense matter, because it would confirm one of the key predictions of general relativity in the strong-field regime and fix the mass of the neutron star in that source, for each assumed equation of state. Given the fundamental significance of the ISCO, it is very important to establish what would constitute strong, rather than merely suggestive, evidence that an ISCO has been detected (for a detailed discussion of various signatures, see [2]). Probably the most convincing signature would be a fairly coherent, kilohertz QPO with a frequency that reproducibly increases steeply with increasing accretion rate but then becomes constant and remains nearly constant as the accretion rate increases further (see Fig. 1d). The constant frequency should always be the same in a given source.

It has been suggested that the similarity of the highest QPO frequencies seen so far indicates that ISCOs are being detected [7] and that the roughly constant frequencies of the 800–900 Hz QPOs seen in 4U 1608–52 [8] and 4U 1636–536 [9] during the first observations of these sources with the Rossi Explorer were generated by the beat of the spin frequency against the frequency of ISCOs in these sources [10]. This would imply that the neutron stars in all the kilohertz QPO sources have masses close to $2.0 \, M_\odot$. However, no strong signatures of the innermost stable circular orbit have so far been seen in any of these sources. Indeed, more recent observations of both 4U 1608–52 [11] and 4U 1636–536 [12] are inconsistent with the suggestion that the QPO frequencies seen initially are related to the frequencies of ISCOs in these sources. Nevertheless, the prospects appear excellent for discovering clear evidence of an ISCO in one or more of the kilohertz QPO sources in the future.

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