Leptonic $CP$ violating Phase in the Yukawaon Model

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Abstract

In the so-called “Yukawaon” model, the (effective) Yukawa coupling constants $Y_f^{eff}$ are given by vacuum expectation values (VEVs) of scalars $Y_f$ (Yukawaons) with $3\times3$ components. In this brief report, we change VEV forms $\langle Y_f \rangle$ in the previous paper into a unified form. Therefore, parameter fitting for quark and lepton masses and mixings is revised. Especially, we obtain predicted values of neutrino mixing $\sin^2 2\theta_{13}$ and a leptonic $CP$ violating phase $\delta_{CP}^\ell$ which are consistent with the observed curve in the $(\sin^2 2\theta_{13}, \delta_{CP}^\ell)$ reported by T2K group recently.

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1 Introduction

Now, measurement of $CP$ violating phase $\delta_{CP}^\ell$ in the lepton sector is within our reach because of the recent development of neutrino physics[1]. The measurement is very important to check quark and lepton mass matrix models currently proposed. At the same time, for model-builders, it is urgently required to predict an explicit value of $\delta_{CP}^\ell$ together with mixing value $\sin^2 2\theta_{13}$ based on their models. So, we estimate a value of $\delta_{CP}^\ell$ based on the so-called “Yukawaon model” [2, 3], which is a unified mass matrix model of quarks and leptons, and which is a kind of flavon model [4].

In the Yukawaon model, the (effective) Yukawa coupling constants $Y_f^{eff}$ are given by vacuum expectation values (VEVs) of scalars $Y_f$ (Yukawaons) with $(8+1)$ of $U(3)$ family symmetry:

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda} (Y_f)_{ij} \quad (f = u, d, \nu, e),$$

where $\Lambda$ is a scale of the effective theory. In understanding flavor physics from a view of a non-Abelian family symmetry, the conventional Yukawa interactions explicitly break its family symmetry. It is only when the conventional Yukawa coupling constants are supposed to be given by Eq.(1) that we can build a model with an unbroken family symmetry.

The characteristic point of the Yukawaon model is the following point: The quark and lepton mass matrices are described by using only the observed values of charged lepton masses ($m_e, m_\mu, m_\tau$) as input parameters with family-number dependent values; thereby, we investigate
whether we can describe all other observed mass spectra (quark and neutrino mass spectra) and mixings (the Cabibbo-Kobayashi-Maskawa [5] (CKM) mixing and the Pontecorvo-Maki-Nakagawa-Sakata [6] (PMNS) mixing) without using any other family number-dependent parameters. Here, terminology “family number-independent parameters” means, for example, coefficients of a unit matrix $1$, a democratic matrix $X_3$, and so on, where

$$
1 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}.
$$

In the previous paper, the form of $\langle Y_d \rangle$ in the down quark sector has been supposed to be unnaturally different from those in other sectors. In this paper, we revise the form of $\langle Y_f \rangle$ so that it takes a unified form for all sectors as given Eq.(3) in the next section. Accordingly, parameter fitting for quark and lepton masses and mixings is also revised as given in Secs.3 and 4. Especially, it is shown in Sec.4 that we obtain predicted values for neutrino mixing $\sin^2 2\theta_{13}$ and a leptonic $CP$ violating phase $\delta^\ell_{CP}$ which are consistent with the observed curve in the $(\sin^2 2\theta_{13}, \delta^\ell_{CP})$ plane reported by T2K group [7] recently.

2 Models

Here and hereafter, for convenience, we use the notation $\hat{A}$, $A$ and $\bar{A}$ for fields with $8 + 1$, $6$ and $6^*$ of $U(3)$, respectively. Explicit forms of VEV relations among the Yukawaon in this paper are given by

$$
\langle \bar{Y}_f \rangle^j_i = k^j_f \left[ (\langle \Phi_f \rangle^ik)(\bar{\Phi}_f^T)^{kj} + \xi_f 1^j_i \right] \quad (f = e, \nu, d, u),
$$

$$
\langle \Phi_f \rangle_{ij} = k^f_f (\Phi_0)^{ia}(\bar{S}_f^T)^{\alpha\beta}(\bar{\Phi}_0)^{\beta j} \quad (f = e, \nu),
$$

$$
\langle E_u \rangle^{ik}(\Phi_u)^{kl}(\bar{E}_u)_{lj} = (\Phi_0)^{ia}(S_u)^{\alpha\beta}(\bar{\Phi}_0^T)^{\beta j}.
$$

$$
\langle \bar{P}_d \rangle^{ik}(\Phi_d)^{kl}(\bar{P}_d)_{lj} = (\Phi_0)^{ia}(S_d)^{\alpha\beta}(\bar{\Phi}_0^T)^{\beta j},
$$

$$
\langle \Phi_f \rangle_{ij} = k^f_f (\Phi_0)^{ia}(\bar{S}_f)^{\alpha\beta}(\Phi_0)^{\alpha\beta} \quad (f = e, \nu, d, u),
$$

$$
\langle S_f \rangle^{\alpha\beta} = (1 + a_f X_3)^{\alpha\beta}, \quad \langle \bar{S}_f \rangle^{\alpha\beta} = (1 + a_f X_3)^{\alpha\beta},
$$

where $\langle E \rangle = 1$, and indexes $\alpha, \beta, \cdots$ are of another family symmetry $U(3)'$. We consider that the form (8) is due to a symmetry breaking $U(3)' \to S_3$ at $\mu = \Lambda'$. The $\xi_f$ terms in Eq.(3) will be discussed later. Here, the VEV matrices $\bar{Y}_e$, $\bar{Y}_\nu$, $\bar{Y}_u$ and $\bar{Y}_d$ correspond to charged lepton mass matrix $M_e$, neutrino Dirac mass matrix $M_{Dirac}$, up-quark mass matrix $M_u$, and down-quark mass matrix $M_d$, respectively. Here and hereafter, we drop flavor-independent factors in those VEV matrices, because we deal with only mass ratios and mixings in this paper.
The VEV structures are essentially the same as the previous paper [3]. However, we have done the following minor changes from the previous paper: (i) In the previous paper, \( \langle \hat{Y}_d \rangle \) and \( \langle \Phi_d \rangle \) were given by \( \langle \hat{Y}_d \rangle = \langle \Phi_d \rangle \langle \Phi_d \rangle \) and \( \langle \Phi_d \rangle = \langle \Phi_0 \rangle \langle S_d \rangle / \langle \Phi_0 \rangle + \xi'_d \mathbf{1} \), respectively, differently from other sectors. However, it is unnatural that such the term \( \xi'_d \mathbf{1} \) appears only in the VEV of \( \Phi_d \). In this paper, we remove the \( \xi'_d \mathbf{1} \) term from the \( \Phi_d \) and unify the appearance place of the \( \mathbf{1} \) terms which appear in \( \langle \hat{Y}_f \rangle \) common to all sectors as shown in Eq.(3). (ii) Along with the changing of the VEV structure in the down-quark sector, a phase matrix \( P_u \) in the previous paper is moved to the down quark sector as shown in Eq.(6). For convenience, \( \bar{E} \) in Eq.(5) and \( \bar{P}_d \) in Eq.(6) were exchanged with \( \bar{P}_u \) and \( \bar{E} \) in the previous paper, respectively.

Neutrino mass matrix \( M_\nu \) is given by a seesaw type
\[
(M_\nu)^{ij} = \langle \hat{Y}_e \rangle^i_k \langle \hat{Y}_e \rangle^{-1} \langle \hat{Y}_e \rangle^j_k ,
\] (9)
as well as in the previous paper [3], where
\[
\langle Y_R \rangle_{ij} = \langle \hat{Y}_e \rangle_{ik} \langle \Phi_u \rangle_{kj} + \langle \Phi_u \rangle_{ik} \langle \hat{Y}_e \rangle^T_{kj}.
\] (10)

In general, we can choose either one in two cases, (a) \( \langle A \rangle = \langle A \rangle^* \) or (b) \( \langle A \rangle = \langle A \rangle \) for VEV matrices \( \langle A \rangle \) and \( \langle A \rangle \) under the \( D \)-term condition. We assume the type (b) for \( \Phi_f \) and \( S_f \), while the type (a) for \( P_d \):
\[
\langle P_d \rangle = v_P \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1), \quad \langle \bar{P}_d \rangle = v_P \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1),
\] (11)

In order to distinguish each Yukawaon from the others, we assume that \( \hat{Y}_f \) have different \( R \) charges from each other together with considering \( R \)-charge conservation (a global U(1) symmetry in \( N = 1 \) supersymmetry). The \( R \) charge assignments are essentially not changed from the previous paper [3] except for \( E_u \) and \( P_d \).

Since we consider that the charged lepton mass matrix is the most fundamental one, we assume \( a_e = 0 \) and \( \xi_e = 0 \). Then, \( \langle \Phi_0 \rangle \) is expressed as follows:
\[
\langle \Phi_0 \rangle = \langle \Phi_0 \rangle \equiv \text{diag}(x_1, x_2, x_3) \propto \text{diag}(m^{1/4}_e, m^{1/4}_\mu, m^{1/4}_\tau),
\] (12)
from the \( D \)-term condition, where \( x_i \) are real and those are normalized as \( x_1^2 + x_2^2 + x_3^2 = 1 \).

Now, let us give a brief review of the derivation of \( \xi_f \) terms. We assume the following superpotential for \( \hat{Y}_f \) \( (f = \nu, e, u, d) \), with introducing flavons \( \hat{\Theta}_f \)
\[
W_{\hat{Y}_f} = \sum_{f=\nu,e,u,d} \left[ (\mu_f (\hat{Y}_f)_i^j + \lambda_f \langle \Phi_f \rangle_{ik} \langle \Phi_f \rangle^{kj}) (\hat{\Theta}_f)_j^i + (\mu'_f (\hat{Y}_f)_i^j + \lambda'_f \langle \Phi_f \rangle_{ik} \langle \Phi_f \rangle^{kj}) (\hat{\Theta}_f)_j^i \right].
\] (13)
(Here, we have assumed that only \( \Theta_f \) can be allowed to appear as a form \( \text{Tr}[\Theta] \) in the superpotential.) Then, a SUSY vacuum condition \( \partial W_{\hat{Y}_f} / \partial \hat{\Theta}_f = 0 \) leads to VEV relation
\[
\langle \hat{Y}_f \rangle = \langle \Phi_f \rangle / \langle \Phi_f \rangle + \xi_f \mathbf{1},
\] (14)
where

$$\xi_f = -\frac{\mu_f'}{\mu_f} \left( \text{Tr}[\langle Y_f \rangle] + \frac{\lambda_f'}{\mu_f'} \text{Tr}[\langle \Phi_f \rangle \langle \Phi_f \rangle] \right) = -\frac{\lambda_f/\mu_f - \lambda_f'/\mu_f'}{1 - 3\mu_f'/\mu_f} \text{Tr}[\langle \Phi_f \rangle \langle \Phi_f \rangle]. \quad (15)$$

Here we have assumed that all VEVs of flavons $\Theta$ take $\langle \Theta \rangle = 0$, so that SUSY vacuum conditions for other flavons do not bring any additional VEV relations. As seen in Eq.(15), if $\langle \Phi_f \rangle$ is complex, then the coefficient $\xi_f$ becomes complex too. Although the derivation discussed above was given in the previous work [3], we considered that the effect of the phase of $\xi_\nu$ is negligibly small, so that we treated $\xi_\nu$ as a real parameter approximately in the previous work. However, in this paper, we found that the phase of $\xi_\nu$ affects not a little on our parameter fitting.

3 Parameter fitting

**General:** We summarize our mass matrices $M_f (\langle Y_f \rangle)$ as follows:

$$Y_e = \Phi_e \bar{\Phi}_e + \xi_e 1, \quad \Phi_e = \bar{\Phi}_e = \Phi_0 (1 + \alpha_e X_3) \Phi_0, \quad (a_e = 0, \xi_e = 0), \quad (16)$$

$$Y_\nu = \Phi_\nu \bar{\Phi}_\nu + \xi_\nu e^{i\beta_\nu} 1, \quad \Phi_\nu = \bar{\Phi}_\nu = \Phi_0 (1 + \alpha_\nu e^{i\alpha_\nu} X_3) \Phi_0, \quad (17)$$

$$Y_u = \Phi_u \bar{\Phi}_u + \xi_u 1, \quad \Phi_u = \bar{\Phi}_u = \Phi_0 (1 + a_u X_3) \Phi_0, \quad (18)$$

$$Y_d = \Phi_d \bar{\Phi}_d + \xi_d e^{i\beta_d} 1, \quad \Phi_d = P_d^* \Phi_0 (1 + a_d e^{i\alpha_d} X_3) \Phi_0 P_d^*, \quad \bar{\Phi}_d = P_d \Phi_0 (1 + a_d e^{i\alpha_d} X_3) \Phi_0 P_d, \quad (19)$$

$$M_\nu = Y_\nu Y_R^{-1} Y_\nu, \quad Y_R = Y_e \Phi_u + \Phi_u Y_e. \quad (20)$$

Here, for convenience, we have dropped the notations “$\langle \cdot \rangle$” and “$\hat{\cdot}$”. Since we are interested only in the mass ratios and mixings, we use dimensionless expressions $\Phi_0 = \text{diag}(x_1, x_2, x_3)$ (with $x_1^2 + x_2^2 + x_3^2 = 1$), $P_d = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, 1)$, and $E = 1 = \text{diag}(1,1,1)$. Therefore, the parameters $a_e, \alpha_\nu, \cdots$ are re-defined by Eqs.(16)-(20).

Since the parameters $a_f$ in Eq.(8) can be complex in general, we denote $a_f$ as $a_f e^{i\alpha_f}$ by real parameters $(a_f, \alpha_f)$. The VEV structure of $Y_u$ in the present paper is practically unchanged from the previous paper [3], so that we inherit the numerical results in the up quark sector in the previous work by assuming $\alpha_u = 0$. Since $\alpha_\nu$ and $\alpha_d$ are complex, $\xi_\nu$ and $\xi_d$ are also complex as discussed in Eq.(14). We have denoted $\xi_\nu$ and $\xi_d$ in Eq.(3) as $\xi_\nu e^{i\beta_\nu}$ and $\xi_d e^{i\beta_d}$, respectively, in Eqs.(17) and (19). Of course, the parameters $\beta_\nu$ are fixed by the values $(a_f, \alpha_f)$, so that $\beta_f$ are not free parameters.

The explicit values of the parameters $(x_1, x_2, x_3)$ are fixed by Eq.(12) as $(x_1, x_2, x_3) = (0.115144, 0.438873, 0.891141)$, where we have normalized $x_i$ as $x_1^2 + x_2^2 + x_3^2 = 1$. Therefore, in the present model, except for the parameters $(x_1, x_2, x_3)$, we have 10 adjustable parameters, $(a_\nu, \alpha_\nu, \xi_\nu), (a_u, \xi_u), (a_d, \alpha_d, \xi_d)$, and $(\phi_1, \phi_2)$ for the 16 observable quantities (6 mass ratios
in the up-quark-, down-quark-, and neutrino-sectors, four CKM mixing parameters, and 4+2 PMNS mixing parameters).

**Quark mass ratios**: First, we fix the parameter values \((a_u, \xi_u)\) from the observed up-quark mass ratios \(r_{12}^u \equiv (m_u/m_c)^{1/2} = 0.045^{+0.013}_{-0.010}\) and \(r_{23}^u \equiv (m_c/m_t)^{1/2} = 0.060 \pm 0.005\) at \(\mu = m_Z\) as follows,

\[
(a_u, \xi_u) = (-1.4715, -0.001521),
\]

Of course, we obtain the same values as those in the previous paper.

Next, we try to fix the parameters \((a_d, \alpha_d, \xi_d)\) in the down-quark sector by using input parameters \(r_{12}^d \equiv m_d/m_s = 0.053^{+0.005}_{-0.003}\) and \(r_{23}^d \equiv m_s/m_b = 0.019 \pm 0.006\). However, since we have three parameters for two input values \(m_d/m_s\) and \(m_s/m_b\), we cannot fix our three parameters. It is more embarrassing that there is no solution of \(m_s/m_b \sim 0.019\) in the \((a_d, \alpha_d, \xi_d)\) parameter region. Nevertheless, we found that the minimal value of \(m_s/m_b\) is \(m_s/m_b \sim 0.03\) at \((a_d, \alpha_d, \xi_d) \sim (-1.5, 16^\circ, 0.004)\) which can give a reasonable value of \(m_d/m_s\) at the same time too. Therefore, we take the following values:

\[
(a_d, \alpha_d, \xi_d) = (-1.4735, 15.7^\circ, 0.00400),
\]

which leads to predictions \(r_{12}^d = 0.0597\) and \(r_{23}^d = 0.0312\). Note that the value \(r_{23}^d = 0.0312\) is considerably large compared with \(r_{23}^d \simeq 0.019\) by Xing et al. \(\S\), while the value is consistent with \(r_{23}^d \simeq 0.031\) by Fusaoka and Koide \(\S\). The values \(m_d(\mu)\) and \(m_s(\mu)\) are estimated at a lower energy scale, \(\mu \sim 1\) GeV, so that we consider that the ratio \(r_{12}^d\) at \(\mu = M_Z\) is reliable. On the other hand, the value \(m_b(\mu)\) is extracted at a different energy scale \(\mu \sim 4\) GeV from \(\mu \sim 1\) GeV, so that the value \(m_b(M_Z)\) is affected by the prescription of threshold effects at \(\mu = m_t\), while the value \(m_s(M_Z)\) affected by those at \(\mu = m_c, \mu = m_b\) and \(\mu = m_t\). We consider that as for the ratio \(r_{23}^d\) at \(\mu = M_Z\) the value is still controversial. Anyhow, we have fixed three parameters \((a_d, \alpha_d, \xi_d)\) only from two values \(m_d/m_s\) and \(m_s/m_b\).

**CKM mixing**: The purpose of the present paper is to discuss PMNS parameters, especially \(CP\) violating phase \(\delta_{CP}\). However, since our model is to give unified description of quarks and leptons, for reference, we give results of CKM parameter fitting, too.

Since the parameters \((a_u, \xi_u)\) and \((a_d, \alpha_d, \xi_d)\) have been fixed by the observed quark mass ratios, the CKM mixing matrix elements \(|V_{us}|\), \(|V_{cb}|\), \(|V_{ub}|\), and \(|V_{td}|\) are functions of the remaining two parameters \(\phi_1\) and \(\phi_2\) defined by Eq.(11). We use the observed CKM mixing matrix elements \(10\) \(|V_{us}| = 0.2254 \pm 0.0006, |V_{cb}| = 0.0414 \pm 0.0012, |V_{ub}| = 0.00355 \pm 0.00015\), and \(|V_{td}| = 0.00886^{+0.00033}_{-0.00032}\). (Two of those are used as input values in the present analysis, and the remaining two are our predictions as references.) All the experimental CKM parameters are satisfied by fine tuning the parameters \(\phi_1\) and \(\phi_2\) as

\[
(\phi_1, \phi_2) = (-42.0^\circ, -15.1^\circ),
\]

which leads to the numerical results as follows: \(|V_{us}| = 0.2255, |V_{cb}| = 0.0429, |V_{ub}| = 0.00359,\) and \(|V_{td}| = 0.00928\) with \(\delta_{CP} = 73.0^\circ\). In spite of our aim described in the Sec. 1, we are forced
to introduce family number-dependent parameters \((\phi_1, \phi_2)\) in the present model, too, as the same as in the previous model [3]. Model-building without using parameter \((\phi_1, \phi_2)\) is left to our future task.

4 Parameter fitting in the PMNS mixing and CP violating phase \(\delta_{CP}\)

We have already fixed our seven parameters as Eqs. (21), (22) and (23). The remaining free parameters are only \((a_\nu, \alpha_\nu, \xi_\nu)\) in the Dirac neutrino sector. We determine the parameter values of \((a_\nu, \alpha_\nu, \xi_\nu)\) as follows:

\[
(a_\nu, \alpha_\nu, \xi_\nu) = (-3.54, -18.0^\circ, -0.0238),
\]

which are obtained so as to reproduce the observed values [10] of the following PMNS mixing angles and \(R_\nu\),

\[
\sin^2 2\theta_{12} = 0.846 \pm 0.021, \quad \sin^2 2\theta_{13} = 0.093 \pm 0.008,
\]

\[
R_\nu = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{m^2_{\nu_2} - m^2_{\nu_1}}{m^2_{\nu_3} - m^2_{\nu_2}} = \frac{(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2}{(2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2} = (3.09 \pm 0.15) \times 10^{-2}.
\]

We show the \(a_\nu\) and \(\alpha_\nu\) dependences of the PMNS mixing parameters \(\sin^2 2\theta_{12}\), \(\sin^2 2\theta_{23}\), \(\sin^2 2\theta_{13}\), and \(R_\nu\) in Fig. 1(a) and Fig. 1(b), respectively. It is found that \(R_\nu\) is very sensitive to \(a_\nu\).

Figure 1: Contour curves of the observed center, upper, and lower values of the lepton mixing parameters \(\sin^2 2\theta_{12}\) (dashed), \(\sin^2 2\theta_{13}\) (dotdashed), and the neutrino mass squared difference ratio \(R_\nu\) (solid). (a): We draw the curves in the \((\alpha_\nu, a_\nu)\) plane with taking \(\xi_\nu = -0.0238\). (b): We draw the curves in the \((\alpha_\nu, \xi_\nu)\) plane with taking \(a_\nu = -3.54\).
Figure 2: $\alpha_\nu$ dependence of the lepton mixing parameters $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$, $R_\nu$, and the leptonic CP violating phase $\delta_{\nu}^{C\!P}$. We draw the curves of those as functions of $\alpha_\nu$ for the case of $\xi_\nu = -0.0238$ with taking $a_\nu = -3.54$ (solid).

As seen in Fig. 2, we obtain two solutions, which are consistent with the neutrino data except for the data of $\delta_{\nu}^{C\!P}$. However, as seen the best fit curve on the $(\sin^2 \theta_{13}, \delta_{\nu}^{C\!P})$ plane in the Fig.5 in the resent T2K report [7], the solution with $0 < \delta_{\nu}^{C\!P} < \pi$ is obviously ruled out. Therefore, we adopt the solution with $-\pi < \delta_{\nu}^{C\!P} < 0$ in our model. Then, we obtain the predictions of our model

$$R_\nu = 0.0310, \quad \sin^2 \theta_{12} = 0.837, \quad \sin^2 \theta_{23} = 0.988, \quad \sin^2 \theta_{13} = 0.0987, \quad \delta_{\nu}^{C\!P} = -125^\circ. \quad (27)$$

We can predict neutrino masses, for the parameters given by (21) and (24), as follows

$$m_{\nu 1} \simeq 0.00037 \text{ eV}, \quad m_{\nu 2} \simeq 0.00868 \text{ eV}, \quad m_{\nu 3} \simeq 0.0501 \text{ eV}, \quad (28)$$

by using the input value $[10] \Delta m_{32}^2 \simeq 0.00244 \text{ eV}^2$. We also predict the effective Majorana neutrino mass $[11] \langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = |m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2| \simeq 6.0 \times 10^{-3} \text{ eV}. \quad (29)$$

Our model also predicts $\delta_{\nu}^{C\!P} = -125^\circ$ for the Dirac $C\!P$ violating phase in the lepton sector, which indicates relatively large $C\!P$ violating effect in the lepton sector.

5 Concluding remarks

We have tried to describe quark and lepton mass matrices by using only the observed values of charged lepton masses ($m_e, m_\mu, m_\tau$) as input parameters with family number-dependent values, except for $P_d$ defined by Eq.(11). Thereby, we have investigated whether we can describe
all other observed mass spectra (quark and neutrino mass spectra) and mixings (CKM and PMNS mixings) without using any other family number-dependent parameters. In conclusion, we have obtained reasonable results. We have predicted the $CP$ violating phase in the lepton sector as $\delta_\ell^{CP} \simeq -125^\circ$ and $\sin^2 2\theta_{13} \simeq 0.099$ in Eq.(27), are consistent with the observed curve in the $(\sin^2 2\theta_{13}, \delta_\ell^{CP})$ plane which has been reported by T2K group [7]. (The predicted value of $\delta_\ell^{CP}$ in the previous paper was $\delta_\ell^{CP} = -26^\circ$.)

The origin of the $CP$ violation in the lepton sector is in the phase factor $\alpha_\nu$ in the Dirac neutrino mass matrix (17). Note that we have taken $\alpha_f = 0$ ($f = e, u$) for economy of the parameters. However, we have been obliged to accept $\alpha_\nu \neq 0$ in order to fit the observed value of $\sin^2 2\theta_{13}$.

Although the present model is a miner improved version of the previous paper [3], the predicted value of $\delta_\ell^{CP}$ have been changed into a more detectable value in near future neutrino observations, and it is consistent with the recent T2K result [7]. We expect that the value of $\delta_\ell^{CP}$ will be confirmed by near future observations.

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