Statistical Entropy of Four-Dimensional Extremal Black Holes

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Abstract

String theory is used to count microstates of four-dimensional extremal black holes in compactifications with $N = 4$ and $N = 8$ supersymmetry. The result agrees for large charges with the Bekenstein-Hawking entropy.
Recently it has been shown \cite{1-5} that string theory can, in some special cases, provide a statistical derivation of the Bekenstein-Hawking entropy \cite{6,7}, by representing the black holes as bound states of D-branes and strings. The statistical entropy is the logarithm of the bound state degeneracy, which was counted using D-technology introduced in \cite{8,9,10}. Curiously the results so far have been limited to five dimensions. The reason for this is that four-dimensional black holes with nonzero horizon area can not be constructed from D-branes alone. Another type of object such as a symmetric fivebrane or Kaluza-Klein monopole is required, and further technology is needed. In this paper we will find the missing piece of technology in references \cite{11,12} and use it to compute the statistical entropy of certain four-dimensional extremal black holes in $N = 4$ and $N = 8$ supergravity theories. The result agrees with the Bekenstein-Hawking entropy, which was computed in a special $N = 4$ case in \cite{13}, more generally for $N = 4$ in \cite{14,15} and for $N = 8$ in \cite{16}.

The statistical entropy of four-dimensional black holes has been recently analyzed in \cite{17} with methods seemingly quite different from those used herein. It would be very interesting to understand the relation between the two approaches.

The required modification of \cite{1} is rather simple and this presentation will be accordingly brief. Let us begin by rederiving the result of \cite{1} in a T-dualized picture with one extra $\hat{S}^1$-compactified dimension. Consider type IIA string theory on $X = Y \times S^1 \times \hat{S}^1$, where $Y$ is $T^4$ for the $N = 8$ case and $K3$ for the $N = 4$ case. A dual description of the D-brane configuration in \cite{1} (obtained by T-dualizing along $\hat{S}^1$) consists of $Q_6$ sixbranes wrapping $X$, $Q_2$ twobranes wrapping $S^1 \times \hat{S}^1$, and right-moving momentum $n$ along the $S^1$. We take $n, Q_2 \gg 1$. The twobranes are marginally bound to the sixbranes \cite{18-20}. For $Q_6 = 1$ the momentum is carried by massless, right-moving modes of $(2, 2)$ open strings that end on the twobranes. It is sufficient to consider the case $Q_6 = 1$ because duality implies the results can depend only on the product $Q_2Q_6$. (This has been explicitly verified in some cases \cite{18-22}.) BPS excitations of these $(2, 2)$ strings correspond to transverse motion of the $Q_2$ twobranes within $Y$ (and the sixbrane).\footnote{Since the two branes are separated in $Y$ the $(2, 2)$ open strings going between different twobranes are massive and do not contribute to the extremal entropy as in \cite{1}. $(2, 6)$ strings also do not contribute in this case ($Q_6 = 1$) because of charge confinement.} Because $Y$ is four-dimensional this means there are $4Q_2Q_6$ bosons and their $4Q_2Q_6$ fermionic superpartners available to
carry the momentum. The number of BPS- saturated states of this system as a function of $Q_2, Q_6$ and $n$ follows from the standard $(1 + 1)$-dimensional entropy formula

$$S = \sqrt{\frac{\pi (2N_B + N_F)EL}{6}},$$

(1)

where $N_B$ ($N_F$) is the number of species of right-moving bosons (fermions), $E$ is the total energy and $L$ is the size of the box. Using $N_B = N_F = 4Q_2Q_6$ and $E = 2\pi n/L$, we find the $L$-independent result for the large $n$ thermodynamic limit [1]

$$S_{stat} = 2\pi \sqrt{Q_2Q_6n}.$$ (2)

The Bekenstein-Hawking entropy was computed from the corresponding four dimensional extremal black hole solutions in [13,14,15,16]. The result, in our notation, for either $N = 4$ or $N = 8$, is

$$S_{BH} = 2\pi \sqrt{Q_2Q_6nm}.$$ (3)

The integer $m$ here is the axion charge carried by a symmetric fivebrane which wraps $Y \times S^1$. Since that charge is absent in this $S^1$ compactification of the configuration of $\mathbb{P}^1$, $S_{BH} = 0$. This is not a contradiction because in four dimensions $S_{BH}$ as computed from the leading low energy effective action always scales like $(\text{charge})^2$, in contrast to five dimensions where it scales like $(\text{charge})^{3/2}$. Since (2) scales like $(\text{charge})^{3/2}$, it appears at leading order in five dimensions but is an invisible subleading correction in four.

In order to get a nonzero area in four dimensions, we must add $m$ fivebranes wrapping $Y \times S^1$. These $m$ fivebranes can be located anywhere on the $S^1$. Each twobrane must intersect all $m$ fivebranes along the $S^1$. The effect of this was explained in [11,12]. A twobrane can break and the ends separate (in $Y$) when it crosses a fivebrane. Hence the $Q_2$ toroidal twobranes break up into $mQ_2$ cylindrical twobranes, each of which is bounded by a pair of fivebranes. The momentum-carrying open strings now carry an extra label

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2 We suppress here the anomalous shift of $Q_2$ for $K3$ [21,22] which is subleading for large $Q_2$.

3 To facilitate comparison with [14,15], we note that under type II-heterotic duality an $m$-wound symmetric fivebrane together with momentum $n$ becomes a fundamental heterotic string with $(\text{winding, momentum}) = (m, n)$ around $S^1$. The twobranes and sixbranes become the magnetic heterotic $S$-duals of a fundamental heterotic string with $(\text{winding, momentum}) = (Q_2, Q_6)$ associated to the $(20, 4)$ part of the Narain lattice.

4 In fact the four dimensional solution with $m = 0$ contains scalar fields that blow up at the horizon, rendering the classical geometry at the horizon singular.
describing which pair of fivebranes they lie in between. The number of species becomes 
\( N_B = N_F = 4mQ_2Q_6 \). Inserting this into (1) together with \( E = 2\pi n/L \) we obtain

\[
S_{stat} = 2\pi \sqrt{Q_2Q_6nm}, \tag{4}
\]

In agreement with the semiclassical result (3) for \( S_{BH} \).

For the \( N = 4 \) case there are, in general, 28 electric charges \( \vec{Q} \) and 28 magnetic charges \( \vec{P} \) which lie in the \( (22, 6) \) Narain lattice. In our notation \( 2Q_2Q_6 = \vec{P}^2 \) and \( 2nm = \vec{Q}^2 \). Duality implies that the entropy depends only on \( \vec{P}^2 \), \( \vec{Q}^2 \) and \( \vec{Q} \cdot \vec{P} \). The general formula for the Bekenstein Hawking entropy is \[14,15\]

\[
S_{BH} = \pi \sqrt{\vec{P}^2 \vec{Q}^2 - (\vec{Q} \cdot \vec{P})^2}. \tag{5}
\]

For our example the last term vanishes. It would be interesting to construct a more general example for which this last term does not vanish, and so verify the general formula.

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