Non-standard Neutrino Oscillations at Icecube

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Abstract
In this talk I review the potential of Icecube for revealing physics beyond the standard model in the oscillation of atmospheric neutrinos [1].

1. Introduction
With its high statistics data [2] Super–Kamiokande (SK) established beyond doubt that the observed deficit in the μ-like atmospheric events is due to oscillations, a result supported by the K2K and MINOS long-baseline (LBL) experiments [3,4].

Mass oscillations are not the only possible mechanism for atmospheric (ATM) $\nu_\mu \to \nu_\tau$ flavour transitions. These can be also generated by a variety of non-standard physics characterized by the presence of an unconventional $\nu$ interaction that mixes neutrino flavours [5]. Examples include violations of the equivalence principle (VEP) [6,7], non-standard neutrino interactions with matter [8], neutrino couplings to space-time torsion fields [9], violations of Lorentz invariance (VLI) [10] and of CPT symmetry [11,12]. In contrast to the $E$ energy dependence of the conventional oscillation length, new physics can produce neutrino oscillations with wavelengths that are constant or decrease with energy. [13,14]. At present these scenarios cannot explain the data[15] and a combined analysis of the ATM and LBL data can be performed to constraint them even as subdominant oscillation effects [16].

IceCube, with energy reach in the $10^3$ to $10^4$ TeV range for ATM neutrinos, is the ideal experiment to search for new physics. For most of this energy interval standard $\Delta m^2$ oscillations are suppressed and therefore the observation of an angular distortion of the ATM neutrino flux or its energy dependence provide a clear signature for the presence of new physics mixing neutrino flavours [1].

2. Propagation in Matter of High Energy Oscillating Neutrinos
We concentrate on $\nu_\mu-\nu_\tau$ flavour mixing mechanisms for which the propagation of $\nu$’s (+) and $\bar{\nu}$’s (−) is governed by the following Hamiltonian [12]:

$$H_{\pm} \equiv \frac{\Delta m^2}{4E} U_\theta \left( \begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \right) U^\dagger_\theta + \sum_n \sigma_n^\pm \frac{\Delta \delta_n}{2} U_{\xi_n,\pm \eta_n} \left( \begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \right) U^\dagger_{\xi_n,\pm \eta_n} ,$$

$\Delta m^2$ is the mass–squared difference between the two neutrino mass eigenstates, $\sigma_n^\pm$ accounts for a possible relative sign of the new physics (NP) effects between $\nu$’s and
\(\nu\)'s and \(\Delta \delta_n\) parametrizes the size of the NP terms. By \(\eta_n\) we denote the possible non-vanishing relative phases.

\[
U_\theta = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}, \quad U_{\xi_n, \pm \eta_n} = \begin{pmatrix}
\cos \xi_n & \sin \xi_n e^{\pm i \eta_n} \\
-\sin \xi_n e^{i \eta_n} & \cos \xi_n
\end{pmatrix}; \quad (2)
\]

If NP strength is constant along the neutrino trajectory the oscillation probabilities take the form [12]:

\[
P_{\nu_{\mu} \rightarrow \nu_{\mu}} = 1 - P_{\nu_{\tau} \rightarrow \nu_{\tau}} = 1 - \sin^2 2\Theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \mathcal{R}\right),
\]

\[
\sin^2 2\Theta = \frac{1}{\mathcal{R}^2} \left(\sin^2 2\theta + R_n^2 \sin^2 2\xi_n + 2R_n \sin 2\theta \sin 2\xi_n c\eta_n\right),
\]

\[
\mathcal{R} = \sqrt{1 + R_n^2 + 2R_n \left(\cos^2 \theta \cos 2\xi_n + \sin^2 \theta \sin 2\xi_n c\eta_n\right)}, \quad R_n = \sigma_n \frac{\Delta \delta_n E_n}{2} \frac{4E}{\Delta m^2},
\]

where, for simplicity, we have assumed scenarios with one NP source characterized by a unique \(\Delta \delta_n\), \(c\eta_n = \cos \eta_n\).

Eq. (1) describes, for example, flavour mixing due to new tensor-like interactions for which \(n = 1\) leading to a contribution to the oscillation wavelength inversely proportional to the neutrino energy. This is the case for \(\nu_\tau\)'s and \(\nu_\nu\)'s of different masses in the presence of violation of the equivalence principle due to non-universal coupling of the neutrinos, \(\gamma_1 \neq \gamma_2\) to the gravitational potential \(\phi\) [6], so \(\Delta \delta_1 = 2|\phi|/(\gamma_1 - \gamma_2) \equiv 2|\phi|\Delta \gamma\). \(\nu_1\) and \(\nu_2\) are related to \(\nu_\mu\) and \(\nu_\tau\) by a rotation \(\xi_1 = \xi_{\text{vlep}}\).

For constant potential \(\phi\), this mechanism is phenomenologically equivalent to the breakdown of Lorentz invariance resulting from different asymptotic values of the velocity of the neutrinos, \(c_1 \neq c_2\), \(\Delta \delta_1 = (c_1 - c_2) \equiv \delta c/c\), with \(\nu_1\) and \(\nu_2\) being related to \(\nu_\mu\) and \(\nu_\tau\) by a rotation \(\xi_1 = \xi_{\text{vli}}\) [10].

For vector-like interactions, \(n = 0\), the oscillation wavelength is energy-independent. This may arise, for instance, from a non-universal coupling of the neutrinos, \(k_1 \neq k_2\), so \(\Delta \delta_0 = Q(k_1 - k_2)\) \((\nu_1\) and \(\nu_2\) is related to the \(\nu_\mu\) and \(\nu_\tau\) by a rotation \(\xi_0 = \xi_{\text{vli}}\), to a space-time torsion field \(Q\) [9]. Violation of CPT resulting from Lorentz-violating effects such as the operator, \(\bar{\nu}_L^{\alpha \beta} L_{\mu}^{\alpha} \gamma_{\mu} \nu_L^{\beta}\), also leads to an energy independent contribution to the oscillation wavelength [11,12] which is a function of the eigenvalues of the Lorentz violating CPT-odd operator, \(b_1, \Delta \delta_0 = b_1 - b_2\), and the rotation angle, \(\xi_0 = \xi_{\text{vli}}\), between the corresponding eigenstates \(\nu_1\) and the flavour states \(\nu_\nu\).

For most of the neutrino energies considered here, \(\Delta m^2\) oscillations are suppressed and the NP effect is directly observed. Thus the results will be independent of the phase \(\eta_n\) and we can chose the NP parameters in the range \(\Delta \delta_n \geq 0, 0 \leq \xi_n \leq \pi/4\).

The Hamiltonian of Eq. (1) describes the coherent evolution of the \(\nu_\mu - \nu_\tau\) ensemble for any neutrino energy. High-energy neutrinos propagating in the Earth can also interact inelastically with the Earth matter either by charged current (CC) and neutral current (NC) and as a consequence the neutrino flux is attenuated. This attenuation is qualitatively and quantitatively different for \(\nu_\tau\)'s and \(\nu_\nu\)'s. \(\nu_\mu\)'s are absorbed by CC interactions while \(\nu_\tau\)'s are regenerated because they produce a \(\tau\) that decays into another tau neutrino before losing energy [17]. As a consequence, for each \(\nu_\tau\) lost in CC interactions, another \(\nu_\tau\) appears (degraded in energy) from the \(\tau\) decay and the Earth never becomes opaque to \(\nu_\nu\)'s. Furthermore, a secondary flux of \(\bar{\nu}_\mu\)'s is also generated in the leptonic decay \(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau\) [18].
Attenuation and regeneration effects of incoherent neutrino fluxes can be consistently described by a set of coupled partial integro-differential cascade equations (see for example [19] and references therein) or by a Monte Carlo simulation of the neutrino propagation in matter [17,18,26]. For astrophysical $\nu$’s, because of the long distance traveled from the source, the oscillations average out and at the Earth the neutrinos can be treated as an incoherent superposition of mass eigenstates.

For ATM neutrinos this is not the case because oscillation, attenuation, and regeneration effects occur simultaneously when the neutrino beam travels across the Earth’s matter. For conventional neutrino oscillations this fact can be ignored because the neutrino energies covered by current experiments are low enough for attenuation and regeneration effects to be negligible. But for non-standard scenario oscillations, future experiments probe high-energy neutrinos for which the attenuation and regeneration effects have to be accounted for simultaneously.

In order to do so it is convenient to use the density matrix formalism to describe neutrino flavour oscillations. The evolution of the neutrino ensemble is determined by the Liouville equation for the density matrix $\rho(t) = \nu(t) \otimes \nu(t)^\dagger$:

$$\frac{d\rho}{dt} = -i[H, \rho],$$

where $H$ is given by Eq. (1). The survival probability is given by $P_{\mu\mu}(t) = \text{Tr}[\Pi_{\nu_{\mu}}\rho(t)]$, where $\Pi_{\nu_{\mu}} = \nu_{\mu}\otimes\nu_{\mu}$ is the $\nu_{\mu}$ state projector, and with initial condition $\rho(0) = \Pi_{\nu_{\mu}}$. An equivalent equation can be written for the $\bar{\nu}$ density matrix.

In this formalism attenuation effects due to CC and NC interactions can be introduced by relaxing the condition $\text{Tr}(\rho) = 1$. In this case

$$\frac{d\rho(E, t)}{dt} = -i[H(E), \rho(E, t)] - \sum_{\alpha} \frac{1}{2\lambda^\alpha_{\text{int}}(E, t)} \{\Pi_{\alpha}, \rho(E, t)\},$$

where $[\lambda^\alpha_{\text{int}}(E, t)]^{-1} \equiv [\lambda^\alpha_{\text{CC}}(E, t)]^{-1} + [\lambda^\alpha_{\text{NC}}(E, t)]^{-1}$, $[\lambda^\alpha_{\text{CC}}(E, t)]^{-1} = n_\tau(x)\sigma^\alpha_{\text{CC}}(E)$, and $[\lambda^\alpha_{\text{NC}}(E, t)]^{-1} = n_\tau(x)\sigma^\alpha_{\text{NC}}(E)$ ($n_\tau(x)$ is the number density of nucleons at the point $x = ct$).

$\nu_\tau$ regeneration and neutrino energy degradation can be accounted for by coupling these equations to the shower equations for the $\tau$ flux, $F_\tau(E_\tau, t)$ (we denote by $F$ the differential fluxes $d\phi/(dE \, d \cos \theta)$). For convenience we define the neutrino flux density matrix $F_\nu(E, x) = F_{\nu_{\mu}}(E, x_0)\rho(E, x = ct)$ where $F_{\nu_{\mu}}(E, x_0)$ is the initial neutrino flux:

$$\frac{dF_\nu(E_\nu, x)}{dx} = -i[H, F_\nu(E_\nu, x)] - \sum_{\alpha} \frac{1}{2\lambda^\alpha_{\text{int}}(E_\nu, x)} \{\Pi_{\alpha}, F_\nu(E_\nu, x)\}$$

$$+ \int_{E_\nu}^{\infty} \frac{1}{\lambda^\tau_{\text{NC}}(E_\tau, x)} F_\tau(E_\tau, x) \frac{dN_{\text{NC}}(E_\nu', E_\nu)}{dE_\nu'} \frac{dE_\nu'}{dE_\nu}$$

$$+ \int_{E_\nu}^{\infty} \frac{1}{\lambda^\tau_{\text{CC}}(E_\tau, x)} F_\tau(E_\tau, x) \frac{dN_{\text{CC}}(E_\nu', E_\nu)}{dE_\nu'} \frac{dE_\nu'}{dE_\nu},$$

$$\frac{dF_\tau(E_\tau, t)}{dx} = -\frac{1}{\lambda^\tau_{\text{dec}}(E_\tau, x)} F_\tau(E_\tau, x)$$

$$+ \int_{E_\tau}^{\infty} \frac{1}{\lambda^\tau_{\text{CC}}(E_\nu, t)} \text{Tr}[\Pi_{\tau} F_\nu(E_{\nu}, t)] \frac{dN_{\text{CC}}(E_\nu', E_\nu)}{dE_\nu'} \frac{dE_\nu'}{dE_\nu}. \tag{6}$$
\( \lambda_{\text{dec}}(E, x) = \gamma_{\tau} c \tau_{\tau} \). \( \tau_{\tau} \) is the \( \tau \) lifetime and \( \gamma_{\tau} = E_{\tau}/m_{\tau} \) is its gamma factor. 
\[
\frac{dN_{\text{NC}}(E', E_{\nu})}{dE'} = \frac{1}{\sigma_{\text{NC}}(E_{\nu})} \frac{dN_{\text{NC}}(E', E_{\nu})}{dE_{\nu}} \\
\frac{dN_{\text{CC}}(E, E_{\nu})}{dE} = \frac{1}{\sigma_{\text{CC}}(E_{\nu})} \frac{dN_{\text{CC}}(E, E_{\nu})}{dE_{\nu}}
\]
can be easily computed. The \( \tau \) decay distributions \( \frac{dN_{\text{NC}}(E', E_{\nu})}{dE'} \) and \( \frac{dN_{\text{CC}}(E, E_{\nu})}{dE} \) can be found in Refs. [19,27].

The third term in Eq. (5) represents the neutrino regeneration by NC interactions and the fourth term represents the contribution from \( \nu_{\tau} \) regeneration, \( \nu_{\tau} \rightarrow \tau^{-} \rightarrow \nu_{\tau} \), describing the energy degradation in the process. The secondary \( \nu_{\mu} \) flux from \( \bar{\nu}_{\tau} \) regeneration, \( \bar{\nu}_{\tau} \rightarrow \tau^{+} \rightarrow \bar{\nu}_{\tau} \mu^{+} \nu_{\mu} \), is described by the last term where we denote by over-bar the energies and fluxes of the \( \tau^{+} \). \( Br_{\mu} = 0.18 \) is the branching ratio for this decay. In Eq. (6) the first term gives the loss of taus due to decay and the last term gives the \( \tau \) regeneration due to CC \( \nu_{\tau} \) interactions. In writing these equations we have neglected the tau energy loss, which is only relevant at much higher energies.

An equivalent set of equations can be written for the \( \bar{\nu} \) flux density matrix and for the \( \tau^{+} \) flux. Both sets are coupled due to the secondary \( \nu \) flux term.

We solve this set of ten coupled evolution equations that describe propagation through the Earth numerically using the matter density profile of the Preliminary Reference Earth Model and obtain the neutrino fluxes in the vicinity of the detector

\[
\frac{d\phi_{\nu_{\mu}}(E, \theta)}{dE d\cos \theta} = \text{Tr}[F_{\nu_{\mu}}(E, L = 2 R \cos \theta) \Pi_{\nu_{\mu}}].
\]

In Fig. 1 we illustrate the interplay between the different terms in Eqs. (5) and (6). The figure covers the example of VLI-induced oscillations with \( \delta c/c = 10^{-27} \) and maximal \( \xi_{\nu_{\mu}} \) mixing. The upper panels show the final \( \nu_{\mu} \) and \( \nu_{\tau} \) fluxes for vertically going neutrinos after traveling the full length of the Earth for the initial conditions \( d\Phi(\nu_{\mu})_{0}/dE_{\nu} = d\Phi(\bar{\nu}_{\mu})_{0}/dE_{\nu} \propto E^{-1} \) and \( d\Phi(\nu_{\tau})_{0}/dE_{\nu} = d\Phi(\bar{\nu}_{\tau})_{0}/dE_{\nu} = 0 \).

The figure illustrates that the attenuation in the Earth suppresses the neutrino fluxes at higher energies. The effect of the attenuation in the absence of oscillations is given by the dotted thin line in the left panel. Even in the presence of oscillations this effect can be well described by an overall exponential suppression [27,24] both for \( \nu_{\mu} \)'s and the oscillated \( \nu_{\tau} \)'s. In other words, we closely reproduce the curve for “oscillation + attenuation” simply by multiplying the initial flux by the oscillation probability and an exponential damping factor:

\[
\frac{d\phi_{\nu_{\tau}}(L = 2 R \cos \theta)}{dE d\cos \theta} = \frac{d\phi_{\nu_{\tau}}(L = 2 R \cos \theta)}{dE d\cos \theta} P_{\nu_{\mu}}(E, L = 2 R \cos \theta) \exp[-X(\theta)(\sigma_{\text{NC}}(E) + \sigma_{\text{CC}}^{\nu}(E))],
\]

where \( X(\theta) \) is the column density of the Earth.

The main effect of energy degradation by NC interactions (the third term in Eq. (5)) that is not accounted for in the approximation of Eq. (7) is the increase of the flux in the oscillation minima (the flux does not vanish in the minimum) because higher energy neutrinos end up with lower energy as a consequence of the NC interactions. The difference between the dash-dotted line and the dashed line is due to the interplay between the \( \nu_{\tau} \) regeneration effect (fourth term in Eq. (5)) and the flavour oscillations. As a consequence of the first effect, we see in the right upper panel that the \( \nu_{\tau} \) flux is enhanced because of the regeneration of higher energy \( \nu_{\tau} \)'s, \( \nu_{\tau}(E) \rightarrow \tau^{-} \rightarrow \nu_{\tau}(E' < E) \), that originated from the oscillation of higher energies \( \nu_{\mu} \)'s. In turn this excess of \( \nu_{\tau} \)'s produces an excess of \( \nu_{\mu} \)'s after oscillation which is seen as the difference between the dashed line and the dash-dotted curve in the left upper panel. Finally the secondary effect of \( \bar{\nu}_{\tau} \) regeneration (last term in Eq. (5)), \( \bar{\nu}_{\tau}(E) \rightarrow \tau^{+} \rightarrow \mu^{+} \bar{\nu}_{\tau} \nu_{\mu}(E' < E) \), results into the larger \( \nu_{\mu} \) flux (seen in the left upper panel as the difference between the dashed and the thick full lines).
This, in turn, gives an enhancement in the $\nu_\tau$ flux after oscillations as seen in the right upper panel.

The lower panels show the final fluxes for an atmospheric-like energy spectrum $d\Phi(\nu_\mu)/dE_\nu = d\Phi(\bar{\nu}_\mu)/dE_\nu \propto E^{-3}$ and $d\Phi(\nu_\tau)/dE_\nu = d\Phi(\bar{\nu}_\tau)/dE_\nu = 0$. In this case regeneration effects result in the degradation of the neutrino energy and the more steeply falling the neutrino energy spectrum, the smaller the contribution to the total flux. As a result the final fluxes can be relatively well described by the approximation in Eq.(7).

3. Example of Physics Reach: VLI-induced Oscillations

The expected number of $\nu_\mu$ induced events at IceCube can be obtained by a semianalytical calculation as:

$$N_{\nu_\mu}^{nu} = T \int_{-1}^{1} d\cos \theta \int_{-l}^{l} dl' \int_{E_\mu^{min}}^{E_\mu} dE_\mu \int_{E_\mu^{fin}}^{E_\mu} dE_\mu^{fin} \int_{E_\mu^{0}}^{E_\mu} dE_\mu^{0} \int_{E_\mu^{0}}^{E_\mu} dE_\mu^{0} \int_{E_\mu^{0}}^{E_\mu} dE_\mu^{0}$$

$$\frac{d\phi_{\nu_\mu}}{dE_\nu d\cos \theta} (E_\nu, \cos \theta) \frac{d\sigma_{CC}}{dE_\mu^{0}} (E_\nu, E_\mu^{0}) n_T F(E_\mu^{0}, E_\mu^{fin}, l) A_{eff}^{0} .$$

$\frac{d\phi_{\nu_\mu}}{dE_\nu d\cos \theta}$ is the differential muon neutrino neutrino flux in the vicinity of the detector after evolution in the Earth matter obtained as described in the previous section. We use the neutrino fluxes from Honda [20] extrapolated to match at higher energies the fluxes from Volkova [21]. At high energy prompt neutrinos from
charm decay are important and it is evaluated for two different models of charm production: the recombination quark parton model (RQPM) developed by Bugaev et al [22] and the model of Thunman et al (TIG) [23] that predicts a smaller rate. 

\[ \frac{d\sigma_{\text{CC}}}{dE_\mu}(E_\nu, E_\mu^0) \]

is the differential CC interaction cross section producing a muon of energy \( E_\mu^0 \). \( T \) is the exposure time of the detector. Equivalently, muon events arise from \( \bar{\nu}_\mu \) interactions that are evaluated by an equation similar to Eq.(9).

After production with energy \( E_\mu^0 \), the muon ranges out in the rock and in the ice surrounding the detector and looses energy. We denote by \( F(E_0^\mu, E_{\text{fin}}^\mu, l) \) the function that describes the energy spectrum of the muons arriving at the detector. We compute the function \( F(E_0^\mu, E_{\text{fin}}^\mu, l) \) by propagating the muons to the detector taking into account energy losses due to ionization, bremsstrahlung, \( e^+e^- \) pair production and nuclear interactions according to Ref. [24].

The details of the detector are encoded in the effective area \( A_{\text{eff}}^0 \) for which we make a phenomenological parametrization to simulate the response of the IceCube detector after events that are not neutrinos have been rejected (referred to as “level 2” cuts in Ref. [25]). The explicit form of \( A_{\text{eff}}^0 \) can be found in Ref.[1].

Together with \( \nu_\mu \)-induced muon events, oscillations also generate \( \mu \) events from the CC interactions of the \( \nu_\tau \) flux which reaches the detector producing a \( \tau \) that subsequently decays as \( \tau \to \mu \bar{\nu}_\mu \nu_\tau \) and produces a \( \mu \) in the detector:

\[ N_{\nu_\tau}^{\nu_\tau} = T \int_{-1}^{1} d\cos \theta \int_{0}^{\infty} dl_{\text{min}}^\nu \int_{l_{\text{min}}^\nu}^{\infty} dl \int_{E_{\text{fin}}^\mu}^{\infty} dE_{\text{fin}}^\mu \int_{E_\mu^0}^{\infty} dE_\mu \int_{E_\tau}^{\infty} dE_\tau \int_{E_0^\mu}^{\infty} dE_0^\mu \int_{E_0^\tau}^{\infty} dE_0^\tau (9) \]

where \( \frac{dN_{\text{hep}}(E_\nu, E_\mu^0)}{dE_\mu^0} \) can be found in Ref. [27]. Equivalently we compute the number of \( \bar{\nu}_\tau \)-induced muon events.

Neutrino oscillations introduced by NP effects result in an energy dependent distortion of the zenith angle distribution of ATM muon events. We quantify this effect in IceCube by evaluating the expected angular and \( E_{\text{fin}}^\mu \) distributions in the detector using Eqs. (9) and (10) in conjunction with the fluxes obtained after evolution in the Earth for different sets of NP oscillation parameters.

For illustration we concentrate on oscillations resulting from VLI that lead to an oscillation wavelength inversely proportional to the neutrino energy. The results can be directly applied to oscillations due to VEP. We show in Fig. 2 the zenith angle distributions for muon induced events for different values of the VLI parameter \( \delta c/c \) and maximal mixing \( \xi_{\nu_\mu} = \pi/4 \) for different threshold energy \( E_{\text{fin}}^\mu > E_{\text{threshold}} \) normalized to the expectations for pure \( \Delta m^2 \) oscillations. The full lines include both the \( \nu_\mu \)-induced events (Eq.(9)) and \( \nu_\tau \)-induced events (Eq.(10)) while the last ones are not included in the dashed curves. We see that for a given value of \( \delta c/c \) there is a range of energy for which the angular distortion is maximal. Above that energy, the oscillations average out and result in a constant suppression of the number of events. Inclusion of the \( \nu_\tau \)-induced events events leads to an overall increase of the event rate but slightly reduces the angular distortion.

In order to quantify the energy-dependent angular distortion we define the vertical-to-horizontal double ratio

\[ \frac{d\phi_{\nu_\tau}}{dE_\nu} \]
Fig. 2. **Upper panels:** Zenith angle distributions for muon induced events for different values of the VLI parameter $\delta c/c$ and maximal mixing $\xi_{vli}=\pi/4$ for different threshold energy $E_{\mu}^{\text{fin}} > E_{\text{threshold}}$ normalized to the expectations for pure $\Delta m^2$ oscillations. **Lower panels:** The predicted horizontal-to-vertical double ratio in Eq.(10) for different values of $\delta c/c$. The data points in the figure show the expected statistical error corresponding to the observation of no NP effects in 10 years of IceCube.

\[ R_{h/v}(E_{\mu}^{\text{fin},i}) \equiv \frac{P_{\text{hor}}(E_{\mu}^{\text{fin},i})}{P_{\text{ver}}(E_{\mu}^{\text{fin},i})} = \frac{\frac{N_{\mu}^{vli}(E_{\mu}^{\text{fin},i}, -0.6 < \cos \theta < -0.2)}{N_{\mu}^{\text{no-vli}}(E_{\mu}^{\text{fin},i}, -0.6 < \cos \theta < -0.2)}}{\frac{N_{\mu}^{\text{vli}}(E_{\mu}^{\text{fin},i}, -1 < \cos \theta < -0.6)}{N_{\mu}^{\text{no-vli}}(E_{\mu}^{\text{fin},i}, -1 < \cos \theta < -0.6)}} , \quad (10) \]

where by $E_{\mu}^{\text{fin},i}$ we denote integration in an energy bin of width $0.2 \log_{10}(E_{\mu}^{\text{fin},i})$ using that IceCube measures energy to 20% in $\log_{10} E$ for muons.

In what follows we will use the double ratio in Eq. (10) as the observable to
determine the sensitivity of IceCube to NP-induced oscillations. We have chosen a double ratio to eliminate uncertainties associated with the overall normalization of the ATM fluxes at high energies. It is worth noticing that using this observable relies on the fact that the zenith angular dependence of the effective area is well understood.

In Fig. 2 we plot the expected value of this ratio for different values of $\delta c/c$. As mentioned above, IceCube measures energy to 20% in $\log_{10} E$ for muons. Accordingly, we have divided the data in $16 E_{\mu}^{\text{fin}}$ bins: 15 bins between $10^2$ and $10^5$ GeV and one containing all events above $10^5$ GeV. In the figure the full lines include both the $\nu_\mu$-induced events (Eq.(9)) and $\nu_\tau$-induced events (Eq.(10)) while the last ones are not included in the dashed curves. As described above, the net result of including the $\nu_\tau$-induced events is a slight decrease of the maximum expected value of the double ratio. The data points in the figure show the expected statistical error corresponding to the observation of no NP effects in 10 years of IceCube. In order to estimate the expected sensitivity we assume that no NP effect is observed and define a simple $\chi^2$ function including only the statistical errors.

We show in Fig. 3 the sensitivity limits in the $[\delta c/c, \xi_{\nu\text{li}}]$-plane at 90, 95, 99 and 3 $\sigma$ CL obtained from the condition $\chi^2(\delta c/c, \xi_{\nu\text{li}}) < \chi^2_{\text{max}}(\text{CL},2\text{dof})$. We show in the figure the results obtained using the RQPM model and the TIG model. The difference is about 50% in the strongest bound on $\delta c/c$.

Fig. 3. Sensitivity limits in the $\delta c/c, \xi_{\nu\text{li}}$ at 90, 95, 99 and 3 $\sigma$ CL. The hatched area in the upper right corner is the present 3$\sigma$ bound from the analysis of SK data in Ref. [16].

The figure illustrates the improvement on the present bounds by more than two orders of magnitude even within the context of this very conservative analysis. The loss of sensitivity at large $\delta c/c$ is a consequence of the use of a double ratio as an observable. Such an observable is insensitive to NP effects if $\delta c/c$ is large enough for the oscillations to be always averaged leading only to an overall suppression.

When data becomes available a more realistic analysis is likely to lead to a further improvement of the sensitivity.
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