An interval interference reliability evaluation method for structural strength

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Abstract. The structural strength reliability analysis can effectively improve the service life of the structure. Based on the interval method and stress-strength interference method, the structural stress and strength at any time can be converted into the standardized interval. According to the relationship between the critical state function and the standardized interval, the interval interference reliability index $\eta$ is defined. And then an interval interference evaluation method is proposed for structural strength. The validity of the method is verified through compared with the interval reliability method and the stress-strength interference reliability method. The results show that the proposed method can analysis the structural strength reliability without the parameter distribution form.

1. Introduction
The structural strength reliability analysis is of great practical significance to ensure the safety of the structure and improve the service life of the structure. However, under the influence of corrosion, aging and external random load, the reliability of structural strength is not a single value under the traditional model, but a value with time-varying characteristic.

At present, the reliability analysis methods based on probability theory, such as the outcrossing rate method[1], the PHI2 method[2], the Monte Carlo method[3], the equivalent stochastic process transformation method[4], the cross-entropy-based adaptive importance sampling method[5] and the stress-strength interference method[6] are usually adopted to analysis the time-varying reliability of the structure. However, these probabilistic reliability analysis methods largely rely on the probability density function of parameters. If the probability density function is assumed subjectively due to lack of data, the analysis results are not convincing[7].

The great advantage of non-probability reliability theory in dealing with uncertainty provided an important reference for structural time-varying reliability analysis. Ben-haim[8] proposed the concept of structural reliability based on convex model theory firstly in the 1990s. In 1995, Elishakoff[9] analysed the non-probability safety by using the interval theory. In 2001, Guo et al[10] proposed a non-probability method that defined the shortest distance from the origin to the limit state plane as the reliability. In 2003, Qiu et al[11] compared the non-probability interval method and probability method to prove the consistency of analysis results between interval theory and probability theory.

Based on interval theory and stress-strength interference theory, an improbability interval interference reliability analysis method applied to analysis the structural strength time-varying reliability is proposed in this paper.

2. Formulation the equivalent stress
Suppose $X = \{X_1, X_2, \ldots, X_n\}$ is the set of parameters related to strength $R$, $Y = \{Y_1, Y_2, \ldots, Y_m\}$ is the set of parameters related to stress $S$. Based on the stress-strength interference method, the structural
The state function can be expressed as
\[ M = R(X) - S(Y) \]  
(1)

The constant strength \( R_s(X) \) and uncertain strength \( R_u(X) \) of structure can be expressed as the interval variables
\[ R_s(X) = [R_s(X), \bar{R}_s(X)] \]  
(2)
\[ R_u(X) = [\bar{R}_u(X), \bar{R}_u(X)] \]  
(3)

Based on the interval method, the upper and lower bound of structural strength \( R(X) \) can be expressed as
\[ \bar{R}(X) = \bar{R}_s(X) + \bar{R}_u(X) \]  
(4)
\[ R(X) = R_s(X) + R_u(X) \]  
(5)

The median and deviation of \( R(X) \) are
\[ R^*(X) = R^*_s(X) + R^*_u(X) \]  
(6)
\[ R'(X) = R'_s(X) + R'_u(X) \]  
(7)
where \( R^*_s(X) \) and \( R^*_u(X) \) are the median and deviation of \( R_s(X) \), \( R'_s(X) \) and \( R'_u(X) \) are the median and deviation of \( R_u(X) \).

The constant stress \( S_s(Y) \) and uncertain stress \( S_u(Y) \) of structure can be expressed as the interval variables
\[ S_s(Y) = [S_s(Y), \bar{S}_s(Y)] \]  
(8)
\[ S_u(Y) = [\bar{S}_u(Y), \bar{S}_u(Y)] \]  
(9)

Based on the interval method, the upper and lower bound of structural stress \( S(Y) \) can be expressed as
\[ \bar{S}(Y) = \bar{S}_s(Y) + \bar{S}_u(Y) \]  
(10)
\[ S(Y) = S_s(Y) + S_u(Y) \]  
(11)

The median and deviation of \( S(Y) \) are
\[ S^*(Y) = S^*_s(Y) + S^*_u(Y) \]  
(12)
\[ S'(Y) = S'_s(Y) + S'_u(Y) \]  
(13)
where \( S^*_s(Y) \) and \( S^*_u(Y) \) are the median and deviation of \( S_s(Y) \), \( S'_s(Y) \) and \( S'_u(Y) \) are the median and deviation of \( S_u(Y) \).

Hence, the structural stress and strength at any time can be converted into the standardized interval
\[ R(X) = R^*(X) + R'(X)\delta_R \]  
(14)
\[ S(Y) = S^*(Y) + S'(Y)\delta_S \]  
(15)
where \(-1 \leq \delta_R \leq 1 \) and \(-1 \leq \delta_S \leq 1 \) are the standardized variables of strength and stress.

The structural state function can be rewritten as
\[ M = R^*(X) + R'(X)\delta_R - S^*(Y) - S'(Y)\delta_S \]  
(16)

And the critical state function \( M = 0 \) is
\[ \delta_R = \frac{S^*(Y) - S'(Y)}{R^*(X) - R'(X)} \]  
(17)

The slope of \( M = 0 \) in standardized interval can be expressed as

\[ \delta_R = \frac{S^*(Y) - S'(Y)}{R^*(X) - R'(X)} \]  

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When $R(X) \geq \bar{S}(Y)$, for any $(\delta_x, \delta_y)$ in standardized interval, the structural state function $M \geq 0$, it means that the structure in completely reliable state, the interval interference reliability index $\eta$ can be defined as

$$\eta = \frac{R^e(X) - R^o(X)}{S^e(Y) + S^o(Y)}$$

When $R(X) < \bar{S}(Y)$, for any $(\delta_x, \delta_y)$ in standardized interval, the $\eta \geq 1$ represents the structural reliability, the structure is in completely reliable state. When $k > 1$, $k = 1$, $0 < k < 1$ and $k = 0$, the relationship between the $M = 0$ and the standardized interval can be shown in Figure 2.

![Figure 1. Critical state function and normalized interval](image)

The ratio between the area of reliable region $A$ and the total area of standardized interval $A = 4$ is defined as the interval interference reliability index $\eta$

$$\eta = \frac{A}{A}$$

According to equation (20), the $\eta \in (0,1)$ represents the structural reliability, the structure is in incompletely reliable state. When $k > 1$, $k = 1$, $0 < k < 1$ and $k = 0$, the $\eta$ can be obtained by the equations (21)-(24).

$$\eta = \begin{cases} 1 - \frac{\left[ R^e(X) + S^o(Y) + S^o(Y) - R^o(X) \right]^2}{8R^e(X)S^o(Y)} & R(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{S^e(Y) + R^e(X) - S^o(Y)}{2S^o(Y)} & \bar{R}(X) < \bar{S}(Y), \bar{S}(Y) < R(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & R(X) \leq \bar{S}(Y) < \bar{R}(X) \\ 1 - \frac{\left[ R^e(X) + S^o(Y) + S^o(Y) - R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{R}(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{S}(Y) < \bar{R}(X) < \bar{S}(Y) \end{cases}$$

$$\eta = \begin{cases} 1 - \frac{\left[ R^e(X) + S^o(Y) + S^o(Y) - R^o(X) \right]^2}{8R^e(X)S^o(Y)} & R(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{R}(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{S}(Y) < \bar{R}(X) < \bar{S}(Y) \end{cases}$$

$$\eta = \begin{cases} 1 - \frac{\left[ R^e(X) + S^o(Y) + S^o(Y) - R^o(X) \right]^2}{8R^e(X)S^o(Y)} & R(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{R}(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{S}(Y) < \bar{R}(X) < \bar{S}(Y) \end{cases}$$

$$\eta = \begin{cases} 1 - \frac{\left[ R^e(X) + S^o(Y) + S^o(Y) - R^o(X) \right]^2}{8R^e(X)S^o(Y)} & R(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{R}(X) < \bar{S}(Y) \leq \bar{R}(X) \\ \frac{\left[ R^e(X) - S^o(Y) + S^o(Y) + R^o(X) \right]^2}{8R^e(X)S^o(Y)} & \bar{S}(Y) < \bar{R}(X) < \bar{S}(Y) \end{cases}$$
The stress obeys normal distribution, and the mean value is $R_S(X) + S(Y)$. The reliability index of stress $S(Y)$, the structure is in failure state, and the reliability index is defined as $\eta = 0$. 

3. Numerical example

The reliabilities of ten cases as shown in Table 1 are analysed. And the results are compared with the stress-strength reliability and the interval reliability method to verify the validity of proposed method.

Table 1. The parameters of storage tank stress interval (MPa)

| Parameter | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|-----------|----|----|----|----|----|----|----|----|----|----|
| $R'(X)$  | 300| 300| 300| 300| 300| 300| 300| 300| 300| 260|
| $R'(Y)$  | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 20 |
| $S'(Y)$  | 260| 260| 260| 260| 260| 260| 260| 260| 260| 300|
| $S'(Y)$  | 10 | 20 | 30 | 10 | 20 | 30 | 10 | 20 | 30 | 20 |

Suppose the strength obeys normal distribution, and the mean value is $R'(X)$ and the standard deviation is $R'(X)/3$. The stress obeys normal distribution, and the mean value is $S'(Y)$ and the standard deviation is $S'(Y)/3$. The reliability index of stress-intensity interference with normal distribution parameters can be expressed as

$$\eta_n = \frac{R'(X) - S'(Y)}{\sqrt{\left[R'(X)/3\right]^2 + \left[S'(Y)/3\right]^2}}$$  \hspace{1cm} (25)

where the $\Phi(\cdot)$ is the standard normal distribution function.

The interval reliability method is an important non-probabilistic reliability analysis method, and the interval reliability index can be expressed as

$$\eta_i = \frac{R'(X) - S'(Y)}{R'(X) + S'(Y)}$$  \hspace{1cm} (26)

Based on the equations (21)-(26), the reliability indexes $\eta$, $\eta_i$, and $\eta_n$ of the structure can be obtained, as shown in Table 2.

Table 2. Parameters of a certain type tank

| Index | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|-------|------|------|------|------|------|------|------|------|------|------|
| $\eta$ | 1.074| 1.036| 1.000| 1.037| 1.000| 0.979| 1.000| 0.979| 0.944| 0.000|
| $\eta_i$ | 2.000| 1.333| 1.000| 1.333| 1.000| 0.800| 1.000| 0.800| 0.667| 0.000|
| $\eta_n$ | 1.000| 1.000| 1.000| 1.000| 1.000| 0.999| 1.000| 0.999| 0.997| 0.017|
In order to clearly show the difference between the $\eta$, $\eta_i$, and $\eta_n$, the results can be figured out in Figure 2.

![Figure 2. The difference between the $\eta$, $\eta_i$ and $\eta_n$.](image)

The Table 2 and Figure 2 show that the $\eta$ is between the $\eta_i$ and $\eta_n$. When the $\eta \geq 1$, the structure is in completely reliability state. When the $0 < \eta < 1$, the structure is in incompletely reliability state. When the $\eta = 0$, the structure is in failure state.

4. Conclusion
In this paper, an interval interference reliability evaluation method is proposed for structural strength, the main conclusions are as follows:

(1) The structural stress and strength at any time can be converted into a standardized interval, and the reliability index $\eta$ can be obtained according to the relationship between the critical state function $M = 0$ and the standardized interval.

(2) When $\eta \geq 1$, the structure is in completely reliability state, and the $\eta$ represents the safety margin. When $0 < \eta < 1$, the structure is in non-completely reliability state, and the $\eta$ represents the reliability. When $\eta = 0$, the structure is in failure state.

(3) The interval interference reliability evaluation method can analysis the structural strength time-varying reliability without the parameter distribution form. And the analysis result is consistent with the results of interval reliability method and stress-strength reliability method.

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