ESCAPING PARTICLE FLUXES IN THE ATMOSPHERES OF CLOSE-IN EXOPLANETS. I. MODEL OF HYDROGEN

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ABSTRACT

A multi-fluid model for an atomic hydrogen–proton mixture in the upper atmosphere of an extrasolar planet is presented when the continuity and momentum equations of each component have already been solved with an energy equation. The particle-number density, temperature distribution, and structure of velocity can be found by using the model. I chose two special objects, HD 209458b and HD 189733b, for discussion and concluded that their predicted mass-loss rates are consistent with those observed. The most important physical process in coupling each component is the charge exchange, which couples atomic hydrogen tightly with protons. Most of the hydrogen escapes from hot Jupiters as protons, especially in the young star–planet system. I found that the single-fluid model can describe the escape of particles when the mass-loss rate is higher than a few times $10^9$ g s$^{-1}$, while below $10^9$ g s$^{-1}$ the multi-fluid model is more suitable because of the decoupling of particles. Assuming an energy limit, I found that the predicted mass-loss rates of HD 189733b are a factor of 10 larger than those calculated by my models because of a high degree of ionization. For ionized wind, which is mainly composed of protons, assuming an energy limit is no longer effective. I fitted the mass-loss rates of the ionized wind as a function of $F_{\text{UV}}$ by calculating the variation of the mass-loss rates with UV fluxes.

Key words: hydrodynamics – planetary systems – planets and satellites: atmospheres – planets and satellites: individual (HD 109458b, HD 189733b)

1. INTRODUCTION

The discovery that the hot Jupiter HD 209458b is losing mass was rather unexpected. (Recently, Lecavelier des Etangs et al. 2010 found atmospheric evaporation in HD 189733b. This is the second extrasolar planet whose mass loss has been detected.) The excess absorption in Ly$\alpha$ first found by Vidal-Madjar et al. (2003, hereafter referred to as VM03) and later confirmed by Linsky et al. (2010) can be explained either by atmospheric mass loss from X-ray and ultraviolet (XUV) energy input from host stars (Lammer et al. 2003, 2009; Lecavelier des Etangs et al. 2004; Yelle 2004, 2006; Tian et al. 2005; Garcia Munoz 2007; Penz et al. 2008; Murray-Clay et al. 2009) or by the charge exchange between stellar wind and the planetary-escaping exosphere (Holmström et al. 2008). For the former, these models describe the thermal particle escape. For the latter, Erkaev et al. (2005) and Holmström et al. (2008) discussed the loss of nonthermal neutral atoms due to the interaction between stellar wind and the exosphere (for more details, see Ekenbäck et al. 2010). The model of Erkaev et al. (2005) clearly underestimates the particle-loss rates, but Ekenbäck et al. (2010) modeled the production of neutral hydrogen and matched the Ly$\alpha$ absorption rate. It is not easy to distinguish which process dominates Ly$\alpha$ absorption more. Ben-Jaffel & Hosseini (2010) found that either energetic H i of stellar origin or thermal H i populations in the planetary atmosphere can fit Ly$\alpha$ observations. Koskinen et al. (2010, hereafter referred to as K10) used an empirical model to analyze UV transit depths, and their results showed that observations can be explained solely by absorption in the upper atmosphere, so the process of charge exchange may not be necessary. Note that properties of the planetary magnetic field are still unclear. Thus, it is important for both thermal and nonthermal models to self-consistently calculate the deflection distance around planets by using magnetohydrodynamics models.

Observations of transit of hot Jupiters have found that mass-loss rates are in the range of $10^8$–$10^{11}$ g s$^{-1}$ (VM03; Linsky et al. 2010; Lecavelier des Etangs et al. 2010). The effect of evaporation has been used to determine the evolution of exoplanets. Baraffe et al. (2004, 2005) considered the effect of mass loss on the evolution of exoplanets and mass-loss rates varying from $10^{-12}$ $M_J$ yr$^{-1}$ to $10^{-8}$ $M_J$ yr$^{-1}$ are obtained from their calculations. However, an escape rate $10^2$ lower than Baraffe et al.’s (2004, 2005) was found by Hubbard et al. (2007a, 2007b). Their results showed that moderate mass-loss rates may be more appropriate for the mass function obtained from observations. Guo (2010) further investigated the influence of mass loss on the tidal evolution of exoplanets and found that it cannot be neglected for planets with initial mass $<1$ $M_J$ and initial orbital distance $<0.1$ AU. It has also been suggested that evaporation can lead to significant modification of the nature of planets, forming planetary remnants (Lecavelier des Etangs et al. 2004, 2007; Penz & Micela 2008; Penz et al. 2008; Davis & Wheatley 2009).

Current theoretical models focus on macroscopic material escape and incorporate photochemistry (Yelle 2004; Tian et al. 2005; Garcia Munoz 2007; Penz et al. 2008; Murray-Clay et al. 2009). These models suggest that the EUV and X-ray energy input by host stars is the main source of energy for atmospheric escape. However, the escape process of different species may be dominated by different physical mechanisms, and the winds around exoplanets whose orbits are distant may be tenuous. Thus, it must be argued whether the heavier species decouples from the light elements in tenuous winds. (O i and CI transit depths of 13% and 7.5% obtained by Vidal-Madjar et al. 2004 indicated that both oxygen and carbon in the atmosphere of HD 209458b are lost.) Moreover, a detailed physical
model may be helpful in determining the origin of hydrogen around HD 209458b. Because scarcely any micro-physics has been included in hydrodynamic simulations, it may be worthwhile to study the behavior of species by using multi-fluid models.

Considering the process of radiative transfer, this paper aims to calculate planetary atomic hydrogen and proton loss rates through solutions to their mass, momentum, and energy equations. Many microscopic physics processes are covered by the mass and momentum equations (Section 2.1). The code is designed for ordinary equations with one or more critical points (Section 2.2). I use the Henyey method to calculate these equations (Section 2.4). The results are presented in Section 3. Of special interest is the decoupling of atomic hydrogen from protons (Section 4.1). In Section 4.2, I discuss properties of ionized wind and fit the mass-loss rate as a function of UV flux. I summarize the results in Section 5.

2. THE MODEL

The model describes the steady-state radial expansion of plasma containing three species: atomic hydrogen (h), protons (p), and electrons (e). Each species has its own continuity and momentum equations and is described by a particle density, n, and velocity, u. In this model, I do not include H$_2$ because the thermosphere of close-in planets should be composed primarily of H and H$^+$. The location of the transition from H$_2$ to H is about 1.1 $R_p$ (Yelle 2004). In addition, assuming that $T_h = T_p = T_e$, only one energy equation for electrons is given in the calculations.

2.1. Equations and Assumptions

As this model deals with a mixture of atomic hydrogen and protons, the following processes are considered: photoionization, recombination, and charge exchange. The most important collision between neutral hydrogen and ions is the charge exchange. Other collisions, such as the hard-sphere collision, between neutral and ionized particles are not important. The continuity and momentum equations for each species, including the source/sink terms that relate to photoionization/recombination and charge exchange for a species $j$, are

$$\frac{1}{r^2} \frac{d}{dr} (r^2 n_j u_j) = \frac{\delta n_j}{\delta r} (j = h, p)$$  

$$u_h \frac{dn_h}{dr} + \frac{1}{m_n n_h} \frac{dP_h}{dr} + \frac{GM_p}{r^2} - \frac{3GM_h}{a^3} = \frac{\delta M_h}{\delta r}$$  

$$u_p \frac{dn_p}{dr} + \frac{1}{m_n n_p} \frac{dP_p}{dr} \frac{-eE}{m_p} + \frac{GM_p}{r^2} - \frac{3GM_h}{a^3} = \frac{\delta M_p}{\delta r},$$

where $G$ is the gravitational constant, $P_j = n_j k_B T$ is partial pressure, $T$ is temperature, $m_n$ is hydrogen atom mass, and $M_p$ is planetary mass. The term $\frac{3GM_j}{m_j a^3}$ denotes the tidal gravity; $M_a$ and $a$ are stellar mass and the semi-major axis, respectively; and $E$ is the charge-separation electric field.

Quasi-neutrality and zero current were assumed in the derivation of the momentum equation; thus, $n_e = n_p$ and $u_e = u_p$. Both inertia and gravity are negligible in the electron momentum equation because of the small mass of electrons, so I can express the electric field $E$ as

$$eE = -\frac{1}{n_e} \frac{dP_e}{dr}.$$  

The two terms $\frac{\delta M_h}{\delta r}$ and $\frac{\delta M_p}{\delta r}$ represent the source and sink terms caused by elastic and inelastic collisions with other species, respectively. The sources and sinks for the particle flux density derive from photoionization and recombination. The term of resonant charge exchange is $H, H^+ \leftrightarrow H^+$, and is included in the momentum equation but not in the continuity equation, because the same particles are present before and after the interaction. With the assumptions and the definitions as above, I write the collision terms as

$$\frac{\delta n_h}{\delta r} = n_p \gamma_{\text{rec}} - n_h \gamma_{\text{pho}}$$  

$$\frac{\delta n_p}{\delta r} = n_h \gamma_{\text{pho}} - n_p \gamma_{\text{rec}}$$

$$\frac{\delta M_h}{\delta r} = -\gamma_{\text{rec}} n_p (u_h - u_p) - \gamma_{\text{pho}} n_h (u_h - u_p)$$  

$$\frac{\delta M_p}{\delta r} = -\gamma_{\text{rec}} n_h (u_p - u_h) - \gamma_{\text{pho}} n_p (u_p - u_h),$$

where $\gamma_{\text{pho}} = \frac{F_{\text{UV}} e^{-1} \sigma_{\text{e0}}}{h_0}$ and $\gamma_{\text{rec}} = 2.7 \times 10^{-13} (\frac{T}{10^4})^{-0.9} n_p s^{-1}$ (Murray-Clay et al. 2009) represent the photoionization and recombination rates, respectively. $\gamma_{\text{rec}} = 1.12 \times 10^{-8} (\frac{T}{10^4})^{1/2} [1 - 0.12 \log (\frac{T}{10^4})]^{-2}$ cm$^3$s$^{-1}$ is the rate of charge exchange (Geiss & Bürgi 1986).

I assumed that ions and atoms have the same temperature, which is justified by the high collision rates. For protons, the temperature equilibration time in atomic hydrogen is

$$t_{eq} = \frac{1}{n_h \gamma_{\text{pho}}},$$

Due to $n_h \sim 10^5-10^{11}$ cm$^{-3}$, $T \sim 10^4$ K, I have $t \sim 10^{-3}-10^{-1}$ s. The ionization time of about $10^5$ s (Yelle 2004) in the upper atmosphere is two orders of magnitude greater than the temperature equilibration time. Following Schunk (1975), the energy equation for electrons is given as

$$\frac{3}{2} n_e u_e k_B \frac{dT}{dr} - k_B u_e \frac{dn_e}{dr} = \sum_{j} \sqrt{2} Z_j^2 \nu_{ee} m_j (u_j - u_e)^2 + H - L,$$

The right-hand-side terms in the equation denote in-sequence elastic collisions with heavy ions, heating and cooling. Assuming charge neutrality and zero current, the elastic collision term equals zero. The heating process in the mixed flows is complex. The stellar radiation ionizes the species to produce high-energy photoelectrons that share their energy with other species via collisions.

A full description of the process is beyond the scope of this paper. I follow the model of Murray-Clay et al. (2009) to describe heating from photoionization, but I use a coefficient, $\alpha = \frac{n_h}{n_e + n_h}$, to denote the fraction that is shared with electrons. Therefore

$$H = \alpha eF_{\text{UV}} e^{-1} \sigma_{\text{e0}} \gamma_{\text{pho}} n_h,$$
where \( \varepsilon = (h\nu_0 - 13.6 \text{ eV}) / h\nu_0 \) is the fraction of photon energy deposited as heat, \( h\nu_0 = 20 \text{ eV} \), and

\[
\tau = \sigma_0 \int_r^\infty n_h dr
\]

(12)
is the optical depth.

Murray-Clay et al. (2009) pointed out that the main contribution to cooling comes from Ly\( \alpha \) radiation. Although the radiation is emitted by atoms and ions, excitation of the atoms and ions is due to electron collisions. Thus, the process is an energy-loss mechanism of the electrons. The cooling is

\[
L = 7.5 \times 10^{-19} n_h n_p e^{-118348/T} \text{ erg cm}^{-3} \text{s}^{-1}.
\]

(13)

2.2. Critical Points

Equations considered here have structures similar to those of the solar wind. From Equations (1)–(8) and (10), I find that there exist two singular (sonic) points where the velocity derivatives cannot be determined by this set of equations. These singularities are important because only the transonic solution can yield flows from subsonic velocities at the base of the wind to supersonic velocities outside the singularity. At these points, velocity derivatives must be obtained by additional conditions, such as regularity conditions.

Eliminating all derivatives other than the velocity derivative from the H atom and proton momentum equations leads to coupled differential equations of the momentum, which can be written in matrix form as

\[
\begin{pmatrix}
  a_{hh} & a_{hp} \\
  a_{ph} & a_{pp}
\end{pmatrix}
\begin{pmatrix}
  \frac{du_h}{dr} \\
  \frac{du_p}{dr}
\end{pmatrix}
= \begin{pmatrix}
  b_h \\
  b_p
\end{pmatrix},
\]

(14)

where

\[
a_{hh} = \left( \frac{k_B T}{m_H u_h} - u_h \right),
\]

(15)

\[
a_{pp} = \left( \frac{10}{3} \frac{k_B T}{m_H u_p} - u_p \right),
\]

(16)

where vector \( \mathbf{b} \) does not contain any functions of the derivatives of the variables \( n_j, u_j \) (\( j = h, p \)), or \( T \).

For the solar wind, the locus of singularity can be defined by the points at which the determinant of matrix \( D = a_{hh}a_{pp} - a_{ph}a_{hp} \) vanishes (B"urgi 1992). In this model, I find \( a_{pp} = 0 \); thus, \( a_{hh} \times a_{pp} = 0 \) can be forced into \( a_{hh} = 0 \) and \( a_{pp} = 0 \). This means that there exist two separate critical points for atomic hydrogen and protons, respectively, at which their velocities equal their sound velocities.

2.3. Boundary Conditions

Six boundary conditions are necessary to solve Equations (1)–(3), (10), and (12). At the bottom of the flow, I set \( T_0 = 1000 \text{ K} \). Here, the particle densities are assumed to be \( n_h + n_p = \rho_0 / m_H \). Considering the model of Murray-Clay et al. (2009), I assumed that the density value is \( 4 \times 10^{-15} \text{ g cm}^{-3} \). In the bottom regions, collisions between atomic hydrogen and protons are frequent; thus, I assumed \( u_h = u_p \). I need three more boundary conditions to close the set of equations. Two boundary conditions are provided by the regularity condition. These conditions can be written as

\[
dH_i/dr = \sum_j \left( \frac{\partial H_i}{\partial y_j} \frac{\partial y_j}{dr} \right) = 0,
\]

(17)

where \( H_i \) is the momentum equation of species \( i \) and \( y_j \) refers to variables \( r, T, n_i, \) and \( u_y(k = h, p) \). These boundary conditions are added to corresponding mesh points. However, in my calculations I find that these conditions are numerically unstable. When the total number of mesh points is increased, the approximate condition \( du_k/dr = 0 \) (\( k = h, p \)) can be successfully used to substitute for Equation (17) (Nobili & Turolla 1988). For the last boundary conditions, I assumed the optical depth in the sonic points of atomic hydrogen to be \( \tau_c^h = 0.0023 \), where \( \tau_c^h \) is the optical depth between the sonic point and the Roche lobe (Murray-Clay et al. 2009).

2.4. Numerical Method

The momentum equations have two singularities at two points, \( u_p^2 = \frac{10/3k_T}{m_H} \) and \( u_h^2 = \frac{\tau}{m_H} \) (see Equations (15) and (16)), for protons and atomic hydrogen, respectively. Equations (2) and (3) can be explicitly written as

\[
y' + \frac{g(x, y)}{f(x, y)} = 0,
\]

(18)

where \( f(x, y) \) vanishes in the sonic points. Normally, the velocity of flow increases with radius. At a specific point (the sonic point), \( f(x, y) \) begins to change sign. Thus, the critical points can be found at the location where the sign of \( f(x, y) \) changes. To solve boundary problems, the Henyey method is convenient, and critical points appearing within the integration domain can be treated accordingly (Nobili & Turolla 1988). As a modified Newton–Raphson method, this method includes the critical points in the set of linearized flow equations. All variables \( Y_i \) at the half-mesh point are interpolated as

\[
Y_{x_{i+1/2}} = \frac{Y_{i+1} + Y_i}{2},
\]

(19)

while derivatives are approximated by finite differences:

\[
Y'_{x_{i+1/2}} = \frac{Y_{i+1} - Y_i}{x_{i+1} - x_i},
\]

(20)

In my calculations, I allow the specification of one condition every time a critical point is met.

3. RESULTS

For comparison with the single-fluid model, the Murray-Clay et al. (2009) model is first solved via their own method and then via the Henyey method. No significant difference is found between these methods, but minor differences may exist due to a detailed numerical configuration. Using the Henyey method is advantageous because it enables one to obtain instant solutions for all domains and to self-consistently handle several critical points in the numerical configuration.

3.1. HD 209458b

In this section, I applied my model to a typical planet sample, HD 209458b, which has a radius of 1.4 \( R_J \), mass of 0.7 \( M_J \), and semi-major axis of 0.05 (Murray-Clay et al. 2009). The upper limit of the observed orbital eccentricity is 0.028 (Ibgui & Burrows 2009), so I assumed a circular orbit in my calculations. The mass of the host star is 1 \( M_\odot \).

Results for HD 209458b, along with the single-fluid results of Murray-Clay et al. (2009), are plotted in Figure 1. It is clear
The transonic wind driven by stellar XUV has two sonic points: one is for atomic hydrogen and the other is for protons. Because protons bear an external electric force, $E$, if we assume the same temperature in both particles, the sonic point extends to outside the wind. The velocities of H and H$^+$ are almost the same. When one particle travels outward, the other particle drags it inward. The collision between these two particles leads to a tight coupling of their kinetics.

I described the photoionization, recombination, and charge-exchange rates in Section 2.1. Evidently, the charge exchange dominates the other two collision processes ($\gamma_{\text{ex}} \gg \gamma_{\text{pho}}, \gamma_{\text{rec}}$). To show the effect of the charge exchange, I calculate a model without it (In fact, I encounter numerical problems when the charge exchange is fully neglected. Thus, I retain the term and multiply it by a factor of $10^{-3}$). Figure 2 displays that protons decouple from atomic hydrogen. If the most important process is neglected, the behavior of each particle looks like a single “fluid.”

Note that the ratios of two velocity components decline with an increase in radius but rise at the point $R = 2R_p$ (see Figure 2). This phenomenon can be explained by the behavior of photoionization and recombination rates. Inside the wind, the amount of momentum transferred by the processes of photoionization and recombination is not enough; therefore,
neutral hydrogen decouples from protons. With decreases in optical depth and temperature, the effect of photoionization and recombination rates on the transfer of momentum increases outside the wind, so the value of $u_b/u_p$ also increases. A similar phenomenon will be discussed in Section 4.1.

The mass-loss rates are defined as

$$
\dot{M} = 4 \pi m_H \left( \left( \frac{r_p}{R_p} \right)^2 n_{bH} + \left( \frac{r_p}{R_p} \right)^2 n_{bH} \right),
$$

where $r_p$ and $r_p^2$ are the sonic points of atomic hydrogen and protons, respectively; $n$ and $u$ are the corresponding number density and velocity. The mass-loss rate of $\dot{M} = 9 \times 10^{10} \, g \, s^{-1}$, obtained by applying the solution over the entire planetary surface, is higher than that of $\dot{M} = 3 \times 10^{10} \, g \, s^{-1}$ obtained by Murray-Clay et al. (2009), but both are approximately in accord with the value in VM03’s observations. To fit the observed profile of Ly$\alpha$, K10 assumed that the mean temperature of the thermosphere is 8000–10,000 K and the upper boundary is located at $R \sim 2.9 R_p$, where the number density is $n = 2.6 \times 10^7 \, cm^{-3}$. My calculation results show that the particle-number density at the radius ($R \sim 2.9 R_p$) is $n = 5 \times 10^6 \, cm^{-3}$. A significant difference between their model and mine is that they assume hydrostatic equilibrium. My results show that the sonic points of H and H$^+$ are 2.57 $R_p$ and 3.67 $R_p$, respectively. This means that the assumption of hydrostatic equilibrium is acceptable in the region ranging from 1 $R_p$ to 2.57 $R_p$. However, the number-density distribution could be different due to the difference in physical detail. For example, my results that about 60% hydrogen is ionized at 2.9 $R_p$ do not support the assumption of K10 that the atmosphere is mostly ionized above 2.9 $R_p$. In addition, with the assumption of hydrostatic equilibrium, the profile of number density is flatter than that of hydrodynamics, which can lead to high optical depth in the wings of the line.

It is convenient to define the mass-loss rates of neutral hydrogen and protons as

$$
\dot{M}_{h,p} = 4 \pi m_H \left( \left( \frac{r_p}{R_p} \right)^2 n_{bH} + \left( \frac{r_p}{R_p} \right)^2 n_{bH} \right).
$$

My results indicate that the mass-loss rates of neutral hydrogen and protons are $3.4 \times 10^{10} \, g \, s^{-1}$ and $5.6 \times 10^{10} \, g \, s^{-1}$, respectively.

To fit the observations of HD 209458b, two scenarios can supply a satisfactory fit. In the first case, thermal hydrogen atoms are enough to fit the Ly$\alpha$ transit profile so that the energetic atoms are not necessary. In the second case, superthermal (hot) hydrogen atoms are required to fit the observations if depleted thermal hydrogen atoms are assumed (Ben-Jaffel et al. 2010). Superthermal hydrogen atoms in the atmosphere can be formed via the absorption of stellar UV radiation (Shematovich 2010). When hydrogen atoms have an excess of kinetic energy, they can be locally thermalized with the surrounding particles if the atmospheric density is high enough. However, in the upper atmosphere, superthermal hydrogen atoms may escape due to excess kinetic energy. A mass-loss rate of $3.4 \times 10^9 \, g \, s^{-1}$ lower than the observed value for HD 209458b has been estimated by Shematovich (2010), whose results showed that superthermal hydrogen atoms have velocities of about 20 km s$^{-1}$. Therefore, it is unlikely that superthermal hydrogen atoms can explain the observed velocities at the wing of the line. Note that the location of transition from H$_2$ to H is about 1.1 $R_p$ (Yelle 2004; Shematovich 2010), implying that superthermal hydrogen atoms cannot directly escape from the atmosphere of planets but must exchange energy and momentum with cool background particles in large-scale ranges. In fact, the influence of superthermal hydrogen should be included in future hydrodynamic models.

### 3.2. HD 189733b

HD 189733b is the second extrasolar planet whose atmospheric evaporation has been detected (Lecavelier des Etangs et al. 2010). According to observations, the authors constrain the escape rate of atomic hydrogen to between $10^9$ and $10^{11} \, g \, s^{-1}$ and EUV flux to 10–40 times the solar value, namely, $F_{EUV} \sim 1-4 \times 10^6 \, erg \, cm^{-2} \, s^{-1}$. The value may be changed to $2.5-10 \times 10^4 \, erg \, cm^{-2} \, s^{-1}$ if the emission of H Ly$\alpha$ is also included. The observed value has been validated by X-ray observations. The luminosities of HD 209458b and HD 189733b in X-rays were measured as log $L_x = 26.12$ and log $L_x = 28.18 \, erg \, s^{-1}$ (Sanz-Forcada et al. 2010) by the XMM-Newton. Because the orbital distance of HD 189733b is 1.5 times closer to its host star than that of HD 209458b, the planet receives X-ray radiation from its host star that is 300 times stronger than that received by HD 209458b. If the X-ray flux is proportional to the UV flux, HD 189733b receives a UV flux of $F_{UV,189733b} = \frac{F_{HD209458b}}{300} \sim 10^8 \, erg \, cm^{-2} \, s^{-1}$.

The planet has a mass $M_p = 1.13 \, M_J$, radius $R_p = 1.16 \, R_J$, and a semi-major axis $a = 0.03 \, AU$ (Bakos et al. 2006; Winn et al. 2007; Southworth 2010). The mass of its host star is $0.8 \, M_\odot$ (Nordström et al. 2004). Compared to HD 209458b, the mass of HD189733b is 1.7 times larger, but its radius is smaller. This means that the potential well of HD 189733b is about twice as deep as that of HD 209458b. Thus, the effect of strong X-ray flux can be balanced to a certain extent by its larger potential well.

Figure 3 displays the results for HD 189733b with different values of $F_{UV}$. The mass-loss rates are $4.8 \times 10^{10}$, $1.1 \times 10^{11}$, and $1.98 \times 10^{11} \, g \, s^{-1}$ for $F_{UV} = 2 \times 10^4$, $5 \times 10^4$, and $10^4 \, erg \, cm^{-2} \, s^{-1}$, respectively. The mass-loss rate is sensitive to the UV flux received from the host star. Lecavelier des Etangs et al. (2010) found that an escape rate between $10^9$ and $10^{11} \, g \, s^{-1}$ can fit the Ly$\alpha$ absorption rate of the observations. However, I find that a value of $10^9 \, g \, s^{-1}$ can be obtained only when the UV flux decreases to $10^9 \, erg \, cm^{-2} \, s^{-1}$, which is one order of magnitude smaller than the lower limit of the observed value ($\sigma$ level). If the value of $F_{UV}$ is $10^9 \, erg \, cm^{-2} \, s^{-1}$, the mass-loss rate of HD 189733b is $2.4 \times 10^{10} \, g \, s^{-1}$. 
Figure 3. Wind model of HD 189733b. From top to bottom, values of the UV flux are $10^5$ (dotted lines), $5 \times 10^4$ (dashed lines), and $5 \times 10^4$ erg cm$^{-2}$ s$^{-1}$ (solid lines). In each panel, number densities (upper left), velocities (upper right), temperatures (lower left), and the ionization fraction (lower right) are plotted as functions of altitude.

In contrast to the results of HD 209458b, as much as 80% of hydrogen at $R = 1.2 R_p$ is ionized, and the wind is almost fully ionized outside $R = 1.6 R_p$. The mass-loss rate of neutral hydrogen is only of the order of magnitude of $10^8$ g s$^{-1}$ due to the photoionization of strong UV irradiation from the host star. (The number density of H is of the order of magnitude of $10^6$ cm$^{-3}$ at $R = 1.6 R_p$ for the model of $F_{UV} = 5 \times 10^4$ erg cm$^{-2}$ s$^{-1}$. The corresponding temperature at the radius is 11,000 K.) Since the wind is mainly composed of protons, I wonder whether neutral hydrogen can produce adequate absorption to be detected. (Due to the steep decline in the number density of neutral hydrogen, the optical depth of the wing of the line can be very low.) If the amount of atomic hydrogen is not adequate to fit the observations, the fact that the transits of HD 189733b in H$_1$ Ly$_\alpha$ have been observed may imply that other mechanisms can also play an important role. A direct comparison with the model of Lecavelier des Etangs et al. (2010) can answer the question. Unfortunately, physical details of their model were not published.

4. DISCUSSION

4.1. Decoupling of Species

I have modeled particle escape with a multi-fluid model. For the planet HD 209458b, I did not find significant differences between the results of the multi-fluid model and the single-fluid model. In most cases, the very close-in planets are bathed in strong XUV radiation from their host stars. The species of gas are tightly coupled by collisions, so the description of the single-fluid model is accurate. However, in certain special circumstances, the description of the single-fluid model should be revisited or substituted, e.g., when the planetary wind is tenuous or the irradiation from the star is weak. For a high number density, the frictional force is able to transfer sufficient momentum from one species to another. However, a decrease in the number density may lead to a lower frictional force, i.e., decoupling is possible.

The case is modeled via the method mentioned in this paper in order to discuss the possibility of the decoupling of H and H+. Figure 4 shows the ratios of two velocity components for HD 209458b and HD 189733b in the standard models. Neutral and ionized hydrogen have the same velocity over the entire region. For testing, HD 209458b and HD 189733b are moved to two times their original separations, namely, $a = 0.1$ and 0.06 AU, and other parameters are retained. As seen in Figure 4, both particles have the same velocity profile. I also note that the number densities of particles are only lower than those of the standard model by a factor of two.

It could be concluded that, even at large separations, the description of the single-fluid model is still realistic. However,
note that the star HD 189733 is younger than HD 209458 (the ages of HD 189733 and HD 209458 are about 1.15 Gyr (Sanz-Forcada et al. 2010) and 4 Gyr (Guo 2010), respectively), which means that more XUV radiation can be emitted by HD 189733. If the age of HD 189733 were the same as that of HD 209458, the conclusion may be different. To test such a theoretical hypothesis, I calculated many cases with different values of $F_{\text{UV}}$ for HD 189733b. As seen in Figure 5, there exists a critical mass-loss rate below which decoupling can occur. (I assumed that decoupling would occur when $u_h/u_p$ is smaller than 95% in the calculations.) The results showed that a significant diffusion between neutral hydrogen and protons occurs when the mass-loss rate is below $1.3 \times 10^9$ g s$^{-1}$. Therefore, the critical value is about $10^7$ g s$^{-1}$.

This conclusion is also verified by testing other cases. To decrease the mass-loss rate of HD 209458b, I artificially increased the mass of the planet to 0.85 $M_\oplus$ and decreased the density of the lower boundary by a factor of 10. The results showed that ionized hydrogen has a higher velocity profile and a clear decoupling occurs throughout the wind. Finally, I obtained a mass-loss rate of $M = 5 \times 10^9$ g s$^{-1}$.

To obtain a common flow, the momentum gained by one species should be shared by other species. This requires that the characteristic timescale for slowing down the fast particle via interaction with the slow particle be smaller than the timescale of the flow. The timescale of the flow can be estimated as $R_1/u$ where $R_1 \sim 0.1 R_p$ and $u$ is $10^5$–$10^6$ cm s$^{-1}$. The decelerating timescale, $t_s$, is approximated as $1/(n_h \gamma_{\text{rec}})$. The condition of decoupling can be estimated by equating two timescales. Therefore, the critical number density of neutral hydrogen is about $10^7$–$10^8$ cm$^{-3}$. Because $n_p/n_h \sim 10$, the critical mass-loss rate is of the order of magnitude of $10^9$ g s$^{-1}$.

As mentioned in Section 3.1, the ratios of two velocity components behave as parabolic curves. To maintain the common flow, a sufficient amount of momentum must be transferred between two components. The most important process of transferring momentum is the charge exchange. Near the bottom of the wind, the wind is relatively dense, so the process of momentum exchange is effective. However, when the radius increases, the rate of charge exchange decreases (the solid line at the bottom of Figure 4). To test the assumption of a single fluid, HD 209458b and HD 189733b are moved to two times their original separation. No significant diffusion was found in either case.

![Figure 4](image1.png)

**Figure 4.** Plot of $u_h/u_p$ for HD 209458b and HD 189733b (dashed line: two times original separation; solid line: original separation). To test the assumption of a single fluid, HD 209458b and HD 189733b are moved to two times their original separation. No significant diffusion was found in either case.

![Figure 5](image2.png)

**Figure 5.** Velocity ratios of atomic hydrogen to protons (top panel) and the photoionization, recombinination, and charge-exchange rates (bottom panel). In the top panel, the mass-loss rates are $8.8 \times 10^8$, $1.3 \times 10^9$, $2.4 \times 10^9$, and $4.2 \times 10^9$, and $5.9 \times 10^9$ g s$^{-1}$. The corresponding UV fluxes are 450, 630, 1080, 1800, and 2500 erg cm$^{-2}$ s$^{-1}$, respectively (from top to bottom). The bottom panel only shows the photoionization, recombinnation, and charge-exchange rates in the model with $F_{\text{UV}} = 450$ erg cm$^{-2}$ s$^{-1}$. For comparison, I defined these rates as $\gamma_{\text{pho}} = \gamma_{\text{pho}}/n_p$, $\gamma_{\text{rec}} = \gamma_{\text{rec}}/n_p$, and $\gamma_{\text{rec}} = \gamma_{\text{rec}}$ cm$^{-3}$ s$^{-1}$.

![Figure 5](image3.png)

**Figure 5.** Thus, the transfer of momentum also decreases with radius. Note that the processes of photoionization and recombination can also redistribute momentum from one component to another, and the variations in photoionization and recombination rates (dashed and dotted lines) are contrary to that of the charge-exchange rate. Therefore, outside the wind, the momentum transferred by photoionization and recombination also plays a role. Finally, there is a minimum value of $u_h/u_p$ in the middle of the wind.

### 4.2. Dependence of $M$ on UV Fluxes

Lammer et al. (2003) presented that the energy deposition of X-ray and UV radiation from the parent star can lead to a high temperature and that a hydrodynamic process can occur in planetary atmospheres. The mass-loss rates of energy deposition depend closely on the fluxes of XUV radiation. In general, young stars can radiate more energy than old ones in the XUV band. The energy-limit mass-loss rate can be written as

$$M = \frac{3 \eta \beta^3 F_\beta}{G \rho K(\xi)},$$

(23)

where $\beta$ is the ratio of the expansion radius $R_1$ to the planetary radius $R_p$, $R_1$ is the altitude where XUV radiation is absorbed, $R_p$ is the distance from the center of the planet to the 1-bar pressure level in the atmosphere, $\eta$ is heating efficiency, and $\rho$ is the mean density of the planet. Here, $K(\xi) = 1 - \frac{1}{2\pi} + \frac{1}{2\pi} \xi$, $\xi = d(M_\star)^{1/3}$ is the Roche lobe boundary distance, where $M_\star$ is the mass of the star and $d$ is the orbital distance.

Based on the hydrodynamic model of Watson et al. (1981), Lammer et al. (2003) estimated $\beta = 3$, but $\beta$ could be equal to unity according to recent hydrodynamic models that showed that the expansion radius could be 1–1.5 $R_p$ (Yelle 2004; Murray-Clay et al. 2009). Lammer et al. (2009) also found that the mass-loss rate was overestimated by Baraffe et al. (2004), who...
reported $\beta = 3$. With the full energy-limited condition, the heating efficiency $\eta = 100\%$. In fact, the heating efficiency is about 25\%. In this paper, I set $\beta = 1.1$ and heating efficiency $\eta = 0.1–0.25$ (Murray-Clay et al. 2009 and references therein). Thus, Equation (23) describes a modified energy-limit approach.

For comparison with the energy-limited mass-loss rate, I calculated the mass-loss rate of HD 189733b as a function of the UV flux, and the results are shown in Figure 6 (left panel). The mass-loss rates given in my models are a factor of 3–10 lower than those calculated from Equation (23), assuming $k(\bar{\xi}) = 1$ and $\eta = 0.1–0.25$. For completeness, I also examined the single-fluid model (Murray-Clay et al. 2009) and found a systemic difference compared to the mass-loss rates predicted by my model. The single-fluid model can predict a comparable value for $M$ when the heating efficiency in the energy-limit method is decreased to $\eta = 0.1$ (the left panel of Figure 6). However, this is not consistent with my results. The mass-loss rates calculated by the single-fluid model are still higher than those of my model. With an increase in $F_{\text{UV}}$, the ratio of $M_{\text{single}}/M_{\text{this paper}}$ decreases from $5$ ($F_{\text{UV}} = 450$) to $2.5$ ($F_{\text{UV}} = 10^5$). As discussed in Section 3.2, my results fit the observations well. Thus, a lower heating efficiency is required for highly ionized wind.

My calculation results showed that the winds are highly ionized and mainly composed of protons, even if the UV fluxes are assumed to be at a low level. A high degree of ionization can occur as a consequence of a low mass-loss rate. Even if the UV flux is at the same level, the mass-loss rate of HD 189733b should be lower than that of HD 209458b due to the larger potential well of HD 189733b. Thus, for HD 189733b, low mass loss leads to low optical depth and high ionization. Given the energy equation, it is clear that photoionization heating is proportional to the number density of neutral hydrogen. Assuming an energy limit, most of the UV radiation energy is deposited as heat (due to low ionization), which is used to lift material out of the gravitational potential well. Thus, the energy-limit condition results in higher mass-loss rates. In the case of ionized wind, the material is mainly composed of protons. Only a little UV radiation can be transformed into heat, and further goes into $PdV$ work. Murray-Clay et al. (2009) found that at high $F_{\text{UV}}$ the flow is radiation-recombination-limited (at low flux, $\dot{M} \propto F_{\text{UV}}^{0.9}$; at high flux, $\dot{M} \propto F_{\text{UV}}^{-6}$), and an almost isothermal wind is predicted. In contrast to the case of a radiation-recombination limit, my results showed a “normal” temperature profile, which suggests that heating is balanced by $PdV$ work rather than radiation cooling. Thus, I can conclude that the modified energy-limit approach can be used in the case of a low or moderate degree of ionization, but is unsuccessful for high-ionization winds. This conclusion can be validated by examining HD 209458b: I found that the mass-loss rate calculated by Equation (23) can predict a reasonable observation value for HD 209458b.

These results motivated us to fit the mass-loss rate as a function of the UV flux in the case of ionized wind. In Figure 6, the straight line depicts the lower part, and the upper part is fitted by using a polynomial (a jump appears at $F_{\text{UV}} = 2000$). Finally, the mass-loss rate can be expressed as

$$M\bar{\rho} = \begin{cases} -0.24 + 0.017F_{\text{UV}}, & F_{\text{UV}} \leq 2 \times 10^4 \text{ erg cm}^{-2} \text{ s}^{-1} \\ 34.1 + 0.018F_{\text{UV}} - 3.92 \times 10^{-8}F_{\text{UV}}^2, & 10^4 \text{ erg cm}^{-2} \text{ s}^{-1} \geq F_{\text{UV}} > 2 \times 10^4 \text{ erg cm}^{-2} \text{ s}^{-1} \end{cases}$$

(24)

where $\bar{\rho}$ is the mean density of the planet.

Note that Equation (24) is appropriate only for ionized planetary wind. However, it is not easy to determine whether the wind is ionized. To maintain an ionized wind, the photoionization rate should be larger than the recombination rate, namely, $\gamma_{\text{pho}} > \gamma_{\text{rec}}$. Thus, I have

$$\frac{F_{\text{UV}}}{h\nu_0}n_h e^{-\tau_{\nu_0}} > n_p^2 \gamma_{\text{rec}}.$$

(25)

At $R = 1.5 R_p$, the optical depth $\tau \sim 0$ and $T \approx 10,000$ K. If the wind is highly ionized at $R = 1.5 R_p$, the value of $n_p/n_h$ is about 10–100. Inequality (25) can be changed to

$$\frac{F_{\text{UV}}}{h\nu_0} \frac{\sigma_{\text{pho}}}{\gamma_{\text{rec}}} \frac{n_p}{n_h} > n_p^2 \gamma_{\text{rec}}.$$

(26)

Assuming $F_{\text{UV}} = 10^3 \text{ erg cm}^{-2} \text{ s}^{-1}$, the left-hand side of inequality (26) is about of the order of magnitude of $10^9$. 

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**Figure 6.** Left: the behaviors of mass-loss rates as a function of UV fluxes. The results of the energy limit (●: heating efficiency $\eta = 10\%$; ■: heating efficiency $\eta = 25\%$) clearly show larger mass-loss rates than those predicted by hydrodynamic models (▲: the model of Murray-Clay et al. 2009; ■: the model of this paper). Right: the mass-loss rates as a function of UV fluxes. For $F_{\text{UV}} \leq 2000 \text{ erg cm}^{-2} \text{ s}^{-1}$, I used a linear fitting (solid line). I fitted the rest of the data with a polynomial (dashed line).
Using the hydrostatic density profile, I can estimate the particle number density at $R = 1.5 \, R_p$. For HD 209458b, $n_p \sim 7 \times 10^9$, so that $\frac{\alpha_{\gamma}}{\gamma} n_p$ is of the order of $10^{11}$. Because the left-hand-side term of inequality (26) is smaller than the right-hand-side term, the wind of HD 209458b is not ionized. For HD 189733b, $n_p \sim 10^8$. Thus, inequality (26) can be roughly fulfilled if $F_{\text{UV}} \geq 10^3$ erg cm$^{-2}$ s$^{-1}$.

4.3. The Effect of Magnetic Fields

Many planets in our solar system have magnetic fields. For example, the magnetic field on the surface of Jupiter is about 4.3 G. What role the magnetic field plays in the upper atmosphere depends on its field strength. For planets without intrinsic magnetic fields, magnetic fields can be induced in the interaction area between the stellar and planetary winds. However, the occurrence of strong magnetic fields on the surfaces of planets can lead to significant changes in their upper atmospheres. Trammell et al. (2011) presented a three-dimensional (3D) isothermal magnetohydrodynamic model based on the stellar wind model (Mestel 1968), although they did not consider some physical details and the interaction between stellar and planetary winds. To include full physical processes, such as magnetic fields and collision of winds, a powerful 3D MHD model is required, which is beyond the scope of this paper. However, the potential role of magnetic fields can be discussed in the current one-dimensional model. In the following discussion, I assume that planetary flows are spherically symmetric; that is, the magnetic field does not change the geometry of the planetary wind within the magnetosphere, but the boundary of the magnetosphere constrains the range of planetary flows.

The most important effect of a magnetic field on hydrodynamic escape is whether the magnetic field can change the locations of sonic points in different species. If the boundary of the magnetosphere is greater than that of the sonic point, the planet can still emit a transonic wind. Otherwise, the planetary wind will be suppressed to subsonic flow or even quenched. Grießmeier et al. (2004) estimated the stand-off distance of the magnetosphere, $R_s$, via pressure balance and found that the stand-off distance of HD 209458b was about 2.6–3.8 $R_p$, which varied with different rotation velocities. My results showed that the locations of sonic points of H and H$^+$ are comparable to the stand-off distance. As discussed in Section 3.1, the flow of atomic hydrogen can be transonic, but the escape of protons can be suppressed to subsonic flow. However, it is clear from Equations (2), (3), (7), and (8) that the drag force of H by H$^+$ is negative if the velocity of H is higher than that of H$^+$. Thus, the subsonic behavior of H$^+$ can result in a decreased velocity of H. In the process, the charge exchange shifts momentum from the species with high velocity to the species with low velocity. One possibility in this case is that both flows become subsonic within the magnetosphere and attain a new equilibrium status. Consequently, the mass-loss rates of H and H$^+$ can decrease by a factor of a few (Murray-Clay et al. 2009). In fact, the stand-off distance depends strongly on the strength of the magnetic field, and the field strength can vary with the mass and evolution of the planet (Reiners & Christensen 2010; Scharf 2010). The capture of a radio signal can explain the intrinsic magnetic field; however, no signal has been found so far in observations (Lazio et al. 2010).

Note that the pressure of gas is omitted in the calculation of Grießmeier et al. (2004), so the stand-off distance can increase if the gas contributes a significant pressure fraction. Johansson et al. (2009) indicated the influence of an expanding atmosphere on stellar interaction. For unmagnetized exoplanets, the ram pressure of the ionosphere can push the bowshock toward the host stars. The results imply that if the contribution of gas to the total pressure is included, the magnetopause of the exoplanet could be outside the sonic points.

The ionization of hydrogen is not controlled explicitly by its velocity structure. Equation (1) clearly shows that the ionization of hydrogen is dominated by photoionization. Magnetic fields indirectly affect ionization by changing the mass-loss rates of planets. If the mass-loss rates decrease by a factor of a few, one can predict that there is a higher degree of ionization in the atmosphere because the optical depth will also decrease.

For an exoplanet with an intrinsic dipole magnetic field, the ions near the equator can be inhibited by the planetary magnetic field, while the neutral particles can freely escape from the atmosphere. In the region of the planetary surface occupied by closed magnetic fields, hydrostatic equilibrium is attained. Ions cannot escape from a closed magnetic field, although neutral particles can do so. Trammell et al. (2011) predicted a mass-loss rate of $10^9$ g s$^{-1}$ when the magnetic field was factored into their model. For details, readers may refer to the corresponding two-dimensional or 3D models (Trammell et al. 2011; Adams 2011). My model should be suitable for the polar regions, where the magnetic lines are open and almost aligned with the radial direction. Thus, the neutrals can escape from the entire surface of the planet. However, the escaping surface for ions could decrease at least by a factor of two (Yelle 2004). For low-ionized wind, the intrinsic dipole magnetic fields only confine a small part of the wind. For high-ionized wind composed mainly of ions, the intrinsic magnetic field can severely decrease the mass-loss rate.

5. CONCLUSIONS

I developed a multi-fluid model to describe particle escape from the upper atmosphere of close-in exoplanets. The continuity and momentum equations for each component were solved together with an energy equation. Detailed micro-physical processes, such as photoionization, recombination, and charge exchange, were included in my models. Based on the Henyey method, the code can treat systems of ordinary differential equations with one or more critical points.

I examined two close-in planets, HD 209458b and HD 189733b. My calculations showed that the mass-loss rates of these planets are of the order of magnitude of $10^{10}$–$10^{11}$ g s$^{-1}$, which agree completely with the observed mass-loss rates. Through detailed testing and verification, I found that the most important physical process in the atmosphere of hot Jupiters is the charge exchange, which couples atomic hydrogen tightly with protons. Most of the hydrogen escaping from hot Jupiters is composed of protons, especially in young star–planet systems. Thus, the transit of Lyα in HD 189733b could be induced by other unknown physical processes.

In the assumption of a single fluid, all species must frequently collide. Otherwise, the description of the single-fluid model cannot be used. My method can be applied to the other case, for example, the tenuous wind. It is found that decoupling may occur if the mass-loss rates are lower than $10^9$ g s$^{-1}$.

I also found that the assumption of an energy limit is not appropriate for ionized winds. My model predicted a mass-loss rate lower than that of the energy limit. By calculating the variations of mass-loss rates with UV fluxes, I fitted the mass-loss rates of ionized wind as a function of $F_{\text{UV}}$. 

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