Quantum secure direct communication with pure entangled states

Jian Wang, Quan Zhang, and Chao-jing Tang

School of Electronic Science and Engineering, National University of Defense Technology, Changsha, 410073, China

We present a quantum secure direct communication protocol where the channels are not maximally entangled states. The communication parties utilize decoy photons to check eavesdropping. After ensuring the security of the quantum channel, the sender encodes the secret message and transmits it to the receiver by using Controlled-NOT operation and von Neumann measurement. The protocol is simple and realizable with present technology. We also show the protocol is secure for noisy quantum channel.

PACS numbers: 03.67.Dd, 03.65.Ud, 42.79.Sz

In recent years, a novel concept, quantum secure direct communication (QSDC) has been proposed [1]. Different from quantum key distribution whose object is to establish a common key between the communication parties, QSDC’s object is to transmit the secret message directly without first establishing a key to encrypt it. QSDC can be used in some special environments, which has been shown in Ref. [2, 3]. The works on QSDC attracted a great deal of attention. We can divide these works into two kinds, one utilizes single photon [4-8], the other is based on entangled state [9-15]. Deng et al. [4] proposed a QSDC protocol using batches of single photons which serves as quantum one-time pad cryptosystem. Cai et al. [2] presented a deterministic secure direct communication protocol using single qubit in a mixed state. We proposed a QSDC protocol based on the order rearrangement of single photons [6]. The QSDC protocol using entanglement state is certainly the mainstream. Boström and Felbinger [2] proposed a Ping-Pong protocol which is quasi-secure for secure direct communication if perfect quantum channel is used. Cai et al. [16, 17] pointed out that the Ping-Pong protocol is vulnerable to denial of service attack or joint horse attack with invisible photon. They also presented an improved protocol which doubled the capacity of the Ping-Pong protocol [17]. Zhang et al. [15] indicated that the ping-pong protocol can be eavesdropped even in an ideal quantum channel. Deng et al. [6] put forward a two-step QSDC protocol using Einstein-Podolsky-Rosen (EPR) pairs. We presented a multiparty controlled QSDC protocol by using Greenberger-Horne-Zeilinger (GHZ) state and a QSDC protocol without using perfect quantum channel [8, 9]. Wang et al. [10, 11] proposed a QSDC protocol with quantum superndense coding and a multi-step QSDC protocol with a sequence of Greenberger-Horne-Zeilinger states. Yan and Zhang [12] presented a QSDC protocol using EPR pairs and teleportation. Man et al. [13] proposed a QSDC protocol by using swapping quantum entanglement and local unitary operations. Gao et al. [14, 15] presented a QSDC protocol by using GHZ states and entanglement swapping and a controlled QSDC protocol using teleportation.

Many QSDC protocols require maximally entangled states. However, we do not have maximally entangled states because of decoherence and noise. Moreover, it can only obtain almost maximally entangled states from partially entangled states by using quantum distillation. In this paper, we present a QSDC protocol using pure entangled states. In the protocol, the communication parties utilize decoy photons to ensure the security of the quantum channel, which is similar to the method used in Ref. [18]. If there is no eavesdropping in the transmission line, the sender encodes the secret message and transmits it to the receiver by using Controlled-NOT (CNOT) operation and von Neumann measurement. According to the sender’s measurement result, the receiver can obtain the secret message. In the protocol, the transmitting photon sequence does not carry the secret message. We also show the present protocol is secure even with a noisy channel.

Suppose the sender, Alice, wants to transmit her secret message to the receiver, Bob directly. The details of our QSDC protocol is as follows:

(S1) Alice prepares an ordered $N$ two-photon states, each of which is in the state

$$|\phi\rangle_{AB} = a|00\rangle_{AB} + b|11\rangle_{AB},$$

where $|a|^2 + |b|^2 = 1$. We denotes the ordered $N$ states with $\{P_1(A), P_1(B)\}$, $\{P_2(A), P_2(B)\}$, $\cdots$, $\{P_N(A), P_N(B)\}$, where the subscript indicates the pair order in the sequence, and $A$ and $B$ represent the two photons of each state. Alice takes one photon from each state to form an ordered partner photon sequence $P_1(A)$, $P_2(A)$, $\cdots$, $P_N(A)$, called A sequence. The remaining partner photons compose B sequence, $P_1(B)$, $P_2(B)$, $\cdots$, $P_N(B)$.

(S2) Alice prepares some decoy photons for eavesdropping check. Each of the decoy photons is randomly in one of the four states $\{|0\rangle, |1\rangle, |+\rangle=\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |\rangle=-\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$. Alice inserts the prepared decoy...
photon in $B$ sequence and sends the new $B$ sequence to Bob.

(S3) After confirming that Bob has received $B$ sequence, Alice publishes the position and basis of the decoy photons in $B$ sequence. Bob performs von Neumann measurement on the decoy photons according to the corresponding measuring basis and announces publicly his measurement result. According to Bob’s result, Alice can then analyze the error rate during the transmission of $B$ sequence. If the error rate is below the threshold they preset, Alice can conclude that there is no eavesdropper in the line. Alice and Bob continue to the next step. Otherwise, they abort the communication.

(S4) Alice prepares a photon $a$ according to the bit value of her secret message. If Alice’s secret message bit is “0” (“1”), she prepares a photon $a$ in the state $|0\rangle$ ($|1\rangle$). Thus Alice prepares $N$ photons for the ordered $N$ states, which we call a sequence, $|P_1(a), P_2(a), \cdots, P_N(a)\rangle$. If the state of photon $P_i(a)$ ($i = 1, 2, \cdots, N$) is $|0\rangle$, then the state of the system composed by photons $P_1(a), P_i(A)$ and $P_1(B)$ is

$$|\Phi_0\rangle_{aAB} = |0\rangle_a \otimes (a|00\rangle + b|11\rangle)_{AB},$$

where the subscript $a$ denotes $P_i(a)$. If the state of $P_i(a)$ is $|1\rangle$, then the state of $[P_1(a), P_i(A), P_1(B)]$ is

$$|\Phi_1\rangle_{aAB} = |1\rangle_a \otimes (a|00\rangle + b|11\rangle)_{AB}.$$ (3)

(S5) Alice sends photons $P_1(a)$ and $P_1(B)$ ($i = 1, 2, \cdots, N$) through a CNOT gate (photons $P_1(A)$ is the controller and photon $P_i(B)$ is the target). Thus $|\Phi_0\rangle_{aAB}$ is changed to

$$|\Phi_0'\rangle_{aAB} = (a|000\rangle + b|111\rangle)_{aAB},$$

and $|\Phi_1\rangle_{aAB}$ becomes

$$|\Phi_1'\rangle_{aAB} = (a|100\rangle + b|011\rangle)_{aAB}.$$ (5)

(S6) After having done CNOT operation, Alice and Bob measures photon $P_i(A)$ and $P_i(B)$ in $Z$-basis, $\{|0\rangle, |1\rangle\}$, respectively. According to equations 1 and 4, although Bob obtains his measurement result, he cannot recover the secret message without Alice’s result.

(S7) Alice publishes her measurement results of the photons in a sequence. Referring to Alice’s result, Bob can recover Alice’s secret message, as illustrated in Table 1. For example, when Bob’s result is $|0\rangle$, he can conclude that the Alice’s secret message is “0” (“1”), if Alice’s result is $|0\rangle$ ($|1\rangle$).

So far we have proposed the QSDC protocol with pure entangled states. Now, let us discuss the security for the present protocol. The crucial point is that the inserted decoy photons does not allow an eavesdropper, Eve, to have a successful attack and Eve’s attack will be detected by the communication parties during the eavesdropping check. Because each of the decoy photons is randomly in one of the four states $\{|0\rangle, |1\rangle, |+\rangle, |−\rangle\}$, the security for the protocol is the same as that for BB84 protocol [20]. If the security of the quantum channel is ensured, the protocol is completely secure.

According to Stinespring dilation theorem [2], Eve’s attack can be realized by a unitary operation $E$ on a large Hilbert space, $H_{AB} \otimes H_E$. The state of decoy photon and Eve’s probe state is

$$E|0, \epsilon\rangle = \alpha|0, \epsilon_{00}\rangle + \beta|1, \epsilon_{01}\rangle,$$

$$E|1, \epsilon\rangle = \beta'|0, \epsilon_{10}\rangle + \alpha'|1, \epsilon_{11}\rangle,$$ (6)

$$E|+\rangle = \frac{1}{\sqrt{2}}(|0, \epsilon_{00}\rangle + |1, \epsilon_{01}\rangle$$

$$+ \beta'|0, \epsilon_{10}\rangle + \alpha'|1, \epsilon_{11}\rangle),$$

$$E|−\rangle = \frac{1}{\sqrt{2}}(|0, \epsilon_{00}\rangle + |1, \epsilon_{01}\rangle$$

$$− \beta'|0, \epsilon_{10}\rangle − \alpha'|1, \epsilon_{11}\rangle),$$ (7)

where $|\epsilon\rangle$ denotes Eve’s probe state. The probe operator can be written as

$$\hat{E} = \begin{pmatrix} \alpha & \beta' \\ \beta & \alpha' \end{pmatrix}.$$ (10)

As $\hat{E}$ is an unitary operation, the complex numbers $\alpha$, $\beta$, $\alpha'$ and $\beta'$ must satisfy $\hat{E}E^\dagger = I$ and we can obtain the relations

$$|\alpha'|^2 = |\alpha|^2, |\beta'|^2 = |\beta|^2.$$ (11)

The error rate introduced by Eve is $\epsilon = |\beta|^2 = 1 − |\alpha|^2$.

The above analysis is based on ideal circumstances and does not take into account noise in the transmission line. In noisy quantum channel, Eve intercepts some transmitting photons in $B$ sequence at step (S2) and sends the others to the receiver using a better quantum channel in which the photon loss will not increase. During the eavesdropping check, Eve’s attack will not be detected in this situation. However, according to our protocol, Eve cannot obtain Alice’s secret message without Alice’s measurement result even if she captures some photons in $B$ sequence. If the eavesdropping check is passed, Bob tells

| Alice’s result | Bob’s result | Secret message |
|----------------|-------------|---------------|
| $|0\rangle$    | $|0\rangle$ | 0             |
| $|0\rangle$    | $|1\rangle$ | 1             |
| $|1\rangle$    | $|0\rangle$ | 1             |
| $|1\rangle$    | $|1\rangle$ | 0             |
Alice which photon he has received and which photon is lost in the transmitting line at step (S6) of the protocol. Alice then only publishes her measurement results of the corresponding photons which Bob has received. For example, if Bob has received photons $P_3(B)$, $P_5(B)$, · · · in $B$ sequence, then Alice publishes her measurement results of photons $P_3(a)$, $P_5(a)$, · · · . As described above, our protocol is also secure for noisy quantum channel.

So far we have proposed a QSDC protocol with pure entangled states and analyzed the security for the present protocol. To check eavesdropping in the transmission line, Alice inserts some decoy photons in the transmitting photon sequence. After ensuring the security of the quantum channel, Alice encodes her secret message on the pure entangled states by using CNOT operation. The communication parties measures each of their photons in $Z$-basis. Alice publishes her measurement result and Bob can then recover Alice’s secret message. The present protocol is efficient in that all pure entangled states are used to transmit the secret message. We also point out that our protocol is secure with noisy quantum channel. As for the experimental feasibility, our protocol can be realized with today’s technologies.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant No. 60472032.

[1] A. Beige, B.-G. Englert, Ch. Kurtsiefer, and H. Weinfurter, Acta Phys. Pol. A 101, 357 (2002).
[2] K. Boström and T. Felbinger, Phys. Rev. Lett. 89, 187902 (2002).
[3] F. G. Deng, G. L. Long, and X. S. Liu, Phys. Rev. A 68, 042317 (2003).
[4] F. G. Deng and G. L. Long, Phys. Rev. A 69, 052319 (2004).
[5] Q. Y. Cai and B. W. Li, Chin. Phys. Lett. 21, 601 (2004).
[6] J. Wang, Q. Zhang and C. J. Tang, Phys. Lett. A, in press.
[7] Q. Y. Cai and B. W. Li, Phys. Rev. A 69, 054301 (2004).
[8] J. Wang, Q. Zhang and C. J. Tang, Opt. Commun., in press.
[9] J. Wang, Q. Zhang and C. J. Tang, quant-ph/0511092
[10] C. Wang, F. G. Deng, Y. S. Li, X. S. Liu and G. L. Long, Phys. Rev. A 71, 044305 (2005).
[11] C. Wang, F. G. Deng and G. L. Long, Opt. Commun. 253, 15 (2005).
[12] F. L. Yan and X. Q. Zhang, Euro. Phys. J. B 41, 75 (2004).
[13] Z. X. Man, Z. J. Zhang and Y. Li, Chin. Phys. Lett. 22, 18 (2005).
[14] T. Gao T, F. L. Yan and Z. X. Wang, J. Phys. A 38, 5761 (2005).
[15] T. Gao T, F. L. Yan and Z. X. Wang, Chin. Phys. 14, 893 (2005).
[16] Q. Y. Cai, Phys. Rev. Lett. 91, 109801 (2003).
[17] Q. Y. Cai, Phys. Lett. A 351, 23 (2006).
[18] Z. J. Zhang, Y. Li and Z. X. Man, Phys. Lett. A 341, 385 (2005).
[19] X. H. Li, F. G. Deng, C. Y. Li, Y. J. Liang, P. Zhou and H. Y. Zhou, quant-ph/0606007.
[20] C. H. Bennett and G. Brassard, in Proceedings of IEEE international Conference on Computers, Systems and signal Processing, Bangalore, India (IEEE, New York), pp. 175 - 179 (1984).