Deterministic generation of parametrically driven dissipative Kerr soliton

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We theoretically study the nature of parametrically driven dissipative Kerr soliton (PD-DKS) in a doubly resonant degenerate micro-optical parametric oscillator (DR-DμOPO) with the interplay of $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities. We show that there is a threshold group velocity mismatch above which single PD-DKS in DR-DμOPO can be generated deterministically. We also find that the exact PD-DKS generation dynamics can be divided into two distinctive regimes depending on the phase matching condition. In both regimes, the perturbative effective third-order nonlinearity resulting from the cascading quadratic process is responsible for the soliton annihilation and the deterministic single PD-DKS generation. We also develop the experimental design guidelines for accessing such deterministic single PD-DKS state. The principle of deterministic single PD-DKS alleviates the need for extensive dispersion engineering and thus it can be easily applied to different material platforms as a competitive ultrashort pulse and broadband frequency comb source architecture at the mid-infrared spectral range.

1. INTRODUCTION

Ultrafast optical parametric oscillator (OPO) has been demonstrated as a versatile and competitive optical frequency comb architecture [1-5] in otherwise difficult-to-access spectral ranges including the molecular fingerprinting mid-infrared region [6]. Recently, intriguing dissipative soliton dynamics in both synchronously pumped and continuous-wave (cw) pumped OPOs are observed and further utilized to enhance the performances of ultrafast OPOs [7-13]. In general, these ultrafast OPOs are restricted to the operation regime near the zero group velocity mismatch (GVM) point such that cascaded quadratic nonlinearity can be efficiently utilized for these quadratic frequency comb generation.

In addition, dissipative Kerr soliton (DKS) formation in fiber-feedback OPO has been observed by balancing the $\chi^{(2)}$ nonlinearity and the group velocity dispersion (GVD) with a meter-long single mode fiber as well as the $\chi^{(2)}$ parametric gain and the cavity loss with a millimeter-long periodically poled lithium niobate (PPLN) [14]. The unique example demonstrates how the combination of $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities in a singly resonant OPO can be utilized to facilitate signal DKS formation that enhances its stability and bandwidth. On the other hand, achieving such a clear separation between the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities in chip-scale micro-OPOs (μOPOs) is very challenging if not impossible. In addition, most chip-scale μOPOs are doubly resonant and thus the effect of resonant pump must also be considered.

In this letter, we theoretically study the nature of signal DKS generation in a doubly resonant degenerate micro-OPO (DR-DμOPO) with the interplay of $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities. In a recent experiment with an aluminum nitride (AlN) DR-DμOPO, a unique deterministic single DKS generation dynamics has been demonstrated [15], but the exact nature of the signal DKS remains obscure and the underlying physics has not been studied in detail. Here we show for the first time that there is a threshold GVM above which single parametrically driven DKS (PD-DKS) in DR-DμOPO can be generated deterministically. With the proper choice of GVM, material Kerr nonlinearity (MKN) will dominate the properties of PD-DKS while effective third-order nonlinearity from the competing cascaded quadratic process becomes soliton perturbation. The exact PD-DKS generation dynamics can be divided into two distinctive regimes depending on the phase matching condition. In both regimes, the perturbative effective third-order nonlinearity resulting from the cascaded quadratic process is responsible for the soliton annihilation and the deterministic single PD-DKS generation. Moreover, with large phase mismatch, deterministic single PD-DKS can be obtained with reduced GVM threshold but at the cost of higher pump power. Our theoretical analysis matches with the recent experimental observation [15] and provides the basis for detailed understanding of the phenomena. Finally, we have developed the experimental design guidelines for accessing such deterministic single PD-DKS state.

Of note, the deterministic single PD-DKS alleviates the need for extensive dispersion engineering and thus it represents an advantageous complement to the pure quadratic soliton mode-locking principle [8-13]. The concept of deterministic single PD-DKS can be easily applied to different material platforms, making it a competitive ultrashort pulse and broadband frequency comb source architecture at the mid-infrared spectral range.

2. THEORETICAL ANALYSIS AND NUMERICAL RESULTS
The field evolution of a cw-pumped DR-DOPO with both $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities obeys the coupled equations in the retarded time frame:

$$\frac{\partial A}{\partial z} = \left[\frac{\alpha_1}{2} - i \frac{k_1}{2} \frac{\partial^2}{\partial \tau^2}\right] A + i \kappa B A^* e^{i\Delta k \tau} + i \left[\gamma_1 |A|^2 + 2\gamma_2 |B|^2\right] A,$$

$$\frac{\partial B}{\partial z} = \left[\frac{-\alpha_2}{2} - \Delta k \frac{\partial}{\partial \tau} - i \frac{k_2}{2} \frac{\partial^2}{\partial \tau^2}\right] B + i \kappa A A^* e^{i\Delta k \tau} + i \left[\gamma_1 |B|^2 + 2\gamma_2 |A|^2\right] B,$$

and the boundary conditions:

$$A_{n+1}(0, \tau) = \sqrt{1 - \theta} A_n(L, \tau) e^{-i\delta},$$

$$B_{n+1}(0, \tau) = \sqrt{1 - \theta} B_n(L, \tau) e^{-i\delta} + \sqrt{\theta} B_n,$$

where $A$ is the signal field envelope, $B$ is the pump field envelope, $B_0$ is the cw pump, $\alpha_{1,2}$ are the propagation losses per unit length, $\Delta k$ is the GVM, and $k_1, k_2$ are the GVD coefficients. Higher-order dispersion and nonlinearity are neglected for simplicity. $\kappa = \sqrt{\xi_{\text{av}} \xi_{\text{ef}}} \left|A_0\right|^2 \sqrt{n_1 n_2}$ is the normalized second-order nonlinearity coupling coefficient, where $d_{ij}$ is the effective second-order nonlinear coefficient, $A_0$ is the effective mode area, $c$ is the speed of light, $\xi_0$ is the vacuum permittivity, and $n_{1,2}$ are the refractive indices. $\Delta k$ is the wave-vector mismatch, $\gamma_1, \gamma_2$ are self-phase modulation (SPM) coefficients and $\Delta \omega$ are signal-resonance phase detuning, respectively.

With near perfect phase matching condition, Eq. (1)-(2) can be simplified into a single mean-field equation for the signal field, by only considering signal SPM effect (see Section S3, Supplement 1) under the mean-field and good cavity approximations:

$$\frac{\partial A}{\partial z} = \left[-\alpha_1 - i \delta_1 - i \frac{k_1}{2} \frac{\partial^2}{\partial \tau^2}\right] A + i \gamma_1 |A|^2 A + i \mu A^* + i \kappa L \left|A^*\right|^2 A + i \mu A^* + i \kappa L \left|A^*\right|^2 A + i \mu A^* - i \left[k L \text{sinc} \left(\frac{\theta}{2}\right)\right] A + i \gamma_1 |A|^2 A + i \mu A^*$$

where $\tau$ is the “slow” time that describes the envelope evolution over successive round-trips, $t_0$ is the signal roundtrip time, $\tau_0$ is the “fast” time that depicts the temporal profiles in the retarded time frame, $\alpha_{1,2}$ are the total linear cavity losses, $\mu = \kappa L \text{sinc} \left(\frac{\theta}{2}\right)$ is the phase-sensitive parametric pump driving term. Here $\varphi = \text{arctan}(\delta_1/\gamma_1)$ is the phase offset between the cw pump field $B_0$ and the signal field $A$, $\xi = \Delta k L / 2$ is the wave-vector mismatch parameter. Equation (3) is the parametrically driven nonlinear Schrödinger equation (PDNLSE) [9-10], with a perturbation term representing cascaded quadratic process induced dispersive effective third-order nonlinearity $J(\Omega) = J' \left(\sqrt{\Omega} \right)$ where $J' = \frac{d J}{d \Omega}$ and $\xi = \sqrt{\Omega^2 + \alpha_2^2}$ is the offset angular frequency with respect to the signal resonance frequency. As shown in Fig. 1 with dotted lines, the validity of PDNLSE is verified and it provides a computationally efficient way to study the deterministic PD-DKS generation in this section.

Figures 2(a) and 2(b) shows the histogram of 100 independent intra-cavity average power traces with and without the perturbation term in Eq. (3), respectively. The pump phase detuning $\delta_1$ is tuned linearly from blue to red side in 180 ns and held constant for another 70 ns to stabilize the PD-DKS generation. Each simulation starts with restarted noises to make sure there is no correlation between consecutive runs. Deterministic single PD-DKS formation is observed in Fig. 2(a), with each scan converging into the same intra-cavity power and single pulse shape (inset). We have to emphasize that this deterministic single PD-DKS generation is observed in both simulations, histogram of 100 intra-cavity average power traces during the frequency scanning process with only EKN and TPA effect, respectively. Therefore, multiple solitons will experience long range interaction due to self-focusing nonlinearity and the signal field, $\varsigma$. Deterministic single PD-DKS generation is observed in both simulations, indicating the contribution of both effects. On the other hand, EKN has a more profound effect on the PD-DKS peak power and pulse duration, due to its direct impact on the phase detuning, while TPA mainly increases the pump threshold because of the elevated loss. The influence of both effects on the pulse is also investigated with a test Gaussian pulse (see Section S5 in Supplement 1) and can be concluded as: (i) a sub-pulse appears right next to the main pulse; (ii) the intensity of the sub-pulse can be adjusted by GVM; smaller GVM results in larger sub-pulse intensity.

As shown in Figs. 3(a) and 3(b), with large GVM $X(\Omega)$ and $Y(\Omega)$ are both narrow-band with maximum values at the center frequency. Therefore, multiple solitons will experience long range interaction due to near-band perturbation. According to Eq. (3), this perturbation

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**A. Deterministic single PD-DKS with perfect phase matching**
from TPA or EKN effect can be viewed qualitatively as amplitude or phase modulation to the pump field \((\mu_\text{i},\mu_\text{s})\), which is a common method for deterministic DKS generation in \(\chi^3\) nonlinear cavities [17, 18]. Furthermore, the last term in Eq. (3) breaks down the phase symmetry \(A \rightarrow -A\), which means solitons with opposite phase can no longer exist. Simulations show that single soliton with opposite phase will disappear with TPA effect or automatically adjust its phase and evolve into a soliton with EKN effect.

Figures 3(c) and 3(d) show how multiple solitons from Fig. 2(b) evolve under the influence of TPA and EKN, respectively. Similar to the avoided mode crossings induced Cherenkov radiation [19], TPA and EKN effect lead to dispersive waves and destabilize the solitons through long range interaction. Multiple solitons interact with each other, experience extra loss from the dispersive waves and finally only single soliton survives. Breathing behaviors are observed during the soliton interaction process, which is common due to the energy exchange between multiple solitons [19]. In addition, soliton interaction is more sensitive to EKN rather than TPA, similar to the more effective pump phase modulation method for conventional DKS generation. In fact, during the pump frequency scanning process in Fig. 2(c) and 2(d), single soliton usually arises from the highest intensity peak in the background (see Section S4, Supplement 1) instead of multiple solitons, resulting in no evident soliton steps for soliton annihilation.

Bandwidth of \(\lambda(\Omega)\) and \(\gamma(\Omega)\) increases with the reduction of GVM, thus causing stronger soliton perturbation. With \(d_{\text{eff}} = 4 \text{ pm/V}, a \text{ GVM larger than 380 fs/mm is required to keep the soliton perturbation manageable so the PD-DKS can still sustain itself. With below-threshold GVM of 300 fs/mm, the PD-DKS breaks up into sub-pulses and eventually evolves into a cw solution [Fig. 3(e)]. Importantly, the GVM threshold increases as a function of \(d_{\text{eff}}\) [Fig. 3(f)] because larger \(d_{\text{eff}}\) means stronger soliton perturbation [Eq. (3)] and in turn requires larger GVM to alleviate the perturbative effect and stabilize the PD-DKS.

**B. Deterministic single PD-DKS with large phase mismatch**

In the case of large phase mismatch, the single mean field equation Eq. (3) fails to describe the system dynamics since the coherent length is smaller than the cavity length. The integration along the cavity length from Eq. (1) to Eq. (S6) (see Section S3, Supplement 1) and the averaging effect of laser fields is invalid due to the strong energy exchange between pump and signal within one roundtrip. The comparative results of Eq. (1) and Eq. (3) in Fig. S5 (see Section S6, Supplement 1) indicate that PDLNLE is a good approximation only around the perfect phase matching point. Therefore, we will apply Eq. (1) along with the boundary conditions Eq. (2) to investigate deterministic PD-DKS generation with large phase mismatch in this section.
Fig. 4. Signal soliton with large phase mismatch, $\xi_0 \approx 0.75\pi$, $|\beta_2| = 100\,\text{mW}$. (a) pulse profiles and (b) spectra. Pulse evolution within three successive roundtrips (indicated by white dashed lines) for signal (c) and pump field (d). The inset of (d) shows the temporal tuning along the axis corresponding to peak powers before and after meeting with pump. (e) Histogram of 100 overlaid intracavity average power traces with same frequency scanning strategy in Fig. 2. (f) GVM threshold versus second-order nonlinearity coefficient.

To experimentally access the deterministic single PD-DKS state, one has to consider how these three parameters co-determine the system’s behavior as well as the experimental restrictions: (i) GVM can be tuned via dispersion engineering [20] but it ultimately limited by material dispersion; (ii) $d_{eff}$ can be changed by choosing different nonlinear crystals or different crystal axes; (iii) $\xi$ is the most controllable parameter in experiment, through temperature tuning, angle tuning, quasi-phase-matching, and more. Admitting the above-mentioned parameter in experiment, through temperature tuning, angle tuning, crystals or different crystal axes; (iii)

According to the experimental guidelines, the deterministic single PD-DKS can be summarized as: (i) for small $d_{eff}$, it is better to operate near the perfect phase matching point to lower the pump threshold; however, the GVM threshold is higher in this regime; (ii) for large $d_{eff}$, it is better to operate with large phase mismatch to lower the GVM threshold; compromise between the GVM threshold and pump threshold should be considered in this regime.

According to the experimental guidelines, the deterministic single PD-DKS in the AlN DR-DμOPO [15] is believed to be the consequence of relatively large MKN, large GVM, and relatively small PD-DKS in the AlN DR-DμOPO [15]. The simulated results (see Section S7, Supplement 1) including the soliton spectrum and corresponding dynamics, agree excellently with the experimental ones in Ref. [15]. What’s more, we numerically obtain the GVM induced small pump spectral peaks, which is clearly shown in the experiment but is not well explained.

3. CONCLUSION

In conclusion, we theoretically study the nature of PD-DKS generation in a DR-DμOPO with the interplay of $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities. With the proper choice of GVM, MKN will dominate the properties of PD-DKS while effective of third-order nonlinearity from the competing cascaded quadratic process becomes soliton perturbation. The exact PD-DKS generation dynamics can be divided into two distinctive regimes depending on the phase matching condition. In both regimes, the perturbative effective third-order nonlinearity resulting from the cascaded quadratic process is responsible for the soliton annihilation and the deterministic single PD-DKS generation. Moreover, with large phase mismatch, deterministic single PD-DKS can be obtained with reduced GVM threshold but at the cost of higher pump power. To access the deterministic single PD-DKS state, it is thus better to operate near the perfect phase matching point with low $d_{eff}$ materials while it is beneficial to operate at large phase mismatch when high $d_{eff}$ materials are available.

Importantly, the deterministic single PD-DKS alleviates the need for extensive dispersion engineering and thus the working principle can be easily applied to different material platforms, making it a competitive ultrashort pulse and broadband frequency comb source architecture at the mid-infrared spectral range.

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See Supplement 1 for supporting content.

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