UNSTEADY MHD SLIP FLOW OF NON NEWTONIAN POWER-LAW NANOFLUID OVER A MOVING SURFACE WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

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Abstract. In this paper, unsteady magnetohydrodynamic (MHD) boundary layer slip flow and heat transfer of power-law nanofluid over a nonlinear porous stretching sheet is investigated numerically. The thermal conductivity of the nanofluid is assumed as a function of temperature and the partial slip conditions are employed at the boundary. The nonlinear coupled system of partial differential equations governing the flow and heat transfer of a power-law nanofluid is first transformed into a system of nonlinear coupled ordinary differential equations by applying a suitable similarity transformation. The resulting system is then solved numerically using shooting technique. Numerical results are presented in the form of graphs and tables and the effect of the power-law index, velocity and thermal slip parameters, nanofluid volume concentration parameter, applied magnetic field parameter, suction/injection parameter on the velocity and temperature profiles are examined from physical point of view. The boundary layer thickness decreases with increase in strength of applied magnetic field, nanoparticle volume concentration, velocity slip and the unsteadiness of the stretching surface. Whereas thermal boundary layer thickness increase with increasing values of magnetic parameter, nanoparticle volume concentration, velocity slip and velocity slip at the boundary.

1. Introduction. Class of fluids in which particles of nanometric size are disseminated in the base fluid is called a nanofluid. Choi [9] first used the term nanofluid and presented the practicability of the concept. Nanofluids got a conspicuous scientific and engineering applications after Eastman et al.[11], extended the idea and studied an unusual thermal conductivity rise in copper-water nanofluid at small nanoparticles volume concentration. Experiments performed by Wang et al.[44] and Keblinski et al. [20] show that the effective thermal conductivity of nanofluids increases under macroscopically stationary conditions. A detailed study on convective transport in nanofluids was done by Buongiorno [8] and discussed seven slip mechanisms that can generate a relative velocity between the nanoparticles and the

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base fluid. The detailed literature survey on transport and heat transfer characteristics of nanofluids is presented in the review articles of Keblinski et al. [21] and Wang et al. [43].

The MHD flow and heat transfer attributes of nanofluids over a stretching sheet have important industrial applications, for example, extrusion processes, optical modulators, magneto-optical wavelength filters, optical switches, magnetic drug targeting, reduction of turbulent drag, MHD flow meters, MHD generators etc. Khan and Pop [22] first presented the numerical results for the boundary layer flow of nanofluid past a stretching sheet. Makined and Aziz [25] extended the idea and include the convective conditions in the boundary layer flow model caused by the linear stretching of flat sheet. MHD boundary layer flow of nanofluids past vertical porous stretching surface was investigated by Kandasamy et al. [19]. The work is further extended by Bhattacharya and Layek [5] for MHD boundary flow of nanofluid over an exponentially stretching surface. The unsteady MHD flow and heat and mass transfer of nanofluid including a nonuniform heat source over a stretching surface is examined numerically by Shankar and Yirga [38]. The thermal radiation effects on the flow and heat transfer of nanofluids over an unsteady stretching surface were studied by Kalidas et al. [10] and Motsumi and Makinde [26].

Nanofluids motion and heat transfer characteristics in the boundary layer is influenced by a number of factors including, viscosity, thermal conductivity, wall slip, flow medium, orientation and intensity of the applied magnetic field etc. The fluid velocity profiles are significantly affected by the presence of slip at the fluid-solid interface. Especially in the case of microfluids and nanofluids, the control and manipulation of fluid require fundamental understanding of slip boundary conditions (for details see, Priezjev [33]). Zheng et al. [45] presented a mathematical model which incorporate velocity slip and temperature jump on MHD flow and heat transfer of nanofluids over a porous shrinking sheet. Zheng et al. [46] included the radiation heat transfer and porous medium effects in the model presented by the previous authors. Uddin et al. [41] analyzed numerically the g-Jitter mixed convective unsteady slip flow of nanofluids past a permeable porous sheet embedded in a Darcian porous media with variable viscosity. Noghrehabadi et al. [31] observed the effects of partial slip boundary conditions on the flow and heat transfer of nanofluids. Bhaskar et al. [4] carried out an analysis to investigate the influence of variable thermal conductivity and partial velocity slip on the hydro-magnetic two-dimensional boundary layer flow of nanofluids over a porous sheet with a convective boundary condition. Noghrehabadi et al. [32] carried out a study on the effects of variable thermal conductivity and viscosity on the natural convective heat transfer of nanofluids over a vertical plate. Comprehensive studies and lists of important references on the wall slip condition and variable thermophysical properties of nanofluids are presented in [2]-[13].

In real situation, nanofluids do not have the characteristics of Newtonian fluids hence it is more reasonable to consider them as Non-Newtonian fluids. There are many models proposed for the non-Newtonian fluids. The theory of boundary layer for each proposed model is also available in the literature. It is beyond the scope of this work to revisit the vast amount of literature on the boundary layer flow of different non-Newtonian fluid models. A careful review of the literature reveals that non Newtonian nanofluids have so far received very little attention. The numerical study of the magnetohydrodynamic (MHD) boundary layer flow of a Maxwell
nanofluid past a stretching sheet was executed by Nadeem et al. [30]. Ramzan and Bilal [35] examined the unsteady (MHD) second grade incompressible nanofluid flow towards a stretching sheet. Santra et al. [37] considered the forced convection flow and heat transfer of cu-water nanofluid in a channel with both Newtonian and Non-Newtonian models. For the non-Newtonian case, the power-law rheology is applied in which the fluid consistency coefficient and the flow behaviour index are interpolated and extrapolated from the experimental results presented in [34]. Ellahi et al. [12] presented the analytical series solutions of third grade nanofluids flow over a stretching surface with reynolds and vogels model. Rashidi et al. [36] illustrated the entropy analysis of convective MHD flow of third grade nanofluid over a stretching sheet. Lin et al. [23] investigated the unsteady, MHD flow of pseudo-plastic nanofluid in a finite thin film over stretching surface with internal heat generation. Influence of heat and mass flux conditions in hydromagnetic flow of Jeffrey nanofluid are examined by Abbasi [1]. Sui et al. [40] analyzed boundary layer heat and mass transfer with Cattaneo-Christov double-diffusion in upper-convected Maxwell nanofluid past a stretching sheet with slip velocity condition. Analysis of thixotropic nano-material in a doubly stratified medium including magnetic field effects is presented by Hayat et al. [14]. Bhatti and Rashidi [6] studied the effects of thermo-diffusion and thermal radiation on Williamson nanofluid over a porous shrinking/stretching sheet. Moreover, Taha et al. [3] found the exact solutions for Stokes flow of a third grade nanofluid model using the Lie similarity approach. In addition to the above, the non-Newtonian nanofluid models are well discussed in [17]-[29]. To the best of authors knowledge no research is conducted to study the unsteady MHD slip flow and heat transfer of power-law nanofluids over a porous nonuniform stretching surface with temperature dependent thermal conductivity.

2. Mathematical model for the problem. In this present paper, we consider an unsteady, two-dimensional laminar flow with heat transfer of an incompressible electrically conducting power-law nanofluid over a porous stretching sheet. The surface of the sheet admit the partial slip condition and thermal conductivity of the nanofluid is to vary with temperature, $T$. The $x$-axis is along the surface of the porous sheet and $y$-axis is perpendicular to it. A uniform magnetic field of strength $B(t) = \frac{B_0}{\sqrt{1-\alpha t}}$ is applied in the transverse direction to the flow and the induced magnetic field is considered negligible as compared to applied magnetic field. The stretching sheet is moving with the non-uniform velocity

$$U(x,t) = \frac{bx}{1-\alpha t},$$

where $b$ is the initial stretching rate and $\frac{1}{1-\alpha t}$ is the effective stretching rate with $(\alpha t < 1)$. $T_w$, is the plate temperature and $T_\infty$ is the temperature outside the boundary layer. The geometry of the flow model is given in Figure (1). The governing equations under boundary layer approximation for the flow of power-law nanofluid along with heat transfer are obtained as (see for example Khadeejha and Asim [2])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}} \frac{\partial \tau_{xy}}{\partial y} - \frac{\sigma_{nf} B^2(t) u}{\rho_{nf}}, \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho C_p)_{nf}} \frac{\partial}{\partial y} \left( \kappa_{nf}(T) \frac{\partial T}{\partial y} \right). \quad (4)$$
Here, $u$ and $v$ are velocity components along the $x$ and $y$ directions respectively, $t$ is the time, $\tau_{xy}$ is the shear stress component of nanofluid, $\rho_{nf}$ is the density, $\sigma_{nf}$ is the electrical conductivity, $(C_p)_{nf}$ is the specific heat capacity and $\kappa_{nf}^*$ is the thermal conductivity of nanofluid. $\tau_{xy}$ for the power-law model is taken as given by Bird et al. [7]

$$\tau_{xy} = \mu_{nf} \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y},$$  

(5)

where $\mu_{nf}$ is the consistency viscosity coefficient and $n$ is the power-law index. The case $n = 1$ corresponds to Newtonian fluids, $0 < n < 1$ represents the pseudo-plastic fluid behaviour while $n > 1$ describes a dilatant fluid. In the present study, the thermal conductivity of power-law nanofluid is assumed as (see for example, Zheng et al.[47])

$$\kappa_{nf}^*(T) = k_{nf} \left| \frac{\partial T}{\partial y} \right|^{n-1},$$  

(6)

where $k_{nf}$ is the constant value of the thermal conductivity. The appropriate slip boundary conditions are

$$u(x, 0) = U_w + A_1 \mu_{nf} \left| \frac{\partial u}{\partial y} \right|^{n-1} \left( \frac{\partial u}{\partial y} \right), \quad v(x, 0) = V_w, \quad T(x, 0) = T_w + D_1 \left( \frac{\partial T}{\partial y} \right),$$  

(7)

$$u \to 0, \quad T \to T_\infty \text{ as } y \to \infty.$$  

(8)

In above equations $A_1 = A_0 \left( \frac{(1-\alpha t)^{2-n}}{x^2} \right)^{\frac{1}{n-1}}$ and $D_1 = D_0 \left( \frac{(1-\alpha t)^{2}}{x^2} \right)^{\frac{1}{n-1}}$ are the velocity and thermal slip factors with $A_0$ and $D_0$ be the initial value of velocity and thermal slip. The surface temperature of the sheet is $T_w(x, t) = T_\infty + \frac{bx}{1-\alpha t}$ and $V_w$ is the the mass transfer at the surface with $V_w > 0$ is for injection and $V_w < 0$ is for suction.

Following Khadeeja and Asim [2] the nanofluid’s physical parameters are defined as

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,$$

$$\rho_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s.$$  

(9)
Here, \( \phi \) is nanoparticle volume fraction coefficient, \( \mu_f, \rho_f \) and \((C_p)_f\) are the dynamic viscosity, density and specific heat capacity of the base fluid, \( \rho_s \) and \((C_p)_s\) are the density and specific heat of the nanoparticles, \( \kappa_{nf} \) is thermal conductivity of the nanofluid, \( \kappa_f \) and \( \kappa_s \) define the thermal conductivity of base fluid and nanoparticles, respectively. The electrical conductivity of the nanofluid is defined by

\[
\sigma_{nf} = \left[ 1 + \frac{3(\frac{\kappa_s}{\sigma_f} - 1)\phi}{(\frac{\kappa_s}{\sigma_f} + 2) - (\frac{\kappa_s}{\sigma_f} - 1)\phi} \right] \sigma_f,
\]

where \( \sigma_s \) is the nanoparticle electrical conductivity and \( \sigma_f \) is the base fluid conductivity.

3. **Solution of the problem.** In order to solve the governing boundary value problem (2)-(11) defined in the previous section, we first employed the similarity transformation technique to transform the governing partial differential equations into a system of nonlinear coupled ordinary differential equation. The resulting ordinary differential equations are then solved numerically using shooting technique. The stream function \( \psi(x, y) \) is introduced which identically satisfies Eq. (2) with

\[
u = \frac{\partial \psi}{\partial y}, \quad \phi = -\frac{\partial \psi}{\partial x},
\]

The dimensionless stream functions \( \psi(\eta), \theta(\eta) \) and the similarity variable \( \eta \) are defined by

\[
\psi(x, y) = (1 - \alpha t) \frac{1-2n}{n+1} f(\eta)(b^{1-2n} \nu_f^{-1}) \frac{1}{n+1} (x \frac{n}{2} n), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},
\]

\[
\eta(x, y) = (1 - \alpha t) \frac{n+2}{n+1} y((b^{-2-n} \nu_f^{-1})) \frac{1}{n+1} (x \frac{n}{2} n).
\]

Using Eqs.(13)-(14) the system of partial differential equation (3)-(4) along with boundary conditions (7)-(8) are transformed into

\[
(|f''|^{n-1} f'')' - (1-\phi)^{2.5} [(1-\phi + \phi \frac{\rho_s}{\rho_f})\{A(f' + \frac{2-n}{n+1} \eta f'' + f'^2 - (\frac{2n}{n+1}) f f''\} + M(1 + \frac{3(\frac{\kappa_s}{\sigma_f} - 1)\phi}{(\frac{\kappa_s}{\sigma_f} + 2) - (\frac{\kappa_s}{\sigma_f} - 1)\phi}) f' = 0,
\]

\[
(|\theta'|^{n-1} \theta')' - (1-\phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f} \frac{\kappa_f}{\kappa_{nf}}) \frac{M}{\kappa_{nf}} \left[A \left( \frac{2-n}{n+1} \eta \theta' + \theta \right) + f' \theta - (\frac{2n}{n+1}) f \theta' \right] = 0,
\]

\[
f(0) = S, \quad f'(0) = 1 + \frac{\delta}{(1-\phi)^{2.5}} f''(0) |f''(0)|^{n-1}, \quad \theta(0) = 1 + \Delta \theta'(0),
\]

\[
f'(|\eta|) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty.
\]

Here, \( A = \frac{\alpha_f}{\alpha_f} \) is the unsteadiness parameter, \( M = \frac{\sigma_f B^2}{\eta_f} \) is the magnetic parameter, \( Pr = \frac{\nu_f}{\alpha_f} \) is the Prandtl number, \( \alpha_f = \frac{\kappa_f}{(\rho C_p)_f} \) is the thermal diffusivity, \( S = \)

\[
\frac{1}{(\frac{\kappa_s}{\sigma_f} + 2) - (\frac{\kappa_s}{\sigma_f} - 1)\phi} \]

1 \) and \( \eta \rightarrow \infty. \)
\[-V_c(b_l-2n(y)^{-1}) \frac{
u_0}{\nu_1} (n+1)\] is the suction/injection parameter, \(\delta = A_0 \mu_f \left( \frac{\nu_0}{\nu_1} \right) \) is the velocity slip parameter and \(\Delta = D_0 \left( \frac{\nu_0}{\nu_1} \right) \) is the thermal slip parameter.

We now use shooting method to solve the system of ordinary differential equation (15)-(16) as well as the associated boundary conditions (17)-(18). Therefore we assume

\[
y_1 = f, \quad y_2 = f', \quad y_3 = f'', \quad y_4 = \theta, \quad y_5 = \theta'.
\]

Using Eqs. (19), the system (15)-(18) transformed into

\[
y_1' = y_2, \quad y_2' = y_3, \quad y_3' = y_4, \quad y_4' = y_5,
\]

\[
y_3' = \frac{(1 - \phi)^2 n}{(n + 1)(n - 1)} \left[ (1 - \phi + \phi \frac{\rho_0}{\rho_f}) A(y_2 + \frac{2 - n}{n + 1} y_3) + y_2^2 \right] - \frac{3(\frac{\sigma}{\sigma_f}) - 1}{(\frac{\sigma}{\sigma_f} + 2) - (\frac{\sigma}{\sigma_f} - 1)\phi} y_2,
\]

\[
y_5' = \frac{(1 - \phi + \phi \frac{\rho_0}{\rho_f}) \left( \frac{\sigma}{\sigma_f} \right) - 1}{(\frac{\sigma}{\sigma_f} + 2) - (\frac{\sigma}{\sigma_f} - 1)\phi} [A(\frac{2 - n}{n + 1} y_5 + y_4) + y_2 y_4 - (\frac{2n}{n + 1}) y_1 y_5 (y_5)^2],
\]

\[
y_1(0) = S, \quad y_2(0) = 1 + \frac{\delta}{(1 - \phi)^2 s y_3(0)} y_3(0)^{n-1}, \quad y_3(0) = a, \quad y_4(0) = 1 + \Delta y_5(0), \quad y_5(0) = b.
\]

Where \(a\) and \(b\) are unknown which are to be determined such that the boundary conditions \(y_2(\infty)\) and \(y_4(\infty)\) are satisfied. To confirm the numerical procedure using the proposed code, the results are compared with the results available in the literature. The test case is the natural convection of magnetohydrodynamic boundary layer slip flow of Newtonian fluid over a porous flat stretching surface. The results obtained by the present code for several values of skin friction coefficient are compared with published results of Hayat et al. [16] and Khadeejha and Asim [2] and are presented in Table (1). There is an excellent agreement between the present results and those of other investigators, thus, we are very much confident that the present results are accurate.

| \(M\) | \(S\) | \(\delta\) | \(A\) | \(-f''(0)\) T.Hayat | \(-f''(0)\) Khadeejah | \(-f''(0)\) Present |
|---|---|---|---|---|---|---|
| 0.25 | 1.0 | 1.0 | 0.2 | 0.60157 | 0.60157 | 0.60160 |
| 1.0 | 0.2 | 1.0 | 0.2 | 0.57563 | 0.57563 | 0.57560 |
| 1.0 | 0.5 | 1.0 | 0.2 | 0.602285 | 0.602265 | 0.60228 |

Table 1. Values of \(-f''(0)\) for the variation of parameters and fixed \(Pr = 6.2, \Delta = 1.0\) and \(\phi = 0.0\).
4. **Numerical results and discussion.** A numerical investigation has been performed to study the effect of power-law index $n$, unsteadiness parameter $A$, magnetic parameter $M$, volume fraction parameter $\phi$, velocity slip parameter $\delta$ and suction/injection parameter $S$ on flow and heat transfer characteristics of Cu–water nanofluid. To demonstrate the meaningful relationship between the parameters, numerical results are presented in the form of graphs for variations in the velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles. Thermophysical properties of the base fluid and nanoparticles are given in Table (2).

| Physical properties | Base fluid | Nanoparticles |
|---------------------|------------|---------------|
| $C_p (J/kgK)$       | 4179       | 385           |
| $\rho (kg/m^3)$     | 997.1      | 8933          |
| $k (W/mK)$          | 0.613      | 400           |
| $\sigma (\Omega.m)^{-1}$ | 0.05   | $5.96 \times 10^7$ |

**Table 2.** Thermophysical properties of the base fluid and nanoparticles.

The effects of power-law index $n$ and unsteadiness parameter $A$ on velocity and temperature profiles of Cu-water nanofluid have been studied in Figures (2)-(3). Figure (2) demonstrates that for the fixed value of power-law index the velocity decreases with the increase in unsteadiness parameter $A$. This behavior indicates that the thickness of the momentum boundary layer is decreasing with the increasing values of $A$. The hierarchy of this trend is seen for the shear-thickening fluids $n > 1$ followed by the Newtonian fluids $n = 1$ and then the shear-thinning fluids $n < 1$. The temperature profiles presented in Figure (3), also for the fixed value of power-law index show a decreasing behavior with increase in unsteadiness parameter $A$ for a given distance from the plate. This shows the enhancement in the rate of heat transfer and the reduction in thickness of thermal boundary layer. The cross-over point is also witnessed in temperature profiles, i.e., the temperature falls with the rising values of $A$ before crossing over point, whereas it rises slightly after this. We may explain this phenomenon near the boundary as increase in unsteadiness parameter accelerates the nanoparticles causing more collisions near the surface. This makes the wall temperature higher than the ambient temperature and the movement of nanoparticles to cooler area causing an increase in the temperature profile. Furthermore Figure (3) also illustrates the thermal boundary layer thickness of power-law nanofluids. A nanofluid with power-law exponent $n = 1.4$ has relatively thin boundary layer as compared to nanofluid with power-law exponent $n = 0.8$.

Figures (4)-(5) depicted the velocity and temperature profiles of power-law nanofluid for variation in magnetic parameter $M$ and power-law index $n$ in the presence of slip at the boundary. A downward trend in the velocity of nanofluid is observed with increase in the strength of applied magnetic field $M$ and as result decreases the thickness of the momentum boundary layer. This behavior is due to Lorentz force, which resists the motion of the fluid and generated by the applied transverse magnetic field. Figure (5) quantifies the increase in temperature $\theta$, with increase in magnetic parameter $M$. The magnetic parameter is inversely proportional to the density of the base fluid; therefore the increase in $M$ reduces the density.
of base fluid and raises the temperature inside the boundary layer. The opposite trend in temperature profiles are witnessed for the increase in the power-law index, $n$. 

**Figure 2.** Velocity profiles for different values of parameter $A$. 

**Figure 3.** Temperature profiles for different values of parameter $A$. 

**Figure 4.** Velocity profiles for different values of parameter $M$. 

$\eta$
Variation in nanoparticle volume concentration $\phi$ and its influence on velocity and temperature profiles is illustrated in Figures (6)-(7). Denser nanoparticle volume concentration causes thinning of momentum boundary layer and the rate of heat transfer also reduces within boundary layer. This general observation is consistent with the fact that, the increase in volume of nanoparticles within the base fluid increases the overall thermal conductivity of nanofluids which leads to the decrease in the thickness of momentum boundary layer and increase in the thickness of the thermal boundary layer.

The comparison of curves in Figure (8) presented the reduction in the thickness of momentum boundary layer with increase in velocity slip at the boundary. This phenomenon indicate the force generated by the stretching of the surface is not completely transferred to the fluid because ascending values of velocity slip parameter allow more fluid to slip at the boundary. Figure (8) also present velocity profiles for Newtonian ($n = 1$), shear-thinning ($n < 1$) and shear-thickening ($n > 1$) for fixed value of slip parameter (say, $\delta = 0.4$). It is evident the boundary layer thickness of shear-thickening fluids is thinnest, followed by Newtonian and then the shear-thinning fluids. The temperature profiles in Figure (9) show the increase in temperature and thermal boundary layer thickness with an increase in velocity slip at the boundary.
Figures (10)-(13) depicted the influence of variation of suction/injection parameter $S$ on temperature and velocity profiles, respectively. The velocity profiles in Figure (10) present decreasing trend for increase in suction velocity ($S > 0$) at the boundary. The opposite behavior is noted for injection ($S < 0$) in Figure (12). This expected trend is because, suction draw more fluid into the boundary and decrease
the overall velocity of the fluid within boundary layer and in case of injection heated fluid is pressed farther from the wall where due to less influence of the viscosity, the flow is accelerated. It is observed from Figs. (11)-(13), the imposition of suction on the surface causes reduction in the thermal boundary layer thickness and the injection causes an increase in the thermal boundary layer thickness.
5. Concluding remarks. In this paper a simplified model is presented to study the magnetohydrodynamic boundary layer slip flow and heat transfer characteristics of power-law nanofluid over an unsteady porous stretching sheet. The magnetic field is applied in the transverse direction to the flow and the thermal conductivity is assumed as a function of temperature. The governing system of partial differential equations are transformed into a system of ordinary differential equations and then solved numerically using shooting method. The numerical results focus on the principal effects of power-law index, unsteadiness, applied magnetic field, nanofluid volume concentration, wall slip and suction/injection velocity on the velocity and temperature profiles non-Newtonian nanofluid. The effects of variation in different governing parameters on velocity and temperature profiles are discussed in the previous section. It is hoped that present simplified model can be utilized to better understand the physics of non-Newtonian nanofluids flow over a flat stretching plate. Moreover, the analysis can be extended to include the results for other water-based nanofluids, different non-Newtonian nanofluid models, the use of temperature dependent viscosity, heat flux and convective boundary conditions, multidimensional MHD slip flow etc. The results presented in this study will serve as a motivation for future experimental work.

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