Bread and butter physics: NLO electroweak corrections to polarized top quark decays

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Abstract. We report on the status of an ongoing calculation of the NLO electroweak corrections to sequential polarized top quark decays $t(↑) \rightarrow X_b + W^+(\rightarrow ℓ^+ + ν_ℓ)$. 

1. Singly produced top quarks: Rates, polarization and integrated luminosities

The $t$-channel production of single top quarks is mediated by parity-violating weak interactions which is a necessary condition for a non-vanishing polarization of the produced top quarks. And, in fact, the polarization of singly produced top quarks is calculated to be $P_t \sim 90\%$ where the polarization is primarily along the spectator quark of the production process. The Standard Model (SM) rates for single top production are given by

$$\sigma^{8\text{ TeV}} = 55 \cdot 10^3 \text{ fb} \quad \sigma^{13\text{ TeV}} = 136 \cdot 10^3 \text{ fb}$$

Typical numbers on integrated luminosities quoted by the CMS and ATLAS Collaborations in their analysis of singly produced top quarks are $20 \text{ fb}^{-1} @ 8 \text{ TeV}$ and $3 \text{ fb}^{-1} @ 13 \text{ TeV}$ which corresponds to $1.1 \cdot 10^6$ and $0.41 \cdot 10^6$ single top quark events, respectively. The present experimental situation is nicely reviewed in Ref. [1]. The experimental analysis of top quark polarization by the ATLAS Coll. is based on $20.2 \text{ fb}^{-1} @ 8 \text{ TeV}$ ($\sim 1.1 \cdot 10^6$ events) [2]. ATLAS quotes a value of $P_t = 0.97 \pm 0.12$ for the polarization of a singly produced top quark [2].

2. The hunt is on

The $t \rightarrow b$ transition matrix element of an effective current $J^{\text{eff}}_μ(t \rightarrow b)$ is given by (see e.g. Ref. [3])

$$M^{\text{eff}}_μ = -\frac{g_\text{w}}{\sqrt{2}} u_b \left\{ \gamma_μ(f_L P_L + f_R P_R) + \frac{iσ^{\muν}q_ν}{m_\text{W}} (g_L P_L + g_R P_R) \right\} u_t$$

In the SM one has $f_R, g_L, g_R = 0$ and $f_L = V_{tb}^\ast \sim 1$. Imaginary parts of the coupling factors can be generated by final state interactions (SM; $CP$-conserving) or by introducing non-SM $CP$-violating imaginary couplings by hand. Real contributions of $f_L, f_R, g_L, g_R$ to the spin density elements of the $W^+$ compete with higher order perturbative corrections.

Large samples of polarized top quarks are presently being produced at the LHC. There is a strong ongoing experimental program (ATLAS and CMS) to unravel the structure underlying polarized top quark decay. In particular, the hunt is on for signals of non-SM physics in polarized top quark decays.
3. Scope of the problem
There are 8 $T$-even and 2 $T$-odd helicity structure function describing sequential polarized top quark decays $t(↑) \rightarrow X_b + W^+ (\rightarrow \ell^+ + \nu_\ell)$. Three of these describe unpolarized top quark decay. As concerns NLO electroweak contributions the following calculations exist in the literature:

i) Total unpolarized rate \[4, 5\]

ii) Three unpolarized helicity fractions \[6\] preceded by a soft photon calculation \[7\]

iii) Electroweak one-loop corrections to the real and imaginary parts of the left and right tensorial coupling factors $g_{R,L}$ \[8, 9, 10, 11\].

The NLO electroweak radiative corrections to 5 $T$-even and to 2 $T$-odd polarized helicity structure functions are still missing and need to be calculated.

The electroweak radiative corrections to the unpolarized structure functions amount to 1.55\% in the rate and to 1.3\%−2.1\% for the longitudinal and transverse structure functions. The NLO electroweak radiative corrections to the polarized structure functions are not likely to be much larger then those for the unpolarized case. One will therefore not win big merits and recognition for such a calculation. We have nevertheless decided to embark on such a “bread and butter physics” enterprise.

4. Size of radiative corrections to unpolarized structure functions
Let us list the radiative corrections to the longitudinal rate $\Gamma_L$ which we normalize to the Born term rate. One has

$$\hat{\Gamma}_L = 0.703 \left( 1 - \frac{9.51\%}{\text{NLO QCD [12, 13]}} - \frac{3.21\%}{\text{NNLO QCD [14]}} + \frac{1.32\%}{\text{NLO EW [6]}} \right)$$  \hspace{1cm} (3)

Contrast this with what is known/unknown for the corresponding polarized longitudinal rate given by

$$\hat{\Gamma}^P_L = 0.703 \left( 1 - \frac{9.62\%}{\text{NLO QCD [12, 13]}} - \frac{?\%}{\text{NNLO QCD}} + \frac{?\%}{\text{NLO EW}} \right)$$  \hspace{1cm} (4)

J.G.K. had applied for a grant to the Deutsche Forschungsgemeinschaft (DFG) proposing, among others, to calculate the NNLO QCD and NLO electroweak radiative corrections to polarized top quark decays to fill in the missing pieces such as in Eq. (4). The money asked for was a mere pittance. One of the referees shot down the grant application with the following argument: “The polarization of singly produced top quarks has been measured by the ATLAS Collaboration with an error of 12\% [2]. Why calculate the above radiative corrections which are likely to be smaller than 12\%?” My answer: The error on $P_t$ is based on pre-2016 data. In the meantime the data base has grown by a factor of 7 and will continue to grow in the next few years. The HL-LHC to come in the next years is expected to collect 250 $fb^{-1}$ per year and detector which corresponds to $34 \times 10^6$ polarized top quarks. The error on the top quark’s polarization is likely to be reduced considerably during the next few years which makes the calculation of radiative corrections to polarized top quark decays indispensable.

5. Counting the number of structure functions
The spin density matrix elements of the $W^+$ (also called helicity structure functions) are hermitian. They satisfy

$$\left(H_{\lambda_w \lambda_w}^{\lambda_t \lambda_t}\right)^\dagger = \left(H_{\lambda_w \lambda_w}^{\lambda_t \lambda_t}\right)$$  \hspace{1cm} (5)
One has to observe the angular momentum constraint \( \lambda_W - \lambda'_W = \lambda_t - \lambda'_t \) which implies \( |\lambda_W - \lambda'_W| \leq 1 \). With the above constraint one counts ten independent double spin density matrix elements:

\[
H_{++}, \quad H_{-+}, \quad H_{++}, \quad H_{--}, \quad H_{-+}^+, \quad H_{00}^+, \quad H_{00}^-, \quad H_{00}^+, \quad H_{00}^-, \quad H_{00}^+.
\]  

(6)

We mention that the counting by covariants is not straightforward because of a nontrivial Schouten identity among the covariants. We sort the \( H_{\lambda_W \lambda'_W} \) into a set of three unpolarized structure functions and a set of seven polarized structure functions by writing:

\[
H_{++} = H_{++}^+ + H_{++}^- \\
H_{00} = H_{00}^+ + H_{00}^- \\
H_{--} = H_{--}^+ + H_{--}^- \\
H_{++}^P = H_{++}^+ - H_{++}^- \\
H_{00}^P = H_{00}^+ - H_{00}^- \\
H_{--}^P = H_{--}^+ - H_{--}^-.
\]  

(7)

It is convenient to consider particular linear superpositions of the spin density elements which feature in the angular decay distribution of the process. These are:

\[
H_U = H_{++} + H_{--} \\
H_{L} = H_{00} \\
H_{F} = H_{++} - H_{--} \\
H_{+}^P = H_{++}^P + H_{--}^P \\
H_{-}^P = H_{00}^P + H_{--}^P \\
H_{+}^P = H_{++}^P - H_{--}^P \\
H_{L}^P = \frac{1}{2} (Re H_{+}^P + Re H_{-}^P) \\
H_{A}^P = \frac{1}{2} (Re H_{0}^P + Re H_{0}^P) \\
H_{L}^P = \frac{1}{2} (Im H_{+}^P - Im H_{-}^P) \\
H_{A}^P = \frac{1}{2} (Im H_{+}^P + Im H_{+}^P).
\]  

(8)

6. Angular decay distribution

We introduce the polar angles \( \theta_P \) and \( \theta \) which are defined in the top quark and \( W^+ \) rest frames, respectively, as depicted in Fig. 1. The azimuthal angle \( \phi \) measures the azimuthal angle between the two planes as shown in Fig. 1.

The angular decay distribution in the sequential decay \( t(\uparrow) \rightarrow X_b + W^+ (\rightarrow \ell^+ + \nu_\ell) \) can then be obtained from the master formula [13]

\[
W(\theta_P, \theta, \phi) \propto \sum_{\lambda_W - \lambda'_W = \lambda_t - \lambda'_t} e^{i(\lambda_W - \lambda'_W)\phi} d_{\lambda_W^{1}}(\theta) d_{\lambda'_W^{1}}(\theta) H_{\lambda_t \lambda'_t} \rho_{\lambda_t \lambda'_t}(\theta_P),
\]  

(9)

where the spin density matrix of the top quark is given by:

\[
\rho_{\lambda_t \lambda'_t}(\theta_P) = \frac{1}{2} \begin{pmatrix} 1 + P_t \cos \theta_P & P_t \sin \theta_P \\ P_t \sin \theta_P & 1 - P_t \cos \theta_P \end{pmatrix}.
\]  

(10)
\( P_t \) is the magnitude of the polarization of the top quark. The second lower index in the small Wigner \( d(\theta) \)-function \( d_{W1}^1 \) is fixed at \( m = 1 \) for zero mass leptons because the total \( m \)-quantum number of the lepton pair along the \( t^+ \) direction is \( m = 1 \). One then obtains the normalized angular decay distribution [13, 15, 16, 17, 18]

\[
W(\cos \theta, \cos \theta_P, \phi) = \frac{1}{4\pi} \frac{3}{8} \left\{ (\hat{H}_U + \hat{H}_D^P P_t \cos \theta_P)(1 + \cos^2 \theta)
+ (\hat{H}_L + \hat{H}_D^P P_t \cos \theta_P)2 \sin^2 \theta + (\hat{H}_F + \hat{H}_D^P P_t \cos \theta_P)2 \cos \theta
+ \hat{H}_T^P P_t \sin \theta_P 2 \sqrt{2} \sin \theta \cos \phi + \hat{H}_A^P P_t \sin \theta_P 4 \sqrt{2} \sin \theta \cos \phi
+ \hat{H}_T^P P_t \sin \theta_P 2 \sqrt{2} \sin \theta \phi + \hat{H}_A^P P_t \sin \theta_P 4 \sqrt{2} \sin \theta \sin \phi \right\}
\]

where we define hatted normalized structure functions by

\[
\hat{H}_i^{(P)} = \frac{H_i^{(P)}}{H_{\text{total}}} = \hat{H}_i^{(P)}/(H_U + H_L)
\]
such that \( \int d\cos \theta d\cos \theta_P d\phi W(\cos \theta, \cos \theta_P, \phi) = 1 \).

7. **T-odd correlations**

The last two terms in the angular decay distribution correspond to \( T \)-odd angular correlations. This becomes evident by rewriting the angular factors in terms of the normalized three-vectors of the process (see Fig. 1):

\[
\hat{P}_t = (\sin \theta_P, 0, \cos \theta_P) \quad \hat{p}_t = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \quad \hat{q} = (0, 0, 1)
\]

One finds

\[
\begin{align*}
\sin \theta_P \sin \theta \sin \phi &= \hat{q} \cdot (\hat{P}_t \times \hat{p}_t) \\
\sin \theta_P \sin 2\theta \sin \phi &= 2(\hat{p}_t \cdot \hat{q}) \hat{q} \cdot (\hat{P}_t \times \hat{p}_t)
\end{align*}
\]

Under time reversal \( t \to -t \) one has \( (\hat{p}, \hat{P}_t) \to (-\hat{p}, -\hat{P}_t) \). One therefore calls the above two angular correlations \( T \)-odd correlations. As mentioned before there are two possible sources of \( T \)-odd correlations:

i) SM source: Imaginary parts from absorptive contributions,

ii) Non-SM source: \( CP \)-violating imaginary couplings.

8. **NLO tree-level Feynman diagrams**

There are four NLO electroweak tree level Feynman diagrams that contribute to \( t \to b + W^+ + \gamma \) since the photon can couple to the top quark, the bottom quark and the \( W^+ \)-boson (three diagrams) plus the diagram where the internal \( W^+ \) is replaced by the Goldstone boson \( \xi^+ \). The procedure to calculate the tree-level diagrams is standard. One splits the tree-level matrix element into a hard and a soft photon part according to

\[
M^\mu M^\nu (\text{hard + soft}) = \left( M^\mu M^\nu (\text{hard + soft}) - |M|^2 \text{(soft)} \right) + M^\mu M^\nu \text{(soft)}
\]

The first piece is infrared and collinear finite and can be safely integrated in \( D = 4 \) dimension. The second piece is universal in the sense that it is proportional to the Born term. The finite piece has to be projected onto the relevant eight \( T \)-even structure functions and then has to be integrated over the photon phase space. This has been done. For the projection onto the structure functions it is convenient to peruse the eight covariant projectors listed in Ref. [13]. The second universal piece has been known since long.
Figure 2. Absorptive parts of the four Feynman diagrams that contribute to the $T$-odd correlations in polarized top decays.

9. NLO electroweak one-loop vertex graphs
In the Feynman 't Hooft gauge there are altogether 18 NLO electroweak one-loop Feynman vertex diagrams that contribute to $t \rightarrow b + W^+$. The corresponding one-loop integrals are not simple in that they have five mass scales: $m_t, m_b, m_W, m_Z, m_H$.

In the following we shall set $m_b = 0$. The number of vertex diagrams is reduced to 13 since $g_{Hbb} = 0$ and $g_{\chi_3bb} = 0$. The complexity of the one-loop integrals is also somewhat reduced because now there are only four mass scales. The calculation of the 13 one-loop diagrams plus the counter-term diagrams needed for renormalization has been done.

In the limit $m_b = 0$ one has the simplifying feature that the longitudinal and transverse projection of the matrix element $M^\mu$ are proportional to the corresponding projections of the Born matrix element, i.e. one has ($x = m_W/m_t$) [7]

$$
\varepsilon^\mu(0)M^\mu = (f_L - x g_R) \varepsilon^\mu(0)(\text{Born})
$$

$$
\varepsilon^\mu(-)M^\mu = (f_L - \frac{1}{x} g_R) \varepsilon^\mu(-)M^\mu(\text{Born})
$$

10. Electroweak one-loop vertex graphs that admit of absorptive cuts
There are four ($m_B = 0$) NLO electroweak one-loop Feynman diagrams for $t \rightarrow b + W^+$ that admit of absorptive cuts (often also referred to as final state interactions/rescattering corrections) which we show in Fig. 2. We have carefully extracted the imaginary parts of the four diagram in analytical form. Numerically one obtains

$$
\text{Im} g_R = -0.217\%
$$

The azimuthal analyzing power is thus given by a tiny

$$
\hat{H}^P_{IA} \frac{3\pi}{4\sqrt{2}} = 0.0711\%
$$
11. Final remarks

Let us discuss some of the features of the NLO electroweak corrections to sequential polarized top quark decays treated in this talk. The soft photon contributions are proportional to the Born terms with a universal proportionality factor $H_i^{(P)}(\text{soft}) = a B_i^{(P)}$ while the hard photon contributions are not proportional to the Born terms. In the limit $m_b = 0$ we have found several simplifications for the contributions of the one-loop vertex graphs. First, the number of contributing NLO one-loop vertex graphs is reduced. Second, the structure functions $H_{--}, H_{-+}^P, H_{00}^P$ and $H_{0+}^P$ vanish, and third, the contributions of the one-loop vertex graphs to the structure functions are proportional to the corresponding Born terms with non-universal proportionality factors $H_i^{(P)}(\text{one-loop}) = b_i B_i^{(P)}$. The non-universal factors $b_i$ are not difficult to obtain.

The happy news is that we have assembled all ingredients necessary to obtain the NLO electroweak corrections to sequential polarized top quark decays. What remains to be done is to put the various pieces together and to evaluate them numerically. A first result was reported on the azimuthal analyzing power of the SM absorptive $T$-odd contribution is a tiny $0.07\%$. If experimentalists discover large $T$-odd effects in polarized top quark decays they would be most certainly due to non-SM $CP$-violating imaginary coupling terms with only a tiny contamination from SM absorptive contributions.

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