Trilinear gauge boson couplings and bilepton production in the SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ models

Hoang Ngoc Long$^a$ and Dang Van Soa$^b$

$^a$Laboratoire de Physique Théorique LAPTH, Chemin de Bellevue, B P 110, F-74941 Annecy-le-Vieux Cedex, France
and
Institute of Physics, NCST, P. O. Box 429, Bo Ho, Hanoi 10000, Vietnam

$^b$The Abdus Salam International Centre for Theoretical Physics, Trieste 34100, Italy

The trilinear gauge boson couplings in the SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ (3 - 3 - 1) models are presented. We find that new $Z_2$ does not interact with the usual (in the standard model) gauge bosons $Z, W^\pm$. Based on these results, production of new heavy gauge bosons at high energy colliders such as $e^+ e^-$ is calculated. We show that the cross sections obtained in the 3 - 3 - 1 model with right-handed neutrinos can be one order bigger than the same in the minimal 3 - 3 - 1 model.

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1. Introduction

Although the standard model (SM) \([1]\) of electroweak interactions has been verified to great precision in the recent years at LEP, SLC and other places, there remain a few unanswered questions concerning mainly the mass spectrum and the generation structure of quarks and leptons. In particular the question of the number of generations remains open and few progress has been made towards the understanding of the interrelation between generations. Amongst the possible models beyond the standard one, from modest extensions to GUTs, few address this question, the generations are usually assumed to be a replicate of the first one. The models based on the SU(3)\(_C\) \(\otimes\) SU(3)\(_L\) \(\otimes\) U(1)\(_N\) gauge group \([2–6]\), are interesting form this point of view. They have the following intriguing features: Firstly the models are anomaly free only if the number of generations \(N\) is a multiple of three. If further one adds the condition of QCD asymptotic freedom, which is valid only if the number of generations of quarks is to be less than 5, it follows that the number of generations is equal to 3. The second characteristics of the 3 - 3 - 1 models is that one generation of quarks is treated differently from two others. This could lead to a natural explanation for the unbalancingly heavy top quark. The possibility of the third generation being different from the first two is not excluded experimentally. While the anomalous behaviour of the parameters \(R_b\) and \(A_b\) in the LEP data \([8]\) has more or less disappeared, the effects are now 1.8 \(\sigma\) away from the SM value for \(R_b\) and 3 \(\sigma\) for \(A_b\), there is still room for generation universality breaking in the third generation. The third interesting feature is that the Peccei-Quinn symmetry naturally occurs in these models \([9]\). Finally, from a phenomenological point of view, the 3 - 3 - 1 models are very interesting, they predict new physics at a scale only slightly above the SM scale (typically TeV) and even give upper bounds on the mass of some new particles. Therefore the models can be confirmed or ruled out in the next generation of collider experiments from Tevatron, LHC, or a future linear collider, in stark contrast with “grand desert” scenarios in Grand Unification Theories.

Despite the extremely precise measurements of the SM parameters, one important component has not been tested directly with precision: the non-abelian self-couplings of the weak gauge bosons. The measurements performed at LEP1 have provided us with an extremely accurate knowledge of the parameters of the \(Z\) gauge boson: its mass, partial widths, and total width. There even is first evidence that the contributions of gauge-boson loops to the gauge-boson self-energies are indeed required \([10]\). Thus, an indirect confirmation of the existence of the trilinear gauge boson couplings (TGC’s) has been obtained. With the excellent performance of the LEP machine at high energy in last couple of years, electroweak physics at LEP2 now trully merits the epithet “precise”. The core measurements of the LEP2 program, the \(W\) mass, and the vector boson self-couplings have been made with precision better, in some cases substantially so, than elsewhere. The mixing in the neutral gauge boson sector and the angular distributions as well as the \(W\) helicities in the final states of \(W^+W^-\) production have been searched for at LEP2 \([11]\). Deviation of non-abelian couplings from expectation would signal new physics. In addition, precise measurements of gauge boson self-interactions will provide important information on the nature of electroweak symmetry breaking. The TGC’s have been investigated by many authors \([12–15]\), and some direct tests of these couplings have
been made in \[16\]. TGC’s in the beyond - the standard models, in which there exist heavy particles with mass much larger \(m_W\) have been investigated in \[17\]. In the 3 - 3 - 1 models, the TGCs have the structure of the standard model couplings, up to a coupling constant. Recent investigations have indicated that signals of new gauge bosons in the models may be observed at the CERN LHC \[18\] or Next Linear Collider (NLC) \[19\].

Our aim in this paper is to present TGC’s in the 3 – 3 – 1 models and use these couplings to discuss new processes that could be measured at future high energy colliders.

This paper is organized as follows. In Sec. II we give a brief review of two models: relation among real physical bosons and gauge fields, which is necessary in getting of TGC’s. TGC’s are given in this section. Sec. III is devoted to bilepton production at high energy collider \(e^+ e^−\) and discussions are given in the last section - section IV.

2. A review of the 3 – 3 – 1 models

To frame the context, it is appropriate to briefly recall some relevant features of two types of 3 – 3 – 1 models: the minimal model proposed by Frampton, Pisano and Pleitez \[3, 4\], and the model with right-handed (RH) neutrinos \[5,6\].

2.1. The minimal 3 – 3 – 1 model

The model treats the leptons as \(SU(3)_L\) antitriplets \[4,4\]

\[
f^a_L = \begin{pmatrix} e^a_L \\ -\nu^a_L \\ (e^c)^a \end{pmatrix} \sim (1, 3, 0),
\]

where \(a = 1, 2, 3\) is the generation index.

Two of the three quark generations transform as triplets and the third generation is treated differently - it belongs to an antitriplet:

\[
Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \\ D_{iL} \end{pmatrix} \sim (3, 3, -1/3),
\]

\[
u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), \ i = 1, 2,
\]

\[
Q_{3L} = \begin{pmatrix} d_{3L} \\ -u_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 2/3),
\]

\[
u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3).
\]

Of the nine gauge bosons \(W^a(a = 1, 2, ..., 8)\) and \(B\) of \(SU(3)_L\) and \(U(1)_N\), four are light: the photon \((A)\), \(Z\) and \(W^\pm\). The remaining five correspond to new heavy gauge bosons \(Z_2\), \(Y^\pm\) and the doubly charged bileptons \(X^{\pm\pm}\). They are expressed in terms of \(W^a\) and \(B\) as \[7\]:

\[
\sqrt{2} W^+_\mu = W^1_\mu - i W^2_\mu, \quad \sqrt{2} Y^+_\mu = W^6_\mu - i W^7_\mu,
\]

\[
\sqrt{2} X^{++}_\mu = W^4_\mu - i W^5_\mu.
\]

\[1\]The leptons may be assigned to a triplet as in \[3\], however the two models are mathematically identical.
\[ A_\mu = s_W W_\mu^3 + c_W \left( \sqrt{3} t_W W_\mu^8 + \sqrt{1 - 3 t_W^2} B_\mu \right), \]
\[ Z_\mu = c_W W_\mu^3 - s_W \left( \sqrt{3} t_W W_\mu^8 + \sqrt{1 - 3 t_W^2} B_\mu \right), \]
\[ Z'_\mu = -\sqrt{1 - 3 t_W^2} W_\mu^8 + \sqrt{3} t_W B_\mu. \]

where we use the following notations: \( s_W \equiv \sin \theta_W \) and \( t_W \equiv \tan \theta_W \). The physical states are a mixture of \( Z \) and \( Z' \):

\[ Z_1 = Z \cos \phi - Z' \sin \phi, \]
\[ Z_2 = Z \sin \phi + Z' \cos \phi, \]

where \( \phi \) is a mixing angle.

Symmetry breaking and fermion mass generation can be achieved by three scalar SU(3) \(_L\) triplets and a sextet

\[
\begin{align*}
\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_N \\
\downarrow \langle \Phi \rangle \\
\text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \\
\downarrow \langle \Delta \rangle, \langle \Delta' \rangle, \langle \eta \rangle \\
\text{SU}(3)_C \otimes \text{U}(1)_Q,
\end{align*}
\]

where the minimally required scalar multiplets are summarized as

\[
\begin{align*}
\Phi &= \begin{pmatrix} \phi^{++} \\ \phi^+ \\ \phi^o \end{pmatrix} \sim (1, 3, 1), \\
\Delta &= \begin{pmatrix} \Delta_1^+ \\ \Delta^o \\ \Delta_2^- \end{pmatrix} \sim (1, 3, 0), \\
\Delta' &= \begin{pmatrix} \Delta_-^o \\ \Delta_-^{--} \end{pmatrix} \sim (1, 3, -1), \\
\eta &= \begin{pmatrix} \eta_1^{++} / \sqrt{2} \\ \eta_1^+ / \sqrt{2} \\ \eta_2^+/\sqrt{2} \end{pmatrix} \sim (1, 6, 0).
\end{align*}
\]

The vacuum expectation value (VEV) \( \langle \Phi^T \rangle = (0, 0, u/\sqrt{2}) \) yields masses for the exotic quarks, the heavy neutral gauge boson \( (Z_2) \) and two charged gauge bosons \( (X^{++}, Y^+) \). The masses of the standard gauge bosons and the ordinary fermions are related to the VEVs of the other scalar fields, \( \langle \Delta^o \rangle = v/\sqrt{2}, \langle \Delta'^o \rangle = v'/\sqrt{2} \) and \( \langle \eta^o \rangle = \omega/\sqrt{2}, \langle \eta'^o \rangle = 0 \). In order to be consistent with the low energy phenomenology we have to assume that \( u \gg v, v', \omega \). The masses of gauge bosons are explicitly:

\[
m_W^2 = \frac{1}{4} g^2 (v^2 + v'^2 + \omega^2), \quad M_Y^2 = \frac{1}{4} g^2 (u^2 + v^2 + \omega^2), \quad M_X^2 = \frac{1}{4} g^2 (u^2 + v'^2 + 4\omega^2), \quad (6)
\]
and
\[ m_Z^2 = \frac{g^2}{4c_W^2} (v^2 + v'^2 + \omega^2) = \frac{m_W^2}{c_W^2}, \]
\[ M_{Z'}^2 = \frac{g^2}{3} \left[ \frac{s_W^2}{1 - 4s_W^2} u^2 + \frac{1 - 4s_W^2}{4c_W^2} (v^2 + v'^2 + \omega^2) + \frac{3s_W^2}{1 - 4s_W^2} v'^2 \right]. \]  

Expressions in (3) yield a splitting on the bileptons masses (4)
\[ |M_X^2 - M_Y^2| \leq 3 m_W^2. \]  

By matching the gauge coupling constants we get a relation between \( g \) and \( g_N \) – the couplings associated with SU(3)_L and U(1)_N, respectively:
\[ \frac{g_N^2}{g^2} = \frac{6 s_W^2(M_{Z_2})}{1 - 4s_W^2(M_{Z_2})}, \]  

where \( e = g s_W \) is the same as in the SM.

Combining constraints from direct searches and neutral currents, one obtains a range for the mixing angle (7) \(-1.6 \times 10^{-2} \leq \phi \leq 7 \times 10^{-4}\) and a lower bound on \( M_{Z_2} \geq 1.3 \) TeV. Such a small mixing angle can safely be neglected. Adding the constraints from “wrong” muon decay experiments one also obtains a range for the new gauge charged bosons: \( M_{Y^+} \geq 230 \) GeV. By computing the oblique parameters \( S \) and \( T \), a lower bound of 367 GeV for the mass of the singly charged bilepton \( Y^+ \) is derived (24). Combining this with the mass splitting given in (8) we obtain a lower bound around 400 GeV for the mass of the doubly charged bilepton \( (X^+) \). However the most stringent limit on the mass of doubly charged bilepton is derived from constraints on fermion pair production at LEP and lepton-flavour violating charged lepton decay (25): \( M_{X^{\pm \pm}} > 740 \) GeV. With the new atomic parity violation in cesium, one gets a lower bound for the \( Z_2 \) mass (26): \( M_{Z_2} > 1.2 \) TeV. From symmetry breaking it follows that the masses of the new charged gauge bosons \( Y^\pm, X^{\pm \pm} \) are less than a half of \( M_{Z_2} \), the allowed decay \( Z_2 \rightarrow X^{\pm} X^{\mp} \) with \( X^{\pm \pm} \rightarrow 2 l \pm \) provides a unique signature in future colliders.

The TGC’s in this model are obtained from the part of the Lagrangian describing the self-interactions of gauge fields.
\[ \mathcal{L}_{TGC} = -g f_{abc} \partial_\mu W^a_\mu W^{b\mu} W^{c\nu}, a, b, c = 1, 2, ..., 8. \]  

Expressing \( W^a \) \((a = 1, 2, ..., 8)\) in terms of physical fields using Eqs (4) and (5), a straightforward but cumbersome calculation leads to
\[
\frac{i}{g} \mathcal{L}_{TGC}^{\min} = s_W \left[ A^\nu (W_{\mu\nu} - W^{+\mu} W^{-\nu} + A_{\mu\nu} W^{-\mu} W^{+\nu}) \right] \\
+ c_W \left[ Z^\nu (W_{\mu\nu} - W^{+\mu} W^{-\nu} + Z_{\mu\nu} W^{-\mu} W^{+\nu}) \right] \\
+ s_W \left[ A^\nu (Y^{-\mu} Y^{+\mu} - Y^{+\nu} Y^{-\mu} + A_{\mu\nu} Y^{-\mu} Y^{+\nu}) \right] \\
- \frac{(c_W + 3 s_W t_W)}{2} \left[ Z^\nu (Y^{-\mu} Y^{+\mu} - Y^{+\nu} Y^{-\mu} + Z_{\mu\nu} Y^{-\mu} Y^{+\nu}) \right] \\
+ 2 s_W \left[ A^\nu (X^{-\mu} X^{+\mu} - X^{+\nu} X^{-\mu} + A_{\mu\nu} X^{-\mu} X^{+\nu}) \right].
\]
\[\frac{(c_W - 3s_W t_W)}{2} \left[ Z^\nu (X^+ X^+ X^+ X^- X^-) + Z_{\mu \nu} X^+ X^- X^+ X^- X^- \right] \]
\[-\frac{\sqrt{3}(1 - 3t_t^2)}{2} \left[ Z^\nu (Y^+ Y^+ Y^+ Y^- Y^-) + Z'_{\mu \nu} Y^+ Y^- Y^+ Y^- \right] \]
\[-\frac{\sqrt{3}(1 - 3t_t^2)}{2} \left[ Z'_{\mu \nu} X^+ X^- X^+ X^- X^- \right] \]
\[+ \frac{1}{\sqrt{2}} \left[ X^- (Y^+ W^+ W^+ - Y^- W^- W^-) + X^+ (Y^+ W^+ W^+) \right] \]
\[+ \frac{1}{\sqrt{2}} \left[ X^+ (W^- Y^- Y^- - Y^- Y^+ Y^+) + X^- (W^- Y^- Y^- - Y^- Y^+ Y^+) \right], \tag{11}\]

where \( W_{\mu \nu} \equiv \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu} \). The coupling constants for all trilinear couplings are summarized in Table 1.

### Table 1

| Vertex       | Coupling constant/e |
|--------------|---------------------|
| \( \gamma W^+ W^- \) | 1                   |
| \( Z W^+ W^- \) | \( 1/t_W \)         |
| \( \gamma Y^+ Y^- \) | 1                   |
| \( Z Y^+ Y^- \) | \( -(1 + 2s_W^2)/\sin 2\theta_W \) |
| \( \gamma X^+ X^- \) | 2                   |
| \( Z X^+ X^- \) | \( (1 - 4s_W^2)/\sin 2\theta_W \) |
| \( Z' Y^+ Y^- \) | \( -\sqrt{3}(1 - 4s_W^2)/\sin 2\theta_W \) |
| \( Z' X^+ X^- \) | \( -\sqrt{3}(1 - 4s_W^2)/\sin 2\theta_W \) |
| \( X^- Y^+ W^+ \) | \( 1/(\sqrt{2} s_W) \) |
| \( X^+ W^- Y^- \) | \( 1/(\sqrt{2} s_W) \) |

As we can see from Table 1, the \( Z' \) does not interact with the usual gauge bosons: photon, \( W^\pm \) and \( Z \). Strictly speaking, the new neutral gauge boson \( Z_2 \) interacts very weakly with the usual SM bosons since the mentioned coupling constants are proportional to \( \sin \phi \). The SM trilinear gauge boson couplings are recovered in (11).

### 2.2. The model with RH neutrinos

In this model the leptons are in triplets, and the third member is a RH neutrino [5,6]:

\[
f^a_L = \left( \begin{array}{c} \nu^a_L \\ e^a_L \\ (\nu_L^a)^c \end{array} \right) \sim (1, 3, -1/3), e^R \sim (1, 1, -1). \tag{12}\]

The first two generations of quarks are in antitriplets while the third one is in a triplet:

\[
Q_{iL} = \left( \begin{array}{c} d_{iL} \\ -u_{iL} \\ D_{iL} \end{array} \right) \sim (3, \bar{3}, 0), \tag{13}\]
\[ u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), \ i = 1, 2, \]

\[ Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3), \quad (14) \]

\[ u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3). \]

The doubly charged bileptons of the minimal model are replaced here by complex neutral ones:

\[ \sqrt{2} W^+ = W^1 - iW^2, \quad \sqrt{2} Y^- = W^6 - iW^7, \]
\[ \sqrt{2} X^o = W^4 - iW^5. \quad (15) \]

For a shorthand notation, hereafter we will use \( X^o \equiv X \).

The physical neutral gauge bosons are again related to \( Z, Z' \) through the mixing angle \( \phi \). Together with the photon, these are defined as follows [6]:

\[ A_\mu = s_W W^3_\mu + c_W \left( -\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \]
\[ Z_\mu = c_W W^3_\mu - s_W \left( -\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \]
\[ Z'_\mu = \sqrt{1 - \frac{t_W^2}{3}} W_\mu^8 + \frac{t_W}{\sqrt{3}} B_\mu. \quad (16) \]

The symmetry breaking can be achieved with just three \( SU(3)_L \) triplets

\[ SU(3)_C \otimes SU(3)_L \otimes U(1)_N \]
\[ \downarrow \langle \chi \rangle \]
\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \]
\[ \downarrow \langle \rho \rangle, \langle \eta \rangle \]
\[ SU(3)_C \otimes U(1)_Q, \quad (17) \]

where

\[ \chi = \begin{pmatrix} \chi^o \\ \chi^- \\ \chi^o \end{pmatrix} \sim (1, 3, -1/3), \quad (18) \]
\[ \rho = \begin{pmatrix} \rho^+ \\ \rho^o \\ \rho^+ \end{pmatrix} \sim (1, 3, 2/3), \quad (19) \]
\[ \eta = \begin{pmatrix} \eta^o \\ \eta^- \\ \eta^o \end{pmatrix} \sim (1, 3, -1/3). \quad (20) \]
The necessary VEVs are

\[ \langle \chi \rangle^T = (0, 0, \omega/\sqrt{2}), \quad \langle \rho \rangle^T = (0, u/\sqrt{2}, 0), \quad \langle \eta \rangle^T = (v/\sqrt{2}, 0, 0). \]  

(22)

Here the electric charge is defined:

\[ Q = \frac{1}{2}\lambda_3 - \frac{1}{2\sqrt{3}}\lambda_8 + N. \]  

(23)

The VEV \( \langle \chi \rangle \) generates masses for the exotic \( \frac{2}{3} \) and \( -\frac{1}{3} \) quarks, while the VEVs \( \langle \rho \rangle \) and \( \langle \eta \rangle \) generate masses for all ordinary leptons and quarks. Neutrinos, however, are still massless. After symmetry breaking the gauge bosons gain masses

\[ m_W^2 = \frac{1}{4}g^2(u^2 + v^2), \quad M_Y^2 = \frac{1}{4}g^2(v^2 + \omega^2), \quad M_X^2 = \frac{1}{4}g^2(u^2 + \omega^2), \]  

(24)

and

\[ m_Z^2 = \frac{g^2}{4c_W^2}(u^2 + v^2) = \frac{m_W^2}{c_W^2}, \]  

\[ M_{Z'}^2 = \frac{g^2}{4(3 - 4s_W^2)} \left[ 4\omega^2 + \frac{u^2}{c_W^2} + \frac{v^2(1 - 2s_W^2)^2}{c_W^2} \right]. \]  

(25)

In order to be consistent with the low energy phenomenology we have to assume that \( \langle \chi \rangle \gg \langle \rho \rangle, \langle \eta \rangle \) such that \( m_W \ll M_X, M_Y \).

In this model the analog of formula (9) for the ratio of coupling constants is:

\[ \frac{g_N^2}{g^2} = \frac{18s_W^2(M_Z_2)}{3 - 4s_W^2(M_Z_2)}. \]  

(26)

The symmetry-breaking hierarchy gives us splitting on the bileptons masses [24]

\[ |M_X^2 - M_Y^2| \leq m_W^2. \]  

(27)

Therefore in the future studies it is acceptable to put \( M_X \simeq M_Y \).

The constraint on the \( Z - Z' \) mixing based on the \( Z \) decay, is given [3]: \(-2.8 \times 10^{-3} \leq \phi \leq 1.8 \times 10^{-4}\), and in this model we have not a limit for \( \sin^2 \theta_W \). From the data on parity violation in the cesium atom, one gets a lower bound on the \( Z_2 \) mass at a range from 1.4 TeV to 2.6 TeV [23]. The muon decay data [25] gives a lower bound for the \( Y \) boson mass: 230 GeV (90 % CL). Analyzing radiative correction based on the \( S \) and \( T \) parameters gives similar results [24]: \( M_{Y^+} \geq 230 \) GeV, \( M_X \geq 240 \) GeV.

Repeating the procedure for deriving the trilinear interactions of gauge bosons, one obtains:

\[ \frac{i}{g} \mathcal{L}_{TGC}^{\text{eff}} = s_W \left[ A^\nu(W_{\mu\nu}W^{+\mu} - W_{\mu\nu}W^{-\mu}) + A_{\mu
u}W^{-\mu}W^{+\nu} \right] 
\ + c_W \left[ Z^\nu(W_{\mu\nu}W^{+\mu} - W_{\mu\nu}W^{-\mu}) + Z_{\mu
u}W^{-\mu}W^{+\nu} \right] 
\ + s_W \left[ A^\nu(Y_{\mu\nu}Y^{+\mu} - Y_{\mu\nu}Y^{-\mu}) + A_{\mu
u}Y^{-\mu}Y^{+\nu} \right] 
\ + \frac{(c_W - s_W t_W)}{2} \left[ Z^\nu(Y_{\mu\nu}Y^{+\mu} - Y_{\mu\nu}Y^{-\mu}) + Z_{\mu
u}Y^{-\mu}Y^{+\nu} \right]. \]
\[
- \frac{(c_W + s_W t_W)}{2} \left[ Z^\nu (X_{\mu\nu} X^{*\mu} - X_{\mu\nu}^* X^\mu) + Z_{\mu\nu} X^\mu X^{*\nu} \right]
- \frac{\sqrt{3} - t_W^2}{2} \left[ Z^\nu (Y_{\mu\nu}^+ Y^{*\mu} - Y_{\mu\nu}^* Y^{+\mu}) + Z_{\mu\nu}^* Y^{-\mu} Y^{+\nu} \right]
- \frac{\sqrt{3} - t_W^2}{2} \left[ Z^\nu (X_{\mu\nu} X^{*\mu} - X_{\mu\nu}^* X^\mu) + Z_{\mu\nu}^* X^\mu X^{*\nu} \right]
+ \frac{1}{\sqrt{2}} \left[ X^\nu (W_{\mu\nu}^+ Y^{*\mu} - Y_{\mu\nu}^+ W^{-\mu}) + X_{\mu\nu} W^{-\mu} Y^{+\nu} \right]
+ \frac{1}{\sqrt{2}} \left[ X^{*\nu} (Y_{\mu\nu}^+ W^{*\mu} - W_{\mu\nu}^+ Y^{-\mu}) + X_{\mu\nu}^* Y^{-\mu} W^{+\nu} \right]. \tag{28}
\]

The coupling constants for the TGC’s in this model are listed in Table 2

| Vertex | coupling constant/e |
|--------|---------------------|
| $\gamma W^+ W^-$ | 1 |
| $ZW^+ W^-$ | $1/t_W$ |
| $\gamma Y^+ Y^-$ | 1 |
| $ZY^+ Y^-$ | $1/\tan 2\theta_W$ |
| $Z XX^*$ | $-1/\sin 2\theta_W$ |
| $Z' Y^+ Y^-$ | $-\sqrt{3 - 4 s_W^2}/\sin 2\theta_W$ |
| $Z' XX^*$ | $-\sqrt{3 - 4 s_W^2}/\sin 2\theta_W$ |
| $XW^- Y^+$ | $1/(\sqrt{2} s_W)$ |
| $X^* Y^- W^+$ | $1/(\sqrt{2} s_W)$ |

As we can see again from Table 2, the $Z'$ does not interact with the usual gauge bosons: photon, $W^\pm$ and $Z$. As expected there is no coupling of the photon with the neutral bosons $X$, and the SM couplings remain undeveloped. It must be noted that the coupling of the $Z_2$ to the new gauge bosons is much stronger than that in the minimal model which was suppressed by a factor $(1 - 4 s_W^2)$.

3. Bilepton production at high energy colliders

Recently production of doubly-charged vector bileptons in high energy collision has been widely discussed both in generic models [26,27] and in the minimal model [19,28]. The production of bileptons in the 3 - 3 - 1 model is particularly relevant for colliders in the TeV range since the models predict new gauge bosons at the same scale. Furthermore the present constraints on bilepton masses are not very stringent [29]: the constraint is only $M_Y \geq 230$ GeV from the muon decay experiment. One of the strongest limits on the bilepton mass comes from the fermion pair production and lepton-flavour violating charged lepton decays [22] at about 750 GeV. The current experimental lower limit on
$M_{X^-}$ is claimed to be 850 GeV \cite{30} (However, such a lower limit can be derived from oblique corrections assuming the Higgs mass to be equal to 300 GeV \cite{21}). In this section, we present the cross sections for the production of two bileptons in both types of 3 - 3 - 1 models. In the models we are considering one has to include, in addition to the photon and $Z$ exchange, the contribution from the new neutral gauge boson $Z_2$ as well as from internal lepton exchange (in $t$ channel). These were not considered in the general calculation of \cite{27} and we will see that in many cases, especially for the model with RH neutrinos, the $Z_2$ has a significant contribution.

There are four modes for the bileptons discovery: $e^+e^-$, $e^-e^-$, $\mu^-\mu^-$ and $\gamma\gamma$. The $e^-e^-$ and $\mu^-\mu^-$ running modes of the linear colliders are particularly suitable for discovering of doubly charged bileptons. However we have to wait for these modes. In this paper we concentrate on production of the bileptons at high energy $e^+e^-$ colliders from 1 TeV (NLC) to 3 TeV, such as the CLIC project designed at CERN.

### 3.1. Production of bileptons in the minimal model

Bileptons (singly and doubly charged) can be produced in high energy $e^+e^-$ colliders and this process has been examined (see report of Dawson in \cite{3}). In this model the intermediate states are photon, $Z$ and new $Z_2$ bosons in the $s$ channel and neutrino/electron in $t$ channel. It must be emphasized that in \cite{27} the intermediate states consist only of the first two neutral gauge bosons, while in our case, the $Z_2$ also gives a contribution.

![Feynman diagram](image)

**Figure 1**: Feynman diagram for $e^+e^- \rightarrow Y^+ Y^-$ in the 3 - 3 - 1 models

First we consider the production of singly charged bileptons:

$$e^- (k, \lambda) + e^+ (k', \lambda') \rightarrow Y^- (p, \tau) + Y^+ (p', \tau'), \quad (29)$$

where the first and the second letters in parentheses stand for the momentum and the helicity of the particle, respectively. The Feynman diagram for the full process is depicted in Fig. 1.

The amplitude for this process due to neutrino, $\gamma, Z$ and $Z_2$ is given (in the notation of \cite{31})

$$M_{fi} = M_{fi}^\nu + M_{fi}^e + M_{fi}^Z + M_{fi}^{Z_2}. \quad (30)$$
Figure 2. Cross section $\sigma(e^+e^- \to Y^+Y^-)$ in the minimal 3 - 3 - 1 model as a function of $M_Y$.

The neutrino exchange part is written as

$$M_{fi}^\nu = -\frac{e^2}{4s_W^2} \bar{v}(k') \gamma^\mu(\bar{k} - \bar{p})\gamma^5 u(k). \tag{31}$$

The diagrams with $\gamma, Z, Z_2$ intermediate lines involve the three-boson vertices defined in Subsect. 2.1 and the new $Z_2 e^+e^-$ (vector and axial) vertices

$$g_{Z_2ee}^V = \frac{\sqrt{3}}{2} \sqrt{1 - 4s_W^2}, \quad g_{Z_2ee}^A = -\frac{1}{2\sqrt{3}} \sqrt{1 - 4s_W^2}. \tag{32}$$

Hence contributions of the diagrams with gauge boson exchange are given by

$$M_{fi}^\gamma = -\frac{e^2}{s} Q_Y \bar{v}(k') \gamma^\mu u(k) V_\mu^\gamma, \tag{33}$$

$$M_{fi}^Z = \frac{e^2}{s - m_Z^2} \bar{v}(k') \gamma^\mu(a - b\gamma^5) u(k) V_\mu^Z, \tag{34}$$

$$M_{fi}^{Z_2} = \frac{e^2}{s - M_{Z_2}^2} \bar{v}(k') \gamma^\mu(a' - b'\gamma^5) u(k) V_\mu^{Z_2}, \tag{35}$$

where

$$a = \frac{(1 - 4s_W^2)(1 + 2s_W^2)}{4c_W^2 \sin 2\theta_W}, \quad b = \frac{(1 + 2s_W^2)}{4c_W^2 \sin 2\theta_W}, \tag{36}$$

$$a' = \frac{3(-1 + 4s_W^2)}{4c_W^2 \sin 2\theta_W}, \quad b' = \frac{(-1 + 4s_W^2)}{4c_W^2 \sin 2\theta_W}. \tag{37}$$
With the notations of Ref. [31] (see fig. 1 there), $\mathcal{V}_\mu^V$ is defined as follows

$$
\mathcal{V}_\mu^V = g V [\epsilon . \epsilon' (p - p')_\mu - 2 \epsilon . p \epsilon' + 2 \epsilon . p' \epsilon'_\mu].
$$

As usual, we have used $s = (k + k')^2 = (p + p')^2$, $t = (k - p)^2 = (p' - k')^2$.

In the high energy-limit, $s \gg m_Z^2, M_{Z_2}^2$, unitarity considerations of partial wave amplitudes imply cancellations among the various diagrams to control the bad high-energy behaviour of each amplitude. The sum of the amplitudes for the production of longitudinal gauge bosons will tend to zero. For the production of singly charged bilepton (29), therefore contributions from $\nu$ and $\gamma$ exchange are the same as in $e^+ e^- \rightarrow W^+ W^-$ in the SM. Therefore contributions from $Z$ and $Z_2$ exchanges should be equal to $Z$ exchange in the SM. From (36) and (37) it is easy to check that such is the case, indeed in this model we have:

$$
a + a' = \frac{-1 + 4 s^2_W}{4 s_W c_W} = a_{SM}, \quad \quad \quad (38)
$$

$$
b + b' = \frac{-1}{4 s_W c_W} = b_{SM}. \quad \quad \quad (39)
$$

It is convenient to decompose the amplitude in the helicity basis: the helicity of the electron (positron) is denoted by $\lambda = \pm \frac{1}{2} (\lambda' = -\lambda)$ while the helicities of the $Y^-$ and $Y^+$ by $\tau = \pm 1, 0, \tau' = \pm 1, 0$), respectively. They are given in Table 3, where the first row corresponds to the lepton exchange diagram and the second row to the gauge bosons exchange diagrams:

| Table 3 |
|-----------------------------|
| The helicity amplitudes for $e^+ e^- \rightarrow Y^+ Y^-$ |

| $\tau = \tau' = \pm 1$ | $\tau = -\tau' = \pm 1$ | $\tau = \tau' = 0$ | $\tau = 0, \tau' = \pm 1, \epsilon = 1$ |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $\frac{2 \lambda - 1}{4 s_W}$ | $\frac{2(\lambda - 2 \lambda')}{s W (s - M_Y^2)}$ | $\frac{-2 \lambda}{s}$ | $\frac{-2 \lambda}{s}$ |
| $\cos \vartheta - \beta_Y$ | $-\cos \vartheta - 2 \tau \lambda$ | $-\beta_Y (1 + \frac{2 M_Y^2}{s})$ | $-\beta_Y (1 + \frac{2 M_Y^2}{s})$ |

where $\beta_Y = (1 - 4 M_Y^2 / s)^{\frac{1}{2}}$ and $\vartheta$ is the center-of-mass scattering angle between the incident electron momentum $k$ and the final $Y^-$ momentum $p$. To obtain the amplitude for definite electron helicity $\lambda = \pm \frac{1}{2}$ and definite helicity of the bilepton ($Y^-$) $\tau = \pm 1, 0$, the elements in the corresponding column have to be multiplied by the common factor on top of the column.

We again stress that due to a factor $(1 - 4 s_W^2)$ (see Table 1) contribution from the $Z_2$ is very small.

Figure 2 shows the dependence of the total cross section $\sigma(e^+ e^- \rightarrow Y^+ Y^-)$ on the $Y^+$ mass. We have taken $M_{Z_2} = 1.2$ TeV and 3 TeV. As we can see from the figure, there is no difference between two lines at $M_Y \approx 450$ GeV.
Now we consider the doubly charged bilepton production:
\[ e^-(k, \lambda) + e^+(k', \lambda') \to X^{--}(p, \tau) + X^{++}(p', \tau'). \] (40)

The vector currents coupled to \( X^{--}, X^{++} \) vanish due to Fermi statistics, therefore suitable Lagrangian for this process is given [28]
\[ \mathcal{L} = -\frac{g}{\sqrt{2}} X^{++}_\mu e^T C \gamma^\mu \gamma_5 e - \frac{g}{\sqrt{2}} X^{--}_\mu \bar{e} \gamma^\mu \gamma_5 C \bar{e}^T. \] (41)

The Feynman diagrams which contribute to this process are shown in Fig. 3.

Figure 3: Feynman diagram for \( e^+ e^- \to X^{++} X^{--} \) in the minimal 3 - 3 - 1 model

The contribution from the electron exchange diagram is given
\[ \mathcal{M}_{fi} = -\frac{e^2}{2t s_W^2} \bar{v}(k') \gamma^\nu (k - \bar{p}) \gamma_\nu u(k). \] (42)

Hereafter the notations are similar to those in the previous case.

Table 4
The helicity amplitudes for \( e^+ e^- \to X^{++} X^{--} \)

| \( \tau = \tau' = \pm 1 \) | \( \tau = -\tau' = \pm 1 \) | \( \tau = \tau' = 0 \) | \( \tau = 0, \tau' = \pm 1, \epsilon = 1 \) |
|----------------|----------------|----------------|----------------|
| \(-\frac{1}{2t s_W^2}\) | \(-\frac{1}{2t s_W^2}\) | \(-\frac{1}{2t s_W^2}\) | \(-\frac{1}{2t s_W^2}\) |
| \(\cos \vartheta - \beta_X\) | \(-\cos \vartheta - 2\tau \lambda\) | \(\frac{s}{2M_X^2} [\cos \vartheta] - \beta_X (1 + \frac{2M_X^2}{s})\) | \(-2\beta_X\) |
| \(-\frac{1}{s} + \frac{2(a_x - 2b_x \lambda)}{t_W(s - M_Z^2)}\) | \(-\frac{1}{s} + \frac{2(a_x - 2b_x \lambda)}{t_W(s - M_Z^2)}\) | \(-\frac{1}{s} + \frac{2(a_x - 2b_x \lambda)}{t_W(s - M_Z^2)}\) | \(-\beta_X \frac{\sqrt{s}}{M_X}\) |

The gauge bosons contributions are the same as for the singly charged bilepton [33]-[35] with the couplings \( a, b, a', b' \) replaced by
\[ a_x = \frac{(-1 + 4s_W^2)^2}{4c_W^2 \sin 2\theta_W}, \quad b_x = \frac{(-1 + 4s_W^2)}{4c_W^2 \sin 2\theta_W}. \] (43)
Figure 4. Cross section $\sigma(e^+e^- \to X^{++}X^{--})$ in the minimal 3 - 3 - 1 model as a function of $M_X$.

\[
a'_x = \frac{3(-1 + 4s_W^2)}{4c_W^2 \sin 2\theta_W}, \quad b'_x = \frac{(1 - 4s_W^2)}{4c_W^2 \sin 2\theta_W}, \quad (44)
\]

It is can be verified that in the high energy limit $s \gg m_Z^2, M_{Z_2}^2$ the full amplitude vanishes.

The helicity amplitudes for the considered process are given in Table 4.

In figure 4 we plot $\sigma(e^+e^- \to X^{++}X^{--})$ as a function of $M_X$ mass.

Production of the bileptons of the minimal version at Hadron Collider was considered in [18]. One found that the vector bileptons of mass $M_Y \leq 1$ TeV could be observable at the LHC. Looking at the Table 1 we see that contributions from the $Z$ and the $Z_2$, due to the factor $(1 - 4s_W^2)$, are very small. This means that the similar processes in the model with RH neutrinos will be much bigger.

3.2. Production of bilepton in the model with RH neutrinos

The amplitude for the singly charged bilepton production are obtained from the amplitude in the minimal model after modifying coupling constants. The $Z_2e^+e^-$ vertex is modified to

\[
g_{Z_{2ee}}^{V} = -\frac{(1 - 4s_W^2)}{2\sqrt{3 - 4s_W^2}}, \quad g_{Z_{2ee}}^{A} = \frac{1}{2\sqrt{3 - 4s_W^2}}. \quad (45)
\]

The amplitudes are simply given in Table 3 replacing $a, b$ by $a_{rhn}, b_{rhn} with

\[
a_{rhn} = \frac{(-1 + 4s_W^2)}{4c_W^2 \tan 2\theta_W}, \quad b_{rhn} = -\frac{1}{4c_W^2 \tan 2\theta_W}, \quad (46)
\]
\[ a'_{rh} = \frac{(-1 + 4s_W^2)}{4c_W^2 \sin 2\theta_W}, \quad b'_{rh} = \frac{-1}{4c_W^2 \sin 2\theta_W}. \]  

Again, it is easy to verify that in the high energy limit the amplitude for longitudinal bileptons will tend to zero.

Figure 5. Cross section \( \sigma(e^+e^- \rightarrow Y^+Y^-) \) in the 3-3-1 model with RH neutrinos as a function of \( M_Y \).

Figure 5 shows the dependence of the total cross section \( \sigma(e^+e^- \rightarrow Y^+Y^-) \) in the 3-3-1 model with RH neutrinos as a function of \( M_Y \).

Next, we consider the production of neutral complex gauge boson in this model

\[ e^-(k, \lambda) + e^+(k', \lambda') \rightarrow X^0(p, \tau) + X^{0*}(p', \tau'). \]  

(48)
For this process we have not only the photons in the $s$ channel but also neutrino in $t$ channel (see Feynman diagram depicted in Fig. 6). The contributions from $Z$ and $Z^2$ to the amplitude are similar with those of the previous process after replacement of the corresponding mass. Helicity amplitudes of this process are given in Table 5.

Table 5

| Helicity Amplitude | $\tau = \tau' = \pm 1$ | $\tau = -\tau' = \pm 1$ | $\tau = \tau' = 0$ | $\tau = 0, \tau' = \pm 1, \epsilon = 1$ |
|---|---|---|---|---|
| $\beta_{Xo}$ | $0$ | $-\beta_{Xo} \left(1 + \frac{s}{2M_{Xo}}\right)$ | $-\beta_{Xo} \frac{s}{\sqrt{2}M_{Xo}}$ | 

where

$$a_{x_o} = \frac{(1 - 4s^2_W)}{4c^2_W \sin 2\theta_W}, \quad b_{x_o} = \frac{1}{4c^2_W \sin 2\theta_W},$$

$$a'_{x_o} = \frac{(-1 + 4s^2_W)}{4c^2_W \sin 2\theta_W}, \quad b'_{x_o} = \frac{-1}{4c^2_W \sin 2\theta_W},$$

Applying formula (120) in [27] we get the cross section for this process

$$\sigma(e^+ e^- \rightarrow X^o X^{*o}) = \frac{\pi \alpha^2}{6s} \pi^{2} \beta^2_X \left(\frac{4}{1 - \beta^2_X} - 1 - 3\beta^2_X\right) \Sigma_{X^o},$$

where

$$\Sigma_{X^o} = |LL| \frac{s^2(1 - 2s^2_W)^2}{4 \sin^2 2\theta_W} \left[\frac{1}{(s - m^2_Z)} - \frac{2}{(s - M^2_{Z^2})}\right]^2$$

$$+ |RR| \left[\frac{s^2 t^2_W}{4} \left[\frac{1}{(s - m^2_Z)} - \frac{1}{(s - M^2_{Z^2})}\right]^2\right],$$

and

$$|RR| = \frac{1 + P_+ + P_+ + P_- + P_-}{4}, \quad |LL| = \frac{1 - P_+ - P_+ + P_- + P_-}{4}.$$

In the limit $|P_{e^-}| = |P_{e^+}| = 1, s^2_W = 0.25$, Eq. (51) becomes:

$$\sigma(e^+_R e^-_R \rightarrow X^o X^{*o}) = \frac{\pi \alpha^2}{24} s \beta^2_X t^2_W \left(\frac{4}{1 - \beta^2_X} - 1 - 3\beta^2_X\right)$$

$$\times \left[\frac{1}{(s - m^2_Z)} - \frac{1}{(s - M^2_{Z^2})}\right]^2.$$
Figure 7. Cross section $\sigma(e^+e^\rightarrow X^0X^{0*})$ in the 3 - 3 - 1 model with RH neutrinos as a function of $M_X$.

The cross section for this process is plotted in Fig. 7. From Table 2 we see that the contribution from $Z_2$ is much bigger than that in the minimal version. As we can see from figures 2, 4, 5 and 7, when $M_{Z_2}$ is not too heavy the cross sections in the model with RH neutrinos can be one order bigger than that in the minimal version.

We ignored the questions connected with the experimental difficulties of identifying the neutral gauge bosons, which interact with neutrinos (and the exotic quarks) only.

4. Discussion and numerical results

From the helicity amplitudes in the previous section it is a simple task to compute the cross section for production of any pair of bileptons in a given helicity state. In figures 2 and 4, we show the total cross section for production of X and Y in the minimal model at 1 TeV. With planned colliders of luminosity $\mathcal{L} = 80fb^{-1}$, one expects several thousand events almost up to the kinematic limit. Note that the $Z_2$ coupling to $Y$ is proportional to $1 - 4\sin^2\theta$ so the pair production process is not very sensitive to the $Z_2$ exchange.

Doubly charged bileptons can also be produced singly in $e^+e^-$ via Weisaker-Williams photons and the subprocess $e\gamma \rightarrow Xe$. In this model where the bilepton coupling is of electromagnetic strength, the pair production is slightly larger in the mass range kinematically allowed. We note here that the coupling of the single charged bileptons are exactly the same as for the W except for the third generation where the $Y$ couples to the $b$ quark and an exotic up-type quark. Therefore if the exotic quark is heavier than the bileptons we have the decay width $\Gamma_Y = \frac{G_Fm_Y^3M_Y}{2\sqrt{2}}$. 
The total cross section for new gauge bosons pair production in the right-handed model are given in Fig. 5. The production cross section is sizeable, and at least for singly charged bileptons, a signal should easily be extracted from either the purely leptonic decay mode or the semileptonic mode. For the complex neutral bileptons, extracting a signal over the background is more troublesome as the leptonic decay mode is exclusively into neutrinos. As for the $Y$, the neutral bileptons couples to the first two generations of quarks and to a pair of top quark/exotic quark with equal strength.

Since the $Z_2$ turn out to be heavier than two bileptons it should be seen four jet final states that is unique signature of the considered minimal model. As expected, the linear collider sensitivity to $Z_2$ properties is best when running near the resonance $\sqrt{s} = M_{Z_2}$. With the planned machine parameters the considered processes should be seen unless the bilepton masses are bigger than 2 TeV.

Finally, it must be emphasized again that: when $M_{Z_2}$ is not too heavy the cross sections in the version with RH neutrinos can be one order bigger than the same in the minimal model.

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REFERENCES

1. S. L. Glashow, Nucl. Phys. 20 (1961) 579; A. Salam, in Elementary Particle Theory, ed. N. Svartholm, (1968); S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264.
2. For earlier 3 - 3 - 1 models see: J. Schechter and Y. Ueda, Phys. Rev. D 8 (1973) 484; H. Fritzsch and P. Minkowski, Phys. Lett. B 63 (1976) 99; P. Langacker and G. Segré, Phys. Rev. Lett. 39 (1977) 259; B. W. Lee and S. Weinberg, Phys. Rev. Lett. 38 (1977) 1237; G. Senjanovic, Nucl. Phys. B 136 (1978) 301; M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D 22 (1980) 738; J. W. F. Valle and M. Singer, Phys. Rev. D 28 (1983) 540.
3. F. Pisano and V. Pleitez, Phys. Rev. D 46 (1992) 410; R. Foot, O.F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47 (1993) 4158.
4. P. H. Frampton, Phys. Rev. Lett. 69, (1992) 2889.
5. R. Foot, H. N. Long and Tuan A. Tran, Phys. Rev. D 50 (1994) R34.
6. H. N. Long, Phys. Rev. D 54 (1996) 4691.
7. D. Ng, Phys. Rev. D 49 (1994) 4805.
8. G. Altarelli, Preprint CERN-TH/96-265 [hep-ph/9611239]. In St. Croix 1996, Tech-
niques and concepts of high-energy physics, IX, pp. 1 - 31;
S. Dawson, Preprint BNL - HET - SD - 95 - 5 [hep-ph/9609340]. In Minneapolis 1996, 
Particles and fields, vol. 1, pp. 129 - 136.
9. P. B. Pal, Phys. Rev. D 52 (1995) 1659.
10. P. Gambino and A. Sirlin, Phys. Rev. Lett. 73 (1994) 621;
Z. Hioki, Phys. Lett. B 340 (1994) 181;
S. Dittmaier et al, Nucl. Phys. B 426 (1994) 249; Nucl. Phys. B 446 (1995) 334.
11. See for example: J. Mnich, in: Proceedings of The European Conference on High 
Energy Physics, Tampere, 1999;
D. G. Charlton, Int. J. Mod. Phys. A 15, (2000) 352;
G. Altarelli, hep-ph/0011078.
12. K. J. F. Gaemers and G. J. Gounaris, Z. Phys. C1 (1979) 259.
13. K. Hagiwara, R. D. Pecei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B 282 (1987) 
253.
14. F. Boudjema and F. M. Renard, in $e^+e^-$ Collisions at 500 GeV: The physical potential,
ed. P. Zerwas, DESY, Hamburg 1992.
15. For recent reviews, see: R. Casalbuoni, S. De Curtis, and D. Guetta, Preprint 
DFF258/10/96 hep-ph/9610377, Phys. Rev. D 55 (1997) 4203;
Gounaris et al, in Physics at LEP2, CERN Yellow preprint CERN 96-01, Eds. G. 
Altarelli, T. Sjostrand and F. Zwirner, p.525.
16. G. Belanger and G. Couture, Phys. Rev. D 49 (1994) 5720.
17. T. Inami, C. S. Lim, B. Takeuchi and M. Tanabashi, Phys. Lett. B 381 (1996) 458.
18. B. Dion, T. Gregoire, D. London, L. Marleau and H. Nadeau, Phys. Rev. D 59 (1999) 
075006.
19. P. Frampton and A. Rasin, Phys. Lett. B 482 (2000) 129, hep-ph/0002133.
20. J. T. Liu and D. Ng, Z. Phys. C 62 (1994) 693;
N. A. Ky, H. N. Long and D. V. Soa, Phys. Lett. B 486 (2000) 140.
21. P. Frampton and M. Harada, Phys. Rev. D 58 (1998) 095013.
22. M. B. Tully and G. C. Joshi, Phys. Lett. B 466 (1999) 333.
23. H. N. Long and L. P. Trung, Phys. Lett. B 502 (2001) 63, hep-ph/0010204.
24. H. N. Long and T. Inami, Phys. Rev. D 61 (2000) 075002.
25. Particle Data Group, D. E. Groom, et al., Eur. Phys. J. C 15 (2000) 1.
26. T. G. Rizzo, Phys. Rev. D 45 (1992) 42; Phys. Rev. D 46 (1992) 910;
N. Lepore, B. Thorndyke, H. Nadeau and D. London, Phys. Rev. D 50 (1994) 2031
and references therein;
See also, J. Agrawal, P. Frampton and D. Ng, Nucl. Phys. B 386 (1992) 267.
27. F. Cuypers and S. Davidson, hep-ph/9609487, Eur. Phys. J. C 2 (1998) 503;
F. Cuypers and M. Raidal, Nucl. Phys. B 501 (1997) 3;
M. Raidal, Phys. Rev. D 57 (1998) 2013;
See also: Proceedings of 3rd Workshop on Electron-Electron Interactions at TeV 
energies, Ed. C. A. Heusch, Int. J. Mod. Phys. A 15 (2000) No. 16.
28. P. H. Frampton and D. Ng, Phys. Rev. D 45 (1992) 4240;
P. Frampton, Int. J. Mod. Phys. A 15 (2000) 2455.
29. E. D. Carlson and P. H. Frampton, Phys. Lett. B 283 (1992) 123;
H. Fujii, S. Nakamura and K. Sasaki, Phys. Lett. B 299 (1993) 342;
H. Fujii, Y. Mimura, K. Sasaki and T. Sasaki, Phys. Rev. D 49 (1994) 559.
30. L. Willman et al., Phys. Rev. Lett. 82 (1999) 49.
31. G. Gounaris, J. Layssac, G. Moultaka and F. M. Renard, Int. J. Mod. Phys. A 8 (1993) 3285;
M. Bilenky, J. L. Kneur, F. M. Renard and D. Schildknecht, Nucl. Phys. B 409 (1993) 22.