Effects of polarisation on study of anomalous $VVH$ interactions at a Linear Collider

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We investigate the use of beam polarisation as well as final state $\tau$ polarisation effects in probing the interaction of the Higgs boson with a pair of heavy vector bosons in the process $e^+e^-\rightarrow f\bar{f}H$, where $f$ is any light fermion. The sensitivity of the International Linear Collider (ILC) operating at $\sqrt{s}=500$ GeV, to such $VVH(V=W/Z)$ couplings is examined in a model independent way. The effects of ISR and beamstrahlung are discussed.

1 Introduction

The particle physics community hopes that the LHC will soon present it with the signal for the Higgs; but, it is to the ILC that we will have to turn to for establishing it as the SM Higgs boson through a precision measurement of its properties. The dominant channel of Higgs production at the ILC, viz. $e^+e^-\rightarrow f\bar{f}H$ where $f$ is any light fermion, proceeds via the $VVH$ interaction with $V=Z(W)$. The most general form of the $VVH$ vertex, consistent with Lorentz–invariance, can be written as:

$$\Gamma_{\mu\nu} = g_{V}^{SM} \left[ a_{V} g_{\mu\nu} + \frac{b_{V}}{m_{V}^{2}} (k_{1\nu}k_{2\mu} - g_{\mu\nu} k_{1\cdot}k_{2\cdot}) + \frac{\tilde{b}_{V}}{m_{V}^{2}} \epsilon_{\mu\nu\alpha\beta} k_{1\alpha}k_{2\beta} \right]$$

(1)

where $k_i$’s denote the momenta of the two $V$’s and, at the tree level in the SM, $a_{V} = 1$ and $b_{V} = \tilde{b}_{V} = 0$. In our analysis we assume $a_{V}$ to be real and retain terms up to linear order in other anomalous parts. In an effective theory, the general structure of $VVH$ coupling can be derived from dimension–six operators.

2 The Final State and Kinematical cuts

We choose to work with a Higgs boson of mass 120 GeV and consider its detection in the $b\bar{b}$ final state with a branching ratio 0.68. Furthermore, we assume the detection efficiency of $b$-quark to be 70%. We impose kinematical cuts designed to suppress dominant backgrounds. Cuts $R_{1}$ ($R_{2}$) on the invariant mass of the $ff$ system: $|m_{ff} - M_{Z}| \leq (\geq) 5\Gamma_{Z}$, can be used to enhance (suppress) the effect of the $s$–channel $Z$–exchange diagram.

Statistical fluctuations in the cross-section or in an asymmetry, for a given luminosity $\mathcal{L}$ and fractional systematic error $\epsilon$, can be written as:

$$\Delta\sigma = \sqrt{\sigma_{SM}/\mathcal{L} + \epsilon^{2}\sigma_{SM}^{2}}$$

and

$$(\Delta A)^{2} = \frac{1}{\sigma_{SM}\mathcal{L}} - \frac{\epsilon^{2}}{2}(1 - A_{SM}^{2})^{2}.\quad (2)$$

We demand that the contribution to the observable coming from the anomalous parts are less than the statistical fluctuation in these quantities at a chosen level of significance and
study the sensitivity of a LC to probe them. We choose $\epsilon = 0.01$, $\mathcal{L} = 500 \text{ fb}^{-1}$ and look for a 3$\sigma$ effect. Note that in the case of polarisation asymmetries the total luminosity of 500 fb$^{-1}$ is divided equally among different polarisation states.

### 3 ZZH couplings

We construct observables ($O_i$) whose behaviour (odd/even) under the discrete transformations $C$, $P$ and $\tilde{T}$ (the pseudo time reversal operator which reverses particle momenta and spins without interchanging initial and final states) is the same as that for a particular operator in the effective Lagrangian. This is achieved by taking the expectation values of signs of various combinations of measured quantities such as particle momenta and spins $C_i$'s, $i \neq 1$. Some of these combinations are listed in Table 1. The observables are cross-sections and various asymmetries with polarised beams and polarised final state $\tau$'s, which we discuss in the following sections and are also listed in the Table.

| ID | $C_i$ | $C$ | $P$ | $CP$ | $\tilde{T}$ | $CPT$ | Observable($O_i$) | Coupling |
|----|-------|-----|-----|------|-------------|-------|-------------------|----------|
| 1  |  $P_e \cdot \vec{p}_H$ | +   | +   | +   | +   | $A_{FB}$ | $A_{FB}$ | $a_2, R(b_z)$ |
| 2a |       | -   | -   | +   | -   | $A_{UD}$ | $A_{UD}$ | $\Im(b_z)$   |
| 2b | $(P_e \times \vec{p}_H) \cdot P_f$ | +   | -   | -   | +   | $A_{comb}$ | $A_{comb}$ | $\Re(b_z)$   |
| 2c | $[P_e \cdot \vec{p}_H] \cdot [(P_e \times \vec{p}_H) \cdot P_f]$ | -   | +   | -   | -   | $A^\prime_{comb}$ | $A^\prime_{comb}$ | $\Im(b_z), \Re(b_z)$ |
| 2d | $[P_e \cdot \vec{p}_f] \cdot [(P_e \times \vec{p}_H) \cdot P_f]$ | $\otimes$ | -   | $\otimes$ | -   | $\otimes$ | $\otimes$ | $\Im(b_z), \Re(b_z)$ |

Table 1: Various possible $C_i$'s, their discrete transformation properties, the anomalous couplings on which they provide information along with observables $O_i$. Symbol $\otimes$ indicates that the corresponding $C_i$'s do not have any definite transformation property under $CP$ or $\tilde{T}$. Here, $P_e \equiv \vec{p}_e - \vec{p}_e^\ast$ and $P_f \equiv \vec{p}_f - \vec{p}_f^\ast$ with $\vec{p}_e$ ($\vec{p}_e^\ast$) is momentum of initial state electron (positron) and analogously $\vec{p}_f$ ($\vec{p}_f^\ast$) is the momentum of final state fermions (anti-fermions).

#### 3.1 Use of Polarised Initial Beams

The preferentially axial coupling of the $Z$ boson with the charged leptons indicate that initial beam polarisation may affect our observables strongly. A similar statement also holds for the $W$-contribution to $\nu_e \bar{\nu}_e H$ production. In our study, we take $e^-/e^+$ beam polarisations to be 80% and 60% respectively and denote $P \equiv (-,+)$ for $P_e^- = -0.8$ and $P_e^+ = 0.6$. The forward-backward (FB) asymmetry in the production of the Higgs boson with respect to (w.r.t.) the $e^-$ direction ($O_{2a}$) is odd under $CP$, even under $\tilde{T}$ and hence can be used to probe $\Im(b_Z)$. The up-down (UD) asymmetry ($O_{2b}$) of the fermion w.r.t the H production plane, is odd under both $CP$ and $\tilde{T}$ and hence can constrain $\Re(b_Z)$. In Table 2 we list the limits of sensitivity on $\Im(b_Z)$ and $\Re(b_Z)$ possible with polarised beams for $E_{cm} = 500$ GeV. We compare these limits with those obtained using unpolarised beams [2]. It is clear from Table 2 that use of longitudinally polarised beams improves the limit of $\Re(b_Z)$ and $\Im(b_Z)$ by a factor of upto 5 or 6. This improvement can be traced to the circumvention of the vanishingly small vector coupling of electron to the Z boson. Our results agree with those.
of Ref. [3] if we remove the kinematical cuts as well as the use of finite b-tagging efficiency implemented in our analysis.

| Polarisated Beams | Unpolarised Beams |
|-------------------|-------------------|
| Limits | Observables used | Limits | Observables used |
| \(|R(\hat{t}_Z)| \leq 0.070 \) | \(O_{Z\mu\mu}^{F}, \) R1-cut; \(\mu^-\mu^+H\) final state | \(|R(\hat{t}_Z)| \leq 0.41 \) | \(A_{UD}, \) R1-cut; \(\mu^-\mu^+H\) final state |
| \(|\Im(\hat{b}_Z)| \leq 0.0079 \) | \(O_{Z\mu\mu}^{F}, \) R1-cut; \(\mu^-\mu^+H, \) \(q\bar{q}H\) final states | \(|\Im(\hat{b}_Z)| \leq 0.042 \) | \(A_{FB}, \) R1-cut; \(\mu^-\mu^+H, \) \(q\bar{q}H\) final states |

Table 2: Limits on anomalous ZZ'H couplings from various observables at 3σ level with polarised and unpolarised beams, for values of different parameters as listed in the text.

### 3.2 Use of Final state \(\tau\) Polarization

Since \(\tau\) polarization can be measured [4, 5, 6] using the decay \(\pi\) energy distribution, one can also construct observables using the final state \(\tau\) polarisation to probe ZZ'H couplings. To demonstrate this, we construct various asymmetries for a sample of (as an example) left handed \(\tau\) in the final state. Using the combination \(C_{2c}\) of Table 1 we construct a mixed polar-azimuthal asymmetry, given by 

\[
A_{\text{comb}} = \frac{(\sigma_{FU} - \sigma_{FD} - \sigma_{BU} + \sigma_{BD})}{\sigma}.
\]

Here \(\sigma\) is total cross section and \(\sigma_{FU}\) is the partial rate with \(H\) in the forward (F) hemi-sphere w.r.t. initial state \(e^-\) along with the \(\tau^-\) above (U) the \(H\) production plane etc. It probes \(\Im(b_Z)\). Similarly we use another combined asymmetry corresponding to combination \(C_{2d}\), defined as 

\[
A'_{\text{comb}} = \frac{(\sigma_{F'U} - \sigma_{F'D} - \sigma_{B'U} + \sigma_{B'D})}{\sigma},
\]

where \(F'\) (\(B'\)) corresponds to the production of \(\tau^-\) in forward (backward) hemi-sphere w.r.t. initial state electron. \(U, D\) have the same meaning as above. One may use this asymmetry to constrain both \(\Im(b_Z)\) and \(R(b_Z)\) simultaneously. The up-down (UD) azimuthal asymmetry for the \(\tau^-\) can probe \(R(b_Z)\).

The important issue of efficiency of obtaining a sample enriched with \(\tau^-\)'s with a particular (say negative) helicity, which we use in the analysis, is beyond the scope of discussion here. Table 3 lists the limits of sensitivity to different anomalous couplings, assuming the net effect of having to isolate a negative helicity \(\tau\), to be just a scaling of asymmetries by 40% and 25% respectively. We also compare these with the limits possible without the use of \(\tau\) polarisation information. The super-

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**Figure 1:** Region in \(R(b_Z) - \Im(b_Z)\) plane corresponding to the 3σ variation of asymmetries with an integrated luminosity of 500 fb\(^{-1}\), corresponding to 40% scaling of the asymmetries as mentioned in the text. The horizontal lines are for 3σ variation in \(A'_{\text{comb}}\), whereas the vertical lines are for the variation in \(A_{UD}^L\). The slant lines are corresponding to variation in \(A'_{\text{comb}}^L\).
Using Polarisation of final state $\tau$

| Coupling | Limits | Observables used | Limit | Observables used |
|----------|--------|------------------|-------|------------------|
| $|\Im(b_z)| \leq$ | 0.10 | $A_{1}^{L\_comb}$ | 0.23 | $A_{1\_comb}$ |
| $|\Re(b_z)| \leq$ | 0.18 | $A_{1}^{L\_UD}$ | 0.41 | $A_{1\_UD}$ |

Table 3: Limits on anomalous $ZZH$ couplings from various observables at $3\sigma$ level with/without using the information of final state $\tau$ polarisation.

scripts $L, 1$ in various asymmetries refer to the helicity of the $\tau$, use of $R1$ cut etc. Table 3 shows that the use of the $\tau$ polarisation can improve the sensitivity to $\Im(b_z)$. Ref. [7] had also pointed out similar improvements on using the $\tau$ polarisation in the context of optimal observable analysis. Figure 1 shows the region in $\Re(bZ) - \Im(bZ)$ plane that can be probed using the above mentioned asymmetries for $\tau$'s in negative helicity state, scaling them by 40% as mentioned earlier.

4 WWH couplings

We study the process $e^+e^- \rightarrow \nu\bar{\nu}H$ with longitudinally polarised beams to constrain the anomalous $WWH$ couplings. In this case, one can not use the momenta of $\nu$'s to construct any $\tilde{T}$-odd observables. We use polarised cross sections and FB-asymmetry w.r.t. polar angle of the Higgs boson to probe the anomalous parts of $WWH$ vertex. Keeping only one anomalous coupling to be nonzero at a time, we obtain individual limits of sensitivity on these couplings. The values for the same for $\tilde{T}$-odd $WWH$ couplings without/with beam polarisation are listed in Table 4. The simultaneous limits of sensitivity, obtained by letting all the anomalous couplings to be nonzero, for $\Im(bW)$ and $\Re(bW)$ with polarised and unpolarised beams are listed in Table 5. It may be noted from the limits given in Table 4 and 5 that although use of beam polarisation improves the sensitivity to $\Im(bW)$ and $\Re(bW)$ by upto a factor 2, there is little reduction in the contamination coming from the anomalous $ZZH$ couplings.

Table 4: Individual limits on anomalous $\tilde{T}$-odd $WWH$ couplings with polarised and unpolarised beams at $3\sigma$ level at an integrated luminosity of 500 $fb^{-1}$.

5 Sensitivity studies at higher c.m. energies.

The $s$ and $t$ channel behave differently with increasing energy. It is therefore interesting to study the energy dependence of the sensitivity of our observables to the anomalous couplings.
Table 5: Simultaneous limits on anomalous $\tilde{T}$-odd $WWH$ couplings with polarised and unpolarised beams at $3\sigma$ level at an integrated luminosity of 500 $fb^{-1}$.

| Coupling  | $3\sigma$ limit with Polarized Beams | $3\sigma$ limit with Unpolarized Beams |
|-----------|-------------------------------------|---------------------------------------|
| $|\Im(b_W)|$ | $0.71$ | $1.6$ |
| $|\Re(b_W)|$ | $1.7$ | $3.2$ |

We have also investigated the reach in sensitivity of CLIC to $VVH$ couplings at five different c.m. energies, namely at 0.5, 0.8, 1, 1.5 and 3 TeV. We found that going to higher energy can improve the sensitivity and best possible sensitivity, for example, for $\Re(b_Z)$ is obtained at $\sqrt{s} = 1$ TeV, with $R2$-cut. This improvement is upto a factor of 2 as compared to the analysis made earlier for an ILC operating at 500 GeV c.m. energy [2]. At higher energies, however, both the initial state radiation (ISR) effect as well as the effect of beamstrahlung which causes energy loss of the incoming electron (or positron) due to its interaction with the electromagnetic field of the opposite bunch, have to be further taken into account. Corrections coming from both are sizable and change the rates. For example, at 500 GeV, the ISR effects change the SM contributions by $\leq 15\%$ whereas the contribution coming from (say) $\Re(b_Z)$ changes by about 9%; with Beamstrahlung at (say) 1 TeV these effects are $\sim 10\%$ and $20\%$ respectively. However, the effect on the limits for sensitivity that may be obtained is less drastic as these affect both the SM as well as anomalous contribution similarly. At 1 TeV, for example, the above mentioned limit changes by 15%.

6 Summary

Thus we show that use of polarised initial beams can yield higher sensitivity to $\Re(b_Z)$, $\Im(b_Z)$ and to both the $\tilde{T}$-odd $WWH$ couplings. The limit on $\Im(b_Z)$ can be improved by a factor of 2 to 3 using $\tau$ polarisation as well, even with pessimistic assumptions on the efficiency of the polarisation measurement. We also study effect of increasing energy on the sensitivity. For example, at $\sqrt{s} = 1$ TeV one obtains an improvement by a factor 2, which further changes by about 15% due to ISR and Beamstrahlung effects.

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