Delay-Enhanced Synchronization in Two Coupled Chaotic Light-Emitting Diodes

Qing Qin*, J Zhang and T Guo
Tianjin Key Laboratory of Information Sensing & Intelligent Control, School of Automation and Electrical Engineering, Tianjin University of Technology and Education, Tianjin, 300222, China
Email: qing@tute.edu.cn

Abstract. In a light-emitting diode (LED) with ac-coupled nonlinear optoelectronic feedback, chaos can occur within a wide range of the voltage threshold parameter. When two identical such LEDs are coupled without delays, with increase of the coupling strength, transitions occur from anti-phase synchronization of periodic spiking to complete synchronization of chaos. Here we present that a coupling time-delay enhances chaos synchronization with small coupling strengths. The numerical simulations with bifurcation diagrams demonstrate the underlying mechanisms.

1. Introduction
Synchronization is a collective phenomenon emerging in coupled oscillatory systems in many physical, biological and engineering fields, which plays important roles in various functions [1]. For example, synchronization is vital in creating a high-power laser system by combining many low-power stable lasers [2]. Chaotic system is a dynamical system that its evolution very sensitive to the initial conditions, which implies that without coupling or external forcing, even two identical chaotic systems with slightly different initial conditions defy synchronization. Since the pioneer work of Pecora and Carrol on synchronization between two identical chaotic systems [3], chaos synchronization has been studied extensively in various systems [4].

Recently, complex dynamics including mixed mode oscillations and chaos have been observed in a GaAs light-emitting diode (LED) with ac-coupled nonlinear optoelectronic feedback [5]. The effects of noises on chaotic synchronization of two instantly coupled such LEDs have been studied [6]. While the information transmission in coupled systems is not instantaneous, coupling delay exists in general. The goal of this paper is to investigate the effects of coupling delays on the chaos synchronization between two bidirectional coupled LED systems. We present numerical bifurcation diagrams to demonstrate that a time-delay can enhance chaos synchronization even with a very small coupling strength.

2. LED System Model and Its Complex Dynamics
From the previous results [5], the LED system dynamics can be expressed in the following dimensionless form:
\begin{equation}
\begin{aligned}
\dot{x}_1 &= x_1(x_2 - 1) \\
\dot{x}_2 &= \gamma (\delta_0 - x_2 + \alpha(x_3 + x_t))/(1 + s(x_3 + x_t) - x_1x_2) \\
\dot{x}_3 &= -\varepsilon(x_3 + x_t)
\end{aligned}
\end{equation}

where \(\delta_0, \gamma, \varepsilon, \alpha, \) and \(s\) are system parameters. The states and the parameters are derived from a physical LED system (see [5] for details).

We have previously shown that the system passes through a cascade of period doubled and chaotic attractors by varying over a small interval (figure 1 in [7]). Figure 1 shows the time series of state \(x_1\) and the corresponding phase portrait on \(x_1 - x_2 - x_3\) space for different values of \(\delta_0\), the other parameters values are \(\gamma = 0.0033, \varepsilon = 0.0004, s = 0.2\) and \(\alpha = 1.002\). Figure 1a and 1b show chaotic dynamics with \(\delta_0 = 1.13\) and (c, d) are periodic oscillations with \(\delta_0 = 1.1\). Here we chose \(\delta_0 = 1.13\) such that the LED is in a chaotic spiking regime.

**Figure 1.** Dynamics of an individual LED system: (a) and (c) time series of state \(x_1\), (b) and (d) phase portraits on \(x_1 - x_2 - x_3\) space.

### 3. Chaos Synchronization of LED Systems

Two LED systems, both in chaotic spiking regime, bidirectionally coupled through the bias current are described as follows:

\begin{equation}
\begin{aligned}
\dot{x}_1 &= x_1(x_2 - 1) \\
\dot{x}_2 &= \gamma (\delta_0 - x_2 + \alpha(x_3 + x_t))/(1 + s(x_3 + x_t) - x_1x_2 + g_c(y_3(t - \tau) - x_2)) \\
\dot{x}_3 &= -\varepsilon(x_3 + x_t) \\
\dot{y}_1 &= \dot{y}_1 \\
\dot{y}_2 &= \gamma (\delta_0 - y_2 + \alpha(y_3 + y_t))/(1 + s(y_3 + y_t) - y_1y_2 + g_c(x_2(t - \tau) - y_2)) \\
\dot{y}_3 &= -\varepsilon(y_3 + y_t)
\end{aligned}
\end{equation}

where \(g_c\) and \(\tau\) are coupling strength and coupling delay, respectively.

Figure 2 shows the bifurcation diagram for the maximum values of \(x_1\) with changes of the coupling strength \(g_c\) when \(\tau = 0\) (figure 2a) and with changes of coupling delay \(\tau\) when \(g_c = 2\) (figure 2b). Figure 3 shows examples of dynamics of the coupled LED system with different coupling strength \((g_c = 0.2, 2\) and 10\). When there is no coupling delay \((\tau = 0)\), figures 3a-3c give the time series of the states \(x_1\) and \(y_1\). Figures 3d-3f show phase portraits of one LED in \(x_1 - x_2 - x_3\) space. Figures 3g-3i demonstrate the relationship between the two LEDs by phase portraits on \(x_1 - y_1\) planes. As shown in figure 2a, without coupling delay, when the coupling strength \(g_c\) is very small, the LED still show chaotic spiking oscillations, but they are not synchronous (figures 3a and 3g). As the coupling strength increases, the LED behaves as periodic oscillations in a very large range of \(g_c\), and the two LEDs exhibit in anti-phase synchronization (figures 3b and 3b). As the \(g_c\) becomes very big, the LED shows chaotic spiking dynamics again, and the two LEDs become synchronous (figures 3c and 3i). Thus, chaos synchronization can only be obtained with very big coupling strength.
The effects of coupling delays on the synchronization are illustrated in figures 2b and 4 with a fixed value of $g_c = 2$. As shown in figure 2b, with increases of tau, the LED system shows transitions from periodic oscillations to chaotic spiking to periodic oscillations through a series of bifurcations. As shown in figures 4a, 4f and 4k, if the coupling delay is not big enough ($\tau = 100$), it cannot change the anti-phase synchronization of periodic oscillations in the coupled LED systems. As further increase of $\tau$ beyond a critical value, the LED exhibits chaotic spiking oscillations and in-phase chaotic synchronization emerges (figures 4b, 4g and 4l, $\tau = 200$). With a further increase of tau, in-phase synchronization of mixed-mode periodic oscillations can be observed (figures 4c, 4h and 4m, $\tau = 250$). Complete chaos synchronization occurs at bigger values of $\tau$ (figures 4d, 4i, and 4n, $\tau = 350$), but if $\tau$ is too big, chaos synchronization will be replaced by complete synchronization of periodic spiking (figures 4e, 4j and 4o, $\tau = 500$).

**Figure 2.** Chaotic dynamics of the individual LED system: (a) time series of state $x_1$, (b) phase portrait on $x_1 - x_2 - x_3$ space.

**Figure 3.** Waveforms of states $x_1$ and $y_1$ without coupling delay and different coupling strength $g_c$: (a), (d), (g) $g_c = 0.2$, (b), (e), (h) and $g_c = 2$, (c) $g_c = 10$. 
Figure 4. Waveforms of states $x_1$ and $y_1$ with a fixed coupling strength $g_c = 2$ and different coupling delays: (a), (f), (k) $\tau = 100$, (b), (g), (l) $\tau = 200$, (c), (h), (m) $\tau = 250$, (d), (i), (n) $\tau = 350$ and (e), (j), (o) $\tau = 500$.

4. Conclusions
In this paper, we have investigated the effects of coupling delays on synchronization of two bidirectionally coupled LEDs. Without coupling delays, anti-phase synchronization of periodic spiking is observed in a large range of relatively small coupling strengths. Complete chaos synchronization only occurs with very big coupling strengths. However, chaos synchronization can be enhanced with proper coupling delays even for small coupling strengths, which helps to reduce energy consumption.

Acknowledgments
This work is supported by the Natural Science Foundation of Tianjin, China (Grant Nos. 18JCYBJC88200, 18JQNCJC74500, 18JYBJC43600 and 17JQNCJC03700), National Natural Science Foundation of China (Grant No. 61801328) and Tianjin Municipal Special Program of Talent Development for Excellent Youth Scholars (Grant No. TJJZJH-QNBJR-C2-21). We would also acknowledge the support of Tianjin University of Technology and Education (KYQD1810, KYQD1811).

References
[1] Pikovsky A, Rosenblum A, Kurths J 2001 *Synchronization: A Universal Concept in Nonlinear Science* (Cambridge: Cambridge University Press)
[2] Winful H G and Rahman L 1990 Phys Rev Lett. 65 1575
[3] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* 64 821
[4] Pecora L M and Carroll T L 2015 *Chaos* 25 097611
[5] Marino F, Ciszak M, Abdalah S F, Al-Naimee K, Meucci R and Arecchi F T 2011 *Phys. Rev. E* 047201
[6] Abdalah S F, Ciszak M, Marino F, Al-Naimee K, Meucci R, and Arecchi F T 2012 *IEEE Systems Journal*, 6 558
[7] Qin Y, Liu B, Wang X, Han C and Che Y 2015 *Proc. Int. Conf. on Material Science and Environmental Engineering (Wuhan)* (Boca Raton: CRC Press) p 609