Possibility of C/2002 CE10 in state of retrograde-polar resonance with Saturn in the future

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Abstract. Many objects in the Solar System which are associated or in resonance state with the giant planets have low orbital inclination. The effect of resonance that occurs on high-inclination objects is not well-known. This study describes results of numerical studies in term of orbital evolution of a high inclination Halley-Type Comet C/2002 CE10 (LINEAR). The ephemeris data was taken from JPL Small Body Database (https://ssd.jpl.nasa.gov/sbdb_query.cgix) at the epoch of MJD 58200. We report that C/2002 CE10 (LINEAR), whose inclination is 145°, will be in a retrograde 1:-1 (co-orbital - trisectrix type) resonance with Saturn for about 600 thousand years in the future. This resonance state is examined using the method FAIR (Fast Identification of Mean Motion Resonance). C/2002 CE10 (LINEAR) exhibits chaotic orbits and may not be stable in the resonance state. While data quality of the provided orbital elements is not quite good, orbital evolutions of clones of C/2002 CE10 (LINEAR) show small possibility that the object will be in the resonance state (retrograde co-orbital) with Saturn in the future.

1. Introduction

The presence of small bodies in mean motion resonance with a giant planet is a common feature in the Solar System. Most of the objects known to be resonance have small inclination \cite{2}. The effect of the resonance that occurs on high-inclination objects (more than 70°) is not well-known. But recently, it was found that mean motion resonance capture with giant planet such as Jupiter and Saturn occurs at large inclination. One of the objects is 2015 BZ509 that orbiting Jupiter with an inclination of 163° and in mean motion 1:1 with its planet. Since the inclination is high, the orbit of the object is retrograde. So, this object is in co-orbital retrograde-polar resonance with Jupiter \cite{3}.

In this paper, we report that Halley-types comet C/2002 CE10 has a possibility in state retrograde-polar resonance with Saturn. C/2002 CE10 is an object in a retrograde elliptical orbit whose inclination is 145.46° orbiting Saturn. This object appears to be inactive comet since there is no coma and only very weak tail was detected during the past perihelion passage \cite{4}. C/2002 CE10 will be in a retrograde co-orbital 1:1 resonance with Saturn. This state is examined using the method FAIR (Fast Identification of Mean Motion Resonance) as described in the following section.
2. Polar Resonance and Retrograde Motion
Mean motion resonance is a dynamical condition when the ratio of the orbital periods of two objects can be expressed as the ratio of two small integers \((j:k)\). If a small body in a mean motion with giant planet and its orbital inclination is high, that mean motion resonance is known as polar resonance. Since the orbital inclination is high, the small bodies orbiting the giant planet are in retrograde motion, and thus the resonance is polar-retrograde resonance. Retrograde motion is clockwise revolution of bodies around the Sun when seen from above the north ecliptic pole of the Solar System [5]. Since this motion is rare, the presence of the object being in state retrograde-polar resonance is also rare. Because the trend of high inclination objects are scattered or jump out of the system.

To study the resonance of the object, we have to develop a disturbing function for nearly polar orbits. The disturbing function is a series expansion of the gravitational interaction of two bodies that revolve around the Sun. The polar disturbing function informs that the \((j:k)\) inclination resonances of arguments are

\[
\theta'_{jk} = k\lambda - j\lambda_p + (j - k)\Omega - l\omega,
\]

where \(\Omega\) and \(\omega\) are the small body’s longitude of ascending node and argument of pericentre, \(\lambda\) and \(\lambda_p\) are the small body and planet’s mean longitudes. The integer \(l\) is even if \((j-k)\) is even and odd if \((j-k)\) is odd. If an object is in resonance state with a giant planet, then the argument of its resonant angle will be varied in a particular value. Since the object is not stable in its orbit through the high inclination orbit, then the resonance occurs just in a few times (temporal resonance).

3. Data and Method
3.1. Data
The ephemeris data was taken from JPL Small Body Database (https://ssd.jpl.nasa.gov/sbdb_query.cgi) at epoch of MJD 58200. The data quality of the provided orbital elements of this object is not quite good (\(U\) parameter > 4), so we generate orbital clones of the object by varying its orbital parameters. The orbital parameters used are semimajor axis, eccentricity, inclination, longitude of ascending node, argument of perihelion, and mean anomaly. There are two types of the orbital clones, the first orbital clones set are simple variation for each orbital elements so the number of the orbital clones are 12. The second set is by using all the possible combination of the orbital elements and give us 729 orbital clones. This orbital clones then integrated by SWIFT integrator for about 1 million years. The SWIFT integration technique is Regularized Mixed Variable Symplectic (RMVS) and use an assumption that the gravitational perturbation is just from giant planets. From the integrated orbital clones, we know that there is a possibility that the object will be in the resonance state (retrograde co-orbital) with Saturn in the future.

3.2. Method
An efficient method to identify the mean motion resonance is described and introduced as FAIR method (Fast Identification of Mean Motion Resonance) [1]. The efficiency of this method is easily find the mean motion resonance without any a priori knowledge on them. To find the mean motion resonance by this method is to plot the resonance variables to each other. The resonance variables are listed in Table 1.

| Resonance Variables | Description |
|---------------------|-------------|
| \(\lambda - \lambda_p\) | \(|\theta'_{jk} - M|\) for Inner co-orbital |
| \(\lambda - \lambda_p\) | \(|\theta'_{jk} - M + \omega|\) for Outer co-orbital |

The resonance variables listed in Table 1 above are headed for eccentric objects (low inclination but high eccentricity orbit objects). Since C/2002 CE10 has high inclination orbit, the resonance variables are modified by replacing \(M\) to \(M + \omega\), and \(\omega\) to \(\Omega\), and the rest is the same. By plotting \(\lambda - \lambda_p\) versus \(M\), or \(\lambda - \lambda_p\) versus \(M + \omega\) depending on the type (Inner or
Table 1. Resonance variables. The third column shows the variables to be plotted versus each other, the fourth and fifth columns show the number of intersecting stripes on the horizontal and vertical axes.

| Type  | Resonant Angle | Plot               | Ver | Hor |
|-------|----------------|--------------------|-----|-----|
| Inner | $j\lambda_p - k\lambda - (j - k)\overline{\omega}$ | $\lambda_p - \lambda$ versus $M$ | $(j - k)$ | $j$ |
| Inner | $j\lambda_p - k\lambda - (j - k)\overline{\omega}_p$ | $\lambda_p - \lambda$ versus $M_p$ | $(j - k)$ | $j$ |
| Outer | $j\lambda - k\lambda_p - (j - k)\overline{\omega}$ | $\lambda - \lambda_p$ versus $M$ | $(j - k)$ | $k$ |
| Outer | $j\lambda - k\lambda_p - (j - k)\overline{\omega}_p$ | $\lambda - \lambda_p$ versus $M_p$ | $(j - k)$ | $j$ |

Figure 1. C/2002 CE10 in outer resonance with Saturn, upper panel for inclination types object, lower panel for eccentricity types.

Outer resonance respect to the planet), the counting of the number of intersecting stripes with the horizontal and vertical axis give the value of the integer resonance ($j$ and $k$). If the integer resonance is known, then substitute it in the resonant angle argument listed above, we can derive the resonant angle which is varied in a particular value by plotting the resonant angle versus time.

4. Result and Analysis
Plot $\lambda - \lambda_p$ versus $M$ for C/2002 CE10 in outer resonance with Saturn is presented in left panel of Figure 1. This panel is one periodic so the stripes on 0.0 and 1.0 are the same. Use FAIR method (Table 1) to identify the mean motion resonance of C/2002 CE10, the number of intersecting stripes with the horizontal and vertical axis based on the Figure 1 is 1 and 0. Since C/2002 CE10 is in outer resonance with Saturn, then the number of $k$ is 1 and the number of $(j-k)$ is 0. Then we can conclude that C/2002 CE10 is in 1:-1 co-orbital retrograde resonance with Saturn. The variation of its resonance angle are relatively stable around 0° for about 600 thousand years based on the right panel of this Figure.
Figure 2. C/2002 CE10 has trisectrix type of motion orbit. Left panel presented the orbital parameters for small segment time, the right panel presented the Cartesian coordinate of the orbit. The Sun and Saturn presented as yellow and blue dot are in positions (0,0) and (10,0), respectively.

Figure 2 presents C/2002 CE10 trisectrix type of co-orbital 1:-1 motion around the Sun perpendicular to the Saturn orbit (right panel). The top left panel is relative distance object to Saturn, and the bottom left panel is resonant angle on the relative distance. This resonance is not stable due to the high inclination orbit of the object and the gravitational perturbation of the Saturn. For this segment of time, the temporal resonance is co-orbital retrograde and its resonant angle varied around 60°.

Based on the orbital clones examination, from the first type, there are 5 orbital clones from 12 that known to be resonance and has trisectrix motion type for about 400 thousand years. So the probability is about 40%, but this number is not accurate since the number of the orbital clones are not representative. The variation for each orbital parameter does not influence another parameter. The more accurate orbital clones set are the second set, but from 729 orbital clones that have been checked up to 800 thousand years, the results are as follows; there are 4 out of 729 that have very long 1:-1 resonant and the probability is about 0.5%, 1.4% long resonant 1:-1 for about 400 thousand years, and 6.1% short resonant 1:-1 for some ten thousand years. There are 2.8% objects that bounded in another resonance but the exact resonance ratio is still unknown and 13.4% objects are discarded.
Figure 3. Two examples of the orbital clones of C/2002 CE10 in trisectrix orbit motion. The upper figure is clone number 10 and the bottom figure is clone number 6. The explanation of this figure are the same as Figure 2.

5. Conclusion
The probability of object C/2002 CE10 in retrograde-polar resonance with Saturn is small. From the orbital clones that have been checked, just 4 out of 729 orbital clones that have very long resonant 1:-1 along 800 thousand years. We just only have less than 1% with very long resonance, and the others also very limited even just in some thousand years. This small probability due to the data quality of the provided orbital elements that not quite good, so the uncertainties are remain large. The resonance still occurs but in a short time (temporal resonance) due to the high inclination and retrograde orbit. The type of the object motion around the Sun is retrograde trisectrix (co-orbital) of Saturn orbit.
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