**Multidisciplinary design optimization on the splitter blade of high head Francis turbine**

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**Abstract.** Hydraulic and structural performances are two main subjects in the optimization design of hydraulic machines. This article gives consideration to both hydraulic and structural performances during the optimization process of hydraulic turbine with main and splitter blades on the basis of Multidisciplinary Design Optimization (MDO). The aim is to improve the optimization methods of hydraulic turbine, shorten the development cycle time and ensure stable and efficient operation of the unit. Computational fluid dynamics (CFD) and finite element method (FEM) analysis are combined to improve the overall performance of the runner in the multidisciplinary optimization process. The optimization considers the whole runner including the crown and band to get more accurate results of stress distribution. Open Grips program was used to parameterize runner blade and Bezier curve was used to match the bone line and it changed the blades through transforming the bone lines of different profile. NSGA-II algorithm was adopted during the optimization calculation. What’s more, super transfer approximation method was used to obtain the weighted coefficients considering the overall performance under three operating conditions, based on this, the final optimization result was obtained. The whole performance of optimized runner gets improved and has a better outflow quality. Also, it provides a feasible method for engineering application in the multidisciplinary optimization design of runner blades.

1. **Introduction**

Efficiency, cavitation and stability are three important indexes to estimate hydraulic performance and structure performance of hydraulic turbine. The optimization of Francis turbine runner will inevitably involve the two disciplines which include fluid mechanics and solid mechanics. In order to balance the influence of hydraulic performance and structure performance, it needs to adopt a multidisciplinary design optimization (MDO) method to perform the optimization problem of Francis turbine runner comprehensively.

In the last decades, many investigations are devoted to the impeller machinery by experiments and computational fluid dynamics (CFD) technology. Xi [1] adopted the response surface method for the multidisciplinary optimization of centrifugal impeller, experimental design technique and response surface method were combined with 3D numerical simulation of the centrifugal impeller. Pan [2] studied the optimization design of a ship according to the conceptual design, and established related optimization model considering such aspects as the quickness and the total longitudinal strength. Based on the NSGA-II algorithm, Guo [3-5] studied the multidisciplinary optimization of the blade of the hydraulic turbine, the activity guide and the blade of the turbine, and established a multidisciplinary optimization design framework. Lyutov [6] studied the single runner and coupled runner-
draft tube (DT) optimization, it is shown that optimized runner-DT geometry outperforms the result of single runner optimization by about 0.3% in terms of average efficiency. Pilev I M\(^7\) chosen the draft tube pulsation criteria as the optimal object during the process of the multi-objective optimal design of runner blade.

The high head Francis turbine has the characteristics of high water head, high rated speed and high flow velocity, so the water flow has a great impact on the blades. In this paper, based on the NSGA-II genetic algorithm, the MDO of a high head Francis turbine is studied, and the optimal geometry is selected through the super transfer approximation method \(^8-9\), it can consider the overall performance under multiple operating points.

2. Multi-objective optimization system

2.1. Parameterized technique of runner blade

Table 1 and table 2 shows the model turbine parameters studied in this paper\(^10\), in order to facilitate comparison, the three operating points-part load, best efficiency point and high load are named opt1, opt2 and opt3 respectively.

| Runner diameter D2/(mm) | Number of stay vane | Number of guide vane | Main blade | Splitter blade |
|-------------------------|---------------------|----------------------|------------|---------------|
| 349                     | 14                  | 28                   | 15         | 15            |

| Operating points | opt1 | opt2 | opt3 |
|------------------|------|------|------|
| Guide vane opening (°) | 3.91 | 9.84 | 12.44 |
| Experiment head | 12.29 | 11.91 | 11.84 |
| Experiment efficiency (%) | 71.7 | 92.6 | 90.6 |
| Rotate speed (r/s) | 6.77 | 5.59 | 6.16 |
| Q(m\(^3\)/s) | 0.07 | 0.20 | 0.22 |
| n_p,(r/s) | 0.22 | 0.18 | 0.20 |
| q_{ef}(m\(^3\)/s) | 0.05 | 0.15 | 0.17 |

In this paper, the optimization is carried out on the basis of modification of the blade, which requires parameterization of the runner blade and a series of parameters to control and express the geometry we want to study. In order to ensure that the reshaped blades are matched with the runner crown and band, the three-dimensional geometry of the runner blade is characterized with eight sections of the profile which were shown in figure 1.

![Figure 1. Parameterization of runner blade](image-url)
Based on the OpenGrip program language, three-dimensional blade profile was converted to two-dimensional profile. A series of inscribed circle in the two-dimensional profile was paint, and then on the basis of the inscribed circle, bone line was done, profile bone line was fitting by the Bezier curve. Its schematic diagram is shown in figure 1 (b). These parameters are treated as dependent variables and can be expressed by equations (1-3):

\[ X_1 = \frac{P_0 P_1}{P_0 Q}, \quad X_2 = \frac{P_2 P_3}{P_3 Q} \]  \hspace{1cm} (1)

\[ \bar{P}_1 = \bar{P}_0 + X_1 (\bar{Q} - \bar{P}_0) \]  \hspace{1cm} (2)

\[ \bar{P}_2 = \bar{P}_3 - X_2 (\bar{P}_3 - \bar{Q}) \quad (X_1, \ X_2 \in [0,1]) \]  \hspace{1cm} (3)

It can control the deflection of both ends of the profile bone line by changing the x1 and x2, which can change the angle of attack at both ends of the profile, and realize the overall change of the bone line. The blades are composed of eight sections of the profile, each of which is controlled by the two deflection variables, so that the design parameters are \( x_i \) (\( i = 1, 2, \cdots, 16 \)) respectively. The thickness variable is \( T_{\text{max}} \) and only the maximum thickness varies in the optimization process, and the other parts are changed according to the proportion. The head offset variable is \( P_0 \). To reduce variable and keep the blade smooth, eight profiles were selected the same head offset and the maximum thickness, each profile relative thickness changes proportionately. Totally, there are 18 parameters to parameterize the runner blade.

2.2. Performance calculation and target selection

In this paper, multidisciplinary optimization design is carried out for the performance of the selected high-water head mixed wheel in small flow conditions, optimal working conditions and large flow conditions. In the optimization process, the individual performance evaluation should be carried out. This paper focuses on the improvement of hydraulic performance and structure performance of hydraulic turbine.

2.2.1. Hydraulic performance calculation. In this paper, in order to save the calculation, cost and increase the calculation speed, the computation domain is show in figure 2. In order to ensure the accuracy of numerical calculation, a structured grid is used to discrete the computation domain, and the same topology is used in each optimization process. The grid is shown in figure 3.

The inlet boundary of calculation domain was given mass flow, the outlet boundary was given static pressure 0 Pa, the reference pressure is one atmosphere, and the wall surface is treated without slip boundary. The Maximum residual is less than \( 10^{-3} \). This paper adopts general grid interface(GGI) grid splicing technique on the static interface. The shear stress transport(SST) model was adopted and the convection of flow field adopts high resolution.

Figure 2. Signal period geometry of runner
This paper focuses on the hydraulic efficiency, and analyses its cavitation performance with the lowest static pressure on the runner blades. Therefore, the total head is selected as the constraint condition, efficiency and the minimum static pressure of the blade as the target of hydraulic performance.

![Runner mesh](image1)

(a) Runner mesh

![DT mesh](image2)

(b) DT mesh

**Figure 3. Mesh of single flow passage**

In order to ensure the accuracy of flow field calculation, based on the mesh independence test, the mesh scheme is shown in figure 4 and the final mesh has 4.34x10⁵ nodes.

![Analysis of grid independence](image3)

**Figure 4. Analysis of grid independence**

![FEA mesh of runner and load condition](image4)

**Figure 5. FEA mesh of runner and load condition**

### 2.2.2. Structure performance calculation

In this paper, finite element calculation was used to analyse the structural stress of each individual in the optimization process. In order to get closer to reality, this paper carries on the structural stress analysis with the model of the runner crown and the band, and the tetrahedral structure element is adopted because the geometry is more complex. The grid cell type adopts 10 nodes solid187, and the number of grid nodes is 28.54 million which is shown in figure 5.

The runner materials are stainless steel, elastic modulus and poison ratio are respectively 201GPa and 0.3, and the material mass density is 7900kg/m³. In the process of finite element calculation, the runner crown coupling shaft with fixed displacement constraint, band under the effect of gravity and centrifugal force. In the optimization process, the maximum static stress is chosen as the target of the structural performance.

In summary, the optimization problem can be expressed as follows:
variable: \( X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, P_0, T_{\text{max}}) \)

objective: \( \min(W_p), \max(\text{Stress}), \max(\text{Efficiency}) \)

\[
\text{constraint} : H \leq \delta
\]

Where \( \delta = \text{head}_{\text{int}} - \text{head}_{\text{opt}} = 0.05 \)

2.3 Optimize process and algorithm

In this paper, the optimization of the splitter blades takes into account both the hydraulic performance and the structural performance \(^{11-12}\). Based on this, multidisciplinary feasible (MDF) optimization strategy was chosen. The design variables and optimization objectives of all disciplines are included in this strategy.

The flow chart is shown in figure 6. The optimization system is mainly composed of fluid domain and solid domain, this flow chart mainly involves the transfer of data between the fluid and the solid which was deal with FORTRAN program during the process of optimization. And NSGA-II algorithm was adopted, because it has a more reasonable allocation method to improve the search efficiency and computational cost is small. Also, maintaining the diversity of the population and reducing the loss of excellent individuals are effective.

![Flow Chart](image)

**Figure 6.** The flow chart of multi-disciplinary optimization

3. Analysis of examples

After parameterization, the initial value of each variable is show in table 3. In the process of optimization, the population number of the NSGA-II \(^{13-14}\) algorithm is 20, the evolutionary algebra is 40, the crossover variation probability is 0.8, and the change scope of the design variable is controlled by 20%. Total 800 steps were performed to obtain the distribution of Pareto-front solutions based on the NSGA-II algorithm, and the optimization solution was chosen as the final solution according to the improvement degree. The optimized design parameters are shown in table 3.

Figure 7 shows that the solution space distribution of the optimization results is better than the initial value, among these three operating points, it can be seen that the solution distribution of high load is better than the other two points and each target is in the upper left corner area basically. In order to visualize the distribution of the optimization solution, taking the opt2 as an example, figure 7(d) shows the 3D solution space distribution and the Pareto-front. And it can be seen that the optimization effect is obvious.
Considering the performance of geometry synthetically, the value of the optimized geometry of three operating points are shown in Figure 8. What’s more, three operation points are weighted by the super transfer approximation method. To facilitate research, it can be assumed that the importance of the opt2 is six times than the opt1, as well as the importance of the opt2 is 4 times than the opt3, the importance of the opt3 is 2 times than the opt1. The main steps to determine the weight factors are as follows: (1) Generate the binary comparison matrix; (2) Generate the complementary matrix; (3) Constructed the super transfer approximation matrix; (4) The weighted coefficients was obtained according to the characteristic vector method. At last, the weighted coefficients of the opt(1-3) are 0.1722, 0.5890 and 0.2388 respectively. So the three weighted objective functions are $e = 0.1722* e1 + 0.5890* e2 + 0.2388* e3$ , $p = 0.1722* p1 + 0.5890* p2 + 0.2388* p3$ , $s = 0.1722* s1 + 0.5890* s2 + 0.2388* s3$. Among them, $e$ is the efficiency; $p$ is the minimum static pressure on blade surface, $s$ is the maximum static stress value of the runner.

**Table 3.** Variation range and value comparison of optimization design variables

| Variable | Upper bound | Lower bound | Initial value | Opt1     | Opt2     | Opt3     |
|----------|-------------|-------------|---------------|----------|----------|----------|
| $X_1$    | 0.62        | 0.52        | 0.57          | 0.597    | 0.591    | 0.529    |
| $X_2$    | 0.83        | 0.73        | 0.78          | 0.817    | 0.735    | 0.731    |
| $X_3$    | 0.47        | 0.37        | 0.42          | 0.393    | 0.389    | 0.415    |
| $X_4$    | 0.45        | 0.35        | 0.41          | 0.366    | 0.351    | 0.37     |
| $X_5$    | 0.53        | 0.43        | 0.48          | 0.52     | 0.479    | 0.466    |
| Variable | Upper bound | Lower bound | Initial value | Opt1 | Opt2 | Opt3 |
|----------|-------------|-------------|---------------|------|------|------|
| X_6      | 0.45        | 0.35        | 0.40          | 0.421| 0.37 | 0.352|
| X_7      | 0.59        | 0.49        | 0.54          | 0.556| 0.5  | 0.538|
| X_8      | 0.58        | 0.48        | 0.53          | 0.569| 0.51 | 0.517|
| X_9      | 0.60        | 0.50        | 0.56          | 0.53 | 0.53 | 0.586|
| X_{10}   | 0.55        | 0.45        | 0.50          | 0.527| 0.531| 0.524|
| X_{11}   | 0.55        | 0.45        | 0.50          | 0.483| 0.47 | 0.480|
| X_{12}   | 0.51        | 0.41        | 0.46          | 0.428| 0.493| 0.430|
| X_{13}   | 0.58        | 0.48        | 0.53          | 0.528| 0.562| 0.514|
| X_{14}   | 0.54        | 0.44        | 0.49          | 0.441| 0.517| 0.452|
| X_{15}   | 0.54        | 0.44        | 0.49          | 0.484| 0.534| 0.470|
| X_{16}   | 0.48        | 0.38        | 0.43          | 0.467| 0.478| 0.446|
| T_{max}  | 1.20        | 0.80        | 1.00          | 0.987| 0.902| 1.094|
| P_{0}    | 0.20        | -0.20       | 0.00          | 0.150| 0.138| -0.193|

(a) Efficiency value  
(b) Minpressure value  
(c) Maxstress value

**Figure 8.** The objective functions in three operating points

(a) Efficiency value  
(b) Minpressure value  
(c) Maxstress value

**Figure 9.** The weighted objective functions in three operating points

Figure 9 shows that the average efficiency and the water pressure of the optimized geometry 3 are increased most, and the maximum static stress is decreased obviously. This indicates that the hydraulic performance and structural performance of the runner blade has been improved. So, the geometry 3 is chosen as the final optimized geometry. Figure 10 shows the comparison of geometry before optimization and after.

Figure 11 shows the pressure distribution on main blade and splitter blade, it can be seen that the pressure distribution of blade surface before and after optimization is basically consistent.
more, before and after the optimization, the static stress is concentrated on the joint of blade and crown from figure 12, biggest static stress is in the suction surface of splitter blade which is close to the crown, the maximum static stress value is reduced clearly than the initial.

![Initial vs Optimized](image)

**(a) Splitter blade comparison**  
**(b) Section profile comparison**

**Figure 10.** The comparison of geometry

![Pressure Distribution](image)

**(a) The pressure distribution of main blade**  
**(b) The pressure distribution of splitter blade**

**Figure 11.** The comparison of pressure distribution

![Stress Distribution](image)

**Figure 12.** The comparison of stress distribution

### 4. Conclusions

Based on the NSGA-II genetic algorithm, the multi-discipline optimization design of a high head Francis turbine is studied, and the optimal geometry is selected through the super transfer approximation method, and the conclusions are as follows:

The efficiency of the optimized runner in three operating points were improved, and the efficiency of the part load increased by a maximum of 0.5626%. The minimum static pressure value of the blade has been improved in three operating points. The stress distribution of the runner is basically the same as the initial runner, and the maximum static stress is significantly lower than the initial value. The optimization effect is obvious. It is proved that the design method of the multi-discipline optimization of the splitter blade based on NSGA-II is feasible and effective.

This paper established a multidisciplinary optimization design system for high head Francis turbine, and it provides an effective way to ensure the overall performance of hydraulic machinery. The super transfer approximation method is also helpful for the multi-point optimization design of hydraulic turbine.
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