Finite element Analysis of Large Span Continuous Two-Way Ribbed Slabs with Some Parametric Studies

Ayad Abdulhameed Sulaibi1 Dhifaf Natiq H. Al-Amiery2

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Abstract

This paper investigates the results of finite element analysis for three proposed full-scale two-way slabs. The aim of this study is to use finite element method (FEM) by using ANSYS-v15 program to analyze the proposed slabs and study the flexural behavior, especially load-deflection relationship and ultimate strength. Some parametric studies on these works are also done to cover the effect of some important parameters on the ultimate load capacity and deflection. Proposed slabs are divided into three groups with different dimensions to study the effect of using continuous large spans on the structural behavior of two-way ribbed (waffle) slabs as compared to solid slabs. In all three groups, each slab consists of three by three panels supported by concrete columns at corners. For the first group, when the void ratio (the ratio of volume of voids between ribs to total volume of ribbed slab) increases, the stiffness of waffle slab also increases. Increasing stiffness for waffle slab is continued up to some limit, and then will decrease with increasing void ratio. The best case in this example occurs when the void ratio equal to (0.667) which gives increase in stiffness of (0.347) as compared to solid slab with the same thickness. The results of ANSYS analysis shows that the best percentage of increase in deflection is (51%) with decreasing in concrete volume of (59%) for long to short span ratio of (1.5) and 4(300)mm thickness. For the third group of proposed models, the stiffness of two-way ribbed (waffle) slab is higher than the solid slab which has the same volume of concrete. The displacement of two-way ribbed (waffle) slab in the elastic range (at first crack) is lower than the solid slab. In this manner, it will give the maximum reduction in concrete weight with higher thickness.

Keywords: Ribbed slab; waffle slab; finite element analysis; ANSYS.


table: التحليل بالعناصر المحددة للبلاتات الممصبة باتجاهين ذات الفضاءات الكبيرة المستمرة مع بعض الدراسات البارامترية

الخلاصة:

بحث هذه الورقة في تأثير تحليل العناصر المحددة للثلاث بلاطات مصحبة باتجاهين ذات فضاءات واسعة مترحة. الهدف من هذه الدراسة هو استخدام طريقة العناصر المحددة (FEM) باستخدام برنامج ANSYS-v15 لتحليل السطوف المترحة ودراسة سلوك الاجسام. خاصة علاقه الخروج بالتحمل والقوة الناتيه. يتم إجراء بعض الدراسات البارامترية حول هذه السطوف لمعرفة تأثير بعض المتغيرات المهمة على السعة التقصي للأخراج. تقسم البلاتات المترحة إلى ثلاث مجموعات ذات أبعاد مختلفة لدراسة تأثير استخدام فضاءات كبيرة مستمرة على السلاسل الهيكلي للبلاتات المصحبة باتجاهين مترحة بالبلاتات الهيكلية. في كل

1 Asst. Prof., PhD Civil Engineering, University of Anbar, Iraq, Email: ayadsulaibi@uoanbar.edu.iq.
2 MSc Student, Civil Engineering, University of Anbar, Iraq, Email: dufafalamiery@gmail.com.
1. Introduction

Two-Way Ribbed slab system can be defined as the slab constructions having a flat flange plate, or deck, and equally spaced parallel beams in two orthogonal direction, or grillage. The main purpose of using two-way ribbed slabs is to reduce the quantity of concrete and reinforcement are decreases. Some of previous studies on analysis and design of two-way ribbed (waffle) slabs will be presented here.

Kennedy (1983) tested three specimens of reinforced concrete waffle slab to study the effect of rib orientation on the carrying capacity of waffle slab. The specimens were different in the shape and construction method, but having the same volume of concrete and the same area of reinforcing steel bars. It was concluded from the experimental results that the shape and method of construction for reinforced concrete slab affected the ultimate load capacity and stiffness. Abdul-Wahab & Khalil (2000)[2] used experimental study and theoretical analysis to discuss the effect of rib spacing and the depth of rib on the flexural rigidity resistance for waffle slabs, and compared between the results of different models. In the experimental work, six specimens of square panels of ribbed flat slabs in 1: 4 scale and two solid flat slabs had been tested. To study the effect of the bending and torsion the slabs were considered isotropic in shape and reinforced in two perpendicular directions, so that the resistant moments were identical in both directions. The test specimen was simply supported along the four edges and its dimensions were (1540 * 1540) mm. It was concluded that increasing the number of ribs, or decreasing their spacing, stiffness of waffle slab was increased and the deflection in elastic uncracked range was decreased. In 2009, Hájek et.al [3] studied the effect of using high performance fiber concrete on the top slab in waffle slab structures, and compared between the results of different models. In this research, 11 various series were tested. The specimens are differed in types of fibers and concrete mixture used. They were subjected to different combinations of flexural and torsion loads. Test results showed higher shear and torsion capacity with using fibre concrete. Therefore, steel fibers can be placed instead of conventional shear reinforcement. Ibrahim (2014) [4] focused on analysis of two-way ribbed slabs with hidden beams. From the obtained results, the researcher concluded that the distribution of moments in two-way slabs with hidden beams was similar to the distribution of moments in slabs without beams if the stiffness of the hidden beams was small. In addition, using of three dimensional modelling by computer software provides a good solution for moment’s determination and distribution. Lau & Clark (2007)[5] tested 20 models consisting major wide beams that are much wider than the supporting columns, wide beams are formed in the two orthogonal directions, while the ribs between beams in only one direction. Experimental work was very important to understand the behavior of punching failure and to help in shear design of wide beam ribbed slabs. This was because of the UK design code, BS 8110.5 does not cover adequately the shear design procedure for wide beam ribbed slabs. In case of the beams are very wide, the punching failure surface could form within the section of full depth, but if the beams are narrower, the punching failure surface could pass through the reduced depth section. As result, a smaller shear failure surface could be mobilized, which, consequently, would lead to a lower punching shear capacity. Olawale & Ayodele (2014) [6] compared the flexural behavior for waffle and solid slab models.
under concentrated load. This work had showed the difference between characteristics of waffle and solid slab models. Twenty test samples were presented to determine the deflections, crack width and bending moments. Each specimen was subjected to an incremental concentrated loading of 1.00 kN interval after 28 days of casting. The samples were divided into two groups, ten samples had been small size panels (900 mm × 300mm) supported on all four sides. While the others had been large size panels (1353 mm × 430 mm), supported on the two short sides. It was shown from the test results, that waffle slabs have a higher structural stiffness than solid slabs. However, through estimation the crack width for both the waffle and solid slabs, the results showed that the waffle slab have upper crack width if compared with solid slabs at service load. While, at the failure load, waffle slabs have lower crack width if compared with solid slabs. This was because of the presence of ribs in the waffle had reduced the effect of load on the slab portion by carrying the tensile forces and the results of flexural cracks were smallest failure load. Alaa & Zainab (2011) [7] presented and discussed the optimum design problem of reinforced concrete two-way ribbed(waffle) slabs by using genetic algorithms. Two cases had been studied, the first was a waffle slab with solid heads, and the second was a waffle slab with band beams. The main objective for the study was to specify the optimum values for the various design variables. The design variables included the effective depth of the slab, ribs width, the spacing between ribs, the top slab thickness, the width of band beams, and the area of steel reinforcement of the beams. The direct design method was used to analyze and design the slabs. It was applied according to requirements of ACI 318-05 code and the ultimate strength design method. The researchers used MATLAB computer program to accomplish the structural analysis and design of waffle slabs by the direct design method. Process of optimization was carried out by using the built-in genetic algorithm toolbox of MATLAB. The researchers concluded that the total cost of waffle slab with band beams was higher than that with solid head for slabs with the same span length.

The purpose of this study is to understand the behavior of two-way ribbed slabs under various loading conditions through the following objectives:

1. Use of finite element method by creating model in ANSYS program, to perform analysis of two way ribbed slabs by using real scale continuous slab with large size and studying the linear response for these slabs.

2. Parametric study using various parameters such as length to width ratio, spacing of ribs and total slab thickness and its influence on the mid span deflection as compared to solid slabs.

In the present study, the proposed slabs are divided into three groups:

(i) Frame consists of three by three panels with different dimensions (solid and two way ribbed slab) with the same thickness and different rib spacing.

(ii) Frame consists of three by three panels with different dimensions (solid and two way ribbed slab) with the same rib spacing and different thickness.

(iii) One panel with different dimensions (solid and two way ribbed slab) for the same volumes of concrete with variable rib spacing.

2- Finite Element Modelling & Analysis by ANSYS

ANSYS program is a general-purpose program for the finite element analysis and design. It contains over 100,000 lines of code and more than 284 different elements conducted in the package. Through the study of some of the general characteristics of the program ANSYS, it turns out that it can be used in many fields of engineering. ANSYS package has the ability to solve static (linear and nonlinear) and dynamic structural problems, steady-state and transient heat transfer problems.[8]
2.1 ELEMENT TYPES

2.1.1 For Concrete: An eight-node solid element, Solid65, was used to model the concrete. This element has eight nodes with three degrees of freedom at each node—translation in the nodal x, y, and z directions. It has been used for the 3-D modelling of concrete solids with or without reinforcing bars (rebar). This element treats the nonlinear material properties. The concrete is capable of cracking (in three orthogonal directions), crushing, plastic deformation, and creep [8]. The geometry for the element type is shown in Figure (1-a).

2.1.2 For Steel Bars: LINK180 element was used for modelling of steel bars. It is a 3-D bar that is useful in a variety of engineering applications. The element can be used to model trusses, sagging cables, links, springs, and so on. The element is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x, y, and z directions. Tension-only (cable) and compression-only (gap) options are supported. As in a pin-jointed structure, no bending of the element is considered. Plasticity, creep, rotation, large deflection, and large strain capabilities are included. The geometry for the element type is shown in Figure (1-b).

![Figure 1. (a) 8-Nodes isoperimetric brick element (solid 65), (b) LINK180 Geometry [8]](image)

2.1.3 For Steel Plate: SOLID185 is used for 3-D modelling of solid structures. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, hyper elasticity, stress stiffening, creep, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyper elastic materials. The geometry, node locations, and the coordinate system for this element are shown in Figure (2).

![Figure 2. 8-Nodes brick element (solid 185)[8]](image)

2.2 Material properties:

ANSYS requires input data to define the material properties of concrete as follows:
Ultimate uniaxial compressive strength ($f'_c$). Elastic modulus ($E_c$). Ultimate uniaxial tensile strength (modulus of rupture, $f_r$). Poisson’s ratio ($\nu$). Shear transfer coefficient ($β_t$). Compressive uniaxial stress-strain relationship for concrete.

Use the following equations from ACI code [9]:

\[ E_c = 4700 \sqrt{f'_c} \]
\[ f_r = 0.62 \sqrt{f'_c} \]

Poisson’s ratio for concrete in this study is taken as (0.2).

The shear transfer coefficient, $β_t$, represents conditions of the crack face. The value of $β_t$ ranges from 0.0 to 1.0, with 0.0 representing a smooth crack (complete loss of shear transfer) and 1.0 representing a rough crack (no loss of shear transfer) [10].

For steel reinforcement, representation of the mechanical properties is very simple and it needs a single stress-strain relation to define the material properties in the analysis of the reinforced concrete members. The behavior of steel bar is the same in compression and tension loading.

In finite element method, representation of steel reinforcement can be implemented by two methods: discrete reinforcement connecting solid elements nodes or smeared reinforcement which means that some of solid elements containing a smeared reinforcement [11]. In this study, discrete model is used for modelling the reinforcement. Figure (3) shows reinforcement representation types.

![Figure 3. Types of reinforcement representation. [11]](image3)

The discrete model of reinforcing bars is generally modeled as separate elements commonly truss or cable elements. Representation of reinforcement bars is shown in Figure (4).

![Figure 4. Discrete Representation of Reinforcement Bars. [11]](image4)

### 2.3 Modelling of Two-Way Ribbed Slabs:

The slabs were modelled according to ACI code [9]. The dimensions of two-way ribbed slabs are illustrated in Figure (5) and their limits as per ACI 318 are summarized below.
Figure 5. Dimensions of the Cross Section of the Ribs.

- Minimum thickness of structural toppings (t) is 50 mm or one-tenth (1/10) of the clear distance between ribs, whichever is greater.
- Clear ribs spacing (S) shall not exceed 750mm.
- Width of ribs (bw) shall be at least 100mm at any location along the depth.

| Slab Model | Long Direction (Lx) | Short Direction (Lz) | Total Thickness (h) | Rib Spacing (S) | Total Load (kN/m²) |
|------------|---------------------|----------------------|---------------------|-----------------|--------------------|
| R1*        | 9200                | 6200                 | 250                 | 600             | 6                  |
| R2         | 9200                | 6200                 | 250                 | 800             | 6                  |
| R3         | 9200                | 6200                 | 250                 | 1000            | 6                  |
| S1**       | 9200                | 6200                 | 250                 | ----            | 6                  |

- The depth of ribs (hw) shall not exceed (3.5) Times the minimum width.

2.4 Modelling of proposed slabs:

2.4.1 First Group: Four slab models have been designed in this group. Arrangement and details of slab models are shown in Table (1).

Table 1. Arrangement and Details of Slab Models for Group -1

| Slab Model | Long Direction (Lx) | Short Direction (Lz) | Total Thickness (h) | Rib Spacing (S) | Total Load (kN/m²) |
|------------|---------------------|----------------------|---------------------|-----------------|--------------------|
| R1*        | 9200                | 6200                 | 250                 | 600             | 6                  |
| R2         | 9200                | 6200                 | 250                 | 800             | 6                  |
| R3         | 9200                | 6200                 | 250                 | 1000            | 6                  |
| S1**       | 9200                | 6200                 | 250                 | ----            | 6                  |

All models are supported by columns with dimensions (400*400*400) mm in (x, y, z) directions. Solid185 element is used for modelling the columns.

Nonlinear analysis by 3D finite elements model is done using ANSYS. The total load applied to finite element model is divided into a series of load increments called load steps. At the completion of each incremental solution, the stiffness matrix of the model is adjusted to reflect nonlinear changes in the structural stiffness before proceeding to the next load increment [8]. The ANSYS program uses Newton-Raphson equilibrium iterations for updating the model stiffness. The real constants for this example are shown in Table (2).

Table 2. Real Constant.

| Real Constant | Set No. | Element Type | Material     |
|---------------|---------|--------------|--------------|
| 1             | Solid65 |              | Concrete     |
| 2             | Link180 |              | Steel Bar(rib) |
| 3             | Link180 |              | Steel Bar(slab) |

Materials properties for specimens as used in ANSYS are summarized in Table (3).
Table 3. Material Properties.

| Material Model Number | Element Type | Properties          | Ex  | PRXY |
|-----------------------|--------------|---------------------|-----|------|
|                       | Solid 65     | Linear Isotropic   | 25743 | .2   |
|                       |              | Concrete            |     |      |
|                       |              | Open Shear Transfer Coef. | .4 |
|                       |              | Close Shear Transfer Coef. | .9 |
| 4                     | Solid185     | Linear Isotropic   | 200000 | .3   |

Modelling of slab models is shown typically in Figure (6).

![Figure 6. Modelling of R1 (Ribs With Hidden Beams) and S1 (Solid Flat Slab)](image)

Table (4) shows the element size in (X-Y-Z) directions for slab models.

Table 4. Element Size in (X-Y-Z) Directions For Slab Models.

| Slab Models | X   | Y   | Z   |
|-------------|-----|-----|-----|
| R1          | 200 | 50  | 200 |
| R2          | 200 | 50  | 200 |
| R3          | 200 | 50  | 200 |
| S1          | 200 | 50  | 200 |

Typical meshing and boundary conditions of slab models are shown in Figure (7) and (8) respectively.

![Figure 7. Typical Meshing for all Slabs.](image)
2.4.2 Second Group: Twelve slab models have been designed in this group. Arrangement and details of slab models are shown in Table (5).

**Table 5. Arrangement and details of Slab Models.**

| slab model | Long direction (Lx) | Short direction (Lz) | Lx/Lz | Total thickness (h) | Rib spacing (S) | Total load (kN/m²) |
|------------|---------------------|---------------------|-------|--------------------|-----------------|-------------------|
| RA1        | 12000               | 8000                | 1.5   | 250                | 800             | 7                 |
| RA2        | 12000               | 8000                | 1.5   | 300                | 800             | 7                 |
| RA3        | 12000               | 8000                | 1.5   | 350                | 800             | 7                 |
| SA1        | 12000               | 8000                | 1.5   | 250                | -               | 7                 |
| SA2        | 12000               | 8000                | 1.5   | 300                | -               | 7                 |
| SA3        | 12000               | 8000                | 1.5   | 350                | -               | 10                |
| RB1        | 16000               | 10000               | 1.6   | 250                | 800             | 6                 |
| RB2        | 16000               | 10000               | 1.6   | 300                | 800             | 6                 |
| RB3        | 16000               | 10000               | 1.6   | 350                | 800             | 6                 |
| SB1        | 16000               | 10000               | 1.6   | 250                | -               | 6                 |
| SB2        | 16000               | 10000               | 1.6   | 300                | -               | 6                 |
| SB3        | 16000               | 10000               | 1.6   | 350                | -               | 6                 |

All slab specimens are supported by columns with dimensions (600*600*600) mm in (x, y, z) directions and (Solid185) element is used for modelling them. Modelling of slab specimens are shown typically for RA1 and SA1 in Figure (9).
2.4.3 Third Group:

In this group, a single one panel solid slab has been transformed into two-way ribbed slab by assuming the volume of concrete to be constant for both. Dimensions of one of the models for solid slab are (12000*8000*300) mm. The thickness of two-way ribbed slab is determined by using the flowing equations:

\[ V_t = (11.4*7.4*.3) = 25.308 \text{ m}^3 \]
\[ V_{slab} = (11.4*7.4*.05) = 4.218 \text{ m}^3 \]  \[\ldots\] Where (\( V_{slab} \)) is the volume of top slab

\[ V_r = V_t - V_{slab} \]  \[\ldots\] Where (\( V_r \)) is the volume of ribs.

\[ =25.308-4.218 =21.09 \text{ m}^3 \]

\[ A_r = A_t - A_v \]  \[\ldots\] Where (\( A_r \)) is the area of rib, (\( A_t \)) is the total area; (\( A_v \)) is the area of voids.

\[ A_v= (N_i A_i) \]  \[\ldots\] Where (\( N_i \)) is the number of voids, (\( A_i \)) is the area for each void.

\[ =4(.2*.3) +36(.4*.3) +22(.2*.4) +11*18*(.4*.4) = 38 \text{ m}^2 \] for (600 mm) Rib spacing.

\[ A_r = (11.4*7.4) - 38 =46.36 \text{ m}^2 \]

\[ t_r = \frac{V_r}{A_r} =.454+\text{thikness of slab (.05)} \]

\[ =.504 \text{ mm or } 504 \text{ mm.} \]

By the same procedure, Thickness of two-way ribbed slab for other models can be calculated. Twelve slab models have been designed in this group. Since the slab models are symmetric, quarter of slab model has been modelled for the analysis. Arrangement and details of slab models are shown in Table (6). Typical modelling of slabs are shown in Figure (10). Finite element meshing of slab models is shown in Figure (11) and element sizes are shown in Table (7).

### Table 6. Arrangement and details of Slab Models for Third Group

| slab models | Long direction (Lx) | Short direction (Lz) | Dimensions (mm) | Total thickness (h) | Rib spacing (S) | Total load (kN/m²) |
|-------------|---------------------|---------------------|----------------|--------------------|----------------|-------------------|
| S250        | 12000               | 8000                | -              | 250                | -              | 15                |
| R414        | 12000               | 8000                | -              | 414                | 600            | 15                |
| R516        | 12000               | 8000                | -              | 516                | 800            | 15                |
| R628        | 12000               | 8000                | -              | 628                | 1000           | 15                |
| S300        | 12000               | 8000                | -              | 300                | -              | 15                |
| R505        | 12000               | 8000                | -              | 505                | 600            | 15                |
| R630        | 12000               | 8000                | -              | 630                | 800            | 15                |
| R774        | 12000               | 8000                | -              | 774                | 1000           | 15                |
| S350        | 12000               | 8000                | -              | 350                | -              | 20                |
| R596        | 12000               | 8000                | -              | 596                | 600            | 20                |
| R750        | 12000               | 8000                | -              | 750                | 800            | 20                |
| R915        | 12000               | 8000                | -              | 915                | 1000           | 20                |
Figure 10. Typical Modelling of Slab Models (Quarter of Slab).

Figure 11. Typical Meshing of Slab Models (Quarter of Slab).
Applying displacement boundary conditions at planes of symmetry which prevent the movement in the direction of (x and z) at the plans (x,z) respectively. This applies for all models.

### 3. Results and Discussions

The twenty-eight models explained in the previous section have been analyzed by using (ANSYS) (version 15.0) to study the effect of several important parameters on the behavior of two-way ribbed slab. **In the first group**, the parameters include the effect of void ratio on stiffness of waffle slab and the effect of rib spacing (S) on the maximum stress under uniform loads. *In the second group*, the parameters include influence of the depth of waffle slab on the maximum deflection for different span to width ratios (L/W) of waffle slab as compared with the solid slab with constant rib spacing (S) and influence of the depth of waffle slab on the maximum stress. *In the third group*, the parameters include the influence of rib spacing (S) on the stiffness and maximum deflection for waffle slab as compared to Solid slab. Span to width ratio (L/W) and concrete volume are kept constant.

#### 3.1. First Group: Figure (12) and (13) show Typical analysis results for first group.

| Slab Models | X  | Y  | Z  |
|-------------|----|----|----|
| S250        | 100| 50 | 100|
| R414 (rib)  | 100| 36.4|100|
| R414 (top slab) | 100| 50 | 100|
| R516 (rib)  | 100| 46.6|100|
| R516 (top slab) | 100| 50 | 100|
| R628 (rib)  | 100| 57.8|100|
| R628 (top slab) | 100| 50 | 100|
| S300        | 200| 150|200|
| R505 (rib)  | 100| 45.5|100|
| R505 (top slab) | 100| 50 | 100|
| R630 (rib)  | 200| 58 |200|
| R630 (top slab) | 200| 50 | 200|
| R774 (rib)  | 100| 72.4|100|
| R774 (top slab) | 100| 50 | 100|
| S350        | 100| 70 |100|
| R596 (rib)  | 100| 54.6|100|
| R596 (top slab) | 100| 50 | 100|

**Table (7) Element Size in (X-Y-Z) Directions For Slab Models.**
3.1.1. **Load-Displacement Response:** From analysis results, the effect of rib spacing on the maximum deflection is observed. Figure (14) and (15) show load-displacement response for slab models with different rib spacing and the effect of this spacing on the maximum deflection.

**Figure 13. Vertical displacement for Model S1.**

**Figure 14. Load-Displacement Response for Ribbed Slab Models**

**Figure 15. Effect of Rib Spacing on the Maximum Deflection.**
From figures above, it is concluded that when the rib spacing increases, the maximum deflection increases. That is because increasing rib spacing will decrease the slab rigidity.

3.1.2. Effect of Void Ratio on Stiffness of Waffle Slab: Figure (16) shows the influence of “void ratio” \((S-W)/S\) that obtained from different rib spacing on the stiffness of waffle slab. From this figure, it is found that when the void ratio increases, stiffness of waffle slab also increases. Increasing stiffness for waffle slab continues up to some limit. Then will decrease with increasing void ratio. The best case in this example occur when the void ratio equal (0.667) which gives increase in stiffness (0.347) as compared to solid slab with the same thickness.

![Figure 16. Effect of Void Ratio on Stiffness.](image)

3.1.3. Effect of Rib spacing \((S)\) on Maximum Stress: Numerical analysis for slab models is carried out by using (ANSYS) to predict the equivalent stress (Von-Mises) for slab models to study the effect of rib spacing\((S)\) on the maximum stress. Figure (17) shows maximum stress for two-way ribbed slab.

![Figure 17. Value and Location of Maximum Stress for R1 Slab (bottom view).](image)
From analysis results for slab models which have different rib spacing (S), it is found that the maximum stress increases when the rib spacing increases. Table (4.5) shows the value and location of maximum stress for slab models.

The stress distribution along the slab models is shown in figures (4.24),(4.25) and (4.26) respectively.

### Table 8. Value And Location of Maximum Stress.

| Slab Model | Maximum Stress (MPa) | X    | Y        | Z        |
|------------|----------------------|------|----------|----------|
| R1         | 17.2142              | 397.981 | 0.439103 | 6400.19  |
| R2         | 18.9115              | 9196.56  | 198.652  | 398.776  |
| R3         | 24.0858              | 398.186  | 0.643365 | 12600.1  |

The stress distribution along the slab models is shown typically for slab R1 in figures (18).

**Figure 18. Von-Mises Stress distribution along the slab (R1).**

### 3.2. Second Group:

In this group, analysis results have been done to study the influence of the depth of waffle slab on the maximum deflection for different span to width ratios (L/W) of waffle slab with constant rib spacing (S) as compared with the solid slab. Also, the percentage of increase in deflection for waffle slab as compared to solid slab is studied to arrive to the case that gives the best percentage of decreasing in concrete volume. The span to width ratios (L/W) for slab specimens are ranged from (1.5) for panel (12*8) m dimensions to (1.6) for (16*10) m dimensions. Rib spacing (S) is taken (800) mm for all models. Figures (19) to (22) show typical analysis results for models.
Figure 19. Deformed Shape of Models (RA1), (RA3) and (SA1), (Sectional View)
Figure 20. Deformed Shape for (RB1), (Sectional View)

Figure 21. Deformed Shape for (RB3), (Sectional View)

Figure 22. Deformed Shape for (SB1), (Sectional View)
After analysis, the maximum deflection values due to the application of uniform load to the twelve models have been determined according to the present ANSYS model. The load-deflection response for all models is shown in figures (23) and (24).

Figure 23. Load-Displacement Response for Span to Width Ratio =1.5

![Graph showing load-displacement response for span to width ratio 1.5](image)

Figure 24. Load-Displacement Response for Span to Width Ratio =1.6

![Graph showing load-displacement response for span to width ratio 1.6](image)

Table (9) shows the influence of the depth of waffle slab on the maximum deflection for different span to width ratios (L/W) of waffle slab as compared with the solid slab with constant rib spacing(S). The best case for this example with span to width ratio (1.5) and (300) mm depth.
study the influence of the depth of waffle slab on the maximum stress, from the results shown typically in Fig(25), it is found that the maximum stress for span to depth ratio = (1.5) is increased with increasing the depth of slab specimens; this is because the distribution and location of maximum stress is different for each specimen. For span to depth ratio = (1.6), all specimens have been the same location of maximum stress approximately. So, the depth of waffle slab will effect on the value of maximum stress where it decreases with increasing of depth.

Table (10) shows values and Locations of maximum Stresses for all slab models.

| Slab Models | Depth (mm) | Percentage of Increase in Max. Deflection | Percentage of Decrease in Concrete Volume |
|-------------|------------|------------------------------------------|------------------------------------------|
| RA1& SA1    | 250        | (88%)                                    | (61%)                                    |
| RA2& SA2    | 300        | (51%)                                    | (59%)                                    |
| RA3& SA3    | 350        | (76%)                                    | (58%)                                    |
| RB1& SB1    | 250        | (73%)                                    | (61%)                                    |
| RB2& SB2    | 300        | (111%)                                   | (60%)                                    |
| RB3& SB3    | 350        | (74%)                                    | (58%)                                    |

Figure 25. Maximum Stress of Two-Way Ribbed Slab.(Typical Results)
### Table 10. Value and Location of Maximum Stress

| Slab Specimens | Total Depth (mm) | Maximum Stress (MPa) | X Location (mm) | Y Location (mm) | Z Location (mm) |
|----------------|-----------------|----------------------|-----------------|-----------------|-----------------|
| RA1            | 250             | 31.3051              | 12007.5         | -35.1926        | 8992.48         |
| RA2            | 300             | 31.7537              | 597.746         | .57447          | 7999.42         |
| RA3            | 350             | 35.4283              | 597.595         | .902402         | 8400.12         |
| RB1            | 250             | 20.1822              | 598.75          | .405867         | 10400.3         |
| RB2            | 300             | 19.5876              | 598.362         | .452881         | 10199.9         |
| RB3            | 350             | 14.8975              | 599.091         | -.813993        | 10200.1         |

3.3. **Third group:** In this group, volume of concrete is considered constant for both waffle and solid slab. One panel with dimensions (12*8) m have been analyzed with different values of rib spacing (S) (600, 800 and 1000 mm) to study the influence of rib spacing on the stiffness and mid-span deflection of waffle slab as compared to solid slab. Span to width ratio (L/W) and concrete volume are kept constant. Results of ANSYS analysis are shown in figures (4.42) to (4.45).

![Table 10. Value and Location of Maximum Stress](image)

**Figure 26. Typical Results of Deformed Shape of Slab Models**
To compare between FE results for slab models, the values of mid-span deflection due to application of uniform load to models are shown in figure (27).

![Figure 27. Load-Midspan Deflection Curves of Slab Models](image)

From figure above, results of analysis shows that the stiffness of two-way ribbed slab is higher than the solid slab that has the same volume of concrete. The displacement of two-way ribbed slab in the elastic range (at first crack) is lower than the solid slab. In this manner, it will give the maximum
reduction in concrete volume with higher thickness. Table (11) shows the comparison between the loads and displacement at the first crack and the load, displacement at the failure load.

Table (11) Comparison Between the Loads and Displacements for Slab Models.

| Slab Models | P crack (kN/m²) | ∆ at first crack (mm) | P failure (kN/m²) | ∆ at failure load (mm) |
|-------------|----------------|-----------------------|------------------|-----------------------|
| S250        | 1.5            | 6.05965               | 3                | 12.8328               |
| R414        | 4.5            | 6.87108               | 4.5              | 6.87108               |
| R516        | 4.5            | 3.98346               | 7.5              | 10.742                |
| R628        | 4.5            | 2.784                 | 10               | 17.1348               |
| S300        | 3              | 6.74125               | 6                | 14.7336               |
| R505        | 4.5            | 3.73425               | 7.5              | 6.39267               |
| R630        | 6              | 3.20786               | 13.5             | 12.4969               |
| R774        | 6              | 2.17413               | 13.5             | 13.8988               |
| S350        | 4              | 6.27508               | 6                | 9.66064               |
| R596        | 6              | 3.32516               | 10               | 5.68073               |
| R750        | 6              | 2.12654               | 14               | 7.51474               |
| R915        | 8              | 1.89258               | 16               | 9.71999               |

4. Conclusions and Recommendations

4.1 Conclusions: Based on the results of Finite Element analysis in this study, the main conclusions can be summarized as follows:

1- Applying the finite element method by using ANSYS to model and analyze the two way-ribbed slabs of large sizes, it is found that when the void ratio increases, stiffness of waffle slab also increases. Increasing stiffness for waffle slab is continued up to some limit. Then it will decrease with increasing void ratio, the best case in this study occurs when the void ratio equal to (0.67) which gives increase in stiffness of (34.69%) as compared to solid slab with same thickness.

2- For the models which have length to width ratio of (1.5), the percentage of increase in deflection is (88%) for (250) mm depth with decreasing in concrete volume of (61%). For (300) mm depth slab, the percentage of increase in deflection is (51%) with decreasing in concrete volume of (59%). For (350) mm depth slab, the percentage of increase in deflection is (76%) with decreasing in concrete volume of (58%).

3- For models which have length to width ratio of (1.6), the percentage of increase in deflection is (73%) for (250) mm depth with decreasing in concrete volume of (61%). For (300) mm depth, the percentage of increase in deflection is (111%) with decreasing in concrete volume of (60%). For (350) mm depth slab, the percentage of increase in deflection is (74%) with decreasing in concrete volume of (58%). The best case for this study occurs with length to width ratio (1.5) and (300) mm depth.

4- Regarding the maximum Von-Mises stress, the maximum stress for length to width ratio of (1.5), increased with increasing the thickness of slab specimens. However, for length to width ratio of (1.6), all specimens have approximately the same location of maximum stress.

5- The stiffness of two-way ribbed slab is higher than the solid slabs that have the same volume of concrete. The deflection of two-way ribbed slab in the elastic range (at first crack) is lower than that of solid slab. In this manner, it will give the maximum reduction in concrete weight with larger thickness.

4.2 Recommendations for Future studies

1- Analysis of skew waffle slab as compared with Right angle slab.
2- Analysis of curved waffle slab.
3- Analysis of waffle slab under low-speed and high-speed impact load.
4- Experimental and theoretical analysis of lightweight concrete waffle slab.

5. References

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