MOUNTAIN BUILDING OF SOLID QUARK STARS

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ABSTRACT

One of the key differences between normal neutron and (bare) quark stars is relevant to the fact that the former are gravitationally bound while the latter self-confined unless their masses approach the maximum mass. This difference results in the possibility that quark stars could be very low massive whereas neutron stars cannot. Mountains could also build on quark stars if realistic cold quark matter is in a solid state, and an alternative estimation of the mountain building is present. As spinning compact objects with non-axisymmetric mass distribution will radiate gravitational waves, the equations of states of pulsars could be constraint by the amplitude of gravitational waves being dependent on the heights of mountains. We then estimate the maximum mountains and thus quadrupole moment on solid quark stars, to be consistent with that by Owen (2005) if the breaking strain is $\sim 10^{-1}$, addressing that a solid quark star with mass $< \sim 10^{-2} M_\odot$ could be “potato-like”. We estimate the gravitational wave amplitude of solid quark stars with realistic mountains to be order of $h_0 \sim 10^{-27}$.

Subject headings: dense matter — gravitational waves — pulsars: general — stars: neutron — elementary particles

1. INTRODUCTION

Neutron stars, with mass of $\sim M_\odot$ and radius of $\sim 10$ km and thus supra-nuclear density, provide as unique astro-laboratory for testing fundamental theories. Due to their high mass to radius ratio, neutron stars with strong gravity could radiate gravitational waves (GWs) in a few potential ways: inspiralling binaries (Sathiyaparakash & Schutz 2009), supernova core collapse (New 2003; Ott 2009), rotating deformed stars and GW radiation induced from oscillations and instabilities (Andersson & Kokkotas 1998). See Andersson (2010) for a review of these topics.

This work is focus on the gravitational waves due to the rotating deformed stars. It is well-known that neutron stars with non-axisymmetric distribution of mass may radiate GWs because of the time-varying quadrupole moment they have. The frequency of the gravitational waves will be twice of rotation frequency of neutron stars. As the the frequencies are of $10^{2-3}$ Hz, the pulsars are good sources of gravitational waves for interferometers detectors, such as LIGO (Abbott et al. 2002) and Virgo (Acerese et al. 2008). Although LIGO hasn’t detect the signal from pulsars yet, a limitation of the amplitude of the gravitational waves has been put forward (Abbott et al. 2005, 2007, 2010).

One of the key motivations to detect GWs from pulsars is to provide constraints on the equation of state of cold matter at supra-nuclear density, a challenge in understanding the fundamental strong interaction between quarks. Pulsars could be neutron or quark stars, even in a solid state (Xu 2003, 2009). Can the LIGO detection be understood in the regime of solid quark stars? What about GWs if pulsars are really solid quark star? We are concerned about these issues in this paper. Ushomirsky et al. (2000) first consider the elastic deforming neutron stars. Their estimation of the maximum quadrupole moment of the normal neutron stars is $2.4 \times 10^{38}$ g·cm$^2$, which has been updated by Owen (2005) via redefining the shear modulus. They also give a general equation to calculate the quadrupole moment induced by shear modulus which can be used to other stars such as solid quark stars, and Owen (2005) estimates the quadrupole moment of the solid quark stars to be $2.8 \times 10^{41}$ g·cm$^2$. The quadrupole moment is also calculated in other stellar models, such as Hybrid crystalline clou-sur-supconducting star (Knippel & Sedrakian 2009, Haskell et al. 2007; Lin 2007) and Hybrid and meson condensate stars (Owen 2005). Pitkin (2011) provides a good review on these.

However, all these studies assume that the equilibrium structure of the neutron stars is almost spherically symmetric. Certainly, some solidified mountains would also be possible on the surfaces of solid quark stars. Unlike shear-modulus-induced mountains (elastic mountains), we suggest here latent-heat-induced mountains (solidified mountains) there. Under this condition, instead of the shear modulus and the breaking strain, latent heat perform an important role that determine the height of the mountains. From this point of view, our model is only one parameter (i.e., latent heat) dependent, rather than than two parameter (i.e., shear modulus and breaking strain) dependent in case of elastic mountains.

After estimating the maximum height of the mountains, we find that if the mass of solid quark stars is small enough that the height is of the same order with the radius of the star, the star may be “potato-like”. The critical mass for potato-like solid quark stars is estimated to be $\sim 10^{-3-2} M_\odot$, which agrees with the mutually independent study of Xu (2010). Also it is found that, in reality, the actual GW amplitude of a pulsar would be too small to be detected with LIGO now since the maximum GW amplitude requires a star to deform into a particular distribution of mountains.

2. GENERAL ARGUMENTS

Mountains on the Earth as an analogy. Before dis-
cussing the latent-heat-induced mountains on the solid quark stars, we present an estimation of the maximum height of mountains on the Earth using the same method. Consider a mountain with the height $H$ and sectional area $A$. The earth is assumed to have mass of $M$ and radius $R$. Local acceleration of gravity is $g$. If the mountain gets $\delta$ height down, the material at the foot of the mountain will get energy of $A\delta g R$. While this energy is enough for this amount of material to melt, the mountain can no longer get higher because the mountain may tend to melt rather than to keep the height if it is higher than this. Certainly, this process of melting is somewhat a virtual one. We can also understand this in another way that this method is comparing energy between two: a mountain with height $H$ and the other with height $H - \delta$. The maximum height of the mountain, $H_m$, will then guarantee an equation of $A\delta g H_m = A\delta \rho \Delta N\Lambda/\mu$, where $\Delta N\Lambda$ is the latent heat per mol (with $N\Lambda$ the Avogadro constant) and $\mu$ is the mass per mol. Therefore we have $H_m = \Delta N\Lambda/(\mu g)$. On the earth the material that construct mountains is SiO$_2$, which has $\mu = 60$ g/mol and $\Delta N\Lambda = 854$ kJ/mol. The estimate of the maximum height of mountains on the earth is thus 14.5 km. The highest mountain known on the Earth is Qomolangma with the height of 8.8 km, which is the same order of the estimated maximum height.

**Mountains on solid quark stars.** In this case, we can also have the equation of

$$\frac{GM}{R^2} H_m A\delta n \chi \rho_0 = A\delta n \Lambda,$$

where $n$ is the number density of the quark clusters, $\chi$ is the number of quarks in a quark cluster, $\rho_0 = 300$ MeV is an assumed mass of every dressed quark, and $\Lambda$ is the melting energy per quark cluster. Michel (1988) proposed that quark-alpha particle with 18 quarks is possible to exist, while the proton and neutron are formed with 3 quarks. We then suggest that the $\chi$ varies from 3 to 18 and choose $\chi = 10$ as fiducial value. Ishii et al. (2007) shows that the nucleon-nucleon potential could be as depth as $\sim 100$ MeV. The latent heat should thus be considered to vary from 1 MeV to 10 MeV, and $\Lambda = 5$ MeV is chosen as the fiducial value for further calculation. The definition of $A$, $H_m$, $M$ and $R$ is the same as above. One can then has

$$H_m \approx 8 \times 10^3 \text{cm} \left(\frac{R}{10\text{km}}\right)^2 \left(\frac{1.4M\odot}{M}\right) \left(\frac{\Lambda}{5\text{MeV}}\right) \left(\frac{10}{\chi}\right).$$

(1)

It seems that there are two parameters, $\Lambda$ and $\chi$, but it is evident that the maximum height is only one parameter dependent, $\Lambda/\chi$, the average latent heat per quark.

Certainly the stars are not spherical because of many mountains there. Stars could only be approximately spherical if $H_m \ll R$, otherwise, if $H_m/R \sim O(1)$, those gravity-free stars might be “potato-like”. For a low mass solid quark star with then approximately constant density, $\rho \sim 4 \times 10^{14} \text{g/cm}^3$, one has from Eq. (1)

$$\frac{H_m}{R} \approx 0.008 \left(\frac{R}{10\text{km}}\right) \left(\frac{1.4M\odot}{M}\right) \left(\frac{\Lambda}{5\text{MeV}}\right) \left(\frac{10}{\chi}\right),$$

$$= 0.013 \left(\frac{10\text{km}}{R}\right)^2 \left(\frac{4 \times 10^{14} \text{g/cm}^3}{\rho}\right) \times \left(\frac{\Lambda}{5\text{MeV}}\right) \left(\frac{10}{\chi}\right).$$

When $R \sim 0.013 \times 10$ km = 1.15 km, we will have $H_m/R \sim O(1)$, and the corresponding mass for potato-like solid quark stars would be $M \lesssim (\sqrt{0.013})^3 \times 1.4M\odot = 0.002M\odot$. This result agrees with that of Xu (2010), who addressed that quark stars with masses as low as $10^{-2} \sim 10^{-3}M\odot$ are gravity-free. An important astrophysical consequence of potato-like solid stars would be of precession with high amplitude, and we expect to test this by future observations. Possible low-mass compact stars is a direct consequence of the suggestion that pulsars could be quark stars, which could be the central remnants left by the detonation of the accretion-induced collapse of white dwarfs (Yu & Xu 2011).

3. GRAVITATIONAL WAVE RADIATION FROM SOLID QUARK STARS

As there are mountains on the stars, these mountains may attribute to the quadrupole moment of the stars and thus influence the amplitude of the gravitational waves the stars radiate. However, if a star accrete mass axisymmetrically, it will have an axisymmetrical distribution of mountains. Thus, the star will not radiate gravitational waves at all. It is difficult for us to know the real distribution of mountains and amplitude of gravitational waves the star radiate, but we can provide an upper bound of the amplitude, by assuming a special distribution of mountains on the star. The first term that attribute to the quadrupole moment is $Y_{22}$, so we can assume the stars having a height distribution of the mountains as

$$h(\theta, \phi) = \frac{1}{2\sqrt{N}} H_m \Re \{Y_{22}(\theta, \phi)\},$$

$$= \frac{1}{2} H_m \sin^2(\theta) \cos(2\phi),$$

(2)

where $N = 15/32\pi$. We give the factor $1/(2\sqrt{N})$ here to ensure the difference between the maximum value and minimum value is $H_m$. We can define $Q_{22}$ in the way of Ushomirsky et al. (2000), $Q_{22} = \int \delta \rho_2(r) r^4 dr$. In our simulation, we simply define a density perturbation of $\delta \rho$ to be of delta function (only to be non zero when $r = R$). By comparing this with our distribution of the mountains, we can obtain $Q_{22} = \rho H_m R^4/2$. We have thus

$$Q_{22,\text{max}} = 4.17 \times 10^{42} \text{g} \cdot \text{cm}^2 \left(\frac{\rho}{4 \times 10^{14} \text{g/cm}^3}\right) \times \left(\frac{R}{10\text{km}}\right)^6 \left(\frac{1.4M\odot}{M}\right) \left(\frac{\Lambda}{5\text{MeV}}\right) \left(\frac{10}{\chi}\right).$$

(3)

Let’s compare this result with that of Owen (2005), who proposed a solid quark star to have a quadrupole moment of $2.8 \times 10^{41} \text{g} \cdot \text{cm}^2$ by suggesting a breaking strain
of $10^{-2}$. Our estimation is one order higher than that of Owen (2003), and shows that the solidified mountains can provide a bigger quadrupole moment than elastic mountains do. Horowitz et al. (2009) have done some work to show that the breaking strain on normal neutron stars can be as big as $10^{-1}$. Our result about the maximum quadrupole moment on the solid quark stars would agree with Owen’s if the breaking strain on solid quark stars can also be as high as $10^{-1}$. It is evident that both the dependence on radius and mass are the same in this work and that of Owen (2003).

What can we constrain the equation of state by the GW observations? To compare the theoretical quadrupole moment with the LIGO S5 data (Abbott et al. 2010), we are to provide the relationship between $Q_{22}$ and GW amplitude $h_0$. Assuming a density perturbation of $\delta \rho = \text{Re} \rho_{22} Y_{22}(\theta, \phi)$ (Ushomirsky et al. 2000) and following the definition of $Q_{22}$ above, one comes to

$$h_0 = 3.87 \times 10^{-27} \left( \frac{f}{100 \text{Hz}} \right)^2 \left( \frac{1 \text{kpc}}{d} \right) \left( \frac{Q_{22}}{10^{38} \text{g} \cdot \text{cm}^{-2}} \right),$$

where a factor $1/\sqrt{2}$ before the amplitude of the trace-reversed perturbation is added for $h_0$ estimation. Similar treatments can also be found in other gravitational wave literatures (Jaranowski et al. 1998). The only difference is applying ellipticity $\epsilon$ rather than quadrupole moment $Q_{22}$. Note the relationship $\epsilon = \sqrt{8\pi/15} Q_{22}/I_{zz}$, it is easy to prove that they are equivalent.

With our estimation of the maximum quadrupole moment into the equation above, we can obtain the maximum amplitude of the gravitational waves,

$$h_0^{\text{max}} = 1.6 \times 10^{-22} \left( \frac{f}{100 \text{Hz}} \right)^2 \left( \frac{1 \text{kpc}}{d} \right) \left( \frac{\rho}{4 \times 10^{14} \text{g/cm}^3} \right) \times \left( \frac{R}{10 \text{km}} \right)^6 \left( \frac{1.4 M_\odot}{M} \right) \left( \frac{\Lambda}{5 \text{MeV}} \right) \left( \frac{10}{\chi} \right).$$

LIGO is focusing on the direct detection of gravitational waves. Recent LIGO S5 data has given an upper bound of the $h_0$ for 116 known pulsars (Abbott et al. 2010). The value of the upper bound of the 116 pulsars varies from $10^{-26}$ to $10^{-25}$. However, from Eq. (4), it seems that solid quark stars can have mountains high enough to radiate gravitational waves with amplitudes as high as $10^{-22}$. Does this mean that these 116 pulsars cannot be solid quark stars?

Should we believe that the mountains on solid quark stars have the maximum heights? In addition, in our estimation of Eq. (5), we assume a distribution of $Y_{22}$, which can contribute most to the gravitational waves. It is surely quite strange if natural mountains on stars have the exact distribution of $Y_{22}$ in order to produce the maximum gravitational waves.

The orogeny in the Earth’s crust is powered by the mantle convection and thus the engagement of tectonic plates. What could be the force to build mountains on solid quark stars? Elastic energy develops when a star evolves, and both bulk-invariable and bulk-variable forces can result in decreases of moment of inertia during a star quake (Peng & Xu 2008). This force would be responsible for mountain building too. We can then estimate the real heights of mountains from the glitch phenomena of pulsars below. We think glitches occur if the moment of inertia change suddenly inside a solid quark stars (Zhou et al. 2004). The pulsars’ angular momentum, under the assumption of sphere, is $\sim 3M R^2 \Omega/5$. Making a differential of it, we have $\delta R/R \sim 5 \times 10^{-7} \delta (\Omega/\Omega)/10^{-6}$. Although this consideration could not be the real heights of mountains on pulsars, we think that actually the height of mountains might be of the same order of $\delta R$. Therefore, replacing $\delta R$ with $H_m$ and inserting this into the estimation of Eq. (4), we have the estimation of amplitude $h_0$ below,

$$h_0 = 1 \times 10^{-26} \left( \frac{f}{100 \text{Hz}} \right)^2 \left( \frac{1 \text{kpc}}{d} \right) \left( \frac{\rho}{4 \times 10^{14} \text{g/cm}^3} \right) \times \left( \frac{R}{10 \text{km}} \right)^5 \left( \frac{\delta \Omega}{\Omega} \right)/10^{-6}.$$  

We draw Fig. 1 to show how $h_0$ varies as the mass of solid quark stars changes in the model of Lai & Xu (2009), for different glitch amplitude $\delta \Omega/\Omega$. In the calculations, we choose the numbers of quarks in a quark clusters to be 18 and the potential $U_0$ to be 50 MeV. We can see that if our estimation of the real height of mountains is valid, it would be natural that LIGO still hasn’t detect the gravitational waves directly, since the maximum GW amplitude presented in Eq. (5) requires maximum height and a particular $Y_{22}$-distribution of mountains. What if the mountains haven’t the $Y_{22}$-distribution? This will be discussed in the next section.

4. GRAVITATIONAL WAVES INDUCED FROM RANDOM DISTRIBUTION

Above we give an approach to the actual amplitude of GWs using the glitch phenomenon. However, the amplitude of GWs from quark stars depends also on another factor. Besides the maximum height of mountains, the distribution of mountains plays a key role as well. In the previous section, we give an estimation of the maximum amplitude of GWs by assuming a specific distribution described in Eq. (2). Such a hypothesis is too strong to approach the physical circumstances. In this section we will discard this assumption and consider another likely distribution.

It is certainly very difficult to know the real distribution of mountains on a solid quark star. Nevertheless, an idea comes out that we can give a random distribution to approach the actual situation. Yet another problem of the definition of random distribution emerges. The most strict and physical definition should satisfy the following requirements. (1) The height of mountains should varies from 0 to the maximum height of mountains. (2) The surface of the star should be continuous, say the function $H(\theta, \phi)$ which describe the distribution of mountains should be infinitely differentiable. (3) The third requirement does not from mathematics, but from physics: the partial derivative of $H(\theta, \phi)$ should not be too high, otherwise a mountain which is quite precipitous would tend to fall down.

With all the considerations above, we find it is impossible or at least very hard to use the most strict definition
of random distribution to approach the real amplitude of GWs. Nonetheless, we would have a order-of-magnitude estimation of the distribution. The only thing we do care is the quadrupole moment the distribution induces. We then use another definition which is not that strict but we expect the latter definition can give a similar prediction or at least in the same order. We simply divide our star into some small pieces, say 100 × 100 for example. Each piece has height varying from zero to the maximum height of mountains in a uniformly random way. This is the definition of \( H_r(\theta, \phi) \) which is discussed below.

As a scatter function defined on sphere, the function of \( H_r(\theta, \phi) \) can be expanded using the spherical harmonics as following,

\[
H_r(\theta, \phi) = \sum_{l,m} C_{lm} Y_{lm}(\theta, \phi).
\]

Each mode corresponds to contributing mode of GWs. The mode with the sphere distribution of \( Y_{22}(\theta, \phi) \) is believed to contribute GWs with the biggest amplitude. To pick up this mode we use the orthogonal property of the sphere harmonics. We multiply the \( H_r(\theta, \phi) \) with Re\[Y_{22}(\theta, \phi)\] and calculate the integral on the sphere. The result is regarded as the factor before \( Y_{22} \) in Eq. [2], and we can calculate the quadrupole moment accordingly.

We divide the star into 100 × 100 pieces, calculate 10000 times in the simulation, take the average of the results and finally obtain the estimation of the quadrupole moment of the order \( 10^{-18} \text{g} \cdot \text{cm}^2 \). Correspondingly, the amplitude of GWs is of the order \( 10^{-27} \). Considering the upper bond given by LIGO S5 data varies from \( 10^{-26} \) to \( 10^{-25} \), our results would be good enough to explain the conflict between maximum estimation and the observations.

5. CONCLUSIONS AND DISCUSSIONS

We present a new method of building mountains on solid quark stars. It is induced by the latent heat rather than shear modulus. The quadrupole moment brought by this kind of solidified mountains is of the same order with that of Owen (2005) if one uses \( 10^{-1} \) (instead of \( 10^{-2} \)) as the fiducial value of the breaking strain on solid quark stars.

Although solid quark stars potentially radiate strong gravitational waves, it is necessary to build high mountains with enough energy to deform the quark stars. It is worth noting that the maximum amplitude of gravitational waves requires the stars to deform into a particular distribution of mountains, \( Y_{22} \). With the estimation of reality, we find that the actual amplitude (with \( h_0 \) of the order \( 10^{-27} \)) would be too small to be detected with LIGO now. We expect also gravitational waves being detected directly in the second or third generation detectors (Pitkin 2011).

As well as the discussion of gravitational wave radiation, we predict “potato-like” quark stars with mass as low as \( < 10^{-2}M_{\odot} \) (the corresponding stellar radius is smaller than a few km), since gravity could be negligible there, as in the case of asteroids. Another independent reason for potato-like quark stars is shown in Fig. 1 of Xu (2010). Rigid body precesses naturally, but fluid one can hardly. The observation of a few precession pulsars may suggest a totally solid state of matter. As discussed above, potato-like quark stars are gravitationally force-free, and thus free or torque-induced precession may easily be excited with larger amplitude in low-mass solid quark stars, which are potato-like.

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Fig. 1.— Estimation of amplitude of gravitational waves from solid quark stars in Lai & Xu (2009) model ($N_q = 18$ and $U_0 = 50$ MeV). This figure shows how GW amplitude, $h_0$, varies while the mass of stars changes. The line on the top is $h_0$ estimated from the maximum height of mountains with the distribution of $Y_{22}$. The five lines under it represent for the $h_0$ estimated from the consideration of glitch. See the text for details.