1. Introduction

As it is generally known, the main task at present of gas scrubbing is to bring content of toxic impurities in gas emissions to maximum permissible concentrations established by sanitary code. The methods used for the removal of aerosols (dust) and gaseous and vaporous impurities from gases vary depending on the nature of harmful impurities. All methods of gas scrubbing are determined primarily by physico-chemical properties of impurities, their aggregate state, dispersity, chemical composition, etc. The variety of harmful impurities in industrial gas emissions and their physico-chemical properties brings about a large number of cleaning methods, devices and reactors used.

The greatest success in the centrifugal capture of particulate dust from gas flows was observed in the field of hardware implementation rather than in scientific designs. This is due to the accumulation of a many years’ experience in operation of industrial devices on the one hand and a high complexity of the gas-solid system in the centrifugal field on the other hand. The theory of operation of cyclones and other devices for dry cleaning of gas flows from dust has not yet been improved and does not provide the possibility for a reasonable choice of the device and its main characteristics. Therefore, study of flow conditions in the dry gas purification device, redistribution of energy under various conditions and influence of physical and chemical characteristics of the gas flow on these parameters is a relevant issue and will improve device efficiency.

2. Literature review and problem statement

The processes of cleaning technological and ventilation emissions taking place at chemical, processing, metallurgical, machine-building enterprises from gas, dust and vapor impurities are characterized in many cases by a number of features [1, 2]:

- gases emitted into atmosphere have a rather high temperature (especially after leaving furnaces, reactors) reaching at times 673–873 K and contain large amounts of dust (more than 5 g/Nm³) which significantly complicates the cleaning process and requires preliminary gas preparation;

- concentration of gaseous and vaporous impurities more often in ventilation and less often in the process emissions is variable and relatively low.

As a rule, NOₓ, SOₓ, CO, HCl, HF, SF₆, NH₃ individually or in a combination, depending on the process type, are present as gaseous impurities in the gas stream. Concentration of each component can range from 50 to 100 mg/Nm³. In many cases, the task set during gas purification is not just
to bring the content of harmful components to the maximum allowable concentrations in the emitted gases but also to use exhaust heat of the purified gas [3, 4]. In other cases, for example in production of soda ash [5], the purified gas leaving shaft furnaces and calciners should not contain more than 5 mg/m$^3$ of aerosol since it must be subjected to compression and then used in subsequent manufacturing operations. In industrial conditions, to achieve maximum allowable aerosol concentrations and eliminate gaseous impurities, combined methods are commonly used in the process of gas cleaning such as dry-wet gas cleaning [6, 7]. Disadvantages of such schemes are well known [8]: high (0.4–4.2 kg/Nm$^3$) consumption of liquids (water) and formation of suspensions and slurries. Virtually no purification from such gaseous impurities as NO$_x$, SO$_x$, CO is ensured. The exhaust heat of gases is not used and installation of additional equipment is required for separation and filtration of slurry suspensions.

The first stage, dry gas cleaning, is carried out in widely used centrifugal-inertial devices, such as cyclones of various designs [9]. Cyclones are notable for high gas productivity, design simplicity and operational reliability. For example, for high-yield cyclones, in particular battery cyclones (with a productivity of more than 20,000 Nm$^3$/h), purification efficiency is 20–30% for $d=2–5 \mu$m average particle size; 40–60% for $d=5–20 \mu$m and 90–92% for $d=30 \mu$m [10].

Resistance of a high-performance cyclone is about 1080 Pa. Obviously, cyclones work most effectively if the particle size is $d=30 \mu$m. This conclusion is valid even for the most effective TsN-11 cyclone [11]. Therefore, many current studies are aimed at improvement of the centrifugal-inertial cleaning of gas flows and corresponding device designs [12–14]. All improvements in the devices can be classified according to the following characteristics:

- a tube or a branch tube at the entry of dusty gas stream to the main unit;
- a main unit where separation takes place;
- an outlet branch tube for the purified gas stream and a hopper and an outlet branch tube for the trapped dust.

Hence, it is advisable to consider such an option that when a dusty gas stream is fed into the main unit like a setting forebunker for dry scrubbing, preparation of the gas before its coming to the forebunker could be carried out in vortex tubes with a partial recirculation of the heated gas due to the Ranque effect. Principal provisions of such an approach were discussed in [15–17] where it was noted that implementation of the vortex tube – setting forebunker scheme can ensure fine cleaning of the incoming gas from impurities and utilization of the purified gas energy. However, there are no experimental data that would confirm the above conclusion for specific industrial conditions [15–17]. Besides, there is no mathematical description of the processes taking place in the vortex tube and in the forebunker for these conditions.

Obviously, the process in the vortex tube can be regarded as a gas-phase chemical process proceeding under non-isothermal conditions. To control this process, it is necessary to clarify the effect of changes in the initial conditions (temperature, concentration ratio, nature of the particle size variation, hydrodynamic conditions) and solve a system of differential equations constituting a mathematical model of the process. The basic choice is the choice of the vortex tube design, i.e. the ratio of dimensions, the tube cross-sectional profile (tapering, presence of bends, etc.) which provides one or another hydrodynamic situation affecting distribution of temperatures, concentrations, and other parameters. The vortex tube under these conditions can be considered as a process control object. As is known, to describe the control object, it is necessary to establish connections between its inputs and outputs taking into account the model of the medium flow in the vortex tube. Therefore, the first part of the studies is based on the fundamental concepts of flow of swirling streams [13–17, 18–20] and on the analysis of known experimental data concerning flow of a swirling stream in a vortex tube [21–24] and is devoted to a mathematical description of a reaction medium motion in the vortex tube.

3. The study objective and tasks

The study objective was to make a mathematical description of flow of aerosol-containing manufacturing waste gases in a vortex tube under the conditions ensuring occurrence of the Ranque effect and analyze “physics” of the processes taking place in the model for various process parameters of the gas flow at the inlet to the vortex tube.

To achieve this objective, the following tasks were set:

- to analyze physico-chemical characteristics of manufacturing and ventilation emissions at enterprises and establish characteristic indicators for dust (fractional composition, temperature of gas-dust flows);
- to study the mathematical model of a gas-dust stream flow in a vortex tube (determine velocity fields, trajectory of movement of dust particles and analyze temperature distribution in the stream);
- based on the developed mathematical model of the gas-dust flow in a vortex tube, to analyze the possibility of aggregation of dust particles and destruction of NO$_x$, SO$_x$, CO gas impurities.

4. Study of the process of cleaning the gas-dust stream

A schematic diagram of the vortex tube is shown in Fig. 1 [25].

![Fig. 1. Schematic diagram of the vortex tube: reaction tube (1); nozzle (connecting tube) for supply of compressed gas from the process source (2); gas swirler (3); control valve (4); gas outlet chamber (5); connecting tube for supply of the compressed gas to the forebunker (6); diaphragm (7)](image-url)
Compressed air (gas) enters the nozzle 3. When tangential stream of air enters the tube 1, it swirls forming a rotating stream inside the tube and acquiring kinetic energy. The air moves in a vortex stream with different angular velocities. The rotation speed at the tube axis is higher than at the periphery. Therefore, having given their kinetic energy to the inner layers of air, the outer layers cool down to a temperature $T_j$ and exit through the diaphragm 2. The air heated to temperatures $T_k$ exits thru the free end of the tube. Flow and temperature of the air are regulated by the throttle valve 4. Temperature of the cooled air depends on its initial parameters, i.e. pressure and temperature as well as on the device design.

Symbols O, I, II, III, A, B in Fig. 2 denote the conditional sections of the vortex tube at which dynamic structure of the flow and loss of kinetic energy are considered. At one end (A-A section), flow exits through the aperture (diaphragm) in the axis of the tube. At the opposite end of the tube, there is an outlet in a form of a peripheral annular gap (B-B section). As experience shows, with an adiabatic flow, there is a flow exiting through the diaphragm. In this case, the return flow over the diaphragm can be considered forced and its intensity depends on the state of the passage areas of the throttle and the diaphragm. When the diaphragm is closed, a return flow characteristic of the swirling flows in common appears at sufficiently high velocities near the flow axis. Thus, two forms of swirling flow are observed inside the vortex tube when the circumferential velocity profile in the return flow zone (I→O→A sections) is close to a quasi-solid type of rotation and close to a quasipotential type of rotation in the peripheral zone (II→III→B sections). Therefore, analysis and calculation of the process are based on a simplified scheme of flow dynamics and an energy equation.

A homogeneous gas is fed to the zero section of the vortex tube under $P_{0\text{m}}$ pressure (most often, studies are carried out for air). As noted above, the gas system to be treated in the vortex tube may contain various gas components in various proportions (for example, $\text{H}_2\text{O}$, $\text{CO}_2$, $\text{H}_2$, $\text{N}_2$, $\text{O}_2$) and in addition, the above toxic gas impurities and aerosols (up to 10 g/Nm$^3$) with various particle sizes. At the same time, the gas at a flow rate of 1000–15,000 Nm$^3$/h, (0.3–4.2 Nm$^3$/s) having temperature of 673–873 K enters the vortex tube under pressure (1.5–2.5 atm). Taking into account that the internal diameters of the vortex tubes for these gas flow rates are within (5–10$^{-2}$–25.0–10$^{-3}$ m) [15–17], the flow velocity can reach 80–100 m/s at the inlet.

The characteristic of the described gas mixture will influence the process in the vortex tube due to the change in viscosity, density, concentration, particle size, chemical composition and specific surface area. Chemical interactions between the medium components are possible. Influence of some of these parameters on the process in the vortex chambers is described in literature. For example, workers [25, 26] studied technological processes and devices in which baking of raw materials, drying and dry hydration of the baked products are fundamental. The aim was to determine chemical composition and dispersity of aerosols in exhaust gases. The results of the studies are presented in a form of nomogram (Fig. 3).

![Fig. 2. Schematic view of the vortex tube: tube (1); diaphragm (2); nozzle (3); valve (4)](image)

Efficiency of temperature separation in the gas flow is expressed by the ratio of the temperatures corresponding to section I-I and II-II. Therefore, analysis and calculation of the process are based on a simplified scheme of flow dynamics and an energy equation.

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![Fig. 3. Classification nomogram for determining dust group:](image)
It gets obvious from Fig. 3 that according to the classification, overwhelming majority of aerosols in exhaust gases belong to the class of fine and very fine dusts which are practically not captured by cyclones. Besides that, dust particles, such as CaO, MgO, Ca(OH)₂, ZnO have a sufficiently large specific surface (according to BET method, up to 10 m²/g) which indicates their sufficiently high adsorption capacity, e.g. with gas impurities, and aggregation capability. Analysis of the obtained data also showed that the particle size distribution in these aerosols obeys multimodal distributions, \( f(x) \), of the following form:

\[
f(x) = \frac{W(x-x_{\min})^{\alpha}(x_{\max}-x)^{\alpha}}{\int_{x_{\min}}^{x_{\max}} (x-x_{\min})^{\alpha}(x_{\max}-x)^{\alpha} dx} + \frac{(1-W)(x-x_{\min})^{\alpha}(x_{\max2}-x)^{\alpha}}{\int_{x_{\min}}^{x_{\max2}} (x-x_{\min})^{\alpha}(x_{\max2}-x)^{\alpha} dx},
\]

where \( W \) is the mass fraction of fine particles; \((1-W)\) is the mass concentration of the coarse fraction; \( x \) is the absolute size, \( x_{\min}, x_{\max} \) are the smallest and the largest absolute sizes.

This type of function makes it possible to track changes in the particle size during processing more accurately and with higher probability. In [27, 28], a possibility of aggregation of dust particles with characteristics close to those shown in Fig. 3, under hydrodynamic conditions corresponding to the I-I section, i.e., to the quasi-solid type of rotation of the gas flow, has been established theoretically and experimentally. It was shown that the amount of a very fine fraction (particle size less than 5 μm) decreases in 2–2.5 times due to aggregation of dust particles occurs even more intensively under the action of the alternating longitudinal pressure gradient that arises in the tube when the gas flow moves to the B-B section of stratification of particles does not depend on the curvature of the channel but rather on the tangential velocity of the aerosol in the circumferential direction; it also shows a similar motion of pure and dusty gases.

Thus, it can be assumed that hydrodynamics and nature of motion of the dust stream in the vortex tube will be the same as for a pure gas. The content of suspended matter (dust) in the gas stream up to 10 g/Nm³ which changes the level of velocities does not affect the character of their distribution, and the change in the tangential velocity profile practically does not affect the trajectory of the aerosol particles if the average level of velocities remains constant [32, 33]. A possibility of a physico-chemical transformation of gaseous impurities (NO, CO, SO₂) in excess of H₂O vapors relative to the mass of impurities was considered for a combustible swirling stream moving in a vortex tube [34, 35]. The excess of H₂O vapors is maintained within the following limits: H₂O vapor mass, mg/gas impurities mass, mg=5–10. A homogeneous catalysis of the following type takes place in a stream:

\[
\text{CO} + \text{H}_2\text{O} \rightarrow \text{CO}_2 + \text{H}_2,
\]

\[
2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O},
\]

at stream temperatures of 673–873 K.

The rate of chemical transformation was considered as a function of intensity of the heat and mass transfer processes in a stream. The resulting rate of chemical reaction “W” for one component was determined as:

\[
W = W_r / K_x,
\]

where \( W_r \) is the rate of the direct reaction; \( 1/K_x = 1 - P_x \), where

\[
P_x = f(\text{Re}, P_{\text{Pr}}, P_{\text{fo}}...),
\]

\( a, b \) are concentrations of gas impurities; \( \mu, \eta, \alpha, \beta \) are stoichiometric coefficients; \( P_x \) is the criterion of equilibrium; \( K_x \) is criterion of contact, \( K_x = (k_1 a^2 b^2) / \left( \partial C / \partial t \right) \); \( C \) is concentration of initial substances; \( k_1, k_2 \) are constants of the direct and reverse reaction rates.

In a general case, the process kinetics is completely determined by the conditions of mass and energy transfer:

\[
P_x = f(\text{Re}, P_{\text{Pr}}, P_{\text{fo}}...),
\]

where \( P_{\text{Pr}} = v / \alpha \) is the Prandtl criterion for heat exchange; \( \alpha \) is coefficient of temperature conductivity; \( P_{\text{fo}} = v / D_i \) is Prandtl’s criterion for material exchange; \( D_i \) is the diffusion coefficient of gas impurities; \( v \) is coefficient of kinematic viscosity, m²/s.

The results of experimental studies have shown that concentration of gaseous toxicants under the abovementioned
conditions can be reduced by a factor of 2–3. This result correlates with the data of industrial tests of reduction of NOx oxides concentration in the exhaust of gas turbine installations due to injection of steam into the combustion chamber [36]. Analysis of the data showed that all considered positive effects which were observed in purification of exhaust gases in the vortex tubes are explained by certain conditions of viscous gas turbulent flow. This occurs in three main sections (areas), namely (O→I, I→II→III, O→A), and accordingly at a quantitative ratio of flow rates. When creating a computational mathematical model for determining velocity field in the working cavity of the vortex tube, it is necessary to take into account not only conditions for formation of flows in the device but also conditions of aerodynamic situation that leads to the Ranque effect. To create the model, use the schematic calculation diagram of the vortex tube which provides for aerosol separation, Fig. 4.

Assume that the flow regime of the gas stream in the device is turbulent and the working substance is a viscous, compressible gas. The stream is strongly turbulentized in the zone of energy separation and the self-similarity of the process is observed according to the Re number (Re>10^5), therefore, Re can be taken constant in the calculations. With a greater degree of probability, steady-state heat and mass transfer in the asymmetric flow of a vortex stream can occur throughout the region at a constant Pr number value of Pr<1 (Pr=0.64–0.7). The walls of the tube are considered to be insulated, so there is no heat exchange between the gas and the external medium. The effect of the Re number on the turbulent layer (flux) at high velocities downstream the swirler in I–III and O–A zones (Fig. 1, 2) is the same as the effect on the flow of incompressible substance [18–20] and therefore it can be assumed that the gas flow is potentially circulating, that is, the particles (the gas moles) rotate around a certain axis that does not intersect the trajectories but do not rotate relative to their own axes, and therefore ω=0.

In the O-I region (Fig. 1, 2), at a "quasi-solid" motion of the rotating flow with Re≥10^5, Taylor vortices and toroidal Hill vortices are formed. In this case, tangential components of velocity of the gas volumes moving towards the center and periphery are arbitrarily set relative to the radius. The turbulent flow of a viscous gas in the region of the swirler causes flow rearrangement and ensures a situation of gas-dynamical energy separation provided that the number Re of the flow (Q) is Re≥10^5 at the inlet. At the same time, the non-uniform distribution of velocities along the radius explains the intensive dissipation of mechanical energy, internal heat release and uneven braking temperature distribution. Therefore, the length of the vortex zone (L) and the velocity distribution in it are of decisive importance for the processes occurring in the gas flow and for the vortex tube calculation.

Thus, the pattern of flows will take the following form. A circular source of quasi-solid rotational flow (Qs flow) is formed at the periphery of the swirler (Fig. 1), near the wall where the diaphragm is located. In the center of the diaphragm, there is a drain (Qo flow rate) at the device axis with a capacity equal to the flow rate of the heated gas. The difference in the flow rates of the working medium between the ring source and the central run-off of the cooled gas (Qs–Qo) is the flow rate Qo of the heated (hot) gas.

The axisymmetric motion with rotation of the ring source is described in [18–20] by a system of Navier-Stokes equations and the continuity of energies and states using an empirical relationship between viscosity coefficients and temperature.

The equation of continuity:

$$\frac{\partial (\rho c)}{\partial t} + \frac{\partial (\rho c)}{\partial z} = 0.$$  \hspace{1cm} (8)

Navier-Stokes equation:

$$\rho \left( \frac{\partial c}{\partial t} + \frac{\partial c}{\partial z} \right) = \frac{\partial p}{\partial r} + \frac{2}{3} \frac{\partial}{\partial r} \left( \mu \frac{\partial c}{\partial z} \right) - \mu \left( \frac{\partial c}{\partial z} + \frac{\partial c}{\partial z} \right) - \frac{2\mu}{r} \frac{\partial c}{\partial r}.$$  \hspace{1cm} (9)

Equation of energy:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ \rho c \left( 1 + \frac{c^2}{2} \right) \frac{\partial}{\partial r} \left( \frac{i}{Pr} \right) - 2\mu c \frac{\partial}{\partial z} \left( \frac{\partial c}{\partial z} \right) - \mu c \left( \frac{\partial c}{\partial z} + \frac{\partial c}{\partial z} \right) + \frac{2\mu}{3} \frac{\partial c}{\partial r} \right] +$$

$$+ \frac{1}{r} \frac{\partial}{\partial z} \left[ \rho c \left( 1 + \frac{c^2}{2} \right) \frac{\partial}{\partial z} \left( \frac{i}{Pr} \right) - 2\mu c \frac{\partial c}{\partial z} - \mu c \left( \frac{\partial c}{\partial z} + \frac{\partial c}{\partial z} \right) + \frac{2\mu}{3} \frac{\partial c}{\partial r} \right] = 0.$$  \hspace{1cm} (10)

The system end equations:

$$\frac{P}{\rho} = RT, \quad \frac{\mu}{\mu_0} = \left( \frac{T}{T_0} \right)^{\gamma/2} \frac{T}{T + \epsilon}.$$  \hspace{1cm} (11)
where \( r, \theta, z \) is cylindrical coordinate system; \( C_r, C_\theta, C_z \) are the velocity components; \( \rho \) is medium density; \( \mu \) is medium viscosity; \( T \) is medium temperature; \( K \); \( R^* \) is universal gas constant; \( i \) is enthalpy; \( z \) axis is along the channel axis; \( r \) axis is normal to \( z \) axis.

The condition of axial symmetry implies absence of gradients in the circumferential direction.

To connect the tensor of turbulent directions with the averaged values of velocities, use the Prandtl hypothesis (the semi-empirical theory of the mixing length).

Then the relationship between the turbulent stress tensor and certain velocities will be written by a system of equations:

\[
\begin{align*}
\rho \partial C_r^2 &= 2A \left( \frac{\partial C_r}{\partial r} + \frac{C_r}{r} \right) - \rho C_r C_z = \Lambda \left( \frac{\partial C_r}{\partial z} + \frac{C_r}{r} \right) \frac{C_z}{r} \\
\rho \partial C_\theta^2 &= 2A \left( \frac{1}{r} \frac{\partial C_\theta}{\partial \theta} + \frac{C_\theta}{r} \right) - \rho C_r C_\theta = \Lambda \left( \frac{1}{r^2} \frac{\partial C_\theta}{\partial r} + \frac{C_\theta}{r} \right) \\
\rho \partial C_z^2 &= 2A \left( \frac{\partial C_z}{\partial z} + \frac{C_z}{r} \right) - \rho C_r C_z = \Lambda \left( \frac{\partial C_z}{\partial r} + \frac{C_z}{r} \right)
\end{align*}
\]  

(12)

where \( \Lambda \) is the coefficient of turbulent viscosity.

Taking into account that the coefficient of turbulent viscosity is a linear function of radius, that is, it increases from the center of the tube to its periphery (wall), then the boundary conditions for solving the systems of equations (8)—(12) will be as follows:

\[
\begin{align*}
C_r &= \omega; C_\theta = \omega \rho; C_z &= 0; P = P_0 \quad \text{at} \quad z = 0, \\
C_r &= 0; C_\theta = 0; C_z = \text{const}; P = \text{const} \quad \text{at} \quad z \to \infty,
\end{align*}
\]

(13)

where \( P \) is the pressure of the carrier medium, \( \rho \omega \) is the angular velocity of the flow rotation downstream the swirler, rad/s.

For the above parameters of the input stream, the system of equations (8)—(12) was computed using the method of finite differences with replacement of the first derivatives with the left differences, and the second derivatives with the right differences. The resulting system of equations with respect to the value of the unknown functions at the grid nodes was solved by the method of coordinate descent.

As a result, the components of the velocity of the first flow (O-I) can be determined from the expressions:

\[
\begin{align*}
C_r &= \rho \omega F_r(z); \\
C_\theta &= \rho \omega F_\theta(z); \\
C_z &= \sqrt{\rho \omega} \cdot F_z(z).
\end{align*}
\]

(14)

where \( B = (1 + A/\mu) \nu; \) \( \mu \) is coefficient of dynamic viscosity \( \text{ns/m}^2; \) \( \nu \) is coefficient of kinematic viscosity \( \text{m}^2/\text{s}. \)

In this case, in accordance with [18–20], the following quantity is considered as the parameter of flow twist:

\[
\Lambda_\theta = \sqrt{\frac{K-1}{2}} \zeta_0,
\]

(15)

where \( K \) is the index of the gas property, \( \zeta_0 = \frac{C_0}{a_0} = \sqrt{(K-1)} \); \( i_0 \) is speed of sound in the state of braking; \( i_0 \) is the specific enthalpy of the moving gas.

The values of the calculation functions \( F_r, F_\theta, F_z \) dimensionless velocity components, are shown in Fig. 5.

Figure 5. Distribution of functions \( F_r(z), F_\theta(z), F_z(z) \) along the length of the vortex zone: \( F_r(z) \) function (a); \( F_\theta(z) \) function (b); \( F_z(z) \) function (c)

Parameter (15) characterizing intensity of flow swirling at each radius in the initial section is explicitly related to the degree of expansion of the working substance in the device. The calculations were carried out for various \( L(z)/D_z \) ratios and the values of 0.05; 0.1; 0.2; 0.25 m were considered as \( D_z \). We admit that the swirlers had diameters analogous to the \( D_z \) dimension and the width of rings was 0.014 m in all cases. Each ring had 8 tangential \( 0.003 \text{ m} \) diameter round holes and tangential \( 0.3-0.8 \text{ m} \) diameter supersonic nozzles. The slots and holes were evenly distributed around the swirler perimeter.

It was also assumed that the area of the annular slot at the B-B section was at least 10 times larger than the total area of the entry slots of the swirler. Thus, at a steady flow rate \( Q_0 \), parameter (15) changed as a function of \( D_z \) due to a change in the linear velocity of the gas flow ahead of the swirler inlet. This velocity varied from 50 to 150 m/s for the abovementioned mass flows.

Analysis of the obtained dependences (graphs) in Fig. 5 allows us to conclude that intensity of motion of the gas flows changes significantly along the tube length and it was observed that rotation along the device was damping with the growth of the distance from the swirler. The tangential velocity component \( C_\theta \) (function \( F_\theta(z) \)) is significantly reduced along the pipe length. The higher intensity of the
working substance twist, the more pronounced is the law of the flow velocity decrease along the zone length (Fig. 5). The nature of this decrease is similar for any value of \( D_r \) and its intensity depends on the law of distribution of the angular velocities along the radius in the initial section and is specified by parameters of the working substance, the twist parameter and the tube geometry. It is characteristic that the axial components of velocity in the section under consideration (Fig. 5) are directed from the mixing zone toward the diaphragm, that is against the motion of the hot stream. The numerical value of \( F(z) \) is small but, nevertheless, this indicates appearance of reverse currents. As can be seen from Fig. 5 (\( F(z_1); F(z_2) \)), the reverse flow exists in a zone close to the vortex chamber. Apparently, the vortex chamber has an annular shape. Further, when the flow moves to the B-B section, distribution of \( C_v \) becomes different, the zone of quasi-solid rotation reduces, the absolute value of \( C_v \) decreases due to friction of the gas flow against the wall and internal friction between the gas layers and the boundary layer grows. Apparently, the bulk of the aerosol moves in the boundary layer or a layer close to it. This aerosol aggregates (coagulates) in the zone of quasi-solid rotation where maximum values of \( C_v \) are attained (Fig. 5) as shown in [27, 28]. Motion of a dispersed phase in the vortex tube is determined by integrating the dynamic equations:

\[
\begin{align*}
\frac{dz}{dt} &= v_z; \quad m \frac{dv_z}{dt} = F_z, \\
\frac{dr}{dt} &= v_r; \quad m \frac{dv_r}{dt} = F_r + \frac{m v^2_r}{r}, \\
\frac{d\phi}{dt} &= v_\phi; \quad m \frac{dv_\phi}{dt} = F_\phi,
\end{align*}
\]

(16)

where \( z, r, \phi \) are cylindrical coordinates of the particle; \( v_z, v_r, v_\phi \) are components of the particle velocity vector; \( F_z, F_r, F_\phi \) are components of the force acting on the particle from the gas side; \( z \) is time; \( m = \rho (\pi d^3/6) \) is the particle mass; \( \rho \) is the aerosol material density; \( d \) is the aerosol particle diameter.

To find the force \( F \) vector, use the following expression:

\[
\vec{F} = \theta \rho v \frac{\pi d^3}{6} \left[ \frac{\vec{W} - \vec{u}}{2} \right] - \frac{\pi d^3}{6} \nabla P,
\]

(17)

where \( \theta \) is the coefficient of aerodynamic resistance,

\[
\theta = \frac{24}{Re} \left( 1 + \frac{Re^{2/3}}{6} \right); \quad Re = \left( \frac{|W - u| d}{v} \right)
\]

and \( v \) is kinematic gas viscosity; \( \rho \) is the gas density; \( \vec{W}(z, r, \phi) \) is the gas velocity field; \( P(z, r, \phi) \) is the field of gas pressures.

The system of equations (16) was solved numerically using the Runge-Kutta method for the following initial conditions: \( z = 0; z = 0; r = r_0; \phi = 0; u_z = 0; u_r = 0; u_\phi = 0 \). The calculated trajectories of motion of the aerosol particles which have entered the zone of the vortex zone at various radii, are presented in Fig. 6.

As can be seen from Fig. 6, the aerosol particles rapidly gain tangential and radial velocity and run under the action of inertial forces along a spiral path to the device periphery (towards the cylinder walls and the B-B section) regardless of the working zone radius. It should be noted that trajectories of the particles correlate with the trajectory of a change in the function \( F_{\text{zr}} \) (Fig. 5) and, apparently, conditions are created in the 0+0.8 m section for aggregation of aerosol particles where addition of one element to another is the dominant mechanism. As a result, aerosol moves in the quasi-potential section of the tube (III-III-B section), near the walls of the device in the boundary layer or in a layer close to it.

If aggregation (enlargement) of the aerosol particles actually takes place, then the jumping motion of the rotating aerosol particles and their adjacent vortices must turbulize flow in the layer close to the wall, that is, an effect equivalent to an increase in the Re number should be observed. If the average velocity \( W_{\text{av}} \) of the flow remains unchanged in this case, then

\[
\Delta P = \rho \frac{L}{D_r} \lambda \rho W_{\text{av}}^2,
\]

(18)

where \( \lambda \) is friction coefficient decreasing with an increase of Re number.

Proceeding from (18), a decrease in resistance with increase in the size of aerosol particles should be expected since only large particles can cause appearance of adjacent vortices. The ash particles will influence mixing of gas particles, the gaseous toxic impurities, their distribution over the volume of the device and kinetics of toxicant conversion. These effects will occur when hydraulic resistance decreases. The ash particles differ in their density from the carrier medium. Thus, at the exit from the vortex tube (B-B section), a layerwise flow is considered with its core of minimum aerosol concentration and the periphery is saturated with aerosol particles. For removal of particles from the volume of the vortex chamber according to Fig. 4, there are a separation tube (5), a forebunker (6) and an aerosol discharge tube (7). These devices should be structurally designed in relation to the concrete operating conditions and concrete vortex tubes.

The energy balance equation (10) together with the equation of radial equilibrium and relationship \( P = (k/k-1) (P/p) \) can be written as:

\[
\begin{align*}
\frac{2}{Pr} \frac{d}{dr} & \left( r \frac{d}{dr} \right) \left( r \frac{d}{dr} \right) = 2 \frac{F^2}{Pr} \frac{d}{dr} \left( \frac{d}{dr} \right) - r^2 \left[ d \left( r / r^2 \right) / dr \right]^2 - \frac{F^2}{Pr} \frac{d}{dr} \left( \frac{d}{dr} \right) - \frac{d}{dr} \left( rC, dC / dr \right),
\end{align*}
\]

(19)

If the radial diagrams of velocities \( C_0(r) \) and \( C_r(r) \) are known, then double integration of equation (19) will give a diagram of thermodynamic temperatures.
In the first integration, an arbitrary constant is equal to zero if adiabaticity and conditions of adhesion on the channel wall are taken as boundary conditions. In the second integration, the condition of maintaining total energy along the channel must be met:

\[ m_{\text{ij}} = 2\pi \int_{\theta_{-}}^{\theta_{+}} i_{\text{ij}} \rho C_{r} r dr, \]  
(20)

where \( i_{\text{ij}} \) is the enthalpy of braking a flow with mass of \( m_{\text{ij}} \rightarrow \left( Q_{\text{ij}} \right) \) at the entrance to the channel.

To analyze the effect of redistribution of the total energy in the annular channel, the following relationships are used:

\[ r = C_{r} r^{m}, \]

\[ \frac{d\Gamma}{dr} = C_{t} \left( 1-m \right) r^{m}, \]

where \(-1 \leq m \leq 1\). In this case, relationship between the axial mass and correspondingly equal to the heat supplied to the hot temperature in the annular channel by the swirler reduces to:

\[ \Delta \Gamma = \frac{\Delta Q}{m} r^{m}. \]

The maximum degree of separation of the braking temperature in the annular channel by the swirler relative to \( 0=2r/(R-r) \) with flow parameters at the middle diameter at

\[ \lambda_{\text{pr}} = \sqrt{\frac{C_{p}^{2} - C_{t}^{2}}{2k/k+1 - R^{2} T_{0}}} \]

is

\[ \Delta T_{\text{pr}} = \frac{T_{10} - T_{1b}}{T_{10}} = \]

\[ = 1 + \left[ \left( 1-S \right) \frac{k-1}{k+1} \lambda_{\text{pr}}^{2} \cos^{2} \beta_{\text{pr}} \left( \frac{\theta_{0}}{\theta_{1}} \right)^{2} \left( 1 - \frac{\theta_{0}}{\theta_{1}} \right)^{2 - 1} \right]^{-1}, \]
(22)

where

\[ S = \left( \frac{1 + 3}{m} \right) \]

\[ \Delta T_{\text{m}} = T_{10} - T_{1b}; \]

\[ \Delta T_{\text{pr}} = T_{10} - T_{1b}; \]

Then the amount of heat attributed to the mass of the flowing gas \( m_{\text{ij}} \rightarrow \left( Q_{\text{ij}} \right) \) taken from the cold fraction \( m_{\text{ij}} \rightarrow \left( Q_{\text{ij}} \right) \) and correspondingly equal to the heat supplied to the hot mass \( m_{\text{ij}} \rightarrow \left( Q_{\text{ij}} \right) \) will be:

\[ q_{\text{ic}} = B \cdot C_{p} \cdot \Delta T_{\text{pr}} = \left( 1 - B \right) C_{p} \cdot \Delta T_{\text{pr}}, \]
(23)

where

\[ B = \frac{m_{\text{ij}} - \left( Q_{\text{ij}} / \rho \right)}{m_{\text{ij}} / \rho}; \]

\[ C_{p} \] is the heat capacity of the gas mixture at a constant \( P \).

5. Discussion of the results obtained in studies of the process of scrubbing a gaseous toxicants.

For the conditions considered in this paper, function \( q_{\text{ic}}(B) \) has a maximum at \( B=0.6 \), therefore the range of \( 0.2 \leq B \leq 0.8 \) can be taken as the working range of the vortex tube where the temperature separation takes place. Calculations performed at \( \rho_{\text{g}}=1.07 \text{ kg/m}^{3}, \rho_{\text{aer}}=3.40 \text{ kg/m}^{3}, k=1.38 \), \( T_{10}=473 \text{ K}, \) \( v_{\text{gas}}=4.25 \times 10^{4} \text{ m}^{2}/s, \) average molecular mass of the gas flow \( \mu_{\text{g}}=28.3 \times 10^{-3} \text{ kg/mole}, \) aerosol content \( m_{\text{aer}}=0.20 \text{ g/s}, \) aerosol particle (spherical shape) size \( d=15-40 \mu m, \) gas flow rate \( 0.8 \text{ m}^{2}/s, \) \( D_{1}=0.05 \text{ m} \) allowed us to obtain the \( \Delta T_{\text{pr}} \) dependence on the value of “B” which is shown in Fig. 7.

As is seen in Fig. 7, with growth of aerosol concentration to 0.6 g/s, the cooling effect of flow \( Q_{\text{c}} \) increases and the maxi- \( \Delta T_{\text{pr}} \) shift slightly toward smaller “B” values. A fur- ther increase in the aerosol concentration leads to pulsating changes of \( \Delta T_{\text{pr}} \) toward lower values. Therefore, the most rational range of “B” values for treatment of the exhaust gas in a vortex tube appears to be within 0.2-0.5.
6. Conclusions

1. Exhaust gases emitted from process installations contain dust in a concentration of 3 to 15–20 g/Nm³. Fractional distribution of dust is described by a function of multimodal distribution and belongs to a class of fine and very fine dusts with a predominant particle size of less than 15 μm. Specific surface of dust particles is 3 to 10 m²/g. To ensure effective operation of dust collectors, preliminary preparation of the gas-dust flow is necessary. It changes physico-chemical characteristics of the flow. As the analysis showed, one of the ways to the problem solution is to pretreat the exhaust process gas streams in vortex tubes under conditions ensuring occurrence of the Ranque effect.

2. Axially symmetrical motion of the gas flow with its rotation in the vortex tube was described by the system of Navier-Stokes equations, equations of continuity, energy and gas state using an empirical relationship between viscosity and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients. When solving a system of equations, relationship between the turbulent stress tensor and temperature coefficients.

Thus, the main conclusion is that the preliminary treatment of the dust-gas flow in the vortex tubes will increase efficiency of the dust collectors and ensure overall purification of exhaust gases.
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