Empirical constraints on the star formation and redshift dependence of the Ly$\alpha$ ‘effective’ escape fraction

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ABSTRACT

We derive empirical constraints on the volume-averaged ‘effective’ escape fraction of Ly$\alpha$ photons from star-forming galaxies as a function of redshift, by comparing star formation functions inferred directly from observations to observed Ly$\alpha$ luminosity functions. Our analysis shows that the effective escape fraction increases from $f_{\text{esc}}^{\text{eff}} \sim 1$–5 per cent at $z = 0$ to $f_{\text{esc}}^{\text{eff}} \sim 10$ per cent at $z = 3$–4 and to $f_{\text{esc}}^{\text{eff}} = 35$–50 per cent at $z = 6$. Our constraint at $z = 6$ lies above predictions by models that do not include winds, and therefore hints at the importance of winds in the Ly$\alpha$ transfer process (even) at this redshift. We can reproduce Ly$\alpha$ luminosity functions with an $f_{\text{esc}}^{\text{eff}}$ that does not depend on the galaxies star formation rates ($\psi$) over up to $\sim$2 orders of magnitude in the Ly$\alpha$ luminosity. It is possible to reproduce the luminosity functions with an $f_{\text{esc}}^{\text{eff}}$ that decreases with $\psi$ – which appears favoured by observations of drop-out galaxies – in models which include a large scatter ($\sigma \gtrsim 1.0$ dex) in $f_{\text{esc}}^{\text{eff}}$ and/or in which star-forming galaxies only have a non-zero $f_{\text{esc}}^{\text{eff}}$ for a fraction of their lifetime or a fraction of sightlines. We provide a fitting formula that summarizes our findings.

Key words: line: formation – radiative transfer – intergalactic medium – galaxies: ISM – cosmology: observations – ultraviolet: galaxies.

1 INTRODUCTION

The Ly$\alpha$ emission line is one of the most prominent features in the intrinsic spectrum of star-forming galaxies (e.g. Partridge & Peebles 1967; Schaerer 2003; Johnson et al. 2009). The presence of a luminous, redshifted Ly$\alpha$ line has been used to spectroscopically confirm – and find – galaxies out to $z \sim 7$ (e.g. Iye et al. 2006; Ota et al. 2010; Rhoads et al. 2012).

Ly$\alpha$ emitting galaxies (LAEs hereafter) are of interest for various reasons, including for example: (i) their continua are typically fainter than – and hence complement samples of – continuum-selected (i.e. drop-out-selected) galaxies; (ii) LAEs at $z > 5$ provide an independent probe of the reionization epoch, as the Ly$\alpha$ line is affected by neutral intergalactic gas (e.g. Haiman & Spaans 1999; Malhotra & Rhoads 2004); (iii) as Ly$\alpha$ photons likely scatter through the interstellar media (ISM) of galaxies, the total distance they travel through the ISM is enhanced compared to that of continuum photons. Ly$\alpha$ photons are therefore thought to provide a sensitive probe of the dust content (and gas kinematics) of the ISM; (iv) Ly$\alpha$ selected galaxies will be used to probe the equation of state of the dark energy at $z = 1.9$–$3.5$ by the HETDEX$^2$ experiment (Hill et al. 2008).

The main uncertainty that affects interpretations of Ly$\alpha$ observations of LAEs relates to the complex radiative transfer (RT) of Ly$\alpha$ photons through both the ISM, the circum galactic medium (CGM) and intergalactic medium (IGM; e.g. Zheng et al. 2010; Cantalupo, Lilly & Haehnelt 2012; Dijkstra & Kramer 2012; Jeeson-Daniel et al. 2012; Laursen, Duval & Ostlin 2012; Verhamme et al. 2012). Moreover, these processes are not independent: RT at the ISM level affects how the RT proceeds at the intergalactic level. In recent years, Ly$\alpha$ RT has been modelled on all these scales, usually

$^2$http://www.hetdex.org

$^3$The simplest way to illustrate this dependence is by considering that scattering of Ly$\alpha$ photons through outflows of H$\text{I}$ gas (on – say – kpc scales) results in an overall redshift of the Ly$\alpha$ spectral line relative to other nebular lines (e.g. Zheng & Miralda-Escudé 2002; Dijkstra, Haiman & Spaans 2006; Verhamme et al. 2008). This overall redshift of the Ly$\alpha$ line reduces the probability that these photons subsequently scatter in the IGM (Dijkstra & Wyithe 2010).
by combining simulated galaxies with Lyα RT calculations (e.g. Tasitsiomi 2006; Laursen & Sommer-Larsen 2007; Barnes et al. 2011; Verhamme et al. 2012; Yajima et al. 2012). These calculations are extremely difficult to carry out from first principles (see Dijkstra & Kramer 2012) and ultimately must be constrained by observations.

The goal of this paper is to provide empirical (i.e. based purely on observations) constraints on the dependence of the effective escape fraction$^4$ of $L_\alpha$ photons, $f_{\text{esc}} = L_\alpha / L_{\alpha,\text{int}}$, where $L_\alpha$ ($L_{\alpha,\text{int}}$) denotes the observed (intrinsic) Lyα luminosity. Our goal is to constrain $f_{\text{esc}}$ as a function of redshift (as in Blanc et al. 2011; Hayes et al. 2011). Furthermore, we investigate whether $f_{\text{esc}}$ depends on the star formation rate of galaxies. Previous works by Hayes et al. (2011) and Blanc et al. (2011) constrained the volume-averaged effective escape fraction $f_{\text{esc}}$ by comparing the star formation rate density, $\rho_\ast$, inferred from the observed Lyα luminosity density to $\rho_\ast$ inferred from other observations. This method is highly non-trivial, because Lyα observations only detect galaxies for which $f_{\text{esc}}$ exceeds some star formation rate-dependent value (at very low star formation rates, the Lyα flux falls below the detection threshold even when all Lyα photons made it to the observer), and one must attempt to account for this. For example, Hayes et al. (2011) use ultraviolet (UV)-luminosity functions of drop-out galaxies to estimate the appropriate value for $\rho_\ast$ at $z \gtrsim 2.5$, and a significant part of their analysis is devoted to choosing the proper lower integration limit when integrating over the UV-luminosity function. Our method uses star formation rate functions to estimate $f_{\text{esc}}$. We show that this allows for more direct constraints which circumvent the difficulties associated with choosing such integration limits.

The outline of this paper is as follows: we describe in Section 2, how we combine observations of Lyα luminosity functions of LAEs with observations of star formation functions to put constraints on the effective escape fraction of Lyα photons, $f_{\text{esc}}$. We present our main results in Section 3 before presenting our conclusions in Section 5.

2 EMPIRICAL CONSTRAINTS ON THE LYα EFFECTIVE ESCAPE FRACTION

Lyα photons – just as H$\alpha$ photons – are emitted following recombination events in H II regions, and they closely trace ongoing star formation. The H$\alpha$ luminosity of a galaxy is related to its star formation rate, denoted with $\psi$, as $L_{\text{H}\alpha} = 1.2 \times 10^{41} \times (\psi / [\text{M}_\odot \text{yr}^{-1}]) \text{erg s}^{-1}$ (Kennicutt 1998, this conversion assumes a Salpeter initial mass function (IMF) in the mass range $0.1 – 100 \text{M}_\odot$). The intrinsic Lyα luminosity, denoted with $L_{\text{Ly}\alpha,\text{int}}$, is $\approx 8 \times$ larger than the H$\alpha$ luminosity (for case-B recombination and $T = 10^4$ K; e.g. Hayes et al. 2011), and we have

$$L_{\text{Ly}\alpha,\text{int}} = k \times \left(\frac{\psi}{\text{M}_\odot \text{yr}^{-1}}\right),$$

where $k = 10^{42} \text{erg s}^{-1}$. The factor $k$ can be higher (or lower) by a factor of $\sim 2$ depending on the assumed IMF and/or stellar metallicity. In the extreme case of a top-heavy IMF and zero-metallicity stars, the factor $k$ can be as high as $k \approx 10$ (Raiter, Schaerer & Fosbury 2010). Our constraints on $f_{\text{esc}}$ scale with our assumed $k$ as $f_{\text{esc}} \propto k^{-1}$. The ‘observed Lyα luminosity’, defined as the observed flux multiplied by $4\pi d_L^2(z)$ [$d_L(z)$ denotes the luminosity distance out to redshift $z$], is

$$L_\alpha = f_{\text{esc}}(\psi,z) \times L_{\alpha,\text{int}},$$

where $f_{\text{esc}}(\psi,z)$ denotes the effective escape fraction of Lyα photons.

We constrain the parameter $f_{\text{esc}}(\psi,z)$ by comparing observed Lyα luminosity functions to observationally inferred star formation functions: the Lyα luminosity function, denoted by $\Phi(z) \frac{\text{d}n}{\text{d} \log L_\alpha}$, measures the comoving number density of galaxies with (the logarithm of) their Lyα luminosities in the range $\log L_\alpha \pm \log L_\alpha/2$. The star formation function, denoted with $\frac{\text{d}n}{\text{d} \log L_\alpha}$, measures the comoving number density of galaxies that are forming stars at a rate (whose logarithm is) in the range $\log L_\alpha \pm \log L_\alpha/2$. We describe the star formation functions used in our analysis and how we convert these into Lyα luminosity functions in Section 2.1. This conversion depends on $f_{\text{esc}}(\psi,z)$, and we use observed Lyα luminosity functions to obtain constraints in Section 3.

2.1 Star formation functions

Star formation functions can be described by Schechter functions:

$$\frac{\text{d}n}{\text{d} \log \psi} = \ln 10 \times \psi \times \Phi_\ast \left(\frac{\psi}{\Phi_\ast}\right)^{-\alpha} \exp(-\psi/\psi_\ast).$$

We adopt the redshift dependent Schechter function parameters from Smit et al. (2012, their table 3).$^5$ Our results are insensitive to this choice (see Section 4.3).

In the absence of scatter, there is a one-to-one relation between $\psi$ and $L_\alpha$. The Lyα luminosity functions then relate to the star formation functions as

$$\frac{\text{d}n}{\text{d} \log L_\alpha} = \frac{\text{d}n}{\text{d} \log \psi} \frac{\text{d} \log \psi}{\text{d} \log L_\alpha} = \frac{\text{d}n}{\text{d} \log \psi} \left|_{\psi=\psi_\ast(k^{f_{\text{esc}}})} \right.,$$

where in the last equality, we used equations (1) and (2).

In reality, we do not expect each galaxy that forms stars at some rate $\psi$ to have exactly the same $f_{\text{esc}}$. It is therefore reasonable to study models in which we assume that there is a dispersion (or scatter) in $f_{\text{esc}}$ at a fixed $\psi$. In the presence of scatter, we generally have

$$\frac{\text{d}n}{\text{d} \log L_\alpha} = \int_{-\infty}^{\infty} \text{d} \log \psi \frac{\text{d} \log \psi}{\text{d} \log L_\alpha} P(\log L_\alpha | \log \psi),$$

where $P(\log L_\alpha | \log \psi)$ denotes the probability that a galaxy that is forming stars at a rate $\psi$ has a Lyα luminosity in the range $\log L_\alpha \pm \log L_\alpha/2$. We assume that the effective escape fraction $f_{\text{esc}}$ has a scatter$^6$ that is described by a (truncated) log-normal

$^4$The term ‘effective escape fraction’ was coined previously by Nagamine et al. (2010) and is often simply referred to as ‘escape fraction’. In Section 4.1, we argue why we caution against universal usage of the term escape fraction and why it helps to distinguish between an escape fraction and an effective escape fraction.

$^5$For the data at $z = 0.35$, we use the values from Bell et al. (2007). For the data at $z = 0.35$, Bell et al. (2007) combine UV and mid-infrared luminosity functions to construct their star formation functions. For the higher redshift star formation functions, Smit et al. (2012) construct star formation functions from UV luminosity functions (LFs), combined with constraints on the slope of the rest-frame continuum $\beta$.

$^6$We assume for simplicity that this scatter is independent of star formation rate $\psi$. Garel et al. (2012) have recently presented a model in which the scatter in $f_{\text{esc}}$ increases with $\psi$. 

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distribution. That is,
\[
P(\log L_\alpha | \log \psi) = \frac{dP}{d\log L_\alpha} = \frac{dP}{d\log f_{\text{esc}}^\alpha | f_{\text{esc}}^\alpha = L_\alpha/(k \psi)} = \begin{cases} 
\mathcal{N}
\end{cases} \left( \frac{-[\log f_{\text{esc}}^\alpha - \log f_{\text{esc}}^\alpha(\psi)]^2}{2\sigma^2}, \right)
\]
where \( \mathcal{N} \) denotes a factor that ensures that the function \( dP/d\log f_{\text{esc}}^\alpha \) is normalized.

2.2 Constraining the effective escape fraction

We first assume that \( f_{\text{esc}}^\alpha(z, \psi) = f_{\text{esc}}^\alpha(z) \). That is, we first assume that \( f_{\text{esc}}^\alpha \) is independent of the star formation rate \( \psi \). We make this assumption because we will show later that the Ly\( \alpha \) luminosity functions are surprisingly consistent with this assumption.

Our analysis focusses on the Ly\( \alpha \) luminosity functions centred on \( z = 0.35 \) from Deharveng et al. (2008, red filled circles) and Cowie, Barger & Hu (2010, blue filled squares), and \( z = 3.1, 3.7 \) and 5.7 from Ouchi et al. (2008, red filled circles) and Cassata et al. (2011, blue filled squares). At each redshift, we compute the posterior probability for a range of \( f_{\text{esc}}^\alpha \) as \( P(f_{\text{esc}}^\alpha) \propto \int dX \mathcal{L}(f_{\text{esc}}^\alpha) P(f_{\text{esc}}^\alpha | P(X), \mathcal{L}[f_{\text{esc}}^\alpha] = \exp[-0.5\chi^2] \) denotes the likelihood, in which \( \chi^2 = \sum N_{\text{data}}(\text{model}, - \text{data})_i^2/\sigma_i^2 \). The function \( P(f_{\text{esc}}^\alpha) \) denotes the prior probability distribution for \( f_{\text{esc}}^\alpha \). We stress however that our results do not depend on our choice of prior.

Finally, the vector \( X \) contains the three Schechter function parameters \( X^T = (\alpha, \psi_\alpha, \Phi_\alpha) \). The function \( P_i(X) \) describes the prior probability for having any combination of parame-

ters: we assumed that \( P_i(X) \) is a multivariate Gaussian, i.e. \( P_i(X) = \mathcal{N}[\mathcal{N} \exp\left[-\frac{1}{2}(X - \mu_i)^T C_i^{-1}(X - \mu_i)\right], \mathcal{N} \) denotes the normalization factor. The vector \( \mu \) contains the best-fitting values for each of the parameters. The covariance matrix \( C \) contains the measured uncertainties on the parameters (the most likely values and their uncertainties were taken from Smid et al. 2012).

3 RESULTS

3.1 No scatter

Fig. 1 shows four panels, each of which corresponds to one redshift bin. The observed Ly\( \alpha \) luminosity functions that we used in our analysis are shown as the data points. The inset in each panel shows \( \mathcal{L}[f_{\text{esc}}^\alpha] \) as a function of \( f_{\text{esc}}^\alpha \). These panels contain two lines, both of which were obtained by fitting to a single data set. For example, we obtained the black solid line (red dashed line) in the \( z = 0.35 \) panel by fitting to the data from Deharveng et al. (2008) (Cowie et al. 2010). At each redshift, we show the model luminosity functions

\[\text{Figure 1.} \quad \text{The figure compares observed Ly} \alpha \text{ luminosity functions (indicated by the data points) of LAEs at } z = 0.35 \text{ (upper left), 3.1 (upper right), 3.7 (lower left) and 5.7 (lower right) with model predictions under the assumption that there is a one-to-one relation between star formation rate } \psi \text{ and observed Ly} \alpha \text{ luminosity } L_\alpha \text{ (see equation 4, i.e. there is no scatter in } f_{\text{esc}}^\alpha) \text{, for the best-fitting observed Ly} \alpha \text{ fraction, } f_{\text{esc}}^\alpha \text{ (as black solid lines and red dashed lines, see the text). The insets show the likelihood } L(f_{\text{esc}}^\alpha) \text{ as a function of } f_{\text{esc}}^\alpha. \text{ This figure illustrates that the models reproduce the Ly} \alpha \text{ luminosity functions well, except at the bright end (which may be contaminated by low-luminosity AGN; see Ouchi et al. 2008). It is worth pointing out that } f_{\text{esc}}^\alpha \text{ is independent of } \psi \text{ in our models.} \]
for which both $L_\alpha$ curves are maximized, using the same line colour and style.

The upper-left panel shows that the data from Deharveng et al. (2008) translate to $f_{\text{esc}} = 8.5 \pm 3\%$, while the data from Cowie et al. (2010) imply $f_{\text{esc}} = 3.7 \pm 1\%$ at $z = 0.35$. Here, the error bars denote 68 per cent confidence levels, where we use the so-called ‘shortest interval’ method (see Andrae 2010, and references therein) to determine the confidence intervals. The upper-right panel shows that the effective escape fraction increases to $f_{\text{esc}} = 17 \pm 5\%$ per cent ($f_{\text{esc}} = 10 \pm 3\%$ per cent) for the Ouchi et al. (2008) (Cassata et al. 2011) data at $z = 3.1$ and to $f_{\text{esc}} = 17_{-3}^{+4}\%$ per cent ($f_{\text{esc}} = 8 \pm 3\%$ per cent) for the Ouchi et al. (2008) (Cassata et al. 2011) data at $z = 3.7$. Finally, we find that the effective escape fraction increases to $f_{\text{esc}} = 57_{-3}^{+34}\%$ per cent ($f_{\text{esc}} = 44_{-23}^{+29}\%$ per cent) for the Ouchi et al. (2008) data (Cassata et al. 2011) at $z = 5.7$.

Our quoted uncertainties are statistical only and do not take into account systematic uncertainties associated with the determination of observed Ly$\alpha$ luminosity functions. The different constraints we obtain on $f_{\text{esc}}$ from different data sets may reflect these systematic uncertainties: in particular, at $z \geq 3.1$, the data from Ouchi et al. (2008) derive from a narrow-band survey for LAEs, while the data from Cassata et al. (2011) derive from a deep spectroscopic survey (see Section 4.3 for a more detailed discussion of systematic uncertainties).

Fig. 1 shows that our models reproduce the individual data sets of observed Ly$\alpha$ luminosity functions well. Different data sets can result in slightly different constraints on $f_{\text{esc}}$. It is striking that at $z \geq 3.1$ (especially $z = 3.1$ and 5.7), a $\psi$-independent $f_{\text{esc}}$ reproduces the observations well over up to two orders Ly$\alpha$ luminosity, and therefore $\psi$. If anything, our models do not produce enough bright LAEs, which could be solved by having $f_{\text{esc}}$ increase with $\psi$. Note, however, that Ouchi et al. (2008) point out that the bright end (i.e. at log $L_\alpha \gtrsim 43.4$) may be contaminated by low-luminosity active galactic nucleus (AGN). The only data set that we cannot reproduce well is that of Cowie et al. (2010): our model predicts significantly fewer LAEs than their two data points at log $L_\alpha \gtrsim 42.3$. This discrepancy could again be (partially) resolved by having $f_{\text{esc}}$ increase with $\psi$. As we show below (in Section 3.2), we also significantly improve the agreement with the data when we introduce a scatter in $f_{\text{esc}}$.

3.2 With Scatter

Fig. 2 shows the same as Fig. 1, but here the models include a dispersion in $f_{\text{esc}}$ (equation 5), which is described by a (truncated) log-normal distribution with a standard deviation of $\sigma = 0.5$. This choice for $\sigma$ is a bit arbitrary, but can be justified by the work of Dijkstra & Westra (2010), who found that the ratio of the Ly$\alpha$ to the UV-derived star formation rate can be described by a log-normal distribution with $\sigma = 0.4$. We stress that changes to our main results are insignificant, even if we adopted $\sigma = 1.0$.

Fig. 2 shows that a dispersion in $f_{\text{esc}}$ flattens the predicted luminosity functions and smoothens out the sharp turnover in the predicted luminosity function. Both these changes help to improve the fit to the data at $z = 0.35$ (and also at the highest $L_\alpha$ data point at $z = 3.1$). Importantly, these models obtain constraints from different data sets that agree better with each other: for example, the best-fitting models to the data from Ouchi et al. (2008) also provide decent fits to the data from Cassata et al. (2011).

In spite of the flattening of the predicted luminosity functions, these models still reproduce the data with a (log $f_{\text{esc}}$) that is independent of $\psi$. It is only when we adopt $\sigma > 1.0$ that the predicted luminosity functions become flatter than the observations. Garel et al. (2012) have recently predicted that the scatter in $f_{\text{esc}}$ increases with $\psi$ and that it may be even larger than this at $\psi \gtrsim 20$ M$_\odot$ yr$^{-1}$. For models that include this large scatter, the data would require

![Figure 2](https://academic.oup.com/mnras/article-abstract/435/4/3333/1031271)

**Figure 2.** Same as Fig. 1, but for models in which we assume that there is a dispersion in $f_{\text{esc}}$, described by a log-normal distribution with a standard deviation of $\sigma = 0.5$, at a fixed $\psi$. This figure shows that a dispersion in $f_{\text{esc}}$ flattens the predicted Ly$\alpha$ luminosity functions, which improves the agreement with the data at $z = 0.35$ and at high $L_\alpha$. These models obtain constraints from different data sets that agree better with each other.
(log $f_{\text{esc}}$) to decrease with $\psi$. Such a requirement would be expected given observations of drop-out galaxies, which show evidence that the fraction of continuum-selected galaxies that have ‘strong’ Ly$\alpha$ emission lines increases towards lower UV-luminosities (e.g. Stark et al. 2010). This suggests that Ly$\alpha$ photons have an easier time escape from galaxies with lower UV luminosities and therefore likely from galaxies with lower star formation rates $\psi$.

The insets in Fig. 2 show $\mathcal{L}$(log $f_{\text{esc}}$)). For example, the inset in the upper-right panel shows that the best-fitting (log $f_{\text{esc}}$) at $z = 3.1$ is $10^{\log f_{\text{esc}}(3.1)} \sim 0.06 - 0.1$, which lies a factor of $\sim 1.7$ below the best-fitting $f_{\text{esc}}$ we derived for the model with no scatter. The best-fitting values of (log $f_{\text{esc}}$) depend on the choice of $\sigma$; the larger $\sigma$, the smaller (log $f_{\text{esc}}$). The reason for this reduction is that at fixed (log $f_{\text{esc}}$), increasing $\sigma$ increases the expectation value $E(f_{\text{esc}}(\sigma))$, which is given by $E(f_{\text{esc}}(\sigma)) = \int d\sigma f_{\text{esc}} f_{\text{esc}} d\sigma$. We have verified that this best-fitting expectation value $E(f_{\text{esc}})$ barely depends on our choice of $\sigma$.

We now practically have three ‘measures’ of $f_{\text{esc}}$ [namely $f_{\text{esc}}$, (log $f_{\text{esc}}$) and $E(f_{\text{esc}}(\sigma))$], which may be a bit confusing. We have therefore briefly summarized the meaning of these symbols in Table 1.

### Table 1. Summary of ‘different’ measures of $f_{\text{esc}}$.

| Symbol | Meaning |
|--------|---------|
| $f_{\text{esc}}$ | Effective escape fraction of Ly$\alpha$ of a galaxy |
| (log $f_{\text{esc}}$) | Mean of log $f_{\text{esc}}$ in a log-normal PDF (equation 6) |
| $E(f_{\text{esc}}(\sigma))$ | Expectation value of $f_{\text{esc}}$ for models with a log-normal PDF |

#### 3.3 Comparing our $f_{\text{esc}}(z)$ with previous works

We compare our inferred redshift evolution of $f_{\text{esc}}(z)$ to the power-law fitting function from Hayes et al. (2011) in Fig. 3. The filled symbols represent the constraints on $f_{\text{esc}}(z)$ that we obtained for the ‘no-scatter’ models in Section 3.1. The open symbols represent our constraints on the best-fitting expectation value $E(f_{\text{esc}}(\sigma))$ (these constraints do not depend on our adopted $\sigma$). We have offset these data points by $\Delta z = 0.2$ for clarity. At each redshift, we have two data points which correspond to different data sets. Including scatter reduces the expectation value of $f_{\text{esc}}$ compared to models that have no scatter, typically by about $\sim 1\sigma$. The reduction is a bit larger at $z = 0.35$. However, here we point out that the models that do not include scatter had difficulties fitting the data to begin with, and the constraints that we inferred from these models were likely less reliable.

Fig. 3 shows that our best-fitting values are consistent with Hayes et al. (2011) at $z \geq 3.1$, albeit on the high end of their quoted range. At $z = 0.35$, our constraints on $f_{\text{esc}}$ lie significantly higher than those of Hayes et al. (2011), who found $f_{\text{esc}} = 1.3 \pm 0.9$ per cent using the data from Deharveng et al. (2008) and $f_{\text{esc}} = 0.3 \pm 0.2$ per cent using the data from Cowie et al. (2010). These values would clearly not allow us to reconstruct the observed Ly$\alpha$ luminosity functions. Our constraint at $z = 0.35$ is in better agreement with the ‘transition model’ fitted by Blanc et al. (2011).

A possible explanation for the lower preferred values for $f_{\text{esc}}(z)$ at $z = 0.35$ by Hayes et al. (2011) is that they compare the observed Ly$\alpha$ luminosity density to a total star formation rate density $\dot{\rho}_s \approx 30 \times 10^{-3} M_{\odot} \text{yr}^{-1} \text{cMpc}^{-3}$. If we consider an extreme example in which all galaxies with $\psi < 10 M_{\odot} \text{yr}^{-1}$ have $f_{\text{esc}} = 3$ per cent, then their observed Ly$\alpha$ luminosity would be $L_{\alpha} \approx 3 \times 10^{41} \text{erg s}^{-1}$, which lies below the minimum detectable Ly$\alpha$ luminosity. The luminosity function presented by Cowie et al. (2010) is therefore consistent with all galaxies $\psi < 10 M_{\odot} \text{yr}^{-1}$ having $f_{\text{esc}} = 3$ per cent. Even if all galaxies with $\psi > 10 M_{\odot} \text{yr}^{-1}$ would have $f_{\text{esc}} = 0$, then $f_{\text{esc}}$ averaged over the population as a whole would be $\sim 2.4$ per cent, which is almost an order of magnitude higher than the value reported by Hayes et al. (2011).

While the example we discussed here is clearly not realistic, it nevertheless shows that for very small $f_{\text{esc}}$, large systematic uncertainties may be associated with estimating $f_{\text{esc}}$ by comparing an observed Ly$\alpha$ luminosity density to a star formation rate density. Another way to phrase this is that for very small $f_{\text{esc}}$, existing observations probe luminosities that are likely close to (or even larger than) $L_{\alpha}$ in the Ly$\alpha$ luminosity function. In these cases, it is difficult and uncertain to estimate the Ly$\alpha$ luminosity density in faint, undetected sources.

We have also indicated a (ad hoc) fitting formula that we found to capture the redshift evolution of our inferred $f_{\text{esc}}$ reasonably well:

$$f_{\text{esc}}(z, \psi) = \exp(-\tau_{\text{eff}}), \quad \tau_{\text{eff}} = a_1 + a_2 z,$$

where $a_1, a_2 > 0$. This formula is not our primary focus, but it does provide a useful way to visualize and compare the constraints on $f_{\text{esc}}(z)$ to the analytic fitting formula of Blanco et al. 2011, who found $f_{\text{esc}}(z) \ll 0.35$ by Hayes et al. (2011). While the ‘no LF integration limit’ fit, which predicts $f_{\text{esc}} \sim 10^{-3}$ per cent at $z = 0.35$. This same fit gives slightly lower values for $f_{\text{esc}}$ at $1 \leq z \lesssim 3.5$. Our constraints at redshifts $z > 2.4$ per cent, which is almost an order of magnitude higher than the value reported by Hayes et al. (2011). While the example we discussed here is clearly not realistic, it nevertheless shows that for very small $f_{\text{esc}}$, large systematic uncertainties may be associated with estimating $f_{\text{esc}}$ by comparing an observed Ly$\alpha$ luminosity density to a star formation rate density. Another way to phrase this is that for very small $f_{\text{esc}}$, existing observations probe luminosities that are likely close to (or even larger than) $L_{\alpha}$ in the Ly$\alpha$ luminosity function. In these cases, it is difficult and uncertain to estimate the Ly$\alpha$ luminosity density in faint, undetected sources.

We have also indicated a (ad hoc) fitting formula that we found to capture the redshift evolution of our inferred $f_{\text{esc}}$ reasonably well: $$f_{\text{esc}}(z, \psi) = \exp(-\tau_{\text{eff}}), \quad \tau_{\text{eff}} = a_1 + a_2 z,$$ where $a_1, a_2 > 0$. This formula is not our primary focus, but it does provide a useful way to visualize and compare the constraints on $f_{\text{esc}}(z)$ to the analytic fitting formula of Blanco et al. 2011, who found $f_{\text{esc}}(z) \ll 0.35$ by Hayes et al. (2011). While the ‘no LF integration limit’ fit, which predicts $f_{\text{esc}} \sim 10^{-3}$ per cent at $z = 0.35$. This same fit gives slightly lower values for $f_{\text{esc}}$ at $1 \leq z \lesssim 3.5$ than ours.

A possible explanation for the lower preferred values for $f_{\text{esc}}(z)$ at $z = 0.35$ by Hayes et al. (2011) is that they compare the observed Ly$\alpha$ luminosity density to a total star formation rate density $\dot{\rho}_s \approx 30 \times 10^{-3} M_{\odot} \text{yr}^{-1} \text{cMpc}^{-3}$. This value corresponds to the total integrated star formation rate density (see table 1 of Bothwell et al. 2011). Bothwell et al. (2011) show that galaxies with $\psi \gtrsim 10 M_{\odot} \text{yr}^{-1}$ account only for $\sim 20$ per cent of $\dot{\rho}_s$. If we consider an extreme example in which all galaxies with $\psi < 10 M_{\odot} \text{yr}^{-1}$ have $f_{\text{esc}} = 3$ per cent, then their observed Ly$\alpha$ luminosity would be $L_{\alpha} \approx 3 \times 10^{41} \text{erg s}^{-1}$, which lies below the minimum detectable Ly$\alpha$ luminosity. The luminosity function presented by Cowie et al. (2010) is therefore consistent with all galaxies $\psi < 10 M_{\odot} \text{yr}^{-1}$ having $f_{\text{esc}} = 3$ per cent. Even if all galaxies with $\psi > 10 M_{\odot} \text{yr}^{-1}$ would have $f_{\text{esc}} = 0$, then $f_{\text{esc}}$ averaged over the population as a whole would be $\sim 2.4$ per cent, which is almost an order of magnitude higher than the value reported by Hayes et al. (2011).
where $a_1 = 4.0 \pm 0.16$ and $a_2 = -0.52 \pm 0.05$. We obtain best-fitting values for $a_1$ and $a_2$ by minimizing $\chi^2$, which we compute using all 16 data points shown in Fig. 3. The redshift evolution of $f_{\text{esc}}^{\text{eff}}$ for the best-fitting combination of $a_1$ and $a_2$ is represented by the black solid line in Fig. 2. The upper/lower boundary of the grey region represents our fitting formula when we simultaneously subtract/add $\sigma/\sqrt{2}$ to both $a_1$ and $a_2$. Here, uncertainties on the parameters $a_1$ and $a_2$ represent marginalized 1σ uncertainties. The fitting formula equation (6) captures our main results well and further ‘predicts’ that $f_{\text{esc}}^{\text{eff}} \sim 5^{+10}_{-5}$ per cent at $z = 2$, which is consistent with Hayes et al. (2010), who found $f_{\text{esc}}^{\text{eff}} = 5.3 \pm 3.8$ per cent by comparing Ly$\alpha$ to Hz luminosity functions. We have also applied our analysis to the more recent $z \sim 1$ data of Barger, Cowie & Wold (2012, not shown here) and found $f_{\text{esc}}^{\text{eff}} = 5 \pm 1$ per cent for the ‘no scatter case’, which is also captured reasonably well by our fitting formula.

4 DISCUSSION

4.1 ‘Effective escape’ fraction versus ‘escape’ fraction

We explicitly differentiate between the term ‘escape’ fraction and ‘effective escape’ fraction, because these two quantities can take on very different values. In theoretical calculations that follow the transport of Ly$\alpha$ photons through a dusty medium, it is straightforward to compute the fraction of photons that are not absorbed by dust, and hence ‘escape’ (e.g. Neufeld 1990; Hansen & Oh 2006; Laursen & Sommer-Larsen 2007; Laursen et al. 2012; Yajima et al. 2012, 2013). However, a large fraction of Ly$\alpha$ photons that escape from this medium can scatter in the surrounding CGM and/or IGM and give rise to a low-surface-brightness Ly$\alpha$ glow around galaxies (e.g. Dijkstra, Lidz & Wyithe 2007; Zheng et al. 2010; Laursen, Sommer-Larsen & Razoumov 2011; Steidel et al. 2011; Zheng et al. 2011; Jeeson-Daniel et al. 2012). The surface brightness of this scattered radiation is typically much fainter than can be observed, and this Ly$\alpha$ radiation would effectively be lost in observations. For example, Zheng et al. (2010) find that Ly$\alpha$ scattering in the ionized IGM at $z = 5.7$ rendered 80–95 per cent of all emitted Ly$\alpha$ radiation undetectable (consistent with the other studies, see Section 4.4). There is no dust in their simulations, and the escape fraction of Ly$\alpha$ photons is 100 per cent. In contrast, the effective escape fraction would only be $f_{\text{esc}}^{\text{eff}} \sim 5$–20 per cent.

There is also observational evidence for the existence of spatially extended low-surface-brightness Ly$\alpha$ emission around galaxies (e.g. Fynbo, Möller & Thomsen 2001; Rauch et al. 2008; Ostlin et al. 2009; Steidel et al. 2011; Matsuda et al. 2012; Hayes et al. 2013). Steidel et al. (2011) detected spatially extended Ly$\alpha$ emission after stacking Ly$\alpha$ observations on 92 $z \sim 2.6$ Lyman-break galaxies, which allowed them to probe Ly$\alpha$ emission down to $\sim 10$ times fainter surface brightness levels. The total flux in their spatially extended haloes significantly exceeded the total Ly$\alpha$ flux coming directly from their galaxies. The observations by Steidel et al. (2011) imply that the escape fraction of Ly$\alpha$ photons can exceed the effective escape fraction significantly for surface brightness thresholds that are typical for current observations. Similarly, Matsuda et al. (2012) detected Ly$\alpha$ haloes around $z = 3.1$ LAEs and found that the size of the haloes (at fixed UV luminosity of the LAEs) increases with local density (measured by the number density of LAEs). This dependence may help explain why other groups have not detected spatially extended Ly$\alpha$ haloes around LAEs (e.g. Feldmeier et al. 2013). In any case, the possibility that there is more Ly$\alpha$ flux in diffuse Ly$\alpha$ haloes than in a compact source illustrates that the effective escape fraction – and previous determinations of this quantity – depend on the surface brightness threshold of the survey of interest (or the size of the photometric aperture in fixed aperture photometry), while the escape fraction does not (also see Yajima et al. 2012).

The universal usage of the term escape fraction complicates comparisons between different studies: for example, Yajima et al. (2013) compute true Ly$\alpha$ escape fractions in simulated galaxies as a function of redshift. Similarly, semi-analytic studies that model LAEs at $z = 3$–6 (e.g. Kobayashi, Totani & Nagashima 2007; Dayal, Maselli & Ferrara 2011; Forero-Romero et al. 2011; Shimizu, Yoshida & Okamoto 2011) introduce an escape fraction, which corresponds to a true escape fraction. Caution must be exercised when comparing these escape fractions to the observationally inferred effective escape fractions (as in Hayes et al. 2011, Blanc et al. 2011, and in this paper). Moreover, in some (but not all) studies, the constraints on $f_{\text{esc}}$ (and/or $f_{\text{esc}}^{\text{eff}}$) involve a ‘correction’ for scattering in the IGM. We stress that this correction is highly uncertain, as it depends on the RT at the interstellar and circum-galactic level (see Section 1).

4.2 Comparison to previous works

We already compared our results to those obtained by Hayes et al. (2011, and also Blanc et al. 2011). Our approach, in which we use star formation functions and Ly$\alpha$ luminosity functions to constrain $f_{\text{esc}}^{\text{eff}}$, is similar to that adopted in theoretical studies. For example, Le Delliou et al. (2006) use semi-analytic models – while e.g. Nagamine et al. (2010) use hydrodynamical simulations – to generate star formation functions and then use Ly$\alpha$ luminosity functions to constrain $f_{\text{esc}}^{\text{eff}}$ at $z = 3$–6. Importantly, the models that are used to generate the theoretical star formation functions are typically constrained by observations. However, these (almost the same) observations can be converted directly into star formation functions, i.e. without generating the intermediate theoretical model. Indeed, our method completely circumvents this intermediate step. The fact that we can sidestep this (substantial) part of the calculations allows us to more efficiently explore a larger suite of models for $f_{\text{esc}}^{\text{eff}}$ and to explore the impact of uncertainties with the observationally inferred star formation functions on our results.

Our results are broadly consistent with these previous theoretical studies: Nagamine et al. (2010) find that $f_{\text{esc}}^{\text{eff}} = 0.1$ at $z = 3.1$, which is in excellent agreement with our results. Nagamine et al. (2010) find $f_{\text{esc}}^{\text{eff}} = 0.15$ at $z = 6$, which is a factor of $\sim 3$ lower than what we find. The origin of this difference is unclear, but the lower-right panel of Fig. 2 shows that the value preferred by

10 For example, the surface brightness threshold for the $z = 5.7$ LAE survey by Ouchi et al. (2008) is $\sim 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$. Rauch et al. (2008) managed to go a factor of $\sim 10$ deeper in an $\sim 100 h$ exposure on the VLT.

11 Recently, Jiang et al. (2013) did not detect spatially extended Ly$\alpha$ emission around stacks of 43 $z = 5.7$ LAEs and 40 $z = 6.5$ LAEs. At these redshifts there is room to hide a significant Ly$\alpha$ flux in the halo, even for the surface brightness threshold of $\sim 10^{-19}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$ that is reached in the stacking analysis. Jiang et al. (2013) comment that these observations indeed still appear broadly consistent with the predictions by Zheng et al. (2011).

12 To be precise, these models generate intrinsic Ly$\alpha$ luminosity functions, which give the number density of galaxies as a function of Ly$\alpha$ luminosity. This intrinsic Ly$\alpha$ luminosity function is practically the same as a star formation function.
Nagamine et al. (2010) \(10^{\log(f_{\text{esc}})} = 0.15\) is not ruled out at great significance. Le Delliou et al. (2006) find \(f_{\text{esc}} \sim 0.02\) at \(z = 3 - 6\). However, a redshift-dependent fraction of stars form in bursts with a top-heavy IMF for which \(k \sim 10\) (see Section 2) in their models. Hayes et al. (2011) show that if this top-heavy IMF is replaced with a standard Salpeter IMF, then the constraints obtained by Le Delliou et al. (2006) agree well with Nagamine et al. (2010) at \(z = 3 - 6\).

Finally, Nagamine et al. (2010) showed that while their models with a constant \(f_{\text{esc}}\) fit the data well (in good agreement with our work), they obtain better fits using the so-called ‘duty cycle’ models, in which \(\frac{dn}{d\log \psi} = \epsilon_{\text{DC}} \frac{dn}{d\log \psi} \big|_{\psi = \psi_0 + k/2}\). These models represent a scenario in which star-forming galaxies only have non-zero \(f_{\text{esc}}\) for a fraction \(\epsilon_{\text{DC}}\) of their lifetimes. We note that this may also represent a scenario in which \(\psi\) escapes anisotropically from galaxies and in which \(f_{\text{esc}} > 0\) only along a fraction \(\epsilon_{\text{DC}}\) of the sightlines from them. The duty cycle parameter \(\epsilon_{\text{DC}}\) can also be incorporated in the \(f_{\text{esc}}\) PDF, simply by adding a Dirac-delta function at \(f_{\text{esc}} = 0\) (after which we must renormalize the full PDF). We have repeated our analysis including a duty cycle of \(\epsilon_{\text{DC}} = 0.25\) into our \(f_{\text{esc}}\) PDF and found that these models flatten the predicted luminosity functions, similarly to models with a non-zero scatter in \(f_{\text{esc}}\). These ‘duty-cycle models’ therefore provide somewhat better fits to the luminosity functions (for models with \(\sigma = 0\), mostly because they improve the fits at the bright ends (just as our models with \(\sigma = 0.5\), in agreement with Nagamine et al. (2010). Moreover, the best-fitting expectation values of \(f_{\text{esc}}\) in these duty cycle models\(^\text{13}\) are consistent with our those obtained previously.

### 4.3 Model uncertainties

A potential caveat is that (some of) our adopted \(\psi\) luminosity functions were constructed from narrow-band surveys. Such surveys do not only impose a \(\psi\) flux cut, but in practice also a cut in \(\psi\) equivalent width (EW). For example, Ouchi et al. (2008) adopt colour–colour criteria to select LAEs at \(z = 3.1\) that translate (roughly) to \(EW \gtrsim 64\) Å. We may worry that this data set therefore misses a significant fraction of LAEs. In practise, however, the EW cut does not appear to affect determinations of the \(\psi\) opacity, because they do not include the impact of outflows of optically thick (to \(\psi\) photons) \(H_1\) gas on the \(\psi\) spectral line profile emerging from galaxies. Winds are known to redshift \(\psi\) photons out of the line resonance, which can strongly increase the fraction of photons transmitted through the IGM (see e.g. Dijkstra et al. 2007; Iliev et al. 2008; Zheng et al. 2010; Dayal et al. 2011; Laursen et al. 2011) of photons through an ionized Universe at \(z \sim 6\). Under the reasonable assumption that dust suppresses the emerging \(\psi\) flux by an additional factor, these models would predict effective escape fractions that appear inconsistent with our inferred fraction (and also that of Hayes et al. 2011). A plausible reason for this discrepancy is that the models overestimate the IGM opacity, because they do not include the impact of outflows of optically thick (to \(\psi\) photons) \(H_1\) gas on the \(\psi\) spectral line profile emerging from galaxies. Winds are known to redshift \(\psi\) photons out of the line resonance, which can strongly increase the fraction of photons transmitted through the IGM (see e.g. Dijkstra & Wyithe 2010). It is interesting that current constraints on \(f_{\text{esc}}\) provide evidence for winds impacting the \(\psi\) RT at \(z \sim 6\).

(ii) Our work has also shown that it is possible to reproduce \(\psi\) luminosity functions with a constant (\(\psi\)-independent) \(f_{\text{esc}}\) in agreement with previous studies (e.g. Nagamine et al. 2010; Shimizu et al. 2011), although we have shown that this applies over a wider range of observed \(\psi\) luminosities (by adding the data from Cassata et al. 2011 to the data from Ouchi et al. 2008 which were used in most previous analyses). We have shown that we ‘flatten’ the predicted luminosity functions by adding a dispersion in \(f_{\text{esc}}\) and/or a ‘duty cycle’ (as in Nagamine et al. 2010). This flattening can improve the fit to the observed luminosity function at the bright end. If we flatten the predicted luminosity functions even more (by increasing the dispersion or reducing the duty cycle), then we need to invoke that \(f_{\text{esc}}\) decreases towards higher \(\psi\), which appears to be favoured by the observed increase ‘\(\psi\) fraction’ towards fainter \(\psi\) galaxies (see Section 3.2).

For a fixed set of parameters (\(\Phi_*, \psi_*, \sigma\)), the Saunders function is identical to the Schechter function for \(\psi \lesssim \psi_*\). However, at \(\psi > \psi_*\), it cuts off as a Gaussian in log space with a standard deviation \(\sigma\), instead of the sharper exponential cutoff of the Schechter function in real space. Salim & Lee (2012) show that Schechter functions typically predict (slightly) fewer galaxies at the largest \(\psi\) compared to the actual observations (see e.g. fig. 5 of Smit et al. 2012 and fig. 2 of Salim & Lee 2012), because of their exponential cutoff at \(\psi > \psi_*\). For \(\sigma = 0.5\), we can boost \(dn/d\psi\) by a factor of \(\sim\) a few at the high-\(\psi\) end, which may help resolve this issue. We repeated our analysis in which we replaced Schechter functions with Saunders functions (using \(\sigma = 0.5\) and keeping the other parameters fixed) and found that this did not change our results at all. However, this may become more relevant in the future with larger LAE surveys which can probe down to larger \(\psi\) luminosities (and likely larger values of \(\psi\)).
5 CONCLUSIONS

In this paper, we have constrained the ‘effective escape’ fraction of Lyα photons, $f_{\text{esc}}^{\text{eff}}$, which is defined as the ratio of the observed Lyα luminosity of a galaxy to its intrinsic Lyα luminosity. This ratio is often referred to in the literature simply as an escape fraction. In Section 4.1, we have argued why we caution against universal usage of the term escape fraction and why it helps to distinguish between an escape fraction and an effective escape fraction.

We have constrained the effective escape fraction by converting observed star formation functions to observed Lyα luminosity functions. This conversion depends directly on $f_{\text{esc}}$, and we use observed Lyα luminosity functions at $z = 0.35$, 3.1, 3.7 and 5.7 to get constraints on $f_{\text{esc}}^{\text{eff}}$ at these redshifts. We have explored models in which $f_{\text{esc}}^{\text{eff}}$ takes on a single value (Section 3.1) and in which $f_{\text{esc}}^{\text{eff}}$ has a dispersion (Section 3.2). Models which include a dispersion predict flatter luminosity functions, which appear to be in better agreement with the observations. We note that the flattening predicted by these models cannot be captured by Schechter functions (a Saunders function in equation 7 would likely be more appropriate).

We found that the effective escape fraction (or its expectation value in a distribution) $f_{\text{esc}}^{\text{eff}} \sim 1-3$ per cent at $z = 0$ and that it increases to $f_{\text{esc}}^{\text{eff}} \sim 10$ per cent at $z = 3-4$ and to $f_{\text{esc}}^{\text{eff}} \sim 30-50$ per cent at $z = 6$ (see Fig. 2). Equation (6) provides a convenient fitting formula that encapsulates our main findings. Our results are consistent with previous work (e.g. Hayes et al. 2010; Blanc et al. 2011; Hayes et al. 2011), except at $z \sim 0.35$ where our inferred $f_{\text{esc}}^{\text{eff}}$ is higher than previous works. We have argued in Section 3 that this difference may be a result of the systematic uncertainty on $f_{\text{esc}}^{\text{eff}}$ becoming increasingly large for very small $f_{\text{esc}}^{\text{eff}}$ in previous analyses. We argued in Section 4.4 that our constraint on $f_{\text{esc}}^{\text{eff}}$ at $z \sim 6$ appears higher than predicted by models that do not include winds. This hints at the importance of winds in the Lyα transfer process even at this high redshift.

We have shown that we can reproduce observed Lyα luminosity functions in individual redshift bins with a constant – i.e. independent of $\psi - f_{\text{esc}}^{\text{eff}}$ over up to two orders in $\psi$ and Lyα luminosity (see Fig. 2), in agreement with previous work. We require $f_{\text{esc}}^{\text{eff}}$ to decrease with $\psi$ – as appears to be favoured by observations of drop-out galaxies (see Section 3.2) – only in models which include a large scatter ($\sigma \gtrsim 1.0$ dex) in $f_{\text{esc}}^{\text{eff}}$ or in which star-forming galaxies only have a non-zero $f_{\text{esc}}^{\text{eff}}$ for a fraction $\epsilon_{\text{DC}}$ of their lifetime and/or a fraction of sightlines (see Section 4.3).

We anticipate that observations of LAEs in the near future (e.g. with MUSE,14 Hyper Suprime-Cam15 and by HETDEX) will determine the Lyα luminosity functions over a wider range of luminosities and will reduce their systematic uncertainties. This may allow for better constraints on $f_{\text{esc}}^{\text{eff}}$ and its PDF. As illustrated by the discussion in Section 4.4, constraints on the $f_{\text{esc}}^{\text{eff}}$ PDF yield valuable basic insights into Lyα transfer process on small scales. Perhaps this is more speculative, but the possible dependence of these luminosity functions on the surface brightness threshold of the survey would shed light on the presence of spatially extended Lyα haloes around star-forming galaxies, which encode valuable information on cold gas around galaxies (e.g. Zheng et al. 2011; Dijkstra & Kramer 2012; Jeeson-Daniel et al. 2012).

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