Introduction. A nonzero positive cosmological constant appears to be the most plausible cause for the observed accelerated expansion of our universe, and thus, in order to be a candidate for a theory of everything, string theory must contain low-energy de Sitter (dS) space solutions.

The best-studied model for a highly-warped region of a flux compactification is the so-called Klebanov-Strassler warped deformed conifold (KS) solution [2], and anti-D3 branes placed in this solution have been argued to be metastable [3] and are the key ingredient in the KKLT mechanism for uplifting AdS vacua and producing a de Sitter landscape [1]. The suitability of anti-D3 branes in KS throats for describing metastable vacua and for uplifting AdS to dS vacua has been recently put into question by the perturbative investigation of the backreaction of smeared anti-D3 branes in a warped deformed conifold (KS) background with positive D3 charge dissolved in fluxes. Furthermore, the only fully-backreacted regular solution with anti-D3 branes in the infrared has anti-D3 charge dissolved in fluxes, and hence it is just the supersymmetric KS solution with a different charge orientation (which we will refer to as anti-KS). This was first conjectured in [4] (based on an analogy with the brane-bending calculation of [7]) and our results confirm this conjecture. The setup is shown on Figure 1.

To make such a statement one may naively try to construct the fully-backreacted anti-D3 solution by solving analytically or numerically the underlying 8 nonlinear coupled second-order differential equations [5], but this is not necessary. We believe there exist at least three ways to demonstrate that imposing regularity near the anti D3-branes cannot give a solution with positive D3 charge at infinity, and in this note we present the three proofs:

1. We solve brute-force the equations in a Taylor expansion around the infrared. Setting to zero all the coefficients that give singular metric and 3-form fluxes, we found that the full solution up to order $\tau^{10}$ (where $\tau$ is the radial coordinate away from the KS tip) has three independent parameters all of which, as we will show below, are singular in the ultraviolet. The only regular solution is hence the BPS anti-KS solution with anti-D3 branes.

2. We explore the boundary conditions for the fields and their derivatives in the infrared, and show that if one imposes singularity-free boundary conditions the right hand sides of some of the equations are zero at all orders in perturbation theory. The remaining equations only have UV-singular solutions, and the only possible regular solution is the supersymmetric one.

3. A more elegant way to prove that there is no regular solution whose D3-brane charge changes sign from IR to UV is to find a topological argument similar to that of [9]. We present an argument along these lines. This argument may be generalizable to the case of localized anti-D3 branes.
The setup. As argued in [4], the Ansatz for the solution describing smeared D3 and anti-D3 branes in the KS solution is [8]:

\[ ds_{10}^2 = e^{2A + 2p - \tau} ds_{1,3}^2 + e^{-2 p - \tau} (d\tau^2 + g_{5}^2) + e^{x+y} (g_1^2 + g_2^2) + e^{-y} (g_3^2 + g_4^2) \]

\[ H_3 = \frac{1}{2} (k - f) \ g_5 \wedge (g_1 \wedge g_3 + g_2 \wedge g_4) \]

\[ F_3 = F g_1 \wedge g_2 \wedge g_5 + (2P - F) g_3 \wedge g_4 \wedge g_5 \]

\[ F_5 = F_5 + *F_5, \quad F_5 = K g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5, \]

with

\[ K = -\frac{\pi}{4} Q + (2P - F) f + kF, \]

where all the functions depend only on the radial variable \( \tau \) and the angular forms \( g_i \) are defined in [2]. The constant \( P \) is proportional to the 5-brane flux of the KS solution and \( Q \) is the number of (anti) D3 branes.

In order to handle the second-order equations of motion for the scalars of the PT Ansatz, we found crucial to define particular combinations of fields, inspired by the GKP [10] notations. The warp factor \( e^{A+4p-2\tau} \) and the five-form flux \( K \vol_5 \) are combined into scalar modes \( \xi^\pm_1 \), defined as

\[ \xi^\pm_1 = -e^{4(p+A)} \left( \dot{x} - 2 \dot{\phi} - 2 \dot{A} \mp \frac{1}{2} e^{-2x} K \right). \]

with \( *g \) the six-dimensional Hodge star and \( G_3 = F_3 + i e^{-\phi} H_3 \). The scalar components of \( G_\pm \) will be called \( \xi^\pm_1, \xi^\pm_2 \) and \( \xi_F^\pm \). This notation follows from the fact that these modes are the conjugate momenta to the fields \( f, k \) and \( F \) in [2], in the reduced one-dimensional system that describes the dynamics of the 8 scalar functions (seven in [1] and [2] plus the dilaton \( \phi \)).

Supersymmetry imposes either that \( G_- = F_{D3} = 0 \) or \( G_+ = F_{T\Omega} = 0 \), depending on which supersymmetries are preserved. We will refer to the solutions with ISD and IASD fluxes as KS and anti-KS respectively. With this notation the KS solution has \( \xi^+_a = 0 \), \( a = 1, f, k, F \), while for the anti-KS solution \( \xi^-_a = 0 \). A crucial fact is that the equations of motion for the scalars \( \xi^{\pm}_a \) are just first-order ODEs. For the \( \xi^-_a \) modes we find:

\[ \dot{\xi}^- + K e^{-2x} \xi^- = 4e^{2x-4(p+A)} \left[ e^{\phi+2y}(\xi_f^+) + e^{\phi-2y}(\xi_f^-) + e^{\phi}(\xi_F^+) \right]^2 + \frac{1}{2} e^{-\phi}(\xi_F^-)^2 \]

and

\[ \dot{\xi}^-_f = \frac{1}{2} e^{-2x}(2P - F) \xi^-_f - \frac{1}{2} e^{-\phi} \xi^-_F \]

\[ \dot{\xi}^-_k = \frac{1}{2} e^{-2x} F \xi^-_k - \frac{1}{2} e^{-\phi} \xi^-_F \]

Remarkably, these are the only equations that we will need in this letter. One can define additional scalars \( \xi^{\pm}_a \) which are the conjugate momenta to the four additional modes \( x, y, p, \phi \), in such a way that the BPS KS solution with \( Q \) mobile D3-branes has all the \( \xi^+_a \) modes equal to zero. The 8 integration constants of the BPS system \( \xi^+_a = 0 \) are fixed as follows:

1. The zero-energy condition of the effective Lagrangian fixes the \( \tau \)-redefinition gauge freedom and is automatically solved when \( \xi^-_a = 0 \), but the constant shift \( \tau = \tau_0 + \alpha \) still remains unfixed, and so \( \tau_0 \) appears as a “trivial” integration constant. 2. The conifold deformation parameter \( \epsilon \) and the constant dilaton \( e^{\phi_0} \) give two other free parameters. 3. An additional parameter renders the conifold metric singular in the IR [11] and has to be discarded. 4. The three equations for the flux functions \( f, k \) and \( F \) appear to have three free parameters [12]. One gives singular BPS fluxes in the IR, the second gives a \((0,3)\) complex 3-form \( G_3 \equiv F_3 + i e^{-\phi} H_3 \) that is singular in the UV, and the third corresponds to a \( B \)-field gauge transformation \((f, k) \to (f + c, k + c)\) that can be absorbed in the redefinition of \( Q \). 5. The warp function \( h = e^{-4(p+A)+2x} \) can only be determined up to a constant, which is fixed requiring that \( h \) vanishes at infinity.

To summarize, the KS solution with \( Q \) mobile D3-branes and the free parameters \( \epsilon \) and \( e^{\phi_0} \) is the only (IR and UV) regular solution with \( \xi^-_a = 0 \), where by IR-regular we denote a solution whose only singularities are those coming from D-branes.
The boundary conditions for anti-D3-branes. The main goal of this letter is to show that there is no IR-regular solution with smeared anti-D3 branes $(Q < 0$, hence $K > 0$) at the tip of the conifold and with KS asymptotics $(K < 0)$ in the UV. Starting with a singularity-free anti-brane solution in the IR, one necessarily ends up with an anti-KS solution in the UV. Moreover, we will prove that the only regular solution with $|Q|$ anti-D3 branes is the anti-KS flip of the solution with $Q$ mobile branes we reviewed above.

To obtain IR-regular solutions we require that:

- the 6d conifold metric has the tip structure of the KS solution: the 2-sphere shrinks smoothly at $\tau = 0$ and the 3-sphere has finite size.

- The warp factor comes from $|Q|$ anti-D3 branes smeared on the 3-sphere, and hence goes like $h \sim |Q|/\tau$. As a result the Taylor expansions of the functions $x, p, A$ and $y$ start with the same logarithmic and constant terms as in the KS solution with mobile branes and can differ only by linear (and higher) terms. The constant term in $A$ cannot be fixed by the regularity condition, since it corresponds to the conifold deformation parameter $\epsilon$.

- There is no singularity in the three-form fluxes; their energy densities, $H^2_3$ and $F^2_3$, do not diverge at $\tau = 0$. Hence, the Taylor expansions of the functions $f, k$ and $F$ start from $\tau^3, \tau$ and $\tau^2$ terms respectively, exactly like in the KS background. To be more precise, in a solution with branes at the tip the functions $f, k$ and $F$ can also start with non-integer powers $(\tau^{9/4}, \tau^{1/4}$ and $\tau^{5/4})$, but one can show that the logarithmic terms in the metric imply that the IR expansion of the solution only has integer powers of $\tau$.

- The dilaton is finite at $\tau = 0$.

It is important to stress that we do not impose any kind of anti-KS IR boundary conditions for the 3-form fluxes, and a-priori the 3-form can be either ISD or IASD (or have both components). On the other hand, we do require the singularities in the warp factor and the five-form flux to correspond to objects that exist in string theory.

These observations are helpful to determine the possible leading-order behaviors of the $\xi^+_a$’s and $\xi^-_a$ (for our argument we mostly need the latter). Let us denote by $n_a$ the lowest possible leading orders of the fields $\xi^+_a$. For small $\tau$, the metric regularity conditions imply that the functions $e^{2\tau}$ and $e^{4(p-1)}$ go like $\tau$ and $\tau^2$ respectively. From the explicit definitions of the $\xi^+_a$ modes we find:

$$ (n_1, n_f, n_k, n_F) = (2, 1, 3, 2) .$$

(9)

The IR obstruction. Our goal is to show that when solving the equation of motions (7), (8) for $\xi^-_1, \xi^-_f, \xi^-_k$ and $\xi^-_F$ in the IR (small $\tau$) and imposing the IR regularity conditions, one finds only trivial solutions for these functions. This essentially means that the IASD conditions $\xi^-_f = \xi^-_k = \xi^-_F = 0$ will be satisfied all the way to the UV and not only at $\tau = 0$. To prove this, a simple counting argument is sufficient, as we will now prove.

Let us assume that $\xi^-_f$ and $\xi^-_k$ start from $\tau^n$ and $\tau^{n+l}$ for some $n \geq 2$. We treat separately the two possibilities:

1. $l > -1$. Recalling that $e^{\varphi} \approx \frac{\tau}{2} + \ldots$, we can see from a simple power analysis that the $\xi^-_k$ term is subleading both in the $|\xi^-_k|$ and $\xi^-_F$ equations in (8). In the latter equation the $\xi^-_F$ term is also subleading. We arrive at the set of two simple equations near $\tau = 0$: $\xi^-_k \approx -\frac{1}{2} e^{-\varphi} \xi^-_F$ and $e^{-\varphi} \xi^-_F \approx -4\tau^{-3} \xi^-_k$. They have only two solutions, $\xi^-_F \sim \tau^{-2}$ and $\xi^-_F \sim \tau$ and both full short of the irregularity conditions (9). Remarkably, in showing that the system has no regular solution we have not used (7).

2. $l \leq -1$. Now the right hand side of (8) is certainly negligible with respect to the left hand side. This means that we have $\xi^-_1 + \tau^{-1}: \xi^-_1 \approx 0$ for small $\tau$ leading to the singular solution $\xi^-_1 \sim \tau^{-1}$.

For non-integer powers, the argument above can be straightforwardly extended, and the two regimes of parameters corresponding to those above are $l > -1/4$ and $l \leq -1/4$.

To conclude, we see that regularity in the IR implies that the functions $\xi^-_1, \xi^-_f, \xi^-_k$ and $\xi^-_F$ vanish identically. Consequently, the solution will remain IASD for any value of $\tau$, and the force on probe anti-D3 branes will remain identically zero.

A second way to see that a solution is not possible is to look at the polarization of the scalar in the PT Ansatz in a power expansion around the origin. Upon eliminating all the singular modes, we find that to order $\tau^{10}$ the space of solutions is parameterized by three constants. None of these constants breaks the IASD condition, which confirms the results of the previous section.

One can also use the fact that $\xi^-_1, \xi^-_f, \xi^-_k, \xi^-_F$ are necessarily zero to identify the three IR modes we find, and to show that they correspond actually to UV singular solutions:

1.  Plugging $\xi^-_1, f, k, F = 0$ into the remaining equations of motion, it is easy to show that there exists an IR regular but UV divergent one parameter family of solution.

2.  A second mode is the $(3, 0)$-form solution of the superpotential $\xi^-_n = 0$ equations, which breaks supersymmetry and diverges in the UV (see [12]).

3.  A third “superpotential mode” is related to the shift of the warp function and, following our previous discussion, has to be excluded.

Summarizing, we see that the only solution with smeared anti-D3 branes at the KS tip that is regular both in the UV and in the IR is the anti-KS solution. Stated differently, the only way to obtain a sensible supersymmetry-breaking solution corresponding to the backreaction of smeared anti-D3’s is to allow for IR singularities in the energy densities of the 3-form fluxes.
The global obstruction. We can also present a "global" argument why the functions $\xi_1^-$, $\xi_f^-$, $\xi_k^-$, and $\xi_F^-$ have to vanish in a regular solution, without focusing on their Taylor expansions. The proof for the remaining four functions proceeds precisely as above.

Our key observation is that the flux functions $f(\tau)$, $k(\tau)$ and $F(\tau)$ appear only in equations (1) and (2). None of the remaining $\xi_n^-$ equations has any flux function in it. Next, the equations in (2) might be derived from the Lagrangian:

$$L_{\text{fluxes}} = 4e^{2x-4(p+A)+\phi} \left[ e^{2y}(\xi_f^-) = e^{-2y}(\xi_k^-) \right]$$

The Lagrangian is

$$\frac{1}{2} e^{-2\phi}(\xi_F^-)^2 + e^{-4p(\phi)}(\xi_1^-)^2.$$ (10)

We treat $L_{\text{fluxes}}$ as the effective Lagrangian only for the fields $f(\tau)$, $k(\tau)$ and $F(\tau)$ with the remaining five fields being free but subject to the proper boundary conditions ensuring the IR regularity. This means that the first three terms in (10) are kinetic terms, while the last one is a potential term. Recall also that the $\xi^-_n$'s are first order in the derivatives of $\phi$'s and so the Lagrangian is of the second order, precisely as it should be.

This Lagrangian has a remarkable property: it is strictly non-negative and vanishes only for $\xi_1^-, \xi_f^-, \xi_k^-, \xi_F^- = 0$. In other words, the global minimum of (10) corresponds to the IASD solution. The only way to arrive at a different solution, which describes only some functions and their conjugate momenta in the IR, is to impose boundary conditions (either in the IR or in the UV) that are at

the three flux functions and their conjugate momenta $\xi_f^-, \xi_k^-, \xi_F^-$ have to vanish at $\tau = 0$ for a regular solution. Similarly $\xi_1^- = 0$ in the IR, thus the IR boundary conditions following solely from the regularity are consistent with the "trivial" IASD solution. We conclude again that requiring regularity forces us into the anti-KS solution.

Note, however, that the equations derived from (10) are singular in the IR, and so our arguments should be taken very cautiously. At the same time, this approach may prove efficient for the localized anti D3 branes, where one cannot use the Taylor expansion argument.

Conclusions. We presented a detailed analysis of the nonlinear backreaction of smeared anti-D3 branes on the KS geometry. In the near-brane (IR) region we impose boundary conditions coming from the presence of a smeared source: singular warp factor and commensurate five-form flux, while in the UV region we require absence of highly divergent modes. We showed that with these assumptions there is a unique solution of the equations of motion, namely the supersymmetric anti-KS solution.

We have thus proven that any supersymmetry-breaking solution associated to the backreaction of smeared anti-D3 branes on the KS geometry has singularities in the IR region not directly associated with the anti-D3 branes themselves. Moreover, these singularities appear in the three-form fluxes, confirming the linearized analysis of (4).

A singularity in a supergravity solution does not necessarily mean that this solution should be automatically discarded. However, physically acceptable singularities are those which are resolved in the full string theory (for example, the singularity in the supergravity solution for a brane is resolved by the open strings on the brane, other singularities are resolved by brane polarization [13, 15] or geometric transitions). Here it is not at all clear that there is any mechanism capable of explaining the present singularities. If there is no such mechanism, then the singularity in the supergravity background is telling us that there is no (meta)stable anti-D3 brane solution, and the whole system of branes with charge opposite to that of the background is unstable. This would invalidate the anti-brane AdS to dS uplifting mechanism, and therefore most of the String Theory de Sitter landscape.

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