Effective algorithm for solving complex problems of production control and of material flows control of industrial enterprise

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Abstract. A universal economical and mathematical model designed for determination of optimal strategies for managing subsystems (components of subsystems) of production and logistics of enterprises is considered. Declared universality allows taking into account on the system level both production components, including limitations on the ways of converting raw materials and components into sold goods, as well as resource and logical restrictions on input and output material flows. The presented model and generated control problems are developed within the framework of the unified approach that allows one to implement logical conditions of any complexity and to define corresponding formal optimization tasks. Conceptual meaning of used criteria and limitations are explained. The belonging of the generated tasks of the mixed programming with the class of NP is shown. An approximate polynomial algorithm for solving the posed optimization tasks for mixed programming of real dimension with high computational complexity is proposed. Results of testing the algorithm on the tasks in a wide range of dimensions are presented.

1. Introduction

The presented work is devoted to solving one of the tasks of managing production and economic systems and processes. Within its framework, a model and an algorithm for synthesis of optimal solutions based on methods of mathematical programming are developed. Such studies are most often devoted to specific topics: tasks of placement, tasks of selecting suppliers, assigning jobs, inventory management, supply chain management, logistics, production. In the present work, a comprehensive, systematic approach to optimizing the management of the industrial assortment and material flows of an industrial enterprise has been applied [1].

At the moment, optimization tasks are widely used in various fields of production [2]. For example, in article [3], the process for finding a solution to the problem of multi-purpose optimization of the movement of material in a logistics network by using a control system based on fuzzy logic, as well as an algorithm for simulating annealing and a genetic algorithm, is described.

In work [4], it is said that research of optimization in the sphere of supply chains is now urgent, because in today's competitive and flexible environment, companies need to plan their activities effectively with the help of modern technologies and calculations. Such technologies are dynamic modeling tools, namely discrete-event simulation (DES).

In the work of researchers from Sweden [5], the issue of optimizing production logistics is described, which is also relevant to Russian enterprises. The article considers the result of the joint use
of discrete-event simulation (DES) and Simulation-Based Multi-objective Optimization (SBO) for analysis and improvement of logistics and production systems.

2. Substantial task assignment

Control actions: selection of suppliers of goods, determination of the volume of purchases throughout the assortment list, transportation, production, storage and marketing [1, 6].

Considered features of productive-economic or commercial activities, the high price of a product unit throughout the assortment (for example, electronic chips, plant seeds, jewelry), this condition does not significantly affect the structure of the formal model; relatively small volumes of supplies in terms of natural product; own production is accounted for in the general scheme as domestic supplies. Transportation costs can be considered insignificant. Only a delivery term of a cargo depends on the remoteness of suppliers, which is compensated by the necessary level of stocks in the warehouse. Supply conditions, characterized by the presence of wholesale discounts, are to be considered as essential, dependence of which on the volume of supply in value terms is shown in Fig. 1.

![Figure 1. Terms of products delivery](image)

Demand is the important factor of the model. In the worst case, it is necessary to have an average estimation of the demand forecast, in the best case - it is assumed that there is a forecast of the demand function throughout the assortment list.

Another feature is presence of several groups of consumers (wholesale customers, retail customers, persons entitled to benefits, holders of discount cards).

Taking into account the above-mentioned circumstances, the control problem can be described as follows.

It is necessary to ensure such a procurement strategy (selection of suppliers, volume of deliveries taking into account discounts), as well as the price policy of sales by consumer groups, that will maximize the criterial index (net income, or working assets at the end of the planning period), taking into account the limitations on working assets at the beginning of the period and capacity of the warehouse. A "procurement strategy" is considered as the totality of the planned volume of purchases of goods for the entire assortment, selected prices and discounts from all potential suppliers, determined for each allocated time interval of the planning period. A "sales strategy" is considered as the totality of planned sales volumes of goods for the entire assortment for all groups of consumers, determined on the basis of demand for each allocated time interval of the planning period.

Limitations of the task are logical conditions that take into account possible discounts in the procurement and sale of goods [1], as well as the demand, capacity of the warehouse, production capacities and financial capabilities of the company [6] in dynamics.

It should also be noted that the duration of the production cycle for the case under consideration is much less than any interval of the planning period.

3. Formal statement

The following notations are used:
t – number of the time interval, with a discreteness to which the model time is determined (hereinafter months); j – number of supplier ( $j = 1, J$ ), i – product number in assortment of deliveries ( $i = 1, I$ ), l – consumer type index ( $l = 1, L$ ), k – number of scale interval of discount ( $k = 1, K$ ), and demand ( $k = 1, K$ );
y$_j(t)$ – volume of purchases in terms of natural product $i$ from supplier $j$ in month $t$;

Q(t) – balance of product stock $i$ in the beginning of month $t$;

C$_j(t)$ – basic wholesale price of product $i$ from supplier $j$ in month $t$;
d$_j(t)$ – volume of purchases in value terms from supplier $j$ in month $t$ at base price (excluding discounts);
h$_{jk}(t)$ – value of right border of interval $k$ of discount scale from supplier $j$ in month $t$;
g$_{jk}(t)$ – discount from supplier $j$ in month $t$ on interval $k$ of corresponding scale (in percent);
w$_{jk}(t)$ – indicator of hit of purchase volume in interval $k$ of scale of discounts from supplier $j$ in month $t$;
x$_{ilk}(t)$ – volume of sales in terms of natural product $i$ to consumer of type $l$ in month $t$ on interval $k$ of scale of demand function;
p$_{ilk}(t)$ – product unit selling price $i$ for consumer of type $l$ in month $t$ on demand $k$ function interval;

Q(t) – size of working capital in month $t$;

N(t) – wages and overhead in month $t$;
s$_{ilk}(t)$ – value of right boundary of interval of $k$ scale of the function of demand for goods $i$ from consumer type $l$ in month $t$.

An economic-mathematical method (EMM) of optimal supply management and marketing of heterogeneous products of the enterprise will look as follows:

$$\sum_{j=1}^{J} C_{ij}(t)y_{ij}(t) = d_{j}(t), \quad j = 1, J, t = 1, T;$$  \hspace{1cm} (1)

$$d_{j}(t) - h_{jk}(t)w_{jk}(t) \geq 0, \quad j = 1, J, t = 1, T;$$ \hspace{1cm} (2)

$$0 \leq w_{jk}(t) \leq 1, \quad w_{jk}(t) - \text{whole, } y_{ij}(t) \geq 0, \quad i = 1, I, \quad j = 1, J, \quad t = 1, T, \quad k = 1, K;$$ \hspace{1cm} (3)

$$\sum_{j=1}^{J} [d_{j}(t) - d_{j}(t)] \sum_{k=1}^{K} g_{jk}(t)w_{jk}(t)] \leq Q(t), \quad t = 1, T, \quad \text{where}$$ \hspace{1cm} (4)

$$g_{jk}(t) = \begin{cases} g_{j1}(t), & \text{if } d_{j}(t) \leq h_{j1}(t), \\ g_{j2}(t), & \text{if } h_{j1}(t) < d_{j}(t) \leq h_{j2}(t), \\ \vdots & \\ g_{jk}(t), & \text{if } d_{j}(t) > h_{jk}(t). \end{cases}$$ \hspace{1cm} (5)

$$x_{ilk}(t) \leq s_{ilk}(t), \quad i = 1, I, \quad l = 1, L, \quad t = 1, T;$$ \hspace{1cm} (6)
\[
\sum_{j=1}^{L} y_{ij}(t) + O_{j}(t-1) \geq \sum_{i=1}^{K} \sum_{k=1}^{K} x_{ik}(t) , \quad i = 1, T ; \quad t = 1, T ;
\]  
(7)

\[
O_{j}(t) = \sum_{j=1}^{L} y_{ij}(t) + O_{j}(t-1) - \sum_{i=1}^{K} \sum_{k=1}^{K} x_{ik}(t) , \quad i = 1, T ; \quad t = 1, T ;
\]  
(8)

\[
Q(t+1) = \sum_{j=1}^{L} \sum_{i=1}^{K} \sum_{k=1}^{K} p_{ik}(t)x_{ik}(t) - N(t) - \sum_{j=1}^{J} \sum_{k=1}^{K} d_{jk}(t) - d_{j}(t) \sum_{k=1}^{K} g_{jk}(t)w_{jk}(t) , \quad t = 1, T ;
\]  
(9)

\[
\sum_{t=2}^{T} \alpha(t)Q(t) \rightarrow \max , \quad \text{under conditions}\ 0 \leq \alpha(t) \leq 1, \sum_{t=2}^{T} \alpha(t) = 1 ;
\]  
(10)

\[
Q(T) \rightarrow \max .
\]  
(11)

With the help of formulas (1), the volumes of purchases in value terms are set without taking into account the discounts in month t of supplier j; formulas (2) and (3) are the logical limitations on the availability of discounts and their sizes; (4) – the limitations on the volume of purchases in value terms, taking into account the discounts in month t for all suppliers; (5) and (6) – the limitations on the demand for each product for all types of consumers in month t. Formulas (7) are logical constraints: total volumes of purchases and balances in the warehouse for each assortment position in each month should not be less than the corresponding sales volumes. Formulas (8) set the dynamics of the stock balances throughout the entire assortment, (9) - the dynamics of net income, (10) - the criteria efficiency indicator with the meaning of the weighted average of net income, (11) - a special case - the value of net income at the end of the planned period.

Taking into account the fact that the task considered in this work takes into account the production component of the process, one more limitation will be added to this set of limitations: limitation on the ways of converting raw materials and components Y into sold goods X:

\[
X = A \times Y,
\]  
(12)

where A is the tensor of technological coefficients. Here it is worth noting that since one calculates several output values for one output product (taking into account the types of consumers and the scale of discounts), calculations will be carried out using the same tensor coefficients A for the given goods for all these values. This is taken into account by the following group of limitations:

\[
\sum_{k=1}^{K} \sum_{l=1}^{L} x_{ik}(t) = \sum_{i=1}^{I} \left[ A_{iy}(t) \sum_{j=1}^{J} y_{ij}(t) \right] , \quad i = 1, T ; \quad t = 1, T .
\]  
(13)

4. Approximate algorithm for solving the problem of optimal control of supply and sales of products

As noted above, without taking into account the specific conditions, task (1) - (13), for specified parameters of computational complexity, is formally undecidable by exact methods. To resolve this problem, let us construct an algorithm which takes into account the maximum of this specificity. Let us note that all discount functions \( g_{jk}(t) \) are non-decreasing functions, respectively, all functions of wholesale prices and demand are nonincreasing ones. Taking into account these circumstances, let us propose the following algorithm for finding the optimal solution of task (1) - (13):

Preliminary step. Define the relaxed task to (1) - (13) as follows. Choose any \( \tilde{g}_{j}(t) \in \{ g_{jk}(t) \} , \quad j = 1, J ; \quad t = 1, T \), and on the basis of task (1) - (13), form a linear subtask.

Let us add the notations introduced above. Let n be the step number of the algorithm. Let us denote by \( Y^{n} \), \( X^{n} \), \( z^{n} \) the solution of the relaxation task at step n (volumes of purchases, sales and the value of the efficiency criterion). Let us denote by \( G^{n} \) the set of intervals of discounts at step n, \( \{ \tilde{g}_{j}(t) \} \) \( G^{n} \).
Below the algorithm for solving assigned task step by step is represented.

First step. Let us assume that \( \bar{g}_j(t) = \max_k \{ g_{jk}(t) \} = g_{jk}(t), \ j=1, J, t=1, T \) and set up a relaxed subtask. Let us denote its solution by \( Y^0, X^0, z^0 \). \( (Y^0 = [y^0_j(t)], \ X^0 = [x^0_{ik}(t)], \ z^0 = z(Y^0)) \) and define matrix identity \( \| \bar{g}_j(t) \| = G^0 \).

Step n. Based on the solution obtained in previous step \( Y^{n-1}, X^{n-1}, z^{n-1} \) with \( G^{n-1} \), let us define new values \( Y^n, X^n, z^n \) and check them for optimality. If \( \varepsilon \leq z^n - z^{n-1} \leq \varepsilon \) (\( \varepsilon \) - some small number that specifies the accuracy of calculations), then the optimal solution of task (1) - (13) is obtained at the current step. If the condition does not hold, go to the next step (n + 1), defining new values \( Y^n \).

It is quite obvious that the algorithm converges in a finite number of steps, which cannot be higher than value \( J \cdot K \cdot T \). This is determined by the specifics of the nondecreasing functions of discounts \( g_{jk}(t) \). For example, \( J \cdot K \cdot T = 90 \). The statistical estimate of the number of steps for this example when the initial data are varied is 5.

Thus, the proposed algorithm transforms task (1)-(13) into a category that is polynomially solvable with respect to dimension. When using it, task (1)-(13) falls into another class of linear models with continuous variables and the complete absence of integer variables.

5. Obtained results

This algorithm was implemented as a program. The universality of the model and the program in addition to the toolkit [7] developed before with respect to types of enterprises is achieved with the help of the tensor of technological coefficients, which determines the ways of converting raw materials and components into goods sold and participates in the group of constraints of the task (12) - (13). A trading enterprise does not produce its own products, but purchases finished goods and resells them. In this case, the corresponding technological coefficients will be units.

An industrial enterprise purchases a number of other products and raw materials for production of certain type of a product. In these tests, they were generated in the range from 0 to 1.

Let us consider the results of testing the developed program in the case of manufacturing enterprises. Here is a description of Table 1 below. It shows the input data and the results of the program’s work for each test, which are located in separate lines. In the right part of the Table, the input conditions are provided, i.e. values showing how many products are there in a supply range in this case (in this task, in this test) \( I \), consumer index type \( L \), suppliers \( J \), intervals discounts volumes scales \( K \) and demand \( K1 \), the number of time intervals \( T \). In the second part of Table 1, the resulting indicators such as the time, during which the program has been completed (in seconds and fractions of seconds), the number of steps, for which the task has been solved \( q \) and the number of restrictions in the task are given.

Columns "Number of continuous variables" and "Number of Boolean variables" indicate the
number of variables involved in solving the task. This includes all the components of tensors $y_g(t)$, $x_{jk}(t)$ and $w_{jk}(t)$, taking into account their difference for each from the time intervals.

The "Number of constraints" column shows the number of constraints in a given task, which depends on the input values of the variables. The calculation of the number of constraints was carried out in the following way: that is, accounted for the restrictions (1), (2), (3) for variables in the volume of purchases, (4), (5), (6), (7), (8) and (13).

**Table 1.** Results of testing the program taking into account the availability of production

| No. | I (TYPES OF GOODS) | J (TYPES OF CONSUMERS) | K (INTERVALS OF SCALE OF DISCOUNTS) | K1 (INTERVALS OF SCALE OF DEMAND) | T (MONTHS) | COUNTING TIME (SECONDS) | Q (STEPS) | NUMBER OF CONTINUOUS VARIABLES | NUMBER OF BOOLEAN VARIABLES | NUMBER OF CONSTRAINTS |
|-----|-------------------|------------------------|------------------------------------|-----------------------------------|------------|-------------------------|-----------|------------------------------|--------------------------|---------------------|
| 1   | 10                | 5                      | 10                                 | 4                                 | 3          | 6                       | 4.25      | 5                           | 1500                     | 240                 |
| 2   | 10                | 5                      | 10                                 | 4                                 | 6          | 4.77                    | 5         | 1800                        | 300                     | 1656                |
| 3   | 7                 | 4                      | 4                                  | 3                                 | 6          | 2.59                    | 3         | 672                         | 96                      | 1020                |
| 4   | 7                 | 4                      | 4                                  | 5                                 | 3          | 2.8                     | 3         | 840                         | 120                     | 1188                |
| 5   | 6                 | 3                      | 4                                  | 3                                 | 6          | 2.69                    | 3         | 432                         | 72                      | 690                 |
| 6   | 6                 | 3                      | 3                                  | 5                                 | 4          | 3.28                    | 4         | 540                         | 90                      | 798                 |
| 7   | 10                | 5                      | 10                                 | 3                                 | 2          | 4.78                    | 5         | 1200                        | 180                     | 1806                |

In this section of testing, three blocks of tests are shown: in each of them there are identical indicators I, L and J and on two different tests for different K and K1. The planning period was the same for all tests in this section. It can be seen how time increases linearly for tests with higher dimensionality, that is, with large K and K1 for rows with the same indices I, L and J, which confirms the efficiency of the algorithm. The number of steps in the tests with a larger dimension is greater than or equal to that of the corresponding paired tests with a smaller dimension.

**6. Findings**

The initial task of management of external material flows of an enterprise is considered and supplemented with the production component. The program that implements the modified algorithm has been developed and tested. The result is a new decision-making support tool. Thus, this program can be successfully applied to solve real-time problems that arise in the production sector, in terms of EMM application in the field of logistics of an enterprise, as well as decision support in planning the volumes of purchases, production and sales, taking into account the criterion of maximizing working capital balances. According to estimation of the experts, the potential to improve performance in the area of finding the best logistics solutions of problems is an average of 30% [7,8].

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