The angular momentum of colliding rarefied preplanetesimals and the formation of binaries

S. I. Ipatov\textsuperscript{1,2}\star

\textsuperscript{1}Catholic University of America, Department of Physics, Washington DC 20064, USA

\textsuperscript{2}Space Research Institute, Moscow, Russia

ABSTRACT
This paper studies the mean angular momentum associated with the collision of two celestial objects in the earliest stages of planet formation. Of primary concern is the scenario of two rarefied preplanetesimals (RPPs) in circular heliocentric orbits. The theoretical results are used to develop models of binary or multiple system formation from RPPs, and explain the observation that a greater fraction of binaries originated farther from the Sun. At the stage of RPPs, small-body satellites can form in two ways: a merger between RPPs can have two centres of contraction or the formation of satellites from a disc around the primary or the secondary. Formation of the disc can be caused by that the angular momentum of the RPP formed by the merger is greater than the critical angular momentum for a solid body. One or several satellites of the primary (moving mainly in low-eccentricity orbits) can be formed from this disc at any separation less than the Hill radius. The first scenario can explain a system such as 2001 QW\textsubscript{322} where the two components have similar masses but are separated by a great distance. In general, any values for the eccentricity and inclination of the mutual orbit are possible. Among discovered binaries, the observed angular momenta are smaller than the typical angular momenta expected for identical RPPs having the same total mass as the discovered binary and encountering each other in circular heliocentric orbits. This suggests that the population of RPPs underwent some contraction before mergers became common.

Key words: Kuiper belt – minor planets, asteroids – planets and satellites: formation – Solar system: formation.

1 INTRODUCTION

1.1 The case for rarefied preplanetesimals
From the 1950s to the 1980s, many authors (e.g. Safronov 1969; Goldreich & Ward 1973) believed that planetesimals originated from rarefied dust condensations (RDCs): overdense regions of the disc that are gravitationally bound, and contain solid particles, but are still gaseous or diffuse. During this period, the mechanisms behind RDC formation and the coagulation of larger ‘secondary condensations’ were studied by several authors (e.g. Safronov 1969, 1995; Eneev & Kozlov 1981, 1982; Pechernikova & Vityazev 1988). The duration of the condensation stage was thought to be longer at greater distances from the Sun, where the RDCs have lower densities. According to an early model by Safronov (1967, 1969), the time required for RDCs to form into solid bodies was about 10\textsuperscript{4} yr in the terrestrial zone and 10\textsuperscript{6} yr at the distance of Jupiter. The corresponding increases in mass were 10\textsuperscript{2} and 10\textsuperscript{3} from the initial values. Safronov concluded that while the rotation of initial RDCs impedes contraction, they also become denser when they combine. In the models of RDC contraction considered by Myasnikov & Titarenko (1989, 1990), collisions between condensations were not taken into account. The times needed for RDCs to become solid planetesimals could exceed several million years, depending on the optical properties of the dust and gas and the type and concentration of short-lived radioactive isotopes in RDCs. Depending on the concentration, the times varied by a factor of more than 10 and could exceed 10 Myr at the concentration greater than 0.02.

In the 1990s, scenarios of planetesimal formation through the gravitational instability of RDCs were frustrated by the phenomenon of self-generated mid-plane turbulence. Models involving the hierarchical accretion of planetesimals from smaller solid objects became more popular (e.g. Weidenschilling 2003). In recent years, however, new arguments (e.g. Makalkin & Ziglina 2004; Johansen et al. 2007; Cuzzi, Hogan & Shariff 2008; Lyra et al. 2010) have suggested that the population of RPPs underwent some contraction before mergers became common.
2008; Johansen, Youdin & Mac Low 2009 have arisen in favour of RDCs, or as they can be also called rarefied preplanetesimals (RPPs) (condensations, clusters and clumps). Unlike earlier models, however, the overdense regions could include metre-sized boulders and smaller solid bodies. Some results from these more recent models are summarized below.

Makalkin & Ziglina (2004) showed that after the subdisc reaches a certain critical density, its inner, equatorial layer could be gravitationally unstable. This is possible because the inner disc, unlike the two subsurface layers, contains no shear turbulence. In their model, RDCs form trans-Neptunian objects (TNOs) with diameters up to 1000 km in \( \sim 10^6 \) yr.

Johansen et al. (2007) found that gravitationally bound clusters can form with masses comparable to dwarf planets. They determined that peak densities in the clusters approached \( 10^4 \rho_{\text{gas}} \) (where \( \rho_{\text{gas}} \) is the average gas density) after only seven orbits. Lyra et al. (2008) noted that strong drag forces in the disc might delay gravitational collapse of the clusters. Cuzzi et al. (2008) showed how self-gravity could stabilize dense clumps of millimetre-sized particles. Such clumps form naturally in three-dimensional turbulence, forming diffuse but cohesive ‘sandpiles’ of the order of 10–100 km in diameter. For chondrules of radius \( r_{\text{c}} = 300 \mu m \), they obtained a ‘sedimentation’ time-scale of about 30–300 orbital periods at 2.5 au. (The time-scale is proportional to \( r_{\text{c}}^{-1.3} \).) Youdin (2008) summarized various mechanisms for particle concentration in gas discs, including turbulent pressure maxima, drag instabilities and long-lived anticylonic vortices.

Ida, Guillot & Morbidelli (2008) studied the accretion and destruction of solid planetesimals in turbulent discs, and found that accretion proceeds only for planetesimals with diameters above 300 km at 1 au and above 1000 km at 5 au. Their analytical arguments were based on fluid dynamical simulations and orbital integrations. They concluded that some mechanism must be capable of producing Ceres-mass planetesimals on very short time-scales. Based on the observed size-frequency distribution and collisional evolution of asteroids, Morbidelli et al. (2009) concluded that the initial planetesimals had to range from one hundred to several hundreds of km in size, probably up to 1000 km.

Prior to these new arguments in favour of RPPs, I (Ipatov 2001, 2007) had suggested that TNOs with mildly eccentric orbits (\( e < 0.3 \)) and diameters greater than 100 km (such as Pluto and Charon) could result from the compression of large RPPs with semimajor axes \( a > 30 \) au rather than the accretion of small, solid planetesimals. I also proposed that some planetesimals with diameters \( d \sim 100–1000 \) km in the feeding zone of the giant planets, some planetesimals with \( d \sim 100 \) km in the terrestrial zone and some large main-belt asteroids could have formed by direct compression. Some smaller objects (TNOs, planetesimals and asteroids) with \( d < 100 \) km could be debris from large objects, while others could have formed by direct compression of preplanetesimals.

1.2 Formation of binaries

The frequency of binary systems and the ratio of secondary mass to primary mass are greater among classical TNOs with inclinations \( i < 5^\circ \) than among main-belt asteroids (Richardson & Walsh 2006; Noll 2006; http://www.johnstonsarchive.net/astro/asteroidmoons.html).

Planetesimal formation models based on the accretion of solid bodies have put forth several hypotheses on the formation of binaries. For example, several papers (e.g. Doressoundiram et al. 1997; Durda et al. 2004; Canup 2005; Walsh 2009) have been devoted to the mechanism of catastrophic collisions. Weidenschilling (2002) studied the collision of two planetesimals within the sphere of influence of a third body. Goldreich, Lithwick & Sari (2002) proposed that the primary body could capture a secondary component passing inside its Hill sphere due to dynamical friction from surrounding small bodies or gravitational scattering by a third large body. Funato et al. (2004) studied a model in which a low-mass secondary component is ejected and replaced by a third body in a wide but eccentric orbit. Astakhov, Lee & Farrelly (2005) studied binary system formation in four-body simulations with solar tidal effects. Gorkavyi (2008) proposed a multi-impact model. Six mechanisms of binary formation were discussed by Doressoundiram et al. (1997). More references can be found in papers by Richardson & Walsh (2006), Noll (2006), Petit et al. (2008), Noll et al. (2008) and Scheeres (2009).

We agree with Čuk (2007), Pravec, Harris & Warner (2007), Walsh, Richardson & Michel (2008), and several other authors that binary systems with small primaries (such as near-Earth objects) arise mainly from the rotational breakup of ‘rubble piles’, for example due to increasing spin from the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect. However, I believe that collisions could also push small bodies beyond their critical spin limits.

Some scientists believe that the formation of small-body binaries could be similar to the formation of binary stars within fragmented discs (Alexander, Armitage & Cuadra 2008). Nesvorný (2008) noted that an excess of angular momentum prevents the agglomeration of all available mass into solitary objects during gravitational collapse.

1.3 Problems studied in this paper

In contrast with models of binary formation that consider solid bodies, in 2004 I proposed that a considerable fraction of trans-Neptunian binaries (including Pluto–Charon) could form while RPPs moving in almost circular orbits undergo compression (Ipatov 2004). In 2009, I set forth two detailed models of binary formation during this stage (Ipatov 2009). They are repeated here and further discussed in Sections 2.2 and 2.3. These models can explain trans-Neptunian binaries, and some asteroid binaries with large primaries (with diameters of at least 50 km). Small asteroid binaries could mainly arise from collisions between larger solid bodies.

The problem of binary formation from RPPs has several aspects (e.g. simulations of contraction, collisions between preplanetesimals). In this paper, I pay particular attention to calculating the angular momentum of two colliding RPPs about their centre of mass (Section 3). I also compare the predicted angular momenta to the observed momenta of discovered binary systems (Section 4). Other aspects of binary formation at the preplanetesimal stage (e.g. mergers of colliding planetesimals) are discussed briefly in Section 5, and may be the subject of future publications.

We show that if RPPs existed during the period permitting encounters to occur between preplanetesimals within their Hill spheres then it is possible to explain the differences between TNO and asteroid binaries as well as some of the peculiarities of observed binaries. The material presented in Section 3 also helps us better understand the angular momenta of planetesimals, TNOs and asteroids, although these momenta could change considerably through gravitational interactions and collisions after all bodies have solidified.
2 SCENARIOS OF BINARY FORMATION DURING THE RAREFIED PREPLANETESIMAL STAGE

2.1 Application of previous solid-body scenarios to preplanetesimals

At present, most small bodies move in orbits with eccentricities exceeding 0.05. Solid planetesimals would have begun with much smaller eccentricities, but mutual gravitation tends to increase the eccentricities and inclinations of their orbits (e.g. Ipatov 1988, 2000). RPPs, on the other hand, would remain in almost circular orbits with very small inclinations before the moment of formation of a binary. The smaller growth of RPP orbital eccentricities could be because the stage of preplanetesimals was short and circulation of an orbit of an object (e.g. a preplanetesimal or a planetesimal) of a fixed mass due to the influence of gas and dust could be greater for a larger size object. Due to their larger sizes, preplanetesimals collided more often than solid planetesimals per unit of time.

If we consider almost circular heliocentric orbits, the typical minimum distance between the centres of mass of the objects could be smaller than that between two objects entering the Hill sphere from eccentric heliocentric orbits. Heliocentric orbits with high eccentricities more often give rise to hyperbolic relative motion. In contrast, for nearly circular orbits, the trajectory of relative motion inside the sphere can be complicated (e.g. Ipatov 1987; Greenzweig & Lissauer 1990; Iwasaki & Ohtsuki 2007). In this case, the objects can move inside the sphere for a long time, and the relative distance can change considerably. The role of tidal forces could also be greater for rarefied objects.

The above discussion supports my belief that models of binary formation due to gravitational interaction or collisions occurring inside the Hill sphere of the future primary, which have been considered by several authors for solid objects, could be even more effective for RPPs.

2.2 Binary formation from two centres of contraction

RPPs contract over time, and may exhibit centrally concentrated radial density functions. It is reasonable to suppose that in some cases the collision of two such objects results in a rotating system with two centres of contraction. The end result of gravitational collapse would be a binary system with almost the same total mass and components separated by a large distance (such as 2001 QW32). For such a scenario, the values of the eccentricity of the present mutual orbit of components can have any values less than 1.

If the original RPPs were similar in size to their Hill spheres, the separation \( L \) of the solid binary resulting from the combined system could range up to the radius \( r_H \) of the Hill sphere. More often, however, the preplanetesimals were much smaller than their Hill spheres. The separation distance \( L \) would then be less than \( r_H \). For discovered binaries with \( L/r_F > 100 \), the ratio \( r_F/r_H \) of the radii of secondary to primary components is greater than 0.5. This fact may imply that when the masses of two RPPs differ greatly only one centre of contraction survives. Note that other authors (e.g. Descamps & Marchis 2008) have suggested different explanations for the above ratio.

2.3 Binary formation due to excessive angular momentum

In my opinion, some binaries could have formed from RPPs that obtained more angular momentum than the maximum possible value for a solid body. Let \( v_s \) denote the velocity of a particle on the surface of a rotating object, and let \( v_f \) be the minimum velocity at which a particle can leave the surface. As a rotating RPP contracts, some of the material with \( v_s > v_f \) could form a cloud (that transforms into a disc) that moves around the primary. For a spherical object, \( v_s \) is greatest at the equator and \( v_f \) can be close to the circular velocity. For relatively condense preplanetesimals, their collision can eject material into a satellite-forming disc. The disc can capture dust and boulders that enter the Hill sphere after the encounter of RPPs.

One or several satellites of the primary could form in this cloud (similar to typical models of formation of satellites of planets; see Woolson 2004, for example). The eccentricities of such satellites would usually be small. As we show at the end of Section 3.3, this scenario allows the formation of satellites at practically any distance from the primary up to the Hill radius. The initial radius of the ‘parent’ preplanetesimal, however, could be greater than the separation between components of the formed binary.

2.4 Hybrid scenario and the formation of elongated bodies

The two scenarios presented in Sections 2.2 and 2.3 could take place at the same time. In addition to massive primary and secondary components, smaller satellites moving around either body could be formed. For binaries formed in such a way, the separation distance between the main components can be large or small.

It is possible that massive yet rarefied binary components merge when their central parts became dense enough. In such a case, the form of the solid body obtained could differ greatly from an ellipsoid. For example, this situation could give rise to an elongated, bone-like body where both of the original components are visible. As in other scenarios, the solid body may have one or more small satellites. For example, (216) Kleopatra (with dimensions of 217 × 81 km) could have formed in this manner.

Even more rarely, several partly compressed components could merge simultaneously. In this case, the form of the solid body would be complicated, to some extent ‘remembering’ the forms of its components.

3 THE ANGULAR MOMENTUM OF COLLIDED RAREFIED PREPLANETESIMALS

3.1 Basic model of an encounter between preplanetesimals

In order to analyse the scenarios discussed in Section 2, Sections 3.2–3.4 will study the angular momentum \( K_s \) of two colliding RPPs relative to their centre of mass and the period \( T_c \) of the solid primary formed by contraction. Maximum and typical values of the separation distance \( L \) between binary components for the above scenarios are discussed in Section 3.3.

Most of the formulae and calculations below are presented for a ‘basic’ model: the merger of two spherical RPPs that before the collision was moving in coplanar, circular and heliocentric orbits. The mass of the resulting object is equal to the sum of the two preplanetesimal masses. The rotating, merged system then contracts and transforms into a solid planetesimal. The angular momenta presented in Table 1 were obtained assuming that the radii of the two preplanetesimals are equal to their Hill radii. Based on the ‘basic’ model, we discuss more complicated models.

Scientists who study the formation of preplanetesimals generally consider almost circular orbits and very small inclinations. These conditions minimize the relative velocities of neighbouring objects.
preplanetesimals. The probability of a merger is smaller for greater eccentricities. In all existing models of planetesimal discs, the mean inclination (in radians) of the orbits is assumed to be smaller than the mean eccentricity. Relaxing this condition on the inclinations will not change the order of magnitude of the angular momentum of two colliding preplanetesimals, but can change its direction (see Sections 4.3 and 5.3).

For formation of a binary, it is not necessary that every encounter between preplanetesimals (i.e. when one object’s centre of mass passes through the Hill sphere of the other) results in a merger. It is enough that for some preplanetesimals at least one such encounter occurred during their lifetimes. For almost circular heliocentric orbits, the minimum distance between encountered material points with masses of preplanetesimals can be several orders of magnitude smaller than the Hill radius $r_H$, and some of the material points can move inside the sphere for a long time. For example, in an earlier work (Ipatov 1987) I presented a case where two planetesimals move together for more than half an orbital period (see also discussions in Sections 2.1 and 5.1). Therefore, the values of $K_s$ and $T_s$ obtained for the ‘basic’ model can also be true for encounters between preplanetesimals much smaller than their Hill spheres. The sizes and mergers of encountered preplanetesimals are discussed in Section 5.1.

Not all boulders could be captured during an encounter, and some material could be lost at the stage of contraction. Therefore, the mass of the formed binary (or a preplanetesimal) may be smaller than the sum of the masses of the two colliding preplanetesimals. Likewise, the final value of $K_s$ may be smaller than the total angular momentum of the RPPs.

Sections 3.2 and 3.3 present formulae for $T_s$ and $K_s$ after a single collision in the basic model. For multiple collisions, the angular momenta are summed. In my opinion (see Section 4.3), most of the angular momentum possessed by a trans-Neptunian binary usually resulted from a single collision.

Naturally, a present-day binary can differ significantly from the primordial binary formed by contracting RPPs. For example, tidal forces can increase the separation between binary components [e.g. the Pluto-Charon and Antiope-S/2000(90) systems] and decrease the period of axial rotation (as occurred for the Earth–Moon system) on time-scales shorter than the age of the Solar system. Although a given primordial binary system might have been more compact than its corresponding discovered system, the total angular momentum could remain the same. The angular momenta of individual components could also change due to collisions with other small bodies and the YORP effect. Finally, the angular momentum of a contracting preplanetesimal could decrease due to tidal interactions with the Sun.

It is likely that most primordial planetesimals were ellipsoidal. The velocities of particles close to the equator of a rotating contracting preplanetesimal were greater than those of particles close to the poles, and more particles could leave the equator. Therefore, some contracting preplanetesimals could be elongated along the axis of rotation. On the other hand, a surrounding disc could feed material to the equator, resulting in an oblate shape. In either case, the equator would be approximately circular. In the ‘rubble pile’ model considered by Walsh et al. (2008), the axial radius was smaller than the equatorial radius. The formation of highly elongated primordial small bodies was discussed in Section 2.4.

### 3.2 Principal formulae for calculation of the angular momentum of two colliding preplanetesimals

Previous papers (e.g. Lissauer et al. 1997; Ohtsuki & Ida 1998) devoted to the formation of axial rotation of celestial bodies have mainly studied the final rotation rates of planets accreted from a disc of solid planetesimals. My own studies on the formation of axial rotation of RPPs, solid planetesimals and planets were presented in two preprints (N 101 and N 102) published in Russian in 1980 by the Institute of Applied Mathematics of the USSR Academy of Sciences. These studies are summarized in Ipatov (2000), which cites many papers devoted to the formation of spin. The preprints focused mainly on the axial rotation of planets, but the formulae used are applicable to a wide variety of cases. Sections 3.2–3.4 present similar formulae. In Sections 4–5, the formulae are applied to the studies of the formation of binaries.

| binary       | Pluto | (90842) Orcus | 2000 CF$_{105}$ | 2001 QW$_{322}$ | (87) Sylvia | (90) Antiope |
|--------------|-------|--------------|----------------|----------------|-------------|-------------|
| $a$ (au)     | 39.48 | 39.3         | 43.8           | 43.94          | 3.94        | 3.156       |
| $d_p$ (km)   | 2340  | 950          | 170            | 108?           | 286         | 88          |
| $d_i$ (km)   | 1212  | 260          | 120            | 108?           | 18          | 84          |
| $m_p$ (kg)   | $1.3 \times 10^{22}$ | $7.5 \times 10^{20}$ | $2.6 \times 10^{18}$ & $6.5 \times 10^{17}$ | $1.478 \times 10^{19}$ | $4.5 \times 10^{17}$ |
| $m_i$ (kg)   | $1.52 \times 10^{21}$ | $1.4 \times 10^{18}$ | $9 \times 10^{17}$ | $6.5 \times 10^{17}$ | $3 \times 10^{18}$ | $3.8 \times 10^{17}$ |
| $L$ (km)     | 19750 | 8700         | 23000          | 120000         | 1356        | 171         |
| $L/r_H$      | 0.0025| 0.0029       | 0.04           | 0.3            | 0.019       | 0.007       |
| $2L/d_p$     | 16.9  | 18.3         | 271            | 22000          | 9.5         | 3.9         |
| $T_s$ (h)    | 153.3 | 10           | 5.18           | 16.5           |             |             |
| $K_{\text{cm}}$ (kg km$^2$ s$^{-1}$) | $6 \times 10^{24}$ | $9 \times 10^{21}$ | $5 \times 10^{19}$ | $3.3 \times 10^{19}$ | $10^{17}$ | $6.4 \times 10^{17}$ |
| $K_{\text{spin}}$ (kg km$^2$ s$^{-1}$) | $10^{23}$ | $10^{22}$ | $1.6 \times 10^{18}$ | $2 \times 10^{17}$ | $4 \times 10^{19}$ | $3.6 \times 10^{16}$ |
| $(K_{\text{cm}} + K_{\text{spin}})/K_{\text{d06p}}$ | 0.07 | 0.2 | at $T_s = 8$ h | at $T_s = 8$ h | 6.6 | 10$^{18}$ |
| $(K_{\text{cm}} + K_{\text{spin}})/K_{\text{d06q}}$ | 0.02 | 0.01 | 0.2 | 0.63 | 0.05 | 0.1 |
| $v_{\text{eq}}$ (m s$^{-1}$) | 6.1 | 2.2 | 0.36 | 0.26 | 2.0 | 0.82 |
| $v_{\text{eq06}}$ (m s$^{-1}$) | 5.5 | 1.8 | 0.3 | 0.26 | 1.3 | 0.82 |
| $v_{\text{esc-pr}}$ (m s$^{-1}$) | 15.0 | 5.8 | 0.8 | 0.53 | 5.3 | 1.7 |

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Consider an inelastic collision between two non-rotating spherical objects (preplanetesimals or planetesimals) with masses \( m_1 \) and \( m_2 \) and radii \( r_1 \) and \( r_2 \), moving around the Sun in coplanar circular orbits. The first object is closer to the Sun. Its semimajor axis is denoted by \( a \); its circular velocity can be calculated from the relationship \( v_i = (GM_S/a)^{1/2} \), where \( G \) is the gravitational constant and \( M_S \) is the mass of the Sun. The angular momentum of the colliding system relative to its centre of mass is

\[
K_s = v_i(r_1 + r_2)m_1m_2/(m_1 + m_2),
\]

where \( v_i \) is the tangential component of the velocity (with respect to the centre of mass of the first object) when the objects collide. If the difference \( \Delta \alpha \) in the orbits’ semimajor axes equals \( \Theta(r_1 + r_2) \), we have

\[
v_i = \kappa_0 v_a(r_1 + r_2)/a = \kappa_0 (GM_S)^{1/2}(r_1 + r_2)a^{-3/2}
\]

and

\[
K_s = K_{\text{coll}} = \kappa_0 (GM_S)^{1/2}(r_1 + r_2)^2 \frac{m_1m_2}{(m_1 + m_2)a^{3/2}}.
\]

For \( r_a = (r_1 + r_2)/a \ll \Theta \), one can obtain

\[
k_{\text{coll}} \approx (1 - 1.5\Theta^2)
\]

and

\[
v_{rel} \approx (1 - 0.75\Theta^2)^{1/2}v_ar_a,
\]

where \( v_{rel} \) is the relative velocity between the two colliding objects.

Below, I derive equations (3) and (4). The angle between the lines connecting the Sun to the two encountering objects is \( \alpha \approx \sin \alpha \approx r_a(1 - \Theta^2)^{1/2} \). Consider the triangle ABC, where points A and B are the positions of the objects and point C is the intersection between the orbit of the first (farther) object and the line from the Sun to the second object (point B). The angle CAB is close to the angle \( y \) between the velocity of the first object and the direction from the first object to the second object (located farther from the Sun). We therefore consider \( \sin y \approx \Theta \). Taking into account that \( v_1 \propto a^{-3/2} \) and \((1 + r_a\Theta)^{-1} \approx 1 - r_a\Theta/2 \), we obtain that the velocity of the second object equals \( v_2 \approx v_1(1 - r_a\Theta)/2 \). For the considered model, \( \cos \Theta \approx 1 \) and \( \cos y \approx (1 - \Theta^2)^{1/2} \). We therefore obtain \( \sin (y + \alpha) \approx \sin y + \alpha \cos y \), and the difference between the tangential velocities of the two encountering objects is

\[
v_{11} - v_{12} = v_1 \sin y - v_2 \sin(y + \alpha)
\]

\[
\approx v_1 \Theta - v_1(1 - r_a\Theta/2) \{\Theta + [r_a(1 - \Theta^2)^{1/2}] (1 - \Theta^2)^{1/2}\}
\]

\[
\approx v_1 r_a(1.5\Theta^2 - 1).
\]

This difference is valid for clockwise motions (negative angular velocities). Therefore, \( k_{\alpha} \approx (1 - 1.5\Theta^2) \).

Let us take into account that the angle between velocities \( v_1 \) and \( v_2 \) is equal to \( \alpha \), \( v_2 \approx v_1(1 - r_a\Theta/2), \sin \alpha \approx r_a(1 - \Theta^2)^{1/2} \) and \( \cos \alpha \approx 1 \). Neglecting the term with \( r_a \), we obtain

\[
v_{rel}^2 \approx (v_1 - v_2 \cos \alpha)^2 + (v_2 \sin \alpha)^2
\]

\[
\approx [\Theta^2/4 + (1 - \Theta^2)](v_1r_a)^2 = (1 - 0.75\Theta^2)(v_1r_a)^2.
\]

Though the above derivation of equation (3) is not valid for small \( \Theta \), below we can use it to estimate the mean values of \( k_{\alpha} \), \( K_s \), and \( v_i \), by evaluating the integrals over \( \Theta \). The reviewer noted that equations (3) and (4) are actually accurate even for \( \Theta = 0 \), as planetesimals ‘colliding’ (or more accurately, touching) on the same circular heliocentric orbit have non-zero angular momentum relative to each other in the inertial frame despite being stationary in the rotating frame. Only at larger \( \Theta \) does the Keplerian shear begin to dominate, resulting in a negative (i.e. retrograde) total angular momentum.

The overall mean value of \( |k_{\alpha}| \) is 0.6. Among positive values of \( k_{\alpha} \), the mean is 2/3, and among negative values it is \(-0.24\). The values of \( K_{\text{coll}} \) and \( k_{\alpha} \) are positive in the range \( 0 < \Theta < 2/3^{1/2} \approx 0.8165 \) and negative in the range \( 0.8165 < \Theta < 1 \). The same range of \( \Theta \) for positive and negative values of \( K_{\text{coll}} \) were used by Eneyev & Kozlov (1982), who obtained them from a numerical integration of the three-body problem. The minimum value of \( k_{\alpha} \) is \(-0.5\), and its maximum value is 1. If \( \Theta \) is uniformly distributed for objects moving in the same plane, the probability of a single collision producing negative rotation is about \( 1/5 \). The ratio of the sum of positive values of \( K_s \) to the sum of all \( |K_s| \) equals 0.925. This ratio, obtained by evaluating the integrals given above, is close to the ratio obtained in numerical experiments with initially circular orbits. The results of the experiments were presented in the aforementioned preprints in 1981.

In the basic model, a new object forms in any encounter, and heliocentric orbits are considered to be circular at the time of encounter up to \( r_1 \). This model could work for some encounters of very RPPs (see discussion in Section 5.1). Compared to the above model, the fraction of collisions with positive angular momentum would be smaller if one considers collisions of solid (or at least higher density) objects and/or supposes that their orbits are circular only so long as the objects are widely separated (i.e. the orbits deviate from a heliocentric circle when separation equals to \( r_1 \)). Under the right conditions, it is even possible for the merged planetesimal to have negative momentum. Therefore, the dominance of positive angular momentum in the basic model does not contradict the negative spins obtained by authors who have used other models and considered collisions of solid bodies (e.g. Giuli 1968).

Below we assume that a new spherical object of radius \( r_f \) and mass \( m_1 = m_1 + m_2 \) forms as the result of a collision, and then contracts to radius \( r_f = r_f/k_f \). If the angular momentum \( K_{\text{col}} \) relative to the centre of mass is the same as that of the system at the time of collision then equation (2) yields the period of axial rotation:

\[
T_s = 2\pi I_s/K_s = k_f (m_1 + m_2)^2 r_f^2 a^{3/2},
\]

where \( k_f = 0.8 \pi \chi k_{\text{v}} (GM_S)^{1/2} \). The moment of inertia is \( I_s = 0.4\pi m_f r_f^2 \), where \( \chi = 1 \) for a homogeneous sphere. At \( r_f \approx r_f^1 + r_f^2, m_1 = m_2, \) and \( r_1 = r_2 \), using equations (1) and (5), we then obtain

\[
T_s = 2^{5/3} k_f a^{3/2} k_f^{-2} \approx 1.67 \pi r_f^3/(v_1 r_1) \approx 6.33 x_{rf}/(v_1 k_f).
\]

For \( m_1 \gg m_2 \) and \( r_f \approx r_f/k_f \), we have

\[
T_s \approx k_f (m_1 + m_2) a^{3/2} k_f^{-2} \approx 0.8 \pi \chi r_f m_1/ (m_2 v_1 k_f^2).
\]

In the case of heliocentric orbits with eccentricity \( e \), we can use \( v_r \approx 2^{-1/2}v_1 e \).

The angular momentum of binary components (with masses \( m_p \) and \( m_r \) and radii \( r_p \) and \( r_r \)) relative to their centre of mass equals

\[
K_{\text{cm}} = k_L m_p m_r/(m_p + m_r),
\]

where \( L \) is the mean distance between the components (i.e. the semimajor axis of their mutual orbit) and \( v_f = (GM_S + m_f)/L \) \( a^{5/2} \) is the mean velocity of their relative motion. The ratio of the momentum \( K_{\text{col}} \) obtained at the collision of two spherical preplanetesimals to
3.3 The spins of colliding preplanetesimals that have radii proportional to their Hill radii

This subsection calculates $K_{ob}$ under the assumption that the radius of each preplanetesimal is proportional to the radius of its Hill sphere, i.e., $r_i = k_{ob}(m_i/3M_s)^{1/3}$ ($i = 1, 2$). The coefficient $k_{ob}$ is arbitrary; as discussed in Section 3.1, we often consider $k_{ob} = 1$.

The radii used to calculate $K_{ob}$ can be much larger than the physical radii of the preplanetesimals.

Let us consider $m_1 = m_2 = m_0 = s = m_1/2 = \frac{2}{3}\pi r^3 \rho$ and $r_0 = r_0 = r$, where $\rho$ is the density of solid binary components of radius $r$. In this case, formulae (2), (8), and (9) yield

$$K_{ob} = \frac{6\sqrt{3}}{\pi} \rho G^{1/2} m_0^{3/2} L_{s/2}^{1/2}$$

and

$$k^2 = \frac{6M_s\rho G^{1/2}}{\pi} \rho L_{s/2}^{1/2}$$

From equation (11) we obtain

$$L \approx 5.5k^2 \rho G^{1/2} r_{HL}^{-2}$$

where $r_{HL} \approx a(m_1/3M_s)^{1/3}$. For $k = k_{ob} = 1$, we have $L \approx 0.5\pi r_H$ at $k_{ob} = 0.6$ and $L \approx 0.8$ at $k_{ob} = 0.816$. Therefore, for an encounter between RPPs moving in almost circular heliocentric orbits, a binary with components of equal mass could form with any separation up to the Hill radius.

In Sections 4.2 and 4.3, I will compare the values of $K_{ob}$ and $K_{scm}$ for several different binaries. Using the dependence (11) of $k_{ob}$ on $\rho$ and $L$, one can estimate the uncertainty on $k_{ob}$ for the binaries where these values are not well known.

At $r_0 = k_{ob} r_H$, the ratio $k = r_0/r_H$ is proportional to $a\rho^{1/3}$ and does not depend on mass. It equals 133 at $a = 1$ au, $k_{ob} = 1$, and $\rho = 1$ g cm$^{-3}$. For a solid planetesimal formed by contraction of a preplanetesimal of radius $r_H$, one obtains

$$T_s \approx a^{2/3} r_H^{-2} \approx a^{-1/2} \rho^{-2/3}$$

At $\chi = 1$, $k_{ob} = 0.6$, $\rho = 1$ au, $k_{ob} = k_{ob} = 1$ and $m_1 = m_2$, the rotation period of a RPP formed from the collision of two Hill sphere-sized planetesimal is $T_s \approx 9 \times 10^3$ h. At $k_{ob} = 133$, on the other hand, $T_s \approx 0.5$ h. For greater values of $a$, $T_s$ is even smaller. Such small periods of axial rotation cannot exist, especially as bodies obtained by the contraction of rotating RPPs are loosely bound and can lose material easier than solid bodies.

The value of $v_{ls} = 2\pi r_0/T_s$ (the velocity of a particle on the surface of a rotating spherical object of radius $r_0$ at the equator) is equal to $v_{ls} = (\sqrt{2}GM_H/r)^{1/2}$ (the minimum velocity of a particle that can leave the surface) at $T_s = T = (3\pi/\sqrt{2}G)^{1/2}$. Thus, $v_{ls}$ is equal to the equatorial escape velocity $v_{ls} = (3\pi/\sqrt{2}G)^{1/2}$. If $k_{ob}$ is the density in g cm$^{-3}$ then $T_s \approx 3.3/k_{ob}^2$ and $v_{ls} \approx 2.3/k_{ob}^2$.

Considering that $T_s = 2\pi J_s/K_{ob}$, the critical radius at which material begins to leave a contracting planetesimal is equal to

$$r_{es} = \frac{K_{es}^2}{0.16\sqrt{2} \rho G^{1/2}} = \frac{k_{es}^2 k_{ob}^2 r_H}{\sqrt{2}}$$

(12)

The coefficient $k_{es}$ in equation (12) is approximately 0.82 at $m_1 = m_2 = m_i/2$, and $k_{es} \approx 2.08(m_2/m_1)^2$ at $m_2 \ll m_1 \approx m_2$. Therefore, at $k_{ob} = k_{hl} = \chi = 1$ and $m_1 = m_2$, the radius of the disc formed around the primary could be as much as $0.82r_H$. In most cases, however, the disc would be relatively small since the masses of colliding preplanetesimals are usually different and the planetesimals are smaller than their Hill spheres.

If preplanetesimals are smaller than their Hill spheres, it may be better to consider the encounter of the preplanetesimals up to the radius of the Hill sphere corresponding to the mass $m_i = m_1 + m_2$, but not up to the sum of the radii of their Hill spheres, as it was in formulae (2), (5) and (10). For two identical planetesimals, this consideration decreases $K_{ob}$ and increases $T_s$ by a factor of $4^{1/3} \approx 2.5$.

3.4 The spin imparted to a planetesimal by the accretion of much smaller objects

A planetesimal can change its spin and mass due to the capture of much smaller objects (e.g. dust and boulders) initially moved outside of its Hill sphere. In this section, we consider only accreted objects initially moving in circular heliocentric orbits. At $\Delta a = \theta_{r12}$, the difference in the velocities of two objects is about $0.5\pi r_{r12}/a$, where $r_{r12} = r_1 + r_2$. The relative distance covered with this velocity during one revolution around the Sun is $(0.5\pi r_{r12}/a)(2\pi/\theta_{r12}) = \pi\Delta a_{r12}$. Thus, about $(2\pi/a)/(\pi\theta_{r12})$ revolutions are needed for an object with radius $r_1$ to sweep up all smaller objects with radii $r_2 < r_1$. For example, taking Pluto’s Hill sphere and semimajor axis for $r_1$, we obtain $2a/r_1 \approx 1.5 \times 10^4$. This value may be similar to the contraction times of trans-Neptunian planetesimals, or smaller by one or two orders of magnitude (see the discussion in Section 1.1).

A cross-section of a sphere is a circle, and the length of a chord at distance $\theta_{r12}$ from its centre is $2r_1(1 - \theta_{r12}^2)/3$. The relative velocity at this distance is proportional to $\theta_{r12}/a$. Therefore, if small objects are uniformly distributed around the orbit of the first object, the number of the small objects captured by a planetesimal per unit of time at $\Delta a = \theta_{r12}$ is proportional to $(1 - \theta_{r12}^2)/2\theta_{r12}$. Multiplying this value by $k_0$ (as $k_0 \propto k_{ob}$), we obtain $K \propto (1 - \theta_{r12}^2)/2\theta_{r12}^2$. By integrating over $\theta_{r12}$ from 0 to 0.8165 and from 0.8165 to 1 for a circular cross-section, I find that the ratio of positive to negative angular momenta is 9.4.

Let us consider a spherical planetesimal of mass $m_{pp}$ and radius $r = r_{pp}$ that grows due to collisions with smaller objects as described above. Typical tangential velocity of collisions is supposed to be $v_r = 0.6v_{ls}/r$, and $\Delta K$ is the difference between fractions of positive and negative angular momenta acquired by the planetesimal. The ratio between the radii of a planetesimal of density $\rho_0$ and the planetesimal formed by its contraction is denoted by $k_0$. Considering an integral over a radius $r$ of a growing planetesimal and supposing $dK_s = d(r_v dm)$ and $dm = 4\pi r^2 dr$, we can calculate $K_s$. Finally, the rotation period of the planetesimal formed by contraction of the planetesimal is

$$T_s = 2\pi J_s/K_s \approx 7\chi \rho G^{1/2} \Delta K^{-1/2} k_0^{-2}$$

If $r_{pp}$ is equal to the radius of the Hill sphere, we have

$$T_s = 5.7\chi M_s^{3/4}(Ga)^{-1/2} \Delta K^{-1/2} k_0^{-2/3}$$

If the planetesimal is much smaller than its Hill sphere and $v_r \approx v_{par}$ (where $v_{par}$ is the parabolic velocity at the surface of the planetesimal) then

$$K_s \approx 0.85\alpha(r_{pp} G^{1/2})^{3/2} \Delta K \approx 0.67\alpha G^{1/2} \rho_0^{-1/6} m_{pp}^{5/3} \Delta K$$

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and
\[ T_\varepsilon \approx 1.45\chi(\alpha \Delta K)^{-1}(\rho, G)^{-1/2} \kappa^{-2}. \]

Based on simulations of a large number of encounters between two objects of density \( \rho \) moving around the Sun, we calculated values of \( \Delta K \) for different elements of heliocentric orbits. The calculations showed that the fraction \( K_+ \) of positive angular momenta (note that \( \Delta K = 2K_+ - 1 \)) tends to decrease with increases in \( a, \rho \) or eccentricity. For RPPs and small eccentricities \( e \), positive angular momentum increments are more frequent than negative increments \((K_+ > 0.9 \text{ at } e = 0)\). For solid objects in orbits with \( a = 1 \) au and \( \varepsilon \cdot (m_1/M_\oplus)^{-1/3} \gtrsim 2-7 \), \( \Delta K \) is positive and about a few hundredths. For circular orbits, \( a \) was typically calculated to be 0.6.

4 COMPARISON OF ANGULAR MOMENTA OF DISCOVERED BINARIES WITH MODEL ANGULAR MOMENTA

4.1 Description of the data presented in Table 1

For six discovered binaries, the angular momenta \( K_{\text{scm}} \) of the systems were estimated and compared to the present model. The primary and secondary components of the binary systems have diameters \( d_p \) and \( d_s \) and masses \( m_p \) and \( m_s \), reported in the first rows of Table 1. Using the formulae presented in Sections 3.2 and 3.3, I also calculate the theoretical angular momentum \( K_{0\text{ps}0}\), of two preplanetesimals with masses \( m_p \) and \( m_s \) encountered from spherical heliocentric orbits at the mean value of \( k_4 \) (equal to 0.6) and the angular momentum \( K_{0\text{eq}0} \) of two preplanetesimals with masses equal to 0.5\((m_p + m_s)\). Thus, each system is compared to a theoretical scenario where two RPPs on different orbits merged then contracted into a pair of solid bodies. All three momenta are relative to the centre of mass of the system.

While \( K_{0\text{ps}0} \) and \( K_{0\text{eq}0} \) are calculated for RPP diameters equal to the Hill radius, binary systems can also be formed by the collision of much smaller preplanetesimals, as discussed in Section 3.3. Naturally, the heliocentric orbits of two preplanetesimals can have other separations, so \( k_4 \) can be smaller or larger than 0.6.

The spin momentum of the primary is \( K_{\text{spin}} = 0.2\pi G m_p d_p^2 / T_{sp} \), where \( T_{sp} \) is the period of axial rotation of the primary. The separation distance between the primary and the secondary is denoted by \( L \). The data used in these calculations were taken from the site http://www.johnstonsarchive.net/astro, which also provides many references for each system. Similar (but older) data can be found in Richardson & Walsh (2006). For calculation of \( r_{kps} = (K_{\text{scm}} + K_{\text{spin}}) / K_{0\text{ps}0} \) and \( r_{kO} = (K_{\text{scm}} + K_{\text{spin}}) / K_{0\text{eq}0} \), only the spin of the primary was considered. However, these ratios would be about the same if they included the spins of all components, because \( K_{\text{spin}} \) is several times smaller than \( K_{\text{scm}} \) even for equal masses. For unequal masses, most of the spin momentum is due to the primary component.

Table 1 also reports the values of \( 2L/d_p \) and \( L/r_{H} \), where \( r_{H} \) is the Hill radius for the total mass \( m_{\text{ps}} \) of the binary. Three velocities are presented in the last lines of Table 1: \( v_{\text{ps}} \) is the tangential velocity \( v_\varepsilon \), of the encounter at \( k_4 = 0.6 \) between Hill spheres with masses equivalent to the present primary and secondary components; \( v_{\text{scm}} \) is the same velocity for Hill spheres of equal masses (0.5\( m_{\text{ps}} \)) and \( v_{\text{esc}} \) is the escape velocity at the edge of the Hill sphere of the primary.

A relative velocity \( v_{\varepsilon} \), equal to 1 m s\(^{-1} \) is obtained for two colliding objects orbiting at \( e \approx 0.0002 \) and \( a = 40 \text{ au} \). The model described above uses perfectly circular heliocentric orbits, and may not be correct at \( e > 0.0002 \) for preplanetesimals corresponding to solid objects of diameter \( d \approx 200 \text{ km} \) because \( v_{\varepsilon} \approx v_{\text{esc}} \) for \( e \approx 0.0002 \) and \( d \approx 200 \text{ km} \). This conclusion is valid for other values of \( a \), since both \( v_{\varepsilon} \) and \( v_{\text{esc}} \) are proportional to \( a^{-1/2} \). It is reasonable to assume that the eccentricities of RPPs are very small because of their interaction with gas and dust.

4.2 Discussion of the data presented in Table 1

For the trans-Neptunian binaries presented in Table 1, neither \( r_{kps} = (K_{\text{scm}} + K_{\text{spin}}) / K_{0\text{ps}0} \) nor \( r_{kO} = (K_{\text{scm}} + K_{\text{spin}}) / K_{0\text{eq}0} \) exceed 0.63. This highest value was obtained for 2001 QW\(_{322} \), which consists of two approximately equal masses separated by a large distance. For the other trans-Neptunian binaries, \( L/r_{H} \) and, therefore, \( r_{kO} \) are smaller (often much smaller) than those for 2001 QW\(_{322} \). The small values of \( r_{kps} \) and \( r_{kO} \) can be explained if the discovered binaries arose from preplanetesimals that were already partly compressed (i.e. smaller than their Hill spheres) and/or denser towards the centre at the moment of collision. Petit et al. (2008) noted that most models of binary formation cannot easily explain the properties of 2001 QW\(_{322} \). For this binary, the equation \( r_{kO} = K_{\text{scm}} \) is fulfilled if \( k_4 \approx 0.4 \) and \( v_\varepsilon \approx 0.16 \text{ m s}^{-1} \). Therefore, formation of this binary system can be explained as the merger of two RPPs in circular heliocentric orbits.

Another property of 2001 QW\(_{322} \) is that its components have a retrograde mutual orbit. As shown in Section 3.2, the angular momentum is negative for encountering RPPs at \( 0.82 < \Theta < 1 \). This condition can also result in the formation of widely separated binaries.

Main-belt asteroid (87) Sylvia (384 × 262 × 232 km) has two satellites, Romulus (\( L_1 \approx 1356 \text{ km}, d_1 \approx 18 \text{ km} \)) and Remus (\( L_2 \approx 706 \text{ km}, d_2 \approx 7 \text{ km} \)). For this system, \( r_{kO} \approx 0.05 \ll r_{kps} \approx 130 \). If the Sylvia system originated in the collision of two RPPs then their masses did not differ by more than an order of magnitude (as \( K_{0\text{eq}0} / K_{\text{scm}} \approx 20 \)).

Main-belt asteroid (90) Antiope belongs to a synchronous double-asteroid system. Its rotational and orbital periods are 16.5 h, and the mutual orbit of the components is almost circular. Before tidal forces induced these orbits, the components may have been separated by a distance even smaller than the present 171 km. Since the present ratio \( 2L/d_s \) is small, this binary likely formed at the stage of solid (or almost solid) bodies. For this binary, \( r_{kps} \approx r_{kO} \approx 0.1 \). Since \( K_{\text{scm}} / K_{\text{scm}} \propto (L/a)^{1/2} \) (see Section 3.3), the model of rarefied Hill sphere preplanetesimals in almost circular heliocentric orbits is capable of producing a binary system with masses of components similar to those for Antiope. However, the separation \( L \) obtained is greater by two orders of magnitude than that of the Antiope binary and is comparable to the radius of the Hill sphere.

4.3 Formation of Pluto’s axial rotation and the inclined mutual orbits of binary systems

Pluto has three satellites, but the contribution of two satellites (other than Charon) to the total angular momentum of the system is small; for the whole system, \( K_{\text{scm}} / K_{\text{ps}} \approx 60 \). The axial tilt of Pluto with respect to its orbit is 119.6. Such a reverse rotation is possible in the collision of two (solid or rarefied) objects, but at the stage of RPPs it is better to use \( |k_4| \sim 0.2–0.3 \) (see Section 3.2). In the proposed model, it is not possible to obtain Pluto’s or any reverse rotation from a large number of collisions with smaller objects (as in Section 3.4).
For a collision of two identical Hill sphere preplanetesimals, the value of $K_{\text{col}}$ obtained for $|k_\text{el}| = 0.3$ exceeds $K_{\text{esc}}$ for the Pluto system by a factor of 23. If the masses of the parent Hill sphere RPPs are equal to the masses of Pluto and Charon then $K_{\text{col}}/K_{\text{esc}} \approx 7$ at $|k_\text{el}| = 0.3$. Furthermore, the angular momentum of colliding preplanetesimals in eccentric orbits can be greater than that obtained for circular orbits. The above estimates imply that the radii of the preplanetesimals that gave rise to the Pluto system were smaller by at least an order of magnitude than their Hill radii.

Most of the angular momentum of the Pluto system could have resulted from a single collision of preplanetesimals moving in different planes. To explain Pluto’s axial tilt (about 120°) and the inclined mutual orbit of 2001 QW$_{122}$ (124° from the ecliptic) in these terms, I first note that the thickness of the disc (i.e. the distances between the centres of mass of the colliding preplanetesimals and the equatorial plane of the disc) was comparable to or greater than the radii of the RPPs themselves. The inclined mutual orbits of many trans-Neptunian binaries support the idea that the momenta of such binaries mainly resulted from single collisions of preplanetesimals rather than the accretion of smaller objects. (In the latter case, the primordial inclinations of mutual orbits to the ecliptic would be relatively small.)

In the models of binary formation considered here, the spin vector of the primary preplanetesimal is almost perpendicular to the plane in which satellites of the primary move (as is the case with Pluto). However, the direction of this vector could vary over time due to collisions of the primary with solid bodies.

5 DISCUSSION

5.1 Radii and mergers of rarefied preplanetesimals

The radii of preplanetesimals need to be comparable to their Hill radii only if the resulting binary is to have a separation $L$ comparable to its own Hill radius $r_{\text{H}}$. For all binary systems, the binary separation $L$ is significantly below $0.04$. Among the binaries considered by Richardson & Walsh (2006) in their fig. 1, this ratio was less than 0.3. RPPs much smaller (by at least an order of magnitude) than $r_{\text{H}}$ suffice to explain the formation of binaries with $L/r_{\text{H}} < 0.04$. Generally speaking, the required radii of the RPPs can be of the same order as the present distance $L$ between the binary components. However, the secondary component could form much closer to the primary than the outer edge of the disc, and $L$ could increase due to tidal forces. This estimate of the preplanetesimal size is therefore not an accurate predictor.

The values of $2L/d_p$ presented in Table 1 vary from 17 to 2200 for trans-Neptunian binaries. The trans-Neptunian binaries considered by Richardson & Walsh (2006) fall inside the same range, but asteroid binaries may have smaller values.uzzi et al. (2008) considered spherical, rarefied clumps of diameter $l = (1 - 5) \times 10^3$ km and mass equivalent to a body of unit density and radius $10-100$ km, orbiting at $a = 2.5$ au. The diameters $l$ are greater than the separations of known asteroid binaries. Rarefied pre-asteroids may therefore have solidified before collision with other pre-asteroids.

The densities of RPPs can be very low, but their relative velocities $v_{\text{rel}}$ during an encounter are also very small. In particular, the relative speed of the collision is typically smaller than the escape velocity $v_{\text{esc}}$ at the Hill radius of the primary. In addition to $v_{\text{esc}}$, Table 1 presents the tangential component $v_{\text{rel}}$ of $v_{\text{rel}}$. In some models of the evolution of Saturn’s rings (Perrine, private communication), colliding objects form a new object if their impact speed is less than the mutual escape speed by a certain factor. If the greater encountered object is much smaller than its Hill sphere, and if both heliocentric orbits are almost circular before the collision, then the velocity of the collision $v_{\text{col}} \approx (v_{\text{esc}}^2 + v_{\text{par}}^2)^{1/2}$ does not differ much from the parabolic velocity $v_{\text{par}}$ at the surface of the primary RPP (radius $r_{\text{pc}}$). Indeed, $v_{\text{par}}$ is proportional to $r_{\text{pc}}^{-1/2}$. Therefore, encounters could result in a merger (followed by formation of a binary) at any $r_{\text{pc}} < r_{\text{H}}$.

For some pairs of objects, the mutual orbit can be more complicated than a parabola or ellipse. In a coplanar model, decreasing $r_{\text{pc}}$ by a factor of 10 can reduce the fraction of encounters resulting in a collision by a factor less than 10. Greenzweig & Lissauer (1990) published plots showing the dependence of the closest approach $r_{\text{min}}$ between objects in initially circular heliocentric orbits separated by an initial distance $\Delta a$. Based on these figures, the maximum value of $\Delta a$ is $0.47r_{\text{H}}$ for $r_{\text{min}} < 0.1r_{\text{H}}$ and $0.16r_{\text{H}}$ for $r_{\text{min}} < 0.01r_{\text{H}}$. In Greenzweig and Lissauer’s study, the integration began at a distance greater than $r_{\text{H}}$ and considered close encounters taking place at $\Delta a < 3.5r_{\text{H}}$. Note that in the above example, while the densities of uniform spherical preplanetesimals ranging from $r_{\text{H}}$ to $0.01r_{\text{H}}$ in size differ by six orders of magnitude, the probability of collision differs only by a factor of ~20 provided all motions occur in the same plane. The difference in the probabilities is much greater in a non-planar model.

Johansen et al. (2007) determined that the mean free path of a boulder inside a cluster-preplanetesimal is shorter than the size of the cluster. This result supports the picture of mergers between RPPs. Lyra et al. (2008) noted that the velocity dispersion of RPPs remains below 1 m s$^{-1}$ in most simulations, so destructive collisions between boulders are avoided.

To illustrate the probability of a merger between two RPPs, consider the following simple model. There is a spherical preplanetesimal of diameter $D$, and mass $M$, consisting of $N$ identical boulders of diameter $d$. After contraction, a solid, spherical preplanetesimal of diameter $D$ is formed. The densities $\rho$ of the boulders and the solid planetesimal are the same. A second preplanetesimal then passes through this cluster. The ratio of the length of its path inside the first preplanetesimal to $D$ is denoted as $k_x$. Because the relative motion of the centres of mass of the preplanetesimals can be complicated, $k_x$ can exceed 1.

The boulders belonging to both preplanetesimals are considered to have identical diameters $d$, and the volume swept by one boulder is $\pi d^2k_x D$. The ratio of this volume to the volume $\frac{2}{3} D^2$ of the Hill sphere, divided by $N = (D/d)^3$, gives the number of collisions $N_{\text{col}} = 6k_x D^3/(D^2d)$. Let us take $D = 2k_x a(M/3M_S)^{1/3}$, $\rho = k_x \rho_0$, $g$ cm$^{-3}$ and $k_x = k_{\text{H}}$. In this case, we obtain

$$D/d \approx N_{\text{col}} \times 3 \times 10^3 \times \frac{k_x^3}{k_{\text{H}}^2}.$$  

For $D = 1000$ km, $d = 0.3$ m and $k_x = k_{\text{H}} = N_{\text{col}} = 1$, equation (13) is fulfilled at $a = 33$ au. Using equation (13), we find that $N_{\text{col}} \approx Dk_{\text{H}}^2$. This verifies the intuition that for small values of the ratio $k_{\text{H}}$ between a preplanetesimal’s radius and its Hill radius $N_{\text{col}}$ can be relatively large. $N_{\text{col}}$ also decreases with $D$, but at $D = 50$ km and $k_{\text{H}} = 0.2$, $N_{\text{col}}$ is almost the same as for $D = 1000$ km and $k_{\text{H}} = 1$. This means that for most binaries in the Solar system with $D > 50$ km their parent preplanetesimals should have had $N_{\text{col}} \geq 1$ provided that $d \leq 1$ m. Boulders (or dust particles) are more likely to be captured if they have smaller diameters $d$. The probability of capture also increases for boulders (particles) closer to the centres of the preplanetesimals, if the density of the preplanetesimal is higher at a smaller distance from the centre. The relative velocities of the boulders in this model are usually smaller than the escape velocities.
of the preplanetesimals, so some collided boulders could remain inside the Hill sphere.

At greater eccentricities, the mean collision velocity between preplanetesimals and the minimum distance of closest approach between material points corresponding to encountering preplanetesimals is greater, and the particles remain inside the Hill sphere for less time. Therefore, the probability of a merger of preplanetesimals is smaller. However, the typical angular momentum of preplanetesimals encountered up to the Hill sphere is greater.

The fraction of trans-Neptunian binaries that formed during the preplanetesimal stage depends on the initial distribution of sizes, densities and contraction times among preplanetesimals. These issues must wait on further studies of the formation and evolution of preplanetesimals.

5.2 Binaries that originated at different distances from the Sun

Given a primary of mass \( m_p \) and a much smaller secondary, both in circular heliocentric orbits, one can obtain \( v_t / v_{esc-pre} \approx 0.666 \sqrt{a} \sqrt{m_p / m_s} \) (see designations and formulae in Sections 3.2 and 4.1). This ratio decreases with increasing \( a \) and \( m_p \). Therefore, preplanetesimals are more likely to merge when the primary is more massive and located farther from the Sun. The total mass of all preplanetesimals and the ratio of the time needed for contraction of preplanetesimals to the period of their rotation around the Sun could be greater for the trans-Neptunian region than for the initial asteroid belt. (Several authors have arrived at the similar conclusion on the ratio for dust condensations; e.g. Safronov 1969.)

The above factors could explain why a larger fraction of binaries are found at greater distances from the Sun, and why the typical mass ratio (secondary to primary) is greater for TNOs than for asteroids. The binary fractions in the minor planet population are about 2 per cent for main-belt asteroids, 22 per cent for cold classical TNOs and 5.5 per cent for all other TNOs (Noll 2006). Note that TNOs moving in eccentric orbits (the third category) are thought to have formed near the giant planets, closer to the Sun than classical TNOs (e.g. Ipatov 1987).

5.3 Asteroid binaries

In our opinion, some asteroid binaries with large primaries could have formed at the preplanetesimal stage. The eccentricities of satellite orbits around large asteroids (including Sylvia and Antiope) are usually (but not always) relatively small. Under the proposed model of RPP mergers, small eccentricities could indicate that the satellites formed from a disc around the primary. The plane of the disc will be close to that defined by the line connecting two preplanetesimals and their relative velocity vector. If the initial preplanetesimal orbits have large eccentricities \( e \) and/or inclinations \( i \), the relative velocity depends on the angle between the planes of heliocentric orbits. For small \( e \) and \( i \), the plane of relative motion inside the Hill sphere can differ significantly from the orbital planes.

If the thickness of the disc of preplanetesimals is of the same order as the diameters of the largest Hill spheres then the proposed model of binary formation permits any inclination \( i_{m} \) between the mutual orbit of a binary and the ecliptic. Some asteroid binaries have highly inclined mutual orbits (e.g. \( i_m = 64^\circ \) for Antiope and \( i_m = 93^\circ \) for Kalliope). For (87) Sylvia, (107) Camilla, and (283) Emma, on the other hand, \( i_m \) does not exceed 3°. High-velocity collisions between solid asteroids may more often yield \( i_m \) of the order of typical inclinations in the asteroid belt, as most of the debris will have neither large nor very small inclinations.

The total angular momentum of two identical Hill sphere preplanetesimals in circular heliocentric orbits often exceeds [in the case of (87) Sylvia, by an order of magnitude] the critical value at which particles leave the surface of the solid primary.

Solid bodies can also attain the critical angular momentum (corresponding to \( T_s \leq 3.3 \sqrt{k_1 / 2} \) h; see the end of Section 3.3) by collisions, but eccentric (and/or inclined) heliocentric orbits are required. Using equation (6) with \( m_1 = m_2, \), \( v_1 / \chi = 3.5 \text{ km s}^{-1} \) (e.g. at a typical velocity \( v_{col} \) of collisions in the asteroid belt equal to \( 5 \text{ km s}^{-1}, \) \( v_1 \approx 0.7 v_{col} \) and \( \chi = 1 \), and \( r_1 \approx 6600 \text{ km} \) I obtain \( T_s \approx 3.3 \text{ h} \). Therefore, the critical angular momentum can also result from the collision of two identical solid asteroids of any radius (<6600 km).

For a given ratio \( m_1 / m_2 \) and eccentricities of heliocentric orbits of solid bodies, the probability of attaining the critical momentum during a collision is greater for smaller values of \( m_1 \) and \( a \) (because \( T_s \propto r_1 / v_1 \), and at smaller \( a \) both orbital velocity and \( v_1 \) are larger). For \( v_1 / \chi = 3.5 \text{ km s}^{-1}, \) \( T_s = 3.3 \text{ h} \) and \( r_1 = 100 \text{ km} \); formula (7) then yields \( m_2 / m_1 \approx 0.006 \).

Solid bodies can be disrupted during a collision, whether or not they reach the critical angular momentum. Most of the material ejected in this fashion would leave the Hill sphere, especially when the primary has a relatively small mass. The spin periods of asteroid primaries with \( d > 50 \text{ km} \) are often several times greater than 3 h. It may be that their original rotation was faster.

Some present-day asteroids (especially those with \( d < 10 \text{ km} \)) may be debris from larger solid bodies. As discussed in Section 1.2, binary asteroids in the near-Earth object population formed at the stage of solid bodies. The spin and shape of these bodies could have changed during the evolution of the Solar system due to collisions with other small solid bodies.

5.4 The colours and total mass of TNOs

In the proposed model of binary formation, two preplanetesimals originate at almost the same distance from the Sun. This point agrees with the correlation between the colours of primaries and secondaries obtained by Benecchi et al. (2009) for trans-Neptunian binaries. In addition, the material within the preplanetesimals could have been mixed before the binary components formed.

The formation of classical TNOs from RPPs could have taken place for a small total mass of preplanetesimals in the trans-Neptunian region, even given the present total mass of TNOs. Models of TNO formation from solid planetesimals (e.g. Stern 1995; Davis & Farinella 1997; Kenyon & Luu 1998, 1999) require a more massive primordial belt and small (∼0.001) eccentricities during the process of accumulation. However, the gravitational interactions between planetesimals during this stage could have increased the eccentricities to values far greater than those mentioned above (e.g. Ipatov 1988, 2007). This increase testifies in favour of formation of TNOs from RPPs.

6 CONCLUSIONS

This analysis has shown that some TNOs could have acquired their axial momenta and/or satellites during a primordial stage as RPPs. In this scenario, most rarefied pre-asteroids could have solidified before colliding with other pre-asteroids.

Models of binary formation due to gravitational interactions or collisions between objects within the Hill radius, which have been studied by several authors for solid objects, could be more effective for RPPs. For example, due to their almost circular heliocentric
orbits, two nearby preplanetesimals remain inside the Hill radius longer. The centres of mass of two RPPs may also be able to approach each other more closely than solid planetesimals because the planetesimals could have greater eccentricities of heliocentric orbits.

During a collision, RPPs can have higher densities closer to their centres. In this case, there could be two centres of contraction inside the rotating preplanetesimal formed as a result of the collision of two RPPs. The result would be a binary with two roughly equal masses, which could be separated by any distance up to the Hill radius. The eccentricity and inclination of the secondary component’s orbit around the primary component can have any value. The observed separation distance can be of the order of the radius of a greater encountered preplanetesimal.

Some binaries could form because the angular momentum of a binary that was obtained at the stage of RPPs was greater than the angular momentum that can exist for solid bodies. Material that left a contracted preplanetesimal formed as a result of a collision of two preplanetesimals could form a disc around the primary. One or more satellites could grow in the disc within the Hill radius, typically much less. These satellites move mainly in low-eccentricity orbits.

The two scenarios described above can take place at the same time. It is thereby possible that, besides massive primary and secondary components, smaller satellites can form around the primary and/or the secondary.

Among discovered trans-Neptunian binaries, the angular momentum is usually considerably smaller than the model’s prediction for two identical RPPs having the same total mass and encountering up to the Hill radius from circular heliocentric orbits. The predictions for two preplanetesimals with unequal masses equivalent to those observed in the trans-Neptunian binaries are also too large. Furthermore, the observed separations between components are usually much smaller than the Hill radii. These facts support the hypothesis that most planetesimals were already partly compressed at the moment of collision, i.e. smaller than their Hill radii and/or centrally concentrated. The contraction of preplanetesimals could be slower farther from the Sun, which would explain the greater fraction of binaries formed at greater distances from the Sun.

The results of this research, which has focused on the angular momentum of colliding preplanetesimals, can also be used to analyse the formation of axial rotation of rarefied and solid bodies.

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