ASPECTS OF D-INSTANTONS

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Abstract. An overview over effects of D-instantons in ten dimensional IIB superstring theory is given, including the supergravity instanton solution, instanton induced effective interaction vertices, the conjectured \( SL(2, \mathbb{Z}) \) invariant completion of such terms and the connection of such terms to a one loop calculation in eleven dimensional supergravity.

1. Introduction

D-instantons are responsible for several important nonperturbative effects in superstring theory.

Firstly, they appear as euclidian Dirichlet \( p \)-branes whose \( p + 1 \) dimensional worldvolume wraps a supersymmetric \( p + 1 \) cycle in the compactification manifold. Such configurations were discussed as euclidian membranes and fivebranes in Calabi-Yau compactifications of M-theory [1], instanton corrections to the hypermultiplet geometry near the conifold in IIA on a Calabi-Yau manifold [2], D-string instanton corrections to \( R^4 \) and \( F^4 \) terms in type I string theory which are important for type I/heterotic duality [3, 4], threshold corrections to \( R^4 \) terms and U-duality in toroidally compactified type II theories [5].

Secondly, even in uncompactified ten dimensional IIB superstring theory they make an appearance as \( p = -1 \) branes [6], i.e D-branes with worldvolumes which are points in spacetime.

Indeed in the context of bosonic string theory such D-instantons were the first 'Dirichlet' objects considered in the past. The introduction of boundaries where the embedding coordinates satisfy Dirichlet boundary conditions in all directions was used to implement point-like partonic behavior in dual model amplitudes which is a desirable feature for a string theory
of strong interactions [7]. The fact that momentum is not conserved in the presence of Dirichlet boundaries was used to define off-shell continuations of string scattering amplitudes [8]. It was also observed that the characteristic softness [9] of string amplitudes in the high energy fixed angle scattering regime, does not hold in the presence of D-instantons [10]. Polchinski [11] showed in the context of bosonic strings that the new singularities which arise because of the pointlike nature of the open string boundary conditions cancel in a generalization of the Fischler-Susskind mechanism. In this scheme the combinatorics of string perturbation theory in the presence of D-instantons changes and one has to include disconnected worldsheets of different topology whose boundaries are mapped into the same spacetime point.

In the context of IIB superstrings D-instantons have several new properties. A D-instanton is a BPS solution which preserves half of the spacetime supersymmetries. In [12] the supergravity solution corresponding to a D-instanton was found. The broken supersymmetries induce fermionic zero modes which have to be integrated over in order to restore spacetime supersymmetry. This induces new effective (t'Hooft) vertices in the low energy effective action. In [13] the instanton induced interactions were derived and the $SL(2, Z)$ duality symmetry of IIB superstring theory was used to conjecture a modular invariant completion of the $R^4$ term which contains tree level, one loop and instanton corrections. In [14, 15, 16] these expressions were related to similar terms in M-theory and in particular it was argued that they arise from a one loop calculation in eleven dimensions. The material presented in this talk was obtained in collaboration with M.B. Green, H.H. Kwon and P. Vanhove and the interested reader is referred to the original papers for a more detailed discussion.

2. IIB supergravity

The massless fields of IIB superstring theory are given by the bosonic fields: a complex scalar $\rho$, two second rank AST potentials $B^{(2)}_a$, $a = 1, 2$, the zehnbein $e^a_\mu$ and a fourth rank AST potential $C^{(4)}$ with selfdual field strength and the fermionic fields: a complex spin 1/2 dilatino $\lambda$ and a complex spin 3/2 gravitino $\psi_\mu$ of opposite chirality. As usual in extended supergravity theories the scalar fields parametrize a coset, which for IIB supergravity is $SL(2, R)/U(1)$. Under $SL(2, R)$ transformations only the second rank AST transform as doublets. The fermions $\lambda$ and $\psi_\mu$ transform under the local $U(1)$ with weight 3/2 and 1/2 respectively. The metric and the fourth rank AST are inert under both transformations. The scalars can be parameter-
ized by a zweibein \[17\]
\[V = \left( \begin{array}{ccc} V^1_+ & V^1_- \\ V^2_+ & V^2_- \end{array} \right) = \frac{1}{\sqrt{2i\rho_2}} \left( \begin{array}{cc} \rho e^{i\phi} & \rho e^{-i\phi} \\ e^{i\phi} & e^{-i\phi} \end{array} \right), \tag{1} \]

The group \(SL(2, R)\) acts by matrix multiplication from the left and the local \(U(1)\) acts from the right and induces a shift \(\phi \to \phi + \alpha\). We will use the \(U(1)\) gauge invariance to set \(\phi = 0\). A compensating gauge transformation accompanies an \(SL(2, R)\) transformation in order to maintain the gauge condition. In this case a \(SL(2, R)\) transformation of the complex scalar and the compensating \(U(1)\) transformation are given by
\[\rho \to \frac{a\rho + b}{c\rho + d}, \quad e^{i\alpha} = \left( \frac{c\bar{\rho} + d}{c\rho + d} \right)^\frac{1}{2}. \tag{2} \]

It is useful to define a scalar field strength \(P_\mu\) and a complex combination of the AST \(H\) which have \(U(1)\) weight 2 and 1 respectively.
\[P_\mu = -\epsilon_{ab} V^a_+ \partial_\mu V^b_+ = \frac{i}{2\rho_2} \partial_\mu \rho, \quad H_{\mu\nu\rho} = V^a_+ \partial_{[\mu} B^a_{\nu\rho]} \tag{3} \]

3. D-instanton in IIB supergravity

The classical D-instanton is a solution of the euclidian IIB supergravity \[12\] which preserves half the euclidian supersymmetry. Only the metric and the scalar fields are nontrivial in this background so the nontrivial part of the action is given by
\[S = \int d^{10}x \sqrt{-g} \left\{ R - \frac{1}{2\rho^2_2} \partial_\mu \rho \partial^\mu \rho \right\}. \tag{4} \]

The BPS constraint on the solution requires that
\[\delta \lambda^* = i\gamma^\mu P_\mu \epsilon + .. = 0, \quad \delta \psi_\mu = D_\mu \epsilon + .. = 0. \tag{5} \]

The classical D-instanton solution is given, in the Einstein frame, by setting \(g^E_{\mu\nu} = \eta_{\mu\nu}\). If the \(R \otimes R\) scalar is expressed as the sum of a constant term and an euclidian fluctuation, \(\rho_1 = \chi + i f(x)\), the BPS condition (5) reduces to \(de^{-\phi} = -df\). The dilaton equation of motion is solved by
\[e^{\phi(x) - \phi_\infty} = 1 + \frac{Ce^{-\phi_\infty}}{|x - y|^8}, \tag{6} \]

where \(y\) is the position of the D-instanton in space time. The string coupling constant is identified with the expectation value of the dilaton at \(|x| \to \infty\),
$g = e^{\phi_{\infty}} = 1/\rho_2$. The constant $C$ is quantized and given by $C = 2N/\pi^{3/2}$ where $N \in \mathbb{Z}$. The action of a charge $N$ D-instanton is given by $S = 2\pi i N \rho$. The charge $N$ D-instanton configuration preserves sixteen of the thirty-two supersymmetries. The broken supersymmetries generate fermionic zero modes in the instanton background which have to be integrated over and they induce new vertices which soak up the zero modes

$$
\int d^{10} y d^{16} \epsilon_0 \langle \psi^1(x_1) \cdots \psi^n(x_n) \rangle_N = e^{2\pi i N \rho} \int d^{10} y d^{16} \epsilon \langle \psi \rangle_{s_1} \cdots \langle \psi \rangle_{s_n},
$$

where $\langle \psi \rangle_{s_i}$ defines a tadpole of a field of IIB supergravity which soaks up $s_i$ fermionic zero modes and $\sum_i s_i = 16$.

4. Instanton induced interactions

In closed string perturbation theory processes in the presence of D-instantons are formulated by including worldsheets with boundaries, where the boundary is mapped to a point in spacetime corresponding to the position of the instanton. Because the boundary conditions relate left-moving and right-moving worldsheet fields, a D-instanton preserves half of the closed string supersymmetry, i.e. is a BPS state. The broken spacetime supersymmetry generators have the form of zero momentum open string fermion vertex operators and have a natural interpretation as generating fermionic zero modes in the instanton background. The simplest open-string world-sheet that arises in a D-brane process is the disk diagram.

![Figure 1](image.png)

Figure 1. An on-shell closed-string state, $\Psi$, coupling to $s$ open-string fermions on a disk with Dirichlet boundary conditions.

A tadpole diagram with one fermionic zero mode attached couples to the dilatino and the calculation in A.1 gives

$$
\langle \lambda \rangle_1 = \langle cF_{\frac{1}{2}}(x) \varepsilon \bar{c} V_{(\lambda)}(z, \bar{z}) \rangle = \bar{\zeta}_\lambda \epsilon_0,
$$

where $\zeta^a_\lambda = \gamma^a_{\mu \nu} (\zeta^{b \mu N}_\nu + i \zeta^{b N}_\mu)$ is the holomorphic combination of the two dilatinos. Similarly one can evaluate the one point functions of the complex
combination of the NS ⊗ NS and R ⊗ R two-forms, \( B = B_{NS} + iC^{(2)} \) with two fermionic zero modes attached \( \langle B \rangle_2 \) and the tadpole \( \langle \psi \rangle_3 \) holomorphic gravitino with three fermionic zero modes attached. Of particular interest is the disk with four fermionic zero modes attached which couples to the graviton and the calculation in A.4 gives

\[
\langle h \rangle_4 = \langle cF_{1/2} (x_1) \int dx_2dx_3dx_4F(x_2) \frac{1}{2} F_{1/2} (x_3)F_{1/2} (x_4)c\bar{c}V^{NN}(z, \bar{z}) \rangle = \epsilon_0 \gamma^\rho \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda.
\]

The results of the calculation in the appendix of tadpoles for the massless fields which absorb up to four fermionic zero modes are summarized with their weight under \( U(1) \) and the coupling in the following table

| Field | \( U(1) \)-wt | no. z.m. | tadp |
|-------|-------------|---------|------|
| \langle \lambda \rangle | 3/2 | 1 | \( \lambda \) |
| \langle H \rangle | 1 | 2 | \( \bar{\epsilon}_0 \gamma^\mu \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda / \epsilon_0 \gamma^\rho \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda \) |
| \langle \psi \rangle | 1/2 | 3 | \( \epsilon_0 \gamma^\rho \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda / \epsilon_0 \gamma^\rho \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda \) |
| \langle h \rangle | 0 | 4 | \( \epsilon_0 \gamma^\rho \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda / \epsilon_0 \gamma^\rho \epsilon_0 \bar{\epsilon}_0 \gamma^\nu \epsilon_0 \zeta_{\mu\nu} k_\rho k_\lambda \) |

There are a whole variety of instanton induced vertices which are in fact related to each other by supersymmetry. The simplest case is given by a sixteen fermion term which is induced by sixteen disks with one holomorphic dilatino \( \lambda \) and one zero mode \( \epsilon \) attached to each disk.

\[
\int d^{10}y \int d^{16} \epsilon_0 \prod_1^{16} \langle \lambda \rangle_i = \epsilon^{a_1 \cdots a_{16}} \chi^{a_1} \cdots \chi^{a_{16}}.
\]

Among many other terms we focus on a \( R^4 \) term which arises from four disks with a graviton and four fermionic zero modes attached to each disk.

\[

A_4(\{ \zeta^{(r)}_h \}) = \int d^{10}y d^{16} \epsilon_0 \prod_1^{4} \langle h^{(1)} \rangle_4 \langle h^{(2)} \rangle_4 \langle h^{(3)} \rangle_4 \langle h^{(4)} \rangle_4 \\

= \int d^{10}y d^{16} \epsilon_0 \prod_1^{4} \left( \bar{\epsilon}_0 \gamma^{\mu \rho} \epsilon_0 \bar{\epsilon}_0 \gamma^{\nu \rho} \epsilon_0 \zeta_{(\mu \nu)} k^\rho k^\rho \right),
\]

\[

= \int d^{10}y \left( \bar{\tau}^{i_1j_1 \cdots i_4j_4} \epsilon_{m_{1} \cdots m_{4}n_{4}} - \frac{1}{4} \epsilon^{i_1j_1 \cdots i_4j_4} \epsilon_{m_{1} \cdots m_{4}n_{4}} \right) R^{m_{1}n_{1}}_{i_1j_1} R^{m_{2}n_{2}}_{i_2j_2} R^{m_{3}n_{3}}_{i_3j_3}
\]

The close connection between \( t^8 t^8 R^4 \) and \( \chi^{16} \) terms is obviously a consequence of their common superspace origin which can be seen already in
the rigid limit of the linearized theory. Consider a linear superfield \( \Phi(x, \theta) \) (where \( \theta \) is a complex chiral \( SO(9,1) \) Grassmann spinor) that satisfies the holomorphic constraint \( D^* \Phi = 0 \) and the on-shell condition \( D^4 \Phi = D^* \Phi^* \) [18] where

\[
D_A = \frac{\partial}{\partial \theta^A} + 2i(\Gamma^\mu \theta^*)_A \partial_\mu, \quad D_A^* = -\frac{\partial}{\partial \theta^{*A}}
\]  

(recall that \( \bar{\theta} = \theta \Gamma^0 \) does not involve complex conjugation) are the holomorphic and anti-holomorphic covariant derivatives that anticommute with the rigid supersymmetries

\[
Q_A = \frac{\partial}{\partial \theta^A}, \quad Q_A^* = -\frac{\partial}{\partial \theta^{*A}} + 2i(\bar{\theta} \Gamma^\mu)_A \partial_\mu.
\]

The field \( \Phi \) has an expansion in powers of \( \theta \) (but not \( \theta^* \)), describing the 256 fields in an on-shell supermultiplet.

\[
\Phi = \rho_0 + \Delta
\]

\[
= \rho_0 + a - \frac{2i}{g} \bar{\theta} \lambda - \frac{1}{24g} \bar{\theta} \Gamma^{\mu \nu \sigma} \theta G_{\mu \nu \sigma} + \frac{i}{6g} \bar{\theta} \Gamma^{\mu \nu \sigma} \theta \bar{\theta} \Gamma_{\nu} \partial_{\sigma} \psi_{\mu}
\]

\[
- \frac{i}{48g} \bar{\theta} \Gamma^{\mu \nu \eta} \theta \bar{\theta} \Gamma^\sigma \theta R_{\mu \nu \sigma \tau} + \cdots,
\]

where \( \Delta \) is the linearized fluctuation around a flat background with a constant scalar, \( \rho_0 = \rho - a = C_0^{(0)} + ig^{-1} \). Note that because of the constraint \( D^4 \Phi = D^* \Phi^* \) the expansion terminates at \( \theta^8 \). The coefficients of the component fields are consistent with the conventions used in [17]. The terms indicated by \( \cdots \) fill in the remaining members of the ten-dimensional \( N = 2 \) chiral supermultiplet, comprising (in symbolic notation) \( \partial \partial C^{(4)} \), \( \partial \psi^*, \partial^2 G_{\mu \nu \sigma}, \partial^3 \lambda^* \) and \( \partial^4 \rho^* \). The linearized supersymmetric on-shell action has the form [16]

\[
S' = \text{Re} \int d^{10} x d^{16} \theta g^4 F[\Phi],
\]

which is manifestly invariant under the rigid supersymmetry transformations, (11). The various component interactions contained in (13) are obtained from the \( \theta^{16} \) term in the expansion,

\[
F[\Phi] = F(\rho_0) + \Delta \frac{\partial}{\partial \rho_0} F(\rho_0) + \frac{1}{2} \Delta^2 \left( \frac{\partial}{\partial \rho_0} \right)^2 F(\rho_0) + \cdots
\]

All terms are F-terms, i.e they are realized as integrals over half the superspace.
5. Modular Invariance

The fact that the Lorentz index structure of the $t_8t_8R^4$ term in the instanton induced effective action is of the same form as the tree level [19] and one loop [20] perturbative terms together with invariance of the graviton under the $SL(2, Z)$ duality of IIB string theory make it very plausible to conjecture for the exact nonperturbative $R^4$ term [13].

$$S_{R^4} = \int d^{10}x \ t_8t_8 \ R^4 \ f(\rho, \bar{\rho}),$$

where the nonholomorphic modular function $f$ is given by

$$f(\rho, \bar{\rho}) = \sum_{(p,n) \neq (0,0)} \frac{\rho_2^{3/2}}{|p + n\rho|^3},$$

In the string frame the large $\rho_2$ expansion gives

$$\rho_2^{1/2} f(\rho, \bar{\rho}) = 2\zeta(3)(\rho_2)^2 + \frac{2\pi^2}{3} + 4\pi^{3/2} \sum_N (N\rho_2)^{1/2} \sum_{N|m} \frac{1}{m^2} \times \left(e^{2\pi iN\rho} + e^{-2\pi iN\bar{\rho}}\right) \left(1 + \sum_{k=1}^{\infty} (4\pi N\rho_2)^{-k} \frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)k!}\right)$$

There are several interesting properties of this expansion. There are terms corresponding to tree level, one loop and instanton contributions. The tree and one loop terms exactly agree with results in string perturbation theory. The fact that no higher perturbative corrections appear in (17) together with the fact that all such terms have the form of integral over half the superspace, lead to the conjecture that the $R^4$ and related terms are protected from higher loop perturbative corrections by a nonrenormalization theorem. Recently substantial evidence for the validity of such a theorem has been obtained [21, 22]. It is also interesting to note that it is possible to infer the value of certain integrals which are important for the calculation of the Witten index for bound states of D0-branes [23] from the modular function $f$ [24]. The nonholomorphic function $f$ is invariant under $SL(2, Z)$ modular transformations; this is because the graviton and hence $t_8t_8R^4$ is invariant under $SL(2, Z)$. For other terms which are related to $R^4$ like the $\lambda^{16}$ the modular function has to be generalized, since such terms transform with a nontrivial weight under the induced $U(1)$ R-symmetry. For a term in the effective action which transforms with weight $2k$ a natural
generalization of (16) is
\[ f^{(k)} = \sum_{(m,n)\neq(0,0)} \frac{\rho_2^{3/2}(m+n\rho)^{2k}}{|m+n\rho|^{2k+3}}. \] (18)

This function transforms under \( SL(2,\mathbb{Z}) \) with a \( U(1) \) weight \( k \)
\[ f^{(k)}(ap+b, cp+d) = \left( \frac{c\rho + d}{cp+d} \right)^k f^{(k)}(\rho). \] (19)

The functions \( f^{(k)} \) with different \( k \) are related to \( f^{(0)} \) by
\[ f^{(k)} = \rho_2^k D_{2(k-1)} \cdots D_2 D_0 f^{(0)}, \] (20)
where
\[ D_l = i(\frac{\partial}{\partial \rho} - \frac{l}{\rho - \rho}). \] (21)

is the nonholomorphic covariant derivatives which maps modular forms of weight \( l \) into modular forms of weight \( l + 2 \).

Comparing the relations in (13) with (20) it becomes clear that the linearized superspace incorporated the action of the ordinary derivatives \( \partial/\partial \rho \) by expanding the function \( F[\Phi] \), but does not provide a covariantization. One indication that the full nonlinear superspace should produce a structure like (20) can be obtained by considering the perturbative fluctuations around the instanton configurations which are suppressed by powers of \( \rho_2 \). For the \( \lambda^{16} \) term the lowest order contribution in the charge one instanton sector comes from sixteen disks with one dilatino and one fermionic zero mode attached. Because the genus of the disk is \(-1\) and the insertion of a closed string vertex operator contributes one power of \( g \) such a term contributes to order \( g^0 \). A term where two dilatinis are on the same disk with two fermionic zero modes attached is of order \( g \) and corresponds to a perturbative fluctuation. There should be a contact term which contributes when the vertex operator of the two dilatinis come close to each other. Such contact terms are suppressed by \( 1/\rho_2 = g \) and should be responsible for the covariantization of the leading D-instanton terms. Such terms with more than one closed string vertex operator on a disk together with disks with handles attached should produce the perturbative fluctuations around the instanton configuration 17.

6. One loop in eleven dimensions

In this section we briefly review the relation of \( t^8 t^8 R^4 \) term (15) by evaluating the four-graviton one loop amplitude of eleven-dimensional supergravity compactified on a torus, \( T^2 \), in the directions 9, 11. The volume of the torus is \( V = R_9 R_{11} \) and its complex structure, \( \Omega = \Omega_1 + i\Omega_2 \), may be expressed in

\(^1\text{Similar considerations can be found in [25]}\)
terms of the components eleven-dimensional metric $G_{\hat{\mu}\hat{\nu}}$ ($\hat{\mu} = 0, \cdots, 9, 11$) as $\Omega_2 = G_{9\,9}/G_{11\,11} = R_9/R_{11}$ and $\Omega_1 = G_{9\,11}/G_{11\,11} = C^{(1)}$, where $C^{(1)}$ is the component of the IIA one-form along the direction $x^9$. M-theory on $T^2$ is identified with IIB on $S^1$ via setting $\Omega = \rho$. The expression for the amplitude, obtained rather efficiently by considering a first-quantized super-particle in the light-cone gauge [15], has the structure,

$$A_{R^4} = \int d^9 p \frac{1}{V} \sum_{m,n} \prod_{r=1}^4 d\tau_r \text{tr}(V^{(1)}_h(k^1)V^{(2)}_h(k^2)V^{(3)}_h(k^3)V^{(4)}_h(k^4)), \quad (22)$$

where $V^{(r)}_h(k^r)$ is the graviton vertex operator for the $r$th graviton with momentum $k^r$ and is evaluated at a proper time $\tau_r$ around the loop (and dimensional quantities are measured in units of the eleven-dimensional Planck scale). The trace is over the fermionic zero modes and the loop momentum in the directions $0, \cdots, 8$ and the integers $m$ and $n$ label the Kaluza-Klein momenta in the directions 9, 11 of $T^2$. The trace over the fermionic modes gives an overall factor, $K$, which contains eight powers of the external momenta and is the linearized form of $t^8t^8R^4$. The $t^8t^8R^4$ term in (22) is obtained by setting the momenta equal to zero in the remainder of the loop integral. It is convenient to perform a double Poisson resummation in order to rewrite the sum over the Kaluza-Klein momenta as a sum over windings, $\hat{m}$ and $\hat{n}$, of the loop around the cycles of $T^2$, giving,

$$\mathcal{V}A_{R^4} = K\mathcal{V} \sum_{\hat{m},\hat{n}} \int d\tau \frac{1}{2} e^{-\tau V^{[\hat{m}+\hat{n}\Omega]^2/12}}$$

$$= K\mathcal{V} C + \pi^2 K\mathcal{V} \frac{1}{2} f(\Omega, \bar{\Omega}), \quad (23)$$

where

$$f(\Omega, \bar{\Omega}) = \pi^{-2} \Gamma(3/2) \sum_{(m,n)\neq(0,0)} \frac{\Omega_2^{3/2}}{|m + n\Omega|^3}$$

is a modular function. The cubic ultraviolet divergence of $A_{R^4}$ is contained in the zero winding term, $\hat{m} = 0 = \hat{n}$, which has the divergent coefficient, $C$. It was argued in [14] that in any regularization that is consistent with T-duality between the IIA and IIB string theories in nine dimensions this has to be replaced by a regularized finite value, $C = \pi/3$ [15]. Presumably a microscopic description of M-theory (such as matrix theory) would reproduce this value. In the limit of zero volume, $\mathcal{V} \to 0$, this term disappears and only the term with coefficient $\mathcal{V}^{-1/2}$ survives. Similar results for the $\lambda^{16}$ and related terms and the corresponding modular functions $f_k$ have been obtained in [16] by evaluating different one loop amplitudes of $M$ compactified on $T^2$. 

D-INSTANTONS
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A. One point functions in the upper half plane

We give two examples of the calculation of one point functions on the half plane which give the tadpoles $\langle \Psi \rangle$ of closed string fields used in section 4.

A.1. DILATINO

Attaching one fermionic zero mode to the boundary leads to a non-vanishing one-point function for a combination of the two dilatinos from the NS $\otimes$ R

$$\langle \lambda^{NR} \rangle_1 = \langle c\overline{eV}(-1,-\frac{1}{2})(v,z)cV_{-\frac{1}{2}}(\epsilon,x_1) \rangle$$

$$= v_{\mu a} \epsilon_b \langle c\overline{e} e^{-\phi^b_{i\mu}} e^{-\frac{1}{2}\phi^b_2 S_a^i} e^{ikX}(z,\bar{z}) c e^{-\frac{1}{2}\phi^b_2 S^b}(x_1) \rangle$$

$$= i \overline{\epsilon} \gamma_{ab} \epsilon \mu a_{\mu} \epsilon_b \frac{(z - x_1)(\bar{z} - x_1)(z - \bar{z})}{(z - x_1)(\bar{z} - x_1)(z - \bar{z})}$$

$$= i \overline{\epsilon} \lambda$$ (25)

and similarly for the R $\otimes$ NS dilatino

$$\langle \lambda^{RN} \rangle_1 = \langle c\overline{eV}(-\frac{1}{2},1)(v,z)cV_{1}(\epsilon,x_1) \rangle = \overline{\epsilon} \lambda.$$ (26)

The calculation of such correlation functions on the upper half plane is most easily done by doubling. One uses the boundary conditions for the D-instanton to map the antiholomorphic fields of the vertex operators living in the upper half plane to holomorphic fields in the lower half plane and evaluates the correlation function using holomorphic correlators in the complex plane. Putting (25) and (26) together the coupling of the dilatinos is given by

$$\langle \lambda \rangle_1 = (\zeta_{a\mu}^{RN} + i\zeta_{a\mu}^{NR})(\gamma^b_{\mu} \epsilon_b) \epsilon_0.$$ (27)

A.2. NS $\otimes$ NS ANTISYMMETRIC TENSOR

Attaching two fermionic zero modes produces a coupling of the D-instanton to the antisymmetric rank 2 tensor fields from the NS $\otimes$ NS and R $\otimes$ R
sector. For the NS $\otimes$ NS antisymmetric tensor the relevant amplitude is

\[
\langle B_{\mu\nu} \rangle_2 = \langle c\overline{c}V_{(0,-1)}(v,z) cV_{-\frac{1}{2}}(\epsilon, x_1) \int dx_2 V_{-\frac{1}{2}}(\epsilon, x_2) \rangle \\
= \zeta_{\mu\nu}^{NN} e^{\frac{1}{2}}_a e^{\frac{1}{2}}_b \left\langle c\overline{c} e^{-\phi} \psi^\mu (\overline{\partial} X^\nu + ik\lambda \overline{\psi}^\nu) e^{ik} (z, \overline{z}) c e^{-\frac{1}{2} \phi} S^a (x_1) \right. \\
\times \left. \int dx_2 e^{-\frac{1}{2} \phi} S^b (x_2) \right\rangle \tag{28}
\]

Note that the $\overline{\partial} X^\mu$ cannot be contracted into anything, the $\overline{\psi}^\nu (\overline{z})$ in $V_{(0,0)}$ acts as a current $\overline{j}^\nu$ insertion which can be evaluated,

\[
\langle B_{\mu\nu} \rangle_2 = i k\lambda \zeta_{\mu\nu}^{NN} e^{\frac{1}{2}}_1 e^{\frac{1}{2}}_2 \int dx_2 \frac{(z-\overline{z})(\overline{z} - x_1)}{(z-x_2)(x_1-x_2)} \\
\times \left( \gamma_{ab}^{\lambda\mu\nu} \frac{\gamma_{ab}^{\lambda\mu\nu}}{\overline{z} - x_1} + \eta^{\lambda\mu\nu}_{ab} - \eta^{\mu\lambda\nu}_{ab} \right) \\
= i k\phi \zeta_{\mu\nu}^{NN} e_{\gamma^{\mu\nu}} \epsilon \int dx_2 \frac{(z-\overline{z})}{(z-x_2)(x_1-x_2)} \\
= \pi H_{\mu\nu}^{NN} \epsilon_{\gamma^{\mu\nu}} \epsilon. \tag{29}
\]

Where we defined $H_{\mu\nu}^{NN} = ik\phi \zeta_{\mu\nu}^{NN}$. Only the first two terms proportional to $\gamma^{\mu\nu}$ contribute because the term proportional to just one $\gamma$ matrix in the first line of (29) $e_1 \gamma^\mu e_2$ vanishes after the spinor wavefunctions $e^1$ and $e^2$ are antisymmetrized. In the last line we used the fact that

\[
\int dx \frac{(z-\overline{z})}{(z-x)(\overline{z}-x)} = \pi \tag{30}
\]

which is true for any $z$ with nonvanishing imaginary part by the residue-theorem.

A.3. $R \otimes R$ ANTISYMMETRIC TENSOR

For the $R \otimes R$ antisymmetric tensor the relevant amplitude is given by

\[
\langle B^{RR} \rangle_2 = \langle c\overline{c} V^{(-\frac{1}{2},-\frac{1}{2})} (F^{(1)}, z) c V_{-\frac{1}{2}} (\epsilon, x_1) \int dx_2 V_{-\frac{1}{2}} (\epsilon, x_2) \rangle \\
= \frac{i}{4} k_{\mu_1} \zeta_{\mu_2\mu_3}^{RR} e^{\frac{1}{2}}_a e^{\frac{1}{2}}_d \left\langle c\overline{c} e^{-\frac{1}{2} \phi} e^{-\frac{1}{2} \phi} S^{\alpha\mu_1\mu_2\mu_3} \overline{S}^{\beta} e^{ik} \right. \\
\times e^{\frac{1}{2} \phi} S^\beta (x_1) \left. \int dx_2 e^{-\frac{1}{2} \phi} S^d (x_2) \right\rangle \\
= -\frac{1}{4} k_{\mu_1} \zeta_{\mu_2\mu_3}^{RR} e^{\frac{1}{2}}_a e^{\frac{1}{2}}_d \int dx_3
\]
\[ \times \left( \frac{\bar{\gamma}_{12}}{z - x_2} \right) \frac{\text{tr}(\gamma^\rho \gamma^{\mu_1 \mu_2 \mu_3}) \gamma^c d}{(z - x_2)(x_1 - x_2)} = i\pi H_{\mu \nu \rho} \bar{\gamma}_{12} \gamma^c d \]  

(31)

Again \[ H_{\mu \nu \rho} = i k_{\rho} \bar{\gamma}_{12} \gamma^c d \]. Note that the first term vanishes because of the trace of \( \gamma \)-matrices and Bose-Fermi symmetry of \( \epsilon_1, \epsilon_2 \). To evaluate the second term we used the identity \( \gamma^\rho \gamma^{\mu_1 \mu_2 \mu_3} \gamma_\rho = -4 \gamma^{\mu_1 \mu_2 \mu_3} \) for ten dimensional \( \gamma \)-matrices and the integral (30).

Putting together (29) and (31) we see that the following couples to the D-instanton with two fermionic zero modes attached.

\[ (B)_2 = \bar{\epsilon}_0 \gamma^{\mu \nu \rho} \epsilon_0 i k_{\mu} \bar{\gamma}_{12} \gamma^c d \]  

(32)

### A.4. GRAVITON

Attaching four fermionic zero modes gives a coupling of the self-dual \( R \otimes R \) four form and the graviton to the D-instanton. We focus here on the graviton where the relevant amplitude is given by

\[ \langle h \rangle_4 = \langle c e V_{(0,0)}(\sigma, \zeta) c V_{(0,0)}(x_1) \int dx_2 dx_3 dx_4 V_{(1,1)}(x_2) V_{(1,1)}(x_3) \int dx_2 dx_3 dx_4 e^{-\phi S}(x_2) e^{-\phi S}(x_3) \rangle. \]

Using doubling and the well known correlation function for the superghosts and spin fields, in particular

\[ \langle j^{\mu}(z) j^{\nu}(z) S^a(x_1) S^b(x_2) S^c(x_3) S^d(x_4) \rangle \]

\[ = \sum_{i=1}^{4} \frac{M^{\rho \mu}(i)}{z - x_i} \sum_{j=1}^{4} \frac{M^{\rho \nu}(j)}{z - x_i} \frac{\gamma^{\rho \nu}(i)}{x_{12} x_{13} x_{24} x_{34} / 4}. \]  

(33)

The operator \( M(i) \) acts on the spinor index associated with the \( i \)-th spin field as a Lorentz transformation. As above only terms like \( \epsilon_i \gamma^{\mu_1 \mu_2 \mu_3} \epsilon_j \) are nonzero after antisymmetrization over the spinor wavefunctions \( \epsilon_i \).

For example the contributions of the form \( \bar{\epsilon}_1 \gamma^{\mu \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\lambda \nu \tau} \epsilon_4 \) is given by \( M^{\rho \mu} \) acting on \( \gamma^\tau_{ab} \) and \( M^{\rho \lambda} \) acting on \( \gamma^\tau_{cd} \), which results in

\[ \bar{\epsilon}_1 \gamma^{\mu \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\lambda \nu \tau} \epsilon_4 \int dx_2 dx_3 dx_4 \frac{(z - \bar{z})(z - x_1)(\bar{z} - x_1)}{(x_{12} x_{13} x_{24} x_{34})}. \]
\[
\times \left( \frac{1}{z-x_1} - \frac{1}{z-x_2} \right) \left( \frac{1}{z-x_3} - \frac{1}{z-x_4} \right) \\
= \bar{\epsilon}_1 \gamma^{\mu \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\lambda \mu \nu} \epsilon_4 \int dx_2 dx_3 dx_4 \frac{(z-\bar{z})(x_1-\bar{x_1})}{x_2 x_3 x_4 (z-x_2)(\bar{z}-x_3)(\bar{z}-x_4)} \\
= \bar{\epsilon}_1 \gamma^{\mu \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\lambda \mu \nu} \epsilon_4 \int dx_2 \frac{(z-\bar{z})}{(z-x_2)(\bar{z}-x_2)} \\
= \pi \bar{\epsilon}_1 \gamma^{\mu \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\lambda \mu \nu} \epsilon_4. \\
\]

There are three other contributions in (33) given by \( \bar{\epsilon}_1 \gamma^{\lambda \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\rho \mu \tau} \epsilon_4 \), \( \bar{\epsilon}_1 \gamma^{\lambda \nu \tau} \epsilon_4 \bar{\epsilon}_2 \gamma^{\rho \mu \tau} \epsilon_3 \) and \( \bar{\epsilon}_1 \gamma^{\rho \mu \tau} \epsilon_4 \bar{\epsilon}_2 \gamma^{\lambda \nu \tau} \epsilon_3 \). After antisymmetrization over all \( \epsilon_i, i = 1, 2, 3, 4 \) these terms add up to give like the result for the graviton four point function

\[
\langle h \rangle_4 = \pi \zeta_{\mu \nu} k_\rho k_\lambda \bar{\epsilon}_1 \gamma^{\mu \nu \tau} \epsilon_2 \bar{\epsilon}_3 \gamma^{\lambda \mu \nu} \epsilon_4. \\
\]

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