Study of a model-independent method for the measurement of the angle $\phi_3$

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This report shows the latest results on the study of the method to determine the angle $\phi_3$ of the unitarity triangle using Dalitz plot analysis of the $D^0 \to K_S^0 \pi^+ \pi^-$ decay from $B^\pm \to D K^\pm$ process in a model-independent way. We concentrate on the case with a limited charm data sample, which will be available from the CLEO-c collaboration in the nearest future, with the main goal to find the optimal strategy for $\phi_3$ extraction. We find that the analysis using decays of $D_{CP}$ only cannot provide a completely model-independent measurement in the case of limited data sample. The procedure involving binned analysis of $B^\pm \to D K^\pm$ and $\psi(3770) \to (K_S^0 \pi^+ \pi^-)_D (K_S^0 \pi^+ \pi^-)_D$ decays is proposed, that allows to obtain the $\phi_3$ precision comparable to unbinned model-dependent fit.

1. Introduction

The measurement of the angle $\phi_3$ ($\gamma$) of the unitarity triangle using Dalitz plot analysis of the $D^0 \to K_S^0 \pi^+ \pi^-$ decay from $B^\pm \to D K^\pm$ process, introduced by Giri et al. [1] and Belle collaboration [2] and successfully implemented by BaBar [3] and Belle [4], presently offers the best constraints on this quantity. This corresponds to $\sim 10^3$ and due to large statistical error does not affect the precision of $\phi_3$ measurement. As the amount of $B$ factory data increases, though, this uncertainty will become a major limitation. Fortunately, a model-independent approach exists (see [1]), which uses the data of the $\tau$-charm factory to obtain the missing information about the $D^0$ decay amplitude.

In our previous study of the model-independent Dalitz analysis technique [3] we have implemented a procedure proposed by Giri et al. involving the division of the Dalitz plots into bins, and shown that this procedure allows to measure the phase $\phi_3$ with the statistical precision only 30-40% worse than in the unbinned model-dependent case. We did not attempt to optimize the binning and mainly considered a high-statistics limit with an aim to estimate the sensitivity of the future super-B factory.

The data useful for model-independent measurement are presently available from the CLEO-c experiment [6]. The integrated luminosity at the $\psi(3770)$ resonance decaying to $DD$ available for the analysis is 400 pb$^{-1}$. By the end of CLEO-c operation this statistic will grow up to 750 pb$^{-1}$ [6]. This corresponds to $\sim 1000$ events where $D$ meson in a $CP$ eigenstate decays to $K_S^0 \pi^+ \pi^-$, and twice as much events of $\psi(3770) \to D^0 \bar{D}^0$ with both $D$ mesons decaying to $K_S^0 \pi^+ \pi^-$. Both of these processes include the information useful for a model-independent $\phi_3$ measurement. In this paper, we report on studies of the model-independent approach with a limited statistics of both $\psi(3770)$ and $B$ data, using both $D_{CP} \to K_S^0 \pi^+ \pi^-$ and $(K_S^0 \pi^+ \pi^-)_D (K_S^0 \pi^+ \pi^-)_D$ final states.

2. Model-independent approach

The density of $D^0 \to K_S^0 \pi^+ \pi^-$ Dalitz plot is given by the absolute value of the amplitude $f_D$ squared:

$$p_D = |p_D(m^2_+, m^2_-)| = |f_D(m^2_+, m^2_-)|^2$$

(1)

In the case of no $CP$-violation in $D$ decay the density of the $D^0$ decay $\bar{p}_D$ equals to

$$\bar{p}_D = |\bar{f}_D|^2 = p_D(m^2_+, m^2_-).$$

(2)

Then the density of the $D$ decay Dalitz plot from $B^\pm \to D K^\pm$ process is expressed as

$$p_{B^\pm} = |f_D + r_B e^{i(\delta_B + \phi_3)} \bar{f}_D|^2 = p_D + r_B^2 \bar{p}_D + 2r_B \sqrt{p_D \bar{p}_D} \langle x_\pm c + y_\pm s \rangle,$$

(3)

where $x_\pm, y_\pm$ include the value of $\phi_3$ and other related quantities, the strong phase $\delta_B$ of the $B^\pm \to D K^\pm$ decay, and amplitude ratio $r_B$:

$$x_\pm = r_B \cos(\delta_B \pm \phi_3); \quad y_\pm = r_B \sin(\delta_B \pm \phi_3).$$

(4)

The functions $c$ and $s$ are the cosine and sine of the strong phase difference $\Delta \delta_D$ between the symmetric Dalitz plot points:

$$c = \cos(\delta_D(m^2_+, m^2_-) - \delta_D(m^2_+, m^2_+)) = \cos \Delta \delta_D;$$
$$s = \sin(\delta_D(m^2_+, m^2_-) - \delta_D(m^2_+, m^2_+)) = \sin \Delta \delta_D.$$  

(5)

The phase difference $\Delta \delta_D$ can be obtained from the sample of $D$ mesons in a $CP$-eigenstate, decaying to $K_S^0 \pi^+ \pi^-$. The Dalitz plot density of such decay is

$$p_{CP} = |f_D \pm \bar{f}_D|^2 = p_D + \bar{p}_D \pm 2 \sqrt{p_D \bar{p}_D} c$$

(6)

(the normalization is arbitrary). Decays of $D$ mesons in $CP$ eigenstate to $K_S^0 \pi^+ \pi^-$ can be obtained in the process, e.g. $e^+e^- \to \psi(3770) \to DD$, where the
other (tag-side) \(D\) meson is reconstructed in the \(CP\) eigenstate, such as \(K^+K^-\) or \(K^{0}_C\omega\).

Another possibility is to use a sample, where both \(D\) mesons (we denote them as \(D\) and \(D'\)) from the \(\psi(3770)\) meson decay into the \(K^{0}_C\pi^+\pi^-\) state \([5]\). Since \(\psi(3770)\) is a vector, two \(D\) mesons are produced in a \(P\)-wave, and the wave function of the two mesons is antisymmetric. Then the four-dimensional density of two correlated Dalitz plots is

\[
p_{\text{corr}}(m^2_x, m^2_y, m^2_{x'}, m^2_{y'}) = |f_Df'_D - f'_Df_D|^2 = p_Dp_D' + p_Dp_D' - 2 \sqrt{p_Dp_D'p_Dp_D'}(cc' + ss'),
\]

This decay is sensitive to both \(c\) and \(s\) for the price of having to deal with the four-dimensional phase space.

In a real experiment, one measures scattered data rather than a probability density. To deal with real data, the Dalitz plot can be divided into bins. In what follows, we show that using the appropriate binning, it is possible to reach the statistical sensitivity equivalent to the model-dependent case.

### 3. Binned analysis with \(D_{CP}\) data

The binned approach was proposed by Giri et al. \([1]\). Assume that the Dalitz plot is divided into \(2N\) bins symmetrically to the exchange \(m^2_x \leftrightarrow m^2_y\). The bins are denoted by the index \(i\) ranging from \(-N\) to \(N\) (excluding 0); the exchange \(m^2_x \leftrightarrow m^2_y\) corresponds to the exchange \(i \leftrightarrow -i\). Then the expected number of events in the bins of the Dalitz plot of \(D\) decay from \(B^\pm \rightarrow DK^\pm\) is

\[
\langle N_i \rangle = h_B[K_i + \nu_B^2K_{-i} + 2\sqrt{K_iK_{-i}}(xc_i + ys_i)],
\]

where \(K_i\) is the number of events in the bins in the Dalitz plot of the \(D^0\) in a flavor eigenstate, \(h_B\) is the normalization constant. Coefficients \(c_i\) and \(s_i\), which include the information about the cosine and sine of the phase difference, are given by

\[
c_i = \frac{\int_{D_i} \sqrt{p_Dp_D'} \cos(\Delta \delta_D(m^2_x, m^2_y))dD}{\sqrt{\int_{D_i} p_DdD' \int_{D_i'} p_D'rdD'}},
\]

\(s_i\) is defined similarly with cosine substituted by sine. Here \(D_i\) is the bin region, over which the integration is performed. Note that \(c_i = c_{-i}\), \(s_i = -s_{-i}\) and \(c_i^2 + s_i^2 \leq 1\) (the equality \(c_i^2 + s_i^2 = 1\) being satisfied if the amplitude is constant across the bin).

The coefficients \(K_i\) are obtained precisely from a very large sample of \(D^0\) decays in the flavor eigenstate, which is accessible at \(B\)-factories. The expected number of events in the Dalitz plot of \(D_{CP}\) decay equals to

\[
\langle M_i \rangle = h_{CP}[K_i + K_{-i} + 2\sqrt{K_iK_{-i}}c_i],
\]

and thus can be used to obtain the coefficient \(c_i\). As soon as the \(c_i\) and \(s_i\) coefficients are known, one can obtain \(x\) and \(y\) values (hence, \(\phi_3\) and other related quantities) by a maximum likelihood fit using equation (8).

Note that now the quantities of interest \(x\) and \(y\) (and consequently \(\phi_3\)) have two statistical errors: one due to a finite sample of \(B^\pm \rightarrow DK^\pm\) data, and due to \(D_{CP} \rightarrow K^{0}_C\pi^+\pi^-\) statistics. We will refer to these errors as \(B\)-statistical and \(D_{CP}\)-statistical, respectively.

Obtaining \(s_i\) is a major problem in this analysis. If the binning is fine enough, so that both the phase difference and the amplitude remain constant across the area of each bin, expressions (9) reduce to

\[
c_i = \cos(\Delta \delta_D)\text{ and } s_i = \sin(\Delta \delta_D),
\]

so \(s_i\) can be obtained as \(s_i = \pm \sqrt{1 - c_i^2}\). Using this equality if the amplitude varies will lead to the bias in the \(x, y\) fit result. Since \(c_i\) is obtained directly, and \(s_i\) is over-estimated by the absolute value, the bias will mainly affect \(y\) determination, resulting in lower absolute values of \(y\).

Our studies \([5]\) show that the use of equality \(c_i^2 + s_i^2 = 1\) is satisfactory for the number of bins around 200 or more, which cannot be used with presently available \(D_{CP}\) data. It is therefore essential to find a relatively coarse binning (the number of bins being 10–20) which a) allows to extract \(s_i\) from \(c_i\) with low bias, and b) has the sensitivity to the \(\phi_3\) phase comparable to the unbinned model-dependent case.

Fortunately, both the a) and b) requirements appear to be equivalent. To determine the \(B\)-statistical sensitivity of a certain binning, let’s define a quantity \(Q\) — a ratio of a statistical sensitivity to that in the unbinned case. Specifically, \(Q\) relates the number of standard deviations by which the number of events in bins is changed by varying parameters \(x\) and \(y\), to the number of standard deviations if the Dalitz plot is divided into infinitely small regions (the unbinned case):

\[
Q^2 = \frac{\sum_i \left(\frac{1}{\sqrt{N_i}} \frac{dN_i}{dx}\right)^2 + \left(\frac{1}{\sqrt{N_i}} \frac{dN_i}{dy}\right)^2}{\int_{D} \left[\frac{1}{\sqrt{|f_B|^2}} \frac{d|f_B|^2}{dx}\right]^2 + \left(\frac{1}{\sqrt{|f_B|^2}} \frac{d|f_B|^2}{dy}\right)^2} dD,
\]

where \(f_B = f_D + (x+iy)f_D\), \(N_i = \int_{D_i} |f_B|^2 dD\).

Since the precision of \(x\) and \(y\) weakly depends on the values of \(x\) and \(y\) \([5]\), we can take for simplicity \(x = y = 0\). In this case one can show that

\[
Q^2|_{x=y=0} = \sum_i (c_i^2 + s_i^2)N_i / \sum_i N_i
\]

Therefore, the binning which satisfies \(c_i^2 + s_i^2 = 1\) (i.e. the absence of bias if \(s_i\) is calculated as \(\sqrt{1 - c_i^2}\)) also has the same sensitivity as the unbinned approach. The factor \(Q\) defined this way is not necessarily the
best measure of the binning quality (the binning with higher $Q$ can be insensitive to either $x$ or $y$, which is impractical from the point of measuring $\phi_3$), but it allows an easy calculation and correctly reproduces the relative quality for a number of binnings we tried in our simulation.

The choice of the optimal binning naturally depends on the $D^0$ model. In our studies we use the two-body amplitude obtained in the latest Belle $\phi_3$ Dalitz analysis [4].

From the consideration above it is clear that a good approximation to the optimal binning is the one obtained from the uniform division of the strong phase difference $\Delta \delta_D$. In the half of the Dalitz plot $m_+^2 < m_0^2$ (i.e., the bin index $i > 0$) the bin $D_i$ is defined by condition

$$2\pi(i - 1/2)/N < \Delta \delta_D(m_+^2, m_0^2) < 2\pi(i + 1/2)/N,$$

(13)
in the remaining part ($i < 0$) the bins are defined symmetrically. We will refer to this binning as $\Delta \delta_D$-binning. As an example, such a binning with $N = 8$ is shown in Fig. 1 (left). Although the phase difference variation across the bin is small by definition, the absolute value of the amplitude can vary significantly, so the condition $c_1^2 + s_1^2 = 1$ is not satisfied exactly. The values of $c_1$ and $s_1$ in this binning are shown in Fig. 1 (bottom left) with crosses.

Figure 1 (right) shows the division with $N = 8$ obtained by continuous variation of the $\Delta \delta_D$-binning to maximize the factor $Q$. The sensitivity factor $Q$ increases to 0.89 compared to 0.79 for $\Delta \delta_D$-binning.

![Figure 1: Divisions of the $D^0 \to K^0 \pi^+ \pi^-$ Dalitz plot. Uniform binning of $\Delta \delta_D$ strong phase difference with $N = 8$ (left), and the binning obtained by variation of the latter to maximize the sensitivity factor $Q$ (right).](image)

We perform a toy MC simulation to study the statistical sensitivity of the different binning options. We use the amplitude from the Belle analysis [4] to generate decays of flavor $D^0$, $D_{CP}$, and $D$ from $B^\pm \to DK^\pm$ decay to the $K^0\pi^+\pi^-$ final state according to the probability density given by 1, 3 and 3, respectively. To obtain the $B$-statistical error we use a large number of $D^0$ and $D_{CP}$ decays, while the generated number of $D$ decays from the $B^\pm \to DK^\pm$ process ranges from 100 to 100000. For each number of $B$ decay events, 100 samples are generated, and the statistical errors of $x$ and $y$ are obtained from the spread of the fitted values. A study of the error due to $D_{CP}$ statistics is performed similarly, with a large number of $B$ decays, and the statistics of $D_{CP}$ decays varied. Both errors are checked to satisfy the square root scaling.

The binning options used are $\Delta \delta_D$-binning with $N = 8$ and $N = 20$, as well as “optimal” binnings with maximized $Q$ obtained from these two with a smooth variation of the bin shape. Note that the “optimal” binning with $N = 20$ offers the $B$-statistical sensitivity only 4% worse than an unbinned technique. For comparison, we use the binnings with the uniform division into rectangular bins (with $N = 8$ and $N = 19$ in the allowed phase space, the ones which are denoted as 3x3 and 5x5 in 3).

The $B$- and $D_{CP}$-statistical precision of different binning options, recalculated to 1000 events of both $B$ and $D_{CP}$ samples, as well as their calculated values of the factor $Q$, are shown in Table 1. In the present study we use the errors of parameters $x$ and $y$ rather than $\phi_3$ as a measure of the statistical power since they are nearly independent of the actual values of $\phi_3$, strong phase $\delta$ and amplitude ratio $r_B$. The error of $\phi_3$ can be obtained from these numbers given the value of $r_B$. The factor $Q$ reproduces the ratio of the values $\sqrt{1/\sigma_2^2 + 1/\sigma_3^2}$ for the binned and unbinned approaches with the precision of 1–2%. While the binning with maximized $Q$ offers better $B$-statistical sensitivity, the best $D_{CP}$-statistical precision of the options we have studied is reached for the $\Delta \delta_D$-binning. However, for the expected amount of experimental data of $B$ and $D_{CP}$ decays the $B$-statistical error dominates, therefore, slightly worse precision due to $D_{CP}$ statistics does not affect significantly the total precision.

We have considered the choice of the optimal binning only from the point of statistical power. However, the conditions to satisfy low model dependence are quite different. Since the bins in the binning options we have considered are sufficiently large, the requirement that the phase does not change over the bin area is a strong model assumption. We have performed toy MC simulation to study the model dependence. While the binning was kept the same as in the statistical power study (based on the phase difference from the default $D^0$ amplitude), the amplitude used to generate $D^0$, $D_{CP}$ and $B^\pm \to DK^\pm$ decays was altered in the same way as in the Belle study of the model-dependence in the unbinned analysis [4]. As a result, the same bias of $\Delta \phi_3 \sim 10^\circ$ is observed as in unbinned analysis. The bias in $x$ and $y$ if demonstrated in Fig. 2. We remind that the cause of this bias is a fixed relation between the $c_1$ and $s_1$. Therefore, proposed binning options, although providing good statistical precision, are not flexible enough to provide also a low model dependence. To minimiz
Table I Statistical precision of \((x,y)\) determination using different binnings and with an unbinned approach. The errors correspond to 1000 events in both the \(B\) and \(D_{CP}\) \((K^0_S\pi^+\pi^-)^2\) samples.

| Binning    | \(Q\) | \(\sigma_x\) | \(\sigma_y\) | \(\sigma_{\text{stat. err.}}\) | \(\sigma_{\text{stat. err.}}\) | \(\sigma_{\text{stat. err.}}\) |
|------------|-------|--------------|--------------|-------------------------------|-------------------------------|-------------------------------|
| \(\mathcal{N} = 8\) (uniform) | 0.57  | 0.0331       | 0.0600       | 0.0053                        | 0.0097                       | 0.0145                        |
| \(\mathcal{N} = 8\) (\(\Delta \delta_D\)) | 0.79  | 0.0273       | 0.0370       | 0.0042                        | 0.0072                       | 0.0050                        |
| \(\mathcal{N} = 8\) (optimal)   | 0.89  | 0.0232       | 0.0324       | 0.0058                        | 0.0114                       | 0.0082                        |
| \(\mathcal{N} = 19\) (uniform)  | 0.69  | 0.0274       | 0.0549       | 0.0042                        | 0.0112                       | -                             |
| \(\mathcal{N} = 20\) (\(\Delta \delta_D\)) | 0.82  | 0.0266       | 0.0350       | 0.0048                        | 0.0074                       | -                             |
| \(\mathcal{N} = 20\) (optimal)  | 0.96  | 0.0223       | 0.0290       | 0.0078                        | 0.0110                       | -                             |
| Unbinned   | -     | 0.0213       | 0.0279       | -                             | -                            | -                             |

4. Binned analysis with correlated \(D^0 \rightarrow K^0_S\pi^+\pi^-\) data

The use of the \(\psi(3770)\) decays where both neutral \(D\) mesons decay to the \(K^0_S\pi^+\pi^-\) state allows to significantly increase the amount of data useful to extract phase information in \(D^0\) decay. It is also possible to detect events of \(\psi(3770) \rightarrow (K^0_S\pi^+\pi^-)_D (K^0_S\pi^+\pi^-)_D\), where \(K^0_L\) is not reconstructed, and its momentum is obtained from kinematic constraints. The number of these events is approximately twice that of \((K^0_S\pi^+\pi^-)^2\) due to combinatorics. However, it is impossible to simply combine these samples since the phases of the doubly Cabibbo-suppressed components in \(\overline{D}^0 \rightarrow K^0_S\pi^+\pi^-\) and \(D^0 \rightarrow K^0_L\pi^+\pi^-\) amplitudes are opposite [6]. In the analysis of \(B\) data only \(K^0_S\pi^+\pi^-\) state can be used, but it is possible to utilize \(K^0_L\pi^+\pi^-\) data to better constrain the \(\overline{D}^0 \rightarrow K^0_S\pi^+\pi^-\) amplitude using model assumptions based on SU(3) symmetry. In what follows, we will consider the use of \(K^0_S\pi^+\pi^-\) data only.

In the case of a binned analysis, the number of events in the region of the \((K^0_S\pi^+\pi^-)^2\) phase space is

\[
\langle M \rangle_{ij} = h_{\text{corr}}[K_i K_{-j} + K_{-i} K_j] - 2\sqrt{K_i K_{-i} K_j K_{-j} (c_i c_j + s_i s_j)].
\]

Here two indices correspond to two \(D\) mesons from \(\psi(3770)\) decay. It is logical to use the same binning as in the case of \(D_{CP}\) statistics to improve the precision of the determination of \(c_i\) coefficients, and to obtain \(s_i\) from data without model assumptions, contrary to \(D_{CP}\) case. The obvious advantage of this approach is its being unbiased for any finite \((K^0_S\pi^+\pi^-)^2\) statistics (not asymptotically as in the case of \(D_{CP}\) data).

Note that in contrast to \(D_{CP}\) analysis, where the sign of \(s_i\) in each bin was undetermined and has to be fixed using model assumptions, \((K^0_S\pi^+\pi^-)^2\) analysis has only a four-fold ambiguity: change of the sign of all \(c_i\) or all \(s_i\). In combination with \(D_{CP}\) analysis, where the sign of \(c_i\) is fixed, this ambiguity reduces to only two-fold. One of the two solutions can be
chosen based on a weak model assumption (incorrect $s_i$ sign corresponds to complex-conjugated $D$ decay amplitude, which violates causality requirement when parameterized with the Breit-Wigner amplitudes).

Coefficients $c_i$, $s_i$ can be obtained by minimizing the negative logarithmic likelihood function

$$-2 \log \mathcal{L} = -2 \sum_{i,j} \log P(M_{ij}, \langle M \rangle_{ij}),$$

where $P(M, \langle M \rangle)$ is the Poisson probability to get $M$ events with the expected number of $\langle M \rangle$ events.

The number of bins in the 4-dimensional phase space is $4N^2$ rather than $2N$ in the $D_{CP}$ case. Since the expected number of events in correlated $K_S^0 \pi^+ \pi^-$ data is of the same order as for $D_{CP}$, the bins will be much less populated. This, however, does not affect the precision of $c_i$, $s_i$ determination since each of the free parameters is constrained by many bins.

The coefficients $c_i$, $s_i$ obtained this way can then be used to constrain $x$, $y$ with the maximum likelihood fit of the $B$ decay data using Eq. (8). To correctly account for the errors of $c_i$, $s_i$ determination, this likelihood should include distributions of these quantities, in addition to Poisson fluctuations in $B$ data bins. A more convenient way is to use the common likelihood function, covering both $B$ and $(K_S^0 \pi^+ \pi^-)^2$ data:

$$-2 \log \mathcal{L} = -2 \sum_{i,j} \log P(M_{ij}, \langle M \rangle_{ij}) - 2 \sum_i \log P(N_i, \langle N \rangle_i),$$

with $x$, $y$, $h_B$, $h_{corr}$, $c_i$ and $s_i$ as free parameters. This approach is also more optimal in the case of large $B$ data sample, since it imposes additional constraints on $c_i$, $s_i$ values.

The toy MC simulation was performed to study the procedure described above. Using the amplitude from the Belle analysis, we generate a large number of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays and several sets of $(K_S^0 \pi^+ \pi^-)^2$ decays (according to the probability density given by Fig. 3). We use the same binning options as in $D_{CP}$ study with $N = 8$. The combined negative likelihood (15) is minimized in the fit to each toy MC sample. We constrain $c_i^2 + s_i^2 < 1$ in the fit to avoid entering unphysical region with negative number of events in the bin. The number of $(K_S^0 \pi^+ \pi^-)^2$ and $B$ decays ranges from $10^3$ to $10^5$. The errors of $x$ and $y$ parameters are calculated from the spread of the fitted values. If the number of $(K_S^0 \pi^+ \pi^-)^2$ decays is comparable or larger than the number of $B$ decays, the $x$ and $y$ errors can be represented as quadratic sums of two errors, each scaled as a square root of $(K_S^0 \pi^+ \pi^-)^2$ and $B$ statistics, respectively. However if the number of $B$ decays is large, the errors of $c_i$ and $s_i$ depend also on $B$ decay statistics, so separating the total error into $B$- and $(K_S^0 \pi^+ \pi^-)^2$-statistical errors becomes impossible.

The best $(K_S^0 \pi^+ \pi^-)^2$-statistical error is obtained for $\Delta \delta_D$-binning and recalculated to 1000 events yields $\sigma_x = 0.0050$, $\sigma_y = 0.0095$, which is only slightly worse than the error obtained with the same amount of $D_{CP}$ data (see Table 4 for comparison). We also check that significant change of the model used to define the binning does not lead to the systematic bias (although it does decrease the statistical precision). Figure 3 demonstrates the precision of the determination of $c_i$, $s_i$ coefficients in our toy MC study and the absence of the systematic bias for both $x$ and $y$ when the model is varied.

The numbers of $(K_S^0 \pi^+ \pi^-)^2$ and $D_{CP}$ decays in $\psi(3770)$ data are comparable, so are the statistical errors due to $\psi(3770)$ data sample for the two approaches. The same binning can be used in both approaches, therefore improving the accuracy of $c_i$ determination. The approach based on $(K_S^0 \pi^+ \pi^-)^2$ data allows to extract both $c_i$ and $s_i$ without additional model uncertainties, so it can be used to check the validity of the constraint $c_i^2 + s_i^2 = 1$ and therefore to test the sensitivity of the particular binning.
5. Conclusion

We have studied the model-independent approach to \(\phi_3\) measurement using \(B^\pm \rightarrow DK^{\pm}\) decays with neutral \(D\) decaying to \(K_S^0\pi^+\pi^-\). The analysis of \(\psi(3770) \rightarrow D\bar{D}\) data allows to extract the information about the strong phase in \(\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-\) decay that is fixed by model assumptions. However, as the \(\psi(3770)\) statistics are of the order of 0.01. For the CLEO-c, statistics of 750 pb\(^{-1}\) and \((K_S^0\pi^+\pi^-)^2\) events the expected errors of parameters \(x\) and \(y\) due to \(\psi(3770)\) statistics are of the order of 0.01. For \(r_B = 0.1\) it gives the \(\phi_3\) precision \(\sigma_{\phi_3} = \sigma_{x,y}/(\sqrt{2}r_B) \simeq 5^\circ\), which is far below the expected error due to present-day \(B\) data sample. Further improvement of \(\phi_3\) precision will require larger charm dataset, which can be provided by BES-III experiment [10]. In our study, we did not consider the experimental systematic uncertainties \(e.g.\) due to imperfect knowledge of the detection efficiency or background composition. We believe these issues can be addressed in a similar manner as in already completed model-dependent analyses.

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