Radiation ‘damping’ in atomic photonic crystals

S. A. R. Horsley, 1, 2, 3 M. Artoni, 3, 4 and G. C. La Rocca 5

1School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, UK
2Department of Physics, University of York, Heslington, York, UK
3European Laboratory for Nonlinear Spectroscopy, Sesto Fiorentino, Italy
4Department of Chemistry and Physics of Materials, University of Brescia, Italy
5Scuola Normale Superiore and CNISM, Pisa, Italy

(Dated: July 13, 2010)

The force exerted on a material by an incident beam of light is dependent upon the speed of light. Here we show that this tiny effect is greatly amplified in multilayer systems composed of resonantly absorbing atoms (e.g. optically trapped 87Rb), which may exhibit ultra–narrow photonic band gaps. The amplification of the effect is shown to be three orders of magnitude greater than previous estimates for conventional photonic—band—gap materials, and significant for material velocities of a few ms−1.

PACS numbers: 42.50.Wk, 37.10.Vz, 42.70.Qs, 67.85.-d, 03.30.+p

The force of radiation pressure is dependent upon the material’s velocity in the laboratory frame of reference. This velocity dependence is known to be difficult to measure, as it is proportional to the incident optical power multiplied by the ratio of the material velocity to the speed of light. Here we show that this tiny effect is greatly amplified in multilayer systems composed of resonantly absorbing atoms (e.g. optically trapped 87Rb), which may exhibit ultra—narrow photonic band gaps. The amplification of the effect is shown to be three orders of magnitude greater than previous estimates for conventional photonic—band—gap materials [3]. We note that our results are not specific to trapped 87Rb, but apply equally well to any system exhibiting a photonic—band—gap on the same frequency scale.

The rate of transfer of four—momentum, dP^0_{MAT}/dt, to a medium which is possibly dispersive and absorbing, may be calculated from the electromagnetic fields in the vacuum region outside of the medium,

\[
\frac{dP^0_{\text{MAT}}}{dt} = -\int_{\partial \text{MAT}} T^{0i}_{\text{FIELD}} dS_j,
\]

where the surface element dS_j points outward from the material surface [5], and where the relevant components of the energy—momentum tensor are \(T^0_i = c_0 (E \times B)_i\), and \(T^{ij} = c_0 [\delta_{ij} (E^2 + c^2 B^2)/2 - E_i E_j - c^2 B_i B_j]\). We start by considering a material planar slab at rest, upon which a beam of linearly polarized radiation of cross sectional area \(A\) is normally incident. Radiation of frequency \(\omega\) enters the material through the surface at \(x = x_1\) and exits through a similar surface at \(x = x_2\), with complex reflection and transmission amplitudes \(r(\omega)\) and \(t(\omega)\). Averaging over a time interval \(\Delta t \gg \omega^{-1}\) yields the average four—force experienced by the slab at rest,

\[
\left\langle \frac{dP^0_{\text{MAT}}}{dt} \right\rangle = \frac{A c_0 E_0^2}{2} \left[ \frac{1 - R(\omega)}{1 + R(\omega)} \right],
\]

where \(R(\omega) = |r(\omega)|^2\) and \(T(\omega) = |t(\omega)|^2\). Upon Lorentz transforming (2), one obtains the corresponding expression for the slab in motion with velocity \(V = \hat{V} \hat{x}\) in the

\[
\left\langle \frac{dP^0_{\text{MAT}}}{dt} \right\rangle = \frac{A c_0 E_0^2}{2} \left[ \frac{1 - R(\omega)}{1 + R(\omega)} \right] \hat{x},
\]
lab frame. In terms of the lab (primed) frame quantities, one has \( \omega = \sqrt{(1 - V/c)/(1 + V/c)} \omega' = \eta \omega' \), \( E_0 = \gamma E_0' \), and the average four-force becomes,

\[
\left\langle \frac{d\mathbf{P}'}{dt} \right\rangle' = \frac{\mathcal{A}cE_0^2}{2\sqrt{1 - (\frac{\omega}{\omega'})^2}} \left[ (1 - R(\eta\omega') - T(\eta\omega')) + \frac{V}{c} (1 + R(\eta\omega') - T(\eta\omega')) \right] \left[ (1 - R(\eta\omega') - T(\eta\omega')) + \frac{V}{c} (1 - R(\eta\omega') - T(\eta\omega')) \right] \left( \mathbf{F} \right) \]  

where \( P' = \mathcal{A}c\omega E_0^2/2 \) is the incident radiation mean power. The force expression \( \left(3\right) \) has a rather involved dependence on the velocity as it stems from \( R \) and \( T \) that depend on the velocity through the Doppler effect. A characteristic velocity range for cold atoms experiments is such that \( \eta \approx 1 - V/c \), so that the reflectivity and transmissivity can be expanded to the leading order in \( V/c \) around the lab frame frequency as, \( R(\eta\omega') \approx R(\omega') - (\omega'V/c) R^{(1)}(\omega') \), and \( T(\eta\omega') \approx T(\omega') - (\omega'V/c) T^{(1)}(\omega') \). This yields the simpler expression on the right hand side of \( \left(3\right) \) where we set,

\[
F_{(0)}^{(0)} = 1 - R(\omega') - T(\omega') \\
F_{(1)}^{(1)} = 1 - 3R(\omega') - T(\omega') - \omega' \left( R^{(1)}(\omega') + T^{(1)}(\omega') \right) \\
F_{(0)}^{(1)} = 1 + R(\omega') - T(\omega') \\
F_{(1)}^{(1)} = 1 + 3R(\omega') - T(\omega') + \omega' \left( R^{(1)}(\omega') - T^{(1)}(\omega') \right).
\]

For non-dispersive materials \( F_{(0)}^{(0)} \) and the the first three terms in \( F_{(1)}^{(1)} \) are the only contributions to the radiation pressure. These are positive and for a lossless medium of fixed reflectivity \( R_0 \) reduce to the familiar results \( \left(1\right) \) \( \left(2\right) \) respectively for the pressure force \( 2R_0P'/c \) and the radiation pressure damping (friction) \( -4R_0P'/c^2 \). The latter is typically much smaller than the former and for non-dispersive mirrors, where \( R(\eta\omega') \approx R(\omega') \) and \( T(\eta\omega') \approx T(\omega') \), velocities as large as \( V \approx 10^6 \) ms\(^{-1}\) are needed to observe a few per cent velocity shift, corresponding to a damping contribution to the force. For dispersive materials, on the other hand, we have an extra contribution to \( F_{(1)}^{(1)} \) whose sign depends on the relative strength of the reflectivity and transmissivity gradients \( R^{(1)}(\omega') \) and \( T^{(1)}(\omega') \). When \( R \) and \( T \) change significantly over a frequency range \( \Delta\omega \ll \omega' \), this contribution could be substantial, and become the dominant term in \( F_{(1)}^{(1)} \) whose sign, unlike other terms in the force expression in \( \left(3\right) \), can then become negative or positive. This amounts, respectively, to either amplification or damping of the medium relevant dynamics \( \left(3\right) \) \( \left(4\right) \). Materials with optical reflectivity changes \( \Delta R \approx 1 \) on the MHz range, \( i.e. \omega \Delta R/\Delta\omega \approx 10^3 \), would yield an appreciable velocity shift in the force even at \( V \approx 1 \) ms\(^{-1}\).

Periodic structures of trapped two-level atoms separated by vacuum fit the aforementioned parameter range quite well \( \left(7\right) \), and typical reflectivities and transmissivities are plotted in Fig. 1 as a function of the scaled detuning \( \delta = (\omega_0 - \omega)/\gamma \) from the atomic resonance transition \( \omega_0 \), where \( \gamma \) denotes the excited state decay rate. These specific profiles correspond to a stack of alternating (complex) refractive indices \( \left(10\right) \) \( n_\alpha(\omega) \approx 1 \) and \( n_\beta(\omega) \approx \sqrt{1 + 3\pi N(N - 1 - i)} \) respectively with thicknesses \( a \) and \( b \), where \( N \) denotes the density of states. These multilayered atomic structures exhibit two pronounced photonic stop bands when the Bragg scattering frequency is not too far from the atomic transition frequency. The first one develops from the polaronic stop band and has one edge at the atomic resonance frequency, the second one corresponds to the usual stop band of more familiar (non resonant) photonic crystals and develops around the Bragg frequency \( \left(7\right) \). For the case of Fig.1 the first and second stop bands lie respectively on the negative and positive detuning regions. At a given frequency \( \omega' \) the radiation pressure is solely determined by the multi-structure optical response and Fig. 2 shows both force components \( F_{(0)}^{(1)} \) and \( F_{(1)}^{(1)} \) in the red-detuned spectral region near the polaritonic stop-band in Fig. 1. In particular, Fig. 2b displays the force velocity-

\[\text{FIG. 1: Reflectivity and transmissivity for a multilayered atomic structures of } 5.3797 \times 10^4 \text{ unit cells with } a=370.873 \times 10^{-9} \text{ m and } b=19.4807 \times 10^{-9} \text{m. Atomic parameters are, } (87\text{Rb } D_2 \text{ line } 5^2 S_{1/2} \rightarrow 5^2 P_{3/2}, \gamma = 2\pi \times 6 \times 10^6 \text{Hz}, \omega_0 = c/\lambda_0 = 2\pi \times 384.02 \times 10^{12} \text{Hz and an atomic density } N/V = 6 \times 10^{18} \text{m}^{-3}. \]
dependent contribution, $F_{(1)}^1$, which largely arises from the term proportional to $R_{(1)}^1(\omega') - T_{(1)}^1(\omega')$. This gradient difference amounts precisely to $2 \times R_{(1)}^1(\omega') + A_{(1)}(\omega')$ and, unlike familiar dielectric photonic crystals structures [11], absorption dispersion $A_{(1)}(\omega')$ makes an appreciable contribution to the force at the edge(s) of the polaritonic stop-band where energy transfer to the sample through absorption is largest [2]. Radiation pressure values can scale with $V/c$ by as much as $|F_{(1)}^1| = 1 \times 10^7$ (ignoring the region very close to resonance) so that an appreciable ten per cent radiation pressure shift is possible with velocities of $\sim 3 \text{ms}^{-1}$, that are now within experimental reach [12]. It is worth noting that the velocity contribution to the radiation pressure turns out to be at least three orders of magnitude larger than those obtained with traditional dielectric photonic crystals [3].

In addition, depending on whether the difference between $R_{(1)}^1(\omega')$ and $T_{(1)}^1(\omega')$ is positive or negative alternating cooling and heating cycles become easily accessible in the appropriate spectral region [19]. We should note that the above analysis is approximate due to the expansion to first order in $V/c$, yet the above conclusions are not fundamentally altered if all orders of $V/c$ are retained. This is shown in the following in the case of a light pulse, or sequence of pulses, of finite extent rather than a (monochromatic) plane light-wave. This is indeed to be examined to better assess the experimental feasibility and becomes furthermore important when opto-mechanical effects of radiation pressure associated with weak quantum light pulses are to be observed [8] [13]. The momentum exchanged between a pulse of a specified extent and the multilayered atomic structure described above is here calculated by keeping all orders of $V/c$ in the resulting expression. At the position of the material surface, $x_1$, the electric field of a pulse of central frequency $\omega_c$ and half–width–half–maximum, $L \sqrt{\ln(2)}$, can be written as a superposition of its incident and reflected parts as follows (the real part of this expression entering the energy–momentum tensor),

$$E(t) = \int_0^\infty \frac{d\omega}{\sqrt{2\pi}} \xi(\omega) \left( e^{i\omega(x_1/c-t)} + r(\omega)e^{-i\omega(x_1/c+t)} \right).$$

(4)

When the pulse is linearly polarized along the $y$–axis and the wavelength associated with the carrier frequency is much shorter than the extent of the pulse $(\omega_c/L/c \gg 1)$, then a Gaussian pulse can be described as: $\xi(\omega) \approx \sqrt{\rho_0 N \hbar \omega_c L/(4\pi^2/\gamma)} e^{-(\omega - \omega_c)^2/2\gamma^2}$, where $N$ is the average number of photons within the pulse. The transferred momentum from a pulse incident on the structure entrance surface at $x_1$, as observed in the material’s rest frame, is calculated as an integral over all time of the relevant integrated energy-momentum tensor component in (1). As an example, in $T_{(1)}^1$ one encounters the integral,

$$\int_{-\infty}^\infty dt \left[ E(t)^2 + c^2B(t)^2 \right] = \int_0^\infty d\omega \left| \xi(\omega) \right|^2 \left[ 1 + R(\omega) \right]$$

(5)

A similar contribution for the momentum transfer at the rear surface at $x_2$, can be obtained starting from (1) where we replace $e^{i\omega(x_1/c-t)} + r(\omega)e^{-i\omega(x_1/c+t)} \rightarrow t(\omega)e^{i\omega(x_2/c-t)}$. The net integrated four-momentum imparted by the pulse as seen in the slab rest frame can be calculated and in turn transformed as a four-vector into the lab frame. If the change in the medium velocity is negligible over the transit time of a single pulse, the net lab (primed) frame momentum transfer per pulse is given by,

$$\Delta P^{\mu\nu} = \frac{e_0 A \eta}{2\sqrt{1 - (V/c)^2}} \int_0^\infty d\omega' \left| \xi(\eta \omega') \right|^2 \left( \frac{P_{0\nu}^{\mu}}{P_{\nu}^{0\mu}} \hat{x} \right)$$

(6)

where in the lab (primed) frame we have: $\mathcal{L} = \mathcal{L}'/\eta$; $\omega_c = \eta \omega_c'$; $\mathcal{P}^{\mu\nu} = 1 - R(\eta \omega) - T(\eta \omega) + (V/c)(1 + R(\eta \omega') - T(\eta \omega'));$ and $\mathcal{T}^{\mu\nu} = 1 + R(\eta \omega') - T(\eta \omega') + (V/c)(1 - R(\eta \omega') - T(\eta \omega')).$ This recovers well known results for a transparent lossless slab [14] in the limit for which $V/c \rightarrow 0$ and $\eta \rightarrow 1$. The plane wave limit in (3) also emerges namely when $\mathcal{L} \rightarrow \infty$, if $\Delta P^{\mu\nu}$ is first divided by the proper time interval for a square pulse containing the same amount of momentum, and of the same maximum amplitude as the incident Gaussian pulse, $\Delta \tau = \sqrt{\pi/2} (\mathcal{L}/c)$. The results presented in figure

![Graph](image-url)
to a stationary structure (ence of length \(D\) of the ground state of the lattice wells. The disks thickness \(d\) of the confining optical lattice \(U\) is determined by the in-well power difference between the two counterpropagating laser beams of the optical lattice to the other which is given by \(\hbar k \Delta N_{ph} = |\Delta \vec{P}|\). Of course, by sending a train of \(N_{ip}\) light pulses equally spaced by the period of the atomic harmonic oscillations so to always push the atoms at the right time, \(\Delta N_{ph}\) would be amplified by the factor \(N_{ip}\). All-optically tailorable light pressure damping and amplification effects can here be attained in the absence of a cavity, which makes multilayered atomic structures not only interesting in their own right but also amenable to a new optomechanical regime. The large per-photon pressures that can be observed compare in fact with state of the art micro [17] and nano [18] optomechanical resonators, yet involving (atomic) masses that are various orders of magnitude smaller.

ACKNOWLEDGMENTS

We thank G. Ferrari and L. Fallani for stimulating discussions. This work is supported by the CRUI-British Council Programs “Atoms and Nanostructures” and “Metamaterials”, and the IT09L244H5 Azione Integrata MIUR grant.

\* Electronic address: Simon.Horsley@st-andrews.ac.uk

[1] V. B. Braginski and A. B. Makunin, J. Exp. Teor. Phys. 52, 998 (1967).
[2] A. B. Matsko, E. A. Zubova, and S. P. Vyatchanin, Opt. Commun. 131, 107 (1996).
[3] K. Karrai, I. Favero, and C. Metzger, Phys. Rev. Lett. 100, 240801 (2008).
[4] D. V. van Coevorden, R. Sprik, A. Tip, and A. Lagendijk, Phys. Rev. Lett. 77, 2412 (1996).
[5] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, Electrodynamics of Continuous Media (Butterworth-Heinemann, Oxford, 2004), 2nd ed.
[6] M. Bajcsy, A. S. Zibrov, and L. M. D., Nature 426, 638 (2003).
[7] M. Artoni, G. C. La Rocca, and F. Bassani, Phys. Rev. E 72, 046604 (2005).
[8] F. De Martini, F. Sciarrino, C. Vitelli, and F. Cataliotti, Phys. Rev. Lett. 104, 050403 (2010).
[9] F. Marquardt and S. M. Girvin, Physics 2, 40 (2009).
[10] M. O. Scully and M. S. Zubairy, Quantum optics (Cambridge University Press, Cambridge, 1997), 1st ed.
[11] I. Favero, C. Metzger, S. Camerer, D. König, H. Lorenz, J. P. Kotthaus, and K. Karrai, Appl. Phys. Lett. 90, 104101 (2007).
[12] S. Schmid, G. Thalhammer, K. Winkler, L. Lang, and J. H. Denschlag, New Journal of. Physics 8, 159 (2006).
[13] D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Phys. Rev. Lett. 98, 030405 (2007).
[14] R. Loudon, Journal of Modern Optics 49, 821 (2002).
[15] G. Raithel, W. D. Phillips, and S. L. Rolston, Phys. Rev. Lett. 81, 3615 (1998).
[16] K. M. R. van der Stam, E. D. van Ooijen, R. Meppelink, J. M. Vogels, and P. van der Straten, Review of Scientific Instruments 78, 013102 (2007).
[17] M. Eichenfield, C. P. Michael, R. Perahia, and O. Painter, Nature Photonics 1, 416 (2007).
[18] Q. Lin, J. Rosenberg, X. Jiang, K. J. Vahala, and O. Painter, Phys. Rev. Lett. 103, 103601 (2009).
[19] We have verified that the results do not change if the effects of a non–uniform atom density towards the edges of the trap are taken into account. If 100 periods on each side of the trap have half the atom density of the centre, then a calculation of the transfer matrix shows that in the region with ($|\delta| > 10$), corrections to $R$ and $T$ are at most $\sim 10^{-2}$.
[20] This is in general the case for optical traps using two counter propagating Gaussian beams. Efficient radial confinement of the atoms may also be achieved by employing optical traps with Bessel beams [12].
[21] This prevents absorption of optical lattice photons from hampering interactions between the light pulse and the coherent atoms during transfer of momentum.