A new procedure for solving linear programming problems in an intuitionistic fuzzy environment

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Abstract. This paper presents a novel approach for solving Linear Programming Problems in an Intuitionistic Fuzzy environment. Here, the cost coefficients in the objective function of an Intuitionistic Fuzzy Number Linear Programming Problem (IFNLPP) are considered to be Symmetric Triangular Intuitionistic Fuzzy Numbers (STriIFNs) and the problem has been solved without converting them to crisp Linear Programming Problem using linear Ranking function. Intuitionistic Fuzzy Simplex algorithm which is a most powerful tool of handling any IFLPP has been used for solution procedure. An example has also been illustrated to reveal the effectiveness of the projected method.

Keywords. Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Number Linear Programming Problems, Symmetric Triangular Intuitionistic Fuzzy Numbers and Ranking Function.

1. Introduction

In decision making problems, the estimation of model parameters are often imprecise and vague. Such phenomena have been very well captured through fuzzy sets in modelling these problems. Appliances of fuzzy sets in decision making problems and in specific to optimization problems have been extensively studied.

Really, most of the facts available are inadequate, obscure or inaccurate. Consequently, it is enviable to deem the information of connoisseurs about the parameters as fuzzy data. Fuzzy sets theory are very restricted in extent and in many cases, do not represent the real problem very well even though fuzziness in decision making problems have been considered by numerous researchers. Intuitionistic fuzzy set (IFS) is one of the generalizations of fuzzy sets theory. IFS have been established to deal with indistinctness, out of various higher-order fuzzy sets.

In fuzzy sets a membership function alone considered but IFS is distinguished by the measure of acceptance and the measure of non-acceptance so that the sum of both values is less than one. Currently IFS are being studied and applied in various fields of science. Intuitionistic fuzzy set was originated by Atanassov [1] and appears to be applicable to real world problems. An open problem in IF Sets theory is discussed by Atanassov and Gargov [2]. Intuitionistic fuzzy soft sets are considered by Maji et al [4]. A symmetric trapezoidal intuitionistic fuzzy number is introduced by Parvathi and Malathi [6] and their desirable properties are also included. Solution of a vector optimization problem has been obtained by the usage of an intuitionistic fuzzy goal programming which is discussed by Pramanik and Roy [7]. Jana and Roy [3] solved a transportation model by using multi-objective IFLP.

This paper presents a new idea to solve the IFLPP in which the cost coefficients in the objective function are STriIFNs and a linear ranking function to rank STriIFNs. A sample problem has also been presented to exhibit the competence of the planned method.
2. Preliminaries

Definition 2.1[6]
An Intuitionistic Fuzzy Set (IFS) \( A \) in \( X \) is defined as an object of the form \( A = \{<x, \mu_A(x), \nu_A(x)>\} \)
where the functions \( \mu_A : X \to [0, 1] \) and \( \nu_A : X \to [0, 1] \) define the degree of membership and the degree of non-membership of the element \( x \in X \) respectively, and for every \( x \in X \) in \( A \), \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) holds.

Definition 2.2[6]
An Intuitionistic Fuzzy Number (IFN) \( \tilde{A} \) is
(i) an Intuitionistic Fuzzy subset of the real line,
(ii) convex, that is, there is some \( x_0 \in R \) such that \( \mu_{\tilde{A}}(x_0) = 1 \), \( \nu_{\tilde{A}}(x_0) = 0 \)
(iii) for the membership function \( \mu_{\tilde{A}}(x) \) that is,
\[ \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \] for every \( x_1, x_2 \in R, \lambda \in [0,1] \)
(iv) concave for the non-membership function \( \nu_{\tilde{A}}(x) \) that is,
\[ \nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)) \] for every \( x_1, x_2 \in R, \lambda \in [0,1] \)

Definition 2.3[7]
An IFS \( \tilde{A} \) in \( R \) is said to be Symmetric Triangular Intuitionistic Fuzzy Number (STriIFN) if there exist real numbers \( \alpha, h, h_0 \) where \( h_0 \leq h_0 \) and \( h, h_0 \geq 0 \), such that membership function and non-membership function as follows:
\[ \mu_{\tilde{A}}(x) = \begin{cases} \frac{x-(\alpha-h)}{h} & \text{for } x \in [\alpha-h, h] \\ \frac{\alpha+h-x}{h_0} & \text{for } x \in [\alpha, \alpha+h] \end{cases} \] and
\[ \nu_{\tilde{A}}(x) = \begin{cases} \frac{x-(\alpha-h)}{h_0} & \text{for } x \in [\alpha-h, h] \\ \frac{\alpha+h-x}{h} & \text{for } x \in [\alpha, \alpha+h] \end{cases} \]

A STriIFN is denoted by \( \tilde{A}_{STriIFN} = [\alpha; h, h_0; h_0, h] \).

3. Arithmetic Operations on STriIFNs [7]
Let \( \tilde{A} = [\alpha; h, h_0; h_0, h] \) and \( \tilde{B} = [\beta; k, k_0; k_0, k] \) be two STriIFNs. Then the basic arithmetic operations on \( \tilde{A} \) and \( \tilde{B} \) are as follows:
The addition of \( \tilde{A} \) and \( \tilde{B} \), denoted by \( \tilde{A} + \tilde{B} \), is defined as
\[ \tilde{A} + \tilde{B} = [\alpha + \beta; h_1 + k_1, h_1 + k_1; h_1 + k_1, h_1 + k_1] \]
The additive image of \( \tilde{A} \) and additive image of \( \tilde{B} \) are defined as
Additive image of \( \tilde{A} = [-\alpha; h_0, h_0; h, h] \)
Additive image of \( \tilde{B} = [-\beta; k, k_0; k_0, k] \)
The subtraction of \( \tilde{A} \) and \( \tilde{B} \), is defined as
\[ \tilde{A} - \tilde{B} = [\alpha - \beta; h_1 + k_1, h_1 + k_1; h_1 + k_1, h_1 + k_1] \]
The scalar multiplication of $\tilde{A}'$ by the scalar $k$ denoted by $k\tilde{A}'$, is defined as

$$k\tilde{A}' = \begin{cases} [k\alpha; kh_1, kh'_1; kh'_1] & \text{if } k > 0 \\ [-k\beta; kh'_1, kh_1; kh_1] & \text{if } k < 0 \\ [0;0;0;0] & \text{if } k = 0 \end{cases}$$

### 3.1 Ranking function [6]

There are many ranking functions for comparing intuitionistic fuzzy numbers. Here, linear ranking function $\mathcal{R}$ has been considered which is defined as a mapping from $F(R)$, the set of STriIFNs into the real line $R$. Now orders on $F(R)$ are defined as follows:

- $\tilde{A}' \geq \tilde{B}'$ if and only if $\mathcal{R}(\tilde{A}') \geq \mathcal{R}(\tilde{B}')$
- $\tilde{A}' \leq \tilde{B}'$ if and only if $\mathcal{R}(\tilde{A}') \leq \mathcal{R}(\tilde{B}')$
- $\tilde{A}' \approx \tilde{B}'$ if and only if $\mathcal{R}(\tilde{A}') = \mathcal{R}(\tilde{B}')$ where $\tilde{A}', \tilde{B}'$ are in $F(R)$.

In this paper, the following linear ranking function has been used:

$$\mathcal{R}(\tilde{A}') = 2\alpha + \frac{1}{2}(h'_1 - h_1)$$

where $\tilde{A}'_{STriIFN} = [\alpha; h_1, h'_1; h'_1]$.

### 4. General structure of IFNLPP

In general, an IFNLPP can be defined as follows:

\[
\max \quad \tilde{z}' = \tilde{c}'x \\
\text{subject to} \quad Ax \leq b \\
x \geq 0.
\]

where $b \in R^m, x \in R^n, A \in R^{m \times n}, \left(\tilde{c}'\right)^T \in (F(R))^n, \tilde{c}' = [\alpha; h_1, h'_1; h'_1]$.

### 5. Simplex method for the IFNLPP problems

**Step: 1** If the problem is of minimization, convert it into the maximization problem by the relation \( \min \tilde{z}' = - \max (-\tilde{z}') \).

**Step: 2** Make all the $b_i$'s positive.

**Step: 3** Modify the constraints into equations by introducing non-negative slack and/or surplus variables.

**Step: 4** Find an IFBFS to the problem in the form $x_B = B^{-1}b$.

**Step: 5** Compute the net evaluations $\tilde{\Delta}'_j$, ($j = 1, 2, \ldots, n$) by using the relation

\[
\tilde{\Delta}'_j = \tilde{z}'_j - \tilde{c}'_j = \tilde{z}'_j - \tilde{c}'_j y_j - \tilde{c}'_j
\]

where $y_j = B^{-1} a_j$.

**Step: 6** Check the sign of $\tilde{\Delta}'_j$

- If all $\tilde{\Delta}'_j \geq 0$, then the initial IFBFS $x_B$ is optimal.
- Otherwise go to next step.

**Step: 7** If $\tilde{\Delta}'_j \leq 0$, for more than one $j$, then select the most negative of them. Let it be $\tilde{\Delta}'_j$. 

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If all \( y_{ir} \leq 0 \), there is an indication of unbounded solution.

If at least one \( y_{ir} > 0 \), then \( y_r \) is the in-coming vector in the basis \( y_B \).

**Step: 8** calculate the ratios \( \left\{ \frac{x_{Bi}}{y_{ir}} : y_{ir} > 0, i=1,2,\ldots,m \right\} \) and take the minimum of these ratios be \( \frac{x_{Bk}}{y_{kr}} \). Then \( y_k \) is the out-going vector in the basis \( y_B \). The element \( y_{kr} \) is known as the pivotal element.

**Step: 9** Change all pivotal element to 1 by dividing its row through the pivotal element itself, and all other elements in its column to zeros by using the relations,

\[
(new \ y_{ij}) = y_{ij} \frac{y_{kj}}{y_{kr}} \quad \text{and} \quad (new \ y_{kj}) = \frac{y_{kj}}{y_{kr}}.
\]

**Step: 10** If the solution is not optimal go to step 6 and repeat the process until an optimum solution obtained.

6. **ILLUSTRATION**

A factory can produce the products of two different types A and B, the profit per unit of product A and product B are closed to Rs.10 and Rs.6 respectively. Each unit of product A requires 5 machine hours and that of product B requires 3 machine hours. Each unit of product requires 1 units of raw substance whereas each unit of product B requires 2 units of raw substance. The maximum available machine hours and raw substance units are 30 and 18 respectively. Find the number of units to be produced of each kind so that the total profit is maximum.

**Solution:**

Let \( x_1 \) and \( x_2 \) denote the number units of A and B respectively. Then the problem can be formulated as the following linear programming with STRiIFNs in the objective function.

Max \( \tilde{z}^f \approx 10^f x_1 + 6^f x_2 \)

subject to \( 5x_1 + 3x_2 \leq 30 \)

\( x_1 + x_2 \leq 18 \)

\( x_1 \geq 0, x_2 \geq 0 \).

The profit per unit of product A and B are close to Rs.10 and Rs.6 are modelled as \((4.5; 1, 1; 3, 3)\) and \((2.75; 1, 1; 2, 2)\) respectively. Now the IFNLP model may be written in the standard form as follows:

Max \( \tilde{z}^f \approx (4.5; 1, 1; 3, 3) x_1 + (2.75; 1, 1; 2, 2) x_2 \)

subject to \( 5x_1 + 3x_2 + x_3 = 30 \)

\( x_1 + 2x_2 + x_4 = 18 \)

\( x_1, x_2, x_3, x_4 \geq 0 \).

The initial basic feasible solution exists and is presented here,

**Initial Table:**

| Basis | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | R.H.S. |
|-------|---------|---------|---------|---------|--------|
| \( x_3 \) | 5       | 3       | 1       | 0       | 30     |
| \( x_4 \) | 1       | 2       | 0       | 1       | 18     |
| \( \tilde{z}^f - \tilde{z}^f \) | (-4.5; 3; 3; 1, 1) | (-2.75, 2; 2; 1, 1) | \( \tilde{0}^f \) | \( \tilde{0}^f \) | \( \tilde{0}^f \) |

Since \( (\tilde{z}^f - \tilde{z}^f, \tilde{z}^f - \tilde{z}^f) \approx ((-4.5, 3, 3, 1, 1), (-2.75, 2, 2; 1, 1)), \) and \( (\tilde{A}_1, \tilde{A}_2) = (9(\tilde{A}_1), 9(\tilde{A}_1)) = (-10, -6), \) then \( x_1 \) is the in-coming vector and the out-going vector is \( x_3. \)
First Iteration:

| Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | R.H.S. |
|-------|-------|-------|-------|-------|--------|
| $x_1$ | 1     | $rac{3}{5}$ | $rac{1}{5}$ | 0     | 6      |
| $x_4$ | 0     | $rac{7}{5}$ | $-rac{1}{5}$ | 1     | 12     |

$\bar{z}_j^I - \bar{z}_j^I (0;2;2;6,6) (-0.05;\frac{8}{5};\frac{8}{5};\frac{19}{5};\frac{19}{5}) (0.9;\frac{1}{5};\frac{1}{5};\frac{3}{5};\frac{3}{5}) \bar{z}^I (27;6,6;18,18)$

From the above table, the optimal solution is obtained and is given by, $x_1 = 6, \ x_2 = 0$ and $\text{Max } \bar{z}^I \approx (27 ; 6 , 6 ; 18 , 18)$ with $\mathcal{R}(\bar{z}^I) = 60$.

7. Conclusion

In this paper, IFNLP had been solved by IF simplex method. The usual simplex algorithm in the crisp environment had been modified as an intuitionistic fuzzy environment. The cost coefficient in the objective function had been taken as STriIFNs and using arithmetic operations on STriIFNs the optimum solution obtained. Finally, a linear ranking function had been used for comparing STriIFNs. Suitable illustration had also been presented to exhibit the efficiency of the proposed work.

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