\( U(1)_A \) symmetry in two-doublet models, 
\( U \) bosons or light scalars, and \( \psi \) and \( \Upsilon \) decays

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Abstract

\( \psi \) and \( \Upsilon \) decays may be used to search for light neutral spin-1 or spin-0 bosons associated with a broken extra-\( U(1) \) symmetry, local or global, acting axially on quarks and leptons, as may be present in supersymmetric theories with a \( \lambda H_1 H_2 S \) superpotential term. Recent data on \( \Upsilon \to \gamma + \text{invisible neutral} \) constrain an axial, pseudoscalar or scalar coupling to \( b \) to \( f_{bA} < 4 \times 10^{-7} m_U \text{(MeV)}/\sqrt{B_{\text{inv}}} \), \( f_{bP} < 4 \times 10^{-3}/\sqrt{B_{\text{inv}}} \) or \( f_{bS} < 6 \times 10^{-3}/\sqrt{B_{\text{inv}}} \), respectively. This also constrains, from universality properties, couplings to electrons to \( f_{eA} < 4 \times 10^{-7} m_U \text{(MeV)}/\sqrt{B_{\text{inv}}} \), \( f_{eP} < 4 \times 10^{-3}/\sqrt{B_{\text{inv}}} \) or \( f_{eS} < 6 \times 10^{-3}/\sqrt{B_{\text{inv}}} \).

The pseudoscalar \( a \) (possibly traded for a light gauge boson, or scalar particle) should then be, for invisible decays of the new boson, for \( \tan \beta > 96 \% \) singlet and \( < 4 \% \) doublet, for \( \tan \beta > 1 \). Or, more generally, \( < 4 \% (\tan^2 \beta B_{\text{inv}}) \) doublet, which implies a very small rate for the corresponding \( \psi \) decay, \( B(\psi \to \gamma + \text{neutral}) B_{\text{inv}} \lesssim 10^{-6}/\tan^4 \beta \).

Similar results are obtained for new spin-1 or spin-0 neutral bosons decaying into \( \mu^+ \mu^- \).

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1. \( U(1) \) symmetries in two-doublet models

Particle physics theories involving two Englert-Brout-Higgs doublets, now denoted as \( (h_1^0, h_1^-) \) and \( (h_2^+ , h_2^0) \), allow for a possible \( U(1) \) symmetry acting as

\[
h_1 \to e^{i\alpha} h_1, \quad h_2 \to e^{i\alpha} h_2,
\]

constraining both their interaction potential and Yukawa couplings to fermions [1] \(^\dagger\). This occurs naturally within supersymmetric extensions of the standard model, which require two doublet superfields \( H_1 \) and \( H_2 \) responsible for the electroweak breaking and the generation of quark and lepton masses [2, 3]. Such a transformation may also be used as a possible way to rotate away \( CP \)-violating effects in QCD [4].

\(^\dagger\)The allowed quartic interactions in \( V(h_1, h_2) \) are \( (h_1^0 h_1^-)^2, (h_2^+ h_2^0)^2, (h_1^0 h_1^-) (h_2^+ h_2^0) \) and \( |h_1 h_2|^2 \) or equivalently \( |h_1 h_2|^2 = (h_1^0 h_1^-) (h_2^+ h_2^0) - |h_1 h_2|^2 \). Within supersymmetry they appear as electroweak gauge interactions, with [2]

\[
V_{\text{quartic}} = \frac{g^2 + g'^2}{8} (h_1^0 h_1^- - h_2^+ h_2^0)^2 + \frac{g^2}{2} |h_1 h_2|^2 .
\]
This $U(1)$ symmetry, broken through $\langle h^0 \rangle = v_1/\sqrt{2}$ with $\tan \beta = v_2/v_1$, would lead, after the Goldstone combination $\text{Im}(\cos \beta h^0_1 - \sin \beta h^0_2)$ gets eliminated by the $Z$, to a quasi-massless “axion” field

$$A = \sqrt{2} \text{Im}(\sin \beta h^0_1 + \cos \beta h^0_2),$$

if it were not broken explicitly as in [2]. This explicit breaking through $f(S)$ superpotential terms provides a mass, proportional to the $\lambda$ parameter of the $\lambda H_1 H_2 S$ superpotential coupling with the singlet $S$ introduced in [2], for the “axion” field $A$ (2) that would otherwise remain quasi massless. This extra-$U(1)$, which acts axially on quarks and leptons and will be referred to as $U(1)_A$, may also be taken (in the absence of $f(S)$ and $\mu H_1 H_2$ superpotential terms that would break it explicitly) as a gauged symmetry, leading to the USSM [3]. It is very much the same as the $U(1)$ considered in [4], excepted that anomalies should in principle be cancelled if this $U(1)$ is to be gauged. The pseudoscalar Goldstone boson eaten away in [3] to give a mass to the new neutral gauge boson of $SU(3) \times SU(2) \times U(1) \times \text{extra-}U(1)$ is very similar to the axion found later in [5, 6]. When the extra-$U(1)$ is gauged, the new gauge boson acquires its mass by eliminating a would-be “axion”.

This neutral gauge boson, referred to as $U$ (also often called $Z'$), did not show up in neutral current phenomenology nor direct searches at particle colliders. It can be made much heavier than the $W^\pm$ and $Z$, say of $\sim$ TeV scale, as the singlet described by $S$ can acquire a large v.e.v., making the $U$ contribution to neutral current effects sufficiently small [7]. The mass $m_U = g'' F_U$, on the other hand, may be small if the extra-$U(1)$ gauge coupling $g''$ is small or very small. However, even in the case of a very small gauge coupling the $U$ could still conserve sizeable interactions, as it would in fact behave very much as the eaten-away Goldstone boson $A$ in (2), in the absence of a singlet v.e.v.; or in a more general way as a doublet-singlet combination $a$, which would make it much harder to detect [7, 8]. This was used long ago to discuss the production of light spin-1 $U$ bosons or of their effectively-equivalent spin-0 pseudoscalars $a$ in the radiative decays $\psi$ and $\Upsilon \rightarrow \gamma U/a$ [8, 9].

A very light $U$ does not decouple in the limit $g'' \rightarrow 0$, but gets produced and interacts very much as the eaten-away axionlike pseudoscalar $a$ [8]. In the absence of a singlet v.e.v. this one is a mixing of $h^0_1$ and $h^0_2$ as defined by $A$ in (2), and would be produced as a standard axion, a possibility that turned out to be excluded. When the extra-$U(1)$ is broken not only by $\langle h^0_1 \rangle$ and $\langle h^0_2 \rangle$ but also by a large singlet v.e.v. $\langle s \rangle$, at a scale possibly significantly larger than the electroweak scale, the spin-1 $U$ boson is produced and interacts as the (eaten-away) axionlike pseudoscalar $a$, now given by the doublet-singlet combination

$$a = \cos \zeta \left(\sqrt{2} \text{Im}(\sin \beta h^0_1 + \cos \beta h^0_2)\right) + \sin \zeta \left(\sqrt{2} \text{Im} s\right).$$

The pseudoscalar $A$ (with the same expression (2) as for the standard axion, or $A$ of the MSSM) mixes with the singlet $s$,
uncoupled to quarks and leptons and to electroweak gauge bosons. The resulting combination $\alpha$ thus interacts essentially through its doublet component $A \cos \zeta$, proportionally to the *invisibility parameter* $r = \cos \zeta$. The branching ratios for $\psi$ or $\Upsilon \rightarrow \gamma + U/\alpha$ are essentially the same as for a standard axion $A$ [5] but multiplied by $r^2 = \cos^2 \zeta$ [8, 9, 10]. If $<s>$ is large, the $U(1)$ is broken “at a large scale”, $r = \cos \zeta$ is small, and the pseudoscalar $\alpha$ (or associated $U$ boson in case of a local $U(1)$ symmetry) becomes largely “invisible”. This mechanism can be used as well for a spin-1 $U$ boson or spin-0 axion or axionlike pseudoscalar $\alpha$ (or even also scalar), then mostly an electroweak singlet as proposed in [7], according to what was called later the “invisible axion” mechanism.

If the extra-$U(1)$ is only global (and possibly anomalous) and broken “almost spontaneously” but with small additional explicit-breaking terms, the would-be Goldstone or quasi Goldstone boson $\alpha$ acquires small mass terms, with its production rates still given by the same formulas, proportionally to $r^2 = \cos^2 \zeta$. This applies, in particular in the N/nMSSM, when the $U(1)$ symmetry considered (which may be a $U(1)_A$ commuting with supersymmetry, or an $R$-symmetry not commuting with it) is explicitly broken in this way through *small* superpotential couplings (such as $\bar{\mu}S^2$, $\bar{\sigma}S$, or $\mu H_1 H_2$) and/or *small* soft supersymmetry-breaking terms, ultimately responsible for a small mass for the pseudoscalar $\alpha$ associated with this “almost spontaneous” breaking of the global $U(1)$.

We shall be especially interested in spin-1 $U$ bosons associated with a local extra-$U(1)$ symmetry as in the USSM [3], which may decay into $\nu\bar{\nu}$ or $e^+e^-$, depending on their mass [8] (or even have dominant decays into light dark matter particles [11, 12]), light pseudoscalars being discussed in [13]. More generally we shall obtain, from $\Upsilon$ decays, new constraints on the pseudovector, pseudoscalar or scalar couplings of the new boson to the $b$ quark. They have important implications for the decay $\psi \rightarrow \gamma +$ invisible neutral, which should be very small, as well as for the couplings of the new spin-1 or spin-0 boson to charged leptons. Finally, we also discuss new constraints obtained from searches for a neutral boson decaying into $\mu^+\mu^-$, and their implications for $\psi$ decays and new boson couplings.

2. Extra $U(1)_A$ and extra singlet from supersymmetric theories

Having two Higgs doublets instead of a single one as in the standard model allows for the possibility of “rotating” them independently, thanks, in addition to the weak hypercharge $U(1)$, to the extra-$U(1)$ symmetry acting as [1]

$$h_1 = \left( \begin{array}{c} h_1^0 \\ h_1^+ \end{array} \right) \xrightarrow{U} e^{i\alpha}h_1, \quad h_2 = \left( \begin{array}{c} h_2^+ \\ h_2^0 \end{array} \right) \xrightarrow{U} e^{i\alpha}h_2,$$

(4)

embedded within supersymmetric models according to [2]

$$H_1 \xrightarrow{U} e^{i\alpha}H_1, \quad H_2 \xrightarrow{U} e^{i\alpha}H_2,$$

(5)

and broken through $<h_1^0>$ and $<h_2^0>$. The $\mu$ parameter of the $\mu H_1 H_2$ superpotential term, not invariant under this extra $U(1)$ [3] (nor under the continuous $R$-symmetry), was then promoted into a dynamical variable $\mu(x, \theta)$, with $\mu H_1 H_2$ replaced in [2] by a trilinear coupling $\lambda H_1 H_2 S$ with the extra singlet $S$, transforming under $U$ as

$$S \xrightarrow{U} e^{-2i\alpha}S.$$

(6)

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\[3\]This $U(1)_A$ symmetry is broken explicitly in the MSSM through $\mu H_1 H_2$ and by a soft susy-breaking term proportional to Re $h_1 h_2$, allowing the pseudoscalar $A$ in (2) to acquire a mass.

Note that there is no specific hierarchy problem associated with the size of the supersymmetric $\mu$ parameter, which may be kept small (compared to large masses like $m_{GUT}$ or $m_{Planck}$) by means of this (broken) extra-$U(1)_A$ symmetry. Or also by a continuous (broken) $U(1)_R$-symmetry, so that $\mu$ may be naturally of the same order as susy-breaking parameters, most notably gaugino masses $m_{1/2}$ [14] which break explicitly this continuous $U(1)_R$. 

3
This $U(1)$ symmetry acts axially on quark and lepton superfields according to [3] 4

\[
(Q, \bar{U}, \bar{D}; L, \bar{E}) \xrightarrow{U} e^{-i \frac{\pi}{3}} (Q, \bar{U}, \bar{D}; L, \bar{E}) ,
\]

and may be referred to as $U(1)_A$ 5. The superpotential $\mathcal{W}$ (with the $\mu H_1 H_2$ term of the MSSM absorbed within the trilinear $\lambda H_1 H_2 S$ superpotential) may be written in a general way (omitting family indices for simplicity) as

\[
\mathcal{W} = \lambda e H_1 \bar{E} L + \lambda_d H_1 \bar{D} Q - \lambda_u H_2 \bar{U} Q + \lambda H_1 H_2 S + \frac{\kappa}{3} S^3 + \frac{\mu_5}{2} S^2 + \sigma S .
\]

$\mathcal{W}_{lq}$, responsible for quark and lepton masses, and the $\lambda H_1 H_2 S$ superpotential coupling with the singlet $S$, are both invariant under the extra-$U(1)$ symmetry, as well as under the continuous $U(1)_R$ symmetry that led to $R$-parity, $R_0 = (-1)^F$ 6. The terms $f(S)$, which provide in the N/nMSSM 7 an explicit breaking of the extra-$U(1)$ so that there is no quasi-massless “axion”, are excluded if this $U(1)$ is gauged, as well as the direct mass term $\mu H_1 H_2$ (that may be dynamically restored from $\lambda H_1 H_2 S$ through $<s>$). Then (8) reduces to the superpotential of the USSM,

\[
\mathcal{W}_{USSM} = \mathcal{W}_{lq} + \lambda H_1 H_2 S ,
\]

the would-be “axion” being eliminated when the new gauge boson acquires its mass [3] 8.

3. Axial and pseudoscalar couplings of $U$ and $a$ to quarks and leptons

The mass $m_U = g^* F_U$ (with $F_U$ representative of new symmetry breaking scale) may be naturally small if the extra-$U(1)$ gauge coupling $g^*$ is small. One might think that the $U$ should then decouple, as the amplitudes for emitting (or absorbing) it, $\mathcal{A}(A \rightarrow B + U) = g^* (\ldots)$, seem to vanish with $g^*$. But the longitudinal polarisation vector $e_L^\nu \propto k^\nu_{lq}/m_U$ then becomes singular, so that

\[
\mathcal{A}(A \rightarrow B + U_{\text{long}}) \propto g^* \frac{k^\mu_{lq}}{m_U} < B | J_{\mu U} | A > = \frac{1}{F_U} k^\mu_{lq} < B | J_{\mu U} | A >
\]

\[\text{This also illustrates the connection between a very weakly coupled } U, \text{ with } g^* \text{ very small, and supersymmetry broken "at a large scale" with a very weakly coupled goldstino/gravitino [15], and how soft susy-breaking terms may be generated spontaneously, when the susy-breaking scale gets very large so that the goldstino decouples.}\]

\[\text{More generally the } U(1)_A \text{ symmetry considered may be replaced by a } U(1) \text{ associated with a linear combination of the } U(1)_A \text{ generator } F_A \text{ with } \alpha B + \beta L_i + \gamma Y.\]

\[\text{By "invariant under } U(1)_R \text{ symmetry" we mean that the superpotential terms } \mathcal{W}_{lq} \text{ and } \lambda H_1 H_2 S \text{ transform according to}

\[
\mathcal{W} \rightarrow e^{2i\alpha} \mathcal{W}(x, \theta e^{-i\alpha}) ,
\]

\[\text{so that their } F \text{-components, proportional to } \text{Re } \int \mathcal{W} d^2 \theta, \text{ are } R\text{-invariant. They are also invariant under any modified } U(1)_R \text{ symmetry combining the original } U(1)_R \text{ with } U(1)_A, \text{ as generated by } R' = R + c F_A \text{ (or } R + c F_A + \alpha B + \beta L_i + \gamma Y) \text{ in which } F_A \text{ is the generator of } U(1)_A.\]

\[\text{In the nMSSM the superpotential (8) is further restricted to}

\[
\mathcal{W}_{\text{nMSSM}} = \mathcal{W}_{lq} + \lambda H_1 H_2 S + \sigma S ,
\]

using the $U(1)_R$-symmetry, acting as $S \xrightarrow{R} e^{2i\alpha} S(x, \theta e^{-i\alpha})$, which restricts $f(S)$ to the linear $\sigma S$ term [2]. This superpotential already allows for electroweak breaking, even in the absence of supersymmetry-breaking terms.}

\[\text{See e.g. [17] for a recent study of neutralino dark matter in the USSM with a heavy } U \text{ boson, and [18] for a discussion of light, very weakly coupled gauge bosons in string compactifications.}\]
has a finite limit. A very light $U$ with longitudinal polarisation couples proportionally to $f_{V,A} k_U^a / m_U$, where $k_U^a$ acting on an axial current $\bar{f} \gamma^\mu \gamma_5 f$ resurrects an effective pseudoscalar coupling to $\bar{f} \gamma_5 f$ with a proportionality factor $2 m_f$. A light $U$ has thus effective pseudoscalar couplings, in particular to quarks and leptons, given in terms of original axial ones by

$$f_{q,l} = f_{q,l}^A \frac{2 m_{q,l}}{m_U}. \quad (11)$$

This equivalence theorem ensures that a light $U$ with non-vanishing axial couplings to fermions behaves very much as the “eaten-away” pseudoscalar $a$ \cite{8} $^9$ $^{10}$. This is perfectly analogous to what happens for a light spin-$\frac{3}{2}$ gravitino, whose $\pm \frac{1}{2}$ polarisation states, although coupled with gravitational strength $\propto \kappa$, continue to behave very much as a spin-$\frac{1}{2}$ goldstino, according to the equivalence theorem of supersymmetry, and with a strength inversely proportional to the supersymmetry-breaking scale parameter \cite{15} $^{11}$.

The couplings of $h_0^a$ and $h_2^a$ to quarks and leptons are $m \sqrt{2}/(v \cos \beta)$ and $m \sqrt{2}/(v \sin \beta)$. The pseudoscalar couplings of $A$ in (2) are thus $(m/v) \times (\tan \beta = 1/x)$ for charged leptons and down quarks, and $(m/v) \times (\cot \beta = x)$ for up quarks, acquiring masses through $h_1$ and $h_2$, respectively. With $v = 2^{-1/4} G_F^{-1/2} \approx 246 \text{ GeV}$ we get the pseudoscalar couplings of the standard axion (or $A$ of the MSSM), $2^4 G_F^4 \frac{1}{2} m_{q,l} \times (\tan \beta$ or $\cot \beta)$. When $A$ in (2) mixes with the imaginary part of $s$ into expression (3) of the pseudoscalar $a$ associated with the extra-$U(1)$ breaking, we get the following pseudoscalar (or effective pseudoscalar) couplings, now also proportional the invisibility parameter $r = \cos \zeta$,

$$f_{q,l} \propto \frac{2^4 G_F^4 \frac{1}{2} m_{q,l}}{4 \times 10^{-6} m_{q,l}(\text{MeV})} \times \left\{ \begin{array}{ll}
r x = \cos \zeta \cot \beta & \text{for } u, c, t \text{ quarks,} \\
r x = \cos \zeta \tan \beta & \text{for } d, s, b \text{ quarks and } e, \mu, \tau \text{ leptons.} \\
\end{array} \right. \quad (12)$$

These couplings may also be determined from the spin-1/spin-0 equivalence \cite{8}, from the axial couplings of the $U$ when the extra-$U(1)$ symmetry is realized locally. The $U$ current is obtained from the initial extra-$U(1)$ current, with an additional contribution proportional to $J_Z = J_3 - \sin^2 \theta \ J_\text{em}$ originating from $Z - U$ mixing effects, typically induced by $v_1$ and $v_2$ when $\tan \beta \neq 1$ \cite{8, 10, 20}. This leads to the axial couplings of the $U$ to quarks and leptons $^{12}$.

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$^9$The $U(1)$ coupling $g^\prime$ may be taken as small or very small, and the $U$ related (in part) to the gravitino as it participates in spontaneous supersymmetry breaking through a non-vanishing $<D>$, contributing in particular to the mass$^2$-splittings $m_\chi^2$ for squarks and sleptons \cite{16}. The spin-1 $U$ boson, which has eaten away the axionlike pseudoscalar $a$, is then partly related to the spin-$\frac{1}{2}$ goldstino eaten away by the spin-$\frac{3}{2}$ gravitino, partner of the spin-2 graviton \cite{7, 8}. Due to this relation with gravity, $g^\prime$ appears as $\propto \kappa$ and possibly very small.

$^{10}$Anomalies associated with the extra-$U(1)$ should in principle be cancelled (using e.g. mirror fermions or $E_6$-like representations, ... ) if it is to be gauged, and we assume this is realized. However, due to the relation with gravity, with $g^\prime$ possibly very small, the cancellation of anomalies may not be necessary within the low-energy field theory \cite{7}, and could involve other sectors related to gravity or strings (see e.g. \cite{19} for a related discussion).

$^{11}$Its effective interactions are $\propto \kappa/m_{3/2}$ i.e. inversely proportional to $d$ or equivalently $A_{3/2}^2$. They may or may not be very small, depending on the scale ($\Lambda_{3/2}$) at which supersymmetry is spontaneously broken. With

$$m_{3/2} = \kappa d / \sqrt{6} = \kappa F / \sqrt{3} = \sqrt{8 \pi G_N / 3} \ F,$$

one has $A_{3/2} = \sqrt{F} = (\frac{7}{3})^{1/4} \sqrt{m_{3/2} m_{\text{Planck}}}$. A large $d$ (i.e. supersymmetry spontaneously broken “at a large scale” $\Lambda_{3/2}$ with a very weakly coupled goldstino component of the gravitino) is then naturally connected with a very small $g^\prime$, corresponding to a very weakly coupled $U$ boson.

$^{12}$The vector couplings of the $U$ are usually expressed as a linear combination of the $B$ and $L$ (or $B - L$) and electromagnetic currents \cite{20}, and may contribute to invisible meson decays, such as those of the $\pi^0$, $\eta$, $\eta'$, $\psi$ or $\Upsilon$ \cite{12, 21, 22}. (Note that $\psi$ and $\Upsilon$ cannot decay invisibly into dark matter particles, according to $\psi(\Upsilon) \rightarrow \chi \chi$, through the virtual production of a spin-0 boson \cite{12}).
\[ f_{q,l} A \simeq \frac{2^{-\frac{3}{2}}}{2 \times 10^{-6} m_U (\text{MeV})} G_F \frac{1}{2} m_{q,l} \times \begin{cases} \frac{r x}{e^2} = \cos \zeta \cot \beta & \text{for } u, c, t \text{ quarks}, \\ \frac{r / x}{e^2} = \cos \zeta \tan \beta & \text{for } d, s, b \text{ quarks and } e, \mu, \tau \text{ leptons}. \end{cases} \] (13)

Using (11) we recover in this way the effective pseudoscalar couplings (12) of the \( U \), the same as for a standard axion or pseudoscalar \( A \) in the MSSM, multiplied by the invisibility factor \( r = \cos \zeta \).

4. New constraints from \( \Upsilon \rightarrow \gamma + \text{ invisible neutral} \), and their consequences

The quarkonium branching ratios, obtained using (13) or (12) which introduce \( r = \cos \zeta \), may be expressed from the ratios of the pseudoscalar (or effective pseudoscalar) couplings to the quarks \( f_{q,P} \) to the elementary charge \( e \) [5, 8]. They are given at lowest order, disregarding \( m_U \) or \( m_a \), by

\[ \frac{B(\text{onium} \rightarrow \gamma U/a)}{B(\text{onium} \rightarrow \mu^+ \mu^-)} = \frac{2 f_{q,P}^2}{e^2} = \frac{G_F m_a^2}{\sqrt{2} \pi \alpha} (r^2 x^2 \text{ or } r^2 \bar{x}^2). \] (14)

The branching ratios, given by

\[ \begin{align*}
\frac{B(\psi \rightarrow \gamma U/a)}{B(\psi \rightarrow \mu^+ \mu^-)} &= \frac{G_F m_a^2}{\sqrt{2} \pi \alpha} r^2 x^2 C_\psi F_\psi \simeq 8 \times 10^{-4} r^2 x^2 C_\psi F_\psi, \\
\frac{B(\Upsilon \rightarrow \gamma U/a)}{B(\Upsilon \rightarrow \mu^+ \mu^-)} &= \frac{G_F m_a^2}{\sqrt{2} \pi \alpha} r^2 \bar{x}^2 C_\Upsilon F_\Upsilon \simeq 8 \times 10^{-3} r^2 \bar{x}^2 C_\Upsilon F_\Upsilon,
\end{align*} \] (15)

and expressed in terms of \( \cos \zeta \cot \beta = r x \) and \( \cos \zeta \tan \beta = r / x \), are approximately equal to [8, 9, 10]:

\[ \begin{align*}
B(\psi \rightarrow \gamma U/a) &\simeq 5 \times 10^{-5} \cos^2 \zeta \cot^2 \beta C_\psi F_\psi, \\
B(\Upsilon \rightarrow \gamma U/a) &\simeq 2 \times 10^{-4} \cos^2 \zeta \tan^2 \beta C_\Upsilon F_\Upsilon.
\end{align*} \] (16)

\( C_\psi \) and \( C_\Upsilon \) take into account QCD radiative and relativistic corrections [23], and are usually expected to be larger than 1/2. \( F \) is a phase space factor, equal to \( 1 - \frac{m_a^2}{m_\psi^2} \) or \( 1 - \frac{m_a^2}{m_\Upsilon^2} \), with \( 1 - \frac{m_a^2}{m_\psi^2} = 2 E_\gamma/m_\psi \).
ψ decays:

For a U with mostly invisible decays (into ν̅ν or light dark matter particles), we get, from \( B(ψ → γ + \text{invisible}) < 1.4 \times 10^{-5} \), \( rx = \cos ζ \cot β < .75 \) [10, 12, 24], which requires \( |f_{cA}| < 1.5 \times 10^{-6} m_U(\text{MeV}) \) for a pseudovector coupling, or \( |f_{cP}| < 5 \times 10^{-3} \) for the pseudoscalar coupling of a massless or light spin-0 boson a that would not decay visibly within the detector. Very much the same limit, but taken as \( |f_{cS}| < 10^{-2} \) as QCD corrections may now be somewhat larger [23], applies, in a similar way [25], to the coupling of a massless or light scalar boson with invisible decays. If the invisible branching ratio \( B_{\text{inv}} \) (within the detector) is not \( ∼ 1% \), the limits get divided by \( \sqrt{B_{\text{inv}}} \), so that

\[
rx = \cos ζ \cot β < .75/\sqrt{B_{\text{inv}}} \iff |f_{cA}| < 1.5 \times 10^{-6} m_U(\text{MeV})/\sqrt{B_{\text{inv}}} , \quad \text{or } |f_{cP}| < 5 \times 10^{-3}/\sqrt{B_{\text{inv}}} . \tag{17}
\]

Υ decays, and consequences for the ψ:

The experimental limit on \( \Upsilon → γ + \text{invisible} \) [26] got improved with the Υ(3S) by more than 4 by the BABAR coll. [27], with a preliminary upper limit increasing from 3.2 to 3.5 \( 10^{-6} \) when the mass of the unobserved neutral grows from 0 to 1 GeV, down to .7 \( 10^{-6} \) for 3 GeV, and less than 4 \( 10^{-6} \) for any mass up to 6 GeV. We then get the new limits

\[
r/x = \cos ζ \tan β < .2/\sqrt{B_{\text{inv}}} \iff |f_{bA}| < 4 \times 10^{-7} m_U(\text{MeV})/\sqrt{B_{\text{inv}}} , \quad \text{or } |f_{bP}| < 4 \times 10^{-3}/\sqrt{B_{\text{inv}}} , \tag{18}
\]

which take into account the invisible branching ratio of the new boson. This remains valid for a new particle mass of up to about 5 GeV (taking into account the phase space factor \( F_{\Upsilon} \)), as long as invisible decay modes are present. The limit (18) on the pseudoscalar (or effective pseudoscalar) coupling \( f_{bP} \) is 5 times smaller than the standard Higgs coupling to \( b, m_b/v \sim 2 \times 10^{-2} \), for an invisibly decaying boson. It may also be applied to a scalar coupling \( f_{bS} \) provided it is slightly relaxed, to \( |f_{bS}| < 6 \times 10^{-3}/\sqrt{B_{\text{inv}}} \).\(^{13}\)

From \( r^2 = \cos^2 ζ < .6 \tan^2 β /B_{\text{inv}} \), and \( < 4 \times 10^{-2} \cot^2 β /B_{\text{inv}} \), we get the upper limit independent of \( β \) on the invisibility parameter,

\[

r^2 = \cos^2 ζ < .15 /B_{\text{inv}} , \tag{19}
\]

so that a should be mostly singlet (\( > 85% \)), rather than doublet (\( < 15% \)), for invisible decays of the new boson. The Υ limit, expressed as a constraint on the doublet fraction

\[
doublet fraction: \quad r^2 = \cos^2 ζ < 4% / (\tan^2 β B_{\text{inv}}) \tag{20}
\]

is stronger than the ψ one for \( \tan β \) larger than \( ∼ .5 \). It requires that a should be (\( < 4% \) doublet, > 96% singlet) for \( \tan β > 1 \); and \( < .5% \) doublet for \( \tan β > 3 \), for invisible decays of the new boson.

The dependence on \( B_{\text{inv}} \) disappears when we evaluate the upper limit for the production, in radiative decays of the ψ, of a new boson decaying invisibly. The non-observation of a signal in \( \Upsilon → γ + \text{invisible neutral} \) decays implies a rather small branching ratio for the similar decay of the ψ,

\[
B (ψ → γ + \text{neutral}) \ B_{\text{inv}} \lesssim 10^{-6}/\tan^4 β , \tag{21}
\]

i.e. \( \lesssim 10^{-8} \) for \( \tan β \gtrsim 3 \), independently of the invisible branching ratio \( B_{\text{inv}} \) (a result also applicable, with little change, to the production of a scalar particle).

\(^{13}\)Considering that in the scalar case the correction factor \( C_γ \) should be larger than \( ∼ .2 \), instead of .5, leads to relax the bound by \( ∼ 1.6 \).
Constraints from $\Upsilon$ decays on the couplings to electrons:

$\Upsilon$ results also have implications on the couplings of the new spin-1 or spin-0 boson to the electron, muon or $\tau$ lepton. Eq. (13) implies universality properties for the axial couplings of the $U$, family-independent and identical for all charged leptons and down quarks [10]. This is also a consequence of the gauge invariance of the Yukawa couplings responsible for their masses in a 2-Higgs-doublet model [20], leading to $f_{eA} = f_{\mu A} = f_{\tau A} = f_{dA} = f_{sA} = f_{bA}$. It also reflects that the corresponding couplings of the pseudoscalar $a$ to down-quarks and charged leptons are proportional to their masses (as expressed by (12)), so that

$$f_{eP} = f_{bP} \frac{m_e}{m_b} .$$  \hfill (22)

The new strong limit (18) on $f_{bA}$ then applies also to $f_{eA}$, severely restricting it to

$$|f_{eA}| < 4 \times 10^{-7} \frac{m_U(\text{MeV})}{\sqrt{B_{\text{inv}}}} , \quad \text{or} \quad |f_{eP}| < 4 \times 10^{-7} / \sqrt{B_{\text{inv}}} .$$  \hfill (23)

The last limit on a pseudoscalar coupling $f_{eP}$ is 5 times smaller than the standard Higgs coupling to the electron, $m_e/v \simeq 2 \times 10^{-6}$, for invisible decays of the new boson. As scalar couplings are also proportional to masses, with $h_1$ alone responsible for charged-lepton and down-quark masses so that

$$f_{eS} = f_{bS} \frac{m_e}{m_b} ,$$  \hfill (24)

the last limit on $|f_{eP}|$ may also be applied to a scalar coupling provided it is slightly relaxed, as for $|f_{bP}|$, the effect of radiative corrections in $\Upsilon$ decays being larger in this case, so that

$$|f_{eS}| < 6 \times 10^{-7} / \sqrt{B_{\text{inv}}} .$$  \hfill (25)

For a spin-1 $U$ boson the strong limit (23) on $f_{eA}$ is in agreement with the results of parity-violation experiments in atomic physics, which imply a strong limit on $|f_{eA} f_{\gamma U}|$ [28]. It has implications on the size of the $e^+ e^- \rightarrow \gamma U$ annihilation cross section, roughly proportional to $f_{eV}^2 + f_{eA}^2$, which should then be very small for a light $U$, unless its vector coupling to the electron is significantly larger than the axial one.

5. Comparison with constraints from $\Upsilon \rightarrow \gamma + \text{(neutral} \rightarrow \mu^+ \mu^-) \text{ decays}$

The analysis applies as well to a relatively light spin-1 $U$ boson or spin-0 pseudoscalar $a$, or scalar, decaying visibly for example into $\mu^+ \mu^-$ with a branching ratio $B_{\mu \mu}$. This was searched for recently by the CLEO [29] and BABAR [30] collaborations with spin-0 particles in mind, but the results may be used to constrain light spin-1 $U$ bosons as well, given the quasi-equivalence between their production rates in radiative $\Upsilon$ decays, pointed out long ago [7, 8, 9] with the pseudoscalar $a$ already a mixing of doublet ("active") and singlet ("inert") components. This is particularly relevant as many theoretical constructions now appeal to such light weakly-coupled neutral bosons.

With $B(\Upsilon \rightarrow \gamma + \text{neutral} \rightarrow \mu^+ \mu^-) B_{\mu \mu}$ taken to be $\lesssim 2 \times 10^{-6}$ in most of the mass range considered (and always less than $5 \times 10^{-6}$ at 90\% c.l. up to nearly 9 GeV, excepted for two small regions around the $\psi$ and $\psi'$), as compared to $B(\Upsilon \rightarrow \gamma + \text{neutral}) B_{\text{inv}} \lesssim 3.5 \times 10^{-6}$, we can rescale (18) into $r/x = \cos \zeta \tan \beta \lesssim .15/\sqrt{B_{\mu \mu}}$, so that

$$|f_{bA}| \lesssim 3 \times 10^{-7} \frac{m_U(\text{MeV})}{\sqrt{B_{\mu \mu}}} , \quad |f_{bP}| \lesssim 3 \times 10^{-3} / \sqrt{B_{\mu \mu}} , \quad \text{or} \quad |f_{bS}| \lesssim 5 \times 10^{-3} / \sqrt{B_{\mu \mu}} .$$  \hfill (26)

With a more conservative experimental upper limit $\lesssim 4 \times 10^{-6}$, the above limits should be slightly relaxed, to

$$r/x = \cos \zeta \tan \beta \lesssim .2 / \sqrt{B_{\mu \mu}} \quad \iff \quad |f_{bA}| \lesssim 4 \times 10^{-7} \frac{m_U(\text{MeV})}{\sqrt{B_{\mu \mu}}} , \quad |f_{bP}| \lesssim 4 \times 10^{-3} / \sqrt{B_{\mu \mu}} , \quad \text{or} \quad |f_{bS}| \lesssim 6 \times 10^{-3} / \sqrt{B_{\mu \mu}} .$$  \hfill (27)
These limits may or may not be more constraining than (18), depending on whether or not \( B_{\mu\mu} \) is larger than \( \approx B_{\text{inv}} \). As an illustrative example a 1 GeV \( U \) boson could have \( B_{\text{inv}} \approx 16\% \) and \( B_{\mu\mu} \approx 10\% \), if we ignore the possibility of light dark matter particles [11] which could make \( B_{\text{inv}} \) very close to 1. See also [31] for a discussion of the upper limit on \( \cos \zeta \tan \beta \) for a spin-0 pseudoscalar \( \alpha \) in the NMSSM, using CLEO results [29].

Eq. (26) translates into

\[
doublet \text{ fraction: } r^2 = \cos^2 \zeta \lesssim 2\% / (\tan^2 \beta B_{\mu\mu}) \ .
\]

The dependence on \( B_{\mu\mu} \) disappears when we evaluate the upper limit for the production, in radiative decays of the \( \psi \), of a new spin-1 or spin-0 boson decaying into \( \mu^+\mu^- \). The non-observation of a signal in \( \Upsilon \rightarrow \gamma + (\text{neutral} \rightarrow \mu^+\mu^-) \) decays thus also implies a small branching ratio in the similar decay of the \( \psi \) (very much as we saw in (21) for the invisible decays):

\[
B \left( \psi \rightarrow \gamma + \text{neutral} \right) B_{\mu\mu} \lesssim 5 \times 10^{-7} / \tan^4 \beta \ , \tag{29}
\]

i.e. \( \lesssim 5 \times 10^{-9} \) for \( \tan \beta \gtrsim 3 \), independently of \( B_{\mu\mu} \), a result also applicable to the production of a scalar particle. Again limits on \( b \) couplings may be translated into limits on the pseudovector, pseudoscalar or scalar couplings to the electron, leading, very much as in (23,25), to

\[
|f_{eA}| \lesssim 3 \times 10^{-7} m_U (\text{MeV}) / \sqrt{B_{\mu\mu}} \ , \quad |f_{eP}| \lesssim 3 \times 10^{-7} / \sqrt{B_{\mu\mu}} \quad \text{or} \quad |f_{eS}| \lesssim 5 \times 10^{-7} / \sqrt{B_{\mu\mu}} \ . \tag{30}
\]

6. Conclusions

Theories with 2 Higgs doublets may allow for a broken extra-\( U(1) \) symmetry, local or global, acting axially on quarks and leptons, and leading to new light neutral spin-1 or spin-0 bosons. This occurs naturally in supersymmetric extensions of the standard model with a trilinear \( \lambda H_1 H_2 S \) superpotential. The extra \( U(1) \) may be gauged, as in the USSM, and the new spin-1 boson \( U \), which eliminates an axionlike pseudoscalar \( \alpha \), could be light if the corresponding gauge coupling is small. This may have a more profound origin with a possible connection of the extra-\( U(1) \) and \( U \) boson with the gravitino and gravity itself, which would not be so surprising as the pseudoscalar \( \alpha \), and associated scalar partner, interact proportionally to masses.

\( \Upsilon \) decays constrain an axial, pseudoscalar or scalar coupling to the \( b \) to \( f_{bA} < 4 \times 10^{-7} m_U (\text{MeV}) / \sqrt{B_{\mu\mu}} \), \( f_{bP} < 4 \times 10^{-3} / \sqrt{B_{\mu\mu}} \) (5 times less than the standard Higgs coupling, for invisible decays of the new boson) or \( f_{bS} < 6 \times 10^{-3} / \sqrt{B_{\mu\mu}} \), respectively, also constraining strongly their couplings to the electron (to e.g. \( < 4 \times 10^{-7} / \sqrt{B_{\mu\mu}} \) for a pseudoscalar). Similar limits have been obtained from searches for \( \Upsilon \rightarrow \gamma + (\text{neutral} \rightarrow \mu^+\mu^-) \), for \( m_{\text{neut.}} > 2 m_\mu \), which, altogether, strongly constrain the rates for both \( \psi \rightarrow \gamma + \text{invisible neutral} \) and \( \psi \rightarrow \gamma + (\text{neutral} \rightarrow \mu^+\mu^-) \).

The results apply, generically, in a large class of theoretical models involving an extra-\( U(1) \) symmetry, either global or local, and whether supersymmetry is present or not, even if 2-higgs doublet susy extensions of the standard model (N/nMSSM, USSM, ...) provide the most natural framework and best motivation. They are relevant for a variety of experimental searches, including quarkonium decays (e.g. at BES III or with a super B factory [22, 32]) and other experiments relying on the coupling of the new neutral boson to the electron, muon or \( \tau \) lepton.

The search for light weakly coupled particles such as goldstinos/gravitinos, \( U \) bosons, axions, axionlike or dilatonlike particles, ..., constitutes a direction to be further explored, in complement of the high-energy frontier at the Tevatron, LHC, and ILC. This may also contribute to the understanding of high-energy physics, with the very weak couplings of such light particles in close relation with the mass spectrum.
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