Floquet engineering of the Luttinger Hamiltonian

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During last years, the control of electronic parameters of condensed-matter structures by a high-frequency electromagnetic field (so-called “Floquet engineering” based on the Floquet theory for periodically driven quantum systems) became the important and established research area which resulted in the discovery of many fundamental effects (see, e.g., Refs. 1–6). It is well known that absorption of an electromagnetic field by an electron system takes place if only a characteristic electronic frequency coincides with a field frequency (resonant field). However, even if the field is non-resonant and cannot be absorbed, it still interacts with electrons. Formally, this non-resonant interaction is described by the nonstationary Schrödinger equation with a periodically time-dependent potential. Solution of this equation is the Floquet wave function which is periodic in time with period of the field. If the field frequency is high enough (i.e. its frequency is much larger than characteristic frequencies of an electronic system), all electronic characteristics obtained with using the Floquet function can be averaged over the field period. As a result, properties of electrons “dressed” by the field can be described by the quantum dynamics equations which are similar to the equations for “bare” electrons but depend on field parameters. As a consequence, behavior of dressed electrons can be considered by analogy with the behavior of bare electrons, stationary physical parameters of which (energy spectrum, effective mass, etc.) are renormalized by the field. Therefore, the theory of renormalization of electronic properties of any structure by a high-frequency field (Floquet engineering) is based on solution of the Floquet problem for the corresponding nonstationary Schrödinger equation.

Historically, investigations of the processes of interaction of electrons with a high-frequency electromagnetic field, which lead to stationary renormalization of physical properties of electronic systems by the field, have started in the middle of XX century. For a long time, the main objects of the investigations were atomic and molecular systems. In particular, the first investigations of non-resonant interaction of electrons with a strong high-frequency field were carried out for isolated atoms in the 50s years of the XX century and lead to observation of atomic energy levels shift caused by the field (the Autler-Townes effect). As to investigations of these effects in solid state structures, they have started in the 60s with the works done by Galitsky, Goreslavsky and Elesin, who theoretically predicted the existence of light-induced band gaps in energy spectrum of semiconductors, which later were observed experimentally. Their pioneering ideas about light-induced modification of band electronic structure of solids were developed later theoretically and experimentally for various crystal structures. However, effects of electromagnetic renormalization of electronic properties of solids were ignored as rule in the most studies for a long time because the scattering of conduction electrons significantly obstructs experimental investigations of them. Situation has changed when it became possible to fabricate solid state structures with very high charge carrier mobility and, correspondingly, with weak electron scattering. As a consequence, during last decade were published many works dedicated to the Floquet engineering of various solid state structures, including quantum rings, quantum wells, topological insulators, graphene and related 2D materials, etc.

Among various condensed-matter structures important to both fundamental science and device applications, it should be noted especially those of them which are based on conventional semiconductor materials (Si, Ge and A\textsubscript{3}B\textsubscript{5} semiconductors) and gapless semiconductors (HgTe and related materials). Particularly, the most of modern nanostructures are fabricated with using them. Since valence band of the conventional semiconductors and band structure of the gapless semiconductors near the band edge (the electronic term \Gamma\textsubscript{5} in the \Gamma point of the Brillouin zone) are described by the well-known Luttinger Hamiltonian, it is necessary to develop the con-
sistent Floquet theory for electronic systems described by the Hamiltonian in order to control electronic properties of the corresponding semiconductor structures by a high-frequency electromagnetic field. The present work is dedicated to solving this theoretical problem.

The article is organized as follows. In Section II, we solve the Floquet problem for the nonstationary Schrödinger equation based on the Luttinger Hamiltonian in the presence of a high-frequency field. In Section III, we calculate electron dispersion of the Luttinger Hamiltonian modified by the field and discuss possible experimental manifestations of the field-induced renormalization of electronic properties. The last sections of the article contain conclusions and acknowledgments.

II. MODEL

Let us consider a semiconductor material with the electron energy spectrum described by the Luttinger Hamiltonian (the electronic term $\Gamma_8$), which is irradiated by a plane electromagnetic wave with the frequency $\omega$ and the electric field amplitude $E_0$. Assuming size of the irradiated semiconductor sample along the direction of the wave propagation, $d$, to be much larger than the interatomic distance and much less than the wave length, $\lambda = 2\pi c/\omega$, we can neglect the size quantization of electron energy spectrum of the sample and consider the wave field inside the sample as uniform. Then electronic states of the irradiated semiconductor sample near the center of the Brillouin zone (the electronic term $\Gamma_8$) within the conventional minimal coupling approach can be described by the time-dependent Hamiltonian,

$$\hat{H}(k, t) = \hat{H}_L(k - eA(t)/\hbar),$$

(1)

where $\hat{H}_L(k)$ is the Luttinger Hamiltonian, $k = (k_x, k_y, k_z)$ is the electron wave vector, and $A(t) = (A_x, A_y, A_z)$ is the vector potential of the wave inside the semiconductor, which periodically depends on the time, $t$. In the present analysis, we will restrict the consideration to the isotropic approximation of the electron dispersion in the semiconductor. Then the Luttinger Hamiltonian takes the form

$$\hat{H}_L(k) = (\gamma_1 + 5\gamma/2) k^2 - 2\gamma(kJ)^2,$$

(2)

where $\gamma = (2\gamma_2 + 3\gamma_3)/5$, $\gamma_{1,2,3}$ are the Luttinger parameters, and $J_{x,y,z}$ are the $4 \times 4$ matrices corresponding to the electron angular momentum $J = 3/2$. To perform calculations, it is convenient to rewrite the Hamiltonian as a $4 \times 4$ matrix in the basis of Luttinger-Kohn wave functions, $\psi_{jz}$, which describe four-fold degenerate electron states of the conduction and valence band in the center of the bulk Brillouin zone (the $\Gamma$ point), and correspond to the four different projections of electron momentum on the $z$ axis, $j_z = \pm 1/2$ and $j_z = \pm 3/2$ (see for more details, e.g., the appendix in Ref. [3]). In this basis, the Hamiltonian [2] reads

$$\hat{H}(k) = \begin{pmatrix} +3/2 & F & H^* & I \\ -1/2 & I^* & 0 & G \\ +1/2 & G & 0 & I \\ -3/2 & I^* & -H^* & F \end{pmatrix},$$

(3)

where the matrix elements are

$$F = (\gamma_1 + \gamma)(k_x^2 + k_y^2) + (\gamma_1 - 2\gamma)k_z^2,$$

$$G = (\gamma_1 - \gamma)(k_x^2 + k_y^2) + (\gamma_1 + 2\gamma)k_z^2,$$

$$I = -\sqrt{3}\gamma(k_x - ik_y)^2,$$

$$H = -2\sqrt{3}\gamma(k_x - ik_y)k_z.$$

(4)

In the following, we will demonstrate that electronic properties of an irradiated semiconductor substantially depend on polarization of the electromagnetic wave. Therefore, it is convenient to analyze the Hamiltonian [3] for the linear and circular polarizations separately.

Linear polarization. Let the electromagnetic wave propagates along the $x$ axis and is linearly polarized along the $z$ axis (see Fig. 1a). Then its vector potential inside the semiconductor can be written as

$$A(t) = \left(0, 0, \frac{E_0}{\omega} \cos \omega t\right).$$

(5)

FIG. 1: Sketch of the semiconductor sample irradiated by an electromagnetic wave with different polarizations: (a) linear polarization; (b) circular polarization.

To simplify calculations, let us subject the Hamiltonian [1] with the vector potential [5] to the unitary transform-
where \( \phi = -3\theta/2 + \pi/4 \), \( \varphi = -\theta/2 - \pi/4 \), \( \theta \) is the polar angle of the electron wave vector in the \( (x,y) \) plane and the electron wave vector as a function of the angle \( \theta \) reads

\[
k = (k_x, k_y, k_z) = \left( \sqrt{k_x^2 + k_y^2 \cos \theta}, \sqrt{k_x^2 + k_y^2 \sin \theta}, k_z \right).
\]

Then the transformed Hamiltonian \( \hat{H}'(k,t) = \hat{U}^{-1} \hat{H}(k,t) \hat{U} \), takes the block-diagonal form,

\[
\hat{H}'(k,t) = \begin{bmatrix} \hat{H}(1)(k,t) & 0 \\ 0 & \hat{H}(2)(k,t) \end{bmatrix},
\]

where the 2 \times 2 matrices are

\[
\hat{H}(1)(k,t) = \begin{bmatrix} F & \tilde{M} \\ \tilde{M}^* & G \end{bmatrix}, \quad \hat{H}(2)(k,t) = \begin{bmatrix} \tilde{G} & -\tilde{M} \\ -\tilde{M}^* & F \end{bmatrix},
\]

and the matrix elements of the Hamiltonian are

\[
\begin{aligned}
F &= F + (\gamma_1 - 2\gamma) \left( \frac{eE_0}{\hbar \omega} \cos \omega t - 2k_z \right), \\
G &= G + (\gamma_1 + 2\gamma) \left( \frac{eE_0}{\hbar \omega} \cos \omega t - 2k_z \right), \\
\tilde{M} &= \frac{\gamma}{|\gamma|} |I| + i \frac{\gamma}{|\gamma|} |H| k_z - \left( \frac{eE_0}{\hbar \omega} \right) \cos \omega t.
\end{aligned}
\]

In the most general form, the nonstationary Schrödinger equation for an electron in a periodically time-dependent field with the frequency \( \omega \) can be written as \( i\hbar \partial_t \psi(t) = \hat{H}(t) \psi(t) \), where \( \hat{H}(t+T) = \hat{H}(t) \) is the periodically time-dependent Hamiltonian and \( T = 2\pi/\omega \) is the field period. It follows from the Floquet theorem that solution of the Schrödinger equation is the Floquet function, \( \psi(t) = e^{-i\epsilon t/\hbar} \varphi(t) \), where \( \varphi(t + T) = \varphi(t) \) is the periodically time-dependent function and \( \epsilon \) is the electron (quasi)energy describing behavior of the electron in the periodic field. The Floquet problem is aimed to find the electron energy spectrum, \( \epsilon(k) \). Let us write down the problem for the Hamiltonian (7).

The two Hamiltonians (8) describe the two spin-degenerated electron states of the semiconductor with the same energy, \( \epsilon(k) \). Therefore, one can consider any of the two Hamiltonians \( \hat{H}'(1,2)(k,t) \) to find the sought energy spectrum of the irradiated semiconductor, \( \epsilon(k) \). For definiteness, let us restrict the consideration by the \( 2 \times 2 \) matrix Hamiltonian \( \hat{H}(1)(k,t) \). The Floquet problem with this Hamiltonian can be solved accurately at \( k = 0 \). Namely, the Hamiltonian \( \hat{H}(1)(k,t) \) at \( k = 0 \) reads

\[
\hat{H}_0^{(1)}(t) = \left( \frac{eE_0}{\hbar \omega} \right)^2 \cos^2 \omega t \begin{bmatrix} \gamma_1 - 2\gamma & 0 \\ 0 & \gamma_1 + 2\gamma \end{bmatrix}.
\]

Exact solutions of the Schrödinger equation with the Hamiltonian (9), \( i\hbar \partial_t \Phi_{\pm} = \hat{H}_0^{(1)}(t) \Phi_{\pm} \), are the two Floquet functions \( \Phi_{\pm} = \exp(-i\epsilon_{\pm} t/\hbar)\varphi_{\pm} \), where

\[
\begin{aligned}
\varphi_- &= e^{-i[(\gamma_1 - 2\gamma)/4\hbar \omega] (eE_0/\hbar \omega)^2 \sin 2\omega t}, \\
\varphi_+ &= e^{-i[(\gamma_1 + 2\gamma)/4\hbar \omega] (eE_0/\hbar \omega)^2 \sin 2\omega t},
\end{aligned}
\]

are the eigenspinors of the Floquet problem with the Hamiltonian (9) and the corresponding electron energies at \( k = 0 \) are

\[
\epsilon_{\pm} = \frac{\gamma_1 \pm 2\gamma}{2} \left( \frac{eE_0}{\hbar \omega} \right)^2.
\]

As to the Floquet problem with the Hamiltonian \( \hat{H}(1)(k,t) \) for \( k \neq 0 \), it can be solved approximately for small electron wave vectors, \( k \), as follows. In the new orthonormal basis (10), the Hamiltonian \( \hat{H}(1)(k,t) \) reads

\[
\hat{H}'(1)(k,t) = \begin{bmatrix} F & F \tilde{M} \\ M^* & G \end{bmatrix},
\]

where its matrix elements are

\[
\begin{aligned}
F &= \epsilon_- + F - 2(\gamma_1 - 2\gamma)k_z \left( \frac{eE_0}{\hbar \omega} \right) \cos \omega t, \\
G &= \epsilon_+ + G - 2(\gamma_1 + 2\gamma)k_z \left( \frac{eE_0}{\hbar \omega} \right) \cos \omega t, \\
M &= \tilde{M} e^{-i\eta \sin 2\omega t},
\end{aligned}
\]

and \( \eta = (\gamma/\hbar \omega)(eE_0/\hbar \omega)^2 \). To find the sought energy spectrum, \( \epsilon(k) \), in the high-frequency limit, one can apply the conventional Floquet-Magnus approach to turn the time-dependent Hamiltonian (12) into the effective time-independent Hamiltonian \( \hat{H}_{\text{eff}}(k) \) written as a series expansion in powers of \( (1/\omega) \). Omitting the terms \( \sim (\gamma k^2/\hbar \omega)^n \) with \( n > 0 \) in the expansion, we arrive at the effective time-independent Hamiltonian describing the electron energy spectrum, \( \epsilon(k) \),

\[
\hat{H}_{\text{eff}}(k) = \frac{1}{T} \int_0^T \hat{H}(1)(k,t) dt,
\]

which is written in the zeroth order of the Floquet-Magnus expansion for small electron wave vectors, \( k \), satisfying the condition \( |\gamma| k^2 \ll \hbar \omega \). Using the well-known Jacobi-Anger expansion, \( e^{i\alpha \sin \gamma} = \sum_{n=-\infty}^{\infty} J_n(\alpha) e^{in\gamma} \), to transform the exponential factor in the Hamiltonian (12), the effective Hamiltonian (13) can be rewritten in the explicit form as

\[
\hat{H}_{\text{eff}}(k) = \begin{bmatrix} F & \tilde{M} \left[ |I| + i \frac{k_z}{|k_z|} |H| \right] J_0(\eta) \\ \tilde{M}^* \left[ |I| - i \frac{k_z}{|k_z|} |H| \right] J_0(\eta) & G + \epsilon_+ \end{bmatrix},
\]
where \( J_0(\eta) \) is the zeroth order Bessel function of the first kind. Diagonalizing the effective Hamiltonian \([14]\), we arrive at the sought electron energy spectrum near the \( \Gamma \) point of the Brillouin zone,

\[
\varepsilon^{(\pm)}(k) = (\gamma/2)(eE_0/\hbar\omega)^2 + \gamma k^2 + \gamma [(2k_z^2 - k_x^2 - k_y^2) + (eE_0/\hbar\omega)^2] + 3(k_x^2 + k_y^2)(k_z^2 + k_x^2 + 4k_y^2)J_0^2(\eta)\]^{1/2},
\]

where the signs “\( \pm \)” correspond to the two branches of the Luttinger Hamiltonian and the electron wave vector, \( k \), is assumed to meet the condition \( |\gamma| k^2 \ll \hbar \omega \).

Certainly, the energy spectrum (15) in the absence of the irradiation \( (E_0 = 0) \) exactly coincides with the spectrum of the unperturbed Luttinger Hamiltonian \([2]\), \( \varepsilon^{(\pm)}(k) = (\gamma \pm 2\gamma)k^2 \).

Circular polarization. Let an electromagnetic wave irradiating a semiconductor propagates along the \( z \) axis and is circularly polarized in the \((x, y)\) plane (see Fig. 1b). Then its vector potential inside the semiconductor can be written as

\[
A = \left( \frac{E_0}{\omega} \cos \omega t, \frac{E_0}{\omega} \sin \omega t, 0 \right).
\]

The Floquet problem with the Luttinger Hamiltonian \([11]\) and the vector potential \([16]\) at \( k = 0 \) can be written in the block-diagonal form as

\[
\hat{H}_0 = \begin{bmatrix}
\hat{H}^{(+)} & 0 \\
0 & \hat{H}^{(-)}
\end{bmatrix},
\]

where the Hamiltonian \( \hat{H}_0^{(\pm)} \) written in the basis \( \{ \psi_{\pm 3/2}, \psi_{\pm 1/2} \} \) reads

\[
\hat{H}^{(\pm)} = (eE_0/\hbar\omega)^2 \begin{pmatrix}
\gamma_1 + \gamma & -\sqrt{3}\gamma e^{\mp i2\omega t} \\
-\sqrt{3}\gamma e^{\mp i2\omega t} & \gamma_1 - \gamma
\end{pmatrix}.
\]

Solving the nonstationary Schrödinger equation with the Hamiltonian \([18]\), \( i\hbar \partial_t \Phi^{(\pm)}_{1,2} = \hat{H}^{(\pm)}\Phi^{(\pm)}_{1,2} \), one can find the four exact Floquet functions, \( \Phi^{(\pm)}_{1,2} = \exp(-i\varepsilon_{1,2}t/\hbar)\psi^{(\pm)}_{1,2} \), and the four eigenspinors of the considered Floquet problem,

\[
\phi^{(\pm)}_1 = \begin{pmatrix}
\gamma_1 & \sqrt{\Omega_{\pm} + \Delta_{\pm}} e^{\mp i\omega t} \\
\sqrt{\Omega_{\pm} + \Delta_{\pm}} e^{\mp i\omega t} & \gamma_1
\end{pmatrix} e^{i\omega t},
\]

\[
\phi^{(\pm)}_2 = \begin{pmatrix}
\gamma_1 & \sqrt{\Omega_{\pm} - \Delta_{\pm}} e^{\mp i\omega t} \\
\sqrt{\Omega_{\pm} - \Delta_{\pm}} e^{\mp i\omega t} & \gamma_1
\end{pmatrix} e^{-i\omega t},
\]

where \( \Omega_{\pm} = \sqrt{\Delta_{\pm}^2 + 3\gamma^2(eE_0/\hbar\omega)^4}, \Delta_{\pm} = \gamma(eE_0/\hbar\omega)^2 \mp \hbar\omega \), and the corresponding electron energies at \( k = 0 \) are

\[
\varepsilon^{(\pm)}_1 = \gamma_1(eE_0/\hbar\omega)^2 + \hbar\omega - \Omega_{\pm},
\]

\[
\varepsilon^{(\pm)}_2 = \gamma_1(eE_0/\hbar\omega)^2 - \hbar\omega + \Omega_{\pm}.
\]

To solve the Floquet problem with the Luttinger Hamiltonian \([11]\) and the vector potential \([16]\) at \( k \neq 0 \), let us rewrite the Hamiltonian in the new orthonormal basis \([19]\). In the high-frequency limit, \( \hbar\omega \gg |\gamma| k^2 \), one can apply the Floquet-Magnus approach to the rewritten Hamiltonian in the way discussed above for a linearly polarized field. As a result, we arrive at the effective time-independent Hamiltonian, \( \hat{H}_{\text{eff}}(k) \), which is similar to the Hamiltonian \([13]\). Namely, the Hamiltonian \( \hat{H}_{\text{eff}}(k) \) is the Hamiltonian \([11]\) with the vector potential \([16]\), which is rewritten in the basis \([19]\) and time-averaged over the field period. In the explicit form, the effective Hamiltonian describing the sought electron energy spectrum, \( \varepsilon(k) \), at small wave vectors \( k \ll \sqrt{\hbar\omega/|\gamma|} \) reads

\[
\hat{H}_{\text{eff}}(k) = \begin{pmatrix}
\phi_{j}^{(+)} & \phi_{j}^{(-)} \\
\phi_{j}^{(-)} & \phi_{j}^{(+)}
\end{pmatrix} A_{j}^{'},
\]

where the matrix elements are

\[
A_{\pm} = \frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} F + \frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} G + \varepsilon^{(\pm)}_{1},
\]

\[
B_{\pm} = \frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} F + \frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} G + \varepsilon^{(\pm)}_{2},
\]

\[
C_{\pm} = \frac{\gamma}{|\gamma|} \left[ \sqrt{\frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} \sqrt{\frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}}} - \frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} H, \right]
\]

\[
D_{\pm} = \frac{\gamma}{|\gamma|} \left[ \sqrt{\frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}} \sqrt{\frac{\Omega_{\pm} \pm \Delta_{\pm}}{2\Omega_{\pm}}} I, \right]
\]

Correspondingly, the energy spectrum, \( \varepsilon(k) \), can be found as a solution of the secular equation,

\[
\det[\hat{H}_{\text{eff}}(k) - 1\varepsilon(k)] = 0,
\]

where \( I \) is the unity matrix. To find the spectrum in the broad range of electron wave vectors, the equation \([23]\) should be solved numerically. However, the spectrum \( \varepsilon(k) \) can be written in analytical form for the high-symmetry directions in the Brillouin zone, \( k = (0, 0, k_z) \) and \( k = (k_x, k_y, 0) \). Namely, we can write the spectrum as the four branches, \( \varepsilon^{(\pm)}(k_z) \) for \( k_x = k_y = 0 \).
and $\varepsilon_{1,2}(k_x,k_y)$ for $k_z = 0$, where

$$
\varepsilon_1^{(+)}(k_z) = \gamma_1 \left( \frac{e E_0}{\hbar \omega} \right)^2 + \hbar \omega - \Omega_\pm + \gamma_1 k_z^2 + \frac{2\gamma k_z^2 \Delta_\pm}{\Omega_\pm},
$$

$$
\varepsilon_2^{(+)}(k_z) = \gamma_1 \left( \frac{e E_0}{\hbar \omega} \right)^2 - \hbar \omega + \Omega_\pm + \gamma_1 k_z^2 - \frac{2\gamma k_z^2 \Delta_\pm}{\Omega_\pm}.
$$

(24)

$$
\varepsilon_1^{(\pm)}(k_x,k_y) = \gamma_1 \left( \frac{e E_0}{\hbar \omega} \right)^2 + \gamma_1 k^2 + \xi_\pm \left( \hbar \omega - \Omega_\pm \right)
- \frac{\gamma \Delta_\pm (k_x^2 + k_y^2)}{\Omega_\pm}
+ \frac{3\gamma^2(\Omega_\pm + \Delta_\pm)^2(\kappa_x^2 + \kappa_y^2)^2}{4\Omega_\pm^2} \right)^{1/2},
$$

$$
\varepsilon_2^{(\pm)}(k_x,k_y) = \gamma_1 \left( \frac{e E_0}{\hbar \omega} \right)^2 + \gamma_1 k^2 - \xi_\pm \left( \hbar \omega - \Omega_\pm \right)
- \frac{\gamma \Delta_\pm (k_x^2 + k_y^2)}{\Omega_\pm}
+ \frac{3\gamma^2(\Omega_\pm + \Delta_\pm)^2(\kappa_x^2 + \kappa_y^2)^2}{4\Omega_\pm^2} \right)^{1/2},
$$

(25)

and $\xi_\pm = (\hbar \omega - \Omega_\pm)/|\hbar \omega - \Omega_\pm|$. In the absence of the irradiation ($E_0 = 0$), the effective Hamiltonian turns into the unperturbed Luttinger Hamiltonian and the solution of the secular equation exactly coincides with the unperturbed electron dispersion, $\varepsilon^{(\pm)}(k) = (\gamma_1 \pm 2\gamma)k^2$.

III. RESULTS AND DISCUSSION

As it was mentioned above, the Luttinger Hamiltonian can describe both valence band of conventional semiconductors (if the two quantities, $\gamma_1 \pm 2\gamma$, are of the same sign) and the band structure of gapless semiconductors near the band edge (if they are of opposite signs) since the both cases correspond to the same electronic term $\Gamma_8$ in the Brillouin zone center. For definiteness, let us restrict the consideration by the cases of valence band of such a conventional semiconductor as GaAs ($\gamma_1 = -6.96 h^2/2m_0$, $\gamma_2 = -2.06 h^2/2m_0$, $\gamma_3 = -2.93 h^2/2m_0$) and the gapless semiconductor HgTe ($\gamma_1 = 15.6 h^2/2m_0$, $\gamma_2 = 9.6 h^2/2m_0$, $\gamma_3 = 8.6 h^2/2m_0$), where $m_0$ is the electron mass.

The energy spectrum of the term $\Gamma_8$ is defined by Eqs. (11) and (24) and plotted for GaAs (Fig. 2) and HgTe (Fig. 3) irradiated by an electromagnetic wave with different polarizations. In the absence of irradiation, the electronic term $\Gamma_8$ consists of the two branches which correspond to the bands of heavy and light holes in GaAs (see the red heavy lines in Fig. 2) and the conduction and valence bands in HgTe (see the red heavy lines in Fig. 3). These branches are degenerated at $k = 0$ and, in addition, their electron states are doubly degenerated in spin at any electron wave vector $k$. It follows from the plots that the irradiation lifts the degeneracy but the lifting strongly depends on the light polarization. Namely, a linearly polarized wave lifts the spin degeneracy of the bands (see the two blue thin lines in Figs. 1a and 2a), whereas a circularly polarized wave lifts also the spin degeneracy at any electron wave vector (see the four blue thin lines in Figs. 2b and 3b). It follows from Eqs. (11) and (24) that the light-induced band splittings marked in the Figs. 2–3 as $\Delta_i$ read

$$
\Delta_{1,5} = 2|\gamma| (e E_0/\hbar \omega)^2,
$$

$$
\Delta_{2,4,6,8} = \sqrt{\gamma^2(e E_0/\hbar \omega)^2 - \hbar \omega^2 + 3\gamma^2(e E_0/\hbar \omega)^2},
$$

$$
\Delta_3 = 2\hbar \omega - 2\sqrt{\gamma^2(e E_0/\hbar \omega)^2 - \hbar \omega^2} + 3\gamma^2(e E_0/\hbar \omega)^2 - 2\hbar \omega,
$$

$$
\Delta_7 = 2\hbar \omega - 2\sqrt{\gamma^2(e E_0/\hbar \omega)^2 - \hbar \omega^2} + 3\gamma^2(e E_0/\hbar \omega)^2,
$$

(26)
Since the band splittings \( \Delta_s \) are of meV scale for the irradiation intensities around \( I \sim \text{kW/cm}^2 \), they can be observed experimentally in optical electron transitions induced by another weak (probing) electromagnetic wave. Particularly, such optical transitions between the split bands will lead to fine structure of the optical spectra. Besides the conventional optical measurements, the modern angle-resolved photoemission spectroscopy (ARPES) can also be applied to study the electron energy spectra plotted in Figs. 2–3. Indeed, ultra-violet laser-based ARPES provides sub-meV resolution of electron dispersion \( \varepsilon(k) \), which is enough for detecting features of them. It should be noted also that the band splittings \( \Delta_s \) appear from exact solutions of the Floquet problem at \( k = 0 \) and, therefore, go beyond the scope of the known simple models based on the direct time-averaging of the Luttinger Hamiltonian.

To clarify physical nature of the light-induced band splitting, it should be noted that a circularly polarized electromagnetic wave breaks the time-reversal symmetry (since the time-reversal turns left-polarized photons into right-polarized ones and vice versa). Therefore, a circularly polarized electromagnetic wave acts similarly to a magnetic field which lifts the spin degeneracy and induced the asymmetry of electronic properties along the field direction and perpendicularly to the field. As to a linearly polarized electromagnetic wave, it acts similarly to an uniaxial mechanical stress along the direction of polarization vector, which both splits the degeneracy of electron states at \( k = 0 \) and induced the anisotropy of electron dispersion. As a consequence, the light-induced band splitting is accompanied by the anisotropy of electronic properties. Indeed, the unperturbed electron dispersion, \( \varepsilon(\pm)(k) = (\gamma_1 \pm 2\gamma)k^2 \), is isotropic, whereas an irradiation results in the anisotropy of electron dispersions along different axes in the \( k \) space (see the blue thin lines in Figs. 2–3). Certainly, the anisotropy of electron dispersion will result in the anisotropy of electron transport which is discussed in the following.

Let charge carriers fill only ground band of the split bands near the band edge. Then the anisotropic transport can be described by effective electron masses. Expanding the electron energy spectra into the series expansion in powers of electron wave vector \( k \), they can be easily rewritten near the band edge in the parabolic form,

\[
\varepsilon(k) = \frac{\hbar^2}{2m} \left( k_x^2 + k_y^2 \right) + \frac{\hbar^2}{2m} \left( k_z^2 \right),
\]

where \( m \) are the electron effective masses of the band. It should be stressed that the anisotropic electron dispersion takes also place in valence band of conventional semiconductors under uniaxial mechanical stress. Therefore, we can apply the approach known from the theory of strained semiconductors, which is based on the relaxation time approximation. Within this approach, the conductivity tensor is

\[
\sigma_{\alpha\beta} = e^2 \nu \int \frac{d^3k}{(2\pi)^3} v_{\alpha}(k)v_{\beta}(k)\tau(\varepsilon) \left[ -\frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right],
\]

where \( \varepsilon(k) \) is the electron energy spectrum in the ground band, \( v(k) = \partial\varepsilon(k)/\hbar\partial k \) is the electron velocity, \( \tau(\varepsilon) \) is the electron relaxation time, \( f_0(\varepsilon) \) is the Fermi-Dirac distribution function, and \( \nu \) is the factor of spin degeneracy of the band (\( \nu = 1, 2 \) for the cases of linear and circular polarizations, respectively). Assuming that the temperature is \( T = 0 \), Eq. (23) yields the sought light-induced anisotropy of conductivity,

\[
\frac{\sigma_{xx}}{\sigma_{xx}} = \frac{\sigma_{zz}}{\sigma_{yy}} = \frac{m_\perp}{m_\parallel},
\]

Particularly, for GaAs irradiated by a linearly polarized electromagnetic wave and the Fermi electron wave vector satisfying the condition \( k_F \ll \omega/E_0 \), the anisotropy
does not depend on the field parameters and reads
\[
\frac{\sigma_{zz}}{\sigma_{xx}} = \frac{\sigma_{zz}}{\sigma_{yy}} = \frac{\gamma_1 - 2\gamma}{\gamma_1 + \gamma}.
\]

Finalizing the discussion, it should be reminded that the present theory is developed under assumption of continuous electron wave vector, \(\mathbf{k}\). However, the effective Hamiltonians \(H_{\text{eff}}(\mathbf{k})\) derived above can also be used to describe electronic properties of nanostructures, where the electron wave vector is discontinuous. To take into account the size quantization in nanostructures, one have to analyze the Schrödinger problem with the Hamiltonian \(H_{\text{eff}}(\mathbf{k}) + U(\mathbf{r})\), where \(\mathbf{k} = -i\partial/\partial\mathbf{r}\) is the electron wave vector operator, \(U(\mathbf{r})\) is the quantizing potential of the nanostructure, and the Hamiltonian \(H_{\text{eff}}(\mathbf{k})\) results from the effective Hamiltonians \(H_1\) and \(H_2\) with the replacement \(\mathbf{k} \rightarrow \mathbf{k}\).

IV. CONCLUSION

Applying the Floquet formalism to electron states described by the Luttinger Hamiltonian, we developed the theory of optical control of the states originated from the electronic term \(\Gamma_8\) (valence band in conventional semiconductors like GaAs and the valence and conduction bands in gapless semiconductors like HgTe). As a main result, the electron energy spectrum of such materials renormalized by light is derived. It follows from analysis of the spectrum that the electronic properties crucially depend on the irradiation which can induce the anisotropy of electronic properties for different directions in the Brillouin zone, band gaps in the spectrum and the spin splitting of the bands. Possible manifestations of the found electronic features in optical spectra and transport measurements are discussed. Since semiconductor materials described by the Luttinger Hamiltonian are actively used in the modern nanotechnology, the present theory can be helpful to describe electronic properties of micro- and nanostructures.

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