Oscillating monopole-antimonopole pair in the Weinberg-Salam model

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We investigate further the properties of axially symmetrical monopole-antimonopole pair in the standard Weinberg-Salam theory. These numerical solutions behave quite differently from their counterparts in the SU(2) Yang-Mills-Higgs theory as the poles are now bounded by a flux string. Our results confirm that the energy of these solutions resides in a range of 13.17 - 21.02 TeV. In addition, we found strong evidence suggesting this configuration is time-dependent and discovered an energy oscillation phenomenon between the monopole and anti-monopole, which is associated with the symmetry of the solution. Finally, we calculate numerically the magnetic charge of the solution and confirm that its value is indeed \( \frac{4}{\pi} \sin^2 \theta_w \), as predicted by Y. Nambu in the 1970s. Counterintuitively, the magnetic charge is spread out and distributed along the string instead of concentrated near the poles.

INTRODUCTION

Since Dirac [1] introduced the magnetic monopole into Maxwell’s theory, the concept has become a subject of extensive studies. Wu and Yang [2] were the first to generalize the idea to non-Abelian gauge theories. The first finite energy solution was found by 't Hooft and Polyakov [3] independently in the SU(2) Yang-Mills-Higgs theory. These solutions are spherically symmetrical. However, for configurations with more than one pole, such as the monopole-antimonopole pair (MAP) solution reported by Kleihaus and Kunz [5], they possess at most axial symmetry. While there exists an exact solution in the Bogomol’nyi-Prasad-Sommerfield (BPS) limit [4], only numerical solutions can be found when the Higgs field is non-vanishing. Unfortunately, these solutions were found in simpler but unrealistic gauge theories. Solutions found in the SU(2)×U(1) Weinberg-Salam model would possess more important physics implications.

It was widely believed that there exists no topological monopole of interest in the Weinberg-Salam model because upon symmetry breaking, the quotient space SU(2)×U(1)/U(1)_{em} allows no non-trivial second homotopy. However, it was demonstrated by Cho and Maison [6] that when the standard Weinberg-Salam model is viewed as a gauged CP^1 model, the Higgs field could then admit a topologically non-trivial second homotopy, \( \pi_2(CP^1) = Z \). This paved the way for constructing realistic magnetic monopole models that are experimentally verifiable. In 1977, Y. Nambu [7] demonstrated through theoretical derivation that a pair of magnetic monopole and antimonopole bound by a flux string is admitted by the Weinberg-Salam model. Essentially, this is the electroweak counterpart of Kleihaus and Kunz’s MAP solution [5]. Such a configuration was proven existent numerically by Teh et al. [8]. However, Weinberg angle, \( \theta_w = \frac{\pi}{4} \), was chosen for ease of calculation in their research and the dimension of quantities investigated was not taken into consideration. For this reason, the work done in Ref. [8] lacks physical significance and therefore in this research, to address this, an appropriate dimensionless transformation is first performed on the model. All solutions obtained are then calculated when the dimensionless Higgs coupling constant, \( \beta = 0.7782 \) and the Weinberg angle, \( \tan \theta_w = 0.5358 \). These parameters are calculated according to the latest data published by Particle Data Group [9], which correspond to \( m_W = 80.379 \text{GeV} \), \( m_Z = 91.1876 \text{GeV} \) and \( m_H = 125.10 \text{GeV} \).

STANDARD WEINBERG-SALAM MODEL

The Lagrangian of the standard Weinberg-Salam model is given by [6, 8]

\[
\mathcal{L} = -\left( D_\mu \phi \right)^\dagger D^\mu \phi - \frac{\lambda}{2} \left( \phi ^\dagger \phi - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}.
\]

Here, \( D_\mu \) is the covariant derivative of the SU(2)×U(1) group, which is defined as

\[
D_\mu = D_\mu - \frac{i}{2} g' B_\mu = \partial_\mu - \frac{i}{2} g A^a_\mu \sigma_a - \frac{i}{2} g' B_\mu,
\]

where \( D_\mu \) is the covariant derivative of SU(2) group only. The SU(2) gauge coupling constant, gauge potential and electromagnetic tensor are given by \( g \), \( A^a_\mu \) and \( F_{\mu\nu}^a \). Their counterparts in the U(1) group are denoted by \( g' \), \( B_\mu \) and \( \sigma_a \).
From the energy-momentum tensor, 
where the unit vector, \( \hat{\xi} \)

\[ \phi = \frac{H}{\sqrt{2}} \xi \left( \xi^i \xi = 1 \right) \]  

(3)
to further simplify the calculation. Metric used in this paper is \((- + + +)\).

Through Lagrangian \( \xi \), \( 3 \) equations of motion can be obtained as the following,

\[ D^\mu F_{\mu} \phi = \lambda \left( \phi^\dagger \phi - \frac{\mu^2}{\lambda} \right) \phi, \]  

(4)

\[ D^\mu F_{\mu} \epsilon = \frac{i g}{2} \left[ \phi^\dagger \sigma^\alpha (D_{\alpha} \phi) - (D_{\alpha} \phi)^\dagger \sigma^\alpha \right], \]  

(5)

\[ \partial^\mu G_{\mu\nu} = \frac{i g'}{2} \left[ \phi^\dagger (D_{\alpha} \phi) - (D_{\alpha} \phi)^\dagger \right]. \]  

(6)

From the energy-momentum tensor, \( T^{\mu\nu} \), the energy density of the system can be obtained as

\[ \varepsilon = T^{00} = \frac{1}{4} F_{ij}^a F^{aij} + \frac{1}{2} F_{i0}^a F^{ai0} + \frac{1}{4} G_{ij} G^{ij} + \frac{1}{2} G_{i0} G^{i0} + \left( D_{\alpha} \phi \right)^\dagger \left( D^\alpha \phi \right) + \left( D_{\alpha} \phi \right)^\dagger \left( D^\alpha \phi \right) + \frac{\lambda}{2} \left( \phi^\dagger \phi - \frac{\mu^2}{\lambda} \right)^2. \]  

(7)

The total energy could then be calculated using the integral, \( E = \int \varepsilon r^2 \sin \theta \; dr \; d\theta \; d\phi \).

To obtain the MAP solutions in Weinberg-Salam model, the following electrically neutral axially symmetrical magnetic ansatz is employed:

\[ gA_0^a = - \frac{1}{r} \psi_1 (r, \theta) \tilde{n}_1^a \hat{\phi}_1 + \frac{n}{r} \psi_2 (r, \theta) \tilde{n}_2^a \hat{\phi}_1 + \frac{1}{r} R_1 (r, \theta) \tilde{n}_1^a \hat{r}_1 - \frac{n}{r} R_2 (r, \theta) \tilde{n}_2^a \hat{r}_1, \]

\[ g'B_0 = \frac{n}{r} B_1 (r, \hat{\phi}_1), \]

\[ gA_0^0 = g'B_0 = 0, \]

\[ \Phi^a = \Phi_1 (r, \theta) \tilde{n}_1^a + \Phi_2 (r, \theta) \tilde{n}_2^a = \frac{H (r, \theta)}{\sqrt{2}} \hat{\Phi}^a, \]

\[ \xi = i \left( \sin \frac{\alpha (r, \theta)}{2} e^{-i m \phi} - \cos \frac{\alpha (r, \theta)}{2} \right), \]  

(8)

where the unit vector, \( \hat{\Phi}^a \), can be expressed as,

\[ \hat{\Phi}^a = \xi^i \sigma^a \xi = - \cos (\alpha - \theta) \tilde{n}_r^a - \sin (\alpha - \theta) \tilde{n}_\theta^a. \]  

(9)

In magnetic ansatz \( \xi \), the spatial spherical coordinate unit vectors are

\[ \hat{r}_1 = \sin \theta \cos \phi \hat{\delta}_{11} + \sin \theta \sin \phi \hat{\delta}_{12} + \cos \theta \hat{\delta}_{13}, \]

\[ \hat{\delta}_{1} = \cos \theta \cos \phi \hat{\delta}_{11} + \cos \theta \sin \phi \hat{\delta}_{12} - \sin \theta \hat{\delta}_{13}, \]

\[ \hat{\phi}_1 = - \sin \phi \hat{\delta}_{11} \cos \phi \hat{\delta}_{12}, \]  

(10)

whereas the unit vectors for isospin coordinate system are given by

\[ \tilde{n}_r^a = \cos \theta \cos n \phi \hat{\delta}_{1}^a + \sin \theta \sin n \phi \hat{\delta}_{2}^a + \cos \theta \hat{\delta}_{3}^a, \]

\[ \tilde{n}_\theta^a = \cos \theta \cos n \phi \hat{\delta}_{1}^a + \cos \theta \sin n \phi \hat{\delta}_{2}^a - \sin \theta \hat{\delta}_{3}^a, \]

\[ \tilde{n}_\phi^a = - \sin \phi \hat{\delta}_{1}^a \cos \phi \hat{\delta}_{2}^a. \]  

(11)

The \( \phi \)-winding number, \( n \), is set to 1. The angle, \( \alpha (r, \theta) \rightarrow p \theta \) asymptotically \( \xi \), where \( p \) is a parameter controlling the number of poles in the solution, which is set to 2 for MAP solutions in this research.

To investigate the magnetic property of the solution, the magnetic ansatz \( \xi \) is transformed by using the gauge,

\[ G = - i \left( \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} e^{-i m \phi} \right) = \cos \frac{-\pi}{2} + i \tilde{\alpha}^a \sigma^a \sin \frac{-\pi}{2}, \]

\[ \tilde{\alpha}^a = \sin \frac{\alpha}{2} \cos \phi \hat{\delta}_{1}^a + \sin \frac{\alpha}{2} \sin \phi \hat{\delta}_{2}^a + \cos \frac{\alpha}{2} \hat{\delta}_{3}^a. \]  

(12)
The transformed Higgs field and the third component of the gauge potential will have the following form,

\[ \Phi'^a = \delta^a_3, \]

\[ gA'^a_\mu = \frac{n}{r} \left( \psi_2 h_2 - R_2 h_1 - \frac{1 - \cos \alpha}{\sin \theta} \right) \hat{\phi}_\mu = \frac{A_1}{r} \hat{\phi}_\mu. \]

The particular gauge in equation (12) was chosen because \( gA'^2_\mu \) produced is precisely the negative gauge potential of the 't Hooft electromagnetic tensor \( E \), \( F_{\mu \nu} = \hat{\Phi}^a F^a_{\mu \nu} - \frac{1}{2} \varepsilon^{abc} \hat{F}^a D_\mu \hat{F}^b D_\nu \hat{F}^c \). For this reason, the U(1) and SU(2) magnetic field can be calculated as

\[ gB_i^{U(1)} = -\frac{g'}{2} \varepsilon^{ijk} G_{jk} = -\varepsilon^{ijk} \partial_j \left\{ n B_1 \sin \theta \right\} \partial_k \phi, \]

\[ gB_i^{SU(2)} = -\frac{g}{2} \varepsilon^{ijk} F_{jk} = \varepsilon^{ijk} \partial_j \left\{ gA'^3_\mu \right\} = -\varepsilon^{ijk} \partial_j \left\{ A_1 \sin \theta \right\} \partial_k \phi, \]

respectively. The magnetic field lines can be constructed by drawing the contour lines of the terms in curly brackets. According to Coleman, there is no unique definition for the magnetic field outside of Higgs vacuum. The above definition views the configuration as a point charge from afar and hence, it is not preferable when investigating the 3D magnetic charge density at small \( r \). For this reason, the Bogomolny definition of SU(2) magnetic field is used

\[ gB_i^{SU(2)} = gB_i^a \hat{\Phi}^a = -\frac{g}{2} \varepsilon^{ijk} F_{\mu \nu} \hat{\Phi}^\mu \hat{\Phi}^\nu. \]

The total magnetic charge of the system is calculated using this definition from the following integral

\[ M = \int \partial^\mu B_i \, d^3 x = \oint B_i \, d^2 A, \]

where \( d^2 A \) is the surface element of an appropriate Gaussian surface. Lastly, the real neutral field, \( Z_\mu \), and electromagnetic field, \( A_\mu \), are related to gauge fields, \( B_\mu \) and \( A'^3_\mu \) through

\[ \begin{pmatrix} \Phi \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ A'^3_\mu \end{pmatrix}. \]

**NUMERICAL METHOD**

To obtain the MAP solutions, the set of magnetic ansatz (8) were substituted into the equations of motion (4) - (6), which were then reduced to seven coupled second-order partial differential equations corresponding to seven profile functions present in the magnetic ansatz, \( \psi_1, \psi_2, R_1, R_2, \Phi_1, \Phi_2 \) and \( B_1 \). The equations were further simplified with the following substitutions,

\[ x = m_w r, \quad \bar{H} = \frac{H}{H_0}, \quad \tan \theta_0 = \frac{g'}{g}, \quad \beta^2 = \lambda \frac{g'}{g^2}, \]

where \( H_0 = \sqrt{2 m_W / 2} \) and \( m_W = g H_0 / 2 \). The new radial coordinate, \( x \), is dimensionless. The rescaled Higgs field, \( \bar{H} \), approaches 1 asymptotically. The rescaled \( \bar{H}, \) approaches 1 asymptotically. Only two parameters were left in the equations after the transformation, namely \( \tan \theta_0 \) and \( \beta \), which were calculated to be 0.5358 and 0.7782 according to the latest data published by Particle Data Group [9].

The seven coupled equations were then solved by fixing boundary conditions at small distances \( (r \to 0) \), large distances \( (r \to \infty) \), along positive and negative \( z \)-axis when \( \theta = 0 \) and \( \pi \). Asymptotically, we have

\[ \begin{align*}
\psi_1(\infty, \theta) &= \psi_2(\infty, \theta) = 2, \\
R_1(\infty, \theta) &= R_2(\infty, \theta) = B_1(\infty, \theta) = 0, \\
\Phi_1(\infty, \theta) &= \cos \theta, \quad \Phi_2(\infty, \theta) = \sin \theta.
\end{align*} \]

For small distances,

\[ \begin{align*}
\psi_1(0, \theta) &= \psi_2(0, \theta) = R_1(0, \theta) = R_2(0, \theta) = B_1(0, \theta) = 0, \\
\Phi_1(0, \theta) &= \sin \theta + \Phi_2(0, \theta) \cos \theta = 0, \\
\partial_r (\Phi_1 (r, \theta) \cos \theta - \Phi_2 (r, \theta) \sin \theta)|_{r=0} &= 0.
\end{align*} \]
FIG. 1. 3D Higgs modulus plots for MAP solutions found in (a) SU(2) Yang-Mills-Higgs theory and (b) SU(2)×U(1) Weinberg-Salam model.

Along positive and negative $z$-axis,
\begin{align*}
\partial_\theta \psi_1(r, \theta)|_{\theta=0} &= \partial_\theta \psi_2(r, \theta)|_{\theta=0} = R_1(r, 0) = R_2(r, 0) = \partial_\theta \Phi_1(r, \theta)|_{\theta=0} = \Phi_2(r, 0) = B_1(r, 0) = 0, \\
\partial_\theta \psi_1(r, \theta)|_{\theta=\pi} &= \partial_\theta \psi_2(r, \theta)|_{\theta=\pi} = R_1(r, \pi) = R_2(r, \pi) = \partial_\theta \Phi_1(r, \theta)|_{\theta=\pi} = \Phi_2(r, \pi) = B_1(r, \pi) = 0.
\end{align*}

The equations were then solved numerically with boundary conditions (21) - (24). Using finite difference approximation method, the set of seven coupled equations of motion were converted into a system of nonlinear equations which was then discretized onto a non-equidistant grid of $70 \times 60$ with a compactification factor of $\tilde{x} = x/(x+1)$. The region of integration covers all space which translates to $0 \leq \tilde{x} \leq 1$ and $0 \leq \theta \leq \pi$. MAP solutions in the SU(2) Yang-Mills-Higgs theory were used as initial guesses to obtain numerical solutions in the standard Weinberg-Salam model.

RESULTS AND DISCUSSION

Figure 1 shows a direct comparison of the 3D Higgs modulus plots between MAP solutions found in different models. In SU(2) Yang-Mills-Higgs theory, the monopole and anti-monopole found, such as those reported by Kleihaus and Kunz [5], are two separate entities as shown in Fig. 1(a) with a well-defined pole separation. In SU(2)×U(1) Weinberg-Salam model, however, the pair is bound and surrounded by a flux string as in Fig. 1(b), the pole separation can no longer be accurately measured. The $\rho$-axis is radial distance as in cylindrical coordinate systems. It has been proven that the flux string is unstable for a range of the Weinberg angle, $0 \leq \sin^2 \theta_W < 0.8$ when the Higgs boson mass is greater than 24 GeV [11]. Since both conditions are satisfied in reality, the numerical results presented here are all unstable staddle point solutions in the Weinberg-Salam model.

The Higgs modulus, $|\Phi|$, is described by the two profile functions in magnetic ansatz [3] and defined as $|\Phi| = \sqrt{\Phi_1^2 + \Phi_2^2}$. Figure 2 shows the surface plots of profile functions, $\Phi_1$ and $\Phi_2$, for the particular solution displayed in Fig. 1(b). Along certain boundaries, the regions that are not smooth in Fig. 2(a) and (b) correspond to small $r$ and along $z$-axis, which is precisely where the flux string is located. These areas are outlined in red and can be seen clearly in the top views of respective surface plots as shown in Fig. 2(c) and (d). Such irregularities only exist in $\Phi_1$ and $\Phi_2$. The flux string constitutes a current, which is dynamic in nature. Investigating such a system using a static model could explain why the irregularities are introduced in the first place. Furthermore, the irregularities render the conventional method used in this field to obtain solutions unviable. In general, an initial guess is picked and used to obtain a solution, said solution would then be used as the new initial guess to calculate subsequent solution, which would then be used as an initial guess again. Unfortunately, due to the irregularities present in our solutions, employing the conventional method would spread them onto other profile functions, creating huge spikes in their respective surface plots. To avoid said effect, each solution presented in this research is obtained from slightly different initial guesses converted from MAP solutions found in the SU(2) Yang-Mills-Higgs theory. These initial
guesses possess small differences with each entry differs by an amount of the order of $10^{-15}$, and could essentially be categorized as the same solution. Such minute differences result in fairly diverse solutions in the Weinberg-Salam model as we shall demonstrate in the following. The wide spectrum of solutions stem from the nature of the model and magnetic ansatz chosen, not from the method used in obtaining the solutions. The method used here, when applied to other models, would produce exactly the same, singular solution, just as the conventional method. This, in fact, is the major difference between solutions found in this research and any solution reported previously.

Figure 3(a) shows the dimensionless total energy, $E$, of the system of a collection of 300 data when $\beta = 0.7782$ in the Weinberg-Salam model, the corresponding value of $\beta$ in the SU(2) Yang-Mills-Higgs theory for the set of initial guesses used is also 0.7782. These data are arranged according to the order by which they are obtained, the dimensionless value of the total energy ranges from 2.683886 to 4.284372 with no apparent pattern at first sight. However, we discovered that higher energy spikes tend to appear if the solution’s Higgs modulus contour is sufficiently asymmetrical with respect to the origin and therefore we devised the following algorithm as an objective approach to quantify the degree of symmetry.

The 3D Higgs modulus plots or contours are generated using MATLAB from a $160 \times 160$ matrix containing values of $|\Phi|$ at each point on the $z$-$\rho$ plane. Row 1 to 80 contain data for the upper hemisphere (positive $z$-axis), row 81 to 160 are in turn for the lower hemisphere (negative $z$-axis). We used the following equation to calculate the relative difference ($RD$) between a point in the upper hemisphere and its corresponding counterpart in the lower hemisphere,

$$RD = \frac{|Z(p,q) - Z(161 - p,q)|}{Z(161 - p,q)} - 1,$$  \hspace{1cm} (25)
where $Z$ denotes the $160 \times 160$ matrix, $(p,q)$ is the location of any point in the upper hemisphere and $(161 - p, q)$ is the location of its counterpart in the lower hemisphere. The value of $RD$ is calculated at each point and rounded down to the nearest tenth with the only exception when $RD \geq 1$, where it will always be rounded down to 1. The values are then summed and averaged over all 12800 points. The resulting number, which would be expressed as a percentage value, is the difference in symmetry, $D$, and is the quantified criterion we used for all subsequent analyses. We then rearranged the above 300 data in Fig. 3(a) according to the values of their difference in symmetry from low to high as shown in Fig. 3(b). The pattern becomes clear, higher energy spikes all shifted to the right where solutions become more and more asymmetrical according to our algorithm.

While analyzing the energy density, we realized that the large amount of terms with $1/r^2$ factor in $\varepsilon$ requires the profile functions to be accurate in order not to blow up to infinity at small distances. The irregularities in $\Phi_1$ and $\Phi_2$ therefore created fake singularities near the origin. For this reason, we plotted the weighted energy density, $(r^2 \sin \theta)\varepsilon$. The $r^2$ factor is used to iron out the spikes at small $r$ and at large distances, it acts as a rescaling and therefore does not affect the qualitative interpretation of energy density plots. The weighted energy density contour plots of selected solutions are shown in Fig. 4(a). In these plots, certain area between the poles rises up forming a bump. This is the result of the $\sin \theta$ in the weighted energy density expression. At small $r$, near $\theta = 0$ and $\pi$, the energy density possesses high value as those are the locations of the monopole and anti-monopole. The large scale obscures minute details of locations where energy density is small. The $\sin \theta$ factor decreases the value of energy density near the poles and highlights concealed details of areas of interest. The location of the bump is directly related to the energy density distribution. When the peak of the bump is located at $z = 0$, the energy density is symmetric across the upper and
lower hemisphere. When the energy density distribution is uneven, the peak will shift towards where energy density value is higher. The bump appears in all solutions. Its location moves downwards passing the $z = 0$ red line in Fig. 4(a), which clearly forms a trajectory across the upper and lower hemisphere. Furthermore, the location of the bump decides the symmetry of the solution, the more it deviates from $z = 0$, the more asymmetrical the solution becomes, namely higher value of $D$. This can be seen clearly from the first and last contour plots in Fig. 4(a).

The plots of $U(1)$ magnetic field lines are presented in Fig. 4(b). The MAP does not appear to be in the $U(1)$ gauge field. However, it is clear that there exists an electric current circulating, where its center is located at around $\rho = 2.5$. The current moves in the same pattern in accordance with the bump in Fig. 4(a) from the upper hemisphere, across $z = 0$, to the other. Figure 4(c) shows the $R_1-\theta$ view of the surface plot of profile function, $R_1$, for the specific solutions displayed in Fig. 4(a). There is a clear oscillation and this is a strong indication that the profile function is time-dependent. Such an oscillation is ubiquitously present in all 7 profile functions, suggesting the dynamic nature of the MAP configuration in the Weinberg-Salam model as stated earlier. The cross sections of 3D magnetic charge density plots at $\rho = 1$ are shown in Fig. 4(d), due to irregularities in $\Phi_1$ and $\Phi_2$ creating spikes near the $z$-axis. The positive magnetic charge moves towards the Southern hemisphere (negative $z$). In addition, the magnetic charge distribution spreads out, changes continuously, indicating the magnetic charge is distributed along the string instead of localized, concentrated near the poles. This is in stark contrast to SU(2) MAPs. These observations all suggest that energy oscillates between the monopole and the anti-monopole. As the weighted energy density includes a factor
of \( r^2 \), the energy oscillation actually appears along the flux string, near the z-axis. This is another striking difference as compared to monopole configurations found previously where solutions are strictly static.

To verify the observation that MAP does not reside in the U(1) gauge field, we first calculated the total magnetic charge of the system in U(1). The result is \( M_{U(1)} = (0.46 \pm 8.65) \times 10^{-7} \). The mean and sample standard deviation are calculated from all data collected. The range corresponds to a 95% confidence interval which is used throughout the paper. We then move on to verify the U(1) magnetic charge of the lower hemisphere. The result is \( M_{U(1)_{LH}} = (-3.31 \pm 3.92) \times 10^{-6} \). Therefore we conclude the MAP indeed does not reside in the U(1) gauge field. For the total magnetic charge, \( M \), the weight of contribution from U(1) and SU(2) is determined by the electromagnetic potential, \( A_\mu \), from equation (19). Through theoretical derivation, Y. Nambu predicted the value of \( |M| \) in either hemisphere is \( \frac{4\pi}{e} \sin^2 \theta_W \) \([7]\). For ease of calculation, we dropped the \( 4\pi \) factor and in natural unit,

\[
\frac{1}{e} \sin^2 \theta_W = \frac{1}{e} \frac{1}{1 + (\tan^2 \theta_W)} = \frac{1}{0.303} \frac{1}{1 + (0.5358)^2} \approx 0.7361.
\]

Figure 5 shows the total magnetic charge enclosed in the upper hemisphere, \( M_{UH} \), calculated from all solutions obtained, arranged according to \( D \) from low to high. Significant drops in magnetic charge appear more frequently in asymmetrical solutions, which correspond to higher \( D \). As shown in Fig. 4(d) above, in asymmetrical solutions, magnetic charge does not distribute evenly across hemispheres. However, the Gaussian surface used in calculating \( M_{UH} \) is drawn from the red line, therefore resulting in the drop. For this reason, when calculating \( M_{UH} \), we only consider the most symmetrical, 1.5%-2% data. The result is \( M_{UH} = 0.7338 \pm 0.006 \). Hence, our numerical results match with Y. Nambu’s prediction.

The related statistics of the data displayed in Fig. 3 are tabulated in Table 1. The data counts form a bell distribution as shown in Fig. 6 with the vast majority of the data centered around 2% to 4% difference in symmetry. However, more symmetrical solutions possess higher accuracy regardless of whether the statistics is exclusive or accumulative as seen in Table 1. Exclusive statistics include all data for a specific range of difference of symmetry, whereas accumulative statistics include all previous entries. As stated earlier, the location of the bump decides the symmetry of the configuration. The difference in symmetry, \( D \), could then be viewed as an indication as to where the solution is in the energy oscillation process. Symmetrical solutions with lower \( D \) have the lowest energy, even energy and magnetic charge distribution, and measurements performed on these solutions possess great accuracy as well. For this reason, again, the most symmetrical, 1.5%-2% data from Table 1 is presented as our estimation of
The double integral is the dimensionless $E$ as stated above and the factor, $2\pi H_0^2/m_w^2$, is found to be 4.907 TeV and hence our estimation of the mass of electroweak MAP is $m_{\text{MAP}} = 14.1673 \pm 0.9618$ TeV, which is just above the upper limit of the Large Hadron Collider at CERN. If we take all 300 data into consideration, the MAP resides in a range of 13.17 - 21.02 TeV.
CONCLUSION

We have studied the electrically neutral axially symmetrical electroweak MAP configurations with $\beta = 0.7782$, $\tan \theta_W = 0.5358$ in the standard Weinberg-Salam model and confirmed the magnetic charge of the monopole matches that predicted by Y. Nambu. These solutions are extremely sensitive to the minute differences in initial guesses used, unlike all previous work done in the field, reflecting the unstable nature of these solutions. The collection of all data, when viewed as a whole, exhibits time-dependent characteristics. Certain area in the weighted energy density contour plots forming a trajectory, combined with observations like the oscillating profile functions and magnetic charge distribution, lead to the conclusion of the presence of an energy oscillation phenomenon. The electric current in U(1) gauge field moves in accordance with this phenomenon could be interpreted as the origin of this mechanism, though more evidences are needed to draw decisive conclusions. In the process, we devised an important algorithm, the difference in symmetry, $D$, which played an integral role in discovering the energy oscillation. $D$ could be viewed as an indication as to where the solution is in the energy oscillation process. Symmetrical solutions possess even energy and magnetic charge density distribution, and the system is at a lower energy state. They could be considered as the initial states. Asymmetrical solutions, on the other hand, possess higher energy. The location of the bump would move towards the area where energy density is higher. Magnetic charge distribution in these solutions would also shift in accordance. In turn, they might be instances of later stages in the evolution of MAP in the Weinberg-Salam model. Finally, we converted the dimensionless measurements to the actual energy of the system, which is found to be just above current experimental capability.

There remains numerous difficulties. Inferring the behaviours of a dynamic solution using a static model has its limitations. For example, we cannot tell the direction in which the energy bump is moving, nor the rate at which the process is taking place. The work done here could also be extended by adding electric charges into the model, forming dyons. It would be interesting to know whether the electric charge from the dyon would interact with the U(1) electric current reported here, considering both are electric in nature. Doing so would inevitably produce configurations with even higher masses. Though, whether magnetic monopoles possess electric charges naturally is currently unknown. We will report theses findings in a separate paper.

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