Unruh Effect Revisited: Poincaré $\theta$-Vacua as Coherent States of Conformal Zero Modes

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Abstract. We report on a group-theoretical revision of the Unruh effect based on the conformal group $SO(4,2)$, which has been developed by the authors and collaborators. Special Conformal Transformations (SCT) are interpreted as transitions to relativistic uniformly accelerated frames. Poincaré invariant $\theta$-vacua (which turn out to be coherent states of conformal zero modes) are destabilized by SCT and radiate as a black body.

1. Introduction

The Fulling-Davies-Unruh effect [1, 2, 3] has to do with vacuum radiation in a non-inertial reference frame and shares some features with the (black-hole) Hawking [4] effect. In simple words, whereas the Poincaré invariant vacuum $|0\rangle$ in QFT looks the same to any inertial observer (i.e., it is stable under Poincaré transformations), it converts into a thermal bath of radiation with temperature

$$T = \frac{h a}{2 \pi v k_B} \quad (1)$$

in passing to a uniformly accelerated frame ($a$ denotes the acceleration, $v$ the speed of light and $k_B$ the Boltzmann constant).

This situation is always present when quantizing field theories in curved space as well as in flat space, whenever some kind of global mutilation of the space is involved (viz, existence of horizons). This is the case of the natural quantization in Rindler coordinates [2, 5], which leads to a quantization inequivalent to the normal Minkowski quantization (see next Section), or that of a quantum field in a box, where a dilatation produces a rearrangement of the vacuum [1].

In the reference [6], it was showed that the reason for the Planckian radiation of the Poincaré invariant vacuum under uniform accelerations (that is, the Unruh effect) is more profound and related to the spontaneous breakdown of the conformal symmetry in quantum field theory. From this point of view, a Poincaré invariant vacuum will be regarded as a coherent state of conformal zero modes, which are undetectable (“dark”) by inertial observers but unstable under Special Conformal Transformations (SCT)

$$x^\mu \to x'^\mu = \frac{x^\mu + c^\mu x^2}{1 + 2 c x + c^2 x^2}. \quad (2)$$
which can be interpreted as transitions to systems of relativistic, uniformly accelerated observers with acceleration $a = 2c$ (see e.g. Ref. [7, 8, 9]). Indeed, let us take for simplicity an acceleration along the “$z$” axis, $c^\mu = (0, 0, 0, \alpha)$, and the temporal path $x^\mu = (t, 0, 0, 0)$. Then the transformation (2) reads:

$$t' = \frac{t}{1 - \alpha^2 t^2}, \quad z' = \frac{\alpha t^2}{1 - \alpha^2 t^2}. \quad (3)$$

Writing $z'$ in terms of $t'$ gives the usual formula for the relativistic uniform accelerated (hyperbolic) motion:

$$z' = \frac{1}{a}(\sqrt{1 + a^2 t'^2} - 1) \quad (4)$$

with $a = 2\alpha$. Let us say that at least two alternative meanings of SCT have also been proposed [10, 11]. One is related to the Weyl’s idea of different lengths in different points of space time [10]: “the rule for measuring distances changes at different positions”. Other is Kastrup’s interpretation of SCT as geometrical gauge transformations of the Minkowski space [11].

The point of view exposed here is consistent with the idea that quantum vacua are not really empty to every observer. Actually, the quantum vacuum is filled with zero-point fluctuations of quantum fields. The situation is similar to quantum many-body condensed matter systems describing, for example, superfluidity and superconductivity, where the ground state mimics the quantum vacuum in many respects and quasi-particles (particle-like excitations above the ground state) play the role of matter. Moreover, we know that zero-point energy, like other non-zero vacuum expectation values, leads to observable consequences as, for instance, the Casimir effect, and influences the behavior of the Universe at cosmological scales, where the vacuum (dark) energy is expected to contribute to the cosmological constant, which affects the expansion of the universe (see e.g. [12] for a nice review). Indeed, dark energy is the most popular way to explain recent observations that the universe appears to be expanding at an accelerating rate.

In the papers [13] and [14] we construct a conformal-\(SO(4,2)\)-invariant quantum theory in 3+1 dimensions, giving the Hilbert space and an orthonormal basis for our conformal particle. The construction is based on an holomorphic square-integrable irreducible representation of the conformal group on the eight-dimensional phase space \(\mathbb{D}_4 = SO(4, 2)/SO(4) \times SO(2)\) inside the complex Minkowski space \(\mathbb{C}^4\). In [15], the Poincaré invariance of the ground state is highlighted and the mean energy, partition function and entropy of the accelerated ground state (seen as a statistical ensemble) is explicitly calculated. This leads us to interpret the accelerated ground state as an Einstein Solid, to obtain a deviation from the Unruh’s formula (1) and to discuss the existence of a maximal acceleration. Here we shall just deal with the second quantized (many-body) theory, where Poincaré-invariant (degenerated) pseudo-vacua are coherent states of conformal zero modes. Selecting one of this Poincaré-invariant pseudo-vacua spontaneously breaks the conformal invariance and leads to vacuum radiation.

2. Vacuum radiation as a spontaneous breakdown of the conformal symmetry

Let us offer an alternative explanation for the Unruh effect based on symmetry grounds. Actually, in Quantum Field Theory, the vacuum state is expected to be stable under some underlying group of symmetry transformations \(G\) (namely, the Poincaré group). Then the action of some spontaneously broken symmetry transformations can destabilize/excite the vacuum and make it to radiate. We argue that this is precisely the case of the Planckian radiation of the Poincaré invariant vacuum under uniform accelerations. Here, the Poincaré invariant vacuum looks the same to every inertial observer but converts itself into a thermal bath of radiation in passing to a uniformly accelerated frame. In fact, we point out that the reason for this radiation
is related to the spontaneous breakdown of the conformal symmetry in quantum field theory (see [6] and [15] for more details).

Let us denote by $\varphi_n$ the energy $E_n$ eigenstates. The Fourier coefficients $a_n$ (and $a_n^*$) of the expansion in energy modes of a state

$$\phi = \sum_n a_n \varphi_n,$$

are promoted to annihilation $\hat{a}_n$ (and creation $\hat{a}_n^\dagger$) operators in second quantization. The fact that the ground state of first quantization, $\varphi_0$, is invariant under Poincaré transformations (see [6] and [15]) implies that the annihilation operator of zero-(“dark”)-energy modes $\hat{a}_0$ commutes with all Poincaré generators. It also commutes with all annihilation operators and creation operators

$$[\hat{a}_0, \hat{a}_n^\dagger] = 0, \quad n > 0$$

of particles with non-zero (“bright”) energy. Therefore, by Schur’s Lemma, $\hat{a}_0$ must behave as a multiple of the identity in the broken theory, which means that Poincaré “$\theta$-vacua” fulfill

$$\hat{a}_0(\theta) = \theta |\theta\rangle \Rightarrow |\theta\rangle = e^{\theta \hat{a}_0 - \theta \hat{a}_0^\dagger} |0\rangle.$$  

That is, Poincaré “$\theta$-vacua” are coherent states of conformal zero modes (see [16] and [17] for a thorough exposition on coherent states).

In the first-quantized theory, the unitary transformation $U(c)$ that implements the acceleration (2) does not leave invariant the ground state $\varphi_0$ (see [15]) . In fact, a decomposition of the “accelerated ground state” in energy modes is obtained:

$$\varphi_0' = U(c) \varphi_0 = \sum_{n=0}^{\infty} \phi_n(c) \varphi_n,$$

where $\phi_n(c)$ denotes the probability amplitude of finding $\varphi_n'$ in $\varphi_n$ for a given acceleration $c$. In the second-quantized theory, this acceleration leads to a transformation of annihilation operators

$$\hat{a}_0' = \sum_{n=0}^{\infty} \phi_n(c) \hat{a}_n,$$

so that accelerated Poincaré $\theta$-vacua become:

$$|\theta'\rangle = e^{\theta \hat{a}_0 - \theta \hat{a}_0^\dagger} |0\rangle.$$  

The average number of particles with energy $E_n$ in the accelerated vacuum (10) is then given by

$$N_n(c) = \langle \theta' | \hat{a}_n^\dagger \hat{a}_n | \theta' \rangle = |\theta|^2 |\phi_n(c)|^2,$$

where $|\theta|^2$ is the total average number of particles in $|\theta\rangle$, and $|\phi_n(c)|^2$ is the probability of finding a particle in the energy state $E_n$ of the accelerated vacuum $|\theta'\rangle$.

In the same way, the probability $P_n(q, c)$ of observing $q$ particles with energy $E_n$ in $|\theta'\rangle$ can be calculated as:

$$P_n(q, c) = |\langle q(n) | \theta' \rangle|^2 = \frac{e^{-|\theta|^2}}{q!} |\theta|^2 q |\phi_n(c)|^2 q = \frac{e^{-|\theta|^2}}{q!} N_n^q(c).$$  

Therefore, the relative probability of observing a state with total energy $E$ in the excited vacuum $|\theta\rangle$ is:

$$P(E) = \frac{\sum_{q_0, \ldots, q_k} \prod_{n=0}^{k} P_n(q_n, c)}{\sum_{n=0}^{k} E_n q_n = E}.$$  

(13)

For the case studied here, this distribution function can be factorized as $P(E) = \Omega(E) e^{-\tau E}$, where $\Omega(E)$ is a relative weight proportional to the number of states with energy $E$ and the factor $e^{-\tau E}$ fits this weight properly to a temperature $T = k_B / \tau$.

One can also compute the mean energy

$$\bar{E}(c) = \langle \theta | \sum_{n=1}^{\infty} E_n a_n^\dagger a_n | \theta \rangle = |\theta|^2 \sum_{n=1}^{\infty} |\phi_n(c)|^2 E_n.$$  

(14)

and see that it is indeed Planckian (see [6] and [15]).

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