COMPLEX ANGULAR MOMENTA
AND THE PROBLEM OF EXOTIC STATES

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Abstract

After some brief personal recollections about V.N. Gribov, I demonstrate that his results and ideas on complex angular momenta may be applied in unfamiliar directions. As an example, it is shown, that any strong interaction amplitude, satisfying dispersion relation (in momentum transfer), has infinite number of energy-plane poles, both for exotic and non-exotic quantum numbers. This result ensures the necessary condition for existence of exotic hadrons. However, without more detailed knowledge of dynamics one cannot secure sufficient conditions for the exotics existence.

1 Introduction

I was lucky to work for 20 years in the close contact with V.N. Gribov, in the Ioffe Physico-Technical Institute and in the Leningrad (now Petersburg) Nuclear Physics Institute. It seems that I was the youngest of his colleagues who called him by his Russian nickname, Volodya. When he changed his home institution, to the Landau Theoretical Physics Institute, our contacts still continued, though became irregular. Later, after his death in 1997, I looked through his list of publications and, quite unexpectedly, was disappointed. For me, both the list, and publications themselves, could hardly reflect his bright personality.

V.N. Gribov was a rare person, with whom one could discuss practically any problem of physics (and not only physics). Discussions with him were always very intensive and passionate. Sometimes he was not right after all, but even then he suggested such non-trivial arguments for his position, which were difficult to reject. Similar discussions always emerged also at our theoretical seminars, even if the presented work had been earlier discussed with Gribov personally. I think that was why many physicists disliked (feared?) to talk at our seminars. However, being very difficult, discussions with Gribov were always very interesting and very useful. They gave to the author a better understanding of own work. Moreover, they provided a beautiful training how to present results of the work.

References to Gribov’s works are rather frequent today in the literature. But they mostly refer to QCD problems only - Gribov copies, Gribov horizon, and, of course, DGLAP equations for the parton evolution (here G means just Gribov). Very intensive Gribov’s activity
on Regge theory seems to be nearly forgotten now. Meanwhile, his numerous papers in this
direction may still generate ideas and arguments on various problems, even if the problems
look totally unrelated to the Reggeology. In the present note I demonstrate this for the prob-
lem of exotic states, i.e., hadrons with such quantum numbers that cannot be composed of 3
quarks (for baryons) or one quark-antiquark pair (for mesons). The corresponding result has
been published earlier [1]. Its derivation and meaning will be discussed here in more detail.

2 Dispersion relations and energy-plane poles

As is well-known, amplitude of any 2-particle-into-2-particle process depends on two kine-
matical variables. They may be taken to be \( s = W^2 \) and \( z = \cos \theta \), with \( W \) and \( \theta \) being the
c.m. energy and scattering angle, respectively. It is convenient in many cases, instead of \( z \), to
use the invariant quantity, squared c.m. momentum transfer. There are two such variables:
\( t \), linear in \( z \), and \( u \), linear in \( -z \) (of course, the sum \( t + u \) is independent of \( z \)).

As function of \( z \), the invariant amplitude \( A(s,z) \) can be decomposed into partial-wave
amplitudes \( f_j(s) \), that correspond to the process at a definite value of the total angular
momentum \( j \), the same in initial and final states. Physical values of \( j \) are (half)integer.
Of course, one could define \( f_j(s) \) as an analytical function for arbitrary values of \( j \), real or
even complex, such that coincides with physical amplitudes at physical values of \( j \). Such
procedure is always possible, but is generally ambiguous (e.g., when continuing from positive
integer points one could add any term proportional to \( 1/\Gamma(1-j) \)). However, under some
conditions, the partial-wave amplitudes \( f_j(s) \) admit an unambiguous analytical continuation
to non-physical values of \( j \).

In what follows, we use three assumptions: 1) possibility of the unambiguous analytical
continuation in the angular momentum \( j \); 2) absence of massless physical hadrons; 3) unitarity
condition. Let us consider their meaning.

For relativistic amplitudes, the possibility of unambiguous analytical continuation in \( j \) is
usually connected with dispersion relations in \( z \) (see, e.g., the monograph [2]). Generally, dis-
persion relations have not been deduced mathematically from axioms of the Quantum Field
Theory. They have not been proved also in terms of the more specific Quantum Chromody-
namics (QCD), which is believed now to provide the physical basis for strong interactions.
However, not all details of QCD have been understood up to now. Even relation between
quarks and gluons, on one side, and mesons and/or baryons, on the other, cannot be de-
scribed today in a model-independent way. That is why various dispersion relations (or their
analogs/generalizations) are widely applied to phenomenological treatment of strong inter-
actions. The dispersion relations are used as input in modern Partial-Wave Analyses (PWA;
see, e.g., the latest pion-nucleon PWA [3]). They are operative to pick out such strong in-
teraction parameters as meson-baryon coupling constants, and/or the pion-nucleon \( \sigma \)-term.
Analytical properties, corresponding to dispersion relations, provide basis for various sum
rules. Analogous analytical properties of inelastic amplitudes are assumed when extracting
the meson-meson scattering amplitude from data on processes like \( \pi \rightarrow 2\pi \). Up to now,
phenomenological applications of dispersion relations have not revealed any inconsistencies,
either theoretical nor experimental, and we still may consider them to be true.

Dispersion relations lead to the integral Gribov-Froissart formula [4, 2] for the continued
partial-wave amplitudes, at least at large positive \( \text{Re} \, j \). Note, however, that the dispersion relations provide a sufficient, but not necessary condition for the analytical continuation of \( f_j(s) \) in \( j \). The continuation survives under much weaker conditions on singularities of the invariant amplitudes in \( t \) and/or \( u \), and the continued partial-wave amplitudes still conserve essentially the same Gribov-Froissart form.

Now, consider the 2nd condition. Let us begin, for simplicity, with the amplitude for elastic scattering of two spinless particles, where the total angular momentum \( j \) coincides with the orbital momentum \( l \). If there are no massless hadrons, the physical partial-wave amplitudes, being described by the Gribov-Froissart formula, reveal the familiar threshold behavior

\[
f_l(s) \sim k^{2l} \quad \text{at} \quad k \to 0,
\]

where \( k \) is the c.m. momentum. The same threshold behavior is true also for the continued amplitudes \( f_l(s) \) with any, real or complex, angular momenta, at least till the Gribov-Froissart formula is applicable. This result has very simple physical meaning. Non-massless exchange, in the non-relativistic limit, induces the Yukawa potential of the form \( \exp(-\mu r)/r \), with non-vanishing \( \mu \) and the limited radius of action \( R \sim 1/\mu \). As is well-known, every quantum mechanical potential with final radius \( R \) generates amplitudes with the threshold behavior \( \sim (kr)^{2l} \).

Evidently, the threshold behavior (1) at real \( l \) may be consistent with the unitarity relation (2) only if \( l \geq -1/2 \). This means that near the threshold (i.e., at \( s \to s_{\text{th}} \), \( k^2 \to 0 \)) the Gribov-Froissart formula cannot stay unchanged (with its integral convergent) at \( \text{Re} \, l \leq -1/2 \).

The analysis, made by Gribov and Pomeranchuk \[5\], shows that at \( k^2 \to +0 \) the analytically continued partial-wave amplitude \( f_l(s) \) should reveal unlimited number of poles in \( l \) at \( \text{Re} \, l > -1/2 \). They are Regge poles (or reggeons), with energy-dependent positions. Further, from more detailed consideration of the small-\( k^2 \) region, Gribov and Pomeranchuk demonstrated \[5\] that the reggeons in the vicinity of \( l = -1/2 \) have trajectories

\[
l_n(s) \approx -\frac{1}{2} + \frac{i\pi n}{\ln(R\sqrt{|k^2|})} + O(\ln^{-2}(R\sqrt{|k^2|})),
\]

with \( R \) being the effective interaction radius. The number \( n \) takes any positive and negative integer values, \( n = \pm 1, \pm 2, \ldots \). We see that there are infinitely many reggeons accumulating to \( l = -1/2 \) at \( k^2 \to 0 \).

The structure of this accumulation, at real or even complex energies, may be investigated more explicitly in the non-relativistic quantum mechanics with final range potential, \textit{e.g.}, with the Yukawa potential \[6, 7\]. Below the threshold (at \( k^2 < 0 \)), the set of accumulating poles consists of infinitely many pairs of reggeons in the left half-plane \( \text{Re} \, l \leq -1/2 \). The reggeons of each pair have the same \( |n| \), and their trajectories are complex conjugate to each other. At \( k^2 \to -0 \), they tend to the limiting point -1/2. Such simple correlation becomes
destroyed above the threshold (at $k^2 > 0$), and some reggeons appear in the right half-plane $\Re l \geq -1/2$.

Up to now we have neglected particle spins. Accounting for them changes the situation, but only quantitatively. For the elastic threshold of two particles with spins $\sigma_1$ and $\sigma_2$, there appear several accumulation points in the $j$-plane, the rightmost one is at

$$j = -1/2 + \sigma_1 + \sigma_2,$$

instead of $j = -1/2$ [8]. The reason is simple: the accumulation points still correspond to $l = -1/2$, but particle spins provide several possible $j$-values for any fixed $l$-value, and vice versa. Correspondingly, structure of the accumulations in the $j$-plane, at thresholds of two spinning particles, are still described by trajectories [3], with the shifted limiting points [4]. The case of multi-particle thresholds has never been really studied (though hypothesized by Gribov and Pomeranchuk [5]).

The above consideration was applied to a purely elastic two-particle case, and the discussed reggeons were seen as poles for the corresponding elastic partial-wave amplitudes. But at other (higher) energies the unitarity relation connects the elastic scattering with different, inelastic processes. After all, we arrive at the conclusion, that partial-wave amplitudes of all processes, which can be related by rescattering, should couple to the same reggeons. In particular, they should contain those accumulating Regge poles.

Essential for our present purpose is the infinite number of the accumulating Regge poles [5], which implies that the total number of Regge poles is certainly infinite as well. Their positions depend on energy and are determined by some relation of the form

$$F(s, j) = 0.$$  

Each of its solution corresponds to a pole of a partial-wave amplitude and may be considered in two ways: either as a pole in $j$ with the position depending on energy (on $s$), or as a pole in energy (in $s$) with the position depending on the total angular momentum $j$. This provides one-to-one correspondence between reggeons and energy-plane poles. Therefore, the infinite number of reggeons corresponds to the infinite number of poles in the energy plane. The same conclusion is true for the non-relativistic case of a finite-range potential, in particular, for the Yukawa potential. This allows to investigate more clearly the pole structure of amplitudes.

At first sight, the infinite number of energy-plane poles for any finite-range potential looks impossible, since the energy-plane poles are related to bound states or resonances. As well known, the bound or resonance states may exist, say, in the Yukawa potential, only if it is attractive and sufficiently strong. However, one can ask here an interesting question: whether every complex-energy pole should be considered as possibly related with a resonance? The problem is that, in terms of complex angular momenta, the Regge poles participating in the Gribov-Pomeranchuk accumulations are clearly separated from physical points, which are, for the $j$-plane, only integer (or half-integer) non-negative points. Thus, the accumulating poles cannot provide physically meaningful (bound or resonance) states, at least near the corresponding threshold. This could be true for non-threshold energies as well.

Moreover, it might be that the total set of Regge poles is split into two (or more?) different subsets: one related, say, with the Gribov-Pomeranchuk accumulations, the other with the bound and/or resonance states. Then there could be infinite number of poles of the former
type, while only few (if any) poles of the latter type. This would be so, e.g., if Eq. (5) could be factorized as

\[ F_1(s, j) F_2(s, j) = 0. \]

However, explicit expressions of Ref. [6] show non-factorizable structure of Eq. (5). Moreover, there is no basic difference between various Regge poles [7]: all Regge trajectories appear to be different branches of the same multi-valued analytical function. Formally, this fact is related to non-trivial analytical properties of the Regge trajectories as functions of the energy; their singularities (branch points) come not only from physical thresholds, but also from coincidence of two (or more) reggeons [7], where those reggeons can be interchanged.

When considering the poles as energy-plane poles, one should have in mind that the energy plane generally has many Riemann sheets. An energy-plane pole corresponds to a bound state or resonance only if it is placed at the physical sheet or near the physical region. Poles placed far from the physical region or at far Riemann sheets are “not seen”.

The infinite number of reggeons for the Yukawa potential can be “visualized”. Evidently, the limit \( \mu \to 0 \) transforms the Yukawa potential \( \exp(-\mu r)/r \), with the finite radius of interaction \( \sim 1/\mu \), into the Coulomb potential, with the infinite radius. As traced in Ref. [6], this limit simultaneously transforms the Yukawa potential reggeons, that realize the Gribov-Pomeranchuk accumulation, into Coulomb reggeons, that realize orbital and radial Coulomb excitations. Therefore, the infinite number of energy-plane poles in the Yukawa potential becomes seen in the limit \( \mu \to 0 \) as the infinite number of the Coulomb levels. The Gribov-Pomeranchuk accumulation of reggeons near the threshold energy transforms in this limit into the well-known accumulation of the Coulomb bound states to the threshold (note that the double limiting transition \( \mu \to 0, k \to 0 \) is not equivalent here to the similar, but reversed limit \( k \to 0, \mu \to 0 \)).

Returning from the non-relativistic quantum mechanics to relativistic amplitudes, we emphasize that the above arguments have not assumed any specific quantum numbers in the scattering channel. Therefore, their conclusion on the infinite number of energy-plane poles should be equally applicable (or non-applicable) to both bosonic and fermionic hadron poles, having any flavor quantum numbers (exotic or non-exotic).

Thus, if we study \( 2 \to 2 \) strong interaction amplitudes, we should admit existence of (infinite number of) complex-energy poles with any exotic quantum numbers, both mesonic and baryonic. Alternatively, one could assume that analytical properties of strong interaction amplitudes, having exotic quantum numbers (at least, for one of the physical channels, \( s-, t-, \) or \( u \)-channel), are essentially different from those of totally non-exotic amplitudes. Such assumption looks very unnatural, and should be rejected. Thus, we obtain an argument for existence of exotic states, absent in previous publications.

Of course, existence of the energy-plane poles is only a necessary condition for the physical existence of exotic hadrons. To guarantee fulfillment of the sufficient condition, i.e., existence of energy pole(s) with particular quantum numbers near the physical region, one should have more detailed knowledge of dynamics.

It is interesting to note that the above consideration uses the reggeons in a non-standard manner. Usually, to apply the complex angular momenta approach for obtaining results in the \( s \)-channel (the channel where the invariant \( s \) has the meaning of the squared c.m. energy), one begins from the crossed channel, where \( t \) and \( s \) are, respectively, the squared energy and
squared momentum transfer (see Ref. [2]). Analytical continuation of partial-wave amplitudes in this \( t \)-channel allows, after returning into the \( s \)-channel, to study behavior of the invariant amplitude (and cross section) at high energy \( s \) at fixed value of momentum transfer \( t \). Now, to obtain the conclusion about energy-plane poles in the \( s \)-channel, we use complex angular momenta in the same \( s \)-channel.

To summarize, earlier results on complex angular momenta imply that any 2-hadron interaction amplitude, under standard assumptions, has infinite number of poles in the energy plane. Those poles may have exotic, as well as non-exotic, quantum numbers. However, the poles may be placed at far Riemann sheets of the energy plane. Therefore, more detailed dynamical information is necessary to guarantee real existence of exotic hadrons.

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