Optimization of energy resolution and Pulse Shape Discrimination for a CLYC detector with integrated digitizers

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ABSTRACT: Sufficient current pulse information of nuclear radiation detectors can be retained by direct waveform digitization owing to the improvement of digitizer’s performance. In many circumstances, reasonable cost and power consumption are on demand while the energy resolution and PSD performance should be ensured simultaneously for detectors. This paper will quantitatively analyse the influence of vertical resolution and sampling rate of digitizers on the energy resolution and PSD performance. The energy resolution and PSD performance can be generally optimized by improving the sampling rate and ENOB (effective number of bits) of digitizers. Several integrated digitizers, with sampling rates varying from 100 MSPS to 500 MSPS and vertical resolution ranging from 12-Bit to 16-Bit, were designed and integrated with a CLYC detector for verifications. Experimental results show good accordance with theoretical calculations. The conclusion can give guidance to designs of digitizers for similar applications in need of optimizing the energy resolution and PSD performance, and help to choose proper digitizers for different requirements.

KEYWORDS: Data acquisition circuits; Particle identification methods; Gamma detectors (scintillators, CZT, HPG, HgI etc); Neutron detectors (cold, thermal, fast neutrons)
1 Introduction

Cs$_2$LiYCl$_6$ (CLYC) scintillator has the ability to discriminate the fast neutron, thermal neutron, and gamma-ray. It is a potential counterpart for $^3$He proportional counter tube [1, 2] and can be used as high resolution gamma-ray and neutron spectrometers due to its exceptional PSD (Pulse Shape Discrimination) performance [3, 4]. Digital PSD algorithms [5] have been widely used in radiation detectors such as LaBr$_3$ scintillator [6], plastic scintillators [7], and organic scintillators [8, 9].

With the rapid development of ADC (Analog-to-Digital Converter) and FPGA (Field Programmable Gate Array), the vertical resolution and sampling rate of an integrated digitizer have been substantially increased [10–12]. Generally, digitizers with higher performance will result in more cost and power consumption. It is critical to understand the correlations between the resolution or/sampling rate of a digitizer and the energy resolution or/PSD FOM of a radiation detector. In this paper, we have used several self-developed digitizer systems to research the influence of sampling properties and ENOB on CLYC scintillator’s energy resolution and PSD performance, and verified the applicability of the theoretical formula based on quantization error. The research can provide a more general recommendations and guidance for the design of digitizer systems.

This paper will demonstrate methods of quantitative analysis on energy resolution and PSD FOM in section 2. Section 3 will introduce the CLYC detection system and setup of experiments. The results will be illustrated in section 4 and the conclusion and future work will be discussed in section 5. The conclusion is useful to choose or design digitizers in specific applications.
2 Method

J. Cang et al. [6] have quantitively analyzed how the sampling properties affect the energy resolution and PSD of LaBr$_3$:Ce scintillators. However, their studies mainly used a high-performance oscilloscope and researched the quantization error with different sampling rates. The influence of ENOB has not been fully verified. In this section, we make use of the theory proposed by J. Cang et al. [6] and ADC quantization noise theory to research how the digitizer’s sampling properties influence the energy resolution of CLYC and corresponding neutron/gamma discrimination performance.

2.1 The quantitative analysis of energy resolution

For a typical $^{137}$Cs gamma energy spectrum, as depicted in figure 1, the energy resolution is defined as equation (2.1). As indicated in the Gaussian fit of the full-energy peak, $E$ is the peak location, and $\Delta E$ is FWHM (Full Width at Half Maximum) of deposited energy distribution, $\sigma_E^2$ is the variance of deposited energy, and $\eta$ is the energy resolution.

![Figure 1. The typical $^{137}$Cs gamma energy spectrum.](image)

The current pulse integration is illustrated in figure 2, which is directly sampled by an ADC followed the current amplifier. $x_i$ (unit : V) is the amplitude for $i$th sampling point, $\Delta T$ (unit : s) is sampling period and $F_S$ is sampling rate ($\Delta T = 1/F_S$). As shown in equation (2.3), the integrated charge $Q$ (unit : C) is obtained by the summation of $N$ sampling points ($N = T/\Delta T$), where $A$
(unit: ohm) is the I-V (current to voltage) gain of preamplifier before ADC.

\[
\eta = \frac{\Delta E}{E} \approx \frac{2.355 \sigma_E}{E} = \frac{2.355 \sigma_Q}{Q} \tag{2.1}
\]

\[
\sigma_Q^2 = \sigma_{\text{digitizer}}^2 + \sigma_{\text{other}}^2 \tag{2.2}
\]

\[
Q = \sum_{i=1}^{N} \Delta T \times \frac{x_i}{A} = \sum_{i=1}^{N} \frac{1}{F_s} \times \frac{x_i}{A} \tag{2.3}
\]

For \( u = u(x_1, x_2, x_3, \ldots, x_N) \), if \( x_1, x_2, x_3, \ldots, x_N \) is uncorrelated, the total variance is \( \sigma_u^2 = \sum_{i=1}^{N} \left( \frac{\partial u}{\partial x_i} \right)^2 \sigma_{x_i}^2 \). According to the ADC quantized error formula \[14\], \( \sigma_{x_i}^2 \) contributed by the digitizer is estimated to be \( \left( \frac{\text{FUS}}{2^\text{ENOB}} \right)^2 \frac{1}{12} \). The variance of charge contributed by the digitizer can be described as equation (2.4), where \( \text{FUS} \) is the full-scale differential input of ADC, \( \text{ENOB} \) is the effective number of bits of ADC and \( T \) is the integration time.

\[
\sigma_{\text{digitizer}}^2 = \left( \frac{\Delta T}{A} \right)^2 \sum_{i=1}^{N} \sigma_{x_i}^2 = \frac{1}{F_s} \left( \frac{\text{FUS}}{A \times 2^{\text{ENOB}}} \right)^2 \frac{\Delta T \times N}{12} = \frac{1}{F_s} \left( \frac{\text{FUS}}{A \times 2^{\text{ENOB}}} \right)^2 \frac{T}{12} \tag{2.4}
\]

The energy resolution contributed by the digitizer is shown in equation (2.5).

\[
\eta \approx \frac{2.355 \sigma_Q}{Q} = \frac{2.355 \left( \sqrt{\sigma_{\text{other}}^2 + \frac{1}{F_s} \left( \frac{\text{FUS}}{A \times 2^{\text{ENOB}}} \right)^2 \frac{T}{12} } \right)}{Q} \tag{2.5}
\]

### 2.2 The quantitative analysis of pulse shape discrimination

The typical pulses of gamma-ray and neutron are shown in figure 3. The PSD ratio \( R \) is calculated as equation (2.6), where \( Q_L \) and \( Q_S \) are long delay and short prompt integrations of charge respectively. The uncertainties of \( Q_L \) and \( Q_S \) caused by digitizers are uncorrelated, substituting equation (2.4), the variance of \( R \) contributed by the digitizer can be calculated using equation (2.7), where \( T_L \) and \( T_S \) are delay and prompt integration time respectively.

\[
R = \frac{Q_L}{Q_L + Q_S} \tag{2.6}
\]

\[
\sigma_{R(\text{digitizer})}^2 = \left( \frac{\partial R}{\partial Q_L} \right)^2 \sigma_{Q_L}^2 + \left( \frac{\partial R}{\partial Q_S} \right)^2 \sigma_{Q_S}^2 = \frac{Q_L^2 \sigma_{Q_L}^2 + Q_S^2 \sigma_{Q_S}^2}{(Q_L + Q_S)^4} = \frac{T_L Q_S^2 + T_S Q_L^2}{12 \times (Q_L + Q_S)^4} F_s \left( \frac{\text{FUS}}{A \times 2^{\text{ENOB}}} \right)^2 \tag{2.7}
\]

Figure 4 demonstrates a typical distribution of PSD ratio for neutron and gamma-ray in CLYC detectors, where \( R_n \) and \( R_{\gamma} \) is the mean value of neutron and gamma-ray respectively. The PSD FOM is defined as equation (2.8), which is generally used to evaluate the performance of the particle.
discrimination, e.g. GRR (Gamma Rejection Ratio) is defined as 0.5 × erfc \left(2\sqrt{\ln 2 \times \text{FOM}}\right) \[4\].

\[
\text{FOM} = \frac{|R_\gamma - R_n|}{2.355 \left(\sigma_{R_\gamma} + \sigma_{R_n}\right)} = \frac{|R_\gamma - R_n|}{2.355 \left(\sqrt{\sigma_{R_\gamma(\text{digitizer})}^2 + \sigma_{R_\gamma(\text{other})}^2} + \sqrt{\sigma_{R_n(\text{digitizer})}^2 + \sigma_{R_n(\text{other})}^2}\right)}
\]

\[
= 2.355 \left(\frac{T_L Q_s^2 + T_s Q_L^2}{120 (Q_L + Q_S)^2 F_s (FUS A \times ENOB)^2} R_\gamma(\text{digitizer}) + \sigma_{R_\gamma(\text{other})}^2\right) + \sqrt{\sigma_{R_n(\text{digitizer})}^2 + \sigma_{R_n(\text{other})}^2}
\]

\[2.8\]

Figure 3. The typical pulses of neutron and gamma-ray.

Figure 4. The PSD ratio distribution.

3 Experiments

The detection system consists of a 25.4 mm diameter and 25.4 mm height CLYC scintillator with 95% enrichment of $^6\text{Li}$ from SCIONIX, a R6231-100 PMT from Hamamatsu and readout electronics. As sketched in figure 5, PMT readout electronics is stacked with the high voltage power board, preamplifier (~ 50 MHz bandwidth), ADC board, ZYNQ board, POE (Power Of Ethernet) board, and User Interface board \[15\]. The PMT is connected with the readout electronics via a dedicated socket. The electronics system is shielded and supported by an aluminum cylinder.

Seven integrated digitizers have been designed for testing. As summarized in table 1, the ENOBs from the datasheets are listed. Moreover, the ENOB at the reference frequency from the datasheet are also measured using a standard sinusoidal signal according to IEEE 1241-2000 standard. They are used in equation (2.5) and (2.8). The measured ENOBs should be used because they can reflect real performance of digitizers. The degradation of the measured ENOB, compared with the manual reference, for integrated digitizers is most likely caused by the jitter of ADC’s sampling clock. The full-scale voltage and the typical power consumptions of ADCs are also listed. Generally, the higher the ADC sampling rate, the greater the power consumption.
Different gamma-ray sources ($^{57}$Co, $^{137}$Cs and $^{60}$Co) are used for energy calibration and resolution calculations, a moderated neutron source with $^{252}$Cf and polyethylene is deployed to evaluate the PSD performance.

![Figure 5. The integrated PMT readout electronics.](image)

| Digitizer     | Speed (MSPS)/Resolution (Bit) | ENOB (datasheet) (Bit) | ENOB measured (Bit) | ADC full-scale voltage (V) | Power dissipation (mW/Channel) |
|---------------|-------------------------------|------------------------|---------------------|-----------------------------|--------------------------------|
| ISLA212P50    | 500/12                        | 11.27 @ 30 MHz         | 10.70               | 1.8                         | 858                            |
| AD9642        | 250/14                        | 11.50 @ 30 MHz         | 10.30               | 1.75                        | 360                            |
| AD9634        | 250/12                        | 11.20 @ 30 MHz         | 10.00               | 1.8                         | 360                            |
| AD9265        | 125/16                        | 12.80 @ 2.4 MHz        | 10.70               | 1.8                         | 439                            |
| AD9255        | 125/14                        | 12.70 @ 2.4 MHz        | 11.00               | 1.8                         | 437                            |
| AD9233        | 125/12                        | 11.40 @ 2.4 MHz        | 10.38               | 1.8                         | 415                            |
| AD9253        | 100/14                        | 12.10 @ 9.7 MHz        | 11.25               | 1.8                         | 101                            |

### 4 Results

#### 4.1 Energy resolution

The energy resolutions of the system at 662 keV measured with various digitizers are delineated as the circle points in figure 6. According to equation (2.5), combined with sampling rate and ENOB (effective number of bits) of digitizers, $F_S \times 4^{\text{ENOB}}$ is defined to represent the performance of digitizers. Basically, the sampling rate of digitizer should be larger than the Nyquist bandwidth of the input signal. Increasing the sampling rate by a factor of four is equivalent to decreasing ENOB by one. As expected, the energy resolution becomes better with the increase of the sampling rate and vertical resolution. The solid line is the fitting result of how the energy resolution varies with $F_S \times 4^{\text{ENOB}}$, of which the fitting parameters are also shown in figure 6. Based on equation (2.5), the fitting parameter $a$ can be calculated as equation (4.1), where the integrated charge $Q$ is $\sim 22$ pC, the integration time $T$ is $\sim 20$ µs, and the I-V gain of preamplifier is $\sim 20,000$. The theoretically calculated result $a = 1.55 \times 10^8$ is in accordance with the fitted parameter $(1.34 \pm 0.83) \times 10^8$.

$$a = \frac{T}{12} \left( \frac{2.355 \times \text{FUS}}{Q \times A} \right)^2 = 20 \times 10^{-6} \left( \frac{2.355 \times 1.8}{22 \times 10^{-12} \times 20,000} \right)^2 = 1.55 \times 10^8 \quad (4.1)$$
Intrinsic energy resolution without the digitizer’s contribution is estimated to be $\sim 4.56\%$ at 662 keV according to the fitting results. When $F_S \times 4^{\text{ENOB}}$ is more than $3 \times 10^8$ MHz (e.g. 300 MSPS 10-Bit ENOB), the energy resolution will approach the intrinsic energy resolution, and further enhancement of digitizer benefits a limited part for energy resolution. For some circumstance, such as portable gamma-ray spectrometer and radiation pagers, where the demand of energy resolution is $\sim 5\%$, a low-cost digitizer (e.g. 100 MSPS 9-Bit ENOB) is sufficient, system cost and power consumption (not only the ADC, but also the associated electronics, such as clock generator, driver, buffer and FPGA) will be dramatically saved.

![Figure 6. The energy resolution varies as $F \times 4^{\text{ENOB}}$.](image)

### 4.2 PSD FOM

Three typical PSD ratio distributions and FOM calculations are portrayed in figure 7 with a moderated neutron source with $^{252}\text{Cf}$ and polyethylene. They are measured by 500 MSPS, 250 MSPS and 125 MSPS digitizers with theoretical 12-Bit vertical resolution. As observed, the FOM becomes better with the increase of the sampling rate. The region associated with thermal neutrons is from $^6\text{Li}(n,\alpha)t$ ($Q = +4.79$ MeV), which has a Gamma Equivalent Energy (GEE) of 3.2 MeV (67% conversion efficiency) approximately. The region of thermal neutrons and the gamma-ray hits with GRR ranged from 1 to 2.5 MeV are selected to calculate the PSD FOM. The digitizer with 250 MSPS has a higher baseline than others and gamma-ray hits with high amplitude are cut, so it curls up at 2.5 GEE.

Similar in section 4.1, $F_S \times 4^{\text{ENOB}}$ is defined to represent the performance of digitizers. The two terms in the denominator of equation (2.8) are selected and fitted separately as figure 8 and figure 9. The fitting results for gamma-ray are shown in figure 8, where the fitting parameter $a$ can be calculated from equation (4.2). The integrated charge $Q_L$ and $Q_S$ are $\sim 8.8$ pC and $\sim 2.7$ pC respectively, the integration time $T_L$ and $T_S$ are 1 µs and 0.1 µs and the I-V gain of preamplifier is $\sim 20,000$. The theoretically calculated parameter $a = 5.8 \times 10^5$ for gamma-ray is in good accordance.
with the fitted parameter \((5.8 \pm 2.6) \times 10^5\).

\[
a = \frac{T_L Q_S^2 + T_S Q_L^2}{12 \times (Q_L + Q_S)^2} \left( \frac{\text{FUS}}{A} \right)^2 = \frac{10^{-6} \times (2.7 \times 10^{-12})^2 + 10^{-7} \times (8.8 \times 10^{-12})^2}{12 \times ((8.8 + 2.7) \times 10^{-12})^4} \left( \frac{1.8}{20,000} \right)^2 \approx 5.8 \times 10^5
\]

(4.2)

Moreover, the fitting results for neutrons are shown in figure 9, where the fitting parameter \(a\) can be calculated by equation (4.3). The corresponding integrated charge \(Q_L\) and \(Q_S\) are \(\sim 21.1\) pC and \(\sim 2.5\) pC respectively, \(T_L\) and \(T_S\) are \(1\) µs and \(0.1\) µs and the I-V gain of preamplifier is \(\sim 20,000\). The theoretically calculated parameter \(a = 1.1 \times 10^5\) for neutron is reasonably in consistence with the fitted parameter \((2.2 \pm 1.1) \times 10^5\).

\[
a = \frac{T_L Q_S^2 + T_S Q_L^2}{12 \times (Q_L + Q_S)^2} \left( \frac{\text{FUS}}{A} \right)^2 = \frac{10^{-6} \times (2.5 \times 10^{-12})^2 + 10^{-7} \times (21.1 \times 10^{-12})^2}{12 \times ((21.1 + 2.5) \times 10^{-12})^4} \left( \frac{1.8}{20,000} \right)^2 \approx 1.1 \times 10^5
\]

(4.3)

The FOM values are calculated from the above two fitting results, and corresponding results are shown in figure 10. From the figure, the FOM improves slightly when \(F_S \times 4^{\text{ENOB}}\) is more than \(3 \times 10^8\) MHz (e.g. 300 MSPS 10-Bit ENOB). If GRR need to be less than \(10^{-12}\), FOM should be larger than 3.0 and the \(F_S \times 4^{\text{ENOB}}\) should be greater than \(2.98 \times 10^7\) MHz (e.g. 100 MSPS 9.1-Bit ENOB). Based on the fitted results, the intrinsic FOM is estimated to be \(\sim 4.1\) assuming an ideal performance of ADC, which means an intrinsic GRR \(\sim 2.346 \times 10^{-22}\). Similarly, appropriate digitizer with moderate cost and power consumption can be selected based on our method for dedicated requirement of FOM or GRR.

5 Conclusions and future work

In this paper, the influences of the vertical resolution and sampling rate of digitizer on the energy resolution and PSD performance are quantitatively calculated. Seven digitizers with sampling rate ranging from 100 MSPS to 500 MSPS and vertical resolution varying from 12-Bit to 16-Bit are designed and integrated with CLYC detector for the verification of our calculation method. Experimental results are fitted and in good accordance with the calculation results. This paper contributes an effective guidance to choose proper digitizers to compromise with cost, power consumption, and practical performance, such as portable neutron and gamma-ray spectrometer, radiation pagers, high-energy particle detection in outer space, and thousands of PMTs used in discrimination of Cherenkov and scintillation light for neutrinos detection, etc.

In the future, in order to improve the energy resolution or PSD performance, intrinsic fluctuation from detectors and electronic noise from preamplifiers or PMTs will be intensive studied.

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Figure 7. The PSD ratio distribution of hits with $1 \sim 2.5$ MeV GEE.
Figure 8. Fitting curve and corresponding parameters for the FWHM of FOM values (gamma-ray) and $F_S \times 4^{\text{ENOB}}$.

Figure 9. Fitting curve and corresponding parameters for the FWHM of FOM values (neutron) and $F_S \times 4^{\text{ENOB}}$. 

$$
2.355 \times \frac{\sigma_{R_z}}{|R_z - R^*_z|} = \sqrt{\frac{a}{F_S \times 4^{\text{ENOB}} + b}} \\
\text{a} = (5.8 \pm 2.6) \times 10^3 \\
\text{b} = (3.0 \pm 0.3) \times 10^{-2}
$$
Figure 10. The FOM varies as $F_S \times 4^{\text{ENOB}}$.

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