Quasi-two-body decays $B \to K \rho \to K \pi\pi$ in perturbative QCD approach

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We analyze the quasi-two-body decays $B \to K \rho \to K \pi\pi$ in the perturbative QCD (PQCD) approach, in which final-state interactions between the pions in the resonant regions associated with the $P$-wave states $\rho(770)$ and $\rho'(1450)$ are factorized into two-pion distribution amplitudes. Adopting experimental inputs for the time-like pion form factors involved in two-pion distribution amplitudes, we calculate branching ratios and direct CP asymmetries of the $B \to K \rho(770), K \rho'(1450) \to K \pi\pi$ modes. It is shown that agreement of theoretical results with data can be achieved, through which Gegenbauer moments of the $P$-wave two-pion distribution amplitudes are determined. The consistency between the three-body and two-body analyses of the $B \to K \rho(770) \to K \pi\pi$ decays supports the PQCD factorization framework for exclusive hadronic $B$ meson decays.

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I. INTRODUCTION

Strong dynamics contained in three-body hadronic $B$ meson decays is much more complicated than in two-body cases, because of entangled nonresonant and resonant contributions, and significant final-state interactions [1]. Nonresonant contributions may not be negligible in these decays, as indicated by the observations made in Refs. [2-7]. Quasi-two-body channels through intermediate scalar, vector and tensor resonances, which produce hadron pairs with final-state interactions, usually dominate total branching fractions. An amplitude for a three-body hadronic $B$ meson decay, as a coherent sum of nonresonant and resonant contributions, leads to nonuniform distributions of events described by differential branching fractions [2-10] and of direct CP asymmetries [17-20] in a Dalitz plot [21]. Dalitz-plot analyses of abundant three-body hadronic $B$ meson decays from different collaborations (BaBar [2-4, 11-16], Belle [5, 6, 8-10] and LHCb [17-19]) have revealed valuable information on involved strong and weak dynamics.

On the theoretical side, substantial progress on three-body hadronic $B$ meson decays by means of symmetry principles and factorization theorems has been made, although rigorous justification of these approaches is not yet available. Isospin, U-spin and flavor SU(3) symmetries were adopted in [22-31], and the role of the CPT invariance in three-body $B$ meson decays was discussed in Refs. [32, 33]. The QCD factorization [34, 35] has been widely applied to studies of three-body charmless hadronic $B$ meson decays [36-48], including, for instance, detailed investigation on factorization properties of the $B^+ \to \pi^+\pi^+\pi^-$ mode in various regions of phase space [49]. The perturbative QCD (PQCD) approach based on the $k_T$ factorization theorem [50, 51] has been employed in Refs. [52-55], where strong dynamics between two final-state hadrons in resonant regions are factorized into a new nonperturbative input, the two-hadron distribution amplitudes. An advantage of the PQCD factorization approach is that both nonresonant and resonant contributions can be accommodated into this new input. A model that combines the heavy quark effective theory and the chiral Lagrangian was proposed in Ref. [57] to compute nonresonant decay amplitudes. The $B$ meson transition to a meson pair has been analyzed in the heavy-mass and large-energy limits [58], and in the light-cone sum rules [59] also in terms of two-meson distribution amplitudes. Nonresonant contributions to the above transition were evaluated in the heavy meson chiral perturbation theory [60] in Refs. [43-44, 46].

In this Letter we will focus on resonant contributions to three-body hadronic $B$ meson decays in the PQCD approach, extending our previous work on $S$-wave resonances to $P$-wave ones. We have determined the Gegenbauer moments of the $S$-wave two-pion distribution amplitudes by fitting our formalism to the $B^0_{(s)} \to J/\psi\pi^+\pi^-$ and $B_s \to \pi^+\pi^-\ell^+\ell^-$ data. Here we will consider the quasi-two-body decays $B \to K \rho \to K \pi\pi$, which receive contributions mainly from the $\rho(770)$ and $\rho'(1450)$ intermediate states. These resonant contributions are parametrized into the time-like pion form factors involved in the two-pion distribution amplitudes, for which there exist experimental inputs from the $e^+e^-$...
annihilation. It will be demonstrated that agreement of theoretical results with data can be achieved by choosing appropriate Gegenbauer moments of the $P$-wave two-pion distribution amplitudes. On one hand, the consistency between the three-body and two-body analyses of the quasi-two-body modes $B \to K\rho(770) \to K\pi\pi$ to be verified below supports the PQCD factorization for exclusive hadronic $B$ meson decays. On the other hand, with both the $S$-wave and $P$-wave distribution amplitudes being ready, we can proceed to predictions for branching ratios and direct $CP$ asymmetries of three-body hadronic $B$ meson decays in various localized regions of two-pion phase space.

The rest of this Letter is organized as follows. The PQCD framework for three-body hadronic $B$ meson decays is reviewed in Sec. II, where the $P$-wave two-pion distribution amplitudes up to twist 3 are parametrized. Numerical results for branching ratios and direct $CP$ asymmetries of the various $B \to K\rho \to K\pi\pi$ modes are presented and compared with those from the two-body analysis in Sec. III. The straightforward extension of the present formalism to other $P$-wave resonant contributions is highlighted. Section IV contains the Conclusion. The factorization formulas for the relevant three-body decay amplitudes are collected in the Appendix.

II. FRAMEWORK

In the rest frame of the $B$ meson, we write the $B$ meson momentum $p_B$ and the light spectator quark momentum $k_B$ as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad k_B = \left(0, \frac{m_B}{\sqrt{2}}x_B, k_{BT}\right),$$

in the light-cone coordinates, with $m_B$ being the $B$ meson mass and $x_B$ the momentum fraction. For the $B \to K\rho \to K\pi\pi$ decays, we define the resonant state momentum $p$ (in the plus $z$ direction) and the associated spectator quark momentum $k$, and the kaon momentum $p_3$ (in the minus $z$ direction) and the associated non-strange quark momentum $k_3$ as

$$p = \frac{m_B}{\sqrt{2}}(1, \eta, 0_T), \quad k = \left(\frac{m_B}{\sqrt{2}}z, 0, k_T\right), \quad p_3 = \frac{m_B}{\sqrt{2}}(0, 1 - \eta, 0_T), \quad k_3 = \left(0, \frac{m_B}{\sqrt{2}}(1 - \eta)x_3, k_{3T}\right),$$

with the variable $\eta = w^2/m_B^2$, $w = \sqrt{p^2}$ being the invariant mass of the resonant state, and the momentum fractions $z$ and $x_3$. The momenta $p_1$ and $p_2$ for the two pions from the resonant state have the components

$$p_1^i = \zeta \frac{m_B}{\sqrt{2}}, \quad p_1^- = (1 - \zeta)w \frac{m_B}{\sqrt{2}}, \quad p_2^i = (1 - \zeta) \frac{m_B}{\sqrt{2}}w, \quad p_2^- = \zeta w \frac{m_B}{\sqrt{2}},$$

in which the momentum fraction $\zeta$ of the first pion runs between 0 and 1.

In Ref. [55] we have introduced the distribution amplitudes for the pion pair [61, 63]

$$\phi_{\nu\bar{\nu}}(z, \zeta, w^2) = \frac{1}{2\sqrt{2N_c}} \int \frac{dy^-}{2\pi} e^{-izp^+y^-} (\pi^+(p_1)\pi^-(p_2)|\bar{\psi}(y^-)\gamma_\nu T\psi(0)|0),$$

$$\phi_{\nu\bar{\nu}}^\perp(z, \zeta, w^2) = \frac{1}{2\sqrt{2N_c}} \rho^+ w \int \frac{dy^-}{2\pi} e^{-izp^+y^-} (\pi^+(p_1)\pi^-(p_2)|\bar{\psi}(y^-)T\psi(0)|0),$$

$$\phi_{\nu\bar{\nu}}^3(z, \zeta, w^2) = \frac{1}{2\sqrt{2N_c}} \rho^+ f_{2\pi}^+ w^2 \int \frac{dy^-}{2\pi} e^{-izp^+y^-} (\pi^+(p_1)\pi^-(p_2)|\bar{\psi}(y^-)\sigma_{\mu\nu}n^\mu n^\nu T\psi(0)|0),$$

where $N_c$ is the number of colors, $n_\perp = (0, 1, 0_T)$ is a dimensionless vector, $T = \tau^3/2$ is chosen for the isovector $I = 1$ state, $\psi$ represents the $u$-d quark doublet, and $f_{2\pi}^+$ is a normalization constant. For $I = 1$, the $P$-wave is the leading partial wave, to which $\phi_{\nu\bar{\nu}}^1$ and $\phi_{\nu\bar{\nu}}^3(\perp)$ contribute at twist 2, and $\phi_{\nu\bar{\nu}}^1(\perp)$, $\phi_{\nu\bar{\nu}}^3(\perp)$, and $\phi_{\nu\bar{\nu}}^3(\perp)$ contribute at twist 3. With $w^2$ being a variable, the above two-pion distribution amplitudes contain both nonresonant and resonant contributions from the pion pair.

The $P$-wave two-pion distribution amplitudes are organized into

$$\phi_{\pi\pi}^{I=1} = \frac{1}{\sqrt{2N_c}} \left[ \rho \phi_{\nu\bar{\nu}}^{I=1}(z, \zeta, w^2) + \omega \phi_{\nu\bar{\nu}}^{I=1}(z, \zeta, w^2) + \frac{\rho_1 \rho_2 - \rho_1 \rho_2}{w(2\zeta - 1)} \phi_{\nu\bar{\nu}}^{I=1}(z, \zeta, w^2) \right],$$
FIG. 1: Typical Feynman diagrams for the quasi-two-body decays $B \to K \rho \to K \pi \pi$, in which the symbol $\otimes$ stands for the weak vertex, $\times$ denotes possible attachments of hard gluons, and the green rectangle represents intermediate states.

whose components are parametrized as

$$
\phi_{i\nu\nu+}^I(z, \zeta, w^2) \equiv \phi^0(z, \zeta, w^2) = \frac{3F_\pi(w^2)}{\sqrt{2N_c}} z(1 - z) \left[ 1 + a_2^0 C^{3/2}_2(1 - 2z) \right] P_1(2\zeta - 1),
$$

$$
\phi_{i\nu\nu-}^I(z, \zeta, w^2) \equiv \phi^a(z, \zeta, w^2) = \frac{3F_\pi(w^2)}{2\sqrt{2N_c}} (1 - 2z) \left[ 1 + a_2^a (1 - 10z + 10z^2) \right] P_1(2\zeta - 1),
$$

$$
\phi_{i\nu\nu+}^{I-1}(z, \zeta, w^2) \equiv \phi^0(z, \zeta, w^2) = \frac{3F_\pi(w^2)}{2\sqrt{2N_c}} (1 - 2z)^2 \left[ 1 + a_2^C C^{3/2}_2(1 - 2z) \right] P_1(2\zeta - 1),
$$

with the Gegenbauer polynomial $C^{3/2}_2(t) = 3(5t^2 - 1)/2$ and the Legendre polynomial $P_1(2\zeta - 1) = 2\zeta - 1$. In principle, the time-like form factors associated with the second Gegenbauer moments $a_{20,1s}^B$ differ from $F_{\pi,s,t}(w^2)$ associated with the leading ones. Here we assume that they are the same, which can then be factored out and serve as the normalization of the two-pion distribution amplitudes. The moments $a_{20,1s}^B$ will be regarded as free parameters and determined in this work. Up to the second Gegenbauer terms, the Legendre polynomial $P_1(2\zeta - 1)$ also contributes. However, more unknown form factors will be introduced, and currently available data are not sufficient for their extraction.

The time-like pion form factor $F_\pi(w^2)$ has attracted considerable theoretical effort and been measured with high precision by the CMD-2, KLOE, BaBar, BESII, ALEPH, CLEO, and Belle Collaborations. The $\rho$ meson dominance model for $F_\pi$ has been established in Ref. [94]. Guaranteed by the Watson theorem, strong interactions between the $\rho$ meson and the pion pair, including elastic rescattering of the two pions, can be factorized into $F_\pi$. In experimental investigations of three-body hadronic $B$ meson decays, the $\rho$ meson contribution is usually parametrized as the Gounaris-Sakurai (GS) model based on the Breit-Wigner (BW) function. Taking into account the $\rho$-$\omega$ interference and excited-state contributions, we write $F_\pi$ as a coherent sum

$$
F_\pi(w^2) = \left[ \frac{1 + c_2 \rho \Gamma}{1 + c_2} \right] F_{\omega}(w^2, m_\omega, \Gamma_\omega) + \sum c_i \left[ \frac{1 + c_i}{1 + c_i} \right] F_{\omega,i}(w^2, m_i, \Gamma_i)^{-1},
$$

with $i = \rho'(1450), \rho''(1700)$ and $\rho''''(2254)$. The explicit expressions of the auxiliary functions GS and BW in Eq. (11) are referred to Refs. [86, 87]. The inputs for the masses $m$ and widths $\Gamma$ of $\rho', \rho''$, and $\rho''''$, and for the complex parameters $c$ can be found in Ref. [86]. We have $c_0 = 0$ for a charged $\rho$ meson, because of no interference between it and a $\omega$ meson. Note that the Gegenbauer moments $a_{20,1s}^B$ are the same for all the resonant states $\rho, \rho', \rho''$, in the above parametrization of the two-pion distribution amplitudes.

The quasi-two-body decays $B \to K \rho \to K \pi \pi$ can be also analyzed in an alternative approach based on two-body decays: the quark pair $qq$ from a hard decay kernel forms the $\rho$ meson, followed by its BW propagator, and then by the $\rho \to \pi \pi$ transition with the strength $g_{\rho\pi\pi}$. The equivalence between the framework with the $\rho$ meson propagator and the present one with the two-pion distribution amplitudes hints the relation

$$
F_\rho(w^2) \approx \frac{g_{\rho\pi\pi} w f_\rho}{D_\rho(w^2)},
$$

where $F_\rho$ represents the $\rho$ component of Eq. (11), $f_\rho$ is the $\rho$ meson decay constant, and $D_\rho$ is the denominator of the BW function for the $\rho$ resonance. We have the similar relations for the $\rho$ components in the other two form factors, $F_{\rho,i}(w^2) \approx g_{\rho\pi\pi} w f_\rho / D_\rho(w^2)$, in which the decay constant $f_\rho$ normalizes the twist-3 $\rho$ meson distribution amplitudes. Due to the dominance of the $\rho$ meson contributions to the time-like form factors, it is legitimate to postulate the approximation $F_{\rho,i}(w^2) \approx (f_\rho^2 / f_\rho) F_\pi(w^2)$. 

\[\text{FIG. 1: Typical Feynman diagrams for the quasi-two-body decays $B \to K \rho \to K \pi \pi$, in which the symbol $\otimes$ stands for the weak vertex, $\times$ denotes possible attachments of hard gluons, and the green rectangle represents intermediate states.}\]
TABLE I: PQCD results for the $f$ distribution amplitudes for a longitudinally polarized $f$ meson and $B$ meson (kaon, two-pion) modes are presented in Table I. The theoretical uncertainties come from the variations $m_{\pi}$ and the kaon mass $m_{K}$. The $B \rightarrow K\rho \rightarrow K\pi\pi$ decay amplitudes $A$ are collected in the Appendix, which are similar to those in Ref. [99] for the two-body $B$ meson decay into a pseudoscalar meson and a vector meson.

III. RESULTS

For the numerical study, we adopt the inputs (in units of GeV) [98]

$$A^{(f=4)}_{MS} = 0.250, \quad m_{B^{\pm,0}} = 5.280, \quad m_{K^{\pm,0}} = 0.494, \quad m_{K^{\pm,0}} = 0.498,$$

$$m_{\pi^{\pm}} = 0.140, \quad m_{\rho^{0}} = 0.135, \quad m_{\rho} = 0.775, \quad \Gamma_{\rho} = 0.149,$$

the mean lifetimes $\tau_{B^{0}} = 1.519 \times 10^{-12}$ s and $\tau_{B^{\pm,0}} = 1.638 \times 10^{-12}$ s, and the Wolfenstein parameters from Ref. [98]. The decay constant $f_{\rho}$ has been extracted from the $\tau^{\pm} \rightarrow \rho^{\pm} \nu$, decay rate for the charged $\rho^{\pm}$ meson and from $\rho^{0} \rightarrow e^{+}e^{-}$ for the neutral $\rho^{0}$ meson. In this work we take their arithmetic average value $f_{\rho} = (0.216 \pm 0.003)$ GeV [100,101]. The decay constant $f_{\rho}$ has been computed in lattice QCD [102,103], for which we choose $f_{\rho} = 0.184$ GeV [102]. The ratio $f_{\rho}^{B}/f_{\rho}$ then determines the ratios $F_{s,t}/F_{\pi}$ postulated in the previous section. The $B$ meson and kaon distribution amplitudes are the same as widely adopted in the PQCD approach [5,104,108].

TABLE I: PQCD results for the $CP$ averaged branching ratios and the direct $CP$ asymmetries of the $B \rightarrow K\rho \rightarrow K\pi\pi$ decays. The corresponding data are quoted from Particle Data Group [98].

| Mode | Results | Data [98] |
|------|---------|------------|
| $B^{+} \rightarrow K^{+}\rho^{0} \rightarrow K^{+}\pi^{+}\pi^{−}$ | $B(10^{-6}) = 3.42^{+0.78}_{−0.55}(\omega_{B}−0.44(a^{2})−0.39(m_{0}^{K}−0.38(a^{2})−0.29(a^{2}))$ | $4.75 \pm 0.03$ |
| $A_{CP}$ | $0.43^{±0.05}(\omega_{B}) ± 0.06(a^{2}) ± 0.03(m_{0}^{K}) ± 0.03(a^{2}) ± 0.01(a^{2})$ | $0.37 \pm 0.10$ |
| $B^{+} \rightarrow K^{0}\rho^{+} \rightarrow K^{0}\pi^{+}\pi^{0}$ | $B(10^{-6}) = 7.43^{−1.31}_{−0.12}(\omega_{B})^{−1.42(a^{2})−0.88(m_{0}^{K}−0.62(a^{2})−0.52(a^{2}))$ | $8.0 \pm 1.5$ |
| $A_{CP}$ | $0.15^{±0.03}(\omega_{B}) ± 0.01(a^{2}) ± 0.01(m_{0}^{K}) ± 0.01(a^{2}) ± 0.01(a^{2})$ | $−12.1 \pm 0.17$ |
| $B^{0} \rightarrow K^{+}\rho^{−} \rightarrow K^{+}\pi^{+}\pi^{−}$ | $B(10^{-6}) = 6.51^{−1.31}_{−1.12}(\omega_{B})^{−1.42(a^{2})−0.88(m_{0}^{K}−0.62(a^{2})−0.52(a^{2}))$ | $7.0 \pm 0.9$ |
| $A_{CP}$ | $0.31^{±0.09}(\omega_{B}) ± 0.02(a^{2}) ± 0.01(m_{0}^{K}) ± 0.01(a^{2}) ± 0.02(a^{2})$ | $0.20 \pm 0.11$ |
| $B^{0} \rightarrow K^{0}\rho^{0} \rightarrow K^{0}\pi^{+}\pi^{−}$ | $B(10^{-6}) = 3.76^{±0.03}(\omega_{B})^{−0.60(a^{2})−0.42(m_{0}^{K}−0.28(a^{2})−0.26(a^{2}))$ | $4.7 \pm 0.6$ |
| $A_{CP}$ | $0.06^{±0.01}(\omega_{B}) ± 0.00(a^{2}) ± 0.00(m_{0}^{K}) ± 0.00(a^{2}) ± 0.00(a^{2})$ | $−$ |

We first single out the $\rho(770)$ component of the time-like pion form factor in Eq. (11). The fit to the data in Table I determines the Gegenbauer moments $a_{0}^{2} = 0.25, a_{2}^{2} = 0.75$, and $a_{4}^{2} = −0.60$, which differ from those in the distribution amplitudes for a longitudinally polarized $\rho$ meson [109,110]. The resultant $CP$ averaged branching ratios ($B$) and direct $CP$ asymmetries ($A_{CP}$) for the $B^{+} \rightarrow K^{+}\rho^{0} \rightarrow K^{+}\pi^{+}\pi^{−}$, $B^{−} \rightarrow K^{0}\rho^{+} \rightarrow K^{0}\pi^{+}\pi^{0}$, $B^{0} \rightarrow K^{+}\rho^{−} \rightarrow K^{+}\pi^{−}\pi^{0}$ and $B^{0} \rightarrow K^{0}\rho^{0} \rightarrow K^{0}\pi^{+}\pi^{−}$ modes are presented in Table II. The theoretical uncertainties come from the variations
of the shape parameter of the $B$ meson distribution amplitude $\omega_B = 0.40 \pm 0.04 \text{ GeV}$, $a_2^B = -0.60 \pm 0.20$, the chiral scale associate with the kaon $m_0^K = 1.6 \pm 0.1 \text{ GeV}$, $a_2^K = 0.25 \pm 0.10$, and $a_2^\pi = 0.75 \pm 0.25$. The uncertainties from $\tau_{B\pi}$, $\tau_{B\rho}$, the Gegenbauer moments of the kaon distribution amplitudes, and the Wolfenstein parameters in [98] are small and have been neglected. It is observed that the uncertainties of $A_{CP}$ are much smaller than those of $B$, and that the consistency between our results and the data is satisfactory.

Examining the distributions of these branching ratios in the pion-pair invariant mass $w$, we find that the main portion of the branching ratios lies in the region around the pole mass of the $\rho$ resonance as expected: the differential branching ratios of the $B^\pm \rightarrow K^\pm \rho^0 \rightarrow K^{\pm} \pi^+ \pi^-$ decays in Fig. 2(a) exhibit peaks at the $\rho$ meson mass. The central values of $B$ are $1.78 \times 10^{-6}$ and $2.46 \times 10^{-6}$ for the $B^+ \rightarrow K^+ \rho^0 \rightarrow K^{\pm} \pi^+ \pi^-$ decay in the ranges of $w$, $[m_\rho - 0.5\Gamma_\rho, m_\rho + 0.5\Gamma_\rho]$ and $[m_\rho - \Gamma_\rho, m_\rho + \Gamma_\rho]$, respectively, which amount to $52\%$ and $72\%$ of $B = 3.42 \times 10^{-6}$ in Table I. The branching fraction $3.27 \times 10^{-6}$ is accumulated in the range $[2m_\rho, 1.5 \text{ GeV}]$ for this mode. Figure 2(b) displays the differential distributions of $A_{CP}$ for the four $B \rightarrow K \rho \rightarrow K \pi \pi$ modes, in which a falloff of $A_{CP}$ with $w$ is seen for $B^+ \rightarrow K^+ \rho^0 \rightarrow K^{\pm} \pi^+ \pi^-$, $B^+ \rightarrow K^0 \rho^+ \rightarrow K^0 \pi^+ \pi^0$, and $B^0 \rightarrow K^+ \rho^- \rightarrow K^+ \pi^- \pi^0$. It implies that the direct CP asymmetries in the above three quasi-two-body decays, if calculated as the two-body decays $B \rightarrow K \rho$ with the $\rho$ resonance mass being fixed to $m_\rho$, may be overestimated. The ascent of the differential distribution of $A_{CP}$ with $w$ for $B^0 \rightarrow K^0 \rho^0 \rightarrow K^0 \pi^+ \pi^-$ implies that its direct CP asymmetry, if calculated in the two-body formalism, may be underestimated.

To verify the above observation, we treat the $B \rightarrow K \rho \rightarrow K \pi \pi$ modes as the two-body decays $B \rightarrow K \rho$ in the PQCD approach [99] by imposing the replacement $\eta \rightarrow r_\eta^2$ for the momenta in Eqs. (2) and (3), with the mass ratio $r_\rho = m_\rho/m_B$. Employing the same Gegenbauer moments $a_{2i}^{B,\pi}$ for the $\rho$ meson distribution amplitudes, we obtain:

$$B^+ \rightarrow K^+ \rho^0 \begin{cases} B = (3.52^{+0.40}_{-0.34} m_0^K)^{+0.42}_{-0.38} (a_2^K)^{+0.25}_{-0.24} (a_2^\pi) \times 10^{-6}, \\ A_{CP} = (5.55^{+0.02}_{-0.02}) (a_2^K)^{+0.00}_{-0.01} (a_2^\pi) \times 10^{-6}, \end{cases}$$

$$B^+ \rightarrow K^0 \rho^+ \begin{cases} B = (7.66^{+1.09}_{-1.19} m_0^K)^{+1.04}_{-0.95} (a_2^K)^{+0.43}_{-0.41} (a_2^\pi) \times 10^{-6}, \\ A_{CP} = (0.22 \pm 0.03) (a_2^K)^{+0.05}_{-0.01} (a_2^\pi) \times 10^{-6}, \end{cases}$$

$$B^0 \rightarrow K^+ \rho^- \begin{cases} B = (6.92^{+1.58}_{-1.04} m_0^K)^{+0.68}_{-0.81} (a_2^K)^{+0.40}_{-0.42} (a_2^\pi) \times 10^{-6}, \\ A_{CP} = (0.34^{+0.06}_{-0.01}) (a_2^K)^{+0.03}_{-0.02} (a_2^\pi) \times 10^{-6}, \end{cases}$$

$$B^0 \rightarrow K^0 \rho^0 \begin{cases} B = (4.01^{+1.07}_{-0.71} m_0^K)^{+0.70}_{-0.63} (a_2^K)^{+0.55}_{-0.50} (a_2^\pi)^{+0.40}_{-0.35} (a_2^\pi) \times 10^{-6}, \\ A_{CP} = (0.04 \pm 0.01) (a_2^K) \times 10^{-6}, \end{cases}$$

The comparison of Table I with Eqs. (17)-(20) confirms that the branching ratios of the four quasi-two-body modes in the three-body and two-body frameworks are close to each other. The tiny distinction between them suggests that the PQCD approach is a consistent theory for exclusive hadronic $B$ meson decays. The total $A_{CP}$ for the decays $B^+ \rightarrow K^+ \rho^0 \rightarrow K^{\pm} \pi^+ \pi^-$, $B^+ \rightarrow K^0 \rho^+ \rightarrow K^0 \pi^+ \pi^0$, and $B^0 \rightarrow K^+ \rho^- \rightarrow K^{\pm} \pi^- \pi^0$ in Table I compared with the corresponding values in Eqs. (17)-(19), have been, as explained above, moderated by the finite width of the $\rho$ resonance appearing in the time-like form factor $F_\pi$. Because $A_{CP}$ in Table I agree better with the data, it may be more appropriate to treat $B \rightarrow K \rho$ as three-body decays.
TABLE II: PQCD predictions for the CP averaged branching ratios and the direct CP asymmetries of the $B \to K \rho' \to K \pi \pi$ decays.

| Mode                  | Results                  |
|-----------------------|--------------------------|
| $B^+ \to K^+ \rho^0 \to K^+ \pi^+ \pi^-$ | $B (10^{-7}) = 4.32_{-0.99}^{+1.17}(\omega_B)^{+0.81}_{-0.79}(a_{2^+})^{+0.09}_{-0.04}(a_2)_{m_K}^{+0.40}_{-0.46}(a_2)_{m_{K'},-0.17}(a_2)$ |
| $B^+ \to K^0 \rho^+ \to K^0 \pi^+ \pi^0$ | $\mathcal{A}_{CP} = 0.32_{-0.04}^{+0.06}(\omega_B)_{m_K}^{+0.03}_{-0.02}(a_2)_{1.26}_{-0.02}(a_2)_{m_K}^{+0.01}_{-0.01}(a_2)$ |
| $B^+ \to K^+ \rho^- \to K^+ \pi^- \pi^0$ | $B (10^{-7}) = 10.37_{-2.36}^{+3.14}(\omega_B)_{m_K}^{+1.03}_{-1.31}(a_2)_{m_K}^{+1.13}_{-0.52}(a_2)$ |
| $B^0 \to K^+ \rho' \to K^+ \pi^- \pi^0$ | $\mathcal{A}_{CP} = 0.12_{-0.02}^{+0.02}(\omega_B)_{m_K}^{+0.04}_{-0.02}(a_2)_{m_K}^{+0.01}_{-0.01}(a_2)$ |
| $B^0 \to K^0 \rho' \to K^0 \pi^+ \pi^-$ | $B (10^{-7}) = 7.61_{-1.90}^{+2.37}(\omega_B)_{m_K}^{+1.32}_{-1.03}(a_2)_{m_K}^{+1.17}_{-0.86}(a_2)_{m_{K'}}_{m_{K'}}^{+0.28}_{-0.22}(a_2)$ |
| $B^0 \to K^0 \rho' \to K^0 \pi^+ \pi^-$ | $\mathcal{A}_{CP} = 0.27_{-0.01}^{+0.01}(\omega_B)_{m_K}^{+0.08}_{-0.01}(a_2)_{m_K}^{+0.02}_{-0.02}(a_2)$ |
| $B^0 \to K^0 \rho' \to K^0 \pi^+ \pi^-$ | $B (10^{-7}) = 4.84_{-1.32}^{+2.01}(\omega_B)_{m_K}^{+0.11}_{-1.05}(a_2)_{m_K}^{+0.48}_{-0.46}(a_2)$ |

FIG. 3: CP averaged differential branching ratios for the decays $B^+ \to K^+ \rho^0 \to K^+ \pi^+ \pi^-$ and $B^+ \to K^+ \rho^0 \to K^+ \pi^+ \pi^-$.

The parametrization of the time-like pion form factor in Eq. (11) also allows to single out the the $\rho'(1450)$ component. Adopting the two-pion distribution amplitudes in Eqs. (8)-(10), we derive the CP averaged branching ratios and the direct CP asymmetries for the decays $B^+ \to K^+ \rho^0 \to K^+ \pi^+ \pi^-$, $B^+ \to K^0 \rho^+ \to K^0 \pi^+ \pi^0$, $B^0 \to K^+ \rho^- \to K^+ \pi^- \pi^0$, and $B^0 \to K^0 \rho' \to K^0 \pi^+ \pi^-$ listed in Table II, whose errors have the same sources as in Table I. We compare the differential branching ratios for the $B^+ \to K^+ \rho^0 \to K^+ \pi^+ \pi^-$ and $B^+ \to K^+ \rho^0 \to K^+ \pi^+ \pi^-$ decays in Fig. 3 whose difference is mainly governed by the corresponding BW functions. All these predictions can be confronted with data in the future.

To extract the branching ratios for the two-body decays $B \to K \rho'$ from the quasi-two-body ones $B \to K \rho' \to K \pi \pi$, we need the branching fraction for $\rho' \to \pi \pi$, which is inferred from the ratio of the widths, $\Gamma_{\pi \pi}/\Gamma_{\rho'}$. The width $\Gamma_{\pi \pi}$ for $\rho' \to \pi \pi$ was evaluated in the Nambu-Jona-Lasinio quark model, and found to be 22 MeV [111], consistent with 17 $\sim 25$ MeV obtained from the $e^+e^-$ annihilation data [112]. Taking $\Gamma_{\rho'} = 0.311 \pm 0.062$ GeV [112], we get the branching fraction $B(\rho' \to \pi \pi) = 4.56\% \sim 10.0\%$. The $\rho' \to \pi \pi$ branching fraction can be also estimated from the relation [113]

$$\Gamma_{\rho' \to \pi \pi} = \frac{g_{\rho' \pi \pi}^2}{6\pi} \frac{m_{\rho'}^3}{m_{\rho'}^3}.$$  (21)

The coupling $g_{\rho' \pi \pi}$ is read off the $\rho'$ component of the time-like form factor $F_\pi$ in Eq. (11) according to $F_{\rho'}(w^2) \approx g_{\rho' \pi \pi} w f_{\rho'}/D_{\rho'}(w^2)$ at $w = m_{\rho'}$, which is similar to Eq. (12) for the $\rho$ component. We adopt the decay constant $f_{\rho'} = 0.185_{-0.035}^{+0.036}$ GeV resulting from the data $\Gamma_{\rho' \to e^+e^-} = 1.6 \sim 3.4$ keV [112], which agrees with $f_{\rho'} = (0.186 \pm 0.005)$ GeV from the perturbative analysis in the large-$N_c$ limit [114], $f_{\rho'} = (0.186 \pm 0.014)$ GeV from the double-pole QCD sum rules [115], and $f_{\rho'} = 0.128$ GeV from the relativistic constituent quark model [116]. Equation (21) then yields $B(\rho' \to \pi \pi) = 10.04_{-2.61}^{+5.23}\%$, compatible with $B(\rho' \to \pi \pi) = 4.56\% \sim 10.0\%$ from the width ratio $\Gamma_{\pi \pi}/\Gamma_{\rho'}$. 

With $\mathcal{B}(\rho' \to \pi\pi) = 10.04\%$, we extract the $B \to K \rho'$ branching ratios from Table II (in units of $10^{-6}$),

$$\mathcal{B}(B^+ \to K^+ \rho^0) = 4.30\pm^{+1.16}_{-0.99}(\omega_B)+0.80(a_2^0)+0.59(a_4^0)+0.40(m_0)+0.13(a_0^0),$$

(22)

$$\mathcal{B}(B^+ \to K^0 \rho^+ ) = 10.33\pm^{+3.71}_{-2.35}(\omega_B)+3.13(a_2^0)+0.52(a_4^0)+0.26(m_0)+0.41(a_0^0),$$

(23)

$$\mathcal{B}(B^0 \to K^+ \rho^- ) = 7.57\pm^{+2.36}_{-1.85}(\omega_B)+1.31(a_2^0)+1.10(a_4^0)+0.86(m_0)+0.38(a_0^0),$$

(24)

$$\mathcal{B}(B^0 \to K^0 \rho^0) = 4.82\pm^{+1.82}_{-1.31}(\omega_B)+1.11(a_2^0)+0.50(a_4^0)+0.47(m_0)+0.14(a_0^0).$$

(25)

Note that the data $\mathcal{B}(B^0 \to K^+ \rho^- ) = (2.4\pm0.1\pm0.6) \times 10^{-6}$ from BaBar [117] by assuming $\mathcal{B}(\rho' \to \pi\pi) \approx 100\%$ is much larger than Eq. (23) based on $\mathcal{B}(\rho' \to \pi\pi) = 4.56\% \sim 10.04\%$ or $10.04\%$, and the data $\mathcal{B}(B^0 \to K^+ \rho^- ) = (6.6\pm0.5\pm0.8) \times 10^{-6}$ in Ref. [117].

The branching ratios and the direct CP asymmetries of the quasi-two-body decays $B \to K(\omega, \rho', \rho'') \to K\pi\pi$ can be predicted by singling out the corresponding components in the time-like form factor $F_\pi$ in principle, since the Gegenbauer moments of the $P$-wave two-pion distribution amplitudes have been determined. This is a merit of our PQCD formalism for three-body hadronic $B$ meson decay. Besides, we can extract, for example, the $B \to K\omega$ branching ratios from the predictions for the $B \to K\omega \to K\pi\pi$ modes, given the $\omega \to \pi\pi$ branching fraction. We will leave the above observables to future studies.

IV. CONCLUSION

In this paper we have applied the PQCD approach to the quasi-two-body decays $B \to K\rho \to K\pi\pi$, which were analyzed in both three-body and two-body factorization formalisms. In the former strong dynamics between the $P$-wave resonances and the pion pair, including two-pion final-state interactions, is parametrized into the two-pion distribution amplitudes. The advantage of this approach is that the time-like pion form factor $F_\pi$ involved in the two-pion distribution amplitudes accommodates both resonant and nonresonant contributions. Inputting $F_\pi$ extracted from the $e^+e^-$ annihilation data, we have calculated the branching ratios and the direct CP asymmetries of the $B \to K\rho \to K\pi\pi$ modes, whose agreement with the data was achieved by tuning the Gegenbauer moments of the $P$-wave two-pion distribution amplitudes. The consistency between the three-body and two-body analyses of the $B \to K\rho \to K\pi\pi$ branching ratios was verified, which supports the PQCD approach to exclusive hadronic $B$ meson decays. The comparison to the results from the two-body framework indicates that the direct CP asymmetries of the $B \to K\rho \to K\pi\pi$ modes have been moderated by the finite width of the $\rho$ resonance, and become closer to the data. It suggests that the three-body framework is more appropriate for studying quasi-two-body hadronic $B$ meson decays.

The contribution from the $\rho'$ intermediate state was simply singled out from the given time-like form factor $F_\pi$ in our formalism. Using the determined Gegenbauer moments of the $P$-wave two-pion distribution amplitudes, we have predicted the branching ratios and the direct CP asymmetries of the $B \to K\rho' \to K\pi\pi$ channels, and compared their differential branching ratios with the $B \to K\rho \to K\pi\pi$ ones. With the estimated $\rho' \to \pi\pi$ branching fraction, the two-body $B \to K\rho'$ branching ratios have been extracted from the results for the $B \to K\rho \to K\pi\pi$ decays. All these predictions can be confronted with future data. The same framework is applicable straightforwardly to other channels $B \to K(\omega, \rho', \rho'') \to K\pi\pi$ in principle. Moreover, with both the $S$-wave and $P$-wave distribution amplitudes being ready, we will proceed to predictions for differential branching ratios and direct CP asymmetries of three-body hadronic $B$ meson decays in various localized regions of two-pion phase space in a forthcoming paper.

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Appendix A: DECAY AMPLITUDES

The quasi-two-body \(B \to K\rho \to K\pi\pi\) decay amplitudes are given, in the PQCD approach, by

\[
A(B^+ \to K^+|\rho^0 \to |\pi^+\pi^-) = \frac{G_F}{2} \left\{ V_{ub}^* V_{us} \left[ \left( \frac{C_1}{3} + C_2 \right) (F_{T\rho}^{LL} + F_{A\rho}^{LL}) + C_1 (M_{T\rho}^{LL} + M_{A\rho}^{LL}) + \left( C_1 + \frac{C_2}{3} \right) F_{T\rho}^{LL} \right] + C_2 M_{T\rho}^{LL} \right\} - V_{tb}^* V_{ts} \left[ \left( \frac{C_3}{3} + C_4 + C_9 + \frac{C_{10}}{3} + C_{10} \right) (F_{T\rho}^{LL} + F_{A\rho}^{LL}) \right] + \left( C_5 + C_7 \right) M_{T\rho}^{LL} + \left( \frac{3C_9}{2} + \frac{C_{10}}{2} \right) F_{T\rho}^{LL}
\]

\[
A(B^+ \to K^+|\rho^0 \to |\pi^0\pi^0) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{us} \left[ \left( \frac{C_1}{3} + C_2 \right) F_{T\rho}^{LL} + C_1 M_{T\rho}^{LL} \right] - V_{tb}^* V_{ts} \left[ \left( \frac{C_3}{3} + C_4 + C_9 + \frac{C_{10}}{3} + C_{10} \right) F_{T\rho}^{LL} \right] + \left( C_4 + C_5 + C_7 \right) F_{T\rho}^{LL}
\]

\[
A(B^+ \to K^+|\rho^0 \to |\pi^0\pi^-) = \frac{1}{\sqrt{2}} \left\{ A(B^+ \to K^0|\rho^0 \to |\pi^+\pi^0) + A(B^+ \to K^+|\rho^0 \to |\pi^0\pi^-) \right\}
\]

in which \(G_F\) is the Fermi coupling constant, \(V's\) are the Cabibbo-Kobayashi-Maskawa matrix elements, and the amplitudes \(F(M)\) denote the factorizable (nonfactorizable) contributions. It should be understood that the Wilson coefficients \(C\) and the amplitudes \(F\) and \(M\) appear in convolutions in momentum fractions and impact parameters \(b\). With the ratio \(r = m_K^b/m_B\), the amplitudes from Fig. 1(a) are written as

\[
F_{T\rho}^{LL} = 8\pi C_F m_B^4 f_K \int dxdz \int db db'db'db' \{ \sqrt{\eta}(1-2z)(\phi^* + \phi^0) + (1+z)\phi^0 \} \times E_{lab}(t_{1a}) h_{1a}(x,z,b,b') E_{lab}(t_{1b}) h_{1b}(x,z,b,b')
\]

\[
F_{T\rho}^{SP} = -16\pi C_F m_B^4 f_K \int dxdz \int db db'db'db' \{ \sqrt{\eta}(2\phi^* - \sqrt{\eta} \phi^0) \} E_{lab}(t_{1a}) h_{1a}(x,z,b,b') \times E_{lab}(t_{1b}) h_{1b}(x,z,b,b')
\]

\[
M_{T\rho}^{LL} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dxdzdx_3 \int db db'db'db' \{ \phi(\phi^* - \phi^0) + (1-\eta)(1-x_3-x_B+z\eta)\phi^0 \} E_{1cd}(t_{1c}) h_{1c}(x,B,z,x_3,b,b,b)
\]

\[
+ \{ \phi(\phi^* + \phi^0) - \phi^0 \} E_{1cd}(t_{1d}) h_{1d}(x,B,z,x_3,b,b,b)
\]
\[ M_{F}^{L/R} = -32\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int b_{B}db_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ \sqrt{\pi}z(\phi_{K}^{0} - \phi_{K}^{0}')(\phi^{+} + \phi') \right] + \sqrt{\pi}(1 - x_{3})(1 - \eta_{0} - x_{B})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') + (-1)(1 - x_{3})(1 - \eta_{0} - x_{B})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') + \pi_{K}(\phi_{K}^{0} - \phi_{K}^{0}')^{2} \} \times E_{1cd}(t_{1d})h_{1d}(x_{B},z,x_{3},b_{d},b_{b}) \}
\]

with the color factor \( C_{F} = 4/3 \) and the kaon decay constant \( f_{K} \). The amplitudes from Fig. 1(b) are written as

\[ F_{\Delta\rho}^{LL} = 8\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ 2r\sqrt{\pi}z(\phi_{K}^{0} - \phi_{K}^{0}')(\phi^{+} + \phi') + r(1 - x_{3})(1 - \eta_{0}) - (1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{1ab}(t_{1a}) \times h_{ab}(z_{1},b_{a},b_{b}) \}
\]

\[ F_{\Delta\rho}^{RP} = 16\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ \sqrt{\pi}(1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') - r(1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{1ab}(t_{1a}) \times h_{ab}(z_{1},b_{a},b_{b}) \}
\]

\[ M_{\Delta\rho}^{LL} = 32\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ 2r(1 - x_{3})(1 - \eta_{0}) - (1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{1ab}(t_{1a}) \times h_{ab}(z_{1},b_{a},b_{b}) \}
\]

\[ M_{\Delta\rho}^{RP} = -32\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ \sqrt{\pi}(1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') + r(2 - x_{3})(1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') + r(1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{1cd}(t_{1d})h_{1d}(x_{B},z_{1},b_{a},b_{b}) \}
\]

with the B meson decay constant \( f_{B} \). The amplitudes from Fig. 1(c) are written as

\[ F_{\Delta\rho}^{LL} = 8\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ 2r(1 - x_{3})(1 - \eta_{0}) - (1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{2ab}(t_{2a})h_{2a}(x_{B},z_{1},b_{a},b_{b}) + [x_{B}(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi')] E_{2ab}(t_{2b}) \times h_{2b}(z_{1},b_{a},b_{b}) \}
\]

\[ M_{\Delta\rho}^{LL} = 32\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ (1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{2cd}(t_{2c})h_{2d}(x_{B},z_{1},b_{a},b_{b}) + [x_{B}(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi')] E_{2cd}(t_{2d})h_{2d}(z_{1},b_{a},b_{b}) \}
\]

\[ M_{\Delta\rho}^{RP} = 32\pi C_{F} m_{B}^{4}/\sqrt{2N_{c}} \int dxdydz \int bdbdb_{B}d_{b}b_{B}(x_{B},b_{B}) \{ \left[ (1 - x_{3})(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi') \right] E_{2cd}(t_{2c})h_{2d}(x_{B},z_{1},b_{a},b_{b}) + [x_{B}(1 - \eta_{0})(\phi_{K}^{0} + \phi_{K}^{0}')(\phi^{+} + \phi')] E_{2cd}(t_{2d})h_{2d}(z_{1},b_{a},b_{b}) \}
\]
The amplitudes from Fig. 1(d) are written as

\[
F_{AK}^{LL} = 8\pi C_F m_B^4 f_B \int dzdx_3 \int bdb_3 d\beta_3
\]

\[
\times \left\{ [x_3(x_1-\eta)-1](1-\eta)\phi_{K}^{\ast} \phi_{A}^{0} + 2\sqrt{\eta}(x_3(1-\eta)(\phi_{K}^{\ast} - \phi_{F}^{0}) - 2\phi_{K}^{0}) \phi_{A}^{0}] E_{3ab}(t_{3a})h_{3a}(x_3, b, b_3) + [z(1-\eta)\phi_{K}^{\ast} \phi_{A}^{0} + 2\sqrt{\eta}\phi_{K}^{0}((1-\eta)(\phi_{K}^{\ast} - \phi_{F}^{0}) + z(\phi_{A}^{0} + \phi_{F}^{0}))] E_{3ab}(t_{3b})h_{3b}(x_3, b, b_3) \right\},
\]

(A17)

\[
F_{AK}^{SP} = 16\pi C_F m_B^4 f_B \int dzdx_3 \int bdb_3 d\beta_3
\]

\[
\times \left\{ [2\sqrt{\eta}(1-\eta)\phi_{K}^{\ast} \phi_{A}^{0} + r(1-x_3)(\phi_{K}^{\ast} + \phi_{F}^{0}) \phi_{A}^{0} + r\eta(1+x_3)\phi_{K}^{0} - (1-x_3)\phi_{F}^{0}) \phi_{A}^{0}] E_{3ab}(t_{3a})h_{3a}(x_3, b, b_3) + [2r(1-(1-\eta))\phi_{K}^{\ast} \phi_{A}^{0} + z\sqrt{\eta}(1-\eta)\phi_{K}^{0} \phi_{A}^{0}] E_{3ab}(t_{3b})h_{3b}(x_3, b, b_3) \right\},
\]

(A18)

\[
M_{AK}^{LL} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dxdzdx_3 \int bdb_3 d\beta_3 \phi_B(x_B, b_B) \phi_B(x_B, b_B) [\sqrt{\eta}(1-\eta)(1-\eta)(1-\eta)\phi_{K}^{0} \phi_{A}^{0} - r(1+x_3)
\]

\[
\times (\phi_{K}^{\ast} - \phi_{K}^{0}) \phi_{A}^{0} - \eta[(1-x_3)(1-\eta) - (x_3 - z)(\phi_{K}^{\ast} + \phi_{F}^{0}) - 2\phi_{K}^{0}] \phi_{A}^{0}] E_{3cd}(t_{3a})h_{3a}(x_B, z, b_3, b_3) - [r(1-\eta)(x_3 - 1)(\phi_{K}^{\ast} - \phi_{K}^{0}) \phi_{A}^{0} + \sqrt{\eta}(x_3 - z)[r\sqrt{\eta}(\phi_{K}^{\ast} + \phi_{F}^{0}) \phi_{A}^{0} - (1-\eta)\phi_{K}^{0} \phi_{A}^{0}]] E_{3cd}(t_{3d})
\]

(A19)

\[
M_{AK}^{LR} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dxdzdx_3 \int bdb_3 d\beta_3 \phi_B(x_B, b_B) \phi_B(x_B, b_B) \left[ \sqrt{\eta}(1-\eta)(2-x_B-z)\phi_{K}^{0} \phi_{A}^{0} - r(1+x_3)
\right.
\]

\[
\times (\phi_{K}^{\ast} - \phi_{K}^{0}) \phi_{A}^{0} - \eta[(1-x_3)(1-\eta) - (x_3 - z)(\phi_{K}^{\ast} + \phi_{F}^{0}) - 2\phi_{K}^{0}] \phi_{A}^{0}] E_{3cd}(t_{3a})h_{3a}(x_B, z, b_3, b_3)
\]

(A20)

The hard functions \( h_{\alpha a} \), the hard scales \( t_{\alpha a} \), and the evolution factors \( E_{iab} \) and \( E_{icd} \), with \( i = 1, 2, 3, 4 \) and \( \alpha = a, b, c, d \), have their explicit expressions in the Appendix of Ref. [54]. Since the Legendre polynomial \( P_{1/2}(2\zeta - 1) \) in the \( P \)-wave two-pion distribution amplitudes appears as an overall factor in decay amplitudes, the integration over \( \zeta \) can be performed trivially, yielding a factor \( \int_{0}^{1} d\zeta (2\zeta - 1)^2 = 1/3 \) to branching ratios.

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[1] I. Bediaga, P. C. Magalhães, arXiv:1512.09284 [hep-ph].
[2] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 80 (2009) 112001.
[3] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 79 (2009) 072006.
[4] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 78 (2008) 052005.
[5] A. Garmash et al., Belle Collaboration, Phys. Rev. D 75 (2007) 012006.
[6] A. Garmash et al., Belle Collaboration, Phys. Rev. Lett. 96 (2006) 251803.
[7] T. Bergfeld et al., CLEO Collaboration, Phys. Rev. Lett. 77 (1996) 4503.
[8] J. Dalseno et al., Belle Collaboration, Phys. Rev. D 79 (2009) 072004.
[9] A. Garmash et al., Belle Collaboration, Phys. Rev. D 71 (2005) 092003.
[10] A. Garmash et al., Belle Collaboration, Phys. Rev. D 65 (2002) 092005.
[11] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 78 (2008) 012004.
[12] B. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. 99 (2007) 221801.
[13] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 74 (2006) 032003.
[14] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 72 (2005) 072003; 74 (2006) 099903(E).
[15] A. Garmash et al., Belle Collaboration, Phys. Rev. D 72 (2005) 052002.
[16] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 70 (2004) 092001.
[17] R. Aaij et al., LHCb Collaboration, Phys. Rev. D 90 (2014) 112004.
[18] R. Aaij et al., LHCb Collaboration, Phys. Rev. Lett. 112 (2014) 011801.
[19] R. Aaij et al., LHCb Collaboration, Phys. Rev. Lett. 111 (2013) 101801.
[20] B. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. 99 (2007) 161802.
[21] R. H. Dalitz, Phil. Mag. 44 (1953) 1068; Phys. Rev. 94 (1954) 1046.
[22] M. Gronau, J. L. Rosner, Phys. Lett. B 564 (2003) 90.
[23] G. Engelhardt, Y. Nir, G. Raz, Phys. Rev. D 72 (2005) 075013.
[24] M. Gronau, J. L. Rosner, Phys. Rev. D 72 (2005) 094031.
[25] M. Imbeault, D. London, Phys. Rev. D 84 (2011) 056002.
[26] M. Gronau, Phys. Lett. B 727 (2013) 136.
[27] B. Bhattacharya, M. Gronau, J. L. Rosner, Phys. Lett. B 726 (2013) 337.
[99] H. Li, S. Mishima, Phys. Rev. D 74 (2006) 094020.
[100] P. Ball, G. W. Jones, R. Zwicky, Phys. Rev. D 75 (2007) 054004.
[101] A. Bharucha, D. M. Straub, R. Zwicky, arXiv:1503.05534 [hep-ph].
[102] K. Jansen, C. McNeile, C. Michael, C. Urbach, Phys. Rev. D 80 (2009) 054510.
[103] C. Allton, et al., RBC and UKQCD Collaborations, Phys. Rev. D 78 (2008) 114509.
[104] V. M. Braun, et al., Bern-Graz-Regensburg Collaboration, Phys. Rev. D 68 (2003) 054501.
[105] D. Becirevic, V. Lubicz, F. Mescia, C. Tarantino, JHEP 05 (2003) 007.
[106] Z. J. Xiao, W. F. Wang, Y. Y. Fan, Phys. Rev. D 85 (2012) 094003.
[107] W. F. Wang, Z. J. Xiao, Phys. Rev. D 86 (2012) 114025.
[108] A. Ali, et al., Phys. Rev. D 76 (2007) 074018.
[109] T. Kurimoto, H. Li, A. I. Sanda, Phys. Rev. D 65 (2001) 014007.
[110] P. Ball, V. M. Braun, Y. Koike, K. Tanaka, Nucl. Phys. B 529 (1998) 323.
[111] M. K. Volkov, D. Ebert, M. Nagy, Int. J. Mod. Phys. A 13 (1998) 5443.
[112] A. B. Clegg, A. Donnachie, Z. Phys. C 62 (1994) 455.
[113] M. Benayoun, et al., Eur. Phys. J. C 2 (1998) 269.
[114] O. Catà, V. Mateu, Phys. Rev. D 77 (2008) 114009.
[115] M. S. M. de Sousa, R. R. da Silva, arXiv:1205.6793 [hep-ph].
[116] D. Arndt, C. R. Ji, Phys. Rev. D 60 (1999) 094020.
[117] J. P. Lees, et al., BABAR Collaboration, Phys. Rev. D 83 (2011) 112010;
B. Aubert, et al., BABAR Collaboration, arXiv:0807.4567 [hep-ex].