Three dimensional point cloud data reduction algorithm and application in tooth modelling

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Abstract. Reverse engineering is an advanced manufacturing technology, which can design, develop and innovate products. The processing technology of 3D point cloud data is the key of reverse engineering technology, and the simplification technology is an important part of data processing, which plays a decisive role in the quality and efficiency of subsequent surface modelling. The advantages and disadvantages of different point cloud data reduction algorithms are analysed. By combining the efficiency of three-dimensional mesh algorithm with the accuracy of curvature reduction method, a second reduction algorithm is proposed. Firstly, the algorithm uses the non-uniform mesh reduction algorithm of octree to simplify it, to ensure the data of feature points is not damaged, and then uses the curvature reduction method of B-spline surface fitting to simplify it. The experiment of tooth point cloud data reduction shows that this method can retain the details of the original data better on the basis of higher reduction ratio.

1. Introduction

Modern measurement technology has been quite advanced, in which measurement technology and measurement accuracy have reached a very high level. Therefore, the amount of point cloud data obtained by scanning measurement is often large and dense. When reconstructing the model, it is difficult to reconstruct the surface by directly using the point cloud data. Because the direct use of these complex data points will bring many problems, such as: computer processing efficiency is relatively low, storage space is insufficient, surface reconstruction time-consuming and so on. In addition, too many data points will affect the smoothness of the reconstructed surface, but the smoothness is very important in the product appearance design. Based on keeping the feature points that can reflect the product, removing many redundant data is helpful to shorten the modelling time and improve the modelling quality. Therefore, in reverse engineering, it is a key technology to simplify the measurement data.

In recent years, scholars at home and abroad have done a lot of work on point cloud data reduction technology, and obtained different algorithms for point cloud data reduction. Sun et al. [1] introduced the method of local surface interpolation, which effectively solved the problem that the size of bounding box was controlled by human, but this method was not suitable for the data with complex surface, and it was more suitable for the simple ruled surface data. Therefore, the method of...
encirclement is suitable for the parts with single surface feature, and the change of surface curvature is gentle. Based on angle method, Wang Zhiqing et al [2] proposed an algorithm to simplify point cloud by angle difference iteration. Compared with angle method, this method is more efficient and easier to identify feature point data. The advantage of this method is that the angle threshold and the angle used for comparison are constantly changing and showing a decreasing law. But the disadvantage of this method is that it needs human participation in data reduction, and the degree of automation is not high.

In reference [3], it is proposed to construct the bounding box first, regard the surface of the bounding box as the segmentation surface, then project the point cloud onto these segmentation surfaces, process the data into a scan line structure, and then use the angle chord height joint criterion method to simplify them one by one. The bounding box of this method cannot simplify the data points, but only "structure" the data points. Therefore, this algorithm is more suitable to simplify the scattered data with large curvature change and complex surface. But the efficiency of this algorithm is not high. In reference [4], put forward the uniform grid algorithm. The principle of the code refers to the principle of "median filter", which has been widely used in image processing. The advantage of this algorithm is that it expands the reduction range, can be applied to the single block data perpendicular to the scanning surface, and solves the defect of spline curve, but the disadvantage is also obvious, that is, using uniform mesh cannot capture the shape features of parts sensitively, even some point clouds with larger curvature will be lost. Considering the shortcomings of the uniform mesh method, Lee et al [5] proposed a three-dimensional mesh reduction method, but it did not solve the problem fundamentally. FAN et al [6] proposed a triangular mesh algorithm, which first triangulates data points, and then reduces the number of these triangular meshes to achieve data point reduction. This algorithm is more suitable for block data points or relatively complete 3D data, but the data points must be processed into a triangular grid first. However, it is very difficult to process complex plane and many scattered data points into a triangular grid, and its efficiency is also very low. Therefore, it will be a little difficult to apply it in practice. LI et al [7] proposed a data reduction algorithm for different plane curves.

To sum up, although many scholars put forward different reduction algorithms, there are some problems. Therefore, it is necessary to study the algorithm of 3D point cloud data reduction.

2. Evaluation method of point cloud reduction algorithm

Using the least number of data points to reflect the geometric characteristics of the three-dimensional surface of the original point cloud set is the standard of simplifying point cloud data. The set of given points is \( P = \{p_1, p_2, \ldots, p_n\} \), \( S \) is the two-dimensional surface represented by point set \( P \). Assuming the sampling rate of cloud data at a given point \( n < |P| \). Then the point set \( P \) needs to be reduced to \( P' \), so that \(|P'| < n \), and \( S' \) is the two-dimensional surface defined by \( P' \). Let \( e = (S, S') \) represent the normal distance between \( S \) and \( S' \). If \( e \) is smaller, point cloud \( P' \) meets the requirement of simplification.

In other words, the goal of reducing point cloud is to obtain a data point set \( P' \) and make \( d(S, S') \leq \varepsilon \) and \(|P'|\) minimum at the same time.

To find this data point set \( P' \), we need a point cloud reduction algorithm. To measure the advantages and disadvantages of this algorithm, not only is the fewer the points, the better, not the faster the reduction speed, the better. The correct idea is whether this algorithm can express the surface features of parts completely with the least data points, but also faster.

(1) Accuracy. That is, the error between the quasi synthetic surface and the real surface. This error value can meet the requirements if it is within the specified range. In addition, the simplified data points should represent the original surface model as much as possible.

(2) Simplicity. Based on ensuring the characteristics of the model represented by the point cloud data, the number of the reduced point cloud should be as small as possible. Point cloud data reduction is to reduce the number of point cloud data. At the same time, it needs to be noted that too few data
points will affect the triangulation and quadrilateral localization of data points. Therefore, data reduction needs to be based on the actual situation.

(3) Speed. that is, the time it takes to simplify point cloud data. The less time it takes, the better, without affecting accuracy and simplicity.

In practical application, it is difficult to achieve accuracy, simplicity and speed at the same time. Most of the algorithms can meet one or two goals. The most ideal reduction effect is that the data can retain the detail features of the parts completely after the rapid reduction, and the amount of data is less. The distribution rule of the data points after simplification should be that there are more regional points with large curvature and fewer regional points with small curvature. Therefore, in order to improve the efficiency and accuracy of surface reconstruction, the data should be reduced unevenly according to the curvature of the surface.

3. The two-time reduction algorithm based on the combination of 3-grid method and curvature method

In this paper, the first reduction of point cloud data is carried out by using the non-uniform grid method of octree. The purpose is to delete data points as much as possible based on ensuring the key features of point cloud data, to improve the efficiency of curvature calculation. Secondly, the point cloud is simplified, and the curvature reduction algorithm of B-spline surface fitting is used.

3.1. One-time non-uniform mesh reduction algorithm

First, the data points are reduced by the non-uniform grid of octree. The data points are divided by the octree structure space, which is a cube space surrounded by a piece of data point cloud and sorted. If the cube is regarded as the root node of the octree, and then the cube is divided evenly, eight small cubes will be divided, so the number of sub cubes is the power of 2. Each cube can be used as a node to divide eight sub cubes, which are evenly subdivided. In the X, y, z direction, half of the side length of the upper cube is the side length of the next cube. After these processes, the first simplification of point cloud data is realized, and the simplification process is shown in Figure 1. The reduced data is used as the initial data of the second curvature reduction.

3.2. Quadric curvature reduction algorithm

Set point \( P \) as a point in the discrete point, \( n \) as the unit normal vector of point \( P \), \( r_u \) as the tangent vector of point \( P \), \( r_v \) as the tangent vector of its direction, and construct the local coordinate system of the surface. The surface in the neighborhood of the point is expressed as a quadric surface of \( N(u, v) = (u, v, s(u, v)) \), where \( s(u, v) = au^2 + buv + cv^2 \), \( a, b, c \) is the coefficient to be determined. \( s_u, s_v, s_{uv}, s_{uu}, s_{vv}, s_{uv} \) is the first and second order differential of function \( s \) with respect to \( u \) and \( v \) respectively, then the unit normal vector \( n \) of the surface is

\[
 n = \left[ \begin{array}{c} 
 s_u \\
 s_v \\
 s_{uv} \\
 s_u \times s_v \\
 \end{array} \right]
\]

from the first basic form of the surface, we get

\[
 l = s_{uy} \cdot n, m = s_{vy} \cdot n, n = s_{uv} \cdot n
\]

from the second basic form of the surface, we get

\[
 l = s_{uy} \cdot n, m = s_{vy} \cdot n, n = s_{uv} \cdot n
\]

according to the Gauss curvature of the surface

\[
 K = \frac{LN - M^2}{EG - F^2}
\]

mean curvature

\[
 H = \frac{EN - 2FM + GL}{2(EG - F^2)}
\]
by formula

\[ k_{1,2} = H \pm \sqrt{H^2 - k} \]

Finally, the main curvatures \(A\) and \(B\) of a point are calculated, and the average curvatures \(C\) are selected to simplify the data points. After the curvature of the point cloud to be simplified is calculated by B-spline surface fitting, the process is as shown in Figure 2.

**Figure 1. Non uniform grid simplification process**

4. **The application of two-time reduction algorithm in tooth point cloud data reduction**

The second reduction algorithm proposed in this paper is applied to the reduction of tooth point cloud data. According to the actual application and accuracy requirements of teeth, the reduction rate of point cloud model of teeth is controlled at 65% to compare different reduction algorithms. Figure 3 (a) shows the original point cloud data, with a total of 15382 data points. Figure 3(b) is the data point simplified by the secondary reduction algorithm. Compared with the original point cloud, more points are reserved at the edge of the teeth, and less points are reserved in the flat area, which is more uniform. See Table 1 for specific data analysis.
Figure 2. Curvature reduction algorithm flow

Table 1. Tooth data

|                        | Bounding box method | Angle chord method | Second reduction method |
|------------------------|---------------------|--------------------|-------------------------|
| Area points with small curvature change | 3428                | 2793               | 2150                    |
| Area points with large curvature change | 2172                | 3758               | 3452                    |

The experimental results show that the point cloud reduction algorithm proposed in this paper is simple and efficient, which can be applied to the scattered and disordered point cloud data measured in reverse engineering. It can directly and effectively simplify many dense data, and retain more detailed information for areas with large curvature changes and more detailed features.
5. Conclusion

Point cloud data reduction is the premise of surface reconstruction, and its quality will directly affect the quality of reconstructed surface. At the same time, the higher the simplification of point cloud data, the faster the construction of surface. At present, there are many methods to simplify point cloud, but there are few methods with moderate reduction. The rapid automatic modeling technology from point cloud to surface has developed rapidly, but the high-precision surface reconstruction still relies heavily on human-computer exchange. In this paper, a second reduction algorithm is proposed, that is, firstly, the non-uniform mesh reduction algorithm of octree is used to simplify it, this time, the feature point data is not damaged, and finally, the curvature reduction method of B-spline surface fitting is used to simplify it. This method can keep the details of the original data better based on high reduction ratio. The experimental results show that the secondary reduction algorithm has high reduction ratio and can completely retain the details of the original data. Compared with curvature method, the efficiency of calculation is obviously improved, which provides great convenience for subsequent work such as surface modeling.

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