Comments on the nucleon spin composition and diquark correlations

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We discuss possible relation between recent lattice results on the flavor composition of the nucleon spin and the topology-induced quark correlations.

The so called “nucleon spin problem” is a part of a wider issue, of understanding of the hadronic structure functions. Multiple experiments have revealed that, contrary to perturbative expectations, the PDFs of the “sea” quarks and antiquarks of a polarized nucleon are rather strongly polarized, both in spin and even the flavor (the distributions of $\bar{u}, \bar{d}, \bar{s}$ are not the same).

Lattice simulations of QCD with realistic quark masses and inclusion of disconnected diagrams, which only recently became feasible, have provided further interesting observations. These comments were, in particular, triggered by Ref. [1], which in particular compared the flavor contributions of $u, d, s$ quarks and gluons to two observables, the nucleon spin and its total momentum. While in the latter case the contributions of the $d$ quark to nucleon’s total momentum is, as expected, close to half of that of $u$, a completely different flavor distribution is found for the spin: the contribution of the $d$ and $s$ quarks are much smaller, being $11 \pm 8\%$ and $9 \pm 4\%$ respectively. It is also interesting that the gluon contribution to both is, on the contrary, the same, $27 \pm 3\%$.

The main point of these comments is to suggest an explanation for the $d$ and $s$ quarks failing to contribute to the nucleon spin: namely that they mostly appear in the nucleon wave function in form of $spin$ zero diquarks, $ud$ and $us$.

Existence of diquark correlations in baryons and their parameters has been discussed in vast literature since the beginning of $SU(3)$ symmetry in 1960’s. During the last decade experimental discoveries of multiquark states with heavy $c, b$ quarks had initiated large activity in hadronic spectroscopy. Many of those papers also use diquarks as a tool, say reducing the number of bodies from five for pentaquark to three. These comments are not aimed at covering any of that, and in particularly not discussing diquarks containing heavy quarks.

Our focus is on the dynamical origins of diquark correlations, their relation to chiral symmetry breaking, and thus dependence on the quark (pion) mass. The points to be made below are all known but scattered in multiple publications. Therefore, we remind here the main points, once again.

Starting with the kinematics, it is obvious from Fermi statistics that scalar (spin zero) diquarks must be $antisymmetric$ in flavor, that is $ud$ and $us$ but not $uu$, while the vector ones (spin one) must be flavor symmetric.

Let me then present the symmetry argument, from [2], based on continuation in the number of colors $N_c$. In the massless two-color $N_c = 2$ QCD there exist additional Pauli-Gursey symmetry, which mixes quarks with anti-quarks. As a result, diquarks (which are of course color singlets) are degenerate with the corresponding mesons. Chiral symmetry breaking is in fact such that scalar diquarks are Goldstone bosons, like pions, and thus must also be massless, small size etc. The vector diquarks are degenerate with the vector $\rho, \omega$ mesons. Now, proceeding from $N_c = 2$ to $N_c = 3$ QCD, one expect to see continuity, in form of not massless but deeply bound scalar diquarks. So to say, the scalar diquark is no longer the pion twin, but remains its half-brother. Strong quark pairing in the scalar channel was then recognized as a basis for $color$ superconductivity in dense quark matter.

To proceed to more quantitative estimates one needs to have certain dynamical information about the binding effects. We will not repeat here discussion in [2] of the full topology-induced ’t Hooft Lagrangian, Fierz-rearranged to the diquark channels, but just note that
for two flavors its main attractive term is the square of the scalar diquark operator

\[ S_0 = \epsilon_{abc} (u_b^T C \gamma_5 d_c) \]  

(1)

where \( C \) is charge conjugation and \( T \) means the transposed spinor. The coefficient have the \( N_c \) dependence in a simple form of \( 1/(N_c - 1) \): which is 1 for \( N_c = 2 \), as required by the symmetry argument given above, and 1/2 for \( N_c = 3 \). This simple factors confirmed finding made in the instanton liquid framework previously \textsuperscript{3}, which concluded that the binding of the scalar diquark is very large, comparable to a single effective quark mass \( \sim 350 - 400 \text{ MeV} \).

In the last two decades there were multiple attempt to calculate properties of various diquarks on the lattice. Again, we do not intend to make a review of this literature, but mention one representative work \textsuperscript{4}, and in particular reproduce their Fig.8 here. It summarizes the dynamical quark results. Two obvious comments on these plots are: (i) the only channel in which significant interaction (attraction) is detected is the scalar diquark we discuss; (ii) the effect becomes more pronounced as the quark mass diminishes. Unfortunately, in this decade old paper the quark mass is still not what it should be, but one can at least extrapolate.

Many other experimental measurements have indications to quark-diquark structure of the nucleon. For example, JLAB experiment \textsuperscript{4} have studied intrinsic parton momenta and found them to be very different for \( u \) and \( d \) quarks, qualitatively consistent with it.

However, one also needs to add a disclaimer. Applications of the diquark model to multi-quark hadrons, such as pentaquarks \( \bar{s}(ud)^2 \) and dibaryons \( (ud)(us)(ds) \) \textsuperscript{5, 6} were based on a simple additive model. Unfortunately, later studies \textsuperscript{7} has revealed strong repulsion between two or more diquarks containing the same quark, pushing masses of exotic hadrons much higher than estimated in these works. Indeed, topology in vacuum is relatively dilute, and a single instanton can only provide one zero mode per flavor, and Pauli principle prevents binding of more than one diquark. Since such effect should be absent for perturbative forces, the issue of multi-quark hadrons needs to be investigated on the lattice more quantitatively.

Now, how one can test the quark-diquark idea on the lattice? One test would be to study the spin structure of hadrons, with a nonzero spin but without scalar diquarks. Such hadrons can be \( \rho \) mesons or baryon decuplet particles such as \( \Delta \). Unfortunately, both are resonances with about 100 MeV width, which complicates this approach.

Much more feasible – and in a way already done multiple times but not properly emphasized – is to systematically comparison of the couplings of the nucleon to all possible currents of the form \( \epsilon_{abc} (u_b^T C T \Lambda d_c) \Gamma^A u_a \) with various spin-color matrices \( \Gamma^A \).

Furthermore, one can do standard three-point correlators, measuring the expectations values of \textit{all} 4-quark operators of the types \( \hat{O}_T = [\epsilon_{abc} (u_b^T C T \Lambda d_c)]^2 \) over the nucleon. The quark-diquark model would predict that the density of scalar diquark, for \( \Gamma = \gamma_5 \), would be significantly enhanced compared to other operators. Such measurements are similar to what was already done for weak decays of the hyperons, although with different operators.

Finally, let us repeat here some arguments of why the topology-induced effects have much better chances to explain the spin and isospin sea polarization than the perturbative \( g \rightarrow q\bar{q} \) vertex. The latter is flavor blind and of vector structure \( \bar{q}LqL + \bar{q}RqR \), while the former is flavor asymmetric and violates chiral symmetry. As noticed in \textsuperscript{8}, the ’t Hooft Lagnagian would induce splitting functions for a \( u_L \) quarks only.

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**FIG. 1:** The measured point-to-point diquark correlators, normalized to such for free massless quarks versus distance. The open squares are for scalar diquark, others for three other diquark types. Two plots are for different quark masses indicated in the plot.
of the type
\[ u_L \rightarrow u_R(\bar{d}_Rd_L) \] (2)
which one needs to add to the perturbative Altarelli-Parisi evolution equation.

Let me end with a more general point. The problems of the nucleon structure and its
spin is just a small part of the problem of the vacuum structure. In particular, the di-
quark correlations cannot be treated in nonrel-
ativistic or other simplified pQCD-like models
which do not provide proper account for chiral
\(SU(N_f)\) and \(U(1)_a\) symmetry breaking and pions. These correlations are known to increase
strongly, as the quark mass decreases to its
physical values, even for rather small masses
as seen in the plot above. The extent to which
those correlations are or are not topology re-
lated remains to be studied on the lattice in
more detail.

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