The optimization of nuclear power plants operation modes in emergency situations

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Abstract. An emergency situations resulting in the necessity for temporary reactor trip may occur at the nuclear power plant while normal operating mode. The paper deals with some of the operation c aspects of nuclear power plant operation in emergency situations and during threatened period. The xenon poisoning causes limitations on the variety of statements of the problem of calculating characteristics of a set of optimal reactor power off controls. The article show a possibility and feasibility of new sets of optimization tasks for the operation of nuclear power plants under conditions of xenon poisoning in emergency circumstances.

1. Introduction

The present advancement of nuclear power allows building nuclear power plants in areas of unstable climatic conditions or with an increased risk of seismic activity, in remotely located areas where the nuclear power plant is the only source of energy, e.g. in the Arctic and Antarctic, as well as regions of political instability and terroristic threats [1]. Such power plants may undergo emergency situations. An example of an emergency is the earthquake and tsunami in Japan on March 11, 2011, which caused the accident at the Fukushima nuclear power plant. Thus, power plants can be exposed to emergencies, which cause abrupt nuclear reactor trips and their consequent idleness.

Almost all nuclear power reactors are currently thermal neutron reactors, for example, VVER, PWR, BWR type reactors. A significant physical feature of these reactors is the presence of xenon poisoning. Shutdowns at certain states of a reactor may cause long-term outages due to very high levels of xenon poisoning [2].

The process of xenon poisoning is described by the system of differential equations [3]

\[
\begin{align*}
\frac{dx(t)}{dt} &= \lambda_I(t) - \lambda_x x(t) - \sigma_x \varphi x(t) \\
\frac{di(t)}{dt} &= \sigma_x \varphi - \lambda_x i(t)
\end{align*}
\]

The reactor control is impossible in case of exceeding the marginal xenon-135 concentration \(x(t)\) level inside the reactor core [4, 5].

After reactor shutdown at \(\tau\), the minimal reactivity margin, needed for xenon poisoning compensation, changes. After the shutdown, the concentration of xenon first increases due to the decay of the iodine nuclei, and then decreases during the decay of xenon nuclei. The reactivity reserve \(\Delta \rho - x(t)\) guarantees the criticality of the reactor until \(t_1\) and after \(t_2\) instances of reactor shutdown transient. At \(t_1 < t < t_2\), the reactor power cannot be increased. In order for the reactor to be able to be
put into operation at any time, the reactivity margin must compensate for the absorption of the maximum concentration by the xenon. This value depends on the neutron flux density change mode before the reactor shutdown. Reserving an additional amount of reactivity leads to a loss in the reactor's power production, since it means that the excessive fuel is not burned up.

The time when emergencies occur cannot be predicted precisely. A significant difference between natural emergencies and accidents caused by the human factor and the mechanism failure is, as a rule, a warning from the special services about the possibility of natural emergencies. There is usually a threatened period, which is declared beforehand, given by its duration and probability distribution for the event time. Different problems may be stated [3] on optimizing xenon poisoning of a nuclear reactor. Some of these problems are discussed below.

2. The task of optimizing the reactivity reserve in case of accidental shutdown of the power unit

Problem statement. Find reactivity margin value to reserve so that the average damage due to random reactor shutdown would be minimal.

The increase of the reactor power level is impossible if reactivity margin value is less than \( \Delta \rho \), even if negative effects of the shutdown are eliminated after \( t_1 \). So the reactor power may be increased not before \( t_2 \).

In this case the reactor loses power production calculated as

\[
\Delta Q_2 = W_{\text{max}} \cdot (t_2 - t_1),
\]

where \( W_{\text{max}} \) - design capacity of a nuclear reactor, \( MW \).

Let the average value of power production loss of the reactor, which is shut down at random instance, be:

\[
\Delta \overline{Q} = p_0 \cdot \rho(t_1) \cdot W_{\text{max}} \cdot (t_2 - t_1),
\]

where \( p_0 \) - the probability of unplanned reactor shutdown;

\( \rho(t_1) \) - the probability of negative effects elimination after \( t_1 \).

Consider the case of the uniform probability distribution of the consequences elimination instance over \((0, t_2)\), that is \( f(t) = \frac{1}{t_2} \). Then the probability of consequences elimination after \( t_1 \) is:

\[
p(t_1) = \int_{t_1}^{t_2} f(t)dt = 1 - \frac{t_1}{t_2}.
\]

In this case the average power production loss is calculated as

\[
\Delta \overline{Q} = p_0 \cdot W_{\text{max}} \cdot \frac{(t_2 - t_1)^2}{t_2}
\]

In (3) \( t_1 \) and \( t_2 \) instances are functions of \( \Delta \rho \). This dependence can be derived from the solution of equations, describing poisoning transient after the reactor shutdown:

\[
\frac{dl}{dt} = -\lambda_I \cdot l
\]

\[
\frac{dX}{dt} = \lambda_I \cdot l - \lambda_x \cdot X
\]

given the initial conditions of steady state reactor poisoning:
\[
I(0) = \frac{\gamma_I \Sigma_f \phi_0}{\lambda_I} \\
X(0) = \frac{\gamma_I \Sigma_f \phi_0}{\lambda_x + \sigma_x \phi_0}
\]

The solution of (5) given the initial conditions is:
\[
X(t) = \frac{\gamma_I \Sigma_f \phi_0}{\lambda_I} \left\{ \frac{\lambda_I}{\lambda_x - \lambda_I} \left[ e^{-\lambda_I t} - e^{-\lambda_x t} \right] + \frac{\lambda_I}{\lambda_x + \sigma_x \phi_0} e^{-\lambda_x t} \right\}
\]

The concentration of xenon is connected with the reactivity loss:
\[
\rho_x = -\theta \frac{\sigma_x}{\Sigma_a} X(t)
\]

where \( \theta \) - thermal neutron utilization factor; \( \Sigma_a \) - macroscopic absorption cross-section, m\(^2\).

The reactivity compensation for xenon poisoning is defined as
\[
\Delta \rho = -\rho_x = \frac{\gamma_I \Sigma_f \phi_0 \sigma_x}{\lambda_I \Sigma_a} \theta \left( \frac{\lambda_I}{\lambda_x - \lambda_I} \left[ e^{-\lambda_I t} - e^{-\lambda_x t} \right] + \frac{\lambda_I}{\lambda_x + \sigma_x \phi_0} e^{-\lambda_x t} \right)
\]

The following estimations are valid for thermal reactors:
\[
\frac{\Sigma_f}{\Sigma_u} \approx 0.5, \quad \sigma_x \cdot \phi_0 \approx 9 \cdot 10^{-5} \text{s}^{-1}, \quad \theta \approx 0.8
\]

The numerical estimation of reactivity margin for any outage duration is calculated as:
\[
\Delta \rho = 0.308 \cdot (e^{-0.0756t} - e^{-0.1t}) + 0.019 \cdot e^{-0.0756t}
\]

It is clear, that \( t_1(\Delta \rho) \) and \( t_2(\Delta \rho) \) can be derived explicitly from (9). The average power production loss is the function of reactivity margin:
\[
\bar{\Delta Q} = p_0 \cdot W_{\text{max}} \cdot \frac{(t_2(\Delta \rho) - t_1(\Delta \rho))^2}{t_2(\Delta \rho)}
\]

The estimations show, that even with \( p_0 = 1 \) the power production loss given reactivity margin of 1\% is never more then \( \bar{\Delta Q} \approx 10^4 \div 10^5 \text{MW} \cdot 24h \). At the same time, the power production loss for maintenance of such reactivity margin is estimated as \( \approx 10^5 \text{MW} \cdot 24h \). Therefore, the reservation of additional reactivity margin is unnecessary. Another case is considered when unplanned shutdown causes serious economic circumstances. For example, if nuclear power plant is located in a remote area, its power production loss can’t be compensated by other sources of electricity. Then, the cost of reserving additional amount of reactivity and the damage, caused by unplanned shutdown, are comparable. Thus, the following optimization problem is stated:

find the amount of reactivity to reserve, so that the average damage, caused by unplanned shutdown, would be minimal; the optimization criterion has the form:
\[
\min_{\Delta \rho} \Delta Q = \frac{\Delta \rho}{a} + C \cdot p_0 \cdot p(t_1) \cdot W_{\text{max}} \cdot (t_2 - t_1)
\]

where \( C \) - the conversion factor of economic damage to the loss of power production. It can be defined as the division of economic damage by the cost of electricity production.

Then the problem takes the following form:
\[
\min_{\Delta \rho} \Delta Q = \frac{\Delta \rho}{a} + C \cdot p_0 \cdot (1 - \frac{t_1}{T}) \cdot W_{\text{max}} \cdot (t_2 - t_1)
\]
In fig. 1 the numerical estimations of power production loss, caused by the reservation of additional amount of reactivity, for various values of unplanned shutdown probability are plot. The calculations were performed given  
\[ a = 4.8 \cdot 10^{-6} \cdot \frac{1}{MW \cdot 24h}, \quad W_{max} = 3200 \text{ MW}. \]

![Figure 1. The dependence of estimated power production loss on the reactivity margin value](image)

The figure shows, that optimal case is relay mode in relation to \( C \cdot p_0 \) coefficient. Physically, it means that optimal value corresponds to either absence of reactivity margin (for \( C \cdot p_0 \leq 0.0471 \)), or having additional reactivity reserve for the reactor fast shutdown (for \( C \cdot p_0 > 0.0471 \)).

3. Optimization of the regime for changing the power of a nuclear reactor during a threatened period in emergency circumstances

The solution of the problem above shows, that it is unnecessary to reserve the maximum amount of reactivity in regular conditions, but there are special cases, when unplanned shutdown causes serious economic circumstances. Earthquake is an example of a situation, when other sources of power can’t compensate the loss. In such case the optimal solution is to reserve the amount of reactivity, sufficient for the reactor shutdown. This measure is extreme and in some cases another way can be considered. Indeed, when a short period of outage is assumed, which starts at random instance, described by some probability distribution, the problem of optimal power change mode can be stated.

Problem statement. Find the power change mode during emergency period to minimize the average amount of reactivity reserved for the compensation of maximum xenon concentration.

The optimization criterion has the form:

\[
\min_{0}^{T} \int \Delta \rho_{\text{max}} (U(t)) f(\tau) d\tau
\]  

(13)

where \( U(t) \) is the control during the emergency period:

\[
U(t) = \begin{cases} 0 \leq \varphi(t) \leq \varphi_{\text{max}}, \text{ for } 0 \leq t < \tau \\ 0, \text{ for } t \geq \tau \end{cases}
\]
\( \tau \) is random instance of emergency shutdown due to force-majeure circumstances, \( 0 \leq \tau \leq T \); 
\( f(\tau) \) is density of probability distribution for the instance of an emergency shutdown;

\[ \beta = \int_{0}^{T} f(\tau) d\tau \] is the probability of the reactor shutdown due to force-majeure circumstances.

Let \( \varphi(t) \) be the solution of (1). Classic optimization theory, applied to the problem of control in case of xenon poisoning (optimal performance problem, minimal or maximal power production problem etc.), gives the optimal solution as the composition of following modes: \( U = 0, \ U = \varphi_{\max}, \ U_{\text{classic}} = \varphi(t), \ U_{\text{bound}} = \eta(t) \). The last two of them can be accurately approximated by step changes of power.

In this research the solution of the optimization problem (13) was carried out for \( f(\tau) \) describing the uniform distribution and for step power change controls (see fig. 2): during the period \( \Delta t \) given, the power is kept at level \( \alpha \varphi_{\max} \), then increased to maximum level \( \varphi_{\max} \) during \( \Delta t \) and so on until shutdown is performed.

![Figure 2. The power change mode](image)

The table shows the values of the average reactivity reserve for various changes in capacity during the threatened period (\( T = 24 \) hours).

**Table 1.** The value of the average reserve of reactivity at various power reduction level (\( \alpha \)).

| \( \alpha \) | \( \Delta t \), hours |
|---|---|
| 1 | 2 | 3 | 4 | 5 | 10 | 24 |
| 0 | 2.04 | 2.03 | 2.11 | 2.24 | 2.39 | 2.65 | 2.65 |
| 0.3 | 2.20 | 2.20 | 2.19 | 2.18 | 2.19 | 2.17 | 1.95 |
| 0.5 | 2.33 | 2.32 | 2.31 | 2.30 | 2.30 | 2.28 | 2.05 |
| 0.8 | 2.52 | 2.52 | 2.51 | 2.51 | 2.51 | 2.49 | 2.40 |

For example, if \( \alpha = 0.3 \), a stepwise change in power was investigated with a power reduction of up to 30% of the nominal value for different durations of the steps. When \( \Delta t = 1 \) hour switching control from \( 0.3 \varphi_{\max} \) to \( \varphi_{\max} \) and vice versa was carried out every hour, and \( \Delta t = 24 \) hours with only a single reduction in power up to \( 0.3 \varphi_{\max} \). As follows from the table, it is this latter regime that requires the smallest average of the reactivity reserve for the possibility of escape from the iodine pit at any time \( \Delta \rho_{\max} = 1.95\% \). In the standard approach, when the reserve of reactivity is reserved for the complete
shutdown of the reactor immediately when the threat period is declared \((\alpha = 0, \Delta t = 24 \text{ hours})\) the reactivity margin \(\Delta \rho_{\text{max}} = 2.65\%\). Thus, the optimization effect is:

\[
S = \frac{\Delta \rho_{\text{max}}(\alpha = 0, \Delta t = 24) - \Delta \rho_{\text{max}}(\alpha = 0.3, \Delta t = 24)}{\Delta \rho_{\text{max}}(\alpha = 0, \Delta t = 24)} = \frac{2.65 - 1.95}{1.95} = 36\%
\]

The above results show that the optimization of the power change modes during the threatened period is appropriate and the optimal is a simple reduction of power to a certain level, the value of which depends on the specific parameters of the nuclear reactor.

4. The search for the interval of possible optimization of the power change mode in emergency circumstances

In the previous problem, the question was raised about the power change mode so as to minimize the average reactivity reserve. However, in practice, the reactivity reserve in the reactor can already be set, based on technological considerations. Then the task is to optimize the regime of capacity change during the threatened period within the existing limitations (on the amount of operational reactivity reserve and power level) [6].

From physical considerations it follows that the problem of optimal control in such a formulation with a restriction on xenon concentration has a solution not for any given duration of the control interval and not for any restriction on the xenon concentration. If the reactivity reserve is positive but does not compensate for the maximum poisoning, then at the beginning of the threatened period there is an interval of control duration values for which the solution of this optimal control problem does not exist because of the absence of solutions that satisfy the constraints.

Problem statement. Is there such a time period in which any management will lead to violation of the restrictions? Let the reactor operate at nominal (maximum possible) power and the reactivity margin for compensation of xenon poisoning is not. Then any decrease in power leads to the entry of the reactor into the iodine pit, that is, any optimization due to power change is impossible.

As it was mentioned above, the reactor shutdown at some set of states may cause a long-term reactor outage. That is why the condition for the values of both iodine-135 and xenon-135 at the end of process is introduced in order to limit the concentration of xenon-135 after the reactor shutdown. Finally, after the reactor shutdown occurs, there is an additional time span, reserved for the elimination of the accident’s consequences, when the reactor can’t operate.

Thus, the optimization problem statement conforms to the problem of optimal control on a limited interval with the limitation of xenon-135 concentration before shutdown and both iodine-135 and xenon-135 at the time of shutdown.

The threatened event time is random; it defines the duration of the process and the control function, which corresponds to the duration value. In practice, for the problem stated the optimal process can’t be determined unless the period of control is given. Nevertheless, if the optimal control search problem is resolved in advance for any process duration value, the functions derived and their characteristics may be useful for applied control modes development.

In the time axis, originating simultaneously with the start of the reactor control process, there exists the time interval, containing instants, when the reactor shutdown doesn't satisfy the conditions. Further this interval is denoted by \(\Delta_x\). The origin of this interval coincides with the origin of the time axis, the ending is defined by the parameters of the system and the limitations stated. The value of \(\Delta_x\) is difficult to obtain analytically, therefore numerical estimates are considered in the article.

One-group point model of a nuclear reactor [7] was used in the research. For the case of the nuclear reactor after having been shut down, the system of differential equations takes the following form

\[
\begin{align*}
\frac{dx(t)}{dt} &= \lambda_i (t) - \lambda_x x(t) \\
\frac{di(t)}{dt} &= -\lambda_i (t)
\end{align*}
\] (14)
Initial conditions: \( i(\tau) = i^0 \), \( x(\tau) = x^0 \).

Two ways of estimating the length of the interval were considered. The first way corresponds with the lower estimation.

Let \( t_0 \) be the origin of the threatened period and \( t_1 \) be the instant of the reactor shutdown. In these terms, if \( t > t_1 \) then \( \varphi(t) = 0 \) and the transients are described by (14).

Let \( \Phi \) be the set of admissible control functions \( \varphi(t) \), such that \( 0 \leq \varphi(t) \leq \varphi_{\text{max}} \), where \( t_0 \leq t \leq t_1 \) is the time span of the control process. The set of states of the model (1) is represented by the quadrant \( Q = \{(i,x) \mid i \geq 0, x \geq 0 \} \) of the phase plane \( Oix \).

Let \( A \) be the set of probable model states at time \( \tau \), given \( i(t_0) = i_0, x(t_0) = x_0 \) for any \( \varphi(t) \in \Phi \). Let \( B \) be the set of acceptable states at \( t = t_1 \) that is the end of the control process. In these terms, if \( A \cap B = \emptyset \) at some instant \( \tau \), then, if \( t_1 = \tau \), the problem is inconsistent. This is to be proved by the following.

To investigate the properties of the set \( B \), consider the function \( F(i,x) \) over the quadrant \( Q \); the function is defined by (14) solution properties as follows. Let \( i(t), x(t) \) be the solution of (14) given initial conditions \( i(t_1) = i_1, x(t_1) = x_1 \), where \( (i_1,x_1) \in Q \) and \( t \geq t_1 \). By construction, \( F(i_1,x_1) = \max_{t \in [t_1,\infty)} x(t) \). Consequently, \( B = \{(i,x) \in Q \mid F(i,x) \leq x_{\text{lim}} \} \).

The equation describing the phase curve for the system (2) has the form:

\[
x = \frac{\lambda_i}{\lambda_1 - \lambda_X} \left( \frac{\lambda_k}{i_m} - i \right), \quad \text{with } 0 \leq i \leq i_m.
\]

Let \( f(i) \) be the right-hand member of this equation; note also that \( f(i) \) is convex upwards at \( 0 < i < i_m \). Then the equiscalar lines of \( F(i,x) \) have the form of \( x = L(i) \), where

\[
L(i) = \begin{cases} f(i^*), & i^* \leq i \leq i_m, \\ f(i^*'), & 0 \leq i < i^* \end{cases}
\]

and \( i^* = i_m \left( \frac{\lambda_k}{\lambda_1} \right)^{\lambda_1/\lambda_k} \) - the maximum point of \( f(i) \) (see figure 3 below).

It is clear that the value of \( F(i,x) \) over the equiscalar line equals the maximal value of \( f(i^*) \).

![Figure 3. The form of the equiscalar line \( x = L(i) \)](image-url)

Consequently, the function \( L(i) \) is also convex upwards at \( 0 < i < i_m \); furthermore, the set \( B \) is also convex. The boundary of the set \( B \) at \( i > 0 \) and \( x > 0 \) is defined by the equiscalar line \( x = L(i) \), corresponding to the value \( F(i,x) = x_{\text{lim}} \).
The set $B$ is closed, and, if $(i_0, x_0) \notin B$, then there exists a circular area $U_0 = \{(i, x) \mid \sqrt{(i - i_0)^2 + (x - x_0)^2} \leq a\} \ (a > 0)$, such that $U_0 \cap B = \emptyset$. So, the estimation [8]

$$(i(t), x(t)) \in U_0 \text{ where } t_0 \leq t \leq a / M \text{ for any } \varphi(t) \in \Phi,$$ 

and $D = \{(i, x, \varphi) \in \mathbb{R}^3 \mid (i, x) \in U_0, \ 0 \leq \varphi \leq 1\}$, which follows from the boundedness of right-hand members of (1) over the set $D$, results in $\Delta_t > a / M > 0$, because for any $t_0 \leq t \leq a / M$ the set $A \subset U_0$ and, consequently, $A \cap B = \emptyset$.

More precise lower estimation of $\Delta_t$ may be obtained by adjusting the shape of sets $A$ and $B$. To simplify the computation procedure convex polygons are considered; they are denoted by $A_c$ and $B_c$, and contain sets $A$ and $B$ respectively. Thus, if $A_c \cap B_c = \emptyset$, then sets $A$ and $B$ do not intersect either.

The conditions, considered in this article, implying that the initial state $(i_0, x_0)$ coincides with the equilibrium point of the system (1) given $\varphi = \varphi_{\max}$ and $(i_0, x_0) \notin B$, then the construction of the set $B_c$ may be based on the following reasoning. In the coordinate system $O_i x_i h$ the tangential plane defined by the equation $h = F(i, x)$ at the point $(i_0, x_0, F(i_0, x_0))$ is considered. The $O_i x_i$ phase plane projection of the intersection line between the tangential plane and the plane $h = x_{\lim}$ may be used as a part of the boundary of the set $B_c$ (line $F$ in figure 4).

The sides of the polygon $A_c$, estimating the set $A$, are formed by $K$ estimations for the solutions of (1) denoted by $i(t), x(t)$ given $i(t_0) = i_0, x(t_0) = x_0$ and any $\varphi(t) \in \Phi$

$s_k(t) \leq \alpha_k (i(t) + \beta_k) x(t), \ |\alpha_k| + |\beta_k| > 0, \text{ for } t_0 \leq t \leq t_1, \ k = 1, \ldots, K.$

The estimations of such type are obtained by the integration of differential inequalities, which follow from (1) given the limitation $0 \leq \varphi(t) \leq \varphi_{\max}$, signs of summands in right-hand members of equations, as well as signs of coefficients $\alpha_k$ and $\beta_k$ for the specific estimation line. For example,

1) $s_1(t) = i_0 \exp(-\lambda_i (t - t_0)), \ \alpha_1 = 1, \ \beta_1 = 0$;

2) $s_2(t) = (x_0 - c_0) \exp(-\lambda_x + \lambda_i (t - t_0)) + c_0 \exp(-\lambda_i (t - t_0)), \ \text{where } c_0 = \frac{\lambda_i i_0}{\lambda_x + \lambda_i - \lambda_i}$,

$A = \sigma_x \varphi_{\max}, \ \alpha_2 = 0, \ \beta_2 = 1$;

3) $s_3(t) = i_0 + x_0 - (\lambda_x + A)((x_0 - d_0 - A/\lambda_x)(1 - \exp(-\lambda_x (t - t_0))) / \lambda_x + A(t - t_0) / \lambda_x + d_0(1 - \exp(-\lambda_i (t - t_0))) / \lambda_i), \ \text{where } d_0 = \frac{\lambda_i i_0 - A}{\lambda_x - \lambda_i}, \ \alpha_3 = 1, \ \beta_3 = 1$;

etc. The choice of the count $K$ of estimations and their coefficients $\alpha_k$ and $\beta_k$ may be varied to adjust the set $A_c$ (see figure 4 below).
Then the relative position of \( A_c \) and \( B_c \) is analyzed for various values of \( \tau \) and the point of time is estimated when the condition \( A_c \cap B_c = \{\emptyset\} \) is violated. The maximal duration of the time interval \( t_0 \leq \tau < t_0 + \Delta \tau \), where each point \( \tau \) satisfies \( A_c \cap B_c = \{\emptyset\} \), defines the lower estimation for \( \Delta \tau \).

The upper estimation of the length of the interval was based on the solution of the problem of optimal speed reactor power decrease with limited xenon concentration. The statement of this problem and analytical principles of the solution construction are stated in [9]. The duration of the numerically calculated process is the upper estimation for the interval. The duration of the process as the function of the xenon limitation value was plot (see figure 6 below).
As results of the research carried two ways of estimation of the time interval observed were developed and applied and the dependencies of the estimations as functions of xenon limitation value were plot. The results obtained may be used in further research of the improvements to the problem statement considered in this article.

5. The search of the best time shift for the quickest reactor recovery to the nominal power after the reactor trip

In the previously considered problems, it was about optimizing the operating mode under emergency circumstances of one power unit. However, as a rule, a nuclear power plant consists of several power units whose maneuvering capabilities may differ. For example, power units have different reactivity reserves, since they were overloaded at different times.

It proposes the formulation and solution of the optimization task [10] of choosing the best time shift between the refuelling of nuclear reactors, which allows the maximum rapidity to reach the rated power after the power plant is shut down.

For simplicity, the NPP consists of two VVER-type reactors; the reactivity margin of each of them for compensation of xenon poisoning depends on the time elapsed since the beginning of the campaign. \( T_0 \) is the operating time of the reactor between the overloads, \( \tau \) is the time shift between the refuelling of the two power units, \( \Delta t \) is the time required for refuelling. The reactivity of the reactors of the two power units is shown in figure 7 below.

![Figure 6. The estimations of the duration of segment \( \Delta \tau \) as the functions of the limitation value for the concentration of xenon: solid and dashed lines for upper and lower estimations respectively](image)

![Figure 7. Dependence of reactivity on time within the campaign](image)
With brief shifts $\tau(1)$ between refuelling after unscheduled trips of both reactors, a period exists when neither can restore nominal power, since both stay in iodine pit. For the rest of time both reactors can restore nominal power in no time. On the contrary, with large time shifts $\tau(2)$ one of the reactors at a certain point would possess reactivity margin to compensate the iodine pit, when the other still stays in the iodine pit.

Thus, the shift of $\tau$ significantly affects the maneuverability of nuclear power plants. Let us find the time shift after the refuelling should be in the reactors, in order to increase the power to the nominal level in the minimum time after the end of the threatened period.

If the reactor stays at such a campaign stage, that it is impossible or undesirable to handle the control rods, to get out of the iodine pit one should alter boric acid concentration in the active zone (letdown). Let $C_0$ is the concentration of boric acid at the beginning of the transient process, $G$ - letdown rate, $V$ - the volume of the primary circuit. Then the concentration of boric acid in the case of make-up by distillate will be calculated by the formula: $C(t) = C_0 \exp(-Gt/V)$. Reactivity margin due to boric acid presence in the coolant is proportional to its concentration. If it is required to release the reactivity margin $\Delta\rho_x$, the concentration of boric acid should be reduced in accordance with the following equation:

$$\rho(T) - \rho(t) = \rho(T)[1 - \exp(-Gt/V)] = \Delta\rho_x$$

Let the boric acid concentration time dependence within single campaign be approximated with enough accuracy by linear function. Thus, if the reactor was stopped at the time of the $T$ campaign, then it takes time for it to exit the iodine pit:

$$t = -\frac{V}{G} \ln(1 - \frac{\Delta\rho_x}{\rho_0 - \alpha t})$$

The further the reactor is from the beginning of the campaign, the more time it will take to purge it to provide a reserve of reactivity sufficient to compensate for xenon poisoning. If the reactor trip occurred in the interval $(0; \tau)$, then the reactivity reserve is greater for the first reactor. Therefore, it must be the first to reach power. If the stop has occurred in the interval $(\tau; T_0)$, then the second reactor.

Since the moment of trip is random, we can speak about average letdown time for starting one reactor at the power level depending on the time shift. Suppose that the moments of occurrence of emergency circumstances within the campaign are equally probable. The average time to power output will be expressed by the ratio:

$$\bar{t} = -\frac{V}{GT_0} \left[ \int_0^\tau \ln(1 - \frac{\Delta\rho_x}{\rho_0 - \alpha T})dT + \int_\tau^{T_0} \ln(1 - \frac{\Delta\rho_x}{\rho_0 - \alpha(T - \tau)})dT \right]$$

If we set the optimization task of finding the time shift $\tau$ so that in a minimum time after the cancellation of the threatened period, at least one power unit could reach its rated power. Then taking the derivative of average time with respect to the parameter $\tau$, we obtain for the case of two identical reactors $\tau = 50\% T_0$ (see figure 8 below). If we set the task of finding the time shift $\tau$ to minimize the time to power output after the end of the threatened period of the entire NPP, then in this situation the minimum will be reached at 0 and 100%, that is, in the absence of a shift (see figure 9 below).
Thus, it has been shown that the presence of a time shift between reactor overloads at nuclear power plants allows one unit to be produced as quickly as possible in the case of force majeure, but not all reactors can be simultaneously outputted to power. In the general, for NPP consisting of \( N \) reactors the optimum shift is \( 1/N \) of the reactor campaign.

6. Conclusions
The problems discussed above show that there is a possibility and feasibility of new sets of optimization tasks for the operation of nuclear power plants under conditions of xenon poisoning in emergency circumstances.

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