Biometric Identification Systems With Both Chosen and Generated Secret Keys by Allowing Correlation*

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SUMMARY We propose a biometric identification system where the chosen- and generated-secret keys are used simultaneously, and investigate its fundamental limits from information theoretic perspectives. The system consists of two phases: enrollment and identification phases. In the enrollment phase, for each user, the encoder uses a secret key, which is chosen independently, and the biometric identifier to generate another secret key and a helper data. In the identification phase, observing the biometric sequence of the identified user, the decoder estimates index, chosen- and generated-secret keys of the identified user based on the helper data stored in the system database. In this study, the capacity region of such system is characterized. In the problem setting, we allow chosen- and generated-secret keys to be correlated. As a result, by permitting the correlation of the two secret keys, the sum rate of the identification, chosen- and generated-secret key rates can achieve a larger value compared to the case where the keys do not correlate. Moreover, the minimum amount of the storage rate changes in accordance with both the identification and chosen-secret key rates, but that of the privacy-leakage rate depends only on the identification rate.

key words: Identification system, secrecy-leakage, privacy-leakage, binary sources, Gaussian sources

1. Introduction

Biometrics based identification and authentication has been drawing public attention increasingly. It is renowned for the features of providing high security and convenience since biological data (bio-data) of our humankind such as finger, eyes, and palm, which cannot be forgotten or lost like password or smart-card, are used [1]. Biometric identification systems (BISs) were first analyzed in [2] and [3], and the identification capacity of the BIS was characterized in [5]. In [4], a constraint (lossless coding) on the helper data stored in a public database was added, and extended work of [4] can be found in [5] to recover noisy reconstruction (lossy coding). Error exponents of the BIS are examined in [6] based on Arimoto’s arguments [7], and in [8] from information-spectrum perspectives [9].

The BIS with estimating both user’s index and secret key was first investigated in [10]. In this work, two common BIS models, generated secret BIS (GS-BIS) model and chosen secret BIS (CS-BIS) model, were analyzed. In the GS-BIS model, a secret key is extracted from a bio-data sequence, while in the CS-BIS model, the secret key is chosen independently of it. Furthermore, the GS-BIS model in the presence of an adversary was analyzed [11]. Recently, Yachongka and Yagi characterized the capacity regions of more general models, where the noise in the enrollment phase was taken into account, in [12] for the GS-BIS model via two auxiliary random variables (RVs) and in [13] for both models via one auxiliary RV. Another trend of studies is the BIS with one user, e.g., [14]–[17]. This system can be viewed as the source model with one-way communication only for the key-agreement problem considered in [15], where eavesdropper has no side information related to the source sequence. However, a privacy constraint, which was not imposed in [18], was added in these studies.

In the previous studies such as [10], [12]–[17], the chosen- and generated-secret keys are assumed in the separate models, namely, GS- and CS-BIS models, respectively. Then, a rising question is when the two keys are used in the same system, how the chosen- and generated-secret key rates affect the fundamental performance of the BIS. One more thing is if a larger amount of information can be conveyed to the decoder by allowing these secret keys to be correlated. The answers to these questions have not yet been known, and they are not trivial from the results of the previous studies.

In this paper, the BIS model in a novel setting, where the chosen- and generated-secret keys are used together, is proposed, and we are first characterize the optimal trade-off of identification, chosen- and generated-secret key rates under privacy and storage constraints for the discrete alphabet sources. Also, we allow the chosen- and generated-secret keys to be correlated at a certain level. In the derivation, it seems hard to bound the privacy-leakage rate directly in the converse part, and we newly establish a lemma to overcome such difficulty. In addition, in the direct part, the degree of correlation for the two keys (proof by cases) is carefully analyzed. As a result, the characterization shows that identification, chosen- and generated-secret key rates are in a trade-off relation, and a larger sum of these rates is achievable. The minimum storage rate (storage space) requires to be larger as the identification and chosen-secret key rates increase, but it is not affected by the generated-secret key rate. Unlike the storage rate, the privacy-leakage rate varies in accordance with only the changes of the identification rate. As special cases, this result reduces to several known characterizations.

*This paper was presented in part at the 2020 International Symposium on Information Theory and Its Applications. This study was supported in part by JSPS KAKENHI Grant Numbers JP20K04462, JP19J13686, JP18H01438, and JP17K00020. DOI: 10.1587/trans.E0.??1
providing in previous studies [13], [17], [19]. Moreover, extending the characterization for the system with discrete alphabets, the optimal trade-off regions for binary and Gaussian sources are derived. To illustrate the behaviors of the capacity region in the case where the two keys are allowed and disallowed to be correlated, some numerical calculations for the binary sources are given.

The organization of this paper is as follows: We describe the basic settings of the system model in Sect. 2. Our main result is presented in Sect. 3 and the proof of the main result is given in the Appendixes. Connections to the results in previous studies and examples are given in Sect. 4. Finally, a short concluding discussion is given in Sect. 5.

2. Notation and System Model

Calligraphic letter $\mathcal{A}$ stands for a finite set and its cardinality is written as $|\mathcal{A}|$. Upper-case such as $\mathcal{A}$ denotes a random variable (RV) taking values in $\mathcal{A}$ and lower-case $a \in \mathcal{A}$ denotes its realization. $\mathcal{A}^n = (A_1, \ldots, A_n)$ represents a string of RVs, taking values in $\mathcal{A}^n$, and subscripts represent the position of an RV in the string. $P_A(a) = \Pr[A = a]$, $a \in \mathcal{A}$, represents the probability distribution on $\mathcal{A}$. For integers $k$ and $t$ such that $k < t$, $[k : t]$ denotes the set $\{k, k + 1, \ldots, t\}$. A partial sequence from the $k$th symbol to the $t$th symbol is represented by $c^k_t$. $T^{(m)}_e(\cdot)$ denotes the strongly $e$-typical set [20], [21]. $H_p(\cdot)$ is the binary entropy, and $H_p^{-1}(\cdot)$ is its inverse function. The $*$-operator is defined as $a \ast b = a(1 - b) + (1 - a)b$. In $x$ is the natural logarithm of $x > 0$. $h(\cdot)$ is differential entropy.

The system model considered in this paper is illustrated in Fig. 1. $P_X$, $P_{Y|X}$, and $P_{Z|X}$ denote the biometric source, enrollment channel, and identification channel, respectively. $P_{Y|X}$ and $P_{Z|X}$ are discrete memoryless channels (DMCs). Let $\mathcal{I} = \{1 : M_I\}$ and $\mathcal{J} = \{1 : M_J\}$ be the sets of user's indexes and helper data. For each user $(i \in \mathcal{I})$, the chosen--;and generated-secret keys of $i$ are denoted by $S_C(i)$ and $S_G(i)$, respectively. Let $S_C = \{1 : M_C\}$ and $S_G = \{1 : M_G\}$ be the sets of the chosen- and generated-secret keys. Lowercase letters $s_C(i) \in S_C$ and $s_G(i) \in S_G$ stand for their realizations.

The BIS consists of two phases: Enrollment Phase and Identification Phase. Random vector $X^n_i$ denotes the source sequences of user $i \in \mathcal{I}$ and each symbol of $X^n_i$ is generated independently and identically distributed (i.i.d.) from $P_X$. Random vectors $Y^n_i$ and $Z^n_i$ are the outputs of the enrollment channel $P_{Y|X}$ and the identification channel $P_{Z|X}$, respectively, with having $X^n_i$ as input. Assume that the chosen-secret key is uniformly distributed on $S_C$, i.e., $P_{S_C}(s) = \frac{1}{M_C}$ for all $i \in \mathcal{I}$ and $s \in S_C$. In the Enrollment Phase, observing $Y^n_i$ and $S_C(i)$, the encoder $e$ generates the pair $(J(i), S_G(i)) = e(Y^n_i, S_C(i))$, where $J(i)$ is called a helper data and takes values in $\mathcal{J}$. This operation is repeated for all users. The $J(i)$ and $S_G(i)$ are stored at position $i$ in

footnote{1}{This assumption of finite alphabet is relaxed in Sect. 4.3 to consider continuous sources.}
One of the main results in this paper is presented below.

**Theorem 1.** The $\Gamma$-capacity region of the BIS is given by

$$
\mathcal{R}(\Gamma) = \left\{ (R_I, R_C, R_J, R_L) \in \mathbb{R}^4_+ : R_I + R_C \leq I(Z; U), \\
R_I + R_G \leq I(Z; U), \\
R_I + R_C + R_G \leq I(Z; U) + \min\{\Gamma, R_C, R_G\}, \\
R_J \geq I(Y; U) - I(Z; U) + R_I + R_C, \\
R_L \geq I(X; U) - I(Z; U) + R_I \quad \text{for some } U \text{ s.t.} \\
Z - X - Y - U \text{ with } |U| \leq |Y| + 2 \right\}.
$$

(9)

**Remark 1.** For the convenience of analysis, in this paper, we focus only on the case where the condition of (normalized) secrecy-leakage (cf. [8]) is imposed under a weak secrecy criterion. The achievability proof of Theorem 1 makes use of random coding arguments [20]. However, a privacy amplification technique developed in [22] based on information-spectrum approaches [9] can be used to show that the secrecy leakage under a strong secrecy criterion is achievable. More specifically, combining [22, Lemma 12] with [22, Lemma 3], the (unnormalized) secrecy-leakage of the BIS can be bounded by a negligible amount regardless of the block length $n$ and the capacity regions of both cases coincide. The readers may refer to [24, Appendix A] for detailed analysis.

Using a similar technique shown in [10, Sect. IV-A], one can easily check that $\mathcal{R}(\Gamma)$ is a convex region. For the detailed proof of Theorem 1, see Appendix A.

**Fig. 2** An explanation of the optimal values of each rate in Theorem 1 for the case where $\Gamma = \min\{R_C, R_G\}$.

In our setting, the decoder is required to reconstruct the user index and both secret keys. In [19], the authors showed that the sum of identification, generated- and secret key rates cannot be larger than $I(Z; U)$ if the right-hand side of (7) is replaced with a negligible amount (vanishing secrecy-leakage rate). However, since we permit the chosen- and generated-secret keys to be correlated (non-vanishing secrecy-leakage rate), the maximum recognizable value for the sum of these rates is $I(Z; U) + \Gamma$, which can exceed the result in [19] (cf. the middle band graph in Fig. 2). More precisely, $R_I$ and $R_C$ can be any value in the range of $[0, I(Z; U)]$ under a constraint that their sum should be less than $I(Z; U)$ as shown in the top band graph. The optimal value of the generated-secret key rate $R_G$ is $I(Z; U) + \Gamma - (R_I + R_C)$, which could be originally achieved at most $I(Z; U) - (R_I + R_C)$ for the case of vanishing secrecy-leakage rate [19].

The optimal amount of the storage rate is shown in the bottom band graph of Fig. 2 which is the sum of the two green parts $(I(Y; U) - I(Z; U) + R_J + R_C)$. The storage rate $R_J$ depends on both the identification rate $R_I$ and the chosen-secret key rate $R_C$, and this value is larger than the one derived in [13, Theorem 1]. This is because the information related to the chosen-secret key is needed to be stored in the DB, so that it can be reconstructed at the decoder reliably.

Since the minimum value of the storage rate of Theorem 1 is larger than the one in [13, Theorem 1], it is somehow expected that a larger storage rate might lead to leaking a more amount of user’s privacy. Nevertheless, the minimum amount of the privacy-leakage rates of Theorem 1 and [13, Theorem 1] is bounded by the same quantity, which is $I(X; U) - I(Z; U) + R_I$. Obviously, the chosen-secret key rate is not involved, and only the changes of the identification rate affect the minimum required amount of the privacy-leakage rate. The reason is because the bits related to the chosen-secret key stored in the DB is made perfectly confidential, e.g., using the one-time pad operation, and this portion makes no contribution to the privacy-leakage.

### 4. Special Cases and Examples

#### 4.1 Connections to Previous Results

We can check that Theorem 1 covers the several results provided in previous studies. For instance, in the case where the chosen- and generated-secret key do not correlate ($\Gamma = 0$), $\mathcal{R}(\Gamma)$ naturally reduces to the one given in [19, Theorem 1]. When there are no provision of secret keys ($R_C = 0$) and no allowance of secret-key correlation ($\Gamma = 0$), $\mathcal{R}(\Gamma)$ coincides with the one given in [13, Theorem 1]. In the case of no secret-key generation ($R_G = 0$) and no allowance of secret-key correlation ($\Gamma = 0$), the capacity region, denoted by $\mathcal{R}'$ in this case, is given in the following corollary.

**Corollary 1.**

$$
\mathcal{R}' = \left\{ (R_I, R_C, R_J, R_L) \in \mathbb{R}^4_+ : \begin{array}{l}
R_I + R_C \leq I(Z; U), \\
R_J \geq I(Y; U) - I(Z; U) + R_I + R_C, \\
R_L \geq I(X; U) - I(Z; U) + R_I \quad \text{for some } U \text{ s.t.} \\
Z - X - Y - U \text{ with } |U| \leq |Y| + 2 \right\}.
$$

(10)

Although the expression of $\mathcal{R}'$ and the one given in [13, Theorem 2] are different, it can be checked that both are identical. One can easily see that $\mathcal{R}'$ is contained in the region of [13, Theorem 2] due to the range of $R_J$. For proving the opposite relation, we choose a new test channel $P_{U'|U}$ satisfying that $R_I + R_C = I(U'; Z)$. We can pick such channel since $I(Z; U) \geq I(Z; U') \geq 0$ and $I(Z; U')$ is a continuous function of $P_{U'|U}$. The bounds on the storage...
and privacy-leakage rates become
\[
R_J \geq I(Y;U) - I(Z;U) + I(Z;U')
\]
\[
\geq I(Y;U') - I(Z;U') + I(Z;U') = I(Y;U')
\]
\[
R_L \geq I(X;U) - I(Z;U) + R_I
\]
\[
\geq I(X;U') - I(Z;U') + R_I,
\]
where (a) and (b) follow from the fact that \(I(Y;U|Z) \geq I(Y;U'|Z)\) and \(I(X;U|Z) \geq I(X;U'|Z)\), respectively. Hence, there always exists an auxiliary \(U'\) where an achievable rate tuple \((R_I, R_C, R_J, R_L)\) in the region of \([13, \text{Theorem 2}]\) is also included in \(\mathcal{R}'\).

Moreover, in the case where we set \(R_I = 0\) (no provision of secret keys), one can easily see that \(\mathcal{R}'\) is equivalent to the one given in \([17, \text{Theorem 1}]\). Moreover, in the case \(R_G = 0\) (no generation of secret keys), it can also be shown that \(\mathcal{R}'\) matches the region provided in \([17, \text{Theorem 2}]\) by a similar argument for proving that \(\mathcal{R}'\) and the region of \([13, \text{Theorem 2}]\) are the same (cf. \([11, \text{and } 12]\)).

**Corollary 2.**
\[
\mathcal{R}' = \left\{ (R_C, R_G, R_J, R_L) \in \mathbb{R}_+^4 : R_C + R_G \geq I(Z;U), R_J \geq I(Y;U) - I(Z;U) + R_C, R_L \geq I(X;U) - I(Z;U) \text{ for some } U \right\}
\]
\[
Z = X - Y - U \text{ with } |U| \leq |Y| + 2.
\]

When \(R_C = 0\) (no provision of secret keys), one can easily see that \(\mathcal{R}'\) is equivalent to the one given in \([17, \text{Theorem 1}]\). Moreover, in the case \(R_G = 0\) (no generation of secret keys), it can also be shown that \(\mathcal{R}'\) matches the region provided in \([17, \text{Theorem 2}]\) by a similar argument for proving that \(\mathcal{R}'\) and the region of \([13, \text{Theorem 2}]\) are the same (cf. \([11, \text{and } 12]\)).

### 4.2 Examples of Binary Sources

In this section, a numerical example of the rate region of the BIS for a binary hidden source is given. We consider the case where \(P_x(0) = P_x(1) = 0.5\), and the enrollment channel \(P_{Y|X}\) and the identification channel \(P_{Z|X}\) of the system are binary symmetric channels with crossover probabilities \(0 \leq p_E \leq 0.5\) and \(0 \leq p_D \leq 0.5\), respectively. First, we simplify the capacity region for this case by applying Mrs. Gerber’s Lemma (MGL) \([25]\) twice into two opposite directions. The simplification of the rate region of the BIS with one user for binary hidden sources was given in \([17]\), too. However, by introducing an additional parameter, our deriving method is simpler than the one shown in \([17]\). In the right-hand side of \([9]\), we have that
\[
I(Z;U) = 1 - H(Z|U),
\]
\[
I(Y;U) - I(Z;U) = H(Z|U) - H(Y|U),
\]
\[
I(X;U) - I(Z;U) = H(Z|U) - H(X|U).
\]

From the above relations, it indicates that to simplify the capacity region, we must maximize \(H(Y|U)\) and minimize \(H(Z|U)\) for fixed \(H(X|U)\). First, observe that since \(1 \geq H(X|U) \geq H(X|Y) = H_b(p_E)\), there must exist a \(\gamma\) satisfying that
\[
H(X|U) = H_b(\gamma p_E).
\]
where \(\gamma \in [0, 0.5]\). By applying MGL to the Markov chain \(U - X - Z\), we have
\[
H(Z|U) \geq H_b(H_b^{-1}(H(X|U)) + p_D) = H_b(\gamma + p_E p_D).
\]
Again, in the opposite direction, if MGL is applied to the Markov chain \(U - Y - X\), it follows that
\[
H(X|U) \geq H_b(H_b^{-1}(H(Y|U)) + p_E).
\]
As \(H(X|U) = H_b(\gamma + p_E)\), \([17]\) yields that
\[
H_b(\gamma + p_E) \geq H_b(H_b^{-1}(H(Y|U)) + p_E),
\]
and thus
\[
\gamma + p_E \leq H_b^{-1}(H(Y|U)) + p_E.
\]

Therefore, we obtain
\[
H(Y|U) \leq H_b(\gamma).
\]

In \([16]\) and \([20]\) for binary symmetric \(P_{U|Y}\) with crossover probability \(\gamma\), the minimum \(H(Z|U) = H_b(\gamma + p_E + p_D)\) and the maximum \(H(Y|U) = H_b(\gamma)\) are achieved. Therefore, the following theorem is obtained. We denote the \(\Gamma\)-capacity region for binary sources as \(\mathcal{R}_b(\Gamma)\).

**Theorem 2.** For binary hidden sources, \([9]\) reduces to
\[
\mathcal{R}_b(\Gamma) = \bigcup_{\gamma \in [0, 0.5]} \left\{ (R_I, R_C, R_G, R_J, R_L) \in \mathbb{R}_+^5 : R_I + R_C \leq 1 - H_b(\gamma + p_E + p_D), R_I + R_G \leq 1 - H_b(\gamma + p_E + p_D) + \min\{\Gamma, R_C, R_G\}, R_I + R_C + R_G \leq 1 - H_b(\gamma + p_E + p_D) + R_I + R_C, R_L \geq H_b(\gamma + p_E + p_D) - H_b(\gamma + p_E) + R_I \right\}.
\]

We calculate the rate region above under two settings; both \(R_I\) and \(\Gamma\) are zero, and \(R_I\) is zero, but \(\Gamma\) is a positive value. The crossover probabilities of enrollment and identification channels are set to be \(p_E = 0.03\) and \(p_D = 0.1\), respectively, which are close to the actual transition probabilities of the channels in the real-life systems \([10, 17]\). To recap how secret-key correlation affects the rate region of BIS, we further fix the chosen-secret key rate to be \(I(Z;U)\) and \(\frac{1}{2}I(Z;U)\), and investigate the optimal values of the generated-secret key and storage rates under these two settings. The numerical results are shown in Figures 3(a) and 3(b). In both figures, the graphs with blue circles and red asterisks represent the boundaries of the pair \((R_J, R_G)\) in
We discuss the case of Gaussian RVs \((X, Y, Z)\), where the alphabets are continuous. Assume that biometric sources \(X \sim \mathcal{N}(0, 1)\), where \(\mathcal{N}(0, 1)\) is the standard Gaussian distribution with mean zero and variance one. The enrollment and identification channels are modeled as \(Y = \rho_1 X + N_1\), and \(Z = \rho_2 X + N_2\), respectively, where \(|\rho_1|, |\rho_2| < 1\), and \(N_1 \sim \mathcal{N}(0, 1 - \rho_1^2)\) and \(N_2 \sim \mathcal{N}(0, 1 - \rho_2^2)\) are independent of each other and other RVs. By a similar procedure of deriving Theorem 1, it can be proved that the single-letter expression in (9) also holds for Gaussian sources and channels. However, since the alphabets, e.g., \(X, Y, Z,\) and \(U,\) are unbounded, the region is not directly computable. In this section, we aim to derive a parametric form for the Gaussian case via the constraints in (9).

Let \(\mathcal{R}_g(\Gamma)\) denote the \(\Gamma\)-capacity region for Gaussian sources and channels.

**Theorem 3.** For i.i.d. Gaussian sources, the \(\Gamma\)-capacity region of the BIS is given by

\[
\mathcal{R}_g(\Gamma) = \bigcup_{\alpha \in (0,1]} \left\{ (R_I, R_C, R_G, R_J, R_L) \in \mathbb{R}^5_+ : \right. \\
R_I + R_C \leq \frac{1}{2} \ln \left( \frac{1}{\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2} \right), \\
R_I + R_G \leq \frac{1}{2} \ln \left( \frac{1}{\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2} \right), \\
R_I + R_C + R_G \leq \frac{1}{2} \ln \left( \frac{1}{\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2} \right) + \min\{\Gamma, R_C, R_G\}, \\
R_J \geq \frac{1}{2} \ln \left( \frac{\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2}{\alpha} \right) + R_I + R_C, \\
R_L \geq \frac{1}{2} \ln \left( \frac{\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2}{\alpha \rho_1^2 + 1 - \rho_1^2} \right) + R_I \bigg\}. \tag{22}
\]

For proving (22), a technique of the converted system introduced in [26] plays an important role. The technique tell us that the joint density of the system \((Y = \rho_1 X + N_1, Z = \rho_2 X + N_2)\) is equivalent to that of

\[
X = \rho_1 Y + N_1', \\
Z = \rho_2 X + N_2 \tag{23}
\]
with $N'_1 \sim N(0, 1 - \rho_1^2)$. To derive the parametric form, we will make use of the relations (23) and (24) to prove (22) instead of the relation of the original system.

**Proof of Achievability:** Let $0 < \alpha \leq 1$. Also, let $U$ be an auxiliary Gaussian RV with mean zero and variance $1 - \alpha$, i.e., $U \sim N(0, 1 - \alpha)$, and $\Phi \sim N(0, \alpha)$. Choose $Y = U + \Phi$. Put this $Y$ into (23), we obtain $X = \rho_1 U + \rho_1 \Phi + N'_1$, and now put $X$ into (24), we have that $Z = \rho_1 \rho_2 U + \rho_1 \rho_2 \Phi + \rho_2 N'_1 + N_2$. Observe that $\text{Var} \left[ \rho_1 \Phi + N'_1 \right] = \rho_1^2 \text{Var} [\Phi] + \text{Var} \left[ N'_1 \right] = \alpha \rho_1^2 + 1 - \rho_1^2$ and $\text{Var} \left[ \rho_1 \rho_2 \Phi + \rho_2 N'_1 + N_2 \right] = \rho_1^2 \rho_2^2 \text{Var} [\Phi] + \rho_2^2 \text{Var} \left[ N'_1 \right] + \text{Var} \left[ N_2 \right] = \alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2$. Now it can be calculated that $I(Y; U) = \frac{1}{2} \ln \frac{1}{\alpha \rho_1^2 + 1 - \rho_1^2}$, and $I(Z; U) = \frac{1}{2} \ln \frac{1}{\rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2}$. Furthermore, it is not difficult to see that $I(Y; U) = I(Z; U) = \frac{1}{2} \ln \frac{1}{\alpha \rho_1^2 + 1 - \rho_1^2}$ and $I(X; U) - I(Z; U) = \frac{1}{2} \ln \frac{1}{\alpha \rho_1^2 + 1 - \rho_1^2}$. From (9), it is easy to see that the right-hand side of (22) is achievable for this choice of $U$.

**Proof of Converse:** Similar to the development of [14], for Gaussian sources, mutual information on the right-hand side of (9) can be expanded as $I(Z; U) = \frac{1}{2} \log(2\pi e) - h(Z|U)$, $I(Y; U) - I(Z; U) = h(Z|U) - h(Y|U)$, and $I(X; U) - I(Z; U) = h(Z|U) - h(X|U)$. To obtain the outer bound of Gaussian sources for the region (9), we need to derive the optimal lower bound on $h(Z|U)$ and the optimal upper bound on $h(Y|U)$ under a fixed condition of $h(X|U)$.

Now let us fix

$$h(X|U) = \frac{1}{2} \ln \left( 2 \pi e (\alpha \rho_1^2 + 1 - \rho_1^2) \right)$$  (25)

for $0 < \alpha \leq 1$. This is an appropriate setting since we have that $\frac{1}{2} \ln \left( 2 \pi e \right) = h(X) \geq h(X|U) \geq h(X|Y) = \frac{1}{2} \ln \left( 2 \pi e (1 - \rho_1^2) \right)$. By applying the conditional entropy power inequality (EPI) [27] to [24], it follows that

$$e^{2h(Z|U)} \geq e^{2h(\rho_2 Y|U)} + e^{2h(N'_1|U)} = \rho_2^2 e^{2h(Y|U)} + e^{2h(N'_1)}$$

$$= \rho_2^2 (2 \pi e (\alpha \rho_1^2 + 1 - \rho_1^2)) + 2 \pi e (1 - \rho_2^2)$$

$$= 2 \pi e (\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2),$$  (26)

and thus,

$$h(Z|U) \geq \frac{1}{2} \ln \left( 2 \pi e (\alpha \rho_1^2 \rho_2^2 + 1 - \rho_1^2 \rho_2^2) \right).$$  (27)

On the other hand, again applying the conditional EPI to (25), we have that

$$e^{2h(X|U)} \geq e^{2h(\rho_1 Y|U)} + e^{2h(N'_1|U)} = \rho_1^2 e^{2h(Y|U)} + e^{2h(N'_1)}.$$  (28)

Next, plugging the value of $h(X|U)$ into (28), it follows that

$$2 \pi e (\alpha \rho_1^2 + 1 - \rho_1^2) \geq \rho_1^2 e^{2h(Y|U)} + 2 \pi e (1 - \rho_1^2),$$  (29)

and finally, it can be concluded that

$$e^{2h(Y|U)} \leq 2 \pi e \alpha \text{ or } h(Y|U) \leq \frac{1}{2} \ln (2 \pi e \alpha).$$  (30)

Now using (25), (27), and (30), one can see that the right-hand side of (9) is contained in the right-hand side of (22), and thus the converse proof is completed.

5. Conclusion and Future Work

In this study, we proposed the BIS with both chosen- and generated-secret keys, and characterized the capacity region among identification, chosen- and generated-secret key, storage, and privacy-leakage rates for the system. The characterization shows that identification, chosen- and generated-secret key rates are in a trade-off relation, and by permitting the correlation of the two secret keys, a larger sum of these rates is achievable. In addition, larger memory space for the DB is required when the sum of identification and chosen-secret key rates increases. Unlike the storage rate, only the identification rate contributes to the minimum required amount of the privacy-leakage rate, but the chosen-secret key rate does not. As special cases, this characterization reduces to the results seen in [13] and [17].

For future work, an interesting problem is application of rate-distortion theory to the BIS. In [5], though lossy source coding was applied to the BIS, however, the model considered in the paper only dealt with user’s identification and requirements on secrecy and privacy were not imposed. Therefore, there are still rooms for discussions about lossy source coding for the BIS in which secrecy and privacy constraints are taken into account. Another problem is to extend the result in Sect. 4.3 to vector Gaussian sources and channels, and clarify the capacity region of the BIS. Actually, as it was mentioned in Sect. 1, there are similarities between the BIS and the key-agreement model. To obtain the capacity region of the BIS for vector Gaussian sources, the technique used in [28] for analyzing the optimal trade-off of the key-agreement model may be useful.

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Appendix A: Proof of Theorem 1

In this appendix, we provide the detailed proof of Theorem 1, the result for discrete sources and channels.

A. Converse Part

We consider a more relaxed case where RV $\mathbf{W}$ is uniformly distributed on $\mathcal{I}$, and (1), (4), and (6–8) are replaced by

$\Pr\{E(\mathbf{W}) \neq \hat{E}(\mathbf{W})\} \leq \delta$, \hspace{1cm} (31)

$\frac{1}{n}H(S_G(\mathbf{W})|W) \geq R_G - \delta$, \hspace{1cm} (32)

$\frac{1}{n}I(X_n^W;J(W)|W) \leq R_L + \delta$, \hspace{1cm} (33)

$\frac{1}{n}I(S_C(W);S_G(\mathbf{W})|W) \leq \Gamma$, \hspace{1cm} (34)

$\frac{1}{n}I(S_C(\mathbf{W}), S_G(\mathbf{W}); J(W)|W) \leq \delta$, \hspace{1cm} (35)

respectively. Note that the capacity region of the BIS under the average error criterion is fundamentally larger than the capacity region evaluated under the maximum error criterion. We demonstrate that even this case, the outer bound of the capacity region coincides with its inner bound derived under the circumstance that the prior distribution of $\mathbf{W}$ is unknown.

We assume that a rate tuple $(R_I, R_C, R_G, R_J, R_L)$ is achievable, implying that there exists a pair of an encoder and a decoder satisfying (2), (3), (5), and (31–35). First, we provide some useful lemmas. For $t \in [1 : n]$, we define an auxiliary RV $U_t = (Z_t^{-1}(W), T(W))$, where $T(W) = (J(W), S_C(W), S_G(W), \mathbf{W})$. We denote strings of RVs by $X_n^W = (X_1(W), \ldots, X_n(W))$ and $Y_n^W = (Y_1(W), \ldots, Y_n(W))$. Also, the partial random sequences $X_t(W) = (X_1(W), \ldots, X_t(W))$ and $Y_t(W) = (Y_1(W), \ldots, Y_t(W))$ represent strings of RVs from the first to $r$th positions in the sequences $X_n^W$ and $Y_n^W$ of user $W$, respectively.

Lemma 1. The following Markov chains hold:

$Z_t^{-1} - (Y_t^{-1}(W), T(W)) - Y_t(W)$, \hspace{1cm} (36)

$Z_t^{-1} - (X_t^{-1}(W), T(W)) - X_t(W)$. \hspace{1cm} (37)

Proof: See the proof of [12, Appendix C-A].

Lemma 2. There exists an auxiliary RV $U$ satisfying $Z - X - Y - U$ and

$\sum_{t=1}^{n} I(Z_t;U_t) = nI(Z;U)$, \hspace{1cm} (38)

$\sum_{t=1}^{n} I(Y_t(W);U_t) = nI(Y;U)$, \hspace{1cm} (39)

$\sum_{t=1}^{n} I(X_t(W);U_t) = nI(X;U)$. \hspace{1cm} (40)

Proof: See the proof in [12, Appendix C-B].

We fix the auxiliary RV $U$ specified by Lemma 2. The
next lemma plays a key role in the analysis of privacy-leakage, which will be seen later.

**Lemma 3.** It holds that

\[ I(Z^n; J, W) \geq n(R_I - (\delta + \delta_n)), \tag{41} \]

where \( \delta_n = \frac{1}{n} (1 + \delta \log M_I M_C M_G) \), and \( \delta_n \downarrow 0 \) as \( n \to \infty \) and \( \delta \downarrow 0 \).

Proof: We expand the left-hand side in (41) as

\[ I(Z^n; J, W) \geq H(W|J) - H(W|Z^n, J) \]

(a) follows because conditioning reduces entropy,

(b) follows because Fano’s inequality is applied,

(c) follows because (35) is used and conditioning reduces entropy,

where

(a) follows because Fano’s inequality is applied,

(b) is due to (2).

Analysis of Identification, Chosen- and Generated-Secret Key Rates: We begin with considering the join entropy of

\[ E(W) = (W, S_C(W), S_G(W)) \]

as

\[ H(E(W)) = H(E(W)|Z^n, J) + I(E(W); Z^n, J) \]

where

(c) holds as \( E(W) \) is a function of \( (Z^n, J) \),

(d) follows because conditioning reduces entropy, and

(e) holds because \( W \) is independent of other RVs and only \( J(W) \) is possibly dependent on \( Z^n, S_C(W), \) and \( S_G(W) \),

(f) follows because (35) is used and conditioning reduces entropy,

(g) holds due to (38) in Lemma 2.

In the opposite direction, we can also derive the following relation:

\[ H(E(W)) = H(W, S_C(W), S_G(W)) \]

= \( H(W) + H(S_C(W), S_G(W)|W) \)

\[ \begin{align*}
& (b) \quad H(W) + H(S_C(W)) + H(S_G(W)|W) \\
& - I(S_C(W); S_G(W)|W) \\
& (i) \quad \log M_I + \log M_C + H(S_G(W)|W) \\
& - I(S_C(W); S_G(W)|W) \\
& \geq n(R_I + R_G - (\Gamma - 3\delta)), \tag{44} 
\end{align*} \]

where

(h) holds because \( W \) and \( S_C(W) \) are independent of each other,

(i) follows since \( W \) and \( S_C(W) \) are uniformly distributed on \( I \) and \( S_C \), respectively,

(j) is due to (2), (3), (32), and (34).

From (43) and (44), we obtain

\[ R_I + R_C + R_G \leq I(Z; U) + \Gamma + 4\delta + \delta_n. \tag{45} \]

Using (43), it is straightforward that

\[ H(W, S_C(W)) \leq n(I(Z; U) + \delta + \delta_n), \tag{46} \]

and

\[ H(W, S_G(W)) \leq n(I(Z; U) + \delta + \delta_n). \tag{47} \]

By a similar manner of (44), it can be shown that

\[ H(W, S_C(W)) \geq n(R_I + R_C - 2\delta) \]

and \( H(W, S_G(W)) \geq n(R_I + R_G - 2\delta) \), and therefore,

\[ R_I + R_C \leq I(Z; U) + 3\delta + \delta_n, \tag{48} \]

\[ R_I + R_G \leq I(Z; U) + 3\delta + \delta_n. \tag{49} \]

From (48) and (49), one can easily see that

\[ R_I + R_C + R_G \leq I(Z; U) + R_G + 3\delta + \delta_n, \tag{50} \]

\[ R_I + R_G + R_C \leq I(Z; U) + R_C + 3\delta + \delta_n. \tag{51} \]

Finally, we have that

\[ R_I + R_G + R_C \leq I(Z; U) + \min\{\Gamma, R_G, R_C\} + 4\delta + \delta_n. \tag{52} \]

where Equation (52) is due to comparing the values on the right-hand sides of (43), (50), and (51), and the smallest one is a valid bound for all these equations.

**Analysis of Storage Rate:** We have that

\[ n(R_I + \delta) \geq \log M_I \geq \max_{w \in \mathcal{I}} H(J(w)) \]

\[ \geq \frac{1}{M_I} \sum_{w=1}^{M_I} H(J(W)|W = w) = H(J(W)|W) \]

\[ = I(Y^n_W, S_C(W); J(W)|W) \]

\[ \begin{align*}
& (k) \quad H(Y^n_W) + H(S_C(W)) \\
& - H(Y^n_W, S_C(W), S_G(W)|J(W), W) \\
& \geq n(R_I + R_C + R_G - \Gamma - 3\delta) \\
& \leq I(Z; U) + \min\{\Gamma, R_G, R_C\} + 4\delta + \delta_n. \tag{52} \]

where Equation (52) is due to comparing the values on the right-hand sides of (43), (50), and (51), and the smallest one is a valid bound for all these equations.

**Analysis of Storage Rate:** We have that
\[ \begin{align*}
(1) & \quad H(Y^n_W) - H(Y^n_W|T(W)) + \log M_C \\
& - H(S_C(W), S_G(W)|J(W), W) \\
& \geq \sum_{i=1}^{n} \left\{ H(Y_i(W)) - H(Y_i(W)|Y^{i-1}(W), T(W)) \right\} \\
& + \log M_C - H(S_C(W), S_G(W)|W) \\
& = \sum_{i=1}^{n} \left\{ H(Y_i(W)) - H(Y_i(W)|Z^{i-1}, Y^{i-1}(W), T(W)) \right\} \\
& + n(R_C - \delta) - H(S_C(W), S_G(W)|W) \\
& \geq \sum_{i=1}^{n} \left\{ I(X_i(W); Z^{i-1}, T(W)) - I(Z_i; U_i) \right\} \\
& + I(Z^n; J(W), W) - n\delta_n \\
& \geq \sum_{i=1}^{n} \left\{ I(X_i(W); Z^{i-1}, T(W)) - I(Z_i; U_i) \right\} \\
& + I(Z^n; J(W), W) - n\delta_n \\
& \geq \sum_{i=1}^{n} \left\{ I(X_i(W); U_i) - I(Z_i; U_i) \right\} \\
& + n(R_I - (\delta + \delta_n)) - n\delta_n \\
& = n(I(X; U) - I(Z; U) + R_I - \delta - 2\delta_n), \quad (54)
\end{align*} \]

where

(p) follows because \( W \) is independent of \( X^n_W \),

(q) follows because the third term and the last term cancel out each other, Fano’s inequality is applied for the fifth term, and the sixth term is eliminated,

(r) follows since each symbol of the sequences \( (X^n_W, Z^n) \) is i.i.d. generated, and thus \( \sum_{i=1}^{n} I(X_i(W); Y^{i-1}(W)) = \sum_{i=1}^{n} I(Z_i; Z^{i-1}) = 0 \),

(s) is due to \( 37 \),

(t) follows from Lemma \( 3 \),

(u) holds due to \( 35 \) and \( 40 \) in Lemma \( 1 \).

For the cardinality bound \( |U| \leq |Y| + 2 \), we can derive the condition by using the support lemma \( 21 \) Appendix C. Finally, by letting \( n \to \infty \) and \( \delta \downarrow 0 \), we complete the proof of the converse part. \( \square \)

B. Direct Part (Achievability)

In the proof, we show only the case where \( \Gamma \leq \min \{ R_C, R_G \} \). The other cases, namely, (I) \( R_C \leq \min \{ \Gamma, R_G \} \) and (II) \( R_C \leq \min \{ \Gamma, R_C \} \), the constraint \( R_I + R_C + R_G \leq I(Z; U) + \min \{ \Gamma, R_C, R_G \} \) in Theorem \( 1 \) reduces to \( R_I + R_C \leq I(Z; U) \) for (I) and \( R_I + R_C \leq I(Z; U) \) for (II), respectively, which already emerge in Theorem \( 1 \). The proof of these cases follows similarly to that of \( \Gamma \leq \min \{ R_C, R_G \} \) with minor adjustments. In this part, we omit the detailed proof although how to prove cases (I) and (II) will be mentioned later.

Parameter Settings: First, fix \( P_{U|Y} \). Let \( \delta \) be a small enough positive value and fix \( n \). We set

\[ R_I > 0, \quad R_C > 0 \quad (R_I + R_C < I(Z; U)), \quad (55) \]

\[ R_G = I(Z; U) + \Gamma - (R_I + R_C) - \delta, \quad (56) \]

\[ R_M = I(Y; U) - I(Z; U) + R_I + 2\delta, \quad (57) \]

\[ R_J = I(Y; U) - I(Z; U) + R_I + R_C + 2\delta, \quad (58) \]

\[ R_L = I(X; U) - I(Z; U) + R_I + 2\delta, \quad (59) \]

where \( R_M \) denotes the rate of a dummy message shared between the encoder and decoder. We set \( I = [1: 2^n R_I], S_C = [1: 2^n R_C], S_G = [1: 2^n R_G], \) and \( J = [1: 2^n R_J] \). We also define four new sets \( S_T = [1: 2^n], S_C[Y] = [1: 2^{n(R_C-\Gamma)}], S_G[Y] = [1: 2^{n(R_G-\Gamma)}], \) and \( M = [1: 2^n R_M] \), representing the sets of shared bits, unshared bits in chosen- and generated-secret keys, and dummy message, respectively.
and $\ominus$ denotes the subtraction modulo $M_C$. After that, the decoder determines the corresponding pair $(s_{C1}(w), s_{C2}(w))$ from the one-to-one mapping table, and uses $s_{C1}(w)$ to estimate the generated-secret key as $\tilde{s}_G(w) = (s_{C1}(w), s_{C2}(w))$.

**Evaluations of Performance:** We shall check that all the conditions \((1)-(8)\) in Definition 1 satisfy with a random codebook $C_m = \{U^n(s_1, s_2, m) : s_1 \in SC, s_2 \in SG, m \in M\}$. We first introduce some useful lemmas, which are often used in the evaluations of these conditions, and then dive into the core part of the discussion. Let $(S_1(i), S_2(i), M(i))$ denote the corresponding index tuple of individual $i$ chosen by the encoder for given $Y^n_i$. For simplicity, $(S_1(i), S_2(i), M(i))$ and the sequence $U^n(S_1(i), S_2(i), M(i))$ are represented by $V(i)$ and $U^n(V(i))$, respectively. Also, define $\epsilon_n = \frac{1}{n} + \delta \log |Y|$, and this quantity goes to zero as the block length $n$ tends to infinity and $\delta$ approaches zero.

**Lemma 4.** It holds that

\begin{align*}
H(Y^n_i | V(i), C_n) &\leq n(H(Y^n_i | U) + \epsilon_n), \\
H(Y^n_i | X^n_i, V(i), C_n) &\leq n(H(Y^n_i | X^n_i, U) + \epsilon_n).
\end{align*}

**Proof:** Since $V(i)$ determines $U^n(V(i))$ directly, the above lemma can be viewed as [29] Lemma 4. For the detailed proof, see [29] Appendix C or [12] Appendix B as a reference.

**Lemma 5.** We have that

\begin{align*}
H(S_1(i) | C_n) &\geq n(R_C - \delta - \epsilon_n), \\
H(S_2(i) | C_n) &\geq n(R_G - \Gamma - \delta - \epsilon_n), \\
I(S_1(i), S_2(i), M(i)) | C_n) &\geq n(I(Z; U) - R_1 - 2\delta - \epsilon_n), \\
I(S_1(i); S_2(i); M(i)) | C_n) &\leq n(\delta + \epsilon_n), \\
I(S_1(i); S_2(i); M(i)) | C_n) &\leq n(\delta + \epsilon_n).
\end{align*}

**Proof:** The proof is provided in Appendix B.

**Analysis of Error Probability:** For $W = i$, an error event possibly happens at the encoder is

$$
E_i \equiv \{(Y^n, U^n(s_1, s_2, m) \notin T^n_e(YU)) \text{ for all } s_1 \in SC, s_2 \in SG, m \in M\},
$$

and those at the decoder are:

$$
E_2 \equiv \{(Z^n, U^n(V(i))) \notin T^n_e(ZU)\},
$$

$$
E_3 \equiv \{(Z^n, U^n(S_1(i), s_2', M(i)) \notin T^n_e(ZU) \text{ for } \exists s_2' \neq S_2(i), s_2' \in SG\},
$$

$$
E_4 \equiv \{(Z^n, U^n(s_1', s_2, M(i)) \notin T^n_e(ZU) \text{ for } \exists s_1' \neq S_1(i), s_1' \in SC\},
$$

$$
E_5 \equiv \{(Z^n, U^n(s_1', s_2', M(i)) \notin T^n_e(ZU) \text{ for } \exists s_1' \neq S_1(i), s_2' \in SG\},
$$

$$
E_6 \equiv \{(Z^n, U^n(s_1, s_2', M(i')) \notin T^n_e(ZU) \text{ for } \exists i' \neq i, i' \in I, s_1 \in SC, s_2 \in SG\}.
$$

Note that $E(W) = (W, S_C(W), S_G(W))$, and then we can further bound the entire error probability as

$$
\Pr\left(\bigcup_{i=1}^{6} E_i | W = i\right) = \Pr\left(E(W) \neq E(W) \mid W = i\right).
$$
\[ \leq \Pr \{ E_1 \} + \Pr \{ E_2 | E_1^c \} + \Pr \{ E_3 \} + \Pr \{ E_5 \} + \Pr \{ E_6 \}. \]

By using the covering lemma [21, Lemma 3.3], \( \Pr \{ E_i \} \) can be made smaller than \( \delta \) since \( R_C + R_G - \Gamma + R_M > I(Y; U) \). The \( \Pr \{ E_2 | E_1^c \} \) can also be made small enough by the Markov lemma [20, Lemma 15.8.1]. The probabilities \( \Pr \{ E_i \}, i = 3, \ldots, 6 \) vanish as well by applying the packing lemma [21, Lemma 3.1] since we have \( R_I \geq 0, R_C \geq 0, R_G \geq 0 \), and \( R_I + R_C + R_G - \Gamma < I(Z; U) \). Overall, the error probability averaged over the random codebook can be bounded by

\[ \Pr \{ \bar{E}(W) \neq E(W) | W = i \} \leq 6\delta \]

for large enough \( n \).

**Analyses of Identification and Chosen-Secret Key Rates:** From the parameter settings, (2) and (3) are trivial.

**Analysis of Generated-Secret Key Rate:** For the left-hand side of (4), we have that

\[ H(S_G(i) | C_n) = H(S_{C1}(i), S_2(i) | C_n) \]

\[ = H(S_{C1}(i) | C_n) + H(S_2(i) | C_n) \] (a)

\[ \geq n(R_G - \delta - \epsilon_n) , \]

where

(a) holds as \( S_{C1}(i) \) is independent of \( S_2(i) \),

(b) follows because \( S_{C1}(i) \) is uniformly distributed on \( S_I \) and (66) is applied.

**Analysis of Storage Rate:** The total required storage rate is

\[ \frac{1}{n} \log M_I \leq R_M + R_C \]

\[ = I(Y; U) - I(Z; U) + R_I + 2\delta + R_C \leq R_I + \delta . \]

**Analysis of Privacy-Leakage Rate:** By invoking the same arguments around [22, Appendix B], we obtain that

\[ I(X^n_i; J(i) | C_n) = I(X^n_i; M(i) | C_n). \]

(74)

From (74), since we denote \( V(i) = (S_1(i), S_2(i), M(i)) \), we have

\[ I(X^n_i; M(i) | C_n) \]

\[ = I(X^n_i; V(i) | C_n) - I(X^n_i; S_1(i), S_2(i) | M(i), C_n) \]

\[ = H(V(i) | C_n) - H(V(i) | X^n_i, C_n) \]

\[ - H(S_1(i), S_2(i) | M(i), C_n) \]

\[ + H(S_1(i), S_2(i) | M(i), X^n_i, C_n) \]

\[ \leq H(V(i) | C_n) - \{ H(Y^n_i | V(i), C_n) \}

\[ - H(Y^n_i | X^n_i, V(i), C_n) \}

\[ - H(S_1(i), S_2(i) | C_n) \]

\[ - I(S_1(i), S_2(i); M(i) | C_n) \} + n\delta_n \]

\[ \leq n(I(Y; U) + \delta) - \{ H(Y^n_i | V(i), X^n_i, C_n) \}

\[ - I(S_1(i), S_2(i); M(i) | C_n) \} + n\delta_n \]

\[ \leq n(I(Y; U) + \delta) - \{ H(Y^n_i | V(i), C_n) \}

\[ - H(Y^n_i | X^n_i, V(i), C_n) \}

\[ - n(I(Z; U) - R_I - 2\delta - \epsilon_n) \] (c)

\[ + n(\delta + \epsilon_n) + n\delta_n \]

\[ \leq n(I(Y; U) - nH(Y | X) + H(Y^n_i | X^n_i, V(i), C_n) \}

\[ - n(I(Z; U) - R_I - 4\delta - 3\epsilon_n - \delta_n) \] (f)

\[ \leq n(I(Y; U) - nH(Y | X) + H(Y^n_i | X^n_i, V(i), C_n) \}

\[ - n(I(Z; U) - R_I - 4\delta - 3\epsilon_n - \delta_n) \] (g)

\[ = n(I(Y; U) - I(Y; U(X)) - n(I(Z; U) \]

\[ - R_I - 4\delta - 3\epsilon_n - \delta_n) \]

\[ = n(I(U) - H(U | Y) - H(U | X) + H(U | Y, X)) \]

\[ - n(I(Z; U) - R_I - 4\delta - 3\epsilon_n - \delta_n) \] (h)

\[ \leq n(I(X; U) - I(Z; U) + R_I + 4\delta + 3\epsilon_n + \delta_n) , \]

(75)

where

(c) follows since \( (S_1(i), S_2(i)) - (M(i), X^n_i) \) \(- Z^n \) holds for given \( C_n \), we have that \( H(S_1(i), S_2(i) | M(i), X^n_i, C_n) \leq H(S_1(i), S_2(i) | M(i), Z^n, C_n) \) \leq n\delta_n \), where the last inequality is obtained by Fano’s inequality with \( \delta_n = \frac{1}{n}(1 + \delta \log M_I M_C M_G) \),

(d) is due to \( H(V(i) | C_n) \leq H(S_1(i) | C_n) + H(S_2(i) | C_n) + H(M(i) | C_n) \leq n(I(Y; U) + \delta) \),

(e) follows because (67) and (69) in Lemma 5 are applied,

(f) follows as \( X^n_i \) and \( Y^n_i \) are independent of \( C_n \),

(g) is due to (66) in Lemma 3,

(h) holds due to the Markov chain \( X - Y - U \) (cf. (9)), that is, \( H(U | Y, X) = H(U | Y) \).

**Analysis of Secret-Key Leakage:** The information leakage between the two secret keys can be bounded as follows:

\[ I(S_C(i); S_G(i) | C_n) \]

\[ = I(S_{C1}(i), S_{C2}(i); S_{C1}(i), S_2(i) | C_n) \]

\[ = H(S_{C1}(i), S_{C2}(i) | C_n) \]

\[ - H(S_{C1}(i), S_{C2}(i) | S_{C1}(i), S_2(i) | C_n) \]

\[ \geq H(S_{C1}(i), S_{C2}(i) | C_n) \]

\[ = H(S_{C1}(i), S_{C2}(i) | C_n) \]

\[ = n\Gamma, \]

(76)

where (i) follows as \( S_{C2}(i) \) is independent of \( (S_{C1}(i), S_2(i)) \).

**Analysis of Secrecy-Leakage:** For (8), it follows that

\[ I(S_C(i), S_G(i); J(i) | C_n) \]

\[ \geq I(S_C(i), S_2(i); M(i), S_C(i) \oplus S_1(i) | C_n) \]

\[ = H(M(i), S_C(i) \oplus S_1(i) | C_n) \]

\[ - H(M(i), S_C(i) \oplus S_1(i) | S_C(i), S_2(i) | C_n) \]

\[ = H(M(i) | C_n) + H(S_C(i) \oplus S_1(i) | M(i), C_n) \] (i)
and (67) hold, and therefore we omit the details.

where

\[ V \]

the conditions in Definition 1 for large enough \( n \), there exists at least one good codebook satisfying all

results shown above (i.e., Eqs. (71), (72), (73), and (75)–(77)), there exists at least one good codebook satisfying all

\[ \Gamma \]

where

\( (j) \) is due to the fact that \( S_C(i) = (S_C1(i), S_C2(i)) \) and \( S_C1(i) \) is the first half of the chosen-secret key \( S_C(i) \).

\( (k) \) holds since \( S_C(i) \) is independent of other RVs.

\( (l) \) follows because (65), (68), and (69) in Lemma 5 are applied.

By applying the selection lemma [30, Lemma II] to all results shown above (i.e., Eqs. (71), (72), (73), and (75)–(77)), there exists at least one good codebook satisfying all the conditions in Definition 1 for large enough \( n \).

\[ \Box \]

Appendix B: Proof of Lemma 5

We begin with checking (65). In view of \( V(i) = (S_1(i), S_2(i), M(i)) \), we have

\[
H(S_1(i)|C_n) = H(V(i)|C_n) - H(S_2(i), M(i)|S_1(i), C_n)
\]

\[ \geq H(Y^n | V(i), C_n) - H(Y^n | V(i), i) \]

\[ \geq n(H(Y) - n(I(Z; U) - R_I - R_C - \delta) - n(I(Y; U) - I(Z; U) + R_I + 2\delta)
\]

\[ n(H(Y) + \epsilon_n) = n(R_C - \delta - \epsilon_n), \quad (78) \]

where

\( (a) \) follows because conditioning reduces entropy and \( Y^n \) is independent of \( C_n \).

\( (b) \) follows because (63) in Lemma 4 is applied.

Similar to the arguments around (78), it can be verified that (66) and (67) hold, and therefore we omit the details.

For (68), it can be shown that

\[
I(S_1(i), S_2(i), M(i)|C_n)
\]

\[ \leq H(S_1(i)|C_n) + H(S_2(i)|C_n) + H(M(i)|C_n)
\]

\[ = H(Y_i^n | V(i), C_n)
\]

\[ \leq nR_C + n(I(Z; U) - R_I - R_C - \delta) + n(I(Y; U) - I(Z; U) + R_I + 2\delta)
\]

\[ - nH(Y) + nH(Y|U) + \epsilon_n
\]

\[ = n(\delta + \epsilon_n), \quad (79) \]

where

\( (c) \) follows because conditioning reduces entropy.

\( (d) \) follows because (63) in Lemma 4 is applied.

Equation (69) can be shown in a similar manner.

\[ \Box \]

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