Can Maxwell’s Equations Describe Elementary Waves and Charges?

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Abstract

The steady Poynting flow defines the elementary Maxwell-Heaviside wave which spirality replicates the helicity of photons in quantum physics. The angular momentum of this classical wave requires non-point classical charges for elementary emission - absorption events. Analytical field solutions of Maxwell’s equations can specify the nonlocal radial source for EM fields instead of the localized point charge. The Maxwell-Mie (astro)electron has the monotonous radial astrodistribution of only negative charge densities, while the positive elementary (astro)charge contains a sharp radial front between positive and negative densities. Non-empty space around visible macroscopic frames of laboratory bodies is filled by only negative fractions of continuous (astro)protons as well as of (astro)electrons.

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1 Spirality of the elementary Maxwell wave

By considering Maxwell’s equations as a very reliable tool for any engineering computations, physicists are not looking anymore on the Mie program [1] to use this classical tool for description of elementary charges. Spins of elementary particles, for example, are often discussed only by way of quantum physics, rather than by the classical theory of fields. As a result, one could assume that spins of elementary particles are related exclusively to their quantum properties, which have to disappear in the classical limit \( \hbar \to 0 \) for the Planck constant. However, Classical Electrodynamics (CED) complies not only with the Einstein principle of relativity for space-time translations of matter, but also with the 10-dimensional Poincaré group. This noncompact Lie group is a semidirect product of the translations with the Lorentz transformations. It was established that the spin of a classical system can be related to the Lorentz little group, namely to the unitary irreducible representations which determine the polarization degrees of freedom [2].

Sooner or later, a complete theory of classical fields should find a way to specify spatial structures of rotating non-point sources and to unify continuous particle and field densities for distributed elementary energy, momentum, angular momentum, charge, current, etc. This conceptual avenue to specific properties of distributed elementary sources beyond delta-operator densities starts from the well-studied Maxwell’s equations, regardless of the fact that new advanced theories, like Quantum Electrodynamics (QED) or Chromodynamics, are more beneficial for matter fundamentals. Our initial search is focused on a basic
CED wave, which ought to replicate the spin ±1 of the QED photon as a strict consequence of only classical laws and equations (due to Lorentz, Poincaré, and Wigner). Then, only the energy quantization of wave fields, rather than their spin or helicity, falls under the exclusive scope of quantum physics.

Recall that the classical continuity equation for non-stationary charged matter,

\[ \text{div} \vec{J} = -\frac{\partial \rho}{\partial t} \neq 0, \quad (1) \]

was used as a main argument for the time-varying modification of the Ampère law for steady currents and static magnetic fields. Indeed, the displacement current had been suggested by Maxwell in order to keep the charge density conservation for any non-stationary current \( \vec{J} \). The latter had been modeled by Lorentz through an averaged ensemble of elementary microscopic currents \( \vec{j}_k \). The postulated existence of the stable microscopic charge enabled the successful Lorentz theory with the steady flow option, \( \text{div} \vec{j}_k = 0 \), for the microscopic current.

In general, an existence of steady states of energy, momentum, angular momentum, charge or current can be used as a universal criterion for the introduction of indivisible or elementary objects in classical physics. As such, a steady power flow ought to be required for the basic or elementary electromagnetic wave. This criterion for the basic wave element in linear superposition of CED fields can be applied to all known periodic electric, \( \vec{E} \), and magnetic, \( \vec{H} \), solutions of Maxwell’s equations.

Vector components of CED waves obey the verified classical equality,

\[ \vec{\nabla} \times \vec{E} + \frac{\partial \mu \vec{H}}{\partial t} = 0. \quad (2) \]

Its plane wave solution, \( \vec{E} = E_o \cos(\omega t - kz)\vec{a}_x \) and \( \vec{H} = (E_o k/\omega \mu) \cos(\omega t - kz)\vec{a}_y \), looks like the simplest free wave. However, the power density flow, or the Poynting vector \( \vec{S} \equiv \vec{E} \times \vec{H} = (kE_o^2/\omega \mu) \cos^2(\omega t - kz)\vec{a}_z \) of this ‘simplest’ wave, is not uniform even in homogeneous space with \( \mu = \text{const}, \epsilon = \text{const} \).

The plane wave solution with \( \text{div} \vec{S} \neq \text{const} \) cannot be called the elementary classical wave because this free electromagnetic field does not satisfy the steady energy flow criterion for the basic element of matter. One may only suppose for the plane wave a linear superposition of basic wave elements resulting in a non-steady Poynting flow and a non-stationary field energy density. Notice that power density variations along the plane wave are strictly periodic and cannot be associated with fluctuations.

It is our contention that basic wave elements in Maxwell’s electrodynamics have even more complicated field compositions,

\[
\begin{align*}
E_{\pm} &= E_o [\cos(\omega t - kz)\vec{a}_x \pm \sin(\omega t - kz)\vec{a}_y] \\
H_{\pm} &= (kE_o^2/\omega \mu) [\cos(\omega t - kz)\vec{a}_y \mp \sin(\omega t - kz)\vec{a}_x],
\end{align*}
\]

than the plane wave. At the same time, both the EM power flow,

\[
\vec{E}_{\pm} \times \vec{H}_{\pm} = (kE_o^2/\omega \mu) [\cos^2(\omega t - kz) + \sin^2(\omega t - kz)]\vec{a}_z \equiv (kE_o^2/\omega \mu)\vec{a}_z, \quad (4)
\]
and the conventional energy density of EM fields,

\[ \frac{\epsilon E_o^2}{2} + \frac{\mu H_o^2}{2} = \left( \frac{\epsilon E_o^2}{2} + \frac{k^2 E_o^2}{2\omega^2\mu} \right) \left[ \cos^2(\omega t - k z)\vec{a}_x + \sin^2(\omega t - k z)\vec{a}_y \right] = \epsilon E_o^2, \]

for these two elementary waves are uniform over space and time. Moreover, the steady angular rotation rate, which is strictly ±360° per every wavelength \( \lambda = k^{-1} = (\omega^2\epsilon\mu)^{1/2} \), also matches the introduced criterion for the basic element of matter. For all of the aforesaid reasons, the paired fields (3) with steady power flow, steady energy density, and steady angular circulations of vector field polarizations can be defined as basic electromagnetic waves in the coherent approach to elementary classical objects.

The wave solutions (3) with clock-wise circulation of electric, \( \vec{E}_+ \), and magnetic, \( \vec{H}_+ \), vectors and counter clock-wise circulation of \( \vec{E}_- \) and \( \vec{H}_- \), are surely well known in Maxwell’s electrodynamics. Originally, this Heaviside signal was postulated as ‘an unchanging slab of ExH energy current traveling forward at the speed of light’ [3]. Such a steady-flow signal, which ‘carries all its properties with it unchanged’, replaced the initially assumed ‘rolling wave’ with Faraday relations between the wave electric and magnetic fields. The rotating classical waves were successfully replicated by QED photons and enforced by quantization. Why should one look again at this ‘trivial’ classical physics?

The point is that simple logical exercises with the Heaviside ‘unchanged energy current’ for the elementary wave directly maintain the Mie’s nontrivial hypothesis that Maxwell’s equations should have conceptual abilities to describe structures of basic material elements, including continuous elementary charges. The classical interpretation of absorption-emission events for the elementary Maxwell-Heaviside wave with angular momentum requires angular momentum for an elementary Maxwell charge. Therefore, the ‘simplest’ Maxwell equation (2) or the covariant equality for CED wave solutions has demanded, in fact, quite complicated classical physics with non-zero angular inertia of a CED charge. Such a charge with angular momentum cannot be a point mechanical object. Can other Maxwell’s equations-equalities self-consistently address non-point elementary charges or ‘obsolete’ Mie’s ideas were incorrect?

2 The radial Maxwell-Mie electron

The postulated point-particle paradigm results not only in the mathematical divergence of the electrostatic self-energy, but also in the physical inconsistency of the microscopic electron theory. Any point source in the microscopic Maxwell-Lorentz equations may be considered as ‘an attempt which we have called intellectually unsatisfying’ [4]. Einstein also criticized his 1915 equation due to the point gravitational source: ‘it resembles a building with one wing built of resplendent marble and the other built of cheap wood’ (translation [5]).

The Dirac delta-operator formalism for the point charge seems as a provisional modeling of physical reality until local analytical charge-field relations can be finally proposed for
Maxwell’s electrodynamics. ‘A coherent field theory’, stated Einstein (translation [5]), ‘requires that all elements be continuous... And from this requirement arises the fact that the material particle has no place as a basic concept in a field theory. Thus, even apart from the fact that it does not include gravitation, Maxwell’s theory cannot be considered as a complete theory.’

The continuously distributed elementary (astro)charge was reasonably initiated by Mie in order to derive properties of charges from properties of fields (and potentials) and to avoid the energy divergence in the Coulomb field center. Unfortunately, the ‘Theory der Matter’ [1] had not found gauge-invariant post-Coulomb (logarithmic) potentials and the promising non-empty space concept had not been timely finalized in 1912-1913. Quantum era of empty-space probabilities for delta-operator (dice) interpretations of physical reality postponed the search for rigorous analytical solutions of Mie’s matter. The problem of the unphysical point source is still unresolved within the available mathematical approaches to the classical theory of fields. This provisional discomfort may be considered as a motivation for our reinforcement of the Mie nonlocal (astro)particle with the continuous radial density over the infinite world volume or the Universe.

We expect that it is possible to relate analytically the electric, \( \vec{e}(x) = \vec{d}(x)/\epsilon \), and magnetic, \( \vec{b}(x) = \mu \vec{h}(x) \), field intensities in the Maxwell-Lorentz equations [6],

\[
\begin{align*}
\text{div } \vec{d}(x) &= \rho(x) \\
\text{div } \vec{b}(x) &= 0 \\
\text{curl } \vec{h}(x) &= \rho(x) \vec{v} + \partial_t \vec{d}(x) \\
\text{curl } \vec{e}(x) &= -\partial_t \vec{b}(x),
\end{align*}
\]

(6)
to the local charge, \( \rho(x) \), and current, \( \rho(x) \vec{v} \), densities of the extended electron with the elementary charge integral \((-e_0) = \text{const})\). Contrary to the recognized Lorentz model, the electron’s charge was not postulated by Mie within a microscopic spatial volume, while the electron’s fields are in charge-free regions outside the charge volume (i.e. in supposed empty space). The mathematical equation \( \text{div} \vec{d}(\vec{x}, t) = \rho(\vec{x}, t) \) can be rigorously resolved for non-empty space, where the charge and its field coexist locally in all space points \( \vec{x} \) in the Universe. In other words, we tend to maintain the Mie (Einstein) idea that the elementary electric (gravitational) charge is to be integrated into its spatial field structure with instantaneous local relations between scalar functions \( \rho(\vec{x}, t) \) and \( \vec{d}(\vec{x}, t)\vec{e}(\vec{x}, t) \).

The mass density \( m_o n(r) \) of the distributed radial electron with the analytical density \( n(r) \) should possess the same active mass-energy density as the passive mass-energy of electron’s gravitational field (due to the Principle of Equivalence). The charge density \( \rho(r) = (-e_0)n(r) \) of the same electron should possess a EM energy density which is equal to the electric field energy density. Therefore, the continuous charge density of the extended particle is to be proportional to the electromagnetic field energy density, \( \rho(\vec{x}, t) = \epsilon \vec{e}^2(\vec{x}, t)/\Lambda \) in the rest frame of reference. Then, the electron’s charge (and its conservation) has the self-energy meaning of the constant \((-e_0) \) in the uniform self-potential \( \Lambda = \text{const} \). Maxwell-Lorentz’s equations for extended electrons can equally be discussed...
for the electric current density and for the electric energy flow densities. One can say that EM self-energy currents are even more fundamental for CED than electric charge currents. Anyway, CED can employ the charge self-energy (justified so far only in QED) in Mie’s non-empty space approach to EM fields (distributed within continuous particle-charges). The delta-operator description of point charges, say on separated capacitor’s plates, can also discuss the electrostatic energy of charges in addition to equal amount of their field energy, spatially separated from such point charges. Contrary to diverging self-energy options for point particles, our self-energy of the spatially distributed charge can be described through regular energy density terms and the Poynting vector for continuous fields.

The local equality of the electron’s charge self-energy density, $\rho \Lambda$, and the electron’s field energy density, $\varepsilon \vec{e}^2$, suggests finite electrostatic integrals, $\mathcal{E}_{11}^{ch}$ and $\mathcal{E}^f$, for both charge and field energy fractions of the elementary continuous carrier of electricity,

$$\mathcal{E}_{11}^{ch} \equiv \Lambda \int \rho(\vec{x}, t) dv = \frac{(-e_o)^2}{4\pi r_o} = \int \varepsilon \vec{e}^2(\vec{x}, t) dv \equiv \mathcal{E}^f.$$  \hspace{1cm} (7)

Here we used $\int \rho(\vec{x}, t) dv = -e_o < 0$ for the negative elementary charge and introduced the electron’s self-potential $\Lambda \equiv (-e_o)/4\pi r_o < 0$ through the phenomenological scale $r_o$, with $\varepsilon = 8.854 \times 10^{-12} C^2/N \cdot m$ for a seclude electron. This electron’s scale, $r_o = Gm_o/c^2 = 7 \times 10^{-58} m$, can be evaluated [7] from gravitational theories of continuous energy sources that might shed some light on the meaning of $\Lambda$. Detail physics for the charge-to-mass ratio is outside of our present goals. We keep $r_o$ as a free parameter in (7), where we prefer to operate formally with the doubled conventional energy-density of EM fields in the self-energy integral $\int \varepsilon \vec{e}^2(\vec{x}, t) dv$. The doubled EM energy-density matches the Lorentz force acceleration for the bi-fractional continuous mass carrier of passive-inertial energy (of a gravitational field) and equal active self-energy (of a distributed source). Then, the passive charge energy density in our analysis of paired active and passive charges in one electron universally depends on the (doubled) self-potential $\Lambda = const$ and on an external interaction potentials $W$ of another radial charge.

The inhomogeneous post-Coulomb potential $W(\vec{x}) \neq const$, with $\vec{e}(\vec{x}) = -\nabla W(\vec{x})$, of the active radial charge should be introduced for interaction with other (passive) radial charges, but not for the charge self-action. The strict identical balance between the EM self-field energy density and the charge self-energy density is required only for the self potential $\Lambda$, but not for the interaction potential $W$, i.e. $\varepsilon \vec{e}^2(\vec{x}) \neq \rho(\vec{x})W(\vec{x})$. However, we shall prove the exact balance between the finite integral energies of the distributed charge in $\Lambda = const$ and in $W(\vec{x}) \neq const$. The principal difference is that the uniform self-potential $\Lambda$ does not generate gradients or local self-forces that explain the stability of the extended energy source or steady nonlocal charge without additional Poincaré-type pressures.

The divergence-free energy distribution (7) for a steady continuous astrolelectron ‘at rest’ ($\vec{v} = \vec{b} = 0$) does exist in $\text{div} \vec{d}(r) = \rho(r)$ for the radial solution of Maxwell’s equations,

$$\begin{cases} 
\vec{d}(r) = (-e_o)\hat{r}/4\pi r(r + r_o) = \varepsilon \vec{e}(r) \\
\rho(r) = (-e_o)r_o/4\pi r^2(r + r_o)^2.
\end{cases} \hspace{1cm} (8)$$
where the steady particle’s density \( n(r) = r_o/4\pi r^2(r + r_o)^2 \) replaces the Dirac operator density \( \delta(r) \) from the point particle model.

The field flux of the radial charge distribution \( \rho(r) \) depends on a selected radius \( R \) for a Gaussian sphere,

\[
4\pi R^2 \mathbf{d}(R) = q(R) = \int_0^R \rho(r)4\pi r^2 dr = \int_0^R \frac{(-e_o)r_o dr}{(r + r_o)^2} = (-e_o)\frac{R}{R + r_o},
\]

where the carrier density scale \( r_o \) is the half-charge radius for the unlimited astrodistribution (8) of the total charge \( (-e_o) \). In other words, any extremely small but finite scale \( r_o \neq 0 \) unavoidably results in the global astrodistribution of the elementary charge over the entire Universe. Therefore, the non-point Maxwell electron (8) cannot be localized in principle within the microscopic (and even macroscopic) volume assumed in the Lorentz model. The Maxwell charge density is firmly bound with the electric field energy density, \( \rho \propto \vec{e}^2 \). This identical charge and its field energy counter-flows ought to fill the non-empty world space continuously together with collinear (residual) energy co-flows of the distributed active mass and its passive gravitation field within the elementary radial carrier [7]. The exact mathematical solutions (8) to equations (6) for overlapping elementary charges conceptually reject the empty space paradigm with separated fields and particles.

The classical Poisson equation, \( \nabla^2 W = -\rho e^{-1} = -\Lambda^{-1}(\nabla W)^2 \), reads the Maxwell (astro)electron as a non-linear field composition with respect to the radial field intensity (8) or the electron self-potential \( W(r) \), with \( e(r) = -\partial_r W(r) \) and

\[
\frac{1}{r^2} \partial_r [r^2 \partial_r W(r)] = -\Lambda^{-1} [\partial_r W(r)]^2 = -\frac{(-e_o)r_o}{4\pi \epsilon r^2(r + r_o)^2}.
\]

This non-linear equation reveals the post-Coulomb solution for the continuous radial carrier of electricity,

\[
W(r) = \frac{(-e_o)}{4\pi \epsilon r_o} \ln \left( 1 + \frac{r_o}{r} \right) = \Lambda \ln \left( 1 + \frac{r_o}{r} \right),
\]

next to the harmonic solution \( C_1 r^{-1} + C_2 \) of the incomplete Laplace equation for the point-particle paradigm and the Coulomb approximation of electric potentials.

The post-Coulomb potential (11) reproduces the regular Coulomb law for point particles when \( r >> r_o \rightarrow 0 \), as \( (-e_o/4\pi \epsilon r_o)\ln[(r + r_o)/r] \approx (-e_o/4\pi \epsilon r)[1 - (r_o/2r)] \). Recall that the sphere with microscopic radius \( r_o \) contains exactly half of the electron charge \( (-e_o) \). The other half of the elementary astrocharge is distributed over micro, macro, and mega scales in the Universe, which is already known [8] as the non-local material system. The electron’s density scale \( r_o = 7 \times 10^{-58} \) m is well beyond the Planck’s length and the current limit \( 10^{-18} \) m of space measurements. This corresponds to the formal success of the \( \delta \)-operator modeling of nonlocal particles for their concentrated (almost point) interactions, formally called local ones.
One can verify that the astrocharge distribution (8) matches the CED integral relation for the logarithmic post-Coulomb potential (11),

\[
\int \frac{(-e_o)n(r')dr'}{4\pi\epsilon|r' - r|} = \int \int \frac{d\phi d\theta' r^2 dr'}{4\pi\epsilon\sqrt{r^2 + r'^2 - 2rr'\cos\theta^2}} \frac{(-e_o)r_o}{4\pi\epsilon(r' + r_o)^2} \left(\frac{rr' - |r' - r|}{2rr'}\right) = \frac{(-e_o)}{4\pi\epsilon r_o} \int_r^\infty dr' \left(\frac{1}{r'} - \frac{1}{r' + r_o}\right) \equiv \int_r^\infty e(r')dr' = W(r).
\]

Notice that \(W(r)\) coincides with the work associated with the displacement of a unit probe (positive) charge from the point \(r\) to \(\infty\) against the negative field \(e(r) = -\partial_r W(r)\). The integration over \(r'\) within \(0 \leq r' \leq r\) vanishes identically in (12) in analogue with the \(\delta\)-source physics, \(n(r') \rightarrow \delta(r')\), of the Coulomb potential.

By taking the post-Coulomb solution (11) with the zero Laplace contribution, \(C_1 = C_2 = 0\), one can also verify the integral equality of the electrostatic energy of the elementary charge density \(\rho(r)\) in its interaction potential \(W(r)\) and in the self-potential \(\Lambda\),

\[
\int \rho(r)W(r)dv = \int \frac{(-e_o)^2ln(1 + r_o/r)}{4\pi\epsilon(r + r_o)^2}dr = \frac{(-e_o)^2}{4\pi\epsilon r_o} = \int \rho(r)\Lambda dv.
\]

An integral electrostatic balance of the field energy and the charge energy in inhomogeneous (interaction) potentials always takes place in Maxwell’s electrodynamics due to the vector identities \(\vec{d} = -d\vec{\nabla} \varphi = -\nabla(\vec{d} \cdot \varphi) + \varphi \nabla \vec{d}\) and the Gauss divergence theorem. Now this general balance equally works in (13) for the electron’s radial self-energy in the constant potential \(\Lambda\), when the charge density is ‘simply’ proportional to the local field energy density. The strict mathematical equality \(\int_0^\infty dx[ln(1 + x^{-1})/(1 + x)^2] = \int_0^\infty dx/(1 + x)^2 = 1\) stands behind our introduction of the stable radial charge in the constant self-potential \(\Lambda\). The local electric current and energy flow densities of the elementary energy carrier are proportional one to another in this Mie-type electrodynamics.

The conventional Lagrange density is contributed by the partially balanced field and charge energy terms, \((\epsilon e^2 - \rho W) \neq 0\) in the Mie original approach to the distributed electron. The electrostatic action integral vanishes, \(\int (\epsilon e^2 - \rho W)dv = E_f - E_{\text{el}}^i = e_o^2/4\pi\epsilon r_o - e_o^2/4\pi\epsilon r_o = 0\), for the extended particle and field carrier of electricity even for \(r_o \rightarrow 0\). We may require even the complete local balance of the radial charge and its field energy densities in the EM Lagrangian. Indeed, \(\rho = -\nabla^2 W\epsilon\) for the distributed charge density and, due to the divergence theorem, the equivalent EM action \(\int [\epsilon e^2 - \epsilon(\nabla W)^2]dv\) has the vanishing Lagrange density. This local balance works for a particle’s equation of motion (after the variations) and completely removes the balanced electromagnetic energies from the electron’s mass-energy density \(m_{\alpha n}(r)\). Again, elementary electric currents are actually electric self-energy flows of the distributed particle-field carrier (8). Therefore, Maxwell-Lorentz’s equations-equalities (6) should equally represent elementary electric currents and elementary self-energy flows (with local identical balance of these EM energy flows for particle and field fractions).
The mutual interaction energy, $E_{12+21}(d)$, of two static electron/positron charges is determined by their continuous local overlap with the external interaction potentials $W_1(\vec{x}) \neq W_2(\vec{x})$ in all world space points, rather than with the uniform self-potentials $\Lambda_1 = \pm \Lambda$. This global interaction depends on the space distance $d = 2z_o$ between the particles’ centers of symmetry at $(0,0,-z_o)$ and $(0,0,+z_o)$,

$$E_{12+21}(d) = \int dv [\rho_1(\vec{x})W_2(\vec{x}) + \rho_2(\vec{x})W_1(\vec{x})]$$

$$= \frac{\mp e_o}{4\pi \epsilon} \int \frac{dv}{4\pi} \left( \frac{ln(1 + r_o/r_-)}{r^2(r_+ + r_o)^2} + \frac{ln(1 + r_o/r_+)}{r^2(r_- + r_o)^2} \right),$$

where $r_\pm = \sqrt{x^2 + y^2 + (z \pm z_o)^2}$. Extreme values, $\pm e^2_o/2\pi\epsilon r_o$ for $d = 0$, of these astrospace interactions are finite even under the strict coincidence of the particles’ geometrical centers. For example, the total electron-positron potential energy (two positive potential self-energies $E_{11}^{ch}$ and $E_{22}^{ch}$, plus the negative interaction energy $E_{12+21}^{ch}$) monotonously changes from zero, when $d = 0$, to $2E_{11}^{ch} = e_o^2/2\pi\epsilon r_o$, when $d \to \infty$. Such a huge electrostatic energy of sole charges is fully compensated by their field energy in the Lagrangian, while the residual electron-positron annihilation energy can be related only to the particles’ mechanical energy and energy of their gravitational field.

The potential self and mutual energies of elementary continuous charges contribute to the potential electric energy of the global charge density overlap, $\rho(x) = \sum_i \rho_i(x)$, in the net local potential, $W(x) = \sum_i W_i(x)$,

$$E_{glo}^{ch} = \int dv \left( \sum_k \rho_k(x) \right) \left( \sum_i W_i(x) \right)$$

$$= \int dv \rho(x) W(x) = \int dv \bar{\rho}(x) \bar{d}(x) = \int dv \left( \sum_i \bar{\epsilon}_i(x) \right) \left( \sum_k \bar{d}_k(x) \right) \equiv E_{glo}^{f}.$$ 

Here we used $\rho = \nabla \epsilon \bar{\epsilon}$ and the divergence theorem for the integral electrostatic balance $\int \epsilon \bar{\epsilon}^2 dv - \int \rho W dv = 0$. Again, equal electrostatic energies of elementary charges and their fields are bound and balanced even locally. Only very small, residual imbalance from the electron’s EM energy balance can be expected for the mechanical part of the elementary Lagrangian with a residual space curvature for the electron’s rest mass-energy $m_o = c^2_o/G << e^2_o/4\pi\epsilon r_o$.

The classical field equations (6) and the rest-frame solution (8) admit Lorentz transformations of fields for moving frames of reference. Therefore, the continuous astrocharge (8) may replace the point electron on covariant CED relations. Moreover, the nonlocal Maxwell electron tends to replicate astrodistributions of the (nine) known quantum electrons, for example in Bohm’s formulation of quantum theory [9].

At first glance, the uniqueness of the analytical solution (8) for the differential Poisson equation seems leave no room for extra positive charge solutions in Maxwell’s theory. The ‘sole charge’ argument was indeed used against Mie’s attempts to re-interpret all classical
charges as continuous field functions. How, for example, could Classical Electrodynamics specify the difference between positron’s and proton’s spatial structures with equal contents of positive electricity?

3 The positive radial charge

The (astro)proton has to exhibit the opposite electron charge \( \int \rho_p dv = +e_o \), while the positive vertex \((+e_o)\) in (8) belongs to the (astro)positron. Fortunately for the Mie theory of continuous matter there are no limitations in Maxwell’s equations, and in their exact solution (8), on the sign (and the magnitude) of the radial parameter \( r_o \). Therefore, one can coherently describe the elementary positive charge (the static proton or the sum of dynamical quarks) by replacing the electron’s radial parameter with a negative number, \( r_o \rightarrow -r_p < 0 \), and \((-e_o)\) with \((+e_o)\) in the electron’s field induction \( \vec{d} \). Then, the charge density \( \rho_p(r) = \nabla \vec{d}_p \) of the classical radial proton can be found analytically at \( r \neq r_p \),

\[
\begin{align*}
\vec{d}_p(r > r_p) = \hat{\mathbf{r}} e_o/4\pi r(r - r_p), \quad \rho_p(r > r_p) &= -e_o r_p/4\pi r^2(r - r_p)^2 < 0 \\
\vec{d}_p(r < r_p) = \hat{\mathbf{r}} e_o/4\pi r (r_p - r), \quad \rho_p(r < r_p) &= +e_o r_p/4\pi r^2(r_p - r)^2 > 0.
\end{align*}
\]

These exact solutions of Maxwell’s equations for negative and positive static charge densities should be agreed with to the (dynamical, fluctuating) neutral spherical front at \( r = r_p \). We employ a complex proton parameter \( z_p \equiv r_p + i\delta \) in order to describe steady states of baryons (with dynamical sub-charges or substructures) without static state peculiarities at the pole point \( r_p \). Contrary to the monotonous radial electron, the radial Maxwell proton has quite complicated spatial substructures. Our description of the steady (astro)proton in terms of complex continuous fields in \( 0 \leq r < \infty \),

\[
\begin{align*}
\vec{d}_p(r) &= \hat{\mathbf{r}} e_o/4\pi r|r - z_p| \\
4\pi r^2 \rho_p(r) &= 4\pi \partial_r r^2 d_p(r) = +e_o [r_p(r_p - r) + \delta^2]/[(r - r_p)^2 + \delta^2]^{3/2},
\end{align*}
\]

(17)
can be interpreted as the quasi-equilibrium approximation of dynamical sub-states with the averaged parameter \( r_p \equiv < r_p(t) >_t \) for the fluctuating neutral front between positive and negative charge substructures.

The (astro)proton’s electric induction takes the regular Coulomb approximation \( 4\pi \varepsilon_o d_r = +e_o/r^2 > 0 \) for \( r \gg r_p \) when the continuous proton density \( \rho_p(r) \approx -e_o r_p/4\pi r^4 \) is negative. This charge density sharply becomes positive only at very short proximities, \((r_p^2 + \delta^2)/r_p \approx r_p \) for \( \delta \ll r_p \), to the proton’s center of symmetry. The radial charge distribution \( dq(r)/dr = 4\pi r^2 \rho_p(r) \) reaches its extreme positive and negative rates, \( \pm 2e_o r_p/\sqrt{27\pi} \delta^2 \) at \( r_p \mp (\delta/\sqrt{2}) \), just next to the neutral charge front at \( r_p \). Such non-monotonous and very sharp distributions of the charge and field densities (17) result in enhanced electrostatic
self-energy content for the continuous proton,

\[ e_o \Lambda_p \equiv \mathcal{E}^{ch} = \mathcal{E}^f = \int \frac{d^2 p}{\epsilon} = \frac{e_o^2}{4\pi \epsilon \delta} \left[ \frac{\pi}{2} + \arctg \left( \frac{r_p}{\delta} \right) \right] \approx \frac{e_o^2}{4\epsilon \delta}, \quad (18) \]

compared to the similar self-energy (7) for the continuous electron.

Now one can apply the Gauss’s flux theorem in order to find an enclosed charge within different spheres R around the proton’s center:

\[ q_p(R) \equiv \int_R \rho_p(r) 4\pi r^2 dr = \oint_R \vec{d}_p d\vec{s} = +e_o \frac{R}{\sqrt{(R - r_p)^2 + \delta^2}}. \quad (19) \]

The enclosed charge (19) is always positive and exhibits its observable macroscopic value \( q_p(\infty) = +e_o \) for \( R \gg r_p \). This means that the central positively charged sphere (with the maximum positive content \( q(r_p) = +e_o r_p / \delta \gg +e_o \)) is screened by microcosm of large negative charge \( -e_o (r_p - \delta) / \delta \approx -e_o r_p / \delta \ll -e_o \). From here the non-empty world space (EM ether) is charged negatively by both astroelectrons and astroprotons everywhere apart from protons’ core spheres of very small radius \( (r_p = GE_p / c^4 \approx 1.2 \times 10^{-54}m) \).

### 4 Conclusions

Our main conclusion is that Mie was conceptually right. Maxwell’s equations-equalities for locally bound particle and field fractions of the elementary radial carrier of electricity can describe self-consistently continuous structures of elementary charges and basic wave modulations within these charged structures. The Dirac operator density \( q_k \delta(r - \vec{r}_k) \) for the formal localization of distributed energy-matter in one point \( \vec{r}_k \) can be replaced with the equilibrium radial density \( q_k n(\vec{r} - \vec{r}_k) = q_k r_o / 4\pi (\vec{r} - \vec{r}_k)^2 (|\vec{r} - \vec{r}_k| + r_o)^2 \), for the nonlocal continuous source in any field point \( \vec{r} \). And the local balance of particle and field EM energy flows (with equal active and passive EM self-energy densities) is the physical meaning of local Maxwell’s equations-equalities.

The nonlocal continuous electron is free from peculiarities and this Maxwell-Mie charge can better address the EM current-vs-field density relations than the point electron, which is ‘a stranger in electrodynamics’ according to Einstein. The considered static radial densities without rotations are not relevant to the electron’s magnetic moment and mechanical spin. Practical observations of charged EM matter are given through dynamics and energy exchanges of inertial (residual) masses-energies. Therefore, CED energy-balances cannot lead to a complete theory without gravimechanical equations. There are no charges without masses and Maxwell’s equations ought to be compatible with the (unknown) mass creation mechanism for the continuous carrier of distributed electromagnetic and gravitational energies.

Stationary rotational solutions with energy-driven spin states might be on the very surface of the classical theory of fields. If General Relativity (GR) employs the analytical
astrodistribution $m_k n(\vec{r} - \vec{r}_k)$ for the rest-mass energy (together with equal gravitational field energy), then properties of the combined Einstein-Maxwell radial sources in non-empty space can be described through classical relativistic notions (distributed angular momentum, magnetic moments of rotating charge densities, gyromagnetic factors, etc.) in agreement with the Lie algebra for the Poincaré group.

The Maxwell-Mie nonlocal astrocharge, distributed over the entire Universe, had been already replicated by the distributed quantum particle. Nonetheless, only exact analytical relations for continuous sources, rather than dice probabilities for them, may conceptually justify the non-empty space paradigm for physical reality. Non-empty space readings of Maxwell’s and Einstein’s equations through locally bound particle-field (source-sink, active-passive, yang-ying) flows of elementary energies may be useful for causal replications of quantum objects. In this way, quantum particles may converge with nonlocal classical particles after proper quantization of their EM-GR energies and angular momentums.

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