Family non-universal $Z'$ models with protected flavor-changing interactions

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We define a new class of $Z'$ models with neutral flavor-changing interactions at tree level in the down-quark sector. They are related in an exact way to elements of the quark mixing matrix due to an underlying flavored $U(1)'$ gauge symmetry, rendering these models particularly predictive. The same symmetry implies lepton-flavor non-universal couplings, fully determined by the gauge structure of the model. Our models allow to address presently observed deviations from the SM and specific correlations among the new physics contributions to the Wilson coefficients $C^{(d)}_{9,10}$ can be tested in $b\to s\ell^+\ell^-$ transitions. We furthermore predict lepton-universality violations in $Z'$ decays, testable at the LHC.

I. INTRODUCTION

The Standard Model (SM) of electroweak interactions [1], based on the gauge group $G_{SM} = SU(2)\times U(1)_Y$, has been tested to very high accuracy over the last decades. This has strong implications for new physics (NP) scenarios, which are then required to involve a very high mass scale, a highly non-trivial flavor structure, or both. Recently the LHCb collaboration has performed two measurements involving semi-leptonic $b\to s\ell^+\ell^-$ transitions which show deviations with respect to the SM expectations: The ratio $R_K = Br(B \to K\mu^+\mu^-)/Br(B \to Ke^+e^-)$ has been measured with a central value indicating a violation of lepton universality at the 25% level [2], implying a 2.6σ deviation, and the observable $P_\epsilon$ obtained from the angular analysis of $B \to K\mu^+\mu^-$ decays differs from the SM expectation with 2.9σ significance [3]. Taking these measurements at face value, they require the NP to have the following features:

(i) Sizable contributions to $b\to s\ell^+\ell^-$ transitions,

(ii) lepton non-universal couplings.

Extensions of the SM gauge group by an additional $U(1)'$ factor are among the possible NP scenarios that could explain these deviations [4–10] (other NP interpretations have been discussed e.g. in Refs. [11–14]). Such $U(1)'$ models have been popular extensions of the SM for many years, see Ref. [15] and references therein. However, the majority of the extensions considered in the literature are family universal in the quark (and charged-lepton) sector, and therefore do not meet condition (ii). Models with departures from family universality can give rise to large flavor changing $Z'$ interactions, which are strongly constrained by current flavor data [9, 16, 17]. The conditions (i) and (ii) therefore limit considerably the viable $U(1)'$ gauge symmetries.

Starting a new construction with the minimal particle content, i.e. the SM one without right-handed neutrinos, the only possible extension is gauging one of the combinations of family-specific lepton number, $L_\alpha - L_\beta$ (with $\alpha, \beta = e, \mu, \tau$ and $\alpha \neq \beta$) [18]. Since the resulting $Z'$ boson couples only to leptons, these models do not meet requirement (i) and have been discussed mostly in the context of neutrino phenomenology, usually considering the $L_\mu - L_\tau$ symmetry [19]. Recent models [4, 5] circumvent this problem by introducing additional fermions (vector-like quarks). This yields the required couplings at an effective level.

In order to avoid such exotic vector-like quarks, the $U(1)'$ symmetry has to involve both quarks and leptons, and should be family-dependent in order to satisfy conditions (i) and (ii). However, such a non-trivial flavor symmetry in the quark sector requires the extension of the scalar sector in order to accommodate the quark masses and mixing angles [20]. It has been shown in Ref. [6] that this option can be realized by including an additional complex scalar doublet. This model generates flavor-changing phenomena in the down-quark sector which are related in a first approximation to the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix [21]. Flavor-changing interactions are also present in the up-quark sector of this model, but these are not related to CKM matrix elements or quark masses.

We would like our model to have all of its fermion couplings related exactly to elements of the CKM matrix. To that aim, we note that there is a class of two-Higgs-doublet models (2HDM) that achieves this, known
as Branco–Grimus–Lavoura (BGL) models \[22\]. These models have flavor-diagonal interactions in the up-quark and charged-lepton sectors, together with flavor-changing neutral currents (FCNCs) in the down-quark sector. The latter are suppressed by quark masses and/or off-diagonal CKM elements in an exact way, thereby providing an alternative solution to the flavor problem in 2HDMs which differs radically from the hypothesis of natural flavor conservation \[23\].

While in the original BGL model the flavor symmetry is global, in this work we promote it to a local one. This is achieved by charging also the leptons under the symmetry, thereby enabling anomaly cancellation. In this gauged BGL framework \((U(1)_{\text{BGL}}')\), the properties of the BGL models are transferred to the gauge boson sector: we obtain FCNCs mediated at tree-level by the neutral scalar and massive gauge vector bosons of the theory, all of which are suppressed by off-diagonal CKM elements and/or fermion masses, and therefore naturally suppressed. This class of models necessarily exhibits deviations from lepton universality due to its gauge structure. Note that this form of flavor suppression in the down-quark sector also appears in certain 3-3-1 models \[7\]. However, in our framework this is not a choice, but a consequence of the gauge symmetry.

This paper is organized as follows: We formulate the \(U(1)_{\text{BGL}}'\) models in Sec. \[1\] in Sec. \[II\] we discuss the constraints from flavor and collider data, testing the strong correlations present in these models, specifically with respect to the deviations observed by LHCb. We conclude in Sec. \[IV\]. The appendices include technical details on anomaly cancellation and the scalar sector.

## II. GAUGED BGL SYMMETRY

A characteristic feature of BGL models \[22\] is the presence of FCNCs at tree level, which are however sufficiently suppressed by combinations of CKM matrix elements. This is a consequence of specific patterns of Yukawa couplings, generated by corresponding charge assignments under a horizontal, family-non-universal (BGL)-symmetry.

The quark Yukawa sector of the model reads \((i = 1, 2)\)

\[
\mathcal{L}^{\text{quark}}_{\text{Yuk}} = q^T_d L \Gamma_i \Phi_i d_R^0 + \bar{q}^T \Delta_i \Phi_i u_R^0 + \text{h.c.},
\]

where \(\Gamma_i\) and \(\Delta_i\) denote the Yukawa coupling matrices for the down- and up-quark sectors, respectively, and \(\Phi_i \equiv i\sigma_2 \Phi_i^*\); with the Pauli matrix \(\sigma_2\). The neutral components of the Higgs doublets acquire vacuum expectation values (vevs) \(|\langle \Phi_i^0 \rangle| = v_i/\sqrt{2}\). Defining \(v = (\sqrt{2}G_F)^{-1/2} \approx 246\) GeV. We further define \(\tan \beta = v_2/v_1\) and use the following abbreviations: \(\cos \beta \equiv c_\beta\), \(\sin \beta \equiv s_\beta\), \(\tan \beta \equiv t_\beta\).

Choosing an abelian symmetry under which a field \(\psi\) transforms as

\[
\psi \to e^{i X^0} \psi,
\]

the most general symmetry transformations in the quark sector yielding the required textures are of the form

\[
X^0 = \frac{1}{2} \left[ \text{diag} (X_{uR}, X_{uR}, X_{dR}) + X_{dR} \mathbb{1} \right],
\]

\[
X^0 = \text{diag} (X_{uR}, X_{uR}, X_{dR}),
\]

\[
X^0 = X_{dR} \mathbb{1},
\]

with \(X_{uR} \neq X_{dR}\). The Higgs doublets transform as

\[
X^\Phi = \text{diag} (X_{\Phi_1}, X_{\Phi_2}) = \frac{1}{2} \text{diag} (X_{uR} - X_{dR}, X_{dR} - X_{uR}).
\]

There are several possible implementations of this symmetry, related by permutations in flavor space and exchanging up- and down-quark sectors; here, for definiteness, the top quark has been singled out, yielding the patterns

\[
\Gamma_1 = \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix},
\]

\[
\Delta_1 = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}.
\]

These textures give rise to FCNCs in the down-quark sector, which are however suppressed by quark masses and off-diagonal elements of the third CKM row \[22\]. This is related to the fact that the CKM mixing matrix \(V = U_{udL}^U U_{dlL}^U\) has a direct correlation in this model with the elements of the third row of the rotation of the left-handed down quarks, \(i.e.\ (U_{dlL})_{3i} = V_{3i}\). The choice of the top quark in Eq. \[3\] implies a particularly strong suppression of flavor-changing phenomena in light-quark systems. Present constraints from \(\Delta F = 1\) and \(\Delta F = 2\) quark flavor transitions are accommodated even when the scalars of the theory are light, with masses of \(\mathcal{O}(10^2)\) GeV \[25\].

While this model can be implemented through an abelian discrete symmetry, this always yields an accidental continuous abelian symmetry which introduces an undesired Goldstone boson into the theory \[22\]. Several solutions to this problem have already been proposed in the literature \[22, 29\]; in the following we provide a new solution, promoting the BGL symmetry to a local one.

When gauging a symmetry, special attention must be paid to whether it remains anomaly-free\[2\] BGL
models are automatically free of QCD anomalies, i.e. $U(1)'^\prime$[SU(3)$_C$]$_Z^2$ [20]. However, we also need to fulfill the anomaly conditions for the following combinations:

$$\begin{align*}
U(1)'[SU(2)_L]^2, & \
U(1)'[U(1)_Y]^2, & \
[U(1)']^2U(1)_Y, & \
[U(1)']_Z^3, & \
U(1)'[\text{Gravity}]^2.
\end{align*}$$

(6)

We find that there is no solution for this system within the quark sector alone with the charge assignments in Eq. (5). Satisfying these anomaly conditions is highly non-trivial and requires, in general, additional fermions. However, the implementation of the BGL symmetry as a local symmetry is possible by adding only the SM leptonic sector when allowing for lepton-flavor non-universal couplings. As in the SM, the cancellation of the gauge anomalies then occurs due to a cancellation between quark and lepton contributions; the cancellation will however involve all three fermion generations, contrary to the SM gauge group, for which the anomaly cancellation occurs separately within each fermion generation.

Gauging the BGL symmetry therefore not only removes the unwanted Goldstone boson, which is “eaten” by the $Z'$, but also provides a consistent $Z'$ model without extending the particle content beyond the additional scalars, i.e. no vector-like quarks are necessary. The flavor structure is extended to the $Z'$ couplings, yielding again strongly suppressed FCNCs in the down-type quark sector, while the up-type-quark couplings remain diagonal. Especially, since the flavored gauge symmetry fixes also the charged-lepton Yukawa matrices to be diagonal, we can obtain large deviations from lepton universality without introducing lepton-flavor violation. Our models provide thereby explicit examples for which the general arguments given in Refs. [11] 28, 29 do not hold.

### A. Lepton sector

As emphasized above, the anomaly constraints can be fulfilled by assuming that leptons are also charged under the $U(1)'$ symmetry. The resulting charge constraints are given in Appendix A.

For $X_{\Phi_2} = 0$ the mixing between the neutral massive gauge bosons is suppressed for large $\tan \beta$. With this choice, we obtain only one possible solution to the anomaly conditions up to lepton-flavor permutations, implying the following charge assignments:

$$\begin{align*}
\lambda_R^d = \mathbb{1}, & \
\lambda_R^u = \text{diag} \left( \frac{7}{2}, \frac{7}{2}, 1 \right), & \
\lambda_L^d = \text{diag} \left( \frac{5}{4}, \frac{5}{4}, 1 \right), & \
\lambda_L^u = \text{diag} \left( \frac{9}{4}, \frac{21}{4}, -3 \right).
\end{align*}$$

(7)

We have normalized all $U(1)'$ charges by setting $X_{dR} = 1$, thereby fixing also the normalization for the gauge coupling $g'$. In addition to the top, this particular model singles out the tau lepton as the only one coupling to $\Phi_2$. Permutations of the charges in $\lambda_L^d$ and $\lambda_R^u$ give rise to 5 more models.

With the charge transformations in Eq. (7), the charged-lepton Yukawa sector takes the form

$$\begin{align*}
-\mathcal{L}_{\text{Yuk}}^{\text{c-leptons}} = \tilde{\Phi}_L \Pi_i \Phi_2 e_R^c + \text{h.c.},
\end{align*}$$

with

$$\begin{align*}
\Pi_1 = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, & \\
\Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix},
\end{align*}$$

(8)

while the quark Yukawa sector remains unchanged.

We call to attention that no reference to the neutrino sector has been made in the previous arguments. In this work we are mostly interested in the properties of the quark and charged-lepton sectors in the presence of a new gauge boson, while we leave a detailed discussion of the neutrino sector to future work. For a possible implementation with the SM fermion content we can build the $d = 5$ Weinberg effective operator ($\tilde{\Phi}_L)(\tilde{\Phi}_R^c \tilde{e}^c$), which after spontaneous symmetry breaking induces a Majorana mass term for the neutrinos. If this effective operator is invariant under the new flavored gauge symmetry in Eq. (7), we are led to a $L_\mu - L_\tau$ symmetric $d = 5$ effective operator and, consequently, Majorana mass term. This structure has been studied in Refs. [19], and while it is not capable of fully accommodating the neutrino data, it can serve as a good starting point.

If we want to improve on this scenario while keeping our solution to the anomaly conditions, we may want a mechanism that at the effective level already breaks the accidental $L_\mu - L_\tau$ symmetry. For instance, we may extend the model with 3 right-handed neutrinos, where one of them is not charged while the other two have opposite charges. In this way the anomaly solutions remain unchanged. Choosing the charges of the right-handed neutrinos appropriately, the Dirac mass term for the neutrinos can be made diagonal and the Majorana mass for the right-handed ones $L_\mu - L_\tau$ symmetric. Along the lines of Ref. [6], coupling an additional scalar to the right-handed neutrinos could break the accidental symmetry at the effective level.

We can also envisage other mechanisms that do change the anomaly conditions and, therefore, introduce new solutions.

### B. Gauge boson sector

In order to avoid experimental constraints on a $Z'$ boson, the $U(1)'$ symmetry must be broken at a relatively
high scale, rendering the new gauge boson significantly heavier than the SM ones \cite{15}. This can be achieved through the introduction of a complex scalar singlet $S$, charged under the BGL symmetry with charge $X_S$, which acquires a vev $\langle S \rangle = v_S/\sqrt{2} \gg v$. The charge of the scalar singlet is fixed once the scalar potential is specified, in our case $X_S = -9/8$, see Appendix \cite{B} for details.

As a consequence of the gauge symmetry breaking, $G_{SM} \times U(1)^{\prime} \to U(1)_{em}$, the neutral massive gauge bosons mix, giving rise to a Lagrangian of the form\cite{3}

$$
\mathcal{L} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu}
+ \frac{1}{2} M^2 Z_{\mu} Z^{\mu} + \frac{1}{2} M^2 \tilde{Z}_{\mu} \tilde{Z}^{\mu} + \Delta^2 \tilde{Z}^\mu \tilde{Z}_\mu \\
- J^a_{\mu} A_{\mu} - J^\mu_{\tilde{Z}} \tilde{Z}_{\mu} - J^{\mu\nu}_{\tilde{Z}} \tilde{Z}^\mu_{\nu},
$$
(10)

where $J^a_{\mu}$ and $J^\mu_{\tilde{Z}}$ are the SM currents of the photon and the $Z$, respectively. The fermionic piece of the new current takes the form

$$
\tilde{J}^\mu_{\tilde{Z}} \supset g' \bar{\psi} \gamma^\mu \left[ (\tilde{X}^\psi_R)_{ij} P_L + (\tilde{X}^\psi_L)_{ij} P_R \right] \psi_j,
$$
(11)

with $P_{L,R} = (1 \mp \gamma_5)/2$ denoting the usual chiral projectors, $g'$ the $U(1)^{\prime}$ gauge coupling, and $\tilde{X}_X$ the phase transformation matrices in the fermion mass basis, i.e.

$$
\tilde{X}_R = X_R, \quad \tilde{X}_L = X_L, \quad \tilde{X}_{\tilde{Z}} = X_{\tilde{Z}} \quad \text{and} \quad \tilde{X}^\mu_{\tilde{Z}} = -\frac{5}{4} + \frac{9}{4} \left[ \frac{V_{td}^2 V_{ts}^* V_{ub} V_{us}^*}{V_{td} V_{ts} V_{ub} V_{us}} \right].
$$
(12)

The neutral gauge boson mass and mass-mixing parameters in this model are given by

$$
M^2_Z = \frac{e^2 v^2}{4 s_W c_W}, \quad M^2_{\tilde{Z}} = g'^2 \left( \sin^2 \theta_W + v^2_s X^2_S \right), \quad \Delta^2 = -\frac{e g'}{2 c_W s_W} X_{\Phi_2} v^2_2,
$$
(13)

where $e = \sqrt{4\pi\alpha}$ is the electromagnetic charge and its ratio with the SU(2)$_L$ coupling $g$ defines $\sin^2 \theta_W = \arcsin (e/g)$, which differs from the experimentally measured weak angle due to mixing effects \cite{30}: $s_W c_W M_Z = s_W c_W M_{\tilde{Z}}$, where $M_{\tilde{Z}}$ is the physical $Z$ mass. We rotate to the physical eigenbasis by performing the following orthogonal transformation:

$$
\begin{pmatrix}
Z_{\mu} \\
\tilde{Z}^\mu_{\nu}
\end{pmatrix} = O \begin{pmatrix}
\tilde{Z}_\mu \\
\tilde{Z}^\mu_{\nu}
\end{pmatrix}, \quad \text{with} \quad O = \begin{pmatrix}
c_\xi & s_\xi \\
-s_\xi & c_\xi
\end{pmatrix}
$$

and

$$
\tan 2\xi = \frac{2\Delta^2}{M^2_Z - M^2_{\tilde{Z}}}.
$$
(14)

Since we assume $v \ll v_S$, we obtain $M^2_{\tilde{Z}} \Delta^2 \ll M^2_Z$, and the resulting mixing angle is small \cite{15}:

$$
g' \xi \simeq \frac{e}{2 c_W s_W} \frac{X_{\Phi_2} v^2_2 + X_{\Phi_3} v^2_3}{X^2_S v^2_S} \simeq -\frac{9e}{8c_W s_W} \left( \frac{g' c_\beta v}{M_Z^2} \right)^2.
$$
(15)

Expanding in this small parameter, we obtain the following leading expression for the hatted weak angle in terms of physical quantities

$$
s^2_W \simeq s^2_2 \simeq s^2_W - \frac{c^2_W s^2_W}{c^2_W - s^2_W} \xi^2 \left( \frac{M^2_{\tilde{Z}}}{M^2_Z} - 1 \right).
$$
(16)

In the physical basis the neutral gauge boson masses read

$$
M^2_{Z,\tilde{Z}} = \frac{1}{2} \left[ \tilde{M}^2_{Z,\tilde{Z}} + \tilde{M}^2_{Z,\tilde{Z}} + \sqrt{\left( \tilde{M}^2_{Z,\tilde{Z}} - M^2_Z \right)^2 + 4\Delta^4} \right],
$$
(17)

and their fermionic currents are given by

$$
J^{\mu}_{Z(\tilde{Z})} \supset \bar{\psi} \gamma^\mu \left[ \epsilon^{(t)\nu}_i P_L + \epsilon^{(t)\nu}_i P_R \right] \psi_j,
$$
(18)

with the couplings (up to $O(\xi^2)$)

$$
\epsilon^{(t)}_{X,i,j} = \frac{e}{c_W s_W} \left[ 1 + \frac{\xi^2}{2} \left( \frac{M^2_{\tilde{Z}}}{M^2_Z} - 1 \right) \right] \times \left[ \left( T^\psi_{3x} - s^2_Q \right) \bar{\psi} - \xi g' c_W s_W \bar{\phi} \right],
$$
(19)

Here $T^\psi_{3x}$ is the third component of weak isospin for the left-handed fields ($T^\psi_{3L} = 0$) and $Q_\psi$ the electric charge. We also define the vector and axial-vector combinations of the $Z^{(t)}$ couplings,

$$
\epsilon^{(t)}_{V,i,j} \equiv \epsilon^{(t)}_{L,i,j} + \epsilon^{(t)}_{R,i,j}, \quad \epsilon^{(t)}_{A,i,j} \equiv \epsilon^{(t)}_{L,i,j} - \epsilon^{(t)}_{R,i,j}.
$$
(20)

The coefficients $\epsilon^{(t)}_{V(i)A,i,j}$ encode corrections to the SM $Z$ couplings due to mixing with the $Z'$ which are proportional to $g' \xi$ at leading order in the mixing angle. Such corrections are restricted to be small ($g' \xi \lesssim 10^{-4}$) \cite{31} \cite{32}. In addition to our initial assumption, $v_S \gg v$, which already gives a small $\xi$, we will assume that the mixing angle receives an additional suppression, because $\tan \beta$ is large\cite{4}. This also guarantees that flavor-changing effects mediated by the $Z$ are negligible compared with those of the $Z'$.

\footnote{This limit is quite natural in our model since $\Phi_2$ is coupled to the top quark, while $\Phi_1$ is coupled to the light first two generations. Therefore, the hierarchy $v_2 > v_1$ accommodates well the quark mass spectrum.}
Another important theoretical constraint is obtained when requiring the absence of a low-energy Landau pole, i.e., an energy scale \( \Lambda_{\text{LP}} \) at which perturbativity is lost. This scale can be found from the renormalization-group running of the \( U(1)' \) coupling, we have:

\[
\frac{d \alpha'}{d \ln q^2} = b \alpha'^2 + O(\alpha'^3),
\]

where the one-loop beta function \( b \) contains the charge information of the model. It reads

\[
b = \frac{1}{4\pi} \left[ \frac{2}{3} \sum_i X_{q_{iL,R}}^2 + \frac{1}{3} \left( 2 \sum_i X_{\eta_i}^2 + X_{\lambda_3}^2 \right) \right],
\]

with \( f \) including all fermion degrees of freedom and \( i = 1,2 \). The Landau pole can be extracted from the pole of \( \alpha'(q^2) = g^2(q^2)/4\pi \), we obtain

\[
\Lambda_{\text{LP}} \approx M_{Z'} \exp \left[ \frac{1}{2 b \alpha'(M_{Z'}^2)} \right].
\]

For the charges in our model we have \( b \approx 12.90 \), see Eq. (7). Taking \( M_{Z'} \approx 4 \) TeV, we get a Landau pole at the Planck scale, \( \Lambda_{\text{LP}} \gtrsim 10^{13} \) GeV, for \( g' \lesssim 0.12 \). Notice that this bound is stronger than the naive perturbativity bound \( \alpha' M_{Z'}^2 \lesssim O(1/\max\{|X_i|^2\}) \), which in our model gives \( g' \lesssim 0.47 \). We can relax this condition by assuming that additional NP will appear at the see-saw or the Grand Unification scale. This way we obtain \( g' \lesssim 0.14 \) for \( \Lambda_{\text{LP}} \gtrsim 10^{14} \) GeV and \( g' \lesssim 0.13 \) for \( \Lambda_{\text{LP}} \gtrsim 10^{16} \) GeV. The extension of the model to these scales is beyond the scope of this work.

\section{Scalar sector}

Here we discuss the main features of the Higgs sector of the model, a detailed derivation of these results is given in Appendix [3]. It consists of two complex Higgs doublets and a complex scalar gauge singlet. Their properties are largely determined by the fact that \( v/v_s \ll 1 \). The spectrum contains four would-be Goldstone bosons giving mass to the electroweak (EW) gauge vector bosons \( \{M_W, M_2, M_{Z'}\} \), a charged Higgs \( H^\pm \) and four neutral scalars (three CP-even \( \{H_1, H_2, H_3\} \) and a CP-odd \( A \)).

We obtain a decoupling scenario with a light SM-like Higgs boson, \( M_{H_3}^2 \sim O(v^2) \), three heavy quasi-degenerate states coming mainly from the scalar doublets \( M_{H_1}^2 \simeq M_A^2 \simeq M_{H_2}^2 \sim O(v_s^2) \) and another heavy CP-even Higgs coming mainly from the scalar singlet \( M_{H_3}^2 \sim O(v_s^2) \). The Yukawa interactions of the scalar bosons reflect this decoupling. They can be written as

\[
-\mathcal{L}_Y \supset \sum_{\varphi=H_k,A} \varphi \left[ \overline{d_L} Y_d \varphi d_R + \overline{u_L} Y_u \varphi u_R + \overline{\ell_L} Y_{\ell} \varphi e_R \right] + \frac{\sqrt{2}}{v} H^+ \left( \overline{u_L} N_d d_R - \overline{u_R} N_{Ud}^d d_L + \overline{\ell_L} N_{\ell e} e_R \right) + \text{h.c.}
\]

Neglecting corrections of \( O(v^2/v_s^2) \) we obtain

\[
Y_f^{H_1} = \frac{D_f}{v}, \quad Y_f^{H_2} = -\frac{N_f}{v}, \quad Y_f^{A} = \frac{i N_u}{v}, \quad Y_{d,\ell}^{A} = -\frac{i N_{d,\ell}}{v},
\]

with \( (f = u, d, \ell) \). Here fermions have been rotated to the mass eigenbasis. The diagonal matrices \( D_f = u, d, \ell \) contain the fermion masses, and the matrices \( N_f \) are given as

\[
(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (V^\dagger)_{i3} (V)_{3j} (D_d)_{jj},
\]

\[
N_u = \frac{v_2}{v_1} \text{diag}(m_u, m_c, 0) - \frac{v_1}{v_2} \text{diag}(0, 0, m_t),
\]

\[
N_{\ell} = \frac{v_2}{v_1} \text{diag}(m_\mu, m_\tau, 0) - \frac{v_1}{v_2} \text{diag}(0, 0, m_\tau).
\]

The couplings of the scalar \( H_3 \) to the fermions are additionally suppressed, since they are generated only through mixing with the doublet degrees of freedom: \( Y_f^{H_3} \sim \mathcal{O}(v/v_s) \). The matrix \( N_d \) is non-diagonal in flavor space, giving again rise to tree-level FCNCs in the down-quark sector controlled by the CKM matrix \( V \). Furthermore, as shown in Appendix [13], the constraints from the \( U(1)' \) symmetry render the scalar potential CP invariant. CP violation in our model is therefore uniquely controlled by the CKM phase, like in the SM.\[6\]

\section{Model variations}

As mentioned in section [1A], we have six model implementations according to the different lepton flavor permutations of the \( U(1)' \) charges, with identical quark and scalar charges given in Eq. (7). These permutations are performed in the basis where the lepton mass matrix is diagonal and the eigenvalues are correctly ordered. This way each permutation corresponds to the simultaneous interchange of the charges of all left- and right-handed leptons. It is straightforward to check that the permutation of just one set of charges, while satisfying the anomaly conditions as well, would give rise to a non-diagonal mass matrix and reduce to one of the six cases considered here after diagonalization.

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5 It is also possible to motivate scenarios that avoid decoupling based on an enhanced Poincaré symmetry protecting the weak scale [29]. These can give rise to a rich scalar sector at the EW scale and a pseudo-Goldstone boson in the spectrum.

6 We do not consider strong CP violation in this work.
In order to present the predictions for each of these models, we introduce the generic lepton charges

\[
X_{1L} = \frac{9}{4}, \quad X_{2L} = \frac{21}{4}, \quad X_{3L} = -3, \quad X_{1R} = \frac{9}{2}, \quad X_{2R} = \frac{15}{2}, \quad X_{3R} = -3,
\]

such that each implementation is labeled by \((e, \mu, \tau) = (i, j, k)\). For instance, the model presented in Eq. (7) is now labeled as (1, 2, 3).

The additional possibilities to implement the quark and neutrino sectors have been discussed in Secs. III and II A, respectively.

### III. DISCUSSION

In this section we discuss the phenomenology of the gauged BGL models introduced in the previous section. The phenomenology of a scalar sector with a flavor structure identical to the one of our model has been analyzed in Ref. 23. Since we are assuming a decoupling scalar sector, we can naturally accommodate a SM-like Higgs at 125 GeV; the heavy scalars are not expected to yield sizable contributions to flavor observables in general. We therefore focus on the phenomenological implications of the \(Z'\) boson for this class of models. In the cases where the constraints depend on the choice of lepton charges in our model, we give here the most conservative bound and discuss the differences in Secs. III C and III D.

#### A. Low-energy constraints

Despite the large mass of several TeV, the \(Z'\) boson yields potentially significant contributions on flavor observables, due to its flavor-violating couplings in the down-type quark sector. However, because of \(M_{Z'}^2/M_2^2 \lesssim 0.1\%\) and the CKM suppression of the flavor-changing \(Z'\) couplings, these contributions can only be relevant when the corresponding SM amplitude has a strong suppression in addition to \(G_F\). This is specifically the case for meson mixing amplitudes and electroweak penguin processes which we will discuss below. Further examples are differences of observables that are small in the SM, for example due to lepton universality or isospin symmetry.

The particular flavor structure of the model implies a strong hierarchy for the size of different flavor transitions: \(|V_{ts}^* V_{td}| \sim \lambda^3 \ll |V_{tb}^* V_{tb}| \sim \lambda^3 \ll |V_{tb}^* V_{td}| \sim \lambda^2\) (where \(\lambda \simeq 0.226\)). However, since the SM amplitudes often show a similar hierarchy, the relative size of the \(Z'\) contribution has to be determined on a case-by-case basis. We obtain for example similar bounds from the mass differences \(\Delta m_{d,s}\) in the \(B_{d,s}\) systems and \(\epsilon_K\). Here we include exemplarily the constraint from \(B_s\) mixing, while leaving a detailed phenomenological analysis for future work.

Given the high experimental precision of \(\Delta m_s\) [31], the strength of the corresponding constraint depends completely on our capability to predict the SM value. The limiting factors here are our knowledge of the hadronic quantity \(f_{B_s} \sqrt{\mathcal{B}_s}\) (for which substantial progress is expected soon, see Ref. [32]) and the relevant CKM elements. Using the corresponding values from Refs. 30 and 37, respectively, we conclude that contributions to \(\Delta m_s\) up to 20% remain possible at 95% confidence level (CL); from this we obtain the limit \(M_{Z'}/g' \gtrsim 16\text{ TeV}\). Tree-level scalar contributions have been neglected, since their effect cancels to a very good approximation in the decoupling limit considered here [35].

Regarding the input for the CKM parameters, we make the following observations in our models: (i) Charged-current tree-level processes receive negligible contributions. (ii) The mixing phase in \(B_{d,s}\) mixing and the ratio \(\Delta m_s/\Delta m_d\) remain SM-like. (iii) The additional direct CP violation in \(B \to \pi\pi, \rho\pi, \rho\rho\) and \(B \to J/\Psi K\) is negligible. These observations imply that in our context the global fits from Refs. 37, 39 remain valid to good approximation.

In our models the \(Z'\) boson couples to muons and will contribute to neutrino trident production (NTP), \(\nu_\mu N \to \nu N \mu^+ \mu^-\). We expect dominance of the \(Z'\) vector current for our models. The NTP cross section normalized to the SM one was calculated in Refs. 4, 9, 40. Using a combination of the latest NTP cross-section measurements we obtain a bound on the parameter combination \(M_{Z'}/g'\) [41], which is however weaker than the one from \(B_s\) mixing.

Atomic parity violation (APV) measurements also place bounds on additional neutral gauge bosons coupling to electrons and light quarks. In particular, the precise measurement of the \(133\text{Cs}\) weak charge from APV experiments is in a reasonable agreement with the prediction of the SM and can be used to set bounds on the \(Z'\) boson of our models [12]. The bound we obtain on \(M_{Z'}/g'\) from the latest determination of the \(133\text{Cs}\) weak charge is however again weaker than the one from \(B_s\) mixing.

Further potentially strong bounds stem from electric dipole moments (EDMs) and the anomalous magnetic moment of the muon. EDMs from the scalar sector occur at the two-loop level; while this alone does not render them necessarily sufficiently small, the additional suppression and cancellations in the decoupling limit imply very small effects [24, 43]. Anomalous magnetic moments can be present at the one loop level, but are again too small in 2HDMs near the decoupling limit [25, 41]. Concerning the new gauge boson, one-loop contributions to EDMs require different phases for left- and right-handed \(Z'\) couplings to quarks and/or leptons [45]; this feature is not present in our model, therefore the \(Z'\) contributions are again at the two-loop level and sufficiently suppressed, given the large \(Z'\) mass. In the case of anomalous magnetic moments we have non-vanishing one-loop
contributions, which for the muon read $^7$  

$$a_{\mu}^{\text{NP}} \simeq \frac{m_\mu^2}{4 \pi^2} \frac{g^2}{M_{Z'}^2} \left( \frac{1}{3} X_{\mu V}^2 - \frac{5}{3} X_{\mu A}^2 \right),$$  

(28)

where $X_{\mu V} \equiv (X_{\mu L} + X_{\mu R})/2$ and $X_{\mu A} \equiv (X_{\mu R} - X_{\mu L})/2$. Using the bound on $M_{Z'}/g'$ from $B^0 - B^0_s$ mixing, we get $a_{\mu}^{\text{NP}} < 1.3 \times 10^{-11}$ for all charge assignments, which is smaller than the current theory uncertainty in the prediction of $a_{\mu}^{\text{SM}}$ $^8$.

B. Direct searches

The obvious way to search directly for a $Z'$ is via a resonance peak in the invariant-mass distribution of its decay products. At the LHC this experimental analysis is usually performed by the ATLAS $^9$ and CMS collaborations for $Z'$ production in the $s$-channel in a rather model-independent way, but assuming validity of the narrow-width approximation (NWA), negligible contributions of interference with the SM $^{10}$, and flavor-universal $Z'$ couplings to quarks. Under these assumptions, the cross section for $pp \to Z'X \to f\bar{f}X$ takes the simplified form $^{11} 12$

$$\sigma = \frac{\pi}{48 s} \left[ c^f_u w_u (s, M_{Z'}^2) + c^f_d w_d (s, M_{Z'}^2) \right],$$  

(29)

where the functions $w_{u,d}$ are hadronic structure factors that encode the information of the Drell-Yan production processes of the $Z'$ and their QCD corrections (for a precise definition of these functions we refer the reader to Ref. $^{12}$). The model-dependent part of the cross section is contained in the coefficients $c_{u,d}$:

$$c^f_u \simeq g'^2 \left( X_{uL}^2 + X_{uR}^2 \right) \text{Br} (Z' \to f\bar{f}) ,$$

$$c^f_d \simeq g'^2 \left( X_{dL}^2 + X_{dR}^2 \right) \text{Br} (Z' \to f\bar{f}) .$$  

(30)

While the assumptions for these expressions are not exactly fulfilled in our model, they are applicable when neglecting the small contributions proportional to the off-diagonal CKM matrix elements. The reason is that under this approximation the $Z'$ couplings to the first two generations - which yield the dominant contribution to the Drell-Yan production of the $Z'$ - are universal and flavor conserving in the quark sector, see Eqs. (7) and (12). Note that $X_{uL} \sim X_{dL}$ up to small corrections proportional to the off-diagonal CKM matrix elements that we are neglecting.

For high values of $g'$ the ratio $\Gamma_{Z'}/M_{Z'}$ can be quite large, spoiling the NWA. The constraint is therefore only applicable for $g' \lesssim 0.2$, such that $\Gamma_{Z'}/M_{Z'}$ does not exceed 15%. However, for the mass range accessible so far such a large coupling is typically excluded by the constraint $R_K$, as will be shown below. Combined exclusion limits on the $Z'$ at 95% CL using the dimon and dielectron channels have been provided by the CMS collaboration in the $(c_u, c_d)$ plane $^{13}$. Using these bounds together with the constraint from $R_K$, we find that a $Z'$ boson is excluded in our models for $M_{Z'} \lesssim 3-4$ TeV, depending on the lepton charge assignments, while $g'$ should be $O(10^{-1})$. These bounds are stronger than those from many $Z'$ benchmark models, for which current LHC data typically give a lower bound of $M_{Z'} \gtrsim 2-3$ TeV $^{14}$ $^{15}$. The reason is a combination of a sizable value for $g'/M_{Z'}$, required by the constraint from $R_K$ and the combination of $Z'$ couplings to fermions appearing in our model, see Eq. (30).

When the $Z'$ is too heavy to be produced at a given collider, $M_{Z'} \gg s$, indirect searches can be carried out by looking for effects from contact interactions, i.e. effective operators generated by integrating out the heavy $Z'$. At the LHC, the corresponding expressions hold for $M_{Z'} \gg 4.5$ TeV, and limits can be extracted e.g. from searches for the contact interactions $(q\bar{q} \gamma_{\mu} P_L q')(\bar{e}\gamma_{\nu} P_R e)$ $^{16}$ $^{17}$. These are presented in terms of benchmark scenarios considering a single operator with a specific chirality structure. From the 95% CL limit provided in Ref. $^{18}$ for the operator $(q\bar{q} \gamma_{\mu} P_L q')(\bar{e}\gamma_{\nu} P_R e)$ we extract $M_{Z'}/g' \gtrsim 22$ TeV. Searches for contact interactions of the type $(e\gamma_{\mu} P_L R e)(f\gamma_{\nu} P_R f)$ were performed at LEP $^{19}$. These are sensitive to a $Z'$-boson coupling to electrons assuming $M_{Z'} \gtrsim 210$ GeV and the resulting limits are again given for benchmark scenarios. In this case we extract $M_{Z'}/g' \gtrsim 16$ TeV from the 95% CL bound on the operator $(e\gamma_{\mu} P_R e)(\bar{\mu}\gamma_{\nu} P_R \mu)$. It is important to note that the extracted bounds from contact interactions using benchmark scenarios do not capture the full dynamics of our models once the $Z'$ boson is integrated out. Interference effects due to operators with different chirality structures will be relevant in general, a detailed analysis of these effects is however beyond the scope of this work. These bounds should therefore be taken as a rough guide to the sensitivity of contact interactions searches at LEP and the LHC to the $Z'$ of our models. However, in general they do not put relevant bounds as long as $g'$ is not too large.

C. Discriminating the different models

The constructed class of models has two important features which imply specific deviations from the SM: controlled FCNCs in the down-quark sector and violations of lepton universality. Thanks to the very specific flavor structure, the two are strongly correlated in each of our models. While the flavor-changing couplings are universal in the models we discuss here, the lepton charges differ. Correspondingly the models can be discriminated

$^7$ Flavor changing $Z'$ couplings between the first two quark generations are suppressed by $|V_{td}|^2 \sim \lambda^5$ while violations of universality are suppressed by $|V_{td}|^2 \sim \lambda^6$ and $|V_{ts}|^2 \sim \lambda^2$, with $\lambda \simeq 0.220$.

$^8$ Applicable for $g' \lesssim 0.2$, such that $\Gamma_{Z'}/M_{Z'}$ does not exceed 15%. However, for the mass range accessible so far such a large coupling is typically excluded by the constraint $R_K$, as will be shown below. Combined exclusion limits on the $Z'$ at 95% CL using the dimon and dielectron channels have been provided by the CMS collaboration in the $(c_u, c_d)$ plane. Using these bounds together with the constraint from $R_K$, we find that a $Z'$ boson is excluded in our models for $M_{Z'} \lesssim 3-4$ TeV, depending on the lepton charge assignments, while $g'$ should be $O(10^{-1})$. These bounds are stronger than those from many $Z'$ benchmark models, for which current LHC data typically give a lower bound of $M_{Z'} \gtrsim 2-3$ TeV. The reason is a combination of a sizable value for $g'/M_{Z'}$, required by the constraint from $R_K$ and the combination of $Z'$ couplings to fermions appearing in our model, see Eq. (30).

$^9$ When the $Z'$ is too heavy to be produced at a given collider, $M_{Z'} \gg s$, indirect searches can be carried out by looking for effects from contact interactions, i.e. effective operators generated by integrating out the heavy $Z'$. At the LHC, the corresponding expressions hold for $M_{Z'} \gg 4.5$ TeV, and limits can be extracted e.g. from searches for the contact interactions $(q\bar{q} \gamma_{\mu} P_L q')(\bar{e}\gamma_{\nu} P_R e)$ we extract $M_{Z'}/g' \gtrsim 22$ TeV. Searches for contact interactions of the type $(e\gamma_{\mu} P_L R e)(f\gamma_{\nu} P_R f)$ were performed at LEP. These are sensitive to a $Z'$-boson coupling to electrons assuming $M_{Z'} \gtrsim 210$ GeV and the resulting limits are again given for benchmark scenarios. In this case we extract $M_{Z'}/g' \gtrsim 16$ TeV from the 95% CL bound on the operator $(e\gamma_{\mu} P_R e)(\bar{\mu}\gamma_{\nu} P_R \mu)$. It is important to note that the extracted bounds from contact interactions using benchmark scenarios do not capture the full dynamics of our models once the $Z'$ boson is integrated out. Interference effects due to operators with different chirality structures will be relevant in general, a detailed analysis of these effects is however beyond the scope of this work. These bounds should therefore be taken as a rough guide to the sensitivity of contact interactions searches at LEP and the LHC to the $Z'$ of our models. However, in general they do not put relevant bounds as long as $g'$ is not too large.

$^{10}$ The constructed class of models has two important features which imply specific deviations from the SM: controlled FCNCs in the down-quark sector and violations of lepton universality. Thanks to the very specific flavor structure, the two are strongly correlated in each of our models. While the flavor-changing couplings are universal in the models we discuss here, the lepton charges differ. Correspondingly the models can be discriminated...
by processes involving leptons, specifically rare leptonic and semileptonic decays of $B$ mesons, which can test both features.

The effective Hamiltonian describing these $b \to s \ell^+ \ell^-$ transitions reads

$$\mathcal{H}_{\text{eff}} = -\frac{G_F^2}{\sqrt{2} \pi} V_{tb} V_{ts}^* \sum_i \left( C^t_i \mathcal{O}^t_i + C^\ell_i \mathcal{O}^\ell_i \right) + \mathcal{H}_{\text{eff}}^{b \to s \gamma \ell},$$

(31)

where

$$\mathcal{O}^t_9 = (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma_\mu \ell), \quad \mathcal{O}^\ell_9 = (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma_\mu \ell),$$

$$\mathcal{O}^t_{10} = (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma_\mu \gamma_5 \ell), \quad \mathcal{O}^\ell_{10} = (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma_\mu \gamma_5 \ell),$$

$$\mathcal{O}^t_S = m_b (\bar{s} P_R b) (\bar{\ell} \ell), \quad \mathcal{O}^\ell_S = m_b (\bar{s} P_L b) (\bar{\ell} \ell),$$

$$\mathcal{O}^t_P = m_b (\bar{s} P_R b) (\bar{\ell} \gamma_5 \ell), \quad \mathcal{O}^\ell_P = m_b (\bar{s} P_L b) (\bar{\ell} \gamma_5 \ell).$$

(32)

The piece $\mathcal{H}_{\text{eff}}^{b \to s \gamma \ell}$ is the effective Hamiltonian for the radiative transitions including dipole operators [17]. New physics contributions to $\mathcal{H}_{\text{eff}}^{b \to s \gamma \ell}$ arise in our models from one-loop diagrams mediated by $Z'$ and Higgs bosons. Both of them are very small due to the assumed mass scale of several TeV [17].

At the $b$-quark scale, the SM contribution to the remaining part reads $C^9_{\text{SM}} \approx -C^9_{\text{SM}} \approx 4.2 \sqrt{\ell}$, while all other contributions are negligible. The chirality-flipped operators $O_{9,10}$ receive negligible contributions also in our models since $\epsilon_{R,10} = 0$, while all others receive contributions from $Z'$ or Higgs exchanges [17] [38]. The $Z'$ contribution to $C^9_{9,10}$ is given by

$$C^9_{\ell} \approx \frac{\pi \epsilon^2}{\alpha V_{tb}^2} \epsilon^t_{L,R,10} \ell_{V,\ell},$$

(33)

$$C^{10}_{\ell} \approx \frac{\pi \epsilon^2}{\alpha V_{tb}^2} \epsilon^t_{L,R,10} \ell_{A,\ell},$$

where $C^i_{\ell} \equiv C^i_{\text{SM}} + C^i_{\text{NP}}$. In Table I we show the correlations between the Wilson coefficients $C^9_{9,10}$ in our models and provide $C^{NP}_{9,10}$ as a function of $g'/M_{Z'}^2$ by introducing a conveniently normalized parameter $\kappa^9_{\ell}$:

$$C^{NP}_{9,10} \equiv \kappa_{\ell}^9 \times 10^4 \left( \frac{g'}{M_{Z'}^2} \right)^2 = \kappa_{\ell}^9 \times 605 \text{ TeV}^2 \left( \frac{g'}{M_{Z'}^2} \right)^2.$$

(34)

This allows the direct estimation of $C^{NP}_{9,10}$ in terms of $g'/M_{Z'}^2$, useful for phenomenological purposes.

The CP-even Higgs $H_2$ and the CP-odd Higgs $A$ can give sizable contributions to $O^t_S$ and $O^t_P$, respectively. For muons we have\(^8\)

$$C_{9,10} \approx -C^{NP}_{9,10} \approx \frac{-2 \pi (t^2 + t^{-2}) (N_{\ell})_{\mu \mu}}{\alpha M_{H}^2}.$$

(35)

\(^8\) The correlation $C_S = -C_P$ was expected given the assumed decoupling in the scalar sector [53].

### Table I. Correlations among the NP contributions to the effective operators $O_{9,10}$.

| Model | $C^{NP}_9/C^{NP}_{10}$ | $C^{NP}_9/C^{NP}_{10}$ | $C^{NP}_9/C^{NP}_{10}$ | $C^{NP}_9/C^{NP}_{10}$ | $\kappa_{9,10}$ |
|-------|------------------------|------------------------|------------------------|------------------------|----------------|
| (1,2,3) | 3/17                  | 9/17                  | 3/17                  | 3/17                  | -1.235         |
| (1,3,2) | 0                     | -9/8                  | -3/8                  | 0                     | 0.581          |
| (2,1,3) | 1/3                   | 17/9                  | 1/3                   | -0.654                |               |
| (2,3,1) | 0                     | -17/8                 | -3/8                  | 0.581                 |               |
| (3,1,2) | 1/3                   | -8/9                  | 0                     | -0.654                |               |
| (3,2,1) | 3/17                  | -8/17                 | 0                     | -1.235                |               |

Here we have denoted the quasi-degenerate masses of $H_2$ and $A$ by $M_H \equiv M_{H_2} \approx M_A$, see Appendix B for details of the scalar sector. The coupling $(N_{\ell})_{\mu \mu}$ is again model-dependent: $(N_{\ell})_{\mu \mu} = t_3 \beta m_{\mu}$ for models (1, 2, 3), (2, 1, 3), (3, 1, 2) and (3, 2, 1), while $(N_{\ell})_{\mu \mu} = -t_3^{-1} m_{\mu}$ for (1, 3, 2) and (3, 2, 1). The suppression by the muon mass renders these contributions negligible, apart from observables in which also the SM and $Z'$ contributions receive this suppression, e.g. $B_s \to \mu^+ \mu^-$. Higgs contributions to $O^t_{S,P}$ will be additionally suppressed by a factor $m_\ell/m_{H_2}$ compared to the corresponding non-primed operators and are neglected in the following.

The angular distributions of neutral-current semileptonic $b \to s \ell^+ \ell^-$ transitions allow for testing these coefficients in global fits. Furthermore, they provide precise tests of lepton universality when considering ratios of the type

$$R_M \equiv \frac{\text{Br} (B \to M^{\mu^+ \mu^-})_{\text{SM}}}{\text{Br} (B \to M^{e^+ e^-})} = 1 + O(m_{\mu}^2/m_{b}^2),$$

(36)

with $M \in \{ K, K^*, X_s, K_0(1430), \ldots \}$ [55], where many sources of uncertainties cancel when integrating over identical phase-space regions.

Additional sensitivity to the dynamics of NP can be obtained via double-ratios [56],

$$\tilde{R}_M \equiv \frac{R_M}{R_{K^*}}.$$

(37)

It was shown in Ref. [57] that the dependence on the NP coupling to left-handed quarks cancels out in $\tilde{R}_M$. Since $C^t_{9,10} = 0$ in our models we have $\tilde{R}_{K^*} = \tilde{R}_{X_s} = \tilde{R}_{K_0(1430)} = 1$, providing an important test of the flavor structure of our models.

The possible hadronic final states for these ratios yield sensitivity to different NP structures, thereby providing complementary information. The discriminating power of these observables has been shown recently in Ref. [14] within the framework of leptoquark models and in more general contexts in Refs. [15, 56, 57].

To analyze the deviations from flavor-universality we define $R_M \approx 1 + \Delta + \Sigma$, where $\Sigma$ is the pure contribution from NP and $\Delta$ the one from the interference with the
SM. These quantities are given by
\[ \Delta = \frac{2}{|C_9^\text{SM}|^2 + |C_{10}^\text{SM}|^2} \left[ \text{Re} \left( C_9^\text{NP} \left( C_9^\text{NP}_\mu \right)^* \right) \right. \]
\[ + \text{Re} \left( C_{10}^\text{SM} \left( C_{10}^\text{NP}_\mu \right)^* \right) \left. - (\mu \rightarrow e) \right], \]
\[ \Sigma = \frac{\left| C_{10}^\text{NP}_\mu \right|^2 + \left| C_{10}^\text{NP}_\mu \right|^2}{\left| C_9^\text{SM} \right|^2 + \left| C_{10}^\text{SM} \right|^2} - (\mu \rightarrow e), \]
and are valid to a very good approximation given the present experimental uncertainties.

Our models also give clean predictions for the rare decay modes \( B \rightarrow \{ K, K^*, X_s \} \nu \bar{\nu} \). Tree-level \( Z' \)-boson contributions to these decays are generically expected of the same size as in \( b \rightarrow s \ell \ell \) transitions. However, the modes with neutrino final states do not distinguish between our different models, since there is no sensitivity to the neutrino species. We obtain again a universal value for all ratios \( R_{\nu}^M = Br(B \rightarrow M \nu \bar{\nu})/Br(B \rightarrow M \nu \bar{\nu})_{\text{SM}} \), due to the fact that \( e'_{\ell b}^d = 0 \). For the same reason the average of the \( K^* \) longitudinal polarization fraction in \( B \rightarrow K^* \nu \bar{\nu} \) is not affected by the \( Z' \) exchange contribution [17]. The enhancement of \( R_{\nu}^M \) turns out to be relatively small, \( O(10\%) \) for \( g' \sim 0.1 \) and \( M_{Z'} \sim O(\text{TeV}) \). The reason is again the sum over the different neutrino species: due to the different charges, the \( Z' \) contribution interferes both constructively and destructively, leaving a net effect that is smaller than in the modes with charged leptons.

The leptonic decays \( B_{\ell \ell}^0 \rightarrow \ell^+ \ell^- \) constitute another sensitive probe of small NP effects. Within the SM, these decays arise again at the loop level and are helicity suppressed. Due to the leptonic final state, the theoretical prediction of these processes is very clean. They receive \( Z' \) and Higgs-mediated contributions at tree-level in our model through the operator \( O_{10,S,P}^\ell [18] \),

\[ \frac{Br(B_s \rightarrow \mu^+ \mu^-)}{Br(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}}} \simeq \left| 1 - 0.24 C_{10}^\text{NP}_\mu - y_\mu C_{10}^\text{NP}_\mu \right|^2 \]
\[ + \left| \mu_{e} C_{10}^\text{NP}_\mu \right|^2, \]
where \( y_{\mu} \simeq 7.7 m_b \). For the range of model parameters relevant here, the \( Z' \) contribution to \( B_s \rightarrow \mu^+ \mu^- \) is very small with respect to the SM, \( \sim 1\% \) for \( M_{Z'} \sim 5 \text{ TeV} \) and \( g' \sim 0.1 \). Larger contributions are possible due to scalar mediation for the models \( (1,2,3), (2,1,3), (3,1,2) \) and \( (3,2,1) \), given that \( C_{10} \simeq - C_P \propto m_b^2 \) in these cases. Taking \( M_H \sim 10 \text{ TeV} \) and \( m_b \sim 30 \) for example, one obtains a suppression of \( Br(B_s \rightarrow \mu^+ \mu^-) \) by about 10% relative to the SM. In our model the ratio \( Br(B_d \rightarrow \mu^+ \mu^-)/Br(B_s \rightarrow \mu^+ \mu^-) \) remains unchanged with respect to the SM to a very good approximation.

If a \( Z' \) boson is discovered during the next runs of the LHC [55], its decays to leptons can be used to discriminate the models presented here. We define the ratios

\[ \frac{\mu_{f/f'} \equiv \sigma(pp \rightarrow Z' \rightarrow f \bar{f})}{\sigma(pp \rightarrow Z' \rightarrow f' f')} \]
\[ \text{for } (f,f') = (\ell^+ \ell^-), (\ell^+ \mu^-), (\ell^+ e^-), (\mu^+ \mu^-), (e^+ e^-), (\mu^+ \ell^-), (\ell^+ \ell^-), \]
of our models is fixed, so half of them cannot accommodate \( R_K < 1 \), namely \((1, 3, 2), (2, 1, 3)\) and \((2, 3, 1)\). This is illustrated in Fig. 1, where it is additionally seen that a large deviation from \( R_K = 1 \), as indicated by the present central value and 1\(\sigma\) interval, can actually only be explained in two of the remaining models. This strong impact shows the importance of further measurements of \( R_M \) ratios.

In Fig. 2 we show the constraints from the \( R_K \) measurement for the remaining models \((1, 2, 3)\), \((3, 1, 2)\) and \((3, 2, 1)\). The allowed regions are consistent with the constraint from \( B_s^0 \) meson mixing. LHC searches for a \( Z' \) boson exclude values of \( M_{Z'} \) below 3-4 TeV, as discussed in Sec. III B the corresponding areas are shown in gray. We also show the theoretical perturbativity bounds obtained from the requirement that the Landau pole for the U(1)' gauge coupling appears beyond the see-saw or the Grand Unification scales, i.e. \( \Lambda_{\text{LP}} > 10^{14} \) GeV and \( \Lambda_{\text{LP}} > 10^{16} \) GeV, respectively.

Regarding the angular analysis in \( B \to K^* \mu^+ \mu^- \), the situation is more complicated. The wealth of information provided in this and related measurements requires a global analysis, which is beyond the scope of this work. However, a number of model-independent studies have been carried out [3, 64, 67]. These analyses differ in terms of the statistical methods used, treatment of hadronic uncertainties (see also Ref. [68]), and consequently in the significance they find for the NP hypothesis over the SM one. However, they agree that a contribution \( C_9^{\text{NP}_\mu} \sim -1 \) can fit the data for \( P_5' \) without conflicting with other observables. Additional contributions from e.g. \( C_{10}^{\text{NP}_\nu} \) can be present, but are less significant and tend to be smaller. When using Table I to translate the \( R_K \) measurement into a bound on \( C_9^{\text{NP}_\mu} \) in our models, see Table III, the ranges are perfectly compatible with the values obtained from \( P_5' \), as also observed for other \( Z' \) models [3, 64, 67].

This is highly non-trivial given the strong correlations in our models and will allow for decisive tests with additional data.

Let us further comment on the implications of the recent measurements of \( B_s^0 \to \mu^+ \mu^- \) decays [69],

\[
\frac{\text{Br}(\bar{B}_s \to \mu^+ \mu^-)^\text{exp}}{\text{Br}(\bar{B}_s \to \mu^+ \mu^-)^\text{SM}} = 0.76^{+0.20}_{-0.18}, \tag{42}
\frac{\text{Br}(\bar{B}_d \to \mu^+ \mu^-)^\text{exp}}{\text{Br}(\bar{B}_d \to \mu^+ \mu^-)^\text{SM}} = 3.7^{+1.6}_{-1.4}.
\]

Both results seem to hint at a deviation from the SM, however in opposite directions. A confirmation of this situation with higher significance would rule out our models which predict these ratios to be equal; however, the uncertainties in the \( B_d \) mode are still large. Regarding the \( B_s \) mode, which is measured consistent with the SM prediction, it should be noted that the result depends sensitively on the value adopted for \( |V_{cb}| \), where the value from inclusive \( B \to X_c \ell \nu \) decays was chosen in the SM calculation [70]. In any case, as discussed in Sec. III C a potential shift could be explained in the context of our models by scalar contributions, which are, however, not directly related to the \( Z' \) contributions discussed above.

The models that accommodate the \( b \to s \ell^+ \ell^- \) data can be further discriminated using the observables described in the last section, notably using the ratios \( \mu_{\ell^\prime/\nu^\prime} \), provided a \( Z' \) boson is discovered at the LHC.

### IV. CONCLUSIONS

The class of family-non-universal \( Z' \) models presented in this article exhibits FCNCs at tree level that are in accordance with available flavor constraints while still inducing potentially sizable effects in various processes, testable at existing and future colliders. This is achieved by gauging the specific (BGL)-symmetry structure, introduced in Ref. [22] for the first time, which renders the resulting models highly predictive.

The particle content of the models is minimal in the sense that the extension of the U(1)' symmetry to the lep-

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**Figure 1.** Model-dependent predictions for \( R_K \) as a function of \( g'/M_{Z'} \). The recent measurement of \( R_K \) by the LHCb collaboration is shown at 1\(\sigma\) and 2\(\sigma\). Constraints from \( B_s \) mixing are also shown at 95% CL.

**Table III.** Model-dependent bound on \( C_9^{\text{NP}_\mu} \) from the \( R_K \) measurement. The constraint from \( B_s \) mixing is taken into account.

| Model | \( C_9^{\text{NP}_\mu}(1\sigma) \) | \( C_9^{\text{NP}_\mu}(2\sigma) \) |
|-------|---------------------------------|---------------------------------|
| (1,2,3) | \( - \) | \( -2.92, -0.61 \) |
| (3,1,2) | \( [-0.93, -0.43] \) | \( [-1.16, -0.17] \) |
| (3,2,1) | \( [-1.20, -0.53] \) | \( [-1.54, -0.20] \) |
ton sector allows to restrict the fermion content to the SM one. The only additional particles are then a second Higgs doublet, a scalar singlet and the $Z'$ boson, all of which are heavy after spontaneous symmetry breaking, due to the large mass scale for the singlet. The anomaly conditions largely determine the charge assignments under the U(1)' symmetry; the remaining freedom is used for two phenomenologically motivated choices, leaving only six possible models which are related by permutations in the lepton sector.

The main phenomenological features of these models can be summarized as follows:

1. FCNCs at tree level in the down-quark sector, which are mediated by heavy $Z'$ gauge bosons and neutral scalars, controlled by combinations of CKM matrix elements and/or fermion mass factors.

2. Non-universal lepton couplings determined by the charges under the additional U(1)' symmetry.

3. No FCNCs in the charged-lepton or up-quark sectors.

4. Complete determination of the U(1)' sector up to two real parameters, $M_{Z'}$ and $g'$, where all observables at the electroweak scale and below depend only on the combination $g'/M_{Z'}$.

Present data already strongly restrict the possible parameter ranges in our models: direct searches exclude $Z'$ masses below $3 - 4$ TeV and the constraint from $B$ mixing implies $M_{Z'}/g' \geq 16$ TeV (95% CL). Theoretical bounds from perturbativity give an upper bound on the value of the gauge coupling, e.g. $g' \lesssim 0.14$ for a Landau pole beyond the see-saw scale. Nevertheless, three of our models can explain the deviations from SM expectations in $b \to s\tau^+\tau^-$ transitions seen in LHCb measurements $^{[2, 3]}$, while the other three are excluded (at 95% CL) by $R_K < 1$. These findings are illustrated in Figs. $^1$ and $^2$.

Any significant deviation from the SM in an observable with $Z'$ contributions allows for predicting all other observables, for instance the values for $C_{9,10}^{NP}$, obtained from $R_K$, see Tables $^1$ and $^III$. Further characteristic predictions in our models include the following:

- The absence of flavor-changing $Z'$ couplings to right-handed quarks implies $\hat{R}_M = 1$, i.e. all ratios defined in Eq. $^{[56]}$ are expected to be equal, $R_K = R_{K'} = R_{X_s} = \ldots$ The same holds for the ratios $R_M^{s,t}$ in $B \to M\nu\nu$ decays.

- If a $Z'$ is discovered at the LHC, measurements of $\sigma(pp \to Z' \to \ell_i\ell_j)/\sigma(pp \to Z' \to \ell_i\bar{\ell}_j)$ can be used to discriminate between our models, due to the specific patterns of lepton-non-universality, see Table $^IV$.

- Leptonic down-quark FCNC decays are sensitive to the Higgs couplings in our model, specifically $B_{d,s} \to \ell^+\ell^-$. Double ratios of $B_s$ and $B_d$ decays are again expected to equal unity.

In the near future we will therefore be able to differentiate our new class of models from other $Z'$ models as well as its different realizations from each other. This will be possible due to a combination of direct searches/measurements at the LHC and high-precision measurements at low energies, e.g. from Belle II and LHCb. Further progress can come directly from theory, e.g. by more precise predictions for $\Delta m_{d,s}$ or $\epsilon_K$.

As a final remark, we recall that in this minimal implementation of gauged BGL symmetry neutrinos are massless. Additional mechanisms to explain neutrinos masses can introduce new solutions to the anomaly equations; these new variants are subject to future work.
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Appendix A: Anomaly cancellation conditions

In this appendix we present the restrictions on the charges derived from the requisite gauge anomaly cancellation for quarks transforming according to Eq. (3) and allowing the leptons to transform in the most general way:

$$X_L^c = \text{diag} (X_{eL}, X_{\mu L}, X_{\tau L})$$

$$X_R^c = \text{diag} (X_{eR}, X_{\mu R}, X_{\tau R}) \ . \quad (A1)$$

As mentioned in Sec. the QCD anomaly condition is automatically satisfied within BGL models. On the other hand, the anomaly conditions involving a single U(1)′ gauge boson together with SU(2) or U(1)′ ones, and the mixed gravitational-U(1)′ anomaly are non-trivial and read

$$A_{22} \sim 3X_{aR} + \frac{3}{2}X_{R} + \frac{9}{2}X_{dR} + \sum_{\alpha=e, \mu, \tau} X_{aL} \ ,$$

$$A_{111} \sim -\frac{5}{2}X_{aR} - \frac{5}{4}X_{R} - \frac{3}{4}X_{dR} + \sum_{\alpha=e, \mu, \tau} \left( \frac{1}{2}X_{aL} - X_{aR} \right) \ , \quad (A2)$$

$$A_{GG1} \sim \sum_{\alpha=e, \mu, \tau} (2X_{aL} - X_{aR}) \ .$$

The triangle diagrams involving two or three U(1)′ gauge bosons result in more complicated conditions for the BGL charges, which include a quadratic and a cubic equation:

$$A_{1111} \sim -\frac{7}{2}X_{aR} - \frac{7}{4}X_{R} - \frac{15}{4}X_{dR} + X_{aR}X_{dR} + \sum_{\alpha=e, \mu, \tau} (X_{aL}^2 - X_{aR}^2) \ ,$$

$$A_{11111} \sim -\frac{9}{4} \left( 2X_{aR}^3 + X_{aR}^3 + 3X_{dR}^3 \right)$$

$$-X_{dR} \left( 2X_{aR}^2 + X_{aR}^2 - X_{dR}^2 \right) \ , \quad (A3)$$

Satisfying these conditions with rational charges is non-trivial and there is only one class of solutions giving an anomaly-free model, up to lepton flavor permutations:

$$X_{aR} = -\frac{1}{3}X_{R} \ , \quad X_{aR} = -4X_{dR} + \frac{2}{3}X_{R} \ ,$$

$$X_{eL} = X_{dR} + \frac{1}{6}X_{R} \ , \quad X_{eL} = -X_{dR} + \frac{1}{6}X_{R} \ ,$$

$$X_{\tau L} = \frac{9}{2}X_{dR} - X_{R} \ ,$$

$$X_{\tau R} = 2X_{dR} + \frac{1}{3}X_{R} \ , \quad X_{\tau R} = 7X_{dR} - \frac{4}{3}X_{R} \ . \quad (A4)$$

At this point we have two free charges in the above relations, i.e. $X_{dR}$ and $X_{R}$. Using $X_{\Phi_2} = 0$, as discussed in Sec. yields one relation between the two charges. The remaining free charge, e.g. $X_{dR}$, is fixed by some normalization convention.

Appendix B: Scalar potential

The Higgs doublets are parametrized as

$$\Phi_i = \left( \Phi^+_i \Phi_i^0 \right) = e^{i\theta_i} \left( \frac{1}{\sqrt{2}} (v_i + \rho_i + i\eta_i) \right) \ (i = 1, 2) \ . \quad (B1)$$

Their neutral components acquire vevs given by

$$\langle \Phi^0_j \rangle = e^{i\phi_j} v_j / \sqrt{2} \quad \text{with} \ v_i > 0 \ \text{and} \ (\phi_1^2 + \phi_2^2)^{1/2} \equiv v \ .$$

In full generality we can rotate away the phase of $\Phi_1$, leaving the second doublet with the phase $\theta = \theta_2 - \theta_1$. We parametrize the complex scalar singlet as

$$S = e^{-i\alpha} \frac{v_S + R_0 + iI_0}{\sqrt{2}} \ , \quad (B2)$$

with $\alpha > 0$. Since we have in our model three scalar fields, $\{\Phi_1, \Phi_2, S\}$, the phase-blind part of the scalar potential has a U(1)′ global invariance. In order to avoid massless Goldstone bosons, we need to charge the $S$ field in such a way that the phase-sensitive part breaks this symmetry down to $U(1)^2 = U(1)_Y \times U(1)'$. Gauge-invariant combinations can be built only from $\Phi_1^0 \Phi_2$ and $S$, leaving us with the possibilities $\Phi_1^0 \Phi_2 S$, $\Phi_1^0 \Phi_2 S^2$ and combinations involving complex conjugates. For concreteness we choose $\Phi_1^0 \Phi_2 S^2$ to be invariant, which imposes $X_S = -9/8$. The scalar potential then reads

$$V = m_1^2 |\Phi_1|^2 + \frac{\lambda_1}{2} |\Phi_1|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2$$

$$+ \lambda_4 |\Phi_2|^4 + \frac{b_2}{2} |S|^2 + \frac{d_0}{4} |S|^4 + \frac{\delta_1}{2} |\Phi_1|^2 |S|^2 \quad (B3)$$

$$- \frac{\delta_3}{4} \left( \Phi_1^0 \Phi_2 S^2 + \Phi_1^0 \Phi_1 (S^*)^2 \right) \ .$$
All the parameters of the scalar potential but \( \delta_3 \) are real due to hermiticity. The parameter \( \delta_3 \) has been chosen to be real and positive by rephasing the scalar field \( S \) appropriately.

The vacuum expectation value of the potential is given by

\[
V_0 = \frac{v^4}{16} \left\{ m_1^2 c_{\beta}^2 + 2\lambda_1 c_{\beta}^4 + 4s_{\beta}^2 \left( \frac{m_2^2}{v^2} + (\lambda_3 + \lambda_4)c_{\beta}^2 \right) \right. \\
+ 2\lambda_2 s_{\beta}^2 + 2\hat{v}^2 \left( \frac{b_2}{v^2} + \delta_1 c_{\beta}^2 + \delta_2 s_{\beta}^2 \right) + d_2 \hat{v}^4 \\
- \left. 2\delta_3 c_{\beta} s_{\beta} \hat{v}^2 \cos(\alpha_S - \theta) \right\} .
\] (B4)

Here we have defined the ratio \( \hat{v} = v_S/v \). The stability of the vacuum requires that

\[
0 = \frac{\partial V_0}{\partial \theta} = -\frac{\delta_3 v^2 v_S^2}{16} s_{2\beta} \sin(\alpha_S - \theta) .
\] (B5)

By convention we choose \( \alpha_S = \theta \), the other possibilities in \( \alpha_S = \theta \mod 2\pi \) simply amount to an unphysical rephasing of the scalar field \( S \). Stability also requires \( \partial V_0 / \partial v_1 = 0 \), to which the only non-trivial solution reads

\[
b_2 = \frac{v^2}{2} \left[ -\delta_1 c_{\beta}^2 + \delta_3 c_{\beta} s_{\beta} - \delta_2 s_{\beta}^2 - d_2 \hat{v}^2 \right] ,
\]

\[
m_1^2 = \frac{v^2}{8} \left[ -4(\lambda_3 + \lambda_4)s_{\beta}^2 - 4\lambda_1 c_{\beta}^2 + \delta_3 t_{\beta} \hat{v}^2 - 2\delta_1 \hat{v}^2 \right] ,
\]

\[
m_2^2 = \frac{v^2}{8} \left[ -4(\lambda_3 + \lambda_4)c_{\beta}^2 - 4\lambda_2 s_{\beta}^2 + \delta_3 t_{\beta}^{-1} \hat{v}^2 - 2\delta_2 \hat{v}^2 \right] .
\] (B6)

The scalar potential is then determined in terms of 10 unknown parameters \( \{v_S, \beta, \lambda_{1-4}, d_2, \delta_{1-3}\} \), since \( \theta = \alpha_S \) does not appear explicitly.

The masses of the physical CP-odd boson \( A \) and the charged scalar are given by

\[
M_A^2 = \frac{\delta_1 v^2}{4} \left( \frac{s_{2\beta} - s_{2\beta}}{s_{2\beta}} \right)^2 \approx \frac{\delta_1 v_S^2}{4s_{2\beta}} ,
\]

\[
M_{H^\pm}^2 = \frac{v_S^2}{4} \left( \frac{\delta_3}{s_{2\beta}} - \frac{2\lambda_4}{\hat{v}^2} \right) \approx \frac{\delta_3 v_S^2}{4s_{2\beta}} .
\] (B7)

respectively. The physical states \( H_{1,2,3} \) are expressed in terms of \( \{\rho_1, \rho_2, R_0\} \) via an orthogonal transformation,

\[
\begin{pmatrix}
H_1 \\
H_2 \\
H_3
\end{pmatrix} = \mathcal{R}_S
\begin{pmatrix}
\rho_1 \\
\rho_2 \\
R_0
\end{pmatrix} .
\] (B8)

The mass matrix for the CP-even bosons can be diagonalized analytically in a perturbative expansion around \( 1/\hat{v} \ll 1 \). Including the leading corrections in \( 1/\hat{v} \) one obtains

\[
\mathcal{R}_S \approx \begin{pmatrix}
c_{\beta} & s_{\beta} & \omega_{13} \\
-s_{\beta} & c_{\beta} & -\omega_{23} \\
\omega_{23}s_{\beta} - \omega_{13}c_{\beta} & -(\omega_{23}c_{\beta} + \omega_{13}s_{\beta}) & 1
\end{pmatrix} ,
\] (B9)

with

\[
\omega_{13} = \frac{2\delta_1 c_{\beta}^2 - \delta_3 s_{2\beta} + 2\delta_2 s_{2\beta}^2}{2\hat{v}d_2} ,
\]

\[
\omega_{23} = \frac{\delta_3 s_{2\beta} + 2(\delta_1 - \delta_2)s_{2\beta}^2}{2\hat{v}(2d_2 s_{2\beta} - \delta_3)} .
\] (B10)

The resulting masses for the CP-even scalars are given by

\[
M_{H_1}^2 \approx \frac{\hat{\lambda} v^2}{2d_2} , \quad M_{H_2}^2 \approx \frac{\delta_1 v_S^2}{4s_{2\beta}} , \quad M_{H_3}^2 \approx \frac{d_2 v_S^2}{2} ,
\] (B11)

with

\[
\hat{\lambda} = (2d_2 \lambda_1 - \delta_1^2)c_{\beta}^4 + (\delta_1 c_{\beta}^2 + \delta_2 s_{2\beta}^2)\delta_3 s_{2\beta} - \frac{2\delta_1 \delta_2 + \delta_3^2}{4s_{2\beta}^2} - (\delta_2 - 2d_2 \lambda_2)s_{\beta}^4
\]

\[
+ d_2(\lambda_3 + \lambda_4)s_{2\beta}^2 .
\] (B12)

The exact expression for their Yukawa couplings in Eq. (24) is

\[
v_{Y_f}^H = [c_{\beta}(\mathcal{R}_S)_{k1} + s_{\beta}(\mathcal{R}_S)_{k2}] D_f
\]

\[
+ [s_{\beta}(\mathcal{R}_S)_{k1} - c_{\beta}(\mathcal{R}_S)_{k2}] N_f .
\] (B13)

Finally, the vacuum solution in our models allow for complex vevs but that is not sufficient to have spontaneous CP violation. In the weak gauge sector CP violation is manifest through the invariant \( \text{Tr} \left[ H_d H_u \right]^3 \). In the CP-invariant scenario, i.e. with real Yukawa matrices, the Hermitian combinations \( H_{d,u} \) are given by

\[
2H_u = v_1^2 \Delta_1 \Delta_1^T + v_2^2 \Delta_2 \Delta_2^T + v_1 v_2(\Delta_1 \Delta_2^T + \Delta_2 \Delta_1^T) c_\theta
\]

\[
+ iv_1 v_2(\Delta_1 \Delta_2^T - \Delta_2 \Delta_1^T) s_\theta ,
\]

\[
2H_d = v_1^2 \Gamma_1 \Gamma_1^T + v_2^2 \Gamma_2 \Gamma_2^T + v_1 v_2(\Gamma_2 \Gamma_1^T + \Gamma_1 \Gamma_2^T) c_\theta
\]

\[
+ iv_1 v_2(\Gamma_2 \Gamma_1^T - \Gamma_1 \Gamma_2^T) s_\theta .
\] (B14)

In our model we get \( \text{Tr} \left[ H_d H_u \right]^3 = 0 \), implying the absence of CP violation in the gauge interactions when the only phase is carried by the scalar vev. As a consequence, the source of CP violation for the weak currents in our model is present in the Yukawa couplings and will appear in the observables through the CKM mechanism.

[1] S. L. Glashow, Nucl. Phys. 22, 579 (1961). S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967). A. Salam, Conf. Proc. C 680519, 367 (1968).
