A Game Analysis of Iron Ore Right Investment between Steel Enterprises and the Government Based on Static Incomplete Information

Yu-shan CHEN¹,* and Tao DUAN²

1School of Business, Nanjing Xiaozhuang University, Jiangsu, P.R. China
2School of Management, Henan University of Technology, Zhengzhou P.R. China

*Corresponding author

Keywords: Mining right, Transfer, Iron ore, Game model, Equilibrium.

Abstract. Against the background of increasing demand for iron ore resources in Chinese steel enterprises and based on game tools, this paper builds a game equilibrium between steel enterprises and the government under the condition of non-competitive mining rights transfer. It is discussed two equalization strategies, and finds that trading opportunities for Linear Strategy are greater than opportunities for the one-price equilibrium strategy between steel enterprises and the governments in Bayesian Nash equilibrium. At the same time, investment efficiency of linear strategy steel enterprises will be better investment efficiency than one-price equilibrium strategy.

Introduction

At present, the supply of iron ore raw materials in our country is relatively low, and domestic mining enterprises cannot meet the demand of steel enterprises' output growth, which results in the opportunism for foreign mining enterprises in iron ore pricing negotiations. In the iron ore negotiations, Chinese steel enterprises are forced to accept high-priced iron ore raw materials upstream, when Overseas monopoly mining enterprises have monopoly pricing and threaten China to interrupt the supply of iron ore raw materials. According to data from the General Administration of Customs of China, as prices of iron ore is risen, Rio Tinto, the international iron ore company, earned net income of $14.324 billion (about 95.5 billion yuan) in fiscal 2010, surpassed China's steel industry's total profit of $85 billion in 2010. Therefore, the Chinese steel enterprise become a major problem how they obtained the iron ore resources in the rise of the price of the raw materials. Under such monopoly pricing and threat, steel enterprises have to take into account the self-guarantee supply of iron ore raw materials. In addition to the current monopoly of the three major mining enterprises, there are still many iron ore resources to be developed in many countries. According to different mining rights transfer systems, steel enterprise can obtain the iron ore resources in the non-competitive situations, they play games with the governments, so that they can guarantee their own supply of the iron ore resources.

Game theory method has become a common method to study incomplete market. Vickrey (1961) made a groundbreaking study on the game theory model of auction, and put forward the hypothesis of independent private value, symmetry and so on[1]. Maskin and Riley (2000) studied the auction model under asymmetric conditions of bidders[2]. Yan Bo (2007) established the auction design model of mining rights when bidders avoid risks, and put forward that the value of exploration rights and mining rights obtained by means of bidding, auction and listing is always uncertain and risky[3]. Tan Xuhong and Tan Mingjun (2008) believe that auction theory and method have been widely used in the design of various market mechanisms in recent years, and it is one of the most challenging problems to introduce this mechanism into the mining right market and set the optimal retention price. Since the evaluation value of mining right is only used as a reference for the pricing of mining right transaction, it is necessary to investigate the formation of mining right transaction price through the transaction process[4]. Duan Tao (2015) analyzes the interaction between local government and mining investors, probes into the equilibrium strategies chosen by different types of the governments, explains that the regulatory government, as the main body of interest, gives mining enterprises greater policy risk by adopting opportunistic behavior [5].
Based on previous studies, this paper expounds the circulation system of mining rights at home and abroad. This paper constructs the game equilibrium of mining right transfer between steel enterprises and the governments by game tools in the case of static incomplete information, probes into the two equilibrium strategies of one-price equilibrium and linear equilibrium, and provides the game strategy Suggestions to Chinese steel enterprises.

**Game Hypothesis under Static Incomplete Information**

It is essentially a static Bayesian game problem, which the game transaction between steel enterprises and the government under static incomplete information. The evaluation of mining rights by buyers and sellers is their own private information, and the evaluation of game parties cannot be fully counted.

In the non-competitive transfer of mining rights, there is usually only one investor which negotiates the price with the government’s geological and mining authorities. The competent authorities of many local governments use the evaluation of mining rights and similar market prices as the basis for bargaining, and based on a certain agreed price to transfer mining rights. The benefit of this procedure is to simplify the transfer process and to increase efficiency, but it is easy to lead to a low transfer price. The two-party quotation transaction is a special market transaction with its typicality. It contains the basic content which the buyer and the seller should buy a mining right for trading, and the rules of the transaction are as follows:

Hypothesis 1: the buyer and seller quote each price respectively, let the buyer's steel enterprise quote $P_b$ and the seller quote $P_s$. If $P_s \geq P_b$, At the price is equal to $P = (P_b + P_s)/2$, they have a deal. This kind of trading rule is similar to the trading rule of electronic automatic trading matching system in securities trading. It is the difference with them that there are many buyers and sellers in securities trading, the buyers and sellers can deal with different trading objects, and there is only one buyer and seller in quotation trading.

Hypothesis 2: Steel enterprises (buyers) value mining rights as $v$ and the government (sellers) value mining rights as $r$, and assume that the standard distribution of mutual knowledge of each other valuation is on the [0,1] interval. If the two parties deal at price $P$, then the profit of the steel company is $v - P$ and the benefit of the government is $P - r$. If the price is not suitable for a deal, both parties gain 0.

Hypothesis 3: in this game, one of the strategies of steel enterprises, that is, a price function of $v$, is $P_b(V)$, which assumes every possible valuation of mining rights traded by steel enterprises. At the same time, one of the government's strategies, that is, a price function of $r$ is $P_g(r)$, which is also a possible valuation of the mining rights of the transaction. If valuations of $[P_b(v), P_g(r)]$ satisfies the Bayes Nash equilibrium, then for any $v \in [0, 1]$, $P_b(v)$ must meet:

$$\max_{P_b} [v - \frac{P_b + E(P_g(r) | P_b \geq P_s(r))}{2} | P_b \geq P_s(r)] P(P_b \geq P_s(r))$$

(1)

Of which, Steel companies expect and accept seller's asking prices of the governments, which $E(P_g(r) | P_b \geq P_s(r))$ is in line with the premise that the bid of the steel company is greater than the asking price of the governments. At the same time, for any $r \in [0, 1]$, $P_g(r)$ must be satisfied that:

$$\max_{P_g} [P_g + E(P_b(v) | P_b \geq P_s) - r | P_b(v) \geq P_s] P(P_b(v) \geq P_s)$$

(2)

In this static Bayesian game, there are multiple Bayes Nash equilibrium. It is well, as long as the equilibrium is satisfied between the form of $P_b(v), P_g(r)$ function and the value of $v, r$, and their distribution probability is satisfied the maximization of the above two formulas. Therefore, this paper is discussed the Bayes Nash equilibrium of the game without restriction.
Analysis of Investment Game between Steel Enterprises and the Government under Static Incomplete Information

In the game between steel enterprises and the government iron ore mining rights under static incomplete information, this paper mainly discusses two equilibrium strategies: one-price equilibrium and one linear equilibrium.

One-price Equilibrium in the Game between Steel Enterprises and the Government under Static Incomplete Information

First of all, consider the special situation, that is, the equilibrium between steel enterprises and the government at a given price level, which is usually called one-price equilibrium. This equilibrium is characterized by a given value of \([0, 1] x\), which can be understood as the prevailing price in the market, the government-directed price, or the theoretical value based on the price index. When Steel companies (buyers) strategy is evaluated valuation for \(v \geq x\), \(p_b\) is equal to \(x\). Otherwise, \(p_b\) is equal to 0, that is, steel enterprises do not buy mining rights. When the government's strategy is evaluated valuation for \(v \leq x\), \(p_b\) is great than and equal to \(x\). Otherwise, \(P_g\) is equal to 1, mining rights will not be sold by the government.

That is given the strategy of the steel enterprise, which is possible transaction conditions, that is \(r \leq x \leq v\), the highest price that the seller's the government can achieve is \(P_g = x\). Any price will not be settled when condition is \(P_g \geq x\). Because the benefit of the transaction is \(P_g - r = x - r \geq 0\), the asking price is \(P_g = x\), it is the best response to make a deal.

When value is \(r > x\) and the price is met by \(P_g = x\), the benefits of the transaction are formula \(P_b = r = x - r \leq 0\). Therefore, the transaction is not established, and the two sides of the transaction will not lose money. The above is the best strategy for the government to bid for steel companies. By the same token, it can be proved that the above countermeasures of steel company is also the best response under the given government strategy. That is to say, it will be concluded at a given price, otherwise it would be better not to do so.

In the equilibrium of one price, steel companies and the government can trade while the condition is \(r \leq x \leq v\). As it is possible to do so in the rectangular trading area of figure 1 and figure 2, the shaded part cannot be traded, so the efficiency of the transaction is not high.

![Figure 1. The one-price Equilibrium.](image1)

![Figure 2. The Linear Equilibrium.](image2)

Linear Strategy Equilibrium in Investment Game between Steel Enterprises and the government under static incomplete Information

When the strategy of bidding between the players and the government is a linear function strategy, the strategy of the steel enterprise is assumed to be \(P_b(v) = a_b + c_b v\). The strategy of the government is \(P_g(r) = a_g + c_g r\). And \(v\), \(r\) are standard distribution in [0, 1], so \(P_b(v)\) and \(P_g(r)\), separately is standard distribution in \([a_b\, a_b + c_b v], [a_g\, a_g + c_g r]\). If \([P_b(v), P_g(r)]\) is the Bayes Nash equilibrium that the Pb must satisfy the following formula:
$$\max \left[ v - \frac{1}{2} \left( P_b + \frac{a_g + P_g}{2} \right) - \frac{P_b - a_g}{c_g} \right]$$

In formula (3) the first order condition is as follows:

$$P_b = \frac{2}{3} v + \frac{1}{3} a_g$$

At the same time, $P_g (r)$ must be satisfied that formula (5):

$$\max \left[ \frac{1}{2} \left( P_g + a_b + \frac{P_g + c_b}{2} \right) - r \right] - \frac{a_b + c_b - P_g}{c_b}$$

In formula (5) the first order condition is formula (6).

$$P_g = \frac{2}{3} r + \frac{1}{3} (a_b + c_b)$$

The above formula is obtained by bringing the linear function and probability distribution of linear strategy into the assumption of maximizing Bays, and the hypothetical linear function is obtained simultaneously with two first-order conditions:

$$\begin{align*}
P_b (v) &= a_b + c_g v \\
P_b &= \frac{2}{3} v + \frac{1}{3} a_g \\
P_g (r) &= a_g + c_g r \\
P_g &= \frac{2}{3} r + \frac{1}{3} (a_b + c_b)
\end{align*}$$

$$\begin{align*}
&\begin{cases}
a_b = \frac{1}{3} \\
c_b = 2/3
\end{cases} \\
\Rightarrow &\begin{cases}
a_g = \frac{1}{3} (a_b + c_b) \\
c_g = 2/3
\end{cases}
\end{align*}$$

According to Equation 7, the formula (8) can be calculated

$$\begin{align*}
P_b &= \frac{2}{3} v + \frac{1}{12} \\
P_g &= \frac{2}{3} r + \frac{1}{3}
\end{align*}$$

This is the Bayesian equilibrium (that is, linear strategy equilibrium) under the linear strategy of steel enterprises and the government. Because in the process of quotation, only when $P_b \geq P_g$, the two sides will conclude a transaction. Therefore, the transaction conditions must meet the following functions:

$$\frac{2}{3} v + \frac{1}{12} \geq \frac{2}{3} r + \frac{1}{3}$$

So if and only if $v$ cost estimate of the steel companies is at least a quarter higher than the government's cost estimate, the two sides will be able to conclude a mining rights deal without competition.

**Mining Right Transaction Case under Static Incomplete Information**

In the transfer of mining rights agreement, there is usually only one investor negotiating with the government's competent geological and mining authorities. The game transaction between Hebei steel enterprises and the government in agreement transfer is essentially a static Bayesian game problem. In 2009, Hebei provincial government and the Ministry of Land and Resources agreed to transfer the mining rights of 1.044 billion tons of iron ore resources to Hebei Iron and Steel Group
Mining Company by agreement. When the iron ore agreement was signed, the two sides reached an agreement.

Now suppose Hebei steel company (buyer) evaluates the mining right to $v$, the government (seller) evaluates the mining right to $r$, and assumes that the criteria for each other's valuation are distributed in the [0, 1] range. If the two sides trade at the price $P$, the benefit of Hebei steel enterprises is $v - P$, the benefit of the government is $P - r$, and if the price is not suitable for transaction, the benefit of both parties is 0.

When we assume that the strategy of the two parties of the Hebei Steel Company and the government is a linear function strategy, we assume that the strategy of the Hebei Steel Company is met by

$$P_b(v) = a_b + c_b v,$$

and the strategy of the government is met by

$$P_g(r) = a_g + c_g r.$$

Both $v$ and $r$ are standard distribution in [0, 1], so both $P_b(v)$ and $P_g(r)$ separately are standard distribution in $[a_b, a_b + c_b v], [a_g, a_g + c_g r]$. If $[P_b(v), P_g(r)]$ is the Bayes Nash equilibrium that the $P_h$ must satisfy the following formula:

Where, the first order condition is formula (10)

$$P_b = \frac{2}{3} v + \frac{1}{3} a_g \tag{10}$$

At the same time, $P_g(r)$ has to meet Conditions which formula (11) and (12) are satisfied.

$$\max_v \left[ v - \frac{1}{2} \left( P_b + \frac{a_g + P_b}{2} \right) \right] \frac{P_b - a_g}{c_g} \tag{11}$$

$$\max_r \left[ \frac{1}{2} \left( P_g + \frac{a_b + P_g + c_b}{2} \right) - r \right] \frac{a_b + c_b - P_g}{c_b} \tag{12}$$

Their first-order condition is formula (13).

$$P_g = \frac{2}{3} r + \frac{1}{3} (a_b + c_b) \tag{13}$$

Since the agreement reached between the steel enterprises and the government, the two parties will only conclude their quotations. Only if the quotations of both sides of $P_b \geq P_g$, the agreed prices must be met as follows :

$$\frac{2}{3} v + \frac{1}{12} \geq \frac{2}{3} r + \frac{1}{3} \tag{14}$$

By solving the simultaneous equation, the Bayesian equilibrium (abbreviated as linear strategy equilibrium) under the linear strategy of steel enterprises and the government can be obtained. Solution is $v = r + 1/4$.

Therefore, to reach an agreement, Hebei steel companies only need to give more than 1/4 above the government floor price to reach an agreement. This agreement he two sides is fast, but the price is relatively low.

**Conclusion**

In the process of non-competitive transfer between steel enterprises and the government, it can be found that in the Bayes Nash equilibrium with linear strategy, the trading opportunities of both sides of the game are greater than those of Nash equilibrium of one-price strategy. First, both equilibrium include the most efficient trading. Second, the one-price equilibrium eliminates many valuable trading opportunities. Third, on the whole, the efficiency of linear strategy equilibrium is better than that of one-price equilibrium, and it has higher expectations than any other Bayes Nash equilibrium.
Therefore, in the iron ore transfer game, when the Chinese iron and steel enterprises as buyers, the linear strategy equilibrium has more trading opportunities and higher investment efficiency than the one-price strategy equilibrium. In order to reduce the adverse effects of insufficient information, it is necessary to employ powerful mining rights evaluation institutions to evaluate the target mining rights objectively and accurately.

Acknowledgment
This work was supported in part by the philosophy and social science research project in universities of Jiangsu in China (No.2017SJB0426), supported in part by Applied economics funding for key disciplines at Nanjing in China (the cultivation of the subject) (Ningjiao Normal University (2017) No. 7).

Reference
[1] Vickrey, William. "Counter speculation, auctions, and competitive sealed tenders." The Journal of finance, 1961, 16(1), pp: 8, 37.
[2] Maskin, Eric, and John Riley. "Asymmetric auctions." The Review of Economic Studies, 2000, 67(3), pp: 413-438.
[3] Yan Bo. “Auction design of mining right when bidder risk averse,” Chinese coal, 2007(2), pp: 28-30.
[4] Tan Xuhong, Tan Mingjun. “Pricing model of mining right auction and design of optimal auction mechanism,” China's mining industry, 2008(12), pp: 21-25.
[5] Duan Tao. “Study on the Policy Risk of Mining Investment based on Dynamic Game Mode,” Research on Technology Economy and Management, 2015 (9), pp: 9-12.