Effect of rolling direction to texture evolution in an aluminium single crystal (011)[100]: a crystal plasticity FEM investigation

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Keywords: crystal plasticity FEM, rolling direction, crystal rotation, single crystal

Abstract
In this study, the crystal plasticity finite element method (CPFEM) was used to predict the texture evolution in a Goss (011)[100] aluminium single crystal after multi-pass reverse rolling and unidirectional rolling. Submodel was also used to improve the mesh resolution in a smaller region, by which micro-subdivision was successfully captured. After the predictions being validated by the experimental observations, the deformation behaviours were investigated, in terms of crystal rotation, shear strain, and slip system activity. Macro-subdivision occurred through the thickness by forming matrix bands. It was found that the deformation was unidirectional after unidirectional rolling and macro- and micro-subdivision were amplified with increasing reduction, while the previously developed deformation pattern was destroyed by the following pass of reverse rolling.

1. Introduction
Single crystals have been widely used in investigating texture evolution and deformation behaviours because of no pre-existent grain boundaries [1–3]. The inhomogeneous deformation in single crystals would lead to macroscopic subdivision by forming deformation bands [1–3], and their formation is strongly associated with slip system activity. The activation of slip systems is determined by both external and internal factors [4]. The external loading is influenced by the geometry of working tools, friction conditions, deformation of neighbouring regions, etc. The internal factor means the local crystal orientation, work-hardening, sub-grain microstructure, etc. The external loading and local material properties are fully coupled during deformation. During rolling, the evolution pattern of texture and microstructure keeps the same if the rolling direction is unidirectional due to the same slip system activation [5, 6], while it is altered after reversing the rolling direction [3, 7–9], i.e., the previously developed texture and microstructure are (partially or completely) destroyed by the following reverse rolling [3, 10]. How the rolling direction influences the deformation behaviours and texture evolution has not been widely researched, and this is the thrust of this study.

Texture modelling has become a well-recognized tool to study the texture evolution and plastic deformation [11], and numerous crystal plasticity models have been proposed. Among these models, no homogenization is assumed in the crystal plasticity finite element method (CPFEM), and accordingly it is called ‘full-field theory’ [11, 12]. In CPFEM, the crystal plasticity constitutive law is incorporated into the finite element method (FEM) framework. The FEM acts as the solver of boundary problems, while the crystal plasticity constitutive law is taken into account at each material point. The stress equilibrium and strain compatibility are achieved by basic principles of mechanics [11]. The main advantage of the CPFEM model is its ability to solve crystal plasticity problems under complicated internal and external boundary conditions [13], and thus the CPFEM model is the ideal one to study the deformation of multi-pass reverse rolling (RR) and unidirectional rolling (UR). This model has successfully predicted the texture evolution after two-pass unidirectional rolling [14], multi-pass accumulative roll-bonding (ARB) [12], and reverse ARB [15].

In the current research, the CPFEM model was used to study the texture evolution after RR and UR. The predictions were validated by the experimental observations. The deformation behaviours after UR and RR were...
2. CPFEM simulation model

The kinematic theory of the crystal plasticity model follows the scheme proposed by Asaro [16]. Crystal slip and lattice spin are the two mechanisms for deformation. Crystal slip is caused by the motion of dislocation along the slip direction, while lattice spin is the reason for crystal rotation. Bassani-Wu hardening model [17] was adopted, which is regarded as the best texture predictor [18]. ABAQUS/Standard was adopted to perform the simulation, and a Lagrangian formulation was used. The crystal plasticity model and hardening model were implemented into the ABAQUS/Standard by the user-defined material (UMAT) subroutine. The kinematic theory is given in appendix A, and the hardening model in appendix B. The crystal plasticity theory, hardening model, material parameters, and CPFEM implementation are described in [12]. The slip plane is \{111\} and slip direction \langle 110 \rangle, and the 12 slip systems in FCC structured aluminium are listed in table 1.

The simulation followed the experiment in [3]. The simulation model shown in figure 1 was two-dimensional under plane strain conditions. The starting material was an aluminium single crystal with an initial orientation of Goss (011) [100]. The 50% reduction was accomplished in four passes with the reduction of 18%, 30%, 42% and 50% in the first, second, third, and fourth pass, respectively. Following the experiment procedure, both UR and RR were simulated. The sheet was introduced into the rolling gap in the same direction in each pass of UR, while it was turned end for end after each pass in RR. The rolling direction (RD) and normal direction (ND) are also shown in figure 1. A remeshing analysis technique, mapping solution, was used between passes [12], by which the solution of the deformed mesh was transferred into a new mesh, but the distorted mesh was completely replaced by a new mesh. To obtain the solution variables at a node of the deformed mesh, the values of integration points in all elements having this node were extrapolated and then the values were averaged over these elements. All necessary variables were interpolated from the nodes of the deformed mesh to the nodes of the new mesh. The initial thickness of the sheet was 2.1 mm. The rolls were considered as rigid bodies with a diameter of 75 mm. The friction between the sheet and rolls was described by Coulomb’s friction law, and a friction coefficient of 0.25 was used to approximate the unlubricated rolling conditions. The element type was 4-node bilinear with reduced integration (element id: CPE4R). The sheet was meshed into 10240 elements in 1-pass, and the element size was 66 \( \mu \text{m} \times 66 \mu \text{m} \). After the simulation of the whole sheet (called Wholemodel), a smaller region (2.1 mm \times 2.1 mm, including 1024 elements) was selected from the Wholemodel of 1-pass, and then it was reconstructed in the Submodel. The element size in the Submodel was 6.6 \( \mu \text{m} \times 6.6 \mu \text{m} \) in 1-pass. This is to say one element in the Wholemodel was remeshed into 100 elements in the Submodel. The deformation history at the boundary of the selected region in the Wholemodel (saved in ABAQUS data) was used to drive the Submodel to deform. Mapping solution was also used between passes for the Submodel.

### Table 1. Notation of slip systems.

| Slip plane | (111) | (\overline{1} 11) | (\overline{1} 10) | (1 11) | (\overline{1} 1 1) | (11 \overline{1}) |
|-----------|-------|-----------------|-----------------|--------|-----------------|-----------------|
| Slip direction | [0 1 1] | [1 0 1] | [\overline{1} 0 1] | [\overline{1} 1 0] | [\overline{1} \overline{1} 0] | [0 1 1] |
| Slip system | a1 | a2 | a3 | b1 | b2 | b3 | c1 | c2 | c3 | d1 | d2 | d3 |

Figure 1. Simulation model.
3. Simulation results and experiment validation

In each element, the crystal rotation was partitioned into rotation about the transverse direction (TD), RD and ND using the exactly same method in the corresponding experimental research \[3\], and this partition method is clearly described in \[19\]. Clockwise TD-rotation in UR and anticlockwise TD-rotation in RR are termed as positive, while anticlockwise TD-rotation in UR and clockwise TD-rotation in RR are termed as negative. The thickness position is defined by \(t/t_0\), where \(t\) is the distance from the upper surface to a thickness location, and \(t_0\) is the total thickness.

\[\{111\}\] pole figures in figures 2(a) and (b) respectively show the deformation textures after 30% and 50% reduction in the UR simulation. It can be seen from the pole figures that the crystal rotation angles are very low, and the rotation is mainly about the TD in both clockwise and anticlockwise directions. The partitioned RD- and ND-rotation are smaller than \(0.2^\circ\), and thus in the following only they are not presented. The distribution of partitioned TD-rotation is shown in figures 2(c) and (d). The TD-rotation is very low even after 50% reduction, with a maximum value of \(\sim 3^\circ\), and it divides the whole thickness into two matrix bands (termed as ‘M’). In each matrix band, the direction of TD-rotation keeps constant, and it is positive and negative in M1 and M2, respectively. In contrast, the distribution of TD-rotation is almost uniform along the RD. The TD-rotation increased slowly in each matrix band with increasing reduction, as shown in figure 2(e). The predicted TD-rotation agrees well with the experimental measurement shown in figures 2(f) and (g).

The distribution of TD-rotation after 30% and 50% reduction in the RR simulation (figures 3(a) and (b)) is different from those after UR. In the two smaller regions marked as M1 and M2 in figure 3(c), the direction of TD-rotation reverses after each pass. Reversal of TD-rotation can also be seen in other regions, but it does not occur after each pass due to the change of rolling bite geometry \[3\]. The continuous reduction of thickness altered the rolling bite geometry, and thus the strain and stress conditions changed. In RR, the previously developed matrix bands were destroyed by following reverse rolling, and the already evolved TD-rotation was lowered. The relatively low TD-rotation can also be seen from the experimental observation in figure 3(d).

The deformed FEM meshes in figures 2 and 3 represent the imposed shear strain, and the shear strain is plotted in figure 4. The cumulative shear strain increased continuously with increasing passes of UR. In contrast, the previously imposed shear strain was reduced by the following pass of RR, which resulted in cumulative shear.

\[\text{Figure 2.}\ \{111\\} \text{pole figures, (c), (d) distribution of TD-rotation and FEM meshes after 30\% and 50\% reduction in the UR simulation. Distribution of TD-rotation after (e) each pass in the UR simulation, and (f) 30\% and (g) 50\% reduction in the UR experiment [3].}\]
strain being low after the fourth pass. In UR, the distribution of TD-rotation (figure 2(e)) and shear strain (figure 4) are in the similar pattern, but they are in the opposite direction.

The simulated slip traces are presented in figure 5. The simulated slip trace of each slip system is presented by a segment of straight line on the RD-ND plane centred at the integration point of the element and the relative magnitude of cumulative shear strain on the slip system is presented by the length of the line. The black, blue, green and red colours refer to the slip systems with the first, second, third and fourth largest magnitude of the cumulative shear strain, respectively. The four highly activated slip systems are a2, a3, b1 and b2. The shear strain on a2 is very close to that on a3, and it is true for b1 and b2. Due to the low non-TD-rotation, the slip traces of a2 and a3 are overlapped on the RD-ND plane, and it is the same for b1 and b2, and thus only slip traces in red (a2–a3 set) and blue (b1–b2 set) can be seen in figures 5(a)–(e). The shear strain on a2-a3 in M1 is higher than that on b1-b2 after each pass of UR. The simulated slip trace agrees well with the scanning electron microscope (SEM) images in the experimental research [3]. The imbalance ratio between $\gamma_{a2}$ and $\gamma_{b2}$ calculated according to $(\gamma_{a2} - \gamma_{b2}) / \max(\gamma_{a2}, \gamma_{b2})$ is shown in figure 5(f). The imbalance ratio is in the same pattern after each pass of UR. In contrast, after 2- and 4-pass of RR the imbalance ratio is almost in the opposite direction to those after UR, since the rolling direction was reversed in 2- and 4-pass of RR. The primary and secondary slip system sets altered two times at the centre after UR (figure 5(a–c)), while 3 and 5 times after 2-pass and 4-pass of RR (figure 5(d, e), respectively. The imbalance ratio distributes in the similar pattern to shear strain (figure 4), but in the opposite direction to TD-rotation (figure 2(e)).
Figure 6 shows the distribution of TD-rotation in the Submodel, and the TD-rotation along the RD is not uniform anymore. The Submodel successfully captured the micro-subdivision, which agrees with the experimentally observed slip traces in figure 5. The micro-subdivision are in the same direction as the simulated primary slip traces in figure 5. The subdivision was amplified after UR (figures 6(b), (c), (d)), while it was destroyed after RR (figures 6(c), (d)). Figure 6(e) shows the TD-rotation in the rolling bite. The TD-rotation was very large at the entry of rolling bite, and then it decreased gradually as rolling proceeded. According to the offset of the circled micro-subdivision in figure 6(e), the deformation history is schematically shown on the right side of figure 6(e). Micro-subdivision along b1–b2 developed firstly (stage 1), and then it was offset by the activation of a2–a3 in stage 2. The alternation of the two sets of activated slip systems resulted in a cell-block structure in rolled single crystals [5, 20].

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Figure 5. SEM images in the experiment [3] and simulated slip traces in the simulation after (a) 18%, (b) 30%, (c) 50% of UR, and (d) 30%, and (e) 50% of RR, (f) Imbalance ratio of shear strain on slip systems, $(\gamma_{ab} - \gamma_{bc})/\max(\gamma_{ab}, \gamma_{bc})$. The slip traces in in M1 and M2 of (a), (b), and (c) are enlarged and shown on the right of the upper panel. The positions at which the primary and secondary slip systems alternate are marked by arrows in (d) and (e).

Figure 6. Distribution of TD-rotation in the Submodel (a) after 18%, and (b) after 30% of UR, and (c) after 30%, and (d) after 50% of RR, and (e) during rolling (in the rolling bite) at 18%.
4. Discussion

In rolling, the imposed stress and strain are basically determined by the surface friction (between the sheet and rolls) and rolling bite geometry. The combined effect of these two factors results in shear stress and shear strain alternating their directions through the thickness. In addition to these two factors, the initial orientations of single crystals also influence the distribution of stress and strain. In this study, two matrix bands developed in the rolled Goss single crystal, while four matrix bands developed in rolled Cube single crystals \[20\] and rotated-Cube single crystals \[5\]. According to the hardening model (equation \(B6\) in appendix B), the activation of slip systems is only determined by the resolved shear stress on them, and the resolved shear stress is determined by the macroscopic stress and crystal orientation according to the Schmid’s law. Therefore, the crystal orientation influences the slip system activation. The inclination of the four slip systems in Goss is less than that in Cube and rotated-Cube \[21\]. This is to say the slip systems in Goss are not as sensitive as those in Cube and rotated-Cube to the imposed shear stress (Schmid’s law), and thus the effect of alternating shear stress to slip system activation is not strong. This is why only two matrix bands developed in Goss. The distribution of TD-rotation (figure 2(e)), shear strain (figure 4), and imbalance ratio (of shear strain on slip systems) are in a similar pattern (figure 5(f)), either in the same or opposite direction. This correlation is a reflection of equation (A4) in appendix A, which has been explained in \[22\]. This correlation also holds for those after RR, although the TD-rotation, shear strain, and imbalance ratio alternate their directions more times than those after UR.

In UR, the shear stress is unidirectional, which results in the same slip system activation and shear strain. Therefore, the crystal rotation is also unidirectional, according to equation (A4) in appendix A. Due to the unidirectional increase of imbalance ratio and crystal rotation, the macro-subdivision (by forming matrix bands) is amplified after increasing the reduction. The imposed shear stress and thus slip system activation are reversed in RR, and accordingly the imbalance ratio alternates after each pass of RR (figure 5(f)). Meanwhile, the introduced shear strain is also reversed after each pass of RR (figure 4), and thus the direction of crystal rotation changes through the thickness. This is why the TD-rotation, shear strain, and imbalance ratio are very low in RR. Due to the change of rolling bite geometry and crystal rotation, the previously developed shear strain and crystal rotation are not completely compensated by the following pass. Because of the different slip system activation and crystal rotation between passes in RR, the already developed microstructure is usually destroyed by the following pass in RR (figures 6(c) and (d)).

5. Conclusions

1. The CPFEM model was used to investigate the deformation of Goss single crystals processed by reverse rolling and unidirectional rolling.

2. The predictions agree well with the experimental observations, in terms of subdivision and texture evolution.

3. Two matrix bands developed through the thickness after unidirectional rolling. The pattern of matrix bands was amplified with increasing reduction of unidirectional rolling.

4. The pattern of matrix bands was altered after each pass of reverse rolling, and the micro-subdivision predicted in the Submodel were also destroyed after reverse rolling.

Declarations of interest

The authors declare no competing interests.

Appendix A: Kinematics

The crystal plasticity model in the current research follows the well-recognized kinematical scheme developed by Asaro \[16\] and Peirce \[23, 24\]. In this scheme, the deformation gradient \(F\) is decomposed into two components as
\[ F = F^e \cdot F^p \]  
where \( F^e \) embodies the elastic deformation and rigid body rotation, and \( F^p \) consists of crystallographic slip on slip systems. The velocity gradient \( L \) is evaluated from the deformation gradient by

\[ L = FF^{-1} = L^e + L^p \]  

The velocity gradient can be uniquely decomposed into a symmetrical part and a skewed-symmetrical part as

\[ W = \frac{1}{2}(L + L^t) \quad (A3a) \]

\[ D = \frac{1}{2}(L + L^t) \quad (A3b) \]

\[ \Omega = \frac{1}{2}(L - L^t) \quad (A3c) \]

where \( D \) and \( \Omega \) are called the stretch rate tensor and spin tensor, respectively. \( \Omega \) can be represented by the rigid rotation of a finite region or redundant shear strain, and it can also be decomposed into the elastic stretching and lattice rotation part \( \Omega^e \) and plastic part \( \Omega^p \), namely

\[ \Omega = \Omega^e + \Omega^p \]  

\( \Omega^e \) is due to distortion and rotation of the crystal lattice, which is the reason for texture evolution. The plastic spin \( \Omega^p \) is caused by the motion of dislocation on slip planes and along slip directions, which is calculated according to

\[ \Omega^p = \sum_{\alpha=1}^{n} \frac{1}{2} (m^{(\alpha)} - m^{(\alpha)} \cdot s^{(\alpha)}) \gamma^{(\alpha)} \]  

where \( s^{(\alpha)} \) and \( m^{(\alpha)} \) are the slip direction and slip plane normal, respectively.

### Appendix B: Hardening model

The Bassani-Wu hardening model \([17]\) is regarded as the best one in a comparative study \([18]\). In this hardening model, the shear strain rate \( \dot{\gamma}^{(\alpha)} \) is related to the resolved shear stress \( \tau^{(\alpha)} \) on slip system \( \alpha \), as expressed by equation (B6), where \( \dot{\gamma}^{(\alpha)}_0 \) is the reference value of the shear strain rate, \( n \) is the rate-sensitive exponent, and \( \tau^{(\alpha)}_c \) is the critical resolved shear stress of the slip system \( \alpha \). The values of \( \dot{\gamma}^{(\alpha)}_0 \), \( n \) and \( \tau^{(\alpha)}_c \) are listed in Table B1.

\[ \dot{\gamma}^{(\alpha)} = \dot{\gamma}^{(\alpha)}_0 \text{sgn}(\tau^{(\alpha)}) \left| \frac{\tau^{(\alpha)}}{\tau^{(\alpha)}_c} \right|^n \quad \text{for} \ \tau^{(\alpha)} \geq \tau^{(\alpha)}_c \]

\[ \dot{\gamma}^{(\alpha)} = 0 \quad \text{for} \ \tau^{(\alpha)} < \tau^{(\alpha)}_c \]  

The \( \text{sgn}(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases} \)

The \( \tau^{(\alpha)}_c \) represents the strength of activating the slip system \( \alpha \), and its increase rate in value, i.e., \( \dot{\tau}^{(\alpha)}_c \), is determined by:

\[ \dot{\tau}^{(\alpha)}_c = \sum_{\beta=1}^{n} h_{\alpha\beta} |\gamma^{(\beta)}| \]  

where \( h_{\alpha\beta} \) is the hardening modulus. As expressed in equation (B8), the activation of all slip systems would affect the hardening of each slip system. It is self-hardening, i.e., \( h_{\alpha\alpha} \), when \( \alpha \) is equal to \( \beta \), while it is latent hardening \( h_{\alpha\beta} \) when \( \alpha \) is not equal to \( \beta \). The \( h_{\alpha\alpha} \) and \( h_{\alpha\beta} \) are expressed by:

\[ h_{\alpha\alpha} = \left( h_0 - h_s \right) \text{sech}^2 \left( \frac{h_0 - h_s \dot{\gamma}^{(\alpha)}_0}{\tau^{(\alpha)}_c - \tau^{(\alpha)}_0} \right) + h_i \left[ 1 + \sum_{\beta=1}^{N} f^{(\beta)} \tanh \left( \frac{\gamma^{(\beta)}_0}{\gamma^{(\beta)}_0} \right) \right] \]

\[ h_{\alpha\beta} = q h_{\alpha\alpha}, \ \alpha \neq \beta \]

| \( n \) | \( \dot{\gamma}^{(\alpha)}_0 \) (s\(^{-1}\)) | \( h_0 \) (MPa) | \( h_s \) (MPa) | \( \tau^{(\alpha)}_c \) (MPa) | \( \tau^{(\alpha)}_0 \) (MPa) | \( q \) |
|---|---|---|---|---|---|---|
| 300 | 0.0001 | 100 | 0.01 | 6.3 | 6 | 1 |

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Table B1. Parameters used in the Bassani-Wu hardening model.
where \( h_0 \) is the hardening modulus after initial yield, \( h_s \) is the hardening modulus of easy slip, \( \tau_0 \) is the critical stress when plastic flow begins, \( \tau_1 \) is the initial critical resolved shear stress, \( q \) is the ratio between latent hardening modulus and self-hardening modulus, and \( f_{\alpha,\beta} \) means the interaction between slip system \( \alpha \) and \( \beta \).

The value of \( f_{\alpha,\beta} \) is determined by the relative position of two slip systems, and thus five constants of \( f_{\alpha,\beta} \) exist. The parameter \( f_{\alpha,\beta} \) is chosen as: \( a_1 = a_2 = a_3 = 1.75, a_4 = 2 \) and \( a_5 = 2.25 \) according to the study in [25]. Other material parameters in equation (B6) and (B8) are listed in table B1, which were evaluated by fitting the simulated stress–strain curve with the experimental results of an aluminium single crystal under plane strain compression [26]. The three elastic moduli are \( C_{11} = 112 \text{ 000 MPa}, C_{12} = 66 \text{ 000 MPa} \) and \( C_{44} = 28 \text{ 000 MPa} \).

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**References**

[1] Afri N et al 2012 Spatial orientations and structural irregularities associated with the formation of microbands in a cold deformed Goss oriented Ni single crystal Acta Mater. 60 6288–300

[2] Yoshida A, Miyajima Y and Onaka S 2014 Evolution of the spread of crystal orientation with plastic deformation in a cold-rolled Cu single crystal J. Mater. Sci. 49 2013–7

[3] Liu Q, Wert J and Hansen N 2000 Location–dependent lattice rotation and shear strain in rolled aluminum single crystals of cube and Goss orientations Acta Mater. 48 4267–79

[4] Raabe D, Zhao Z and Mao W 2002 On the dependence of in-grain subdivision and deformation texture of aluminum on grain interaction Acta Mater. 50 4379–94

[5] Li Z, Godfrey A and Liu Q 2004 Evolution of microstructure and local crystallographic orientations in rolled Al–1%Mn single crystals of \{ 0 0 1 \} \{ 1 1 0 \} orientation Acta Mater. 52 149–60

[6] Deng G Y et al 2013 Influence of cold rolling reduction on the deformation behaviour and crystallographic orientation development Comput. Mater. Sci. 81 2–9

[7] Gu C F, Töth L S and Davies C H J 2011 Effect of strain reversal on texture and grain refinement in route C equal channel angular pressed copper Scr. Mater. 65 167–70

[8] Gu C F and Töth L S 2011 The origin of strain reversal texture in equal channel angular pressing Acta Mater. 59 5749–57

[9] Li S, Beyerlein I J and Necker C T 2006 On the development of microstructure and texture heterogeneity in ECAE via route C Acta Mater. 54 1397–408

[10] Vega M C V et al 2015 The influence of deformation path on strain characteristics of AA1050 aluminium processed by equal-channel angular pressing followed by rolling Mater. Sci. Eng. A 646 154–62

[11] Van Houtte P et al 2005 Deformation texture prediction: from the taylor model to the advanced lamel model Int. J. Plast. 21 589–624

[12] Wang H et al 2019 Texture modeling of accumulative roll-bonding processed aluminum single crystal \{ 1 2 3 \} \{ 6 3 4 \} by crystal plasticity FE Adv. Eng. Mater. 21 1800827

[13] Roters F et al 2010 Overview of constitutive laws, kinematics, homogenization and multiscale methods in crystal plasticity finite-element modeling: Theory, experiments, applications Acta Mater. 58 1152–211

[14] Wang H et al 2019 Coupled effects of initial orientation scatter and grain-interaction to texture evolution: a crystal plasticity FE study Int. J. Mater. Form. 12 161–71

[15] Wang H, Lu C and Tieu K 2019 A combined experiment and crystal plasticity FEM study of microstructure and texture in aluminium processed by reverse and unidirectional accumulative roll-bonding Crystals 9 119

[16] Bassani L J and Wu T-Y 1991 Latent hardening in single crystals: II. Analytical characterization and predictions P. Roy. Soc. A 435 21

[17] Lin G and Hannver K S 1996 A comparative study of hardening theories in torsion using the Taylor polycrystal model Int. J. Plast. 12 695–718

[18] Wert J A, Liu Q and Hansen N 1997 Dislocation boundary formation in a cold-rolled cube-oriented Al single crystal Acta Mater. 45 2565–76

[19] Liu Q and Hansen N 1998 Macroscopic and microscopic subdivision of a cold-rolled aluminium single crystal of cubic orientation P. Roy. Soc. A 454 2553–91

[20] Wert J A 2002 Macroscopic crystal rotation patterns in rolled aluminium single crystals Acta Mater. 50 3125–39

[21] Wang H et al 2019 Correlation between crystal rotation and redundant shear strain in rolled single crystals: a crystal plasticity FE analysis Acta Metall. Sin.-Engl 32 452–60

[22] Peirce D, Asaro R J and Needleman A 1982 An analysis of nonuniform and localized deformation in ductile single crystals Acta Metall. 30 1087–119

[23] Peirce D, Asaro R J and Needleman A 1983 Material rate dependence and localized deformation in crystalline solids Acta Metall. 31 1951–76

[24] Francis P., Berveiller M and Zaoui A 1980 Latent hardening in copper and aluminium single crystals Acta Metall. 28 273–83

[25] Liu Q et al 1998 Heterogeneous Microstructures and Microtextures in CubeOriented Al crystals after channel die compression Metall. Mater. Trans. A 29 2333–44