DIPOLE AND QUADRUPOLE MOMENTS OF
MIRROR NUCLEI $^8$B AND $^8$Li

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Magnetic dipole and electric quadrupole moments of the mirror nuclei $^8$Li and $^8$B are analysed in the framework of the multiparticle shell model by using two approaches: i) the one-particle spectroscopic factors and ii) the one-particle fractional parentage coefficients. These two approaches are compared both each to other and with a microscopic multicluster model. The one-particle nucleon states are calculated taking into account the continuum by the method of the expansion of the Sturm-Liouville functions. The experimental magnetic and quadrupole moments of $^8$Li and $^8$B are reproduced well by using fractional parentage coefficients technique. The root mean-square radii and the radial density distributions are obtained for these nuclei.

1. Introduction

From a known proton-rich nuclei the nucleus $^8$B has an extremely low binding energy of the valence proton and is one of the best candidates for the proton-halo nucleus. The $^8$B proton-halo problem was considered in many experimental and theoretical works (see [1] and the references listed there). At present wide experimental material has been accumulated: the total interaction cross section of $^8$B with the different nuclei in the wide range of the incident energies, the quasielastic scattering data on different targets, the fragmentation data of $^8$B to $^7$Be and proton, as well as the measurement of the electric quadrupole moment of $^8$B [2]. And although the latest publications [1] can explain the existing experimental data by a proton halo in $^8$B, it is necessary to understand, whether spatially extended density distribution of the loosely bound proton is the property of the structure $^8$B itself or the introduction of the halo is consequence of our insufficient knowledge of the
reaction mechanisms.

Of course, when the investigations of nucleus-nucleus collisions are performed, the questions of structure of the interacting nuclei and the reaction dynamics are closely interlaced. Therefore, it would be the ideal variant to have the form factors of elastic electron scattering on $^8\text{Li}$ and $^8\text{B}$. As the electron-nucleus interaction is, mainly, electromagnetic and, in principle, well known one, the most precise information on the nuclear structure may be derived from the analysis of these form factors together with the data of electric and magnetic moments for these nuclei. And if it is realized, then the trustworthy of the reaction mechanisms for different processes can be deduce.

As far as the obtaining of electron scattering data on unstable nuclei - is the problem of the future investigations [3], one has to carry out the careful analysis of the static data: electric quadrupole and magnetic dipole moments of nuclei $^8\text{Li}$ and $^8\text{B}$. The description of these data depends completely on the structure of these nuclei.

A ”nucleon halo” - is a new form of nuclear matter, characterized by low and nonuniform density distribution. The description of the halo nuclei needs the extra caution. It is necessary to pay the special attention to the following, very important, factors : I) the asymptotic behavior of the wave function of the valence nucleon; II) the correct consideration of the continuum effects; III) taking into account of the nucleon associations or the clustering phenomena; IV) the exact treatment of the Pauli principle or the antisymmetrization effects. At once one should point out that all these factors are intercorrelated. The different models of the resonating-group method (RGM) (or the generator-coordinate method (GCM)) [4,5] are the most promising ones from the point of view of the clustering phenomena and the Pauli principle. It may be the nucleon association model [6] or the microscopic multicluster model intensively developing now using the stochastic variational method [7]. These models assume the wide use of powerful computer facilities. Moreover, so far as in these models the basis function are chosen in Gaussian form, for the achievement of the correct asymptotic form of the wave functions and for the taking into account the continuum (the high partial wave channels) it is necessary to take a very large basis of the trial functions. The calculation becomes too complicated and computer time consuming. In spite of all respect to the valuable results, obtained by such a way, one should be point out, that the
big heuristic power of the analytical method being a characteristic feature for the traditional theoretical physics, here is reduced noticeably.

In present work the multiparticle shell model [8,9], for the description of magnetic dipole and electric quadrupole moments $^8\text{Li}$ and $^8\text{B}$. The Pauli principle is treated exactly in this model. For obtaining the correct asymptotic behavior in our approach we used the single-particle wave functions calculated in the Woods-Saxon potential both with and without the spin-orbit interaction [10]. If the energy and the wave function of the quasi-bound $1p_{1/2}$ - state are calculated taking into account the spin-orbit interaction, then the method of the expansion of the Sturm-Liouville functions is used, i.e the calculations are performed with an appropriate treatment of continuum effects [11,12]. Concerning to the clusterization, in the shell model the construction of the shell wave function by itself is connected with the associative structure of the nuclear state. For instance, in the Young’s scheme $[f1f2f3]=[444]$, corresponding to the structure from three $\alpha$-clusters, the attraction within a separate line (i.e the attraction of nucleons inside the association) and the repulsion between nucleons of the different clusters (lines) dominates. Let us mention also the well-known fact, that if the harmonic oscillator size parameters of the wave functions of the internal and relative motions are equal, then there is the identity between the wave functions of RGM and the shell model. If the oscillator parameter of the wave function of the relative motion of the clusters differs from that of the motion of nucleons inside the cluster, then one can expand the relative wave function on the principal quantum number with respect to the intrinsic wave functions. Then every term of the expansion will represent the wave function of the shell model and the expansion coefficient will define the weight of the corresponding configuration into the common wave functions of RGM. And this means that such approach allows to determine ”non-obviously” the influence of continuum on the RGM wave function. This multiparticle shell model with the realistic radial wave functions and the correct asymptotic behavior, which takes into account exactly of the continuum effects contains the aforesaid factors, that is necessary for considering the weakly bound halo nuclei.

2. Formalism
The wave functions for the nuclei with $A=8$ in ground state are as follows:

$$\Psi_{\alpha T}(\text{gr.st}) = \sum_i \beta_i [(1p)^4[\lambda]^{(2T+1)(2S+1)}L_j >,$$

(1)

where $\beta_i$ - configuration mixing coefficients, determining state weights with given values of orbital -$L$, spin -$S$ and isospin -$T$ moments for Young’s scheme $[\lambda]$ in the complete wave function of 4 nucleons in the 1p-shell, $J$ - the total momentum of nucleus. If the values of coefficients $\beta_i$ are known, then it is very convenient to calculate the matrix elements of the single-particle operators with such wave functions (1), separating the single-particle states with the fractional parentage coefficients technique. For the considered nuclei $^8B$ and $^8Li$ we have $-J^\pi = 2^+, T = 1$. The nuclear quadrupole and magnetic moments are defined by the proton-neutron formalism as:

$$eQ = \frac{8}{5} \sqrt{\frac{2}{7}} \sum_{\alpha=p,n} e^{eff}_\alpha \left[ < \varphi^{(\alpha)}_{1p_{1/2}}(r)|r^2|\varphi^{(\alpha)}_{1p_{3/2}}(r) > X^{(\alpha)}_{1,1,2} - < \varphi^{(\alpha)}_{1p_{3/2}}(r)|r^2|\varphi^{(\alpha)}_{1p_{3/2}}(r) > X^{(\alpha)}_{3,3,2} \right],$$

(2)

$$\mu_{\text{Nuc.}} = \frac{2\sqrt{5}}{15} \left\{ e[2X^{(p)}_{1,1,1} + \sqrt{10}X^{(p)}_{3,3,1} - 2\sqrt{2}X^{(p)}_{1,3,1} < \varphi^{(p)}_{1p_{1/2}}(r)|\varphi^{(p)}_{1p_{3/2}}(r) >] + \sum_{\alpha=p,n} \mu_\alpha [ - X^{(\alpha)}_{1,1,1} + \sqrt{10}X^{(\alpha)}_{3,3,1} + 4\sqrt{2}X^{(\alpha)}_{1,3,1} < \varphi^{(\alpha)}_{1p_{1/2}}(r)|\varphi^{(\alpha)}_{1p_{3/2}}(r) >] \right\},$$

(3)

where $\alpha = -$ for protons and $\alpha = n$ - for neutrons;

$e^{eff}_\alpha$ - the effective charge of nucleon;

$\mu_\alpha$ - the magnetic moment of nucleon;

$\varphi^{(\alpha)}_{1p_j}(r)$ - the radial function of nucleon in $1p_j$ - state;

$X^{(\alpha)}_{j',j,k}$ - the nucleon spectroscopic amplitude, which is expressed through the weight coefficients $\beta_i$ and the fractional parentage coefficients.
The one-particle root-mean-square (rms) radius may be presented in these notations as follows:

\[ \langle r^2_{1l_j} \rangle_{\alpha}^{1/2} = \sqrt{\langle \varphi_{1l_j}^{(\alpha)}(r) | r^2 | \varphi_{1l_j}^{(\alpha)}(r) \rangle}. \]  \hspace{1cm} (4)

For the nucleon occupation numbers of 1l-j - states we have [10]:

\[ \eta_{1l_j}^{(\alpha)} = \left( \frac{2j + 1}{2} \right) X_{j,j,0}^{(\alpha)}. \]  \hspace{1cm} (5)

Here l=p for 1p-shell, l=s for 1s-shell.

For the nuclear rms radii (R_{p,r.m.s}, R_{n,r.m.s} and R_{m,r.m.s} – the proton, neutron and matter rms radii, respectively) we have following expressions:

\[ R_{r.m.s}^p = \left\{ \frac{\eta_{1s1/2}^{(p)} < r_{1s1/2}^2 >_p + \eta_{1p3/2}^{(p)} < r_{1p3/2}^2 >_p + \eta_{1p1/2}^{(p)} < r_{1p1/2}^2 >_p}{Z} \right\}^{1/2}, \]  \hspace{1cm} (6a)

\[ R_{r.m.s}^n = \left\{ \frac{\eta_{1s1/2}^{(n)} < r_{1s1/2}^2 >_n + \eta_{1p3/2}^{(n)} < r_{1p3/2}^2 >_n + \eta_{1p1/2}^{(n)} < r_{1p1/2}^2 >_n}{N} \right\}^{1/2}, \]  \hspace{1cm} (6b)

\[ R_{r.m.s}^m = \left\{ \frac{Z \cdot (R_{r.m.s}^p)^2 + N \cdot (R_{r.m.s}^n)^2}{A} \right\}^{1/2}, \]  \hspace{1cm} (6c)

where Z – the number of protons, N – the number of neutrons in the nucleus with the number of nucleons A.

If a nucleus is not strongly excited, then

\[ \eta_{1s1/2}^p = \eta_{1s1/2}^n = 2. \]
At once it should be noted, if a spin-orbit interaction is not taken into account for calculation of the radial wave function in the mean field, then

\[
\varphi^{(\alpha)}_{1p_{3/2}}(r) = \varphi^{(\alpha)}_{1p_{1/2}}(r) = \varphi^{(\alpha)}_{1p}(r), \quad < r_{1p_{3/2}}^2 >^\alpha = < r_{1p_{1/2}}^2 >^\alpha = < r_{1p}^2 >
\]

and the expressions (6) and (6b) will take the following forms:

\[
R_{r.m.s}^p = \left\{ \frac{2 < r_{1s}^2 >^p + (Z-2) < r_{1p}^2 >^p}{Z} \right\}^{\frac{1}{2}}, \quad (7a)
\]

\[
R_{r.m.s}^n = \left\{ \frac{2 < r_{1s}^2 >^n + (N-2) < r_{1p}^2 >^n}{N} \right\}^{\frac{1}{2}}. \quad (7b)
\]

It follows from these expressions and (6), that if the spin-orbit interaction is absent, then the nuclear rms radii are independent from the nuclear structure and defined completely by the single-particle rms radii or the one-particle radial wave functions.

The nuclear quadrupole and magnetic moments can be expressed by the one-particle spectroscopic factors [13]:

\[
eQ = 5\sqrt{\frac{21}{21}} \sum_{\alpha=p,n} e_{\alpha}^{eff} \sum_{JcTc(E_c)} \sum_{jj'} [S_{JcTc(E_c),1p_j,\alpha}^{JT(0)}]^{\frac{1}{2}} [S_{JcTc(E_c),1p_{j'},\alpha}^{JT(0)}]^{\frac{1}{2}} \times
\]

\[
< \varphi^{(\alpha)}_{1p_j}(r, E_c)|r^2|\varphi^{(\alpha)}_{1p_j}(r, E_c) > (-1)^{Jc-j-\frac{1}{2}} \sqrt{(2j+1)(2j'+1)} \times
\]

\[
\begin{cases}
1 & \frac{1}{2} & j \\
\frac{1}{2} & 1 & j'
\end{cases}
= \begin{cases}
J & j' & Jc
\end{cases}, \quad (8)
\]

\[
\mu_{Nucl.} = 5\sqrt{\frac{5}{6}} \sum_{JcTc(E_c)} \sum_{jj'} [S_{JcTc(E_c),1p_j,p}^{JT(0)}]^{\frac{1}{2}} [S_{JcTc(E_c),1p_{j'},p}^{JT(0)}]^{\frac{1}{2}} \times
\]

\[
\begin{cases}
1 & \frac{1}{2} & j \\
\frac{1}{2} & 1 & j'
\end{cases}
= \begin{cases}
J & j' & Jc
\end{cases}, \quad (8)
\]

6
\[ \langle \varphi_{1p,j}^{(p)}(r, E_c) | \varphi_{1p,j}^{(p)}(r, E_c) \rangle \rangle = (-1)^{2j+J_c+\frac{3}{2}} \sqrt{(2j+1)(2j'+1)} \times \]

\[ \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & j \\ j' & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} J & j' & J_c \\ j' & J & 1 \end{array} \right\} + \]

\[ \sum_{\alpha=p,n} \mu_{\alpha} \sum_{J_cT_c(E_c), j,j'} [S^{JT(0)}_{J_cT_c(E_c),1p,j',\alpha}]^{\frac{1}{2}} [S^{JT(0)}_{J_cT_c(E_c),1p,j,\alpha}]^{\frac{1}{2}} \times \]

\[ \langle \varphi_{1p,j}^{(\alpha)}(r, E_c) | \varphi_{1p,j}^{(\alpha)}(r, E_c) \rangle \rangle = (-1)^{j+j'+J_c+\frac{3}{2}} \sqrt{(2j+1)(2j'+1)} \times \]

\[ \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & j \\ j' & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} J & j & J_c \\ j' & J & 1 \end{array} \right\} , \] (9)

where \( S^{JT(0)}_{J_cT_c(E_c),1p,j,\alpha} \) is the spectroscopic factor of the separation of the nucleon (proton - \( \alpha = p \) or neutron - \( \alpha = n \)) from 1p-shell with the total angular moment \( j \) from nucleus \( A \) in the ground state with the quantum numbers \( J \), when the residual nucleus (A- or A-n) is in the state with energy \( E_c \) and the quantum numbers \( J_cT_c \).

The expression (8) is the detailed presentation of expression (2) from the work [13], where the shell-model of Cohen-Kurath and Millener-Kurath (C-K and M-K) for the p-shell nuclei with the effective interaction CKPOT was used. The radial single-particle wave functions were calculated in the Woods-Saxon (WS) potential taking into account the centrifugal and Coulomb (for proton) potentials and neglecting the spin-orbit interaction. All the core exited states of A-1 nucleus were taken into account. For each core exited state with the excitation energy \( E_c \), the separation energy of the valence nucleon \( S_{\nu} \) was determined by the formula

\[ S_{\nu} = B(A) - B(A+1) + E_c , \] (10)

where \( B(A) \) is the binding energy of the nucleus \( A \).
The valence single-particle wave function \( \varphi^{(\alpha)}_{1pj}(r, E_c) \) was calculated by adjusting the depth of the WS potential to obtain the separation energy for each core exited state.

The expression (4) for the one-particle rms radius is rewritten as:

\[
<r^2_{1pj}>^2_{\alpha,E_c} = \sqrt{<\varphi^{(\alpha)}_{1pj}(r, E_c)|r^2|\varphi^{(\alpha)}_{1pj}(r, E_c)}. \tag{11}
\]

And for the nuclear rms radii we obtain:

\[
R_{r.m.s}^p = \left\{ \frac{1}{Z}[2<r^2_{1s_{1/2}}>_p + 5\frac{\sqrt{5}}{24} \sum_{JcTc(E_c)} S^{JT(0)}_{JcTc(E_c),1p_{3/2},p} <r^2_{1p_{3/2}}>_p,E_c + \frac{5\sqrt{10}}{24} \sum_{JcTc(E_c)} S^{JT(0)}_{JcTc(E_c),1p_{1/2},p} <r^2_{1p_{1/2}}>_p,E_c] \right\}^{1/2}, \tag{12a}
\]

\[
R_{r.m.s}^n = \left\{ \frac{1}{Z}[2<r^2_{1s_{1/2}}>_n + 5\frac{\sqrt{5}}{24} \sum_{JcTc(E_c)} S^{JT(0)}_{JcTc(E_c),1p_{3/2},n} <r^2_{1p_{3/2}}>_n,E_c + \frac{5\sqrt{10}}{24} \sum_{JcTc(E_c)} S^{JT(0)}_{JcTc(E_c),1p_{1/2},n} <r^2_{1p_{1/2}}>_n,E_c] \right\}^{1/2}, \tag{12b}
\]

and for \( R_{r.m.s}^m \) the expression (6) remains valid taking into account (12) and (12b).

If the spin-orbit interaction is not taken into account, then we have:

\[
R_{r.m.s}^p = \left\{ \frac{1}{Z}[2<r^2_{1s_{1/2}}>_p + 5\frac{\sqrt{5}}{24} \sum_{JcTc(E_c)} (S^{JT(0)}_{JcTc(E_c),1p_{3/2},p} + \sqrt{2} \sum_{JcTc(E_c)} S^{JT(0)}_{JcTc(E_c),1p_{1/2},p}) <r^2_{1p}>_p,E_c] \right\}^{1/2}, \tag{13a}
\]
\[ R_{r.m.s}^n = \left\{ \frac{1}{N} \langle r_1^2 \rangle_p + \frac{5\sqrt{5}}{24} \sum_{J_cT_c(E_c)} \langle S_{J_cT_c(E_c),1p_3/2,n}^{JT(0)} \rangle \right\}^{1/2} \]

Unlike (7) and (7), in these expressions the dependence of the nuclear rms radii on the nuclear structure (the spectroscopic factors) remains.

As the mean field, determining the one-particle levels and wave functions, in our work we take the potential in the form:

\[ V(r) = V_{WS} + V_{Centrif} + V_{Coul} + V_{ls}, \]

where

\[ V_{WS} = -V_0 \left\{ 1 + \exp \left( \frac{r - r_0 A^{1/3}}{a} \right) \right\}^{-1} \]

- the Woods-Saxon potential,

\[ V_{Centrif} \] - the centrifugal potential,

\[ V_{Coul} \] - the Coulomb potential,

\[ V_{ls} = -\kappa (\vec{\sigma} \cdot \vec{l}) \frac{1}{r} \frac{dV_{WS}}{dr} \] - the spin-orbital interaction,

\[ \vec{\sigma} \cdot \vec{l} = \begin{cases} l & j = l + \frac{1}{2} \\ -(l + 1) & j = l - \frac{1}{2} \end{cases} \]

\[ \kappa \] - the constant of the spin-orbital interaction.

It is more convenient to rewrite the expressions (1) and (7) for the quadrupole moments as

\[ eQ = e_p^{eff} q_p + e_n^{eff} q_n. \]
3. Results and discussion

The analysis of the experimental data was started from reproducing the results of ref. [13] and adding information on the magnetic moments. The proton single-particle wave functions in the nucleus $^8$B and the neutron ones in $^8$Li were calculated, taking into account the formula (10). The nucleon radius parameter $r_0$ and the surface diffuseness $a$ were taken as standard values: $r_0 = 1.27$ fm, $a = 0.65$ fm. The results of calculations of the single-particle rms radii in accordance with expression (11) and with $\kappa = 0$ (i.e. without spin-orbit interaction, see (14)), are listed in Table 1. The harmonic oscillator parameter $b = 1.6$ fm was taken for the neutron radial wave functions in $^8$B and the proton ones in $^8$Li (both for 1s- and 1p- state), like in [13].

The results of calculations of the quadrupole and magnetic moments (the expressions (8), (9) and (15)) are presented in Table 2 (variant I). In this table the empirical values of the quadrupole moment $Q(^8B) = 6.83$ fm$^2$ and $Q(^8Li) = 3.27$ fm$^2$ [2] correspond to the adduced values $q_\alpha$ and $e_{\alpha}^{eff}$. And for the experimental values of the magnetic moments we have $\mu_{Nucl.}(^8B) = 1.0355$ and $\mu_{Nucl.}(^8Li) = 1.65335$ [14]. Table 2 shows that the theoretical $\mu_{Nucl.}$ are somewhat smaller than the empirical values, especially for $^8$Li. The values of the nuclear rms radii (the expressions (13a), (13b) and (6c)) are listed in Table 3 (variant I). As follows from Tables 2 and 3, we have reproduced the results of the calculations from [13].

And now let us use the multiparticle shell model, where the matrix elements are calculated by the method of fractional parentage coefficients via the weight coefficients $\beta_i$ (expressions (1)–(5)). From the single-particle rms radii we will use only the values at $E_c = 0$ (the first line in Table 1). In the framework of this model the coefficients of the configurations $\beta_i$ were best fitted to the experimental values of $Q$ and $\mu_{Nucl.}$ for $^8$B and $^8$Li. The values of $\beta_i$ determined in this way are presented in Table 4 (variant A). The values $e_{\alpha}^{eff}$ corresponding to these $\beta_i$ are very close to ones from [13], and good description is obtained for the experimental values $\mu_{Nucl.}$ by this approach (Table 2 (variant II)). The values of the nuclear rms radii, which in given variant of the calculations are independent from the nuclear structure, i.e. from the spectroscopic amplitudes
(see expressions (7a) and (7b)) are presented in Table 3 (variant II). The comparison with the variant I in this table shows the considerable increase of $R_{r.m.s}^p$ for $^8$B and $R_{r.m.s}^n$ for $^8$Li.

The results of calculations in the framework of two approaches require a detailed analysis of the considered models. We should point out the defect of the Cohen-Kurath and Millener-Kurath model [8] especially for nuclei $^8$Li and $^8$B with $J=2$: the main component with $j = j' = \frac{3}{2}$ in the first term of the expansion on the states of the residual nuclei $^7$Li and $^7$Be (expression (8)) is suppressed by chance in this case because for $^7$Li (gr. st.) and $^7$Be (gr. st.) $J_c = \frac{3}{2}$, resulting to zero value of 6-j symbol,

$$\left\{ J \ j \ J_c \right\} = \left\{ \begin{array}{ccc} 2 & 3/2 & 3/2 \\ 3/2 & 2 & 2 \end{array} \right\} = 0.$$

This component brings the dominant contribution in the calculations of the magnetic moments and the nuclear rms radii (expressions (9), (13), (13)). It should be noted also somewhat artificial character of the expansion method on the spectroscopic factors, since the last depend both on the structure of the parental and residual nuclei. Moreover, using the expression (10) for the separation energy of the valence nucleon gives rise to the decreasing of the nuclear rms radii as compared with the results of the multiparticle shell model (Table 3, variants I and II). This is a consequence of the single-particle rms radii dependence on the excitation energies $E_c$. If we try to reduce the expressions (13) and (13b) to the form like (7) and (7b), then it is necessary to take out of summation an average value of the single-particle rms radius $\langle r_{1p}^2 \rangle_{av.}$ independent on $E_c$. Then we obtain:

$$R_{r.m.s}^p = \left\{ \frac{2< r_{1s}^2 >_p + (Z - 2)< r_{1p}^2 >_{p_{av.}}}{Z} \right\}^{\frac{1}{2}}, \quad \text{(16a)}$$

$$R_{r.m.s}^n = \left\{ \frac{2< r_{1s}^2 >_n + (N - 2)< r_{1p}^2 >_{n_{av.}}}{N} \right\}^{\frac{1}{2}}, \quad \text{(16b)}$$

The average values of $\langle r_{1p}^2 \rangle_{p_{av.}}^{1/2} = 3.575$ fm and $\langle r_{1p}^2 \rangle_{n_{av.}}^{1/2} = 3.136$ fm correspond to the values $R_{r.m.s}^p = 3.034$ fm for $^8$B and $R_{r.m.s}^n = 2.727$ fm.
for $^8\text{Li}$, respectively. We choose the values $r_0$ and $a$ (the depth of the Wood-Saxon potential corresponds to the separation energy of the valence proton in $^8\text{B}$) so as to obtain $\langle r^2_{1p} \rangle^{1/2}_{p\text{av.}} = 3.575$ fm. For $r_0 = 1.17$ fm and $a = 0.37$ fm we have obtained: $\langle r^2_{1p} \rangle^{1/2}_{p\text{av.}} = 3.576$ fm for $^8\text{B}$ and $\langle r^2_{1p} \rangle^{1/2}_{n\text{av.}} = 2.872$ fm for $^8\text{Li}$. Given these data in the framework of the multiparticle shell model, we fitted the coefficients $\beta_i$ describing the experimental values $\mu_{\text{Nucl.}}$ and $Q$. The values of the coefficients $\beta_i$ are represented in Table 4 (variant B). The values $\mu_{\text{Nucl.}}$ and $e^{eff}_\alpha$ listed in Table 2 (variant III) correspond to these $\beta_i$. Tables 2 and 3 (variants I and III) demonstrate reproduction the results of the work [13], and besides sufficiently good description of the magnetic moments in the given case. But the values $r_0$ and $a$ become much less than the standard ones and the nuclear wave function becomes more clustered: $|\beta_1| = 0.906$ (table 4, variant B) $|\beta_1| = 0.859$ (variant A). All aforesaid results were obtained with the radial one-particle wave functions calculated in the mean field potential without the spin–orbit interaction. But for such loosely-bound system as nucleus $^8\text{B}$, having the binding energy of the valence proton $E_{1l_j}^{p} = E_{1p_{3/2}}^{p} = -0.137$ MeV, the probability to discover proton in the $1\frac{1}{2}$ state is not zero. And it is possible when taking into account the spin–orbit interaction specifically for nucleus $^8\text{B}$. In general case for a weakly-bound halo nuclei the value of spin-orbital interaction is an open problem at present. And in most cases for such nuclei, the levels of energies and the single-particle wave functions of $1\frac{1}{2}$ - states are not defined by the conventional methods, because the Levinson’s theorem is not executed. In present work the energy and the wave function of the single-particle $1\frac{1}{2}$ - state of proton in $^8\text{B}$ were calculated by the method of the expansion of the Sturm-Liouville functions, thereby taking into account the influence of the continuum [11,12]. The constant of the spin–orbit interaction $\kappa$ was chosen so that the calculated single-particle energy proved to be lower than the centrifugal + Coulomb + spin-orbital barrier. Then we can consider this state with the positive energy as quasibound one. The wave function of such a state is convergent and can be normalized to 1. The one-particle energies and wave functions of the valence nucleons in $^8\text{B}$ and $^8\text{Li}$ were calculated for the ground and first excited states of the residual nuclei $^7\text{Li}$ and $^7\text{Be}$, respectively (see expressions (10)). The spin-orbital constant $\kappa = 0.101$ was chosen. Having the wave functions obtained in the framework of
C-K and M-K model, were calculated the single-particle rms radii. The results of the calculations are listed in Table 5. Taking these rms radii (and the others represented in Table 1) we calculated the quadrupole and magnetic moments (the Table 2 (variant IV)) and the nuclear rms radii (Table 3 (variant IV)). As expected, we obtained just a small increase of \( R_{r.m.s}^p(8B) \) and \( R_{r.m.s}^n(8Li) \) and the related decrease of \( e_{eff}^n \) and \( e_{eff}^p \) (as compared with the calculations without the spin-orbital interaction (variant I in tables 2 and 3). The results of calculations of the wave functions, including the spin-orbit coupling (see Table 5, the values for \( E_c = 0 \) MeV only), in the framework of the multiparticle shell model are represented in Tables 2 and 3 (variants V) and Table 4 (variant C). The occupation numbers of the nucleon one-particle states are listed in Table 6. The distributions of the proton density in \(^8\)B (solid curve) and the neutron density in \(^8\)Li (dashed curve) are shown in Fig. 1. In this figure the dash-dot curve corresponds to the distribution of the neutron density in \(^8\)B or the proton density in \(^8\)Li, as far as in both cases we used the harmonic oscillator wave functions with the equal values of the oscillator parameter. The comparison with the results of the calculations of the wave functions in the mean field potential without the spin–orbit interaction (variants II in tables 1 and 2, variant A in table 4) shows only a slight increase in \( R_{r.m.s}^p(8B) \) and \( R_{r.m.s}^n(8Li) \). It is caused by the small value of the spin-orbit potential \( V_{ls} \). \( V_{ls} = 4 \) MeV corresponds to the value \( \kappa = 0.101 \), while \( V_{ls} \) varies within \( 8 \div 13 \) MeV for the stable nuclei of 1-shell [15]. The values of the single-particle rms radii in \(^8\)B \( < r_{1p3/2}^2 >_p^{1/2} = 4.360 \text{ fm} \) and \( < r_{1p1/2}^2 >_p^{1/2} = 4.677 \text{ fm} \) (Table 5) are significantly smaller than the corresponding value \( < r_{1p3/2}^2 >_p^{1/2} = 6.83 \text{ fm} \), calculated by the quasiparticle random phase approximation model [16], but larger than \( < r_{1p3/2}^2 >_p^{1/2} = 4.02 \text{ fm} \) — a pure mean-field plus pairing correlation calculation [16]. It follows from Table 3 (variants II and V) and Fig. 1 that nucleus \(^8\)B has a significant proton halo, and \(^8\)Li has a less neutron halo. And in this case, a neutron (proton) in \(^8\)B (\(^8\)Li) is found in \( 1_{1/2} \) - state with a low probability ~12%, whereas one of the three 1p-protons(neutrons) in \(^8\)B (\(^8\)Li) can be found in this state already with a higher probability ~46% (Table 6). Our values of the nuclear rms radii, calculated in the framework of the multiparticle shell model exceed those available in the literature including [7] (Table 3, variant VI). The authors of [7] reproduced the binding energy
of the valence proton and the large quadrupole moment of $^8$B, using the microscopic multicluster model, however they could not describe the magnetic dipole moments of $^8$Li and $^8$B (Table 2, variant VI). Of course, in spite of the successful description of the magnetic and quadrupole moments of the mirror nuclei $^8$Li and $^8$B in the framework of our approach, the obtained results are preliminary yet. And it would be very desirably to have the experimental data of the electron scattering on these nuclei and also the data of the polarization measurements. As far as the electron scattering on unstable nuclei has not been realized yet, it would be actual to carry out experiments on scattering of secondary beams of $^8$Li and $^8$B on polarized protons.

**4. Conclusion**

We used the multiparticle shell model with: the radial wave functions, calculated in a realistic Woods-Saxon potential; the exact taking into account of a continuum; the use of the method of fractional parentage coefficients – for analysis of the experimental electric quadrupole and magnetic dipole moments of nuclei $^8$Li and $^8$B. The obtained values of the nuclear rms radii and the nucleon density distributions show that nucleus $^8$B has a significant proton halo, and $^8$Li has a less neutron halo. We compared applicability of the multiparticle shell model and the microscopic multicluster model to the weakly bound halo nuclei. For the obtaining more correct wave functions of the considered nuclei, it is necessary to have the experimental scattering data of secondary beams of $^8$Li and $^8$B on polarized protons.
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Table 1. The one-particle root-mean-square (rms) radii for the proton one-particle state in $^8B$ and the neutron one-particle state in $^8Li$.

| $E_c(^7Be)$, [MeV] | $<r_{1p}^2>_{p,E_c}(^8B)$, [fm] | $E_c(^7Li)$, [MeV] | $<r_{1p}^2>_{n,E_c}(^8Li)$, [fm] |
|-------------------|-------------------------------|-------------------|-------------------------------|
| 0.0               | 4.359                         | 0.0               | 3.620                         |
| 0.4292            | 3.911                         | 0.4776            | 3.463                         |
| 4.5700            | 2.923                         | 4.6330            | 2.851                         |
| 6.7300            | 2.750                         | 6.6800            | 2.709                         |
| 7.2100            | 2.719                         | 7.4670            | 2.666                         |
| 9.2700            | 2.610                         | 9.6100            | 2.560                         |
| 9.9000            | 2.582                         | 10.250            | 2.541                         |
| 11.010            | 2.538                         | 11.250            | 2.505                         |
Table 2. Matrix elements $q_\alpha$ (fm$^2$), the empirical effective charge of nucleon $e_{\alpha}^{eff}$ and theoretical magnetic moments, (experimental $\mu_{\text{Nucl.}}(^8B) = 1.0355$ and $\mu_{\text{Nucl.}}(^8Li) = 1.65335$ [14]).

| Variant | Nuclei | $q_n$ | $q_p$ | $e_{n}^{eff}$ | $e_{p}^{eff}$ | $\mu_{\text{Nucl.}}$ (theor.) |
|---------|--------|-------|-------|---------------|---------------|-------------------------------|
| I       | $^8Li$ | 4.163 | 0.800 | 0.577         | 1.095         | 1.367                         |
|         | $^8B$  | 0.800 | 5.813 |               |               | 0.988                         |
| II      | $^8Li$ | 3.914 | 0.936 | 0.570         | 1.109         | 1.604                         |
|         | $^8B$  | 0.936 | 5.677 |               |               | 1.021                         |
| III     | $^8Li$ | 3.592 | 1.107 | 0.567         | 1.114         | 1.584                         |
|         | $^8B$  | 1.107 | 5.567 |               |               | 1.016                         |
| IV      | $^8Li$ | 4.574 | 0.800 | 0.537         | 1.022         | 1.367                         |
|         | $^8B$  | 0.800 | 6.263 |               |               | 0.988                         |
| V       | $^8Li$ | 4.024 | 0.879 | 0.570         | 1.109         | 1.603                         |
|         | $^8B$  | 0.879 | 5.705 |               |               | 1.020                         |
| V       | $^8Li$ | 4.024 | 0.879 | 0.570         | 1.109         | 1.603                         |
|         | $^8B$  | 0.879 | 5.705 |               |               | 1.020                         |
| VI [7]  | $^8Li$ | –     | 2.23  | –             | –             | 1.42                          |
|         | $^8B$  | –     | 6.65  | –             |               | 1.17                          |

Attention. In the work [7] effective charges have not used.

$e_{n}^{eff} = e_n = 0, \ e_{p}^{eff} = e_p = 1$.  

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Table 3. The proton, neutron and matter rms radii. (fm)

| Variant | Nucleus | $R_{p/r.m.s}$ | $R_{n/r.m.s}$ | $R_{m/r.m.s}$ |
|---------|---------|---------------|---------------|---------------|
| I       | $^8B$   | 3.034         | 2.164         | 2.740         |
|         | $^8Li$  | 2.164         | 2.727         | 2.531         |
| II      | $^8B$   | 3.597         | 2.164         | 3.138         |
|         | $^8Li$  | 2.164         | 3.065         | 2.763         |
| III     | $^8B$   | 3.034         | 2.164         | 2.741         |
|         | $^8Li$  | 2.164         | 2.547         | 2.411         |
| IV      | $^8B$   | 3.082         | 2.164         | 2.775         |
|         | $^8Li$  | 2.164         | 2.854         | 2.617         |
| V       | $^8B$   | 3.634         | 2.164         | 3.164         |
|         | $^8Li$  | 2.164         | 3.079         | 2.772         |
| VI [7]  | $^8B$   | 2.830         | 2.260         | 2.630         |
|         | $^8Li$  | 2.190         | 2.600         | 2.450         |

Table 4. The weights of the corresponding configurations

| Configuration | $\beta_i$ |
|---------------|-----------|
|               | Variant A | Variant B | Variant C |
| [31]^{35}P    | -0.859    | -0.906    | -0.850    |
| [31]^{31}D    | -0.378    | -0.269    | -0.388    |
| [31]^{35}D    | -0.241    | -0.185    | -0.241    |
| [31]^{33}F    | -0.124    | -0.069    | -0.131    |
| [22]^{33}D    | -0.193    | -0.236    | -0.202    |
| [211]^{33}P   | -0.055    | 0.049     | -0.065    |
| [211]^{35}P   | -0.072    | 0.093     | -0.085    |
Table 5. Energies and root-mean-square states (energies (MeV), radii (fm)).

|        | $^8B$ |        |        | $^8Li$ |        |        |
|--------|-------|--------|--------|--------|--------|--------|
| $E_c(\text{\ci{Be}c})$ | $1l_j$ | $E_{1l_j}^p$ | $<r_{1l_j}^2>^{1/2}$ | $E_c(\text{\ci{Li}i})$ | $1l_j$ | $E_{1l_j}^n$ | $<r_{1l_j}^2>^{1/2}$ |
| 0.0    | 1$p_{3/2}$ | -0.137 | 4.360  | 0.0    | 1$p_{3/2}$ | -2.000 | 3.521  |
|       | 1$p_{1/2}$ | 1.004  | 4.667  |        | 1$p_{1/2}$ | -0.832 | 4.229  |
| 0.4292 | 1$p_{3/2}$ | -0.566 | 3.888  | 0.4776 | 1$p_{3/2}$ | -2.4776 | 3.380  |
|       | 1$p_{1/2}$ | 0.648  | 4.172  |        | 1$p_{1/2}$ | -1.234 | 3.875  |

Table 6. The occupation numbers of the nucleon 1$p_j$ – states in $^8B$.

|        |        |        |        |        |
|--------|--------|--------|--------|--------|
| $\eta_{1p_{3/2}}^{(p)}$ | $\eta_{1p_{1/2}}^{(p)}$ | $\eta_{1p_{3/2}}^{(n)}$ | $\eta_{1p_{1/2}}^{(n)}$ |
| 2.543  | 0.457  | 0.878  | 0.122  |
Fig. 1. The distributions of the proton density in $^8B$ (solid curve), the neutron density in $^8Li$ (dashed curve), the neutron density in $^8B$ or the proton density in $^8Li$ (the dash-dot curve).