Possible Deviation from the Tri-bimaximal Neutrino Mixing in a Seesaw Model

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Abstract

We propose a simple but suggestive seesaw model with two phenomenological conjectures: three heavy (right-handed) Majorana neutrinos are degenerate in mass in the symmetry limit and three light Majorana neutrinos have the tri-bimaximal mixing pattern $V_0$. We show that a small mass splitting between the first generation and the other two generations of heavy Majorana neutrinos is responsible for the deviation of the solar neutrino mixing angle $\theta_{12}$ from its initial value $35.3^\circ$ given by $V_0$, and the slight breaking of the mass degeneracy between the second and third generations of heavy Majorana neutrinos results in a small mixing angle $\theta_{13}$ and a tiny departure of the atmospheric neutrino mixing angle $\theta_{23}$ from $45^\circ$. It turns out that a normal hierarchy of the light neutrino mass spectrum is favored in this seesaw scenario.

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Thanks to the enormous progress made in solar, atmospheric and terrestrial neutrino experiments [1–4], now we have got very robust evidence for the existence of neutrino oscillations, a new window to physics beyond the standard model. While the atmospheric neutrino deficit still points toward a maximal mixing between the tau and muon neutrinos \( \theta_{23} \simeq 45^\circ \), the solar neutrino anomaly favors a large but non-maximal mixing between the electron and muon neutrinos \( \theta_{12} \simeq 33^\circ \). In addition, the third neutrino mixing angle \( \theta_{13} \) is constrained to the range \( 0^\circ \leq \theta_{13} \leq 10^\circ \) and its best-fit value is actually around zero [5]. These results motivate a number of authors to consider the so-called tri-bimaximal neutrino mixing pattern [6]

\[
V_0 = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]

(1)

It corresponds to \( \sin^2 \theta_{12} = \frac{1}{3} \), \( \sin^2 \theta_{23} = \frac{1}{2} \) and \( \sin^2 \theta_{13} = 0 \). Such a special neutrino mixing pattern is most likely to arise from an underlying flavor symmetry (e.g., the discrete non-Abelian symmetry \( A_4 \) [7]). The spontaneous or explicit symmetry breaking is in general unavoidable, because a flavor symmetry itself cannot reproduce the observed lepton mass spectra and lepton mixing angles simultaneously [8].

Given the successful seesaw mechanism [9], which naturally explains the smallness of three (left-handed) neutrino masses, it will be interesting to anticipate that \( V_0 \) just results from diagonalizing the effective neutrino mass matrix \( M_\nu \) in the flavor basis where the charged-lepton mass matrix \( M_l \) is diagonal and real (positive):

\[
M_\nu = M_D M_R^{-1} M_D^T = V_0 \overline{M}_\nu V_0^T,
\]

(2)

where \( M_R \) is symmetric and denotes the heavy (right-handed) Majorana neutrino mass matrix, and \( \overline{M}_\nu \equiv \text{Diag}\{m_1, m_2, m_3\} \) with \( m_i \) (for \( i = 1, 2, 3 \)) being three light Majorana neutrino masses. Then the tri-bimaximal neutrino mixing can be achieved in three possible ways in such a seesaw model:

- Taking \( M_D \) to be diagonalized by \( V_0 \) and \( M_R = M_0 I \) with \( M_0 \) being a common mass scale and \( I \) being the identity matrix;

- Taking \( M_D = m_0 I \) with \( m_0 \) being a common Dirac fermion mass scale and \( M_R \) to be diagonalized by \( V_0 \);

- Taking very simple but non-trivial textures of \( M_D \) and \( M_R \) such that \( V_0 \) results from diagonalizing both of them.

Among these three possibilities, the first one is the simplest and most interesting from a phenomenological point of view, which will be clearer later. The mass degeneracy of three heavy Majorana neutrinos, which corresponds to the \( S(3) \) flavor symmetry, has actually been speculated in some interesting seesaw models [10–13] so as to generate large neutrino mixing angles. Although the well-motivated \( V_0 \) is fairly compatible with all today’s neutrino oscillation data, a tiny departure of the realistic neutrino mixing pattern \( V \) from \( V_0 \) remains allowed. In particular, possible deviation of \( \theta_{23} \) from \( 45^\circ \) and non-vanishing \( \theta_{13} \) will be an
important issue to probe at the upcoming neutrino experiments. Thus it is worthwhile to consider the deviation of $V$ from $V_0$ and to examine its possible origins [14]. In this paper, we aim to establish a direct relationship between the breaking of heavy Majorana neutrino $S(3)$ symmetry and the deviation of $V$ from $V_0$. A similar idea has recently been proposed in Ref. [15] to investigate the small correction, induced by the mass splitting of heavy Majorana neutrinos, to the well-known bi-maximal neutrino mixing pattern [16] in a simple seesaw scenario.

We are going to begin with the first conjecture made above and propose a simple scenario, in which the mass splitting between the first generation and the other two generations of heavy Majorana neutrinos is responsible for a departure of the solar neutrino mixing angle $\theta_{12}$ from its initial value $35.3^\circ$ given by the tri-bimaximal neutrino mixing pattern $V_0$. Furthermore, we shall show that the slight breaking of the mass degeneracy between the second and third generations of heavy Majorana neutrinos gives rise to a small mixing angle $\theta_{13}$ and a tiny deviation of the atmospheric neutrino mixing angle $\theta_{23}$ from $45^\circ$. These results imply that our seesaw model has much more room to fit the present and future neutrino oscillation data, and it provides some useful hints towards further model building to obtain a dynamical picture of neutrino mass generation and lepton flavor mixing.

The remaining part of this paper is organized as follows. Section 2 is devoted to some analytical calculations of how the breaking of the mass degeneracy among heavy Majorana neutrinos is related to the deviation of lepton flavor mixing from the tri-bimaximal mixing pattern $V_0$. A numerical illustration of the typical parameter space is given in section 3. Some further discussions and conclusion are presented in section 4.

Let us conjecture that $M_R = M_0 I$ holds in the symmetry limit and $M_D$ can be diagonalized by the transformation $V_0^\dagger M_D V_0^{*} = \text{Diag}\{x, y, z\}$ with $x$, $y$ and $z$ being real and positive. Then Eq. (2) holds automatically. Note that it is possible to decompose the tri-bimaximal neutrino mixing matrix $V_0$ into a product of two rotation matrices $R_{23}(\theta_{23}^0)$ and $R_{12}(\theta_{12}^0)$; i.e., $V_0 = R_{23}(\theta_{23}^0) \otimes R_{12}(\theta_{12}^0)$, where

$$R_{12}(\theta_{12}^0) = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{3} & 2/\sqrt{6} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_{23}(\theta_{23}^0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix},$$

with $\theta_{12}^0 = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ and $\theta_{23}^0 = 45^\circ$. In the following, we consider two mass splitting scenarios for three heavy Majorana neutrinos and calculate the corresponding light neutrino mass spectra and flavor mixing angles.

**A. Scenario with $M_1 \neq M_2 = M_3$**

In this case, we define the mass splitting parameter $\delta_{12} \equiv (M_2 - M_1)/M_2$ and assume $|\delta_{12}| \ll 1$. The effective neutrino mass matrix $M_\nu$ turns out to be
\[ M_\nu = M_D M_{R}^{-1} M_D^T \]
\[ = V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T \left( \begin{pmatrix} M_1^{-1} & 0 & 0 \\ 0 & M_2^{-1} & 0 \\ 0 & 0 & M_3^{-1} \end{pmatrix} \right) V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T \]
\[ = V_0 \left( \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \right) R_{12}^T \left( \begin{pmatrix} M_1^{-1} & 0 & 0 \\ 0 & M_2^{-1} & 0 \\ 0 & 0 & M_3^{-1} \end{pmatrix} \right) R_{12} \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T \equiv V_0 M'_\nu V_0^T, \quad (4) \]

where
\[ M'_\nu = \frac{z^2}{3 M_1} \begin{pmatrix} \omega \eta^2 (3 - \delta_{12}) & \sqrt{2} \omega \eta^2 \delta_{12} & 0 \\ \sqrt{2} \omega \eta^2 \delta_{12} & \eta^3 (3 - 2 \delta_{12}) & 0 \\ 0 & 0 & 3 (1 - \delta_{12}) \end{pmatrix} \quad (5) \]

with \( \omega \equiv x/y \) and \( \eta \equiv y/z \). It is obvious that \( M'_\nu \) can be diagonalized by the orthogonal transformation \( O_{12}^T M'_\nu O_{12} = M_{\nu} \), in which \( O_{12}(\theta_{12}) \) denotes the Euler rotation matrix analogous to \( R_{12}(\theta_{12}) \), and \( M_{\nu} \) has been defined in Eq. (2). A straightforward calculation yields

\[ m_1 = \frac{z^2 \eta^2}{3 M_1} \left[ \omega^2 (3 - \delta_{12}) \cos^2 \theta_{12} + (3 - 2 \delta_{12}) \sin^2 \theta_{12} - \sqrt{2} \omega \delta_{12} \sin 2 \theta_{12} \right], \]
\[ m_2 = \frac{z^2 \eta^2}{3 M_1} \left[ \omega^2 (3 - \delta_{12}) \sin^2 \theta_{12} + (3 - 2 \delta_{12}) \cos^2 \theta_{12} + \sqrt{2} \omega \delta_{12} \sin 2 \theta_{12} \right], \]
\[ m_3 = \frac{z^2}{M_1} (1 - \delta_{12}) ; \quad (6) \]

and
\[ \tan 2 \theta_{12}' = \frac{2 \sqrt{2} \omega \delta_{12}}{3 (1 - \omega^2) + (\omega^2 - 2) \delta_{12}}. \quad (7) \]

Then we obtain the lepton flavor mixing matrix
\[ V = V_0 O_{12} = R_{23}(\theta_{23}) \otimes R_{12}(\theta_{12}), \quad (8) \]

where \( \theta_{12} = \theta_{12}^0 + \theta_{12}' \). One can see that the original mixing angle \( \theta_{12}^0 \) gets modified as a consequence of the mass splitting between the first generation and the other two generations of heavy Majorana neutrinos. In the limit \( \delta_{12} \to 0 \), \( \theta_{12} \to \theta_{12}^0 \) is therefore restored. Because of \( \theta_{23} \approx 45^\circ \) and \( \theta_{13} \approx 0^\circ \), the deviation of \( V \) from \( V_0 \) is fairly slight.

**B. Scenario with \( M_1 \neq M_2 \neq M_3 \)**

In this more general case, the mass degeneracy between the second- and third-generation heavy Majorana neutrinos is lifted too. The corresponding mass splitting parameter is defined as \( \delta_{23} \equiv (M_3 - M_2)/M_3 \). Of course, \( |\delta_{23}| \ll 1 \) is required to hold, because it is expected to be responsible for the tiny deviation of \( \theta_{23} \) from \( 45^\circ \) and for a small value of \( \theta_{13} \). The effective neutrino mass matrix \( M_\nu \) then reads
\[ M_\nu = M_D M_R^{-1} M_D^T \]
\[ = V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T \begin{pmatrix} M_1^{-1} & 0 & 0 \\ 0 & M_2^{-1} & 0 \\ 0 & 0 & M_3^{-1} \end{pmatrix} V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T = V_0 M'_\nu V_0^T, \quad (9) \]

where
\[ M'_\nu = \frac{z^2}{6M_1} \begin{pmatrix} \omega^2 \eta^2 (6 - 2\delta_{12} - \delta_{23}) & \sqrt{2} \omega \eta^2 (2\delta_{12} + \delta_{23}) & -\sqrt{3} \omega \eta \delta_{23} \\ \sqrt{2} \omega \eta^2 (2\delta_{12} + \delta_{23}) & 2\eta^2 (3 - 2\delta_{12} - \delta_{23}) & \sqrt{6} \eta \delta_{23} \\ -\sqrt{3} \omega \eta \delta_{23} & \sqrt{6} \eta \delta_{23} & 3(2 - 2\delta_{12} - \delta_{23}) \end{pmatrix}, \quad (10) \]
in which the terms of \( O(\delta_{12}^2), O(\delta_{23}^2) \) and \( O(\delta_{12} \delta_{23}) \) have safely been neglected. One can diagonalize \( M'_\nu \) by using three Euler rotation transformations \( O_{12}(\theta'_{12}), O_{23}(\theta'_{23}) \) and \( O_{13}(\theta'_{13}) \) in the following way: \( O_{13}^T O_{23}^T O_{12}^T M'_\nu O_{12} O_{23} O_{13} = \overline{M}_\nu \). After a lengthy but straightforward calculation, we arrive at
\[ m_1 \simeq \frac{z^2 \omega^2 \eta^2}{M_1} \left( 1 - \frac{1}{3} \delta_{12} - \frac{1}{6} \delta_{23} \right), \]
\[ m_2 \simeq \frac{z^2 \eta^2}{M_1} \left( 1 - \frac{2}{3} \delta_{12} - \frac{1}{3} \delta_{23} \right), \]
\[ m_3 \simeq \frac{z^2}{M_1} \left( 1 - \delta_{12} - \frac{1}{2} \delta_{23} \right); \quad (11) \]

and
\[ \tan 2\theta'_{12} \simeq \frac{2\sqrt{2} \omega (2\delta_{12} + \delta_{23})}{6(1 - \omega^2) + 2(\omega^2 - 2) \delta_{12} + (\omega^2 - 2) \delta_{23}}, \]
\[ \tan 2\theta'_{23} \simeq \frac{2\sqrt{6} \eta \delta_{23}}{6(1 - \eta^2) + 2(2\eta^2 - 3) \delta_{12} + (2\eta^2 - 3) \delta_{23}}, \]
\[ \tan 2\theta'_{13} \simeq \frac{-2\sqrt{3} \omega \eta \delta_{23}}{6(1 - \omega \eta) + 2(\omega^2 \eta^2 - 3) \delta_{12} + (\omega^2 \eta^2 - 3) \delta_{23}} \quad (12) \]
in a good approximation. The neutrino mixing matrix is given by
\[ V = V_0 O_{12} O_{23} O_{13} \]
\[ = R_{23}(\theta^0_{23}) \otimes R_{12}(\theta^0_{12} + \theta'_{12}) \otimes R_{23}(\theta'_{23}) \otimes R_{13}(\theta'_{13}). \quad (13) \]

We see that the original mixing angles \( \theta^0_{12} \) and \( \theta^0_{23} \) are both modified, and the third mixing angle \( \theta_{13} \) becomes non-vanishing.

It is interesting to take a look at the approximate analytical dependence of \( \theta'_{12}, \theta'_{23} \), and \( \theta'_{13} \) on \( \delta_{12} \) and \( \delta_{23} \). For simplicity, let us assume that the magnitudes of \( \omega \) and \( \eta \) are around 0.5 or smaller. Then Eq. (12) can be simplified to
\[ \tan 2\theta'_{12} \simeq \frac{\sqrt{2} \omega}{3(1 - \omega^2)} (2\delta_{12} + \delta_{23}), \]
\[ \tan 2\theta'_{23} \simeq \frac{\sqrt{6} \eta}{3(1 - \eta^2)} \delta_{23}, \]
\[ \tan 2\theta'_{13} \simeq \frac{-\omega \eta}{\sqrt{3}(1 - \omega \eta)} \delta_{23}. \quad (14) \]
Combining Eq. (13) with Eq. (14), we obtain

\[ \theta_{12} \simeq \theta_{12}^0 + \frac{\sqrt{2} \omega}{6 (1 - \omega^2)} (2\delta_{12} + \delta_{23}) , \]

\[ \theta_{23} \simeq \theta_{23}^0 + \frac{\eta (2 + \omega)}{3} \delta_{23} , \]

\[ \theta_{13} \simeq \frac{\eta (1 - \omega)}{3\sqrt{2}} \delta_{23} . \]  

This approximate result indicates that the mass splitting \( \delta_{23} \) is responsible for both the non-vanishing \( \theta_{13} \) and the slight deviation of \( \theta_{23} \) from its original value \( \theta_{23}^0 = 45^\circ \). In comparison, the small deviation of \( \theta_{12} \) from its initial value \( \theta_{12}^0 \approx 35.3^\circ \) depends on both \( \delta_{12} \) and \( \delta_{23} \). In the limits \( \delta_{12} \to 0 \) and \( \delta_{23} \to 0 \), \( V \to V_0 \) is immediately restored.

Scenario A is of course regarded as the special case of scenario B in the limit \( \delta_{23} \to 0 \). The latter totally involves six independent parameters: three eigenvalues of \( M_D \) and three eigenvalues of \( M_R \); or equivalently, \( z, \omega, \eta, M_1, \delta_{12} \) and \( \delta_{23} \). Correspondingly, there are six observable parameters: three neutrino masses and three lepton mixing angles. Our present knowledge on the neutrino mass spectrum and the flavor mixing pattern is inadequate to fully determine the free parameters of our seesaw model, but some useful constraints on its parameter space ought to be obtainable.

To the leading order in our above calculation, two neutrino mass-squared differences are given by

\[ \Delta m_{21}^2 \equiv m_2^2 - m_1^2 \simeq \frac{z^4 \eta^4}{M_1^2} (1 - \omega^4) , \]

\[ \Delta m_{32}^2 \equiv m_3^2 - m_2^2 \simeq \frac{z^4}{M_1^2} (1 - \eta^4) . \]  

The solar and atmospheric neutrino oscillation data yield \( \Delta m_{21}^2 = (7.2 \cdots 8.9) \times 10^{-5} \text{ eV}^2 \) and \( |\Delta m_{32}^2| = (1.7 \cdots 3.3) \times 10^{-3} \text{ eV}^2 \) at the 99% confidence level [5]. Then we arrive at \( 0 < \omega < 1 \), but both \( 0 < \eta < 1 \) and \( \eta > 1 \) are allowed. If \( \eta > 1 \) is assumed, \( \omega \) must be close to 1 in order to assure \( \Delta m_{21}^2 / |\Delta m_{32}^2| \sim O(10^{-2}) \) to hold. Note that \( \theta_{12}^0 \) serves as a perturbation to \( \theta_{12}^0 \) in our model, hence its reasonable magnitude is \( \leq 5^\circ \). Note also that \( \omega \sim 1 \) is undesirable, because it may lead to a rather large value of \( \theta_{12}^0 \) unless \( \delta_{12} \) and \( \delta_{23} \) are negligibly small. These arguments motivate us to choose \( 0 < \eta < 1 \) instead of \( \eta > 1 \); in other words, the light neutrino mass spectrum favors a normal hierarchy in our scenario.

In the standard parametrization of \( V \), a global analysis of current neutrino oscillation data yields \( 30^\circ \leq \theta_{12} \leq 38^\circ, 36^\circ \leq \theta_{23} \leq 54^\circ \) and \( \theta_{13} \leq 10^\circ \) [5] at the 99% confidence level. One can see that the tri-bimaximal neutrino mixing pattern \( V_0 \) is actually in good agreement with these data. Thus the perturbations \( \theta_{12}^0, \theta_{23}^0 \) and \( \theta_{13}^0 \) in scenario B of our seesaw model should be well constrained.

Now we proceed to scan the whole parameter space relevant for two neutrino mass-squared differences and three neutrino mixing angles by using Eqs. (11), (12) and (13). Our numerical results are shown in Figs. 1 and 2. Some comments are in order.
(1) The mass-squared differences $\Delta m^2_{21}$ and $\Delta m^2_{32}$ are mainly determined by the parameters $\omega, \eta$ and $z^2/M_1$. Fig. 1 shows that the allowed ranges of $\eta$ and $z^2/M_1$ are very restrictive. In contrast, $\omega$ is essentially unrestricted, although the region of $\omega \to 1$ is apparently disfavored. Note that $m_3 \approx z^2/M_1 \approx 0.05$ eV holds in the leading-order approximation, as a straightforward consequence of the normal neutrino mass hierarchy. Typically taking $\omega \approx 0.5$ and $\eta \approx 0.45$, we obtain $m_2 \approx 0.01$ eV and $m_1 \approx 0.0025$ eV. Given $z \sim 175$ GeV, a mass scale close to the electroweak scale or the top-quark mass, it turns out that $M_1 \sim 6 \times 10^{14}$ GeV, which is just the typical seesaw scale.

(2) Because both $\delta_{12}$ and $\delta_{23}$ serve as small perturbations, we have taken $|\delta_{12}| \leq 0.2$ and $|\delta_{23}| \leq 0.2$ in our numerical calculations. The signs of these two parameters are crucial to control the departures of $\theta_{12}$ and $\theta_{23}$ from $\theta^0_{12}$ and $\theta^0_{23}$. Note that $\theta_{13}$ has been arranged to lie in the first quadrant even for $\delta_{23} < 0$, as one can see in Fig. 2. It is likely to obtain $\theta_{13} \geq 1^\circ$, provided $\delta_{23} \geq 0.1$ is taken. Note also that a small departure of $\theta_{13}$ from $\theta^0_{13}$ can be achieved from non-vanishing $\delta_{23}$ in the limit $\theta_{12} \to 0$, as shown in Eq. (15). In other words, the mass splitting $\delta_{23}$ can simultaneously affect $\theta_{12}, \theta_{23}$ and $\theta_{13}$, while the mass splitting $\delta_{12}$ mainly affects $\theta_{12}$ in our seesaw model.

We remark that the uncertainties of the values on $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$ are expected to reduce to about 20% at $3\sigma$ in future experiments [17], which would constrain our parameters $\delta_{23}$ and $\delta_{13}$ further up to $13 \sim 18\%$. As for the mixing angle $\theta_{13}$, we point out that the reasonable magnitude of $\theta_{13}$ in this seesaw scenario ($\theta_{13} \simeq 2^\circ$ for $|\delta_{23}| \leq 0.2$) remains too small to be measured in the near future. Indeed, the sensitivity of a few currently-proposed reactor neutrino oscillation experiments to $\theta_{13}$ is at the level of $\theta_{13} \sim 3^\circ$ or $\sin^2 2\theta_{13} \sim 0.01 [18]$. In contrast, our scenario allows the mixing angles $\theta_{12}$ and $\theta_{23}$ to deviate from their original (tri-bimaximal) values up to $5^\circ$ for $|\delta_{12}| \sim |\delta_{23}| \sim 0.2$. Such remarkable deviations are expected to be observable in the upcoming long-baseline neutrino oscillation experiments [19]. Thus it is possible to test the validity of our seesaw model even before the neutrino factory era.

4 We have proposed a very simple seesaw model based on two phenomenological assumptions: (a) the masses of three heavy Majorana neutrinos are exactly degenerate; and (b) three light Majorana neutrinos have the tri-bimaximal mixing pattern $V_0$ in this limit. A small mass splitting between the first generation and the other two generations of heavy Majorana neutrinos is found to be responsible for the departure of the solar neutrino mixing angle $\theta_{12}$ from its initial value set by $V_0$. It is also shown that a small mass splitting between the second and third generations of heavy Majorana neutrinos gives rise to a non-vanishing mixing angle $\theta_{13}$ and a slight departure of the atmospheric neutrino mixing angle $\theta_{23}$ from $45^\circ$. In this seesaw scenario, the mass spectrum of three light Majorana neutrinos has a normal hierarchy, leading to a tiny effective mass of the neutrinoless double-beta decay (of $O(10^{-3})$ eV or smaller).

It is worth remarking that radiative corrections to three neutrino mixing angles, which can be evaluated by using the renormalization-group equations from the seesaw scale to the electroweak scale, are expected to be very small in our model. On the one hand, the small mass splitting between any two generations of three heavy Majorana neutrinos implies that the seesaw threshold effects [20] are negligible. On the other hand, the normal mass
hierarchy of three light Majorana neutrinos assures that the renormalization-group effect on three mixing angles is insignificant when they run down to the electroweak scale. These arguments are valid for both the tri-bimaximal neutrino mixing pattern $V_0$ and its modified form $V$ in the framework of the standard model or its minimal supersymmetric extension, unless $\tan \beta$ is extremely large [21].

Note that CP violation is not taken into account in our work. It is well known that the exact mass degeneracy of three heavy Majorana neutrinos forbids CP violation in their lepton-number-violating decays [22], hence there is no thermal leptogenesis [23] in this case. If that mass degeneracy is lifted and non-trivial CP-violating phases are introduced into $M_D$ and (or) $M_R$ [24], then it is possible to achieve successful leptogenesis at the seesaw scale. It is in turn likely to generate large CP violation in the neutrino mixing matrix $V$ via the seesaw relation. While such ideas are certainly interesting, they should be realized in a simple and suggestive way. We shall explore the possibilities to combine our present seesaw model with CP violation elsewhere.

Finally, we emphasize that the tri-bimaximal neutrino mixing pattern is just a typical example in our seesaw scenario. One may consider to embed other interesting (constant) patterns of lepton flavor mixing, such as those proposed in Ref. [25], into our model in a similar way. In this sense, our phenomenological scenario does provide a simple, useful and flexible framework to understand the observed features of lepton flavor mixing. It might also help provide some enlightening hints towards further model building, in order to obtain a dynamical picture of neutrino mass generation and leptonic CP violation.

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FIG. 1. Allowed parameter space of ($\omega$, $\eta$), ($z^2/M_1$, $\Delta m^2_{31}$) and ($z^2/M_1$, $\Delta m^2_{32}$).
FIG. 2. Allowed parameter space of $(\theta_{12}, \delta_{12}), (\theta_{23}, \delta_{23})$ and $(\theta_{13}, \delta_{23})$. 