Predicting Shear Fracture in Deep Drawing: Combined Yld2000-3D and Hosford-Coulomb Fracture Model

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Abstract. The proper numerical treatment of strain and stress distributions are an indispensable prerequisite of predicting the fracture initiation in deep drawing operations. The formability of sheet metal is generally limited by the so-called Forming Limit Curve (FLC) that is broadly accepted in forming community. However, fracture actually occurs at higher strains than the membrane instability predicted by the FLC. In the present paper, the results of the well-known Nakazima experiments (which are traditionally used to determine a FLC) are explored beyond the necking limit to determine the multi-axial fracture response. Indeed, the strains at the fracture initiation have been determined using a hybrid experimental-numerical approach, i.e. through the numerical simulation of each experiment. To increase the robustness of the hybrid experimental-numerical technique, we also measured the post-mortem thickness of all failed specimens. Moreover, in view of characterizing the local plasticity, the non-quadratic full stress constitutive model along a recently Hosford-Coulomb fracture criterion are employed. The candidate tests to determine the fracture parameters are the Nakazima test and a special configuration of the cup drawing test, which fails under the out-of-plane shear loading range. The obtained complex constitute model are then applied to predict the fracture in a new designed triangle shape part for deep drawing. This complex new triangular deep drawing shape is fabricated to validate the constitutive model and predict a crack initiation.

1. Introduction
The quest of reliable models for predicting ductile failure is constantly growing due to the never ending minimization of structural weight. The Forming Limit Curve (FLC), initially conducted by Keeler and Goodwin [1-2], is the default engineering tool to predict necking in deep drawing simulations. Ductile fracture usually occurs after necking, but the introduction of advanced high strength steels also generated numerous examples where ductile fracture within the “safe domain” defined by the forming limit curve. Min et al. proposed the new approach to resolve the difference of FLC measured by two well-known experimental methods Nakazima and Marciniak. Their method takes into account the effect of a curvature and a non-linear strain path that happens during the deformation, by means of optical measurement system [3]. The classical FLC has been extended from the localization determination to fracture characterization in principal strain space by introducing the new experimental approach so-called Localized Level Forming Limit Diagram (LL-FLD); calculating the elastic constraints by a Digital Image Correlation (DIC) observations and considering the local strain rate in the remaining plastic regions [4]. Furthermore, to achieve predicting fracture limit, the study of Wierzbicki and his co-researchers has significantly contributed to develop ductile fracture
theory in three-dimensional equivalent strain and deviatoric state variables [5-7]. Those works have been continued with the new phenomenological approach so-called Hosford-Coulomb [8] for prediction of ductile fracture at low stress triaxialities. This method improved the agreement between the model and experimental out-puts by combining the Hosford equivalent stress and normal stress. Despite of showing promising agreement of this model under different loading scenarios, this model has never been questioned in deep drawing applications. The present conference proceedings paper is an abridged version of [17]. In the framework of this research, we challenged this fracture model to predict the crack initiation in complex geometrical con-figurations. Aside from characterizing the fracture response within materials, it has been concluded that only non-quadratic yield functions are suitable to represent plastic behaviour of metals having anisotropy coefficients less than unity, such as aluminium alloys [9-10]. More advanced model presented afterwards to more accurately characterize the plastic behaviour [11-12]. All of these functions as well as a well-established Yld2000-2d yield locus [13], are presented under the plane stress condition that can be just considered for shell elements in finite element simulations. The Yld2000-2d yield criterion has later been modified by the same authors in order to take into account the three-dimensional full stress state yield criterion [14]. This improvement has been accomplished by applying the associated linear transformation on the stress deviator with totally eighteen coefficients. In the way of remedying the deficiencies of large amount of experiments, Dunand et al. introduced the new model motivated by the needs of 3D solid elements applications [15]. They proposed four additional variables than eight Yld2000-2d coefficients; converts the plane stress yield function to the three-dimensional stress states one. This research extends the previous work of the authors by introducing the numerical model capable of predicting the fracture in deep drawing applications with the non-quadratic full stress constitutive model along a recently Hosford-Coulomb fracture criterion. The candidate tests to determine the fracture parameters are the well-known Nakazima test and a special configuration of the cup drawing test, which fails under the out-of-plane shear loading range. The obtained complex constitute model were then validated for shear-dominated zones to predict the fracture in a new designed triangle shape part for deep drawing [16]. This complex new triangular deep drawing shape is fabricated to validate the constitutive model and predict a crack initiation.

2. Experimental procedures
All specimens are extracted from 1.0mm thick AA6016-T4 sheets using wire EDM cutting. The performed experiments are as following:

Uniaxial tension: The basic strain-stress response of the material is determined through standard uniaxial tension experiments. The specimen gage section is 20mm wide and 80mm long; the tensile axis is aligned with either the Rolling (RD), the Diagonal (DD) or Transverse Direction (TD). Two extensometers are employed to measure the average axial strain and the average width strain at the centre of the specimen; all experiments are done under displacement-control with a constant axial engineering strain rate of 0.002/s to 0.06/s (machine maximum). The measured force-displacement and consequently engineering stress-strain curve show a negligible effect of strain rate on the AA6016-T4 response for the range of strain rates of interest in this study.

Hydraulic bulge: A hydraulic bulge test with an average strain rate of about 0.02/s is performed on a bulge testing system to identify the stress-strain curve for equi-biaxial tension. A 200x200mm sheet specimen is clamped onto a circular 103.2mm diameter die using a mechanical blank holder. A uniform pressure is applied onto the lower specimen surface using hydraulic oil. The displacement field of the speckle-painted upper surface is measured through the DIC. The surface strain and curvature fields are then computed by DIC software ARAMIS-3D. Under the assumption of small ratio between a current thickness \( t \) of a work-piece and local curvature radius \( \rho \), the principal stresses can be estimated according to the curvature and hydraulic oil pressure \( P \) by using the vessel formula \( \sigma_{1} = \sigma_{2} = P\rho/2t \). Obtained stresses are then used to calculate the plastic work \( \Omega(t) = \)
The post-ability of the experiments; the experiments are performed with two-

s measured using

A cup drawing test is used to characterize the out-

II) Bottom crack: a crack occurs on the side

the purpose of this test is validating the computational models at the

Nakazima fracture: Strip-like Nakazima specimens are clamped onto a 102.2mm diameter circular die with a 5.5mm edge radius and subjected to out-of-plane loading using a 100mm diameter hemispherical punch. By varying the width of the specimen gage section, stress states ranging from uniaxial to equi-biaxial tension are achieved in the Nakazima experiments. A sandwich layer of two Teflon-like foils and three layers of grease is used to minimize the friction between the die and the sheet. The experiments are carried out at a constant punch velocity of 1mm/s (with an average strain rate of about 0.02/s). The Forming Limit Curve (FLC) is determined using the so-called Cross Section Method (CSM). The strains at the instant of fracture initiation are therefore determined using a hybrid experimental-numerical approach. To increase the robustness of the hybrid experimental-numerical technique, we also measured the post-mortem thickness of all failed specimens. Each specimen is cut in the transverse direction along a straight line that goes through the point where the crack initiated. They are then embedded into an epoxy resin, ground and polished. The thickness is measured using optical microscopy.

Cup drawing: A cup drawing test is used to characterize the out-of-plane shear fracture; using a cylindrical punch with a 75mm diameter and 5mm edge radius as well as a 77mm diameter die with 3mm edge radius. A double-sided greased blank is drawn at a velocity of 1mm/s until rupture occurs (with an average strain rate of about 0.025/s). Depending on the blank’s layout and blank holder force, three different scenarios can be obtained: I) No failure. II) Bottom crack: a crack occurs on the side wall, near the punch-side of the work-piece. III) Shear crack: it occurs at the die profile, when the material is traveling over the corner of the die. Here, we chose a blank holder force of 200kN and squared-blank size 140 x 140mm to achieve failure through out-of-plane shear cracking.

Triangular shape deep drawing: the purpose of this test is validating the computational models at the structural level. A deep drawing die-punch-binder system is employed to form a prismatic cup of triangular shape from the flat 6016-T4 aluminium sheets (the geometry of the forming tool set are shown in [16]). The particular convex-concave form of the stamping tool and three rounded corners with different radii results in large strains, especially in the vertical walls of the deforming work-piece. The blank holder is subjected to a total force of 400kN and an average strain rate of about 0.5/s, pushing the work-piece against the die. Note that all specimens are raster laser-printed for the post-experimental evaluation with the DIC system ARGUS.

3. Material model

3.1. Constitutive equations
The isotropic hardening of the material is described through the Swift-Voce law [18-19] which expresses the evolution of the deformation resistance as a function of the work-conjugate anisotropic equivalent plastic strain $\dot{\varepsilon}_p$. 

$$\int_0^t (\sigma_1 \dot{\varepsilon}_p^1 + \sigma_2 \dot{\varepsilon}_p^2) dt : \text{plastic strains } \dot{\varepsilon}_p^1 \text{ and } \dot{\varepsilon}_p^2 \text{ were obtained from the optical measurement}. \text{ After evaluating the anisotropic equivalent stress } \bar{\sigma} \text{ (explained in section 3), the equivalent plastic strain in bulge test is computed throughout the work-conjugacy } (d\dot{\varepsilon}_p = d\Omega/\bar{\sigma}). \text{ With this approach, the hardening curve is obtained from hydraulic bulge test (see transferred flow curve in Fig. 1a).} \text{ Stack compression: The biaxial r-value } (r_b) \text{ is determined from stack compression experiments. Nine circular discs of an initial diameter of 10mm are stacked on top of each other to form of a cylindrical compression specimen. A layer of grease and thin Teflon is applied to the top and bottom surfaces of the stacks to ensure the reliability of the experiments; the experiments are performed with two different cross-head speeds (0.01 and 0.02mm/s) and up to global strains of 0.2 and 0.3 (the average strain rate is in the range of 0.003/s to 0.01/s). The biaxial Lankford ratio } r_b \text{ is then defined by the ratio of the strain in the transverse direction over the strain in the rolling direction.} \text{ Nakazima fracture: Strip-like Nakazima specimens are clamped onto a 102.2mm diameter circular die with a 5.5mm edge radius and subjected to out-of-plane loading using a 100mm diameter hemispherical punch. By varying the width of the specimen gage section, stress states ranging from uniaxial to equi-biaxial tension are achieved in the Nakazima experiments. A sandwich layer of two Teflon-like foils and three layers of grease is used to minimize the friction between the die and the sheet. The experiments are carried out at a constant punch velocity of 1mm/s (with an average strain rate of about 0.02/s). The Forming Limit Curve (FLC) is determined using the so-called Cross Section Method (CSM). The strains at the instant of fracture initiation are therefore determined using a hybrid experimental-numerical approach. To increase the robustness of the hybrid experimental-numerical technique, we also measured the post-mortem thickness of all failed specimens. Each specimen is cut in the transverse direction along a straight line that goes through the point where the crack initiated. They are then embedded into an epoxy resin, ground and polished. The thickness is measured using optical microscopy.} \text{ Cup drawing: A cup drawing test is used to characterize the out-of-plane shear fracture; using a cylindrical punch with a 75mm diameter and 5mm edge radius as well as a 77mm diameter die with 3mm edge radius. A double-sided greased blank is drawn at a velocity of 1mm/s until rupture occurs (with an average strain rate of about 0.025/s). Depending on the blank’s layout and blank holder force, three different scenarios can be obtained: I) No failure. II) Bottom crack: a crack occurs on the side wall, near the punch-side of the work-piece. III) Shear crack: it occurs at the die profile, when the material is traveling over the corner of the die. Here, we chose a blank holder force of 200kN and squared-blank size 140 x 140mm to achieve failure through out-of-plane shear cracking.} \text{ Triangular shape deep drawing: the purpose of this test is validating the computational models at the structural level. A deep drawing die-punch-binder system is employed to form a prismatic cup of triangular shape from the flat 6016-T4 aluminium sheets (the geometry of the forming tool set are shown in [16]). The particular convex-concave form of the stamping tool and three rounded corners with different radii results in large strains, especially in the vertical walls of the deforming work-piece. The blank holder is subjected to a total force of 400kN and an average strain rate of about 0.5/s, pushing the work-piece against the die. Note that all specimens are raster laser-printed for the post-experimental evaluation with the DIC system ARGUS.} \text{ 3. Material model} \text{ 3.1. Constitutive equations} \text{ The isotropic hardening of the material is described through the Swift-Voce law [18-19] which expresses the evolution of the deformation resistance as a function of the work-conjugate anisotropic equivalent plastic strain } \dot{\varepsilon}_p,
\[ k_f \varepsilon_p = \alpha A \left( \varepsilon_p + \varepsilon_{0f} \right)^n + (1 - \alpha) \left( \varepsilon_p + q \left( 1 - e^{-\beta \varepsilon_p} \right) \right) \]

(1)

where \( \{A, n, \varepsilon_{0f}\} \) and \( \{k_v, Q, \beta\} \) denote the Swift and the Voce parameters, respectively. “\( \alpha \)” is the weighting factor of these functions.

Assuming associated plastic flow, the effect of anisotropy is entirely incorporated into the definition of the anisotropic equivalent stress. The 3D extension of the Yld2000-2d model [13, 15] is used to model the sheet material response with solid elements. The Yld2000-3D function has been constructed such that (1) it features the same parameters as the Yld2000-2D, and (2) it collapses to the Yld2000-2d model for plane stress conditions. This Yld2000-3D anisotropic equivalent stress is defined as

\[ \tilde{\sigma} = \frac{1}{2^{1/m}} \left( \phi \left[ X' \right] + \phi'' \left[ X'' \right] \right)^{1/m} \]

(2)

with the exponent \( m \) related to crystallographic structure of the material, and the transformed deviatoric stress vectors \( X' = \{X'_{11}, X'_{22}, X'_{33}, X'_{12}, X'_{23}, X'_{31}\} \) and \( X'' = \{X''_{11}, X''_{22}, X''_{33}, X''_{12}, X''_{23}, X''_{31}\} \), see [15] for details.

3.2. Fracture initiation model

The micromechanically-motivated Hosford-Coulomb failure criterion is employed to predict the ductile fracture initiation. The stress state has been characterized by the stress triaxiality \( \eta \) (\( \eta = \sigma_m / \tilde{\sigma} \)) and the Lode angle parameter \( \bar{\theta} \) (\( \bar{\theta} = 1 - \frac{2}{\pi} \arccos \left( \frac{2J_3 - J_2}{(J_2)^{3/2}} \right) \); \( J_2 \) and \( J_3 \) are stress deviator tensor invariants). For proportional loading paths, the equivalent plastic strain at the instant of fracture initiation is:

\[ \varepsilon_p \left[ \eta, \bar{\theta} \right] = b(1 + c) \left( \frac{1}{2} (f_1 - f_2)^2 + \frac{1}{2} (f_2 - f_3)^2 + \frac{1}{2} (f_3 - f_1)^2 \right)^{1/2} + c(2\eta + f_1 + f_3) \]

(3)

with the transformation exponent \( n=0.1 \) and the Lode angle parameter-dependent trigonometric functions

\[ f_1 \left[ \bar{\theta} \right] = \frac{2}{3} \cos \left( \frac{\pi}{6} (1 - \bar{\theta}) \right) ; \quad f_2 \left[ \bar{\theta} \right] = \frac{2}{3} \cos \left( \frac{\pi}{6} (3 + \bar{\theta}) \right) ; \quad f_3 \left[ \bar{\theta} \right] = -\frac{2}{3} \cos \left( \frac{\pi}{6} (1 + \bar{\theta}) \right) \]

(4)

The model is embedded into a damage indicator framework to predict ductile fracture initiation after non-proportional loading. Assuming the initial condition and the damage indicator evolution law is \( (dD = d\varepsilon_p/\varepsilon_f^p(\eta, \theta) \). Fracture is assumed to initiate when \( D=1 \).

4. Results

4.1. Experimental results from plasticity experiments

Experimental results showed that the stress-strain responses of the diagonal (DD) and transverse (TD) specimens are similar; they exhibit a maximum engineering stress of 235MPa, while the maximum stress for rolling (RD) reaches 242MPa. The comparison of the stress-strain curves for different material orientations reveals about 5% higher stresses for the RD (as compared to the TD and DD orientations). The true stress-strain curve of the material is then calculated by using standard formulas. The obtained hardening curve for the AA6016-T4 material is shown in Fig. 1a. Necking occurs at an axial plastic strain of about 0.2 and a true stress of 295MPa. The average Lankford ratios between 2% and 20% strain are \( r_{DD}=0.69 \), \( r_{TD}=0.50 \), and \( r_{RD}=0.67 \). The equi-biaxial r-value obtained from the stack compression test is found to be equal to unity. Figure 1 compares the hardening behaviour of the AA6016-T4 from the hydraulic bulge and the uniaxial tensile experiments; the equivalent strain in the bulge test reaches 0.6 and lies on top of the curve obtained from uniaxial tension, which are only valid up to a strain of about 0.2.
4.2. Experimental results from Nakazima experiments

Figure 2a shows the measured surface strain histories for the seven Nakazima experiments. They are represented in terms of the minor- and major strains on the specimen surface. In all experiments, we observe a slight biaxial loading phase at the beginning. Subsequently, all surface strain paths are approximately linear before drifting towards a plane strain path shortly before fracture initiation. Note that this drift is the characteristic signature of localized necking prior to fracture initiation. The FLCs obtained from applying the cross-section method is also shown in Fig. 2a; the lowest major strain is observed for plane strain loading.

Figure 2b shows selected sequences of the surface strain field for the Nakazima experiments with width 20mm (corresponding to nearly uniaxial tension), width 100mm (corresponding to transverse plane strain tension), and width 200mm (corresponding to equi-biaxial tension). In all experiments, the strain fields remain approximately symmetric with respect to the principal specimen axes until the instant of fracture initiation. The solid green lines depict the major strain along the centre line for the instant where the CSM criterion is met.

4.3. Experimental results from deep drawing experiments

The squared-layout size $140 \times 140$mm of AA6016-T4 sheet sample encountered by a $200kN$ binder force tends to shear failure for all levels of the applied binder forces ($20kN \sim 300kN$). Figure 3 represents the strain distribution in $\varepsilon_1$-$\varepsilon_2$ space as well as the mapping of the major strain on the inner layer of the work-piece. Most strains are located between the uniaxial and the shear loading directions. The maximum principal strain in corner "A" increased to 0.7 ($\varepsilon_{1\text{max}}$ at drawing 35mm is 0.6). Finally, a crack initiates on the wall of a sample, slightly above the bended curvature and between 40 and 45 mm drawing depth. The experiments are carried out up to three distinct drawing depths: 35, 40 and 45 mm. The experimental observations demonstrate that the work piece is still crack free at a punch.
displacement of 40\,mm, while the 45\,mm configuration shows a crack near the die radius at corner A (see Fig. 3).

![Image of triangular-shape deep, representing two critical corners.]

**Figure 3.** Strain distribution of triangular-shape deep, representing two critical corners.

4.4. **Plasticity model**

The eight Yld2000 coefficients ($\alpha_1$-$\alpha_8$) are determined using the Lankford coefficients and yield stresses as determined from the uniaxial tension and bulge experiments (see Table 1. In addition, comparison of the yield loci between von-Mises and anisotropic Yld2000 yield criteria is sketched in Fig. 1b). After identifying the Yld2000 coefficients, the equivalent stress versus equivalent strain curve is extracted from the bulge experiment. The corresponding Swift-Voce fit is shown as a red curve in Fig. 1a. A quick check demonstrates that the identified hardening curve and its derivative cross each other at the end of the uniform elongation in the uniaxial tensile test. The final set of the Swift-Voce parameters are given in Table 2.

**Table 1.** Yld2000-2d parameters of the AA6016-T4 sheet sample.

| $\alpha_1$ [-] | $\alpha_2$ [-] | $\alpha_3$ [-] | $\alpha_4$ [-] | $\alpha_5$ [-] | $\alpha_6$ [-] | $\alpha_7$ [-] | $\alpha_8$ [-] |
|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.947         | 1.017          | 0.961          | 1.032          | 1.021          | 1.013          | 0.967          | 1.153          |

**Table 2.** Hardening parameters of the AA6016-T4 sheet sample.

| ratio | Swift parameters | Voce parameters |
|-------|------------------|-----------------|
| $\alpha$ | $A$ [MPa] | $n$ [-] | $\varepsilon_0$ [-] | $k_0$ [MPa] | $Q$ [MPa] | $\beta$ [-] |
| 0.739 | 286.15          | 0.229          | 0.0161          | 160.1         | 464.50       | 9.89           |

4.5. **Loading paths to fracture**

All Nakazima experiments are simulated using explicit solver of LS-DYNA with six solid elements through the thickness with 0.5\,mm element size. The numerical simulation is stopped as soon as the critical zone of each Nakazima sample reaches the experimental fracture thickness, measured by microscopy (see Table 2). Figure 4 shows a comparison of the surface strain paths as obtained from the simulations with the experimental results. Different from the von Mises model, the simulations with the anisotropic Yld2000 function are capable to provide reasonable predictions of the ratio of major to minor strain on the specimen surface. The end of the dashed curves (simulations) and solid curves (experiments) does not coincide. This is partly attributed to the high sensitivity of the simulation results to changes in the large strain hardening behaviour and the limited local accuracy of the strain measurements on the highly curved specimen surface within the localized necks. Stopping the simulation when the specimen thickness equals the experimentally-measured fracture thickness is expected to add the robustness of the identified fracture strains. Table 3 lists the calculated fracture
strain. The strain path obtained from the cup drawing simulation is also included in Fig. 4. Here, the simulation is stopped at the corresponding experimental fracture drawing depth (27.4mm). At this instant, the sheet thickness in the simulation and experiment are also equal (0.77mm). The surface strains accumulate linearly in shear loading range, drifting towards a plane strain path after the small compression loading. In the equivalent strain versus stress triaxiality plane, the loading path to fracture for cup drawing exhibits a stress triaxiality close to zero up to an equivalent plastic strain of about 0.2, before the stress triaxiality increases to 0.22. Fracture occurs at an equivalent plastic strain of 0.7, i.e. the strain to fracture for shear-dominant loading is higher than other loading scenarios. The loading paths to fracture, i.e. the evolution of the stress triaxiality as a function of the equivalent plastic strain, are shown in Fig. 4c. The most highly strained integration point has been chosen for extracting the loading path to fracture. The forming limit curve is shown as dashed line in these figures. Once the loading paths have crossed the FLC, the loading paths are inclined in a way that they approach the stress state of plane strain tension, i.e. a stress triaxiality of about 0.58 and a Lode angle parameter of zero. The highest equivalent plastic strain of about 0.60 (at the assumed instant of fracture initiation) is observed for uniaxial tension and equi-biaxial tension, while the lowest of 0.40 is observed for plane strain tension.

Table 3. Fracture thickness and fracture strain obtained by Yld2000-3D of the Nakazima and cup specimens.

| Width [mm] | CupDD 20 | 50 | 80 | 90 | 100 | 120 | 200 |
|------------|----------|----|----|----|-----|-----|-----|
| Thickness [mm] | 0.77 | 0.635 | 0.679 | 0.636 | 0.652 | 0.634 | 0.598 | 0.536 |
| Major fracture strain $\varepsilon^f_1$ [-] | 0.61 | 0.60 | 0.41 | 0.34 | 0.37 | 0.34 | 0.41 | 0.34 |
| Minor fracture strain $\varepsilon^f_2$ [-] | -0.35 | -0.15 | -0.05 | 0.00 | 0.01 | 0.02 | 0.07 | 0.27 |

Figure 4. Comparison of experimental observations and numerical responses, (a) J2 plasticity, and (b) Yld2000-3D. (c) Hosford-Coulomb fracture model: plane stress FLC as a function of the triaxiality.

4.6. Identified fracture model parameters
Based on the loading paths to fracture from the Nakazima and cup drawing experiments, the Hosford-Coulomb fracture model parameters \{a,b,c\} are identified. Due to the non-linearity of the loading paths to fracture, an inverse identification procedure is applied. The solid dots shown in Fig. 4c represent prediction of instants of fracture initiation of the identified fracture initiation model for the respective loading paths. Even though the Hosford-Coulomb model features only three free parameters, the maximum error in the equivalent plastic strain to fracture (observed for the Nakazima experiment with $w=100mm$) is smaller than 5%. As for other aluminium alloys (e.g. [20]), we obtain a low a-parameter in comparison to advanced high strength steels [8] which indicates a high Lode angle.
sensitivity of the fracture response of the aluminium 6016 alloy. The underlying Hosford-Coulomb fracture locus for proportional loading is shown in Fig. 5 in the space of fracture strain, stress triaxiality and Lode angle parameter. The black line highlights the fracture limit for plane stress conditions which includes the special stress states of (1) pure shear, (2) uniaxial tension, (3) plane strain tension, and (4) equi-biaxial tension.

Figure 5. (a) 3D fracture locus for proportional loading as a function of the stress states, (b) Hosford–Coulomb fracture limit for plane stress loading.

4.7. Validation of the fracture model at the structural level
The triangular shape deep drawing experiment is also simulated by the explicit non-linear FE-package LS-DYNA in conjunction with the Yld2000-3D. The first-ordered reduced integration solid elements with an average mesh size of 1mm and with six solid elements along the thickness-direction are used to discretize the work-piece; total element number is over 400,000. Tools (die, punch, and blank-holder) are modelled as rigid bodies and punch stroke is defined as 75mm/s (like experimental case). Figures. 6a to 6d show selected snapshots of the equivalent plastic strain and damage contour in two critical zones in drawing depth \( H = 40 \text{mm} \) (slightly before the crack initiation). The red fringes show higher value while the blue ones show zero amount of the strain or the damage. Comparing the two regions of the deep drawing illustrates that equivalent strain in “corner B” is more critical than “corner A”, whereas accumulation of the damage in “corner A” is larger i.e., crack initiates earlier. Ultimately and as shown in Fig. 6e, the failed simulated stamped sample is located in “corner A”, which has an agreement with the experimental observation. The shear crack is initiated at one corner, near the die radius and on the wall where the sheet between the binder and the die is held back due to the highest contact pressure.

Figure 6. Numerical analysis of triangular shape deep drawing; comparing (a) damage and plastic strain in two critical corners at drawing depth \( H = 40 \text{mm} \) (depth about to fracture), (b) exemplarily experiment work-piece versus the numerical prediction.
5. Conclusions
In the present work, a new methodology is proposed to extract the fracture properties from forming experiments. In other words, the same set of experiments is used to determine both the forming limit diagram and the parameters of the stress-state dependent Hosford-Coulomb model. Nakazima experiments are performed for seven different biaxial loading paths along with a cup drawing experiment. The latter is intentionally designed such that the sheet material fails through out-of-plane shear fracture, thereby providing crucial information on the strain to fracture at nearly zero stress triaxiality. Experiments are performed on specimens extracted from 1mm thick aluminium 6016-T4 sheets. The parameters of the non-quadratic Yld2000 plasticity model are identified based on uniaxial and equi-biaxial experiments. Simulations are performed of all forming experiments to identify the loading paths to fracture in terms of equivalent plastic strain, stress triaxiality and Lode angle parameters. It is shown that the Hosford-Coulomb model can describe the fracture response for eight different non-linear loading histories with good accuracy. Note that this model showed its capability in prediction of additively-manufactured structures [21]. In addition, independent of all calibration experiments, the material model is validated through the accurate prediction of the onset of failure during the deep drawing of a diamond-shaped part. For more details on the work presented in this conference paper, the readers are referred to the corresponding full-length journal paper [17].

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