Pumped shot noise in adiabatically modulated graphene-based double-barrier structures

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Abstract
Quantum pumping processes are accompanied by considerable quantum noise. Based on the scattering approach, we investigated the pumped shot noise properties in adiabatically modulated graphene-based double-barrier structures. It is found that compared with the Poisson processes, the pumped shot noise is dramatically enhanced where the dc pumped current changes flow direction, which demonstrates the effect of the Klein paradox.

1. Introduction
Quantum pumping is a transport mechanism which induces dc charge and spin currents in a nanoscale conductor in the absence of a bias voltage by means of a time-dependent control of some system parameters. Research on quantum pumping has attracted continued interest since its prototypical proposition due to its importance in quantum dynamic theory and potential application in various fields [1–40]. The pumped current (PC) and noise properties in various nanoscale structures were investigated, for example the magnetic-barrier-modulated two-dimensional electron gas [5], mesoscopic one-dimensional wire [7, 23], quantum-dot structures [6, 12, 13, 29, 33, 41], mesoscopic rings with Aharonov–Casher and Aharonov–Bohm effect [8], magnetic tunnel junctions [11], chains of tunnel-coupled metallic islands [26], the nanoscale helical wire [27], the Tomonaga–Luttinger liquid [25], and graphene-based devices [21, 22, 34–40].

Graphene continues to attract intense interest, especially as an electronic system in which charge carriers are Dirac-like particles with linear dispersion and zero rest mass [42]. Quantum pumping properties of graphene-based devices have been investigated by several groups [21, 22, 34–38]. It is found that the direction of the PC can be reversed when a high potential barrier demonstrates stronger transparency than a low one as an effect of the Klein paradox [21]. The shot noise properties of a quantum pump are important in two aspects: understanding the underlying mechanisms of the shot noise may offer possible ways to improve pumping efficiency and achieve optimal pumping. On the other hand, the shot noise reflects current correlation and is sensitive to the pump source configuration [43]. The pumped shot noise (PSN) properties may provide further information of the correlation between Dirac fermions. However, this topic has not ever been looked into. In this work, we focus on the PSN properties in adiabatically modulated graphene-based double-barrier structures using the scattering matrix method. The effect of the Klein paradox on the PSN is illuminated.

2. Theoretical formulation
The crystal structure of undoped graphene layers is that of a honeycomb lattice of covalent-bond carbon atoms. One valence electron corresponds to one carbon atom and the structure is composed of two sublattices, labeled by A and B. In the vicinity of the K point and in the presence of a potential U, the low-energy excitations of the gated graphene monolayer are described by the two-dimensional (2D) Dirac equation

\[ v_F (\boldsymbol{\sigma} \cdot \hat{\mathbf{p}}) \Psi = (E - U) \Psi, \]

where the pseudospin matrix \( \sigma \) has components given by Pauli’s matrices and \( \hat{\mathbf{p}} = (p_x, p_y) \) is the momentum operator. The ‘speed of light’ of the system is \( v_F \), i.e. the Fermi velocity \( (v_F \approx 10^6 \text{ m s}^{-1}) \). The eigenstates of equation (1)
are two-component spinors $\Psi = [\psi_A, \psi_B]^T$, where $\psi_A$ and $\psi_B$ are the envelope functions associated with the probability amplitudes at the respective sublattice sites of the graphene sheet.

In the presence of a 1D confining potential $U = U(x)$, we attempt to find solutions of equation (1) in the form $\psi_A(x, y) = \phi_A(x)e^{i\theta y}$ and $\psi_B(x, y) = \phi_B(x)e^{i\theta y}$ due to the translational invariance along the $y$ direction. The resulting coupled, first-order differential equations read

$$\frac{d\phi_A}{d\xi} + \beta\phi_A = (\varepsilon - u)\phi_A, \quad (2)$$

$$\frac{d\phi_B}{d\xi} - \beta\phi_A = -(\varepsilon - u)\phi_B. \quad (3)$$

Here $\xi = x/L$, $\beta = k_bL$, $u = U_L/h\nu_F$, and $\varepsilon = E_M/h\nu_F$ ($L$ is the width of the structure). The incident angle $\theta$ is given by $\sin(\theta) = \beta/\varepsilon$. We consider a double-barrier structure with two square potentials of height $U_1$ and $U_2$, which can be time-dependent modulated by ac gate voltages (see figure 1). Equations (2) and (3) admit solutions which describe electron states confined across the well and propagating along it. Typical values $L/4$ for the barrier widths and the inter-barrier separation $L/2$ are used. The transmission and reflection amplitudes $t$ and $s$ can be determined by matching $\phi_A$ and $\phi_B$ at region interfaces.

Following the standard scattering approach [3, 4] we introduce the fermionic creation and annihilation operators for the scattering states of the carriers. The operator $\hat{a}_L(E, \theta, t)$ or $\hat{a}_L(E, \theta, t)$ creates or annihilates particles with total energy $E$ and incident angle $\theta$ in the left lead at time $t$, which are incident upon the sample. Analogically, we define the creation $\hat{b}_L(E, \theta, t)$ and annihilation $\hat{b}_L(E, \theta, t)$ operators for the outgoing single-particle states. Considering a particular incident energy $E$ and incident angle $\theta$, the scattering matrix $\hat{s}$ follows from the relation

$$\begin{pmatrix} \hat{b}_L \\ \hat{b}_R \end{pmatrix} = \begin{pmatrix} R & T \\ T^* & R^* \end{pmatrix} \begin{pmatrix} \hat{a}_L \\ \hat{a}_R \end{pmatrix}, \quad (4)$$

where $T$ and $R$ are the scattering elements of incidence from the left reservoir and $T^*$ and $R^*$ are those from the right reservoir.

The frequency of the potential modulation is small compared to the characteristic times for traversal and reflection of electrons. The pump is thus adiabatic. In this case one can employ an instant scattering matrix approach, i.e. $\hat{s}(t)$ depends only parametrically on the time $t$. To realize a quantum pump one varies simultaneously two system parameters, e.g. [3, 4]

$$\begin{align*}
X_1(t) &= X_{10} + X_{01}e^{i(\varphi_1 - \varphi)} + X_{02}e^{-i(\varphi_1 - \varphi)}, \\
X_2(t) &= X_{20} + X_{01}e^{i(\varphi_2 - \varphi)} + X_{02}e^{-i(\varphi_2 - \varphi)}. \quad (5)
\end{align*}$$

Here $X_1$ and $X_2$ are measures for the two-time-dependent barrier heights $U_1$ and $U_2$ (see figure 1), which can be modulated by applying two low-frequency ($\omega$) alternating gate voltages. $X_{01}$ and $X_{02}$ are the corresponding oscillating amplitudes with phases $\varphi_1/2$; $X_{10}$ and $X_{20}$ are the static (equilibrium) components. The scattering matrix $\hat{s}$ being a function of parameters $X(t)$ depends on time.

In adiabatic conditions, the external parameter changes so slowly that up to corrections of order $h\nu_F/\gamma$ ($\gamma$ measures the escape rate) we can apply an instant scattering description using the scattering matrix $\hat{s}(t)$ frozen at some time $t$. Usually time variation of the driving wave is sufficiently smooth on the scale of the dwell time. And we assume that the amplitude $X_{0,j}$ is small enough to keep only the terms linear in $X_{0,j}$ in an expansion of the scattering matrix [4] $\hat{s}(t) \approx \hat{s}^{(0)} + \hat{s}^{(\omega)}e^{i\omega t} + \hat{s}^{(-\omega)}e^{-i\omega t}. \quad (6)$

In the limit of small frequencies the amplitudes $\hat{s}^{(\omega)}$ can be expressed in terms of parametric derivatives of the on-shell scattering matrix $\hat{s}$,

$$\hat{s}^{(\omega)} = \sum_j X_{0,j} e^{i\omega j} \frac{\partial \hat{s}}{\partial X_j}. \quad (7)$$

The expansion, equation (6), is equivalent to the nearest sideband approximation which implies that a scattered electron can absorb or emit only one energy quantum $h\nu_F$ before it leaves the scattering region.

The problem of current noise in a quantum pump is closely connected with the problem of quantization of the charge pumped in one cycle. On the other hand, the noise in mesoscopic phase-coherent conductors is interesting in itself because it is very sensitive to quantum-mechanical interference effects and can give additional information about the scattering matrix [4]. To describe the current–current fluctuations we will use the correlation function [44]

$$S_{\alpha\beta}(t, t') = \frac{1}{\hbar} \langle \Delta I_\alpha(t) \Delta I_\beta(t') + \Delta I_\alpha(t') \Delta I_\beta(t) \rangle, \quad (8)$$

with $\Delta I = \hat{I} - \langle \hat{I} \rangle$ and $\hat{I}_\alpha(t)$ is the quantum-mechanical current operator in the lead $\alpha$ as

$$\hat{I}_\alpha(t) = \frac{e}{\hbar} \langle \hat{b}_L^\dagger(\hat{a}_L^\dagger - \hat{a}_L)(\hat{a}_L^\dagger - \hat{a}_L)(\hat{b}_L^\dagger - \hat{a}_L^\dagger - \hat{a}_L)(\hat{a}_L^\dagger - \hat{a}_L)(\hat{a}_L^\dagger - \hat{a}_L) \rangle. \quad (9)$$

The time-dependent operator is $\hat{a}_L(t) = \int dE \hat{a}_L(E)e^{-iEt}$ and $\hat{b}_L(t) = \sum_{\beta} s_{\alpha\beta}(t) \hat{a}_\beta(t)$ with $s_{\alpha\beta}$ an element of the instant scattering matrix $\hat{s}$. Note that in the case of a time-dependent scatterer the correlation function depends on two times $t$ and $t'$. Here we are interested in the noise averaged over a long time.
time ($\Delta t \gg 2\pi/\omega$) and we investigate

$$S_{\alpha\beta}(t) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt' S_{\alpha\beta}(t, t').$$  \hspace{3cm} (10)

In addition we restrict our consideration to the zero-frequency component of the noise spectra $S_{\alpha\beta} = J \int dt S_{\alpha\beta}(t)$. Substituting the current operator equation (9), and taking into account equations (4) and (6) we can write the time-averaged zero-frequency PSN properties as [4]

$$S_{\alpha\beta} = \frac{\epsilon^2 \omega}{\pi} \left[ \delta_{\alpha\beta} (T_{-\omega,0} + T_{+\omega,0}) - T_{\alpha\beta}^{\text{(corr)}} \right].$$  \hspace{3cm} (11)

with $T_{\omega,0} = \sum_\beta | S_{\omega,\beta} |^2$ and $T_{\alpha\beta}^{\text{(corr)}} = | \sum_\gamma \epsilon_\beta \epsilon_\gamma e^{-i\omega t} |^2 + | \sum_\gamma \epsilon_\beta \epsilon_\gamma e^{i\omega t} |^2$. Equation (11) can be used to investigate the time-averaged zero-frequency PSN properties in different nanoscale adiabatic pumping structures. Detailed derivation is provided in appendix A as a pedagogic introduction.

The PC and PSN could be expressed in terms of the scattering matrix as follows [4, 21]:

$$I_L = \frac{2e\omega}{\pi} \sum_\beta \text{Im} \left( \frac{\partial S_{\beta L}}{\partial \chi_1} \frac{\partial S_{\beta R}}{\partial \chi_2} \right) X_{\omega,1} X_{\omega,2} \sin(\varphi_2 - \varphi_1),$$

$$S_{LL} = \frac{2e^2 \omega}{\pi} \sum_{j_1,j_2} X_{\omega,j_1} X_{\omega,j_2} \cos(\varphi_{j_1} - \varphi_{j_2})$$

$$\times \left[ \frac{\partial S_{1L}}{\partial \chi_{1j_1}} \frac{\partial S_{2L}}{\partial \chi_{2j_2}} - \sum_{\gamma_1,j_2} S_{\gamma_11} L_{\gamma_2} \frac{\partial S_{1L}}{\partial \chi_{1j_1}} \frac{\partial S_{2L}}{\partial \chi_{2j_2}} \right].$$  \hspace{3cm} (12)

Due to current conservation, it can be seen that for a two-lead (left and right) quantum pump (see figure 1), $I_L = I_R$ and $S_{LL} = S_{RR} = -S_{LR} = -S_{RL}$. It is reasonable to consider only $I_L$ and $S_{LL}$. The symbols $I_p$ and $S_p$ are used for the PC $I_p$ and PSN $S_{LL}$, respectively. A convenient measure for the relative noise strength is the Fano factor defined by $F_p = S_p / 2eI_p$, which characterizes the noise with respect to the Poisson processes. The Poissonian shot noise in the configuration of a quantum pump is discussed in appendix B.

3. Numerical results and interpretations

We consider the PSN properties in a graphene-based conductor modulated by two ac gate voltages as sketched in figure 1. In numerical calculations, the parameters are $U_{10} = U_{20} = 100$ meV, $L = 200$ nm, and $U_{180} = U_{20} = 0.01$ meV. The phase difference of the two oscillating gate potentials is $\phi = \varphi_2 - \varphi_1$ in radians.

The PC, PSN, and Fano factor as functions of the incident angle $\theta$ for different Fermi energies are shown in figure 2. Electrons at the Fermi levels of the reservoirs are driven to flow in one direction by modulating the two barriers with a phase lag, which results in a dc PC at zero bias. The direction of the PC can be reversed when a high potential barrier demonstrates stronger transparency than a low one, which results from the Klein paradox [21]. As a result of electron–hole conjugation, transmission above the electron–hole degenerate point (Dirac point) demonstrates a mirror image of that below the Dirac point. The PC can be reversed accordingly for Fermi energies above the Dirac point (see the dotted line of figure 2).

The PSN is nonnegative as it measures the PC–PC correlation flowing in the same direction. It can be seen that the PSN increases when the PC is increased. The Poisson shot noise demonstrates the process governed by uncorrelated electrons and barrier gates without conduction structure (see appendix B). The most significant difference between shot noise of bias transport and that of quantum pumping is the time-reversal symmetry. No time-dependent parameter is involved in bias transport, which secures a time-reversal-symmetric transport. However, time-reversal symmetry is required to be broken for effective quantum pumps within a cyclic period. For the time-reversal-symmetric parameter setup, such as in-phase variation of the parameters and with a phase difference of $\pi$, the PC is exactly zero. The PC-direction-reversal occurs crossing the zero-PC point. At these configurations, the heat current is maximized [4], with evidence of virtual particle transport.

In graphene conductors, quantum states below potential barriers are hole states. Transmission from electron states outside the potential barriers into the hole states inside the potential barriers is characterized by the Klein paradox. For some incident angles and certain potential heights when chirality meets, the potential barrier is transparent. For other situations violating chirality alignment, the potential barrier is
The PC, PSN, and Fano factor as functions of the Fermi energy of the two reservoirs $E$ for the incident angle $\theta = 0.01$ are shown in figure 3. The absolute value of the PC is a maximum at transmission peaks of the two-barrier graphene structure. Around the transmission peaks, the PC reverses direction. In our pumping configuration, $\phi_1 < \phi_2$. The right gate opens in advance of the left gate. In quantum pumps constructed by other conductors, the PC always flows from the right to the left reservoir at the $\phi_1 < \phi_2$ phase lag. As a result of the Klein paradox, a higher potential barrier demonstrates stronger transmission when the chirality alignment meets and the PC reverses direction. The chirality consistency favoring transmission is different between the incident energy above and below the peak energy. When the Fermi energy is smaller than the Dirac point 100 meV, above the peak energy, a higher potential barrier demonstrates stronger transmission and the PC flows from the left reservoir to the right. Below the peak energy, a higher potential barrier demonstrates weaker transmission and the PC flows from the right reservoir to the left. When the Fermi energy is larger than the Dirac point, the PC direction is reversed as the transmission configuration is reversed. Larger PCs have relatively stronger current–current correlation. The shot noise demonstrates peaks at the PC peaks as shown in figure 3(b). The shot noise is positive since the rightward current flow correlates with the rightward current flow and vice versa. The Fano factor is above 1 due to the Klein paradox induced virtual correlation between electrons and holes. The dc PC is a net charge flow. For all configurations, transmission to-and-fro occurs simultaneously and generates finite heat current [4]. For small PC and large to-and-fro transmission, the PSN is greatly enhanced beyond the Poisson value. At energies when the PC reverses direction, zero-PC occurs, to-and-fro transmission contributing no PC is strong, so the shot noise is extraordinarily enhanced beyond the Poisson value.

The PC, PSN, and the Fano factor as functions of the driving phase difference are shown in figure 4. The PC varies with the driving phase $\phi$ in a sinusoidal function and the PSN in a cosinusoidal function, which can be already seen in equation (12). From figure 4(c) we can see that for all the Fermi energies considered the Fano factor varies with $\phi$ in similar forms. When the Fermi energy $E$ and the incident angle $\theta$ are fixed, transmission features of the conductor are fixed. Variation of the pumping phase lag would not change the transmission features. For all Fermi energies and incident angles, the pumping properties as functions of the driving phase difference are similar. For configurations of $E$ and $\theta$ in which higher potential barriers have stronger transmission, the PC and Fano factor are positive at $\phi_2 - \phi_1 \in [\pi, 2\pi]$ and negative at $\phi_2 - \phi_1 \in [0, \pi]$. And for configurations of $E$ and $\theta$ in which lower potential barriers have stronger transmission, the sign of the PC and Fano factor is reversed. At phase lag 0, $\pi$, and $2\pi$, the PC changes direction as a result of the swap of the opening order of the two gates. When the PC changes direction, interaction of electrons and holes in virtual processes is enhanced and the Fano factor demonstrates a sharp rise.

The KL paradox virtually correlates the hole states with the electron states. Therefore, the PSN is remarkably enhanced beyond the Poisson value, the latter of which indicates uncorrelated transport. The PSN relative to the Poisson value measured by the Fano factor is presented in figure 2(c). It can be seen that the Fano factor is above 1. Klein paradox induced virtual correlation between electrons and holes enhances the PSN beyond the Poisson value. It is also revealed in figure 2 that the PSN and Fano factor are extremely large at the incident angle when the PC reverses direction. The pumping configuration at these incident angles is time-reversal-symmetric. In one cyclic period, particle transport along $+x$ and $-x$ directions cancels out, inducing no PC. Concurrently, energy is transported by heat current and the opposite transmission contributes to the shot noise. As a result, the shot noise is dramatically enhanced compared with the Poisson noise measured by $2eI_p$.

The inset is an enlargement of the circled area at one of the zero pumped current points, at which the pumped current is 0.

Figure 3. Pumped current (a), shot noise (b), and Fano factor (c) as functions of the Fermi energy. Driving amplitude $U_{\text{d1}} = U_{\text{d2}} = 0.01$ meV. Driving phase $\phi_1 = 0.1$ and $\phi_2 = 0.6$. Incident angle $\theta = 0.01$. The inset is an enlargement of the circled area at one of the zero pumped current points, at which the pumped current is very large relative to the Poisson value.
with $\Delta \hat{I} = \hat{I} - \langle \hat{I} \rangle$ and $\hat{I}_a(t)$ is the quantum-mechanical current operator in the lead $a$. The zero-frequency pumped shot noise (PSN) averaged over a long time ($\Delta t \gg 2\pi/\omega$) is the time integral of $S_{a\beta}(t, t')$ as follows:

$$S_{a\beta} = \frac{\omega}{2\pi} \int_{-\infty}^{+\infty} \int_{0}^{2\pi} S_{a\beta}(t, t') \, dt' \, dt. \quad (14)$$

The first term in the PSN is

$$\frac{1}{2} \frac{\omega}{2\pi} \int_{-\infty}^{+\infty} \int_{0}^{2\pi} \langle \hat{I}_a(t) \hat{I}_\beta(t') \rangle \, dt' \, dt$$

with

$$\hat{I}_a(t) = \frac{\hbar}{\omega} [\hat{b}_a(t) \hat{b}_a(t) - \hat{a}_\alpha(t) \hat{a}_\alpha(t)]. \quad (16)$$

Substituting $\hat{I}_a(t) = \sum_{\mu \nu} s_{a\beta}(t) \hat{a}_\beta(t)$ into the above equation, we have

$$\hat{I}_a(t) \hat{I}_\beta(t') = \frac{\hbar}{\omega} \sum_{\mu \nu} \langle \hat{a}_\beta(t) \rangle \langle \hat{a}_{\mu}(t) \rangle \langle \hat{s}_{\alpha\nu}(t) \rangle \hat{a}_\alpha(t)$$

and

$$\langle \hat{I}_a(t) \hat{I}_\beta(t') \rangle = \frac{\hbar}{\omega} \sum_{\mu \nu} \langle \hat{a}_\beta(t) \rangle \langle \hat{a}_{\mu}(t) \rangle \langle \hat{s}_{\alpha\nu}(t) \rangle \hat{a}_\alpha(t)$$

The following equations were obtained using Wick's theorem in the quantum statistical expectation value of products of four operators.

$$\langle \hat{a}_\beta(t) \hat{a}_\alpha(t) \rangle = \frac{\hbar}{\omega} \sum_{\mu \nu} \langle \hat{a}_\beta(t) \rangle \langle \hat{a}_{\mu}(t) \rangle \langle \hat{s}_{\alpha\nu}(t) \rangle \hat{a}_\alpha(t)$$

Using $\hat{a}_\alpha(t) = \int dE \hat{a}_\alpha(E) e^{-iEt/\hbar}$, the first term in equation (17) reads

$$\frac{\hbar}{\omega} \sum_{\mu \nu} \int dE_1 dE_2 dE_3 dE_4 \hat{a}_\mu(E_1) e^{iEt_1/\hbar}$$

with

$$\hat{a}_\mu(E_1) = \int dE \hat{a}_\mu(E) e^{-iEt/\hbar}$$

and

$$\langle \hat{a}_\beta(t) \rangle = \int dE \hat{a}_\beta(E) e^{-iEt/\hbar}$$

Wick's theorem gives the quantum statistical expectation value of products of four operators. For a Fermi gas at equilibrium this expectation value is [44]
\[
\begin{align*}
\langle \hat{a}^\dagger_\mu(E_1) \hat{a}_\nu(E_2) \hat{a}^\dagger_\kappa(E_3) \hat{a}_\eta(E_4) \rangle \\
-\langle \hat{a}^\dagger_\mu(E_1) \hat{a}_\nu(E_2) \rangle \langle \hat{a}^\dagger_\kappa(E_3) \hat{a}_\eta(E_4) \rangle \\
= \delta_{\mu\nu} \delta_{\kappa\eta} \delta(E_1 - E_4) \delta(E_2 - E_3) f_{\nu}(E_1)[1 - f_{\nu}(E_2)].
\end{align*}
\]

(20)

\(f_{\alpha}(E)\) is the Fermi distribution function of the \(\alpha\) reservoir connected to the adiabatically modulated conductor. Substituting equation (20), in total five plus five terms contribute to the time-averaged zero-frequency PSN. Using the expansion of the scattering matrix (equations (6) and (7)), we reach the general expression of the time-averaged zero-frequency PSN:

\[
S_{\mu\nu} = \frac{e^2 \omega}{2\pi} \sum_{\mu\nu j j^\prime} X_{\mu j j^\prime} X_{\nu j j^\prime} s_{10}^{j j^\prime} \frac{\partial s_{\mu u}}{\partial X_{j j}} \frac{\partial s_{\nu u}}{\partial X_{j j}} \cos(\psi_{j j} - \psi_{j j^\prime}) + \frac{e^2 \omega}{2\pi} \sum_{\mu\nu j j^\prime} X_{\mu j j^\prime} X_{\nu j j} s_{10}^{j j^\prime} \frac{\partial s_{\mu u}}{\partial X_{j j}} \frac{\partial s_{\nu u}}{\partial X_{j j}} \cos(\psi_{j j^\prime} - \psi_{j j}) + \frac{e^2 \omega}{2\pi} \sum_{\mu\nu j j^\prime} X_{\mu j j^\prime} X_{\nu j j} s_{10}^{j j^\prime} \frac{\partial s_{\mu u}}{\partial X_{j j^\prime}} \frac{\partial s_{\nu u}}{\partial X_{j j^\prime}} \cos(\psi_{j j^\prime} - \psi_{j j^\prime}) + \frac{e^2 \omega}{2\pi} \sum_{\mu\nu j j^\prime} X_{\mu j j^\prime} X_{\nu j j} s_{10}^{j j^\prime} \frac{\partial s_{\mu u}}{\partial X_{j j^\prime}} \frac{\partial s_{\nu u}}{\partial X_{j j^\prime}} \cos(\psi_{j j^\prime} - \psi_{j j^\prime})
\]

× \cos(\psi_{j j} - \psi_{j j^\prime}) + \frac{e^2 \omega}{2\pi} \sum_{\mu j j^\prime j j^\prime j j^\prime} X_{\mu j j^\prime j j^\prime j j^\prime} X_{\nu j j^\prime j j^\prime j j^\prime} \frac{\partial s_{\mu u}}{\partial X_{j j^\prime}} \frac{\partial s_{\nu u}}{\partial X_{j j^\prime}} \frac{\partial s_{\eta u}}{\partial X_{j j^\prime}} \frac{\partial s_{\eta u}}{\partial X_{j j^\prime}} \cos(\psi_{j j^\prime} - \psi_{j j^\prime} - \psi_{j j^\prime} - \psi_{j j^\prime}).
\]

The last term of equation (21) is a product of four pumping amplitudes, four derivatives of the scattering matrix elements relative to the oscillating parameter, and a cos 2\(\phi\) function. As small pumping amplitudes are considered in our approach, the magnitude of this term is negligible. Inserting the following unitarity of the scattering matrix:

\[
\sum_{\gamma} s_{\mu\gamma} s_{\nu\gamma}^* = \delta_{\mu\gamma}, \quad (s_{\mu\gamma}^*) = s_{\nu\gamma}^*,
\]

\[
\sum_{\gamma} \frac{\partial s_{\mu\gamma}}{\partial X_{j j}} s_{\nu\gamma}^* + \sum_{\gamma} s_{\mu\gamma} \frac{\partial s_{\nu\gamma}}{\partial X_{j j}} = 0,
\]

we can find that

\[
S_{\mu\nu} = \frac{e^2 \omega}{\pi} \left[ \delta_{\mu\nu} (T_{\omega, a} + T_{\omega, a^\dagger}) - T_{\omega, a} \right]
\]

(23)

with \(T_{\omega, a} = \sum_{\beta} |s_{\beta\omega}^{\omega\omega}|^2 \) and \(T_{\omega, a^\dagger} = \sum_{\beta} |s_{\beta\omega}^{\omega\omega}|^2 + \sum_{\beta} |s_{\beta\omega}^{\omega\omega^\dagger}|^2\). Thus, we give a detailed derivation of the result of [4].

**Appendix B. Discussion of the Poissonian PSN**

Schottky’s result [44, 45] for the shot noise corresponds to the uncorrelated arrival of particles with a distribution function of time intervals between arrival times which is Poissonian, \(P(\Delta t) = \tau^{-1} \exp(-\Delta t/\tau)\) with \(\tau\) being the mean time interval between carriers. \((P(\Delta t))\) is normalized with \(\int_0^{\infty} P(\Delta t) d(\Delta t) = 1\) and \(\int_0^{\infty} (\Delta t)P(\Delta t) d(\Delta t) = \tau\). With the Poissonian time interval distribution function, we could consider the Poissonian current and shot noise. It is convenient to look at a single-electron tunneling process with \(P(\Delta t)\) normalized to 1 and the complete relevant time range is in the order of \(\tau\).

We take an infinitesimal time segment \([t, t + dt]\) from the continuous time flow in \([0, +\infty)\). The time-dependent current generated by the reservoir could be expressed

\[
I(t) = \frac{\int_t^{t+dt} eP(t') dt'}{dt} = e^{-t/\tau}.
\]

(24)

The mean current follows

\[
\bar{I}(t) = \lim_{T \to \infty} \frac{1}{T} \int_0^T I(t) dt = \frac{1}{\tau} \int_0^{\infty} I(t) dt = e^{-t/\tau}.
\]

(25)

Here the single-electron tunneling picture is used. The mathematical object which allows us to characterize the duration of the current pulse is called the autocorrelation function and is defined by

\[
R_I(t') = \lim_{T \to \infty} \frac{1}{T} \int_0^{T/2} I(t)(t + t') dt.
\]

(26)

From the time-dependent current, we can obtain the autocorrelation function as

\[
R_I(t') = \left. \frac{\int_0^{T/2} I(t)(t + t') dt'}{I(t)} \right|_{t = t'} = \frac{e^{-2t/\tau}}{\tau^2} e^{-t/\tau}.
\]

(27)

The subscript \(t\) means the mean value is evaluated relative to the variable \(t\). Using the following relation coming from the result of equation (25):

\[
\frac{e^{-2t/\tau}}{\tau^2} e^{-t/\tau} = \left[ \frac{1}{2} e^{-t/\tau} \right]_{t'} = \frac{1}{\tau} e^{-t/\tau} = \frac{e^{-t/\tau}}{\tau}.
\]

(28)

we have

\[
R_I(t') = \frac{e^2}{\tau^2} e^{-t/\tau}.
\]

(29)

The Wiener–Khinchin theorem states that the noise spectrum is the Fourier transform of the autocorrelation function:

\[
S_I(f) = 2 \int_0^{\infty} R_I(t') e^{-i2\pi ft'} dt'.
\]

(30)

Therefore, the zero-frequency shot noise

\[
S_I(0) = 2 \int_0^{\infty} \frac{e^2}{\tau^2} e^{-t/\tau} dt' = 2 \int_0^{\infty} e^2 dt' = 2\tau I.
\]

(31)

which is just the Poisson shot noise.

Following that, we consider the pumping configuration to achieve the Poissonian quantum PSN. To achieve a pure Poisson process, we should exclude all conducting structures and let the conductance be totally governed by two Poisson-distributed random emitters at the left and right leads.
since any scattering structure would induce interactions and break the Poissonian picture. The pumping mechanism is thus reduced to a semi-classical one with two modulating gates and a single-particle level between the two gates. The two gates are modulated with a phase lag $\phi = \pi/2$. We assume the gates to be two oscillating semi-classical potential barrier with the time dependence of their heights as follows:

$$U_1 = \sin \left( t + \frac{\pi}{2} \right), \quad U_2 = \sin(t).$$  \hspace{0.5cm} (32)

In typical quantum pumps, the oscillation period $T = 2\pi/\omega$ is much larger than the mean time interval between carriers $\tau$. Here the pumping frequency $\omega$ is set to be 1 without blurring any physics. We divide one pumping period into four quarters. When $t \in [0, \pi/2]$, $\sin(t)$ changes from 0 to 1 and $\sin(t + \pi/2)$ changes from 1 to 0. Considering the integral effect, the two gates are equally high and the system could be approximated by two identical emitters shooting electrons at each other with a possible emission phase lag. The time-dependent current could be formulated as

$$I_p(t) = \frac{e^{i\tau_q}}{\tau} - \frac{e^{-i\tau_R}}{\tau}. \hspace{0.5cm} (33)$$

For two uncorrelated emitters, $\tau_q$ and $\tau_R$ are possibly different. When $t \in [\pi/2, \pi]$, $\sin(t)$ changes from 1 to 0 and $\sin(t + \pi/2)$ changes from 0 to $-1$. In this quarter, the gate $U_1$ is open and the gate $U_2$ is closed. The electron has some probability of being emitted from the left reservoir to the middle single-electron level and fill it. There is a current flow from the left reservoir to the middle level. The time-dependent current flow from the left emitter to the middle level could be formulated as

$$I_p(t) = \frac{e^{-i\tau_q}}{\tau}.$$

For adiabatic quantum pumps, $T/\Delta \gg \tau$. Therefore, the time average in one period could be approximated as the time average in the infinite time interval $[0, +\infty)$. Following a similar derivation as the ordinary conductor, we could obtain

$$\bar{I}_p(t) = \frac{e}{\tau}, \hspace{0.5cm} (36)$$

and the zero-frequency shot noise

$$S_p(0) = 2 \frac{e^2}{\tau} = 2e\bar{I}_p,$$

which is the Poisson PSN.

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