Application of lattice Boltzmann method for studying interaction dynamics of parallel plane minijets

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Abstract. In this work, the method of lattice Boltzmann equations was applied to simulate the systems of two and three parallel laminar plane jets. Simulation was carried out for different Reynolds numbers (Re = 20-200). For the case of two jets, the character of jet interaction has been determined to be, in general, non-stationary, but for small Re values, the process becomes stable due to the action of viscosity forces. For the case of three jets, the character of jets interaction is found to differ significantly from the case of two jets and is non-stationary even for small Re numbers.

1. Introduction

Parallel minijets are widely used in industry: for example, in fuel supply systems for chip cooling, waste disposal, and ventilation, which increases the need to study their behavior. Their interaction was studied in [1-2] where the experiments were carried out with a turbulent plane double jet. The parallel jets attract each other. The article [3] continued the experimental study of such jets and confirmed the nature of interaction obtained in [1-2]. The jets were found to merge into one combined jet, whose behavior was consistent with the case of a single jet. The process of jet interaction was considered in more detail in [4], which identified three main areas of interaction: (a) convergence; (b) merging and (c) combination. In the first two areas, two jets were still separated and interacted with each other, and in the combination area, there was one jet, and this was consistent with the result of [3]. In [5], the authors performed successful numerical simulation of a system of two parallel plane jets for the turbulent case with Re = 10,000 using the RANS method and described the dependence of periodic vortex flows on the distance between the jets. The case of laminar jets was considered in [6], numerically simulating a similar system with Re = 56 using the method of lattice Boltzmann equation and establishing the character of jet interaction, which was consistent with the above results. The dependence of such jet interaction upon varying the distance between the inlet slits was also identified.

In contrast to numerous works on two-jet systems, there are relatively few studies on the systems with three or more jets. In [7] the authors experimentally consider a system of five plane parallel jets using the two-dimensional PIV method. It is shown that the jets are also combined into one, but the nature of their interaction differs from the two-jet system because of the presence of several neighboring jets.
1. In this paper, the method of lattice Boltzmann equations is applied to simulate the systems of two and three plane parallel jets. The case of constant distance between jet axes $s/d = 5$ with the Reynolds number of 20-200 is considered.

2. Mathematical model

2.1. Main equations

At the mesoscopic level, the behavior of gases can be described using the Boltzmann kinetic equation [9]:

$$\frac{\partial f}{\partial t} + \vec{\xi} \cdot \nabla f + \frac{\vec{F}}{m} \nabla \cdot f = \Omega_f, \quad (1)$$

where $f = f(x, v, t)$ is the probability density function of particle distribution by coordinates and velocities, $\vec{v}$ is the microscopic velocity, $m$ is the molecule mass, and $\vec{F}$ is the body force. The right side of equation (1) includes the so-called collision integral, for which the Bhatnagar - Gross - Crook approximation is used [8]:

$$\Omega_f = -\frac{f - f^eq}{\tau}, \quad (2)$$

where $f^eq = \frac{\rho}{(2\pi RT)^{D/2}} \exp \left[ -\frac{(\vec{\xi} - \vec{u})^2}{2RT} \right]$ is Maxwell-Boltzmann equilibrium distribution function, $\tau$ is the time of relaxation. It is shown in [10] that the Navier-Stokes equation can be derived from (1)-(2).

The discrete Boltzmann equation can be obtained from (1) - (2) by following [9]:

$$f_i(x + \delta t, \vec{v} + \delta t) - f_i(x, \vec{v}) = -\frac{1}{\tau} (f_i - f_i^eq), \quad (3)$$

where $\vec{v}_i$ is the lattice velocity (discrete kind of microscopic velocity $\vec{v}$), $\delta t$ is the time step, $f_i$ is the discrete distribution function, and $f_i^eq$ is the discrete equilibrium distribution function. It is assumed in this work that there is no field of external forces.

2.2. Method of lattice Boltzmann equations

The method of lattice Boltzmann equations consists in solving the discrete Boltzmann equation (3) for simulating the liquid motion. In the previous section, during the transition from the continuous form of equation to the discrete one, the transition from the microscopic velocity $\vec{v}$ to the set of lattice velocities $\{\vec{v}_i\}$ was made: these were the velocities of particles in the lattice points, which were the same for each of the points. Usually the velocities of particles in the lattice are chosen in such a way that in one time step, the particle travels a distance of one lattice step along the coordinate, and their number depends on the chosen lattice type.

For lattices, a notation in the form $DnQm$ was taken, where $n$ is the dimension of space, and $m$ is the number of lattice vectors. In this paper, the $D2Q9$ lattice was chosen, which gives the following form for $\vec{v}_i$:

$$\vec{v}_i = \begin{cases} (0,0), & i = 0 \\ (\cos(i-1)\pi/2, \sin((i-1)\pi/2)), & i = 1...4 \\ \sqrt{2}(\cos((2i-9)\pi/4, \sin((2i-9)\pi/4)), & i = 5...8 \end{cases} \quad (4)$$

In addition to discretization of the microscopic velocity, in the previous section there was transition $f^eq \rightarrow f_i^eq$ to a set of discrete equilibrium distribution functions. This process is described in detail in [9]; we give the final form:


\[ f_i^{eq} = w_i \rho \left( 1 + \frac{\tilde{e}_i \cdot \bar{u}}{c_s^2} + \frac{(\tilde{e}_i \cdot \bar{u})^2 - \bar{u}^2}{2c_s^4} \right), \]  

where \( w_i \) is the weight function, \( c_s = \frac{1}{\sqrt{3}} \) is the sound velocity in the lattice for D2Q3.

The type of weight functions depends on the specific type of lattice, for the case of D2Q9 it is:

\[
w_i = \begin{cases} 
4/9, & i = 0 \\
1/9, & i = 1...4 \\
1/36, & i = 5...8
\end{cases}
\]  

Similar to the continuous case (1), it is possible to obtain the macroscopic velocity and density of liquid through the distribution functions:

\[
\rho = \sum_i f_i, \quad \rho \bar{u} = \sum_i \tilde{e}_i f_i.
\]  

The relaxation time \( \tau \) is associated with kinematic viscosity:

\[
\bar{v} = \left( \tau - \frac{1}{2} \right) c_s^2.
\]  

Boundary conditions for equation (1) were set by the scheme proposed in [11]. Uniform fixed velocity was used for jet inlets, no-slip condition was applied to the wall between jets and fixed pressure was set at other boundaries.

The numerical solution of above described system was performed using the code developed by the authors. For every case studied, grid independence test was performed at the highest Reynolds number, showing optimal grids up to several millions of nodes.

### 2.3. Model validation

To validate the model, we simulated the outflow of a single laminar plane jet into free space. For such a problem, there is an analytical solution, which can be used for comparison. As a result of simulation, the velocity distribution in space and the velocity profiles at different distances from the inlet slit were obtained and compared (figure 1) with analytical solution:

\[
u = \left( \frac{3M^2}{52\rho u_x} \right)^{1/3} \text{sech}^2 \zeta,
\]  

where \( M = \int_{-\infty}^{+\infty} u^2 dy \), \( \zeta = \left( \frac{M}{48\rho u^3} \right)^{2/3} \frac{y}{x^{2/3}} \) is the generalized coordinate, and \( x \) is the distance from the inlet.

![Figure 1. Comparison of velocity profiles for the analytical solution (9) and simulation result. Profile of longitudinal velocity at \( x = 20d \).](image)
As it can be seen in figure 1, there is a satisfactory agreement between the analytical solution and the result.

3. Results

3.1. A system of two plane parallel jets

The method of lattice Boltzmann equations was used to model a system of two plane parallel laminar jets with \( \text{Re} = 20-200 \). Reynolds number was varied by changing viscosity while slit width and initial velocity were fixed. The size of simulation area was \( 50d \times 100d \), where \( d \) is the slit width from which one jet flows out. Boundary conditions

As a result, the picture of flow dynamics was obtained (figure 2, 3), and the main areas of interaction were determined: convergence area with two approaching separate jets; merging area, where the jets begin to merge, but still behave separately and interact with each other; and combining area, where two jets are finally combined into one. This corresponds to the results obtained in [1-4].

![Figure 2](image1.png)

**Figure 2.** Distribution of instantaneous longitudinal velocity at \( \text{Re} = 30 \) (a), \( 60 \) (b). Here and below velocity was normalized by inlet velocity, coordinates by slit width.

![Figure 3](image2.png)

**Figure 3.** Distribution of instantaneous (a) and average (b) longitudinal velocity at \( \text{Re} = 166 \).

It has been also determined that the process of interaction of two jets at low Reynolds numbers is stationary, but with its increase, a transition to non-stationary interaction occurs. This is especially pronounced in the character of dependence of the merging point position on the \( \text{Re} \) number (figure 4). The merging point is defined on the axis of symmetry where the mean streamwise velocity is zero. The data from [6] for \( \text{Re} = 56 \) are also given in this figure for comparison.
3.2. System of three plane parallel jets

Further, this method was used to simulate a system of three jets with the same Reynolds numbers. Analysis of results shows that the flow character in this case is significantly different from the case of two jets due to the presence of a middle jet, which changes its direction in the non-stationary regime, being attracted to one of the neighboring jets (figure 5). This is similar to the behavior of jets that differ from the extreme ones, in other words, internal jets considered in [7].

Conclusion

A mathematical model has been constructed using the method of lattice Boltzmann equations and validated using a single plane laminar jet as an example. The resulting velocity profiles have been compared with the analytical solution.

The method was used to simulate the systems of two and three plane parallel laminar jets. For the case of two jets, velocity distributions in space were obtained for different Reynolds numbers in the range from 20 to 200. The nature of motion, consistent with the results of [1-4], was established; the process of jet interaction was found to be non-stationary, excluding the range Re <120. The flow of
three parallel jets was analyzed. It has been established that the character of interaction in this case differs significantly from the case of two jets. It has been also obtained that the process of interaction of such a number of jets is highly non-stationary even at low $Re \sim 50$ values.

Acknowledgements
The study was financially supported by the Russian Science Foundation (grant 19-79-30075).

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