Needlet bispectrum asymmetries in the WMAP 5-year data

Davide Pietrobon,1,2⋆ Paolo Cabella,1,3 Amedeo Balbi,1,4 Robert Crittenden,2
Giancarlo de Gasperis1 and Nicola Vittorio1

1 Dipartimento di Fisica, Università di Roma ‘Tor Vergata’, via della Ricerca Scientifica 1, 00133 Roma, Italy
2 Institute of Cosmology and Gravitation, Dennis Sciama Building, Barnaby Road, Portsmouth PO1 3FX
3 Dipartimento di Fisica, Università La Sapienza, P.le A. Moro 2, Roma, Italy
4 INFN Sezione di Roma ‘Tor Vergata’, via della Ricerca Scientifica 1, 00133 Roma, Italy

Accepted 2009 November 26. Received 2009 November 20; in original form 2009 May 22

ABSTRACT

We apply the needlet formalism to the Wilkinson Microwave Anisotropy Probe 5-year data, looking for evidence of non-Gaussianity in the bispectrum of the needlet amplitudes. We confirm earlier findings of an asymmetry in the non-Gaussianity between the Northern and Southern Galactic hemispheres. We attempt to isolate which scales and geometrical configurations are most anomalous and find that the bispectrum is most significant on large scales and in the more co-linear configurations and also in the ‘squeezed’ configurations. However, these anomalies do not appear to affect the estimate of the non-linear parameter $f_{NL}$, and we see no significant difference between its values measured in the two hemispheres.

Key words: methods: data analysis – cosmic microwave background – early Universe.

1 INTRODUCTION

Since the first release of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite data (Bennett et al. 2003), there have been many claims of anomalies in the statistical distribution of cosmic microwave background (CMB) temperature fluctuations in the sky (see e.g. Eriksen et al. 2004). For example, there appear to be localized areas which are hotter or colder than would be expected in the concordance Λ cold dark matter (ΛCDM) cosmological model with Gaussian statistics (see Cruz et al. 2005). Also, power seems to be preferentially aligned along a certain direction (dubbed the ‘axis of evil’; Land & Magueijo 2007) and the quadrupole and octopole power appears to be correlated (de Oliveira-Costa et al. 2004). These anomalies were subsequently confirmed with new releases of the WMAP data (Spergel et al. 2007; Nolta et al. 2009). Many other studies have highlighted a marked difference in the statistics of the northern and southern galactic skies. Park (2004) found an asymmetry in the Minkowski functional values in the Northern and Southern galactic hemispheres. Eriksen et al. (2005) detected anomalies at large angular scales comparing the amplitudes of temperature power spectra in the two hemispheres and confirmed that the anomalies are present in the n-point correlation function. Vielva et al. (2004) studied the kurtosis of Spherical Mexican Hat Wavelet coefficients, discovering a strong non-Gaussian signal in the Southern hemisphere. Hansen et al. (2004b) reported that the local curvature of the CMB sky exhibited asymmetric behaviour as well. McEwen et al. (2008) and Pietrobon et al. (2008) applied two different wavelet constructions to the 5-year WMAP data, confirming many of these results; they have also been seen using scaling indices (Rossmanith et al. 2009). Copi et al. (2007) pointed out a lack of power in the Northern hemisphere in the two-point correlation function. The presence of these anomalies has been tested against mask effect and foreground contamination by Bernui et al. (2007). Lew (2008) constrains the direction of the anomaly axis using a generic maximum a posteriori method. Very recently, Hansen et al. (2009) reported that the power asymmetry spans a very large range of angular scales (corresponding to multipoles $2 \leq \ell \leq 600$): this result is based on an angular power spectrum analysis of the WMAP sky maps. A summary of most of these anomalies can be found in Bernui & Reboucas (2009).

Here, we investigate the CMB anomalies using the needlet bispectrum (Lan & Marinucci 2008) in the WMAP 5-year data; this technique was recently used to constrain primordial non-Gaussianity in the same data set by Pietrobon et al. (2009) and Rudjord et al. (2009a). For the first time, we analyse the contribution of different triangular configurations, grouped according to their size and shape. The paper is organized as follows: in Section 2, we describe the needlet framework; the data set and the simulations we use are discussed in Section 3 where we present our results on the north–south asymmetry and its configuration dependence; finally, in Section 4 we draw our conclusions.

2 FORMALISM OF THE NEEDLET BISPECTRUM

We perform our analysis of the non-Gaussianity of WMAP 5-year data by means of needlets, which are isotropic wavelets with many

*E-mail: davide.pietrobon@roma2.infn.it
Needlet bispectrum asymmetries in the WMAP

Next, we describe the simulations used in our needlet bispectrum analysis. We start by producing simulated Gaussian CMB maps taking into account the beam and the noise properties of each WMAP-5\(^1\) channel Q, V and W. From these single-channel maps we construct an optimal map \(T(\hat{\gamma}) = \sum_{nm} T_{nm}(\hat{\gamma}) w_{nm}(\hat{\gamma})\) (see Jarosik et al. 2007), where \(\hat{\gamma}\) represents a direction on the sky and \(w_{nm} = n_{0}(\hat{\gamma}) / \sigma_{\theta,0} / \sum_{nm} n_{0}\), where \(n_{0}\) is the number of observations of a given pixel and \(\sigma_{\theta,0}\) the nominal sensitivity of the channel (Hinshaw et al. 2009). We apply the \(j=1, j=4'Kq75\) combined mask described by Pietrobon et al. (2008) and degrade the resulting map to the resolution of \(N = 256\). We extract the needlet coefficients \(\beta_{jk}\) from the simulated maps for a given \(B\) and compute the needlet bispectrum of the reconstructed coefficient maps according to equation (3). Finally, we calculate \(S_{\ell_1\ell_2\ell_3}\) from the real data of the foreground-reduced WMAP 5-year Q, V and W channels, using the same procedure applied to the simulated maps. To test the Gaussianity of WMAP 5-year data, we compare the distribution of the \(\chi^2 = X^{-1} X^T\) of the simulated data set to the value obtained from data, where \(X\) is the array consisting of the needlet bispectrum values calculated via equation (3). We consider the needlet bispectrum values (indicated by ‘all’ in the tables) and, to identify where the anomalies are concentrated, we split the analysis into different branches according to the geometry of the triangles. For the

\[^1\]http://lambda.gsfc.nasa.gov/product/map/dr3/m_products.cfm

Figure 1. Needlet coefficients of the combined Q, V and W map at the resolution \(j = 4\). The \(B\) parameter is fixed to 2.
Furthermore, considering the triangle configurations as classified (see Table 2). The results are shown in the histogram plots in Fig. 2.

| Conf. | Full sky (per cent) | North (per cent) | South (per cent) |
|-------|---------------------|-----------------|-----------------|
| All (115) | 29 | 96 | 2 |
| Equi (9) | 20 | 11 | 45 |
| Iso (56) | 5 | 96 | 0.5 |
| Scal (50) | 60 | 90 | 7 |
| Open (50) | 3 | 85 | 2 |

Table 2. Percentage of the simulations with a $\chi^2$ larger than WMAP 5-year data for the different triangular configurations of the needlet bispectrum. An asymmetry is present in each triangle configuration except for the equilateral and is significant when all the configurations are combined.

Table 3. $\chi^2$ for the WMAP 5-year QVW data compared to simulations. The top panel is for the large-scale study and the bottom panel is for the small-scale one.

| Conf. | Full sky | Large scales ($j \leq 5$) |
|-------|-----------|---------------------------|
| All (28) | 11 | 93 | 14 |
| Equi (5) | 86 | 26 | 45 |
| Iso (16) | 70 | 90 | 22 |
| Scal (7) | 37 | 62 | 15 |
| Open (7) | 3 | 38 | 2 |

Figure 2. Needlet bispectrum $\chi^2$ distribution of the three WMAP 5-year temperature data. The Southern hemisphere is barely compatible with the Gaussian hypothesis, shown by the blue line which marks the real data $\chi^2$ in the tail of the distribution.

On the full CMB sky, the $\chi^2$ of the data is compatible with the Gaussian hypothesis, shown by the blue line which marks the real data $\chi^2$ in the tail of the distribution.

3.1 Large–small-scale analysis

Going more deeply, we focused on small and large angular scales separately. In particular, with the choice $B = 2$, we define the subset of needlets $j = 1$ to $j = 5$ as large scale, corresponding roughly to scales larger than 1° while the subset $j = 6$ to $j = 9$ corresponds to the sub-degree scales (see Table 1). We then perform the same analysis we carried out on the whole needlet set. The results are shown in Table 3. The isosceles configurations still show a large difference between the two hemispheres, but the significance is lower than the whole set analysis. The open configuration result is still anomalous. No open configurations exist for the small-scale subset $6 \leq j < 9$; however for the large scales these co-linear configurations are most significantly non-zero for the biggest contribution of the power. For the sub-degree set, we did not find a high degree of anomaly, as summarized in Table 3, though the isosceles configurations are still significantly different between the two hemispheres. Dividing the analysis between the two sets at large and small scales, we miss the important contribution given by the correlation between the two, which is indeed crucial for the squeezed triangles. We then consider two more sets as follows. One set is formed by triangles...
of asymmetry. A pattern is clearly visible, but the direction which maximizes the anomaly seems to be orthogonal to the one reported by other authors with different estimators. This may suggest that either the three-point correlation function couples differently to the dipole modulation or the nature of the asymmetry we measured is different. No matter whether this direction depends upon the shape and the angular scale we consider, its significance and the link with the direction found in the literature are interesting topics, which require a dedicated study and they will be addressed in a forthcoming paper.

Finally, we checked whether the sky asymmetries we detected affect the measure of the primordial non-Gaussianity parameter. A complete review on the nature of this parameter may be found in Bartolo et al. (2004) and Fergusson & Shellard (2009); recent constraints from CMB experiments can be found in Smith, Senatore & Zaldarriaga (2009), Curto et al. (2008), Komatsu et al. (2009) and de Troia et al. (2007) while Slosar et al. (2008) constrained $f_{NL}$ through the galaxy distribution. Limits on $f_{NL}$ using wavelets are discussed in Curto, Martínez-Gonzalez & Barreiro (2009), Cabella et al. (2004) and Mukherjee & Wang (2004). We estimate $f_{NL}$ performing the same analysis described in Pietrobon et al. (2009), applying the estimator

$$f_{NL} = \frac{X^d T C^{-1} X^n}{X^{d+n} T C^{-1} X^n}$$

(4)

to the WMAP 5-year data set. Here $X$ is a vector composed of the values of the needlet bispectrum for a given triangular configuration according to equation (3). The covariance matrix $C$ is calculated from 20000 Gaussian simulations, since its dependency on $f_{NL}$ is negligible (e.g. see Spergel & Goldberg 1999). The theoretical non-Gaussian template $X^{th}$ was calculated via Monte Carlo simulations over the 100 primordial non-Gaussian maps (Liguori et al. 2007). Since we know that the CMB sky is asymmetric, showing more non-Gaussianity in the Southern hemisphere, we carried out a split north–south analysis to see if the asymmetry extends to differences in the $f_{NL}$ estimate. Recently, Curto et al. (2009) and Rudjord et al. (2009b) targeted the same issue, finding no evidence of $f_{NL}$ varying on the sky. We do not find a significant deviation between the values measured in the two hemispheres, while the error bars become significantly larger due to the reduced number of pixels used to calculate the needlet bispectrum.

3.2 Further analysis

Since the first anomalies were found, several methods have been applied to search for a specific direction in the sky which maximizes the asymmetry. Indeed, many works identify a direction very close to the ecliptic poles (Hansen et al. 2004a; Land & Magueijo 2005; Hansen et al. 2009; Ráth et al. 2009). In particular, a dipole modulation has been proposed as a possible explanation for such a pattern in Hofuft et al. (2009). In order to see whether a similar modulation underlies the asymmetry we detect, we rotated the reference frame, spanning uniformly the sky, and recomputed our statistics for the WMAP data with the new north–south definition. Since the set of simulations we used assume isotropy and homogeneity, the rotation of the reference frame does not affect their statistics and the covariance matrix we applied in our previous analysis.

The result is shown in Fig. 3, where we plotted the reduced $\chi^2$ for the Southern hemisphere in the particular case of the isosceles configurations, as a function of the North Pole direction. We chose the isosceles triangles since they show the highest degree

| Conf. | Full sky (per cent) | Correlation (LSS) North (per cent) | South (per cent) |
|-------|---------------------|-----------------------------------|-----------------|
| Iso (20) | 23 | 78 | 0.4 |
| Scal (26) | 76 | 40 | 51 |
| Open (4) | 32 | 35 | 54 |
| Conf. | Full sky (per cent) | Correlation (LSS) North (per cent) | South (per cent) |
| Iso (8) | 47 | 94 | 20 |
| Scal (13) | 62 | 98 | 15 |
| Open (39) | 3 | 88 | 2 |

Figure 3. Reduced $\chi^2$ of the Southern hemisphere as a function of the North Pole definition in the most anomalous case of the isosceles configurations. The grey squares mark the standard $z$-axis.

Table 4. $\chi^2$ for the WMAP 5-year QVW data compared to simulations: correlation large–small scale. The top panel is for the LSS set and the bottom panel is for the LLS set.

4 CONCLUSIONS

In this paper, we used the needlet bispectrum to investigate the presence of anomalies in the WMAP 5-year data. For the first time, we exploited the bispectrum formalism analysing the triangular configurations according to their shape. By splitting the $\chi^2$ analysis of the needlet bispectrum for the Northern and Southern hemispheres, we found that the Southern sky is barely compatible with the Gaussian hypothesis while the Northern hemisphere is characterized by a lack of global bispectrum signal. This is complementary to what was found by applying different statistics: power spectra (Hansen et al. 2009), bispectrum (Land & Magueijo 2005) and $n$-point correlation functions (Eriksen et al. 2005). We distinguished equilateral, isosceles, scalene and open configurations and compared the power present in the data to random Gaussian simulations. The most anomalous signals in the Southern Galactic hemisphere arise in the squeezed configurations (isosceles, large–small–small) and in the very co-linear configurations (open, large–large–small). This kind of information should be useful as a means of finding out the physical origin of the anomalies. While the large squeezed signal hints

© 2009 The Authors. Journal compilation © 2009 RAS, MNRAS 402, L34–L38

Downloaded from https://academic.oup.com/mnrasl/article-abstract/402/1/L34/1747473 on 27 July 2018
at a local type of non-Gaussianity, this is not confirmed when an optimal estimator tuned specifically to this type of non-Gaussianity is used. We investigated the effect of hemispherical asymmetry on the measurement of $f_{NL}$ finding no significant discrepancy between north and south. As consistency check, we performed the same tests (anomalies and $f_{NL}$ estimates) with a different needlet parameter ($B = 3.5$) and for the channels Q, V and W separately and found consistent results.

ACKNOWLEDGMENTS

We thank Frode K. Hansen, Michele Liguori and Sabino Matarrese for providing us with the primordial non-Gaussian map data set. We are grateful to Domenico Marinucci and Marcella Veneziani for useful discussions. The ASI contract LFI activity of Phase2 is acknowledged.

REFERENCES

Ackerman L., Carroll S. M., Wise M. B., 2007, Phys. Rev. D, 75, 083502
Baldi P. et al., 2006, Ann. Statistics, 37, 1150
Bartolo N. et al., 2004, Phys. Rep., 402, 103
Bennett C. L. et al., 2003, ApJS, 148, 1
Bernui A., Reboucas M. J., 2009, Phys. Rev. D, 79, 063528
Bernui A. et al., 2007, Int. J. Modern Phys. D, 16, 411
Cabella P. et al., 2004, Phys. Rev. D, 69, 063007
Copi C. et al., 2007, Phys. Rev. D, 75, 023507
Cruz M. et al., 2005, MNRAS, 356, 29
Curto A. et al., 2008, A&A, 486, 383
Curto A., Martinez-Gonzalez E., Barreiro R. B., 2009, ApJ, 706, 399
de Oliveira-Costa A. et al., 2004, Phys. Rev. D, 69, 063516
de Troia G. et al., 2007, ApJ, 670, L73
Ericcek A. I., Kamionkowski M., Carroll S. M., 2008, Phys. Rev. D, 78, 123520
Eriksen H. K. et al., 2004, ApJ, 605, 14
Eriksen H. K. et al., 2005, ApJ, 622, 58
Fay G. et al., 2008, Phys. Rev. D, 78, 083013
Fergusson J. R., Shellard E. P. S., 2009, Phys. Rev. D, 80, 043510
Geller D., Marinucci D., 2008, preprint (arXiv:0811.2935)
Groeneboom N. E., Eriksen H. K., 2009, ApJ, 690, 1807
Guilhoux F., Fay G., Cardoso J.-F., 2007, preprint (arXiv:0706.2598)
Hansen F. K., Banday A. J., Górski K. M., 2004a, MNRAS, 354, 641
Hansen F. K. et al., 2004b, ApJ, 607, L67
Hansen F. K. et al., 2009, ApJ, 704, 1448
Hinshaw G. et al., 2009, ApJS, 180, 225
Hofu J. et al., 2009, ApJ, 699, 985
Jarosik N. et al., 2007, ApJS, 170, 263
Komatsu E. et al., 2009, ApJS, 180, 330
Lan X., Marinucci D., 2008, Electron. J. Statistics, 2, 332
Land K., Magueijo J., 2005, MNRAS, 357, 994
Land K., Magueijo J., 2007, MNRAS, 378, 153
Lew B., 2008, J. Cosmology Astroparticle Phys., 09, 023
Liguori M. et al., 2007, Phys. Rev. D, 76, 105016
McEwen J. D. et al., 2008, MNRAS, 388, 659
Marinucci D. et al., 2008, MNRAS, 383, 539
Moudden Y. et al., 2005, EURASIP J. Application Signal Processing, 15, 2437
Mukherjee P., Wang Y., 2004, ApJ, 613, 51
Narcowich F. J., Petrushev P., Ward J. D., 2006, SIAM J. Math. Analysis, 38, 574
Nolta M. R. et al., 2009, ApJS, 180, 296
Park C.-G., 2004, MNRAS, 349, 313
Pietrobon D., Balbi A., Marinucci D., 2006, Phys. Rev. D, 74, 043524
Pietrobon D. et al., 2008, Phys. Rev. D, 78, 103504
Pietrobon D. et al., 2009, MNRAS, 396, 1682
Ráth C. et al., 2009, Phys. Rev. Lett., 102, 131301
Rossmanith G. et al., 2009, MNRAS, 399, 1921
Rudjord Ø. et al., 2009a, ApJ, 701, 369
Rudjord Ø. et al., 2009b, preprint (arXiv:0906.3232)
Slosar A. et al., 2008, J. Cosmology Astroparticle Phys., 8, 31
Smith K. M., Senatore L., Zaldarriaga M., 2009, J. Cosmology Astroparticle Phys., 09, 06
Spergel D. N., Goldberg D. M., 1999, Phys. Rev. D, 59, 103001
Spergel D. N. et al., 2007, ApJS, 170, 377
Vielva P. et al., 2004, ApJ, 609, 22

This paper has been typeset from a TeX/LaTeX file prepared by the author.