Global and local perspectives of gravitationally assisted negative–phase–velocity propagation of electromagnetic waves in vacuum

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Abstract

Consistently with the Einstein equivalence principle and using an electromagnetic formulation first suggested by Tamm, we show that a local observer cannot observe negative–phase–velocity (NPV) propagation of electromagnetic waves in vacuum, whereas a global observer can appreciate that phenomenon. Using the specific example of the Kerr metric, we also demonstrate the possibility of NPV propagation within the ergosphere of a rotating black hole.

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1 Introduction

Recently, we have shown that electromagnetic plane waves can propagate in classical vacuum in such a way that the phase velocity vector has a negative projection on the time–averaged Poynting vector, provided that the vacuum is nontrivially affected by a gravitational field [1]. Our approach is based upon Tamm’s electromagnetic formulation [2], involving a gravito–magnetic metric. The negative–phase–velocity (NPV) propagation characteristic depends

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on the direction of the propagation wavevector but not on the frequency. NPV propagation characteristics in certain homogeneous material mediums lead to the phenomenon of negative refraction [3]–[5], which suggests the possibility of gravitationally assisted negative refraction of electromagnetic waves by vacuum, with attendant scientific implications [1],[6].

Researchers have studied electromagnetic wave propagation in terrestrial environments for several centuries, and our planet is certainly affected by the solar gravitational field. Yet, NPV propagation in classical vacuum has never been reported in the terrestrial context. Although the solar gravitational field evidently does not satisfy the conditions for NPV propagation that were reported elsewhere [1], evidence accumulated from the Pioneer 10 mission has prompted suggestions that new physics may be found even at the outer reaches of our solar system [7]. These and other considerations beg the question: Are global and local perspectives of gravitationally assisted NPV propagation different? In this communication, we answer that question in the affirmative.

2  Maxwell equations in universal spacetime

In the absence of charges and currents, electromagnetic fields obey the covariant Maxwell equations

\[ f_{\alpha\beta,\nu} + f_{\beta\nu,\alpha} + f_{\nu\alpha,\beta} = 0, \quad h^{\alpha\beta}{}_{\beta} = 0, \quad (1) \]

where \( f_{\alpha\beta} \) and \( h^{\alpha\beta} \) are, respectively, the covariant and the contravariant electromagnetic field tensors whereas the subscript \( _\nu \) indicates the covariant derivative with respect to the \( \nu \)th spacetime coordinate. The spacetime — denoted by the vector \( x^\alpha \) containing the normalized time coordinate \( x^0 = ct \) (with \( c \) as the maximum speed of light in the absence of the gravitational field) and the space coordinates \( x^1, x^2, \) and \( x^3 \) — is Riemannian, with the metric \( g_{\alpha\beta} \) being a function of spacetime and carrying the signature \((+,-,-,-)\) [8].

It is commonplace to follow up on a suggestion of Tamm [2],[9]–[11] and change the form of (1) for application to electromagnetic fields in vacuum. The Maxwell equations (1) may be expressed in noncovariant form as

\[ f_{\alpha\beta,\nu} + f_{\beta\nu,\alpha} + f_{\nu\alpha,\beta} = 0, \quad \left[ (-g)^{1/2} h^{\alpha\beta} \right]_{\beta} = 0, \quad (2) \]

wherein \( g = \det[g_{\alpha\beta}] \) and the subscript \( _\nu \) denotes ordinary differentiation with respect to the \( \nu \)th spacetime coordinate. We note that generalizing the Maxwell equations from noncovariant to covariant formulations is not totally unambiguous [12],[13]. In the absence of experimental evidence to eliminate this ambiguity, we adopt the standard generalization (1).

Let us introduce the electromagnetic field vectors \( E_\ell, B_\ell, D_\ell \) and \( H_\ell \) via the standard decompositions

\[ E_\ell = f_{\ell 0}, \quad B_\ell = (1/2)\varepsilon_{\ell mn} f_{mn}, \quad D_\ell = (-g)^{1/2} h^{\ell 0}, \quad H_\ell = (1/2)\varepsilon_{\ell mn} (-g)^{1/2} h^{mn} \quad (3) \]

\( \text{§} \) Roman indexes take the values 1, 2 and 3; Greek indexes take the values 0, 1, 2, and 3; summation is implied over any repeated index; and Gaussian units are used.
with $\varepsilon_{\ell mn}$ being the three–dimensional Levi–Civita symbol. Thereby the noncovariant Maxwell equations (2) assume the familiar form

$$
\begin{align*}
B_{\ell,\ell} &= 0, \\
B_{\ell,0} + \varepsilon_{\ell mn}E_{m,n} &= 0, \\
D_{\ell,\ell} &= 0, \\
-D_{\ell,0} + \varepsilon_{\ell mn}H_{m,n} &= 0
\end{align*}
$$

(4)

In vacuum, the components of the electromagnetic field tensors are connected by the constitutive relations

$$
\begin{align*}
h^{\alpha\beta} &= g^{\alpha\mu} g^{\beta\nu} f_{\mu\nu}, \\
f_{\alpha\beta} &= g^{\alpha\mu} g^{\beta\nu} h_{\mu\nu}.
\end{align*}
$$

(5)

These constitutive relations of vacuum can be stated for the electromagnetic field vectors as

$$
\begin{align*}
D_{\ell} &= \gamma_{\ell m} E_m + \epsilon_{\ell mn} \Gamma_m H_n \\
B_{\ell} &= \gamma_{\ell m} H_m - \epsilon_{\ell mn} \Gamma_m E_n
\end{align*}
$$

(6)

where

$$
\begin{align*}
\gamma_{\ell m} &= -(-g)^{1/2} \frac{g_{\ell m}}{g_{00}}, \\
\Gamma_m &= \frac{g_{0m}}{g_{00}}.
\end{align*}
$$

(7)

Equations (1) and (5) employ curved spacetime. So do (4) and (6), but the difference is that they look like the familiar electromagnetic equations in flat spacetime applied to an instantaneously reacting, bianisotropic medium. Techniques commonly employed to handle electromagnetic problems in the absence of gravitational fields should therefore be useful for solving (4) and (6). Before proceeding further with our analysis, let us therefore recast (4) and (6) using the conventional 3–vectors and 3×3 dyadics as

$$
\begin{align*}
\nabla \times E(ct, r) + \frac{\partial}{\partial t} B(ct, r) &= 0 \\
\nabla \times H(ct, r) - \frac{\partial}{\partial t} D(ct, r) &= 0
\end{align*}
$$

(9)

and

$$
\begin{align*}
D(ct, r) &= \varepsilon_0 \gamma(ct, r) \cdot E(ct, r) - \frac{1}{c} \Gamma(ct, r) \times H(ct, r) \\
B(ct, r) &= \mu_0 \gamma(ct, r) \cdot H(ct, r) + \frac{1}{c} \Gamma(ct, r) \times E(ct, r)
\end{align*}
$$

(10)

wherein $\gamma(ct, r)$ is the dyadic–equivalent of $\gamma_{\ell m}$, $\Gamma(ct, r)$ is the vector–equivalent of $\Gamma_m$ and SI units are adopted.

### 3 Global perspective

Suppose that we wish to solve these equations in a certain region $\mathcal{X}$ of spacetime, subject to specific boundary conditions. A fairly standard procedure would be to partition $\mathcal{X}$ into
subregions \( ^{(n)} \mathcal{X} \), \((n = 1, 2, 3, \ldots)\), in each of which we would replace the nonuniform metric \( g_{\alpha \beta} \) by the uniform metric \( \tilde{g}_{\alpha \beta} \). Correspondingly, the nonuniform vector with components \( \Gamma^\ell_\mu \) would be replaced by the uniform vector with components \( \tilde{\Gamma}^\ell_\mu \) in each subregion \( ^{(n)} \mathcal{X} \). The spacetime \( x^a \) would still remain curved. After solving (4) and (6) in each subregion, we could stitch back the subregional solutions into the regional solution. This piecewise uniform approximation technique is very common for solving differential equations with nonhomogeneous coefficients [14].

Accordingly, let us examine the electromagnetic fields in the \( n \)th subregion. A three-dimensional Fourier transform of the electromagnetic field vectors can be taken, with the wavevector \( \mathbf{k} \) denoting the Fourier variable corresponding to \( r \); thus,

\[
\begin{align*}
E(ct, \mathbf{r}) &= \frac{1}{c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\omega/c, \mathbf{k}) \exp \left[i(\mathbf{k} \cdot \mathbf{r} - \omega t)\right] d\omega dk_1 dk_2 \\
H(ct, \mathbf{r}) &= \frac{1}{c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\omega/c, \mathbf{k}) \exp \left[i(\mathbf{k} \cdot \mathbf{r} - \omega t)\right] d\omega dk_1 dk_2
\end{align*}
\]

where \( i = \sqrt{-1} \) and \( \omega \) is the usual temporal frequency. The wavevector component \( k_3 \) is determined by first substituting (11) in (9), and then formulating a \( 4 \times 4 \) matrix ordinary differential equation which is then solved as an eigenvalue problem [15].

As our focus lies here on propagating plane waves, evanescent (nonuniform) solutions are eliminated from further consideration by enforcing the restriction \( k_3 \in \mathbb{R} \). We remark that \( E(\omega/c, \mathbf{k}) \) and \( H(\omega/c, \mathbf{k}) \) are the complex–valued phasors of the electric and magnetic fields, respectively, associated with a plane wave, for which a time–averaged Poynting vector \( \langle \mathbf{P}(\omega/c, \mathbf{k}) \rangle_t \) can be derived. When the projection of \( \mathbf{k} \) on \( \langle \mathbf{P}(\omega/c, \mathbf{k}) \rangle_t \) is negative, i.e., \( \mathbf{k} \cdot \langle \mathbf{P}(\omega/c, \mathbf{k}) \rangle_t < 0 \), we say that the phase velocity is negative.

The crucial quantity \( \mathbf{k} \cdot \langle \mathbf{P}(\omega/c, \mathbf{k}) \rangle_t \) is derived as follows [1]: Combining the Fourier representations (11) with the constitutive relations (10) and the Maxwell curl postulates (9), we get

\[
\begin{align*}
p \times E(\omega/c, \mathbf{k}) &= \omega \mu_0 \tilde{\gamma} \cdot H(\omega/c, \mathbf{k}), \\
p \times H(\omega/c, \mathbf{k}) &= -\omega \epsilon_0 \tilde{\gamma} \cdot E(\omega/c, \mathbf{k})
\end{align*}
\]

where

\[
p = \mathbf{k} - \frac{\omega}{c} \tilde{\Gamma}.
\]

The use of (12) to eliminate \( H(\omega/c, \mathbf{k}) \) from (13) provides, after some manipulation, the eigenvector equation

\[
\left\{ \left[ \left( \frac{\omega}{c} \right)^2 |(n)\tilde{\gamma}| - p \cdot (n)\tilde{\gamma} \cdot p \right] I + pp \cdot (n)\tilde{\gamma} \right\} \cdot E(\omega/c, \mathbf{k}) = 0
\]

for representing \( E(\omega/c, \mathbf{k}) \), and the corresponding dispersion relation

\[
\left[ p \cdot (n)\tilde{\gamma} \cdot p - \left( \frac{\omega}{c} \right)^2 |(n)\tilde{\gamma}| \right]^2 = 0.
\]
Herein, \( \mathbb{I} \) is the identity dyadic, and \(|(n)\tilde{\gamma}|\) is the determinant of \((n)\tilde{\gamma}\). Considering (16) in light of (17), we see that all \( \mathbf{E}(\omega/c, \mathbf{k}) \) eigenvectors solutions must satisfy the relation

\[
p \cdot (n)\tilde{\gamma} \cdot \mathbf{E}(\omega/c, \mathbf{k}) = 0.
\] (18)

Let us introduce the eigenvalues \((n)\tilde{\gamma}_{1,2,3}\) and corresponding eigenvectors \((n)\tilde{\nu}_{1,2,3}\) of \((n)\tilde{\gamma}\). For the purposes of illustration, suppose we consider planewave propagation along the direction parallel to \((n)\tilde{\nu}_3\), while the vector \((n)\tilde{\gamma}\) lies at an angle \(\theta\) in the plane of the eigenvectors \((n)\tilde{\nu}_1\) and \((n)\tilde{\nu}_3\), as per

\[
\begin{align*}
(n)\tilde{\Gamma}_1 &= (n)\tilde{\Gamma} \sin \theta \\
(n)\tilde{\Gamma}_2 &= 0 \\
(n)\tilde{\Gamma}_3 &= (n)\tilde{\Gamma} \cos \theta
\end{align*}
\] (19)

After exploiting the orthogonality condition (18), the general solution to (16) may be expressed as the sum of two independent modes as

\[
\mathbf{E}(\omega/c, \mathbf{k}) = A_a(\omega/c, \mathbf{k}) \mathbf{e}_a(\omega/c, \mathbf{k}) + A_b(\omega/c, \mathbf{k}) \mathbf{e}_b(\omega/c, \mathbf{k})
\] (20)

where

\[
\begin{align*}
\mathbf{e}_a(\omega/c, \mathbf{k}) &= \frac{1}{(n)\tilde{\mu}_0} \cdot \left[ \left( \frac{\omega}{c} (n)\tilde{\Gamma} \cos \theta - |\mathbf{k}| \right) \mathbf{e}_1 + \frac{\omega}{c} (n)\tilde{\Gamma} \sin \theta \mathbf{e}_3 \right] \\
\mathbf{e}_b(\omega/c, \mathbf{k}) &= \frac{1}{(n)\tilde{\mu}_0} \cdot \left[ \left( |\mathbf{k}| - \frac{\omega}{c} (n)\tilde{\Gamma} \cos \theta \right)^2 \mathbf{e}_3 + \left( \frac{\omega}{c} (n)\tilde{\Gamma} \sin \theta \right)^2 \mathbf{e}_1 \right]
\end{align*}
\] (21)

The complex–valued scalars \(A_{a,b}\) are unknown amplitude functions that can be determined from initial and boundary conditions. The corresponding general solution for \(\mathbf{H}(\omega/c, \mathbf{k})\) follows straightforwardly by combining (20) with the Maxwell curl postulates (9) as

\[
\mathbf{H}(\omega/c, \mathbf{k}) = A_a(\omega/c, \mathbf{k}) \mathbf{h}_a(\omega/c, \mathbf{k}) + A_b(\omega/c, \mathbf{k}) \mathbf{h}_b(\omega/c, \mathbf{k})
\] (22)

with

\[
\begin{align*}
\mathbf{h}_a(\omega/c, \mathbf{k}) &= \frac{1}{(n)\tilde{\mu}_0} \cdot \left[ \left( \frac{\omega}{c} (n)\tilde{\Gamma} \cos \theta - |\mathbf{k}| \right) \mathbf{h}_1 + \frac{\omega}{c} (n)\tilde{\Gamma} \sin \theta \mathbf{h}_3 \right] \\
\mathbf{h}_b(\omega/c, \mathbf{k}) &= \frac{1}{(n)\tilde{\mu}_0} \cdot \left[ \left( |\mathbf{k}| - \frac{\omega}{c} (n)\tilde{\Gamma} \cos \theta \right)^2 \mathbf{h}_3 + \left( \frac{\omega}{c} (n)\tilde{\Gamma} \sin \theta \right)^2 \mathbf{h}_1 \right]
\end{align*}
\] (23)

The dispersion equation (17) reduces to a \(|\mathbf{k}|\)–quadratic expression which yields the two wavenumbers

\[
|\mathbf{k}| = \frac{\omega}{c} \left( (n)\tilde{\Gamma} \cos \theta \pm \sqrt{(n)\tilde{\gamma}_1 (n)\tilde{\gamma}_2 - \frac{(n)\tilde{\gamma}_1}{(n)\tilde{\gamma}_3} (n)\tilde{\Gamma}^2 \sin^2 \theta} \right).
\] (24)
Finally, combining (20), (22) with (24), we find that

\[
\begin{align*}
\mathbf{k} \cdot \langle \mathbf{P}(\omega/c, \mathbf{k}) \rangle_t &= \frac{1}{2\mu_0} \left[ \langle \langle (\mathbf{n})\tilde{\gamma}_1 \rangle (\mathbf{n})\tilde{\gamma}_2 - \langle (\mathbf{n})\tilde{\gamma}_1 \rangle (\mathbf{n})\tilde{\gamma}_3 \rangle \rangle \langle (\mathbf{n})\tilde{\gamma}_1 \rangle (\mathbf{n})\tilde{\gamma}_2 - \langle (\mathbf{n})\tilde{\gamma}_1 \rangle (\mathbf{n})\tilde{\gamma}_3 \rangle \rangle \right] \\
&\pm \langle \langle \langle \mathbf{n} \rangle \tilde{\Gamma} \rangle \rangle \langle (\mathbf{n})\tilde{\gamma}_1 \rangle (\mathbf{n})\tilde{\gamma}_2 - \langle (\mathbf{n})\tilde{\gamma}_1 \rangle (\mathbf{n})\tilde{\gamma}_3 \rangle \rangle \right] \\
&\times \left( |\mathbf{A}_a(\omega/c, \mathbf{k})|^2 + \langle (\mathbf{n})\tilde{\gamma}_3 \rangle \omega^2 |\mathbf{A}_b(\omega/c, \mathbf{k})|^2 \right). \
\end{align*}
\] (25)

Let us probe the physical significance of (25). To do so, we focus upon a specific gravito-magnetic metric, namely the Kerr metric. We begin by noting that for a rotating black hole of geometric mass \(m_{rbh}\) and angular velocity \(a_{rbh}\) in conventional units [16], the Kerr metric yields

\[
\langle (\mathbf{n})\tilde{\gamma} \rangle = \frac{\langle (\mathbf{n})\tilde{R}^4 + (a_{rbh})^2 (\mathbf{n})\tilde{z}^2 \rangle^2}{\langle (\mathbf{n})\tilde{R}^4 + (a_{rbh})^2 (\mathbf{n})\tilde{z}^2 - 2m_{rbh} (\mathbf{n})\tilde{R}^3 \rangle^2},
\] (26)

and

\[
\langle (\mathbf{n})\tilde{\gamma}_1 \rangle = \langle (\mathbf{n})\tilde{\gamma}_2 \rangle = \frac{\langle (\mathbf{n})\tilde{R}^4 + (a_{rbh})^2 (\mathbf{n})\tilde{z}^2 \rangle}{\langle (\mathbf{n})\tilde{R}^4 + (a_{rbh})^2 (\mathbf{n})\tilde{z}^2 - 2m_{rbh} (\mathbf{n})\tilde{R}^3 \rangle}, \quad \langle (\mathbf{n})\tilde{\gamma}_3 \rangle = 1,
\] (27)

where \((\mathbf{n})\tilde{x}\), \((\mathbf{n})\tilde{y}\), and \((\mathbf{n})\tilde{z}\) are the values of the three space coordinates at some representative point in the subregion \((\mathbf{n})\mathcal{X}\). The quantity \((\mathbf{n})\tilde{R}\) is defined implicitly via

\[
\langle (\mathbf{n})\tilde{R}^2 \rangle = \langle (\mathbf{n})\tilde{x}^2 \rangle + \langle (\mathbf{n})\tilde{y}^2 \rangle + \langle (\mathbf{n})\tilde{z}^2 \rangle - a_{rbh}^2 \left[ 1 - \left( \frac{\langle (\mathbf{n})\tilde{z} \rangle}{\langle (\mathbf{n})\tilde{R} \rangle} \right)^2 \right].
\] (28)

Since both \((\mathbf{n})\tilde{\gamma} \rangle > 0\) and \((\mathbf{n})\tilde{\gamma}_3 \rangle > 0\), we see from (25) that NPV propagation in \((\mathbf{n})\mathcal{X}\) arises provided that the inequality

\[
\langle (\mathbf{n})\tilde{\gamma} \rangle - \langle (\mathbf{n})\tilde{\Gamma} \rangle \cdot \langle (\mathbf{n})\tilde{\gamma} \rangle \cdot \langle (\mathbf{n})\tilde{\Gamma} \rangle < 0
\] (29)

is satisfied.

It is illuminating to recast (29) in terms of the metric components as per

\[
\frac{\langle (\mathbf{n})\tilde{g}^{00} \rangle}{\langle (\mathbf{n})\tilde{g}_{00} \rangle} < 0.
\] (30)

For the Kerr metric we have [16]

\[
\frac{\langle (\mathbf{n})\tilde{g}^{00} \rangle}{\langle (\mathbf{n})\tilde{g}_{00} \rangle} = \frac{(\langle (\mathbf{n})\tilde{R}^4 + (a_{rbh})^2 (\mathbf{n})\tilde{z}^2 + 2m_{rbh} (\mathbf{n})\tilde{R}^3 \rangle)}{(\langle (\mathbf{n})\tilde{R}^4 + (a_{rbh})^2 (\mathbf{n})\tilde{z}^2 - 2m_{rbh} (\mathbf{n})\tilde{R}^3 \rangle)^2}. \quad (31)
\]
Within the ergosphere — which lies between the outer event horizon and the stationary limit surface — we have

\[ m_{rbh} + \sqrt{m_{rbh}^2 - a_{rbh}^2} < (n)\tilde{R} < m_{rbh} + \sqrt{m_{rbh}^2 - a_{rbh}^2 \left(\frac{\tilde{z}}{(n)\tilde{R}}\right)^2}, \]  

(32)

and \( m_{rbh}^2 > a_{rbh}^2 \). Therefore, inside the ergosphere

\[ \begin{align*}
(n)\tilde{R}^4 + a_{rbh}^2 (n)\tilde{z}^2 - 2m_{rbh} (n)\tilde{R}^3 &< 0 \\
(n)\tilde{R} &> 0
\end{align*} \]

(33)

and the NPV condition (30) is evidently satisfied for the chosen wave-propagation case.

Thus, when the traversal of an electromagnetic signal is traced from point \( P \in (p)\mathcal{X} \) to point \( Q \in (q)\mathcal{X} \), with \( p \neq q \), the possibility of NPV propagation in some subregions of \( \mathcal{X} \) cannot be ruled out \textit{a priori}.

4 Local perspective

The previous two sections are from a global perspective. Let us now attend to the local perspective of an observer located at some point \( \wp \). This observer is constrained to formulate an electromagnetic theory valid only in some small neighborhood of \( \wp \), and this theory must emerge from observations made only in that neighborhood. The gravitational field in this neighborhood can be held to be virtually uniform, the metric from the global perspective being \( g_{\alpha\beta}\big|_\wp \). However, given the admissibility of the uniform-gravity theory for all local observers in this neighborhood, we show here that the local observer would end up formulating a local metric — which is the same as in the special theory of relativity.

By virtue of the Einstein equivalence principle [17], \( g_{\alpha\beta}\big|_\wp \) is constrained to be such that there exists a matrix \( \Lambda^\alpha_\beta \) yielding

\[ \eta_{\mu\nu} = \Lambda^\alpha_\mu \ g_{\alpha\beta}\big|_\wp \ \Lambda^\beta_\nu, \]

(34)

where \( \eta_{\mu\nu} = \text{diag} \ [1, -1, -1, -1] \) is the Lorentzian spacetime metric [8]. The matrix \( \Lambda^\alpha_\beta \) can be used to construct a flat spacetime \( x'^\alpha \) for all \( x^\alpha \) in the specific neighborhood of \( \wp \) as per

\[ \Lambda^\alpha_\beta = \frac{\partial x'^\alpha}{\partial x^\beta}, \]

(35)

\[ \left\{ \begin{align*}
dx^0dx^1dx^2dx^3 &= \det \left[ \Lambda^\alpha_\beta \right] \ dx^0dx^1dx^2dx^3.
\end{align*} \]

We must note that, as \( \Lambda^\alpha_\mu \) depends upon \( g_{\alpha\beta}\big|_\wp \) and therefore on \( x^\alpha\big|_\wp \), a global coordinate transformation which simultaneously transforms \( g_{\alpha\beta} \) into \( \eta_{\mu\nu} \) for all \( x^\alpha \in \mathcal{X} \) cannot be realized unless gravity is totally ignored [8].
In the local coordinate system $x'^\alpha$, the spacetime metric in the specific neighborhood is given as $\eta_{\alpha\beta}$. Repeating the steps outlined in Section 2 with $\eta_{\alpha\beta}$ substituting for $g_{\alpha\beta}$, we find that the local constitutive relations for the neighborhood reduce to the trivial form

\[
\begin{align*}
D'_\ell &= E'_\ell \\
B'_\ell &= H'_\ell
\end{align*}
\]

with respect to $x'^\alpha$. NPV propagation is not possible for the medium characterized by the constitutive relations (36) [18].

5 Concluding remarks

Thus, we have shown that gravitationally assisted negative–phase–velocity propagation in vacuum can be appreciated only from a global perspective based on curved spacetime; whereas a local observer, constrained to a flat spacetime, must conclude NPV propagation is impossible in vacuum. The diversity of the local and the global perspectives is in full accord with the Einstein equivalence principle, and can be explained as follows: The subregional metric $(m)\tilde{g}_{\alpha\beta}$ appears from a suitable subregional averaging of $g_{\alpha\beta}$. Whereas $g_{\alpha\beta}$ must satisfy the dictates of the Einstein equivalence principle at every $x^\alpha \in \mathcal{X}$, there is no reason for $(m)\tilde{g}_{\alpha\beta}$ to satisfy the same dictates at even one $x^\alpha \in (m)\mathcal{X}$.

The feasibility of NPV propagation within the ergosphere of a rotating black hole has been demonstrated in this communication. A detailed numerical study is called for in order to further explore the circumstances and implications of such type of propagation, which we hope to report in due course of time.

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