the quantum vacuum, fractal geometry, 
and the quest for a new theory of 
gravity

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Abstract

In this letter recent developments are shown in experimental and theoretical physics which brings into question the validity of General Relativity. This letter emphasizes the construction of a fractal $3 + \phi^3$ spacetime, in $N$-dimensions in order to formalize a physical and consistent theory of ‘quantum gravity.’ It is then shown that a ‘quantum gravity’ effect could arise by means of the Strong Equivalence Principle. Which is made possible through a pressure of the form $-\kappa(R_{ab}^{ca} - \frac{1}{2}g_{ab}R^c) = \kappa T_{ab}^{\phi}$. Where it is seen that nuclear pressures can be added to the gravitational field equations by means of twistor spaces.

Keywords: Fractal Geometry, New Relativity, quantum vacuum, EPR, zeropoint field, quantum gravity, Mach’s Principle, Holographic Principle, Fermat’s Last Theorem, alternative gravity, Bohmian Mechanics.

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1 Introduction

Recently the validity of General Relativity (GR) has been brought to question by Yilmaz [4], et al. Although such interpretations allow for gravitation to be mathematically consistent and singularity free. Such revisions fail to describe

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the behavior of test particles as adequately as GR, elevating GR as the correct theory. Today certain questions about GR remain relevant, such as how does it relate to vacuum energy and quantum mechanics in general. It has been shown in previous works that GR remains self consistent when including the quantum vacuum, or zero-point field. However, the search for a self consistent theory of “quantum gravity,” remains a major theoretical challenge today. Among the theoretical arguments against the standard interpretation of GR is the choice of mathematical coordinate systems. Special Relativity (SR), is based upon the structure of a flat Minkowski spacetime given in a four-dimensional coordinate system. Recently attempts have been made describing coordinate systems with fractal spaces as opposed to natural ones. Such an adaptation as the case with the Yilmaz approach eliminates singularities within the field equations.

Recent observational and experimental data have also put into question the validity of GR. The National Aeronautics and Space Administration (NASA) has reported an anomalous acceleration of \(\pm 8.5 \times 10^{-8}\) cm on spacecraft on the outer edge of the solar system. This data was obtained from information gathered by the Jet Propulsion Laboratories (JPL), and the Deep Space Network (DSN). Thus far, no satisfactory conclusion has been given to explain the so called “anomalous acceleration towards the sun.” Not only have spacecraft provided some fundamental flaws with gravitation, but laboratory results as well. Dr. Eugene Podkletnov, has reported a “gravitational shielding” effect with composite bulk \(YBa_2Cu_3O_{7-x}\) ceramic plates. In light of all of these developments it is hard to consider GR as the correct theory. It is the opinion of the author that GR is a theory that “works,” however it doesn’t necessarily make it the correct theory.

The goal of this letter is to show that GR is not the correct theory of gravitation, but just works exceptionally well. Just as previously Newton’s law of universal gravitation worked exceptionally well. This letter is not intended to be a replacement for GR, nor is it intended to present theoretical flaws of the that theory. This letter is only presented as an introductory work for an alternative theory of gravitation. The general theme of this letter is given by the following postulates:

**Postulate 1** *(virtual gravitation).* Spacetime is not a null energy field, it consist of asymptotic vacuum fluctuations, and behaves as a virtual “energy-sheet.”

**Postulate 2** *(planckian invariance).* The Planck length is a gauge invariant function for all (interacting) brane observers. [an adaptation to the postulate of New Relativity.]

This letter is presented in the following format in section a brief introduction into unified field theories are given. In section a few quantum gravity approaches are introduced. In section fractal geometry is introduced and its relations to a complex system are given. In section the meaning of fractal geometry for QED is discussed. In section the meaning of a fractal geometry is discussed for QCD. In section a new theoretical particle is introduced utilizing
fractal geometry. In section 4.1 a relationship between N-dimensional and two-dimensional systems are given. In section 3 a philosophy of geometry is given. In section 4.1 the effects of the quantum vacuum are discussed. In section 5.2 a relationship between fractal geometries and the quantum vacuum are discussed. In section 5.3 the meaning of Feynman diagrams are discussed. In section 5.4 the validity of quantum mechanics is brought into question. In section 5.5 an alternative description of gravity is given which may explain the EPR paradox. In section 6 an overview of a canonical non Riemannian gravitational field is given. In section 6.1 the planck length results as a function in canonical quantum gravity. In section 7 a possible alternative for the “anomalous acceleration” of spacecraft is given. In section 7.1 pseudo geodesic equations are presented. In section 8 a general discussion of this work is presented. In section 8.1 a discussion of the meaning of the planck length is given. The conclusions of this work is drawn in section 9, which gives stronger definitions to equivalence principle in Appendix A. Finally it is suggested that there may exist a detectable from of “Yang-Mills Gravity” in Appendix B.

2 Unification a brief history

“I am convinced that He [God] does not play dice.”

–A. Einstein

“Einstein, quit telling God what to do.”

–N. Bohr

The unification of gravitation with quantum mechanics began with Einstein’s objections to the newly developed quantum theory. Although acknowledging the successes of the new theory he believed it to be incomplete. Einstein was convinced that there was a deeper theory involved, one which would also include GR, a unified field theory was christened. Soon came the work of Kaluza and Klien, giving a pseudo mathematical unification of electromagnetism and gravitation. The theory would soon die out and lose interest, until quantum mechanics came around. And asked the simple question, how does gravity behave at the quantum level; the answer Kalzua-Klien gravity [10]. Research in this area soon exploded, extra dimensions were soon added to the field equations, superstring theory was born. Particle physics began unifying fundamental forces as well, the weak force, the strong force, the electromagnetic force. But no gravitational force, Cosmologists helped out, the big bang and nuclearsynthesis would help to explain the problem. Soon physics became littered with Grand Unified Theories (GUTs) and Theories of Everything (TOEs), they all have the approach of a “unified field theory.” However, they missed the simple point Einstein was trying to make, how do quantum mechanics and relativity relate? This is a hierarchical question, the relevant question is how do macroscopic and microscopic worlds communicate?
2.1 quantum gravity

Historically there have been two models formulated for the construction of a consistent quantum gravity theory. They are the canonical and Hamiltonian approaches [11]. These two approaches have had limited success, however more recently the theory of loop quantum gravity [12] has been introduced into the sea. Out of the three approaches presented loop quantum gravity is generally accepted as the correct approach. However, for a more accurate description of the historic developments of quantum gravity see [13]. In this letter I will focus on the canonical approach as it relates to gauge invariance.

3 Fractal Geometry

“...If we’re built from Spirals while living in a giant Spiral, then is it possible that everything we put our hands to is infused with the Spiral?”

–Max Cohen in the motion picture π

The presence of matter within GR disturbs the field equations by the existence of singularities or “point-particles.” How can one avoid this eyesore in the equations, simple fractal geometry. If matter is fractal it can not condense into points, however this allusion can still take place above the planck energy scale. It is hard to believe that this simple approach has only been attempted in recent times, fractal sets are more common in nature than simple polygons. First let us begin with a simple construction of a fractal set with the simple equation $Z(n) = (Z(n-1))^2 + C$. One must also realize that a fractal is composed of a complex number system, i.e. $a+ib$. Using this form one may wish to construct a complex averaging of the mean, which results from the golden mean $\phi = \sqrt{5-1}/\sqrt{(5-1)}$. Thus we have constructed a complex mean which has two possible solutions as seen below:

$$C_m = \frac{\sqrt{a-1}}{\sqrt{(a-1)}} + \frac{\sqrt{b-1}}{\sqrt{(b-1)}} = \begin{cases} \frac{\sqrt{a+2}}{\sqrt{(a+2)}}; \frac{\sqrt{b}}{\sqrt{(b)}} & \Leftrightarrow a = c \\ \frac{\sqrt{a}}{\sqrt{(a)}}; \frac{\sqrt{b+2}}{\sqrt{(b+2)}} & \Leftrightarrow a \neq c \end{cases}$$

(1)

this complex mean thus has non communcating solutions. Which from the standpoint of imaginary numbers yields the general statements:

$$\frac{\sqrt{-n}}{\sqrt{(n)}} = i \Leftrightarrow \frac{\sqrt{(n)}}{\sqrt{-n}} = -i$$

(2)

Thus these statements would appear to be in agreement of the theory of quaternions. Which is interesting enough in itself, a four-dimensional version of the complex number system.

With this preliminary work set we can now construct a Minkowski spacetime which that takes advantage of fractal dimensions. First one can construct the generalized three-dimensional manifold as a three-brane, and thus incorporating
time as a fractal set. Thus the fractal construction on spacetime is presented in the form \(3 + \phi^3\) a similar approach was made in Ref. [4]. It is here postulated the origin for this three-dimensional brane arises from planck scaling. The reason for this postulate is seen when the golden mean is applied to \(N\)-dimensions \(\phi_n = \sqrt{N-1}/\sqrt{(N-1)}\), the higher \(n\) the closer the mean is to 1. Thus only when \(n\) approaches infinity will we see that \(3 + \phi^3\) will yield the standard Minkowski space of \(3+1\) dimensions. This is of course in agreement with any system that we apply \(c=\hbar=\kappa=K=1\) more evidently this corresponds to a time dilation effect in terms of special relativity. Such that we have the following revision to the flat four-dimensional Minkowski space:

\[
(\omega, z^2) = \omega e^2 - (\phi)z^2_1 - (\phi)z^2_2 - (\phi)z^2_3
\]  

It is interesting to note that this pseudo metric appears to be an inverse of the standard Minkowski spacetime, this importance is seen in section 5.5. So far this method has only left intriguing consequences, however it diverges from the point made earlier in this section.

In string theory the atomism view of matter is replaced with vibrating strings, these vibrating strings correspond to a real geometry. However, fractal geometries are allowed to break of these strings into imaginary components. These imaginary components thus make the string a complex function, yielding a pseudo point-like string. To analyze this premise allow us to view the Nambu-Goto action \(S = \mu_0 \int d^2 \xi \sqrt{-g}\). If strings can indeed be made to subside with fractal geometry, then they would break off into imaginary components by the empirical action:

\[
S = \aleph_0 \int \omega^2 \phi \sqrt{-g}
\]  

where \(g = \lambda^2 dz^2 \otimes d\bar{z}^2 \equiv x + iy\). This pseudo string forms many more string components in \(N\)-dimensions, that is the string fragment continues in infinitum. However, in the physical sense the string exist a pseudo point-like particle, do to the scaling nature of the planck length. These fractal strings then interact within a field, known as gravitational or zero-point fields. When the fractal strings converge with other fractals, a self organization takes place, i.e. the production of virtual particles. This production is made possible through the non communicating mathematics associated with quantum mechanics. As the particles are produced they destroy one another, such that their world-sheets reverberate in a complex form. This complex reverberations in \(N\)-dimensions is responsible for the vibration of the string, which we naively associate with mass.

### 3.1 QED fractality

Electromagnetic waves are the result of four-dimensional interactions, and its real wave would correspond to the results found in Classical Mechanics. However, it would intern have a fractal complex field, which would cause the field to break periodically yielding a string fragment, a quanta. This quanta in non
self regulating, i.e. it is the nature of the real wave which causes the string to reverberate. The above consideration may carry some controversy in the well known theory of Quantum Electrodynamics (QED). If a quanta is just a fractal string, then what is the proper approach for the exchange of energy between the two systems? Well, the result appears as classical approximation, the quanta hits the string as a solid body, causing a change in geometry, which virtual particles oppose. This causes the string to “bounce” back to its original form, emitting a real wave, but not necessarily a quanta, remember a quanta is given by a complex field. Since quantum mechanics is sketched out onto a point particle-like environment, its consequences would agree with the QED model. In fact the fractal model yields a much physical picture for non communicating relationships than the quantum theory.

3.2 QCD fractality

The above result would agree with QED, however, its definitions are quite weak, in fact one may expand these definitions to Quantum Chromodynamics (QCD). Thus a nucleon may be made to reflect electromagnetic radiation as well, however, this dose not defy the documented experiments in any magnitude. It would be the interaction of the system, i.e. what string perimeters give way to the colored, and other gauged forces, that yields its “particles”. Each string fragment consist of its own local vibrations (gauge invariance), which attributes its mass, i.e. differentiates between a Higgs particle and a quark. These states then have their own local statistics, their real waves would then correspond differently than the electromagnetic field. Which results in the production of the celebrated Yang-Mills field, and thus yields the production of colored particles such as the gluon.

4 Cardinal Strings

Cantor pioneered the study of infinities with his new theory of Cardinal numbers, however he faced opposition in his time for this new theory [14]. Cardinal numbers offer the best insight into to the study of fractal strings in N-dimensions, these interpretations in fact have direct physical consequences. Most notably it can explain the situation unleashed by the infamous EPR-Bell paradox, where Faster Than Light (FTL) communication appears possible (depending on planckian scaling). For recent theoretical implications and interpretation of the EPR-Bell paradox see [15]. In each scaling the laws of physics would be very different, and hence superluminal velocities would seem to appear in lower branes. Thus the traditional interaction of strings should not be limited on a specified dimension, but behave as a set of Cardinal numbers. Thus a slight revision of the Nambu-Goto action should be given which yields

$$S = \kappa_0 \int \omega^2 \phi \sqrt{-\tilde{g}}$$  \hspace{2cm} (5)
where \( \tilde{g} = -dt' \otimes dt' + [dr' \otimes dr' + \frac{1}{4}(\sin(2r'))^2 \Omega - 2]. \)

### 4.1 Fermat’s Last Theorem

"To divide a cube into two or other cubes, a fourth power, or, in general, any power whatever into two powers of the same denomination above the second is impossible..."

—Fermat

Fermat’s Last Theorem can be associated with fractal geometry in one respect, there is no general real solution to fractal geometry above dimension 2. This may be a simple coincidence and may have no deeper meaning, however, this is contrary to the recently created Holographic Principle (HP) \[\boxed{}\]. The HP relates that the Universe may exist in dimensions of infinitum status. However, the laws of physics are best projected onto a pseudo two-dimensional screen, and our three-dimensional world is only a pseudo manifestation of an \(N\)-dimensional continuum. In string theory we can view our universe as made up of two-dimensional branes (described by type IIA D2 membranes). Thus any other dimension outside the holographic conjecture yields no physical meaning and no solution. Just as what is suggested by Fermat’s Last Theorem, therefore our four dimensional slice of the brane is a pseudo physical manifestation of the holographic screen. There is only one explanation for this result, there must exist a physical constant for specified energy scales, i.e. the planck scale. Here another coincidence appears to arrive, the two dimensional wave equation for string theory \( \left( \frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) \chi^\mu(\sigma, \tau) = 0. \) It seems that both mathematically and physically there is a special importance with dimension 2. This discussion is largely philosophical, however it is interesting to note that cardinal strings are given by complex numbers. In fact a cardinal string in four-dimensions is remarkably similar to the two-dimensional form, and seems to correspond to a torus:

\[
S^2 = \aleph_0 \int d^2 \xi \sqrt{-g} + \aleph_0 \int d^2 \xi \sqrt{-g} \equiv \aleph_0 \int d^4(x \frac{1}{\lambda_{IIA}^2 t_{IIA}^4} M)
\]

(6)

From this complex structuring, and properties of cardinal numbers it can be seen why quaternions were alluded to in section \[\boxed{}\].

### 5 a matter of geometry

Einstein’s theory of GR transformed Newton’s theory of a gravitational force, to a direct consequence of geometry. However, although the idea of a force was replaced with a geometry, the geometry still yields a force when explained in Riemannian geometry. An even more radical approach to gravitation as a geometry was produced by Roger Penrose in his theory of twistor spaces. The geometry itself is more important than physical masses, in fact masses only come
important when one expands this theory. This is true for a fractal revision of string theory, it exists as a pure geometry, the interacting geometry in fact produces mass. This seems almost a radical stance from the point of view of GR, however geometry remains a key factor as the ideal of a Force to GR.

5.1 the vacuum

The vacuum exist from a state of virtual particles being produced via cardinal string fragments. Since virtual particles are a pure construction of “particles” in N-dimensions, they are not true strings (i.e. they violate the HP). These particles thus carry no mass-energy equivalent within our universe, never the less they still posses a geometry. This situation only holds true when the system is localized, however, when interacting with non cardinal strings can induce an energy exchange. By the well known Casimir effect the energy of the vacuum should be given by [20]:

$$\rho_{ZP}(\omega) = \frac{\hbar \omega^3}{2\pi^2 c^3}$$

and when interacting with an inertial mass system we have:

$$m_i = \frac{V_0}{c^2} \int \eta(\omega) \rho_{ZP}(\omega) d\omega.$$ 

This relates the fact that as mass is accelerated it pushes the quantum vacuum energy (which is analogous to the assumption made be postulate one). In other words it reacts in the same fashion as air molecules do when inertial masses accelerate on earth (producing pressure on the system). Furthermore, it can be assumed that a material body increases its rest mass by absorbing this zero-point-energy (this assumption must be given in order to satisfy conservation laws).

Moreover, since they are cardinal strings they are unified in a manner, thus the vacuum is a geometrical manifestation of string particles. That is the geometrical patterns formed through string interactions is what we call a gravitational field, i.e. a virtual gravitational field. Since these string interactions are only virtual there is no reason to modify the Einstein Field Equation, unless one wishes to discuss quantum string effects. Furthermore N-dimensional spacetime metrics have shown to be very similar to the structure of four-dimensional spacetimes [17]. Therefore the classical gravitational field is removed from quantum mechanics as it exist in a virtual sense, thus quantum mechanics is a property of matter, i.e. interacting geometries.

5.2 the stage

The vacuum however is not currently treated as the geometry of a fractal spacetime system, and hence is incompatible with other vacuum theories [18][19]. However a fractal model for quantum mechanics appears to agree with at least one interpretation of the quantum vacuum [20]. In fact this interpretation goes
right along with loop quantum gravity, and string theory see Ref. [21]. The Hausdorff (or fractal) dimension suggest that dimensions maybe confined to a $D = 3$ spacetime, with fractal string scaling. This principle was postulated earlier in this letter as the consequence of the “planckian scaling,” and is the leading postulate in New Relativity. Here the importance now becomes what is the meaning of de-Broglie phenomenon. The Einstein de-Broglie equation $\hbar \omega C = m_0 c^2$ can be seen as a representation of the relativistic wave equation, i.e. mass is a quantifiable measure of energy. Which can be applied directly to string theory, the vibrations of the string are given in a fractal frequency comparable to the Compton wavelength.

5.3 the Feynman stage

¿From the above consideration it can be equally applied the the origin of a bodies mass intern comes from the gravitational field itself. This requires the use of Feynman diagrams, and believe it or not this approach is indeed correct, if strings are represented by cardinal numbers (and if quantum mechanics is considered to be correct). Since this allows for FTL communication at the classical level it can be interpreted at least at the quantum level that mass originates from spacetime (when measured at the planckian scale). In fact at the quantum level the production of virtual particles may be responsible for a light paths geodesic curvature, yielding a quantum gravity theory. More correctly it may be viewed that the quantum theory is in reality a classical approximation of string theory.

Theorem 1 (Brussels approximation). Quantum mechanics exist as a classical approximation of string interactions which possesses an apparent time reversed symmetry.

5.4 quantum mechanics?

Deriving this theorem let us consider the following thought experiment: If mass is composed of vibrating stings, and intern these strings produce gravitational fields in N-dimensions then space is vibrating. However, such an approach would imply that geometrically speaking the two systems are unaware of their own vibrations under gauge invariance. On the other hand, the interaction of fractal strings are given in a complex field, which itself is anti-communcating. Thus spacetime, or string space is subjected to Uncertainty Principles as shown in Ref. [22]. Since at the classical level, the fractal space can give way to real solutions I now make the assertion that the quantum particles are at flux, and not the space itself (for argumentative purposes only). Therefore when a quantum is observed, it is the space which becomes “fuzzy,” not the particle, and when not observed the inverse follows. Thus the theorem leads to two possible out comes during an observation sequence. i) the fuzzy quantum particle becomes a point particle, when time symmetries are reversed. ii) spacetime is fuzzy, however when collapsed by a point particle elucidates to a “natural” state.
Thus it is seen that only when time symmetries are reversed does one obtain the laws commonly associate with quantum mechanics. Making the only valid approximation of quantum mechanics the Brussels Interpretation \(23\), this may also explain the EPR paradox. However our thought experiment does yield one solution which interpretation i and ii are consistent.

When a particle is observed by a frame, it is in reality observed by the local states (brane) of the cardinal string, which may be in any number of states. As a point particle (non local string) enters the system it begins to collapse the wave function of the local state. This makes it appear that a fuzzy quantum particle has entered the system, much as a star appears to twinkle in the night sky. This collapsing of the state I will call the observational’s frame “present,” before the action the observational state was fuzzy. However, after the event a self organization took place, an event occurred, producing a present state. After the quanta is observed by the observational frame, its present state then becomes certain in the terminology of classical quantum mechanics. Therefore local brane string interactions can not take place until a non local (cardinal) string collapses the wave function of the system.

5.5 Bohmian Mechanics, gravitation, and EPR

From the Feynman interpretation of the time reversed symmetries of the gravitational field, new conclusions about the nature of spacetime can be made. It is here postulated that classical mechanics is in reality a description of a quantum system given under an approximation of a time reversed symmetry. Such that the following statements become true:

- Reversal of Bohmian Mechanics (BM) yields Classical Mechanics
- Reversal of Brussels Interpretation (BI) yields Standard Quantum Mechanics (SQM)

With the fractalization of spacetime given in section \(3\), we may conclude that (with the use of Bohmian Mechanics) that inertial mass yields an expansion of spacetime. Thus as a body gains mass as it accelerates in classical spacetime it causes the fractalization of the Bohmian system to increase (which is analogous to a Lorentz transformation). Which thus gives the allusion that spacetime is contracting in the classical real frame. The gravitational force, thus is an inertial acceleration which radiates a pseudo center of gravity vector in terms of Newtonian mechanics. However, this is how we interpret the events classically, in reality it is the expansion of the fractal Bohmian space (the \(\phi^3\) term in section \(3\), e.g. the time dimension) which yields inertial acceleration.

Therefore, it is the Brussels interpretation of quantum mechanics which yields the EPR paradoxes, the connection of the particles is created by the (incorrect) approximation of time reversed symmetries. That is to say the EPR paradox only includes simple (non fractal) states, which by time reversal appears to yield FTL communication (see figure \(3\)).
This appearance of FTL communication shouldn’t be taken to seriously since recent experiments (cfr. Wang, et al [33]) appear to yield FTL communication. However, it is the string interactions which yield quantum mechanics, in fact it yields the same interpretations as Bohmian Mechanics [24]. Thus Bohmian Mechanics adequately describes the behavior of “particles” while, the Brussels Interpretations yields standard quantum mechanics [meaning that this system is only an approximation].

6 quantum gravity?

Since complex spaces have been presented as a solution to the singularity problem, it is natural for a formulation of a complex spacetime. To proceed in this manner one must neglect the cherished Einstein-Hilbert action

\[ s = \int d^4x \sqrt{-g} R \]

and replace it with the Tucker-Wang action:

\[ s = \int \lambda^2 R \star 1 \]

Therefore we can now discuss a complex gravitational field, without the traditional Riemannian geometry. The classical Einsteinian relativity gives the generic field for a spacetime geometry as \( \nabla^\mu G_{\mu\nu} = 0 \). Such that I now wish to make the generalized statement \( \nabla^\mu G_{\mu\nu} \neq 0 \), or in canonical terms \( \nabla_a G^{ab} = 0 \). As such a generalization of a purely idealistic spacetime governed by perfect fluid becomes:

\[ G^{ab} = 16\pi G T_{ab} \] (7)

where under ideal cases one can have the geometry \( R^a_b + \frac{1}{2} g_{ab} R \). The reason the field takes on the term \( R^a_b \), as opposed to \( R_{ab} \), can be seen with the use Riemannian metrics. First, let us begin with a Ricci symmetric tensor of the form:

\[ \frac{\partial \Lambda^a_\sigma}{\partial x^\nu} = \eta_{\mu\nu} \Phi_\nu \Lambda^a_\mu \]

which within a constant field becomes \( R_{\mu\nu} = \Lambda^a_\mu F_{a\nu} \). This field can thus transpose to \( F_{\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu \), and couple to an opposing electromagnetic field
by the connection
\[ \Gamma^\sigma_{\mu\nu} = \Lambda^\sigma_{\mu} \Phi_{\nu} \]
Which therefore leads to the following antisymmetric Riemannian field
\[ R_{\mu\nu} \left( \frac{\partial \Phi_{\sigma}}{\partial x^\sigma} - \frac{\partial \Phi_{\sigma}}{\partial x^\nu} + C_{\nu\sigma} \Phi_{\alpha} \Phi_{\beta} \right) \Lambda^\nu_{\mu} = F_{\sigma\nu} \Lambda^\sigma_{\mu} \]
Whence therefore means that there must be an equivocal orthonormal action taking place, such that one has \( R^a_{\mu\nu} = d \Lambda^a_{\mu} + \Lambda^a_{\mu} \wedge \Lambda^a_{\nu} \). In which the generic geometry for a perfect fluid becomes that of
\[ G^{ab} = 16\pi G T_{ab} \]
which translates to the field equation:
\[ R^a_{\mu\nu} - \frac{1}{4} g(\hat{e}_{(a)}, \hat{e}_{(b)}) R = -\frac{16\pi G}{c^4} T_{ab} \quad (9) \]
This equation must be modified when given in an \( N \)-dimensional system such that on has:
\[ R^a_{\mu\nu} - \frac{1}{2(n)} g(\hat{e}_{(a)}, \hat{e}_{(b)}) R = -\frac{8\pi (n) G}{c^{2(n)}} T_{ab} \quad (10) \]
The above equation is in essence a canonical gravitational field equation, which appears to be a good candidate for a quantum gravity theory. Where the geodesic equations become
\[ \frac{d}{dl} \left[ \left( 1 + \frac{\gamma_{ab(n)}}{2(n)} \right) \frac{dx^\mu}{dl} \right] - \Gamma^a_{[\mu\nu]} \frac{dx^\mu}{dl} \frac{dx^\nu}{dl} \left( 1 + \frac{\gamma_{ab(n)}}{2(n)} \right) \neq 0 \quad (11) \]
However, this interpretation suggest that spacetime is quantitized by a canonical action however, quantum particles are given as classical particles. Hence this interpretation would be an inverse of understood quantum mechanics. However, here a paradox opens up, when time is reversed particles remain in one quantum state, thus SQM is not retrieved. Thus, it is seen that there exist no true “quantum gravity” theory. In fact if one applies this formulation with the planck length it destroys the principle of \( \text{planckian invariance} \) and gives an allusion to the existence of an \( \text{Æther} \).

6.1 canonical approach fails
Therefore the planck length no longer remains a constant but becomes a dynamical function. First let us write the planck length in terms of \( N \)-dimensions and apply it to the above field equation such that we have:
\[ l_{\text{pc}} = \left( \kappa h^n / m_p^n c^{3n} \right)^{1/2n} \cdot \psi. \quad (12) \]
Momentum must be reevaluated from \( E = \pm (p^n c^n + m_0^n c^{2n})^{1/n} \), such that \( m_{\text{po}} = \mp (p^n c^n / c^{-2n})^{1/n} \). Thereby the planck length, and mass are actually
given by a particle’s rest momentum. Such that the Planck length is in reality given by the function:

\[ l_p = \pm \left( \frac{\kappa \hbar^n}{\gamma m_p c^{3n}} \right)^{1/2n} \psi. \]  

(13)

This thereby has major implications, that the Planck length is not really a constant at all but a function of momentum. Such that as an object increases in speed with respect to its rest momentum, its Planck energy becomes larger. That is as a material body is subjected to length contraction, its Planckian energy is modified to compensate for the effect. Since the momentum is measured at rest \( m \) remains a constant, it is the velocity of the system which changes. Thereby meaning that length contraction in special relativity is not given by Lorentz transformations, but by the local rest momentum of the Planck barrier. This results when we interpreted this action canonically however under BM it yields expected results. Therefore it is seen that a canonical formulation fails to keep “Planckian invariance” which represents a failed attempt at a quantum gravity theory.

### 7 anomalous accelerations?

Finally I bring light to an alternative explanation for the acceleration of spacecraft [8]. Since the findings of the “anomalous acceleration towards the sun,” there have been a number of possible explanations given [27, 28, 29]. With the construction of a fractal \( N \)-dimensional spacetime, I view this as a quantum gravity effect. As an object accelerates its fractal geometry changes [by means of BM], thus resulting in pressure on the system. Pressures as the source of a gravitational field were pioneered long ago by Einstein [30]:

\[ R_{\mu\nu} = -\kappa(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \]  

(14)

\[ \frac{2}{a^2} \gamma_{\mu\nu} = \kappa(\frac{\sigma}{2} - p) \]  

(15)

\[ 0 = -\kappa(\frac{\sigma}{2} + p) \]  

(16)

This method is not ad hoc, gravitational pressures for atomic gases and radiation can be given by [37]:

\[ p_{\text{gas}} = \frac{\kappa}{\mu H} qT \]  

and  

\[ p_{\text{rad}} = \frac{1}{3} \alpha T^4 \]  

(17)

which lends the general results

\[ T = \left( \frac{\kappa}{\mu H} \frac{1 - \beta}{\beta} \right)^{1/3} q^{1/3} \]  

(18)

\[ p = \left[ \frac{\kappa}{\mu H} \frac{4}{3} \frac{1 - \beta}{\alpha} \right]^{1/3} q^{4/3} \]  

(19)

\[ q^{4/3} = c(\beta) q^{4/3} \]  

(20)
Thus after a slight modification of eq. (10), one can obtain the following gravitational pressure:

\[ R_{i}^{ca} = -\kappa (T_{i}^{ca} - \frac{1}{2} g_{i}^{ca} T) + \frac{2}{a^2} \gamma_{i}^{ca} = \kappa (\sigma^2 - p) \] (21)

Therefore the flat field equations can be given by:

\[ \Gamma_{ab}^{c}(z) \neq 0, \quad R_{i}^{ca}(z) \neq 0 \] (22)

thus a line elements trajectory would be given by

\[ (\omega, z^2) = g_{ca} \omega z_c \omega z_a \] (23)

therefore a metric in a complex fractal spacetime can be given by:

\[ \sum_{ab=1,2} \delta \omega z_a \omega z_b \equiv 0 \] (24)

Since this quantum gravity effect originates from the Planck length it is very unlikely that the Yukawa interaction [31] :

\[ V(r) = -\int dr_1 \int dr_2 \frac{G \rho_1 (r_1 X r_2)}{r_{12}} [r + \alpha \exp(-r_{12}/\lambda)] \] will take place (unless special conditions arise). However, if such an effect does arise, it may yield peculiar motion for an object's geodesic path.

### 7.1 SQM pseudo geodesic paths

First let us begin with the two-dimensional Lagrangian Hamiltonian, so that we have an equation of motion from the simple action

\[ \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{q_i} \] (25)

In canonical terms motion is given by

\[ q_i - \frac{\partial H}{\partial p_i}, \quad p_i = \frac{\partial H}{q_i} \] (26)

lending a four-vector of the form \( p = \dot{m} x + \xi \dot{a}(x) \). In such that a Hamiltonian wave within a gravitational field would be in motion according to the geodesic path:

\[ \frac{\partial H}{\partial q^i} - 1/r \frac{dx^i}{ds} \frac{dx^j}{ds} \neq 0 \] (27)

This geodesic unlike the prior for a classical particle, will not differentiate and thus its motion need not transverse through classical Euclidean space. Therefore it can be seen that complex spaces could impose unseen forces which would effect
a geodesic path for a body (or wave) in motion. A proposal made in Ref. [35], made a like was case in the relativistic sense so that one would have:

$$F^\mu = \hat{m}_0 \frac{W'}{W} \frac{dy}{d\lambda} \frac{dx_\mu}{d\lambda}$$

(28)

Although there is no direct physical evidence of this, it is still however an intriguing explanation. After all the so called “anomalous acceleration” is only experienced by small bodies, not massive ones such as planets. Thus a quantum interpretation of this effect seems to fit the observed data better than any other approach. Alternatively Modanese has also predicted a macroscopic quantum gravity effect [32], however it is limited to the Podkletnov experiment [9].

8 Discussion of theory

The formulation of this theory was based on a desire for a reformulation of GR in order to describe a singularity free theory; in which a fractal formulation of the field equations were derived. The second desire for this theory was the formulation of a quantum construction of GR, however the end result is a gravity theory which describes quantum mechanics. Therefore the gravitational field and matter can be considered to be molded into the following form. Matter exist as a pseudo point particle whose field of movement is restricted onto a two-dimensional (complex) frame. This two-dimensional frame’s movement is governed by BM, and in part by the HP. Matter, is thus in reality a fractal vibrating string fragment which continues on into \(N\)-dimensions. The fractalization of this “cardinal string” produces virtual particles which possess a geometry, it is this (virtual) fractal geometry that is responsible for the gravitational field.

In light of future studies it is likely that an adequate formulation for an alternative to GR be given in the following forms. One the acceptance of a fractal (even if only quasi fractal) structure of matter and space as an adequate formulation for the geometry of spacetime. Two the acceptance of complex systems into the equations, e.g. quaternions, octonions, C* algebras, etc. And finally three, the acceptance of physical conditions which may not be “popular,” but yield results that are not contradictory to known data. The mathematical conditions are the most intriguing to author because there seems to be a hidden mechanism in the mathematics. However, my advanced mathematics skills are mediocre at best so these avenues are still left open in this letter.

8.1 what layith beyond the planck length?

Several physical arguments against the existence of singularities have been given by Loinger [25, 26], as well as Einstein’s classic objections. Thus one may inquire what happens at the planck length, i.e. what are the laws of physics? Here I now quote Kip Thorne, on our current understanding of singularites and ‘quantum foam.’
“How probable is that a black hole’s singularity will give birth to ‘new universes?’ We don’t know. It might well never happen, or it might be quite common—or we might be on completely the wrong track in believing that singularities are made of quantum foam.”

–Thorne (1994)

This now leads a discussion to recent attempts to model gravity in terms of $N$-dimensional spaces (Arkani-Hamed, et al 62-69), in which the planck length varies with the number of dimensions. Of course the planck length could be infinitely small in an infinite system, clearly a challenge to the principle of planck invariance. However, we note that with Mach’s Principle (MP) the planck length must be observed by an external mechanism to remain invariant. Thus the planck length exist as a fractalization of BM, which becomes an observational frame in classical real mechanics. Furthermore, from this it may be seen that the laws of physics as we understand them are in direct consequence of the planck length. We may also assume the chosen string field is quantitized (i.e. given by BM), because its mass is attributed to a complex pseudo oscillation (vibration). Where I now quote David Bohm (cfr. Bohm, 22):

“We may conclude that all systems which oscillate are quantitized with $E = n\hbar\nu$ whether these systems be mathematical oscillators, sound waves, or electromagnetic waves.”

So what does physics look like beyond the planck length, remember the (local) laws of physics are given by two complex D2-branes. When we interpret these interactions we receive the traditional GR effects at the macroscopic level. However, at the planck scale singularities don’t exist such that the frame interacts via “cardinal strings” and not classical GR. Thus interactions on local branes cease, and supersymmetry takes over. However, only strings which are connected to a form of the D2-brane will have observable physical manifestations, this deals with “planckian invariance”. In fact each dimension may have its own unique planck length which governs its own local laws (explaining the limitation of classical string theory to a set number of dimensions). Which leaves open several areas in $N$-dimensional black hole mechanics, and planck length physics.

9 Conclusion

I have shown that there is enough evidence at present to challenge GR as the correct theory for gravitation. I have also introduced the study of a complex fractal spacetime system and its possible relationship to the planck length. The given formulation for a canonical gravitational field resulted in contradictory conclusions, thus ruling out a canonical approach to “quantum gravity.” Finally if my hypothesizes hold valid then SQM will begin to make invalid predictions for the behavior of particles near the planck length. Thus a fractal correction for SQM will be needed under certain gravitational fields, which may be comparable to BM.
A  the equivalence principle

A new Weak Equivalence Principle (WEP) for the gravitational field can be postulated utilizing a Complex Fractal Minkowski Spacetime (CFMS) system (see eq.(3)). Since there is no spatial acceleration for the gravitational field (in respect to MP), it is the acceleration of the pseudo time dimension in the CFMS which produces a gravitational curvature. Therefore a material body would have the traditional Minkowski spacetime, acting as a Lorentz frame. Since gravitational fields extend indefinitely, this should cause time to continually progress within an inertial acceleration frame. This therefore means that as an object enters a gravitational field it becomes less massive, in terms of a Lorentz transformation. Equivocally it can be stated that energy is lost in curved spacetime. A similar effect is already known, known as a “gravitational time delay,” i.e. the Shapiro Effect.

“... according to the general theory, the speed of a light wave depends on the strength of the gravitational potential along its path.”

–Shapiro (1964)

This is however contradictory to SR, because it fails to describe inertial acceleration within a gravitational field correctly. However, the principle of relativity is still preserved, because the curvature of spacetime corrects for the CFMS. Therefore the reason the equivalence principle is fundamental in GR is because it is the only priori condition which satisfies the principle of relativity. Thus without an equivalence principle, there would be no relativistic theory for the gravitational field.

The logarithm gravitational time delay, may also be responsible for the apparent “anomalous acceleration” of spacecraft. David Crawford has offered a similar explanation, where the gravitational term arises from interplanetary dust [27].

B  Yang-Mills gravity

Here I now hit upon a topic hinted upon in section [4.1]; converting fractal geometry in the terminology of QCD. Let us now rewrite eq.(21), so that we have an equation of the form:

\[-\kappa (R^c_a - \frac{1}{2}g^c_{ab} R^b) = \frac{8\pi}{\sqrt{-g}} T^c_{ab}\]

with this equation a Yang-Mills gravitational pressure can arise under the following field:

\[ S_E = \frac{1}{4g^2} \int \omega^2 z F^\sigma_\mu F^\sigma_\mu + \frac{1}{\alpha_o} \int K^a_b K^b_a \sqrt{-g} \omega^2 \phi \]
which must be given in a conformal field, i.e. \( g_{ab} = p\delta_{ab} \), thus we have:

\[
S = \frac{1}{2\alpha_0} \left[ \int \omega c^2 \phi p^{-1}(\phi)(\partial^2 z)^2 + \chi_{ab}(\partial_{az}\partial_{bz} - p\delta_{ab}) \right] + \aleph_0 \int_p \omega^2 \phi \tag{31}
\]

From this it is now seen that a “cardinal string,” is in fact an \( N \)-dimensional world line. Which can communicate with other world lines, where we have a self-organization of the system by

\[
v(\hat{s}) = \frac{1}{4\pi} \int \omega^2 \phi \sqrt{g} g_{ab} \epsilon^{jklm} \partial_a t_{ab} \partial_{jklm} \tag{32}
\]

It is these interactions which generate a spinor space, which attributes mass to the geometry. Therefore the Yang-Mills field is added to the gravitational field, by means of a gravitational pressure. Here a less restrictive form of the Strong Equivalence Principle (SEP) can be applied:

\[
\text{(spinor pressure)} \cdot \text{(energy density)} = \text{(strength of gravitational field)} \cdot \text{(gravitational pressure)}.
\]

Thus the interaction of two or more “cardinal strings,” produces a twistor like action, represented by \( Z^\infty \) at \( P \in \mathcal{M} \). This also means that certain gravitational anomalies may not only arise at the planck length, and may result in experimental verification.

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