Numerical simulation of crack growth process in Brazilian disk test using finite element method

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Abstract. Accurately simulating the compressive and tensile mechanical behavior of rock is one of the challenges faced by numerical simulation. In this paper, the rock's triaxial compression experiment and the Brazilian disk simulation were simulated, and the propagation process of the rock crack was simulated in Brazilian disk test in the finite element method. According to the experimental data, the KCC model was completely modified and the triaxial compression experiment was carried out with the modified model. The modified model was applied to the Brazilian disk test simulation, and the crack propagation in Brazilian disk test was simulated by inducing the erosion algorithm, which verifies the applicability of the erosion algorithm in the finite element method of material continuity loss.

Keyword: sandstone; numerical simulation; the KCC model; erosion algorithm

1. Introduction
Sandstone is ubiquitous in underground engineering, so studying the behavior of sandstone in underground engineering is of great significance to its application in mining and civil engineering[1]. Sandstone is a brittle material, and its characteristic is that its tensile strength is much lower than its compressive strength. Therefore, both compressive strength and tensile strength need to be considered when studying the mechanical properties of sandstone.

It is difficult to perform the direct tensile experiment of brittle materials. Because the Brazilian disk test is economy and simplicity, it has become a common method for measuring the indirect strength of brittle materials. A large number of numerical simulations have been applied to the study of Brazilian disk test, and many of these studies have focused on the location and propagation process of cracks in the Brazilian disk test. Many researchers use discrete element method (DEM) [2] or the finite element method (FEM) coupled with the DEM [3] to study the crack propagation in the model. These studies generally focus on tension behavior of rock, without considering the compression of the rock at the same model. To be able to consider the compression and tension of the rock at the same time, it is necessary to select an appropriate constitutive model. Mohr-Coulomb[4] and Hoek-Brown[5] is commonly used in the constitutive model of brittle materials. Although these constitutive models have good performance when applied to the compression of materials, they usually overestimate tensile...
strength of the brittle material. The advanced material models, namely Karagozian and Case concrete (KCC) models, is implemented in conjunction with the finite element code LS-DYNA. The KCC model is developed by Malvar et al.[6-8]. After multiple versions upgrades, it can effectively simulate the key behaviors of concrete post-peak softening, shear dilation and strain rate effect. It can also automatically generate a set of parameters based on uniaxial compression strength, or be calibrated according to the laboratory data.

In this paper, the KCC model was modified according to the laboratory data to be applicable to rock compression and tension simulation. The modified model was verified by simulating the tests and the validity of the model was confirmed. The erosion algorithm was implemented in the FEM, and the crack propagation process in the Brazilian disk test was realized in the simulation.

2. Description of KCC model
The KCC model consists of three fixed and independent strength surfaces: the yield strength surface $\Delta \sigma_y$, the maximum strength surface $\Delta \sigma_m$ and the residual strength surface $\Delta \sigma_r$. They can be expressed by equation 1:

$$\begin{align*}
\Delta \sigma_y &= a_{by} + p / (a_{1y} + a_{2y}p) \\
\Delta \sigma_m &= a_{b_1} + p / (a_{1} + a_{2_1}p) \\
\Delta \sigma_r &= p / (a_{1f} + a_{2f}p)
\end{align*} \tag{1}$$

where $\Delta \sigma = \sqrt{\lambda J_2}$ ; $\lambda$ = the second invariant of the deviatoric stress tensor $\sigma'$ ; $a_i$ = the user-defined material strength parameters.

As shown in Figure 1 and Figure 2, the stress state of the model is obtained by interpolating between three strength points. The formula is shown in equation 2.

$$\begin{align*}
\Delta \sigma &= \eta (\Delta \sigma_m - \Delta \sigma_y) + \Delta \sigma_y \quad \lambda \leq \lambda_m \\
\Delta \sigma &= \eta (\Delta \sigma_m - \Delta \sigma_r) + \Delta \sigma_r \quad \lambda > \lambda_m
\end{align*} \tag{2}$$

where $\lambda$ = the damage parameter, which is a function of equivalent plastic strain; $\eta$ = the interpolation parameter, when $0 \leq \lambda \leq \lambda_m$ , $\eta$ increases monotonically from 0 to 1, when $\lambda_m < \lambda$ , $\eta$ decreases monotonously from 1 to 0.

![Figure 1. Shear failure surfaces.](image1)

![Figure 2. Uniaxial stress-strain relation in KCC model.](image2)

The KCC model decouples the deviatoric and volumetric responses. The volumetric response is determined by equation of state (EOS). In LS-DYNA, the EOS is determined by the keyword TABULATED_COMPACTION:

$$p^\text{EOS} = C(e_r) + \gamma T(e_r)U \tag{3}$$
where \( C(\varepsilon_p) \) and \( T(\varepsilon_p) \) are functions of volumetric strain \( \varepsilon_p \); \( \gamma \) = the parameter related to temperature, which are defined by the user.

3. The Procedure of modifying the KCC model

The strength parameters firstly need to be modified for the material properties of sandstone. According to the pressure condition of the triaxial compression test, \( \sigma_z=\sigma_1 \), the confining pressure and deviator stress can be expressed respectively as equation 4 and equation 5:

\[
p = 1/3(\sigma_1 + \sigma_3 + \sigma_3) = 1/3(\sigma_1 + 2\sigma_3)
\]

\[
\Delta \sigma = [\sigma_1 - \sigma_3]
\]

According to the triaxial compression test data of sandstone under 10MPa, 20MPa, 30MPa, 40MPa confining pressure conditions, the maximum strengths and yield strengths of sandstone under different confining pressures were obtained. Use the least square method to fit the above data to equation 1. The 8 material parameters of the modified strength parameters were obtained in Table 1.

| Table 1. Modified KCC model strength surfaces parameters. |
| --- |
| \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_{0y} \) | \( a_{1y} \) | \( a_{2y} \) | \( a_{0r} \) | \( a_{1r} \) | \( a_{2r} \) |
| 2.7 | 0.3584 | 3.115E-04 | 4.375 | 0.4607 | 1.367E-04 | 6.94E-06 | 1.958E-03 | 3.115E-04 |
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The EOS was modified based on test data. The equation of state obtained according to the test is shown in Table 2.

| Table 2. The EOS adopted in numerical simulations of sandstone. |
| --- |
| Volume strain (MPa) | Pressure (MPa) | Bulk unloading modulus (MPa) |
| 0 | 0 | 29000 |
| -0.01 | 290 | 29000 |
| -0.017 | 500 | 30000 |
| -0.0235 | 700 | 30769.23 |
| -0.031 | 3950 | 33333.33 |
| -0.05 | 1600 | 34210.53 |
| -0.1 | 3500 | 38000 |
| -0.12 | 4300 | 40000 |
| -0.15 | 5600 | 43333.33 |
| -0.2 | 7800 | 44000 |

The failure surface interpolation function \( \eta(\lambda) \) also needs to be modified. In the KCC model, the increment of the damage parameters \( \dot{\lambda} \) is defined as a function of the rate of plastic strain tensor \( \dot{\varepsilon}^p \). In the case of quasi-static loading, \( \dot{\lambda} \) and \( \dot{\varepsilon}^p \) respectively expressed as equation 6 and equation 7:

\[
\dot{\lambda} = \begin{cases} \frac{\dot{\varepsilon}^p}{(1 + p / f_j)^{\beta_j}} & p \geq 0 \\ \frac{\dot{\varepsilon}^p}{(1 + p / f_j)^{\beta_j}} & p < 0 \end{cases} \]

\[
\dot{\varepsilon}^p = \sqrt{2 / 3 \dot{\varepsilon}^p : \dot{\varepsilon}^p} = \sqrt{2 / 3 [\dot{\varepsilon}^p + 2(\dot{\varepsilon}^p)^{\bar{\varepsilon}}]} \]
where $f_t$ = tensile strength; $\varepsilon_a^p$ = axial plastic strain; $\varepsilon_l^p$ = lateral plastic strain; $b_l$ = the scale coefficient. The full set (13 pairs) for sandstone is provided in Table 3.

4. Numerical simulation results

The numerical simulation is carried out in the explicit non-linear FEM program LS-DYNA. The pre-processing and post-processing of the model were carried out with LS-PrePost, and single-point integral solid hexahedral elements were used in the simulation. Then the triaxial compression test was simulated by using the modified KCC model. The comparison with the experimental data proves the effectiveness of the modified model in simulating the mechanical behavior of sandstone from low to high confining pressure conditions. And the Brazilian disk test were simulated to verify effectiveness of the modified model in tensile simulation.

4.1. Triaxial compression simulation

The KCC model can generate a set of parameters based on the uniaxial compressive strength. By setting the A0 value in the "MAT072R3(KCC)" material card to "$f_c^m$", the complete strength parameters, damage parameters and state equations of the model can be generated in the LS-DYNA software, and users can view them in the "messag" file. The results of the triaxial compression simulation the modified and original KCC model are shown in figure 3. The modified model simulates the mechanical behavior of sandstone under triaxial compression better than the original model.

![Figure 3. The result of the triaxial compression simulation](image)

4.2. Brazilian disk test simulation

The Brazilian disk simulation model consisted of three parts, namely the loading plate, the disk model and the carrying plate in figure 4. The disk model used the modified KCC model with a diameter of 40 mm and a thickness of 20 mm. The loading plate and the carrying plate used linear elasticity model, $\rho = 7.85 \times 10^{-9}$ T/m$^3$, $\nu$ (Poisson’s ratio) = 0.3, consistent with the properties of steel. The carrying plate was fixed. Due to the quasi-static loading, it was considered appropriate for the upper loading plate to move downward at a speed of 10 mm/s. Set the contact between the upper loading plate, the rock and the carrying plate.

The parameters used in the Brazilian disk test simulation are the same as those of the triaxial compression simulation model, according to equation 6, it can be seen that $b_2$ affects the tensile behaviour of the model. The sensitivity analysis of $b_2$ shows that as $b_2$ increases in Figure 5, the tensile strength of the model gradually decreases. According to the equation 8:

$$\sigma_t = \frac{2P}{\pi DT}$$

*MERGEFORMAT (8)
Where $\sigma_t$ = tensile strength; $P$ = Maximum loading force; $D$ = the diameter of the disk; $T$ = the thickness of the disk. According to the uniaxial tensile strength of sandstone as 2MPa, it is more appropriate that $b_2$ should be equal to 4.

The element erosion in LS-DYNA will remove elements from the simulation when elements meet the set conditions. Since the KCC models do not specify damage and erosion in their formulas, this option provides a way to achieve model damage and erosion. The algorithm also has the ability to deal with excessive deformation of the element. When the mechanical response of the element reaches the set erosion condition, the element is deleted[9].

The external erosion algorithm can be realized through ADD_EROSION. The generation of cracks was achieved by determining the maximum effective strain at failure. The hexahedral solid elements that meet the failure criteria were deleted, which reduced the bearing capacity of the model, and simulated the occurrence and penetration of cracks in the Brazilian disk test. Since the effective plastic strain is represented by the cumulative damage of the model, it may be used to determine the maximum effective strain. The maximum effective strain at failure value of 0.03 obtained. According to the results (Fig. 6), as time increases, rock cracks first occur in the center of the disk and then expand to both ends.

![Figure 4](image1.png)
Figure 4. The model of the Brazilian disk test simulation.

![Figure 5](image2.png)
Figure 5. Parameter $b_2$: effect on Brazilian disk test simulation.

![Figure 6](image3.png)
Figure 6. Effective plastic strain of disc at different time

5. Conclusion
The original KCC concrete model was modified and applied to simulate the triaxial compression test and the Brazilian disk test of the sandstone. A complete process of modifying the KCC model is formed, and the effectiveness of the process was verified by comparing simulation results with experimental results. And the erosion algorithm was applied to the finite element simulation to simulate the crack generation and propagation process in the Brazilian disk test, which verified the applicability of the erosion algorithm in the finite element simulation of material continuity loss.
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