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A New Simple Method for Estimating Exergy Destruction in Heat Exchangers

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Received: 3 December 2012; in revised form: 11 January 2013 / Accepted: 22 January 2013 / Published: 28 January 2013

Abstract: This paper presents an abbreviated method for estimating exergy destruction in a heat exchanger, requiring only black-box data of the exchanger’s inputs and outputs, and eliminating part of the mathematical difficulties associated with the calculations. A well-known model for temperature distributions in an exchanger is adapted for this case, and is used to distinguish between the contributions of the three major causes of the total exergetic loss: heat transfer, fluid friction and energy dissipation to the surroundings. This provides insight into the relative importance of the three, allowing for identification of potential improvements to a given design.

Keywords: irreversibility; second law analysis; heat exchanger; thermodynamics; heat transfer

Nomenclature

\[ \dot{B} \] Exergy of the system [kW].

\[ \dot{e}_j \] Exergy flow \( j \) [kJ/kg].

\[ \dot{I} \] Exergetic destruction of system[kW].

\[ \dot{L} \] General balance of exergy \( j \) [kW].

\[ \dot{m}_j \] Mass flow rate of stream \( j \) [kg/s].
1. Introduction

Characterizing the exergetic behaviour of a heat exchanger is important for the design and evaluation of a power plant [1]. A good exergetic design of a heat exchanger would allow for an increase in the global efficiency of the plant, by defining a thermodynamic cycle in which the exergetic losses would be limited to the strictly unavoidable [2]. A complete exergetic analysis of an existing heat exchanger could help to identify sources of exergy loss and possible improvements. Analyzing exergy in a heat exchanger, either for design or for analysis, is not straightforward. The three main causes of irreversibility are heat transfer between the flows, pressure losses due to fluid friction, and dissipation of energy to the environment; the three phenomena can also occur simultaneously. There exist other problems, which have minor effects, such as streamwise conduction in the walls of the heat exchanger [2].

The work of Bejan [1] is regarded as the basis of exergetic analysis of heat exchangers. The problem has been studied enthusiastically by other authors, leading to a number of methods and indicators for expressing exergetic efficiency; these are are summarized in [3]. It is worth mentioning, however, that, as pointed out in [4], most new methods differ only in the way that entropy generation is non-dimensionalized.

In particular, many studies are based directly or indirectly on the concept of rational efficiency [2], the ratio between the amount of exergy received by the cold fluid and that released by the hot fluid. In essence, the ratio between the useful effect and its cost is expressed in terms of exergy. Irreversibility is the reason why the exergy received by the cold fluid and that released by the hot fluid are not equal. In fact, the total exergy loss in an exchanger is another characteristic of its exergetic behavior commonly referred to in the literature [5,6]. However, neither total exergy loss nor rational efficiency provide information about the causes of irreversibility. It would be very useful when designing large processes
involving a heat exchanger, or when analyzing an existing device, to learn which cause of exergy loss or destruction is dominant. This could allow improving it whenever possible, given the fact that only part of the exergy loss can be avoided in practice [2].

The three main causes of irreversibility in a heat exchanger are associated to each of the major processes taking place in the device, as previously mentioned: heat transfer between the hot and cold fluids; heat transfer between the exchanger and its surroundings; and the movement of the fluids. Of the three, heat transfer between the exchanger and its surroundings is usually neglected, given that it frequently represents a small fraction of the transfer as in [7].

Irreversibility associated with heat transfer between the flows is due to the temperature drop between them during the heat exchange, which can be measured in a laboratory [7] or simulated using finite element techniques. Knowing the temperature distribution along the tubes of the exchanger, added to detailed constructive data of the exchanger, allows calculating the associated exergy destruction. A theoretical temperature distribution can be used alternatively, as in [8,9], but detailed constructive data of the exchanger are still required, such as: heat transfer coefficients, area of exchange surfaces, tube cross sections, fluid velocities, etc.

While theoretical approximations for the temperature distribution might represent the real one with sufficient accuracy, as a result of the complexity of the problem, irreversibility due to fluid friction is not generally studied. Fluid densities and pressures have to be modeled at each point along the tubes and integrated between the inputs and outputs. Logically, apart from the fluid modeling challenge, it is absolutely necessary to know the geometry and the characteristics of the materials of the exchanger to be able to calculate the integral. The problem of calculating the friction contribution to irreversibility is formulated from a purely thermodynamic approach in [10], which suggests integrating the momentum equation adequately formulated as in [11]. The same philosophy underlies the expression offered by [12], formulated from the point of view of heat transfer. More applied formulations are used in [9]. Apart from the strong requirements of input data, applying any of these methods often requires iteration, to assume simplifying hypotheses or both.

It might not always be possible to use these procedures for analyzing a heat exchanger exergetically. For instance, the usual approximation of neglecting exergy losses to the surroundings might not hold if the exchanger would operate at temperatures very different to the ambient, as in cryogenic or high temperature processes, in accordance with the discussion about the influence of ambient temperature in [8]. This approximation would not hold either if analyzing an existing heat exchanger that had insulation problems. On the other hand, fluid friction might not be negligible when using dense fluids or when the fluids circulate at high speeds, also in accordance with [7,8]; in fact, the Reynolds number can be used explicitly for modeling the exchanger as in [4,9]. Moreover, at the initial design stages of thermodynamic cycles that include heat transfers, detailed constructive details of heat exchangers cannot even exist, given that it is when all operating conditions and mass flow rates are being defined.

In this paper, a new simple method for analyzing the exergetic performance of heat exchangers is going to be developed. The main novelties with respect to the previous methods are:

- The only necessary data are the temperatures and thermodynamic properties of the fluids at the inputs and outputs of the heat exchanger;
• The exergetic effect of heat transfer between the heat exchanger and its surroundings is not neglected; and
• The three major contributions to exergy loss are distinguished quantitatively

It is clear that in absence of specific data, the degree of accuracy is limited. However, given the few data required and the generality of the methodology, the results should be representative of an order of magnitude, and of the relative importance of the three major sources of irreversibility mentioned earlier.

2. Loss and Destruction of Exergy in Heat Exchangers

A heat exchanger is used to pass energy from one fluid, which we shall call hot, to another one, cold, which will receive it. The energy is passed in the form of heat, which, due to the temperature difference between both fluids, will flow from one to the other spontaneously. The temperature of the fluids will be, in general, different to that of the surroundings of the exchanger, so a heat transfer between them and the outside of the exchanger is inevitable.

These energy transfers carry some exergy, so that the hot fluid releases exergy to the cold fluid which absorbs it, and then out into the environment, where it is wasted. During the movements of the fluids across the tubes of the exchanger, part of the exergy will be consumed by fluid friction.

The exergetic losses and destruction in a heat exchanger can therefore be summarized as follows:

• The hot fluid passes exergy to the cold fluid, but some is destroyed in this process due to the temperature difference between the fluids;
• The movement of the fluids along the ducts of the exchanger entails some degree of exergy destruction; and
• The heat transfer between the exchanger and its surroundings also entails an exergy transfer.

The first point is the exergy destruction due to temperature drop, $\dot{I}_{\Delta T}$; the second, the result of head losses (or pressure drop), $\dot{I}_{\Delta P}$. The heat transfer to the environment does not actually mean that exergy is destroyed, but rather that it abandons the heat exchanger, as denoted by $\dot{B}_Q$. Resulting from all destructions and releases of exergy to the environment, a certain amount of exergy will be lost in the exchanger, $\dot{L}$:

$$\dot{L} = -\dot{B}_Q + T_0\dot{\sigma}_{\Delta T} + T_0\dot{\sigma}_{\Delta P}$$

where $\dot{I}_{\Delta T} = T_0\dot{\sigma}_{\Delta T}$ and $\dot{I}_{\Delta P} = T_0\dot{\sigma}_{\Delta P}$. $\sigma$ denotes entropic generation.

A complete exergetic analysis of a heat exchanger means calculating all four terms independently, but this may prove difficult, even when experimental data about the actual temperatures inside the heat exchanger are available (as in [13], for example). In many cases, the exchanger is only at the design stage, so no data actually exist, except for the approximate input and output conditions of both fluids. It therefore becomes necessary to devise a black-box method in order to characterize the exergy.

In order to do so, each term in Equation (1) must be developed; a well-known theoretical approach is sufficient [10]:
\[ \dot{L} = \sum_{vj} \dot{E}_j \]  

\[ \dot{B}_Q = \int d\dot{Q}_{surr} \left( 1 - \frac{\theta_0}{T} \right) \]  

\[ \dot{\sigma}_{\Delta T} = \int d\dot{Q}_{exch} \left( \frac{1}{T_c} - \frac{1}{T_h} \right) \]  

\[ \dot{\sigma}_{\Delta P} = -\dot{m}_c \int c v_c dP_c - \dot{m}_h \int h v_h dP_h \]

The following notation has been introduced: subindex ‘c’ indicates the cold fluid, ‘h’ the hot, \( v \) is specific volume. The integral in Equation (3) extends across the outer surface of the exchanger. \( dQ_{surr} \) is the differential heat exchanged between the device and its surroundings through an element of the surface. \( T \) is the temperature at which the exchange takes place. \( Q_{exch} \) is the heat actually absorbed by the cold fluid, \( \dot{m} \) stands for mass flow.

Equation (2) illustrates that the total exergy loss in the heat exchanger is the balance of the exergy contained in the fluids coming in and out, through all its inputs and outputs, \( \dot{E}_j \).

It is worth mentioning that \( Q_{surr} \) will be negative if the heat passes from the exchanger to its surroundings (this is the case if the exchanger is hotter than its environment); it is otherwise positive.

The irreversibility due to heat transfer between the fluids, term \( \dot{\sigma}_{\Delta T} \), is related to the temperature difference between the two streams.

The integrals of \( \dot{\sigma}_{\Delta P} \) must be calculated for each stream independently, as the temperature distribution is different and their pressures and specific volumes, \( v_c \) and \( v_h \), may also be. Solving this integral may be impossible in practice, for it would require knowing the fluid properties of both flows in detail, added to the temperature distribution across the exchanger.

We can observe that \( \dot{L} \) could be obtained directly, from a black box exergy balance of the heat exchanger, once inlet and outlet conditions and mass flows are known. The integrals of Equations (3) and (4) could be solved if the hot and cold temperature distributions were known across the exchanger. Then, Equation (5) could be worked out from Equation (1).

### 3. A New Simple Method

The proposed method consists of calculating \( \dot{L} \) first and then the theoretical temperature distribution across the heat exchanger, prior to solving Equations (3) and (4), thus obtaining \( \dot{B}_Q \) and \( \dot{\sigma}_{\Delta T} \). This allows one to work out \( \dot{I}_{\Delta P} \) from Equation (1).

Although this seems straightforward and many heat transfer text books explicate how temperature distributions can be formulated in heat exchangers (see for example [14]), the calculations here are somewhat different. In heat transfer books, the problem is to design a heat exchanger. Here, it is to analyze an existing one (or a possible design). Therefore there exist boundary conditions: inlet, outlet temperatures and two heat transfers. This forces one to carry out a careful interpretation of energy balances, and to introduce one unknown variable into the calculations, which must then be solved by forcing the heat condition.

Thus, before calculating temperature distributions, it is necessary to work out the global, black-box energy, and exergy balances of the heat exchanger. The following sections describe how this is done.
and how to calculate the temperature distributions for heat exchangers in which both fluids remain in the same phase throughout, and also for the case when one fluid changes its phase. All other possible combinations can be analyzed analogously.

3.1. Energy and Exergy Balances to the Exchanger

As we mentioned in section 2, energy in a heat exchanger flows in two ways: the main flow occurs between the fluids, while secondary energy flows between the exchanger itself and its surroundings. It is necessary to distinguish both, which we have denoted, respectively by $\dot{Q}_{\text{exch}}$ and $\dot{Q}_{\text{surr}}$. This can be done with the energy balance. Indeed, we know, by considering each flow independently:

$$\dot{Q}_c = \dot{m}_c \cdot (h_{c,o} - h_{c,i})$$

$$\dot{Q}_h = \dot{m}_h \cdot (h_{h,o} - h_{h,i})$$

Note that $\dot{Q}_c$ is positive while $\dot{Q}_h$ is negative, because the cold fluid absorbs heat and the hot fluid releases it. The energetic losses to the surroundings, $\dot{Q}_{\text{surr}}$, are therefore:

$$\dot{Q}_{\text{surr}} = \dot{Q}_c + \dot{Q}_h$$

On the other side, the heat exchanged by both fluids is given by Equation (9).

$$\left| \dot{Q}_{\text{exch}} \right| = \min \left( \left| \dot{Q}_c \right|, \left| \dot{Q}_h \right| \right)$$

It can be observed that if the heat exchanger was hotter than its environment, $\dot{Q}_{\text{exch}}$ would be the heat absorbed by the cold fluid.

The total exergy losses in the exchanger, $\dot{L}$, are given by Equation (2), which can be developed further by considering the specific exergy of the incoming and outgoing fluids. These, in turn, could be expressed as a function of the temperature of the surroundings, specific enthalpy and entropy as in Equation (10):

$$\dot{L} = \sum_{\forall j} \dot{E}_j = \dot{m}_h \cdot (e_{h,i} - e_{h,o}) + \dot{m}_c \cdot (e_{c,i} - e_{c,o}) =$$

$$\dot{m}_h \cdot (h_{h,i} - h_{h,o} - T_0 \cdot (s_{h,i} - s_{h,o})) + \dot{m}_c \cdot (h_{c,i} - h_{c,o} - T_0 \cdot (s_{c,i} - s_{c,o}))$$

3.2. Temperature Distributions in Monophase Heat Exchangers

The formulation consists in expressing the differential heat absorbed or released by each fluid in terms of its enthalpy increment on one side, and as a function of the temperature difference between the fluids on the other.
Figure 1. Diagram of counterflow, monophase (left) and phase-change (right) heat exchangers and the corresponding temperature distributions.

For example, let us consider a counterflow heat exchanger as the shown in Figure 1 (left). The differential heat exchanged by the cold and hot streams is:

\[ d\dot{Q}_c = \dot{m}_c \cdot c_{p,c} \cdot dT_c \]  \hspace{1cm} (11)

\[ d\dot{Q}_h = \dot{m}_h \cdot c_{p,h} \cdot dT_h \]  \hspace{1cm} (12)

By equalling Equation (11) and Equation (12) it can be observed that there exists a linear relation between both temperatures:

\[ T_c = K_1 \cdot T_h + K_2 \]  \hspace{1cm} (13)

where \( K_1 = \frac{\dot{m}_h \cdot c_{p,h}}{\dot{m}_c \cdot c_{p,c}} \) and \( K_2 = T_{c,i} - K_1 T_{h,o} \). On the other hand, these must be equal to the differential heat transferred from the hot stream due to the temperature difference:

\[ d\dot{Q}_c = K \cdot (T_h - T_c) dl \]  \hspace{1cm} (14)

where \( K \) is an unknown proportionality constant, and \( l \) an auxiliary variable which will indicate the location within the stretch, and which has to vary from \( l = 0 \) at the beginning to \( l = 1 \) at the end of the exchanger. By equalling Equation (14) to Equation (12) and integrating, we get the theoretical temperature distribution for \( T_h \), and consequently that of \( T_c \) by Equation (13):

\[ T_h = \left( T_{h,i} + \frac{K_2}{K_1 - 1} \right) \exp \left( \frac{K(K_1 - 1)}{\dot{m}_h c_{p,h}} l \right) + \frac{K_2}{K_1 - 1} \]  \hspace{1cm} (15)

By integrating (14) between \( l = 0 \) and \( l = 1 \), and equalling the result to \( Q_c \), obtained from Equation (6), we can work out \( K \):

\[ K = K_1 \frac{\dot{m}_c c_{p,c}}{1 - K_1} \ln \left( \frac{T_{c,o} - K_2}{T_{c,i} - K_2} \right) \]  \hspace{1cm} (16)
The average specific heats for both streams are given by Equations (17) and (18).

\[ c_{p,h} = \frac{\dot{Q}_c}{\dot{m}_h(T_{h,i} - T_{h,o})} \]  
\[ c_{p,c} = \frac{\dot{Q}_c}{\dot{m}_c(T_{c,o} - T_{c,i})} \]  

Note that the numerator for approximating the specific heat of the hot fluid, \( c_{p,h} \), is the heat absorbed by the cold fluid. This is to exclude the heat losses to the environment; more will be said in section 3.6.

3.3. Temperature Distributions in Heat Exchangers with Change of Phase

Let us consider a heat exchanger in which the cold fluid passes from saturated liquid to saturated vapor, such as the one in Figure 1 (right).

In essence, the calculations are the same as in the previous case, only that \( T_c = T_{2c} = T_{3c} \) and the stream with change of phase is formulated in terms of vapor quality \( x \). The heat absorbed by the evaporating fluid is proportional to its latent heat, \( \lambda \), and its vapor quality, \( x \). On the other hand, it must be equal to the heat exchanged by the hot fluid:

\[ d\dot{Q}_{exch} = \dot{m}_c \lambda \cdot dx = \dot{m}_h \cdot c_{p,h} \cdot dT_h \]  

Integrating from saturated liquid throughout the change of phase, the solution of Equation (19) is:

\[ T_h = \frac{\dot{m}_c \cdot \lambda}{\dot{m}_h \cdot c_{p,h}} x + T_{h,o} \]  

Again, integrating (14) between \( l = 0 \) and \( l = 1 \) we can work out \( K \):

\[ K = \dot{m}_h \cdot c_{p,h} \cdot \ln \left( \frac{T_{3h} - T_{3c}}{T_{2h} - T_{2c}} \right) \]  

3.4. Entropy Generation Due to Heat Transfer

Equation (4) can be particularized for the stretch:

\[ \dot{\sigma}_{\Delta T} = \int_0^1 K(T_h - T_c) \left( \frac{1}{T_c} - \frac{1}{T_h} \right) dl \]  

The integral can be solved in a number of ways. By applying Gaussian numerical integration, it is possible to reduce the integral to simple calculations (it is a standard method for numerical integration, so it will be summarized here very briefly; an explanation can be found at [15]):

\[ \int_{-1}^{1} f(t) dt \approx \sum_{i=1}^{n} w_i f(t_i) \]  

where \( n \) is an arbitrary integer; \( f(t_i) \) are the values of the subintegral at specific points \( t_i \) in the \((-1, 1)\) interval; \( w_i \) are coefficients.
Finally, because $t_i \in (-1, 1)$, a change of variable is needed for adjusting the limits of integration to $(0, 1)$, the limits that appear in Equation (22). The change $t = 2l - 1$ leads to Equation (24).

$$\dot{\sigma}_{\Delta T} = \sum_{i=1}^{n} w_i f(t_i)$$  \hspace{1cm} (24)

3.5. Complex Heat Exchangers

By the term ‘complex’ we understand a heat exchanger in which one fluid may be in a single phase part of the time, then change phase and then continue in a different one.

The way of analyzing a complex heat exchanger is straightforward: the global energy and exergy balances must be calculated, then the monophase and phase-change stretches must be considered separately by applying the calculations of the previous sections to each one.

We assume that the only data available from the heat exchanger are temperatures and mass flows of both fluids at all inputs and outputs. Therefore, we ignore the exact points where the fluid changes phase.

Hence, the first step is finding these phase-change points so that we can divide the black-box exchanger in stretches. Let us consider, for example, the counterflow heat exchanger of Figure 2, in which the notation of the previous sections has been adapted. Three areas can be distinguished: preheater (P), evaporator (E) and superheater (S). The preheater and the superheater will be analyzed as monophase exchangers (Section 3.2). The evaporator, as a phase-change heat exchanger (Section 3.3).

**Figure 2.** Temperature diagram of a complex heat exchanger.

The first step to finding the phase change points is to decide its temperature. We shall assume that the pressure remains constant along the cold stream, so the temperature of phase change is the saturation
temperature at that pressure, $P_c$, which can be consulted in the tables of properties of the cold fluid. This also gives the saturated liquid and vapor enthalpies and entropies, thus determining completely points $2c$ and $3c$.

By knowing $T_{2c}$, we also know the heat absorbed by the cold fluid between the inlet and the phase change. By knowing the saturated enthalpies, we know the heat absorbed during phase change. This allows us to calculate the relative position at which the phase change starts as shown in Equation (25), and where it finishes, by Equation (26).

\[
\frac{h_{2h} - h_{1h}}{h_{4h} - h_{1h}} = \frac{h_{2c} - h_{1c}}{h_{4c} - h_{1c}} \quad (25)
\]

\[
\frac{h_{3h} - h_{1h}}{h_{4h} - h_{1h}} = \frac{h_{3c} - h_{1c}}{h_{4c} - h_{1c}} \quad (26)
\]

With this, the initial and final relative positions of the phase change are known, so the exchanger can be divided as a series of three black boxes: P+E+S. The losses of each stretch will calculated independently. The total is the sum:

\[
\dot{\sigma}_{\Delta T} = \dot{\sigma}_{\Delta T,P} + \dot{\sigma}_{\Delta T,E} + \dot{\sigma}_{\Delta T,S} \quad (27)
\]

\[
\dot{\sigma}_{\Delta P} = \dot{\sigma}_{\Delta P,P} + \dot{\sigma}_{\Delta P,E} + \dot{\sigma}_{\Delta P,S} \quad (28)
\]

\[
\dot{B}_Q = \dot{B}_{Q,P} + \dot{B}_{Q,E} + \dot{B}_{Q,S} \quad (29)
\]

where the subscripts $P$, $E$ and $S$ indicate preheater, evaporator, and superheater, respectively.

3.6. Exergy Losses to the Surroundings

The exergy losses to the environment are given by Equation (3). We may observe that solving this integral is not straightforward, because two of its terms are unknown: the differential heat loss to the environment, $dQ_{\text{surr}}$, and the temperature at which it is released, $T$, for every point of the outer surface of the heat exchanger. The problem of modeling these two terms is of such magnitude that an accurate calculation would require finite element simulations and detailed plans of the particular heat exchanger being analyzed. However, these data do not exist for our study, so a different approach must be made, allowing at least for a gross estimation.

Regarding the heat loss to the surroundings, we may see that it is the difference between the heat really being released by the hot stream and that being absorbed by the cold one at each $dl$, so a reasonable estimation would be given by:

\[
dQ_{\text{surr}} = -\dot{m}_h (c^T_{p,h} - c_{p,h})dT_h \quad (30)
\]

Where $c^T_{p,h}$ represents the real approximation to the average specific heat of the hot stream, given by $c^T_{p,h} = \frac{\dot{Q}_h}{\dot{m}_h(T_{1h} - T_{2h})}$ instead of Equation (17).

Modeling the temperature of the enclosure, $T$, at which this $dQ_{\text{surr}}$ is released, requires careful consideration. We could consider two extreme cases: making $T$ equal to the lowest and highest temperatures in the exchanger, respectively. This would give us two extreme values for $\dot{B}_Q$, thus defining an expectable range:

\[
\dot{B}_{Q,T_{2h}} = \dot{Q}_{\text{surr}} \left(1 - \frac{T_0}{T_{2h}}\right); \quad \dot{B}_{Q,T_{1c}} = \dot{Q}_{\text{surr}} \left(1 - \frac{T_0}{T_{1c}}\right) \quad (31)
\]
Other models for $T$ can also be taken, such as the average cold and hot temperatures $\bar{T}_c$ and $\bar{T}_h$, or the $\bar{T}_c$ and $\bar{T}_h$ distributions themselves.

3.7. Exergy Destruction Due to Pressure Drop

The entropy generation that results from the pressure drop is given by Equation (5). However, as was mentioned earlier, solving these integrals would require very precise and specific data about the fluids, as well as proposing a number of hypotheses. It is therefore more reliable, in general, to work this term out from Equation (1). By using the two extreme values of $\dot{B}_Q$, derived from the previous section, the following expressions give the corresponding limits for $\dot{I}_{\Delta P}$:

\[
\begin{align*}
\dot{I}_{\Delta Pmax} &= T_0 \dot{\sigma}_{\Delta Pmax} = \dot{L} + \dot{B}_{Qmin} - T_0 \dot{\sigma}_{\Delta T} \\
\dot{I}_{\Delta Pmin} &= T_0 \dot{\sigma}_{\Delta Pmin} = \dot{L} + \dot{B}_{Qmax} - T_0 \dot{\sigma}_{\Delta T}
\end{align*}
\] (32)

It must be remarked that other authors have studied the problem of calculating $\dot{I}_{\Delta P}$ from alternative points of view. An interesting approach was developed by Bejan in [12]. It suggests a new exergetic parameter for heat exchangers called the irreversibility distribution ratio, $\phi$, defined as:

\[
\phi = \frac{\dot{\sigma}_{\Delta P}}{\dot{\sigma}_{\Delta T}} = \frac{\dot{I}_{\Delta P}}{\dot{I}_{\Delta T}}
\] (33)

By theoretical studies of this parameter for different cases of heat exchangers, it was concluded that the optimum in exergy destruction takes place for $\phi_{opt} = 0.168$, although the exergetic performance remains near to the optimum in a range of values around $\phi_{opt}$, especially lower ones, and up to $\phi = 1$ on the higher end.

It is necessary to clarify some points regarding this subject. First, $\phi_{opt} = 0.168$ is a general, theoretical value, and thus virtually impossible to achieve in practice, depending on the fluids being used and the operating conditions. Second, the actual range of $\phi$ in real exchangers might be far off from this value. However, the idea of comparing the temperature and pressure drops destruction terms is interesting.

4. Numerical Aplication

The new simple method is going to be applied to two cases:

- The first example (A) is inspired in devices and data described in [16,17], assuming that an oil-based solar field wanted to be integrated with a combined cycle at its intermediate pressure;
- The second example (B) is a condenser from a power plant in which the cooling water is always below the ambient temperature.

The purpose of the first example is of the method’s application. On the second example, some limitations of this methodology are going to be analyzed.
4.1. Example A: Oil-Water Evaporator

Consider an oil-water counterflow exchanger for steam generation with the same structure as in Figure 2. The oil flow is $\dot{m}_h = 249$ kg/s, and the water flow $\dot{m}_c = 26.2$ kg/s. The inlet and outlet thermodynamic data are given in Table 1. The pressure of each flow has been assumed constant for simplicity. The ambient temperature is $T_0 = 298$ K.

The energy balance has the following results:

$$\dot{Q}_c = \dot{m}_c (h_{4c} - h_{1c}) = 54811.71 \text{ kW}$$
$$\dot{Q}_h^T = \dot{m}_h (h_{1h} - h_{4h}) = -61801.8 \text{ kW}$$

The energetic losses to the surroundings are $\dot{Q}_{surr} = \dot{Q}_c + \dot{Q}_h = -6990.09 \text{ kW}$, approximately 11% of $\dot{Q}_h$.

The global balance of Equation (10) results:

$$\dot{L} = \sum \dot{E}_j = 9945.654 \text{ kW} \quad (34)$$

By consulting the properties of water, the saturation temperature at $P_{1c} = 31$ bar is 509.62 K. With this value, property tables and by applying equations (25) and (26) the properties of the intermediate points are calculated, and shown in Table 2.

| Table 1. Black box, thermodynamic data of inlets and outlets in the oil-water exchanger. |
|-----------------------------------------------|
| T (K) | 571.15 | 666.15 | 487.15 | 580.07 |
| P (bar) | 11 | 11 | 31.45 | 31.45 |
| h (kJ/kg) | 541.80 | 790 | 916.38 | 3008.43 |
| s (kJ/kgK) | 1.55 | 1.93 | 2.46 | 6.45 |

| Table 2. Estimates for the thermodynamic properties of the intermediate points of the oil-water exchanger. |
|-----------------------------------------------|
| T (K) | 575.91 | 656.79 | 509.62 | 509.62 |
| h (kJ/kg) | 554.24 | 765.54 | 1021.19 | 2802.32 |

Entropy generation due to heat transfer can be calculated solving the integral (22) by Gaussian numerical integration as shown in Equation (23). The integer $n$ determines the accuracy. A value of 3 is used in this case. The corresponding values of $t_i$ and $w_i$ are given in Table 3. The numerical results are shown in Table 4. The exergetic destruction due to heat transfer is 54.32% of the total exergetic destruction.
Table 3. Points \( t_i \) in the \((-1, 1)\) interval and coefficients \( w_i \) for the Gaussian integration method using \( n = 3 \).

| \( t_i \)  | \( w_i \)          |
|-------|---------------------|
| -0.7745966692 | 0.5555555556 |
| 0     | 0.8888888889       |
| 0.7745966692  | 0.5555555556     |

Table 4. Global exergetic analysis for the exchanger. Example A.

| kW       |
|----------|
| \( \dot{L} \) | 9945.65 |
| \( \dot{I}_{\Delta T} \) | 5432.91 |
| \( \dot{I}_{\Delta P \text{max}} \) | 1619.54 |
| \( \dot{I}_{\Delta P \text{min}} \) | 712.15  |
| \( \dot{I}_{\Delta P \text{Bejan}} \) | 912.73  |
| \( \dot{B}_{Q \text{min}} \) | -2893.20 |
| \( \dot{B}_{Q \text{max}} \) | -3800.60 |
| \( \dot{B}_{Q \text{Bejan}} \) | -3600.01 |

To determine the exergy losses to surroundings, we shall consider the two extreme cases explained in Section 3.6, choosing \( T \) Equation (31) equal to the lowest and highest temperatures in the exchanger, respectively. The numeric results are shown on Table 5. The maximum and minimum exergetic losses of the surroundings mean 28.93% and 38% of the losses due to this phenomena.

Table 5. Losses to the surroundings for the exchanger. Example A.

| \( T \) [K] | \( \dot{B}_{Q} \) [kW] |
|-------------|------------------------|
| 666.15      | -2893.20               |
| 487.15      | -3800.60               |

From these estimates of \( \dot{B}_{Q} \) and \( \dot{I}_{\Delta T} \), we can proceed as explained in Section 3.7. Equation (32) gives the range for \( \dot{I}_{\Delta P} \). It would be between 7.12% and 16.19% of the total destruction.

Alternatively, by Bejan’s optimum, we could calculate \( \dot{I}_{\Delta P} \) and work out \( \dot{B}_{Q} \). Thus, destruction due to pressure drop to \( \dot{I}_{\Delta P \text{Bejan}} \) would be 912.63 kW, 9.12% of the total destruction. It can be observed in Table 4 that it falls within the expected range.
4.2. Example B: Cooling Tower.

This example is a condenser from a power plant. The input and output parameters are indicated in Table 6. It is a two-phase exchanger, similar to that of Section 3.3, except that, in this case, the fluid-changing phase is the hot fluid that condenses. It must be observed that the cold fluid is cooler than the ambient, at $T_0 = 298$K. As we shall see, this will make it necessary to interpret calculations carefully.

Table 6. Black-box data for a condenser in a power plant. Example B.

|       | 1h  | 2h  | 1c  | 2c  |
|-------|-----|-----|-----|-----|
| T (K) | 299.35 | 299.35 | 288.15 | 295.75 |
| P (bar) | 0.034 | 0.034 | 4.0 | 4.0 |
| h (kJ/kg) | 109.79 | 2231.30 | 63.32 | 95.10 |
| s (kJ/kgK) | 0.3838 | 7.4708 | 0.2243 | 0.3331 |
| $\dot{m}$ (kg/s) | 14.936 | = | 1015.0 | = |

In this case, the total exergetic destruction and the exergetic losses are $\dot{L} = 813.77$ kW and $\dot{I}_{\Delta T} = 801.36$ kW. The fact that the heat exchanger absorbs heat from its environment, $d\dot{Q}_{surr} > 0$, makes it necessary to reason on the results for $\dot{B}_Q$. Proceeding as in the previous example would lead, on one side, to $\dot{B}_{Qmax} = 2.58$ kW > 0, which cannot be, because in Equation (3) $d\dot{Q}_{surr} > 0$, and necessarily $T < T_0 < T_{h,max}$. Thus, the maximum admissible value for $\dot{B}_Q$ would be 0, therefore resulting in $\dot{I}_{\Delta P_{max}} = 12.41$ kW. On the other hand, $\dot{B}_{Qmin}$ would lead to a negative $\dot{I}_{\Delta P_{min}}$, which is also impossible, so the minimum admissible value for $\dot{I}_{\Delta P_{min}}$ would be 0, making $\dot{B}_{Qmin} = -12.41$ kW. All these values are shown in Table 7.

Table 7. Global exergetic analysis for the condenser. Example B.

|                        | kW       |
|------------------------|----------|
| $\dot{L}$              | 813.77   |
| $\dot{I}_{\Delta T}$   | 801.361  |
| $\dot{I}_{\Delta P_{max}}$ | 12.41   |
| $\dot{I}_{\Delta P_{min}}$ | 0       |
| $\dot{I}_{\Delta P_{Bejan}}$ | Not applicable |
| $\dot{B}_{Qmin}$       | -12.41   |
| $\dot{B}_{Qmax}$       | 0        |
| $\dot{B}_{Q_{Bejan}}$  | Not applicable |

The contribution of the heat transfer destruction is 98.5% of the total losses. The ranges and relative importance of $I_{\Delta P}$ and $-\dot{B}_Q$, both within $\in (0, 12.41)$ kW may represent a maximum of 1.5% of the
total exergetic losses. In comparison to the previous example, the total exergetic losses can be assumed equal to heat transfer losses. In this case, Bejan’s optimum, \( I_{\Delta P,\text{Bejan}} = 134.63 \text{ kW} \), cannot be applied because it would imply an impossible distribution of \( T \) in Equation (3).

5. Conclusions

The exergetic value of the heat that an exchanger interchanges with its surroundings is usually neglected, due to the fact that it usually is relatively small in energetic terms (\( \sim 10\% \)). It is possible to find a range for it by simple assumptions based on the temperatures of operation of the exchanger. The exergetic equivalent of heat dissipation might become significant for applications in which both streams are far apart from \( T_0 \), and it can be neglected when close by. There are two reasons for this; first, that the Carnot factor at the exergy losses Equation (3) tends to zero, and second, that the heat transfer between exchanger and environment is slower due to their temperatures being closer.

By assuming reasonable hypotheses and having basic, black-box data of an exchanger, it is possible to produce a reasonable characterization of its exergetic performance, useful for analyzing and optimizing plant cycles and processes. The accuracy of the estimation will depend on how well the hypothetical temperature distributions adjust to the real ones in a specific exchanger, although the methodology proposed here should be representative of an order of magnitude.

Acknowledgements

We would like to thank Laura Fernández and José Luis Rapún for their help and advice throughout the many difficult parts in the research leading to this paper.

References

1. Bejan, A. General criterion for rating heat-exchanger performance. *Int. J. Heat Mass Transf.* 1978, 21, 655–658.
2. Kotas, T.J. *The Exergy Method of Thermal Plant Analysis*; 1995 Ed.; Krieger Publishing Company: London, UK, 1985.
3. Yilmaz, M.; Sara, O.N.; Karsli, S. Performance evaluation criteria for heat exchangers based on second law analysis. *Exergy Int. J.* 2001, 4, 278–294.
4. Wu, S.T.; Yuan, X.F.; Li, Y.R.; Xiao, L. Exergy transfer effectiveness on heat exchanger for finite pressure drop. *Energy* 2006, 32, 2110–2120.
5. Franco, A.; Russo, A. Combined cycle plant efficiency increase based on the optimization of the heat recovery steam generator operating parameters. *Int. J. Therm. Sci.* 2002, 41, 843–859.
6. Saidur, R.; Ahamed, J.; Masjuki, H. Energy, exergy and economic analysis of industrial boilers. *Energy Policy* 2010, 38, 2188–2197.
7. Pandey, S.D.; Nema, V. An experimental investigation of exergy loss reduction in corrugated plate heat exchanger. *Energy* 2011, 36, 2997–3001.
8. Haseli, Y.; Dincer, I.; Naterer, G. Optimum temperatures in a shell and tube condenser with respect to exergy. *Int. J. Heat Mass Transf.* 2008, 51, 2462–2470.
9. Hajabdollahi, H.; Ahmadi, P.; Dincer, I. Thermoeconomic optimization of a shell and tube condenser using both genetic algorithm and particle swarm. *Int. J. Refrigeration* 2011, 34, 1066–1076.

10. Lacalle, J.M.; Turet, J.; Nieto, R.; Honduvilla, J.; Teijeiro, A.; González, M.C.; Herrero, F. Termodinámica [Thermodynamics], 2nd Ed.; Number ISBN 84-7484-083-X, Escuela Técnica Superior de Ingenieros Industriales de Madrid, 1997.

11. Huang, F.F. *Engineering Thermodynamics*; Macmillan Publishing Co.: New York, NY, USA, 1976.

12. Bejan, A. *Advanced Engineering Thermodynamics*; John Wiley & Sons: Hoboken, NJ, USA, 1988.

13. Jin, Z.; Dong, Q.; Liu, M. Exergy study of fouling factors in heat exchanger networks. *J. Heat Transf.* 2010, 132, pp. 011802-1–011802-4

14. Incropera, F.P.; DeWitt, D.P.; Bergman, T.L.; Lavine, A.S. *Fundamentals of Heat and Mass Transfer*, 6th Ed.; John Wiley & Sons: Hoboken, NJ, USA, 2006.

15. Holistic Numerical Methods. Chapter 07.06: Textbook chapter of gauss-quadrature method. Available online: http://numericalmethods.eng.usf.edu/mws/gen/07int/mws_gen_int_txt-gaussquadrature.pdf (accessed on 25 January 2013).

16. Baghernejad, A.; Yoghoubi, M. Exergy analysis of an integrated solar combined cycle system. *Renew. Energy* 2010, 35, 2157–2164.

17. Woudstra, N.; Woudstra, T.; Pirone, A.; van der Stelt, T. Thermodynamic evaluation of combined cycle plants. *Energy Convers. Manag.* 2010, 51, 1099–1110.

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