Thermal convection in an inclined cavity under the influence of partial magnetic field

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Abstract: This work is focused on the implication of a partial magnetic field on a square cavity heated isothermally at the bottom wall and cooled at the sidewalls. The top wall is insulated. The enclosure is filled with electrically conducting fluid. The coupled transport equations are solved numerically using a written computing code adopting FVM and SIMPLE algorithm applying specific boundary conditions. The analysis is carried out for different involved parameters like Rayleigh number (Ra), Hartmann number (Ha), and angle of inclination (γ). The results are illustrated using contours of streamline, isotherms, and heat transfer characteristics. It is found that the flow structure, the temperature distribution is severely affected by the above-mentioned parameters. The rate of heat transfer is significantly reduced with the increasing Ha.

Keywords: Natural convection; Partial magnetic field; Inclined cavity; Heat transfer.

1. Introduction

Natural convection has high relevance in natural as well as engineering applications. Mantle convection within the earth’s mantle is the driving force of the tectonic plates, sea breeze formation and climatic advances also depend on the rate of convection. Besides its implementation in nature, it consumes a variety of applications in thermal engineering, nuclear reactors, heating and cooling systems, solar power, and many more. It provides immense scope for the researches to conduct wide ranges of analysis and experiments. A detailed review of free convection inside an enclosure can be found in Refs. [1,2].

The application of the magnetic field has numerous aspects in the field of heat transfer and fluid flow. Magnetic fields are highly used in biomedical, thermal designs to control the dissipation of heat in microprocessors, electronics, aviation, and other engineering processes. In this context, fundamentals about the subject topic magnetic convection can be found in [3]. The consequence of the imposed magnetic field is associated with the Lorentz force and it is known as magnetohydrodynamics (MHD). Several researchers have investigated the thermo-magnetic convection. For example, Rudraiah et al. [4] studied the influence of the imposed magnetic field during natural convective heat transfer from a rectangular enclosure. One of the results was as the Hartmann number increased Nusselt number decreased. Tagawa et al. [5] examined the influence of the direction of a magnetic field on free convective heat transfer from a square cavity. Magnetic convection in a heated cubical cavity filled with paramagnetic fluid is observed by Bednarz et al. [6]. Ganguly et al. [7] investigated...
the magneto-hydrodynamic convective heat transfer process in an enclosure adopting variable magnetic fields utilizing line dipole. The application of a partial magnetic field on thermal convection has received considerable attention for controlling various engineering processes. The thermo-magnetic convection in a differentially heated square cavity subjected to partially active magnetic fields has been examined by Jalil et al. [8]. It is observed that the thermo-fluid flow structure is severely affected by the imposed magnetic fields. The impact of non-uniform magnetic fields during the thermal convection from a cavity has been studied by Szabo and Früh [9]. In this study, it is found that buoyancy-induced thermal convection dominates the magnetic force and associated rate of heat transfer. Very recently, Geridonmez and Oztop [10] have investigated the partial magnetic fields effect on thermal convection in a differentially heated air filled porous cavity. It is observed that the strength, location of the partial magnetic fields can alter the convective heat transfer process. A study related to Magnetohydrodynamic (MHD) influence on natural convection inside a closed cavity was conducted by Matt et al. [11].

Taking into account an extensive literature survey, it is observed that there are no works about the thermal convection in an inclined chamber subject to a partial magnetic field. Thus, this study aims to explore the application of a partial magnetic field on thermal convection in an inclined cavity heated isothermally at the bottom and cooled at the sidewalls. The investigation is conducted numerically and the results are presented and described with the help of streamlines, isotherms, and average Nusselt numbers. The inclination of the cavity along with the magnetic field has a finite amount of research which creates a new horizon for more exploration. In due course, the present work could be helpful is thermal engineering and medicine.

2. Problem description and numerical procedure

The geometry of the problem is comprised of a square enclosure \((L \times H)\) with isothermally heated bottom wall (at temperature \(T_h\)). The sidewalls are kept cold (at temperature \(T_c\), where \(T_c < T_h\)) and top wall is insulated as described in Figure1. The fluid within the cavity is taken as air (\(Pr = 0.71\)) and it is assumed that the fluid is steady, laminar, Newtonian, and incompressible within the validity of Boussinesq approximation. An external magnetic field acts (perpendicular to the sidewalls) partially over the effective length \(0.5H\) (about the middle point of the sidewalls). It is important to note that, for obtaining the magnetic field along the \(x\) direction, electric field or voltage is applied perpendicular to the \(x\)-\(y\) plane. The study involves the inclination of the cavity (\(\gamma\)) with respect to the horizontal direction. As the cavity inclination angle is changing with an angle \(\gamma\) with respect to the horizontal direction, magnetic field remains perpendicular to the sidewalls. However, the gravity force acts into sin and cos components of the buoyancy force as given in the momentum equations.

![Figure 1. Schematic diagram of the computational domain of the physical problem.](image)

Defining the following non-dimensional parameter, the dimensional transport equations are transformed into the dimensionless form:
\[(X,Y) = (x, y)/H; (U, V) = (u, v)H / \alpha; \theta = (T - T_y)/(T_u - T_y); P = ((p + \rho g y) - p_c)H^2 / \rho \alpha^2\]

Where \(u, v\) are dimensional velocity components, \((U, V)\) are dimensionless velocities. The dimensional and non-dimensional axes are \((x, y)\) and \((X, Y)\) respectively. The non-dimensional pressure \(P\) constitute of ambient pressure \((p_a)\) and pressure change along the vertical \((p g y)/g\), respectively. The non-dimensional governing equations in the Cartesian coordinate system are mentioned below:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y}) = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + Pr Ra \theta \sin \gamma
\]

\[
(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y}) = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \lambda_b Ha^2 Pr V + Pr Ra \theta \cos \gamma
\]

\[
(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y}) = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

where,

\[Pr = \frac{\nu}{\alpha}; \quad Ra = \frac{g \beta (T_u - T_y) H^3}{\nu^2 \rho \alpha}; \quad Ha = BH \sqrt{\sigma / \mu}\]

The fluid properties (such as thermal diffusivity \(\alpha\), kinematic viscosity \(\nu\), and volumetric expansion coefficient) are assumed to be constant except density \((\rho)\) variation, which is taken care by the Boussinesq approximation. The partial magnetic fields adopted through the term \(B \lambda\) by setting 1 and 0 for the active and inactive zones, respectively.

The thermal condition of the top wall is adiabatic \((\partial \theta / \partial Y = 0)\), sidewalls are isothermally cold \((\theta = 0)\) and the bottom wall is isothermally heated \((\theta = 1)\). All the walls are set at zero cavity velocity \((U = V = 0)\). Along with the mentioned boundary conditions, an in-house computing code adopting the Finite Volume Method (FVM), and SIMPLE algorithm [12], has been employed iteratively to evaluate the governing equations. The above-mentioned code has been validated extensively under natural and mixed convective heat transfer [13–19]. After conducting a grid independence study, the grid size for the present numerical study is chosen as 200 x 200. The grids are distributed uniformly to capture the partial magnetic field effect correctly. The maximum residual and continuity mass defect is chosen as 10^{-8} for the convergence of the result.

The equations of streamfunction have been evaluated and availed to present the flow field pattern in terms of streamlines. The stream function \((\psi)\) is expressed as

\[\frac{\partial \psi}{\partial X} = V, \quad \frac{\partial \psi}{\partial Y} = U\]

In order to analyze the heat transfer characteristics for the bottom heated inclined square enclosure, the average Nusselt number is calculated on the hot wall as

\[Nu = \int_0^1 \left( \frac{\partial \theta}{\partial Y} \right)_{y=0} dX\]

3. Results and discussion

In the present study, thermomagnetic convection of a square inclined enclosure with heated bottom, an adiabatic top wall, and cold sidewalls is examined under the impact of a partial magnetic field. The analysis is carried out for a wide range of parameters like Rayleigh number \((Ra = 10^3, 10^4, 10^5, 10^6)\), Hartmann number \((Ha = 0, 30, 50, 70, 100)\), and cavity inclination angle \((\gamma = 0^\circ, 30^\circ, 60^\circ, 90^\circ, 150^\circ, 180^\circ)\).
180°). The results of the study are illustrated by the visualization of streamlines, isotherms, and average Nusselt number.

3.1. Effect of Rayleigh number (Ra)

Figure 2 illustrates the streamlines, isotherms and average Nu for Ra = 10^3, 10^4, 10^6 keeping Ha = 50 and γ = 0° fixed. For the different magnitudes of evolved parameters, two numbers convective circulation cells are formed inside the enclosure, namely, clockwise (CW) at the right half and anticlockwise (CCW) at the left half of the cavity. These circulation cells are formed due to the heating at the bottom and cooling at the sidewalls. Due to the bottom heating, heated fluid rises upwards and then obstructed by the top adiabatic wall and then directed towards the side walls, where it cools and goes downward. As magnetic fields act partially over the length 0.5H (about the middle point of the sidewalls), fluid flow faces resistance (due to the Lorentz force) over this area of imposed magnetic fields. As a result, flow structure as well as static temperature distribution distorted in this zone. At the lower value of Ra (= 10^3), conduction mode dictates the heat transfer mechanism. Thus, fluid circulation is poor. Two strong counter-rotating circulations cells appear in the lower part and two weaker counter-rotating circulations cells appear in the upper part of the enclosure. The contours of isotherm are concentrated near the heated bottom wall in a certain symmetrical pattern. As the convection mechanism increases with the increasing Ra and becomes maximum at Ra = 10^6, the circulation strength increases significantly. The buoyancy effect dominates the Lorentz force at higher Ra. Thus, upper counter-rotating circulations cells become weaker. The isotherm contours remains clustered near the active walls. There is a significant rise in the heat transfer by the fluid as indicated by the increasing average Nusselt number as Rayleigh number is increased.

| Ra = 10^3 | Ra = 10^4 | Ra = 10^6 |
|-----------|-----------|-----------|
| Streamlines | Streamlines | Streamlines |
| Isotherms | Isotherms | Isotherms |

Nu = 8.221 (Ra = 10^3)  Nu = 10.079 (Ra = 10^4)  Nu = 16.052 (Ra = 10^6)

Figure 2. Effect of Ra on the contours of streamlines (top row), and isotherms (bottom row) for Ha = 50, γ = 0.

3.2. Effect of Hartmann number (Ha)
The effect of partially imposed magnetic field intensity is described by the Hartman number (Ha). The contours of streamlines, isotherms, and average Nu are shown in Figure 3 for Ha = 0, 30, 70 keeping Ra = 10^5 and $\gamma = 0^\circ$ fixed. In order to understand the effect of the imposed magnetic field, the case of without magnetic field (Ha = 0) is introduced first. Eventually, the magnetic field strength is increased by increasing the value of Ha = 30 and 70. In the absence of a magnetic field, the fluid flow shows two symmetrically cells (without any distortion) and isotherms distributed symmetrically about the mid vertical plane. When the magnetic field is imposed with intensity Ha = 30, two circulation cells stretched in a vertical direction (with a reduction in the circulation strength) and divided into two pairs of counter-rotating circulation cells (one pair at the upper and other pair at the lower part of the cavity). The isotherm lines are distorted about the mid-horizontal zone of the cavity. The associated heat transfer rate is decreased due to a reduction in fluid circulation. Although the heat transfer is symmetrical in the cavity but declines as the partial magnetic strength is further increased to Ha = 70. Flow structure affected significantly with a reduction in the circulation strength. The average Nusselt number decreases substantially with the increasing magnetic field strength. Therefore, a partial magnetic field, when used wisely, is able to control the heat propagation in an enclosure.

| Ha = 0 | Ha = 30 | Ha = 70 |
|--------|--------|--------|
| Streamlines | Streamlines | Streamlines |
| Isotherms | Isotherms | Isotherms |

Figure 3. Impact of Ha on the streamlines (top row) and isotherms (bottom row) contours for Ra = 10^5, $\gamma = 0^\circ$.

3.3. **Effect of cavity inclination ($\gamma$)**

The influence of the cavity inclination angle on the thermo-fluid phenomena is investigated by varying the inclination $\gamma = 30^\circ, 60^\circ, 90^\circ, 150^\circ$ keeping the Ha = 30 and Ra = 10^5 fixed as shown in Figure 4. Now, with the change in the inclination angle ($\gamma$) from 0 to 30° flow-structure alters significantly compared $\gamma = 0^\circ$ (as in Fig. 3, second column). Left CCW circulation becomes larger and merged into a single cell; whereas right circulation cells split into two weaker cells. Corresponding isotherm lines change and show an asymmetric pattern. Further increase in $\gamma$ from 30° to 60° all the circulating cells merged into single-cell rotating in CCW direction. The contours of isotherms have accumulated towards the right half of the cavity due to the inclination of the cavity in the counter-clockwise direction. Now, an increase in $\gamma$ to 90°, the heated wall of the cavity is oriented like a right
vertical wall, whereas cold walls are positioned as horizontal top and bottom walls. As a result, isotherms of high temperature are clustered near the heated wall and low-temperature lines distributed throughout the cavity. Here single circulating cells occupy the entire cavity. With the continual increase in $\gamma$ heat transfer rate decreases gradually. This happens due to the change in the flow structure as well as temperature distribution. Further increase in $\gamma$ to 150°, the heated wall of the cavity is positioned towards the top horizontal direction, whereas the cold walls are positioned as vertical walls. As a result, a single circulating cell shifts towards the right portion of the cavity and breaks into multiple cells. Heat transfer is decreased significantly.

$$\gamma = 30^\circ$$  $$\gamma = 60^\circ$$  $$\gamma = 90^\circ$$  $$\gamma = 150^\circ$$

Figure 4. Effect of $\gamma$ on the streamlines (top row) and isotherms (bottom row) contours at $Ra = 10^5$, $Ha = 30$.

3.4. Heat transfer characteristics

Now, the global thermal performance of the cavity is evaluated using the average Nu with the variation of $Ra (= 10^5, 10^6)$ and is depicted in Figure 5 (a and b) for different $Ha$ and $\gamma$ keeping other parameters at fixed values. In Figure 5a, the average Nu shows a consistently decreasing trend with the increasing $Ha$ for both $Ra = 10^5, 10^6$. This happens due to the presence of negative source terms associated with $Ha$ as in the $Y$ momentum equation (3). Thus, the magnetic force weakens the buoyancy effect. Furthermore, the Nu curve at $Ra = 10^6$ remains above that of $Ra = 10^5$. The reason behind this fact may be understood by observing the flow structure for Ra (as in Fig. 2). Higher Ra, results in stronger buoyancy force. As a result, heat transfer is higher. Furthermore, the selection of the $Ha$ value influences the heat transfer process. In Figure 5b, the average Nu decreases continually as the inclination angle ($\gamma$) gradually increases. When the angle rises, fluid circulation velocity within the cavity reduces leading to lower heat transfer. Thus, the decreasing trend for Ra= $10^5$ and $10^6$ is depicted in the figure. The Nu curve at $Ra = 10^5$ remains well above $Ra = 10^6$ with a steeper slope. It is interesting to note that, as the as the inclination angle ($\gamma$) increases till 180°, the heated wall of the cavity is oriented like a top horizontal wall, whereas cold walls are positioned as vertical sidewalls. It results in reduced heat transfer. Thus, average Nu decreases continually.
4. Conclusions
The present study investigates the effect of a partial magnetic field on the thermal convection in an inclined cavity heated from the bottom and cooled from the sidewalls. The thermomagnetic convection is explored under different parameters like Ra, Ha and $\gamma$. The thermal propagation and fluid flow were analyzed thoroughly using streamlines, isotherms, and average Nusselt number. Mentioned below are some of the major observations:

a) An increase in the partial magnetic field strength significantly alters the convective motion and it results in a decrement in heat transfer in the cavity. Hence, the usage of appropriate magnetic field strength can effectively control the thermal convection within the enclosure.

b) The gradual increase in the inclination angle significantly alters the thermo-fluid structure and associated heat transfer characteristics. The fluid flow becomes asymmetrical and the heat propagation inside the cavity reduces significantly.

c) Along with a change in Ha, the heat transfer rate can also be altered by changing the Ra.

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Nomenclature

| Symbol | Description |
|--------|-------------|
| B      | magnetic fields (Tesla, N A⁻¹ m⁻²) |
| g      | acceleration due to gravity (m s⁻²) |
| H      | height of the cavity/length scale (m) |
| Ha     | Hartmann number |
| L      | length of the cavity (m) |
| Nu     | average Nusselt number |
| p      | pressure (Pa) |
| P      | dimensionless pressure |
| Pr     | Prandtl number |
| Ra     | Rayleigh number |
| T      | temperature (K) |
| u, v   | velocity components (m s⁻¹) |
| U, V   | dimensionless velocity components |
| x, y   | Cartesian coordinates (m) |
| X, Y   | dimensionless coordinates |

Greek symbols

| Symbol | Description |
|--------|-------------|
| α      | thermal diffusivity (m² s⁻¹) |
| β      | thermal expansion coefficient (K⁻¹) |
| θ      | dimensionless temperature |
| γ      | inclination angle of the cavity |
| λm     | magnetic field parameter |
| μ      | dynamic viscosity |
| ν      | kinematic viscosity (m² s⁻¹) |
| ρ      | density (kg m⁻³) |
| σ      | electrical conductivity (µ S cm⁻¹) |
| ψ      | dimensionless stream function |

Subscripts

| Symbol | Description |
|--------|-------------|
| a      | ambient |
| c      | cold |
| h      | hot |