Effect of Tensor Correlations on Gamow-Teller States in $^{90}$Zr and $^{208}$Pb

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Abstract

The tensor terms of the Skyrme effective interaction are included in the self-consistent Hartree-Fock plus Random Phase Approximation (HF+RPA) model. The Gamow-Teller (GT) strength function of $^{90}$Zr and $^{208}$Pb are calculated with and without the tensor terms. The main peaks are moved downwards by about 2 MeV when including the tensor contribution. About 10% of the non-energy weighted sum rule is shifted to the excitation energy region above 30 MeV by the RPA tensor correlations. The contribution of the tensor terms to the energy weighted sum rule is given analytically, and compared to the outcome of RPA.

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Nuclei far from the stability line open a new test ground for nuclear models. Many experimental and theoretical efforts have been put to study the structure and the reaction mechanisms in the nuclei near the drip lines. Studies of exotic nuclei far from the $\beta$-stability line have elucidated unexpected trends for the shell closures [1]. The shell model can explain it if a finite range tensor force is introduced, which mainly originates from one-pion exchange [2]. In the mean field theory, almost 30 years ago, the role played by tensor interactions for the position of the single-particle states was first discussed [3] within the HF framework based on Skryme interactions [4]. Then, tensor force was included in the Skyrme-Landau parametrization and RPA (Random Phase Approximation) calculation in Ref. [5]. However, the tensor force was essentially dropped in most Skyrme parameter sets which have been fitted and which are still widely used in nuclear structure calculations. Recently, in Ref. [6], a Skyrme interaction which includes the tensor contribution was fitted. Then, tensor terms were added perturbatively in Refs. [7] and [8] to the existing standard parametrizations SIII [9] and SLy5 [10], respectively. Eventually, several new parameter sets have been fitted in Ref. [11] and used for systematic investigations within the Hartree-Fock-Bogoliubov (HFB) framework. The inclusion of tensor terms in the Skyrme HF calculations achieved considerable success in explaining some features of the evolution of single-particle states [12]. However, there has been, so far, no RPA or QRPA (Quasiparticle Random Phase Approximation) program available to study the effect of the tensor terms on the excited states of nuclei.

The present work is devoted to including the tensor terms of the Skyrme effective interaction in the self-consistent HF plus RPA calculations. In particular, we are interested in the GT transitions, which should be affected in keeping with the fact that the corresponding operator is spin-dependent [13,14]. In the study of GT transitions, the quenching problem is of some relevance. The experimentally observed strength from 10 to 20 MeV excitation energy (with respect to the ground state of the target nuclei) is about 50% of the model-independent non-energy weighted sum rule (NEWSR); this percentage becomes about 70% if the whole strength in the neighboring energy region is collected [15]. We deem interesting to study if the tensor force has an effect in shifting the strength already at one particle-one hole (1p-1h) level. Coupling the GT with two particle-two hole states is essential to describe the resonance width but it is not expected to affect strongly the position of the main GT peak; the effect of the tensor force in connection with the 2p-2h coupling was studied in
In this letter, we employ the triplet-even and triplet-odd zero-range tensor terms, which have the form originally postulated in the pioneering work by Skyrme and read \[3, 4\]

\[
V^T = \frac{T}{2} \left\{ \left[ (\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k'^2 \right] \delta (r_1 - r_2) 
+ \delta (r_1 - r_2) \left[ (\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k^2 \right] \right\} 
+ \frac{U}{2} \left\{ ((\sigma_1 \cdot k') \delta (r_1 - r_2) (\sigma_2 \cdot k) + (\sigma_2 \cdot k') \delta (r_1 - r_2) (\sigma_1 \cdot k) 
- \frac{2}{3} [(\sigma_1 \cdot \sigma_2) k' \cdot \delta (r_1 - r_2) k] \right\}
\tag{1}
\]

In the above expression, the operator \( k = (\nabla_1 - \nabla_2) / 2i \) acts on the right and \( k' = -(\nabla'_1 - \nabla'_2) / 2i \) acts on the left. The coupling constants \( T \) and \( U \) denote the strengths of the triplet-even and triplet-odd tensor interactions, respectively. The calculation employs, consistently with the choice of the Skyrme force SIII and with Ref. \[7\], the values \( T = 1008 \, \text{MeV fm}^5 \) and \( U = -432 \, \text{MeV fm}^5 \). Similar values of the tensor interactions have been suggested in Ref. \[8\] in the study of spin—splitting of Sb isotopes. In Refs. \[7, 8\], the parameters \( T \) and \( U \) are chosen in such a way to improve the absolute values and the isotopic(or isotonic) trends of single-particle energies. This criterion limits the possible choice on both the magnitude and sign of \( T \) and \( U \). In this way, one obtains an effective tensor interaction which does not necessarily correspond to the result of a G-matrix calculation, since effects from three-body force and nuclear correlations can have been effectively included. This point has been discussed in Ref. \[6\].

The main effect of the tensor terms on HF calculations is a modification of the spin-orbit potential(the total binding energies and radii being, as a rule, less affected). The spin-orbit potential is given by

\[
U_{S.O.}^{(q)} = \frac{W_0}{2r} \left( 2 \frac{d \rho_q}{dr} + \frac{d \rho_{q'}}{dr} \right) + \left( \alpha \frac{J_n}{r} + \beta \frac{J_p}{r} \right),
\tag{2}
\]

In this expression, \( q=0(1) \) labels neutrons (protons). \( J_n \) and \( J_p \) are the so-called spin-orbit densities of neutrons and protons respectively. Their definition can be found in Ref. \[17\]. The first term in the r.h.s comes from the Skyrme two-body spin-orbit interaction, whereas the second term includes both a central exchange and a tensor contribution, that is, \( \alpha = \alpha_C + \alpha_T \)
and $\beta = \beta_C + \beta_T$ with

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 - t_2 x_2),$$  \hspace{1cm} (3)

$$\beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2),$$  \hspace{1cm} (4)

$$\alpha_T = \frac{5}{12}U,$$  \hspace{1cm} (5)

$$\beta_T = \frac{5}{24}(T + U)$$  \hspace{1cm} (6)

It should be noted that $J_q$ gives essentially no contribution in the spin-saturated cases. Therefore, we choose $^{90}\text{Zr}$ and $^{208}\text{Pb}$ as examples to be calculated. $^{90}\text{Zr}$ is a proton spin-saturated nucleus, with a spin-unsaturated neutron orbit $1g_{9/2}$. $^{208}\text{Pb}$ is chosen as it is not saturated either in protons or neutrons. The two examples should allow elucidating separately the role of triplet-even and triplet-odd terms.

The HF plus RPA model is described in many textbooks and papers; accordingly, we give only few details about our numerical implementation. We start by solving the HF equations in coordinate space with a radial mesh extending up to 20 fm in a step of 0.1 fm. When the Skyrme HF potential is calculated, the single-particle energies and wave functions of the occupied and unoccupied levels can be solved by using an expansion over a harmonic oscillator basis. This basis is large enough to ensure that our results are stable and it extends up to a maximum value of the main quantum number $N_{\text{max}} = 10$ and 12, for $^{90}\text{Zr}$ and $^{208}\text{Pb}$, respectively.

Since the tensor force is spin-dependent and affects the spin-orbit splitting, the spin mode is very likely to receive strong influence. we study hereafter the GT excitation as the well-known spin mode. The operator for GT transitions is defined as

$$\hat{O}_{\text{GT}} = \sum_{im} t_{\pm}^i \sigma_m^i$$  \hspace{1cm} (7)

in terms of the standard isospin operators, $t_\pm = \frac{1}{2}(t_x \pm it_y)$. In the charge-exchange RPA, the $t_-$ and $t_+$ channels are coupled and the corresponding eigenstates emerge from a single diagonalization of the RPA matrix.

In self-consistent charge-exchange HF+RPA calculations, the NEWSRs $m_\pm(0)$ and the Energy-Weighted Sum Rules (EWSR) $m_\pm(1)$ (associated with the two different isospin chan-
nels) satisfy the following relations

\[ m_-(0) - m_+(0) = \sum_\nu (|\langle \nu | O_- | 0 \rangle|^2 - |\langle \nu | O_+ | 0 \rangle|^2) \]
\[ = \langle 0 | [O_-, O_+] | 0 \rangle, \]  
\[ m_-(1) + m_+(1) = \sum_\nu (|\langle \nu | O_- | 0 \rangle| + |\langle \nu | O_+ | 0 \rangle|) E_\nu \]
\[ = \langle 0 | [O_+, [H, O_-]] | 0 \rangle, \]  
where \( O_+ (O_-) \) is a generic charge-changing operator proportional to \( t_+ (t_-) \). In the GT case, the difference of NEWSRs (8) is model-independent and turns out to be

\[ m_-(0) - m_+(0) = 3(N - Z), \]  

The sum of the EWSRs (9) is model-dependent and it receives a contribution from the tensor interaction, which is obtained by replacing the total Hamiltonian \( H \) in the double commutator of (9) with \( V^T \). If there is enough neutron excess, and the contributions from the \( t_+ \) channel to the sum rules, \( m_+(0) \) and \( m_-(0) \), are small, then we can estimate the effect of the tensor interaction on the GT centroid in the \( t_- \) channel by writing

\[ \Delta E_{GT} = \frac{m_-(1)}{m_-(0)} \]
\[ \approx \frac{m_-(1) + m_+(1)}{m_-(0) - m_+(0)} \]
\[ = \frac{4\pi}{3(N - Z)^2} \int d^2 r \left[ -\left( \frac{2}{5} U + \frac{5}{6} T \right) J_n J_p - \frac{5}{3} U (J_n^2 + J_p^2) \right], \]  
where the last line comes from a lengthy but straightforward evaluation of the double commutator.

In the present work, we do not include the two-body spin-orbit residual interaction in RPA. Consequently our calculations are not, strictly speaking, fully self-consistent. However, this term of the residual interaction has been shown to be very small \[18\] in the case of the GT. Therefore, we can claim that self-consistency is not seriously broken. We do not make any further approximation, and, in particular, we include in HF the central exchange terms associated with \( \alpha_C \) and \( \beta_C \).

Only the values reported in Table \[I\] are, however, calculated by dropping completely the spin-orbit contribution, both at HF and RPA level. This calculation (with the Skyrme parameter \( W_0 \) set at 0) is not expected to be compared with the experimental findings but respects self-consistency in a strict sense. The shift in the GT centroid caused by the inclusion of tensor terms, \[\text{calculated by using either RPA or the analytical formula (11)},\]
and the EWSR \( m_-(1) + m_+(1) \) obtained from RPA, are listed in Table I for the two nuclei \(^{90}\text{Zr}\) and \(^{208}\text{Pb}\). One should notice the good agreement between the RPA results and the analytical results for the shift.

The GT- strength distributions in \(^{90}\text{Zr}\) and \(^{208}\text{Pb}\) are shown in Figs. 1 and 2, respectively. The calculated results are smoothed by averaging the sharp RPA peaks with Lorentzians weighting function having 1 MeV width. The tensor force affects these results in two ways. Firstly, it changes the single-particle energies (s.p.e.) in the HF calculation; secondly, it contributes to the RPA residual force. We do three different kind of calculations to analyze separately these effects. In the first one, the tensor terms are not included at all. In the second one, we include tensor terms in HF but drop them in RPA. This calculation is not self-consistent, but it displays the effects of changes in single-particle energies on the strength distribution. In the last one, the tensor terms are included both in HF and RPA calculations. For simplicity, results of the three categories of calculations are labeled by 00, 10 and 11, respectively.

We have evaluated the amounts of NEWSR \( m_-(0) \) and EWSR \( m_-(1) \) in different excitation energy regions, and listed them in Table II. When the tensor terms are not included in the residual interaction (i.e., the calculations labeled by 00 and 10), the values of NEWSR in the energy region between 30-60 MeV for both \(^{90}\text{Zr}\) and \(^{208}\text{Pb}\) are small, namely few percent of the total NEWSR (see the Figs. 1(a) and 2(a)). But in the case 11, about 10% of NEWSR is shifted from the lower energy region to the higher energy region. Moreover, we can see that essentially no unperturbed strength appears in this region (see the Figs. 1(b) and 2(b)). This means that including tensor terms in simple RPA calculation shifts about 10% of the GT strength to the energy region 30-60 MeV. While 2p-2h couplings will increase further these high energy strength, we would like to stress that the tensor correlations move substantial GT strength from the low energy region 0-30 MeV to the high energy region 30-60 MeV even within the 1p-1h model space.

The EWSR in the energy region below 30 MeV is of course also decreased after the inclusion of the tensor terms. From Table II, we see that an appreciable amount of EWSR (that is, 25% and 29% of EWSR in \(^{90}\text{Zr}\) and \(^{208}\text{Pb}\), respectively) is shifted from the lower energy region (0-30 MeV) to the higher energy region (30-60 MeV) by including tensor terms in HF plus RPA calculations.

In \(^{90}\text{Zr}\), from Fig 1(a) one can notice that the GT strength is concentrated in two peaks
in the region below 30 MeV. There are only two important configurations involved which are \((\pi 1g_{9/2} - \nu 1g_{9/2}^{-1})\) and \((\pi 1g_{7/2} - \nu 1g_{9/2}^{-1})\) (see Fig 1(b)). When the tensor terms are included only in HF and neglected in RPA, the centroid in the energy region of 0-30 MeV is moved upwards by about 1.5 MeV, and the high energy peak at \(E \sim 16\) MeV is moved upwards by only 0.5 MeV, as compared with the results without tensor terms. When the tensor terms are included in both HF and RPA, the centroid of the GT strength in the energy region 0-30 MeV is moved downwards by about 1 MeV, and the high energy peak is moved downwards about 2 MeV, as compared with the results obtained without tensor terms. Including tensor terms in RPA makes the two main separated peaks closer (this situation also happens for \(^{48}\text{Ca}\)). This result can be attributed from the HF and RPA correlations of the tensor terms. From the typical effect of the tensor correlations on HF field \(^{2,8}\), when the \(\nu 1g_{9/2}\) orbit is filled by neutrons, the tensor correlations give a quenching on the spin–orbit splitting between \(\pi 1g_{9/2}\) and \(\pi 1g_{7/2}\) orbits so that the unperturbed energies of the two main \(p - h\) configurations \((\pi 1g_{7/2} - \nu 1g_{9/2}^{-1})\) and \((\pi 1g_{9/2} - \nu 1g_{9/2}^{-1})\) are closer in energy as it is shown in Fig. 1(b). The RPA results in Fig. 1(a) labelled by (00) and (10) reflect these changes of HF single particle energies due to the tensor correlations and the energy difference between two peaks is narrower. Meanwhile, the RPA correlation associated with tensor terms move the higher energy peak downwards, and this effect can be seen in the results in Fig.1(a) labelled by (10) and (11). For GT transitions in the energy region of 30-60 MeV, several dominant configurations are expected and they receive some strength from the low excitation energy region due to tensor correlations.

In \(^{208}\text{Pb}\), from Fig 2(a) we see that the GT strength is concentrated in two peaks in the low energy region of 0-30 MeV. There are eleven important configurations which do contribute to these peaks. When the tensor terms are only included in HF and neglected in RPA, the centroid of these peaks is moved upwards about 0.5 MeV, and the higher energy peak at \(E \sim 18\) MeV is also raised by about 0.8 MeV. When the tensor terms are included in both HF and RPA calculation, the centroid of these peak moves downwards by about 1.5 MeV, and the higher energy peak moves also downwards by about 3.3 MeV, compared with the result obtained without tensor terms. By including tensor terms in the RPA calculation, the GT strengths in the energy region of 30-60 MeV are increased substantially by the shift of the strength in the energy region of 0-30 MeV through the tensor force.

We have calculated the GT strength in \(^{90}\text{Zr}\) by adding the presently used tensor terms
to SGII and obtained the same result that about 10% of the NEWSR appears in the high energy region of 30-60 MeV.

In conclusion, we have studied the effect of the tensor correlations on the GT excitations in $^{90}$Zr and $^{208}$Pb in the HF plus RPA framework with the Skyrme interaction SIII. If the tensor terms are included only in HF but neglected in RPA, the strength distribution is only slightly shifted to higher energy. But if the tensor terms are included in both HF and RPA, the centroid of GT strength in the energy region below 30 MeV is moved downwards by about 1 MeV for $^{90}$Zr and 3.3 MeV for $^{208}$Pb. At the same time, the dominant peak at $E \sim 16$ MeV (18 MeV) in $^{90}$Zr ($^{208}$Pb) is also moved downwards by about 2 MeV (3 MeV). It is pointed out for the first time that about 10% of NEWSR is moved to the high energy region of 30-60 MeV by the tensor correlations in RPA even within the 1p-1h model space. We give the analytical formula to estimate the effect of the tensor force on the mean GT energy. These formulas predict the upwards energy shift of the average excitation energy due to the tensor correlations. It agrees quite well with our numerical RPA results. It is interesting to point out that the main GT peak, contrarily, gets an energy shift downwards because of the peculiar features of the tensor correlations. In fact, the upwards shift of the average energy is the outcome of the GT strength appearing in the high energy region between 30-60 MeV, but does not correspond to the energy shift of main GT peak. Since the tensor interaction is spin-dependent, we expect that it can have important effects not only on the GT transitions, but on spin-dipole and other spin dependent excitation modes as well. These issues will be discussed in a forthcoming paper.

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TABLE I: Values of the EWSR \( m_-(1) + m_+(1) \) obtained from self-consistent HF plus RPA calculations with and without the tensor terms. \( \delta E_{RPA} \) and \( \delta E_{DC} \) are the contributions of the tensor terms to the GT centroid energy calculated, respectively, by using RPA and the analytical formula\(^{[11]}\). In the case of the numbers reported here (not for the other results in this paper), the spin-orbit term is dropped both at HF and RPA level. See also the main text.

|          | \( m_+ + m_+(1; \text{no tensor}) \) | \( m_+ + m_+(1; \text{with tensor}) \) | \( \delta E_{RPA} \) | \( \delta E_{DC} \) |
|----------|-------------------------------------|-------------------------------------|-------------------|-------------------|
| \(^{90}\text{Zr}\) | 271.45 | 338.68 | 2.241 | 2.276 |
| \(^{208}\text{Pb}\) | 1854.12 | 2000.76 | 1.111 | 1.118 |
TABLE II: Values of the NEWSR $m_{-}(0)$ and EWSRs $m_{-}(1)$ for $^{90}\text{Zr}$ and $^{208}\text{Pb}$ in different excitation energy regions. The two-body spin-orbit interaction is included in HF but neglected in RPA calculation. The results labeled by 00 correspond to neglecting the tensor terms both in HF and RPA; 10 corresponds to including the tensor terms in HF but neglecting them in RPA; 11 corresponds to including the tensor terms both in HF and RPA. See the text for a discussion of the effects of the tensor terms.

| Type of calculation | $m_{-}(0)$ 0-30MeV | $m_{-}(0)$ 30-60MeV | $m_{-}(1)$ 0-30 MeV | $m_{-}(1)$ 30-60 MeV | Total total |
|---------------------|---------------------|---------------------|---------------------|---------------------|-----------|
| **$^{90}\text{Zr}$** |
| 00                  | 29.16               | 0.71                | 395                 | 26.2                | 421.8     | 10.1     |
| 10                  | 29.16               | 0.79                | 444                 | 22                  | 466       | 11.1     |
| 11                  | 27.00               | 2.89                | 366.9               | 122                 | 493.2     | 10.3     |
| **$^{208}\text{Pb}$** |
| 00                  | 127.54              | 3.43                | 2080                | 124.5               | 2212.8    | 18.8     |
| 10                  | 127.38              | 3.68                | 2176                | 93                  | 2269      | 21       |
| 11                  | 114.10              | 16.58               | 1658                | 694                 | 2370      | 19.3     |
FIG. 1: The GT strength in $^{90}\text{Zr}$. In panel (a), the RPA results are displayed, by smoothing them with Lorentzians having 1 MeV width. As explained in the text, result labelled by 00 corresponds to neglecting the tensor terms in both HF and RPA; 10 corresponds to including the tensor terms in HF but neglecting them in RPA; finally, 11 corresponds to including the tensor terms in both HF and RPA. The arrow denotes the experimental energy. In panel (b), the unperturbed strength is shown.
FIG. 2: The same as Fig. 1 in the case of $^{208}$Pb.