The optimal government size in the kingdom of Saudi Arabia: an ARDL bounds testing approach to cointegration

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Abstract: This study attempts to estimate the optimum government size in the kingdom of Saudi Arabia (KSA) using annual data covering the 1971–2019 period by applying the linear and nonlinear Autoregressive Distributed Lag (ARDL) Model. The main focus is whether the Armey curve is valid for KSA. The statistical diagnostic tests provide an evidence for the model adequacy and that the estimation results are reliable. Moreover, the ARDL short-run estimation results revealed that the speed of adjustment is (−0.82) indicating that it takes about 14 months to correct toward the long-run equilibrium due to a short-run shock. The NARDL estimation results revealed asymmetric relationship between government expenditures and economic growth. Further, a positive shock has a positive impact while a negative shock reduces economic growth. Based on the long-run estimation results, the optimum government size is 26.9 as a share of GDP, which is greater than the average share (24.2) during the study period. Based on such result, it is obvious that Saudi Arabia has a room to increase the expenditures share up to the optimal size estimated in the study.

Subjects: Social Sciences; Economics, Finance, Business & Industry; Economic; Macroeconomics; Finance; Public Finance

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PUBLIC INTEREST STATEMENT

Nowadays, countries are very concerned about achieving an acceptable economic growth that would maintain a high living standard. Following the economic theory, the involvement of government though increasing its expenditures is believed to achieve this goal. Hence, there is a direct link between economic growth and government expenditures; however, this link is nonlinear one. This means that up to a certain level of government share of GDP, the relation inversed from positive to negative. Therefore, understanding the true nature of such relation and the maximum government expenditure helps government in dealing with budget deficit. This article studies the validity of Armey curve in the Saudi Arabia and estimates the level of government share of GDP that achieve the maximum-level economic growth. The empirical findings show that for the economic growth to maximum, the government share of GDP should be 26.4.
1. Introduction

No doubt that the government size and economic growth relationship have been crucial and important matters in economics for policy makers. However, this relation is still a controversial issue among economic schools regarding the role of government size in achieving and stimulating economic growth. According to Keynesian school, government expenditure is considered as one component of GDP, and hence, increase in its government expenditures increases GDP level, which is in turn translated into economic growth. However, the classical school, increasing government expenditures reduce economic growth particularly in the long run. Since the end of the 20th century, the debate among policy makers and economists centers on the optimal and the negative impact of government size on economic growth (Richard Armey, 1995; Scully, 1994). The increasing government expenditures share, the relatively high budget deficit ratio, and public debt have motivated this debate.

It is well documented in the applied research that the relation between government size and maximum economic growth is uncertain, and there must be a certain size that can be seen in the mix applied results (Altunc & Celil, 2013; Shaboot, 2018). This means that the effect will not be always positive infinitely. The applied economic research provided that the effect will be positive for developing countries since they are still in the early stages of economic process, whereas it is negative for developed countries since they are in very advanced stages. At the end of the 20th century, a new strand of economic research led by (Armey, 1995; Barro, 1990; Scully, 1994) arose to determine the optimal government size that would achieve the maximum economic growth. Initially, effect was positive on economic growth up to a certain level, and then, any size beyond this caused economic growth to decline. This new strand of research enables policy makers to stimulate economic growth through controlling this ratio (size).

A huge body of the previous applied research on this matter confirms the existence of Armey curve, where the relationship between the two variable is a nonlinear one, and it takes the inverted U-shaped. This conclusion implies that at lower government expenditures share, the relation is positive but at a diminishing pattern until it reaches a maximum share at which economic growth is optimal. Exceeding the optimal level, then the relation becomes negative. This kind of research uses different estimation approaches, different data set, different time horizon. Nevertheless, they reach the same conclusion that is the inverted U-shaped. Moreover, the optimal size in not unique for the countries (Turan, 2014).

As for KSA case, the government is putting a maximum effort to achieve a high sustainable economic growth rate that exceeds population growth rate. However, this effort is faced with few problems, such as the ratio of budget deficit to GDP, and the volatile oil prices, which affects government revenues. Nevertheless, the Saudi government tries to cut the budget deficit ratio through implementing government expenditures rationale policy and restructuring the tax system laws.

The importance of the study arises from the fact that determining the optimal government size is a crucial issue in achieving the pursued maximum economic growth. In addition, it differs from previous studies in KSA in few aspects, they used different estimation and approaches, for example, Barri (2001) used the Standard OLS estimation and Johansen cointegration, Aly and Strazicich (2000) used Barro (1990) and Karras (1997) approaches, and Ahmad (2020) used Khan and Senhadji (2001). Moreover, they utilized data length ranged from 1970 to 2016. Hence, the present study contributes to the present literature regarding Saudi Arabia through investigating the validity of The Army Curve (1995) by applying a modern estimation technique (ARDL) model.
(Pesaran et al., 2001), using longer time horizon from 1971 to 2019, and adds variables that are believed to affect economic growth. Additionally, the study will suggest some policy implications.

The study is constructed of five sections. In addition to the introduction, Section 2 presents the literature review including theoretical background and a survey of previous applied work. Section 3 presents a brief statistical descriptive analysis on the historical development of government expenditures in KSA over the study period. Section 4 contains the study’s methodology which describes the econometric model and data. Section 5 presents and discusses the estimation results. Finally, Section 6 contains a conclusion and policy implementation.

2. Review of literature
The relation between government expenditures and economic growth has been investigated theoretically and empirically since the end of the 20th century. Furthermore, the debate is still going on among different economic doctrines. This section consists of twofolds. The first fold presents the theoretical framework of the economic growth—government expenditures nexus as depicted in the existing economic theories. The second fold surveys the empirical research work carried out on this issue.

2.1. Theoretical framework
This section reviews the debate and proposed models to measure the optimal government size that maximizes economic growth.

2.1.1. Wagner's law
The core theme of Wagner’s law introduced by Adolph Wagner (1886) is that government sector grows faster than the economy (Al-Abdulrazag & Azoubi, 2005). Additionally, the law proposed that causation runs from economic growth to government expenditures through the increase in demand for public goods and service.

2.1.2. The Keynesian theory
The Keynesian theory relied on the concept of aggregate demand that would positively affect the level economic growth. This idea goes back to great depression period where Keynes argued that increasing government expenditures coupled with interest rate reduction would stimulate and encourage private sector to increase investment, and hence increase economic growth through the increase in aggregate demand. According to multiplier effect, increases in government spending encourage producers to produce more.

2.1.3. The army curve concept
The assumed nonlinearity feature between government expenditures GE and economic growth EG was initially investigated by Richard Armey (1995). The Armey Curve is based on the theories of market failure, which support government intervention to provide public goods and correct for negative externalities (Magana, 2015). Moreover, the theory focuses on the possible adverse effects of government size. Armey (1995) introduces the optimal government size concept. The core of his work points out that the relation is an inverted U-shapes, and since then, it is referred to as Armey Curve. Moreover, at low levels of government size increases economic growth till it reaches an optimal size that maximizes economic growth, after which economic growth declines. Armey translates this relation into graphical representation as shown in the Figure 1.

At point (A), low government expenditures increase economic growth up to point (B), which represents the optimum size GE’ maximizes economic growth. Further increase in government expenditure beyond the optimum level economic growth declines.
The justification for the relation's behavior stems from the fact that as the share of GE increases the EG in response also increases. Unfortunately, this behavior pattern of GE does not sustain infinitely. The positive correlation prevails up to GE* where the marginal productivity of public expenditures equals to private sector's where this maximum EG is associated with GE*. Any level of GE beyond the, the marginal effect of GE becomes negative, and hence, exerts negative impact on maximum EG.

The initial positive relationship between GE and EG stems from the fact at early stages of development process, and increase in GE acts as a stimulus to private investors to engage in industrialization activities. At this stage, both GE and private sector jointly exert a positive effect on EG as expected. However, this positive relation is reversed after GE* level of GE where the EG is maximum. Any increase in GE beyond GE* triggers the crowding-out effect on private investment, and hence, GE becomes inefficient because of the diminishing returns. This discussion concludes that an increase in GE beyond the threshold level GE* declines growth rate (Abounori & Nademi, 2010).

The Armey curve model is as follows:

\[ y_t = \beta_0 + \beta_1 GE_t + \beta_2 GE_t^2 + \beta_3 X_t + \epsilon_t \]

Where \( GE_t^2 \) is the quadratic form, \( y_t \) represents economic growth, and \( X_t \) is a vector of some economic control variables, and \( \epsilon_t \) is the error term assumed to be normal distributed with zero mean and constant variance. The estimated results are used later to calculate the optimum government size by differentiating equation (1) with respect to GE as follows:

\[ GE^* = -\frac{\beta_1}{2\beta_2} \]

2.1.4. The Scully model
Scully (1994) proposed estimated the optimal government size. His model takes the Cobb-Douglas production function as follows:
\[ Y = a(G_{t-1})^b[(1 - \tau)Y_{t-1}]^c \]

Where \( \tau \) is the tax rate measured as the share of government expenditures (\( \tau = GE/GDP \)). Furthermore, the model assumes the budget-balanced case, and then, the government expenditures equal to the value of tax returns (\( GE = \tau Y \)). After modifying the model to include the balanced-budget assumption, the model becomes as follows:

\[ Y = a(\tau_{t-1}Y_{t-1})^b[(1 - \tau)Y_{t-1}]^c \]

The optimal size of government is determined by maximum \( \tau^* \) by differentiating the equation with respect to \( \tau \). The maximum size of government is calculated by using the following formula:

\[ \tau^* = b/b + c \]

The Scully model suffers from a weakness in that it relationship produces spurious estimate of an optimal tax (Magana, 2015).

### 2.2. Empirical literature

The issue of the relationship between the optimal government expenditures and economic growth is still controversial. Some economists argued that this relation is positive where economic growth causes government expenditure, whereas, others believe it is negative. Nevertheless, the solution to this problem is an applied application matter. Another issue concerning this relation is the optimum size of government sector. A huge body of literature relied on Armey Curve that advocates the existence of nonlinear relation using various estimation techniques such as OLS, FMOLS, and DOLS, and indicate that the optimal government size varies among countries. Another research predicted the optimal share of government that maximizes economic growth avenue followed the Barro model (1990). Accordingly, economic growth reaches its maximum when the government expenditure level is at its optimal provided that the value of marginal productivity equals 1.

Ahmad (2020) estimated the threshold of optimal government size (share of GDP) to be 27.2% in Saudi Arabia over the period 1970–2016 using the ARDL estimation technique. Garcia (2019) confirm the Armey curve’s validity for Spain where the optimal size is 40.07% over the period 1980–2016 using the OLS estimation method. For the Algerian case, Rennane (2019) applying Sully model for 1973–2018 period using the (DOLS) and the (FMOLS) estimation methods, he showed that the estimated optimal government size is 29%. Moreover, applying Barro model using FMOLS and DOS methods, Nuredin (2019) found the optimum government spending between 23.6% and 34.9% in Algeria over the 1970–2017. Duasa (2018) could not confirm the optimum size of government for 49 Muslim countries over the period 2009–2013 using pool OLS and GMM estimations techniques. Shaboot (2018) estimated the optimal government size to be 37% in Algeria by applying the ARDL approach to Armani curve and Barro model over the 1980–2016 period. Murshed et al. (2017) investigated the validity of Armey Curve in south and Southeast Asian countries for 1980–2016 period using panel data estimation (FE). They estimated that the optimal government size was 148,627.5 and 57,765.7 million US dollars. Tabaghua (2017) estimated government optimal size to be 21% in Georgia by applying the correlation method for the period 2002–2014. Magana (2015) examine Army curve’s validity in Kenya over the period 1963–2012 using the OLS estimation method, estimated optimal government size was 23% of GDP. Turan (2014) examined the application of Armey curve for Turkey over the periods 1950–2012 and 1970–2012. The results for the two periods were different; 8.8–9.1% and 15.4–17% of GDP for the periods 1950–2012 and 1970–
2012, respectively, indicating that Armey curve is sensitive to the sample size. Altunc and Celil (2013) examined the validity of Armey curve over the period 1995–2011 using the OLS estimation method. They found the optimum size was 25.1%, 20.44%, and 22.45% for Turkey, Bulgaria, and Romania, respectively. Applying the standard OLS estimation method and Johansen cointegration to Saudi Arabia data over the period 1970–1998, Barri (2001) showed that the optimal government size is 29%. Aly and Strazicich (2000) findings revealed that the optimal government size is 2% in KSA.

3. Statistical descriptive analysis

Table 1 reports the descriptive data analysis. Looking at the share of general government expenditures (LGE), one can observe that it averaged about 24.1% annually, reached its maximum at 35.23% in year 1987, and a minimum share at 8.432% in year 1974. Concerning the economic growth (EG), it averaged annually at 2.57%, reached a maximum value of 24.17% in year 1973, and a minimum equal to (~20.73) in year 1982.

(Figure 2) shows the historical development of both economic growth and government size over the study period. It seems that government expenditures exhibit less fluctuation over time than economic growth. This could be attributed to the oil price fluctuations. Since GDP is the total oil and nonoil GDP; hence, any fluctuation in oil prices will affect GDP and then economic growth. For the government status, this can be explained by the effort of government to keep government expenditures steady and stable as possible. Another observation is that there is a kind of inverse relation between the two variables over the study period, where there are years where an increase in government, especially high share, is associated with a decrease in economic growth.

| Table 1. The statistical descriptive of the model data |
|------------------------------------------------------|
| **LEG** | **LGE** | **GFCF** | **CPI** | **OPEN** | **FDISH** |
| Mean | 3.575625 | 24.10201 | 6.23E+10 | 79.20242 | 4.329146 | 1.150417 |
| Maximum | 24.170000 | 35.22240 | 1.95E+11 | 120.9313 | 4.792648 | 8.500000 |
| Minimum | ~20.73000 | 8.431734 | 1.18E+09 | 24.49945 | 4.026929 | ~8.220000 |
| Std. Dev. | 8.926783 | 5.874672 | 1.04E+10 | 22.11164 | 0.164521 | 2.962749 |
| Jarque-Bera | 0.768473 | 0.664211 | 9.799309 | 0.335970 | 1.023006 | 5.002392 |
| Probability | 0.680970 | 0.717411 | 0.007449 | 0.845367 | 0.599594 | 0.081987 |

Figure 2. Economic growth-Gov. size (1971–2019).
(Figure 3) provides a better picture for the comovement of the two variables, government size on the horizontal Axis, while the economic growth on the vertical axis.

It can be seen from the graph the association pattern between the variables. Generally speaking, high levels of government expenditures is associated with low (even negative) economic growth. It is worth noting that this association cannot be attributed to the government expenditures only, but there are other factors that could have attributed to this association, in particular oil prices fluctuation among other factors.

4. Econometric methodology and data source
The article utilizes the ARDL model (Pesaran et al., 2001) to investigate the long-run pass-through of government expenditures into economic growth in KSA. Moreover, the NARDL is employed to test the asymmetric relationship between government expenditure and economic growth. In economic literature, the economic growth-government expenditures nexus is usually investigated by employing the widely used estimation techniques: cointegration, error-correction model ECM, VCEM, Granger-Causality, FMOLS, and DOLS.

4.1. The econometric model
Following the discussion of the previous applied research, the relationship is:

\[ \text{LEG}_t = \alpha_0 + \beta_1 \text{LGE}_t + \beta_2 \text{LZ}_t + \epsilon_t \]  

(1)

Where LGE is the economic growth proxy by real GDP, LGE is the government size (share of GDP), LZ is a set of control variables thought to affect LGE, they are trade openness (OPEN) is the trade share, capital (LK) measured by gross fixed capital formation, the foreign direct investment share (FDISH), and the Consumer Price Index (LCPI, 2010 = 100), \( \epsilon_t \) is the error term, and L refers to the natural logarithm. Expressing the estimation model in the logarithm, transforms the variables' parameters into elasticities. The expected positive sign of LGE parameter \( \beta_1 \) means that there is a direct relation with LGE. To estimate the Armey curve, a quadratic form of government expenditures LGESQ has been added to account for the nonlinearity of LGE, and it becomes as follows:

\[ \text{LEG}_t = \alpha_0 + \lambda_1 \text{LGE}_t + \lambda_2 \text{LGESQ}_t + \lambda_3 \text{LZ}_t + \epsilon_t \]  

(2)

The sign of \( \lambda_1 \) and \( \lambda_2 \) are expected to be positive and negative impacts on LGE, meaning that government expenditures effect economic growth positively but at decreasing rate, that is the
effect diminishes with government expenditures increase to reach its maximum level. The raw data needed to carry out the estimation are collected from World Development Indicator (WDI) over the period 1971–2019.

4.1.1. The estimation approach
To achieve the study’s goal in KSA, this section presents the estimation model utilized in the study such as the linear and nonlinear ARDL model and bounds test approach to cointegration.

4.1.2. ARDL bounds test
Recently, the ARDL model has been widely used for its advantages compared to Johansen (Johansen & Juselius, 1990) and (Engle & Granger, 1987) cointegration approaches. First, it does not require variables to be of the same order of integration, but not I(2). Second, it has been widely used in small sample studies. (for more on ARDL advantages, see Davoud et al., 2013; Sami, 2013). The unrestricted ARDL model specification for the long run between \( EG_t \) and government expenditures in KSA is expressed as the following:

\[
\Delta \text{LEG}_t = \alpha_0 + \lambda_1 \text{LGE}_{t-1} + \lambda_2 \text{LGSQ}_{t-1} + \lambda_3 \text{LEG}_{t-1} + \lambda_4 \text{LZ}_{t-1} + \sum_{i=1}^{q_1} \beta_1 \Delta \text{LEG}_{t-i} + \sum_{i=0}^{q_1} \beta_2 \Delta \text{LGE}_{t-i} \\
+ \sum_{i=0}^{q_2} \beta_3 \Delta \text{LEG}_{t-i} + \sum_{i=0}^{q_2} \beta_4 \Delta \text{LZ}_{t-i} + \epsilon_t
\]  

(3)

Where \( \Delta \) indicates the first difference of the variable, \( \alpha_0 \) is the constant, \( \lambda_1, \lambda_2, \lambda_3, \text{and} \lambda_4 \) refer to the long-run elasticities, while \( \beta_1, \beta_2, \beta_3, \text{and} \beta_4 \) represent the short-run elasticities. Equations (3) is a linear ARDL of order (p, q) for economic growth. The appropriate lags length structure is established by the minimum (AIC).

Using the ARDL requires that the variables are cointegrated using the F-bounds test, which is sensitive to the model lag length selection (Shahbaz & Lean, 2012). The null hypothesis of no long-run relationships is \( H_0 : \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0 \) versus the alternative hypothesis, \( H_1 : \lambda_1 \neq \lambda_2 \neq \lambda_3 \neq \lambda_4 \neq 0 \). However, Accepting or rejecting \( H_0 \) depends on calculated F-statistic compared with the tabulated values. Accordingly, \( H_0 \) cannot be rejected if the calculated F-statistic is less than the lower bound critical value or otherwise it can be rejected (Davoud et al, 2013).

5. ARDL estimation
This section presents the ARDL validity of Armey Curve model and estimates the optimal government size in KSA.

5.1. Unit root test results
Table 2 reports the breakpoint unit root test results. As Table 2 shows, all variables are either I(0) or I(1) but none is I(2), and hence, the ARDL model is suitable for estimating purposes.

5.2. ARDL estimation results
Table 3 reports the estimation results of the linear ARDL (2, 3, 2, 3, 3). Referring to the breakpoint test results, they show that there is a structural break point in (LEG) in 1983; therefore, the estimated equation included a dummy variable (DUM1), where it takes zero value for the period 1971–1979 and one otherwise. The control variables include trade openness (LOPEN) is the trade share, capital (LK) measured by gross fixed capital formation, the foreign direct investment share (FDISH), and the Consumer Price Index (LCP, 2010 = 100).
| Breakpoint | Variable      | ADF | C.V | L. | ADF  | C.V | L  |
|------------|---------------|-----|-----|----|------|-----|----|
| 1983       | LEG           | −3.65 | −4.27 | 0  | −6.34* | −4.85 | 0  |
| 1979       | LCPI          | −2.26 | −4.27 | 0  | −5.81* | −5.07 | 0  |
| 1994       | LGE           | −2.26 | −4.27 | 0  | −8.34* | −4.87 | 0  |
| 1994       | LGESQQQQ Filtered | -2.26 | -4.27 | 0  | -8.34* | -4.85 | 0  |
| 1979       | Lk            | −0.80 | −4.27 | 0  | −4.40  | −4.27 | 0  |
| 1979       | LOPEN         | −2.89 | −4.27 | 1  | −9.59* | −4.85 | 0  |
| 2001       | FDISH         | −5.2* | −4.85 | 9  | -     | -    | -  |

*Significant at 1% level. Otherwise the significance level is 5%, C.V is the Critical Value; L: Number of lags.
Table 3. The results of estimated selected model: ARDL(2, 3, 2, 3, 3)

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| C        | 11.38575    | 3.271339   | 3.480457    | 0.0029|
| LEG(−1)* | −0.823045   | 0.152105   | −5.411031   | 0.0000|
| LGEX(−1) | 2.910130    | 1.428312   | 2.037461    | 0.0575|
| LGEXSQ(−1) | −0.441963  | 0.225009   | −1.964206   | 0.0661|
| LOPEN(−1) | −0.812071   | 0.101723   | −7.983151   | 0.0000|
| LK**     | 0.457200    | 0.057948   | 7.889861    | 0.0000|
| FDISH(−1) | −0.012911   | 0.004346   | −2.970710   | 0.0086|
| LCPI(−1) | −0.447623   | 0.140487   | −3.186214   | 0.0054|
| D(EG(−1)) | 0.077840    | 0.122282   | 0.636557    | 0.5329|
| D(EG(−2)) | 0.278869    | 0.104197   | 2.676376    | 0.0159|
| D(EG(−3)) | 0.319301    | 0.103039   | 3.098821    | 0.0065|
| D(LGEX)  | −4.550557   | 1.362741   | −3.339268   | 0.0039|
| D(LGEX(−1)) | −5.748764  | 0.967200   | −5.943719   | 0.0000|
| D(LGEX(−2)) | −2.987535  | 0.568488   | −5.255227   | 0.0001|
| D(LGEXSQ) | 0.704959    | 0.212598   | 3.315932    | 0.0041|
| D(LGEXSQ(−1)) | 0.867478  | 0.149845   | 5.789184    | 0.0000|
| D(LGEXSQ(−2)) | 0.456049   | 0.089613   | 5.089083    | 0.0001|
| D(LGEXSQ(−3)) | 0.017087   | 0.006696   | 2.551963    | 0.0206|
| D(OPEN)  | −0.478416   | 0.108813   | −4.396695   | 0.0004|
| D(OPEN(−1)) | 0.380876   | 0.080809   | 4.713315    | 0.0002|
| D(FDISH) | −0.016130   | 0.003834   | −4.207426   | 0.0006|
| D(FDISH(−1)) | 0.009726    | 0.003957   | 2.457721    | 0.0250|
| D(FDISH(−2)) | −0.012537   | 0.003700   | −3.388646   | 0.0035|
| D(LCPI)  | 0.631098    | 0.300453   | 2.100486    | 0.0509|
| D(LCPI(−1)) | 1.354038   | 0.315901   | 4.286273    | 0.0005|
| D(LCPI(−2)) | 1.159827   | 0.329127   | 3.523949    | 0.0026|
| D(LCPI(−3)) | 0.430813   | 0.162359   | 2.653452    | 0.0167|
| DUM1     | −0.059631   | 0.031673   | −1.882741   | 0.0770|

The estimated results show that the error correction term (ECM) is less than one (−0.823045) and significant at 1% level of significance. This results provide that the model corrects for the short-run shock toward long-run equilibrium with about 15 months.

Table 4 shows that the F-Bounds Test equals 19.44 and it is greater than the upper bound at 1% significance; hence, the variables are cointegrated.

Having established the long-run relation, the next step is to perform the estimation of short-run and long-run relationships.

Table 5 presents the statistical diagnostic tests results required to ensure the validity and the reliability of the estimated results. Jarque-Bera test indicates that the residuals are normally distributed, LM test indicates that the model is free of the autocorrelation problem, and the variance of the error term is homoscedastic indicated by BPG test. Therefore, estimation results
are valid and reliable. Moreover, as Figures 1 and 2, the CUMS and CUSMUSQ indicate that there are no structural breaks. Generally speaking, the model is free of any statistical problem.

5.3. The long-run estimation results
The long-run estimation results are presented in Table 6. It shows that LGE variables have the correct signs and significant, the linear form is positive while the quadratic form is negative. The negative quadratic form provides that the effect diminishes over time and reaches its maximum, then starts to decline beyond this level. This behavior means that the effect of government size is positive but at diminishing rate, then after reaching its maximum, it becomes negative. Generally speaking, the relation is nonlinear.

Moreover, the long-run results in Table 6 show that the control variables are significant at less than 5% level of significance.

To calculate the optimal government size, the long-run results are differentiated with respect to LGE and equate it to zero as follows:

\[
EC = EG - (3.5358*LGE - 0.5370*LGESQ - 0.9867*OPEN + 0.5555*K - 0.0157*FDISH - 0.5439*LCPI + 13.8337).
\]

Table 4. Null hypothesis: no levels relationship

| Test statistic | Value | Significance | I(0) | I(1) |
|----------------|-------|--------------|------|------|
| F-statistic    | 19.44 | 10%          | 1.99 | 2.94 |
| k              | 6     | 5%           | 2.27 | 3.28 |
|                |       | 2.5%         | 2.55 | 3.61 |
|                |       | 1%           | 2.88 | 3.99 |

Table 5. Statistical test

| Statistical test                                | Breusch-Godfrey serial correlation LM test | Heteroskedasticity test: Breusch-Pagan-Godfrey | Jarque-Bera |
|-------------------------------------------------|-------------------------------------------|-------------------------------------------------|-------------|
| Value                                           | 2.9202                                    | 0.74249                                         | 1.4826      |
| F-statistic                                     | 0.0849                                    | 0.7618                                          | 0.4765      |

Table 6. Long-run estimation results of linear ARDL

| Variable | Coefficient | Std. error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| LGE      | 3.535808    | 1.642678   | 2.152465    | 0.0460|
| LGESQ    | -0.536985   | 0.253694   | -2.116667   | 0.0493|
| LOPEN    | -0.986666   | 0.120425   | -8.193226   | 0.0000|
| FK       | 0.555498    | 0.061761   | 8.994375    | 0.0000|
| FDISH    | -0.015687   | 0.004150   | -3.780143   | 0.0015|
| LCPI     | -0.543861   | 0.216644   | -2.510386   | 0.0225|
| C        | 13.83369    | 2.645235   | 5.229662    | 0.0000|
\[
\frac{\partial \text{LEG}}{\partial \text{LGE}} = 3.535808 - 2(0.536985)\text{LGE} = 0
\]

Hence, optimal government Expenditure size is:

\[
\text{LGE}^* = \frac{3.535808}{2(0.536985)} = 3.29228
\]

Taking the unit log of 3.29228, the optimal ratio of general government expenditure is 26.9%. Therefore, applying the Armey curve procedure, the optimal government size is estimated to be 26.9% that maximizes the economic growth in KSA. Any size beyond the optimum size causes economic growth to decline, and the relation becomes negative. The result is very close to those for KSA, for example, (27.2%) by Ahmad (2020) using threshold approach, (29%) found by Barri (2001), and 2% estimated by Aly and Strazicich (2000). Moreover, the estimated government size for countries other than Saudi Arabia is with close range. For example, it was 29% for Algeria (Rennane, 2019), 21% for Georgia (Tabaghua, 2017), and 25% for Turkey (Altunc, 2013).

### 5.4. NARDL model

One feature of the linear ARDL approach is that it does not take into consideration the asymmetric relationship among model variables (that is the possibility that negative and positive variations of the explanatory variables have different effect on the dependent variable). On the other hand, The NARDL approach is an asymmetric extension of the linear autoregressive distributed lag (ARDL) cointegration model proposed by Pesaran et al. (2001), which can be applied to test the presence of asymmetric long-run relationships among a set of variables. The NARDL model not only allows us to detect the existence of asymmetric effects that independent variables may have on the dependent variable, but also permits testing for cointegration in a single equation framework. Moreover, this model poses some advantages, which have been previously explained, over other cointegration techniques used frequently, such as its flexibility regarding the order of integration of the variables involved, the possibility of testing for hidden cointegration, avoiding to omit any relationship which is not visible in a conventional linear setting and a better performance in small samples (Rocher, 2017).

Following Shin et al. (2014), a specification of asymmetric long-run model describing the relation economic growth and the general government expenditures is constructed in the following form:

\[
\text{GE}_t = \theta_0 + \theta_1x_t + \theta_2\text{LGE}_t^+ + \theta_3\text{LGE}_t^- + \mu_t
\]  

(4)

Where \( \text{LEG}_t \) is the economic growth, \( \text{LGE}_t \) is the general government expenditures, \( x_t \) is a \((k \times 1)\) vector of control variables, and \( \theta = (\theta_0, \theta_1, \theta_2, \theta_3) \) is a cointegrating a vector of long run parameters to be estimated. In addition, \( \text{LGE}_t^+ \) the positive effects and \( \text{LGE}_t^- \) the negative effects on the dependent variable are partial sums process, which accumulates positive and negative changes.

According to Shin et al. (2014), the NARDL model is built around the following asymmetric long-run equilibrium relationship The effect of \( \text{LGE}_t \) is decomposed into these effects as:

\[
\text{LGE}_t = x_0 + \text{LGE}_t^+ + \text{LGE}_t^-
\]  

(5)

\( x_0 \) is random initial variable,
\[ LGE_t^+ = \sum_{i=1}^t \Delta LGE_{i-1}^+ = \sum_{i=1}^t \max(\Delta LGE_i, 0) \] 

(6)

\[ LGE_t^- = \sum_{i=1}^t \Delta LGE_{i-1}^- = \sum_{i=1}^t \min(\Delta LGE_i, 0) \] 

(7)

The description of the NARDL allows for joint investigation of the issues of nonstationarity and nonlinearity in the setting of an unrestricted error correction model. From Equation (4), the long-run relation between economic growth and government expenditures is captured by \( \theta_2 \), which is expected to be positive. Meanwhile, \( \theta_3 \) which captures the long-run relation between economic growth and government expenditures decreases is expected to be positive. It is posited that the government expenditures increases will result in higher long-run changes in economic growth as compared to the impact of government expenditures decreases of the same magnitude, i.e. \( \theta_2 > \theta_3 \). Thus, the long-run association presented by Equation (4) indicates asymmetric long-run government expenditures pass through to economic growth.

Following the general form of NARDL model introduced by Shin et al. (2014), equation (1) can be written in the context of ARDL model proposed by Pesaran et al. (2001) as:

\[ \Delta LEG_t = \beta_0 + \beta_1 LGE_{t-1} + \beta_2 X_{t-1} + \beta_3 LGE_{t-2}^+ + \beta_4 LGE_{t-2}^- + \sum_{j=1}^q \gamma_j \Delta LEG_{t-j} + \sum_{i=1}^K \delta_i \Delta X_{t-i} \]

\[ + \sum_{i=0}^q (\delta_i^+ \Delta LGE_{t-i}^+ + \delta_i^- \Delta LGE_{t-i}^-) + \sum_{j=1}^k ECT_{t-j} + \epsilon_t \] 

(8)

Where all variables described in equation (1), \( K, p \) and \( q \) are lags order. The long-run coefficients \( (\beta_3 = -\delta_i^+ / \beta_4, \beta_4 = -\delta_i^- / \beta_1) \) are the long-run effects of LGE increases and decreases on EG, respectively. \( \sum_{i=0}^q \delta_i^+ \) captures the short-run effect of increases in LGE on LEG, whereas \( \sum_{i=0}^q \delta_i^- \) captures the short-run effect of decreases in LGE on LEG. The model specification of Equation (8) indicates the asymmetric short-run impact as well as the asymmetric long-run effects of government expenditures on economic growth.

After finding none of the variables is I(2), the second step involves estimating equation (8) by applying the NARDL approach based on the linear ARDL approach. In the third step, we test for the presence of long-run equilibrium relationships among variables using a bounds testing approach of Pesaran et al. (2001). This can be done by using the Wald F-statistic test of the null hypothesis of no cointegration \( H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0 \), opposed to the alternative \( H_0 : \beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_4 \neq 0 \).

The long-run asymmetry can be tested by applying the Wald-F statistic to the null hypothesis \( H_0 : \beta_3 = \beta_4 \), while for the short-run adjustment to a positive and negative shocks of inflation affecting economic growth, the Wald F-statistic is applied to the null hypothesis of no asymmetry \( H_0 : \delta_i^+ = \delta_i^- \) against the alternative hypothesis of the existence of asymmetry \( H_1 : \delta_i^+ \neq \delta_i^- \). Moreover, we can also develop the asymmetric cumulative dynamic multiplier effects of a one percent change in \( LGE_{t-1}^+ \) and \( LGE_{t-1}^- \) respectively as:

\[ m_k^+ = \sum_{j=0}^k \frac{\delta_{t-j}^+}{\sum_{j=0}^k \Delta LGE_{t-j}} \]

\[ m_k^- = \sum_{j=0}^k \frac{\delta_{t-j}^-}{\sum_{j=0}^k \Delta LGE_{t-j}} \]

(9)
Note that as $k \to \infty, m_k^+ \to \beta^+$ and $m_k^- \to \beta^-$.

### 5.4.1. Analyzing and discussing NARDL results

The purpose of applying the NARDL methodology is to examine the response of economic growth to positive and negative shocks in general government expenditures, in other words, is there a long-run asymmetric relationship between economic growth and government expenditures in KSA over the study period. If the relation is asymmetric, then there is a nonlinear relation between the two variables. On the other hand, if relation is symmetric, then there is a linear association between the two variables.

Table 7 presents the results of the bound test of cointegration. The results provide evidence on the rejection of the null hypothesis which states that there is no cointegration and the acceptance of the alternative hypothesis of the existence of a long-run relationship between model variables as shown by the significant F-bound test at 1% level of significance which is equal to 13.36.

Having found that the model is valid and stable, the next step is to analyze the NARDL estimation results to examine the nature of the relationship between government expenditures and economic growth, that is whether it is asymmetric or symmetric. Table 8 presents the NARDL the positive and negative short-run government shocks estimation results.

The results of statistical diagnostic tests required to ensure that the estimated model is free of such problem, and hence, the reliability of the estimated results are presented in Table 9.

The statistical test results provide evidence that the model is free of statistical problems, i.e. the residuals are normally distributed show by the Jarque-Bera test, the model is free of the autocorrelation problem indicated by the LM test, and the variance of the error term is homoscedasticity and indicated by BPG test. Therefore, estimation results are valid and reliable. Moreover, the CUMUS and CUSMUSQ stability tests indicate the parameters are free of any structural break.

Further, Table 10 presents the Wald-test results of the long-run and short-run asymmetries between the LGEX_POS and LGEX_NEG variables. Accordingly, the Wald-test statistics indicates that there is a difference between the effects of positive and negative shocks in government expenditures on economic growth long run as well as short run. Hence, one can conclude that there is a long-run and a short-run asymmetric relationship between economic growth and general government expenditures.

The long-run effects of the model variables on economic growth are reported in Table 11. It shows that all variables are statistically significant at less than 5% significant level except LOPEN and LCPI variables. Since the main objective of the paper is to examine the effect of positive and negative shocks in government expenditure, the discussion focuses on these two variables. The

| Table 7. The F-bound test results of NARDL |
|------------------------------------------|
| F-bounds test                           | Null hypothesis: No levels relationship |
| Test statistic | Value | Sig. | I(0) | I(1) |
| F-statistic   | 13.36 | 10%  | 1.99 | 2.94 |
| $k$           | 6     | 5%   | 2.27 | 3.28 |
|              |       | 2.5% | 2.55 | 3.61 |
|              |       | 1%   | 2.88 | 3.99 |
The negative relationship between positive shock in government expenditures and economic growth means that an increase in government expenditure reduces economic growth. A 1% increase in Government expenditures causes a 0.156% decrease in economic growth. Such relation reflects the crowding-out effect on the private-sector investment. The increase in government expenditures put upward pressure on government to finance through either taxes or bond-

long-run NARDL results presented in Table 11 indicate that both the positive and the negative shocks have negative signs.
| Statistical test                          | Value | F-Statistic |
|------------------------------------------|-------|-------------|
| Breusch-Godfrey serial correlation LM    | 1.57  | 0.235       |
| Breusch-Pagan-Godfrey Heteroskedasticity test | 0.787 | 0.723       |
| Jarque-Bera Test:                       | 0.647 |             |
| CUSUM (S)                                | 0.716 |             |
| CUSUM(SQD)                               | 0.723 |             |
financing methods. Bond-financing, if exceeds high values, will compete with private sector leading to crowding-out effect, which in turn reduces private investment, and hence, reducing output. On the other hand, if the government chooses to finance its expenditures through taxes, this leads to increase tax rate which is close to average propensity to consume, leading to a decrease in the aggregate demand, and then economic growth. The negative change in government expenditure decreases government expenditure and economic growth. A 1% decrease in government expenditure leads to a 0.34% decrease in economic growth.

This result implies that there is certain level of government expenditure that maintains a maximum level of economic growth, and any deviation from this level results in a decline in economic growth level. Further, this result supports the maximum level that was found by ARDL estimation.

The estimation results show that both trade openness (OPEN) and price level (LCPI) indicate insignificant negative impacts on economic growth; foreign direct investment exerts a significant negative impact on economic growth, and capital (LK) has a significant positive impact on economic growth. The dynamic multiplier shows that there is a difference between positive and negative impacts on economic growth.

6. Conclusion and remarks

The relationship between government expenditures and economic growth is a controversial issue among schools of economic thought. The Keynesians advocate the positive relation, while the classical doctrine believes it has a negative impact. As a result, a new strand of applied research has investigated whether a certain level of government size that maximizes economic growth exists.

Recently, Saudi Arabia put forward the 2030 vision as a strategy to achieve economic goals as well as social goals. One major objective of the 2030 vision is to reduce the budget deficit as possible. Hence, the study estimates the long-run government optimum size in Saudi Arabia using annual data over the 1971–2019 period by applying the linear and nonlinear ARDL bounds-test approach to cointegration. The main focus is whether the Armey-curve is valid for KSA. The statistical diagnostic test provide evidence on the model adequacy. Moreover, the ARDL short-run results revealed that the speed of adjustment is (~0.82), and it take the model to correct toward the long-run equilibrium in about 14 months as a results of a short-run shock. The long-run estimation results provide that 26.9% is the optimal government size, which exceeds the average size (24.2%) over the study period. The NARDL estimation results revealed asymmetric relationship between government expenditures and economic growth. Further, a positive shock reduces economic growth as well as a negative shock. Hence, there is a certain level of government expenditure associated with maximum economic

| Variable | Coefficient | Std. error | t-Statistic | Prob |
|----------|-------------|------------|-------------|------|
| LGEX_POS | –0.156791   | 0.066062   | –2.373410   | 0.0283 |
| LGEX_NEG | –0.345262   | 0.105348   | –3.277358   | 0.0040 |
| LOPEN    | –0.360051   | 0.194659   | –1.849653   | 0.0800 |
| LK       | 0.211548    | 0.090341   | 2.341660    | 0.0302 |
| LFDISH   | –0.019086   | 0.003204   | –5.957059   | 0.0000 |
| LCPI     | 0.363970    | 0.271371   | 1.341227    | 0.1957 |
| C        | 21.19255    | 0.472055   | 44.89426    | 0.0000 |

EC = EG(–0.1568*LGEX_POS –0.3453*LGEX_NEG –0.3601*OPEN + 0.2115
*K – 0.0191*FDISH + 0.3640*LCPI + 21.1926)
growth level. Further, any deviation away from this level reduces economic growth. Based on such result, Saudi Arabia has the chance to expand expenditures to its optimal. Hence, the study recommends that Saudi Arabia can increase its government expenditures share of GDP up to 26.9%.

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Note
1. The quadratic form of government expenditures (GE) is added in the estimated model in order to compute the estimated optimum government size as well as to contain statistical problems related to the nonlinearity in government expenditures.

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