CONSTRaining the cosmological constant from large-scale redshift-space clustering

Takahiko Matsubara
Department of Physics and Astrophysics, Nagoya University, Chikusa, Nagoya 464-8602, Japan; taka@aphys.nagoya-u.ac.jp

and

Alexander S. Szalay
Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, MD 21218; szalay@jhu.edu

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ABSTRACT

We show how the cosmological constant can be estimated from redshift surveys at different redshifts, using maximum likelihood techniques. The apparent redshift-space clustering on large scales ($\geq 20 h^{-1}$ Mpc) is affected in the radial direction by infall, and curvature influences the apparent correlations in the transverse direction. The relative strengths of the two effects will strongly vary with redshift. Using a simple idealized survey geometry, we compute the smoothed correlation matrix of the redshift-space correlation function and the Fisher matrix for $\Omega_m$ and $\Omega_L$. These represent the best possible measurement of these parameters given the geometry. We find that the likelihood contours are turning, according to the behavior of the angular diameter distance relation. The clustering measures from redshift surveys at intermediate to high redshifts can provide a surprisingly tight constraint on $\Omega_L$. We also estimate confidence contours for real survey geometries, using the Sloan Digital Sky Survey Luminous Red Galaxy and QSO surveys as specific examples. We believe that this method will become a practical tool to constrain the nature of dark energy.

Subject headings: cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

Dark energy, which is the cosmological constant $\Lambda$ in its simplest form, currently has turned out possibly to be a dominant component in the universe (e.g., Bahcall et al. 1999). It is one of the central issues in cosmology to reveal the quantitative nature of this mysterious form of energy. One of the mysteries of the cosmological constant is its smallness. No evidence for the existence of the cosmological constant has been detected on Earth. The cosmological constant is so small that it affects phenomena only on cosmologically large scales.

There are several traditional tests for the cosmological constant. The expected frequency of multiple-image lensing events for high-redshift sources is quite sensitive to the cosmological constant (Fukugita, Futamase, & Kasai 1990; Turner 1990). Luminosity-volume and redshift-volume relations can also be used to measure the geometry of the universe to constrain the cosmological constant (Rowan-Robinson 1968; Loh 1988). Alcock & Paczyński (1979) proposed an evolution-free test for the cosmological constant using statistically spherical objects. The Type Ia supernova Hubble diagram is used to constrain the cosmological constant (Schmidt et al. 1998; Perlmutter et al. 1999) through the luminosity distance $d_L$. Acoustic peaks of cosmic microwave background (CMB) anisotropies constrain the curvature of the universe (e.g., Hu, Sugiyama, & Silk 1997). Recent observational developments of the Type Ia supernova and the CMB anisotropies (Balbi et al. 2000; de Bernardis et al. 2000) suggest a flat, low-density universe with positive cosmological constant, $\Omega_m \sim 0.3$, $\Omega_\Lambda \sim 0.7$.

The effect of a cosmological constant on the clustering properties of objects in the nearby universe ($z \ll 1$) is so weak that redshift surveys have not been used to constrain $\Omega_\Lambda$ so far. As the depth and the sampling rate of redshift surveys increase, redshift-space clustering depends on the cosmological constant through the cosmological redshift distortions (Ballinger, Peacock, & Heavens 1996; Matsubara & Suto 1996; Matsubara 2000). Several applications of this effect are proposed (Nair 1999; Nakamura, Matsubara, & Suto 1998; Popowski et al. 1998; Yamamoto & Suto 1999; Yamamoto, Nishioka, & Suto 1999). Maximally to extract the cosmological information from the survey data, likelihood analysis combined with a data reduction technique such as the Karhunen-Loève transform has been quite successful at low redshifts (Vogeley & Szalay 1996; Szalay, Matsubara, & Landy 1998; Matsubara, Szalay, & Landy 2000). We expect it to be just as useful at intermediate to high redshifts here.

In this Letter, we combine the two methods, i.e., likelihood analysis on pixelized data and cosmological distortions. Specifically, we compute the Fisher matrix for simple geometries to illustrate how the cosmological distortions constrain the cosmological constant and the density parameter in given redshift survey data.

2. FROM CORRELATIONS TO FISHER MATRIX

To investigate generically how a given redshift survey can constrain the cosmological constant, we construct a rectangular box, in which objects such as galaxies or quasars in redshift space are observed. We apply a Gaussian smoothing window to the objects in the survey to get a smoothed estimate of the local density. With a sufficiently large smoothing radius, we do not have to deal with the nonlinearities of the density field. The Gaussian-smoothed cells are placed on lattice sites $i$ in the box so as to have the smoothed density vector $\rho_i$. In the following, we assume that the mean value of the density vector $\langle \rho \rangle$ is known so that we can define a density-fluctuation vector $d_i = \rho_i / \langle \rho \rangle - 1$.

In standard theories of structure formation, the linear density field is a random Gaussian process. In this case, all clustering properties of the universe are represented by two-point correlations. Thus, a correlation matrix, $C_{ij} = \langle d_i d_j \rangle$, theoretically specifies all the statistical information for a given data set. This matrix is related to a smoothed two-point correlation function...
In this equation, \( C_y = \) the shot-noise term:

\[
C_y = \int d^3k \! W(s_t - s_p) W(s_t - s_p) \xi(s_t, s_p) \\
+ \int d^3k W(s_t - s_p) W(s_t - s_p)/\bar{n}(s),
\]

where \( W(s) = \exp[-s^2/(2R^2)]/(\sqrt{2\pi}R) \) is a Gaussian smoothing window, \( \xi \) is a two-point correlation function, \( s_t \) is the position vector of a lattice site \( i \), and \( \bar{n}(s) \) is the mean number density field. The position vectors are all in observable redshift space, which consists of the redshift \( z \) and the angular distortion \( \theta, \phi \). In the distant-observer approximation, we can approximately use the Cartesian coordinates in redshift space and the two-point correlation function is a function of the relative vector \( s_t - s_p \). The correlation function does depend on the direction of this relative vector, because of the redshift distortions. An analytic form of the linear two-point correlation function in redshift space including high-redshift effects is given by Matsubara & Suto (1996). One of the equivalent forms given in Matsubara & Suto (1996) is (see also Ballinger, Peacock, & Heavens 1996):

\[
\xi(s_t - s_p) = b^2(z)D^2(z) \int \frac{d^3k}{(2\pi)^3} e^{i\cdot k(x_t - x_p)} \\
\times [1 + \beta(z) k^2/2] P(k),
\]

where \( P(k) \) is the linear mass power spectrum at \( z = 0 \), \( D(z) \) is the linear growth rate normalized as \( D(0) = 1 \), \( b(z) \) is the bias factor at redshift \( z \), and \( \beta(z) \) is the redshift distortion parameter, which is approximately related to the redshift-dependent mass density parameter as \( \beta(z) \sim \Omega_m^3(z)/b(z) \). In the above equation, the third axis is taken as the direction of the line of sight. The vectors \( x_t \) and \( x_p \) are the comoving positions of the two points that are labeled \( s_t \) and \( s_p \) in redshift space (see Matsubara & Suto 1996). Assuming the distant-observer approximation, a Gaussian window function, and that the mean number density is effectively constant, \( \bar{n} \), we find that equation (1) finally reduces to

\[
C_y = \frac{1}{2} b^2(z)D^2(z) \int_0^\pi \int_0^\pi \sin \theta d\theta d\phi \\
\times \exp[-k^2R^2(c_t^2 - c_p^2) \cos^2 \theta + c_t^2] \\
\times [1 + \beta \cos^2 \theta] I_0(kR \sin \theta) \sin \theta \\
\times \cos(kR \cos \theta \cos \phi) + \frac{\exp[-x^2/(4R^2)]}{\pi^{3/2}R^2}.
\]

In this equation, \( c_t(z) = H_t/H(z) \), \( c_p(z) = H_t/s_k(z)/c \) are the distortion factors parallel and perpendicular to the line of sight, respectively, where \( H(z) \) and \( s_k(z) \) are the Hubble parameter and the comoving angular diameter distance at \( z \) and \( H_t \) is the Hubble constant. A line-of-sight component of the redshift-space distance \( s_k \) between the centers of \( i \)-cell and \( j \)-cell is related to that of the comoving distance \( x_t \) by \( x_t = c_t(z)s_k \). Similarly, for a component perpendicular to the line of sight we have \( x_p = c_p(z)s_k \). In this notation, the quantities in equation (3) can be written as \( x = (c_t^2 s_k^2 + c_p^2 s_k^2)^{1/2} \) and \( \theta \rightarrow \cos^{-1}(c_t s_k/x) \). Integration over \( \theta \) remains because a spherical Gaussian smoothing kernel in observable redshift space is no longer spherical but is ellipsoidal in comoving space. The second term in equation (3) is a shot-noise term, convolved with the Gaussian kernel.

Once the correlation matrix can be theoretically calculated in any cosmological model, one can calculate the Cramér-Rao bound, which gives an estimate of how well the model parameters can be measured. This is one of the most powerful results in estimation theory (Therrien 1992). The Fisher information matrix is a key quantity in this theory (Kendall & Stuart 1969):

\[
F_{\alpha\beta} = -\left\langle \frac{\partial^2 \ln L}{\partial \theta_\alpha \partial \theta_\beta} \right\rangle,
\]

where \( L(d; \theta) \) is a probability distribution for the data vector \( d \), which depends on a vector of model parameters \( \theta \). In our case, the data vector is density fluctuations on lattice sites and the model parameters are the cosmological parameters. The Cramér-Rao bound states that the maximal likelihood estimate constrains the model parameters with a minimum variance, \( \langle \theta, \theta \rangle \geq (F^{-1})_{\alpha\beta} \) where \( F^{-1} \) is the inverse matrix of \( F \). When the number of data, i.e., the dimension of the data vector, is very large, the Cramér-Rao bound becomes equality. A contour \( \langle \theta, \theta \rangle^{1/2} = A \) in parameter space gives a concentration ellipsoid, which indicates a region where the likelihood density for model parameters is most concentrated. The threshold \( A = 1, 2, 3 \) corresponds to maximally attainable confidence levels of 1 \( \sigma \), 2 \( \sigma \), 3 \( \sigma \), respectively, in a likelihood analysis, if the likelihood function is Gaussian. The concentration ellipsoids are useful even when the likelihood function is not Gaussian to give a rough idea of the spread of the density (Therrien 1992).

The probability distribution of the linear density field is considered to be Gaussian so that the likelihood function has the form

\[
-2 \ln L = \ln \det C + d^T C^{-1} d + \text{const},
\]

where \( C \) is the correlation matrix, which depends on model parameters, and \( d \) is the data vector. In this case, the Fisher information matrix reduces to (see, e.g., Vogeley & Szalay 1996)

\[
F_{\alpha\beta} = \frac{1}{2} \text{Tr} (C^{-1} C_{\alpha\beta} C^{-1} C_{\beta\alpha}),
\]

where \( C_{\alpha\beta} = \partial C/\partial \theta_\alpha \), etc. Thus, the Fisher matrix or the concentration ellipsoids for any model parameters are straightforward to calculate from the correlation matrix of equation (3).

### 3. RESULTS FOR A SIMPLE CUBIC BOX

In this Letter, the mass density parameter \( \Omega_m \) and the normalized cosmological constant \( \Omega_\Lambda \) both at the present time, are the model parameters to be constrained. These parameters have the primary importance in high-redshift clustering distortions. For simplicity, the power spectrum \( P(k) \) and the bias parameter \( b(z) \), on which the correlation matrix of equation (3) also depends, are fixed throughout. We use the cold dark matter--type spectrum with a fixed shape parameter \( \Gamma = 0.2 \) with a fixed normalization \( \sigma_8 = 1 \). Throughout this Letter, we take a fiducial model \( (\Omega_m, \Omega_\Lambda) = (0.3, 0.7) \) at which the Fisher matrix is evaluated.

Comoving coordinates are not actually observable in redshift surveys and depend on the cosmological models that we are seeking. Thus, we use a coordinate system such as (\( z, \theta, \phi \)) in
Fig. 1.—Concentration ellipses from the Fisher matrix for generic boxes. Contour lines correspond to confidence levels 1, 2, 3 attainable from a single 200 $h^{-3}$ Mpc box placed at redshift $z = 0$–6.0 as indicated in each panel. This figure contains no shot noise and a bias factor of 1.

In polar coordinates. The clustering scale we are interested in is too small in figures when the distance is represented by $z$ itself, so we invent a new radial coordinate $s = cz/H_0 = 2997.9z$, i.e., the linear extrapolation of the distance-redshift relation of the nearby universe to all values of $z$. For example, a redshift interval $\Delta z = 0.1$ around any redshift $z$ corresponds to $\Delta s = 300$. Although the unit of this coordinate system is still $h^{-1}$ Mpc, we use the new notation $h^{-1}$ Mpc, to avoid confusion with the comoving coordinate system.

In this coordinate system, a 200 $h^{-1}$ Mpc cubic box is considered to obtain generic estimates for the Cramér-Rao bound, and we compute the Fisher matrix for this sample, varying the mean redshift $z$ of this box. The density fluctuations are sampled on regular $10 \times 10 \times 10$ lattice sites in the box. The Gaussian smoothing radius is set as $R = 10 h^{-1}$ Mpc.

Figure 1 shows the resulting concentration ellipses at mean redshift $z = 0$–6.0. The contour lines correspond to the maximally attainable confidence levels of 1, 2, 3. The shot noise is neglected in this figure, while it would be difficult to reduce shot noise for $z \geq 2$ in reality. As is well known, a low-redshift sample ($z \sim 0$) constrains the mass density parameter only through its dependence on the redshift distortion parameter $\beta \sim \Omega_M/b$. As the mean redshift increases, the concentration ellipses rotate clockwise and the major axis becomes shorter, and thus the cosmological constant becomes constrained. The higher the mean redshift is, the more the cosmological constant is constrained. Around $z \sim 1.7$, the concentration ellipses begin to rotate counterclockwise. This is consistent with the fact that the angular diameter distance–redshift relation turns over at $z \sim 1.7$ in our fiducial model ($\Omega_M, \Omega_\Lambda = (0.3, 0.7)$. For particularly high redshift boxes ($z > 5$), $\Omega_\Lambda$ is constrained much better. This is because the velocity-distortion parameter $\beta(z)$ approaches $1/b$, so the clustering pattern depends on $\Omega_M$ and $\Omega_\Lambda$ only through the cosmological distortions.

On the one hand, the number density one can sample is smaller for high-redshift objects, which dilute the constraints on cosmological parameters. On the other hand, there is a larger volume to be sampled than for low-redshift objects. To obtain the concentration ellipses in realistic samples, one should take into account both the shot-noise effect and the total volume in a given sample. We again set the 200 $h^{-1}$ Mpc $\Omega_M$ boxes and estimate the Cramér-Rao bound with the shot-noise effect included. In Figure 2, the bound for the normalized cosmological constant $\Omega_\Lambda$ is plotted. In the left panel, the volume is given by just one 200 $h^{-1}$ Mpc boxes as above. The shot noise is varied as $(20 h^{-1}$ Mpc $)^2 n = 0.1, 0.3, 1, 3, 10$, and $\infty$, from top to bottom lines. In the right panel, the Cramér-Rao bound is scaled by the number of independent 200 $h^{-1}$ Mpc boxes in a $\pi$ sr region with a redshift interval $z/2$ around $z$, to obtain a rough idea of how our error bound is affected by the survey volume. We can see how densely the objects should be sampled to constrain the cosmological constant with a certain accuracy both for a sample with a fixed volume and a sample with a fixed solid angle.

Fig. 2.—Cramér-Rao bound for the normalized cosmological constant $\Omega_\Lambda$ as a function of the mean redshift $z$. Shot noise is varied as $(20 h^{-1}$ Mpc $)^2 n = 0.1, 0.3, 1, 3, 10$, and $\infty$, from top to bottom lines. Left: Single 200 $h^{-1}$ Mpc box. Right: For a survey of $\pi$ sr, a redshift interval of $z/2$, centered at $z$. The number of independent boxes is increased by the volume of the shell; correspondingly the accuracy improves. A bias factor of 1 has been used throughout this figure.

4. POSSIBILITIES FOR REALISTIC SURVEY VOLUMES

We have considered several different survey layouts for both galaxies and quasars. The best survey to perform these tests seems to be the Luminous Red Galaxy (LRG) sample of the Sloan Digital Sky Survey (SDSS). This sample consists of
100,000 galaxies selected for spectroscopic observations on the basis of their very red rest-frame colors, using photometric redshifts, down to a limiting magnitude of $r = 19.5$. They form an approximately volume-limited sample, where the outer edge lies at around $z = 0.45$ and the total surface area is 10,000 deg$^2$.

We consider this geometry as a composite of the generic 200 $h^{-1}$ Mpc$^3$ boxes at the mean redshift $z = 0.3$. There are about 220 boxes out to $z = 0.45$ in a $\pi$ sr region, so the shot noise is approximately given by $(20 h^{-1} \text{Mpc})^3 n = 0.5$. We assume two bias factors, $b = 1.5$ and $b = 2$. The resulting concentration ellipses are shown in Figure 3. We can see that the result is quite remarkable. The Crame´r-Rao bound for $\Omega_L$ is only $[(F^{-1})_{\Lambda\Lambda}]^{1/2} = 0.04$ for $b = 1.5$ and $[(F^{-1})_{\Lambda\Lambda}]^{1/2} = 0.03$ for $b = 2$. This shows that the shot-noise level and the depth of the survey volume are suitably balanced to constrain the geometry of the universe in the SDSS LRG survey.

We have considered various QSO surveys to possibly measure the dark energy at higher redshifts. Unfortunately, the currently ongoing QSO redshift surveys, such as the SDSS (York et al. 2000) and Two Degree Field QSO redshift survey (Boyle et al. 2001), have lower sampling rates for QSOs, $n \sim 10^{-3}/(40 h^{-1} \text{Mpc})^3$. They typically give a Crame´r-Rao bound of order $\Delta \Omega_L \sim 1$, almost regardless of the smoothing radius. To constrain the cosmological constant with QSO surveys, one should sample QSOs more densely than these current QSO surveys. This fact is in agreement with Popowski et al. (1998), who analyzed nonlinear clustering to constrain the geometry of the universe and indicated an advantage of a dense sampling.

5. DISCUSSION

We have shown that large-scale clustering of galaxies at intermediate redshifts $z \sim 0.5$ is surprisingly suitable for constraining the cosmological parameters of $\Omega_M$ and $\Omega_L$, and thus the geometry of the universe. The QSOs in currently ongoing surveys are too sparse to give comparable constraints.

The apparent redshift-space–clustering method used in this Letter is a completely self-contained test for $\Omega_M$ and $\Omega_L$. The results from this method can be further combined with any of the other independent tests to obtain stricter constraints or to check the consistency of our standard picture of the cosmology.

One can use Figure 2 to aid designs of future surveys at various redshifts. The lines indicate the statistical uncertainty in the cosmological constant corresponding to different sampling rates. They should be scaled, noting that the Crame´r-Rao error bound roughly scales as the inverse of the square of survey volumes.

In this work, we have considered only two parameters, $\Omega_M$ and $\Omega_L$. We still need to measure the evolution of the bias parameter, which is not obvious. Moreover, there is a possibility that the dark energy has a more complex behavior than the cosmological constant (Wang et al. 2000). There are many other cosmological parameters, such as baryonic density $\Omega_b$, primordial spectral index $n$, neutrino mass density $\Omega_\nu$, etc., which more or less depend on the apparent redshift-space clustering.

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