Oscillating Color Transparency in $\pi A \rightarrow \pi p(A - 1)$ and $\gamma A \rightarrow \pi N(A - 1)$

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Abstract: The energy dependence of $90^\circ$ cm fixed angle scattering of $p p \rightarrow p' p'$ and $\gamma p \rightarrow \pi^+ n$ at large momentum transfer are found to be well described in terms of interfering short and long distance amplitudes with dynamical phases induced by Sudakov effects. We calculate the color transparency ratio for the corresponding processes in nuclear environments $\pi A \rightarrow \pi p p(A - 1)$ and $\gamma A \rightarrow \pi N (A - 1)$ taking nuclear filtering into account. A prediction that the transparency ratio for these reactions will oscillate with energy provides an important test of the Sudakov phase shift and nuclear filtering hypothesis which is testable in upcoming experiments.

The strong interactions remain a mystery and a phenomenon. Color transparency separates conventional strong interaction physics from perturbative QCD. The perturbative calculation predicts suppression of strong interactions in certain exclusive reactions containing a large momentum transfer $Q^2 >> GeV^2$ subprocess. Suppression is supposed to occur in initial or final state interactions with nuclear targets. The perturbative QCD ($pQCD$) prediction is dramatic because it apparently contradicts the older theory in a domain of its validity. Indeed it is not clear whether color transparency is capable of being described using hadronic coordinates. At the same time, the many shortcomings of the $pQCD$ description at the moderate $Q^2$ values of experiments are well known: Hence the phenomena of color transparency play a pivotal role from either point of view.

The BNL E850 experiment of Carroll \textit{et al.}\cite{1} compared proton-proton elastic collisions with corresponding quasi-elastic nuclear processes $p A \rightarrow p' p'' (A - 1)$. A transparency ratio oscillating with energy was observed. The origin of the oscillations remains controversial, and the underlying mechanism has generally been assumed to be unique to proton-proton reactions. Here we show that oscillatory color transparency is also expected in several processes involving the pion. Only recently have the full perturbative kernels needed for a $pQCD$ description of color transparency been completely evaluated so that the calculations have a workable paradigm. Here we report calculations predicting new phenomena observable in experiments currently underway at CEBAF, which will provide fundamental information on how $pQCD$ may be applied to exclusive processes both in free space and in a nuclear medium. Other experimental predictions can be checked at BNL or other hadron beam laboratories. The predictions are rather distinctive, and tests of the entire framework of color transparency become available.

Consider the reaction $p p \rightarrow m' p'$ compared to $\pi A \rightarrow m' N'(A - 1)$, where $m'$ represents a meson and $N'$ represents a nucleon. These processes contain Landshoff pinch singularities, and in $pQCD$ are expected to show oscillations about power-law energy dependence at fixed angle. The wavelength of the oscillations (imaginary anomalous dimensions) are calculable, but have not yet been calculated. These important calculations have only been performed so far for selected diagrams occurring in $pp$ scattering. Now considering the $p p$ reactions, the same physics of pinch singularities again demands a factorization scheme more general than the short-distance “quark-counting” method, which is sometimes misunderstood to define $pQCD$. We re-iterate that among the various competing renditions—the short distance factorization of Brodsky-LePage, the asymptotic impact-parameter factorization of Botts and Sterman, and the finite $Q^2$ factorization of Gousset \textit{et al} incorporating spin-effects, all represent $pQCD$ and concepts such as “asymptotic” or “short distance” are not synonymous with “$pQCD$”.

To represent all the diagrams and integration regions, our calculations include the transverse spatial separation between quarks. Several remarkable things emerge: First, the naive association of $b \sim 1/Q$ breaks down, and hard reactions depend on the entire region $1/Q < b < 1/\Lambda_{QCD}$. There is generic violation of the short-distance selection rule known as “hadron-helicity conservation”, a model-independent test of the short-distance framework, and $pQCD$ predicts non-trivial transverse and helicity-violating spin effects for large $Q^2 >> GeV^2$. Next, Sudakov factors regulate the approach to the pinch configurations and must be included among the kernels. The Sudakov factors re-instate the geometrical strong-interaction $Fm$-scale by drastically cutting off amplitudes at distances larger than $1/\Lambda_{QCD}$. The Sudakov-improved amplitudes must obey analyticity, exhibited in $pQCD$ by color and flavor matrix phase factors of the form $\exp[-i\pi\ln(\ln(s)/\Lambda_{QCD}^2)]$, handled by extending the notion of anomalous dimensions to purely imaginary numbers. In a nuclear medium large-$b$ regions interact inelastically with exponential attenuation, while those regions of small $b$ interact proportional to $b^2 \rightarrow 0$, resulting in transparency. By depleting the long distance amplitudes, “nuclear filtering” quantum mechanically favors short distance processes in
large nuclei [15][13].

All of these elements are seen in free-space pp reactions and the BNL color transparency experiment. In particular the free space cross section \( s^{10}d\sigma/dt \) at fixed cm angle 90\(^\circ\) oscillates with \( \ln(\ln(s/\Lambda_{QCD}^2)) \). (Here \( s, t \) are the Mandelstam variables for cm energy-squared and momentum transfer-squared). The color transparency ratio was found to show oscillations 180\(^\circ\) out of phase with the free-space oscillations. In a two-component model this rather unambiguously indicates strong attenuation or “filtering” in the nuclear medium of one long-distance amplitude, and little attenuation of another short-distance component. In addition, attenuation cross sections extracted from the data [21] are substantially smaller than the traditional 40 \( mb \) of conventional strong interaction physics at these energies, and scaling in the variable \( Q^2/A^{1/3} \) was observed. Consistently, the cross section in the nuclear target shows negligible oscillations with energy [4] and apparently conforms to predictions of short-distance physics [22][44]. In contrast, a model based on the hadronic basis (Farrar et al Ref. [3]) fails to describe the data by many standard deviations.

Correlating these observations of pp reactions with the dynamical similarity of \( \pi p \) reactions suggests similar phenomena should be observed. Parts of the calculations are stymied by a major difficulty: no systematic method exists to find the relative phases of exclusive amplitudes. It is not enough to calculate the phase of the asymptotically largest amplitude (the procedure of Ref. [22]) but it is necessary to find any sizable phase coefficient of any sizable amplitude.

As a practical resolution we have fit the 90\(^\circ\)cm fixed angle \( s^8d\sigma/dt \) data [21] for \( pp \rightarrow \pi'p' \) with a two-component model. The existence of this data for fixed angle scattering compiled by Blazey [21] (Fig. 1a) appears not to be widely appreciated. Oscillations in this data show much the same features as the free-space pp data. With \( M \) denoting the 2 \( \rightarrow \) 2 amplitude for the reaction, our fit is given by:

\[
\frac{d\sigma}{dt} = \frac{s^8}{A} \left[ A_0 + \frac{A_1}{\log(s)^{d_1}} - ic_1 \log q^2/\Lambda^2_{QCD} \right] + \frac{A_2}{\log(s)^{d_2}} - ic_2 \log q^2/\Lambda^2_{QCD} \right]^2
\]

(1)

where \( A_0, A_1, A_2, c_1, c_2, d_1, d_2 \) are real parameters. The functional form of our exponents have been updated compared to Ref. [17] and come from expanding the imaginary parts of Sudakov exponents. In accord with the discussion, the two components, \( A_1 \) and \( A_2 \) represent regions of large \( b \), associated Sudakov effects, and logarithmically-varying phases, while small-\( b \sim 1/Q \) regions are described by short-distance theory. The best fit gives \( A_0 = -0.638, A_1 = 5.1, c_1 = 25.6, d_1 = 5.13, A_2 = -0.065, c_2 = -26.3, d_2 = -1.16 \) with \( \chi^2/dof = 1.97 \). If we include only one long distance amplitude setting \( A_2 = 0 \), then the best fit gives \( A_0 = -0.661, A_1 = -7.67, c_1 = 23.2, d_1 = 6.03 \) with \( \chi^2/dof = 5.01 \). In comparison the short-distance model \( s^{-8} \) fit gives \( \chi^2/dof = 99 \).

Now turn to the corresponding pion-initiated reaction with a nuclear target. In the two-amplitude model each component interacts with the nuclear target by a different rule. For the long distance pieces, the target measures the integration region (“transverse size”) via attenuation by the rule \( I_f = exp(-\int \sigma_n dz) \), where \( z \) is the straight-line propagation distance across the target, and \( n \) is the nucleon density; \( \sigma_{tot} \) is the absorptive cross section for particles \( a, b \). We used \( \sigma_{ab} = 40mb \), and \( \sigma_{pp} = 26mb \). The short distance amplitude is attenuated with a model inspired by \( \sigma_S = k/(x_1x_2Q^2) \), where \( x_1, x_2 \) are the momentum fractions of the quarks inside the proton. Since this amplitude is short-distance we set \( x_1 = x_2 = 0.5 \) and so \( \sigma_S = k(1.6mb)/(GeV^2/Q^2) \). Short-range nuclear correlations are included [22] in both cases. We then calculate the cross section in the nuclear case by

\[
\frac{d\sigma}{dt} = \int d^2x_n(x) \left[ A_0I_S^fI_S^f + A_1\sqrt{s}e^{-ic_1} \log q^2/\Lambda^2_{QCD} + i\phi_A \right]
\]

\[
+ A_2\sqrt{s}e^{-ic_2} \log q^2/\Lambda^2_{QCD} + i\phi_A \right]
\]

(2)

where \( A \) is the nuclear number; superscripts \( i \) and \( f \) refer to initial and final state attenuation factors, respectively. The formula indicates we took into account a potential relative phase \( \phi_A \) between the two amplitudes due to interaction with the nucleus.

We assume Fermi-motion is taken out experimentally by overdetermined kinematic reconstruction (such as possible at BNL) and so this has not been included in the calculations. We treat \( \phi_A \) and \( k \) as parameters subject to considerable uncertainty. However for the entire range of \( 0 < \phi_A < 2\pi \) and varying \( 5 < k < 10 \) the calculations are sufficiently robust to predict rather dramatic effects. In Fig. 1b we show the results for the transparency ratio, \( T = d\sigma(\pi A \rightarrow n'N'(A - 1); 90\circ)/dt \) for the two different models. The plots (Fig 1, b-c) show a striking 180\(^\circ\) phase shift between the oscillations of the transparency ratio and those seen in the free-space reaction. \( T \) is less sensitive to variations of \( \phi_A \) compared to \( k \): for all values of the \( \phi_A \) we find that \( T \) shows significant oscillations with energy. Only for very large values of \( k \gg 10 \) do these oscillations disappear, a limit in which no short distance contribution effectively exists. The plots are given for large nuclei where the calculation indicates filtering will be effective: for \( A >> 1 \), short distance physics predicts scaling in the variable \( Q^2/A^{1/3} \). The theory may be extended to smaller \( A \approx 12 \), where our calculations also show a substantial effect, with less confidence regarding
the importance of the short-distance component. As in the \( pp \) case, the measurement of the transparency ratio as a function of \( s \), or the \( A \) dependence at fixed large \( s \), would be capable of ruling out the hadronic-basis predictions for the same reaction, which are either monotonic (Glauber theory) or linear in the energy (expinging point-like classical expansion theory (Farrar et al., [3]).

We fit the experimental data for \( \gamma p \rightarrow \pi^+ n \) with center of mass scattering angle 90° and \( \sqrt{s} > 2 \text{ GeV} \). The best fit to the 17 data points available is shown in Fig. (2). We use the same amplitude ansatz as Eq (1) for \( s^2 d\sigma/dt \), obtaining \( A_0 = 0.90, A_1 = 2.65/s, c_1 = 64.5, A_2 = 8.0/s, c_2 = -126.4 \) with \( \chi^2/dof = 0.69 \). Here we have set \( d_1 = d_2 = 4 \), as the quality of fit does not depend substantially on these parameters. The values of \( d_1 \) and \( d_2 \) were chosen to obtain a relatively flat free space behavior beyond \( \sqrt{s} = 3.0 \) GeV, where the presence or absence of oscillations remains experimentally unstudied. We arbitrarily imposed a model of short-distance physics for this region. If we set \( A_2 = 0 \) then the best fit gives, \( A_0 = 0.89, A_1 = -4.15 \) and \( c_1 = 79.8 \) with \( \chi^2 \) per degree of freedom of 1.09. For comparison the short-distance \( s^{-7} \) model gives \( \chi^2/dof = 2.9 \). While our fit is favored statistically, including effects of extra parameters, the short-distance model is not ruled out in comparison. Cutting the experimental uncertainties in half would be pivotal. We mention this because the uncertainties are expected to decrease with the experiments imminent.

For the nuclear process \( \gamma A \rightarrow \pi^+ n + (A-1) \), we calculate \( s^2 d\sigma/dt \) with the same format as Eq (2). Results for the transparency ratio for \( A = 12, 56, 197 \) are shown in Fig. 2. In calculating filtering factors we conservatively assume that the incident photon does not attenuate significantly. While there are many models to attenuate the photon somewhat, this allows a conservative presentation, because the effects of filtering which generate the oscillating transparency ratio are minimized. Let us note that experimentally the final state \( N \) can be a proton or a neutron, but to predict the neutron case definitively we would need free space neutron scattering data that we do not currently have.

Observing Fig. (2), the predicted transparency ratio \( T \) again oscillates 180° out of phase with the free space cross-section. This simple fact has so far not been appreciated as generic, and previous hadronic-basis estimates for the transparency ratio have not taken the oscillations in free space data into account, yielding monotonically increasing energy dependence. The upcoming photon-initiated experiments, then, may be on the verge of confirming a third case of oscillating fixed angle data, and oscillating color transparency.

Color transparency with a photon beam remains significantly different from hadron initiated processes. The distinction becomes clear when the \( Q^2 \) dependence of a virtual photon is used as an experimental tool. In the limit of large \( Q^2 \gg \text{GeV}^2 \), experimental evidence from deeply-inelastic scattering overwhelming supports the concept of a point-like photon interaction, with negligible attenuation and pertubatively understood hadronic components in scattering. The lack of pinch singularities of the large-\( Q^2 \) framework predicts power-law fading.
of oscillations in both free space cross section and transparency ratio in the limit of large-$Q^2$. The regime of large-$Q^2$ for photons should coincide with the regime of Bjorken scaling, so that the moderate $Q^2$ of existing electron beams should suffice. This would be extremely interesting and productive area to explore experimentally.

Direct tests of the hadron-helicity non-conserving character of the pinch-singularity regions are very interesting. A pion beam suggests studying reactions involving a final-state $\rho$ meson: again the process has pinch singularities. The $pQCD$ analysis indicates that oscillations of fixed-angle scattering with energy will occur, and indeed one of the points of this paper is that such oscillations are generic. The failure of short-distance models, and dynamical importance of the pinch regions for $\pi p \rightarrow \pi p$ is supported by observations $^{[1]}$ of final-state $\rho$-polarization density matrix elements $\rho_{1-\perp}$ of order unity. If this is due to the pinch regions, as expected $^{[11]}$, then filtering in a large nucleus should remove them. Oscillating polarization effects would be very dramatic: $\rho_{1-\perp}$ oscillating with energy at fixed angle is expected if the dynamical phases are correlated with exchange of orbital angular momentum. Counting powers of the internal co-ordinate $b$ and the units of orbital angular momentum, we can predict that at fixed large $Q^2$, each power of $b^2$ in amplitude calculations will scale like $A^{-1/3}$ due to nuclear filtering.

To conclude, oscillating color transparency is a generic prediction of $pQCD$, testable with imminent experiments. We believe that the observation of oscillations in experimental data for the transparency ratio, consistently $180^\circ$ out of phase with the free space counterparts, and in three independent reactions, will be strong confirmation of nuclear filtering and the basic $pQCD$ understanding of color transparency.

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