Charged Perfect Fluid Cylindrical Gravitational Collapse

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Abstract

This paper is devoted to study the charged perfect fluid cylindrical gravitational collapse. For this purpose, we find a new analytical solution of the field equations for non-static cylindrically symmetric space-time. We discuss physical properties of the solution which predict gravitational collapse. It is concluded that in the presence of electromagnetic field the outgoing gravitational waves are absent. Further, it turns out that when longitudinal length reduces to zero due to resultant action of gravity and electromagnetic field, then the end state of the gravitational collapse is a conical singularity. We also explore the smooth matching of the collapsing cylindrical solution to a static cylindrically symmetric solution. In this matching, we take a special choice of constant radius of the boundary surface. We conclude that the gravitational and Coulomb forces of the system balance each other.

Keywords: Gravitational collapse; Junction conditions; Cylindrical symmetry; Electromagnetic field.

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1 Introduction

Gravitational collapse of a massive star occurs when all the thermonuclear reactions in the interior of a star could not favor the pressure against gravity. Gravitational collapse is one of the most important problems in general relativity. The singularity theorems 1) state that there exist spacetime singularities in the realistic gravitational collapse. To investigate the nature of spacetime singularity, Penrose 2) suggested a hypothesis known as Cosmic Censorship Hypothesis (CCH). It states that the final fate of gravitational collapse of a massive astrophysical object is always a black hole. This is equivalent to saying that the singularities appearing in gravitational collapse are always clothed by an event horizon.

Many attempts predicted that final fate of gravitational collapse of the massive star might be a black hole or naked singularity depending upon the choice of initial data. In this chain, Virbhadra et al. 3) introduced a new theoretical tool using the gravitational lensing phenomena. In a recent paper 4), Virbhadra used the gravitational lensing phenomena to find an improved form of the CCH. The classical paper of Oppenheimer and Snyder 5) is devoted to study dust collapse according to which singularity is neither locally or globally naked. In other words, the final fate of the dust collapse is a black hole. Many people 6) extended this work for physically existing form of fluid with cosmological constant in spherically symmetric background.

In order to generalize the geometry of the star, people worked on gravitational collapse using cylindrical symmetry. The existence of cylindrical gravitational waves provides a strong motivation in this regard. Bronnikov and Kovalchuk 7) were the pioneers to the work on gravitational collapse with non-spherical symmetry. Later on, the same authors 8) extended it for some non-spherical exact model. Nolan 9) investigated the naked singularities in the cylindrical gravitational collapse of counter rotating dust shell.

Hayward 10) studied gravitational waves, black holes and cosmic strings in cylindrical symmetry. Sharif and Ahmad 11) analyzed cylindrically symmetric gravitational collapse of two perfect fluids using the high speed approximation scheme. They investigated the emission of gravitational radiation from cylindrically symmetric gravitational collapse. Di Prisco et al. 12) discussed the shear free cylindrical gravitational collapse using junction conditions. Nakao et al. 13) studied gravitational collapse of a hollow cylinder composed of dust.

Gravity is the weakest interaction among all the natural forces. The be-
behavior of electromagnetic field in gravitational field has been the subject of
interest for many years. Thorne\textsuperscript{14} developed the concept of cylindrical en-
ergy and investigated that a strong magnetic field along the symmetry axis
may halt the cylindrical collapse of a finite cylinder before it reached to sin-
gularity. Oron\textsuperscript{15} studied the relativistic magnetized star with the poloidal
and toroidal fields. Thirukkanesh and Maharaj\textsuperscript{16} found that the inclusion
of electromagnetic field in gravitational collapse would counterbalance the
gravitational attraction by the Coulomb repulsive force along with pressure
gradient. In recent papers\textsuperscript{17,19}, we have investigated the effects of elec-
tromagnetic field on the perfect fluid collapse by using junction conditions
in spherically symmetric background with positive cosmological constant.
It has been found that electromagnetic field reduces the pressure and favors
the naked singularity formation but cannot play a dominant role. Thus black
hole was formed as a final state of the gravitational collapse.

In this paper, we study the cylindrically symmetric charged perfect fluid
gravitational collapse. The main objective of this work is to study the final
fate of charged perfect fluid gravitational collapse in the cylindrically sym-
metric background. The plan of the paper is as follows: In the next section,
we discuss the solution of the Einstein-Maxwell field equations. The physi-
cal properties of the solution are discussed in section 3. Section 4 gives the
derivation of the matching conditions. We summarize the results in the last
section.

Geometrized units (i.e., the gravitational constant $G=1$ and speed of light
in vacuum $c=1$) are used. All the Latin and Greek indices run from 0 to 3,
otherwise, it will be mentioned.

\section{Solution of the Einstein Field Equations}

This section is devoted to the solution of the Einstein field equations coupled
with the charged perfect fluid as the source of gravitation distributed per
unit length of the cylinder. The general cylindrically symmetric spacetime
is given by the following line element\textsuperscript{12}

$$ds^2 = A^2(dt^2 - dr^2) - B^2d\theta^2 - C^2dz^2,$$

where $A$, $B$ and $C$ are functions of $t$ and $r$. Here we take the following
restrictions on the coordinates in order to preserve the cylindrical symmetry

\begin{align}
\end{align}
of the spacetime

\[-\infty \leq t \leq \infty, \quad r \geq 0, \quad 0 \leq \theta \leq 2\pi, \quad -\infty < z < \infty. \quad (2.2)\]

The proper unit length of the cylinder for the line element (2.1) is defined by

\[l = 2\pi BC. \quad (2.3)\]

The Einstein field equations are given by

\[G_{\mu}^{\nu} = \kappa(T_{\mu}^{\nu} + T_{\mu}^{\nu(\text{em})}). \quad (2.4)\]

The energy-momentum tensor for perfect fluid is

\[T_{\mu}^{\nu} = (\rho + p)u_{\mu}u^{\nu} - p\delta_{\mu}^{\nu}, \quad (2.5)\]

where \(\rho\) is the energy density, \(p\) is the pressure and \(u_{\mu} = A\delta_{\mu}^{0}\) is the four-vector velocity in co-moving coordinates. The energy-momentum tensor for the electromagnetic field is given by

\[T_{\mu}^{\nu(\text{em})} = \frac{1}{4\pi}(-F_{\nu\lambda}F_{\mu\lambda} + \frac{1}{4}\delta_{\mu}^{\nu}F_{\pi\lambda}F^{\pi\lambda}), \quad (2.6)\]

where \(F_{\mu\nu}\) is the Maxwell field tensor. Now we solve the Maxwell’s field equations

\[F_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu}, \quad F^{\mu\nu}_{\nu,\nu} = 4\pi J^{\mu}, \quad (2.7)\]

where \(\phi_{\mu}\) is the four potential and \(J^{\mu}\) is the four current. In co-moving coordinate system, the charge per unit length of the cylinder is assumed to be at rest so that the magnetic field will be zero. Thus we can choose the four potential and four current as follows

\[\phi_{\mu} = (\phi(t, r), 0, 0, 0), \quad J^{\mu} = \sigma u^{\mu}, \quad (2.8)\]

where \(\sigma\) is charge density. The only non-zero component of the field tensor is

\[F_{01} = -F_{10} = -\frac{\partial \phi}{\partial r}. \quad (2.9)\]

Thus the Maxwell field equations take the following form

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{\partial \phi}{\partial r} [\frac{B'}{B} + \frac{C'}{C} - 2\frac{A'}{A}] = 4\pi \sigma A^3, \quad (2.10)
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{A^3} \frac{\partial \phi}{\partial r} \right) + \left( \frac{1}{A^3} \frac{\partial \phi}{\partial r} \right) \left[ \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + 2\frac{\dot{A}}{A} \right] = 0, \quad (2.11)
\]
where dot and prime indicate derivatives with respect to time $t$ and radial coordinate $r$, respectively. Integration of Eq. (2.10) implies that

$$\frac{\partial \phi}{\partial r} = 2qA^2 \frac{B}{C},$$

(2.12)

where $q(r) = 2\pi \int_0^r \sigma(ABC)dr$, being the consequence of conservation law of charge, i.e., $J^\mu_\mu = 0$ is known as the total amount of charge per unit length of the cylinder. It is mentioned here that Eq. (2.11) is identically satisfied by the solution of Eq. (2.10). We can write Eq. (2.9) as follows

$$F_{01} = -F_{10} = -\frac{2qA^2}{BC}. \quad (2.13)$$

The non-zero components of $T^{\nu(\text{em})}_{\mu}$ turn out to be

$$T_{0}^{0(\text{em})} = T_{1}^{1(\text{em})} = -T_{2}^{2(\text{em})} = -T_{3}^{3(\text{em})} = \frac{1}{2} \frac{q^2}{(BC)^2}.$$  

The electric field intensity is defined by

$$E(r, t) = q \frac{2}{\pi(BC)}. \quad (2.14)$$

We assume that the charged perfect fluid distributed per unit length of the cylinder follows along the geodesics in the interior of the cylindrical symmetry. This requires that velocity should be uniform and acceleration must be zero which is only possible if $A$ is constant and in particular, we take $A = 1$ (for simplicity). Thus the field equations (2.14) take the following form

$$-\frac{B''}{B} - \frac{C''}{B} + \frac{C'}{C} + \frac{B'C''}{BC} + \frac{B\dot{C}'}{BC} = 8\pi(\rho + 2\pi E^2), \quad (2.15)$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 0, \quad (2.16)$$

$$\frac{B'C'}{BC} - \frac{B\dot{C}}{BC} - \frac{B}{B} - \frac{C'}{C} = 8\pi(p - 2\pi E^2), \quad (2.17)$$

$$\frac{\dot{C}}{C} + \frac{C''}{C} = 8\pi(p + 2\pi E^2), \quad (2.18)$$

$$\frac{\dot{B}}{B} + \frac{B''}{B} = 8\pi(p + 2\pi E^2). \quad (2.19)$$
We note that there are five equations and five unknowns $B$, $C$, $p$, $\rho$ and $E$, thus we can find a unique solution.

For this purpose, we adopt the method of separation of variables. The comparison of Eqs. (2.18) and (2.19) give

$$\frac{-\ddot{C}}{C} + \frac{C''}{C} = -\frac{\ddot{B}}{B} + \frac{B''}{B},$$

(2.20)

which yields the necessary condition for pressure to be isotropic. We take

$$B = f(r)g(t), \quad C = h(r)k(t).$$

(2.21)

Using Eq. (2.21) in (2.16), we get

$$f = \alpha h^L, \quad k = \delta g^{-L},$$

(2.22)

where $L (\neq 0$, for non-trivial solution) is a separation constant while $\alpha$ and $\delta$ are integration constants. Using Eq. (2.22) in (2.20), it follows that

$$\frac{\ddot{g}}{g} - \frac{\ddot{k}}{k} = f'' - \frac{h''}{h}.$$  

(2.23)

Since both sides are functionally independent, we put them equal to constant say $M (\neq 0)$

$$\frac{\ddot{g}}{g} - \frac{\ddot{k}}{k} = M = f'' - \frac{h''}{h}.$$  

(2.24)

Application of Eq. (2.22) to Eq. (2.24) leads to

$$\frac{\ddot{g}}{g} - \frac{\dot{g}^2}{g^2} = \frac{M}{L + 1}, \quad \frac{h''}{h} + \frac{h'^2}{h^2} = \frac{M}{L - 1}.$$  

(2.25)

The solution to these equations is

$$g(t) = \beta_0 \cos \frac{1}{\sqrt{L+1}}(Wt + t_0), \quad h(r) = \beta_1 \cosh \frac{1}{\sqrt{L+1}}(Sr + r_0),$$  

(2.26)

where $\beta_0$, $\beta_1$, $t_0$ and $r_0$ are constants of integration. Further, $W$ and $S$ are given by the following relations

$$W = \sqrt{\frac{M(L - 1)}{L + 1}}, \quad S = \sqrt{\frac{M(L + 1)}{L - 1}}.$$  

(2.27)
Using Eq. (2.26) in (2.22), it follows that

\[ k(t) = \beta_2 \cos \frac{L}{L-1} (Wt + t_0), \quad f(r) = \beta_3 \cosh \frac{L}{L-1} (Sr + r_0), \]  

(2.28)

where \( \beta_2 \) and \( \beta_3 \) are constants of integration. Thus the metric coefficients, given by Eq. (2.21), turn out to be

\[ B = \Omega \cosh \frac{L}{L-1} (Sr + r_0) \cos \frac{L}{L-1} (Wt + t_0), \]  

(2.29)

\[ C = \Psi \cosh \frac{L}{L-1} (Sr + r_0) \cos \frac{L}{L-1} (Wt + t_0), \]  

(2.30)

where \( \Omega = \beta_0 \beta_3 \), \( \Psi = \beta_1 \beta_2 \). Consequently, the spacetime (2.1) takes the form

\[ ds^2 = dt^2 - dr^2 - \frac{\Omega^2}{L} \cosh \frac{L}{L-1} (Sr + r_0) \cos \frac{L}{L-1} (Wt + t_0) d\theta^2 - \frac{\Psi^2}{L} \cosh \frac{L}{L-1} (Sr + r_0) \cos \frac{L}{L-1} (Wt + t_0) dz^2. \]  

(2.31)

Using the following transformations

\[ Sr' = Sr + r_0, \quad Wt' = Wt + t_0, \quad \theta' = \Omega \theta, \quad z' = \Psi z, \]

this metric reduces to

\[ ds^2 = dt'^2 - dr'^2 - \cosh \frac{L}{L-1} (Sr') \cos \frac{L}{L-1} (Wt') d\theta'^2 - \cosh \frac{L}{L-1} (Sr') \cos \frac{L}{L-1} (Wt') dz'^2. \]  

(2.32)

By assuming \( \Omega = 1 \), it is clear that the above metric preserves cylindrical symmetry with the restriction on coordinates given by Eq. (2.2). Here we take

\[ \tilde{B} = \cosh \frac{L}{L-1} (Sr') \cos \frac{L}{L-1} (Wt'), \quad \tilde{C} = \cosh \frac{L}{L-1} (Sr') \cos \frac{L}{L-1} (Wt'). \]

### 3 Physical Properties of the Solution

Here, we discuss some physical and geometrical properties of the solution. The physical parameters, i.e., pressure \( p \), density \( \rho \), and the electric field
intensity $E$ for the metric (2.32) are given by

$$
p = \frac{1}{16\pi} \left[ \frac{S^2}{(1 + L)} - \frac{4\tan^2(W't')W^2L}{(1 - L)^2} + \frac{W^2}{(L - 1)(2L - 1)} \right], \quad (3.1)$$

$$
E = \left[ \frac{1}{32\pi^2} \right]^{\frac{1}{2}} \left\{ \frac{2L(1 + L)W^2 \sec^2(W't') - (1 + L)^3W^2 - (L - 1)^3S^2}{(1 - L)^2} + \frac{2LS^2 \sec^3h(Sr')}{(L + 1)^2} \right\}, \quad (3.2)
$$

$$
\rho = \frac{1}{8\pi} \left[ \frac{-S^2(1 + L + L^2 + L\sec^2(Sr'))}{(L + 1)^2} + \frac{ LW^2\tan^2(W't')}{(L - 1)^2} \right]. \quad (3.3)
$$

We would like to mention here that Eqs. (2.29), (2.30) and (3.1)-(3.3) satisfy all the field equations with the restriction on constants given by Eq. (2.27). The proper unit length of the cylinder for the new metric is given by

$$l = 2\pi\tilde{B}\tilde{C} \equiv 2\pi\cosh(Sr')\cos(W't') \quad (3.4)$$

and the longitudinal length in this case is

$$\tilde{l} = \tilde{B}\tilde{C} \equiv \cosh(Sr')\cos(W't'). \quad (3.5)$$

The rate of change of longitudinal length is

$$\dot{l} = -W\cosh(Sr')\sin(W't'), \quad (3.6)$$

where negative sign shows that motion is directed inward. Thus such motion represents gravitational collapse of the charged perfect fluid distributed per unit length of the cylinder.

In order to analyze the nature of singularity of the solution, we use the curvature invariants. Many scalars can be constructed from the Riemann tensor but symmetry assumption can be used to find only a finite number of independent scalars. Some of these are

$$R_1 = R = g^{ab}R_{ab}, \quad R_2 = R_{ab}R^{ab}, \quad R_3 = R_{abcd}R^{abcd}, \quad R_4 = R_{cd}^{\ ab}R_{ab}.$$

Here, we give the analysis for the first invariant commonly known as the Ricci scalar. For the metric (2.32), it is given by

$$R = \frac{2}{l} (\ddot{B}\dot{C} - \dot{B}\ddot{C} - \ddot{B}'\dot{C}' - \dot{B}'\ddot{C}' + \dot{B}\dot{C}' - \dot{B}'\dot{C}''), \quad (3.7)$$
where $\tilde{l}$ is given by Eq. (3.5). We see that Ricci scalar as well as all the other curvature invariants and physical parameters of the solution are finite for $r' \to 0$. Thus $r' = 0$ is the conical singularity of the metric (2.32).

Now we analyze the values of the constants for which the solution is physical. In this solution, $L$ and $M$ are non-zero separation constants for the non-trivial solution while the rest are integration constants that are removed by applying the transformations to Eq. (2.31) and by evaluating the physical parameters from the field equations. From Eq. (2.27), it is clear that the constants $W$ and $S$ cannot be chosen arbitrarily. These are non-zero because $M \neq 0$ for non-trivial solution. Further, for $W$ and $S$ to be real, there are following four possible solutions:

1. $L < -1$, $M > 0$; 2. $L > -1$, $M < 0$;
3. $L > 1$, $M > 0$; 4. $L < 1$, $M < 0$.

Keeping in mind these restrictions on the constants, we find that the cases 1 and 2 lead to non-physical solutions (i.e., negative energy density for the arbitrary choice of coordinates). In the case 3, for $0 < M \leq 0.5$ and $1 < L \leq 1.9$, there exists a physical solution which represents the gravitational collapse. The graphs 1-4 in this case indicate that all the physical quantities become homogeneous. Thus the geodesic model with charged perfect fluid distributed per unit length of the cylinder is free of initial inhomogeneities.

It is interesting to mention here that pressure remains function of time only for the geodesic model that is analogous to the spherical case. In the case 4 for $0.63 \leq L \leq 0.95$ and $-1 < M \leq -0.10$, all quantities except pressure behave like the case 3, in this case pressure is negative indicating a dark energy solution. As long as the realistic energy condition $\rho + 3p > 0$ holds, the gravity remains attractive. However, the violation of this condition i.e., $\rho + 3p < 0$ due to negative pressure, leads to the repulsive gravitational effects. Thus in the relativistic physics, negative pressure acting as a repulsive gravity plays the role of preventing the gravitational collapse. We are interested to study the gravitational collapse which is the consequence of attractive gravity, so the case 4 is not interesting here. Thus the only interesting case is the case 3.

Now we proceed to discuss the energy conditions for case 3 which are given by the following relations

1. Weak energy condition:

$$\rho \geq 0, \quad \rho + p \geq 0, \quad (3.8)$$

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2. Dominant energy condition:
\[ \rho + p \geq 0, \quad \rho - p \geq 0, \quad (3.9) \]

3. Strong energy condition:
\[ \rho + 3p \geq 0. \quad (3.10) \]

For the pressure and energy density given by Eqs. (3.1) and (3.3) respectively, it follows that
\[
\begin{align*}
\rho + p &= \frac{1}{16\pi} \left[ \frac{S^2}{1 + L} \{1 - 2(1 + L + L^2 + L\sec^2 h(Sr')) \} ight. \\
&\left. \quad + \frac{W^2}{L - 1} \left( \frac{1}{2L - 1} - 2L\tan^2 Wt' \right) \right], \\
\rho - p &= \frac{1}{16\pi} \left[ -\frac{S^2}{1 + L} \{1 + 2(1 + L + L^2 + L\sec^2 h(Sr')) \} \\
&\quad + \frac{W^2}{L - 1} \left( 6L\tan^2 Wt' - \frac{1}{2L - 1} \right) \right], \\
\rho + 3p &= \frac{1}{16\pi} \left[ \frac{S^2}{1 + L} \{3 - 2(1 + L + L^2 + L\sec^2 h(Sr')) \} \\
&\quad + \frac{W^2}{L - 1} \left( \frac{1}{2L - 1} - 10L\tan^2 Wt' \right) \right].
\end{align*} \quad (3.11-3.13)
\]

Notice that all these equations satisfy Eqs. (3.8)-(3.10) for \( 0 < M \leq 0.5, \ 1 < L \leq 1.9, \ 0 \leq t' \leq 1 \) and \( 0 < r' \).

The rate of change of longitudinal length in Figure 1 shows that the longitudinal length is a decreasing function of time, thus the resulting solution represents the gravitational collapse. The collapse starts at some finite time and ends at \( t' = 1 \), where longitudinal length of the cylinder reduces to zero. Further, energy density is an increasing function of time shown in Figure 2. This is the strong argument for a model to collapse. The pressure in the interior of cylinder starts decreasing as shown in Figure 3. This causes to initiate the gravitational collapse, more matter is concentrated in the small volume, hence density goes on increasing.

It is to be noted that decrease in the proper unit length of the cylinder, increases the interaction between the electric charges and a strong electromagnetic tension inside the cylinder is created. This is an increasing function of time as shown in Figure 4. The coupled action of electromagnetic and gravitational forces play a dominant role to reduce longitudinal length of cylinder to zero.
Figure 1: Decrease in longitudinal length with the passage of time for $0 < M \leq 0.5$ and $1 < L \leq 1.9$ (Color online).

Figure 2: Increase of density with the passage of time for $0 < M \leq 0.5$ and $1 < L \leq 1.9$ (Color online).

Figure 3: Decrease in pressure with the passage of time for $0 < M \leq 0.5$ and $1 < L \leq 1.9$. 

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Figure 4: Increase of electric intensity with the passage of time for $0 < M \leq 0.5$ and $1 < L \leq 1.9$ (Color online).

The nature of the collapse can be seen as follows: When latitudinal and vertical lengths of the cylinder reduce to zero, there is a complete collapse. From the metric (2.32), we have $g_{\theta\theta} = \tilde{B}^2$, $g_{zz} = \tilde{C}^2$. Since singularity analysis implies that the Ricci scalar diverges at a point where the longitudinal lengths $\tilde{l} = \tilde{B}\tilde{C} = 0$. Thus when the longitudinal length as well as the latitudinal and vertical lengths reduce to zero, we obtain a conical singularity at $r' = 0$.

The conical singularity has effects on the gravitational collapse of non-zero mass objects. The conical singularity belongs to the class of line singularities which are gravitationally weaker than point singularities and stronger than the plane singularities. The tidal forces for point and line singularities are so strong that these can crush an object of finite size and non-zero mass to zero volume. However, it was pointed out by Nakao et al. \cite{24} that only conical singularity is the exceptional line singularity which does not crush an object to zero volume that collapses onto it. Hence, it is concluded that massive objects of finite size are collapsed on it without crushing to zero volume. The reason of non-crushing to zero volume does not imply that tidal forces are weak but it may be due to the geometric structure of the conical singularity.

4 Matching Conditions

Following,\cite{21, 25} we proceed to cut the spacetime and match it with another spacetime, which represents the exterior region of the collapsing cylinder. We
match the charged perfect fluid solution with the electro-vacuum solution. For this purpose, we consider the Darmois junction conditions, which require that the first and second fundamental forms (that are the line elements and the extrinsic curvature respectively) must be continuous over the boundary surface \( \Sigma \).

We assume that the 3D timelike boundary surface \( \Sigma \) splits the two 4D cylindrically symmetric spacetimes \( V^+ \) and \( V^- \). The metric which describes the internal region \( V^- \) is the charged perfect fluid solution given by Eq.(2.32). For the representation of the exterior region \( V^+ \), a charged cylindrically symmetric electro-vacuum solution is taken as

\[
d s_+^2 = H dT^2 - \frac{1}{H} dR^2 - R^2 (d\theta^2 + dz^2),
\]

where \( H(R) = \frac{2a^2}{R^2} - \frac{4M}{R} \), \( M \) and \( Q \) are the mass and charge per unit length of the cylinder, respectively. This choice of the exterior solution in \( V^+ \) region is compatible with the charged perfect fluid solution in the interior region \( V^- \) for their smooth matching over the boundary surface \( \Sigma \).

Now the boundary surface \( \Sigma \) in terms of interior and exterior coordinates can be described by the following equations

\[
\begin{align*}
 f_-(r', t') &= r' - r'_{\Sigma} = 0, \quad (4.2) \\
 f_+(R, T) &= R - R_{\Sigma}(T) = 0, \quad (4.3)
\end{align*}
\]

where \( r'_{\Sigma} \) is a constant. Using these equations, the interior and exterior metrics on \( \Sigma \) take the following form

\[
\begin{align*}
 (ds^2)_\Sigma = dt'^2 - \tilde{B}^2 d\theta'^2 - \tilde{C}^2 dz'^2, \\
 (ds^2_\Sigma) = [H(R_{\Sigma}) - \frac{1}{H(R_{\Sigma})} (\frac{dR_{\Sigma}}{dT})^2] dT^2 - R_{\Sigma}^2 (d\theta^2 + dz^2). \quad (4.5)
\end{align*}
\]

We assume \( g_{00} > 0 \) in Eq.(4.5) so that \( T \) is a timelike coordinate.

The continuity of the first fundamental form gives

\[
\begin{align*}
 (\tilde{B})_{\Sigma} &= R_{\Sigma}, \quad (\tilde{C})_{\Sigma} = R_{\Sigma}, \quad (4.6) \\
 [H(R_{\Sigma}) - \frac{1}{H(R_{\Sigma})} (\frac{dR_{\Sigma}}{dT})^2]^{\frac{1}{2}} dT &= (dt')_{\Sigma}. \quad (4.7)
\end{align*}
\]

The components of extrinsic curvature \( K^\pm_{ij} \) in terms of interior and exterior
coordinates are
\[
\begin{align*}
K_{00}^- &= 0, \quad K_{22}^- = K_{33}^- = (\tilde{B}\bar{B})_{\Sigma}, \\
K_{00}^+ &= (R^tT^\dagger - T^tR^\dagger - \frac{H}{2} \frac{dZ}{dR}T^\dagger t^3 + \frac{3}{2H} \frac{dH}{dR} T^t t^2)_{\Sigma}, \\
K_{22}^+ &= K_{33}^+ = (HRT^\dagger)_{\Sigma},
\end{align*}
\]
where dagger $\dagger$ and bar $\bar{}$ represent differentiation with respect to the new coordinates $t'$ and $r'$ respectively. The continuity of the extrinsic curvature components with Eqs. (4.6) and (4.7) leads to
\[
(\tilde{B}^\dagger)_{\Sigma} = 0,
\]
\[
M = \frac{Q^2}{2B} + \frac{\tilde{B}}{4}(\tilde{B}^t t^2 - \tilde{B}^2)_{\Sigma}.
\]
Here Eq. (4.9) implies that the boundary surface $\Sigma$ represents a cylinder with constant proper unit length which behaves as boundary of the interior charged perfect fluid distributed per unit length of the cylinder. Thus it connects the interior charged perfect fluid solution to the exterior electrovacuum solution. Using this equation, Eq. (4.10) reduces to $M = (\frac{Q^2}{2B})_{\Sigma}$. The parametric representation of this equation implies that the gravitational and Coulomb forces of the system balance each other on the boundary surface $\Sigma$. This consequence in the absence of pressure on the boundary, is equivalent to the result found by Thirukkanesh and Maharaj.\(^{16}\)

## 5 Discussion

In this paper, we find an analytical solution to the Einstein field equations coupled with the charged perfect fluid distributed per unit length of the cylinder. It has been found that all the physical parameters become homogenous but isotropic pressure is function of time only. This property (i.e., pressure being only time dependent) of cylindrically symmetric geodesic model is similar to the spherical case. Further, all the energy conditions are satisfied for a range of separation parameters and coordinates for which the solution is physical. All sorts of singularities of the solutions are discussed in detail. It is found that physical singularity of the solution occurs at a point where proper areal radius of the cylinder reduces to zero.
The physical and geometrical properties of the solution such as increase of density and decrease in proper unit length of the cylinder with respect to time represents the gravitational collapse. Also, the time interval for the collapse has been investigated. It is found that the electric field intensity of the system increases with time. This implies that as longitudinal length of the cylinder decreases, charges come close to each other and the interaction between the charges is increased.

In general, it is known that the presence of an electromagnetic field in a geometry causes to disturb its generic properties which may result in the form of oscillation of spacetime. Here, we discuss the absence of oscillation in geometry by showing the absence of outgoing gravitational waves.

Following Pereira and Wang, the component of the Weyl tensor for our solution, is $\Psi_0 = -C_{\mu\nu\lambda\sigma}L^{\mu}M^{\nu}L^{\lambda}M^{\sigma} = 0$, where $L^{\mu}$ and $M^{\nu}$ are null vectors. It means that there does not exist outgoing gravitational waves implying that oscillations are absent in the geometry of the spacetime. Thus there is no energy loss and hence no bouncing. Further, the absence of fluctuations in the energy density graph indicates the absence of bounce, oscillation and gravitational waves. The prediction that the fluctuation in energy density represents gravitational waves is recently made by Hussain et al.

We would like to remark here that in this case electromagnetic field is weak as compared to matter field which is obvious from Figures 2 and 4. This condition ($E^2 < \rho$) for the weak electromagnetic field was stated by Tasagas and used by us to preserve the generic properties of the Friedmann universe models. In a paper, $\Psi_0 \neq 0$ implies that there exist outgoing gravitational waves from the cylindrical symmetric radiating fluid gravitational collapse. It would be interesting to find solution of the field equation with radiating fluid and electromagnetic field in order to predict collective role of the electromagnetic field and radiation flux for the existence of the gravitational waves.

The Darmois criteria for the smooth matching of the cylindrically symmetric charged perfect fluid solution to an electro-vacuum charged static cylindrically symmetric solution leads to the following consequences: (a) boundary surface represents a cylinder with constant proper unit length and behaves as the boundary of the charged perfect fluid distributed per unit length of the cylinder; (b) the gravitational and Coulomb forces of the system balance each other on the boundary surface $\Sigma$.

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