The \( \omega \rho \pi \) coupling and the influence of heavier resonances

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Abstract. We determine the value of the \( \omega - \rho - \pi \) mesons coupling \( (g_{\omega \rho \pi}) \), in the context of the vector meson dominance model, from radiative decays, the \( \omega \rightarrow 3\pi \) decay width and the \( e^+e^- \rightarrow 3\pi \) cross section. For the last two observables we consider the effect of either a heavier resonance \( (\rho'(1450)) \) or a contact term. A weighted average of the results from the set of observables yields \( g_{\omega \rho \pi} = 13.9 \pm 0.1 \text{ GeV}^{-1} \) in absence of those contributions, and \( g_{\omega \rho \pi} = 11.9 \pm 0.2 \text{ GeV}^{-1} \) or \( g_{\omega \rho \pi} = 11.4 \pm 0.1 \text{ GeV}^{-1} \) when including the \( \rho' \) or contact term respectively. Improved measurements of these observables and the \( \rho'(1450) \) meson parameters are needed to give a definite answer on the pertinence of the inclusion of this last one in the considered processes.

1. Introduction

The high precision experiments have become a very powerful tool to test the standard model and to look for physics beyond it. For example, the measurement of the muon magnetic moment performed by the BNL-E821 [1] experiment is able to scrutiny contributions from different natures: The electromagnetic, weak and strong contributions simultaneously. While the contributions from the first two are known within the current experimental precision, a lack of knowledge of the strong interaction do not allow to draw definite conclusion on the observation of new physics [2]. The strong interaction regime involved here is not other but the hadronic one. The experimental data on hadronic production from electron-positron annihilation and tau decays [3, 4, 5] have increasing accuracy, thus providing a rich information that can lead to a deeper understanding of the strong interaction.

In particular, \( 2\pi 3\pi \) and \( 4\pi \) decays are the dominant decay modes. These are mainly driven by the low mass mesons interactions. Here we are interested in the strong interaction between the \( \omega, \rho \) and \( \pi \) mesons which can be encoded in a single parameter, denoted by \( g_{\omega \rho \pi} \).

The direct determination of the magnitude of \( g_{\omega \rho \pi} \) would require the observation of the \( \omega \) decaying into the others, which is not allowed, since there is not enough phase space for the three particles to be on the mass shell. Therefore, it must be extracted by indirect means. For example, from the above mentioned annihilation process and tau decays [3, 4, 5] have increasing accuracy, thus providing a rich information that can lead to a deeper understanding of the strong interaction.

The Chiral approach based on the low energy symmetries of Quantum Chromodynamics (QCD) and the so called vector meson dominance model (VMD) are able to describe them. Although they have different spirit, they both manage to resume the low energy manifestation of the strong interaction.
In this work we determine the value of $g_{\omega\rho\pi}$, in the context of VMD, from several processes: i) radiative decays of the form $V \rightarrow \pi\gamma$ ($V$: vector meson), ii) the $\omega \rightarrow 3\pi$ decay and iii) the $e^+e^- \rightarrow 3\pi$ cross section. These two last are known to be dominated by the $\rho\pi \rightarrow 3\pi$ channel [5, 6], here we also take into account the heavier resonance $\rho'$ (1450) and a $\omega \rightarrow 3\pi$ contact term. We find the effective value of $g_{\omega\rho\pi}$ which is consistent with all the considered observables.

The VMD Lagrangian including the $\rho$, $\pi$ and $\omega$ mesons can be set as:

$$
\mathcal{L} = g_{\rho\pi\pi}\epsilon_{abc}\rho_\mu^a\pi_\mu^b\pi_\mu^c + g_{\omega\rho\pi}\delta_{ab}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu\omega_\nu\partial_\lambda\rho_\sigma^a\pi^b + \frac{em_\gamma^2}{g_V}\rho_\mu A^\mu,
$$

where we have made explicit the notation regarding the couplings and the corresponding fields and, in the last term, $V$ refers in general to vector mesons and $A^\mu$ refers to the photon field. The couplings in this context are free parameters to be determined from experiment. Relations between them and those coming from low energy theorems can be drawn by building models which incorporate vector mesons into the chiral symmetric lagrangians [7, 8, 9, 10, 11]. In the following we will determine such couplings from the experimental data and draw some comparisons whenever possible.

### 2. Radiative decays

The $g_{\omega\rho\pi}$ coupling can be obtained from vector mesons radiative decays considering that the photon emission is mediated by a neutral vector meson [12]:

- $\omega \rightarrow \pi\rho \rightarrow \pi\gamma$ decay
- $\rho \rightarrow \pi(\omega, \phi) \rightarrow \pi\gamma$ decay
- $\pi^0 \rightarrow (\omega, \phi)\rho \rightarrow \gamma\gamma$ decay.

In Table 1, we exhibit the numerical values obtained for the above processes. Neglecting small correlations induced by the VMD vector mesons parameters, we can compute a weighted average, which yields $11.9 \pm 0.2$ with the error dominated by the most precise $\omega \rightarrow \pi^0\gamma$ channel. A standard average gives $g_{\omega\rho\pi} = 12.2 \pm 1.3$ GeV$^{-1}$, with errors added in quadratures and is dominated by the uncertainty in the $\rho^0 \rightarrow \pi^0\gamma$ decay width. In the following we will refer to the weighted average from radiative decays as VMDr.

As a way of comparison, let us get the expected value from the agreement between VMD and low energy theorems [13] for the $\pi \rightarrow \gamma\gamma$ decay [9]: Considering only the $\rho - \omega$ channel (the $\rho - \phi$ channel makes a small effect), it requires that

$$
|g_{\omega\rho\pi}| = |g_\rho g_\omega/8\pi^2f_\pi| = 11.5\text{ GeV}^{-1}.
$$

If, in addition, we impose universality ($g_\rho = g_{\rho\pi\pi}$) and SU(3) symmetry ($g_\rho = 3g_\omega$) then

| Decay | $|g_{\omega\rho\pi}|$ [GeV$^{-1}$] |
|-------|------------------|
| $\rho^- \rightarrow \pi^-\gamma$ | 11.3 $\pm$ 0.9 |
| $\rho^0 \rightarrow \pi^0\gamma$ | 13.1 $\pm$ 0.9 |
| $\omega \rightarrow \pi^0\gamma$ | 11.4 $\pm$ 0.2 |
| $\pi^0 \rightarrow \gamma\gamma$ | 12.8 $\pm$ 0.3 |
| Weighted Average | 11.9 $\pm$ 0.2 |

**Table 1.** Determination of $|g_{\omega\rho\pi}|$ from radiative decays.
\[ |g_{\omega\rho\pi}| = |3g_{\rho\pi\pi}^2/8\pi^2 f_\pi| = 14.4 \text{ GeV}^{-1}. \quad (3) \]

By using the KSFR relation \(g_{\rho\pi\pi} = m_\rho/(\sqrt{2}f_\pi)\) [14] it takes the form
\[ |g_{\omega\rho\pi}| = |3m_\rho^2/16\pi^2 f_\pi^3| = 14.2 \text{ GeV}^{-1}, \quad (4) \]
where \(f_\pi = 0.093 \text{ GeV}\). Therefore, at this stage, the value from radiative decays favors the one expected from the strict agreement between VMD and low energy theorems.

3. The \(\omega \to 3\pi\) decay

Let us consider the process \(\omega(\eta, q) \to \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\). Where the \(p_i\) are the corresponding 4-momenta and \(\eta\) is the \(\omega\) meson polarization. The contributions to the amplitude from the \(\rho\) channel (Fig. 1a-c) and the contact term (Fig. 1d) can be set as follow:
\[ M_D = i\epsilon_{\mu\alpha\beta\gamma} p_1^\mu p_2^\alpha p_3^\beta \mathcal{A}, \quad (5) \]
where \(\mathcal{A}\) is given by:
\[ \mathcal{A} = 6g_{3\pi} + 2g_{\omega\rho\pi}g_{\rho\pi\pi} \left(D^{-1}[\rho^0, p_1 + p_2] + D^{-1}[\rho^+, p_1 + p_3] + D^{-1}[\rho^-, p_2 + p_3]\right), \quad (6) \]
where, \(D[\rho, Q] = Q^2 - m_\rho^2 + im_\rho \Gamma_\rho\) and the factors of 6 and 2 in \(\mathcal{A}\) come from the cyclic permutations and momentum conservation used to bring the amplitude into the current form. The coupling \(g_{\rho\pi\pi} = 5.95 \pm 0.02\) is fixed by the decay width of \(\rho \to \pi\pi\), \(\Gamma_\rho = 149.1 \pm 0.8\) MeV. Using these values and \(g_{\omega\rho\pi}\) from radiative decays we can check that the prediction for the width, without taking into account the contact term \((g_{3\pi} = 0)\), is \(\Gamma_{\omega \to 3\pi}^0 = 4.4 \pm 0.2\) MeV, which is 58% of the experimental value \((\Gamma_{\omega \to 3\pi}^{exp} = 7.56 \pm 0.13\) MeV [5]). The correction by using an energy dependent width of the \(\rho\) is negligible compared with the error bars and the radiative corrections have been also estimated to be negligible [15].

In order to reach the 100% of the experimental width it may be necessary either to increase the coupling constant value up to 15.7 GeV\(^{-1}\) or to keep the value from radiative decays and include additional contributions like the contact term. Let us make a blind inclusion of this last in order to have an idea of its expected strength, needed to account for the observed decay width, this yields a quadratic equation for the coupling strength whose solutions are: \(g_{3\pi} = -62 \pm 7\) and \(+409 \pm 10\) GeV\(^{-3}\).
3.1. The contact contribution

The contact term is not really a free parameter, it is fixed from the axial anomaly [16]. Let us recover some previous results, obtained by linking the low energy theorems to the corresponding amplitudes in VMD, while respecting the KSFR relation. It has been found [17, 18] that when the vector channel is saturated by the $\rho$ meson, this contribute with

$$A^\rho = e^2 \frac{6}{g_\omega} g_{\omega\rho\pi} m_\rho^2 = \frac{3 e^2}{2 4\pi^2 f_\pi^3} = \frac{3}{2} A^{\text{anomaly}}$$

that is, three halves of the total amplitude as obtained from the Chiral anomaly [16]. Therefore, the contact term contributes with minus one half of the total amplitude. This was found by Rudaz [17] as a consistency requirement and by Cohen [18] as the one which satisfies axial Ward identities.

$$A^{\text{contact}} = e^2 \frac{6}{g_\omega} g_{3\pi} = -\frac{1}{2} A^{\text{anomaly}}$$

this fixes the corresponding coupling to be:

$$-g_{3\pi} = -\frac{g_{\rho\pi\pi}}{16\pi^2 f_\pi^3} = -47 \text{ GeV}^{-3}. \tag{9}$$

where we have made use of the relationship among the couplings as discussed in the previous section. The result is close to one of the above solutions obtained to account for the experimental decay width. Considering this value we get $\Gamma_{\omega\rightarrow3\pi} = 6.8 \pm 0.2 \text{ MeV}.$

3.2. The $\rho'$ channel

So far, we have considered the $\rho$ channel and a Wess-Zumino fixed contact term as the only ways the decay can go through. However, decays via radial excitations may be also important, provided the mass suppression factor is not extremely large compared to the energies involved in the process. The $\rho'(1450)$ meson ($m_{\rho'} = 1465 \text{ MeV}$ and $\Gamma_{\rho'} = 400 \pm 60 \text{ MeV}$), satisfies this condition for the $\omega$ decay regime. Let us explore the role of such contribution: The heavy mass of the $\rho'$ allows to simplify its propagator leading to identify the global coupling as an effective contact term (Fig.2), in full analogy to Eqn. (1):

$$|g'_{3\pi}| \approx \frac{g_{\omega\rho'\pi} g_{\rho'\pi\pi}}{m_{\rho'}^2}. \tag{10}$$

The couplings involved in the right hand side are not settled, neither in the theoretical side nor experimentally [4, 19]. In order to make an estimate of their magnitudes, we assume that

$$g_{\omega\rho'\pi}/g_{\rho'\pi\pi} = g_{\omega\rho\pi}/g_{\rho\pi\pi} = 2. \tag{11}$$

Studies on the value of $|g_{\omega\rho'\pi}|$ have found it to lay in the interval 10-18 GeV$^{-1}$ [19]. Under these assumptions we get $|g'_{3\pi}| \approx 46 \pm 23 \text{ GeV}^{-3}.$ We have evaluated the deviations from this value in Eqn. (10) due to momentum and width dependence of the propagators, which combined produce an increase of 6%. Thus, our estimate for the coupling is

$$|g'_{3\pi}| = 49 \pm 24 \text{ GeV}^{-3} \tag{12}$$

This is also close to the smallest value of the contact coupling obtained by requiring to get the 100% of the width, and to the result in Eqn. (9). In accordance, we choose its sign to be negative.
In Table 2, we have collected a set of values for the contact coupling computed in the literature from different approaches. [17] uses the VMD approach being consistent with low-energy theorems. [20] Extended the previous idea by including an infinite number of radial excitations. [7] works within a framework of a chiral effective lagrangian. [8] uses an extension to the chiral lagrangian adding spin-1 fields, and our results obtained from different approaches. It can be argued that given the different nature of these approaches a direct comparison between them is meaningless. However, we consider that it is interesting to quote their magnitudes as a way to exhibit that, besides the model dependence, there exist a regularity on these contributions.

To estimate the decay width we will not take the $\rho'$ contribution simultaneously with the contact term, since its inclusion in the current form breaks the consistency between low energy theorems and VMD, which was already achieved by considering only the $\rho$ meson [21].

4. The $e^+e^-\rightarrow \omega \rightarrow 3\pi$ cross section

Now, we explore the implications of the contact term or the $\rho'$ in the $e^+e^-\rightarrow 3\pi$ cross section. Following the same notation as in the previous section, we can write the amplitude for the $\omega$ channel as follows:

$$M = \frac{e^2 m_\omega^2}{g_\omega} \bar{v} \gamma^\mu u \epsilon_{\mu\alpha\beta\gamma} p_1^\alpha p_2^\beta p_3^\gamma A$$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Reference & $|g_{3\pi}|$ [GeV$^{-3}$] \\
\hline
Rudaz, Cohen [17, 18] & 47 \\
Dominguez [20] & 29 ± 3 \\
Kuraev [7] & 123 \\
Kaymakcalan [8] & 37 \\
This work from $\Gamma(\omega \rightarrow 3\pi)$ & 65 ± 7 \\
This work from $\rho'$ & 49 ± 24 \\
\hline
\end{tabular}
\caption{Determination of $|g_{3\pi}|$-like terms from several approaches. See text for details.}
\end{table}
Figure 3. $e^+e^- \rightarrow \omega \rightarrow 3\pi$ cross section. Experimental data [3] (circle symbols) and the prediction by VMD using $g_{\omega\rho\pi}$ from radiative decays (solid line), and including either the $\rho'$ term (decreased by 1 stdv., dashed line) or the contact term (dot-dashed line).

A fit to the CMD2 data considering only the $\rho$ channel, leaving $g_{\omega\rho\pi}$ as a free parameter leads to $|g_{\omega\rho\pi}| = 13.1 \pm 0.1$ GeV$^{-1}$.

5. Discussion
We have performed an analysis to obtain the $g_{\omega\rho\pi}$ coupling in the VMD approach, considering radiative decays, the $\omega \rightarrow 3\pi$ decay width and the $e^+e^- \rightarrow \omega \rightarrow 3\pi$ cross section. Our global result from all these observables are presented within two possible scenarios (See Table 3): i) Without contact nor $\rho'$ term: In this case we average (weighted) the value predicted from radiative processes, the value which reproduces the experimental decay width for $\omega \rightarrow 3\pi$ and the value for the coupling obtained to fit the experimental $e^+e^- \rightarrow \omega \rightarrow 3\pi$ cross section. ii) With either $\rho'$ or contact term: In this case we average the value predicted from radiative processes, the values from the decay width for $\omega \rightarrow 3\pi$ and the $e^+e^- \rightarrow \omega \rightarrow 3\pi$ cross section considering the existence of a contact term or the effect produced by the $\rho'$ meson.

There is no ambiguity on the $\rho'$ resonance contributions, besides the uncertainties associated to its corresponding mass, decay width and couplings. The inclusion of the contact term and the $\rho$ channel alone in the $\omega \rightarrow 3\pi$ decay is consistent with the low energy theorems while fulfilling the KSFR relation. At this stage, we have not considered the contact and $\rho'$ contributions simultaneously to avoid possible inconsistencies.

The value obtained for $g_{\omega\rho\pi}$ is sensitive to the inclusion of other contributions, in the context discussed above. Estimates, based on different approaches, yield similar values [17, 20, 22, 23], pointing out to the small model dependence of our results. Improved measurements of these observables and the $\rho'(1450)$ meson parameters are needed to settle the issues above mentioned.

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References
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Table 3. The $\omega \rho \pi$ coupling (GeV$^{-1}$) from different scenarios.