Nanoflow: Scalable Normalizing Flows with Sublinear Parameter Complexity

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Abstract

Normalizing flows (NFs) have become a prominent method for deep generative models that allow for an analytic probability density estimation and efficient synthesis. However, a flow-based network is considered to be inefficient in parameter complexity because of reduced expressiveness of bijective mapping, which renders the models prohibitively expensive in terms of parameters. We present an alternative of parameterization scheme, called Nanoflow, which uses a single neural density estimator to model multiple transformation stages. Hence, we propose an efficient parameter decomposition method and the concept of flow indication embedding, which are key missing components that enable density estimation from a single neural network. Experiments performed on audio and image models confirm that our method provides a new parameter-efficient solution for scalable NFs with significantly sublinear parameter complexity.

1 Introduction

Recently, flow-based models have become a prominent approach for density estimation and generative models. These models are based on normalizing flows (NFs) [24], wherein a deep invertible model is trained with an analytically estimated likelihood of data. The model learns a bijective mapping between the data and a known prior (typically isotropic Gaussian), and its reverse operation synthesizes realistic samples from the prior. Compared with the variational autoencoder [16] and generative adversarial network [8], NFs exhibit the distinct characteristic of an exact probability density estimation from a principled maximum likelihood training. When combined with non-autoregressive coupling methods [5,14], NFs become a powerful generative model that offer a significantly simplified training and efficient inference.

Since the introduction of the framework into neural networks, the current studies on flow-based models have been focused on improving the expressiveness of the bijective operation [1,6,9,14,21,22]. However, parameter complexity and memory efficiency are less emphasized by the research community. The lack of focus in maximizing the expressiveness under a specified amount of capacity of the neural network has become problematic recently when scaling up a flow-based model to real-world applications. A notable example is the waveform synthesis model [12,21,22]. Although the aforementioned studies have achieved audio generation faster than real-time (thereby removing the slow inference bottleneck of WaveNet [25]), they resulted in an increase in the number of parameters by an order of magnitude, which is prohibitively expensive in terms of memory. Hence, building a complex, scalable, and memory-efficient flow-based model remains challenging.

This motivates us a question, is it true that NFs do require a significantly larger network capacity to perform expressive bijections, or are we utilizing the representational power of the deep neural networks inefficiently? We argue that studies regarding NFs should consider the parameter complexity,
where the expressiveness of multiple flows is not necessarily accompanied by a linearly growing number of parameters.

In this study, we challenge the typical assumption in building flow-based models and aim to decouple the required number of parameters and the expressiveness of multiple bijective operations for flow-based models. We present NanoFlow, an alternative of parameterization scheme for NFs that operates on a single neural density estimator. Because the shared density estimator is applied to multiple stacks of flows, the parameter requirement is no longer proportional to the number of flows, and the memory footprint is reduced significantly. Consequently, NanoFlow can consistently improve its expressiveness by stacking flows without sacrificing parameter efficiency.

Our results show that using a conventional notion of weight sharing did not yield a good performance on flow-based models, which nullifies the potential benefits. To achieve the concept of a shared neural density estimator, we demonstrate several parameter-efficient solutions for increasing the flexibility of NanoFlow. We show that decomposing a deep hidden representation estimated by the shared neural network and the projected densities from the representation can significantly enhance the expressiveness of NanoFlow with the addition of a few parameters. Furthermore, we also demonstrate that conditioning the shared estimator with our flow indication embedding can remedy the modeling difficulties of multiple densities from a single estimator without dissatisfying any invertibility constraints.

Additionally, we also provide a deeper analysis of the condition under which our method yields the most benefits. Specifically, we assess the effectiveness of the single density estimator by varying the amount of autoregressive structural bias into the model. Our results demonstrate that our method performs best on bipartite flows, which provide an expanded narrative on our intuition regarding the performance gap between non-autoregressive and autoregressive models. In summary, our study is the first that focuses on a systemic assessment for enabling scalable NFs with an almost constant parameter complexity.

2 Background

NFs learn the bijective mapping between data and a known prior. The prior is typically constructed as an isotropic Gaussian, and the reverse of the bijective mapping can synthesize the data from the noise sampled from the prior. Formally, NFs learn the bijective function \( f(x) = z \) that transforms a complex data probability distribution \( P_X \) into a simple known prior \( P_Z \) with the same dimension. We can analytically compute the probability density of real data \( x \) using the change of variables formula:

\[
\log P_X(x) = \log P_Z(z) + \log |\det(\frac{\partial f(x)}{\partial x})| \tag{1}
\]

where \( |\det(\frac{\partial f(x)}{\partial x})| \) is a Jacobian determinant of the function \( f(x) = z \). To enhance the expressiveness of \( f \), NF models decompose the function into multiple flows as follows:

\[
f = f^K \circ f^{K-1} \circ ... \circ f^1(x) \tag{2}
\]

where \( K \) is the number of flows defined by the model. Using the notations \( x = z^0 \) and \( z = z^K \), each \( f^k(z^{k-1}) = z^k \) learns the intermediate densities between \( x \) and \( z \), and \( \log P_X \) can be re-expressed as follows:

\[
\log P_X(x) = \log P_Z(z) + \sum_{k=1}^{K} \log |\det(\frac{\partial f^k(z^{k-1})}{\partial z^{k-1}})| \tag{3}
\]

Because the determinant typically requires \( O(n^3) \) computing time (where \( n \) is the dimension of the data), NF models are designed to maintain a triangular Jacobian \([4,15]\). By maintaining a triangular Jacobian, the determinant becomes easy to compute, and the model becomes computationally tractable for both forward and inverse functions.

Our mathematical notation for the coupling transformation follows that of WaveFlow \([22]\). Although the study focused on waveform synthesis, it provides a unified view from bipartite to autoregressive flows, which subsumes a wide range of flow-based models.

Formally, for training data \( x \), assume that we split \( x \) into \( G \) groups as \( \{X_1,...,X_G\} \). The model is trained to learn the bijective mapping between \( X \) and a prior \( Z \) with the same dimension. This is
Figure 1: High-level overview of NanoFlow. (a) Conventional NFs employ separate neural networks as a density estimator for each flow. (b) NanoFlow-naive shares an entire part of the neural network for density estimation with multiple flow steps. (c) NanoFlow decomposes the estimator into two parts, one for the deep shared latent space representation augmented by flow indication embedding, and separate projection layers employed to each flows.

achieved by applying an affine transformation $f : X \rightarrow Z$ which models a sequential dependency between the grouped data as follows:

$$Z_i = \sigma_i(X_{<i}; \theta) \cdot X_i + \mu_i(X_{<i}; \theta), \quad i = 1, ..., G \quad (4)$$

where $X_{<i}$ refers to all the partitions of the data before the $i$-th group, $X_i; \sigma$ and $\mu$ are the scale and shift variables estimated by the neural networks, respectively. From the sampled noise $Z$, the inverse of the affine transformation $f^{-1} : Z \rightarrow X$ generates $X$ sequentially as follows:

$$X_i = \frac{Z_i - \mu_i(X_{<i}; \theta)}{\sigma_i(X_{<i}; \theta)}, \quad i = 1, 2, ..., G \quad (5)$$

The model becomes a purely autoregressive flow when $G = \dim(x)$. Conversely, the equations theoretically represent bipartite flows when $G = 2$. Increasing the number of groups introduces a higher amount of autoregressive structural bias into the model, at a cost of $O(G)$ inference latency.

As mentioned above, the entire bijective function $f : X \rightarrow Z$ is decomposed into $K$ flows as $f = f^K \circ f^{K-1} \circ ... \circ f^1(X)$, where we use the notation $X = Z^0$ and $Z = Z^K$. Each $f^k : Z^{k-1} \rightarrow Z^k$ is parameterized by separate neural networks $\theta^k$, whereas each $\theta^k$ estimates the intermediate density of $Z^k$ by computing $\sigma^k$ and $\mu^k$ for the flow operation. For clarity, we consider the notation of $X$ as the input and $Z$ as the output for $f^k$. We re-write $f^k$ for completeness as follows:

$$Z_i = \sigma_i(X_{<i}; \theta^k) \cdot X_i + \mu_i(X_{<i}; \theta^k) \quad (6)$$

3 NanoFlow

In this section, we present NanoFlow, a new alternative parameterization scheme for a flow-based model (Figure 1). The main goal of NanoFlow is to decouple the expressiveness of the bijections and the parameter efficiency of density estimation from neural networks. We first describe a core change in the design of the neural architecture by decomposing the parameters for neural density estimation and sharing parameters across flows.

3.1 Parameter sharing and decomposition

We reformulated $f^k_{\mu, \theta}$ using a single shared neural network $f_{\mu, \theta}$, parameterized by $\theta$. Based on this framework, all $\sigma^k$ and $\mu^k$ for each flow were estimated by the shared $f_{\mu, \theta}$. This formulation can reduce the number of parameters by a fraction of the number of flows by $\frac{1}{K}$, and we call this variant the NanoFlow-naive. However, as our experimental results suggest, this aggressive re-use
Table 1: Comparison of parameterization scheme of $f^k$ between methods for bijection. $K$ is the total number of flows defined by the model, and $\cdot | \cdot$ is the number of parameters of the neural network with the designated letters.

| Method                  | $f^k : X_i \rightarrow Z_i = \sigma^k_i \cdot X_i + \mu^k_i, i = 1, \ldots, G$ | Parameters            |
|-------------------------|--------------------------------------------------------------------------------|-----------------------|
| WaveFlow(baseline)      | $\mu^k_i, \sigma^k_i = f^k_{\mu,\sigma}(X < x_i; \theta)$                     | $\sum_{k=1}^{K} |\theta^k|$ |
| NanoFlow-naive          | $\mu^k_i, \sigma^k_i = f_{\mu,\sigma}(X < x_i; \theta)$                       | $|\theta|$            |
| NanoFlow-decomp         | $\mu^k_i, \sigma^k_i = f^k_{\mu,\sigma}(g(X < x_i; \hat{\theta}) ; \epsilon^k)$ | $|\hat{\theta}| + \sum_{k=1}^{K} |\epsilon^k|$ |
| NanoFlow(proposed)      | $\mu^k_i, \sigma^k_i = f^k_{\mu,\sigma}(g^k(X < x_i; \hat{\theta}; \epsilon^k) ; \epsilon^k)$ | $|\hat{\theta}| + \sum_{k=1}^{K} (|\epsilon^k| + |\epsilon^k|)$ |

of parameters is unsuitable for modeling multiple densities that suffer from severe degradation in performance, as it completely nullifies the potential benefit of the $O(1)$ memory footprint.

Based on these observations, we propose to relax the constraint of the shared estimator by decomposing the shared model into a network that computes a hidden representation and a projection layer that estimates the densities. The function is decomposed as $f^k_{\mu,\sigma}$ into $f^k_{\mu,\sigma} \circ g$, where $g$ is the shared estimator parameterized by $\hat{\theta}$, excluding the projection layer, i.e. each $f^k_{\mu,\sigma}$ has separate parameters for the projected intermediate density by computing $\sigma^k$ and $\mu^k$.

Assuming that $\hat{\theta}$ has sufficient capacity for density estimation, the projection layer can be as shallow as a $1 \times 1$ convolution; hence, the number of parameters is negligible in comparison with $\hat{\theta}$. Using this decomposition, we can construct the NanoFlow with an arbitrary number of flows. Interestingly, this alternative scheme was pivotal for achieving the competitive performance of NanoFlow. We observed that the likelihood of the data continuously increased as we stacked additional flows without sacrificing the efficiency of weight sharing. Hence, we can re-write $f^k$ as follows:

$$Z_i = \sigma_i(X < x_i; \hat{\theta}, \epsilon^k) \cdot X_i + \mu_i(X < x_i; \hat{\theta}, \epsilon^k) \quad (7)$$

where $\epsilon^k$ is the separate projection layer assigned to each flow.

### 3.2 Flow indication embedding

Even when the parameter decomposition described above is used, the shared estimator $\hat{\theta}$ must learn multiple intermediate densities of bijective transformations without context. Hence, we introduce a key missing module, which we term flow indication embedding, to enable the shared model to simultaneously learn multiple bijective transformations. Because the flow-based model is based on the bijective function, the embedding must be available simultaneously to learn multiple bijective transformations. Because the flow-based model is based on the bijective function, the embedding must be available simultaneously to learn multiple bijective transformations.

For each $f^k$, we define an embedding vector $\epsilon^k \in \mathbb{R}^D$, where $D$ is the dimension of the embedding. Subsequently, we feed the embedding to the shared model $g(\cdot; \hat{\theta})$ as an additional context. From the embedding $\epsilon^k$, we can further guide $g(\cdot; \hat{\theta})$ to learn multiple intermediate densities with minimal addition of parameters, transforming it into $g^k(\cdot; \hat{\theta}, \epsilon^k)$. Because the order of flow operations is pre-defined, we can use $\epsilon^k$ by feeding it to the shared parameter in the reverse order during inference, i.e., our embedding does not dissatisfy any invertibility constraints.

The optimal injection of the embedding into $\hat{\theta}$ can depend on the neural architecture. We investigated three candidates: 1. concatenative embedding, where we augment the input with the embedding vector at the start of each flow; 2. additive bias, where for each layer inside $\hat{\theta}$, $\epsilon^k$ provides a channel-wise bias projected from additionally defined $1 \times 1$ convolutional layers; 3. multiplicative gating, where we employ independent channel-wise scalars inside $\hat{\theta}$ for each flow that controls the propagation of the convolutional feature map according to the specified flow steps. It is noteworthy that the aforementioned methods involve a negligible number of additional parameters, do not dissatisfy the invertibility, and impose a minimal effect on the inference latency.

Table 1 summarizes the parameterization scheme of $f^k$ and its complexity. The parameter efficiency of NanoFlow is due to employing a single neural density estimator, $\theta$, for multiple flow operations.
Method | Channels | Params (M) | LL | MOS | kHz
--- | --- | --- | --- | --- | ---
WaveFlow (Re-impl) | 128 | 22.468 | 5.2059 | 4.11 ± 0.08 | 347
WaveFlow (Re-impl) | 64 | 5.958 | 5.1357 | 3.52 ± 0.09 | 828
NanoFlow-naive | 128 | 2.798 | 5.1247 | 3.23 ± 0.09 | 376
NanoFlow | 128 | 2.818 | 5.1586 | 3.63 ± 0.09 | 362
NanoFlow (K=16) | 128 | 2.845 | 5.1873 | 3.82 ± 0.08 | 186

Table 2: Model comparison. We report the number of model parameters in millions (M), a log-likelihood (LL) on the test set, a subjective five-scale mean opinion score (MOS) on naturalness with 95% confidence interval, and a synthesis speed using a single Nvidia V100 GPU with half-precision arithmetic. MOS on ground-truth audio is 4.58 ± 0.06.

From NanoFlow-naive which incorporates a conventional weight sharing, NanoFlow-decomp relaxes the constraint of intermediate density estimations by employing separate $\epsilon^k$ for each flow. The final proposed model, NanoFlow, further increases the parameter efficiency of $\hat{\theta}$ by incorporating the flow indication embedding $e^k$. We emphasize that $\hat{\theta}$ embodies the majority of parameters from the model.

4 Experiments

In this section, we present a systemic assessment of the effectiveness of NanoFlow. We first present the experimental results from an audio generative model with WaveFlow [22] as the baseline architecture, combined with an extensive ablation study. Next, we provide a likelihood ratio analysis of NanoFlow by varying the amount of autoregressive structural bias into both models, which evaluates under a condition where NanoFlow yields the most benefit. Finally, we investigate the generalizability of our methods by performing density modeling on the image domain, with Glow [14] as the reference model.

4.1 Audio generation results

For the performance evaluation of waveform generation, we used the LJ speech dataset [11], which is a 24-h single-speaker speech dataset containing 13,100 audio clips. We used the first 10% of the audio clips as the test set and the remaining 90% as the training set. We used the audio preprocessing and mel-spectrogram construction pipeline provided by the official WaveGlow implementation [23]. Specifically, we used an 80-band log-scale mel-spectrogram condition with an FFT size of 1,024, a hop size of 256, and a window size of 1,024. We used a maximum frequency of 8,000Hz for the STFT without audio volume normalization, and we set the noise distribution to $Z_i \sim N(0, 1)$, which is the default setting from the open-source WaveGlow.

We used the default architecture described in [22] with $G = 16$ for WaveFlow and NanoFlow. We constructed the models with eight flows unless otherwise specified, and we used the permutation strategy of reversing the order of the group dimensions per flow for both models. Our selection of flow indication embedding is a combination of additive bias and multiplicative gating, as WaveNet-based [25] architecture features a natural method of utilizing additive bias as global conditioning augmented by a gated residual path [26]. We used $D = 512$ for $\epsilon^k \in \mathbb{R}^D$ in the default eight-flows model and $D = 1024$ in the 16-flows variant.

We trained all models for 1.2 M iterations with a batch size of eight and an audio clip size of 16,000, using an Nvidia V100 GPU. We used the Adam optimizer [13] with an initial learning rate of $10^{-3}$, and we annealed the learning rate by half for every 200 K iterations. For the evaluation, we applied checkpoint averaging over 10 checkpoints with 5 K iteration intervals. We sampled the audio at a temperature of 1.0.

Table 2 shows an objective performance measure of log-likelihood (LL) on the test set, as well as a subjective and relative audio quality evaluation with a five-scale mean opinion score (MOS) on naturalness using the Amazon Mechanical Turk. Furthermore, we also provide the audio synthesis speed in kilohertz using a single Nvidia V100 GPU with half-precision arithmetic.
Table 3: Log-likelihood (LL) ratio results with varying amount of autoregressive structural bias on the number of groups. Lower value means higher similarity in probability density modeling performance between two models.

The results show that our method can synthesize waveforms with a slight quality degradation against the baseline while using only approximately $1/8$ of the parameters. However, the NanoFlow-naive failed to perform competitively even against a 64-channel variant of WaveFlow. This suggests that for flow-based models, a strict constraint of $O(1)$ memory requirement severely degenerates the modeling capability. On the contrary, NanoFlow provided significantly enhanced expressiveness, with a negligible number of additional parameters from the decomposition technique with flow indication embedding. By stacking double the amount of flows, we further verified that the enhanced expressiveness of the flows was no longer proportional to the capacity of the deep generative model. Consistent with the results from a previous work [22], we discovered that the subjective MOS scores aligned well with the objective likelihood scores.

4.2 Likelihood ratio analysis with autoregressive structural bias

Our reference model, WaveFlow [22], provided a unified view of the expressiveness of flow-based models by incorporating a fixed amount of autoregressive structural bias into the architecture. The model provides a hybrid method where the autoregressive bias is proportional to the number of group dimensions. In this section, we provide an expanded narrative on the performance gap between the non-autoregressive and autoregressive flows by adjusting the amount of bias for both WaveFlow and NanoFlow. We trained each model with 64 channels for 500 K iterations with a batch size of two for varying degrees of the group dimension. We used $D = 128$ for the NanoFlow embedding.

Table 3 quantitatively shows the expressiveness of the autoregressive bias. As we enforce a higher amount of the autoregressive structure into the model, we can achieve a more expressive model under the same network capacity. However, this is at the expense of sequential inference, which has been reported in previous studies [14, 19, 25].

In addition, we provide the log-likelihood ratio between WaveFlow and NanoFlow, where we measure the gap in modeling capability by introducing a shared neural density estimator. Most importantly, we observed a nearly monotonic decrease in the performance gap of NanoFlow as we decreased the number of groups. This further provides an insight into our effectiveness in utilizing the capacity of the deep generative network: if we impose a lesser amount of the explicit dependency between partitions of data, we can extract a deep shared latent representation that is easier to manipulate by our flow indication embedding. In other words, we can expect a wide range of flow-based models with bipartite coupling to benefit significantly from the parameterization scheme of NanoFlow.

4.3 Image density modeling results

To demonstrate that our method is applicable to any configuration of NF and data domains, we assessed the effectiveness of NanoFlow’s parameterization scheme to Glow [14]. We used the training configurations of an open-source implementation identically as described in [14]. We trained the models on CIFAR10 dataset for 1000 epochs. Although this did not provide a fully converged model, it was sufficient to evaluate the relative performance difference, as reported by [9]. We used 256 channels and a batch size of 64 for all configurations for an extensive ablation study under a fixed computational budget.

Because NanoFlow is designed to leverage deeper density estimations from a shared model, we increased the number of convolutional layers to six, and modified the kernel size to $3 \times 3$ for all layers. We changed the kernel size of the separate density projection layers to $1 \times 1$ to maintain a nearly constant memory footprint of NanoFlow. We refer to the model with this modified architecture without
the application of our method as Glow-large. This model serves as an upper bound on modeling performances but the parameter complexity is increased. We trained the original model with the exact topology from [14] and with Glow-large to completely assess the capability of NanoFlow.

Because Glow uses a multi-scale architecture [5], NanoFlow is applied by sharing the estimator separately for each scale. Regarding the selection of flow indication embedding, we discovered that the concatenative method performed better than the additive bias, likely because of the better compatibility with the activation normalization technique [14] which stabilizes the training of flow-based models with extreme depth. For $e^k \in \mathbb{R}^D$, we used $D = 64$ for the default 32 flows per scale, and $D = 192$ for a scaled-up model with 48 flows per scale, respectively. $e^k$ is reshaped to match the input for each flow as $e^k \in \mathbb{R}^{C \times H \times W}$, where $C = \frac{D}{H \times W}$. Furthermore, multiplicative gating together with the concatenative embedding performed well in this configuration.

As in Figure 2 and Table 4, we discovered that NanoFlow-naive failed to perform competitively, even with the increased capacity of the shared estimator. Additionally, the model could not compete with the baseline model using the reference network topology. This suggests that even if a more powerful neural network is introduced, a critical bottleneck exists when modeling multiple flows from a single model when our method is not applied. On the contrary, NanoFlow began to outperform the baseline architecture by applying only the decomposition technique, suggesting that the last projection layer for the transformation is pivotal for the expressiveness of flow-based models.

The performance was further improved using the flow indication embedding. Surprisingly, NanoFlow with the default number of flows (32 steps per scale) started to outperform Glow-large which has over 28 times more parameters. The results show that NanoFlow can achieve competitive performance with significantly fewer parameters, even without the additional flow depth, suggesting that the deep shared latent space representation, augmented by the methods proposed by NanoFlow, is easier to train and more scalable than employing separate estimators where each neural network should learn the intermediate probability densities from scratch.

When we scaled up the model to 48 flows per scale, NanoFlow began to significantly outperform Glow-large with approximately 27x fewer parameters. Overall, the density estimation results in bits per dimension were consistent with the audio generation results. The effectiveness of our method was further highlighted in this setup with bipartite coupling, which further confirms our findings from the likelihood ratio analysis in the preceding section. See Appendix A for sampled images from the models.

### 5 Related Work

#### 5.1 Improving coupling transformations

Since the introduction of the NFs into neural networks [4, 24], most studies have focused on composing a flexible bijection for better expressiveness [5, 6, 9, 10, 14, 18]. Building a more complex bijection can also achieve better memory efficiency by attaining the desired level of complexity under fewer
flow operations. Our study provides an orthogonal perspective on this topic with a specific focus on the parameterization of a scalable NF under a specified network capacity, where we systemically assess the feasibility of employing a single shared neural density estimator for multiple flow steps. Because our parameterization scheme is agnostic to any setup of flow-based models and coupling operator, we can apply any off-shelf bijections into our framework, together with improved methods for training NFs [9].

5.2 Parameter sharing

The concept of parameter sharing has been studied previously in various domains, from the core foundation of the design principle of CNNs and RNNs to parallel sequence models, such as the Transformer [2, 17, 21]. The most notable example is [17] in the natural language processing domain, which demonstrated a significantly reduced memory footprint of BERT [3] using a cross-layer parameter sharing of the self-attention block. We investigated the effectiveness of the weight sharing concept on different granularities for flow-based models. We applied parameter sharing on a model-level, where a shared neural density estimator was applied to multiple stages of bijective transformation that performed bijective operations. Contrary to [17], our study revealed the following findings: In NFs, sharing an entire block failed to model the probability density competitively, whereas minimal relaxation from the decomposition was critical to the performance.

6 Discussion

In this study, we presented an extensive and systemic analysis of the feasibility and potential benefits of using a single shared neural density estimator for multiple flow operations. Based on the analysis, we developed a novel parameterization scheme, i.e., NanoFlow, which enabled scalable NFs with a nearly constant memory complexity and competitive performances as both generative model and density estimation model, owing to the compact network capacity. This enables direct control over the tradeoff between expressiveness and inference latency, which is beneficial in domains where compact parameterization is desired. One can explicitly design the performance target using NanoFlow as a building block depending on the task requirements, which can be useful for practitioners who incorporate NFs to applications.

The decomposed view on building flow-based models with NanoFlow suggests that two directions can be endeavored in future research: composing more expressive bijections, which has been the primary focus in existing literature, and building an optimized neural density estimator that can potentially provide a more adaptive computation path leveraged by flow indication embedding. Furthermore, these proposed future studies can be expanded from [9], which investigated better neural architecture designs for building flow-based models using self-attention for the estimator. Combined with increasing evidence in other research domains applying similar architecture [17], we expect the self-attention-based estimator to provide more expressive density estimations [7, 20], where the attention mechanism could be directly augmented from flow indication embedding. We leave this research direction as future work.

We demonstrated that the flow indication embedding from a look-up table enables multiple densities to be estimated from a single neural network. However, the look-up table approach we used assumes a factorized distribution over the embedding space, where the method does not consider an explicit dependency between the embedding. Whether we can construct a better embedding scheme remains an open question: for example, one can consider imposing an autoregressive structure to the embedding, such as employing smaller-scale RNNs using hidden states from the embedding as the augmented ones.

In summary, NanoFlow, which is a bijection-agnostic and generalized solution that achieves significant savings in network capacity, provides an alternative method for parameterizing NFs. Extensive experiments on real-world data domains have provided deep insights into the relationship between the capacity of deep generative models and the expressiveness of flow operations, along with possible future research directions. We hope that the modular scheme of NanoFlow will motivate researchers to further develop flexible and scalable flow-based models.
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Appendix

A Samples generated from image models

Figure 3: Unconditional samples generated from image models in Section 4.3 trained on CIFAR10 with 1000 epochs. The temperature is set to 1.0. Models with lower bpd tend to generate sharper and detailed textures, which is consistent with the existing literature.