Study and simulation with VHDL-AMS of the electrical impedance of a piezoelectric ultrasonic transducer

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ABSTRACT

Ultrasonic transducers are a key element that governs the performances of both generating and receiving ultrasound in an ultrasonic measurement system. Electrical impedance is a parameter sensitive to the environment of the transducer; it contains information about the transducer but also on the medium in which it is immersed. Several practical applications exploit this property. For this study, the model is implemented with the VHDL-AMS behavioral language. The simulations approaches presented in this work are based on the electrical Redwood model and its parameters are deduced from the transducer electroacoustic characteristics.

Keywords: Impedance, Piezoelectric, Transducer, Ultrasonic, VHDL-AMS

1. INTRODUCTION

Ultrasound systems are widely used. They find many applications in engineering, medicine, biology, and other areas. The one indispensable part in these systems is the transducer. These will use the properties of magnetostrictive or piezoelectric materials to convert electrical energy into ultrasonic mechanical energy [1]. Piezoelectric materials have the advantage over other systems of having good performance and being available in very diverse geometries. The electromechanical interaction of piezoelectric transducer, represented by electrical equivalent circuits, was first introduced by Mason [2]. He proposed an exact equivalent circuit that separated the piezoelectric material into an electrical port and two acoustical ports through the use of an ideal electromechanical transformer. The problems with the model are that it required a negative capacitance at the electrical port. Redwood [3] improved this electromechanical model by incorporating a transmission line, making possible to extract useful information on the temporal response of the piezoelectric component.

The Electrical impedance is a parameter sensitive to the environment of the transducer. Several practical applications exploit this property[4, 5]. The measurement of the impedance makes it possible, for example, to detect the physical or structural modifications of the medium due in particular to damage. This approach is used for non-destructive testing to monitor the condition of structures such as aging and corrosion [6, 7]. Frequency analysis of the impedance makes it possible to precisely locate the resonance zone of the transducer. This location can be exploited to control and stabilize the operating frequency of high power systems such as ultrasonic welding devices [8]. Real-time knowledge of the electrical impedance also makes it possible to determine and optimize the power emitted by a transmitter or the sensitivity in reception of a piezoelectric sensor [9].
Different approaches were proposed to predict the piezoelectric transducer behavior such as a numerical resolution of piezoelectric equations. Another approach is based on the equivalent electrical circuit simulation using an electric simulator like SPICE (Simulation Program with Integrated Circuit Emphasis) \[10, 11\]. However, SPICE presents some limitations; SPICE is on a continuous basis; it cannot support discrete representations, and as a result, it is not suitable for mixed modeling \[12, 13\]. VHDL-AMS (Very High speed integrated circuit Hardware Description Language Analog and Mixed Signal) is a high-level language that enables numerical and analog simulations, while giving the possibility to simulate systems with different physical magnitudes: mechanical, thermal, and electrical. The use of an analysis tool, such as the VHDL-AMS behavioral description language, can be a solution to the limitations caused by the use of the SPICE simulator.

2. THEORY OF PIEZOELECTRIC ULTRASONIC TRANSDUCER

The piezoelectric material is the principal element in an ultrasonic transducer. The piezoelectric materials have the advantage compared to the other systems of presenting good performances and to be available in very diverse geometries. These materials are generally appeared as a disc, a ring or plate. Our piezoelectric element has the characteristic to vibrate in thickness mode, on only one direction axis \(z\). This modelling study is thus limited to one geometrical dimension (1-D), with the boundary conditions at the acoustic ports Figure 1 \[14\].

\[ F = -AT \]

Where \(A\) is the area of the transducer and \(T\) is the internal stress.

Let’s consider \(e\) the thickness of the piezoelectric plate and \(V_3\) is the excitation voltage. \(F_1\) and \(F_2\) presented the forces transmitted to the propagation medium on the front and back face of the transducer. \(v_1\) and \(v_2\) are the acoustic particles velocities at the front and the back faces of the transducer.

In these conditions, the transfer matrix form (2) which describes the global behavior between the electric excitation port and the two acoustic ports.

\[
\begin{bmatrix}
F_1 \\
F_2 \\
V_3
\end{bmatrix} = \begin{bmatrix}
Z_C/\tan \theta & Z_C/\sin \theta & h_{33}/\omega \\
-j \times Z_C/\sin \theta & Z_C/\tan \theta & h_{33}/\omega \\
h_{33}/\omega & h_{33}/\omega & 1/\omega C_0
\end{bmatrix} \times \begin{bmatrix}
v_1 \\
v_2 \\
i_3
\end{bmatrix}
\]

\(v_1\) and \(v_2\) (m/s) are the acoustic particle velocities at the front and the back faces of the plate, \(F_1\) and \(F_2\) are the acoustic forces at the transducer faces, \(Z_C\) is the acoustic impedance of the piezoelectric material, \(h_{33} = e_{33}/\varepsilon_{33}\) is the piezoelectricity constant and \(C_0\) is the capacitance value of the plate, and \(i_3\) is...
the electrical current. \( \theta = \omega e \sqrt{\rho / C^D} \). The dephasage generates by the propagation with \( \omega = 2\pi f \) is the electrical pulsation where \( f \) is the frequency, \( e \) is the thickness of the piezoelectric plate, \( \rho \) is the material density, \( C^D \) is the elasticity modulus with constant displacement field.

**Determination of the electrical impedance of the piezoelectric transducer**

The transducer is assumed here charged to the acoustic ports by homogeneous mediums. We determine the input electrical impedance of the transducer by the impedances of the acoustic loads \( Z_1 \) and \( Z_2 \). At ports acoustic given the sense of speed in the diagram, we have:

\[
F_1 = -Z_1 v_1 \\
F_2 = -Z_2 v_1
\]

We report these relations in the first two equations of the matrix (2) which makes it possible to determine the electrical impedance

\[
\frac{V_3}{I_3} = \frac{1}{\jmath \omega C_0} \left[ 1 + \frac{C_0 h_3^2}{\omega} \frac{-2Z_1(1-\cos \theta) + j(Z_1 + Z_2) \sin \theta}{(Z_1^2 + Z_2^2) \sin \theta - jZ_2(Z_1 + Z_2) \cos \theta} \right]
\]

The expression of electrical impedance is generally found in the literature in the following form:

\[
Z_T(j\omega) = \frac{1}{\jmath \omega C_0} \left[ 1 + \frac{K_T^2}{\omega} \frac{-2Z_1(1-\cos \theta) + j(Z_1 + Z_2) \sin \theta}{(Z_1^2 + Z_2^2) \sin \theta - jZ_2(Z_1 + Z_2) \cos \theta} \right]
\]

Where \( K_T^2 = \frac{C_0 h_3^2}{\omega} \). For longitudinal waves, the parameter \( K_T \) is often defined as the piezoelectric coupling constant for a transversely clamped material; for it is the effective piezoelectric constant used when there is no motion transverse to the electric field.

3. **ELECTRO-AcouSTIC MODEL OF PIEZOELECTRIC ELEMENT**

For reasons of simplicity, we chose to use Redwood's equivalent electrical circuit. The equivalent circuit of a thickness–mode piezoelectric transducer can be represented by the Redwood model as shown in Figure 2.

![Figure 2. Equivalent circuit of Redwood's model](image-url)
and a voltage source noted $V_V$. The second part is composed by the two acoustic ports, $T$ is an ideal electro-acoustic transformer with a ratio $h_{33}C_0$. Piezoceramic layer is assimilated to a propagation line characterized by its characteristic impedance $Z_0$ ($Z_0 = Z_C, A$), $A$ is the area and the propagation time $T_d$ ($T_d = e/v^D$), with $v^D$ is the acoustic velocity and $e$ is the thickness.

One branch of the piezoceramic layer is in contact with the back medium $Z_{back}$ and the other is in contact with the propagation medium $Z_{front}$.

4. VHDL-AMS BEHAVIORAL MODEL OF TRANSDUCER

Before discussing the modeling of transducers in VHDL-AMS, we quickly present this programming language. The VHDL-AMS is an IEEE standard [15]. It was developed as an extension of VHDL to enable the modeling and simulation of circuits and analog and digital - analog mixed systems.

The VHDL-AMS implementation of the previous model as shown in Figure 2 is divided in two parts. First is the declaration of the entity which is composed of the physical characteristics of the transducer and the different terminals used in connection. Each TERMINAL depends of the physical nature of the relation to be implemented to describe the element. Using the statements ELECTRICAL in the electrical domain and KINEMATIC_V in the acoustic domain. The second part of the model is the architecture which establishes the physic laws related to the mathematical relation between each terminal [16].

In this approach, electrical and acoustic elements are described. Electrical element is defined with voltage and current quantities and acoustic element with force and velocity quantities [17, 18]. The VHDL-AMS implementation of Redwood's equivalent model as shown in Figure 2 is given by:

```vhdl
ENTITY Redwood IS
  GENERIC (C0, kt, Z0, Td: real);
  PORT (TERMINAL p, m: electrical;
          TERMINAL t11, m11, t22, m22: kinematic_v);
END ENTITY Redwood;
ARCHITECTURE structure OF Redwood IS
  TERMINAL p1: electrical;
  TERMINAL t1, t1x, t2x : kinematic_v;
  QUANTITY v1 across i1 through p TO m;
  QUANTITY v2 across i2 through p TO p1;
  QUANTITY vte across ite through p1 TO m;
  QUANTITY pti across uti through t1 TO kinematic_v_ground;
  QUANTITY p1xr across u1xr through t1x TO t1;
  QUANTITY p2xr across u2xr through t2x TO t1;
  QUANTITY p1x across u1x through t1x TO t11;
  QUANTITY p2x across u2x through t2x TO t22;
  QUANTITY p11 across t11 TO t1;
  QUANTITY p22 across t22 TO t1;
BEGIN
  i1 == C0 * v1'dot;
  i2 == -C0 * v2'dot;
  pti == kt * vte;
  Uti == -ite/kt;
  p1xr == p22'DELAYED (Td) – p1x;
  p2xr == p11'DELAYED (Td) – p2x;
  p1x == (u1x+u2x'DELAYED (Td))*Z0/2.0;
  p2x == (u2x+u1x'DELAYED (Td))*Z0/2.0;
END ARCHITECTURE structure;
```

5. SIMULATION RESULTS

We presentions in this part the results of simulation with VHDL-AMS of the input electrical impedance of the transducer. The studied transducer is built with PZT ceramic of P1-88 type, produced by Quartz and Silice society, vibrating in thickness mode at a frequency of 2.25 MHz, 1 mm thick and area $A=132.73 \text{ mm}^2$. These characteristics are recalled in Table 1. These characteristics are recalled in Table 1.

\[
    C_0 = \varepsilon_{33}^S \frac{A}{e} ; \quad v^D = \sqrt{\varepsilon_{33}^S/\rho} ; \quad K_t = h_{33}\sqrt{\varepsilon_{33}^S/C_{13}^N} ; \quad Z_{e} = \rho v^D
\]

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Table 1. Transducer acoustic characteristics [19]

| Paramètres  | Definition           | Value       | Unit       |
|-------------|----------------------|-------------|------------|
| \( \rho \)  | Density              | 7700        | Kg/m\(^3\) |
| \( v^B \)   | Acoustic velocity    | 4530        | m/s        |
| \( Z_0 \)   | Acoustic impedance   | 34.9        | Mrayls     |
| \( C_0 \)   | Capacitor of the ceramic | 759   | PF         |
| \( K_t \)   | Coupling factor      | 0.49        | -          |
| \( C_{33}^E \) | Elastic constant      | 15.8x10\(^{10}\) | N/m\(^2\) |
| \( \varepsilon_{33} \) | Dielectric constant  | 870\(\varepsilon_0\) | F/m       |
| \( h_{33} \) | Piezoelectric constant | 1.49x10\(^9\) | -          |
| \( \tan \delta \) | Dielectric loss factor | 0.02       | -          |

The frequential transducers response study is essential to predict the sensitivity of the system for the various analyzed mediums. Table 2 gives acoustic characteristics of simulated mediums. The amplitude of the transducer is fixed at 1 Volt with a frequency of 2.25MHz.

Table 2. Acoustic characteristic of different mediums [20]

| Mediums     | Acoustic impedance (Mrayls) | Acoustic velocity (kW) |
|-------------|-----------------------------|------------------------|
| Water       | 1.49                        | 1469                   |
| Ethanol     | 0.91                        | 1158                   |
| Paraffin oil | 1.86                       | 1420                   |

5.1. Study of the electrical impedance of the Redwood model

The complex impedance of the Redwood model is of the form: \( Z_T(jw) = R + jX = |Z_T|e^{i\theta} \). We denote \( f_1 \) and \( f_2 \) the frequencies for which the impedance module reaches respectively its minimum and maximum values, the frequencies of resonance \( f_r \) and antiresonance \( f_a \) are the values for which the global reactance \( X \) of the transducer is zero (\( \theta = 0 \)). The simulation code used to obtain the transducer input electrical impedance is given by:

```
ENTITY Impedance_Simulation IS
END Impedance_Simulation;
ARCHITECTURE struct OF Impedance_Simulation IS
TERMINAL n1, n2: ELECTRICAL;
TERMINAL n3, n4: KINEMATIC_V;
CONSTANT e: real:=1.0e-3
CONSTANT A: real:=132.73e-3
CONSTANT v^B: real:=4530.0;
CONSTANT fo: real:=2.25e6
CONSTANT Zc: real:=34.9e6;
CONSTANT kt: real:=0.49;
CONSTANT \varepsilon_0: real:=8.8542e-12;
CONSTANT \varepsilon_{33}: real:=650.0;
CONSTANT \rho: real:=3300.0;
CONSTANT h: real:=kt*v^B*sqrt(red/(\varepsilon_0*\varepsilon_{33}));
CONSTANT Co: real:=A*\varepsilon_{33}/e;
QUANTITY vac: real spectrum 1.0, 0.0;
QUANTITY vinput across ie through n1 to electrical_ground;
BEGIN
Vinput==vac;
R: ENTITY resistanc (bhv) GENERIC MAP (50.0) PORT MAP (n1, n2);
T1: entity Redwood (bhv) generic map (Co, K, A*Zc, e/v^B) PORT MAP (n3, kinematic_v_ground, n4, kinematic_v_ground, n2, electrical_ground);
Rfront: ENTITY resistanc (bhv) GENERIC MAP (1.5e6*A) PORT MAP (n4, ground);
Rback: ENTITY resistanc (bhv) GENERIC MAP (445.0 A*) PORT MAP (n3, electrical_ground);
END ARCHITECTURE struct;
```

Figure 3 represents different characteristic curves that can be extracted from the impedance \( Z_T(jw) \). In practice the frequencies \( f_1 \) and \( f_r \) on the one hand and the frequencies \( f_a \) and \( f_2 \) on the other hand are very close.
5.2. Influence of the propagation medium on the impedance modulus

The vibration modes of the piezoelectric transducers are strongly influenced by the medium in which they are immersed or by the structure of which they are integral. In a homogeneous liquid or gaseous fluid, the corresponding acoustic load modifies the vibration velocity of the transducer and dampens its resonance [21]. Retrodiffused echoes to the transducer generate a reception current. In the case of a thin piezoelectric blade bonded to a solid structure under stress, the influence of the latter is preponderant. These considerations show that the electrical impedance of the transducer is very sensitive to its environment [22]. The transducer is loaded on the front face by three different mediums: water, ethanol and paraffin oil. Their acoustic characteristics are shown in Table 2. The simulation result is shown in the following figure. Figure 4 clearly shows the influence of the medium on the electrical impedance modulus and precisely on $Z_{\text{max}}$. We note a concordance between the result obtained by VHDL-AMS and the experimental result presented in [23], so a good modeling by VHDL-AMS.

![Figure 4. Transducer impedance modulus and the phase for different propagation mediums](image)

5.3. Influence of coupling factor on the impedance modulus

From the formula (6) it is found that the electrical input impedance of the transducer is directly related to the coupling factor $K_t$. The following figure shows the variation of the impedance modulus for different coupling factors. The transducer is loaded on the front face by water as propagation medium.

As shown in Figure 5, the difference between the resonant and antiresonance frequencies is strongly influenced by $K_t$. The influence of $K_t$ is important on $Z_{\text{max}}$ but negligible on $Z_{\text{min}}$. 

![Figure 5. Characteristic curves of the impedance of the Redwood model and Impedance phase](image)
5.4. Influence of intrinsic dielectric losses on the impedance modulus

It should be noted that the form (6) of the electrical impedance, does not show the dielectric losses in the ceramic. These can be taken into account when considering a transmission line with losses.

Electrical losses is well known to be the resistance leak of the ceramic capacitance considered as a resistance which depends on the frequency described by relation (7) [24, 25].

\[ R_e = \frac{1-K_t^2}{\omega C_0 \tan(\delta_e)} \]  

(7)

With \( K_t \) the coupling factor in thickness mode, \( C_0 \) is the capacitance of the ceramic, \( \tan(\delta_e) \) is the losses factor and \( \omega \) the pulsation.

Figure 6 shows a growth with \( R_e \) of the maximum value \( Z_{\text{max}} \) of the electrical impedance at the antiresonance frequency, like we can notice a reduction of the phase (the inductive effect decreases in the resonance zone when \( R_e \) increases). The antiresonance and resonance frequencies as well as the minimum value \( Z_{\text{min}} \) are independent of \( R_e \).

6. CONCLUSION

In this paper, a new approach to the ultrasonic transducer modeling system is presented. Our study begins with the presentation of an electric and electroacoustic model of the transducer. From this model we determine the analytical expression of the input electrical impedance. We modeled and simulated the input electrical impedance of the piezoelectric transducer with VHDL-AMS, as well as the influence of the various parameters such as the coupling factor, the propagation medium and the electrical losses on the modulus of the latter. We have shown that the frequency analysis of the electrical impedance can precisely locate the
resonance zone to control and stabilize the operating frequency of the system. In addition, the use of the VHDL-AMS language has the advantage of combining multi-physical domains, and indicates that the simulation of an ultrasonic detection device, comprising both electronic components and (electromechanical) transducers, is possible with VHDL-AMS. Finally, this study makes it possible to obtain a good representation of the state, the behavior and the performances in real time of the transducer.

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