A Flaring Magnetar in FRB 121102?

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Abstract

The persistent radio counterpart of fast radio burst (FRB) 121102 is estimated to have \( N \approx 10^{32} \) particles, energy \( E_N \approx 10^{49} \) erg, and size \( R \approx 10^{17} \) cm. The source can be nebula inflated and heated by an intermittent outflow from a magnetar—a neutron star powered by its magnetic (rather than rotational) energy. The object is young and frequently liberating energy in magnetic ﬂares driven by accelerated ambipolar diﬀusion in the neutron star core, feeding the nebula and producing bright millisecond bursts. The particle number in the nebula is consistent with ion ejecta from giant ﬂares. The nebula may also contain the freeze-out of electron–positron pairs \( N_e \approx 10^{51} \) created months after the neutron star birth; the same mechanism offers an explanation for \( N_e \) in the Crab Nebula. The persistent source around FRB 121102 is likely heated by magnetic dissipation and internal waves excited by the magnetar ejecta. The volumetric heating by waves explains the nebula’s enormous eﬃciency in producing radio emission. The repeating radio bursts are suggested to occur much closer to the magnetar, as a result of ultrarelativistic internal shocks in the magnetar wind, which are launched by the magnetospheric ﬂares. The shocks are mediated by Larmor rotation, which forms a GHz maser with the observed ms duration. Furthermore, the flare ejecta can become charge-starved and then convert to electromagnetic waves.

Key words: dense matter – magnetic ﬁelds – radiation mechanisms: general – relativistic processes – stars: magnetars – supernovae: general

1. Introduction

The repeating source of fast radio burst (FRB) 121102 has been active since its discovery four years ago (Spitler et al. 2016). The detection of its persistent counterpart (Chatterjee et al. 2017) led to its accurate localization and the discovery of its host dwarf galaxy at redshift \( z = 0.193 \) (Tendulkar et al. 2017). This establishes the distance to the source, its persistent radio luminosity \( L \approx 10^{59} \) erg s\(^{-1}\), and the ms burst energies \( \approx 10^{38} \) erg. The bursts and persistent emission are co-located— the upper limit on their projected separation is 40 pc (Marcote et al. 2017), and the upper limit on the persistent source size is 0.7 pc. The persistent spectrum has a sharp break at frequency \( \nu \approx 10 \) GHz. A plausible age of the source is between 10 and 100 years (Metzger et al. 2017).

2. The Persistent Synchrotron Source

Synchrotron emission from particles with Lorentz factor \( \gamma_e \) in a magnetic ﬂeld \( B \) peaks at frequency \( \nu \approx 0.2\gamma_e^2eB/2\pi m_e c \), and the radiated spectral luminosity \( L_\nu \propto \nu^{-\alpha} \) is related to the particle distribution over \( \gamma_e \),

\[
L_\nu \approx \frac{3e^2B}{m_ec^3} \frac{dN}{d\ln \gamma_e}. \quad (1)
\]

The observed \( L_\nu \approx 10^{29} \nu_0^{1.2} \) erg at \( \nu < 10 \) GHz gives

\[
NB \approx 2 \times 10^{50} \text{ G}. \quad (2)
\]

The 10 GHz spectral break (from \( \alpha \approx -0.2 \) to \( \alpha \approx -1 \)) implies that the energy of the emitting plasma \( E_N \) peaks at \( \gamma_e \approx \gamma_{br} \approx 10^7 B^{-1/2} \) (where \( B \) is in Gauss). Then the characteristic magnetization parameter of the source of size \( R, \sigma = R^3B^2/3BN_e \), is

\[
\sigma \approx \frac{R^3B^2}{3N\gamma_{br} m_ec^2} \approx 3 \times 10^{54} \frac{B^{5/2}}{N} R_{17}^3. \quad (3)
\]

Equations (2) and (3) now yield

\[
B \approx 0.06 \sigma^{2/3} \gamma_r^{-6/7} \text{ G} \quad (4)
\]

\[
N \approx 3 \times 10^{51} \sigma^{-2/3} \gamma_r^{6/7} \quad (5)
\]

\[
E_N \approx 10^{48} \sigma^{-3/7} \gamma_r^{9/7} \text{ erg}. \quad (6)
\]

One can test the value of \( B \) by looking at its implications for self-absorption and cooling breaks. The transition to efficient cooling occurs at the Lorentz factor

\[
\gamma_e = \frac{6\pi m_e c}{\sigma t B t} \approx 200 t_9^{1} \sigma^{-4/7} \gamma_r^{12/7}. \quad (7)
\]

where \( t \) is the source age. The corresponding frequency is

\[
\nu_c \approx \frac{10 e m_e c}{\sigma t B t} \approx 2 t_9^{-2} \sigma^{-6/7} R_{17}^{18/7} \text{ GHz}. \quad (8)
\]

\( R \approx 10^{17} \) cm and age \( t \approx 10^9 \) s \( \approx 30 \) years are consistent with the cooling break being near 10 GHz.

Emission at frequency \( \nu \) is dominated by electrons with energy \( E_e \approx (\nu/10^9 B)^{1/2} m_e c^2 \). Equating this energy to a few times the brightness temperature of the source \( kT_e = c^2 L_\nu/8\pi^2 R^2 \nu^2 \), one ﬁnds the frequency at which the observed \( L_\nu \) must become self-absorbed,

\[
\nu_{abs} \approx 2 B^{1/5} R_{17}^{-4/5} \text{ GHz} \approx 1 \sigma^{3/5} R_{17}^{-3/5} \text{ GHz}. \quad (9)
\]

\( R \approx 10^{17} \) cm is consistent with self-absorption being marginally important at \( \nu \approx 1 \) GHz.

3. Origin of the Nebula

The possibility of an FRB association with neutron star activity was discussed in previous works (e.g., Lyubarsky 2014; Katz 2016; Dai et al. 2017; Kashtyan & Murase 2017). Our estimate for the nebula energy \( E_N \approx 10^{48} \) erg, the sporadic ms activity at its center, and the probable age \( t \approx 10^9 \) s all point to
a young bursting magnetar, the older counterparts (ages \( \sim 10^{11}\) s) of which are observed in our galaxy (Kaspi & Beloborodov 2017). They have magnetic dipole moments \( \mu \sim 10^{33}\) G cm\(^2\), hidden internal fields \( B_r \sim 10^{16}\) G, and energies \( E_r \sim (R_p^3 B_r^2 / 6) \sim 2 \times 10^{49} B_r^2 / 16\) erg. Magnetars generate multiple bursts of different energies, which cluster in time. Such intermittent activity is typical for evolving magnetic fields (as, e.g., in solar flares).

The magnetar spindown power decreases with time,\(^1\)

\[
L_{sd} \approx \frac{c^2 I}{4 \mu^2 t^2} \approx 7 \times 10^{36} \mu_{33}^{-2} t_{16}^{-2} \text{ erg s}^{-1},
\]

where \( t \approx 10^{45}\) g cm\(^2\) is the stellar moment of inertia. At \( t \sim 10^9\) s the expected \( L_{sd}\) is two orders of magnitude below the observed radio luminosity of the nebula.

However, the magnetar is capable of releasing its magnetic energy with a much higher rate,

\[
L \sim \frac{E_r}{t_{amb}} \sim 10^{40} E_{40} t_{16}^{-1} t_{amb,9}^{-1} \text{ erg s}^{-1}.
\]

The magnetic energy is expelled from the magnetar core due to ambipolar diffusion on a timescale \( t_{amb} \). It was previously estimated as \( t_{amb} \sim 10^{11}\) s (Thompson & Duncan 1996), but recent work suggests a much shorter timescale (Beloborodov & Li 2016). It is controlled by proton friction against neutron liquid and sensitive to the core temperature, which can be calculated self-consistently. For minimum (modified URCA) cooling, \( t_{amb} \sim 10^8 (R_t / 3 \times 10^{16}) G^{-1/2} k^{-5/6}\) year, where \( k \sim 2 \pi / R_t\), describes the field gradient in the core. In addition to an ultrastrong \( B_r > 10^{16}\) G, \( t_{amb}\) may be reduced by enhanced neutrino cooling due to the Cooper pairing of neutrons or direct URCA reactions; the latter are activated in sufficiently massive neutron stars.

The magnetar was born in a supernova explosion ejecting mass \( M \sim 10^{34}\) g. The ejecta, with current ballistic expansion speed \( V\), has a mass density \( \rho \sim (3M / 4\pi V^3)^{1/3}\). The magnetar activity, with average power \( L\), inflates a pressure bubble of radius \( R\) inside the ejecta.

\[
R \sim (3c L V^{3/6}/M)^{1/5} \approx 10^{17} e^{1/3} L_{40}^{1/5} V_{0.5}^{3/5} t_{16}^{1/5} \text{ cm},
\]

where \( e < 1\) is the power reduction factor due to radiative losses in the nebula. This rough estimate suggests that the radio source with the estimated energy density \( U \sim 3E_r / 4\pi R^3 \sim 10^{-4}\) erg cm\(^{-3}\) is consistent with \( L \sim 10^{39}\) erg s\(^{-1}\) and \( R \sim 10^{17}\) cm. The bubble inside the ejecta with realistic \( V(r)\) and \( \rho(r)\) requires detailed calculations deferred to a future work. The inner edge of the nebula (the wind termination shock) is at

\[
R_{TS} \approx (3L / 4\pi c U)^{1/2} \approx 3 \times 10^{16} L_{40}^{1/2} U_{-4.2}^{-1/2} \text{ cm}.
\]

The obtained number of particles \( N \sim 10^{52}\) cannot be supplied by the usual mechanism invoked for pulsar wind nebulae (PWN). Pulsars create \( e^\pm\) pairs with the rate \( N_{e^\pm} = 2\lambda M / e\), where \( M\) is the pulsar's multiplicity and \( f = \mu^2 / e\) is the electric current circulating through the magnetosphere rotating with rate \( \Omega\). Then the \( e^\pm\) number ejected over the spin-down time \( t\)

\[
N_{e^\pm} \sim \dot{N}_{sd} t \sim \mathcal{M}_0, \quad \mathcal{M}_0 \sim \frac{c^2 I}{e \mu} \sim 2 \times 10^{42} \mu_{33}^{-1}. \tag{14}
\]

which is insufficient for the nebula of FRB 121102.

### 3.1. Pair Freeze-out

The nebula can be loaded with \( e^\pm\) pairs at an early expansion stage, when it was mainly powered by \( L_{sd} \propto t^{-2}\) (Equation (10)). The early \( e^\pm\) creation was noted in the context of superluminous supernovae (SLSN; Metzger et al. 2014); below we estimate the number of pairs that survive annihilation and remain in the nebula.

The power \( L_{sd}\) is delivered to the termination shock and converts to high-energy particles, which are quickly cooled through inverse Compton (IC) and synchrotron emission. The IC cooling is accompanied by an \( e^\pm\) cascade when the nebula has a large “compactness parameter” \( \ell = \sigma T L_{SD} / R m_e c^3 > 1\), where \( L_{SD} = \dot{L}_{sd}\) is the power fraction deposited into the IC cascade. For the early times, the nebula radius \( R(t)\) (Equation (12)) should be estimated with \( L = L_{sd}\) and \( e \sim 1\), which yields

\[
\ell(t) \approx 2f_1^{1/3} V_0^{3/5} t_9^{14/5}.
\]

When \( \ell > 10\), a “saturated” pair cascade occurs, converting \( Y \sim 10\%\) of energy into \( e^\pm\) rest mass (Svensson 1987). The pair yield \( Y\) decreases at \( \ell < 10\); it depends on the self-consistent spectrum of soft radiation that absorbs the IC photons, converting them to \( e^\pm\) pairs. Below we use a rough estimate \( Y \sim 0.1(\ell/10)\).

The annihilation balance in the nebula gives

\[
\frac{3Y L_{SD}}{4\pi R^3 m_e c^2} = \frac{3}{4} n_{e^\pm} n_{e^\mp} \sigma_\text{t} c \Rightarrow Y \ell = \frac{\pi \ell^2}{4},
\]

where \( \tau_\pm = \sigma_\text{t} R_{\pm}\). This balance is still (marginally) satisfied at the freeze-out transition, when the expansion time \( R(dR/dt)^{-1} \sim t\) becomes equal to the annihilation time \( t_{amb} = 8/3n_{e^\pm} \sigma_\text{t} c\). Thus, at freeze-out time \( t_\pm\) we have both Equation (16) and

\[
\tau_\pm = \frac{16 c}{3 \dot{R}} \approx 0.014 \mu_{33}^{-2/5} V_0^{3/5} t_9^{-1/5}.
\]

Equations (15)–(17) can be solved for \( \tau_\pm, t_\pm, \) and \( \ell(t_\pm)\)

\[
\ell_\pm \approx 0.1 f_1^{-1/3} \mu_{33}^{-4/13} V_0^{9/13} \tag{18}
\]

\[
t_\pm \approx 3 \times 10^6 f_1^{5/13} \mu_{33}^{-6/13} V_0^{6/13} \text{ s}. \tag{19}
\]

The pair yield at freeze-out \( Y_\pm \sim 10^{-2} \ell_\pm\) then determines the number of \( e^\pm\) that remain in the nebula,

\[
N_{e^\pm} \approx Y_\pm f_{sd} t_{sd} \frac{\ell_\pm}{m_e c^2} \approx 2 \times 10^{51} f_7^{7/13} \mu_{33}^{-24/13} V_0^{5/13}. \tag{20}
\]

### 3.2. Matter Ejection in Magnetar Flares

Magnetars eject plasma during their giant flares. The most powerful flare observed to date occurred in SGR 1806-20 in 2004. It radiated \( E_r \sim 2 \times 10^{46}\) erg in gamma-rays (Palmer et al. 2005) and was followed by radio afterglow emitted by mildly relativistic ejecta (Gaensler et al. 2005). Granot et al. (2006) estimated a lower limit on the ejecta mass

\footnote{1 This standard estimate for \( L_{sd}\) (with the braking index of 3) neglects variations of \( \mu\) in active magnetars and may be missing a numerical factor of a few.}
\( M_{\text{ej}} > 3 \times 10^{24} \text{ g} \), which corresponds to a minimum number of ejected ions \( N_{\text{ej}} \approx 2 \times 10^{48} \). The ejecta are dominated by electron-ion plasma (annihilation limits \( e^\pm \) ejection, see Section 5).

The lower limit on \( N_{\text{ej}} \) in SGR 1806-20 corresponds to \( N_{\text{ej}}/E_{\ell} > 10^2 \) particles per erg emitted in gamma-rays. An upper limit is set by the condition that the flare energy per proton exceeds the gravitational binding energy \( GM_p/R_s \approx 10^{-4} \text{ erg} \). This gives an estimate,

\[
10^2 < \xi \equiv \frac{N}{E} < 10^4 \text{ erg}^{-1}.
\]

(21)

An active magnetar, releasing a total of \( E \sim 10^{49} \text{ erg} \) in flares, supplies a large number of particles to the nebula,

\[
N \sim 10^{52} \left( \frac{\xi}{10^3 \text{ erg}^{-1}} \right) E_{49}.
\]

(22)

4. Heating of the Nebula

Observations of SGR 1806-20 show that giant flares eject chunks of matter with speeds \( v \sim 0.3-0.7c \). Velocity dispersion within the ejecta \( \delta v \sim v \) implies a spread in time of arrival to a radius \( R \). \( \delta t(R) \sim v/dv \sim 10^{6}R_{15} \text{ s} \). Because of this spreading, frequent impulsive flares create a quasi-continual wind before reaching the termination shock (if the flare rate exceeds \( 10^{-6}\Omega_{0}^{1} \text{ s}^{-1} \), where \( \Omega_{0} \) is the ejecta solid angle). The baryon-rich ejecta, which carry a fraction of the magnetar power, interact and mix with the more energetic high-\( \sigma \) flow from the flares before they reach the nebula. This creates a wind with average energy per particle \( \sim E/N \sim 1 \text{ GeV} \) (corresponding to \( \xi \sim 10^{7} \text{ erg}^{-1} \) in Equation (22)). The variable wind is reheated by internal shocks before the termination shock.

The wind is decelerated by a termination shock, or by a smooth pressure gradient, and joins the nebula. The deceleration of the variable wind occurs with variable compression, which implies a persistent source of sound waves with frequencies \( \nu \gtrsim c_{l}/R_{\text{TS}} \) and wavelengths \( \lambda = c_{l}/\nu \).

These waves will propagate through the nebula and dissipate over time. They dissipate due to heat conduction and viscosity, controlled by the particle diffusion coefficient \( D \sim l_{\text{c}} \), where \( l_{\text{c}} \) is the particle mean free path. The waves dissipate on a scale \( l_{\text{abs}} > \lambda \) if \( D < c\lambda \),

\[
l_{\text{abs}} \sim \frac{\lambda^2}{l}, \quad l < \lambda.
\]

(23)

Particles follow the magnetic field lines in the nebula, as their Larmor radii are small, \( r_{l} = \gamma m_{e} c^{2}/eB \sim 10^{8} \text{ cm} \). Their effective \( l \) does not exceed the correlation length of the magnetic field, which is a fraction of the nebula radius. The magnetic field structure may be complicated by surviving high-\( \sigma \) stripes from the flare ejecta.

A plausible \( l < R_{\text{TS}} \) allows sound waves with \( \lambda > (lR_{\text{TS}})^{1/2} \) to propagate and transport energy through the nebula. In the opposite case, \( l \gtrsim R_{\text{TS}} \), effective energy transport occurs as well, now due to heat diffusion. We conclude that wave excitation by the variable magnetar wind provides efficient volumetric heating of the nebula. It was not studied before in normal PWN (Gaensler & Slane 2006), where most of the dissipated energy is deposited near the steady termination shock into a small number of high-energy particles radiating at high frequencies.

The released heat is partitioned between ions and electrons, with an energy distribution that may be broadened by magnetic reconnection events and stochastic particle acceleration by the turbulence. At low energies, the electron distribution is affected by synchrotron self-absorption, which must create a low-energy break. Electrons with energies exceeding \( \gamma_{e} m_{e} c^{2} \) (which happens to be comparable with the mean particle energy \( \sim 0.1 \text{ GeV} \)) will cool faster than the age of the nebula, increasing its radiative losses. The characteristic \( B \sim 0.03 \text{ G} \), the volumetric heating sustaining \( \gamma_{e} \sim 300 \), and the proximity of \( \gamma_{e} \) to \( \gamma_{e} \) are what makes the nebula so efficient at radiating its energy in synchrotron radio waves.

If the magnetar releases energy in rare energetic flares, the ejecta do not mix into a continual wind at \( r < R_{\text{TS}} \). Then the leading high-\( \sigma \) part of the flare ejecta preserves its high power and suddenly applies a huge pressure to the nebula, pushing out the termination shock. The impact power is \( L_{t} = E_{t}/\tau_{t} \), where \( E_{t} \) is the flare energy and \( \tau_{t} \) is its duration. The dynamics of such an impact and accompanying radiation was discussed by Lyubarsky (2014) and Murase et al. (2016). A sufficiently high \( L_{t} \) launches a forward shock into the nebula with Lorentz factor \( \Gamma_{0} \approx (L/4\pi r^{2}cU)^{1/4} \) (where \( U \sim 10^{-4} \text{ erg cm}^{-3} \) is the nebula energy density) while the reverse shock continues to propagate through the ejecta. The reverse shock crosses the ejecta after time \( \tau_{0} = \Gamma_{0}^{2} / \tau_{0} \), and the forward shock begins to decelerate. Its Lorentz factor \( \Gamma \) decreases with approximate energy conservation,

\[
4\pi r^{2} c \tau U T^{2} \approx E_{t}, \quad \Gamma(\tau) \approx \left( \frac{\tau}{\tau_{0}} \right)^{-1/2}.
\]

(24)

The shock becomes a sound wave when \( \Gamma_{t} \) decreases to \( \Gamma_{t} \sim 1 \) (unless the upstream \( \sigma \gg 1 \); then \( \Gamma_{t} \approx \sigma^{-1/2} \)). This occurs after time \( \tau_{s} = \tau_{0}(\Gamma_{0}/\Gamma_{t})^{2} \), which determines the length traveled by the shock,

\[
l_{s} = c_{\tau} \sim 10^{16} E_{t,48} \Gamma_{t,2}^{-2} U_{-4}^{-1} R_{15}^{-2} \text{ cm}.
\]

(25)

The shocked plasma cools on a timescale \( \tau_{\text{syn}} \sim \Gamma^{-2} \sim \tau_{s} \) and loses energy fraction \( \tau_{s}/\tau_{\text{syn}} = \tau_{s}/\tau_{\text{syn}} \ll 1 \), where \( \tau_{\text{syn}} \sim 10^{9} \text{ s} \) is the cooling time in the nebula ahead of the shock. At times \( \tau \gg \tau_{s} \), most of the impact energy \( E_{t} \) is carried by a strong sound wave. Its wavelength is \( \lambda \sim l_{s} \), and its initial amplitude is \( \sim 1 \). Its heating effect is similar to that of sound waves generated by the mixed mildly relativistic wind described above.

5. Fast Radio Bursts

A synchrotron maser must form at the termination shock of a pulsar wind (Gallant et al. 1992). Lyubarsky (2014) proposed that FRBs could be emitted by a similar maser in a magnetar wind nebula (MWN), when its power is suddenly boosted by a giant flare. A shortcoming of the model is the assumed flare energy \( E_{1} \sim 10^{48} \text{ erg} \) and the huge power \( L_{1} \sim 10^{52} \text{ erg s}^{-1} \) required to push the nebula shock to \( \Gamma_{0} \sim 10^{7} \) orivate the Doppler-compressed FRB duration exceeded 1 ms (the minimum duration of emission arriving from a flashing spherical shock is \( r/\Gamma_{0}^{2}c \)). In addition, following a flare the termination shock cannot recover quicker than \( R_{\text{TS}}/c \), contradicting the short times between bursts in FRB 121102. Below we suggest
an alternative scenario, with moderate \( E_1 \sim 10^{44} \) erg and \( L_1 \sim 10^{47} \) erg s\(^{-1}\), which is consistent with the large number of bursts from FRB 121102.

Magnetars produce flares when their magnetospheres are over-twisted (Parfrey et al. 2013). Flares on magnetic field lines extending to \( r_0 \sim 10^7 \) cm release \( E_1 \sim \mu_0^2 r_0^3 \) erg. At the flare onset a magnetic island of size \( \sim r_0 \) disconnects and accelerates away from the star on a timescale \( r_0/c \), creating a shell of thickness \( \Delta \sim r_0 \) and carrying energy \( E \) comparable to \( E_1 \).

This first ejected shell is unlikely to be significantly polluted by baryons from the magnetar surface. The magnetospheric twist that triggered the flare is supported by electric current \( j_{w0} \sim c \mu_0/\Delta \), which gives a minimum number of \( e^\pm \) in the flare region \( N \sim M/(c j_{w0}/e)(r_0/c) \), with an expected \( e^\pm \) multiplicity \( M \sim 10^2 \) (Beloborodov 2013). Thus, a minimum number of particles in the \( \Delta \)-shell is \( N_{\text{min}} \sim M/\sigma_0 \Delta \sim 2 \times 10^{37} M_2 \mu_3 r_0^{-3} \).

More pairs are loaded if the shell ejection occurs with partial dissipation of \( E \), creating a thermal fireball with equipartition temperature \( T_0 \sim (\alpha c)^4 E/\Sigma_j^3 \). The expanding fireball cools from \( k T_0 \sim 200 \) keV to \( e^\pm \) freeze-out \( k T_{\text{fl}} \sim 20 \) keV (Paczynski 1986) after expansion to \( R_0 \sim 10^8 \) cm and acceleration to Lorentz factor \( \Gamma_0 \sim 10 \). The freeze-out occurs when \( \tau_{w} \sim n_0 \sigma_T R_0 / \Gamma_0^2 \sim 1 \) which determines the maximum number of survived pairs \( N_{\text{max}} \sim \Gamma_0^2 R_0/\sigma_T \sim 10^{41} \).

5.1. Internal Shocks

The energy per particle in the leading \( \Delta \)-shell ejected by the flare is high, \( \eta_\Delta = E/\Sigma m_e c^2 \sim 10^{41} \Sigma_{14} N_{-39}^{-1} \). As the shell expands, its energy remains concentrated within a radial thickness \( \Delta \sim r_0 \) while its Lorentz factor grows as \( \Gamma_\Delta \sim (\eta_\Delta \Gamma_0/r_0)^{1/3} \) (Lyutikov 2010; Granot et al. 2011). The fast \( \Delta \)-shell drives a blast wave into the pre-flare magnetar wind.

The blast Lorentz factor \( \Gamma \) is given by pressure balance,

\[
\Gamma \approx \Gamma_w \left( \frac{L_{147}}{L_{w,39}} \right)^{1/4} = 10^4 \Gamma_w \left( \frac{L_{147}}{L_{w,39}} \right)^{1/4},
\]

where \( L \sim \epsilon c/\Delta = 3 \times 10^{47} \Sigma_{14} \Delta^{-1} \) erg s\(^{-1}\), \( L_w \) is the wind power, and \( \Gamma_w \) is the wind Lorentz factor. The pre-flare \( L_w \) is likely far above the nominal \( L_{sd} \sim 10^{37} \) erg s\(^{-1}\), as the twisted magnetosphere is inflated and its dipole moment \( \mu \) is increased (Parfrey et al. 2013). An unknown parameter of the pre-flare wind is

\[
\sigma_w = \frac{L_w}{\Gamma_w \Sigma m_e c^2 N_w} > 1,
\]

where \( N_w \) is the particle outflow rate. The enhanced \( L_w \) may involve enhanced \( e^\pm \) loading; therefore \( \sigma_w \) may be much lower than in ordinary pulsars.

Energy transferred from the \( \Delta \)-shell to the blast wave grows with radius,

\[
\mathcal{E}_\text{bw}(r) \approx \frac{r}{2 \Gamma^2 \Delta} \mathcal{E} \approx \frac{r}{2 c} (L_1 L_w)^{1/2} \quad (r < 2 \Gamma^2 \Delta).
\]

Most of the transferred energy is stored in the swept-up wind magnetic field, and a fraction \( \sigma_w \) is deposited into the shocked wind plasma. The wind magnetic field is transverse to the radial direction and the shock is mediated by Larmor rotation. The shocked particles gyrate in the fluid frame with Lorentz factor \( \gamma_c \sim \Gamma / \Gamma_w \) and Larmor radius \( r_\ell \sim \Gamma_w m_e c^2 / eB_w \), where \( B_w = (L_w/2c)^{1/2} \) is the pre-shock magnetic field measured in the lab frame. The gyrating particles form the synchrotron maser, and a fraction of their energy \( \varepsilon \sim 10^{-2} \) converts to semi-coherent electromagnetic waves (Gallant et al. 1992) with a characteristic observed frequency

\[
\nu_{\text{obs}} \sim \frac{\Gamma c}{2 \pi r_L} = \frac{e(L_{147} L_w)^{1/4}}{2 \pi \Sigma m_e c^2 r_L/\gamma_c} \approx 3 \text{ GHz} \left( \frac{L_{147} L_{w,39}}{r_13} \right)^{1/4}. \tag{29}
\]

The coherent radiation carries energy \( \mathcal{E}_\text{FRB} \sim \varepsilon^{-1} \mathcal{E}_\text{bw}(r) \),

\[
\mathcal{E}_\text{FRB} \sim 10^{39} r_13 \varepsilon^{-2} \sigma_w^{-1} \Gamma_w^{-2} (L_{147} L_{w,39})^{-2} / 2 \text{ erg}, \tag{30}
\]

and observed duration

\[
\tau_{\text{obs}} \sim \frac{r}{\Gamma c^2} \approx 3 \times 10^{-6} r_13 \Gamma_4^{-2} \text{ s}. \tag{31}
\]

A lower \( \Gamma \) gives a longer duration; then \( \tau_{\text{obs}} \) can become related to the \( \Delta \)-shell thickness and the observed time of its energy transfer to the blast wave, \( \Delta/c \sim 1 \) ms.

As the shock expands to radius \( r \) it sweeps up wind material that was emitted by the magnetar during a short time \( \delta t \) just before the flare: \( \delta t \sim r/c \Gamma_0 \sim 3 \sigma_0 \Gamma_0^{-2} \). This time could exceed the magnetar spin period \( P \), which imprints periodicity on the swept-up wind; then the FRB emission is modulated with period \( (\Gamma_0/P)^2 P \).

5.2. Charge Starvation

The ideal (force-free) picture of the expanding \( \Delta \)-shell is valid as long as it sustains its current density \( j \) demanded by the gradient of its magnetic field \( B = \Gamma_\Delta (B / \Delta) \) (the tilde indicates the shell rest frame). The maximum possible \( j \) is \( ec \delta n_\perp / \Gamma_\Delta \), where \( \delta n_\perp = n_\perp / \Gamma_\Delta \) is the proper density and \( n_\perp = \Sigma/4\pi r^2 \Delta \). It falls short of the required current \( \tilde{j} \sim (c/4\pi) \tilde{B} / \Delta \) if

\[
\Gamma_\Delta < \Gamma_{\text{cr}}(r) = \frac{r}{e \delta N_{-39}^{1/4} \epsilon_{14}^{1/2} \Delta^{-1/2}}, \tag{32}
\]

where \( B^2 \Delta^2 \sim \epsilon/\Delta \) was used. A freely expanding \( \Delta \)-shell with \( \Gamma_\Delta \sim (\eta_\Delta \Gamma_0/r_0)^{1/3} = 10^{14} \Sigma_{14} \delta_{13}^{1/3} \Delta^{-1/3} \) will not experience charge starvation.

However, the initial free expansion is inevitably followed by deceleration. At the latest, this occurs at the termination shock \( R_{TS} \). Deceleration at smaller \( r \) occurs when the \( \Delta \)-shell runs into the slow baryonic ejecta tail from a previous magnetar flare.

The deceleration will trigger charge starvation, and the \( \Delta \)-shell energy will partially convert to vacuum electromagnetic waves. The observed duration of this event is

\[
\tau_{\text{obs}} \sim \frac{r}{\Gamma_{\text{cr}} c} \sim 10^{-3} \delta_1^{1/4} \Sigma_{-39}^{1/2} \epsilon_{14}^{-1} \text{ s}. \tag{33}
\]

The main wave frequency \( \epsilon/\Delta \sim 1 \text{ kHz} \) is too low to be interesting, however a fraction \( f_{\text{GHz}} \) of wave power might emerge at GHz frequencies. A GHz burst with energy \( \sim 10^{38} \Sigma_{14} \) erg would require \( f_{\text{GHz}} \sim 10^{-4} \).

6. Discussion

The energy and number of particles \( N \sim 10^{52} \) in the nebula of FRB 121102 are consistent with magnetar ejecta, based on
the observations of SGR 1806-20. This explanation of $N$ implies that the nebula is made of $\sim 3 \times 10^{-6} M_\odot$ of the magnetar crustal material. The electron-ion nebula could create Faraday rotation for linearly polarized FRBs. A correlation length of the magnetic field $l \lesssim R$ gives, with the nebula parameters estimated in Section 2, a modest rotation measurement $\sim 10/(l/R)$ rad m$^{-2}$, depending on the electron distribution at low $\gamma < 100$.

The frequent bursts of FRB 121102 imply many flares, many more than observed from the local magnetar population. This may not be surprising, as the local magnetars are about two orders of magnitude older and often dormant (Kaspi & Beloborodov 2017). Local MWN are hardly detectable because of their age and weaker activity. The only observed MWN, in Swift J1834.9-0846 (Younes et al. 2016), is consistent with a magnetar flare origin (Granot et al. 2017). The evolution of old magnetars is likely driven by the (relatively slow) Hall drift of their crustal magnetic fields rather than ambipolar diffusion in the core.

In addition to the young age, the hyper-active FRB 121102 probably has an unusual progenitor. The nebula size $R \gtrsim 10^{17}$ cm is consistent with a hyper-energetic supernova shell accelerated to $V \gtrsim 10^9$ cm s$^{-1}$. The shell energy $M V^2/2 \sim 10^{52}$ erg may come from the magnetar birth with rotational energy $\Theta T^2/2 \sim 2 \times 10^{52} P^{-2} m^{-2}$ erg, where $P \sim 1$ ms is the spin period. By contrast, local magnetars have supernova shells with energies $\lesssim 10^{51}$ erg (Vink & Kuiper 2006).

Ultra-fast initial rotation likely generates exceptionally strong magnetic fields (Duncan & Thompson 1992). This implies faster ambipolar diffusion in the magnetar core, resulting in enhanced energy release through magnetic flares. In addition, the magnetar might have a large mass, which can enhance its neutrino cooling, further accelerating the ambipolar drift (Beloborodov & Li 2016). The progenitor of FRB 121102 likely had a low metallicity, consistent with the rare type of its host galaxy; such hosts are also typical for GRBs and hydrogen-poor SLSN, suggesting a connection (Metzger et al. 2017).

The frequent bursting partially compensates for the low birth rate of objects like FRB 121102 and allows them to contribute to (or perhaps dominate) the observed FRB rate. Magnetars produced by ordinary progenitors are less active but may be capable of emitting FRBs by the same mechanism. The non-detection of FRB from the SGR 1806-20 giant flare (Tendulkar et al. 2016) might be explained by a limited solid angle of the blast wave with sufficiently high $\Gamma$. In addition, its pre-flare wind may be weaker compared with hyper-active younger magnetars, and a low $L_\nu$ makes the frequency $\nu_{\text{obs}}$ low (Equation 29)). Furthermore, collisions between ejecta from rare flares do not occur.

The magnetar should be spun down to $P \sim 2 \mu_{33}^{-1/2} \nu_9^{1/2}$ s, and its current $L_{\text{rad}}$ is small. However, $L_{\text{rad}}$ was high in the past. As a result, at an age $t \sim$ 1 month, the energetic compact nebula must have experienced the freeze-out of $e^\pm$ pairs with a significant $N_e$ (Equation 20)). The relict pairs are likely mixed with later ejecta from flares.

The same calculation of freeze-out $N_e$ should apply to ordinary PWN with $V \sim 10^6$ cm s$^{-1}$ and $\mu \sim 10^{-3} \text{G cm}^3$, which gives a large $N_e$. This offers a solution to the old puzzle of $\gtrsim 10^{51}$ low-energy particles inferred from radio observations of the Crab Nebula (Shklovskij 1969). The relict pairs may be reheated by later magnetic dissipation.

The internal shocks described in Section 4 generate a train of multiple ms bursts at small radii, well before the ejecta reach the nebula. The clustering of bursts in time is of particular interest; it implies more efficient collisions between the flare ejecta. FRB 121102 has demonstrated multiple bursts separated by minutes to hours, and an intermittent pattern of enhanced magnetar activity.

In the picture suggested in this Letter, two factors should control the future evolution of FRB 121102: the evolution of the magnetar flaring activity, and the ballistic expansion of the supernova shell confining the nebula. Both should evolve with age (likely $\sim 10^8$ s timescale). The heating of the nebula by a single flare may not strongly boost its persistent radio emission; a period of enhanced magnetar flaring can create a stronger heating impact accumulating on the sound crossing time $R/c_s \sim 0.5$ year.

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