Tunneling exponents sensitive to impurity scattering in quantum wires

M. Kindermann

1 School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA
(Dated: March 2007)

We show that the scaling exponent for tunneling into a quantum wire in the “Coulomb Tonks gas” regime of impenetrable, but otherwise free, electrons is affected by impurity scattering in the wire. The exponent for tunneling into such a wire thus depends on the conductance through the wire. This striking effect originates from a many-body scattering resonance reminiscent of the Kondo effect. The predicted anomalous scaling is stable against weak perturbations of the ideal Tonks gas limit at sufficiently high energies, similar to the phenomenology of a quantum critical point.

PACS numbers: 73.21.Hb, 71.10.Pm, 72.10.-d, 72.10.Fk

Introduction: The “spin-incoherent” limit of interacting quantum wires has attracted much recent attention [1]. Generically this limit is reached at low electron densities, when the Coulomb interaction suppresses spin-exchange between electrons. It appears at temperatures higher than the spin-exchange energy \( |J| \), when the spin configuration of the wire becomes effectively static. Many of its properties [1, 2, 3] differ qualitatively from those of the asymptotic low-energy limit described by the Luttinger liquid [4]. It has been shown by Fogler that spin-incoherent physics can also be observed at high electron densities, when the Coulomb interaction induces the Coulomb Tonks gas of impenetrable, but otherwise free, electrons in ultra-thin quantum wires [5].

In this Letter we show that, rather than being just one of many realizations of the spin-incoherent electron gas, the Coulomb Tonks gas exhibits its own and qualitatively different low-energy physics. Like most many-body effects observed in quantum wires with spin-incoherence [3, 6, 8, 9] this shows in experiments where electrons tunnel from an external probe, such as a scanning tunneling microscope tip, into the wire. We consider the situation depicted in Fig. 1, panel a, where electrons tunnel into the wire near a static impurity. For a generic repulsively interacting electron gas, such an impurity cuts the wire into two at low energies [10]. Electrons then effectively tunnel into the end of a half-infinite conductor without any signatures of spin-incoherence [1]. In the Coulomb Tonks gas, in contrast, interactions are weak except for a “hard core” that renders the electrons impenetrable. Electron transmission through the impurity here remains possible at low energies, ideally with an energy-independent amplitude \( t \). As a consequence, a many-body resonance develops that can be anticipated already from a perturbative analysis at weak transmission \( t \ll 1 \). One finds that a perturbative calculation is invalidated at low voltages \( V \) by contributions that diverge as \( \ln(eV/\Lambda) \), where \( \Lambda \) is the bandwidth of the wire (we take the zero temperature limit at even smaller \( |J| \)). The physics of these divergencies is analogous to that of similar divergencies in the Kondo model [11], and it is illustrated in panels b and c of Fig. 1. Two processes contribute to the correction of the tunneling current at lowest order in \( t \). In the first, electron-like one an electron tunnels into the wire and subsequently transmits through the impurity (Fig. 1b). In the second, hole-like process an electron first scatters across the impurity and leaves a hole that then is filled by an electron tunneling in (Fig. 1c). The static spins of the Coulomb Tonks gas follow the motion of charge, and thus, electrons scattering within the wire shift the spatial spin ordering. In general, the above two processes, where this shift of the spin state occurs either before or after the addition of a spin by the tunneling electron, therefore result in different final spin states as illustrated in Fig 1. This prevents cancellations between the corrections to the tunneling amplitude due to these two processes that occur in the absence of spin. After a summation over intermediate electron and hole energies, the mentioned divergencies result.

Below we resum these divergencies in the perturbation series and find that they result in a power law divergency of the conductance \( G_{\text{tun}} \) for tunneling into the wire,

\[
G_{\text{tun}} \propto \left( \frac{\Lambda}{eV} \right)^{\alpha}; \quad \alpha = \frac{2 \arcsin|t|}{\pi} \left( 1 - \frac{\arcsin|t|}{\pi} \right),
\]

(1)

where arcsin is the inverse of the sin-function. The resonance described here has the peculiar effect of induc-
ing tunneling power laws with exponents that depend on the scattering properties of and thus on the conduction through the wire. While tunneling exponents that depend on the strength of electron-electron interactions \[10\] or the magnetic field \[12\] had been found before for one-dimensional quantum wires, this effect is, to the best of our knowledge, characteristic of the Coulomb Tonks gas. Eq. \[1\] reproduces the exponent \(\alpha = 1/2\) of the impurity-free case at \(t = 1\) \[2,3\].

**Ideal Tonks Gas:** A wire in the ideal (fermionic) Tonks gas limit is described by a one-dimensional Hubbard model with an infinite onsite repulsion \(U\). This model is solved exactly in terms of a static spin background and spinless fermions \(c\), describing holes in the charge configuration \[13,14\]. The electron annihilation operator \(\psi_\sigma\) in this solution is decomposed into the fermions \(c(x)\) and operators \(S_\sigma(x)\) that add a spin \(\sigma\) to the spin background at position \(x\) as \(\psi_\sigma(x) = c^\dagger(x)S_\sigma(x)\). We study such a wire with an impurity at \(x = 0\) at zero temperature and \(eV \ll \Lambda\), such that the spectrum may be linearized,

\[
H = v_F \int \frac{dq}{2\pi} q \left( \frac{c^\dagger(x)c_q + c^\dagger_q c(x)}{2} \right) + H_S,
\]

\[
H_S = \lambda_S c^\dagger_0 c_0 + h.c. \quad (2)
\]

Here, \(c^\dagger_q\), \(c_q\) create electrons with momentum \(q\) to the left and right of the impurity, \(c_\mu = \int (dq/2\pi)c_{\mu q}\) \((\mu \in \{L,R\}\), \(\lambda_S\) is the amplitude for scattering across the impurity, and \(v_F\) the Fermi velocity.

We study the current \(I_{\text{tun}}\) from a noninteracting tunnel probe into the interacting wire at \(x = 0^+\) (we set \(\hbar = 1\)),

\[
I_{\text{tun}} = 2|v|^2 \sum_\sigma \int d\tau e^{-i\tau v_F} \left[ G^<_{\sigma}(\tau)G^>_{\text{probe},\sigma}(\tau) - G^>_{\sigma}(\tau)G^<_{\text{probe},\sigma}(\tau) \right],
\]

(3)

where \(v\) is the amplitude for tunneling between wire and probe, and \(G_\sigma\), \(G^>_{\text{probe},\sigma}\) are the Green functions of the interacting wire and the tunnel probe, respectively. We take \(v \to 0\) to be arbitrarily small. The Green function \(G^>_{\sigma}(\tau) = -i\langle\psi_\sigma(0^+\tau)\psi_\sigma^\dagger(0^+\tau)\rangle\) at the point of tunneling in terms of hole and spin operators takes the form

\[
G^>_{\sigma}(\tau) = -i\langle S_{\sigma}(0^+\tau)S^\dagger_{\sigma}(0)S_{\sigma}(0^+\tau)S^\dagger_{\sigma}(0) \rangle.
\]

(4)

The spin expectation value in Eq. \[4\] is non-vanishing only if all spins between the one at \(x = 0^+\) at time \(\tau\) and the spin that is at \(x = 0^+\) at time \(0\) have orientation \(\sigma\). This occurs with probability \(p_{N(\tau)N(0)}\), where \(N = \int (dq/2\pi)c^\dagger_q c_q\) is the number of electrons to the right of the impurity and \(p_1 = 1 - p_0 = \exp(-\beta E_Z) + 1^{-1}\) is the probability for a spin at inverse temperature \(\beta\) to point along the magnetic field with Zeeman energy \(E_Z \ll eV\).

Consequently the spin and the charge expectation values in Eq. \[4\] do not factorize and we obtain \[2\]

\[
G^>_{\sigma}(\tau) = z_\tau^{-1} \sum_k p^{|k|}_\sigma \int_0^\tau \frac{dk}{2\pi} e^{ik\pi} G^>_{\sigma}(\tau),
\]

\[
G^>_{\xi}(\tau) = -i(e^{iH_F}\xi R_c e^{i\xi N_\alpha} - e^{-iH_F}e^{-i\xi N_\alpha} R_c^\dagger) \quad (6)
\]

after a particle-hole transformation. Here, \(z_\tau = \exp(-iE_Z\sigma\tau/2)\) and \(G^>_{\sigma}(\tau) = p_\sigma^* G^>_{\sigma}^* (\tau)^*\).

**Weak Transmission:** At low bias voltages a perturbative evaluation of \(I_{\text{tun}}\) in \(\Lambda\) is invalidated by the logarithmic divergencies motivated in the introduction. We thus start with a perturbative renormalization group (RG) approach valid at weak \(\lambda_S \ll v_F\), summing all contributions to the perturbation series of leading order in the diverging logarithms at a given order in \(\Lambda\). To compute \(G^>_{\xi,\sigma}\), and thus \(I_{\text{tun}}\) via Eq. \[3\], we add the vertices

\[
H_v = \sum_{k,\mu \in \{L,R\}} v_{\mu k} c_{\mu} e^{i\xi(N_R-k)} \quad (7)
\]

to \(H\). While not directly needed for \(G^>_{\xi,\sigma}\), the vertices with \(\mu = L\) or \(k \neq 0\) are included as they are generated by the RG flow. In terms of the logarithm \(l = \ln(\omega/\Lambda)\) of the running cut-off \(\omega\) we then find the scaling equations

\[
dv_{\mu k}/dl = (2\pi v_F)^{-1}\lambda_S(v_{Lk} - v_{Lk+1}),
\]

\[
dv_{Lk}/dl = (2\pi v_F)^{-1}\lambda_S(v_{Rk} - v_{Rk-1}).
\]

(8)

The Green function \(G^>_{\xi,\sigma}\), Eq. \[6\], averages a product of two of the vertices contained in \(H_v\). Using the solution of Eq. \[7\] in Eqs. \[8\] and \[9\] we arrive, up to prefactors, at the scaling of the tunneling conductance \(G_{\text{tun}} = I_{\text{tun}}/V\),

\[
G_{\text{tun}} \sim \left(\frac{\Lambda}{\pi v_F}\right)^{2|l|/\pi} \quad (10)
\]

Here, \(|l| = |\lambda_S|/v_F + \mathcal{O}(|\lambda_S|/v_F)^2\) is the transmission amplitude through the impurity introduced before. For small \(t\) the logarithmic divergencies encountered in a perturbation series in \(t\) thus indeed lead to the \(t\)-dependent tunneling exponent advertised in the introduction.

**Good Transmission:** Eq. \[10\] is valid only at small \(t \ll 1\) and thus its scaling exponent is small and hard to measure. To access also the nonperturbative regime of good transmission \(t \approx 1\) we employ a method that was introduced by Abanin and Levitov in Ref. \[15\]. To this end we use time translation invariance to rewrite Eq. \[8\] in a form directly corresponding to Eq. \[6\] of Ref. \[15\],

\[
G^>_{\xi,\sigma}(\tau) = -i(c^\dagger c e^{-iH_F} c_R^\dagger e^{iH_F}) \quad (11)
\]
We may thus closely follow Ref. \[15\] in expressing Eq. (11) in a basis of time-dependent scattering states with indices \(s\) for their scattering time and \(\gamma\) for their direction and rewriting it in terms of single-particle operators,

\[
G_{\xi_\sigma}^{(s)}(\tau) = -i \det \{ 1 + (T - 1)f \} \sum_{\gamma^\prime} u_{\gamma} u_{\gamma'}^*, \tag{12}
\]

\[
\times \left\{ (1 - f) \left[ f + T^{-1} (1 - f) \right]^{-1} \right\}_{0\gamma, \gamma'} ,
\]

where \(u_{\gamma}\) is the amplitude for an electron in state \(\gamma\) to be at \(x = 0^+, f_{ss'} = 1/2\pi i(s - s' - i\delta)\) with \(\delta \approx 1/\Lambda\), and

\[
T = e^{ih\tau} e^{i\xi_n n} e^{-i\theta} e^{-i\xi R}. \tag{13}
\]

Here, \(h\) and \(nR\) are the first-quantized forms of \(H\) and \(N_R\), respectively. The matrix \(T\) takes the form \(T_{ss'} = \delta_{ss'} R\) for \(-\tau < s < 0\) and \(T = \delta_{ss'}\) otherwise, where

\[
R = S^\dagger e^{i\xi_R n} S e^{-i\xi_R n}. \tag{14}
\]

(At \(\tau \geq 0\); similarly for \(\tau < 0\).) \(\sigma_R = (\sigma_x + i\sigma_y)/2\), with the Pauli matrices \(\sigma_{x/y}\) projects onto amplitudes to the right of the scatterer that has a scattering matrix \(S\). Evaluated in the eigenbasis of \(R\), Eq. (12) yields

\[
G_{\xi_\sigma}^{(s)}(\tau) = \frac{-i}{2\pi \delta} \sum_j |\tilde{u}_j|^2 e^{i2\chi_j / \pi} \sum_k (-\alpha)^k I_{2k} \left( \frac{2|\tau| w^2}{\pi} \right). \tag{15}
\]

with \(w^2 = \ln(|\tau - i\delta|)/(-i\delta)\). The indices \(j, k\) label the two eigenvalues of \(R\) and the \(\tilde{u}_j\) correspond to their eigenvectors. The phases \(\chi_j\) take the form

\[
\chi_j = (-1)^j \arcsin \left[ |t| \sin \left( \frac{\xi}{2} \right) \right]. \tag{16}
\]

In the regime of weak transmission \(t \ll 1\) the Green function implied by Eq. (15) has a closed form expression in terms of modified Bessel functions of the first kind \(I_k\),

\[
G_{\sigma}^{(s)}(\tau) = \frac{2 - (2 - |t|^2) \cos \varphi}{2\pi i z \tau \delta e^{-w^2}} \sum_k (-\alpha)^k I_{2k} \left( \frac{2|\tau| w^2}{\pi} \right). \tag{17}
\]

Here, \(\varphi\) is the phase that electrons incident on the scatterer from the right pick up when backscattering. Without spin-polarization, when \(\alpha \approx 1/2\), the \(k\)-summation in Eq. (17) terminates after a few terms. Using the asymptotic form of the Bessel functions \(I_k(x) \sim \exp(x)/\sqrt{2\pi x}\) at \(|t| w^2 \gg \pi k^2\), that is at low voltages, the sum over \(k\) may be performed explicitly and one finds the same scaling as with our RG approach, Eq. (19).

For general \(t\) we carry out the \(\xi\)-integration of Eq. (5) in saddle point approximation, again valid at low voltages, \(|t| w^2 \gg \pi\). In the absence of spin-polarization, when \(1 - \alpha\) is not small, the dominant saddle point is located at \(\xi = (-1)^j \pi\), where \(j\) is the summation index in Eq. (18). The corresponding contribution to the integral is

\[
G_{\sigma}^{(s)}(\tau) = \frac{1 - \cos \varphi \cos \theta}{4\pi i z \tau \delta \sqrt{1/2 - \varphi / \pi}} \sum_k \left( \frac{(-\alpha)^k}{1 + \alpha} \right) \left( \frac{-i\delta}{\tau - i\delta} \right)^{1-\alpha}, \tag{18}
\]

\[
E_Z^* = -2\beta - \frac{1}{\pi} \left( 1 - \frac{\theta}{\pi} \right) \ln \left( \frac{\Lambda}{eV} \right). \tag{19}
\]

The crossover occurs at decreasing magnetic fields as the transmission through the wire decreases until at \(t = 0\) the tunneling current is Ohmic already in zero field.

**Coulomb Tonks Gas:** In the Coulomb Tonks gas formed by real electrons in quantum wires, the tails of the Coulomb potential introduce weak forward scattering interactions in addition to the hard core repulsion that renders the electrons impenetrable [5]. The dynamics of the charge carrying holes in the interacting wire may then be described by a Luttinger liquid with interaction parameter \(g\). The ideal Tonks gas considered above is recovered at \(g = 1\). At sufficiently high energies the forward scattering interactions are weak also on the Coulomb Tonks gas, \(|1 - g| \ll 1\). Hence we may include these interactions in our earlier perturbative RG approach valid at \(t \ll 1\). We first take \(g\) to be energy-independent. The flow equations [6] in that case have to be complemented by an additive contribution \((g^{-1} - 1)\psi_{\mu k}/2\) to \(d\psi_{\mu k}/dl\)
and an additional equation \( dt/dl = (g^{-1} - 1)t \), yielding
\[
G_{\text{tunnel}} \sim \left( \frac{eV}{\Lambda} \right)^{1-g} \exp \left[ \frac{2|t|}{\pi} \frac{1 - \left( eV/\Lambda \right)^{1-g}}{1 - g} \right]
\] (21)
at \([1 - g] \ll 1\) (up to a prefactor). Expanding the power law inside the exponential of Eq. (21), we find that the scaling of Eq. (11) is still observable over \( \lesssim [1 - g]^{-1} \) decades, with a renormalized transmission \( t_V \) and an additional renormalization of the tunneling amplitude \( v \),
\[
\frac{G_{\text{tunnel}}(V)}{G_{\text{tunnel}}(V_0)} \sim \left( \frac{V}{V_0} \right)^{1-g-2|t_V|/\pi}, \quad t_V = \left( \frac{eV}{\Lambda} \right)^{1-g} t
\] (22)
for \(|V/V_0| \ll [1 - g]^{-1}\). We expect that similarly also Eq. (11) at strong scattering is robust against the weak forward scattering interactions in the Coulomb gas with a restricted scaling range of voltages. At \( V = 0 \), \( G_{\text{tunnel}}/G_{\text{tunnel}}|_{t=0} \) has an essential singularity at \( g = 1 \) [see Eq. (21)]. This formally supports our earlier statement that the ideal Tonks gas limit qualitatively differs from the general spin-incoherent electron gas with \( g \neq 1 \). The above nonanalyticity at \( V = 0 \) is an indication of possible nonanalyticities in ground state properties and thus of a quantum phase transition in our model at \( g = 1 \). This conjecture finds further support in the scaling behavior of \( G_{\text{tunnel}} \). Perfect scaling has been found only at the (possibly critical) point \( g = 1 \). But for sufficiently high energies, \( eV \gg eV_g = \Lambda \exp(-1/[1 - g]) \), scaling persists to \( g \neq 1 \), the typical behavior around a quantum critical point [10]. One may thus speculate that there is a phase transition in our model between two qualitatively different ground states, where i) all spins in the interacting wire are entangled with the noninteracting probe \( g > 1 \) and ii) this holds only for one half of the wire that is cut into two by the impurity \( g < 1 \). Definite statements about the ground state, however, are not possible on the basis of the above calculation, because it is perturbative in the amplitude \( e \) that grows large in the limit \( V \to 0 \).

The Coulomb Tonks gas is realized in wires whose radius \( R \) is much smaller than their effective Bohr radius \( a_B = \hbar^2/\kappa m_e e^2 \), such that the parameter \( \mathcal{L} = \ln(a_B/R) \) grows large [8]. Here, \( m_e \) is the effective mass of the conduction electrons, and \( \kappa \) is the dielectric constant of the substrate that supports the wire. The regime appears at high electron density \( n \), when \( r_s = 1/2na_B \) is small, \( \mathcal{L}^{-1} \ll r_s \ll 1 \). The electrons in such a wire are impenetrable, with \( J \approx \Lambda/r_s(\mathcal{L} + \ln r_s)^{-1} \), but otherwise free at energies \( \omega \gg \omega^* \) exceeding an exponentially small scale \( \omega_* = (\Lambda/r_s)\exp(-\pi^2/2r_s) \). The forward scattering part of the Coulomb potential is indeed a small perturbation described by an interaction parameter \( 1 - g_c(\omega) \approx (2r_s/\pi^2)\ln(\Lambda/\omega) \ll 1 \) [8]. Repeating our above RG-analysis for this energy-dependent interaction, we find \( v_{\text{deck}} \sim \exp[-r_s^2/2\pi^2 - |t|\sqrt{\pi/4r_s} \text{erf}(t\sqrt{r_s}/\pi)] \). For frequency ratios \( \ln(\omega/\omega_0) \ll \min\left\{1 - g_c(\omega)\right\}^{-1}, \pi/\sqrt{r_s} \) we may expand the error function erf around \( l = \ln(\omega_0/\Lambda) \) to find for \( G_{\text{tunnel}} \) an expression like Eq. (22) with \( g = g_C(eV) \) and \( t_V = (eV/\Lambda)^{(1-g)/2}t \). At \( t \ll 1 \) the predicted scaling is thus observable over \( \lesssim \min\{\pi^2/2r_s, \ln(\Lambda/eV), \pi/\sqrt{r_s}\} \) decades with \( eV \gg \beta^{-1} \). For the above effects to be observable the spin-incoherent condition \( \beta^{-1} \gg J \) has to be satisfied. If tunneling takes place remote from the scatterer, at \( x = x_0 \neq 0 \), \( eV \ll v_F/x_0 \) is required additionally. Our calculation further assumes \( eV \ll L_0 \), where \( L_0 \) is the width of the energy window around the Fermi level in which the energy-dependence of the scattering amplitude \( \lambda_s \) may be neglected. The above scaling thus occurs at \( \Lambda/r_s(\mathcal{L} + \ln r_s) \ll \beta^{-1} \approx eV \approx \min\{v_F/x_0, L_0\} \). Alternatively, the Coulomb Tonks gas is found in gated wires at \( \mathcal{R} \lesssim a_B \) and low densities, \( n \ll a_B/D^2 \ll a_B^{-1} \), where \( D \) is the distance between wire and gate [11, 12].

The physics described above is relevant also for ultracold clouds of fermionic atoms. There the Tonks gas regime \( g \approx 1 \) is naturally realized since atoms are charge neutral and interact only via a local, “hardcore” potential. The bosonic Tonks gas [18, 19] as well as one-dimensional Fermi gases [20] have already been demonstrated experimentally. Progress is being made in cooling Fermi gases substantially below the Fermi temperature [21, 22], as required to observe the scaling predicted above. Also the experimental techniques for transport measurements with local probes as described above have been proposed [23, 24] and are expected to be experimentally implemented in the near future.

Conclusions: We have predicted a many-body scattering resonance for the strongly correlated Tonks gas regime of quantum wires. It is reminiscent of the Kondo resonance and possibly marks the critical point of a quantum phase transition. The effect occurs in the presence of static impurities and it has a clear and intriguinig experimental signature: The tunneling current into the wire obeys a power law with an exponent that depends on the conductance through the wire. This anomalous scaling is observable in ultra-thin wires at high electron density, in gated conductors at low density, and in ultracold gases of fermionic atoms.

The author thanks M. M. Fogler very much for helpful correspondence.

[1] G. A. Fiete, Rev. Mod. Phys. 79, 801 (2007).
[2] V. V. Cheianov and M. B. Zvonarev, Phys. Rev. Lett. 92, 176401 (2004).
[3] G. A. Fiete and L. Balents, Phys. Rev. Lett. 93, 226401 (2004).
[4] F. D. M. Haldane, J. Phys. C 14, 2585 (1981).
[5] M. M. Fogler, Phys. Rev. B 71, 161304(R) (2005).
[6] M. Bockrath, et al., Nature 397, 598 (1999).
[7] O. Auslaender, et al., Science 295, 825 (2002).
[8] B. J. LeRoy, et al., Nature 432, 371 (2004).
[9] Z. Yao, et al., Nature 402, 273 (1999).
[10] C. L. Kane and M. P. A. Fisher, Phys. Rev. B 46, 15233 (1992).
[11] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, 1993).
[12] M. Kindermann, P. W. Brouwer, and A. J. Millis, Phys. Rev. Lett. 97, 036809 (2006).
[13] M. Ogata and H. Shiba, Phys. Rev. B 41, 2326 (1990).
[14] M. Kindermann and P. W. Brouwer, Phys. Rev. B 74, 125309 (2006).
[15] D. A. Abanin and L. S. Levitov, Phys. Rev. Lett. 93, 126802 (2004).
[16] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, 1999).
[17] W. Häusler, L. Kecke, and A. H. MacDonald, Phys. Rev. B 65, 085104 (2002).
[18] B. Paredes, et al., Nature 429, 277 (2004).
[19] T. Kinoshita, T. Wenger, and D. Weiss, Science 305, 1125 (2004).
[20] H. Moritz, et al., Phys. Rev. Lett. 94, 210401 (2005).
[21] B. DeMarco and D. S. Jin, Science 285, 1703 (1999).
[22] A. G. Truscott, et al., Science 291, 2570 (2001).
[23] A. Micheli, et al., Phys. Rev. Lett. 93, 140408 (2004).
[24] J. H. Thywissen, R. M. Westervelt, and M. Prentiss, Phys. Rev. Lett. 83, 3762 (1999).