Optimization of the blade trailing edge geometric parameters for a small scale ORC turbine

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Abstract. In general, the method proposed by Whitfield and Baines is adopted for the turbine preliminary design. In this design procedure for the turbine blade trailing edge geometry, two assumptions (ideal gas and zero discharge swirl) and two experience values ($W_R$ and $\gamma$) are used to get the three blade trailing edge geometric parameters: relative exit flow angle $\beta_6$, the exit tip radius $R_{6t}$ and hub radius $R_{6h}$ for the purpose of maximizing the rotor total-to-static isentropic efficiency. The method above is established based on the experience and results of testing using air as working fluid, so it does not provide a mathematical optimal solution to instruct the optimization of geometry parameters and consider the real gas effects of the organic, working fluid which must be taken into consideration for the ORC turbine design procedure. In this paper, a new preliminary design and optimization method is established for the purpose of reducing the exit kinetic energy loss to improve the turbine efficiency $\eta_{ts}$, and the blade trailing edge geometric parameters for a small scale ORC turbine with working fluid R123 are optimized based on this method. The mathematical optimal solution to minimize the exit kinetic energy is deduced, which can be used to design and optimize the exit shroud/hub radius and exit blade angle. And then, the influence of blade trailing edge geometric parameters on turbine efficiency $\eta_{ts}$ are analyzed and the optimal working ranges of these parameters for the equations are recommended in consideration of working fluid R123. This method is used to modify an existing ORC turbine exit kinetic energy loss from 11.7% to 7%, which indicates the effectiveness of the method. However, the internal passage loss increases from 7.9% to 9.4%, so the only way to consider the influence of geometric parameters on internal passage loss is to give the empirical ranges of these parameters, such as the recommended ranges that the value of $\gamma$ is at 0.3 to 0.4, and the value of $r$ is at 0.5 to 0.6.

1. Introduction

In recent years, the automotive industry has made great progress in improving the engine efficiency. Current produced gasoline engines are working with a top efficiency of 30%-36%, while diesel engines already achieve about 40%-47%. In the internal-combustion engine, more than half of the fuel energy is wasted in the form of heat. Organic rankine cycle (ORC) is identified as a favorable approach for the recuperation of waste heat [1-3]. The most critical component of the ORC system is the turbine, therefore its feasibility needs to be evaluated by performing the preliminary design of a radial turbine for the best system resulting from the thermodynamic cycle optimization [4].
In general, the method proposed by Whitfield and Baines [5] is adopted for the preliminary design. In this design procedure for the turbine blade trailing edge geometry, three main parameters need to be determined: relative exit flow angle $\beta_6$, the exit tip radius $R_{6t}$, and hub radius $R_{6h}$. According to this method, $\beta_6$ is designed to minimize the relative Mach number at the shroud radius. Under the assumption of zero discharge swirl and ideal gas model, the optimal value of $\beta_6$ is set to about -55 degrees. With the relative exit flow angle fixed, the exit velocity triangle is complete once the magnitude of a velocity vector has been derived. The convenient method to fix the exit velocity triangle is to set the relative velocity ratio $W_R = W_{3s}/W_2$ to a value based on the test data or experience to ensure a good expansion through the rotor. Then the radius ratio $r_3/r_2$ is given by a simple function related to $W_R$, $\beta_6$, and relative inlet flow angle $\beta_4$. Finally, the appropriate value of hub to shroud radius ratio $\gamma$ relies on the experience and the results of testing.

The design procedure shows that when choosing the turbine blade trailing edge geometric parameters, the main purpose is to maximize the rotor efficiency. Two kinds of loss need to be considered; one is the internal passage loss, the other is the exit kinetic energy loss. In the method proposed by Whitfield and Baines, two assumptions (ideal gas and zero discharge swirl) and two experience values ($W_R$ and $\gamma$) are used to get the blade trailing edge geometry. Relative exit flow angle $\beta_6$ is designed to minimize the relative Mach number with the purpose of decreasing the internal passage loss as much as possible, and with the relative velocity vector $W_6$ fixed, the assumption of zero discharge swirl can minimize the exit kinetic energy. Hence, $\beta_6$ is a compromise to be made between a low relative Mach number to reduce the internal passage loss, and a low absolute Mach number to reduce the exit kinetic energy loss. What’s more, two experience values ($W_R$ and $\gamma$) have the same effect.

The method above is established based on the experience and results of testing, so it does not provide a mathematical optimal solution to instruct the optimization of geometry parameters. And then, the real gas effects of the organic fluid must be taken into consideration when designing the ORC turbine [6-8]. It means that most of the empirical values employed in the procedure using air a kind of ideal gas as working fluid, are not available for the high-expansion ratio typical of ORC turbines. What’s more, the assumption of zero discharge swirl is the best choice in consideration of fixed relative velocity vector $W_3$ to reduce the absolute Mach number. If the exit velocity triangle is determined to minimize the exit kinetic energy and $W_3$ can be adjusted for this purpose, the zero discharge swirl assumption is no longer the best choice.

In this paper, a new preliminary design and optimization method is established for the purpose of reducing the exit kinetic energy loss to improve the turbine efficiency $\eta_{ts}$. The rotor exit velocity triangle analysis is carried out to derive the mathematical optimal solution for minimizing the exit velocity. And then with consideration of organic working fluid R123, the influence of exit geometry parameters on the turbine efficiency performance is analyzed and the suitable ranges of these parameters are given. In the end, the blade trailing edge geometric parameters of an existing turbine are optimized using the method above.

2. The equation of mathematical optimal solution

Figure 1 below shows the rotor exit velocity triangle at the Root-Mean-Square radius. The exit absolute velocity can be determined by three variables, the exit blade speed $U_6$, the exit relative speed $W_6$, and the exit flow angle $\beta_6$. Along the exit radius height, each radius value has a unique velocity triangle figure. The selected standard radius should reflect the average status of the velocity triangle. In this procedure, the Root-Mean-Square radius is selected as the standard one. Parameters influencing these three variables are listed below:

- Exit blade speed $U_6$: the exit Root-Mean-Square radius $R_{\text{RMS}}$, the rotating speed $N$;
- Exit relative speed $W_6$: the exit volume flow rate $Q_6$, the exit tip radius $R_{6t}$, and hub radius $R_{6h}$, the exit flow angle $\beta_6$;
- Exit flow angle $\beta_6$: the exit blade angle $\beta_{6b}$ and incident angle $i_6$;
The velocity triangle in Figure 1 shows that given the exit blade speed $U_6$ and flow angle $\beta_6$, the minimum value of exit absolute velocity $C_6$ exists in the solution that $C_6$ is perpendicular to $W_6$, as the red dotted line illustrates. That also means if the rotating speed $N$ and exit flow angle $\beta_6$ are known, the minimum value of $C_6$ is already determined. The way to achieve this value is to adjust the value of $W_6$. $W_6$ is mainly determined by the exit volume flow rate $Q_6$ and exit area $A_6$ (the function of exit tip radius $R_{6t}$ and hub radius $R_{6h}$).

The main purpose is to find out the mathematical optimal solution of exit absolute velocity, so the inlet conditions are kept constant, especially the rotor inlet radius $R_4$, which can be used as a standard to determine other radii. Two dimensionless parameters are defined to substitute for the exit tip radius $R_{6t}$ and hub radius $R_{6h}$, they are defined as:

$$\tau = \frac{R_{RMS}}{R_4}$$  \hspace{1cm} (1)

$$\gamma = \frac{R_{6h}}{R_{6t}}$$  \hspace{1cm} (2)

Where $R_{RMS} = \sqrt{\frac{1}{2}(R_{6t}^2 + R_{6h}^2)}$ is the exit RMS radius.

When the condition that $C_6$ is perpendicular to $W_6$ is satisfied, equations (3)-(4) are set up. Then the mathematical optimal solution can be derived, given $\tau$, $N$ and $\beta_6$, the minimum value of $C_6$ is:

$$C_6 = U_6 \cdot \cos \beta_6$$  \hspace{1cm} (3)

$$C_6 = \frac{C_{e6}}{\sin \beta_6}$$  \hspace{1cm} (4)

Where $U_6 = \alpha R_{RMS} = \frac{\pi R_4}{30} N \tau$ and $C_{e6} = \frac{Q_6}{A_6} = \frac{Q_6}{\pi (R_{6t}^2 - R_{6h}^2)}$.

To eliminate $C_6$ in equations (3)-(4), and using parameters $\tau$, $\gamma$ and $R_4$ to substitute for $R_{RMS}$, $R_{6t}$ and $R_{6h}$, then an equation concluding $\tau$, $\gamma$, $\beta_6$ is achieved, as equation (5) shows:

$$\frac{1 + \gamma^2}{1 - \gamma^2} = Z \tau^3 \sin 2\beta_6$$  \hspace{1cm} (5)

Where $Z = \frac{\pi^2 R_4^3 N}{30 Q_6}$ is a new defined parameter.
When designing a turbine, the specific speed \( N_s \) and specific diameter \( D_s \) are significant parameters to be considered firstly, they are described as:

\[
N_s = \frac{\omega \sqrt{Q_6}}{\Delta h_0^{3/4}} 
\]

(6)

\[
D_s = \frac{D \Delta h_0^{1/4}}{\sqrt{Q_6}}
\]

(7)

Where \( D \) is a representative diameter, conventionally taken to be the rotor inlet diameter \( D_i \).

Combining equations (6)-(7) to eliminate the total enthalpy drop \( \Delta h_0 \), it is found that the parameter \( Z \) is a simple function of \( N_s \) and \( D_s \),

\[
Z = \frac{1}{8} \pi N_s D_s^3
\]

(8)

Until now, the equation containing turbine blade trailing edge geometric parameters is achieved (equation 5), and the parameter \( Z \) is derived to be the combination of \( N_s \) and \( D_s \) (equation 8). The equation (5) can be used for the design procedure as well as the parametric optimization. In the design procedure, given the specific speed \( N_s \) and based on the experience relation \( N_s D_s = 2 \), parameter \( Z \) can be determined. Then, the three exit geometry parameters have the certain relation according to equation (5). Given two of them, the third parameter can be calculated. In the parametric optimization procedure, all of the parameters are known already, if the exit kinetic energy loss is large, the exit geometry parameters can be modified based on equation (5).

3. Influence of blade trailing edge dimensionless geometric parameters on the turbine efficiency

In this section, based on an existing turbine, large amount of cases are calculated to discuss influence of exit geometry parameters on the turbine efficiency performance using the commercial software Concepts NREC. The turbine efficiency \( \eta_t \) contour map is shown below in Figure 2. In each efficiency contour map, \( N \) and \( \beta_{60} \) are given, and the efficiency is the function of \( \tau \) and \( \gamma \). In the efficiency contour maps below, the horizontal ordinate represents \( \tau \), and the Y-axis represents \( \gamma \).

From the turbine efficiency contour maps in figure 2, several results can be achieved. First of all, the exit blade angle has the biggest influence on the turbine efficiency contour map. It determined the turbine efficiency contour map type and maximum value of the turbine efficiency. From -20 degree to -60 degree, the cosine value changing from 0.94 to 0.5, the maximum efficiency increases from 78% to 87%; however the high efficiency regions become a long and narrow region (see figure 2 c, f, i), which indicates that when the exit blade angle becomes bigger, the efficiency is more sensitive to exit tip and hub radius. From equation (3), it is known that exit blade angle is an independent parameter, and the changing of exit blade angle from -20 to -60 degree leads to about 47% decreasing of the cosine value of this angle, which means almost 47% decreasing of exit absolute velocity. Hence, it is reasonable that the exit blade angle has big influence on the efficiency contour map.

Secondly, the rotating speed have little influence on the efficiency contour map. From the contour maps on each column, the rotating speed changing from 31,000 to 39,000 rpm, it shows that the maximum efficiencies are almost the same, the efficiency distributions have limited difference. This result indicates that in different rotating speed there almost exits the same high efficiency region, which means the rotating speed has no influence on the optimal combination of \( \tau \) and \( \gamma \).

Finally, the exit geometry parameters \( \gamma \) and \( \tau \), which determine the exit hub and shroud radius, are discussed. From the efficiency contour maps, it is shown that large \( \gamma \) value and small \( \tau \) value always lead to low efficiency, which is in the top left corner of the maps, the main reason of this result is the increasing of internal flow passage loss. In general, there exits high efficiency in all these rotating speed and exit blade angle combination when the value of \( \gamma \) is at 0.3 to 0.4, and the value of \( \tau \) is at 0.5
to 0.6. The suitable ranges get from the efficiency contour maps are the compromise to be made between a low relative Mach number to reduce the internal passage loss, and a low absolute Mach number to reduce the exit kinetic energy loss. And these ranges will also be regarded as the recommended working ranges of the equation derived in section 2.

![Efficiency Contour Maps](image)

**Figure 2.** Turbine efficiency $\eta_{ts}$ contour map in different rotating speed $N$ and exit blade angle $\beta_6$ combination (The figure above gives 9 combinations of $N$ and $\beta_6$. The rotating speed of first row is 31,000 rpm, and the second and third row are 35,000 and 39,000 rpm. The exit blade angle of first column is -20 degree, and the second and third column are -40 and -60 degree.)

4. **Parametric optimization based on the mathematical optimal solution**

In this section, an original turbine performance parameters are shown in table 1. Based on the system requirement, the original turbine using R123 as working fluid is designed, and after some preliminary optimizations, such as the adjustment of inlet incident angle to the common best range, the value of which is set to -22 degree. The performance of the turbine and loss distribution are achieved. The turbine efficiency $\eta_{ts}$ is 80.4%, but the exit kinetic energy loss accounts for 11.7% of the total loss 19.6%, which is much too large. Then the relation of mathematical optimal solution achieved in section 2 is used to give an instruction on the parametric optimization procedure. The results in table 1 are obtained from the Concepts NREC software, Rital module.
Table 1. Performance parameters of original and modified cases.

| Parameters            | Original case | Modified case |
|-----------------------|---------------|---------------|
| $\beta_{th}$ (degree) | -45.0         | -55.0         |
| $\beta_6$ (degree)    | -32.5         | -45.8         |
| $\alpha_6$ (degree)   | 54.8          | 43.8          |
| $\tau$               | 0.550         | 0.525         |
| $\gamma$             | 0.42          | 0.33          |
| $C_6$ (m/s)           | 77.0          | 60.4          |
| Exit kinetic energy loss (%) | 11.7  | 7.0         |
| $\eta_t / \eta_{in}$ (%) | 92.1 / 80.4  | 90.6 / 83.6  |
| Output power (kW)     | 20.5          | 21.3          |

The original geometry and the dimensionless parameters are list in table 1. According to equation (3), the minimum value of $C_6$ is 76.6m/s, which is almost the same with original case result 77m/s. The original case is already working on the place near the mathematical optimal solution with relative flow angle $\beta_6$ fixed, the potential to reduce the exit kinetic energy loss is to increase the value of $\beta_6$. Hence, the optimal solution is to improve the exit blade angle $\beta_6$ and reduce parameter $\tau$. However, $\tau$ cannot make a big reduction because of the blade stress limitation. Based on the optimal ranges in section 3, the values selected are that $\beta_6$ is set to -55 degree and $\tau$ is set to 0.525. And then, $\gamma$ can be calculated through equation (5). The value of $\gamma$ is 0.33. In the end, the exit tip and hub radius can also be calculated by inlet radius $R_4$, $\tau$ and $\gamma$. The performance parameters of the modified turbine is also shown in table 1.

Comparing the two case, it is found that the exit kinetic energy loss can be reduced largely from 11.7% to 7%, and the exit relative angle plus exit absolute angle is almost equal to 90 degree. The results indicate that the relation of mathematical optimal solution is a good method for selecting the exit tip and hub radius, aiming at minimizing the exit kinetic energy loss. However, the total-to-total efficiency drops from 92.1% to 90.6%, which also indicates that the exit geometry has an influence on the rotor internal passage loss. The internal passage loss cannot be considered in this optimal solution, so even the theoretical solution shows that bigger $\beta_6$ and smaller $\tau$ are good choice, this may also lead to the bigger internal passage loss, and finally the total-to-static efficiency may drop because this trade-off relationship. The only way to consider the influence of geometric parameters on internal passage loss is to give the empirical ranges of these parameters.

Based on the equation (3), given designed rotating speed, the mathematical optimal exit absolute velocity monotonically decreases as parameter $\tau$ decreases and the exit relative flow angle $\beta_6$ increases theoretically; hence, the first and best choice to decrease the exit absolute velocity should be to decrease $\tau$ and increase $\beta_6$. However, two important factors limit the changing ranges of $\tau$ and $\beta_6$: rotor passage loss and rotor blade stress, especially the rotor passage loss. It is the fact that when selected the extreme values of $\tau$ and $\beta_6$, although the exit kinetic energy loss can be reduced vastly, total-to-total efficiency will also decrease dramatically. In the end, this trade-off relationship will lead to the largely dropping of the total-to-static efficiency not increasing. From the results in section 3, there exits high efficiency in all these rotating speed and exit blade angle combination when the value of $\gamma$ is at 0.3 to 0.4, and the value of $\tau$ is at 0.5 to 0.6. Thus, this method can be recommended to be used when $\tau$ and $\beta_6$ are in this optimal ranges. And the parametric optimization can only be effective in this limited range.

5. Conclusions
In this paper, a new preliminary design and optimization method is established for the purpose of reducing the exit kinetic energy loss to improve the turbine efficiency $\eta _{ts}$, and the blade trailing edge geometric parameters for a small scale ORC turbine are optimized according to the equations derived based on the mathematical optimal solution for minimizing the exit kinetic energy.

Firstly, the equations containing the dimensionless parameters $\tau$, $\gamma$, $N_s$, $D_s$ and exit blade angle $\beta_6$ are derived in section 2. And then, the total-to-static efficiency performance analysis with consideration of organic fluid R123 is discussed in section 3. The results show that the exit blade angle has the biggest influence on the turbine efficiency contour map, it determined the turbine efficiency contour map type and maximum value of the turbine efficiency; the rotating speed has almost no influence on the efficiency counter map, especially on the optimal combination of $\tau$ and $\gamma$; there exists high efficiency in all these rotating speed and exit blade angle combination when the value of $\gamma$ is at 0.3 to 0.4, and the value of $\tau$ is at 0.5 to 0.6. The optimal ranges get from the efficiency contour map will also be regarded as the recommended working range of the equations derived in section 2. The results above can be used as basis and instruction for preliminary design and optimization of turbine blade trailing edge geometric parameters.

In the end, the method is used to modify the exit kinetic energy loss of an existing turbine from 11.7% to 7%, and the exit relative angle plus exit absolute angle is almost equal to 90 degree. The improvement of the modified case can be a good proof, which indicates that the equations is an appropriate method for blade trailing edge geometric parameters optimization. However, the internal passage loss increases from 7.9% to 9.4%, which indicates that this relation cannot consider any passage loss, so it cannot reflect to the change of total-to-total efficiency in theory. The only way to consider the influence of geometric parameters on internal passage loss is to give the empirical ranges of these parameters. Hence, this method can only be used in limited range of $\gamma$, $\tau$ and $\beta$, such as the recommended ranges that the value of $\gamma$ is at 0.3 to 0.4, and the value of $\tau$ is at 0.5 to 0.6.

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