Fault Estimations and Non-Fragile Control Design for Fractional-Order Multi-Weighted Complex Dynamical Networks

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\textbf{ABSTRACT} This paper deals with the robust fault estimation based synchronization problem for a class of fractional-order multi-weighted complex dynamic networks subject to external disturbances. Specifically, the synchronization problem is converted to an equivalent uniformly ultimately boundedness problem of the corresponding error system by use of spectral decomposition of the Laplacian and proper scalings of the faults. First, by introducing an intermediate estimator technique and using monochromatic Lyapunov function method, the sufficient condition for uniformly ultimately boundedness of the resulting error system via fault estimation based non-fragile controller is presented. Next, by using the linear matrix inequality (LMI) technique and optimization approach, the existence condition in the form of LMIs for designing the fault estimation based robust non-fragile controller is derived. Finally, two numerical examples including financial model are provided to demonstrate the validity and feasibility of the developed theoretical results.

\textbf{INDEX TERMS} Fractional-order complex dynamical networks, multi-weights, non-fragile control, fault estimation.

I. INTRODUCTION

Recently, complex dynamical networks (CDNs) have been applied in various areas such as transportation networks, power grids, unmanned aerial vehicles, image processing and so on [1]–[8]. Precisely, most of the existing results are formulated as CDNs with single weight, but for actual application, real-time networks might be represented by CDNs with multi-weights, such as the transportation networks, electricity distribution networks, aviation networks and social networks. Currently, few tremendous results for the synchronization criterion of CDNs with multiple weights have been explored, for instance, [9]–[11]. In [10], the authors derived a criterion for the passivity analysis of multi-weighted CDNs (MWCDNs) on the basis of edges-based pinning adaptive control strategy and Lyapunov stability theory. More recently, the fractional-order dynamical systems have been widely utilized in various fields of science and engineering due to its lower order, less parameters and higher accuracy [12]–[16].

In [14], the authors investigated the global synchronization problem for fractional-order CDNs with discontinuous nodes via finite-time approach. However, because of increasing difficulty of the fractional-order CDNs, it is not easy to ensure the synchronization criteria. So, it is necessary and significant to develop a more reliable control algorithm for the synchronization of fractional-order multi-weighted complex dynamic networks. The problem of robust synchronization of fractional-order CDNs with unknown bounded uncertainty and time delay, has been discussed in [17], where the fractional uncertainty and disturbance estimator approach has been presented to obtain the required theoretical result.

As we know, in the procedure of controller execution in many real-time systems, it is hard or even not possible to attain accurate controller due to the existence of some inevitable uncertainties in control design parameters. Further, it is proved that the vanishingly small perturbations in control parameters can produce poor performance or damage the control system. Therefore, it is significant to design a controller that can admit uncertainties and disturbances in its...
design. This inspires the study of non-fragile control problems for CDNs in the recent decades [18]–[20]. On the other hand, the fault estimation method plays an important role in the development of synchronization criteria for CDNs, whose purpose is to track the fault signals occurred in the network and give their exact sizes by constructing an appropriate fault estimator. Recently, very few results about the fault estimation problem for CDNs can be found in [21]. In [21], the authors investigated an integrated fault estimation and accommodation problem for a class of CDNs via distributed adaptive estimator approach. In [22], the joint state and fault estimation problem of a nonlinear stochastic systems in the presence of sensor saturations and randomly occurring faults has been reported. However, to the best level of authors’ knowledge, distributed intermediate fault estimation based synchronization problem for fractional-order MWCDNs via non-fragile control law has not been reported yet in the existing works.

Motivated by the aforementioned works, in this paper we focus on the design of a fault estimation based non-fragile control for synchronization of fractional-order MWCDNs subject to external disturbances and the lack of full-state measurements. The main features and contributions of this paper can be summarized as follows:

- Based on fault estimation and distributed intermediate estimator approach, a new observer-based synchronization control scheme is proposed for fractional-order multi-weighted CDNs with external disturbances.
- Specifically, by using spectral decomposition of the Laplacian and proper scalings of the faults, the synchronization problem is converted to an equivalent simultaneous uniformly ultimately bounded problem of the corresponding error systems.
- Simulation result reveals that the performance of the proposed non-fragile control design is effective since the distributed intermediate estimators needs only less information for communications.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

In this section, first we define the Riemann-Liouville derivative, then, discuss the construction of fault estimation based non-fragile control design by employing fractional-order Luenberger-type state observer.

Definition 1 [23]: The definition of fractional integral is described by

$$D^{-\alpha}_t f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f'(s) ds, \quad \alpha > 0,$$

where $\alpha$ denotes the fractional-order and $\Gamma(\cdot)$ denotes the Gamma function defined by $\Gamma(\alpha) = \int_{0}^{\infty} e^{-z} z^{\alpha-1} dz$.

Definition 2 [23]: The Riemann-Liouville derivative is defined by

$$D^\alpha f(t) = D^m D^m_{-\alpha} f(t), \quad \alpha \in [m - 1, m),$$

where $m \in Z^+$, $D^m$ is the classical m-order derivative.

Consider a class of fractional-order MWCDNs consisting of $N$ identical nodes with actuator faults and external disturbances, where the dynamics of the $i$-th node is represented by the following nonlinear descriptor equations:

$$D^\alpha_{x_i}(t) = A_1 x_i(t) + B g(t, x_i(t)) + \sum_{j=1}^{N} a_{ij} G^\nu_{ij} y_j(t) + C u_i(t) + D f_i(t) + E \omega_i(t),$$

where

$$y_i(t) = A_2 x_i(t),$$

$$D^\alpha_t x_i(t_0) = \phi_i(t_0), \quad i = 1, 2, \ldots, N,$$

where $D^\alpha_t$ denotes the Riemann-Liouville derivative; $0 < \alpha < 1$ is the fractional order; $x_i(t) \in \mathbb{R}^n$ is the state vector of the $i$th node; $y_i(t) \in \mathbb{R}^m$ is measured output vector of the $i$-th node; $u_i(t) \in \mathbb{R}^m$ denotes the control input vector of the $i$-th node; $f_i(t) \in \mathbb{R}^l$ is the fault signal of the $i$-th node; $\omega_i(t)$ is the network external disturbance which belongs to $L_2[0, \infty)$; $\phi_i(t_0)$ is the continuous initial vector function; $g(t, x_i(t))$ is a vector-valued time-varying nonlinear function, which satisfies bounded sector constraint which will be defined later; $A_1$, $A_2$, $B$, $C$, $D$ and $E$ are known constant matrices with appropriate dimensions; $a_{ij} \in \mathbb{R}^+$ ($v = 1, 2, \ldots, r$) denote the coupling strength of the $v$th coupling form; $\Gamma^\nu \in \mathbb{R}^{n \times n}$ represent the positive diagonal inner coupling matrix of the $v$th coupling form; $G^\nu = [G^\nu_{ij}]_{N \times N}$ denote the outer coupling configuration matrix of the $v$th coupling form, where the non-diagonal elements of $G^\nu$ satisfy the conditions if there is a connection between node $i$ and node $j$, then $G^\nu_{ij} > 0$; otherwise, $G^\nu_{ij} = 0$, and the diagonal elements of matrices $G^\nu$ are defined by $G^\nu_{ii} = - \sum_{j=1, j \neq i}^{N} G^\nu_{ij} (i = 1, 2, \ldots, N$ and $v = 1, 2, \ldots, r)$.

Let $e_i(t) = x_i(t) - s(t)$ be the synchronization error, where $s(t) \in \mathbb{R}^n$ represent the state trajectory of the unforced target node and which satisfies $\eta D^\rho s(t) = A_1 s(t) + B g(t, s(t))$. Further, $y_i(t) = A_2 s(t)$ is the undisturbed isolate output vector. Then, using (1), the dynamic equations of the fractional-order error system can be expressed as follows:

$$D^\rho_{x_i} e_i(t) = A_1 e_i(t) + B \tilde{g}(t, e_i(t)) + \sum_{j=1}^{N} a_{ij} G^\nu_{ij} \Gamma^\nu y_j(t) + C u_i(t) + D f_i(t) + E \omega_i(t),$$

where

$$\tilde{g}(t, e_i(t)) = g(t, x_i(t)) - g(t, s(t))$$

and $$\tilde{y}_i(t) = y_i(t) - y_j(t).$$

Further, let us define the intermediate variable as follows:

$$z_i(t) = \eta_i(t) - \delta D^\rho_{e_i(t)}, \quad i = 1, 2, \ldots, N,$$

where $$\eta_i(t) = C^* D^\rho_{f_i(t)}$$ in which $C^*$ is any matrix with appropriate dimension such that $(I - CC^*)D = 0$ and $\delta$ is a known positive scalar. Then by using (2) and (3), it can be
written as

\[
D_t^\alpha \hat{z}_i(t) = D_t^\alpha \eta_i(t) - \delta D^\tau \left[ A_1 e_i(t) + B\hat{g}_i(t), e_i(t) \right] + \sum_{i=1}^{r} \sum_{j=1}^{N} a_{ij} G_{ij}^\tau \hat{e}_j(t) + C u_i(t) + \Delta C D^\tau \hat{e}_i(t) + E \omega_i(t),
\]

(4)

In this study, the state information of all nodes are assumed to be unknown. To achieve synchronization, state and fault signal estimation, the intermediate state estimators of the systems (2), (3) and (4) are taken respectively in the following form [24]:

\[
D_t^\alpha \hat{e}_i(t) = A_1 \hat{e}_i(t) + B\hat{g}_i(t, \hat{e}_i(t)) + \sum_{i=1}^{r} \sum_{j=1}^{N} a_{ij} G_{ij}^\tau \hat{e}_j(t) + C u_i(t) + \Delta C D^\tau \hat{e}_i(t) + L[\tilde{y}_i(t) - \tilde{y}_i(t)],
\]

(5)

\[
\tilde{y}_i(t) = \tilde{z}_i(t) + \delta D^\tau \hat{e}_i(t),
\]

(6)

\[
D_t^\alpha \tilde{z}_i(t) = -\delta D^\tau \left[ A_1 \hat{e}_i(t) + B\hat{g}_i(t, \hat{e}_i(t)) + \sum_{i=1}^{r} \sum_{j=1}^{N} a_{ij} G_{ij}^\tau \hat{e}_j(t) \right] + G v^\tau \hat{e}_j(t) + C u_i(t) + C\tilde{z}_i(t) + \delta C D^\tau \hat{e}_i(t),
\]

(7)

\[
\tilde{y}_i(t) = A_2 \tilde{e}_i(t),
\]

(8)

where \( \hat{e}_i(t) \in \mathbb{R}^a \), \( \tilde{z}_i(t) \in \mathbb{R}^d \) and \( \tilde{y}_i(t) \in \mathbb{R}^p \) are the estimator vectors of \( e_i(t), \tilde{z}_i(t) \) and \( \eta_i(t) \), respectively; \( \tilde{y}_i(t) \) is the output estimation vector of the \( i \)-th node; \( L \in \mathbb{R}^{a \times p} \) is the gain matrix of the observer. Let us introduce some error vectors \( \xi_i(t) = e_i(t) - \hat{e}_i(t) \), \( \Phi_i(t) = \tilde{z}_i(t) - \tilde{z}_i(t) \), \( \nabla \tilde{y}_i(t) = \tilde{y}_i(t) - \tilde{y}_i(t) \) and \( \nabla \eta_i(t) = \eta_i(t) - \hat{y}_i(t) \). Then the overall dynamics of the distributed intermediate estimators can be represented by

\[
D_t^\alpha \xi_i(t) = (A_1 + \delta C D^\tau - L A_2) \hat{z}_i(t) + B\hat{g}_i(t, \xi_i(t))
\]

\[
+ \sum_{i=1}^{r} \sum_{j=1}^{N} a_{ij} G_{ij}^\tau \hat{e}_j(t) \quad + C \Phi_i(t) + E \omega_i(t),
\]

(9)

\[
D_t^\alpha \Phi_i(t) = D_t^\alpha \eta_i(t) - \delta D^\tau \left[ (A_1 + \delta C D^\tau) \xi_i(t) + C \Phi_i(t) \right]
\]

\[
+ B G(t, \xi_i(t)) + \sum_{i=1}^{r} \sum_{j=1}^{N} a_{ij} G_{ij}^\tau \xi_j(t) + E \omega_i(t),
\]

(10)

\[
\nabla \eta_i(t) = \Phi_i(t) + \delta D^\tau \xi_i(t),
\]

(11)

\[
\nabla \tilde{y}_i(t) = A_2 \xi_i(t),
\]

(12)

where \( G(t, \eta_i(t)) = \hat{g}_i(t, e_i(t)) - \hat{g}_i(t, \hat{e}_i(t)) \). In this paper, we are interested in designing of the intermediate estimator and non-fragile state-feedback controller are expressed as follows:

\[
u_i(t) = (K + \Delta K(t)) \hat{e}_i(t) - \hat{y}_i(t),
\]

(13)

where \( K \) is the feedback controller gain matrix which to be determined in the forthcoming section, \( \Delta K(t) \) denotes the control gain matrix with the structure \( \Delta K(t) = M \Delta(t) N \), where \( M \) and \( N \) are known real constant matrices and \( \Delta(t) \) is an unknown time-varying matrix satisfying \( \Delta T(t) \Delta(t) = \mathbb{I} \).

Now, by substituting the control law (13) into (2), we can obtain as follows:

\[
D_t^\alpha e_i(t) = A_1 e_i(t) + B\hat{g}_i(t, e_i(t)) + \sum_{i=1}^{r} \sum_{j=1}^{N} a_{ij} G_{ij}^\tau \hat{e}_j(t) + C u_i(t) + \Delta C D^\tau \hat{e}_i(t) + E \omega_i(t) + C(K + M \nabla(t) N) \hat{e}_i(t) - C(K + M \nabla(t) N) \hat{e}_i(t),
\]

\[
+ C \Phi_i(t) + \delta C D^\tau \xi(t) + E \omega_i(t),
\]

(14)

By the virtue of Kronecker product and the systems (9), (10) and (14) are expressed as follows:

\[
D_t^\alpha \eta_i(t) = (I \otimes A_1) \xi_i(t) + (I \otimes B) \xi_i(t) + (I \otimes C) \Phi_i(t)
\]

\[
+ \sum_{i=1}^{r} a_i G^\tau(t) \otimes \xi_i(t) + \delta (I \otimes CD^\tau(t)) \xi(t)
\]

\[
+ (I \otimes E) \omega_i(t) - (I \otimes L A_2) \xi_i(t),
\]

(15)

\[
D_t^\alpha \Phi_i(t) = D_t^\alpha \eta_i(t) - \delta (I \otimes D^\tau(t) A_1) \xi_i(t) - \delta (I \otimes D^\tau(t) B)
\]

\[
\times G(t, \xi_i(t)) - \delta \sum_{i=1}^{r} a_i G^\tau(t) \otimes D^\tau(t) \xi_i(t)
\]

\[
- \delta (I \otimes D^\tau(t) C) \times \Phi_i(t) - \delta (I \otimes D^\tau(t) CD^\tau(t)) \xi_i(t)
\]

\[
- \delta (I \otimes D^\tau(t) E) \omega_i(t),
\]

(16)

\[
D_t^\alpha \xi_i(t) = (I \otimes A_1) \xi_i(t) + (I \otimes B) \hat{g}_i(t, e_i(t)) + (I \otimes C \tilde{K}) \xi_i(t)
\]

\[
- (I \otimes C \tilde{K}) \xi_i(t) + \sum_{i=1}^{r} a_i G^\tau(t) \otimes \xi_i(t)
\]

\[
+ (I \otimes C) \Phi_i(t) + \delta (I \otimes CD^\tau(t)) \xi_i(t) + (I \otimes E) \omega_i(t),
\]

(17)

where \( \xi_i(t) = [\xi_1^T(t), \ldots, \xi_r^T(t), \Phi_i(t)]^T \), \( \xi_i(t) = [\Phi_1^T(t), \ldots, \Phi_r^T(t), \Phi_i(t)]^T \), \( e_i(t) = [e_1^2(t), e_2^2(t), \ldots, e_r^2(t)]^T \), \( \omega_i(t) = [\omega_1^2(t), \ldots, \omega_r^2(t)]^T \), \( G(t, \xi_i(t)) = [G_1^2(t, \xi_i(t)), \ldots, G_r^2(t, \xi_i(t))]^T \), \( \hat{g}_i(t, e_i(t)) = [\hat{g}_1^2(t, e_i(t)), \ldots, \hat{g}_r^2(t, e_i(t))]^T \), \( D_t^\alpha \eta_i(t) = [D_1^\alpha \eta_1^2(t), \ldots, D_r^\alpha \eta_r^2(t)]^T \) with \( K = (I_N \otimes K) + (I_N \otimes M \Delta(t) N) \).

This study aims at designing a non-fragile controller, intermediate estimator in the form of (13) to fractional-order MWCDNs (1) with actuator faults. To achieve this goal, it is sufficient to show that the closed-loop systems (15)-(17) is asymptotically synchronized. Before presenting the main results, we provide some preliminaries, which are more indispensable for the later development.

**Assumption 1:** For the nonlinear function \( g(t, x(t)) \), there exists a known real constant matrix \( V \) such that: \( ||g(t, x(t))|| \leq ||V x(t)|| \), for any \( x(t) \in \mathbb{R}^a \), where \( \cdot \) refers to the Euclidean vector norm.

**Assumption 2:** The fault signals \( f_i(t) \), \( i = 1, 2, \ldots, N \), are assumed to satisfy \( ||D_t^\alpha f_i(t)|| \leq \kappa \) with \( \kappa \geq 0 \).
Lemma 1: [25] The fractional-order nonlinear differential equation $D_\alpha^\tau x(t) = f(x(t))$ can be written as
\[
\begin{aligned}
\frac{\partial \mathcal{Y}(w, t)}{\partial t} &= -w \mathcal{Y}(w, t) + f(x(t)), \\
x(t) &= \int_0^\infty \zeta(w) \mathcal{Y}(w, t) dw,
\end{aligned}
\]
where $\mathcal{Y}(w, t)$ is the infinite dimensional distributed state variable, $w$ is the elementary frequency and $\zeta(w) = \frac{\sin(\alpha \pi)}{\pi} w^{-\alpha}$.

III. MAIN RESULTS

In this section, we will design a non-fragile controller (13) to achieve synchronization for fractional-order MWCDNs (1). First we show that if the control gain is known then the closed-loop system (15)-(17) is uniformly bounded.

Theorem 1: Let Assumptions 1 and 2 be true. For given positive scalars $\delta, \varphi, \upsilon$ and positive diagonal matrices $V_1$ and $V_2$, the closed-loop system (15)-(17) is uniformly bounded if there exist symmetric positive matrices $P_1, P_2$ and $P_3$, and a positive scalar $\epsilon$ such that the following matrix inequality holds:
\[
\tilde{\Lambda} = \begin{bmatrix} [\Lambda]_{6 \times 6} & \epsilon \varphi & \upsilon^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0,
\]
where $\Lambda_{1,1} = (I \otimes P_3 A_1) + (I \otimes A_1^T P_3) + \sum_{v=1}^r a_v (G^v \otimes P_3 \Gamma^v) + \sum_{v=1}^r a_v (G^v \otimes \Gamma^v T P_3) + (I \otimes P_3 CK) + (I \otimes V_1) (I \otimes V_1)^T$, $\Lambda_{1,2} = - (I \otimes P_3 CK) + \delta (I \otimes P_3 CD^T)$, $\Lambda_{1,3} = (I \otimes P_3 C)$, $\Lambda_{1,4} = (I \otimes P_3 B)$, $\Lambda_{1,6} = -(I \otimes A_1^T) + (I \otimes P_3 E)$, $\Lambda_{2,2} = (I \otimes P_1 A_1) + (I \otimes A_1^T P_1) + (I \otimes V_2) (I \otimes V_2)^T + \sum_{v=1}^r a_v (G^v \otimes P_1 \Gamma^v) + \sum_{v=1}^r a_v (G^v \otimes \Gamma^v T P_1) + \delta (I \otimes P_1 CD^T) - (I \otimes P_1 LA_2)$, $\Lambda_{2,3} = (I \otimes P_1 C) - \delta (I \otimes A_1^T DP_2) - \sum_{v=1}^r a_v (G^v \otimes P_1 \Gamma^v D^T) - \delta^2 (I \otimes D^T D_{P_2})$, $\Lambda_{2,5} = (I \otimes P_1 B)$, $\Lambda_{2,6} = (I \otimes P_1 E)$, $\Lambda_{3,3} = \upsilon^{-1} (I \otimes P_2 D_B) + (I \otimes V_2) (I \otimes V_2)^T - \delta (I \otimes P_2 D^T E) - \delta (I \otimes C^T D^T P_2)$, $\Lambda_{3,5} = -\delta (I \otimes P_2 D^T E)$, $\Lambda_{3,6} = -\delta (I \otimes P_2 CM)$, $\Lambda_{4,4} = -I$, $\Lambda_{5,5} = -I$, $\Lambda_{6,6} = -\varphi I$, $\varphi = [(I \otimes P_3 CM) (I \otimes P_3 CM) 0 0 0 0]^T$, $\upsilon = [(I \otimes N) (I \otimes N) 0 0 0 0]$ and the remaining elements are zero.

Proof: It follows from Lemma 1 in [25] that the fractional-order MWCDNs (15)-(17) can be written as follows:
\[
\begin{aligned}
\frac{\partial \tilde{X}(w, t)}{\partial t} &= -w \tilde{X}(w, t) + (I \otimes A_1) \xi(t) + (I \otimes C) \Phi(t) \\
&\quad + \sum_{v=1}^r a_v (G^v \otimes \Gamma^v) \xi(t) + \delta (I \otimes C D^T) \xi(t) \\
&\quad + (I \otimes E) \omega(t) + (I \otimes B) \mathcal{G}(t, \xi(t)) \\
&\quad - (I \otimes LA_2) \xi(t),
\end{aligned}
\]
\[
\begin{aligned}
\frac{\partial \tilde{Y}(w, t)}{\partial t} &= -w \tilde{Y}(w, t) + D_\alpha^\tau \eta(t) - \delta (I \otimes D^T A_1) \xi(t) \\
&\quad - \delta (I \otimes D^T B) \mathcal{G}(t, \xi(t)) - \sum_{v=1}^r a_v (G^v \otimes D^T \Gamma^v) \xi(t) \\
&\quad \times (I \otimes C D^T) \Phi(t) - \delta^2 (I \otimes D^T C D^T) \xi(t) \\
&\quad \times (I \otimes D^T E) \omega(t),
\end{aligned}
\]
where $\tilde{X}(w, t) = [X_1(w, t), X_2(w, t), \ldots, X_n(w, t)]^T$, $\tilde{Y}(w, t) = [Y_1(w, t), Y_2(w, t), \ldots, Y_m(w, t)]^T$ and $\tilde{Z}(w, t) = [Z_1(w, t), Z_2(w, t), \ldots, Z_n(w, t)]^T$. Then, we choose the Lyapunov function for the systems (19)-(21) as $v_{11}(w, t) = \tilde{X}^T(w, t)(I \otimes P_1) \tilde{X}(w, t)$, $v_{22}(w, t) = \tilde{Y}^T(w, t)(I \otimes P_2) \tilde{Y}(w, t)$, $v_{33}(w, t) = \tilde{Z}^T(w, t)(I \otimes P_3) \tilde{Z}(w, t)$ where $(I \otimes P_1)$, $(I \otimes P_2)$ and $(I \otimes P_3)$ are positive symmetric matrices. This function is called as the monochromatic Lyapunov function corresponding to the elementary frequency $w$. Based on $v_{11}(w, t)$ ($a = 1, 2, 3$), we construct the global monochromatic Lyapunov function for the systems (19)-(21), we have
\[
V(t) = \int_0^\infty \xi(w) \mathcal{X}^T(w, t)(I \otimes N_1) \mathcal{X}(w, t) dw \\
+ \int_0^\infty \xi(w) \mathcal{Y}^T(w, t)(I \otimes N_2) \mathcal{Y}(w, t) dw \\
+ \int_0^\infty \xi(w) \mathcal{Z}^T(w, t)(I \otimes N_3) \mathcal{Z}(w, t) dw.
\]
This Lyapunov function integrates all the monochromatic $v_{11}(w, t)$ ($a = 1, 2, 3$) with a weighting function $\xi(w)$ on the whole spectral range. Now, by calculating the time derivative of (22) along the solution of the systems (19)-(21), we can get
\[
\begin{aligned}
\dot{V}(t) &= -2 \int_0^\infty \xi(w) \mathcal{X}^T(w, t)(I \otimes P_1) \mathcal{X}(w, t) dw + 2 \mathcal{X}^T(w, t) \\
&\quad \times (I \otimes P_1)(I \otimes A_1) \xi(t) + (I \otimes B) \mathcal{G}(t, \xi(t)) + \sum_{v=1}^r a_v (G^v \otimes \Gamma^v) \xi(t) \\
&\quad \times (I \otimes C) \Phi(t) + \delta (I \otimes C D^T) \xi(t) \\
&\quad + (I \otimes E) \omega(t) - (I \otimes LA_2) \xi(t) - 2 \int_0^\infty \xi(w) \mathcal{X}^T(w, t) \\
&\quad + \xi(w) \mathcal{X}^T(w, t) + (I \otimes A_1) \xi(t) + (I \otimes B) \mathcal{G}(t, \xi(t)) + \sum_{v=1}^r a_v (G^v \otimes D^T \Gamma^v) \xi(t) \\
&\quad \times (I \otimes C D^T) \Phi(t) - \delta^2 (I \otimes D^T C D^T) \xi(t) \\
&\quad \times (I \otimes D^T E) \omega(t),
\end{aligned}
\]
\[ \left( I \otimes P_2 \right) \tilde{Y}(w, t) \, dw + 2 \Phi^T(t) \left( I \otimes P_2 \right) [D^\mu_0 \eta(t) + \delta (I \otimes D^T A_1) \xi(t) - \delta (I \otimes D^T B) \mathbb{G}(t, \xi(t)) - \delta \sum_{v=1}^{r} a_v \right. \\
\times (G^v \otimes D^T C) \Phi(t) \\
\left. - \delta^2 (I \otimes D^T CD^T) \times (t - \delta (I \otimes D^T E) \omega(t)) \right] \\
- 2 \int_0^T (\tilde{w}(w, t) (I \otimes P_3) \tilde{Z}(w, t) \, dw \\
+ 2 \tilde{e}(t) (I \otimes P_3) \tilde{e}(t) + (I \otimes B) \tilde{e}(t)) \\
+ \sum_{v=1}^{r} a_v (G^v \otimes \Gamma^v) \xi(t) + (I \otimes C \tilde{K}) \xi(t) - \xi(t) (I \otimes E) \omega(t) \right]. \tag{23} \\
\end{align*}

From Assumption 2, it can be obtained that there exist scalar \( \kappa \) such that \( \| D^\mu_0 \eta(t) \| \leq \kappa \). By considering this fact, for any chosen positive scalar \( v \). Then, the following inequality always hold:

\[ 2 \Phi^T(t) \left( I \otimes P_3 \right) (I \otimes P_3) \Phi(t) + v \Phi^T \tag{24} \]

Moreover, according to Assumption 1, we can obtain the following inequalities

\begin{align*}
\xi^T(t) (I \otimes \mathcal{V}_1) (I \otimes \mathcal{V}_1) \xi(t) - \mathcal{G}^T(t, \xi(t)) \mathbb{G}(t, \xi(t)) &\geq 0, \\
\mathcal{G}^T(t, \xi(t)) \mathbb{G}(t, \xi(t)) &\geq 0, \\
\end{align*}

where \( \mathcal{V}_1 \) and \( \mathcal{V}_2 \) are known positive matrices. By considering (23)-(26), it is easy to obtain that

\[ \hat{V}(t) - 2 \tilde{e}^T(t) \omega(t) - \varphi^T(t) \omega(t) \leq \xi^T(t) \Lambda \xi(t) + v \kappa^2, \tag{27} \]

where \( \xi(t) = \left[ e^T(t) \xi(t) \Phi^T(t) \tilde{w}(t) \tilde{e}(t) \tilde{e}(t) \right] \omega(t), \Lambda = \Lambda + \varrho (t) (\Delta(t) + v)^T \) and the elements of \( \Lambda, \varrho \) and \( v \) are defined in the theorem statement. Moreover, based on Lemma 2 in [26], for any scalar \( \epsilon > 0 \), the RHS of (27) can equivalently be written as

\[ \hat{V}(t) - 2 \tilde{e}^T(t) \omega(t) - \varphi^T(t) \omega(t) \leq -\psi V(t) + \Omega, \tag{29} \]

Based on the Schur complement, it is noted that RHS of (28) is equivalent to the LHS of LMI of (18). Hence, it follows from (18) that \( \Lambda < 0 \). On the other hand, it is noted that \( V(t) \leq \max \{ \lambda_{\max}(P_1), \lambda_{\max}(P_2) \} \| \xi(t) \| + \| \Phi(t) \| + \| \omega(t) \|^2 \}. \) Now let us denote \( \Lambda = \Lambda - \Lambda \), it is obvious that if \( \Lambda < 0 \) i.e., \( \Lambda > 0 \), then

\[ \hat{V}(t) - 2 \tilde{e}^T(t) \omega(t) - \varphi^T(t) \omega(t) \leq -\psi V(t) + \Omega, \tag{29} \]

where \( \psi = \frac{\lambda_{\min}(\Lambda)}{\kappa}, \chi = \max \{ \lambda_{\max}(P_1) \| \xi(t) \| + \lambda_{\max}(P_2) \| \Phi(t) \| + \lambda_{\min}(P_2) \| e(t) \| \}, \Omega = v \kappa^2. \) Moreover, we consider the set \( \mathcal{W} = \{ (\xi(t), \Phi(t), e(t)) : \lambda_{\min}(P_1) \| \xi(t) \|^2 + \lambda_{\min}(P_2) \| \Phi(t) \|^2 + \lambda_{\min}(P_2) \| e(t) \|^2 \leq \frac{\Omega}{\psi} \}. \) If \( (\xi(t), \Phi(t), e(t)) \in \mathcal{W} \), then it follows that

\[ V(t) \geq \lambda_{\min}(P_1) \| \xi(t) \|^2 + \lambda_{\min}(P_2) \| \Phi(t) \|^2 + \lambda_{\min}(P_2) \| e(t) \|^2 \geq \frac{\Omega}{\psi}. \tag{30} \]

From the inequalities (29) and (30), it can be observed that \( V(t) - 2 \tilde{e}^T(t) \omega(t) - \varphi^T(t) \omega(t) \leq 0 \) if \( (\xi(t), \Phi(t), e(t)) \in \mathcal{W} \). Obviously, the pair \( (\xi(t), \Phi(t), e(t)) \) is uniformly bounded and exponentially converges to the set \( \mathcal{W} \) at a rate greater than \( e^{-\psi t} \) from (29). Hence, the proof is completed. 

It should be noted that if the controller gain matrix are unknown, then the constraint in (18) cannot be solved directly via MATLAB LMI control toolbox due to the existence of nonlinear terms. To rectify this problem, we apply linear congruence transformation to the derived conditions in the Theorem 1. The following theorem presents the necessary LMI constraints to attain the control gain matrix.

**Theorem 2:** Suppose that Assumptions 1 and 2 hold. Then the intermediate estimator approach based non-fragile state-feedback controller can ensure that all the signals in the closed-loop systems (15)-(17) are uniformly bounded for given positive scalars \( \varrho, \varphi, v, \lambda \) and positive diagonal matrices \( \mathcal{V}_1 \) and \( \mathcal{V}_2 \), if there exist symmetric positive matrices \( P_1, P_2, P_3 \), any suitable dimensioned matrices \( X, Y, \) and \( \mathbb{I} \), and a positive scalar \( \epsilon \) such that the following LMIs are satisfied:

\[ \begin{bmatrix} \hat{\Lambda}_1 & \epsilon \end{bmatrix}_{9 \times 9} < 0, \tag{31} \]

\[ \begin{bmatrix} -\lambda(I \otimes I) & (I \otimes CX) - (I \otimes P_2 C) \\ -\lambda(I \otimes I) & -\lambda(I \otimes I) \end{bmatrix} < 0, \tag{32} \]

where \( \hat{\Lambda}_1 = (I \otimes P_3 A_1) + (I \otimes A_1^T P_3) + \sum_{v=1}^{r} a_v (G^v \otimes P_1 \Gamma^v) + \sum_{v=1}^{r} a_v (G^v \otimes \Gamma^v T P_3) + (I \otimes CY) + (I \otimes \mathcal{V}_1) (I \otimes \mathcal{V}_1)^T, \]

\[ \hat{\Lambda}_2 = -((I \otimes CY) \delta(I \otimes P_3 CD^T), \hat{\Lambda}_{2.1} = (I \otimes CX) + \delta(I \otimes P_3 CD^T), \hat{\Lambda}_{2.1} = (I \otimes CX) + \delta(I \otimes P_3 CD^T), \hat{\Lambda}_{2.1} = (I \otimes CX) + \delta(I \otimes P_3 CD^T), \hat{\Lambda}_{2.1} = (I \otimes CX) + \delta(I \otimes P_3 CD^T). \]

**Proof:** In order to prove the proof for this theorem, we consider the linear congruence transformations \( P_3 C = CX \), \( Y = XK \) and \( L = P_1 L \). Then, by applying Schur complement, the inequalities in (18) can be equivalently written as (31). It is noted that the assumption \( P_3 C = CX \) is not a linear inequality and so it is difficult to solve via MATLAB LMI toolbox. To resolve this difficulty, \( P_3 C = CX \) can be replaced by relatively equivalent inequality constraint.
The inner coupling of non-delay matrices are selected as follows:

\[ \begin{bmatrix} -15 & 1 & -1 \\ -5 & -2 & 3 \\ 1 & 3 & -7 \end{bmatrix}, \quad \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}, \]

\[ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 1 & 2 \end{bmatrix}, \]

\[ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma^1 = \text{diag}(0.1, 0.4, 0.2), \]

\[ \Gamma^2 = \text{diag}(0.3, 0.1, 0.4), \quad \Gamma^3 = \text{diag}(0.2, 0.3, 0.5), \]

\[ a_1 = 0.5, \quad a_2 = 0.2, \quad a_3 = 0.4. \]

The inner coupling of non-delay matrices are selected as follows:

\[ \begin{bmatrix} -0.5 & 0.1 & 0.2 \\ 0.1 & -0.4 & 0.2 \\ 0.1 & 0.1 & -0.8 \end{bmatrix}, \quad \begin{bmatrix} 0.1 & 0.1 & -0.3 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}, \]

\[ \begin{bmatrix} 0.1 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.3 \end{bmatrix}, \quad \begin{bmatrix} 0.2 & -0.6 & 0.2 & 0.2 \\ 0.1 & 0.1 & -0.3 & 0.1 \end{bmatrix}, \]

\[ \begin{bmatrix} 0.3 & 0.1 & -0.5 \\ 0.3 & 0.1 & 0.2 \end{bmatrix}, \quad \begin{bmatrix} -0.5 & 0.6 & 0.2 \\ 0.3 & -0.7 & 0.3 \end{bmatrix}, \]

\[ \begin{bmatrix} 0.1 & 0.2 & -0.5 \\ 0.3 & 0.3 & 0.2 \end{bmatrix}. \]

(32) for any small positive scalar \( \lambda \). Hence, the proof is completed.

**Remark 1:** It is worth pointing out that, a great number of research works regarding the issue of synchronisation of CDN's have been reported in the recent literature, for instance see [3]–[8]. Nevertheless, only very few works have been focused on the problem of synchronisation of fractional-order CDN's with external disturbances [16], [17]. It should be noted that, all the aforementioned works do not consider multi-weights in the system. Very recently, some interesting results on synchronisation of fractional-order CDN's with multi-weights have been discussed in [12]. However, the issue of the robust fault estimation based non-fragile control design for fractional-order CDN's subject to multi-weights, actuator faults and external disturbances has not yet been discussed in the literature. Thus, the main contribution of this paper is to fill such a gap through employing a non-fragile control law based on intermediate estimator for achieving robust synchronization in fractional-order CDN's in the presence of multi-weights and external disturbances, which makes this work different form the existing works on fractional-order CDN's.

**IV. NUMERICAL EXAMPLES**

In this section, two numerical examples including the financial model are given to demonstrate the effectiveness and superiority of the proposed non-fragile controller design (13).

**Example 1:** Consider the fractional-order MWCDNs (1) with four nodes and three coupling weights and its parameters are taken as follows:

\[ A_1 = \begin{bmatrix} -15 & 1 & -1 \\ -5 & -2 & 3 \\ 1 & 3 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}, \]

\[ C = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}, \quad D = \begin{bmatrix} 5 \\ 2 \\ 5 \end{bmatrix}, \quad E = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 1 & 2 \end{bmatrix}, \]

\[ A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma^1 = \text{diag}(0.1, 0.4, 0.2), \]

\[ \Gamma^2 = \text{diag}(0.3, 0.1, 0.4), \quad \Gamma^3 = \text{diag}(0.2, 0.3, 0.5), \]

\[ a_1 = 0.5, \quad a_2 = 0.2, \quad a_3 = 0.4. \]

The first state responses of fractional-order MWCDNs (1) with non-fragile controller (13) are given in Figure 1.

**FIGURE 1.** First state responses of fractional-order MWCDNs (1) with non-fragile controller (13).

Further, the nonlinear function is chosen as

\[ g(t, x_i(t)) = \begin{bmatrix} 0.35 \exp(-0.3t) \tanh(x_{i1}(t)) \\ 0.35 \exp(-0.2t) \tanh(x_{i2}(t)) \\ 0.35 \exp(-0.5t) \tanh(x_{i3}(t)) \end{bmatrix} \]

and according to Assumption 1, we obtain \( \mathcal{V} = \text{diag}(0.35, 0.35, 0.35) \). The rest of parameters involved in the simulation are set to be \( \delta = 5.95, \varrho = 0.05, \nu = 0.5 \) and \( \lambda = 0.5 \). The uncertain matrices \( M = \begin{bmatrix} 0.1 & 0.2 & 0.3 \end{bmatrix} \) and

\[ N = \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.2 \end{bmatrix}. \]

Then, by solving the LMIs (31) and (32) in Theorem 2 via MATLAB LMI toolbox, we can obtained the set of feasible solution from which the non-fragile state feedback control and observer gain matrices are given by

\[ A^{\hat{}} = \begin{bmatrix} -15 & 1 & -1 \\ -5 & -2 & 3 \\ 1 & 3 & -7 \end{bmatrix}, \quad B^{\hat{}} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}, \]

\[ C^{\hat{}} = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}, \quad D^{\hat{}} = \begin{bmatrix} 5 \\ 2 \\ 5 \end{bmatrix}, \quad E^{\hat{}} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 1 & 2 \end{bmatrix}, \]

\[ A^{\hat{}}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma^{\hat{}}^1 = \text{diag}(0.1, 0.4, 0.2), \]

\[ \Gamma^{\hat{}}^2 = \text{diag}(0.3, 0.1, 0.4), \quad \Gamma^{\hat{}}^3 = \text{diag}(0.2, 0.3, 0.5), \]

\[ a^{\hat{}}_1 = 0.5, \quad a^{\hat{}}_2 = 0.2, \quad a^{\hat{}}_3 = 0.4. \]
$K = \begin{bmatrix} -0.2395 & -1.8063 & -1.1694 \end{bmatrix}$ and 
$L = \begin{bmatrix} 70.8338 & 323.1983 \\ 28.9458 & 111.5322 \\ 76.5372 & 295.1444 \end{bmatrix}$. 

For the simulation purposes, the fault $f_i(t)$ ($i = 1, 2, 3, 4$) are chosen as follows:

- $f_1(t) = \begin{cases} 2, & 0.3 \leq t \leq 0.6, \\ 0, & \text{elsewhere}, \end{cases}$
- $f_2(t) = \begin{cases} 0, & 0 \leq t \leq 0.3, \\ 2 \sin(5\pi t), & \text{elsewhere}, \end{cases}$
- $f_3(t) = \begin{cases} 0, & 0.3 \leq t \leq 0.6, \\ 3t, & \text{elsewhere}, \end{cases}$
- $f_4(t) = \begin{cases} 2 \cos(7\pi t), & 0.3 \leq t \leq 0.6, \\ 0, & \text{elsewhere}, \end{cases}$

and the external disturbance of all four nodes, the initial conditions for the states of nodes and the isolated node are respectively chosen as follows: $\omega_i(t) = [0.25 \sin(\pi t) \ 0.3 \sin(5\pi t)]$.
seen that the given fault signals are exactly estimated within the short time due to the the proposed controller. Thus, it can be seen from the simulations that intermediate estimator-based non-fragile controller effectively synchronize the fractional-order MWCDNs in the presence of external disturbances.

**Example 2:** The fractional-order financial chaotic system is borrowed from [27], which is described as follows:

\[ D^\alpha_s x_i(t) = A x_i(t) + g(t, x_i(t)), \quad 0 < \alpha \leq 1, \]  

where \( x_i(t) = [x_{i1}(t) \ x_{i2}(t) \ x_{i3}(t)]^T \) is the state variable in which \( x_{i1}(t) \) represents the interest rate; \( x_{i2}(t) \) denotes the investment demand and \( x_{i3}(t) \) is the price index;

\[
A = \begin{bmatrix}
-\delta_1 & 0 & 1 \\
0 & -\delta_2 & 0 \\
-1 & 0 & -\delta_3 \\
\end{bmatrix}, \quad g(t, x_i(t)) = \begin{bmatrix}
x_{i1}(t)x_{i2}(t) \\
1 - x_{i1}^2(t) \\
0 \\
\end{bmatrix}
\]

and positive constants \( \delta_1, \delta_2 \) and \( \delta_3 \) represent the saving amount, per-investment cost and elasticity of demands for commercials, respectively.

Here, the system parameters are taken as follows: \( \delta_1 = 3, \delta_2 = 0.1, \) and \( \delta_3 = 1. \) Now, we show that our proposed control can be applied to fractional-order financial system (33), which contains six identical nodes \( N = 6 \) and four coupling weights \( (\nu = 1, 2, 3, 4) \) given by

\[
D^\alpha_s x_i(t) = A x_i(t) + g(t, x_i(t)) + \sum_{i=1}^{4} \sum_{j=1}^{6} a_{ij} G^\nu_j x_j(t)
\]

\[ + C u_i(t) + D f_i(t) + E \omega_i(t), \]

\( \gamma_i(t) = x_i(t), \quad i = 1, 2, \ldots, 6, \) (34)

where the fractional-order of the system (34) is chosen as \( \alpha = 0.95, C = D = [6 \ 2 \ 3]^T, E = \text{diag}(0.5, 0.5, 0.5), \)
\( \Gamma^1 = \text{diag}(0.2, 0.3, 0.1), \Gamma^2 = \text{diag}(0.1, 0.1, 0.2), \Gamma^3 = \text{diag}(0.3, 0.2, 0.5), \Gamma^4 = \text{diag}(0.5, 0.1, 0.2) \) \( a_1 = 0.2, a_2 = 0.1, a_3 = 0.3 \) and \( a_4 = 0.5. \) The outer coupling matrices are taken as follows:

\[
G^\nu = \begin{bmatrix}
-0.9 & 0.1 & 0.1 & 0.2 & 0.2 & 0.3 \\
0.1 & -0.7 & 0.1 & 0.3 & 0.1 & 0.1 \\
0.2 & 0.1 & -0.8 & 0.1 & 0.2 & 0.2 \\
0.1 & 0.1 & 0.1 & -0.6 & 0.2 & 0.1 \\
0.1 & 0.1 & 0.1 & -0.5 & 0.1 & 0.1 \\
0.1 & 0.2 & 0.1 & 0.1 & 0.2 & -0.7 \\
\end{bmatrix}
\]

\[
G^2 = \begin{bmatrix}
-0.8 & 0.2 & 0.2 & 0.2 & 0.1 & 0.1 \\
0.1 & -0.5 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.1 & 0.2 & -0.7 & 0.2 & 0.1 & 0.1 \\
0.2 & 0.1 & 0.2 & -0.8 & 0.1 & 0.2 \\
0.2 & 0.1 & 0.1 & 0.1 & -0.6 & 0.1 \\
0.1 & 0.1 & 0.1 & 0.1 & 0.1 & -0.5 \\
\end{bmatrix}
\]

\[
G^3 = \begin{bmatrix}
-0.7 & 0.2 & 0.2 & 0.1 & 0.1 & 0.1 \\
0.2 & -0.6 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.1 & 0.1 & -0.5 & 0.1 & 0.1 & 0.1 \\
0.2 & 0.2 & 0.2 & -0.9 & 0.2 & 0.2 \\
0.3 & 0.1 & 0.1 & 0.2 & -0.8 & 0.1 \\
0.1 & 0.1 & 0.1 & 0.1 & 0.2 & -0.6 \\
\end{bmatrix}
\]
FIGURE 7. Fault $\eta_4(t)$ and its estimate $\hat{\eta}_4(t)$.

$$G^4 = \begin{bmatrix} -0.5 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & -0.6 & 0.1 & 0.1 & 0.2 & 0.1 \\ 0.2 & 0.2 & -0.8 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.2 & -0.9 & 0.2 & 0.3 \\ 0.2 & 0.1 & 0.3 & 0.1 & -0.8 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & -0.6 \end{bmatrix}.$$ 

Let the fault $f_i(t)$ ($i = 1, 2, 3, 4, 5, 6$) are selected as given below;

$$f_1(t) = \begin{cases} 0, & 0 \leq t < 1, \\ 0.2 \sin(40(t - 2)), & \text{elsewhere}, \end{cases}$$

$$f_2(t) = \begin{cases} 2 \sin(6.5 \pi t), & 0 \leq t \leq 6, \\ 0, & \text{elsewhere}, \end{cases}$$

$$f_3(t) = \begin{cases} 0.5 - 0.1(t - 5), & 2 \leq t \leq 5, \\ 0.2t, & \text{elsewhere}, \end{cases}$$

$$f_4(t) = \begin{cases} 5 \cos(0.2 \pi t), & 1 \leq t \leq 5, \\ 0, & \text{elsewhere}, \end{cases}$$

$$f_5(t) = \begin{cases} 0.18t, & 0 < t \leq 2, \\ 0.1, & 4 \leq t \leq 6, \\ 1 + 0.5 \sin(2\pi(t - 4)), & \text{elsewhere}, \end{cases}$$

$$f_6(t) = \begin{cases} 0, & 3 \leq t \leq 5, \\ 0.2 \cos(10\pi t), & \text{elsewhere}. \end{cases}$$

The rest of parameters involved in the simulation are set to be $\delta = 0.95$, $\varrho = 0.5$, $\nu = 0.1$ and $\lambda = 0.001$. Take, the uncertain matrices $M = \begin{bmatrix} 0.1 & 0.2 & 0.3 \end{bmatrix}$ and $N = \begin{bmatrix} 0.1 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}$.

Then, by solving the LMIs (31) and (32) in Theorem 2 via MATLAB LMI toolbox, we can obtain a set of feasible solutions from which the non-fragile state feedback control and observer gain matrices are computed by

$$K = \begin{bmatrix} -0.7500 & -1.7793 & -1.8458 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 46.5193 & 15.0933 & 20.3858 \\ 14.4223 & 19.4942 & 16.4488 \\ 20.1673 & 2.6178 & 23.7571 \end{bmatrix}.$$
For the simulation purposes, the external disturbance of all six nodes, the initial conditions for the states of nodes and the isolated node are respectively chosen as 

\[ \omega_i(t) = [0.1 \sin(2\pi t) \ 0.3 \cos(\pi t) \ 0.5 \sin(t)]^T, \]

\[ x_1(0) = [-10 \ 15 \ 10]^T, \]

\[ x_2(0) = [13 \ -12 \ -10]^T, \]

\[ x_3(0) = [15 \ -14 \ 12]^T, \]

\[ x_4(0) = [-17 \ 18 \ -11]^T, \]

\[ x_5(0) = [17 \ -17 \ 18]^T, \]

\[ x_6(0) = [20 \ -10 \ -15]^T \] and \[ s(0) = [23 \ 20 \ 19]^T. \] Based on the above values, the simulation
results are shown in Figs. 8-14. The first, second and third state curves of six nodes together with the isolate nodes state under the proposed control are plotted in Figs 8(a), 8(b) and 8(c), respectively. It can be easily observed from these figures that the states of the nodes are exactly synchronized with the states of the isolate node within a short period which demonstrates the effectiveness of the proposed controller. In Figs 9-14, the actual faults and their corresponding estimations are plotted, where it can be observed that the given signals are perfectly estimated from the beginning itself. Thus, from this simulation results, we can conclude that the proposed controller effectively synchronize the financial model even in the presence of external disturbances. Therefore, the results reveal that the proposed control design is more suitable for synchronizing the addressed financial model.

V. CONCLUSION

In this work, the non-fragile control problem has been analyzed for a class of fractional-order MWCDNs via intermediate estimator approach. Particularly, the Luenberger fractional-order state observer has been created to estimate the states of the addressed system. By using proper scalings of the faults, the synchronization problem is converted to an equivalent simultaneous uniformly ultimately boundedness problem for the corresponding error systems. Further, the uniformly ultimately boundedness criterion has been developed for the error system under consideration in terms of LMIs. At last, two numerical examples have been provided to demonstrate the effectiveness of the proposed controller. It is worth noting that the stochastic synchronization of fractional-order CDNs with multi-weights based on the intermediate estimator approach will be the topic of our future research work.

REFERENCES

[1] T. H. Lee, J. H. Park, H. Y. Jung, S. M. Lee, and O. M. Kwon, “Synchronization of a delayed complex dynamical network with free coupling matrix,” *Nonlinear Dyn.*, vol. 69, no. 3, pp. 1081–1090, Jan. 2012.

[2] Z.-G. Wu, J. H. Park, H. Su, B. Song, and J. Chu, “Exponential synchronization for complex dynamical networks with sampled-data,” *J. Franklin Inst.*, vol. 349, no. 9, pp. 2735–2749, Nov. 2012.

[3] Y. Wan, J. Cao, G. Chen, and W. Huang, “Distributed observer-based cyber-security control of complex dynamical networks,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 64, no. 11, pp. 2966–2975, Nov. 2017.

[4] X. Wu and Z. Nie, “Synchronization of two nonidentical complex dynamical networks via periodically intermittent pinning,” *IEEE Access*, vol. 6, pp. 291–300, 2018.

[5] M. J. Park, S. H. Lee, O. M. Kwon, and A. Seuret, “Closeness-centrality-based synchronization criteria for complex dynamical networks with interval time-varying coupling delays,” *IEEE Trans. Cybern.*, vol. 48, no. 7, pp. 2192–2202, Jul. 2018.

[6] N. Ma, Z. Liu, L. Chen, and F. Xu, “Finite-time $H_{\infty}$ fault-tolerant synchronization control for complex dynamical networks with actuator faults,” *IEEE Access*, vol. 7, pp. 128925–128935, 2019.

[7] Z. Xu, P. Shi, H. Su, Z.-G. Wu, and T. Huang, “Global $H_{\infty}$ pinning synchronization of complex networks with sampled-data communications,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 5, pp. 1467–1476, May 2018.

[8] X.-J. Li and G.-H. Yang, “Adaptive fault-tolerant synchronization control of a class of complex dynamical networks with general input distribution matrices and actuator faults,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 3, pp. 559–569, Mar. 2017.

[9] J.-L. Wang, M. Xu, H.-N. Wu, and T. Huang, “Passivity analysis and pinning control of multi-weighted complex dynamical networks,” *IEEE Trans. Neural Netw. Sci. Eng.*, vol. 6, no. 1, pp. 60–73, Jan. 2019.

[10] X.-X. Zhang, J.-L. Wang, Y.-L. Huang, and S.-Y. Ren, “Analysis and pinning control for passivity of multi-weighted complex dynamical networks with fixed and switching topologies,” *Neurocomputing*, vol. 275, pp. 958–968, Jan. 2018.

[11] Q. Wang, J.-L. Wang, Y.-L. Huang, and S.-Y. Ren, “Generalized lag synchronization of multiple weighted complex networks with and without time delay,” *J. Franklin Inst.*, vol. 355, no. 14, pp. 6597–6616, Sep. 2018.

[12] Y. Jia, H. Wu, and J. Cao, “Non-fragile robust finite-time synchronization for fractional-order discontinuous complex networks with multi-weights and uncertain couplings under asynchronous switching,” *Appl. Math. Comput.*, vol. 370, p. 124929, Apr. 2020.

[13] M. Chen, S.-Y. Shao, P. Shi, and Y. Shi, “Disturbance-observer-based robust synchronization control for a class of fractional-order chaotic systems,” *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 64, no. 4, pp. 417–421, Apr. 2017.

[14] Y. Jia and H. Wu, “Global synchronization in finite time for fractional-order complex networks with discontinuous dynamic nodes,” *Neurocomputing*, vol. 358, pp. 20–32, Sep. 2019.

[15] R.-J. Liu, Z.-Y. Nie, M. Wu, and J. She, “Robust disturbance rejection for uncertain fractional-order systems,” *Appl. Math. Comput.*, vol. 322, pp. 79–88, Apr. 2018.

[16] H.-L. Li, J. Cao, H. Jiang, and A. Alsaedi, “Finite-time synchronization of fractional-order complex networks via hybrid feedback control,” *Neurocomputing*, vol. 320, pp. 69–75, Dec. 2018.

[17] P. Selvaraj, O. M. Kwon, and R. Sakthivel, “Disturbance and uncertainty rejection performance for fractional-order complex dynamical networks,” *Neural Netw.*, vol. 112, pp. 73–84, Apr. 2019.

[18] Y. Liu, B.-Z. Guo, J. H. Park, and S.-M. Lee, “Nonfragile exponential synchronization of delayed complex dynamical networks with memory sampled-data control,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 1, pp. 118–128, Jan. 2018.

[19] Z. Zhang, H. Zhang, Z. Wang, and Q. Shan, “Delay-dependent robust $H_{\infty}$ filter design for uncertain neutral stochastic system with time-varying delay,” *IEEE Trans. Cybern.*, vol. 47, no. 8, pp. 2008–2019, Jul. 2017.

[20] Z.-G. Wu, J. H. Park, H. Su, and J. Chu, “Non-fragile synchronization control for complex networks with missing data,” *Int. J. Control*, vol. 86, no. 3, pp. 555–566, Mar. 2013.

[21] S. Cheng, H. Yang, and B. Jiang, “An integrated fault estimation and accommodation design for a class of complex networks,” *Neurocomputing*, vol. 216, pp. 797–804, Dec. 2016.

[22] J. Hu, Z. Wang, and H. Gao, “Joint state and fault estimation for time-varying nonlinear systems with randomly occurring faults and sensor saturations,” *Automatica*, vol. 97, pp. 150–160, Nov. 2018.

[23] Y.-H. Lan and Y. Zhou, “Non-fragile observer-based robust control for a class of fractional-order nonlinear systems,” *Syst. Control Lett.*, vol. 62, no. 12, pp. 1143–1150, Dec. 2013.

[24] J.-W. Zhu and G.-H. Yang, “Robust distributed fault estimation for a network of dynamical systems,” *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 14–22, Mar. 2018.

[25] Y.-H. Lan, H.-B. Gu, C.-X. Chen, Y. Zhou, and Y.-P. Luo, “An indirect Lyapunov approach to the observer-based robust control for fractional-order complex dynamic networks,” *Neurocomputing*, vol. 136, pp. 235–242, Jul. 2014.

[26] B. Kaviarasana, R. Sakthivel, and Y. Lim, “Synchronization of complex dynamical networks with uncertain inner coupling and successive delays based on passivity theory,” *Neurocomputing*, vol. 186, pp. 127–138, Apr. 2016.

[27] W. K. Wong, H. Li, and S. Y. S. Leung, “Robust synchronization of fractional-order complex dynamical networks with parametric uncertainties,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 17, no. 12, pp. 4877–4890, Dec. 2012.
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