Gauge field mimetic cosmology

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Abstract. We extend the mimetic cosmology to models containing gauge invariant $p$-forms. The 0-form case reproduces the well-known results of the mimetic dark matter, the 1-form corresponds to the gauge field mimetic model while the 2-form model is the Hodge dual of the 0-form model in 4 spacetime dimensions. We study the cosmological applications of the new gauge field mimetic model and show that it generates an energy density component which mimics the roles of spatial curvature. In the presence of the Maxwell term, the model also supports the flat, open and closed de Sitter-like cosmological backgrounds while the spatial geometry is flat for all three cases. We perform the cosmological perturbations analysis and show that the model is stable in the case of open de Sitter-like solution while it suffers from ghost instabilities in the case of the closed de Sitter-like solution.

Keywords: cosmological perturbation theory, modified gravity

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1 Introduction

Recently, the mimetic gravity has been suggested as a modification of general relativity such that one isolates the conformal degree of freedom of the metric by means of a scalar field [1, 2]. The setup can be equivalently realized from degenerate conformal/disformal transformation. The number of degrees of freedom then increases such that the longitudinal mode of gravity turns out to be dynamical [3–6]. Evidently, this new degree of freedom can play the roles of dark matter in cosmological setup [7]. Apart from these interesting features of the mimetic dark matter model (see refs. [8–18] for the recent works), the scalar mode is plagued with instabilities [19–21] and without taking into account some coupling between the second derivative of the scalar field and curvatures, one cannot find healthy perturbations [22–25]. The model also suffers from caustics that are formed virtually everywhere in the universe [26, 27].

Here, we would like to seek for the possibility of isolating the conformal degree of freedom of gravity by means of gauge fields rather than a scalar field. Some attempts have been made in this direction. In ref. [26], it is argued that the usual scalar-tensor mimetic gravity cannot admit rotating dark matter while the vector-tensor counterpart can simulate the rotating flows of dark matter. It is also shown that the vector-tensor extension of the mimetic gravity would be free of ghost instabilities [28]. In order to construct mimetic vector-tensor model via gauge fields, we briefly review the basic idea of the mimetic scalar-tensor gravity from the conformal/disformal transformation point of view.

Consider the conformal transformation $g_{\mu\nu} = A(\phi, K)\tilde{g}_{\mu\nu}$ between the physical metric $g_{\mu\nu}$ and an auxiliary metric $\tilde{g}_{\mu\nu}$ in which the coefficient $A > 0$ being the function of scalar
field $\phi$ and its canonical kinetic term $K = -\tilde{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$. Then the nontrivial singular limit of the transformation uniquely fixes the functional form of the conformal coefficient as $A = K [3, 4, 29]$ (up to an overall factor independent of $K$) which implies the following transformation

$$g_{\mu\nu} = \left(-\tilde{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi\right) \tilde{g}_{\mu\nu}. \quad (1.1)$$

It is easy to see that the physical metric $g_{\mu\nu}$ is invariant under the conformal transformation of the auxiliary metric $\tilde{g}_{\mu\nu}$ and also satisfy the following constraint

$$g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi = -1. \quad (1.2)$$

The transformation (1.1) is a particular (conformal) case of more general transformations known as disformal transformations [30]. The number of degrees of freedom does not change under non-degenerate disformal transformation [31]. However, the mimetic transformation (1.1) is degenerate and the number of degrees of freedom increases such that the scalar field $\phi$ is present even in the absence of usual matter and makes the longitudinal mode of gravity dynamical.

On the other hand, one can replace $\partial_\mu \phi$ with a vector field $A_\mu$ and construct a mimetic vector-tensor gravity [26] (see also refs. [32, 33]). These models can be equivalently obtained from the following mimetic (singular) vector transformation $g_{\mu\nu} = (-\tilde{g}^{\alpha\beta} A_\alpha A_\beta) \tilde{g}_{\mu\nu}$, provided that $A_\mu$ (instead of $A^\mu$) is transformed to itself. The model then implies constraint $g^{\mu\nu} A_\mu A_\nu = -1$ which is the well-known constraint for Einstein-Aether model [34]. This model however clearly breaks the local gauge invariance in the corresponding action. In the present paper we are interested in a gauge invariant generalization of the mimetic scenario. We therefore consider conformal transformation $g_{\mu\nu} = A(X) \tilde{g}_{\mu\nu}$ in which $X = -\tilde{g}^{\alpha\beta} \tilde{g}^{\sigma\rho} F_{\alpha\beta} F_{\rho\sigma}$ is the standard Maxwell term and $F_{\mu\nu}$ is the strength tensor associated to the gauge field. Following the same step as in the case of scalar field, it is straightforward to show that the singular limit of the transformation is given by (see the appendix A for the detail)

$$g_{\mu\nu} = \left(-\tilde{g}^{\alpha\beta} \tilde{g}^{\sigma\rho} F_{\alpha\beta} F_{\rho\sigma}\right)^{\frac{1}{2}} \tilde{g}_{\mu\nu}. \quad (1.3)$$

Note that the physical metric $g_{\mu\nu}$ is invariant under the conformal transformation of the auxiliary metric $\tilde{g}_{\mu\nu}$ in (1.3). Apart from systematic derivation of (1.3) in the appendix A, it is easy to understand why the square root of the scalar $F^2$ has appeared in (1.3): the inverse metric enters twice in $\tilde{g}^{\alpha\beta} \tilde{g}^{\sigma\rho} F_{\alpha\beta} F_{\rho\sigma}$ while it appears once in the case of $\tilde{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$.

The inverse of the metric (1.3) is given by $g^{\mu\nu} = (-\tilde{g}^{\alpha\beta} \tilde{g}^{\sigma\rho} F_{\alpha\beta} F_{\rho\sigma})^{-\frac{1}{2}} \tilde{g}_{\mu\nu}$. Multi-plying it with another inverse metric and then contracting the result with two strength tensors, it is easy to show that the conformal mimetic gauge field transformation (1.3) implies the following constraint

$$g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} F_{\mu\nu} = -1. \quad (1.4)$$

The above constraint is the gauge-invariant vector extension of the scalar mimetic constraint (1.2).

Note that the scalar mimetic constraint (1.2) is invariant under the shift symmetry $\phi \rightarrow \phi + c$ where $c$ is a constant while the new proposed mimetic constraint (1.4) is invariant

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1 We work in the mostly positive metric signature $(-, +, +, +)$. 

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under the gauge symmetry $A_\mu \rightarrow A_\mu - \partial_\mu \Lambda$, where $\Lambda$ is an arbitrary function. Considering $\phi$ to be a 0-form and $A_\mu$ to be the components of a 1-form, the goal of this work is to generalize the results (1.2) and (1.4) to a general $p$-form in a unified manner.

The rest of the paper is organized as follows. In section 2 we extend the mimetic constraint to the general case of gauge invariant $p$-form. In section 3 we study the cosmological applications of the gauge field mimetic model with global $O(3)$ symmetry, corresponding to the case $p = 1$. We study both the background dynamics and the cosmological perturbations. The summary of the paper and some discussions are presented in section 4. The analysis of singular conformal transformation which motivates our mimetic constraints for $p$-form gauge fields are presented in appendix A while the equivalence of the two models with $p = 0$ and $p = 2$ is demonstrated in appendix B.

2 Gauge-invariant mimetic $p$-forms

In this section, our aim is to formulate the general mimetic $p$-form constraint which reduces to the constraints (1.2) and (1.4) for the special cases $p = 0$ and $p = 1$ respectively.

Consider the $p$-form potential $A$ on the Lorentzian manifold $(\mathcal{M}, g)$ with $g$ being the metric. The associated field strength will be $F = dA$ where $d$ denotes the exterior derivative. The gauge-invariant Yang-Mills Lagrangian is given by $F \wedge \star F$. We define $\langle F, F \rangle$ by $F \wedge \star F = \langle F, F \rangle \star 1/(p + 1)!$. For $p = 0$ and with the scalar potential $\phi$, we find $\langle F, F \rangle = \partial^\mu \phi \partial_\mu \phi$ which is the standard kinetic term for the scalar field. In the same manner and for $p = 1$ we find $\langle F, F \rangle = F_{\mu \nu} F^{\mu \nu}$ where $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The standard mimetic constraint for the scalar field is given by $\partial^\mu \phi \partial_\mu \phi = -1$. Having this in mind we propose mimetic $p$-form constraint to be

$$\langle F, F \rangle = \mp 1. \quad (2.1)$$

For $p = 0$ and minus sign, the above constraint reduces to the standard scalar mimetic constraint (1.2) and for $p = 1$ and minus sign, it reduces to the gauge field mimetic constraint (1.4). The plus sign corresponds to other possibilities.

Having the $p$-form mimetic constraint (2.1) in hand, we can easily define the gauge-invariant mimetic $p$-form model as

$$S_p = \frac{1}{2} \int_\mathcal{M} [\star R - \lambda_p (\mathcal{F} \wedge \star \mathcal{F} \pm \star 1)] = \frac{1}{2} \int_\mathcal{M} d^4x \sqrt{-g} \left[ R - \lambda_p (\langle \mathcal{F}, \mathcal{F} \rangle \pm 1) \right], \quad (2.2)$$

where $\lambda_p$ are auxiliary fields which enforce the mimetic constraint (2.1).

Note that apart from the cosmological constant term, we can also add a general function $f(\langle \mathcal{F}, \mathcal{F} \rangle)$ to the above action which is forced to be constant by the mimetic constraint (2.1). In some sense, this term plays the roles of cosmological constant. We will explicitly see this fact in the next section.

Let us consider the action (2.2) for various values of $p$. For the case $p = 0$, the above action gives

$$S_0 = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R - \lambda_0 (\partial^\mu \phi \partial_\mu \phi \pm 1) \right]. \quad (2.3)$$

With the plus sign, the above action is nothing but the action of the mimetic dark matter model [35]. In the case of minus sign, the vector $\partial_\mu \phi$ is space-like while it is time-like in standard mimetic scenario and plays the roles of the gradient of the velocity potential [1, 2].
For the case of $p = 1$, we have
\[ S_1 = \frac{1}{2} \int d^4 x \sqrt{-g} \left[ R - \lambda_1 \left( F_{\mu\nu} F^{\mu\nu} \pm 1 \right) \right] . \] (2.4)

Since the electric field gives negative contribution to the $F^2$ term and the magnetic field gives positive contribution, depending on the situation we can consider both the positive and negative signs. We will see that considering three U(1) gauge-invariant vector fields and assuming a global $O(3)$ symmetry in the field space, the negative sign with purely electric contribution admits cosmological solution.

For the case of $p = 2$ and the potential 2-form $B = \frac{1}{2!} B_{\mu\nu} dx^\mu \wedge dx^\nu$, we have
\[ S_2 = \frac{1}{2} \int d^4 x \sqrt{-g} \left[ R - \lambda_2 \left( J_{\mu\nu\gamma} J^{\mu\nu\gamma} \pm 1 \right) \right] , \] (2.5)

where $J_{\mu\nu\gamma} = \partial_\mu B_{\nu\gamma} + \partial_\nu B_{\gamma\mu} + \partial_\gamma B_{\mu\nu}$ is the component of the field strength 3-form $J = dB$.

Finally, in the case of $p = 3$ the field strength is a 4-form, known as top form, and is proportional to the Levi-civita tensor. The corresponding mimetic constraint then only fixes the magnitude of a non-dynamical field which does not give any special result.

An interesting fact here is that, similar to the free field p-form models [36], the cases $p = 0$ with the action (2.3) and $p = 2$ with the action (2.5) are physically equivalent. In appendix B, we have shown that, at the level of equations of motion, these two models are related to each other through the Hodge duality. Therefore, the models (2.3) and (2.5) with $p = 0$ and $p = 2$ describe the well-known mimetic dark matter scenario which was extensively studied in the literature and we do not consider them any further here.

What remains is the case (2.4) with $p = 1$ which we are interested in this paper. This model is gauge-invariant and can be considered as the mimetic extension of the standard Einstein-Maxwell model. However, we are interested in cosmological applications of the model and for this purpose it turns out that the setup can be extended to accommodate a global $O(3)$ symmetry as we shall study in the next section.

### 3 1-form model: cosmological implications

There is only one gauge field in the mimetic 1-form model (2.4) which intrinsically carries a privileged direction and therefore inevitably introduces anisotropy.\(^2\) We are interested in cosmological applications of the model (2.4) but the model does not admit the isotropic FRW background solution. A natural extension of (2.4) that makes the model compatible with homogeneity and isotropy is to consider an orthogonal triplet of gauge fields [37] such that the field space enjoys a global $O(3)$ symmetry [38]

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} R - \lambda \left( \sum_{a=1}^{3} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + 1 \right) - \frac{1}{4} \mathcal{E} \sum_{a=1}^{3} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} \right] , \] (3.1)

where
\[ F_{\mu\nu}^{(a)} = \partial_\mu A_{\nu}^{(a)} - \partial_\nu A_{\mu}^{(a)} , \] (3.2)

\(^2\)One interesting case is to consider Bianchi geometries in this setup which is not the purpose of this paper.
\(\mu, \nu = 0, \ldots, 3\) are spacetime indices while \(a = 1, 2, 3\) denotes field space indices. The auxiliary field \(\lambda\) enforces the mimetic constraint
\[
\sum_{a=1}^{3} g^{\alpha \mu} g^{\beta \nu} F^{(a)}_{\alpha \beta} F^{(a)}_{\mu \nu} = -1. \tag{3.3}
\]

The generalization of the calculations of the appendix A to the case of global \(O(3)\) symmetry in field space is straightforward such that one can obtain the above constraint from the singular limit of the conformal transformation \(g_{\mu \nu} = A(X)\tilde{g}_{\mu \nu}\), with \(X = - \sum_{a=1}^{3} g^{\alpha \mu} g^{\beta \nu} F^{(a)}_{\alpha \beta} F^{(a)}_{\mu \nu}\).

In the action (3.1) we have also considered the Maxwell-like term (the term containing \(E\)) which, as we have already mentioned, would play the roles of cosmological constant through the mimetic constraint (3.3). Indeed, at the level of the action, it is obvious that one can replace the Maxwell-like term by a constant term via a redefinition of the Lagrange multiplier \(\lambda\). Having a term like a cosmological constant term signals the existence of de Sitter-like solution. We will explicitly show this fact in the rest of this section by means of the Raychaudhuri and the Friedmann equations.

Varying the action (3.1) with respect to \(A^{(a)}_{\mu}\), we obtain Maxwell-like equations
\[
\nabla_{\mu} ((E + 4\lambda) F^{(a)\mu \nu}) = 0.
\]
Varying the action with respect to the metric, one obtains the Einstein field equations \(G^{\mu \nu} = T^{\mu \nu}\) (in the unit where the reduced Planck mass \(M_P\) is set to unity), where \(G^{\mu \nu}\) is the Einstein tensor and \(T^{\mu \nu}\) is the effective energy momentum tensor given by
\[
T^{\mu \nu} = \frac{E}{4} \delta^{\mu \nu} + (E + 4\lambda) \sum_{a=1}^{3} F^{(a) \mu \alpha} F^{(a) \nu \alpha}.
\tag{3.4}
\]
Here, we have used the mimetic constraint (3.3) to simplify the expression. The auxiliary field can be easily obtained from the trace of Einstein’s equations as \(\lambda = -G/4\), where \(G = G^{\mu \mu}\) is the trace of the Einstein tensor.

### 3.1 Raychaudhuri equation

Before restricting ourselves to a particular metric, we can understand singularity properties of the spacetime by considering the Raychaudhuri equation
\[
\frac{d\theta}{d\tau} + \frac{1}{3} \theta^2 = -\sigma_{\mu \nu} \sigma^{\mu \nu} + \omega_{\mu \nu} \omega^{\mu \nu} - R_{\mu \nu} u^\mu u^\nu,
\tag{3.5}
\]
where \(\sigma_{\mu \nu}\) is the shear tensor, \(\omega_{\mu \nu}\) is the vorticity tensor, \(\theta = \nabla_{\mu} u^\mu\) is the expansion scalar and \(u^\mu\) is the four-velocity that is a timelike vector field satisfying \(u^\mu u_\mu = -1\). The expansion, shear, and vorticity would be obtained just after choosing a particular metric. On the other hand, according to the Penrose-Hawking singularity theorem [39], the singularity properties can be understood from the last term in (3.5). As long as the strong energy condition is satisfied, this term is always positive and therefore we conclude that the expansion rate diverges at some point, signalling the existence of a spacetime singularity. Therefore, we focus on this term to see whether it can be negative in our setup. From the Einstein equations we can write \(R_{\mu \nu} u^\mu u^\nu = (T_{\mu \nu} - \frac{1}{2} T g_{\mu \nu}) u^\mu u^\nu\) which, after substituting (3.4), gives the following result
\[
R_{\mu \nu} u^\mu u^\nu = -\frac{E}{4} - 2\lambda + (E + 4\lambda) \sum_{a} F^{(a) \mu \alpha} F^{(a) \nu \alpha} u^\mu u^\nu.
\tag{3.6}
\]
Employing the ADM decomposition of the metric
\[ ds^2 = -N^2 dt^2 + q_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]  
the mimetic constraint (3.3) can be decomposed into the electric and magnetic parts as follows
\[ -2 \sum_a q^{ij} F^{(a)}_{\perp i} F^{(a)}_{\perp j} + \sum_a q^{ik} q^{jl} F^{(a)}_{ij} F^{(a)}_{kl} = -1, \]  
where \( F^{(a)}_{\perp i} \equiv n^\mu F^{(a)}_{\mu i} \) with \( n^\mu \partial_\mu = (1/N)(\partial_t - N^i \partial_i) \), \( F^{(a)}_{\perp i} \equiv q^{ij} F^{(a)}_{\perp j} \) and \( q^{ij} \) is the inverse of \( q_{ij} \). This relation allows us to rewrite the electric part \( \sum_a q^{ij} F^{(a)}_{\perp i} F^{(a)}_{\perp j} \) in terms of the magnetic part \( \sum_a q^{ik} q^{jl} F^{(a)}_{ij} F^{(a)}_{kl} \). In general \( u^\mu \) is expanded as \( u^\mu = -u^i n_\mu + \tilde{u}^i (\partial_i \partial x^i)^\mu \) with \( (u^\perp)^2 - q_{ij} \tilde{u}^i \tilde{u}^j = 1 \), where \( u^\perp \equiv u^\mu n_\mu \) and \( \tilde{u}^i \equiv q^{ij} u_j \). It is then straightforward to show that
\[ F^{(a) \alpha} F^{(a) \nu \alpha} u^\mu u^\nu = q^{ij} F^{(a)}_{\perp i} F^{(a)}_{\perp j} - 2 u^i \tilde{u}^j q^{ik} F^{(a)}_{\perp k} F^{(a)}_{\perp j} + \tilde{u}^i \tilde{u}^j q^{kl} F^{(a)}_{\perp k} F^{(a)}_{\perp l} + \tilde{u}^i \tilde{u}^j q^{ij} \left( q_{ij} F^{(a)}_{\perp i} F^{(a)}_{\perp j} - F^{(a)}_{\perp i} F^{(a)}_{\perp j} \right). \]  

For simplicity let us consider a vorticity-free flow with \( u^\mu = n^\mu \). Using eq. (3.8), eq. (3.6) is then rewritten as
\[ R_{\mu \nu} u^\mu u^\nu = \frac{\mathcal{E}}{4} + (\mathcal{E} + 4\lambda) \sum_a q_{ij} B^{(a) i} B^{(a) j}, \]  
where we have defined the magnetic field \( B^{(a) i} \equiv \epsilon^{ijk} F^{(a)}_{jk}/2 \) and \( \epsilon^{ijk} \) is the 3-dimensional Levi-Civita tensor with \( \epsilon^{123} = 1/\sqrt{\det q} \). This is interesting if we note that in the case of vanishing magnetic part \( B^{(a) k} = 0 \) (or equivalently \( F^{(a)}_{ij} = 0 \)), the expression (3.10) reduces to
\[ R_{\mu \nu} u^\mu u^\nu = \frac{\mathcal{E}}{4} = \text{constant}. \]  

In the standard Einstein-Maxwell theory (with non-vanishing vorticity in general), both the electric part \( F^{(a)}_{\perp i} \) and the magnetic part \( F^{(a)}_{ij} \) contribute to \( R_{\mu \nu} u^\mu u^\nu \). In our setup, the mimetic constraint (3.8) relates electric and magnetic parts to each other such that one can substitute one in terms of another. In general the second line of (3.9) remains but if it vanishes then (3.11) holds in the case of vanishing magnetic part \( B^{(a) k} = 0 \) (or equivalently \( F^{(a)}_{ij} = 0 \)). The condition (3.11) also holds for a maximally symmetric spacetime with constant curvature, if it admits a configuration for which the magnetic part of the field strength vanishes and the second line of (3.9) also vanishes. For the case of curved spacetime with Lorentzian signature, in which we are interested here, there are two possibilities that are de Sitter and anti-de Sitter spacetimes. Regarding our setup with (3.11), de Sitter and anti-de Sitter cases are corresponding to \( \mathcal{E} < 0 \) and \( \mathcal{E} > 0 \) respectively. The de Sitter case \( \mathcal{E} < 0 \) implies negative values for the last term in Raychaudhuri equation (3.5) which shows that our model (3.1) can be nonsingular in this case. The negative values \( \mathcal{E} < 0 \) for the coupling constant of the Maxwell term in (3.1) would not introduce any disastrous features. This can be easily deduced if we note that the Maxwell term in (3.1) is forced to be constant through the mimetic constraint. Therefore, as we have explicitly shown in (3.11), the coupling constant of the Maxwell term in mimetic gauge field model plays the role of the cosmological
constant with the effective cosmological constant $\Lambda_{\text{eff}} = -\frac{\mathcal{E}}{4}$. For a de Sitter-like spacetime we require $\mathcal{E} < 0$.

The above discussions show that demanding the magnetic part of the gauge fields to vanish $F^{(a)}_{ij} = 0$, we can achieve a de Sitter-like solution in our model (3.1) which can also be nonsingular. We will see this fact in cosmological chart in the next section.

### 3.2 Cosmological background equations

In order to have a homogeneous and isotropic solution, we consider the following form for the components of the vector field [40–44]

$$A^{(a)}_{\mu} = A(t) \delta^a_{\mu},$$

in which the function $A(t)$ completely determines the magnitudes of the vector fields. Although we have three different vector fields, we demand that their magnitudes to be exactly the same. In order to preserve this choice, which admits a homogeneous and isotropic solution, we have considered the global $O(3)$ symmetry in the field space.

The components of the field strength tensors (3.2) are given by

$$F_{0i}^{(a)} = \dot{A} \delta^a_i, \quad F_{ij}^{(a)} = 0.$$  

Therefore, the magnetic part of the field strength tensors vanishes within the choice (3.12) for the vector fields. This is the desired condition to avoid the singularity in the presence of Maxwell term in our model (3.1) which was already shown by means of the Raychaudhuri equation in (3.10).

We consider the spatially flat FRW spacetime with background metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij}dx^i dx^j,$$

where $a(t)$ denotes the scale factor. Substituting eq. (3.13) into the mimetic constraints (3.3) (or equivalently in eq. (3.8)), we find that

$$\dot{A} = \frac{a(t)}{\sqrt{6}},$$

where we have assumed that $\dot{A} \geq 0$ without loss of generality.

We now consider the Einstein’s equations in the cosmological background (3.14) under the condition (3.15). The 00 component gives the Friedmann equation

$$3H^2 = 2\lambda + \frac{\mathcal{E}}{4},$$

where $H(t) = \dot{a}(t)/a(t)$ denotes the Hubble parameter.

The $ii$ components give the dynamical equation

$$2\dot{H} + 3H^2 = \frac{2}{3} \lambda - \frac{\mathcal{E}}{12}.$$  

From the above equations, we find the following solution for the auxiliary field $\lambda$

$$\lambda = -\frac{\mathcal{E}}{4} - \frac{3}{2} \dot{H}.$$  

In the following we consider two cases of $\mathcal{E} = 0$ and $\mathcal{E} \neq 0$. The first is the pure mimetic effect while the latter includes the effect of the Maxwell term which would mimic the roles of cosmological constant in our model through the mimetic constraint.
3.2.1 \( \mathcal{E} = 0 \): mimetic energy density as a spatial curvature

In the case of \( \mathcal{E} = 0 \), from eqs. (3.16) and (3.17), we easily find that the setup describes a perfect fluid with energy density \( \rho = 2\lambda \) and equation of state parameter \( w = -\frac{1}{3} \) which decays as

\[
\rho \sim a^{-2}. \tag{3.19}
\]

Thus the mimetic 1-form model behaves as an effective spatial curvature. This is in a sharp contrast to the mimetic 0-form model (2.3), i.e. the original mimetic model [1, 2], that reproduces a dark matter-like energy density component \( \rho \sim a^{-3} \) [1, 2].

Therefore, although we have started from a spatially flat FRW universe (3.14), the mimetic constraint, the second term in (3.1), mimics the roles of an effective spatial curvature.

3.2.2 \( \mathcal{E} \neq 0 \): de Sitter universes

The Maxwell term in (3.1) is forced to be constant through the mimetic constraint (3.3) and therefore it is natural to expect that \( \mathcal{E} \) plays the roles of cosmological constant as we have already shown in eq. (3.11). Indeed, through a redefinition of the Lagrange multiplier \( \lambda \), the Maxwell term can be replaced by a constant term at the level of the action. In order to explicitly see this fact at the level of the equations of motion, we substitute (3.18) into the Friedmann equation (3.16) which gives

\[
\dot{H} + H^2 = \frac{\Lambda_{\text{eff}}}{3}, \tag{3.20}
\]

where the effective cosmological constant is defined as

\[
\Lambda_{\text{eff}} = -\frac{\mathcal{E}}{4}. \tag{3.21}
\]

By integrating eq. (3.20) once, we obtain

\[
3 \left( H^2 + \frac{K_{\text{eff}}}{a^2} \right) = \Lambda_{\text{eff}}, \tag{3.22}
\]

where \( K_{\text{eff}} \) is an integration constant.

We have started with a spatially flat FRW universe in eq. (3.14), but eq. (3.22) is the same for the flat, closed and open de Sitter universes with \( K_{\text{eff}} = 0, > 0 \) and \( < 0 \), respectively. This is not surprising if we note that, as demonstrated in the previous subsection, the mimetic constraint term behaves exactly the same as the spatial curvature. The integration constant \( K_{\text{eff}} \) plays the role of the effective spatial curvature and determines the sign of \( \dot{H} \) such that \( K_{\text{eff}} = 0, < 0 \) and \( > 0 \) correspond respectively to flat, open and closed de Sitter-like universes which we study in turns below.

**Flat de Sitter universe.** From equation (3.18), we can see that the flat de Sitter solution \( K_{\text{eff}} = 0 \) corresponds to \( \lambda = -\frac{\mathcal{E}}{4} \). Solving eq. (3.22) in this case, we obtain the well-known results

\[
a(t) = \exp(H_\Lambda t), \tag{3.23}
\]

\[
H(t) = H_\Lambda, \tag{3.24}
\]
where

\[ H_\Lambda^2 = \frac{\Lambda_{\text{eff}}}{3}. \tag{3.25} \]

We have chosen the origin of the time coordinate \( t \) such that \( a(0) = 1 \). This solution is maximally symmetric and is nothing but the de Sitter spacetime.

**Open de Sitter-like universe.** Solving eq. (3.22) with \( K_{\text{eff}} = -1 \), we obtain

\[
a(t) = \frac{1}{H_\Lambda} \sinh(H_\Lambda t),
\]

\[
H(t) = H_\Lambda \coth(H_\Lambda t),
\]

where \( H_\Lambda \) is defined in (3.25) and, similar to the flat de Sitter-like case, we have chosen the origin of the time coordinate \( t \) such that \( a(0) = 0 \).

The time derivative of the Hubble expansion rate then turns out to be

\[
\dot{H}(t) = -H_\Lambda^2 \text{csch}^2(H_\Lambda t) < 0.
\]

The Hubble expansion rate is bounded from below but unbounded from above as \( H_\Lambda \leq H < \infty \). At the time \( t = 0 \), the universe hits the singularity such that the scale factor eq. (3.26) vanishes and the Hubble expansion rate eq. (3.27) diverges. This solution is not maximally symmetric since \( K_{\text{eff}} \) does not stem from the spatial curvature.

**Closed de Sitter-like universe.** Now, we focus on the case with \( K_{\text{eff}} > 0 \) in which the singularity can be avoided. Solving (3.22) with \( K_{\text{eff}} = 1 \), we obtain

\[
a(t) = \frac{1}{H_\Lambda} \cosh(H_\Lambda t),
\]

\[
H(t) = H_\Lambda \tanh(H_\Lambda t),
\]

in which \( H_\Lambda \) is given by eq. (3.25) and we have chosen the origin of the time such that \( H(0) = 0 \). Again, this solution is not maximally symmetric since \( K_{\text{eff}} \) does not stem from the spatial curvature.

In order to study the bouncing behavior of the model, it is useful to look at the time derivative of the Hubble expansion rate which is given by

\[
\dot{H}(t) = H_\Lambda^2 \text{sech}^2(H_\Lambda t) = H_\Lambda^2 - H^2 > 0.
\]

From eq. (3.30), it is clear that the Hubble expansion rate is bounded as

\[-H_\Lambda < H < H_\Lambda, \tag{3.32}\]

and it approaches \( \pm H_\Lambda \) as \( t \to \pm \infty \), where eq. (3.31) shows that \( \dot{H} \to 0 \). In this limit the universe undergoes an exponential expansion with constant Hubble expansion rate. On the other hand, the Hubble expansion rate vanishes at \( t = 0 \) where the scale factor (3.29) approaches its nonzero minimum value \( a_{\text{min}} = H_\Lambda^{-1} \). The universe starts from the past infinity \( t \to -\infty \) with \( H = -H_\Lambda < 0 \) and then bounces from the contracting phase with \( -H_\Lambda < H < 0 \) into the expanding era with \( 0 < H < H_\Lambda \) at \( t = 0 \). Finally, at the future infinity \( t \to +\infty \), the Hubble expansion rate approaches the constant maximum value...
$H = H_\Lambda$ with $\dot{H} = 0$ and a de Sitter phase with effective cosmological constant (3.21) and exponentially growing scale factor $a(t) = e^{H_\Lambda t}$ arises in this scenario.

From eq. (3.31), it is clear that the Hubble expansion rate always grows, $\dot{H} > 0$. On the other hand, $\dot{H} = -\frac{1}{2}(\rho + p)$ and therefore we have $\rho + p < 0$ so the null energy condition (NEC) is violated. Indeed, the violation of NEC is the common feature of bouncing models [45, 46] and it is commonly associated with ghost or gradient instabilities in scalar modes at the linear perturbations level [47]. It is also possible to construct some bouncing models which violate the NEC without introducing pathologies [48–53]. In the next section, we will see that the scalar and vector modes suffer from ghost instabilities whenever $\dot{H} > 0$.

### 3.3 Cosmological perturbations

In order to study the stability of our setup with the action eq. (3.1), in this section we perform its linear perturbations analysis.

The metric perturbations around the background geometry eq. (3.14) are given by

$$\delta g_{00} = 2\alpha, \quad \delta g_{0i} = a^2(\partial_i \beta + B_i), \quad \delta g_{ij} = a^2(2\psi \delta_{ij} + 2\partial_i \partial_j E + \partial_i F_j + \partial_j F_i + h_{ij}),$$

while the perturbations in gauge field with the global $O(3)$ symmetry are given by [38]

$$\delta A_{0}^{(a)} = Y_a + \partial_a Y, \quad \delta A_{i}^{(a)} = a(t)\left[\delta Q \delta_{ia} + \partial_i (\partial_a M + M_a) + \epsilon_{iab} \partial_b U + U_b \right] + t_{ia},$$

where $(\alpha, \beta, \psi, E, Y, \delta Q, M, U)$ are scalar modes, $(B_i, F_i, Y_a, M_a, U_a)$ are vector modes, and $(h_{ij}, t_{ia})$ label tensor modes. The vector and tensor modes satisfy the transverse and traceless conditions

$$\partial_i B_i = \partial_i F_i = \partial_i Y_i = \partial_i M_i = \partial_i U_i = 0, \quad \partial_i h_{ij} = \partial_i t_{ij} = 0, \quad h_{ii} = t_{ii} = 0.$$

The diffeomorphism invariance associated with the general coordinate transformation fixes two scalar modes and two vector modes. For the scalar modes, we work in the spatially flat gauge with $\psi = E = 0$ and for the case of vector modes we fix the gauge as $F_i = 0$.

Moreover, the model (3.1) enjoys the local gauge symmetry $A_{\mu}^{(a)} \rightarrow A_{\mu}^{(a)} - \partial_\mu \Lambda^a$. Decomposing $\Lambda^a$ into $\Lambda^a = \partial_a \Lambda + \Lambda^a_\perp$ with $\partial_a \Lambda^a_\perp = 0$, at the level of linear perturbations we have

$$\delta A_{\mu}^{(a)} \rightarrow \delta A_{\mu}^{(a)} - \partial_\mu \partial_a \Lambda - \partial_\mu \Lambda^a_\perp,$$

which for perturbations in eq. (3.34) implies

$$Y \rightarrow Y - \dot{\Lambda}, \quad M \rightarrow M - a^{-1} \Lambda,$$

$$Y_a \rightarrow Y_a - \dot{\Lambda}^a_\perp, \quad M_a \rightarrow M_a - a^{-1} \Lambda^a_\perp.$$

All the other perturbations in eq. (3.34) are invariant under the local gauge transformation (3.36). The above relations show that one scalar mode and two vector modes are not real physical degrees of freedom and can be removed through the local transformation (3.36). Thus, without loss of generality, we choose $M = 0$ and $M_a = 0$.

Apart from the above gauge degrees of freedom, the perturbations are also restricted by the mimetic constraint (3.3), which at the linear order implies

$$\sqrt{\frac{3}{2}} a \alpha + \partial^2 Y - 3\delta Q - \partial^2 M + \partial_i Y_i - \partial_i \dot{M}_i + \frac{a}{2\sqrt{6}} h_{ii} - t_{ii} = 0.$$
Imposing the transverse and traceless conditions (3.35), we can easily see that the mimetic constraint induced by (3.38) gives a relation among scalar modes in which we can write one scalar mode in terms of others as

$$\alpha = \sqrt{\frac{2}{3}} a^{-1} \left( -\partial^2 Y + 3\delta \dot{Q} + \partial^2 \dot{M} \right).$$  (3.39)

Setting $M = 0$ by the local gauge transformation eq. (3.37), this yields

$$\alpha = \sqrt{\frac{2}{3}} a^{-1} \left( -\partial^2 Y + 3\delta \dot{Q} \right).$$  (3.40)

In conclusion, after fixing all gauge freedoms and imposing the mimetic constraint we are left with four scalar modes ($\beta, Y, \delta Q, U$), six vector modes ($B_i, Y_a, U_a$), and four tensor modes ($h_{ij}, t_{ij}$). It turns out that the scalar modes ($\beta, Y$) and the vector modes ($B_i, Y_a$) are non-dynamical. Therefore, the quadratic action takes the following form

$$S^{(2)} = S^{(2)}_{S} (\delta Q, U) + S^{(2)}_{V} (U_a) + S^{(2)}_{T} (h_{ij}, t_{ij}).$$  (3.41)

Note that the scalar, vector, and tensor modes are decoupled from each other, thanks to the rotational symmetry of the background and the global $O(3)$ symmetry in the field space of the model given by the action (3.1).

Below we study each type of perturbations separately.

### 3.3.1 Scalar perturbations

Going to the Fourier space, after some calculations, it is straightforward to show that the quadratic action for the scalar modes is given by\(^3\)

$$S^{(2)}_{S} (\delta Q, U) = 6 \int d^3k d^3t a^3 \left( -\dot{H} \right) \left[ 3 \left( \delta Q^2 - \frac{k^2}{3a^2} \dot{\delta Q}^2 + \frac{\dot{H}}{H^2} (2\dot{H} + 3H^2) \delta Q^2 \right) + k^2 \left( \dot{U}^2 - \frac{k^2}{a^2} U^2 - \dot{H} U^2 \right) \right].$$  (3.42)

From the above action, we can easily see that both scalar modes ($\delta Q, U$) become ghost if we consider the bouncing (closed de Sitter-like) solution eq. (3.31) with $\dot{H} > 0$. On the other hand, both of them are free of any ghost and gradient/Laplacian instabilities for $\dot{H} < 0$. The mode $U$ may exhibit a tachyonic instability in infrared (IR) regime $k \to 0$ even for $\dot{H} < 0$. Such infrared instabilities are similar to the standard Jeans gravitational instabilities and are not necessarily a real pathology of the model [54]. Moreover, in the ultraviolet (UV) regime, the scalar mode $\delta Q$ propagates with the squared sound speed $c_s^2 = \frac{1}{3}$ while $U$ propagates with the speed of light. Finally, for $\dot{H} = 0$, the coefficients of the kinetic terms for the two scalar modes vanish and thus they may signal a strong coupling, depending on the behavior of nonlinear interactions.

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\(^3\)For the sake of simplicity, we denote the Fourier amplitude $A_k(t)$ by $A$ for all of the perturbations.
3.3.2 Vector perturbations

With the same manner as in scalar perturbations, we can find the quadratic action for the vector modes in the Fourier space as

\[
S^{(2)}_V(U_a) = 9 \sum_{a=1,2} \int d^3k dt a^3 \left( -\dot{H} \right) \left[ U_a^2 - \frac{k^2}{3a^2} U_a^2 - \frac{5}{3} H^2 U_a^2 \right].
\]  

(3.43)

The two vector modes become ghost for the bouncing (closed-de Sitter like) solution (3.31) with \( \dot{H} > 0 \) while they are free of any ghost and gradient/Laplacian instabilities for \( \dot{H} < 0 \). They also propagate with the squared sound speed \( c_v^2 = \frac{1}{3} \) in the UV regime and they have mass proportional to the Hubble expansion rate in the IR regime.

3.3.3 Tensor perturbations

The tensor sector of the model consists of \( h_{ij} \) from the metric and \( t_{ij} \) from the gauge fields and they couple with each other. It is straightforward to show that the quadratic action for the tensor modes in the Fourier space is

\[
S^{(2)}_T(h_{ij}, t_{ij}) = \frac{1}{2} \int d^3k dt a^3 \left( X^T K X + \dot{X}^T N X - X^T M X \right),
\]  

(3.44)

where \( X \) is a column vector with the component values \( X^i = (h_{11}, h_{12}, t_{11}, t_{12})^T \) and \( X^T \) denotes its transpose. The 4 \( \times \) 4 symmetric kinetic matrix \( K \) determines the coefficients of all kinetic terms which has nonzero components

\[
K_{11} = K_{22} = 1, \quad K_{33} = K_{44} = -24\dot{H},
\]  

(3.45)

and the 4 \( \times \) 4 matrix \( N \) has nonzero components

\[
N_{13} = N_{24} = -4\sqrt{6}\dot{H}.
\]  

(3.46)

The coefficients of the gradient and mass terms are determined by 4 \( \times \) 4 symmetric diagonal matrix \( M \) with the following nonzero components

\[
M_{11} = M_{22} = \frac{k^2}{a^2} + 2\dot{H}, \quad M_{33} = M_{44} = -24\dot{H} \left( \frac{k^2}{a^2} + \dot{H} \right).
\]  

(3.47)

Regarding the ghost instabilities, all of the information are encoded in the kinetic matrix \( K \) which is diagonal. Therefore each diagonal element in eq. (3.45) determines whether the corresponding mode is plagued with ghost-like instability or not. Since all of the elements are positive in the case of \( \dot{H} < 0 \), there would be no ghost instabilities for the tensor modes for \( \dot{H} < 0 \). However, for the bouncing (closed-de Sitter like) solution (3.31) with \( \dot{H} > 0 \), two of tensor modes associated with the gauge field \( t_{ij} \) become ghost.

The computation of the sound speeds is also easy. In the subhorizon limit, \( k^2/a^2 \gg H^2 \), all components of the friction matrix \( N \) are \( \mathcal{O}(k^0) \) and thus do not contribute to the squared sound speeds. In the same limit the mass matrix \( M \) is

\[
M = \frac{k^2}{a^2} K + \mathcal{O}(k^0).
\]  

(3.48)

Therefore, all four tensor modes propagate with the speed of light in the subhorizon limit and there is no Laplacian instability in the tensor sector.
4 Summary and discussions

In the original mimetic dark matter scenario, the conformal degree of freedom of the metric is isolated by means of a scalar field. In this work, we have extended the original mimetic scenario to general gauge invariant $p$-form scenarios. The 0-form case with scalar potential reproduces the original mimetic dark matter scenario. The 1-form case corresponds to a vector potential in which the conformal degree of freedom is isolated by the strength tensor of the vector potential. We have explicitly confirmed this fact by looking at the singular limit of the associated disformal transformation. We have shown that the remaining 2-form mimetic model is equivalent to the standard mimetic 0-form model through the Hodge duality.

We then studied the cosmological implications of the 1-form model. Considering a global $O(3)$ symmetry for the gauge field to allow for isotropic FRW backgrounds, we have obtained the associated cosmological background solutions. In comparison with the standard mimetic 0-form model where the scalar field produces a dark matter-like energy density component, we have found that the 1-form model produces energy density component like the spatial curvature. Due to the mimetic constraint, the Maxwell term (and any function of it) behaves like the cosmological constant term. Adding the Maxwell term we then found, flat, open and closed de Sitter-like solutions. Moreover, for the closed de Sitter-like setup we can obtain bouncing solution. Performing perturbations analysis, however, we have shown that the closed de Sitter-like solution suffers from ghost instabilities while the open de Sitter-like solution is stable. In the case of flat de Sitter-like solution, we found that the quadratic actions for the scalar and vector modes together with gauge field tensor modes vanish which, depending on the behavior of nonlinear interactions, may signal that the model is strongly coupled.

While our analysis show that the setup with $\dot{H} > 0$ suffers from ghost instabilities, one may wonder whether the Horndeski’s non-minimal coupling term \[ \sum_{a=1}^{3} \left( R F^{(a)\mu
u} F^{(a)\mu\nu} - 4 R_{\mu\nu} F^{(a)\mu} F^{(a)\nu} + R_{\mu\nu\alpha\beta} F^{(a)\mu\nu} F^{(a)\alpha\beta} \right), \] (4.1)
can remove the instabilities and provide stable bouncing solution. We have considered the effects of the above term on the stability analysis of our model, and have found that it is not possible to construct a stable bouncing model in our scenario. Specifically, our results show that we can find a region in the parameter space in which the scalar and vector modes are free of any ghost and gradient/Laplacian instabilities but the tensor sector is always sick in the bouncing background. With the mimetic constraint (3.3), we could generalize the Horndeski’s non-minimal coupling (4.1) by allowing the coefficient of the first term to be arbitrary but this generalization is equivalent to a redefinition of the Lagrange multiplier $\lambda_1$ and thus does not change the conclusion.

As we have seen, the effective energy density from the gauge field mimetic constraint mimics the roles of the spatial curvature in our setup. This inspires us to extend the setup to the spatially curved FRW universe. In order to do this, a simple analysis shows that we need to consider the SU(2) gauge symmetry instead of the $\text{U}(1) \times \text{U}(1) \times \text{U}(1)$ gauge symmetry. In the limit of zero gauge coupling constant, the model would reduce to the current global $O(3)$ model and therefore we expect to recover all of our results here. We would like to come back to this question in the near future.
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A Singular limit of gauge-invariant conformal transformation

In the Introduction section, we have mentioned that eq. (1.4) would be the natural gauge field extension of the scalar mimetic constraint eq. (1.2). Here we prove our claim through the same line that eq. (1.2) has been realized as the singular limit of a conformal/disformal transformation [3, 4].

In order to do this, we consider the following conformal transformation between the physical metric $g_{\mu \nu}$ and auxiliary metric $\tilde{g}_{\mu \nu}$

$$g_{\mu \nu} = A(X) \tilde{g}_{\mu \nu}, \quad (A.1)$$

where

$$X = -\tilde{g}^{\alpha \beta} F_{\alpha \beta} F_{\mu \nu}.$$

Note that the conformal transformation (A.1) is gauge-invariant. Our task is to see whether the transformation eq. (A.1) is invertible, i.e. if we can find the auxiliary metric in terms of the physical metric $\tilde{g}_{\mu \nu}(g_{\mu \nu})$. Since the coefficient $A$ in eq. (A.1) is a function of the auxiliary metric, we should look at the Jacobian of the transformation. Equivalently, we can study the following eigenvalue problem for the determinant of the Jacobian [29]

$$\left( \frac{\partial g_{\mu \nu}}{\partial \tilde{g}_{\alpha \beta}} - \eta_i \delta_{\alpha}^{\mu} \delta_{\beta}^{\nu} \right) \xi^i_{\alpha \beta} = 0, \quad (A.3)$$

in which $\eta_i$ are the eigenvalues and $\xi^i_{\alpha \beta}$ are the associated eigentensors.

From eq. (A.1), we can easily find that

$$\frac{\partial g_{\mu \nu}}{\partial \tilde{g}_{\alpha \beta}} = A \delta_{\mu}^{\alpha} \delta_{\nu}^{\beta} - 2 A X F^{\alpha \lambda} F_{\lambda}^{\beta} \tilde{g}_{\mu \nu}, \quad (A.4)$$

where the indices in the r.h.s. are raised and lowered by auxiliary metric $\tilde{g}_{\mu \nu}$. Substituting the above result in eq. (A.3) gives

$$(A - \eta_i) \xi^i_{\mu \nu} - 2 A X F^{\alpha \lambda} F_{\lambda}^{\beta} \xi^i_{\alpha \beta} \tilde{g}_{\mu \nu} = 0. \quad (A.5)$$

There are two sets of solutions for the above eigenvalue problem

$$\eta_C = A; \quad \xi^C_{\mu \nu} = w_{\mu \nu}, \quad \text{with} \quad F^{\alpha \lambda} F_{\lambda}^{\beta} w_{\alpha \beta} = 0, \quad (A.6)$$

$$\eta_X = A - 2 X A X; \quad \xi^X_{\mu \nu} = \tilde{g}_{\mu \nu}, \quad (A.6)$$
where, following the convention of [5, 6], the superscripts $C$ and $X$ denote the conformal-type and the kinetic-type eigenvalues. Note that the conformal-type eigenvalue is degenerate with multiplicity of nine since there is one constraint while the kinetic-type eigenvalue is non-degenerate. The singular limit of the transformation eq. (A.1) is given by $\eta = 0$. In the case of conformal-type eigenvalue $\eta_C = 0$, we obtain the trivial solution $A = 0$ while for the case of kinetic-type eigenvalue $\eta_X = 0$ we find the following nontrivial solution

$$A = \sqrt{X},$$ \hspace{1cm} (A.7)

where, without loss of generality, we have set the constant of integration to be unity. Substituting the above result in the conformal transformation eq. (A.1), we obtain eq. (1.3).

Therefore, the transformation eq. (1.3) is the natural extension of the scalar mimetic transformation eq. (1.1) to the case of gauge field. We have used this analogy in section 2 in order to investigate the mimetic constraint eq. (2.1) in terms of a general $p$-form.

B The equivalence of $p = 0$ and $p = 2$ models

In this appendix, we show that the mimetic 0-form and 2-form models defined by the actions (2.3) and (2.5) are equivalent to each other. More precisely, using the Hodge duality, we show that the field strengths of the two models satisfy the same equations of motion.

The 1-form field strength for the $p = 0$ model with 0-form scalar potential $\phi$ is given by $\Phi = d\phi$ and the Bianchi identity $d\Phi = 0$ implies

$$\nabla_\mu \Phi_\nu - \nabla_\nu \Phi_\mu = 0,$$ \hspace{1cm} (B.1)

where $\Phi_\mu = \partial_\mu \phi$ are the components of the field strength tensor. The variation of the action (2.3) with respect to $\phi$ gives the following modified Klein-Gordon equation

$$\nabla_\alpha (\lambda_0 \Phi^\alpha) = 0.$$ \hspace{1cm} (B.2)

Moreover, the variation with respect to the metric gives the energy-momentum tensor

$$T^{\mu}_\nu = 2\lambda_0 \Phi^\mu \Phi_\nu,$$ \hspace{1cm} (B.3)

in which we have used the mimetic constraint eq. (2.1) in the case of $p = 0$.

In the case of $p = 2$ with 2-form potential $B = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu$, the associated 3-form field strength is given by $J = dB$ and the Bianchi identity $dJ = 0$ then implies that

$$\nabla_\alpha J_{\beta\mu\nu} - \nabla_\beta J_{\mu\alpha\nu} + \nabla_\mu J_{\nu\alpha\beta} - \nabla_\nu J_{\alpha\beta\mu} = 0.$$ \hspace{1cm} (B.4)

Varying the action (2.5) with respect to the 2-form potential $B$, we obtain

$$\nabla_\alpha (\lambda_2 J^{\alpha\mu\nu}) = 0,$$ \hspace{1cm} (B.5)

while the variation with respect to the metric gives the corresponding energy-momentum tensor as

$$T^{\mu}_\nu = \lambda_2 J^{\mu\alpha\beta} J_{\nu\alpha\beta},$$ \hspace{1cm} (B.6)

where $J_{\mu\alpha\beta}$ are the components of the field strength tensor.
Rewriting eqs. (B.4), (B.5), and (B.6) in terms of the dual of the strength tensor $J$ which is defined as $J_\nu = \frac{1}{3!} \epsilon_{\alpha\beta\mu\nu} J^{\alpha\beta\mu}$, we have

\begin{align}
\nabla_\alpha J^\alpha &= 0, \quad \text{(B.7)} \\
\n\nabla_\mu (\lambda_2 J_\mu) - \nabla_\nu (\lambda_2 J_\nu) &= 0, \quad \text{(B.8)} \\
T^\mu_\nu &= 2\lambda_2 J^\mu J_\nu. \quad \text{(B.9)}
\end{align}

Comparing (B.8) with (B.1), we see that the two equations are the same if we consider the identification

$$\Phi_\mu \leftrightarrow \lambda_2 J_\mu. \quad \text{(B.10)}$$

Substituting the above relation into the equations (B.7) and (B.9) and then comparing the results with eq. (B.2) and eq. (B.3), we find the following correspondence between the Lagrange multipliers

$$\lambda_0 \leftrightarrow \frac{1}{\lambda_2}. \quad \text{(B.11)}$$

In some sense, the above relation shows the weak and strong duality between the 0-form and 2-form models.

In conclusion, we have shown that the 2-form model is the Hodge dual of the standard mimetic scenario which is given by the 0-form model in our classification.

References

[1] A.H. Chamseddine and V. Mukhanov, *Mimetic Dark Matter*, *JHEP* 11 (2013) 135 [arXiv:1308.5410] [nSPIRE].
[2] A.H. Chamseddine, V. Mukhanov and A. Vikman, *Cosmology with Mimetic Matter*, *JCAP* 06 (2014) 017 [arXiv:1403.3961] [nSPIRE].
[3] N. Deruelle and J. Rua, *Disformal Transformations, Veiled General Relativity and Mimetic Gravity*, *JCAP* 09 (2014) 002 [arXiv:1407.0825] [nSPIRE].
[4] F. Arroja, N. Bartolo, P. Karmakar and S. Matarrese, *The two faces of mimetic Horndeski gravity: disformal transformations and Lagrange multiplier*, *JCAP* 09 (2015) 051 [arXiv:1506.08575] [nSPIRE].
[5] H. Firouzjahi, M.A. Gorji, S.A. Hosseini Mansoori, A. Karami and T. Rostami, *Two-field disformal transformation and mimetic cosmology*, arXiv:1806.11472 [nSPIRE].
[6] J. Ben Achour, D. Langlois and K. Noui, *Degenerate higher order scalar-tensor theories beyond Horndeski and disformal transformations*, *Phys. Rev. D* 93 (2016) 124005 [arXiv:1602.08398] [nSPIRE].
[7] L. Mirzagholi and A. Vikman, *Imperfect Dark Matter*, *JCAP* 06 (2015) 028 [arXiv:1412.7136] [nSPIRE].
[8] Z. Haghani, M. Shiravand and S. Shahidi, *Energy conditions in mimetic-f(R) gravity*, *Int. J. Mod. Phys. D* 27 (2018) 1850049 [arXiv:1507.07726] [nSPIRE].
[9] R. Myrzakulov, L. Sebastiani, S. Vagnozzi and S. Zerbini, *Static spherically symmetric solutions in mimetic gravity: rotation curves and wormholes*, *Class. Quant. Grav.* 33 (2016) 125005 [arXiv:1510.02284] [nSPIRE].
[10] F. Arroja, N. Bartolo, P. Karmakar and S. Matarrese, *Cosmological perturbations in mimetic Horndeski gravity*, *JCAP* 04 (2016) 042 [arXiv:1512.09374] [nSPIRE].
[11] L. Sebastiani, S. Vagnozzi and R. Myrzakulov, *Mimetic gravity: a review of recent developments and applications to cosmology and astrophysics*, *Adv. High Energy Phys.* **2017** (2017) 3156915 [arXiv:1612.08661] [inSPIRE].

[12] N. Sadeghnejad and K. Nozari, *Braneworld Mimetic Cosmology*, *Phys. Lett. B* **769** (2017) 134 [arXiv:1703.06269] [inSPIRE].

[13] K. Takahashi and T. Kobayashi, *Extended mimetic gravity: Hamiltonian analysis and gradient instabilities*, *JCAP* **11** (2017) 038 [arXiv:1708.02951] [inSPIRE].

[14] J. Dutta, W. Khyllep, E.N. Saridakis, N. Tamanini and S. Vagnozzi, *Cosmological dynamics of mimetic gravity*, *JCAP* **02** (2018) 041 [arXiv:1711.07290] [inSPIRE].

[15] M.H. Abbassi, A. Jozani and H.R. Sepangi, *Anisotropic Mimetic Cosmology*, *Phys. Rev. D* **97** (2018) 123510 [arXiv:1803.00209] [inSPIRE].

[16] M. Rinaldi, L. Sebastiani, A. Casalino and S. Vagnozzi, *Mimicking dark matter and dark energy in a mimetic model compatible with GW170817*, arXiv:1803.02620 [inSPIRE].

[17] S. Brahma, A. Golovnev and D.-H. Yeom, *On singularity-resolution in mimetic gravity*, *Phys. Lett. B* **782** (2018) 280 [arXiv:1803.03955] [inSPIRE].

[18] J. de Haro, L. Aresté Saló and E. Elizalde, *Cosmological perturbations in a class of fully covariant modified theories: Application to models with the same background as standard LQC*, arXiv:1806.07196 [inSPIRE].

[19] S. Ramazanov, F. Arroja, M. Celoria, S. Matarrese and L. Pilo, *Living with ghosts in Hořava-Lifshitz gravity*, *JHEP* **06** (2016) 020 [arXiv:1601.05405] [inSPIRE].

[20] A. Ijjas, J. Ripley and P.J. Steinhardt, *NEC violation in mimetic cosmology revisited*, *Phys. Lett. B* **760** (2016) 132 [arXiv:1604.08586] [inSPIRE].

[21] H. Firouzjahi, M.A. Gorji and S.A. Hosseini Mansoori, *Instabilities in Mimetic Matter Perturbations*, *JCAP* **07** (2017) 031 [arXiv:1703.02923] [inSPIRE].

[22] S. Hirano, S. Nishi and T. Kobayashi, *Healthy imperfect dark matter from effective theory of mimetic cosmological perturbations*, *JCAP* **07** (2017) 009 [arXiv:1704.06031] [inSPIRE].

[23] Y. Zheng, L. Shen, Y. Mou and M. Li, *On (in)stabilities of perturbations in mimetic models with higher derivatives*, *JCAP* **08** (2017) 040 [arXiv:1704.06834] [inSPIRE].

[24] M.A. Gorji, S.A. Hosseini Mansoori and H. Firouzjahi, *Higher Derivative Mimetic Gravity*, *JCAP* **01** (2018) 020 [arXiv:1709.09988] [inSPIRE].

[25] D. Langlois, M. Mancarella, K. Noui and F. Vernizzi, *Mimetic gravity as DHOST theories*, arXiv:1802.03394 [inSPIRE].

[26] A.O. Barvinsky, *Dark matter as a ghost free conformal extension of Einstein theory*, *JCAP* **01** (2014) 014 [arXiv:1311.3111] [inSPIRE].

[27] F. Capela and S. Ramazanov, *Modified Dust and the Small Scale Crisis in CDM*, *JCAP* **04** (2015) 051 [arXiv:1412.2051] [inSPIRE].

[28] M. Chaichian, J. Kluson, M. Oksanen and A. Tureanu, *Mimetic dark matter, ghost instability and a mimetic tensor-vector-scalar gravity*, *JHEP* **12** (2014) 102 [arXiv:1404.4008] [inSPIRE].

[29] M. Zumalacárregui and J. García-Bellido, *Transforming gravity: from derivative couplings to matter to second-order scalar-tensor theories beyond the Horndeski Lagrangian*, *Phys. Rev. D* **89** (2014) 064046 [arXiv:1308.4685] [inSPIRE].

[30] J.D. Bekenstein, *The Relation between physical and gravitational geometry*, *Phys. Rev. D* **48** (1993) 3641 [gr-qc/9211017] [inSPIRE].
[31] G. Domènech, S. Mukohyama, R. Namba, A. Naruko, R. Saitou and Y. Watanabe, 
Derivative-dependent metric transformation and physical degrees of freedom, 
Phys. Rev. D 92 (2015) 084027 [arXiv:1507.05390 [INSPIRE]].

[32] A. Vikman, Superconducting Dark Matter, arXiv:1712.10311 [INSPIRE].

[33] V. Papadopoulos, M. Zarei, H. Firouzjahi and S. Mukohyama, 
Vector disformal transformation of cosmological perturbations, 
Phys. Rev. D 97 (2018) 063521 [arXiv:1801.00227 [INSPIRE]].

[34] T. Jacobson and D. Mattingly, Gravity with a dynamical preferred frame, 
Phys. Rev. D 64 (2001) 024028 [gr-qc/0007031 [INSPIRE]].

[35] A. Golovnev, On the recently proposed Mimetic Dark Matter, 
Phys. Lett. B 728 (2014) 39 [arXiv:1310.2790 [INSPIRE]].

[36] M. Thorsrud, Balancing Anisotropic Curvature with Gauge Fields in a Class of Shear-Free 
Cosmological Models, Class. Quant. Grav. 35 (2018) 095011 [arXiv:1712.02778 [INSPIRE]].

[37] A. Golovnev, V. Mukhanov and V. Vanchurin, Vector Inflation, 
JCAP 06 (2008) 009 [arXiv:0802.2068 [INSPIRE]].

[38] R. Emami, S. Mukohyama, R. Namba and Y.-l. Zhang, 
Stable solutions of inflation driven by vector fields, 
JCAP 03 (2017) 058 [arXiv:1612.09581 [INSPIRE]].

[39] S.W. Hawking and G.F.R. Ellis, The Large Scale Structure of Space-Time, Cambridge 
University Press (1973) [INSPIRE].

[40] J. Cervero and L. Jacobs, Classical Yang-Mills Fields in a Robertson-walker Universe, 
Phys. Lett. B 78 (1978) 427 [INSPIRE].

[41] M. Henneaux, Remarks on space-time symmetries and nonabelian gauge fields, 
J. Math. Phys. 23 (1982) 830 [INSPIRE].

[42] D.V. Galtsov and M.S. Volkov, Yang-Mills cosmology: Cold matter for a hot universe, 
Phys. Lett. B 256 (1991) 17 [INSPIRE].

[43] M.C. Bento, O. Bertolami, P.V. Moniz, J.M. Mourao and P.M. Sa, 
On the cosmology of massive vector fields with SO(3) global symmetry, 
Class. Quant. Grav. 10 (1993) 285 [gr-qc/9302034 [INSPIRE]].

[44] C. Armendariz-Picon, Could dark energy be vector-like?, 
JCAP 07 (2004) 007 [astro-ph/0405267 [INSPIRE]].

[45] J. Khoury, B.A. Ovrut, N. Seiberg, P.J. Steinhardt and N. Turok, 
From big crunch to big bang, 
Phys. Rev. D 65 (2002) 086007 [hep-th/0108187 [INSPIRE]].

[46] D. Battefeld and P. Peter, A Critical Review of Classical Bouncing Cosmologies, 
Phys. Rept. 571 (2015) 1 [arXiv:1406.2790 [INSPIRE]].

[47] J.M. Cline, S. Jeon and G.D. Moore, The Phantom menaced: Constraints on low-energy 
effective ghosts, 
Phys. Rev. D 70 (2004) 043543 [hep-ph/0311312 [INSPIRE]].

[48] P. Creminelli, M.A. Luty, A. Nicolis and L.Senatore, Starting the Universe: Stable Violation of the Null Energy Condition and Non-standard Cosmologies, 
JHEP 12 (2006) 080 [hep-th/0606090 [INSPIRE]].

[49] A. Ijjas and P.J. Steinhardt, Classically stable nonsingular cosmological bounces, 
Phys. Rev. Lett. 117 (2016) 121304 [arXiv:1606.08880 [INSPIRE]].

[50] D.A. Easson, I. Sawicki and A. Vikman, G-Bounce, 
JCAP 11 (2011) 021 [arXiv:1109.1047 [INSPIRE]].

[51] M. Osipov and V. Rubakov, Galileon bounce after ekpyrotic contraction, 
JCAP 11 (2013) 031 [arXiv:1303.1221 [INSPIRE]].
[52] M. Libanov, S. Mironov and V. Rubakov, Generalized Galileons: instabilities of bouncing and Genesis cosmologies and modified Genesis, JCAP 08 (2016) 037 [arXiv:1605.05992] [inSPIRE].

[53] D.A. Dobre, A.V. Frolov, J.T.G. Gheri, S. Ramazanov and A. Vikman, Unbraiding the Bounce: Superluminality around the Corner, JCAP 03 (2018) 020 [arXiv:1712.10272] [inSPIRE].

[54] A.E. Gümrukçüoğlu, S. Mukohyama and T.P. Sotiriou, Low energy ghosts and the Jeans’ instability, Phys. Rev. D 94 (2016) 064001 [arXiv:1606.00618] [inSPIRE].

[55] A. Maleknejad and M.M. Sheikh-Jabbari, Gauge-flation: Inflation From Non-Abelian Gauge Fields, Phys. Lett. B 723 (2013) 224 [arXiv:1102.1513] [inSPIRE].

[56] A. Maleknejad and M.M. Sheikh-Jabbari, Non-Abelian Gauge Field Inflation, Phys. Rev. D 84 (2011) 043515 [arXiv:1102.1932] [inSPIRE].

[57] E. Dimastrogiovanni and M. Peloso, Stability analysis of chromo-natural inflation and possible evasion of Lyth’s bound, Phys. Rev. D 87 (2013) 103501 [arXiv:1212.5184] [inSPIRE].

[58] P. Adshead, E. Martinec and M. Wyman, Perturbations in Chromo-Natural Inflation, JHEP 09 (2013) 087 [arXiv:1305.2930] [inSPIRE].

[59] R. Namba, E. Dimastrogiovanni and M. Peloso, Gauge-flation confronted with Planck, JCAP 11 (2013) 045 [arXiv:1308.1366] [inSPIRE].

[60] G.W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, Int. J. Theor. Phys. 10 (1974) 363 [inSPIRE].