On Moduli Stabilization Scheme in Type IIB Flux Compactifications

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ABSTRACT: We revisit the two-stage procedure for moduli stabilization in Type IIB orientifolds at light Kähler-modulus limit. In view of the necessity to keep the Kähler geometry structure of the moduli space during the stabilization, we define a holomorphic quantity called effective superpotential. The KKLT superpotential as well as the superpotential proposed by Villadoro and Zwirner are then examined with respect to this holomorphic effective superpotential. The mechanism is also illustrated with a simple toy model of one complex structure modulus.

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Searching for de Sitter or Minkowski vacua in 4-dimensional spacetime with spontaneously broken $\mathcal{N} = 1$ supersymmetry is undoubtedly a challenge in superstring phenomenology, which is directly related to the problem of moduli stabilization. Recently, some promising progress has been made in this direction, particularly in understanding moduli stabilization of Type IIB flux compactifications on Calabi-Yau orientifolds. The first encouraging proposal along this line was made by Kachru et al.\cite{1}, now known as KKLT in the literature. In the context of type IIB theory with D-branes, this mechanism can be used for stabilizing all closed string moduli and constructing de Sitter vacua through the incorporation of miscellaneous contributions of the closed string background fluxes, the D-brane related nonperturbative effects and the possible D-brane interactions. The idea of moduli stabilization is realized by a two-stage decoupled procedure. In the first stage, we have an incomplete F-term potential which is obtained by turning suitable 3-form fluxes on some 3-cycles of the Calabi-Yau orientifold and is independent of the Kähler-moduli of the compactification, in particular the overall volume modulus. This potential is then used to stabilize the complex structure moduli (including dilaton-axion field) at their extrema at a high scale while the remaining light Kähler moduli are kept free, therefore a low-energy no-scale effective $\mathcal{N} = 1$ supergravity theory with a constant superpotential $W_0$ is formulated for the remaining light Kähler moduli. In the second stage, the possible non-perturbative contributions from gauge theory on D7-branes and/or from Euclidean D3-instantons, that can produce an exponential dependence on the Kähler moduli in the superpotential, are included. These are in turn used to stabilize the light Kähler moduli at their extrema. So long as $W_0 \neq 0$ and the pullbacks of threefold Calabi-Yau divisors on a fourfold $X$ have arithmetic genus one\cite{2}, the stabilization of these light Kähler moduli is possible. The resulting vacuum happens to be purely supersymmetric and anti-de Sitter. KKLT further suggested to add the effect of anti D3-branes to the model for the purpose of lifting the SUSY preserving AdS vacuum to a SUSY breaking de Sitter vacuum.

KKLT proposal has raised a great enthusiasm in the research of string moduli stabilization and de Sitter vacua recently. However there still exist a few issues regarding the logical validity of this proposal and its successful implementation within a realistic string model. The ad hoc uplifting of the vacuum energy has been questioned since the supersymmetry breaking introduced by an anti-D3 brane is explicit and by hand. So far no consistent supergravity description of such an approach has been found. There are a few attempts trying to resolve this issue. The first unsuccessful effort was by considering a D-term uplifting in Ref.\cite{3} but it turned out not to work since the D-term considered there actually vanishes due to the vanishing F-term. However, this is remedied in a recent paper \cite{4} by Achúcarro et al (ACD) where an anomalous D-term can be generated due to an anomalous U(1). Other efforts for the uplifting have been considered in Ref.\cite{5} and in Ref.\cite{6}. In either case, the crucial thing is the gauged U(1) symmetry which gives rise to a consistent D-term responsible for the uplifting.

In almost all the models discussed above, the described two-stage procedure of KKLT has been used for the moduli stabilization. The validity of this two-stage procedure has been raised recently in \cite{7,8,9} in which it was argued that the nonperturbative contributions to the superpotential, if they exist at all, should be considered logically throughout the
whole stabilization process. The KKLT two-stage procedure, if reliable, must be a genuine light-Kähler-modulus approximation of such an one-stage procedure. Including the possible nonperturbative effects from the outset does generically violate the imaginary self-duality (ISD) of the 3-form fluxes\footnote{In $\mathcal{N} = 1$ supergravity, $G(T, \bar{T})$ is defined as $G(T, \bar{T}) = K(T, \bar{T}) + \ln|W(T)|^2$, which is invariant under the Kähler transformation $K(T, \bar{T}) \rightarrow K(T, \bar{T}) + \ln |f(T)|^2$, $W(T) \rightarrow \frac{1}{f(T)}W(T)$.} (which is the necessary condition for preserving $\mathcal{N} = 1$ supersymmetry during the stabilization of complex structure moduli and dilaton by turning suitable 3-form fluxes\footnote{In $\mathcal{N} = 1$ supergravity, $G(T, \bar{T})$ is defined as $G(T, \bar{T}) = K(T, \bar{T}) + \ln|W(T)|^2$, which is invariant under the Kähler transformation $K(T, \bar{T}) \rightarrow K(T, \bar{T}) + \ln |f(T)|^2$, $W(T) \rightarrow \frac{1}{f(T)}W(T)$.}) so the supersymmetry is spontaneously broken at the expected F-term potential level. In general, the one-stage procedure is too complicated and it is hard to deal with. Fortunately, in Ref.\cite{7} de Alwis proposed a modified version of the two-stage procedure, in which the imaginary self-duality of the 3-form fluxes is persisted and then the supersymmetry should not be broken during the moduli stabilization (if we ignore the contributions of massless squark condensation\footnote{In $\mathcal{N} = 1$ supergravity, $G(T, \bar{T})$ is defined as $G(T, \bar{T}) = K(T, \bar{T}) + \ln|W(T)|^2$, which is invariant under the Kähler transformation $K(T, \bar{T}) \rightarrow K(T, \bar{T}) + \ln |f(T)|^2$, $W(T) \rightarrow \frac{1}{f(T)}W(T)$.}, for example). The difference between KKLT original procedure and de Alwis’ improvement is that in de Alwis’ approach, the nonperturbative corrections to the superpotential are considered at both stages for moduli stabilization. de Alwis examined the KKLT procedure within his modified prescription and concluded that the original decoupled two-stage procedure can not be viewed as a proper approximation (at the light-Kähler-modulus limit) to the exact one-stage procedure.

Given that many models for uplifting (to the metastable deSitter vacua) are based on the KKLT two-stage moduli stabilization procedure, the justification of this decoupled procedure within the strict one-stage approach seems really imperative. In this short note we are going to make such a justification. As in Ref.\cite{7} by de Alwis, we take light Kähler-modulus approximation (light-$T$ limit for short from now on), insist on the imaginary self-duality of 3-form fluxes during the moduli stabilization and pay main attention to Kähler function $G(T, \bar{T}) = -3\ln(T + \bar{T}) + \ln(\Lambda(Ce^{-hT}, \bar{C}e^{-h\bar{T}}))$. Here $\Lambda(Ce^{-hT}, \bar{C}e^{-h\bar{T}})$ is defined as a power series expansion in terms of nonperturbative superpotential $Ce^{-hT}$. (See below). The new ingredient in our approach is that we put forward a concept of effective superpotential. This effective superpotential with holomorphicity is necessary if the two-stage moduli stabilization procedure at the light-$T$ limit preserves the Kähler geometry structure of the moduli space. The mathematical structure of the KKLT superpotential, a constant plus a term proportional to the nonperturbative superpotential, can be interpreted as this effective superpotential if we keep only the leading order terms in $\Lambda(Ce^{-hT}, \bar{C}e^{-h\bar{T}})$. Moreover, we observe that the Villadoro-Zwirner superpotential\footnote{In $\mathcal{N} = 1$ supergravity, $G(T, \bar{T})$ is defined as $G(T, \bar{T}) = K(T, \bar{T}) + \ln|W(T)|^2$, which is invariant under the Kähler transformation $K(T, \bar{T}) \rightarrow K(T, \bar{T}) + \ln |f(T)|^2$, $W(T) \rightarrow \frac{1}{f(T)}W(T)$.}, which consists of a single $T$-dependent exponential and was regarded to be very difficult to have a stringy realization\footnote{In $\mathcal{N} = 1$ supergravity, $G(T, \bar{T})$ is defined as $G(T, \bar{T}) = K(T, \bar{T}) + \ln|W(T)|^2$, which is invariant under the Kähler transformation $K(T, \bar{T}) \rightarrow K(T, \bar{T}) + \ln |f(T)|^2$, $W(T) \rightarrow \frac{1}{f(T)}W(T)$.}, will be realized as an effective superpotential if the power series $\Lambda(Ce^{-hT}, \bar{C}e^{-h\bar{T}})$ is approximated up to the second order terms $O(C^2e^{-2hT})$.

We now begin to report our results in detail. Firstly, let us give a brief review of the modified KKLT mechanism\footnote{In $\mathcal{N} = 1$ supergravity, $G(T, \bar{T})$ is defined as $G(T, \bar{T}) = K(T, \bar{T}) + \ln|W(T)|^2$, which is invariant under the Kähler transformation $K(T, \bar{T}) \rightarrow K(T, \bar{T}) + \ln |f(T)|^2$, $W(T) \rightarrow \frac{1}{f(T)}W(T)$.} and the improved uplifting prescription proposed in Ref.\cite{12}. As was shown in Ref.\cite{12}, the low energy gauge-invariant action for $\mathcal{N} = 1, D = 4$ supergravity with chiral and vector multiplets coming from Type IIB string theory compactified on a Calabi-Yau orientifold is determined by four ingredients: the real Kähler potential $K$, the holomorphic superpotential $W$, the holomorphic gauge kinetic function $f_{ab}$ and
the holomorphic Killing vectors. It is remarkable that although the D-term part of the supergravity scalar potential is governed by all these ingredients, the F-term potential is solely determined by the real Kähler potential $K$ and the holomorphic superpotential $W$, 

$$V_F = e^K (K^{i\bar{j}} D_i W D_j \bar{W} - 3|W|^2)$$ (1)

or equivalently determined by the invariant Kähler function $G$, 

$$V_F = e^G (G^{i\bar{j}} G_i G_j - 3)$$ (2)

where $G = K + \ln |W|^2$. For simplicity, here we only consider the orientifolds with just one overall Kähler modulus $T$. The classical Kähler potential$^2$ and the superpotential coming from fluxes and the nonperturbative contributions of the Euclidean instanton or gaugino condensation are then,

$$K = -3 \ln (T + \bar{T}) - \ln (S + \bar{S}) + N_f |M|^2 + k(z^i, \bar{z}^{\bar{j}})$$

$$W = A(z^i) + SB(z^i) + Ce^{-hT}$$ (3)

where $C = (N - N_f)(\frac{2}{M^{2N_f}})^{\frac{1}{N - N_f}}$ and $h = -\frac{2N_f(\sigma + \bar{\sigma})}{\delta_{GS}(N - N_f)}$. Here the nonperturbative contribution to the superpotential is from the condensation $(M^2)_j^i = 2Q^i \bar{Q}_j$ of $N_f$ massless squark pairs $\{Q^i, \bar{Q}_j\}$ with color group $U(N) = SU(N) \times U_X(1)$. $\delta_{GS}$, $q$ and $\bar{q}$ are the charges of Kähler modulus $T$ and the squark and anti squark under the anomalous gauge group $U_X(1)$, respectively. Under a $U_X(1)$ transformation the Kähler modulus transforms as $T \rightarrow T + i\frac{\delta_{GS}}{2N_f}$ while $Q^i \rightarrow e^{i\sigma} Q^i$ and $\bar{Q}_i \rightarrow e^{i\bar{\sigma}} \bar{Q}_i$. The Kähler potential is manifestly invariant under $U_X(1)$ transformation, and the anomaly cancelation condition further guarantees the invariance of the superpotential. Provided that the superpotential is reduced to the KKLT type $W = W_0 + Ce^{-hT}$ with $W_0$ an effective constant (after the complex structure moduli and dilaton-axion are integrated out) in the light-$T$ limit, the authors of Ref. showed that the unbroken supersymmetry conditions $D_T W = D_M W = 0$ cannot be simultaneously fulfilled for the F-term potential $V_F$, therefore indicating that the minimum of $V_F$ is at a supersymmetry breaking point in moduli space. A nonvanishing D-term potential can then be added to uplift the vacuum to be de Sitter.

Now the question is whether the superpotential can be cast as $W = W_0 + Ce^{-hT}$ in general in the light-$T$ limit. In the original KKLT scheme, such a superpotential was obtained via a decoupled two-stage procedure. KKLT first considered the 3-form fluxes $G_3 = F_3 + iSH_3$ to give rise to the superpotential $W_{flux} := \frac{1}{(2\pi)^2 \alpha'} \int \Omega \wedge G_3 = A(z^i) + SB(z^i)$ and used it to fix the complex structure moduli and dilaton-axion field via the supersymmetric extreme conditions $D_z W_{flux} = D_S W_{flux} = 0$. The validity of these conditions indicates that in the process of fixing the complex structure moduli (including dilaton-axion) the imaginary self-duality $G_3 = -i \ast_6 G_3$ of 3-form fluxes is preserved.

After that, the nonperturbative corrections to superpotential was added to define the “full”

$^2$For simplicity here we only consider the tree-level Kähler potential. However, inclusion of the possible perturbative and nonperturbative corrections to Kähler potential into the present scenario is straightforward.
superpotential $W = W_0 + Ce^{-hT}$, with $W_0$ the value of $W_{\text{flux}}$ at the supersymmetric minimum of F-term potential energy. The reliability of this ad hoc two-stage procedure was assumed in Ref.\cite{8} on the argument that in some region of moduli space the complex structure moduli and dilaton-axion field are heavy enough so they could be integrated out with the partial superpotential purely from the flux contributions. However, de Alwis pointed out that such an argument is untrustworthy if one takes the light-T approximation seriously\cite{9}. The point in de Alwis’ analysis is that the full superpotential in Eqs.\cite{3} should be used to solve the vanishing F-term conditions

$$D_S W = B(z^i) - (S + S)^{-1} W = 0$$
$$D_j W = \partial_j A(z^i) + S \partial_j B(z^i) + \partial_j k(z^i, \bar{z}^i) W = 0$$

(4)

The non-perturbative term $Ce^{-hT}$ considered explicitly in the full superpotential means that the dilaton-axion field $S$ and the complex structure moduli $z^i \ [i = 1, 2, \cdots, h^{(2,1)}]$ can not be integrated out holomorphically through the solutions of Eqs.\cite{3}, which in general depend on both of $Ce^{-hT}$ and its conjugate. Technically, due to the highly nonlinearity of Eqs.\cite{4}, exact solutions are in general not expected. Fortunately, at the light-T limit for which $h^T \gg 1$, approximated solutions can be found through a power series expansion as follows,

$$S = \alpha + \beta Ce^{-hT} + \gamma \bar{C}e^{-hT} + \zeta C^2 e^{-2hT} + \chi \bar{C}^2 e^{-2hT} + \psi|C|^2 e^{-h(T+T)} + \cdots$$
$$z^i = \alpha^i + \beta^i Ce^{-hT} + \gamma^i \bar{C}e^{-hT} + \zeta^i C^2 e^{-2hT} + \chi^i \bar{C}^2 e^{-2hT} + \psi^i|C|^2 e^{-h(T+T)} + \cdots$$

(5)

In either one-stage procedure\cite{8, 9} or the modified two-stage procedure of moduli stabilization developed by de Alwis\cite{7}, the heavy complex structure moduli and the dilaton-axion field are integrated out through Eqs.\cite{5}. Different from the original KKLT two-stage procedure, the non-Kähler moduli have not been fixed at their extreme values hereunto, instead they are integrated out as functions of both $Ce^{-hT}$ and $\bar{C}e^{-hT}$. The resultant superpotential does generically become a nonholomorphic function of the Kähler modulus $T$, i.e., $W = W(Ce^{-hT}, \bar{C}e^{-hT})$. In the one-stage procedure, the F-term potential $V_F$ is found to be

$$V_F = e^K \left[ \frac{1}{(T + \bar{T})^2} D_T W^2 - 3|W|^2 \right]$$
$$= e^K \left[ \frac{1}{(T + \bar{T})^2} \cdot \frac{1}{(S + S)^2} e^{k(z^i, \bar{z}^i)} \left[ \frac{1}{3}(T + \bar{T}) h^2 |C|^2 e^{-h(T+T)} + h(W \bar{C}e^{-hT} + W Ce^{-hT}) \right] \right]$$

(6)

where the Kähler derivative of superpotential with respect to the Kähler modulus $T$

$$D_T W = -hCe^{-hT} - \frac{3W}{(T + \bar{T})}$$

has been used. By Eqs.\cite{3} the superpotential is expressed as,

$$W = \xi_0 + \xi_{10} Ce^{-hT} + \xi_{01} \bar{C}e^{-hT} + \xi_{20} C^2 e^{-2hT} + \xi_{02} \bar{C}^2 e^{-2hT} + \xi_{11} |C|^2 e^{-h(T+T)} + \cdots$$

(7)

Similarly, the invariant Kähler function that depends only upon modulus $T$ and its conjugate after the non-Kähler moduli are integrated out becomes\cite{7} \footnote{The $U_X(1)$ symmetry still remains.},

$$G = -3 \ln(T + \bar{T}) + N_f |M|^2$$
$$+ \ln \left[ \nu + b Ce^{-hT} + \bar{b} C \bar{e}^{-hT} + c C^2 e^{-2hT} + \bar{c} \bar{C}^2 e^{-2hT} + d |C|^2 e^{-h(T+T)} + \cdots \right]$$

(8)
Based on these two series solutions to $W$ and $G$, we can formulate the F-term potential energy as:

\[
V_F = \frac{e^{N_f|M|^2}}{|\xi_0|^2(T+\bar{T})^2} \left[ h\nu\xi_0 Ce^{-hT} + h\nu\xi_0 \bar{C}e^{-h\bar{T}} + h\nu\bar{\xi}_0 C^2 e^{-2hT} + h\bar{b}\xi_0 \bar{C}^2 e^{-2h\bar{T}} - \frac{h\nu(\xi_0 + \xi_0\bar{\xi})}{\xi_0} C^2 e^{-2hT} + h\nu(\xi_0 + \bar{\xi}_0) \bar{C}^2 e^{-2h\bar{T}} + \frac{1}{3}(T + \bar{T}) h^2 |C|^2 e^{-h(T+\bar{T})} + h(b\xi_0 + \bar{b}\bar{\xi}_0) |\bar{C}|^2 e^{-h(T+\bar{T})} - \frac{h\nu(\xi_0 + \bar{\xi}_0)}{\xi_0} |\bar{C}|^2 e^{-h(T+\bar{T})} + \delta(C^3 e^{-3hT}) \right] \tag{9}
\]

Recall that the supersymmetric extremes are determined by conditions $D_3 W = D_3 \bar{W} = D_T W = 0$. These vacua (if exist) are bounded to be anti-deSitter spaces with potential energy $V_F = -3e^K|W|^2 < 0$. At such a supersymmetric vacuum the Kähler modulus $T$ is frozen by the solutions of the equation $\frac{3W}{(T+\bar{T})} = hC e^{-hT}$ that becomes

\[
\frac{3}{(T+\bar{T})} \left[ \xi_0 + \xi_{10} C e^{-hT} + \xi_{01} \bar{C} e^{-\bar{h}\bar{T}} + \xi_{20} C^2 e^{-2hT} + \xi_{02} \bar{C}^2 e^{-2h\bar{T}} + \xi_{11} |C|^2 e^{-h(T+\bar{T})} + \cdots \right] = hC e^{-hT}
\]

in the light-$T$ limit. Of course, there are probably some non-supersymmetric vacua determined by conditions $D_3 W = D_3 \bar{W} = \partial_T V_F = 0$.

Despite reliable in principle, the one-stage procedure is generically too fussy in technique to be used in practical moduli stabilization. The two-stage procedure suggested by KKLT, on the other hand, is much simpler. From the perspective of two-stage procedure, however, the nonholomorphicity of the superpotential (as the function of light Kähler modulus) in the second stage is absolutely unacceptable. Non-holomorphicity of the superpotential implies the spoilage of the Kähler geometry structure of the moduli space (in view of the two-stage procedure). Although the invariant function $G$ itself is allowed to be nonholomorphic, the equivalence between the two expressions of the F-term potential, $V_F = e^G(G^{TT}G_T G_T - 3)$ and $V_F = e^K(K^{TT}|D_T W|^2 - 3|W|^2)$, depends crucially upon the holomorphicity of the superpotential $W$ appearing in the relation $G = K + \ln |W|^2$. For this reason, in the two-stage procedure, it is still preferable to have an effective holomorphic superpotential $W_{\text{eff}}(T, M)$ so that

\[
G = -3 \ln(T + \bar{T}) + N_f |M|^2 + \ln |W_{\text{eff}}(T, M)|^2 \tag{10}
\]

and then the usual practice applies:

\[
V_F = \frac{e^{N_f|M|^2}}{(T + \bar{T})^2} \left[ \frac{1}{3}(T + \bar{T}) |\partial_T W_{\text{eff}}|^2 - (W_{\text{eff}} \partial_T W_{\text{eff}} + \bar{W}_{\text{eff}} \partial_T W_{\text{eff}}) \right] \tag{11}
\]

\(^4\text{The general supersymmetry breaking vacua are determined by } \partial_5 V_F = \partial_j V_F = \partial_T V_F = 0. \text{ However, we are interested in those vacua in which the supersymmetries are minimally broken.}\)}
The consistency of two expressions (8) and (11) for the F-term potential energy $V_F$ up to the second order of $Ce^{-hT}$ implies that the effective superpotential $W_{\text{eff}}(T, M)$, if it exists, should take the KKLT form,

$$W_{\text{eff}}(T, M) \approx W_0 + gCe^{-hT}$$

(12)

$W_{\text{eff}}(T, M)$ is in principle determined by the equivalence

$$|W_{\text{eff}}(T, M)|^2 = \left[v + bCe^{-hT} + \bar{b}Ce^{-h\bar{T}} + cC^2e^{-2hT} + \alpha C^2e^{-2h\bar{T}} + \bar{C}^2e^{-2hT} + d|C|^2e^{-h(T+\bar{T})} + \cdots\right]$$

(13)

between the two expressions (8) and (11) of the invariant Kähler function $G = G(T, \bar{T})$. The verification of these equivalences can actually be fulfilled order by order with respect to $Ce^{-hT}$ (and its complex conjugate) in light $T$ limit. To the first order of $Ce^{-hT}$, the coincidence of the two expressions of the F-term potential energy calculated within two different procedures demands $v = b\xi_0$, under which the effective superpotential exists and is given by Eq.(12) with $|W_0|^2 = v$ and $gW_0 = b$ (Here we suppose $v \neq 0$).\(^5\) To the second order of $Ce^{-hT}$, the equivalence of Eqs.(9) and (11) imposes more severe constraints $b\xi_0 = d\xi_0^2 = v$, $2c\xi_0 = b(1 - \xi_{10})$ and $\xi_0 = 0$ on the parameters of the series solutions (7) and (8). Eq.(13) is also very stringent that has no solutions unless $b^2 = vd^6$. In particular, if the parameters $v = b = c = 0$, $\xi_0 = 0$, $\xi_{10} = 1$, $\xi_{01} = 0$ but $d > 0$, the two procedures are equivalent up to $O(C^2e^{-2hT})$, with the effective superpotential in the two-stage procedure given by

$$W_{\text{eff}}(T, M) = \sqrt{d}Ce^{-hT}$$

(14)

It has the very form suggested in Ref.[3] for having a nonvanishing D-term uplift, thought difficult there in having an explicit stringy realization[11, 6].

We are now at the position to give a toy model to illustrate how the effective superpotential works. Consider a Type IIB orientifold with one overall Kähler modulus and just one complex structure modulus. The tree-level Kähler potential and the superpotential (from 3-from flux contribution plus nonperturbative corrections) read\[^6\]

$$K = -3\ln(T + \bar{T}) - \ln(S + \bar{S}) - \ln(U + \bar{U})$$

$$W = \alpha_0 + \alpha_1U + \alpha_2S + \alpha_3SU + Ce^{-hT} \quad (\alpha_i \in \mathbb{R}, \ h > 0)$$

(15)

For simplicity we ignore the contributions from the open string moduli, i.e., a constant prefactor $C$ is assumed in the above equations\[^7\]. We also assume that all of the flux parameters $\alpha_i$ ($i = 0, \cdots, 3$) are non-vanishing.

In terms of the full superpotential given in Eq.(14), we can explicitly write down the supersymmetry-preserving extreme conditions $D_UW = D_SW = 0$:

$$\alpha_0 + \alpha_1U - \alpha_2\bar{S} - \alpha_3\bar{SU} + Ce^{-hT} = 0$$

$$\alpha_0 - \alpha_1\bar{U} + \alpha_2S - \alpha_3SU + Ce^{-hT} = 0$$

(16)

\(^5\)If $v = 0$, the effective superpotential of the KKLT type does not exist.

\(^6\)The condition $b^2 = vd$ is generally an independent constraint although it has been implied by $b\xi_0 = d\xi_0^2 = v$ when $\xi_0 \neq 0$.

\(^7\)Though the massless squark pair condensation $M$ is crucial for having a supersymmetry-broken F-term vacuum[4], it is not important here in searching for the effective superpotential $W_{\text{eff}}(T, M)$. 

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By setting $U = u + iv$, $S = s + i\sigma$, $X = \frac{1}{2}(Ce^{-hT} + \bar{C}e^{-h\bar{T}})$ and $Y = \frac{1}{2i}(Ce^{-hT} - \bar{C}e^{-h\bar{T}})$, we can recast Eqs.(16) as,

$$\begin{align*}
\alpha_0 - \alpha_3(su + \sigma v) + X &= 0 \\
sv - \sigma u &= 0 \\
\alpha_2 s - \alpha_1 u &= 0 \\
\alpha_1 \nu + \alpha_2 \sigma + Y &= 0
\end{align*}$$

(17)

From Eqs.(17) we see that to ensure both $s$ and $u$ being fixed at the acceptable positive values $\alpha_1$ and $\alpha_2$ should have the same sign. Having a meaningful solution of this set of equations in the light-$T$ limit does also require both $\alpha_0$ and $\alpha_3$ have the same sign. Eqs.(17) are reduced to $\nu = -\frac{Y}{2\alpha_1}$, $\sigma = -\frac{Y}{2\alpha_2}$, $s = \frac{\alpha_1 u}{s}$ and

$$4\alpha_1^2\alpha_3 u^2 - 4\alpha_0 \alpha_1 \alpha_2 - 4\alpha_1 \alpha_2 X + \alpha_3 Y^2 = 0$$

In the light-$T$ limit, by solving the last equation we can write $u$ as a power series expansion in $X$ and $Y$,

$$u = \zeta \left[ 1 + \frac{X}{2\alpha_0} - \frac{X^2}{8\alpha_0^2} - \frac{\alpha_3 Y^2}{8\alpha_0 \alpha_1 \alpha_2} \right] + \cdots$$

(18)

with $\zeta = \sqrt{\frac{\alpha_0 \alpha_2}{\alpha_1 \alpha_3}}$ a real and positive parameter. The remaining equations determine the rest and up to the second order of $Ce^{-hT}$ the moduli $S$ and $U$ are expressed as:

$$\begin{align*}
S &\approx \frac{\alpha_1}{\alpha_3} \zeta \left[ 1 + \frac{X}{2\alpha_0} - \frac{X^2}{8\alpha_0^2} - \frac{\alpha_3 Y^2}{8\alpha_0 \alpha_1 \alpha_2} \right] - i \frac{Y}{2\alpha_2} \\
U &\approx \zeta \left[ 1 + \frac{X}{2\alpha_0} - \frac{X^2}{8\alpha_0^2} - \frac{\alpha_3 Y^2}{8\alpha_0 \alpha_1 \alpha_2} \right] - i \frac{Y}{2\alpha_1}
\end{align*}$$

(19)

These equations are used to integrate out the complex structure modulus and dilaton-axion field. After that, the superpotential acquires the form of Eq.(7) with the parameters given by,

$$\begin{align*}
\xi_0 &= 2(\alpha_0 + \alpha_1 \zeta) \\
\xi_{10} &= 1 + \frac{\zeta}{2\alpha_0 \alpha_2}(\alpha_1 \alpha_2 - \alpha_0 \alpha_3) \\
\xi_{01} &= 1 + \frac{\zeta}{2\alpha_0 \alpha_2}(\alpha_1 \alpha_2 + \alpha_0 \alpha_3) \\
\xi_{20} &= \frac{\alpha_2}{8\alpha_0 \alpha_2} - \frac{\zeta}{16\alpha_0 \alpha_2}(\alpha_1 \alpha_2 + \alpha_0 \alpha_3) \\
\xi_{02} &= \frac{\alpha_3}{8\alpha_0 \alpha_2} - \frac{\zeta}{16\alpha_0 \alpha_2}(\alpha_1 \alpha_2 - 3\alpha_0 \alpha_3) \\
\xi_{11} &= -\frac{\alpha_1}{4\alpha_1 \alpha_2} - \frac{\zeta}{8\alpha_0 \alpha_2}(\alpha_1 \alpha_2 + \alpha_0 \alpha_3)
\end{align*}$$

(20)

As expected, this naive superpotential depends only on the Kähler modulus $T$ but in a non-holomorphic manner. Plugging Eqs.(19), (7) and (20) into $G = \tilde{K} + \ln |W|^2$ we get the anticipated result

$$\begin{align*}
G(T, \bar{T}) &= -3 \ln(T + \bar{T}) + \ln \Lambda(Ce^{-hT}, \bar{C}e^{-h\bar{T}}), \\
\Lambda(Ce^{-hT}, \bar{C}e^{-h\bar{T}}) &= \frac{|W|^2}{(S + S)(U + U)} \\
&\approx \left[ v + b(Ce^{-hT} + \bar{C}e^{-h\bar{T}}) + c(C^2 e^{-2hT} + \bar{C}^2 e^{-2h\bar{T}}) + d |C|^2 e^{-h(T + \bar{T})} + \cdots \right],
\end{align*}$$

(21)
with parameters as follows,

\[
\begin{align*}
v &= \frac{\alpha_2}{\alpha_0} (\alpha_0 + \alpha_1 \zeta)^2 \\
b &= \frac{\alpha_2}{2\alpha_0} (\alpha_0 + \alpha_1 \zeta) \\
c &= -\frac{\alpha_3}{16\alpha_0^2} (\alpha_1 \alpha_2 - \alpha_0 \alpha_3) \\
d &= -\frac{\alpha_3}{8\alpha_0^2} (\alpha_1 \alpha_2 + \alpha_0 \alpha_3)
\end{align*}
\]

Therefore, for the model under consideration, the expansion of \( \Lambda(\text{Ce}^{-hT}, \text{Ce}^{-h0}) \) to the first order of \( \text{Ce}^{-hT} \) admits a KKLT-like effective superpotential

\[
W_{\text{eff}}(T) = W_0 + g \text{Ce}^{-hT}
\]

through \( \Lambda \approx |W_{\text{eff}}(T)|^2 \), with \( W_0 \approx \sqrt{\alpha_3/\alpha_0} (\alpha_0 + \alpha_1 \zeta) \) and \( g \approx \frac{1}{2} \sqrt{\alpha_3/\alpha_0} \). The requirement \( b \xi_0 = v \) for having a correct \( V_F \) (up to the first order of \( \text{Ce}^{-hT} \)) in the two-stage procedure is also in saturation. Notice if the original KKLT two-stage procedure is employed, the above two constants would be given as \( W_0 = 2(\alpha_0 + \alpha_1 \zeta) \) and \( g = 1 \) instead. Because the concept of effective superpotential in the modified two-stage procedure has its root in the exact one-stage procedure developed by Lüst et al[8], the similar form between \( W_{\text{eff}}(T) \) in Eq.(23) and the superpotential assumed in the original decoupled KKLT procedure may indicate the validity of the original KKLT procedure in sense of the effective superpotential \( W_{\text{eff}}(T) \) in this modified light-T limit.

The existence of the effective superpotential at the \( \Theta(C^2 e^{-2hT}) \) level of the \( \Lambda \)’s expansion is a genuine challenge because of the obligatory constraints \( v^2 = bd \) and \( \xi_{01} = 0^8 \). These constraints turn out to be so stringent that they are not satisfied in general. One interesting exception we find occurs if the flux parameters accidentally satisfy \( \zeta = -\alpha_0/\alpha_1 \) in which \( v = b = c = \xi_{01} = 0 \) and \( d = \frac{\alpha_2^2}{4\alpha_1 \alpha_2} > 0 \). Consequently, there exists an effective superpotential of the exponential type

\[
W_{\text{eff}}(T) = \frac{\alpha_3}{2\sqrt{\alpha_1 \alpha_2}} C e^{-hT}.
\]

This superpotential does not yield any minima in F-term potential \( V_F \), for which it was regarded to be unacceptable in the original KKLT procedure. However, it provides an appealing mechanism to break (spontaneously) supersymmetry. As mentioned earlier, such a holomorphic superpotential has been used by Villadora and Zwirner to establish a consistent D-term uplift scenario[7], where both \( V_F \) and \( V_D \) are monotonic functions of the Kähler modulus \( T \) and their sum (the full potential) has a possibility of giving a de Sitter vacuum where the modulus \( T \) is fixed.

In conclusion, we have revisited de Alwis’ modified two-stage procedure at light-T limit and reexamined the KKLT superpotential within such a scenario. This modified approach is found to be useful if the invariant Kähler function, after the non-Kähler moduli are integrated out, can be expressed in terms of an effective superpotential with holomorphicity. The invariant Kähler function consists of two terms of which both are logarithmic functions

\[\text{– 8 –}\]

\[^8\text{The remaining conditions } b \xi_0 = v \text{ and } 2 \xi_0 = b(1 - \xi_{10}) \text{ have been satisfied. See Eqs. (20) and (22).}\]
of the Kähler modulus. The first logarithmic function is nothing but the tree-level Kähler potential for Kähler modulus. The second logarithmic function, on the other hand, depends upon the Kähler modulus through a variable Λ which is generically a power series expansion of the nonperturbative superpotential and its complex conjugate. The KKLT-like superpotential turns out to appear as the leading-order approximation of the effective superpotential if Λ contains a nonvanishing constant term, otherwise it can not be understood within de Alwis’s approach. Besides, Villadora-Zwirner superpotential may be produced as an effective superpotential in this scenario if we approximate Λ to the second order of the nonperturbative superpotential. The analysis in the context can be straightforwardly extended to the multi-Kähler moduli orientifolds, from which the so-called “better race-track” superpotentials\[2, 16\] are expected to be reproduced as some appropriately defined effective superpotentials.

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