Optical turbulence simulations at Mt Graham using the Meso-NH model

S. Hagelin, 1,2★ E. Masciadri1★ and F. Lascaux1

1INAF Osservatorio Astrofisico di Arcetri, Largo Enrico Fermi 5, I-501 25 Florence, Italy
2Uppsala Universitet, Department of Earth Sciences, Villavägen 16, S-752 36 Uppsala, Sweden

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ABSTRACT

The mesoscale model Meso-NH is used to simulate the optical turbulence at Mt Graham (Arizona, USA), site of the Large Binocular Telescope. Measurements of the \( C^2_N \) profiles obtained with a generalized scidar from 41 nights are used to calibrate and quantify the model’s ability to reconstruct the optical turbulence above the site. The measurements are distributed over different periods of the year, permitting us to study the model’s performance in different seasons. A statistical analysis of the simulations is performed for all the most important astroclimatic parameters: the \( C^2_N \) profiles, the seeing \( \varepsilon \), the isoplanatic angle \( \theta_0 \) and the wavefront coherence time \( \tau_0 \).

The model shows a general good ability in reconstructing the morphology of the optical turbulence (the shape of the vertical distribution of \( C^2_N \)) as well as the strength of all the integrated astroclimatic parameters. The relative error (with respect to measurements) of the averaged seeing on the whole atmosphere for the whole sample of 41 nights is within 9.0 per cent. The median value of the relative error night by night is equal to 18.7 per cent, so that the model still maintains very good performances. Comparable percentages are observed in partial vertical slabs (free atmosphere and boundary layer) and in different seasons (summer and winter). We prove that the most urgent problem, at present, is to increase the ability of the model in reconstructing very weak and very strong turbulence conditions in the high atmosphere. This evidence in the model mainly affects, at present, the model’s performances for the isoplanatic angle predictions, for which the median value of the relative error night by night is equal to 35.1 per cent. No major problems are observed for the other astroclimatic parameters. A variant to the standard calibration method is tested but we find that it does not provide better results, confirming the solid base of the standard method.

Key words: turbulence – atmospheric effects – methods: numerical – site testing.

1 INTRODUCTION

Mt Graham International Observatory is located in south-eastern Arizona, USA, at 3200 m above sea level. The observatory consists of three telescopes: the Large Binocular Telescope (LBT; with two 8.4-m mirrors), the Heinrich Hertz Submillimetre Telescope (SMT; \( D = 10 \) m) and the Vatican Advanced Technology Telescope (VATT; \( D = 1.83 \) m). Some studies that aim to characterize the optical turbulence above Mt Graham have been carried out in the past with measurements mainly retrieved from a generalized scidar (GS; Egner & Masciadri 2007; Egner, Masciadri & McKenna 2007; Masciadri et al. 2010). In this paper we investigate the possibility of characterizing and predicting the optical turbulence (vertical distribution and integrated values) at Mt Graham using an atmospheric mesoscale model called Meso-NH. In a previous paper (Hagelin, Masciadri & Lascaux 2010), we used the same model to investigate the possibility of predicting the vertical wind speed distribution at Mt Graham. We proved that the Meso-NH model provides reliable estimates of the vertical distribution of the wind speed at all heights from the ground up to 20 km. This wind speed can therefore be used for the calculation of the wavefront coherence time \( \tau_0 \) on the Mt Graham summit.

The Meso-NH model has already been used to study the optical turbulence (OT) at different astronomical sites. It was first used by Masciadri, Vernin & Bougeault (1999a,b), who also developed the code for the optical turbulence in the Meso-NH, the so-called Astro-Meso-NH package, including the algorithms for the \( C^2_N \) parametrization. These were, to our knowledge, the first \( C^2_N \) profiles ever simulated with a mesoscale model in an astronomical context. In those studies, the model was proved to be sensitive to orographic effects and to be able to reconstruct \( C^2_N \) profiles well correlated to measurements provided by a scidar. Also, it was

★E-mail: hagelin@arcetri.astro.it (SH); masciadri@arcetri.astro.it (EM)
able to discriminate between the worst and the best seeing of the measurement campaign. Masciadri et al. (1999a,b) discussed the main limitations encountered in the $C_N^2$ modelling and proposed methods to overcome them.

Some improvements in the model reliability have been achieved more recently thanks to the method for the model calibration introduced by Masciadri & Jabouille (2001, hereafter MJ01). Such a calibration reduces some systematic errors and it is based on the tuning of a free parameter called the minimum turbulence kinetic energy ($E_{min}$). The authors proved that $C_N^2$ can be related to $E_{min}$ and the calibration aims to fix the value of $E_{min}$ using as a reference the measured $C_N^2$ profiles. MJ01 proved that the calibrated Meso-NH model reduced some of the previous systematic errors in the model, and the shape of the resulting $C_N^2$ profiles fitted better with measurements than the uncalibrated model. The statistic reliability of this method was proved only later by Masciadri, Avila & Sánchez (2004), who used the calibrated Meso-NH to simulate the optical turbulence at the San Pedro Mártir Observatory on a sample of 10 nights, for which there were $C_N^2$ profiles obtained with a GS and micro-thermal sensors mounted on radiosondes. Measurements provided by different instruments permitted the authors to prove that the dispersion between measurements and simulations was of the same order as the dispersion between measurements provided by different instruments. This qualified the numerical technique and the Meso-NH model as potentially useful to perform autonomous estimates of the optical turbulence. More recently (Masciadri & Egner 2006), the Meso-NH has been used in an autonomous way (after calibration) on a sample of 80 nights to simulate the optical turbulence at San Pedro Mártir.

The algorithms for the optical turbulence parametrization introduced by Masciadri et al. (1999a) as well as the calibration procedure (MJ01) were later implemented in other mesoscale models such as the Weather Research and Forecasting model (WRF; Kemp, Felton & Allis 2008).1

The goal of this paper is to use the Meso-NH model to simulate the optical turbulence at Mt Graham using, as a reference, a sample of measurements associated to 43 nights. A subsample of this rich statistical sample is used to calibrate the model. The outputs of the model are then compared to measurements obtained with a GS (Masciadri et al. 2010). The sample of measurements is around four times larger than the previous samples (Masciadri et al. 2004) and the largest ever used at present for this purpose. For this reason we can carry out the following tasks. (i) We can verify if the calibration permits the model to still obtain a typical vertical distribution well correlated with measurements and/or if this correlation increases/decreases. (ii) We can study how the correlation between numerical calculations and measurements deteriorates when we consider in the sample data not used for the calibration. (iii) We can study the ability of the Meso-NH model to reproduce the seasonal differences in the optical turbulence. The measurements are, indeed, quite evenly distributed over different periods. In this paper we also test a variant of the method of model calibration proposed by MJ01.

In Section 2 we briefly present the measurements used as a reference in this study. In Section 3 we present the Meso-NH model and describe the model configuration used in this study. In Section 4 we describe the calibration procedure. In Section 5 we present the results of this study in four subsections dedicated, respectively, to the vertical distribution of the optical turbulence ($C_N^2$ profiles), the seeing $\epsilon$, the isoplanatic angle $\theta_0$ and the wavefront coherence time $\tau_0$. In Section 6 we present the conclusions of this study.

2 MEASUREMENTS OF REFERENCE

$C_N^2$ profiles obtained with a GS (Egner et al. 2007; Masciadri et al. 2010) mounted on the VATT have been used as a reference for this study. The GS is based on the observation of binaries with a typical separation $\theta$ within $(3–10)$ arcsec; the binary magnitude $m_1$, $m_2 \leq 5$ mag and $\Delta(m_{1,2}) \leq 1$ mag. The GS needs a telescope with a pupil size $\geq 1.5$ m. The $C_N^2$ profiles are obtained from the inversion of the autocorrelation of the scintillation map of binaries (Avila, Vernin & Masciadri 1997; Fuchs, Tallon & Vernin 1998). This instrument provides a vertical distribution of the optical turbulence on the whole 20 km from the ground with a vertical resolution that scales as $0.78\sqrt{\lambda(h-h_{gs})/\theta}$, where $h$ is the height from the ground and $h_{gs}$ is the height under ground at which the conjugated plane is optically placed. The GS provides a vertical resolution, which, considering the parameter space just described, is typically of the order of 1 km on the whole atmosphere, reaching the best resolution near the ground (of the order of some hundreds of metres). GS measurements are the best choice (with respect to other vertical profilers) for a mesoscale model validation for a few reasons: (i) measurements are obtained with a remote sensing technique and they are therefore available for an extended period of time during a night; (ii) the $C_N^2$ is obtained with a completely independent and autoconsistent method requiring no calibration; (iii) we can access directly the $C_N^2$ (i.e. the prime element from which all the integrated astroclimatic parameters can be calculated autonomously). In Appendix A we report the analytical equations that describe how the seeing, the isoplanatic angle and the wavefront coherence time are calculated from the $C_N^2$.

3 MODEL CONFIGURATION

The Meso-NH is a non-hydrostatic mesoscale model developed jointly by the Centre National des Recherches Météorologiques (CNRM, Météo-France) and Laboratoire d'Aérogologie in Toulouse, France (Lafore et al. 1998). It is a grid-point model based on the anelastic approximation to efficiently filter out the acoustic waves. A Gal-Chen & Sommerville (1975) coordinate on the vertical and a C-grid in the formulation of Arakawa & Mesinger (1976) for the spatial digitalization are used. The temporal scheme is an explicit three-time-level leap-frog scheme with a time filter (Asselin 1972). The turbulent scheme is a one-dimensional 1.5 closure scheme (Cuxart, Bougeault & Redelsperger 2000). The model permits the use of different mixing lengths. In this paper, we have used the one-dimensional Bougeault & Lacarrère (1989) mixing length (BL89). The surface exchanges are computed in an externalized surface scheme (SURFEX) including the physical package Interactions Soil Biosphere Atmosphere (ISBA; Noilhan & Planton 1989), which controls the air/ground turbulent fluxes budget of Meso-NH. The model can simulate the temporal evolution in three dimensions of the classical meteorological parameters, such as wind speed and direction, potential temperature, pressure, and so on. Meso-NH uses the code implemented by Masciadri et al. (1999a,b) to forecast the optical turbulence ($C_N^2$ three-dimensional maps) and all the astroclimatic parameters deduced from the $C_N^2$. We refer to the ‘Astro-Meso-NH code’ to indicate this package. The integrated astroclimatic parameters are calculated integrating $C_N^2$ with respect to the zenith in the Astro-Meso-NH code.

1 See also the web page for the Mauna Kea Weather Center, http://mkwc.ifa.hawaii.edu.
turbulence layers at the different heights of the atmosphere appear well reconstructed by the model.\(^2\) The spatial variability of the simulated turbulence is, in general, smoother than that measured. This effect is more evident in the high part of the atmosphere. This is what we expect from the calculations of a parametrized parameter that is not explicitly resolved. Also, the temporal variability of the simulated turbulence appears smoother with respect to that observed. It is difficult to say that the value of a parametrized parameter can be predicted at a precise time \(t = t_0\). For this reason, so far we have preferred to provide averaged estimates of the optical turbulence.

We calculate the mean of the \(C^2_N\) profiles simulated over the whole night (with a sampling of the \(C^2_N\) profile every 2 min in the interval \(19:00–05:00\) LT) and we associate the result with the mean of the observed \(C^2_N\) profiles obtained during the same night with the GS. Therefore, we compare typical measured and simulated \(C^2_N\) profiles. Hereafter, we discuss the performance of the Meso-NH model under this assumption. This logic is the same as used in all previous studies carried out on the modelling of the optical turbulence with Meso-NH (Masciadri et al. 1999b; Masciadri & Garfias 2001; Masciadri, Vermin & Bougeault 2001; Masciadri et al. 2002, 2004; Masciadri & Egner 2006; Lascaux et al. 2009, 2010).

### 4 MODEL CALIBRATION

#### 4.1 MJ01 method

The method for the model calibration implemented in Meso-NH was proposed by MJ01, and later validated by Masciadri et al. (2004). It is based on the idea that the model depends on a free parameter \(E_{\text{min}}\), i.e. the minimum kinetic energy, that can be considered as a sort of background climatological noise. In the regions in which the dynamic turbulence is well developed, the model rapidly forgets the \(E_{\text{min}}\) value and the kinetic energy overcomes this value. \(C^2_N\) is therefore not dependent on \(E_{\text{min}}\) in these regions. However, in stable regions, the authors proved that \(C^2_N \propto E^{2/3}_{\text{min}}\). In each \(C^2_N\) profile, it is possible to identify typically more than one vertical slab in which turbulence is in stable regimes (see fig. 1 of MJ01). If we change the value of \(E_{\text{min}}\), it means that we assume that the atmosphere has more or less inertia in different vertical slabs and the

\(^{2}\)LT denotes Local Time.

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**Table 1.** Meso-NH configuration. The second column gives the horizontal resolution \(\Delta X\) of the imbricated models, the third column the grid points, the fourth column the surface covered by the models and the fifth column the time-steps used by the model.

| Model   | \(\Delta X\) (km) | Grid points | Surface (km) | Time-step (s) |
|---------|-------------------|-------------|--------------|---------------|
| Model 1 | 10                | 80 × 80     | 800 × 800    | 30            |
| Model 2 | 2.5               | 64 × 64     | 160 × 160    | 6             |
| Model 3 | 0.5               | 120 × 120   | 60 × 60      | 3             |

The Meso-NH model is run, in this paper, in a grid-nesting mode using three imbricated models with different resolutions (Table 1). These are all centred on the Mt Graham Observatory (32.7013°N, 109.8919°W). The size of the outermost model (model 1) is 800 × 800 km, which covers most of south-eastern Arizona, south-west New Mexico and also a part of north-western Mexico (see Fig. 1). The middle model (model 2) covers 160 × 160 km, using a grid size of 2.5 km. The innermost model (model 3) has a resolution of 500 m and covers an area of 60 × 60 km. The vertical resolution is the same for all three models with 54 vertical grid points, reaching up to 3.5 km above the ground. Above 3.5 km, the resolution is almost constant and equal to \(\sim 600\) m.

The model is initialized and forced every 6 h at the synoptic hours (00:00, 06:00, 12:00 UTC) with analyses from the European Centre for Medium-Range Weather Forecasts (ECMWF). The model runs for 12 h; the first 2 h of every simulation are rejected because the model is still adapting to the orography. The output from the remaining 10 h (19:00–05:00 LT)\(^2\) is used for the characterization of the optical turbulence at Mt Graham. The Astro-Meso-NH model provides the vertical profile of \(C^2_N\) for every 2 min at the grid point located at the astronomical Observatory. Fig. 2 (left) shows an example of the temporal evolution of \(C^2_N\) extended on 20 km obtained with the Astro-Meso-NH package. Fig. 2 (right) shows the corresponding temporal evolution of the measured \(C^2_N\) profiles obtained with the GS (Masciadri et al. 2010). In these figures, we can appreciate the characteristics of the simulated \(C^2_N\) profiles. Most of the
thermodynamic instabilities require more or less energy to trigger turbulence in some regions of the atmosphere. As a consequence, the threshold (the seed $E_{\text{min}}$) should be different in the respective vertical slabs. To identify the optimal values of $E_{\text{min}}$, the main idea in the MJ01 method was therefore to divide the whole atmosphere (~20 km) into a finite number (5–6) of vertical slabs and to optimize the value of $E_{\text{min}}$ in each vertical slab, minimizing the differences between simulated and measured $C_N^2$ profiles (i.e. minimizing the $\chi^2$ function for each night $m$):

$$\chi_{m,k}^2 = \sum_{i=1}^{N_i} [a_{m,k} x_{m,i} - y_{m,j}]^2.$$  

(1)

Here, $N_i$ is the number of levels in the vertical slab $k$, $y_{m,j}$ is the average of the measured $C_N^2$ sampled on $N_i$ levels for each night $m$, $x_{m,i}$ is the simulated $C_N^2$ sampled on $N_i$ levels for each night $m$ and $a_{m,k}$ is the free coefficient that has to be fixed minimizing the function $\chi_{m,k}^2$. We refer the reader to MJ01 for details; we simply summarize here that, after an average of all $a_{m,k}$ with respect to the number of nights $m$, we obtain a coefficient $a_k$ for each vertical slab. Knowing the value of $a_k$ for $k = (1, K)$, where $K$ is the number of vertical slabs, the kinetic energy is modified as

$$E_{\text{min},k} = E_{\text{min},k}^{1/2}, \quad k = 1, K.$$  

(2)

Once the optimized $E_{\text{min}}$ is identified, characterized by $K$ steps, this is implemented in the model that is run again for each night. We highlight that the same new $E_{\text{min}}$ is used to simulate the new $C_N^2$ for all the nights. This is what we call ‘an output obtained with a calibrated model’. These are the results discussed in the next section, with respect to measurements.

The MJ01 method reduces some systematic error and reconstructs a typical mean $C_N^2$ profile that agrees better with the measurements. The method has been validated statistically by Masciadri et al. (2004) on a sample of 10 nights using measurements provided by different instruments and taken simultaneously. It is worth noting that the method optimizes $E_{\text{min}}$ with respect to the mean $C_N^2$ profiles simulated on the total sample used to calibrate the model. The calibration is based basically on the conservation of the turbulent energy ($J = C_N^2 \Delta H$) in each vertical slab. The differences between measurements and simulations calculated on each night are, in general, obviously larger than the difference between the statistical values (average in this case), as we discuss later in the paper. The interest in increasing the statistical sample for the calibration is to verify how the reliability of the model changes, increasing the number of nights.

In this study, the model calibration started with a selection of the sample of nights on which we calibrated the model. A brief digression is necessary here. In an ideal case, assuming that we have a very rich sample of measurements (e.g. a year of measurements), we should calibrate the model using as many different nights as possible in order to approach a profile of the $E_{\text{min}}$ characteristic of the site. Besides, once the model is calibrated, the ideal case should be to have an equally rich sample of independent measurements (and associated simulations) to investigate the performance of the model after calibration. It is also worth highlighting that the goal of the calibration is to reduce/eliminate a precise systematic error, and therefore it is our interest to eliminate from the sample used for the calibration all cases that correspond to unusual results, which might be associated with a failure of the model for whatever reason. These data, if introduced into the calibration sample, might bias the calibration. In other words, for the calibration, a subjective selection is not only allowed but it is also suggested without diminishing confidence in the results.

In this study, we can access a sample of 43 nights. In spite of the great number of nights with respect to previous studies, it is still too small a number to be able to perform the strategy just described using two independent samples equally statistically rich. We therefore decided on an alternative solution, which is a sort of compromise that maximizes the progress (answers to open questions) we can make in this field with this data set. First, we are interested in limiting the sources of uncertainties in our sample of reference. The assumption made is necessarily to consider the measurements as a reference. In other words, we perform the approximation that measured $C_N^2$ profiles match with the true $C_N^2$ profiles. For this reason, we eliminated two nights (2008 March 2 and 3) from the original sample of 43 nights, because the relative shape of the mean of the

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**Figure 2.** Left: $C_N^2$ temporal evolution predicted by the Meso-NH model on 2008 March 1 from 00 to 12 UTC. The simulated $C_N^2$ profiles extend to 20 km. The temporal sampling is of one $C_N^2$ profile each 2 min. Right: $C_N^2$ profiles measured by the GS during the same night.
$C_n^2$ profile on each night was characterized by unusual features that could affect the calibration process. Among the remaining 41 nights, we subtracted a subsample of seven nights (~17 per cent of the total sample) in which the model provided unusual results that might let us think that for some reason the model did not work correctly, independently of the algorithm used for the $C_n^2$ parametrization. We therefore calibrate the model on the remaining sample of 34 nights, which is the richest sample achievable with this data set. We therefore study the statistical performances of the model calibration carried out on 34 nights, separating the calibration aspect from the model score of success. After this, we quantify the model performances on the whole sample of 41 nights (i.e. a sample including also the values of the seven nights that have not been considered in the calibration). As said before, this sample contains many cases that might be associated with bad model performances. This strategy permits us to achieve some progress in this research field. At the same time, we are able (i) to optimize the calibration process, (ii) to discuss the model performance, including an independent sample of nights not considered in the calibration, and (iii) to investigate the model performance when it contains a realistic sample of failure cases.

After the identification of the samples, we divided the atmosphere into six vertical slabs above (0–400 m), (400 m–2 km), (2–7 km), (7–11 km), (11–13 km) and (13–20 km) and we applied the model calibration for $h \geq 400$ m. Below this height, we observed that the model no longer depends on $E_{\text{min}}$ and the seed is quickly forgotten by the model during the simulation. As already explained in previous papers, the thickness and the number of vertical slabs are arbitrary. We have selected regions in which the turbulence seems to be characterized by similar trends.

4.2 Variant to the MJ01 method

In this paper, we test whether the calibration procedure for $E_{\text{min}}$ carried out on each model level instead of vertical slabs (as is the case for the MJ01 method) might produce substantially better results or not. The minimization of the $\chi^2$ function is carried out for each model level,

$$E_{\text{min},i}^* = E_{\text{min},i}^{3/2}, \quad i = 1, N,$$

where $N$ is the total number of model levels. This variant has been suggested by several colleagues in private communications after the publication of the first results obtained with the MJ01 method (Masciadri et al. 2004; Masciadri & Egner 2006). From a purely mathematical point of view, such a method might in theory work better on the calibration sample (34 nights) because the mathematical fit has more constraints (a total number of measurements equal to the number of levels) on which to constrain the fit. However, from a physical point of view, this method is characterized by a questionable physical assumption. It is indeed hard to justify it because it would be as to admit that the climatological noise $E_{\text{min}}$ is different at each model level and the values of $E_{\text{min}}$ are the same in each model level for all the nights. This is the reason why we think that, at present, the original MJ01 version is the most solid approach from a physical point of view. However, in this paper, taking advantage of the rich statistical sample, we have decided, independently from the arguments that might justify (or not) the MJ01 variant, to simply test this variant, first, to verify if the gain is effective and, secondly, to check if this gain is maintained with samples containing nights not included in the calibration sample (in our case the sample of 41 nights). To simplify the discussion, we hereafter call this method MJ01*.

5 RESULTS

To discuss the results, we divide the solar year into two periods: the summer (April–September) and the winter (October–March). We investigate the optical turbulence vertical distribution ($C_n^2$ profiles) and the three major integrated astroclimatic parameters (seeing, isoplanatic angle and wavefront coherence time). For $C_n^2$ and the seeing, we show first the results obtained with the sample of 34 nights used for the calibration and, afterwards, the results obtained with the total number of 41 nights. For the isoplanatic angle and the wavefront coherence time, we treated directly the total sample of 41 nights.

When simulated versus measured $C_n^2$ profiles are treated, it is important to define how to statistically analyse the data. This is not univocal. As an example, Fig. 3 shows, for a pedagogic approach, four $C_n^2$ profiles calculated from the sample of observed $C_n^2$ profiles: (i) the median of all the individual $C_n^2$ profiles of the whole sample of 41 nights (15 956 profiles; red line); (ii) the median of the 41 $C_n^2$ profiles associated with each night (each $C_n^2$ profile is obtained by averaging the $C_n^2$ of a night; green line); (iii) the mean of all the individual $C_n^2$ profiles (15 956 profiles) of the whole sample of 41 nights (light blue line); (iv) the average of the 41 $C_n^2$ profiles associated with each night (each $C_n^2$ profile is obtained by averaging the $C_n^2$ of a night; dark blue line). The difference between these $C_n^2$ profiles tells us that, depending on how we treat the data statistically, we obtain different results. In studies on site characterization carried out with measurements taken with the monitor approach, (i) is frequently used. Our context, however, is different.

![Figure 3. Observed $C_n^2$ profiles. Red line: median of all the $C_n^2$ profiles (15 956) of the total sample of 41 nights. Green line: median of the 41 $C_n^2$ profiles, each being the average of the $C_n^2$ of one night. Light blue line: mean of all the observed $C_n^2$ profiles (15 956) of the total sample of 41 nights. Dark blue line: mean of the 41 $C_n^2$ profiles associated with each night and obtained by averaging the $C_n^2$ of one night.](https://academic.oup.com/mnras/article-abstract/412/4/2695/1022536)
As we have said, it is hard to predict \( C_N^2 \) at a precise time \((t = t^*)\) of the night. At present, in order to investigate quantitatively the performances of the model, our goal is to associate the average of the \( C_N^2 \) profiles simulated during one night with the average of the measured \( C_N^2 \) profiles of the same night. For this reason, we are forced to eliminate approaches (i) and (iii). Moreover, the calibration is based basically on the fitting of the average of \( M C_N^2 \) profiles (\( M \) is the number of nights). We selected therefore the criterion (iv) and we considered the ‘mean’ as a statistical operator to quantify the model performances of \( C_N^2 \). This is the same approach used in Masciadri et al. (2004). For homogeneity, we considered the same criterion with the integrated astroclimatic parameters (seeing, isoplanatic angle and wavefront coherence time).

5.1 \( C_N^2 \) profiles: optical turbulence vertical distribution

Fig. 4 shows the average of the \( C_N^2 \) profiles measured with the GS and simulated with the calibrated model with the methods MJ01 and MJ01*. In the same picture, we also report the average \( C_N^2 \) before the calibration as well as those calculated in the two seasons: summer and winter. Fig. 5 shows the same for the 41 nights.

The agreement between the morphology (shape) of the averaged \( C_N^2 \) profiles measured by the GS (red lines) and calculated with the Meso-NH (light blue lines, MJ01; dark blue lines, MJ01*) is very good when looking at the average of all 34 nights (Fig. 4, left). The model can reconstruct all the major typical features of the \( C_N^2 \) profile, such as the position (at \( \sim 10 \) km) and shape of the secondary peak. Between 4 and 8 km above the ground, the MJ01* method seems slightly better than the MJ01 method. The vertical distribution of the optical turbulence in the winter (Fig. 4, right) is also very well described by the Meso-NH model. However, in summer (Fig. 4, centre) the model seems to overestimate \( C_N^2 \) in the free atmosphere. However, the MJ01 method is better than the MJ01* method. The MJ01 method is indeed well correlated to measurements up to around 8 km from the ground while the MJ01* method overestimates \( C_N^2 \) starting at 4 km from the ground. The ‘\( \alpha \) effect’ (Masciadri & Egner 2006; Masciadri et al. 2010) is well reconstructed by the model too: in summer, the secondary peak shifts to a higher altitude, as shown by the measurements. At the same time, the strength of the secondary peak slightly decreases, as observed in measurements. No major differences are appreciated in the shape of the averaged \( C_N^2 \) profiles between the calibration case (34 nights) and the whole sample (41 nights). Therefore, the inclusion in the sample of some more nights does not seem to produce a major impact on the morphology (shape) of the averaged \( C_N^2 \) profile. What is the cause of the model’s overestimate of \( C_N^2 \) in the free atmosphere in summer? Looking at Fig. 6, we note that, in the free atmosphere, the model variation between the minimum and the maximum values of \( C_N^2 \) is substantially smaller than observed with measurements. The higher the altitude, the higher the model inertia and, as a consequence, the model has more difficulties in reconstructing the extreme values (minimum and maximum) in the high part of the atmosphere. We could expect therefore a slight underestimate of the model in winter and a slight overestimate in summer. The fact that a model overestimate is observed only in summer could be because, in this season, the number of nights for the samples of 34 and 41 nights are, respectively, 14 and 15 nights, while in winter we have 20 and 26 nights, respectively. It is
Figure 6. Minimum and maximum values of the $C_n^2$ observed and simulated in the sample of 41 nights. Red lines refer to measurements from the GS. Light blue and dark blue lines refer to the MJ01 and MJ01* models, respectively. The values of $C_n^2$ are shown for $h \geq 400$ m.

therefore possible that the winter data have a more important weight in the calibration. Also, in summer we obtained all nights with extremely weak turbulence in the free atmosphere. With a richer and more homogeneous statistical sample, we should therefore expect a decrease in strength of the discrepancy and a more symmetric discrepancy with respect to the two seasons. We do not think that there is a specific problem for the summer period. Nevertheless, it is a fact that the model variability needs to be improved in the high part of the atmosphere to be able to detect well all the cases of very strong as well as very weak turbulence conditions. We are working on this topic at present.

5.2 Seeing: $\epsilon$

The seeing depends on $C_n^2$, as shown by equation (A2) in Appendix A. Fig. 7 shows the total seeing ($\epsilon_{\text{tot}}$), the boundary layer ($\epsilon_{\text{BL}}$) and the free atmosphere ($\epsilon_{\text{FA}}$) simulated by the model (MJ01 and MJ01*) plotted against the respective seeing observed by the GS for the sample of 34 nights after calibration. The boundary layer is defined as the first kilometre above the surface and the free atmosphere from the top of the boundary layer up to 20 km above the surface. All values of the seeing are calculated by subtracting from the measurements from the GS the dome seeing contribution and by subtracting from the simulations the contribution of first 20 m of the $C_n^2$ profiles (the equivalent of the telescope height). Fig. 7 (bottom right of each panel) reports the correlation coefficient (c.c.) calculated for the total sample of 41 nights. Tables 2 and 3 (left) report the values of the average seeing and the relative error for the calibration sample (34 nights). Tables 3 (right) and 4 report the same for the total sample of 41 nights. The correlation coefficients are indicated for the MJ01 and MJ01* cases in each panel of Fig. 7. The total seeing and the seeing in the boundary layer show a good correlation (c.c. = 0.78–0.82) with measurements in the calibration sample with no major differences between the MJ01 and MJ01* cases. It is very encouraging that the turbulence near the ground, which represents most of the turbulence developed in the atmosphere, is well predicted by the model. The correlation decreases for the seeing in the free atmosphere. The explanation for such an effect is that, in the free atmosphere, visibly the model inertia is still too high and the parameters predicted by the model vary in a smaller range than the measurements show, as discussed in Section 5.1.

The model has, at present, some problems in identifying the best (the minimum observed $\epsilon_{\text{FA}}$) and the worst (the maximum observed $\epsilon_{\text{FA}}$) conditions in this part of the atmosphere. Looking at Fig. 4, it appears clear that the problem concerns $C_n^2$ at $h > 10$ km. However, we note that the model (with both methods, MJ01 and MJ01*) is able to reconstruct a weaker $\epsilon_{\text{FA}}$ in summer than in winter, as we expect and also as is observed (Table 2). For the calibration sample, no substantial and systematic differences are noted on results obtained with the MJ01 and MJ01* methods. The very encouraging result is that the relative error for the seeing in all three regions of the atmosphere (Table 3, left) is very good even in the free atmosphere. Results are equally very good if we look at the individual seasons. The important conclusion is that the relative error for the seeing on the whole sample is within 9.2 per cent for the best method (MJ01) and within 17.6 per cent if we consider also the
MJ01° method. If we consider the two subsamples (summer and winter) the relative error decreases slightly, still maintaining good performances. In conclusion, we observe remarkably good relative errors, in spite of a modest correlation in the free atmosphere. We also note that the correlation is very good for seeing smaller than 1 arcsec, and deviations are more typical when there is strong turbulence. Finally, we can say that the MJ01 method is confirmed as effective and it improves the model’s reliability, reducing some systematic effects.

If we look at the total sample of 41 nights (Tables 3, right, and 4), including nights not used for the calibration, we observe that the correlation decreases for $\epsilon_{BL}$ and $\epsilon_{BL}$ (from 0.82 to 0.61). For $\epsilon_{FA}$, the correlation decreases in a less consistent way and the MJ01 method seems better than the MJ01° method in this region. Most of the new nights included in this sample belong to the winter period. The reason for the decrease of the correlation is that the model reconstructs a seeing in the boundary layer that is statistically too strong. In this case, the MJ01 method is visibly better than the MJ01° method in both cases: the total sample and the summer and winter periods. The MJ01 method achieves a remarkably good result with a relative error within 9.0 per cent for the total sample and a maximum relative error of 13.9 per cent if we look at the subsamples (summer and winter). The MJ01 method is also better than the MJ01° method in reconstructing the weakest and the strongest values in the two seasons in the partial $\epsilon_{FA}$ and $\epsilon_{BL}$. Also, in this case, the model reconstructs $\epsilon_{FA}$ that varies in a smaller range with respect to that observed. As for the calibration sample, we can however observe that the model can reconstruct a weaker seeing in summer than in winter, as expected.

Finally, to study the performance of the model night by night in a compact way, we have calculated the cumulative distribution of the relative errors with respect to measurements for $\epsilon_{tot}$, $\epsilon_{BL}$ and $\epsilon_{FA}$ obtained with the MJ01 and MJ01° methods (Fig. 8). We note that the relative error in the three regions of the atmosphere is absolutely remarkably good. The median value (50 per cent of times) of the relative error is within 26.3 per cent in the three regions of the atmosphere for the MJ01 method, which is visibly better than the MJ01° method in this case. This means that, even if the correlation for $\epsilon_{FA}$ is not as good as for $\epsilon_{tot}$ and $\epsilon_{BL}$, the relative error night by night is maintained within 21 per cent. The largest relative error is obtained in the boundary layer. Even if this is the region in which the correlation is the best, the impact of an error of the model can be more important than an error produced in the free atmosphere.

In conclusion, we can state that, for the total sample of nights, the MJ01 method provides globally better results than the MJ01° method. This means that the MJ01 method, which is based on a minor number of constraints than the MJ01° method, has a better performance probably because it is based on more robust physical assumptions. A larger number of constraints can be effective on the calibration sample but, when an independent sample of nights is taken into account, this theoretical advantage seems to lose its effect.

### 5.3 Isoplanatic angle: $\theta_0$

Fig. 9 shows the simulated versus measured isoplanatic angle $\theta_0$ for the sample of 41 nights. $\theta_0$ depends on $C_N^2$ as shown by

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3 This indicates that by increasing the number of nights the calibration attains more accurate results. For a sample of 10 nights, Masciadri et al. (2004) obtained a relative error on average of the order of 20 per cent.
Optical turbulence simulations at Mt Graham

Figure 8. Cumulative distribution of the relative errors of each single night of the total seeing ($\varepsilon_{\text{TOT}}$), the seeing in the boundary layer ($\varepsilon_{\text{BL}}$) and the seeing in the free atmosphere ($\varepsilon_{\text{FA}}$) for the total sample of 41 nights. Bold line, MJ01 method; thin line, MJ01* method.

Figure 9. Isoplanatic angle ($\theta_0$) from the simulations (stars, MJ01; circles, MJ01*) plotted against the measurements related to the sample of 41 nights. The values reconstructed by the model span a much smaller range (1.7–2.2 arcsec) than has been observed (0.9–5.6 arcsec). The average of the values obtained for all the nights is in good agreement with measurements. The average of the simulations is equal to 2.06 arcsec for MJ01 (1.82 arcsec for MJ01*) versus 2.65 arcsec measured by the GS. The difference between simulations and measurements is a result of a small offset in the model calibration. As we have already discussed in the previous sections, the calibration sample is slightly biased in the high part of the atmosphere because measurements are not completely homogeneously distributed in the two seasons, and in the summer we have many nights with extremely weak turbulence in the high part of the atmosphere. Looking at Fig. 5 (left), we can appreciate a generally very good agreement between measurements and simulations, but a very weak overestimate by the model is visible for $\theta_0$ quantified by measurements is not because of the calibration but because, at present, the model shows a larger inertia in the high part of the atmosphere, as discussed in Section 5.1. This produces a more modest correlation between simulations and measurements for $\theta_0$. As already said, we are at present working on this issue to improve the model variability.

Fig. 10 shows the cumulative distribution of the relative error (night by night) for $\theta_0$ obtained with both methods (MJ01 and MJ01*). The median value of relative error for $\theta_0$ is slightly larger (35.1 per cent for the best method, MJ01) than is observed for the seeing (described in the previous section).

5.4 Wavefront coherence time: $\tau_0$

$\tau_0$ depends on $C_n^3$ and the wind speed vertical profiles as shown by equation (A5) in Appendix A. How do we calculate $\tau_0$ for measurements and simulations? For the wind speed profiles, we have shown in previous studies (Egner et al. 2007; Masciadri et al. 2010) that the best solution is to consider a composite profile: the wind speed profiles retrieved from the ECMWF analyses (i.e. analyses from the General Circulation Model) in the nearest grid point (~12 km away) for $h$ above 1 km and the wind speed as retrieved from the GS for $h$ below 1 km. This is necessary because the analyses are not...
sampled with enough high horizontal resolution and they are not reliable near the ground. However, in a more recent study (Hagelin et al., 2010), it has been proved that the wind speed profiles provided by the Meso-NH model at the summit of Mt Graham are very well correlated to the wind speed estimated by the ECMWF analyses above 1 km and are well correlated with measurements taken with a GS and an anemometer near the ground. To calculate $\tau_0$, we considered therefore the wind speed reconstructed by Meso-NH above the summit. Because for both measured and simulated $\tau_0$ we used the wind speed profiles as retrieved from the Meso-NH, when comparing $\tau_0$ we are basically comparing the effects of the simulated and measured $C_N^2$ on the simulated and measured $\tau_0$. We consider the average of the wind speed profiles during each night and we calculate $\tau_0$ for each night. Fig. 11 (left) shows the simulated versus measured wavefront coherence time ($\tau_0$) for the sample of 41 nights. The data set is well distributed along a straight line, showing a very good correlation (0.95 for MJ01 and 0.96 for MJ01*). Up to 10 ms, the values are very well correlated. A very small bias is evident for the extremely good values of $\tau_0$ ($\geq 10$ ms).

To verify which part of the $C_N^2$ (low or high atmosphere) mainly affects $\tau_0$ in Fig. 11 (left), we calculated $\tau_0$ in the partial 0–10 and 10–20 km ranges (Fig. 11, centre and right). We observe that while the simulated and measured values are well correlated in the 0–10 km range, the model underestimates $\tau_0$ in the 10–20 km range. However, the contribution coming from the 0–10 km region has a much more important effect on the calculation of $\tau_0$ on the whole atmosphere than the contribution coming from the 10–20 km region. This means that $\tau_0$ on the whole 0–20 km is very well reconstructed by the model globally. This is a confirmation that, also for this parameter, the small bias of $C_N^2$ in the high part of the atmosphere produces some effect on $\tau_0$. However, differently from $\theta_0$, this effect is almost negligible (Fig. 11, left).

To study the performance of the model night by night, Fig. 12 shows the cumulative distribution of the relative error for $\tau_0$, simulated with both the MJ01 and MJ01* methods, and calculated by taking the measurements as a reference. The median of the relative error is very good for $\tau_0$, as good as for the seeing: 22.5 per cent for the MJ01 method and 21.8 per cent for the MJ01* method.

6 CONCLUSIONS

In this paper, we have discussed the abilities of the Meso-NH model in simulating the optical turbulence above Mt Graham, the site of the LBT. Simulated $C_N^2$ profiles are compared with a large statistical sample of measured $C_N^2$ profiles related to 41 nights obtained with a GS. This large sample of measurements has allowed us to study the performances of the model calibration in great detail and to investigate the performance of the model in different seasons, discussing statistically how well the model reconstructs $C_N^2$, the seeing in different regions of the atmosphere, the isoplanatic angle and the wavefront coherence time.

Two different methods of model calibration have been investigated. The calibration is carried out for a sample of 34 nights and the model performances are discussed for the whole sample of 41 nights.
The most important results obtained are the following.

(i) We have proved that the model calibration definitely improves the model performance. The morphology of the vertical distribution of the optical turbulence (average of all $C_N^2$ of the 34 nights) matches very well with the corresponding measured $C_N^2$ profile. If we look at the model behaviour in different seasons, we show evidence of a model overestimate of $C_N^2$ in the high part of the atmosphere ($h > 8\,\text{km}$) in summer. This bias is highly probable because the sample of the investigated nights is not completely uniformly distributed between summer and winter. With a reasonably richer sample (of the order of 1 yr) this small bias can be corrected. This evidence proves the necessity of a very rich statistical sample to carry out an efficient model calibration.

(ii) We have observed that the model basically always reconstructs the most important features of the shape of the measured $C_N^2$ profile, but it still has some difficulties in reconstructing the very extreme values (very good and very bad turbulence conditions) in the free atmosphere. We are working at present on improving these model performances.

(iii) For the model calibration, the total and boundary layer seeing reconstructed by the Meso-NH model are well correlated with measured values with a correlation coefficient (c.c.) of the order of 0.78–0.82. The seeing in the free atmosphere is more weakly correlated to measurements (c.c. $\sim$0.58). The relative errors are, however, extremely good in all three regions (total seeing, in the boundary layer and free atmosphere). The best method MJ01 provides a relative error within 9.2 per cent on the total sample and within 14.9 per cent if we consider the two seasons. These percentages remain basically the same if we consider the whole sample of 41 nights.

(iv) When we consider the total sample of 41 nights, the correlation between simulations and measurements decreases slightly for $\varepsilon_{\text{tot}}$ and $\varepsilon_{\text{BL}}$ (c.c. $\sim$0.60), and much less for $\varepsilon_{\text{FA}}$ (c.c. $\sim$0.47).

(v) If we consider the cumulative distribution of the relative errors night by night (i.e. the typical conditions of the operational mode), we find extremely encouraging results. The median value of the relative errors is indeed extremely small with some small variations for the seeing in the three regions of the atmosphere, but typically of the order of 20 per cent. Therefore, even if we look at the most difficult conditions typical of the operational mode, the model maintains a reasonably good result.

(vi) For the isoplanatic angle, the relative error of the average is still very good ($\sim$18 per cent) but the median of the relative errors calculated night by night is 35.1 per cent. This is the parameter with the poorest performance between $\varepsilon$, $\theta_i$ and $\tau_0$. This is not surprising because it is the parameter that, more than others, is very sensitive to the turbulence in the high part of the atmosphere.

(vii) The wavefront coherence time reconstructed by the model shows a very good correlation with measurements (c.c. $\sim$0.95). We have proved that the small bias in the high atmosphere on $C_N^2$ produces a negligible effect on $\tau_0$. The first 0–10 km represent the most important contributions to the final $\tau_0$ value. The cumulative distribution of the relative error calculated night by night is still very good with a median value equal to 22.5 per cent.

(viii) The MJ01 method to calibrate the model is confirmed as a solid method and it shows a general better performance of the model than the variant MJ01*, which consists of fitting measurements and simulations on each model level instead of the vertical slabs of a few kilometres. It is worth noting that the calibration of the high atmosphere is more delicate because a small discrepancy from the measured $C_N^2$ profile can produce a large discrepancy on some astroclimatic parameters (such as $\theta_0$).

The general conclusions are that the Meso-NH model is able to describe the optical turbulence distribution above Mt Graham, showing very good performances with small relative errors with respect to measurements for most of the astroclimatic parameters. The most urgent problem to be solved is the improvement of the model’s ability in reconstructing the very weak and very strong turbulent conditions in the high atmosphere (i.e. the ability to reconstruct isoplanatic angles in better agreement with measurements). Finally, we have shown evidence of the importance of having a very rich statistical sample of $C_N^2$ profiles to efficiently calibrate the model. In future, we would like to access even richer samples of measurements carried out preferably with a GS in order to be able to consider a completely independent sample for the model validation. In future, it would also be interesting to test the model performance when it is initialized with forecasts from the ECMWF instead of the operational analyses in order to use the Meso-NH model as part of a system to forecast the optical turbulence at Mt Graham, allowing the implementation of a flexible-scheduling management of the LBT.

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APPENDIX A: INTEGRATED
ASTROCLIMATIC PARAMETERS
AS A FUNCTION OF $C_N^2$

The seeing ($\varepsilon$), the isoplanatic angle ($\theta_0$) and the wavefront coherence time ($\tau_0$) are defined as

$$r_0 = \left[ 0.423 \left( \frac{2\pi}{\lambda} \right)^2 \int_0^\infty C_N^2(h) \, dh \right]^{-3/5},$$  \hspace{1cm} (A1)

$$\varepsilon = 0.98 \frac{\lambda}{r_0},$$ \hspace{1cm} (A2)

$$\theta_0 = 0.057 \lambda^{6/5} \left[ \int_0^\infty h^{5/3} C_N^2(h) \, dh \right]^{-3/5},$$ \hspace{1cm} (A3)

$$V_0 = \left[ \frac{\int_0^\infty V(h)^{3/5} C_N^2(h) \, dh}{\int_0^\infty C_N^2(h) \, dh} \right]^{3/5},$$ \hspace{1cm} (A4)

$$\tau_0 = 0.31 \frac{r_0}{V_0}.$$ \hspace{1cm} (A5)

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