Improved limit on electron neutrino charge radius through a new evaluation of the weak mixing angle

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Abstract

We have obtained a new limit on the electron neutrino effective charge radius from a new evaluation of the weak mixing angle by a combined fit of all electron-(anti)neutrino electron elastic scattering measurements. Weak mixing angle is found to be $\sin^2 \theta_W = 0.259 \pm 0.025$ in the low energy regime below 100 MeV. The electron neutrino charge radius squared is bounded to be in the range $-0.13 \times 10^{-32} \text{ cm}^2 < \langle r_{\nu_e}^2 \rangle < 3.32 \times 10^{-32} \text{ cm}^2$ at 90\% C.L. Both results improve previously published analyses. We also discuss perspectives of future experiments to improve these constraints.

\textit{Key words:} Neutrinos, weak mixing angle, electromagnetic form factors

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1 Introduction

The search for neutrino electromagnetic properties is as old as neutrino theories \cite{1}. The electromagnetic interaction of Dirac neutrino is described by four form-factors: vector, axial vector, magnetic and electric ones \cite{2,3}. In the

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The gauge-invariant definition of the neutrino effective charge radius as a physical observable has been discussed since a long time \[4, 5, 6, 7, 8, 9\]. Recently it was shown that gauge-dependent terms cancel each other and therefore neutrino charge radius can be defined as a gauge-invariant physical observable \[10, 11, 12, 13\]; however, theoretical expectations \[10\] for electron neutrino charge radius are one order of magnitude smaller than present experimental limits. On the other hand it was shown in Ref. \[14\] that it is not possible to reach a good sensitivity to neutrino charge radius through astrophysical and cosmological observations and therefore it can be constrained only in terrestrial experiments.

At the low energy scale, besides atomic parity violation \[15\] and Moller scattering \[16\], neutrino-electron scattering experiments are sensitive to the weak mixing angle and they have also been used to search for neutrino effective charge radius. Searches for a non-zero neutrino magnetic moments have also been a challenge in this kind of experiments \[17, 18, 19\].

Recently, the need for a precise measurement of the weak mixing angle at low energies has been encouraged by the NuTeV collaboration \[20\], in which a $3\sigma$ deviation of the Weinberg angle from the Standard Model prediction was found in deep inelastic neutrino scattering.

In this paper we introduce a new limit on the weak mixing angle, $\sin^2 \theta_W$, and on the electron neutrino effective charge radius, $\langle r_{\nu_e}^2 \rangle$, obtained from the combined analysis of all available low energy (anti)neutrino-electron elastic scattering experiments.

We have analyzed all available measurements of the (anti)neutrino-electron scattering from the following reactor and accelerator experiments: first measurement of neutrino-electron scattering made by Reines, Gurr and Sobel (Irvine) \[21\], the Kurchatov institute group at the Krasnoyarsk reactor \[22\], the group from Gatchina at the Rovno reactor \[23\], MUNU at the Bugey reactor \[17\], LAMPF \[24\] and LSND \[25\].

We haven’t included into our analysis the recent reactor neutrino results from TEXONO \[18\] and GEMMA \[19\] experiments, which have put very strong limits on neutrino magnetic moments, $\mu_\nu < 7.4 \times 10^{-11}\mu_B$ and $\mu_\nu < 5.8 \times 10^{-11}\mu_B$ at 90% CL, respectively. Both experiments are working at very low
energy thresholds searching for neutrino magnetic moments and are not yet sensitive to the weak mixing angle and neutrino charge radius. The discussion of theoretical and experimental issues on muon- and tau-neutrino effective charge radii [10,14] is also out of the scope of this paper.

The paper is organized as follows: in Section II we derive limits to the weak mixing angle and to the neutrino charge radius, in Section III we discuss future perspectives of precise measurements at reactor neutrino experiments. The Summary is presented in Section IV.

2 Limits on weak mixing angle and neutrino charge radius

The differential weak cross section for electron (anti)neutrino scattering off electron is given by

\[
\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 \left( 1 - \frac{T}{E_\nu} \right)^2 - \left( g_V^2 - g_A^2 \right) \frac{m_e T}{E_\nu^2} \right],
\]  

(1)

here \( E_\nu \) is the incoming neutrino energy, \( T \) is the recoil electron energy. The vector and axial weak couplings are given as \( g_V = 1/2 + 2 \sin^2 \theta_W \) and \( g_A = 1/2 \) for neutrinos, while in the case of anti-neutrinos \( g_A = -1/2 \).

We will concentrate first in the determination of the weak mixing angle from (anti)neutrino electron scattering experiments and we will discuss the neutrino charge radius in detail afterwards. To extract the allowed region for the weak mixing angle from the experimental data we have done a \( \chi^2 \) analysis (details can be found in Ref. [26]) for each experiment and then we have performed a global fit. In particular, we have consider two different \( \nu_e \) experiments, LAMPF and LSND. In both cases we have confronted the theoretical cross section with the reported measurement to obtain our value of \( \sin^2 \theta_W \).

We also considered the experimental results reported in four different reactor experiments. In this case we compute the total antineutrino cross section

\[
\sigma = \int dT' \int dT \int dE_\nu \frac{d\sigma}{dT} \lambda(E_\nu) R(T, T')
\]  

(2)

with \( \lambda(E_\nu) \) the neutrino energy spectra and \( R(T, T') \) the detector energy resolution function. We use an anti-neutrino energy spectrum given by

\[
\lambda(E_\nu) = \sum_{k=1}^{4} a_k \lambda_k(E_\nu),
\]  

(3)
where \( a_k \) is the abundance of \( ^{235}\text{U} \) \((k = 1)\), \( ^{239}\text{Pu} \) \((k = 2)\), \( ^{241}\text{Pu} \) \((k = 3)\) and \( ^{238}\text{U} \) \((k = 4)\) in the reactor, \( \lambda_k(E_\nu) \) is the corresponding neutrino energy spectrum which we take from the parametrization given in [27], with the appropriate fuel composition. For energies below 2 MeV there are only theoretical calculations for the antineutrino spectrum which we take from Ref. [28]. For the case of the Irvine experiment (that reports two energy bins) we prefer to use the neutrino energy spectrum used by the experimentalists at that time [29]. With this cross section evaluation we can perform a \( \chi^2 \) analysis either comparing with the expected number of events, for the case of the MUNU collaboration, or with the reported cross section in the rest of the experimental results. We neglect correlations between experiments; this is a good approximation as the only possible correlation comes from the reactor neutrino energy spectrum, estimated to be less than 2\% [27], small in view of the statistical errors. The results are shown in Fig. 1. The minimum \( \chi^2 \) was found to be \( \chi^2_{\text{min}}/N_{\text{d.o.f.}} = 2.17/6 \). As an additional check of our global result we have also computed the predicted allowed values of the weak mixing angle excluding one single experiment at a time from our global fit. In all the cases the results are similar.

The present analysis of a weak mixing angle generalizes the recent result \( \sin^2 \theta_W = 0.27 \pm 0.03 \) [26] by adding the data from the LAMPF collaboration [24] and from the Krasnoyarsk reactor [22]. From the global fit shown in Fig. 1 we have derived a new value on the weak mixing angle in the low energy range (below 100 MeV):

\[
\sin^2 \theta_W = 0.259 \pm 0.025 \tag{4}
\]

This value is 1.45 standard deviations larger than the value of the weak mixing angle obtained from a global fit to electroweak measurements without neutrino-nucleon scattering data, \( \sin^2 \theta_W = 0.2227 \pm 0.00037 \) [20], that will be used in our numerical calculations of the neutrino charge radius.

The obtained precision of the weak mixing angle in our analysis is about 10\% which is close to the expected sensitivities of several proposals aiming to detect neutrinos in the same low energy range [30][31][32].

The result (4) is not competitive with the current best measurements at low energies obtained from atomic parity violation [15] and from Moller scattering by SLAC E158 [16] which have a precision better than 1\%. However, it is derived from a different channel and it could therefore give new information about other effects, such as the electron neutrino effective charge radius.
Fig. 1. Global $\Delta \chi^2$ for weak mixing angle obtained from all discussed $\nu_e e$ and $\bar{\nu}_e e$ scattering experiments are plotted. The contribution of each experiment to the $\Delta \chi^2$ is also shown.

The gauge-invariant process-independent expression for the Standard Model neutrino charge radius has been derived in one-loop approximation in Refs [10,11,12]

$$\langle r^2_{\nu_i} \rangle_{SM} = \frac{G_F}{4\sqrt{2}\pi^2} \left[ 3 - 2 \log \left( \frac{m_i^2}{m_W^2} \right) \right],$$

where $m_W$ is the $W$-boson mass and $m_i$ denote the lepton masses for $i = e, \mu, \tau$. A numerical evaluation gives the value for the electron neutrino charge radius squared [10]

$$\langle r^2_{\nu_e} \rangle_{SM} = 0.4 \times 10^{-32} \text{ cm}^2.$$
Fig. 2. Global $\Delta \chi^2$ for electron neutrino charge radius, $\langle r^2_{\nu_e} \rangle$, in units of $10^{-32}$cm$^2$, obtained from all discussed $\nu_e e$ and $\bar{\nu}_e e$ scattering experiments is plotted. The contribution of each experiment to the $\Delta \chi^2$ is also shown.

We consider therefore an effective approach and, following previous literature \cite{33,34,35}, we denote the vector coupling as

$$g_V = 1/2 + 2 \sin^2 \theta_W + (2\sqrt{2}\pi\alpha/3G_F)\langle r^2_{\nu_e} \rangle,$$

where $\langle r^2_{\nu_e} \rangle$ contains all the contributions discussed above. The expression translates into an effective displacement in the value of the weak mixing angle $\sin^2 \theta_W = \sin^2 \bar{\theta}_W + \delta$ with the radiative correction $\delta = (\sqrt{2}\pi\alpha/3G_F)\langle r^2_{\nu_e} \rangle = 2.3796 \times 10^{30}$ cm$^2 \times \langle r^2_{\nu_e} \rangle$. In the last expression we have taken the fine structure constant, $\alpha$, and the Fermi coupling constant, $G_F$, as reported by the Particle Data Group \cite{37}. As previously referred, $\sin^2 \bar{\theta}_W$ is the value of the weak mixing angle without taking into account the contribution from $\langle r^2_{\nu_e} \rangle$.

We have done a $\chi^2$ analysis for every experiment and combined the results into a global fit as shown in Fig. 2. The obtained allowed region for the electron neutrino effective charge radius is

$$-0.13 \times 10^{-32}\text{cm}^2 < \langle r^2_{\nu_e} \rangle < 3.32 \times 10^{-32}\text{cm}^2 \text{ at 90\% C.L.,}$$

\footnote{A different notation is used in some articles ($g_V = \bar{g}_V + \delta$) that implies a definition of $\langle r^2_{\nu_e} \rangle$ smaller by a factor two.}
or \( \langle r_{\nu_e}^2 \rangle = 1.69^{+1.01}_{-1.09} \times 10^{-32} \text{cm}^2 \) for one standard deviation. Although the precision of present experimental measurements is not enough to conclude that electron neutrino charge radius is non-zero, better sensitivity is expected in future proposed experiments and they will be discussed below.

In Table 1 we have shown the list of experiments along with their cross section measurements, the obtained weak mixing angle and the neutrino charge radius constraint, if available. Our combined global result for weak mixing angle and electron neutrino charge radius is also shown for comparison.

| Experiment | Energy | Events | Measurement, \( \sigma \) | \( \sin^2 \theta_W \) | \( \langle r_{\nu_e}^2 \rangle > \) | \( \langle r_{\nu_e}^2 \rangle < \) |
|------------|--------|--------|--------------------------|---------------------|---------------------|---------------------|
| LAMPF [24] | 7-60   | 236    | \([10.0 \pm 1.5 \pm 0.9] E_{\nu_e} \cdot 10^{-45} \text{cm}^2\) | 0.249 ± 0.063      | −3.56               | 5.44                |
| LSND [25]  | 10-50  | 191    | \([10.1 \pm 1.5] \cdot E_{\nu_e} \cdot 10^{-45} \text{cm}^2\) | 0.248 ± 0.051      | −2.97               | 4.14                |
| Irvine [21] |        |        | \(0.86 \pm 0.25 \pm \sigma_{V-A}\) | 0.29 ± 0.05        | N/A                 | N/A                 |
| Krasnoyarsk [22] | 3.15-5.175 | N/A    | \([4.5 \pm 2.4] \cdot 10^{-46} \text{cm}^2/\text{fission}\) | 0.22^{+0.7}_{-0.8} | −7.3                | 7.3                 |
| Rovno [23]  | 0.6-2.0| 41     | \([1.26 \pm 0.62] \cdot 10^{-44} \text{cm}^2/\text{fission}\) | N/A                 | N/A                 | N/A                 |
| MUNU [17]   | 0.7-2.0| 68     | \(1.07 \pm 0.34 \text{ events/day} \) | N/A                 | N/A                 | N/A                 |
| Global      |        |        |                           | 0.259 ± 0.025      | −0.13               | 3.32                |

Table 1

Current experimental data on electron-(anti)neutrino electron scattering including measurements of \( \sin^2 \theta_W \) and limits on \( \langle r_{\nu_e}^2 \rangle \), given by the collaborations. 'N/A' means that a collaboration hasn’t published the value. Note that MUNU has provided number of events, but not cross-section. Global limits are obtained from the combined analysis of cross-section measurements of all experiments. Neutrino effective charge radius, \( \langle r_{\nu_e}^2 \rangle \), limits are in units of \( 10^{-32} \text{cm}^2 \) for 90% C.L. range. All energies are in MeV.

3 Future reactor experiments

Finally we discuss the perspectives to improve the neutrino charge radius constraints. It was recently proposed that future reactor experiments aiming to measure the neutrino mixing angle \( \theta_{13} \) will be able to reach a very good sen-
sitivity to neutrino-electron scattering cross section [36]. Therefore the weak mixing angle could be measured with 1% precision at very low energies with reactor neutrinos.

We have estimated the sensitivity of such measurements to the electron neutrino charge radius. In order to get our estimates we have considered a fixed sensitivity to the weak mixing angle and derived the corresponding resolution for a neutrino charge radius. The results are given in Table 2.

| Experimental 1σ resolution of sin² θ_W | 1σ sensitivity to ⟨r²_ν_e⟩ in 10⁻³² cm² | 3σ sensitivity to ⟨r²_ν_e⟩ in 10⁻³² cm² |
|--------------------------------------|----------------------------------------|----------------------------------------|
| 10% (present)                        | 1.05                                   | 2.9                                    |
| 5%                                   | 0.47                                   | 1.4                                    |
| 3%                                   | 0.29                                   | 0.84                                   |
| 1%                                   | 0.10                                   | 0.28                                   |

Table 2

Present (this paper) resolution to weak mixing angle, sin² θ_W, and the corresponding 1σ and 3σ sensitivity ranges to ⟨r²_ν_e⟩ (in units 10⁻³² cm²) are shown. Expected future sensitivity is given for different values (1%, 3% and 5%) of estimated sin² θ_W resolution.

One can conclude, that from the measurement of the weak mixing angle with 1% resolution it is possible to find a strong evidence for electron neutrino charge radius, which is estimated theoretically to be of the order of 0.4 × 10⁻³² cm² [10].

4 Summary

We have obtained the weak mixing angle with 10% precision at energies below 100 MeV from (anti)neutrino electron scattering off electrons. To get this result we have combined all available data from accelerator (LSND and LAMPF) and reactor (Irvine, Rovno, Krasnoyarsk and MUNU) experiments.

This analysis was also used to set a new limit to the electron neutrino effective charge radius squared which improves previously published bounds [37].

Future reactor experiments with the estimated precision of δ(sin² θ_W) ~ 1% will be able to find a strong evidence for theoretically predicted electron neutrino effective charge radius.
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