Unitary Structure of the QCD Sum Rules and $KYN$ and $KYΞ$ Couplings

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Abstract. New relations between QCD Borel sum rules for strong coupling constants of K-mesons to baryons are derived. It is shown that starting from the sum rule for the coupling constants $g_{πΣΣ}$ and $g_{πΣΛ}$ it is straightforward to obtain corresponding sum rules for the $g_{KYN}$, $g_{KYΞ}$ couplings, $Y = Σ, Λ$.

INTRODUCTION

Meson-baryon couplings were studied for years thoroughly either for pion-baryon couplings or kaon-baryon baryon ones as these couplings are important parameters of strong interaction physics.

Since the advent of the $SU(3)$ symmetry all the meson-baryon coupling constants were usually expressed in terms of $F$ and $D$ constants which gave possibility to construct a reliable phenomenological approach.

As soon as in [1] QCD sum rules (SR’s) were proposed, they were used not only for baryon masses and magnetic moments starting from the works [2] but also for baryon-meson coupling constants. Naturally, a pion-nucleon coupling attracted the most attention (see, for example, [3], [4], [5]). Coupling constants of $π^0$- and $η$- mesons to baryons were studied recently in various QCD SR approaches [6], [7]. Also QCD sum rule for the $η$ coupling to the $Λ$ hyperon was written [8] which was usually absent in these approaches.

As for K-mesons they were also studied in the framework of the QCD sum rules (see, e.g., [9], [10], [11]). Usually these sum rules are not related straightforwardly to those treating $π$ and $η$ couplings to baryons.

We would like to propose here QCD sum rules for octet meson-baryon couplings through some universal $F$ and $D$ functions written in a unified manner. In order to be clear we choose for methodical reasons as a basis for our reasoning $SU(3)$ breaking QCD Borel sum rules proposed in [6].

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RELATION BETWEEN $\pi^0 \Sigma$ AND $\pi \Sigma \Lambda$ CONSTANTS IN SU(3)

We begin as in [12] with a simple example. In the unitary model all the pion-baryon coupling constants can be expressed in terms of $F$ and $D$ coupling constants. But coupling of the $\Sigma$-like baryons $B(qq, q')$, $q, q' = u, d, s$ to $\pi^0$ meson related in the quark model to the current $j^{\pi^0} = \frac{1}{\sqrt{2}}[\bar{u}\gamma_5 u - \bar{d}\gamma_5 d]$ can be put in the form

$$g(\pi^0 BB) = g_{\pi qq}2F + g_{\pi q'q}(F - D),$$

or, particle per particle:

$$g(\pi^0 pp) = g_{\pi uu}2F + g_{\pi dd}(F - D) = \sqrt{\frac{1}{2}}(F + D);$$

$$g(\pi^0 \Sigma^0 \Sigma^0) = g_{\pi uu}2F + g_{\pi ss}(F - D) = \sqrt{2}F,$$

and so on, where $g_{\pi uu} = +\sqrt{\frac{1}{2}}$, $g_{\pi dd} = -\sqrt{\frac{1}{2}}$ and $g_{\pi ss} = 0$ are just read off the quark current.

The only coupling which cannot be written immediately in this way is $\pi^0 \Sigma^0 \Lambda$. To overcome this difficulty let us write for $\pi^0 \Sigma^0 \Sigma^0$ coupling (which is equal to zero!):

$$g(\pi^0 \Sigma^0 \Sigma^0) = g_{\pi uu}F + g_{\pi dd}F + g_{\pi ss}(F - D) = 0$$

and change $(d \leftrightarrow s)$ and $(u \leftrightarrow s)$ to form two auxiliary quantities

$$g(\pi^0 \Sigma^0, ds \Sigma^0, ds) = g_{\pi uu}F + g_{\pi ss}F + g_{\pi dd}(F - D) = \sqrt{\frac{1}{2}}D;$$

$$g(\pi^0 \Sigma^0, us \Sigma^0, us) = g_{\pi dd}F + g_{\pi ss}F + g_{\pi uu}(F - D) = -\sqrt{\frac{1}{2}}D.$$}

The following relation holds:

$$g(\pi^0 \Sigma^0, ds \Sigma^0, ds) - g(\pi^0 \Sigma^0, us \Sigma^0, us) = \sqrt{3}g(\pi^0 \Sigma^0 \Lambda).$$

The origin of this relation lies in the structure of $\Sigma^0 (ud, s)$ and $\Lambda$ wave functions in the NRQM. With the exchanges $d \leftrightarrow s$ and $u \leftrightarrow s$ one arrives at the corresponding $U$-spin and $V$-spin quantities, so

$$\begin{pmatrix} \tilde{\Sigma}^0_{ds} \\ \tilde{\Lambda}_{ds} \end{pmatrix} = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} \Sigma^0 \\ \Lambda \end{pmatrix}, \quad \begin{pmatrix} \tilde{\Sigma}^0_{us} \\ \tilde{\Lambda}_{us} \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} \Sigma^0 \\ \Lambda \end{pmatrix}. $$

It is easy now to show that the relation Eq.(4) follows and it shows us the way to proceed with the QCD sum rules.
**KYN, KYΞ AND πΣΛ COUPLINGS IN THE SU(3)**

Now we consider kaon and charged pion couplings to baryons. They are given by SU(3) symmetry formulae but we rewrite it in a way suitable for derivation of the corresponding Borel sum rules. Let us write coupling of pion to $\Sigma^+$ and $\Lambda_{ds}$ given by the Eq.(5):

$$2[g(\pi^- \Sigma^+ \bar{\Lambda}_{ds})] = -\sqrt{3}g(\pi^- \Sigma^0) + g(\pi^- \Sigma^+ \bar{\Lambda}) =$$

$$= -\sqrt{3}(-\sqrt{2}F) + \sqrt{2}D = \sqrt{\frac{2}{3}}(3F + D).$$

Now we perform $d \leftrightarrow s$ exchange. Our auxiliary baryon $\Lambda_{ds}$ returns to real $\Lambda$ while $\pi^-(\bar{d}u)$ changes to $K^-(\bar{s}u)$ and $\Sigma^+(uu, s)$ changes to $-p(uu, d)$, so that

$$2[g(\pi^- \Sigma^+ \Lambda_{ds})]_{ds} = -2[g(K^- p \Lambda)] = \sqrt{\frac{2}{3}}(3F + D).$$

This is the unitary symmetry result. In the same way we write the formal coupling of pion to $\Sigma^+$ and $\Lambda_{us}$ given by the Eq.(5) and then perform $u \leftrightarrow s$ exchange to obtain

$$2[g(\pi^- \Sigma^+ \bar{\Lambda}_{us})]_{us} = 2[g(K^0 \Xi^0 \bar{\Lambda})] =$$

$$= -\sqrt{\frac{2}{3}}(3F - D).$$

This is again the unitary symmetry result. Similarly one can show that

$$-2[g(\pi^- \Sigma^+ \Sigma^0_{ds})]_{ds} = 2[g(K^- \Sigma^0)] = \sqrt{2}(-F + D),$$

$$-2[g(\pi^- \Sigma^+ \Sigma^0_{us})]_{us} = 2[g(K^0 \Xi^0 \bar{\Sigma}^0)] = -\sqrt{2}(F + D).$$

Derivation of these coupling constants indicates us the way to proceed in the formalism of QCD sum rules.

**QCD SUM RULES**

We use as the example QCD sum rules based on the formalism developed in [6] where unitary symmetry is broken but formulae are rather transparent. The sum rule for the $M \Sigma^0 \Sigma^0$ coupling reads:

$$\frac{1}{\sqrt{2}}m^2_{\not\Delta} \lambda^2_{\Sigma} g(M \Sigma^0 \Sigma^0) e^{-\frac{m^2}{M^2}}[1 + A \Sigma M^2] =$$

$$g_{\not\Delta ss} m^2_{\not\Delta} M^4 E_0(x) \left[ \frac{\langle \bar{s} s \rangle}{12 \pi^2 f_{\not\Delta}} + \frac{3 f_{3.\not\Delta}}{4 \sqrt{2} \pi^2} \right]$$

$$- g_{\not\Delta ss} \frac{1}{f_{\not\Delta}} M^2 (m_d \langle \bar{u} u \rangle + m_u \langle \bar{d} d \rangle) \langle \bar{s} s \rangle$$

$$- g_{\not\Delta ss} \frac{m^2_{\not\Delta}}{72 f_{\not\Delta}} \langle \bar{s} s \rangle \left( \frac{\alpha_s G^2}{\pi} \right)$$
where \( m_q \), \( q = u, d, s \) are current quark masses, \( f_{\mathcal{M}} \) is a \( \mathcal{M} \)-meson decay constant, \( \mathcal{M} = \pi^0, \eta \), quark condensates are \( \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -0.23^3 \text{ GeV}^3 \), \( \langle \bar{s}s \rangle / \langle \bar{d}d \rangle = 0.8 \), while \( m_0^2 = 0.8 \text{ GeV}^2 \), \( \langle \bar{q}q \mathbf{\sigma} \cdot \mathbf{g} q \rangle \equiv m_0^2 \langle \bar{q}q \rangle \). The factor \( E_0(x) = (1 - e^{-x}) \) is used to subtract the continuum contribution, \( x = W^2 / M^2 \) [2] (we take \( W^2 = 2.0 \text{ GeV}^2 \)). The overlap amplitude is taken as \( \lambda_\Sigma^2 = C \cdot M_\Sigma^6 \text{ GeV}^6 \) [6], with \( C = 5.48 \times 10^{-4} \). We neglect in calculations \( f_{A_\mathcal{M}} \). Parameter \( A_B \) accounts for high-resonance contributions.

We define \( \mathcal{D}^{(0)}(\mathcal{M}; M^2; u, d; s) \) and \( \mathcal{F}^{(0)}(\mathcal{M}; M^2; u, d; s) \) (this shorthand notation means that they depend on \( M^2 \), all quark masses and all condensates: \( \mathcal{D}^{(0)}(\mathcal{M}; M^2; u, d; s) \equiv \mathcal{D}^{(0)}(\mathcal{M}; M^2; m_u, \langle \bar{u}u \rangle, ...; m_d, \langle \bar{d}d \rangle, ...; m_s, \langle \bar{s}s \rangle, ...) \), similar for \( \mathcal{F}^{(0)} \) :

\[
\mathcal{F}^{(0)}(\mathcal{M}; M^2; u, d; s) = \frac{1}{6 f_{\mathcal{M}}} m_0^2 [\langle \bar{s}s \rangle \langle m_d \bar{u}u \rangle + m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle],
\]

\[
\mathcal{D}^{(0)}(\mathcal{M}; M^2; u, d; s) - \mathcal{F}^{(0)}(\mathcal{M}; M^2; u, d; s) = -[m^2_{\mathcal{M}} M^4 E_0(x) \left( \frac{\langle \bar{s}s \rangle}{12 \pi^2 f_{\mathcal{M}}} + \frac{3 f_{3_{\mathcal{M}}}}{4 \sqrt{2} \pi} \right) - \frac{1}{f_{\mathcal{M}}} M^2 (m_d \langle \bar{u}u \rangle + m_u \langle \bar{d}d \rangle) \langle \bar{s}s \rangle - \frac{m_{\mathcal{M}}}{72 f_{\mathcal{M}}} \langle \bar{s}s \rangle \langle \alpha_s / \pi \rangle],
\]

The right-hand side (RHS) of the Eq.(10) can be written in a form

\[
\text{RHS}(\mathcal{M} \Sigma^0 \Sigma^0) = g_{\mathcal{M} uu} \mathcal{F}^{(0)}(\mathcal{M}; M^2; u, d; s) + g_{\mathcal{M} dd} \mathcal{F}^{(0)}(\mathcal{M}; M^2; d, u; s) + \frac{1}{2} g_{\mathcal{M} ss} [ \mathcal{D}^{(0)}(\mathcal{M}; M^2; s, d; u) + \mathcal{F}^{(0)}(\mathcal{M}; M^2; s, u; d) ] - \frac{1}{2} g_{\mathcal{M} ss} [ \mathcal{D}^{(0)}(\mathcal{M}; M^2; u, d; s) + \mathcal{D}^{(0)}(\mathcal{M}; M^2; d, u; s) ].
\]

With isotopic invariance we construct Borel sum rule for the \( \pi^- \Sigma^+ \Sigma^0 \):

\[
-m^2_{\pi} \lambda_{\Sigma}^2 g (\pi^- \Sigma^+ \Sigma^0) e^{-m^2_{\Sigma} / M^2} [1 + A_{\Sigma} M^2] = \frac{m_0^2}{6 f_{\pi}} [(m_u \langle \bar{s}s \rangle + m_s \langle \bar{u}u \rangle)(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle)] \equiv \sqrt{2} \mathcal{F}^{(-)}(\pi^- ; M^2; u, d; s)
\]

and a similar sum rule for \( \pi^+ \Sigma^- \Sigma^0 \) coupling (upon \( u \leftrightarrow d \)).

Using analogue of the Eq.(4)

\[
\sqrt{3} \text{RHS}(\pi^0 \Sigma^0 \Lambda) = \text{RHS}(\pi^0 \Sigma^0_{ds} \Sigma^0_{ds}) - \text{RHS}(\pi^0 \Sigma^0_{us} \Sigma^0_{us})
\]

we construct QCD Borel sum rule for \( \pi^0 \Sigma \Lambda \) coupling [8]

\[
\sqrt{3} \frac{m^2_{\pi}}{M^2_{\Sigma}} \lambda_{\Sigma} g (\pi^0 \Sigma^0 \Lambda) M^2 \left( e^{-M^2_{\Lambda} / M^2} - e^{-M^2_{\Sigma} / M^2} \right) [1 + A_{\Sigma \Lambda} M^2] =
\]
Performing transformation:

Isotopic invariance allows to deduce the corresponding expression for the auxiliary couplings upon using quantities $\alpha_s g^2$

And now we are able to derive Borel sum rules for $K$-meson couplings to octet baryons starting from those for $\pi \Sigma \Lambda$ and $\pi \Sigma \Sigma$ couplings given by the Eqs.(13,16). We shall form auxiliary couplings upon using quantities $\Lambda_{ds}, \Sigma_{ds}^0$ and $\Lambda_{us}, \Sigma_{us}^0$ given by the Eq. (5), and then return to those usual ones performing transformations $d \leftrightarrow s$ and $u \leftrightarrow s$. First we construct a formal sum rule for the case where $\Lambda$ is changed to $\Lambda_{ds}$ just by using Eq.(5), and we retain for a moment only RHS of the corresponding sum rules:

$$RHS(\pi^- \Sigma^+ \bar{\Lambda}_{ds}) = -\frac{\sqrt{3}}{2}RHS(\pi^- \Sigma^+ \bar{\Sigma}^0) + \frac{1}{2}RHS(\pi^- \Sigma^+ \bar{\Lambda}) =$$

$$\sqrt{\frac{1}{6}}(3\mathcal{D}(-)(\pi^-; M^2; u, d; s) + \mathcal{D}(-)(\pi^-; M^2; d; u)).$$

(17)

Performing transformation $(d \leftrightarrow s)$ we should change $\pi^-$ to $K^-$ and $\Sigma^+$ to $-p$ to obtain:

$$RHS((g(\pi^- \Sigma^+ \bar{\Lambda}_{ds})_{ds}) = -RHS(g(K^- p \bar{\Lambda}) =$$

$$\sqrt{\frac{1}{6}}(3\mathcal{D}(-)(K^-; M^2; u, s; d) + \mathcal{D}(-)(K^-; M^2; d, s; u))$$

$$\rightarrow |_{exact} SU(3) \frac{1}{\sqrt{6}}(3F + D),$$

(18)
Upon interchange 

\[ \pi^+ \] or in full notation

and Eqs.(13,16) we construct RHS of the sum rule involving

and then return to real

As the last one we construct sum rule for the formal quantity involving

finally

Analogous sum rules can be constructed for

In a similar way constructing a formal sum rule with

one transforms it into the sum rule for

one get the sum rule for the coupling constant

Interchanging \((u \leftrightarrow d)\) one transforms it into the sum rule for \(g_{K^0\pi}\):

In a similar way constructing a formal sum rule with \(\Lambda_{us}\) we obtain:

Upon interchange \((u \leftrightarrow d)\) one get the sum rule for the coupling constant \(K^0\pi\) coupling with kaon. First using Eq.(5) and Eqs.(13,16) we construct RHS of the sum rule involving \(\Sigma^0\):

and then return to real \(\Sigma^0\) with the 2nd transformation \((d \leftrightarrow s)\) changing \(\Sigma^+ \) to \(-p\) and \(\pi^- \) to \(K^-\):

As the last one we construct sum rule for the formal quantity involving \(\Sigma^0_{us}\) to obtain finally

or in full notation

\[
\begin{align*}
2m_K^2g_{K^0\pi^0} \frac{\lambda_N\lambda_NM^2}{(M^2_\Lambda - M^2_N)} (e^{-M^2_N/M^2} - e^{-M^2_N/M^2}) (1 + A_{\Sigma N} M^2) = \\
-2 \cdot RHS((\pi^-\Sigma^+\Sigma^0)_{us}) = 2 \cdot RHS(K^-\pi^0) = \\
-\sqrt{2} \mathcal{D}^-(K^-; d; s) + \sqrt{2} \mathcal{D}^-(K^-; d; s) \\
\rightarrow |_{\text{exact}} SU(3) \quad -\sqrt{2} (F - D), \tag{22}
\end{align*}
\]
\[-\sqrt{2}\mathcal{D}^{(-)}(K^0, M^2; s, d; u) - \sqrt{2}\mathcal{D}^{(-)}(K^0, M^2; u, d; s) \rightarrow |_{\text{exact SU}(3)} - \sqrt{2}(F + D),\]

(23)

Sum rules for other $g_{K\Sigma}$ and $g_{K\Xi}$ couplings are obtained with isotopic transformations.

**SUMMARY AND RESULTS**

Thus we have constructed QCD sum rules with the Lorenz structure $i\gamma_5$ for K meson - baryon coupling constants $g_{K\Sigma}$ and $g_{K\Xi}$, $Y = \Sigma, \Lambda$ starting from those for $g_{\pi\Sigma}$ and $g_{\pi\Lambda}$ ones. We have calculated (absolute values of) coupling constants of $K$-mesons to octet baryons. The results are presented in the Tables 1,2. In order to control our results we recalculate sum rules for $\pi$ couplings to baryons obtaining values close to those of [6]. As the Lorenz structure $i\gamma_5$ was chosen mostly for methodical reasons the results do not pretend to account for real quantities [6].

The sum rules confirm a known result that unitary picture in terms of the D and F constants is not suitable for meson-baryon couplings due to large symmetry breaking. At the same time these sum rules when expressed in terms of the generalized functions $\mathcal{F}$ and $\mathcal{D}$ reveal indeed a simple $SU(3)_f$ pattern, and this is one of the main results we present here. The relations obtained here indicate in what way one can change and use the concept of the unitary symmetry in the framework of QCD sum rules.

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**Table 1.** The best-fitted values of the coupling constants \( g_{YN} \), \( g_{KY} \) and corresponding values of \( A_{YN}, A_{KY} \) are given together with the Borel windows for each sum rule, \( Y = \Lambda, \Sigma \)

| Coupling     | Borel Window \( M^2, \text{GeV}^2 \) | \( g \) | \( A_g, \text{GeV}^{-2} \) | \( A, \text{GeV}^{-2} \) |
|--------------|-------------------------------------|--------|------------------|-----------------|
| \( \pi^0 pp \) | 1.0-1.4                             | 13.4/\( \sqrt{2} \) | 5.75             | 0.62            |
| \( \bar{K}^0 \Xi^0 \Lambda \) | 1.3-2.3                             | -1.35  | 0.8              | -0.59           |
| \( K^- p \Sigma^0 \) | 1.1-2.1                             | 1.18   | 2.53             | 2.14            |
| \( \bar{K}^0 \Xi^0 \Sigma^0 \) | 1.5-2.5                             | -3.09  | -0.94            | 0.30            |

**Table 2.** The values of the coupling constants \( g_{YN} \), \( g_{KY} \), \( Y = \Lambda, \Sigma \) of this work as well as of several recent works are given

| Coupling | \( |g| \) [11] | \( g \) [9],[10] | \( g \), this work |
|----------|--------------|-----------------|-------------------|
| \( \pi^0 pp \) | -            | -               | 13.4/\( \sqrt{2} \) (input) |
| \( KN \Lambda \) | 2.37±0.09 | -3.47           | 0.77              |
| \( \bar{K} \Xi \Lambda \) | -          | -               | 1.35              |
| \( KN \Sigma \) | 0.025±0.015 | 1.17            | 1.18              |
| \( \bar{K} \Xi \Sigma \) | -          | 7.02            | -3.09             |