UPGRADING OF PRECISION CALCULATIONS FOR ELECTROWEAK OBSERVABLES

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Abstract

A critical assessment is given of the comparison between the new versions of the programs TOPAZ0 40i and ZFITTER 510. The relevance for precision calculations around the $Z$ resonance is briefly discussed.
1 Introduction

In 1995 the Z phase of LEP came to an end and at present the ultimate analysis of the data is imminent. This involves in particular the completion of the line-shape analysis, including the final LEP energy calibration. Consequently, the safest possible estimate of the theoretical accuracy is of the utmost importance. It should be noted that the LEP 1 data (1990–1995) were taken in the energy range $|\sqrt{s} - M_Z| < 3$ GeV and consist of the hadronic and leptonic cross sections, the leptonic forward–backward asymmetries, the various polarization asymmetries, the partial widths, and the quark forward–backward asymmetries. All this makes it mandatory to assess the theoretical precision of the available programs for different channels and for different pseudo-observables.

In this note we focus on the calculation of the pseudo-observables. Independently of the renormalization procedure that is used, the matrix element for $Z \rightarrow f \bar{f}$ will be written as

$$M_{Zf\bar{f}} = i f_{Z} \left( G_{V}^{f} + G_{A}^{f} \gamma_{5} \right) v_{f}. \quad (1)$$

With the above results we can now define the pseudo-observable quantities that are relevant for the phenomenology of LEP 1/SLC. The pseudo-variables are related to measured cross sections and asymmetries by some deconvolution or unfolding procedure. The concept of pseudo-observability is introduced by saying that the experiments measure some primordial (basically cross sections and thereby asymmetries also) quantities, which are then reduced to secondary quantities under some set of specific assumptions. Within these assumptions the secondary quantities, the pseudo-observables, also deserve the label of observability.

In 1995 the CERN Report [1] on ‘Precision Calculations for the Z resonance’ provided as basic documentation the theoretical basis for upgrading the ’89 Report on Z Physics at LEP 1’ [2].

Although the previous analyses remain quite comprehensive, an update of the discussion of radiative corrections has become necessary for one very good reason: a sizeable amount of theoretical work has appeared following the CERN report of 1995. In particular, a crucial amount of work has been performed in providing higher-order QCD corrections, mixed electroweak and QCD corrections [3], and sub-leading two-loop corrections of $O(\alpha^{2}m_{t}^{2})$ [4].

In ref. [4] the two-loop $O(\alpha^{2}m_{t}^{2})$ corrections are incorporated in the theoretical calculation of $M_{w}$ and $\sin^{2} \theta_{W}$. More recently the complete calculation of the decay rate of the Z has been made available to us [5]. The only case that is not covered is the one of final $b$-quarks, because it involves specific $O(\alpha^{2}m_{t}^{2})$ vertex corrections.

Another recent development in the computation of radiative corrections to the hadronic decay of the Z is contained in two papers, which, together, provide complete corrections of $O(\alpha \alpha_{s})$ to $\Gamma(Z \rightarrow q\bar{q})$ with $q = u, d, s, c$ and $b$. In the first reference of [3] the decay into light quarks is treated. In the second one the remaining diagrams contributing to the decay into bottom quarks are considered and thus the mixed two-loop corrections are complete.

2 Numerical results and comparison

Two of the programs described in the ’95 CERN Report have been constantly updated [6], and we focus, in this note, on a comparison between TOPAZ0 [7] and ZFITTER [8], with an update of the
predictions of $Z$-resonance observables within the minimal standard model.

In Table 1 we compare the prediction of TOPAZ0 and ZFITTER for $M_Z = 91.1867$ GeV, $m_t = 175.6$ GeV, $\alpha_s(M_Z^2) = 0.120$ and $M_H = 100$ GeV. The results are from the new versions of the programs, and we have also shown absolute and relative (in per mille) deviations for a set of 25 pseudo-observables. The relative deviation is defined as

$$\delta = 2 \frac{\text{TOPAZ0} - \text{ZFITTER}}{\text{TOPAZ0} + \text{ZFITTER}}. \quad (2)$$

For quantities such as the asymmetries, we report the absolute deviation, which is the only relevant one.

A similar comparison for $M_H = 200$ GeV is shown in Table 2. We observe a deviation in $\sin^2 \theta_{\text{eff}}^l$ of $2.7 \times 10^{-5}$ ($2.3 \times 10^{-5}$) for $M_H = 100$ GeV (200 GeV). For the total $Z$ width, the difference between the two programs is of 0.19 MeV (0.08 MeV) for $M_H = 100$ GeV (200 GeV).

In the hadronic $Z$ width the difference is 0.07 MeV at $M_H = 100$ GeV, which roughly corresponds to a variation of $\Delta \alpha_s(M_Z^2) = 0.00013$ in the predictions for $\alpha_s(M_Z^2)$ from the two programs. Variations for the $W$ mass are everywhere below 0.5 MeV.

The level of agreement that is reached is highly satisfactory, especially if we take into account the fact that the implementation of the new correction factors has been performed in a completely independent way, different renormalization schemes and, more important, absolutely different strategies.

It is also interesting to compare the present situation with the differences registered between the two codes at the time of the ’95 CERN Report. For this reason we have taken again $M_Z = 91.1888$ GeV, $m_t = 175$ GeV, $\alpha_s(M_Z^2) = 0.125$ and $M_H = 300$ GeV (the ’95 input parameter set) and compared some of the predictions. In Tables 3-4 we give the comparison showing, at the same time, the old–old and new–new deviations and the shifts old–new.

It is worth noticing that new–new deviations are always less (or much less) than the corresponding old–old ones. This fact induces a sizeable reduction of the theoretical uncertainty, achieved after the implementation of the new correction factors. We have also compared our results without the sub-leading terms $O(\alpha^2 m_t^2)$ and found again the same level of agreement as reached in ’95.

In conclusion, we have achieved an important goal: after a substantial upgrading, TOPAZ0 and ZFITTER continue to agree with each other extremely well, in most cases better than they ever have.

An important consequence of this fact is that the central value of the Higgs boson mass in the famous $\chi^2(M_H)$ curve moves down from 115 GeV obtained with old versions to approximately 87 GeV. The difference in predictions between TOPAZ0 and ZFITTER is less than 5 GeV in any of the fits performed so far [9]. Moreover, a substantial reduction is expected of the entire blue band that is giving the theoretical uncertainty in the same curve.

3 Acknowledgements

We would like to express special thanks to the TOPAZ0 and ZFITTER teams [10]. Without their contributions the two programs would not be what they are. We both would like to express special
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[6] The updated versions of programs might be accessed from the following web-sites:
   TOPAZ0: http://wftp.to.infn.it/pub/giampiero.
   ZFITTER: http://www.ifh.de/theory/publist.html#software.
   The new descriptions are in preparation.

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[9] The LEP Collaborations: ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavour Group. Prepared from Contributions of the LEP and SLD experiments to the 1998 winter conferences, in preparation.

[10] At present, the following physicists are active members of the two teams:
   TOPAZ0: G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini.
   ZFITTER: D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olshevski, S. Riemann, T. Riemann.
|                | TOPAZ0 | ZFITTER | Rel. dev. (per mille) | Abs. dev. |
|----------------|--------|---------|------------------------|-----------|
| $M_W$ [GeV]    | 80.3864| 80.3860 | 0.005                  | 0.4 [MeV] |
| $\Gamma_\nu$ [MeV] | 167.235| 167.262 | -0.16                  | -0.027 [MeV] |
| $\Gamma_e$ [MeV] | 84.0028| 84.0140 | -0.13                  | -0.011 [MeV] |
| $\Gamma_\mu$ [MeV] | 84.0021| 84.0133 | -0.13                  | -0.011 [MeV] |
| $\Gamma_\tau$ [MeV] | 83.8110| 83.8237 | -0.15                  | -0.013 [MeV] |
| $\Gamma_u$ [MeV] | 300.372| 300.387 | -0.05                  | -0.015 [MeV] |
| $\Gamma_d$ [MeV] | 383.161| 383.187 | -0.068                 | -0.026 [MeV] |
| $\Gamma_c$ [MeV] | 300.315| 300.329 | -0.047                 | -0.014 [MeV] |
| $\Gamma_b$ [MeV] | 376.100| 376.082 | 0.048                  | 0.018 [MeV] |
| $\Gamma_Z$ [MeV] | 2496.62| 2496.81 | -0.076                 | -0.19 [MeV] |
| $\Gamma_h$ [MeV] | 1743.10| 1743.17 | -0.040                 | -0.07 [MeV] |
| $\Gamma_{inv}$ [MeV] | 501.706| 501.787 | -0.16                  | -0.081 [MeV] |
| $\sin^2 \theta_{eff}$ | 0.231489| 0.231516 | -0.12                  | -0.000027 |
| $\sin^2 \theta_{eff}$ | 0.232788| 0.232902 | -0.49                  | -0.000114 |
| $A_{FB}^l$ | 0.0162774| 0.0162225 | - | 0.000549 |
| $A_{FB}^b$ | 0.103232| 0.103094 | - | 0.000138 |
| $A_{FB}^c$ | 0.0738276| 0.0736708 | - | 0.0001568 |
| $A_{LR}$ | 0.147320| 0.147071 | - | 0.000249 |
| $\sigma_{had}$ [nb] | 41.4717| 41.4734 | -0.041                 | -0.0017 [nb] |
| $R_l$ | 20.7505| 20.7486 | 0.092                  | 0.0019 |
| $R_b$ | 0.215765| 0.215746 | 0.088                  | 0.000019 |
| $R_c$ | 0.172288| 0.172289 | -0.006                 | -0.000001 |
| $A_{LR}^l$ | 0.934703| 0.934638 | 0.070                  | 0.000065 |
| $A_{LR}^c$ | 0.667961| 0.667892 | 0.103                  | 0.000069 |
| $s_W$ | 0.222855| 0.222862 | 0.031                  | -0.000007 |

Table 1: Comparison of TOPAZ0 40i and ZFITTER 510 for $M_\mu = 100$ GeV. Here $M_Z = 91.1867$ GeV, $m_t = 175.6$ GeV, $\alpha_s(M_Z^2) = 0.120$, $1/\alpha_{em}(M_Z^2) = 128.896$. 


Table 2: Comparison of TOPAZ0 40i and ZFITTER 510 for $M_H = 200$ GeV. Here $M_Z = 91.1867$ GeV, $m_t = 175.6$ GeV, $\alpha_s(M_Z^2) = 0.120$, $1/\alpha_{em}(M_Z^2) = 128.896$. 

| NEW versions | TOPAZ0 | ZFITTER | Rel. dev. (per mille) | Abs. dev. |
|--------------|--------|---------|-----------------------|-----------|
| $M_W$ [GeV]  | 80.3417| 80.3416 | 0.001                 | 0.1 [MeV] |
| $\Gamma_{\nu}$ [MeV] | 167.181| 167.198 | -0.10                 | -0.017 [MeV] |
| $\Gamma_e$ [MeV] | 83.9576| 83.9635 | -0.070                | -0.006 [MeV] |
| $\Gamma_{\mu}$ [MeV] | 83.9569| 83.9629 | -0.071                | -0.006 [MeV] |
| $\Gamma_{\tau}$ [MeV] | 83.7658| 83.7733 | -0.090                | -0.008 [MeV] |
| $\Gamma_{\nu}$ [MeV] | 300.083| 300.080 | 0.010                 | 0.003 [MeV] |
| $\Gamma_{\mu}$ [MeV] | 382.863| 382.866 | -0.008                | -0.003 [MeV] |
| $\Gamma_{\tau}$ [MeV] | 300.026| 300.022 | 0.013                 | 0.004 [MeV] |
| $\Gamma_{\nu}$ [MeV] | 375.785| 375.785 | 0                     | 0 [MeV] |
| $\Gamma_{\mu}$ [MeV] | 2494.83| 2494.91 | -0.032                | -0.08 [MeV] |
| $\Gamma_{\tau}$ [MeV] | 1741.61| 1741.62 | -0.006                | -0.01 [MeV] |
| $\Gamma_{\nu}$ [MeV] | 501.542| 501.593 | -0.10                 | -0.051 [MeV] |
| $\sin^2\theta^l_{\text{eff}}$ | 0.231849| 0.231872 | -0.099                | -0.000023 |
| $\sin^2\theta^b_{\text{eff}}$ | 0.233150| 0.233247 | -0.42                 | -0.000097 |
| $A^l_{FB}$ | 0.0156576| 0.0156088 | -0.42                 | -0.000488 |
| $A^b_{FB}$ | 0.101185| 0.101101 | -                    | -0.000084 |
| $A^l_{LR}$ | 0.0722734| 0.0721303 | -0.006                | -0.0001431 |
| $A^b_{LR}$ | 0.144488| 0.144263 | -                    | -0.000225 |
| $\sigma_{\text{had}}$ [nb] | 41.4734| 41.4746 | -0.029                | -0.0012 [nb] |
| $R_l$ | 20.7439| 20.7426 | 0.063                 | 0.0013 |
| $R_b$ | 0.215769| 0.215768 | 0.005                 | 0.000001 |
| $R_c$ | 0.172269| 0.172266 | 0.017                 | 0.000003 |
| $A^l_{LR}$ | 0.934471| 0.934416 | 0.059                 | 0.000055 |
| $A^b_{LR}$ | 0.666715| 0.666657 | 0.087                 | 0.000058 |
| $s^2_W$ | 0.223719| 0.223721 | 0.009                 | -0.000002 |
### NEW versus OLD

| TOPAZ0 | ZFITTER | Rel. dev. (per mille) | Abs. dev. |
|--------|---------|-----------------------|-----------|
| $M_{\nu}$ [GeV] |         |                       |           |
| Old    | 80.310  | 80.317                | -0.09     | -7 [MeV] |
| New    | 80.308  | 80.308                | -0.01     | -0.1 [MeV] |
| Rel. shift (per mille) | -0.03 | -0.11 | |
| Abs. shift | -2.1 [MeV] | -9.0 [MeV] | |
| $\Gamma_e$ [MeV] |         |                       |           |
| Old    | 83.931  | 83.941                | -0.12     | -0.01 [MeV] |
| New    | 83.915  | 83.920                | -0.07     | -0.006 [MeV] |
| Rel. shift (per mille) | -0.19 | -0.25 | |
| Abs. shift | -0.016 [MeV] | -0.021 [MeV] | |
| $\sin^2 \theta^l_{\text{eff}}$ |         |                       |           |
| Old    | 0.23200 | 0.23205               | -0.22     | -5.0×10^{-5} |
| New    | 0.23209 | 0.23211               | -0.09     | -2.0×10^{-5} |
| Rel. shift (per mille) | 0.39 | 0.26 | |
| Abs. shift | 8.9×10^{-5} | 6.1×10^{-5} | |
| $A_{\text{FB}}^l$ |         |                       |           |
| Old    | 0.015360 | 0.015310             | -         | 5.0×10^{-5} |
| New    | 0.015249 | 0.015204             | -         | 4.5×10^{-5} |
| Abs. shift | -1.1×10^{-4} | -1.1×10^{-4} | |
| $A_{\text{LR}}^l$ |         |                       |           |
| Old    | 0.14327 | 0.14289 | - | 3.8×10^{-4} |
| New    | 0.14259 | 0.14238 | - | 2.1×10^{-4} |
| Abs. shift | -6.8×10^{-4} | -5.1×10^{-4} | |
| $\Gamma_Z$ [MeV] |         |                       |           |
| Old    | 2497.4  | 2497.5                | -0.04     | -0.1 [MeV] |
| New    | 2496.1  | 2496.2                | -0.04     | -0.1 [MeV] |
| Rel. shift (per mille) | -0.52 | -0.52 | |
| Abs. shift | -1.29 [MeV] | -1.30 [MeV] | |

Table 3: Comparison of TOPAZ0 and ZFITTER. Here $M_Z = 91.1888$ GeV, $m_t = 175$ GeV, $M_H = 300$ GeV, $\alpha_s(M_Z^2) = 0.125$ and $1/\alpha_{\text{em}}(M_Z^2) = 128.896$. 
|               | TOPAZ0 | ZFITTER  | Rel. dev. (per mille) | Abs. dev. |
|---------------|--------|----------|-----------------------|-----------|
| $R_l$         |        |          |                       |           |
| Old           | 20.782 | 20.781   | 0.05                  | 0.001     |
| New           | 20.773 | 20.773   | 0                     | 0         |
| Rel. shift(per mille) | -0.41  | -0.39    |                       |           |
| Abs. shift    | -0.0085| -0.008   |                       |           |
| $R_b$         |        |          |                       |           |
| Old           | 0.21567| 0.21571  | -0.19                 | -4.0×10^{-5}|
| New           | 0.21579| 0.21580  | -0.05                 | -1.0×10^{-5}|
| Rel. shift(per mille) | 0.57   | 0.42     |                       |           |
| Abs shift     | 1.2×10^{-4}| 9.0×10^{-5}|                       |           |
| $R_c$         |        |          |                       |           |
| Old           | 0.17237| 0.17236  | 0.06                  | 1.0×10^{-5}|
| New           | 0.17235| 0.17235  | 0                     | 0         |
| Rel. shift(per mille) | -0.09  | -0.06    |                       |           |
| Abs. shift    | -1.6×10^{-5}| -1.0×10^{-5}|                       |           |
| $A^B_{FB}$    |        |          |                       |           |
| Old           | 0.10033| 0.10013  | -                     | 2.0×10^{-4}|
| New           | 0.099815| 0.099767 | -                     | 4.8×10^{-5}|
| Abs. shift    | -5.1×10^{-4}| -3.6×10^{-4}|                       |           |
| $A^C_{FB}$    |        |          |                       |           |
| Old           | 0.071590| 0.071380 | -                     | 2.1×10^{-4}|
| New           | 0.071235| 0.071100 | -                     | 1.3×10^{-4}|
| Abs. shift    | -3.6×10^{-4}| -2.8×10^{-4}|                       |           |

Table 4: Comparison of TOPAZ0 and ZFITTER. Here $M_Z = 91.1888$ GeV, $m_t = 175$ GeV, $M_H = 300$ GeV, $\alpha_s(M_Z^2) = 0.125$ and $1/\alpha_{em}(M_Z^2) = 128.896$. 