Practical Method for Calculating Settlement of underlying stratum in composite ground

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Abstract. In the current settlement calculation methods of composite ground, there are considerable discrepancies between calculating data and measuring data. Based on Mindlin displacement solution, a simple and practical approach is presented for the calculation of settlement of underlying stratum of the reinforced area in composite ground. According to this method, the calculated result agrees well with measured data.

1. Introduction

Generally, the settlement of the composite foundation consists of two parts: the compression of the reinforced area and the compression of the underlying layer[1-3]. The accuracy of the composite foundation deformation calculation depends largely on the deformation calculation of the underlying layer in the reinforced area[4]. The calculation methods of compression in the reinforced area include composite modulus method, stress correction method and pile compression method[5]. Among the three commonly used methods for calculating the deformation of the underlying layer in the reinforced area, the improved Geddes method is complicated to calculate and very inconvenient to apply, and its rationality needs to be further studied [6-7]. Both the stress diffusion method and the equivalent solid method are based on finding the magnitude of the stress $p_b$ at the top surface of the weak underlying layer and its action range, and then calculating the additional stress in the underlying layer according to the Boussinesq solution. It is derived on the condition of a half-space homogeneous elastic body surface (equivalent to the surface), and $p_b$ acts inside the half-space elastic body (at a considerable depth under the surface). The loading conditions do not match the Brinell solution assumptions. In this paper, based on the Mindlin’s solution under concentrated force, the calculation formula of elastic theoretical settlement under rectangular uniformly distributed flexible load is derived, which can be used in the deformation calculation of the underlying layer in the reinforced area of composite foundation. This method can consider the influence of the burial depth of the load acting surface and the foundation soil condition (Poisson's ratio) on the foundation stress and deformation. Both theoretically and logically from the calculation results are more reasonable than the current method using Brinell solution.

2. Formula derivation

2.1. Settlement of the corner and center of the action surface
According to the research of Mindlin (1936) [8], the concentrated force $P$ acts on the depth $h$ of the elastic half-space body (Figure 1), and the vertical displacement of any point $M$ in the foundation from the ground depth $Z$ is:

$$W = \frac{P(1+\mu)}{8\pi E(1-\mu)} \left[ \frac{a_1}{R_1} + \frac{a_2}{R_2} + \frac{a_3}{R_1^3} + \frac{a_4}{R_2} + \frac{a_5}{R_2^3} \right]$$

where $r$ is the horizontal distance from the line of action of the force to the point under consideration; $\mu$ is the Poisson's ratio of the soil; $E$ is the elastic modulus of the soil (estimated instantaneous settlement of cohesive soil, generally expressed by $E$) or deformation modulus (estimated Final settlement, expressed by $E_0$), MPa;

![Figure 1. The vertical point load acts at a certain depth below the surface of ground.](image)

$$a_1 = 3 - 4\mu; \ a_2 = 5 - 12\mu + 8\mu^2; \ a_3 = (Z - h)^2;$$

$$a_4 = (3 - 4\mu)(Z + h)^2 - 2hZ; \ a_5 = 6hZ(Z + h)^2;$$

$$R_1 = \left[ r^2 + (Z - h)^2 \right]^{1/2}; \ R_2 = \left[ r^2 + (Z + h)^2 \right]^{1/2}.$$

Now we suppose that the thickness of the reinforced area of the composite foundation is $h$, and the additional stress $p_h$ at the top surface of the underlying layer is evenly distributed on the rectangular area of $a \times b$, as shown in Figure 2. Since $p_h$ is an evenly distributed flexible load, the settlement $(WC)_{Min}$ of the load action corner o can be obtained by integrating the load distribution area using equation (1). Now take the corner point o of the rectangle as the origin for the rectangular coordinate, and then take the differential area $dF = dx dy$ at any point on the area of the rectangle, then the load on this area is $p_h dx dy$. The vertical displacement caused by this load to the ground $Z$ below point o is:

$$dW = \frac{p_h(1+\mu)}{8\pi E(1-\mu)} \left[ \frac{a_1}{R_1} + \frac{a_2}{R_2} + \frac{a_3}{R_1^3} + \frac{a_4}{R_2} + \frac{a_5}{R_2^3} \right] dx dy$$

![Figure 2. The loading on substratum of improved region composite foundation.](image)
The vertical displacement caused at the corner \( o \) due to the uniformly distributed load on the rectangle \( a \times b \) is:

\[
\begin{align*}
(W_c)_{\min} &= \frac{p_x(1+\mu)}{8\pi E(1-\mu)} \left\{ (3-4\mu)[a\ln b+\sqrt{a^2+b^2}]+b\ln a+\sqrt{a^2+b^2} \right\} \\
&+ (5-12\mu+8\mu^2)[a\ln b+\sqrt{a^2+b^2+4h^2}]+b\ln a+\sqrt{a^2+b^2+4h^2} \\
&+ \frac{2abh^2}{(a^2+b^2+4h^2)} \left[ \frac{1}{a^2+b^2+4h^2} - 4h(1-4\mu+4\mu^2)\tan^{-1}\left( \frac{1}{2h} \sqrt{\frac{a^2+b^2+4h^2}{a^2+b^2}} \right) \right]
\end{align*}
\]

Equation (2) can be further written as follows:

\[
\begin{align*}
(W_c)_{\min} &= \frac{p_x(1+\mu)}{8\pi E(1-\mu)} \left\{ (3-4\mu)[a\ln n+\sqrt{n+1+n^2}]+n\ln \frac{1+\sqrt{n+1+n^2}}{n} \right\} \\
&+ (5-12\mu+8\mu^2)[n\ln \frac{n+\sqrt{1+n^2+4t^2}}{\sqrt{1+4t^2}}]+n\ln \frac{1+\sqrt{1+n^2+4t^2}}{\sqrt{1+n^2+4t^2}} \\
&+ \frac{2nt^2}{(1+4t^2)(n^2+4t^2)(1+n^2+4t^2)^{\frac{1}{2}}} - 4t(1-4\mu+4\mu^2)\tan^{-1}\left( \frac{n}{2t(1+n^2+4t^2)^{\frac{1}{2}}} \right)
\end{align*}
\]

where \( n=b/a \), which is the length-to-width ratio of the rectangular load acting surface; \( t=h/a \), the ratio of the buried depth of the rectangular load acting surface to the width of the load acting surface, that is, the relative buried depth of the load acting surface.

When the load acts on the ground \( (h=0) \), according to the Boussinesq solution, the vertical displacement \( (W_c)_{\text{bou}} \) at the corner \( o \) is:

\[
(W_c)_{\text{bou}} = \frac{p_x(1-\mu^2)}{\pi E} [n\ln \frac{1+\sqrt{1+n^2}}{n}+\ln(n+\sqrt{1+n^2})]
\]

The symbols in the formula have the same meanings as above.

Dividing Formula (3) by Formula (4), the ratio of the two is as follows:

\[
K_c = \frac{(W_c)_{\min}}{(W_c)_{\text{bou}}} = \frac{1}{8(1-\mu^2)} \left\{ (3-4\mu)[5-12\mu+8\mu^2]\frac{1}{\sqrt{1+4t^2}} \\
+n\ln \frac{1+\sqrt{1+n^2+4t^2}}{\sqrt{1+4t^2}} + \frac{2nt^2}{(1+4t^2)(n^2+4t^2)(1+n^2+4t^2)^{\frac{1}{2}}} \\
- 4t(1-4\mu+4\mu^2)\tan^{-1}\left( \frac{n}{2t(1+n^2+4t^2)^{\frac{1}{2}}} \right) \right\}
\]

\( K_c \) is the settlement correction factor of the corner of the flexible rectangular uniformly distributed load:

\[
K_c = f(\mu,n,t)
\]

Substituting different values of \( \mu, n, \) and \( t \) into equation (5), a series of \( K_c \) values can be obtained.
and a coefficient table can be formulated. Due to the length limit of the paper, this will not be elaborated here.

Since it is an absolutely flexible load, the settlement \((W_0)_{\text{Min}}\) of the center point of the rectangular flexible uniformly distributed load acting surface can be obtained by Formula (3) using the corner point method:

\[
(W_0)_{\text{Min}} = \frac{p_a(1+\mu)\alpha}{4\pi E(1-\mu)} \left\{ (3-4\mu)\ln(n+\sqrt{1+n^2}) + n\ln\frac{1+\sqrt{1+n^2}}{n} ight. \\
+ (5-12\mu+8\mu^2)\left[\ln\frac{n+\sqrt{1+n^2}+16t^2}{\sqrt{1+16t^2}} + n\ln\frac{1+\sqrt{1+n^2}+16t^2}{\sqrt{1+n^2}+16t^2} \right] \\
+ \frac{8nt^2(1+n^2+32t^2)}{(1+16t^2)(n^2+16t^2)(1+n^2+16t^2)^{\frac{3}{2}}} - 8t(1-4\mu+4\mu^2)\tan^{-1}\frac{n}{4t(1+n^2+16t^2)^{\frac{3}{2}}} \right\} \quad (6)
\]

When the load \(P_b\) acts on the ground \((h=0)\), according to the Boussinesq solution, the vertical displacement \((W_0)_{\text{Bou}}\) at the center of the rectangular load acting surface is:

\[
(W_0)_{\text{Bou}} = \frac{2p_a(1-\mu)^2\alpha}{\pi E} \left[ n\ln\frac{1+\sqrt{1+n^2}}{n} + \ln(n+\sqrt{1+n^2}) \right] \quad (7)
\]

Divide Equation (6) by Equation (7), and the ratio of the two is obtained:

\[
K_0 = \frac{(W_0)_{\text{Min}}}{(W_0)_{\text{Bou}}} = \frac{1}{8(1-\mu)^2} \left\{ (3-4\mu) + [(5-12\mu+8\mu^2)\ln\frac{n+\sqrt{1+n^2}+16t^2}{\sqrt{1+16t^2}} \right. \\
+ n\ln\frac{1+\sqrt{1+n^2}+16t^2}{\sqrt{n^2+16t^2}} + \frac{8nt^2(1+n^2+32t^2)}{(1+16t^2)(n^2+16t^2)(1+n^2+16t^2)^{\frac{3}{2}}} \\
- 8t(1-4\mu+4\mu^2)\tan^{-1}\frac{n}{4t(1+n^2+16t^2)^{\frac{3}{2}}} \left[ n\ln\frac{1+\sqrt{1+n^2}}{n} + \ln(n+\sqrt{1+n^2}) \right] \} \quad (8)
\]

We call \(K_0\) the settlement correction factor at the center of the flexible rectangular uniformly distributed load, which is expressed as:

\[
K_0 = F(\mu,n,t)
\]

Substituting different values of \(\mu, n,\) and \(t\) into equation (8), a series of \(K_0\) values can be obtained, and a coefficient table is formulated. Due to the length limit of the paper, this will not be elaborated here.

2.2. Calculation of average settlement
It is mathematically more difficult to deduce the formula for the average settlement of the underlying layer in the reinforced area under the rectangular flexible load \(P_b\) using the Mindlin's solution. However, it can be seen from the calculation results of \(K_c\) and \(K_0\) that under the condition that \(\mu, n,\) and \(t\) are the same, the settlement correction coefficient \(K_c\) of the corner point of the rectangular load acting surface is not much different from the settlement correction coefficient \(K_0\) of the center point.

Therefore, we can approximately take the mean value of \(K_c\) and \(K_0\) for the same \(\mu, n,\) and \(t\) as the correction factor of the average settlement in the corresponding case, expressed with \(K_n\), that is \(K_n = (K_c + K_0) / 2\), the \(K_n\) value table will be omitted here. The resulting errors generally do not exceed the allowable range of the project.

In the actual settlement calculation, the settlement amount \(W_{\text{Bou}}\) can be calculated according to the
elastic settlement calculation formula (4) or (7) of the foundation based on the Brinell displacement solution. This method determines the corresponding correction factor $K$. Finally, the required settlement of the underlying layer of the composite foundation reinforcement area $s_2$ is:

$$s_2 = W_{Min} = K_m \cdot W_{Bou}$$  \hspace{1cm} (9)

3. Engineering example calculation

Considering that it is best to have actual settlement observations, the data is more comprehensive, and there are certain calculations to facilitate comparison, this article selects the examples in [9] for comparative analysis. The relevant content is excerpted as follows:

A multi-storey residential complex in Shanghai is reinforced with powder-sprayed piles. The composite foundation has a design bearing capacity of 140 kPa, a reinforced area of $74 \times 16$ m$^2$, a design pile length of 14.0 m, a pile diameter of 0.5 m, and a replacement rate of 15%. Sex parameters are shown in Table 1. After the completion of the building, the measured settlement averaged 102 mm.

Table 1. The parameters of strata.

| Layer number | Name of soil layer     | Average thickness (m) | Severe (kN·m$^{-3}$) | Compression modulus (N·mm$^{-2}$) | Bearing capacity (kPa) |
|--------------|-----------------------|-----------------------|----------------------|-----------------------------------|------------------------|
| 1            | Ploughing             | 1.5                   |                      |                                   |                        |
| 2            | Brownish yellow silty clay | 1.5                | 18.6                 | 3.68                             | 85                     |
| 3            | Grey silt             | 1.5                   | 18.7                 | 8.79                             | 90                     |
| 4            | Gray silty silty clay | 4.0                   | 18.0                 | 2.94                             | 75                     |
| 5            | Gray silty clay       | 9.5                   | 17.2                 | 1.93                             | 60                     |
| 6            | Gray clay             | 1.5                   | 17.5                 | 2.74                             | 70                     |
| 7            | Gray silty silty clay | 4.5                   | 18.1                 | 4.01                             | 80                     |
| 8            | Gray silty clay       | 10.0                  | 18.4                 | 5.09                             |                        |

Considering the critical pile length of flexible piles, for the settlement calculation of cement-soil mixing pile composite foundation with a design pile length greater than the critical pile length, the compression layer is divided into three parts: ① The critical pile length part, the pile body compression is greater, its compression The amount is $S_1$; ② The part of the pile body range below the critical pile length can be regarded as a non-compressed pile body, which is equivalent to a "rigid pile", which "sinks" with the deformation of the underlying layer, and its compression amount is $S_2 = 0$; The compression of the lying layer is $S_3$ (Figure 3). We assume $l_c = 9$m, and then calculate $S_1$ using the composite modulus method, calculate the additional stress on the top surface of the underlying layer according to the principle of the stress correction method, and then calculate $S_3$ using the layering method to obtain the total settlement of the composite foundation is 119.3 mm [9].

Based on [9], the method of this paper is used to calculate the settlement of the underlying layer, $K_m = 0.8038$, and the settlement value of the composite foundation is 106.6mm. It can be seen that the calculated settlement value obtained by the method in this paper is very close to the measured value.

The comparison of the calculated values of composite foundation settlement obtained by various methods is listed in Table 2.

![Figure 3. The settlement calculation model of considering the critical length of column.](image-url)
Table 2. Comparison of results with different calculation methods.

|                          | Reinforcement layer compression (mm) | Lower layer compression (mm) | Total settlement (mm) | Actual settlement (mm) |
|--------------------------|-------------------------------------|-----------------------------|-----------------------|----------------------|
| $E_p$ method[9]          | 38.0                                | 160.0                       | 198.0                 |                      |
| $E_s$ method[9]          | 264.3                               | 160.0                       | 424.3                 |                      |
| $E_{sp}$ method[9]       | 78.8                                | 160.0                       | 238.8                 | 102                  |
| Reference [9] method     | 54.4                                | 64.9                        | 119.3                 |                      |
| This article method      | 54.4                                | 52.2                        | 106.6                 |                      |

4. Conclusion
(1) Whether it is based on the Brinell displacement solution or the elastic settlement formula based on the Mindlin's solution proposed in this paper, $E$ (or $E_0$) is assumed to be unchanged in the entire foundation soil layer, but in fact the elasticity of the foundation soil The parameters change with depth, so these methods are only approximate when the local base soil layer is relatively uniform. For layered foundations, the weighted average of the $E$ (or $E_0$) values of each layer of soil can be approximated.

(2) Engineering practice shows that the theoretical calculation value of settlement based on the elastic mechanics formula based on the Brinell displacement solution is often significantly larger than the actual settlement observation value, especially for the case where the burial depth of the load acting surface is large [10]. In this paper, based on the Ming's displacement solution, the settlement calculation of the underlying layer in the reinforced area of the composite foundation is carried out. And the method of this article is quite convenient and quick.

References
[1] JGJ79-2012 (2012) Technical code for ground treatment of buildings. S.
[2] GB/T50783-2012 (2012) Technical code for composite foundation. S.
[3] DBJ 15-38-2005 (2005) Technical code for ground treatment of buildings, Guangdong. S.
[4] YANG G.H. (2008) New computation method for soil foundation settlement. J. Chinese Journal of Rock Mechanics and Engineering., 27(4): 679–686.
[5] YAN M.L., QU X.L., LIU W. et al. (2004) Composite modulus analysis of composite foundation. J. Building Science., (4): 27-32.
[6] WANG W., WANG S.J., ZHU C.Z. et al. (2008) Study on Settlement Calculation of the Rigid Pile Composite Foundation. J. Soil Eng. and Foundation., 22(1): 36-39.
[7] Gong X.N. Composite Foundation Design and Construction Guide. China Communications Press, Beijing.
[8] R. D. Mindlin. (1936) Force at a Point in the Interior of a Semi- Infinite Solid. J. Physicss., 5: 195–202.
[9] Yao X.Q., He J.H. Settlement Analysis of Composite Foundation of Cement-Soil Mixing Pile. Zhejiang University Press. Zhejiang.
[10] Wang S.J., Zhang M., Zhang J.Z. (2001) ON mindlin stress formulas. J. Engineering Mechanics., 18(6): 141–147.