Metamaterial Lenses and Their Applications at Microwave Frequencies

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Abstract

Metamaterials are artificial structures characterized by flexible and designable electromagnetic properties, including permittivity, permeability, and refractive index. Because of their subwavelength-scale composing units, metamaterials are able to manipulate the propagation of EM waves at extremely fine resolutions, and offer solutions for new-concept devices and components with novel functionalities and challenge-breaking properties. From among the attractive applications of metamaterials, this review paper focuses on metamaterial lenses (also termed as meta-lenses) developed in recent years for the purpose of microwave engineering, including beam forming, beam steering, and imaging. Bulk lenses made of three-dimensional metamaterials and planar lenses made of two-dimensional transmission-type metasurfaces are both involved and investigated. Different categories of meta-lenses, together with the theory, principles, methods, and advantages of their designs, are introduced and discussed. Potential and future applications of meta-lenses at microwave frequencies are also presented.

1. Introduction

Metamaterials (MTMs) are artificial media composed of subwavelength electromagnetic (EM) particles whose response to outer EM fields can be designed and engineered. For metamaterials, such particles are arranged periodically or non-periodically as composing unit cells, instead of as
natural atoms or molecules, as in the case of natural materials. The concept of metamaterials originated in the last century from Russian scientists\textsuperscript{[1]}, whereas rapid expansion of related research began at the beginning of this century, since the conception and documentation of the first electric and magnetic resonant unit cells in 1996 and 1999, respectively \textsuperscript{[2,3]}. In its early stages, research on metamaterials was focused on the realization of negative (refractive) index materials (NIMs), which were expected to exhibit extraordinary physical properties that could not be delivered by natural materials \textsuperscript{[4-8]}. In general, negative refraction indicates both negative permittivity ($\varepsilon$) and negative permeability ($\mu$), which are rarely found in natural materials. The artificial MTMs offer us with new solution of realizing negative (refractive) index because they are composed of sub-wavelength unit cells with designable EM resonances, and an induced opposite EM fields may overcome an incident EM field at specific frequencies.

In fact, according to the theory of effective medium \textsuperscript{[9]}, the macroscopic permittivity and/or permeability of MTMs follow the Lorentz-Drude dispersion curve, as is plotted in Figure 1(a). It is known that permittivity of normal bulk metals such as gold and copper follows the Drude dispersion curve with the plasma frequency $f_p$ (which is related to the oscillation of plasmon) located in the ultraviolet (UV) region of the spectrum\textsuperscript{[2]}. The value of permittivity is negative below the plasma frequency, and gradually increases to zero as frequency goes up towards the plasma frequency. At lower frequencies (e.g. microwaves), the permittivity gets huge negative values, making the impedance of the structure incompatible with that of free space or dielectrics. In view of this, array of continuous metallic wires was proposed to bring the plasma frequency into lower-than-UV region, making the negative permittivity values in microwave to be applicable. Moreover, when the thin wires are cut along the direction of electric field, opposite charges exist at the two ends of the cut wire and thus an additional restoring force (besides the force of the external electric field) brings in a resonance frequency ($f_r$) larger than zero, and the Drude dispersion curve becomes a Lorentz-Drude one depicted in Figure 1(a). In Region III of the dispersion curve, negative
permittivity is observed around the resonance frequency ($f_r$). On the other hand, although there exist few types of natural magnetic materials in the gigahertz range, their magnetic responses are usually not tunable or designable. Therefore, magnetically resonant particles have also been proposed to compose MTMs with Lorentz-Drude dispersion of permeability.

Different types of MTM particles have been created and verified, some of which are listed in Figure 1(b-h). Figure 1(b) depicts an array of metallic wires, which reduces effective plasma frequency and helped to realize negative permittivity below UV for the first time [2]. Figure 1(c) presents an improved form of the metallic wire, the I-shaped resonator, in which the wire is cut off, and two vertical bars are added at the two ends [10]. I-shaped resonators are commonly used in MTMs with negative permittivities. Figure 1(d) shows another improved electric resonator, the electric-LC (ELC) resonator [11]. Compared with the I-shaped resonator, the ELC resonator exhibits a lower resonance frequency and stronger self-contained energy. Figure 1(e) shows the split ring resonator (SRR) [3], which is the first proposed magnetic particle for negative permeability. Furthermore, for a more compact volume, planar metallic SRRs are printed on dielectric substrates, as shown in Figure 1(f). Other types of electric or magnetic resonant particles, including isotropic and anisotropic particles, have also been presented in past reports. For example, the spiral resonator in Figure 1(g) is a magnetic resonator with a relatively low resonance frequency [12]. Aside from metallic resonators, dielectric particles with electric-dipole responses have also been adopted as composing units for MTMs [13]. These particles are arranged in air or in the host medium with subwavelength spacings, as illustrated in Figure 1(h). Apart from bulk three-dimensional (3D) metallic or dielectric MTMs, other set-ups that exhibit negative refractive indices include periodically LC loaded networks or transmission lines (TLs) [14]. In particular, composited right/left-handed (CRLH) TLs has been developed and applied in microwave circuits to enhance bandwidth and left-handed/right-handed properties [15, 16]
The novel properties and functionalities of NIMs, including negative refraction, evanescent-wave amplification, and super lensing [17-19], have been intensively investigated. Among them, negative refraction was first demonstrated at microwave frequencies in 2001 using the prototype shown in Figure 2(a) [8]. Negative permeability and permittivity were due to the SRRs and metallic wires, respectively, around a designed frequency of 10.5 GHz, resulting in a negative refractive index of approximately −2.7. Compared with those of a normal medium characterized by a positive refractive index, e.g., the comparative medium known as Teflon, negative refraction angles are clearly observed for the NIM (also termed as left-handed materials (LHM)), as plotted in Figure 2(b). Another attractive application of NIM is “perfect lensing” as illustrated in Figure 2(c), which...
is based on the idea that both the propagating and evanescent waves emitted from a source can be recovered completely at the exterior image point of an NIM slab\(^{[20]}\). Although the NIM slab has been criticized as being lossy (which is mainly due to the strong resonance of the structure) and therefore cannot really function as a perfect lens, the slab could still function as a super lens that overcomes the conventional diffraction limits associated with normal materials. Figure 2(d) shows a two-dimensional (2D) demonstration of the sub-wavelength focusing on microwave frequencies, in which the LC-loaded network in the center serves as an NIM slab.

Figure 2. (a) A prototype of the negative refractive index MTM composed of SRRs and wires. Reproduced with permission\(^{[8]}\) (b) Negative refraction of LHM (or NIM) observed in experiment. Reproduced with permission\(^{[8]}\) (c) Scheme of perfect lensing. Reproduced with permission\(^{[20]}\) (d) Microwave experiment demonstrating that sub-wavelength focusing is possible. Reproduced with permission\(^{[20]}\)

In the year of 2005, the idea of gradient index (GRIN) MTMs was proposed\(^ {21}\). Since then, the concept of MTMs has been extended and is no longer restricted to materials with negative permittivities and/or permeabilities, but can also refer to other materials with flexible EM properties.
According to the Lorentz-Drude dispersion curve in Figure 1(a), MTMs can exhibit designable and wide-ranged values of permittivity and/or permeability, both positive (Regions I and II) and negative (Regions III and IV), including some extreme values that can hardly be found in nature, e.g., near-zero or extremely high refractive indices. GRIN MTMs have been used to develop a series of devices with novel functionalities and featured performances, including the Luneburg lens, carpet cloak, EM black-holes\cite{22,23}. In the meantime, the theory of transformation optics (TO) (also termed as optical transformation (OT)) was proposed to manipulate the propagation of EM waves with high flexibility\cite{24, 25}. Through the application of this theory, a virtual space in which coordinates are distorted and the travelling traces of EM wave is tailored accordingly (for example, as shown in Figure 3(a)) can be equally realized in a physical space filled with transformation media whose refractive index distribution is calculated based on the form invariance of Maxwell’s equations\cite{26, 27}. The transformation media usually require a gradient distribution of the refractive index (e.g., as shown in Figure 3(d)) and can, therefore, be realized using GRIN MTMs. Figure 3(b) illustrates that in a cloak of invisibility, the incident waves are diverted within the cloak and emerge on the far side without deviating from their original course, whereas Figure 3(c) shows details of the first prototype of a cloak of invisibility at microwave frequency\cite{28}. GRIN MTMs were also adopted to realize the carpet cloak shown in Figures 3(d) and (e), which conceals objects on the ground from outer detection\cite{29, 30}. A large number of TO devices with interesting functionalities and outstanding performances, such as illusion devices and different types of lenses and antennas\cite{31-33}, have thus far been presented, and among these, TO lenses will be discussed in detail in the following sections.

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With the rapid development of and high requirement for compact integrated circuits in wireless communication systems, the 2D version of MTMs, the metasurface (MS), has been intensively investigated in recent years because of its planar profile, light weight, low cost, and easy fabrication. The generalized Snell’s laws were presented, showing that when a metasurface is made of 2D array of subwavelength resonators (e.g., the optical antennas or the EM cavities) whose phase responses to the incidence are spatially varying, the phase discontinuity along the surface could be designed through tailoring the geometry of the resonators, and in this way the wavefront of reflected or refracted wave could be engineered in nearly arbitrary ways, as illustrated in Figure 4(a) [34]. The metasurfaces include the transmission-type and reflection-type, and refracted and reflected waves can be manipulated at will in terms of their scattering patterns, polarization, beam direction, etc [35–44]. Most recently, digitally and programmable coding MTMs/MSs have been proposed to
build a novel connection between the digital world and the physical one \cite{45, 46}, through which the EM waves can be controlled digitally by coding sequences composed of “0” and “1”, which represent phase or amplitude information in a unit or unit cells of a metasurface. Figures 4(c)-(d) illustrate a 1-bit digital coding metamaterial composed of two types of meta-atoms with “0” and “1” states. Using the technology of coding and programmable metasurface, microwave imaging systems and wireless communication systems with compact profiles, easy integration, and excellent performance have been developed \cite{47, 48}. In this review paper, we focus on the transmission-type metasurfaces with amplitude and phase modulations, which can be considered as a special type of planar lenses: the meta-lens.

![Figure 4](image-url)

Figure 4. (a) Schematic of ordinary and anomalous reflection and refraction on a metasurface. Reproduced with permission.\cite{34} (b) Scanning electron microscope image of the optical metasurface sample. Reproduced with permission.\cite{34} (c) The 1-bit digital coding metamaterial composed of two types of meta-atoms with “0” and “1” states. (d) The unit of a 1-bit digital coding meta-atom and the corresponding phase responses. Reproduced with permission \cite{46}.

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In this paper, we present a thorough review of the development of MTM lenses in recent years, based mainly on research performed by our group. We start with 2D lenses made of bulk MTMs, including GRIN lenses, TO lenses, and zero-index lenses. These 2D MTM lenses are designed to be able to tailor the travelling of EM wave in a plane vertical to the electric field or magnetic field. Three-dimensional (3D) versions of these meta-lenses are then presented, and their applications in microwave engineering are introduced and summarized. Subsequently, we introduce some interesting designs of transmission-type metasurfaces and demonstrate their broad prospects in the manipulation of EM waves and multi-physics modulations. Finally, future directions for the research and application of meta-lenses are discussed.

2. Two-Dimensional Metamaterial Lenses

Lenses such as convex and concave lenses are widely used for beam forming, focusing, imaging, etc. For example, glass lenses are widely adopted in terahertz and optical circuits, and dielectric lenses are used to generate directive beams in microwave antennas. The profiles of the lenses and their combinations are decided based on the theory of geometrical optics (GO). Currently, the accuracy and functionality of lenses are limited by the precision of fabrication and assembly and by the range and resolution of the refractive index of the material in use. Because bulk MTMs are able to provide flexible refractive indices in a designable frequency band, they have great potentials in the creation of new types of lenses with remarkable properties. In this section, we discuss 2D MTM lenses.

2.1 2D Gradient Index (GRIN) Lenses

2.1.1 MTM Lenses with Analytically Described Refractive Index Profile

GRIN MTMs are especially suitable for constructing lenses with a specific distribution of refractive index, e.g., the Luneburg lens\(^{49}\) and Maxwell’s fisheye lens\(^{50}\), the refractive index profiles of
which can be described analytically. Usually, these lenses are manufactured from layers of dielectrics and are tend to be heavy and expensive. MTM lenses, however, are alternatives with lower cost, lighter weight and good performance.

Luneburg Lens

The Luneburg lens illustrated in Figure 5(a) is very important in microwave antennas because it can focus incident waves from all directions to a point on the spherical surface of the lens, and conversely, a point source on the surface can produce a highly directive beam to the free space (with the ray trajectories shown in Figure 5(a)), mainly through the gradient refractive index distribution in the sphere. The refractive index profile of a Luneburg lens is described as follows:

\[ n = \sqrt{2 - r^2/R^2}, \]

where \( R \) is the radius of the lens. The refractive index is distributed concentrically according to Equation (1). Different types of MTM units, including metallic and dielectric units, can be adopted to fulfill the required range of refractive index from 1 to 1.414. Here, the I-shaped structure \(^{[51]}\) shown in Figure 5(d) is used for demonstration. Two methods of discretization of the refractive index profile, shown in Figure 5(b) and (c), are chosen. In Figure 5(b), each discretized slab exhibits gradient permittivity and is shown with the corresponding refraction index, and is realized by I-shaped metallic structures printed on a dielectric substrate with gradually changing dimensions. By contrast, in Figure 5(c), each discretized annulus contains uniform I-shaped metallic structures.

The slabs and annuli are fixed inside the foam. The I-shaped structures are noted to be polarization-sensitive, and therefore, the desired refractive index is attained only in the vertical plane. In this demonstration, the MTM lens is in fact a 2D Luneburg lens. Two prototypes of the 2D MTM Luneburg lenses were tested in the 2D scanning system at Southeast University (SEU) in China, which included a vector network analyzer (VNA) (Agilent N5230C) and three electronic motor steppers in the \( x \), \( y \), and \( z \) directions, respectively. In the measurement setup, shown in Figure 5(e), a probe placed on the side of the MTM lens served as the source. Directive beams and high-gain
performance from 7 to 8.5 GHz were observed through the near-electric-field distribution and the far-field radiation pattern.

Figure 5. (a) Ray trajectories of the Luneburg lens. (b-c) Discretized distributions of the refractive index of the Luneburg lens. (d) The I-shaped structure and the dispersion curves. (e) The experiment sample of the MTM Luneberg lens in the 2D scanning system. Reproduced with permission \[51\]

**Maxwell’s Fisheye Lens**

In imaging systems, Maxwell’s fisheye lens is a widely used lens that can transform a point source on the surface into a focus on the diametrically opposite side. The half Maxwell’s fisheye (HMFE) lens has also been applied in microwave antennas \[52, 53\] because the point source on the spherical surface of the HMFE is transformed into a highly directive beam, as indicated by the ray tracing shown in Figure 6(a). The refractive index profile of a Maxwell’s fisheye lens is described as

\[
n(r) = \frac{n_0}{1+(r/R)^2}
\]

where \(R\) is the radius of the lens and \(n_0\) is the refractive index of the air. I-shaped structures are designed to produce refractive indices covering the required range from 1 to 2 (as shown in Figure 6(c)) \[54\]. In Figure 6(b), the refractive index is distributed concentrically and discretized such that in each grid, the index has a uniform value. The I-shaped metallic structures are printed on dielectric substrates and fixed inside the foam, as shown in Figure 6(d). High directivity from 10 to 12 GHz has been verified experimentally.

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2.1.2 Flat MTM Lenses Based on Geometric Optics

Most conventional lenses are made of uniform dielectrics or layered dielectrics. They are specifically designed according to the theory of geometrical optics, with specific profiles and layouts to control ray trajectories for expected functionalities. Usually, conventional lenses are heavy and have curved surfaces and therefore require high-precision polishing. GRIN MTMs provide a novel way of conceiving flat lenses with designable functionalities because of the flexible and wide range of refractive indices that could be produced using MTMs. Fermat’s principle states that the optical path length is related to the refractive index \(^{55}\). Figure 7(a) shows the process of ray refraction in GRIN media \(^{56}\), which can be discretized into thin layers along the radial direction.

Figure 6. (a) Ray trajectories of the MTM HMFE lens. (b) Distribution of the refractive index and the discretized profile. (c) The relationship between the geometry of the I-shaped structure and the refractive index \(n\). (e) The manufactured sample of the 2D HMFE lens for test. Reproduced with permission \(^{54}\).
with a thickness of $dr$ (as described in Figure 7(b)). The refraction angle at the adjacent surface of different layers is determined using

$$n_i \cdot \sin \theta_i = n_{i+1} \cdot \sin \theta_{i+1},$$

(3)

whereas the final phase distribution on the flat aperture of each ray can be determined using

$$\left( \frac{dn_i dz_i}{d\bar{n}_{i+1} d\bar{z}_{i+1}} \right)^2 \left[ \left( \frac{dz_i}{d\bar{r}_i} \right)^2 + 1 \right] = \left( \frac{dz_{i+1}}{d\bar{r}_{i+1}} \right)^2 + 1,$$

(4)

where $n_i$ is the refractive index of the $n$-th layer. Through tailoring of the distribution of $n_i$, ray tracing in an MTM lens can be controlled, and the source on one side of the lens can be transformed to an EM wave with a designed wavefront on the other side. Thus, novel functional lenses with flat profiles can be achieved. In addition to that, impedance matching layers (IMLs) are necessary for flat MTM lens, as illustrated in Figure 7(a), to compensate for the mismatch in impedance between the air and MTMs and consequently reduce reflection on the MTM lens. The thickness of an IML is $\lambda_0/4$, according to our knowledge of microwave technology, and may be optimized for real applications.

Different types of 2D MTM lenses have been created for beam forming with different purpose. For example, the lens shown in Figure 7(c) is composed of closed ring structures [57]. Here, two functionalities are demonstrated: the first one is the beam scanning that proved in Figure 7(d) with a monotonically increasing refractive index from 1 to 1.68; and the second one is the beam focusing in Figure 7(e) with a maximum refractive index of 1.68 located at the center of the lens. Another design of beam scanning lens antenna is also delivered to provide highly directive beam in a designed direction [58]. The composing unit cells, which are modified from ELC resonators, are depicted in Figure 7(f). In this demonstration, a 2D GRIN MTM lens was placed in front of a 2D metallic horn antenna, and increased directivity and depressed side lobes of the horn antenna were observed experimentally within a 1-GHz bandwidth.
2.2 2D Transformation Optics Lenses

As discussed in the Introduction, transformation optics helps in the derivation of the relationship between the electromagnetic properties of a distorted virtual space and the constitutive parameters of filling media in a physical space. In the virtual space, the coordinates are distorted, as illustrated in Figure 8, and the propagating trace of the EM waves is manipulated in correspondence with the coordinates. This technique is based essentially on the foundational knowledge that Maxwell’s equations have a form-invariant nature in different coordinate systems, where the only change is the normalization of the electromagnetic parameters (e.g., permittivity and permeability) of the background media. In this regard, MTMs are especially suitable for producing the flexible parameters required in TO media.

Figure 7. (a) The ray tracing distribution in the GRIN MTM lens. Reproduced with permission [56]. (b) The schematic diagram of the ray tracing in the lens. Reproduced with permission [56]. (c) Fabricated sample of a 2D GRIN MTM lens composed of closed rings. Reproduced with permission [57]. (d-e) The beam steering and beam focusing performance of the MTM lenses. Reproduced with permission [57]. (f) The experimental sample of the 2D beam-scanning MTM lens antenna. The composing unit cells are modified from the ELC resonators. Reproduced with permission [58].
Figure 8. Left: the Cartesian coordinates in the physical space. Right: the distorted coordinates in the virtual space. The red arrows indicate the propagating traces of EM waves in the two coordinates. Reproduced with permission [59].

The two coordinate systems in TO are related through a general transformation function
\[ x' = x'(x, y, z), \quad y' = y'(x, y, z), \quad z' = z'(x, y, z). \]
Maxwell’s equations have exactly the same form in both spaces:
\[
\nabla \times \mathbf{E} + \mu \frac{\partial \mathbf{H}}{\partial t} = 0, \quad \nabla \times \mathbf{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} = 0 \quad \text{and} \quad \nabla' \times \mathbf{E'} + \mu' \frac{\partial \mathbf{H'}}{\partial t} = 0, \quad \nabla' \times \mathbf{H'} - \varepsilon' \frac{\partial \mathbf{E'}}{\partial t} = 0. \tag{5}
\]

The virtual space is considered to be the free space with \( \varepsilon_0 \) and \( \mu_0 \); therefore the resulting permittivity and permeability tensors in the physical space are given by
\[
\varepsilon = \frac{i\varepsilon' \mathbf{J}^T}{\det(\mathbf{J})}, \quad \mu = \frac{i\mu' \mathbf{J}^T}{\det(\mathbf{J})}. \tag{6}
\]
Here \( \varepsilon' \) and \( \mu' \) are the permittivity and permeability tensors, respectively, in the virtual space, and \( \mathbf{J} \) is the Jacobian transformation matrix between the two coordinate systems. This deduction provides the primary tool for constructing a physical space that mimics the electromagnetic environment of a desired virtual space and therefore acts as a microwave device with novel functionalities. Figure 9 shows some interesting examples of TO devices, apart from the cloak of invisibility. As shown in Figure 9(a–b), a TO lens is created to convert cylindrical waves in the center to plane waves in four orthogonal directions [60]. TO lenses with different numbers of beams have also been created in the
same way \cite{61, 62}. Figure 9(c) presents the working principle of the radar illusion device, which can make the EM image of a target gathered by radar look like a different target \cite{63}. 2D prototypes of illusion devices, e.g., a “ghost” illusion device and an optically controllable transformation-direct current (DC) illusion device, shown in Figure 9(d) \cite{64, 65}, have also been demonstrated, whereas other types of TO devices, such as planar reflectors, field concentrators, and field rotators, have also been reported \cite{66}.

![Figure 9.](image)

Equation (6) indicates that the EM parameters of background media in a physical space may be inhomogeneous and isotropic, and sometimes may require extreme values approaching zero or infinity, depending on the distortion of the virtual space. Although MTMs can provide inhomogeneous and isotropic permittivity and permeability, and also a zero index, the construction of an isotropic MTM device may be very complicated, and the bandwidth of zero-index metamaterials (ZIM) is usually narrow. To circumvent these limitations, the idea of a simplified yet limited-performance cloak was suggested together with a method for discrete coordinate transformation with quasi-conformal (QC) mapping \cite{29}. In quasi-conformal transformation optics (QCTO), both the physical and virtual spaces are discretized with quasi-orthogonal grids, and

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transformation optics are implemented between a pair of near-conformal local coordinates. QCTO can minimize the anisotropy of MTMs in the physical space and results in more applicable devices with broadband performance and easy design and implementation. Based on the QCTO method, 2D isotropic MTM lenses with broadband performance and high directivity have been developed at microwave frequencies \(^{[67-69]}\). Figure 9(e) shows how a convex lens in a virtual space is transformed into a flat lens in a physical space using QCTO, whereas Figure 9(f) shows the permittivity map of the flat lens \(^{[59]}\). The permittivity map can be discretized and simplified into a few blocks that are smaller than half the wavelength, and with simplification, the focusing performance of the lens is not significantly reduced. This type of TO lens can be easily realized using MTMs at non-resonant frequencies, and is therefore essentially broadband and low-cost.

### 2.3 2D Zero Index Metamaterial Lenses

The Lorentz–Drude dispersion curves shown in Figure 1(a) demonstrate that MTMs are able to exhibit permittivities and/or permeabilities approaching zero, which are rarely found in natural materials. Such extreme values of EM parameters reveal unique properties of MTMs. For example, because of the constant phase inside a ZIM, the refracted rays radiating from it are always normal to the surface while the wavefront is parallel to the surface, leading to extraordinary applications that can hardly be realized without ZIMs. An important demonstration of ZIM is the tunneling effect of EM waves through a narrow channel in a waveguide \(^{[70]}\), in which complementary split ring resonators (CSRRs) are adopted to deliver zero permittivity \(^{[71]}\). A CSRR is a metallic screen with a negative image of an SRR, whose fundamental resonance is electric resonance due to the duality of the EM field. In this study, ZIMs provide a new scheme for raising the characteristic impedance of a narrow channel and consequently reduce the reflection between different sections of a waveguide.

In free space, ZIM helps to create highly directive radiation, as explained in Figures 10(a) and (b) \(^{[72]}\). A line source placed in a free space radiates omnidirectional cylindrical waves, as visualized in
Figure 10(a). When the line source is embedded in a ZIM slab with a thickness of $t$ (defined in Figure 10(b)), the parameters $\varepsilon_z$, $\mu_x$, and $\mu_y$ of the anisotropic ZIM (AZIM) slab are relevant to the propagation and scattering of transverse-electric (TE) waves according to Maxwell’s equation

$$\frac{\partial E_z}{\partial x} = -i\omega \mu_y \mu_0 H_y.$$ \hspace{1cm} (7)

Here $\mu_y$ is approaching zero, the electric field $E_z$ inside the slab is constant along the $x$ axis and varies only along the $y$ direction. Therefore, the wave vector on the surfaces of the slab become parallel to the $y$ axis, and the wave emitted from the line source is transformed into a plane wave. We denote that the transformed plane wave propagates in the $y$ direction, and the corresponding wave impedance is $\eta_0 \sqrt{\mu_x / E_z}$ (where $\eta_0$ is the wave impedance in free space). Whenever the wave impedance matches that in free space, non-reflective radiation is achieved. A 2D ZIM lens was realized using an SRR array, as depicted in Figure 10(c), with $\mu_y$ approaching zero and a wave impedance close to $\eta_0$. Transformation from the cylindrical wave of the source to the plane wave has been observed experimentally and plotted to be at 8.1 GHz, as shown in Figure 10(d). Another demonstration of a 2D ZIM is a directivity-enhanced Vivaldi antenna using compactly anisotropic ZIM (as shown in Figure 10(e)) \cite{73}. The meander-line resonators are printed periodically on a dielectric substrate to provide near-zero permittivity in the propagation direction of the $x$ axis, and to tailor the wave front of the radiation from the original Vivaldi antenna. A significant increase in gain was verified for a wide frequency band from 8 to 12 GHz.
Figure 10. The radiation of a line source in (a) free space and (b) anisotropic metamaterial with the permeability component $\mu_y \to 0$. Reproduced with permission \cite{72}. (c) A ZIM lens composed of SRRs and the dispersion curves of permittivity and permeability components. Reproduced with permission \cite{72}. (d) Measured beaming forming performance of the ZIM lens. Reproduced with permission \cite{72}. (e) A ZIM loaded Vivaldi antenna. Reproduced with permission \cite{73}. (f) Measured gains of the Vivaldi antenna with and without the ZIM. Reproduced with permission \cite{73}.

3. 3D Metamaterial Lenses

In Section 2, we introduce some categories of 2D MTM lenses, including the GRIN lens, TO lens, and ZIM lens. These lenses are made of bulk MTMs and are designed to manipulate the propagation of EM waves in a specific plane. On the other hand, 3D bulk MTMs are achieved when the composing unit cells are arranged periodically along the three axes of the Cartesian coordinates, i.e., $(x, y, z)$ coordinates, as shown in Figure 11(a). 3D MTMs may be isotropic if the EM responses in the three directions are identical, or anisotropic if they are different (e.g., as shown in Figure 11(a)). In addition to metallic unit cells printed on substrates, dielectric resonators, such as the spheres shown in Figure 11(b), are also widely used in 3D bulk MTMs. Dielectric spheres are periodically embedded in host media and become an artificial material bulk as a whole. The sizes of the spheres and the spacings between neighboring spheres together determine the EM parameters of the MTM. Moreover, a dielectric cylinder array in host media, as depicted in Figure 11(c), can be considered as a simpler substitute for 3D nearly isotropic MTMs. A series of functional devices...
have been delivered using 3D MTMs \cite{74-76}, among which the 3D ground-plane carpet cloak shown in Figure 11(d) has inspired great expectations for novel-functional MTM devices \cite{77}. In this section, we introduce some interesting and inspiring lenses made of 3D MTMs for microwave engineering.

![Figure 11.](image)

Figure 11. (a) A unit cell of the 3D MTM. (b) Dielectric sphere arrays embedded in host media. (c) Dielectric cylinder arrays embedded in host media. (d) A 3D ground-plane cloak made of dielectric plates with drilled cylindrical holes. Reproduced with permission \cite{77}.

### 3.1 3D Gradient Index Lenses

#### 3.1.1 3D Luneburg Lens and Fisheye Lens

Similar to its 2D equivalent, the refractive index profile for a 3D Luneburg lens is also described by Equation (1). For demonstration, a 3D half spherical Luneburg lens is realized in Figure 12 using the dielectric MTM with drilled cylindrical holes \cite{78}. Two types of unit cells, as given in Figure 12(b), are designed using the drilled-hole F4B dielectric substrate ($\varepsilon_r=2.65$) with different sizes of $2\times2\times2$ mm$^3$ and $2\times2\times1$ mm$^3$, to cover the required ranges of refractive indices from 1.63 to 1.18 and from 1.18 to 1.1, respectively. These unit cells are arranged in the form of drilled-hole dielectric plates and assembled piece by piece using glue or tape (which may slightly harm the...
efficiency), as shown in Figure 12(a). In the experimental setup, the half Luneburg lens was placed on a perfect electric conductor (PEC) ground, and a rectangular waveguide was placed on the surface of the lens as the point source. When the source moves on the surface, the directive beam scans in the corresponding direction. To verify the isotropic performance of an MTM Luneburg lens, two different polarizations of incidence, i.e., the HPP polarization in which the magnetic field is parallel to the ground, and the EPP polarization in which the electric field is parallel to the ground, were both investigated in an experiment. Beam scanning with a 60° span was achieved with low side lobes and high gain for both polarizations.

![Figure 12](image)

Figure 12. (a) Photograph of the half spherical Luneburg lens made of MTMs. (b) Two types of unit cells made of F4B substrate with drilled cylindrical holes. (c) A half spherical Luneburg lens combined with the PEC ground. (d) Two different polarizations of the source of the lens. (e) The measurement setup in a fully anechoic microwave chamber. Reproduced with permission [78].

Another dielectric lens with an analytically described refractive index profile, the Fisheye lens, has also been verified using 3D MTMs in a similar way. For the purpose of application in microwave antennas, a half Maxwell’s fisheye lens is developed using the dielectric MTMs with drilled cylindrical holes again to produce a highly directive beam according to the ray tracing illustrated in Figure 13(a). A rectangular waveguide is located on the spherical surface of the lens.
as the feed, and the IML shown in Figure 13(d) (which is also made of drilled-hole dielectric MTMs) is added to depress reflections caused by the mismatch of impedance between the fisheye lens body and air. Two types of unit cells, with the first made of the dielectric of F4B and the second made of FR4, are adopted to cover the refractive index range from 1 to 2. This 3D MTM lens was manufactured, assembled and tested, showing highly directive beam with lower side lobes compared with the conventional dielectric-loaded horn antenna.

![Figure 13. Sketch and fabrication of the half-spherical fisheye lens. (a) Sketch of the lens with IML. (b) Photograph of the fabricated lens and the feeding waveguide. (c) Photograph of the fisheye lens body. (d) Photograph of the IML. Reproduced with permission [78].](image)

### 3.1.2 3D GRIN Lenses

According to Section 2, ray tracing can be pre-designed according to the distribution of refractive indices in bulk MTMs. Several 2D MTM lenses are shown in Figure 7. When the refractive index profile is rotated along an axis, e.g., the z-axis in Figure 7(a), the 2D lens becomes a 3D lens with axial symmetry. Axial symmetry lenses are widely adopted in microwave engineering to create directive beams and to focus EM energy. For 3D MTM lenses, the isotropic
property should be considered with regard to the polarization of incident EM waves. Both isotropic and anisotropic MTM unit cells, including metallic and dielectric cells, may be used in different circumstances.

**Figure 14**(a-c) shows the design of a flat MTM lens that can be loaded inside a horn antenna. Such a flat lens has three layers, including a core layer in the middle and two coat layers acting as IMLs on the two sides of the core layer. The core layer transforms spherical waves from the horn, into plane waves radiating into free space through the formation of a uniform wavefront on the outer surface of the lens, whereas the two coat layers help to ease impedance mismatches between the core layer and air. Closed square rings (CSRs) have been chosen as the composing unit cells to attain the required EM parameters. Measured results have verified that when such a flat GRIN lens is added to the horn, the antenna obtains a relative bandwidth of 40% and can work over the full X-band with significantly improved directivity and low side-lobe level.

In addition to the phase distribution on the aperture of a flat GRIN lens, the aperture distribution of the amplitude is also vital to the performance of radiation. To achieve a lens antenna with high gains and low side lobes, the amplitude distribution on the lens aperture should approach a Taylor circular array distribution, whereas the phase distribution should be uniform. Optimization algorithm, e.g., the particle swarm optimization, can be adopted to optimize the refractive index distribution of the GRIN lens to achieve the required distributions of the phase and the amplitude simultaneously. In a previous study, a feeding horn antenna with identical E- and H-plane radiation patterns was used as the source of the lens, and dielectric plates with drilled cylindrical holes were fabricated to construct the lens, as shown in Figure 14(d). In that experiment, the MTM GRIN lens antenna exhibited very low side lobes and high gains across the entire Ku-band, demonstrating the low cost, flexible design, and good performance of the MTM lens.

Anisotropic MTMs have also been applied in microwave lens antennas. For example, anisotropic unit cells, the cut wires of which have an electric response in the E-plane of a horn
antenna, are arranged periodically and loaded inside a horn antenna, as illustrated in Figure 14(e)\cite{83}.

This anisotropic MTM lens modifies the aperture field distribution in the E direction, tapering the amplitude distribution and making the phase distribution uniform. In this way, a significant reduction in side-lobe and back-lobe radiation is realized from 4 to 6 GHz.

Figure 14. (a) The schematic diagram of the GRIN Lens with IMLs. Reproduced with permission\cite{79}. (b) Photograph of a MTM layer for the GRIN lens. Reproduced with permission\cite{79}. (c) Photograph of the lens antenna setup. Reproduced with permission\cite{79}. (d) Photograph of a MTM lens whose aperture fields (including the phase and the amplitude) are optimized. Reproduced with permission\cite{82}. (e) An empty horn loaded with the anisotropic MTM lens composed of metallic strips. Reproduced with permission\cite{83}.

3.2 3D Transformation Optics (TO) Lenses

The concept of transformation optics, which offers a flexible and powerful tool for conceiving new types of lenses, is discussed in Section 2.2, and a series of 2D MTM lenses based on this method are presented. For actual application, 3D MTM lenses with novel functionalities and outstanding features are achievable using the TO technique. Because many lenses are symmetrical or work at a specified polarization, 3D TO lenses can be produced through the extension or rotation
of 2D models by one axis. The analytical TO method with full EM parameters and the quasi-conformal TO method with simplified parameters are both applied to produce the refractive index distribution in the 3D volume. Different types of MTMs can be used to produce the required index profiles of the permittivity and/or permeability distribution.

Figure 15(a) depicts a TO lens whose distribution of permittivity in the xoy plane is the same as in the simplified map given in Figure 9(f)\textsuperscript{84}. Strips with gradient widths, which are indicated by the black lines, are designed to attain permittivity distribution. Six identical layers are then arranged along the z direction to construct the 3D lens. When a line source is placed at the focal point of the 3D lens, a cylindrical wave from the source is transformed into a plane wave on the other side of the lens. Another demonstration of an MTM TO lens is a four-beam emission device made of capacitor-loaded ring resonators (CLRRs), which provide the required anisotropic permeability created using the analytical TO method (Figure 15(b))\textsuperscript{62}. In addition, the permittivity distribution deduced by the quasi-conformal TO method can be isotropic and without extreme values. Artificial dielectrics manufactured in the laboratory may be used to fulfill the permittivity map. For instance, the 3D permittivity map of the flat hyperbolic lens in Figure 15(c) is created via the rotation of the 2D permittivity map by the z-axis and implemented using nano- and micro-sized titanates dispersed into a polymeric matrix material, as depicted in Figure 15(d)\textsuperscript{85}. The synthetic dielectrics are inexpensive and reproducible, but are less flexible or easy-accessible than the MTMs at microwave frequency. Synthetic dielectrics are inexpensive and reproducible but are less flexible or less easily accessible than MTMs at microwave frequencies.
TO has also been used to produce 3D Luneburg lenses with designable profiles. A conventional Luneburg lens has a spherical profile, and therefore, is not easily integrated on flat surfaces or in low-profile systems. In addition, to generate a directive beam in different directions, the point source should rotate on the surface accordingly, which increases the difficulty of beam control. In this context, flattened Luneburg lenses have been created and demonstrated [86]. Figure 16(a–d) illustrate the design procedure of a 3D TO-based Luneburg lens with a flattened focal plane.

The original Luneburg lens in the virtual space (in Figure 16(a)) is transformed into a flattened lens in the physical space (in Figure 16(b)) using the QCTO method. The 3D Luneburg lens is designed via the rotation of the 2D profile in Figure 16(b) around the z-axis, as shown in Figure 16(c). This lens is developed using non-resonant MTMs composed of multilayered dielectric plates, as shown in Figure 16(d). In each layer, different inhomogeneous cylindrical holes were drilled to realize the refractive index map achieved in Figure 16(c). Experimentally, this TO-based Luneburg lens can radiate from 15 to 18 GHz, and when the feed is moved from the center of the focal plane by 30
mm, the beam is steered to an angle of 50° off the z-axis. Another TO-based Luneburg lens is shown in Figures 16(e) and (f) \cite{87}. The spherical Luneburg lens is transformed into a cylindrical lens with a symmetric distribution for the refractive index. Analytical TO is then performed, and the anisotropic permittivity and permeability distributions are decided accordingly. Approximation and discretization are applied to reduce the complexity of the creation of the TO lens. Finally, a flat Luneburg lens is manufactured using 17-layer GRIN MTMs. This lens exhibits a broad bandwidth, high gain, and wide scanning angle from 9 to 11 GHz.

Figure 16. (a) The 2D original Luneburg lens in the virtual space. Reproduced with permission \cite{86}. (b) The 2D flattened Luneburg lens and its refractive index distribution in the physical space. Reproduced with permission \cite{86}. (c) The simplified refractive index distribution in the xoz plane. The 3D lens is generated by rotating the profile around the z axis. Reproduced with permission \cite{86}. (d) Photograph of the fabricated 3D lens. Reproduced with permission \cite{86}. (e) A spherical Luneburg lens is transformed to a cylindrical one. Reproduced with permission \cite{87}. (f) Construction of the TO based 17-layer cylindrical Luneburg lens. Reproduced with permission \cite{87}.

3.3 3D ZIM Lens

Similar to 2D ZIMs, 3D ZIMs are characterized by wavefront forming due to a zero refractive index. Because impedance matching is difficult to achieve simultaneously along all axes, ZIMs are commonly anisotropic and sensitive to polarization. A series of ZIM lenses with interesting
functionalities have thus far been accomplished. Figure 17(a) shows a scheme for composing a spatial power combination device for omnidirectional radiation [88]. Two or more sources are embedded inside a ZIM lens with a permeability approaching zero and a wave impedance equal to that of free space. CLRRs are then carefully designed to produce near-zero permeability at approximately 10.4 GHz, as shown in Figure 17(b–c). Waves emitted from all sources inside the ZIM lens are transformed into perfectly cylindrical waves because the in-phase or nearly in-phase conditions are satisfied on the profile of the ZIM lens. In this way, power from different sources can be combined to enhance omnidirectional radiation. ZIM lenses have also been demonstrated to improve the performance of horn antennas through their phase-tailoring ability. As demonstrated in a previous study [89], the ZIM lens shown in Figure 17(d) was designed to decrease the beam width of the main lobe in the E-plane, whereas the lens in Figure 17(e) was designed to decrease the beam width of the main lobe in the H-plane. If the two lenses are packed together and positioned in front of a horn antenna, the directivity in both the E- and H-planes is significantly improved.
Figure 17. (a) Schematic diagram of an anisotropic ZIM lens for omni-directional radiations and power combination. Reproduced with permission [88]. (b) Retrieved radial component of the permeability, which is nearly zero at 10.4 GHz. Reproduced with permission [88]. (c) Fabricated lens sample made of circular rings. Reproduced with permission [88]. (d) The anisotropic ZIM lens composed of meander-line unit cells. Reproduced with permission [89]. (e) The anisotropic ZIM lens composed of CLRR unit cells. Reproduced with permission [89].

4. Planar Metasurface Lenses

As mentioned previously, the rapid development of MTM lenses in microwave engineering started in 2005, showing the great potential of this artificial EM material. On the other hand, as integrated circuits and systems become highly required in future communications, new types of lenses with compact sizes and planar profiles become extremely in demand. In this context, the application...
The scope of 3D MTMs is limited, and 2D versions of MTMs, known as metasurfaces, have been proposed and intensively investigated in recent years.

4.1 Phase Modulations

The concept or technology of metasurfaces involves the generalized Snell’s law of reflection and refraction, Huygens metasurfaces, cascaded transmit-array metasurfaces, etc \[34, 90, 91\]. Both transmission-type and reflection-type metasurfaces have been studied for their fulfilment of the requirements for spatial wave modulations in different circumstances and configurations. The transmission-type metasurface can be considered as a special type of meta-lens (or MTM lens) with flexible beamforming, high transmission, low cost, easy fabrication, and easy integration.

In the early stages, wave front construction was achieved using specific designs of phase discontinuities on a metasurface. Considering that an antenna is located close to the metasurface as the feed, as shown in Figure 18(a). The metasurface is composed of a \( M \times N \) rectangular array of unit cells. The principle for predicting the far-field scattering pattern of the metasurface follows the antenna array summation:

\[
E(\theta, \varphi) \approx \sum_{m=1}^{M} \sum_{n=1}^{N} \cos^{q_f} \theta_f(m, n) e^{-jk(\overrightarrow{r_{mn}} - \overrightarrow{r_f})} \cos^{q_e} \theta_e(m, n) e^{j\varphi_{mn}}
\]

\[
\hat{\mu} = \hat{x} \sin \theta \cos \varphi + \hat{y} \sin \theta \sin \varphi + \hat{z} \cos \theta
\]

where \( \overrightarrow{r_{mn}} \) is the position vector of \( mn \)-th element on metasurface and \( \overrightarrow{r_f} \) is that of feed antenna, and \( \varphi_{mn} \) represents the phase shift of the \( mn \)-th unit cell. Using the scalar approximation with a cosine \( q \) model for the radiation pattern of the feed antenna, which is symmetrical and the 2D form can be approximated as \( \cos^q \theta \), where \( q \) is obtained by the best approximation. The initial amplitude and phase response of \( mn \)-th element from feed antenna are \( \cos^{q_f} \theta_f(m, n) \) and \( e^{j\theta} \) respectively, in which \( \theta_f \) represents the spherical angle in the coordinate system of the feed antenna. Similarly, the radiation pattern of \( mn \)-th element can be approximated by \( q \) model as \( \cos^{q_e} \theta_e(m, n) \) with
corresponding phase shift being \( e^{j\phi_{mn}} \). From Equation (8), the far-field radiation pattern is inferred to be dependent on phase shifts on the metasurface. For traditional antenna arrays such as the patch array, modifications and optimizations for creating far-field radiation on demand are applied. These modifications and optimization may require large amounts of time to accomplish. For coding metasurfaces, however, if the states of a unit cell (in terms of phase or amplitude) are defined by a \( N \)-bit code, in other words a unit cell should be able to present \( 2^N \) possible states in maximum, the number of different unit cells are limited to \( 2^N \) as well. Therefore, the coding metasurface is especially suitable to serve as a beam forming lens through phase modulations of the unit cells.

In contrast to the originally proposed reflection-type metasurface, which has a metallic ground that reflects all incident energy to radiation space, transmission-type metasurfaces require carefully designed layouts that depress reflections and achieve high radiation efficiencies. For example, in Figure 18(b), a double-layer transmission-type metasurface that can create programmable transmission patterns for single-sensor and single-frequency imaging systems is presented. Two voltage-controlled switchable diodes are included in the two layers of the unit cell, resulting in four reconfigurable states as 2-bit binary codes "11," "01," "10," and "00." High-amplitude transmissions and sufficient phase shifts for the four states are observed in the figure between 9 and 10 GHz. In an experiment, a \( 5 \times 5 \) programmable metasurface lens was fabricated and positioned in front of a horn antenna to create 25 different programmable patterns, which transformed the masks of modulators for a single-sensor image reconstruction, as shown in Figure 18(c). The appearances and gains of the transmission patterns are both discriminative, demonstrating that programmable coding metasurfaces offer a fast and low-cost method of pattern creation and switching.
Figure 18. (a) The sketch graphic of the transmission-type metasurface. Reproduced with permission [92]. (b) The diagram of a 2-bit coding unit cell and (c) the corresponding programmable coding metasurface for single-sensor and single-frequency imaging, in which FPGA can randomly deliver the binary codes (0 or 3.3 V voltage) on rows (green) and columns (red), simultaneously. (d-e) The simulated amplitudes (d) and phases (e) of transmission characteristics of the 2-bit coding unit cell with different states. (f) The schematic of a single-sensor and single-frequency microwave imaging system using the metasurface. $\sigma_{r_{ti}}, \sigma_{r_{tj}}$, and $\sigma_{r_{tk}}$ are the sub-RCSs in sub-areas of the image scene (which is divided into N sub-areas) related to the radius vectors of $r_{ti}, r_{tj}$ and $r_{tk}$, respectively. (b-f) Reproduced with permission [93].
Different types of transmission-type metasurfaces have been created to realize desired beamforming functionalities. For example, Figure 19(a) illustrates a broadband 1-bit coding metasurface in the X-band [94]. This metasurface contains five metallic layers to ensure good transmission amplitude and sufficient phase responses that are insensitive to the polarization of incidence. The two patches shown in Figure 19(b) are designed to exhibit states “0” and “1,” respectively, and produce high transmittance and near-180° phase difference. With this isotropic metasurface, high-directivity pencil beams and two symmetrical deflecting beams can be generated toward pre-designed directions, as shown in Figure 19(c-d). Isotropic unit cells have been widely used in metasurfaces with different-bit coding and show great potential for flexible and easy manipulation of transmitted EM waves, including beam deflection, multi-beam radiation, and Fourier operations [95].

Figure 19. (a) The sketch graphic of the unit cell of a 1-bit transmission-type metasurface. (b) The amplitude and phase distribution of transmission characteristics for different layers in (a). (c) Detailed sketch of the experimental setup. (d) Measured E-plane radiation patterns of the beam-forming performance (the first line) and the beam-scanning performance (the second line) at 8.5, 10.5, and 12.5 GHz, respectively. Reproduced with permission [94].

4.2 Anisotropic Phase Modulations

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In microwave communication, polarization is one of the most important pieces of information carried by EM waves. Isotropic metasurface unit cells, such as the square ones shown in Figure 18, are insensitive to polarization, and therefore may limit the scope of application of metasurfaces and its capacity for carrying information. To involve more flexibility in the modulation of EM waves, anisotropic unit cells in metasurfaces have been studied and developed. These anisotropic unit cells are usually non-central-symmetrical in geometry and exhibit different EM responses to differently polarized incidences. Figure 20(a) shows an example of an anisotropic unit cell that contains five metallic layers and four dielectric substrates, among which the cross-slot in the center metallic layer is designed to determine the operating frequency, and the four patches sized $l_p \times \omega_p$ are designed to define the digital code $^{92}$. Based on a comparison with the unit cell shown in Figure 19(a), anisotropy is inferred to have been introduced because the squares occur together with rectangles. The amplitude and phase plotted in Figure 20(b) demonstrate a high transmission efficiency and a nearly $180^\circ$ difference in phases for different polarizations. Through the careful optimization of the geometric parameters of such a unit cell, 3-bit anisotropic codes can be defined for eight different digital states, as shown in Figure 19(c), and a $45^\circ$ phase difference can be attained between two adjacent states. Such anisotropic metasurfaces present different phase modulations for differently polarized incidences and correspondingly result in independent and designable beams at large angular scales. In an experiment, for example, a $30\lambda \times 30\lambda$ metasurface lens at 60 GHz was fabricated and fed with a horn antenna.
Anisotropic metasurfaces are characterized by more powerful and flexible modulations of the transmitted EM wave than those of the isotropic ones because anisotropic metasurfaces offer more degrees of freedom and, therefore, make it easier to accomplish wide-range phase control and polarization control. In particular, a Huygens’ metasurface, which allows for control over both electric and magnetic responses, can be constructed using anisotropic unit cells. Figure 21(a) illustrates an ultra-thin (i.e., approximately one-fifteenth the working wavelength) Huygens’ metasurface, which is composed of three layers of Jerusalem crosses and two layers of F4B substrate. The geometrical dimensions of the unit cell are designed differently in the x and y directions to produce different electromagnetic responses for differently polarized incident waves. When the metasurface is positioned in front of a linearly polarized horn antenna, the radiating beam is steered in a predetermined direction. Furthermore, the direction and polarization of the main beam are reconfigurable through the rotation of the anisotropic Huygens metasurface. Another Huygens’ metasurface, where a split ring and rectangular metallic stub are both applied in a double-layer unit cell to produce magnetic and electric resonances at the same time, thereby increasing the efficiency, is presented in Figure 21(b). Such metasurfaces have been manufactured and demonstrated to
result in very high aperture efficiencies (up to 61.04%) with very small thicknesses (only 0.033 wavelengths at 13 GHz).

Apart from beam forming and radiation control, anisotropic metasurfaces are characterized by an attractive ability for polarization conversion. In contrast with traditional polarization-converting lenses, which are usually thick and heavy, meta-lenses can be much lighter and thinner. Figure 21(c) shows a broadband polarization-converting metasurface constructed with three layers of CSRRs\textsuperscript{[98]}. Because of the different orientations of the CSRRs in different layers, an $x$-polarized wave incident on the metasurface is perfectly transformed into a $y$-polarized transmitted wave, and vice versa. The thickness of the metasurface is only 3.2 mm in the X band, with a conversion efficiency of up to 96\%, covering a bandwidth of 24\% of the central wavelength. Elsewhere, a series of polarization-converting metasurfaces, including the double-layer opposite-D-shaped slot metasurface in Figure 21(d)\textsuperscript{[99]} and those implemented for terahertz beaming control in Figure 21(e–f)\textsuperscript{[100, 101]}, have also been devised.

![Figure 21](image_url)

Figure 21. (a) The anisotropic unit cell containing three metallic layers and the corresponding ultra-thin metasurface located in front of a horn antenna for beam control. Reproduced with permission\textsuperscript{[96]}. (b) A two-layer Huygens’ metasurface containing the split ring and the rectangular metallic stub in a unit cell. Reproduced with permission\textsuperscript{[97]}. (c) A three-layer metasurface containing twisted SRRs for broadband polarization conversion. Reproduced with permission\textsuperscript{[98]}. (d) A double-layer polarization conversion metasurface containing through-via holes. Reproduced with permission\textsuperscript{[99]}. (e) Photographs of a free-standing and flexible metasurface sample peeled off from the silicon wafer. The receiving angle of transmitted beam can be designed and the efficiency of this transmission-type metasurface is much higher than those of the designs fabricated on bulk substrate.
4.3 Amplitude-Phase Modulations

Metasurfaces introduced in the previous subsections permit the phase modulation of transmitted EM waves. In addition, amplitude modulation, together with phase modulation, offers more degrees of freedom in wave manipulation. An amplitude–phase (A–P) controllable metasurface may then be able to synthesize arbitrary wavefronts and full-space propagation directions. Generally speaking, independent control of the amplitude and phase of a transmission coefficient is vital in the design of A–P modulation, which can be a great challenge because both amplitude and phase responses are related to resonances in the unit cell, and mutual interference is commonly exhibited. To solve this problem, intensive studies have been conducted on the creation of A–P metasurfaces, and different types of unit cells, including C-shaped structures\textsuperscript{102} and complicated units constructed with two horizontal SRRs and a vertical double-SRR\textsuperscript{103}, have been reported to be the composing elements. Both electric and magnetic responses to incident waves may be involved and engineered into one unit in accordance with the structure of the metasurface.

With the rapid development of digital coding metasurfaces in recent years, digital transmission-type metasurfaces with phase and amplitude codes for flexible EM wave manipulation have been conceived and verified, attracting significant attention from both academic and engineering societies\textsuperscript{104,105}. By comparison, for a reflection-type or transmission-type metasurface, radiation beams can be implemented only in half space. In this context, a reflection–transmission (R–T) metasurface is proposed and presented in Figure 22(a–b) to accomplish full-space radiation control and EM manipulation\textsuperscript{102}. The unit cell of such a metasurface includes five metallic layers and four substrate layers, among which the middle layer with a rectangular slot decides the transmission or reflection under different polarizations. With impingement by $x$-polarized EM waves, perfect reflection (defined as the “0” state of the amplitude code) will occur, whereas when...
the incidence changes to be y-polarized, the corresponding bit will be “1.” In this way, the radiating beams can exist in full space in terms of the amplitude code. When positive–intrinsic–negative (PIN) diodes are embedded in the unit cells, a dynamic R–T amplitude code is created, and real-time control of far-field radiation characteristics is realized. It was verified experimentally that the R–T amplitude code can be associated with the phase code to improve the information entropy of a digital metasurface.

Figure 22. (a) The unit cell for 3-bit A-P code. Reproduced with permission [102]. (b) Intuitive display of the digital metasurface with phase and R-T amplitude codes. Reproduced with permission [102]. (c) The structure of C-shaped particle with independent controls of phase and amplitude. Reproduced with permission [106]. (d) An anisotropic A-P controllable metasurface and the multi-layer composing unit cell. Reproduced with permission [107].

Another example of the amplitude–phase modulation of a metasurface is in a near-field modulation lens, which is composed of the C-shaped particles shown in Figure 22(c) [106]. This is a commonly used metasurface unit cell that provides stable and continuous control of amplitude and phase independently through variations in the opening and orientation angles, as plotted in the figure. The full transmission phase range $2\pi$ and amplitude range from 0 to 0.5 are attained using the unit cells. Through the manipulation of the orientation angle $\alpha$ and opening angle $\beta$ shown in
Figure 22(b), the electric field distribution in the near-field region can be controlled. Moreover, differential and integral operations on predesigned electric field distributions can be completed using this kind of metasurface. A more recent study on A–P modulation is on the anisotropic A–P controllable metasurface illustrated in Figure 22(d) \[^{107}\]. Independent control of the phase and amplitude under two orthogonal polarizations are accomplished on the transmission-type anisotropic metasurface, via an especially designed unit structure containing tightly stacked cross-shaped patches and slots, as shown in the figure. This anisotropic configuration results in new degrees of freedom, and has been proved to generate multiple equal-power vortex-beam radiation patterns and to serve as a polarization-reconfigurable multi-focal meta-lens.

### 4.4 Multi-Physics Modulations

For most digital coding metasurfaces, dynamic control of the EM wave (including reflected and transmitted waves) is realized using actively tunable building blocks such as diodes, varactors, and biased lines. In most digital coding metasurfaces, active components are individually embedded in each digital coding unit to accomplish real-time control of the phase and amplitude. However, an external direct current (DC) supply device and connecting wires are required for each unit cell, which inevitably reduces the performance and increases the complexity of the metasurface. Because of this limitation, multi-physics modulation, or in particular, optical-controlled modulation, has been developed recently to achieve remote control of reflection-type and transmission-type metasurfaces with fewer or no connecting wires \[^{108-110}\].

An optically controlled transmission-type metasurface can be developed using photodiodes, as shown in Figure 23(a) \[^{108}\]. In each unit cell, a varactor diode was loaded into the structure. A silicon PIN photodiode series array is used to provide DC reverse voltage for tuning the capacitance of the embedded varactor in real time, and to control the working frequency of the metasurface. The manufactured sample, where 70 photodiodes are implemented in series and located on the side of
the metasurface, is depicted in the figure. When the intensity of the light projected onto the photodiodes changes, the states of corresponding digital units are controlled dynamically, resulting in different coding patterns on the metasurface and reconfigurable radiation patterns in the far field.

In addition, more complicated optical control has been applied in reflection-type metasurfaces. For example, the optically integrated digital platform (OIDP) in Figure 23(b) has already been constructed and verified to be functional \(^{[110]}\). Such a platform can be programmed optically to implement EM functions. In particular, visible light illumination is converted to voltage and applies bias to the metasurface elements, generating specific microwave reflection phase distributions. Functionalities such as external cloaking, illusions, and dynamic vortex beam generation have also been verified experimentally. Similar applications with transmission-type meta-lenses are expected in the future.

5. Conclusions

In this paper, we introduce the applications of MTMs, including 3D bulk MTMs and 2D metasurfaces, at microwave frequencies for the purpose of lensing and beamforming. On the one hand, MTMs are characterized by extremely flexible EM parameters such as permittivity, permeability, and refractive index, and by ultra-fine manipulation of EM waves via designable composing units. With theories such as geometrical optics and transformation optics, MTMs may offer new-concept devices with unprecedented functionalities or that break challenges that cannot be solved properly using current design methods. On the other hand, digital coding metasurfaces and programmable metasurfaces have been introduced recently in the design of meta-lenses, and control of EM waves through a meta-lens has been shown to be possibly more complicated and dynamic, with different functionalities being involved in a single meta-lens \(^{[111]-[114]}\). Therefore, we conclude that new-concept meta-lenses are highly expected to be further developed in the next stage of MTMs, offering new solutions for compact devices and systems, low-profile antennas, high-resolution imaging, and smart information/communication systems.

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Although the reviewed works were completed mostly at Southeast University in China and have been focused on microwaves, the methods and concepts introduced in this review can be extended to higher frequencies.

Figure 23. (a) (left) A transmissive digital unit cell that is controlled by light illumination; (middle) Simulated amplitudes of transmission coefficient of designed digital unit with different capacitance values. The two orange areas represent two working frequency bands in which the state of the digital unit can be switched dynamically between “0” and “1”; (right) Photograph of the fabricated sample of the light-controlled metasurface. Reproduced with permission [108]. (b) The optically integrated digital platform (OIDP) and its programmable EM functions. Reproduced with permission [110].

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This review paper focuses on microwave metamaterial lenses developed in recent years, including bulk lenses made of 3D metamaterials and planar lenses made of 2D transmission-type metasurfaces, for the purpose of microwave engineering. Metamaterials are able to manipulate the propagation of EM waves at extremely fine resolutions, and offer solutions for new-concept lenses with novel functionalities and challenge-breaking properties.