Structure constants in $\mathcal{N}=2$ superconformal quiver theories at strong coupling and holography

Marco Billò $^{a,b}$, Marialuisa Frau $^{a,b}$, Alberto Lerda $^{a,b}$, Alessandro Pini $^b$, and Paolo Vallarino $^{a,b}$

$^a$ Università di Torino, Dipartimento di Fisica, Via P. Giuria 1, I-10125 Torino, Italy
$^b$ Istituto Nazionale di Fisica Nucleare - sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy
$^c$ Università del Piemonte Orientale, Dipartimento di Scienze e Innovazione Tecnologica, Viale T. Michel 11, I-15121 Alessandria, Italy

We consider a supersymmetric quiver theory in four dimensions with gauge group $SU(N)\times SU(N)$ and bi-fundamental matter, schematically represented in Fig. 1. This model, which arises as a $\mathbb{Z}_2$ orbifold of $\mathcal{N}=4$ Super Yang-Mills (SYM) theory, has $\mathcal{N}=2$ supersymmetry, is conformally invariant at the quantum level and admits a holographic dual description in terms of Type II B string theory on $AdS_5 \times S^5/\mathbb{Z}_2$ [1][2]. As such it is one of the simplest four-dimensional $\mathcal{N}=2$ theories to explore the strong-coupling regime and probe the holographic correspondence in a non-maximally supersymmetric set-up (see also [3]).

With this aim, we study 2- and 3-point functions of protected gauge-invariant operators defined as

\begin{align}
U_k(x) &= \frac{1}{\sqrt{2}} \left[ \text{tr} \phi_0(x)^k + \text{tr} \phi_1(x)^k \right], \\
T_k(x) &= \frac{1}{\sqrt{2}} \left[ \text{tr} \phi_0(x)^k - \text{tr} \phi_1(x)^k \right].
\end{align}

Here $k$ is an integer $\geq 2$ and $\phi_{0,1}$ are the chiral scalar fields of the adjoint vector multiplets in the two nodes of the quiver. Replacing $\phi_{0,1}$ with their complex conjugates $\bar{\phi}_{0,1}$ yields the anti-chiral operators $\bar{U}_k$ and $\bar{T}_k$. All primary operators with conformal dimension $\Delta = k$ and charge $Q = \pm k$ in the chiral or anti-chiral case. We call the operators [1] and [2] untwisted ($U$) and twisted ($T$), since they are, respectively, even and odd under the $\mathbb{Z}_2$ symmetry exchanging the nodes of the quiver.

Conformal invariance, charge conservation and $\mathbb{Z}_2$ symmetry fix the form of the 2-point functions to be

\begin{align}
\langle U_k(x) \bar{U}_k(y) \rangle &= \frac{G_{U_k}}{|x-y|^{2k}}, \\
\langle T_k(x) \bar{T}_k(y) \rangle &= \frac{G_{T_k}}{|x-y|^{2k}},
\end{align}

where the coefficients $G_{U_k}$ and $G_{T_k}$ depend on $k$, $N$ and the 't Hooft coupling $\lambda$. Also the 3-point functions are constrained by the symmetries of the theory. Here we will consider the following correlators:

\begin{align}
\langle U_k(x) U_\ell(y) \bar{U}_p(z) \rangle &= \frac{G_{U_k U_\ell U_p}}{|x-y|^{2k} |y-z|^{2\ell}} , \\
\langle U_k(x) T_\ell(y) \bar{T}_p(z) \rangle &= \frac{G_{U_k T_\ell T_p}}{|x-z|^{2k} |y-z|^{2\ell}} ,
\end{align}

with the understanding that $p = k + \ell$ for charge conservation. Again the coefficients in the numerators are functions of $N$, $\lambda$ and of the conformal dimensions. One could consider also the conjugate 3-point functions where chiral and anti-chiral operators are exchanged, as well as $\langle T_k(x) T_\ell(y) \bar{U}_p(z) \rangle$ and their conjugates. For simplicity in this Letter we focus on the above cases.

The coefficients $G$ in (3)–(6) are sensitive to the normalization of the operators. To remove such dependence we define the structure constants

\begin{align}
C_{U_k U_\ell U_p} &= \frac{G_{U_k U_\ell U_p}}{\sqrt{G_{U_k} G_{U_\ell} G_{U_p}}}, \\
C_{U_k T_\ell T_p} &= \frac{G_{U_k T_\ell T_p}}{\sqrt{G_{U_k} G_{T_\ell} G_{T_p}}},
\end{align}

which, together with the spectrum of conformal operators, are part of the conformal field theory data. We study these structure constants in the large-$N$ 't Hooft limit. Using supersymmetric localization, we analytically obtain the exact dependence on $\lambda$ and predict the strong-coupling behavior, which is then obtained also with a holographic calculation using the Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence [4][6][7].
LOCALIZATION RESULTS

The flat-space correlators discussed above can be conformally mapped to correlators defined on a 4-sphere. These, in turn, can be evaluated [8–15] in terms of a matrix model determined by localization techniques [16].

The matrix model has the quiver structure of Fig. 1 with two Hermitian $N \times N$ matrices $a_0$ and $a_1$ defined in the two nodes. Neglecting instanton contributions which are exponentially suppressed in the planar limit, its partition function is

$$Z = \int da_0 da_1 e^{-\text{tr} a_0^2 - \text{tr} a_1^2 - S_{\text{int}}} = \langle e^{-S_{\text{int}}} \rangle_0 ,$$

where $\langle \cdot \rangle_0$ indicates the vacuum expectation value with respect to the Gaussian measure, and $S_{\text{int}}$ is a perturbative series in $\lambda$ given in Eq. (C.5) of [14]. The gauge theory operators $U_k$ and $T_k$ in [14] and [2] are represented in the matrix model, respectively, by $O_k^+$ and $O_k^-$ with

$$O_k^\pm = \frac{1}{\sqrt{2}} : (\text{tr} a_0^k \pm \text{tr} a_1^k) : ,$$

where the normal-ordering $: :$ means that one has to subtract the contractions with all operators of lower conformal dimension. As proven in [8–15], in the large-$N$ limit this amounts to perform a Gram-Schmidt diagonalization within the set of single-trace operators only. Also the anti-chiral operators $\bar{U}_k$ and $\bar{T}_k$ are represented by $O_k^+$, and thus the coefficients $G$ in the 2- and 3-point functions can be computed by evaluating the vacuum expectation value of products of such operators. For example,

$$G_{U_k T_k T_p} = \langle O_k^+ O_\ell^- O_p^- \rangle = \frac{1}{Z} \langle O_k^+ O_\ell^- O_p^- e^{-S_{\text{int}}} \rangle_0 .$$

In [14] it has been shown that the interaction action $S_{\text{int}}$ only contains the twisted operators $O_k^-$. This allowed us to evaluate the partition function as

$$Z = \det \left( 1 - X \right)^{-\frac{1}{2}},$$

where $X$ is an infinite matrix whose elements are

$$X_{i,j} = -8(-1)^{i+j+2ij} \sqrt{i} \int_0^\infty dt e^{t} \int_0^\infty dt e^{t} J_i \left( \frac{t\sqrt{\lambda}}{2\pi} \right) J_j \left( \frac{t\sqrt{\lambda}}{2\pi} \right).$$

Here $J_i$ is the Bessel function of the first kind and the indices $i$ and $j$ are both even or both odd. Remarkably, using the full Lie Algebra approach developed in [10–11], one can show that at large $N$ also the expectation values as the one in (11) can be written in closed form in terms of the matrix $X$. This means that their full dependence on $\lambda$ is known through integrals of Bessel functions.

The correlators involving only untwisted operators, which do not appear in $S_{\text{int}}$, are actually $\lambda$-independent. Indeed, at large $N$ we find

$$G_{U_k} = k \left( \frac{N}{2} \right)^k \equiv G_k ,$$

$$G_{U_k U_l T_p} = \frac{k \ell p}{\sqrt{2}} \left( \frac{N}{2} \right)^{k+l+1} \equiv G_{k,\ell,p} ,$$

for all values of $\lambda$. From this it easily follows that

$$C_{U_k U_l T_p} = \frac{\sqrt{k \ell p}}{\sqrt{2} N} ,$$

which, apart from the factor of $\sqrt{2}$ due to the $\mathbb{Z}_2$ orbifold, is the same expression of the $\mathcal{N} = 4$ SYM theory [17].

On the contrary, the correlators with twisted operators depend on $\lambda$, as one can see already at the first perturbative order by using Feynman diagrams (see [12–14–18]). Exploiting the matrix model formulation, one can go to very high orders in perturbation theory and, with limited computational effort, generate long series in $\lambda$. As shown in [14] for the 2-point correlators, these series can be resummed in closed-form in terms of the matrix $X$, thus obtaining the $\lambda$-dependence beyond perturbation theory. From the asymptotic behavior of $X$ for $\lambda \to \infty$ [13–19], we can then determine the coefficients $G_{k\ell}$ at strong coupling. In [15] these methods have been generalized to the 3-point correlators in an orientifold model. Building on these results, we have further extended these calculations to quiver theories [20] and managed to obtain an analytic expression of $G_{U_k T_k T_p}$ for any value of $\lambda$ by resumming the perturbative expansions in terms of the matrix $X$ which can be extrapolated to strong coupling. For the two-node quiver of Fig. 1 our findings can be summarized as

$$G_{T_k} = \begin{cases} G_k & \text{for } \lambda \to 0 , \\ \frac{4\pi^2 k(k-1)}{\lambda} G_k & \text{for } \lambda \to \infty , \end{cases}$$

and

$$G_{U_k T_k T_p} = \begin{cases} G_{k,\ell,p} & \text{for } \lambda \to 0 , \\ \frac{4\pi^2 (\ell-1)(p-1)}{\lambda} G_{k,\ell,p} & \text{for } \lambda \to \infty , \end{cases}$$

where $G_k$ and $G_{k,\ell,p}$ are defined in [14] and [15]. From these expressions, it follows that

$$C_{U_k U_l T_p} = \begin{cases} \frac{\sqrt{k \ell p}}{\sqrt{2} N} & \text{for } \lambda \to 0 , \\ \frac{\sqrt{k (\ell-1)(p-1)}}{\sqrt{2} N} & \text{for } \lambda \to \infty . \end{cases}$$

We emphasize that our methods allow to evaluate the structure constants for all values of $\lambda$, and not only in the asymptotic regimes. As an example, in Fig. 2 we show how the structure constant $C_{U_k T_k T_p}$ varies in a region of intermediate values of $\lambda$, where we have done two Padé resummations of the perturbative series.

HOLOGRAPHIC DERIVATION

We now derive the structure constants (7) and (8) at strong coupling using the AdS/CFT correspondence [4–6]. Since we are interested in the large-$N$ results, we can
work in the classical supergravity approximation. However, it is useful to start from the string set-up.

We consider the $\mathbb{Z}_2$ orbifold projection from a stack of $2N$ regular D3-branes in Type II B string theory that engineer a $\mathcal{N} = 4$ SYM theory with gauge group $\text{SU}(2N)$. Breaking this configuration into two stacks of $N$ fractional D3-branes located at the orbifold fixed-point \cite{21}, we obtain the quiver theory of Fig.\ [11].

The fractional D3-branes are soliton configurations emitting the metric and a 4-form potential $C_4$ with a self-dual field strength, together with the scalars $b$ and $c$ corresponding to the wrapping of the 2-forms $B_2$ and $C_2$ around the exceptional 2-cycle of the orbifold \cite{22}. We therefore have an untwisted sector comprising the metric and the 4-form $C_4$, which propagate in ten dimensions, and a twisted sector with the scalars $b$ and $c$, which propagate in the six-dimensional space defined at the orbifold fixed point. In the near horizon limit these spaces become, respectively, $\text{AdS}_5 \times S^5/\mathbb{Z}_2$ and $\text{AdS}_5 \times S^3$ \cite{11,2].

The dynamics of the untwisted fields is governed by the equations of Type II B supergravity in $\text{AdS}_5 \times S^5/\mathbb{Z}_2$ with the field-strength of $C_4$ proportional to the volume form of the $\text{AdS}_5$ space. The fluctuations about this background are bulk fields dual to the untwisted operators of the quiver theory. We set \cite{23}

$$G_{mn} = g_{mn} + h_{mn},$$

$$C_4 m_1 \ldots m_4 = c_{m_1 \ldots m_4} + a_{m_1 \ldots m_4} ,$$

where $g_{mn}$ and $c_{m_1 \ldots m_4}$ are the background fields, while the fluctuations are as in \cite{17,24}, namely

$$h_{\mu \nu} = h'_{(\mu \nu)} - \frac{3}{25} h \delta_{\mu \nu} \text{ with } g'^{\mu \nu} h'_{(\mu \nu)} = 0 ,$$

$$h_{\alpha \beta} = h'_{(\alpha \beta)} + \frac{1}{3} h g_{\alpha \beta} \text{ with } g^{\alpha \beta} h'_{(\alpha \beta)} = 0 ,$$

$$a_{\mu_1 \mu_2 \mu_3 \mu_4} = - \epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \partial^\nu a ,$$

$$a_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} = \epsilon_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \partial^\nu a .$$

We now expand the fluctuations in the spherical harmonics of $S^5$. Since we are interested in those Kaluza-Klein (KK) modes which can couple to the fields of the twisted sector that are localized at the $\mathbb{Z}_2$ orbifold fixed point, we can restrict our attention to harmonics of the form

$$Y^k = \frac{1}{2^{\frac{k(k+1)}}} e^{i k \theta} \cos^k \phi \text{ for } k \in \mathbb{Z} ,$$

where $\theta \in [0, 2\pi]$ parametrizes the circle $S^1$ transverse to $\text{AdS}_5$ at the orbifold fixed point, and $\phi$ is one of the $S^5$ coordinates selected in such a way that $\phi = 0$ corresponds to the fixed point. The relevant expansions are

$$h_2 = \sum_k h_{2,k} Y^k , \quad a = \sum_k a_k Y^k ,$$

$$h'_{(\mu \nu),k} = \sum_k h'_{(\mu \nu),k} Y^k .$$

Doing the same analysis as in \cite{17,24}, one can show that the combinations

$$s_k = \frac{1}{20(k+2)} \left[ b_{2,k} - 10(k+4) a_k \right] ,$$

for $k > 0$, and the complex conjugates $s^*_k$, correspond to KK excitations in $\text{AdS}_5$ with mass squared $m^2 = k(k-4)$ that are dual to the untwisted operators $U_k$ and $\bar{U}_k$ of the quiver theory. Furthermore, as in \cite{17} we have

$$h'_{(\mu \nu),k} = \frac{4}{k+1} \nabla_{(\mu} \nabla_{\nu)} s_k , \quad h_{(\mu \nu),k} = \frac{4}{k+1} \nabla_{(\mu} \nabla_{\nu)} s^*_k ,$$

for $k > 0$. The linearized equations for $s_k$ and $s^*_k$ can be derived from the quadratic action

$$S\text{untw} = \frac{4}{(2\pi)^2} \int_{\text{AdS}_5} d^5 z \sqrt{g} \sum_{k > 0} A_k \left[ \partial s_k \cdot \partial s^*_k + k(k-4) s_k^2 \right] \frac{\pi^3}{2} ,$$

where the prefactor is the rewriting of the gravitational constant using the parameters of the quiver theory in units where the radius of $\text{AdS}_5$ is set to one. The last factor of $\pi^3/2$ is the volume of $S^5/\mathbb{Z}_2$, and finally

$$A_k = \left[ \frac{32}{2k+1} \left( \frac{k(k-1)(k+2)}{2(k+1)} \right)^2 \frac{2}{2k(k+1)(k+2)} \right] ,$$

where the first bracket was derived in \cite{17} and the second bracket comes from the normalization of the spherical harmonics \cite{22}.

Let us now turn to the twisted sector. As shown in \cite{2}, the dynamics of the scalars $b$ and $c$ is described by

$$S_b = \frac{1}{2 G_6} \left[ \int d^6 x \sqrt{G_6} \left( \frac{1}{2} \partial b \cdot \partial b + \frac{1}{2} \partial c \cdot \partial c \right) + 4 \int C_4 \wedge db \wedge dc \right] ,$$

where we have adopted the conventions of \cite{14}, with $G_6$ being the determinant of the metric and $2 k_b^2$ the gravitational constant. Assuming $ds^2 = ds^2_{\text{AdS}_5} + db^2$, and expanding $b$ and $c$ in harmonics of $S^1$, namely

$$b = \sum_k b_k e^{i k \theta} , \quad c = \sum_k c_k e^{i k \theta} ,$$

FIG. 2. Plot of the Padé approximations of degree 59 (blue line) and 70 (red line) of the structure constant $C_{U_3 T_2 T_3}$. For larger values of $\lambda$ they tend towards the predicted strong coupling value (dashed black line).
one can show \cite{11, 20} that the combination
\[ \eta_k = c_k - i b_k \] (30)
for \( k > 0 \), and its complex conjugate \( \eta_k^* \), are KK excitations in AdS\( _5 \) with mass-squared \( m^2 = k(k - 4) \) which are dual to the twisted operators \( T_k \) and \( T_k \) of the quiver theory. Their dynamics is governed by the action
\[
S_{tw} = \frac{4(2N)^2}{(2\pi)^3 2 \lambda} \int_{AdS_5} d^5z \sqrt{g} \sum_{k>0} \left[ \frac{1}{2} \partial \eta_k \cdot \partial \eta_k + k(k - 4) \eta_k \eta_k^* \right] 2\pi , \tag{31}
\]
where the prefactor comes from the gravitational constant \( 1/2\kappa_6^2 \) of the orbifold using the AdS/CFT dictionary, and the last factor of \( 2\pi \) is just the length of \( S^1 \).

The actions \cite{26} and \cite{31} can be used to obtain the 2-point functions \cite{3} and \cite{4} at large \( N \) and in strong coupling with the AdS/CFT methods \cite{6}. Using Eq. (17) of \cite{26} and the correction factor in Eq. (95), from \cite{26} we deduce that
\[
G_{U_k} = \frac{4(2N)^2}{(2\pi)^3} A_k \frac{1}{\pi^2} \frac{\Gamma(k + 1)}{\Gamma(k - 2)} \frac{2(k - 2)}{k} \pi^3 2 . \tag{32}
\]
In a similar fashion, from \cite{31} we find
\[
G_{T_k} = \frac{4(2N)^2}{(2\pi)^3} \frac{2^2}{\lambda} 2 \frac{1}{\pi^2} \frac{\Gamma(k + 1)}{\Gamma(k - 2)} \frac{2(k - 2)}{k} \pi 2 \tag{33}
\]
In writing these formulas we have not taken into account the possible presence of an arbitrary proportionality constant in the coupling between the quiver operators and the supergravity modes on the boundary of AdS\(_5\) since, as shown in \cite{17}, this constant drops out in the normalized structure constants.

To compute the 3-point correlators we have to work out the cubic interactions of the KK modes. For the untwisted ones, we can rely again on the analysis of \cite{17} which, translated in our notations, leads to
\[
S_{untw} = \frac{4(2N)^2}{(2\pi)^3} \int_{AdS_5} d^5z \sqrt{g} \sum_{k,\ell, p>0} \left[ V_{k\ell p} s_k s_\ell s_p^* \right. \left. \times \delta_{k+\ell-p, 0} + c.c. \right] \frac{\pi^3}{2} , \tag{34}
\]
where the cubic coupling \( V_{k\ell p} \) can be read from Eqs (3.39) and (3.40) of \cite{17}. From this action, using the AdS/CFT formulas of \cite{26}, we obtain
\[
G_{U_k T_\ell T_p} = \frac{N^2}{2^{\ell+\ell+7-\pi^6}} \frac{k(k - 1)(k - 2)}{k + 1} \times \frac{\ell(\ell - 1)(\ell - 2) p(p + 1)(p - 2)}{\ell + 1 p + 1} \tag{35}
\]
where the \( \delta \)-function imposing charge conservation is understood. Combining \cite{35} and \cite{32}, it follows that
\[
C_{U_k T_\ell T_p} = \frac{\sqrt{k \ell p}}{\sqrt{2 N}} \tag{36}
\]
in agreement with the localization result \cite{16}.

We now consider the twisted sector. In this case we have to expand the twisted action \cite{28} to first order in the fluctuations to obtain the couplings involving one untwisted mode and two twisted ones. Using \cite{21} and the relations \cite{24} and \cite{25}, up to a boundary term we obtain
\[
S'_{tw} = \frac{4(2N)^2}{(2\pi)^3} \frac{2^2}{\lambda} \int_{AdS_5} d^5z \sqrt{g} \sum_{k,\ell, p>0} \frac{1}{2} \left[ W_{k\ell p} s_k \eta_\ell \eta_p^* \times \delta_{k+\ell-p, 0} + c.c. \right] \pi 2 , \tag{37}
\]
where \cite{24}
\[
W_{k\ell p} = -(k + \ell - p)(k + p - \ell) \tag{38}
\times (k + \ell + p - 2)(k + \ell + p - 4) \frac{2^2(k + 1)}{2^2(k + 1)} \]
We observe that if one uses the \( \delta \)-function that imposes charge conservation, this cubic coupling vanishes. This is a well-known feature of all couplings related to extremal correlators \cite{26, 27}, and is not in contradiction with the fact that the final correlators are non-vanishing. Indeed, the zero in the coupling coefficient is compensated by a pole in the cubic Witten diagram of the 3-point function, so that the product yields a finite result. This can be clearly seen \cite{17, 26, 27} if one imposes the charge-conserving \( \delta \)-function only at the end, as we are going to do \cite{28}. With this understanding, using the AdS/CFT formulas of \cite{26}, we obtain
\[
G_{U_k T_\ell T_p} = \frac{N^2}{2^{\ell+\ell+7-\pi^6}} \frac{k(k - 1)(k - 2)}{k + 1} \times \frac{\ell(\ell - 1)(\ell - 2) p(p + 1)(p - 2)}{\ell + 1 p + 1} \tag{39}
\]
Then, from \cite{33} and \cite{52}, it follows that
\[
C_{U_k T_\ell T_p} = \frac{\sqrt{k \ell p}}{\sqrt{2 N}} \tag{40}
\]
which confirms the localization result \cite{19} at strong coupling.

**CONCLUSIONS**

By exploiting the power of supersymmetric localization we were able to obtain the exact \( \lambda \)-dependence of the structure constants of single-trace operators in the 2-node quiver theory of Fig. [1] in the large-\( N \) limit. When all operators are untwisted, the structure constants are \( \lambda \)-independent, like in the \( \mathcal{N} = 4 \) SYM theory, but when two of the operators are twisted they depend on \( \lambda \) in a highly non-trivial way. These twisted structure constants are therefore observables which, remarkably, can be followed from weak to strong coupling in an analytic way. The strong-coupling behavior of the structure constants
predicted by localization is confirmed by a holographic calculation based on the AdS/CFT correspondence. This agreement can be seen either as a validation of the strong-coupling extrapolation of the localization results or, alternatively, as an explicit check of the AdS/CFT correspondence for a non-maximally supersymmetric theory in four dimensions. We finally mention that these results can be generalized to quiver theories with more than two nodes, as well as to orientifold models \(^{20}\).

We thank F. Fucito, F. Galvagno and J. F. Morales for useful discussions.

\(^*\) This work is partially supported by the MIUR PRIN Grant 2020KR4KN2 “String Theory as a bridge between Gauge Theories and Quantum Gravity”.

\(^{†}\) billo.frau,lerda.apini,vallarin@to.infn.it

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\[23\] Our conventions are the following: ten-dimensional indices are denoted by Latin letters $\mu, \nu, ...$, five-dimensional indices along the 5-dimensional space are denoted by Greek letters from the middle part of the alphabet $\alpha, \beta, ...$, whereas five-dimensional indices along the 5-sphere are denoted by Greek letters from the beginning part of the alphabet $\alpha, \beta, ..., \eta$, whereas five-dimensional indices along the 5-sphere are denoted by Greek letters from the beginning part of the alphabet $\alpha, \beta, ..., \eta$.

\[24\] H. J. Kim, L. J. Romans, and P. van Nieuwenhuizen, The Mass Spectrum of Chiral $\mathcal{N} = 2$ D = 10 Supergravity on $S^5$, Phys. Rev. D 32, 389 (1985)

\[25\] D. Z. Freedman, S. D. Mathur, A. Matusis, and L. Rastelli, Correlation functions in the CFT$_d$/AdS$_{d+1}$ correspondence, Nucl. Phys. B 546, 96 (1999) [arXiv:hep-th/9804058]

\[26\] E. D’Hoker, D. Z. Freedman, S. D. Mathur, A. Matusis, and L. Rastelli, Extremal correlators in the AdS/CFT correspondence (1999), arXiv:hep-th/9908160

\[27\] L. Rastelli and X. Zhou, How to Succeed at Holographic Correlators Without Really Trying, J. High Energy Phys. 04, 014 (2018), arXiv:1710.05923 [hep-th]

\[28\] If one wishes to impose $p = k + \ell$, one has to consider the boundary term, which otherwise does not contribute. These two approaches lead to the same results \(^{20}\).