Multi-agent time-varying formation control based on consistency

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Abstract. This paper studies the dynamic time-varying formation control of multi-agent systems. Aiming at the unmanned vehicle systems, based on the static time-varying formation in literature [18], a multi-agent dynamic time-varying formation control based on consistency is put forward. Additionally, the sufficient conditions for the agreement and the steps for solving the control parameters are given. The simulation is conducted to prove the effectiveness and robustness of the theoretical results.

Keywords: Multi-agent system, Time-varying formation control.

1. Introduction

1.1. Background and purpose
A coordinated, orderly, and consistent movement can be formed, when quantities of individuals gather in nature, such as the migration of birds, the swarm of fish, and the gathering of ants. The interaction topologies between individuals of the biological groups are dynamically changing, and information is obtained and exchanged through the topologies. Inspired by this phenomenon, scholars of the engineering have put forward the concept of multi-agents.

Multi-agent system is that the entire group interacts with the environment, acquires the information and cooperates with each other to achieve their goals respectively, thereby completing system tasks [1,2]. The advantage of multi-agent systems is that the tasks are shared by the collaboration of multiple agents, reducing the performance requirements of a single agent, so the stability and robustness of the system are improved. At the same time, when completing more complex tasks, multi-agents are more efficient than a single agent, and the methods for solving problems are more diverse. Multi-agents are widely used in military [3], aerospace [4], and transportation [5].

The formation control is a major study in terms of multi-agent systems. Internationally, research teams such as the University of Pennsylvania and the Massachusetts Institute of Technology have continuously in-depth researched on stable and adaptable formation methods and structures [6], some research results have been used in agricultural testing and commercial transportation.
1.2. Research status of formation problem

Formation is that the multi-agent system maintains the target formation, avoid obstacles in the process of completing the task, and realizes the target efficiently without collision when complying tasks.

At present, formation control is a crucial technology in the application of multi-agent system. The main research is formation, formation maintenance, formation switching and formation avoidance, etc. Three typical methods have been come up with in the past few years, which are Leader-Follower method, Behavior-Based method and Virtual-Leader method.

The basic idea of Leader-Follower method is that one agent is selected as the leader, and the rest are followers in the process of formation. The leader agent maintains the set track, and the following agents the leader’s position and direction by the control strategy, and keep a certain distance from the leader agent [1,7]. Desai et al. proposed a feedback linearization control method using the Leader-Follower method to form formations, and two following rules: $l - q$ rule and $l - l$ rule [8,9]. Mastellone S et al. used Lyapunov’s theory to design the motion control law of a single individual in the formation, and extended the law to the entire formation through the Leader-Follower method [10].

The basic idea of Behavior-Based method is that the agent obtains information through its own sensing or communication, then choose the corresponding behaviour according to the obtained information. Set expected behaviour for each agent, including obstacle avoidance, target finding, and formation maintenance, etc. Each agent completes its own tasks independently, through the weighted average of each behaviour or its own design weighting factors [7,11]. Lafferriere G, Williams A et al. designed the control rules of car formation by the Behavior-Based method [12], in which they set the basic behavior of the formation as directional driving, obstacle avoidance and formation maintaining; Reif J H, Wang H et al. further refined the individual behaviour in the formation [13], and took into account the existence of packet loss in individual data or control information, conducting the research and evaluation of formation control; Monteiro et al. used a nonlinear induced dynamic model architecture to generate the asymptotically stable expected behaviour sequence, so that the formation behavior is asymptotically stable and robust to external disturbances [14].

The basic idea of Virtual-Leader method is that regarding the entire formation as a stationary structure, each individual is a relatively fixed point on the virtual structure. When the formation moves, the relative position of the points is fixed, making the desired formation. The concept of virtual structure was first proposed by Tan K.H et al. [15]; JA.Fax et al. designed a virtual structure formation based on a constant system [16], and used Laplacian matrix and Nyquist criterion to prove that the formation is stable in communication; Kostic et al. used Virtual-Leader method to study the dynamic formation control problem [17], and proposed a saturation control law.

1.3. Chapter arrangement

Part 1 introduces the research background of multi-agent systems and formation control methods. The knowledge of graph theory is illustrated, and the motion control process of unmanned vehicle system is also modeled in part 2. In part 3, time-varying formation control is represented, and simulation results are shown. In part 4, dynamic time-varying formation is proposed, and simulation results are given. Part 5 summarizes the whole study.

2. Problem description and system modelling

2.1. Graph theory

The interaction topology among the agents can be represented by a weighted undirected graph $G = \{V, E, W\}$, where $V = \{v_1, v_2, \ldots, v_m\}$ is the node set, $E \subseteq \{(v_i, v_j): v_i, v_j \in V\}$ is the edge set, furthermore, $W = [w_{ij}] \in R^{m \times m}$ is a non-negative symmetric adjacency matrix, where $i,j \in \{1,2,\ldots,m\}$. For $i,j \in \{1,2,\ldots,m\}$, the agent $i$ and $j$ can be denoted by $v_i$ and $v_j$, and the interaction path between the agent $i$ and the agent $j$ can be represented by $e_{ij} = (v_i, v_j)$. If there exists an edge $e_{ij}$, the agent $i$ is regarded as the neighbor of the agent $j$. Define $m_i = \{v_i \in V: e_{ij} \in E\}$ as the neighbor set.
The interaction weight $w_{ij}$ meets that $w_{ij} > 0 (i \neq j)$ if and only if $e_{ij} \in E$ and $w_{ii} = 0$. The Laplacian matrix $L$ of graph $G$ is defined as $L = D - W$, where $D = \text{diag} \left\{ \sum_{j=1}^{m} w_{ij}, i = 1, 2, \ldots, m \right\}$ represents a degree matrix. When a channel from each node to any other node existd, the graph $G$ is called to be connected, otherwise it is disconnected [18].

2.2. Mathematic symbol

$I_m$ and $\otimes$ represents an identity matrix with dimension $m$ and the Kronecker product respectively. Denoted by $R^{m \times m}$ and $R^n$ a real matrix of size $m \times m$ and real vector space with dimension $n$.

2.3. Modeling

Assuming that there are $m$ unmanned vehicles participating in the formation, the formation control process for each one can be decomposed into an inner loop control and outer loop control, where the inner loop controls motors of vehicles and the outer loop makes the unmanned vehicles towards the target formation [19-21]. The double-loop structure for formation control is shown in Figure 1. This passage mainly discusses the outer loop control of the formation process. The consistency of position and speed among agents should be considered when it comes to the formation of unmanned vehicles, so the outer-loop is modelled by a second-order system [20,22-24], which is as follows:

\[
\begin{align*}
\dot{x}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= u_i(t),
\end{align*}
\]

where $i = 1, 2, \ldots, m$, $x_i(t) \in R^n$, $v_i(t) \in R^n$, $u_i(t) \in R^n$ represent the location, speed and control input vectors of the $i$th unmanned vehicle in respective.

![Figure 1. The double-loop structure for formation control.](image)

3. Time-varying formation control protocol and simulation results

Let $\beta_i(t) = [x_i(t), v_i(t)]^T$, $C_1 = [1,0]^T$, $C_2 = [0,1]^T$. Then unmanned vehicle system (1) can be expressed as

\[
\dot{\beta}_i(t) = C_1^T \beta_i(t) + C_2^T u_i(t). \tag{2}
\]

Denote by $p(t) = [p_1^T(t), p_2^T(t), \ldots, p_m^T(t)]^T \in R^{2m}$ the time-varying formation, where $p_i(t) = [p_{ix}(t), p_{iv}(t)]^T$ ($i = 1, 2, \ldots, m$) are differentiable vectors.

3.1. Formation control protocol

From the reference [18], for the $i$th unmanned vehicle, the following protocol for time-varying formation control can be obtained

\[
\begin{align*}
u_i(t) &= Q_1(\beta_i(t) - p_i(t)) + Q_2 \sum_{j=1}^{m} w_{ij} ((\beta_j(t) - p_j(t)) \nonumber \\
&= - (\beta_i(t) - p(t)) + \dot{p}_i(t), \tag{3}
\end{align*}
\]
Where $Q_1 \in R^{1 \times 2}$ and $Q_2 \in R^{1 \times 2}$ are constant gain matrices. The way for determining $Q_1$ and $Q_2$ would be described in detail later.

Let $\beta(t) = [\beta_1^T(t), \beta_2^T(t), \ldots, \beta_m^T(t)]^T$, $p_x(t) = [p_{1x}(t), p_{2x}(t), \ldots, p_{mx}(t)]^T$ and $p_v(t) = [p_{1v}(t), p_{2v}(t), \ldots, p_{mv}(t)]^T$. Under protocol (3), unmanned vehicle system (2) can be depicted as

\[
\dot{\beta}(t) = (I_m \otimes (C_2Q_1 + C_1C_2^T)) - L_{\sigma(i)} \otimes (C_2Q_2)) \beta(t)
- (I_m \otimes (C_2Q_1) - L_{\sigma(i)} \otimes (C_2Q_2)) p(t) + (I_m \otimes C_2) \dot{p}_v(t)
\] (4)

Unmanned vehicles system complies the time-varying formation $p(t)$ if and only if, for all $i \in \{1,2,\ldots,m\}$

\[
\lim_{t \to \infty}((p_i(t) - p_j(t)) - (\dot{p}_i(t) - \dot{p}_j(t))) = 0, j \in m'_{\sigma(i)}.
\] (5)

The steps for determining $Q_1$ and $Q_2$ are as follows [18]:

Step 1: According to the formula (5), judge whether the given formation can be realized. If yes then continue; else the unmanned vehicles system (2) cannot achieve this formation under protocol (3) and stop.

Step 2: Select $Q_1$ to describe moving patterns of formation reference $r(t)$ by allocating the eigenvalues of $C_2Q_1 + C_1C_2^T$ at desired positions in the complex plane.

Step 3: Solve the algebraic Riccati equation

\[
N(C_2Q_1 + C_1C_2^T) + (C_2Q_1 + C_1C_2^T)^T N - NC_2C_2^TN + I = 0
\] (6)

for $N$ and then select

\[
Q_2 = (2\lambda_{\text{min}})^{-1}C_2^TN.
\] (7)

3.2. Simulation results

According to the previous formation control protocol, simulation is carried out for the formation of 4 unmanned vehicles starting from any initial position. The 4 unmanned vehicles are numbered 1-4 sequentially. The system interaction topology is shown in Figure 2.

![Figure 2. Interaction topology](image)

Consider the following time-varying formation:

\[
p_i(t) = \begin{bmatrix}
    r \cos \left(\omega t + \frac{(i-1)\pi}{2}\right) \\
    -r \sin \left(\omega t + \frac{(i-1)\pi}{2}\right) \\
    r \sin \left(\omega t + \frac{(i-1)\pi}{2}\right) \\
    \omega r \cos \left(\omega t + \frac{(i-1)\pi}{2}\right)
\end{bmatrix}
\] (8)

\[
(i = 1,2,3,4),
\]
Where \( r = 10m \) and \( \omega = 0.6 \text{ rad/s} \).

The moving patterns of \( r(t) \) are set to be steady by selecting \( Q_1 = I_2 \otimes [-1 \quad -0.8] \) to allocate the eigenvalues of \( C_2 Q_1 + C_1 C_2^T \) at \(-0.4 + 0.9165j, -0.4 + 0.9165j, -0.4 - 0.9165j, -0.4 - 0.9165j\) with \( j^2 = -1 \). The smallest nonzero eigenvalues of the four Laplacian matrices can be got, which are 0.5858, 0.5858, 1, and 0.5858 respectively [18]. So \( \lambda_{\text{min}} = 0.5858 \). From the method of deciding \( Q_1 \) and \( Q_2 \), one can obtain matrix \( N \) as

\[
N = I_2 \otimes \begin{bmatrix} 1.4219 & 0.4142 \\ 0.4142 & 0.7711 \end{bmatrix},
\]

(9)

And matrix \( Q_2 = I_2 \otimes [0.3535 \quad 0.6528] \) to make the unmanned vehicles system form the target formation.

Set the primary condition of four unmanned vehicles as

\[
\beta_1(0) = [2.28 \quad -2.39 \quad 2.16 \quad 2.07];
\]

\[
\beta_2(0) = [-4.41 \quad -3.05 \quad 4.61 \quad 1.22];
\]

\[
\beta_3(0) = [-1.47 \quad 3.38 \quad 5.48 \quad 1.62];
\]

\[
\beta_4(0) = [-1.93 \quad 4.08 \quad 3.11 \quad -2.25];
\]

The simulated motion trajectories of the four unmanned vehicles are shown in Figure 3, and the formation error diagram is shown in Figure 4.

**Figure 3.** Each agent position in simulation
Figure 4. Formation error in simulation

From the simulation results, one can prove that the formation control protocol is effective. However, the protocol can only form a static time-varying formation, which has certain limitations, so the dynamic time-varying formation control protocol would be proposed later.

4. Dynamic formation protocol and simulation results

4.1. Dynamic time-varying formation protocol

For the $i$th unmanned vehicle, the following protocol for dynamic time-varying formation control can be obtained

$$u_i(t) = Q_1(\beta_i(t) - P_i(t)) + Q_2 \sum_{j \neq i} w_{ij}(\beta_j(t) - P_j(t)) -$$

$$\dot{P}_i(t) = P_i(t) + \dot{f}_i(t),$$

$$\dot{\beta}_i(t) = \beta_i(t) + \dot{f}_i(t),$$

where $Q_1 \in R^{1 \times 2}$ and $Q_2 \in R^{1 \times 2}$ are constant gain matrices, $P_i(t)$ represents dynamic formation control function, $p_i(t)$ represents static formation control function, $\dot{f}_i(t)$ represents center motion track function.

4.2. Simulation results

Based on the dynamic time-varying formation protocol, a simulation for 4 unmanned vehicles from any initial position is performed.

Consider the following time-varying formation:
\[ P_i(t) = \begin{bmatrix} r \cos \left( \omega t + \frac{(i-1)\pi}{2} \right) \\ -\omega r \sin \left( \omega t + \frac{(i-1)\pi}{2} \right) \\ r \sin \left( \omega t + \frac{(i-1)\pi}{2} \right) \\ \omega r \cos \left( \omega t + \frac{(i-1)\pi}{2} \right) \end{bmatrix} (i = 1,2,3,4), \]  

\[ f_i(t) = \begin{bmatrix} \sin (0.15kT) \\ 0.15 \cos (0.15kT) \\ 0.15kT \\ 0.15 \end{bmatrix} (i = 1,2,3,4), \]  

\[ P_i(t) = \begin{bmatrix} r \cos \left( \omega t + \frac{(i-1)\pi}{2} \right) + \sin(0.15kT) \\ -\omega r \sin \left( \omega t + \frac{(i-1)\pi}{2} \right) + 0.15 \cos(0.15kT) \\ r \sin \left( \omega t + \frac{(i-1)\pi}{2} \right) + 0.15kT \\ \omega r \cos \left( \omega t + \frac{(i-1)\pi}{2} \right) + 0.15 \end{bmatrix} (i = 1,2,3,4), \]  

where \( r = 15m \) and \( \omega = 0.6 \text{ rad/s} \).

The moving patterns of \( r(t) \) is the same as the \( r(t) \) for the static time-varying formation, so the parameters are equal.

\[ Q_1 = I_2 \otimes [-1 \ -0.8], \ Q_2 = I_2 \otimes [0.3535 \ 0.6528], \ N = I_2 \otimes [1.4219 \ 0.4142 \ 0.4142 \ 0.7711]. \]

Set the primary conditions of four unmanned vehicles as

\[ \beta_i(0) = [2.28 \ -2.39 \ 2.16 \ 2.07]; \]
\[ \beta_i(0) = [-4.41 \ -3.05 \ 4.61 \ 1.22]; \]
\[ \beta_i(0) = [-1.47 \ 3.38 \ 5.48 \ 1.62]; \]
\[ \beta_i(0) = [-1.93 \ 4.08 \ 3.11 \ -2.25]; \]

The simulated motion trajectories of the four unmanned vehicles are shown in Figure 5, and the formation error diagram is shown in Figure 7. Compared it with the static time-varying formation protocol, the stability performance for dynamic formation protocol is well.

Additionally, after changing the radius of the desired formation, the motion trajectories and error simulation results are displayed in Figure 6 and Figure 8 respectively, verifying the robustness of the protocol.

**Figure 5.** Each agent position in simulation
The simulation results show that the dynamic formation protocol is effective, and has good stability and robustness.
5. Conclusions
This paper takes the unmanned vehicle system as the research object, and reproduces the time-varying formation control protocol proposed in literature [18], moreover, the sufficient conditions, which is for the unmanned vehicle system to accomplish desired time-varying formation, are put forward too. Furthermore, it is proposed to change the target reference function to form dynamic time-varying formation. In the end, the effectiveness of the dynamic formation control is demonstrated through the simulation.

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