Block Method for the Solution of First Order Nonlinear ODEs and Its Application to HIV Infection of CD4+T Cells Model

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Abstract Some of the issues relating to the human immunodeficiency virus (HIV) epidemic can be expressed as a system of nonlinear first order ordinary differential equations. This includes modelling the spread of the HIV virus in infecting CD4+T cells that help the human immune system to fight diseases. However, real life differential equation models usually fail to have an exact solution, which is also the case with the nonlinear model considered in this article. Thus, an approximate method, known as the block method, is developed to solve the system of first order nonlinear differential equation. To develop the block method, a linear block approach was adopted, and the basic properties required to classify the method as convergent were investigated. The block method was found to be convergent, which ascertained its usability for the solution of the model. The solution obtained from the newly developed method in this article was compared to previous methods that have been adopted to solve same model. In order to have a justifiable basis of comparison, two-step length values were substituted to obtain a one-step and two-step block method. The results show the newly developed block method obtaining accurate results in comparison to previous studies. Hence, this article has introduced a new method suitable for the direct solution of first order differential equation models without the need to simplify to a system of linear algebraic equations. Likewise, its convergent properties and accuracy also give the block method an edge over existing methods.

Keywords HIV Infection, CD4+T Cells, Numerical, Block Method, First Order

1. Introduction

The recent trend in research has delved into mathematical modelling of real-life cases, and problems existing in the field of biological sciences have not been left out. One of the major issues which are still being investigated intensely is the human immunodeficiency virus (HIV). In the investigation by [1], it was stated that an estimated number of 35 million adults worldwide are HIV positive. The CD4+T cells in human body, also known as leukocytes or T helper cells, are quite vital to the immune system, as it joins other cells in the human immune system to fight diseases. The effect of the HIV on CD4+T cells is that it depletes and infects these cells, thus most important to the production of acquired immunodeficiency syndrome (AIDS). This development from HIV to AIDS happens due to the inability of human body to defend itself against other infections as the CD4+T cells are destroyed in the blood. However, if the virus is detected on time and treated, the human immune system can still be protected and hence controlling the spread or growth of the HIV by determining which CD4+T cells are infected, and which are not [2].
[3] developed an HIV infection of the human immune system model of three variables that include the population sizes of the infected cells, the uninfected cells and the free particles of the virus. Further study [4] expanded the paradigm in a manner that replicates various of the scientifically acquired signs of AIDS. Four factors consisting of latently infected cells, uninfected cells, free particles of viruses and actively infected cells were considered in this expansion. However, it was considered by [5] that the assumption should follow that all the infected cells are able to produce the virus, thus defining the dynamic model required to observe the CD4+T cells, as a system of three nonlinear ordinary differential equations defined as:

\[
\begin{align*}
\frac{dT(t)}{dt} &= \gamma_I T(t) + \gamma_V (T(t) - T(t)) - \gamma_T V(t) T(t), \\
\frac{dI(t)}{dt} &= \gamma_I V(t) T(t) - \gamma_I I(t), \\
\frac{dV(t)}{dt} &= N \gamma_I I(t) - \gamma_V V(t).
\end{align*}
\] (1)

In Equation (1), the concentration of susceptible CD4+T cells, CD4+T cells infected with HIV viruses and free HIV virus particles in the blood at time t, denoted as, and respectively, are the three basic component models. The definition of each parameter in Equation (1) is shown in Table 1, as described by [5].

| Parameter | Description |
|-----------|-------------|
| \(\gamma_I\) | Level of body-produced CD4+T cells |
| \(\gamma_I\) | Normal rates of uninfected T cells' turnover |
| \(\gamma_I\) | CD4+T cell growth rate |
| \(\gamma_I\) | Level of infection |
| \(\gamma_I\) | Infected T cells' normal turnover rate |
| \(\gamma_I\) | Normal turnover rate of particles from viruses |
| \(T_{max}\) | In the body, the maximum CD4+T cell concentration |
| \(N\) | Particles of viruses manufactured by contaminated CD4+T cells |

Over the years, many studies have considered solutions to the model in Equation (1). These include [6] the suggestion of an updated variational iteration method, [7] the implementation of the Bessel collocation method, [8] the adoption of the Laplace Adomian decomposition method, [9] the use of the differential transformation method, [10] the introduction of the multi-stage Laplace Adomian decomposition method, and [11] the suggestion of the perturbation iteration algorithm. More recent investigations to obtain a numerical solution to the system of nonlinear ordinary differential equations have been explored by [2] with the use of the Shifted-Lagrangian Jacobi polynomials, [12] adopting a collocation method based on Bernoulli polynomials, [13] providing the model's approximate solution using orthonormal Bernstein polynomials, [14] developing a novel biologically-inspired computing framework, [15] combining the traditional homotopy analysis method with the Laplace transformation approach, among others. In general, the definition adopted by these authors was a reduction to the corresponding system of nonlinear algebraic equations of the nonlinear system of ordinary differential equations, which is then rectified by following some reasonable computational solution, such as the well-known method of Newton. This approach of the reduction which involves more computations can be bypassed with the application of direct methods for solving ordinary differential equations, such as the block method.

The vast applicability of the block method alongside its impressive accuracy in terms of evaluation to the accurate solutions of first order ODE models, has been documented in various studies [16-23]. However, the application of the block method to solve dynamic model required to examine the CD4+T cells in this article is quite uncommon, besides the recent study by [24] who proposed a separately diagonally implicit block backward differentiation formula designed for order two to approximate the solution to the model in Equation (1). Whereas, the solution of the model in this article will consider the use of two block methods for first order differential equations with impressive accuracy, despite their order, in comparison to previous solutions in literature.

### 2. Methodology

This section details the derivation of the block methods in addition to discussing their individual convergence properties. Deriving the block methods to be utilised as a new approach to numerically solve HIV infection model of CD4+T cells involves considering the block method scheme of the form

\[
\begin{align*}
\phi_{i+1/2} &= \frac{1}{\Delta t} \left( \phi_2 - \phi_0 + \sum_{k=1}^{m} \phi_{i+k} \left( \gamma_I - \gamma_T \phi_{i+k} + \gamma_T \phi_{i+k} \right) \phi_2 \right), \\
\phi_{i+1/2} &= \frac{1}{\Delta t} \left( \phi_{i+1} - \phi_{i-1} + \sum_{k=1}^{m} \phi_{i+k} \left( \gamma_I - \gamma_T \phi_{i+k} + \gamma_T \phi_{i+k} \right) \phi_{i+1} \right), \\
\phi_{i+1/2} &= \frac{1}{\Delta t} \left( \phi_{i+1} - \phi_{i-1} + \sum_{k=1}^{m} \phi_{i+k} \left( \gamma_I - \gamma_T \phi_{i+k} + \gamma_T \phi_{i+k} \right) \phi_{i+1} \right)
\end{align*}
\] (2)

and also

\[
\begin{align*}
\phi_{i+1/2} &= \frac{1}{\Delta t} \left( \phi_2 - \phi_0 + \sum_{k=1}^{m} \phi_{i+k} \left( \gamma_I - \gamma_T \phi_{i+k} + \gamma_T \phi_{i+k} \right) \phi_2 \right), \\
\phi_{i+1/2} &= \frac{1}{\Delta t} \left( \phi_{i+1} - \phi_{i-1} + \sum_{k=1}^{m} \phi_{i+k} \left( \gamma_I - \gamma_T \phi_{i+k} + \gamma_T \phi_{i+k} \right) \phi_{i+1} \right), \\
\phi_{i+1/2} &= \frac{1}{\Delta t} \left( \phi_{i+1} - \phi_{i-1} + \sum_{k=1}^{m} \phi_{i+k} \left( \gamma_I - \gamma_T \phi_{i+k} + \gamma_T \phi_{i+k} \right) \phi_{i+1} \right)
\end{align*}
\] (3)
The block scheme is presented as a system of equations to conform with the model as defined in equation (1). In Equations (2) and (3), $m$ is the order of the differential equation model, $k$ denotes the step number chosen as 1 in Equation (2) and 2 in Equation (3), with $\xi = 1, 2, ..., k$.

These $k$ values were selected in order to obtain suitable block methods of equal or lower order to the previously developed numerical methods for comparison. Following the linear block approach by [25], the unknown coefficients for the $k = 1$ block method are derived from the expression

$$
\varphi_{j\xi} = A_j^{-1}B_j, \quad j = T, I, V \quad \text{with} \quad A_j = \begin{pmatrix} 1 & 1 \\ 0 & h \end{pmatrix}
$$

and

$$
B_j = \begin{pmatrix} (\xi h) \\ \frac{(\xi h)^2}{2} \end{pmatrix}.
$$

while the coefficients for the $k = 2$ block method are derived from the expression

$$
\varphi_{j\xi} = A_j^{-1}B_j, \quad j = T, I, V \quad \text{with} \quad A_j = \begin{pmatrix} 1 & 1 & 1 \\ 0 & h & 2h \\ 0 & \frac{(h)^2}{2} & \frac{(2h)^2}{3} \end{pmatrix}
$$

and

$$
B_j = \begin{pmatrix} (\xi h) \\ \frac{(\xi h)^2}{2} \\ \frac{(\xi h)^3}{3} \end{pmatrix}.
$$

This implies that, for the one-step block method, Equation (2) takes the form

$$
\begin{aligned}
T_{n+1} &= T_n + \varphi_{T01} T_n + \varphi_{T11} T_{n+1} \\
I_{n+1} &= I_n + \varphi_{I01} I_n + \varphi_{I11} I_{n+1} \\
V_{n+1} &= V_n + \varphi_{V01} V_n + \varphi_{V11} V_{n+1}
\end{aligned}
$$

while Equation (3) takes the form

$$
\begin{aligned}
T_{n+1} &= T_n + \varphi_{T02} T_n + \varphi_{T12} T_{n+1} + \varphi_{T22} T_{n+2} \\
I_{n+1} &= I_n + \varphi_{I01} I_n + \varphi_{I11} I_n + \varphi_{I21} I_{n+1} + \varphi_{I22} I_{n+2} \\
V_{n+1} &= V_n + \varphi_{V01} V_n + \varphi_{V11} V_n + \varphi_{V21} V_{n+1} + \varphi_{V22} V_{n+2}
\end{aligned}
$$

where

$$
\begin{aligned}
X_{T0} &= \gamma_1 - \gamma_2 T_n + \gamma_3 T_{n+1} \left(1 - T_{\max}^{-1} (I_n + L_j - T_n)\right) - \gamma_4 V_n \\
X_{T1} &= \gamma_1 - \gamma_2 T_{n+1} + \gamma_3\left(1 - T_{\max}^{-1} (I_{n+1} + L_j)\right) - \gamma_4 V_{n+1} \\
X_{T2} &= \gamma_1 - \gamma_2 T_{n+2} + \gamma_3\left(1 - T_{\max}^{-1} (I_{n+2} + L_j)\right) - \gamma_4 V_{n+2} \\
X_{I0} &= \gamma_5 V_n - \gamma_1 I_n \\
X_{I1} &= \gamma_5 V_{n+1} - \gamma_1 I_{n+1} \\
X_{I2} &= \gamma_5 V_{n+2} - \gamma_1 I_{n+2} \\
X_{V0} &= \gamma_6 V_n - \gamma_5 I_n \\
X_{V1} &= \gamma_6 V_{n+1} - \gamma_5 I_{n+1} \\
X_{V2} &= \gamma_6 V_{n+2} - \gamma_5 I_{n+2}
\end{aligned}
$$

(6).

Hence,

$$
\begin{aligned}
\varphi_{T01} &= \begin{pmatrix} 1 & 1 \end{pmatrix}, \\
\varphi_{T11} &= \begin{pmatrix} h \end{pmatrix}, \\
\varphi_{T21} &= \begin{pmatrix} \frac{5h}{12} \\ -\frac{h}{12} \end{pmatrix},
\end{aligned}
$$

$$
\begin{aligned}
\varphi_{T02} &= \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \\
\varphi_{T12} &= \begin{pmatrix} h \frac{2h}{3} \frac{2h}{3} \frac{2h}{3} \end{pmatrix}, \\
\varphi_{T22} &= \begin{pmatrix} \frac{5h}{12} \frac{2h}{3} \frac{2h}{3} \frac{2h}{3} \frac{2h}{3} \end{pmatrix}.
\end{aligned}
$$

Similarly, $(\varphi_{I01}, \varphi_{I11})^T = (\varphi_{I02}, \varphi_{I12})^T = (\varphi_{I03}, \varphi_{I13})^T = (\varphi_{V01}, \varphi_{V11})^T = (\varphi_{V02}, \varphi_{V12})^T = (\varphi_{V03}, \varphi_{V13})^T$.

Substituting the obtained $\varphi_{j\xi}, \varphi_{j\xi}$ values back in Equations (4) and (5) gives the required block methods to solve the model as defined in equation (1).

Now, considering the convergence properties of the block methods in Equations (2) and (3). For these block methods to be convergent, they must satisfy the property of linear multistep system $s$ which is consistent and zero-stable [26].

Firstly, the consistency of the methods is checked, where a linear multistep system is constant if it has an order of $p \geq 1$. Collectively merging all the variables in the model rewrites the block expressions as:
with \( j = T, I, V \). The individual terms \( j_n \), \( j_{n+1} \), and \( j_{n+2} \) in Equations (7)- (9), including terms within the previously defined \( \mathcal{X}_{j0} \), \( \mathcal{X}_{j1} \), and \( \mathcal{X}_{j2} \), are expanded using Taylor series expansion about \( t = t_n \). Let \( C_m \) be the value obtained on equating coefficients of \( h^m f^{(m)}(t_n) \) in Equations (7)-(9). Equation (7) has
\[
C_0 = C_1 = C_2 = 0, C_3 = -\frac{h^3}{12}, \quad \text{while in Equation (8)},
\]
\[
C_0 = C_1 = C_2 = C_3 = 0, C_4 = \frac{h^4}{24}, \quad \text{and for Equation (9)},
\]
\[
C_0 = C_1 = C_2 = C_3 = 0, C_4 = 0, C_5 = -\frac{h^5}{90}.
\]
Following the definition by [27], a linear multistep system for solving first order differential equations is of order \( p \) if \( C_0 = C_1 = C_2 = \ldots = C_p \) and \( C_{p+1} \neq 0 \). This implies that the block methods are consistent as the order \( p = \{2,3,4\} \) satisfies \( p \geq 1 \).

Considering the second condition for convergence, this is the zero-stability criterion. The block method in the subsequent matrix difference equation form
\[
A^0 j_{n+k} = A^1 j_{n-k} + h \left[ B^0 j_{n+k} + B^1 j_{n-k} \right],
\]
(10)
is zero-stable if the characteristic polynomial takes the form
\[
\rho(R) = \det \left( Rj_A^0 - A^1 \right); \quad R_j = R^i \delta_{ji} \quad (i = 1,2, \ldots, k)
\]
(11)
and the roots of \( \rho(R) = 0 \) satisfies the root condition that every its roots lie in or on the unit circle, with those on the boundary being simple, that is, all roots satisfy \( |R| \leq 1, j = 1, \ldots, k \), and a few that satisfy \( |R| = 1, j = 1, \ldots, k \), are common, where \( V \) is said to be a simple root of \( \rho(R) \) if \( (v - R) \) is a factor, but \( (v - R)^2 \) is not. The coefficients \( A^0 \) and \( A^1 \) of the matrix difference equation form for the block methods are substituted in Equation (11) to obtain the characteristic polynomial \( \rho(R) \) as \( R - 1 \) and \( R^3 - R \), for the one-step and two-step block methods respectively. The absolute value of the roots of \( \rho(R) \) gives values of 0 and 1. Therefore, all the roots satisfy \( |R| \leq 1 \) and the block methods are zero-stable. Hence, since the block methods have satisfied both conditions of consistency and zero-stability, this implies that both block methods are convergent.

3. Results

In this section, the block methods will be adopted to solve the dynamic model that deals with the HIV infection of CD4\(^+\)T cells as defined in Equation (1). In the absence of the exact solution, comparison will be made to the converged solution obtained by [2] and [12]. The conditions at the initial point for the model in Equation (1) are \( T(0) = 0.1 \), \( I(0) = 0 \) and \( V(0) = 0.1 \) with the unknown parameters set as \( \gamma_1 = 0.1 \), \( \gamma_2 = 0.02 \), \( \gamma_3 = 3 \), \( \gamma_4 = 0.0027 \), \( \gamma_5 = 0.3 \), \( \gamma_6 = 2.4 \), \( T_{\text{max}} = 1500 \), \( N = 10 \). The tables detailing the results obtained demonstrate the numerical value of \( T(t) \), \( I(t) \) and \( V(t) \) within the interval \([0,1]\) in comparison to recent studies exploring an estimated solution to the model.

| Table 2. Numerical results for \( T(t) \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( t \) | Converged Solution | Computed Solution [28] | Computed Solution [24] | Computed Solution [13] | One-Step Block Method | Two-Step Block Method |
| 0.1 | - | - | - | - | 0.1463630478 | 0.1463590814 |
| 0.2 | 0.2088080843 | 0.2073594783 | 0.2090205532 | 0.2129282162 | 0.2088187668 | 0.2088080818 |
| 0.3 | - | - | - | - | 0.2929509957 | 0.292940899 |
| 0.4 | 0.4062405428 | 0.4034685968 | 0.4065545236 | 0.4110169992 | 0.4062792966 | 0.4062405336 |
| 0.5 | - | - | - | - | 0.5589285924 | 0.5586633248 |
| 0.6 | 0.7644238985 | 0.7595171444 | 0.7648420298 | 0.7757846339 | 0.7645293020 | 0.7644238736 |
| 0.7 | - | - | - | - | 1.0414263729 | 1.0412607830 |
| 0.8 | 1.4140468519 | 1.4050224494 | 1.4145067965 | 1.4347539521 | 1.4143015118 | 1.4140467917 |
| 0.9 | - | - | - | - | 1.9163467361 | 1.9159611404 |
| 1.0 | 2.5915948717 | 2.5753421941 | 2.5840259154 | 2.7432245704 | 2.5921710130 | 2.5915947158 |
Table 3. Absolute error (AE) results of computed $\tilde{T}(t)$ solutions with converged solution

| $t$ | $AE$ [28]     | $AE$ [24]     | $AE$ [13]     | $AE$ (One-Step Block Method) | $AE$ (Two-Step Block Method) |
|-----|----------------|----------------|----------------|-------------------------------|------------------------------|
| 0.2 | 1.4000e-03     | 2.1247e-04     | 4.1000e-03     | 1.0682e-05                    | 2.5000e-09                   |
| 0.4 | 2.8000e-03     | 3.1398e-04     | 4.8000e-03     | 3.8754e-05                    | 9.2000e-09                   |
| 0.6 | 4.9000e-03     | 4.1813e-04     | 1.1400e-02     | 1.0540e-04                    | 2.4900e-08                   |
| 0.8 | 9.0000e-03     | 4.5994e-04     | 2.0700e-02     | 2.5466e-04                    | 6.0200e-08                   |
| 1.0 | 1.6300e-02     | 7.6000e-03     | 1.5160e-01     | 5.7616e-04                    | 1.3590e-07                   |

Figure 1. Absolute error (AE) results of computed $\tilde{T}(t)$

Table 4. Numerical results for $I(t)$

| $t$ | Converged Solution | Computed Solution [28] | Computed Solution [24] | Computed Solution [13] | One-Step Block Method | Two-Step Block Method |
|-----|--------------------|------------------------|------------------------|------------------------|-----------------------|-----------------------|
| 0.1 | -                  | -                      | -                      | -                      | 0.0000028649          | 0.0000028649          |
| 0.2 | 0.0000060327       | 0.0000093532           | 0.0000060459           | 0.0000059037           | 0.0000060327          | 0.0000060327          |
| 0.3 | -                  | -                      | -                      | -                      | 0.0000094715          | 0.0000094714          |
| 0.4 | 0.0000131583       | 0.0000108470           | 0.0000131824           | 0.0000129974           | 0.0000131586          | 0.0000131583          |
| 0.5 | -                  | -                      | -                      | -                      | 0.0000170793          | 0.0000170787          |
| 0.6 | 0.0000212238       | 0.0000207830           | 0.0000212573           | 0.0000212334           | 0.0000212246          | 0.0000212238          |
| 0.7 | -                  | -                      | -                      | -                      | 0.0000255910          | 0.0000255898          |
| 0.8 | 0.0000301774       | 0.0000266389           | 0.0000302185           | 0.0000302700           | 0.0000301790          | 0.0000301774          |
| 0.9 | -                  | -                      | -                      | -                      | 0.0000349929          | 0.0000349909          |
| 1.0 | 0.0000400378       | 0.0000388399           | 0.0000399547           | 0.0000394304           | 0.0000400404          | 0.0000400378          |

Table 5. Absolute error (AE) results of computed $\tilde{I}(t)$ solutions with converged solution

| $t$ | $AE$ [28]     | $AE$ [24]     | $AE$ [13]     | One-Step Block Method | Two-Step Block Method |
|-----|----------------|----------------|----------------|-----------------------|-----------------------|
| 0.2 | 3.3205e-06     | 1.3200e-08     | 1.2900e-07     | 0.0000e+00            | 0.0000e+00            |
| 0.4 | 2.3113e-06     | 2.4000e-08     | 1.6000e-07     | 3.0000e-09            | 0.0000e+00            |
| 0.6 | 4.4080e-07     | 3.3500e-08     | 9.6000e-09     | 8.0000e-10            | 0.0000e+00            |
| 0.8 | 3.5385e-06     | 4.1100e-08     | 9.2600e-08     | 1.6000e-09            | 0.0000e+00            |
| 1.0 | 1.1979e-06     | 8.3100e-08     | 6.0740e-07     | 2.6000e-09            | 0.0000e+00            |
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Figure 2. Absolute error (AE) results of computed \( I(t) \)

Table 6. Numerical results for \( V(t) \)

| \( t \) | Converged Solution | Computed Solution [28] | Computed Solution [24] | Computed Solution [13] | One-Step Block Method | Two-Step Block Method |
|---|---|---|---|---|---|---|
| 0.1 | - | - | - | - | 0.0786622699 | 0.0786631764 |
| 0.2 | 0.0618798432 | 0.0618799249 | 0.061925726 | 0.0616038027 | 0.0618784173 | 0.0618798434 |
| 0.3 | - | - | - | - | 0.048678065 | 0.0486784892 |
| 0.4 | 0.0382948878 | 0.0382947904 | 0.0383177394 | 0.0381135846 | 0.0382931232 | 0.0382948880 |
| 0.5 | - | - | - | - | 0.0301261389 | 0.0301278741 |
| 0.6 | 0.0237045500 | 0.0237043306 | 0.0237164782 | 0.0236278850 | 0.023709125 | 0.0237045503 |
| 0.7 | - | - | - | - | 0.0186514182 | 0.0186529210 |
| 0.8 | 0.0146803637 | 0.0146803895 | 0.0146863853 | 0.0146194704 | 0.0146790131 | 0.0146803639 |
| 0.9 | - | - | - | - | 0.0115554996 | 0.0115566945 |
| 1.0 | 0.0091008440 | 0.0091008815 | 0.0091490338 | 0.0081082206 | 0.0090998013 | 0.0091008452 |

Table 7. Absolute error (AE) results of computed \( V(t) \) solutions with converged solution

| \( t \) | AE [28] | AE [24] | AE [13] | AE (One-Step Block Method) | AE (Two-Step Block Method) |
|---|---|---|---|---|---|
| 0.2 | 8.1700e-08 | 4.2729e-05 | 2.7604e-04 | 1.4259e-06 | 2.0000e-10 |
| 0.4 | 9.7400e-08 | 2.2852e-05 | 1.8130e-04 | 1.7646e-06 | 2.0000e-10 |
| 0.6 | 2.1940e-07 | 1.1928e-05 | 7.6665e-05 | 1.6375e-06 | 3.0000e-10 |
| 0.8 | 2.5800e-08 | 6.0216e-06 | 6.0893e-05 | 1.3506e-06 | 2.0000e-10 |
| 1.0 | 3.7500e-08 | 4.8190e-05 | 9.9262e-04 | 1.0427e-06 | 1.2000e-09 |
4. Conclusions

This article has developed one and two-step block methods for the solution of a system of first order nonlinear ordinary differential equations. The properties of the methods as shown in the methodology section have defined the methods to be of low orders, hence the expectation of low accuracy. Rather, the block methods have displayed impressive performance in comparison to results from previous studies. The methods that were compared to the one and two-step block methods, as shown in Tables 2-7 include the optimization method by [28] which is based on generalized polynomials of degrees 5 and 7, the order 2 singly diagonally implicit backward block method by [24], and [13] collocation method implemented with degree 8 orthonormal Bernstein polynomials. The lower and equal order of the block methods in this article gives a good basis of comparison to these selected studies. As seen in Table 2, Table 4, and Table 6, the previous studies presented their results for \( t = [0:0.2:1] \), while the solutions from the block methods were shown for \( t = [0:0.1:1] \), which gives room for better comparison for future studies. When considering the absolute error values as shown in Table 3, Table 5, and Table 7, the one-step method shows improved accuracy than the method having same order 2 as itself in Table 3 and Table 5. Likewise, Figure 1, Figure 2, and Figure 3 clearly show the block method having absolute error tending to zero. In general, the two block methods performed better than the other approaches for the solutions of \( T(t) \) and \( I(t) \), particularly the two-step block method which obtained the same values as the converged solution of \( I(t) \) despite having low order. For the solution for \( V(t) \), the absolute error values in Table 7 shows the one-step method is still performing better than the order 2 method of [24], with the method by [28] showing good accuracy but the two-step block method gave the most accurate results in terms of absolute error comparison. This has shown that the new block methods, despite being methods of low order perform favourably when adopted for the solution of a system of nonlinear first order ordinary differential equations, which can be sued in modelling various real-life scenarios, including the model for HIV Infection of CD4\(^+\)T considered in this article.

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