Self-organization effects and light amplification of collective atomic recoil motion in a harmonic trap

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Self-organization effects related to light amplification in the collective atomic recoil laser system with the driven atoms confined in a harmonic trap are investigated further. In the dispersive parametric region, our study reveals that the spontaneously formed structures in the phase space contributes an important role to the light amplification of the probe field under the atomic motion being modified by the trap.

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I. INTRODUCTION AND SIMULATION RESULTS

Optical amplification in a strongly driven sodium atoms system without population inversions has been observed in the experiment [1] where the atomic recoil effect and Doppler shift were purposely avoided by a specific systematic configuration. When the pumping field is off-detuned from the atomic transition frequency $\omega_0$, the experimental pump-probe gain spectrum exhibits a normal absorption-gain profile similar to that of Fig.1(a) which is drawn from a direct simulation and has been obtained by Bonifacio et al in their original work [2]. The vertical ordinate of the gain profile $G(\tau, \Delta_{21})$ of the probe field versus pump-probe detuning $\Delta_{21}$ with parameters $\nu = 0$, $\Gamma = 1$, $\kappa = 0.01$, $\rho = 3$, $A_2 = 2$, $\Delta_{20} = -15$ and $\tau = 5$. (a) the atomic motion is totally removed from the model; (b) the motion effect is included (traditional CARL).

Fig.1 stands for a relative gain of the probe field and the horizontal ordinate $\Delta_{21}$ denotes the detuning between the pump and the probe field. However, this anti-symmetry gain profile around $\Delta_{21} = 0$ can be drastically changed (Fig.1(b)) by the atomic recoil motion which plays an important role in the amplification mechanism of the collective atomic recoil laser (CARL) [2]. As shown in Fig.1(b), the gain peak within the traditional Maday region is greatly enhanced, and which has been a signature of light amplification induced by cooperative atomic recoil effect. Being modified by the recoil motion and exchanging energies with one common pump field, the atoms will exhibit some kinds of self-organization behavior or dynamic synchronization which induces phase transitions in the relevant system. Indeed, in the traditional CARL system, the atomic density grating as well as the population inversion

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grating on the scale of the electromagnetic wavelength emerges spontaneously during the light amplification process. However, some theoretical simplicities of the traditional CARL model, as have been pointed by some authors, impede a CW gain output of the system. Therefore, the steady-state CARLs were proposed and have been experimentally demonstrated, but, with a relative weak gain. In this paper, we report a result in order to give a more detailed theoretical analysis to a modified CARL system which produces a CW laser output with a higher gain. In our previous scheme, the active cold atoms are trapped in a harmonic potential, which can be realized by loading the atoms into an optical lattice or an overall magnetic trap, instead of introducing a light molasses or adding a thermal collision/momentum damping mechanism to the atomic motion. We suppose the trap to be strong in radial direction and loose in axial direction (cigar-shaped trap), which reducing our system effectively to one-dimension. Our modification by this configuration produces a very different laser gain profile illustrated by Fig.2. Fig.2 displays a discrete spectral structure which is very similar to that of a nonlinear Jaynes-Cummings model of one atom in a high-Q cavity. When the pump-probe detuning satisfies a Raman condition $\Delta_{21} = n\nu$, where $n$ is an arbitrary integer and $\nu$ is the effective axial trapping frequency, a distinct light amplification process occurs for certain $n$ as displayed by Fig.2. In this case, a self-organization of all the atomic oscillators along the trap axis is demonstrated by atomic distribution pattern and atomic orbit bunching in the phase space in this paper. The following contains our model with a more rigorous analysis of all the above interesting results in a framework of semiclassical method.

II. SIMPLE MODEL AND GAIN SPECTRUM ANALYSIS

The relevant system is supposed to be enclosed in a ring cavity and the operating medium is composed of a collection of $N$ two-level atoms of mass $m$, moving along the axial direction $z$ of a trap described by $V_{\text{trap}} = \frac{1}{2}m[\nu_{\perp}^2(x^2 + y^2) + \nu_z^2z^2]$. The atoms are driven by a strong far off-detuned pump field propagating along $z$ and probed by a counterpropagating weak probe field. If we set $\nu_{\perp} \gg \nu_z$, the radial trapping mode will be hardly excited in the recoil process and a one-dimensional trap, $V' = \frac{1}{2}m\nu_z^2z^2$, should be a promising approximation. With above simplification, the explicit form of the Hamiltonian of this system is

$$\hat{H} = \sum_{j=1}^{N} \left( \frac{\hat{p}_j^2}{2m} + \frac{1}{2}m\nu_z^2\hat{z}_j^2 \right) + \hbar\omega_0 \sum_{j=1}^{N} \hat{\sigma}_{zj} + \hbar\omega_1\hat{a}_1^\dagger\hat{a}_1 + \hbar\omega_2\hat{a}_2^\dagger\hat{a}_2 + i\hbar \left( g_1\hat{a}_1 \sum_{j=1}^{N} \hat{\sigma}_j^z e^{-ik_1\hat{z}_j} + g_2\hat{a}_2^\dagger \sum_{j=1}^{N} \hat{\sigma}_j^z e^{+ik_2\hat{z}_j} - \text{H.c.} \right), \quad (1)$$

where the operators $\hat{p}_j$ and $\hat{z}_j$ are the canonically conjugate momentum and center-of-mass position of the $j$th atom, $\hat{\sigma}_j^z$ and $\hat{\sigma}_{zj}$ denote the usual spin operators of the atomic internal degrees of freedom with a transition frequency of $\omega_j = \omega_{0j} + \nu$ for $j=1,2$. Fig.2 shows the long time gain profile $G(\tau, \Delta_{21})$ of the probe field in a trap with frequency of $\nu = 2$ and the time $\tau = 100$. The other parameters are the same as Fig.1 for a convenient comparison.
where the atomic polarization and population damping $\Gamma$ as well as the probe field damping $\kappa$ have been phenomenologically added. The strong pump field $A_2$ is treated as a real undepleted field and removed from Eq. (2) as an external condition $[2]$. Some dimensionless variables or transformations have been introduced in above derivations, i.e., $p_j = \frac{\langle \hat{p}_j \rangle}{\hbar}$, $\theta_j = 2k \langle \hat{z}_j \rangle$, $A_i = \frac{\langle \hat{A}_i \rangle}{\hbar} e^{i \omega_{\lambda} t}$ ($i = 1, 2$), $\sigma_j = \langle \hat{\sigma}_j \rangle e^{ikz_j \hat{z}_j^2} e^{i \omega_{\lambda} t}$, $\sigma_{zj} = -2 \langle \hat{\sigma}_{zj} \rangle$, $\Delta_{21} = (\omega_2 - \omega_1)/(\omega_r \rho)$, $\Delta_{20} = (\omega_2 - \omega_0)/(\omega_r \rho)$ and $\nu = \nu_2/(\omega_r \rho)$ under the assumption $k_1 \approx k_2 = k$ and $g_1 \approx g_2 = g$, for simplicity. The scaling parameters $\omega_r$ and $\rho$ used above are two-photon recoil frequency $\omega_r = 2\hbar k^2/m$ and the dimensionless CARL parameter $\rho = \left( \frac{\hbar k}{\omega_r} \right)^2$, respectively. It should be noted that Eq. (2) are identical to those derived in the Ref. $[2, 6, 9]$ except for one single term $-\nu^2 \theta_j$ for the harmonic trapping force, and all the simulations throughout this paper will be based on these equations.

In order to analyze the unique discrete gain profile in Fig. 2, we focus on the long-time evolution of $A_1$ when the probe damping is weak so as to be enclosed in a high-finesse ring cavity $[13]$. The gain quantity depicted in Fig. 2 is defined by $G(\Delta_{21}, \tau) = \frac{\langle A_2(\tau) \rangle^2 - \langle A_1(0) \rangle^2}{\langle A_1(0) \rangle^2} \Delta_{20}$ with $A_1(0)$ being the initial value of the weak probe field. By formally integrating Eq. (2), the probe field can be expressed as

$$A_1(\tau) = A_1(0)e^{i(\Delta_{21} - \kappa)\tau} + e^{i(\Delta_{21} - \kappa)\tau} \int_0^\tau \left( \frac{1}{N} \sum_j \sigma_j(\tau') e^{-i \theta_j(\tau')} \right) e^{(\kappa - i \Delta_{21})\tau'} d\tau',$$

and the probe gain

$$G(\kappa, \Delta_{21}, \tau) = e^{-2\kappa \tau} \left[ \frac{\bar{C}(\kappa - i \Delta_{21}, \tau)}{A_1(0)} \right]^2 + 2Re \left( \frac{\bar{C}(\kappa - i \Delta_{21}, \tau)}{A_1(0)} \right) + (e^{-2\kappa \tau} - 1),$$

where $\bar{C}(\kappa - i \Delta_{21}, \tau)$ is defined by

$$\bar{C}(\kappa - i \Delta_{21}, \tau) = \int_0^\tau C(\tau') e^{(\kappa - i \Delta_{21})\tau'} d\tau'$$

with $C(\tau) = \frac{1}{N} \sum_j \sigma_j(\tau) e^{-i \theta_j(\tau)}$ being defined as a collective atomic coherence parameter by Ref. $[2]$. For an enough long time of evolution, provided $\tau \gg 2\pi/\nu$ with $\nu = \sqrt{\kappa \Delta_{20}}$ $[12]$, all the atoms in the trap will approximately perform a harmonic oscillation with different amplitudes $\theta_j(\tau)$ and phases $\phi_j(\tau)$, described by $\theta_j(\tau) \approx \theta_{j0}(\tau) \cos[\nu \tau + \phi_{j0}(\tau)]$. Then $e^{-i \theta_j(\tau')}$ in Eq. (3) will be expanded as

$$\bar{C}(\kappa - i \Delta_{21}, \tau) \approx \int_0^\tau \left[ \frac{1}{N} \sum_{j=1}^N J_n[\theta_{j0}(\tau)] e^{in[\phi_{j0}(\tau) - \frac{\pi}{2}]} \int_0^\tau d\tau' \sigma_j(\tau') e^{[\kappa - i (\Delta_{21} - n \nu)]\tau'} \right],$$

where the function $J_n[\theta_{j0}(\tau)]$ is the $n$-th order Bessel function of the first kind. In above derivations, we have assumed that the amplitude $\theta_{j0}(\tau')$ and phase $\phi_{j0}(\tau')$ are slowly varying quantities after a long time evolution and can be
moved out from the integral of Eq.(1). If we extend the upper integral limit of Eq.(6) to infinity \((\tau \to \infty)\), the integral defines a Laplace transformation of \(\sigma_j(\tau')\), i.e., \(L[\sigma_j(\tau')] = \int_0^{\infty} dr' \sigma_j(\tau') e^{-\Omega (\tau-\tau')}\), on condition that \(|\sigma_j(\tau')|\) doesn’t increase much more than \(e^r\). Generally \(\sigma_j(\tau)\) is limited by Bloch sphere of two-level atom and can be approximately expanded by a Fourier series with \(\nu\) as a fundamental frequency

\[
\sigma_j(\tau) \approx \langle \sigma_j(\tau) \rangle + \sum_{n=1}^{\infty} c_n e^{\imath n \nu \tau},
\]

where \(c_n\) is the Fourier amplitude of the \(n\)-th order harmonic components. Furthermore, if the pump-atom detuning satisfies \(\Delta_{20} \gg \Gamma\), the long-time average polarization \(\langle \sigma_j(\tau) \rangle\) will be

\[
\langle \sigma_j(\tau) \rangle \approx S_0 - \frac{\Omega \Gamma}{2(\Omega^2 + \Gamma^2 + \Delta_{20}^2)} - i \frac{\Omega \Delta_{20}}{2(\Omega^2 + \Gamma^2 + \Delta_{20}^2)},
\]

where \(\Omega = 2\rho A_2\) is the Rabi frequency of the pump field and \(S_0\) can be determined by taking an adiabatic approximation of atomic internal freedom as done in [13]. If we only take the dc term of \(\sigma_j(\tau)\) in Eq. (7) for a qualitative spectrum analysis, Eq. (6) reads

\[
\overline{G}(\kappa - i \Delta_{21}, \tau) \approx S_0 \sum_{n=-\infty}^{\infty} \left[ \frac{1}{N} \sum_{j=1}^{N} J_n[\theta_j(\tau)] e^{\imath \nu \phi_{j0}(\tau) - \frac{\tau}{\kappa}} \int_0^{\tau} dr' e^{\kappa (\tau - r') \kappa - i (\Delta_{21} - \nu \nu) \tau - 1 \kappa - i (\Delta_{21} - \nu \nu)} \right]
\]

As the probe-field damping rate \(\kappa > 0\), we have \(\Delta_{21} - \nu \nu \to 0\) \(-\kappa \kappa - i (\Delta_{21} - \nu \nu) \tau - 1 \kappa - i (\Delta_{21} - \nu \nu)\) = \(-\kappa - 1 \kappa - 1\), then the gain under resonant condition is

\[
G(\kappa, \tau) = \left( 1 - e^{-\kappa \tau} \right)^2 \left| \frac{C_0(\tau)}{A_1(0)} \right|^2 + 2 e^{-\kappa \tau} \frac{1 - e^{-\kappa \tau}}{\kappa} \Re \left( \frac{C_0(\tau)}{A_1(0)} \right) + (e^{-2\kappa \tau} - 1),
\]

where \(C_0(\tau) = \frac{1}{N} \sum_{j=1}^{N} S_0 e^{\imath \theta_{j0}(\tau)} \cos[\phi_{j0}(\tau)]\) is the asymptotic coherence parameter which is determined by the secular average atomic polarization \(S_0\) and the instantaneous atomic distribution at \(\tau\). The coefficient \(1 - e^{-\kappa \tau} / \kappa\) of Eq.(10) shows here that when the damping rate \(\kappa\) is small enough and the evolution time \(\tau \gg 1 / \kappa\), the intensity gain of the probe field will be extremely high in the neighborhood of \(\Delta_{21} = \nu \nu\), and thus results in a uniform discrete gain structure as shown in Fig.2 simulated directly on Eq.(2). Although the Raman gain condition will not be changed if the higher harmonic components of Eq.(7) is included, except for a more complicated sum formula for Eq.(9), the height of the gain peak for a specific \(n\) in Fig.2 can’t be interpreted by above analytical method. As shown by Fig.2, for example, the light amplification mostly for positive multiple of the trap frequency and approximately up to \(n = 10\) is still a problem. The further simulations only indicate that the light amplification for positive detuning is a feature owing to the red-detuned of the pump field from the atom frequency. However, this regular gain spectrum still can be developed as an efficient detection method to analyze the micro-trap potential formed by a local interference field.

III. SELF-ORGANIZATION IN PHASE SPACE

Under the Raman resonant condition \(\Delta_{21} = \nu \nu\), another ingredient that enhances the maximum gain of probe field is the synchronization of the atomic motion in the harmonic trap, which is indicated by the term of \(C_0(\tau)/A_1(0)\) in Eq.(10) and can be demonstrated through a bunching of the atomic trajectories in the phase space. To some extent, the present problem described by Eq. (2) under the adiabatic approximation is similar to the famous Kuramoto model of ensembles of coupled oscillators [2], in which the complex order parameter \(R(\tau)e^{\imath \Phi(\tau)} = \frac{1}{N} \sum_{j=1}^{N} e^{\imath \theta_j(\tau)}\) plays an equivalent role as that of the bunching parameter \(b\) in the traditional CARL system [14]. For the complex order parameter, \(R(\tau)\) measures the phase coherence of all the oscillators and \(\Phi(\tau)\) describes their average phase. A main result of Kuramoto model is: When the coupling intensity (in present model the coupling intensity is related to the Rabi frequency, \(\rho A_2\), of the pump field) exceeds a certain threshold, the incoherent oscillators are mutually synchronized and \(R(\tau)\) grows exponentially. In present case, when the resonant conditions \(\Delta_{21} = n \cdot \nu\) and \(\Delta_{20} \gg \Gamma\) are both satisfied, the long-time output gain of \(A_1\) reduces to

\[
G(\kappa, \tau) \approx \left( 1 - e^{-\kappa \tau} \right)^2 \left| \frac{S_0}{A_1(0)} \right|^2 R_0(\tau) + 2 e^{-\kappa \tau} \frac{1 - e^{-\kappa \tau}}{\kappa} \Re \left( \frac{S_0 e^{\imath \Phi(\tau)}}{A_1(0)} \right) R_0(\tau) + (e^{-2\kappa \tau} - 1),
\]
where \( R_0(\tau) e^{i\Phi_0(\tau)} = \frac{1}{n} \sum_{j=1}^{N} e^{i\theta_j(\tau) \cos[\phi_j(\tau)]} \) is the secular complex order parameter determined only by the atomic distribution in the phase space. Eq. (11) indicates that the field gain is explicitly dependent on the increase of collective atomic coherence, \( R(\tau) \). At an early stage of evolution, as the atoms incoherently scatter the pumping photon into probe mode and their spatial distribution in the trap almost keeps uniform with \( R(\tau) \sim 0 \) (Fig.3(a)), the probe field fluctuates around a small value. After a period of lethargy, a self-organization structure is established through the recoil motion modified by the trap and the atoms in phase space concentrate into several clusters (Fig.3(b)), resulting in an exponential growth of the probe field. When this process is saturated (Fig.3(c)) by a counteraction of the spontaneous damping processes \[17\], the coherent parameter reaches its maximum value, or extremely \( R(\tau) \rightarrow 1 \) (all the atoms moving synchronously as one big atom), and a stable output of \( A_1 \) will be set up. Fig.3(d), which is directly simulated from Eq. (2), displays a typical amplification process of the probe intensity under this resonant condition and, finally, establishes a saturated output with an efficiency nearly to 18% (Here the efficiency is defined by the intensity rate between pumping field and the output probe field). According to a further Fourier analysis, the fast oscillation observed in Fig.3(d) at saturation is induced by the harmonic motion of the atoms in the trap.

During the lasing process, the self-organization structure related to the light amplification corresponds to a spontaneous bunching of atomic trajectories in the phase space (Fig.4), which, actually, is similar to the atomic density grating in the free space CARL system when \( b \) is large. In order to demonstrate the spatial features of this atomic self-organization behavior, the initial spatial distribution of the atoms are set uniform in a region of \([0, \lambda]\) and the initial atomic momentum follows a Gaussian distribution (showed by Fig.3(a)) with a thermal spread of width above recoil temperature (about \( 0.8\rho \hbar k \)) \[18\]. Before the increment of \( A_1 \), the atomic trajectories are almost ubiquitous in the phase space and, after a stable output of \( A_1 \) is established, most atomic trajectories converge into the antinode area (around \( 0.25\lambda \)) and the other ones into the node area (around \( 0.5\lambda \)) of a standing wave induced by an interference between the amplified probe field and the strong pump field, with its wavelength being defined by \( \lambda = 4\pi/(k_1 + k_2) \).
FIG. 4: The atomic trajectories in phase space during (a) $500 < \tau < 510$ and (b) $1700 < \tau < 1710$ with parameters $\nu = 2$, $\Gamma = 1$, $\kappa = 0.01$, $\rho = 3$, $A_2 = 2$, $\Delta_{20} = -15$ and $\Delta_{21} = 4$.

The bunching positions of the most atomic trajectories into the antinode area are a clear result due to the red-detuned of the pump field $\Delta_{20} = -15 < 0$ in Fig.4.

According to more simulations, we find that the amplification and the pattern formation of the atoms in phase space under the Raman resonant condition are robust to other parametric values provided that the gas medium is operated on a far-off red-detuned region and the trap is strong enough [14]. Following above conditions the change of collective coupling constant $\rho$ in the resonant gain region will not qualitatively change the self-organization effect and the long time high CW gain output of this model.

IV. DISCUSSIONS

The experimental realization of above CW CARL system can be carried out by loading the laser-cooled atoms into an overall magnetic trap or into a far-off detuned optical lattice. Thereafter, the trap is displaced along its axis for about one light wavelength $\lambda$ to trigger the atomic motion, and then the atoms are pumped by a strong red-detuned field. A gain of a seed probe field in the backward direction under the resonant conditions $\Delta_{21} = 0, \pm \nu, \pm 2\nu, \pm 3\nu \cdots$ will be detected then. This process will be enhanced further by a collective behavior of the atoms due to the synchronization of their recoil motion under Raman resonant condition. The strong pump field feeds the energy to the system (excite the trapping oscillating mode by atomic recoil motion) and the dipole force, generated by the interference of the probe and pump light, bunches the atoms into several groups to adjust them synchronically extracting kinetic energy from the trap mode into the probe field [14]. This scheme will also present a clue to introduce a new configuration of free electron lasers (FEL) where the high speed electrons are injected into a (or an array of) strong quantum trap (e.g. quantum dots). By stimulating the electrons’ motion through electromagnetic field on the electrode with appropriate frequency, the high gain field with certain frequency will be generated when the electrons oscillate collectively in (or through) these quantum traps.

In conclusion, we have investigated the light amplification in a CARL with the atomic center-of-mass motion influenced by an external harmonic potential. Under appropriate parametric condition that the pump field is red-detuned from the atomic transition frequency, our results show that CARL confined by a robust trap can give rise to a CW laser with a higher lasing gain than that of the CARL in the free space. The optical gain profile as a function of the pump-probe detuning displays a discrete structure and another kind of atomic self-organization related to the light gain is demonstrated in the phase space. This work suggests a new way to promote a CW CARL device with a higher gain and, perhaps, analogically implies a new configuration of FEL.

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7

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