Sub- and Super-Synchronous Self-Excited Vibrations of a Columnar Rotor Due to Axial Clearance Flow

H. Nishimura¹, H. Horiguchi², T. Suzuki³, K. Sugiyama² and Y. Tsujimoto²

¹ School of Engineering Science, Osaka University, 1-3 Machikaneyama, Toyonaka, 560-8531 Osaka, Japan,
² Graduate School of Engineering Science, Osaka University, 1-3 Machikaneyama, Toyonaka, 560-8531 Osaka, Japan,
³ Toshiba Corporation, 2-4, Suehiro-cho, Tsurumi-ku, Yokohama, 230-0045 Kanagawa, Japan,
haruka.nishimura@flow.me.es.osaka-u.ac.jp

Abstract. Sub- and super-synchronous self-excited vibrations due to axial clearance flows were observed in a columnar rotor with an upstream seal in experiments. A smaller clearance on the downstream seal had a larger effect of stabilizing the rotor. In computations, it was found that the rotordynamic fluid force tangential to the whirling orbit, which is caused as a response to the vibrations (whirling motions), destabilizes the rotor in the case of the upstream seal and stabilizes the rotor in the case of the downstream seal. It was clarified in the 1-D flow model that the tangential rotordynamic fluid force is mainly caused by an inertia of the clearance flow.

1. Introduction

In the operation of pumps and hydro turbines, self-excited vibrations of the rotor can occur [1,2] due to fluid forces and moments [3,4]. It has been considered that one of the fluid forces is caused by the leakage flow between a front shroud and a casing. The fluid force caused as a response to vibrations is called a rotordynamic fluid force, which has large influences on the self-excited vibration of the rotor. The rotordynamic fluid forces on conical and contoured rotors modeling a front shroud of a centrifugal pump in whirling motions were measured [5,6] and the effects of a swirling flow on the fluid forces were revealed with the same experimental equipment [7,8]. However, the generation mechanism of the rotordynamic fluid force has not been clarified yet.

In centrifugal pumps, a seal is set at the downstream of the leakage flow between a front shroud and a casing. On the other hand, in francis turbines, a seal is set at the downstream or the upstream of the leakage flow. It has been indicated by a simple flow model [1] that a position of the seal is related to the occurrence of the vibrations. The rotordynamic fluid force in the case with the upstream seal can be quite different from that with the downstream seal. However, the fluid force with the upstream seal has not been investigated and its characteristics are not clear.

One of the causes of vibrations is an interference of the swirl of the leakage flow generated by the rotation of the rotor to the rotor itself. Another cause is thought to be an inertia of the leakage flow in a meridian direction. We focused on the inertia of the leakage flow and firstly performed experiments with a columnar rotor modeling a front shroud of francis turbines to investigate the effects of positions of the seal, the width of the flow passage, a pre-swirl flow, and the clearance of the seal on characteristics of vibrations in the present study. The vibrations observed in experiments were...
whirling motions. The whirling motion is caused by the rotordynamic fluid force tangential to a trajectory of the whirling motion. Therefore, computations were carried out to evaluate the rotordynamic fluid force on the whirling rotor. Moreover, the clearance flow between the rotor and the casing was modeled as an axial one-dimensional flow and the generation mechanism of the tangential rotordynamic fluid force was discussed.

2. Experimental Equipment and Methods

2.1. Free vibration equipment

Figure 1 shows the schematic of experimental equipment to observe free vibrations. The columnar rotor (No.1) is attached to the lower end of the shaft (No.3). The shaft is rotated by the main motor (No.5) and supported by two bearings, which are fixed on the sleeve (No.7). The sleeve can swing by the support of the gimbal (No.6). As the length of 1000[mm] of the shaft is much longer than the maximum lateral displacement of about 0.5[mm] of the shaft, the angular displacement of the shaft is negligible. The lower edge of the sleeve is supported by four springs (No.9), and the natural frequency of the shaft can be adjusted by the springs. Two laser displacement sensors (No.8) are used to measure the displacements of $x$, $y$-directions of the shaft in the Cartesian coordinate system having $z$-axis upward along the shaft. As the protection ring (No.10) is attached to avoid the contact of the rotor and the inner casing (No.12), the lateral displacement of the shaft is limited to about 0.5[mm].

For generating the downward flow in the clearance between the rotor and the inner casing, water as a working fluid is flowed from the upper port (No.11) to the lower port (No.14), and for the upward flow, it is inversely flowed from the lower port to the upper port. For generating a pre-swirl flow, the swirl generator (No.13) is used. Its cross-section is shown in figure 2. The swirl generator has eight inlet ports and the fluid was flowed into the test section from one of the inlet ports.

![Figure 1. The schematic of the experimental equipment.](image1)

![Figure 2. Cross-section of the swirl generator.](image2)
2.2. Test sections and experimental methods

2.2.1. Experiment of characteristics of vibrations

Figure 3 shows the details of test sections. The fluid flows between the rotor (No.1) and the inner casing (No.6). As an example, flow directions in the case of the downward flow are indicated by arrows. For the mean radial clearance $C=4\,[\text{mm}]$, we used the rotor with a diameter of $234\,[\text{mm}]$ and the inner casing with an inner diameter of $242\,[\text{mm}]$, which are shown in figure 3 (a). The heights of the rotor and the inner casing are $60\,[\text{mm}]$. The height of the seal set on the upper end of the inner casing is $5\,[\text{mm}]$, and the radial clearance $C_1$ of the seal is $1\,[\text{mm}]$. For the case of $C=8\,[\text{mm}]$, the rotor with a diameter of $230\,[\text{mm}]$ and the inner casing with an inner diameter of $246\,[\text{mm}]$, which were shown in figure 3 (b). The other specifications are the same as those of the test section shown in figure 3 (a).

At first, in order to obtain the natural frequency $f_n$ of the rotor, maximum initial displacement in $x$-direction was given to the shaft and the displacement of the shaft was measured at $Q=0\,[\text{L/min}]$ and $N=0\,[\text{min}^{-1}]$. In the case with the downward flow, the flow rate $Q$ was increased and the critical flow rate at which the self-excited vibration (the whirling motion) started to occur was investigated at each rotational speed $N$ of 0, 30, and $50\,[\text{min}^{-1}]$. The critical flow rate at which the whirling motion in the direction opposite to the direction of a spontaneously generated whirling motion started to occur was also investigated, when the initial velocity in the opposite direction was given to the rotor. The frequencies of whirling motions were also examined in every condition. In the case with the upward flow, the displacements of the shaft were observed at various flow rates and rotational speeds.

In the case of $C=8\,[\text{mm}]$, the effect of the pre-swirl flow on vibrations was also examined in the downward flow. The critical flow rates at which the whirling motion started to occur and the frequencies were investigated at $N=0\,[\text{min}^{-1}]$ in both cases that the rotational direction of the rotor was the same as or the opposite to the direction of the pre-swirl flow at $N=50\,[\text{min}^{-1}]$.

![Diagram of test sections](image-url)
2.2.2. Experiment of the downstream seal

Figure 4 shows the details of the test section with upper and lower seals. Specifications except the lower seal are the same as those of test section shown in figure 3(b). The Upper seal clearance $C_1$ is $1\text{[mm]}$, and lower seal clearances $C_2$ are $1$, $1.5$, $2$, and $3\text{[mm]}$.

To examine the effect of the clearance of the downstream seal, the experiment using the test section shown in figure 4 was conducted. The direction of the flow is downward and the rotational speed $N$ is $0\text{[min}^{-1}\text{]}$. In the cases with $C_2=1.5\text{[mm]}$ and $3\text{[mm]}$, the effect of the pre-swirl flow was also examined. As the vibrations did not occur in many cases of $Q=0\sim20\text{[L/min]}$, maximum initial displacement in $x$-direction was given to the shaft and the displacement of the shaft was measured. Figure 5 shows an example of time history of the lateral displacement of the shaft. We evaluated damping ratio $\zeta$ and the natural frequency $f_n$ of the rotor respectively defined by,

$$\zeta = \frac{1}{\pi} \ln \left( \frac{a_0 - x_0}{a_1 - x_0} \right),$$  \hspace{1cm} (2.1)

$$f_n = \frac{1}{T},$$  \hspace{1cm} (2.2)

where the amplitudes $a_0-x_0$, $a_1-x_0$ and the period $T$ are determined from the measured waveform as indicated in figure 5.
3. Experimental Results and Discussion

3.1. Characteristics of self-excited vibrations (whirling motions)

Figures 6 (a), (b) show the frequencies of the whirling motion in the experiment of the downward flow, namely the upstream seal. In the case of the mean radial clearance $C=4[mm]$, the natural frequency $f_n$ of the rotor was $1.8[Hz]$, which was measured at the condition of $Q=0[L/min]$ and $N=0[\text{min}^{-1}]$. When the flow rate was increased, the whirling motion started to occur at $Q=9.0[L/min]$ and $N=0[\text{min}^{-1}]$. This whirling frequency was lower than the natural frequency, as shown in figure 6 (a). At $N=30[\text{min}^{-1}]$, the backward whirling motion spontaneously started to occur at $Q=9.0[L/min]$, and this frequency was lower than the natural frequency. When a forward initial velocity was given to the rotor, the forward whirling motion started to occur at $Q=10.5[L/min]$. This frequency was the same as the natural frequency. At $N=50[\text{min}^{-1}]$, the backward whirling motion spontaneously started to occur at $Q=9.5[L/min]$, and this frequency was lower than the natural frequency. When the forward initial velocity was given to the rotor, the forward whirling motion started to occur at $Q=11.5[L/min]$. This frequency was higher than the natural frequency. Therefore, as the critical flow rate at which the whirling motion started to occur increased as the rotational speed increased, it was found that the whirling motion becomes harder to occur as the rotational speed increases. As the flow rate increases, the frequencies of vibrations increased. The backward whirling motion easily occurred in comparison with the forward whirling motion. At higher flow rates, vibrations (whirling motions) were observed, but their frequencies were not measured because the rotor strongly contacted with the protection ring and the frequencies were not proper as frequencies of natural vibrations.

In the case of the mean radial clearance $C=8[mm]$, the natural frequency $f_n$ of the rotor was $1.98[Hz]$. As shown in figure 6 (b), characteristics of the whirling motion in the case of $C=8[mm]$ were the same as those of the case of $C=4[mm]$.

In the case with the upward flow, namely the downstream seal, no vibrations including the whirling motion occurred. When an initial displacement was given, the vibration decreased immediately. Such a damping feature was observed in both the case of $C=4[mm]$ and $8[mm]$.

![Diagram](a) Mean radial clearance $C=4[mm]$. (b) Mean radial clearance $C=8[mm]$.

Figure 6. Frequencies of the whirling motion in the downward flow.

Figure 7 shows the frequencies of the whirling motion in the case of $C=8[mm]$ with the pre-swirl flow. In this experiment, the natural frequency $f_n$ of the rotor was $1.97[Hz]$. The whirling motion in the same direction as the direction of the pre-swirl flow spontaneously started to occur regardless of the rotational direction of the rotor. Therefore, in presence of the pre-swirl flow, whirling motion of the same direction as that of the pre-swirl flow becomes easier to occur, while that of the opposite direction becomes harder to occur. At the flow rates at which vibrations started to occur, the frequencies of the backward whirling motion were lower than the natural frequency, while those of the
forward whirling motion were higher. As the flow rate increases, the frequencies of whirling motion increased.

3.2. Effect of the clearance of the downstream seal

Figure 8 shows damping ratios estimated in the experiment of the downstream seals with the clearance $C_2$ shown in figure 4. As the value of $C_2$ decreased, the damping ratio increased. Therefore, the damping effect of the downstream seal increases as the clearance of the seal decreases. The damping ratios in the case with the pre-swirl flow are almost the same as those in the case of no swirl flow. Therefore, the pre-swirl flow does not affect the damping ratio.

Figure 9 shows natural frequencies of the rotor. As the value of $C_2$ increases, the natural frequency increases. This is thought to be caused by the decrease of an added mass.

4. Computations and 1-D Flow Model

4.1. Computations

Numerical simulations were performed to reveal rotodynamic fluid forces on the whirling rotor using ANSYS CFX 15.0. Governing equations are the equation of continuity and the Navier-Stokes equations. Shear Stress Transport model was used as a turbulent model. The computational domain is shown in figure 10. The geometry of the computational domain is the same as that of the test section.
with the upper seal and without the lower seal shown in figure 3 (a). The mean radial clearance $C$ is 4[mm]. Unlike the experiment, the computation does not treat the whirling motion as a solution to the freely rotational problem. Instead, the eccentricity of the rotor axis is prescribed to be 0.5[mm] away from a centre of the casing, and the fluid force on the rotor is obtained. The computational grid is generated by Gridgen V15. The number of the computational grid points is 3.2 million. The flow rate of 15[L/min] was given at the inlet boundary and the pressure of 0[Pa] was set at the outlet boundary. Computations were carried out for both the downward and upward flows.

The basic equations are discretized on a fixed mesh attached to the whirling body at the angular speed of $\Omega$. The adaptation of the Navier-Stokes equation, which is described on a rotating frame of reference, is taken into consideration of the fictitious force terms (i.e. the centrifugal and Coriolis force terms). In order to produce the whirling motion with the angular velocity $\Omega$, the rotor with the angular velocity $\omega$, the computational domain was rotated at the angular velocity $\Omega$ surfaces of the rotor was rotated at the angular velocity $\omega = \omega - \Omega$, and walls of the casing were rotated at the angular velocity $-\Omega$. The rotational angular velocity $\omega$ was fixed at 50[min⁻¹].

4.2. Computational results and discussion
The tangential component of the rotordynamic fluid force pointing to the direction same as (respectively opposite to) the whirling direction implies that the force has an excitation (respectively a damping) effect on the whirling motion.

Figure 11 shows the numerical results of the tangential fluid force component $F_t$ on the rotor. The symbols ○ and □ indicated the downward and upward flows, respectively (namely, upstream and downstream seal, respectively). $F_t$ for the downward (respectively upward) flow is positively (respectively negatively) correlated with the angular velocity $\Omega$, revealing that the whirling motion is likely to be excited (respectively damped).

4.3. 1-D flow model
To clarify the generation mechanism of the tangential rotordynamic fluid force on the whirling rotor, we tried a modelling of the clearance flow. At first, we consider the case with the upstream seal in the clearance between the rotor and the casing.

As shown in figure 12, the axial flow in the clearance between a whirling columnar rotor and a cylindrical casing is considered. Figure 12 (a) shows a $z$ cross-section through the seal shown in figure 12 (b), and figure 12 (b) shows a meridian cross-section. The direction of the eccentricity is taken to be a positive direction of $x$-axis. $z$-axis is taken along the center axis of the cylinder, and points to the direction of the clearance flow. The origin of $z$-axis is at the outlet of the seal. The axial length of the
Rotor is denoted by $l$. The radial seal clearance $C_{\text{seal}}$ at an angle $\theta$ from the $x$-axis is given by

$$C_{\text{seal}} = C_0 - \epsilon \cos \theta.$$  \hfill (4.1)

where $C_0$ is a circumferentially averaged seal clearance and $\epsilon$ is an amount of the eccentricity. The use of Bernoulli's principle gives the relation between the pressure decrease $\Delta p$ through the seal and the fluid velocity $v_{\text{seal}}$ in the clearance.

$$v_{\text{seal}} = \sqrt{\frac{2\Delta p}{\rho}}. \hfill (4.2)$$

For simplicity, a circumferential deviation of $\Delta p$ has been neglected here. Upon using equations (4.1) and (4.2), the local flow rate $dq$ at $\theta$ is expressed as

$$dq = C_{\text{seal}} \cdot Rd\theta \cdot v_{\text{seal}} = C_0 Rd\theta \sqrt{\frac{2\Delta p}{\rho}} - \epsilon Rd\theta \sqrt{\frac{2\Delta p}{\rho}} \cos \theta,$$  \hfill (4.3)

where $R$ is a radius of the rotor. The first term on the right-hand-side of equation (4.3) accounts for the effect of the average seal clearance $C_0$, and the second term including $\epsilon \cos \theta$ for the deviation therefrom owing to the eccentricity of shaft. The circumferential coordinate $\theta_I$ in the inertial frame shown in figure 13 is given by

$$\theta_I = \theta + \Omega t.$$  \hfill (4.4)

Substituting equation (4.4) into equation (4.3) gives the deviation $\bar{dq}$ of the local flow rate.

$$\bar{dq} = -\epsilon Rd\theta \sqrt{\frac{2\Delta p}{\rho}} \cos(\theta_I - \Omega t).$$  \hfill (4.5)

Hence, the deviation $\bar{v}$ of the flow velocity in the clearance is expressed as

$$\bar{v} = \frac{\bar{dq}}{C Rd\theta} = -\frac{\epsilon}{C} \sqrt{\frac{2\Delta p}{\rho}} \cos(\theta_I - \Omega t).$$  \hfill (4.6)

The equation of motion of the fluid ranging from $z = 0$ to $z = l$ is represented by

$$\rho \frac{d\bar{v}}{dt} = p(0) - p(l),$$  \hfill (4.7)

$$\Leftrightarrow p(0) = \rho l \frac{d\bar{v}}{dt}, \quad (\therefore p(l) = 0)$$

Here, the pressure $p_{\text{ave}}$ averaged over $0 < z < l$ is estimated approximately by the arithmetic mean, namely

$$p_{\text{ave}} = p(l) = \frac{p(0) + p(l)}{2}. \hfill (4.8)$$
Using equations (4.6), (4.7), and (4.8), we can obtain a theoretical expression of the tangential fluid force as a function of the eccentricity of shaft, namely

\[ F_t = -2\pi \int_0^{2\pi} p_{ave} \cdot \sin(\theta_l - \Omega t) \cdot IRd\theta = \frac{\rho l^2 \varepsilon \varepsilon \Omega}{2C} \frac{Q}{2C_0}, \]  

(4.9)

where \( Q \) is the average flow rate given by

\[ Q = 2\pi RC_0v_{seal}. \]  

(4.10)

In a similar manner, the tangential fluid force for the downstream seal can be derived by replacing \( p(l) = 0 \) with \( p(0) = 0 \) in equation (4.7), namely

\[ F_t = -\frac{\rho l^2 \varepsilon \varepsilon \Omega}{2C} \frac{Q}{2C_0}. \]  

(4.11)

Note that the sign of equation (4.11) for the downstream seal is opposite to that of equation (4.9) for the upstream seal, immediately implying that the force direction is dependent merely upon the seal position.

4.4. Results of the 1-D flow model and discussion

The tangential fluid forces calculated by equations (4.9) and (4.11) are shown as the lines in figure 11. The variations of \( F_t \) predicted by the 1-D model are comparable to those by the numerical simulation. Therefore, a conclusion drawn from the consistency in figure 11 is that the tangential fluid force is mainly caused by inertia of the flow in the clearance since the present model is developed within a framework of the Bernoulli principle.

5. Conclusion

The results of the present study can be summarized as follows.

1. In the case with the upstream seal, the backward whirling motion of the rotor spontaneously occurred. When initial forward velocities were given to the rotor, the forward whirling motion occurred. At the flow rates at which the whirling motion started to occur, the frequency of the backward whirling motion was lower than the natural frequency, while that of the forward whirling motion was nearly the same as the natural frequency.

2. Characteristics of the whirling motion mentioned above are not different in both cases of the mean radial clearance of the flow passage \( C=4[\text{mm}] \) and \( 8[\text{mm}] \).

3. In the case with the upstream seal, when the pre-swirl flow was given, the whirling motion in the same direction as the direction of the pre-swirl flow occurred regardless of the rotational direction of the rotor.

4. In the case with the downstream seal, the clearance flow has the damping effect on vibrations.

5. As the clearance of the downstream seal decreased, the damping effect increased. The pre-swirl flow did not have any effects on the damping ratio.
The numerical results show that tangential rotordynamic fluid force on the rotor with the upstream (respectively downstream) seal has an excitation (respectively a damping) effect on the whirling motion.

The variations of the tangential fluid force predicted by the 1-D model are comparable to those by the numerical simulation. Therefore, it was found that the tangential fluid force is mainly caused by inertia of the flow in the clearance.

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**Nomenclature**

| Symbol | Description |
|--------|-------------|
| C      | Mean radial clearance between the rotor and the inner casing |
| R      | Maximum displacement of the rotor or the radius of rotor |
| C₀     | Mean radial clearance of a seal |
| C₁     | Mean radial clearance of the upper seal |
| C₂     | Mean radial clearance of the lower seal |
| Cₛₑᵃˡ | Radial clearance of a seal |
| T      | Period of vibrations |
| ṷ      | Deviation of an axial velocity |
| vₛᵉᵃˡ | Axial flow velocity in a seal |
| ω      | Rotational angular velocity of the rotor [rad/s] |
| f      | Frequency [Hz] |
| ρ      | Density of water |
| fₙ     | Natural frequency [Hz] |
| ω'     | Relative rotational angular velocity [rad/s] |
| l      | Length of rotor |
| N      | Rotational speed of the rotor [min⁻¹] |
| Q      | Mass flow rate [L/min] |
| ζ      | Damping ratio |
| Ψ      | Whirling angular velocity [rad/s] |

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