Baryonic matter in holographic QCD

Kanabu Nawa\textsuperscript{1}, Hideo Suganuma\textsuperscript{2} and Toru Kojo\textsuperscript{3}

\textsuperscript{1}RCNP, Osaka University, Ibaraki, Osaka 567-0047, Japan
\textsuperscript{2}Department of Physics, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{3}RBRC, Brookhaven National Laboratory, Upton, NY11973, USA

We study baryons and baryonic matter in holographic QCD with D4/D8/D8\textsuperscript{-}brane system. In large-\(N_c\) holographic QCD, the baryon appears as a topologically non-trivial chiral soliton, which is called “brane-induced Skyrmion”. We also analyze the features of the baryonic matter in holographic QCD by investigating the system of single brane-induced Skyrmion on a three-dimensional closed manifold \(S^3\). We propose a new interesting picture of “pion dominance” near the critical density.

§1. Baryons in holographic QCD

“Holography” is a new concept of the superstring theory proposed by Maldacena in 1997\textsuperscript{1} as the duality between a \((p+1)\)-dimensional gauge theory and a \(\{(p+1)+1\}\)-dimensional supergravity, which are related through the \(D_p\) brane. One of the most essential properties of the holography is the “strong-weak duality” between the gauge theory and the supergravity, and the holography provides a remarkable possibility that non-perturbative aspects of one side can be analyzed by the other dual side just with the tree-level calculations. Then, if QCD is constructed on appropriate D brane configurations, non-perturbative aspects of QCD can be examined by the tree-level dual supergravity side. This is the strategy of the holographic QCD.

In 2005, Sakai and Sugimoto succeeded in constructing QCD with massless quarks and gluons on the D4/D8/D8\textsuperscript{-}brane configurations in type IIA superstring theory.\textsuperscript{2} With this model, many phenomenological properties of mesons belonging to non-perturbative QCD are uniquely derived from the tree-level dual supergravity calculations. However, since the actual classical supergravity is obtained for strong-coupling “large-\(N_c\)” QCD, baryons do not directly appear as the dynamical degrees of freedom, as a general property of large-\(N_c\) QCD.\textsuperscript{3} In this sense, there occurs a problem of how to describe the baryons in the large-\(N_c\) holographic model.

In 2006, we performed the first study of the baryon as a non-trivial topological soliton in holographic QCD.\textsuperscript{4} This topological soliton is called as a “brane-induced Skyrmion”. We derive the four-dimensional meson effective action from holographic QCD with pions and \(\rho\) mesons, considering the consistency with the ultra-violet cutoff scale of the Kaluza-Klein mass \(M_{KK} \sim 1\text{GeV}\) in the holographic approach. We obtain a stable topological soliton solution as a baryon in the holographic QCD, and investigate the baryon properties (its mass, radius and energy density distribution) starting from the superstring theory.\textsuperscript{4} (When infinite tower of the color-singlet modes are included even beyond the cutoff \(M_{KK}\), the soliton solution shrinks in the leading order.\textsuperscript{5}) We find the \(\rho\)-meson field appearing in the core region of baryons.
§2. Baryonic matter in holographic QCD

Now we consider an application of the holographic model to dense QCD. According to the non-abelian nature of QCD, various interesting phases are expected to appear at finite temperature and density, and it should be important to make clear the structure of this “QCD phase diagram” based on QCD.

However, perturbative QCD breaks down at the low-energy scale where the QCD coupling becomes strong. Even with lattice QCD numerical studies as the first principle calculation of the strong interaction, its applicability is severely restricted only near zero-density at finite-temperature in the wide QCD phase diagram, because of the “sign problem”. With these theoretical backgrounds, the holographic QCD would be a new important analytical tool starting from QCD to analyze the finite density regime of QCD. This is the aim of our study.

Here, we consider the baryonic matter with large $N_c$, because the holographic QCD is formulated as a large-$N_c$ effective theory. In the large-$N_c$ scheme of the baryonic matter, the kinetic energy, the $N$-$\Delta$ mass splitting and quantum fluctuations (apart from $O(1)$ zero-point quantum fluctuations) are $O(1/N_c)$, so that they are suppressed relative to the static baryon mass of $O(N_c)$, and the baryonic matter reduces to the static Skyrme matter. To analyze such static Skyrme matter, we take a mathematical trick proposed by Manton and Ruback. To represent the high density state of the many Skyrmion system, one unit cell shared by a Skyrmion in physical coordinate space $\mathbb{R}^3$ is compactified into a three-dimensional closed manifold $S^3$ with finite radius $R$, as shown in Fig.1. The single Skyrmion placed on the surface volume $2\pi^2 R^3$ of the manifold $S^3$ corresponds to the finite-density baryonic matter with $\rho_B = 1/(2\pi^2 R^3)$, and smaller radius $R$ of $S^3$ represents larger total baryon-number density of the matter. Then, we study the baryonic matter in holographic QCD, through the single brane-induced Skyrmion on the closed manifold $S^3$.

For the topological description of the baryon, we take the hedgehog configuration for the chiral (pion) field $U(x) = e^{i\pi(x)} \in SU(2)_A$ and the $\rho$-meson field $\rho_\mu(x)$ in the meson effective action derived from holographic QCD as

$$U^*(x) = e^{i\tau_a \tilde{x}_a F(r)} \quad \rho_0^* (x) = 0 \quad \rho_1^* (x) = \rho_1^* (x) \tilde{\tau}_a = \left\{ \varepsilon_{iab} \tilde{x}_b \tilde{G}(r) \right\} \tilde{\tau}_a, \quad (2.1)$$

with $r \equiv |x|$. The pion profile $F(r)$ is a dimensionless function with the boundary...
Fig. 2. The total energy density of single baryon on $S^3$ with various values of radius $R$ in ANW unit: (left) the brane-induced Skyrme (BIS) model; (right) the Skyrme model.

condition of $F(0) = \pi$ and $F(\pi R) = 0$ on $S^3$, which gives the topological charge equal to unity as a unit baryon number. The $\rho$-meson profile $\tilde{G}(r)$ has no such boundary conditions. By the geometrical projection from the flat three-dimensional space $R^3$ onto the surface of the closed manifold $S^3$, we obtain the energy functional of the brane-induced Skyrmion on $S^3$ for the hedgehog configuration as follows:

$$E[F(r), \tilde{G}(r)] = \int_0^{\pi R} 4\pi dr R^2 \sin^2 \frac{r}{R} \cdot \varepsilon[F(r), \tilde{G}(r)],$$

$$R^2 \sin^2 \frac{r}{R} \cdot \varepsilon[F(r), \tilde{G}(r)] = \left( R^2 \sin^2 \frac{r}{R} \cdot F'^2 + 2 \sin^2 F \right) + \sin^2 F \left( 2F'^2 + \frac{\sin^2 F}{R^2 \sin^2 \frac{r}{R}} \right)$$

$$+ 2 \left( \frac{m_\rho}{f_\pi} \right)^2 \left[ 4R^2 \sin^2 \frac{r}{R} \cdot \tilde{G}^2 \right] - (2e^2) g_{3\rho} \left[ 16R \sin \frac{r}{R} \cdot \tilde{G}^3 \right] + (2e^2) \frac{1}{2} g_{4\rho} \left[ 16R^2 \sin^2 \frac{r}{R} \cdot \tilde{G}^4 \right]$$

$$+ (2e^2) \frac{1}{2} \left[ 8 \left( 2 + \cos^2 \frac{r}{R} \right) \tilde{G}^2 + 2R \sin \frac{r}{R} \cos \frac{r}{R} \cdot \tilde{G}(\tilde{G}') + R^2 \sin^2 \frac{r}{R} \cdot \tilde{G}^2 \right]$$

$$+ (2e^2) g_1 \left[ 16 \left( F' \sin F \cdot \left( \cos \frac{r}{R} \cdot \tilde{G} + R \sin \frac{r}{R} \cdot \tilde{G}' \right) + \sin^2 F \cdot \tilde{G} / \left( R \sin \frac{r}{R} \right) \right] \right]$$

$$- (2e^2) g_2 \left[ 16 \sin^2 F \cdot \tilde{G}^2 \right] - (2e^2) g_3 \left[ 16 \sin^2 F \cdot (1 - \cos F) \tilde{G} / \left( R \sin \frac{r}{R} \right) \right]$$

$$- (2e^2) g_4 \left[ 16 (1 - \cos F) \tilde{G}^2 \right] + (2e^2) g_5 \left[ 16R \sin \frac{r}{R} \cdot (1 - \cos F) \tilde{G}^3 \right]$$

$$+ (2e^2) g_6 \left[ 16R^2 \sin^2 \frac{r}{R} \cdot F'^2 \tilde{G}^2 \right] + (2e^2) g_7 \left[ 8 (1 - \cos F)^2 \tilde{G}^2 \right],$$

where $F' \equiv \frac{dF(r)}{dr}$ and $\tilde{G}' \equiv \frac{d\tilde{G}(r)}{dr}$, and we use Adkins-Nappi-Witten (ANW) unit$^8$ for the unit of length and energy. All the coupling constants ($e$, $g_{3\rho}$, $g_{4\rho}$, $g_1$-$g_7$) in Eq.$^{2,3}$ are uniquely determined by just two experimental inputs, $f_\pi = 92.4$ MeV and $m_\rho = 776$ MeV. Such uniqueness is one of the remarkable consequences of the holographic framework.

Solving the field equations with the topological boundary condition of $F(0) = \pi$ and $F(\pi R) = 0$, we obtain the pion profile $F(r)$ and the $\rho$-meson profile $\tilde{G}(r)$ as
the hedgehog soliton solution for the energy functional (2.2) at each $R$. In Fig. 2, we show the $R$-dependence of the total energy density of single Skyrmion on the manifold $S^3$ for the brane-induced Skyrme (BIS) model, together with the result in the Skyrme model (without $\rho$ mesons). As the radius $R$ of $S^3$ decreases, i.e., as the baryon number density $\rho_B = 1/(2\pi^2 R^3)$ increases, there appears delocalization of the energy density of the baryon in both models. We find the “delocalization phase transition” into the uniform phase at $R = R_{\text{BIS crit}} = 1.19$ for the BIS model and at $R = R_{\text{Skyrme crit}} = \sqrt{2}$ for the Skyrme model. Such delocalization phase transition is expected to relate to the deconfinement in the presence of baryons. It is also related to the chiral restoration in the bulk hadronic matter in terms of the spatially-averaged chiral condensate as the global order parameter of the chiral symmetry.\(^6,9\)

Taking the two experimental inputs, $f_\pi = 92.4\text{MeV}$ and $m_\rho = 776\text{MeV}$, we get the critical density of the delocalization phase transition as $7.12\rho_0$ for the BIS model and $4.26\rho_0$ for the Skyrme model.\(^6\) For the BIS model, the heavy $\rho$ meson appearing in the core region of the baryon is to provide the attraction with the pion field, which leads to the shrinkage of the total size of the baryon.\(^4\) Owing to the shrinkage of the baryon due to $\rho$ mesons, larger baryon number density is needed for the BIS model to give the delocalization phase transition.

In Fig. 3, we compare the total energy density of the brane-induced Skyrmion and each contribution from the $\rho$-meson interaction terms in Eq. (2.3) for $R = 4.0$, $\sqrt{2}$, and 1.19. For $R = 4.0$ and $\sqrt{2}$, the $\rho$-meson field appears in the core region of the baryon. On the other hand, at $R = R_{\text{BIS crit}} = 1.19$, the $\rho$-meson field and its contributions disappear in the uniform phase.

We conjecture that such disappearance of the $\rho$-meson field in high density phase around the critical density can be generalized to all the other (axial) vector mesons even including the heavier mesons, $\rho'$, $\rho''$, $\cdots$, denoted by the field $B^{(n)}_\mu(x)$ with the mass $m_n$ ($m_1 < m_2 < \cdots$), by the following reasons:

1) The kinetic term of the (axial) vector meson field $B^{(n)}_\mu$ appears on $S^3$ as $\frac{1}{2} \text{tr} \left( \partial_\mu B^{(n)}_\nu - \partial_\nu B^{(n)}_\mu \right)^2 \propto R^{-2}$. Therefore, the spatial variation of the field $B^{(n)}_\mu(x)$ is suppressed for small $R$, i.e., at high density.

2) The mass term $m^2_n \text{tr} \{ B^{(n)}_\mu B^{(n)}_\mu \}$ suppresses the absolute value of the $B^{(n)}_\mu$ field because of its large mass $m^2_n$.

3) The coupling between pions and heavier (axial) vector mesons $B^{(n)}_\mu$ with larger index $n$ is found to become smaller both in the phenomenological dimensional deconstruction model\(^10\) and in the holographic QCD.\(^3\) Hence, the effect of heavier (axial) vector mesons becomes smaller for the baryon as the chiral soliton, which basically consists of a large-amplitude pion field.

These considerations 1), 2), 3) would support our conjecture. In other words, only the pion field survives in the baryonic matter near the critical density, which we call “pion dominance”. The effect of (axial) vector mesons only appears through their interaction with the pion field, which affects the actual value of the critical density.

Note that the pion field cannot disappear because of the boundary condition, $F(0) = \pi$ and $F(\pi R) = 0$ to maintain the unit baryon number on each closed manifold $S^3$. On the other hand, there is no constraint for the (axial) vector meson
fields, and they can disappear in high density phase. Such pion dominance would be somehow related with the importance of chiral soft modes in the strong-coupling QGP (sQGP) at finite temperature in Refs. 11) and 12).

In summary, we have studied baryons and baryonic matter in holographic QCD. We have investigated the baryonic matter in terms of single brane-induced Skyrmion on a three-dimensional closed manifold $S^3$, and have found a new interesting picture of “pion dominance” near the critical density.

Fig. 3. The total energy density and each contribution from the $\rho$-meson interaction terms in Eq. (2.3) for the single brane-induced Skyrmion on $S^3$ with the radius (a) $R = 4.0$, (b) $R = \sqrt{2}$, and (c) $R = 1.19$ in ANW unit. The vertical dashed line in (b) and (c) denotes $r = \pi R$, the maximal value on $S^3$. For $R = R_{\text{BIS}} = 1.19$, the total energy density becomes a uniform distribution as the identity map, where the $\rho$-meson field disappears and only the pion field survives, indicating “pion dominance”.

Note Added: After completing this study, we noticed a related paper,13) where the authors used instantons on the D8 brane in the Wigner-Seitz approximation.

Acknowledgement: We thank YITP at Kyoto University for useful discussions during the Int. Symp. on “Fundamental Problems in Hot and/or Dense QCD”.

References
1) J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998).
2) T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005); ibid., 114, 1083 (2006).
3) G. ’t Hooft, Nucl. Phys. B 72, 461 (1974); B 75, 461 (1974).
4) K. Nawa, H. Suganuma and T. Kojo, “Baryons in holographic QCD”, hep-th/0612187.
5) H. Hata, T. Sakai, S. Sugimoto and S. Yamato, Prog. Theor. Phys. 117, 1157 (2007); D. K. Hong, M. Rho, H.-U. Yee and P. Yi, Phys. Rev D 76, 061901 (2007).
6) K. Nawa, H. Suganuma and T. Kojo in preparation.
7) N. S. Manton and P. J. Ruback, Phys. Lett. B 181, 137 (1986).
8) G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B 228, 552 (1983).
9) H. Forkel et al., Nucl. Phys. A 504, 818 (1989).
10) D. T. Son and M. A. Stephanov, Phys. Rev. D 69, 065020 (2004).
11) T. Hatsuda and T. Kunihiro, Phys. Rev. Lett. 55, 158 (1985).
12) C. DeTar and J. Kogut, Phys. Rev. Lett. 59, 399 (1987).
13) K.-Y. Kim, S.-J. Sin and I. Zahed, arXiv:0712.1582 [hep-th].