A mathematical model for predicting the indoor moisture variation by using moisture buffering theory

Kan Zu¹ and Menghao Qin¹

¹ Department of Civil Engineering, Technical University of Denmark, Lyngby, 2800, Denmark
Corresponding email: menqin@byg.dtu.dk

Abstract: Indoor air humidity evaluation plays an of great importance role on the thermal comfort and building energy consumption. The utilization of hygroscopic materials as building materials acts on the indoor air humidity by regulating its humidity fluctuations, and then reduces a certain fraction of energy consumption on the air conditioning systems. Based on the Fick’s law, the physical process inside these hygroscopic materials requires the determinations of hygrothermal properties, which signify the extensive and reiterative experiments. While in many building simulation toolboxes, moisture buffering behavior has been evaluated by either simple approximations or complicated heat and mass model. In this case, we developed a mathematical model about the moisture transport with acceptable solution time and accuracy in terms of the moisture buffer value (MBV) theory. Considering that MBV originally represents the moisture buffering capacity of those hygroscopic materials, we did some mathematical deduction about MBVs under different boundary conditions. Then the definition of time-average MBV has been used, and all the required parameters was obtained from the practical MBV test. By comparing the new moisture buffer value model (MBM) with HAMT model, the results in indicated that MBM could provide reasonably accurate prediction for indoor moisture variation.

1. Introduction

In the past decades, more and more buildings are supplied with heating, ventilation and air conditioning (HVAC) systems to reach the desired thermal comfort of the built environment, and the applications of these systems are estimated to consume more than 40% of total building energy consumption [1–3]. The most commonly employed system is the vapor-compression type air-conditioner, which features having a low coefficient of performance (COP) due to the sub-cooling process, and the extreme weather (i.e. humid and hot climate) may even worsen its operation states.

To mitigate the indoor moisture fluctuation readily within a desired range, some research have indicated that a passive technology can reach the goal by taking advantage of the moisture buffering effect of the hygroscopic materials. In 2005, Rode et al. [4] have proposed the concept of moisture buffer value (MBV) to characterize the moisture buffering capacity of the hygroscopic materials used in the built environment. By using these materials, energy saving reported can approach to 19.57% [5]. Although the definition and test protocols of MBV are widely used and performed [1, 6–8], little research [9] have reported its application in the building simulation, and the drawbacks of MBV theory (ideal) can be concluded as: 1) Square-wave moisture variation (rare in the real conditions); 2) Ignorance of the surface resistance. In this regard, it is required to build the connection between the MBV and the surroundings, and this can be achieved by the Fourier transformation, which can decompose wave equations into the sum of trigonometric functions.
Herein, a mathematic deduction has been conducted to achieve the transformation of square-wave moisture variation to the harmonic-wave one, and the measurement of practical MBV can lump the surface resistance into the fitting parameters to obtain the lumped total vapor transfer coefficient. A time-average MBV has been defined and then revised by the lumped coefficient. Finally, the developed moisture buffer value model (MBM) has been integrated into the calculation of indoor vapor balance to predict the indoor moisture variation.

2. Methods

2.1. MBV tests
By referring to NORDTEST protocol test, the test sample with 26mm of diameter and 20mm of thickness was prepared, and all sides of the sample were sealed except the top side exposed to the surrounding air in the climate chamber. The climate chamber has a constant temperature at 23°C. Fig.1 shows the schematic of the climate chamber. The RH level of the chamber can be regulated through the proportional mixing of dry and saturated air. The temperature and humidity was measured by the built-in sensor, and then the measuring signal was sent to the PID regulator to achieve the precise hygrothermal control. The fan here provided an even temperature and humidity distribution by stirring the chamber air.

Prior to the test, the test sample was first dried at 50°C in the oven, and then cooled to the ambient temperature in the climate chamber. During the MBV tests, the sample was hung on a saucer connected to the balance, and the measured weight of the sample against was recorded every minute in the computer. As for the square wave humidity variation, the test cycle is made of 12h of high RH (75%) followed by 12h of low RH (33%). While for the harmonic wave humidity variation, the RH varies in the range between 33% and 75%. The test ran for several cycles until the equilibrium state was reached.

Figure 1 Schematic of the climate chamber.

Figure 2. Variation of RH in square wave and harmonic wave.
2.2. Mathematical deduction

2.2.1. MBV under a square wave of RH (Ideal MBV) According to the NORDTEST protocols, the ideal MBV represents the mass variation of the test sample with a cyclic RH variations, which is usually a $\gamma T$ of RH$_{high}$ and $(1-\gamma)T$ of RH$_{low}$. The given boundary conditions can be written as:

$$RH_0 = \left\{ \begin{array}{ll} RH_{high} & , 0 < t < \gamma T \\ RH_{low} & , \gamma T < t < T \end{array} \right. \quad (1)$$

According to the theory of Fourier transformation, a square wave function, i.e. Eq.(1), can be calculated through the sum of sines and cosines functions. Thus, the square-wave expression in Eq. (1) can be expressed as:

$$RH_0 = C_0 + \sum_{n=1}^{\infty} \left( A(t) \cdot \cos\left(\frac{n\pi t}{T}\right) + B(t) \cdot \sin\left(\frac{n\pi t}{T}\right) \right) \quad (2)$$

$$C_0 = \gamma RH_{high} + (1-\gamma)RH_{low} \quad (3)$$

$$A(t) = \frac{RH_{high} - RH_{low}}{n\pi} \left( \sin(\gamma n\pi) - \sin(1-\gamma)n\pi \right) \quad (4)$$

$$B(t) = \frac{RH_{high} - RH_{low}}{n\pi} \left( 1 + \cos(n\pi) - \cos(\gamma n\pi) - \cos((1-\gamma)n\pi) \right) \quad (5)$$

Based on the harmonic function items in Eq.(2), the moisture flux over the interface between the material and air is presented as:

$$\phi_h(t) = \sqrt{\frac{\pi}{T}} \delta \rho \xi \sum_{n=1}^{\infty} \left( A(t) \cdot \cos\left(\frac{n\pi t}{T} + \frac{\pi}{4}\right) + B(t) \cdot \sin\left(\frac{n\pi t}{T} + \frac{\pi}{4}\right) \right) \quad (6)$$

Considering that the hygroscopic material will approach to the equilibrium state after several cycles, the amount of the uptake equals to that of release. In this regard, the ideal MBV under a square-wave RH variation can be evaluated as:

$$MBV_{ideal} = \frac{1}{RH_{high} - RH_{low}} \int_0^{\gamma T} \phi_h(t) dt = 0.0127(\gamma(1-\gamma))^{0.535} \sqrt{\delta \rho \xi T} \quad (7)$$

2.2.2. MBV under a harmonic wave of RH Without the assistance of mechanical systems, it is hard to keep a constant RH in the built environment, and the RH will vary more likely in a fluctuate wave but not in a piecewise function, which means that ideal MBV is improper in this case. Thus, to build the relationship between MBV under square-wave RH and MBV under climate conditions, some mathematical deduction was performed. The transformation from square-wave to harmonic-wave RH is shown in this section. By referring to the square-wave moisture variation, the assumed boundary conditions can be expressed as:

$$RH_0 = \left\{ \begin{array}{ll} \overline{RH} + (RH_{high} - \overline{RH}) \sin(\frac{\pi}{\gamma T}), 0 < t < \gamma T \\ \overline{RH} + (RH - RH_{low}) \sin(\frac{\pi}{(1-\gamma)T}), \gamma T < t < T \end{array} \right. \quad (8)$$

The moisture flux of adsorption process with harmonic change can be expressed as:

$$\phi_{0, ad}(t) = \left( RH_{high} - \overline{RH} \right) \sqrt{\frac{\pi \delta \rho \xi}{\gamma T}} \cdot \sin\left(\frac{\pi t}{\gamma T} + \frac{\pi}{4}\right) \quad (9)$$
\[ \phi_{0,de}(t) = \frac{(RH - RH_{low})}{\sqrt{1 - \gamma}^T} \left( \frac{\pi \delta \rho \xi}{(1 - \gamma)^T} \sin \left( \frac{\pi (t - \gamma T) + \pi}{4} \right) \right) \]  

(10)

After several cycles, both the moisture uptake and release reach to the equilibrium state, which means there is the same net change of the accumulated moisture mass in the process of adsorption and release.

\[
\int_0^{\gamma T} \phi_{0,ad}(t) dt = \int_0^{\gamma T} \phi_{0,de}(t) dt \rightarrow \frac{RH_{high} \cdot \sqrt{\gamma + RH_{low} \cdot \sqrt{1 - \gamma}}}{\gamma + \sqrt{1 - \gamma}}
\]

(11)

Thus, the MBV\textsubscript{har} can be written as:

\[
MBV_{\text{har}} = \frac{1}{RH_{high} - RH_{low}} \int_0^{\gamma T} \phi_{0,ad}(t) dt = 0.888 \frac{(\gamma(1 - \gamma))^{-0.035}}{\sqrt{\gamma + \sqrt{1 - \gamma}}} MBV_{\text{ideal}}
\]

(12)

In Eq. (12), it can be found that the correlation between MBV\textsubscript{har} and MBV\textsubscript{ideal} depends on the ratio factor \( \mu \), which is a function of time interval \( \gamma \).

\[
\mu(\gamma) = 0.888 \frac{(\gamma(1 - \gamma))^{-0.035}}{\sqrt{\gamma + \sqrt{1 - \gamma}}}
\]

(13)

2.2.3. *Time-average MBV*  
Assumed that the cycle is divided as multiply of \( \Delta t \), the time-average MBV at time \( t_1 \) here has been defined as the unit time amount of water vapor exchanged during the adsorption or release process:

\[
MBV_t = \frac{1}{\Delta t \cdot RH} \int_{n \Delta t}^{t + \Delta t} \phi_0(t) dt
\]

(14)

3. Results

In figure 3, the measured isotherm of MIL-100(Fe) indicates its remarkable adsorption capacity owing to the even intrinsic crystal structure, which also shows 0.55 g/g of water uptake at 0.75P/P0. Theoretically, the maximum net change in the uptake can approach to 0.09 g g\(^{-1}\) in the RH range between 33% and 75%. According to the correlation between MBV\textsubscript{har} and MBV\textsubscript{ideal}, figure 4 shows the variation of the ratio factor along the time interval \( \gamma \), and the minimum \( \mu(\gamma) \) is supposed to locate at the 0.5 cycle (24h as a cycle).

**Figure 3.** Isotherms of MIL-100(Fe) at 298K.  
**Figure 4.** Variation of the ratio factor \( \mu \) against \( \gamma \).
Table 1. Measured parameters under square and harmonic wave moisture variation.

|         | Square             | Harmonic           |
|---------|--------------------|--------------------|
| $\delta_{lumped}$ ($\times 10^{-7}$kg/m/s) | 1.52±0.12          | 1.69±0.29          |
| MBV$_{measured}$ | 11.9±0.33         | 8.3±0.31           |
| MBV$_{calculated}$ | 13.1±0.51         | 9.1±0.75           |
| MBV$_{ideal}$    | 20.8±0.73          | 13.7±0.62          |

In order to calculate the vapor transfer coefficient inside the moisture flux (i.e. Eqs.(3) and (6)), the variation of moisture flux under different climate conditions is measured as shown in figure 5. From figure 5(a), it is clear that a sharp change appeared around the switching point (75%→33% or 33%→75%). While in figure 5(b), it exhibits a continuous change in the moisture flux under a harmonic-wave vapor concentration variation, which is more practical for the real condition. Based on the measured moisture flux, the lumped total vapor transfer coefficient can be obtained by fitting with Eqs. (3) and (6). The measured $\delta_{lumped}$ and the corresponding MBV under different climate conditions were shown in Table 1. Here, it can be found that the MBV using the lumped total vapor transfer coefficient verges on the measured MBV.

Figure 5. Moisture flux measured under a 24h of cyclic vapor concentration variation. a) Square; b) Harmonic.

4. Discussion

According to the theory of moisture buffer capacity, the value of MBV for the hygroscopic materials can affect the indoor moisture balance, which indicates adsorbing the extra vapor out of the indoor air or releasing the adsorbed water vapor to the indoor air based on the indoor vapor concentration. Ideal MBV is not suitable to the building simulation due to the ignorance of the surface resistance and the environmental effect, and is much larger than the practical one (Table 1), thus can only be used to characterize the material properties. However, the time-average MBV obtained from the practical MBV
tests can build the connection between the material and the surroundings. It is known that the moisture buffer behavior at the room level is directly related to the building materials, moisture load, indoor air exchange and indoor air conditions etc. By inserting the time-average MBV into the vapor balance equation, the indoor vapor balance can be simplified as:

\[
\frac{dC_i}{dt} = a(C_0 - C_i) + \frac{G}{V} + \frac{A}{V} MBV \cdot \Delta RH
\]  

(15)

Where \(a\) is the air exchange rate [h\(^{-1}\)]. \(G\) is the moisture gain inside the building [kg h\(^{-1}\)]. \(A\) and \(V\) are the surface area contacting with the indoor air [m\(^2\)] and indoor volume [m\(^3\)], respectively.

The simulation uses the BESTEST benchmark building as the reference presented in figure 6. And the conditions of the simulation was shown in Table 2.

**Table 2. Parameters adopted in the simulation.**

| Parameter                        | Values                      |
|----------------------------------|-----------------------------|
| Room size                        | 8mx6mx2.7m                  |
| Surface area of the panel         | 12 m\(^2\)                  |
| \(\delta_{lumped}\)              | 1.6\times10^{-7} kg/m/s     |
| Air exchange rate                 | 0.5 1/h                     |
| Initial indoor air concentration  | 6.15g/m\(^3\)              |
| Moisture gain                     | 0.5kg/h (between 9:00 am and 5:00 pm) |

![Figure 6. Schematic of BESTEST benchmark building.](image-url)
It is known that the HAMT model considering the complete coupling of heat and mass transfer has been validated though the experiments and simulations. It is therefore assumed as the correct results calculated from HAMT model. Figure 8 shows that MBM can well predict the trend of the indoor moisture variation under the given operation conditions. The accuracy was quantified by comparing the fraction of error distribution, which shows that the results from MBM mainly centered within ±2.5%. As for the analysis on the computing time, MBM is still based on the linear functions, which means that its computing time is the same order of magnitude to that of EC model, and much faster than HAMT model. In this regard, MBM model modified by the lumped total vapor transfer coefficient can provide a fast and reasonable predicting results for the indoor humidity level.

5. Conclusion
Hygroscopic materials used as moisture buffering materials can play an important role on the control of indoor latent load. According to the theory of moisture buffer value (MBA), a new moisture prediction model (MBM) has been developed. Firstly, the mathematical deduction has been conducted to show the feasibility of transforming moisture variation functions from square-wave type (used in the ideal MBV) to harmonic-wave type. Secondly, the classic MBV tests have been conducted to lump the surface resistance into the vapor transfer coefficient through the fitting calculation. By using this lumped total vapor transfer coefficient, the time-average MBV been employed and then integrated into the vapor balance. Finally, the simulated results demonstrated that MBM model could provide a fast and reasonable prediction in comparison with other moisture prediction models (i.e. EC, EMPD and HAMT) [10, 11]. Although the MBM is still derived from the isothermal conditions, the accuracy of this model is guaranteed. It is promising to examine the MBM by inserting into the building simulation if the temperature effect and even more complicated air conditions are considered in the future work.

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