Spin waves in quasi-equilibrium spin systems.

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Using the Landau Fermi liquid theory we have discovered a new regime for the propagation of spin waves in a quasi-equilibrium spin systems. We have determined the dispersion relation for the transverse spin waves and found that one of the modes is gapless. The gapless mode corresponds to the precessional mode of the magnetization in a paramagnetic system in the absence of an external magnetic field. One of the other modes is gapped which is associated with the precession of the spin current around the internal field. The gapless mode has a quadratic dispersion leading to some interesting thermodynamic properties including a $T^{3/2}$ contribution to the specific heat. We also show that these modes make significant contributions to the dynamic structure function.

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There has been much renewed interest in studying non-equilibrium (NEQ) spin polarized paramagnetic Fermi liquids and gases. The NEQ spin system can be defined as a polarized spin system which can be obtained by maintaining an unequal population of different spin species to create an off-balanced Fermi level in the absence of an external field. This is both an old subject\textsuperscript{[1, 2]}, and a rather new one\textsuperscript{[3, 4, 5, 6, 7]}. The NEQ spin systems have been created in a wide variety of materials using many novel techniques. An example of the most recent one would be the creation of steady state spin polarization in a system of cold and ultra-cold Fermi gases of alkali metals. In these systems one uses optical trapping and atomic manipulation to get unequal populations of the fermions in two different states\textsuperscript{[8, 9, 10, 11]}. The NEQ spin systems have also been created in many materials which comprise the study of one of the most promising areas of physics, namely spintronics\textsuperscript{[12, 13, 14, 15, 16]}. In this area of study a couple of methods are in practice to achieve the NEQ state of these spin systems.

Widely used methods for achieving NEQ states are optical pumping and polarized spin injection. In the optical pumping method\textsuperscript{[12]}, electron orbital momentum is directly oriented by a strongly circularly polarized laser light and through the spin-orbit interaction the electron spin is polarized. In the polarized spin injection\textsuperscript{[10]} method one connects a magnetic electrode to a normal metal sample and drives a polarized current using external biasing which creates unequal population of spins in the normal metal. In such a ferromagnet - metal bilayer structure, it has been suggested that if the direction of the magnetization in the ferromagnet is made to precess (by applying an external magnetic field on the ferromagnet) we can inject polarized spin into the normal metal\textsuperscript{[11]}. It has also been suggested in the literature that we can use the interaction of the electron spin with the strain-induced potential to create the NEQ system in a nano-mechanical system\textsuperscript{[16, 17]}. The NEQ system of polarized liquid helium is one example of an old but still interesting subject\textsuperscript{[18, 19, 20]}. The rapid melting method suggested by Castaing and Nozieres\textsuperscript{[21, 22, 23]}, and the optical pumping\textsuperscript{[24, 25, 26]} were widely used to create the NEQ systems of liquid helium.

In this Letter we are interested in studying the thermodynamics and the spin dynamics of the NEQ systems in the limit of small spin polarization. We know that for a paramagnetic spin system of free fermions in the presence of an external magnetic field, the magnetization, $m$, precesses around the applied magnetic field at the Larmor frequency. For a small transverse perturbation $\delta B = B_x \hat{x} + B_y \hat{y}$, and defining $\delta B^\pm = B_x \pm i B_y$ and $m^\pm = m^x \pm im^y$, the evolution of $m$ for non interacting Fermi system can be written as\textsuperscript{[27, 28]},

$$\frac{\partial m^\pm}{\partial t} = 2im^\pm \gamma B$$

where $m = m_0 e^{-i\omega_0^+ t}$ and $\omega_0^\pm = \pm 2\gamma B$ is the Larmor frequency and $B$ is the external magnetic field. The fact that the magnetization precesses around the external field with the Larmor frequency also holds true for a system of interacting fermions, with spin conserving interactions. Now, for the NEQ systems, since the external field is zero, the precessional mode should be gapless. To understand the details of the dispersion of the gapless mode we need to study the thermodynamics and the dynamics of a Fermi liquid away from equilibrium.

The general theory of Fermi liquids and gases far from equilibrium is beyond our current understanding. We will develop here the theory for weakly spin polarized paramagnetic Fermi liquids with $B = 0$. In this limit we can argue that the Fermi liquid will be in a quasi-equilibrium(QEQ) state. It is well known that the polarization dependent chemical potential has the form $\mu(m) = \mu(0) + O(m^2)$\textsuperscript{[4, 17]}, hence for weak polarization, i.e. for $\frac{m}{n} \ll 1$ where $n$ is the particle density, $\mu(m) \approx \mu(0)$. This suggests that for small $m$ the system will be in a QEQ phase and we can study the dynamics of the spin system around this phase. We know that the variation of the energy per-unit volume due to a variation in the distribution function from the ground state is written as\textsuperscript{[4, 15, 16]},

$$\varepsilon = \sum_{\rho \sigma} \varepsilon_{\rho \sigma}^0 \delta n_{\rho \sigma} + \sum_{\rho \sigma \rho' \sigma'} f_{\rho \rho' \sigma \sigma'} \delta n_{\rho \sigma} \delta n_{\rho' \sigma'} + \ldots$$


So the quasi-particle energy can be written as:
\[ \varepsilon_{pa} = \varepsilon_{p}^{0} + \sum_{p' \sigma'} f_{pp'}^{\sigma \sigma'} \delta n_{p' \sigma'} + \ldots \] (3)
where for a spin conserving interaction with \( \frac{n}{m} \ll 1 \),
\[ f_{pp'}^{\sigma \sigma'} \approx f_{pp'}^{\sigma} + f_{pp'}^{\sigma} \sigma' \cdot \sigma' \] (4)

Using Eq. (3) and expanding \( f_{pp'}^{\sigma} \) in Legendre polynomials we can derive that the chemical potential of the up and the down spins will have the forms \( \mu = \varepsilon_{p}^{0} + \frac{f_{pp}^{0} m}{N(0)} + O(m^{2}) \) and \( \varepsilon = \varepsilon_{p}^{0} - \frac{f_{pp}^{0} m}{N(0)} + O(m^{2}) \), where \( N(0) = \frac{2m}{m} + \frac{\pi}{2} \), \( \mu = \varepsilon_{p}^{0} - \frac{f_{pp}^{0} m}{N(0)} + O(m^{2}) \) is the \( m = 0 \) density of states of the quasi-particles at the Fermi level. So in leading order in \( m, \varepsilon_{p}^{0} - \varepsilon_{F}^{0} = \frac{-2F_{0}^{\alpha}}{N(0)} \).

For the EQ system, \( \mu = \varepsilon_{p}^{0} - B - \frac{f_{pp}^{0} m}{N(0)} + O(m^{2}) \) and \( \mu = \varepsilon_{p}^{0} + B - \frac{f_{pp}^{0} m}{N(0)} + O(m^{2}) \). Again in leading order in \( m, \varepsilon_{p}^{0} - \varepsilon_{F}^{0} = 2B - \frac{2f_{pp}^{0} m}{N(0)} \). Contrary to the EQ system, we see that the change in the Fermi energy of the up spin and the down spin fermions of the QEQ system depends explicitly on the Fermi liquid parameter \( F_{0}^{\alpha} \). For the QEQ and EQ systems with a majority of up spins, we have shown, in Fig.1a, the change in the Fermi energy of the up spin \( (\varepsilon_{p}^{0}) \) and the down spin \( (\varepsilon_{F}^{0}) \) fermions, and in Fig.1b, the energy spectrum of these spin species. Here \( B_{eff} \) is the effective field. For the EQ system \( B_{eff} = \frac{mF_{0}^{\alpha}}{N(0)} \) and for the EQ system \( B_{eff} = \frac{mF_{0}^{\alpha}}{N(0)} \). For a QEQ system, Fig.1a corresponds to a steady state with constant number density which can be obtained, for example, either by taking out some down spins and putting in equal number of up spins or by flipping the down spin over. To maintain such steady states we need to continuously supply energy to the system (e.g., continuously shining light in the system in the optical pumping method) against the loss in the polarization due to spin flip scattering mechanism. In what follows we want to fix the number density of the Fermions to avoid any complication that would be introduced due to the coupling between the charge density fluctuations and the spin density fluctuations.

To study the collective excitations of the QEQ system we investigate the oscillations of the transverse component of the magnetization \( \delta \mathbf{m}_{p} \). The linearized kinetic equations for the evolution of \( \delta \mathbf{m}_{p} \) can be written as \([12, 16, 17]\):

\[ \frac{\partial \delta \mathbf{m}_{p}}{\partial t} + \mathbf{v}_{p} \cdot \nabla (\delta \mathbf{m}_{p}) - \frac{\partial n_{p}}{\partial t} \mathbf{h}_{p} \delta \mathbf{m}_{p} = -2(\delta \mathbf{m}_{p} \times \mathbf{h}_{p} + \delta \mathbf{m}_{p} \times \mathbf{h}_{p}) + \mathcal{I}[m_{p}] \] (5)

where \( \delta \mathbf{h}_{p} = -\delta \mathbf{B} + \sum_{p'} f_{pp'}^{\sigma} \delta m_{p'} \) is the fluctuation in the effective magnetic field and \( \mathcal{I}[m_{p}] \) is the collision integral. The equilibrium field \( \mathbf{h}_{p}^{0} = -B + \sum_{p'} f_{pp'}^{\sigma} m_{p'} \). For the QEQ system \( B = 0 \). We can use the relaxation time approximation for the collision integral
\[ \mathcal{I}[m_{p}] = -\frac{\delta m_{p}^{0}}{T_{D}} + \mathcal{I}_{Q}^{0} \] (6)

which gives the net magnetization conservation law.

For the \( l = 0 \) moment, which gives the motion of the magnetization current, we get,
\[ \omega \nu_{0}^{\pm} - \frac{1}{3}(1 + F_{0}^{\alpha}) \nu_{0}^{\pm} = 0 \] (7)

which can be negligible for low \( q \). We also find that the exclusion of the \( l = 2 \) moment also does not change the important aspect of the dispersion. To see the main features of the modes we will first work with the projection of the \( l = 0 \), and \( l = 1 \) moments and truncate the distortions of the Fermi surface, \( \nu_{l}^{ \pm } \), and the Landau parameters, \( F_{l}^{\alpha} \), for \( l \geq 2 \).

For the \( l = 0 \) moment, we get,
\[ (\omega + \frac{\nu_{0}^{\pm}}{3} + q_{F} \nu_{0}^{\pm}) = 0 \] (8)

\[ \nu_{0}^{\pm} - \nu_{0}^{\pm} (1 + \frac{F_{0}^{\alpha}}{3}) = 0 \] (9)

\[ (\omega + \nu_{0}^{\pm} ) - q_{F} (1 + F_{0}^{\alpha}) = 0 \] (10)

\[ (\omega + \nu_{0}^{\pm} ) = q_{F} (1 + F_{0}^{\alpha}) \] (11)

\[ (\omega + \nu_{0}^{\pm} ) = q_{F} (1 + F_{0}^{\alpha}) \] (12)

\[ \omega \nu_{0}^{\pm} - \frac{1}{3}(1 + F_{0}^{\alpha}) \nu_{0}^{\pm} = 0 \] (13)

\[ \omega \nu_{0}^{\pm} - \frac{1}{3}(1 + F_{0}^{\alpha}) \nu_{0}^{\pm} = 0 \] (14)
where \( \omega_1^\pm = \pm \frac{2m}{\Delta N(0)}(F_0^a - F_1^a) \) is related to the internal field due to the interacting fermions.

Solving the Eqs. (10, 11), we get,
\[
\omega(\omega + \omega_1^\pm) - c_2^2 q^2 = 0
\]
\[(9)\]

where \( c_2^2 = \frac{1}{3}(1 + F_0^a)(1 + F_2^a)\nu_\sigma^2 \). We solve Eq. (9) for \( \omega \).

The solutions are:
\[
\omega_0^\pm(q) = -\frac{c_2^2 q^2}{\omega_1^\pm}
\]
\[(10)\]
\[
\omega_1^\pm(q) = \omega_1^\mp + \frac{c_2^2 q^2}{\omega_1^\pm}
\]
\[(11)\]

If we take \( F_0^a \) and \( \nu_\sigma^\pm = 0 \) for \( l \geq 3 \) and include the projection of \( l = 2 \), Eq. (11) will be modified to,
\[
(\omega(\omega + \omega_1^\pm) - c_2^2 q^2)(\omega + \omega_1^\pm) - \frac{4\omega^3}{5} c_2^2 q^2 = 0
\]
\[(12)\]

where \( c_2^2 = \frac{1}{3}(1 + F_0^a)(1 + F_2^a)\nu_\sigma^2 \). From Eq. (12) we can show that the contribution of order \( q^2 \) to the dispersion relations will be only to \( \omega_1^\pm(q) \), which now reads as,
\[
\omega_1^\pm(q) = \omega_1^\mp + \frac{c_2^2 q^2}{\omega_1^\pm} + \frac{4N(0)\nu_\sigma^2}{30m} \left( \frac{3}{F_1^a} + 1 \right) q^2
\]
\[(13)\]

For the EQ system, Eq. (11) will have the form
\[
(\omega \pm \omega_0 - \vec{v}_p \cdot \vec{q} \pm 2\mu_0^m)\nu_\sigma^\pm - (\vec{v}_p \cdot \vec{q} N(0) \pm 2\mu_0^m) \int \frac{d\Omega}{4\pi} P_{ph}^\pm_{\nu_\sigma^\pm} \delta B^\pm = -(\vec{v}_p \cdot \vec{q} + 2\mu_0^m N(0)) \frac{\Delta \epsilon}{N(0)} \delta \phi^\pm
\]
\[(14)\]

which modifies the Eqs. (10) and (11) as follows:
\[
\omega_0^\pm(q) = \pm \omega_0 + \frac{c_2^2 q^2}{\omega_1^\pm}
\]
\[(15)\]
that both the collective modes of the EQ system are also propagating. But contrary to the QEQ system, none of the modes are gapless. The uniform mode in the EQ system corresponds to the precession of the magnetization around the external field at the Larmor frequency, $\pm \omega_0$. The current mode corresponds to the precession of the spin current in the transverse direction with the Larmor frequency $\pm \omega_1$.

The presence of the $f_1^a$ in the kinetic equation has an interesting consequence in the dynamical structure function, $S(q, \omega)$. From Eq. (6), we find an expression for the dynamical spin susceptibility using $\chi^{\pm}(q, \omega) = \frac{-N(0)}{\sigma_{\Omega_{q,\omega}^B}}$ and used it to obtain

$$S^\pm(q, \omega) = -\frac{1}{2\pi} (\text{Im}[\chi^+(q, \omega)] - \text{Im}[\chi^-(q, -\omega)])$$

(17)

to study the frequency dependence of $S^\pm(q, \omega)$ and the $f$-sum rule. For experimental purpose $S^\pm(q, \omega)$ is defined in relation to the scattering cross section as $\frac{d^2 \sigma}{d \Omega d \omega} = \sigma_0 S^\pm(q, \omega)$, where $\sigma_0$ contains all the elements related to the probe particle interaction and $S^\pm(q, \omega)$ characterizes the many-body system. The $f$-sum rule states that $\int d\omega S^\pm(q, \omega) = (1 + \frac{f_{1a}}{m^*}) a$, where $n$ is the total density, and $m^*$ is the effective mass. For the QEQ system we see that the gapless and the gapped collective modes exhaust the sum rule if we include the $f_{1a}^q$ in the kinetic equation. In this case the $h$-contribution to the sum will be proportional to $q^4$. But if we did not include $f_{1a}^q$ the $h$-continuum would also have a contribution proportional to $q^2$. The gapped mode, which comes out of the $h$-continuum due to the inclusion of $f_{1a}^q$ in the kinetic equation, takes over most of the spectral weight of the $h$-continuum contribution to the sum rule. We observe a similar behavior for the EQ phase, as was found earlier in the ferromagnetic system.

In addition to the dynamical consequences of the gapless mode there may be interesting thermodynamic properties that can be observed in the QEQ system. Perhaps the most dramatic would be a $T^{3/2}$ contribution to the specific heat. Whereas, for the EQ system, because all of the transverse collective modes are gapped, the specific heat contribution would decreases exponentially with the decrease in temperature.

As mentioned earlier, to study the effect of the temperature on the collective modes, we solved the Landau kinetic equation using the relaxation-time approximation for the collision integral. We find that the real part of the dispersion relations remains dominant at low temperature. The dispersion relation shows a transition of the spin system from the collision less region to a hydrodynamic region as has been discussed in the literature.

In summary using the Landau Fermi liquid theory we explored the thermodynamics and the dynamics of a paramagnetic spin system which is out of equilibrium. We report for the first time the prediction of a gapless collective mode in such systems. We discuss the difference in the nature of the collective modes in the QEQ and the EQ systems. We point out the importance of including the $f_{1a}^q$ Landau parameter in the kinetic equation. We believe that some of the qualitative features of the collective modes, in particular the gapless mode, will also be seen in systems far from equilibrium. We want to emphasize that this study is applicable in a wide variety of interesting materials.

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