A two-particle simulation of nonunitary Newtonian gravity

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Abstract

The result of a numerical simulation of two interacting particles in the framework of Nonunitary Newtonian Gravity is presented here. Particles are held close together by a 3d harmonic trap and interact with each other via an 'electrical' delta-like potential and via the ordinary Newtonian term, together with a fluctuational nonunitary counterpart of the latter. Fundamental nonunitarity can be seen as arising from the interaction of the physical degrees of freedom with (gravitational) hidden copies of them. Starting from an energy eigenstate within the ordinary setting, it is shown that, while energy expectation remains constant, a slow net variation of the von Neumann entropy for the system as a whole takes place, with a small modulation induced on the relative entanglement entropy of the two particles. Besides, the simulation shows explicitly how fundamental gravity-induced entropy can be clearly distinguished from the subjective notion of coarse-grained entropy, i.e. from the entropy of one particle with respect to the ‘environment’ of the other.

1. Introduction

The possible role of gravity in producing a fundamental nonunitary time evolution of the state vector in quantum mechanics (QM) has been invoked in the past few decades by a number of authors, on different grounds [1–5]. The long-standing difficulties posed by the two basic processes of QM, i.e. the Schrödinger deterministic evolution and the nonunitary process associated with the act of measurement lead to the quest of how to reconcile these radically different processes and explain the transition to classicality.

It is worth noting that the origin of the quantum-to-classical transition is today widely investigated and the emergence of classicality has been attributed mainly to environmental decoherence [6]. A different route to a gravity-related decoherence has been also proposed, which is entirely due to the presence of relativistic time dilation, whose effect on a composite quantum particle is to induce a universal coupling between the center of mass and its internal degrees of freedom. The resulting decoherence of the particle’s position takes place even for isolated composite systems and does not involve any modification of QM [7]. The important point here is that in both the frameworks, environmental and time dilation decoherence, global unitarity is preserved and some fundamental problems, at least in our view, remain.

Within this context, in the past years De Filippo introduced a nonunitary non-Markovian model of Newtonian gravity, NNG from now on (see e.g. [8, 9], references therein). On the one hand this model can be seen as the non-relativistic limit of a classically stable version of higher derivative gravity, whose heuristic application to black hole singularity is in agreement with Bekenstein–Hawking entropy [10–12], given the smoothed singularity of the black hole (see section VI of [8]). On the other hand, it presents several appealing features to become a natural candidate as an effective low-energy model of gravity. For example, while reproducing at a macroscopic level the ordinary Newtonian interaction, it presents a mass threshold for localization, which for ordinary matter densities is about $10^{11}$ proton masses [13, 14]. As a key result, it provides a mechanism for the evolution of macroscopic coherent superpositions of states into ensembles of pure states. Furthermore the evolution of the density matrix is compatible with the expectations leading to the phenomenological spontaneous localization models [4], as it was argued that they should be both nonlinear and

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nonunitary. However, at variance with them, it does not present obstructions consistent with its special-relativistic extension [15].

As to the non-Markovian character of the model, it is interesting to mention the works of Unruh, who studied some nonunitary toy models mimicking black hole evaporation, pointing out the quest for non-Markovianity in order to avoid net dissipation while keeping decoherence [16, 17]. In the NNG model such a feature is built-in, together with a number of other appealing properties, like the natural relativistic extension.

On the black hole entropy side, in the last decades it has been shown how the decoherence associated to black hole formation and evaporation [18–20] appears to be a fundamental one, requiring a modification of the unitary time evolution of QM in a nonunitary sense: that would allow a non vanishing probability for the evolution from pure states to mixed states. In this sense black hole entropy has been hypothesized [21] as due to the entanglement between matter-fields inside and outside the event horizon, the ultimate meaning of the degrees of freedom involved in its definition remaining quite elusive. Likewise, starting from suitable initial conditions, only a nonunitary quantum dynamics could allow for a microscopic derivation of the Second Law of thermodynamics for a closed system by resorting to the concept of von Neumann entropy, as pointed out in [22]. In this way it should be possible in principle to shed new light on the long standing and still open problem of quantum foundations of the Second Law of thermodynamics [23–29] without resorting to the ambiguous and subjective procedure of coarse graining [30, 31].

Indeed, deep contradictions arise when dealing with the problem of the physical nature and origin of the law of increasing entropy in a closed system, as pointed out by Landau and Lifshitz [24]. In particular, difficulties show up when one tries to apply statistical physics to the entire Universe taken as a single closed system. A solution could be found by resorting to the general theory of relativity, in which the Universe as a whole must be regarded as a system in a variable gravitational field, but within such a context the application of the law of increase of entropy does not imply relaxation of statistical equilibrium. Furthermore, another problem in formulating the law of increasing entropy arises when defining the direction of time as that of increasing entropy. This should be valid also in QM. But QM involves a physical non equivalence of the two directions of time, when considering the interaction of a quantum object with a classical one, i.e. the measurement process.

Aim of this work is to make a first step in demonstrating the ability of NNG to reproduce a gravity-induced relaxation towards thermodynamic equilibrium even for a perfectly isolated system. To do so, a simple system of two interacting particles is considered. While of course localization mechanism is not acting in this case, due to the far from threshold mass of the particles, some of the peculiar characteristic of the model can be verified in a more general context with respect to the single-lump problem.

More specifically we consider the two particles in a harmonic trap, interacting with each other through ‘electrostatic’ and gravitational interaction. Starting from an eigenstate of the physical Hamiltonian, we carry out a numerical simulation and calculate the time evolution of the von Neumann entropy. Incidentally, this entropy can be interpreted as an entanglement entropy with some (fictitious) hidden degrees of freedom, which recalls the mentioned approach to black hole entropy as entanglement entropy. As a result, we find that entropy fluctuations take place, owing to the (nonunitary part of) gravitational interactions, with the initial pure state evolving into a mixture. This constitutes a first necessary step: an extension of this work can be envisaged, which could show explicitly within a realistic setting the way in which thermodynamics could emerge in the framework of NNG, shedding new light on the possible role of gravity in the quantum foundations of thermodynamics in ordinary low-energy physics [8, 9, 13, 32, 33].

The plan of the paper is as follows. In section 2 we give a brief general description of the basic NNG model. In section 3 we introduce the specific model which is the subject of our study, and set the parameter values. Section 4 is then devoted to the application of NNG model to the two particles system via numerical simulations, and contains the main results of the paper. Finally, in section 5, we draw some conclusions and outline future perspectives of this work. Explicit expressions of the energy matrix elements are contained in the appendix.

2. A brief survey on nonunitary Newtonian gravity

The aim of this section is to briefly recall the key features of the NNG model. On the basis of a number of considerations (among which, the consistency with the basic formal relations of QM, energy conservation, classical and quantum behavior of matter, suggestion that non-unitary terms could have a gravitational origin), it is possible to isolate a two-parameter class of non-unitary gravity models, as discussed in detail in [8, 13, 32, 33]. We will comment later in the section on these parameters. Let us start here by giving a definition of the model in its simplest form, suitably oriented to the problem we are going to discuss below.

Let \( H_0[\psi^g, \psi] \) be the non-relativistic ‘physical’ Hamiltonian of a finite number of particle species, like electrons, nuclei, ions, atoms and/or molecules, where \( \psi^g, \psi \) denote the whole set \( \psi^g_j(x), \psi_j(x) \) of creation-annihilation
operators, i.e. one couple per particle species and spin component. $\mathcal{H}_0[\psi^\dagger, \psi]$ includes the usual electromagnetic interactions accounted for in atomic, molecular and condensed-matter physics.

Denoting by $\mathcal{H}_P$, the ‘physical’ energy operator including also the usual Newtonian interaction,

$$
\mathcal{H}_P[\psi^\dagger, \psi] = \mathcal{H}_0[\psi^\dagger, \psi] - \frac{G}{2} \sum_{j,k} m_j m_k \int \mathrm{d}x \mathrm{d}y \frac{\psi_j^\dagger(x) \psi_j^\dagger(y) \psi_k(x) \psi_k(y)}{|x-y|},
$$

(2.1)
to incorporate that part of gravitational interactions responsible for non-unitarity one has to introduce complementary creation-annihilation operators $\bar{\psi}_j(x), \bar{\psi}_j^\dagger(x)$ and the overall (meta-)Hamiltonian:

$$
\mathcal{H}_{\text{TOT}} = \mathcal{H}_P[\psi^\dagger, \psi] + \mathcal{H}_P[\bar{\psi}^\dagger, \bar{\psi}]
$$

$$
- \frac{G}{4} \sum_{j,k} m_j m_k \int \mathrm{d}x \mathrm{d}y \left[ \frac{2 \psi_j^\dagger(x) \psi_j^\dagger(y) \bar{\psi}_k(x) \bar{\psi}_k(y)}{|x-y|} \right] + \frac{G}{4} \sum_{j,k} m_j m_k \int \mathrm{d}x \mathrm{d}y \left[ \frac{\psi_j^\dagger(x) \psi_j^\dagger(y) \bar{\psi}_k(x) \bar{\psi}_k(y)}{|x-y|} \right].
$$

The above operators act on the product $F_\psi \otimes F_{\bar{\psi}}$ of Fock spaces of the $\psi$ and $\bar{\psi}$ operators, where $m_i$ is the mass of the $i$th particle species and $G$ is the gravitational constant. The $\bar{\psi}$ operators obey the same statistics as the corresponding operators $\psi$, while $[\psi, \bar{\psi}] = [\psi, \bar{\psi}^\dagger] = 0$.

The meta-particle state space $S$ is the subspace of $F_\psi \otimes F_{\bar{\psi}}$, including the meta-states obtained from the vacuum $|0\rangle = |0\rangle_\psi \otimes |0\rangle_{\bar{\psi}}$ by applying operators built in terms of the products $\psi_j^\dagger(x) \psi_j(x)$ and symmetrical with respect to the interchange $\psi^\dagger \leftrightarrow \bar{\psi}^\dagger$; as a consequence they have the same number of $\psi$ (physical) and $\bar{\psi}$ (hidden) meta-particles of each species. Since constrained meta-states cannot distinguish between physical and hidden operators, the observable algebra is identified with the physical operator algebra. In view of this, expectation values can be evaluated by preliminarily tracing out the $\bar{\psi}$ operators. In particular, the most general meta-state corresponding to one particle states is represented by

$$
|\langle f \rangle\rangle = \int \mathrm{d}x \int \mathrm{d}y \langle f(x, y) \bar{\psi}_j^\dagger(x) \bar{\psi}_j(y) |0\rangle\rangle,
$$

(2.3)

with

$$
\langle f(x, y) = f(y, x).
$$

This is a consistent definition since $\mathcal{H}_{\text{TOT}}$ generates a group of (unitary) endomorphisms of $S$.

If we prepare a pure $n$-particle state, represented in the original setting, excluding gravitational interactions, by

$$
|\langle g \rangle\rangle = \int \mathrm{d}^n x g(x_1, x_2, \ldots, x_n) \psi_{j_1}^\dagger(x_1) \psi_{j_2}^\dagger(x_2) \ldots \psi_{j_n}^\dagger(x_n) |0\rangle\rangle,
$$

its representation in $S$ is given by the meta-state

$$
\int \mathrm{d}^n y \int \mathrm{d}^n x g(x_1, x_2, \ldots, x_n) \psi_{j_1}^\dagger(x_1) \psi_{j_2}^\dagger(x_2) \ldots \psi_{j_n}^\dagger(x_n) \bar{\psi}_{j_1}(y) \bar{\psi}_{j_2}(y) \ldots \bar{\psi}_{j_n}(y) |0\rangle\rangle.
$$

It should be stressed that the model could be formulated, through an Hubbard–Stratonovich transformation, without any explicit reference to the hidden degrees of freedom, as a dynamic equation for the density matrix of the observed degrees of freedom [8].

Note that $\mathcal{H}_{\text{TOT}}$ and $\mathcal{H}_P[\psi^\dagger, \psi] + \mathcal{H}_P[\bar{\psi}^\dagger, \bar{\psi}]$ differ only due to correlations. In fact, because of the state constraint, the hidden degrees of freedom show the same average energy of the observed ones, while the two last terms in $\mathcal{H}_{\text{TOT}}$ have approximately equal expectation values and fluctuate around the classical gravitational energy. While these energy fluctuations have to be present in any model leading to dynamical wave function localization, which in itself requires a certain injection of energy [34], on the other side the same fluctuations, though irrelevant on a macroscopic scale, are precisely what can lead to thermodynamic equilibrium in a closed system if thermodynamic entropy is identified with von Neumann entropy [35]. Indeed, due to the interaction with the hidden degrees of freedom, a pure eigenstate of the ordinary energy $\mathcal{H}_P$ is expected to evolve into a mixture. It is just this feature, inferred on the above general grounds, that we will check numerically on a specific (simple) physical system.

A further comment is in order. Within a more general version of the model, we have $N$ interacting identical copies of the system under study: indeed such a symmetry requirement, which has to be imposed as a constraint on meta-states, is mandatory in order to avoid a priori a net energy flux between observable and hidden degrees of freedom and to guarantee energy conservation. That happens in strict analogy with the nonunitary toy model.
introduced by Unruh and Wald [16] and the simple quantum mechanical model recently analyzed by Unruh [17], which takes into account the scattering of two particles off each other, mediated by some hidden degrees of freedom (represented as a certain number of spin $\frac{1}{2}$ objects). It is also interesting to note that the limit $N \to \infty$ reproduces the famous nonlinear Newton–Schrödinger equation, sometimes considered in the literature as a possible candidate equation for the inclusion of self-gravity in QM, relevant to the quantum–classical transition [4, 13].

3. The physical system: interacting particles in a trap

In this section we introduce the system model that will be the subject of our study in the following. Then we look for a range of physical parameters, where the two-body interaction term in the Hamiltonian can be considered at most comparable with the kinetic and harmonic terms.

Let us start by considering a single particle of mass $\mu$, confined by a spherically symmetric harmonic trap [36]. The complete wave-functions are

$$\psi_{\text{line}}(r, \theta, \phi) = R_{n\ell}(\xi) Y_{\ell m}^{\star}(\theta, \phi),$$

where $\xi = r \sqrt{\mu \omega / \hbar}$, $R_{n\ell} = A_{n\ell}^{\frac{1}{2}} \xi^{\frac{1}{2}} e^{-\xi^{2}} F(-n, 1 + \frac{3}{2}, \frac{\xi^{2}}{2})$ with $A_{n\ell}$ the normalization factor and $F$ the confluent hypergeometric function and $Y_{\ell m}^{\star}$ the spherical harmonics. The corresponding energy levels are

$$E_{n\ell} = \hbar \omega \left(2n + 1 + \frac{3}{2}\right)$$

(3.2)

Let us now introduce a system of $n$-interacting particles in the same spherical trap, whose ‘physical’ Hamiltonian, in the ordinary (first-quantization) setting, is

$$H_{\text{ph}}(x_1, x_2) = -\frac{1}{2} \sum_{i=1}^{n} \frac{\hbar^2}{\mu} \Delta x_i - \mu \omega^2 x_i^2 + \frac{4\pi\hbar^2}{\mu} \sum_{i<j=1}^{n} \delta^{(3)}(x_i - x_j) - G\mu^2 \sum_{i<j=1}^{n} |x_i - x_j|^{-1};$$

(3.3)

here $\mu$ is the mass of the particles, $\omega$ is the frequency of the trap and $l_{\ell}$ is the $s$-wave scattering length. We are considering a ‘dilute’ system, such that the electrical interaction can be assumed to have a contact form with a dominant $s$-wave scattering channel. We take numerical parameters that make the electrical interaction at most comparable with the oscillator’s energy, while gravity enters the problem as an higher order correction. Reliable values of the parameters could be estimated such that the condition

$$\eta = \frac{U}{(3/2)\hbar \omega} \lesssim 1$$

(3.4)

holds, where $U$ is the mean value of the delta interaction potential evaluated on the system’s initial state. Furthermore, if we assume that only the first levels above the ground are excited, then $U$ can be evaluated approximately on the 0th order ground state

$$U \simeq \frac{4\hbar^2 l_{\ell}}{\mu \sqrt{\pi}} \left(\frac{\mu \omega}{\hbar}\right)^{1/2}.$$

(3.5)

In our case, then, $\eta = 0.98$. From now on we fix $n = 2$ in equation (3.3), which corresponds to the simplest case of two interacting particles. In the following section, we apply NNG model to this system and study its time evolution, starting from an initial pure state, particularly from an energy eigenstate. For simplicity, we perform our computation within a reduced Hilbert space built on the lowest energy eigenvalues of the system, corresponding to the tensor product of the single-particle space spanned by the first 4 states, i.e. the ground state and three degenerate excited states of the harmonic oscillator. The consistency of this huge dimensionality reduction is verified a posteriori by the fact that higher energy levels are not excited at all throughout the time evolution.

4. Numerical results—time evolution of the von Neumann entropy

The aim of this section is to study the time evolution of the two-particles system introduced above in the framework of the NNG model [8, 13, 32, 33] starting from a pure state for each particle, which we identify with the lowest energy state defined by the quantum number $n = 0$ in (3.2). We will define the general meta-Hamiltonian by coupling via gravitational interaction only the physical system with an hidden system which is the identical copy of it. Then the time dependent physical density matrix is computed by tracing out the hidden degrees of freedom and the corresponding von Neumann entropy is derived as the entanglement entropy with such hidden degrees of freedom. The nonunitary physical theory obtained will produce an entropy fluctuation
in the closed system chosen and then a fundamental gravitationally-induced decoherence, which can be clearly
distinguished from the usual coarse-grained entropy. We carry out numerical simulations by fixing for
simplicity a number of particles \( n = 2 \), which is the simplest nontrivial case one can consider in order to show
some of the most important peculiarities of NNG. Our model, in the (first-quantization) ordinary setting, is
defined by

\[
H_{\text{TOT}} = H_{\text{ph}}(\mathbf{x}_1, \mathbf{x}_2) + H_{\text{ph}}(\mathbf{x}_3, \mathbf{x}_4) + H_{\text{NNG}}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{x}_3, \mathbf{x}_4),
\]

with

\[
H_{\text{NNG}} = G\mu^2 \sum_{i=1}^{n} \left(-\frac{1}{|x_i - x_1|^3} + \frac{1}{|x_i - x_2|^3} + \frac{1}{|x_i - x_3|^3} + \frac{1}{|x_i - x_4|^3}\right)
\]

in agreement with (2.2).

We write the components of the generic basis meta-state \( \Psi \) as

\[
\langle \Psi | \rangle = |i_1 i_2 \ldots i_n\rangle \otimes |j_1 j_2 \ldots j_m\rangle = \left[ \bigotimes_{a=1}^{n} |i_a\rangle \right] \otimes \left[ \bigotimes_{b=1}^{m} |j_b\rangle \right],
\]

where \( \alpha \) defines the following cumulative index \( \alpha \equiv (i_1 i_2 \ldots i_n; j_1 j_2 \ldots j_m) \) and \( i_a = 1, 2, 3, 4 \) is one of the four
basis states of the single molecule (the quantum numbers we are dealing with are \( n = 0; l = 1, 2; m = -1, 0, 1 \)).
In the following we carry out numerical simulations for a system of \( n = 2 \) particles, so we get 16 basis
states and a general meta-state with 256 components respectively, while the matrix dimension of the whole
meta-Hamiltonian \( H_{\text{TOT}} \) is \( 64 \times 64 \). The evaluation of the matrix elements for the relevant (nonunitary)
gravitational interaction between the basis meta-states (4.1) is given in detail in the appendix.

Let us now outline the main steps of our simulation. A numerical diagonalization gives the eigenvalues and
the eigenvectors of \( H_{\text{TOT}} \), so we get all the ingredients to study the time evolution of the system. By choosing as
initial state the following general unentangled meta-state:

\[
|\Psi(0)\rangle = \sum_{l=1}^{2^n} \alpha_l |\Psi_l\rangle,
\]

where \( |\Psi_l\rangle \) are the meta-eigenstates of \( H_{\text{TOT}} \), the time evolution will produce the entangled meta-state:

\[
|\Psi(t)\rangle = \sum_{l=1}^{2^n} \alpha_l \mathrm{exp}[-(i/\hbar)H_{\text{TOT}} t]|\Psi_l\rangle.
\]

It is now possible to calculate the density matrix of only one molecule of the physical system by tracing out
the remaining \( 2n - 1 \) degrees of freedom:

\[
\langle i | \rho_m(t) | j \rangle = \sum_{i_{l_0} \ldots i_{l_m} = 1}^{4^n} \langle \{ i_1, i_2, \ldots, i_{l_m} \} | | \Psi(t)\rangle \langle \Psi(t) | \{ j_1, j_2, \ldots, j_m \} \rangle.
\]

But the key quantity for our work is the density matrix of the physical system, which is obtained by tracing out
the hidden degrees of freedom. That corresponds to split the whole meta-system into two parts, as every bipartite
system, identified with the physical and hidden degrees of freedom respectively. We get:

\[
\langle \{ i_1, \ldots, i_n \} | \rho_{\text{PH}}(t) | \{ j_1, \ldots, j_m \} \rangle
\]

\[
= \sum_{i_{l_0}, \ldots, i_{l_n} = 1}^{4^n} \langle \{ i_1, i_2, \ldots, i_n, i_{l_0} \ldots, i_{l_n} \} | | \Psi(t)\rangle \langle \Psi(t) | \{ j_1, j_2, \ldots, j_m, i_{l_0} \ldots, i_{l_n} \} \rangle.
\]

Starting from the von Neumann entropy definition \( S(\rho) = -k_B \text{Tr}(\rho \log \rho) \), where \( k_B \) is the Boltzmann
constant, we compute the single-particle von Neumann entropy \( S(\rho_m) \), and the two-particle system von
Neumann entropy \( S(\rho_{\text{PH}}) \). The results shown in the following refer to an initial state, \( |\phi_2\rangle \), equal to the eigenstate
of the physical energy associated with the 2nd highest energy eigenvalue \( E_2 \) shown in figure 1. The results for
the entropies are shown in figure 2 respectively for the following values of the physical parameters \( l , \mu \) and \( \omega = l \times 10^{-8} \text{ m}, \mu = 1.2 \times 10^{-24} \text{ Kg} \) and \( \omega = 4\pi \times 10^3 \text{ s}^{-1} \) (which are compatible with current experiments
with trapped ultracold atoms [37] and complex molecules [38]) and with an artificially augmented ‘gravitational constant’ \( G = 6.67408 \times 10^{-8} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} (10^4 \text{ times the real constant}).

To begin with, due to gravity-induced fluctuations the system entropy shows a net variation over very long
times, at variance with the case \( G = 0 \), in which it would have been a constant of motion. At the same time,
single particle entropy, which in the ordinary setting is itself constant, shows now a modulation by the system
entropy itself. Incidentally, it has been verified that the expectation of physical energy is a constant, meaning that
the non-unitary term has no net energy associated with itself, but is purely fluctuational.

Besides the density plot (see figure 3) shows that another energy level is slightly excited, leading to the initial
entropy growth.
To ensure physical consistency with our assumption that the whole (meta-)dynamics is confined within the truncated 64-dimensional Hilbert space, one can assume suitable anharmonic corrections to the harmonic trap potential, leading to a sufficient spacing of higher energy levels for these levels not to be excited at all by all the interactions in the system.

Figure 1. Energy levels of the $n = 2$-particle system. In red the energy level of the eigenstate $|\phi_2\rangle$.

Figure 2. Time evolution of one-particle and two-particle entropies (respectively, curve above and below) for the initial state $|\phi_2\rangle$.

Figure 3. Density plot of the energy levels in the case of an initial state of the system equal to $|\phi_2\rangle$. It is possible to observe the involvement of another energy level starting from a time $\sim 10^{12}$ s. This value is in agreement with the time–energy uncertainty relation $\Delta t \sim \frac{\hbar}{\Delta E}$ applied in the meta-Hilbert space and evaluated on the initial meta-state. An estimate in terms of the basic parameters of the problem is given by $\Delta t \sim \hbar^{3/2}G^{-1/3}a^{-1/2}$, obtained by considering the gravitational energy corresponding to the length scale of the fundamental state of the harmonic oscillator.

To ensure physical consistency with our assumption that the whole (meta-)dynamics is confined within the truncated 64-dimensional Hilbert space, one can assume suitable anharmonic corrections to the harmonic trap potential, leading to a sufficient spacing of higher energy levels for these levels not to be excited at all by all the interactions in the system.
A further comment is in order about the employed value of gravitational constant throughout the computation. The computation in fact has been carried out keeping experimentally accessible values of particle masses and other parameters like scattering length and trap frequency, while using a significantly higher value of the real gravitational constant. Using the real value for the last constant would have produced a numerical noise overcoming the small interesting effect we wanted to see.

Alternatively, one can use the real value of $G$ and far from reachable values of the other parameters, adopting for example the following scaling of the Schroedinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(X, Y; t) = -iGT_{\text{TOT}}\Psi(X, Y; t),$$

$$G \rightarrow \lambda G, \quad t \rightarrow t, \quad \mu \rightarrow \lambda^{-2/5}\mu, \quad X \rightarrow \lambda^{1/5}X, \quad l_r \rightarrow \lambda^{1/3}l_r$$

$$\Psi(X, Y; t) \rightarrow \Psi(\lambda^{1/5}X, \lambda^{-1/5}Y; t).$$

Incidentally, starting from a specific numerical solution, the scaling above provides also for a continuous one-parameter family of solutions.

It is worth noting that the simple multiperiodic behavior observed for the entropy is due to the very elementary structure of our system’s energy levels, together with the fact that the involved gravitational energy is small compared to the separation between levels. Taking a system with a macroscopic number of particles, gravitational energy would very easily overcome the tiny level spacings of the order of $10^{-n}$ [24]. As this condition is met, and so many different frequencies enter into play, we can expect that entropy fluctuations would be strongly suppressed within a timescale proportional to $n^{-2}$ (as can be inferred from the time-uncertainty relation).

5. Conclusions and perspectives

The main feature introduced by NNG model is a fundamental non-unitarity arising from the gravitational interaction between the physical and hidden degrees of freedom, the last being an identical copy of the physical system, and with a symmetry constraint on the state-space. As a consequence, taking the simplest nontrivial (almost realistic) physical system of two-interacting particles the initially pure state taken as an eigenstate of the physical Hamiltonian (including the ordinary Newtonian potential) ‘diffuses’ and develops over a very long time in a mixture of the states involving neighboring energy levels. As a consequence both the system entropy and the single particle entropy manifest a net variation, while the expectation of energy remains constant over all the time. The very long time scale of the system studied here is compatible with the up to now experience with very small and isolated systems like the systems used to implement basic quantum computation, in which quantum coherence is preserved as long as environmental decoherence is controlled.

One may easily expect that using a more complex system with many particles one would get a much more rapid growth and subsequent stabilization of the system entropy, following the formation of a microcanonical ensemble around the initial energy level. In fact the accessible Hilbert space is rapidly growing as new degrees of freedom are added. The entropy is expected to grow consequently, while the spacing of energy levels, being proportional to $\exp(-S)$, becomes infinitesimally small. This last circumstance would allow for the excitation of a huge number of surrounding energy levels, and transitions among them would be induced even by the very weak gravitational interactions.

Of course, on the basis of the Poincare’s recurrence theorem applied in the meta-Hilbert space, one may expect that after a sufficiently long time the system entropy would diminish, going back to zero. But the time at which this would happen becomes longer and longer as the number of particles gets thermodynamic relevance.

It is important to stress that the analysis performed above gives also an (in principle) operational way to clearly distinguish between the ordinary coarse-graining (subjective) entropy and the ‘fundamental’ entropy due to the peculiar non unitary gravitational interactions of the model.

As a future perspective of the present work, a system of thermodynamic size should be analytically solved and studied in order to ensure a definite and real ability of the NNG model to reproduce the spontaneous relaxation of mesoscopic and macroscopic systems toward thermodynamic equilibrium, paving the way to a new foundational basis for the Second Law of thermodynamics.

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Appendix. Gravitational interaction matrix elements

This appendix is devoted to the evaluation of the matrix elements of the gravitational interaction term

\[ H_G = -\mu^2 \sum_{i<j} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \]

in the basis of meta-states with \( m^2 \) components, as defined in equation (4.1), where \( m \) is the number of single-particle basis states chosen. We have

\[ \langle (i_1, i_2, \ldots, i_{2n}) | H_G | (j_1, j_2, \ldots, j_{2n}) \rangle = -\mu^2 \sum_{n=1}^{2n} \sum_{\beta=n+1}^{2n} \int dr_1 dr_2 \psi_i^{(\mathbf{r})_i}(\mathbf{r}_1) \psi_j^{(\mathbf{r})_j}(\mathbf{r}_2) \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} \psi_{j_1}(\mathbf{r}_1) \psi_{j_2}(\mathbf{r}_2). \quad (A.2) \]

In order to evaluate the general matrix element let us concentrate on the integral and introduce spherical coordinates. We get

\[
\int dr_1 dr_2 \psi_i^{(\mathbf{r})_i}(\mathbf{r}_1) \psi_j^{(\mathbf{r})_j}(\mathbf{r}_2) \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} = \sum_{l_1=0}^{\infty} \int_0^\infty d\eta_1 \eta_1 (\eta_1 r_1)^2 \frac{1}{r_1^{l_1+1}} R_i^{l_1}(\eta_1) R_j^{l_1}(\eta_1) \int d\Omega_1 d\Omega_2 Y_i^{m_1}(\theta_1, \phi_1) Y_j^{m_1}(\theta_2, \phi_2) X_{l_1}^{m_1}(\theta_1, \phi_1) Y_{l_1}^{m_1}(\theta_2, \phi_2),
\]

where we have used the multipolar expansion \( \frac{1}{1 + (\mathbf{r}_1 \cdot \mathbf{r}_2)} = \sum_{l=0}^{\infty} \frac{r_1^l}{r_2^{l+1}} P_l(\cos \gamma) \). By substituting the expression for \( P_l(\cos \gamma) \)

\[ P_l(\cos \gamma) = \frac{4\pi}{2l + 1} \sum_{m=-l}^{l} Y_i^{m_1}(\theta_1, \phi_1) Y_j^{m_1}(\theta_2, \phi_2), \]

we obtain

\[
\frac{4\pi}{l+1} \int d\Omega Y_i^{m_1}(\theta_1, \phi_1) Y_i^{m_1}(\theta_2, \phi_2) = \frac{(2l_1 + 1)(2l_2 + 1)}{4\pi (2l + 1)} C_{l_1 l_2 00} C_{l_1 l_2 m_1 m_2}.
\]

The two angular integrations can be performed by means of the general formula

\[ \int d\Omega Y_i^{m_1}(\theta, \phi) Y_i^{m_1}(\theta, \phi) = \frac{(2l_1 + 1)(2l_2 + 1)}{4\pi (2l + 1)} C_{l_1 l_2 00} C_{l_1 l_2 m_1 m_2}. \quad (A.5) \]

(Where \( C \) symbols correspond to Clebsch–Gordan coefficients) and of the property \( Y_i^{m_1}(\theta, \phi) = (-1)^{m_1} Y_i^{m_1}(\theta, \phi) \).

To put better in evidence the symmetries Clebsch–Gordan coefficients can be rewritten in terms of 3j-Wigner symbols:

\[ C_{l_1 l_2 1 m}^{l_1 l_2} = (-1)^{l_1 - l_2 + m} \sqrt{2l_1 + 1} \left( \begin{array}{ccc} l_1 & l_2 & l \\ m_1 & m_2 & -m \end{array} \right). \quad (A.6) \]

By collecting all these properties and substituting in equation (A.4) we obtain for the matrix element (A.2):

\[
\sum_{l=0}^{\infty} \left( \begin{array}{ccc} l_1 & l_2 & l \\ m_1 & m_2 & -m \end{array} \right) \left( \begin{array}{ccc} l_1 & l_2 & l \\ 0 & 0 & 0 \end{array} \right) \frac{(-1)^{l_1 - l_2}}{l_1} \frac{(-1)^{l_2 - l_1}}{l_2} \int_{0}^{\infty} dr_1 \left( \begin{array}{ccc} l_1 & l_2 & l \\ -m_1 & m_2 & m \end{array} \right) \int_{r_1}^{\infty} dr_2 \left( \begin{array}{ccc} l_1 & l_2 & l \\ r_1 & r_2 & r_3 \end{array} \right) \right] R_r^{l}(\eta_1) R_r^l(\eta_2).
\]

\[ \left[ \begin{array}{ccc} l_1 & l_2 & l \\ 0 & 0 & 0 \end{array} \right] \]
Here the sum over \( l \) has been restricted to the first 2 terms by using the triangular property of 3j-Wigner symbols:

\[
|l_r - l| \leq l \leq l_r + |l|, \\
|l_r' - l| \leq l \leq l_r' + |l|.
\]

Finally, the last integrals can be evaluated numerically.

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