Prediction of the sound insulation of panel structures based on the virtual testing room

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Abstract. The sound transmission through an elastically bounded panel between rooms is presented in this paper. A fully-coupled modal is used to investigate the effect of elastic boundary of the panel on the sound transmission by using Spectro-Geometric Method (SGM) which is a meshless and parametric modeling technique for general vibro-acoustic analysis. Unlike the traditional modal superposition methods, the continuity of the normal velocities is faithfully enforced on the interfaces between the flexible panel and the (interior and exterior) acoustic media. The sound insulation structures can represent a simple panel or a complex structure with various structural and/or acoustical design features. This model can be effectively used to virtually test the acoustic performance of sound insulation designs under different environments and application conditions.

1. Introductions

Structural-acoustic coupling is of interest to automotive, aerospace and other industries. The sound transmission through an elastic panel between two rooms is of significance to the calculation of the sound transmission loss of panel-like structures. A literature survey of the structural-acoustic coupling shows that a large amount of work has been done in calculating free and forced responses of cavity-panel-cavity systems.

The standards for the determination of sound insulation of building elements are derived assuming that the sound fields in both rooms are perfectly diffuse, i.e., with equal probability of energy flow in all directions. The diffuse field assumption is only valid in medium and high frequency ranges, since at low frequencies the sound field in the reverberation chambers is dominated by a few normal modes. It has been concluded that the actual standards did not provide essential improvements of the accuracy at very low frequencies. To better understand the sound transmission through the two rooms via the panel, analytical and numerical methods have been used. Modal analysis is adopted to study the errors in sound transmission measurements. Maluski and Gibbs have employed the FEM to study how the configuration of two identical reverberant rooms influences the transmission loss of a partition at low frequencies. Jo and Elliott studied the sound insulation properties of the panel mounted between adjacent rooms and give a strategy for active control of the low-frequency sound transmission between rooms. Xiang, et al. studied the sound energy decays consisting of multiple exponential decay rates in the coupled-volume systems. Papadopoulos proposed a virtual laboratory to represent a real laboratory consisting of two reverberation rooms to study the TL of the panel based on the technology of acoustic measurements and standards.

Compared to the conventional level difference method to measure the TL of the panel, newer technique base on the measurements of sound intensity on the receiving room side of the partition are
widely used due to the advances in instrumentation and technology. Halliwell and Warnock\cite{8} compared the conventional method and the intensity method to measure the TL of the panel and found significant differences between the two measurement techniques at low frequencies and high frequencies.

Recently, Spectro-Geometric Method (SGM)\cite{9,10} is adopted to study the cavity with impedance boundary conditions and the coupled cavity-panel system. The model allows accurate accounting for the strong coupling between the vibrating panel and the sound field in the cavity. Wang et al.\cite{11} study the TL of an elastic panel in cavity-panel system by the SGM method and the TL is estimated in a way similar to the intensity method.

In this paper, a vibro-acoustic model is proposed for determining the sound transmission characteristics of a flexible panel. The model consists of a reverberant room, a flexible panel, and a room with dissipative wall surface. The panel can be mounted elastically onto the window that connecting the two rooms which allows considering the effects on sound transmission of mounting conditions. Unlike most of previous techniques, the continuity of the normal velocity is truthfully enforced on the interface between the flexible panel and the acoustic cavities. Numerical examples are presented to validate the model and to demonstrate the impacts on the transmission loss of different panel mounting conditions and different sound sources.

2. Theoretical Formulation

As shown in Fig. 1, the vibroacoustic system consists of a (source) room 1 of dimensions $a_1 \times b_1 \times c_1$, a sound transmission panel of dimensions $L_1 \times L_y$, and a (receiving) room 2 of dimensions $a_2 \times b_2 \times c_2$. For the sake of generality, the two rooms are not necessarily of the same sizes. The panel may only partially cover the interface between the two rooms. The coordinate systems are also illustrated in Fig. 1, and the panel lies in the $z_1=0$ and $z_2=0$ in room 1 and room 2, respectively. This general configuration is chosen so that the sound transmission predicted by the model can be directly compared with that measured using a sound transmission test suite wherein the two rooms are not perfectly aligned.

![Figure 1](image_url)

For the elastically supported rectangular panel, its flexural displacement will be invariantly expanded into a modified Fourier series as\cite{9}

$$w(x, y) = \sum_{m=0}^{M} \sum_{n=0}^{N} A_{mn} \cos \lambda_m x \cos \lambda_n y + \sum_{j=1}^{4} (\xi_{j}(y) \sum_{m=0}^{M} c_{m}^{j} \cos \lambda_m x + \xi_{j}(x) \sum_{n=0}^{N} d_{n}^{j} \cos \lambda_n y),$$

(1)
where $\cos \lambda_n x = m \pi / L_x$, $\lambda_n = n \pi / L_y$, and $A_{mn}$, $c_m^i$ and $d_m^j$ represent the unknown Fourier expansion coefficients to be determined. The supplementary functions $\xi_i(x)$ and $\xi_j(y)$ are introduced to overcome the potential discontinuities encountered when the displacement function and its lower-order derivatives are periodically extended onto the entire $x$-$y$ plane as mathematically implied by the Fourier expansion. As an immediate numerical benefit, the Fourier series in Eq. (1) will converge uniformly at an accelerated rate.

The pressure field in the source room and the receiving room will also be similarly expressed as modified Fourier series expansions. Specifically, the pressure inside the rooms, $p_1$ and $p_2$, are expanded as

$$p_1(x, y, z) = \sum_{m=0}^{M_1} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} P_{m, n, l}^i \cos \lambda_{mn} x \cos \lambda_{nl} y \cos \lambda_{ml} z + \xi_i(z) \sum_{m=0}^{M_1} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} b_{m, n, l} \cos \lambda_{mn} x \cos \lambda_{nl} y,$$

$$p_2(x, y, z) = \sum_{m=0}^{M_2} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} P_{m, n, l}^j \cos \lambda_{mn} x \cos \lambda_{nl} y \cos \lambda_{ml} z + \left( \xi_j(x) c_{m, n, l}^1 + \xi_j(x) d_{2m, n, l} \right) \sum_{m=0}^{M_2} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} \cos \lambda_{mn} x \cos \lambda_{nl} y,$$

$$+ \left( \xi_j(y) e_{2m, n, l}^1 + \xi_j(y) f_{2m, n, l} \right) \sum_{m=0}^{M_2} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} \cos \lambda_{mn} x \cos \lambda_{ml} z,$$

where $P_{m, n, l}^i$ and $b_{m, n, l}$ denote the complex Fourier expansion coefficients, $\lambda_{mn} = m \pi / a$, and $\xi_i(s) = s_i (s / s_i)^2 (s / s_i - 1)$ in which $s=x, y, z$ and $i=1, 2$.

The pressure representations in Eqs. (2) may appear to be similar to the traditional superposition of the modes for the (rigidly walled) cavities. However, the cosine functions here are simply used as a complete set of basic functions which do not necessarily have any physical meanings. The supplementary terms are introduced to account for the fact that the derivative of pressure at the boundaries does not identically vanish on the wall where the elastic panel is mounted or the impedance boundaries are placed. It should be noted that each of the walls in the receiving room can be specified as impedance boundary conditions.

The Rayleigh-Ritz procedure will be employed to calculate the unknown expansion coefficients in Eqs. (1) and (2). The Lagrangian for the panel structure can be written as

$$L_{Panel} = U_{Panel} - T_{Panel} + W_{Room1&Panel} - W_{Room2&Panel},$$

where $U_{Panel}$ is the total potential energy associated with the transverse vibration of the panel and the deformations of the restraining springs; $T_{Panel}$ denotes the total kinetic energy of the panel, $W_{Room1&Panel}$ and $W_{Room2&Panel}$ represents the works done by the sound pressures in room 1 and room 2.

The Lagrangians for the room 1 and room 2 are

$$L_{Room1} = U_{Room1} - T_{Room1} - W_{Panel&Room1} - W_{wall},$$

and

$$L_{Room2} = U_{Room2} - T_{Room2} - W_{Panel&Room2} - W_{wall},$$

where

$$p_1(x, y, z) = \sum_{m=0}^{M_1} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} P_{m, n, l}^i \cos \lambda_{mn} x \cos \lambda_{nl} y \cos \lambda_{ml} z + \xi_i(z) \sum_{m=0}^{M_1} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} b_{m, n, l} \cos \lambda_{mn} x \cos \lambda_{nl} y,$$

$$p_2(x, y, z) = \sum_{m=0}^{M_2} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} P_{m, n, l}^j \cos \lambda_{mn} x \cos \lambda_{nl} y \cos \lambda_{ml} z + \left( \xi_j(x) c_{m, n, l}^1 + \xi_j(x) d_{2m, n, l} \right) \sum_{m=0}^{M_2} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} \cos \lambda_{mn} x \cos \lambda_{nl} y,$$

$$+ \left( \xi_j(y) e_{2m, n, l}^1 + \xi_j(y) f_{2m, n, l} \right) \sum_{m=0}^{M_2} \sum_{n=0}^{M_2} \sum_{l=0}^{M_3} \cos \lambda_{mn} x \cos \lambda_{ml} z,$$

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The Lagrangian for the room 1 and room 2 are

$$L_{Room1} = U_{Room1} - T_{Room1} - W_{Panel&Room1} - W_{wall},$$

and

$$L_{Room2} = U_{Room2} - T_{Room2} - W_{Panel&Room2} - W_{wall},$$

where
where \( U_{\text{room}} \) and \( T_{\text{room}} \) are the potential energy and the kinetic energy for the room \( t \) (\( t=1, 2 \)), \( W_{\text{Panel} \& \text{Room}} \) is the work due to the panel vibration (actually, \( W_{\text{Panel} \& \text{Room}} = W_{\text{Room} \& \text{Panel}} \)), \( W_s \) is the work done by the acoustic source, and \( W_{\text{wall}} \) is the energy dissipation on the wall surfaces.

Take a monopole source for example. The acoustic work can be calculated from

\[
W_s = -\frac{1}{2} \int V \frac{pQ}{j\omega} dV,
\]

and

\[
W_{\text{wall}} = -\frac{1}{2} \int S \frac{1}{j\omega} Z dS
\]

where \( Q_0 \) is the volume velocity amplitude of the monopole source, and \( Z \) is the complex acoustic impedance of the wall surface in room 2.

Substituting Eqs. (1, 2) into Eqs. (3) (4) and (5) and minimizing the resulting equations against the unknown Fourier coefficients will lead to a set of couple system

\[
\begin{bmatrix}
K_{\text{room1}} & 0 & 0 \\
0 & K_{\text{Panel}} & C_{r_1 k_1 p} \\
0 & 0 & K_{\text{room2}}
\end{bmatrix}
+ \omega^2
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
+ \omega^2
\begin{bmatrix}
M_{\text{room1}} & -C_{r_1 k_1 p}^T & 0 \\
0 & M_{\text{Panel}} & 0 \\
0 & -C_{r_2 k_2 p}^T & M_{\text{room2}}
\end{bmatrix}
= \begin{bmatrix}
P_1 \\
P_2 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
P_1 \\
P_2 \\
0
\end{bmatrix}
= \begin{bmatrix}
F_p \\
F_p \\
0
\end{bmatrix}
\]

where \( P_1 \) and \( P_2 \) represent the vector of the Fourier expansion coefficients for the sound pressure in room 1, in room 2 and the plate displacement, respectively. \( F_p \) is derived from Eq. (6), which is expressed as

\[
F_{p_{m_1 n_1 p_{m_1}}} = -j\omega \rho_c Q_0 \cos \lambda_{m_1 x_1} \cos \lambda_{n_1 y_1} \cos \lambda_{p_{m_1} z_1}, \quad \text{where} \quad (x_1, y_1, z_1) \text{ is the location of the monopole source.}
\]

It should be point out that Eq. (8) represents a strong coupled vibroacoustic system, that is, the interaction between the vibration of the panel and the pressure fields on the both sides have been faithfully taken into account. Thus, this model can be used to predict the responses of the coupled vibroacoustic system to the excitation of acoustic sources in any room. The sound transmission through the panels or apertures could be fully studied by this generally established model.

3. Results and Discussion

In order to validate the proposed model, a system consisting two rectangular cavities and a panel is first considered. Since this configuration has no analytical solution available for comparison the current results are validated by the FEM results obtained using LMS Virtual Lab. The dimensions of the source and receiving rooms are: \( a_1 \times b_1 \times c_1 = 2 \times 2.2 \times 2.4 \text{ m} \) and \( a_2 \times b_2 \times c_2 = 2.2 \times 2.4 \times 2.6 \text{ m} \), and they are connected via an elastic panel of dimensions \( 1 \times 1.2 \text{ m} \). The panel is positioned centrally between the two rooms with the \((x, y)\) coordinates of the corner of the panel closest to the origin of \((1 \text{ m}, 1.1 \text{ m})\) for the source room, and of \((0.8 \text{ m}, 0.9 \text{ m})\) for the receiving room. The speed of sound is \( c_0 = 340 \text{ m/s} \), air density \( \rho_0 = 1.2 \text{ kg/m}^3 \) and loss factor 0.02. The panel has a thickness 0.005 m, a Young’s modulus 70.3 GPa, a density 2700 kg/m\(^3\) and a Poisson’s ratio of 0.3. The panel is assumed to be simply supported (SSSS) which is considered a special case of elastic restraints by setting the transverse and rotational springs to infinity and zero, respectively. A point sound source is applied at \((0.2 \text{ m}, 0.2 \text{ m}, 2.2 \text{ m})\) in the source room. The sound pressure levels (SPL) at the observation points \((1.3 \text{ m}, 1.4 \text{ m}, 0.9 \text{ m})\) in the source room and \((0.8 \text{ m}, 0.9 \text{ m}, 1.6 \text{ m})\) in the receiving rooms are calculated. In the present approach, the series expansions are truncated to: \( M \times N = 10 \times 12 \), \( M_{x1} \times M_{y1} \times M_{z1} = 9 \times 9 \times 10 \) and \( M_{x2} \times M_{y2} \times M_{z2} = 10 \times 10 \times 11 \). In the FEM, the element size is chosen as 50 mm.

First, the calculated natural frequencies of this coupled system are compared between the present and FEM models. Overall, these two models agree well in terms of the prediction. It is seen that the difference is relatively larger between the first three sets of frequencies, which is not easy to explain.
Next, the SPL curves obtained from the present and FEM method are compared in Fig. 2. While the pressures compare very well for the point in the source room, the agreement is noticeably deteriorated which appears that the damping effects in the FEM model are somehow underestimated.

Table 1. Coupled system resonant frequencies predicted from the present approach compared with FEM (Hz).

| Modal | Present | FEM  | Difference (%) | Modal | Present | FEM  | Difference (%) |
|-------|---------|------|----------------|-------|---------|------|----------------|
| 1     | 20.83   | 21.09| 1.24           | 13    | 101.30  | 101.32| 0.02           |
| 2     | 45.28   | 45.71| 0.94           | 14    | 104.81  | 104.84| 0.03           |
| 3     | 56.32   | 56.87| 0.97           | 15    | 104.81  | 104.89| 0.07           |
| 4     | 65.53   | 65.58| 0.07           | 16    | 110.62  | 110.69| 0.06           |
| 5     | 70.82   | 70.86| 0.06           | 17    | 114.82  | 114.88| 0.05           |
| 6     | 71.03   | 71.09| 0.08           | 18    | 117.39  | 117.76| 0.32           |
| 7     | 77.28   | 77.33| 0.07           | 19    | 123.50  | 123.50| 0            |
| 8     | 77.29   | 77.33| 0.06           | 20    | 124.60  | 123.86| 0.60           |
| 9     | 81.92   | 82.03| 0.14           | 21    | 130.88  | 130.96| 0.06           |
| 10    | 85.01   | 85.05| 0.05           | 22    | 134.89  | 134.97| 0.06           |
| 11    | 87.82   | 88.12| 0.34           | 23    | 141.56  | 141.47| 0.06           |
| 12    | 96.45   | 96.48| 0.03           | 24    | 141.81  | 141.88| 0.05           |

Figure 2: SPL obtained by the present method and FEM (a: the observation point in the source room; b: the observation point in the receiving room)

The transmission loss (TL) of a panel can be calculated as

$$\text{TL}(\omega) = 10\log_{10} \left( \frac{\Pi_{\text{inc}}(\omega)}{\Pi_{\text{rad}}(\omega)} \right) \text{ (dB)},$$

where $\Pi_{\text{inc}}(\omega)$ and $\Pi_{\text{rad}}(\omega)$ are the sound powers impinging upon and radiated from the panel, respectively. The source room is often treated as a reverberate one, and the impinging sound power is approximately calculated using

$$\Pi_{\text{inc}}(\omega) = \frac{1}{K} \sum_{k=1}^{K} \left| p_{\text{inc}}^{k} \right|^2 S_{\text{panel}},$$

where $p_{\text{inc}}^{k}$ is the sound pressure measured at the $k$th of the randomly selected locations in the source room and $S_{\text{panel}}$ is the area of the panel.
The wall surfaces of the receiving room are assumed to be the classical pressure release boundary with no sound wave being reflected from them. Then, the radiation sound power can be determined by the intensity method

\[
\Pi_{\text{rad}}(\omega) = \frac{1}{2} \int \int_{S_{\text{panel}}} \text{Re}\left[ u'(x, y) p_2(x, y) \right] dx dy,
\]

(11)

where Re and * denote the real part and complex conjugate of a complex number, respectively.

It is well known that the mounting condition and the thickness of an elastic panel can significantly affect its sound insulation/radiation characteristic. As an example, we consider four panels with different thicknesses under two classical mounting conditions: SSSS, CCCC (C: clamped, S: simple supported). The dimensions of these two rooms are \(a_1 \times b_1 \times c_1 = 0.3 \times 1.5 \times 0.4 \, \text{m and } a_2 \times b_2 \times c_2 = 0.6 \times 3 \times 0.8 \, \text{m.} \) The panel parameters are chosen as \(a=0.3 \, \text{m, } b=1.5 \, \text{m, mass density } \rho = 2770 \, \text{kg/m}^3, \) Poisson ratio \(\nu=0.33, \) Young’s modulus \(E=71 \, \text{GPa and with four thickness values: } h=0.01 \, \text{m, } 0.03 \, \text{m, } 0.05 \, \text{m and } 0.01 \, \text{m.} \) The TL curves for these two boundary conditions are compared in Fig. 3. While TL is often used as a measure of sound insulation performance of a panel, the results in Fig. 3 clearly indicate that it can be meaningfully affected by the mounting conditions. Thus, the mounting condition should be considered an important design factor in practical applications. It could also been seen that the thicker panel has a larger TL value especially at the low frequencies.

**Figure 3:** TL of the panel with different thickness (a: SSSS, b: CCCC)

**Figure 4:** TL of the panel obtained by different locations of the sound sources (a. CCCC; b. SSSS)
To determine the sound powers impinging upon the panel, the source room should be built as a reverberate one. However, the number of and the location of the sound sources could also influence the reverberation condition. To illustrate this point, four different sound sources are placed at four randomly selected locations in source room. Fig. 4a and 4b shows the TL of panel obtained by one sound source which is placed at four randomly selected locations each time. Fig. 5a and 5b plots the TL of the panel when 1, 2, 3 and 4 sound sources are use separately. Those two situations are both considered when the panel is mounted with CCCC and SSSS boundaries. The results also clearly indicate that the mounting conditions affect the sound insulation performance of the panel significantly. From Figs. 4 and 5, it is also seen that the number of and the placement of the sound sources can also affect the estimated TL.

4. Conclusions
A vibro-acoustic model is developed for predicting the sound transmission between two rooms through an elastic panel. The model is numerically validated by comparing with the results obtained by FEM. Transmission loss of a panel is calculated for different mounting conditions, indicating that the mounting condition has a meaningful impact on the sound transmission loss. This suggests that selecting proper mounting conditions represents a viable design option for improving the transmission characteristics of sound insulation panels. Also, it is demonstrated that the calculation of the panel TL can be significantly affected by the number of the sound sources and their locations. This modeling capability provides an effective and alternative means for evaluating the transmission characteristic of a sound insulation panel, and the effects on it of certain design variables.

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