The mode II delamination toughness of arc unidirectional fiber-reinforced composites with ENF test

Y S Kang1,*, X Z Zhang1, L M Wang2 and C Jiang2

1 Department of Engineering Physics, Tsinghua University, Beijing 100084, China
2 Institute of Physical and Chemical Engineering of Nuclear Industry, Tianjin 300180, China

*E-mail: kys16@mails.tsinghua.edu.cn

Abstract. To obtain the mode II delamination toughness (the energy release rate $G_{IIC}$) of arc unidirectional fiber-reinforced composites, the end notched flexure (ENF) test with arc specimen was used to study. In terms of theory, the formulas of compliance and $G_{IIC}$ for Arc-ENF specimen were derived by using energy method and Irwin-Kies formula, and the crack length correction and friction coefficient were respectively introduced to correct these formulas, with the verification by the numerical simulation. As the ENF test, the compliance curve of the Arc-ENF specimen was recorded under the condition of the constant loading speed, and the crack tip position was determined by the strain diagram of the specimen in 1-3 plane with the optical strain gauge. Finally, the friction coefficient and the $G_{IIC}$ of the Arc-ENF specimen in the ENF test were determined using the corrected formulas, the numerical simulation method and the ENF test data.

Keywords: Arc-ENF Specimen; Delamination Toughness; Mode II

1. Introduction
The delamination is one of the main damage forms of fiber-reinforced composite structures. In order to prevent the occurrence of the delamination, it is necessary to evaluate the interlaminar properties of material during the design for material structures, and researchers have designed various test methods for analysis. For mode II delamination toughness test, the ENF specimen is common, with the test methods including the ENF test, four-point bending end notched flexure (4ENF) test and End load separation (ELS) test, etc. Brunner et al. [1] and Tabiei et al.[2] made review of these, and pointed out that the friction effect in the 4ENF test was obvious, which lead to results higher, and the clamp in ELS test will introduce many other factors, in which the test data are difficult to process, and the unstable crack propagation process in ENF test leads to a large error in the measurement results. Nevertheless, considering the advantages of ENF test in terms of test methods and data processing, standard D7905/D7905M-19e1 [3] still proposes it as the main test method for mode II delamination toughness test of unidirectional fiber-reinforced composite materials, for which the ENF test is adopted on the Arc-ENF specimen.

For the analytical solution of the ENF test, Russell and Street [4] firstly obtained the compliance and $G_{IIC}$ formulas based of the simple beam theory (SBT):

$$C_{SBT}^{IIC} = \frac{2L^3 + 3a^3}{8E_Bh^3}$$  \hspace{1cm} (1)
\[ G_{IIC}^{SBT} = \frac{9P^2}{16E_B B^3 h^3} \]  

where \(2L, B, \) and \(2h\) are respectively the length, width, and thickness of the ENF specimen, and \(a\) is the crack length, \(E_1\) is the elastic modulus in the fiber direction. Then, the compliance formula are modified based on the Timoshenko beam theory (TBT) by considering the effect of shear deformation.

\[ C_{IIC}^{TBT} = \frac{2L^3 + 3a^4}{8E_i B h^3} + \frac{3L}{10G_{ij} B h} \]  

where \(G_{ij}\) is the shear modulus of the 1-3 plane, and for \(G_{IIC}\) formulas, \(G_{IIC}^{SBT} = G_{IIC}^{TBT}\). The reason that the expressions of \(G_{IIC}\) are same is that the TBT is the first-order shear-deformation beam theory. And the effect of shear deformation on \(G_{IIC}\) can be seen with the application of elasticity or higher-order shear-deformation beam theories [6-8], alternatively, enhanced beam theory (EBT) models with elastic or cohesive interfaces [9] are another effective way, with the same expressions,

\[ G_{IIC} = \frac{9P^2}{16E_B B^3 h^3} [(a + \chi h)^2 + \Delta a] \]  

where \(\chi\) is the crack-length correction parameter related to the modulus of materials, and \(\Delta a\) is a constant. Besides, the effect of friction on the \(G_{IIC}\) formula is also significant. Zebar et al. [10] studied the mixed 4ENF test of two types of composites, and obtained the \(G_{IIC}\) formula considering the friction effect. Based on the point friction hypothesis and the beam theory, Wang et al. [11] used the energy method to derive a \(G_{IIC}\) formula considering the friction effect, and the calculation results were consistent with the results of the finite element analysis. Liu et al. [12] obtained a similar formula under the same assumption, and proposed a new cohesion model for the simulation of 4ENF test, and the simulation results were in good agreement with the test results. Mencattelli et al. [13] studied the effect of friction in 4ENF test and found that the friction effect between the loading clamp and the specimen was the main factor influencing the test, and suggested the rolling supporting to reduce the influence of friction. Parrinello et al. [14-16] used the mechanical behavior of ideal rigid plastic materials to describe the friction behavior, and combined with Euler-Bernoulli beam theory to establish an analytical method for analysing four-point bending test, and the final theoretical results were consistent with the numerical simulation results.

With the development of computer technology, the numerical analysis has been widely used in the study of delamination of composites, in which cohesion zone model (CZM) [17, 18] is widely concerned by researchers. Dourado [19] used a bilinear CZM to analyse the mode II delamination in composites with considering the fiber bridging effect in the process of test, and the CZM parameters were obtained by using a genetic algorithm to compare the result of simulation and test, and then De Morais [20] used this method on the mixed-mode (I+II) delamination. Nguyen, et al. [21] proposed a new mixed CZM for numerical analysis, and the simulation result had a good agree with the test of double cantilever beam (DCB), ENF and mixed-mode bending (MMB), which show the applicability of this model. Xie et al. [22] obtained the cohesion model by using the closed form solution method on the classical lamination theory (CLT), and applied it to the numerical analysis of DCB, ENF and MMB to study the influence of the crack length, interface strength, fracture energy and cohesive model form on the calculation results. In general, researchers can use the mechanical properties of matrix of composites as the CZM parameters in the numerical analysis.

The geometrical shape in the longitudinal direction of ENF specimen in different test introduced above is straight, however, in engineering applications, composites will have different geometric shapes according to design requirements or forming processes, such as the cylinder structure formed by winding forming process. For those structure with curvature, people want to get performance parameters as far as possible under the same processing conditions. Thus, based on the content introduced above, the \(G_{IIC}\) of Arc-ENF specimen will be obtained, and the content is divided into three
parts. The first part introduces the derivation, correction and verification of the formulas of compliance and $G_{IIc}$ for Arc-ENF specimen, and the second part introduces ENF test, finally, in the third part, $G_{IIc}$ for Arc-ENF specimen will be obtained by analysing the ENF test data with the modified formulas and CZM numerical simulation method.

2. Theory

2.1. Basic formulas of compliance and $G_{IIc}$ for Arc-ENF specimen

Compared with the ENF test of standard specimen, the biggest difference of arc specimen is that their geometrical shape in the longitudinal direction is arc (Figure 1), and there is no difference in test methods. And there are two normal ways to derive the compliance formula. The first one is that, based on the beam theory, the displacement of the loading point can be determined by calculating the deformation of the specimen in the loading and boundary condition, and then the compliance expression will be obtained. And the another one is to determine the strain energy of the specimen based on the energy method, and then calculate the loading point displacement by using the Castigliano theorem, and finally obtain the expression of the compliance. However, since the bending deformation equation of the arc specimen based on the beam theory is a nonlinear equation (the first derivative of deflection cannot be ignored), the analytic solution of deformation cannot be obtained. Thus, the second way based on the energy method will be used to give the compliance formula.

![Figure 1. Arc-ENF specimen model in 1-3 plane.](image)

The strain energy $U$ of the Arc-ENF specimen (Figure 1) in ENF test is combined with bending deformation energy $U_1$ and shear deformation energy $U_2$,

$$U = U_1 + U_2 = \int_{-a}^{a} \frac{M(\theta)^2}{2E_1I} Rd\theta + \int_{-h}^{h} \frac{\tau(\theta, x_i)^2}{2G_{13}} B dx_i \cdot Rd\theta$$

(5)

where $M$ is flexure moment and $\tau$ is shear force, and their expressions are described in Appendix A; $E_1$ is the elastic modulus in the 1 direction, $I$ is the second moment of area of the specimen section, $R \approx R_0 + h$, $B$ is the width of the specimen, $G_{13}$ the shear modulus of the 1–3 plane. Then, considering the Castigliano theorem $\delta = dU/dF$ and Irwin-Kies formula, the expression of compliance and $G_{IIc}$ can be written as

$$C = \frac{3R^3}{8E_1Bh^2} f_i(\alpha_0) + \frac{3R}{20G_{13}Bh} \cdot g(\alpha_0)$$

(6)
\[ G_{IC} = \frac{F^2}{2B} \int_{d_0} \, dC = \frac{9F^2R^2(\sin(\alpha) + (-1)^i \sin(\alpha - \alpha_0))^2}{16E_iB^4h^3} + \frac{9F^2\cos(\alpha_0 - \alpha)^2}{40G_{ij}B^2h} \]  

where \( i=1, \ 0 < \alpha_0 \leq \alpha; \ i=2, \ \alpha < \alpha_0 < 2\alpha \), and \( U, \ f_i(\alpha_0), \ g(\alpha_0) \) are described in Appendix A.

2.2. The correction of the basic formulas of compliance and \( G_{IC} \) for Arc-ENF specimen

The correction of the basic formulas can be realized in two aspects based on the research in the introduction section. The first one is the crack length correction in Eq. (4), reflecting the effect of shear deformation, and observing the form of \( f_i(\alpha_0) \), the corrected term with a assumption that correction is related to the crack length can be written as

\[ f_{\text{shear}}(\alpha_0) = \sin^2(\Delta \alpha \cdot \frac{\alpha_0}{\alpha}) \]  

where \( \Delta \alpha \) is constant. And the corrected formulas are

\[ C_{\text{shear}} = \frac{3R^2}{8E_iBh^3} \sin^2(\Delta \alpha \cdot \frac{\alpha_0}{\alpha}) \]  

\[ G_{\text{IC, shear}} = \frac{3F^2R^2}{16E_iB^4h^3} \frac{\Delta \alpha}{\alpha} \sin(2\Delta \alpha \cdot \frac{\alpha_0}{\alpha}) \]  

Another one is to consider the effect of friction. Due to the friction occurring between the upper and lower layers after delamination during the ENF test, the correction expression need to be derived from the energy of friction,

\[ U_f = k_f \cdot F_i \cdot X_i \]  

where \( k_f \) is a friction correction factor for the approximation of friction force and displacement, and their approximate expressions are

\[ F_i = \mu \frac{\alpha_0}{2\alpha} F, \ X_i = \int_{\alpha}^{\alpha + \alpha_0} 2\varepsilon_i(\theta) \cdot Rd\theta, \]  

where \( \mu \) is the coefficient of friction. Thus, the corrected term of compliance and \( G_{IC} \) are

\[ C_{\text{friction}} = -\frac{6k_f \mu R^2}{\alpha E_iBh^3} \cdot g(\alpha_0) \]  

\[ G_{\text{IC, friction}} = -\frac{3k_f \mu F^2R}{\alpha E_iB^4h^3} \cdot g_i(\alpha_0) \]  

where \( i=1, \ 0 < \alpha_0 \leq \alpha; \ i=2, \ \alpha < \alpha_0 < 2\alpha \), and \( g(\alpha_0), \ g_i(\alpha_0) \) are described in Appendix A.

2.3. Verification of the formulas of compliance and \( G_{IC} \) for Arc-ENF specimen

In this section, the numerical simulation will be used with CZM based on the ABAQUS for the verification of the formulas in section 2.2. The model is 2D with geometric parameters listed in table 1, and the materials of Arc-ENF specimen have two different properties [23] (table 2). As for the interface model, the bilinear CZM is used with its parameters (table 3) referred to the matrix of composites, and the interface failure adopts the maximum energy criterion. Besides, when the initial crack length is small, a larger load is required to cause the initial crack propagation, and at the same time, the load will suddenly drop, so the initial crack length set during simulation is large. This phenomenon can occur referred to the maximum load-displacement curve in Figure 2 or Figure 3.

Firstly, the influence of the crack length correction on the compliance and \( G_{IC} \) is studied. In this part, two different types of composite materials are used under two different energy release rates (\( G_{IC} \) =0.7, 1.4 kJ/m²), and finally the load F-displacement \( \delta \) curve is given. For the theoretical calculation results, the load force at each point on the curve represents the maximum force value under the corresponding displacement, and the crack propagation occurs when the load reaches this value. So the simulation results should be consistent with the theoretical during the crack propagation. By observing
the curves with two different materials in Figure 2 and Figure 3, it can be seen that there is a big difference between the theory and the simulation results before correction, but the corrected results are in good agreement, which indicates the reasonability of the correction. However, the value of $\Delta \alpha$ is related not only to the energy release rate of the interface, but also to the mechanical properties of the material. And when the difference between the longitudinal direction modulus and the transverse modulus of the material is small, the value of $\Delta \alpha$ is larger. Besides, with the increase of $G_{IIc}$, $\Delta \alpha$ becomes larger. So referring to the values of $\Delta \alpha$ showing in Figure 2 and Figure 3, $\Delta \alpha = 4.0^\circ$ is suggested for M1 material, and $\Delta \alpha = 3.0^\circ$ is suggested for M2 material, or revising the value with the numerical simulation results.

**Table 1.** Geometric parameters of the Arc-ENF specimen model.

| Radius, $R_0$ (mm) | Angle, $\alpha$ | Thickness, $h$ (mm) | Width, $b$ (mm) | Initial crack angle, $\alpha_0$ |
|-------------------|-----------------|---------------------|-----------------|------------------|
| 70                | 16.13$^\circ$   | 1                   | 10              | 14.13$^\circ$    |

**Table 2.** Mechanical properties of the Arc-ENF specimen model.

| Material types | Longitudinal modulus, $E_1$ (GPa) | Transverse modulus, $E_3$ (GPa) | In-plane shear modulus, $G_{13}$ (GPa) | Major Poisson's ratio, $\nu_{12}$, $\nu_{13}$ | Through thickness Poisson's ratio, $\nu_{23}$ |
|----------------|----------------------------------|----------------------------------|----------------------------------------|----------------------------------|----------------------------------|
| M1: T300/BSL914C | 138                              | 11                               | 5.5                                    | 0.28                             | 0.4                              |
| M2: Silkena E-Glass 1200tex/MY750 | 45.6                             | 16.2                             | 5.83                                   | 0.278                            | 0.4                              |

**Table 3.** Parameters of the interface of the Arc-ENF specimen model.

| Interface types | penalty stiffness ($\times 10^5$ N/m$^3$) | Tensile strength (MPa) | Thickness (mm) |
|-----------------|-------------------------------------------|------------------------|----------------|
| M1-interface    | 4                                         | 75                     | 0.01           |
| M2-interface    | 3.35                                      | 80                     | 0.01           |

**Figure 2.** Load-displacement curves of M1 material (Left: $G_{IIc} = 0.7$ kJ/m$^2$; Right: $G_{IIc} = 1.4$ kJ/m$^2$).

And based on the crack length correction, the influence of friction on the compliance and $G_{IIc}$ is also studied. The model is same with the above, and the energy release rate of the interface is set as 1 kJ/m$^2$ with $\Delta \alpha = 3.3^\circ$ for M1 material and $\Delta \alpha = 4.4^\circ$ for M2 material. Because of the approximation of friction force and displacement, the value of the friction correction factor $k_f$ is 0.25 by comparing the
theoretical calculation results with simulation results. Therefore, the load-displacement curves are given in Figure 4 under the different values of friction coefficient ($\mu$). It can be seen that the theoretical calculation results are in good agreement with the simulation results, which indicates the reasonability of the correction.

![Figure 3. Load-displacement curves of M2 material (Left: $G_{IC}=0.7$ kJ/m$^2$; Right: $G_{IC}=1.4$ kJ/m$^2$).](image)

![Figure 4. Load-displacement curves under different friction coefficients (Left: M1; Right: M2).](image)

3. The ENF test
The ENF test was used to obtain the $G_{IC}$ of the Arc-ENF specimen, seen in Figure 5. And the geometric parameters are given in table 4 and the mechanical properties are given in table 5. To ensure the quasi-static crack propagation, the loading speed was set at $4\times10^{-6}$ m/s, and the radius of the cylinders used for bearing and loading was 1.5 mm. As for the position of the crack tip, it’s difficult to observe with eyes. So the optical strain gauge was used to record the deformation in 1-3 plane of the specimen, and after processing, the strain diagram in any moment could be obtained, such as Figure 5. And the position of the crack tip (Point A in Figure 5) was determined by 5% strain, empirically, in the longitudinal direction. Finally, the value of load and displacement of the specimen were recorded.

| Table 4. Geometric parameters of the Arc-ENF specimen in ENF test. |
|---------------------------------|-------------------------------|-----------------------------|-------------------|------------------|------------------|
| Radius, $R_0$(mm)              | Angle, $\alpha$ (°)           | Displacement between left and right bearing point (mm) | Thickness, $h$(mm) | Width, $b$(mm)   | Initial crack angle, $\alpha_0$ |
| 71.2                           | 15.485°                      | 40                          | 1.05              | 8                | 14.485°          |
Table 5. Mechanical properties of the Arc-ENF specimen in ENF test.

| Longitudinal modulus, $E_1$(GPa) | Transverse modulus, $E_3$(GPa) | In-plane shear modulus, $G_{13}$(GPa) | Poisson's ratio | penalty stiffness of interface ($\times 10^5$ N/m$^3$) | Tensile strength of interface (MPa) | Thickness of interface (mm) |
|----------------------------------|---------------------------------|--------------------------------------|----------------|---------------------------------------------------|------------------------------------|--------------------------|
| 160                              | 8                               | 5                                    | 0.3           | 4                                                 | 50                                 | 0.01                     |

Figure 5. The ENF test facility (Left: the physical picture; Right: the strain diagram of the specimen).

4. Result analysis
Although the corrected formulas of the compliance and $G_{IIC}$ for the Arc-ENF specimen verified above were effective, and the necessary data in the ENF test had been recorded, there are still two problems to be answered. The first one is whether the crack tip position determined by the 5% strain is near or not to the $\alpha_0$ in the formulas, and another one is that the value of $\mu$ for the specimen is not sure. The following content will give answers and finally, the $G_{IIC}$ value and friction coefficient will be determined.

Figure 6. The numerical simulation results of the ENF test.

The problem about the crack tip position derives from the formula derivation, in which the deformation of the specimen was divided into two parts, where the uncrack part of specimen should...
have little deformation in the crack tip. But in fact, the interface has a certain deformation, like the part between point A and B, or C and D in Figure 6. And around the point B and D, the specimen has little deformation determined by 5% of the maximum Mises stress. So, the recorded crack tip position has a difference from the $\alpha_i$ in the formulas. Considering the length AB is 4.2 mm, and the length CD is 4.7 mm, the final correction of the crack tip position recorded in the ENF test is 4.45 mm. The parameters used in simulation come from table 4 and table 5 with $\mu=0$ and $G_{Ic}=1.0$ kJ/m$^2$, and $\Delta \alpha =2.9^\circ$.

As for the friction coefficient $\mu$, there were four different groups of $\mu$ (0.1, 0.3, 0.5 and 0.7) used as the known parameter in the formulas to calculate the Arc-ENF specimen’s $G_{Ic}$ (1.0944, 1.0411, 0.9878, 0.9345, unit: kJ/m$^2$), combined with the corrected recorded test data, seen in Figure 7. And then, the results of the numerical simulation of the ENF test with these four group parameters, in form of load-displacement curve, are shown in Figure 8. Finally, by comparing the simulation results and test data, $\mu=0.5$ and $G_{Ic}=0.9878$ kJ/m$^2$ are determined.

![Figure 7. The $G_{Ic}$ results under different $\mu$ conditions.](image)

![Figure 8. Load-displacement curves of the numerical simulation result and test data.](image)

5. Discussion

The method to obtain the corrected formulas of the compliance and $G_{Ic}$ for the Arc-ENF specimen is similar to the normal ENF specimen in the ENF test, which contains two terms, the crack length correction and the effect of friction correction.

For the first one, the correction reflecting the effect of shear deformation is expressed with an additional expression (Eq. 10) for the Arc-ENF specimen, which is different from that for the normal ENF specimen, expressed by the change of the crack length (Eq.4). It’s notable that both of the corrections are small, compared with the bigger crack length correction found by comparing the strain diagrams from the optical strain gauge (Figure 5) and the numerical simulation results (Figure 6). The bigger crack length correction drives from the deformation of interlamination at the crack tip, and as the fracture extensibility of the interlamination increasing, the value of the correction will be bigger. To obtain the expression of this correction, it should be derived from the basic elastic theory, and the finding is also valuable for the further correction on the formulas for the normal ENF specimen.

For the effect of friction correction, there are many research results for the normal ENF specimen introduced in the section 1, which are based on the point-friction assumption. However, due to the smooth surface of the cylinders of bearing and loading, this assumption can be ignored in the ENF test for the Arc-ENF specimen, instead the friction occurring between the upper and lower layers after delamination is considered. And comparing the results of the numerical simulation and the ENF test,
the value of $G_{IIc}$ calculated by the corrected formulas is reasonable, which has reference value on the correction of formulas for the normal ENF specimens.

6. Conclusion
In this paper, a corrected formulas of the compliance and $G_{IIc}$ for Arc-ENF specimen in the ENF test are introduced, with the verification by comparing the theoretical results with numerical simulation results. And for the test method, the optical strain gauge is used to determine the position of the crack tip. Finally, $G_{IIc} = 0.9878$ kJ/m² of the Arc-ENF specimen is obtained from the use of the corrected formulas, the numerical simulation method and the ENF test data.

Appendix A
The flexure moment $M$ is

$$ M(\theta) = \begin{cases} 0.5FR \cdot (\sin(\alpha) + \sin(\theta)) & -\alpha \leq \theta \leq 0 \\ 0.5FR \cdot (\sin(\alpha) - \sin(\theta)) & 0 < \theta \leq \alpha \end{cases} $$

(A.1)

The shear force $\tau$ is

$$ \tau(\theta, x_i) = \frac{F_i(\theta)}{2I} \cdot \left( \frac{h_i^2}{4} - x_i^2 \right), \text{ where } F_i(\theta) = \frac{F}{2\cos(\theta)} \quad -\alpha \leq \theta \leq \alpha $$

(A.2)

The strain energy $U$ is

$$ U = U_1 + U_2 = \frac{F^2R}{8E_I} \cdot f_1(\alpha_0) + \frac{3F^2R}{40G_{IIc}Bh} \cdot g(\alpha_0) $$

(A.3)

where

$$ f_1(\alpha_0) = \left( \frac{1}{2} + \sin(\alpha_0) \right)(3\alpha_0 + 2\alpha) + 2\sin(\alpha_0) \cdot (5\cos(\alpha_0) - 3\cos(\alpha_0 - \alpha)) - 2 + \frac{1}{4} \left( 3\sin(2\alpha_0 - 2\alpha) - 5\sin(2\alpha) \right) $$

(A.4)

$$ f_2(\alpha_0) = \left( \frac{1}{2} + \sin(\alpha_0) \right)(3\alpha_0 + 2\alpha) + 2\sin(\alpha_0) \cdot (5\cos(\alpha_0) + 3\cos(\alpha_0 - \alpha)) - 8 + \frac{1}{4} \left( 3\sin(2\alpha_0 - 2\alpha) - 5\sin(2\alpha) \right) $$

(A.5)

$$ g(\alpha_0) = \frac{2\alpha + 3\alpha_0}{2} + \frac{5\sin(2\alpha) + 3\sin(2(\alpha_0 - \alpha))}{4} $$

(A.6)

The expressions in formulas corrected by the effect of friction are

$$ g_{f1}(\alpha_0) = \alpha_0 \cdot [\sin(\alpha)\alpha_0 - \cos(\alpha_0 - \alpha) + \cos(\alpha)] $$

(A.7)

$$ g_{f2}(\alpha_0) = \alpha_0 \cdot [\sin(\alpha)\alpha_0 + \cos(\alpha_0 - \alpha) + \cos(\alpha) - 2] $$

(A.8)

$$ g'_{f1}(\alpha_0) = \alpha_0 \cdot [2\sin(\alpha) + \sin(\alpha_0 - \alpha)] + \cos(\alpha) - \cos(\alpha_0 - \alpha) $$

(A.9)

$$ g'_{f2}(\alpha_0) = \alpha_0 \cdot [2\sin(\alpha) - \sin(\alpha_0 - \alpha)] + \cos(\alpha) + \cos(\alpha_0 - \alpha) - 2 $$

(A.10)

References
[1] Brunner A J, Blackman B R K and Davies P 2008 A status report on delamination resistance testing of polymer–matrix composites Eng. Fract. Mech. 75(9) 2779-94
[2] Tabiei A and Zhang W 2018 Composite laminate delamination simulation and experiment: A review of recent development Appl. Mech. Rev. 70(3) 030801
[3] ASTM International 2019 Standard Test Method for Determination of the Mode II Interlaminar Fracture Toughness of Unidirectional Fiber-Reinforced Polymer Matrix Composites (USA: ASTM International)
[4] Russell A J and Street K N 1985 Moisture and temperature effects on the mixed-mode delamination fracture of unidirectional graphite/epoxy (USA: Pittsburgh)
[5] Silva M A L, de Moura M F S F and Morais J J L 2006 Numerical analysis of the ENF test for mode II wood fracture Compos. Part A-Apppl. S. 37(9) 1334-44
[6] Whitney J M 1990 Analysis of interlaminar mode-II bending specimens using a higher-order beam theory J. Reinf. Plast. Comp. 9(6) 522-36
[7] Chatterjee S N 1991 Analysis of Test Specimens for Interlaminar Mode II Fracture Toughness, Part I. Elastic Laminates J. Compos. Mater. 25(5) 470-93
[8] Wang Y and Williams J G 1992 Corrections for mode II fracture toughness specimens of composites materials Compos. Sci. Tech. 43(3) 251-6
[9] Wang J and Qiao P 2004 Novel beam analysis of end notched flexure specimen for mode-II fracture Eng. Fract. Mech. 71(2) 219-31
[10] Zebar M E M, Hattali M L and Mesrati N 2020 Interfacial fracture toughness measurement in both steady state and transient regimes using four-point bending test Int. J. Fract. 222(1-2) 123-35
[11] Wang W X, Nakata M, Takao Y and Matsubara T 2009 Experimental investigation on test methods for mode II interlaminar fracture testing of carbon fiber reinforced composites Compos. Part A- Appl. S. 40(9) 1447-55
[12] Liu W and Chen P 2019 A simple procedure for the determination of the cohesive law in 4-ENF test with consideration of the friction and R-curve effect Eng. Fract. Mech. 220 106651
[13] Mencattelli L, Borotto M, Botsis J and Lazzeri R 2018 Analysis and evaluation of friction effects on mode II delamination testing Compos. Struct. 190 127-36
[14] Parrinello F, Failla B and Borino G 2009 Cohesive–frictional interface constitutive model Int. J. Solids. Struct. 46(13) 2680-92
[15] Parrinello F, Marannano G, Borino G and Pasta A 2013 Frictional effect in mode II delamination: Experimental test and numerical simulation Eng. Fract. Mech. 110 258-69
[16] Parrinello F 2018 Analytical solution of the 4ENF test with interlaminar frictional effects and evaluation of Mode II delamination toughness J. Eng. Mech. 144(4) 01433
[17] Dugdale D S 1960 Yielding of steel sheets containing slits J. Mech. Phys. Solids 8(2) 100-4
[18] Barenblatt G I 1962 The mathematical theory of equilibrium cracks in brittle fracture Adv. Appl. Mech. 7(C) 55-129
[19] Dourado N, de Moura M F S F, de Morais A B and Pereira A B 2012 Bilinear approximations to the mode II delamination cohesive law using an inverse method Mech. Mater. 49 42-50
[20] de Morais A B, Pereira A B, de Moura M F S F, Silva F G A and Dourado N 2015 Bilinear approximations to the mixed-mode I–II delamination cohesive law using an inverse method Compos. Struct. 122 361-6
[21] Nguyen N and Waas A M 2016 A novel mixed-mode cohesive formulation for crack growth analysis Compos. Struct. 156 253-62
[22] Xie J, Waas A M and Rassaian M 2016 Closed-form solutions for cohesive zone modeling of delamination toughness tests Int. J. Solid. Struct. 88-89 379-400
[23] Soden P D, Hinton M J and Kaddour A S 1998 Lamina properties, lay-up configurations and loading conditions for a range of fibre-reinforced composite laminates Compos. Sci. Tech. 58(7) 1011-22