Probing Andreev reflection reach in semiconductor-superconductor hybrids by Aharonov-Bohm effect

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Recent development in fabrication of hybrid nanostructures allows for creation of quantum interferometers that combine semiconductor and superconductor materials [S. Gazibegovic et al., Nature 548, 434 (2017)]. We show that in those nanostructures the joint phenomena of Aharonov-Bohm effect and Andreev reflections can be used to determine the length on which the electron is retro-reflected as a hole. We propose to exploit this feature for probing of the quasiparticle coherence length in semiconductor-superconductor hybrids by a magnetoconductance measurement.

Recently there is a great interest in semiconductor-superconductor hybrids that under proper tuning of the external magnetic field can realize topological superconducting phase [1, 2] that hosts quasi-particles equivalent of Majorana fermions [3, 4]. In those hybrids, experimentally realized in the form of quasi one-dimensional [5–7] or two-dimensional nanowires [8, 9] contacted with a thin superconducting shell, the coupling between electron-like and hole-like quasi-particles is induced by the proximity effect microscopically governed by the Andreev reflections. Experimental studies of those devices rely on electronic transport measurements where the charge carriers are transferred between the normal and proximitized part. The electrons propagating with Fermi velocity $v_f$ from the semiconducting part undergo Andreev reflection at the normal-superconducting (NS) interface on a distance corresponding to wave function decay length. In the proximitized part $\Psi \sim \exp[-x/\xi]$, where

$$\xi = \frac{\hbar v_f}{\Delta},$$

mimics the coherence length of quasi-particles penetrating into the ordinary superconductor. The reach of Andreev reflection $\xi$ is interesting not only for fundamental reasons, but also it is crucial for the analysis of transport measurements as it determines the extent of the structure accessible for probing by the means of tunneling spectroscopy. Specifically, recent experiments reported measurement of conductance quantization in Majorana nanowires [10–14] that signifies ballistic transport over the coherence length [15, 16].

In this letter we propose a unique method to determine the quasiparticle coherence length $\xi$ of the proximitized semiconductor. We exploit recent developments in bottom-up synthesis that allows for fabrication of crossing hybrid nanowires [17]. Nanowire branches can be formed into closed loops creating quantum phase-coherent interferometers with predefined number of epitaxial superconductor-semiconductor interfaces. Utilizing combined Aharonov-Bohm [18] effect and Andreev reflection we exploit this novel generation of hybrid structures to indirectly visualize the length on which the Andreev reflection takes place.

The proposed concept can be explained on an example of quantum ring proximitized by a superconductor [Fig. 1(a)]. In the presence of the magnetic field the transport properties of an ordinary quantum rings is mainly determined by the interference effect as the phases

Figure 1. a) Cartoon of the quantum interferometer in a form of semiconductor ring proximitized by a superconductor with two semi-infinite leads. The gray and pink regions correspond to normal and proximitized parts, respectively. The arrows show propagating electron and hole trajectories with the region where the quasiparticle wave function decays in the superconducting part over the coherence length $\xi$. b) Numerically obtained local density of states of proximitized interferometer for $\Delta = 0.25$ meV and $B = 5$ mT.
\[ \phi = \frac{\pi}{2} \int \mathbf{A} \cdot d\mathbf{l} \] acquired from the magnetic field by the charged particles traveling through the upper and lower arm differ from each other. Consequently, the conductance of the structure undergoes Aharonov-Bohm oscillations with the period of \( \phi_0 = h/e \), as reported experimentally for metallic [19] and semiconducting [20] quantum rings already in late 80’s. The Aharonov-Bohm effect has been reported only recently for hybrid nanowire networks [17] which allow to combine this effect with the scattering processes that occur at the NS interface.

Let us consider the nanostructure with a NS interface depicted schematically in Fig. 2 (a). When the excitation energy of the incoming electron lies inside the superconducting gap the electron undergoes Andreev reflection at the NS interface. As a result, the period of the Aharonov-Bohm oscillations is determined by the phase accumulated both by the propagating electron and the retro-reflected hole. The particles acquire the phase during propagation on the total length \( L_i = L_i + \xi \), where \( L_i \) is twice the distance of propagation through the half of one arm of the interferometer – one for scattering of electron, one for the hole – for the case of half-covered ring. Most importantly the length \( L_i \) is increased by \( \xi \) that stems from the phase accumulated by the electron and hole evanescent modes in the proximitized part where \( \langle e^{-x/\xi} | \mathbf{A} | e^{-x/\xi} \rangle \sim \xi/2 \). The elongation of the effective propagation length of the particles by Andreev reflection will lead to a decrease of the Aharonov-Bohm oscillation period.

\[ H = (\hbar^2 k^2/2m^* - \mu) \sigma_0 \tau_z + \Delta \sigma_0 \tau_x + \alpha (\sigma_x k_y - \sigma_y k_z) \tau_z \]

acting on the spinor \( \Psi = (\psi^e_r, \psi^e_i, \psi^h_r, -\psi^h_i)^T \), where \( e \) (\( h \)) corresponds to electron-like (hole-like) component and \( \uparrow \) (\( \downarrow \)) denotes spin up (down) component. \( \Delta \) is the effective induced pairing potential, \( \sigma_i \) and \( \tau_i \) with \( i = x, y, z \) are the Pauli matrices acting on spin- and electron-hole degrees of freedom, respectively. The orbital effects of the magnetic field are included through the canonical momentum, \( \mathbf{k} = -i \nabla + e \mathbf{A}/\hbar \cdot \tau_z \) with the vector-potential in the Lorentz gauge \( \mathbf{A} = [-yB, 0, 0] \). As the considered magnetic field are of order of mT we neglect Zeeman effect and include spin-orbit coupling whose strength is controlled by the parameter \( \alpha \). The adopted material parameters correspond to recently studied InSb nanostructures [17] with \( m^* = 0.014m \). The numerical problem is solved by discretizing Eq. (2) on a square grid with grid spacing \( \delta x = \delta y = 4 \) nm using Peierls substitution of the hopping elements \( t_{nm} \rightarrow t_{nm} \exp \left[-ie \int |A|d|l|/\hbar \right] \) to account for the orbital effects of the magnetic field. We assume that the interferometer is connected to semi-infinite leads and calculate the scattering matrix using Kwant [21] package that implements wave function matching method. Finally, we obtain the conductance in the linear response regime at zero temperature as \( G = e^2/h \cdot (N - R_{ee} + R_{he}) \), where \( N \) is the number of transverse modes in the leads and \( R_{ee} \) (\( R_{he} \)) correspond to the backscattering probability of electrons into electrons (holes).

Let us start by considering the semiconducting ring with the channel width 80 nm and the radius \( R = 640 \) nm depicted in Fig. 2 (b). For simplicity we neglect the spin-orbit coupling by setting \( \alpha = 0 \). Firstly we consider the case of pure semiconducting ring and assume the chemical potential \( \mu = 5 \) meV such there is one occupied spin-degenerate conducting mode. The map in Fig. 2(a) presents conductance as a function of the magnetic field and incoming electron energy. We observe that the conductance oscillates in \( B \) independently on the electron energy, with the period \( B_p \approx 3.2 \) mT which corresponds to the flux quanta \( \phi = BL^2/\pi \) with \( L = \pi R \). Now let us consider that the ring is half-covered by the superconductor – however the symmetric coverage of the ring is not a vital assumption of the model. We set \( \Delta = 0.25 \) meV and present the corresponding conductance map in Fig. 2 (b). We can clearly subdivide the map into two regions. For \( E > \Delta \) where the excitation energy \( E \) exceeds the superconducting gap we see the same pattern in conductance oscillations as in the panel (a) but overlaid with resonances on Andreev bound states that are most pronounced at the proximity of the gap edge. For \( E < \Delta \) (below the white dashed line) where we expect the elongation of the effective propagation length of the particles by \( \xi \) due to the Andreev reflection, we observe

![Figure 2. Conductance map of a quantum ring versus the excitation energy E of incoming electron and the magnetic field B calculated for \( \Delta = 0 \) (a) and \( \Delta = 0.25 \) meV (b). The dashed horizontal line in (b) denotes the energy of the superconducting gap.](image-url)
the conductance oscillation pattern with much less period than the one found for $E > \Delta$. The magnitude of the maximal conductance is doubled for $E < \Delta$ due to transfer of 2e charge through Andreev reflection process [15, 16, 22]. Finally, inspecting the local density of states for $B = 5$ mT and $E = 1$ meV in Fig. 1 (b) we observe decaying probability density in the proximitized part as expected from Eq. (1).

Finally we turn our attention to the implementation of the discussed concept in state-of-the-art experimental devices. Namely, we consider a structure formed by crossing nanowires such they form a hashtag – a square interferometer [17] as presented in the inset of Fig. 3 (b). The Aharonov-Bohm conductance oscillations have been already measured in those devices and the development in the superconductor deposition allows for an arbitrary arrangement of the superconducting leads – either during the growth stage [23] or later by the means of litographical deposition [18]. In our calculations we assume that the device is connected to a normal and superconducting leads through two protruding ends of the wires and that two arms of the structure are proximitized by a superconductor that opens the energy gap $\Delta$ therein – see inset in Fig. 3 (b). We assume the arm length 600 nm, the width 80 nm and spin-orbit coupling with the strength comparable to the one reported experimentally i.e. $\alpha = 50$ meVnm [23–25].

Figure 3 (a) shows conductance traces calculated at $E = 0$ for several values of the gap parameter $\Delta$ ranging from 0.2 meV to 1.9 meV. The traces are sequentially shifted by 0.08 $e^2/h$ for clarity of the presentation. We see that as the gap is increased the Aharonov-Bohm oscillation period – the distance between the two red dots – also raises. Note that for the gap 0.2 meV the wave function tail in the proximitized region is more than the length of the arms and there are concurrent processes of interference at the input and output connectors which leads to the change of the conductance trace character.

To qualitatively analyze the correspondence between the induced gap parameter $\Delta$ and the Aharonov-Bohm oscillations we compare the coherence length predicted theoretically with the ones that result from the conductance oscillation period. The theoretical estimation of the coherence length is obtained for each transverse mode as,

$$\xi = \frac{\hbar}{\Delta} \sqrt{\frac{2E_g}{m^*}},$$  \hspace{1cm} (3)$$

where we estimate $E_g = \mu - E_0$ with $E_0$ being the excitation energy of the ground-state transverse mode. The analytically estimated coherence length is plotted with the blue curve in Fig. 3 (b). At the same time we extract the coherence length from Aharonov-Bohm oscillation period according to formula

$$\xi = \sqrt{\frac{\hbar}{eB_p} - L_i},$$ \hspace{1cm} (4)$$

obtained by comparing flux quanta $\hbar/e$ with the flux pen-
etrating the device, i.e. $(L_q^2)^2 B$ and plot it in Fig. 3 with the red dots. Taking $L_q$ as twice the length of the uncovered arm, i.e. 1200 nm we observe exceptional agreement between the theoretically predicted and probed coherence length.

The experimental estimation of $E_q$ is cumbersome due to the inability to directly measure the ground state energy of each conduction mode, which makes it difficult to directly apply the formula Eq. (3) to deduce the coherence length. On the contrary, the method that we propose here bases on easily achievable conductance measurement of the Aharonov-Bohm oscillations which renders it as a feasible mean for experimentally probing the quasiparticle coherence length in proximitized nanostructures.

In summary we considered the effect of Andreev reflection in the quantum interferometer partially proximitized by superconductor in the presence of the external magnetic field. We showed that the period of the Aharonov-Bohm conductance oscillations depends on the electron and hole wavefunctions decay length $\xi$ in the proximitized part allowing to probe the length on which the Andreev reflection occur in experimentally realizable devices. We proposed to exploit this feature for direct probing of quasiparticle coherence length by magnetoconduction measurement and confronted this idea with numerical quantum transport simulations of quantum rings and nanowire hashtags.

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