Di Liberti, Ivan; Loregian, Fosco
Accessibility and presentability in 2-categories. (English) Zbl 07567646
J. Pure Appl. Algebra 227, No. 1, Article ID 107155, 25 p. (2023)

Summary: We outline a definition of accessible and presentable objects in a 2-category \( \mathcal{K} \) endowed with a “KZ context”, that is to say a pair of lax-idempotent monads interacting in a prescribed way; this perspective suggests a unified treatment of many “Gabriel-Ulmer like” theorems, asserting how presentable objects arise as reflections of generating ones. We outline the notion of (Gabriel-Ulmer) envelope for a KZ context, sufficient to concoct Gabriel-Ulmer duality. We end the paper with a roundup of examples, involving classical (set-based and enriched), low dimensional category theory, and a perspective for future work, rooted in higher category theory and homotopy theory.

MSC:
18C35 Accessible and locally presentable categories
18A05 Definitions and generalizations in theory of categories
18D05 Double categories, 2-categories, bicategories and generalizations (MSC2010)
18D65 Proarrow equipments, Yoneda structures, KZ doctrines (lax idempotent monads)

Full Text: DOI

References:
[1] Adámek, J.; Borceux, F.; Lack, S.; Rosicky, J., A classification of accessible categories, J. Pure Appl. Algebra, 175, 1-3, 7-30 (2002) · Zbl 1010.18005
[2] Adámek, J.; Rosicky, J., Locally Presentable and Accessible Categories, London Mathematical Society Lecture Note Series, vol. 189 (1994), Cambridge University Press: Cambridge University Press Cambridge · Zbl 0803.18001
[3] Adámek, J.; Rosicky, J.; Vitale, E. M., Algebraic theories, Camb. Tracts Math., 184, 1 (2011) · Zbl 1209.18001
[4] Borceux, F.; Dejean, D., Cauchy completion in category theory, Cah. Topol. Géom. Différ. Catég., 27, 2, 133-146 (1986), (en). MR 850528 · Zbl 0595.18001
[5] Bunge, M.; Funk, J., On a bicomma object condition for KZ-doctrines, J. Pure Appl. Algebra, 143, 1-3, 69-105 (1999) · Zbl 0935.18006
[6] Borceux, F., Handbook of Categorical Algebra. 1: Basic Category Theory, Encyclopedia of Mathematics and Its Applications, vol. 50 (1994), Cambridge University Press: Cambridge University Press Cambridge · Zbl 0803.18001
[7] Borceux, F.; Quinteiro, C., Enriched accessible categories, Bull. Aust. Math. Soc., 54, 3, 489-501 (1996) · Zbl 0881.18011
[8] Borceux, F.; Quinteiro, C.; Rosicky, J., A theory of enriched sketches, Theory Appl. Categ., 4, 3, 47-72 (1998) · Zbl 0981.18006
[9] Centazzo, C., Generalised Algebraic Models (2004), Presses Universitaires de Louvain
[10] Centazzo, Claudia; Vitale, E. M., A duality relative to a limit doctrine, Theory Appl. Categ., 10, 20, 486-497 (2002) · Zbl 1019.18003
[11] Day, B. J.; Lack, S., Limits of small functors, J. Pure Appl. Algebra, 210, 3, 651-663 (2007) · Zbl 1120.18001
[12] Groth, M.; Ponto, K.; Shulman, M., The additivity of traces in monoidal derivators, J. K-Theory, 14, 3, 422-494 (2014) · Zbl 1349.18015
[13] Groth, M.; Derivators, pointed derivators and stable derivators, Algebra Geom. Topol., 13, 1, 313-374 (2013) · Zbl 1266.55009
[14] Groth, M.; Šťovíček, J., Tilting theory via stable homotopy theory, J. Reine Angew. Math. (2014)
[15] Groth, M.; Šťovíček, J., Abstract tilting theory for quivers and related categories, Ann. K-Theory, 3, 1, 71-124 (2018) · Zbl 1382.55012
[16] Gabriel, P.; Ulmer, F., Lokal präsentierbare Kategorien, Lecture Notes in Mathematics, vol. 221 (1971), Springer-Verlag: Springer-Verlag Berlin · Zbl 0225.18004
[17] Kelly, G. M., Structures defined by finite limits in the enriched context, I, Cah. Topol. Géom. Différ. Catég., 23, 1, 3-42 (1982) · Zbl 0538.18006
[18] Kelly, G. M., Basic concepts of enriched category theory, Repr. Theory Appl. Categ. (2005), vi+137 pp. (electronic), Reprint of the 1982 original [Cambridge Univ. Press, Cambridge; MR0651714] · Zbl 1086.18001
[19] Kock, A., Monads for which structures are adjoint to units, J. Pure Appl. Algebra, 104, 1, 41-59 (1995), MR 1359690 · Zbl 0849.18008
[20] Kashiwara, Masaki; Schapira, Pierre, Categories and Sheaves, Grundlehren der Mathematischen Wissenschaften, vol. 332
