Application of Expurgated PPM to Indoor Visible Light Communications—Part II: Access Networks

Mohammad Noshad, Student Member, IEEE, and Maïté Brandt-Pearce, Senior Member, IEEE

Abstract—Providing network access for multiple users in a visible light communication (VLC) system that utilizes white light emitting diodes (LED) as sources requires new networking techniques adapted to the lighting features. In this paper we introduce two multiple access techniques using expurgated PPM (EPPM) that can be implemented using LEDs and support lighting features such as dimming. Multilevel symbols are used to provide \( M \)-ary signaling for multiple users using multilevel EPPM. In the first technique, the \( M \)-ary data of each user is first encoded using an optical orthogonal code assigned to the user, and the result is fed into an EPPM encoder to generate a multilevel signal. The second multiple access method uses subsets of the EPPM constellation to apply MEPPM to the data of each user. While the first approach has a larger Hamming distance between the symbols of each user, the latter can provide higher bit-rates for users in VLC systems using bandwidth-limited LEDs. Both techniques are able to support up to 15 simultaneous users transmitting 200 Mb/s with a bit error rate of \( 10^{-3} \) under normal lighting conditions.

Index Terms—Balanced incomplete block designs (BIBD), expurgated pulse position modulation (EPPM), optical code division multiple access (OCDMA), optical networks, visible light communications (VLC).

I. INTRODUCTION

Visible light communications (VLC) is an appealing technology for network access in indoor environments since it is immune to radio-frequency (RF) interference, has low impact on human health, and is able to provide a high data-rate connection [2]. It has been proposed as an alternative to Wi-Fi to provide high-speed access for tablets, phones, laptops and other devices in indoor spaces such as offices, homes, airplanes, hospitals, and convention centers. The application of VLC to indoor networking not only requires the capability to provide simultaneous connection for a large number of users, but also to meet the requirements of the lighting system, mainly dimming. This paper is Part II of a two-part paper. Part I addresses single-user communications, while Part II introduces two techniques to provide high-speed multiple access for simultaneous users in a VLC system.

Integrating VLC networks with illumination systems imposes limitations on the modulations and networking techniques that can be used. White light emitting diodes (LEDs) are the most common optical sources that are used in VLC systems, and modulation schemes that can be used with these devices are limited. Because of the structure of these LEDs and their inherent nonlinearity, implementing modulation and multiple-access approaches that require frequency-domain processing may be expensive and complicated. Therefore, time-based modulations, specially pulsed techniques, are the preferred modulation technique in LED-based VLC systems. Dimming is an important feature of indoor lighting systems through which the illumination level can be controlled. Including dimming in VLC system requires further constraints on the multiple-access schemes that can be used. A practical VLC network should support various optical peak to average power ratios (PAPR) so that, for a fixed peak power, the average power, which is proportional to the illumination, can be regulated.

Optical code division multiple access (OCDMA) is a networking technique that provides multiple access by assigning binary signature patterns to users [3]. Among various OCDMA forms that have been proposed, direct-sequence OCDMA is of most interest for indoor VLC system, since it can be implemented by simply turning the LEDs on and off [4]. In this type of OCDMA, binary sequences with special cross-correlation constraints, such as optical orthogonal codes (OOC), are used to encode the data of users in the time-domain [5]. Codewords of an OOC are binary sequences that meet a given correlation constraint [6]. The application of OOCs to VLC networks requires codes with a wide range of parameters for different dimming levels, which may not be practical for a network with a large number of users.

In part I of this two-part paper [7], we propose to apply expurgated pulse-position modulation (EPPM) [8] and multilevel-EPPM (MEPPM) [9], both based on balanced incomplete block designs (BIBD), to indoor VLC systems in which the dimming can be done by simply changing the generating BIBD code. Only single-user systems are studied. In this part II of the paper, two networking methods using MEPPM, which can be considered synchronous OCDMA methods, enable users in a VLC network to have high-speed access to the network. In the first method, we assign one OOC codeword to each user in order to encode its \( M \)-ary data. For each user, every bit of this encoded binary sequence is multiplied by a BIBD codeword, and then the OOC-encoded BIBD codewords are added to generate a multilevel signal. Hence, the PAPR of the transmitted data can be controlled by changing the code-length to code-weight ratio of the BIBD code. In the second technique, a subset of BIBD...
codewords is assigned to each user, and then the MEPPM scheme is used to generate multi-level symbols using the assigned codewords. In this approach, users can have different bit-rates by partitioning the BIBD code into unequal-size subsets. This paper is an extension of the techniques proposed in [1]. Different detectors are proposed for the first networking technique, and their performance is analyzed for different noise statistics. Analytic expressions are also derived for the error probability of these two techniques, and the performances are compared using this analysis.

The organization of the rest of the paper is as follows. In Section II we describe the indoor VLC network. The two networking methods to provide simultaneous access for multiple users are proposed in Section III. The performance of the proposed techniques are compared using numerical results in Section IV. Finally, Section V concludes the paper.

II. SYSTEM DESCRIPTION

This section describes the principles of a VLC network. In LED-based VLC systems, both lighting and communications needs can be addressed at the same time. The downlink configuration of a VLC network is shown in Fig. 1, where arrays of white LEDs are used as sources. Here, we consider LED arrays as access points, and we assume that all LED arrays in a room are synchronous and transmit the same data. For each user, usually the strongest received power is the one that corresponds to the signal received from the direct path of the closest LED array, and therefore, it is considered as the main signal and the main data source. In situations where the direct path to the main source is blocked, the data can be retrieved using the multipath signals received from non-line-of-sight paths [7]. In this paper, VLC is assumed to be used only for the downlink, and another independent system, such RF or infrared (IR) communications, is used for the lower data-rate uplink channel to avoid self-interference from the full-duplex communication.

In [9] multilevel signals are constructed by combining multiple BIBD codes, which are then used as modulation symbols in multilevel EPPM (MEPPM). Each symbol-time is divided into equal time-slots, and then subsets of LEDs within the array are turned ON and OFF according to a BIBD codeword. In this study, BIBDs are used to generate the multiple-access codewords. We utilize a BIBD code with parameters \((Q, K, \lambda)\), where \(Q\), \(K\), and \(\lambda\) denote the code-length, the Hamming weight, and the cross-correlation between any two codewords, respectively. The \(m\)th codeword is the binary vector \(c_m = (c_{m1}, c_{m2}, \ldots, c_{mQ}) = c_m^{(m)}\), \(m = 1, 2, \ldots, Q\), where the notation \(x^{(m)}\) is the \(m\)th cyclic shift of the vector \(x\). The following relation holds between \(c_m\)'s and is referred to as the fixed cross-correlation property [10]:

\[
\sum_{i=1}^{Q} c_{mi} c_{nj} = \begin{cases} K; & m = n, \\ \lambda; & m \neq n. \end{cases}
\]

Four different MEPPM schemes are introduced in [9], two of which (called type-I and type-II MEPPM) have a PAPR of \(Q/K\) and are used in this paper. An implementation using on-off modulated LEDs is shown in Fig. 2(a).

The structure of a simple receiver for single user EPPM and MEPPM systems using a shift-register is shown in Fig. 3. In this receiver, for symbol-epoch \(k\), the sampled data at the output of a pulse-matched filter, \(r_k\), is stored in a shift register, and then is circulated inside it to generate vector \(z_k = (z_{k1}, z_{k2}, \ldots, z_{kQ})\), \(z_{kj} = \langle r_k, c_j \rangle\), at the output of the differential circuit, where \(\langle x, y \rangle\) denotes the dot product of the vectors \(x\) and \(y\). In this figure, \(T_s\) is the symbol time and \(\Gamma = \lambda/(w - \lambda)\). The wires of the lower branch are matched to the first codeword of the BIBD code, \(c_1\), and those of the upper branches are matched to its complement. This receiver is equivalent to the correlation decoder, which is shown to be the optimum decoder for additive-white-Gaussian-noise (AWGN) channels in [8] and for shot-noise channels in [7].

III. MULTIPLE ACCESS FOR INDOOR OPTICAL NETWORKS

Indoor optical networks must be able to provide simultaneous access for multiple users. Optical code division multiple access
(OCDMA) can be used to fulfill this need. In VLC systems, since the access points are illumination sources and are the same for all users, synchronous OCDMA techniques can be used for the downlink. In [4], synchronous time-spreading OCDMA using OOC and on-off keying (OOK) was studied. Because of the limited bandwidth of LEDs and the long length of OOCs needed, the data-rate for each user is low. An efficient technique to increase the data-rate in OCDMA systems is \( M \)-ary modulation using cyclic shifts of the OOC codewords, which is called code cycle modulation (CCM) in [11]. In this modulation, any cyclic shift of an OOC codeword with length \( L \) is considered as a symbol, and therefore, the bit-rate is increased by a factor of \( \log_2 L \). This technique is highly susceptible to synchronization errors, as is any technique using cyclic codes.

A limitation of using OOCs in VLC systems is their incompatibility with the dimming feature. For a CCM OCDMA system that uses an OOC with length \( L \) and weight \( w \), the PAPR is \( L/w \). Furthermore, for an OOC code with length \( L \), weight \( w \), and cross-correlation \( \alpha \), the number of codewords, which must be at least as large as the number of users, \( N \), is bounded by the Johnson bound [6]

\[
N \leq \left\lfloor \frac{1}{w} \left( \frac{L-1}{w-1} \left[ \frac{L-2}{w-2} \cdots \left( \frac{L-\alpha}{w-\alpha} \right) \cdots \right] \right) \right\rfloor. \tag{2}
\]

Changing the pulse duty-cycle is not possible for bandwidth-limited sources, while changing the pulse amplitude requires a complex tunable circuit due to the source nonlinearity. Therefore, in an OCDMA network with a given number of users, changing the PAPR requires employing a new OOC code, which, considering (2), may not be possible for low PAPRs. As an example, for a PAPR of 2, we should have \( L = 2w \), which implies that \( \alpha > w^2/L = w/2 \) and corresponds to systems with large \( N \)'s. Codes with these parameters are not only difficult and highly complicated to design, but also have high interference collision probability and poor performance. Furthermore, to change either the number of user or the dimming level it is necessary to change the OOC code, which requires a large database of OOC codes.

In this section, two networking methods based on MEPPM are introduced in order to not only provide multiple access for different users in an indoor VLC network, but also provide \( M \)-ary transmission for each user so that a higher data-rate can be achieved.

### A. Networking Using Coded-MEPPM

OOCs are used in OCDMA networks to provide simultaneous access for users by assigning signature pattern to each user [6]. In our proposed technique that we call coded-MEPPM (C-MEPPM), we combine OOCs with MEPPM to provide multi-access and high data-rate for each user. Unlike traditional OCDMA systems where OOCs are applied in the time domain, in our approach the OCDA codewords are implemented in the code domain, and are applied on the codewords of a BIBD code in code-space.

Let \( d_n = [d_{n1}, d_{n2}, \ldots, d_{nL}], n = 1, 2, \ldots, N, \) \( d_{n\ell} \in \{0, 1\} \), be the \( n \)th codeword of an OOC with length \( L \), weight \( w \) and cross-correlation \( \alpha \). By assigning the \( n \)th OOC codeword to user \( n \) and assuming \( N \) active users, the \( m \)th symbol in the \( Q \)-ary constellation is given by

\[
u_{m,n} = \frac{1}{Nw} \sum_{\ell=1}^{L} d_{n\ell} e_{\ell}^{(m)}. \tag{3}
\]

In this manner, the symbols of user \( n \) are cyclic shifts. The factor \( \frac{1}{Nw} \) in (3) guarantees a PAPR of \( Q/K \) for LED arrays. From (3), the length of the OOC should be no larger than the number of BIBD codewords, i.e., \( L \leq Q \). In this study, for a fixed \( Q \) we use an OOC with \( L = Q \), since the larger the OOC-length, the higher performance it can achieve. Fig. 2(b) shows the transmitter structure using shift-registers for coded-MEPPM. To illustrate the concept, Fig. 4 shows the multilevel symbol generated using a \((13,3,1)\) OOC codeword and a \((13,4,1)\) BIBD code. The first four symbols of CCM-OCDMA using the OOC codeword and coded-MEPPM described in Fig. 4 are shown in Fig. 5. Note that CCP-OCDMA is two-level while C-MEPPM is multilevel.

Using the C-MEPPM technique, the dimming level can be controlled by changing the BIBD code as discussed in [7], and the maximum number of users can be increased by switching the OOC code or by using a time-sharing technique. Therefore, a significantly smaller code database is needed compared to the case when only OOCs are used for networking in VLC systems.

1) Decoders for Multiuser C-MEPPM System: In a VLC system, due to the short range of the channel, the received power is usually high, and therefore, the system is limited by shot-noise. In [7] we show that in single-user systems the optimum decoder for EPPM in the shot-noise limited regime is the correlation receiver, and hence, C-MEPPM symbols can be suboptimally decoded using two successive shift registers as shown in Fig. 6. Both consider a single-user environment ignoring the multiple-access interference (MAI). To implement this sub-optimum correlation detector, at the receiver side a
time-chip, and \( \mathbf{u}_{m,n} \) is component \( j \) of vector \( \mathbf{u}_{m,n} \). \( \Lambda_b \) can be written as 
\[
\Lambda_b = \eta P_0 \frac{h f}{\pi},
\]
where \( P_0 \) is the peak received power, \( h \) is Planck’s constant, \( f \) is the central optical frequency, and \( \eta \) is the efficiency of the photodetector. Considering that both OOC and BIBD are fixed-weight codes, the optimum decoder can be simplified to
\[
\hat{m}_k = \arg \max_{1 \leq m \leq Q} \left( \sum_{j=1}^{Q} r_{kj} \log \left( N \mathbf{u}_{m,n} + \Lambda_b \right) \right)
\]  
\[+ (N - 1) \frac{K}{Q} \right) .
\]  

Defining \( \mathbf{v}_n := \langle v_1, v_2, \ldots, v_Q, n \rangle \) as
\[
v_{j,n} := \log \left( \frac{1}{L} \sum_{\ell=1}^{L} d_{n,\ell} \mathbf{c}_{\ell,j} \right) + \Lambda_b \frac{N}{\Lambda_0} + \left( N - 1 \right) \frac{K}{Q} \right)
\]
for \( j = 1, 2, \ldots, Q \) and \( n = 1, 2, \ldots, N \), the optimum detector for user \( n \) can be written as
\[
\hat{m}_k = \arg \max_{1 \leq m \leq Q} \left( \mathbf{r}_k, \mathbf{v}_n^{(m)} \right)
\]
and therefore, the optimum SUD is also a correlation detector that can be implemented using a shift-register.

2) Error Probability Analysis: As we show below, the symbol error rate of the sub-optimum correlation receiver is close to the error rate of the optimum SUD, and acts as a bound on its performance. Therefore, and for mathematical simplicity, we only analyze the error probability of the sub-optimum correlation receiver (Fig. 6) in this section.

We derive an approximate mathematical expression for the error-probability of the sub-optimum correlation receiver. The performance of the receiver can be evaluated in three cases: 1) Shot noise limited case, 2) weak MAI scenario, and 3) strong MAI. For the first case we consider a high SNR regime and calculate an upper bound on the symbol error probability. For the second and third cases, MAI is the main limiting factor, and the effect of the shot-noise is less important. For case (3), the performance of C-MEPPM is similar to CCM-OCDMA, and the analytical error probability presented in [11] can be used to calculate the bit error probability (BER). For \( \alpha = 1 \), which corresponds to the weak interference case (case (2)), the approximation given in [11] is not accurate. In order to get a more accurate analysis for this scenario, we model the interference as a Bernoulli trial, which has binomial distribution.

For the first case, without loss of generality we assume that the desired user is user 1 and it transmits \( s_{k,1} \) at symbol-time \( k \). The overall system using OOC and MEPPM encoders and decoders is depicted in Fig. 6. Then the symbol error probability, \( P_s \), is bounded by
\[
P_s \leq \sum_{m_1=1}^{Q} \mathbb{P}(s_{k,1} = \mathbf{u}_{m_1,1}) \times \sum_{m' \neq m_1} \mathbb{P}(y_{km'} > y_{km_1} | s_{k,1} = \mathbf{u}_{m_1,1})
\]
which is derived using the union bound. For equiprobable transmission,

\[
\Pr(y_{km} > y_{km_1} | s_{k,1} = u_{m_1,1}) = \frac{1}{LN-1} \sum_{m_2 = 1}^{L} \ldots \sum_{m_N = 1}^{L} \Pr(y_{km} > y_{km'} | s_{k,n} = u_{m_{n-1},n}, n = 1, \ldots, N).
\]

Here,

\[
y_{km} - y_{km'} = \sum_{j=1}^{Q} r_{kj} \left( \sum_{l=1}^{L} d_{1l} (c_{(m_2 @ \ell_j)} - c_{(m_2 @ \ell_j)}) \right)
\]

where \( @ \) is the sum modulo \( Q \). Since \( r_{kj} \)'s are Poisson distributed random variables, \( (y_{km} - y_{km'}) \) is the sum of weighted Poisson random variables. According to [14], its probability distribution function (pdf) can be approximated by a Gaussian distribution. Thus, in order to calculate the error probability in (10), we need to calculate the conditional mean and variance of \( (y_{km} - y_{km'}) \).

Given that \( s_{k,n} \) is transmitted for user \( n, n \in \{1, 2, \ldots, N\} \), in symbol-time \( k \), the mean value of the output of the OOC-correlator that matches the OOC-codeword \( d_1 \) in symbol-time \( k, y_{km}, m' = 1, 2, \ldots, L \), is [1]

\[
E[\{y_{km}\} | s_{k,n} = u_{m_{n-1},n}, n = 1, 2, \ldots, N] = \left( \frac{\alpha_0 K}{N w} \right) \langle d_{1}^{(m_1)}, d_{1}^{(m')} \rangle + \left( \frac{\alpha_0 K}{N w} \sum_{n=2}^{N} \langle d_{1}^{(m_n)}, d_{1}^{(m')} \rangle \right) MAI
\]

and thus,

\[
E[\{y_{km_1} - y_{km'}\} | s_{k,n} = u_{m_{n-1},n}, n = 1, 2, \ldots, N] = \left( \frac{\alpha_0 K}{N w} \right) \left( w - \langle d_{1}^{(m_1)}, d_{1}^{(m')} \rangle \right) + \left( \frac{\alpha_0 K}{N w} \sum_{n=2}^{N} \langle d_{1}^{(m_n)}, d_{1}^{(m_1)} - d_{1}^{(m')} \rangle \right).
\]

(13)

In high SNR regimes, an approximate value can be calculated for \( \Pr(y_{km} > y_{km_1} | s_{k,1} = u_{m_1,1}) \) by only considering the largest terms in (10), which correspond to the smallest mean and maximum variance of \( (y_{km} - y_{km'}) \) given \( s_{k,1} = u_{m_1,1} \).

\[
E[y_{km} - y_{km'} | s_{k,1} = u_{m_1,1}] \text{ takes its minimum value when } \langle d_{1}^{(m_1)}, d_{1}^{(m')} \rangle = 0 \text{ and } \langle d_{1}^{(m_n)}, d_{1}^{(m')} \rangle = \alpha, 2 \leq n \leq N.
\]

For each \( n \) at most \( w^2/\alpha m' \) 's exist that satisfy the second condition. Defining \( \alpha_{ij} := \langle d_{1}^{(i)}, d_{1}^{(j)} \rangle \), in the worst case we have

\[
E[y_{km} - y_{km'} | s_{k,1} = u_{m_1,1}] \geq \left( \frac{\alpha_0 K}{N w} \right) (w - \alpha_{m_1,m'}) + \alpha - \alpha N
\]

and this is minimized when \( \alpha_{m_1,m'} = \alpha \). We thus define

\[
\mu := \min_{m_1 \neq m'} E[y_{km} - y_{km'} | s_{k,1} = u_{m_1,1}] \geq \left( \frac{\alpha_0 K}{N w} \right) (w - \alpha N).
\]

(14)

The variance of \( (y_{km} - y_{km'}) \) can be calculated as

\[
\var{y_{km} - y_{km'} | s_{k,1} = u_{m_1,1}} = \sum_{j=1}^{Q} \var{y_{km'} | s_{k,n} = u_{m_{n-1},n}, n = 1, \ldots, N} \left( \sum_{l=1}^{L} d_{1l} (c_{(m_2 @ \ell_j)} - c_{(m_1 @ \ell_j)}) \right)^2
\]

\[
\times \left( \sum_{l=1}^{L} d_{1l} (c_{(m_2 @ \ell_j)} - c_{(m_1 @ \ell_j)}) \right)^2.
\]

(16)

The maximum \( \var{y_{km} - y_{km'} | s_{k,1} = u_{m_1,1}} \) can be written as follows:

\[
\sigma^2 := \max_{m_1 \neq m'} \var{y_{km} - y_{km'} | s_{k,1} = u_{m_1,1}} = \frac{\alpha_0 K}{N w} \sum_{n=2}^{N} \langle d_{1}^{(m_n)}, d_{1}^{(m')} \rangle^2 + \alpha - \alpha N
\]

(17)

Assuming a high SNR regime, we get

\[
P_s \leq \frac{w^2}{2\sigma^2} \text{erfc} \left( \frac{\mu}{\sqrt{2\sigma}} \right)
\]

(18)

where \text{erfc}(. \) is the complementary Gaussian error function. This approximation is not valid when \( \alpha N \geq \omega \) since in that case \( E[y_{km} - y_{km'}] \) can be smaller than zero, which means \( \Pr(y_{km'} > y_{km} | s_{k,1} = u_{m_1,1}) \) approaches 1. This corresponds to the second and third cases, where the MAI is the limiting factor.

As mentioned above, for the second scenario we use a Bernoulli model to approximate the symbol error probability. Denote the probability that a pulse from an interfering user overlaps with one of the pulses of symbol \( m' \) and not with any pulses from symbol \( m_1 \) as \( p := \omega (w - \alpha) / L \). Then, assuming no shot-noise, \( \Pr(y_{km'} > y_{km} | s_{k,1} = u_{m_1,1}) \) can be written as

\[
\Pr(y_{km'} > y_{km} | s_{k,1} = u_{m_1,1}) \approx \sum_{j=0}^{N-w} \left( \begin{array}{c}
N - 1 \\j
\end{array} \right) p^j (1 - p)^{N-1-j}
\]

\[
\times \sum_{i=w-1}^{N-w} \left( \begin{array}{c}
N - 1 \\i
\end{array} \right) p^i (1 - p)^{N-1-i}
\]

(19)

which, for \( p \ll 1 \), becomes

\[
\Pr(y_{km'} > y_{km} | s_{k,1} = u_{m_1,1}) \approx \left( \begin{array}{c}
N - 1 \\w
\end{array} \right) p^{(w-1)}
\]

\[
\times (1 - p)^{(N-w)}.
\]

(20)

As mentioned before, at most \( w^2 \) symbols of user 1 can have a cross-correlation of 1 with symbol \( m_1 \). Thus,

\[
P_s \approx \frac{w^2 (N - 1)}{w - 1} p^{(w-1)} (1 - p)^{(N-w)}
\]

(21)

Simulation and analytic results for the two detectors are given in Fig. 8 for a coded-MEPPM system using a (341,5,1)-OOC code [6] and a (341,85,21)-BIBD [15], which can support up to
17 simultaneous users. The optimum SUD derived in (8) is simulated assuming Poisson statistics, and the correlation detector in Fig. 2 is simulated using both Gaussian and Poisson noise. The analytical BER expression from (21) is also plotted. In these results, the effective area of the photodetector is 1 cm², the background light power is 10 μW and the peak received signal power is 40 μW, which corresponds to an illumination level of 100 lx (a typical minimum illumination level for indoor environments [16]) assuming a PAPR of 4; the data is transmitted using ideal LEDs with a center wavelength of 650 nm at a bit-rate of 200 Mb/s through an ideal channel. According to these results, the performance of the optimum SUD is better than the correlation receiver and has a slightly lower BER compared to the latter. The Gaussian approximation for the statistics of the received photoelectron count is accurate, as the simulated BER are shown to be close. As can be seen, the analytical result calculated in (21) provides a good approximation to the BER of the C-MEPPM system.

### B. Networking Using Divided-MEPPM

In the second proposed technique, which we call divided-MEPPM (D-MEPPM), the generating BIBD code is divided into several smaller codes, and a different set of codewords is assigned to each user. Then, as in the MEPPM scheme, each user uses its codeword set to generate multilevel symbols.

Let \( q_n \) be the number of BIBD codewords that are assigned to user \( n \), such that \( q_1 + q_2 + \ldots + q_N = Q \), and let \( C_n, |C_n| = q_n \), be the set of codewords that are assigned to user \( n \), such that \( C_n \cap C_m = \emptyset \) for any \( n \neq m \), and \( C_1 \cup C_2 \cup \ldots \cup C_N = \{ c_1, c_2, \ldots, c_Q \} \). This can be considered as a kind of CDMA, where distinct codeword sets with cross-correlation \( \lambda \) are assigned to users. Using this definition, user \( n \) can utilize an \( \ell_n \)-branch MEPPM, \( 1 \leq \ell_n < q_n \), for \( M \)-ary transmission using \( C_n \). It can use either MEPPM type-I or type-II, yielding a constellation of size \( \left( q_n + \ell_n \right) / \ell_n \) for type-I MEPPM [9]. Symbol \( m \) of user \( n \) can be expressed as

\[
\mathbf{u}_{m,n} = \frac{1}{Q} \sum_{\ell \in C_{m,n}} \mathbf{c}_{\ell}
\]

(22)

where \( C_{m,n} \) is the set of \( \ell_n \) codewords assigned to symbol \( m \) of user \( n \). For type-I MEPPM, all \( \ell_n \) elements of \( C_{m,n} \) are distinct, and for type-II, a codeword can be repeated more than once in \( C_{m,n} \). Each user can generate these symbols using the transmitter shown in Fig. 2(a). The correlation receiver shown in Fig. 3 is used to decode the received signal, and then the detection techniques described in [9] are used to estimate the received symbol.

An advantage of this networking technique over the one presented in Section III-A is its potential to provide different data-rates for different users, which can be done by assigning unequal size subsets to users. Thus, a larger number of BIBD codewords is provided to users requiring a higher bit-rate. Table I gives examples of how a system using two possible BIBD codes, (63, 31, 15) and (57, 8, 1) can provide PAPR levels of 2 and 7.125, respectively, while simultaneously assigning each of 5 users in the network various data-rates. Data-rates of users can be changed by assigning code sets of different sizes.

An analytical expression can be calculated for D-MEPPM similar to the one above for C-MEPPM. Without loss of generality we calculate the symbol error probability for the first user given that \( s_{k,n} = \mathbf{u}_{m,n} \) is sent for user \( n \). In this case, the variance of \( z_{kj} \) is

\[
\text{Var}[z_{kj}] = \sum_{i=1}^{Q} c_{ji} \text{Var}[r_{ki}] + \Gamma^2 \sum_{i=1}^{Q} (1 - c_{ji}) \text{Var}[r_{ki}]
\]

\[
= \sum_{i=1}^{Q} c_{ji} E[r_{ki}] + \Gamma^2 \sum_{i=1}^{Q} (1 - c_{ji}) E[r_{ki}]
\]

(23)

### Table I

**Allocations of Codewords to Provide Different Data-Rates and PAPRs in a VLC Network**

| (63, 31, 15) BIBD | (57, 8, 1) BIBD |
|------------------|-----------------|
| User  | 1 | 2 | 3 | 4 | 5 |
| Number of Codes | 13 | 13 | 13 | 12 | 12 |
| Data Rate (Mb/s) | 26 | 26 | 24 | 24 | 24 |
| Number of Codes | 21 | 21 | 7  | 7  | 7  |
| Data Rate (Mb/s) | 43.3 | 43.3 | 13.3 | 13.3 | 13.3 |
| Number of Codes | 35 | 9  | 5  | 5  | 5  |
| Data Rate (Mb/s) | 73.7 | 17.6 | 17.6 | 9.2 | 9.2 |

![Fig. 8. BER versus the number of active interfering users for optimum detector from (8) for Poisson statistics, and correlation receiver for both Poisson and Gaussian statistics. (21) is used for the analytical results.](image-url)
where
\[ E[r_{ki}|s_{k,n} = u_{m_{}\alpha,n}, n = 1, 2, \ldots, N] = \frac{1}{Q} \sum_{n=1}^{N} \sum_{\ell \in C_{m_{\alpha,n}}} c_{\ell i}. \]  

Since \( C_{m_{\alpha,n}} \)'s are distinct sets we have
\[ \text{Var}\left[z_{kj}|s_{k,n} = u_{m_{\alpha,n}, n = 1, 2, \ldots, N}\right] = \text{Var}\left[z_{kj}|s_{1,1} = u_{m_{\alpha,1,1}}\right] = \frac{\Lambda_{0}}{Q} (\ell_{1} + \ldots + \ell_{N})(\lambda K - \lambda + n_{e} + (K - 2\lambda)K) \]  
\[ \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{\ell \in C_{m_{\alpha,n}}} c_{\ell i}. \]  

We can then use the following expressions to approximate the BER [9]:
\[ P_{b} \approx M' \text{erfc}\left(\sqrt{\frac{(K-\lambda)^{2}\Lambda_{0}/Q}{(\ell_{1} + \ldots + \ell_{N})\lambda K + (K - 2\lambda)K}}\right) \]  
\[ \text{erfc}(x) = \frac{1}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^{2}} \, dt \]  
\[ M' = \frac{\ell_{1}(q_{1} - \ell_{1})}{8} \]  
\[ M' = \frac{\ell_{1}q_{1}(q_{1} - 1)^{2}}{8(q_{1} + \ell_{1})(q_{1} + \ell_{1} - 1)(q_{1} + \ell_{1} - 2)} \]

IV. NUMERICAL RESULTS

In this section numerical results using the analytical BER expressions derived above are presented to compare the performance of the proposed networking techniques to each other, and also to that of a CCM-OCDMA system. Unless otherwise stated, the parameters used in the results are: receiver effective area of 1 cm², peak received power \( P_{0} = 40 \mu W \), background light power of 10 \( \mu W \), center wavelength \( \lambda_{0} = 650 \text{ nm} \), photodetector quantum efficiency \( \eta = 0.8 \), and bit-rate of 200 Mb/s.

Analytical results for symbol error probability of a coded-MEPPM system using a \( (341,5,1) \)-OOC code and a \( (341,85,21) \)-BIBD [15] is given in Fig. 9 versus the received peak power, \( P_{0} \), for different number of active users, \( N \). Analytical results from (18) and bounds from (21).

Fig. 10 compares the BER of a CCM-OCDMA system with that of a coded-MEPPM for different numbers of active users and for two PAPR values. For coded-MEPPM, the BIBD code is chosen to satisfy the PAPR condition, and the OOC is chosen to provide the lowest collision probability between users, i.e., the smallest MAI effect. For a PAPR = 4 and a constellation size of 101, the CCM-OCDMA system uses a \( (101,25,7) \)-OOC and the coded-MEPPM uses a \( (101,11,2) \)-OOC and a \( (101,25,6) \)-BIBD, for a maximum of 10 users. Since the parameters of the OOC must satisfy \( \alpha > w^{2}/L \) [6], for the CCM-OCDMA with \( L = 101 \) and \( w = 25 \), \( \alpha = 7 \) is the smallest cross-correlation that can be chosen. As shown in the figure, coded-MEPPM can achieve a lower BER compared to CCM-OCDMA. For a PAPR = 2, the BER of a CCM-OCDMA system using a \( (83,41,20) \)-OOC is compared to that of a coded-MEPPM system using a \( (83,41,20) \)-BIBD and \( (83,15,3) \)-OOC for different number of active users up to 13 users. The coded-MEPPM is shown to have a better performance compared to CCM-OCDMA.

A comparison between the analytical BER of the coded-MEPPM using a \( (63,31,15) \)-BIBD and a \( (63,7,2) \)-OOC, and divided-MEPPM using the same BIBD code is shown in Fig. 11. In this figure the BER is plotted versus the number of users for three different peak received power levels and for a PAPR of 2. The C-MEPPM system can provide simultaneous multiple access for up to 16 users. For divided-MEPPM, distinct sets of 4 BIBD codewords are assigned to users, and therefore, the maximum number of users is again 16. For this technique, each user...
utilizes 4-level type-II MEPPM and has 70 symbols. For weak peak powers the error probability of coded-MEPPM is lower than divided-MEPPM since it has larger distance between its symbols and its performance is only limited by MAI, while the performance of divided-MEPPM is limited by the shot noise. According to these results, D-MEPPM is able to support up to 15 simultaneous users transmitting 200 Mb/s with a BER $\leq 10^{-3}$ under normal lighting conditions.

V. CONCLUSION

Two multiple-access methods for use in VLC systems are introduced and compared. The first method, using both OOC and BIBD codes to encode the user data, is shown to have a large distance between symbols. Unlike CMC-OCDMA systems, the parameters of the OOC that is used in C-MEPPM can be chosen independent from the given PAPR, and hence, C-MEPPM is able to provide a lower BER compared to CCM-OCDMA when MAI is the dominating factor. The second technique only uses BIBD codes to construct MEPPM symbols, and, therefore, can have a large constellation size and consequently a high bit-rate for each user. It is also able to provide flexible and unequal data-rates to users. According to the numerical results, for low SNR cases the coded-MEPPM technique achieves a lower BER compared to divided-MEPPM, while the latter is preferred in high SNR regimes since it has a lower MAI effect.

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Mohammad Noshad is currently working toward the Ph.D. degree in the Department of Electrical and Computer Engineering, University of Virginia. He received the “Best Paper Award” at IEEE Globecom 2012. His research interests include free-space optical communications, visible light communications, and information and coding theory.

Maité Brandt-Pearce received the Ph.D. degree in electrical engineering in 1993 from Rice University. She is currently a Full Professor in the Department of Electrical and Computer Engineering, University of Virginia. Her research interests include optical and wireless communications, and biomedical, and radar signal processing. She is a member of Tau Beta Pi andEta Kappa Nu. She has more than hundred and fifty major publications.