Models of hysteresis oscillation damping at pulse loadings

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Abstract. The article describes the results of modeling the oscillation of the impact device elements in the presence of hysteresis damping together with dissipative. The influence of the symmetrical and asymmetric power characteristics on the oscillation mode is considered. For a single-mass model, initial problems are formulated and solved. Under the condition of continuity, an analytical solution is obtained that practically coincides with the numerical one. Initial-boundary value problems are formulated for the tool of an impact device, as a rod of variable section. Problem solving is found by the finite difference method. The properties of two-layer and three-layer difference schemes are estimated. The limiting amplitudes of the oscillations of the rod cross-sections are controlled by the initial-boundary-value problem, which is obtained by the Fourier method. The stress distribution over the rod length is obtained.

1. Introduction

The problems of damping mechanical oscillation with pulse loadings were studied by a number of researchers and are given in [1-9] works. A method of oscillation suppression using a heterogeneous lyophobic system (HLS) is presented in [2, 10] works. The effectiveness of such a system is confirmed experimentally. The design of pulsed dynamic systems involves the creation of mathematical models for an adequate description of oscillatory processes. The damping characteristics are non-linear in impact-type device and therefore the use of analytical research methods is very difficult. The use of combined methods, that is, analytical and numerical, is effective. The nonlinearity of the power characteristic lies in the dependence of the resistance force on the displacement and direction of motion. In the work [9], a numerical solution of the initial problem was obtained in the presence of a nonlinear hysteresis characteristic. In the present work, we obtain an analytical solution to the initial problem using the example of a single-mass system in combination with dissipative resistance and constant loading force. The analytical solution is compared with a numerical solution by the Runge-Kutta method (Mathcad programming system). The single-mass model allows to study the device oscillations as a reduced mass (portable motion). To obtain a picture of the tool cross section oscillations, the equation of the rod oscillations with a variable cross section is used. The power characteristic of the HLS is obtained as a result of experimental studies at a special stand. The study of the mathematical model was carried out by the finite difference method using two-layer and three-layer difference schemes. The calculation control in the stability domain of the difference scheme were controlled by solving the linear problem (adapted to the parameters of the
original problem) using the Fourier method. Various options for determining the average speed were studied, and the influence of the difference scheme form was evaluated taking into account the established fact of the influence of high-frequency oscillations of the contact rod end on the change of the design speed sign and, therefore, on the power characteristic.

2. Discrete model and solution of the initial problem

The dynamic system is represented by a single mass model with reduced mass $m$, dissipation coefficient $b$, constant stiffness $c_1$, and variable stiffness $C_g$. The diagram of the dynamic system and its asymmetric power characteristic are shown in figure 1. The rigidity of the additional structural element $c_3$ is taken into account in the power characteristic.

![Figure 1](image)

**Figure 1.** (a) Dynamic system diagram: $m = 65$ kg, $V_0 = 4.5$ m/s; (b) asymmetric power characteristic with sections under conditions:

- $1 - V > 0$;
- $2 - V < 0$;
- $3 - x < 0$.

$c_1 = 789900$ N/m, $c_2 = 87000$ N/m, $c_3 = 6000$ N/m.

2.1. The problem of dissipation

Let us consider the case of oscillations taking into account the dissipation. In [9], a numerical solution of a similar problem was considered without the dissipative element with a symmetric force characteristic. The initial problem has the form

$$m \frac{d^2x}{dt^2} + R\left(x, \frac{dx}{dt}\right) + b \frac{dx}{dt} = 0, \quad x(0) = 0, \quad \frac{dx}{dt}(0) = V_0. \quad (1)$$

The impact pulse is modeled by determining the value of the initial velocity $V_0$. The power characteristic has the form

$$R(x, V) = \begin{cases} c_1x, & \text{if } 0 < x < L \land V \geq 0, \\ c_2x, & \text{if } 0 < x < L \land V < 0, \\ c_3x, & \text{if } x \leq 0. \\ \end{cases} \quad (2)$$

To find a solution to the nonlinear initial problem (1), auxiliary linear problems connected with the help of the initial conditions are formulated and their continuous solutions are found. Typical instant of times for the initial problems are presented in table 1.

| Instant of time, $t$ | 0 | $t_1$ | $t_2$ | $t_3$ |
|---------------------|---|-------|-------|-------|
| Displacement, $x$   | 0 | $x(t_1)$ | 0     | 0     |
| Velocity, $V$       | $V_0$ | 0     | $V(t_2)$ | $V(t_3)$ |
We write the characteristic equation under the condition \( V = \frac{dx}{dt} > 0; \ k^2 + \frac{b}{m} k + \frac{c}{m} = 0 \).

If \( \frac{b^2}{4m^2} - \frac{c}{m} < 0 \), the general solution of the differential equation is written as

\[
x(t) = e^{\frac{b}{2m}t} \left( A \cos \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \cdot t \right) + B \sin \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \cdot t \right) \right).
\]

(3)

With the initial conditions we obtain

\[
x(t) = \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \right)^{\frac{1}{2}} \arctg \left( \frac{\sqrt{4mc_1 - b^2}}{b} \right) \sin \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \cdot t \right), \text{ if } t < t_1.
\]

(4)

Here \( t_1 = \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \right)^{\frac{1}{2}} \arctg \left( \frac{\sqrt{4mc_1 - b^2}}{b} \right) \) is determined from the condition \( V(t) = \frac{dx}{dt} = 0 \).

When \( t > t_1 \) consider the problem

\[
m \frac{d^2x}{dt^2} + c_2x + b \frac{dx}{dt} = 0, \ t \in [t_1, t_2],
\]

(5)

\[
x(t_1) = V_0 \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \right)^{\frac{1}{2}} e^{\frac{b}{2m}t_1} \sin \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \cdot t_1 \right), \ \frac{dx}{dt}(t_1) = 0.
\]

(6)

If the condition is satisfied \( \frac{b^2}{4m^2} - \frac{c}{m} < 0 \), the general solution of the differential equation (5) is written in the form

\[
x(t) = e^{\frac{b}{2m}t} \left( A_1 \cos \left( \frac{c_2}{m} - \frac{b^2}{4m^2} \cdot t \right) + B_1 \sin \left( \frac{c_2}{m} - \frac{b^2}{4m^2} \cdot t \right) \right).
\]

(7)

The constants \( A_1 \) and \( B_1 \) are determined from the system of equations:

\[
\begin{align*}
-A_1 \left( \alpha_1 b + \sqrt{4mc_2 - b^2} \cdot \beta_1 \right)^{-1} + B_1 \left( -\beta_1 b + \sqrt{4mc_2 - b^2} \cdot \alpha_1 \right)^{-1} = 0, \\
A_1 \alpha_1 + B_1 \beta_1 = \frac{2V_0m}{\sqrt{4mc_1 - b^2} \cdot \sin \left( \frac{c_1}{m} - \frac{b^2}{4m^2} \cdot t_1 \right)};
\end{align*}
\]

where \( \alpha_1 = \cos \left( 2m \right)^{-1} \sqrt{4mc_2 - b^2} \cdot t_1 \), \( \beta_1 = \sin \left( 2m \right)^{-1} \sqrt{4mc_2 - b^2} \cdot t_1 \).

Formulas for constants \( A_1 \) и \( B_1 \) have the form:

\[
A_1 = \frac{2mV_0}{\sqrt{4mc_1 - b^2} \cdot \alpha_1} - \left( \frac{\beta_1}{\sqrt{4mc_2 - b^2}} \cdot \beta_1 \right), \quad B_1 = \frac{2mV}{\sqrt{4mc_1 - b^2} \cdot \alpha_1} + \left( \frac{\alpha_1 b}{\sqrt{4mc_2 - b^2}} + \beta_1 \right) \beta_1.
\]

The initial data and graphs of solutions are presented in figure 2.

We give the formula for solving the general initial problem (1):
To compare the solutions, an approximate problem solution was found by the Runge-Kutta method. Figure 2 (a) gives the analytical and approximate solutions to the initial problem on the interval \([0, t_2]\), which practically coincide. The solution of the initial problem on the longer interval of time is shown in Figure 2 (b). The damping of the oscillation amplitude occurs with a large damping decrement (averagely \(\Delta > 2\)).

![Figure 2](image)

**Figure 2.** (a) short interval of time; (b) long interval of time

1 – numerical solution; 2 – analytical solution;

Parameters: \(b=500\ \text{Ns/m}\), \(c_1=789900\ \text{N/m}\), \(c_2=87000\ \text{N/m}\), \(c_3=40000\ \text{N/m}\).

2.2. The problem with the additional action of constant force

The initial problem was considered in the form:

\[
m \frac{d^2x}{dt^2} + R \left( x, \frac{dx}{dt} \right) - P = 0, \quad x(0) = 0, \quad \frac{dx}{dt}(0) = V_0. \tag{9}
\]

To find a solution to the nonlinear initial problem (9), we consider the sequence of initial problems and their analytical solutions.

Problem (1a): \[ \frac{d^2x}{dt^2} = -\frac{c_1}{m} x + \frac{P}{m}, \quad t \in [0, t_1], \quad x(0) = 0, \quad \frac{dx}{dt}(0) = V_0. \tag{10} \]

A solution to the problem (1a):

\[
x(t) = V_0 \sqrt{\frac{m}{c_1}} \sin \sqrt{\frac{c_1}{m}} t + \frac{P}{c_1} \left( 1 - \cos \sqrt{\frac{c_1}{m}} t \right), \quad \varphi_0 = \arcsin \left( \frac{P}{c_1} \left( \frac{V_0^2 m + P^2}{c_1} \right)^{\frac{1}{2}} \right). \tag{11}
\]

where \( t_1 = \sqrt{\frac{m}{c_1}} \left( \frac{\pi}{2} + \varphi_0 \right) \).

Problem (2a): \[ \frac{d^2x}{dt^2} = -\frac{c_2}{m} x + P, \quad t \in [t_1, t_2], \quad x(t_1) = \sqrt{\frac{m}{c_1}} + \frac{P^2}{c_1}, \quad \frac{dx}{dt}(t_1) = 0. \]
A solution to the problem (2a):

\[ x(t) = \left( \frac{Pc_2 - c_1}{c_1c_2} + \sqrt{\frac{V_0^2m + \frac{P^2}{c_1^2}}{c_1} + \frac{\frac{\sqrt{c_2^2m(t-t_1)} + P}{c_2}}{\sqrt{c_2^2m(t-t_1)}}} \right) + \frac{P}{c_2}, \tag{12} \]

\[ \frac{dx}{dt}(t) = \sqrt{\frac{c_2}{m}} \left( \frac{Pc_1 - c_2}{c_1c_2} + \sqrt{\frac{V_0^2m + \frac{P^2}{c_1^2}}{c_1} + \frac{\frac{\sqrt{c_2^2m(t-t_1)} + P}{c_2}}{\sqrt{c_2^2m(t-t_1)}}} \right) \sin \left( \frac{\sqrt{c_2^2m(t-t_1)}}{m} \right). \tag{13} \]

We find the timepoint \( t_2 \), when \( x(t_2) = 0 \). We have the equation

\[ x(t) = \left( \frac{Pc_2 - c_1}{c_1c_2} + \sqrt{\frac{V_0^2m + \frac{P^2}{c_1^2}}{c_1} + \frac{\frac{\sqrt{c_2^2m(t-t_1)} + P}{c_2}}{\sqrt{c_2^2m(t-t_1)}}} \right) + \frac{P}{c_2} = 0, \]

from which we obtain

\[ t_2 = t_1 + \frac{m}{c_2} \arccos \left( \frac{Pc_1 - c_2}{c_1c_2} - c_2 \sqrt{\frac{V_0^2m + \frac{P^2}{c_1^2}}{c_1} + \frac{\frac{\sqrt{c_2^2m(t-t_1)} + P}{c_2}}{\sqrt{c_2^2m(t-t_1)}}} \right). \tag{14} \]

When \( t > t_2 \) we come to the initial problem:

Problem (3a):

\[ \frac{d^2x}{dt^2} = -\frac{c_3}{m}x + \frac{P}{c_3}, \quad t \in [t_2, t_4], \quad x(t_2) = 0, \]

\[ \frac{dx}{dt}(t_2) = \sqrt{\frac{c_2}{m}} \left( \frac{Pc_1 - c_2}{c_1c_2} + \sqrt{\frac{V_0^2m + \frac{P^2}{c_1^2}}{c_1} + \frac{\frac{\sqrt{c_2^2m(t-t_1)} + P}{c_2}}{\sqrt{c_2^2m(t-t_1)}}} \right) \sin \left( \frac{\sqrt{c_2^2m(t-t_1)}}{m} \right) = V_1. \tag{15} \]

We have a solution to the problem (3a):

\[ x(t) = \sqrt{A^2 + B^2} \cos \left( \frac{c_3 t - \psi_1}{c_3} \right) + \frac{P}{c_3}, \quad \frac{dx}{dt} = -\sqrt{A^2 + B^2} \sqrt{\frac{c_3}{m}} \cos \left( \frac{c_3 (t - t_2)}{m} \right), \]

\[ A = -\frac{P}{c_3} \cos \left( \frac{c_3 t_2 - V_1}{c_3} \right) \frac{m}{c_3} \sin \left( \frac{c_3}{m} \right), \quad B = V_1 \frac{m}{c_3} \cos \left( \frac{c_3 t_2}{m} \right) - \frac{P}{c_3} \sin \left( \frac{c_3}{m} \right), \quad \psi_1 = \arcsin \left( \frac{B}{\sqrt{A^2 + B^2}} \right). \tag{16} \]

From the condition \( x(t_3) = 0 \) we find \( t_3 = -\frac{m}{c_3} \arccos \left( -\frac{P}{c_3 \sqrt{A^2 + B^2}} \right) + \frac{m}{c_3} (\psi_1 + 2 \pi) \).

We give the general formula for solving problem (9):

\[ x(t) = \begin{cases} \sqrt{\frac{m}{c_1}} \sin \left( \frac{c_3}{m} \cdot t + \frac{P}{c_1} \left( 1 - \cos \left( \frac{c_3}{m} \cdot t \right) \right) \right), & \text{if } 0 < t \leq t_1, \\
\frac{Pc_2 - c_1}{c_1c_2} + \sqrt{\frac{V_0^2m + \frac{P^2}{c_1^2}}{c_1} + \frac{\frac{\sqrt{c_2^2m(t-t_1)} + P}{c_2}}{\sqrt{c_2^2m(t-t_1)}}} \cos \left( \frac{c_3}{m} (t - t_1) \right) + \frac{P}{c_2}, & \text{if } t_1 < t \leq t_2, \\
\sqrt{A^2 + B^2} \cos \left( \frac{c_3}{m} (t - \psi_1) \right) + \frac{P}{c_3}, & \text{if } t_2 \leq t \leq t_3. \end{cases} \tag{17} \]

Figure 3(a) shows the analytical and approximate solutions to the initial problem (9) (excluding dissipation) on the interval \([0, t_1]\), which practically coincide.
The combined action of dissipation and additional force leads to the result, which is shown in Figure 3(b).

**Figure 3.** (a) The additional force without dissipation; (b) Combined description for additional force and dissipation; 1 – numerical solution; 2 – analytical solution ((b) taking into account the dissipation).

The obtained analytical and numerical solutions of the initial problems demonstrate the correctness of the applied oscillation modeling approaches with for power characteristics of hysteresis type.

3. Framed structure and the initial-boundary value problem

3.1. Approximation of the power characteristic

It is known that today thermotechnical devices are limited by thermodynamic compactness (TC), it isn't higher than 600 $J/m^3 K$. New thermodynamic working bodies are investigated and practically applied in National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" in order to reach a new level of TC values and increase the accumulating and dissipative abilities of working bodies. Working bodies are synthesized based on the use of surface phenomena in highly dispersed systems. It was suggested to use as a working body not a volume phase (gas, steam), but a surface phase in a condensed lyophobic “liquid-solid” system [2, 9, 10]. An increase in energy intensity in such a heterogeneous lyophobic system (HLS) is achieved through the synthesis of “liquid-capillary-porous matrix” systems with a maximum contact angle.

In the compression process, due to the forced intrusion of a liquid into a capillary-porous solid (CPS), the development of the “liquid-solid” interphase surface occurs. In this case, the system accumulates energy due to the formation of an interphase. The system is practically incompressible until the pressure in the system reaches the capillary pressure of Laplace. After this, the intrusion process and, accordingly, the isothermal compression process of the system begins. In the reverse process of expanding the system (after removing the external load), extrusion occurs. Spontaneous liquid exit from the CPS and the reduction of interphase complete hysteresis with energy dissipation.

It is the interphase that is the working body of a new class (in the thermodynamic interpretation of this term). An energy dissipation level of about 90% (unreachable value for hydraulic shock absorbers) is achieved due to the experimental studies of the HLS hysteresis.

The “hydrophobized silica gel system with a highly developed internal surface plus water as a working body” was used as an HLS substance. The system is packaged in a special container (50x23x8) mm, which has been degassed with air removed from the mixture. As a process liquid, AMG-40 mineral oil was used. A detailed description of the installation is given in [9] work. The diagram of the stand installation and the obtained power characteristic of the HLS element are shown in figure 4. In the multiplication chamber (3), the HLS (7) is installed in a flexible shell-container (8). To register process parameters, pressure (9, 10), temperature (11), and motion sensors (12) were used. Figure 4(b) shows fragments of the oscillosgram, which characterizes loading and unloading processes of HLS and the approximation of these curves by a piecewise linear function. The design scheme "rod of variable cross section-HLS" (impact device tool) and power characteristic are shown in figure 5.
Figure 4. (a) Structural diagram of the pressure intensifier:
1 – frame; 2 – piston and rod; 3 – animation chamber; 4, 5, 6 – hydraulic lines; 7 – HLS; 8 – shell-container; 9, 10 – pressure sensors; 11 – temperature sensor; 12 – motion sensor. (b) Fragments of the oscillogram and its approximation, characterizing the loading and unloading processes of HLS: 0-1-2 – the process of loading (intrusion); 2-3-0 – the process of unloading (extrusion).

In the multiplication chamber (3), the HLS (7) is installed in a flexible shell-container (8). To register process parameters, pressure (9, 10), temperature (11), and motion sensors (12) were used. Figure 4 (b) shows fragments of the oscillogram, which characterizes loading and unloading processes of HLS and the approximation of these curves by a piecewise linear function. The design scheme "rod of variable cross section-HLS" (impact device tool) and power characteristic are shown in figure 5.

Figure 5. (a) Design model: 1 – HLS; 2 – rod of variable cross section; 3 – hard element; 4 – dissipative element; (b) power characteristic of HLS.

The change of the radius and cross-sectional area of the rod was given by the formulas:

\[ R(x) = R_1 \left( \frac{\chi}{4} \cos \frac{m \pi x}{L} + \frac{3}{4} \right), \quad S(x) = \pi R(x)^2, \]

with equivalent cylinder radius of constant cross section

\[ R_1 = R_0 \sqrt{L \left( \int_0^L \left( \frac{\chi}{4} \cos \frac{m \pi x}{L} + \frac{3}{4} \right)^2 \ dx \right)^{1/2}}. \]

Parameters \( \chi \) and \( m \) allow you to change the distribution of the cross-sectional area of the rod along the length for modeling various profiles of the impact device tool. The characteristics of the power resistance of the working body, which were obtained experimentally, are implemented in the form of a program block, i.e. the function
Fourier method to the calculation results, which are compared with the analytical solution of a linear problem (19). The initial shock pulse is transmitted through the material of the rod; \( u \) is considered in the form:

\[
\begin{align*}
R(x, z, z_2) &= \begin{cases} 
   c_{1, x}, & \text{if } 0 \leq x < x_1 \land z \geq 0, \\
   c_{1, x} + c_2(x - x_1), & \text{if } x_1 \leq x < x_2 \land z \geq 0, \\
   c_{1, x} + c_2(x - x_1) + c_3(x - x_2), & \text{if } x_2 \leq x < x_3 \land z < 0, \\
   [c_{1, x} + c_2(x - x_1) + c_3(x - x_2)]x_1^+, & \text{if } x < x_3 \land z < 0, \\
   c_{0, x}, & \text{if } x \leq 0.
\end{cases}
\end{align*}
\]

(18)

The resisting force depends on the displacement \( x \) and the motion direction (the speed sign of the left end of the rod - \( z \)). A simplified graph of the resisting force dependence on displacement and motion direction is shown in figure 5(b). Such a power characteristic is realized with simplified calculations. When the sign of speed changes, a jump in the resistance force occurs (transition to the lower part of the broken one when the speed is negative). More accurate calculations that provide a transition on the characteristic to the lower part of the curve involve the construction of an iterative process for determining the coordinate \( x_2 \).

3.2. Initial boundary value problem and its solution

The Initial boundary value problem is considered in the form:

\[
\frac{\partial^2 U(t, x)}{\partial t^2} = a^2 \left[ \frac{1}{S(x)} \frac{\partial S(x)}{\partial x} \frac{\partial U(t, x)}{\partial x} + \frac{\partial^2 U(t, x)}{\partial x^2} \right], t > 0, \quad x \in [0, L],
\]

(19)

\[
\frac{\partial U}{\partial x}(t, 0) = 0, \quad ES(L) \frac{\partial U}{\partial x}(t, L) = -C_1 U(t, L) - R \left( U(t, L), \frac{\partial U(t, L)}{\partial t} \right) - B \frac{\partial U(t, L)}{\partial t},
\]

(20)

\[
U(0, x) = 0, \quad \frac{\partial U}{\partial t}(0, x) = F(x), \quad x \in [0, L],
\]

(21)

\[
F(x) = \begin{cases} 
   F \left( \int_0^x S(x) dx \right)^{-1}, & \text{if } 0 \leq x \leq \varepsilon, \\
   0, & \text{if } \varepsilon < x \leq L.
\end{cases}
\]

(22)

In system (19) - (22): \( U(t, x) \) - the displacement of the cross section of the rod with the coordinate \( x \); \( a = \sqrt{EP} \) - the speed sound in the material of the rod; \( E \) - elastic model; \( \rho \) - the density of the material; \( \varepsilon \) - the parameter of the initial shock load (the length of the rod part to which the initial shock pulse is transmitted [11]); \( P \) - shock pulse. To find an approximate solution to problem (19) - (22), a mixed difference scheme is used [12]. The parameter \( \sigma \) is selected according to the calculation results, which are compared with the analytical solution of a linear problem by the Fourier method [9; 11; 13]. The difference problem is written in the form:

\[
\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^{n-1}}{\tau^2} = \sigma a^2 \left[ \frac{1}{S(x_i)} \frac{\partial S(x_i)}{\partial x} \left( \frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2h} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{h^2} \right) \right] +
\]

\[
+ (1 - \sigma) a^2 \left[ \frac{1}{S(x_i)} \frac{\partial S(x_i)}{\partial x} \left( \frac{u_{i+1}^{n} - u_{i-1}^{n}}{2h} + \frac{u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}}{h^2} \right) \right] +
\]

\[
+ (\sigma - 1) a^2 \left[ \frac{1}{S(x_i)} \frac{\partial S(x_i)}{\partial x} \left( \frac{u_{i+1}^{n-1} - u_{i-1}^{n-1}}{2h} + \frac{u_{i+1}^{n-1} - 2u_i^{n-1} + u_{i-1}^{n-1}}{h^2} \right) \right],
\]

(23)

\[
u_i^{n+1} - u_i^{n+1} = 0, \quad ES(L) \left( u_{i+1}^{n+1} - u_{i-1}^{n+1} \right) h^{-1} = -C u_i^n \cdot H \left( u_i^n \right) - R(u_i^n, V_i^n) - B \left| V_i^n \right| \cdot H_1 \left( V_i^n \right),
\]

(24)
\[ V_N^n = \left( u_N^n - u_N^{n-1} \right) (k \tau)^{-1}, \]  

where \( \zeta \) – a parameter determining the type of difference scheme. When \( \zeta = 1 \) we get a two-layer scheme, and when \( \zeta = 2 \) – a three-layer scheme. The parameter \( k \) allows to find the "average" velocity of the contact end of the rod with the HLS element over an interval \( \Delta t = k \tau \). The functions \( H(u_N^n) \) and \( H_1(V_N^n) \) are intended to determine the sign of the elastic and dissipative resistance forces depending on the movement and the displacement velocity. The application of these functions allows us to solve problems with more general power characteristics.

At each time layer \( t_{n+1} \), the system of equations (23) - (24) is solved by the sweep method. We give the formulas of the sweep method for finding boundary values. From the system of equations

\[ \alpha 0 = \alpha_0 u_0 + \beta_0, \quad u_0 = u_1, \quad \text{we get} \quad \alpha_0 = 1, \quad \beta_0 = 0. \]

To calculate \( u_{N}^{n+1} \), we consider the system of equations

\[ u_{N-1}^{n+1} = u_N^{n+1} \left(1 + h C \cdot H(V_N^n \cdot (ES(L))^{-1}) + h R(u_N^n, V_N^n) + B \cdot H_1(V_N^n) \cdot V_N^n \right) \left[ ES(L) \right]^{-1}, \]

\[ u_{N-1}^{n+1} = \alpha_{N-1} u_N^{n+1} + \beta_{N-1}. \]

From this system we find

\[ u_{n+1}^{n+1} = \left( \beta_{N-1} - \frac{h R(u_N^n, V_N^n) + B H_1(V_N^n) V_N^n}{ES(L)} \left(1 + \frac{h C H(u_N^n)}{ES(L)} - \alpha_{N-1} \right) \right)^{-1}. \]

The functional diagram of the program in the Mathcad system is shown in figure 6.

**Figure 6.** Functional diagram of the program. Purpose of the main functional blocks:

1) \( DN(N, T, M, F, f, k, \ell, S) \) – control block; 2) \( trdag \{a, b, c, f, N, Z0, Z1, Z2, S\} \) – realizes the sweep method; 3) \( R(U, V, x_2) \) – power characteristic; 4) \( f(x, S) \) and 5) \( F(x, S) \) – the corresponding distributions of the initial displacement and velocity of the rod sections; 6), 7), 8) - implement the Fourier method for a linear problem; 9) the cross-sectional area of the rod; 10) determination of the maximum deviation of the contact end of the rod in the iterative cycle.
4. Results and discussions

The dependence of the displacement and velocity of the left end in time is considered under the condition $R(U, V) = 0$ (there is no HLS). The solutions obtained by the Fourier method and the difference method is almost the same. Thus, the parameters of the difference scheme are selected, for example, when $P = 400$ Ns: $\tau = 6.25 \cdot 10^{-5}$ s, $h = 9.6 \cdot 10^{-3}$ m (coarse grid), $\tau = 6.25 \cdot 10^{-7}$ s (fine grid), $\sigma = 0.9$.

Figure 7 shows the solutions obtained over a short interval (fine grid). In this case, we have oscillations of high frequency and small amplitude ($c, d$). They are the cause of high-frequency oscillation of the rod end velocity ($b$). For comparison, the rod oscillation without an HLS element is given (the solution is found by the Fourier method). The use of a three-layer scheme showed a better approximation to the solution obtained by the Fourier method only for high-frequency oscillations of small amplitude. Such oscillations lead to significant frequency changes in the velocity sign, which is taken into account in the power characteristic. The use of an average velocity sign partially removes this undesirable effect. The experiment results do not show oscillations of this type. It turned out that this problem is solved automatically for a two-layer scheme, therefore, the use of a two-layer scheme can be considered more preferable in solving problems of this type.

![Graphs and diagrams](image)

Figure 7. (a) Change in cross-sectional area of the rod; (b) Displacement velocity of the contact end of the rod: 1 – two-layer scheme; 2 – three-layer scheme

High-frequency oscillations of the rod ends with constant ($c$) and variable ($d$) area:

1 – Fourier method; 2 – three-layer scheme; 3 – two-layer scheme.

Low-frequency oscillations of the contact rod end over a long interval including a dissipative element (coarse grid, two-layer scheme) are presented in figure 8.
Figure 8. Oscillations of the contact rod end with the restriction: 1 - without HLS and dissipation (Fourier method); 2 - including HLS and dissipation (difference method) (a) $P = 400$ Ns; $k_1 = 1$; $b = 3000$ Ns / m; (b) $P = 600$ Ns; $k_1 = 1$; $b = 6000$ Ns / m.

The voltage distribution along the length of the variable cross-section rod at different points of time (see table 2) is shown in figure 9.

Table 2. Values of time points.

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|
| $t_i$, $10^6$ c | 2.5 | 35 | 75 | 160 | 200 | 325 |

The comparison of two difference schemes confirms the advantage of a two-layer scheme for solving this problem. The solution obtained using the three-layer scheme is characterized by the presence of high-frequency voltage oscillations, and this is not consistent with the experimental data.

Figure 9. Voltage distribution over the cross sections of the rod at specific points of time: 1 – three-layer scheme.

The conducted numerical experiments can justify the use of heterogeneous lyophobic systems (HLS) in conjunction with dissipative elements in a variety of hydraulic drives of technical systems. HLS can physically exist in the form of any capillary-porous body with open porosity immersed in non-wetting fluid.

5. Conclusion

1. Mathematical models of damping oscillations using hysteresis including a dissipative element are suggested. An increase in the efficiency of the damping device is shown.
2. Taking into account the solution continuity of initial problems with nonlinear force characteristics including dissipative resistance and additional force, analytical solutions are found on the example of a single-mass model at small time intervals.

3. The coincidence of approximate solutions (Runge-Kutta method) with exact analytical solutions including a nonlinear force characteristic, additional force, and a dissipative element is shown for a single-mass model.

4. A model of the impact device tool as a rod of variable cross section in the presence of hysteretic damping (HLS element) together with a dissipative one is constructed. The efficiency of the two-layer difference scheme in comparison with the three-layer difference scheme for control calculations by the Fourier method to solve the linear problem is shown. The use of oscillation equations for a rod of variable cross-section allowed us to determine the voltage distribution along the rod length.

5. Modeling of oscillations taking into account the power characteristic obtained experimentally is provided by an iterative process in which the power characteristic is adjusted with the result of the first iteration.

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