Dynamical Dark Energy model parameters with or without massive neutrinos.

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Abstract: We use WMAP5 and other cosmological data to constrain model parameters in quintessence cosmologies, focusing also on their shift when we allow for non-vanishing neutrino masses. The Ratra–Peebles (RP) and SUGRA potentials are used here, as examples of slowly or fastly varying state parameter $w(a)$. Both potentials depend on an energy scale $\Lambda$. Here we confirm the results of previous analysis with WMAP3 data on the upper limits on $\Lambda$, which turn out to be rather small (down to $\sim 10^{-9}$ in RP cosmologies and $\sim 10^{-5}$ for SUGRA). Our constraints on $\Lambda$ are not heavily affected by the inclusion of neutrino mass as a free parameter. On the contrary, when the neutrino mass degree of freedom is opened, significant shifts in the best-fit values of other parameters occur.

Keywords: Dark Energy theory, Dark Matter, Cosmological Neutrinos, neutrino properties, cosmology of theories beyond the SM.
1. Introduction

Recent data require an accelerated expansion of the Universe. High-redshift supernovae [1], CMB [2] and deep samples [3] essentially agree with requiring a density parameter for non–relativistic matter $\Omega_m \sim 0.27$, while the total density parameter $\Omega_0 \sim 1$. A fluid, dubbed Dark Energy (DE), filling the gap in the cosmological energy balance, can account for cosmic acceleration if its state parameter $w \equiv \rho_{de}/P_{de}$ is close to -1.

In the minimal model, dubbed ΛCDM (or cosmic concordance) cosmology, DE has a state parameter $w \equiv -1$ as though being a false vacuum, and this is equivalent to introducing Einstein’s cosmological constant. All data available up to now can be accommodated in this cosmology. Still there are several problems related to the nature of the DE. Infact, two critical issues plague such model: the fine tuning of the present DE energy density to the vacuum expectation value of quantum field theories, and the coincidence between DE domination and the advent of the non–linear growth of matter fluctuations.

In the attempt to approach a better understanding of DE nature, a varying state parameter $w(a)$ has been tentatively allowed. Here two patterns can be followed: either phaenomenological fitting expressions for $w(a)$ can be introduced or, if we assume that DE is a self–interacting scalar field $\phi$ [4], tracking potential expressions $V(\phi)$ [5], depending on suitable parameters, can be tested. This latter approach has the advantage of allowing for larger energy scales of DE, thus easing the fine tuning problem.

Analysis of such dynamical Dark Energy (dDE) scenario were previously performed when WMAP3 data became available [6]. The first aim of this work is to inspect how far their output are modified when WMAP5 data are considered, in analogy to the modifications occurring for ΛCDM cosmologies. As in [6], we shall do so by using RP and SUGRA potentials (see below).

Here we shall however further focus on the impact on parameters arising from allowing for a neutrino mass. As a matter of fact, cosmology is sensitive to $\nu$ masses (see [7], [8]) even at a level below current limits set by $\beta$–decay experiments [9], so much that the very opening of the degree of freedom of neutrino mass, causes appreciable shifts in the best–fit
values of some cosmological parameters. It should be outlined that non-vanishing neutrino masses are required by flavor mixing results \cite{10}. Even though such results set quite a low limit to the sum $M_\nu$ of neutrino masses, it would be therefore unappropriate to neglect the neutrino mass option.

This fact was recently exploited by several researchers to show that a larger coupling between CDM and DE is compatible with data \cite{11, 12, 13} when allowing for a non-zero neutrino mass. Here, however, no such coupling shall be considered.

Let us finally outline that further alternatives to a cosmological constant or a scalar field have been considered. Among them, modifications to Hilbert–Einstein Lagrangian density, adding \( f(R) \) terms (\( R \): Ricci scalar), or the hypothesis that DE arises as a back reaction to the development of inhomogeneities \cite{14}. Such options make the need of an extra \textit{substance} superfluous, but give rise to other theoretical and observational problems. We shall not deal with them in this work.

The plane of the paper is as follow. In Sec. 2 we describe the models. Data and methods used are reviewed in Sec. 3, while in Sec. 4 we present results and discuss our findings.

2. Models

In a dDE scenario and in the reference frame where the metric is \( ds^2 = a^2(\tau)[d\tau^2 - d\ell^2] \) (\( \tau \): conformal time; \( d\ell \): spatial line element), the energy density and pressure of the field \( \phi \) read

\[
\rho_{\text{de}} = \rho_k + V(\phi), \quad P_{\text{de}} = \rho_k - V(\phi), \quad \text{with} \quad \rho_k = (\phi')^2/2a^2, \quad (2.1)
\]

prime indicating differentiation in respect to \( \tau \). Accordingly, if \( \rho_k \gg V \), the DE state parameter approaches +1 (\textit{stiff matter}) so that DE energy density rapidly dilutes during expansion (\( \rho \propto a^{-6} \)). In the opposite case \( V \gg \rho_k \), the state parameter approaches \(-1\) and DE is suitable to explain the observed cosmic acceleration.

The equations

\[
\rho_{\text{de}}' + 3\frac{a'}{a}(P_{\text{de}} + \rho_{\text{de}}) = 0, \quad (2.2)
\]

and

\[
\phi'' + 2\frac{a'}{a}\phi' + a^2V_{\phi} = 0 \quad (2.3)
\]

are then clearly equivalent. By integrating (2.3) together with the Friedmann equations, we obtain the time evolution of \( \phi \) and, thence, of \( \rho_k, V(\phi), \rho_{\text{de}}, P_{\text{de}} \) and, finally, \( w(a) \).

Quite in general, the solution of a differential equation depends on initial conditions. \textit{Tracker potentials}, however, have attractor solutions (\textit{tracking solutions}) on which they converge (almost) independently of the initial conditions.

Among tracker potentials, we consider here:

\[
V(\phi) = \Lambda^{\alpha+4}/\phi^\alpha \quad \text{RP} \quad (2.4)
\]

\[
V(\phi) = (\Lambda^{\alpha+4}/\phi^\alpha)\exp(4\pi \phi^2/m_p^2) \quad \text{SUGRA} \quad (2.5)
\]
Figure 1: Comparison between the variation of the state parameter for RP and SUGRA potentials and the usual parametrization $w(a) = w_o + (1 + a)w_a$. Here, $\Lambda$ is 100 GeV for SUGRA and $10^{-8}$ GeV for RP, while $(w_o, w_a) = (-0.8908, 0.525)$. These values are selected to yield a behavior close to SUGRA, at low $z$. Even renouncing to a full coincidence at $z = 0$, the fast variability of $w(a)$ in SUGRA cannot be met by any polynomial $w(a)$. Notice also that, for a RP potential, $w(a)$ varies more slowly. The plot is for $h = 0.7$, $\Omega_b = 0.046$, $\Omega_c = 0.209$.

The RP potential yields a slowly varying $w(a)$ state parameter, while the variation of $w(a)$ for SUGRA potential is faster, as shown in Figure 1. In both cases, for any choice of $\Lambda$ and $\alpha$ these potentials yield a precise DE density parameter $\Omega_{de}$. Here we use $\Lambda$ and $\Omega_{de}$ as free parameters in flat cosmologies; the related $\alpha$ value is then suitably fixed.

Let us then remind the constraints on RP and SUGRA potentials obtained when using WMAP3 data [6]. They confirmed that a RP cosmology requires an energy scale $\Lambda$ below $\sim 10^{-7}$ GeV (and then $\alpha \sim 0.5$); this is coherent with the fact that $w(a)$ is then steadily below -0.85 ($\sim 95\%$ C.L.) as required from observations for a constant $w$ [17]. On the contrary, in the SUGRA case, while their best fit still leads to low $\Lambda$‘s, they found that a value $\Lambda \sim 100$ GeV is within $\sim 2\sigma$’s from the best-fitting model; such scale is then compatible with the ones of SUSY or EW transition.

One significant result of our analysis is that these upper limits are badly lowered when using WMAP5 data.

3. Methods

For this work we use a modified version of the cosmological Boltzmann code CAMB [18, 19] to calculate theoretical CMB and matter power spectra. In our modified code, a precursor program is called before running CAMB, evolving the background solutions in order to determine iteratively the value of $\alpha$ consistent with the assigned $\Omega_{de}$ and $\Lambda$. The scalar field $\phi$ evolves according to the Klein–Gordon equation (2.3). Its initial value $\phi_0$ is obtained
considering that $\rho_{de,0} \sim V(\phi_0)$. In the code we also took into account spatial fluctuations of the DE field.

Our code is embedded into the publicly available Monte Carlo Markov Chain (MCMC) engine CosmoMC \[21\]. The following set of free parameters are used: \{\omega_b, \omega_c, \theta, \tau_o, n_s, \ln(10^{10} A_s), \lambda, M_\nu\}. Here $\omega_{b,c} = \Omega_{b,c} h^2$ are the reduced density parameters for baryonic and cold dark matter, respectively, $h$ being the present value of the Hubble constant in units of 100 km/s/Mpc; $\theta$ is then the ratio of the sound horizon to the angular diameter distance at recombination, $\tau_o$ is the optical depth, $n_s$ is the scalar spectral index, $A_s$ is the primordial amplitude of scalar fluctuations at a scale $k = 0.05 \text{Mpc}^{-1}$ and $\lambda \equiv \log_{10}(\Lambda/\text{GeV})$. In addition we marginalize over the SZ amplitude. We assume three massive neutrino species with degenerate masses.

In the MCMC runs we combined the five year data from the WMAP measurements of the CMB radiation (WMAP5) with the galaxy power spectrum from the 2dF survey \[21\], supernovae type 1a data from the SNLS survey \[22\], and added the priors: $h = 0.72 \pm 0.08$ (from the HST key project \[23\]), $\omega_b = 0.022 \pm 0.002$ (from BBN results \[24, 25, 26\]).

4. Results and discussion

In Table 1 we report best fit parameter values and 1–σ errors for different models. In the case of neutrino masses, whose sum $M_\nu$ is not determined by data, we give its (95% C.L.) upper limit. Also for the scales $\lambda = \log(\Lambda/\text{GeV})$, in the RP and SUGRA potentials, an upper limit is given.

Let us first comment on the $\Lambda$CDM and $w = \text{const.}$ column, which was added here for the sake of comparison. In both of them neutrino masses are allowed. As expected, the values we find for all parameters are strictly coherent with those found by the WMAP team when combining the WMAP5 data with measurements of the position of the baryonic acoustic oscillation (BAO) peak in the galaxy power spectrum and SNLS data. The main discrepancy concerns error bars, being systematically slightly greater here than in the outputs from the WMAP team.

The reason for this can be easily understood. Deep sample data as 2dF and SDSS are substantially coherent, as far as the BAO location is concerned. Some discrepancy between them is however present in the spectral details. Also because of that, the WMAP5 fit takes into account the BAO position only. On the contrary, here we took into account the whole information contained in the 2dF spectral data. In addition we have used slightly different priors than the WMAP team did, so that the (small) discrepancies found are fully justified. In particular, our results for a constant $w = -1$ are largely consistent with the analysis of the WMAP team.

Infact, for $w = -1$ we find an upper limit of $M_\nu < 0.66\text{eV}$, while WMAP5+SNLS+BAO yields $M_\nu < 0.61\text{eV}$. However, when $w \neq -1$ models are considered, the upper limit exhibits a sharp increase, up to 0.94 eV. WMAP5+SNLS+BAO outputs show only a $\sim 10\%$ shift in the upper limit when passing from $\Lambda$CDM to $w \neq -1$. Here the shift is $\sim 4$ times greater. This is related to the widening of the $w$ interval, as greater $M_\nu$ values become compatible with data if we delve into the phantom regime \[27\]. Accordingly, as RP and
| Parameter | $\Lambda$CDM | $w = \text{const.}$ | RP | SUGRA |
|-----------|-------------|----------------|-----|-------|
| $10^2 \omega_b$ | 2.258 ± 0.061 | 2.247 ± 0.062 | 2.278 ± 0.060 | 2.271 ± 0.060 | 2.278 ± 0.060 | 2.274 ± 0.060 |
| $\omega_c$ | 0.1098 ± 0.0040 | 0.1132 ± 0.0069 | 0.1051 ± 0.0050 | 0.1062 ± 0.0047 | 0.1043 ± 0.0051 | 0.1055 ± 0.0050 |
| $H_o$ (km/s/Mpc) | 70.1 ± 2.1 | 69.7 ± 2.2 | 70.9 ± 1.7 | 69.8 ± 2.0 | 70.6 ± 1.9 | 69.6 ± 2.0 |
| $\tau_o$ | 0.087 ± 0.017 | 0.085 ± 0.017 | 0.089 ± 0.017 | 0.090 ± 0.018 | 0.089 ± 0.017 | 0.091 ± 0.018 |
| $w$ | -1 | -1.06 ± 0.10 | - | - | - | - |
| $M_\nu$ (eV) (95% C.L.) | $< 0.66$ | $< 0.94$ | - | $< 0.58$ | - | $< 0.59$ |
| $\lambda$ (95% C.L.) | - | - | $<-8.4$ | $<-7.8$ | $<-3.4$ | $<-4.7$ |
| $n_s$ | 0.962 ± 0.014 | 0.958 ± 0.015 | 0.968 ± 0.014 | 0.967 ± 0.014 | 0.970 ± 0.014 | 0.967 ± 0.014 |
| $\ln(10^{10}A_s)$ | 3.045 ± 0.040 | 3.049 ± 0.040 | 3.047 ± 0.041 | 3.041 ± 0.042 | 3.045 ± 0.041 | 3.041 ± 0.041 |
| $\Omega_m$ | 0.270 ± 0.022 | 0.280 ± 0.027 | 0.254 ± 0.017 | 0.266 ± 0.021 | 0.255 ± 0.018 | 0.266 ± 0.020 |
| $\sigma_8$ | 0.713 ± 0.056 | 0.711 ± 0.059 | 0.749 ± 0.039 | 0.697 ± 0.052 | 0.737 ± 0.046 | 0.689 ± 0.054 |

| $-2 \ln(L)$ | 1407.25 | 1407.38 | 1407.35 | 1407.38 | 1407.44 | 1407.31 |

Table 1: Constraints on $\Lambda$CDM and $w = \text{const}$ models, with massive $\nu$'s, and RP and SUGRA models, with and without massive $\nu$'s. Limits obtained using the combination of data sets described in the text. The last row gives the likelihood of the best-fit sample for each model.

SUGRA models yield an effective equation of state $w(a) > -1$, this leads to more stringent limits on $M_\nu$. 
As far as RP and SUGRA models are concerned, the decrease of errors in respect to WMAP3 outputs in \[6\] has direct consequences on the limits on the energy scale $\Lambda$. In the case $M_\nu \equiv 0$, treated in \[6\], an upper limit $\lambda < -7.7 (2.1)$ was found for RP (SUGRA) at 95% C.L.. Such limits are now lowered to -8.4 for RP and to a severe -3.4 in the case of SUGRA. Limits are even more restrictive if $M_\nu \neq 0$ is allowed.

The reason for this strong improvement in the limits on $\lambda$ can partly be understood by inspecting Figures 2 and 3. Here we show 2D contours for $\lambda$ versus $\omega_c$, $\sigma_8$, $H_0$ and $M_\nu$, which are parameters that are strongly correlated with $\lambda$. In the plots we compare the results from the models with $M_\nu = 0$, to those with a free neutrino mass. There is, for example, strong anticorrelations between $\sigma_8$ and $\lambda$ and $\omega_c$ and $\lambda$. As our analysis gives a preference of significantly larger $\omega_c$ and $\sigma_8$ than the WMAP3 analysis in \[6\], we also find much tighter limits on $\lambda$. Parts of the improved limits can also be attributed to our general decrease in error bars on the parameters.

As far as the other parameters are concerned, all shifts in respect to $\Lambda$CDM are within 1–$\sigma$. It may be however worth considering the 2–$\sigma$ upper limit for $n_s$. When using WMAP3 data there was a discrepancy between the $\Lambda$CDM and the RP or SUGRA cases. In the former case $n_s = 1$ was excluded at the 95% C.L.; on the contrary, $n_s = 1$ kept consistent with data, at the same C.L., for RP and SUGRA. Using WMAP5 data the discrepancy disappears and $n_s = 1$ is above the 95% C.L. for all models considered. This means that the related constraint on inflationary models is apparently much more independent from DE nature.

Figure 2: 2D contour plots with $\lambda$ vs $\omega_c$, $\sigma_8$, $H_0$ and $M_\nu$ for models with a RP potential. Solid black lines are for models free $M_\nu$, and dotted red lines show the contours for models with $M_\nu = 0$.
Another significant shift worth outlining concerns $\sigma_8$ values. Let us remind that

$$\sigma_8^2 = \frac{A_s}{2\pi^2} \int dk \ k^{2+n_s} T^2(k) W^2(kR_8) \ .$$

(4.1)

Here: $T(k)$ is the transfer function, $W(kR)$ is the Fourier transform of a (top–hat) filter, $R_8 = 8 \, h^{-1}\text{Mpc}$.

Although in the absence of major shifts on $A_s$ and $n_s$, it is sufficient to open the degree of freedom of neutrino mass, to allow for significant shifts in $T$ which, in turn, reflect onto a smaller best–fit $\sigma_8$ value.

Also likelihood values for the best–fitting models do not exhibit major changes and cannot be used to discriminate among the cosmologies considered here. This is coherent with the general conclusion we draw, that current data are still unable favor any kind of DE nature.

To summarize, we have studied dDE models with RP and SUGRA potentials, and used a compilation of cosmological data sets to constrain parameters of these models. We have also studied the effect of opening an extra degree of freedom, by allowing for non-zero neutrino masses.

Our basic result is that the energy scale of the DE model can be constrained to $\lambda < -8.4 \ (-3.4)$ if $M_\nu \equiv 0$ and to $\lambda < -8.7 \ (-4.7)$ in presence of massive $\nu$, when using a RP (SUGRA) potential. These limits are significantly stronger than what was found in [6], where they used WMAP3 data and vanishing neutrino masses only. Some of the improvement can be attributed to the inclusion of neutrino masses, as non-zero neutrino masses tend to shrink the allowed parameter space for $\lambda$ (see Figures 2 and 3). However, the most of the improvement is caused by our use of more precise data, and that these data have a
preference of slightly different values of parameters like $\sigma_8$ and $\omega_c$, which in turn drives $\lambda$ to smaller values.

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