Robust composite nonlinear feedback control for uncertain robot manipulators

Yuan Jiang1, Ke Lu2, Chengkap Gong3 and Hao Liang2

Abstract
On the basis of the classical computed torque control method, a new composite nonlinear feedback design method for robot manipulators with uncertainty is presented. The resulting controller consists of the composite nonlinear feedback control and robust control. The core is to use the robust control for online approximation of the system’s uncertainty as a compensation term for the composite nonlinear feedback controller. The design method of the new controller is given, and the convergence of the closed-loop system is proved. The simulation results show that the proposed scheme can make the uncertain robot system have strong robustness and anti-interference ability.

Keywords
Robot, uncertainty, composite nonlinear feedback control, robust control, system convergence

Introduction
Robots have various control methods in the literature.1–6 There are two main control purposes. One is how to realize the stability of the closed-loop system, so that the trajectory tracking error tends to zero as fast as possible. The other is how to suppress the interference of the uncertainty factor to the system and minimize the influence of the interference signal on the tracking accuracy.

Composite nonlinear feedback (CNF) theory7 is an effective method for solving fixed-point tracking tasks in saturated systems. Eren et al.8 verify that the CNF control has better control performance than the time optimal control in fixed-point tracking tasks. However, when there is disturbance in the system or the system model is not accurate (hereinafter referred to as “uncertainty”), the system under the control of CNF would no longer be able to match the reference input accurately. Recently, some progresses are reported on the application of CNF theory. Saleh and Fairouz9 proposed a CNF technique for robust tracking control of switched systems with unmatched uncertainties and input saturation. This scheme guarantees robustness against uncertainties, removes reaching phase, and avoids chattering problem. Saleh and Ma10 proposed a combination of finite-time robust-tracking theory and CNF approach for the finite-time and high-performance synchronization of the chaotic systems in the presence of the external disturbances, parametric uncertainties, Lipschitz nonlinearities, and time delays. In this work, a new finite time robust tracking model following control approach is developed based on the CNF scheme. Saleh11 proposed a combination of CNF and integral sliding mode techniques for fast and accurate chaos synchronization of uncertain
chaotic systems with Lipschitz nonlinear functions, time-varying delays, and disturbances. Jafari and Binazadeh\textsuperscript{12} proposed an observer-based improved CNF controller for output tracking of general time-varying reference signals in descriptor systems subject to actuator saturation. Jafari and Binazadeh\textsuperscript{13} studied the robust output regulation in discrete-time singular systems with actuator saturation and matched uncertainties. The main contribution of this communication lies in designing the CNF control law for uncertain discrete-time singular systems with actuator saturation which guarantees the output regulation against system uncertainties and/or external disturbances.

There are two basic control strategies for uncertain robots: adaptive control\textsuperscript{14} and robust control.\textsuperscript{15} Although Erhart and Hirche\textsuperscript{16} use the adaptive control to obtain better suppression of the uncertainty factor, it only targets the external disturbance factor. If other uncertain factors are considered, the structure of the controller will inevitably change. Wang\textsuperscript{17} proposes that when the control system parameters change, adaptive control can achieve certain performance indicators through timely identification, learning, and adjustment control law, but, when there is non-parametric uncertainty, adaptive control is difficult to guarantee the stability of the system. Although the method by Shaker\textsuperscript{18} has improved the method of Wang,\textsuperscript{17} the real-time requirements are more stringent and the implementation process is more complicated. The robust control can ensure the stability of the system and maintain certain performance indicators within a certain range of uncertainty factors; it is a kind of fixed control and easy to implement. A robust control method can be employed when the adaptive controller does not have the ability to identify changes in the system uncertainty to correct the control law.

As an essential subject of robots, the trajectory tracking control has attracted considerable attention over the last few years. In the study of Elmali and Olgac,\textsuperscript{19} a new methodology of sliding mode control with perturbation estimation offers a robust feedback control with much lower gains than its conventional counterparts against slowly varying perturbations. Zeinali and Notash\textsuperscript{20} presented a new approach for tracking control of robot manipulators. Yin and Pan\textsuperscript{21} proposed a robust adaptive control method to significantly reduce the relatively tracking errors of six-degree-of-freedom (DOF) industrial robots under both external disturbances and parametric uncertainties. Chen et al.\textsuperscript{22} proposed a new scheme that combines a computed torque control and a novel model-assisted extended state observer. In the simulation, a two-DOF manipulator is performed to verify the effectiveness and superiority of the proposed controller. Xia et al.\textsuperscript{23} proposed a control strategy that combines the double power reaching law with the modified terminal sliding mode for tracking the tasks of rigid robotic manipulators quickly and accurately. Chen et al.\textsuperscript{24} consider finite-time trajectory tracking control problem for robotic manipulators with parameter uncertainties and external disturbances. A finite-time controller that achieves high precision and strong robustness is proposed without the requirement of the exact dynamic model. The validity of the control scheme is demonstrated by experiments. In the study of Bezak et al.\textsuperscript{25} an intelligent hand–object contact model is developed for a coupled system assuming that the object properties are known. The control is simulated in the MATLAB Simulink, Neural Network Toolbox, and Computer Vision System Toolbox. Based on the intrinsic properties of the plane, Bozek et al.\textsuperscript{26} proposed a new method of calculation of Jordan curves trajectory of the robot movement.

In this short communication, the robust CNF is proposed to the design of a controller for robot manipulators. It is combined with the traditional computed torque design method to improve tracking performance and to enable systematic stability analysis in the presence of the uncertainties. To effectively suppress the adverse effects of uncertain factors on the robot system, realize the stability of the uncertain robot system, and accurately track the reference input, a combined control strategy of CNF control and robust control is proposed. The core is to use the robust control for online approximation of the system’s uncertainty as a compensation term for the CNF controller. Then the design method of the new controller is given, and the convergence of the closed-loop system is proved. The simulation results show that the proposed scheme can make the uncertain robot system have strong robustness and anti-interference ability.

The main contribution of this article lies in robustifying the CNF control law for the uncertain robot manipulators which improves the tracking performance and guarantees the systematic stability against system uncertainties and/or external disturbances. In this regard, the robust CNF is designed by adding the robust control for online approximation of the system’s uncertainty as a compensation term to the nominal CNF control law. For this purpose, a theorem is given to prove that the proposed control law can drive the output of the uncertain robot manipulators to track a reference input with an ultimately bounded tracking error. Compared with the original CNF controller and robust controller, the new controller has better control performance, which not only fully retains the fast response and overshoot of the CNF control method but also retains the advantage of robust control for effective suppression of uncertainties. Furthermore, computer simulations are done for a practical example to verify the theoretical results.

The article is organized as follows. The second section formulates the problems and includes some preliminaries. The third and fourth sections present combined robust control and CNF control design method and carry out the stability analysis of the closed-loop system, in the presence of uncertainties, respectively. Simulation results, in comparison with the original CNF control and the robust control method, are given in the fifth section. The last section concludes the article.
\textbf{Problem formulation}

The equations of motion of an $n$-link rigid robot can be expressed in the form of

\begin{equation}
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + d
\end{equation}

where $q \in \mathbb{R}^n$ is the joint position, $\dot{q} \in \mathbb{R}^n$ is the joint velocity, $\ddot{q} \in \mathbb{R}^n$ is the joint acceleration, $M(q) \in \mathbb{R}^{n \times n}$ is the inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the coriolis matrix, $G(q) \in \mathbb{R}^{n \times 1}$ is the gravity vector, $d \in \mathbb{R}^{n \times 1}$ is a time-varying disturbance satisfying $\|d\| \leq \rho$, and $\tau \in \mathbb{R}^n$ is the applied torque.

Motion tracking of robot manipulators in joint space can be described as follows. For a given reference input $q_d : R_+ \rightarrow \mathbb{R}^n$, design a feedback law such that

\begin{equation}
\lim_{t \rightarrow \infty} q(t) = q_d(t)
\end{equation}

and in the presence of the uncertainty, $q(t)$ approaches a small neighborhood of $q_d(t)$. Assume that $q_d(t)$ and its first and second time derivatives, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$, are continuous and bounded, respectively.

Jiang et al.\cite{Jiang07} has been developed for the tracking control of linear systems under actuator controls. It consists of linear and nonlinear feedback controls and is a nonlinear design method. The linear feedback control has a small damping ratio, which ensures a fast response without saturation occurrence. While the nonlinear feedback control increases the damping ratio, the response approaches the reference input and thus helps to avoid overshoot.

In practical engineering, the precise model of the robot object is difficult to obtain, and the controller based on the calculated torque can only be designed according to the ideal nominal model, which can be expressed as

\begin{equation}
\tau = M_0(q) \left[ \text{sat}_b\left( \dot{q}_d - k_v \dot{e} - k_p e - \rho(e, \dot{e})B^T P e \right) \right] \\
+ C_0(q, \dot{q})\dot{q} + G_0(q)
\end{equation}

where $\text{sat}_b(\cdot)$ is saturation function in the form of $\text{sat}(u) = \text{sign}(u) \min\{u_{\text{max}}, |u|\}$, which can effectively overcome the chattering of the system. $\text{sat}_b(\cdot)$ is the saturation vector value function of each channel inside the robot. The saturation function has some beneficial characteristics and can improve the control performance in the control community. $e = q - q_d$ and $\dot{e} = \dot{q} - \dot{q}_d$ are the position and velocity error vectors, respectively. $\rho(e, \dot{e}) = \text{diag}\{\rho_1(e, \dot{e}), \rho_2(e, \dot{e}), \ldots, \rho_n(e, \dot{e})\}$ with $\rho_i(e, \dot{e})$ being non-negative function, bounded, and locally Lipschitz in $e$ and $\dot{e}$.

In equation (3), when the gain matrices

\begin{equation}
k_p = \text{diag}(\gamma_v^2, \gamma_v^2), k_v = \text{diag}(2\gamma_v, 2\gamma_v), \gamma_v > 0
\end{equation}

are the $n \times n$ symmetric positive definite matrices, the robot system can be decoupled; $B = [0 \ I]^T$ with 0 and $I$ being a zero matrix and identity matrix of appropriate dimensions, respectively; $P > 0$ is the solution of the following Lyapunov equation

\begin{equation}
\begin{bmatrix}
0 & I \\
-k_p & -k_v
\end{bmatrix}
\begin{bmatrix}
0 & I \\
-k_p & -k_v
\end{bmatrix}^T = -Q
\end{equation}

We use $M_0(q)\ddot{q} + C_0(q, \dot{q})\dot{q} + G_0(q)$ to subtract the left and right sides of equation (3), and let $\Delta M = M_0 - M$, $\Delta C = C_0 - C$, and $\Delta G = G_0 - G$, then we can obtain

\begin{equation}
\text{sat}_m\left( \ddot{e} + k_v \dot{e} + k_p e + \rho(e, \dot{e})B^T P e \right) = M_0^{-1}(\Delta M \ddot{q} + \Delta C \dot{q} + \Delta G + d)
\end{equation}

where $|\text{sat}_m(\cdot)| \geq |\text{sat}_b(\cdot)|$. Let $f(x) = M_0^{-1}(\Delta M \ddot{q} + \Delta C \dot{q} + \Delta G + d)$ be the uncertainty term. $\text{sat}_m(\cdot)$ is the saturation vector value function, which can guarantee the stability of the whole closed-loop system in the presence of large uncertainty. It can be seen from equation (4) that the control performance is degraded due to the external disturbances in the system. Therefore, it is necessary to compensate the uncertainty term in the system.

\textbf{New controller design}

The new control law can be designed as follows

\begin{equation}
\tau = \text{sat}_b\left( \dot{q}_d - k_v \dot{e} - k_p e - \rho(e, \dot{e})B^T P e \right) - \hat{f}(x) + C_0(q, \dot{q})\dot{q} + G_0(q)
\end{equation}

Substituting equation (5) into equation (1), we can obtain

\begin{equation}
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \text{sat}_b\left( \dot{q}_d - k_v \dot{e} - k_p e - \rho(e, \dot{e})B^T P e \right) - \hat{f}(x) + C_0(q, \dot{q})\dot{q} + G_0(q) + d
\end{equation}

Because $\Delta M = M_0 - M$, $\Delta C = C_0 - C$, and $\Delta G = G_0 - G$, we have
\[ (M_0 - \Delta M)\ddot{q} + (C_0 - \Delta C)\dot{q} + G_0 - \Delta G \]
\[ = \text{sat} \left[ M_0(q) \left[ \text{sat}_h \left( \hat{q}_d - k_i\dot{e} - k_pe e - \rho(e, \dot{e})B^TP \left[ e \atop \dot{e} \right] \right) - \hat{f}(x) \right] + C_0(q, \dot{q})\dot{q} + G_0(q) \right] + d \]  \hfill (7)

Then, we have
\[ M_0\ddot{q} - \text{sat} \left[ M_0(q) \left[ \text{sat}_h \left( \hat{q}_d - k_i\dot{e} - k_pe e - \rho(e, \dot{e})B^TP \left[ e \atop \dot{e} \right] \right) - \hat{f}(x) \right] \right] = \Delta M\ddot{q} + \Delta C\dot{q} + \Delta G + d \]  \hfill (8)

Then after some algebraic manipulations, we have
\[ \ddot{q} - \text{sat} \left[ \text{sat}_h \left( \hat{q}_d - k_i\dot{e} - k_pe e - \rho(e, \dot{e})B^TP \left[ e \atop \dot{e} \right] \right) - \hat{f}(x) \right] = M_0^{-1}(\Delta M\ddot{q} + \Delta C\dot{q} + \Delta G + d) \]  \hfill (9)

So
\[ \text{sat}_m \left( \dot{e} + k_i\dot{e} + k_pe e + \rho(e, \dot{e})B^TP \left[ e \atop \dot{e} \right] \right) + \hat{f}(x) = f(x) \]  \hfill (10)

where \( \text{sat}_m(\cdot) = \ddot{q} - \text{sat}_h(\cdot) \), \( |\text{sat}(\cdot)| \geq |\text{sat}_m(\cdot)| \geq |\text{sat}_h(\cdot)| \), and \( \hat{f}(x) = -\Delta v \) is the compensation term for the uncertainty in the system. Let \( f(x) = \tilde{\eta} \), then we can obtain \( f(x) - \hat{f}(x) = \tilde{\eta} + \Delta v \).

The robot dynamics equation has the following properties\cite{1}:

1. The inertia matrix \( M(q) \) is a symmetric positive definite bounded matrix, satisfying
\[ \lambda_1 I \leq M(q) \leq \lambda_2 I \]  \hfill (11)

where \( \lambda_1 \) and \( \lambda_2 \) are known positive numbers. In addition, \( M(q) \) is non-singular, that is, \( M^{-1}(q) \) exists.

2. There is a known function \( \Phi(x, t) \) that is bounded by \( t \) such that
\[ \|\Delta C\dot{q} + \Delta G\|_2 \leq \Phi(x, t), \forall q, \dot{q} \in \mathbb{R}^n \]  \hfill (12)

3. For any vector \( q \in \mathbb{R}^n \) and \( \dot{q} \in \mathbb{R}^n \), the time derivative \( M(q) \) and the matrix \( C(q, \dot{q}) \) of the inertia matrix satisfy
\[ \dot{q}^T \left[ \frac{1}{2} \dot{M}(q) - C(q, \dot{q}) \right] \dot{q} = 0 \]  \hfill (13)

that is, the matrix value function \( [\frac{1}{2} \dot{M}(q) - C(q, \dot{q})] \) is obliquely symmetric.

When the above properties are satisfied, the following lemma can be proved.

**Lemma 1.** If the robot system satisfies equation (11), there exists an estimated value \( M_0(q) \) of \( M(q) \) and a corresponding constant \( \alpha \), such that
\[ \|M^{-1}(q)M_0(q) - I\|_2 \leq \alpha < 1, \forall q \in \mathbb{R}^n \]  \hfill (14)

If the robot system satisfies equation (12), there exists an estimated value \( M_0(q) \) of \( M(q) \) and a continuous function \( \delta(x, t) \) which is bounded, such that \( \Delta v \) and \( \tilde{\eta} \) satisfy
\[ \|\Delta v\|_2 \leq \delta(x, t), \|\tilde{\eta}\|_2 \leq \delta(x, t) \]  \hfill (15)

**Proof.** If the inertia matrix \( M(q) \) of the robot satisfies equation (11), then there exist two positive numbers \( \alpha_1 \) and \( \alpha_2 \) such that
\[ \alpha_1 \leq \frac{1}{\lambda_1} \leq \|M^{-1}(q)\|_2 \leq \frac{1}{\lambda_2} \leq \alpha_2, \forall q \in \mathbb{R}^n \]  \hfill (16)

We choose an estimate value \( M_0(q) \) in the following equation
\[ M_0(q) = \frac{2}{\alpha_1 + \alpha_2} I \]

Through simple calculations, we can find such an \( \alpha \) such that
\[ \|M^{-1}(q)M_0(q) - I\|_2 \leq \frac{\alpha_2 - \alpha_1}{\alpha_1 + \alpha_2} \triangleq \alpha < 1, \forall q \in \mathbb{R}^n \]

So if the robot satisfies equation (11), there exists the estimated value \( M_0(q) \) and the constant \( \alpha \) satisfies the equation (14) by using property 1.

If the robot satisfies equation (12), under the condition of equation (15), then the following can be obtained
\[ \|\tilde{\eta}\|_2 = \|M^{-1}M_0[\tilde{q}_d + Kx + \Delta v] - M^{-1}[\Delta C\dot{q} + \Delta G]\|_2 \leq \alpha \|\tilde{q}_d\|_2 + \|K\|_2 \|x\|_2 + \delta(x, t) + \alpha_2\Phi(x, t) \]  \hfill (17)

where \( \tilde{q}_d^M \) is the maximum value of \( \tilde{q}_d \) and \( K = [k_p \ k_v] \).

Because \( 0 < \alpha < 1 \), we let
\[ \alpha \|\tilde{q}_d\|_2 + \|K\|_2 \|x\|_2 + \delta(x, t) + \alpha_2\Phi(x, t) = \rho(x, t) \]

Therefore, we can obtain
\[ \delta(x, t) = \frac{1}{1 - \alpha} \left[ \alpha \|\tilde{q}_d\|_2 + \|K\|_2 \|x\|_2 + \alpha_2\Phi(x, t) \right] \]  \hfill (18)

Let \( v = B^TPx \), so the robust compensation term can be designed as
\[
\Delta v = \begin{cases} 
-\delta(x, t) \frac{v}{\|v\|_2}, & \text{when } \|v\|_2 \geq \varepsilon \\
-\delta(x, t) \frac{v}{\varepsilon}, & \text{when } \|v\|_2 < \varepsilon
\end{cases}
\] (19)

where \( \varepsilon > 0 \) and \( \Delta v \) satisfy equation (15). The proof is completed.

**System stability analysis**

**Theorem 1.** Given the robot system (1) and the new control law (5), if the robot closed-loop systems satisfy the following conditions

1. There exists \( c > 0 \) such that

\[
[e \  \dot{e}]^T \in L_v(c) = \left\{ \begin{bmatrix} e \\ \dot{e} \end{bmatrix} : \begin{bmatrix} e \\ \dot{e} \end{bmatrix}^T P \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \leq c \right\}
\]

\[
\Rightarrow \|\ddot{q}_d - k_{vi} \dot{e} - k_{pi} e\| \leq u_{i,\text{max}}, i = 1, 2, \ldots, n
\]

where \( k_{pi} \) and \( k_{vi} \) are row vectors of matrices \( k_p \) and \( k_v \), respectively; \( L_v(c) \) is the estimated ellipsoid invariant set of the system stability attraction domain and \( c \) is its radius; \( u_{i,\text{max}} \) is the maximum output amplitude of each manipulator.

2. If the initial condition \([e(0) \  \dot{e}(0)]^T\) satisfies

\[
[e(0) \  \dot{e}(0)]^T \in L_v(c).
\]

Then the trajectories of the closed-loop system starting from \( L_v(c) \) will asymptotically converge to the origin. For any reference input \( q_d \) and any initial state \([q(0) \ \dot{q}(0)]^T\) satisfying \([e(0) \ \dot{e}(0)]^T \in L_v(c)\), the trajectory starting from \([q(0) \ \dot{q}(0)]^T\) will asymptotically converge to \([q_d(t) \ \dot{q}_d(t)]^T\).

**Proof.** Combining equations (1), (5), and (10), after cancellation of nonlinear functions \( M(q), C(q, \dot{q}) \dot{q} \), and \( G(q) \), the closed-loop system is represented as

\[
\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -k_p & -k_v \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + B \begin{bmatrix} s_{th} \left( \ddot{q}_d - k_v \dot{q} - k_p e - \rho(e, \dot{e})B^T P \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \right) \end{bmatrix} + \ddot{q} + \Delta v - \ddot{q}_d + k_v \dot{e} + k_p e
\]

Let \( x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \), then choosing a Lyapunov function candidate

\[
V = \frac{1}{2} x^T P x
\]

The time derivative of the Lyapunov function \( V \) along the trajectories of the closed-loop system shows that

\[
\dot{V} = -\frac{1}{2} x^T Q x + x^T \begin{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} s_{th} \left( \ddot{q}_d - k_v \dot{q} - k_p e - \rho(e, \dot{e})B^T P \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \right) \end{bmatrix} + \ddot{q} + \Delta v - \ddot{q}_d + k_v \dot{e} + k_p e
\]

\[
= -\frac{1}{2} x^T Q x + \sum_{i=1}^{n} v_i [s_{th}(k_i - \rho_i(e)v_i) - k_i] + v^T (\ddot{q} + \Delta v)
\]

where \( v_i \) is the \( i \)th element of \( B^T P x \), and \( k_i = \ddot{q}_d - k_v \dot{e} - k_p e, i = 1, 2, \ldots, n \).

Considering the item \( \sum_{i=1}^{n} v_i [s_{th}(k_i - \rho_i(e)v_i) - k_i] \) for each robot manipulator, if \( |k_i - \rho_i(e)v_i| \leq u_{i,\text{max}} \), induces

\[
v_i [s_{th}(k_i - \rho_i(e)v_i) - k_i] = -\rho_i(e)v_i^2 \leq 0 \quad \text{while} \quad |k_i - \rho_i(e)v_i| > u_{i,\text{max}}, \text{ gives rise to } |k_i| \leq u_{i,\text{max}}.
\]

Therefore, we have

\[
\begin{cases}
0 < s_{th}(k_i - \rho_i(e)v_i) - k_i < -\rho_i(e)v_i, k_i > u_{i,\text{max}} \\
-\rho_i(e)v_i < s_{th}(k_i - \rho_i(e)v_i) - k_i < 0, k_i < -u_{i,\text{max}}
\end{cases}
\]

It makes \( v_i \) and \( s_{th}(k_i - \rho_i(e)v_i) - k_i \) have opposite signs, indicating that

\[
v_i [s_{th}(k_i - \rho_i(e)v_i) - k_i] \leq 0
\]
We always have
\[ \sum_{i=1}^{n} v_i [\text{sat}_{h_i}(k_i - \rho_i(v_i) - k_i)] \leq 0 \]

It means
\[ \dot{V} \leq -\frac{1}{2} v^T Q x + v^T (\bar{\eta} + \rho \frac{v}{\|v\|_2}) \]

According to equation (19), when \( \|v\|_2 \geq \varepsilon \), substituting \( \Delta v \) into equation (21), we can obtain
\[ \dot{V} \leq -\frac{1}{2} v^T Q x < 0 \tag{22} \]

When \( \|v\|_2 < \varepsilon \), using the same method, we can obtain
\[ \dot{V} \leq -\frac{1}{2} v^T Q x + v^T \left( -\frac{\delta}{\varepsilon} v + \frac{\delta}{\|v\|_2} \right) \]

According to Huo,\(^{27}\) we can prove
\[ v^T \left( -\frac{\delta}{\varepsilon} v + \frac{\delta}{\|v\|_2} \right) \leq \frac{\varepsilon \delta}{4} \tag{23} \]

Substituting equation (23) into equation (21), we can obtain
\[ \dot{V} \leq -\frac{1}{2} v^T Q x + \frac{\varepsilon}{4} \frac{\delta}{\varepsilon} \leq \lambda_{\text{min}}(Q) x^T x + \frac{\varepsilon}{4} \delta \tag{24} \]

where \( \lambda_{\text{min}}(Q) \) is the minimum eigenvalue of \( Q \) and \( \delta \) is the upper bound in closed region \( \|v\|_2 < \varepsilon \). It can be seen from equation (24) that when \( \|v\|_2 < \varepsilon \), to make \( \dot{V} < 0 \), we choose
\[ \|x\|_2 \geq \sqrt{\frac{\varepsilon \delta}{4\lambda_{\text{min}}(Q)}} \]

From equations (22) and (24), we deduce that the condition for system convergence is \( |x|_2 \geq \sqrt{\frac{\varepsilon \delta}{4\lambda_{\text{min}}(Q)}} \).

That is, for any reference input \( q_d \) and initial state \( [e(0) \dot{e}(0)]^T \in L_v(c) \) the trajectory starting from \([q(0) \dot{q}(0)]^T \) will converge to \([q_d(t) \dot{q}_d(t)]^T \).

From equation (20), using the method of Lagrange multiplier, the convergence radius \( c \) satisfying the stability requirement \( L_v(c) = \{x | \|x\|_2 \leq c \} \) can be obtained by the following equation
\[ c = \min \left\{ e^T P e : [-k_{pu} - k_{vi}] \begin{bmatrix} e \end{bmatrix} = (1 - \Delta) h_i \right\} \]

\[ = \frac{[(1 - \Delta) h_i]^2}{[-k_{pu} - k_{vi}]^T P^{-1} [-k_{pu} - k_{vi}]} , i = 1, 2 \]

(25)

This completes the proof.

In summary, the new control scheme can ensure that the solution of the closed-loop system is ultimately uniformly bounded, and when the control algorithm allows the selected \( \varepsilon \) to be small enough, the control scheme can effectively overcome the influence of uncertain factors in the robot system.

**Simulation results and analysis**

For the PUMA560 robot, the dynamic equation is in the form of equation (1), which is expressed as follows
\[ \begin{bmatrix} 3.46 + 2.4\cos(q_2) & 0.96 + 1.2\cos(q_2) \\ 0.96 + 1.2\cos(q_2) \end{bmatrix} [\ddot{q}_1] \]
\[ + \begin{bmatrix} -1.2\sin(q_2) & \ddot{q}_2 \\ 1.2\sin(q_2) & 0 \end{bmatrix} [\ddot{q}_1] \]
\[ = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + d \tag{26} \]

The error disturbance is selected as
\[ d_1 = 2I, d_2 = 3I, d_3 = 6I, d = d_1 + d_2 \|e\| + d_3 \|\dot{e}\| \]

(27)

The desired trajectory is
\[ q_{id} = 1 + 0.2\sin(0.5\pi t), q_{2d} = 1 - 0.2\cos(0.5\pi t) \]

Let \( \Delta M, \Delta C, \Delta G \) as 20% of the whole, by the robot dynamics equation (26) and the inequality (12), we can calculate \( \Phi(x) \) as
\[ \Phi(x) = \left( 12\cos(q_1 + q_2) + 1.2\dot{q}_1 \dot{q}_2 \sin(q_2) \right)^2 + \left( 12\cos(q_1 + q_2) + 2.5\dot{q}_1 \dot{q}_2 - 1.2\dot{q}_1 \dot{q}_2 \sin(q_2) \right)^2 \frac{1}{2} \]

(29)

To design \( k_p \) and \( k_v \), letting \( \gamma = 3 \), we have \( \|K\|_2 = 10.8 \), and \( \|\dot{q}_d\|_2 \leq \frac{\dot{q}_M}{\sqrt{2}} = 1.56 \). Let \( \alpha = 0.2 \) and \( \alpha_2 = 3.5 \), we can obtain \( \delta(x) = 1.25[0.2(1.56 + 10.8 \|x\|_2) + 3.5\Phi(x)] \). Then let \( \varepsilon = 0.1 \), we can design the robust compensation term \( \Delta v \). The nonlinear function is selected\(^{28}\) as follows \( \rho(e) = \text{diag}(600e^{-60|e_1|}, 1500e^{-47|e_2|}) \).

Using the same method,\(^{29}\) we can obtain \( h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 47.5 \\ 30 \end{bmatrix} \), and let \( \text{sat}_{\mu}(\cdot) = \text{sat}_{\mu}(\cdot) \).

To guarantee the output of the controller operating in the linear range of actuators at the start, we choose \( \tau(0) = M(0)u_{\text{max}} \leq \tau_{\text{max}} \), where \( u_{\text{max}} \) is the maximum saturation value. After calculation, we can obtain
\[ u_{\text{max}} = \begin{bmatrix} u_{1, \text{max}} \\ u_{2, \text{max}} \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix} \]
In the simulation, we choose $\rho = 2.33$. Let $Q = 50I$, via level set estimation, and it follows that the boundary value $c = 24$ can be obtained, so that the stability of the original system can be guaranteed.

Some simulation results of the closed-loop system under feedback laws are shown in Figures 1 to 8.

Figures 1 to 4 are the simulation results of the closed-loop system under the original CNF controller, the robust controller, and the new controller. From Figures 1 to 4, it can be seen that the proposed controller has the fast response, almost no overshoot and the fastest settling time. From the three indices, the proposed new CNF controller has the best transient performance.

Figure 1. Position tracking for link 1 under different controllers.

Figure 2. Position tracking for link 2 under different controllers.

Figure 3. Trajectory tracking error of joint 1 under different controllers.

Figure 4. Trajectory tracking error of joint 2 under different controllers.

Figure 5. Estimation and compensation of uncertainties under the robust controller.

Figure 6. Estimation and compensation of uncertainties under the new controller.
Figures 5 and 6 show the estimation and compensation results of the torque generated by the uncertainties of the system under the robust controller and the proposed new controller. Taking joint one as an example, it can be seen from the compensation curve that the proposed new controller is more accurate in estimating and compensating for uncertainty when $T > 3$. In contrast, the robust controller can obtain more accurate estimation and compensation when $T > 6$, but the closed-loop system still cannot accurately track the reference trajectory. Therefore, the proposed new controller can provide a strong guarantee for the uncertain system to obtain better control performance.

Figures 7 and 8 show the control inputs for the system under different controllers.

Figures 5 and 6 show the estimation and compensation results of the torque generated by the uncertainties of the system under the robust controller and the proposed new controller. Taking joint one as an example, it can be seen from the compensation curve that the proposed new controller is more accurate in estimating and compensating for uncertainty when $T > 3$. In contrast, the robust controller can obtain more accurate estimation and compensation when $T > 6$, but the closed-loop system still cannot accurately track the reference trajectory. Therefore, the proposed new controller can provide a strong guarantee for the uncertain system to obtain better control performance.

Figures 7 and 8 show the control inputs for the system under different controllers.

**Conclusion**

The new controller proposed in this communication not only enables the robot system to obtain good trajectory tracking ability but also makes the system have good stability and robustness. Compared with the original CNF controller and robust controller, the new controller has better control performance, which not only fully retains the fast response and overshoot of the CNF control method but also the advantage of robust control for effective suppression of uncertainties. Therefore, the scheme can be extended to the space trajectory tracking task of the robot manipulators. The controller proposed in this communication has a good control effect through computer simulation. But for the specific robot, the control effect of the proposed algorithm needs to be further verified by specific experiments. In the next step, we will verify whether the algorithm in this communication is effective for the actual robot.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was partially supported by the National Natural Science Foundation of China [nos 61663030 and 61663032], the Natural Science Foundation of Jiangxi Province [no. 20142BAB207021], the Foundation of Jiangxi Educational Committee [no. GJJ150753], the Open Fund of Key Laboratory of Image Processing and Pattern Recognition of Jiangxi Province (Nanchang Hangkong University; no. TX201404003), the Key Laboratory of Nondestructive Testing (Nanchang Hangkong University), the Ministry of Education [no. ZD29529005], and the reform project of degree and postgraduate education in Jiangxi [no. JXYJG-2017-131].

**ORCID iD**

Yuan Jiang https://orcid.org/0000-0002-4821-3545

**References**

1. Mahmoud MS and Nasir MT. Robust control design of wheeled inverted pendulum assistant robot. *IEEE/CAA J Autom Sinica* 2017; 4(4): 628–638.

2. Guerin J and Thiery S. Locally optimal control under unknown dynamics with learnt cost function: application to industrial robot positioning. *Journal of Physics: Conference Series* 2017; 783(1): 012036.

3. Rong X, Jian-Bo Z, Qing T, et al. Time optimal control of mobile robots under constraints of velocity and acceleration limits. *Contr and Deci* 2014; 1: 118–122.

4. Mozei SA, Rafeeyan M, and Zakeri E. Simulation and experimental control of a 3-RPR parallel robot using optimal fuzzy controller and fast on/off solenoid valves based on the PWM wave. *ISA Trans* 2016; 61: 265–286.

5. Zeng W, Wang Q, and Liu F. Learning from adaptive neural network output feedback control of a unicycle-type mobile robot. *ISA Trans* 2016; 61: 337–347.

6. Santos L and Rui C. Computed-torque control for robotic-assisted tele-echography based on perceived stiffness estimation. *IEEE T Autom Sci Eng* 2018; 99: 1–18.

7. Chen BM, Lee TH, and Peng K. Composite nonlinear feedback control for linear systems with input saturation: theory and an application. *IEEE T Automat Contr* 2003; 48(3): 427–439.

8. Eren S, Pahlevaninezhad M, and Bakhshai A. Composite nonlinear feedback control and stability analysis of a grid-
connected voltage source inverter with LCL filter. *IEEE Trans Ind Electron* 2013; 60(11): 5059–5074.

9. Saleh M and Fairouz T. Composite nonlinear feedback integral sliding mode tracker design for uncertain switched systems with input saturation. *Commun Nonlinear Sci* 2018; 65: 173–184.

10. Saleh M and Ma J. Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay. *Chaos Soliton Fract* 2018; 114: 46–54.

11. Saleh M. Chaos synchronization of uncertain chaotic systems using composite nonlinear feedback based integral sliding mode control. *ISA Trans* 2018; 77: 100–111.

12. Jafari E and Binazadeh T. Observer-based improved composite nonlinear feedback control for output tracking of time-varying references in descriptor system. *ISA Trans* 2019; 91: 1–10.

13. Jafari E and Binazadeh T. Robust output regulation in discrete-time singular systems with actuator saturation and uncertainties. *IEEE T Circuits II* 2019; 67: 1–1.

14. Jia YH, Hu Q, and Xu SJ. Dynamics and adaptive control of a dual-arm space robot with closed-loop constraints and uncertain inertial parameters. *Acta Mech Sinica* 2014; 30(1): 112–124.

15. Kim CS, MO EJ, and Han SM. Robust visual servo control of robot manipulators with uncertain dynamics and camera parameters. *Int J Control Autom* 2010; 8(2): 308–313.

16. Erhart S and Hirche S. Adaptive force/velocity control for multi-robot cooperative manipulation under uncertain kinematic parameters. In: *IEEE/RSJ international conference on intelligent robots and systems*, Tokyo, Japan, 3–7 November 2013, pp. 307–314. IEEE.

17. Wang H. Adaptive control of robot manipulators with uncertain kinematics and dynamics. *IEEE T Automat Contr* 2017; 62(2): 948–954.

18. Shaker HR. Adaptive control for revolute joints robot manipulator with uncertain/unknown dynamic parameters and in presence of disturbance in control input. In: *International conference on mechatronics and robotics engineering*, Paris, France, 8–12 February 2017, pp. 23–29. USA: ACM.

19. Elmali H and Olgac N. Sliding mode control with perturbation estimation (SMCPE): a new approach. *Int J Control* 1992; 56(4): 923–941.

20. Zeinali M and Notash L. Adaptive sliding mode control with uncertainty estimator for robot manipulators. *Mech Mach Theor* 2010; 45(1): 80–90.

21. Yin X and Pan L. Enhancing trajectory tracking accuracy for industrial robot with robust adaptive control. *Robot CIM Int Manuf* 2018; 51: 97–102.

22. Chen C, Zhang C, and Hu T. Model-assisted extended state observer-based computed torque control for trajectory tracking of uncertain robotic manipulator systems. *Int J Adv Robot Syst* 2018; 15(5).

23. Xia Y, Xie W, and Ma J. Research on trajectory tracking control of manipulator based on modified terminal sliding mode with double power reaching law. *Int J Adv Robot Syst* 2019; 16(3).

24. Chen C, Zhang CR, Hu TL, et al. Finite-time tracking control for uncertain robotic manipulators using backstepping method and novel extended state observer. *Int J Adv Robot Syst* 2019; 16(3).

25. Bezak P, Bozek P, and Nikitin Y. Advanced robotic grasping system using deep learning. *Procedia Eng* 2014; 96: 10–20.

26. Bozek P and Pokorny P, Svetlik J, et al. The calculations of Jordan curves trajectory of the robot movement. *Int J Adv Robot Syst* 2016; 16(3).

27. Huo W. *Robot dynamics and control*. Beijing: Higher Education Press, 2005, pp. 174–178.

28. Peng WD. *Torque constrained robotic combination nonlinear feedback control*. Shanghai: Shanghai Jiaotong University, 2009.

29. Peng WD and Su JB. Robust control method based on combination nonlinear feedback technology. *High Technol Lett* 2009; 19(6): 591–595.