Higgs Couplings in NonCommutative
Standard Model

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Abstract
We consider the Higgs and Yukawa parts of the Non-Commutative Standard Model (NCSM). We explore the NC-action to give all Feynman rules for couplings of the Higgs boson to electro-weak gauge fields and fermions.

1 Introduction

After discovery of the Higgs particle on 4 July 2012 at LHC [1], it is tentatively confirmed to have zero spin and positive parity [2]. Nonetheless, it is yet to be determined if the particle discovered is the prediction of the Standard Model, or whether, as predicted by the other theories beyond the SM. However, the observed properties of the Higgs boson, such as its mass, couplings and decay rates, can constrain models beyond SM. In string theory, which is a candidate for explaining gravity, the endpoints of an open string on D-branes in a constant B-field background, live on a noncommutative space [3,4]. Therefore, noncommutative field theory (NCFT) appears as a low-energy limit of open string theory with constant antisymmetric background field. The NC field theories and their phenomenological aspects have been explored for many years [5]. There are two approaches to constructing the gauge theories in NC-space. In the first one, the gauge group is restricted to $U(n)$ and the symmetry groups such as a $SU(n)$ can be achieved by an appropriate symmetry breaking [6]. In the second one, one can construct directly a gauge theory based on a $SU(n)$ gauge group-and consequently, the standard model-in noncommutative space [7]. After introducing the Feynman rules for the noncommutative standard model (NCSM) in [8] the phenomenological aspects of NCSM have been considered by many authors [9]. Here we would like to complete the Feynman rules for the the Higgs and

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Yukawa sectors of the NCSM. In fact, in the both sectors there are many new couplings for the Higgs boson with the other particles of the SM. These new interactions lead to many new interesting decay and production modes for the Higgs particle. Therefore, examining this part of NCSM can provide new bounds on the noncommutative space-time.

In section 2 we give a brief review on the NCSM. In section 3 and 4, Feynman rule for the Higgs and Yukawa part is derived, respectively. In Section 5, we summarize our results and give concluding remarks.

2 NCSM

In noncommutative space-time coordinates are operators which in the canonical version satisfy the following commutation relation

\[ [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = i\frac{C^{\mu\nu}}{\Lambda^2_{NC}}, \tag{1} \]

where \( C^{\mu\nu} \) is a real antisymmetric tensor and \( \Lambda_{NC} \) is noncommutative scale. According to the Weyl-Moyal correspondence, in a noncommutative field theory the ordinary product between the fields should be replaced by the star product which can be defined as \[10\]

\[ (f \star g)(x) = f(x) \exp\left(\frac{i}{2} \frac{\partial}{\partial \mu} \theta^{\mu\nu} \frac{\partial}{\partial \nu}\right) g(x). \tag{2} \]

Using this correspondence, however, there are some problems to construct non-Abelian gauge theories on noncommutative space. In noncommutative QFT the field strength tensor is defined as follows

\[ F_{\mu\nu} = [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \tag{3} \]

where the last term doesn’t vanish even in the Abelian U(1) gauge group. In this framework the allowed particles have \( \{0, \pm1\} \) charges and quarks with the fractional charges can not be accommodated in the model \[11\]. The second problem appears in the commutator of two infinitesimal gauge transformations, \( \Lambda = \Lambda_a T^a \) and \( \Lambda' = \Lambda'_b T^b \), where \[12,13\]

\[ [\Lambda, \Lambda'] = \frac{1}{2} [T^a, T^b] \{\Lambda^a(x), \Lambda'^b(x)\}_* + \frac{1}{2} \{T^a, T^b\} [\Lambda^a(x), \Lambda'^b(x)]_* \tag{4} \]

Non-vanishing anticommutator in the last term reproduce all the higher powers of the generators in a non-Abelian gauge group such as a \( SU(N) \). It seems an enveloping algebra, consisting of all ordered tensor powers of the generators, can be a proper choice to solve this problem. Meanwhile, the infinite
number of degrees of freedom of the NC-gauge parameters and fields can be restricted demanding that they depend on the algebra-valued quantities and their space-time derivatives only. This requirement, based on the equivalence between ordinary and noncommutative gauge fields to any finite order in $\theta_{\mu\nu}$ and the existence of the Seiberg-Witten map to all orders, leads to expansions for matter field $\hat{\psi}$, gauge parameters $\hat{\Lambda}$, gauge fields $\hat{A}_\mu$ and scalar field $\hat{\phi}$ as follows [4, 13]

\[
\hat{\psi} = \psi + \frac{1}{2} \theta^{\mu\nu} A_\nu \partial_\mu \psi + \frac{i}{8} \theta^{\mu\nu} [A_\mu, A_\nu] + \mathcal{O}(\theta^2),
\]

\[
\hat{A}_\mu = A_\mu + \frac{1}{4} \theta^{\rho\nu} \{ A_\nu, (\partial_\rho A_\mu - F_{\rho\mu}) \} + \mathcal{O}(\theta^2),
\]

\[
\hat{\Lambda} = \Lambda + \frac{1}{4} \theta^{\mu\nu} \{ A_\nu, \partial_\mu \Lambda \} + \mathcal{O}(\theta^2),
\]

\[
\hat{\phi} = \phi + \frac{1}{2} \theta^{\mu\nu} A_\nu \left( \partial_\mu \phi - i \left( A_\mu \phi - \phi A'_\mu \right) \right) + \frac{1}{2} \theta^{\mu\nu} \left( \partial_\mu \phi - i \left( A_\mu \phi - \phi A'_\mu \right) \right) A'_\nu + \mathcal{O}(\theta^2),
\]

where the scalar field can be transformed on the left and right under two different gauge groups with the corresponding gauge fields $A$ and $A'$. Now the NC-standard model based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)$, can be constructed in two steps:

1. Replacing the ordinary products and fields in the action of the standard model with the star products and the corresponding NC-fields, respectively.

2. Substituting the noncommutative fields for each corresponding commutative one via the Seiberg-Witten map given in (5).

The minimal NCSM based on the Seiberg-Witten maps has been introduced in [7]. The action of noncommutative standard model can be separated into four parts as

\[
S_{\text{NCSM}} = S_{\text{fermions}} + S_{\text{gauge}} + S_{\text{Higgs}} + S_{\text{Yukawa}}.
\]

Meanwhile, the Feynman rules for $S_{\text{fermions}}$ and $S_{\text{gauge}}$ is fully introduced in [8]. In the next sections we explore the remaining parts to find the Feynman rules for the Higgs interactions.
3 Higgs part of the NCSM action

The Higgs part of the NCSM action up to the first order of $\theta$ can be written as follows [8]:

$$S_{\text{Higgs}} = \int d^4x \left( (D_\mu \Phi)\dagger(D^\mu \Phi) - \mu^2 \Phi\dagger\Phi - \lambda (\Phi\dagger\Phi)^2 \right) + \frac{1}{2} \theta^{\alpha\beta} \int d^4x \Phi \left( U_{\alpha\beta} + U_{\alpha\beta}^\dagger + \frac{1}{2} \mu^2 F_{\alpha\beta} - 2i \lambda \Phi (D_\alpha \Phi)\dagger D_\beta \Phi \right),$$

(7)

where $D_\mu = \partial_\mu 1 - i V_\mu$ and $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$ are the ordinary covariant derivative and field strength tensor, respectively, and

$$U_{\alpha\beta} = \left( \partial^\mu + i V^\mu \right) \left( - \partial_\mu V_\alpha \partial_\beta - V_\alpha \partial_\mu \partial_\beta + \partial_\alpha V_\mu \partial_\beta ight) + i V_\mu V_\alpha \partial_\beta + \frac{i}{2} \partial_\mu (V_\alpha V_\beta) + \frac{1}{2} \partial_\mu \{V_\alpha, V_\beta + F_{\beta\mu}\},$$

(8)

in which $1$ is a unit matrix and the gauge field $V_\mu = g'A_\mu Y_\Phi 1 + gB^a_{\mu} T^a_L$ can be easily rewritten as

$$V_\mu = \begin{pmatrix} g'A_\mu Y_\Phi + gT_{3,\phi_{\text{up}}} B^3_{\mu} & \frac{g}{\sqrt{2}} W^+_\mu \\ \frac{g}{\sqrt{2}} W^-_\mu & g'A_\mu Y_\Phi + gT_{3,\phi_{\text{down}}} B^3_{\mu} \end{pmatrix},$$

(9)

where $Y_\Phi = 1/2$, $T_{3,\phi_{\text{up}}} = 1/2$, $T_{3,\phi_{\text{down}}} = -1/2$. Consequently, in terms of the physical fields $Z_\mu$ and $A_\mu$,

$$Z_\mu = \frac{-g'A_\mu + gB^3_{\mu}}{\sqrt{g^2 + g'^2}} = -\sin \theta_W A_\mu + \cos \theta_W B^3_\mu,$$

$$A_\mu = \frac{gA_\mu + g'B^3_{\mu}}{\sqrt{g^2 + g'^2}} = \cos \theta_W A_\mu + \sin \theta_W B^3_\mu,$$

(10)

the diagonal components of the gauge field $V_\mu$ can be obtained as follows

$$V_{11,\mu} = eA_\mu + \frac{g}{2 \cos \theta_W} (1 - 2 \sin^2 \theta_W) Z_\mu,$$

$$V_{22,\mu} = -\frac{g}{2 \cos \theta_W} Z_\mu.$$

(11)
Meanwhile, the Higgs field in the unitary gauge is $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) + v \end{pmatrix}$, where $v = \sqrt{-\mu^2/\lambda}$ is the Higgs vacuum expectation value and $h(x)$ is the physical Higgs field. Now the Higgs action (7), by using (8) to (11), can be rewritten in terms of the physical fields to obtain all Lagrangian densities containing the Higgs interactions with the gauge fields as follows

- one Higgs 2 neutral gauge bosons, $hZZ$:

$$L_{hZZ} = L_{hZZ}^{SM} + O(\theta^2).$$ (12)

- 2 Higgs’s 2 neutral gauge bosons, $hhZZ$:

$$L_{hhZZ} = L_{hhZZ}^{SM} + O(\theta^2).$$ (13)

- 2 Higgs’s a neutral gauge boson, $hhZ$:

$$L_{hhZ} = \left( \frac{g}{4 \cos \theta_w} \right) \theta^{\alpha \beta} \left[ -(\partial_\mu \partial^\mu h)(\partial_\beta h) Z_\alpha + (\partial_\alpha \partial^\mu h)(\partial_\beta h) Z_\mu + 2\lambda v^2 h(\partial_\alpha h) Z_\beta \right].$$ (14)

- 3 Higgs’s a neutral gauge boson, $hhhZ$:

$$L_{hhhZ} = \left( \frac{3g}{4 \cos \theta_w} \right) \theta^{\alpha \beta} \lambda v h^2 (\partial_\alpha h) Z_\beta.$$ (15)

- 4 Higgs’s a neutral gauge boson, $hhhhZ$:

$$L_{hhhhZ} = \left( \frac{g}{4 \cos \theta_w} \right) \theta^{\alpha \beta} \lambda h^3 (\partial_\alpha h) Z_\beta.$$ (16)

- one Higgs 2 charged gauge bosons, $hW^+W^-$:

$$L_{hW^+W^-} = \frac{g^2}{2} vhW^{-\mu}W^\mu_{-\mu}$$

$$+ \frac{ig^2}{8} v \theta^{\alpha \beta} \left[ -(\partial_\beta h)(W^{-\mu}(\partial_\mu W^\alpha_{\alpha}) - W^{+\mu}(\partial_\mu W^-_{\alpha})) - W^{-\mu}(\partial_\alpha W^\mu_{-\mu} + W^{+\mu}(\partial_\mu W^-_{\mu})) - (\partial_\mu \partial_\beta h)(W^{-\mu}W^\alpha_{\alpha}) - W^{+\mu}W^-_{\alpha} + (\partial^\mu h)((\partial_\mu W^-_{\alpha})W^\beta_3 - (\partial_\alpha W^\mu_{\alpha})W^-_{\beta}) + 2\lambda v^2 hW^\alpha_{\alpha}W^-_{\beta} \right].$$ (17)
- 2 Higgs’s 2 charged gauge bosons, $hhW^+W^-$:

\[
\mathcal{L}_{hhW^+W^-} = \frac{g^2}{4} h^2 W^{+\mu} W^-_{\mu} \\
+ \frac{i g^2}{8} \theta^{\alpha\beta} \left[ -h(\partial_\beta h)(W^{-\mu}(\partial_\mu W^+_{\alpha}) - W^{+\mu}(\partial_\mu W^-_{\alpha}) \\
- h(\partial_\mu \partial_\beta h)(W^{-\mu} W^+_{\alpha} - W^{+\mu} W^-_{\alpha}) \\
+ h(\partial^\mu h)(\partial_\mu W^-_{\alpha} W^+_{\beta} - (\partial_\mu W^+_{\alpha}) W^-_{\beta}) \\
+ h(\partial^\mu h)(\partial_\beta h)(W^-_{\alpha} W^+_{\alpha} - W^+_{\alpha} W^-_{\alpha}) \\
+ h(\partial^\mu h)(\partial_\beta h)(W^-_{\alpha} W^+_{\beta} + 5\lambda v^2 h^2 W^+_{\alpha} W^-_{\beta}). \right] \tag{18}
\]

- 3 Higgs’s 2 charged gauge bosons, $hhhW^+W^-$:

\[
\mathcal{L}_{hhhW^+W^-} = \frac{ig^2}{2} \lambda \theta^{\alpha\beta} h^3 W^+_{\alpha} W^-_{\beta}. \tag{19}
\]

- 4 Higgs’s 2 charged gauge bosons, $hhhhW^+W^-$:

\[
\mathcal{L}_{hhhhW^+W^-} = \frac{ig^2}{8} \lambda \theta^{\alpha\beta} h^4 W^+_{\alpha} W^-_{\beta}. \tag{20}
\]

- one Higgs 2 charged gauge bosons one photon, $hW^+W^-\gamma$:

\[
\mathcal{L}_{hW^+W^-A} = \frac{eg^2}{8} v \theta^{\alpha\beta} \left[ -h(\partial_\beta h) A^\mu \\
+ 2h(\partial^\mu A_\beta - \partial_\beta A^\mu) (W^-_{\alpha} W^+_{\alpha} + W^+_{\alpha} W^-_{\alpha}) \\
+ 2h(\partial_\beta A_\alpha) W^{+\mu} W^-_{\mu} \right]. \tag{21}
\]

- 2 Higgs’s 2 charged gauge bosons one photon, $hhW^+W^-\gamma$:

\[
\mathcal{L}_{hhW^+W^-A} = \frac{eg^2}{8} \theta^{\alpha\beta} \left[ -h(\partial_\beta h) A^\mu \\
+ h^2(\partial^\mu A_\beta - \partial_\beta A^\mu) (W^-_{\alpha} W^+_{\alpha} + W^+_{\alpha} W^-_{\alpha}) \\
+ h^2(\partial_\beta A_\alpha) W^{+\mu} W^-_{\mu} \right]. \tag{22}
\]
– one Higgs 3 neutral gauge bosons, $hZZZ$:

$$\mathcal{L}_{hZZZ} = \frac{1}{16} \left( \frac{g}{\cos \theta_W} \right)^3 v \theta^{\alpha \beta} [ (\partial_\beta h)(Z^\mu Z_\mu Z_\alpha) + 2hZ^\mu Z_\alpha (2\partial_\beta Z_\mu - \partial_\mu Z_\beta)].$$  \hspace{1cm} (23)

– 2 Higgs’s 3 neutral gauge bosons, $hhZZZ$:

$$\mathcal{L}_{hhZZZ} = \frac{1}{16} \left( \frac{g}{\cos \theta_W} \right)^3 \theta^{\alpha \beta} [ h(\partial_\beta h)(Z^\mu Z_\mu Z_\alpha) + h^2 Z^\mu Z_\alpha (2\partial_\beta Z_\mu - \partial_\mu Z_\beta)].$$  \hspace{1cm} (24)

– one Higgs 2 charged gauge bosons one neutral gauge boson, $hW^+W^-Z$:

$$\mathcal{L}_{hW^+W^-Z} = \frac{g^3 v}{8 \cos \theta_W} \theta^{\alpha \beta} \{ (\sin^2 \theta_W Z_\mu (W^{-\mu} W^+_\alpha + W^{+\mu} W^-_\alpha) + Z_\alpha W^{-\mu} W^+_\mu)(\partial_\beta h) $$

$$- h[Z^\mu (W^+_\mu (\partial_\beta W^-_\alpha) + W^-_\mu (\partial_\beta W^+_{\alpha}))] $$

$$+ (Z^\mu W^+_\alpha + Z_\alpha W^{+\mu})(\partial_\mu W^-_\beta - \partial_\beta W^-_\mu) $$

$$+ (Z^\mu W^-_\alpha + Z_\alpha W^{-\mu})(\partial_\mu W^+_\beta - \partial_\beta W^+_\mu) $$

$$- \cos 2\theta_W [(W^{+\mu} W^-_\alpha + W^{-\mu} W^+_{\alpha})(\partial_\mu Z_\beta - \partial_\beta Z_\mu) $$

$$+ (\partial_\beta Z_\alpha) W^{+\mu} W^-_\mu]\}.$$  \hspace{1cm} (25)

– 2 Higgs’s 2 charged gauge bosons one neutral gauge boson, $hhW^+W^-Z$:

$$\mathcal{L}_{hhW^+W^-Z} = \frac{g^3 v}{8 \cos \theta_W} \theta^{\alpha \beta} \{ (\sin^2 \theta_W Z_\mu (W^{-\mu} W^+_{\alpha} + W^{+\mu} W^-_{\alpha}) + Z_\alpha W^{-\mu} W^+_\mu)(h\partial_\beta h) $$

$$- h[Z^\mu (W^+_\mu (\partial_\beta W^-_{\alpha}) + W^-_\mu (\partial_\beta W^+_{\alpha}))] $$

$$+ (Z^\mu W^+_\alpha + Z_\alpha W^{+\mu})(\partial_\mu W^-_\beta - \partial_\beta W^-_\mu) $$

$$+ (Z^\mu W^-_\alpha + Z_\alpha W^{-\mu})(\partial_\mu W^+_\beta - \partial_\beta W^+_\mu) $$

$$- \cos 2\theta_W [(W^{+\mu} W^-_{\alpha} + W^{-\mu} W^+_{\alpha})(\partial_\mu Z_\beta - \partial_\beta Z_\mu) $$

$$+ (\partial_\beta Z_\alpha) W^{+\mu} W^-_\mu]\}.$$  \hspace{1cm} (26)
\(-\) one Higgs 2 charged gauge bosons one neutral gauge boson one photon, \(hW^+W^-Z\gamma\):

\[
\mathcal{L}_{hW^+W^-Z} = \left( -\frac{i e v g^3}{8 \cos \theta_w} \right) \theta^{\alpha \beta} h^2 \left[ 2 Z^\mu A_\alpha W^\mu_\beta - 2 Z^\mu A_\alpha W^+_- \right.
\]
\[
+ 2 Z^\mu A_\mu W^-_\alpha + Z_\beta A_\mu W^-_\alpha + Z_\beta A_\mu W^-_\alpha \\
- Z_\beta A_\mu W^+_-] \right].
\] (27)

\(-\) 2 Higgs’s 2 charged gauge bosons one neutral gauge boson one photon, \(hhW^+W^-\gamma\):

\[
\mathcal{L}_{hhW^+W^-} = \left( -\frac{i e g^3}{16 \cos \theta_w} \right) \theta^{\alpha \beta} h^2 \left[ 2 Z^\mu A_\alpha W^\mu_\beta - 2 Z^\mu A_\alpha W^+_- \right.
\]
\[
+ 2 Z^\mu A_\mu W^-_\alpha + Z_\beta A_\mu W^-_\alpha + Z_\beta A_\mu W^-_\alpha \\
- Z_\beta A_\mu W^+_-] \right].
\] (28)

\(-\) one Higgs 2 charged gauge bosons 2 neutral gauge bosons, \(hW^+W^-ZZ\):

\[
\mathcal{L}_{hW^+W^-Z} = -i v \left( \frac{g^2}{4 \cos \theta_w} \right)^2 \theta^{\alpha \beta} h^2 \left[ (1 - 2 \sin^2 \theta_W) \left[ -Z^\mu Z_\beta W^-_- \right.
\]
\[
+ Z_\beta W^-_- W^\mu_\alpha + 2 Z^\mu Z_\beta W^-_- \right] + 3 Z_\beta [Z_\alpha W^-_- W^\mu_\alpha \\
+ Z_\beta W^-_- W^\mu_\alpha - Z_\beta W^-_- W^\mu_\alpha \right].
\] (29)

\(-\) 2 Higgs’s 2 charged gauge bosons 2 neutral gauge bosons, \(hhW^+W^-ZZ\):

\[
\mathcal{L}_{hhW^+W^-Z} = -i \left( \frac{g^2}{2 \cos \theta_w} \right)^2 \theta^{\alpha \beta} h^2 \left[ (1 - 2 \sin^2 \theta_W) \left[ -Z^\mu Z_\beta W^-_- \right.
\]
\[
+ Z_\beta W^-_- W^\mu_\alpha + 2 Z^\mu Z_\beta W^-_- \right] + 3 Z_\beta [Z_\alpha W^-_- W^\mu_\alpha \\
+ Z_\beta W^-_- W^\mu_\alpha - Z_\beta W^-_- W^\mu_\alpha \right].
\] (30)

\(-\) one Higgs 4 charged gauge bosons, \(hW^+W^+W^-W^-\):

\[
\mathcal{L}_{hW^+W^+W^-W^-} = \frac{i g^4 v}{8} \theta^{\alpha \beta} h^2 [W^-_- W^\mu_\alpha W^-_- W^\mu_\beta].
\] (31)

\(-\) 2 Higgs’s 4 charged gauge bosons, \(hhW^+W^+W^-W^-\):

\[
\mathcal{L}_{hhW^+W^+W^-W^-} = \frac{i g^4}{16} \theta^{\alpha \beta} h^2 [W^-_- W^\mu_\alpha W^-_- W^\mu_\beta].
\] (32)
3.1 Gauge Terms

In this subsection, we list Feynman rules for the NCSM Higgs-gauge couplings up to the first order of $\theta$. To derive the rules, it is assumed that all particles momenta are incoming into the vertices.

Equation (14):

\[ H(k) \quad Z_\rho \quad H(k') \]

\[ -\frac{M_Z}{2v}[(k'^2k_\beta + k'^2k'_\beta)\theta^\beta\rho + k'^\rho k_\beta(k' - k)^\rho\theta^\alpha\beta - m_H^2(k' + k)_\alpha\theta^\alpha\rho], \quad (33) \]

Equation (15):

\[ H(k) \quad H(k') \quad Z_\rho \quad H(k'') \]

\[ \frac{3}{2} \frac{M_Z m_H^2}{v^2} \theta^\mu\nu[k + k' + k'']_\alpha, \quad (34) \]

Equation (16):

\[ H(k') \quad H(k) \quad Z_\rho \quad H(k'') \quad H(k''') \]

\[ \frac{3}{2} \frac{M_Z m_H^2}{v^3} \theta^\mu\nu[k + k' + k'' + k''']_\alpha, \quad (35) \]
Equation (17):

\[ H(k) W^+(k_1) - W^-(k_2) \]

\[ \frac{M_W^2}{2v^2} \left[ 4ig^{\sigma\gamma} + (k_1 + k_2)\theta^{\sigma\gamma} + k_1k_2\theta^{\sigma\beta} - k_1^2\theta^{\gamma\beta} \right] \]

\[ + k_2(k_1\theta^{\sigma\beta} - k^\sigma\theta^{\gamma\beta}) + k_\beta(k_1 - k_2)g^{\sigma\alpha\beta} - m^2_H \theta^{\gamma\alpha} \}

Equation (18):

\[ H(k') W^+(k_1) - W^-(k_2) \]

\[ \frac{M_W^2}{2v^2} \left[ 4ig^{\sigma\gamma} + (k' - k)\theta^{\sigma\gamma} - (k - k')\gamma\theta^{\sigma\beta} \right] \]

\[ - k_1^\beta(k + k')\theta^{\sigma\beta} + k_2(k + k')\theta^{\sigma\beta} + (k + k')\beta(k_1 - k_2) \]

\[ \times g^{\gamma\sigma\beta} + [2k.k' + (k + k').(k_1 + k_2)]\theta^{\gamma\beta} - 5m^2_H \theta^{\gamma\alpha} \}

Equation (19):

\[ H W^+ - W^- \]

\[ \frac{6m^2_H M_W^2}{v^3} \theta^{\sigma\gamma} \]
Equation (20):

\[ H H H H W^+ + \gamma W - \sigma M^2 W^2 v^2 \theta, \quad (39) \]

Equation (21):

\[ H(k) W^+ (k_1) \quad A_\rho(k_3) \quad W^- (k_2) \]

\[ e \frac{M_W^2}{2v^2} \{ (k + k')_\beta (\theta^\beta \gamma g^{\sigma \rho} + \theta^\beta \sigma g^{\gamma \rho}) + 2(k_3^2 \theta^\sigma \rho + k_3^3 \theta^\gamma \rho) \}
\]

Equation (22):

\[ H(k') W^+ (k_1) \quad A_\rho(k_3) \quad W^- (k_2) \]

\[ e \frac{M_W^2}{2v^2} \{ (k + k')_\beta (\theta^\beta \gamma g^{\sigma \rho} + \theta^\beta \sigma g^{\gamma \rho}) + 2(k_3^2 \theta^\sigma \rho + k_3^3 \theta^\gamma \rho) \}, \quad (41) \]
Equation (23):

\[ H(k) \]

\[ \begin{array}{c}
Z_\gamma(k_1) \\
Z_\sigma(k_2) \\
Z_\rho(k_3)
\end{array} \]

\[
\frac{M_Z^3}{v^2} \left\{ k_\beta(\theta^{\rho\beta} g^{\gamma\sigma} + \theta^{\sigma\beta} g^{\rho\gamma} + \theta^{\gamma\beta} g^{\rho\sigma}) + 2k_{1\beta}(\theta^{\rho\beta} g^{\gamma\sigma} + \theta^{\sigma\beta} g^{\gamma\rho}) \\
+ 2k_{2\beta}(\theta^{\rho\beta} g^{\gamma\sigma} + \theta^{\sigma\beta} g^{\rho\gamma}) + 2k_{3\beta}(\theta^{\gamma\beta} g^{\rho\sigma} + \theta^{\sigma\beta} g^{\rho\gamma}) \\
+ (k_1 - k_3)^\rho \theta^{\gamma\rho} + (k_3 - k_2)^\gamma \theta^{\rho\sigma} + (k_2 - k_1)^\sigma \theta^{\rho\gamma} \right\}, \quad (42)
\]

Equation (24):

\[ H(k) \]

\[ \begin{array}{c}
Z_\gamma(k_1) \\
Z_\sigma(k_2) \\
Z_\rho(k_3)
\end{array} \]

\[
\frac{M_Z^3}{v^2} \left\{ (k + k')_\beta(\theta^{\rho\beta} g^{\gamma\sigma} + \theta^{\sigma\beta} g^{\rho\gamma} + \theta^{\gamma\beta} g^{\rho\sigma}) + 2k_{1\beta}(\theta^{\rho\beta} g^{\gamma\sigma} + \theta^{\sigma\beta} g^{\gamma\rho}) \\
+ 2k_{2\beta}(\theta^{\rho\beta} g^{\gamma\sigma} + \theta^{\sigma\beta} g^{\rho\gamma}) + 2k_{3\beta}(\theta^{\gamma\beta} g^{\rho\sigma} + \theta^{\sigma\beta} g^{\rho\gamma}) \\
+ (k_1 - k_3)^\rho \theta^{\gamma\rho} + (k_3 - k_2)^\gamma \theta^{\rho\sigma} + (k_2 - k_1)^\sigma \theta^{\rho\gamma} \right\}, \quad (43)
\]
Equation (25):

\[ H(k) \rightleftharpoons W^+_{\gamma}(k_1) \]

\[ H(k) \rightleftharpoons Z_{\rho}(k_3) \]

\[ W^-_{\sigma}(k_2) \]

\[
\frac{M_Z M_W^2}{v^2} \left\{ (k_3 g^\gamma g^\sigma g^\rho g^\beta + \sin^2 \theta_W (g^{\sigma \rho} g^{\gamma \beta} + g^{\gamma \rho} g^{\sigma \beta})) - k_{2\beta} \right. \\
\times (g^{\gamma \rho} g^{\sigma \beta} - g^{\gamma \sigma} g^{\rho \beta} - g^{\rho \sigma} g^{\gamma \beta}) + k_{1\beta} (g^{\gamma \sigma} g^{\rho \beta} + g^{\gamma \rho} g^{\sigma \beta} - g^{\rho \sigma} g^{\gamma \beta}) \\
\left. - \theta^\gamma (k_2 - k_1)^\rho + k_3^\sigma g^{\rho \gamma} + k_3^\rho g^{\sigma \gamma} + \cos 2\theta_W [k_3^\gamma g^{\rho \gamma} + k_3^\gamma g^{\sigma \gamma}] \right\}, \quad (44)
\]

Equation (26):

\[ H(k) \rightleftharpoons W^+_{\gamma}(k_1) \]

\[ H(k') \rightleftharpoons Z_{\rho}(k_3) \]

\[ W^-_{\sigma}(k_2) \]

\[
\frac{M_Z M_W^2}{v^2} \left\{ (k + k')_\beta (g^{\gamma \sigma} g^{\rho \beta} + \sin^2 \theta_W (g^{\sigma \rho} g^{\gamma \beta} + g^{\gamma \rho} g^{\sigma \beta})) - k_{2\beta} \right. \\
\times (g^{\gamma \rho} g^{\sigma \beta} - g^{\gamma \sigma} g^{\rho \beta} - g^{\rho \sigma} g^{\gamma \beta}) + k_{1\beta} (g^{\gamma \sigma} g^{\rho \beta} + g^{\gamma \rho} g^{\sigma \beta} - g^{\rho \sigma} g^{\gamma \beta}) \\
\left. - \theta^\gamma (k_2 - k_1)^\rho + k_3^\sigma g^{\rho \gamma} + k_3^\rho g^{\sigma \gamma} + \cos 2\theta_W [k_3^\gamma g^{\rho \gamma} + k_3^\gamma g^{\sigma \gamma}] \right\}, \quad (45)
\]
Equation (27):

\[ eM_Z M_W^2 \left\{ 2 g^{\sigma \rho} \theta^{\tau \gamma} + 2 g^{\gamma \rho} \theta^{\sigma \tau} + 2 g^{\rho \tau} \theta^{\gamma \sigma} + g^{\tau \sigma} \theta^{\rho \gamma} + g^{\gamma \tau} \theta^{\rho \sigma} \right\}, \quad (46) \]

Equation (28):

\[ eM_Z M_W^2 \left\{ 2 g^{\sigma \rho} \theta^{\tau \gamma} + 2 g^{\gamma \rho} \theta^{\sigma \tau} + 2 g^{\rho \tau} \theta^{\gamma \sigma} + g^{\tau \sigma} \theta^{\rho \gamma} + g^{\gamma \tau} \theta^{\rho \sigma} \right\}, \quad (47) \]

Equation (29):

\[ 2 \frac{M_W^2 M_Z^2}{v^3} \left\{ (1 + \cos^2 \theta_W)(\theta^{\tau \gamma} g^{\sigma \rho} + \theta^{\sigma \tau} g^{\rho \gamma} + \theta^{\rho \gamma} g^{\sigma \tau} + \theta^{\gamma \sigma} g^{\rho \tau}) + (1 + 4 \cos^2 \theta_W) \theta^{\gamma \sigma} g^{\rho \tau} \right\}, \quad (48) \]
Equation (30):

\[ 2M_W^2 M_Z^2 \frac{v^4}{v^4} \{(1 + \cos^2 \theta_W)(\theta^\gamma g^{\sigma \rho} + \theta^\sigma g^{\gamma \rho} + \theta^\rho g^{\gamma \sigma}) + (1 + 4 \cos^2 \theta_W)\theta^\gamma g^{\sigma \rho}\gamma^\tau\}, \quad (49) \]

Equation (31):

\[ 2M_W^4 \frac{v^4}{v^4} \{ g^{\gamma \sigma} \theta^{\gamma \rho} + g^{\sigma \gamma} \theta^{\sigma \rho} + g^{\gamma \rho} \tau^\gamma + g^{\gamma \sigma} \tau^\rho \}, \quad (50) \]

Equation (32):

\[ 2M_W^4 \frac{v^4}{v^4} \{ g^{\gamma \sigma} \theta^{\gamma \rho} + g^{\gamma \sigma} \theta^{\sigma \rho} + g^{\gamma \rho} \tau^\gamma + g^{\gamma \sigma} \tau^\rho \}. \quad (51) \]
4 Yukawa part of the NCSM Action

In this section, we explore the Yukawa part of the NCSM-action up to the first order of $\theta$ to derive all corresponding Feynman rules. The Yukawa part of the action in terms of the physical fields is [8]:

$$S_{\psi, \text{Yukawa}} = \int d^4x \sum_{i,j=1}^{3} \left[ \bar{\psi}_{\text{down}}^{(i)} \left( N_{dd}^{V(ij)} + \gamma_5 N_{dd}^{A(ij)} \right) \psi_{\text{down}}^{(j)} ight. \\
\left. + \bar{\psi}_{\text{up}}^{(i)} \left( N_{uu}^{V(ij)} + \gamma_5 N_{uu}^{A(ij)} \right) \psi_{\text{up}}^{(j)} ight. \\
\left. + \bar{\psi}_{\text{down}}^{(i)} \left( C_{ud}^{V(ij)} + \gamma_5 C_{ud}^{A(ij)} \right) \psi_{\text{down}}^{(j)} \\
\left. + \bar{\psi}_{\text{up}}^{(i)} \left( C_{du}^{V(ij)} + \gamma_5 C_{du}^{A(ij)} \right) \psi_{\text{up}}^{(j)} \right].$$

(52)

The neutral currents are:

$$N_{dd}^{V(ij)} = -M_{\text{down}}^{(ij)} \left( 1 + \frac{h}{v} \right) + N_{dd}^{V,\theta(ij)} + O(\theta^2),$$

$$N_{dd}^{A(ij)} = N_{dd}^{A,\theta(ij)} + O(\theta^2),$$

$$N_{uu}^{V(ij)} = -M_{\text{up}}^{(ij)} \left( 1 + \frac{h}{v} \right) + N_{uu}^{V,\theta(ij)} + O(\theta^2),$$

$$N_{uu}^{A(ij)} = N_{uu}^{A,\theta(ij)} + O(\theta^2),$$

(53)

where

$$N_{dd}^{V,\theta(ij)} = -\frac{1}{2} \theta_{\mu\nu} M_{\text{down}}^{(ij)} \left\{ i \frac{\partial_{\mu} h}{v} \partial_{\nu} \right. \right.$$  

\begin{align*}
- & \left[ e Q \hat{\psi}_{\text{down}} A_\mu + \frac{g}{2 \cos \theta_W} (T_3 \hat{\psi}_{\text{down},L} - 2 Q \hat{\psi}_{\text{down}} \sin^2 \theta_W) Z_\mu \right] \left( \frac{\partial_{\nu} h}{v} \right) \\
+ & \left[ e Q \hat{\psi}_{\text{down}} (\partial_{\nu} A_\mu) + \frac{g}{2 \cos \theta_W} (T_3 \hat{\psi}_{\text{down},L} - 2 Q \hat{\psi}_{\text{down}} \sin^2 \theta_W) (\partial_{\nu} Z_\mu) \right] \\
- & i \left. \frac{g^2}{2} W_\mu^+ W^-_{\nu} \right) \left( 1 + \frac{h}{v} \right) \right\},
\end{align*}

(54)

$$N_{dd}^{A,\theta(ij)} = \frac{g}{4 \cos \theta_W T_3 \hat{\psi}_{\text{down},L}} \theta_{\mu\nu} M_{\text{down}}^{(ij)} \left( 1 + \frac{h}{v} \right) Z_\mu$$

\begin{align*}
& \times \left[ (\partial_{\nu} - \bar{\partial}_{\nu}) + 2 i e Q \hat{\psi}_{\text{down}} A_\nu \right],
\end{align*}

(55)
The charged currents are given by

\[
C_{ud}^{V(ij)} = C_{ud}^{V,\theta(ij)} + \mathcal{O}(\theta^2),
\]

\[
C_{ud}^{A(ij)} = C_{ud}^{A,\theta(ij)} + \mathcal{O}(\theta^2),
\]

(57)

where

\[
C_{ud}^{V,\theta(ij)} = - \frac{g}{4\sqrt{2}} \theta^{\mu\nu} \left( 1 + \frac{h}{v} \right) \left\{ \left[ \left( V_f M_{down} \right)^{(ij)}_{ud} + \left( M_{up} V_f \right)^{(ij)}_{ud} \right] \left( \partial_{\nu} W_{\mu}^+ \right) + \left( \left( V_f M_{down} \right)^{(ij)}_{ud} \partial_{\nu} + \left( M_{up} V_f \right)^{(ij)}_{ud} \partial_{\nu} \right) W_{\mu}^+ \right\}
\]

\[
+ ie \left[ \left( V_f M_{down} \right)^{(ij)}_{ud} Q_{\psi_{up}} - \left( M_{up} V_f \right)^{(ij)}_{ud} Q_{\psi_{down}} \right] A_\mu W_{\nu}^+ 
\]

\[
+ i \frac{g}{\cos \theta_W} \left[ \left( V_f M_{down} \right)^{(ij)}_{ud} \left( 2T_3, \psi_{down, L} - Q_{\psi_{down}} \sin^2 \theta_W \right) - \left( M_{up} V_f \right)^{(ij)}_{ud} \left( 2T_3, \psi_{down, L} - Q_{\psi_{down}} \sin^2 \theta_W \right) \right] Z_{\mu} W_{\nu}^+ \right\},
\]

(58)

and

\[
C_{ud}^{A,\theta(ij)} = C_{ud}^{V,\theta(ij)} \left( M_{up} \rightarrow -M_{up} \right),
\]

(59)

while

\[
C_{du}^{V(ij)} = \left( C_{ud}^{V(ij)} \partial_{\nu} \partial_{\mu} \right)^ \dagger,
\]

\[
C_{du}^{A(ij)} = - \left( C_{ud}^{A(ij)} \partial_{\nu} \partial_{\mu} \right)^ \dagger,
\]

(60)

where \( \partial_{\rho} ( \tilde{\partial}_{\rho} ) \) denotes the partial derivative which acts only on the fermion fields on its right (left) side:

\[
\partial_{\rho} \psi \equiv \tilde{\partial}_{\rho} \psi \quad \partial_{\rho} \overline{\psi} \equiv \overline{\tilde{\partial}_{\rho} \psi}.
\]

(61)

Now, the Yukawa-action (52) can be simplified to find all lagrangian densities containing the Higgs couplings with fermions as follows:
• Higgs-Fermion-Antifermion:

\[ \mathcal{L}_{h\bar{f}f} = -\frac{m_f}{v} \bar{f}(h + \frac{i}{2} \theta^{\mu\nu} (\partial_\mu h) \rightarrow \partial_\nu f), \]  

(62)

• Higgs-Photon-Fermion-Antifermion:

\[ \mathcal{L}_{h\bar{f}fA} = \frac{eQ_fm_f}{2v} \theta^{\mu\nu} [A_\mu(\partial_\nu h) - h(\partial_\nu A_\mu)] \bar{f}f, \]  

(63)

• Higgs-Z boson-Fermion-Antifermion:

\[ \mathcal{L}_{h\bar{f}fZ} = \frac{g m_f}{4v \cos \theta_W} \theta^{\mu\nu} [2(\partial_\nu h) \bar{f}f + h(\partial_\nu f)\bar{f} + \gamma^5 c_{A,\nu} h((\partial_\nu \bar{f}) f - (\partial_\nu f) \bar{f}) Z_\mu], \]  

(64)

• Higgs-Z boson-Photon-Fermion-Antifermion:

\[ \mathcal{L}_{h\bar{f}fZA} = \frac{ie\gamma^5 Q_f g m_f}{2v \cos \theta_W} \theta^{\mu\nu} c_{A,\mu} h Z_\mu A_\nu \bar{f}f, \]  

(65)

• Higgs-\( W^+ \) boson-\( W^- \) boson-Fermion-Antifermion:

\[ \mathcal{L}_{h\bar{f}fW^+W^-} = \frac{im_f g^2}{4v} \theta^{\mu\nu} h f \bar{f}W^+ \mu W^- \nu, \]  

(66)

• Higgs-\( W^+ \) boson-d-\( \bar{u} \):

\[ \mathcal{L}_{hW^+d\bar{u}} = -\frac{g}{4\sqrt{2}v} \theta^{\mu\nu} V^{ij}_f h \bar{u}^i \{(m_d)_{ij} + m_u \}
\times (\partial_\mu W^+ \nu) + (m_d \rightarrow \partial_\nu + m_u \rightarrow \partial_\nu)
\times W^+ \mu + \gamma^5 [(m_d - m_u) (\partial_\nu W^+ \mu) + (m_d \rightarrow \partial_\nu - m_u \rightarrow \partial_\nu) W^+ \mu]\} d^j, \]  

(67)

• Higgs-\( W^- \) boson-u-\( \bar{d} \):

\[ \mathcal{L}_{hW^-u\bar{d}} = -\frac{g}{4\sqrt{2}v} \theta^{\mu\nu} V^{*ij}_f \bar{d}^i \{(m_d)_{ij} + m_u \}
\times (\partial_\mu W^- \nu) + (m_d \rightarrow \partial_\nu + m_u \rightarrow \partial_\nu)
\times W^- \mu - \gamma^5 [(m_d - m_u) (\partial_\nu W^- \mu) + (m_d \rightarrow \partial_\nu - m_u \rightarrow \partial_\nu) W^- \mu]\} hu^i, \]  

(68)
- Higgs-$W^+$ boson-photon-d-$\bar{u}$:

\[ L_{hW^+Ad\bar{u}} = -\frac{ieg}{4\sqrt{2}v} \theta^{ij} \bar{u}^i \{ (m_d Q_u - m_u Q_d) A_\mu W^\mu_{0} h A_d \bar{u}^i, \] (69)

- Higgs-$W^-$ boson-photon-u-$\bar{d}$:

\[ L_{hW^-A_d\bar{d}} = \frac{ieg}{4\sqrt{2}v} \theta^{ij} \bar{d}^j \{ (m_d Q_u - m_u Q_d) A_\mu W^\mu_{-} h A_d \bar{d}^j, \] (70)

- Higgs-$W^+$ boson-Z boson-d-$\bar{u}$:

\[ L_{hW^+Zd\bar{u}} = -\frac{ig^2}{8\sqrt{2}v \cos\theta_W} \theta^{ij} \bar{u}^i \{ (m_d Q_u + c_{V,f}) (1 + \gamma^5) Z_d h A_d \bar{u}^i, \] (71)

- Higgs-$W^-$ boson-Z boson-u-$\bar{d}$:

\[ L_{hW^-Zu\bar{d}} = \frac{ig^2}{8\sqrt{2}v \cos\theta_W} \theta^{ij} \bar{d}^j \{ (m_d Q_u + c_{V,f}) (1 - \gamma^5) Z_u h A_d \bar{d}^j, \] (72)

where $u$ and $d$ indicate \{u, c, t\} and \{d, s, b\}, respectively, and

\[ c_{V,f} = T_{3,f_L} - 2 Q_f \sin^2 \theta_W, \]
\[ c_{A,f} = T_{3,f_L}. \] (73)

### 4.1 Feynman Rules for Yukawa Terms

In this subsection, we list the Feynman rules for the Yukawa terms of the NCSM action up to the first order of $\theta$:

Equation (62):

\[ \frac{im_f}{v} (1 - \frac{i}{2} \theta_{\mu\nu} k^\mu k^\nu), \] (74)
Equation (63):

\begin{equation}
H(k) \xrightarrow{f} A_{\mu}(k')
\end{equation}

\begin{equation}
\frac{e Q_f}{2 v} m_f \theta^{\mu\nu} (k - k')_{\nu},
\end{equation} (75)

Equation (64):

\begin{equation}
H(k) \xrightarrow{f} Z_{\mu}(k')
\end{equation}

\begin{equation}
\frac{m_f M_Z \theta^{\mu\nu}}{2 v^2} [c_{V,f}(2k + k_{in} - k_{out})_{\nu} - \gamma_5 c_{A,f}(k_{in} + k_{out})_{\nu}],
\end{equation} (76)

Equation (65):

\begin{equation}
H \xrightarrow{f} A_{\nu} \xrightarrow{f} Z_{\mu}
\end{equation}

\begin{equation}
- \frac{e m_f M_Z}{v^2} Q_f c_{A,f} \gamma_5 \theta^{\mu\nu},
\end{equation} (77)
Equation (66):

\[ f_u W^+ + \mu W^+ - \nu f_u H f_d W^- + \mu W^- - \nu f_d H M^2 W m f v^3 \theta \mu \nu, \]

\[ -M_W^2 m_f \theta \mu \nu, \]

(78)

Equation (67):

\[ \begin{aligned}
H(k) & \xrightarrow{d^j} W^+_{\mu}(k') \\
& \xleftarrow{w^i}
\end{aligned} \]

\[ - \frac{M_W}{2\sqrt{2}v^2} V_f^{ij} \theta^{\mu\nu} [m_{f_d}(k' + k_{in})_\nu (1 + \gamma) + m_{f_d}(k' - k_{out})_\nu (1 - \gamma)], \]

(79)

Equation (68):

\[ \begin{aligned}
H(k) & \xrightarrow{d^j} W^-_{\mu}(k') \\
& \xleftarrow{w^i}
\end{aligned} \]

\[ - \frac{M_W}{2\sqrt{2}v^2} V_f^{ij} \theta^{\mu\nu} [m_{f_d}(k' - k_{out})_\nu (1 - \gamma) + m_{f_d}(k' + k_{in})_\nu (1 + \gamma)], \]

(80)
Equation (69):

\[ d_j A \mu W + \nu u_i H(k) \left( \frac{eM_W}{2\sqrt{2}v^2} V_f^{ij} \theta^{\mu\nu} [m_{f_d} Q_{f_u}(1+\gamma_5) - m_{f_u} Q_{f_d}(1-\gamma_5)] \right) \]  

Equation (70):

\[ u_i W - \nu A \mu d_j H(k) \left( \frac{eM_W}{2\sqrt{2}v^2} V_f^{*ij} \theta^{\mu\nu} [m_{f_d} Q_{f_u}(1-\gamma_5) - m_{f_u} Q_{f_d}(1+\gamma_5)] \right) \]  

Equation (71):

\[ d_j W + \nu Z \mu u_i H(k) \left( \frac{M_Z M_W}{2\sqrt{2}v^3} V_f^{ij} \theta^{\mu\nu} [m_{f_d} (c_{V,f}+3c_{A,f})_u(1+\gamma_5) - m_{f_u} (c_{V,f}+3c_{A,f})_d(1-\gamma_5)] \right) \]
Equation (72):
\[
-H(k)\begin{array}{c}
\begin{array}{c}
v^i \\
\downarrow \\
d^j \\
\end{array} \\
\begin{array}{c}
\uparrow \\
W^- \\
\end{array} \\
\begin{array}{c}
\downarrow \\
\nu \\
\end{array} \\
\begin{array}{c}
\mu \\
\end{array} \\
\end{array}
\end{array}
\]
\[
-\frac{M_Z M_W}{2\sqrt{2}v^3} V^{*ij}_f \theta^{\mu\nu} \left[ m_f (c_{V,f} + 3c_{A,f}) u (1 - \gamma_5) - m_f (c_{V,f} + 3c_{A,f}) d (1 + \gamma_5) \right],
\]
(84)

5 Summery

We examined the Higgs and Yukawa parts of the NCSM-action to find all the Higgs couplings with gauge and fermion fields, see (12)-(32) and (62)-(72), respectively. We obtained the corresponding Feynman rules as is given in (33)-(51) and (74)-(84). One can easily see that besides the usual standard model interactions, there are new couplings between the Higgs and fermions and the electroweak gauge bosons such as \( Z_{hh}, Z_{hhh}, h_{hh} W^+ W^-, h_{\gamma W^+ W^-}, h_{ZW^+ W^-}, Z_{Z\gamma h}, h_{\gamma Z W^+ W^-}, \ldots \) from the Higgs part and \( h \bar{f} f \gamma, h \bar{f} f Z, h \bar{f} f \gamma Z, h \bar{f} f W^+ W^-, h \bar{u} d W^+, \bar{h} \bar{u} d W^+ \gamma, \bar{h} \bar{u} d W^+ Z \) and so on from the Yukawa part. These new vertices, if exist, lead to new production and decay channels that will be very important to consider in LHC or the future ILC colliders. For instance, in the decay of Higgs to four leptons, \( H \to llll \), which is one of the main discovery channels for the Higgs boson [14], there are new vertices in the NCSM such as \( h \bar{f} f \gamma \) and \( h \bar{f} f Z \). Meanwhile, in the NC space the Higgs can be produced through new channels such as \( qq \to WW^* \to qq H \gamma, qq \to WW^* \to qq H Z, qq \to ZZ^* \to qq H Z, \) and so on.
References

[1] The ATLAS Collaboration, Phys. Lett. B 716, 1 (2012), ATLAS-CONF-2012-162; The CMS Collaboration, Phys. Lett. B 716, 30 (2012), CMS-PAS-HIG-12-045.

[2] The CMS Collaboration, S. Chatrchyan et al., Phys. Rev. Lett. 110, 081803 (2013), arXiv:1212.6639.

[3] F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, JHEP 02, 016 (1999).

[4] N. Seiberg and E. Witten, JHEP 09, 032 (1999).

[5] Everton M. C. Abreu, and M.J. Neves, Nucl. Phys. B 884, 741 (2014); C.P. Martin, Phys. Rev. D 89, 065018 (2014); Josip Trampetic, and Jiangyang You, SIGMA 10, 054 (2014); C.P. Martin, Class. Quant. Grav. 30, 155019 (2013); Elisabetta Di Grezia, Giampiero Esposito, Marco Figliolia, and Patrizia Vitale, Int. J. Geom. Meth. Mod. Phys. 10, 1350023 (2013); Everton M.C. Abreu, and Mario J. Neves, Int. J. Mod. Phys. A 28, 1350017 (2013); C.P. Martin, Phys. Rev. D 86, 065010 (2012); H. Falomir, F. Vega, J. Gamboa, F. Mendez, and M. Loewe, Phys. Rev. D 86, 105035 (2012); R. Horvat, A. Ilakovac, P. Schupp, J. Trampetic, and J. You, JHEP 1204, 108 (2012); Raul Horvat, Amon Ilakovac, Peter Schupp, Josip Trampetic, and Jiang-Yang You, Phys. Lett. B 715, 340 (2012); Raul Horvat, and Josip Trampetic, JHEP 1101, 112 (2011); M. Chaichian, P. Presnajder, M. M. Sheikh-Jabbari, and A. Tureanu, Phys. Lett. B 683, 55 (2010); M. M. Ettefaghi and M. Haghighat, Phys. Rev. D 77, 056009 (2008); M. M. Ettefaghi, M. Haghighat, and R. Mohammadi, Phys. Rev. D 82, 105017 (2010); C.P. Martin, Phys. Rev. D 82, 085020 (2010); L. Bonora and M. Salizzoni, Phys. Lett. B504, 80 (2001); C. P. Martin and D. Sanchez-Ruiz, Phys. Rev. Lett. 83, 476 (1999); M. M. Ettefaghi and M. Haghighat, Phys. Rev. D 75, 125002(2007); M. Haghighat and M. M. Ettefaghi, Phys. Rev. D 70, 034017 (2004); M. M. Sheikh-Jabbari, JHEP 9906, 015 (1999); T. Krajewski and R. Wulkenhaar, Int. J. Mod. Phys. A15, 1011 (2000); S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP 0002, 020 (2000); A. Matusis, L. Susskind and N. Toumbas, JHEP 0012, 002 (2000); M. Hayakawa, Phys.Lett. B478, 394 (2000); A. A. Bichl, J. M. Grinstein, L. Popp, M. Schweda and R. Wulkenhaar, Int. J. Mod. Phys. A17 2219 (2002); B. Jurco, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C17, 521(2000); M. Buric, D. Latas and V. Radovanovic, JHEP 0602, 046 (2006); M. Buric, V. Radovanovic and J. Trampetic,
JHEP **0703**, 030 (2007); R. Wulkenhaar, JHEP **0203**, 024 (2002); R. Amorim, and Everton M. C. Abreu, Phys. Rev. D **80**, 105010 (2009); R. Amorim1, Everton M. C. Abreu, and W. G. Ramirez, Phys. Rev. D **81**, 105005 (2010); M. Haghighat and F. Loran, Phys. Rev. D. **67**, 096003 (2003); M. Haghighat and F. Loran, Mod. Phys. Lett. A **16**, 1435 (2001); M. Haghighat, S. M. Zebarjad and F. Loran, Phys. Rev. D. **66**, 016005 (2002).

[6] M. Chaichian, P. Presnajder, M.M. Sheikh-Jabbari, and A. Tureanu, Eur. Phys. J. C **29**, 413 (2003).

[7] X. Calmet, B. Jurco, P. Schupp, J. Wess, and M. Wohlgenannt, Eur. Phys. J. C **23**, 363 (2002); T. Ohl and J. Reuter, Phys. Rev. D **70**, 076007 (2004).

[8] B. Melic, K. Passek-Kumericki, J. Trampetic, P. Schupp, and M. Wohlgenannt, Eur. Phys. J. C **42**, 499 (2005); B. Melic, K. Passek-Kumericki, J. Trampetic, P. Schupp, and M. Wohlgenannt, Eur. Phys. J. C **42**, 483 (2005).

[9] M. Haghighat, and M. Khorsandi, arXiv:1410.0836; M. Ghasemkhani, R. Goldouzian, H. Khanpour, M. Khatiri, and M. Mohammadi, Prog. Theor. Exp. Phys. 081B01(2014); S. Aghababaei, M. Haghighat, and A. Kheirandish, Phys. Rev. D **87**, 047703, (2013); Weijian Wang, Jia-Hui Huang, and Zheng-Mao Sheng, Phys. Rev. D **88**, 025031 (2013); Weijian Wang, Feichao Tian, and Zheng-Mao Sheng, Phys. Rev. D **84** 045012 (2011); A. Jafari, Eur. Phys. J. C **73**, 2271 (2013); M. M. Etteteaghi, Phys. Rev. D **86**, 085038 (2012); Weijian Wang, Jia-Hui Huang, and Zheng-Mao Sheng, Phys. Rev. D **86**, 025003 (2012); G. Deshpande, and Sumit K. Garg, Phys. Lett. B **708**, 150 (2012); Josip Trampetic, Int. J. Geom. Meth. Mod. Phys. **09**, 1261016 (2012); Horvat, A. Ilakovac, P. Schupp, J. Trampetic, and J. You, JHEP **1204**, 108 (2012); Kai Ma, and Sayipjamal Dulat, Phys. Rev. A **84**, 012104 (2011); M.M. Etteteaghi, and T. Shakouri, JHEP **1011**, 131 (2010); M. Haghighat, Phys. Rev. D **79**, 025011 (2009); M. M. Etteteaghi , Phys Rev. D **79**, 065022 (2009); R. Horvat and J. Trampetic, Phys. Rev. D **79**, 087701 (2009); A. Joseph, Phys. Rev. D **79**, 096004 (2009); M. Haghighat, M. M. Etteteaghi, and M. Zeinali, Phys. Rev. D **73**, 013007 (2006); Mansour Haghighat, Nobuchika Okada and Allen Stern, Phys. Rev. D **82**, 016007 (2010); M. Zarei, E. Bavarsad, M. Haghighat, I. Motie, R. Mohammadi and Z. Rezaei, Phys. Rev. D **81**, 084035 (2010); C. P. Martín, D. Sanchez-Ruiz and C. Tamarit, JHEP **0702**, 065 (2007); A. Al-boteanu,
T. Ohl and R. Rckl, Phys. Rev. D 74, 096004 (2006); Seyed Yaser Ayazi, Sina Esmaeili, Mojtaba Mohammadi-Najafabadi, Phys. Lett. B 712, 93 (2012); V. Nazaryan and Carl E. Carlson, Int. J. Mod. Phys. A 20, 3495 (2005); A. Devoto, S. Di Chiara and W. W. Repko, Phys. Rev. D 72, 056006 (2005); S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane, and T. Okamoto, Phys. Rev. Lett. 87, 141601 (2001); S. Godfrey and M. A. Doncheski, Phys. Rev. D 65, 015005 (2002).

[10] J. E. Moyal, Proc. Cambridge Phil. Soc. 45, 99 (1949).

[11] M. Hayakawa, Phys. Lett. B478, 394 (2000); M. Hayakawa, hep-th/9912167.

[12] J. Wess, Commun. Math. Phys. 219, 247 (2001); B. Jurco, L. Möller, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C 21, 383 (2001).

[13] J. Madore, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C 16, 161 (2000).

[14] CMS Collaboration, Phys. Rev. Lett. 108, 111804 (2012), [hep-ex/1202.1997].