A Simple Measure of Product Substitutability Based on Common Purchases

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Abstract: We propose a measure of product substitutability based on correlation of common purchases, which is fast to compute and easy to interpret. In an empirical study of a drugstore retail chain, we demonstrate its properties, compare it to a similarly simple measure of product complementarity, and use it to find small clusters of substitutes.

Keywords: Retail Business, Scanner Data, Market Structure Analysis, Substitutes, Complements, Hierarchical Clustering.

JEL Codes: C38, D12, M31.

1 Introduction

Understanding relationships between products is crucial in retail marketing. Products are commonly divided into substitute, complementary, and independent products. In a broad definition, product A is a substitute for product B if A is bought and used instead of B for the same purpose. Product A is a complement for product B if A is bought and used together with B for a common purpose. Product A is independent of product B if neither of the previous relationships exist.

Classical microeconomics textbooks such as Mankiw (2015) define substitutability, complementarity, and independence with respect to cross-price elasticity of demand. In this narrow definition, product A is a substitute for product B if an increase in the price of B, ceteris paribus, leads to an increase in the demand for A. Product A is a complement for product B if an increase in the price of B, ceteris paribus, leads to a decrease in the demand for A. Product A is independent of product B if a change in the price of B, ceteris paribus, does not lead to a change in the demand of A. To determine the relationship between two products and measure its strength, a complex model relating prices, demands, and possibly other variables for multiple products is required. As the number of products increases, so does the dimension of the model causing difficulties in estimation (see e.g. Chernozhukov et al., 2019). Furthermore, prices of products may not be observed for sufficiently long period and detail or may remain constant (see e.g. Ruhm et al., 2012). Finally, there are other factors besides prices which influence substitutability and complementarity (defined in the broad sense) such as seasonality and product unavailability (see e.g. Vulcano et al., 2012). For these reasons, other measures of substitutability and complementarity are often adopted.

From a marketing perspective, substitute and complementary products are often approached by market structure analysis using scanner data (see e.g. Elrod et al., 2002). The recent literature offers several models of market structure explaining relationships between a potentially large number of products. Gabel et al. (2019) use a neural network language model to represent products as points in a two-dimensional map, in which the similarity of products is measured by the Euclidean distance. Ruiz
et al. (2020) propose a sequential probabilistic model of shopping data and define the exchangeability measure as the symmetrized Kullback–Leibler divergence to identify substitutes. Chen et al. (2020) follow the previous two approaches and use the complementarity and exchangeability measures based on a low-dimensional space of products. Tian et al. (2021) study product relationships in a bipartite product-purchase network and define the substitutability and complementarity measures as cosine similarity.

In this short paper, we focus on finding substitutes and propose a straightforward measure based on correlation of common purchases. It is similar to the above mentioned measures but much more simpler, one could even say elementary, as we do not assume any model of market structure. This makes our measure fast to compute and easy to interpret.

2 Scanner Data

Scanner data contains information on every product sold by a retail store. A set of products bought together by a single customer at a given time is labeled as a shopping basket. The total number of baskets is denoted as \( n \), the number of baskets containing product \( i \) as \( n_i \), \( i = 1, \ldots, m \), and the number of baskets containing both products \( i \) and \( j \) as \( n_{ij} \), \( i, j = 1, \ldots, m \). The total number of products is denoted as \( m \).

In our empirical analysis, we analyze a Czech drugstore retail chain. The dataset contains \( n \approx 30 \) million shopping baskets and \( m = 13303 \) products. The products are hierarchically divided into 68 categories and 472 subcategories. We focus on the category Products for Men, which contains 12 subcategories and 566 products. Most drugstore customers are women and the category Products for Men is rather a small world of its own.

3 Correlation of Common Purchases

We propose to measure product substitutability between products \( i \) and \( j \) by the Pearson correlation coefficient between counts \( n_{ik} \) and \( n_{jk} \) for \( k \neq i, j \). Specifically, we define

\[
\rho_{ij} = \frac{\sum_{k \neq i, j} (n_{ik} - \mu_i) (n_{jk} - \mu_j)}{\sqrt{\sum_{k \neq i} (n_{ik} - \mu_i)^2} \sqrt{\sum_{k \neq j} (n_{jk} - \mu_j)^2}}, \quad i, j = 1, \ldots, m, \tag{1}
\]

where \( \mu_i \) is given by

\[
\mu_i = \frac{1}{m - 1} \sum_{k \neq i} n_{ik}, \quad i = 1, \ldots, m. \tag{2}
\]

As \( \rho_{ij} \) is a correlation coefficient, it takes values in \([-1, 1]\). High values of \( \rho_{ij} \) are caused by similar purchase patterns of products \( i \) and \( j \) with respect to the other products \( k \); more specifically, by similar sets of complements of products \( i \) and \( j \). Similarly to Ruiz et al. (2020) and Tian et al. (2021), we attribute this behavior to substitutes. Values of \( \rho_{ij} \) near zero indicate independent products. Low values of \( \rho_{ij} \) indicate products with opposite purchase patterns. Note that expression (1) is identical to the expression when the summations are taken over the whole set of products \( k = 1, \ldots, m \) with \( n_{ii} \) set to \( \mu_i \). Using this identity, it is then clear that \( \rho_{ij}, i, j = 1, \ldots, m \) form a positive-semidefinite correlation matrix.

We estimate \( \rho_{ij} \) for the pairs of products within the Products for Men category but let \( k \) take values from the set of all products. In our data, negative values of \( \rho_{ij} \) do not occur at all. Figure 1 shows the kernel density of the estimated \( \rho_{ij} \) with the fitted density of the symmetric beta distribution for comparison. The kernel density of \( \rho_{ij} \) is concentrated around mean value 0.51 and is slightly positively skewed. Concerning tails, 0.28 percent of \( \rho_{ij} \) is lower than 0.20 and 3.63 percent is higher than 0.80. Densities of \( \rho_{ij} \) for other product categories are fairly similar.

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2 We include only products with at least 500 units sold during the analyzed period.

3 First, the kernel density is estimated using the Epanechnikov kernel. Second, the beta distribution is made to be symmetric by setting \( \alpha = \beta \) and then parameter \( \alpha = \beta = 5.31 \) is estimated by the method of moments.
From the perspective of product pairs, it may be interesting to look at the closest substitute of product \( i \), i.e. for product \( j \) with the highest \( \rho_{ij} \) value. The chord diagram in Figure 2 shows the closest substitutes by subcategories according to \( \rho_{ij} \). For the majority of products (74.73 percent), the closest substitute is within the same subcategory, which is expected. Exceptions are, for example, some deodorants of the same brand differing in form (spray and roll-on) or disposable shaving razors and shaving systems of the same brand. However, such a simple explanation does not always exist. This is the case, for example, of blood stoppers and old-school stainless razor, which are nowadays bought only by a very particular type of customers.

4 Relation to Product Complementarity

We investigate the relation between the proposed measure of product substitutability and a similar correlation-based measure of product complementarity. Specifically, we use the Yule phi coefficient (also known as the mean square contingency coefficient) of occurrence of products \( i \) and \( j \) in the same basket, defined as

\[
\phi_{ij} = \frac{n_{ij} - n_i n_j}{\sqrt{n_i n_j (n - n_i)(n - n_j)}}, \quad i, j = 1, \ldots, m.
\]

In the case of \( i = j \), we set \( n_{ii} \) to be equal to \( n_i \) and obtain \( \phi_{ii} = 1 \). Note that coefficient \( \phi_{ij} \) is identical to the Pearson correlation coefficient based on binary variables indicating presence of products in shopping baskets. High values of \( \phi_{ij} \) indicate high product complementarity. For a survey of complementary choice models, see Berry et al. (2014).

The relationship between two given products can be both substitutive (with above average \( \rho_{ij} \)) and complementary (with above average \( \phi_{ij} \)) at the same time. An example from our dataset are pairs of shower gels of the same brand that customers often buy in multiple units with different scents. Different brands of shower gels show different relationships between products of the same brand – some have similar levels of both substitution and complementarity, some only complementarity, and some only substitution. These findings have important implications for marketing. High complementarity but low substitutability is exemplified by deodorants of the same brand, where some are focused on sport and others on length of effect. Comparison of the relationship between \( \rho_{ij} \) and \( \phi_{ij} \) for all product pairs is shown in Figure 3. A substantial part of the products are independent of each other in terms of complementarity.

In the extreme case of two products always bought together, both \( \rho_{ij} \) and \( \phi_{ij} \) would be equal to 1. In our data, however, products with high \( \phi_{ij} \) have moderate \( \rho_{ij} \) at most and vice versa, as shown in Figure 3. This behavior can be illustrated by the following example. Consider the case of three products only – A and B are substitutes, A and C are complements, and B and C are also complements.
Figure 2: The chord diagram of the closest substitute product by subcategories according to $\rho_{ij}$. 
Figure 3: The relation of substitutability $\rho_{ij}$ and complementarity $\phi_{ij}$ for pairs of products.

C is then frequently bought with either A or B but not with both. High complementarity of A and C then prevents high substitutability of A and C as A is rarely bought with B while C is often bought with B.

5 Distance and Its Use in Clustering

Clustering of products is a common task in marketing analysis. For example, Lingras et al. (2014) and Ammar et al. (2016) iteratively cluster products and customers based on their interactions. Holý et al. (2017) and Sokol and Holý (2021) cluster products using penalty for joint occurrence of products in a shopping basket.

The proposed measure of product substitutability can be used for clustering of products as well. First, we transform correlation coefficient $\rho_{ij}$ into a distance using the law of cosines as

$$d_{ij} = \sqrt{2 - 2\rho_{ij}}, \quad i, j = 1, \ldots, m.$$  

This is an Euclidean distance with the following properties:

(i) Identity of indiscernibles, i.e. $d_{ii} = 0$ and $d_{ij} > 0$ for $i \neq j$.

(ii) Symmetry, i.e. $d_{ij} = d_{ji}$.

(iii) Triangle inequality, i.e. $d_{ij} \leq d_{ik} + d_{kj}$.

Positively correlated products have small distance while negatively correlated products have high distance. Distance $d_{ij}$ can then be used as a dissimilarity measure in various clustering methods such as k-means and hierarchical clustering (see e.g. Hastie et al., 2008).

With regard to marketing, e.g. product listing decisions or discount promotion planning, the natural goal of clustering is to look for small groups of substitutes rather than fewer large groups. A hierarchical structure can be advantageous for the decision-making process, where we can observe, for example, whether a deodorant brand or its scent or other characteristic plays a larger role in the

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4Here, we neglect the possibility of two products always bought together, as they would be truly indiscernible.
successive clustering. Thus, the behavior of the average customer can be revealed such as whether they prefer a different scent from the same brand or the same (or similar) scent from a different brand. This can be subsequently used when there is a shortage of product for which the customer came.

Applying hierarchical clustering using the Ward’s criterion, the products are clustered into 566 hierarchical cuts (levels) that form a tree. For example, at the 30th cut we find a cluster of deodorants of all forms and shower gels with the same brand. Following the cut hierarchy, the shower gels are first separated from the deodorants. Subsequently, deodorants are also separated by form type into spray, stick and roll-on. The individual fragrances are then grouped together in a further subdivision. The described progress is shown in Figure 4.

6 Note on Computation

Computational simplicity of the proposed approach is obvious. Both correlation of common purchases $\rho_{ij}$ and correlation of occurrence $\phi_{ij}$ are based only on counts $n$, $n_i$, and $n_{ij}$. When a new basket arrives, it is needed to increase the appropriate counts by one and then recompute equations (1) and/or (3). Besides the resulting correlations, it is required to store $m^2 + 1$ integers in a database.

Our approach is therefore a streaming algorithm, which can examine a sequence of inputs in a single pass only. This is a great computational advantage as the number of rows in any transaction database increases rapidly. For a survey of the pioneering data stream literature, see Muthukrishnan (2005). Recent studies in this field include e.g. Černý (2019) who deal with estimation and diagnostics of linear regression and Holý and Tomanová (2021) who focus on volatility estimation using financial high-frequency data.
7 Conclusion

We propose to measure substitutability between two products by a correlation coefficient of common purchases. In contrast to existing models of market structure, our approach is simple, straightforward, and computationally inexpensive, yet powerful. We identify two situations in which it can be well used:

(i) When a retail company does not wish to invest in a more complex market structure analysis, our approach can bring a basic insight.

(ii) When a market structure analysis is indeed conducted, our approach can serve as a benchmark for alternative methods.

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