Constraining Cosmological Parameters in the FLRW Metric with Lensed GW+EM Signals

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Abstract

We proposed a model-independent method to constrain cosmological parameters using the Distance Sum Rule of the Friedmann–Lemaître–Robertson–Walker metric by combining the time delay distances and the comoving distances through a multi-messenger approach. The time delay distances are measured from lensed gravitational wave (GW) signals together with their corresponding electromagnetic wave (EM) counterparts, while the comoving distances are obtained from a parameterized fitting approach with independent supernova observations. With a series of simulations based on the Einstein Telescope, Large Synoptic Survey Telescope, and The Dark Energy Survey, we find that only 10 lensed GW+EM systems can achieve the constraining power comparable to and even stronger than 300 lensed quasar systems due to the more precise time delay from lensed GW signals. Specifically, the cosmological parameters can be constrained to $k = 0.01^{+0.05}_{-0.05}$ and $H_0 = 69.7^{+0.35}_{-0.35}$ (1σ). Our results show that more precise time delay measurements could provide more stringent cosmological parameter values, and lensed GW+EM systems therefore can be applied as a powerful tool in the future precision cosmology.

Key words: cosmological parameters – gravitational lensing: strong – gravitational waves

1. Introduction

In 2015, the Laser Interferometer Gravitational Wave Observatory announced the first gravitational wave (GW) event, GW150914, indicating a new window had been opened for astronomical observations. Later in 2017, both the GW and its electromagnetic (EM) counterpart of the binary neutron star merger GW170817 marked the arrival of multi-messenger astronomy (Abbott et al. 2016). Combining the advantages of different messengers, multi-messenger astronomy will provide more information on the universe and thus enhance our understanding of the physical nature of the universe.

One application is to test the cosmological metric and to constrain the cosmological parameters. In cosmology, the Friedmann–Lemaître–Robertson–Walker (FLRW) metric is the most basic hypothesis, and it describes a homogeneous, isotropic universe. In this respect, testing its validity is of great significance. There have been a number of methods proposed to test the FLRW metric. Clarkson et al. (2008) proposed testing it by comparing observational measurements of the expansion rate and cosmological distances. Räslänen (2014) offered to use the parallax distance and angular-diameter distance. Räslänen et al. (2015) found that lensing systems with independent supernova observations can be used to test the FLRW metric using the equivalent Distance Sum Rule (DSR). If the DSR is violated, the FLRW metric can be ruled out. However, if the data are consistent with the FLRW metric, we can go a step further to constrain the cosmological parameters (Räslänen et al. 2015). In this work, we assume the FLRW metric is valid and no other cosmology model is adopted.

Additionally, a number of studies on constraining the cosmic curvature through cosmic background radiation (CMB) and baryon acoustic oscillations (BAO) have emerged in the past decades (Eisenstein et al. 2005; Tegmark et al. 2006; Planck Collaboration 2016). Shafieloo & Clarkson (2010) used the Hubble rate measurements together with supernovae distances; Mörtell & Jönhsson (2011) combined distance and lookback time observations; Sapone et al. (2014) proposed data binning with direct error propagation, principal component analysis, genetic algorithms, and Padé approximants; Cai et al. (2016) applied a model-independent smoothing technique, and Gaussian processes. More recently, Liao et al. (2017b) developed a new model-independent approach to test the FLRW metric and to constrain cosmic curvature using both the strong-lens time delay systems and independent supernovae observations, based on the DSR, and it turns out to be much more accurate than previous works.

The detection of GWs accompanied by electromagnetic counterparts offers new paths of inquiry. For example, strong-lensed GW–EM systems have been proposed as powerful astronomy tools in a series of studies (e.g., Piórkowska et al. 2013; Biesiada et al. 2014; Baker & Trodden 2017; Collett & Bacon 2017; Ding et al. 2017; Fan et al. 2017; Liao et al. 2017a; Wei & Wu 2017). Ding et al. (2015) concluded that once third-generation GW detectors (e.g., the Einstein Telescope, ET) start to operate, $10^5–10^6$ GW events will be detected per year according to their sensitivity, and 50–100 of these are expected to be lensed. The time delay measurements from lensed GWs can be quite accurate, with negligible observation error, and the time delay is an important data point for cosmology research with lensing systems. Thus, lensed GW–EM systems could provide stringent constraints on cosmological parameters (Liao et al. 2017a). Therefore, it is
reasonably to consider that by combining the redshift and the Fermat potential difference observed from lensed EM with the high-accuracy time delay obtained from lensed GW, we can put tighter constraints on the cosmological parameters under the DSR of the FLRW metric. To achieve this purpose, we carried out a series of simulations. We present the results in this paper.

This paper is organized as follows. In Section 2, we describe the methodology. The simulation and fitting results are shown in Section 3. A brief summary of the results is presented in Section 4.

2. Methodology

The method is based on the time delay distance $D_{\Delta t}$, which is usually defined as

$$D_{\Delta t} = \frac{D_A(z_i)D_A(z_s)}{D_A(z_l, z_s)}$$

where $z_l$, $z_s$ are redshifts at the lens and source, respectively. By equating the observational and theoretical time delay distances, we can obtain the cosmic curvature $k$ given the observed quantities of lensed GW+EM systems and supernovae. Next, let us look at the exact expressions for the time delay distance from observational and theoretical perspectives, respectively.

2.1. $D_{\Delta t}$ in Theory

A homogeneous and isotropic universe can be described by the FLRW metric:

$$ds^2 = -c^2dt^2 + \frac{a(t)^2}{1 - Kr^2}dr^2 + a(t)^2r^2d\Omega^2.$$  

Note the cosmic curvature $k = K/(H_0)^2 = -\Omega_k$, where $\Omega_k$ represents the spatial curvature density parameter, and $H_0$ is the Hubble constant. The dimensionless distance $d(z_l, z_s) = (1 + z_s)H_0D_A(z_l, z_s)/c$ can be interpreted as

$$d(z_l, z_s) = \frac{1}{\sqrt{-k}}\sinh\left(\sqrt{-k}\int_{t(z_l)}^{t(z_s)} \frac{H_0}{a(t)} dt\right).$$

According to the dimensionless distance, we have

$$d_{ls} = \frac{c}{H_0(1 + z_l)} \frac{D_A(z_l, z_s)}{D_A(z_l)}$$

We denote $d_{ls} = d(z_l, z_s)$, $d_{i} = d(0, z_l)$ and $d_{s} = d(0, z_s)$, where 0 means redshift is 0. According to Equation (3), $d_{ls}$ can be redefined in terms of $d_{i}$ and $d_{s}$ as in the following formula (namely DSR; see Räsänen et al. 2015):

$$d_{ls} = \epsilon_i d_s \sqrt{1 - kd_l^2} - \epsilon_l d_i \sqrt{1 - kd_s^2},$$

where $\epsilon_i = \pm 1$. For $k \leq 0$, $\epsilon_i = 1$. For $k \geq 0$, the signs depend on the three-dimensional hypersphere location of the source, the lens, and the propagation direction of the light (Räsänen 2014). Assuming $t$ and $z$ are in a one-to-one relation and $d'(z) > 0$, then $\epsilon_l = 1$ (Räsänen et al. 2015). Liao et al. (2017b) rewrote the DSR as follows:

$$\frac{d_{ls}}{d_i d_s} = T(z_i) - T(z_s),$$

where

$$T(z) = \frac{1}{d(z)} \sqrt{1 - kd(z)^2}. $$

In this paper, the comoving distances $d(z)$ are obtained from supernovae observations in a cosmological, model-independent way (for details see Section 3). Therefore, by combining Equations (1), (4), and (6), the time delay distance can be theoretically expressed as:

$$D_{\Delta t} = \frac{c}{H_0(1 + z_l)(T(z_i) - T(z_s))}.$$  

2.2. $D_{\Delta t}$ by Observation

The time delay distance $D_{\Delta t}$ and the lensing system observations can be linked by the following formula:

$$\Delta t_{ij} = \frac{(1 + z_l)D_{\Delta t}}{c} \Delta \phi_{ij},$$

where $\Delta t_{ij}$ is the time delay between images of the lensed source obtained from lensed GW, $z_l$ is the redshift of the lens, and $\Delta \phi_{ij}$ is the Fermat potential difference between image positions that can be obtained by the EM counterpart of the lensed GW:

$$\Delta \phi_{ij} = \frac{1}{2} (\theta_i - \beta)^2 - \psi(\theta_i) - \frac{(\theta_j - \beta)^2}{2} + \psi(\theta_j),$$

where $\theta_i$, $\theta_j$ represents the position of images of the lensed source, $\beta$ is the source position, and $\psi$ is the two-dimensional lensing potential that is related to the mass distribution of the lens.

3. Simulations and Results

In order to constrain the cosmological parameters, we performed simulations for 10 lensed GW+EM systems and 300 lensed quasar systems. The latter is adopted in accordance with Liao et al. (2017b). For both systems, the theoretical values of cosmological parameters $(k, H_0)$ and $d(z)$ can be derived simultaneously in a model-independent way from the observed quantities $(z_l, z_s, \Delta t, \Delta \phi)$ of lensing systems and $D_L$ of supernovae observations based on the time delay distance in the following steps.

(1) The comoving distances $d(z)$ in Equation (7) are assessed in a cosmological, model-independent way. The simulated $d(z)$ data are obtained according to the following formula in the flat $\Lambda$CDM model:

$$D_L(z) = (1 + z)d(z),$$

where the luminosity distances $D_L$ of SNe Ia are from 3540 supernovae simulated based on the 10-field hybrid strategy (Bernstein et al. 2012) of DES. The redshifts of these

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6 The Dark Energy Survey (DES) is an international and collaborative effort that will carry out a deep optical and near-infrared survey of 5000 deg$^2$ of the South Galactic Cap using a new 3 deg$^2$ Charge Coupled Device (CCD) camera to be mounted on the Blanco 4 m telescope at the Cerro Tololo Inter-American Observatory (CTIO; Bernstein et al. 2012). The 10-field hybrid strategy consists of two deep fields and three shallow fields from the 5-field hybrid strategy, plus additional shallow fields clustered around Chandra Deep Field-South, and it offers an attractive balance among all important considerations (Bernstein et al. 2012; Liao et al. 2017b).
supernovae are $z < 1.7$, which are consistent with the lensed GW+EM systems. We direct the reader to Liao et al. (2017b) for details on those simulated SNe Ia.

Then, the theoretical $d(z)$ used in Equations (7) and (13) are assumed following a fourth-order polynomial:

$$d(z) = z + a_1z^2 + a_2z^3 + a_3z^4.$$  

(12)

There is no much difference as long as $d(z)$ is more flexible than a second-order polynomial (Räsänen et al. 2015; Liao et al. 2017b).

(2) For 10 simulated lensed GW+EM systems, we used a typical lensing model that consists of a distant GW source accompanied by its EM counterpart and a foreground elliptical galaxy to obtain the observed quantities $(z_i, z_e, \Delta t_{i,j}, \Delta \phi_{i,j})$. The SIS model is adopted as the universal lens model. Specifically, $(z_i, z_e)$ of 10 lensed GW events are generated based on the expected redshift probability distribution functions (PDFs) of NS–NS systems in Ding et al. (2015). The redshift PDFs are calculated after considering the intrinsic merger rates of the whole class of double compact objects located at different redshifts (Dominik et al. 2013), the designed sensitivity of the ET, and the probability that individual GW signals from inspiralling double compact objects could be lensed by an early-type galaxy (Ding et al. 2015). The source redshift is cut by 2, covering the supernovae redshift range. As for another quantity, the Fermat potential, Liao et al. (2017a) found that with lensed GW+EM signals, the lens modeling yields a Fermat potential with 0.6% uncertainty according to current techniques, which will directly cause a few percent systematic error on the time delay distance. Therefore, we take 0.6% as the uncertainty of the Fermat potential difference $(\Delta \phi_{i,j})$.

(3) In addition to the above redshifts and the Fermat potential difference, we still need a time delay $\Delta t$ to simulate the observational time delay distance. The time delay obtained from the lensed GW is supposed to be very accurate, with negligible error (Liao et al. 2017a). In this paper, we simply assume that the uncertainty of the time delay obtained from a lensed GW is 0 (Wei & Wu 2017).

(4) Finally, we generated the observational time delay distance in the flat ΛCDM model with matter density $\Omega_M = 0.3$, $k = 0$, and Hubble constant $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, and then took into account the observational error propagated from the Fermat potential difference Equation (9) as discussed in step (2).

(5) To compare with traditional lensed quasar systems, following Liao et al. (2017b), we also simulated 300 lensed quasar systems from the OM10 catalog (Oguri & Marshall 2010) which provides mock observations of the upcoming Large Synoptic Survey Telescope based on realistic distributions of quasars and elliptical galaxies, and we have taken the parameter set $(z_i, z_e)$ for the same lensed systems in this paper. As for the Fermat potential in lensed quasar systems, we considered its uncertainty to be the same as that in time delay measurements. The uncertainty of the time delay measurement of lensed quasar systems in the EM domain is $\sim 3\%$, which, combined with the Fermat potential, would result in a $\sim 5\%$ uncertainty in the time delay distance (Liao et al. 2017b).

(6) Given the above simulated data, $(k, H_0, d(z))$ and their relative uncertainties can be derived in a model-independent way based on the theoretical time delay distance (Equation (8)) by minimizing the following equation:

$$\chi^2 = \sum_{i=1}^{10} \frac{(D_{\Delta t}(i) - D_{\Delta t}(ob))^2}{\delta D_{\Delta t}^2} + \sum_{i=1}^{3540} \frac{(d(z)_{ob} - d(z)_{in})^2}{\delta d(z)_{ob}^2},$$  

(13)

where “th” and “ob” represent theory and observation, respectively. This process is achieved by the minimization function in python. Eventually, the five parameters $(a_1, a_2, a_3, k, H_0)$ can be fitted simultaneously.

As a demonstration, the constraint contours with 5000 realizations for $(a_1, a_2, a_3, k, H_0)$ for 10 lensed GW+EM systems are presented in Figure 1. Clearly, we can recover the injection values of $(k, H_0)$ within a certain uncertainty ($\sim 1\%$). For comparison, we showed the 1σ numerical uncertainty of five cosmological parameters obtained from 300 lensed quasars in the EM domain and from 10 lensed GW+EM systems both in Table 1. It can be seen clearly from Table 1 that all five parameters (especially the Hubble constant) can be constrained more precisely with lensed GW+EM systems. To be specific, the constraining power of only 10 lensed GW+EM systems is comparable and even stronger than that of 300 lensed quasar systems. For example, the Hubble constant from lensed GW+EM systems is $H_0 = 69.7^{+0.35}_{-0.33}$, which is comparable to the one from the lensed quasar systems, $k = 0.01^{+0.05}_{-0.05}$. Therefore, we can draw the conclusion that lensed GW+EM systems will place much more stringent constraints on cosmological parameters than lensed quasar systems.

4. Discussion and Summary

With the coming era of GWs, we are excitedly looking forward to gaining new insights into unsolved astrophysical problems through multi-messenger systems. Interestingly, some researchers have considered the lensing effects on GW signals for advanced detectors (Biesiada et al. 2014; Cao et al. 2014; Baker & Trodden 2017; Collett & Bacon 2017; Ding et al. 2017; Liao et al. 2017a; Piórkowska et al. 2013). Moreover, in Fan et al. (2017) and Liao et al. (2017a), lensed GW+EM systems were discussed in detail as a powerful tools to provide stringent constraints on cosmological parameters. In this context we apply the more precise time delays obtained from lensed GWs to cosmology research. Specifically, we constrain cosmological parameters in the FLRW metric. Our results are presented in Figure 1. A comparison of the results obtained from lensed quasars and lensed GW+EM systems is shown in Table 1. The results clearly show that only 10 lensed GW+EM systems could give comparable and even better constrained parameters than 300 lensed quasar systems. In particular, the 1σ uncertainty range of the cosmic curvature parameter is $-0.04 < k < 0.06$.

This paper is a preliminary attempt to apply lensed GW+EM multi-messenger systems to cosmology research. We have proven that accurate time delay measurements from lensed GW are a new tool for precision cosmology that can be widely applied in the future, such as for studying the mass density slope of elliptical galaxies and its evolution with redshift, and dark matter substructure in galaxy-scale halos (Keeton & Moustakas 2009; Liao et al. 2017a). In a word, the more constrained parameters obtained from lensed GW+EM
Figure 1. One-dimensional marginalized distributions and the constraint contours with 5000 realizations for \((a_1, a_2, a_3, k, H_0)\) for 10 lensed GW+EM systems. The black solid line in each contour represents the 1\(\sigma\), 2\(\sigma\), and 3\(\sigma\) confidence intervals. The numerical uncertainties at the 1\(\sigma\) confidence level are presented at the top of the figure.

Table 1

| No.      | \(a_1\)            | \(a_2\)            | \(a_3\)            | \(k\)            | \(H_0\)         |
|----------|---------------------|---------------------|---------------------|-------------------|----------------|
| lensed quasar | \(-0.24^{+0.04}_{-0.04}\) | \(0.06^{+0.02}_{-0.02}\) | \(0.01^{+0.03}_{-0.02}\) | \(0^{+0.05}\) | \(69.86^{+0.53}\) |
| lensed GW+EM | \(-0.25^{+0.02}_{-0.02}\) | \(0.02^{+0.02}_{-0.02}\) | \(0.01^{+0.03}_{-0.01}\) | \(0.01^{+0.05}_{-0.05}\) | \(69.7^{+0.35}_{-0.35}\) |

Note. From top to bottom, 300 lensed quasar systems and 10 lensed GW+EM systems, respectively. The difference between these two systems is that the time delay uncertainties are 3\% and 0\%, respectively.
systems will make contributions to future cosmology studies and thus to our understanding of the evolution of the universe.

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