1. Introduction.

Let $\g$ be a complex finite dimensional simple Lie algebra with the root datum $(Y, X, \ldots)$, see \cite{L2}. Let $W_f$ denote the Weyl group, $R$ denote the root system, $R_+$ denote the set of positive roots. Let $X_+$ denote the set of dominant integral weights. Let $h$ denote the Coxeter number of $\g$.

Let us fix $l \in \mathbb{N}$, $l > h$. We assume that $l$ is odd (and not divisible by 3, if $\g$ is of type $G_2$). Let $W$ denote the corresponding affine Weyl group.

Let $\rho \in X$ denote the halfsum of positive roots. We will denote by dot (for example $w \cdot \lambda$) the action of $W$ (and $W_f \subset W$) centered in $(-\rho)$.

Let $q$ be a primitive $l$–th root of unity and let $U_q$ be the quantum group with divided powers as defined in \cite{L2}. Let $C$ denote the category of finite dimensional $U_q$–modules of type 1 (see e.g. \cite{APW}).

In \cite{A} H.Andersen has studied a tensor subcategory $Q \subset C$ formed by tilting modules. He has introduced a tensor ideal $K \subset Q$ formed by negligible tilting modules. The quotient tensor category $Q/K$ is semisimple. For certain values of $l$ it is tensor-equivalent to a category of integrable modules over affine Lie algebra $\hat{\g}$ equipped with a fusion tensor structure (see e.g. \cite{F}).

Let us recall the definition of $K$. Indecomposable tilting modules are numbered by their highest weights $\lambda \in X_+$; we will denote them by $Q(\lambda)$. The set of dominant weights $X_+$ is covered by the closed alcoves numbered by $W_f \subset W$ — the set of shortest elements in the right cosets $W/W_f$. For $w \in W_f$ the corresponding closed alcove will be denoted by $\overline{C}_w$. For example, the alcove $\overline{C}_e = \overline{C}$ containing the zero weight is given by

$$\overline{C} = \{\lambda \in X | 0 \leq \langle \lambda + \rho, \alpha^\vee \rangle \leq l \text{ for all } \alpha \in R_+\}.$$ 

Now $K$ is formed by the direct sums of tiltings $Q(\lambda)$, where $\lambda$ is dominant and $\lambda \in \bigcup_{w \neq e} \overline{C}_w$. 

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In this note we propose the following generalization of H.Andersen’s result. We recall that G.Lusztig and N.Xi have introduced a partition of $W^f$ into canonical right cells along with the right order $\leq_R$ on the set of cells, see [L1] and [LX]. In particular, $\{e\} \subset W^f$ forms a single right cell, maximal with respect to $\leq_R$. Thus $W^f - \{e\} = \bigsqcup_{A < R\{e\}} A$ — the union of right cells.

**Main Theorem.** Let $A \subset W^f$ be a right cell. The full subcategory $Q_{\leq A}$ formed by the direct sums of tiltings $Q(\lambda)$, $\lambda \in \bigcup_{w \in B_{\leq R}A} C_w$, is a tensor ideal in $Q$.

There is a well-known correspondence between the right cells in $W$ and the right ideals in the affine Hecke algebra $H$ (see [KL1]). Our result is completely parallel to this correspondence, and even the proof is. In fact, the proof is an application of a deep result by W.Soergel who has connected the characters of $Q(\lambda)$ with Kazhdan-Lusztig-type combinatorics of $H$.

In general, the right cells in $W^f$ are infinite, but some are finite, e.g. $\{e\} \subset W^f$. The first nontrivial example is a ”subregular” cell $D_1$ for $\mathfrak{g}$ of type $G_2$ (see the pictures and notations in [L1]) consisting of 8 alcoves. Then the subcategory $Q_{< D_1}$ formed by the direct sums of $Q(\lambda)$ such that $\lambda \in \bigcup_{w \in B_{< R}D_1} C_w$ is a tensor ideal, and we can consider the quotient subcategory $Q/Q_{< D_1}$ with finitely many isomorphism classes of indecomposable objects. This subcategory is non-semisimple, as opposed to Andersen’s fusion category $Q/K$. For example, when $l = 7$, $Q/K$ is equivalent to $\mathbb{C}$–vector spaces, while $Q/Q_{< D_1}$ has 24 isomorphism classes of indecomposable objects. Its Grothendieck ring is a 24-dimensional algebra with nontrivial nilpotent radical, as opposed to the classical fusion rings which are always semisimple. To our knowledge, this is a first example of a nonsemisimple tensor category without fiber functor with finitely many indecomposable objects.

As we already mentioned, for certain values of $l$, $Q/K$ is tensor equivalent to a category of integrable $\widehat{\mathfrak{g}}$–modules of positive central charge. It is a subcategory of a larger category $\mathcal{O}$ of all $\widehat{\mathfrak{g}}$–integrable $\widehat{\mathfrak{g}}$–modules of positive central charge, but the Kazhdan-Lusztig construction of fusion tensor structure in this larger category encounters serious problems (see [KL2]). Still we believe that the quotient categories like $Q/Q_{< D_1}$ are closely related to the would-be fusion structure on $\mathcal{O}$.

The idea of this note is essentially due to J.Humphreys: it was he who suggested the important role played by the right cells in the study of tilting modules [H]. I learnt of his ideas from M.Finkelberg. I am grateful to Catharina Stroppel for her beautiful patterns of tilting characters for $G_2$ which provided a further insight into the connection between right cells and tilting modules. Thanks are also due to
D. Timashov who acquainted me with LIE package; it was very useful for me at the first stage of my work. I am indebted to H.H. Andersen and J. Humphreys for the valuable suggestions which improved the exposition. Finally, I would like to thank the referee for extremely useful comments which simplified the original proof drastically.

2. Preliminaries.

2.1. For any \( \lambda \in \mathbb{C} \) let \( \mathcal{C}(\lambda) \) denote a full subcategory of \( \mathcal{C} \) consisting of modules whose composition factors have highest weights in \( W \cdot \lambda \). The category \( \mathcal{C} \) is a direct sum of the subcategories \( \mathcal{C}(\lambda) \) (linkage principle; see e.g. [APW], §8)

\[
\mathcal{C} = \bigoplus_{\lambda \in \mathbb{C}} \mathcal{C}(\lambda).
\]

For any \( \lambda \in X_+ \) one defines Weyl module \( V(\lambda) \) and module \( H^0(\lambda) \) (see [A] §1). Then the irreducible module \( L(\lambda) \) is the socle of \( H^0(\lambda) \) as well as the head of \( V(\lambda) \).

2.2. Let \( \mathbb{Z}[X] \) be the group algebra of abelian group \( X \). It is generated by elements \( e^\lambda, \lambda \in X \), with relations \( e^{\lambda_1} \cdot e^{\lambda_2} = e^{\lambda_1+\lambda_2} \forall \lambda_1, \lambda_2 \in X \). There is a natural action of \( W_f \) on \( \mathbb{Z}[X] \) given by the formula \( we^\lambda = e^{w \cdot \lambda} \). Let \( \mathcal{A} := \mathbb{Z}[X]^{W_f} \) be the invariants of this action. It is a subalgebra of \( \mathbb{Z}[X] \).

Let \( ch : K(\mathcal{C}) \to \mathbb{Z}[X] \) be the map associating to a module \( M \in \mathcal{C} \) its character \( ch(M) \). It is known that its image is \( \mathcal{A} \). Moreover the elements \( ch([V(\lambda)]) \) where \( \lambda \) runs through \( X_+ \) form a basis of \( \mathcal{A} \). It is known that \( ch([V(\lambda)]) = ch([H^0(\lambda)]) \) is given by the Weyl character formula (see e.g. [APW] §8):

\[
ch([V(\lambda)]) = \frac{\sum_{w \in W_f} (-1)^{l(w)} e^{w \cdot \lambda}}{\sum_{w \in W_f} (-1)^{l(w)} e^{w \cdot 0}}.
\]

Now for any \( \lambda \in X \) let

\[
ch(\lambda) = \frac{\sum_{w \in W_f} (-1)^{l(w)} e^{w \cdot \lambda}}{\sum_{w \in W_f} (-1)^{l(w)} e^{w \cdot 0}}.
\]

Lemma. (i) If stabilizer (in \( W_f \)) of \( \lambda \) with respect to dot action is nontrivial then \( ch(\lambda) = 0 \).

(ii) Suppose the stabilizer of \( \lambda \) is trivial and let \( w \in W_f \) be such that \( w \cdot \lambda \in X_+ \). Then \( ch(\lambda) = (-1)^{l(w)} ch(w \cdot \lambda) \).

Proof. Clear. □
2.3. Let $W \rightarrow W_f, w \mapsto w$ be the standard homomorphism with the kernel consisting of translations.

**Lemma.** For any $\lambda, \mu \in X$ and $w \in W$ we have $w(\lambda + \mu) = w\lambda + w\mu$ and $w \cdot (\lambda + \mu) = w \cdot \lambda + w\mu$.

**Proof.** The first identity is obviously true for $w \in W_f \subset W$ and for translations. Since $W$ is a semidirect product of $W_f$ and the subgroup of translations we get our result. The second identity is a simple consequence of the first one. □

2.4. **Lemma.** (see e.g. [D] 2.2.3) Let $P$ be a multiset (set with multiplicities) of weights invariant under $W_f$ action. Then for any $\lambda \in X$ we have

$$(\sum_{\omega \in P} e^{\omega})ch(\lambda) = \sum_{\omega \in P} ch(\lambda + \omega)$$

**Proof.** Straightforward computation. □

2.5. A filtration of $U_q$–module is called Weyl filtration (respectively good filtration) if all the associated factors are Weyl modules (respectively modules $H^0(\lambda)$).

2.6. **Definition** (see [A], definition 2.4) A tilting module is a module $M \in \mathcal{C}$ which has both a Weyl filtration and a good filtration.

Let $\mathcal{Q} \subset \mathcal{C}$ be a full subcategory formed by all tilting modules. The main properties of this category are collected in the following (see [A] §2)

**Theorem.** (i) The category $\mathcal{Q}$ is closed under tensor multiplication.

(ii) Any tilting module is a sum of indecomposable tilting modules.

(iii) For each $\lambda \in X^+$ there exists an indecomposable tilting module $Q(\lambda)$ with highest weight $\lambda$.

(iv) The modules $Q(\lambda), \lambda \in X^+$, form a complete set of nonisomorphic indecomposable tilting modules.

(v) A tilting module is determined up to isomorphism by its character.

Let $\mathcal{Q}(\lambda)$ be the full subcategory of $\mathcal{Q}$ consisting of modules contained in $\mathcal{C}(\lambda)$. Then obviously

$$\mathcal{Q} = \bigoplus_{\lambda \in \overline{\mathcal{C}}} \mathcal{Q}(\lambda).$$

2.7. For any $\lambda, \mu \in \overline{\mathcal{C}}$ one defines the translation functor $T^\mu_\lambda : \mathcal{C}(\lambda) \rightarrow \mathcal{C}(\mu)$ (see e.g. [APW] §8). The following Proposition is proved as in [P],II,7.13.
2.7.1. **Proposition.** Suppose \( \lambda, \mu \in \mathcal{C} \) and \( w \in W \) is such that \( w \cdot \lambda \in X_+ \). Then \( T^0_\lambda V(w \cdot \lambda) \) has a filtration with the associated factors \( V(\nu) \) such that \( \nu \in X_+ \) and \( \nu = w w_1 \cdot \mu \) with \( w_1 \in Stab(\lambda) \). Each one of the above factors occurs exactly once.

In particular it follows that translation functors preserve the category \( \mathcal{C} \).

2.7.2. **Corollary.** For any \( w \in W \) such that \( w \cdot \lambda \in X_+ \) the module \( T^0_\lambda T^0_0 V(w \cdot 0) \) has a filtration with associated factors \( V(\nu) \) with \( \nu \in X_+ \) and \( \nu = w w_1 \cdot \mu \) with \( w_1 \in Stab(\lambda) \).

**Proof.** Evident. \( \square \)

3. **Construction of tensor ideals.**

3.1. Recall that \( W \) denotes the affine Weyl group and \( W_f \) denotes the ordinary Weyl group. Let \( W_f \) denote the set of minimal length representatives of right cosets. The multiplication defines a bijection \( W_f \times W_f \to W \). Let \( \mathcal{L} \) be the sign representation of \( W_f \). We will consider it as right \( W_f \)-module. Let us define a right \( W \)-module \( \mathcal{N}^1 := \mathcal{L} \otimes_{\mathbb{Z}[W_f]} \mathbb{Z}[W] \). As \( \mathbb{Z} \)-module it is isomorphic to a free abelian group with generators numbered by \( W_f \). Let \( N^1_x = 1 \otimes x \) for any \( x \in W_f \). These elements form a \( \mathbb{Z} \)-basis of \( \mathcal{N}^1 \). For any \( s \in S \) we have \( N^1_s s = N^1_{xs} \) if \( xs \in W_f \) and \( N^1_s \) otherwise.

3.2. Let \( K(C) \) denote the Grothendieck group of the category \( C \). For any \( \lambda \in \mathcal{C} \) define the map \( \alpha_\lambda : K(C) \to \mathcal{N}^1 \) by \( \alpha_\lambda([V(\mu)]) = 1 \otimes \left( \sum_{x \in W_f; x \cdot \lambda = \mu} x \right) \). In particular \( \alpha_\lambda \) annihilates every object outside of the block \( \mathcal{C}(\lambda) \) of \( C \).

3.2.1. Let us identify \( K(C) \) with the character ring \( \mathcal{A} \).

**Lemma.** For any \( w \in W \) we have

\[
\alpha_\lambda(ch(w \cdot \lambda)) = |Stab(\lambda)|^{-1} \otimes \left( \sum_{x \in Stab(\lambda)} wx \right)
\]

**Proof.** For \( w \in W \) such that \( w \cdot \lambda \in X_+ \) the Lemma is clear from definitions. For other \( w \) use 2.2. \( \square \)

3.2.2. **Lemma.** For any \( \lambda \in \mathcal{C} \) and \( V \in \mathcal{C}(\lambda) \) we have

\[
\alpha_\lambda(V) = \alpha_0(T^0_\lambda V)
\]

**Proof.** Obvious. \( \square \)
3.2.3. Lemma. For any $\lambda \in C$ and $V \in C(0)$ we have

$$\alpha_0(T_0^\lambda T_\lambda^{-1}V) = \alpha_0(V) \sum_{x \in \text{Stab}(\lambda)} x$$

Proof. It is enough to verify the Lemma for $V = V(w \cdot 0)$. Now if $w \cdot \lambda \in X_+$ the result follows from 2.7.2; if $w \cdot \lambda \not\in X_+$ then RHS and LHS both vanish. □

3.3. Proposition. For any $\lambda, \mu \in C$ and $M \in C$ there exists $c(M) = c_{\lambda\mu}(M) \in \mathbb{Z}[W]$ such that for all $V \in C(\lambda)$ we have

$$\alpha_\mu(V \otimes M) = \alpha_\lambda(V)c(M).$$

Proof. (see also [1], II, 7.5) It is enough to check the claim on the level of characters; moreover we can suppose that $\text{ch}(V) = \text{ch}(w \cdot \lambda)$.

Let $P(M)$ be a multiset of weights of module $M$. It is invariant under $W_f$–action. We have by 2.2 and 2.3

$$\text{ch}(V(w \cdot \lambda) \otimes M) = \sum_{\omega \in P(M)} \text{ch}(w \cdot \lambda + \omega) = \sum_{\omega \in P(M)} \text{ch}(w \cdot (\lambda + \omega))$$

Now let us define a multiset $W_{\lambda\mu}(M) := \{x \in W| \lambda + \omega = x \cdot \mu; \omega \in P(M)\}$. It is easy to see that $W_{\lambda\mu}(M)$ is invariant under left multiplication by elements of $\text{Stab}(\lambda)$ and right multiplication by elements of $\text{Stab}(\mu)$. So $W_{\lambda\mu}(M)$ is a union of left and right cosets; let $W_{\lambda\mu}(M)'$ be a set of representatives of right cosets. We claim that we can choose $c_{\lambda\mu}(M) = \sum_{z \in W_{\lambda\mu}(M)'} z$.

Indeed, let $P_{\lambda\mu}(M) := \{\omega \in P(M)| \lambda + \omega \in W \cdot \mu\}$. For any $\omega \in P_{\lambda\mu}(M)$ let $w(\omega)$ be any element of $W$ such that $w(\omega)^{-1} \cdot (\lambda + \omega) = \mu$. It is evident that $\{w(\omega)\}$ is the set of representatives of left cosets in $W_{\lambda\mu}(M)$. We have

$$\alpha_\mu(V \otimes M) = \alpha_\mu(\text{ch}(w \cdot \lambda)\text{ch}(M)) = \alpha_\mu(\sum_{\omega \in P(M)} \text{ch}(w \cdot (\lambda + \omega))) = \alpha_\mu(\sum_{\omega \in P_{\lambda\mu}(M)} \text{ch}(w \cdot (\lambda + \omega))) = \alpha_\mu(\sum_{\omega \in P_{\lambda\mu}(M)} \sum_{x \in \text{Stab}(\mu)} 1 \otimes w\omega x) = \sum_{t \in W_{\lambda\mu}(M)} 1 \otimes wt = \sum_{y \in \text{Stab}(\lambda)} \sum_{z \in W_{\lambda\mu}(M)'} 1 \otimes wyz = \alpha_\lambda(\text{ch}(w \cdot \lambda)) \sum_{z \in W_{\lambda\mu}(M)'} z.$$

The Proposition is proved. □
3.4. **Definition.** A subcategory $C' \subset C$ is called a weak tensor ideal if for any $V \in C'$ and $M \in C$ we have $V \otimes M \in C'$.

We define weak tensor ideals in any subcategory of $C$ closed under tensor multiplication in the same way.

**Corollary.** If $U \subset N^1$ is a $\mathbb{Z}[W]$-submodule, then $C_U := \{ V \in C | \alpha(V) \in U \ \forall \lambda \in C \}$ is a weak tensor ideal of $C$ and $Q_U := Q \cap C_U$ is a weak tensor ideal of $Q$.

**Proof.** Clear. □

4. **Realization of $K(Q(0))$ as a module over Hecke algebra.**

In this section we follow [S1].

4.1. Let $l : W \rightarrow \mathbb{N}$ be the length function and let $\leq$ be the standard Bruhat order on $W$. We will write $x < y$ if $x \leq y$ and $x \neq y$. Let $\mathcal{L} = \mathbb{Z}[v, v^{-1}]$ denote the ring of Laurent polynomials over $\mathbb{Z}$ in variable $v$. Let $\mathcal{H}$ be the Hecke algebra corresponding to $(W, S)$

$$\mathcal{H} = \bigoplus_{x \in W} \mathcal{L}T_x$$

with multiplication given by the rule: $T_xT_y = T_{xy}$ if $l(xy) = l(x) + l(y)$ and $T_s^2 = v^{-2}T_e + (v^{-2} - 1)T_s$ for all $s \in S$ (see [S1] §2).

Let $H_x = v^{l(x)}T_x$ be a new basis of Hecke algebra. There exists unique involutive automorphism of Hecke algebra $d : \mathcal{H} \rightarrow \mathcal{H}$, $H \mapsto \overline{H}$ such that $\overline{v} = v^{-1}$ and $\overline{H_x} = (H_{x^{-1}})^{-1}$. We will call $H \in \mathcal{H}$ selfdual if $\overline{H} = H$.

The following theorem was proved by Kazhdan and Lusztig in [KL1].

**Theorem.** For any $x \in W$ there exists unique selfdual $H_x \in \mathcal{H}$ such that $H_x \in H_x + \sum_{y < x} v\mathbb{Z}[v]H_y$.

The coefficients of $H_x$ in the basis $\{ H_x \}$ are essentially Kazhdan-Lusztig polynomials.

4.2. Let $\mathcal{H}_f$ be the Hecke algebra corresponding to the group $W_f$. We have an obvious embedding $\mathcal{H}_f \subset \mathcal{H}$. Let $\mathcal{L}(-v)$ be a free right $\mathcal{L}$-module of rank 1 with the right action of $\mathcal{H}_f$ given by the following rule: for any $s \in S_f$ the element $H_s$ acts as $(-v)$. We define a right $\mathcal{H}$-module $\mathcal{N} := \mathcal{L}(-v) \otimes_{\mathcal{H}_f} \mathcal{H}$. For any $x \in W^f$ let us define $\overline{N}_x := 1 \otimes H_x \in \mathcal{N}$. Let $\beta : \mathcal{N} \rightarrow N^1$ denote the specialization map: $v \mapsto 1$. We define $N_x^{1} := \beta(N_x) \in N^1$.

4.3. The following statement was conjectured in [S1] (Vermutung 7.2) and then proved in [S2].

**Theorem.** $\alpha(Q(x \cdot 0)) = N_x^{1}$. 7
4.4. We will say that an \( \mathbb{Z}[W] \)-submodule of \( \mathcal{N}^1 \) is a \( KL \)-submodule if it admits a base consisting of elements \( \mathcal{N}^1 \) for some subset of \( W^f \).

5. **Right cells in affine Weyl group.**

5.1. In [KL1] Kazhdan and Lusztig defined three partitions of any Coxeter group into subsets called right, left and two-sided cells respectively. We refer the reader to loc. cit. for the definitions of preorders \( \leq_R, \leq_L, \leq_{LR} \) on Coxeter groups. The right (left, two-sided) cells are the classes of equivalence generated by preorder \( \leq_R \) (respectively \( \leq_R \) and \( \leq_{LR} \)). Let \( w \in W \) and \( A \) be a right cell in \( W \). We will write that \( w \leq_R A \) if \( w \leq_R w' \) for any \( w' \in A \) (and similarly for left and two-sided cells).

5.2. There is a correspondence between cells and ideals in the Hecke algebra. Namely, for any right (left or two-sided) cell \( A \) the \( L \)-submodule \( I_{\leq A} \) of \( H \) generated by \( H_{w}, \ w \leq_R A \) (and similarly for left and two-sided cells) is a right (respectively left and two-sided) ideal of \( H \) (see [KL1]). Moreover any KL-ideal (i.e. ideal admitting a base consisting of some elements \( H_{w} \)) is a sum of such ideals.

5.3. Let \( A \) be a two-sided cell of \( W \). The main result of [LX] is the following

**Theorem.** The intersection \( A \cap W^f \) forms a right cell of \( W \).

5.4. **Definition.** A weak tensor ideal \( \tau \subset \mathcal{Q} \) is called a tensor ideal if for any \( Q_1, Q_2 \) such that \( Q_1 \oplus Q_2 \in \tau \) we have \( Q_1, Q_2 \in \tau \).

For any two-sided cell \( A \) of \( W \) we define the full subcategory \( \mathcal{Q}_{\leq A} \) of \( \mathcal{Q} \) as follows: \( \mathcal{Q}_{\leq A} \) is the additive subcategory of \( \mathcal{Q} \) and indecomposable objects of \( \mathcal{Q}_{\leq A} \) are all the modules \( Q(w \cdot \lambda) \) where \( \lambda \in \mathcal{C}, \ w \in W^f \) and \( w \leq_R A \).

5.5. **Main Theorem.** For any two-sided cell \( A \) of \( W \) the subcategory \( \mathcal{Q}_{\leq A} \) is a tensor ideal.

**Proof.** For any two-sided cell \( A \) we define a \( \mathbb{Z}[W] \)-submodule \( U_{\leq A} \) of \( \mathcal{N}^1 \) to be \( \mathcal{L} \otimes I_{\leq A \cap W^f} \).

We will show that for any \( \lambda \in \mathcal{C} \) \( \alpha_\lambda(Q(w \cdot \lambda)) \in U_{\leq A} \) if and only if \( \alpha_0(Q(w' \cdot 0)) \in U_{\leq A} \) where \( w' \) is the longest element of coset \( wStab(\lambda) \).

We have

\[
\alpha_\lambda(Q(w \cdot \lambda)) = |Stab(\lambda)|^{-1} \alpha_0(T^0_\lambda Q(w \cdot \lambda))
\]

Note that \( T^0_\lambda Q(w \cdot \lambda) \) contains a direct summand \( Q(w' \cdot 0) \). So we proved that \( \alpha_\lambda(Q(w \cdot \lambda)) \in U_{\leq A} \) implies that \( \alpha_0(Q(w' \cdot 0)) \in U_{\leq A} \).
Now note that $T_0^3Q(w' \cdot 0)$ contains a direct summand $Q(w \cdot \lambda) = Q(w \cdot 0)$. Further $\alpha_0(T_0^3T_0^3Q(w' \cdot 0)) = \alpha_0(Q(w' \cdot 0)) \sum_{x \in Stab(\lambda)} x \in U_{\leq A}$ by 3.2.3 and we proved our claim in another direction.

So the proof of theorem is finished. □

5.6. **Remark.** It is easy to see that theorem above establishes bijection between KL-submodules of $N^1$ and tensor ideals in $Q$. Further note that all KL-submodules of $N^1$ are the sums of submodules $U \leq A$. So we describe all tensor ideals in a category of tilting modules.

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Independent Moscow University, 11 Bolshoj Vlasjevskij per., Moscow 121002 Russia
E-mail address: ostrik@nw.math.msu.su