QUANTUM DILATON GRAVITY
IN THE LIGHT–CONE GAUGE

X. Shen

Theory Division, CERN
CH-1211, Geneva 23, Switzerland

ABSTRACT

Recently, models of two-dimensional dilaton gravity have been shown to admit classical black-hole solutions that exhibit Hawking radiation at the semi-classical level. These classical and semi-classical analyses have been performed in conformal gauge. We show in this paper that a similar analysis in the light–cone gauge leads to the same results. Moreover, quantization of matter fields in light–cone gauge can be naturally extended to include quantizing the metric field à la KPZ. We argue that this may provide a new framework to address many issues associated to black-hole physics.

* Work supported partially by a World Laboratory scholarship.

CERN-TH.6633/92
September 1992
1. Introduction

The reconciliation between the principles of quantum mechanics and general relativity has been a long-standing challenge in theoretical physics. Conventional quantization of Einstein’s general relativity leads to unrenormalizability of the theory and thus to the breakdown of any predictive power. An indirect way to quantize gravity came surprisingly from an unsuccessful attempt to describe strong interactions in terms of string dynamics. It is generally believed that a consistent quantum theory of gravity, as well as all other gauge interactions, emerges in the framework of string theory [1].

Prior to the advent of modern string theory, in the seminal work of Hawking [2], it has been shown that a semi-classical treatment of four-dimensional black holes—one of the focal points of quantum gravity—leads to Hawking radiation. However this leaves a series of issues that beg for further elucidation, such as quantum coherence, back reaction on the metric from the radiation, evolution of singularity, and the endpoint of black-hole evaporation [3]. Naturally one may ask if the string picture of quantum gravity can offer some insights in these issues. In [4], Witten obtained a two-dimensional black-hole solution as an exact solution of string theory, which triggered a great deal of interest in studying black holes from the viewpoints of string theory. It was conjectured [4] that the endpoint of Hawking radiation for this two-dimensional black hole should be described by the matrix model formulation of two-dimensional string theory ($c = 1$).

Following Witten’s discovery of two-dimensional stringy black hole, an interesting two-dimensional model was proposed by Callan, Giddings, Harvey and Strominger (CGHS) [5] to describe the dynamics of Hawking radiation. This model admits a classical solution of the black hole, with a singularity shielded by horizon, and at semi-classical level exhibits Hawking radiation. It is therefore believed that it serves as a promising toy model to discuss issues of back reaction, quantum coherence, singularity, and the fate of black hole. There have been a lot of investigations of this model [6–14], as well as some well-motivated variants [15–17].
It has been shown in these models that Hawking radiation at semi-classical level leads to either a naked singularity \([6–9]\) or thunderbolt \([14]\).

The analysis carried out in the original CGHS model, and later in its variants, has been performed in conformal gauge. There the classical equations of motion for dilaton, conformal field of gravity and matter fields in these models are exactly solvable. In particular they give black-hole solutions that can be formed classically from collapsing matter. To discuss the quantum aspects of black holes, one may at the semi-classical level introduce a conformal anomaly term into the original action that arises from quantizing the matter fields. Remarkably this leads to Hawking radiation \([5]\)! Unfortunately all the models considered so far exhibit indefinite Hawking radiation, or absence of a ground state, which is believed to be a consequence of ignoring the back-reaction of radiation on the metric. Moreover a full quantum discussion would require quantizing all fields, including the metric field and the dilaton field. Work in this direction have been discussed in \([15,16]\).

Although a direct quantization of the metric field in four-dimensional space-time is difficult, in two-dimensional space-time it is possible to successfully quantize the induced gravity of Polyakov that arises from matter conformal anomaly \([18,19]\). Since, in the field theory formulation of two-dimensional-induced gravity of Polyakov, quantum regulator terms such as \(R^2\) give non-trivial interactions in conformal gauge, while in the light–cone gauge \(R^2\) is a quadratic kinetic term, it is arguably more advantageous to quantize the Polyakov gravity in the light–cone gauge, as was done in the analysis of \([18,19]\). For in the CGHS-type of models the quantization of matter fields yields the Polyakov-induced gravity, it is natural to think that it might also be more advantageous to choose the light–cone gauge instead of the conformal gauge here.

In this paper we will carry out such an analysis in the light–cone gauge. Section 1 contains the classical solutions of black holes, found first in \([4,5]\), and their analogues in a different coordinate system that corresponds to light–cone gauge choice for the metric. In Sec 2, we will calculate, in the light–cone gauge, the
semi-classical matter stress tensor induced from the conformal anomaly (or rather
gravitational anomaly), which gives a rate of Hawking radiation that agrees with
that of the previous calculation. Finally we quantize the metric field in the light–
cone gauge à la KPZ [19], but in this case Fadeev-Popov ghosts that correspond
to gauge-fixing couple to a (flat) metric, instead of the metric of the black-hole
background geometry. This is necessary so as to avoid Hawking radiation of ghosts
[11]. We argue that this may serve as a useful framework to formulate a full
consistent quantum theory of dilaton gravity.

2. Classical Black-Hole Solutions

The action of dilaton gravity coupled conformal matter of the CGHS model is
given by $S = S_D + S_M$ with

$$S_D = \frac{1}{2\pi} \int d^2\sigma \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + 4 \lambda^2 \right],$$

$$S_M = -\frac{1}{4\pi} \int d^2\sigma \sqrt{-g} \sum_{i=1}^{N} (\nabla f_i)^2.$$  

Classically this theory is exactly solvable [5]. It is conveniently seen in conformal
gauge, given by

$$g_{++} = 0 = g_{--}, \quad g_{+-} = g_{-+} = -\frac{1}{2} e^\rho,$$  

with $\sigma^\pm = \sigma^0 \pm \sigma^1$. In this gauge the action becomes

$$S = \frac{1}{\pi} \int d^2\sigma \left[ e^{-2\phi} \left( -2 \partial_+ \partial_- \rho + 4 \partial_+ \phi \partial_- \phi - \lambda^2 e^{2\rho} \right) - \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right].$$

The equations of motion of dilaton and matter are given by

$$-4 \partial_+ \partial_- \phi + 4 \partial_+ \phi \partial_- \phi + 2 \partial_+ \partial_- \rho + \lambda^2 e^{2\rho} = 0,$$
$$\partial_+ \partial_- f_i = 0; \quad i = 1, 2, \ldots, N.$$
The $\rho$ field equation of motion, corresponding to $T^c_{+-}$, is given by
\begin{equation}
T^c_{+-} = e^{-2\phi}(2\partial_+\partial_-\phi - 4\partial_+\partial_-\phi - \lambda^2 e^{2\rho}) = 0. \tag{5}
\end{equation}

In addition there are two constraints (that come from the equations of motion of $g_{++}$ and $g_{--}$) given by
\begin{align*}
T^c_{++} &= e^{-2\phi}(4\partial_+\rho\partial_+\phi - 2\partial^2_+\phi) + \frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_+ f_i = 0, \\
T^c_{--} &= e^{-2\phi}(4\partial_-\rho\partial_-\phi - 2\partial^2_-\phi) + \frac{1}{2} \sum_{i=1}^N \partial_- f_i \partial_- f_i = 0. \tag{6}
\end{align*}

Given an arbitrary solution $f_i = f_i^+(\sigma^+) + f_i^- (\sigma^-)$ of the matter equations, the general solutions for $\rho$ and $\phi$ are given by
\begin{equation}
\begin{aligned}
e^{-2\phi} &= \frac{M}{\lambda} - \lambda^2 \int d\sigma^+ e^{w_+} \int d\sigma^- e^{w_-} - \frac{1}{2} \int d\sigma^+ e^{w_+} \int d\sigma^- e^{w_-} \sum_{i=1}^N (\partial_+ f_i^+)^2 \\
&\quad - \frac{1}{2} \int d\sigma^- e^{w_-} \int d\sigma^+ e^{w_+} \sum_{i=1}^N (\partial_- f_i^-)^2, \\
\rho - \phi &= \frac{w_+ + w_-}{2}, \tag{7}
\end{aligned}
\end{equation}

where $w_+(\sigma^+)$ and $w_-(\sigma^-)$ are arbitrary gauge functions, and $M$ is an integration constant.

Consider solutions with $f_i = 0$. Fixing the residual conformal subgroup of diffeomorphism in this gauge by setting $w_+ = w_- = 0$, one has the following solution:
\begin{equation}
e^{-2\phi} = \frac{M}{\lambda} - \lambda^2 \sigma^+ \sigma^- = e^{-2\rho}, \tag{8}
\end{equation}
up to constant shifts in $\sigma$. In the case when $M$ is non-vanishing, this corresponds to a two-dimensional black hole with an ADM mass $M \ [4,5]$, in which the event horizon is given by $\sigma^+ \sigma^- = 0$, and the singularity lies behind it on $M/\lambda - \lambda^2 \sigma^+ \sigma^- = 0$.

* The superscript $c$ indicates that it is a classical stress tensor.
The future time infinity $i^+$, space infinity $i^0$ and past time infinity $i^−$ correspond to $(∞, 0)$, $(∞, −∞)$ and $(0, −∞)$ respectively, while the past and future null infinity, $\mathcal{I}^−$ and $\mathcal{I}^+$, correspond to $σ^− → −∞$ and $σ^+ → ∞$ respectively.

When $M = 0$, the metric goes over to a flat metric under the coordinate transformation $σ^± = ±e^±σ^\hat{σ}$, while the dilaton field becomes linear in $\hat{σ}$. This is nothing but the linear dilaton vacuum.

These black hole solutions with non-vanishing $M$ can be generated from the linear dilaton vacuum as a result of matter perturbation [5]. Take a shock wave travelling in the $σ^−$ direction described by the stress tensor

$$ \hat{T}^{f}_{++} = \frac{1}{2} \sum_{i=1}^{N} \partial_+ f \partial_+ f = a\delta(σ^+ − σ^+_0). \quad (9) $$

In the gauge $w_+ = w_− = 0$, one finds that

$$ e^{-2φ} = −a(σ^+ − σ^+_0)\Theta(σ^+ − σ^+_0) − λ^2σ^+σ^− = e^{-2ρ}, \quad (10) $$

where $\Theta(σ^+ − σ^+_0)$ is the step function. This solution thus joins the linear dilaton vacuum together with a black hole of mass $aσ^+_0λ$ along the line $σ^+ = σ^+_0$ of the $f$ wave.

This classical analysis of black-hole solutions in the CGHS model can be performed equally well in the light–cone gauge. Here we will carry it out to set up the stage for a semi-classical analysis as well as a full quantum treatment of the theory in the light-cone gauge, when the metric field is quantized.

Consider the following gauge choice for the metric $^*$:

$$ ds^2 = dx^+ dx^- + h_{++}(dx^+)^2. \quad (11) $$

$^*$ We use $x$ as coordinate so as to distinguish it from the $σ$ in the conformal gauge.
The action (1) reduces to the following in this gauge choice:

\[
S = \frac{1}{\pi} \int d^2x \left[ e^{-2\phi} (\partial_-^2 h_{++} + 4 \partial_+ \phi \partial_- \phi - 4h_{++} (\partial_- \phi)^2 + 2\lambda^2) \right] \\
- \frac{1}{\pi} \int d^2x \left[ \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i + \frac{1}{2} h_{++} \sum_{i=1}^{N} (\partial_- f_i)^2 \right].
\]  

(12)

The dilaton and matter equations of motion are thus given by

\[
e^{-2\phi} (\partial_-^2 h_{++} - 4 \partial_+ \phi \partial_- \phi + 4h_{++} (\partial_- \phi)^2 + 4\partial_+ \partial_- \phi - \lambda^2 - 4h_{++} \partial_-^2 \phi - 4\partial_- h_{++} \partial_- \phi) = 0 \\
\partial_- (\partial_+ f_i - h_{++} \partial_- f_i) = 0; \quad i = 1, 2, \cdots N,
\]

(13)

while the \( h_{++} \) equation of motion is given by

\[
T_{--}^c = -2e^{-2\phi} \partial_-^2 \phi + \frac{1}{2} \sum_{i=1}^{N} (\partial_- f_i)^2 = 0,
\]

(14)

supplemented by two constraints (eqs. of motion for \( g_{+-} \) and \( g_{-+} \)) given by

\[
T_{++}^c = e^{-2\phi} (2 \partial_+ \partial_- \phi - 2 \partial_- h_{++} \partial_+ \phi + 2 \partial_- \phi \partial_+ h_{++} - 4h_{++} \partial_- \partial_- \phi \\
+ \frac{1}{2} h_{++} \sum_{i=1}^{N} (\partial_- f_i)^2 = 0,
\]

\[
T_{++}^c = e^{-2\phi} (-2\partial_+^2 \phi - 2 \partial_- h_{++} \partial_+ \phi + 2 \partial_- \phi \partial_+ h_{++} - 4h_{++} \partial_- \partial_- \phi \\
+ 8h_{++} \partial_+ \partial_- \phi - 8h_{++}^2 \partial_-^2 \phi - 8h_{++} \partial_+ \phi \partial_- \phi + 8h_{++}^2 (\partial_- \phi)^2 - 2h_{++} \lambda^2 \\
- h_{++} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i + h_{++}^2 \sum_{i=1}^{N} (\partial_- f_i)^2 + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_+ f_i = 0.
\]

(15)
By certain linear combinations of (15), a set of simplified equations reads as follows:

\[ 4e^{-2\phi}\partial^2\phi = \sum_{i=1}^{N} (\partial_- f_i)^2, \]

\[ \partial_-(\partial_- h_{++} + 2\partial_+ \phi - 2h_{++}\partial_- \phi) = 0, \]

\[ \partial^2 h_{++} + 4\partial_+ \phi \partial_- \phi - 4h_{++}(\partial_- \phi)^2 - \lambda^2 = 0, \]

\[ \partial_+ \partial_- f_i = \partial_-(h_{++}\partial_- f_i); \quad i = 1, 2, \cdots N, \]

\[ \partial_+(e^{-2\phi}\partial_-^2 h_{++}) = h_{++}\partial_-(\phi) \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i + \frac{1}{2} \partial_- \phi \sum_{i=1}^{N} (\partial_+ f_i)^2 - h_{++}\partial_+ \phi \sum_{i=1}^{N} (\partial_- f_i)^2. \]

(16)

The general solutions of (13)–(15) can presumably be given in closed form. They are related to the general solutions in conformal gauge (7) via an appropriate coordinate transformation. The gauge functions in (7) will have their analogs in the general solutions in light–cone gauge. The presence of these gauge functions reflects the fact that there is a residual symmetry that leaves invariant the light–cone gauge choice (11) [18], given by

\[ \tilde{x}^+ = \alpha(x^+), \]

\[ \tilde{x}^- = \frac{x^-}{\alpha'} + \beta(x^+), \]

where \( \alpha' = d\alpha/dx \), while \( h_{++} \) transforms as

\[ h_{++} = (\alpha')^2 \tilde{h}_{++} - \frac{x^- \alpha''}{\alpha'} + \alpha' \beta'. \]

(18)

The precise form of the general solutions is not of importance to us for the moment. We will focus only on solutions of the type shown in (8) in the absence of matter.
In particular, solutions of our interest take the following form:

$$\phi = -\frac{\lambda}{2}(x^+ + x^-),$$

$$h_{++} = \frac{M}{\lambda}e^{2\phi}. \quad (19)$$

When $M$ is non-vanishing, this is exactly the black-hole solution of mass $M$ in conformal gauge given in (8), which has been transformed into the light–cone gauge via a coordinate transformation given by

$$x^+ = \frac{1}{\lambda} \log(\lambda \sigma^+),$$

$$x^- = \frac{1}{\lambda} \log\left(\frac{M}{\lambda} - \lambda^2 \sigma^+ \sigma^- \right) - \frac{1}{\lambda} \log(\lambda \sigma^+). \quad (20)$$

Again the $M = 0$ case corresponds to the linear dilaton vacuum.

Comparing the solutions (19) with (8), one finds that in light–cone coordinates, the horizon lies on $\lambda(x^+ + x^-) = \log(M/\lambda)$, and the singularity on $x^+ + x^- = -\infty$, behind the horizon, where the curvature $4\partial^2 h_{++}$ indeed blows up. The future time infinity $i^+$, space infinity $i^0$ and past time infinity $i^-$ are at $(\infty, -\infty), (\infty, \infty)$ and $(-\infty, \infty)$ respectively, while the past and future null infinity, $\mathcal{I}^-$ and $\mathcal{I}^+$, correspond to $x^- \to \infty$ and $x^+ \to \infty$ respectively.

3. Semi-Classical Analysis

It was proposed [5] that quantum effects such as Hawking radiation can be computed in a fixed-background geometry (8) by including the trace anomaly term that arises from the quantization of the matter fields $f$. At the level of action it amounts to adding the well-known Polyakov–Liouville term, which is often referred to as the induced action for two-dimensional gravity. It is non-local in terms of metric fields given by

$$S_{\text{ind}} = -\frac{c}{96\pi} \int \sqrt{-g}R \frac{1}{\Box} R, \quad (21)$$

where $c$ represents the central charge of the quantized matter system and the ghost system. As discussed in [5], the inclusion of such a term at the semi-classical
level incorporates both Hawking radiation and its back-reaction on the background geometry.

To discuss Hawking radiation, consider the solution of (10), which is generated by a shock-wave perturbation. It suffices to calculate the expectation value of the energy-momentum tensor for the quantized matter fields, denoted by $T^{F,*}$, in an asymptotic flat coordinate $\sigma^{\pm}$ via, for instance, $e^{\lambda \hat{\sigma}^{\pm}} = \lambda \sigma^{\pm}, e^{-\lambda \hat{\sigma}^{-}} = -\lambda \sigma^{-} - a/\lambda$. In conformal gauge, where the induced action (21) goes over to a free action $\frac{c}{12\pi} \int \partial_{\pm} \rho \partial_{\pm} \rho$, the well-known trace anomaly relation gives:

$$T^{F}_{+-} = -\frac{c}{12} \partial_{+} \partial_{-} \rho.$$  \hspace{1cm} (22)

$T^{F}_{++}$ and $T^{F}_{--}$ can be obtained by the conservation law given by

$$\partial_{\pm} T^{F}_{\mp \pm} + \partial_{\mp} T^{F}_{\pm \mp} - 2 \partial_{\mp} \rho T^{F}_{++} = 0.$$  \hspace{1cm} (23)

They take the form given by

$$T^{F}_{\pm \pm} = -\frac{c}{12} \left( \partial_{\pm} \rho \partial_{\pm} \rho - \partial_{\pm}^{2} \rho + t_{\pm}(\hat{\sigma}^{\pm}) \right).$$  \hspace{1cm} (24)

where $t_{\pm}(\hat{\sigma}^{\pm})$ can be fixed by the boundary conditions.

Imposing that $T^{F}_{++}$ vanishes in the linear dilaton region, and there is no incoming radiation, namely $T^{F}_{++} \to 0, T^{F}_{+-} \to 0$ as $\hat{\sigma}^{-} \to -\infty$, one may fix the form of $t_{\pm}(\hat{\sigma}^{\pm})$ to be given by

$$t_{+}(\hat{\sigma}^{+}) \to 0, \quad t_{-}(\hat{\sigma}^{-}) \to -\frac{1}{4} c \lambda^{2} \left[ 1 - (1 + a e^{\lambda \hat{\sigma}^{-}} / \lambda)^{2} \right].$$  \hspace{1cm} (25)

Thus one reads off the values of the stress tensor for the outgoing radiation as

* Here $F$ indicates some quantum nature of matter fields $f$, its physical meaning becomes clear presently.
\( \bar{\sigma}^+ \to \infty:\)

\[
T_{++}^{F} \to 0, \quad T_{+-}^{F} \to 0
\]

\[
T_{--}^{F} \to \frac{c \lambda^2}{48} \left[ 1 - \frac{1}{(1 + ae^{\lambda \bar{\sigma}^{-}}/\lambda)^2} \right].
\]  \( (26) \)

At future time infinity (\( \bar{\sigma}^- \to \infty \)), \( T_{--}^{F} \) approaches the constant value \( c\lambda^2/48 \) \( [5] \). This gives the rate of Hawking radiation.

This calculation can be performed equally well in light–cone gauge. The trace anomaly relation and conservation law read:

\[
\nabla_+ T_{--}^{F} = -\frac{c}{24} \partial^3_- h_{++},
\]

\[
T_{+-}^{F} = h_{++} T_{--}^{F} + \frac{c}{24} \partial_-^2 h_{++},
\]

\[
T_{++}^{F} = h_{++} T_{+-}^{F} + \frac{c}{12} h_{++} \partial_+^2 h_{++} - \frac{c}{48} (\partial_- h_{++})^2 - \frac{c}{24} \partial_+ \partial_- h_{++} + B(x^+),
\]  \( (27) \)

where \( \nabla_+ \equiv \partial_+ - h_{++} \partial_- - 2 \partial_- h_{++} \), and \( B(x^+) \) is an arbitrary function.

For simplicity, let us consider the fixed-background geometry of (8). One may calculate \( T^F \) by solving (27). With the parametrization of \( h_{++} \)

\[
h_{++} = \frac{\partial_+ F}{\partial_- F}.
\]  \( (28) \)

\( F \) is solved easily to be given by

\[
F(x^+, x^-) = F(\zeta); \quad \zeta = -\frac{M}{\lambda} e^{-\lambda x^+} + e^{\lambda x^-},
\]  \( (29) \)

where \( F \) is now an arbitrary function of \( \zeta \). The general solution of \( T_{--}^{F} \) to (27) can be written as a Schwartzian derivative of \( F \) with respect to \( x^- \):

\[
T_{--}^{F} = -\frac{c}{24} \{ F, x^- \},
\]  \( (30) \)
where the Schwartzian is given by
\[ \{F, x^-\} \equiv \frac{\partial^3 F}{\partial_+^3} - \frac{3}{2} \left( \frac{\partial^2 F}{\partial_- F} \right)^2. \] (31)

Using (29), we obtain the following $T^-_F$:
\[ T^-_F = \frac{c\lambda^2}{48} \left( 1 - 2e^{2\lambda x^-} \{ F, \zeta \} \right); \] (32)

$T^+_F$ and $T^-_F$ can also be solved in this background. For completeness, they are given by
\[
T^+_F = \frac{M\lambda c}{16} e^{-\lambda x^+ - \lambda x^-} - \frac{M\lambda c}{24} e^{-\lambda x^+ + \lambda x^-} \{ F, \zeta \},
\]
\[
T^-_F = \frac{M^2 c}{8} e^{-2\lambda x^+ - 2\lambda x^-} - \frac{M\lambda c}{24} e^{-\lambda x^+ - \lambda x^-} - \frac{M^2 c}{24} e^{-2\lambda x^+} \{ F, \zeta \} + B(x^+). \] (33)

The requirement that there is no incoming radiation can be satisfied by choosing appropriate functions $F$ and $B(x^+)$ so that $T^-_F$, $T^-_F$ and $T^+_F$ vanish on the past null infinity $I^-$. On the future null infinity $I^+$, the metric becomes flat, $T^+_F = 0$, $T^+_F = 0$, and $T^-_F$ is given by (32) with $\zeta = e^{\lambda x^-}$. The rate of Hawking radiation is thus obtained by the value of $T^-_F$ at future time infinity, i.e. $x^+ \to \infty, x^- \to -\infty$, in which case $\zeta = 0$. We thus have the desired value $T^-_F = \frac{c\lambda^2}{48}$, provided that $\{ F, \zeta \}$ is regular at $\zeta = 0$.

The physical meaning of $F$ is that it is a semi-classical description of the quantum matter fluctuation. This becomes clear once we compare the parametrization (28), which defines $F$ for certain $h^{++}$, with the classical equation of motion for matter field $f$ in (13). However, $F$ is not completely determined by Hawking radiation in a fixed background geometry. This indeterminacy is reminiscent of the loss of information and quantum coherence in the process of Hawking radiation [3].

Next is the issue of the back-reaction of Hawking radiation on the metric. Semi-classically this can be discussed by solving a new set of equations of motion.
that arise from the induced Polyakov–Liouville action to the classical stress tensor. In the original CGHS model, the new set of equations are not exactly solvable, and special kinds of solutions to these equations, as well as numerical solutions, indicate [6-9,14] that a singularity occurs. Models with additional conserved currents [15–17] do have solvable semi-classical equations of motion, but they either suffer from a naked singularity or inevitably evolve into a thunderbolt [14].

To address these issues in the light–cone gauge, one may start by writing down the set of semi-classical equations. Here we will limit our considerations to the case without classical matter \( f \). The dilaton equation of motion in (13) is not modified by the induced action. The matter equation in (13) becomes null in the absence of matter. However, quantum matter fluctuation \( F \) takes a form (28) similar to the classical equation of motion. The equation of motion for \( h_{++} \) (14) and two constraints (15) turn into

\[
T^c + T^F = 0, \tag{34}
\]

where the \( T^c \)'s are those given (14), (15) with \( f \) turned off; \( T^F_{--} \) is given in (30), while the \( T^F_{++} \) and \( T^F_{+-} \) are related to the \( T^F_{--} \) by (27), thus expressed in terms of \( F \) as follows:

\[
T^F_{++} = \frac{7c}{24} \frac{(\partial_+ F)^2 (\partial_- F)^2}{(\partial_- F)^4} - \frac{c}{6} \frac{(\partial_+ F)^2 (\partial_+ F) (\partial_- F)^3}{(\partial_- F)^3} - \frac{c}{8} \frac{\partial_+ \partial_- F (\partial_- F)^3}{(\partial_- F)^3} + \frac{c}{6} \frac{\partial_+ \partial_+ F}{(\partial_- F)^2} + \frac{c}{24} \frac{\partial_+ \partial_+ F (\partial_- F)^2}{(\partial_- F)^2} + \frac{7c}{24} \frac{\partial_+ \partial_- F^2 (\partial_- F)^2}{(\partial_- F)^2} - \frac{c}{48} \frac{\partial_+ \partial_+ F (\partial_- F)^2}{(\partial_- F)^2} - \frac{c}{24} \frac{\partial_+ \partial_- F (\partial_- F)^2}{(\partial_- F)}.
\tag{35}
\]

Exact solutions to (34) would presumably be rather difficult to find, and will be left for future work.

4. Full Quantum Treatment
To quantize the metric field $h_{++}$ in the light–cone gauge, we follow [19]. The quantum analogues of the constraint equations (15) are rather tricky to implement. They can be done as follows [19]. Changing from the gauge choice (11) to a new gauge choice given by

$$h_{--} = h_{--}(x), \quad h_{+-} = h_{+-}(x),$$

where $h_{--}$ and $h_{+-}$ are certain fixed but unspecified functions.† Gauge invariance implies that the partition function $Z$, as well as any gauge invariant quantity, must be independent of the choice of gauge functions $h_{--}$ and $h_{+-}$, and in particular it implies that

$$\frac{\delta Z}{\delta h_{--}}\bigg|_{h_{--}=0} = 0, \quad \frac{\delta Z}{\delta h_{+-}}\bigg|_{h_{+-}=0} = 0.$$  

These conditions are just the quantum constraints that

$$T_{tot}^{++} = 0, \quad T_{tot}^{+-} = 0.$$  

The vanishing of $T_{tot}^{++}$ requires that the total central charge be zero.

In order to calculate the gravitational contribution $T_{++}[h_{++}]$ to the total stress tensor, the authors of [19] made use of an observation by Polyakov [18] that there exists a hidden $SL(2, R)$ Kac–Moody symmetry in the induced two-dimensional gravity. The gravitational stress tensor $T_{++}[h_{++}]$ is constructed to be a constrained Sugawara form in terms of the currents of the underlying $SL(2, R)$ Kac–Moody algebra. Thus the gravitational contribution the total central charge is given by

$$c(h_{++}) = -6(k + 2) - \frac{6}{k + 2} + 15,$$  

where $k$ is the level of the underlying $SL(2, R)$ Kac–Moody algebra. The geometrical meaning of this $SL(2, R)$ symmetry is precisely the residual symmetry of the

† Note that they are not quantum fields like $h_{++}$.  

Note: The asterisked notes are not part of the main text and are provided for reference only.
light–cone gauge, given in (17) and (18) [18]. Since in the CGHS model in the
light–cone gauge this residual symmetry remains, we expect that there exists at
least an $SL(2, R)$ symmetry. Thus we assume that (39) also holds in the present
case.

It is necessary to introduce two pairs of Fadeev–Popov ghosts $(\epsilon, \eta)$ and $(c, b)$,
corresponding to the gauge-fixing conditions in (36). The ghost contribution to the
total central charge is a subtle issue, because of the fact that there is an ambiguity
in defining the metric to which the ghosts may couple. This was addressed in [11],
and it was argued that it is the (flat) metric $g^{\text{flat}} = e^{-2\phi} g$ to which the ghosts are
coupled, so as to avoid Hawking radiation of ghosts. Here we will proceed with
this assumption†. The gauge-fixing term is thus given by

$$\mathcal{L}_{\text{gh}} = b \nabla_+ c + \eta (\nabla_+ c - \nabla_- \epsilon),$$  \hspace{1cm} (40)

where $\nabla_\pm$ are covariant derivatives with respect to the flat metric $g^{\text{flat}}$. Integrating
out these ghost fields, one obtains the ghost determinant that is essentially the
induced action (21) for the flat metric $g^{\text{flat}}$, with an appropriate overall constant
indicating that the ghost central charge is $-26 - 2 = -28$. Expressed in terms of
$h_{++}$ and $\phi$, it is thus given by

$$\frac{7}{12} \Gamma[h_{++}] + \frac{7}{6\pi} \int (\phi \partial^2 h_{++} + \phi \partial_-(\partial_+ - h_{++} \partial_-) \phi),$$  \hspace{1cm} (41)

where $\Gamma[h_{++}]$ can be viewed as a gravitational analogue of WZW action, given in
a non-local form in $h_{++}$ by

$$\Gamma[h_{++}] = \frac{1}{2\pi} \int \partial_-^2 h_{++} \frac{1}{\partial_-(\partial_+ - h_{++} \partial_-)} \partial_+^2 h_{++}.$$  \hspace{1cm} (42)

Thus the second term of (41) modifies the kinetic term of the dilaton field $\phi$ and
as well as introduces a background charge for it.

† Semi-classical quantization of dilaton gravity considered in [20] allows more general ansätze.
We will denote the central charge that comes from the dilaton by $c_\phi$. The matter central charge $c_{\text{matter}}$ is $N$ in the present case with $N$ scalars. The vanishing of the total central charge, $0 = c_{\text{tot}} = c_\phi + N - 28 + c(h_{++})$, thus yields a relation, given by $\dagger$

$$k + 2 = \frac{c_\phi - N - 13 + \sqrt{(c_\phi + N - 1)(c_\phi + N - 25)}}{12}.$$

In the case of gravity without dilaton, the coefficient $k$ is nothing but the overall renormalization constant for the quantum effective action $\Gamma_{\text{eff}}[h_{++}]$ $[19,23]$. It is conjectured in [23] that the full quantum effective action to all loops for the induced gravity (21) is given by

$$\Gamma_{\text{eff}}[h_{++}] = \frac{k}{2} \Gamma\left[\frac{k + 2}{k} h_{++}\right],$$

where $\Gamma[h_{++}]$ is given in (42). The renormalization constant $(k + 2)/k$ of $h_{++}$ was checked by performing a 1-loop calculation using either a determinant argument [23] or Feynman diagram perturbative expansion [24]$\S$. We expect that these results carry over to the present case of gravity with dilaton.

In [15,16] it was shown in conformal gauge that a sequence of field re-definition reduces the kinetic terms for the dilaton $\phi$ and the conformal mode $\rho$ to two free fields. One of which has a background charge that dictates the string susceptibility of the theory [20], while the other contributes one unit to the central charge. In our analysis, if we assume that $c_\phi = 1$, then $k = (N - 24)/6$ in the large $N$ limit.

A few remarks are in order. Owing to the correspondence between the KPZ quantization in light–cone gauge [19] and that of conformal field theory adopted in [21], it is natural to think that there might exist a conformal field theory for the full quantum dilaton gravity in conformal gauge. Indeed this has been pursued

---

$\dagger$ The choice made here between the two roots of $k$ corresponds to its correct classical limit as $N \to \infty$.

$\S$ Many of these results of two-dimensional Polyakov gravity have their analogues of higher-spin gravity, namely $W$ gravity [22].
in [15,16], and further elucidated in [20]. It may well be that it is a necessary requirement to have a conformal field description.

We have been rather cavalier in postulating the quantum implications (39) and (44) of the $SL(2, R)$ residual symmetry (17) and (18). A rigorous proof would require more careful analysis. In [18], the light-cone metric field $h_{++}$ there was expanded into $h_{++} = J^+ - 2J^0x^- + J^-(x^-)^2$, in which $J^0, J^\pm$ generate the underlying $SL(2, R)$ symmetry. In the present case, one might hope that the metric field $h_{++}$ for the black-hole background could be expanded similarly. This does not seem to be the case. A possible choice is to expand the flat metric field $g^{\text{flat}}$ that couples to the ghosts, in which case the underlying $SL(2, R)$ generating current describe quantum fluctuation around this flat background metric, instead of the black-hole background.

A quantum treatment of the dilaton field is yet lacking so far. With the assumption that the ghosts couple to the flat metric, quantization scheme of dilaton will need to incorporate modification terms in (41). It is plausible that dilaton quantization may also be dictated by some symmetry, as that of the metric field by $SL(2, R)$. It may well be that there exist some analogue of Polyakov’s analysis in the presence of black-hole geometry with the dilaton field. And one may discover bigger hidden symmetries, for example, some $W$ symmetry advocated in [25]. The exact physical meaning and implication of these considerations are beyond the scope of the present paper.

5. Conclusions

In this paper we have investigated the CGHS model in the light–cone gauge. At the classical level, solutions of black holes are found, corresponding to those found in conformal gauge via coordinate transformation. We have calculated the rate of Hawking radiation, which agrees with the result in conformal gauge. Other related issues, such as back-reaction, singularity and information loss would be analysed in this gauge by looking for solutions to (34) and studying their behaviour.
Quantization of the metric $h_{++}$ has been discussed along the lines of [19]. A proper quantization of the dilaton field still needs to be incorporated into the picture, before some of the fundamental issues of quantum gravity can be better understood.

Other models [15–17,26] that are closely related to the CGHS one can be studied similarly in the light–cone gauge. The models of [15,16] are presumably equivalent to the fully quantized CGHS model, such as that quantized here in light–cone gauge. This wisdom comes from the equivalence between the KPZ [19] and DDK [21] formalism. It would be very interesting to make this more transparent in the case of two-dimensional dilaton gravity. The model of [17], with additional conserved currents, may also be quantized in the light–cone gauge. It would be very interesting to see the implications of the conserved currents, if any, in this language.

Acknowledgements: I am very grateful to F. Bastianelli, E. Bergshoeff, A. Bilal, P. Candelas, M. J. Duff, L. J. Romans, R. Williams and G. Zemba for discussions and valuable help.

REFERENCES

1. M. Green, J. H. Schwarz and E. Witten, Superstring theory (Cambridge University Press, Cambridge, 1987).

2. S. W. Hawking, Commun. Math. Phys. 43 (1975) 199.

3. S. W. Hawking, Phys. Rev. D14 (1976) 2460; for a recent review, see R. Wald, lectures given at 1991 Erice.

4. E. Witten, Phys. Rev. D44 (1991) 314.

5. C. G. Callan Jr., S. B. Giddings, J. A. Harvey and A. Strominger, Phys. Rev. D45 (1992) R1005.

6. J. G. Russo, L. Susskind and L. Thorlacius, Stanford University preprint SU-ITP-92-4.
7. L. Susskind and L. Thorlacius, Stanford University preprint SU-ITP-92–12.

8. T. Banks, A. Dabholkar, M. R. Douglas and M. O’Loughlin, Rutgers preprint RU-91–54 (January, 1992).

9. S. W. Hawking, Caltech preprint CALT-68-1774, hepth@xxx/923052.

10. B. Birnir, S. B. Giddings, J. A. Harvey and A. Strominger, UCSB/Chicago preprint UCSBTH-92-08=EFI-92-16, hepth@xxx/9203042.

11. A. Strominger, Santa Barbara ITP preprint UCSBTH-92–18, hep@xxx/9205028.

12. S. B. Giddings and A. Strominger, Santa Barbara preprint UCSBTH-92-28, hepth@xxx/9207034.

13. S. P. de Alwis, CERN-TH.COLO-HEP-288, hepth@xxx/9207095.

14. S. W. Hawking and J. M. Stewart, DAMTP preprint, hepth@xxx/9207105, July 1992.

15. C. Callan and A. Bilal, Princeton University preprint PUPT-1320, May 1992, hepth@xxx/9205089.

16. S. P. de Alwis, University of Colorado preprint COLO-HEP-280, hepth@xxx/9205069, May 1992.

17. J.G. Russo, L. Susskind and L. Thorlacius, Stanford University preprint SU-ITP-92–17, June 1992.

18. A. M. Polyakov, Mod. Phys. Lett. A2 (1987) 893.

19. V. G. Knizhnik, A. M. Polyakov and A. B. Zamolodchikov, Mod. Phys. Lett. A3 (1988) 819.

20. Y. Tanii, preprint STUPP-92-130, August 1992

21. F. David, Mod. Phys. Lett. A3 (1988) 1651;
   J. Distler and H. Kawai, Nucl. Phys. B321 (1989) 509.
22. M. Bershadsky and H. Ooguri, Commun. Math. Phys. **126** (1989) 49;
   K. Schoutens, A. Sevrin and P. van Nieuwenhuizen, contribution to Trieste Summer School on High Energy Physics and Cosmology, Trieste, Italy, Jun 17-Aug 9, 1991;
   K. Schoutens, A. Sevrin and P. van Nieuwenhuizen, Nucl. Phys. **B364** (1991);
   M. Grisaru and P. van Nieuwenhuizen, CERN-TH.6388/92;
   X. Shen, preprint CERN-TH.6404/92.

23. A. M. Polyakov, in Proceedings of the Les Houches 1988 meeting on *Fields, Strings and Critical Phenomena* (North-Holland, Amsterdam, 1989).

24. K. A. Meisner and J. Pavelchik, Mod. Phys. Lett. **A5** (1990) 763.

25. J. Ellis, D. V. Nonapolous and N. E. Mavromatos, Phys. Lett. **B267** (1991) 465; **B272** (1991) 261; CERN-TH.6595/92, and the references therein;
   F. Yu and Y. S. Wu, Phys. Rev. Lett. **68** (1992) 2996;
   S. Chaudhuri and J. D. Lykken, preprint FERMI-PUB-92/169-T.

26. S. Nojiri and I. Oda, preprint NDA-FP-4/92, OCHA-PP-26, June 1992.