Are optical quantum information processing experiments possible without beamsplitter?

Kishore Thapliyal†,∗ and Anirban Pathak§,†

‡RCPTM, Joint Laboratory of Optics of Palacky University and Institute of Physics of Academy of Science of the Czech Republic, Faculty of Science, Palacky University, 17. listopadu 12, 771 46 Olomouc, Czech Republic
§Jaypee Institute of Information Technology, A-10, Sector-62, Noida, UP-201309, India

Abstract

The significance of beamsplitter in experimental optical quantum information processing and quantum technology is discussed with a focus on the role of a beamsplitter-type Hamiltonian in the recent development in this field of research. Here, we follow a new approach to briefly describe quantum measurement, Bell measurement, quantum state engineering, quantum teleportation, cryptography, and computation using both discrete and continuous variables to establish the wide applications of beamsplitter-type operation. Finally, we also discuss the limitations of this linear optical element.

Keywords: Beamsplitter operation, quantum computation, quantum communication, quantum state engineering, applications of beamsplitter operation

1 Introduction

As reflected from the title page of this issue of the journal, all the articles of this issue are dedicated to Prof. Ajoy Ghatak who has just become an octogenarian. Many of us (including the authors of this work) have learned the basic ideas of optics and quantum mechanics from the excellent books [1–4] authored by him. A characteristic of his books that mesmerized us over the years was their simplicity. Motivated by that and the fact that most of his research works [5,6] and books [1–4] involve traditional optics, fiber optics, and quantum mechanics, we planned to write this article on the modern applications of a very simple component that connects all the three domains of his interest. Specifically, we want to focus this paper on beamsplitter (BS) and its modern applications. BS is a well-known and simple linear optical component that every interferometer contains, be it a simple Michelson interferometer (MI) described in Chapter 15 of Prof. Ghatak’s famous book entitled Optics [3], or a more sophisticated version of MI used in the famous LIGO experiment to detect gravitational wave [7,8]; be it a single photon-based Mach-Zehnder interferometer (MZI) that can be used to establish the existence of quantum superposition and collapse on measurement postulate of quantum mechanics [9] or a nested version of MZI used in the recent proposals for counterfactual quantum communication [10]. In fact, any piece of glass can be viewed as a BS. Of course, it will not be a 50:50 (symmetric) BS, but will indeed be a BS. For example, a piece of pure glass which reflects only 4% of the incident light can be viewed as 4:96=1:24 (asymmetric) BS. Further, in what follows, we will see that optical couplers (which are primary component of the majority of the optical fiber based experiments and any integrated-optic device) are equivalent to BS. The relevance of BSs in designing interferometers, like MI or MZI was known since long, but recently it is realized that the sensitivity of an MI can be enhanced considerably if squeezed vacuum is inserted from the free mode of the BS in an MI, and that enhanced sensitivity can be used to detect gravitational wave [7,8]. A true random number generator can be built by using a single photon source (or an approximate version of that which uses weak coherent pulse) and a BS [11]. Beyond these applications, BSs have been observed to be used in most of the fascinating experiments of quantum optics (e.g., Hanbury Brown-Twiss (HBT) experiment [12,13], homodyne detection [14,15], characterization of squeezed [14] and antibunched [16] states, Bell’s inequality [17,18], higher-order nonclassicality [19], quantum information (quantum teleportation [20], densecoding [21], photon subtraction in decoy state quantum key distribution [22], measurement device independent quantum cryptography [23], continuous variable quantum cryptography [24,25], cryptanalysis in quantum cryptography [26,27]), quantum state

∗Email: kishore.thapliyal@upol.cz
†Email: anirban.pathak@jiit.ac.in
engineering (photon subtraction \cite{28-30}, quantum scissors \cite{31}, and entanglement generation \cite{32}), which will be further discussed in the following sections. This observation led to the question “Is it possible to design an optical quantum information processing experiment without using a BS?”. We aim to address this question in the remaining part of this paper.

The rest of the paper is organized as follows. In Section 2 we introduce the mathematical details of BS operation and its role in quantum optics and information. Significance of BS in discrete and continuous variable quantum communication is discussed in Sections 3 and 5, respectively. Subsequently, the role of BS in discrete and continuous variable quantum computation is summarized in Section 6. Finally, applications of BS operation in other areas of research in the field of quantum foundations, quantum information processing, and quantum technology are discussed in Section 7 before concluding the paper in Section 7.

2 Mathematical modeling of beamsplitter and relevance in quantum optics and information

A BS is a semitransparent thin film which transmits (reflects) a part of the incident beam of light of amplitude $E$ with transmission (reflection) amplitude $t$ ($r$), i.e., $tE$ ($rE$). In case of quantized fields, field amplitudes can be denoted by corresponding field operator $a$. Two output modes of the BS in terms of the input modes (as shown in Fig. 1(a)), reflection and transmission coefficients can be defined as \cite{33}

$$
\begin{pmatrix}
   a' \\
   b'
\end{pmatrix} = \begin{pmatrix}
   t & r \\
   r & t
\end{pmatrix} \begin{pmatrix}
   a \\
   b
\end{pmatrix} = U_{BS} \begin{pmatrix}
   a \\
   b
\end{pmatrix},
$$

(1)

where without loss of generality we have assumed transmission and reflection amplitudes are the same for both the inputs of the BS. Here, $a$ ($a^\dagger$) and $b$ ($b^\dagger$) correspond to the annihilation (creation) operators of two modes of the BS. The requirement for the validity of commutation relation $[A_i, A_j^\dagger] = \delta_{ij}$ is same as the conservation of energy on a lossless BS, $|t|^2 + |r|^2 = 1$ and $t^* r + r^* t = 0$. Using these conditions, we can parameterize transmission and reflection coefficients as $t = \cos \theta$ and $r = \sin \theta \exp (i\Phi)$. In the present work, we assume $\Phi = \frac{\pi}{2}$. Thus, $a' = a (\theta)$ and $b' = b (\theta)$ in Eq. (1) can be interpreted as the solution of differential equations that can be incidentally interpreted as Heisenberg’s equations of motion with the effective Hamiltonian

$$
H_{BS} = -\hbar \ (a^\dagger b + ab^\dagger).
$$

(2)

Thus, a unitary operator which represents a BS can be defined as

$$
U_{BS} = \exp \left\{ i\theta (a^\dagger b + ab^\dagger) \right\}.
$$

(3)

Note that the same Hamiltonian \cite{2} also describes another optical system, namely linear optical directional coupler. In fact, it describes a family of physical systems of practical relevance. For example, it describes an atom-atom two-component BEC system \cite{34, 35}. However, here we wish to restrict to optical systems and note that the linear optical coupler forms an integral part of the integrated waveguide system used in optical quantum information processing experiments.

2.1 Role of beamsplitter in quantum optics and measurements

Homodyne measurements allow us to measure one of the quadrature variables $X = \frac{1}{2} (a + a^\dagger)$ and $Y = \frac{i}{2} (a - a^\dagger)$ analogous to dimensionless position and momentum in the classical phase space. It involves mixing the single-mode to be measured with a strong classical coherent field $|\alpha| \exp (i\phi)$ (called local oscillator) at a BS as input modes $a$ and $b$, respectively (cf. Fig. 1(b)). The difference in the output currents of the BS can be defined in terms of output $a'$ and $b'$ as $\langle a^\dagger a' - b^\dagger b' \rangle = i\langle a^\dagger b - b^\dagger a \rangle$. This can be simplified for coherent field initially in mode $b$ as $2|\alpha| \langle X \cos \phi + \frac{\pi}{2} \rangle + Y \sin (\phi + \frac{\pi}{2})$. Notice that by choosing $\phi = -\frac{\pi}{2}$ and $\phi = 0$ we can measure quadrature $X$ and $Y$, respectively. On top of that, by choosing different values of parameter $\phi$ marginal distributions along rotated quadrature in the phase space can be measured. Repeated measurement of such marginals with corresponding $\phi$, known as optical tomography \cite{36}, allows one to reconstruct a distribution function in the phase space, i.e., Wigner function \cite{14}. In fact, BS plays a significant role in reconstructing the Wigner function, characterization of entanglement and steering as well \cite{15}. Further, homodyne measurement is important in the continuous variable quantum information processing.
Discrete variable quantum communication and computation (specially BB84 types schemes of quantum key distribution and other cryptographic tasks [37-39]) often desire a source which can generate single photon at will. Characterization of such sources of light are based on HBT experiment, which measures second-order intensity correlation defined as
\[ g^{(2)}(\tau) = \frac{\langle n_{a'}(t) n_{b'}(t+\tau) \rangle}{\langle n_{a'}(t) \rangle \langle n_{b'}(t+\tau) \rangle}, \]
where \( n_A = A\dagger A \) is the number operator. It corresponds to the detection of a photon in the output mode \( a' \) of the beamplitter at time \( t \) followed by a photon detected in mode \( b' \) at time \( t + \tau \), which is normalized such that \( g^{(2)}(\tau) = 1 \) for a coherent state. The light is antibunched if \( g^{(2)}(0) < g^{(2)}(\tau) \). A detailed discussion of antibunching can be found in our recent works [40-42], but it would be sufficient to mention here that variation of \( g^{(2)}(\tau) \) with time delay \( \tau \in [-T, T] \) between detection in two detectors must have a correlation dip at \( \tau = 0 \) for an ideal single photon source.

### 2.2 Role in quantum state engineering

BS plays an important role in quantum state engineering. The field of generation of desired quantum state by performing different unitary and non-unitary operations is called quantum state engineering [30, 31, 43-46]. The desired states are usually not available naturally and are required in many quantum information processing tasks. For example, the output of the BS, with an input \( |\psi\rangle \) and vacuum \( |0\rangle \) states sent through two inputs ports, can be written as [33]

\[ |\psi_{\text{out}}\rangle = U_{\text{BS}} |\psi\rangle |0\rangle, \]
\[ = |\psi\rangle |0\rangle + i\theta (a |\psi\rangle) |1\rangle, \]
where we have assumed a highly transmissive BS. Notice that conditioned on a single photon detection in the second output port, a single photon is subtracted from the input state \( |\psi\rangle \). This method of photon subtraction is a probabilistic process. Some of these engineered quantum states are found useful in quantum communication schemes ([23, 47] and references therein). Additionally, a BS is an integral optical element for implementation of quantum scissors in generating finite dimensional nonclassical states [31] and entanglement generation [32].

### 2.3 Idea of an optical qubit

Assuming one of the inputs of the BS in vacuum and other as single photon \( |1\rangle \) initially, the output modes can be described as
\[ |\psi_2\rangle = U_{\text{BS}} |1, 0\rangle, \]
\[ = (\cos \theta |1, 0\rangle + i \sin \theta |0, 1\rangle). \]

In the Fock basis, \( |\psi_2\rangle \) can be described as an Entangled state (for \( \theta \neq \frac{n\pi}{2} \) with integer \( n \)). Fock (number) basis is the set of orthonormal functions which are eigen functions of Harmonic oscillator. One can define logical bit values \( |0\rangle \) as \( |0\rangle_L \) and \( |0, 1\rangle \) as \( |1\rangle_L \), and thus \( |\psi_2\rangle = (\cos \theta |0\rangle_L + i \sin \theta |1\rangle_L) \) represents an optical qubit in path degree of freedom. This is also known as a dual-rail qubit. It is noteworthy here that a qubit can be defined in other degrees of photon as well, such as polarization, orbital angular momentum.

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**Figure 1:** (Color online) (a) BS with input-output relation, (b) homodyne detection with photon detectors \( D_i \), and (c) Bell measurement (BM) using a BS and two polarizing beamsplitters (PBSs).
This also plays a significant role in introducing the idea of measurement postulate of quantum mechanics. For instance, $|\psi_2^\prime\rangle = |\psi_2\rangle_{\theta_2=\pi} = \frac{1}{\sqrt{2}} (|0\rangle_L + i |1\rangle_L)$ for a symmetric BS, and a detector on each output port of the BS destroys this superposition of paths and gives us a single photon detection at one of the detectors with probability 1/2 each. This forms the theoretical basis of commercially available quantum random number generators \cite{11} as the randomness is intrinsic in this case.

Further, it is straightforward to understand the idea of quantum computation and MZI/MI in which case a mirror, i.e., $t = 0$ and $r = i$ in Eq. (1), is applied on both $|0\rangle_L$ and $|1\rangle_L$ in $|\psi_2\rangle$ to result in $i |\psi_2\rangle$. This further evolves to $|\psi_3\rangle = i U_{BS} |\psi_2\rangle = i (\cos 2\theta |0\rangle_L + i \sin 2\theta |1\rangle_L)$, which reduces to $|\psi_3\rangle = - |1\rangle_L$ for the symmetric BS. We have already mentioned that gravitational wave detection uses an MI with squeezed vacuum inserted through the second input port of the BS \cite{7,8}. As an MI is primarily built using a BS and two mirrors, in view of the above, we can comment that gravitational wave detection setup in LIGO was essentially built using 3 BSs only.

### 2.4 Linear optical Bell state discrimination

Bell basis has four maximally entangled orthogonal two-qubit states. Bell states in the polarization degree of freedom can be defined as $|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(a_H^\dagger a_H^\dagger \pm a_V^\dagger a_V^\dagger) |0,0\rangle$ and $|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(a_H^\dagger a_V^\dagger \pm a_V^\dagger a_H^\dagger) |0,0\rangle$, where $|0,0\rangle$ is the two-mode vacuum state, and the subscripts represent horizontal ($H$) and vertical ($V$) polarization. To understand the idea of Bell measurement (BM) with linear optics and the role of BS in that, we can apply the symmetric BS operation on the input Bell state. For instance, on application of a BS $|\psi^{\pm}\rangle$ would become

$$U_{BS} |\psi^{\pm}\rangle = \frac{i}{\sqrt{2}} (a_H^{12} + b_H^{12} \pm a_V^{12} \pm b_V^{12}) |0,0\rangle ;$$

while $|\phi^{+}\rangle$ and $|\phi^{-}\rangle$ would be transformed to

$$U_{BS} |\phi^{+}\rangle = \frac{i}{\sqrt{2}} (a_H^\dagger a_V^\dagger + b_H^\dagger b_V^\dagger) |0,0\rangle$$

and

$$U_{BS} |\phi^{-}\rangle = \frac{1}{\sqrt{2}} (a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger) |0,0\rangle ,$$

respectively. Notice that $|\psi^{\pm}\rangle$ and $|\phi^{+}\rangle$ result in both photons in the same output of the BS (cf. Eqs. (6) - (7)), whereas only $|\phi^{-}\rangle$ gives one photon in each output port of the BS (in Eq. (8)). This behavior can be attributed to the fact that singlet state shows fermionic behavior at a BS, while rest of the triplet Bell states show bosonic nature \cite{48}. Therefore, a BS is able to identify one of the Bell states, i.e., $|\phi^{-}\rangle$, out of total four Bell states.

From Eqs. (6)-(7) it can be observed that both photons in one of the output ports of the BS have same (orthogonal) polarization for Bell state $|\psi^{+}\rangle$ ($|\phi^{+}\rangle$). Exploiting this fact, we can further identify $|\phi^{+}\rangle$ if we place a polarizing beamsplitter (PBS) at both the outputs of the BS (as shown in Fig. 1 (c)). Unlike a polarization independent BS introduced in Eq. (1), a PBS is a particular type of polarization dependent BS which reflects (transmits) vertically (horizontally) polarized photons. Thus, a photon is detected each at $D_{1V}$ and $D_{2H}$ (or $D_{3V}$ and $D_{4H}$) in Fig. 1 (c) for $|\phi^{-}\rangle$, whereas a photon is detected each at $D_{1V}$ and $D_{4H}$ (or $D_{3V}$ and $D_{2H}$) for $|\phi^{+}\rangle$.

It is noteworthy that a single BS is sufficient to identify one of the Bell states, while a single PBS can be used to check parity of the Bell states which is useful in quantum error correction codes \cite{49}.

### 3 Beamsplitter in discrete variable quantum communication

Using the optical resources and photon number measurements discussed in Section 2, we will briefly introduce discrete variable insecure and secure quantum communication.

#### 3.1 Quantum teleportation

We may now discuss the teleportation \cite{50} of a qubit $|\psi\rangle_L = (\alpha c_H^\dagger + \beta c_V^\dagger) |0\rangle$ with the help of shared bipartite quantum channel $|\phi^{-}\rangle$ between sender Alice and receiver Bob. The combined state of channel, after passing the qubit to be
3.2 Quantum cryptography

The idea of quantum cryptography can be understood by a quantum key distribution scheme \cite{51-54}. To begin with Alice (sender) prepares a string of an entangled state $|\psi^+\rangle$ and shares the second mode (qubit in this case) with Bob. Both Alice and Bob pass their individual modes through a BS, namely BS$_A$ and BS$_B$, respectively. Therefore, the combined state $|\psi^+\rangle$ is transformed to

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( A_H^a B_H^b + A_H^b B_H^a - X_H Y_H^1 - X_V Y_V^1 + i \left( A_H^a Y_H^1 + A_V^a Y_V^1 + X_H^1 B_H^1 + X_V^1 B_V^1 \right) \right) |0,0\rangle,$$

which can be further transformed by applying a quarter wave plate on mode $X$ and $Y$ to transform photons in the rectilinear basis to diagonal basis (i.e., $H \rightarrow \frac{H+V}{\sqrt{2}}$, and $V \rightarrow \frac{H-V}{\sqrt{2}}$) as

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( A_H^a B_H^b + A_H^b B_H^a - X_H^0 Y_H^1 - X_V^1 Y_V^1 + D \right) |0,0\rangle.$$

Here, we have written $H \equiv 0, V \equiv 1, \frac{H+V}{\sqrt{2}} \equiv 0$, and $\frac{H-V}{\sqrt{2}} \equiv 1$; and rest of the cases are discarded (shown as $D$). Notice that in all the cases which are not discarded, Alice’s and Bob’s bit values are symmetric. Therefore, this scheme enables Alice and Bob to share a symmetric key, which provides security not conditioned on some computationally complex problem like classical cryptography. Schematic diagram of this quantum key distribution scheme is shown in Fig. 2(b). Both Alice and Bob check half of the obtained string to ensure that an adversary Eve has not tried to eavesdrop, which
would have left detectable traces in the form of errors in the measurement outcomes. The security is further enhanced by error correction and privacy amplification.

The present scheme is same as quantum cryptography scheme proposed in [56]. It is possible for Alice to measure her qubit in $|\psi^+\rangle$ before sending the second qubit to Bob, the present scheme reduces to the first quantum cryptography scheme, BB84 scheme, proposed by Bennett and Brassard [37].

This idea can be further extended to measurement device independent quantum key distribution scheme [57] where both Alice and Bob prepare a string analogous to BB84 scheme and send to a third party Charlie, midway between Alice and Bob. Charlie performs BM as described in Section 3 and announces the successful cases of measurement outcomes $|\phi^+\rangle$ and $|\phi^-\rangle$. These two cases correspond to orthogonal states prepared by Alice and Bob, and thus Alice and Bob obtain a symmetric key once Bob flips all the bit values in his key.

4 Beamsplitter in continuous variable quantum communication

Homodyne/Heterodyne measurement, instead of single photon detectors in the discrete variable communication schemes (in Section 3), is central idea for continuous variable communication. There are certain advantages of this type of quantum communication as it allows one to use existing optical technology to perform metropolitan quantum communication, which exempts us from expensive single photon source and detector (47, 58 and references therein).

4.1 Quantum teleportation

The idea of continuous variable quantum teleportation [59] is analogous to that described in Section 3.1. As probability amplitudes of a quantum state are transferred in discrete variable teleportation, canonically conjugate continuous variable quadratures $x_A$ and $p_A$ of an unknown coherent state are transmitted here. Alice and Bob are expected to share bipartite continuous variable entanglement with quadrature variables $(x_A, p_A)$ and $(x_B, p_B)$, respectively. Alice passes the mode of bipartite entanglement and unknown coherent state through a symmetric BS and measures quadratures $x' = \frac{1}{\sqrt{2}}(x_A + x_B)$ and $p' = \frac{1}{\sqrt{2}}(p_A - p_B)$ in each output of the BS to perform BM. Subsequently, she announces the measurement outcomes $\hat{x}$ and $\hat{p}$ to Bob, who performs displacement operator to obtain $x_{out} = (x_B + G\hat{x})$ and $p_{out} = (p_B + G\hat{p})$ with gain factor $G = \sqrt{2}$. Notice that $x_{out} = x_{in} + (x_A + x_B)$ and $p_{out} = p_{in} - (p_A - p_B)$ therefore for a perfect teleportation of continuous quantum variables the initial bipartite entanglement Alice and Bob share should minimize the noise $(x_A + x_B)$ and $(p_A - p_B)$. This property can be satisfied by two-mode squeezed vacuum state as $\langle (x_A + x_B)^2 \rangle = \langle (p_A - p_B)^2 \rangle = \exp(-2r)$ which tends to zero in case of infinitely strong squeezing, i.e., $r \to \infty$.

Interestingly, a complete description of $n$-mode Gaussian states (a state fully characterized by its first and second moments only) can be provided by corresponding $2n$ dimensional covariance matrix $\sigma$ [60] with $\sigma_{ij} = \langle \{\Delta R_i, \Delta R_j\} \rangle$, where $(A, B)_+ = \frac{1}{2} (AB + BA)$. Here, the vector $R = (x_1, p_1, \ldots, x_n, p_n)^T$ is defined in terms of quadrature variables ensuring $[R_{jk}, R_{kl}] = i\Omega_{jk}$ with $\Omega = \sum_{k=1}^n \omega$ and $\omega = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, and $\Delta R_i = R_i - \langle R_i \rangle$. Necessary and sufficient condition for a matrix to be a covariance matrix based on uncertainty relation is $\sigma + i\Omega > 0$.

Thus, teleportation of a Gaussian state with covariance matrix $\sigma_{in}$, can be performed by using prior shared entanglement $\sigma_{AB}$ [61]. Alice performs BM by using a BS. The BS (in general, any unitary) operation is represented by a symplectic transformation $S_{BS} = \begin{pmatrix} \cos \theta I_2 & \sin \theta S_p \\ -\sin \theta S_p^T & \cos \theta I_2 \end{pmatrix}$ with symplectic matrix for phase shift $S_p = \begin{pmatrix} \cos \Phi & \sin \Phi \\ -\sin \Phi & \cos \Phi \end{pmatrix}$, which satisfies $S_{BS} \Omega S_{BS}^T = \Omega$ analogous to unitary condition. Following the teleportation scheme (discussed above) with $\theta = \frac{\pi}{4}$ and $\Phi = \frac{\pi}{4}$, Bob obtains $R_{out} = R_B + G\hat{R}$ and corresponding covariance matrix $\sigma_{out} = \sigma_{in} + 2N$, where the additional term $2N$ is noise introduced [61]. Assuming the Gaussian state to be teleported as a coherent state $\sigma_{in} = \frac{1}{2} I_2$, and two-mode squeezed vacuum state as shared channel $\sigma_{AB} = \begin{pmatrix} A & C \\ CT & B \end{pmatrix}$ with $A = B = \frac{1}{2} \cosh(2r)I_2$ and $C = \text{diag}(-\frac{1}{2} \sinh (2r), \frac{1}{2} \sinh (2r))$. In that case, the noise is minimum, i.e., $2N = \exp(-2r) I_2$ which becomes zero for $r \to \infty$. Two-mode squeezed vacuum state is an example of Einstein-Podolsky-Rosen (EPR) entanglement [59], which can be generated by sending two single-mode states equally squeezed in different (say $x_a$ and $p_b$) quadratures through two input ports of the BS. Advantage in the performance of continuous variable teleportation is proposed by using local squeezing operations on the bipartite entanglement shared by Alice and Bob [61].
4.2 Quantum cryptography

Analogous to the discrete variable quantum cryptography scheme discussed in Section 3.2, Alice can prepare and share two-mode squeezed vacuum state with covariance matrix \( \sigma_{AB} \) with Bob. Alice and Bob can measure one of the quadratures on their part of the state and thus obtain a string of bits corresponding to measurement outcomes when they measured the same quadrature using homodyne technique. They check their measurement outcomes in one-half of these cases to check and if they are not correlated it can be attributed to the eavesdropping attempts by Eve. Thus, using error correction and privacy amplification a secure key can be generated. However, note that a continuous quantum variable is used to encode a discrete quantum key in this case and thus this type of quantum key distribution schemes is categorized as hybrid continuous variable quantum key distribution schemes [62].

More recently quantum key distribution schemes are presented where Alice performs a single mode squeezing operation \( s \) on the mode sent to Bob [62, 63]. Thus, \( \sigma_{AB} \) transforms to \( \sigma'_{AB} \) with \( B = \text{diag} \left( \frac{1}{2} \cosh (2r) e^{-2s}, \frac{1}{2} \cosh (2r) e^{2s} \right) \) and \( C = \text{diag} \left( -\frac{1}{2} \sinh (2r) \exp (-s), \frac{1}{2} \sinh (2r) \exp (s) \right) \). After Alice’s measurement of quadrature \( x_A \) or \( p_A \) the reduced covariance matrix for Bob can be obtained as \( \sigma''_{B_s} = B - 2 \text{sech} (2r) C^T \Pi C \) or \( \sigma''_{B_p} = B - 2 \text{sech} (2r) C^T \Pi' C \), respectively, with \( \Pi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \) and \( \Pi' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \).

BS and homodyne detection are also required in continuous variable quantum key distribution using non-Gaussian channels [47] and direct secure quantum communication (which allow us to perform secure quantum communication without generating and distributing a quantum key) schemes [58].

5 Beamsplitter in discrete and continuous variable optical quantum computation

Quantum computation using linear optical resources can be performed using BS, phase shifter, and mirror with single photon detectors and quantum memory [64, 65]. While introducing the idea of qubit in Section 2.3 we have already shown that \( U_{BS} \left| 0 \right>_L = (\cos \theta \left| 0 \right>_L + i \sin \theta \left| 1 \right>_L) \), which reduces to \( U_{BS} \left| 0 \right>_L = \frac{1}{\sqrt{2}} (\left| 0 \right>_L + i \left| 1 \right>_L) \) for symmetric BS. Thus, a BS and two phase shifters are sufficient to perform hadamard gate [64] (shown in Fig. 3(a)). However, quantum computing requires feasibility of library of universal quantum gates, preparation of the initial quantum states, and measurement of the final state. Hong-Ou-Mandel effect [66], i.e., two indistinguishable single photons mixing at a symmetric BS \( U_{BS} a^\dagger b^\dagger \left| 0,0 \right> = \frac{1}{\sqrt{2}} (a^{12} + b^{12}) \left| 0,0 \right> \) coalesce to the same output arm of the BS, plays an important role in the implementation of two-qubit gates in quantum computing. For instance, CNOT gate on the spatial qubits uses two BSs, to employ Hong-Ou-Mandel effect, and a controlled phase gate as shown in Fig. 3(b). Therefore, a set of universal quantum gates [67], including Hadamard, phase and CNOT gates, can be performed using BS. Further, an \( n \)-port unitary can be implemented by phase shifters and only \( n (n - 1) / 2 \) BSs [64]. Similarly, BS is relevant in designing several other optical gates, like controlled phase and nonlinear sign gates as well as CNOT with polarization qubits and hyperentanglement (see [64, 68] for detail). CNOT with optical fiber is also implemented experimentally [69]. Significant contributions in the field were performed using ancilla photons by KLM approach [70], which was further improved in [71].
Similarly, BS plays an important role in continuous variable quantum computation [72,73], we refrain us from discussing it further here.

6 Other applications in the field of quantum optics and technology

Significance of BS in several aspects of quantum optical and information processing experiments is difficult to summarize in this article. Therefore, here we briefly mention some of these applications of BS in entanglement concentration protocols [74], quantum repeaters [75], quantum simulation [76], cryptanalysis in quantum cryptography [26,27], linear optical coupler–equivalent to BS operation–is found relevant in the study of non-Hermitian physics or parity-time symmetry [77], implementation of quantum cryptography [78], computation [79], and technology [79], etc. Further, BS, as an ingredient of MZI and MI, is used in the studies of quantum Zeno effect [80] and its use in counterfactual quantum communication [81–83] and computation [84–85]; Elitzur-Vaidman bomb testing or interaction free measurement [86] which is useful in Guo-Shi [87] quantum cryptography scheme; Goldenberg-Vaidman quantum key distribution [88], quantum phase estimation [89,90] used in quantum metrology [91] and quantum radar [92]; experiments relevant in foundations of quantum mechanics [93] as delayed choice measurement [94], realization of Hardy’s paradox [95], wave-particle duality [96,97], violation of Bell’s inequality [17], device independence [94,98], and weak measurements [97]; and gravitational wave detection [7,8].

As far as the Hamiltonian (2) is concerned, it describes Bose-Einstein condensates [34,35], optomechanical systems [99], and plasmonic circuits [100] as well. Additionally, evolution after taking into consideration weak nonlinearity for BS, optical coupler or other physical systems is also studied in the past [41,101–104]. The applications are further extended to slow light beam splitters as well [105].

7 Summary and concluding remarks

The dynamical evolution of the quantum state of a quantum system plays a significant role in quantum mechanics and its experiments. Thus, all the optical elements used in experimental implementation are represented by unitary operations. One of the most important Hamiltonians, often used in quantum optics and information processing, governing dynamics of the optical states (defined using different properties of photon, such as polarization, frequency, orbital angular momentum) is BS operation. Here, we discuss in detail the significance of BS operation in experimental studies ranging from foundational verification of principles of quantum mechanics to quantum optics, quantum information processing, and technology.

Specifically, BS is useful in characterization of nonclassical–antibunched, squeezed, entangled, steerable, Bell nonlocal–states, studies of higher-order nonclassicality, measurement of continuous variable quantum states, quantum state engineering for photon subtraction and entanglement generation, linear optical Bell state discrimination, discrete and continuous variable quantum teleportation and cryptography, cryptanalysis of secure quantum communication schemes, discrete and continuous variable quantum computation, quantum phase estimation, gravitational wave detection to name a few. However, there are some limitations of BS operation, for instance, generation of entanglement using classical resources, optical CNOT, deterministic Bell state discrimination cannot be performed using linear optics solely.

In brief, in the present work, we have tried to reveal the inherent symmetry present in many physical processes of relevance and interest. The inherent symmetry is investigated here by using one of the simplest possible optical components (BS). This investigation is performed from a new approach, and it is expected to complement a set of earlier studies [43,106–108] focused on properties and applications of BS. This article is also expected to be of use in teaching/training young students about the relation between optics, quantum mechanics, and quantum information. If it succeeds in that then that would be our greatest possible tribute to Prof. Ghatak who has spent most part of his life in writing books and articles for young students with a clear focus on clarifying complex ideas in a lucid manner.

It is fascinating to observe that an optical element which was known and used in some form or others in the early civilizations, is still used to produce new results and to obtain new insights into the physical world. We hope the journey will continue and BS-type simple physical systems will continue to help us in enriching our understanding of the nature. Keeping the earlier stated points and this hope in mind, we conclude this article by noting (in analogy with Keats) that simple (BS) is beauty, and beauty is truth.
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