Tau2000 Conference Summary

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This is a review of some of the highlights of the 6th workshop on the physics of the \( \tau \) lepton and its neutrino. This includes the test of lepton universality, measurement of Lorentz Structure, study of hadronic decays, direct evidence of tau neutrino, status of neutrino oscillations, and search for neutrinoless decays. This review concludes with a look at the prospect for \( \tau \) physics in the future.

1. Introduction

The \( \tau \) lepton provides an unique laboratory to test the Standard Model. The lepton can be studied as the decay product of the \( W \) and \( Z \) bosons. Comparison of the decay rates to those for \( e \) and \( \mu \) allows a test of lepton universality. Alternatively, we can examine the decay products of the \( \tau \) lepton. Its large mass allows decays into many channels, providing many venues to challenge the Standard Model. This includes the test of lepton universality and measurement of Lorentz Structure in the leptonic decays, tests of QCD, CVC and isospin symmetry in the hadronic decays, and the search for neutrinoless decays. In this paper, I will review the tests of lepton universality, measurement of Lorentz Structure, study of hadronic decays, and search for neutrinoless decays.

Study of the neutral partner, \( \nu_{\tau} \), is also of great interest. However, the study is greatly complicated by the fact the cross section for neutrinos interacting with matter is very small. Nevertheless, \( \nu_{\tau} \) has been observed for the first time. Super-Kamiokande has also observed the evidence for \( \nu_{\mu} \) oscillates into \( \nu_{\tau} \). In this paper, I will review the evidence of \( \nu_{\tau} \) observation and status of the \( \nu_{\mu} \rightarrow \nu_{\tau} \) oscillation and then discuss the prospects on \( \nu_{\tau} \) oscillation experiments in the future.

There are many more results being presented at this workshop. Space limitation precludes a comprehensive review of all the results. Please accept my apology.

2. Test of Lepton Universality

In the Standard Model, the coupling of the \( \tau \) lepton to the \( W \) and \( Z \) bosons are the same as those for the lighter leptons, \( e \) and \( \mu \). The couplings to the neutral current can be studied in the decay of the \( Z \) boson. However, the couplings to the charged current can be studied in both the decay of the \( W \) boson and the decay of the \( \tau \) lepton into the (virtual) \( W \) boson. This allows a test of the universality of the couplings at vastly different \( Q^2 \). In this section, I will review the test of lepton university in the charged and neutral current couplings.

2.1. Charged Current Couplings

Lepton universality can be tested at low \( Q^2 \) by comparing the measurement of the \( \tau \) lifetime (\( \tau_{\tau} \)) and leptonic branching fractions using the relation [1]:

\[
B(\tau^- \rightarrow l^- \bar{\nu}_l \nu_{\tau}) = \frac{G_{\tau l}^2 m_{\tau}^2}{192 \pi^3 f(m_{\tau}^2)} (1 + \delta) \tag{1}
\]

where the phase space factor is given by

\[
f(x) = 1 - 8x + 8x^3 - x^4 - 12x \ln x \tag{2}
\]

and is 0.9726 for the muon final state; \( \delta = -0.4\% \) is the electroweak correction; \( G_{\tau l} \) is the coupling:

\[
G_{\tau l} = \frac{g_\tau g_l}{4 \sqrt{2} m_W^2} = G_F \tag{3}
\]

where \( m_l, m_{\tau}, \) and \( m_W \) are the mass of the light lepton, \( \tau \) lepton, and \( W \) boson.
Both L3 and ALEPH have reported new measurements of the leptonic branching fractions and L3 and DELPHI have reported new measurements of the \(\tau\) lifetime. These new results can be combined with other measurements to compute new averages for lifetime and leptonic branching fractions so that they can be compared with the corresponding measurements for an analogous decay \(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu\) to test the lepton universality. The results on the ratios of couplings are:

\[
\frac{g_\tau}{g_\mu} = 0.9994 \pm 0.0023 \\
\frac{g_\tau}{g_e} = 1.0000 \pm 0.0023
\]

Both ratios are consistent with the Standard Model expectation to a precision of better than one quarter of a percent, a remarkable achievement.

We can also compare the coupling of \(\tau\) to the average coupling for the light leptons with the assumption of \(e\)-\(\mu\) universality. This is a reasonable assumption since we expect any deviation from lepton universality to occur in the heavier lepton. The measured branching fractions for \(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau\) and \(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau\) are used to compute an effective branching fraction for the \(\tau\) decay into massless leptons, after correcting for phase space. Figure 1 shows the \(\tau\) lifetime vs the effective leptonic branching fraction. The Standard Model expectation as given by Eq. 1 is also shown as a diagonal line with the width given by the uncertainty in the \(\tau\) mass. It is evident that the \(\tau\) mass uncertainty is not negligible compared to the errors on the lifetime and leptonic branching fraction. The ratio of couplings is

\[
\frac{g_\tau}{g_{\text{e,}\mu}} = 0.9997 \pm 0.0020
\]

This is consistent with the Standard Model expectation of lepton universality.

Lepton universality can be tested at high \(Q^2\) by comparing the branching fractions of \(W\) into various leptons. The ratio of the couplings to different leptons are related to the branching fractions by:

\[
\frac{g_\tau}{g_l} = \left( \frac{B(W^- \rightarrow \tau^- \bar{\nu}_\tau)}{B(W \rightarrow l^- \bar{\nu}_l)} \right)^{\frac{1}{2}}.
\] (4)

The measured branching fractions as averaged over the four LEP experiments at the \(e^+e^-\) collider at CERN yields:\n
\[
\frac{g_\tau}{g_\mu} = 1.022 \pm 0.016 \\
\frac{g_\tau}{g_e} = 1.021 \pm 0.016
\]

For comparison, the measurements averaged over CDF and D0 experiments at the \(p\bar{p}\) collider at Fermilab are:\n
\[
\frac{g_\tau}{g_\mu} = 0.996 \pm 0.024 \\
\frac{g_\tau}{g_e} = 0.988 \pm 0.021
\]

The two sets of measurements with very different systematic errors are consistent with each other. Averaging over the two colliders yields:\n
\[
\frac{g_\tau}{g_\mu} = 1.014 \pm 0.013 \\
\frac{g_\tau}{g_e} = 1.009 \pm 0.013
\]

The two coupling ratios are consistent with unity as expected from lepton universality. The precision of the measurements from the real \(W\) decays
is about a factor of six less than those from the
τ decay into the virtual W. However, the two
measurements probe lepton universality at very
different $Q^2$. In summary, lepton universality is
respected from $Q^2$ of 3 to 6500 GeV$^2$.

2.2. Neutral Current Couplings

Lepton universality implies that the vector and
axial-vector couplings of all leptons to the Z boson
are identical. The couplings can be mea-
sured from the leptonic partial widths, the
forward-backward asymmetry, the left-right asym-
metry and, in the case of polarized beam at SLC, left-right asymmetry and
left-right forward-backward asymmetry.

The leptonic partial widths of the Z boson
are related to the vector ($g_{Vl}$) and axial-vector ($g_{Al}$)
couplings by:

$$\Gamma_l = \frac{G_F m_Z^3}{6\pi\sqrt{2}} (g_{Vl}^2 + g_{Al}^2),$$

where $m_Z$ is the mass of the Z boson. The partial
width is therefore sensitive to the quadratic sum
of the coupling constants.

The parity-violating forward-backward asym-
metry in the angular distribution of the final state
leptons is given by

$$A^{FB} = \frac{3}{4} A_l A_e,$$

with the asymmetry parameter related to the vec-
tor and axial-vector couplings via:

$$A_l = \frac{2g_V g_A}{g_{Vl}^2 + g_{Al}^2} = \frac{2(\frac{2L}{g_{Ae}})}{1 + (\frac{2L}{g_{Ae}})^2}.$$

The asymmetry parameter is therefore sensitive
to the ratio of couplings.

The final state leptons from the Z decay are
polarized, with a polarization that depends on
the scattering angle with respect to the beams,

$$P_l(\cos \theta) \equiv \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{\langle R_l \rangle + \frac{8}{3} A^{FB}_{pol} \cos \theta}{1 + \frac{8}{3} A^{FB}_{pol} \cos \theta},$$

where $\sigma_{+(-)}$ is cross section for producing a lep-
ton of positive (negative) helicity, $\langle R_l \rangle = -A_l$ is the polarization averaged over all angles, and

$A^{FB}_{pol} = -\frac{3}{4} A_e$ is the forward-backward polarization
asymmetry. The polarization can be mea-
sured for the case where the final state lepton is
the τ lepton by analyzing the distortions in the
angular and momentum distributions of the decay
products.

At SLC, the electron asymmetry parameter $A_e$
is measured from the left-right asymmetry in the
cross sections for both left- and right-handed electron
beams:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e.$$

The asymmetry parameters for the three
leptons can be measured from left-right forward-
backward asymmetry:

$$A^{FB}_{LR} = \frac{(\sigma_L - \sigma_R) - (\sigma_R - \sigma_L)}{\sigma_L + \sigma_R + \sigma_R + \sigma_L} = \frac{3}{4} A_l.$$

The LEP Electroweak Working Group has
combined the results from ALEPH, DELPHI, L3,
OPAL, and SLD to extract the vector and axial-
vector couplings of the three leptons to the Z
boson as shown in Fig. 2. The three contours
overlap with each other as expected from lepton
universality. The results are also consistent with
the Standard Model expectation, which depends
on the Higgs and top masses. The ratios of the
couplings are:

$$\frac{g_V^e}{g_V^\mu} = 0.962 \pm 0.063$$

$$\frac{g_V^\mu}{g_V^\tau} = 0.958 \pm 0.029$$

$$\frac{g_A^e}{g_A^\mu} = 1.0002 \pm 0.0014$$

$$\frac{g_A^\mu}{g_A^\tau} = 1.0019 \pm 0.0015$$

The precision of the measurements from the τ
lepton is better than that for the muon because of
the possibility of measuring the polarization from
the τ decay products. The ratios are all consistent
with unity as expected from lepton universality.

3. Lorentz Structure

The most general, local, derivative-free, lepton-
number conserving, Lorentz invariant four-lepton
such as those mediated by the charged Higgs of right symmetric models or to the scalar currents.

In many extensions to the Standard Model, additional interactions can modify the Lorentz structure. For example, there can be couplings to a right-handed $W_R^-$ of the leftright symmetric models or to the scalar currents such as those mediated by the charged Higgs of the Minimum Supersymmetric Standard Model (MSSM).

The coupling constants can be measured from the energy spectra of the daughter charged leptons from the $\tau$ decay. Integrating over the two unobserved neutrinos, the scaled energy spectrum of the daughter lepton is:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx \cos \theta} = \frac{x^2}{2} \times \left\{ 12(1-x) + \frac{4\rho}{3}(8x-6) + 24\eta \frac{m_\ell}{m_\tau} \frac{1-x}{x} \right\}$$

where $P_\tau$ is the average $\tau$ polarization, $\theta$ is the angle between the $\tau$ spin and the daughter charged lepton momentum in the $\tau$ rest frame, and $x = E_l/E_{max}$ is daughter charged lepton energy scaled to the maximum energy $E_{max} = (m_\ell^2 + m_\tau^2)/2m_\tau$ in the $\tau$ rest frame.

The Michel parameters $[7]$ are bilinear combinations of the coupling constants from Eq. (11):

$$\rho = \frac{3}{16} (4|g_{VLL}|^2 + 4|g_{VRR}|^2 + |g_{SLL}|^2 + |g_{SSS}|^2)$$

$$+ |g_{VLR}|^2 - 2g_{VLR}^T|g_{VLR}| - 2g_{VLR}^T|g_{VLR}|$$

$$\eta = \frac{1}{2} Re(6g_{VLL}^Tg_{VLR} + 6g_{VLL}^Tg_{VLR} + g_{SLL}^Tg_{SLL})$$

$$+ g_{SLL}^Tg_{SLL} + g_{SLL}^Tg_{SLL} + g_{SLL}^Tg_{SLL}$$

$$\xi = -\frac{1}{4} (|g_{VRR}|^2 + |g_{SRR}|^2 - |g_{SRR}|^2 - |g_{SLL}|^2)$$

$$+ 5(|g_{VLR}^T|^2 - |g_{VLR}|^2)$$

$$- (|g_{VRR}|^2 - 3|g_{VLL}|^2 + 3|g_{VLR}|^2 - |g_{VRR}|^2)$$

$$+ 4Re(g_{VLL}^Tg_{VLR} - g_{VLL}^Tg_{VLR})$$

$$\xi\delta = \frac{3}{16} (4|g_{VLL}|^2 - 4|g_{VRR}|^2 + |g_{SLL}|^2 - |g_{SRR}|^2)$$

$$+ |g_{VLR}^T|^2 - |g_{VLR}|^2 - |g_{VLR}|^2 - |g_{VLR}|^2$$

In the Standard Model, the values of the Michel parameters are:

$$\eta = 0 \quad \rho = \frac{3}{4} \quad \xi = 1 \quad \delta = \frac{3}{4}$$

The spectrum shape parameter $\rho$ can be measured from the momentum spectrum of the
daughter charged lepton. The low-energy parameter $\eta$ can only be measured in the $\tau$ decay to muon because of the factor $m_l/m_\tau$ in Eq. (12) for the helicity flipping of the daughter charged lepton. This parameter also can affect the leptonic decay widths and hence can be constrained from the measured lifetime and leptonic branching fractions using lepton universality. On the other hand, the parameters $\xi$ and $\xi\delta$ can only be measured for a non-zero $\tau$ polarization. At the $Z$ resonance, the small natural polarization ($P_\tau \sim 0.14$) provides some sensitivity. The sensitivity can be enhanced with a polarized beam as the case at SLC. The sensitivity can also be greatly improved using the spin-spin correlation of the $\tau$ leptons, with the opposite $\tau$ decay as a polarization analyzer.

The result [8] on the Michel parameters extracted with the assumption of $e/\mu$ universality is shown in Fig. 3. All the measurements are consistent with each other as indicated by the $\chi^2$ per degree of freedom. The measurements are also consistent with the Standard Model expectations. The limit on the mass of the charged Higgs extracted from the $\eta$ measurement is not competitive compared with the direct search. Since the CLEO result on the Michel parameters is quite competitive compared with the LEP/SLC experiments, one can expect significant improvements in the measurements in the near future from the $b$ factories. The Michel parameters averaged over all experiments without the assumption of $e/\mu$ universality are shown in Fig. 4. The results are consistent with lepton universality for a $\chi^2$ of 3.6 for 7 degrees of freedom.

The measured Michel parameters can be used to extract limits on the absolute values of the coupling constants $g_{\gamma j}$ for the three types of interactions as shown in the complex planes in Fig. 5. Most of the measurements impose significant limits on the coupling constants involving right-handed current $W_R^{-}$ as evident from the small areas shaded in dark gray over the light gray circles, the theoretically allowed regions. There are no limits on the “LL” couplings because the scalar and vector interactions can only be distinguished in the reaction of inverse $\tau$ decay, $\nu_\tau \bar{e} \rightarrow \tau \nu_e$. The limits from the muon are significantly more strin-
gent. As stated above, we can expect significant improvements in sensitivity for new interactions from the $\tau$ decay in the near future from $b$ factories.

4. Hadronic Decays

The $\tau$ lepton can decay into many hadronic states due to its large mass. The hadronic decay products have distinctive charge conjugation ($C$) and isospin (and hence $G$-parity) signatures, a reflection of the quantum number of the charged hadronic weak current. The weak current is classified according to its $G$-parity:

- Vector: $G = +1, J^P = 1^-$ e.g. $\rho^-$
- Axial: $G = -1, J^P = 0^-, 1^+$ e.g. $\pi^-, a_1^-$

These are known as the first class currents. Currents with opposite $G$-parity are called the second class currents $\xi$ and are suppressed by the order $\alpha^2$ or $10^{-4}$ in the Standard Model. Examples of second class current decays are $\tau^- \to a_0^0 \nu_\tau$ and $\tau^- \to b_1^- \nu_\tau$.

The large number of hadronic channels allow the study of many aspects of the Standard Model, including test of the Conserved-Vector-Current (CVC) hypothesis $\eta$, isospin symmetry, QCD sum rules, study of light meson resonances, and measurements of the strong coupling constant, strange quark mass, and hadronic contribution of the muon anomalous magnetic moment. In this section, I will review the topics in which there are significant progress since Tau98.

4.1. Test of CVC

The CVC hypothesis relates the coupling strength of the weak charged vector current in the $\tau$ decay to the electromagnetic neutral vector current in $e^+e^-$ annihilation. This allows the calculation of the branching fraction for the $\tau$ decays into the vector final state from the $e^+e^-$ cross section for the corresponding isovector final state ($I = 1$). For example, the branching fraction for
\[ \tau^- \to \pi^- \pi^0 \nu_\tau \] is given by:
\[
B_{\pi\pi} = \frac{G^2_F \tau \cos^2 \theta_c}{12\pi^4} \int_{4m_\pi^2}^{m_{\tau}^2} dq^2 \, q^2
\]
\[
(m_{\tau}^2 - q^2)^2 (m_{\tau}^2 + 2q^2) \sigma_{\pi^- \nu_\tau}^{-1} \rightarrow \pi^+ \pi^- (q^2) \quad (17)
\]
where \( q \) is the center-of-mass energy of the \( e^+ e^- \) collision, \( m_\pi \) is the \( \pi \) mass, \( \theta_c \) is the Cabbibo angle, and \( \alpha \) is the fine structure constant.

At Tau98, the CVC prediction [11] for \( B_{\pi\pi} \) was:
\[
B_{\pi\pi}^{CVC} = (24.52 \pm 0.33)\%
\]
This is to be compared with the current world average [12] of
\[
B_{\pi\pi} = (25.31 \pm 0.18)\% .
\]

The fractional difference between the two branching fractions is \((3.1 \pm 1.5)\%\), which is slightly over \(2\sigma\) (standard deviation).

At this Workshop, there is a new measurement [13] by L3 on the branching fraction for \( \tau^- \to h^+ \pi^0 \nu_\tau \) of \((25.38 \pm 0.18 \pm 0.14)\%\). Correcting for the kaon contribution [4] of \((0.449 \pm 0.034)\%\) yields:
\[
B_{\pi\pi} = (24.93 \pm 0.23)\% .
\]
This is consistent with the world average. Including this branching fraction into the world average yields a new world average of
\[
B_{\pi\pi} = (25.17 \pm 0.14)\% .
\]
This is to be compared with the new CVC expectation [14] presented at this Workshop based on new data on \( e^+ e^- \to \pi^+ \pi^- \):
\[
B_{\pi\pi}^{CVC} = (24.94 \pm 0.23)\% .
\]

The fractional difference between the two branching fractions is \((0.9 \pm 1.1)\%\) and hence there is good agreement between the measured branching fraction and the CVC prediction.

The agreement between the measured branching fraction for the \(4\pi\) final state and the CVC expectation is not as good. Figure 6 shows the spectral functions [14] for the \(4\pi\) final state extracted from \( \tau^- \to 2\pi^- \pi^+ \pi^0 \nu_\tau \) by CLEO and from \( e^+ e^- \to 2\pi^- 2\pi^+ \) and \( \pi^+ \pi^- 2\pi^0 \) by CMD2 and DM2. The recent CMD2 data from the VEPP-2M collider has significantly better precision than the vintage DM2 data. The spectral function extracted from the \( \tau \) decay is consistently higher across the entire energy range. Since branching fraction is related to the integral of the spectra function, the discrepancy is also expected between the measured branching fraction [15] and the CVC prediction [14]:
\[
B_{3\pi\pi^0} = (4.19 \pm 0.10 \pm 0.21)\%
\]
\[
B_{4\pi}^{CVC} = (3.55 \pm 0.20)\%
\]
The discrepancy is \(\sim 18\%\) with a statistical significant of \(\sim 2\sigma\).
\( B^{CVC}_{\pi\omega} = (1.73 \pm 0.06)\% \)

As expected, the magnitude of the discrepancy is somewhat smaller, \( \sim 10\% \), but the statistical significant remains the same, \( \sim 2\sigma \). It should be noted that \( B_{\pi\omega} \) includes a small contribution from \( \tau^- \rightarrow K^-\omega\nu_\tau \) which is both phase-space and Cabbibo-suppressed.

\[ \text{Figure 7. Cross section for the process } e^+e^- \rightarrow \pi^0\omega. \]

### 4.2. Measurement of \( \rho(1450) \) Parameters

The hadronic \( \tau \) decay provides a clean environment to study the light meson resonance parameters. The CLEO experiment [15] has extracted the \( \rho(1450) \) parameters from a fit to the \( \pi\omega \) mass spectrum in the decay \( \tau^- \rightarrow \pi^-\omega\nu_\tau \):

\[ M_{\rho'} = 1523 \pm 10 \text{ MeV} \]
\[ \Gamma_{\rho'} = 400 \pm 35 \text{ MeV} \]

The mass is significantly different from the fit to the mass spectrum [4] in the decay \( \tau^- \rightarrow \pi^-\pi^0\nu_\tau \) although the width is consistent:

\[ M_{\rho'} = 1406 \pm 15 \text{ MeV} \]
\[ \Gamma_{\rho'} = 455 \pm 41 \text{ MeV} \]

The parameters are also different from those extracted by PDG [6] from a combination of data from fixed target experiments, \( e^+e^- \rightarrow \pi^+\pi^- \) and \( e^+e^- \rightarrow \eta\pi^+\pi^- \), as well as from earlier results from the decay \( \tau^- \rightarrow \pi^-\pi^0\nu_\tau \):

\[ M_{\rho'} = 1465 \pm 25 \text{ MeV} \]
\[ \Gamma_{\rho'} = 310 \pm 60 \text{ MeV} \]

The origin of these differences is unknown.

### 4.3. Test of CVC and Isospin Symmetry in Six-Pion Decays

The CLEO experiment [17] has presented new results on two six-pion decays, \( \tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau \) and \( \tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau \). There is no experimental information on the decay \( \tau^- \rightarrow \pi^-5\pi^0\nu_\tau \) due to the difficulty in extracting a signal from the large combinatoric background. The decays may proceed through the \( \rho, \omega \) or \( \eta \) intermediate states. The six-pion decays are therefore a mixture of vector and axial-vector current decays. The contribution from the axial-vector current decay, \( \tau^- \rightarrow (3\pi^-\eta\nu_\tau) \), must be subtracted in the test of CVC and isospin symmetry.

The new results on the six-pion decay branching fractions are:

\[ B(\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau) = (2.2 \pm 0.3 \pm 0.4) \times 10^{-4} \]
\[ B(\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau) = (1.7 \pm 0.2 \pm 0.2) \times 10^{-4} \]

The results represent significant improvement in precision over previous measurements [4].

The experiment has also searched for \( \eta \) or \( \omega \) intermediate states. For the decay \( \tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau \), the branching fractions with the intermediate decays \( \eta \rightarrow 3\pi^0, \eta \rightarrow \pi^+\pi^-\pi^0 \) and \( \omega \rightarrow \pi^+\pi^-\pi^0 \) are

\[ B(\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau) = (2.9 \pm 0.7 \pm 0.5) \times 10^{-4} \]
\[ B(\tau^- \rightarrow 2\pi^-3\pi^0\nu_\tau) = (1.5 \pm 0.6 \pm 0.3) \times 10^{-4} \]
\[ B(\tau^- \rightarrow \pi^-\pi^0\omega\nu_\tau) = (1.5 \pm 0.4 \pm 0.3) \times 10^{-4} \]

In the decay \( \tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau \), the branching fractions for the intermediate decays \( \eta \rightarrow \pi^+\pi^-\pi^0 \) and \( \omega \rightarrow \pi^+\pi^-\pi^0 \) are

\[ B(\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau) = (1.9 \pm 0.4 \pm 0.3) \times 10^{-4} \]
\[ B(\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau) = (1.2 \pm 0.2 \pm 0.1) \times 10^{-4} \]

This constitutes the first observation of the decay \( \tau^- \rightarrow 2\pi^-\pi^+\nu_\tau \). The results are consistent with
saturation of the two six-pion decays by $\eta$ and $\omega$ intermediate states. The branching fractions for the two decays with $\omega$ in the final states are somewhat smaller than the recent calculations by Gao and Li [19]:
\[
B(\tau^- \rightarrow \pi^- 2\pi^0 \omega \nu_\tau) = 2.16 \times 10^{-4}
\]
\[
B(\tau^- \rightarrow 2\pi^- \pi^+ \omega \nu_\tau) = 2.18 \times 10^{-4}
\]

The results on the six-pion decays can be compared with the isospin symmetry and CVC predictions, after correcting for the contributions from the axial-vector current $\tau^- \rightarrow (3\pi^-)\eta \nu_\tau$, which also violates isospin conservation with the decays $\eta \rightarrow 3\pi^0$ and $\pi^+ \pi^- \pi^0$. To reduce the uncertainty in the corrections, CLEO uses measurements from these decays and the decay $\eta \rightarrow \gamma \gamma$ [19] to obtain the average branching fractions:
\[
\bar{B}(\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau) = (2.4 \pm 0.5) \times 10^{-4}
\]
\[
\bar{B}(\tau^- \rightarrow \pi^- 2\pi^0 \eta \nu_\tau) = (1.5 \pm 0.5) \times 10^{-4}
\]

This yields the vector current branching fractions:
\[
B_V(\tau^- \rightarrow 2\pi^- \pi^+ 3\pi^0 \nu_\tau) = (1.1 \pm 0.4) \times 10^{-4}
\]
\[
B_V(\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau) = (1.1 \pm 0.2) \times 10^{-4}
\]

This corresponds to $\sim 50\%$ and $\sim 65\%$, respectively, of the six-pion branching fractions.

The isospin model [20] classifies $n$-pion final states into orthogonal isospin partitions and determines their contributions to the branching fractions. The partitions are labeled by three integers $(n_1, n_2, n_3)$, where $n_3$ is the number of isoscalar subsystems of three pions, $n_2 - n_3$ is the number of isovector systems of two pions, and $n_1 - n_2$ is the number of single pions. For $n = 6$ there are four partitions: $510$ ($4\pi\rho$), $330$ ($3\rho$), $411$ ($3\pi\omega$), and $321$ ($\pi\rho\omega$), denoted according to the lowest mass states. The isospin model imposes constraints on the relative branching fractions, which can be tested by comparing the following two fractions
\[
f_{2\pi^- \pi^+ 3\pi^0} = \frac{B_V(\tau^- \rightarrow 2\pi^- \pi^+ 3\pi^0 \nu_\tau)}{B_V(\tau^- \rightarrow (6\pi^-)\nu_\tau)}
\]
\[
f_{3\pi^- 2\pi^+ \pi^0} = \frac{B_V(\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau)}{B_V(\tau^- \rightarrow (6\pi^-)\nu_\tau)}
\]
where $B_V(\tau^- \rightarrow (6\pi^-)\nu_\tau)$ is the sum of the branching fractions for the three six-pion vector decays. Figure 8 shows $f_{2\pi^- \pi^+ 3\pi^0}$ vs. $f_{3\pi^- 2\pi^+ \pi^0}$ with the new measurement of the branching fractions. The measurement is presented as a line because $B_V(\tau^- \rightarrow \pi^- 5\pi^0 \nu_\tau)$ has not been measured yet. The result is consistent with the isospin expectation since the experimental measurement overlaps with the isospin triangle. The result indicates the $321$ ($\pi\rho\omega$) partition is dominant because the decays $\tau^- \rightarrow \pi^- 2\pi^0 \omega \nu_\tau$ and $\tau^- \rightarrow 2\pi^- \pi^+ \omega \nu_\tau$ saturate the six-pion (vector) decays.

A significant fraction of the experimentally allowed area is still outside the isospin allowed region. This can only be improved if the branching fraction for the experimentally challenging decay $\tau^- \rightarrow \pi^- 5\pi^0 \nu_\tau$ has been measured. A similar analysis on the five-pion decays should be performed.

The measured branching fractions can be compared with the CVC expectations [21] based on the measured cross sections for $e^+e^- \rightarrow 6\pi$:
\[
B(\tau^- \rightarrow 2\pi^- \pi^+ 3\pi^0 \nu_\tau) \geq (2.5 \pm 0.4) \times 10^{-4}
\]
The predictions are significant larger than the measured branching fractions for the six-pion vector decays. The discrepancy is even more significant if we compare the predicted inclusive branching fraction

\[ B(\tau^- \to 3\pi^- 2\pi^0 \nu_\tau) \geq (2.5 \pm 0.4) \times 10^{-4} \]

with the sum of the measured six-pion vector branching fractions under the assumption that \( B(\tau^- \to \pi^- 5\pi^0 \nu_\tau) \) is comparable with or smaller than \( B(\tau^- \to 2\pi^- \pi^+ 3\pi^0 \nu_\tau) \) and \( B(\tau^- \to 3\pi^- 2\pi^+ \pi^0 \nu_\tau) \) as expected by isospin symmetry:

\[ B(\tau^- \to \pi^- 5\pi^0 \nu_\tau) \leq \frac{9}{26} |B(\tau^- \to 2\pi^- \pi^+ 3\pi^0 \nu_\tau) + B(\tau^- \to 3\pi^- 2\pi^+ \pi^0 \nu_\tau)| \]

This assumption is consistent with the observation that the six-pion vector decays are saturated by intermediate states with an \( \omega \) meson, which implies a small decay width for the \( 510 \ (4\pi\rho) \) state, the only state that contributes to the decay \( \tau^- \to \pi^- 5\pi^0 \nu_\tau \). The discrepancy might be explained by a sizable presence of \( I = 0 \) states in the \( e^+e^- \) annihilation data that should be subtracted before calculating the CVC prediction.

### 5. Anomalous Magnetic Moment

The g-2 experiment at CERN is currently being repeated at Brookhaven National Laboratory with a precision that is sensitive to new physics comparable to the electroweak corrections. The largest uncertainty in calculating the muon anomalous magnetic moment is the contribution from hadronic vacuum polarization. The contribution is calculated using the \( e^+e^- \) cross section of hadronic final states and is dominated by \( \sigma(e^+e^- \to \pi^+\pi^-) \). There is a new estimate of the \( \pi\pi \) contribution with the addition of the recent data from CMD-2 [22]:

\[ a_\mu^{\pi\pi} = (498.8 \pm 5.0) \times 10^{-10} \]

The hadronic decays of the \( \tau \) lepton provide an alternative estimate of the hadronic vacuum polarization with different systematic error. There are estimates of the \( \pi\pi \) contribution using the ALEPH [22] and CLEO [16] measurements of the decay rate for \( \tau^- \to \pi^- \pi^0 \nu_\tau \):

- \( a_\mu^{\pi\pi} = (502.2 \pm 6.9) \times 10^{-10} \) (ALEPH)
- \( a_\mu^{\pi\pi} = (513.1 \pm 5.8) \times 10^{-10} \) (CLEO)

The two measurements are not inconsistent with each other or the measurement using \( \sigma(e^+e^- \to \pi^+\pi^-) \) given the errors. However, the difference between the two measurements is three times the expected precision of \( 4 \times 10^{-10} \) from the new g-2 experiment [23]. If we combined the three measurements despite the possible discrepancy, then:

\[ a_\mu^{\pi\pi} = (504.3 \pm 3.3) \times 10^{-10} \]

The uncertainty is still not negligible in comparison with the expected precision of the g-2 experiment. New measurements from b-factory and other LEP experiments will provide a powerful consistency check. However, it should be noted that the extraction of \( a_\mu^{\pi\pi} \) from the \( \tau \) decay requires the assumption of CVC which should be closely scrutinized because the combined \( \tau \) measurement has now reached a precision of \( \sim 1\% \).

### 6. Strange Quark Mass

The strange quark mass is one of the fundamental parameters of the Standard Model. For example, it is needed in the extraction of the CKM matrix elements from the measurement of the direct CP violation parameter, \( c'/c \), in the kaon system. In the \( \tau \) decay, the strange quark decay rate depends on the strange quark mass, unlike the non-strange decay, which is insensitive to the up and down quark masses. The strange quark mass suppresses the decay rate by \( \sim 10\% \). Extraction of the mass requires the calculation of the somewhat controversial non-perturbative corrections. Nevertheless, there has been significant progress in the past few years in the technique for extracting the mass.

In principle, the strange quark mass can be extracted from the inclusive strange decay rate. However, a better technique is to extract the mass from the difference between the non-strange and strange spectral moments so that the mass-independent non-perturbative corrections cancel to first order. Using the latest results on the
branching fractions for the strange decays by CLEO and OPAL together with the published strange spectral functions measured by ALEPH, Davier et al. \[24\] extracts a new estimate of the mass in the MS scheme:

\[ m_s(m_\tau) = 112 \pm 23 \text{ MeV} \]

which, using four-loop running, yields a mass at a slightly higher scale of

\[ m_s(2 \text{ GeV}) = 107 \pm 22 \text{ MeV} \]

This can be compared with the mass extracted by Maltman and Kambor \[24\]

\[ m_s(2 \text{ GeV}) = 115.1 \pm 13.6 \pm 11.8 \pm 9.7 \text{ MeV} \]

using the ALEPH data with an optimized finite energy sum rule, where the first error is due to the experimental uncertainty, the second due to uncertainty in the CKM matrix element \( V_{us} \), and the third due to the theoretical uncertainty. It is reassuring to see that both techniques give consistent results.

The estimates can be compared with the result from the lattice calculation \[26\]

\[ m_s(2 \text{ GeV}) = 110 \pm 25 \text{ MeV} \]

The \( \tau \) result therefore provides an independent estimate of this fundamental parameter with a very different systematic error. With the advent of the b-factory experiments with excellent kaon identification, we expect more precise measurements of the strange spectral functions and hence a better determination of the strange quark mass.

7. Direct Measurement of Electric Dipole Moment

There are three kinds of dipole moments from the couplings of the \( \tau \) lepton to the \( \gamma, Z, \) and \( W \) bosons: anomalous (electromagnetic) magnetic (\( a_\tau \)) and electric dipole (\( d_\tau \)) moments, anomalous weak magnetic (\( d^W_\tau \)) and electric dipole (\( d^{W'}_\tau \)) moments, and electric dipole (\( \kappa \)) moments. In the Standard Model, radiative corrections produce a non-zero anomalous magnetic dipole moment \[27\]

\[ a_\tau = \frac{g - 2}{2} = (1.1773 \pm 0.0003) \times 10^{-3} \]

The electric dipole moment \( d_\tau \) is zero for pointlike fermions; a non-zero value would violate both \( T \) and \( P \) and hence \( CP \) invariance. The weak magnetic dipole moment is very small \[28\]:

\[ d^W_\tau = -(2.10 + 0.61i) \times 10^{-6} \]

The weak electric dipole moment is also very small but non-zero due to \( CP \) violation in the CKM matrix:

\[ d^{W'}_\tau \approx 3 \times 10^{-37} \text{ e} \cdot \text{cm} \]

The charged weak magnetic and electric dipole moments are expected to be very small. Since all the dipole moments are expected to be small or zero, measurement of the dipole moments provides a sensitive probe of physics beyond the Standard Model.

The ARGUS experiment \[29\] has searched for \( CP \) violation due to a non-zero electric dipole moment in the production of \( \tau \) pairs. The \( CP \) violation produces a charge dependent momentum correlation. The experiment constructed optimized observables which took into account all available information on the observed decay products. No evidence for \( CP \) violation was found, resulting in the following 95% CL upper limit on the electric dipole moment:

\[ Re(d_\tau) < 4.6 \times 10^{-16} \text{ e} \cdot \text{cm} \]

\[ Im(d_\tau) < 1.8 \times 10^{-16} \text{ e} \cdot \text{cm} \]

These are the first direct measurement of the electric dipole moment. For comparison, the limit extracted \[30\] by the L3 experiment from an analysis of the reaction \( e^+e^- \to \tau^+\tau^-\gamma \) is

\[ |d_\tau| < 3.1 \times 10^{-16} \text{ e} \cdot \text{cm} \]

with the assumption of zero anomalous magnetic dipole moment.

8. Direct Observation of the \( \tau \) Neutrino

The existence of the \( \tau \) neutrino was postulated after the discovery of the \( \tau \) lepton in 1975. Much direct evidence for its existence has accumulated in the intervening 25 years. It has been assumed that \( \nu_\tau \) is a sequential neutrino to \( \nu_\mu \).
and $\nu_e$ in the Standard Model. This elusive particle has finally been observed directly [31] by the DONUT experiment at Fermilab.

Evidence for $\nu_\tau$ relies on identification of the $\tau$ leptons produced in the charged current $\nu_\tau$ interactions. The $\tau$ neutrinos are produced by directing 800 GeV/c protons onto a tungsten dump via the decay $D_s^+ \rightarrow \tau^+ \nu_\tau$ with an average energy $\langle E_{\nu_\tau} \rangle = 54$ GeV. The $e$ and $\mu$ neutrinos are also produced in the $\mu$, $\pi$, $K$, and $D$ decays. These neutrinos could interact via a charge or neutral current to produce background events.

Critical to the $\nu_\tau$ identification is the emulsion target which contains $\sim 200$ plastic substrate sheets with $\sim 100$ $\mu$m of emulsion coated on both sides. Each track leaves a trail of emulsion grains, providing the spatial location with excellent resolution, 0.3-0.4 $\mu$m. A track segment is constructed from the emulsion grains in each emulsion sheet. Combining the track segments from the emulsion sheets allows the observation of the kink in a $\tau$ decay. A $\nu_\tau$ candidate from a charge current interaction could produce several tracks emanating from the (primary) interaction vertex, including a $\tau$ candidate track with a kink. The experiment requires both the $\tau$ candidate track and its daughter track to have impact parameter $< 5$ $\mu$m, taking advantage of the high spatial resolution of the emulsion for background suppression. The background includes random association of background tracks, re-scattering of a track emanating from the primary interaction vertex, and charm decay from a charged current interaction. The probability of re-scattering decreases rapidly with traverse momentum $p_t$, in contrast to $\tau$ decays in which $p_t$ peaks at 400 MeV/c. Figure 9 shows the $p_t$ spectrum of the candidate tracks. There are five $\tau$ candidate tracks with $p_t > 250$ MeV/c. One of the high $p_t$ event is identified as a charm background from a charged current interaction by the presence of an electron emanating from the primary vertex. The remaining four events are $\tau$ candidates with no evidence for a lepton from the primary vertex. Figure 10 shows a picture of one of the candidates.

The sample of four events is consistent with the expected signal of $4.1 \pm 1.4$ events. The total estimated background is $0.41 \pm 0.15$ events from mistagged charm decays and secondary interactions. The Poisson probability that the four events are due to the background processes is $8 \times 10^{-4}$. The $\tau$ neutrino has therefore been directly observed after the first sighting of the $\tau$ lepton 25 years ago! This completes the picture of the Standard Model of three generations of fermions.

The DONUT experiment has proven the principle of directly detecting the $\tau$ neutrino. This is an important milestone for the experiments currently being constructed to search for $\nu_\tau$ appearance to verify the signal of $\nu_\mu \rightarrow \nu_\tau$ oscillation observed by Super-Kamiokande. This will be reviewed in the next section.

9. Neutrino Oscillations

In the Standard Model, neutrinos are assumed to be massless. However, it can also accommodate neutrinos with mass. If neutrinos have mass, then they can mix with one another, thereby violating lepton family number conservation. In a two-flavor oscillation between $\nu_\mu$ and $\nu_\tau$, for example, the probability for oscillation is given by

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta_{\mu\tau}) \sin^2(\pi L/L_0),$$

(20)
where \( \theta_{\mu\tau} \) is the mixing angle, \( L \) is distance traversed by \( \nu_\mu \) in meter, and the oscillation length is given by

\[
L_0 = \frac{2.48E_\nu}{\Delta m^2},
\]

with \( \Delta m^2 = m_\tau^2 - m_\mu^2 \) being the difference of the mass squares in eV\(^2\) and \( E_\nu \) being the energy of the neutrino in GeV.

Evidence for neutrino oscillations has been seen in the solar \( \nu_e \) deficit (\( \nu_e \) disappearance), atmospheric \( \nu_\mu \) deficit (\( \nu_\mu \) disappearance), and \( \bar{\nu}_e \) excess in \( \pi^+ \) decays (\( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) appearance). It is difficult to accommodate all the apparent oscillations in a three neutrino generation model. However, an additional sterile neutrino (\( \nu_s \)) can easily explain all the oscillations.

In this Section, I will review the new result presented at this Workshop from Super-Kamiokande \([32]\) on the atmospheric \( \nu_\mu \) deficit and from CHORUS and NOMAD on the search \([33]\) for \( \nu_\mu \rightarrow \nu_\tau \) oscillation. I will then discuss the current status and future prospect of long baseline experiments on the \( \nu_\tau \) oscillation.

### 9.1. Observation of Atmospheric \( \nu_\mu \) Deficit

Atmospheric neutrinos are produced by collisions of cosmic rays with the upper atmosphere. If there is no oscillation, the neutrino flux should be relatively uniform with no up-down asymmetry. The neutrinos originate from the decay chain: \( \pi^+ \rightarrow \mu^+\bar{\nu}_\mu \) with \( \mu \rightarrow e\bar{\nu}_e\nu_\mu \). The ratio of the muon to electron neutrino flux should be \( \sim 2 \) in the no oscillation scenario. Super-Kamiokande observed a very significant \( \nu_\mu \) deficit. This can be attributed to the oscillation of \( \nu_\mu \) to \( \nu_e, \nu_\tau \) or \( \nu_s \). The oscillation cannot be pure \( \nu_\mu \rightarrow \nu_e \) because there is no significant excess of \( \nu_e \) originated from below. In addition, the CHOOZ \([34]\) and Palo Verde \([35]\) experiments have ruled out disappearance of reactor \( \bar{\nu}_e \) for similar parameters.

The \( \nu_\mu \rightarrow \nu_s \) oscillation scenario has been investigated using two techniques based on the fact that \( \nu_s \) has no neutral current coupling. First, there is an MSW-like effect for \( \nu_s \) in the Earth that effectively suppresses oscillation. The effect is more prominent at higher energies and no distortion in the angular distribution of high energy events is observed. Second, there is no deficit of up-going neutral current events in a neutral current enriched sample of multi-ring events. These observations exclude the \( \nu_\mu \rightarrow \nu_s \) oscillation at the 99% CL.

The zenith angle dependent of the atmospheric neutrino events of various visible energies is shown in Fig. 11. For the \( e^-\)like events, events most likely produced by \( \nu_e \) interactions, there is no evidence of any deficit or excess. However, for the \( \mu^-\)like events, there is both zenith angle (path length) and energy dependent as expected from Eq. (20) in the \( \nu_\mu \rightarrow \nu_\tau \) oscillation scenario. A fit to the oscillation hypothesis yields \( \sin^2 2\theta = 1.01 \) and \( \Delta m^2 = 0.0032 \text{ eV}^2 \), with a \( \chi^2 \) of 135 for 152 degrees of freedom. This corresponds to maximum mixing with very small mass difference. The various confidence intervals on the oscillation parameters are shown in Fig. 12.
Figure 11. Zenith angle distributions of $e$-like (left) and $\mu$-like (right) events of various visible energies. The lines show the best fit with $\nu_\mu \rightarrow \nu_\tau$ oscillation. The hatched lines indicate the expectations without oscillation.

Figure 12. The 68, 90, and 99% confidence intervals on the $\nu_\mu \rightarrow \nu_\tau$ oscillation parameters.

9.2. Search for $\nu_\tau$ Oscillation in Short Baseline Experiments

CHORUS and NOMAD are two short baseline neutrino oscillation experiments at CERN. The experiments are designed to search for $\nu_\mu \rightarrow \nu_\tau$ oscillation through the observation of charged current interactions $\nu_\tau N \rightarrow \tau^- X$. The neutrino beam also contains a small fractions of $\bar{\nu}_e, \nu_e,$ and $\bar{\nu}_e$ plus a tiny contamination of prompt $\nu_\tau$, well below the detectable level. The experiments can therefore also search for $\nu_e \rightarrow \nu_\tau$ oscillation. The search is sensitive to very small mixing angles for large mass differences, $\Delta m^2 > 1 \text{ eV}^2$. This is a cosmologically interesting region: a neutrino with a mass in this region is a good candidate for the hot dark matter in the universe.

The two experiments deploy very different techniques for the search of $\tau$ appearance. CHORUS uses an emulsion target whose excellent spatial resolution allows a three dimensional visual reconstruction of the decays $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ and $h^- (\pi^0) \nu_\tau$. NOMAD exploits a purely kinematic technique to identify the decays $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau, h^- (\pi^0) \nu_\tau$, and $h^- h^+ h^- (\pi^0) \nu_\tau$. Both experiments do not observe any evidence of neutrino oscillation and exclude the mixing parameters shown in Fig. 13.
9.3. Search for $\nu_\tau$ Oscillation in Long Baseline Experiments

The best fit to the atmospheric $\nu_\mu$ deficit is based on the $\nu_\mu \to \nu_\tau$ oscillation hypothesis. This is a major challenge to the Standard Model and the result must be confirmed with an artificial neutrino beam from an accelerator with beam parameters that can be varied and monitored. The observed small mass difference (Eqs. (20) and (21)) dictates that the experiments be located hundreds of kilometers away from the neutrino source in order to get appreciable $\nu_\mu \to \nu_\tau$ conversion.

The K2K experiment [38] is the first long baseline experiment conceived for the propose. A $\nu_\mu$ beam with an average $E_\nu \sim 1$ GeV from KEK is directed toward the Super-Kamiokande detector located 250 km away. There is a miniature Super-Kamiokande detector at the near end at KEK to monitor the beam. The experiment searches for a $\nu_\mu$ deficit at the distant detector because the average beam energy is significantly below the 3.5 GeV threshold to produce the $\tau$ lepton. A signal of 27 $\nu_\mu$ events is observed. The expected signal is $40.3^{+4.7}_{-4.6}$ events under the no oscillation scenario. This disfavors the no oscillation scenario at the $2\sigma$ level. We can expect new result based on a significantly larger data sample in the near future.

The MINOS experiment [39] will also look for $\nu_\mu$ disappearance. The 3-20 GeV beam from Fermilab will be directed toward the SOUDAN detector at 730 km away. The detector is based on a magnetic iron tracker. The higher energy and rate allow MINOS to cover much more than K2K of the oscillation parameter region favored by the Super-Kamiokande data. The experiment is expected to start taking data in the year 2003.

Two experiments [40], ICANOE and OPERA, have been proposed to search for $\nu_\tau$ appearance under the CNGS project (CERN Neutrinos to Gran Sasso). The $\nu_\mu$ beam with an average energy of 17 GeV will be directed toward the detector 732 km away. ICANOE uses a liquid argon Time Projection Chamber (TPC) to reconstruct the $\tau$ decay with a kinematic technique similar to that of NOMAD. OPERA reconstructs the $\tau$ decay using an emulsion detector similar to that
used successfully by DONUT to directly observe $\nu_\tau$ for the first time. Both experiments are expected to start collecting data in the year 2005.

10. Search for Lepton Number Violating $\tau$ Decays

In the Standard Model, there is no symmetry associated with lepton number and therefore there is no fundamental conservation law for lepton number; lepton number conservation is an experimentally observed phenomenon. Lepton number violation is expected in many extensions of the Standard Model such as leptoquarks, SUSY, superstrings, left-right symmetric models and models which include heavy neutral leptons. The predictions typically depend on one or two unknown masses of new particles and one or two unknown couplings. Therefore any null result from a search can only constrain the parameter space but cannot rule out a particular model. Nevertheless the search should be pursued vigorously because of its profound implication on the Standard Model, should a positive signal be observed.

The $\tau$ lepton is an excellent laboratory for the search of physics beyond the Standard Model. Its large mass allows for searches at high momentum transfer with many decay channels. The sensitivity may be enhanced because the $\tau$ lepton is a third generation lepton. In some models, the coupling may have a mass dependence, e.g. $\propto m_\tau^5$, resulting in higher sensitivity than searches using the $\mu$ decay.

10.1. Implication from Neutrino Observation

The observed atmospheric $\nu_\mu$ deficit suggests the $\nu_\mu \rightarrow \nu_\tau$ oscillation scenario as discussed in the previous section. The effect of neutrino oscillation on the charged lepton number violating decay has been calculated by Bilenkii and Pontecorvo [11] for $\nu_e \rightarrow \nu_\mu$ oscillation. For the $\tau$ decay, the decay rate for $\tau^- \rightarrow \mu^- \gamma$ can be calculated using the Feynmann diagram shown in Fig. 14:

$$\Gamma = 5.5 \times 10^{-48} \sin^4(2\theta)(\Delta m_{\nu}^2)^2$$

Using the oscillation parameters extracted by Super-Kamiokande, this corresponds to a decay rate of $\sim 6 \times 10^{-53}$. The impact of neutrino oscillation on the neutrinoless decay is therefore negligible.

![Feynmann diagram for the decay $\tau^- \rightarrow \mu^- \gamma$ via neutrino oscillation.](image)

There is a new limit on this decay from the CLEO experiment [12]:

$$B(\tau^- \rightarrow \mu^- \gamma) < 1.1 \times 10^{-6}$$

at the 90% CL. We can expect an order of magnitude improvement of sensitivity in the decay with the b-factory experiments in the near future.

10.2. Status of Search for Lepton Number Violating Decays

The large $\tau$ mass allows the search for lepton number violating decays in many channels: purely leptonic, radiative decay with a lepton or proton in the final state, lepton or proton plus pions or kaons. The experimenters have searched in 49 decay modes and the 90% CL upper limits [4] are typically $\sim 10^{-6}$. The limits on the decays that violate both lepton and baryon numbers (but conserve baryon minus lepton number) are new since Tau98. There are two recent 90% CL limits from BELLE [13] with $K^0$:

$$B(\tau^- \rightarrow e^- K^0) < 7.7 \times 10^{-6}$$

$$B(\tau^- \rightarrow \mu^- K^0) < 8.8 \times 10^{-6}$$

These represent significant improvements over the previous limit [14] set by MARK II in 1982.
At this Workshop, Ilakovac presented the predictions on the branching fractions for lepton number violating decays based on a model with additional heavy Dirac neutrinos. Some of the predictions are tantalizingly close to the current experimental limits, in unit of $10^{-6}$:

\[
\begin{align*}
B(\tau^- \to e^- e^+ e^-) &< 2.9 \times 10^{-6} \\
B(\tau^- \to e^- \mu^+ \mu^-) &< 1.8 \times 10^{-6} \\
B(\tau^- \to e^- \pi^0) &< 3.7 \times 10^{-6} \\
B(\tau^- \to e^- \rho) &< 2.0 \times 10^{-6} \\
B(\tau^- \to e^- \phi) &< 6.9 \times 10^{-6} \\
B(\tau^- \to e^- \pi^+\pi^-) &< 2.2 \times 10^{-6} \\
B(\tau^- \to e^- K^+ K^-) &< 6.0 \times 10^{-6} \\
B(\tau^- \to e^- K^0 \bar{K}^0) &< 0.66 \times 10^{-6} \\
B(\tau^- \to \mu^- K^0 \bar{K}^0) &< 0.13 \times 10^{-6}
\end{align*}
\]

where $y_{\tau e}^2$, $z_{\tau e}^2$, and $z_{\tau \mu}^2$ are parameters that depend on the masses of the heavy Dirac neutrinos and their mixings with the ordinary Dirac neutrinos. It is evident that for two of the related decays, $\tau^- \to e^- \rho$ and $\tau^- \to e^- \pi^+\pi^-$, the experimental limits have already constrained some of the parameters in the theory. The last two decays have no experimental limits and are within the reach of the current b-factory experiments.

10.3. Future Prospect on Search for Lepton Number Violating Decays

We can expect another order of magnitude improvement in sensitivity for lepton number violating $\tau$ decays in the near future from the b-factory experiments. This is based on the assumption that the search is not background limited as was the case with CLEO II which provides all but four of the 49 limits.

LHCb is an experiment at the Large Hadron Collider (LHC) at CERN that may have good sensitivity for neutrinoless $\tau$ decays produced in $pp$ collisions at a center-of-mass energy of 14 TeV. The detector is a single arm spectrometer covering the pseudo-rapidity region $1.8 < \eta < 4.9$. The main source of $\tau$ leptons is from the $D_s$ decays, accounting for 77% of the total rate. LHCb plans to collect data at low luminosity, $2 \times 10^{-32}$ cm$^{-2}$s$^{-1}$, in order to reduce occupancy and radiation damage. This corresponds to the production of $2.5 \times 10^{11}$ $\tau$'s, of which $0.7 \times 10^{11}$ $\tau$'s are in the LHCb acceptance. This production rate has a large uncertainty because the branching fraction for $D_s^+ \to \tau^+ \nu_\tau$ is poorly known. The experiment has investigated the sensitivity on the gold-plated decay, $\tau^- \to \mu^- \mu^+ \mu^-$, by requiring three muons with a detached vertex. The expected mass resolution is excellent, 5.4 MeV. However, the potential signal is on a large combinatoric background of the true muon from $b \to \mu X$ and random tracks. Under the assumption of no observed signal, the experiment can set a 90% CL limit of $1.8 \times 10^{-7}$ after one year of data collection. This is not very competitive in view of the fact that the b-factory experiments will reach this sensitivity in the near future and LHCb is not scheduled to commence data collection until the year 2005. The estimate of the sensitivity is preliminary and the search technique is currently being refined. Possible modification to the trigger to enhance the sensitivity is being considered. The sensitivity to other neutrinoless $\tau$ decays with pions and kaons will be investigated.

I would like to repeat the suggestion I made at Tau98: Both CDF and D0 may have good sensitivity to neutrinoless $\tau$ decays and I would like to urge the experimenters to perform the search!

11. Future Prospect

In this section, I will discuss the future prospects of the physics of the $\tau$ lepton. The future prospect for the $\tau$ neutrino has already been discussed in the section on neutrino oscillation.

In the next few years, we expect most of the results will come from the b-factory experiments. Both BABAR at SLAC and BELLE at KEK have collected a data sample comparable to the CLEO II experiment at Cornell. The CLEO II experiment collected 13.5 fb$^{-1}$ of data, corresponding to $1.23 \times 10^7 \, \tau^+ \tau^- \, \text{produced events}$. We expect most of the analyses performed by CLEO II will be repeated with smaller statistical errors. It will require care and hard work to reduce the systematic errors in the systematic limited analyses. In the following, I will discuss a few analyses that we can expect to see significant progress in the near future:
• Michel Parameters: We can expect significant improvement in the measurement of the Michel parameters with the much larger data samples. In particular, the measurement of the low-energy parameter $\eta$ will be greatly improved. As the name implied, the biggest sensitivity to this parameter is at low lepton energy and the parameter can only be measured in the muon decay channel because of the helicity flipping factor $m_l/m_\tau$ for the daughter charged lepton in Eq. (12) as discussed in Section 3. The CLEO experiment can only identify muons with momentum above 1 GeV. The muon detector of BABAR, for example, can identify muons with momentum as low as 0.6 GeV, greatly increasing the sensitivity to $\eta$.

• Cabbibo-Suppressed Decays: The single most important difference between the CLEO II and b-factory detectors is the addition of an excellent kaon identification system. This will allow precision measurements of major kaon channels and observation of new rare kaon modes, reminiscence of the advance in the study of decays with $\pi^0$ and $\eta$’s provided by the CsI calorimeter of the CLEO II detector.

• Second Class Current: Observation of the second class current decay $\tau^- \rightarrow \pi^- \eta \nu_\tau$ is one of the major goals of $\tau$ physics of the b-factory experiments. The branching fraction of the decay is expected to be $(1.2 - 1.5) \times 10^{-5}$ [17]. The upper limit on the branching fraction as extracted by the CLEO II experiment [18] is $B(\tau^- \rightarrow \pi^- \eta \nu_\tau) < 1.4 \times 10^{-4}$ at 95% CL. In the analysis, the major background with an $\eta$ signal is from the hadronic events ($e^+e^- \rightarrow q\bar{q}$) and the decays $\tau^- \rightarrow \pi^-\pi^0\eta \nu_\tau$ and $K^-\eta \nu_\tau$. The latter decay has the smallest contribution and can be easily eliminated with the excellent kaon identification capacity. The other two backgrounds can be suppressed with a much tighter photon veto. This selection criterion was not imposed in the CLEO II analysis because the analysis was designed to study the decay $\tau^- \rightarrow K^-\eta \nu_\tau$, which had a different background contamination. The second class current decay can therefore be observed in the near future.

• $\nu_\tau$ Mass: The atmospheric $\nu_\mu$ deficit observed by Super-Kamiokande favors the solution of $\nu_\mu \rightarrow \nu_\tau$ oscillation with $\Delta m^2 = 0.0032$ eV$^2$. Since the upper limit on the $\nu_\mu$ mass is 0.19 MeV at the 90% CL [4], this implies that the $\nu_\tau$ mass must also be less than 0.19 MeV, beyond the sensitivity of any b-factory experiment. However, as discussed in Section 9, the confirmation of the $\nu_\mu \rightarrow \nu_\tau$ oscillation hypothesis with the detection of $\nu_\tau$ appearance will have to wait until the year 2005. The experimentalists should therefore continue to measure the $\nu_\tau$ mass with the much larger data sample. With the excellent kaon identification, the use of kaon decay modes such as $\tau^- \rightarrow \pi^-K^+K^-\nu_\tau$ and $K^-K^0\nu_\tau$ could greatly enhance the sensitivity. With luck, a sensitivity to $\nu_\tau$ mass as low as 10 MeV is quite possible.

12. Conclusion

A large number of results were presented at Tau2000. The Standard Model is being tested both with the $\tau$ lepton as a decay product and with the decay products of the $\tau$ lepton. This includes the test of lepton universality, measurement of the Lorentz structure, and search for lepton number violating decays. There is no hint of physics beyond the Standard Model. The $\tau$ lepton is also used as a laboratory to measure the strong coupling constant, strange quark mass, and the hadronic vacuum polarization, in addition to the test of QCD.

The neutral partner of the $\tau$ lepton, $\nu_\tau$, on the other hand, points to physics beyond the Standard Model via the observation of $\nu_\mu \rightarrow \nu_\tau$ oscillation. This implies the violation of lepton number conservation, a fundamental assumption of the Standard Model. This challenge to the Standard Model needs to be confirmed with the direct observation of $\nu_\tau$ appearance in an $\nu_\mu$ beam, an observation that is now proven to be possible by
the direct observation of $\nu_\tau$ by DONUT.

This is the ten-year anniversary of the Workshop dedicated to examine the $\tau$ lepton and its neutrino. There has been tremendous progress in the field. The first Workshop marked the transition from DORIS/PEP/PETRA to CESR/LEP. This Workshop marks the transition to the b-factory and neutrino oscillation experiments. LEP is a laboratory where the major decay modes of the $\tau$ lepton can be studied with high efficiency and hence low systematic error. CLEO II excels in decays with $\pi^0$ and $\eta$ and dominates the search for lepton number violating decays via its large data sample. The b-factory experiments will collect a significantly larger data sample with excellent kaon identification, opening a new window of opportunity to challenge the Standard Model.

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