Research on Location Selection of UAV Distribution Center Based on Improved Gravity Method

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Abstract. In order to speed up logistics distribution efficiency and reduce distribution costs, logistics companies around the world began to use drones for related distribution work. In addition to upgrading the hardware technology of UAV, optimizing the distribution path is also one of the ways to reduce the distribution cost of UAV distribution center. In this paper, the location of UAV distribution center will be studied theoretically based on the economic optimization and equipment maintenance optimization. Finally, the feasibility of the algorithm is verified by simple simulation application.

Keywords: Drone logistics; Location Algorithm; Center of gravity algorithm.

1. Introduction
The drone as an emerging, multi-purpose intelligent tool has been used in many industries[1]. The use of drones for logistics distribution activities has become possible in recent years[2]. With the deepening of the vertical take-off and landing of fixed-wing aircraft and the production, the drone distribution will shift towards higher speeds and higher loads[3]. The use of vertical take-off and landing drones for logistics distribution has solved the problem of untimely delivery caused by subjective and objective reasons such as road congestion and traffic control. At the same time, based on the flying nature of the drone, it is sent from the distribution center to the receiving place. The route is bound to be a "point-to-point" straight route[4]. With these significant advantages, it is possible to use new drones to deliver vaccines, medicines, etc and solve related distribution problems caused by traffic problems in urban distribution in the future[5].

In the construction of distribution center, in addition to updating the equipment, the cost can be reduced in the use process, and optimizing the distribution path will further reduce the cost. As the UAV distribution adopts the "point-to-point" straight traveling route, and because of its low single freight volume, it can only be delivered to a single receiving place, so it needs to return to the distribution center immediately after unloading the goods for the next distribution work. Because of its characteristics, it shows that the process of finding distribution center is similar to the way of finding the center of gravity on geometric objects, so we can borrow the method of finding the center of gravity of geometric objects and improve and apply it in combination with the characteristics of the research itself.

2. Introduction to the Theory of Gravity Center Model for the Location of UAV Distribution Center
The center of gravity method originated from geometry and the original purpose was to find the center of gravity of irregular geometric objects[6-8]. After determining the location of each point, this method
is used to study the location of distribution centers that the sales cost is regarded as a linear function of the transportation distance and the number of transportation.

2.1. General Assumptions of the Model
There are many complex problems to be considered in UAV logistics, such as aircraft condition and weather condition. The existence of these problems will directly lead to errors in the calculation results, which should be arranged according to the actual situation. The theoretical process shown in this paper is based on the optimal environment of any execution conditions, so no general assumptions are set separately.

2.2. Constraints of the Model
The location model of UAV logistics base needs to meet certain constraints, and this part also needs to be carried out according to the actual situation. Similarly, this part is carried out under the theoretical optimal condition, so it is not set separately.

2.3. Model-related Parameter Settings
This article will set related parameters that generally exist from a theoretical perspective, as shown in Table 1:

| Parameter symbol | parameter |
|------------------|-----------|
| $O$              | Location of drone distribution center |
| $X$              | UAV’s own weight |
| $M = \{i = 1, 2, \ldots, n\}$ | Distribution area |
| $x = \{i = 1, 2, \ldots, n\}$ | The abscissa of the demand area in the two-dimensional map |
| $y = \{i = 1, 2, \ldots, n\}$ | The ordinate of the demand area in the two-dimensional map |
| $xx$             | The abscissa of the UAV distribution center in the two-dimensional map |
| $yy$             | Vertical coordinate of UAV distribution center |
| $R$              | Transportation rate from drone distribution center to demand area and when returning |
| $j$              | Number of unmanned aerial vehicles |
| $V = \{i = 1, 2, \ldots, n\}$ | Shipment volume from drone distribution center to demand area |
| $D = \{i = 1, 2, \ldots, n\}$ | Straight line distance from drone distribution center to demand area |
| $T = \{i = 1, 2, \ldots, n\}$ | Transportation time from drone distribution center to demand area |
| $C = \{i = 1, 2, \ldots, n\}$ | Transportation cost required from drone distribution center to demand area |
| $FC$             | Unmanned aerial vehicle unit daily maintenance cost |
| $FCZ$            | Unmanned aerial vehicle maintenance cost |
| $ZC$             | Total transportation cost |
| $P$              | total cost |
| $A$              | Unmanned aerial vehicle maximum cargo capacity |
| $TS$             | Maximum flying speed of UAV |
| $TZ$             | The total daily delivery time of the UAV |

2.4. Drone Logistics Cost Analysis

2.4.1. Single transportation cost. The transportation of this scheme refers to the costs incurred by the drone during the flight. Generally, the drone flying costs the electricity or (and) fuel, and the single one-way transportation cost:
2.4.2. Total maintenance cost. The maintenance cost of this solution refers to the maintenance cost of unmanned aerial vehicles, as well as the cost of human resources. For the convenience of calculation, the total maintenance cost is expressed as:

\[ FCZ = jFC \]  

(2)

2.4.3. Total transportation cost. The total transportation cost refers to the total transportation cost from the drone distribution center to each demand area, which is expressed as:

\[ ZC = \sum_{i=1}^{n} C_i + \sum_{i=1}^{n} RXD_i \]  

(3)

2.4.4. Total cost. Total cost in this scheme, the total cost only includes the total transportation cost and the total maintenance cost, which is expressed as:

\[ P = FCZ + ZC \]  

(4)

2.5. Model Establishment

2.5.1. Optimal model of transportation costs. According to the distance formula between two points, the distribution distance from the distribution center to a single demand place is obtained:

\[ D_i = \sqrt{(x_i - xx)^2 + (y_i - yy)^2} \]  

(5)

Substitute equations (1) and (5) into equation (3). At this time, the function is determined as the transportation cost change function. The formula is:

\[ ZC = \sum_{i=1}^{n} C_i + \sum_{i=1}^{n} RXD_i = \sum_{i=1}^{n} R(V_i + X) \sqrt{(x_i - xx)^2 + (y_i - yy)^2} + \sum_{i=1}^{n} RX \sqrt{(x_i - xx)^2 + (y_i - yy)^2} \]  

(6)

2.5.2. Optimal model of maintenance cost. With an appropriate number of aircraft, the mission can be completed exactly in one day. At this time, the minimum number of aircraft can be determined, which meets the following mathematical relationship:

\[ \sum_{i=1}^{n} (2VD_i)/(Aj \times B) = TZ \]  

(7)

Then the function to determine the number of unmanned aerial vehicles (must be rounded) according to the formula is:

\[ j = \frac{\sum_{i=1}^{n} VD_i}{2ABTZ} \]  

(8)

According to the objective function, the lowest total maintenance cost is obtained:

\[ FCZ = jFC = FC \times \frac{\sum_{i=1}^{n} VD_i}{2ABTZ} \]  

(9)

2.5.3. Optimal model of total cost. Combining equations (6), (9) and (4), we get a new equation:
Here, we have the objective function, which adds the condition of maintenance cost to the traditional site selection problem. After further integration, its formula is:

\[ P = \sum_{i=1}^{n} R(V_i + X)\sqrt{(x_i - xx)^2 + (y_i - yy)^2} + \sum_{i=1}^{n} RX\sqrt{(x_i - xx)^2 + (y_i - yy)^2} + FC \times \frac{\sum_{i=1}^{n} V_i D_i}{2ABTZ} \]  

(10)

When the P value is the smallest and the P value is 0, the theoretical total coordinates of the UAV base with the optimal total cost and the machinery and equipment needed to meet the one-day flight mission are determined. At this time, the coordinates of the UAV base O are determined for:

\[ xx(yy) = \frac{\sum_{i=1}^{n} [(2RX + RV_i + V_i)(x_i(y_i) - xx(yy))] + D_i}{\sum_{i=1}^{n} [(2RX + RV_i + V_i) + D_i]} \]  

(11)

2.6. Subsequent Calculation

2.6.1. Find the partial derivative. According to equation (12), the equation contains unknown numbers \( D_i \), \( xx \) and \( yy \). Because there is a correlation between these three unknown numbers, the final total cost will be a function of \( xx \) and \( yy \). In order to minimize the total cost, in equation (11), find the partial derivatives of \( xx \) and \( yy \), and the result is 0, which is:

\[ \frac{\partial P}{\partial x(y)} = \sum_{i=1}^{n} (2RX + RV_i + V_i)(x_i(y_i) - xx(yy)) + D_i = 0 \]  

(13)

2.6.2. Iterat. Because equation (13) still contains the unknown number \( D_i \), that is, it contains the required unknown numbers \( xx \), \( yy \), so further iterations are needed to calculate.

Step 1: Use the center of gravity of all areas as the initial coordinates of the drone base \((xx^0, yy^0)\).

Step 2: Use equation (11) to calculate its corresponding cost \( P^0 \).

Step 3: After that \((xx^r, yy^r)\) Substitute into the following formula (14) \( D_{i(k-1)} \), Calculate the preferred address of the drone base \((xx^k, yy^k)\). Its substitution formula is as follows:

\[ xx^k(yy^k) = \frac{\sum_{i=1}^{n} [(2RX + RV_i + V_i)(x_i(y_i) + D_{i(k-1)})]}{\sum_{i=1}^{n} [(2RX + RV_i + V_i) + D_{i(k-1)}]} \]  

(14)

Step 4: Use equation (11) again to calculate the total cost \( P^k \) corresponding to \((xx^k, yy^k)\).

Step 5: Compare \( P^k \) with \( P^0 \). If \( P^k < P^0 \), return to the third step to calculate again. At this time, substitute \((xx^r, yy^r)\) into formula (11) to calculate the UAV base and then optimize the coordinate \((xx^k, yy^k)\) to \( P^{r+1} \geq P^r \).

Step 6: When \( P^{r+1} \geq P^r \), then find the optimal coordinates of UAV base \((xx^r, yy^r)\).
This method tends to use the center-of-gravity points between the regions as the initial address, and uses any primary coordinates to calculate. The speed of convergence and the optimal solution will be affected. At the end of step, the entire algorithm that uses the center of gravity method has been relatively improved for the characteristics of UAV distribution has been completely completed in theoretical modeling and it can be directly applied to real problems.

3. Simulation and Simulation Calculation
After the establishment of the model, it is necessary to verify the availability of the model through simulation operation. In this paper, 22 demand places are randomly generated, and simulation generation includes delivery address coordinates and demand quantity. Specific generated data are shown in Table 2. At the same time, according to the actual situation, the model constants are reasonably set, among which \( A = 15, TS = 40, TZ = 9 \).

| Place | Coordinates | Demand |
|-------|-------------|--------|
| M1    | 103.269447,28.034531 | 2475   |
| M2    | 103.304884,28.043158 | 960    |
| M3    | 103.276321,28.054926 | 795    |
| M4    | 103.241414,28.000887 | 675    |
| M5    | 103.228056,28.051116 | 780    |
| M6    | 103.235924,28.026716 | 495    |
| M7    | 103.197695,28.075174 | 540    |
| M8    | 103.213861,28.07315 | 660    |
| M9    | 103.195186,28.059137 | 570    |
| M10   | 103.35618,27.968944 | 1575   |
| M11   | 103.363165,27.980755 | 1230   |

According to the above simulation data, use MATLAB to solve the model, and finally get the result after 11 iterations: When the coordinates of O point are (103.3377, 28.0070), the result is the best. At this time, there are 61 unmanned aerial vehicles, and the total cost can be calculated by multiplying the distance at this time by the actual unit price. Figure 1 shows the simulated result image, from which it can be intuitively judged that it is almost located at the center of gravity of the two-dimensional plan, so this method can be applied to the actual calculation in the future.

![Simulation result diagram](image)

4. Conclusions and Suggestions
It is based on the characteristics of point-to-point straight distribution route of unmanned aerial vehicle (UAV) to solve the optimization problem of UAV distribution center location by gravity method. However, when the objective factors such as airspace management change, the distribution route may not satisfy the point-to-point transportation. However, it is only necessary to change the distribution...
route to a certain extent according to the limitation of airspace, so it is necessary to improve the existing barycenter model to a certain extent according to the actual situation. In addition, if the area where the final distribution center is located cannot meet the basic conditions of distribution center construction, for example, the final result of the model is in water or deep mountains, multiple areas that can be built can be selected within a certain distance near the theoretical point, and the place with the best construction cost can be found by using the inverse operation of the barycenter method. However, with the further upgrade of UAV technology in the next few years, for example, the UAV with larger cargo capacity built for accurate airdrop will no longer be a single-point to single-point distribution form, so it will be more difficult to optimize the location of distribution center by using the barycenter method again. At this time, it may be necessary to consider another algorithm as the best algorithm for studying the location of new distribution center.

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