RADIUS AND STRUCTURE MODELS OF THE FIRST SUPER-EARTH PLANET

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ABSTRACT

With improving methods and surveys, the young field of extrasolar planet studies has recently expanded into a qualitatively new domain—terrestrial (mostly rocky) planets. The first such planets were discovered during the past year and a half, judging by their measured masses of less than 10 $M_{\oplus}$ (“super-Earths”). They are introducing a novel physical regime that has not been explored before, as such planets do not exist in our solar system. Their compositions can be completely terrestrial, or they may harbor an extensive ocean (water and ices) above a rocky core. We model the structure and properties of the first super-Earth (mass $\sim 7.5 M_{\oplus}$), discovered in 2005, illustrating the possible compositions and providing radius evaluations in view of future detection of similar planets by transits. We find that there exists a threshold in radius above which a super-Earth most certainly has an extensive water content. In the case of GJ 876d, this threshold is at about 12,000 km. Our results show that unique characterization of the bulk composition of super-Earths will be possible with future transit studies.

Subject headings: Earth — planetary systems — planets and satellites: individual (GJ 876d)

1. INTRODUCTION

In the last 11 years, more than 200 exoplanets have been discovered, but only in the last year and a half has the minimum mass for detection been pushed down to allow for terrestrial planets. The first extrasolar planet with a mass lower than 10 $M_{\oplus}$, that is, a “super-Earth,” was discovered by Rivera et al. (2005), orbiting an M4 V star named GJ 876. Its estimated mass is 7.5 ± 0.7 $M_{\oplus}$ and it has an orbital period of 1.94 days. It is close to the host star, and the surface temperature is calculated to lie between 430 and 650 K (Rivera et al. 2005). The star is known to have two gas giant planets, GJ 876b and GJ 876c, in 30 and 61 day orbits, respectively (Marcy et al. 1998, 2001). Hence, the planetary system of star GJ 876 has both an architecture and a planet—the super-Earth—that are unfamiliar to our solar system experience. Two other low-mass planets have been discovered in the past year: OGLE-2005-BLG-390Lb (̃5 $M_{\oplus}$ at 5 AU; Beaulieu et al. 2006) and HD 69830b (̃10 $M_{\oplus}$ at 0.08 AU; Lovis et al. 2006), exemplifying the variety in super-Earths that will be discovered in the near future with missions such as Kepler.

Calculating the internal structure of a super-Earth can help one determine how different or similar to Earth such a planet might be. One of our goals is to provide mass-radius relations that will help characterize super-Earths discovered by the transit method. We describe in this study the tools we have to characterize super-Earths, and we use GJ 876d as an example of our model’s capabilities and limitations. We first describe the numerical method used to obtain density, pressure, and temperature profiles as a function of radius. For more details, we refer the reader to Valencia et al. (2006), where we provided the first theoretical grid of models for super-Earth planets. To calculate a planet’s radius and internal structure, we need to make reasonable assumptions about its composition. It might be completely rocky or it might have accumulated a substantial amount of ices, depending on the material available during formation. We show the results and implications for different likely compositions of GJ 876d as an example that can be replicated for any super-Earth. In addition, we explore the effects of tidal heating that might be present in short-period orbits (such as that of GJ 876d) with nonzero eccentricities.

2. MODEL

2.1. Numerical Method and Equations of State

We model the planet as being composed of distinct spherical layers that are homogeneous in composition. For a terrestrial planet, these are the mantle, which is divided into lower mantle and upper mantle, and the core, which, depending on the temperature structure of the planet, might be divided into a liquid outer core and a solid inner core, as is the case for Earth. The composition for the different layers is taken from the mineralogical composition known for Earth (McDonough & Sun 1995). The upper mantle is taken to be olivine (ol) and higher pressure forms of olivine (wadsleyite [wd] and ringwoodite [rw]); the lower mantle develops when olivine transforms to perovskite (pv) and ferromagnesiowustite (fmw), with an additional shell at high pressures when pv transforms to post-perovskite (ppv). The solid inner core of Earth is composed of Fe and small quantities of Ni, and the outer liquid core is composed of Fe and a light alloy (see Fig. 1). The candidates for this alloy are S, Si, O, C, and H, and to this day there is no consensus on which one(s) or in what amount(s). We have used in this study compositions of pure Fe and Fe$_{0.8}$(FeS)$_{0.2}$ to show the uncertainties in the radius of a planet due to lack of knowledge of the composition of the core.

For planets that harbor a substantial H$_2$O/ice content (“ocean planets”), there is an additional layer overlying the rocky interior (mantle and core), composed of two shells—water above high-pressure phases of ice. In some cases the pressure is not
high enough (the amount of H$_2$O is not large enough) to solidify water, and the ice layer is absent (Fig. 1). The thickness of the water layer is determined by the intersection between the pressure-temperature ($P$-$T$) curve of the planet and the melting curve of ice. In the case of low surface temperature, the planet will have an additional shell above the water layer composed of two silicate shells (ppv+fmw, pv+fmw); and an upper mantle composed of two silicate shells (ppv+fmw, pv+fmw). The thickness of the shells will depend on the $P$-$T$ profile for the planet and the amount of mass in the core. An ocean planet (right) will have an additional water/ice layer above the rocky core.

\[
\frac{d\rho}{dr} = -\frac{\rho(r)g(r)}{\phi(r)}, \quad \frac{dg}{dr} = 4\pi G\rho(r) - \frac{2Gm(r)}{r^3}, \quad \frac{dm}{dr} = 4\pi r^2 \rho(r), \quad \frac{dP}{dr} = -\rho(r)g(r),
\]

where $\phi(r) = K_S\rho(r)/\rho_0$ is the seismic parameter, which can be calculated with an equation of state (EOS) for $K_S$, the adiabatic bulk modulus; $G$ is the gravitational constant, and $r$ is the radius.

The model integrates from the surface inward with boundary conditions for $P$, $T$, $\rho$, and total mass ($M$). It starts with a guess for the planet's radius ($R$), which yields a surface gravity value, and determines the structure to the last shell where there could be an excess or deficit in mass depending on whether the starting $R$ is too small or too large, respectively. The model then uses a bisection/Newton approach to determine the radius that yields zero mass and zero gravity at the center of the planet (within 5 m).

The EOS that we have implemented for all layers except the water layer is the Vinet equation (Vincent et al. 1987, 1989), because it is the best analytical EOS for extrapolation:

\[
K_T(\rho, 300) = 3K_0(x^{1/3} - x^{1/3}) \exp \left[ \frac{3}{2}(K_0' - 1)(1 - x^{-1/3}) \right],
\]

(Hama & Suito 1996), where $x = \rho/\rho_0$ and $K_0$ and $K_0'$ are the isothermal bulk modulus and its first pressure derivative at a reference state—zero pressure and 300 K. Here $K_S = K_T(1 + \alpha\gamma T)$, where $\alpha$ is the thermal expansion coefficient and the Grünneisen parameter $\gamma = \gamma_0 \alpha^{-1}$, with $q = -\partial \ln \rho$ in $\gamma$. Most high-pressure experiments fit their data to the third-order Birch-Murnaghan (BM3) EOS, which is derived from a truncation of a power series of the Helmholtz free energy with strain (Poirier 2000). The extrapolation of this EOS is highly uncertain, since the fourth term in the expansion might not be smaller than the third term in some cases. We use values for $K_0$ and $K_0'$ obtained from the literature corresponding to the BM3 EOS to obtain $P(V)$ and refit the data to the Vinet EOS within a volume compression between 1 and $\frac{3}{2}$. The general trend is that $K_0$ is smaller by a few GPa and $K_0'$ is larger by $\sim 6\%$--$8\%$ in the Vinet fit. We used the Rankine-Hugoniot EOS from Stewart & Ahrens (2005) within the water region.

To incorporate the effects of temperature into the EOS, we added a thermal pressure term (due solely to deviations in temperature from 300 K) for core and mantle regions, which translates into a thermal bulk modulus correction. With $R$ the universal gas constant, $\theta = \theta_0 \exp \left[ \frac{(\gamma - \gamma_0)}{\gamma_0} \right]$ the Debye temperature, and $n$ the number of atoms in a unit cell, the thermal correction is expressed as

\[
K_T(\rho, T) = K_T(\rho, 300) + \Delta K_{th}(\rho, T),
\]

where

\[
\Delta K_{th} = 3nR\gamma(\rho(f(T) - f(T_0)))
\]

with

\[
f(T) = (1 - q - 3\gamma)\frac{T^4}{\theta^3} \int_0^{\theta/T} \exp \frac{\xi}{1 - \xi} \frac{d\xi}{1 + 3\gamma \exp(\xi/T) - 1}.
\]

For the icy region, we incorporated the temperature effects in the density through the thermal expansion coefficient:

\[
\rho(P, T) = \rho(P, 300) \exp \left[ \int_{T_0}^{T} \alpha(P, T')dT' \right],
\]

\[
\alpha(P, T) = (a_0 + a_1 T) \left( 1 + \frac{K_0'}{K_0} P \right)^{-b},
\]

where $a_0$, $a_1$, and $b$ are coefficients determined at zero pressure.

2.2. Thermal Model

In order to incorporate temperature effects into the EOS, we need a model describing the temperature profile in the planet. With Earth as our starting point, we use parameterized convection theory to model the temperature regime of super-Earths. The surface heat flow on Earth (44 TW; Pollack et al. 1993) reflects the heat from radioactive sources in the mantle and crust (some authors also place potassium in the core) and secular cooling from an initial hot state. If we account for the concentration of heat sources in the crust (generating about 5--12 TW [Vaucquer 1991; Taylor & McLennan 1995; Davies 2000]; we take 8 TW) and assume a bulk silicate concentration of uranium, potassium, and thorium, radioactive heat in the mantle accounts for $\sim 58\%$ of the total heat flow. In Valencia et al. (2006), we reported on scaling the heat flow with mass for Earth-like planets as a first attempt to scale radioactive heat sources and secular cooling together. Here we refine this approach by separating the
to the two sources. If we assume that the material that makes super-Earths is the same as the bulk material for Earth, then the radioactive heat sources will scale with mantle mass. Planets that have larger mantles will have higher heat flows. It is difficult to scale secular cooling for a super-Earth because we do not know the role that mass plays in the evolution of a planet. With the scaling laws derived in Valencia et al. (2006) and parameterized convection, we have found (D. Valencia et al., 2007, in preparation) that a massive planet is likely to convect in a plate tectonic regime similar to Earth. Intuitively, the more massive the planet is, the higher the Rayleigh number (Ra) that controls convection, the thinner the top boundary layer (lithosphere), and the faster the convective velocities. We find that despite the high Ra, the heat flux of a massive rocky planet scales proportionally with mass, so that the temperature beneath the lithosphere, in a temperature-dependent viscosity case, is almost independent of planetary mass (in contrast to the effect of surface insulation explored by Lenardic et al. [2004]). This scenario is favorable to the subduction of the lithosphere and onset of plate tectonics (Moresi & Solomatov 1998). Thus, we assume that the evolution of a super-Earth might be such that it leads to the same proportion of secular cooling to radioactive heating as for Earth. This is the first attempt to scale secular cooling with mass.

We define the Rayleigh number Ra (the dimensionless parameter that controls convection) in terms of the heat flux ($q_i$):

$$Ra = \frac{\rho g a q_i / k}{\kappa \eta} D^4,$$

where $g$, $k$, $\kappa$, and $\eta$ are the average gravity, conductivity, diffusivity, and viscosity of the mantle. The thickness of the top boundary layer is

$$\delta = a D \left( \frac{Ra}{Ra_{\text{crit}}} \right)^{-1/4}$$

independent of the size of the mantle, where $Ra_{\text{crit}} \approx 1000$ is the critical Rayleigh number and $a$ is a coefficient of order unity. We treat viscosity in two ways: (1) the isoviscous case and (2) using a temperature-dependent treatment $\eta(T) = \eta(T/T_0)^{-30}$ (a power-law fit to an Arrhenius law for viscosity within the temperature range; Davies 1980). The planet modeled in this study has a convective core, mantle, and ice/water layer where the temperature can be described as adiabatic:

$$\frac{dT(r)}{dr} = -\frac{\rho(r)g(r)T(r)}{K_S(r)} \gamma(r).$$

(13)

All interfaces between the chemically distinct layers develop boundary layers that transfer heat conductively,

$$\frac{dT}{dr} = -\frac{q}{k}.$$

(14)

Their thickness depends strongly on the local viscosity. Owing to the low viscosity of the core and water, we only model conductive boundary layers at the top and bottom of the mantle. The top boundary layer’s thickness is described by equation (12). In an isoviscous internally heated case, the bottom boundary layer of the mantle would vanish. Conversely, in an isoviscous heated-from-below case it would have the same thickness as the top layer. Since we assume these planets are in an intermediate regime (the same as Earth), the bottom boundary layer is taken to be half the top one, and the heat flux reflects the heat coming out of the core.

Even though there are two critical assumptions in determining the temperature structure—the constant ratio between internal heating and secular cooling, and the thickness of the lower boundary layer in the mantle—the effects of temperature on the internal structure of a planet are small, particularly the effect on radius. The temperature profile is needed mostly to determine the location of the phase changes that determine the different regions in the planet. A temperature-dependent viscosity increases the radius by $\sim20$ km compared with the isoviscous case, showing that neither assumption is critical.

The numerical model iterates several times until convergence is achieved between the boundary layer thickness and the radius (i.e., surface heat flux), so that the temperature profile is determined self-consistently. The model needs as input the composition of the different regions, as well as the ice mass fraction (IMF) and core mass fraction (CMF), and surface $P$ and $T$. The output is density, pressure, temperature, mass, and gravity as functions of radius, the total radius, and the locations of the phase changes and composition boundaries. Table 1 shows the different compositional values used in this study. The compositions

| Layer               | Composition | $\rho_0$ (kg m$^{-3}$) | $K_0$ (GPa) | $K_0'$ | $\gamma_0$ | $q$ | $\theta_0$ | Refs. |
|---------------------|-------------|-------------------------|-------------|---------|-------------|----|-------------|-------|
| Ocean shell         | H$_2$O      | 998.23                  | 2.18        | ...     | ...         | ...| ...         | 1     |
|                     | Ice VII+X   | 1463                    | 2.308       | 4.532   | 1.2         | 1  | 1470        | 2, 3  |
| Upper mantle        | ol          | 3347                    | 126.8       | 4.274   | 0.99        | 2.1| 809         | 4     |
|                     | wd+rw       | 3644                    | 174.5       | 4.274   | 1.20        | 2.0| 908         | 4     |
| Lower mantle        | pv+fmw      | 4152                    | 223.6       | 4.274   | 1.48        | 1.4| 1070        | 4     |
|                     | ppr+fmw      | 4270                    | 233.6       | 4.524   | 1.68        | 2.2| 1100        | 5     |
| Core                | Fe           | 8300                    | 160.2       | 5.82    | 1.36        | 0.91| 998         | 6, 7  |
|                     | Fe$_{0.8}$FeS$_{0.2}$ | 7171                  | 150.2       | 5.675   | 1.36        | 0.91| 998         | 6, 7  |

Note.—The data used in each shell of the model have been taken from the different references and refitted to the Venet EOS (Vinet et al. 1987, 1989).

References.—(1) Stewart & Ahrens 2005 (Rankine-Hugoniot EOS); (2) Hemley et al. 1987; (3) Fei et al. 1993; (4) Stixrude & Lithgow-Bertelloni 2005 (a Reuss average was performed according to a mixture of 10% Fe and 30% fmw for the bulk modulus; the thermodynamic parameters $\gamma_0$, $a_1$, and $b_1$ were taken from the most dominant phase in each shell); (5) Tsuchiya et al. 2004 (an increase in density was needed to account for 20% Fe in pv according to $\partial p/\partial x = 0.3$, where $x$ is the iron content [Mao et al. 2004]); a Reuss average was then used with fmw); (6) Williams & Knittle 1997 (we also tried Uchida et al. [2001] parameters for density and bulk modulus and found a discrepancy in the radius of only 60 km for a composition of CMF = 80%); (7) Uchida et al. 2001.
Compositions for GJ 876d

We consider a few basic scenarios for the composition of GJ 876d that are representative but probably not exhaustive. They are (1) a simple Earth-like composition—a planet with a core that makes up 33% of the total mass with a mantle that is 10% (by mole) iron-enriched, and a lower mantle composed of 30% fmw and the rest pv or ppv; (2) Earth-like, but with a very large core (mass fraction of 80%); (3) an ocean planet with a 20% by mass water/ice layer on top of a terrestrial (Earth-like) core; and (4) an ocean planet with a 40% water/ice layer. Our reasons for introducing the latter three scenarios are that case 2 is analogous to Mercury—with its very close orbit, GJ 876d might have a super-Earth with a pure Fe core, its radius would extend to 10,800 km, or 1.70 \( R_\oplus \). The second scenario looks at the effect of having a larger CMF of 80% (Fig. 2, \( top \); lines with stars), perovskite transforms to post-perovskite at a radius of \(~9800\) km, so most of the mantle is actually composed of ppv+fmw. This brings attention to this relatively newly discovered phase present in the lowermost mantle on Earth (only from 125 to 136 GPa) and the importance of determining with accuracy its behavior at high pressures. If this planet had a pure Fe core, its radius would extend to 10,800 km, or 1.70 \( R_\oplus \) (Earth radii), with an additional 130 km (1.71 \( R_\oplus \)) if the core were composed of Fe\(_{0.8}\)(FeS)\(_{0.2}\). Despite the large differences in core density (300 kg m\(^{-3}\)) between the two core compositions, the different locations of the core-mantle boundary that satisfy the CMF cause the radius to differ by very little. This implies that the radius for planets with medium-sized cores is a robust parameter.

The second scenario looks at the effect of having a larger CMF of 80% (Fig. 2, \( top \)), in which case we expect the composition of the core to have a larger effect. A planet that has a higher iron content, presumably mostly in its core, can accommodate more of its mass in the central region and therefore have a smaller radius compared with an iron-poor planet. This family of planets (super-Mercurias; Valencia et al. 2006) exhibit comparatively large bulk densities, as Mercury does in our solar system. If GJ 876d had a core of 80% by mass, its radius would only be 9200 km (or 1.45 \( R_\oplus \)) for a pure Fe core and 9600 km (or 1.5 \( R_\oplus \)) for a sulfur-enriched one.

Next, consider the possibility that this planet has a water/ice layer amounting to 20% or 40% of the total mass on top of a terrestrial core (Fig. 2, \( bottom \)). Despite the high compressibility of ice and water (bulk moduli 1–2 orders of magnitude smaller, respectively, than for silicates), their densities at initial compression are low, and the relation of the density gradient to density is quadratic. Consequently, a large amount of water makes the planet larger. If Earth had a substantial amount of water, its radius would

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2.3. Compositions for GJ 876d

Table 2 shows the different phase boundaries between the silicate mantle minerals used in this study. P is in GPa and \( T \) is in kelvins.

| Transition | Boundary |
|------------|----------|
| ol → wd+rw | \( T = 400^\circ \text{C} \rightarrow 4287 \) |
| rw → pv+fmw | \( P = 22.6 \text{ if } T > 1750 \text{ K} \) |
| \( P = 13,573 \text{ if } T \leq 1750 \text{ K} \) |
| pv+fmw → ppv+fmw | \( T = 1335 \text{ K} \rightarrow 1392 \) |

Note.—The table shows the different phase boundaries between the silicate mantle minerals used in this study. P is in GPa and \( T \) is in kelvins.
extend to 7100 km (1.11 $R_\oplus$) for 20% water content, and to 7600 km (1.19 $R_\oplus$) for 40% water content. Figure 2 shows the density structure, and Figure 3 the pressure-temperature structure, for the different composition cases considered above. The ocean planets have a layer of water above a layer of denser ice, due to the positive Clapeyron slope of ice VII (note the melting curve for ice VII in Fig. 3). Hemley et al. (1987) showed that ice VII transforms gradually to ice X with pressure and proposed an EOS for the combined system. We have adopted those values for this calculation. Below the ice layer, the pressure is ~150 GPa, beyond the transition pressure between pv and ppv ($P = 125$ GPa at $T = 2750$ K; Tsuchiya et al. 2004); therefore, the silicate mantle is made of ppv and fmw—perovskite and upper mantle silicate phases are absent. If GJ 876d had a water/ice layer that accounted for 20% and a core that accounted for 33% of the total mass, its radius would be 11,900 km (or 1.87 $R_\oplus$). An additional 20% of water would extend the radius 550 km more (to 1.95 $R_\oplus$). These radii are 11% and 15% larger, respectively, than the result for an Earth-like composition. Such planets will be easier to detect when transiting their stars. Adding sulfur to the core would extend the radius of both ocean planets by ~130 km. The results for all compositions are shown in Table 3.

In summary, the four bulk compositions produce differences in the super-Earth mean densities and radii that are large enough—from 3% to 30% in radius—to be observationally measurable. For example, if GJ 876d were transiting its star, photometry from the Microvariability and Oscillations of Stars (MOST) satellite could achieve 3% precision in its radius (Rowe et al. 2006), with uncertainty dominated by the radius of the star ($R_\ast = 0.32 R_\odot$).

Our main result is that with a measurement of both mass and radius (even with a precision of 10%), one could certainly tell an iron-rich differentiated terrestrial planet ($R < 9600$ km) from a water-rich (IMF > 20%) ocean planet ($R > 12,000$ km). This is because there is a limited family of compositions that would fit both. To illustrate this point, we have calculated the trade-off curve between the amount of ice (IMF) and core (CMF) that would satisfy $M = 7.5 M_\oplus$ and $R = 11,900$ km (see Fig. 4). The maximum amount of ices such a planet could harbor is ~49%, with CMF = 51% and no mantle. The minimum could be zero if the planet had no core. This extreme case (no core) is so unlikely that a radius detection of 11,900 km or more would necessarily mean that the planet has a water layer. Hence, knowledge of just the radius and mass of a planet—in combination with internal-structure models—can yield valuable structural and

![Density Profile](image1.png)

**Fig. 2.**—Internal structure of GJ 876d: density profile. Four different compositions are illustrated. The surface is to the right and the center of the planet is to the left. The solid lines show the cases in which the composition of the core is taken to be pure Fe; dashed lines are for the case of Fe0.2(FeS)0.8. Lines with stars show the internal structure of GJ 876d if the composition is Earth-like. Squares show the density profile if this planet has 80% of the mass in the core. Circles show the structure if the planet formed outside the snow line and retained 20% of its mass as a water/ice layer. Diamonds show the density structure if GJ 876d retained 40% of water/ice. The Preliminary Reference Earth Model (PREM; Dziewonski & Anderson 1981) is shown for reference.

**Fig. 3.**—Internal structure of GJ 876d: pressure-temperature profile. The $P-T$ structure of GJ 876d is illustrated considering the four compositions presented in this study and pure Fe in the core. The ocean compositions cross the ice VII solidus at ~55 GPa, which means the water layer is 1200 km deep for both ocean planets (IMF = 20% and IMF = 40%).

![Pressure-Temperature Profile](image2.png)

**Table 3**

| Composition | Fe Core (km) | Fe0.2(FeS)0.2 Core (km) |
|-------------|-------------|------------------------|
| Earth-like  | 10786       | 10914                  |
| Rocky, CMF 100% | 9228       | 9580                   |
| Ocean, IMF 20%  | 11890      | 12014                  |
| Ocean, IMF 40%  | 12448      | 12576                  |

**Note.**—Results for the four different bulk compositions, for a pure Fe core and a core with Fe and S. The only appreciable difference, of 400 km, happens in a core-dominated planet.
compositional information about the planet and its formation environment.

4. TIDAL HEATING EFFECTS

The super-Earth’s extremely close proximity to its star GJ 876 means that even small perturbations to its orbit might induce significant additional heating from tides and change its internal structure. We check this by a simple analysis, with a surface heat flux consisting of energy from radioactive heat sources, secular cooling, and tidal heating. We add different amounts of tidal heating $Q_{\text{tidal}}$ to the heat flow $Q$ (see §2.3), which translate into higher Rayleigh numbers and thinner lithospheres, and calculate the thermal structure:

$$Q_{\text{total}} = Q + Q_{\text{tidal}}.$$  \hspace{1cm} (15)

For the calculation of the effects of internal heating, we assume a temperature-dependent viscosity. A simple analysis states that the energy dissipation rate produced by an eccentric orbit in a synchronous planet is

$$\frac{dE}{dt} = \frac{63}{4} e^2 n \left( \frac{R_p}{a} \right)^5 \frac{G M_*^2}{a},$$

(Murray & Dermott 2000), where $G$ is the gravitational constant, $M_*$ is the mass of the star, $a$ is the semimajor axis, $n$ is the mean motion, $\tilde{\mu}$ is the ratio of elastic to gravitational forces [$\tilde{\mu} \approx (10^4 \text{ km}/R_p)^2$], and $Q_o$ is the specific dissipation parameter. In other words, the energy available for tidal heating of the 7.5 $M_\oplus$ Earth-composition planet GJ 876d is

$$1.2 \times 10^{-22} e^2 \text{ watts},$$  \hspace{1cm} (16)

where $Q_o = 280$ as for Earth (Ray et al. 2001). Figure 5 shows the effects of different values of $dE/dt$ on the $P$-$T$ regime for this planet. The figure also shows the solidus (first stage of melting in a two-component system) for MgO, the lower mantle mineral with the lowest melting temperature. With an increasing amount of tidal heating, the internal temperature rises, and in some cases the lowermost mantle might be partially molten. Our calculations show that large increments in tidal heating cause small increments in internal temperature. Thus, in order to melt the lowermost mantle the tidal heating would need to exceed $\sim 6.8 \times 10^{17}$ W for this 7.5 $M_\oplus$ planet (or $\approx 10^{17}$ W per Earth mass), meaning 2500 times the heat flow without tidal heating. Even though the internal temperature is much higher than without tidal heating, the radius only increases by $\sim 100$ km. For planet GJ 876d, melting could commence for eccentricities above 0.008. While we do not yet know the dynamical environment of this planet very well, such eccentricities are not excluded (Rivera et al. 2005). We note that the newly discovered $\sim 10$ $M_\oplus$ super-Earth HD 69830b (Lovis et al. 2006), orbiting at 0.08 AU from a 0.86 $M_\oplus$ star, is also in a three-planet system and appears to have a nonzero $e = 0.10 \pm 0.04$. Hence its tidal heating could exceed $10^{17}$ W per Earth mass. HD 69830b is expected to be mainly rocky (Lovis et al. 2006), and we estimate its radius to be $1.84 R_\oplus$ (Valencia et al. 2006).

5. UNCERTAINTIES

The uncertainties in the model come from uncertainties in the EOS and the composition of the planet, which needs to be known at high pressures and temperatures. Any phase change has to be known a priori in order to be incorporated in the model. Therefore, when we extrapolate to high pressures it is possible that the material might change into a different phase that we cannot account for without experimental evidence. If there are high-pressure phases unaccounted for in the model, the radius obtained here for different compositions would be an overestimate, owing to the denser character of high-pressure minerals. Benoit et al. (1996) describe a transition of H$_2$O to ice XI at 300 GPa. GJ 876d with IMF = 40% reaches a maximum pressure of 350 GPa at the bottom of the ice shell. Therefore, the effects of ice XI would be to decrease the radius by a few tens of kilometers for this planet, and more for planets with larger masses and IMF.
The likely compositions chosen for GJ 876d yield a central pressure between 2500 and 5000 GPa, values that exceed any laboratory experiment to date (Cohen et al. 2000). The extrapolation of the EOS — Vinet and third-order Birch-Murhaghan — has yet to be tested at these pressures. Nevertheless, the uncertainty in composition is greater than the errors in this extrapolation. The difference in radius from using the two different EOSs is ~100 km, less than the difference due to the different compositions considered here. There are also uncertainties in the determination of the parameters in the EOS, which translate into uncertainties in the internal structure. We considered two different data sets for the EOS of pure Fe (Uchida et al. 2001; Mao et al. 1990) and find that the difference in radius is only ~65 km for a planet dominated by core composition. Finally, the different treatments of viscosity (isoviscous vs. temperature-dependent) only yield a difference in radius of ~20 km, though the temperature regime is different for the four compositions described herein. Our lack of knowledge of the temperature structure is offset by the small effect of temperature on the total radius of the planet. This means that despite the uncertainties in the thermal model (viscosity assumptions and heat-flow scaling assumptions), the radius is a robust parameter. Therefore we place an estimate on the error in radius of ~200 km due to the lack of knowledge associated with the EOS and the thermal profile.

6. CONCLUSIONS AND DISCUSSION

In conclusion, new surveys and improved detection techniques have opened up the study of a new class of objects, super-Earths, that brings us closer to characterizing the exoplanets most similar to Earth. In particular, we can model the internal structure of the first discovered super-Earth by looking at likely compositions. We find values for the radius that vary from ~9200 to 12,500 km. If the orbital geometry allowed a transit follow-up to GJ 876d, the expected flux drop would be (1.9–3.6) × 10^{-3}, large enough to be observed. If observed with a large-aperture telescope (Holman et al. 2007) or from space, for example, with MOST (Rowe et al. 2006), a transiting planet such as GJ 876d would yield a radius determination with ~3% precision and one would be able to distinguish between all four scenarios/models presented here, especially between an iron-rich differentiated planet and an ocean planet. Moreover, for a given mass there is a radius that delimits the boundary between ocean planets and terrestrial planets. If the radius of GJ 876d were larger than 12,000 km, it would indicate that it is an ocean planet.

With the upcoming space mission Kepler this ability will improve, especially with expected advances in radial velocity measurements (and hence planetary masses) for Kepler’s fainter targets. Therefore, when radius measurements of terrestrial planets become available, we will be able to place constraints on the bulk composition of a planet (given its mass) and begin to understand the conditions on distant terrestrial planets. A follow-up article will treat this problem by showing in a simple manner the relation between radius, mass, and composition.

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