A microscopic modeling of the instant coffee effect

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Abstract. So-called the instant coffee effect is well known in the field of the physics education. The effect is explained that the sound yielded by touching the cup with a spoon is shifted to low-pitched by adulterating bubble owing to putting a spoon of instant coffee into hot water. The phenomenon has been interpreted with the averaged density and compressibility of the fluid in the macroscopic relation for the sound velocity, \( v = \sqrt{\kappa \rho} \). We introduce the linear coupled oscillator model with finite oscillators including the impurity air-mass oscillator. The model may well reproduce the increase in the shift of the eigen frequency accompanying with the amount of bubble.

1. Introduction

The instant coffee effect is well recognized in the field of physics education [1]. The effect is described as follows: As a spoon of instant coffee is put into the hot water in a cup or a glass, the sound yielded by touching the cup with a spoon gradually changes to higher in pitch. The effect was, as far as we know, initially reported by Farrell et al. [2]. More than ten years later, Crawford has performed the minute experiment and given a quantitative discussion based on the macroscopic oscillation theory [3]. Following the theory, this sound effect due to bubble sprung out from the granular instant coffee comes from the fundamental mode, which stands with one fourth of the wavelength on the water column in the vessel, where the bottom of water works as a fixed edge and the top of water as a free edge. Two studies have also been performed as the some kind of advanced studies at high school in Japan. There the time evolution of frequency spectrum was observed [4, 5], and it was clarified that the sound velocity decreases with increasing the quantity of instant coffee [5]. Recently, the time evolution of the fundamental frequency mode was clearly observed through a clever experiment by using a tall glass and the experiment shows that the mode is the fundamental eigen frequency standing on the water column in the tall glass [6].

On the other hand, such effect due to bubbles in sound propagation is expected to have been investigated in the field of fluid technology. As far as we know, van Wijngaarden [7] derived the sound velocity in bubbly water as a function of the volume ratio of bubble based on the phenomenological theory and had shown the concave behavior of it [8].

The consideration of the effect by a microscopic modeling will be interested for getting the insight for the properties of each frequency mode. For the purpose, we introduce a linear coupled oscillator model as a simplified microscopic model. As a result, it is obtained that the eigen frequency is softened by adulterating bubbles.
2. A linear coupled oscillator model

We introduce a microscopic model to describe the sound wave propagating along the principal axis of the water column in a glass. There the bottom of the column is considered as a fixed end for oscillation and the top of it is free end.

The model is given as a linear coupled oscillator with $N$ oscillators, which are connected via spring each other. The oscillators are assumed to be constituted with $N-1$ water-masses with the mass $M$ and a single air-mass with the mass $m$. In this setting, the number of oscillators $N$ determines the volume ratio of bubbles in the water as $N^{-1}$, which is called as the void-ratio.

The air-mass is positioned at $j = \lfloor N/2 \rfloor$ with the Gauss symbol $\lfloor \rfloor$, where $j$ is numbered from the bottom. The spring constant connecting two water-mass to its both edges or connecting on the water-mass and the fixed end at the bottom is denoted by $K$ and it connecting one water-mass and one air-mass by $k$. The spring constant at the top is distinguished as $K'$, because the top end is assumed to be free. The model oscillator is depicted in Fig.1.

![Figure 1. A linear coupled oscillator model of water including bubbles for describing the impurity effect of sound propagation. The symbol ● denotes the water-mass with the mass $M$ and the symbol ○ the impurity air-mass with the mass $m$. The spring constants, $k$, $K$, and $K'$ are given in text.](image)

The equation of motion on the displacement $x_j$ of $j$-th oscillator is expressed as

$$M_j \ddot{x}_j = -K_j(x_j - x_{j-1}) + K_{j+1}(x_{j+1} - x_j),$$

(1)

where $M_j$ and $K_j$ denote the mass of $j$-th oscillator and the spring constant of $j$-th spring, as a general expression, both counted from the fixed end corresponding to the bottom of the water column. Under the assumed condition in the present model, $M_j$ and $K_j$ are represented as

$$M_j = m\delta_{j,(\lfloor N/2 \rfloor)} + M(1 - \delta_{j,(\lfloor N/2 \rfloor)}),$$

(2)

$$K_j = k(\delta_{j,(\lfloor N/2 \rfloor)} + \delta_{j,(\lfloor N/2 \rfloor)+1}) + K'\delta_{j,N+1} + K(1 - \delta_{j,(\lfloor N/2 \rfloor)})(1 - \delta_{j,(\lfloor N/2 \rfloor)+1})(1 - \delta_{j,N+1}).$$

(3)

The dimensionless parameters like $\hat{m} \equiv m/M$ and $\hat{k} \equiv k/K$ are introduced.

The differential equation (1) is actually solved for $N = 3 \sim 7$. By assuming the solution of type $x_j = c_j \exp^{i\omega t}$ with the time variable $t$. The obtained $N$ algebraic equations are solved numerically, under the condition of $c_0 = 0$ and $c_{N+1} = 0$. And for the free end, $K' = 0$ is assumed hereafter. The eigen frequency $\omega$ is normalized as $\hat{\omega} = \omega/\omega_0$ by $\omega_0 = \sqrt{K/M}$, which...
Figure 2. Seven eigen frequencies for $N = 7$ system as a function of $\hat{m}$ at $\hat{k} = 0.1$ (a) and of $\hat{k}$ at $\hat{m} = 0.1$ (b).

is the angular frequency in the harmonic oscillator with a single water-mass. The ratio of mass $\hat{m}$ is chosen as the value smaller than unity, because the density of air is 800 times smaller than that of water. On the other parameter $\hat{k}$, it is not directly estimated. Here we consider the quantity of the spring constant $k$ divided by the suitable length like the natural length of the spring $\ell$,

$$
k/\ell = \Delta F/\ell \Delta x = (\Delta F/\ell^2)/(\ell^2 \Delta x/\ell^3) \sim \Delta p/(\Delta V/V) \sim \kappa^{-1}, \tag{4}
$$

where $V \equiv \ell^3$, $\Delta F$ is the force applied to the spring and $\Delta x$ the displacement. Therefore, $\hat{k}$ is proportional to the ratio of the compressibilities of water $\kappa_{\text{water}}$ and air $\kappa_{\text{air}}$, as $\hat{k} = k/K \propto \kappa_{\text{water}}/\kappa_{\text{air}}$. The compressibility of air $\kappa_{\text{air}}$ is about 15,000 times larger than that of water $\kappa_{\text{water}}$, then the $\hat{k}$ may be expected to be sufficiently smaller than unity. The calculated frequencies $\hat{\omega}$ for $N = 7$ are shown in Fig.2(a) and (b), as a function of $\hat{m}$ at fixed $\hat{k} = 0.1$ and of $\hat{k}$ at fixed $\hat{m} = 0.1$, respectively.

We here come back to our original purpose to get the void-ratio dependence of eigen frequencies. In the present model, the length of coupled oscillator is different for the different number of oscillators $N$, which corresponds to $N\ell$ for the case of free top end of $K' = 0$. To unify the system length, that is the wavelength of standing wave, the obtained eigen frequencies for $N$-oscillator system $\omega_N$ are rescaled as $\hat{\Omega}_N \equiv N/\hat{\Omega}_N$. To compare with the experiment, the deviation of frequency from the purely water system without the impurity air-mass has to be estimated. Then the eigen frequencies of $N$-oscillator system without air-mass are calculated and the rescale procedure has been done as same as the air-mass included system. The rescaled frequency for purely water system is denoted as $\hat{\Omega}_N^\times$. With this quantity, the deviation of frequency from the purely water system is obtained as $\Delta \hat{\Omega}_N = \hat{\Omega}_N - \hat{\Omega}_N^\times$. In Fig.3, the quantity for the lowest frequency mode, designated by the superscript (1), is shown as a function of $N^{-1}$, for the case of $\hat{k} = 0.1$ and $\hat{m} = 0.1$.

The decreasing tendency of the deviation of frequency is, as shown in Fig.3, reproduced in the present finite size calculation under the present model. This decrease is qualitatively consistent with the experimental result [5] and the phenomenological theory [7].
3. Discussions and conclusions

The linear coupled oscillator model including the air-mass as an impurity has been proposed to give the microscopic description of the instant coffee effect. The systems with finite number of oscillators have been calculated to get the eigen frequencies, numerically. The decrease of the fundamental frequency by adulterating bubbles increases with the volume-ratio of bubble, in accord with the experiment. Although the present calculated results are given only for the case of the central position of the impurity air-mass, it has been confirmed that the result is not modified remarkably.

The present coupled oscillator model may be discussed by connecting with the macroscopic relation for the velocity \( v = (\sqrt{\kappa \rho})^{-1} \) by applying the effect of the impurity oscillator into the compressibility \( \kappa \) and the density \( \rho \). The procedure and the discussion will be given in the separate issue.

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