Construction of deformation diagrams in experiments on impact compression of tablets-specimens with allowance for radial inertia

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Abstract. By means of the numerical modelling of the deformation process in axisymmetric formulation, using the LS-DYNA software complex, the influence of radial inertia on construction of dynamic deformation diagrams of different metals and alloys (12Kh18N10T steel, aluminium) at strain rates from $10^4$ to $10^5$ sec$^{-1}$ is studied in this article. Using the experimental-computational method, the verification of Dharan and Hauser analytical method was carried out. It allowed to define twice precisely the contribution of the radial inertia forces in construction of the true dynamic deformation diagrams at impact loadings.

Dynamic deformation diagrams that could be obtained experimentally by direct impact method or split-Hopkinson-bar technique for strain rates between $10^4$ and $10^7$ sec$^{-1}$ [1] are necessary for modeling of dynamic processes in constructions. Direct impact method is rather flexible to allow the variation of parameters during the experimental phase and to achieve the high strain rates. The diameter of the projectile in original Dharan and Hauser test [2] was 6.26 times higher, than the incident bar of the Hopkinson apparatus. Herewith, the most of elastic waves were dispersed in the projectile and the amplitude of the wave, reflected from the disengaged end of the projectile significantly decreased.

The theoretical assessment of the radial inertia of the tablet specimen under the impact compression with velocity $V_x(t)$ was carried out in Dharan and Hauser paper [2]. The cylindrical specimen was studied, with initial dimensions: radius $R_0$ and length $L_{s0}$. During the deformation process the radius $R(t)$ and length $L_s(t)$ were determined with the material incompressibility assumption. The friction on the moving and on stationary contact surfaces was neglected. With these assumptions, stresses and strains would vary only within the radius due to the radial inertia forces, than are known, since the strain rate is measured in the experiment. By means of the integrating of the inertia forces along the disengaged external surface ($r = R$) up to the axis of rotation ($r = 0$), the radial stress was defined as [2]:

$$
\sigma_r(t) = \frac{\rho}{4L_{s0}(1-e_n(t))} \left[ \frac{3V_x^2(t)}{2L_{s0}(1-e_n(t))} + \frac{dV_x}{dt} \right] \left[ \frac{R_0^2}{(1-e_n(t))} - r^2 \right],
$$

(1)

where $\rho$ - material density, $e_n(t)$ - axial engineering strain of the specimen.

The maximum value of $\sigma_r(t)$ corresponds to $r = 0$, i.e. at the specimen axe. Dharan and Hauser [2] considered that radial stress does not depend on $r$ and is equal to $\sigma_r(t) = a[\sigma_r(t)_{\max}, a_{\min} \leq a \leq 1$. Later
the authors [2] consider, that \( \alpha = 1 \) is a satisfactory approximation. Thereby for experiment on compression with a constant velocity \( V_x = \text{const} \) the equation (1) could be given as:

\[
\sigma_r(t) = \frac{3}{8} \rho \left( \frac{R_0}{L_{s0}} \right)^2 \frac{V_x^2}{(1 - \varepsilon中新网(t))^2}.
\]

(2)

The corrected true stress would be equal to:

\[
\sigma(t) = \sigma_x(t) - \sigma_r(t), \quad (\sigma_r(t) = \sigma_{\theta}(t)).
\]

Here \( \sigma_x(t) \) - average axial stress obtained from the experiment, \( \sigma_{\theta}(t) \) - circumferential stress.

In Dharan and Hauser article tests were conducted on pure polycrystalline aluminum with impact velocity \( V_x = 110 \text{ m/s} \). The axial velocity obtained in their experiment is given in Figure 1. It’s rather difficult to verify the experimental-calculation method [2] of dynamic deformation diagrams construction with consideration of the radial inertia forces with existing instrumental measures. At the moment, the verification of its reliability is unknown. Therefore we’ll carry out the analysis of experimental-calculation method [2] error by numeric modeling of the tablet specimen deformation process in axisymmetrical formulation by means of the software complex LS-DYNA.

Based on the experimental-calculation method [3-6] and experimental data (force-displacement) for the impact compression of aluminum [2], the dynamic deformation diagram of aluminum with consideration of the radial inertia with constant impact velocities was obtained by iteration method. The specimen dimensions were chose according to [2] with initial radius \( R_0 = 3.2 \text{ mm} \) and initial length \( L_{s0} = 1.6 \text{ mm} \). The deformation diagrams with corresponding radial stresses, obtained with experimental-calculation method [3-6] and Dharan and Hauser method with coefficient \( \alpha = 1 \) are illustrated in Figure 1.

The obtained dependence of axial stress \( \sigma_x(\varepsilon) \) in Figure 1 represents the average value within the transverse cross section of the specimen. Considering (1), the average value of the radial stress could be obtained as:

\[
\alpha = \frac{1}{A} \int \sigma_r(t) \, dA = \frac{2}{R^2} \int_0^R \sigma_r(t) \, rdr = 0.5,
\]

where \( A \) - area of the transverse cross section of the specimen. Thereby, the adopting of the condition \( \sigma_r(t) = \sigma_r(t)_{\max} \), that is corresponding to \( \alpha = 1 \), lead to overstating of the radial inertia forces in two times and to sufficient error (Figure 1) in true deformation diagram construction.

![Figure 1](image)

**Figure 1.** The true deformation diagrams and the corresponding radial stresses in an aluminum specimen: 1 - axial stress \( \sigma_x(\varepsilon) \) from the experiment [2]; 2 - the true deformation diagram obtained by the experimental-calculation method; 3 - the true deformation diagram obtained by the method of [2] with the coefficient \( \alpha = 1 \); 4 and 5 - radial stresses corresponding to diagrams 2 and 3.

If Dharan and Hauser methodic is applied with coefficient \( \alpha = 0.5 \) (\( \sigma_r(t) = 0.5 \cdot \sigma_r(t)_{\max} \)), the congruence of constructed deformation diagram with diagram, obtained by experimental-calculation
method is observed (Figure 1). Thus, the modified method [2] application allows to construct the dynamic deformation diagrams of elastoplastic materials with a high level of accuracy.

The study of the influence of the radial inertia consideration on the dynamic deformation diagrams was carried out for 12Kh18N10T steel and aluminum. The specimen dimensions were set with radius $R_0 = 3.2$ mm and with different initial lengths $L_{s0}$. The constantly acting axial velocity of the impact compression was set as $V_x = 110$ m/s. To eliminate the oscillations of the numerical solution within the rotation axe of the specimen, the linear distribution of initial impact velocity was set. Due to the incompressibility condition, the linear distribution of the initial velocity $V_{r0} = \frac{r}{2L_{s0}} V_x$ ($0 \leq r \leq R_0$) was set within the radius. The calculations were carried out without consideration of friction forces.

Based on the obtained numerical results and equations (1)-(3), the dependencies characterizing the level of the dimensionless radial stresses $\sigma_r / (\rho V_x^2)$ in experiments with impact compression of tablet specimens with strain rate of axial compression $V_x / L_s$ when $V_x = const$ was constructed in Figure 2.

![Figure 2](image.png)

The obtained numerical results analysis allowed to make a conclusion, that radial accelerations in experiments with impact compression at deformations exceeding 2% are almost independent of material deformation properties, that is determined by the low compressibility of elastoplastic steels and alloys. The constructed enveloping curve in Figure 2 is characterizing the level of the radial inertia under minor radius change of specimen. The dominating influence on the radial inertia forces is caused by the material density, strain rate and the tablet specimen radius value.

The error level in deformation diagrams construction without radial inertia consideration for 12Kh18N10T steel and aluminum is given in Figure 3. The definitions were introduced in Figure 3: $\Delta \sigma = \sigma' - \sigma$, where $\sigma'$ and $\sigma$ – deformation diagrams, constructed without radial inertia and with radial inertia respectively, $\sigma_{0.2} –$ yield strength of material.

![Figure 3](image.png)

The consequence of Figure 3 is that with increasing of degree of deformation, the error in determination of true deformation diagrams without consideration of radial inertia is increasing with the specimen radius increasing.
The illustrated dependencies of radial and circumferential stresses within the tablet specimen radius match with numerical results of the problem solution in axisymmetrical formulation with a high accuracy. From practical point of view the analytical dependencies [2] are rather simple and accurate to assess the level of radial inertia influence in experiments with impact compression of tablet specimens of metals and alloys with consideration of proposed modification.

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