Effects of two-body currents in the one-particle one-hole electromagnetic responses within a relativistic mean-field model

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Longitudinal and transverse responses from inclusive electron scattering are computed within an independent-particle relativistic mean-field model to describe the initial and final states, and one- and two-body current operators leading to the one-particle one-hole response. We find that the two-body contributions have no effect on the longitudinal response but they increase the transverse response by up to 30%, depending on the energy and momentum transfer, improving very significantly the agreement with experimental data. Our calculation is fully relativistic and considers within the full quantum mechanical description both the initial and final nucleon states involved in the process, incorporating realistic dynamics. We also show that it is essential to go beyond the plane-wave approach, as incorporating the distortion of the nucleons while making the initial and final states orthogonal, allows to reproduce both the shape and magnitude of the responses. The good agreement with the electron scattering experimental data supports the use of this approach to describe neutrino-induced scattering reaction.

Electron scattering is one of the most precise and efficient methods to determine the internal structure of atomic nuclei. Electron scattering experiments are regaining interest nowadays [1, 2] due to the exponentially growing experimental effort in neutrino-nucleus scattering, related to neutrino oscillation experiments, and thus the need to understand the nuclear interactions in the target nuclei which otherwise would prevent the extraction of neutrino properties from these experiments [3, 4]. At the very least, the theory employed to describe nuclear effects when analyzing neutrino-nucleus experiments should compare fairly well when put at test against the available electron scattering data under similar kinematics. Of particular interest are the few experimental data available on which not only the cross sections, but also the different contributions from the nuclear responses are separated, as these, while should be similar in electron and neutrino scattering, would be combined differently in both processes. That means, good agreement to electron scattering cross-sections would not be enough to ascertain the reliability of the model, while agreement to every separate nuclear response would be a much more compelling evidence of adequacy of the model.

The description of lepton-nucleus scattering is a complex many-body problem. The energy region of interest for accelerator-based neutrino experiments (e.g. MiniBooNE [7], MINERvA [8], NOvA [9], HyperK [10] and DUNE [11]) corresponds to incident neutrino energies ranging from a few hundreds of MeVs to tens of GeVs. Among all the mechanisms involved in the energy regime of the above mentioned neutrino experiments, the quasielastic (QE) channel, is the dominant one in T2K and MicroBooNE, and a major contribution in MINERvA, NOvA and DUNE. In this work we focus on the QE process, it corresponds to the lepton being scattered by a single nucleon that is consequently ejected from the target nucleus.

The modeling of the scattering process in this region is virtually always performed within the first-order Born approximation, in which one considers that only one boson is exchanged between the lepton and the nuclear system. Further, most often the impulse approximation is employed in the QE region, meaning that the lepton interacts only with one nucleon, the one that is knocked out from the nucleus. It is also important to stress that due to the fact that the energy ($\omega$) and momentum ($q$) transfer between the lepton and the nucleus are in some cases comparable or larger than the mass scale set by the nucleon mass, relativistic effects are relevant.

The residual interaction of relevance is mainly that between a low-energy hole and a high-energy particle, expecting pions to play an important role. Therefore, we extend the usual treatment of QE scattering, based on a one-body current operator,
and include one-pion exchange effects by incorporating a two-body meson-exchange current operator. In this work, meson-exchange currents (MEC) include the dominant Delta-resonance mechanism (Fig. 1) electromagnetic excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$ and the background contributions deduced from the chiral perturbation theory Lagrangian of the pion-nucleon system [12] (Fig. 2 ChPT background or, simply, background terms in what follows). In particular, we have studied the contribution of the two-body meson-exchange currents to the one-particle one-hole (1p-1h) electromagnetic responses.

Previous works have computed the contribution of pion exchange currents to the 1p-1h and 2p-2h responses within different frameworks. There is a consensus that the effect of MEC in the 2p-2h sector leads to a significant contribution in the dip region between the QE and the delta resonance peaks [13–18]. The role of MEC in the 1p-1h responses has been, however, much less explored. In [19], within a non-relativistic shell model that incorporates final-state interactions, it was obtained that the two-body current produced a small decrease of the transverse response ($R_T$). In [20], using a similar nuclear model, it was found that the two-body currents enhance $R_T$ by around 20-30%. In both approaches, by construction, the two-body operator does not affect the longitudinal response ($R_L$). More recently, the ab initio model of [21] has confirmed the essential role of two-body mechanisms to describe the electromagnetic responses of light nuclei. These previous works employed non-relativistic approaches subjected to hold only at relatively low momentum transfer. Hence, MEC contributions to 1p-1h final states have also been studied within fully relativistic frameworks, but over-simplifying the complexity in the nuclear structure and dynamics, for example, the approaches in [22–25] which are based on the relativistic Fermi gas model.

The results presented in this work are computed within a fully relativistic and quantum mechanical framework, where the initial state is described by an independent-particle relativistic mean-field (RMF) model [26], and the final-state is described as a solution of the Dirac equation for the final nucleon in the presence of relativistic potentials. This way we obtain a realistic description of the scattering process that can be applied in the entire kinematical region of interest for electron and neutrino scattering. We compare our calculation of the electromagnetic responses of the $^{12}$C nucleus with the available experimental data. We find the contribution of MEC negligible in $R_L$ while it increases $R_T$ by around 30%. The agreement with data is good in general and very good where the kinematics suppresses the contributions of other processes not considered here, such as real pion production.

In our model, the inclusive responses are computed from the hadron tensor by integration over the variables of the final nucleon and sum over initial nucleons. The hadronic tensor is given by

$$H_{\kappa}^{\mu\nu} = \sum_{m_j, s} [J_{\kappa, m_j, s}^{\mu}]^* J_{\kappa, m_j, s}^{\nu},$$

where the hadronic current is

$$J_{\kappa, m_j, s}^{\mu} = \int dp \overline{\Psi}(p + q, p_N) \Gamma^{\mu} \Psi_{\kappa}^{m_j}(p).$$

\(p\) is the momentum of the bound nucleon, \(\kappa\) represents the nuclear shell and \(m_j\) the third component of its total angular momentum \(j\). \(p_N\) is the asymptotic momentum of the final nucleon and \(s\) its spin.

The bound wave function \(\Psi_{\kappa}^{m_j}\) is obtained with the RMF model of [27]. For describing the final nucleon wave function \(\Psi^s\), we use the energy-dependent relativistic mean-field (ED-RMF) potential, which is real, so that no flux is lost due to the imaginary part of the potential. The ED-RMF is the RMF potential used in the bound state but multiplied by a phenomenological function that weakens the potential for increasing nucleon momenta (see details in [28, 29]). The main advantage of this choice is that it preserves the orthogonality between the initial and final states at low energies of the final nucleon, while approaching the behavior of the phenomenological optical potentials at larger energies.

The hadronic current operator \(\Gamma^{\mu}\) of eq. (2) includes all the processes that lead to a final 1p-1h state. Apart from the usual one-body current operator, we include a two-body current operator that accounts for one-pion exchanged between interacting nucleons inside the nucleus. Thus, the hadronic operator reads

$$\Gamma^{\mu} = \Gamma^{\mu}_{1b} + \Gamma^{\mu}_{2b}. \quad \text{(3)}$$

The one-body operator is given by the usual CC2 prescription [30, 32]. The two-body operator is the sum of the contributions corresponding to the diagrams shown in Figs. 1 and 2. They are discussed in detail in what follows.

We distinguish two different contributions to the two-body current: i) diagrams where a delta is involved, and ii) the background terms. When
the 1p-1h excitation occurs through a two-body current, one of the outgoing nucleons of the two-particle two-hole interaction remains bound to the nucleus, appearing an intermediate bound-nucleon state. Hence, the hadronic final state consists in just a nucleon and the 1p-1h matrix element is obtained via the integration of the intermediate state over all occupied levels in the ground state. The description of the intermediate state is approximated by using free Dirac spinors in a relativistic Fermi gas (RFG), as done in infinite nuclear matter [25], but modified by an effective mass and energy that account for the relativistic interaction of nucleons with the mean-field potential [33]. We have verified that this computational scheme yields essentially the same results as the full calculation in which the intermediate states are described as bound states, only that at a fraction of the computational cost.

The two-body current operator is computed as

$$\Gamma_{2b}^\mu = \int \frac{dp_{ph}}{(2\pi)^3} \Theta(p_F - p_{ph}) [\Gamma_{\text{ChPT}}^\mu + \Gamma_{\Delta}^\mu],$$  \hspace{1cm} (4)

where $\Gamma_{\Delta}^\mu$ and $\Gamma_{\text{ChPT}}^\mu$ are the contributions from the diagrams in figures 1 and 2. The explicit expressions of the operators can be found in [33].

In an independent-particle shell model, $^{12}$C is made of 2 and 4 nucleons in the 1$s_{1/2}$ and 1$p_{3/2}$ states. Each shell has a unique binding energy, which means that the missing energy distribution $E_m$ predicted by the model would be the sum of two Dirac deltas, each one normalized to the occupancy of the given shell. However, this would be a very crude approximation to the missing energy distribution of the strength, which has been measured in $(e,e'p)$ experiments for carbon and other nuclei [34, 35]. It is clear from the data that the energy response of each shell has a finite width, wide for deeper shells and narrower for the ones near the Fermi level. It is also observed that the occupancy of the shells is depleted with respect to the independent-particle shell-model predictions, and that these ‘missing nucleons’ re-appear in deeper missing-energy ($E_m$) and missing-momentum ($p_m$) regions [36, 37]. This behavior is due to effects beyond the independent-particle approach, ascribed to short- and long-range correlations [38–41]. To incorporate that in our formalism, we use a continuous missing-energy profile and reduce the occupation of the shells to 3.3 ($p$) and 1.8 ($s$) nucleons, respectively, taking as reference the missing energy distribution of the Rome spectral function [42, 43]. Furthermore, the high missing energy and momentum region of the spectral function coming due short-range correlations is modeled as a s-wave fitted to reproduce the momentum distribution of the Rome spectral function and normalized so that after summing over all shells the 12 nucleons are recovered; its contribution starts at the two-nucleon emission threshold, it has a soft maximum at around 100 MeV and an exponential fall that extend to high momentum [44, 45].

In Fig. 3 we show our results for the inclusive longitudinal and transverse responses, computed using the one-body and two-body operators. The theory is compared to experimental data extracted by means of a Rosenbluth-type analysis by Jordan [46] and Barreau et al. [47], as well as with the ab initio non-relativistic Green’s function Monte Carlo (GFMC) responses of [21]. We highlight the following salient features. The main effect of the two-body currents with respect to the one-body approach appears in the transverse channel, while in the longitudinal one the 1p-1h MEC contribution is hardly visible. The transverse response increases by up to 30% for ED-RMF, being the relative increase larger for smaller values of $q$. The agreement of our results with data is outstanding, and also it is remarkable the good agreement between both ED-RMF and GFMC calculations in

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1 By missing energy we refer to the part of transferred energy that transforms into internal energy of the residual nucleus.
spite of the fact that they represent completely different theoretical approaches.

Additionally, in Fig. 3 we show the result computed with the two-body current operator only. For the transverse and longitudinal responses, it is nearly two and four orders of magnitude smaller than the one computed with a one-body current, respectively. Therefore, it is clear that the increase observed in the transverse sector is due to the interference between the one- and two-body contributions.

FIG. 3. $^{12}$C longitudinal (up) and transverse (bottom) electromagnetic inclusive response functions. The transferred momentum $q$ is 380 MeV/c. We show our ED-RMF results and the GFMC responses taken from [21]. The longitudinal and transverse responses computed with only two-body currents are shown separately and multiplied by a factor 100 and 10, respectively.

In Fig. 4 we study the effect of using different potentials to describe the final-state nucleon. To address the effect of the distortion of the knocked out nucleon and of the spurious contributions arising from the non-orthogonality between initial and final states [28], we show results with the so-called relativistic plane wave impulse approximation (RPWIA) model, in which the final nucleon is described by a relativistic plane wave. As expected, RPWIA calculations overestimate the data. This is attributed to the lack of orthogonality of the initial and final state, giving rise to spurious contributions to the responses, and of to the distortion of the final nucleon that is not being included in RPWIA. The effect of these two ingredients (which are present in the ED-RMF approach) is to shift the peak to the right position, according to the data, to reduce the total strength and to redistribute it from the peak to the tails. Further, we compare the ED-RMF results with those from (the real part of) the energy-dependent A-independent carbon relativistic optical potential EDAI-C [48]. This phenomenological potential was extracted by fitting elastic proton-carbon scattering data in the range $30 < T_p < 1040$ MeV, $T_p$ being the proton kinetic energy. The two approaches (ED-RMF and EDAI-C) provide very similar results for large enough values of the momentum transfer, $q > 300$ MeV/c [19]. However, the EDAI-C, unlike the ED-RMF, does not preserve exact orthogonality between the initial and final states; hence, when the momentum of the final nucleon is comparable to the momentum of the bound nucleon (i.e., approximately $p_N < 300$ MeV/c), the overlap between the two states is significant, and as a consequence the spurious non-orthogonality contributions become an issue for EDAI-C as well as for RPWIA. This is confirmed by our results, in which one observes that even though EDAI-C and ED-RMF are very similar both in shape and magnitude, the agreement with the data is slightly better for ED-RMF, specially, at lower energies.

In view of the results, our relativistic mean-field based model, with one- and two-body current contributions to the 1p-1h QE peak, can simultaneously describe the longitudinal and transverse electromagnetic responses of $^{12}$C in the quasielastic regime. The key contribution of this work is the incorporation of the two-body meson-exchange current contribution to the 1p-1h channel. It includes the delta resonance mechanism and background terms. We find that the effect of the two-body currents is only significant in the transverse channel, where the response is increased up to a 30%, leading to an improved description of the data compared to the one-body case. The delta resonance mechanism is the main responsible of this result, giving the larger contribution.

This work paves the way for the leap to neutrino-nucleus interaction processes. We point out that in the case of charge-current quasielastic (anti)neutrino reactions the transverse response is clearly the dominant one [50] [51], except at very low four-momentum transfer. Therefore, we expect the two-body current mechanisms to play an important role in the neutrino sector.

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FIG. 4. $^{12}\text{C}$ L (left) and T (right) responses with the ED-RMF, RPWIA and EDAI-C models. The transferred momentum $q$ is (from up to bottom) 300, 380 and 570 MeV/c.

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