Effect of friction on nominal stress results in a single tooth bending fatigue test

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Abstract. One of the main reasons for gear failure is bending fatigue, which occurs due to cyclic stresses at the tooth root region. Bending fatigue can cause crack initiation which, in turn, may propagate and result in catastrophic tooth breakage. Single tooth bending fatigue tests are frequently employed to generate statistically significant fatigue data at relatively low price. Hence, they are often employed as screening tests for bending fatigue behaviour of actual running gear pair. Since relatively small alteration of nominal tooth root stress values can produce significant change in bending fatigue lives, it is important to adequately consider the effect of friction and test fixture deformation when conducting single tooth bending fatigue tests. In this paper, numerical investigation of tooth flank friction effect on spur gear load capacity during single tooth bending fatigue test is carried out. Two different test fixtures are considered. Linear elastic finite element analysis is carried out to obtain nominal tooth root stresses. The results show that friction at gear tooth flank as well as fixture deformation can affect nominal tooth root stress results.

1. Introduction
Single-tooth bending fatigue (STBF) test is often employed for bending fatigue testing of a gear tooth by applying a load normal to the flank at a fixed point. Main goal of this test, which is used as a screening test for operating gears, is to generate a statistically relevant quantity of bending fatigue data. The test itself is usually done via electrohydraulic universal test machine at a relatively low price. The overall cost of an experiment can further be reduced by conducting multiple tests on a single gear specimen.

Various types of STBF test fixture arrangements are currently being used in the field of mechanical gear testing and many studies have been conducted on them. Gasparini et al. [1] studied the effects of material, design and manufacturing parameters on bending fatigue life of helicopter gears. They employed a fixture design in which both test tooth and the reaction tooth are loaded at the same time with two linearly balanced forces, eliminating the need for the gear’s support. Therefore, following the positioning of the gear, the support pin can be removed. Same fixture was used by Gorla et al. [2] for bending fatigue strength study of case carburized and nitride gear steels for aeronautical applications. Handschuh et al. [3] conducted STBF tests on steel spur gears. The fixture arrangement used in this study consists of one actuating arm that loads the test tooth and one fixed anvil that supports the reaction tooth, while the specimen gear is supported in the fixture casing. Here, the load is also applied at the HPSTC. In the Boeing flexural design [4], which appears to have found favor in the aerospace sector, the specimen gear is rigidly supported on a shaft while the load is applied through a carbide block contacting the test tooth at the HPSTC. The loading block is held in the fixed orientation by a flexural loading arm and the reaction is carried through a block contacting the reaction tooth.

This
flexural design ensures accurate loading of the gear tooth with minimal migration of the loading point. Akata et al. [5] investigated performances of differently manufactured spur gears using a single-tooth bending principle in combination with three-point bending loading. They used a fixture design in which the specimen gear is placed on a lower test fixture and bending loads are applied via its upper part by means of needle elements placed between the tooth flank and the upper part. The distance between the loading centers of the needle elements is the diameter of HPSTC. In a similar setup, without using needle elements while the specimen gear sits on his own shaft rather than the lower test fixture, Bian et al. [6] conducted STBF tests on 38SiMnMo alloy steel. The last fixture design considered here, more commonly known as SAE (Society of Automotive Engineers) test fixture [7], primarily consists of a specimen gear, actuating arm and support anvil, all of which are mounted on a common shaft. The actuating arm is L-shaped, and it rotates about the shaft axis, contacting the test tooth at its tip.

According to Vučković et al. [8], friction can significantly affect the bending fatigue life of a gear. To investigate the effect of friction at the results of a single-tooth bending fatigue tests, two of the abovementioned fixture designs are considered. The first one is the three-point bending fixture according to Bian [6] and the second one is the SAE fixture design according to the official standard [7]. Furthermore, a computational model based on the elastic FEM analysis is proposed. The proposed model simulates contact between gear tooth flank and the fixture surface to obtain nominal tooth root stresses. Predicted behavior of the specimen gear tooth with regard to the fixture rigidity is compared to the numerical simulations given by the finite element method (FEM).

2. Materials and methods

Finite element analysis is carried out on AISI 8620H specimen gear, whose geometry is based on the SAE standard [7]. Geometrical and material parameters employed for FEM simulations of both fixtures are presented in Table 1.

| Parameter                        | Value          |
|----------------------------------|----------------|
| Number of teeth, z               | 34             |
| Module, m                        | 4.23 mm        |
| Normal pressure angle of basic rack, $\alpha$ | $20^\circ$ |
| Pitch diameter, $d$              | 143,934 mm     |
| Base diameter, $d_b$             | 135,252 mm     |
| Root diameter, $d_r$             | 133,245 mm     |
| Tip diameter, $d_a$              | 152,28 mm      |
| Modul of Elasticity, $E$         | 210 GPa        |
| Poisson coefficient, $\nu$      | 0.3            |
| Density, $\rho$                  | 7850 kg/m$^3$  |
| Thickness of the gear, $b$       | For three-point loading fixture: $b = 10$ mm |
|                                  | For SAE fixture [7]: $b = 25.4$ mm |

As shown in Table 1, thickness of the specimen gear is varied according to the type of simulated fixture. For the three-point loading fixture, the thickness is determined according to the estimated fixture/gear thickness ratio of 3:1 [6]. For both simulations, same load-time history based on sinusoidal load amplitude variation is applied (Figure 1). The load-time history is divided into five steps with equal time periods. The initial loading is increased from zero to the maximum load, followed by two cycles of unloading from maximum to minimum load (taken as 10% of maximum load) and then loading back to
the maximum load. For three-point loading fixture maximum load is equal to $F_{\text{max}} = 5 \text{ kN}$, while for the SAE fixture maximum load is $F_{\text{max}} = 10 \text{ kN}$.

**Figure 1.** Applied load-time history.

2.1. *Three-point loading model*

The three-point loading model consists of two main parts: the loading anvil and the specimen gear (Figure 2a). Boundary conditions and the loading point are assigned to a numerical model according to Figure 2b. Geometry of the loading anvil is determined to ensure the loading force is vertically tangent to the base diameter of the gear.

**Figure 2.** Three-point loading model: (a) geometric model, (b) numerical model with boundary conditions and loading point.

Load is applied at the center of the loading anvil. Two cases are considered. First case assumes elastic behavior of the loading anvil, while the second one assumes ideally rigid behavior. When the loading anvil behaves as an elastic body (Figure 3a) while in contact with the tooth flank, horizontal plane of the anvil should slide in the outwards motion, meaning friction force $F_f$ should act in the opposite direction on the anvil. The friction force acting upon the tooth flank will then act in the outwards direction and consequently increase the nominal stresses at the tooth root fillet.
For the second case, depicted in the Figure 3b, the rigidity of the anvil should cause the sliding motion of the tooth flank in the outwards direction, meaning friction force on the tooth flank will act in the opposite direction. This should lead to an overall decrease of principal stress at the tooth root fillet.

2.2. SAE fixture model
Second load model, based on the SAE fixture design, consists of three main parts depicted in the Figure 4b. Specimen gear is loaded via an upper anvil; at the same time the gear is supported by a fixed lower anvil.

Load is applied at the top of the L-shaped upper anvil which rotates about the center $S_1$ (Figure 5) while the test tooth bends around its root. Since upper anvil rotates about greater radius than the tooth
itself, it causes the friction force on the tooth flank to act against its direction of rotation, thereby decreasing the principal stress at the tooth root fillet. Directions of the corresponding forces at the contact point between the tooth flank and the changeable plate on the upper anvil are depicted in the Figure 5b.

**Figure 5. SAE loading model: behaviour prediction upon loading and forces at the contact point.**

Numerical analysis for both presented models is conducted within the commercially available FEM software Abaqus 6.13 [9]. According to Pehan et al. [10], plane stress can be assumed if the gear thickness is less than six times its module. Therefore, two-dimensional FE analysis is carried out. Parts within both models are discretized by 8-node bi-quadratic plane stress quadrilateral elements, designated as CPS8 in Abaqus-Standard. For both FE models, convergence test is carried out to determine the required number of finite elements. The mesh is refined, and the number of finite elements is increased in the regions of contacting flanks and corresponding root fillets to detect the slightest changes and increase the accuracy of the results.

3. **Results and discussion**

Numerical simulations are conducted for both proposed loading models by varying friction coefficients and loading.

3.1. **Three-point loading fixture model results**

The effect of friction on the stress-time history for the three-point loading model with elastic behavior of the loading anvil is shown in Figure 6. As predicted, the principal stress at the tooth root fillet increases with the friction coefficient, which causes higher friction force at the tooth flank.

**Figure 6. Principal stress-time history for the tooth root fillet and elastic behaviour of the loading anvil.**
Contrary to the elastic behavior of the anvil, when ideal rigidity of the loading anvil is assumed, the principal stress at the tooth root fillet decreases with an increase of friction coefficient (Figure 7).

Figure 7. Principal stress-time history for the tooth root fillet and rigid behaviour of the loading anvil.

Since the elastic behavior of the loading anvil is more realistic, numerical simulation for various load values are conducted only for this case. As it can be observed from the Figure 8, with an increase in the friction coefficient, the maximum cyclic stress is increased (Figure 8a) and the minimum cyclic stress is decreased (Figure 8b), i.e. the stress range is enlarged. This will, in turn, reduce bending fatigue life of the gear.

Figure 8. The effect of friction and different loads on cyclic stresses at tooth root fillet for elastic behaviour of the loading anvil: (a) maximum cyclic stress, (b) minimum cyclic stress.

3.2. SAE fixture model results

Stress-time history for the SAE fixture design model is provided in Figure 9. As shown in the diagram, the maximum principal stress at the tooth root fillet is decreased with an increase of the friction coefficient.
Contrary the behavior for three-point loading model observed in Figure 8, with an increase in the friction coefficient, the maximum cyclic stress is decreased (Figure 10a) and the minimum cyclic stress is increased (Figure 10b), i.e. the stress range is reduced. This will, in turn, increase the bending fatigue life of the gear.

For both load models, it can be observed that the friction force influences tooth root stress results. Even though variation of maximum principal stresses is relatively small, according to [11], amplitude change of 10% can cause an increase or a decrease in fatigue life by a factor of three.

4. Conclusion
In this paper, numerical analysis of the principal stresses at tooth root fillet while considering friction force and fixture rigidity was presented. By employing the elastic finite element analysis, contact between tooth flanks and fixture for two different load models was simulated and maximum principal stresses at the tooth root fillet were obtained. The results were compared with the initial predictions and were found to be in a good agreement. Furthermore, it was found that friction can impact nominal tooth
root stress results, which can influence bending fatigue life of a gear. Since STBF tests are often used as screening tests for actual bending fatigue behavior of the gear, friction coefficient should be determined before the test and properly compensated for, or reduced to minimum for the most accurate results.

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