The lower limits of the fast ionization waves velocity in a high-intensity laser radiation field

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Abstract. The kinetic processes before the front of an ionizing wave in a gas are investigated. The lower limits of the propagation velocity of a plane ionization wave in the field of high-intensity laser radiation are determined. The results of calculations of the lower boundaries of the velocity of fast ionization waves in argon in the field of a CO₂-laser are given. The calculation results are compared with experimental data and other theories.

1. Introduction

In 1963, a spark flashed for the first time in the air in a focused lens of a ruby laser [1]. This phenomenon is called the laser spark or optical breakdown [1,2]. A plasma focus that forms in the focal region of a lens usually rapidly increases in size during a laser pulse. Occurs, as they say, the distribution of the discharge [2].

Despite the fact that optical discharges are investigated relatively recently, they are studied not much less than discharges at lower frequencies of the electromagnetic field. This is primarily due to the needs of the practice. Optical discharges are used as light sources of very high brightness and for the continuous generation of high temperature plasma (optical plasmatrons). Various variants of rocket engines are also proposed in which an optical discharge, for example, is used to obtain high plasma outflow rates and large specific recoil pulses.

Recently, projects are being developed to supply satellites with energy by converting the laser radiation energy sent from the Earth into electrical energy. Such a conversion, as shown by experimental studies, can be carried out using a continuously maintained optical discharge burning near two electrodes, one of which, for example, is cooled. The potential difference arising at the electrodes makes it possible to maintain a significant current in the circuit. In addition, for the direct conversion of laser energy into electrical energy, it has been proposed to use the optical analog of the MHD generator.

Of great interest is the study of optical discharges for problems of nonlinear atmospheric optics. It was found that the optical discharge in the air is initiated by aerosol particles at radiation intensities substantially lower than in pure (dust-free) gases. The occurrence of a plasma focus leads to a strong attenuation of radiation. Thus, optical breakdown limits the intensity of the radiation that can be transmitted in the atmosphere or gas without much attenuation. On the other hand, the initiation of optical breakdown by aerosol particles can be used to develop remote methods for analyzing the elemental composition of aerosol particles.

At moderate laser radiation intensities of 10-100 MW/cm², characteristic of the breakdown of gases containing aerosol particles of several microns, radiation and light-detonation waves of optical
discharges were observed [3-7]. Radiation waves of an optical discharge in gases propagate due to the heating of cold gas by thermal radiation of the gas. Light detonation waves propagate as a result of the gas being ionized by the shock wave. The speed of a light-detonation wave and radiation waves at such intensities differ several times. However, in a number of experiments, optical discharges propagating at much higher speeds were observed.

Waves of optical discharges in dense gases, which in some cases propagate with speeds exceeding the speed of light-detonation waves under the same conditions, are tens of times and are usually called fast ionization waves (FIW). FIW remain the least studied mode of propagation of optical discharges [3-12]. FIW was observed in both atomic and molecular gases [3-7].

In [8-11], a FIW model for inert gases was proposed in which the process of propagation of an optical discharge is caused by the development of an electron avalanche before a wave front, and the avalanche is initiated as a result of ionization of the gas by UV quanta emitted by the optical discharge plasma. Under the assumption of instantaneous ionization of excited atoms in this model, it was possible to obtain satisfactory agreement between theoretical calculations and the experimentally observed dependences of the propagation velocities of stationary FIW in xenon and argon on the intensity of the CO$_2$-laser radiation.

In [12], a theory was developed that allows one to determine the lower bounds of the velocity of ionizing waves in the field of high-intensity laser radiation. The calculations showed [12] that if the excited argon atoms were really instantly ionized, as was supposed in [8-11], then the propagation velocity of stationary FIW would, at low intensities (less than 60 MW/cm$^2$), be significantly higher than the predicted model FIW [8-11] and experimentally observed speeds. Thus, the question of the physical processes causing the mechanism of propagation of FIW is not entirely clear, and the theory of FIW is not sufficiently developed even for atomic gases, for which the description of the nonequilibrium kinetics of ionization phenomena has been developed quite well.

In the general case, the process of plasma propagation in the field of laser radiation should be described by a complex system of nonlinear equations, including: the Navier-Stokes equation, energy transfer, the kinetic equation for the electron energy distribution function, the propagation of laser and intrinsic plasma radiation, etc. The solution of all these equations, as a rule, can only be obtained using numerical methods and is a complex computational problem.

When studying the structure of stationary ionizing waves, it is necessary, as a rule, to solve the limits value problem by numerical methods, with the photoionization speed and electron concentration ahead of the wave front approach to zero when $x = +\infty$. This leads to the need to arbitrarily set the speed of photoionization ahead of the wave front [8]. Behind the wave front, until its velocity is found, the plasma parameters are also not known, in this case the numerical calculation is also difficult and allows some arbitrariness. In this connection, reliable criteria are required, which must be satisfied by any theory of ionizing waves.

It should be noted that the results obtained in [12] are valid only if the distribution of electrons in energy (or velocity) far ahead of the wave front does not depend on the spatial coordinate $x$, or the transport cross section for collisions of an electron with gas atoms is proportional to its velocity.

The purpose of this work is a theoretical study devoted to determining the lower bounds of the speed of optical discharges in gases with arbitrary dependences of the transport cross section for electron scattering by atoms on energy and taking into account the spatial dependence of the electron velocity distribution ahead of the wave front.

2. Basic equations
Consider the process of propagation of a gas ionization wave in the field of laser radiation far ahead of the wave front. In this case, the plasma can be considered weakly ionized and consisting of mobile electrons, as well as non-mobile ions and neutral atoms.

Electrons diffuse from the region of the wave front into non-ionized region ahead of the discharged front. They increase their kinetic energy in the laser electromagnetic field; lose energy in inelastic and non-elastic collisions and also multiply due to ionization of atoms (molecules) by electron impact.
Let we take moving reference frame with the origin at the front of ionizing wave and \( x \)-axis directed along laser beam. Energy flux of laser radiation is directed from point \( x = +\infty \) to point \( x = -\infty \). In this reference frame we consider equations for the electron energy distribution function and for the flux density of the irradiated by plasma UV-quantas. We will seek a quasistationary solution of the equations. In the region of space ahead of the discharge front, where the electron concentration is low, the absorption of the radiation energy by the plasma is also low. In this case, the equations describing plasma propagation can be linearized.

For a plane ionizing wave, provided that the wave velocity \( U \) is much lower than the root-mean-square electron velocity \( V_{eq} \), \( U \ll V_{eq} \), the linearized system of equations describing the kinetics of ionization before the wave front has the form

\[
-U \frac{\partial f(x,\varepsilon)}{\partial x} = d(\varepsilon) \frac{\partial^2 f(x,\varepsilon)}{\partial x^2} + \hat{L}[f(x,\varepsilon)] + 2\pi \int_{-1}^{1} \Sigma_i (\varepsilon + \varepsilon_i) F(x,\varepsilon + \varepsilon_i, \mu) d\mu; \tag{1}
\]

\[
\frac{\partial F(x,\varepsilon, \mu)}{\partial x} = -\Sigma_a(\varepsilon) F(x,\varepsilon, \mu) + \int_{\varepsilon_i}^{\infty} \beta(\varepsilon', \varepsilon) f(x, \varepsilon') d\varepsilon'; \tag{2}
\]

where \( f(x,\varepsilon) \) is the energy distribution of electrons, \( U \) is the wave velocity, \( d(\varepsilon) = 2e^2/[3m(\varepsilon)] \) - can be interpreted as the diffusion coefficient of electrons with energy \( \varepsilon \), \( \Sigma_i \) - is macroscopic cross-sections of photoionization, \( \varepsilon_i \) is the energy corresponding to the photoionization threshold, \( F(x,\varepsilon, \mu) \) is the flux density of UV quanta with energy \( \varepsilon \) propagating at an angle \( \mu \) to the axis \( x \) \( (\mu \equiv \cos \theta) \), \( \beta(\varepsilon', \varepsilon) \) is the number of quanta with energy \( \varepsilon' \) emitted by the electron with energy \( \varepsilon \) per unit time in a unit solid angle due to collisions of electrons with atoms, \( \Sigma_a \) - macroscopic cross-sections of photoabsorption. Operator \( \hat{L} \) describes quanta propagation in plasma

\[
\hat{L}[f(x,\varepsilon)] = \left[ a(\varepsilon - h\omega) f(x,\varepsilon - h\omega) - [a(\varepsilon) + b(\varepsilon)] f(x,\varepsilon) \right. \\
\left. + b(\varepsilon + h\omega) f(x,\varepsilon + h\omega) \right] F \cdot N + S_m[f(x,\varepsilon)], \tag{3}
\]

\( \omega \) - is laser radiation frequency, \( a(\varepsilon), b(\varepsilon) \) - absorption coefficients and stimulated emission of quanta calculated for one electron and atom

\[
a(\varepsilon) = \frac{8\pi e^2}{3m_e c \omega^2} \frac{e^{2\omega}}{h\omega} \left( \frac{\varepsilon + h\omega}{\varepsilon} \right)^{1/2} \left( \frac{2\varepsilon}{m_e} \right)^{1/2} \sigma \left( \frac{\varepsilon + h\omega}{2}\right), \tag{4}
\]

\[
b\left( \varepsilon + h\omega \right) = \left( \frac{\varepsilon}{\varepsilon + h\omega} \right)^{1/2} a(\varepsilon), \tag{5}
\]

\( \sigma(\varepsilon) \) - cross-section of elastic scattering of an electron on an atom (molecule) of a gas, \( N \) - concentration of atoms (molecules) of a gas, \( S_m[f(x,\varepsilon)] \) - integral of electron-atom collisions

\[
S_m = S_{el} + S_i + S_j. \tag{6}
\]

In this expression, each term describes a certain inelastic process (excitation of gas atom, excitation of vibrational levels of gas molecule) and ionization of atom or molecule.

\[
S_{el} = -f(x,\varepsilon) \sum_k \nu_{el,k}(\varepsilon) + \sum_k f(x,\varepsilon + I_k) \nu_{el,k}(\varepsilon + I_k), \tag{7}
\]
takes into account the excitation of gas atoms in electron-atom collisions, $I_\nu^k$ - the excitation potential of the $k$-th level of a gas atom, $\nu_{\alpha,k} (\nu) = N \cdot V (\nu) \sigma_{\alpha,k} (\nu)$ - the frequency of excitation of the $k$-th level of an atom (molecule) of a gas with electrons with energy $\nu$, $V (\nu) = (2\nu/m)^{1/2}$ - electron velocity with energy $\nu$, $\sigma_{\alpha,k} (\nu)$ - microscopic cross section of the excitation of the $k$-th level atom (molecule) gas electron with energy $\nu$.

$$S_\nu \left[ f(x,\nu) \right] = -f(x,\nu) \sum_k \nu_{\nu,k} (\nu) + \int f(x,\nu + \hbar \omega_k) \nu_{\nu,k} (\nu + \hbar \omega_k) \, d\nu,$$  \hspace{1cm} (8)

takes into account the excitation of the vibrational levels of gas molecules (for molecular gases), $\omega_k$ - the oscillation frequency of the $k$-th vibrational level of a gas molecule, $\nu_{\nu,k} (\nu) = N \cdot V (\nu) \sigma_{\nu,k} (\nu)$ - the excitation frequency of the $k$-th vibrational level of a gas molecule by electrons with energy $\nu$, $\sigma_{\nu,k} (\nu)$ - the microscopic cross-section of the excitation of the $k$-th vibrational level of a gas molecule by an electron with energy $\nu$.

$$S_\nu \left[ f(x,\nu) \right] = -f(x,\nu) \nu_\nu (\nu) + \int f(x,\nu + I) \left[ \nu_\nu (\nu + I, \nu) + \nu_\nu (\nu + I, \nu - \nu) \right] d\nu', \hspace{1cm} (9)$$

takes into account the ionization of a gas, $I$ - the ionization potential of an atom (molecule) of a gas, $\nu_\nu (\nu) = \int \nu_\nu (\nu, \nu') d\nu'$, $\nu_\nu (\nu, \nu') = N \cdot V (\nu) \sigma_\nu (\nu, \nu')$ - the ionization frequency of an atom (molecule) of a gas by electron impact, $\sigma_\nu (\nu, \nu')$ - a microscopic ionization cross section of an atom (molecule) of a gas by electron impact, $\nu, \nu'$ - electron energy before and after a collision, respectively.

Using boundary conditions

$$F(x,\nu,\mu) \bigg|_{\nu=0,\mu=0} = F_0 (\nu, \mu),$$

$$F(x,\nu,\mu) \bigg|_{x \to \infty, \mu=0} \to 0,$$

$$f(x,\nu) \bigg|_{\nu=0} = f_0 (\nu),$$

$$\frac{\partial f(x,\nu)}{\partial x} \bigg|_{\nu=0} = f'_0 (\nu),$$

you can get the expression for the flow of UV quanta irradiated by plasma from the equation (2)

$$F(x,\nu',\mu) = \begin{cases} 
  F_0 (\nu', \mu) \exp \left[ -\frac{\Sigma_\nu (\nu') x}{\mu} \right] 
  + \frac{1}{\mu^2} \int d\nu' \beta (\nu', \nu) f(x,\nu) \exp \left[ \frac{\Sigma_\nu (\nu')(x'-x)}{\mu} \right] d\nu', \text{ if } \mu > 0; \\
  -\frac{1}{\mu^2} \int d\nu' \beta (\nu', \nu) f(x,\nu) \exp \left[ \frac{\Sigma_\nu (\nu')(x'-x)}{\mu} \right] d\nu', \text{ if } \mu < 0.
\end{cases} \hspace{1cm} (11)$$

After substitution (11) in (1) the following equation is obtained
\[-U \frac{\partial f(x, \varepsilon)}{\partial x} = d(\varepsilon) \frac{\partial^2 f(x, \varepsilon)}{\partial x^2} + \hat{L}[f(x, \varepsilon)]\]

\[+2\pi \int_0^\infty \frac{d\mu}{\mu} \left[ \frac{\Sigma_0(\varepsilon + \varepsilon_i, \varepsilon + \varepsilon_j, \varepsilon + \varepsilon_k, \varepsilon + \varepsilon_l, \mu)}{\mu} \right] d\mu\]

\[-\frac{2\pi}{\mu} \int_0^\infty \frac{d\mu}{\mu} \int_{-1}^1 \beta(\varepsilon + \varepsilon_i, \varepsilon + \varepsilon_j, \varepsilon + \varepsilon_k, \varepsilon + \varepsilon_l, \mu) \right] d\varepsilon'\]

\[+\frac{2\pi}{\mu} \int_0^\infty \frac{d\mu}{\mu} \int_{-1}^1 \beta(\varepsilon + \varepsilon_i, \varepsilon + \varepsilon_j, \varepsilon + \varepsilon_k, \varepsilon + \varepsilon_l, \mu) \right] d\varepsilon'.\]

Using replacement

\[x = \frac{y d(\varepsilon)}{U},\]

the equation (12) is brought to mind

\[-\frac{\partial f(y, \varepsilon)}{\partial y} = \frac{\partial^2 f(y, \varepsilon)}{\partial y^2} + \frac{d(\varepsilon)}{U^2} \frac{\partial f(y, \varepsilon)}{\partial y} + \hat{L}[f(y, \varepsilon)]\]

\[+\frac{2\pi}{\mu} \int_0^\infty \frac{d\mu}{\mu} \left[ \frac{\Sigma_0(\varepsilon + \varepsilon_i, \varepsilon + \varepsilon_j, \varepsilon + \varepsilon_k, \varepsilon + \varepsilon_l, \mu)}{\mu} \right] d\mu\]

\[-\frac{2\pi}{\mu} \int_0^\infty \frac{d\mu}{\mu} \int_{-1}^1 \beta(\varepsilon + \varepsilon_i, \varepsilon + \varepsilon_j, \varepsilon + \varepsilon_k, \varepsilon + \varepsilon_l, \mu) \right] d\varepsilon'\]

\[+\frac{2\pi}{\mu} \int_0^\infty \frac{d\mu}{\mu} \int_{-1}^1 \beta(\varepsilon + \varepsilon_i, \varepsilon + \varepsilon_j, \varepsilon + \varepsilon_k, \varepsilon + \varepsilon_l, \mu) \right] d\varepsilon'.\]

Consider the operator \(\hat{L}' = d(\varepsilon) \cdot \hat{L}\). The eigenfunctions \(g_k(\varepsilon)\) and eigenvalues \(\lambda_k\) of the operator \(\hat{L}'\) are determined from the equation

\[\hat{L}'[g_k(\varepsilon)] = \lambda_k g_k(\varepsilon).\]

The operator \(\hat{L}'\) is non-Hermitian, the spectrum of its eigenvalues has the following property: there is a maximum positive eigenvalue \(\lambda_{\max}\), the remaining eigenvalues have a real part less than \(\lambda_{\max}\) [13].

Consider also the adjoint equation to (15)

\[\hat{L}'^*[g_k^*(\varepsilon)] = \lambda_k^* g_k^*(\varepsilon),\]

the function \(g_k^*\) is normalized by the following relation \(\int_0^\infty g_k g_k^* d\varepsilon = 1\), and the eigenvalues corresponding to the functions \(g_k\) and \(g_k^*\) are complexly conjugate to each other.

We will seek a solution of the equation (14), decomposing \(f(x, \varepsilon)\) into the eigenfunctions of the operator \(\hat{L}'\)

\[f(x, \varepsilon) = \sum_k g_k(\varepsilon) Y_k(y),\]

the corresponding coordinate part of the functions is
\[ Y_i(y) = \exp \left\{-\frac{1}{2} \pm \left(\frac{1}{4} - \frac{\lambda_i}{U^2}\right)^{1/2}\right\} y. \] (18)

Then from the equation (14) in the coordinates \( x, \epsilon \) you can get the following integral equation

\[ f(x, \epsilon) = f_0(x, \epsilon) + G(x, \epsilon) - \int_0^\infty K(x, x', \epsilon) f(x', \epsilon) \, dx', \] (19)

where

\[ f_0(x, \epsilon) = \sum_k g_k(\epsilon) \left( C_k \exp \left\{-\frac{x}{2} \frac{U}{d(\epsilon)} \left[ 1 - \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2}\right] \right\} + B_k \exp \left\{-\frac{x}{2} \frac{U}{d(\epsilon)} \left[ 1 + \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2}\right] \right\} \right); \] (20)

the solution of a homogeneous equation, \( C_k, B_k \) - constants determined from the boundary conditions(10),

\[ K(x, x', \epsilon) = \left\{ \begin{array}{ll}
2\pi \int_0^\infty \frac{d\mu}{\mu} \int \beta(\epsilon, \epsilon') \exp \left[ \frac{\Sigma_a(\epsilon)(x' - x)}{\mu} \right] \, d\epsilon', & \text{при } x > x'; \\
-2\pi \int_0^\infty \frac{d\mu}{\mu} \int \beta(\epsilon, \epsilon') \exp \left[ \frac{\Sigma_a(\epsilon)(x' - x)}{\mu} \right] \, d\epsilon', & \text{при } x < x';
\end{array} \right. \] (21)

\[ G(x, \epsilon) = \sum_k \frac{4\pi g_k(\epsilon)}{U} \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2} \int_0^\infty \frac{d(\epsilon)}{d\epsilon} \Sigma_a(\epsilon + \epsilon_i) F_0(\epsilon + \epsilon_i, \mu) g_k(\epsilon) \]
\[ \times \left[ \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2} - \frac{2\Sigma_a(\epsilon + \epsilon_i)d(\epsilon)}{\mu U} \right]^{-1} \exp \left\{-\frac{x}{2} \frac{U}{d(\epsilon)} \left[ 1 - \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2}\right] \right\}, \]
\[ - \left[ \left(1 + \frac{4\lambda_k}{U^2}\right)^{1/2} - \frac{2\Sigma_a(\epsilon + \epsilon_i)d(\epsilon)}{\mu U} \right]^{-1} \exp \left\{-\frac{x}{2} \frac{U}{d(\epsilon)} \left[ 1 + \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2}\right] \right\}, \]
\[ -2\left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2} \left[ \left(1 - \frac{2\Sigma_a(\epsilon + \epsilon_i)d(\epsilon)}{\mu U} \right)^2 - 1 + \frac{4\lambda_k}{U^2}\right] \exp \left[ -\frac{\Sigma_a(\epsilon + \epsilon_i)x}{\mu} \right] \] \( \, d\epsilon. \) (22)

It should be noted that the core of the integral transform, \( K(x, x', \epsilon), \) is positive for all values of its variables. Therefore, for positivity of \( f(x, \epsilon), \) it is necessary (but not enough) the positivity of \( f_0(x, \epsilon) + G(x, \epsilon). \)

We introduce two characteristic values for this problem:

\( L_u = \max \left(\Sigma_a^{-1}\right) - \) maximum mean free path of UV quanta;

\( L_D = \max \left[ d(\epsilon)\lambda_{\text{max}}^{-1/2}\right] - \) maximum diffusion length.

If the parameters of the problem are such that \( L_D > L_u, \) then the asymptotic of the solution when \( x \to +\infty \) is determined by the term \( \exp \left\{-\frac{x}{2} \frac{U}{d(\epsilon)} \left[ 1 - \left(1 - \frac{4\lambda_{\text{max}}}{U^2}\right)^{1/2}\right] \right\} \) in (22), and for the positive definiteness of the function \( f(x, \epsilon), \) the following condition

\[ \frac{x}{d(\epsilon)} \left[ 1 - \left(1 - \frac{4\lambda_k}{U^2}\right)^{1/2}\right] > \frac{\Sigma_a(\epsilon)}{\mu U} \]
If \( L_a > L_0 \), then the asymptotic behavior is determined by the term at \( \exp[-\Sigma_a(\varepsilon + \varepsilon_i)x] \) in (22), with an energy corresponding to the minimum of \( \Sigma_a(\varepsilon + \varepsilon_i) \), and it is necessary to fulfill a more stringent condition arising from the positivity of the corresponding term

\[
1 - \frac{2\Sigma_a(\varepsilon + \varepsilon_i)d(\varepsilon)}{U} - 1 + \frac{4\lambda_{\text{max}}}{U^2} < 0.
\]  

From here you can get a speed limit.

\[
U > \Sigma_a(\varepsilon_i + \varepsilon)d(\varepsilon) + \frac{\lambda_{\text{max}}}{\Sigma_a(\varepsilon_i + \varepsilon)d(\varepsilon)}.
\]  

In this ratio should take the energy \( \varepsilon \) corresponding to the minimum of \( \Sigma_a(\varepsilon_i + \varepsilon) \). This expression can be used only in the case when \( d(\varepsilon) \), with the corresponding energy, does not vanish. Otherwise, which takes place, for example, in argon, in the initial equations (1) and (2) should be taken into account that in a moving reference frame the electrons, produced near the photoionization threshold, in the laboratory coordinates, have an energy differ from zero and equal to \( \varepsilon_u = mU^2/2 \), where \( m \) is the electron mass. In this case, the expression \( d(\varepsilon) \) should be replaced by \( d(\varepsilon_u) \). Then (25) take the form

\[
U > \Sigma_a(\varepsilon_i)d(\varepsilon_u) + \frac{\lambda_{\text{max}}}{\Sigma_a(\varepsilon_i)d(\varepsilon_u)}.
\]  

Solving (26) for \( U \) by iteration, one can get the velocity limit in this approximation.

The system of inequalities (23), (25) allows us to indicate the lower bounds of the rate of discharge propagation without a cumbersome numerical solution of the equations of radiation transfer and ionization kinetics.

3. Results

To determine the lower limits of the speed of propagation of discharges, a software package was developed. The program calculates the eigenfunctions and eigenvalues of the discrete analogue of the operator \( \hat{L} \), as well as the calculation of the lower limits of the rate of propagation of the discharge. The operator \( \hat{L} \) is approximated by a square matrix in the energy range up to 52.2 eV with an energy resolution of 360 to 540 points. The eigenvalues of the matrix are determined by its diagonalization. In some cases, calculations were also performed on a more detailed grid of 720 points. The speed values calculated on these grids are practically independent of the number of points. The cross sections for collisions of electrons with argon atoms are taken from [14-17].

In the present work, calculations were made of the dependence of the intensity of the emission of a \( \text{CO}_2 \) laser on the lower limit of the propagation velocity of a plane ionizing wave in argon. The results are shown in the figure 1: 1 - the results of this work, 2 - the lower limit of the ionization wave velocity according to calculations [12]. The minimum cross section of photoionization in the vicinity of the threshold is assumed to be \( 1.4 \times 10^{-18} \text{ cm}^2 \). In this case, below the points A and \( A_1 \) the limit of velocity is determined by the ratio (23), above by the ratio (25). Points - experimental data taken from: 3 - [18] at atmospheric pressure, 4 - [6] at atmospheric pressure, 5 - [3] at argon pressure \( 1/75 \) atmospheric, in all cases a \( \text{CO}_2 \)-laser with a wavelength of \( \lambda = 10.6 \) microns was used.
4. Conclusion
In conclusion, we list the main results of this work:

1. A theory has been developed that makes it possible to formulate strict inequalities that limit the lower bounds of the velocity of fast ionization waves in gases, depending on the wavelength of the laser radiation, its intensity and the composition of the gaseous medium.

2. Developed a method for the numerical solution of the equations obtained. Created a set of programs to solve this problem.

3. Calculations of the lower boundaries of the velocities of fast ionization waves for argon were carried out. In the calculations, the real dependence of the cross sections for a large number of elastic and inelastic collisions of electrons with atoms and molecules was taken into account.

4. Comparison of the results of calculations with experimental data and the results of other theoretical works. The calculation results do not contradict the experimental data. Further, it is necessary to consider in more detail the kinetics of ionization, in particular, to take into account the associative ionization of highly excited states of the atoms of the medium.

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