Circular Hall Effect in a rod conductor

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We discover the circular Hall Effect in a conducting rod sample. The uniform current flow in a rod can be viewed as a carrier drift in crossed the radial Hall electric field and the azimuthal magnetic field caused by the current itself. Arguing the diaphragm currents could define the correct boundary condition at the inner rod surface we find the nonuniform density of longitudinal current taking into account the finite viscosity of 3D electron liquid. Unexpectedly, under certain condition the resistivity of the rod sample vanishes exhibiting the transition to zero-resistance state. At the transition the magnetic field is shown to be pushed out of the sample bulk. The critical temperature of the transition is calculated.

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I. CIRCULAR HALL EFFECT: UNIFORM CURRENT FLOW

Neglecting the term \((\vec{v})\nabla)\vec{v}\), the routine Euler hydrodynamic equation for 3D electronic liquid placed in arbitrary electric and magnetic fields yields

\[
\frac{\partial \vec{v}}{\partial t} = -\frac{\vec{v}}{\tau} + \frac{e\vec{E}}{m} + \left[\vec{v} \times \omega_e \right],
\]

(1)

where \(e\) is absolute value of the the electronic charge, \(\vec{E}\) is the electric field, \(\omega_e = \omega_c \vec{B}/B\) and \(\omega_c = \frac{e\vec{B}}{mc}\) is the cyclotron frequency, \(m\) is the effective mass, \(\tau\) is the momentum relaxation time due to collisions with impurities and phonons. For steady state \(\frac{\partial \vec{v}}{\partial t} = 0\) one obtains the vector equation

\[
\vec{v} = \frac{\mu \vec{E} + ([\vec{v} \times \omega_e \tau] + \omega_e \tau \left[\vec{E}, \omega_e \right])}{1 + (\omega_e \tau)^2},
\]

(2)

where \(\mu\) is the mobility of the electron gas.

For actual case of a rod conductor(see Fig.1) it is convenient to use the cylindrical geometry(\(\rho, \varphi, z\)). Let us suppose that the longitudinal current density \(j_z = nev_z\) is uniform(i.e. \(v_z \neq v_z(\rho)\)) and, moreover, assume no current in radial direction \(v_r = 0\). Under the above assumptions the only longitudinal \(E_z\) and radial( Hall ) \(E_\rho\) components of the electric field could persist. According to Maxwell equations, at certain \(\rho\) the longitudinal current provides the azimuthal magnetic field as \(B_\rho = \frac{2\pi j_z}{\rho}\). Since \(B_\rho \perp E_z,\rho\), the third term in Eq. (2) can be disregarded. Using Eq.(2), we finally obtain the \(z, \rho\)-axis related equations as it follows

\[
v_z = \frac{\mu E_z + \omega_e \tau E_\rho}{1 + (\omega_e \tau)^2},
\]

(3)

\[
0 = \mu E_\rho - \omega_e \tau E_z.
\]

Finally, we obtain the relationship between the components of the radial and longitudinal electric field as it follows \(E_\rho = \omega_e \tau E_z\), and therefore \(v_z = \mu_ee E_z\). Actually, the longitudinal current can be viewed as a drift of the carriers in a crossed azimuthal magnetic field and radial Hall electric field \(v_z = \frac{E_\rho}{\mu_ee}\).

It is instructive to introduce the ultimate value the azimuthal magnetic field \(B_0 = \frac{2\pi j_z}{\rho R}\), hence \(B_\rho = B_0 \frac{\rho}{R}\), where \(R\) is the rod radius. The radial electric field yields the linear in \(\rho\) dependence as well \(E_\rho = E_0 \frac{\rho}{\rho^2}\), where \(E_0 = \frac{\mu_ee^2}{2\pi R^2}\) is the maximal value of the electric field at the rod surface. Finally, one can find the respective volumetric charge density \(Q = \div \vec{E}/4\pi = ne \left(\frac{\mu_ee}{c}\right)^2\). As expected, the rod is extra charged, while \(Q/ne \ll 1\).

II. NONUNIFORM CURRENT FLOW

Note, under the assumption of the nonuniform current flow \(v_z(\rho)\) the solution of Eq.(1) provides the nonzero azimuthal component of the vortex rot \(\vec{E} = -\frac{1}{\rho} \frac{\partial}{\partial \rho} (\vec{E}_\rho) \neq 0\) which is prohibited within stationary Maxwell equations formalism. Nevertheless, this problem can be resolved using the Stokes equation for viscous flow of 3D electronic liquid[2] (for 2D electrons, see [3]) as

\[
\frac{\partial \vec{v}}{\partial t} = -\frac{\vec{v}}{\tau} + \frac{e\vec{E}}{m} + [\vec{v} \times \omega_e] + \nu \Delta \vec{v},
\]

(4)

Here, \(\nu = v_F^2 7\tau_{ee}/5\) is the kinematic viscosity coefficient, \(v_F\) is the Fermi velocity. Then, \(\tau_{ee} = \tau_{ee}^{*} 2^4[4]\) is the
Here, we introduce the dimensionless velocity for fixed longitudinal electric field and viscosity parameter $\eta$ = 5:50 for wall adhesion boundary condition $v_0 = 0$ (blue); diamagnetic current boundary condition $v_0 = d(\text{red})$. The Poiseuille flow for $\eta = 1$ is shown by the dashed line. Dotted line represents the drift velocity $v = 1$.

electron-electron collisions time, $\xi = \frac{\hbar T}{\mu}$ is the degeneracy parameter, $\mu$ is the Fermi energy of 3D electronic liquid. Note that the hydrodynamic approach is valid when the e-e mean free path $l_{ee} = v_F \tau_{ee}$ is less than the typical length scale of the problem.

For steady state the Eq.(4) decouples into equations associated with longitudinal and radial current flow as

$$\begin{align*}
e\frac{E_z}{m} - \frac{v_z}{\tau} + \nu \nabla v_z & = 0, \quad (5) \\
e\frac{E_\rho}{m} - \frac{v_\rho}{\tau} + eB_z \frac{v_\rho}{mc} & = 0.
\end{align*}$$

With the help of the second equation we conclude that the azimuthal magnetic filed yields $B_z = \frac{\Delta \varepsilon_{ee}}{\mu} \int_0^\rho v_\rho d\rho$ is related to the Hall electric field via relationship $E_\rho = \frac{v_\rho}{\tau} B_z$.

We now attempt to solve Eq.(5) for fixed value of the longitudinal electric field, i.e. $E_z = \text{const}$. Using the textbook form of Laplace operator, we re-write Eq.(5) as

$$v'' + \frac{1}{\rho} v' - v + 1 = 0. \quad (6)$$

Here, we introduce the dimensionless velocity $v = v_z/v_d$ and the radial coordinate $\tilde{\rho} = \rho/\lambda$. Then, $\lambda = \sqrt{BT}$ is the length scale of the problem and $v_d = \mu_c E_z$ is the carrier drift velocity. Eq.(6) obeys a textbook solution as

$$v = 1 + C_1 J_0 + C_2 Y_0, \quad (7)$$

where $J_0$ and $Y_0$ are the zero-order Bessel functions of the first and second kind respectively. Arguing that the flux velocity remains finite we find $C_2 = 0$, since $Y_0 \to \infty$ at $\rho \to 0$. For a moment, we use a general condition for flux velocity at the lateral rod surface in the form $v(R/\lambda) = v_0$. Consequently, Eq.(7) yields

$$v = 1 + (v_0 - 1) \frac{J_0(\eta)}{J_0(\eta)}, \quad (8)$$

where we introduce the reduced coordinate $r = \rho/R$ and the dimensionless viscous parameter $\eta = R/\lambda$. Using Eq.(8) we find the total current $I = ne \int_0^\rho 2\pi v_d p d\rho$ as it follows

$$I = I_d \left[ 1 + (v_0 - 1) \frac{2J_1(\eta)}{\eta J_0(\eta)} \right]. \quad (9)$$

where $I_d = \pi R^2 ne v_d$ is the total ohmic current.

Let us now perform the detailed analysis of the boundary condition at the rod inner surface, i.e. at $r = 1$. Firstly, we discuss the routine condition $v_0 = 0$ of the wall adhesion [2], known in conventional hydrodynamics for viscous Poiseuille flow. In Fig.2 the blue curves depict the spatial dependence of the flux velocity for different values of the viscous parameter $\eta$. For small viscosity $\eta \gg 1$ the 3D fluid velocity is uniform throughout the sample cross section, except the ultra-narrow layer $\sim \lambda$ closed to the rod surface. Actually, the solution of Eq.(8) at $v \to 0$ coincides with that obtained in Sec.I. Then, in the opposite case highly viscous liquid $\eta \leq 1$ the flux velocity follows the Poiseuille law $v(r) = \frac{\nu}{\tau}(1 - r^2)$ shown by the dashed line in Fig.2.

The special interest of the present paper concerns the possibility of absolutely unusual boundary condition $v_0 > 1$, which, however, presume the transparent underlying physics. Let the 3D electron liquid manifests the diamagnetic properties. For certain value of the total current $I$ the azimuthal magnetic field $B_0 = \frac{M_0}{2\pi R}$ at the inner rod surface results in [5] an extra longitudinal diamagnetic current $I_M = 4\chi |I|$, where $\chi = \frac{1}{2} \eta \mu_B < 0$ is the magnetic susceptibility of 3D electrons[6], $\mu_B$ is the Bohr magneton. Let the diamagnetic current exhibits the uniform flow within the narrow layer $\delta \sim \eta^{-1/3}$. One can roughly find out the diamagnetic current density $j_M = \frac{2\chi I}{\delta^2} \sim 1/\delta$, which could be high enough. With the help of the above findings we are able to set the condition at the rod boundary $r = 1$ as

$$v_0 - 1 = \frac{j_M}{}\frac{ne v_d}{I_d} = \frac{I}{I_d} \frac{2\pi R |\chi|}{\delta}, \quad (10)$$

Using the reduced total current $i = I/I_d$, the Eq.(9) becomes self-consistent, thus provides the elementary solution

$$i = \frac{1}{1 - k_d} \quad (11)$$

where $\kappa = \frac{2\pi R |\chi|}{\delta}$ is the sample-size dependent dimensionless parameter associated with diamagnetic properties of 3D liquid. The factor $d(\eta) = \frac{2J_1(\eta)}{\eta J_0(\eta)}$ is related to
flux velocity distribution governed by viscous parameter $\eta$. Eq.(11) defines the total current at fixed longitudinal electric field $E_L$. This is the central result of the present paper. As expected, for low viscosity case $\eta \gg 1$ the total current is given by the sum of the ohmic component superimposed by small diamagnetic correction, i.e. $I = I_d(1 + 4\pi|\chi|)$. Viscosity enhancement leads to total current increase caused by flux density re-distribution (see the red curves in Fig.2).

Introducing the average current density at fixed total current as $j = I/\pi R^2$ and, moreover, assuming the ultra-low current measurements $I \to 0$ the calculations of the sample effective resistivity $\rho_{\text{eff}} = E_z/j$ is straightforward:

$$\rho_{\text{eff}} = \rho_d(1 - \kappa d), \quad (12)$$

where $\rho_d = \frac{m}{ne^2\tau}$ is the conventional Drude resistivity. In absence of viscous effects the effective resistivity is given by the Drude component. Surprisingly, the effective resistivity vanishes and leads to zero-resistance state (ZRS)[7] when condition $\kappa d = 1$ is satisfied. Note that $d(\eta)$ is smooth descending function and, moreover, $d(\eta) < 1$ for $\eta > 0$. Hence, for certain $\kappa > 1$ there exists a critical value of viscous parameter $\eta_{cr}$, which defines the ZRS domain as $\eta < \eta_{cr}$. In general, the ZRS critical diagram can be viewed as $\eta_{cr}(\kappa)$.

Let us now find out the condition of ZRS feasibility, i.e. when $\kappa \geq 1$:

$$\frac{1}{2(9\pi)^{1/3}} \frac{e^2}{m_e c^2} \kappa^{2/3} R \geq 1. \quad (13)$$

For example, for Tl-sample carrier density $n = 2.6 \times 10^{22}\text{cm}^{-3}$ ($\mu = 2.9\text{eV}$) the ZRS is possible when $R \geq 0.25\text{mm}$. Once Eq.(13) is satisfied, one can find the critical viscous parameter $\eta_{cr}$ which defines in turn the critical temperature of ZRS transition. To confirm this, we reassign the viscosity parameter as $\eta = 2\kappa\Theta$, where $\Theta = \frac{1}{4\pi \kappa \lambda}$ plays the role of the dimensionless temperature. Indeed, using e-e scattering time notation $\tau_{ee} = \tau_{ee}^0 \xi^{-2}$, we

obtain $\Theta = T/T_c$, where

$$T_c = T_F \frac{4\pi |\chi|}{\delta} \left( \frac{1}{5} \frac{1}{\rho c} \xi_{ee}^0 \right)^{1/2} \quad (14)$$

is the critical temperature of the bulky sample, which exhibits an enhancement for cleaner and(or) higher electron density samples, i.e. $T_c \sim \rho^{5/2} \xi^{1/2}$. With the help of the above notations we represent the ZRS condition $\kappa d = 1$ as the transcendental equation $\Theta = \frac{f_1(2\kappa\Theta)}{f_2(2\kappa\Theta)}$, whose solution gives the sought-for critical diagram $\Theta_{cr}(\kappa)$. The result is represented in Fig.3. The area below the bold line corresponds to zero resistance state. We argue that the critical temperature depends on the sample size. According to drawings in Fig.3, the bigger is the sample, the higher is the critical temperature. For bulky sample $\kappa \gg 1$ the critical temperature is exactly that specified by Eq.(14). Our findings is exactly that observed in recent experiments[7].

We now able to reconstruct the effective resistivity specified by Eq.(12) as a function of temperature. For Tl sample in question (see Fig.4, insert) we precise the low-T Drude resistivity as $\rho_d = \rho_0(1 + \alpha \xi^2)$, where $\alpha$ is the dimensionless constant. Consequently, the inverse momentum relaxation time yields $\frac{1}{\tau} = \frac{2e^2}{m_e c^2} \frac{1}{\rho_0(1 + \alpha \xi^2)}$. The related viscous parameter takes the form $\eta = 2\kappa\Theta = \beta\xi\sqrt{1 + \alpha \xi^2}$, where $\beta = R \frac{5\pi^2 c^2}{2m_e^3 \mu}$. The dotted line represents the Drude resistivity with viscous effects disregarded. Insert: the typical experimental data for T-dependent resistivity for bulky Tl,Hg,Sn,Pb.

FIG. 3: The critical diagram of the zero-resistance state: the dimensionless temperature $\Theta_{cr} = T/T_c$ as a function of the size-dependent sample parameter $\kappa$.

FIG. 4: The effective resistivity given by Eq.(12) for Tl parameters specified in text and the sample radius $R[\text{mm}] = 1; 5; 10$. ZRS critical temperatures are $0.23; 1.2; 2.4\text{K}$ respectively. The dotted line represents the Drude resistivity with viscous effects disregarded. Insert: the typical experimental data for T-dependent resistivity for bulky Tl,Hg,Sn,Pb.
The current density $j_z(r)$ and magnetic field $B(r)$ (insert) specified by Eq.(15) for $\kappa = 10$ (corresponds to $\eta_{cr} = 7.44$), fixed total current $I$ and viscosity parameter $\eta = \eta_{cr}$; 14.9; 100. Dashed line on the main panel depicts the uniform density of the ohmic current. Dotted line in insert corresponds to magnetic field screening asymptote described in text.

The result is plotted in Fig.5. At fixed constant $\kappa$ the viscosity increase leads to current pinched nearby the inner rod boundary. Simultaneously, the magnetic field is pushed out of the sample bulk. One can easily check that in massive samples ($\kappa \gg 1$) the screening of the magnetic field yields the universal form $B_0 = B_0 \exp \left( -\frac{r}{\eta_{cr}} \right)$, where $l_B \sim \frac{\delta}{\delta \eta_{cr}}$ is the B-screening length. For Tl sample we estimate $l_B \sim 100 \mu$m. Finally, with the help of Eq.(15) the calculations of radial electric field is straightforward $E_{r}(r) = \frac{B_0}{\eta_{cr}}$. We emphasize that the longitudinal current at $T \leq T_c$ could be considered as a non-dissipative drift of carriers in a crossed radial electric and azimuthal magnetic fields, thus provides ZRS.

Remind that the all previous discussion concerned the measurements carried out within zero-current limit. Consequently, we disregard the influence of current-induced magnetic field on the zero-resistance state. For actual cylindrical geometry case the possible influence of the magnetic field can be accounted by the components of 3D liquid viscosity $[8]$ of 3D liquid viscosity $[8]$

$$v_{xx} = \frac{\nu}{1 + 4\omega_c^2 \tau^2}, v_{yx} = \nu_{xx} 2\omega_c \tau$$  \hspace{1cm} (16)

similar to those considered $[3]$ for current flow in a 2D stripe. Here, $\omega_c = \frac{\delta B}{\delta r}$ is the cyclotron frequency. Note that viscosity components specified by Eq.(16) depend on the lowest transport time of the problem, namely the momentum relaxation time, since $\tau \ll \tau_{cr}$. For typical value of the magnetic field $B \sim 100$Gs and $\tau = 10^{-14}$s we obtain $\mu_{ee}^* \sim 1.8 \times 10^{-5} \approx 1$. We conclude that $v_{xx} \approx \nu$ and $v_{yx} \rightarrow 0$, therefore our previous results remain valid.

In conclusion, we discover the circular Hall Effect in a conducting rod sample taking into account both the diamagnetism and finite viscosity of 3D electron liquid. We demonstrate that under certain condition the resistivity of the sample vanishes exhibiting the transition to zero-resistance state. The to current is pinched nearby the inner rod boundary while the magnetic is pushed out of the sample bulk. The critical temperature of the transition is calculated.

### III. ACKNOWLEDGMENTS

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