Flavor structure from flavored Higgs mixing

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Abstract

In this paper, we suggest a simple model which induces realistic flavor structure from mixing of flavored Higgs doublets. The idea is based on the decoupling limit. In a model with many Higgs doublets, the mass eigenstates of scalars are linear combinations of Higgs doublets. If the mass matrix of Higgs fields has only one massless mode, and if the linear combination has flavor dependence, the induced Yukawa coupling will have nontrivial flavor structure.

We construct a realistic flavor structure from a mass matrix of flavored Higgs fields in a $U(2)_L \times U(2)_R$ toy model. It suggests several gauge singlet flavons with characteristic vacuum expectation values.

Strong advantage of this model is that all of the elements in Yukawa matrix can be determined from renormalizable Higgs potential.

1 Introduction

The discovery of the Higgs boson [1, 2] and its coupling analysis [3] reinforce the experimental validity of the Standard Model (SM). However, the model still has several mysteries, e.g., theoretical origin of Higgs, the hierarchy problem, and the flavor puzzle.

If some flavored Higgs field exists [4, 5], flavor structure is mediated to the Higgs sector by radiative or mixing mechanisms [6]. In the mixing context, several ideas induce flavor structures from mixing between SM fermions and heavy partners, such as universal seesaw mechanism [7, 8], partial compositeness [9, 10] and $E_6$ twist mechanism [11, 12]. However, the flavor structure perhaps originates in the scalar mixing. This idea cannot be found as far as author knows.

In this paper, we suggest a simple model which induces realistic flavor structure from mixing of flavored Higgs doublets. The idea is based on the decoupling limit [13, 14]. In a model with many Higgs doublets (nHDM), the mass eigenstates of scalars are linear combinations of Higgs doublets. If the mass matrix of Higgs fields has only one massless mode, and if the linear combination has flavor dependence, the induced Yukawa coupling will have nontrivial flavor structure. We construct a realistic flavor structure from a mass...
matrix of flavored Higgs fields in a $U(2)_L \times U(2)_R$ toy model. It suggests several gauge singlet flavons with characteristic vacuum expectation values (vevs).

Strong advantage of this model is that all of the elements in Yukawa matrix can be determined from renormalizable Higgs potential. This feature is substantially different from Froggatt–Nielsen mechanism \cite{13} with many undetermined $O(1)$ couplings. A drawback is number of Higgs bosons. A two flavor toy model becomes eight Higgs doublet model (8HDM). Naively simple model requires $4n_f^2$ Higgs doublets. However, this number is not so large because 126 representation in SO(10) model has (2, 2, 15) Higgs field in the Pati–Salam model and then it is 30HDM.

Meanwhile, if the determinant of the mass matrix $\det \mathcal{M}_{\mu\nu} < 0$ holds, mass of the lighter boson becomes negative and triggers the electroweak symmetry-breaking (bosonic seesaw mechanism) \cite{10, 18}. Interplay between these ideas also seems to be interesting possibility.

2 Basic idea: the decoupling limit

The basic idea of the paper is based on the “decoupling limit” \cite{13}. In the two Higgs doublet model (2HDM), the mass eigenstates of Higgs fields are linear combinations of two Higgs doublets $\phi \sim a \phi_1 + b \phi_2$. If one of the mass eigenstates have enough large mass, the heavy state will be integrated out from the theory and overall factor of the Yukawa matrices will be changed. In this section, we review this mechanism.

For the Higgs doublets $\phi_1, \phi_2$, the Higgs potential of the 2HDM is written by \cite{13}

$$V_H(\phi_1, \phi_2) = \sum_{a,b}^{1,2} m_{ab} \phi_a^\dagger \phi_b + \sum_{a,b,c,d}^{1,2} \lambda_{ab,cd} (\phi_a^\dagger \phi_b) (\phi_c^\dagger \phi_d).$$

Assuming zero CP phase $m_{21}^2 = m_{21}^\ast$ and $\det \mathcal{M} = m_{11}^2 m_{22}^2 - m_{12}^4 = 0$, the mass matrix is given by

$$\phi_a^\dagger \mathcal{M}_{ab} \phi_b \equiv (\phi_1^\dagger \phi_2^\dagger) \begin{pmatrix} m_{11}^2 & m_{11} m_{22} \\ m_{11} m_{22} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$ (2)

The matrix is diagonalized as

$$\begin{pmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & m_{11}^2 + m_{22}^2 \end{pmatrix} \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix},$$ (3)

Then, the mass eigenstates $\phi, \phi_H$ are found to be

$$\begin{pmatrix} \phi \\ \phi_H \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi \\ \phi_H \end{pmatrix}.$$ (4)

Then the massive eigenstate $\phi_H$ decouples from the low-energy theory (decoupling limit). If the full potential of $\phi_{ij}$ has symmetry breaking minima and $\phi_{1,2}$ develop vevs $v_{1,2}$, the decoupling is realized as far as $m_{ij} \gg v_i(i, j = 1, 2)$ holds.
Yukawa interactions of quarks (with natural flavor conservation [20]) are

\[ \mathcal{L}_Y = Y_u \tilde{q}_L u_R \tilde{\phi}_1 + Y_d \tilde{q}_L d_R \phi_2 + h.c.. \]  

(5)

When the massive state \( \phi_H \) is integrated out, it leads to

\[ \mathcal{L}_Y = Y_u \tilde{q}_L u_R (c_\beta \tilde{\phi} - s_\beta \tilde{\phi}_H) + Y_d \tilde{q}_L d_R (s_\beta \phi + c_\beta \phi_H) + h.c. \]  

\[ \rightarrow c_\beta Y_u \tilde{q}_L u_R \tilde{\phi} + s_\beta Y_d \tilde{q}_L d_R \phi + h.c., \]  

(6)

and the isospin violation of Yukawa interactions is introduced naturally. In some sense, the decoupling limit of 2HDM induces the simplest flavor structure. Therefore, if we extend this idea to the flavor space, the nontrivial flavor structure will be induced from Higgs mixing.

3 \( U(2)_L \times U(2)_R \) toy model

For simplicity, we consider a toy model with \( U(2)_L \times U(2)_R \) flavor symmetry. The Higgs doublets are enlarged to a \( U(2)_L \times U(2)_R \) bi-doublet scalar \( \phi_{aij} \) with Higgs family and flavor indices \( a, i, j = 1, 2 \). Then it is an eight Higgs doublet model (8HDM). Left-(right-)handed quarks are assigned to doublets of \( U(2)_L(R) \). For these fields, the Yukawa interactions are given by

\[ \mathcal{L}_Y = Y \tilde{q}_L \phi_{aij} q_R j = Y \tilde{q}_L \phi_{aij} \begin{pmatrix} u_{Rj} \\ d_{Rj} \end{pmatrix} + h.c.. \]  

(8)

Here, \( Y \) does not have any flavor structures. For simplicity, the rotation between \( \phi_1 \) and \( \phi_2 \) is assumed to be the same form to Eq. (4):

\[ \begin{pmatrix} \phi_{1ij} \\ \phi_{2ij} \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_{ij} \\ \phi_{Hij} \end{pmatrix}. \]  

(9)

When the heavy eigenstate \( \phi_{Hij} \) is integrated out, the massless \( \phi_{ij} \) remains the theory. Then, the isospin violation is introduced as

\[ \mathcal{L}_Y \rightarrow Y \tilde{q}_L \begin{pmatrix} s_\beta & c_\beta \end{pmatrix} \phi_{ij} \begin{pmatrix} u_{Rj} \\ d_{Rj} \end{pmatrix}. \]  

(10)

The general mass matrix of \( \phi_{ij} \) is written as

\[ \phi_{\mu}^\dagger M_{\mu \nu} \phi_{\nu} \equiv \begin{pmatrix} \phi_0^\dagger \\ \phi_i^\dagger \end{pmatrix} \begin{pmatrix} m_{00}^2 & m_{0j}^2 \\ m_{i0}^2 & m_{ij}^2 \end{pmatrix} \begin{pmatrix} \phi_0 \\ \phi_j \end{pmatrix}. \]  

(11)

Here, we rewrite the bi-doublet Higgs as

\[ \phi_{ij} = \sigma_{ij}^\dagger \phi_\mu, \quad \phi_\mu = \frac{1}{2} \text{tr}[\sigma^\nu \phi_{ij}], \quad \sigma^\mu \equiv (1, \sigma^i). \]  

(12)
If $\phi_{ij}$ is a hermitian matrix, $\phi_\nu$ becomes real field. In this case, $\phi_{ij}$ is a general complex matrix and then $\phi_\nu$ is also complex field.

The mass matrix is diagonalized by a unitary matrix $U$:

$$\phi_\mu^\dagger \mathcal{M}_{\mu\nu} \phi_\nu = \phi_\mu^\dagger U_{\mu\rho} \mathcal{M}_\rho^{\text{diag}} U_{\rho\nu} \phi_\nu. \quad (13)$$

Then, the mass eigenstates $\Phi_\rho$ are found to be $\Phi_\rho = U_{\rho\nu} \phi_\nu$ (and then $\phi_\mu = U_{\mu\nu} \Phi_\nu$). Here we assume the eigenvalues of $\mathcal{M}$ has only one zero, such as $\mathcal{M}_{\rho}^{\text{diag}} \sim \text{diag}(0, M, M, M)$. Then $\Phi_0$ is massless mode and massive modes $\Phi_i$ will be integrated out. The Yukawa interactions are modified as

$$\mathcal{L}_Y = Y_{\bar{q}L_i} \phi_{ij} q_{Rj} \rightarrow \bar{Y}_{\bar{q}L_i} (s_\beta \sigma_{ij}^\mu \phi_\mu, c_\beta \sigma_{ij}^\mu \phi_\mu) \begin{pmatrix} u_{Rj} \\ d_{Rj} \end{pmatrix} \quad (14)$$

$$\rightarrow \bar{q}_{Li} (Y s_\beta \sigma_{ij}^\mu U_{\mu0}, Y c_\beta \sigma_{ij}^\mu U_{\mu0}) \Phi_0 \begin{pmatrix} u_{Rj} \\ d_{Rj} \end{pmatrix}. \quad (15)$$

If we identify $\Phi_0$ as the SM Higgs boson, the Yukawa matrices found to be

$$Y_u = Y s_\beta \sigma_{ij}^\mu U_{\mu0}, \quad Y_d = Y c_\beta \sigma_{ij}^\mu U_{\mu0}. \quad (16)$$

Or, in components,

$$Y_u = Y s_\beta \sigma_{ij}^\mu U_{\mu0} = Y s_\beta \begin{pmatrix} U_{00} + U_{30} & U_{10} - iU_{20} \\ U_{10} + iU_{20} & U_{00} - U_{30} \end{pmatrix}. \quad (17)$$

Therefore, when the heavy Higgs fields $\Phi_i$ are integrated out, the massless Higgs field $\Phi_0$ will have nontrivial flavor structures. In particular, hierarchical Yukawa structure requires $U_3 \sim -U_0 \gg U_1, U_2$. In the following, we consider a concrete mass matrix which induces realistic flavor structures.

### 3.1 $U(2)_L \times U(2)_R$ breaking mass terms

The hierarchical Yukawa texture indicates large mixing between $\phi_0$ and $\phi_3$, that is realized by $U(2)_L \times U(2)_R$ breaking mass terms. For example,

$$\begin{pmatrix} m_{00}^2 & m_{03}^2 \\ m_{30}^2 & m_{33}^2 \end{pmatrix} = \begin{pmatrix} M^2 & M^2 \\ M^2 & M^2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 2M^2 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \quad (18)$$

The rotation matrix $U$ is

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} U_{00} & U_{03} \\ U_{30} & U_{33} \end{pmatrix}, \quad (19)$$

and other matrix components $U_{ij}$ are trivial. The Yukawa matrix found to be

$$Y_u = Y s_\beta \begin{pmatrix} U_{00} + U_{30} & U_{10} - iU_{20} \\ U_{10} + iU_{20} & U_{00} - U_{30} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{2}Y s_\beta \end{pmatrix}. \quad (20)$$
Then, the hierarchical structure is realized in the first approximation.

The general analysis of $4 \times 4$ matrix is troublesome because there are so many free parameters. Then, we tentatively reconstruct a desirable mass matrix from a proper eigenstates and mixing.

If we assume texture of $Y_u$ as the Fritzsch type texture [21], $Y_u \sim \begin{pmatrix} 0 & \lambda \\ \lambda & 1 \end{pmatrix}$, it requires $U_{10}, U_{20} \sim \lambda$. This Yukawa matrix is realized by the following mass and mixing matrix:

$$
\left( \begin{array}{cccc}
\frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\
\frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\
\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\
\frac{-1}{\sqrt{2}} & 0 & 0 & 0 \\
\end{array} \right) \left( \begin{array}{cccc}
m^2 & 0 & 0 & 0 \\
0 & M^2 & 0 & 0 \\
0 & 0 & M^2 & 0 \\
0 & 0 & 0 & 2M^2 \\
\end{array} \right) \left( \begin{array}{cccc}
\frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\
\frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\
\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\
\frac{-1}{\sqrt{2}} & 0 & 0 & 0 \\
\end{array} \right) \equiv \mathcal{M}_{\mu\nu},
$$

(21)

with $m^2/M^2 \sim \lambda$.

This matrix can be recast to that of $\phi_{ij}$ by the relation $\phi_{ij} = \frac{1}{2} \sigma^\mu_{ij} \phi_{ji}$:

$$
\phi^\dagger_{\mu} \mathcal{M}_{\mu\nu} \phi_{\nu} = \frac{1}{2} \sigma^\mu_{ij} \phi^\dagger_{ji} \mathcal{M}_{\mu\nu} \frac{1}{2} \sigma^\nu_{kl} \phi_{lk}.
$$

(23)

Diagonal part of $\mathcal{M}_{\mu\nu}$ gives mass term invariant under diagonal subgroup $U(2)_L+R$. The relation of $SU(N)$ generator of fundamental representation $t^a_{ij} t^a_{kl} = \frac{1}{2} (\delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl})$ leads to

$$
\sigma^\mu_{ij} \sigma^\nu_{kl} = \delta_{ij} \delta_{kl} + 2(\delta_{il} \delta_{kj} - \frac{1}{2} \delta_{ij} \delta_{kl}) = 2 \delta_{il} \delta_{kj},
$$

(24)

and then, we found

$$
\frac{1}{2} \sigma^\mu_{ij} \phi^\dagger_{ji} M^2 \phi_{lk} = \frac{1}{4} 2 \delta_{il} \phi^\dagger_{ji} M^2 \delta_{kj} \phi_{lk} = M^2 \frac{1}{2} \phi^\dagger_{ji} \phi_{ij} = M^2 \frac{1}{2} \text{tr}[\phi^\dagger \phi].
$$

(25)

Similarly, off-diagonal part gives $U(2)$ breaking mass term:

$$
(\mathcal{M}_{\mu\nu})^{\text{off-diag}} = \frac{1}{4} M^2 \text{tr}[\sigma^0 \phi^\dagger] \text{tr}[\sigma^3 \phi] + \frac{1}{4} M^2 \text{tr}[\sigma^3 \phi^\dagger] \text{tr}[\sigma^0 \phi] \quad (26)
$$

$$
+ \frac{1}{4} m^2 \text{tr}[\sigma^0 \phi^\dagger] \text{tr}[(\sigma^1 - i \sigma^2) \phi] + \frac{1}{4} m^2 \text{tr}[(\sigma^1 + i \sigma^2) \phi^\dagger] \text{tr}[\sigma^0 \phi].
$$

(27)

This form suggests several bi-doublet flavons whose vevs break $U(2)_L \times U(2)_R$ symmetry:

$$
\langle \Delta \rangle \sim M \sigma^3, \quad \langle \Delta' \rangle \sim M \sigma^0, \quad \langle \theta \rangle \sim m'(\sigma^1 + i \sigma^2).
$$

(28)
If these flavons $\Delta, \Delta', \theta$ are gauge singlet, they cannot couple to SM fermions (except right-handed neutrinos).

This example is rather artificial, nevertheless, it shows nontrivial flavor structure from mixing of flavored Higgs doublet. The Yukawa matrix can be interpreted as a result of higher dimensional operator $[22]$

$$
\begin{pmatrix}
0 & \lambda \\
\lambda & 1
\end{pmatrix} \sim \begin{pmatrix}
0 & \frac{\theta}{\langle \Delta \rangle} \\
\frac{\theta}{\langle \Delta \rangle} & 1
\end{pmatrix}, \quad \Rightarrow \quad \mathcal{L} = \frac{\theta}{\langle \Delta \rangle} \tilde{H}(\bar{q}_L u_R^2 + \bar{q}_L u_R^1) + h.c. .
$$

However, this flavor structure is in principle originated from renormalizable Higgs potential. Meanwhile, if $\det \mathcal{M}_{\mu\nu} < 0$ holds, mass of the lighter boson becomes negative and triggers the electroweak symmetry-breaking (bosonic seesaw mechanism) $[16–18]$. Interplay between these ideas also seems to be interesting possibility.

## 4 Conclusions

In this paper, we suggest a simple model which induces realistic flavor structure from mixing of flavored Higgs doublets. The idea is based on the decoupling limit. In a model with many Higgs doublets, the mass eigenstates of scalars are linear combinations of Higgs doublets. If the mass matrix of Higgs fields has only one massless mode, and if the linear combination has flavor dependence, the induced Yukawa coupling will have nontrivial flavor structure. We construct a realistic flavor structure from a mass matrix of flavored Higgs fields in a $U(2)_L \times U(2)_R$ toy model. It suggests several gauge singlet flavons with characteristic vevs.

Strong advantage of this model is that all of the elements in Yukawa matrix can be determined from renormalizable Higgs potential. This feature is substantially different from Froggatt–Nielsen mechanism with many undetermined $O(1)$ couplings. A drawback is number of Higgs bosons. A two flavor toy model becomes 8HDM. Naively simple model requires $4n_f^2$ Higgs doublets. However, this number is not so large because 126 representation in SO(10) model has $(2, 2, 15)$ Higgs field in the Pati–Salam model and then it is 30HDM.

Meanwhile, if the determinant of the mass matrix $\det \mathcal{M}_{\mu\nu} < 0$ holds, mass of the lighter boson becomes negative and triggers the electroweak symmetry-breaking (bosonic seesaw mechanism). Interplay between these ideas also seems to be interesting possibility.

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