Improving mathematical problem solving ability through problem-based learning and authentic assessment for the students of Bali State Polytechnic

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Abstract. This research is aimed at determining: 1) the differences of mathematical problem solving ability between the students facilitated with problem-based learning model and conventional learning model, 2) the differences of mathematical problem solving ability between the students facilitated with authentic and conventional assessment model, and 3) interaction effect between learning and assessment model on mathematical problem solving. The research was conducted in Bali State Polytechnic, using the 2x2 experiment factorial design. The samples of this research were 110 students. The data were collected using a theoretically and empirically-validated test. Instruments were validated by using Aiken’s approach of technique content validity and item analysis, and then analyzed using anova stylistic. The result of the analysis shows that the students facilitated with problem-based learning and authentic assessment models get the highest score average compared to the other students, both in the concept understanding and mathematical problem solving. The result of hypothesis test shows that, significantly: 1) there is difference of mathematical problem solving ability between the students facilitated with problem-based learning model and conventional learning model, 2) there is difference of mathematical problem solving ability between the students facilitated with authentic assessment model and conventional assessment model, and 3) there is interaction effect between learning model and assessment model on mathematical problem solving. In order to improve the effectiveness of mathematics learning, collaboration between problem-based learning model and authentic assessment model can be considered as one of learning models in class.

1. Introduction

Mathematics is a language using terms which are defined carefully, clearly, accurately, its representation uses symbols, it is more like a symbol language about idea rather than sound \cite{1}. Mathematical calculation becomes a basis for engineering science\cite{2}.

It is stated in the 22\textsuperscript{nd} Regulation of the Minister of Education of Indonesia which was issued in 2016, that mathematical proficiency expected in mathematics learning includes: concept understanding, reasoning and communication, and problem solving. The aim of mathematics learning in colleges including polytechnics is to help the students in building and developing mathematical power. Mathematical power according to NCTM is five process standards in mathematics learning process, such as: understanding, reasoning, communication, connection, and problem solving.

The ability of problem solving is one of important aspects to make the students literate in mathematics. This ability is very necessary to the community, and it is very important in mathematics, not only for those who will explore or learn mathematics, but also for those who will apply in other...
studies and in everyday life. For the students of polytechnics especially in engineering field, this ability is a potential needed in their education.

Mathematical problem solving ability is very important; many efforts have been intensively and sustainably implemented by the government of Indonesia to improve the education. However, many results of researches show that the abilities in solving and understanding mathematical problems from the students in Indonesia, including those who are in polytechnics are still low. For example, a report from Research and Development Agency (Balitbang) issued in 2011 states that the result of survey of Trends International Mathematics and Science Study (TIMSS) in 2003 shows that learning achievement of the Indonesian 8th graded students was ranked 34th out of 45 countries. Learning achievement of Indonesian students in TIMSS 2007 was more alarming since Indonesia was ranked 39th out of 49 countries [3].

As well as in TIMSS, in Programme for International Student Assessment (PISA), learning achievement of Indonesian students aged about 13 years old was still low. In PISA 2009, Indonesia was just ranked 61st out of 65 countries, with the average score of 371, while the international average score was 496. The result of PISA in the assessment of reading literacy, mathematics and nature sciences in 2015 ranked Indonesia 69th out of 76 countries. These data show that the quality of education in Indonesia is still low [4].

The low quality of education in Indonesia as indicated in TIMSS and PISA is obviously caused by many factors. The one is that Indonesian students in general are poorly trained in solving mathematical problems (non-routine questions) in which the core becomes the characteristics of the questions in TIMSS and PISA. It could happen because the problem solving activity in mathematics learning has not become the main activity yet. As what is obtained from the result of the preliminary observation done by researcher in the engineering field of Bali State Polytechnic. The researcher found that the mathematics learning process in this class tends to be based on the achievement of curriculum material targets, more attentive to the concept memorization than the understanding. This can be seen from the learning process in class which is always dominated by the lecturers. In delivering the material, the lecturers apply lecture method, where the students just sit down, note, and listen to what is being delivered and there is little chance for the students to ask questions. That supports some findings of the curriculum and learning team of the general directorate of higher education, such as: there are still some lecturers who don’t understand very well or who are less concerned about learning achievement, learning methods and strategies, proper way of assessment, and the other findings state that most of the recent learning processes are held in form of lecturing, or unidirectional delivery process (from the lecturers to the students). [5]

Another factor resulting in the lack of the students’ mathematical problem solving ability is the learning models applied by the lecturers. Hudoyo stated that one of the factors that causes the students’ outcome of mathematics learning to become low is the learning models applied by teachers or lecturers [1]. Learning model has important role in creating the success of learning process.

The implementation of PBL in learning process provides benefits, such as: (1) preparing the students better to implement their learning into real-world situation; (2) allowing the students to become knowledge producers rather than just being consumers; and (3) helping the students to develop communication, reasoning, and critical thinking skills [7]. Through PBL, college students in
groups will discuss intensively, therefore, orally, they will be asking each other, answering, criticizing, correcting, and clarifying every concept or mathematical argument arisen throughout the discussion. In such discussion, it will be also developed the ability of the students to create, refine, and explore conjectures, so it will consolidate their understanding about the mathematical concept they are learning, or toward the solved mathematical problems. Finally, the students also have to be able to communicate their ideas, either orally or in writing, to solve the problem given [8].

PBL gives many benefits to students to develop high-leveled thinking abilities such as critical thinking, finding and using learning sources, develop cooperative working ability, and lifelong learning. Barell stated that PBL can prepare students to become inquirers, problem solvers, critical thinkers, and creative thinkers in facing complex challenges. PBL helps students to develop skills needed to succeed both in colleges and in workplaces [9].

PBL, compared to the other learning models, is a learning model which is constructivist, complex, and providing great opportunity to form independence in learning. PBL is very good used in developing high-leveled thinking abilities, such as critical and creative thinking. Complex, contextual, and ill-structured problems will give opportunity to the students to develop analytical, evaluative, and reflective thinking abilities as well as their creativity in finding many kinds of information, developing possible solutions, and creating many sources to solve problems which have to be solved [6].

Alfred shows that there is significant difference in the post test of learning achievement of advanced mathematics between the students taught by implementing PBL and those who are taught by using traditional method; the advanced mathematics learning outcome of the students taught by applying PBL is actually better compared to the students taught by applying traditional method. Padmavathy shows that problem-based learning affects mathematics teaching and improves students’ understanding and ability to use concepts in real life. PBL is more effective applied for teaching mathematics [10].

Assessment is an integral part of learning process. Assessment is often considered as one of three main pillars which are crucial for the learning activities. Those three pillars are planning, implementation, and assessment. If those pillars are synergistic and sustainable, they will determine the quality of learning very much. Therefore, assessment has to be planned and held in accordance with the planning and implementation of learning. The assessment system has to be developed in line with the development of learning model and strategy.

The implementation of learning model in class in order to develop the mathematical problem solving ability of the students obviously will be more optimal with proper assessment. Wolf states that good learning quality must also be followed by good implementation of assessment [11]. Assessment is an important part of teaching and good teaching will not work without good assessment. Ridwan states that good assessment in general is with the teaching and learning process. Generally, teaching and learning process will take place effectively if they are supported by effective assessment as well. Therefore, the assessment activity should serve as a means to improve the effectiveness of teaching and learning process [12]. It is needed evaluation system to monitor the whole learning process and steps related with the establishment of the competence. The evaluation practice needed is the one that is meaningful, that involves college students, and fulfills the function of students’ improvement and empowerment so it has to be avoided the evaluation practice focusing on targets of individual work only.

Authentic assessment is a process of collection, reporting, and usage of information about the learning outcome of college students applying the principles of assessment, continuous implementation, authentic, accurate, and consistent evidences as public accountability [13]. Ridwan states that authentic assessment is a kind of assessment which leads the students to demonstrate skills and competencies needed to solve the problems and situations encountered in real life. The competencies are combination of skills based on cognition and implemented with the proper attitude. A person cannot yet be said to be competent if his attitude in demonstrating skills is not in accordance with the attitude as it should be [12]. Eggen and Kouchak state that authentic assessment is suitable to use in assessing the learning process of PBL [14].
The PBL model and authentic assessment become so appropriate to be implemented in vocational schools, since one of the functions of these educational institutions is to prepare the students to face the real world by awakening the students to their wishes, the challenges they will face, and the abilities they need to master\textsuperscript{[15]}. In problem-based learning, the students are given contextual problem. Through contextual problem, teacher relates the material taught with the situation of the students’ real world. This approach will encourage students to make connections between their knowledge and the application in daily life. As a result, learning will become livelier, students are motivated to solve the problems they face. Thus students will more easily understand the concepts in the problems. As a result, students’ mathematics achievement may increase.

Through PBL which is combined with authentic assessment, students are expected to be able to solve the problems given as a process to master the existing mathematics concepts. Through PBL, the students are invited to solve contextual problems. Students are encouraged to make connections between their knowledge and the application in daily life. Their discussion results are then made in form of simple reports and presented through presentation which is one form of authentic assessment. Authentic assessment combined with PBL model aims at increasing students’ activities and motivation in learning and by knowing the relation between mathematics at school and in the real world, students will more easily understand the existing concepts. Hence, students’ mathematics learning achievement may increase.

This research aims to determine: 1) differences of mathematical problem solving ability between the students facilitated with problem-based learning model and conventional learning model, 2) differences of mathematical problem solving ability between students facilitated with authentic assessment model and conventional assessment model, and 3) interaction effect between learning model and assessment on mathematical problem solving ability.

2. Methodology

This research was conducted in mechanical engineering major of Bali state polytechnic by using design of pseudo experiment with two factors measurement in 2x2 factorial version. The sample taken were 110 persons, distributed in 4 classes. Random sampling was implemented for the selection of samples and experiment groups. The experiment design is shown in figure 2.1

| Learning Model | PBL | PKV |
|----------------|-----|-----|
| Assesment      | AOT | Group 1 | Group 2 |
|                | AKV | Group 3 | Group 4 |

Description:
PBL = Model Pembelajaran Berbasis Masalah (Problem-based Learning Model)
PKV = Model Pembelajaran Konvensional (Conventional Learning Model)
AOT = Asesmen Otentik (Authentic Assessment)
AKV = Asesmen Konvensional (Conventional Assessment)

Figure 1. 2x2 Factorial Experiment Design

Data were collected by implementing mathematical problem solving ability test. The instrument had been tested well both theoretically and empirically. Theoretically it was tested the validity of the content by applying Aiken’s statistic index approach, while empirically by applying classical test approach.
Data were analyzed descriptively, qualitatively, and univariate analysis (analisis univariat/ anova) of 2x2 factorial. Before the analysis, it was conducted analysis requirement tests, such as: multivariate normality test, variant homogeneity between groups, homogeneity of variant-covariant matrixes, and multicollinearity test. The results of the tests showed that the assumptions had been fulfilled.

3. Results and Discussion

Median (M) and standard deviation (SD) of concept understanding and mathematical problem solving ability as well as category of each variable after the treatment were presented in Table 2.

| Group     | Problem Solving Ability | Category |
|-----------|-------------------------|----------|
| PBL-AOT   | 78,11                   | Good     |
|           | 4,49                    |          |
| PKV-AOT   | 76,28                   | Good     |
|           | 4,41                    |          |
| PBL-AKV   | 75,642                  | Good     |
|           | 4,7                     |          |
| PKV-AKV   | 66,174                  | Enough   |
|           | 4,75                    |          |

In table 2, it can be seen that after the treatment, the PBL-AOT group show the highest achievement of mathematical problem solving ability. The students facilitated with conventional learning model and authentic assessment is better compared to the ones facilitated with problem-based learning model and conventional assessment in the achievement of mathematical problem solving ability. Meanwhile, the students facilitated with conventional learning model and conventional assessment get the lowest achievement of mathematical problem solving ability compared to the other groups.

The hypothesis testing used ANOVA analysis gradually. All hypotheses were tested in sequence, summary of the test result can be seen in table 3 as follows.

| Source     | Type III Sum of Squares | df | Mean Square | F      | Sig. |
|------------|-------------------------|----|-------------|--------|------|
| Corrected Model | 2457,487a               | 3  | 819,162     | 37,551 | 0,000|
| Intercept  | 607100,708              | 1  | 607100,708  | 27830,221 | 0,000|
|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| X   | 869,216 | 1  | 869,216 | 39,846 | 0,000 |
| A   | 1246,671 | 1  | 1246,671 | 57,149 | 0,000 |
| X \* A | 303,035 | 1  | 303,035 | 13,891 | 0,000 |
| Error | 2312,331 | 106 | 21,814 |     |     |
| Total | 611578,000 | 110 |     |     |     |
| Corrected Total | 4769,818 | 109 |     |     |     |

**Description:**

X = Learning Model  
A = Assessment

Based on the summary of the analysis result shown in Table 3, it can be informed the research findings as follows.

From the source of the effect of learning model (X) on mathematical problem solving ability, it is found the statistic value \( F = 39,846 \) (sig.0.014 <0.05). These results show that \( H_0^1 \) is rejected. It means that there is significant effect of learning model on mathematical problem solving ability. Problem-based learning model is superior to conventional learning model, mathematical problem solving ability (MPBL = 77,11; Mkv = 71,49).

From the source of the effect of assessment (A) on mathematical problem solving ability, it is found the statistic value \( F = 57,149 \) (sig.0.000 <0.05). These results show that \( H_0^2 \) is rejected. It means that there is significant effect of the assessment on mathematical problem solving ability. Authentic assessment model is superior to conventional assessment model, in the achievement of mathematical problem solving ability (AO = 77,67; Akv = 70,94).

From the source of the effect of X\* A on mathematical problem solving ability, it is found the statistic value \( F = 13,891 \) (sig.0.000 < 0.05). These results show that \( H_0^3 \) is rejected. It means that there is significant effect of interaction between learning models and assessment models on mathematical problem solving ability.

Pedagogically, Duff states that problem-based learning is based on theoretical framework of constructivism, with the following characteristics: 1) understanding is gained through interaction and scenario of problems and learning environment; 2) struggles with problems and problem inquiry process create cognitive dissonance that stimulates the students to learn; and 3) knowledge occurs through process of collaboration of social negotiation and evaluation to the existence of a point of view\[16\].

In problem-based learning model, the focus of the learning is on the chosen problem so the students learn not only about the concepts related to the problem but also scientific method to solve the problem. Therefore, students have to understand not only the concepts relevant with the problem that becomes the center of attention but also to gain learning experiences relevant to the skill on applying scientific method in solving problems, and develop critical mindset. Pedagogy of problem-based learning helps the learners to show and clarify the way of thinking as well as the richness of the structure and the cognition process inside. Besides, in problem-based learning, students can become a ‘self directed learner’ who wants to understand and learn, formulate the learning needs, the ability to choose and use best learning resources. Self directed learner is an individual who directs his own self in teaching and learning processes. Sadia confirms that in the PBL situation, students gain learning experiences to develop high-level thinking skill, skill in analyzing and solving problems, as well as building self-reliance and competitiveness. In such situation, students learn how to apply an interactive process or evaluation on their knowledge, identify what they know, gather information, and collaborate in evaluating a hypothesis based on the data collected \[6\]. The same thing is stated by...
Ibrahin and Nur that in PBL situation, students can be involved in high-leveled thinking and problem solving in a situation oriented to the problem in real life, including learning how to learn [17]. Students are expected to have complete understanding on a material formulated in a problem, mastery of positive attitudes, and skills gradually and sustainably. Students’ mental activities are required to understand a concept, principle, and skill through a situation or problem presented at the beginning of a lesson. Students understand the concept and principle of a material through working and learning on the situation and problem given through investigation, inquiry, and problem solving. Students build concept or principle using their own abilities which integrates skills and knowledge which has been understood before [17].

Meanwhile from the situation, conventional learning is a teaching-centered learning model. Coleman states that conventional learning is an information assimilation or classroom learning, with characteristics such as: 1) information gathering through symbolic resources, like teachers or reading activity; 2) assimilation or organization of information in order to make a general principle understandable, 3) the usage of general principle in specific cases, and 4) the implementation of general principle on new circumstances [19]. Santyasa gives the most important record on conventional learning, placed in a source of information in form of symbolic, like listening to teacher explanations or reading. Sources of information greatly affect the learning process [18].

In the implementation of conventional learning model, the authority of lecturers tends to dominate the learning process. Lecturers have to demonstrate knowledge or skills that will be trained to students gradually. The role of a lecturer is required to be an interesting model to the students, the material developed is fitted according to the students’ preferences. Since all activities are arranged and centered to the lecturers and the students can only accept the material given passively, students’ power of reason and knowledge can be only developed to the knowledge of the lecturer. This causes the students’ activity to be limited and they cannot improve their learning outcomes optimally. Based on the description above, it is clear that PBL model is better to be applied to students than conventional learning since in PBL model, all of students’ senses are involved in learning process. Therefore, learning outcome in form of abilities of concept understanding and mathematical problem solving of the students who learn by following PBL model is better than those who learn by following conventional model.

From the two learning models, it seems that PBL model provides facilities for the accomplishment of four pillars of education such as: learning to know, learning to do, learning to be, and learning to live together. The first component (knowing) will be obtained in the same portion in each model. However, the second component (doing), the third component (becoming), and the fourth component (living together) have larger portions in the PBL model compared to the conventional model. Besides, the conventional model badly needs the help of lecturer as the information source. The knowledge formed follows the paradigm of knowledge transmission applied by the lecturer throughout the learning process. This will affect students’ perception that learning is not entirely their responsibility, but partly is the responsibility of the lecturer. This perception will lead to reduce students’ effort in doing, becoming, as well as living together. Therefore, the conventional one gives fewer probabilities of the occurrence of meaningful learning compared to the PBL.

Based on the explanation above, it can be seen that problem-based learning model tends to be superior to the conventional learning in the achievement of mathematical problem solving ability.

The result of hypothesis test shows that authentic assessment significantly affects the concept understanding and mathematical problem solving ability. The students subjected to authentic assessment are superior to those who are subjected to mathematical problem solving ability ($A_{	ext{O}} = 77.84$; $A_{	ext{C}} = 70.53$). This result is in line with the results of the researches of: 1) Kwamina that, authentic assessment learning improve the ability of mathematical problem solving, students’ confidence in doing mathematics increases since they feel more competent [19], and 2) Kinay & Birsen that, authentic assessment approach affects the problem solving skills of prospective teachers.

Assessment is a process of collecting various kinds of data which can give description about students’ learning process [20]. Authentic assessment is a process of collecting, reporting, and using information about students’ learning outcomes by implementing principles of assessment, continuous implementation; authentic, accurate, and consistent evidences as public accountability [21].

Authentic assessment emphasizes on students’ ability to demonstrate their knowledge in meaningful and real way. The activity of assessment is not only inquiring or intercepting the students’ knowledge, but also implementing their real knowledge. Besides, authentic assessment requires the students more to demonstrate their knowledge, skills, and strategies by creating answers or products.
Students are not only asked to respond answers as in conventional test, but required to be able to create and result answers based on theoretical knowledge.

The implementation of authentic assessment in classroom learning can give a good psychological effect for the students. Students’ involvement in assessment process can increase the social interaction and mutual trust between one student and another as well as between students and lecturers. The perception of students that with their involvement in assessment activities in general has increased their confidence, responsibility, and motivation. Social interaction can be mutual control between the outcome of the assessment done by the students and the one which is done by the lecturers toward the students’ self-assessment, thus it can be generated an assessment which is valid, objective, and acceptable for both parties i.e. the students and the lecturers.

Authentic assessment gives many contributions in improving students’ learning achievements. Through authentic assessment in learning process, students feel that the assignments they have done are really meaningful and they know immediately about their knowledge level toward a problem. This is because in authentic assessment there are three main components which have to be considered, such as performance task, performance rubrics, and scoring guide. Then through self evaluation which is done in the end of each learning process, students can see their excellences and weaknesses, to be further become the improvement goal. This results in an increase of students’ responsibility to the process and the achievement of their learning objectives.

Conventional assessment is often referred to paper and pencil assessment. That kind of assessment is separated from the students’ learning process. Paper and pencil assessment cannot be considered as an integral part of a learning process. Giving feedback for the students will be late, even will not be able to be done, so the objectives of learning improvement as an assessment result cannot be done immediately even become uncertain whether it can be done [21]. This assessment model does not provide information for the students during the learning process. Based on this discussion, it seems that authentic assessment model tends to be superior to conventional assessment model in the achievement of mathematical problem solving ability. This is in line with what is stated by Salvia and Ysseldike that self reflection and evaluation are ways to grow ownership, which means that it will arisen an understanding that what have been done and generated by the students are useful for themselves and their lives.

There is an interaction between learning model and assessment with mathematical problem solving ability. Mathematical problem-based learning gives positive effects to the process of learning as follows:

On the students facilitated with PBL learning treatment with authentic assessment, the average value of their mathematical problem solving ability is 79,16; while the average value on those who are facilitated with conventional learning treatment with authentic assessment is 76,52. Mathematical problem solving ability of the students facilitated with PBL with authentic assessment is better than those who are facilitated with conventional learning with authentic assessment.

On the students facilitated with PBL learning treatment with conventional assessment, the average value of their mathematical problem solving ability is 75,35. While on those who are facilitated with conventional learning treatment with conventional assessment, the average value is 65,71. Mathematical problem solving ability on the students facilitated with PBL learning treatment with conventional assessment is better than the ability of those who are facilitated with conventional learning treatment with conventional assessment.

On the students facilitated with PBL learning treatment with authentic assessment, the average value of their abilities of concept understanding and mathematical problem solving is 79,16; while the average value on those who are facilitated with conventional learning treatment with conventional assessment is 65,71. The mathematical problem solving ability of the students facilitated with PBL learning treatment with authentic assessment is better than the ability of those who are facilitated with conventional learning model with conventional assessment.

On the students facilitated with PBL learning treatment with conventional assessment, the average value of their mathematical problem solving ability is 75,35, while the average value on those who are facilitated with conventional learning treatment with authentic assessment is 76,52. Mathematical problem solving ability of the students facilitated with PBL learning treatment with conventional assessment is better than the ability of those who are facilitated with conventional learning model with authentic assessment.

Students facilitated with PBL learning treatment with authentic assessment have the highest average value of mathematical problem solving ability among the students’ groups.
Students in the class facilitated with PBL and authentic assessment discuss intensively in groups, so orally they will ask each other, answer, criticize, correct, and clarify each mathematical concept or argument appeared in the discussion. In such discussion, it is also developed the students’ abilities to create, refine, and explore conjectures so as to establish their understanding on the mathematical concept they are learning or on the mathematical problem they are solving. As a result of the authentic assessment, they have opportunity to construct their learning outcome. The assessment does not only ask the students to restate what they have learned. The students are asked to construct what they have gained when they were faced with concrete situation. In this way, students will select and compile answers based on their knowledge and situation analysis so that their answers will be relevant and meaningful. Such situation of mathematics learning will improve students’ ability in understanding concepts and mathematical problem solving.

Students facilitated with conventional learning treatment with authentic assessment also have higher average value of mathematical problem solving ability compared to both the students facilitated with PBL model and conventional assessment and the students facilitated with conventional learning and conventional assessment.

For the students in class facilitated with conventional learning model and authentic assessment, the learning process tends to be centered to the lecturer. On the learning program plan, the lecturer does not pay attention to the prior knowledge of the students. The learning process takes place in one direction, the role of the lecturer is no longer as a good motivator, facilitator, or mediator; however the lecture holds the authority of learning. In the learning activity, the lecturer tends to use a few lecture method followed by discussion. The lecturer tries to move or transmit his/ her knowledge to the students. This condition tends to make the students becoming passive in receiving the lesson. This kind of learning activity does not create active learning situation, even the students will become so passive. However, as the effect of the assessment model applied; in which it is the authentic assessment, students’ passivity can be reduced and even they will become active. Students are given opportunity to construct their learning outcomes, since the assessment does not just ask the students to repeat what they have been learned. They are asked to construct what they have obtained when they were faced to concrete situation. Through this way, the students will select and arrange answers based on their knowledge and the situation analysis done so the answers will be relevant and meaningful. Such learning situation will facilitate the students to improve their ability in understanding concept and mathematical problem solving.

The students facilitated with PBL learning model with authentic assessment have higher average value of concept understanding and mathematical problem solving ability compared to the students facilitated with PBL model and conventional assessment. The students facilitated with PBL and conventional assessment discuss in groups intensively, so orally they will ask each other, answer, criticize, correct, and clarify every concept or mathematical argument appeared in discussion. In such discussion, it will be also developed the students’ ability to create, refine, and explore conjectures in order to consolidate their understanding on the mathematical concept they are learning, or on the mathematical problem they solve. As the result of conventional assessment, students do not have opportunity to construct their learning outcomes, since the assessment only ask the students to repeat what they have learned. The students are not asked to construct what they have gained when they were faced with concrete situation. Through this way, students do not select and arrange answers based on their knowledge and the situation analysis done so the answers will be relevant and meaningful. Such situation of mathematics learning tends to be less able to improve students’ ability in understanding concept and mathematical problem solving.

From the explanation above, each learning and assessment model has a same direction that is achievement of learning purposes. Learning purposes will be achieved if lecturers and students feel a meaningful learning process. This happens if the process of learning follow the good and correct learning steps based on the characteristics/ syntax of the learning model applied. Thus the optimization of the achievement of learning purposes can be achieved maximally. Based on that explanation, it is seen the advantages and disadvantages of each of learning model depending on the assessment model applied. Therefore, in learning process, lecturer should consider the assessment model used. Learning process will take place effectively if it is supported by effective assessment as well, assessment activity should be seen as a means to improve the effectiveness of the learning process.

The first alternative i.e. problem-based learning model collaborated with authentic assessment is the most appropriate to be applied. The second alternative i.e. conventional learning model collaborated with authentic assessment is also appropriate to be applied in an attempt to improve the
concept understanding and mathematical problem solving ability of the students. Meanwhile the other two models i.e. problem-based learning collaborated with conventional assessment and conventional learning collaborated with conventional assessment are less appropriate to be applied.

The implications of this study are that collaboration of learning models with assessments needs to be considered in learning concepts and solving mathematical problems. PBL implementation is very effective when collaborated with authentic assessment. Conventional learning becomes effective when collaborated with authentic assessment. PBL collaborated with conventional assessments are less effectively applied in an effort to improve concept understanding and mathematical problem solving ability.

4. Conclusion
In conclusion, there are differences of mathematical problem solving ability between the students facilitated with problem-based learning model and conventional learning model. There are differences of problem solving ability between the students facilitated with authentic assessment model and conventional assessment model. There is an effect of the interaction between learning models and assessment on mathematical problem solving ability.

Based on the study result successfully found, it can be suggested that problem-based learning model and authentic assessment based on theory of constructivist learning are appropriate to be referred as an alternative of applied mathematics learning especially in the achievement of concept understanding and mathematical problem solving ability. In the implementation of such model, it should be used form of performance assessment, self evaluation, and essay test; even it can be developed in the other forms of authentic assessment, such as project assessment and portfolio. In addition, our learning frameworks that need to be referenced, are (1) the development understanding and problem solving dimensions based on students’ learning needs in certain topic of learning, (2) determination of understanding goals and problem solving abilities; (3) prediction of comprehension performance and problem solving ability; and (4) comprehensive and sustainable assessment.

5. Acknowledgments
The author would like to thank Directorate Research and Social Service, Ministry of Research, Technology, and Higher Education for their financial support that this research was successfully undertaken.

6. References
[1] Kline, Morris. 1997. “Matematika.” Ilmu dalam Perspektif, ed. Jujun S. Suriasumantri. Jakarta: Yayasan Obor Indonesia.
[2] Jujun S. Suriasumantri, 1997. Ilmu dalam Perspektif, Jakarta: Yayasan Obor Indonesia
[3] Badan Penelitian dan Pengembangan (Balitbang). 2011. Laporan hasil TIMSS 2007. Kementrian Pendidikan dan Kebudayaan
[4] Kemendikbud. 2014. Buku Kurikulum Pendidikan Tinggi. Jakarta: Direktorat Pembelajaran dan Kemahasiswaan Dirjen DIKTI
[5] I Wayan Sadia. 2014. Model Pembelajaran Sains Konstruktivistik. Yogyakarta: Graha Ilmu.
[6] CIDR (Center for Teaching and Learning Teaching and Learning Bulletin). Problem-Based Learning. [Online]. Vol 7, 2004, h. 3
[7] Barrows, H. 1996, Problem-based learning in medicine and beyond: a brief overview. New direction for teaching and learning. Jossey: Bass Publisher.
[8] Djamilah Bondan Widjajanti 2011. “Problem Based Learning dan Contoh Implementasinya”. Makalah. Yogyakarta: FMIPA UNJ
[9] Barel, J. Excerpts from “Problem-Based Learning: The Foundation for 21st Century Skills”, (Online), (http://www.morecuriousminds.com/docs/21stCSummary2.pdf), 2010. diakses pada tanggal 13 Desember 2010
[10] R.D.Padmavathy & Mareesh .K., “Effectiveness of Problem Based Learning In Mathematics” International Multidisciplinary e-Journal, Vol-II, Issue-I, Jan -2013. hh. 47-51
[11] Badmus, G. A., Changing Nature of Technical and Vocational Education and Students’ Assessment Methods, (ganiyubdms@yahoo.com www.iaea.info, 2007), (diakses tanggal, Januari 2010). h.10
[12] Ridwan Abdulah Sani. 2016. Penilaian Autentik. Jakarta: Bumi Akasara.
[13] Depdiknas. 2009. Kurikulum Berbasis Kompetensi. Jakarta: Pusat Kurikulum Balitbang
[14] Paul Eggen & Don Kauchak. 2012. Strategi dan Model Pembelajaran Menegajarkan Konten dan Keterampilan Berpikir. Jakarta: PT Indek.
[15] Dryden, G. 2002. Revolusi Cara Belajar. Cet. Ke-3. Bandung: Kaifa.
[16] Rusman. 2012. Model-Model Pembelajaran Mengembangkan Profesionalisme Guru, Jakarta: PT Raja Grafindo Persada
[17] Ibrahim, M., dan Nur, M. 2000. Pengajaran Berdasarkan Masalah. Surabaya: Universitas Negeri Surabaya
[18] I Wayan Santyasa. 2014. Asesmen dan Evaluasi Pembelajaran Fisika. Yogyakarta: Graha Ilmu
[19] Ato Kwamina Arhin. “The Effect of Performance Assessment-Driven Instruction on the Attitude and Achievement of Senior High School Students in Mathematics in Cape Coast Metropolis, Ghana”, (Journal of Education and Practice. Vol.6, No.2, 2015). h. 109
[20] Kunandar. 2012. Penilaian Autentik Edesi Revisi. Jakarta: PT Raja Grafindo Persada.
[21] Depdiknas, 2009. Kurikulum Berbasis Kompetensi, Jakarta: Pusat Kurikulum Balitbang