Exactly Soluble Quantum Wormhole in Two Dimensions

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Abstract

We are presenting a quantum traversable wormhole in an exactly soluble two-dimensional model. This is different from previous works since the exotic negative energy that supports the wormhole is generated from the quantization of classical energy-momentum tensors. This explicit illustration shows the quantum-mechanical energy can be used as a candidate for the exotic source. As for the traversability, after a particle travels through the wormhole, the static initial wormhole geometry gets a back reaction which spoils the wormhole structure. However, it may still maintain the initial structure along with the appropriate boundary condition.

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I. INTRODUCTION

In a gravitational system, a black hole has the event horizon and the curvature singularity, while a wormhole characterized by the throat is everywhere regular. Interestingly, in the latter case, our universe can be connected to other universes in terms of its throat. However, the key ingredient in threading the two universes is the violation of the energy theorem [1, 2, 3], which provides a flaring-out condition near the throat. It requires more or less an unusual source called exotic matter. It is sometimes described by the negative energy for simplicity. On the other hand, exactly soluble classical wormhole models in two dimensions have been extensively studied by adding the negative energy source in Refs. [4, 5, 6]. However, the origin of the source is still unknown. Therefore, it will be interesting to study some candidates of the exotic source.

We would like to present an exactly soluble traversable wormhole model without the classical exotic source. The exotic matter violating the energy theorem naturally arises from the quantization of real scalar fields so that the quantum-mechanically induced energy may be a candidate [7, 8]. Motivated by these scenarios, we would like to explicitly show that the necessary exotic source to support the wormhole can be obtained from the quantum stress tensors. The D-particle [9] will be introduced as a test particle, whether it passes through the wormhole from our universe to the other universe or not. The reason why we use the D-particle instead of the usual particle is due to the exact solubility of our model.

In Sec. II the geodesic of the D-particle and its energy-momentum tensor are determined and the general solution of metric is found without applying any boundary conditions. This solution describes a wormhole or a black hole by the choice of the boundary condition. We obtain the solution of the traversable wormhole by the proper boundary condition in Sec. III. As a result, it will be given that the formation of the wormhole is possible at the quantum regime with the help of the quantum mechanically induced negative energy. After the particle travels through the wormhole, the static initial wormhole geometry gets a back reaction which spoils the wormhole structure. However, it is able to maintain the initial wormhole structure along with the consistent vacuum state. In Sec. IV we choose another boundary condition to give a black hole solution as a final state after the D-particle passes through the wormhole. This is a different type of solution from the conventional Russo-Susskind-Thorlacius(RST) model. Finally, in Sec. V discussions and summary are
II. THE RST MODEL COMBINED WITH A D-PARTICLE

We now consider the Callan-Giddings-Harvey-Strominger (CGHS) model \[10\] combined with the scalar fields and a D-particle, whose action is given by

\[
S_{cl} = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + 4\lambda^2 \right] + \frac{\epsilon}{2\pi} \int d^2x \sqrt{-g} \sum_{i=1}^{N} \left[ -\frac{1}{2} (\nabla f_i)^2 \right] -m \int d^2x \int d\tau \delta^2(x - z(\tau)) e^{-\phi(x)} \sqrt{-g_{\mu\nu}(x)} \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau},
\]

(1)

where \( g, \phi, \lambda^2, \) and \( m \) are a metric, a dilaton field, a cosmological constant, and the mass of a D-particle, respectively. The scalar fields \( f_i \) are the real conformal fields satisfying the energy condition for \( \epsilon = 1 \). They are also the ghost fields giving the negative energy density for \( \epsilon = -1 \). The D-particle action instead of the conventional particle action was introduced in order to solve the model exactly without any approximations. For the case of \( \epsilon = 0 \) and \( \epsilon = 1 \), the model describes a collapsing D-black hole, which has been classically studied in Ref. \[11\]. On the other hand, when \( \epsilon = -1 \), a classical wormhole geometric structure appears due to the wrong sign of the kinetic term in the action \(1\). This plays the role of the exotic matter. A traversable wormhole is later obtained which the particle can safely travel through. However, this may still open the problem concerning the origin of the classical negative energy density.

Returning back to our model, we now place the real scalar case of \( \epsilon = 1 \) in the action \(1\). Of course, the wormhole solution in this case does not exist because of the absence of the exotic source. But, we semiclassically quantize the action by adding the one-loop effective action of the real matter in the large \( N \)-limit as was done in the RST model \[12\],

\[
S = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + 4\lambda^2 \right] + \frac{1}{2\pi} \int d^2x \sqrt{-g} \sum_{i=1}^{N} \left[ -\frac{1}{2} (\nabla f_i)^2 \right] -m \int d^2x \int d\tau \delta^2(x - z(\tau)) e^{-\phi(x)} \sqrt{-g_{\mu\nu}(x)} \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau} -\frac{\kappa}{2\pi} \int d^2x \sqrt{-g} \left[ \frac{1}{4} R R + \frac{1}{2} \phi R \right],
\]

(2)

where \( \kappa = (N - 24)\hbar/12 \). By introducing an auxiliary variable \( \eta(\tau) \), the Born-Infeld type action \[9, 13\] for a D-particle in the action \(2\) can be rewritten as

\[
S_D = \frac{1}{2} \int d^2x \int d\tau \delta^2(x - z(\tau)) \left[ \eta^{-1}(\tau) g_{\mu\nu}(x) \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau} - \eta(\tau) m^2 e^{-2\phi(x)} \right],
\]

(3)
where the massless limit is well-defined. In order to solve our model, we define new fields as

\[
\chi = \sqrt{\kappa} \rho - \frac{\sqrt{\kappa}}{2} \phi + \frac{1}{\sqrt{\kappa}} e^{-2\phi},
\]

(4)

\[
\Omega = \frac{\sqrt{\kappa}}{2} \phi + \frac{1}{\sqrt{\kappa}} e^{-2\phi},
\]

(5)

\[
\xi = m \int^\tau \eta(\tau) d\tau.
\]

(6)

Subsequently, in the conformal gauge, \( g^+ - g^- = -e^{2\rho}/2, \ g_{\pm\pm} = 0, \) where \( x^\pm = x^0 \pm x^1, \) the action (2) takes the form of

\[
S = \frac{1}{\pi} \int d^2 x \left[ \partial_+ \Omega \partial_- \Omega - \partial_+ \chi \partial_- \chi + \lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\chi - \Omega)} + \frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_- f_i \right]
\]

\[
- m \int d^2 x \int d\xi \delta^2(x - z(\xi)) e^{-2\phi(x)},
\]

(7)

with the constraints,

\[
\kappa t_\pm = (\partial_\pm \Omega)^2 - (\partial_- \chi)^2 + \sqrt{\kappa} \partial_\pm^2 \chi + \frac{1}{2} \sum_{i=1}^N (\partial_\pm f_i)^2 + T_{\pm\pm}^D,
\]

(8)

where \( t_\pm(x^\pm) \) reflects the nonlocality of the conformal anomaly in the action (2), which is fixed by some boundary conditions. Then, the equations of motion may be found in the action (7) as

\[
\partial_+ \partial_- \chi + \frac{\lambda^2}{\sqrt{\kappa}} e^{\frac{2}{\sqrt{\kappa}}(\chi - \Omega)} = T_{+\pm}^D,
\]

(9)

\[
\partial_+ \partial_- \Omega + \frac{\lambda^2}{\sqrt{\kappa}} e^{\frac{2}{\sqrt{\kappa}}(\chi - \Omega)} = T_{+\pm}^D,
\]

(10)

\[
\partial_+ \partial_- f_i = 0,
\]

(11)

\[
\frac{dz^+}{d\xi} \frac{dz^-}{d\xi} - e^{-2(\rho + \phi)} = 0,
\]

(12)

\[
\frac{d^2 z^\pm}{d\xi^2} + 2 \frac{\partial \rho}{\partial z^\mp} \left( \frac{dz^\mp}{d\xi} \right)^2 = -2e^{-2(\rho + \phi)} \frac{\partial \phi}{\partial z^\mp},
\]

(13)

where the energy-momentum tensors for the D-particle are given by

\[
T_{\pm\pm}^D = \frac{\pi m}{2} \int d\xi \delta^2(x - z)e^{2\rho} \left( \frac{dz^\mp}{d\xi} \right)^2,
\]

(14)

\[
T_{+\pm}^D = \frac{\pi m}{2} \int d\xi \delta^2(x - z)e^{-2\phi}.
\]

(15)

From Eq. (11), the solutions of the conformal matter fields are simply \( f_i = f_i^+(x^+) + f_i^-(x^-). \) Combining Eqs. (9) and (10) yields the reduced equation, \( \partial_+ \partial_- (\chi - \Omega) = 0. \) In the Kruskal
gauge which fixes the residual spacetime symmetry, a relation \( \chi = \Omega \), \( i.e. \), \( \rho = \phi \), is obtained. On the other hand, the einbein equation (12) in the Kruskal gauge is written as

\[
\frac{dz^+ \, dz^-}{d\xi \, d\xi} = e^{-4\rho(z)},
\]

and using Eq. (16), the geodesic equation (13) becomes

\[
\frac{1}{A^\pm} \frac{dz^\pm}{d\xi} = e^{-2\rho(z)}.
\]

The particle geodesic is simply obtained as \( z^+ = (A^+/A^-)(z^- + B) \), where \( A^\pm \) and \( B \) are constants. Inserting Eq. (17) into Eq. (16) yields the relation \( A^+ A^- = 1 \).

For simplicity’s sake, if we set \( A = A^+ \), the trajectory of the D-particle is written as

\[
z^+ = A^2(z^- + B),
\]

describing the straight line in the Kruskal diagram. If the incident D-particle starts from our universe, then it is effectively described by the restriction, \( A^2 < 1 \). Note that the simple motion of these particles is due to the exact solubility of our model. For convenience, the energy-momentum tensors for the D-particle are rewritten by substituting Eqs. (16)–(18) into Eqs. (14) and (15),

\[
T^D_{++} = \frac{\pi m}{2 A^3} \delta \left( \frac{x^+}{A^2} - x^- - B \right),
\]

\[
T^D_{+-} = \frac{\pi m}{2 A} \delta \left( \frac{x^+}{A^2} - x^- - B \right),
\]

\[
T^D_{-+} = \frac{\pi m A \delta}{2} \left( \frac{x^+}{A^2} - x^- - B \right),
\]

where they are all singular along with the geodesic of the particle. By substituting Eq. (20) into Eq. (10), we get the geometric solution,

\[
\Omega = a_+(x^+) + a_-(x^-) - \lambda^2 x^+ x^- - \frac{\pi}{2} m A \left( \frac{x^+}{A^2} - x^- - B \right) \theta \left( \frac{x^+}{A^2} - x^- - B \right),
\]

where \( \theta(x) = 0 \) for \( x < 0 \) and \( \theta(x) = 1 \) for \( x > 0 \), and \( a_\pm(x^\pm) \) should be determined by the constraints (8),

\[
k\mathcal{T}_\pm = \partial_\pm^2 a_\pm + \frac{1}{2} \sum_{i=1}^{N} (\partial_\pm f^i_\pm)^2.
\]
Integrating the constraints (23), we obtain the general solution as

\[
\Omega = -\frac{\lambda^2}{\sqrt{\kappa}} x^+ x^- + \int x^+ dx^+ \int x^+ dx^+ \left[ \sqrt{\kappa t_+} - \frac{1}{2\sqrt{\kappa}} \sum_{i=1}^{N} (\partial_+ f^i_+)^2 \right] 
+ \int x^- dx^- \int x^- dx^- \left[ \sqrt{\kappa t_-} - \frac{1}{2\sqrt{\kappa}} \sum_{i=1}^{N} (\partial_- f^i_-)^2 \right] 
- \frac{\pi m A}{2\sqrt{\kappa}} m A \left( x^+ - x^- - B \right) \theta \left( \frac{x^+}{A^2} - \frac{x^- - B}{A^2} \right) + C_+ x^+ + C_- x^- + D, 
\tag{24}
\]

where \(C_\pm\) and \(D\) are the constants of integration. Note that there are two large kinds of geometric solutions in our model. The first one is the well-known RST black hole solution, of which asymptotic geometric structure is Minkowskian. We know it is given by the boundary condition of no incoming quantum radiation. This incoming radiation is calculated by

\[
<T_{\pm\pm}^I> = \kappa [\partial_\pm^2 \rho - (\partial_\pm \rho)^2 - t_\pm]. 
\tag{25}
\]

The boundary conditions require \(< T_{\pm\pm}^I > = 0\) at \(x^+ \to -\infty\) so that \(t_\pm = 1/4(x^\pm)^2\). The time-dependent solution may be found by patching the linear dilaton vacuum, and the black hole across an infall-line:

\[
\Omega = -\frac{\lambda^2}{\sqrt{\kappa}} x^+ x^- - \frac{\sqrt{\kappa}}{4} \ln(-\lambda^2 x^+ x^-) - \frac{M}{\lambda \sqrt{\kappa} x^+_0} (x^+ - x^+_0) \theta(x^+ - x^+_0) 
- \frac{\pi m A}{2\sqrt{\kappa}} \left( \frac{x^+}{A^2} - \frac{x^- - B}{A^2} \right) \theta \left( \frac{x^+}{A^2} - \frac{x^- - B}{A^2} \right), 
\tag{26}
\]

where \(\frac{1}{2} \sum_{i=1}^{N} \partial_+ f^i_+ \partial_- f^i_- = M/(\lambda x^+_0) \delta(x^+ - x^+_0)\) and \(M > 0\) is the energy carried by the incoming shock wave.

### III. TRAVERSABLE WORMHOLE FROM THE QUANTUM SOURCE

We would like to construct the wormhole solution in the quantized theory by imposing a different boundary condition from the previous black hole case. It means that in our soluble model, the past and the future horizon curves are coincident with each other at the throat. In particular, the static wormhole appears at \(x^+ = x^-\). The apparent horizon curves are also given by the definition,

\[
0 = \partial_+ \Omega = -\frac{\lambda^2}{\sqrt{\kappa}} x^- + \int x^+ dx^+ \left[ \sqrt{\kappa t_+} - \frac{1}{2\sqrt{\kappa}} \sum_{i=1}^{N} (\partial_+ f^i_+)^2 \right] 
- \frac{\pi m A}{2\sqrt{\kappa}} \theta \left( \frac{x^+}{A^2} - \frac{x^- - B}{A^2} \right) + C_+, 
\tag{27}
\]
0 = \partial_{+}\Omega = -\frac{\lambda^2}{\sqrt{\kappa}}x^+ + \int^{x^+} x^- \left[\sqrt{\kappa}t_- - \frac{1}{2\sqrt{\kappa}} \sum_{i=1}^{N} (\partial_- f_+^i)^2 \right] + \frac{\pi m A}{2\sqrt{\kappa}} \theta \left(\frac{x^+}{A^2} - x^- - B\right) + C_+. \tag{28}

From the boundary condition of the static wormhole geometry at the asymptotic past time, the unknowns \(C_\pm\) and \(t_\pm\) are completely fixed as

\[C_\pm = \lambda^2 x_1, \quad t_\pm = \frac{\lambda^2}{\kappa},\]  \tag{29}

where \(x_1\) is the coordinate just before the infalling particle appears. Note that we redefined the constant \(D\) as

\[D = \frac{M}{\lambda} + \frac{\sqrt{\kappa}}{4} \left(1 - \ln \frac{\kappa}{4}\right) - \frac{\lambda^2}{\sqrt{\kappa}} x_1^2,\]  \tag{30}

for convenience. The constant \(M > 0\) was chosen from singularity-free condition of the curvature, which will be discussed later.

If the D-particle travels through the static wormhole, then the spacetime is perturbed by the backreaction of the geometry. Now, we require that the initial wormhole structure be recovered after travelling at the later time, specifically, \(x^\pm = x_1\). This is easily realized by modifying the function \(t_\pm\) in Eq. \(29\) as

\[t_\pm = \frac{\lambda^2}{\kappa} \left[1 + \beta_\pm (\theta(x^\pm - x_1) - \theta(x^\pm - x_2))\right], \tag{31}\]

where the constants \(\beta_\pm\) are chosen as,

\[\beta_+ = \frac{\pi m A}{2\lambda^2(x_2 - x_1)}, \quad \beta_- = -\frac{\pi m A}{2\lambda^2(x_2 - x_1)}. \tag{32}\]

They come from the static wormhole boundary condition of the coincidence of the past and future horizons,

\[0 = \partial_{+}\Omega = \lambda^2(x^+ - x^-) - \lambda^2 \beta_+(x_1 - x_2) - \frac{\pi m A}{2}, \tag{33}\]

\[0 = \partial_{-}\Omega = -\lambda^2(x^+ - x^-) - \lambda^2 \beta_-(x_1 - x_2) + \frac{\pi m A}{2}, \tag{34}\]

at \(x^\pm > x_2\). Note that \(x^\pm = x_1\) is the splitting point of the two horizons caused by the incident particle and \(x^\pm = x_2\) is the point where the split horizons rejoin after the incident particle(Fig. 1). The incident travelling particle is defined between \(x_1\) and \(x_2\), and it perturbs the wormhole geometry in this region. The disturbed geometry is eventually
FIG. 1: A test D-particle passes through the wormhole. The geometry of the initial wormhole is perturbed by the infalling D-particle. However, it is recovered by the selection of the new vacuum state described by $t_\pm$.

stabilized through the appropriately chosen $t_\pm$ in Eq. (31). We are now able to figure out the exact time-dependent wormhole solution,

\[
\Omega = \frac{M}{\lambda} + \frac{\sqrt{\kappa}}{4} \left(1 - \ln \frac{\kappa}{4}\right) + \frac{\lambda^2}{2\sqrt{\kappa}} (x^+ - x^-)^2 + \frac{\lambda^2}{2\sqrt{\kappa}} \left[\beta_+(x^+ - x_1)^2\theta(x^+ - x_1)ight.
\]
\[
+ \beta_-(x^- - x_1)^2\theta(x^- - x_1) - \beta_+(x^+ - x_2)^2\theta(x^+ - x_2)
\]
\[
- \beta_-(x^- - x_2)^2\theta(x^- - x_2)\left[ -\frac{\pi m A}{2\sqrt{\kappa}} \left(\frac{x^+}{A^2} - x^- - B\right) \theta \left(\frac{x^+}{A^2} - x^- - B\right) \right],
\]

(35)

which naturally yields the static wormhole,

\[
\Omega = \frac{M'}{\lambda} + \frac{\sqrt{\kappa}}{4} \left(1 - \ln \frac{\kappa}{4}\right) + \frac{\lambda^2}{2\sqrt{\kappa}} (x^+ - x^-)^2
\]

(36)

satisfying the boundary conditions at $x^+ < x_1$ and $x^+ > x_2$. The new constant $M'$ is defined as

\[
M' = M + \frac{\pi \lambda m A}{2\sqrt{\kappa}} \left[ B - \frac{(1 - A^2)(x_2 + x_1)}{2A^2} \right] \delta,
\]

(37)

where $\delta$ is 0 for $x^+ < x_1$ and 1 for $x^+ > x_2$. To make $M'$ positive definite, it should be

\[
\frac{x_1 + x_2}{2} < \frac{BA^2}{1 - A^2}.
\]

(38)
The constants \( M \) and \( M' \) characterize the sizes of the initial and final wormhole throat, respectively. In connection with the wormhole throat, \( e^{-2\phi} \) is assumed to be analogously related to the higher-dimensional radial coordinate \([14]\). It can be used to check whether the wormhole is closed or not. Its radial size is defined by

\[
r^2 = \frac{e^{-2\phi_r}}{\lambda^2} > \frac{\kappa}{4\lambda^2},
\]

where \( \phi_r \) is found in

\[
\frac{\sqrt{\kappa}}{2} \phi_r + \frac{1}{\sqrt{\kappa}} e^{-2\phi_r} = \frac{M'}{\lambda} + \frac{\sqrt{\kappa}}{4} \left(1 - \ln \frac{\kappa}{4}\right),
\]

for the two regions of static wormholes, \( x^+ < x_1 \) and \( x^+ > x_2 \). The size of the throat is larger than \( \kappa/(4\lambda^2) \) from Eq. (40). This means the minimal size exists even for \( M = 0 \) and \( M' = 0 \). In the quantum mechanical sense, as seen from the Planck constant in Eq. (2), the static wormhole is always open due to the quantum correction.

Compared to the previous black hole case, there should be one more constraint which is nothing but the regularity condition. In this model, a curvature singularity may appear at \( d\Omega/d\phi = 0 \) since \( R = 8e^{-2\phi}/\Omega'[\partial_+\partial_\Omega-(\Omega''/\Omega')\partial_+\partial\Omega\partial\Omega] \), where ' denotes a derivative with respect to \( \phi \), and the singularity curve is given by \( \Omega(x_+, x_-) = \sqrt{\kappa}/4(1 - \ln \kappa/4) \). For the first time, the singularity curves at the regions of the static wormholes are given by

\[
(x^+ - x^-)^2 + \frac{2M\sqrt{\kappa}}{\lambda^3} = 0, \quad x^+ < x_1 \quad (41)
\]

\[
(x^+ - x^-)^2 + \frac{2M\sqrt{\kappa}}{\lambda^3} + \frac{\pi mA}{\lambda^2} \left[ B - \frac{(1-A^2)(x_2 + x_1)}{2A^2} \right] = 0, \quad x^+ > x_2 \quad (42)
\]

where \( M \) should be positive for having no singularity from Eq. (41) at \( x^+ < x_1 \). The spacetime at \( x^+ > x_2 \) is regular as far as Eq. (38) is satisfied. Next, to examine the singularity at \( x_1 < x^+ < x_2 \), we assume that the case of \( x_2 - x_1 = \pi mA/[2\lambda^2(1-A^2)] \), which gives the singularity curve as

\[
a \left( x^+ - x^- \right)^2 + \frac{b}{2a} + \frac{2M\sqrt{\kappa}}{\lambda^3} + \frac{\pi mA}{\lambda^2} \left[ B - \frac{(1-A^2)(x_2 + x_1)}{2A^2} \right] \theta \left( \frac{x^+}{A^2} - x^- - B \right) = 0, \quad (43)
\]

where \( a = 1 + \beta_+ > 0 \) and \( b = -2\beta_+ [x_1 + (x_2 - x_1)\theta(x^+/A^2 - x^- - B)] \). This equation of the singularity curve has no roots as long as \( M > 0 \) and \( M' > 0 \). Therefore the intermediate spacetime is also regular. Our calculation was based on the very restricted case rather than on general grounds because we wanted to show the possibility avoiding the curvature singularity.
IV. TRANSITION FROM A WORMHOLE TO A BLACK HOLE

We have shown that there are two kinds of solutions in the quantized theory. The first one is the well-known RST black hole solution, and the second is the present dynamical wormhole solution. In the latter case, the final state is the same with the initial wormhole, whose size is a little bit different from that of the initial one. If this is the case, one might ask whether the end state of our wormhole can be the black hole solution or not. To patch a black hole solution at \( x^\pm = x_3 > x_2 \), we should consider the appropriate boundary conditions for the black hole. Since there is no incoming quantum radiation, \( < T^f_{\pm \pm} >= 0 \) at \( x^\pm \to -\infty \) for \( x^\pm > x_3 \), we set \( t_\pm = 0 \). So, the consistent boundary condition gives

\[
t_\pm = \frac{\lambda^2}{\kappa} [1 + \beta_- (\theta(x^+ - x_1^+) - \theta(x^+ - x_2^+))] - \frac{\lambda^2}{\kappa} \theta(x^+ - x_3),
\]

which yields the solution by using Eq. (24),

\[
\Omega = \frac{M}{\lambda} + \frac{\sqrt{\kappa}}{4} \left( 1 - \ln \frac{\kappa}{4} \right) + \frac{\lambda^2}{2\sqrt{\kappa}} (x^+ - x^-)^2 + \frac{\lambda^2}{2\sqrt{\kappa}} \left[ \beta_+(x^+ - x_1)^2 \theta(x^+ - x_1) \\
+ \beta_- (x^- - x_1)^2 \theta(x^- - x_1) - \beta_+(x^+ - x_2)^2 \theta(x^+ - x_2) - \beta_- (x^- - x_2)^2 \theta(x^- - x_2) \\
- (x^+ - x_3)^2 \theta(x^+ - x_3) - (x^- - x_3)^2 \theta(x^- - x_3) \right] - \frac{\pi m A^2}{2 \sqrt{\kappa}} \left( \frac{x^+}{A^2 - x^- - B} \right) \theta \left( \frac{x^+}{A^2 - x^- - B} \right),
\]

where the apparent horizon is \( \partial_\pm \Omega = \frac{\lambda^2 (-x^+ + x_3)\sqrt{\kappa}}{2\sqrt{\kappa}} = 0 \). The patched black hole is unfortunately different from the previous RST one since the boundary condition \( t_\pm \) is different from that of the RST model. Therefore, the new type of the black hole can be the final state of our dynamical wormhole. In this case, the wormhole is no more traversable due to the size of the throat that is shrunk to zero. Simultaneously, the infalling particle meets the curvature singularity.

V. DISCUSSION

Now, let us discuss the quantum energy-momentum tensors, which are of relevance to the formation of the wormhole geometry. Essentially, the exotic source in contrast to the normal matter satisfying the energy condition should exist in order to support the wormhole structure. In our model, the negative energy source has been obtained by the quantization of the conformal matter fields instead of introducing by hand. To make it explicit, let us first consider the static wormhole geometry before the infalling D-particle, which is achieved
by letting $m \to 0$ in Eq. (35) as $\Omega = M/\lambda + \sqrt{\kappa}/4(1 - \ln(\kappa/4)) + \lambda^2(x^+ - x^-)^2/(2\sqrt{\kappa})$. Especially for a weak coupling, $\Omega \approx e^{-2\phi}/\sqrt{\kappa}$, we get the exotic source $< T^{f}_{\pm \pm} > \approx -\lambda^2(1 + \beta_{\pm})$ at $x^+ \to -\infty$. It corresponds to the Casimir vacuum of a quantum state violating the energy condition discussed in Ref. [7]. After the D-particle travels, the final geometric structure approaches the locally static wormhole. In that case, the energy-momentum tensors are similarly calculated as $< T^{f}_{\pm \pm} > \approx -\lambda^2(1 + \beta_{\pm})$ at $x^+ \to -\infty$. Therefore, the quantum-mechanically induced energy is the exotic source that supports the wormholes in our model.

The final comment is in order. The exoticity for the wormhole solution (36) defined in Ref. [1] can be easily checked. For this purpose, the proper reference frame of a set of observers who remain always at rest in the coordinate system is introduced as $e_0^\hat{0} = e^{-\rho}e_0$ and $e_1^\hat{0} = e^{-\rho}e_1$. In this basis, the metric locally looks like an Minkowskian, $ds^2 = -(d\hat{x}^0)^2 + (d\hat{x}^1)^2$. The energy momentum tensors, $< T^{f}_{\mu \nu} >$ written by $(\hat{0}, \hat{0})$- and $(\hat{1}, \hat{1})$-components in this frame are $< T^{f}_{00} > \approx 2\lambda^2 e^{-2\phi_r}(3+4e^{-2\phi_r}/\kappa)/(1-4e^{-2\phi_r}/\kappa)$ and $< T^{f}_{11} > = -2\lambda^2 e^{-2\phi_r}$ near the throat, $|x^+ - x^-| \to 0$, where $\phi_r$ satisfies Eq. (40). The specific dimensionless function $\zeta$ defined as $\zeta = (-T^{f}_{00} - T^{f}_{11})/|T^{f}_{00}| > 0$ characterizes the exoticity of matter [1]. Now, it reads as $\zeta \approx [4\lambda^2 e^{-2\phi_r}(e^{-2\phi_r} + \kappa/4)]/(e^{-2\phi_r} - \kappa/4) > 0$ near the throat since $e^{-2\phi_r} > \kappa/4$ in Eq. (39).

In summary, we have studied how the D-particle can travel through the wormhole in the two-dimensional dilaton gravity coupled to the D-particle. The crucial key to the formation and maintenance of the wormhole is to set the appropriate vacuum in the quantized theory, which corresponds to appropriate choice of $t_\pm$ in our model. As a result, we have shown that in a simplified model calculation the quantum-mechanically induced energy may be a candidate of the exotic source for the wormhole.

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