Robust ILC of Nonlinearly Parametric Time-Delay Systems with Input Deadzone

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Abstract. The trajectory tracking problem for a class of nonlinearly parametric systems with input deadzone and time-delay is studied in this work. An adaptive iterative learning control (ILC) scheme is developed by using Lyapunov synthesis. The alignment condition is applied to solve the initial position problem of ILC. Robust control and ILC are used to deal with input deadzone, delay and nonlinearly parametric uncertainties, together. As the iteration learning cycle increases, the system state may precisely track the reference signal over the whole interval. A numerical simulation is presented to verify the efficacy of the proposed adaptive iterative learning control method.

Keywords. Adaptive iterative learning control; robust control; input deadzone; time-delay;

1. Introduction

For pursuing better control performance, researchers exploit new control technologies in industrial applications. Iterative learning control (ILC) is a fantastic control technology for two advantages, including low demand on system modelling and good control precision [1]. ILC came out in the early 1980s, which is suitable to tackle repetitive control tasks during a finite time interval. In recent years, ILC has earned a great deal of attention due to the excellent tracking control performance during whole time interval. In recent years, it has been widely applied in the controller design of industrial applications [2-7].

Adaptive ILC is actually a combination of adaptive control and ILC, which has been a hot issue in the field of learning control since this century. In [8], French et al. developed a differential learning law for the parametric systems within known constant parameters during a finite time interval. Xu et al. investigated the controller design for systems within known time-varying but iteration-independent parameters, with difference learning method being used to deal with such uncertainties [1]. In [9], the ILC design for time-iteration-varying parametric uncertain system was studied. In recent years, the ILC design for more complicated systems has been exploited, such as nonparametric systems and nonlinear parametric systems. In most existing ILC results, a common assumption is the initial state error of controlled system should be zero [10]. This initial error condition is so strict that it cannot be satisfied in almost all practical industries. Up to now, some remedies have been reported to remove the zero initial error assumption, inclusive of time-varying boundary layer method, initial rectification action, initial state learning and so on [11-13]. Among them, the alignment condition is useful for the controller design in the cases that the desired trajectories are spatially closed [14].

Time delays are abundant in many industrial applications inherently, which degrade the performances of control systems, more or less, and even result in system error divergences in serious cases. Due to the damages caused by time delays, researchers the corresponding remedies for long.
Time delays have become rising concerns in many ILC applications and systems nowadays. In [15], Li et al. investigated the 2-D theory based ILC for linear continuous multivariable time-delay systems. In [16], Meng et al. proposed a robust ILC algorithm for uncertain time-delay systems, with LMI approach used for controller design. The adaptive ILC algorithms for nonlinearly parametric systems with time-delay have been discussed in [17-19].

On the other hand, deadzone is a typical non-smooth nonlinearity, which exists in the actuators of various industrial applications, such as valves, DC servo motors and so on. Because deadzone nonlinearities do harm to the stability of control systems, it should be dealt with properly during the controller design. Recker, Tao et al. respectively exploited adaptive inverse model approach to directly estimate the deadzone parameters [20-22]. Later, Wang and Ibrir proposed robust adaptive compensating method to deal with symmetric/nonsymmetric deadzone nonlinearity [23, 24]. Besides, neural networks [25] or fuzzy systems [26] were also considered in deadzone remedies. In the adaptive learning field, researchers investigated robust adaptive learning compensating method to tackle deadzones nonlinearities.

In the above-mentioned ILC results, the ILC algorithms for uncertain systems with time-delay systems or for uncertain systems with input deadzone have been studied, but the ILC results for time-delay systems with input deadzone is few. In this work, we propose a robust adaptive ILC algorithm for a class of nonlinearly parametric systems with time-delay and input deadzone, with alignment condition adopted to remove the zero initial error requirement. Adaptive iterative learning mechanism and robust feedback compensating mechanism are used to deal with uncertainties.

2. Problem Formulation
A class of uncertain with time-delay and input deadzone is considered as follows:

\[
\begin{align*}
\dot{x}_k &= x_{i,k}, \quad i=1,2,\ldots,n-1 \\
\dot{x}_k &= \eta(x_k(t-\tau),\theta(t),t) + g(t)u(v_i) \\
x_0(t) &= \sigma(t), \quad \forall t \in [-\tau_{\text{max}},0]
\end{align*}
\]

where \( t \in [0,T] \), \( k \) represents iteration index, system state \( x_k \triangleq [x_{1,k},x_{2,k},\ldots,x_{n,k}]^T \in \mathbb{R}^n \) is measurable, and \( \eta(x_k(t-\tau),\theta(t),t) \in \mathbb{R} \) and \( \tau \in [-\tau_{\text{max}},0] \) meet Assumption 1 and Assumption 2, respectively. \( u(v_i) \) and \( v_i \) respectively denote the input and output of an unknown deadzone:

\[
u_i = \begin{cases} 
m_i(v_i - b_i) & v_i \geq b_i \\
0 & b_i \leq v_i \leq b_i \\
m_i(v_i - b_i) & v_i < b_i
\end{cases}
\]

The deadzone nonlinearity in (2) is similar to the one discussed in [23], i.e., \( u(v_i) \) is assumed to be not available for measurement, \( m_r = m_l = m \), and the exact values of \( b_r > 0, b_l < 0 \) and \( m > 0 \) are all unknown.

Assumption 1:

\[
\left| \eta(\xi_1,\theta(t),t) - \eta(\xi_2,\theta(t),t) \right| \leq \|\xi_1 - \xi_2\| h(\theta,t), \quad \forall \xi_1, \xi_2 \in \mathbb{R}^n
\]

holds, where the smooth function \( h(\theta,t) \) is unknown.

Assumption 2: The time delay \( \tau \) meets \( \dot{\tau} \leq \phi < 1 \), i.e.,

\[
\frac{1 - \dot{\tau}}{1 - \phi} \leq -1.
\]
For the reference trajectory \( x_d(t) = [x_d, ẋ_d, \ldots, x_d^{(n-1)}]^T \) with \( x_d \in C^n[0, T] \), the control task is to let \( x_k(t) \) track \( x_d(t) \) over \([0, T]\) where \( x_k(0) = x_{k-1}(T) \) and \( x_d(T) = x_d(0) \).

In the remainder of this paper, arguments sometimes may be omitted for brevity. Also \( \eta_k(t-\tau) \) denotes \( \eta_k(x_k(t-\tau), \theta(t), t) \).

### 3. Control System Design

Let \( e_k(t) = [e_k, \ldots, e_n]^T = x_k(t) - x_d(t) \) and

\[
s_k = \left(\frac{d}{dt} + \lambda\right)^{-1} e_k
\]

with \( \lambda > 0 \). From (4), we have

\[
s_k = c_1 e_{k,1} + \cdots + c_{n-1} e_{n-1,k-1} + e_{n,k}
\]

in which \( c_j = \frac{(n-1)!}{(j-1)!(n-j)!} \lambda^{n-j}, j = 1, 2, \ldots, n-1 \).

It follows from (1) that

\[
\begin{align*}
\dot{e}_{k,1} &= \dot{e}_{i,1,k}, \quad i=1, 2, \ldots, n-1 \\
\dot{e}_{n,k} &= \eta_k(t-\tau) + gu_k - x_d^{(n)}
\end{align*}
\]

Then, defining a Lyapunov function

\[
V_k = \frac{1}{2gm} s_k^2
\]

and taking its time derivative, we have

\[
\dot{V}_k = \frac{1}{gm} s_k \left( gu_k + \eta_k(t-\tau) - x_d^{(n)} \right)
\]

Based on Assumption 1, the following conclusion may be drawn:

\[
\begin{align*}
\frac{1}{gm} s_k \eta_k(t-\tau) &= \frac{1}{gm} s_k \left( \eta_k(t-\tau) - \eta_k(t-\tau) + \eta_k(t-\tau) \right) \\
\leq |\eta_k(t-\tau)| \left[ \frac{1}{gm} h(\theta, t) + \frac{1}{gm} s_k \eta_k(t-\tau) \right] &\leq \frac{s_k^2}{2gm} + \frac{1}{2} e_k^2(t-\tau) + \frac{s_k}{gm} \eta_k(t-\tau)
\end{align*}
\]

Combining (8) with (9), we get

\[
\dot{V}_k \leq \dot{V}_k + s_k \max(h, |p_k|) + \frac{s_k^2}{2gm} + \frac{1}{2} e_k^2(t-\tau) + \frac{s_k}{gm} \eta_k(t-\tau)
\]

with \( \varphi = \left( \frac{1}{2gm} h^2(\theta, t), \frac{1}{gm} \eta_k(t-\tau) - \frac{x_d^{(n)}}{gm} \right)^T \) and \( \varphi_k = \left( s_k, 1 \right)^T \). Then, we design the controller as follows:

\[
v_k = -\gamma_0 v_k - \varphi_k \varphi - \frac{2s_k}{s_k^2 + \varepsilon^2} \left[ \rho_k + \frac{1}{2(1-\phi)} e_k^2(t) e_k(t) \right]
\]
in which
\[ \sigma_k = \text{sat}_{\sigma}(\sigma_{k-1}) + \gamma_s s_k \phi_k, \sigma_{k-1} = 0 \] (12)
and
\[ \rho_k = \text{sat}_{\rho}(\rho_{k-1}) + \gamma_s |s_k|, \rho_{k-1} = 0 \] (13)

In the above learning laws (12)-(13), \( \text{sat}_{\sigma, \rho}(t) \) is defined as follows: for a scalar \( \hat{a} \),
\[ \text{sat}_{\sigma, \rho}(\hat{a}) \triangleq \begin{cases} \hat{a} & \hat{a} > \bar{a} \\ a & a \leq \hat{a} \leq \bar{a} \\ \bar{a} & \hat{a} < \bar{a} \end{cases} \] (14)
for a vector \( \hat{a} = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_p] \in \mathbb{R}^p \), \( \text{sat}_{\sigma, \rho}(\hat{a}) \triangleq [\text{sat}_{\sigma, \rho}(\hat{a}_1), \text{sat}_{\sigma, \rho}(\hat{a}_2), \cdots, \text{sat}_{\sigma, \rho}(\hat{a}_p)]^T \).

4. Convergence Analysis

**Theorem 1:** As far as the dynamic system (1) meeting Assumptions 1-2 is concerned, the proposed ILC law and learning laws (11)-(13) may ensure that \( |s_k(t)| \leq \epsilon \) and \( e_{i,k}^{(i)}(t) \leq (2 \lambda)^i \frac{e}{\lambda^{n-1}} \), \( i = 0, 1, \cdots, n-1 \) as the learning cycle increases. The boundedness of all signals is guaranteed.

**Proof:** Let us define
\[ V_{2,k} = V_k + \frac{1}{2(1-\phi)} \int_{t-\tau}^{t} e_k^T e_k d\sigma \] (15)
whose time derivative is
\[ \dot{V}_{2,k} \leq s_k (v_k + \sigma^T \phi_k) + |s_k| \rho + \frac{1}{2(1-\phi)} e_k^T (t) e_k (t) \] (16)
Substituting (11) to (16), we have
\[ \dot{V}_{2,k} \leq -\gamma_s s_k^2 + s_k \sigma^T \phi_k + \frac{1}{2(1-\phi)} e_k^T (t) e_k (t) + |s_k| \rho - \frac{2s_k^2}{s_k^2 + \epsilon^2} \left[ \rho_k + \frac{1}{2(1-\phi)} e_k^T (t) e_k (t) \right] \] (17)

While \( |s_k| > \epsilon \), \( \frac{2s_k^2}{s_k^2 + \epsilon^2} > 1 \) holds. By this property, we deduce
\[ \frac{1}{2(1-\phi)} e_k^T (t) e_k (t) - \frac{2s_k^2}{s_k^2 + \epsilon^2} \frac{1}{2(1-\phi)} e_k^T (t) e_k (t) \leq 0 \] (18)
and
\[ |s_k| \rho - \frac{2s_k^2}{s_k^2 + \epsilon^2} \rho_k = |s_k| \rho - |s_k| \rho_k + |s_k| \rho_k - \frac{2s_k^2}{s_k^2 + \epsilon^2} \rho_k \] (19)
Note that \( \rho_k \geq 0 \). Therefore, \( |s_k| \rho_k \leq \frac{2s_k^2}{s_k^2 + \epsilon^2} \rho_k \leq 0 \) and
\[ |s_k| \rho - \frac{2s_k^2}{s_k^2 + \epsilon^2} \rho_k \leq |s_k| \tilde{\rho}_k \]  

(20)

hold while \(|s_k| > \epsilon\), where \(\tilde{\rho}_k = \tilde{\rho} - \tilde{\rho}_k\). Substituting (18) and (20) into (17) leads to

\[ \dot{V}_{2,k} \leq -\gamma_0 s_k^2 + s_k \tilde{\omega}_k^T \varphi_k + |s_k| \tilde{\rho}_k \]  

(21)

Let us define a Lyapunov functional as

\[ L_k = V_{2,k} + \frac{1}{2\gamma_1} \int_0^t (\tilde{\omega}_k^T \tilde{\omega}_k + \rho_k^2) d\sigma + \frac{1}{2\gamma_2} \int_0^t \tilde{\rho}_k^2 d\sigma \]  

(22)

with \(\tilde{\omega}_k = \omega - \omega_k\). While \(k > 0\),

\[ L_k - L_{k-1} \leq V_{2,k} (0) + \int_0^t (\gamma_0 s_k^2 + |s_k| \tilde{\rho}_k) d\sigma - V_{2,k-1} \]

\[ + \frac{1}{2\gamma_1} \int_0^t (\tilde{\omega}_k^T \tilde{\omega}_k - \tilde{\omega}_{k-1}^T \tilde{\omega}_{k-1}) d\tau + \frac{1}{2\gamma_2} \int_0^t (\tilde{\rho}_k^2 - \tilde{\rho}_{k-1}^2) d\tau \]  

(23)

Applying (12), we get

\[ \frac{1}{2\gamma_1} (\tilde{\omega}_k^T \tilde{\omega}_k - \tilde{\omega}_{k-1}^T \tilde{\omega}_{k-1}) + s_k \tilde{\omega}_k^T \varphi_k \]

\[ \leq s_k \tilde{\omega}_k^T \varphi_k + \frac{1}{2\gamma_1} ((\omega - \omega_k)^T (\omega - \omega_k) - (\omega - sat_{z,\sigma}(\omega_{k-1}))^T (\omega - sat_{z,\sigma}(\omega_{k-1}))) \]

\[ - \frac{1}{2\gamma_1} (2\omega - \omega_k - sat_{z,\sigma}(\omega_{k-1}))^T (sat_{z,\sigma}(\omega_k) - \omega_k) + s_k \tilde{\omega}_k^T \varphi_k \]

\[ \leq \frac{1}{\gamma_1} (\omega - \omega_k)^T (sat_{z,\sigma}(\omega_k) - \omega_k + \gamma_1 s_k \varphi_k) \]

\[ = 0 \]  

(24)

Combining (23) with (24) leads to

\[ L_k - L_{k-1} \leq V_{2,k} (0) + \int_0^t (\gamma_0 s_k^2 + |s_k| \tilde{\rho}_k) d\sigma - V_{2,k-1} + \frac{1}{2\gamma_2} \int_0^t (\tilde{\rho}_k^2 - \tilde{\rho}_{k-1}^2) d\sigma \]  

(25)

It follows from (13) that

\[ \frac{1}{2\gamma_1} (\tilde{\rho}_k^2 - \tilde{\rho}_{k-1}^2) + |s_k| \tilde{\rho}_k \leq \frac{1}{2\gamma_1} (2\rho_k - \rho_k - sat_{z,\sigma}(\rho_{k-1}))(sat_{z,\sigma}(\rho_{k-1}) - \rho_k) + |s_k| \tilde{\rho}_k \]

\[ \leq \frac{1}{\gamma_1} (\rho_k - \rho_k)(sat_{z,\sigma}(\rho_{k-1}) - \rho_k) + |s_k| \tilde{\rho}_k = 0 \]  

(26)

From (26) and (25), we get

\[ L_k - L_{k-1} \leq V_{2,k} (0) - \gamma_0 \int_0^t s_k^2 d\sigma - V_{2,k-1} \]  

(27)

It follows that

\[ L_k (t) \leq V_{2,k} (0) - \gamma_0 \int_0^t s_k^2 d\sigma - V_{2,k-1} \leq L_{k-1} (T) - \gamma_0 \int_0^T s_k^2 d\sigma \]  

(28)

It follows from \(x_{k-1} (T) = x_k (0)\) and \(x_d (T) = x_d (0)\) that \(e_k (0) = e_{k-1} (T)\). By using this conclusion and (28), we obtain

\[ L_k (T) - L_{k-1} (T) \leq -\gamma_0 \int_0^T s_k^2 d\sigma \]  

(29)
which further yields
\[ L_k(T) \leq L_{k-1}(T) \leq \ldots \leq L_{\min}(T) \] (30)

Combining (29) with (30) gives
\[ L_k(t) \leq L_{\min}(T) - \int_0^t s_i^2(\sigma) d\sigma \] (31)

According to the continuity of Lyapunov functional, we deduce that \( L_k(T) \) is bounded. By using this conclusion, it follows from (31) that
\[ 0 \leq L_k(t) < +\infty \] (32)

Then, it follows from the definition of \( L_k \) that \( s_k \) is bounded, which can further yield the boundedness of \( e_k \). \( \|s_k\| \). Then, we can assert that \( |s_k| < +\infty \) while \( |s_k| > \epsilon \), which means that \( s_k \) is continuous while \( |s_k| > \epsilon \). Further, the boundedness of other system signals can be deduced. According to (30) and (31), we have
\[ L_k(T) \leq L_{\min}(T) - \sum_{i=1}^k \int_0^t s_i^2(\sigma) d\sigma \] (33)

Because \( L_{\min}(T) \) is a bounded positive number, and \( L_k(T) \) is positive, if \( |s_k(t)| > \epsilon \) holds as the learning cycle increases, then \( L_k(T) \) would be negative, which is contrary to the positiveness of \( L_k(T) \). Hence, as the learning cycle increases, \( |s_k(t)| \leq \epsilon \) may hold, which leads to
\[ |e^{(i)}_{k,i}(t)| \leq (2\lambda)^i \frac{\epsilon}{\lambda^{n-i}} \] (34)
for \( i,0,1,\ldots,n-1 \).

5. Illustrative Example
Let us consider the uncertain system as follows:
\[ \begin{align*}
    x_{1,k} &= x_{2,k} \\
    x_{2,k} &= e^{-\theta(s_{1,k}(t)-s_{2,k}(t))} + g(t)u_v(t) \\
    \left[ x_{1,i}(t), x_{2,i}(t) \right]^T &= [1.5, 0.2], t \in [-\tau_{\max}, 0]
\end{align*} \] (35)

where \( \theta = 0.5 + |\sin(2t)| \), \( \tau(t) = 1 - 0.5 \sin^2 t \), \( g(t) = 1 + 0.5 \sin^2 t \), \( \tau_{\max} = 1 \) and \( \dot{\tau}(t) \leq 0.5 \), \[ x_{1,d}, x_{2,d} \] \Rightarrow \cos(\pi t), -\pi \sin(\pi t) \] .

We put the control algorithm (11)-(13) into effect with \( T = 4 \), \( \gamma_0 = 10 \), \( \gamma_1 = 4.5 \), \( \gamma_2 = 0.1 \), \( \epsilon = 0.01 \), \( \lambda = 2 \). In figures 1-2, \( x_1 \) and \( x_2 \) respectively follow \( x_{1,d} \) and \( x_{2,d} \) at the 50th cycle, with the corresponding tracking error provided in figures 3-4. Figure 5 shows the control input signal at the 50th cycle. The convergence history of \( s_k \) is given in figure 6, where \( J_k \triangleq \max_{t\in[0,T]}|s_k(t)| \).
Figure 1. The system state $x_1$.

Figure 2. The system state $x_2$.

Figure 3. State error $e_1$. 
Figure 4. State error $e_2$.

Figure 5. Control input.

Figure 6. History of $s_k$ convergence.
6. Conclusion
An adaptive ILC scheme is developed in this work, with alignment condition being used to deal with the initial position problem of ILC. Robust control technology and ILC technology are jointly applied to deal with input deadzone, time-delay and nonlinearly parametric uncertainties.

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