Implications of mirror dark matter kinetic mixing for CMB anisotropies

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Mirror dark matter is a dissipative and self-interacting multiparticle dark matter candidate which can explain the DAMA, CoGeNT and CRESST-II direct detection experiments. This explanation requires photon-mirror photon kinetic mixing of strength \( \epsilon \sim 10^{-9} \). Mirror dark matter with such kinetic mixing can potentially leave distinctive signatures on the CMB anisotropy spectrum. We show that the most important effect of kinetic mixing on the CMB anisotropies is the suppression of the height of the third and higher odd peaks. If \( \epsilon \gtrsim 10^{-9} \) then this feature can be observed by the PLANCK mission in the near future.
A large variety of observations have lead to a simple picture of the Universe. In a nutshell, we live in a spatially flat, expanding Universe consisting of dark energy (∼ 70%), non-baryonic dark matter (∼ 26%) and ordinary baryons (∼ 4%). In the last two decades, detailed observations of the cosmic microwave background (CMB) by COBE[1], WMAP[2], SPT[3] and many other missions have provided important tests of this basic picture. A key question concerns the identity of non-baryonic dark matter. Although it is popular to assume that dark matter consists of a single species of weakly interacting massive particles (standard cold dark matter model), in actuality the particle physics underlying the non-baryonic dark matter in the Universe is currently unknown, as is the physics responsible for the dark energy.

One thing we do know, though, is that the standard model has been very successful in describing the interactions of the ordinary particles. In fact, it is possible that such a structure might also be responsible for the non-baryonic dark matter in the Universe as well. That is, dark matter might consist of a hidden (mirror) sector with particles and interactions exactly isomorphic to the ordinary ones[4, 5] (for a review, see e.g.[6]). Provided that initially the mirror sector temperature is much less than in the ordinary sector, i.e. $T' \ll T$ in the early Universe, such a scenario can explain the large scale structure of the Universe in a way completely analogous to standard cold dark matter[7, 8].

On much smaller scales, mirror dark matter is radically different to standard cold dark matter. It is self-interacting, dissipative and multi-component$^2$. These properties might help explain some puzzling aspects of dark matter on small scales, such as the inferred cored central density profiles in galaxies [c.f. ref.[11]]. At the current epoch dark matter needs to be roughly spherically distributed in spiral galaxies to be consistent with various observations$^3$. However, within galaxies mirror dark matter would be expected to collapse into a disk, analogous to the way in which ordinary matter collapses, unless a significant heat source exists. It turns out that ordinary supernovae can potentially supply the required energy provided that photon-mirror photon kinetic mixing[12, 13],

$$\mathcal{L}_{\text{mix}} = \frac{\epsilon}{2} F_{\mu\nu} F'_{\mu\nu}$$

of strength $\epsilon \sim 10^{-9}$ exists[14]. [In the above equation, $F_{\mu\nu}$ ($F'_{\mu\nu}$) is the ordinary (mirror) photon field strength tensor]. That is, the ordinary and dark matter components

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$^2$There are potential limits on self interactions of dark matter from observations of the Bullet cluster[9]. These observations can set stringent limits on dark matter self interactions provided that the bulk of the dark matter particles are distributed throughout the cluster and not bound to individual galaxies. However mirror dark matter is dissipative and in clusters (or at least in some of them) the bulk of the dark matter particles might be confined in galactic halos [c.f. ref.[10]]. Under this assumption mirror dark matter is consistent with Bullet cluster observations.

$^3$Deviations from perfectly spherical halos are also required and might be due to various sources including, e.g. partial collapse of the halo due to dissipative processes, mirror magnetic fields, asymmetric heating from ordinary supernovae which are distributed in the disk (i.e. they are not spherically distributed), possible existence of a dark disk subcomponent etc.
of galaxies might be in a kind of dynamical equilibrium where the energy supplied to the halo by ordinary supernovae balances the energy lost to the halo due to radiative cooling. Since the former is related to the galactic luminosity and the latter the dark matter density, that is, $v_{\text{rot}}$, it has been speculated[15] that this might potentially explain puzzling regularities on small scales, such as the Tully-Fisher relation[16].

Photon-mirror photon kinetic mixing of strength $\epsilon \sim 10^{-9}$ is also implicated[17] by the positive results of the direct detection experiments, DAMA[18], CoGeNT[19] and CRESST-II[20]. It has been shown in ref.[21] that these experiments can be explained by the interactions of a halo $F\nu'$ component with $\epsilon\sqrt{\xi_{F\nu'}} \approx 2 \times 10^{-10}$. Here $\xi_{F\nu'}$ is the abundance by mass of the halo $F\nu'$ component (at the Earth’s location) normalized to $0.3 \text{ GeV/cm}^3$. Naturally, it is very difficult to predict $\xi_{F\nu'}$ with any certainty, but of course we expect $\xi_{F\nu'} < \sim 1$, and thus $\epsilon \sim 2 \times 10^{-10}$.

Kinetic mixing can also have important implications for cosmology. Successful big bang nucleosynthesis (BBN) and large scale structure (LSS) require the initial condition $\rho_{\nu'} \ll \rho_{\nu}$ and $n_{\nu} \approx 5n_{\nu}$. How such initial conditions might have arisen has been discussed in the literature[22]. However, if photon-mirror photon kinetic mixing exists then this will generate entropy in the mirror sector via the process $e\bar{e} \rightarrow e'\bar{e}'$ when $T_\gamma \approx m_e[23]$. That is $T'_\gamma$ will be generated even if we start with $T'_\gamma \ll T_\gamma$. In fact, it has been shown that the asymptotic value of the ratio: $T'_\gamma/T_\gamma$, which we here define as $x$, is given by[24, 25]

$$x \simeq 0.31 \left(\frac{\epsilon}{10^{-9}}\right)^{1/2}. \tag{2}$$

The value $x \sim 0.3$ is close to the limit estimated from the matter power spectrum i.e. successful large scale structure[8]. We will estimate that the upper bound on $x$ from such considerations is conservatively around $x \sim 0.5$. Non-zero $T'_\gamma/T_\gamma$ will also lead to important effects for the CMB as previously discussed in ref.[8] (see also ref.[7]). In view of the forthcoming results from the PLANCK mission it is pertinent to examine thoroughly the possible effects that kinetic mixing will induce for the CMB. This is the purpose of this paper. In fact, we will show that mirror dark matter can potentially leave a distinctive imprint on the tail of the CMB anisotropy spectrum. Our most important observation is that the height of the third and higher odd peaks can be suppressed. This should be observable by PLANCK provided $\epsilon \sim 10^{-9}$.

To summarize, we assume a mirror sector exactly isomorphic to the ordinary one, except with initial conditions $T'_\gamma \ll T_\gamma$. Ordinary and mirror particles can interact via photon - mirror photon kinetic mixing, which can excite the mirror degrees of freedom in the early Universe. In particular the process: $e\bar{e} \rightarrow e'\bar{e}'$ generates the mirror particles until the $e\bar{e}$ have annihilated at $T_\gamma \sim m_e$, the final $T'_\gamma/T_\gamma$ value given in Eq.(2). In fact, most of the mirror entropy generation occurs after the neutrinos have decoupled. One effect of this is to induce a slight cooling of the ordinary photons relative to the ordinary neutrinos. The net effect is that there is additional neutrino energy density and also an additional relativistic component comprised of mirror
photons. These two additional components to the relativistic energy density can be parameterized in terms of extra neutrino degrees of freedom:[25]

\[
\delta N_{eff}^a(\epsilon)[CMB] = 3 \left( \frac{T_{\nu}(\epsilon)}{T_{\nu}(\epsilon = 0)} \right)^4 - 1
\]

\[
\delta N_{eff}^b(\epsilon)[CMB] = \frac{8}{7} \left( \frac{T^*_\gamma(\epsilon)}{T_{\nu}(\epsilon = 0)} \right)^4 .
\]  

(3)

Here, the temperatures are evaluated at the time when photon decoupling occurs, i.e. when \( T_\gamma = T_{dec} \approx 0.26 \text{ eV} \). Using the result from Eq.(2), together with the usual \( T_\nu/T_\gamma \) relation, we have

\[
\delta N_{eff}^b(\epsilon)[CMB] \approx 8 \times 4 \times \frac{11}{4} \frac{4/3}{0.041} \left( \frac{\epsilon}{10^{-9}} \right)^2 .
\]  

(4)

Also, numerical work[25] has found that \( \delta N_{eff}^a(\epsilon)[CMB] \approx 0.8 \delta N_{eff}^b(\epsilon)[CMB] \). This additional energy density can directly affect the predicted CMB anisotropies. In fact, it is known that additional relativistic energy density can dampen the CMB tail[26, 27]. However, there is another important effect for the CMB. If dark matter consists of mirror particles, then they experience significant pressure prior to mirror photon decoupling. If \( T_\gamma^* < T_\gamma \) then this epoch occurs prior to the familiar hydrogen recombination. One can anticipate that the small scale inhomogeneities in the mirror matter density should be suppressed, since the Fourier modes which enter the horizon before the time of mirror hydrogen recombination undergo acoustic oscillations due to the pressure of the mirror baryon-photon fluid. In other words, we expect a suppression of power on small scales when compared with standard non-interacting cold dark matter. The previous study[8] has indeed observed this effect on the matter power spectrum. This suppression of power on small scales will also influence the CMB spectrum, and one would anticipate that this might also dampen the CMB anisotropies at high multipoles. This effect, is of course, in addition to the effect of the increased relativistic energy density due to \( \delta N_{eff}^a + \delta N_{eff}^b \). [Both effects will be included in our numerical work]. It turns out that the mirror baryon acoustic oscillation effect is not only larger in magnitude, but has the distinctive feature of suppressing the higher odd peaks more than the even ones.

Although the effect of additional relativistic energy density has been well studied in the literature (see e.g. ref.[26, 27]), and can be explored using existing CMB codes, the mirror baryon acoustic oscillation effect on the mirror dark matter perturbations

\[4\text{The mirror photons can undergo diffusion (Silk damping). This would wash out small scale inhomogeneities in the mirror radiation field just before mirror photon decoupling. However it should have very little effect on the ordinary CMB or matter power spectrum since } \rho_{\gamma'} \text{ is a very small component to the overall energy density (assuming } x \lesssim 0.5). \text{ Of course mirror photon diffusion would be expected to significantly damp the tail of the mirror CMB anisotropies, just like ordinary photon diffusion does for the ordinary CMB.}\]
requires modifications. The relevant equations, though, are a straightforward generalization to the equations governing the perturbations of the ordinary baryons and photons. Our strategy is to numerically solve these equations, essentially using techniques developed in refs.[28] (see also [29, 30]). A very clear and helpful review is the one given by Dodelson[31].

Recall, the anisotropy spectrum today is characterized in terms of $C_\ell$. These quantities are the variance of the coefficients, $a_{\ell m}$ in the expansion of the photon temperature field in terms of spherical harmonics, $Y_{\ell m}$, i.e.

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell .$$

The terms $C_\ell$ can be related to the $\ell^{th}$ multipole moment, $\Theta_\ell$, in the Legendre expansion of the Fourier transformed photon temperature field via the equation:

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk k^2 P_\phi(k) |\Theta_\ell(k, \eta_0)|^2$$

where $P_\phi$ is the initial power spectrum of the metric perturbation with initial value $\Phi^i$. Finally the moments $\Theta_\ell(k, \eta_0)$ today can be related to the perturbations $\Theta_0(k, \eta)$, $v_b(k, \eta)$, $\Phi(k, \eta)$, $\Psi(k, \eta)$, $\Pi(k, \eta)$ near photon decoupling [where $v_b(k, \eta)$, $\Psi(k, \eta)$, $\Phi(k, \eta)$, $\Pi(k, \eta)$ are the baryonic velocity, metric perturbations, and polarization tensor respectively]. The critical equation is[31]:

$$\Theta_\ell(k, \eta_0) = \int_0^{\eta_0} d\eta g(\eta) \left( \Theta_0 + \Psi + \frac{1}{4} \Pi + \frac{3}{4 k^2} \frac{d^2}{d\eta^2} g(\eta) \Pi \right) j_\ell[k(\eta_0 - \eta)] + \int_0^{\eta_0} d\eta \left( j_{\ell-1}[k(\eta_0 - \eta)] - \frac{(\ell + 1) j_\ell[k(\eta_0 - \eta)]}{k(\eta_0 - \eta)} \right) j_\ell[k(\eta_0 - \eta)]$$

where $\tau$ is the optical depth for Thomson scattering and $g(\eta) \equiv -\tau e^{-\tau}$ is the visibility function, which peaks near photon decoupling. The evolution of the quantities $\Theta_0$, $v_b$, $\Psi$, $\Phi$, and $\Pi$ are governed by a set of linear equations, arising from the Boltzmann-Einstein equations. We assume standard adiabatic scalar initial conditions. The relevant equations are given in the appendix.

It is important to note that the mirror dark matter model introduces only one additional parameter, $x \equiv T'_\gamma/T'_{\gamma}$ which is related to the fundamental Lagrangian kinetic mixing parameter, $\epsilon$ via Eq.(2). The cosmological evolution of mirror dark matter, in the limit where $x \rightarrow 0$ (i.e. $\epsilon \rightarrow 0$) exactly mimics cold dark matter. This is because mirror particles feel negligible pressure after the mirror photon decoupling epoch, $t'_{\text{dec}}$, and $t'_{\text{dec}} \rightarrow 0$ as $x \rightarrow 0$. As $x$ increases from zero, differences begin to appear. Our job now is to determine what the observable differences are. To study these effects for the CMB one cannot simply choose a particular point for the parameters $\Omega_m h^2$, $\Omega_b h^2$, $h$, ... from a fit assuming standard cold dark matter
and vary $x$. Doing this, for example, would modify the epoch of matter radiation equality, $z_{EQ} + 1 = \Omega_m/\Omega_r$, due to the additional contributions [Eq.(3)] to $\Omega_r$. The matter radiation equality has been precisely constrained by the data and thus any modification to $z_{EQ}$ by new physics needs to be compensated for by adjustments to the parameters (in this case, $\Omega_m h^2$). In fact, what needs to be done is to examine parameter space where not only $z_{EQ}$ is fixed, but also $\Omega_b h^2$ and $\theta_s$ (the angular size of the sound horizon at decoupling), since these quantities have also been precisely determined by the data. A similar situation has been noted when considering the effect of additional relativistic neutrino degrees of freedom[27, 26]. In this parameter space direction, the observable effects from varying $x$ occur at small angular scales.

It is reasonably straightforward to write a code to numerically solve the relevant set of equations to obtain the CMB anisotropy spectrum. For a given set of parameters, $\Omega_b h^2, \Omega_m h^2, h, ...$, comparison of our code with existing high accuracy codes, e.g. CMBFAST[32], confirms that our computation of the $C_\ell$ values are accurate to within a few percent. This is sufficient for making a comparison of mirror dark matter with standard cold dark matter.

![Figure 1: The anisotropy spectrum for mirror dark matter versus standard cold dark matter. The solid line is standard cold dark matter model with parameters described in the text (equivalent to mirror dark matter with $x = 0$), while mirror dark matter with $x = 0.3$ (dashed line), $x = 0.5$ (dotted line) and $x = 0.7$ (dashed-dotted line) are also shown.](image-url)
In figure 1,2,3 we give our results for the CMB spectrum. We consider a flat Universe with the reference parameters $\Omega_m h^2 = 0.14$, $\Omega_b h^2 = 0.022$, $\Omega_{\Lambda} = 1 - \Omega_m$, $h = 0.70 \left[ \Omega_m \equiv \Omega_b + \Omega_{b'} \right]$. These reference parameters are defined at $x = 0$. As discussed above, these parameters are adjusted as $x$ is varied such that $z_{EQ}$, $\Omega_b h^2$ and $\theta_s$ are held fixed. [We also adjust the overall normalization by fixing the height of the first peak]. A scale invariant initial perturbation spectrum (Harrison-Zel’doovich and Peebles spectrum) is assumed and we have neglected reionization effect. Since we are interested in comparing mirror dark matter versus standard non-interacting cold dark matter (cosmologically equivalent to mirror dark matter with $x = 0$) small effects due to primordial tilt or reionization are not important to leading order. Figure 1 illustrates the expected agreement at large angular scales, as we vary $x$. In figure 2, we consider the small angular scale region of interest. In figure 3 we plot $F_l(x) \equiv C_l(x)/C_l(x = 0)$ for several several values of $x$.

Figure 2: The CMB tail. The curves correspond to the same parameters as figure 1.
Figure 3: $F_\ell(x) \equiv C_\ell(x)/C_\ell(x=0)$ for $x = 0.3$ (dashed line) and $x = 0.5$ (dotted line), and $x = 0.7$ (dash-dotted line) are shown.

Figures 2, 3 clearly show the expected suppression of anisotropies at small angular scales, starting around the third peak. Interestingly, we see that the suppression is larger for the higher odd peaks than the even ones. These features can be readily understood. Odd peaks arise from compressions of the baryon-photon fluid, even peaks are rarefactions. When the gravitational driving force is suppressed, one expects the odd peaks (the compressions) to be more affected than the even peaks (related effects occur when $\Omega_b^2 h^2$ is reduced). Furthermore, the differences only become apparent for the higher peaks because the suppression of power only occurs at small scales.

Currently the most accurate measurement of the CMB damping tail has been made with the South Pole telescope[3]. These measurements show a slight damping, around $\sim 2.5\%$ at $\ell \sim 2000$ c.f. predictions of the standard cold dark matter model. This damping provides an interesting hint that $x \approx 0.4$ [i.e. $\epsilon \approx 2 \times 10^{-9}$ from Eq.(2)]. In any case, these observations limit $x \lesssim 0.5$ [or $\epsilon \lesssim 3 \times 10^{-9}$]. It is anticipated that the PLANCK mission should improve the precision, which will probe $\epsilon$ in the range: $10^{-9} \lesssim \epsilon \lesssim 3 \times 10^{-9}$.

In addition to CMB anisotropies the matter power spectrum can also be used to constrain parameters. However since small scales $k \lesssim 0.1 \ Mpc^{-1}$ have gone
nonlinear today, we consider the matter power spectrum on larger scales than this [linear regime]. It is straightforward to compute the power spectrum of matter,

\[ P(k) = 2\pi^2 \delta^2 \frac{k}{H_0^2} T^2(k) \]  

where \( H_0 = 100h \, \text{km sec}^{-1} \text{Mpc}^{-1} \) is the Hubble rate today and \( T(k) \) is the transfer function (see e.g. ref.[31] for details). In figure 4 we compare the obtained matter power spectrum for the various \( x \) values considered, for the same parameters used in figures 1-3. [Recall, \( \Omega_m, \Omega_b \) and \( h \) are varied as \( x \) changes such that \( z_{EQ}, \Omega_b h^2 \) and \( \theta_s \) are fixed]. As expected, deviations only occur on small scales as \( x \) increases from zero. This figure indicates that a rough bound of \( x < \sim 0.3 - 0.5 \) could be extracted from galaxy surveys. Also note that similar results to our figure 4 have been obtained in the earlier study by Ciarcelluti[8].

![Figure 4: Power spectrum of matter for the same parameters as figure 1. As in figure 1, \( x = 0 \) (solid line), \( x = 0.3 \) (dashed line), \( x = 0.5 \) (dotted line) and \( x = 0.7 \) (dashed-dotted line).](image)

In conclusion, we have examined the implications of kinetically mixed mirror dark matter for CMB anisotropies. This dark matter candidate can potentially leave distinctive signatures on the CMB spectrum. We have found that the most important effects of kinetic mixing on CMB anisotropies is the suppression of the height of the third and higher odd peaks. This effect will be sensitively probed by the PLANCK mission in the near future.
Appendix - Linear perturbation theory with mirror dark matter

The relevant equations governing the linear evolution of scalar perturbations in the Universe has a rich history starting with the work of Lifshitz in 1946[33] and developed by many others, e.g. ref.[34]. For an up to date review see ref.[31]. As summarized in that review, the relevant equations governing the moments of the photon distribution (including the polarization field), which we consider numerically up to order $\ell = 5$ together with corresponding moments for neutrinos and baryonic matter perturbations, in the conformal Newtonian gauge, are:

\[
\dot{\Theta}_0 + k\Theta_1 &= -\Phi \\
\dot{\Theta}_1 - \frac{k}{3}\Theta_0 + \frac{2k}{3}\Theta_2 &= \frac{k}{3}\Psi + \dot{\tau}\left[\Theta_1 - \frac{iv_b}{3}\right] \\
\dot{\Theta}_\ell - \frac{k\ell}{2\ell + 1}\Theta_{\ell-1} + \frac{k(\ell + 1)}{2\ell + 1}\Theta_{\ell+1} &= \dot{\tau}\left[\Theta_\ell - \delta_{2}\frac{\Pi}{10}\right], \quad \ell \geq 2 \\
\Pi &= \Theta_2 + \Theta_{P2} + \Theta_{P0} \\
\dot{N}_0 + kN_1 &= -\dot{\Phi} \\
\dot{N}_1 - \frac{k}{3}N_0 + \frac{2k}{3}N_2 &= \frac{k}{3}\Psi \\
\dot{N}_\ell - \frac{k\ell}{2\ell + 1}N_{\ell-1} + \frac{k(\ell + 1)}{2\ell + 1}N_{\ell+1} &= 0, \quad \ell \geq 2 \\
\dot{\delta}_b + ikv_b &= -3\dot{\Phi} \\
v_b + \frac{\dot{a}}{a}v_b &= -ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1] \\
\dot{\Theta}_{P0} + k\Theta_{P1} &= \dot{\tau}\left[\Theta_{P0} - \frac{\Pi}{2}\right] \\
\dot{\Theta}_{P\ell} - \frac{k\ell}{2\ell + 1}\Theta_{P(\ell-1)} + \frac{k(\ell + 1)}{2\ell + 1}\Theta_{P(\ell+1)} &= \dot{\tau}\left[\Theta_{P\ell} - \delta_{2}\frac{\Pi}{10}\right], \quad \ell \geq 1 \quad (9)
\]

where $\dot{\tau} \equiv -X_{\epsilon}(1 - Y_p)\rho_0\sigma_T a$, $Y_p \simeq 0.24$ is the primordial helium mass fraction, $\sigma_T$ is the Thomson cross-section and $R \equiv \frac{3\rho_0}{4\gamma\epsilon}$. For the mirror sector, we have an analogous set of equations with $\Theta_\ell \rightarrow \Theta_\ell^\prime$, $\Theta_{P\ell} \rightarrow \Theta_{P\ell}^\prime$, $N_\ell \rightarrow N_\ell^\prime$ ($\ell \geq 0$), $\delta_b \rightarrow \delta_b^\prime$, $v_b \rightarrow v_b^\prime$ and $\dot{\tau}, R \rightarrow \dot{\tau}^\prime, R^\prime$. Here $\dot{\tau}^\prime \equiv -X_{\epsilon'}(1 - Y_p^\prime)\rho_{b\gamma}'\sigma_T a'$. Compared with the standard cold dark matter model, the only additional parameter introduced is $x \equiv T_\gamma^p/T_\gamma$ which is related to $\epsilon$ via Eq.(2). [Recall these equations reduce to the equations governing standard cold dark matter when $x \rightarrow 0$ and $\rho_{b\gamma} \rightarrow \rho_{c\gamma}$]. Finally, we have the two relevant Einstein equations:

\[
k^2(\Phi + \Psi) &= -32\pi G a^2(\rho_\gamma\Theta_2 + \rho_{b\gamma}N_2 + \rho_{b\gamma}'N_2') + \rho_{c\gamma}'\Theta_2' \\
k^2\Phi + \frac{3\dot{a}}{a}\left(\Phi - \frac{\dot{a}}{a}\right) &= 4\pi G a^2[\rho_b\delta_b + \rho_{b\gamma}\delta_{b\gamma} + 4\rho_\gamma\Theta_0 + 4\rho_{b\gamma}N_0 + 4\rho_{c\gamma}'\Theta_0' + 4\rho_{b\gamma}'N_0'] . \quad (10)
\]
where $\rho_{\gamma'} = x^4 \rho_{\gamma}$ and $\rho_{\nu} = N_{\text{eff}} (7/8)(4/11)^{4/3} \rho_{\gamma}$, $N_{\text{eff}} = 3.046 + \delta N_{\text{eff}}$. For our application we can neglect $N_{\ell}'$, $\rho_{\nu}$ because we have negligible excitation of the mirror neutrino degrees of freedom. All derivatives in Eqs.(9,10) are with respect to conformal time, $\eta$. The quantity, $X_e$ is the free electron fraction $[X_e \equiv n_e / n_H$ where $n_H$ is the total number of hydrogen nuclei$]$. It obeys the Boltzmann equation[35, 31]

$$\frac{1}{a} \frac{dX_e}{d\eta} = \left[(1 - X_e) \beta - X_e^2 (1 - Y_p) n_b \alpha^{(2)}\right] C$$

where

$$\beta = \langle \sigma v \rangle \left(\frac{m_e T_\gamma}{2\pi}\right)^{3/2} e^{-\epsilon_0/T_\gamma}$$

$$\alpha^{(2)} = \langle \sigma v \rangle \simeq 9.78 \frac{\alpha^2}{m_e^2} \left(\frac{\epsilon_0}{T_\gamma}\right)^{1/2} \ln \left(\frac{\epsilon_0}{T_\gamma}\right)$$

$$C = \frac{\Lambda_\alpha + \Lambda_{2\gamma}}{\Lambda_\alpha + \Lambda_{2\gamma} + \beta e^{3\epsilon_0/4T_\gamma}}.$$  \hspace{1cm} (12)

Here $\epsilon_0 = 13.6$ eV is the binding energy of Hydrogen, $\Lambda_{2\gamma} = 8.227 \text{ sec}^{-1}$ and $\Lambda_\alpha = H(3\epsilon_0)^3/[(8\pi)^2(1 - X_e)n_b(1 - Y_p)]$. A similar set of equations will govern $X_{e'}$ (with $Y_p \rightarrow Y_{p'}$, $T_\gamma \rightarrow T_{\gamma'}$, $n_b \rightarrow n_{b'}$). Evidently, the latter depends on the primordial mirror helium mass fraction, $Y_{p'}$. This quantity can be computed solving the relevant mirror BBN equations, and for $\epsilon \sim 10^{-9}$, is[36] $Y_{p'} \approx 0.85$. [Note that $Y_{p'}$ is a slowly varying function of $\epsilon$ if $\epsilon \sim 10^{-9}$].

The equations must be supplemented with initial conditions. We consider the standard adiabatic scalar perturbations. We further assume that the initial perturbation of $\Phi$ is drawn from a scale invariant Gaussian distribution with mean zero and variance, parameterized in the usual way: $P_\Phi^i = (50\pi^2/9k^3)\delta_H^2(\Omega_m/D_1(a = 1))^2$.

The above set of equations, together with the Friedmann equation are numerically solved for $k$ values on a logarithmically spaced grid between $[k_{\text{min}}, k_{\text{max}}]$. For our numerical work $k_{\text{min}} = 20/\eta_0$ and $k_{\text{max}} = 6000/\eta_0$. The $C_\ell$ values are then obtained from Eqs.(6,7).

Acknowledgments

This work was supported by the Australian Research Council.

References

[1] G. F. Smoot et al., Astrophys. J. 396, L1 (1992).

[2] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011) [arXiv:1001.4538].

[3] R. Keisler et al., Astrophys. J. 743, 28 (2011) [arXiv:1105.3182].
[4] T. D. Lee and C. -N. Yang, Phys. Rev. 104, 254 (1956). I. Kobzarev, L. Okun and I. Pomeranchuk, Sov. J. Nucl. Phys. 3, 837 (1966); M. Pavsic, Int. J. Theor. Phys. 9, 229 (1974); S. I. Blinnikov and M. Yu. Khlopov, Sov. J. Nucl. Phys. 36, 472 (1981); Sov. Astron. 27, 371 (1983).

[5] R. Foot, H. Lew and R. R. Volkas, Phys. Lett. B272, 67 (1991); Mod. Phys. Lett. A7, 2567 (1992).

[6] R. Foot, Int. J. Mod. Phys. D 13, 2161 (2004) [astro-ph/0407623]; Int. J. Mod. Phys. A 19, 3807 (2004) [astro-ph/0309330]; P. Ciarcelluti, Int. J. Mod. Phys. D 19, 2151 (2010) [arXiv:1102.5530].

[7] Z. Berezhiani, D. Comelli and F. L. Villante, Phys. Lett. B503, 362 (2001) [hep-ph/0008105]; A. Yu. Ignatiev and R. R. Volkas, Phys. Rev. D68, 023518 (2003) [hep-ph/0304260]; Z. Berezhiani, P. Ciarcelluti, D. Comelli and F. L. Villante, Int. J. Mod. Phys. D 14, 107 (2005) [astro-ph/0312605]. P. Ciarcelluti, Int. J. Mod. Phys. D14, 187 (2005) [astro-ph/0409630].

[8] P. Ciarcelluti, Int. J. Mod. Phys. D 14, 223 (2005) [astro-ph/0409633].

[9] D. Clowe et al., Astrophys. J. 648, L109 (2006) [astro-ph/0608407].

[10] Z. K. Silagadze, ICFAI U. J. Phys. 2, 143 (2009) [arXiv:0808.2595].

[11] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. 84, 3760 (2000).

[12] R. Foot and X. -G. He, Phys. Lett. B 267, 509 (1991).

[13] B. Holdom, Phys. Lett. B 166, 196 (1986).

[14] R. Foot and R. R. Volkas, Phys. Rev. D 70, 123508 (2004) [astro-ph/0407522]. See also R. Foot and Z. K. Silagadze, Int. J. Mod. Phys. D 14, 143 (2005).

[15] R. Foot, Phys. Rev. D 82, 095001 (2010) [arXiv:1008.0685].

[16] R. B. Tully and J. R. Fisher, Astron. Astrophys. 54, 661 (1977).

[17] R. Foot, Phys. Rev. D 69, 036001 (2004) [hep-ph/0308254]; Mod. Phys. Lett. A 19, 1841 (2004) [astro-ph/0405362]; Phys. Rev. D 78, 043529 (2008) [arXiv:0804.4518]; Phys. Lett. B 692, 65 (2010) [arXiv:1004.1424]; Phys. Lett. B 703, 7 (2011) [arXiv:1106.2688].

[18] R. Bernabei et al. [DAMA and LIBRA Collaborations], Eur. Phys. J. C 67, 39 (2010) [arXiv:1002.1028]; Eur. Phys. J. C 56, 333 (2008) [arXiv:0804.2741]; Riv. Nuovo Cim. 26N1, 1 (2003) [astro-ph/0307403].

[19] C. E. Aalseth et al., Phys. Rev. Lett. 107, 141301 (2011) [arXiv:1106.0650]; Phys. Rev. Lett. 106, 131301 (2011) [arXiv:1002.4703].

11
[20] G. Angloher et al., Eur. Phys. J. C 72, 1971 (2012) [arXiv:1109.0702].

[21] R. Foot, Phys. Rev. D 86, 023524 (2012) [arXiv:1203.2387].

[22] E. W. Kolb, D. Seckel and M. S. Turner, Nature 314, 415 (1985); H. M. Hodges, Phys. Rev. D 47, 456 (1993); Z. G. Berezhiani, A. D. Dolgov and R. N. Mohapatra, Phys. Lett. B 375, 26 (1996) [hep-ph/9511221]; L. Bento and Z. Berezhiani, Phys. Rev. Lett. 87, 231304 (2001) [hep-ph/0107281]; R. Foot and R. R. Volkas, Phys. Rev. D 68, 021304 (2003) [hep-ph/0304261]; Phys. Rev. D 69, 123510 (2004) [hep-ph/0402267].

[23] E. D. Carlson and S. L. Glashow, Phys. Lett. B 193, 168 (1987).

[24] P. Ciarcelluti and R. Foot, Phys. Lett. B 679, 278 (2009) [arXiv:0809.4438].

[25] R. Foot, Phys. Lett. B 711, 238 (2012) [arXiv:1111.6366].

[26] S. Bashinsky and U. Seljak, Phys. Rev. D 69, 083002 (2004) [astro-ph/0310198].

[27] Z. Hou, R. Keisler, L. Knox, M. Millea and C. Reichardt, arXiv:1104.2333; R. Bowen et al., Mon. Not. Roy. Astron. Soc. 334, 760 (2002) [astro-ph/0110636].

[28] U. Seljak and M. Zaldarriaga, Astrophys. J. 469, 437 (1996) [astro-ph/9603033].

[29] U. Seljak, Astrophys. J. 435, L87 (1994) [astro-ph/9406050].

[30] W. Hu and N. Sugiyama, Astrophys. J. 471, 542 (1996) [astro-ph/9510117].

[31] S. Dodelson, “Modern cosmology,” Amsterdam, Netherlands: Academic Pr. (2003) 440 p.

[32] http://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

[33] E. Lifshitz, J. Phys. (USSR) 10, 116 (1946).

[34] P. J. E. Peebles and J. T. Yu, Astrophys. J. 162, 815 (1970); J. R. Bond and A. S. Szalay, Astrophys. J. 274, 443 (1983); J. R. Bond and G. Efstathiou, Astrophys. J. 285, L45 (1984). N. Vittorio and J. Silk, Astrophys. J. 285, L39 (1984). S. Dodelson and J. M. Jubas, Astrophys. J. 439, 503 (1995) [astro-ph/9308019]; C. -P. Ma and E. Bertschinger, Astrophys. J. 455, 7 (1995) [astro-ph/9506072].

[35] P. J. E. Peebles, Astrophys. J. 153, 1 (1968).

[36] P. Ciarcelluti and R. Foot, Phys. Lett. B 690, 462 (2010) [arXiv:1003.0880].