NON-SINGULAR SOLUTIONS IN MULTIDIMENSIONAL COSMOLOGY WITH A PERFECT FLUID: ACCELERATION AND VARIATION OF G

V.D. Ivashchuk\textsuperscript{1,a,b}, S.A. Kononogov\textsuperscript{2,a}, V.N. Melnikov\textsuperscript{3,a,b} and M. Novello\textsuperscript{4,c}

\textsuperscript{a} Centre for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow 119361, Russia
\textsuperscript{b} Institute of Gravitation and Cosmology, Peoples’ Friendship University of Russia, 6 Miklukho-Maklaya St., Moscow 117198, Russia
\textsuperscript{c} ICRA-Brasil, CBPF, Rua Dr. Xavier Sigaud 150, Rio de Janeiro, Brazil

Exact solutions with an exponential behaviour of the scale factors are considered in a multidimensional cosmological model describing the dynamics of $n + 1$ Ricci-flat factor spaces $M_i$ in the presence of a one-component perfect fluid. The pressures in all spaces are proportional to the density: $p_i = w_i \rho, \ i = 0, \ldots, n$. Solutions with accelerated expansion of our 3-space $M_0$ and a small enough variation of the gravitational constant $G$ are found. These solutions exist for two branches of the parameter $w_0$. The first branch describes superstiff matter with $w_0 > 1$, the second one may contain phantom matter with $w_0 < -1$.

1. Introduction

Fundamental physical constants, relations between them and their possible variations are a reflection of the situation with unification [1,3,4,5,6].

Here we are mainly interested in the gravitational constant and its possible variations. The oldest problem is that of possible temporal variation of $G$, which arose due to papers by Milne (1935) and Dirac (1937). In Russia, these ideas were developed in the 60s and 70s by K.P. Staniukovich [7,5], who was the first to consider simultaneous variations of several fundamental constants.

Our first calculations based on general relativity with a perfect fluid and a conformal scalar field [8] gave $\dot{G}/G$ at the level of $10^{-11} - 10^{-13}$ per year. Our calculations in string-like [9] and multidimensional models with a perfect fluid [10] gave the level of $10^{-12}$, those based on a general class of scalar-tensor theories [13] and a simple multidimensional model with p-branes [12,14] gave for the present values of cosmological parameters $10^{-13} - 10^{-14}$ and $10^{-13}$ per year, respectively. Similar estimations were made by Miyazaki within Machian theories [15] giving for $\dot{G}/G$ the estimate $10^{-13}$ per year and by Fujii — on the level of $10^{-14} - 10^{-15}$ per year [16]. Analysis of one more multidimensional model with two curvatures in different factor spaces gave an estimate on the level of $10^{-12}$ [17]. Here we continue our studies of variation of $G$ in one more multidimensional cosmological model with perfect fluid.

2. The model

We consider a cosmological model describing the dynamics of $n$ Ricci-flat spaces in the presence of a one-component “perfect-fluid” matter [18]. The metric of the model

$$g = -\exp[2\gamma(t)]dt \otimes dt + \sum_{i=0}^{n} \exp[2x^i(t)]g^i$$

is defined on the manifold

$$M = \mathbb{R} \times M_0 \times \ldots \times M_n,$$

where $M_i$ with the metric $g^i$ is a Ricci-flat space of dimension $d_i, \ i = 0, \ldots, n; \ n \geq 2$. The multidimensional Hilbert-Einstein equations have the form

$$R^M_N - \frac{1}{2}\delta^M_N R = \kappa^2 T^M_N,$$

where $\kappa^2$ is the gravitational constant, and the energy-momentum tensor is adopted as

$$(T^M_N) = \text{diag}(\rho, p_0 \delta^{00}, \ldots, p_n \delta^{nn}),$$

describing, in general, an anisotropic fluid.

We assume the pressures of this “perfect” fluid in all spaces to be proportional to the density,

$$p_i(t) = (1 - u_i/d_i)\rho(t),$$

where $u_i = \text{const.}, \ i = 0, \ldots, n$. We also put $\rho \neq 0$.

We also impose the following restriction on the vector $u = (u_i) \in \mathbb{R}^{n+1}$:

$$\langle u, u \rangle_s \neq 0.$$ (6)

Here, the bilinear form $\langle \cdot, \cdot \rangle_s : \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \to \mathbb{R}$ is defined by the relation

$$\langle u, v \rangle_s = G^{ij} u_i v_j,$$

$u, v \in \mathbb{R}^{n+1}$, where

$$G^{ij} = \delta^{ij} + \frac{1}{2 - D}$$

are components of the matrix inverse to the matrix of the minisuperspace metric [22,23]

$$G_{ij} = d_i \delta_{ij} - d_i d_j.$$

(9)
In (5), \( D = 1 + \sum_{i=0}^{n} d_i \) is the total dimension of the manifold \( M \).

The restriction (6) reads

\[
\langle u, u \rangle_* = \sum_{i=0}^{n} \left( \frac{u_i}{d_i} \right)^2 + \frac{1}{2 - D} \left( \sum_{i=0}^{n} u_i \right)^2 \neq 0. \tag{10}
\]

3. Solutions with exponential scale factors

Here, we consider a special family of solutions with an exponential behaviour of the scale factors from [18] with the metric written in the synchronous time parametrization

\[
g = -dt \otimes dt + \sum_{i=0}^{n} a_i^2(t_s) g^i. \tag{11}
\]

Solutions with an exponential behaviour of the scale factors take place for

\[
\langle u^{(\Lambda)} - u, u \rangle_* = 0. \tag{12}
\]

Here and below the vector

\[
u_i^{(\Lambda)} = 2d_i \tag{13}
\]

corresponds to the \( \Lambda \)-term “fluid” with \( p_i = -\rho \) (vacuum-like matter).

In this case, the solutions are determined by the metric (11) with the scale factors

\[a_i = a_i(t_s) = A_i \exp(\nu^i t_s),\tag{14}\]

and the density \( \rho = \text{const}. \tag{15}\)

Here

\[
\nu^i = \varepsilon u^i \sqrt{-\frac{2\kappa^2 \rho}{\langle u, u \rangle_*}}, \tag{16}\]

where \( \varepsilon = \pm 1 \), \( u^i = G^{ij} u_j \) and \( A_i \) are positive constants, \( i = 0, \ldots, n \). Here \( \rho > 0 \) for \( \langle u, u \rangle_* < 0 \) and \( \rho < 0 \) for \( \langle u, u \rangle_* > 0 \).

The model under consideration was integrated in [18] for \( \langle u, u \rangle_* < 0 \). The solutions from [18] were generalized in [19] to the case when a massless minimally coupled scalar field was added. Families of exceptional solutions with power-law and exponential behaviours of the scale factors in terms of synchronous time were singled out in [19] and correspond to a constant value of the scalar field: \( \varphi = \text{const} \). When the scalar field is omitted, we are led to solutions presented in [4] and above for power-law and exponential cases, respectively (in [18] these solutions were originally written in the harmonic time parametrization). It may be verified that the exceptional solutions with an exponential dependence of the scale factors are also valid when the restriction \( \langle u, u \rangle_* < 0 \) is replaced by (6).

4. Acceleration and variation of \( G \)

In this section, the metric \( g^0 \) is assumed to be flat, and \( d_0 = 3 \). The subspace \((M_0, g^0)\) describes “our” 3-dimensional space and \((M_i, g^i)\) the internal factor spaces.

We are interested in solutions with accelerated expansion of our space and small enough variations of the gravitational constant obeying the present experimental constraints, see [12]:

\[|\dot{G}/(GH)|(t_{sc}) < 0.1, \tag{17}\]

where

\[H = \frac{\dot{a}_0}{a_0} \tag{18}\]

is the Hubble parameter. We suppose that the internal spaces are compact. Hence our 4-dimensional constant is (see [10])

\[G = \text{const} \cdot \prod_{i=1}^{n} (a_i^{-d_i}). \tag{19}\]

We will use the following explicit formulae for the contravariant components:

\[u^i = G^{ij} u_j = \frac{u_i}{d_i} + \frac{1}{2 - D} \sum_{j=0}^{n} u_j, \tag{20}\]

and the scalar product reads

\[
\langle u^{(\Lambda)} - u, u \rangle_* = -\sum_{i=0}^{n} \left( \frac{u_i}{d_i} \right)^2 - \frac{2}{D} \sum_{i=0}^{n} u_i + \frac{1}{D} \left( \sum_{i=0}^{n} u_i \right)^2 \tag{21}\]

4.1. Exponential expansion with acceleration

For solutions under consideration, an accelerated expansion of our space takes place for

\[\nu^0 > 0. \tag{22}\]

**Remark: \( \mathbf{D=4 \ case} \).** For \( D = 4 \), when the internal spaces are absent, we get \( u^0 = -u_0/6 \) and

\[
\langle u, u \rangle_* = -\frac{1}{6} u_0^2, \tag{23}\]

\[
\langle u^{(\Lambda)} - u, u \rangle_* = \frac{1}{6} (u_0 - 6) u_0 = 0, \tag{24}\]

which implies \( u_0 = 6 \), or, equivalently,

\[p = -\rho. \tag{25}\]

We get

\[\nu^0 = -\varepsilon \sqrt{\frac{\kappa^2 \rho}{3}}, \tag{26}\]

which agrees with the well-known result for \( D = 4 \): the de-Sitter solution with the cosmological constant \( \Lambda = \kappa^2 \rho > 0 \). The condition \( \nu^0 > 0 \) is equivalent to \( \varepsilon = -1 \).
For our exponential solutions we get
\[
\frac{\dot{G}}{G} = -\sum_{j=1}^{n} \nu^j d_i, \quad H = \frac{a_0}{a_0} = \nu^0,
\]
(27)
and hence
\[
\frac{\dot{G}}{(GH)} = -\frac{1}{\nu^0} \sum_{j=1}^{n} \nu^j d_i \equiv \delta.
\]
(28)
The constant parameter \( \delta \) describes variation of the gravitational constant and, according to (17),
\[|\delta| < 0.1.\]
(29)
It follows from the definition of \( \nu^j \) in (16) that
\[
\delta = -\frac{1}{u^0} \sum_{i=1}^{n} u^i d_i,
\]
(30)
or, in terms of covariant components (see (20)),
\[
\delta = -\frac{(D-4)u_0 - 2 \sum_{i=1}^{n} u_i}{3(5-D)u_0 + \sum_{i=1}^{n} u_i}.
\]
(31)
Thus the relations (21), (22), (29), (31) and the constraint (12) determine a set of parameters \( u_i \) compatible with the acceleration and tests on \( G \)-dot.

In what follows we will show that these relations do really determine a non-empty set of parameters \( u_i \) describing the equations of state.

4.1.1. The case of constant \( G \)

Consider the most important case \( \delta = 0 \), i.e., when the variation of \( G \) is absent: \( \dot{G} = 0 \).

Indeed, there is a tendency of lowering the upper bound on \( \dot{G} \). Moreover, according to arguments of (21), \( \delta < 10^{-4} \). This severe constraint just follows from the identity
\[
\dot{G}/G = \dot{\alpha}/\alpha
\]
(32)
that takes place in some multidimensional models. Here \( \alpha \) is the fine structure constant.

**Isotropic case.** We first consider the isotropic case when the pressures coincide in all internal spaces. This takes place when
\[
u_i = \nu d_i, \quad i = 1, \ldots, n.
\]
(33)
For pressures in internal spaces we get from (5)
\[
p_i = (1-v)\rho, \quad i = 1, \ldots, n.
\]
(34)
Then we get from (10) and (21)
\[
\langle u, u \rangle_s = \frac{1}{2-D} \left[ -\frac{1}{3}(d-1)u_0 + 2u_0 v - 2d v^2 \right],
\]
(35)
\[
\langle u^{(A)} - u, u \rangle_s = \frac{1}{2-D} \left[ 2u_0 + 2dv + \frac{1}{3}(d-1)u_0^2 - 2u_0 v + 2d v^2 \right].
\]
(36)
Here and in what follows we denote \( d = D - 4 \).

For \( \delta = 0 \), we get in the isotropic case
\[
v = u_0/2,
\]
(37)
or, in terms of pressures,
\[
p_i = (3p_0 - \rho)/2, \quad i = 1, \ldots, n.
\]
(38)
Substituting (37) into (35) and (36), we get
\[
\langle u, u \rangle_s = -\frac{u_0^2}{6},
\]
(39)
\[
\langle u^{(A)} - u, u \rangle_s = u_0(u_0 - 6)/6.
\]
(40)
Here, we obtain the same relations as for \( D = 4 \).

For our solution, we should put \( u_0 \neq 0 \) and hence, due to (12),
\[
u_0 = 6,
\]
(41)
i.e.,
\[
p_0 = -\rho, \quad p_i = -2\rho, \quad i > 0.
\]
(42)
Using (33) and (37), we get \( u^0 = -u_0/6 = -1 \) and \( u^i = 0 \) for \( i > 0 \), hence \( \nu_i = 0 \) for \( i = 1, \ldots, n \), i.e., all internal spaces are static.

The metric (11) reads in our case
\[
g = -dt_s \otimes dt_s + A_s^2 \exp(2\nu^0 t_s)g^0 + \sum_{i=1}^{n} A_i g^i.
\]
(43)
where \( A_i \) are positive constants, and
\[
\nu^0 = -\varepsilon \sqrt{\kappa^2 \rho/3}.
\]
(44)
For accelerated expansion we get \( \varepsilon = -1 \). We see that the power \( \nu^0 \) is the same as in case \( D = 4 \).

**Anisotropic case.** Consider the anisotropic (w.r.t. internal spaces) case with \( \delta = 0 \), or, equivalently (see (31)),
\[
(D-4)u_0 = 2 \sum_{i=1}^{n} u_i.
\]
(45)
This implies
\[
\langle u^{(A)} - u, u \rangle_s = \frac{1}{2}u_0(u_0 - 6) - \Delta,
\]
(46)
\[
\langle u, u \rangle_s = -\frac{1}{6}u_0^2 + \Delta,
\]
(47)
where
\[
\Delta = \sum_{i=1}^{n} \frac{u_i^2}{d_i} - \frac{1}{d} \left( \sum_{i=1}^{n} u_i \right)^2 \geq 0, \quad d = D - 4.
\]
(48)
The inequality in (48) can be readily proved using the well-known Cauchy-Schwarz inequality:
\[
\left( \sum_{i=1}^{n} b_i^2 \right) \left( \sum_{i=1}^{n} c_i^2 \right) \geq \left( \sum_{i=1}^{n} b_i c_i \right)^2.
\]
(49)
Indeed, substituting \( b_i = \sqrt{d_i} \) and \( c_i = u_i/\sqrt{d_i} \) into (49), we get (48). The equality in (49) takes place only
when the vectors \((b_i)\) and \((c_i)\) are linearly dependent, which for our choice reads: \(u_i/\sqrt{d_i} = v\sqrt{d_i}\) where \(v\) is constant. Thus \(\Delta = 0\) only in the isotropic case \((33)\).

In the anisotropic case we get \(\Delta > 0\).

In what follows we will use the relation

\[
\langle u^{(A)} - u, u \rangle_* = \frac{1}{6}(u_0 - u_0^*)(u_0 - u_0^+),
\]

where

\[
u_0^+ = 3 \pm \sqrt{9 + 6\Delta}
\]

are roots of the quadratic trinomial \((60)\) obeying

\[
u_0^+ < 0, \quad u_0^+ > 6 \quad \text{for} \quad \Delta > 0.
\]

It follows from \((65)\) that \(u^0 = -u_0/6\) and hence

\[
u^0 = -\frac{\varepsilon u_0}{6} \sqrt{\frac{12\kappa^2 \rho}{u_0^2 - 6\Delta}}.
\]

Here (see \((12)\))

\[
u_0 = u_0^\pm.
\]

Accelerated expansion of our space takes place when \(\nu^0 > 0\), or, equivalently, when either

- \((A)\) \(\nu_0 = u_0^\pm\), \(\varepsilon = 1\) or
- \((B)\) \(\nu_0 = u_0^\pm\), \(\varepsilon = -1\).

In terms of the parameter \(u_0\),

\[
p_0 = u_0 \rho, \quad \nu_0 = 1 - u_0/3,
\]

these two branches read:

- \((A)\) \(\nu_0 = u_0^\pm = \sqrt{1 + \frac{2}{3}\Delta}\),
- \((B)\) \(\nu_0 = u_0^\pm = -\sqrt{1 + \frac{2}{3}\Delta}\).

The first branch \((A)\) describes super-stiff matter \((u_0 > 1)\) with negative density \(\rho < 0\).

The second branch \((B)\) corresponds to matter with positive density (since \(\langle u, u \rangle_* < 0\)). This matter is phantom (i.e., \(u_0 < -1\)) when \(\Delta > 0\).

**4.1.2. The case of varying \(G\)**

Now we consider the case \(\delta \neq 0\), i.e., when \(G \neq 0\). In what follows, we use the observational bound \((29)\): \(|\delta| < 0.1\), stating the smallness of \(\delta\).

Using \((31)\), we get

\[
\sum_{i=1}^{n} u_i = \frac{1}{2}dbu_0,
\]

where \(d = D - 4\) and

\[
b = b(\delta) = \frac{1 + \delta(1 - \delta)/(3d)}{1 - \delta/2}.
\]

For the scalar product we get from \((60)\)

\[
\langle u^{(A)} - u, u \rangle_* = \frac{1}{6}Au_0^2 - Bu_0 - \Delta,
\]

\[
\langle u, u \rangle_* = -\frac{1}{6}Au_0^2 + \Delta,
\]

where \(\Delta\) was defined in \((18)\) (see \((10)\) and \((21)\)),

\[
\frac{A}{6} = \frac{1}{d + 2}\left((\frac{1 + \delta}{2})^2\right) - \frac{d}{4}b^2 - \frac{1}{3},
\]

\[
B = \frac{1}{d + 2}(2 + db).
\]

Using \((61)\), we obtain the explicit formulae

\[
A = A(\delta) = 1 - \frac{(d + 2)\delta^2}{12d(1 - \delta/2)^2},
\]

\[
B = B(\delta) = \frac{1 - \delta/3}{1 - \delta/2}.
\]

Due to \(|\delta| < 0.1\), \(A\) is positive, \(A > 0\), and close to unity: \(|A - 1| < \frac{1}{3}10^{-2}\).

For the contravariant component \(u^0\) we get from \((20)\) and \((60)\):

\[
u_0 = -Cu_0/6,
\]

where

\[
u_0 = -C\frac{u_0}{6} \left(1 + \frac{2}{3}\Delta\right).
\]

It follows from \((63)\) and \((68)\) that (see \((15)\))

\[
u_0 = -\frac{C}{6} \sqrt{\frac{12\kappa^2 \rho}{Au_0^2 - 6\Delta}}.
\]

Here (due to \((12)\))

\[
u_0 = u_0^\pm(\delta) = \frac{1}{A}(3B \pm \sqrt{9B^2 + 6A\Delta})
\]

are roots of the quadratic trinomial \((62)\).

**Isotropic case.** Let us consider the isotropic case \((33)\). Then we obtain from \((60)\)

\[
v = dbu_0/2,
\]

or, in terms of pressures

\[
p_i = \frac{1}{2}[3bp_0 + (2 - 3b)p], \quad i = 1, \ldots, n.
\]

For scalar products we get

\[
\langle u, u \rangle_* = -Au_0^2/6,
\]

\[
\langle u^{(A)} - u, u \rangle_* = au_0(Au_0 - 6B)/6.
\]

For our solution, we should put \(u_0 \neq 0\) and hence \(u_0 = 6B/A > 0\). The metric \((11)\) reads in our case

\[
g = -dt^2 + i^2 + \sum_{i=1}^{n} A_i^2g^i,
\]

where

\[
\sum_{i=1}^{n} b_i = 1/2du_0.
\]
where $A_i$ are positive constants,

$$
\nu^0 = -\frac{C u_0}{6} \sqrt{\frac{12 \kappa^2 \rho}{A u_0^2}}, \quad \text{and} \quad (77)
$$

$$
\nu = \nu^i = \varepsilon \frac{\delta u_0}{6d(1 - \delta/2)} \sqrt{\frac{12 \kappa^2 \rho}{A u_0^2}}, \quad (78)
$$

$i = 1, \ldots, n$. The last formula follows from (16) and

$$
u^i = \frac{u_0 \delta}{6d(1 - \delta/2)}. \quad (79)$$

We see that the power $\nu^0$ does not coincide, for $\delta \neq 0$, with that in case $D = 4$.

The accelerated expansion condition for our 3D space, $\nu^0 > 0$, reads in this case

$$
u_0 = \frac{6B(\delta)}{A(\delta)} \varepsilon = -1 \quad (80)
$$

or, equivalently, in terms of $u_0$ \((57)\) ($p_0 = w_0\rho$)

$$
w_0 = w_0^+(\delta) = 1 - \frac{2B(\delta)}{A(\delta)}. \quad (81)
$$

For $\delta > 0$, we get isotropic contraction of the whole internal space $M_1 \times \ldots \times M_n$. In this case, $w_0^+(\delta) < -1$, and hence phantom matter occurs with the equation of state close to the vacuum one since

$$
w_0^+(\delta) + 1 = -\frac{\delta(1 + \delta/d)}{3[1 - d + (d - 1)\delta^2/(6d)]}. \quad (82)
$$

For small $\delta$ we have $w_0^+(\delta) = -1 - \delta/3 + O(\delta^2)$.

For $\delta < 0$ we obtain isotropic expansion of the whole internal space. Then $w_0^+(\delta) > -1$, and phantom matter does not occur.

**Anisotropic case.** Now we consider the anisotropic case $\Delta > 0$ when $\delta \neq 0$. Here $u_0 = w_0^+(\delta)$, see (44).

Accelerated expansion of our 3-dimensional space takes place when $\nu^0 > 0$, or, equivalently, when either

(A) $u_0 = u_0^+(\delta), \quad \varepsilon = 1$ or

(B) $u_0 = u_0^+(\delta), \quad \varepsilon = -1. \quad (84)$

In terms of the parameter $w_0$ ($p_0 = w_0\rho$, $w_0 = 1 - u_0/3$) these two branches read

(A) $w_0 = w_0^+(\delta), \quad (85)$

(B) $w_0 = w_0^+(\delta), \quad (86)$

where

$$
w_0^+(\delta) = 1 - w_0^+(\delta)/3. \quad (87)
$$

For small $\delta$ we have

$$
w_0^+(\delta) = w_0^+(0) - \frac{\delta}{6} \left(1 \pm \frac{3}{\sqrt{9 + 6\Delta}}\right) + O(\delta^2). \quad (88)
$$

Thus for small $\delta$ the parameter $w_0$ has a small deviation from that obtained for $\delta = 0$. For small $\delta$, $w_0$ shifts by $O(\delta)$ term.

The first branch (A) describes super-stiff matter, $w_0 > 1$, since $w_0^+(\delta) > 1$ due to $u_0^+(\delta) < 0$. It may be shown that the density is negative in this case since $\langle u, u \rangle > 0$.

For branch (B) we find that

$$
w_0^+(\delta) < -1 \quad (89)
$$

only if

$$
\Delta > 6[A(\delta) - B(\delta)] = -\delta/(1 - \delta/2)^2. \quad (90)
$$

This is a condition on the appearance of “phantom” matter. For $\delta > 0$ this inequality is valid, but for $\delta < 0$ it is satisfied only for a big enough value of the anisotropy parameter $\Delta$, see (50).

5. Conclusions

We have considered multidimensional cosmological models describing the dynamics of $n + 1$ Ricci-flat factor spaces $M_i$ in the presence of a one-component anisotropic (perfect) fluid with pressures in all spaces proportional to the density: $p_i = w_i\rho$, $i = 0, \ldots, n$. Solutions with accelerated expansion of our 3-dimensional space $M_0$ and small enough variation of the gravitational constant $G$ were found. In the general non-isotropic case, these solutions exist for two branches of the parameter $w_0$: (A) from (85) and (B) from (86). Branch (A) describes super-stiff matter with $w_0 > 1$ while branch (B) may corresponds to phantom matter with $w_0 < -1$.

Acknowledgement

The work of V.D.I. and V.N.M. was supported in part by the DFG grant Nr. 436 RUS 113/807/0-1 and also by the Russian Foundation for Basic Research, grant Nr. 05-02-17478.

References

[1] V.N. Melnikov, "Gravity as a key problem of the millennium", in: Proc. of 2000 NASA/JPL Conference on Fundamental Physics in Microgravity, Solvang, CA, USA, 2000; NASA Document D-21522, 2001, pp. 4.1-4.17; gr-qc/0007067

[2] V.N. Melnikov, "Gravity and cosmology as key problems of the millennium", in: Proc. "Einstein Siecle" Conference in Paris, June 2005, Herman Publ., 2006.

[3] S.A. Kononogov and V.N. Melnikov, "Fundamental physical constants, gravitational coupling and space SEE Project", Izmer. Teknika 6, 3 (2005).

[4] J.M. Alio, V.D. Ivashchuk, S.A. Kononogov and V.N. Melnikov, Grav. & Cosmol. 12, No. 2-3, 173–178 (2006); gr-qc/0611015

[5] K.P. Staninovich and V.N. Melnikov, "Hydrodynamics, Fields and Constants in the Theory of Gravitation", Moscow, Energoatomizdat, 1983 (in Russian).
V.N. Melnikov, “Fields and Constants in Gravitation Theory”, CBPF-MO-02/02, Rio de Janeiro, 134 pp. 2002.

[6] V.N. Melnikov, in: “Gravitational Measurements, Fundamental Metrology and Constants”. Eds. V. de Sabbata and V.N. Melnikov, Dordrecht, Kluwer Academic Publ., 1988, p. 283.

[7] K.P. Staniukovich, “Gravitational Field and Elementary Particles”, Moscow, Nauka, 1966 (in Russian).

[8] N.A. Zaitsev and V.N. Melnikov, in: “Problems of Gravitation Theory and Particles Theory”, 10th issue, p. 131, Atomizdat, Moscow, 1979 (in Russian).

[9] V.D. Ivanchuk and V.N. Melnikov, Nuovo Cim. B 102, 131 (1988).

[10] K.A. Bronnikov, V.D. Ivanchuk and V.N. Melnikov, Nuovo Cim. B 102, 209 (1988).

[11] V.N. Melnikov, Int. J. Theor. Phys. 33, 1569 (1994).

[12] V.N. Melnikov and V.D. Ivanchuk, “Problems of G and multidimensional models”, In: Proc. JGRG11, Eds. J. Koga et al., Waseda Univ., Tokyo, 2002, pp. 405-409; gr-qc/0208012.

[13] K.A. Bronnikov, V.N. Melnikov and M. Novello, “Possible time variations of G in scalar-tensor theories of gravity”, Grav. Cosmol., 8, Suppl. II, 18-21 (2002).

[14] V.N. Melnikov, “Time Variations of G in Different Models”, Int. J. Mod. Phys. A 17, 4325 (2002).

[15] A. Miyazaki, “Varying cosmological constant and the Machian solution in the generalized scalar-tensor theory”, gr-qc/0103003.

[16] Y. Fujii, Astrophys. Space Sci. 283, 559 (2003).

[17] H. Dehnen, V.D. Ivanchuk, S.A. Kononogov and V.N. Melnikov, “On time variation of G in multidimensional models with two curvatures”, Grav. & Cosmol. 11, 340-344 (2005); gr-qc/0602108.

[18] V.D. Ivanchuk and V.N. Melnikov, "Multidimensional cosmology with m-component perfect fluid" Int. J. Mod. Phys. D 3, 795-811 (1994); gr-qc/9403063.

[19] V.D. Ivanchuk and V.N. Melnikov, “Multidimensional classical and quantum cosmology with perfect fluid”, Grav. & Cosmol. 1, 133-148 (1995); hep-th/9503223.

[20] V.R. Gavrilyov, V.D. Ivanchuk and V.N. Melnikov, “Integrable pseudo-Euclidean Toda-like systems in multidimensional cosmology with multicomponent perfect fluid”, J. Math. Phys. 36, 5829 (1995), gr-qc/9407019.

[21] V. Baukh and A. Zhuk, “Sp-brane accelerating cosmologies”, Phys. Rev. D 73, 104016 (2006); A.I. Zhuk, “Conventional cosmology from multidimensional models”, hep-th/0609126.

[22] V.D. Ivanchuk and V.N. Melnikov, “Perfect-fluid type solution in multidimensional cosmology”, Phys. Lett. 135A, 465 (1989).

[23] V.D. Ivanchuk, V.N. Melnikov and A.I. Zhuk, “On WDW equation in multidimensional cosmology”, Nuovo Cim. B 104, 575 (1989).

[24] V.D. Ivanchuk, “Multidimensional cosmology and Toda-like systems”, Phys. Lett. 170A, 16 (1992).