Superconducting-semiconductor quantum devices: from qubits to particle detectors

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Abstract—Recent improvements in materials growth and fabrication techniques may finally allow for superconducting semiconductors to realize their potential. Here we build on a recent proposal to construct superconducting devices such as wires, Josephson junctions, and qubits inside and out-of single crystal silicon or germanium. Using atomistic fabrication techniques such as STM hydrogen lithography, heavily-doped superconducting regions within a single crystal could be constructed. We describe the characteristic parameters of basic superconducting elements—a 1D wire and a tunneling Josephson junction—and estimate the values for boron-doped silicon. The epitaxial, single-crystal nature of these devices, along with the extreme flexibility in device design down to the single-atom scale, may enable lower-noise or new types of devices and physics. We consider applications for such super-silicon devices, showing that the state-of-the-art transmon qubit and the sought-after phase-slip qubit could both be realized. The latter qubit leverages the natural high kinetic inductance of these materials. Building on this, we explore how kinetic inductance based particle detectors (e.g., photon or phonon) could be realized with potential application in astronomy or nanomechanics. We discuss super-semi devices (such as in silicon, germanium, or diamond) which would not require atomistic fabrication approaches and could be realized today.

Index Terms—Quantum effect semiconductor devices, Semiconductor devices, Superconductor devices, Superconducting materials

I. INTRODUCTION

The emergence of superconductivity in heavily-doped semiconductors has long been predicted [1]. In recent years, group-IV semiconductors such as diamond [2], silicon [3], and germanium [4] have been successfully doped with acceptors reaching high enough hole density to turn superconducting. The more recent experiments in silicon are of significant interest as they show highly uniform, highly predictable superconductivity (versus density), while still maintaining the expitaxial nature of the crystal [5]. With respect to device physics, a recent proposal has raised the possibility of using novel material preparation and atomistic fabrication techniques [6], [7] to realize superconducting (SC) devices in purified silicon crystals [8].

Superconducting circuits are very versatile for various classical and quantum device applications, from sensors to computation. The Josephson junction offers a loss-less non-linearity of particular relevance to quantum information processing and technology [9]. Qubits based on advanced circuit designs such as the transmon [10] have reached remarkably long coherence times [11], [12] and continue to improve. To realize a practical quantum computer, still better gate fidelities are needed to implement quantum error correction toward fault-tolerant quantum computing. In present-day, heterogeneous superconducting devices (metals, insulators, substrates), device performance is limited by loss, often at the surfaces or interfaces [13]–[16]. Although it is unclear what the ultimate limits of loss in superconducting silicon might be, that devices could be constructed inside a single, purified crystal may offer improved performance someday.

Compared to conventional superconductor devices, super-silicon devices could be fabricated with atomic precision (though that is not required). Recently developed scanning tunneling microscope (STM) lithography [6], [7] has been used to create electronic devices down to the single dopant level [17]. This STM lithography technique (combined with atomic layer doping) has been used only for electron doping so far, reaching very high density of electrons, up to one in four crystal atoms replaced with a dopant [18]. There is high motivation to extend this technique for acceptor doping [8], [19].

In this paper, we theoretically investigate basic characteristics of super-silicon for device applications and propose a few possible applications. We consider two types of qubits, the now “standard” transmon [11] and the more elusive phase-slip flux qubit [20], which may be a natural fit for this system. Also of potential interest for device applications, we calculate the kinetic inductance (KI) of super-silicon, show that it is naturally high, and consider its use for KI particle detectors, of potential interest to astronomy (photon, phonon) and nanomechanics applications. We note that there are device geometries that won’t require precision doping, which may enable other materials—most intriguingly superconducting diamond or whole wafers of super-silicon—to be utilized for experiments today.

II. SUPERCONDUCTIVITY CHARACTERISTICS

In this section, we refine the estimation of SC characteristic parameters from [8]. We start with only experimental data and assumptions that the holes can be described by free electron like energy dispersion and that it is a Bardeen-Cooper-Schrieffer (BCS) [21] superconductor in dirty local limit. The effective mass of hole bands can be obtained from numerical simulations or from experiments, but the usually cited values are for low density cases. In super-silicon, the hole density is much higher (≥ 10^{21} cm^{-3}) and the effective mass significantly overestimates the Fermi energy and it leads to
some inconsistencies in estimating SC parameters. So we don’t use the low density effective mass and derive the effective mass from experimental data regarding superconductivity. The highest observed critical temperature for boron-doped silicon (Si:B) is $T_c=0.6K$ for hole density $n_h=4\times 10^{21}cm^{-3}$ [23]. The upper critical field at zero temperature $B_{c2}(0)=0.1T$ and the normal resistivity was measured to be $\rho_n=100\mu \Omega cm$.

Since we are mostly interested in low temperature quantum applications ($T \ll T_c$), zero temperature parameters will be calculated. The Ginzburg-Landau (GL) coherence length is obtained from $B_{c2}(T)=\Phi_0/2\pi \xi_{GL}^2(T)$ where $\Phi_0=h/2e$ is the SC flux quantum,

$$\xi_{GL}(0) = \sqrt{\frac{\Phi_0}{2\pi B_{c2}(0)}} = 57nm. \quad (1)$$

The effective penetration depth $\lambda$ can be calculated from $\mu_0 \lambda^2(0)=\hbar \rho_n/\pi \Delta_0$ where $\Delta_0$ is the SC energy gap at zero temperature, which is estimated to be $\Delta_0=1.76k_B T_c=91\mu eV$,

$$\lambda(0) = \sqrt{\frac{\hbar \rho_n}{\mu_0 \pi \Delta_0}} = 1.35\mu m. \quad (2)$$

The GL parameter $\kappa_{GL}=\lambda(0)/\xi_{GL}(0)=23.6$, which is consistent with the observed type II superconductivity in super-silicon. The mean free path $l_m$ is obtained from an expression that can be derived from $l_m=\nu F$ and $1/\rho_n=\nu F\hbar^2/\pi m_h$ where $\nu F$ is the Fermi velocity of holes, $\tau$ is the relaxation time, and $m_h$ is the hole effective mass;

$$l_m = (3\pi^2)^{1/3} \frac{\hbar}{e^2 \rho_n \hbar^2/\pi} = 5.04nm. \quad (3)$$

The BCS coherence length $\xi_0$ and London penetration depth $\lambda_L(0)$ are obtained from $\xi_{GL}(0)/\xi_0=\pi \lambda_L(0)/2\sqrt{3}/\lambda(0)$ and $\lambda(0)\lambda_L(0)\xi_0/l_m^{1/2}$,

$$\xi_0 = \frac{12 \pi^2 \xi_{GL}(0)}{l_m} = 793nm, \quad (4)$$

$$\lambda_L(0) = \lambda(0) \left(\frac{l_m}{\xi_0}\right)^{1/2} = 108nm. \quad (5)$$

The effective mass $m_h$ is now calculated from $\xi_0=\hbar \nu F/\pi \Delta_0$ and $\nu F=\hbar k_F/m_h=\hbar^2/(3\pi^2 n_h)^{1/3}/m_h$;

$$m_h = \frac{\hbar^2}{\pi \Delta_0 \xi_0}^{1/3} = 1.65m_e. \quad (6)$$

This effective mass is about three times larger than the low density heavy hole band effective mass. It gives a Fermi energy of $\epsilon_F=0.557eV$, which is more consistent with the Fermi energy from simulations for a high hole density [23]. The thermodynamic critical field is

$$B_c(0) = B_{c2}(0)/\sqrt{2\kappa_{GL}} = 3mT. \quad (7)$$

The estimated mean free path $l_m$ is much less than $\xi_{GL}(0)$ and $\lambda(0)$, which is consistent with the assumption of super-silicon being in the local limit.

III. TUNNELING JOSEPHSON JUNCTION

Now we define a few characteristic parameters for the tunneling JJ and estimate their values for the Si:B device. All estimations will be for the hole density $n_h=4.0\times 10^{21}cm^{-3}$ and the SC parameters determined in the previous section. A JJ is typically characterized by a capacitance $C_J$ and an inductance $L_J$. The capacitance is simply the geometric capacitance given by

$$C_J = \frac{\varepsilon_\varepsilon_0 A}{d}. \quad (8)$$

where the dielectric constant $\varepsilon_r \approx 12$ for silicon. The JJ inductance $L_J$ is given by

$$L_J = \frac{\hbar}{2e I_c} = \frac{\Phi_0}{2\pi I_c}, \quad (9)$$

where $I_c$ is the JJ critical current given by [24],

$$I_c = \frac{\pi \Delta(T)}{2e R_n} \tanh \left[ \frac{\Delta(T)}{2k_B T} \right]. \quad (10)$$

$R_n$ is the normal resistance, which is the inverse of the tunneling conductance $G_n$. $G_n$ is proportional to the area $A$ and can be obtained by

$$G_n = \frac{m_h \epsilon_F^2}{2\pi^2 \hbar^2} \int d\varepsilon \Delta(T) \csc \theta \right] \left[ \frac{\Delta(T)}{2k_B T} \right]. \quad (11)$$

where $T(\varepsilon)$ is the transmission coefficient of a square barrier of height $V_b$ and width $d$,

$$T(\varepsilon) = \frac{4\varepsilon \Delta(\varepsilon)}{4\varepsilon \Delta(\varepsilon) + \sqrt{\Delta^2(\varepsilon) - 4\varepsilon^2 \Delta(\varepsilon)}} \csc \theta \right] \left[ \frac{\Delta(T)}{2k_B T} \right]. \quad (12)$$

A crude estimate for the barrier height of the undoped region is $V_b=\varepsilon_F + E_g/2$. Figure 3a shows the tunneling resistance as a function of the distance $d$, for $m_h=1.65m_e$ and $V_b=\varepsilon_F + E_g/2$. The tunneling resistance increases exponentially with increasing distance $d$ and the barrier height $V_b$. The barrier distance $d$ is controlled in the crystal growth process. On the other hand, the barrier height $V_b$ can be tuned by, e.g., additional gates such as doped regions with hole density less than the critical density for the SC transition (normal metal) as was depicted in Fig. 3b), connected to outside voltages.
The two characteristic energies of the JJ are

\[ E_C = \frac{(2e)^2}{2C_f} = 3.02 [\text{eV} \cdot \text{nm}] \frac{d}{A}, \quad (13) \]

\[ E_J = \frac{h I_c}{2e} = \frac{\pi h}{4 e^2} R_n \tanh \left( \frac{\Delta(T)}{2 k_B T} \right), \quad (14) \]

where the last line is for zero temperature.

Another important parameter is the JJ plasma frequency which is defined as

\[ \omega_p = \sqrt{\frac{2e^2}{\hbar}} J_c = \sqrt{\frac{2e}{\hbar} \frac{d}{\varepsilon_r \varepsilon_0}} J_c = 2.02 \times 10^{15} [\text{Hz} \cdot \Omega^{1/2} \cdot \text{nm}^{1/2}], \quad (16) \]

where \( J_c = I_c / A \) is the critical current density.

As an example for the tunneling JJ, we take \( A = 1 \mu m^2 \), \( d = 2 \text{nm} \), \( V_b = 1.988 eV \). Then \( C_f = 53.13 \text{ fF} \), \( L_2 = 3.54 \mu H \), \( J_c = 93.1 \text{ A/m}^2 \), \( E_C = 6.03 \mu eV \), \( E_J = 0.219 \mu eV \), and \( \omega_p = 2.31 \text{GHz} \). By changing the geometry (\( A \) and \( d \)) and the barrier \( V_b \), we can tune these parameters to suit various applications.

IV. KINETIC INDUCTANCE OF A SUPERCONDUCTING WIRE

Superconducting wires can be used in many parts of a circuit. They can be a lossless connecting wire between different elements, an inductor, a resonator, or even a phase slip junction. One of the potentially very useful properties of the SC semiconductors is their high kinetic inductance. Consider a SC wire with length \( l \), width \( w \), and depth \( d \), as depicted in Fig. 1a. The kinetic energy due to the supercurrent, from GL description, is

\[ E_K = \frac{1}{2} m^* n^*_c d w l = \frac{1}{2} L_K I_c^2, \quad (17) \]

where the supercurrent \( I_c = n^*_c (2e) v_s w d \). Here \( m^* \), \( n^*_c \), and \( v_s \) are effective mass, density, and velocity of a Cooper pair, from GL picture. The kinetic inductance \( L_K \) is \([25]\)

\[ L_K = \frac{m^* l}{4 n^*_c e^2 w d} = \mu_0 \lambda^2 \frac{l}{w d}, \quad (18) \]

where \( \lambda = \sqrt{m^*/(4 \mu_0 e^2 n^*_c)} \) is the effective penetration depth. An alternative way of finding an expression for the kinetic inductance based on BCS \([21]\) and Mattis-Bardeen \([25]\) theories is from the complex conductivity \( \sigma = \sigma_1 - i \sigma_2 \) of the superconductor, by equating \( \sigma_2 (w d / l) \) with \( 1 / \omega L_K \). In the low frequency limit,

\[ L_K = \frac{\hbar}{\pi w d} \frac{\rho_n}{\Delta(T) \tanh \left( \frac{\Delta(T)}{2 k_B T} \right)}. \quad (19) \]

This expression is valid for all range of temperature. At zero temperature,

\[ L_K(T = 0) = \frac{\hbar}{\pi w d} \frac{\rho_n}{\Delta_0} \approx 0.18 \frac{\hbar R_n}{k_B T_c}. \quad (20) \]

and near the critical temperature \( T \ll T_c \), it reduces to the GL result. \([18]\). We characterize the low temperature kinetic inductance of various SC materials, by comparing the material parameter \( \rho_n / \pi \Delta_0 \) (see Table I). The actual kinetic inductance will be determined by the geometry of the wire. The critical temperature \( T_c \) and the resistivity \( \rho_n \) are from the references cited and \( \Delta_0 \) was calculated using BCS relation \( \Delta_0 = 1.76 k_B T_c \). Kinetic inductance \( L_K \) is proportional to \( \rho_n / \pi \Delta_0 \), from \([20]\), and we calculated this geometry-independent material parameter for different SC materials. Super-silicon has a kinetic inductance comparable to the most promising SC materials for applications requiring high kinetic inductance. Combined with the potential for much narrower quantum wire through STM lithography, super-silicon could be very useful for phase slip junctions and kinetic inductance detector devices. We will give a few examples in the following sections. In Table I we also estimate the kinetic inductance for hole-doped superconducting diamond and germanium, which are also of promise for various device applications.

V. APPLICATIONS FOR QUANTUM COMPUTATION

A. Transmon Qubit

The transmon qubit \([11, 33]\) is probably the most promising qubit implementation using SC circuits so far. It is a type of Cooper pair box (CPB) charge qubit designed to be insensitive to the charge noise which is the main cause of decoherence in the charge qubit. The Hamiltonian is

\[ H_{\text{transmon}} = E_C (\hat{n} - n_q) - E_J \cos \phi, \quad (21) \]

where \( \hat{n} \) and \( \phi \) are the Cooper pair number operator and the phase of the SC order parameter, respectively. \( n_q \) is the effective offset charge that can be tuned by applying external conditions.
voltage $V_g$ to the coupling capacitor $C_g$ [see Fig. 3(b)]. By tuning the parameters such that $E_J \gg E_C$ by increasing the total capacitance with a large shunting capacitor, the qubit levels become essentially flat as functions of the effective gate charge $n_g$ making the qubit states insensitive to the charge fluctuations in $n_g$. One consequence of making $E_J$ much larger than $E_C$ is that the anharmonicity of the energy levels is reduced. So there is a compromise and typically $E_J \approx 10$ is chosen, where the anharmonicity is still about 10%, large enough to allow qubit operations. This is accomplished by increasing the capacitance with large superconducting metals attached to the JJ. The qubit states energy difference approaches the JJ plasma frequency $\hbar \omega_p \approx \sqrt{2E_CE_J}$. The qubit is coupled to a resonator and the strong coupling regime, where the qubit-resonator coupling is larger than the decay rates, could be achieved in this circuit QED architecture taking advantage of the small mode volume of the quasi-1D transmission line resonator and/or the large dipole matrix elements between qubit states $\{12\}$, $\{35\}$. This strong coupling with resonating modes is utilized for gate operations and measurements.

Now we estimate a geometry suitable for a transmon. If we don’t want to use a shunting capacitor for the JJ as is typical, and instead use a “parallel-plate” design as is possible with our epitaxial approach, then for a given target frequency $\omega_p$, we can get required $d/R_n A$ from $\{16\}$. Then from the relation between $R_nA$ and $d$ [see Fig. 3(a)], we can read the necessary $d$ and $R_nA$. Since $\omega_d$ does not depend on the junction area $A$, we can directly find the necessary junction barrier distance $d$, as in Fig. 3(c). In addition to the target frequency, we want to tune $E_J$ and $E_C$ to be in the transmon regime. This determines optimal junction area. For example, for $\omega_p/2\pi=5$GHz and $E_J/E_C \approx 10$, the necessary geometry for the JJ is $d \approx 1.46$nm and $A=0.95 \mu m^2$, so a large junction area is needed which comes from the requirement of large capacitance. If we use a shunting capacitance $C_B$ [see Fig. 3(b)], from target frequency $\omega_p$ and $E_J/E_C$, we obtain necessary values for $E_C$ and $E_J$, which can be tuned separately using the shunting capacitance. For $\omega_p/2\pi=5$GHz and $E_J/E_C \approx 10$, we need $E_J=10E_C=46.2$µeV. To allow a smaller $A$, we need smaller $d$. If we choose $d=1.2$nm, we need $A=6.07 \times 10^4 \mu m^2$. Then $C_J \approx 5.38$µF and we need a shunting capacitance $C_B \approx 63.9$µF, so we would need a shunt capacitor much larger than the JJ. Note that the shunting capacitance is not a simple parallel plate capacitor, but it should take into account the entire capacitance matrix of the device.

In principle, the super-silicon JJ can be used to make a transmon qubit with similar design and geometry as the Al-based JJ. However, one must note that here the JJ critical current will be smaller than the Al-based JJ’s, so physically it can’t be any smaller for the same target frequency.

### B. Phase Slip Flux Qubit

The high kinetic inductance and the possibility of making a very narrow wire makes the super-silicon a good candidate for the implementation of the phase slip qubit. In very narrow SC nanowires, fluctuations of the SC order parameter can be significant. Of particular interest here is when the amplitude of the SC order parameter reduces to near zero in a region of size $\xi_{GL}$ allowing the phase to change by $2 \pi$. This phase slip can be considered as a macroscopic tunneling in the parameter space between two SC states with phase difference $\Delta \phi=2 \pi$. This is responsible for the wide tail of resistance in SC transition in thin wires well below the critical temperature $\{36\}$, $\{37\}$. At very low temperatures, this macroscopic tunneling can still occur due to quantum fluctuations $\{38\}$–$\{42\}$. This quantum phase slip (QPS) is dual to the Cooper pair tunneling through a JJ $\{43\}$, $\{44\}$, and devices dual to JJ devices were demonstrated $\{45\}$–$\{48\}$. A new type of qubit $\{20\}$, called the phase slip (PS) flux qubit, was proposed in a SC loop with a QPS junction, where coherent QPS connects two states with different fluxoid quantum numbers. It is exactly dual to the charge qubit in CPB geometry, and the coherent superposition of qubit states was experimentally demonstrated $\{49\}$, $\{50\}$. Full qubit operations have not yet been demonstrated.

A schematic diagram of a PS flux qubit is in Fig. 4 (a). The thin section of length $l$ and cross section area $A$ enables the coupling with different flux states via quantum phase slip. The remaining loop is much thicker and has a length of $l'$ and a cross section area $A'$. (b) Phase slip rate $\Gamma_{QPS}$ in the phase slip junction as a function of $\rho_0/A$ for a superconducting silicon device as described in the text, assuming $a=b=1$ in $\{22\}$. From bottom to top, the length $l=100$, 200, 300nm. (c) Energy spectrum for a loop with $l'=1000nm$, $A'=1000nm^2$, and $A=64nm^2$. The black solid curves are for $l=100nm$ and the red dashed ones are for $l=200nm$. If we choose $d=1.2$nm, we need $A=6.07 \times 10^4 \mu m^2$. Then $C_J \approx 5.38$µF and we need a shunting capacitance $C_B \approx 63.9$µF, so we would need a shunt capacitor much larger than the JJ. Note that the shunting capacitance is not a simple parallel plate capacitor, but it should take into account the entire capacitance matrix of the device.

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larger cross section areas, longer junctions would be needed to get the same phase slip rate.

The Hamiltonian for the PS qubit is

\[
H_{\text{QPS}} = E_L (f - N)^2 + \frac{E_s}{2} |N+1 \rangle \langle N+1| + |N \rangle \langle N|, \tag{23}
\]

where \(N\) is the fluxoid quantum number, and \(E_L = \Phi_0^2 / 2L_K^\text{tot}\), and \(E_s/2 = h \Gamma_{\text{QPS}}\). \(L_K^\text{tot}\) is the total kinetic inductance:

\[
L_K^\text{tot} = L_K + L'_K = \frac{\hbar}{\pi \Delta_0} \left( \frac{l}{A} + \frac{l'}{A'} \right). \tag{24}
\]

The kinetic inductance is typically a few nH, while the geometric inductance is in pH range for the geometries considered. So we neglected geometric inductance here. The energy spectrum of the Hamiltonian in (23) around a half-integer external flux is given in Fig. 4(c), for two different lengths \(l=100\) and \(200\) nm.

Flux is the relevant variable for the PS qubit, and the flux can be measured by a SQUID coupled to the phase qubit loop. However, near the optimal value of the external flux \(f=1/2\) where the qubit states are insensitive to the flux noise up to the first order, the two eigenstates are linear combination of the two different flux states and can’t be measured by directly measuring the flux. Instead we can use a resonator such as a lumped element LC resonator or a transmission line coupled to the qubit and perform the dispersive readout [51], [52]. As can be seen from Fig. 4(c), for longer \(l\), phase slip energy \(E_s\) increases while inductance energy \(E_L\) decreases, which leads to flatter energy spectrum near the operation point \(f=1/2\). This is analogous to the charge qubit moving towards the transmon regime. Due to the exact duality with the charge qubit by a transformation [43],

\[
(\hat{q}, \hat{\phi}) \leftrightarrow (-\hat{\phi} / 2\pi, 2\pi \hat{q}), E_s \leftrightarrow E_l, E_L \leftrightarrow E_C, \tag{25}
\]

the transmon regime \(E_l / E_C \gg 1\) corresponds to \(E_s / E_L \gg 1\). This regime can be achieved, e.g. by increasing the total inductance while keeping \(\Gamma_{\text{QPS}}\) constant.

PS qubit has a few advantages over conventional SC qubits. A flux qubit implemented with a simple rf-SQUID loop needs very high loop and JJ inductances to form a well defined bound states, but this will suppress the tunneling between different flux states. Therefore, a widely adopted design for a JJ flux qubit is a SC loop with three JJs [53], [54]. PS qubit does not have this problem, and a simple loop with a single PS junction is enough. The excited states have a much higher energy than the qubit states compared to the transmon qubit, so much faster gate operations are possible without exciting the qubit out of the qubit subspace (leakage). It is insensitive to the charge noise since there is no SC island, and the phase slip rate is determined by the geometry of the thin wire on the scale of the coherence length, making it insensitive to microscopic fluctuations of individual atoms [20]. A main source of decoherence could be the flux noise, but we expect it could be less of a problem for the PS qubit embedded in a pure silicon crystal. Also, we can increase the total inductance to move into the transmon-like regime to suppress the flux noise, at the cost of reduced anharmonicity. In the context of super-silicon devices, the simple washer-like geometry eases fabrication requirements and the need for a precisely controlled tunnel gap.

VI. APPLICATIONS FOR KI DETECTORS

Finally we consider the microwave kinetic inductance particle detector (MKID) devices [55], [56]. If a photon (or phonon) with energy larger than the SC gap is absorbed into a SC resonator, it breaks Cooper pairs and creates quasiparticles. This increases the residual resistance and the kinetic inductance and therefore changes the resonator quality factor and the resonance frequency. This change can be detected by measuring the transmitted signal through a nearby transmission line capacitively coupled to the resonator. A schematic circuit diagram for an MKID detector is shown in Fig. 5(c). This scheme allows easy frequency division multiplexing and it can be deployed in a large array. MKID is currently being developed for a large range of frequencies, from IR to X-ray photons, and especially for astronomy applications.

The coplanar waveguide (CPW) [see Fig. 5(a)] has become the preferred choice for the resonator design [56]. Given the properties of super-silicon, we will consider a CPW resonator in the thin film and local limit (\(w \ll \lambda\)). The thin film CPW resonator of length \(l\) has a surface impedance \(Z_s = R_s + iX_s\) where \(R_s\) is the surface resistance due to the thermally excited or photon-created quasiparticles, and \(X_s = \omega L_s\) with

\[
\Gamma_{\text{QPS}} = 1.5a \frac{\xi_G}{\xi_L(0)} \sqrt{\frac{R_Q}{R_s}} \frac{bT_c}{\hbar} \exp \left( -0.3b \frac{R_Q}{R_s} \right), \tag{22}
\]

where \(a, b\) are fitting constants of order unity and \(R_s = \rho_s \xi_G / A\) is the normal resistance for a wire of length \(\xi_G\). In Fig. 4(b), the phase slip rate is plotted as a function of \(\rho_s / A\) for different lengths, \(l=100\), 200, 300 nm. The target range of the rate would be of the order of a few GHz. For a phase slip junction with \(l=100\) nm, we need \(A \simeq 64\)nm\(^2\) for 10GHz. For larger cross section areas, longer junctions would be needed
the surface inductance $L_s$ being related to the kinetic inductance $L_k=L_s/w$. The resonator has an internal quality factor $Q_i$, and the coupling quality factor $Q_c$ due to the coupling to the transmission line. The total resonator Q-factor $Q_t$ is $1/Q_t=1/Q_i+1/Q_c$. $Q_c$ can be tuned by changing the coupling capacitance $C_c$. The resonator has distributed geometrical capacitance $C'$, geometrical inductance $L'$, and resistance $R'$. Geometrical capacitance and inductance of a CPW resonator can be calculated using analytical expressions derived by conformal mapping [57]. The total inductance is the sum of geometrical and kinetic inductances. For a short-ended quarter-wave resonator, the change in the transmission coefficient near the resonance frequency is given by [57]

$$\delta f_{21}|_{f=f_i} = \frac{\delta Q^2}{Q_c} \left[ \delta \left( \frac{1}{Q_i} \right) - 2i \frac{\delta f_c}{f_c} \right] \approx \alpha \frac{\delta Q^2}{Q_c} \frac{\delta Z_s}{|Z_s|}, \quad (26)$$

where $\alpha$ is the fraction of kinetic inductance and $f_c=1/\sqrt{L'/C'}$ is the resonance frequency of the CPW resonator. $Q_c^2/Q_e$ is maximized if $Q_c$ is set to be equal to $Q_i$, $\alpha$ is larger for smaller resonators, i.e. thinner and narrower CPW resonator has a higher fraction of kinetic inductance.

We calculated the shift in the resonance frequency and the responsivity of the MKID for super-silicon. Fig. 6 shows the change of resonance frequency [[a]] and the phase angle of the transmission coefficient [[b]] as quasi-particles are created by incident photons. The efficiency of the super-silicon MKID is expected to be quite good, compared to other metallic superconductors currently used in MKID devices [29], [30], [58], [59]. We assumed a internal quality factor of $Q_i=10^5$ which was achieved in other materials [58]. Due to the available semiconductor technologies for preparing very ideal, chemically purified, and even isotopically enriched semiconductor crystals, we expect even higher $Q_i$ could be possible. The STM lithography technique could enable much smaller devices than current MKID devices, which could also enhance the sensitivity of the detector.

The historical path of MKID devices has progressed toward CPW structures as they are easier to fabricate, requiring only a single layer of SC metal, and have shown very high $Q$'s, especially in the last decade. However, one of the originally considered geometries, the microstrip resonator [shown in Fig. 6(b)], may be of interest in the context of SC semiconductor devices. It consists of a ground plane at the bottom, a substrate separation layer with height $h$, and a transmission line resonator on top. Although more complicated, the theoretical kinetic inductance fraction can approach unity in such structures while simultaneously having a small active volume (though losses in the dielectric separation layer or the thickness of the separation layer have limited performance in the past [56]). In the super-semi approach, the separation layer could be fully epitaxial and quite thin. And, practically, the bottom ground plane does not require nanoscale precision (STM lithography): the SC region can be doped by other 2D methods, such as gas immersion laser doping (GILD) [53].

If we need larger SC energy gap to detect higher energy photons, boron-doped diamond might be suitable. It has much higher critical temperature ($\sim 7$K) and larger normal resistivity than boron-doped silicon (see Table I). The kinetic inductance is smaller but comparable. SC diamond is also a type II BCS superconductor in the dirty limit like super-silicon, and clean single crystal SC diamond epilayers have been successfully grown and the vortex lattice structure was observed [60].

Most interestingly, the super-semi approach allows one to tune the superconducting gap, $\Delta=1.76 \ k_B T_c$, by changing the density of holes in the superconducting region since the critical temperature $T_c \propto (n_B/n_c - 1)^{1/2}$ as experimentally observed for C:B [60], Si:B [22], and Ge:Ga [33] where $n_B$ is the doped acceptor density and $n_c$ is the critical acceptor density for SC transition. This way, the device can be directly engineered for specific target particle energies or for quasiparticle trapping or movement.

These type of detectors are also being used in the search for weakly interacting massive particles (WIMPs) which have been proposed to comprise dark matter [61]. The Cryogenic Dark Matter Search (CDMS) detectors use bulk silicon or germanium as a target to absorb WIMPs with MKID phonon detectors covering the top [62], [63]. MKIDs can be tuned to detect the phonons created by absorbed WIMPs [64], [65]. Super-silicon or germanium seem to be an attractive material for this since the MKID detector and the target are made of the same material and the SC energy gap can be easily tuned by tuning the hole density in the SC regions.

Phonon detectors may also have direct relevance to the rapidly growing field of opto-/nanomechanics [66]–[69]. It’s interesting to estimate the possibility of use of super-semi MKIDs for acoustic phonon detection. Here, one would want the $T_c$ sufficiently high to limit the thermal noise due to thermal activation of quasi-particles, while having $T_c$ low enough to allow for acoustic phonons to create quasiparticles that are then detected. Taking as an example phonons of 10 GHz that one would wish to detect in a silicon device, one would want to engineer $T_c \approx 0.12$ K such that $2\Delta_0 \lesssim 10$ GHz. This would correspond to a hole density of roughly $2 \times 10^{21}$ cm$^{-3}$. Signal noise to sensitivity would improve with increasing phonon energy.

VII. Conclusions

We have derived the characteristic superconducting parameters for heavily boron-doped silicon as well as relevant JJ
parameters for precision-doped super-silicon devices. Applications of this system for qubits (the transmon qubit and the phase-slip qubit) and for MKID particle detectors have been proposed, and possible device designs were suggested. STM hydrogen lithography-enabled acceptor doping is not yet realized, but it appears that there is no fundamental reason against its implementation in the near future. Once realized, but it appears that there is no fundamental reason against its implementation in the near future. Once realized, realizations of this system for qubits (the transmon qubit and singlet qubit) via the gas immersion laser doping technique [3]. Single electron-doped semiconductors [1], [23], but there has been no experimental evidence yet [70].

If atomic-layer doping is available, but precision hydrogen lithography is not, it should be possible to make a parallel-plate transmon device of Section V.A above by growing two superconducting layers epitaxially separated by a distance d, then etching devices of area A for a given target frequency using standard lithographic techniques. Although in this case there are clearly interfaces near the device, the electric field could be localized in the epitaxial region with possibly improved device performance. This double layer device might also be created via the gas immersion laser doping technique [3]. Single layer superconducting devices such as the phase-slip qubit are simpler, requiring only a single superconducting region followed by patterned etching. We have estimated the kinetic inductance of these materials and have shown how they could be used as particle detectors, perhaps in the very near term. The true potential of superconducting semiconductor devices, as discussed only specifically here, lies in the possibility of completely new device designs and of a testbed for new science due to the flexibility of the bottom-up fabrication and the huge physics-space of SC circuits and systems.

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