Learning Multiple Tasks with Deep Relationship Networks

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Abstract

Deep networks trained on large-scale data can learn transferable features to promote learning multiple tasks. As deep features eventually transition from general to specific along deep networks, a fundamental problem is how to exploit the relationship across different tasks and improve the feature transferability in the task-specific layers. In this paper, we propose Deep Relationship Networks (DRN) that discover the task relationship based on novel tensor normal priors over the parameter tensors of multiple task-specific layers in deep convolutional networks. By jointly learning transferable features and task relationships, DRN is able to alleviate the dilemma of negative-transfer in the feature layers and under-transfer in the classifier layer. Extensive experiments show that DRN yields state-of-the-art results on standard multi-task learning benchmarks.

1 Introduction

Supervised learners trained with limited labeled samples are prone to overfitting, while manual labeling of sufficient training data for new domains is prohibitive. Thus it is imperative to design algorithms for reducing the labeling cost, typically by leveraging off-the-shelf labeled data from relevant tasks. Multi-task learning is based on the idea that the performance of one task can be improved using related tasks as inductive bias [Caruana, 1997]. Knowing the task relationship should enable the transfer of shared knowledge from relevant tasks such that only task-specific features need to be learned. This fundamental idea has motivated a variety of methods, including multi-task feature learning that learns a shared feature representation [Ando and Zhang, 2005, Argyriou et al., 2008, Chen et al., 2011] Chen et al., 2013, Hernández-Lobato et al., 2015], and multi-task relationship learning that models the inherent task relationship [Evgeniou and Pontil, 2004, Jacob et al., 2009, Zhang and Schneider, 2010, Zhang and Yeung, 2010, Kang et al., 2011, Kumar and Daume III, 2012, Ciliberto et al., 2015].

Learning inherent task relatedness is a hard problem, since the training data of different tasks may be sampled from different distributions and fitted by different models. Without prior knowledge on the task relatedness, the distribution shift may pose a major difficulty in transferring knowledge across different tasks. Unfortunately, if cross-task knowledge transfer is impossible, then we will overfit each task due to limited amount of labeled data. One way to circumvent this dilemma is to use an external data source, e.g. ImageNet, to learn transferable features through which the shift in the inductive biases can be reduced such that different tasks can be correlated more effectively. This idea has motivated some latest deep learning methods for learning multiple tasks [Srivastava and Salakhutdinov, 2013, Ouyang et al., 2014, Zhang et al., 2014, Chu et al., 2015], which learn a shared representation in the feature layers and multiple independent classifiers in the classifier layer but without inferring the task relationships. This may result in under-transfer in the classifier layer as knowledge can not be transferred across different classifiers. The literature’s latest findings reveal that deep features eventually transition from general to specific along the network, and feature transferability drops significantly in higher layers with increasing task discrepancy [Yosinski et al., 2014], hence the sharing of all feature layers may be risky to negative-transfer. Therefore, it remains an open problem how to exploit the task relationship across different tasks while accounting for the feature transferability in task-specific layers of the network.

This paper presents a Deep Relationship Network (DRN) architecture for learning multiple tasks, which discovers the inherent task relationship based on multiple task-specific layers of deep convolutional neural networks. For the first time, the tensor normal distribution [Ohlson et al., 2013] is explored for multi-task learning, which is imposed as the prior distribution over network parameters of all task-specific layers to learn the task relationship. By jointly learning the transferable features and task relationships, DRN is able to circumvent the dilemma of negative-transfer in the feature layers and under-transfer in the classifier layer. Since deep models pre-trained with large-scale repositories such as ImageNet are representative for general-purpose perception tasks [Girshick et al., 2014, Yosinski et al., 2014, Hoffman et al., 2014], the DRN model is trained by fine-tuning from the AlexNet model pre-trained on ImageNet [Krizhevsky et al., 2012]. Empirical study shows that DRN learns reasonable task relationships and yields state-of-the-art performance on standard datasets.

2 Related Work

Multi-task learning (MTL) is a learning paradigm that learns multiple tasks jointly by exploiting the shared structures to
improve generalization [Caruana, 1997] and mitigate manual labeling consumption. There are generally two categories of approaches: (1) multi-task feature learning, which learns a shared feature representation such that the distribution shift across different tasks can be reduced [Ando and Zhang, 2005; Argyriou et al., 2008; Chen et al., 2011; Chen et al., 2013; Hernandez-Lobato et al., 2015]; and (2) multi-task relationship learning, which explicitly models the task relationship in the forms of task grouping [Jacob et al., 2009; Kang et al., 2011; Kumar and Daume III, 2012] or task covariance [Evgeniou and Pontil, 2004; Zhang and Schneider, 2010; Zhang and Yeung, 2010; Chiberto et al., 2015]. While these methods have achieved state-of-the-art performance, they may be restricted by their shallow learning paradigm, which cannot suppress task-specific variations to embody the task relationships.

Deep networks learn abstract representations that disentangle and hide explanatory factors of variation behind data [Bengio et al., 2013; Krizhevsky et al., 2012]. Deep representations manifest invariant factors underlying different populations and are transferable across similar tasks [Yosinski et al., 2014]. Thus deep networks are explored for domain adaptations and are transferable across similar tasks [Yosinski et al., 2014; Chu et al., 2011; Collobert et al., 2011] and multi-task learning [Srivastava and Salakhutdinov, 2013; Ouyang et al., 2014; Zhang et al., 2014; Chu et al., 2015], where significant performance gains have been witnessed. Most multi-task deep learning methods [Collobert et al., 2011; Ouyang et al., 2014; Zhang et al., 2014; Chu et al., 2015] learn a shared representation in the feature layers and multiple independent classifiers in the classifier layer without inferring the task relationships. However, this may result in under-transfer in the classifier layer as knowledge cannot be adaptively propagated across different classifiers, while the sharing of all feature layers may still be vulnerable to negative-transfer in the feature layers, as the higher layers of deep networks are tailored to fit task-specific structures and may not be safely transferable [Yosinski et al., 2014]. This paper presents a deep relationship network based on novel tensor normal priors to learn transferable features and task relationships that mitigate both under-transfer and negative-transfer.

3 Tensor Normal Distribution

3.1 Probability Density Function

Tensor normal distribution is a natural extension of multivariate normal distribution and matrix-variate normal distribution [Gupta and Nagar, 2000] to tensor-variate distributions. The multivariate normal distribution is order-1 tensor normal distribution, and matrix normal distribution is order-2 tensor normal distribution. Before defining tensor normal distribution, we first introduce the notations and operations of order-K tensor. An order-K tensor is an element of the tensor product of K vector spaces, each of which has its own coordinate system. A vector \( \mathbf{x} \in \mathbb{R}^{d_1} \) is an order-1 tensor with dimension \( d_1 \). A matrix \( \mathbf{X} \in \mathbb{R}^{d_1 \times d_2} \) is an order-2 tensor with dimension \( (d_1, d_2) \). An order-K tensor \( \mathbf{X} \in \mathbb{R}^{d_1 \times \cdots \times d_K} \) with dimensions \( (d_1, \ldots, d_K) \) has elements \( x_{i_1, \ldots, i_K} : i_k = 1, \ldots, d_k \). The vectorization of \( \mathbf{X} \) is unfolding the tensor into a vector, denoted by \( \text{vec}(\mathbf{X}) \). The matricization of \( \mathbf{X} \) is a generalization of vectorization, reordering the elements of \( \mathbf{X} \) into a matrix. In this paper, we use the mode-\( n \) matricization and denote by \( \mathbf{X}_{(n)} \) the mode-\( n \) matrix of tensor \( \mathbf{X} \), where row \( i \) of \( \mathbf{X}_{(n)} \) contains all elements of \( \mathbf{X} \) that have the \( n \)-th index equal to \( i \).

Consider an order-K tensor \( \mathbf{X} \in \mathbb{R}^{d_1 \times \cdots \times d_K} \). Since we can vectorize \( \mathbf{X} \) to a \( (\prod_{k=1}^{K} d_k) \times 1 \) vector, the normal distribution on a tensor \( \mathbf{X} \) can be considered as a multivariate normal distribution on vector \( \text{vec}(\mathbf{X}) \) of \( \prod_{k=1}^{K} d_k \) dimensions. However, such an ordinary multivariate distribution ignores the special structure of \( \mathbf{X} \) as a \( d_1 \times \cdots \times d_K \) tensor, and as a result, the covariance characterizing the elements of \( \mathbf{X} \) is of size \( (\prod_{k=1}^{K} d_k) \times (\prod_{k=1}^{K} d_k) \), which is usually prohibitive for modeling and estimation. To exploit the structure of \( \mathbf{X} \), tensor normal distributions assume that the \( (\prod_{k=1}^{K} d_k) \times (\prod_{k=1}^{K} d_k) \) covariance matrix \( \Sigma_{1:K} \) can be decomposed as the Kronecker product \( \Sigma_{1:K} = \Sigma_1 \otimes \cdots \otimes \Sigma_K \), and elements of \( \mathbf{X} \) follow

\[
\text{vec}(\mathbf{X}) \sim \mathcal{N}(\text{vec}(\mathcal{M}), \Sigma_1 \otimes \cdots \otimes \Sigma_K),
\]

where \( \Sigma_k \in \mathbb{R}^{d_k \times d_k} \) is a positive definite matrix indicating the covariance between the \( d_k \) rows of mode-\( k \) matricization \( \mathbf{X}_{(k)} \). \( \Sigma \) is the Kronecker product, and \( \mathcal{M} \) is a mean tensor of the same size as \( \mathbf{X} \) containing the expectation of each element of \( \mathbf{X} \). Due to the decomposition of covariance as the Kronecker product, the tensor normal distribution of an order-\( K \) tensor \( \mathbf{X} \), parameterized by mean tensor \( \mathcal{M} \) and covariance matrices \( \Sigma_1, \ldots, \Sigma_K \), has a density [Ohlson et al., 2013] of

\[
p(\mathbf{x}) = (2\pi)^{-d/2} \prod_{k=1}^{K} \Sigma_k^{-d/2} \exp\left( -\frac{1}{2} (\mathbf{x} - \mu)^T \Sigma_k^{-1} (\mathbf{x} - \mu) \right),
\]

where \( |\cdot| \) is the determinant of a square matrix, and \( \Sigma_{1:K} = \Sigma_1 \otimes \cdots \otimes \Sigma_K, \mathbf{x} = \text{vec}(\mathbf{X}), \mu = \text{vec}(\mathcal{M}), d = \prod_{k=1}^{K} d_k \). The tensor normal distribution corresponds to the multivariate normal distribution with Kronecker decomposable covariance tensor. \( \mathbf{X} \) following tensor normal distribution is denoted by

\[
\mathbf{X} \sim \mathcal{N}_{d_1 \times \cdots \times d_K}(\mathcal{M}, \Sigma_1, \ldots, \Sigma_K).
\]

3.2 Maximum Likelihood Estimation

Consider a set of \( n \) samples \( \{\mathbf{X}_i\}_n \) where each \( \mathbf{X}_i \) is an order-3 tensor generated by a tensor normal distribution as \( \mathcal{N} \). The maximum likelihood estimation (MLE) of mean tensor \( \mathcal{M} \) is

\[
\hat{\mathcal{M}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{X}_i.
\]

The MLE of covariance matrices \( \hat{\Sigma}_1, \ldots, \hat{\Sigma}_3 \) are computed by iteratively solving following system [Ohlson et al., 2013]

\[
\hat{\Sigma}_1 = \frac{1}{nd_1d_2} \sum_{i=1}^{n} (\mathbf{X}_i - \mathcal{M})_1 (\hat{\Sigma}_3 \otimes \hat{\Sigma}_2)^{-1} (\mathbf{X}_i - \mathcal{M})^T_1,
\]

\[
\hat{\Sigma}_2 = \frac{1}{nd_3d_4} \sum_{i=1}^{n} (\mathbf{X}_i - \mathcal{M})_2 (\hat{\Sigma}_3 \otimes \hat{\Sigma}_1)^{-1} (\mathbf{X}_i - \mathcal{M})^T_2,
\]

\[
\hat{\Sigma}_3 = \frac{1}{nd_5d_6} \sum_{i=1}^{n} (\mathbf{X}_i - \mathcal{M})_3 (\hat{\Sigma}_2 \otimes \hat{\Sigma}_1)^{-1} (\mathbf{X}_i - \mathcal{M})^T_3.
\]
This flip-flop algorithm is efficient to solve until convergence. Covariance matrices $\Sigma_1, \ldots, \Sigma_3$ are not identifiable and the solutions to maximizing the density are not unique, while only the Kronecker product $\Sigma_1 \otimes \cdots \otimes \Sigma_K$ is identifiable.

4 Deep Relationship Networks

In this work, we model multiple tasks by jointly learning transferable features and task relationships. Given $T$ tasks with training data $\{X_t, Y_t\}_{t=1}^T$, where $X_t = \{x_{t1}, \ldots, x_{tn_t}\}$ and $Y_t = \{y_{t1}, \ldots, y_{tn_t}\}$ are the $N_t$ training examples and associated labels of the $t$-th task, respectively drawn from $D$-dimensional feature space and $C$-cardinality label space, i.e. each training example $x_{tn_t} \in \mathbb{R}^D$ and $y_{tn_t} \in \{1, \ldots, C\}$. Our goal is to construct a deep neural network for multiple tasks $y_{tn_t} = f_t(x_{tn_t})$ to learn transferable features and adaptive task relationships to bridge different tasks effectively and robustly.

4.1 Model

We start with the deep convolutional neural network (CNN) [Krizhevsky et al., 2012], a strong model to learn transferable features that are well adaptive to multiple tasks [Zhang et al., 2014, Hoffmann et al., 2014]. The main challenge is that in multi-task learning, each task is provided with a limited amount of labeled data, which is insufficient to build reliable classifiers without overfitting. In this sense, it is vital to model the task relationships through which each pair of tasks can help with each other if they are related, and can remain independent if they are unrelated to mitigate negative transfer. With this philosophy, we design a Deep Relationship Network (DRN) that exploits both feature transferability and task relationship to establish robust multi-task learning. Figure 1 shows the architecture of the proposed DRN model.

Our model is built upon AlexNet [Krizhevsky et al., 2012], which has convolutional layers (conv1–conv5) and fully connected layers (fc6–fc8). The $t$-th fc layer learns a nonlinear mapping $h_{t,\ell} = a^\ell(W_t \cdot h_{t,\ell-1} + b_{t,\ell})$ for task $t$, where $h_{t,\ell}^n$ is the hidden representation of $x_{tn_t}$, $W_t \cdot \ell$ and $b_{t,\ell}$ are the weight and bias parameters, and $a^\ell$ is the activation function, taken as Rectifier Linear Units (ReLU) $a^\ell(x) = \max(0, x)$ for hidden layers or softmax units $a^\ell(x) = e^x / \sum_{j=1}^{\mid x \mid} e^{x_j}$ for the output layer. Denote by $y_t = f_t(x)$ the CNN classifier of the $t$-th task, then the empirical error of CNN on $\{X_t, Y_t\}$ is

$$J = \min_{f_t} \sum_{n=1}^{N_t} J (f_t(x_{tn_t}), y_{tn_t}),$$

where $J$ is the cross-entropy loss function, and $f_t(x_{tn_t})$ is the conditional probability that the CNN assigns $x_{tn_t}$ to label $y_{tn_t}$. We will not describe how to compute the convolutional layers since these layers can learn transferable features [Yosinski et al., 2014] and we will simply share the network parameters of these layers across different tasks, without explicitly modeling the task relationships in these layers. To benefit from pre-training and fine-tuning, we copy these layers from a model pre-trained from ImageNet 2012 [Yosinski et al., 2014, Jia et al., 2014], and fine-tune all the conv1–conv5 layers.

As revealed by the latest literature findings [Yosinski et al., 2014], the deep features in standard CNNs must eventually transition from general to specific along the network, and the feature transferability decreases while the task discrepancy increases, making the features in higher layers $f_{t\ell}$ unsafely transferable across different tasks. In other words, the $f_{t}$ layers are tailored to their original task at the expense of degraded performance on the target task, which may deteriorate multi-task learning based on deep neural networks. Most previous methods generally assume that the multiple tasks can be well correlated given the shared representation learned by the feature layers $\text{conv}1$–$\text{fc}7$ of the deep neural network [Collobert et al., 2011, Srivastava and Salakhutdinov, 2013, Ouyang et al., 2014, Zhang et al., 2014]. However, it may be vulnerable if different tasks are not well correlated in the deep features, which is common as higher layers are not safely transferable and tasks may be dissimilar. Moreover, existing multi-task learning methods are natively designed for binary-class tasks. It remains an open problem to explore task relationships for multi-class classification in multi-task learning. In this work, we jointly learn transferable features and task relationship for multiple task-specific layers $L$ in a Bayesian framework. In AlexNet, the features in the last feature layer $f_{t\ell}$ is 4096-dimension, which is relatively high for multi-task relationship learning. Hence we attach a bottleneck layer $f_{t\ell}'$ of 256-units between the last feature layer $f_{t\ell}$ and the classifier layer $f_{c8}$ to reduce the feature dimension to 256. Then the task-specific layers $L$ are set to $\{f_{t\ell}', f_{c8}\}$. Denote by $X = \{X_t\}_{t=1}^T, Y = \{Y_t\}_{t=1}^T$ the training data of $T$ tasks, by $W_{t,\ell} \in \mathbb{R}^{D_{\ell}' \times D_{\ell}'}$ the network parameter of the $t$-th task in the $\ell$-th layer, where $D_{\ell}'$ and $D_{\ell}''$ are the rows and columns of matrix $W_{t,\ell}$. In order to capture the task relationship in the network parameters of all $T$ tasks, we construct the $\ell$-th layer parameter tensor as $W_{\ell} = [W_{1,\ell} \ldots ; W_{T,\ell}] \in \mathbb{R}^{D_{\ell}' \times D_{\ell}'' \times T}$. Denote by $W = \{W_{t,\ell} : \ell \in L\}$ the set of parameter tensors of all the task-specific layers $L = \{f_{t\ell}', f_{c8}\}$. The Maximum a Posteriori (MAP) estimation of network parameters $W$ given training data $\{X, Y\}$ to learn multiple tasks is formulated as

$$p(W | X, Y) = p(W) \cdot p(Y | X, W) = \prod_{\ell \in L} p(W_{\ell}) \prod_{t=1}^{T} \prod_{n=1}^{N_t} p(y_{tn_t} | x_{tn_t}, W), \quad (7)$$

where we assume that for prior $p(W)$, the parameter tensor of each layer $W_{\ell}$ is independent on the parameter tensors of the other layers $W_{\ell' \neq \ell}$, which is a common assumption made by most neural network methods [Bengio et al., 2013]. Finally, we assume when the network parameter is sampled from the prior, all tasks are independent. These independence assumptions lead to the factorization of the posterior in Equation (7).

The maximum likelihood estimation (MLE) part in Equation (7) is modeled by deep CNN in Equation (6), which can learn transferable features in lower layers for multi-task learning. We opt to share the network parameters of all these layers ($\text{conv}1$–$\text{fc}7$). This parameter sharing strategy is a relaxation of existing methods [Ouyang et al., 2014, Zhang et al., 2014, Chu et al., 2015], which share all the feature layers except for the classifier layer. We do not share task-specific layers (the bottleneck feature layer $f_{t\ell}'$ and classifier layer $f_{c8}$) so as to mitigate negative-transfer potentially [Yosinski et al., 2014].
The prior part in Equation (7) is crucial since it should be able to model the task relationship across parameter tensors. This paper defines the prior for the $l$-th layer parameter tensor based on tensor normal distribution \cite{Ohlson et al., 2013} as

$$p(W_l) = TN_{D_{l1} \times D_{l2} \times T} \left( \mathbf{0}, D_{l1}, D_{l2}, \Sigma_l \right),$$

where $\Sigma_l \in \mathbb{R}^{T \times T}$ is the mode-3 covariance matrix. Specifically, in the tensor normal prior, the row covariance matrix $I_{D_{l1}}$ models the relationships between features (feature covariance), the column covariance matrix $I_{D_{l2}}$ models the relationships between classes (class covariance), and the mode-3 covariance matrix $\Sigma_l$ models the relationships between tasks in the $l$-th layer network parameters $\{W_{1,l}, \ldots, W_{T,l}\}$. By using identity covariance for feature covariance and class covariance, we assume independent features and classes implicitly, which is a common assumption made by multi-class classifiers. All model parameters, including the task covariance $\Sigma_l$, are learned from data to build adaptive task relationships.

Integrating CNN loss (6) and tensor normal prior (8) into MAP estimation (7) and taking negative logarithm, we obtain the MAP estimate of $W$ by solving optimization problem as

$$\min_{f_t, \Sigma_l \in \mathcal{L}} \sum_{t=1}^{T} \sum_{i=1}^{N_t} J(f_t(x_i^{t}), y_i^{t}) + \frac{1}{2} \sum_{t \in \mathcal{L}} \text{tr}(W_{t}(\Sigma_l^{-1}W_{t}(\Sigma_l^{-1}))) + \frac{1}{2} \text{tr}(\Sigma_l),$$

where $\Sigma_{1:3,l} = I_{D_{l1}} \otimes I_{D_{l2}} \otimes \Sigma_l$ is the Kronecker product of the feature covariance, class covariance, and task covariance. Moreover, we can assume shared task relationship across different layers as $\Sigma_l = \Sigma$, which enhances the connection between task relationships on features $f_{l}^{T}$ and classifiers $f_{c}$.8.

### 4.2 Algorithm

The optimization problem (7) involves Kronecker product, which is hard to solve. We reduce the Kronecker product by

$$w^T \Sigma_{1:K}^{-1} w = w^T (I_{1:K-1} \otimes \Sigma_K^{-1}) w$$

$$= w^T \text{vec}(\Sigma_K^{-1}W_{l}(K)I_{1:K-1})$$

$$= \text{tr} \left( W_{l}(K)^T \Sigma_K^{-1}W_{l}(K) \right),$$

where $w = \text{vec}(W_l)$. Plugging this in problem (8), we obtain

$$\min_{f_t, \Sigma_l \in \mathcal{L}} \sum_{t=1}^{T} \sum_{i=1}^{N_t} J(f_t(x_i^{t}), y_i^{t}) + \frac{1}{2} \sum_{t \in \mathcal{L}} \text{tr}(W_{t}(\Sigma_l^{-1}W_{t}(\Sigma_l^{-1}))) + \frac{1}{2} \text{tr}(\Sigma_l).$$

where $W_{l}(\Sigma_l)$ is the mode-3 matricization of parameter tensor $W_l$. As optimization problem (9) is jointly non-convex with respect to the parameter tensors $W$ and task covariances $\Sigma_l$, we alternatively optimize one set of variables with the others fixed. We first update $W_{l,t}$, the parameter of task-$t$ in layer-$l$. When training deep CNN by back-propagation, we only need the gradient of (12) w.r.t $W_{l,t}$ on each data point $(x_i^{t}, y_i^{t})$ as

$$\frac{\partial O(x_i^{t}, y_i^{t})}{\partial W_{l,t}} = \frac{\partial J(f_t(x_i^{t}), y_i^{t})}{\partial W_{l,t}} + \text{mat} \left( \Sigma_l^{-1}W_{l}(\Sigma_l)_{t,*} \right),$$

where $A_{t,*}$ is the $t$-th row of matrix $A$, and mat[a] is folding a vector $a$ into its corresponding matrix using tensor folding. Since training a deep CNN requires a large amount of labeled data, which is prohibitive for many multi-task learning problems, we fine-tune from an AlexNet model pre-trained on ImageNet 2012 as \cite{Yosinski et al., 2014}. In each epoch, with updated $W$, we can update $\Sigma_l$ by the flip-flop algorithm as

$$\Sigma_l = \frac{W_{l}(\Sigma_l^{-1}W_{l}(\Sigma_l))}{D_l D_l^T} + \varepsilon I_T, \Sigma_l = \frac{\text{tr}(\Sigma_l)}{T},$$

where $\varepsilon$ is a small penalty and the second equation normalizes the covariance matrix, both improving the numerical stability.

Our algorithm scales linearly to sample size $N = \sum_{t=1}^{T} N_t$. For each iteration, the cost of all convolutional layers is same as standard CNNs, and the cost of all fully connected layers is $O(\sum_{t \in \mathcal{L}} N(D_l D_l + T D_l D_l^T))$. Finally, the cost for updating task relationship matrices is $O(\sum_{l \in \mathcal{L}} (T^2 D_l D_l^T + T^3))$.

### 4.3 Discussion

Our work contrasts from prior relationship learning \cite{Zhang and Schneider, 2010, Zhang and Yeung, 2010, Hernández-Lobato et al., 2015} and multi-task deep learning \cite{Zhang et al., 2014, Chu et al., 2015} methods in two key aspects. (1) **Tensor normal prior**: Our work is the first
to explore tensor normal distribution as priors of network parameters in different layers to learn task relationships in deep networks. Since the network parameters of multiple tasks natively stack into an order-3 tensor (will be order-4 tensor if convolutional layers are considered), previous matrix normal distribution [Gupta and Nagar, 2000] cannot be used as priors of network parameters to learn task relationships. (2) Deep task relationship. We define tensor normal prior on multiple task-specific layers \( L = \{ f_{c7'}, f_{c8} \} \). Prior deep learning methods do not learn task relationships and assume that the shared deep features are transferable for multi-task learning, which may not be true [Yosinski et al., 2014]. Prior relationship learning methods are not designed in deep net, which are not able to learn transferable features for multi-task learning.

5 Experiments

5.1 Setup

Office-Caltech [Saenko et al., 2010; Gong et al., 2012] This dataset is the standard benchmark for multi-task learning and transfer learning. The Office part [Saenko et al., 2010] consists of 4,652 images in 31 categories collected from three distinct domains (tasks): Amazon (A), which contains images downloaded from amazon.com, Webcam (W) and DSLR (D), which are images taken by Web camera and digital SLR camera under different environmental variations. This dataset is organized by selecting the 10 common categories shared by the Office dataset and the Caltech-256 (C) dataset [Gong et al., 2012], hence it has four multi-class domains (tasks).

ImageCLEF-DA [1] This dataset is the benchmark for ImageCLEF domain adaptation challenge, organized by selecting the 12 common categories shared by the following four public datasets (tasks): Caltech-256 (C), ImageNet ILSVRC 2012 (I), Pascal VOC 2012 (P), and Bing (B). Both datasets are evaluated using DeCAF; [Donahue et al., 2014] features for shallow methods and original images for deep methods.

We compare DRN with standard and state-of-the-art methods: Single-Task Learning (STL), Multi-Task Feature Learning (MTFL) [Argyriou et al., 2008], Multi-Task-Relationship Learning (MTFL) [Zhang and Schneider, 2010], Robust Multi-Task Learning (RMTL) [Chen et al., 2013], and Multi-Task Deep Convolutional Neural Network (MTCNN) [Zhang et al., 2014]. STL is performed separately on each task. MTFL learns the low-rank shared feature covariance. RMTL extends MTFL to capture the task relationships using a low-rank structure and identify the outlier tasks using a group-sparse structure. MTRL captures the task relationships using task covariance matrix of a matrix normal distribution. MTCNN is originally applied to heterogeneous tasks (e.g. face landmark detection and head pose estimation) by sharing all feature layers and learning multiple classifiers, while we apply it to multiple homogeneous multi-class tasks.

To study the efficacy of learning transferable features and task relationships jointly, we evaluate two DRN variants: (1) DRN using only one network layer, i.e. \( f_{c8} \) for relationship learning, termed DRN\(_5\); (2) DRN using one-vs-rest binary classifier for relationship learning, termed DRN\(_{bi} \). While using one-vs-rest binary classifier for multi-class problems is often less natural and less effective, all comparison methods follow this paradigm, possibly because it is not easy to extend these methods to multi-class setting by matrix normal priors, where parameters involving multi-class are order-3 tensors.

We follow the evaluation protocol [Zhang and Schneider, 2010; Chen et al., 2013] for multi-task learning and randomly select 5%, 10%, and 20% samples from each task as training set and use the rest of the samples as test set. We compare the average classification accuracy for all tasks based on five random experiments, where standard errors are insignificant and are not reported. We conduct model selection for all

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Table 1: Multi-class classification accuracy on the Office-Caltech dataset with standard evaluation protocol.

| Method | STL | MTFL | RMTL | MTCNN | DRN
|-------|-----|------|------|-------|-----
| A     | B   | C    | D    | Avg   |
|       |     |      |      |       |
| A     | B   | C    | D    | Avg   |
|       |     |      |      |       |
| 5%    | 88.9| 73.0 | 80.4 | 88.7  | 82.2 |
| 10%   | 82.0| 88.9 | 87.2 | 86.3  | 90.1 |
|        |     |      |      |       |       |
| 20%   | 91.3| 83.3 | 93.7 | 94.9  | 90.8 |

Table 2: Multi-class classification accuracy on the ImageCLEF-DA dataset with standard evaluation protocol.

| Method | STL | MTFL | RMTL | MTCNN | DRN
|-------|-----|------|------|-------|-----
| C     | I   | P    | B    | Avg   |
|       |     |      |      |       |
| C     | I   | P    | B    | Avg   |
|       |     |      |      |       |
| 5%    | 77.4| 60.3 | 48.0 | 53.4  | 57.7 |
| 10%   | 78.9| 70.5 | 48.1 | 41.8  | 59.8 |
|        |     |      |      |       |       |
| 20%   | 83.3| 74.9 | 49.2 | 47.1  | 63.6 |

1 ImageCLEF-DA: 
http://imageclef.org/2014/adaptation

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[1] ImageCLEF-DA: 
http://imageclef.org/2014/adaptation
methods using five-fold cross-validation on the training set. For CNN-based methods, we adopt AlexNet [Krizhevsky et al., 2012], fine-tune conv1–conv5 and fully connected layers fc6–fc7, train bottleneck layer fc7’ and classifier layer fc8, via back propagation. As the bottleneck and classifier layers are trained from scratch, we set their learning rate to be 10 times that of the lower layers. We use mini-batch stochastic gradient descent (SGD) with 0.9 momentum and the learning rate strategy in Caffe [Jia et al., 2014], and select the learning rate between 10^{-5} and 10^{-2} by multiplicative stepsize \sqrt{10}.  

### 5.2 Results and Discussion

The multi-task classification results on the Office-Caltech and ImageCLEF-DA datasets based on 5%, 10%, and 20% sampled training data are shown in Tables 1 and 2 respectively. We can observe that the proposed DRN model significantly outperforms the comparison methods on most multi-task problems. The substantial performance improvement clearly validates that our deep relationship networks via multi-layer and multi-class relationship learning is able to learn both transferable features and adaptive task relationships. DRN enables more effective and robust multi-task deep learning.

We can make the following observations from the results. (1) Shallow multi-learning methods MTFL, RMTL, and MTRL significantly outperform single-task learning method STL, which confirms the efficacy of learning multiple tasks by exploiting shared structures. Among the shallow multi-task methods, MTRL gives the best accuracies, showing that exploiting task relationship may be more effective than extracting shared feature subspace for multi-task learning. (2) Deep multi-task learning method MTCNN either under-performs (Office-Caltech) or outperforms (ImageCLEF-DA) shallow multi-task learning methods with deep features as input, which confirms the importance of learning deep transferable features to enable knowledge transfer across tasks. However, MTCNN assumes shared network parameters for all feature layers (conv1–fc7) and learns the classifier layer fc8 independently without inducing the task relationship. This may result in negative transfer in the feature layers [Yosinski et al., 2014] and under-transfer in the classifier layers.

We go deeper into DRN by reporting the results of the two DRN variants: DRN_{50} and DRN_{8}, all significantly outperform the comparison methods but generally underperform DRN, which verify our motivation that jointly learning transferable features and adaptive task relationships can bridge multiple tasks more effectively. (1) The disadvantage of DRN_{50} is that it does not learn the task relationship in the lower layers fc7’, which are not safely transferable and may result in negative transfer [Yosinski et al., 2014]. (2) The shortcoming of DRN_{8} is that it does not learn the task relationship based on multiple classes, hence the learned class-wise task relationship may be inaccurate when labeled data is very limited, however, its performance will catch up with DRN when the class-wise training data grows large. The proposed DRN model addresses all these issues and gives the best accuracies.

### 5.3 Visualization Analysis

We show that DRN can learn more reasonable task relationships with deep features than MTRL with shallow features, by visualizing the task covariance matrices \( \Sigma \) learned by MTRL and DRN in Figures 2(a) and 2(b) respectively. Prior knowledge on task similarity in the Office-Caltech dataset [Saenko et al., 2010] describes that tasks A, W and D are more similar with each other while they are significantly dissimilar to task C. DRN successfully captures this prior task relationship and enhances the task correlation across dissimilar tasks, which enables stronger transferability for multi-task learning. Furthermore, all tasks are positively correlated (green color) in DRN, implying that all tasks can better reinforce each other. However, some of the tasks (D and C) are still negatively correlated (red color) in MTRL, implying these tasks should be drawn far apart and cannot improve with each other.

We illustrate the feature transferability by visualizing in Figures 2(c) and 2(d) the t-SNE embeddings [Jia et al., 2014] [Long et al., 2015] of the images in the Office-Caltech dataset with MTCNN features and DRN features, respectively. Compared with MTCNN features, the data points with DRN features are discriminated better across different categories, i.e. each category has small intra-class variance and large inter-class margin; and the data points are also aligned better across different tasks, i.e. the embeddings of different tasks overlap well, implying that different tasks reinforce each other effectively and improve category discrimination performance. This verifies that with joint relationship discovery, DRN can learn even more transferable features for multi-task learning.

### 6 Conclusion

This paper presented deep relationship networks (DRN) that integrate deep networks with tensor normal priors over the network parameters of all task-specific layers, which model the covariance structure over tasks and enable transfer across related tasks. A learning algorithm was devised to learn trans-
ferable features and task relationships jointly. Experiments show that DRN yielded superior results on standard datasets.

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