Effects of the Consistent Interaction on Kaon Photoproduction with Spin 5/2 Nucleon Resonances

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Abstract. Theoretical models for kaon photoproduction with spin 5/2 nucleon resonances have been plagued with the problem of interaction consistency. A number of studies predicted that a model with a consistent interaction leads to a better agreement with data. In this study a model with consistent interaction (model 2) is compared to the old model, which utilizes an inconsistent interaction (model 1), as well as to experimental data. The unknown parameters in scattering amplitude are extracted from fitting to 7400 experimental data points. This is performed by minimizing the $\chi^2/N$ value. It is found that model with a consistent interaction (model 2) is more suitable for explaining experimental data.

1. Introduction
Kaon photoproduction is one of the possible reactions used to study the baryon structure. It is of interest because it has an additional degree of freedom, i.e., the strangeness. Therefore, this reaction will pave the way for searching the missing resonances, since they could have small branching ratio to the main reaction channels, i.e., pion-nucleon scattering. Another application of this reaction is for studying the hypernuclear properties. In the past, investigation of kaon photoproduction was dormant, due to the lack of experimental data. At present, this reaction has abundant experimental data, that originate from many experimental collaborations such as CLAS, GRAAL, MAMI, LEPS, etc.

A number of analyses and methods have been proposed in the literature. One of them is the isobar model. Isobar model requires a more complicated calculation than multipole analysis for higher spin resonance. Adelseck et. al. [1] has constructed a model which includes higher spin resonances, up to 5/2. In the higher spin case there is an issue concerning the interaction consistency, which appears as the lower spin background. Pascalutsa [2] and Vranckx [3] have overcome this problem by using the Rarita-Schwinger theory of free massive spin-3/2 field. However, the physical significance of this prescription has not been emphasized in these studies. It is the aim of the present study to test this prescription for kaon photoproduction, as well as to compare it with other prescriptions.

In this paper, we will compare the result obtained by the model based on the Adelseck prescription with that of the Pascalutsa method. We will calculate all observables and extracted the unknown parameters by minimizing the $\chi^2/N$ value. All results from calculation will be presented in the form of tables and figures.
2. Isobar Model

To model the kaon photoproduction process $\gamma + p \rightarrow K^+ + \Lambda$ we make use of a number of Feynman diagrams for the background and resonance terms, as shown in Fig. 1. From these diagrams we can calculate the scattering amplitude. Background terms include the non-resonant particles, which are called the Born terms, along with the $K^+(892)$ and $K_1(1270)$ intermediate states as well as two hyperon resonances that were found to be important for maintaining the reasonable value of hadronic form factor cut off of the Born terms [4]. The resonance terms consist of the nucleon resonances that have spin 1/2 and spin 5/2, for which their status according to the Particle Data Group (PDG) [5] is at least two-star rating as can be seen in Table 1. All important properties of these particle are shown in Table 1.

To extract the scattering amplitude from the Feynman diagrams shown in Fig. 1 we use the standard procedure. The formulation of the scattering amplitude for spin 1/2 resonance is well known and, in fact, has been established. However, the formulation for spin 5/2 resonances is rather complicated. As stated before in introduction, there is an intrinsic problem for the higher spin states, i.e., the existence of the lower spin background. This background should be removed in order to maintain the consistency. Pascalutsa [2] and Vrancx [3] have proposed certain methods to achieve the consistent interaction of higher spin. Pascalutsa formulates the spin 3/2 propagator by using the Rarita-Schwinger theory for free massive spin-3/2 field [2]. Based on this work Vrancx constructed a model for higher spin nucleon resonance and proved that the model has a consistent interaction [3].

### Table 1. Status, mass and width of nucleon resonances used in our calculation [5].

| Resonance | Status | Mass (MeV) | Width (MeV) |
|-----------|--------|------------|-------------|
| $N(1440)P_{11}$ | **** | 1430 ± 20 | 350 ± 100 |
| $N(1535)S_{11}$ | **** | 1535±20 | 150 ± 25 |
| $N(1650)S_{11}$ | **** | 1655±10 | 140 ± 30 |
| $N(1675)D_{15}$ | **** | 1675 ± 5 | 150±15 |
| $N(1680)F_{15}$ | **** | 1685 ± 5 | 130 ± 10 |
| $N(1710)P_{11}$ | **** | 1710 ± 30 | 100±150 |
| $N(1860)F_{15}$ | ** | 1860±100 | 270±140 |
| $N(1880)F_{15}$ | ** | 1870 ± 35 | 235 ± 65 |
| $N(1895)S_{11}$ | ** | 1895 ± 15 | 90±30 |
| $N(2000)F_{15}$ | ** | 2050 ± 100 | 198 ± 2 |
| $N'(2060)D_{15}$ | ** | 2060 | 375 ± 25 |
2.1. Scattering Amplitude

In the present work we use two models that have different formulations of spin 5/2 nucleon resonance scattering amplitude. The formulation of scattering amplitude for Model 1 is obtained from Ref. [1], whereas for Model 2 we obtain all factors from Refs. [2, 3, 6, 7]. For the hadronic and electromagnetic vertices, along with the corresponding propagator, the formulation reads

\[
\Gamma_{\text{had}}^{\mu\nu}(p, q) = \frac{g_{\text{KY}N^*}}{m_N^3} \Gamma_{\mu\nu} \left[(p \cdot q - p_A q)^\mu + p_A q^\mu - q\eta_A \right] p_A^\nu, \quad (1)
\]

\[
\Gamma_{\text{em}}^{\alpha\beta}(p, k) = \frac{-i}{m_N^3} \left[ g^4 \alpha(\epsilon^\mu \epsilon - k^\mu) + g^2(k^\alpha p - \epsilon^\alpha p \cdot k) + g^3(\epsilon^\alpha k - k^\alpha \epsilon) \right] \Gamma^\alpha \Gamma^\beta, \quad (2)
\]

\[
P_{\mu\nu}(p, k) = \frac{s^2}{m_N^4} \left[ (\bar{p} + \bar{k} + m_N) \left\{ 5 P_{\mu\nu} P_{\alpha\beta} - 2 P_{\mu\nu} P_{\alpha\beta} + 5 P_{\mu\beta} P_{\nu\alpha} + P_{\mu\nu} \gamma^\rho P_{\alpha\beta} + P_{\mu\nu} \gamma^\rho P_{\alpha\beta} P_{\mu\alpha} + P_{\mu\nu} \gamma^\rho P_{\alpha\beta} P_{\mu\alpha} \right\} \right] \gamma^\alpha P_{\sigma\alpha} P_{\beta\gamma} + P_{\mu\nu} \gamma^\rho P_{\alpha\beta} P_{\mu\alpha} + P_{\mu\nu} \gamma^\rho P_{\alpha\beta} P_{\mu\alpha} + P_{\mu\nu} \gamma^\rho P_{\alpha\beta} P_{\mu\alpha} \right\}, \quad (3)
\]

with the parity \(\Gamma_+ = i\gamma_5\) and \(\Gamma_- = 1\), and \(P_{\mu\nu} = -g_{\mu\nu} + \frac{1}{2}(p + k)_\mu(p + k)_\nu\).

Note also that we have defined

\[
\begin{align*}
    g^1 &= ig_{\gamma N^* N} - ig_{\gamma N^* N}^2 \\
g^2 &= -2i g_{\gamma N^* N} + g_{\gamma N^* N}^2 - 2g_{\gamma N^* N}^2 \\
g^3 &= -2i g_{\gamma N^* N} + 3ig_{\gamma N^* N}^2 + g_{\gamma N^* N}^2 \\
g^4 &= -ig_{\gamma N^* N}^2 + ig_{\gamma N^* N}^3 \\
g^5 &= 2ig_{\gamma N^* N}^3 - ig_{\gamma N^* N} + g_{\gamma N^* N}^4
\end{align*}
\]

where \(g_{\gamma N^* N}^i\) is coupling constant along with \(g_{\gamma N^* N}\), where \(i = 1, 2, 3, 4\).

By multiplying all these factors we obtain the scattering amplitude. To simplify the numerical calculation we decompose the scattering amplitude into six gauge and Lorentz invariant matrices \(M_i\), where \(i\) runs from 1 to 6 [8]. Since in the present work we only focus on the photoproduction process, we only list four invariant amplitudes, that are necessary for photoproduction, i.e.,

\[
A_1 = \left[ -s\{(5c_2^2 + c_2c_3 + 2c_1(\frac{1}{4}c_2m_A^2 - \frac{1}{4}c_2c_3)) + m_Nm_A\{5c_2^2 + c_2c_3 + 2c_1(\frac{1}{4}c_2c_3 - b_p\)} \right] 4G^1 + \left[ s\{(m_p + m_A)c_1(m_A - \frac{1}{4}m_Ak^2 - \frac{1}{4}c_Am_p) - \frac{2}{s}c_Ab_pc_1 + (c_1 - b_pc_3)\} + m_Nm_A\{5c_2 - \frac{1}{4}c_2c_A + m_p m_A\} + (m_p + m_A)c_1(m_A m_p - c_A) + 2c_1^2(m_p + m_A)\} \right] G^2 + \left[ s\{(m_p + m_A)c_1c_2 + 2b_p m_A c_1\} \right] G^3
\]

\[
A_2 = \frac{1}{t - m^2} \left[ c_1c_2k^2 = m_Nm_A(k^2 m_A) \right] 16G^1 + \frac{1}{t - m^2} \left[ s\{(b_p c_s - c_1)(5c_1 - \frac{1}{4}c_k m_p m_A + \frac{1}{4}c_A b_p) + \frac{1}{2}k^2(c_p c_2 + c_1 c_m m_A) + m_N \{ b_pc_s - c_1)(5c_1 m_p + \frac{1}{2}c_k m_p m_A - b_p m_A) + \frac{1}{2}m_p k^2(c_p c_2 - c_1 c_A) \} \right] G^2 + \frac{1}{t - m^2} \left[ -k^2 m_A c_1 + 2c_k m_A (c_1 - b_p c_s) + k^2 c_s (c_p m_A + c_A m_p) \right] m_N
\]
In this work all unknown parameters are extracted by fitting the available experimental data. Based on Ref. [9], the dominant contribution comes from the first and second peaks. Therefore, all calculations shown in Fig. 2 are purely prediction. We also note that predicted observables with the corresponding experimental data would be useful.

By combining this result with Table 2 we can determine the resonances that have important contributions to the first and second peaks. Based on Ref. [9], the dominant contribution comes from the first and second peaks. Therefore, all calculations shown in Fig. 2 are purely prediction. We also note that predicted observables with the corresponding experimental data would be useful.

\[
A_3 = \frac{1}{2m_p G^3} \left[ \sum \left( (c_1 - b_p c_s) \right) \left( 5 \left( 1 + \frac{1}{s} c_p c_A \right) + k^2 c_s \left( \frac{1}{s} c_p c_A + m_A m_p - 5c_1 \right) + k^2 \left( \frac{1}{s} c_1 c_A - \frac{1}{s} c_2 c_p \right) \right) \right]
\]

where

\[
G^i = \frac{\kappa}{10m_N^0 \left( s - m_N^0 + i m_N \right)^4}
\]

while \( i \) runs from 1 to 3. We have used the following definitions:

\[
\begin{align*}
&b_p = p \cdot k ; \quad b_A = p_A \cdot k ; \quad b_q = q \cdot k \\
&c_p = (p + k) \cdot p ; \quad c_A = (p + k) \cdot p_A ; \quad c_k = (p + k) \cdot k ; \quad c_s = 1 - \frac{1}{s} c_A \\
&c_1 = b_A - \frac{1}{s} c_A c_k ; \quad c_2 = m_A^2 - \frac{1}{s} c_A^2 ; \quad c_3 = \frac{1}{s} c_k^2 - k^2 ; \quad c_4 = 2b_p + k^2 ; \quad c_5 = 4b_p + k^2
\end{align*}
\]

3. General Results and Discussion

In this work all unknown parameters are extracted by fitting the available experimental data. The fitting process is performed by minimizing the \( \chi^2 / N \) values from 7500 experimental data points. From the fits we found that Model 2 (\( \chi^2 / N = 4.35 \)) has a better agreement with the experimental data compared to Model 1 (\( \chi^2 / N = 4.70 \)). However, this difference seems to be less significant if we want to determine the best model. To this end, checking the agreement of all predicted observables with the corresponding experimental data would be useful.

Figure 2 shows the comparison between total cross section from experimental data and model calculations. Note that the total cross section data shown in Fig. 2 is not included in our fitting database. Therefore, all calculations shown in Fig. 2 are purely prediction. We also note that both models have a similar pattern, i.e., they indicate two peaks in the cross section, albeit with different positions and heights.

By combining this result with Table 2 we can determine the resonances that have important contributions to the first and second peaks. Based on Ref. [9], the dominant contribution comes from the first and second peaks. Therefore, all calculations shown in Fig. 2 are purely prediction. We also note that both models have a similar pattern, i.e., they indicate two peaks in the cross section, albeit with different positions and heights.
from the $S_{11}(1650)$ state. This is in agreement with our results as exhibited in Fig. 2 and Table 2. The second peak is rather complicated, since in this case contribution from two resonances is comparable. Table 2 indicates the resonances for this purpose, i.e., the $S_{11}(1895)$ and $P_{11}(1880)$ states. The $S_{11}(1895)$ resonance turns out to have a bigger contribution than the $P_{11}(1880)$ state, because the second peak of Model 1 has the position around 1880 MeV.

Since the angle dependency has been integrated out, total cross section cannot reveal any information on the reaction mechanism. Therefore, other observables are required. The simplest observable in this case is the differential cross section, which is plotted as functions of the kaon angle and total c.m. energy in Fig. 3.

From Fig. 3 (left), it appears that both models have almost a similar pattern, except at the forward and backward angles. By looking at Fig. 3 closely in the backward region it is found that only Model 2 can fit the experimental data nicely, whereas Model 1 fits the data better in the forward direction. From the energy distribution of the differential cross section shown in the right panel of Fig. 3, it is obvious that both models have almost a similar pattern.

Both models show a slight variance in the differential cross section. Therefore, we should investigate other observables, which are more sensitive to the reaction mechanism. The first candidate is the single polarization observables, which comprises of the photon asymmetry ($\Sigma$),

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**Figure 2.** Comparison between the calculated total cross section from Model 1, Model 2 and Kaon-Maid [10] with experimental data from CLAS [11].

**Table 2.** Mass and width obtained from fit to experimental data.

| Resonance   | Model 1 Mass (MeV) | Model 1 Width (MeV) | Model 2 Mass (MeV) | Model 2 width (MeV) |
|-------------|--------------------|---------------------|--------------------|---------------------|
| $P_{11}(1440)$ | 1470               | 200                 | 1470               | 200                 |
| $S_{11}(1535)$ | 1525               | 175                 | 1525               | 125                 |
| $S_{11}(1650)$ | 1670               | 180                 | 1670               | 120                 |
| $D_{15}(1675)$ | 1680               | 130                 | 1680               | 130                 |
| $F_{15}(1680)$ | 1690               | 120                 | 1680               | 120                 |
| $P_{11}(1710)$ | 1680               | 250                 | 1740               | 161                 |
| $F_{15}(1860)$ | 1960               | 299                 | 1960               | 271                 |
| $P_{11}(1880)$ | 1837               | 170                 | 1889               | 297                 |
| $S_{11}(1895)$ | 1880               | 120                 | 1901               | 120                 |
| $F_{15}(2000)$ | 1989               | 510                 | 1950               | 410                 |
| $D_{15}(2060)$ | 2004               | 351                 | 1960               | 350                 |
Figure 3. Differential cross section at fixed total energy (left) and fixed angle (right). Experimental data are from CLAS ( ■ [11] and □ [12]) and Crystal Ball ( ⊙ [13]) collaborations.

Figure 4. Photon and target asymmetries at fixed angle. Experimental data are taken from GRAAL [14].

target asymmetry ($T$) and recoil polarization ($P$). Due to the limited experimental data on photon and target asymmetries, both models suffer from the problem to fit these data as shown in Fig. 4. For the target asymmetry the agreement with experimental data is worst. This problem happens because the limited data for these observables must compete with other dominant data, such as the differential cross section data. Actually, this problem can be solved by introducing an additional weighing factor. However, such factor is not our present interest can could be addressed in the future work. In both photon and target asymmetries both models display a

Figure 5. Recoil polarization at fixed total energy (left) and fixed angle (right). Experimental data are from CLAS ( ■ [11] and □ [12]) and LEPS ( △ [15]) collaborations.
large variance. Nevertheless, as stated before, this difference has a little effect for distinguishing the two models.

On the other hand, the recoil polarization observable data are more abundant as compared to the photon and target asymmetries. Therefore, this observable would have a more significant impact on the $\chi^2/N$ value and as a consequence, recoil polarization appears to be a more sensitive probe to distinguish the two models. Model 2 appears to have a better agreement with the data than Model 1, as shown in Fig. 5. However, in the case of angular distribution both models fit the data equally. This is similar to the case of differential cross section. The only difference appears in the backward angle region. The origin of this phenomena presumably because both models have the same reaction mechanism, besides their spin 1/2 and 5/2 nucleon resonances.

In order to get a full picture of our work, we need to analyze the double polarization observables. Double polarization observables $O_{x'}$ and $O_{z'}$ have less data and larger error bars, as compared to those of $C_x$ and $C_z$. From Fig. 6, we can see that both models have significant difference. However, this large difference has a little contribution to the $\chi^2$ value, because the corresponding data are limited and have large error bars. Such phenomenon also appears in the photon and target asymmetries. The important result appears from the double polarization $C_x$ and $C_z$, because they have more data points and smaller error bars. Moreover, there exist a relation between the recoil polarization and the double polarization $C_x$ and $C_z$, i.e., $C_x^2 + C_z^2 + P^2 = 1$.

From Fig. 7 (left), it appears that Model 2 has a better agreement with the data than Model 1, but Model 2 cannot reproduce the data in the backward region. For the double polarization
it appears that Model 2 is able to nicely fit all data. The angular distribution of these observables shows a large variance for both models. This indicates that the double polarization $C_2$ is sensitive to the difference of the prescription used. This probably also explains, why there is a discrepancy between our models in the angular distribution of the differential cross section.

4. Summary and Conclusions
In this paper we have presented comparison between two models, which have different formulations of the spin 5/2 resonance. It is found that Model 2, which has the consistent interaction formulation, has a better agreement with experimental data, rather than Model 1, that does not imply the consistent interaction. Furthermore, the consistent interaction formulation is found to be important in explaining the double polarization $C_2$ data.

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References
[1] David J C, Fayard C, Lamot G H and Saghai B 1996 Phys. Rev. C 53 2613
[2] Pascalutsa V 2001 Phys. Lett. B 503 85
[3] Vranct T, De Cruz L, Ryckebus V and Vancraeyveld P 2011 Phys. Rev. C 84 045201
[4] Mart T and Nurhadiansyah N 2013 Few-Body Syst. 54 1729
[5] Olive K A et al 2014 Chin. Phys. C 38 090001
[6] Pascalutsa V 1998 Phys. Rev. D 58 096002
[7] Pascalutsa V and Timmermans R 1999 Phys. Rev. C 60 042201
[8] Mart T 1996 Electromagnetic Production of Kaons off the Nucleon and $^3$He (Universitaet Mainz)
[9] Mart T and Kholili M J 2012 Phys. Rev. C 86 022201
[10] Mart T and Bennhold C 1999 Phys. Rev. C 61 012201
[11] Bradford R et al [CLAS Collaboration] 2006 Phys. Rev. C 73 035202
[12] McCracken M E et al [CLAS Collaboration] 2010 Phys. Rev. C 81 025201
[13] Jude T C et al [Crystal Ball at MAMI Collaboration] 2014 Phys. Lett. B 735 112
[14] Lleres A et al [GRAAL Collaboration] 2007 Eur. Phys. J. A 31 79
[15] Sumihama M et al [LEPS Collaboration] 2006 Phys. Rev. C 73 035214
[16] Lleres A et al [GRAAL Collaboration] 2009 Eur. Phys. J. A 39 149
[17] Bradford R et al. 2007 Phys. Rev. C 75 035205