Semi Analytical Model for the Dynamic Behavior of Offshore Wind Turbine with Flexible Foundation

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Abstract. A new semi analytical formulation is developed which models the dynamic behavior of offshore wind turbine with variable cross section and flexible foundation. The wind turbine tower is modeled using Euler-Bernoulli beam model while the soil-tower interaction is modeled by rotational and lateral springs at the lower end of the tower. Recursive differential method is used to calculate natural frequencies which is found to be efficient in terms of accuracy as well as the computational cost. The predicted natural frequencies of the current proposed mathematical technique were validated and compared with those of two other analytical and experimental techniques, for four selected offshore wind turbines, and results were found to be in very good agreement. Furthermore, the results of the current model, which accurately models the variation of the cross section properties, were found to be in a better agreement with the experimental results compared to those of the constant cross-section approximation.

1. Introduction

The fast growing demand of energy, accompanied with the depletion of the traditional oil resources, and the increasing restrictions on the environmental echo of the proposed technologies, made the green energy resources as a worldwide strategic goal. The most influential limitation to invest in such clean energy resources, is its relatively high operational cost. Wind energy can be considered as one of the most appealing energy resources due to its availability and its promising cost efficiency. Offshore wind turbines, in particular, is gaining more interest due to its high-energy density, lower turbulence, lower wind shear, and fewer civil complaints. Despite of all such attractive aspects, compared to onshore wind turbine, its maintainability still present a significant constraint which calls for more research on the factors that affect its dynamic behavior and hence its service life.

The stability, performance, and lifetime of wind turbine systems can be fundamentally traced back to their dynamic behavior, and most importantly its natural frequencies. Consequently, much attention has been paid to the dynamic analysis and simulation of such turbines which include experimental modal and theoretical modal analyses. The main challenge in simulating the dynamic behavior of wind turbines, using theoretical models, is the proper consideration and estimation of all the parameters which will have a significant effect on the accuracy of the proposed dynamic model.

An extensive research work has been found in the literature in topics related to wind turbine dynamics; either in terms of the dynamics of its main components, their dynamic interaction, and the main parameters presumed significant; or in terms of different proposed mathematical and experimental techniques which strived toward better accuracy along with being computationally and experimentally...
economic. Thresher [1] reviewed the various analysis methods of the dynamic simulation of horizontal axis wind turbines, and compared it with experimental data. Halfpenny [2] used the finite element method to develop a new frequency domain model, able to analyze fully flexible on and offshore wind turbines. Ahlstrom [3] presented a finite element model to simulate the aeroelastic dynamic response of horizontal axis wind turbines. Kessentini et al. [4] used the differential quadrature method (DQM) to develop a mathematical model of a horizontal axis turbine with flexible tower and blades. The model incorporates the nacelle pitch angle and structural damping. Since this paper will focus, particularly, on the dynamic behavior of wind turbine tower – nacelle interaction of offshore wind turbines with flexible foundation, a literature review on the different aspects of such problem will be presented hereinafter.

Abrate [5] presented simple formulas that can be used to predict the fundamental frequency of non-uniform beams with various boundary conditions. Li et al. [6] investigated the free flexural vibration of a non-uniform bar, under the effect of various cases of axial loads, by reducing its differential equation to Bessel’s equations. It was found that the fundamental frequency of cantilevered tall structures, which was simulated by their model, proved to have values closer to the measured field data than that of the computed without considering the existing axial forces. Mehmet et al. [7] presented an analytical solution for the modal analysis of variable cross-section isotropic beams with different boundary conditions. They came to the conclusion that the non-uniformity of the cross-section would have an influence on the natural frequencies and mode shapes. Coşkun et al. [8] presented a transverse vibration analysis of uniform and non-uniform Euler-Bernoulli beams by applying analytical approximate techniques. Zamorska [9] provided a series solution for the fourth order differential equation with various coefficients occurring in the vibration problem of Euler-Bernoulli beam, which can be extended to model the dynamic behavior variable cross wind turbine towers. Taha [10] introduced a recursive differentiation method which can be useful as an analytical solution of static and dynamic stability problems of beams resting on two-parameter foundation. It was found that, based on the elasticity of foundation, the critical buckling load of one of the higher buckling modes can happen to have values lower than those of the lower buckling modes, and the same was found in the natural frequencies of beams subject to axial loads. Abohadima et al. [11] implemented the recursive differentiation method to obtain general analytical solutions for the free and forced vibration of an axially loaded Timoshenko beam resting on two-parameter foundation and subject to non-uniform lateral excitation.

Molenaar [12] provided a comprehensive review on the theoretical basics and the design options of wind turbine dynamics, including offshore machines under wave action. Oh et al. [13] provided a comprehensive literature review on the different types of wind turbine foundation, in addition to the different modelling techniques of the structure-soil interaction. Bhattacharya et al. [14] presented a summary the results from a series of 1:100 scale tests of a V120 Vestas turbine supported while being supported by two types of foundation; monopiles and tetrapod suction caissons. The results were found useful in terms of providing an insight into the long term performance and in identifying some issues related to the soil-structure interaction. Adhikari et al. [15] presented a closed form approximate expression that predicts the fundamental frequency of wind turbine towers with flexible foundation. Their analytical model is based on Euler-Bernoulli beam-column theory with elastic end supports. Bhattacharya et al. [16] introduced novel experimental techniques to obtain the parameters needed for the dynamic model of offshore wind turbines. Experimental results showed that the natural frequencies and the damping factors of the wind turbine tower vary, significantly, based on the model of the soil-foundation interaction. Adhikari et al. [17] characterized the dynamic behavior of offshore wind turbines using a closed form solution based on Euler-Bernoulli beam-column with elastic end supports. Arrany et al. [18] provided an analytical model of offshore wind turbine with flexible foundation in order to have a fast and reasonably accurate estimate for its fundamental frequency. Their model was found suitable for preliminary design or to verify the finite element results. Alamo et al. [19] studied, experimentally, the dynamic effect of foundation parameters of offshore wind turbine systems. Their results simulated the effect of considering the structure – soil interaction on the accuracy of the estimated fundamental frequency and equivalent damping of the soil-structure system. Kumar and Nasar [20] modeled the dynamic behavior offshore wind turbines subject to environmental and rotor vibration forces. The tower was modelled as Euler-Bernoulli beam - column with elastic end supports. Wang et al. [21] investigated the wind turbine dynamic behavior by employing an improved model based on vibration signal analysis.
This model is formulated using the Euler–Lagrangian approach in which the nacelle – tower and tower – foundation dynamic interactions were considered. So far, and to the authors knowledge, the tower – nacelle dynamic interaction, under the combined effect of the flexible foundation and variable tower cross-section, has not been investigated yet neither by analytical nor numerical simulation.

In this paper, a new semi analytical formulation is developed which models the dynamic behavior of offshore wind turbine with variable cross section and flexible foundation. The wind turbine tower is modeled using Euler-Bernoulli beam model while the soil-tower interaction is modeled by rotational and lateral springs at the lower end of the tower. Recursive differential method will be used to calculate fundamental frequencies of some selected real wind turbines and the results will be compared with their experimentally measured values.

2. Modelling

2.1. Physical model

We consider a typical wind turbine tower as shown in Fig. 1. This system is modeled by an Euler Bernoulli beam and the idealization process is explained in the diagram. The bending stiffness of the beam is \( EI(x) \) and it is attached to the foundation. Here \( x \) is the spatial coordinate, starting at the bottom and moving along the height of the structure. The interaction of the structure with the foundation is modeled using two springs. The rotational spring with spring stiffness \( k_r \) and the lateral spring with spring stiffness \( k_l \) constrains the system at the bottom \((x = 0)\). The beam has a top mass with rotary inertia \( J \) and mass \( M_t \). This top mass is used to idealize the rotor and blade system. The mass per unit length of the beam is \( m \), \( r(x) \) is the radius of gyration and the beam is subjected to a constant compressive axial load (Buckling Load) which denoting as force \( P \) that may be generally be a function of time due to the rotational motion of the hub, in this model \( P \) is the weight of the nacelle which equal \( M_t g \).

![Figure 1. Idealization of a tower – nacelle system with flexible foundation](image)

2.2. Mathematical models

The equation of motion of the tower – nacelle system, modeled as Euler beam with variable cross-section and tip concentrated mass, can be stated as follows ([22], [23]):

\[
\frac{\partial}{\partial x} [Q] + m \ddot{w}(x, t) = f(x, t)
\] (1)
\[
Q = \frac{\partial}{\partial x} [M] + P(x) \frac{\partial w(x,t)}{\partial x} - m r^2(x) \frac{\partial \dot{w}(x,t)}{\partial x}, \quad M = EI(x) \frac{\partial^2 w(x,t)}{\partial x^2}
\]

where \( Q \) is the shear force, \( M \) is the Bending Moment, \( w(x,t) \) is the transverse deflection of the beam, \( t \) is time, \((\cdot)\) denotes derivative with respect to time and \( f(x,t) \) is the applied time-varying load on the beam. The height of the tower is considered to be \( L \). Therefore, the equation of motion of the beam can be written as:

\[
\frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right] + \frac{\partial}{\partial x} \left[ P(x) \frac{\partial w(x,t)}{\partial x} \right] - \frac{\partial}{\partial x} \left[ m r^2(x) \frac{\partial \dot{w}(x,t)}{\partial x} \right] + m \ddot{w}(x,t) = f(x,t)
\]

which can be expanded to take the following form:

\[
EI(x) \frac{\partial^4 w(x,t)}{\partial x^4} + 2EI^{(1)}(x) \frac{\partial^3 w(x,t)}{\partial x^3} + \left[ EI^{(2)}(x) + P(x) \right] \frac{\partial^2 w(x,t)}{\partial x^2} \\
+ P^{(1)}(x) \frac{\partial w(x,t)}{\partial x} - m r^{2(1)}(x) \frac{\partial \dot{w}(x,t)}{\partial x} - m r^2(x) \frac{\partial^2 \dot{w}(x,t)}{\partial x^2}
\]

where \( Z^{(n)} \) denotes the \( n \)th derivative of the function \( Z \), and the forcing function is assumed as:

\[
f(x,t) = f_o(x)e^{i\omega t}
\]

where \( \omega \) is related to the rotor frequency or blade passing frequency and the number of blades. The function \( f_o(x) \) is the forcing amplitude as a function of \( x \).

Equation (4) is a general equation of equation which models the dynamic behavior of the tower – nacelle system while considering the variation of tower cross-section and the compressive and lateral effect of the nacelle.

2.3. Boundary Conditions

At \( x=0 \) Bending Condition

\[
M - K_r \dot{\theta} = 0|_{x=0}
\]

\[
EI(x) \frac{\partial^2 w(x,t)}{\partial x^2} - K_r \frac{\partial w(x,t)}{\partial x} = 0|_{x=0}
\]

At \( x=0 \) Shear Condition

\[
Q - K_L w(x,t) = 0|_{x=0}
\]

\[
\frac{\partial}{\partial x} \left[ EI(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right] + P(x) \frac{\partial w(x,t)}{\partial x} - m r^2(x) \frac{\partial \dot{w}(x,t)}{\partial x} + K_L w(x,t) = 0|_{x=0}
\]

At \( x=L \) Bending Condition

\[
M + J \alpha = 0|_{x=0}
\]

\[
EI(x) \frac{\partial^2 w(x,t)}{\partial x^2} + J \frac{\partial \dot{w}(x,t)}{\partial x} = 0|_{x=L}
\]

At \( x=L \) Shear Condition

\[
\frac{\partial}{\partial x} \left[ EI(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right] + P(x) \frac{\partial w(x,t)}{\partial x} - m r^2(x) \frac{\partial \dot{w}(x,t)}{\partial x} - M \ddot{w}(x,t) = 0|_{x=L}
\]
2.4. Solution

Assuming harmonic solution we have
\[ w(x, t) = W(\xi)e^{i\omega t}, \text{ where } \xi = \frac{x}{L} \]  
(10)

Substituting eqn. (10) into eqn. (4) and the boundary conditions, the equation of motion takes the form: results in the following:
\[ W^{(4)} + 2 \frac{EI^{(1)}(\xi L)}{EI(\xi L)} W^{(3)} + \left[ \frac{1}{EI(\xi L)} \left( EI^{(2)}(\xi L) + L^2 P(\xi L) + \omega^2 L^2 m r^2(\xi L) \right) \right] W^{(2)} + \left[ \frac{L^2}{EI(\xi L)} \left( p^{(1)}(\xi L) + \omega^2 m r^{2(1)}(\xi L) \right) \right] W^{(1)} - \frac{m \omega^4 L^4}{EI(\xi L)} W^{(0)} = 0 \]  
(11)

, and the boundary conditions can be written as:

At \( \xi = 0 \)
Bending Condition
\[ EI(0)W^{(2)} - LK_1 W^{(1)} = 0 |_{\xi=0} \]  
(12)

At \( \xi = 0 \)
Shear Condition
\[ EI(0)W^{(3)} + EI^{(1)}(0)W^{(2)} + L^2 [P(0) + \omega^2 m r^2(0)] W^{(1)} + L^3 K_L W^{(0)} = 0 |_{\xi=0} \]  
(13)

At \( \xi = 1 \)
Bending Condition
\[ EI(1)W^{(2)} - L \omega^2 W^{(1)} = 0 |_{\xi=1} \]  
(14)

At \( \xi = 1 \)
Shear Condition
\[ EI(1)W^{(3)} + EI^{(1)}(1)W^{(2)} + L^2 [P(1) + \omega^2 m r^2(1)] W^{(1)} + L^3 K_L W^{(0)} = 0 |_{\xi=1} \]  
(15)

Applying the recursive differential method (RDM) [10], which is considered as of the semi analytical technique to solve partial differential equations (PDE), where the PDE can be written the form:
\[ y^{(n)}(x) = \sum_{i=0}^{n-1} A_{i,0} y^{(i)}(x) + f_0(x) \]  
(16)

\[ W^{(4)} = A_{1,1}(\xi)W^{(0)} + A_{1,2}(\xi)W^{(1)} + A_{1,3}(\xi)W^{(2)} + A_{1,4}(\xi)W^{(3)} \]  
(17)

, where
\[ A_{1,1}(\xi) = \frac{m \omega^2 L^4}{EI(\xi L)} \]
\[ A_{1,2}(\xi) = -\frac{L^2}{EI(\xi L)} \left[ p^{(1)}(\xi L) + \omega^2 m r^{2(1)}(\xi L) \right] \]
\[ A_{1,3}(\xi) = -\frac{1}{EI(\xi L)} \left[ EI^{(2)}(\xi L) + L^2 P(\xi L) + \omega^2 L^2 m r^2(\xi L) \right] \]
\[ A_{1,4}(\xi) = -2 \frac{EI^{(1)}(\xi L)}{EI(\xi L)} \]  
(18)
Therefore, the solution in the stated in the following form:

\[
W(\xi) = \sum_{i=1}^{n} T_i R_i(\xi) + R_f(\xi)
\]  

(19)

where

\[
R_m(\xi) = \frac{\xi^{m-1}}{(m-1)!} + \sum_{i=1}^{N-n} A_{i,m}(a) \frac{\xi^{n+i-1}}{(1+n-1)!}
\]

(20)

and

\[
R_f(\xi) = \sum_{i=1}^{N-n} F_i(a) \frac{\xi^{n+i-1}}{(1+n-1)!}
\]

in which \( F_i(\xi) = 0 \) and hence \( R_f(\xi) = 0 \) and the solution takes the form:

\[
W(\xi) = T_1 R_1(\xi) + T_2 R_2(\xi) + T_3 R_3(\xi) + T_4 R_4(\xi)
\]

(21)

In order to get the boundary equations in terms of \( T_i \) and \( R_i \), eqn. (21) is substituted in the boundary conditions equations (12-15) as follows:

\[
\text{EI}(0) \left\{ T_1 R_1^{(2)} + T_2 R_2^{(2)} + T_3 R_3^{(2)} + T_4 R_4^{(2)} \right\} \\
- L K_r \left\{ T_1 R_1^{(1)} + T_2 R_2^{(1)} + T_3 R_3^{(1)} + T_4 R_4^{(1)} \right\} = 0
\]

(22)

\[
\begin{align*}
\text{EI}(0) & \left\{ T_1 R_1^{(3)} + T_2 R_2^{(3)} + T_3 R_3^{(3)} + T_4 R_4^{(3)} \right\} \\
& + \text{EI}(1)^{(0)} \left\{ T_1 R_1^{(2)} + T_2 R_2^{(2)} + T_3 R_3^{(2)} + T_4 R_4^{(2)} \right\} \\
& + L^2 [P(0) + \omega^2 mr^2(0)] \left\{ T_1 R_1^{(1)} + T_2 R_2^{(1)} + T_3 R_3^{(1)} + T_4 R_4^{(1)} \right\} \\
& + L^3 K_L \left\{ T_1 R_1^{(0)} + T_2 R_2^{(0)} + T_3 R_3^{(0)} + T_4 R_4^{(0)} \right\} = 0
\end{align*}
\]

(23)

\[
\begin{align*}
\text{EI}(1) & \left\{ T_1 R_1^{(2)} + T_2 R_2^{(2)} + T_3 R_3^{(2)} + T_4 R_4^{(2)} \right\} \\
& - L \omega^2 \left\{ T_1 R_1^{(1)} + T_2 R_2^{(1)} + T_3 R_3^{(1)} + T_4 R_4^{(1)} \right\} = 0
\end{align*}
\]

(24)

\[
\begin{align*}
\text{EI}(1) & \left\{ T_1 R_1^{(3)} + T_2 R_2^{(3)} + T_3 R_3^{(3)} + T_4 R_4^{(3)} \right\} \\
& + \text{EI}(1)^{(1)} \left\{ T_1 R_1^{(2)} + T_2 R_2^{(2)} + T_3 R_3^{(2)} + T_4 R_4^{(2)} \right\} \\
& + L^2 [P(1) + \omega^2 mr^2(1)] \left\{ T_1 R_1^{(1)} + T_2 R_2^{(1)} + T_3 R_3^{(1)} + T_4 R_4^{(1)} \right\} \\
& + L^3 \omega^2 M \left\{ T_1 R_1^{(0)} + T_2 R_2^{(0)} + T_3 R_3^{(0)} + T_4 R_4^{(0)} \right\} = 0
\end{align*}
\]

(25)

The boundary condition equations are function in \( T_i \) and \( \omega \) the natural frequencies, we get

\[
M \mathbf{a} = 0
\]

(26)

where

\[
\mathbf{a} = \{T_1, T_2, T_3, T_4\}^T
\]

(27)

where the matrix \( M \) is 4x4 and the matrix elements can be stated as follows:

\[
\begin{align*}
M_{1i} &= \text{EI}(0) R_i^{(2)} - L K_r R_i^{(1)} \\
M_{2i} &= \text{EI}(0) R_i^{(3)} + \text{EI}(1)^{(0)} R_i^{(2)} + L^2 [P(0) + \omega^2 mr^2(0)] R_i^{(1)} + L^3 K_L R_i^{(0)} \\
M_{3i} &= \text{EI}(1) R_i^{(2)} - L \omega^2 J R_i^{(1)} \\
M_{4i} &= \text{EI}(1) R_i^{(3)} + \text{EI}(1)^{(1)} R_i^{(2)} + L^2 [P(1) + \omega^2 mr^2(1)] R_i^{(1)} + L^3 \omega^2 M R_i^{(0)}
\end{align*}
\]

(28)
Since the constant vector $\mathbf{a}$ can't be zero, therefore, the governing equation of the system dynamics is given by:

$$|\mathbf{M}|=0$$  \hspace{1cm} (29)

Accordingly, the natural frequencies of the system can be obtained by solving Eq. (29) for $\omega$. Due to the nonlinearity of this transcendental equation, a numerical solution has been implemented using a Matlab function.

3. Results and Discussion

3.1. Validation study

To validate the current proposed recursive differential method, modal analysis will be conducted using the real wind turbine data presented in Table 1 and the results of the current model are compared with the analytical results presented by Adhikari et al. [17]. Noting that, and for the sake of validation, we followed Adhikari et al. [17] in their assumption of constant cross-section. Three non-dimensional parameters is defined here to utilized here and after; which are non-dimensional axial load $\nu = \frac{P H^2}{E I}$, non-dimensional rotational foundation stiffness $\eta_r = \frac{K_r H}{E I}$, and non-dimensional lateral stiffness ($\eta_l = \frac{K_l H^3}{E I}$).

Table 1. Material and geometric properties of the turbine tower (Tempel and Molenaar [12])

| Turbine Structure Properties | Numerical Values |
|-----------------------------|-----------------|
| Length, $H$                 | 81 m            |
| Average Diameter            | 3.5 m           |
| Thickness                   | 0.075 m         |
| Mass Density                | 7800 Kg/m3      |
| Young's Modulus             | 2.1e11 Pa       |
| Top Mass                    | 130000 Kg       |

Four study cases will be presented in order to assess the validity of the current proposed model using ten terms solution. The first case is to study the variation of the fundamental frequency with the non-dimensional axial load $\nu$, while keeping the non-dimensional rotational foundation stiffness and non-dimensional lateral stiffness constant to be $\eta_r = 2$ and $\eta_l = 1$, respectively. Figure 2.a shows the results of the first case and it is found that the results of the current formulation deviate from those of Adhikari et al. [17] by a maximum of 2.1 %. The second case is similar to the first one but the values of the non-dimensional rotational foundation stiffness and non-dimensional lateral stiffness constant are changed to $\eta_r = 500$ and $\eta_l = 100$, respectively. Figure 2.b shows the results of the second case and the maximum deviation found to be 0.21 %. The third case represents the variation of the fundamental frequency with the non-dimensional rotational foundation stiffness $\eta_r$, while assuming $\nu = 0.001$ and $\eta_l = 100$. Figure 2.c shows the results of the second case and the maximum deviation found to be 0.012 %. The fourth case represents the variation of the fundamental frequency with the non-dimensional lateral foundation stiffness $\eta_l$, while assuming $\nu = 1$ and $\eta_r = 5$. Figure 2.d shows the results of the fourth case and the maximum deviation found to be 0.76 %. Therefore, the results of the current formulation almost coincide with those of Adhikari et al. [18].

The natural frequency of the system increases with the increasing values of the stiffness parameters $\eta_r$ and $\eta_l$. This is due to the fact that the increase in the rotational and lateral stiffness properties stabilizes the system. Note that after certain values of $\eta_r$ and $\eta_l$ (typically above 100), a further increase in their values do not change the natural frequency. This is because after these values, the foundation structure interaction can be essentially considered as fixed so that further increase in $\eta_r$ and/or $\eta_l$ has no effect. The natural frequency of the system decreases with the increasing value of $\nu$. This is expected as the increase in the downward axial force essentially drives the system closer to buckling.

The design should be such that the natural frequency of the system should avoid the rotor and blade passing frequency.
3.2. Variable Cross section

The main aim of the current formulation is to present a computationally efficient, yet economic, technique and to study the quantitative effect, and hence the worthiness, of including cross-sectional variation of wind turbine towers on the simulated values of the fundamental frequency. Toward this end, the results of the current formulation, which considers the variation of tower cross-section, will be compared with other simulations which assumed constant cross-section tower. The data of three real wind turbines, presented in Table 2 and provided by Arany et al. [18], will be used as study cases while their experimental values, provided by [18], will be used as a reference for comparison.

A convergence study is performed for the three wind turbines in order to stand at the proper number of terms to use in the recursive differential model. The convergence of the current model is depicted in Table 3 and based on which, the usage of ten terms was found adequate.

Table 4 presents a comparison between the results of the current formulation and the measured fundamental frequency of the three real wind turbines presented in Table 2. It was found that the current model is in good agreement with the measured values of Irene Vorrink 600 kW and Walney 1 S 3.6 MW, showing a maximum error of 4.7 %. Therefore, the current formulation result proved to be closer to the experimental data compared to those presented in [17, 18] which assumes constant cross-section tower. While the result of Lely A2: NM41 2-bladed did not show the same but, surprisingly, it is case with all the other models found in the literature. When this case was further studied, it was found that, all of the simulated frequencies are closer to the frequency of the infinitely stiff foundation, which might question the accuracy of the measured value in this particular case.

In order to demonstrate the advantage, in terms of accuracy, of modeling the tower as variable cross-section, the “Walney 1 S 3.6 MW” wind turbine as study case for comparison. Table 4 presents the results of the current recursive differential method (RDM) of modelling this wind turbine, once modeled as of constant cross section tower with average diameter 4m, then with a variable cross-section and both results are compared with the measured frequency. This comparison shows the worthiness of simulating the tower as of variable cross-section rather than being of constant cross-section, which is reflected by decreasing the error from being 19 %, in the case of constant cross-section, to be 4.2 % in the variable cross-section one.
### Table 2. Technical Data of Three Real Wind Turbines [18]

| Wind Turbine           | Lely A2: NM41 2-bladed | Irene Vorrink 600 kW | Walney 1 S 3.6 MW |
|------------------------|------------------------|----------------------|-------------------|
| **Given and calculated geometric and material data** |                        |                      |                   |
| Equivalent bending stiffness – EI [GNm] | 22                     | 21.5                 | 274               |
| Young's modulus of the tower material – E [GPa] | 210                    | 210                  | 210               |
| Shear modulus of the tower material – G [GPa] | 79.3                   | 79.3                 | 79.3              |
| Tower height – L [m] | 41.5                   | 51                   | 83.5              |
| Bottom diameter – D_b [m] | 3.2                    | 3.5                  | 5                 |
| Top diameter – D_t [m] | 1.9                    | 1.7                  | 3                 |
| Tower wall thickness range – t [mm] | 12                     | 8-14                 | 20-80             |
| Lateral foundation stiffness – K_L [GN/m] | 0.83                   | 0.76                 | 3.65              |
| Rotational foundation stiffness – K_R [GNm/rad] | 20.6                   | 15.5                 | 254.3             |
| Cross-coupling foundation stiffness – K_LR [GN] | -2.22                  | -2.35                | -20.1             |
| Top mass (rotor-nacelle assembly) – [kg] | 32,000                 | 35,700               | 234,500           |
| Tower mass – [kg] | 31,440                 | 31,200               | 260,000           |
| Average wall thickness – t_b [mm] | 12                     | 11                   | 40                |
| Shear coefficient – k [-] | 0.5328                 | 0.5326               | 0.5327            |
| **Non-dimensional parameters** |                        |                      |                   |
| Non-dimensional lateral stiffness – η_l | 2698                   | 5880                 | 7763              |
| Non-dimensional rotational stiffness – η_R | 38.88                  | 39.64                | 77.49             |
| Non-dimensional cross-coupling stiffness – η_LR | -174                   | -284                 | -511.7            |
| Non-dimensional axial force – ν | 0.033                  | 0.03                 | 0.043             |
| Mass ratio – α | 1.018                  | 1.144                | 0.9               |
| Non-dimensional rotary inertia – β | 0                      | 0                    | 0                 |
| Frequency scaling parameter – c_0 | 3.13                   | 2.035                | 1.3454            |
| Non-dimensional radius of gyration – μ | 0.0214                 | 0.01795              | 0.0168            |
| Non-dimensional shear parameter – γ | 4.97                   | 4.97                 | 4.97              |

### Table 3. Convergence Test of RDM Model

| No. of Terms | Lely A2: NM41 2-bladed | Irene Vorrink 600 kW | Walney 1 S 3.6 MW |
|--------------|------------------------|----------------------|-------------------|
| f (Hz)       | R.E. (%)               | f (Hz)               | R.E. (%)          | R.E. (%)          |
| 5            | 0.726148116            | 0.331699881          |                   |
| 6            | 0.737809775            | 1.580578009          |                   |
| 7            | 0.744717471            | 0.92755927           | 0.5025316         | 0.335403036      | 1.10409104 |
| 8            | 0.744717471            | 0                  | 0.5129731         | 2.03547576       | 0             |
| 9            | 0.5203461             | 1.41695532           |                   |
| 10           | 0.5203461             | 0                   |                   | 0               |

### Table 4. Results of the current formulation compared with measured values

| Type                             | Lely A2: NM41 2-bladed | Irene Vorrink 600 kW | Walney 1 S 3.6 MW |
|----------------------------------|------------------------|----------------------|-------------------|
| Measured                         | 0.634                  | 0.546                | 0.35              |
| RDM Current Formulation (Error %) | 0.74 (17.5%)           | 0.52 (4.7%)          | 0.335 (4.2%)      |
| Analytical solution [17]         | 0.74 (16.7%)           | 0.457 (16.3%)        | -                 |
| Euler–Bernoulli [18]             | 0.735 (15.9%)          | 0.456 (16.5%)        | 0.331 (5.9%)      |
| Timoshenko [18]                  | 0.734 (15.8%)          | 0.456 (16.5%)        | 0.331 (5.9%)      |
| Fixed Base frequency             |                        |                      |                   |
| (infinitely stiff foundation)    | 0.765                  | 0.475                | 0.345             |
Table 5. Comparison between the natural frequency of turbine Walney 1 S 3.6 MW, modeled as constant cross-section and variable cross-section tower

| Model | Constant Cross-Section | Variable Cross-Section | Measured Frequency | Relative Error for Variable | Relative Error for Constant |
|-------|-------------------------|-------------------------|--------------------|----------------------------|----------------------------|
| RDM   | 0.28362                 | 0.33540                 | 0.35               | 4.2 %                      | 19 %                       |

4. Conclusion

In this paper, a new semi-analytical formulation is developed which models the dynamic behavior of offshore wind turbine with variable cross section and flexible foundation. The wind turbine tower is modeled using Euler-Bernoulli beam model while the soil-tower interaction is modeled by rotational and lateral springs at the lower end of the tower. Recursive differential method is used to calculate fundamental frequencies of some selected real wind turbines and the results will be compared with their experimentally measured values.

The main aim of the current recursive differential method is to study the quantitative effect, and hence the worthiness, of including cross-sectional variation of wind turbine towers on the simulated values of the fundamental frequency.

The results of the current formulation, was compared with other simulations which assumed constant cross-section tower using the data of three real wind turbines. The results showed that the effect of simulating the tower as of variable cross-section rather than being of constant cross-section, can be significant as the error, of a certain wind turbine case, dropped from being 19 % when using constant cross-section assumption, to be 4.2 % in the variable cross-section one.

Finally, it can be stated that the recursive differential method proved efficient in modelling the dynamics of variable cross-section tower – nacelle systems along with being a promising tool in terms of computational cost.

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