Applications of the numerical methods in mechanical engineering experimental studies

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Abstract. Tremendous advances in computer science led to important progresses in science and technology. The significant feature of a computer, to rapidly and accurately perform calculi, was used for several purposes; the most important influence regarded the applied mathematics concepts where it was lifted the constraint concerning the necessary volume of calculi to be minimal when solving a complex problem. Conceptual advances also influenced the experimental mechanics domain, where numerical methods or numerical solving principles were applied to solve specific problems. The paper presents three original applications of the numerical methods in experimental mechanics. Reusability of the experimental projects’ results saves time and effort. For relevancy purposes many experimental studies’ results are expressed as diagrams. In the first example the approximation theory is used to solve interpolation problems. The output of the data processor consists of the spline function coefficients stored as CSV files or expressed as automatically generated computer codes, ready to be included in upper level applications. The second example is a particular study included in a hybrid model of an internal combustion engine, which outputs the states of strains and stresses in its cylinder block in running conditions. The particular study presents a method to compute the linear thermal expansion coefficient in the points where the temperatures are measured and to compute the temperatures in the inaccessible points, using the values of the previously computed coefficient. The experiment used strain gage rosettes bonded in 10 measurement points. The last original example presents a method to compute the analytical functions of the isostatics using as input data the pattern of isoclinic fringes of a photoelastic model or coating. Deep integration of the analytical, numerical and experimental studies is an important target in hybrid modelling.

1. Introduction
Advances in computer science in the latest 75 years led to a technological leap which produced profound changes. Computers’ main feature, to perform fast and accurate calculi, was used not only in engineering sciences, but it partially removed an important constrain in science, the one regarding the minimization of the volume of calculi when a new concept or theory is being developed. This was beyond the dreams of the creative minds around the world, who are now able to conceive new solving concepts, new theories and new investigation methods. In this way, new computer based calculus instruments were conceived and new computer based experimental equipment was designed. If the theory of the classic technical sciences relied on simplifying hypotheses or assumptions, which were required to minimise the overall volume of calculi, new methods, even based on approximation theory,
proved to be more reliable, i.e. simple, accurate, flexible and applicable, for a wide range of problems. The analogies between fields of engineering and between fields of sciences were reconsidered, this time from the new perspective based on the advances in computer science.

2. Theoretical background

One of the most important aspects regarding the use of the computer was the lifting of the constraints regarding the volume of calculi in the solution of a given problem. This aspect offered the grounds for the development of the computing methods based on the approximation theory. In this way, the numerical methods are employed in those cases where ‘exact’ and direct mathematical methods cannot be used, either because they don’t exist, or because their applicability is limited to some simple case studies.

Numerical methods may be regarded as a new ‘philosophy’ in the development of the computer based scientific methods. Even the computer based approaches are deterministic or randomness based, i.e. seminumerical methods, [1], the extensive use of the computers led to a synergy which generated new modelling concepts.

Figure 1. Directions of development of the knowledge based on numerical methods.

The basic features of the numerical models were used in two directions, figure 1. The first direction is using to the best the effective computing capabilities of the numerical methods. These basic and direct computing capabilities are used for two purposed, i.e. in another two directions. The first one is to develop concepts, theories and applications for general problems, direction which is close to the applied mathematics field. The second direction was to escalate the levels of generality of the simple numerical methods by conceiving general numerical theories, such as the finite element method that uses computational geometry for domain discretization, interpolation functions, integration, methods to solve systems of particular equations etc. This is why the finite element method may be considered a quasi-numerical method which may numerically solve models of the complex phenomena whose governing equations are unknown, in comparison to the finite difference method. These methods are based on general concepts and they are useful to solve a wide range of problems in engineering. Riding the wave of success of the computer science field, this direction of development recorded tremendous advances. Every single stage of data processing was analysed, optimised from the mathematical standpoint, then algorithmically re-invented and finally implemented using the latest concepts and technologies in order to fulfil several optimisation goals. The according software for both, general numerical methods and for the engineering-applied numerical methods became very popular over the time, each university having its own original code, developed by the professors and the PhD students. An additional advance was recorded when the applications were integrated in the same suite of applications which offers software instruments for computer aided activities, such as: design, finite element method based analysis, manufacturing, documents’ management along the entire lifetime of the product and for the project management. The integration of the seminumerical methods was another step on the way to generalization. The market tough competition of the according software companies which release new versions of their products every year is the best indicator regarding the continuous progress in this direction of development.

The second direction, designated ‘solving's principles' applications’, figure 1, is emphasizing a subtler evolution, that is the metaphoric thinking development, starting from the principles of the numerical methods. As it was previously remarked, an important feature of the numerical methods resides in the simplicity of the basic idea used to search an ‘approximative’ solution in the seek-for-
solutions domain. The criteria used to search the solution may be elevated at a metalevel of abstract understanding. A first example is suggested by the bisection method which may be used to find a ‘target’ in a ‘field’ or to diagnose a system made of successive units, using simple criteria. Another example is to progressively narrow the seek-for-solutions domain by applying more accurate stop conditions at each step where a solutions’ domain is being identified and which becomes the seek-for-solutions domain of the next data processing stage, [2]. Error minimisation conditions suggest another ideas regarding the solutions in other fields where the candidate solutions are iteratively evaluated. This approach is more imaginative than the first direction of development, figure 1, and it can be virtually applied in any solver and in any field of science.

3. Original numerical approaches for experimental mechanics’ studies
Both directions of development originating in the numerical methods solutions may be used to solve various problems, in various fields of science, including experimental mechanics.

Three examples are given regarding the original numerical approaches employed in experimental mechanics’ projects.

3.1. Digitization of the experimental studies’ results expressed as diagrams
The data in the graphs must be used to perform calculi, for instance to design a mechanical part. The dimensioning software cannot request the data from the diagrams to be manually inputted at each iteration, therefore the graph must be digitized, i.e. it must be expressed as an analytic function.

The mathematical background and the main features of its implementation are presented in reference [3], the latest updates being presented in [4]. The interpolation uses spline functions, i.e. third degree polynomials. The application has as input data sets of coordinates of the points on the diagram which must be digitized. The output consists of several information: digitised data, coefficients of the spline functions stored in CSV files or expressed as automatically generated computer code in several programming languages.

Let us consider that a diagram identified by the $k$ index is digitized using $N_I^{(k)}$ intervals. The $N_I^{(k)} + 1$ points at the ends of the intervals are $x_i^{(k)}$. For the $i$, $i + 1$ interval, i.e. for $x \in [x_i^{(k)}, x_{i+1}^{(k)}]$, the coefficients of the spline function are $c_{i,j}^{(k)}$, $j \in [0, 3]$ and the according spline function has the form

$$s_i^{(k)}(x) = c_{i,3}^{(k)} \cdot (x-x_i^{(k)})^3 + c_{i,2}^{(k)} \cdot (x-x_i^{(k)})^2 + c_{i,1}^{(k)} \cdot (x-x_i^{(k)}) + c_{i,0}^{(k)} = \sum_{j=0}^3 c_{i,j}^{(k)} \cdot (x-x_i^{(k)})^j.$$ (1)

The $f_j(x)$ function used to analytically model diagram $#k$ is

$$f_j(x) = \sum_{i=1}^{N_I^{(k)}} \left[ \frac{H(x-x_i) - H(x-x_{i+1})}{x_{i+1} - x_i} \sum_{j=0}^3 c_{i,j}^{(k)} \cdot (x-x_i^{(k)})^j \right],$$ (2)

where $H(x-a) = \begin{cases} 0 & x < a \\ \frac{1}{2} & x = a \quad \text{is the Heaviside’s step function.} \\ 1 & x > a \end{cases}$

The results of one of the studies based on the previously presented method are presented in figure 2, [5]. One of the factors which influence the endurance limit is the surface finish factor. Each diagram in figure 2 was drawn for a given type of surface of the mechanical part, i.e. $C1$ – ideally / mirror polished surface; $C2$ – super-finishing or finish turning; $C3$ – grinding or rough turning; $C4$ – hot rolled; $C5$ – corroded in freshwater; $C6$ – corroded in saltwater.
The resulting CSV file of the spline functions’ coefficients store on each line the following values: $i, \alpha_i, \beta_i, \gamma_i, \delta_i, \epsilon_i, \zeta_i, \eta_i, \theta_i, \varphi_i$. Being a text file which uses a general format, i.e. CSV, format this type of file may be read in any programming language.

Using this method, the results of the old experimental projects expressed as diagrams may be used in the actual modern computing context.

3.2. Experimental data reduction technique which uses the bisection method

The experiment is presented in [6] and it was used to measure the strains of the cylinder block of an internal combustion engine in 10 points where three-gage rosettes were bonded. Some of the results were used to calibrate the finite element model by creating an accurate model of the elastic supports. Another results of the experiment were used to verify the finite element model’s results accuracy.

The cylinder block was made of aluminium alloy while the temperature self-compensation of the strain gage rosettes was considered for steel. This means that if the temperature is known, using a given algorithm the results may be corrected for a given linear thermal expansion coefficient considered by the manufacturer of the strain gage transducers. The difference between the material of the cylinder block and the material used for the self-compensation was beneficial since in most of the measurement points the temperature was measured using a non-contact infrared thermometer.

The idea of the algorithm is presented in figure 3. For this particular study, the strains were measured in stationary conditions of the internal combustion engine, i.e. the ICE is stopped. In the stationary conditions the measured strain values represent an ‘apparent’ strain produced by the difference between the linear thermal expansion coefficients and the small errors which may be corrected using the calculus relations provided by the manufacturer. A general function needed to correct the measured strains was defined, $g(\varepsilon, T, \ldots)$, which uses as input data: $\varepsilon$ - the measured strain, $T$ - the temperature in the measurement point. It was also defined $\Delta\alpha = \alpha_{ICE} - \alpha_{Steel}$, where $\alpha_{Steel}$ was provided by the strain gage manufacturer and $\Delta T = T - T_0$, where $T_0$ is the temperature.
measured at a given moment when the engine was not running and all the $e_i$ strains were measured (calibration process). The equation to be solved is

$$g(e, T, \ldots) - \Delta \alpha \cdot \Delta T = 0.$$  \hspace{1cm} (3)

If we know $T$, the temperature, it results $\alpha_{\text{ICE}}$. The temperature is measured in the $R_i, i = 1..7$ points, therefore $\alpha_{\text{ICE}}$ may be computed using several data, the redundancy being useful to check the accuracy of the results.

![Algorithm based on the bisection method which operates using the experimental data.](image)

Figure 3. Algorithm based on the bisection method which operates using the experimental data.

Once we find $\alpha_{\text{ICE}}$, we are able to use the equation (3), this time in order to find $T$, the temperature in the $P_j, j = 1..3$ measurement points.

The experimental measuring procedure conceived and inserted in the planning of the experiment was to stop the engine which was running in steady state conditions for a brief period of time, to immediately record the temperatures and the $e_i$ strains and to restart the engine. In this way is computed the $T$ temperature in the $P_j$ measurement points and used for the correction of the strains measured in dynamic conditions, i.e. in steady state running conditions of the internal combustion engine.

An important target was not to influence the running conditions and the according measured strains in running conditions, being respected the principle “Prima non nocere”. Another target was to gather data, as many as possible, in order to check, double check and over check the results. Moreover, in a measurement point there are 3 strain gages belonging to the same rosette, therefore we can use the common sense hypothesis that the temperatures are close one to the other in all 3 strain transducers. Other common sense assumption is that the material is isotropic and homogeneous, therefore we compute the same value of $\alpha_{\text{ICE}}$ linear thermal expansion coefficient in a given measurement point for all the three strain gages.

This was the first complex study which required a profound integration of the lower level studies and the according original interfaces between them and it is considered a relevant example of hybrid modelling, [7].

3.3. Finding the analytical functions of the isostatics in photoelasticimetry

Photoelasticimetry is an optical method useful for both qualitative and quantitative evaluation of the strains and for both full field evaluation and local strains’ analysis due to the stress concentrators. Along time several experimental data reduction techniques were conceived. A quantitative analysis must firstly assess the directions of the principal strains and then deduce their values using one of the various methods available, [8].
Starting from the recorded pattern of isoclines of a photoelastic model or coating, the analytical functions of the isostatics may be computed using a computer aided original method, [9], which is automatizing a graphical method.

\[ P_{a_j,d} \xrightarrow{\text{Data processor}} f_{a_j} \xrightarrow{\cup} I_{k,m} \xrightarrow{\text{Data processor}} f_k \]

**Figure 4.** Data processing stages of the experimental data.

The ‘\( j \)’ isoclinic of angle \( \alpha_j \), \( j = 1..NI \), is divided by the \( P_{a_j,d} \) points, \( i = 1..NP^{(j)} + 1 \) in \( NP^{(j)} \) line segments. The \( P_{a_j,d} \) points are interpolated using a data processor, figure 4, and it results the \( f_{a_j} \) analytic functions, their form being presented by relation (2). We consider the \( \Delta_{a_j} \) lines of \( \alpha_j \) angles. After the intersection between \( f_{a_j} \) and \( \Delta_{a_j} \) it results the \( I_{k,m} \) intersection points which are interpolated in order to compute the analytical functions of the isostatics, \( f_k \).

**Figure 5.** Results of the original software.

The results of the data processing method may be visualised by giving values to the \( f_{a_j} \) and \( f_k \) functions, the according values being saved in CSV files which are accessed from a spreadsheet application in order to generate the diagrams, figure 5.

In order to have precise results, the initial pattern of isoclinic fringes must be accurately measured and recorded.

Beside this method, there are also ideas regarding the measurement of the principal strains using the results of a finite difference method data processing technique, i.e. another procedure based on a numerical method employed to process experimental data.
4. Discussion
A thorough analysis may reveal the nowadays large spread of the numerical methods, which may be found in original data processing techniques or embedded in smart electronics. In this broad context the experimental data analysis is an important domain where the numerical methods may be also successfully applied.

Experimental studies are paramount for models’ calibration and for the verification of their output data accuracy. However, complex problems require advanced modelling concepts, such as the hybrid modelling where all the studies are highly integrated. The integration is useful for the effective use of the numerical methods in a wide range of studies. For instance, the ‘Data processor’, figure 4, used for spline interpolation, [3, 4], is used to solve the problem presented in section 3.1 as well as the one presented in section 3.3. Moreover, the ‘Same software library’, figure 3, is used to solve the both equations, firstly having the $\alpha$ unknown in some measurement points where the $T$ temperature was measured and secondly to find the $T$ temperature in some inaccessible points, using the previously computed values of $\alpha$.

An important aspect for the successfully use of the software instruments based on numerical methods consists of their reusability, which requires a general mathematical solution, an intelligent algorithm and a flexible implementation, some practical aspects being presented in [10]. Beside these solvers, their integration in the context of the software instruments of a hybrid model requires interfaces and, if necessary, data converters. Moreover, the ‘common database’ concept, mainly considered for the theoretical models, [10], may be reconsidered in order to enhance and intensify the data exchange with the experimental studies that may access/download and provide/upload specific information.

5. Conclusions
We live the age of the computer’s omnipresence. Most of the activities are computer aided or computer based and many original modern concepts rely on the computer as the only calculation instrument to be considered. Few centuries of the computer age are yet to come, and the computer science is the key ferment of development because its science context offers a richer inspiring environment. Many computing modern algorithms are inspired by life, and they become sources of inspiration in the development of life, i.e. of a new reality based on intelligent and metaphoric learning.

In this context, the numerical methods applied in experimental studies is a natural development direction, which was already discovered and emphasized thanks to the vision of the great professors and scientists, [8]. The original approaches presented in the paper are the naturally results of this approach and the directions of development will generalise the actual solutions. For instance, the hybrid model whose experimental study is presented in section 3.2 may be redesigned using more profound approaches that may take into account the processes in an internal combustion engine, the deflected shape of the mechanical parts in running conditions, which may be also modelled using original software instruments, [11-13].

70 years ago Herbert Alexander Simon was conceiving a general problem solver which is today a source of inspiration in several science domains. The conception of an integrating set of principles which links the underlying rules used to conceive a model and the general solving algorithm is a natural demand. The next generation of the numerical methods and of the experimental studies will stimulate creativity and it will bring new original modelling integrating concepts.

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Acknowledgement

The original concepts, methods and software instruments in the paper are subsequent results of the “Computer Aided Advanced Studies in Applied Elasticity from an Interdisciplinary Perspective” 2007-2010 scientific research project supported by the National University Research Council (CNCSIS), Romania, [14], and of its follow-up scientific research project “Mathematical Models for Inter-Domain Approaches with Applications in Engineering and Economy” MIEC 2010-2012, under the supervision of the National Authority for Scientific Research (ANCS), Romania, [15].