General dissipative coefficient in warm intermediate inflation in loop quantum cosmology in light of Planck and BICEP2

Ramón Herrera, Marco Olivares and Nelson Videla

Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile.

(Dated: September 2, 2014)

Abstract

In this paper, we study a warm intermediate inflationary model with a general form for the dissipative coefficient $\Gamma(T, \phi) = C_\phi T^m/\phi^{m-1}$ in the context of loop quantum cosmology. We examine this model in the weak and strong dissipative regimes. In general, we discuss in great detail the characteristics of this model in the slow-roll approximation. Also, we assume that the modifications to perturbation equations result exclusively from Hubble rate. In this approach, we use recent astronomical observations from Planck and BICEP2 experiments to restrict the parameters in our model.

PACS numbers: 98.80.Cq
I. INTRODUCTION

In cosmology our concepts concerning the early universe have introduced a new ingredient, the inflationary phase of the universe, which provides an attractive approach for resolving some of the problems of the standard model of the universe, as the flatness, horizon, etc. [1–6]. Also, it is well known that inflation provides a graceful mechanism to clarify the large-scale structure [7–11] and the observed anisotropy of the cosmic microwave background (CMB) radiation [12–15]. Recently, the effects from BICEP2 experiment of gravitational waves in the B-mode has been analyzed in Ref. [16]. An important observational quantity obtained in this experiment, is the tensor-to-scalar ratio \( r \), which \( r = 0.2^{+0.07}_{-0.05} \) (68 \% C.L.) and takes out the value \( r = 0 \) (at a significance of 7.0 \( \sigma \)). Therefore, the tensor mode should not be neglected.

On the other hand, warm inflation differs from the cold inflation since evades the reheating period at the end of the accelerated evolution of the universe [17]. During warm inflation the process of radiation production could take place under strong enough dissipation [17–26]. In this form, the dissipative effects are important and these emerge from a friction term since the inflaton field is dissipated into a thermal bath. Also, an interesting feature of the warm inflationary model is that the thermal fluctuations constitute a dominant character in producing the primary density fluctuations essential for Large-Scale Structure (LSS) formation [27–31].

In the context of the dissipative effects, a fundamental quantity is the dissipation coefficient \( \Gamma \). In particular, for the scenario of low-temperature, the parameter \( \Gamma \) was analyzed in supersymmetric models. In these models, there is a scalar field together with multiplets of heavy and light fields that give different expressions for the dissipation coefficient, see Refs. [32–37]. A general form for the dissipative coefficient \( \Gamma \), is given by [35, 36].

\[
\Gamma = C_\phi \frac{T^m}{\phi^{m-1}},
\]

where the constant \( C_\phi \) is related with the dissipative microscopic dynamics and the constant \( m \) is an integer. Various elections of \( \Gamma \) or equivalently of \( m \) have been assumed in the written works [35, 36]. In special, for the value of \( m = 3 \), \( C_\phi \) corresponds to \( C_\phi = 0.64 h^4 \mathcal{N} \) in which \( \mathcal{N} = \mathcal{N}_X \mathcal{N}_{X_{\text{decay}}}^2 \). Here, \( \mathcal{N}_X \) is the multiplicity of the \( X \) superfield and \( \mathcal{N}_{X_{\text{decay}}} \) represents the number of decay channels available in \( X \)'s decay [32, 38–40]. For the special case \( m = 1 \),
i.e., the dissipation coefficient $\Gamma \propto T$ corresponds to the high temperature supersymmetry (SUSY) case. For the value $m = 0$, then $\Gamma \propto \phi$ and the dissipation coefficient represents an exponentially decaying propagator in the high temperature SUSY model. For the case $m = -1$, i.e., $\Gamma \propto \phi^2/T$, it agrees with the non-SUSY case [33, 41].

On the other hand, Loop Quantum Gravity (LQG) is a proceeding of nonperturbative background autonomous approach to quantize gravity [42]. In LQC the geometry is discrete and the continuum space-time is found from quantum geometry in a large eigenvalue limit (see, Refs. [43–47]). Different cosmological models have been studied, in particular the Friedmann-Robertson-Walker (FRW) model [48]. Here, the loop quantum effect modifies the Friedmann equation by adding a correction term in the energy density, specifically $\rho^2$ at the scale when $\rho$ becomes similar to a critical density $\rho_c \approx 0.82 G^{-2}$ ($G$ is the Newton’s gravitational constant) [49, 50]. In this way, the effective Friedmann equation becomes

$$H^2 = \frac{\kappa}{3} \rho \left[1 - \frac{\rho}{\rho_c}\right],$$

where $H = \dot{a}/a$ is the Hubble parameter, $a$ is the scale factor, $\kappa = 8\pi G$, $\rho$ is the total energy density, $\rho_c = \sqrt{3} \rho_p/(16\pi^2 \gamma^3)$ is the critical loop quantum density, and $\rho_p = G^{-2}$ is the Planck density. We note that a rigorous numerical test of the Eq. (2) have been performed recently in Ref. [51].

The inflationary universe model in the context of LQC has been analyzed in Refs. [52, 53]. In particular, the inflationary model has been studied in great detail for power-law and multiple fields in the context of LQC [54], while in the Ref. [55] the authors have studied different isotropic and anisotropic space-times for avoiding singularities in LQC. By the other hand, the model of the warm inflation in LQC scenario was studied in Ref. [56], in which the author studied the inflationary scenario described by a standard scalar field coupled to radiation, see also Ref. [57]. For a review of inflationary LQC models, see Refs. [58–62].

On the other hand, exact solutions in inflationary models can be obtained from an exponential potential, frequently called power-law inflation. Here, the scale factor has an expansion power law type, where $a(t) \sim t^p$, where the constant $p > 1$ [63]. As well, an exact solution can be found by using a constant scalar potential which is often called de Sitter inflationary universe [1]. Nevertheless, exact solutions can also be found from intermediate inflation [64]. In this inflationary model, the scale factor grows as

$$a(t) = \exp[At^f],$$

(3)
where $A$ and $f$ are two constants; $A > 0$ and the value of $f$ varies between $0 < f < 1$ [64]. In intermediate inflation the evolution of the scale factor, $a(t)$, is slower than de-Sitter expansion, but quicker than power law, hence the name “intermediate”. This intermediate evolution was originally elaborated as an exact solution, but this model may be best explained from the slow-roll approximation. In the slow-roll approximation, it is possible to obtain a spectral index $n_s \sim 1$ and for the special value of $f = 2/3$, the spectral index correspond to Harrizon-Zel’dovich spectrum, where $n_s = 1$. Also, the quantity obtained in this model, for the tensor-to-scalar ratio is $r \neq 0$ [65, 66].

Thus the goal of the paper is to study an evolving intermediate scale factor during warm inflation in the framework of LQC model together with a generalized form of dissipative coefficient $\Gamma$. We will study the warm intermediate inflationary model in LQC for different values of $m$, and also we will consider this model for two regimes, the weak and the strong dissipative scenarios. In the context of the cosmological perturbations, we will consider for simplicity the procedure of Refs. [53, 56, 57, 67] for warm inflation in LQC, where the perturbation equations arise only from Hubble rate. Also, we only study the standard inflation scenario, that occurs after the superinflation epoch. For a review of superinflation epoch in LQC, see Refs. [53, 67].

The outline of the paper is the follows: The next section presents the basic equations for warm inflation in the framework of LQC model. In the sections III and IV, we discuss the weak and strong dissipative regimes in the intermediate model. In both sections, we give explicit expressions for the scalar field, the dissipative coefficient, the scalar potential, the scalar power spectrum and the tensor-to-scalar ratio. Also, the Planck and BICEP2 data are used to constrain the parameters in both regimes. Finally, our conclusions are presented in section V. We use units in which $c = \hbar = 1$.

II. WARM-LQC INFLATION: BASIC EQUATIONS.

We consider that during warm inflation, the universe is filled with a self-interacting scalar field of energy density $\rho_\phi$ and a radiation field with energy density $\rho_\gamma$. In fact, the total energy density of the universe $\rho$ is given by $\rho = \rho_\phi + \rho_\gamma$.

In the following, we will regard that the energy density associated to the standard scalar field $\rho_\phi$ is given by $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ and the pressure as $P_\phi = \dot{\phi}^2/2 - V(\phi)$. Here, $V(\phi)$
represents the effective potential. Dots mean derivatives with respect to time.

The evolution equations for $\rho_\phi$ and $\rho_\gamma$ in warm inflation are given by [17]

$$\dot{\rho}_\phi + 3H (\rho_\phi + P_\phi) = -\Gamma \dot{\phi}^2,$$

and

$$\dot{\rho}_\gamma + 4H \rho_\gamma = \Gamma \dot{\phi}^2,$$

where, we recall that $\Gamma$, where $\Gamma > 0$, is the dissipation coefficient and it is responsible of the decay of the field $\phi$ into radiation. This dissipation coefficient can be established to be a constant or a function of the temperature of the thermal bath $\Gamma(T)$, the scalar field $\Gamma(\phi)$, or both $\Gamma(T, \phi)$ [17].

During the evolution of warm inflation, the energy density related to the field $\phi$ dominates over the energy density $\rho_\gamma$ [17, 27–30] and, then the Eq.(2) results

$$H^2 \approx \frac{\kappa}{3} \rho_\phi \left[ 1 - \frac{\rho_\phi}{\rho_c} \right] = \frac{\kappa}{3} \left( \frac{\dot{\phi}}{2} + V(\phi) \right) \left[ 1 - \frac{\frac{\dot{\phi}}{2} + V(\phi)}{\rho_c} \right].$$

(6)

Considering Eqs.(4) and (6), we find

$$\dot{\phi}^2 = \frac{2(-\dot{H})}{\kappa(1 + R)} \left[ 1 - \frac{12H^2}{\kappa \rho_c} \right]^{-1/2},$$

(7)

where the ratio between $\Gamma$ and the Hubble parameter $H$ is denoted by $R = \frac{\Gamma}{3H}$. In this sense, for the case of the weak or strong dissipation regime, we make $R < 1$ or $R > 1$, respectively.

We consider that during warm inflation the radiation production is quasi-stable, in which $\dot{\rho}_\gamma \ll 4H \rho_\gamma$ and $\dot{\rho}_\gamma \ll \Gamma \dot{\phi}^2$, see Refs. [17, 27–30]. In this way, utilizing Eqs.(5) and (7), the energy density of the radiation field, yields

$$\rho_\gamma = \frac{\Gamma \dot{\phi}^2}{4H} = \frac{\Gamma (-\dot{H})}{2\kappa H(1 + R)} \left[ 1 - \frac{12H^2}{\kappa \rho_c} \right]^{-1/2} = C_\gamma T^4,$$

(8)

where the constant $C_\gamma = \pi^2 g_s/30$ and $g_s$ denotes the number of relativistic degrees of freedom. Using the above expression for $\rho_\gamma$, we derive that the temperature of the thermal bath $T$, results

$$T = \left[ \frac{\Gamma (-\dot{H})}{2\kappa C_\gamma H(1 + R)} \right]^{1/4} \left[ 1 - \frac{12H^2}{\kappa \rho_c} \right]^{-1/8}.$$

(9)

Moreover, considering, Eqs.(11) and (9) we get that

$$\Gamma \frac{1}{(1 + R)^{\frac{m}{4}}} = C_\phi \left[ \frac{1}{2\kappa C_\gamma} \right]^{\frac{m}{4}} \phi^{-m} \left[ \frac{-\dot{H}}{H} \right]^{\frac{m}{4}} \left[ 1 - \frac{12H^2}{\kappa \rho_c} \right]^{-\frac{m}{4}}.$$
We note that Eq. (10) establishes the dissipation coefficient $\Gamma$ in the weak (or strong) dissipative regime in terms of the scalar field (or the cosmological time).

Otherwise, the scalar potential from Eqs. (2), (7) and (8), becomes

$$V = \frac{\rho_c}{2} \left[ 1 - \sqrt{1 - \frac{12H^2}{\kappa \rho_c}} \right] + \frac{\dot{H}}{\kappa (1 + R)} \left( 1 + \frac{3}{2} R \right) \left[ 1 - \frac{12H^2}{\kappa \rho_c} \right]^{-1/2},$$  \hspace{1cm} (11)

we note that this potential, could be expressed explicitly in terms of the field $\phi$, for the weak (or strong) dissipative regime.

In the following, we will study the warm-LQC model in the context of intermediate expansion for a general form of the dissipation coefficient $\Gamma = C\phi T^m \phi^{-m-1}$ for the cases $m = 3$, $m = 1$, $m = 0$, and $m = -1$. In our analysis, we will restrict ourselves to the weak (or strong) dissipation scenario.

A. The weak dissipative regime.

In the following, we will consider that our model develops according to the weak dissipative regime, in which $\Gamma < 3H$ or equivalently $R < 1$. In this approach, the solution for the standard scalar field $\phi = \phi(t)$, from Eqs. (3) and (7), becomes

$$\dot{\phi}(t) = \phi_0 + \frac{1}{B} \mathcal{F}[t],$$  \hspace{1cm} (12)

where the constant $B \equiv \frac{3}{2} \left( \frac{\kappa (1-f)}{2Af} \right)^{1/2} \left( \frac{\kappa \rho_c}{12A^2 f^2} \right)^{f/4(1-f)}$ and the function $\mathcal{F}[t]$ is given by the expression

$$\mathcal{F}[t] \equiv \left( 1 - \frac{12A^2 f^2}{\kappa \rho_c t^{2(1-f)}} \right)^{3/4} 2F_1 \left[ \frac{3}{4}, \frac{4 - 3f}{4(1-f)}, \frac{7}{4}, 1 - \frac{12A^2 f^2}{\kappa \rho_c t^{2(1-f)}} \right],$$

here, $2F_1$ is the hypergeometric function [68] and $\phi(t = 0) = \phi_0$ is an integration constant that can be taken as $\phi(t = 0) = \phi_0 = 0$ (without loss of generality). Combining Eqs. (3) and (12), the Hubble parameter in terms of the inflaton field, $\phi$, becomes $H(\phi) = \frac{A}{\mathcal{F}^{-1}[B \phi]^{2(1-f)}}$, where $\mathcal{F}^{-1}$ corresponds to the inverse of the hypergeometric function $\mathcal{F}$.

From Eqs. (3), (11) and (12), the scalar potential in this scenario is given by

$$V(\phi) = \frac{\rho_c}{2} \left[ 1 - \sqrt{1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}^{-1}[B \phi])^{2(1-f)}}} \right],$$  \hspace{1cm} (13)
and now considering Eq. (10) the dissipation coefficient $\Gamma$ as a function of the inflaton field, can be written as

$$\Gamma(\phi) = \gamma_0 \phi^{\frac{4(1-m)}{4-m}} \left( \mathcal{F}^{-1}[B \phi] \right)^{\frac{m}{4-m}} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}^{-1}[B \phi])^{2(1-f)}} \right]^{\frac{m}{2(4-m)}}, \quad (14)$$

where the constant $\gamma_0 = C^{\frac{4}{4+m}} \left[ \frac{(1-f)}{2\kappa C_\gamma} \right]^{\frac{m}{4-m}}$ and $m \neq 4$.

On the other hand, considering the dimensionless slow-roll parameters, we have

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1-f}{A \mathcal{F}^{-1}[B \phi] \mathcal{T}}, \quad \text{and} \quad \eta \equiv -\frac{\ddot{H}}{H \dot{H}} = \frac{2-f}{A \mathcal{F}^{-1}[B \phi] \mathcal{T}}.$$

So, the requirement for inflation to occur $\varepsilon < 1$ is satisfied when $\phi > \frac{1}{B} \mathcal{F} \left( \left( \frac{1-f}{A} \right)^{1/f} \right)$. Also, if we consider that inflationary scenario begins at the earliest possible stage, that occurs when $\varepsilon = 1$ (see Ref. [64]), we get that the value for the scalar field at the beginning of inflation is given by $\phi_1 = \frac{1}{B} \mathcal{F} \left( \left( \frac{1-f}{A} \right)^{1/f} \right)$.

In fact, the number of e-folds $N$ between two different values of cosmological time $t_1$ and $t_2$, or equivalently between $\phi_1$ and $\phi_2$, using Eq. (12) is

$$N = \int_{t_1}^{t_2} H \, dt = A \left[ (t_2)^f - (t_1)^f \right] = A \left[ (\mathcal{F}^{-1}[B \phi_2])^f - (\mathcal{F}^{-1}[B \phi_1])^f \right]. \quad (15)$$

In the following, we will analyze the scalar and tensor perturbations for our model in the scenario $R = \Gamma / 3H < 1$. The complex treatment of scalar perturbations of the effective Hamiltonian in LQC can be viewed in Refs. [69, 70]. This analysis is beyond the range of our article and for simplicity we will follow the procedure of Refs. [53, 56, 57] for warm inflation in LQC model. In this form, following Refs. [17, 53, 56, 57] the density perturbation could be written as $P_R^{1/2} = \frac{H}{\phi} \delta \phi$. During warm inflation, a thermalized radiation component is present and the fluctuation $\delta \phi$ is mostly thermal rather than quantum [17, 27–30]. In the weak dissipation regime, i.e., when $R = \Gamma / 3H < 1$, the fluctuation of the inflaton field, $\delta \phi$, is given by $\delta \phi^2 \simeq HT$ [27–30, 71]. In this way, from Eqs. (7), (9), and (10), the power spectrum of the scalar perturbation $P_R$, results

$$P_R = \frac{\sqrt{3\pi}}{4} \left( \frac{C_\phi}{2\kappa C_\gamma} \right)^{\frac{1}{4-m}} \phi^{\frac{1-m}{4-m}} \frac{1}{4-m} \frac{1}{4-m} H^{\frac{11-3m}{4-m}} \left( -\dot{H} \right)^{\frac{3-m}{4-m}} \left( 1 - \frac{12H^2}{\kappa \rho_c} \right)^{\frac{3-m}{2(4-m)}}. \quad (16)$$

Combining Eqs. (12) and (16), we obtain that the power spectrum as function of the scalar field becomes

$$P_R = k_1 \phi^{\frac{1-m}{4-m}} \left( \mathcal{F}^{-1}[B \phi] \right)^{\frac{2f(4-m) + m - 5}{4-m}} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}^{-1}[B \phi])^{2(1-f)}} \right]^{\frac{3-m}{2(4-m)}}, \quad (17)$$
where the constant $k_1$, is given by $k_1 = \frac{\sqrt{\frac{2\pi m}{3}}}{4} \left( \frac{C_\phi}{2\kappa C_g} \right)^{\frac{1}{4-m}} (A f)^{\frac{3-2m}{4-m}} (1-f)^{\frac{m-3}{4-m}}$ or equivalently $P_R$ in terms of the number of e-folds $N$, can be written as

$$P_R(N) = k_2 (\mathcal{F}[J(N)])^{\frac{1-m}{4-m}} (J[N])^{\frac{2f(4-m)+m-5}{4-m}} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}[J[N]])^{2(1-f)}} \right]^{\frac{3-m}{2(4-m)}}, \quad (18)$$

where $J(N)$ and $k_2$ are given by $J(N) = \left[ \frac{1+f(N-1)}{Af} \right]^{\frac{1}{f}}$ and $k_2 = k_1 B^{-\frac{1-m}{4-m}}$, respectively.

The scalar spectral index $n_s$ is defined by $n_s = \frac{d \ln P_R}{d \ln k}$. In this way, from Eqs. (15) and (18) the scalar spectral index $n_s$, yields

$$n_s = 1 - \frac{5-m-2f(4-m)}{A f (4-m)(\mathcal{F}^{-1}[B \phi])^f} + n_2 + n_3, \quad (19)$$

where

$$n_2 = \frac{1-m}{4-m} \sqrt{\frac{2(1-f)}{\kappa A f} (\mathcal{F}^{-1}[B \phi])^{-f/2}} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}^{-1}[B \phi])^{2(1-f)}} \right]^{-1/4},$$

and

$$n_3 = \frac{12A f(1-f)(3-m)}{\kappa \rho_c (4-m)} (\mathcal{F}^{-1}[B \phi])^{-2(1-f)} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}^{-1}[B \phi])^{2(1-f)}} \right]^{-1}.$$

This spectral index $n_s$, also can be written in terms of $N$, results

$$n_s = 1 - \frac{5-m-2f(4-m)}{(4-m)[1+f(N-1)]} + n_{2N} + n_{3N}, \quad (20)$$

where

$$n_{2N} = B \frac{1-m}{4-m} \sqrt{\frac{2(1-f)}{\kappa A f} (J[N])^{-f/2}} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right]^{-1/4},$$

and

$$n_{3N} = \frac{12A f(1-f)(3-m)}{\kappa \rho_c (4-m)} (J[N])^{-(2-f)} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right]^{-1}.$$

On the other hand, the generation of tensor perturbations during the inflationary period would generate gravitational waves [16, 72]. The spectrum of the tensor perturbations is defined by $P_T = 8 \kappa (H/2\pi)^2$. In order to confront this model with observations, we need to consider the tensor-to-scalar ratio, defined as $r = \frac{P_T}{P_R}$. In this way, from Eq. (18), we found that the tensor-to-scalar ratio $r$ is given by

$$r = \frac{P_T}{P_R} = \frac{A^2 f^2}{2 \pi^2 M_p^2 k_1} \phi \frac{1-m}{4-m} (\mathcal{F}^{-1}[B \phi])^{-\frac{3-m}{4-m}} \left[ 1 - \frac{12A^2 f^2}{\kappa \rho_c (\mathcal{F}^{-1}[B \phi])^{2(1-f)}} \right]^{-\frac{3-m}{2(4-m)}}. \quad (21)$$

Now, the ratio $r$, in terms of the number of e-folds $N$, results
\[ r = \frac{A^2 f^2}{2\pi^2 M_p^2 k^2} (\mathcal{F}[J(N)])^{-\frac{4-m}{4}} (J[N])^{-\frac{4-m}{4}} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right]^{-\frac{4-m}{2(4-m)}}. \quad (22) \]

As well, we can find a relation between the ratio \( R = \Gamma / 3H \) and the number of e-folds \( N \). In this form, combining Eqs. (14) and (19), we get

\[ R(N) = \frac{\gamma_0 B^{-\frac{4}{4-m}} (\mathcal{F}[J(N)])^{-\frac{4}{4-m}} (J[N])^{-\frac{4-2m-f(4-m)}{4-m}} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right]^{-\frac{4-m}{2(4-m)}}. \quad (23) \]

In Fig. 1 we show the evolution of \( R = \Gamma / 3H \), the tensor-to-scalar ratio \( r \), and the quantum geometry effects given by the ratio \( \rho / \rho_c \) on the primordial tilt \( n_s \) for the special case in which we set \( m = 3 \) (in which \( \Gamma = C_\phi T^3 / \phi^2 \)), in the warm intermediate LQC for the weak dissipative regime. In all panels we have fixed three different values of the parameter \( C_\phi \). The upper left panel indicates the dependence of \( R = \Gamma / 3H \) on the spectral index during inflation and we also check that the decay of the ratio \( R < 1 \). In the upper right panel, we exhibit the two-dimensional marginalized constraints (68% and 95% CL) from Planck data in combination with Planck + WP Planck CMB temperature likelihood complemented by the WMAP large-scale polarization likelihood (grey), Planck + WP + highL (red), and Planck + WP BAO (blue) [15]. In the lower panel we show the development of the quantum geometry effects in LQC given by the ratio \( \rho / \rho_c \) during the inflationary scenario on the scalar spectral index \( n_s \). In order to write down values for \( R \), \( r \), \( \rho / \rho_c \) and \( n_s \) for the cases \( \Gamma \propto T^3 / \phi^2 \) (\( m = 3 \)), we numerically manipulate Eqs. (2), (14), (19), and (21), in which \( C_\psi = 70 \), \( \rho_c = 0.82 m_p^4 \), and \( \kappa = 1 \). Additionally, we numerically solve Eqs. (18) and (20), and we find that \( A = 4.79 \times 10^{-2} \) and \( f = 0.54 \) for the case of \( C_\phi = 5 \times 10^4 \), in which \( N = 60 \), \( \mathcal{P}_R = 2.43 \times 10^{-9} \) and the scalar spectral index \( n_s = 0.96 \). Similarly, for the value of \( C_\phi = 10^5 \), corresponds to \( A = 3.58 \times 10^{-2} \) and \( f = 0.55 \), and for the value of \( C_\phi = 5 \times 10^6 \) corresponds to \( A = 2.31 \times 10^{-2} \) and \( f = 0.55 \). From the upper left panel we verify that the decay of the rate \( R = \Gamma / 3H < 1 \) for the different values of the parameter \( C_\phi \). From the upper right panel we note that for \( 10^4 < C_\phi < 10^6 \) the model well supported from the Planck data for the case \( m = 3 \), in the weak dissipative regime. Also, from the lower panel we observe that the ratio \( \rho / \rho_c \), which gives the quantum geometry effects in LQC, becomes \( \rho / \rho_c < 5 \times 10^{-8} \). Here, we observe that this value for \( \rho / \rho_c \) becomes small by 2 orders of magnitude when it is compared with the case of standard LQC, in which \( \rho / \rho_c < 10^{-9} \).
In Fig. 2 we show the evolution of the tensor-to-scalar ratio \( r \) on the spectral index \( n_s \), for the cases \( m = 1 \), \( m = 0 \) and \( m = -1 \) in the warm LQC intermediate weak dissipative scenario. In all panels we use three different values of the parameter \( C_\phi \). In the upper left panel we use \( m = 1 \), in the upper right panel \( m = 0 \), and in the lower panel \( m = -1 \). In the upper right and lower panels, we exhibit the two-dimensional marginalized constraints (68% and 95% CL) from BICEP2 data in combination with Planck + WP + highL [16]. We note that the BICEP2 data places stronger limits on the tensor-to-scalar ratio \( r \) versus \( n_s \) compared with the Planck data. In order to write down values that relate \( n_s \) and \( r \), as before, we numerically solve (19) and (21), where \( C_\gamma = 70 \), \( \rho_c = 0.82m_p^4 \), and \( \kappa = 1 \). For the special case \( m = 1 \), i.e., \( \Gamma \propto T \), we numerically solve Eqs. (18) and (20), and we obtain that \( A = 2.44 \) and \( f = 0.28 \) correspond to \( C_\phi = 10^{-11} \), in which \( N = 60 \), \( \mathcal{P}_\mathcal{R} = 2.43 \times 10^{-9} \), and the scalar spectral index \( n_s = 0.96 \). Similarly, for the value of \( C_\phi = 10^{-10} \) corresponds to \( A = 2.01 \) and \( f = 0.29 \), and for \( C_\phi = 10^{-4} \) corresponds to \( A = 0.78 \) and \( f = 0.28 \). We note that for the value of the parameter \( C_\phi > 10^{-11} \) the model is well supported by the Planck data (upper left panel) in the warm LQC intermediate weak regime. Here, we note that for the value of \( C_\phi = 10^{-4} \) the tensor-to-scalar ratio becomes \( r \sim 0 \). Also, we observe that for the case \( m = 1 \) the value \( C_\phi < 10^{-4} \) is well supported by the condition of the weak dissipative regime, i.e., \( R = \Gamma/3H < 1 \) (figure not shown). Thereby, for the special case \( m = 1 \) the constraint obtained for \( C_\phi \) is \( 10^{-11} < C_\phi < 10^{-4} \). In order to describe the quantum geometric effect in LQC for the special case \( m = 1 \), we numerically find that the rate \( \rho/\rho_c \) becomes \( \rho/\rho_c \sim 10^{-7} \) for \( C_\phi = 10^{-11} \) evaluated at \( n_s = 0.96 \). For the value of the parameter \( C_\phi = 10^{-10} \) corresponds to \( \rho/\rho_c \sim 10^{-8} \) and for value of \( C_\phi = 10^{-4} \) corresponds to \( \rho/\rho_c \sim 10^{-11} \) (figure not shown).

For the value \( m = 0 \) in which \( \Gamma \propto \phi \), as before we numerically solve Eqs. (18) and (20), and we find that \( A = 3.12 \) and \( f = 0.26 \) correspond to \( C_\phi = 10^{-18} \), where as before \( N = 60 \), \( \mathcal{P}_\mathcal{R} = 2.43 \times 10^{-9} \), and the scalar spectral index \( n_s = 0.96 \). Similarly, for the value of \( C_\phi = 10^{-16} \) we obtain \( A = 2.51 \) and \( f = 0.25 \). For the value of \( C_\phi = 10^{-10} \) we find \( A = 1.21 \) and \( f = 0.26 \). As before, we observe that for the value of the parameter \( C_\phi > 10^{-19} \) the model for \( m = 0 \) is well confirmed by the BICEP2 data (middle panel). Here, we observe that for the value of \( C_\phi = 10^{-10} \) the tensor-to-scalar ratio becomes \( r \sim 0 \). Additionally, we note that for this case of \( m \) the value of the parameter \( C_\phi < 10^{-10} \) is well supported by the condition of the weak dissipative regime, in which the rate \( R = \Gamma/3H < 1 \) (not shown). Therefore,
for the case $m = 0$ we obtain for the parameter $C_\phi$ the constraint $10^{-19} < C_\phi < 10^{-16}$ from BICEP2 data. Also, we numerically obtain that the correction term $\rho/\rho_c$ that gives the notion of the quantum geometric effects in LQC, becomes $\rho/\rho_c \sim 10^{-8}$ for $C_\phi = 10^{-18}$ evaluated at $n_s = 0.96$. For the value of $C_\phi = 10^{-16}$ corresponds to $\rho/\rho_c \sim 10^{-9}$ and for $C_\phi = 10^{-10}$ corresponds to $\rho/\rho_c \sim 10^{-11}$ (figure not shown).

For the case $m = -1$ or equivalently $\Gamma \propto \phi^2/T$, as before we numerically solve Eqs. (18) and (20), and we obtain the values $A = 4.2$ and $f = 0.24$ for the parameter $C_\phi = 10^{-26}$. For the value of $C_\phi = 10^{-22}$ corresponds to $A = 2.8$ and $f = 0.23$, and for $C_\phi = 10^{-16}$ corresponds to $A = 1.6$ and $f = 2.4$. We find that for the value of the parameter $C_\phi > 10^{-27}$ the model for $m = -1$ is well confirmed by BICEP2 data (lower panel). Also, we note that for this dissipation coefficient the value $C_\phi < 10^{-16}$ is well supported by the condition of the weak dissipative regime, i.e., $R = \Gamma/3H < 1$ (not shown). Therefore, for the special case $m = -1$, we find for the parameter $C_\phi$ the constraint $10^{-27} < C_\phi < 10^{-22}$ from BICEP2 data. As before, we numerically get that the correction term $\rho/\rho_c$ becomes $\rho/\rho_c < 10^{-8}$, and we note that this value is the same order of magnitude when it is compared with the obtained by standard LQC scenario [53].

**B. The strong dissipative regime.**

In this section we analyze the strong dissipative regime ($R = \Gamma/3H > 1$), and as before our model will remains in this regime until the end of inflation. In the following, we will consider the exact solutions for the separate cases in which $m = 3$ and $m \neq 3$. Combining Eqs. (3), (7) and (10), the solution for the scalar field $\phi(t)$ in the case $m = 3$ is given by

$$\phi(t) - \phi_0 = \exp \left[ \frac{F[t]}{C} \right],$$

where the constant $C = \frac{15}{8} \left( \frac{\kappa C_\phi}{6} \right)^{1/2} \left( \frac{3}{2\kappa C_\gamma} \right)^{3/8} \left( \frac{1}{A f} \right)^{5/8} (1 - f)^{7/8} \left( \frac{\kappa \rho_c}{12A^2T^2} \right)^{(2+5f)/16(1-f)}$. The function $F[t]$ is proportional to the hypergeometric function and it is defined as $F[t] \equiv \left( 1 - \frac{12A^2f^2}{\kappa \rho_c T^2(1-f)} \right)^{15/16} 2F_1 \left[ \frac{15}{16}, \frac{18-11f}{16(1-f)}, \frac{31}{16}, 1 - \frac{12A^2f^2}{\kappa \rho_c T^2(1-f)} \right]$.

For values of $m \neq 3$, the new solution for the scalar field and redefining $\varphi(t) = \frac{2}{3-m} \phi(t)^{\frac{3-m}{m}}$, yields

$$\varphi(t) - \varphi_0 = \frac{F_m[t]}{C_m},$$

(25)
FIG. 1: The evolution of the ratio $R = \Gamma/3H$ versus the primordial tilt $n_s$ (upper left panel), the evolution of the tensor-to-scalar ratio $r$ versus $n_s$ (upper right panel), and the evolution of the ratio $\rho/\rho_c$ versus $n_s$ (lower panel) in the warm-LQC intermediate weak dissipative regime for the case $m = 3$ in which $\Gamma \propto T^3/\phi^2$. In all panels we have taken three different values of the parameter $C_\phi$ and also we have used, $\rho_c = 0.82m_p^4$, $C_\gamma = 70$, and $\kappa = 1$. In the upper right panel, we show the two-dimensional marginalized constraints (68% and 95% CL) on inflationary parameters $r$ and $n_s$, derived from Planck data [15].

where $C_m = \left(\frac{12+m}{8}\right)\left(\frac{\kappa C_\phi}{6}\right)^\frac{1}{2}\left(\frac{3}{2\kappa C_\gamma}\right)^\frac{m}{8}\left(\frac{1}{Af}\right)^\frac{(8-m)}{8}(1-f)^\frac{(m+4)}{8}\left(\frac{\kappa \rho_c}{12A^2f^2}\right)^\frac{[2m-4+f(8-m)]}{16(1-f)}$ and the new function $F_m[t] \equiv \left(1 - \frac{12A^2f^2}{\kappa_r \ell^2(1-f)}\right)^{(12+m)/16}$, $2F_1\left[\frac{12+m}{16}, \frac{12+2m-f(8+m)}{16(1-f)}, \frac{28+m}{16}, 1 - \frac{12A^2f^2}{\kappa_r \ell^2(1-f)}\right]$. As before, without loss of generality, we will consider $\varphi(t = 0) = \varphi_0 = 0$. The Hubble parameter as function of the inflaton field for the case $m = 3$, is given by $H(\phi) = \frac{Af}{(F_1^{-1}(C_m \varphi))]^{1-\frac{1}{f}}}$, and for the case in which $m \neq 3$, results $H(\phi) = \frac{Af}{(F_m^{-1}(C_m \varphi))]^{1-\frac{1}{f}}}$. Here, in both cases, $F^{-1}$ (or $F_m^{-1}$) represents the inverse function of $F[t]$ (or $F_m[t]$).
FIG. 2: Evolution of the tensor-to-scalar ratio $r$ versus the primordial tilt $n_s$ for the cases $m=1$ (upper left panel), $m=0$ (upper right panel), and $m=-1$ (lower panel) in the warm-LQC intermediate weak dissipative regime. As before, in all panels we have used three different values of the parameter $C_\phi$ and also we have taken $\rho_c = 0.82m_p^4$, $C_\gamma = 70$, and $\kappa = 1$. Also, in all panels, we show the two-dimensional marginalized constraints (68% and 95% CL) on inflationary parameters $r$ and $n_s$, derived from Planck (upper left panel) [15] and BICEP2 (upper right and lower panels) in combination with other data sets [16].

The scalar potential as function of the scalar field, considering Eqs. (11) and (25) yields

$$V(\phi) = \frac{\rho_c}{2} \left[ 1 - \sqrt{1 - \frac{12A^2 f^2}{\kappa \rho_c(F^{-1}[C\ln \phi])^2(1-f)}} \right], \text{ for } m = 3$$

(26)

and

$$V(\phi) = \frac{\rho_c}{2} \left[ 1 - \sqrt{1 - \frac{12A^2 f^2}{\kappa \rho_c(F^{-1}_m[C_m\phi])^2(1-f)}} \right], \text{ for } m \neq 3.$$  (27)

Analogously, as in the weak dissipative regime, the coefficient $\Gamma = \Gamma(\phi)$ considering
Eqs. [14], (24) and (25) becomes

\[ \Gamma(\phi) = v \phi^{-2} \left( F^{-1}[C \ln \phi] \right)^{-\frac{3}{2}(f-1)} \left[ 1 - \frac{12A^2f^2}{\kappa \rho_c (C \ln \phi)^{2(1-f)}} \right]^{-\frac{3}{8}}, \quad \text{for } m = 3, \]  

(28)

where \( v = C_\phi \left( \frac{3Af(1-f)}{2\kappa C_\gamma} \right)^{3/4} \). The dissipation coefficient for the case \( m \neq 3 \) is given by

\[ \Gamma(\phi) = v_m \phi^{1-m} \left( F_m^{-1}[C_m \varphi] \right)^{-\frac{m(2-f)}{4}} \left[ 1 - \frac{12A^2f^2}{\kappa \rho_c (F_m^{-1}[C_m \varphi])^{2(1-f)}} \right]^{-\frac{m}{8}}, \]  

(29)

where \( v_m = C_\phi \left( \frac{3Af(1-f)}{2\kappa C_\gamma} \right)^{m/4} \).

As before, for the dimensionless slow-roll parameters, we write \( \varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1-f}{AF(F^{-1}[C \ln \phi])^f}, \) for \( m = 3 \) and \( \tilde{\varepsilon} = \frac{1-f}{AF(F_m^{-1}[C_m \varphi])^f}, \) for \( m \neq 3 \). The \( \eta \) parameter becomes

\[ \eta \equiv -\frac{\dot{H}}{H^2} = \frac{2-f}{AF(F^{-1}[C \ln \phi])^f}, \]  

(30)

for \( m = 3 \) and \( \tilde{\eta} = \frac{2-f}{AF(F_m^{-1}[C_m \varphi])^f}, \) for \( m \neq 3 \).

The inflation scenario is only satisfied when the scalar field becomes \( \phi > \exp \left[ \frac{1}{C} F \left( \left( \frac{1-f}{A} \right)^{1/f} \right)^{f} \right] \) (for \( m = 3 \)), and \( \varphi > \frac{1}{C_m} F_m \left[ \left( \frac{1-f}{A} \right)^{1/f} \right] \) (for \( m \neq 3 \)).

The number of e-folds \( N \) between two different values of the scalar field \( \phi_1 \) and \( \phi_2 \), from Eqs. (31), (24) and (25) results in

\[ N = A \left[ F^{-1}[C \ln \phi_2]^f - (F^{-1}[C \ln \phi_1])^f \right], \]  

(32)

for \( m = 3 \) and \( N = A \left[ F_m^{-1}[C_m \varphi_2]^f - (F_m^{-1}[C_m \varphi_1])^f \right], \) for \( m \neq 3 \). As in the weak regime, we consider that the inflationary scenario begins at the earliest possible, then the value \( \phi_1 = \exp \left[ \frac{1}{C} F \left( \left( \frac{1-f}{A} \right)^{1/f} \right)^{f} \right] \) (for the case \( m = 3 \)), and \( \varphi_1 = \frac{1}{C_m} F_m \left[ \left( \frac{1-f}{A} \right)^{1/f} \right] \) (for \( m \neq 3 \)).

On the other hand, as in the weak regime, the density perturbation could be written as \( \mathcal{P}_R^{1/2} = \frac{H}{\phi} \delta \phi [17], \) where \( \delta \phi \) in the case of strong dissipation is given by \( (\delta \phi)^2 = HT \sqrt{3R/2\pi^2} [36]. \) In this form, combining the Eqs. (7), (9) and (10) in the regime \( R \gg 1 \), the expression for the power spectrum of the scalar perturbation becomes

\[ P_R \simeq \sqrt{\frac{\pi}{2}} C_\phi^3 \frac{3}{8} \left( \frac{3}{2\kappa C_\gamma} \right)^{\frac{3}{8}} \phi^{\frac{3(1-m)}{2}} H^\frac{3}{2} \left( -\dot{H} \right)^{\frac{3m-6}{8}} \left( 1 - \frac{12A^2f^2}{\kappa \rho_c} \right)^{\frac{3m-6}{8}}. \]  

(33)

As for the previous expressions, we need to separate the cases \( m = 3 \) and \( m \neq 3 \). Replacing Eqs. (3), (24), and (25) in Eq. (33), we can express the power spectrum in terms of the scalar field for the two cases, and we obtain

\[ \mathcal{P}_R = K_1 \phi^{-3} (F^{-1}[C \ln \phi])^{\frac{3(5-f)}{8}} \left[ 1 - \frac{12A^2f^2}{\kappa \rho_c (F^{-1}[C \ln \phi])^{2(1-f)}} \right]^{-\frac{3}{16}}, \]  

for \( m = 3, \)

(34)

where \( K_1 = \frac{\sqrt{\pi}}{2} C_\phi^\frac{3}{8} \left( \frac{3}{2\kappa C_\gamma} \right)^{\frac{11}{8}} (Af)^\frac{11}{8} (1-f)^\frac{3}{8} \) and
\[ \mathcal{P}_R = K_m \phi \frac{3(1-m)}{2} \left( F^{-1}[C_m \phi] \right)^{\frac{3f(2+m-2m)}{8}} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (F_m^{-1}[C_m \phi])^{2(1-f)}} \right]^{(3m-6)_{16}}, \text{ for } m \neq 3, \]  

where \( K_m = \frac{\sqrt{2}}{2} C^\phi_0 \left( \frac{n}{6} \right)^{\frac{3m}{2}} \left( Af \right)^{\frac{3m+6}{8}} \left( 1 - f \right)^{\frac{3m}{8}}. \)

By other hand, it is possible rewrite the scalar power spectrum in terms of the number of e-folds \( N \), then using Eqs. (31) and (32) we get

\[ \mathcal{P}_R = K_1 \exp \left( -\frac{3}{C} F[J[N]] \right) \left( J[N] \right)^{\frac{3(5f-6)}{8}} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right]^{-\frac{3}{16}}, \text{ for } m = 3, \]  

and

\[ \mathcal{P}_R = K_m \left[ \frac{(3 - m)}{2} \frac{F_m[J[N]]}{C_m} \right]^{\frac{3(1-m)}{2}} \left( J[N] \right)^{\frac{3f(2+m-2m)}{8}} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right]^{\frac{(3m-6)}{16}}, \text{ for } m \neq 3. \]  

Now, the scalar spectral index as a function of the scalar field, considering Eqs. (3), (24), (25), and (30), becomes

\[ n_s = 1 + \frac{3}{8 Af (F^{-1}[C \ln \phi])^f} \bar{n}_2 + \bar{n}_3, \text{ for } m = 3, \]  

where

\[ \bar{n}_2 = -3 \left( \frac{6}{\kappa C_\phi} \right)^{\frac{1}{2}} \left( \frac{3Af}{2\kappa C_\phi} \right)^{-\frac{3}{8}} (1 - f)^{\frac{3}{8}} \left( F^{-1}[C \ln \phi] \right)^{2-3f} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (F^{-1}[C \ln \phi])^{2(1-f)}} \right]^{-\frac{1}{16}}, \]  

and

\[ \bar{n}_3 = -\frac{9}{2} \frac{Af(1-f)}{\kappa \rho_c} \left( F^{-1}[C \ln \phi] \right)^{f-2} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (F^{-1}[C \ln \phi])^{2(1-f)}} \right]^{-1}. \]  

The expression for \( m \neq 3 \) yields

\[ n_s = 1 + \frac{3}{8 Af (F_m^{-1}[C_m \phi])^f} \bar{n}_2 + \bar{n}_3, \]  

where

\[ \bar{n}_2 = -3 \left( \frac{m-1}{2} \right) \left( \frac{6}{\kappa C_\phi} \right)^{\frac{1}{2}} \left( \frac{3Af}{2\kappa C_\phi} \right)^{-\frac{m}{8}} (1 - f)^{\frac{m}{8}} \left( F_m^{-1}[C_m \phi] \right)^{\frac{(m-2)+4}{8}} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (F_m^{-1}[C_m \phi])^{2(1-f)}} \right]^{-\frac{(4-m)}{16}}, \]  

and

\[ \bar{n}_3 = -\frac{3(1-m)}{2} \frac{Af(1-f)}{\kappa \rho_c} \left( F_m^{-1}[C_m \phi] \right)^{f-2} \left[ 1 - \frac{12 A^2 f^2}{\kappa \rho_c (F_m^{-1}[C_m \phi])^{2(1-f)}} \right]^{-1}. \]
Analogously as in the weak regime, the scalar spectral index can be expressed in terms of the number of e-folds $N$, obtaining

$$n_s = 1 + \frac{3(5f - 6)}{8Af(J[N])^f} + \tilde{n}_2 + \tilde{n}_3,$$

for the case $m = 3$. Here, $\tilde{n}_2$ and $\tilde{n}_3$ are given by

$$\tilde{n}_2 = -3 \left(\frac{6}{\kappa C_{\phi}}\right)^\frac{3}{2} \left(\frac{3Af}{2\kappa C_{\phi}}\right)^{-\frac{3}{2}} (1 - f)^\frac{3}{8} \left(J[N]\right)^{2-3f} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(J[N])^{2(1-f)}}\right]^{-\frac{3}{16}},$$

and

$$\tilde{n}_3 = -\frac{9}{2} \frac{Af(1 - f)}{\kappa \rho_c} (J[N])^{f-2} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(J[N])^{2(1-f)}}\right]^{-1}.$$

The spectral index for the case $m \neq 3$, results

$$n_s = 1 + \frac{3[f(m + 2) - 2m]}{8Af(J[N])^f} + \tilde{n}_2 + \tilde{n}_3,$$

where

$$\tilde{n}_2 = -\frac{3(m - 1)(12 + m)(1 - f)}{8} \frac{\kappa \rho_c}{Af} \left(\frac{2A^2f^2}{12(1-f)}\right)^\frac{12(1-f)}{4m^2} \left(J[N]\right)^{-\frac{m(m-2)+6m}{8}} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(J[N])^{2(1-f)}}\right]^{-\frac{(4-m)}{16}}$$

and

$$\tilde{n}_3 = -\frac{3(1-m)}{2} \frac{Af(1 - f)}{\kappa \rho_c} (J[N])^{f-2} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(J[N])^{2(1-f)}}\right]^{-1}.$$

On the other hand, for the strong dissipative regime, the power spectrum of the tensor perturbations is given by $P_g = 8\kappa(H/2\pi)^2$. Using Eqs.(31) and (32) we may write the tensor-to-scalar ratio $r$ as

$$r = \frac{P_g}{P_R} = \frac{A^2f^2}{2\pi^2 M_p^2 K} \phi^3 (F^{-1}[C \ln \phi])^\frac{f+2}{4} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(F^{-1}[C \ln \phi])^{2(1-f)}}\right]^{\frac{1}{16}}, \text{ for } m = 3 \quad (39)$$

and

$$r = \frac{A^2f^2}{2\pi^2 M_p^2 K} \phi^\frac{3(m-1)}{2} (F^{-1}[C_m \varphi])^\frac{16 + (10 - 3m) + 6m}{16} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(F^{-1}[C_m \varphi])^{2(1-f)}}\right]^{\frac{3m-6}{16}}, \text{ for } m \neq 3. \quad (40)$$

Analogously, the tensor-to-scalar ratio as function of the number of e-folds $N$ results as

$$r(N) = \frac{A^2f^2}{2\pi^2 M_p^2 K} \exp \left(\frac{3}{C}F[J[N]]\right) (J[N])^\frac{f+2}{8} \left[1 - \frac{12A^2f^2}{\kappa \rho_c(J[N])^{2(1-f)}}\right]^{\frac{1}{16}}, \text{ for } m = 3 \quad (41)$$
FIG. 3: The evolution of the ratio $R = \Gamma/3H$ versus the primordial tilt $n_s$ (upper left panel), the evolution of the tensor-to-scalar ratio $r$ versus $n_s$ (upper right panel), and the evolution of the ratio $\rho/\rho_c$ versus $n_s$ (lower panel) in the warm-LQC intermediate-strong dissipative regime for the case $m = 3$, i.e., $\Gamma \propto T^3/\phi^2$. In all panels we used three different values of the parameter $C_\phi$ and also we have used, $\rho_c = 0.82m_p^4$, $C_\gamma = 70$, and $\kappa = 1$. In the upper right panel, we show the two-dimensional marginalized constraints (68% and 95% CL) on inflationary parameters $r$ and $n_s$, derived from BICEP2 data [16].

and

$$r(N) = \frac{A^2f^2}{2\pi^2M_p^2K_m} \left[ \frac{(3 - m)}{2 \frac{F_m[J[N]]}{C_m}} \right] \frac{3(1-m)}{3-m} \frac{(J[N])^{1-16+(10-3m)\frac{+6m}{8}}}{1 - \frac{12A^2f^2}{\kappa\rho_c(J[N])^{2(1-f)}}} \right\}^{\frac{3m-6}{16}},$$

for $m \neq 3$.

As in the weak regime, the analytical relation for the dissipation ratio $R = \Gamma/3H$ between the number of e-folds is given by

$$R(N) = \frac{\delta}{3Af} \exp \left( -\frac{2}{C}F[J[N]] \right) (J[N])^{-\frac{(2+\gamma_f)}{4}} \left[ 1 - \frac{12A^2f^2}{\kappa\rho_c(J[N])^{2(1-f)}} \right]^{-\frac{3}{8}}, \text{ for } m = 3,$$
and

\[
R(N) = \frac{\delta}{3Af} \left[ \frac{(3 - m) F_m[J[N]]}{C_m} \right] \frac{2(1-m)}{3m} \left( J[N] \right)^{\frac{4(1-f)-m(2-f)}{2(1-f)}} \left[ 1 - \frac{12A^2f^2}{\kappa \rho_c (J[N])^{2(1-f)}} \right] \frac{1}{(1-f)^{\frac{1}{2}}} , \text{ for } m \neq 3.
\]

In Fig.3 we show the dependence of \( R = \Gamma / 3H \), the tensor-to-scalar ratio \( r \), and the ratio \( \rho / \rho_c \) on the primordial tilt \( n_s \) for the special case in which we fix \( m = 3 \), in the warm LQC strong dissipative regime. In all panels we consider three different values of the parameter \( C_\phi \). In the upper left panel we show the evolution of \( R = \Gamma / 3H \) during the inflationary epoch and we also check that the decay of the ratio \( R > 1 \). In the upper right panel, we exhibit the two-dimensional marginalized constraints (68% and 95% CL) from Planck [15] and BICEP2 in combination which other data sets [16]. In the lower panel we show the development of the quantum geometry effects in LQC given by \( \rho / \rho_c \) during the inflationary scenario. In order to write down values for \( R \), \( r \), \( \rho / \rho_c \), and \( n_s \) for the value \( m = 3 \), i.e., \( \Gamma \propto T^3/\phi^2 \), we manipulate numerically the Eqs. (2), (28), (35), and (39) in which \( C_\gamma = 70 \), \( \rho_c = 0.82m_p^4 \), and \( \kappa = 1 \). Additionally, we numerically solve Eqs. (33) and (41) and we obtain \( A = 1.7 \times 10^{-7} \) and \( f = 0.96 \) for the case of \( C_\phi = 5 \times 10^8 \), in which \( N = 60 \), \( P_R = 2.43 \times 10^{-9} \) and \( n_s = 0.96 \). Similarly, for the value of \( C_\phi = 10^9 \), we get \( A = 3.6 \times 10^{-7} \) and \( f = 0.9 \), and for the value of \( C_\phi = 2 \times 10^9 \) corresponds to \( A = 5.2 \times 10^{-7} \) and \( f = 0.85 \). From the upper left panel we observe that the value \( C_\phi > 5 \times 10^8 \) is well confirmed by the strong regime \( (R > 1) \) and this value corresponds to an upper bound for \( C_\phi \). From the upper right panel we observe that for \( C_\phi < 2 \times 10^9 \) is well supported by the BICEP2 data. In this form for the value \( m = 3 \), the constraint for the parameter \( C_\phi \) becomes \( 5 \times 10^8 < C_\phi < 2 \times 10^9 \) for the strong regime in warm intermediate model in LQC. Also, from the lower panel we note that the quantum geometry effects in LQC given by the ratio \( \rho / \rho_c \) is \( \rho / \rho_c < 3 \times 10^{-14} \). Additionally we note that this inequality for \( \rho / \rho_c \) becomes small by 5 orders of magnitude when it is compared with the case of standard LQC, in which \( \rho / \rho_c < 10^{-9} \) [53].

For the case \( m = 1 \), in which \( \Gamma \propto T \), we find that the value of \( C_\phi > 0.03 \) is well supported by the strong dissipative regime, i.e., \( R > 1 \), but at the same time the tensor-to-scalar ratio \( r \sim 0 \). In particular, for the value \( C_\phi = 0.1 \) we numerically obtain that \( A = 1.08 \), \( f = 0.21 \) and the tensor-to-scalar ratio \( r \sim 5.4 \times 10^{-8} \). For the other values of \( m \)-parameter, we note that for the cases \( m = 0 \) and \( m = -1 \) i.e., \( \Gamma \propto \phi \) and \( \Gamma \propto \phi^2 / T \), the models of the warm intermediate LQC in the strong dissipative regime are ruled out from the Planck data and BICEP2, because the spectral index \( n_s > 1 \) and hence the models do not work.
TABLE I: Results for the constraints on the parameter $C_\phi$ and the quantum geometry effects in LQC given by $\rho/\rho_c$, in the weak and strong regimes.

| Regime | $\Gamma = C_\phi T_m/\phi^{m-1}$ | Constraint on $C_\phi$ | Constraint on $\rho/\rho_c$ |
|--------|----------------------------------|-------------------------|-----------------------------|
| Weak   | $m = 3$                          | $5 \times 10^4 < C_\phi < 5 \times 10^5$ | $< 1.47 \times 10^{-8}$     |
|        | $m = 1$                          | $10^{-11} < C_\phi < 10^{-4}$ | $< 9.12 \times 10^{-8}$     |
|        | $m = 0$                          | $10^{-19} < C_\phi < 10^{-16}$ | $< 3.73 \times 10^{-8}$     |
|        | $m = -1$                         | $10^{-27} < C_\phi < 10^{-22}$ | $< 7.62 \times 10^{-8}$     |
| Strong | $m = 3$                          | $5 \times 10^8 < C_\phi < 2 \times 10^9$ | $< 2 \times 10^{-14}$      |
|        | $m = 1$                          | The model does not work | -                           |
|        | $m = 0$                          | The model does not work | -                           |
|        | $m = -1$                         | The model does not work | -                           |

Table I indicates the constraints on the parameter $C_\phi$ and the quantum geometry effects in LQC given by $\rho/\rho_c$, in the weak and strong regimes and different choices of the parameter $m$, for a general form of $\Gamma = C_\phi T_m/\phi^{m-1}$, in the context of warm-intermediate LCQ inflationary universe models.

III. CONCLUSIONS

In this paper we have analyzed the intermediate inflationary scenario in the context of warm inflation in LQC. During the slow-roll approximation and considering a general form of the dissipative coefficient $\Gamma(\phi, T) = C_\phi T^m/\phi^{m-1}$, we have found solutions of the Friedmann equations for a flat universe filled with a self-interacting scalar field and a radiation field in the weak and strong dissipative regimes. In special, we researched the values $m = 3$, $m = 1$, $m = 0$, and $m = -1$. From the warm-intermediate inflationary model in LQC, we have found explicit relations for the corresponding scalar potential $V(\phi)$, spectrum of the scalar perturbations $P_R$, scalar spectral index $n_s$, and tensor-to-scalar ratio $r$ in the weak and strong dissipative regimes.

In order to bring some explicit results we have considered the constraint in the $n_s - r$
plane given by the two-dimensional marginalized constraints (68% and 95% C.L.) derived from Planck and BICEP2 in combinations with other data sets. Here, we noted that the BICEP2 data places stronger limits on the tensor-to-scalar ratio $r$ versus $n_s$ compared with the Planck data. Also, we obtained a constraint for the value of the parameter $C_\phi$ analyzed in the weak and strong regimes, and from these scenarios we have found an upper bound for $C_\phi$. Additionally, we observed that when we reduce the parameter $m$ the value of the parameter $C_\phi$ also decreases. In particular, for the strong dissipative regime, we found that for the cases in which $m = 0$ and $m = -1$, i.e., for $\Gamma \propto \phi$ and $\Gamma \propto \phi^2/T$, these models of the warm-intermediate LQC are ruled out from Planck and BICEP2 data, since the spectral index $n_s > 1$, and hence the models do not work. On the other hand, for the weak dissipative regime, the quantum geometry effects in LQC, given by the correction term $\rho/\rho_c$ becomes similar than the reported in the standard LQC scenario. For the strong dissipative regime the results found indicate that the effect of the correction term $\rho/\rho_c$ on the warm inflationary model is marginal. Nevertheless, it cannot be rejected that future experiments uncover it. Our results for both regimes are summarized in Table I. Also, given that the rate $R = \Gamma/3H$ will also evolve during inflation, we may have also models which start in the weak dissipative regime $R < 1$ but end in the strong regime, in which $R > 1$, or the other way round. In this paper, we have not studied these dynamics. Besides, we should mention that we have not addressed a complex treatment of the scalar perturbations of the effective Hamiltonian in LQC, in this sense, we have considered that the modifications to perturbation equations arise exclusively from Hubble rate $\rho/\rho_c$. We hope to return to these points in the near future.

Acknowledgments

R.H. was supported by COMISION NACIONAL DE CIENCIAS Y TECNOLOGIA through FONDECYT grant N° 1130628 and by DI-PUCV N° 123724. N.V. was supported by Proyecto Beca-Doctoral CONICYT N° 21100261.

[1] A. Guth, Phys. Rev. D 23, 347 (1981).
[2] A.A. Starobinsky, Phys. Lett. B 91, 99 (1980).
[3] A.D. Linde, Phys. Lett. B 108, 389 (1982).
[4] A.D. Linde, Phys. Lett. B 129, 177 (1983).
[5] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); A. Linde, Particle Physics and inflationary cosmology, Gordon and Breach, New York, 1990.
[6] K. Sato, Mon. Not. Roy. Astron. Soc. 195, 467 (1981).
[7] V.F. Mukhanov and G.V. Chibisov, JETP Letters 33, 532 (1981).
[8] S. W. Hawking, Phys. Lett. B 115, 295 (1982).
[9] A. Guth and S.-Y. Pi, Phys. Rev. Lett. 49, 1110 (1982).
[10] A. A. Starobinsky, Phys. Lett. B 117, 175 (1982).
[11] J.M. Bardeen, P.J. Steinhardt and M.S. Turner, Phys. Rev. D 28, 679 (1983).
[12] D. Larson et al., Astrophys. J. Suppl. 192, 16 (2011).
[13] C. L. Bennett et al., Astrophys. J. Suppl. 192, 17 (2011).
[14] G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. Suppl. 208, 19 (2013).
[15] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].
[16] P. A. R. Ade et al. [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO]; P. A. R. Ade et al. [BICEP2 Collaboration], arXiv:1403.4302 [astro-ph.CO].
[17] A. Berera, Phys. Rev. Lett. 75, 3218 (1995).
[18] A. Berera, Phys. Rev. D 55, 3346 (1997).
[19] J. Mimoso, A. Nunes and D. Pavon, Phys. Rev. D 73, 023502 (2006).
[20] R. Herrera, S. del Campo and C. Campuzano, JCAP 10, 009 (2006).
[21] S. del Campo, R. Herrera and D. Pavon, Phys. Rev. D 75, 083518 (2007).
[22] S. del Campo and R. Herrera, Phys. Lett. B 653, 122 (2007).
[23] M. A. Cid, S. del Campo and R. Herrera, JCAP 10, 005 (2006).
[24] J. C. B. Sanchez, M. Bastero-Gil, A. Berera and K. Dimopoulos, Phys. Rev. D 77, 123527 (2008).
[25] R. Herrera, Phys. Rev. D 81, 123511 (2010).
[26] R. Herrera, E. San Martin, Eur. Phys. J. C 71, 1701 (2011).
[27] L.M.H. Hall, I.G. Moss and A. Berera, Phys. Rev. D 69, 083525 (2004).
[28] I.G. Moss, Phys. Lett. B 154, 120 (1985).
[29] A. Berera and L.Z. Fang, Phys. Rev. Lett. 74, 1912 (1995).
[30] A. Berera, Nucl. Phys. B 585, 666 (2000).
[31] A. Berera, Phys. Rev.D 54, 2519 (1996).
[32] I. G. Moss and C. Xiong, arXiv:hep-ph/0603266.
[33] A. Berera, M. Gleiser and R. O. Ramos, Phys. Rev. D 58 123508 (1998).
[34] A. Berera and R. O. Ramos, Phys. Rev. D 63, 103509 (2001).
[35] Y. Zhang, JCAP 0903, 023 (2009).
[36] M. Bastero-Gil, A. Berera and R. O. Ramos, JCAP 1107, 030 (2011).
[37] M. Bastero-Gil, A. Berera, R. O. Ramos and J. G. Rosa, JCAP 1301, 016 (2013).
[38] J. C. Bueno Sanchez, M. Bastero-Gil, A. Berera and K. Dimopoulos, Phys. Rev. D 77, 123527 (2008); R. O. Ramos and L. A. da Silva, JCAP 1303, 032 (2013); R. Cerezo and J. G. Rosa, JHEP 1301, 024 (2013).
[39] A. Berera, I. G. Moss and R. O. Ramos, Rept. Prog. Phys. 72, 026901 (2009); M. Bastero-Gil and A. Berera, Int. J. Mod. Phys. A 24, 2207 (2009).
[40] M. Bastero-Gil, A. Berera and R. O. Ramos, JCAP 1109, 033 (2011); S. del Campo and R. Herrera, JCAP 0904, 005 (2009); R. Herrera and M. Olivares, Int. J. Mod. Phys. D 21, 1250047 (2012); M. Bastero-Gil, A. Berera, I. G. Moss and R. O. Ramos, arXiv:1401.1149 [astro-ph.CO].
[41] J. Yokoyama and A. Linde, Phys. Rev D 60, 083509, (1999); R. Herrera, M. Olivares and N. Videla, Phys. Rev. D 88, 063535 (2013).
[42] T. Thiemann , Lect. Notes Math. 631, 41 (2003); A. Ashtekar and J. Lewandowski , Class. Quant. Grav. 21, R53 (2004).
[43] A. Ashtekar and P. Singh, Class. Quant. Grav. 28, 213001 (2011) .
[44] M. Bojowald , Living Rev. Rel. 8, 11 (2005).
[45] M. Bojowald , Phys. Rev. Lett. 86, 5227 (2001).
[46] A. Ashtekar , M. Bojowald and J. Lewandowski, Adv. Theo. Math. Phys. 7, 233 (2003).
[47] M. Bojowald, G. Date, K. Vandersloot, Class. Quantum Grav. 21, 1253 (2004).
[48] A. Ashtekar, T. Pawlowski, P. Singh and K. Vandersloot, Phys. Rev. D 75, 024035 (2007); P. Singh, Class. Quant. Grav. 26, 125005 (2009) ; P. Singh and F. Vidotto, Phys. Rev. D 83, 064027 (2011) ; P. Singh, Phys. Rev. D 85, 104011 (2012); A. Joe and P. Singh, arXiv:1407.2428 [gr-qc].
[49] P. Singh, Phys. Rev. D 73, 063508 (2006) gr-qc/0603043.
[50] A. Ashtekar, T. Pawlowski and P. Singh, Quantum nature of the big bang, Phys. Rev. Lett. 22
96, 141301 (2006).

[51] P. Diener, B. Gupt and P. Singh, Class. Quant. Grav. 31, 105015 (2014).
[52] P. Singh, K. Vandersloot and G. V. Vereshchagin, Phys. Rev. D 74, 043510 (2006).
[53] Zhang X. and Ling Y., JCAP 0708, 012 (2007).
[54] E. Ranken and P. Singh, Phys. Rev. D 85, 104002 (2012).
[55] B. Gupt and P. Singh, Class. Quant. Grav. 30, 145013 (2013).
[56] R. Herrera, Phys. Rev. D 81, 123511 (2010).
[57] X. -M. Zhang and J. -Y. Zhu, Phys. Rev. D 87, no. 4, 043522 (2013).
[58] A. A. Sen, Phys. Rev. D 74 043501 (2006).
[59] Xiong H. H. and Zhu J. Y., Phys. Rev. D 75 084023 (2007).
[60] Xiao K. and Zhu J. Y., Phys. Lett. B 699 217 (2011).
[61] Chen S., Wang B. and Jing J., Phys. Rev. D 78, 123503 (2008); Wu P. and Zhang S. N., JCAP 0806, 007 (2008).
[62] J. Mielczarek, T. Cailleteau, J. Grain and A. Barrau, Phys. Rev. D 81, 104049 (2010); M. Bojowald, G. Calcagni and S. Tsujikawa, JCAP 1111, 046 (2011); L. Linsefors and A. Barrau, Phys. Rev. D 87, 123509 (2013); M. Artymowski, A. Dapor and T. Pawlowski, JCAP 1306, 010 (2013).
[63] F. Lucchin and S. Matarrese, Phys. Rev. D32, 1316 (1985).
[64] J. D Barrow, Phys. Lett. B 235, 40 (1990); J. D Barrow and P. Saich, Phys. Lett. B 249, 406 (1990); A. Muslimov, Class. Quantum Grav. 7, 231 (1990); A. D. Rendall, Class. Quantum Grav. 22, 1655 (2005); J. D Barrow and N. J. Nunes, Phys. Rev. D 76 043501 (2007); J. D Barrow and A. R. Liddle, Phys. Rev. D 47, R5219 (1993); A. A. Starobinsky JETP Lett. 82, 169 (2005); S. del Campo, R. Herrera, J. Saavedra, C. Campuzano and E. Rojas, Phys. Rev. D 80, 123531 (2009); R. Herrera and N. Videla, Eur. Phys. J. C 67, 499 (2010); M. Bastero-Gil and A. Berera, Int. J. Mod. Phys. A 24, 2207 (2009); R. Herrera and E. San Martin, Eur. Phys. J. C 71, 1701 (2011); R. Herrera and M. Olivares, Mod. Phys. Lett. A 27, 1250101 (2012); R. Herrera, M. Olivares and N. Videla, Eur. Phys. J. C 73, 2295 (2013); R. Herrera, M. Olivares and N. Videla, Eur. Phys. J. C 73, 2475 (2013).
[65] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, Phys. Rev. D 74, 023502 (2006).
[66] J. D. Barrow, A. R. Liddle and C. Pahud, Phys. Rev. D, 74, 127305 (2006); R. Herrera and E. San Martin, Int. J. Mod. Phys. D 22, 1350008 (2013).
[67] M. Bojowald, Phys. Rev. Lett. 89, 261301 (2002); S. Tsujikawa, P. Singh and R. Maartens, Classical Quantum Gravity 21, 5767 (2004); E. J. Copeland, D. J. Mulryne, N. J. Nunes and M. Shaeri, Phys. Rev. D 77, 023510 (2008).

[68] Abramowitz, M. and Stegun, I. A. (Eds.). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 9th printing. New York: Dover, 1972; Arfken, G. "The Incomplete Gamma Function and Related Functions." Mathematical Methods for Physicists, 3rd ed. Orlando, FL: Academic Press, 1985.

[69] M. Bojowald, H. H. Hernandez, M. Kagan, P. Singh and A. Skirzewski, Phys. Rev. D 74, 123512 (2006); E.W. Ewing, Classical Quantum Gravity 29, 085005 (2012); M. Bojowald and G. M. Hossain, Phys. Rev. D 78, 063547 (2008); T. Cailleteau, J. Mielczarek, A. Barrau, and J. Grain, Classical Quantum Gravity 29, 095010 (2012).

[70] M. Bojowald, G. Calcagni and S. Tsujikawa, Phys. Rev. Lett. 107, 211302 (2011).

[71] A. Berera, Nucl. Phys. B 585, 666 (2000).

[72] K. Bhattacharya, S. Mohanty and A. Nautiyal, Phys.Rev.Lett. 97, 251301 (2006).