Remarks on an arbitrage–free condition for XVA

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Abstract
After the financial crisis, it has been widely recognized that counterparty default risks have serious consequences and that there are remarkably large differences in various interest rates. As a result, the methodology for pricing derivative securities has been modified: Currently, the price of a derivative security is expressed as the so-called XVA, which is the risk-neutral price plus total valuation adjustment. In this paper, we aim to provide a theoretical interpretation of XVA: Some valuation adjustments are interpreted as the “0th-order” approximation of XVA. Further, we describe a sufficient condition to ensure the arbitrage-free property of the 0th-order price approximation.

Keywords backward stochastic differential equation, nonlinear market model, xva

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1. Setup

In this paper, we use a model similar to the one in [1].

1.1 Market Model

On the probability space \((\Omega, \mathcal{F}, \mathbb{Q})\), the 1-dimensional Brownian motion \(W := (W_t)_{t \geq 0}\) is given. Let \(\mathcal{F}_t := \sigma(W_s; s \in [0, t]) \vee \mathcal{N}, t \geq 0\), where \(\mathcal{N}\) is the totality of null sets. Let \(\tau_I\) be the default time of an investor and \(\tau_C\) be the default time of a counterparty, which are exponentially distributed random variables with parameters \(h_I\) and \(h_C\), respectively. We assume that \(W, \tau_I, \text{ and } \tau_C\) are mutually independent. For \(i \in \{I, C\}\), we define the processes

\[
H_i^t := 1_{\{\tau_i \leq t\}} \quad \text{and} \quad M_i^t := H_i^t - \int_0^t (1 - H_i^s) h_i \, ds,
\]

where the latter is a martingale with respect to \(\mathcal{H}_t := \sigma(H_i^s, H_C^s, s \in [0, t]), t \geq 0\). Moreover, we define the filtration \(\mathcal{G}_t := \mathcal{F}_t \vee \mathcal{H}_t, t \geq 0\), which is interpreted as the market information flow. On the filtered probability space \((\Omega, \mathcal{F}, \mathbb{Q}, \mathcal{G})\), where \(\mathcal{G} := (\mathcal{G}_t)_{t \geq 0}\), \(W\) is a \(\mathcal{G}\)-Brownian motion and \(M_I^t\) and \(M_C^t\) are \(\mathcal{G}\)-martingales. We see that \(W, M_I^t, \text{ and } M_C^t\) are mutually orthogonal.

Next, we define the price processes of a non-defaultable risky asset, the defaultable bond of the investor’s firm, and the defaultable bond of the counterparty’s firm, which are denoted by \(S := (S_t)_{t \geq 0}\), \(P_I := (P_I^t)_{t \geq 0}\), and \(P_C := (P_C^t)_{t \geq 0}\), respectively. We assume these are the respective solutions to the following stochastic differential equations (SDEs) on \((\Omega, \mathcal{F}, \mathbb{Q}, \mathcal{G})\).

\[
dS_t = r_D S_t \, dt + \sigma S_t \, dW_t, \quad S_0 \in \mathbb{R}_{++},
\]

\[
dP_I^t = r_D P_I^t \, dt - P_I^t \, dM_I^t, \quad P_I^0 \in \mathbb{R}_{++},
\]

\[
dP_C^t = r_D P_C^t \, dt - P_C^t \, dM_C^t, \quad P_C^0 \in \mathbb{R}_{++},
\]

where \(r_D > 0\) and volatility \(\sigma > 0\) are constants. We regard the probability measure \(\mathbb{Q}\) as the pricing measure, and \(r_D\) is the risk-free rate, which is used for discounting in the valuation. Further, we define cash account processes \(B_I^t := (B_I^t)_{t \geq 0}\), \(B_C^t := (B_C^t)_{t \geq 0}\), and \(B_0 := (B_0^t)_{t \geq 0}\) by

\[
dB_I^t = r_I(\xi^t_I) B_I^t \, dt, \quad B_I^0 = 1,
\]

where \(r_I(x) = r_I^- 1_{x<0} + r_I^+ 1_{x>0}\). Here, \(r_I^- \in \mathbb{R}_{++}\) is the borrowing rate, \(r_I^+ \in \mathbb{R}_{++}\) is the lending rate, and \(\xi^t_I\) is the number of shares of the corresponding account at time \(t\). We denote by \(\gamma_I(\xi^t_I)\) the funding rate at time \(t\); \(r_I(\xi^t_I)\), the repo rate at time \(t\); and \(r_C(\xi^t_C)\), the collateral rate at time \(t\).

1.2 Derivative: Close Out Value and Collateral

Consider a European derivative contract between an investor and a counterparty. Let \(\Phi : \mathbb{R} \to \mathbb{R}\) be a payoff function that satisfies \(|\Phi(x)| \leq C(|x|^k + 1)\) for all \(x \in \mathbb{R}\) and for some \(C > 0, k \geq 1\). We assume this condition for \(\Phi\) throughout the paper. For a constant \(T \in \mathbb{R}_{++}\), let \(\tau := \tau_I \wedge \tau_C \wedge T\), which defines the maturity of a derivative contract, and let \(\tilde{V}_t := \mathbb{E}[e^{-\tau_D(T-t)} \Phi(S_T)|\mathcal{F}_t]\), which is used to calculate a collateral value at time \(t\). Further, we assume

\[
\xi_t^C B_C^t = -\alpha \tilde{V}_t, \quad \alpha \in [0, 1] \text{ is a given constant.}
\]

where \(\alpha \in [0, 1]\) is a given constant. On the set \(\{\tau_I \wedge \tau_C > T\}\), the investor pays (receives) \(\Phi(S_T)\) to (from) the counterparty at the prescribed horizon \(T\), and on the set \(\{\tau_I \wedge \tau_C \leq T\}\), the investor pays (receives) the value of a derivative netted of the posted collateral to (from) the investor at the default time \(\tau_I \wedge \tau_C\), as given by

\[
\Theta(\tilde{V}_t) := 1_{\{\tau_I \wedge \tau_C \leq T\}} \theta(\tilde{V}_t \wedge \tau_C) + 1_{\{\tau_I \wedge \tau_C > T\}} \Phi(S_T).
\]
\[
\theta(\tilde{V}_{\tau_t \wedge \tau_c}) := [\tilde{V}_{\tau_t \wedge \tau_c} + 1_{\{\tau_c < \tau_t\}} L_t((1 - \alpha)\tilde{V}_{\tau_c})^{-1} - 1_{\{\tau_t < \tau_c\}} L_t((1 - \alpha)\tilde{V}_{\tau_t})^{-1}]
\]

where \(L_t, L_C \in [0, 1]\) are given constants, which are the loss rate of the investor and the counterparty, respectively.

2. Wealth Process and BSDE

Let \(\varphi = (\xi_t, \xi^I_t, \xi^C_t, \xi^f_t, \xi^v_t; t \geq 0)\) be an investment strategy of a self-financing investor. Here, \(\xi_t\) denotes the number of shares of the risky asset \(S\) at time \(t\), \(\xi^I_t\) and \(\xi^C_t\) denote the number of shares of investor’s risky bond \(P^I\) at time \(t\) and the counterparty’s risky bond \(P^C\) at time \(t\), respectively. The wealth process of the self-financing investor \(V_t(x, \varphi)\) is given by \(V_0(x, \varphi) = x\),

\[
dV_t(x, \varphi) = \xi_t dS_t + \xi^I_t dP^I_t + \xi^C_t dP^C_t + \xi^f_t dB^f_t + \xi^v_t dB^v_t,
\]

subject to

\[
V_t(x, \varphi) = \xi_t S_t + \xi^I_t P^I_t + \xi^C_t P^C_t + \xi^f_t B^f_t + \xi^v_t B^v_t - \xi^a_t B^a_t,
\]

For the derivative described in Section 1.2, we want to find a self-financing strategy \((x, \varphi)\) such that

\[
V_t(x, \varphi) = \Theta(\tilde{V}_t)
\]

and that (1), (3) and (4) hold. Now, we set \(V_t := V_t(x, \varphi), Z_t := \sigma^\xi S_t, \) and \(Z^i_t := -\xi^I_t P^I_t, \) \(i \in \{I, C\}\).

Using (1), (3) and (4), we rewrite (2) and (5) as the following backward stochastic differential equations (BSDEs):

\[
\begin{align*}
\begin{cases}
-dV_t = f \left(t, V_t, Z_t, Z^I_t, Z^C_t : \tilde{V}_t \right) dt - Z_t dW_t - Z^I_t dM^I_t - Z^C_t dM^C_t \\
V_T = \Theta(\tilde{V}_T)
\end{cases}
\end{align*}
\]

Here, the driver function \(f\) is defined by

\[
f(t, v, z, z^I, z^C : \tilde{V}) :=
\begin{align*}
&[-r^v_f(v + z + z^C - \alpha \tilde{V})^+ - r^f(v + z^I + z^C - \alpha \tilde{V})^- \\
&+ (r_d - r^*_v) \sigma^{-1} z^I + (r_d - r^*_f) \sigma^{-1} z^C \\
&- r_d z^I - r_d z^C + r^*_f (\alpha \tilde{V})^+ + r^*_c (\alpha \tilde{V})^-]
\end{align*}
\]

with \(x^+ := \max(0, x)\) and \(x^- := \min(0, -x)\). Note that the terminal value \(\Theta(\tilde{V}_T)\) and the driver function \(f\) satisfy Assumption 12. Hence, from Theorem 1, BSDEs (6) have a unique solution \((V, Z, Z^I, Z^C) \in \mathbb{H}^{2,3}_{\beta,T} \times \mathbb{H}^{2,3}_{\beta,T}\) for any sufficiently large \(\beta > 0\), where

\[
\mathbb{S}^{2,3}_{\beta,T} := \{(Y_t)_{t \in [0,T]} \mid \text{\(G\)-adapted, RCLL-process, } E\int_0^T e^{\beta t} |Y_t|^2 dt < \infty \}
\]

\[
\mathbb{H}^{2,3}_{\beta,T} := \{(U_t)_{t \in [0,T]} \mid \text{\(d\)-dim. } \mathbb{G}\text{-predictable process, } E\int_0^T e^{\beta t} |U_t|^2 dt < \infty \}
\]

3. Arbitrage-free Condition and Approximated Price

Results in Subsection 3.1 have been already seen in [1], although we need to modify the proofs and the definition of admissible trading strategies as we are in “nonlinear” wealth dynamics. Results in Subsection 3.2, that is, Theorem 9 and 10, are main contributions of this paper. Throughout this section, we assume the following.

Assumption 1

(i) \(r^v_f \leq r^f \leq r^v_c\).

(ii) \(r^v_f \leq r^f \).

(iii) \(r^v_f - r_d < h_i \wedge h_C\).

(iv) \(r^v_c \leq r^v_c\).

Remark 2

In Assumption 1, (i) and (ii) seem to be natural conditions from a practitioner’s viewpoint, whereas (iii) and (iv) seem to be restrictive conditions. In fact, we require (iii) for utilizing a comparison theorem for BSDEs. (See Appendix, where we can show that Assumption 1 (iii) implies Assumption 12 (iv).) Moreover, forcing condition (iii) might be unnatural from a practitioner’s viewpoint: for example, the referee of this paper suggests as follows: “(iii) means that the investor can make a profit by borrowing money from the funding account and investing into his own defaultable bond. Although this is not an arbitrage trading strategy, there is nothing to limit this”. We are interested in seeking to derive more realistic conditions, or developing a theory, where certain “small arbitrages” are admitted. They are important candidates of future research topics.

3.1 Arbitrage-free Condition

We introduce BSDEs for a seller (denoted by the superscript “+”) and BSDEs for a buyer (denoted by the superscript “−”),

\[
\begin{align*}
-dV_t^\pm &= f^\pm \left(t, V_t^\pm, Z_t^\pm, Z_t^I, Z_t^C ; \tilde{V} \right) dt - Z_t^- dW_t - Z_t^I dM_t^I - Z_t^C dM_t^C \\
V_T^\pm &= \Theta(\tilde{V}_T),
\end{align*}
\]

where

\[
\begin{align*}
f^+(t, v, z, z^I, z^C ; \tilde{V}) := f(t, v, z, z^I, z^C ; \tilde{V}), \\
f^-(t, v, z, z^I, z^C ; \tilde{V}) := -f^+(t, -v, -z, -z^I, -z^C ; -\tilde{V}).
\end{align*}
\]

Using Theorem 14, we obtain the following.

Proposition 3

It holds that \(V_T^- \leq V_0^+\).

Next, we state the following definitions.

Definition 4

A self-financing trading strategy \(\varphi := \left(\xi, \xi^I, \xi^C, \xi^f, \xi^v; t \geq 0\right)\) which satisfies (2), (4), and (5), is called admissible if \((\xi^I_S, \xi^I_P^I, \xi^C_P^C)_{t \in [0,T]} \in \mathbb{H}^{2,3}_{\beta,T}\) for any sufficiently large \(\beta > 0\).

Definition 5 (Seller’s arbitrage)

We say that the derivative price \(p \in \mathbb{R}_+\) admits a seller’s arbitrage if there exists an admissible strategy \(\varphi\) such that \(\mathbb{Q}(V_T(p, \varphi) \geq \Theta(\tilde{V}_T)) = 1\).

Definition 6 (Buyer’s arbitrage)

We say that the derivative price \(p \in \mathbb{R}_+\) admits a buyer’s arbitrage if
there exists an admissible strategy $\phi = -\psi$ such that
\[ \mathbb{Q}(V_r(-p, \phi) \geq -\Theta(V_r)) = 1, \quad \mathbb{Q}(V_r(-p, \phi) > -\Theta(V_r)) > 0. \]

We then obtain the following proposition.

**Proposition 7** $V_0^+$ is the maximal arbitrage-free price
for sellers and $V_0^-$ is the minimal arbitrage-free price
for buyers. Hence, the price $p \in \mathbb{R}$ of the derivative admits
no arbitrage for both buyers and sellers if and only if $V_0^- \leq p \leq V_0^+$.

**Remark 8** $V_0^+ - \hat{V}_0$ is called the seller’s XVA and
$V_0^- - \hat{V}_0$ is called the buyer’s XVA at time 0.

**Proof (Sketch of proof.)** The proof consists of two parts.

(i) Assuming $p \notin [V_0^-, V_0^+]$, we show the price $p$ admits
a seller’s or a buyer’s arbitrage. The proof is similar
to that of [2, Proposition 2.1].

(ii) Assuming that $p \in [V_0^-, V_0^+]$ and that $p$ admits
a seller’s or a buyer’s arbitrage, we derive a
contradiction from (1), (2), (3), (4) and Definition 4 by
using Girsanov’s theorem. \(\text{(QED)}\)

### 3.2 Approximated Prices

In this subsection, we construct an approximation of the
derivative price via a perturbation method of
the associated BSDEs (see [3]). For the approximated price,
we give an interpretation from a practical viewpoint and,
moreover, discuss its arbitrage-free property. Let $\tau_i \in \mathbb{R}_{++}$ and $0 < \epsilon_i < 1, i \in \{f, r\}$. Suppose $r_f = \tau_f \neq \epsilon_f$; that is, the borrowing and the lending rate are
decided from a reference rate. Then, we can decompose the driver function $f^\pm$ of the BSDEs (7) as follows.

\[
f^\pm(t, v, z, z^C, \hat{V}) :=
\begin{align*}
&f^\pm_0(t, v, z, z^C, \hat{V}) + \epsilon_f f^\pm_{1,r}(t, v, z^C, \hat{V}) + \epsilon_r f^\pm_{1,f}(z),
\end{align*}
\]
where $f^\pm_0$, $f^\pm_{1,f}$ and $f^\pm_{1,r}$ are defined by
\[
f^\pm_0(t, v, z, z^C, \hat{V}) :=
\begin{align*}
&-|r_f(v + z + z^C - \alpha \hat{V}_t) + (r_D - r_f)\sigma^- z
- r_D z^t - r_D z^C \pm r_c^+ (\pm \alpha \hat{V}_t \pm r_c^- (\pm \alpha \hat{V}_t^-)),
\end{align*}
\]
\[
f^\pm_{1,f}(t, v, z^C, \hat{V}) :=\pm \sigma^- |z|,
\]
further, we consider the following linear BSDEs.

\[
\begin{align*}
-dV^\pm_0 &=
\begin{cases}
-f^\pm_0(t, V^\pm_0, Z^\pm_0, Z^\pm_1, Z^\pm_2, \hat{V}) dt \\
-Z^\pm_0 dW_t - Z^\pm_0 dM^I_t - Z^\pm_2 dM^C_t \\
V^\pm_0 &= \Theta(\hat{V}_r),
\end{cases}
\end{align*}
\]
\[
\begin{align*}
-dV^\pm_{1,f} &=
\begin{cases}
f^\pm_{1,f}(t, V^\pm_{1,f}, Z^\pm_{1,f}, Z^\pm_2, \hat{V}) dt \\
+ f^\pm_{1,f}(t, V^\pm_{1,f}, Z^\pm_{1,f}, Z^\pm_2, \hat{V}) dt \\
-Z^\pm_{1,f} dW_t - Z^\pm_{1,f} dM^I_t - Z^\pm_2 dM^C_t \\
V^\pm_{1,f} &= 0,
\end{cases}
\end{align*}
\]
\[
\begin{align*}
-dV^\pm_{1,r} &=
\begin{cases}
f^\pm_{1,r}(t, V^\pm_{1,r}, Z^\pm_0, Z^\pm_1, Z^\pm_2, \hat{V}) dt \\
+ f^\pm_{1,r}(t, V^\pm_{1,r}, Z^\pm_0, Z^\pm_1, Z^\pm_2, \hat{V}) dt \\
-Z^\pm_{1,r} dW_t - Z^\pm_{1,r} dM^I_t - Z^\pm_2 dM^C_t \\
V^\pm_{1,r} &= 0.
\end{cases}
\end{align*}
\]

From an a priori estimation of BSDEs, we obtain following.

**Theorem 9** For any $\beta \in \mathbb{R}_{++},$
\[
||V^\pm - V_0^0 ||_\beta = O(\varepsilon),
\]
\[
||V^\pm - V_0^0 ||_\beta = O(\varepsilon^2),
\]
where $\varepsilon := \epsilon_f \vee \epsilon_r$ and the norm $|| \cdot ||_\beta$ is defined by
\[
||V^\pm ||^2_\beta = E \int_0^T e^{{\beta}{\hat{V}^2_s}} ds.
\]

Note that we can obtain a similar error estimation for the $Z$-parts of BSDEs (7). Hence, we refer to the solution of BSDEs (9) as the 0th-order approximation in terms of $\varepsilon$ for the solution of BSDEs (7) and we refer to the solution of the BSDEs (10) and (11) as the 1st-order approximation for the solution of BSDE (7), that is,
\[
V^\pm_t \approx V_t^0 + \varepsilon_f V^1_{t,f} + \varepsilon_r V^1_{t,r},
\]
\[
Z^\pm_t \approx Z_t^0 + \varepsilon_f Z^1_{t,f} + \varepsilon_r Z^1_{t,r},
\]
\[
Z^\pm_t \approx Z_t^0 + \varepsilon_f Z^1_{t,f} + \varepsilon_r Z^1_{t,r}, \quad i \in \{I, C\}.
\]

Now we consider the solution of BSDEs (7) and the 0th-order approximation. Using a comparison theorem for BSDEs, we obtain the following theorem.

**Theorem 10** Let $(V^\pm, Z^\pm, Z^1, Z^C, \hat{V})$ be the solution
of BSDEs (7) and $(V^0, Z^0, Z^1, Z^C)$ be the
solution of BSDEs (9). It holds that $V^{-}_0 \leq V^{-}_0 \leq V^+_0 \leq V^+_0.$
Moreover, assume that $r_D \in [r_f, r_f]$ and $r_D \in [r_r, r_r]$.
Then, we obtain the following explicit representation:
\[
V^0_0 = \hat{V}_0 + A^{(1)}(\tau) - A^{(2)}(\tau) + A^{(3)}(\tau),
\]
where
\[
A^{(1)}(\tau) := \mathbb{Q}(\tau) - 1 E \int_t^{\infty} \int_t^{\infty} e^{-r_d(x-t)}
\times L_c((1 - \alpha)\hat{V}_x) h_t e^{-h_t r_t h_c e^{-h_t r_c} dxdy} dF_t[1](x-y),
\]
\[
A^{(2)}(\tau) := \mathbb{Q}(\tau) - 1 E \int_t^{\infty} \int_t^{\infty} e^{-r_d(x-t)}
\times L_t((1 - \alpha)\hat{V}_x) h_c e^{-h_t r_t h_c e^{-h_t r_c} dxdy} dF_t[1](x-y),
\]
\[
A^{(3)}(\tau) := \pm E \int_0^T e^{-r_d(x-t)} \{r_D - r_c\}(\pm \alpha \hat{V}_t) +
\times (r_c - r_D)(\pm \alpha \hat{V}_t) - d\mu[dG_t].
\]

**Proof** Using Assumption 1, we have
\[
f^{-}(t, v, z, z^C, \hat{V}) \leq f^+_0(t, v, z, z^C, \hat{V}) \leq f^+_0(t, v, z, z^C, \hat{V}),
\]
\[
f^-_0(t, v, z, z^C, \hat{V}) \leq \hat{f}^-_0(t, v, z, z^C, \hat{V}),
\]
\[
f^+_0(t, v, z, z^C, \hat{V}) \leq \hat{f}^+_0(t, v, z, z^C, \hat{V}),
\]
\[
f^-_0(t, v, z, z^C, \hat{V}) \leq f^-_0(t, v, z, z^C, \hat{V}),
\]
\[
f^+_0(t, v, z, z^C, \hat{V}) \leq \hat{f}^+_0(t, v, z, z^C, \hat{V}),
\]
It holds that

\[ < h_1(z_1^i - z_1^j) + K(z_1^i - z_1^j)^+, \]

\[ < h_2(z_1^i - z_1^j) + K(z_1^i - z_1^j)^+. \]

Under Assumption 12, we can show the following.

**Theorem 13** Assume that Assumption 12 (i)-(iii) holds. Then, the BSDEs (12) admit a unique solution \((V, Z, Z^1, Z^2)\) such that

\[ ||V||_\beta + ||Z||_\beta + ||Z^1||_\beta + ||Z^2||_\beta < \infty \]

for any sufficiently large \(\beta \in \mathbb{R}_+.\)

Since Assumption 12 satisfies an assumption in [6], the proof is similar to that of [6, Theorem 2.4]. Further, we can show the following.

**Theorem 14** Let \((f_1, \phi^1, \phi^2, \xi)\) and \((f_2, \phi^2, \phi^2, \eta)\) satisfy Assumption 12 (i)-(iii), let \((V^1, Z^1, Z^{1^1}, Z^{1^2})\), \(i \in \{1, 2\}\) be two solutions of BSDEs with \((f_1, \phi^1, \phi^2, \xi)\) and \((f_2, \phi^2, \phi^2, \eta)\), and let at least one of \(f_1\) and \(f_2\) satisfy Assumption 12 (iv). If

- \(f_t(v, t, v, z, z^1, z^2) \geq f_t(v, t, v, z, z^1, z^2)\) a.s. for all \((t, v, z, z^1, z^2) \in [0, T] \times \mathbb{R}^4\),
- \(\phi^r_i \geq \phi^r_i\), a.s. for \(i \in \{1, 2\}\),
- \(\xi \geq \eta\) a.s.,

then \(V^1_t \geq V^2_t\) a.s. for \(t \in [0, T]\). In addition, if there exists \(l_0 \in [0, T]\) such that \(V^1_{t_0} = V^2_{t_0}\) a.s. then \(V^1_t = V^2_t\) a.s. for \(t \in [l_0, T]\).

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