Study of Polarization in $B \to VT$ Decays

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(Dated: June 28, 2008)

In this paper, we examine $B \to VT$ decays ($V$ is a vector and $T$ is a tensor meson), whose final-state particles can have transverse or longitudinal polarization. Measurements have been made of $B \to \phi K^*_2$, and it is found that $f_T/f_L$ is small, where $f_T$ ($f_L$) is the fraction of transverse (longitudinal) decays. We find that the standard model (SM) naively predicts that $f_T/f_L \ll 1$. The two extensions of the naive SM which have been proposed to explain the large $f_T/f_L$ in $B \to \phi K^+ –$ penguin annihilation and rescattering – make no firm predictions for the polarization in $B \to \phi K^*_2$. The two new-physics scenarios, which explain the data in $B \to \phi K^*_2$ only if the $B \to T$ form factors obey a certain hierarchy. Finally, we present the general angular analysis which can be used to get helicity information using two- and three-body decays.

PACS numbers: 13.25.Hw, 13.88.+e, 11.30.Er

INTRODUCTION

An interesting effect has been observed in some $B \to V_1V_2$ decays ($V_i$ is a light charmless vector meson) which are dominated by $b \to s$ penguin transitions in the SM. Because the final-state particles are vector mesons, this decay is in fact three separate decays, one for each polarization of the vector mesons (one longitudinal, two transverse). Naively, the transverse amplitudes are suppressed by a factor of size $m_V/m_{b}$ ($V$ is one of the vector mesons) with respect to the longitudinal amplitude. As such, one expects the fraction of transverse decays, $f_T$, to be much less than the fraction of longitudinal decays, $f_L$.

The polarizations were first measured in $B \to \phi K^*$ decays [1]. There was found that the two fractions $f_T$ and $f_L$ are roughly equal: $f_T/f_L(B \to \phi K^*) \simeq 1$. This was also seen in some $B \to pK^*$ decays [2, 3]. The latest data are shown in Table I [2–12].

The fact that there is a discrepancy between the observed $f_T/f_L$ and the naive expectation could be a signal of physics beyond the standard model (SM) [13]. Indeed, to date, there have been several hints of such new physics (NP) in $b \to s$ transitions, though none has been statistically significant. On the other hand, there are explanations within the SM. Assuming that there is a single explanation for the large $f_T/f_L$’s, there are two proposed SM solutions [14]: penguin annihilation [15] and rescattering [16, 17].

| Mode                  | $B \ (10^{-6})$ | $f_L$          | $f_T$          |
|-----------------------|-----------------|----------------|----------------|
| $\phi K^*$(892)$^0$ [4–6] | 9.5 ± 0.9       | 0.49 ± 0.03    | 0.25 ± 0.03    |
| $\phi K^*$(892)$^+$ [5, 7] | 10.0 ± 1.1      | 0.50 ± 0.05    | 0.20 ± 0.05    |
| $\rho^+ K^*$(892)$^0$ [2, 3] | 9.2 ± 1.5       | 0.48 ± 0.08    |               |
| $\rho^0 K^*$(892)$^0$ [3] | 5.6 ± 1.6       | 0.57 ± 0.12    |               |
| $\rho^- K^*$(892)$^+$ [3] | < 12.0 (5.4±1.1) | 0.12 (0.05±0.02) |               |
| $\rho^0 K^*$(892)$^+$ [3] | < 6.1 (3.6±1.9) | 0.9 (0.2)      |               |
| $\omega K^*$(892)$^0$ [8, 9] | < 2.8 (1.6±0.7) | 0.8 (0.3)      |               |
| $\omega K^*$(892)$^+$ [8] | < 3.4 (0.6±1.8) |               |               |
| $\phi K^*$(1680)$^0$ [10] | < 3.5         |               |               |
| $\phi K^*$(1430)$^0$ [4] | 7.8 ± 1.3      | 0.85 ± 0.08    | 0.05 ± 0.05    |
| $\phi K^*$(1780)$^0$ [10] | < 2.7         |               |               |
| $\phi K^*$(2045)$^0$ [10] | < 15.3        |               |               |

However, one can also look at $B \to VT$ decays ($T$ is a tensor meson). Here too there are three polarizations, and $f_T/f_L$ can be measured. This has been done in $B \to \phi K^*_2$ [4], and the results are shown in Table I. In this...
case $f_\tau/f_L(B \to \phi K^*_2)$ is found to be small.

The various explanations must account for the $f_\tau/f_L$ data in both $B \to V_1V_2$ and $B \to VT$ decays. In this paper we examine this question, both in the SM and
with NP.

In Sec. 2, we look at the prediction of the naive SM, based on factorization, for the polarizations in $B \to VT$ decays. In Sec. 3, we examine how the expectations change for $B \to \phi K^*_2$, $B \to \rho K^*_2$ and $B \to \omega K^*_2$ in the presence of penguin annihilation or rescattering. In Sec. 4, we discuss new-physics explanations/predictions for polarizations in $B \to V_1V_2$ and $B \to VT$. Sec. 5 contains a discussion of the angular analysis to three-body and two-body decays of the $B$-meson daughters. This is relevant to $B \to VT$, and is applicable to the Chen-Geng explanation [18] of polarizations in $B \to V_1V_2$ and $B \to VT$. We conclude in Sec. 6.

**STANDARD MODEL PREDICTION**

As detailed in the introduction, the SM naively predicts that $f_\tau/f_L \ll 1$ in $B \to V_1V_2$ decays. In this section, we examine the SM prediction for $f_\tau/f_L$ in $B \to VT$ decays. As we will see, the analysis of $B \to VT$ decays is very similar to that in $B \to V_1V_2$.

We begin by describing the kinematics involved in $B(p_B) \to V(q)T(p)$, where we have explicitly labeled the momenta of the participating mesons. We work in the limit of heavy mass for the initial hadron and large energy for the final state [19]:

$$\left(\Lambda_{QCD}, m_{V,T}\right) \ll \left(m_B, E_{V,T}\right),$$

(1)

where $m_{V,T}$ and $E_{V,T}$ are the masses and energies of the vector and tensor mesons, $m_B$ is the $B$ meson mass and $\Lambda_{QCD}$ is the QCD scale. In the rest frame of the $B$ we can therefore write

$$p_B \equiv m_B(1,0,0,0)$$

$$p \equiv E_T(1,0,0,1)$$

$$q \equiv E_V(1,0,0,-1).$$

(2)

Here we have dropped terms of order $(m_{V,T}/E_{V,T})^2$ which is reasonable as $E_{V,T} \sim m_B/2$.

We now specify the polarization vectors of the final-state particles. We define the polarization of the vector meson as

$$\eta^\mu(0) = \frac{1}{m_V}(q_\perp,0,0,E_V) \approx \frac{1}{m_V}(-E_V,0,0,E_V),$$

$$\eta^\mu(\mp) = \frac{1}{\sqrt{2}}(0,\mp 1,-i,0),$$

(3)

where we assume the vector meson is moving along the negative $z$-axis. The polarization of the spin-2 tensor meson $s^{\mu\nu}$ which satisfies

$$s^{\mu\nu}(p,h) = s^{\nu\mu}(p,h),$$

$$s^{\mu\nu}(p,h)p_\nu = s^{\mu\nu}(p,h)p_\mu = 0,$$

$$g_{\mu\nu}s^{\mu\nu} = 0,$$

(4)

where $h$ is the meson helicity. The states of a massive spin-2 particle can be constructed in terms of two spin-1 states as

$$s^{\mu\nu}(\pm 2) = e^\mu(\pm)e^\nu(\pm),$$

$$s^{\mu\nu}(\pm 1) = \frac{1}{\sqrt{2}}[e^\mu(\pm)e^\nu(0) + e^\nu(0)e^\nu(\pm)],$$

$$s^{\mu\nu}(0) = \frac{1}{\sqrt{6}}[e^\mu(+)e^\nu(-) + e^\nu(-)e^\nu(+)]$$

$$+ \frac{\sqrt{2}}{3}e^\mu(0)e^\nu(0),$$

(5)

where $e^\mu(0,\pm)$ denote the polarization vectors of a massive vector state, and their explicit structures are chosen as

$$e^\mu(0) = \frac{1}{m_T}(p_T,0,0,E_T),$$

$$e^\mu(\pm) = \frac{1}{\sqrt{2}}(0,\mp 1,-i,0),$$

(6)

where $m_T(p_T)$ is the mass (momentum) of the particle and $E_T$ its energy. Since the $B$ meson is a spinless particle, the helicities carried by decaying particles in the two-body $B$ decay must be the same. Thus, although the tensor meson contains 5 spin degrees of freedom, only $h = 0$ and $\pm 1$ give nonzero contributions. The $B \to T$ transition form factors then involve the “polarization” vector $\varepsilon_\mu$, defined as

$$\varepsilon_\mu(h) = s^{\mu\nu}(p,h)v^\nu(m_T/p_T),$$

(7)

where $v$ is the velocity of the $B$ meson and $p_T$ is the magnitude of the momentum of the tensor meson. In the rest frame of the $B$ meson we have

$$\varepsilon^\mu(\pm 2) = 0,$$

$$\varepsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}}m_Tv(0)\cdot ve^\mu(\pm),$$

$$\varepsilon^\mu(0) = \sqrt{\frac{2}{3}}m_Tv(0)\cdot ve^\mu(0),$$

(8)

with $v(0)\cdot v = p_T/m_T \approx E_T/m_T$. Note that the polarization vector $\varepsilon_\mu(h)$ has the same energy scaling as the polarization vector of a vector meson. The structure of the $B \to T$ form factors is the same as that of $B \to V$ with $\varepsilon_\mu(h)$ replacing the $V$ polarization vector.

The next step involves the SM effective Hamiltonian for $B$ decays [20]:

$$H_{eff}^q = \frac{G_F}{\sqrt{2}}[V_{tb}V_{td}\bar{c}_t^*O_{1f}^T + c_2O_{2f}^T]$$

$$- \sum_{i=3}^{10}(V_{ub}V_{cd}\bar{c}_d^* + V_{cb}V_{cd}\bar{c}_d^* + V_{tb}V_{td}\bar{c}_t^*)O_{1f}^T + h.c.,$$

(9)
where the superscript \( u, c, t \) indicates the quark which is internal or involved in rescattering from the tree diagram, \( f \) can be the \( u \) or \( c \) quark, \( q \) can be either a \( d \) or \( s \) quark, and the \( c_i \) are Wilson coefficients. In the following, we focus on the specific decay \( B \to \phi K^* \) [21, 24]:

\[
A[B \to \phi K^*_{r}] = \frac{G_F}{\sqrt{2}} X P_\phi ,
\]

\( X = - \sum_{q=u,c,t} V_{q\phi} V_{qs} \times \)

\[
\left[ a_1^2 + a_2^2 + a_5^2 - \frac{1}{2}(a_1^2 + a_2^2 + a_5^2) \right] ,
\]

\[ P_\phi = m_\phi g_\phi \eta_\phi \langle K^*_2 | \bar{b} \gamma_\mu (1 - \gamma_5) s | B \rangle , \]

where

\[
a_i = \begin{cases} 
   c_i + c_{i-1}/N_c , & \text{even} , \\
   c_i + c_{i+1}/N_c , & \text{odd} .
\end{cases}
\]

The quantities \( m_\phi, g_\phi \) and \( \eta_\phi^{\mu\nu} \) represent the mass, decay constant and the polarization four-vector of the \( \phi \) meson. The various form factors and decay constants are defined as

\[
\langle \phi | \bar{s} \gamma^\mu s | 0 \rangle = g_\phi m_\phi \eta_\phi^{\mu\nu} ,
\]

\[
\langle K^*_2 | \bar{b} \gamma^\mu (1 - \gamma_5) s | B \rangle \eta_\mu^{\nu} = \frac{2i}{m_B + m_{K^*_2}} \bar{V} \epsilon^{\mu\nu\alpha\beta} p_\nu q_\alpha \epsilon^\beta_\eta_\nu \]

\[
\pm (m_\phi + m_{K^*_2}) \tilde{A}_1 \epsilon^\beta \cdot \eta^\beta ,
\]

\[
\mp \tilde{A}_2 \frac{2}{m_B + m_{K^*_2}} (p \cdot \eta^*)(q \cdot \epsilon^*) .
\]

The form factors defined above are easily related to those defined in Refs. [22, 23] using the definition in Eq. (7).

We can now write down the various polarization amplitudes from Eq. (11). Using the matrix elements given above, the polarization amplitudes are given by [24]

\[
A_0 \approx \frac{G_F}{\sqrt{2}} 2m_\phi \eta_\phi \sqrt{2} \left[ (\tilde{A}_1 - \tilde{A}_2) \right] + \frac{m_{K^*_2}}{m_B} \left( A_1 + A_2 \right) \frac{m_B^2}{4m_\phi m_{K^*_2}} ,
\]

\[
A_{\parallel} \approx -\frac{G_F}{\sqrt{2}} 2m_\phi \frac{1}{\sqrt{2}} \left[ \frac{m_\phi g_\phi}{m_B} \left( 1 + \frac{m_{K^*_2}}{m_B} \right) \tilde{A}_1 X \right] ,
\]

\[
A_{\perp} \approx -\frac{G_F}{\sqrt{2}} 2m_\phi \frac{1}{\sqrt{2}} \left[ \frac{m_\phi g_\phi}{m_B} \left( 1 - \frac{m_{K^*_2}}{m_B} \right) \tilde{V} X \right] .
\]

In the large-energy limit, the tensor and vector form factors are expressible in terms of two universal form factors [19]. This is due to the simplified structure for the various currents in the effective theory [19], which takes the form

\[
\bar{V}_n b_v = v_\mu \bar{q} \gamma_\mu b_v , \quad (15)
\]

\[
\bar{g}_n \gamma^\mu b_v = n^\nu \bar{q} \gamma_\nu b_v + i e^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{q} \gamma_\sigma b_v , \quad (16)
\]

\[
\bar{g}_n \gamma^\mu \gamma_5 b_v = -n^\nu \bar{q} \gamma_\nu \gamma_5 b_v + i e^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{q} \gamma_5 \gamma_\sigma b_v , \quad (17)
\]

\[
\bar{g}_n \sigma^{\mu\nu} b_v = i [n^\nu \bar{q} \gamma_\nu b_v - n^\nu \bar{q} \gamma_\nu \gamma_5 b_v - (\mu \leftrightarrow \nu)] + e^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{q} \gamma_5 b_v , \quad (18)
\]

\[
\bar{g}_n \sigma^{\mu\nu} \gamma_5 b_v = i [n^\nu \bar{q} \gamma_\nu \gamma_5 b_v + n^\nu \bar{q} \gamma_\nu \gamma_5 b_v - (\mu \leftrightarrow \nu)] + e^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{q} \gamma_5 b_v .
\]

The above relations are valid for both \( B \to V \) and \( B \to T \) form factors. Given this, and given the fact that the transition form factors have the same structure, with proper redefinition of the polarization vector for the tensor meson, we expect that the \( B \to T \) form factors should also be expressible in terms of the form factors in the large-energy limit.

Hence, in the large-energy effective theory (LEET), ignoring possible power-suppressed and \( \alpha_s \) corrections, we have

\[
\tilde{A}_1 \approx \tilde{\zeta}_\perp \left( 1 - \frac{m_{K^*_2}}{m_B} \right) ,
\]

\[
\tilde{A}_2 \approx \tilde{\zeta}_\parallel \left( 1 + \frac{m_{K^*_2}}{m_B} \right) - \frac{2m_{K^*_2}}{m_B} \tilde{\zeta}_\parallel ,
\]

\[
\tilde{V}_1 \approx \tilde{\zeta}_\parallel \left( 1 + \frac{m_{K^*_2}}{m_B} \right) ,
\]

where \( \tilde{\zeta}_\parallel \) and \( \tilde{\zeta}_\parallel \) are the two universal form factors. Note that in the effective theory there exists no relation between \( \tilde{\zeta}_\parallel \) and \( \tilde{\zeta}_\parallel \). For \( B \to V \) form factors, most models find \( \tilde{\zeta}_\parallel \) and \( \tilde{\zeta}_\parallel \) to be of similar size, but this may not be true for the \( B \to T \) form factors. However, there is not much literature on the calculation of \( B \to T \) form factors, and often the model predictions are in large disagreement with each other [23, 25]. The model of Ref. [23] is not expected to be reliable in the low-\( q^2 \) region and predates the form-factor relations obtained in Ref. [19]. The model of Ref. [25] has form factors very different from those in Ref. [23] and appears to be inconsistent with the form-factor relations in LEET.

Given this, we will employ a general analysis assuming only the form-factor relations from LEET. We will consider two cases described below:

Case (a): We assume \( \tilde{\zeta}_\parallel \approx \tilde{\zeta}_\parallel \). With the help of Eqs. (14) and (20) we find that

\[
\frac{A_T}{A_0} \sim \frac{m_\phi}{m_B} , \quad T = \perp, \parallel ,
\]

\[
\frac{A_T}{A_0} \approx 1 .
\]

This follows from the fact that for this case

\[
\tilde{A}_2 = \tilde{A}_1 + O(m_{K^*_2}/m_B) ,
\]

\[
\tilde{V} = \tilde{A}_1 + O(m_{K^*_2}/m_B) .
\]
Case (b): We assume $\zeta_\perp \ll \zeta_\parallel$ with $\zeta_\parallel \sim (m_\phi/m_{K^*_2})\zeta_\perp$. In this case, even though $A_1$ and $\overline{V}$ differ only by terms of $O(m_{K^*_2}/m_b)$, the form factor $A_2$ can be very different. Consequently, with the help of Eqs. (14) and (20), we find that

$$\frac{A_2}{A_0} \sim \frac{m_\phi m_{K^*_2}}{m_0^2}, \quad T = \parallel, \perp,$$

$$\frac{A_1}{A_0} \approx 1.$$  (23)

In both cases the longitudinal polarization dominates and the transverse polarizations are of the same size, so that $f_T/f_L$ is small. Although the decay $B \to \phi K^*_2$ was used to derive this result, it holds for all $VT$ final states. We therefore conclude that the SM predicts that $f_T/f_L \ll 1$ in $B \to VT$ decays.

Finally, we can consider a third case, case (c), in which $\zeta_\parallel \ll \zeta_\perp$ with $\zeta_\perp \sim (m_\phi/m_{K^*_2})\zeta_\parallel$. However, it is clear from Eqs. (14) and (20) that one obtains $A_1/A_0 \sim (m_\phi/m_0)(\zeta_\parallel/\zeta_\perp) \sim (m_\phi/m_{K^*_2})$, so that $f_T/f_L \approx 1$. This is in contradiction with the experimental results for $B \to \phi K^*_2$ (Table I).

Note that, in the presence of new physics, the predictions of the three cases can be altered. Two sections below we turn to the effect of NP.

**PENGUIN ANNIHILATION AND RESCATTERING**

Earlier, it was noted that the SM (naively) predicts that $f_T/f_L \ll 1$ in $B \to V_1V_2$ decays, in contrast to experimental results. It was also noted that there are effects within the SM – penguin annihilation [15] or rescattering [16, 17] – that, if large, could explain the observed value of $f_T/f_L$. In this section, we review the action of penguin annihilation and rescattering in $B \to V_1V_2$ decays, and establish their prediction for $f_T/f_L$ in $B \to VT$ decays, specifically $B \to \phi K^*_2$, $B \to \rho K^*_2$ and $B \to \omega K^*_2$.

$B \to \phi K^*_2$

We begin by examining penguin annihilation (Fig. 1). $B \to \phi K^*$ receives penguin contributions, $b\bar{O}s\bar{q}Oq$, where $q = u, d$ ($O$ are Lorentz structures, and color indices are suppressed). Applying a Fierz transformation, these operators can be written as $b\bar{O}'q\bar{q}O's$. A gluon can now be emitted from one of the quarks in the operators which can then produce an $s\bar{s}$ quark pair. These then combine with the $s, q$ quarks to form the final states $\phi K^{*+}$ ($q = u$) or $\phi K^{*0}$ ($q = d$).

All annihilation contributions are usually expected to be small as they are higher order in the $1/m_b$ expansion, and thus ignored. However, within QCD factorization (QCDf) [26], it is plausible that the coefficients of these terms are large [15]. In QCDf, penguin annihilation is not calculable because of divergences which are parameterized in terms of unknown quantities. One may choose these parameters to fit the polarization data in $B \to \phi K^*$ decays. (Within perturbative QCD (pQCD) [27], the penguin annihilation is calculable and can be large, though it is not large enough to explain the polarization data in $B \to \phi K^*$ [28].)

The second explanation of $f_T/f_L$ in $B \to \phi K^*$ is rescattering (Fig. 2). It has been suggested that rescattering effects involving charm intermediate states, generated by the operator $b\bar{O}'c\bar{c}O's$, can produce large transverse polarization in $B \to \phi K^*$. A particular realization of this scenario is the following [16]. Consider the decay $B^+ \to D^+_s\bar{D}^{*0}$ generated by the operator $b\bar{O}'c\bar{c}O's$. Since the final-state vector mesons are heavy, the transverse polarization can be large. The state $D^+_s\bar{D}^{*0}$ can now rescatter to $\phi K^{*+}$. If the transverse polarization $T$ is not reduced in the scattering process, this will lead to $B^+ \to \phi K^{*+}$ with large $f_T/f_L$. (A similar rescattering effect can take place for $B^0 \to \phi K^{*0}$.)

These conclusions are based on the assumption that the decay $B \to \phi K^*$ is not reduced in the scattering process. The key question now is: what do these predictions for $f_T/f_L$ in $B \to \phi K^*_2$? In order to answer this question, we must establish whether or not the individual explanations depend on the final-state particles. If they do not, then the prediction for $f_T/f_L$ in $B \to \phi K^*_2$ will be
the same as that in $B \rightarrow \phi K^*$, which is in disagreement with experiment.

The calculation of penguin annihilation does depend on the final-state wave function. Thus, it is possible to choose the parameters such that $f_c/f_L$ is small in $B \rightarrow \phi K^*_2$ for the three cases discussed above, in agreement with experiment. For rescattering, the same mechanism that takes place in $B \rightarrow \phi K^*$ can occur in $B \rightarrow \phi K^*_2$: one requires that $D^+_s D^{*0}$ rescat to $\phi K^*_2$. However, any such rescattering has its own set of parameters. It is again possible to choose parameters such that $D^+_s D^{*0}$ rescat to $\phi K^*_2$ results in a small $f_c/f_L$ in $B \rightarrow \phi K^*_2$ for all the three cases. Thus, rescattering can account for the small $f_c/f_L$ result in $B \rightarrow \phi K^*_2$.

It is therefore possible that both explanations agree with the $f_c/f_L$ measurements in $B \rightarrow \phi K^*$ and $B \rightarrow \phi K^*_2$. However, this is not very satisfying, since there is a new set of parameters for each final state, and it is virtually impossible to calculate the values of the parameters. We thus conclude that both penguin annihilation and rescattering are viable, but not very convincing.

\[ B \rightarrow \rho K^*_2 \]

Within the diagrammatic approach [29], amplitudes for $b \rightarrow s$ processes can be written in terms of eight diagrams: the color-favored and color-suppressed tree amplitudes $T'$ and $C'$, the gluonic penguin amplitudes $P'$, the color-favored and color-suppressed electroweak penguin amplitudes $P'_{EW}$ and $P'_{EW}^C$, the annihilation and exchange amplitudes $A'$ and $E'$, and the penguin-annihilation diagram $PA'$. (The primes on the amplitudes indicate $\bar{b} \rightarrow s$ transitions.) There are also other diagrams, but they are much smaller and in general can be neglected. Note that $P'$ includes rescattering contributions from tree-level operators with up- and charm-quark intermediate states. However, using the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one can write

\[
P' = V_{ub} V_{us} P'_{c} + V_{ub} V_{cs} P'_{c} + V_{ub} V_{ts} P'_t
\]

Thus, $P'$ includes the two quark amplitudes $P'_{tc} \equiv (P'_t - P'_{c})$ and $P'_{uc} \equiv (P'_u - P'_c)$.

In principle, $P'_{EW}$, $P'_{EW}^C$ and $PA'$ also receive contributions from the $u$ and $c$ quarks. However, these are negligible – the three diagrams are dominated by the intermediate $t$ quark. Hence all $b \rightarrow s$ $B$-decay amplitudes can be written in terms of the nine diagrams $T'$, $C'$, $P'_{tc}$, $P'_{uc}$, $P'_{EW}$, $P'_{EW}^C$, $A'$, $E'$ and $PA'$.

From here on, we will redefine all nine diagrams by taking them to have absorbed their associated CKM matrix elements. In Ref. [29], the relative sizes of the (redefined) amplitudes were estimated to be roughly

\[
\begin{align*}
1 : |P'_{tc}| , \quad & \mathcal{O}(\tilde{\lambda}) : |T'|, \quad |P'_{EW}| \quad \mathcal{O}(\tilde{\lambda}^2) : |C'|, \quad |P'_{uc}|, \quad |P'_{EW}^C|, \quad |PA'| \quad \mathcal{O}(\tilde{\lambda}^3) : |A'|, \quad |E'|, \quad (25)
\end{align*}
\]

where $\tilde{\lambda} \sim 0.2$. These SM estimates are often used as a guide to neglect diagrammatic amplitudes and reduce the number of parameters.

Now, $B \rightarrow \rho K^*_2$ [30] actually represents four processes. $B^+ \rightarrow \rho^+ K^*_2$ is governed by the underlying quark transition $b \rightarrow s d d$; $B_d^0 \rightarrow \rho^- K^*_2$ is dominated by $b \rightarrow s u t$; $B^0 \rightarrow \rho^0 K^*_2$ and $B^+ \rightarrow \rho^+ K^*_2$ have both transitions. Neglecting all diagrams of $\leq \mathcal{O}(\tilde{\lambda}^2)$ [Eq. (25)], the amplitudes for these decays are given by

\[
A(B^+ \rightarrow \rho^+ K^*_2) = P'_{tc},
\]

\[
\sqrt{2} A(B^0_d \rightarrow \rho^- K^*_2) = P'_{tc} - P'_{EW},
\]

\[
\sqrt{2} A(B^+ \rightarrow \rho^0 K^*_2) = -P'_{tc} - T'_{e^+} - P'_{EW},
\]

\[
A(B^0 \rightarrow \rho^- K^*_2) = -P'_{tc} - T'_{e^+}. \quad (26)
\]

The weak phase information in the CKM matrix is conventionally parameterized in terms of the unitarity triangle, in which the interior CP-violating angles are known as $\alpha$, $\beta$ and $\gamma$ [12]. In the above amplitudes, we have explicitly written the dependence on the weak phase $\gamma$, but the diagrams contain strong phases.

In naive factorization, at the mesonic level, the penguin ($P'$) and tree ($T'$) contributions take the form

\[
\langle K^*_2 | \bar{q} O s | 0 \rangle (\rho | b \bar{O} q | B). \quad (27)
\]

This vanishes, because one cannot produce the tensor meson $K^*_2$ from the vacuum. Thus, the branching ratios for $B^+ \rightarrow \rho^+ K^*_2$ and $B^0 \rightarrow \rho^0 K^*_2$ arise only due to nonfactorizable effects, and are small. (The branching ratios for the other two decays are not small due to the presence of $P'_{EW}$ in the amplitudes.) If rescattering from the tree-level operators is perturbatively calculable, as in QCDf, the size (and phase) of $P'_{tc}$ is changed, but not the Lorentz structure. As such, the branching ratios for $B^+ \rightarrow \rho^+ K^*_2$ and $B^0 \rightarrow \rho^0 K^*_2$ will remain small. (If there are long-distance in calculable rescattering effects then it is possible that the branching ratios of these decays may not be small.) In penguin annihilation there are nonfactorizable contributions to $B^+ \rightarrow \rho^+ K^*_2$ and $B^0 \rightarrow \rho^0 K^*_2$ may not be small. The measurement of these quantities will allow us to test if large penguin annihilation or large nonperturbative rescattering effects are present in these decays.

Turning to polarizations, within penguin annihilation and rescattering, the prediction of $f_c/f_L$ for each of the four decays $B \rightarrow \rho K^*_2$ is arbitrary, as it depends on the unknown set of parameters for each of the final states. It is tempting to try to get information from $B \rightarrow \phi K^*_2$ by using flavor SU(3) for the final states. However, this
does not work. Consider only the contribution of the gluonic penguin to \( B \to \phi K^*_2 \) and \( B \to \rho K^*_2 \). As detailed above, the matrix element of this operator vanishes for \( B \to \rho K^*_2 \), but it does not for \( B \to \phi K^*_2 \). Thus, these two decays are not in fact related by SU(3).

We therefore conclude that there are no firm predictions for \( f_L/f_T \) in any of the \( B \to \rho K^*_2 \) decays from penguin annihilation and rescattering.

\[ B \to \omega K^*_2 \]

The decay \( B \to \omega K^*_2 \) represents two processes: \( B^0_d \to \omega K^{*0}_2 \) and \( B^+ \to \omega K^{*+}_2 \). The amplitudes are given by

\[
\sqrt{2} A(B^0_d \to \omega K^{*0}_2) = P'_{tc} + 2P'_{tc,dir} + \frac{1}{3} P'_{EW}, \tag{27}
\]

\[
\sqrt{2} A(B^+ \to \omega K^{*+}_2) = P'_{tc} + 2P'_{tc,dir} + T' e^\gamma_1 + \frac{1}{3} P'_{EW}.
\]

At the quark level, \( \omega = (d\bar{d} + u\bar{u})/\sqrt{2} \). Since the \( \omega \) is an isosinglet, the gluon can decay directly to it (unlike the \( \rho^0 \)). However, this requires the exchange of two additional gluons to absorb color factors and is expected to be somewhat suppressed according to the OZI rule. We denote this diagram by \( P'_{tc,dir} \) and note that this does not vanish in naive factorization.

Due to the presence of \( P'_{tc,dir} \) and \( P'_{EW} \) in the amplitudes, the branching ratios for these decays may not be small in the SM. Still, it is interesting to note that the branching fraction of the \( B \to \omega K^* \) decays may not vanish in naive factorization.

As in the case of \( B \to \rho K^*_2 \), the prediction of \( f_L/f_T \) for \( B \to \omega K^*_2 \) within penguin annihilation and rescattering depends on the unknown set of parameters for each decay, and is arbitrary.

**NEW PHYSICS**

The CP measurements in many penguin decays that proceed through \( b \to s \) transitions appear to be in conflict with naive SM expectations. For example, the combined branching-ratio and CP-asymmetry measurements in \( B \to \pi K \) decays are at odds with the SM predictions \cite{31}. However, these measurements are not precise enough to draw any firm conclusions about the existence of new physics. As already noted, the polarization measurements in some \( B \to V_1 V_2 \) (\( b \to s \)) decays also disagree with naive SM estimates. It is therefore not unreasonable to attempt to understand the data assuming NP. The important question to ask is then the following: can we find a unified new-physics explanation for all the discrepancies so far reported in measurements of pure-penguin or penguin-dominated decays? An attempt to answer this question was presented in Ref. [13]. In this section we carry the analysis of that paper to \( B \to VT \) decays.

The basic philosophy of this approach is the following: we assume that the naive SM predictions for the CP measurements and polarizations are generally correct. In particular, annihilation contributions are taken to be negligible, and we assume that nonperturbative SM rescattering through charm intermediate states is suppressed by \( O(1/m_b) \). The annihilation contributions are power suppressed in the \( 1/m_b \) expansion and should be small. Indeed, there is no clear experimental evidence of large annihilation effects. The existence of large nonperturbative SM rescattering, not suppressed by \( O(1/m_b) \), is controversial and highly model-dependent with very little predictive power. In any case, a new-physics explanation of the polarization data in \( B \to V_1 V_2 \) decays is only called for if one assumes that annihilation effects and nonperturbative SM rescattering effects are power suppressed. We also neglect rescattering from NP amplitudes. This can be justified by explicit calculation of rescattering effects in QCDF and is expected to be valid even in nonperturbative models of rescattering \cite{32}.

We begin with a general parametrization of NP as in Ref. [13]. It is

\[
\frac{4G_F}{\sqrt{2}} \sum_{A,B=L,R} \{ f_q^{AB} b_{\gamma \lambda} s \bar{q} \gamma_\lambda q + g_q^{A0} b_{\gamma \mu} \gamma_\lambda s \bar{q} \gamma_\mu q \}.
\tag{28}
\]

There are a total of 24 contributing operators \((A,B = L,R, q = u,d,s)\); tensor operators do not contribute to \( B \to \pi K \). For simplicity, we assume that a single operator contributes, and we analyze their effects one by one. By doing a fit to the \( \pi K \) data and the \( \rho K^* \) polarization measurements, the authors of Ref. [13] were able to conclude that operators with coefficients \( f^{AB} \) or \( f^{A0} \) have to be present in any NP model to explain the present data [13, 31]. Such operators may easily arise in multi-Higgs models [33, 34].

The question now is whether these operators can explain NP. The important question to ask is then the following: can we find a unified new-physics explanation for all the discrepancies so far reported in measurements of pure-penguin or penguin-dominated decays? An attempt to answer this question was presented in Ref. [13]. In this section we carry the analysis of that paper to \( B \to VT \) decays.

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factorization it may appear that this operator does not contribute to $B_d \to \phi K_S^*$. However, it can affect this process once we perform a Fierz transformation of this operator (both fermions and colors):

$$
\frac{4}{N_c} G_F f_{RS} \left[ \frac{1}{2} \bar{b} \gamma_{\mu} s \bar{s} \gamma_{\mu} s - \frac{1}{8} \bar{b} \sigma_{\mu\nu} \gamma_{\mu} s \bar{s} \sigma_{\mu\nu} \gamma_{\mu} s \right].
$$

(30)

In order to estimate the effects of the NP operators on $B_d \to \phi K_S^*$, we have to evaluate matrix elements of the type

$$
M_{NP} = \langle \phi K_S^* | H_{NP} | B_d \rangle \sim D_{\mu\nu} F_{\mu\nu},
$$

$$
D_{\mu\nu} = \langle \phi | \bar{s} \sigma_{\mu\nu} \gamma_5 s | 0 \rangle,
$$

$$
F_{\mu\nu} = \langle K_S^* | \bar{s} \sigma_{\mu\nu} \gamma_5 b | B_0 \rangle,
$$

(31)

where $\sigma_{\mu\nu} \equiv (1/2)[\gamma_\mu, \gamma_\nu]$. We can calculate the NP matrix elements as [13]

$$
Z_d^{RR} \left\{ 2 \hat{T}_2 \left( 1 - \frac{m_{K_S}^2}{m_b^2} \right) (\epsilon^* \cdot \eta^*) + \frac{4}{m_b^2} \left( \hat{T}_2 + \hat{T}_3 \right) \left( \frac{m_{K_S}^2}{m_b^2} \right) (\epsilon^* \cdot p_\pi, \eta^* \cdot p_{K_S}^2) - \frac{4x}{m_b^2} \hat{T}_1 e^{\mu\alpha\beta} \eta^* p_\pi \epsilon_\alpha \epsilon_\beta \right\},
$$

(32)

where the $\hat{T}_i$ are form factors and

$$
Z_d^{RR} \equiv - \frac{1}{4N_c} G_F f_{RS}^2 q_1^2 m_b^2.
$$

(33)

The various hadronic quantities are defined as

$$
\langle \phi | \bar{s} \sigma_{\mu\nu} s | 0 \rangle = -i g_5^{\mu} (\eta^* \gamma^\mu - \eta^* \gamma^\mu) + \epsilon^* \cdot q_p, \eta^* \cdot p_{K_S}^2,
$$

$$
\langle K_S^* | \bar{b} \sigma_{\mu\nu} \gamma_5 s | B \rangle \approx -2i T_1 e^{\mu\alpha\beta} \eta^* p_\pi \epsilon_\alpha \epsilon_\beta,
$$

$$
\langle K_S^* | \bar{b} \sigma_{\mu\nu} \gamma_5 s | B \rangle \approx -i T_2 \left( \frac{m_{K_S}^2}{m_b^2} \right) \epsilon^* \cdot q_p, \eta^* \cdot p_{K_S}^2
$$

$$
- \left( \epsilon^* \cdot q_p, \eta^* \cdot p_{K_S}^2 \right) - i T_3 (\epsilon^* \cdot q_p, \eta^* \cdot p_{K_S}^2)
$$

$$
- \left( \frac{m_{K_S}^2}{m_b^2} \right) \left( \epsilon^* \cdot q_p, \eta^* \cdot p_{K_S}^2 \right)
$$

$$
- \left( \frac{m_{K_S}^2}{m_b^2} \right) \left( \epsilon^* \cdot q_p, \eta^* \cdot p_{K_S}^2 \right)
$$

(34)

Working in the large-energy limit, we can then write

$$
\hat{T}_1(q^2) \approx \tilde{\zeta}_\perp,
$$

$$
\hat{T}_2(q^2) \approx \tilde{\zeta}_\parallel \left( 1 - \frac{q^2}{m_b^2 - m_{K_S}^2} \right),
$$

$$
\hat{T}_3(q^2) \approx \tilde{\zeta}_\perp - \frac{2m_{K_S}^2}{m_b^2} \tilde{\zeta}_\parallel,
$$

(35)

where $\tilde{\zeta}_\parallel$ and $\tilde{\zeta}_\perp$ are the same two universal form factors that appear in the SM predictions (Sec. 2). One can then write the various polarization amplitudes as

$$
A_0 = -2 \sqrt{2} m_b \tilde{\zeta}_\parallel Z_d^{RR},
$$

$$
A_\parallel = 2 \tilde{\zeta}_\parallel Z_d^{RR},
$$

$$
A_\perp = 2 \tilde{\zeta}_\perp Z_d^{RR}.
$$

(36)

This leads to

$$
\frac{A_0}{A_\parallel} = - \sqrt{2} \frac{m_b}{m_b \tilde{\zeta}_\parallel},
$$

$$
\frac{A_0}{A_\parallel} = \sqrt{2} \frac{m_b}{m_b \tilde{\zeta}_\parallel},
$$

$$
\frac{A_0}{A_\perp} = 1.
$$

(37)

It is therefore clear that the ratio of transverse to longitudinal amplitudes depends on the value of the form-factor ratio $\tilde{\zeta}_\parallel/\tilde{\zeta}_\perp$. For the case (a) discussed in Sec. 2 with $\tilde{\zeta}_\parallel \approx \tilde{\zeta}_\parallel$, the NP contribution to the longitudinal polarization is suppressed relative to the transverse amplitudes. We then have the prediction

$$
\frac{A_T}{A_0} = \frac{A_T^{SM} + A_T^{NP}}{A_0^{SM} + A_0^{NP}} \sim \frac{A_T^{NP}}{A_0^{SM}} \sim O(1),
$$

(38)

where $T = \perp, \parallel$. Here we have used the fact that the NP amplitude is of similar size to the SM amplitude [13]. This prediction is consistent with data for the $B \to \phi K^*$ decay but not for $B \to \phi K_S^*$. Note that for the $V_1V_2$ final state, most models do find the universal form factors to be of similar size. Indeed, Ref. [13] finds that NP can explain $f_T/f_L$ in $B \to \phi K^*$ with case (a). Hence either our assumption about the values of the universal form factors for the $B \to VK_S^*$ transition is wrong or we need a different kind of new physics.

Let us now turn to case (b), which has $\tilde{\zeta}_\parallel \ll \tilde{\zeta}_\parallel$ with $\tilde{\zeta}_\parallel \sim (m_b/m_\pi) \tilde{\zeta}_\parallel$. Here, from Eq. (37), the NP contribution to $A_0$ is not suppressed and all NP polarization amplitudes are of the same size. We then have

$$
\frac{A_T}{A_0} = \frac{A_T^{SM} + A_T^{NP}}{A_0^{SM} + A_0^{NP}} \sim \frac{A_T^{NP}}{A_0^{SM}}
$$

$$
\sim \frac{(m_b/m_\pi)A_T^{SM}}{(m_b/m_\pi m_{K_S}^2)A_T^{SM}} \sim \frac{m_{K_S}^2}{m_b^2}.
$$

(39)

This prediction is consistent with experiment.

Finally, for case (c), which has $\tilde{\zeta}_0 \ll \tilde{\zeta}_\perp$ the NP longitudinal amplitude is very suppressed. Hence, in this case we obtain

$$
\frac{A_T}{A_0} \sim \frac{A_T^{SM} + A_T^{NP}}{A_0^{SM} + A_0^{NP}} \sim 1,
$$

(40)

where we have assumed no cancellation between the SM and NP transverse amplitudes. This case is, therefore, inconsistent with experiment.

It appears that the NP scenario is the same as that of the SM – the prediction of $f_T/f_L$ in $B \to \phi K_S^*$ depends on the values of unknown parameters. Here it is $\tilde{\zeta}_\parallel/\tilde{\zeta}_\perp$. However, the difference is that, with penguin annihilation and rescattering, the parameters are essentially in calculable, while the NP prediction depends on form factors.
Although the values of these form factors are not very well known at the moment, they can be calculated. We strongly urge that the $B \to T$ form factors be computed.

It should be pointed out that an actual NP calculation of $f_T/f_L$ in $B \to \phi K_2^*$ will require knowledge of the form factors as well as the relative sizes of the NP and SM amplitudes. However, we believe our naive estimate of the polarization fractions will hold even when a detailed calculation is carried out.

We now turn to $B \to \rho K_2^*$ decays. The decays $B^+ \to \rho^+ K_2^{*0}$ and $B_0^+ \to \rho^- K_2^{*+}$ are particularly interesting because they vanish within naive factorization in the SM. The NP contributions to these amplitudes take the form

$$M_{NP} = \langle \rho K_2^* | H_{NP} | B \rangle \sim D_{\mu \nu} F^{\mu \nu},$$

$$D_{\mu \nu} = \langle K_2^* \bar{\sigma}_{\mu \nu} \gamma_5 s | 0 \rangle,$$

$$F^{\mu \nu} = \langle \rho | \bar{\sigma}_{\mu \nu} \gamma_5 d | B \rangle.$$  

(41)

However, the factor $D_{\mu \nu} = 0$ as we cannot construct an antisymmetric tensor out of the symmetric polarization tensor or the momentum vector of the tensor meson. Hence there is no NP contribution to $B^+ \to \rho^+ K_2^{*0}$ and $B_0^+ \to \rho^- K_2^{*+}$, and so they vanish within factorization. It will be very interesting to measure these branching ratios.

By the same logic, the NP does not affect the other two decays, $B_0^0 \to \rho^0 K_2^{*0}$ and $B^+ \to \rho^0 K_2^{*+}$, either. However, this is less important since the SM prediction for these branching ratios is not precise.

Finally, since the NP cannot affect any of the $B \to \rho K_2^*$ decays, the prediction for $f_T/f_L$ here is the same as that of the SM. Specifically, $f_T/f_L$ is expected to be small in $B_0^0 \to \rho^0 K_2^{*0}$ and $B^+ \to \rho^0 K_2^{*+}$ in naive factorization. It will be important to measure the polarization in these decays in order to test the SM and this type of NP.

**ANGULAR ANALYSIS**

In the previous sections, we have concentrated on the process $B \to VT$ without paying attention to how the $T$ decays. Here we focus on this issue. If the spin-2 $T$ has positive parity, it decays principally to two pseudoscalars, e.g. $K \pi$, though a three-pseudoscalar final state is also possible, e.g. $K \pi \pi$. For example, the branching fraction of $K_2^*(1430)$ decay to $K \pi$ is roughly 1/2 and to $K \pi \pi$ is roughly 1/3 [12]. However, if the spin-2 $T$ has negative parity (or the spin-1 $A$ in $B \to VA$ has positive parity $-A$ is an axial-vector meson), its decay to two pseudoscalars is forbidden and it decays principally to three pseudoscalars. These two cases of two-body and three-body decays can be treated separately.

Indeed, recent experimental studies have concentrated on the two-body decays of the $K^*$ meson in $B \to \phi K^*$. However, this limits the study only to the $J^P = 1^+, 2^+$, etc. strange-meson states with $P = (-1)^J$, see Table I.

Here we point out that new information can be obtained from polarization studies of the $B$-meson decays to states with $J^P = 1^+, 2^-$, etc., i.e. with $P = (-1)^{J+1}$, such as $K_1$ and $K_2$ mesons. An example of such a decay is $B \to \phi K_j$, and one needs to reconstruct $K_j$, final states with at least three pseudoscalar mesons, such as $K_3 \to K \pi \pi$. The angular distribution of the $B \to \phi K_j$ decay products becomes more complex and a full angular analysis requires a new formalism.

The angular distribution of the $B \to V K_3^{(*)}$ decay can be expressed as a function of $\theta_1$, $\theta_2$, and $\Phi$, see Fig. 3. Here, $\theta_1$ and $\theta_2$ are the helicity angles of the $V$ and the $K_3^{(*)}$ resonances, defined as the angles between the direction of the daughter meson (e.g. $K$ in $\phi \to K \pi \pi$) or normal to the three-body decay plane (e.g. for $K_j \to K \pi \pi$) and the direction opposite the $B$ in the $V$ or $K_3^{(*)}$ rest frame. The $\Phi$ is the angle between the decay planes of the two systems, defined by the $B$ meson decay axis and the direction of the daughter or normal as discussed above.

The analysis of the two-body angular distribution of the particles has been widely used in polarization measurements [35], see also Ref. [36] for application to $B \to V_1 V_2$ decays. It was pointed out in Ref. [37] that in the three-body decay of a particle, the normal to the decay plane replaces the center-of-mass momentum as the analyzer of the polarization. There are more degrees of freedom in the three-body decay, such as a Dalitz-plot structure. However, the dynamical degrees of freedom can be integrated out and simple results can be obtained from rotational and inversion invariance. We therefore proceed by deriving the angular distributions of various two-body $B$ meson decays to mesons with different spin and parity, which in turn decay strongly to two or three pseudoscalar mesons. This will have direct application...
to a number of polarization measurements discussed in this paper.

We start by extending the angular formalism of a $B$-meson decay (or any other spinless particle for that matter) to two particles $X_1$ and $X_2$ with spins $J_1$ and $J_2$ and parity $P_i = (-1)^{J_i}$. Each of the two particles decays strongly to two pseudoscalars $X_1 \to P_\pi P_\mathcal{B}$ and $X_2 \to P_\mathcal{B} P_\pi$, thus conserving parity. Following the two-body decay formalism [35], we obtain (see also Refs. [4, 10]):

\[
\frac{1}{Y} \frac{d^3 \Gamma}{d \cos \theta_1 d \cos \theta_2 d \Phi} = \frac{1}{\sum |A_\lambda|^2} \left| \sum_\lambda A_\lambda Y_{J_1}^\lambda(\pi - \theta_1, -\Phi) Y_{J_2}^\lambda(\theta_2, 0) \right|^2 , \tag{42}
\]

where $Y_{J}^\lambda$ are the spherical harmonics and the sum is over the helicity values ($\lambda$ takes all discrete values between $-j$ and $+j$, with $j$ being the smaller of the two spins $J_1$ and $J_2$). The $A_\lambda$ is the complex helicity amplitude in the $B$ decay, where $A_0$ corresponds to longitudinal polarization.

Here are the decays described by the above formula:

- $B \to \phi K^*_J$ and $B \to \rho K^*_J$ with $J_1 = 1$ and $J_2 = 2$.

In all of these cases we have three complex amplitudes $A_3$ and six real terms $\alpha_i$ which appear in the angular distribution:

\[
\begin{align*}
\alpha_1 &= \frac{|A_0|^2}{\Sigma |A_\lambda|^2} = f_L \\
\alpha_2 &= \frac{|A_1|^2 + |A_\perp|^2}{\Sigma |A_\lambda|^2} = \frac{|A_{-1}|^2 + |A_{-\perp}|^2}{\Sigma |A_\lambda|^2} = (1 - f_L) \\
\alpha_3 &= \frac{|A_\perp|^2 - |A_{-\perp}|^2}{\Sigma |A_\lambda|^2} = 2 \cdot \frac{\text{Re}(A_{-1}A_{-\perp}^*)}{\Sigma |A_\lambda|^2} = (1 - f_L - 2 \cdot f_{\perp}) \\
\alpha_4 &= \frac{\text{Im}(A_\perp A_{-\perp}^*)}{\Sigma |A_\lambda|^2} = \frac{\text{Im}(A_{-1}A_{-\perp}^*)}{\Sigma |A_\lambda|^2} = \sqrt{f_{\perp} \cdot (1 - f_L - f_{\perp})} \cdot \sin(\phi_{\perp} - \phi_{\mathcal{B}}) \\
\alpha_5 &= \frac{\text{Re}(A_\perp A_0^*)}{\Sigma |A_\lambda|^2} = \frac{\text{Re}(A_{-1}A_0^* + A_{-\perp}A_{-\perp}^*)}{\sqrt{2} \cdot \Sigma |A_\lambda|^2} = \sqrt{f_{\perp} \cdot (1 - f_L - f_{\perp})} \cdot \cos(\phi_{\perp}) \\
\alpha_6 &= \frac{\text{Im}(A_\perp A_0^*)}{\Sigma |A_\lambda|^2} = \frac{\text{Im}(A_{-1}A_0^* - A_{-\perp}A_{-\perp}^*)}{\sqrt{2} \cdot \Sigma |A_\lambda|^2} = \sqrt{f_{\perp} \cdot f_L} \cdot \sin(\phi_{\perp})
\end{align*}
\]

Examples of the decays described by the above formula are $B \to \phi K^*_1$ and $B \to \rho K^*_2$ with $J_1 = 1$ and $J_2 = 2$. In all of these cases we have three complex amplitudes $A_3$, and six real terms $\alpha_i$ which appear in the angular distribution.

Here we use the definition adopted in the literature [12] with four convenient real terms:

\[
\begin{align*}
&f_L = |A_0|^2/\Sigma |A_\lambda|^2 , \tag{49} \\
f_{\perp} = |A_\perp|^2/\Sigma |A_\lambda|^2 , \tag{50} \\
&\phi_{\mathcal{B}} = \text{arg}(A_0/A_{\perp}) , \tag{51} \\
&\phi_{\perp} = \text{arg}(A_{-\perp}/A_0) . \tag{52}
\end{align*}
\]

We do not discuss CP violation in the angular distributions here.

There are two special cases we discuss in more detail, namely $J_1 = 1$ and $J_2 = 1$ or $2$, corresponding to $B \to V_1V_2$ or $B \to V\mathcal{T}$ decays. From Eq. (42), it follows for $J_1 = J_2 = 1$ (e.g. $B \to \phi K^*$, $\phi \to K \mathcal{K}, K^* \to K \pi$) that
The angular distribution for states with any other integer value of $J_1$ and $J_2$ can be easily obtained from Eq. (42).

Next, we turn to the angular formalism of a spinless particle decay to two particles with spins $J_1$ and $J_2$, $B \to V_1 V_2$, where the first particle decays to two pseudoscalars $X_1 \to P_a P_b$ and the second decays to three pseudoscalars $X_2 \to P_c P_d P_e$. We assume parity conservation in the $X_1$ and $X_2$ decays, i.e. $P_1 = (-1)^{J_1}$. However, there is no requirement on the parity $P_2$ of the second particle. We have an additional phenomenological amplitude $F_m$ for the decay $X_2 \to P_c P_d P_e$. It depends on the $X_2$ spin eigenvalue $m$ but not on $\lambda$. Interference between different $F_m$ amplitudes vanishes when integrated over the rotation angle around the normal to the decay plane and we are left with $2J_2 + 1$ real parameters $|R_m|^2$. Additional symmetry considerations could put constraints on $|R_m|^2$ values, as discussed below and in Ref. [37]. Following the three-body decay formalism [37], we obtain

$$
\frac{1}{\Gamma} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \sum_{|\lambda|^2} \sum_{m} |A_{\lambda}|^2 |R_m|^2 \sum_{\lambda} A_{\lambda} V_{J_1}^{-\lambda}(\pi - \theta_1, -\Phi) d_{\lambda,m}(\theta_2), \tag{55}
$$

where we omit the normalization factor for simplicity (in general it would include a combination of the $|R_m|^2$ parameters). The index $m$ runs from $-J_2$ to $+J_2$ and $\lambda$ runs from $-\lambda$ to $+\lambda$ (with $\lambda$ again being the smaller of $J_1$ and $J_2$).

Additional constraints appear in Eq. (55) from parity conservation in the $X_2 \to P_c P_d P_e$ decay: for even (odd) parity of $X_2$ only odd (even) values of $m$ contribute, that is $P_2 = (-1)^{m+1}$ [37]. This results in only $m = 0$ contributing to the decays with $J_{2}^{*} = 1^{+}$, such as $B \to K_s^* \omega$, with $K_s^* \to K \pi$ and $\omega \to \pi^+ \pi^- \pi^0$, and Eq. (55) reduces to Eq. (42) due to the simple relationship of the $d_{\lambda,m}^J$ functions with $m = 0$ and the spherical harmonics $Y_{\lambda}^J$. It has been pointed out earlier, e.g. in Ref. [36], that Eq. (53) applies to $B \to V_1 V_2$ decays with $V_2 \to P_c P_d P_e$, though it was incorrectly stated in Ref. [36] that either one of the three daughter momenta or the normal to the decay plane could define $\theta_2$ and $\Phi$ in this formula. Consequently, Eq. (42) applies to the analysis of the $B \to TV$ decay with $T \to P_a P_b$ and $V \to P_c P_d P_e$, and in particular Eq. (54) describes the $B \to K_\omega^* \omega$ decay.

In a general case of $J_2^{*} \neq 1^-$ quantum numbers, more than one $|R_m|^2$ parameter contributes. In the following, let us consider $J_{1}^{*} = 1^-$ and $J_{2}^{*} = 1^+, 2^+, 2^-$ states. Other final states could be considered by analogy. In this case we still have three complex amplitudes $A_{\lambda}$, but new real terms $\alpha_i$ may appear in the angular distribution
in addition to those shown in Eqs. (43–48):

\[
\alpha_7 = \frac{\Re(e(A_\perp A_\perp^*)}{\Sigma|A_\perp|^2} = \frac{|A_{\perp+}|^2 - |A_{\perp-}|^2}{2 \cdot \Sigma|A_\perp|^2} = \sqrt{f_\perp \cdot (1 - f_\perp - f_\perp) \cdot \cos(\phi_\perp - \phi_0)} \\
\alpha_8 = \frac{3m(A_\perp A_\perp^*)}{\Sigma|A_\perp|^2} = \frac{3m(A_{\perp+} A_\perp^* + A_{\perp-} A_\perp^*)}{\sqrt{2} \cdot \Sigma|A_\perp|^2} = \sqrt{f_\perp \cdot (1 - f_\perp - f_\perp) \cdot \sin(\phi_0)} \\
\alpha_9 = \frac{\Re(e(A_\perp A_\perp^*)}{\Sigma|A_\perp|^2} = \frac{\Re(e(A_{\perp+} A_\perp^* - A_{\perp-} A_\perp^*)}{\sqrt{2} \cdot \Sigma|A_\perp|^2} = \sqrt{f_\perp \cdot f_\perp \cdot \cos(\phi_\perp)}
\]

Let us redefine some of the parameters:

\[
r_m \equiv \frac{|R_m|^2 - |R_{-m}|^2}{|R_m|^2 + |R_{-m}|^2} ; \quad r_{02} \equiv \frac{|R_0|^2}{|R_{02}|^2 + |R_{-2}|^2}.
\]

From Eq. (55), it follows for \(J_1 = 1, J_2 = 2,\) and \(P_2 = +1\) (e.g. \(B \rightarrow \phi K^*_2, \phi \rightarrow K K, K^*_2 \rightarrow K \pi \pi\)) that

\[
\frac{64\pi}{45\Gamma} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \alpha_1 \times \cos^2 \theta_1 \sin^2 2\theta_2 \\
+ \alpha_2 \times \frac{1}{3} \sin^2 \theta_1 (\cos^2 \theta_2 + \cos^2 2\theta_2) \\
- \alpha_3 \times \frac{1}{3} \sin^3 \theta_1 (\cos^2 \theta_2 - \cos^2 2\theta_2) \cos 2\Phi \\
+ \alpha_4 \times \frac{2}{3} \sin^2 \theta_1 (\cos^2 \theta_2 - \cos^2 2\theta_2) \sin 2\Phi \\
- \alpha_5 \times \frac{1}{\sqrt{6}} \sin 2\theta_1 \sin 4\theta_2 \cos \Phi \\
+ \alpha_6 \times \frac{1}{\sqrt{6}} \sin 2\theta_1 \sin 4\theta_2 \sin \Phi \\
+ \alpha_7 \times r_1 \frac{4}{3} \sin^2 \theta_1 \cos \theta_2 \cos 2\theta_2 \\
+ \alpha_8 \times r_1 \frac{2}{3} \sin 2\theta_1 \cos \theta_2 \sin 2\theta_2 \sin \Phi \\
- \alpha_9 \times r_1 \frac{2}{3} \sin 2\theta_1 \cos \theta_2 \sin 2\theta_2 \cos \Phi.
\]

It is worth noting that three new angular terms \(\alpha_7, \alpha_8,\) and \(\alpha_9\) appear in Eq. (61), and will appear below in Eq. (62) together with the asymmetry term \(r_1\) from Eq. (59). These terms would vanish if we had a symmetry with respect to the inversion of the normal to the decay plane for \(X_2 \rightarrow P_\ell P_\ell P_\pi,\) that is between the \(m\) and \(-m\) terms. As was pointed out in Ref. [37], examples of such cases are two identical pseudoscalar particles or pions in an eigenstate of isotopic spin. For example, \(r_1 = 0\) for a sequential decay like \(K_1 \rightarrow \rho K \rightarrow \pi \pi K.\) However, in the more general case, \(r_1\) is bound to \(-1 \leq r_1 \leq +1\) and is a priori unknown without the study of the \(X_2 \rightarrow P_\ell P_\ell P_\pi\) dynamics.

A nonzero value of \(r_1\) may allow the determination of the new angular terms in Eqs. (56–57), which would resolve discrete phase ambiguities for \((\phi_\perp, \phi_\perp),\) or, equivalently, the ambiguity hierarchy of the \(A_+\) and \(A_-\) amplitudes. For any given values of \((\phi_\perp, \phi_\perp),\) the simple transformation \((-\phi_\perp, \pi - \phi_\perp)\) preserves the values of Eqs. (43–48) and leads to an ambiguity in experimental measurements. This difficulty has been resolved for the final states with \(K^+(892)\) with the help of additional angular terms which appear in the interference between the \(K^+(892)\) and \(K^0(1430)\) [4, 7, 38]. However, the presence of nonzero terms in Eqs. (56–57) in the angular distribution may allow this within a single decay mode.

We continue with the application of Eq. (55) to the case of \(J_1 = J_2 = 1\) and \(P_2 = +1\) (e.g. \(B \rightarrow \phi K_1, \phi \rightarrow K K, K_1 \rightarrow K \pi \pi\)) and obtain

\[
\frac{16\pi}{9\Gamma} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \alpha_1 \times \cos^2 \theta_1 \sin^2 2\theta_2 \\
+ \alpha_2 \times \frac{1}{4} \sin^2 \theta_1 (1 + \cos^2 2\theta_2) \\
- \alpha_3 \times \frac{1}{4} \sin^2 \theta_1 \sin^2 2\theta_2 \cos 2\Phi \\
+ \alpha_4 \times \frac{1}{2} \sin^2 \theta_1 \sin^2 2\theta_2 \sin 2\Phi \\
- \alpha_5 \times \frac{1}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \Phi \\
+ \alpha_6 \times \frac{1}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \Phi \\
+ \alpha_7 \times r_1 \frac{1}{\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \Phi \\
- \alpha_8 \times r_1 \frac{1}{\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \Phi.
\]
Next, we apply Eq. (55) to the case of $J_1 = 1$, $J_2 = 2$, and $P_2 = -1$ (e.g. $B \to \phi K_2$, $\phi \to K\bar{K}$, $K_2 \to K\pi\pi$) and obtain

$$
\frac{64\pi(1+r_{02})}{45\Gamma} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \alpha_1 \times \left\{ \cos^2\theta_1 \sin^4\theta_2 + r_{02} \frac{2}{3} \cos^2\theta_1 (3\cos^2\theta_2 - 1)^2 \right\} \\
+ \alpha_2 \times \left\{ \frac{1}{3} \sin^2\theta_1 \sin^2\theta_2 (1 + \cos^2\theta_2) + r_{02} \frac{1}{2} \sin^2\theta_1 \sin^22\theta_2 \right\} \\
- \alpha_3 \times \left\{ \frac{1}{3} \sin^2\theta_1 \sin^4\theta_2 - r_{02} \frac{1}{2} \sin^2\theta_1 \sin^22\theta_2 \right\} \cos 2\Phi \\
+ \alpha_4 \times \left\{ \frac{2}{3} \sin^2\theta_1 \sin^4\theta_2 - r_{02} \sin^2\theta_1 \sin^22\theta_2 \right\} \sin 2\Phi \\
- \alpha_5 \times \left\{ \sqrt{\frac{2}{3}} \sin 2\theta_1 \sin^3\theta_2 \cos\theta_2 - r_{02} \sqrt{\frac{2}{3}} \sin 2\theta_1 \sin 2\theta_2 (3\cos^2\theta_2 - 1) \right\} \cos \Phi \\
+ \alpha_6 \times \left\{ \sqrt{\frac{2}{3}} \sin 2\theta_1 \sin^3\theta_2 \cos\theta_2 - r_{02} \sqrt{\frac{2}{3}} \sin 2\theta_1 \sin 2\theta_2 (3\cos^2\theta_2 - 1) \right\} \sin \Phi \\
+ \alpha_7 \times r_2 \frac{4}{3} \sin^2\theta_1 \sin^2\theta_2 \cos\theta_2 \\
+ \alpha_8 \times r_2 \sqrt{\frac{2}{3}} \sin 2\theta_1 \sin^3\theta_2 \sin \Phi \\
- \alpha_9 \times r_2 \sqrt{\frac{2}{3}} \sin 2\theta_1 \sin^3\theta_2 \cos \Phi 
$$

(63)

The $r_{02}$ and $r_2$ parameters in Eq. (63), just like $r_1$ in Eqs. (61) and (62), are a priori unknown. However, if the dynamics of the $X_2 \to P_d P_e$ decay is known, these parameters could be further constrained, for example for a sequential two-body decay chain such as $X_2 \to P_e X_3$ with $X_3 \to P_d P_e$.

Finally, we study the angular formalism of a spinless particle decay $B \to X_1 X_2$, where both particles decay to three pseudoscalars $X_1 \to P_a P_b P_c$ and $X_2 \to P_d P_e P_f$. An example of such a decay is $B \to \omega K$, or $\omega \pi$ with $\omega \to \pi^+ \pi^- \pi^0$ and $K_\pi \to K\pi\pi$. Again, following the three-body decay formalism [37], we obtain

$$
\frac{1}{\Gamma} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi} \times \sum |A_\lambda|^2 \sum_{m} |R_m|^2 |R'_m|^2 \sum_{\lambda} A_\lambda \exp(i\lambda\Phi) d^4J_{\lambda,m}(\pi - \theta_1) d^2\lambda_{m'}(\theta_2) 
$$

where $|R_m|^2$ and $|R'_m|^2$ are the phenomenological parameters for $X_1 \to P_a P_b P_c$ and $X_2 \to P_d P_e P_f$, respectively, with $2J_1 + 1$ values of $m$ and $2J_2 + 1$ values of $m'$, as discussed with reference to Eq. (55). The same parity-conservation rules apply: $P_1 = (-1)^{m+1}$ and $P_2 = (-1)^{m'+1}$. Eq. (64) reduces to Eq. (55) for the decays with $J_{1}^{P_1} = 1^-$, such as $B \to \omega K^{*0}$ and $\omega\pi$. The former is described by Eqs. (61–63) for $K_2^*$, $K_1$, and $K_2 \to K\pi\pi$, and the latter by Eq. (53).

The above angular formalism should facilitate experimental analysis and measurements of $f_\perp$, $f_\parallel$, $\phi_\parallel$, and $\phi_\perp$ in various $B \to VT$ and $VA$ decays.

The above results are useful in the analysis of the work by Chen and Geng [18], who proposed an explanation of the $B \to V_1 V_2$ and $B \to VT$ polarization results. Briefly, it goes as follows. Consider the decay $B_2^0 \to \phi K^{*0}$. In addition to the naive SM contribution to $B_2^0 \to \phi K^{*0}$, Chen and Geng consider annihilation from the operator $bO_{\text{Odd} O_8}$, which appears in the effective Hamiltonian [Eq. (9)]. They use generalized factorization, which implies that this term is factorizable; we denote it as $FA'$. If $FA'$ is sizeable, one has to consider $P_{t,-} FA'$ interference.
in computing $f_L$, $f_T$, etc. Chen and Geng find
\begin{equation}
|A_L(\phi K^*)|^2 \propto 1 + C_{CG}(m_\phi^2 - m_{\bar{K}}^2).\tag{65}
\end{equation}
Here the first term is due to $P_{tc}^*$, while the second is due to $P_{tc}^*FA'$ interference. $C_{CG}$ depends, among other things, on the $B_d^0 \rightarrow K^{*0}$ form factors. These form factors, and $C_{CG}$, are fixed by the measured value of $f_L$ in $B_d^0 \rightarrow \phi K^{*0}$.

The expression for $|A_L(\phi K^*)|^2$ is identical to that above, with the substitution $K^* \rightarrow K^*_2$. The key point is that $(m_\phi^2 - m_{\bar{K}}^2)$ and $(m_\phi^2 - m_{K^*_2}^2)$ have opposite signs. Thus, assuming that the $B_d^0 \rightarrow K^{*0}$ form factors have the right size, if $f_L$ is small in $B_d^0 \rightarrow \phi K^{*0}$, it will be large in $B_d^0 \rightarrow \phi K^*_2$, in agreement with observation. (The explanation for $B^+ \rightarrow \phi K^{*+}$ is similar; the operator $b\bar{c}Q\bar{u}u\bar{c}$ contributes here.) Chen and Geng assume that the time-like form factor in $B_d^0 \rightarrow \phi K^{*0}$ is related to the form factor in $B_d^0 \rightarrow \phi K^*_2$, with the result that the $B \rightarrow V_1V_2$ and $B \rightarrow VT$ polarization results are reproduced.

Now, there can be objections to this explanation. First, all annihilation effects are thought to be small, and there is no experimental evidence for a large $FA'$. Thus, the suggestion that $FA'$ could be sizeable is somewhat arbitrary. Second, Chen and Geng consider only one type of annihilation, neglecting a second type which is also found in QCD. (It is questionable whether this second type of annihilation amplitudes is negligible compared to the first one.) Third, the polarization results depend on (unknown) form factors. Chen and Geng assume values for these form factors which work, but this might not be the case.

However, putting aside these objections, the question is: can we test the Chen-Geng explanation? Referring to Eq. (65), the reader could propose the following possibility: it might be useful to consider decays in which the two final-state particles have the same mass. Examples include $B_s \rightarrow K^*K^*$ and $B_s \rightarrow \phi\phi\phi$. In this case, $P_{tc}^*FA'$ interference apparently vanishes, so that $f_T/f_L$ is small. This can be tested. Unfortunately, this idea does not work. In Eq. (65), $C_{CG}$ includes in the denominator $m_{\phi}^2 - m_{q,s}$, where the decay is dominated by the $b \rightarrow q$ penguin amplitude, and $q,s$ is the spectator quark. In the above decays, this difference vanishes ($q = s$). Thus, $P_{tc}^*FA'$ interference is of the form $0/0$. After careful evaluation, it is found that this interference is in fact nonzero.

Therefore, $f_T/f_L$ is arbitrary (it depends on unknown form factors). This same argument applies to all decays in which the two final-state particles have the same mass. Thus, there are no decay modes to test the Chen-Geng explanation. The only thing to do is to compute the annihilation form factors in $B \rightarrow VT$ decays, $\langle VT|O_i|0\rangle$, to see if they agree with the Chen-Geng estimates.

In addition, Chen, Geng and collaborators have considered annihilation contributions in $B \rightarrow VA$ decays, $\langle VA|O_i|0\rangle$. Assuming these contributions to be factorizable, they discussed the decay $B \rightarrow \phi K_1$ [39]. In their scenario, they conclude that the annihilation contributions can be neglected, which implies that $f_T/f_L$ in $B \rightarrow \phi K_1$ is small. However, their conclusion assumes that the $B \rightarrow VA$ annihilation form factors are similar to those of $B \rightarrow V_1V_2$. Unfortunately, the form factors relevant to $VA$ final states are unknown – if one takes any values for them, $f_T/f_L$ in $B \rightarrow \phi K_1$ can be large or small. Thus, this is not a real test of the Chen-Geng explanation.

Penguin annihilation and rescattering make no prediction for the polarization in $B \rightarrow \phi K_1$, since this is a different final state from $\phi K^*$. NP also makes no prediction, since its result depends on the unknown $B \rightarrow A$ form factors. Despite the fact that there is no firm prediction for the polarization in $B \rightarrow \phi K_1$ – or perhaps because of it – this is an important measurement to make. Now, $K_1$ decays only to 3 bodies, e.g. to $K\pi\pi$. Thus, the angular analysis of the 3-body decay described above will be necessary to get helicity information.

CONCLUSIONS

In $B \rightarrow V_1V_2$ decays ($V_i$ is a light charmless vector meson), the final-state particles can have transverse or longitudinal polarization. Within the standard model (SM), the naive expectation is that the fraction of transverse-polarization decays is much less than the fraction of longitudinal decays: $f_T/f_L \ll 1$. However, it was found in $B \rightarrow \phi K^*$ that $f_T/f_L \approx 1.3$. This is the “polarization puzzle.” It is not necessary to invoke new physics (NP), though one can. There are two extensions beyond the naive SM in which the polarization puzzle is explained: penguin annihilation and rescattering. One can also look at $B \rightarrow VT$ decays ($T$ is a tensor meson). In $B \rightarrow \phi K^*_2$, $f_T/f_L$ is observed to be small. In this paper, we look at the polarizations in $B \rightarrow VT$ decays, both in the SM (naive and extended) and with NP. The idea is to examine the prediction for polarization in $B \rightarrow VT$, to see if it is in agreement with the measurement.

We begin by considering the naive prediction of the SM. It depends on the $B \rightarrow T$ form factors, which are unknown at present. These form factors can be expressed in terms of two independent, universal form factors, $\zeta_\parallel$ and $\zeta_\perp$. We consider $\zeta_\parallel \approx \zeta_\parallel$ [case (a)] and $\zeta_\perp \ll \zeta_\parallel$ with $\zeta_\parallel \sim (m_t/m_b)\zeta_\parallel$ [case (b)]. In both cases we find that $f_T/f_L(B \rightarrow \phi K^*_2) \ll 1$. We therefore conclude that the naive SM reproduces the polarization measurement in $B \rightarrow \phi K^*_2$.

The polarization predictions of both penguin annihilation and rescattering are not certain. That is, the predictions depend on a new set of parameters for each final state. Thus, the final-state polarization in $B \rightarrow \phi K^*_2$ can be large or small. It is therefore possible that both expla-
nations agree with the $f_+/f_-$ measurements in $B \to \phi K^*$ and $B \to \phi K^*_2$.

The combined branching-ratio and CP-asymmetry measurements in $B \to \pi K$ decays are also in disagreement with the SM predictions (the “$K\pi$ puzzle”). In Ref. [13] it was found that only two new-physics operators can account for the discrepancies in both the $\pi K$ data and the $\phi(\rho)K^*$ polarization measurements. We examine the predictions of these operators for polarization in $B \to \phi K^*_2$. We find a dependence on the $B \to T$ form factors. If they obey case (a), then NP cannot explain the $B \to \phi K^*_2$ polarization data. However, if they are described by case (b), then NP can account for the measurements in $B \to \pi K$, $B \to \phi K^*$ and $B \to \phi K^*_2$. This can be tested by explicit computations of the $B \to T$ form factors.

Finally, most of the polarization measurements to date use the two-body decays of particles. In this paper, we present the general angular analysis. In particular, we show how to get helicity information using three-body decays. This is important for vector, tensor, and axial-vector final-state mesons.

Acknowledgments: This work was financially supported by NSERC of Canada (DL, MN & AS), and the U.S. NSF (YG & AG) and A. P. Sloan Foundation (AG).

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