High-z Supernova Type Ia Data: non-Gaussianity and Direction Dependence

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ABSTRACT
We use the Δχ² statistic introduced in Gupta, Saini & Laskar (2008); Gupta, Saini (2010) to study directional dependence, in the high-z supernovae data. This dependence could arise due to departures from the cosmological principle or from direction dependent statistical systematics in the data. We apply our statistic to the gold data set from Riess et al. (2004) and Riess et al. (2007), and Union2 catalogue from Amanullah et al. (2010). Our results show that all the three data sets show a weak but consistent direction dependence. In 2007 data errors are Gaussian, however other two data sets show non-Gaussian features.

Key words: cosmology: cosmological parameters — cosmology: large-scale structure of universe — supernovae: general

1 INTRODUCTION

A large and diverse variety of cosmological observations (Perlmutter et al. 1999; Riess et al. 1998, 2002, 2004, 2007; Benoît et al. 2003; Page et al. 2007), during the last two decades (notably the Nobel prizes in 2006 and in 2011) have established that we live in a flat universe with an accelerated expansion. This is consistent with the Einstein’s general theory of relativity with a cosmological constant term. The Cosmological Constant (Λ), also known as dark energy, is treated as an ideal fluid with negative pressure; and with equation of state p = wρ, where w = −1. Dark energy density dominates over that of baryonic and dark matter; and it constitute two third of the Universe. This model of the Universe is known as the ΛCDM model or standard model of cosmology.

The foundation of the standard model of cosmology (ΛCDM cosmology) is the Cosmological principle (hereafter CP), which states that the Universe is homogeneous and isotropic on the large scales (Peebles 1993). CP along with the fact that two third of the constituents of the Universe is dark energy can explain many cosmological observations and is the most successful model till date. Despite its success, ΛCDM model has its shortcomings; few of which are summarized below:

- Observations indicate significantly larger amplitude of flows than what ΛCDM predicts. [Watkins et al. 2000] found the large scale peculiar velocity larger than 400km/Sec at scales up to 100h⁻¹ Mpc. This is in the direction l = 282°, b = 6°. The probability of finding such a flow in the ΛCDM cosmology is less than 1%.
  - The planes normal to the quadrupole (l = 240°, b = 63°) and octopole (l = 308°, b = 63°) are aligned with each other and with the direction of the dipole (l = 264°, b = 48°). This indicates a preferred axis; and is inconsistent with the Gaussian random, statistically isotropic skies at about 99%.
  - Using HST key project data [McClure & Dyer 2007] showed that a statistically significant variation of at least 9 km/s/Mpc exists in the observed value of H₀ with a directional uncertainty of about 10-20%. [Gupta, Saini 2011] also found directional dependence in the HST key project data.
  - Recent work provides some evidence for what is known as the Hubble Bubble ([Zehavi et al. 1998; Jha et al. 2007] which suggests that we might be living inside a large void. Value of the Hubble constant inside the bubble is different from what is outside the bubble. There is evidence for such large scale voids in the CMB maps as well ([Cruz et al. 2006; de Oliveira-Costa & Tegmark 2006; Cruz et al. 2006, 2007], suggesting that such large voids are not implausible. If our position is near the center of such a void one can explain the dimming of SNe without invoking dark energy. However, this challenges the Copernican principle that we do not occupy a special place in the Universe. If we assume that our position
is off-centered then one can explain the preferred axis (or tiny departures from CP) also\cite{Blomqvist2010}.

In summary, some cosmological observations show departure from CP and hence are in contradiction to the standard model of cosmology. Another threat to the model comes from the fact that Observed value of $\Lambda$ does not match its theoretical value and requires fine tuning. To avoid this several alternative explanations have been suggested \cite{Silvestri2009,Frieman2008,Sahni2006}. In many of these alternative models, $w$ is allowed to be different from $-1$ in the past; and approaches $w \simeq -1$ at low red-shifts. It is different from cosmological constant, since there the value of $w$ is always $-1$. Cosmological observations suggest $w \simeq -1$ at the present epoch, which is consistent with $\Lambda$CDM as well as the alternative cosmologies. To be able to distinguish among various models, we require data that is precise enough to discern tiny variations in the dark energy. It is also required that data be available at a large number of redshifts to constrain the detailed behavior of dark energy with time. At the present the only data that comes reasonably close to these requirements is provided by the observations of the high-redshift supernovae, which are believed to be standard candles. Besides, the quality of the SNe Ia data maybe doubtful due to the following reasons:

- The physics of SNe Ia is relatively poorly understood.
- The possibility of physical mechanisms, such as dust in the inter-galactic medium that systematically dims them.
- The supernova data are usually collated from several different sources that might have slightly different systematics, due either to instrumental effects or to the fact that they occur in different directions in the sky. Since we have to correct for the galactic dust extinction, which might not get completely removed from the samples, this might produce anisotropy in data.

These considerations imply that to have precise information about the behavior of dark energy we should have a good knowledge of the statistical properties of supernovae, both random as well as systematic. There have been several attempts to search for the direction dependence in the SNe Ia data \cite{Gupta2008,Gupta2010} (hereafter GSL08) and \cite{Gupta2010} (hereafter GS10) used the extreme value statistics to show that the two supernova data sets,\cite{Riess2004} (GD04) and \cite{Riess2007} (GD07), do show some evidence for direction dependence.\cite{Antoniou2010} have also shown a preferred axis using the Union2 catalogue from \cite{Amanullah2010}. Several other works have also indicated either systematic problems with the high-redshift supernova data or directional dependence in the supernova data and other probes \cite{Nesseris2004,Nesseris2007,Jain2006,Jain2007}.

In this paper our main task is 1) to look for direction dependent systematic effects in the latest SNe Ia data (Union 2 catalogue) and 2) to compare the quality of this data with the previous data sets (Gold data 2004 and 2007). The plan of the paper is as follows. In § 2 we introduce the statistic we have used, in § 3 we provide our results and end with conclusions in § 4.

## 2 METHODOLOGY: THE $\Delta \chi^2$ STATISTIC

We use the Gold data (GD04 and GD07) \cite{Riess2004,Riess2007} and the Union2 data \cite{Amanullah2010} for our analysis. The Gold data GD04 and GD07 contain 157 and 182 SNe respectively; while the most recent and largest set Union2 contains 557 Supernovae. For a given supernova the measured quantity, the distance modulus $\mu$, is the difference between the apparent and the absolute magnitude

$$\mu(z) = m(z) - M,$$

where the apparent magnitude $m(z)$ depends on the intrinsic luminosity of a supernova, the redshift $z$ and the cosmological parameters; and $M$ is the absolute magnitude of a type Ia supernova. It can be expressed in terms of the luminosity distance $D_L$ as

$$\mu(z) = 5 \log (D_L(z)/\text{Mpc}) + 25,$$

where the luminosity distance is given by

$$D_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dx}{h(x)},$$

where $h(z;\Omega_M,\Omega_X) = H(z;\Omega_M,\Omega_X)/H_0$, and thus depends only on the cosmological parameters matter density $\Omega_M$ and the dark energy density $\Omega_X$. We assume that the prescription for the variation of dark energy with redshift is separately specified, for example in the $\Lambda$CDM model the energy density in the dark energy remains a constant. In Eq (1) the dependence of the measured quantity $\mu$ on $M$ is linear. Since $\mu$ depends on the logarithm of the luminosity distance it is clear that it depends linearly on the logarithm of the Hubble parameter $H_0$. Usually the data is given in terms of Eq (2) where the constant $M$ has already been marginalized over. Thus, instead of two nuisance parameters we are left with only one parameter, the Hubble constant $H_0$.

We now give an introduction of the $\Delta$ statistic previously introduced and used in GSL08 and GS10. A generalization was given in GS10, where we had numerically marginalized over the Hubble constant $H_0$. Some of the details are repeated here to make this work self-contained. For our analysis we have assumed a flat $\Lambda$CDM universe. A similar analysis could be carried out for a more general model of dark energy.

We consider subsets of the full data set to construct our statistic consisting of $N_{\text{subset}}$ data points. Since the $\Lambda$CDM model fits the gold data sets GD04, GD07 and Union2 well, we first obtain the best fit to the full data sets by minimizing the $\chi^2$, which we define as:

$$\chi^2 = \sum_{i=1}^N \left( \frac{\mu - \mu_{\Lambda\text{CDM}}(z;\Omega_M)}{\sigma_i} \right)^2,$$

where $\sigma_i$ is observed standard error in $\mu_i$. By this we obtain the best fit values of the parameters $\Omega_M$ and $H_0$. Then for each supernova we calculate $\chi_i = |\mu - \mu_{\Lambda\text{CDM}}(z;\Omega_M)|/\sigma_i$, where $\mu_{\Lambda\text{CDM}}(z;\Omega_M)$ is calculated using the best fit values of $\Omega_M$ and $H_0$. We assume that all the supernovae are statistically uncorrelated.

We define $\chi^2_M = \sum_i \chi_i^2$ and the normalized quantity $\chi^2_R = \chi^2_M/N_{\text{subset}}$, $\chi^2_R$ indicates the statistical scatter of the subset from the best fit $\Lambda$CDM model and its expectation value is unity that $(\chi^2_R)^{1/2} = 1$. If CP holds then the apparent magnitude of a supernova should not depend on the
3 RESULTS

We divide the data into two hemispheres labeled by the direction vector $\hat{n}$, and take the difference of the $\chi^2_R$ computed for the two hemispheres separately to obtain $\Delta \chi^2_R = \chi^2_{1R} - \chi^2_{2R}$, where label '1' corresponds to that hemisphere towards which the direction vector $\hat{n}$ points and label '2' refers to the other hemisphere. We take the absolute value of $\Delta \chi^2_R$, since we are interested in the largest value of this quantity and it is obvious that for every value of $\Delta \chi^2_R$, the antipodal point has the negative of that value. We then vary the direction $\hat{n}$ across the sky to obtain the maximum absolute difference

$$\Delta \chi^2 = \max(\{ |\Delta \chi^2_R| \}) \ .$$

As shown in GSL08, the distribution of $\Delta \chi^2$ follows a simple, two parameter Gumbel distribution, characteristic of extreme value distribution type I (Kendall & Stuart 1977),

$$P(\Delta \chi^2) = \frac{1}{s} \exp \left[ - \frac{\Delta \chi^2 - m}{s} \right] \exp \left[ - \exp \left( - \frac{\Delta \chi^2 - m}{s} \right) \right] \ .$$

where the position parameter $m$ and the scale parameter $s$ completely determine the distribution. To quantify departures from isotropy we need to know the theoretical distribution, which is calculated numerically by simulating several sets of Gaussian distributed $\chi_i$ on the gold set supernova positions and obtaining $\Delta \chi^2$ from each realization. And as in GSL08, we compute a bootstrap distribution by shuffling the data values $z$, $\mu(z)$ and $\sigma_{\mu}(z)$ over the supernova positions (for further details see GSL08). A modified version of this statistic was introduced in GS10, which marginalizes over the Hubble Constant. However, by this we loose all the information about $H_0$ and hence, here we do not marginalize.

3 RESULTS

In GSL08 and GS10 we discussed a specific bias in the bootstrap distribution, showing that it is shifted slightly to the left of the theoretical distribution due to the fact that theoretical distribution is obtained by assuming $\chi_i$ to be Gaussian random variates with a zero mean and unit variance.
Therefore theoretical $\chi^2$ is unbounded. However, the bootstrap distribution is obtained by shuffling through a specific realization of $\chi$, and they have a maximum value for some supernova. It is clear that this should, on average, produce slightly smaller values of $\Delta \chi^2$ in comparison with what one expects from a Gaussian distributed $\chi$. For reference we plot the results for simulations in Figure 1 with a total of 157 and in Figure 2 with a total of 557 supernovae. Our results in this paper should be interpreted with respect to Figure 1 for GD04 and GD07, and Figure 2 for Union2 data. Concerns regarding the small number of supernovae in the Gold data and its effect on the efficacy of our method can be addressed by the fact that overall behavior of Figure 1 is repeated in Figure 2 which is plotted for more than 500 of SNe.

This statistic is similar to the one presented in GSL08 and GS10 except for the fact that in GS10 we had marginalized over the Hubble parameter. Our results are different from those presented in GSL08 also due to the fact that we have corrected the numerical bug mentioned in the GS10 produced by the fact that the theoretical distribution was produced slightly differently from the way bootstrap method thereby creating a greater discrepancy between them then should have been the case. In Table 1 we give the best fit values of $\Omega_M$ and $H_0$ for all the three data sets. We note that Union2 gives slightly lower value of $\Omega_M$ and higher value of $H_0$ compared to the Gold data. Reduced $\chi^2$ is smallest for GD07 which indicates that errors are overestimated for this set. Union2 is best among all the three sets in terms of $\chi^2$. The direction of maximum discrepancy and the value of $(\Delta \chi^2)$ is presented in Table 2.

GD04: In Fig 3 we plot the bootstrap and the theoretical distribution expected for GD04 and mark the position of GD04. Comparison with Figure 1 shows a signature of non-Gaussianity since the theoretical distribution instead of being to the right of the bootstrap is shifted to the left. Position of GD04 is about 2 sigma away from the peak of the bootstrap distribution, indicating direction dependence.

GD07: Fig 4 for GD07 is same as for GD04. A comparison with Figure 1 shows a signature of non-Gaussianity since the theoretical distribution instead of being to the right of the bootstrap is shifted to the left. Position of GD04 is about 2 sigma away from the peak of the distribution, indicating direction dependence.

Union2: In Fig 5 we plot both the distributions for Union2, which is to be compared with Fig 2. Comparison shows a weak signature of non-Gaussianity, since position of the bootstrap distribution is not to the left of the theoretical distribution as expected by simulated data in Figure 1, which uses Gaussian deviates suggesting evidence for non-Gaussianity.

**Table 1.** The model parameters ($\Lambda$CDM) for the three data sets are tabulated here.

| Set    | # SNe | $\Omega_M$ | $H_0$ | $\chi^2$/dof |
|--------|-------|------------|-------|--------------|
| GD04   | 157   | 0.32       | 64.5  | 1.14         |
| GD07   | 182   | 0.33       | 63.0  | 0.88         |
| Union2 | 557   | 0.27       | 70    | 0.97         |

**Table 2.** Direction for maximum $\Delta$ in the three data sets are tabulated here.

| Model  | Set | $\Delta \chi^2$ | longt | lat  |
|--------|-----|------------------|-------|------|
| ACDM   | GD04| 0.83             | 96.5  | 44.5 |
| ACDM   | GD07| 0.53             | 347.1 | 27.  |
| ACDM   | Union2| 0.22             | 65.5  | 55.8 |

Figure 4. The theoretical and the bootstrap probability distributions for GD07 for the $\Delta \chi^2$ statistic. Theoretical distribution is to the right as expected by simulated data in Figure 1, which uses Gaussian deviates suggesting evidence for non-Gaussianity.

Figure 5. The theoretical and the bootstrap probability distributions for Union2 for the $\Delta \chi^2$ statistic. Theoretical distribution is not compatible with Figure 2 for the simulated data, which implies slight non-Gaussianity for the residuals.
cal distribution. Position of Union2 is about one sigma away from the peak of the bootstrap distribution which is a sign of direction dependence.

4 CONCLUSIONS

We have presented results for the GD04, GD07 and Union2 data using the statistic introduced in GSL08 and GS10. Our main conclusions for this part of our work are:

(i) GD04 shows some evidence for non-Gaussianity, however, GD07 is entirely consistent with a Gaussian distribution of residuals. Union 2 data again shows some evidence for non-Gaussianity although weaker compared to GD04. Thus, in terms of Gaussianity, GD07 is best among the three data sets.

(ii) GD04 is different from the peak of the bootstrap distribution by slightly more than one sigma; while GD07 and Union2 both are different from the peak by about a sigma. There seems a weak but consistent direction dependence in all three data sets. This consistency indicates a physical effect or a preferred direction.

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