Timetable planning of projects scheduling with account of uniform distribution of financial revenues on the basis of double-transport problem

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Abstract. In this paper a specific algorithm of optimization of timetable of realizing projects in the framework of multi-project organization is considered. It allows to achieve the uniformity of labor costs and financial supplies (revenues). The algorithm is developed on the basis of solution of the double-transport problem.

1. Introduction
Problems of the optimal distribution of resources with respect to projects (or projects with respect to resources) have been considered in a number of publications (papers) ([1–5] and others).

The main purpose of solving these problems reduces to constructing the optimal timetable of realizing all the work of the project. This plan should allow realizing the project by the scheduled date with minimal expenses. The limitations on consumption of the main resource have to be satisfied during the time period.

As a rule the problem of finding the optimal timetable is reduced to solving a variational problem on a network.

Multi-projects are complex projects or programs realized in the framework of large organizations. Multi-project, as well as a project is an object of management (control). However, unlike a simple project, it requires special methods of coordination and multi-project management which ensure achieving the aim of the multi-project while satisfying the imposed limitations and conditions of its realization [6].

The problem of determining the sequence (order) of realizing of constituent projects which will allow obtaining the maximal profit from the multi-project as a whole is characteristic for multi-projects.

The so-called project management structures are employed for high-quality management of multi-projects in factories (companies, enterprises).

The project structures are the structures of management of complex activities that are of utmost importance for the company and therefore require continuing coordination and integration with strict limitations of expenses (expenditures), schedule and quality of work.
The meaning of the project organizational structure consists in selecting the most qualified people of different specializations into one team for realizing a complex project within the deadline with the given quality of work and within the framework of material, financial and labor resources that have been designated for this purpose. Project structures are dealing with operative and strategic management only within the framework of the project.

Thus the project organizational structure implies the use of appropriate approaches (methods) for optimizing management of structural units as mini-factories (projects).

2. Models for determination the optimal sequence of realization of multi-project

There are several models for determination the optimal sequence of realization of the projects within multi-project. Each one of these models is characterized by the criterion allowing making a decision about the order of implementation of the projects. Here the optimization of the sequence means the maximization of the profit (net profit) originating from this particular sequence.

Classical models for determination of the optimal sequence for implementing projects are as follows [6, 7]:

1. Project ordering is made according to the criterion of expended resources. In this model the projects are ordered in the order of increasing expended resources. The main deficiency of the method is that in the project evaluation their revenues are not taken into account. Therefore it is possible that the most profitable projects will be the last ones in the sequence.

2. For gaining the maximal income the investor is ordering the projects in order of diminishing profit. The main deficiency of this method is that in calculation of the profit from the project it can happen that the project with the largest income is less profitable than the projects with lower income due to larger expenditures, which are not taken into account in this model.

3. Model "expenditures-effectiveness" is a kind of a compromise between the first two models and takes into account both the expenditures on the project implementation as well as xxxx effect. The effect to expenditures ratio is used within this method as an ordering criterion. This method is quite simple for implementing, however, as has been shown in [5, 6] it provides a rough estimate because of the lack of timescale.

All the methods described above use the maximization of the profit from the multi-project as a whole as an optimization criterion. The purpose of our study is to find the mechanism of optimization of multi-project implementation, which is based on the condition of rhythmical functioning of the company.

3. General statement of the problem

The company considered produces \( m \) sorts of products. It is necessary to work out (develop) the timetable of production scheduled for \( n \) periods. Let us define:

- \( \hat{A}_i \) — the volume of production of the \( i \)-th product in arbitrary units, \( i = 1, m \)
- \( t_i \) — laboriousness of producing of a single unit of \( i \)-th product, \( i = 1, m \)
- \( c_i \) — working costs for producing a single unit of \( i \)-th product, \( i = 1, m \).

It is necessary to determine the volume of production during each one of the periods, which ensures uniform distribution of the work content (labor intensity) and costs. The total work content and the total cost of the produced products (goods) is evaluated from the formulae:

\[
T = \sum_{i=1}^{m} t_i \hat{A}_i,
\]
\[ C = \sum_{i=1}^{m} c_i \hat{A}_i. \]  

(2)

The uniform distribution is understood as the desired volume (amount) of labor intensity and cost in each one of the periods:

\[ T_j = \alpha_j T, \quad \sum_{j=1}^{n} \alpha_j = 1, \quad \alpha_j \geq 0, \quad j = 1, n \]  

(3)

\[ C_j = \beta_j C, \quad \sum_{j=1}^{n} \beta_j = 1, \quad \beta_j \geq 0, \quad j = 1, n. \]  

(4)

The quantities \( \alpha_j \) and \( \beta_j \) are the weighting coefficients, which are given by consumer. They may be limited to certain periods (seasons) or uniformly distributed, i.e. equal to \( \frac{1}{n} \).

4. **Mathematical model**

The mathematical model can be formulated on the basis of the stated problem:

\[ \sum_{j=1}^{n} \hat{x}_{ij} = \hat{A}_i, \quad i = 1, m \]  

(5)

\[ \sum_{i=1}^{m} t_i \hat{x}_{ij} = T_j, \quad j = 1, n \]  

(6)

\[ \sum_{i=1}^{m} c_i \hat{x}_{ij} = C_j, \quad j = 1, n \]  

(7)

\[ \hat{x}_{ij} \geq 0, \quad i = 1, m \quad j = 1, n \]  

(8)

where \( \hat{x}_{ij} \) — the volume of production of the \( i \)-th product during the \( j \)-th period, \( i = 1, m \), \( j = 1, n \).

Limitation (condition) (5) implies that the year plan of production of each one of the products has to be met.

Limitation (condition) (6) implies that the plan of labor intensity in each one of the periods has to be met.

Limitation (condition) (7) implies that the plan of production costs in each one of the periods has to be met.

We note that for the model the following conditions are satisfied:

\[ \sum_{j=1}^{n} T_j = \sum_{i=1}^{m} t_i \hat{A}_i = T, \]  

(9)

\[ \sum_{j=1}^{n} C_j = \sum_{i=1}^{m} c_i \hat{A}_i = C. \]  

(10)

Equations (5)–(8) can be transformed by the following variables change:

\( x_{ij} = \hat{x}_{ij} t_j \), \( x_{ij} \) — the labor intensity of producing the \( i \)-th product during the \( j \)-th period;

\( A_i = t_i \hat{A}_i \), \( A_i \) — the total labor intensity of producing the \( i \)-th product;

\( a_i = \frac{c_i}{t_i} \), \( a_i \) — the cost of a unit of labor intensity of the \( i \)-th product.
Then the model equations (5)–(8) will have the form:

\[ \sum_{j=1}^{n} x_{ij} = A_i, \; i = \overline{1}, m \]  
\[ \sum_{i=1}^{m} x_{ij} = T_j, \; j = \overline{1}, n \]  
\[ \sum_{i=1}^{m} a_i x_{ij} = C_j, \; j = \overline{1}, n \]  
\[ x_{ij} \geq 0, \; i = \overline{1}, m, \; j = \overline{1}, n. \]  

(11)  
(12)  
(13)  
(14)

Thus we have the problem of distribution of the labor intensity over the periods, where the limitations (11), (12), (14) represent the limitations of the transport problem, while the limitations (11), (13), (14) represent the limitations of the distributional problem.

It should be noted that the problem at hand is not the problem of optimization, and any legitimate solution of the system of equations (9)–(12) will constitute the correct solution.

The following balance conditions (15)–(16) are obviously satisfied for the problem (11)–(14).

\[ \sum_{i=1}^{m} A_i = \sum_{j=1}^{n} T_j, \]  
\[ \sum_{i=1}^{m} a_i A_i = \sum_{j=1}^{n} C_j. \]  

(15)  
(16)

Let us consider the derived system of equations.

The balance condition (15) is the necessary and the sufficient condition of compatibility for the problem (11), (12), (14), while the balance condition (16) is the necessary and sufficient condition of compatibility with the limitations (11), (13), (14) [8].

The problem (11)–(14) we will call the double-transport problem. For it the conditions (15)–(16) constitute only the necessary, but not sufficient conditions for compatibility [9, 10].

Thus in general, if the balance conditions (15)–(16) are satisfied, one cannot say whether the system of equations (11)–(14) is compatible or it is not. However, there exists a simple algorithm for finding an acceptable solution of the problem, which is based on decomposition of the system. The decomposition implies that on each step for \( j = \overline{1}, n \) the following system of equations is solved:

\[ \sum_{i=1}^{m} x_{ij} = T_j, \; j = \overline{1}, n \]  
\[ \sum_{i=1}^{m} a_i x_{ij} = C_j, \; j = \overline{1}, n \]  
\[ 0 \leq x_{ij} \leq A_i, \; i = \overline{1}, m, \; j = \overline{1}, n. \]  

(17)  
(18)  
(19)

We will assume that all coefficients \( a_i \) are organized in ascending order: \( a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_k \leq \cdots a_n \) and will find the solution of the system (17)–(19), which has two non-vanishing (basic) components:

\[ x_{s_j} + x_{k_j} = T_j, \]
\[ a_{s_j} x_{s_j} + a_{k_j} x_{k_j} = C_j. \]  

(20)
The derived system will be solved in the following manner. We choose \( s_j < k_j \) and solve the system (20). Its solutions are given by:

\[
\begin{align*}
  x_{sj} &= \frac{a_{k_j} T_j - C_j}{a_{k_j} - a_{s_j}}, \\
  x_{kj} &= \frac{C_j - a_{s_j} T_j}{a_{k_j} - a_{s_j}}.
\end{align*}
\] (21)

For the obtained values of \( x_{sj} \) and \( x_{kj} \) we check whether the condition (19) is satisfied.

Since \( s_j < k_j \), so \( a_{k_j} > a_{s_j} \), and therefore in order that condition (19) is satisfied it is necessary and sufficient that \( a_{k_j} T_j - C_j \geq 0 \) and \( C_j - a_{s_j} T_j \geq 0 \). Thus the numbers \( s \) and \( k \) must exist, such that

\[
\begin{align*}
  a_{s_j} &\leq \frac{C_j}{T_j} \leq a_{k_j}.
\end{align*}
\] (22)

It is evident that if for a particular value of \( j \) either \( a_{s_j} \) or \( a_{k_j} \) that satisfy the condition (22) do not exist, then the system (20) and consequently all the system of equations (11)–(14) is incompatible.

If \( x_{sj} \leq A_{s_j} \) and \( x_{kj} \leq A_{k_j} \), then (21) constitutes the solution of the system (20) and it is possible to switch to calculation of the variables for the next value of \( j = j + 1 \), but before doing it it is necessary to correct the values of \( A_{k_j} \) and \( A_{s_j} \) according to the rule:

\[
\begin{align*}
  A_{k_j}^H &= A_{k_j} - x_{kj}, \\
  A_{s_j}^H &= A_{s_j} - x_{sj}.
\end{align*}
\]

If \( x_{sj} \geq A_{s_j} \), then \( x_{sj} = A_{s_j} \) is not a basic component. In this case one should assume that:

\[
\begin{align*}
  A_{s_j} &= 0, \\
  T_{j}^H &= T_j - A_{s_j}, \\
  C_{j}^H &= C_j - a_{s_j} A_{s_j},
\end{align*}
\]

and after that \( x_{kj} \) and \( x_{sj} \) are once again calculated using the formula (21) for the same index \( j \) and new values of \( T_j \) and \( C_j \).

If \( x_{kj} \geq A_{k_j} \), then \( x_{kj} = A_{k_j} \) is not a basis component. In this case one should assume that

\[
\begin{align*}
  A_{k_j} &= 0, \\
  T_{j}^H &= T_j - A_{k_j}, \\
  C_{j}^H &= C_j - a_{k_j} A_{k_j},
\end{align*}
\]

and after that \( x_{kj} \) is calculated once again with the same index \( j \) and new values of \( T_j \) and \( C_j \).

It follows from (22) that \( s_j \) and \( k_j \) may not be defined uniquely. In the case of a "bad" choice of \( s_j \) and \( k_j \) the absence of solution under decomposition does not necessarily imply the incompatibility of the system.

In order to remove (avoid) this problem we will accept to statements without proof:

**Statement 1.**

If the numbers are chosen as:

\[
\begin{align*}
  s_j &= \max(i : a_i T_j \leq C_j, \ A_i > 0) \quad (23) \\
  k_j &= \min(i : a_i T_j \geq C_j, \ A_i > 0), \quad (24)
\end{align*}
\]
then the solution of the system (11)–(14) using decomposition method does not destroy its compatibility [11].

**Statement 2 (sufficient condition for compatibility).**

Let \( a_1 \leq a_2 \leq \ldots \leq a_n \). If for \( \forall j \ s_j = s \) and \( k_j = k \), where \( s \) and \( k \) are defined from formulae (23), (24) then the system (11)–(14) is compatible [9, 10].

5. **Mathematical model for the problem of the project timetable planning with account of uniformity of labor intensity and financial revenues**

Let us assume that there are \( m \) — projects that should be realized during a year.

For each project the labor intensity is known (in men/month) as well as the cash return (in rubles).

It is necessary to schedule the implementation of projects according to months in such a way that the financial return (revenues) and labor intensity are uniformly distributed.

Let us construct the mathematical model for the problem stated above.

Let \( \hat{x}_{ij} \) be the fraction (part) of the \( i \)-th project realized during \( j \)-th month.

\( \hat{t}_i \) — the labor intensity in men/month,
\( c_i \) — the cash revenue in rubles,
\( n = 12 \) months
\( \alpha = 1/12, \beta = 1/12 \)

\[
T = \sum_{i=1}^{12} t_i, \quad T_j = \frac{T}{12} \text{ for } \forall j
\]

\[
C = \sum_{i=1}^{12} c_i, \quad C_j = \frac{C}{12} \text{ for } \forall j
\]

We obtain the following model:

\[
\sum_{j=1}^{n} \hat{x}_{ij} = 1, \ i = 1, m
\]  \hspace{1cm} (25)

\[
\sum_{i=1}^{m} t_i \hat{x}_{ij} = T_j, \ j = 1, n
\]  \hspace{1cm} (26)

\[
\sum_{i=1}^{m} c_i \hat{x}_{ij} = C_j, \ j = 1, n
\]  \hspace{1cm} (27)

\( \hat{x}_{ij} \geq 0, \ i = 1, m, \ j = 1, n. \)

Restriction (condition) (25) means, that each project has to be completed.

Restriction (condition) (26) means, that the labor intensity plan has to be realized for each month.

Restriction (condition) (27) means, that the cash revenues plan has to be realized for each month.

We reduce the problem to the double-transport problem. Let

\( x_{ij} = t_i \hat{x}_{ij} \) — labor intensity in men/months of the \( i \)-th project for the \( j \)-th month;

\( t_i \) — total labor intensity of the \( i \)-th project;

\( a_i = \frac{c_i}{t_i} \) — financial (cash) revenue for the labor intensity unit of the \( i \)-th project.
The model has the following form:

\[
\begin{align*}
\sum_{j=1}^{n} x_{ij} &= t_i, \quad i = 1, m \\
\sum_{i=1}^{m} x_{ij} &= T_j, \quad j = 1, n \\
\sum_{i=1}^{m} a_i x_{ij} &= C_j, \quad j = 1, n \\
x_{ij} &\geq 0, \quad i = 1, m; j = 1, n.
\end{align*}
\] (28) (29) (30) (31)

Thus we have a particular case of the double-transport problem. Since for \( \forall j \ T_j = T/12 \) and \( C_j = C/12 \), then according to Statement 2 the problem (28)–(31) is compatible, i.e. the given values of the labor intensity and of the cash revenues in each particular period will be achieved (obtained).

We will solve the problem using the decomposition method.

After the problem will be solved, it will be necessary to return to the initial formulation of the problem, i.e. to determine \( \hat{x}_{ij} \) — the part of \( i \)-th project, which is scheduled for the \( j \)-th month.

\[ \hat{x}_{ij} = \frac{x_{ij}}{t_i} \]

6. Algorithm of search of the acceptable solution of the problem using decomposition method

\textbf{Step 0.} \( n = 12, m \) — the number of projects. \( A_i = 1, t_i \) — labor intensity in men/days, \( c_i \) — cash revenue from the \( i \)-th project.

\textbf{Step 1.} Calculate \( t_i \) in men/days \( \frac{t_i}{30} \).

\textbf{Step 2.} Calculate \( C = \sum_{i=1}^{m} c_i, \ T_j = \frac{T}{12}, \ C_j = \frac{C}{12}, \ A_i = t_i, \ a_i = \frac{c_i}{t_i} \).

\textbf{Step 3.} Arrange \( i \) in order of increasing \( a_i \).

\textbf{Step 4.} Construct the initial table for the decomposition method.

\[
\begin{array}{cccccc}
\hline
i & a_i & 1 & 2 & 3 & 12 & A_i \\
\hline
1 & & & & & & \\
2 & & & & & & \\
3 & & & & & & \\
\hline
T_j & T/12 & T/12 & & & T/12 \\
C_j & C/12 & C/12 & & & C/12 \\
\hline
\end{array}
\]

\textbf{Step 5.} \( j = 1 \).

\textbf{Step 6.} For \( j \leq n - 1 \)

6.1. Search of \( s_j = \max(i : a_i T_j \leq C_j) \);

6.2. \( k_j = s_j + 1 \);

6.3. Calculate \( x_{kj} = \frac{C_j - a_s T_j}{a_{kj} - a_{s_j}}, x_{s_j} = T_j - x_{k_j} \);

6.4. If \( x_{k_j} \leq A_{kj} \) and \( x_{s_j} \leq A_{s_j} \), then

\[ A_{kj}^H = A_{kj} - x_{kj}, \ A_{s_j}^H = A_{s_j} - x_{s_j}, \ j = j + 1 \]
6.5. If \( x_{kj} > A_{kj} \), then \( x_{kj} = A_{kj} \), \( A_{kj} = 0 \),

\[
T^H_{j} = T_{j} - x_{kj}, \quad C^H_{j} = C_{j} - a_{kj} x_{kj}, \quad k_{j} = k_{j} + 1,
\]

Return to 6.3.

6.6. If \( x_{sj} > A_{sj} \), then \( x_{sj} = A_{sj} \), \( A_{sj} = 0 \),

\[
T^H_{j} = T_{j} - x_{sj}, \quad C^H_{j} = C_{j} - a_{sj} x_{sj}, \quad s_{j} = s_{j} - 1,
\]

Return to 6.3.

**Step 7.** For \( \forall i \) \( x_{im} = x_{i12} = A_{i} \)

**Step 8.** Results of calculations are put into the table, all \( x_{ij} \) on empty place. There is no necessity to write the zero values.

**Step 9.** Results of calculations in parts of the implemented projects (in the same order as in the previous table) \( \hat{x}_{ij} = \frac{x_{ij}}{t_{ij}} \)

| \( i \) | \( t_{i} \) | \( c_{i} \) | \( 1 \) | \( 2 \) | \( 12 \) | \( A_{i} \) |
|---|---|---|---|---|---|---|
| 1 | \( t_{1} \) | \( c_{1} \) | | | 1 | |
| 2 | \( t_{2} \) | \( c_{2} \) | | | 1 | |

\( m \)

| \( T_{j} \) | \( T/12 \) | \( T/12 \) |
| \( C_{j} \) | \( C/12 \) | \( C/12 \) |

**Step 10.** Check the results

\[
\sum_{i=1}^{12} \hat{x}_{ij} = 1, \quad i = 1, m
\]

\[
\sum_{i=1}^{m} t_{i} \hat{x}_{ij} = \frac{T}{12}, \quad j = 1, 12
\]

\[
\sum_{i=1}^{m} c_{i} \hat{x}_{ij} = \frac{C}{12}, \quad j = 1, 12.
\]

7. Computational experiment of the algorithm

We have 4 projects, where \( t_{i} \) — labor intensity (in men/days) of the \( i \)-th project \( c_{i} \) — cash revenue of the \( i \)-th project (table 1). It is necessary to arrange a 6 months’ timetable of realization of these projects under condition of uniform distribution of labor intensity and cash revenues.

| Project | Parameters |
|---|---|---|
| Logus | 2 | 10 | 5 |
| Olsam | 3 | 6 | 2 |
| Zarechnoye | 5 | 4 | 0.8 |
| KIS | 2 | 4 | 2 |

**Table 1.** Initial data.
After calculation of $t_i$ in months, $T_j$, $C_j$, $a_i$, we arrange $i$-th in order of increasing $a_i$. In this way the table of intermediate results (table 2) is obtained:

**Table 2. Intermediate results of application of the algorithm.**

| $i$ | $a_i$ | 1 | 2 | 3 | 4 | 5 | 6 | $A_i$ | $A_i^H$ |
|-----|-------|---|---|---|---|---|---|------|--------|
| 1   | 0.8   | 0.715 | 1.43 | 1.43 | 1.425 | 5 | 5 | 5 | 4.285 | 2.855 | 1.425 | 0 |
| 2   | 2     | 2   | 1   | 3   | 3   | 2 | 0 | 0 | 0 | 0 |
| 3   | 2     | 2   | 0.285 | 0.57 | 0.57 | 0.575 | 2 | 2 | 2 | 1.715 | 1.145 | 0.575 | 0 |
| 4   | 5     | 2   | 2   | 2   | 2   | 2 | 0 | 0 | 0 |
| $T_j$ | 2     | 2   | 2   | 2   | 2   | 2 | 0 | 0 |
| $C_j$ | 4     | 4   | 4   | 4   | 4   | 4 | 0 | 0 |

After calculation of $\hat{x}_{ij}$-th final results in parts are filled into the table 3.

**Table 3. Final result of application of the algorithm.**

| Project | Month |
|---------|-------|
|         | 1     | 2     | 3     | 4     | 5     | 6     |
| Logus   | 0,1425 | 0,285 | 0,285 | 0,285 |
| Olsam   | 0,66   | 0,33  |       |       |
| Zarechnoye | 0,143 | 0,286 | 0,286 | 0,285 |
| KIS     | 1     |       |       |       |       |       |

8. Conclusions

The choice of the double-transport problem for solving the problem of timetable of project scheduling with account of uniform distribution of labor intensity and financial revenues is justified:

1) The decomposition algorithm is quite simple.
2) In each time period the minimal possible number of projects is accomplished.
3) In the case, when it takes several periods to accomplish a project, continuity is ensured.

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