An application of stochastic processes for analyzing risks in highway projects

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Abstract

The successes on highway projects are uncertain because of organizational features, improper scope definitions and long lasting complicated processes. Highway projects under uncertain environment can effectively be managed with the application of risk management throughout their life cycles. Risk management within highway projects, therefore, has been recognized vital to improve their performances and increase the success of these projects. Processes of the projects are dynamic by nature. Therefore, commonly used static techniques do not analyze the potential risks properly. The stochastic process is a highly effective tool to quantitatively deal with the risk analysis. In this paper, a new approach based on Markov chain is proposed to assess the potential risks of highway projects in a dynamic framework. The approach takes advantage of the capability of probabilistic tools. Furthermore, using an application example in highway projects, the proposed approach is demonstrated in detail. Finally, the risk management effectiveness of using the stochastic processes is illustrated.

Keywords: Risk analysis, stochastic process, Markov chain, highway projects

1 Introduction

Risks within highway projects can be defined as an uncertain event that has a positive or negative effect on the objectives (e.g., time, cost, scope, and quality) [1, 2]. Risk management process must be considered to manage different risks in highway projects. The processes involve planning, identification, analysis, response planning, monitoring and control on highway projects. Appropriate measures can be taken in the project risk management in order to [1]:

- minimize negative effects on the projects objectives (e.g., schedule, cost, scope and quality);
- maximize chances to improve the objectives by considering the low cost, short schedules, enhanced scope and high quality; and
- minimize management by crisis.
One of the major steps in risk management is to analyze the different risks of projects [3]. The process of the risk analysis can be complicated by considering the complexity of the needs and the subjective nature of the data within highway projects. However, the complexity of the risk analysis process is not overwhelming and the advantages of the results can be highly valuable [4].

For the review of related literature, Suijiao [5] proposed a Markov chain method to assess both short-term and long-term risks in construction projects. In qualitative risk analysis, the likelihood and consequences can be assessed qualitatively, e.g., on a scale from A to E (A very low, B low, C medium, D high, E very high) and later the likelihood can be assessed in percentage and subsequently in monetary terms. The qualitative risk assessment on an A to E scale can be used to transfer non-transparent lists of risks into a priority list of risks by using a scoring-risk matrix. Once risk can be assessed in more detail or exact in percentage (likelihood) and monetary terms (consequences), the applied scores can be adjusted. Quantitative risk analysis determines the impact of project risks on major cost and revenue centers for financial or economic issues in highway projects [1].

Recently, Ebrahimnejad et al. [6] presented a fuzzy decision making model for risk assessment with an application to mega projects. Also, Ebrahimnejad et al. [7] identified the important risks in build-operate-transfer projects and designed a multi-attribute analysis under a fuzzy environment to handle the risks. Mojtabahi et al. [8] introduced the concept of safety to the risk identification and analysis simultaneously by focusing on the health, safety and environment. Makui et al. [9] provided an approach for identifying and analyzing risks of mega projects concurrently by fuzzy group decision analysis in mega projects. Mousavi et al. [10] proposed an approach based on the resampling methods with interval analysis for the risks of engineering projects. In addition, Mousavi et al. [11] applied jackknife technique for risk assessment in highway projects. Mousavi et al. [12] also developed a multi-attribute analysis approach under fuzzy-stochastic environment for risk selection problems with an application to highway projects. Hashemi et al. [13] extended bootstrap technique for risk analysis with interval numbers in construction projects. Further, Hashemi et al. [14] presented a compromise ranking approach with bootstrap confidence intervals for risk assessment in port projects.

Due to the inherent features of the activities within highway projects (e.g., complicated processes, financial intensity and dynamic organization structures), these projects are subject to constant changes in the uncertain environment. The application of traditional methods to assess risks involved in the projects in a static way has been criticized for failing to take into account the sequential nature of the processes in highway projects. In a realistic risk model, project uncertainty must be modeled as a dynamic process [5, 15].

Stochastic process (e.g., Markov chain approach) is a powerful approach introduced for analyzing the time-dependent behavior of numerous dynamic systems. This paper contributes to this area by providing a new framework for the application of Markov chain approach based on the risk data obtained from experts’ judgments in highway projects.

The remainder of this paper is structured as follows. In Section 2, the definitions and basic properties of Markov chains are provided in detail. In Section 3, we describe the proposed stochastic process approach for highway projects. In Section 4, in a highway project as an application example, computational results are conducted. In Section 5, conclusion and further researches are presented.

2 Markov chains

In this section, we consider a stochastic process \( \{X_n, n = 0, 1, 2,...\} \) that takes on a finite or countable number of possible values [15]. If \( X_n = i \), then the process is said to be in state \( i \) at time \( n \). We suppose that whenever the process is in state \( i \), there is a fixed probability \( P_{ij} \) that it will next be in state \( j \). That is, we suppose that...
\[ P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0 \} = P_{ij} \]  

(2.1)

for all states \(i_0, i_1, \ldots, i_{n-1}, i, j\) and all \(n \geq 0\). Such a stochastic process is known as a Markov chain. Equation (2.1) may be interpreted as stating that, for a Markov chain, the conditional distribution of any future state \(X_{n+1}\) given the past states \(X_0, X_1, \ldots, X_{n-1}\) and the present state \(X_n\), is independent of the past states and depends only on the present state.

The value \(P_{ij}\) represents the probability that the process will, when in state \(i\), next make a transition into state \(j\). Since probabilities are nonnegative and since the process must make a transition into some state, we have that

\[ P_{ij} \geq 0, \quad i, j \geq 0; \quad \sum_{j=0}^{\infty} P_{ij} = 1, \quad i = 0, 1, \ldots \]  

(2.2)

Let \(P\) denote the matrix of one-step transition probabilities \(P_{ij}\), so that

\[
P = \begin{pmatrix}
P_{00} & P_{01} & P_{02} & \cdots \\
P_{10} & P_{11} & P_{12} & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
P_{i0} & P_{i1} & P_{i2} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

(2.3)

2.1. Chapman–Kolmogorov equations

We have already defined the one-step transition probabilities \(P_{ij}\). We now define the \(n\)-step transition probabilities \(P_{ij}^n\) to be the probability that a process in state \(i\) will be in state \(j\) after \(n\) additional transitions. That is,

\[ P_{ij}^n = P\{X_{n+k} = j \mid X_k = i\}, \quad n \geq 0, \quad i, j \geq 0 \]  

(2.4)

of course \(P_{ij}^1 = P_{ij}\). The Chapman–Kolmogorov equations provide a method for computing these \(n\)-step transition probabilities [15]. These equations are

\[ P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^n P_{kj}^m \quad \text{for all } n, m \geq 0, \text{ all } i, j \]  

(2.5)

and are most easily understood by noting that \(P_{ik}^n P_{kj}^m\) represents the probability that starting in \(i\) the process will go to state \(j\) in \(n + m\) transitions through a path which takes it into state \(k\) at the \(n\)th transition. Hence, summing over all intermediate states \(k\) yields the probability that the process will be in state \(j\) after \(n + m\) transitions.

If we let \(P^{(n)}\) denote the matrix of \(n\)-step transition probabilities \(P_{ij}^n\), then Equation (2.5) asserts that \(P^{(n+m)} = P^{(n)} \cdot P^{(m)}\) where the dot represents matrix multiplication. Hence, in particular,

\[ P^{(2)} = P^{(1+1)} = P \cdot P = P^2 \]

and by induction

\[ P^{(n)} = P^{(n-1+1)} = P^{n-1} \cdot P = P^n \]  

(2.6)

That is, the \(n\)-step transition matrix may be obtained by multiplying the matrix \(P\) by itself \(n\) times [15].

2.2. Ergodicity

If state \(i\) is recurrent, then it is said to be positive recurrent if, starting in \(i\), the expected time until the process returns to state \(i\) is finite. It can be shown that positive recurrence is a class property [15]. While
there exist recurrent states that are not positive recurrent, it can be shown that in a finite-state Markov chain all recurrent states are positive recurrent. Positive recurrent, aperiodic states are called ergodic.

**Theorem 2.1.** For an irreducible ergodic Markov chain \( \lim_{n \to \infty} P^n_{ij} \) exists and is independent of \( i \). Furthermore, letting

\[
\pi_j = \lim_{n \to \infty} P^n_{ij}, \quad j \geq 0
\]  

then \( \pi_j \) is the unique nonnegative solution of

\[
\pi_j = \sum_{i=0}^{\infty} \pi_i P_{ij}, \quad j \geq 0,
\]

\[
\sum_{j=0}^{\infty} \pi_j = 1
\]  

if and only if the Markov chain is positive recurrent. If a solution exists then it will be unique, and \( \pi_j \) will equal the long run proportion of time that the Markov chain is in state \( j \). If the chain is aperiodic, then \( \pi_j \) is also the limiting probability that the chain is in state \( j \).

### 3 Proposed approach

Risks in highway projects have been the object of attention because of time and cost overrun associated with these projects. A highway project often is divided into three major phases [1]: (1) planning; (2) engineering; and (3) construction. Potential risks change continuously in the life-cycle of highway projects. Therefore, highway project risks may be modeled in a stochastic environment (e.g., random variables or in a dynamic framework, a stochastic process) [5]. Furthermore, because the progress of highway projects mainly depends on the project situations, the potential risks within highway projects have the characteristics of the stochastic process. It can be modeled by Markov chains. Two stages of the proposed approach for the risk analysis in highway projects based on Markov chain are provided as below.

#### 3.1 Project potential risk data gathering

In this stage, risks of highway projects are collected by applying historical information, lessons learned and NGT technique in order to provide potential risk break down structure (PRBS). Numerous approaches have been suggested in the literature for classifying risks [1-10]. New practical approach based on [8] is considered for classifying potential risks. Potential risks of the highway projects are grouped in adhere to project WBS in order to consider potential risks in different levels of the project and scope of work. The solution proposed for structuring problem of risk management within highway projects, to adopt the complete hierarchical approach applied in the WBS, which as numerous levels as are needed to obtain the necessary understanding of risk exposure to allow effective management [8]. In this stage, the PRBS is defined as a source-oriented grouping of potential risks within highway projects that organizes and defines the entire risk exposure.

#### 3.2. Project Potential Risk Analysis

Stage two try to deeper understanding of potential project problems after identifying the risks of highway project. This stage is the process of appraising the potential risks documented in the preceding stage and ranking the risks. In the second stage, we utilized the model presented in [5] to analyze the risks of highway projects. We pre-define the levels of the potential risk denoted by \( PRL_1, PRL_2, ..., PRL_m, ..., PRL_j \).
In planning phase of a highway project, experts or decision makers with the total number as \( k \) are gathered to analysis potential risks of the projects. If there are \( K_i \) experts who determine the risk level as \( PRL_i \), the probability of this project exposed to risk level \( PRL_i \) can be computed as \( S^{(0)} = \frac{K_i}{K} \). In this way, we obtain the initial distribution of potential risk probabilities

\[
S^{(0)} = (S_1^{(0)}, S_2^{(0)}, \ldots, S_{ij}^{(0)}, \ldots, S_j^{(0)})
\]

(3.9)

In the following phase of the proposed approach, the potential risks of the projects are analyzed again through experts’ judgments. If \( k_{ij} \) experts who think the overall project risk change from level \( PRL_i \) to \( PRL_j \), the corresponding transition probability is \( P = \frac{k_{ij}}{K_i} \). Therefore, we can obtain the one step transition probability matrix as in Equation (2.3).

Once the initial distribution and transition matrix are decided, the risk distribution at later stages can be computed by using relations (2.6) and (3.9).

\[
S^{(1)} = S^{(0)}. P, \quad S^{(2)} = S^{(1)}. P = S^{(0)}. P^2, \quad \ldots, \quad S^{(k)} = S^{(0)}. P^k, \ldots
\]

(3.10)

where, \( S^n \) is the risk distribution after \( n \)-step transition from the initial stage.

We suppose the Markov chain for highway project risks is ergodic. Namely, the risk distribution at each risk level remains constant after a long enough time. This distribution can be solved through Equation (2.8).

4 Application

In this section, an application example is presented for risk analysis of highway project in order to demonstrate the potential of the proposed stochastic approach. The purposes of work are as follows:

- Identifying the potential risks for highway projects.
- Designing a project risk analysis model for the potential risks by using Markov chains.

4.1. Pre-stage: Establishing project risk management team

A risk management team for the highway project is formed to manage risks in the planning phase of the highway project. Establishing the team is the main predecessor of risk analysis process with the highway project. It is concerned with extending a structure for the risk tasks to follow. It deserves more attention because it reduces preventable problems. The team establishment steps are needed to provide the organizational and highway project environment in which analyzing the risks is taking place and to decide the main vision, goals, objectives and outcomes required. To set the whole objectives together play an indispensable part of empowering the team for analyzing the project risks. The main goal of the team is to identify and analyze potential risks of the highway project to find their priorities for further measures. Size, budget, location, available resources and unique aspects of the highway project are some factors that influence the selection of a risk management team for the highway project.

4.2. Stage one: Potential risk data gathering

The potential risk breakdown structure (PRBS) based on the highway project WBS is then extended to summarize the different categories of risks. To improve the risk identification process, potential risks can be categorized by considering the different levels of WBS. The risk structure demonstrates the risk groups, risk categories and risk events at the lowest level. Project potential risks are divided into four groups, engineering, procurement, construction, and management. The PRBS is depicted in Table 1.
Table 1: Potential risk breakdown structure for the highway project

| Highway project phase | Potential risks                                      |
|-----------------------|-----------------------------------------------------|
| Engineering           | • Changes in design                                 |
|                       | • Defective design, errors, and rework              |
|                       | • Deficiency in drawing                             |
| Procurement           | • Delay of material supply                          |
|                       | • Imperfect data transmission to vendors             |
|                       | • Material quality problems                         |
| Construction          | • Change in scope of work                           |
|                       | • Delay in equipment delivery to site               |
|                       | • Delay in paying subcontractors invoices           |
|                       | • Health, Safety and Environment (HSE) matters       |
| Management            | • Corruptions & bribes                              |
|                       | • Delay in paying and receiving project’s invoices  |
|                       | • Insufficient time to plan                         |
|                       | • Lack of communication                             |

4.3. Stage two: Project potential risk analysis

In this stage, we classified the potential risks of the highway project into five levels in order to increase the precise of the risk assessment approach. These levels (i.e., Very Low (VL), Low (L), Medium (M), High (H), Very High (VH)) are depicted in Table 2. Twenty five experts with high qualification regarding this subject are selected to form a risk analysis group for undertaking the risk assessment by using the proposed stochastic approach. The assessment results in the planning phase of the highway project are illustrated in Table 2.

Table 2. Project potential risk analysis

| Analysis in the second phase | Potential risk level | VL | L | M | H | VH | Total |
|------------------------------|----------------------|----|---|---|---|----|-------|
|                             | VL                   | 2  | 1 | 1 | 0 | 0  | 4     |
|                             | L                    | 2  | 3 | 1 | 0 | 0  | 6     |
|                             | M                    | 2  | 1 | 3 | 1 | 0  | 7     |
|                             | H                    | 0  | 1 | 1 | 2 | 1  | 5     |
|                             | VH                   | 0  | 1 | 0 | 1 | 1  | 3     |
|                             | Total                | 6  | 7 | 6 | 4 | 2  | 25    |

In the following, we analyze the potential risks in the future. The initial potential risk distribution can be derived as $S^{(0)} = (0.16, 0.24, 0.28, 0.20, 0.12)$, and then, the one-step transition probability matrix is as follows:
\[
P = \begin{bmatrix}
0.50 & 0.25 & 0.25 & 0 & 0 \\
0.33 & 0.50 & 0.17 & 0 & 0 \\
0.29 & 0.14 & 0.43 & 0.14 & 0 \\
0 & 0.20 & 0.20 & 0.40 & 0.20 \\
0 & 0.33 & 0 & 0.33 & 0.33
\end{bmatrix}
\]

By using Equation (3.10), the potential risk distribution in the second phase of the project is

\[S^{(1)} = S^{(0)} \times P\]

\[
= (0.16, 0.24, 0.28, 0.20, 0.12) \times 
\begin{bmatrix}
0.50 & 0.25 & 0.25 & 0 & 0 \\
0.33 & 0.50 & 0.17 & 0 & 0 \\
0.29 & 0.14 & 0.43 & 0.14 & 0 \\
0 & 0.20 & 0.20 & 0.40 & 0.20 \\
0 & 0.33 & 0 & 0.33 & 0.33
\end{bmatrix}
\]

\[= (0.2404, 0.2788, 0.2412, 0.1588, 0.0796)\]

The risk distribution in the next phase is

\[S^{(2)} = S^{(1)} \times P = S^{(0)} \times P^2\]

\[= (0.2822, 0.2913, 0.2430, 0.1236, 0.0580)\]

Generally, the risk distribution after \(k\)-step transition from the initial phase is \(S^{(k)} = S^{(0)} \times P^k\).

When the inherent situations and outer environment of the highway project come to a stable state, the risk level will remain unchanged [5]. To compute potential risk probabilities, let the risk distribution at the stable stage be \(S = (S_1 + S_2 + \ldots + S_5)\). Since the potential project risks are modeled by an ergodic Markov chain, we obtain the \(S_i\) according to Equation (2.8). After solving the Equation, we obtain the potential risk probabilities in equilibrium:

\(S_1 = 0.3308, S_2 = 0.2786, S_3 = 0.2531, S_4 = 0.0707, S_5 = 0.0211\)

The computational results illustrate that after the long run proportion of time, the probabilities of potential risks in the highway project at five levels are 33.1%, 27.9%, 25.3%, 7.1%, and 2.1% respectively.

With the comparison between levels of potential risks in the planning phase of the highway project and after a long time, we have figured out that the severity of the risks has been decreased during the life-cycle of the highway project, particularly for the risks in medium, high, and very high levels. We also have found that the risk response planning brought the overall risk of the project down to an acceptable level and therefore, risk management has been effective way for managing these projects.

5 Conclusions

Highway projects have been becoming complicated and dynamic in their nature. To handle uncertainties, risk identification and analysis are critical subjects within these projects. The main objective of this paper was to understand the different risks in highway projects and to propose a stochastic model of analysis in an uncertain environment. Thus, the stochastic process approach based on Markov chain was presented. Another objective of this paper was to illustrate the application of the proposed approach in highway projects by using risk data. The main steps of the proposed stochastic approach have been implemented in
the highway project as an application example. The computational results have illustrated that this approach based on probabilistic models can be utilized during the life-cycle of highway projects. Also, it can be applied in a dynamic framework without having historical different risk data. For further research, as uncertainties in highway projects may result in imprecise and subjective risk data being present, which makes the risk assessment process complicated, we may work on the topic that considers the proposed stochastic process approach in a fuzzy environment in these projects.

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