Heat flux measurement in a high enthalpy plasma flow

Stefan Löhle, Jean-Luc Battaglia and Jean-Laurent Gardarein
TREFLE Université de Bordeaux I, Esplanade des Arts et Métiers, 33405 TALENCE, France.

stefan.loehle@dlr.de

Pierre Jullien, Bruno van Ootegem
European Aeronautic Defense and Space Company, Avenue du Général Niox, 33165 Saint-Médard en Jalles, France.

Abstract. It is a widely used approach to measure heat flux in harsh environments like high enthalpy plasma flows, fusion plasma and rocket motor combustion chambers based on solving the inverse heat conduction problem in a semi-infinite environment. This approach strongly depends on model parameters and geometrical aspects of the sensor design. In this work the surface heat flux is determined by solving the inverse heat conduction problem using an identified system as a direct model. The identification of the system is performed using calibration measurements with modern laser technique and advanced data handling. The results of the identified thermo-physical system show that a non-integer model appears most adapted to this particular problem. It is concluded that the new method improves the heat flux sensor significantly and furthermore extend its application to very short measurement times.

1. Introduction

Transient heat flux measurements are widely used when time resolution is required or harsh environments do not allow steady-state sensor technique [1], [2]. The principle of the present sensor is to measure the temperature inside an appropriate material which is part of the heat flux sensor and to estimate the heat flux from inverse heat conduction calculation. Solving the inverse heat conduction problem requires a model of heat transfer in order to deduce the heat flux from the temperature measurement at the corresponding measurement location inside the sensor. Considering a particular sensor and linear heat transfer, i.e. constant material properties for the measurement time, such a model can be viewed according to Duhamel’s theorem as the impulse response. This is the transient temperature at the sensor due to a heat flux impulse in form of a Dirac function [3]. Highest reliability in the heat flux estimation is reached when the distance between the surface and the measurement location is as small as possible where the limiting case is a surface temperature measurement [4]. However, when very high heat flux is expected, the temperature measurement itself is at its operational limit as for example thermocouples.

The sensor used throughout this work is known as a null-point calorimeter [1]. It is consistent with the standard test method for high heat flux measurements [5]. A photography of the sensor is shown in Fig. 1. It consists of a rather big copper part, with conical flanks and a spherical tip. The interior of the heat flux sensor is shown in Fig. 2. The tip itself is originally manufactured as a separate part. The heat
flux sensor is then revised after the tip has been inserted to finalize the spherical form. The advantage of such a two-piece design is that the positioning of the thermocouple, which is an essential parameter for the inversion, can be carried out properly before it is inserted in the bigger part. Furthermore, an additional air gap shall strengthen the assumption of one-dimensional heat transfer. This sort of sensor has been developed in the beginning of the 1970s and further extensive studies were performed in the 1990s [6], [7].

Figure 1. Photography of the heat flux sensor.

Figure 2. Cross-sectional view of the heat flux gauge, the tip is enlarged to show the details (lengths are in mm).

Heat transfer in the sensor is classically treated as a one dimensional heat transfer in a semi infinite homogeneous medium. Assuming constant material properties and an initial temperature of $T(0)= 0$, the temperature at the thermocouple is known to be [4]:

$$T(t) = \frac{\varphi}{\sqrt{k \rho C_p}} \left[ 2 \sqrt{\frac{t}{\pi}} e^{-\frac{x_0^2}{4at}} - \frac{x_0}{\sqrt{4at}} \text{erfc}\left(\frac{x_0}{\sqrt{4at}}\right) \right],$$

(1)

Where $\varphi$ is the constant heat flux density applied on the sensor, $x_0$ is the distance from the tip to the thermocouple, $a$ is the thermal diffusivity, $k$ is the thermal conductivity, $\rho C_p$ is the volumic heat and erfc() denotes the complementary error function.

The practical use of the sensor is to measure high heat flux in high enthalpy plasma flows. Given to the very high enthalpy in the plasma, the sensor can not be exposed in a steady-state manner to the flow. The heat flux is therefore measured from the signal recorded while passing the jet radially. Measurement times down to 0.03s with a high resolution are required. An improvement of this sensor is needed because measurements at very short times are not specified and recent calculations showed that relation (1) is no longer valid [2], [7]. It appears that the thermocouple’s junction never reaches a homogeneous temperature for each sampling acquisition time during exposure duration [8]. Indeed, the rise time of the thermocouple junction is 0.06 s which is two times higher than the exposure time of the sensor to the plasma during the whole experiment. In consequence, this means that the measured signal at the thermocouple, that is a voltage drop, can not be related to the junction temperature using the classical thermoelectric calibration. Hence, the extent of this technique to very high heat fluxes in the range of several tens of MW/m² has to be further investigated.

In order to check the sensitivity of the temperature at the thermocouple according to specific geometric parameters, the heat transfer at the tip of the sensor has been investigated using finite element modeling (Comsol© Multiphysics 3.3 with Heat Transfer module). A very detailed model including the thermocouple wire, isolation, and fitting adhesive has been realized. The material properties are taken from databases (National Institute of Standards and Technology). It was demonstrated in [8] that an error in the estimation of the position or a deviation from the specified position of only 0.05 mm corresponds to an error in temperature of 5% with respect to the temperature
at the nominal position. While this error is not huge, its influence on the estimated heat flux is significant. Using an inverse algorithm as for example the function specification method [3], it is shown that an error of 5% in temperature of the sensor leads to an error in heat flux of about 20%. While the positioning and the inertia of the thermocouple are serious disadvantages for the use of this sensor at very short measurement times, an advantage is that the conical part of the probe is minor influenced by the hot plasma flow and hence the temperature rise at the thermocouple due to heat flux applied to the conical flanks is negligible [8]. As will be shown later, this result will have an important impact with respect to the experiment devoted to the sensor calibration which is presented in section 3.

The present paper will show that the use of Non-Integer System Identification (NISI) permits a significant improvement of the rather classical heat flux measurement technique. In particular, the knowledge of the exact location of the thermocouple as well as its time constant are not required applying this method. The basic idea is a calibration of the sensor by applying a transient heat flux in the time domain of interest using a pulsed laser source. Starting from the non-integer system identification method, the parameters of a model that expresses the temperature of the sensor and its fractional time derivatives of the order \( \frac{1}{2} \) according to those of the heat flux are identified [9]. The impulse response is then calculated from this identified system. Furthermore, since the calibration equipment is capable to give a very short heat flux pulses, a direct estimation of the impulse response can be assumed and compared to the non-integer model.

For this new approach, it is not necessary to measure the absolute temperature of the sensor. Since during the calibration step the same quantity, a voltage drop, is measured as during the experiment, the model can be based on the voltage measurement itself.

The following section introduces theoretical aspects concerning the NISI method. The second section gives a short overview on the experimental aspects of the sensor used in a high enthalpy plasma wind tunnel. A conclusion summarizes the progress.

2. Theory

The classical methodology for inverse heat conduction problems rests on the heat diffusion model that involves the material thermo-physical properties (thermal conductivity and diffusivity) as well as the geometrical properties of the system to be investigated [1], [4]. The unknown heat flux occurs in the associated boundary conditions. The more complex this system is the more complex is the formulation and the higher are the parameters to put into the model. On the other hand, the heat exchange coefficients at the free boundaries as well as the thermal resistances at the interfaces between each component of the system also must be considered. All of these parameters are generally not known with a sufficient accuracy [3].

Facing these difficulties a different approach is considered. It is based on the system identification method [10]. It consists in identifying the parameters that enter in the relation between the heat flux density \( \phi(t) \) applied on the sensor and the temperature \( T(t) \) at the thermocouple on the form of successive derivatives of these two quantities. Recent works (e.g. [11]) have shown that the most reliable model, with respect to the classical heat diffusion equation with associated boundary and initial conditions, involves derivatives of the order of \( \frac{1}{2} \) and the model is thus expressed in the form

\[
\sum_{n=0}^{M} \alpha_n D^{\eta/2} T(t) = \sum_{n=0}^{L} \beta_n D^{\eta/2} \phi(t), \quad \text{with } \alpha_M = 1. \tag{2}
\]

The operator \( D^\eta \) stands for the time-differentiation of the order indicated by the superscript \( \eta = n/2 \) where \( n \) is an integer. The definition of such an operator in the sense of Riemann and Liouville [12] is

\[
D^\eta f(t) = D^\eta \left[ I^{n-\eta} f(t) \right], \quad n \in \mathbb{N} \quad \text{and} \quad n-1 \leq \text{Re}(\eta) < n. \tag{3}
\]
\[
I^\nu f(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-u)^{\nu-1} f(u) \, du,
\]

(4)

where \( I^\nu f(t) \) denotes the integral of order \( \nu \) \((0 \leq \nu \leq 1)\) of the function \( f(t) \). Obviously, it is admitted that this function satisfies all the required conditions (especially the continuity). The integral of real or more generally complex order is

\[
\Gamma(\nu) = \int_0^\infty u^{\nu-1} e^{-u} \, du.
\]

(5)

Fractional derivatives to solve heat conduction problems have been discussed also by Oldham and Spanier as well as Kulish [13], [14]. They applied fractional calculus in order to solve the heat equation. It has been demonstrated that in one dimensional heat diffusion configurations, parameters \( \alpha_n \) and \( \beta_n \) can be directly expressed from the thermo-physical properties of the system [15]. From these analytical solutions, it appears that derivatives of the order \( \frac{1}{2} \) are representative of the semi-infinite behavior of a system. Indeed, even if the domain of diffusion is bounded, the system can be viewed as a semi-infinite medium for the small times. For long times, when the temperature becomes dependent from the boundary conditions of the system, the contribution of the non-integer derivatives becomes negligible and it only remains the classical derivatives of integer order. Thus, relation (2) is a generalization of results obtained in 1D to 3D configurations; even this generalization is not mathematically proved yet. The link between \( \alpha_n \) and \( \beta_n \) with thermo-physical properties can not be written anymore in the form of a simple analytical relation. Nevertheless, as for the heat diffusion equation with associated boundary and initial conditions, relation (2) is suitable to describe heat transfer by diffusion in the system [15]. Exact solutions showed that the series in (2) involve an infinite number of terms. In practice, it is shown that the number of terms in each sum can be significantly reduced without making a significant gap with respect to the exact solution. In practice, \( M \) and \( N \) never exceed 3 to 5 [9]. The computation of the fractional derivative is performed using the discrete method of Grünwald [12]:

\[
D^\nu f(t) = \lim_{h \to 0} \frac{\Delta^\nu f(t)}{h^\nu}, \nu > 0,
\]

(6)

where \( \Delta^\nu f(t) \) denotes the non-integer increase of \( f(t) \) as

\[
\Delta^\nu f(t) = \sum_{j=0}^{\nu} (-1)^j \binom{\nu}{j} f(t-jh), t = Nh
\]

(7)

with:

\[
\binom{\nu}{j} = \frac{\nu(\nu-1)\cdots(\nu-j+1)}{j!}.
\]

(8)

In relation (6) \( h \) denotes the constant sampling period. In order to apply this approach, the methodology consists first in identifying the thermal behavior of the sensor according to relation (2). This stage can be viewed as a calibration of the sensor. During this experiment, a known transient heat flux is imposed on the sensor and the transient voltage at the thermocouple is measured. The unknown parameters, \( \alpha_n \) and \( \beta_n \), are then identified from these measurements. The mathematical procedure for
the parameter identification is based on a recursive least square method and is completely described in [15]. The objective function is the quadratic gap $\|D^{\alpha}Y(t) - D^{\alpha}T(t)\|_2$ where $Y(t)$ denotes the temperature measurement at time $t$ and $i$ is chose in order to minimize the residuals at the end of the minimization. From this calibration step, it is then possible to calculate the impulse response $h(t)$ analytically using the fractional derivative of the Dirac function

$$D^\nu \delta(t-\tau) = \frac{(t-\tau)^{\nu-1}}{\Gamma(-\nu)}.$$

Replacing relation (9) in relation (2) lead to

$$\sum_{n=0}^{M} \alpha_n D^{\alpha} h(t) = \sum_{n=0}^{L} \beta_n \frac{\Gamma(i/2)}{\Gamma(-i/2)}.$$

It is thus clear that the variance of the calculated impulse response is essentially related to those of the identified parameters $\alpha_n$ and $\beta_n$.

### 3. Experiments

The main idea in the calibration stage is to heat the null-point calorimeter using a laser on the same area as that concerned during measurement in the plasma wind tunnel. Here, a bow shock forms at the tip of the probe such that substantially the tip of the sensor, i.e. the spherical part, gets in contact to the hot plasma flow. As previous calculation showed, it can be assumed that heat flux on the conical flanks can be neglected [8]. Hence the calibration has to be performed for an area of the diameter of the spherical tip which is 7 mm. Fig. 3 shows a schematic view of the experimental setup for the calibration measurements. As known in the system identification framework (for example in [10],[16]), generating laser pulse of variable length within the timescale of interest have to be considered in order to resolve each characteristic time of the sensor system: diffusion in the tip, response of the thermocouple, influence of the interfaces. Obviously, the lower pulse duration is more than the rise time of the sensor.

![Figure 3. Schematic view of the experimental setup for the calibration measurements](image)

In the present case, a pulsed laser source with pulse duration down to 1ms has been used [8]. The sequence and length of the laser pulses are generated randomly and the laser system is controlled by the function generator (see Fig. 3). An additional fast photodiode records this sequence additionally. In order to get high signal to noise ratio of the thermocouple voltage, the laser pulse energy has to be
sufficiently high. The laser in use is a diode laser system DILAS DFx20-980 that can provide 200 W. It leads to a maximum heat flux density of approximately $20 \times 10^6$ W/m$^2$. However, a constant amplification of the thermocouple data with a constant factor of 250 has been integrated. This is particularly necessary for the wind tunnel experiments. All measured data is recorded using a fast oscilloscope. The heat flux density provided by the laser is deduced from the calibration curve of the manufacturer based on blackbody absorption. An important factor particularly in dissociated flows is the surface state. In the present case, the surface of the copper sensor has been oxidized in a furnace in order to reach high catalytic efficiency [8]. Its emissivity has then been measured by measuring the hemispherical reflection using calibrated spectrometers for the wavelength range of interest. For the used calibration laser at 980 nm, the emissivity is $\varepsilon = 0.7$.

4. Results

Fig. 4 shows one calibration measurement. The oscilloscope is triggered on the first photodiode signal. Then, up to 15 laser pulses have been recorded in a row. As mentioned in the previous section, the sequence and the length of laser pulses have been generated randomly. The maximum heat flux density measured for this experience is $4 \times 10^6$ W/m$^2$. When the theoretical approach as described in section 2 is applied, the system can be identified from these calibration measurements. This means, that the parameters $\alpha_n$ and $\beta_n$ are chosen such that the calibration curve is rebuilt. In Fig. 4, the curve from the identification procedure is also plotted. In fact, the identification needed only 4 parameters for $\alpha_n$ and 3 parameters for $\beta_n$ in order to reach good agreement. The identified system corresponding to equation (2) is

\[
\left(1 - 0.053 D^{0.5} + 1.2 \times 10^{-3} D - 1.16 \times 10^{-5} D^{1.5} + 5.04 \times 10^{-8} D^3\right) V(t) \\
= \left(-9.335 \times 10^3 + 3.063 \times 10^6 D^{0.5} + 0.205 \times 10^7 D\right) \varphi(t).
\] (11)

The standard deviation of each parameter calculated as described in [2] is respectively

\[
\begin{pmatrix}
0 & 0.65 \times 10^{-9} & 2.55 \times 10^{-11} & 3.8 \times 10^{-13} & 2.42 \times 10^{-15} & 0.53 & 0.121 & 4 \times 10^{-3}
\end{pmatrix}.
\] (12)

The residual between the measured temperature and the simulated one using the identified system is plotted in Fig. 5. The deviations are small and the bias is also low which means good system identification.

![Figure 4](image1.png) \hspace{1cm} ![Figure 5](image2.png)

**Figure 4.** Calibration measurement for sensor AQ24 \hspace{1cm} **Figure 5.** Residual of the calibration measurement.
Fig. 6 shows the calculated impulse response from relation (10). Moreover, the result of one very short laser pulse is plotted. This result can be viewed as an experimental impulse response considering the short laser pulse of 1 ms length. The fit between the two responses is good at the first times. The gap increases slightly from 20 ms after the maximum because of the duration of the pulse. On the same figure, it is also reported the impulse response calculated for the analytical solution considering a heat pulse as a Dirac function, i.e.

$$T(t) = \frac{\varphi}{\sqrt{k \rho C_p \pi t}} e^{-\frac{x^2}{4at}}$$

where the thermal properties of copper ($\rho = 8700 \text{ kg m}^{-3}$, $C_p = 285 \text{ J kg}^{-1} \text{K}^{-1}$ and $k = 400 \text{ Wm}^{-1} \text{K}^{-1}$) have to be inserted and the nominal distance of the thermocouple from the tip of the sensor is set to the specified value of $x_0 = 0.3$ mm. It appears in Fig. 6 that this analytical solution is very far from the real behavior of the system. Furthermore, the impulse response is derived from the half space solution, which can also cause the error. However, this result is a clear demonstration of the reliability of the NISI method, because an appropriate model of the heat transfer in the real system has been deduced and neither the assumption of the position nor the temporal behavior of the system had to be assumed.

In order to clearly show the application of such a calibration stage, Fig. 7 shows an exemplary result. Data from one plasma wind tunnel experiment where heat fluxes of the order of 60 MW/m² are expected is treated using the impulse response calculated from the identified system. For comparison the data is treated also assuming a semi-infinite heat conduction problem (relation (1)). The time scale on the abscissa corresponds to the time the sensor passes the jet radially. A correlation to a length parameter has not been done yet. However, the heat flux profile can be clearly seen. The heat flux level calculated using both approaches is similar, but the semi-infinite heat conduction assumption leads to an asymmetrical heat flux profile which is not reasonable with regards to the free jet flow. The measurement error due to the calibration procedure is 16%. As could be seen from the standard deviation of the identification process (see the beginning of this section), the error of this data analysis can be neglected. However, the laser energy is only measured within an accuracy of 10%. An improvement of the laser equipment can therefore significantly reduce the measurement error.

A possible explanation for the asymmetry is that while the sensor is passing the jet, radial heat fluxes inside the sensor body become crucial. If this is the case, the analytical approach for the null-point calorimeter does not hold anymore, because the sensor behaves not as a semi-infinite system.
The NISI method however takes into account the radial influences at least for the time domain the sensor has been calibrated for. Thus, possible radial heat fluxes inside the sensor body are taken into account and a symmetrical profile is measured as expected.

5. Conclusion
This paper shows an application of the fractional calculus to a heat conduction problem. It is proposed to use an identified system describing the transient thermal behavior of the null-point calorimeter as the direct model in the inversion procedure. The non-integer system identification procedure has been presented from the mathematical and experimental point of view. It consists in identifying the parameters of a linear relationship which relies the fractional derivatives of the heat flux to those of the voltage drop at the thermocouple. It is used the fractional order \( \frac{1}{2} \) that has been found from the theory of heat transfer by diffusion. Calibration measurements have been performed using a pulsed laser source.

An important feature of this approach is that the operating range of the conventional null-point calorimeter is enlarged to shorter measurement times. The thermocouple inertia is taken into account already within the calibration step. Moreover, the manufacturing tolerances are accounted for, as for example the thermocouple positioning.

However, the most important aspect of this approach is that the calibration of the sensor using the non-integer system identification (NISI) method allows taking into account the physical phenomena inside the sensor such as radial heat fluxes or thermal resistances the one of which are not known with sufficient accuracy. It is shown that unphysical results of the conventional approach disappear using this new technique.

References
[1] Diller, T.E, *Advances in Heat Flux Measurements*, in: Advances in Heat Transfer, Vol.23, Academic Press, 1993

[2] Löhle, S., Battaglia, J.-L., Batsale, J.-C., Enouf, O., Dubard, J., Filtz, J.-R., *Characterization of a Heat Flux Sensor Using Short Pulse Laser Calibration*, Rev. Sci. Instrum., 78, 053501, 2007

[3] Beck J., Blackwell B., St.Clair C., *Inverse Heat Conduction: Ill-posed Problems*, Wiley Interscience, 1985.

[4] Carslaw H., Jaeger J., *Conduction of Heat in Solids*, Oxford University Press, 1959.

[5] E598, *Standard Test Method for Measuring Extreme Heat-Transfer Rates from High-Energy Environments Using a Transient, Null-Point Calorimeter*, Technical Report no E598-96, American Society for Testing and Materials (ASTM), 2002.

[6] Kennedy W., Rindal R., Powars C., *Heat Flux Measurement Using Swept Null Point Calorimetry*, J. of Spacecrafts and Rockets 9, 668, 1972.

[7] Kidd C., *High Heat Flux Measurements and Experimental Calibration/Characterizations*, Technical Report no NASA CP-3161, NASA, 1990.

[8] Löhle, S. Battaglia, J.-L., van Ootegem, B., Jullien, P., Couzi, J., Lasserre, J.-P., “Improvement of high heat flux measurement using a null-point calorimeter”, *Journal of Spacecrafts and Rockets*, Vol. 45, No. 1, 2008

[9] Battaglia J.-L., Cois O., Puigsegur L., Oustaloup A., *Solving an inverse heat conduction problem using a non-integer identified model*, Int. J. Heat and Mass Transfer, No.44, 2671, 2001.

[10] Ljung L., *System identification: Theory for the user*, Prentice Hall, 1987.
[11] Battaglia J.-L., Puigsegur L. and Kusiak A., Représentation non entière du transfert de chaleur par diffusion. Utilité pour la caractérisation et le contrôle non destructif thermique Int. J. Therm. Sci. , No. 43, 69, 2004.

[12] Oldham, K. B. and Spanier J., The fractional calculus, Academic Press, New York, 1974.

[13] Oldham, K.B., Spanier, J., A General Solution of the Diffusion Equation for Semi-infinite Geometries, J. Math. Anal. Appl., No. 39, 1972

[14] Kulish, V.V., Lage, J.L., Fractional-Diffusion Solutions for Transient Local Temperature and Heat Flux, J. Heat Transfer, Vol. 122, 2000

[15] Battaglia J.-L., Méthodes d’Identification de Modèles à Dérivées d’ordre non entier et de réduction modale, Habilitation à Diriger des Recherches, Université Bordeaux 1, Laboratoire TREFLE, 2002.

[16] Söderstrom T., Stoïca P., System identification, Prentice Hall, 1989.