Entropy spectra of black holes from resonance modes in scattering by the black holes

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Abstract
Since Bekenstein’s proposal that a black hole has an equally spaced area spectrum, the quasinormal modes (QNM) as the characteristic modes of a black hole have been used for obtaining the horizon area spectrum of the black hole. However, the area spectrum of the Kerr black hole in some previous works was inconsistent with Bekenstein’s proposal. In this paper, noting that black holes can have three types of resonance modes which are QNM, total transmission modes (TTM), and total reflection modes (TRM), we propose that all of these modes in a highly damped regime should be used in quantizing the black hole. Although the QNM and the TTM of the Kerr black hole give us complicated quantization conditions from the Bohr–Sommerfeld quantization of an action variable, we find a very simple result from the TRM. It gives an equally spaced outer horizon area. Therefore, by the Bekenstein–Hawking area law, we find that the Kerr black hole has universal behavior of the equally spaced entropy spectrum. With the same argument, we find that the Reissner–Nordström black hole also has the equally spaced entropy spectrum.

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1. Introduction
As a quantum property of black holes, it is believed that the black hole horizon area is quantized. It was first proposed by Bekenstein [1] that the horizon area is an adiabatic invariant and should be quantized, based on the Ehrenfest principle that any classical adiabatic invariant corresponds to a quantum entity with a discrete spectrum. By considering the minimum change of the horizon area in the process of the assimilation of a test particle into a black hole, it was obtained that the area spectrum should be linearly quantized, i.e. \( A = \gamma n \hbar \), where \( \gamma \) is an undetermined dimensionless constant [1]. However, there are other approaches which suggest
a more complicated area spectrum [2] and some attempts to make a point of contact with Bekenstein’s idea via proper interpretation [3].

When a perturbation on a black hole is given, the black hole undergoes damped oscillations which are called quasinormal modes (QNM). By using these QNMs of a black hole, it was realized that the area spectrum of the black hole can be obtained in the semiclassical limit [4–11]. By considering the real part of the asymptotic QNM of a black hole as a transition frequency in the semiclassical limit, the area spectrum of the Schwarzschild black hole was obtained as \( A = (4 \ln 3) \hbar \) [4]. However, Hod’s original conjecture [4] was shown to be inconsistent for other black holes [12–14]. The area spectrum of the Schwarzschild black hole was reproduced by considering an adiabatic invariant of the system with energy \( E \) and vibrational frequency \( \omega(E) \), given by the real part of the asymptotic QNM, via the Bohr–Sommerfeld quantization [5]. For a rotating black hole, a modified form of the adiabatic invariant was suggested as \( \mathcal{I} = \int \frac{dH}{\omega_c} = n \hbar \) via the Bohr–Sommerfeld quantization [6]. Later, it was proposed that a perturbed black hole should be described as a damped harmonic oscillator and a transition in a black hole should be considered as the transition between quantum levels \( (\omega_0)_k \equiv \sqrt{\left( \frac{\omega_R^2}{\omega_0^2} + \omega_I^2 \right)_{k}} \), where \( \omega_R \) and \( \omega_I \) are the real and imaginary parts of the asymptotic QNM [7]. Therefore, the characteristic classical frequency \( \omega_c \) should be identified with the transition frequency between the quantum levels \( (\omega_0)_k \) in the semiclassical limit, i.e. \( \omega_c = (\omega_0)_k - (\omega_0)_{k-1} \simeq |(\omega_0)|_{k} - |(\omega_0)|_{k-1} \) for the highly damped QNM, where \( |k| \gg 1 \) [7]. By using this transition frequency \( \omega_c \), the area spectrum of the Schwarzschild black hole was obtained as \( A = 8\pi n \hbar \) [7, 9, 10]. For the Kerr black hole, the asymptotic QNM have been first investigated numerically in [14, 15]. The transition frequency \( \omega_c \) can be obtained from the asymptotic QNM which is analytically calculated as

\[
\omega = \tilde{\omega}_0 - i4\pi T_0(a)(n + 1/2),
\]

where \( n \) is an integer, \( \tilde{\omega}_0 \) is a function of the black hole parameters whose real part asymptotically approaches \( \text{Re}(\tilde{\omega}_0) \propto m \), and \( T_0(a \equiv J/M) \approx -T_M(a = 0)/2 \) within \( \sim 3\% \) accuracy [16, 17]. The explicit expression of the imaginary part of the QNM is given in terms of the elliptic integrals [18], and it is too complicated to calculate the area spectrum by using an adiabatic invariant. In the previous works [8, 9] on the area spectrum of the Kerr black hole, with the transition frequency approximately taken as \( \omega_c \approx 2\pi T_M(a = 0) = 1/(4M) \), the quantization of the modified adiabatic invariant, i.e. \( \mathcal{I} = \int \frac{dH}{\omega_c} = n \hbar \), was calculated. But this gives the non-equally spaced area spectrum for the Kerr black hole, which is inconsistent with Bekenstein’s proposal. Only for the slowly rotating case with the small angular momentum \( J \) compared to the mass \( M \) of the Kerr black hole, it was found that the area spectrum was approximately equally spaced as \( A = 8\pi n \hbar \) [9, 19]. In our recent work [20], it was recalled that an action variable is an adiabatic invariant, but not every adiabatic invariant is an action variable, and that only the action variable can be quantized via the Bohr–Sommerfeld quantization in the semiclassical limit. Therefore, not an adiabatic invariant but an action variable of the classical system should be identified in order to apply the Bohr–Sommerfeld quantization [20]. By Bohr’s correspondence principle which states that the transition frequency at a large quantum number equals the classical oscillation frequency of the corresponding classical system, a black hole with the transition frequency \( \omega_c \) can be considered as the classical system of periodic motion with oscillation frequency \( \omega_c \) in the semiclassical limit. Therefore, the action variable of the classical periodic system with the oscillation frequency \( \omega_c \) was identified and finally quantized via the Bohr–Sommerfeld quantization in the semiclassical limit as follows [20]:

\[
\mathcal{I} = \int \frac{dE}{\omega_c} = \int \frac{dM}{\omega_c} = n \hbar \quad (n \in \mathbb{Z}, |n| \gg 1),
\]
where the transition frequency $\omega_c$ in the semiclassical limit is given by $\omega_c = (|\omega_I|)_k - (|\omega_I|)_{k-1}$ for highly damped modes, and the change of the energy $E$ of a black hole is considered as the change of the ADM mass $M$ (or the ADT mass $\mathcal{M}$ according to the gravity theory [21]). This formula can also be applied for a rotating black hole with a transition frequency. For example, the area and entropy spectra of the BTZ and warped AdS$_3$ black holes were obtained in [20, 21], where it was found that there is the universality that the entropy spectrum of a black hole is equally spaced, even though the area spectrum is not equally spaced [21].

In this paper, we would like to apply formula (2) for the Kerr black hole and to find if the entropy has the universal behavior of the equally spaced spectrum. Until now, in spite of the several attempts [6, 8, 9] for the area spectrum of the Kerr black hole, the results were not consistent with Bekenstein’s original proposal. Recently, by considering the scattering problem on the Kerr black hole, the highly damped QNM of the Kerr black hole, which are given by equation (1), were obtained from the poles of the transmission and reflection amplitudes [17]. However, we note that there are other resonance modes of the Kerr black hole, which are total transmission modes (TTM) and total reflection modes (TRM). These special modes of black holes were first considered by Chandrasekhar [22], and more investigated in some works [23–26]. The TRM and TTM can be obtained from the zeros of the transmission and reflection amplitudes, respectively. In order to find out the property of a black hole, we have to perform the scattering experiment on the black hole. In the scattering problem of a black hole, the wave equation becomes the Schrödinger-like equation in quantum mechanics. Then, the incident waves from infinity are reflected and transmitted because of the effective potential which plays a role of the potential barrier in quantum mechanics. So, when we consider the scattering problem on a black hole, the black hole can have other resonance modes of TTM and TRM as well as QNM. The QNM only depend on the black hole parameters such as mass, charge, and angular momentum which characterize a black hole. Therefore, it has been considered that QNM are the characteristic modes of the black hole as a fingerprint in directly identifying the existence of a black hole [27] and carry some information about quantum structure of the black hole [7]. More specifically, it was proposed that the imaginary part of the highly damped QNM represents the energy levels as the quantum structure of a black hole in the semiclassical limit, so that the transition frequency between the quantum levels is corresponding to the energy of emitted quanta from the black hole [7]. We note that TTM and TRM are also the characteristic modes of a black hole, since they only depend on black hole parameters. In this sense, we propose that the highly damped TTM and TRM also have the quantum structure of a black hole as the highly damped QNM. Therefore, we should promote TTM and TRM to the equivalent position of QNM, so that they play the same role as QNM in quantizing a black hole. Then, we can obtain the transition frequencies corresponding to each of them in the semiclassical limit. By considering other resonance modes for the Kerr black hole, we will find that the area spectrum is consistent with Bekenstein’s proposal [1] and the entropy spectrum has the universality of the equally spaced spectrum. Moreover, with the same argument, we will also find that the Reissner–Nordström black hole has the same behavior in area and entropy spectra. Throughout this paper, the Planck units with $c = G = \hbar = 1$ are used.

2. Resonance modes of black hole in the graybody factor and Hawking radiation

In this section, we will briefly review some features of resonance modes of black holes. The perturbations of black hole spacetimes are represented by the radial Schrödinger-like wave equations of the form

$$\partial^2_z f(z) + (\omega^2 - V_z(z)) f(z) = 0,$$

(3)
where \( z = z(r) \) is a tortoise coordinate which has the behavior of \( z \sim r \) as \( r \to \infty \) and \( z \to -\infty \) as \( r \to r_+ \) with an outer horizon radius \( r_+ \). The scattering problem for the incident wave from spatial infinity gives the wavefunction as the solution of the above wave equation, which satisfy the following boundary conditions:

\[
f(z)_w \sim \begin{cases} 
e^{-i\omega z} + R(\omega) e^{i\omega z}, & \text{as } z \to \infty \\
T(\omega) e^{-i\omega z}, & \text{as } z \to -\infty
\end{cases}
\]

(4)

where \( T(\omega) \) and \( R(\omega) \) are, respectively, the transmission and reflection amplitudes, and the purely ingoing wave at horizon is imposed as a boundary condition. We can also consider the wavefunction for \(-\omega\) which solves the wave equation (3):

\[
f(z)_{-\omega} \sim \begin{cases} e^{i\omega z} + \tilde{R}(-\omega) e^{-i\omega z}, & \text{as } z \to \infty \\
\tilde{T}(-\omega) e^{i\omega z}, & \text{as } z \to -\infty
\end{cases}
\]

(5)

where \( \tilde{T}(-\omega) \) and \( \tilde{R}(-\omega) \) are some other transmission and reflection amplitudes. Then, the conserved flux is given by

\[
\mathcal{F} = \frac{1}{2i} \left( f(z)_w \partial_z f(z)_w - f(z)_w \partial_z f(z)_{-\omega} \right).
\]

(6)

By calculating the conserved flux at both limits of \( z \), we obtain the following relation:

\[
T(\omega) \tilde{T}(-\omega) + R(\omega) \tilde{R}(-\omega) = 1,
\]

(7)

where \( T(\omega) \tilde{T}(-\omega) \) represents the absorption (transmission) probability which is associated with the graybody factor of a black hole. By considering the scattering problem on a black hole, we can find the resonance modes of a black hole, i.e. TTM, TRM, and QNM, from the transmission and reflection probabilities. The TTM and TRM are respectively obtained from the zeros of \( R(\omega) \tilde{R}(-\omega) \) and \( T(\omega) \tilde{T}(-\omega) \), while the QNM correspond to their poles.

In order to grasp some physical meaning of the resonance modes in some way, let us point out that there is the relation between Hawking radiation and the graybody factor with the information of the resonance modes of a black hole. It is well known that a black hole emits Hawking radiation which has the spectrum of the blackbody radiation [28]. The decay rate of a black hole at event (outer) horizon is given by [28]

\[
\Gamma(\omega) = \frac{1}{e^{(\omega - m/\Omega)/\mathcal{T}_H} + 1} \equiv n_H(\omega).
\]

(8)

The minus (plus) sign corresponds to bosons (fermions). This Hawking formula for the emission spectrum indicates that the black hole is a thermal object. To a static observer at spatial infinity, however, the spectrum of Hawking radiation is not thermal [29]. Since the curvature of the spacetime geometry outside event horizon plays a role of the potential barrier, it filters Hawking radiation in such way that some of radiations are transmitted to infinity and the rest are reflected into the black hole. Therefore, the decay rate at infinity is given by multiplying Hawking radiation at horizon by a factor, i.e. the so-called graybody factor which is dependent on frequency, as follows:

\[
\Gamma(\omega) = \gamma(\omega)n_H(\omega) = \gamma(\omega) \frac{\gamma(\omega)}{e^{(\omega - m/\Omega)/\mathcal{T}_H} + 1}.
\]

(9)

It means that Hawking radiation of the black hole does not ‘blackbody’ radiate, but ‘graybody’ radiate, when it is measured at infinity. When we consider the scattering problem where the incident wave originates from infinity, the graybody factor is defined as the absorption (transmission) probability of a black hole. Therefore, the decay rate of equation (9) is given by

\[
\Gamma(\omega) = \frac{\gamma(\omega)}{e^{(\omega - m/\Omega)/\mathcal{T}_H} + 1} = \frac{T(\omega)\tilde{T}(-\omega)}{e^{(\omega - m/\Omega)/\mathcal{T}_H} + 1}.
\]

(10)
Therefore, from equation (10) we find that the TTM are the resonance modes which give the decay rate in the form of blackbody radiation even at infinity. For the TRM, it seems that equation (10) vanishes since the TRM correspond to $T(\omega)\tilde{T}(-\omega) = 0$. However, it is pointed out in [17] that the graybody factor has zeros only where $n_H$ has poles in general, in particular for the spherical black holes in [29]. Therefore, the black holes we will consider in this paper have the non-vanishing emission spectrum $\Gamma(\omega)$ which is not affected by the TRM. In other words, the TRM are corresponding to the poles of the spectrum of Hawking radiation measured at outer horizon. In this sense, the TRM are associated with the outer horizon. On the other hand, the QNM are corresponding to the poles of spectrum of Hawking radiation measured at infinity.

3. Area and entropy spectra from TRM

For the quantization of the Kerr black hole, we will consider all of the resonance modes (i.e. TTM, TRM, and QNM) and apply them to formula (2). Before that, let us consider the Schwarzschild black hole case first as a warm-up exercise. In the highly damped regime, the transmission and reflection probabilities are given by [29]

\[
T(\omega)\tilde{T}(-\omega) \approx \frac{e^{\frac{n_H}{T_s} - 1}}{e^{\frac{n_H}{T_s}} + (1 + 2 \cos \pi j)} \quad \text{and} \quad R(\omega)\tilde{R}(-\omega) \approx \frac{2(1 + \cos \pi j)}{e^{\frac{n_H}{T_s}} + (1 + 2 \cos \pi j)}.
\]

The scalar, electromagnetic, and gravitational perturbations correspond to $j = 0$, $j = 1$, and $j = 2$, respectively. Since the effective potential in the Schrödinger-like wave equation has the term $(1 - j^2)/r^4$, it becomes singular for the electromagnetic perturbation ($j = 1$) [29]. Therefore, the above results hold except for that case. For the scalar and gravitational perturbations, we can find QNM and TRM of the Schwarzschild black hole, but there is no TTM. The QNM and TRM for the highly damped modes are easily obtained as follows:

\[
\omega_{\text{QNM}}^{\text{TRM}} = -i2\pi T_s^H \frac{k}{k+1/2} \quad (k \in N \text{ and } k \gg 1)
\]

where $T_s^H$ is the Hawking temperature of the Schwarzschild black hole, and we take the time dependence of the wavefunction as $e^{-i\omega t}$. We find that the TRM give the same transition frequency $\omega_c = 2\pi T_s^H = 1/(4M)$ as one from the QNM. In most cases, QNM were enough in finding the horizon area spectrum of a black hole. But for some cases it is not sufficient. In particular, for black holes with two horizons, other resonance modes may be needed to obtain two transition frequencies of a black hole, and from which the spectra of the both inner and outer horizon areas can be obtained. In our previous works [20, 21], for example, we have seen that the spectra of the both inner and outer horizon areas are obtained from the two transition frequencies which are read off from two families of the QNM.

The linearized and massless perturbation of the Kerr black hole is described by Teukolsky’s equation [30]. The wave equation for the Kerr black hole can be solved in the highly damped regime by using the WKB approximation along specific contours in the complex $\tilde{r}$-plane [17]. Using the monodromy matching method along two different contours, the transmission and reflection probabilities of the Kerr black hole can be obtained [17]. Three resonance modes, which are QNM, TTM and TRM, correspond to their poles and zeros. The resonance modes in the highly damped regime have the form

\[
\omega^j = \tilde{\omega}^j + i4\pi T^j (n + \mu^j / 4),
\]
where \( n \) denotes an integer, \( \mu_j \) are Maslov indices, and \( j \) denotes the three resonance modes. The highly damped QNM for the wavefunction with the time dependence of \( e^{-i\omega_0 t/2} \) are given by [17]
\[
\omega_{\text{QNM}} = \tilde{\omega}_0 + i 4\pi T_0 (k + 1/2) \quad (k \in \mathbb{N} \text{ and } k \gg 1),
\]
(15)
where \( \tilde{\omega}_{\text{QNM}} \equiv \tilde{\omega}_0 \) is a function of black hole parameters. Note that \( |T_0| \) is a monotonically increasing function of \( a \equiv J/M \) and \( T_0(a) \approx -T_H(a = 0)/2 \) within about 3% accuracy with \( T_0(a \rightarrow 0) = -T_H(a = 0)/2 \). In [18], it is shown that the explicit expression of the QNM can be given in terms of the elliptic integrals. The TTM and TRM can be obtained from the exact relations between the parameters \( T_j \) and \( \tilde{\omega}_j \) of the three resonance modes [17]:
\[
\frac{1}{2T_{\text{TTM}}} - \frac{1}{2T_{\text{QNM}}} = \frac{1}{2T_{\text{TRM}}} = \frac{1}{T_H},
\]
\[
\frac{\tilde{\omega}_{\text{TTM}}}{2T_{\text{TTM}}} - \frac{\tilde{\omega}_{\text{QNM}}}{2T_{\text{QNM}}} = \frac{\tilde{\omega}_{\text{TRM}}}{2T_{\text{TRM}}} = \frac{m \Omega}{T_H} + i 2\pi s,
\]
(16)
where \( \Omega \) is the angular velocity at horizon, \( T_H \) is the Hawking temperature of the Kerr black hole, and \( s \) is the spin of the fields, i.e. gravitational \( (s = -2) \), electromagnetic \( (s = -1) \), and scalar \( (s = 0) \) fields. Note that \( T_{\text{TTM}} \) and \( T_{\text{TRM}} \) are positive and \( T_{\text{QNM}} \) is negative [17]. The origin of this relation can be most clearly understood when we consider the specific anti-Stokes lines, along which the solutions of the wave equation are purely oscillatory, associated with each of the resonance modes.

For the quantization of the Kerr black hole, we would like to consider TRM and TTM as well as QNM in using formula (2). However, the expressions of \( T_{\text{QNM}} \) and \( T_{\text{TTM}} \) are given in very complicated forms with the elliptic integrals [18]. These give some difficulty in calculating the action variables in equation (2). Nevertheless, we find the area spectrum of the Kerr black hole from the other resonance modes, i.e. TRM. The interesting thing is that we simply get the highly damped TRM of the Kerr black hole from relations (16) and (17) as follows:
\[
\omega_{\text{TRM}} = m \Omega - i 2\pi T_0(k - s) \quad (k \in \mathbb{N} \text{ and } k \gg 1).
\]
(18)
Therefore, the transition frequency is given by
\[
\omega_{\text{TRM}} = 2\pi T_H = \frac{\sqrt{M^4 - J^2}}{2M(M^2 + \sqrt{M^4 - J^2})}.
\]
(19)
With this transition frequency, we obtain the quantization condition from formula (2):
\[
\gamma_{\text{TRM}} = \int \frac{dM}{\omega_{\text{TRM}}^2} = M^2 + \sqrt{M^4 - J^2} = n_r \hbar,
\]
(20)
where \( n_r \) are positive integers. Therefore, we find that the outer horizon area are quantized as follows:
\[
A_{\text{out}} = 8\pi n_r \hbar.
\]
(21)
By the Bekenstein–Hawking area law [28, 31], this implies that the entropy spectrum is also equally spaced as \( \Delta S = 2\pi \). We find that unlike other black hole cases with single horizon [7, 10, 11], which were studied before, where the QNM gives the equally spaced outer horizon spectrum, in the case of the Kerr black hole with two horizons the TRM gives the equally spaced outer horizon spectrum. From relation (16) for the gaps in the imaginary parts of the resonance modes which are associated with the transition frequencies of the Kerr black hole, we can find that the sum of action variables for TTM and QNM is equal to the action variable
for TRM, which gives the quantization of the outer horizon area. In other words, we obtain the following relation:

\[ I_{TTM} + I_{QNM} = I_{TRM} = \frac{A_{out}}{8\pi} = n_r \hbar. \]  

(22)

On account of the difficulty in calculations of action variables for TTM and TRM, it is not clear what quantities the TTM and QNM quantize. For the slowly rotating case, however, the roles of TTM and QNM become clearer. In that case, we can take \( T_{QNM} \approx -T_{H}(a = 0)/2 = -T_{H}^* /2 \), where \( T_{H}^* \) is the Hawking temperature of the Schwarzschild black hole. Using relations (16) and (17) with this, we find the TTM and TRM, and from which the transition frequencies are given by

\[ \omega_{QNM}^c = \frac{2\pi T_{H}}{T_{H}^*} \]  

and \( \omega_{\pm}^c = \pm \frac{4\pi T_{H}}{T_{H}^* + T_{H}^*} \).  

(23)

The plus (minus) sign denotes TRM (TTM), and \( T_{H}^* \equiv T_{H}/2 \) and \( T_{H}^* \equiv T_{in} \), where \( T_{in} \equiv \kappa_0/(2\pi) = T_{H}T_{H}/(T_{H} - T_{H}^*) \) with the negative surface gravity \( \kappa_0 \) at inner horizon \[32\]. Therefore, we find that the action variables for three resonance modes are proportional to horizon areas as follows:

\[ I_{QNM} = \frac{A_{+} + A_{-}}{8\pi} \]  

and \( I_{\pm} = \pm \frac{A_{\pm}}{8\pi} \).  

(24)

where \( A_{+} \equiv A_{out} \) and \( A_{-} \equiv A_{in} \). Therefore, we find that the spectra of the both inner and outer horizon areas are equally spaced as \( \Delta A_{out} = \Delta A_{in} = 8\pi \hbar \). By the Bekenstein–Hawking area law \[28, 31\], the entropy spectrum is also equally spaced: \( \Delta S = 2\pi \). Therefore, for the slowly rotating case, we find that while the quantization of the action variable for QNM is associated with the quantization of the total horizon area, the TTM and TRM lead to the quantizations of the inner and outer horizon areas, respectively. Therefore, for black holes with multiple horizons, we conjecture that TRM rather than QNM are associated with the quantization of the outer horizon area. Indeed, in the previous work for the BTZ black hole with two horizons, we have seen that the action variables for the two transition frequencies from the two families of the QNM lead to the quantization conditions of the total horizon area and the difference between two horizon areas \[20\].

We can also consider the quantization of other black holes in this manner. The three types of resonance modes can be obtained from the zeros and poles of transmission (\( T \)) and reflection (\( R \)) amplitudes for waves traveling from spatial infinity to the black hole horizon. As an example, we consider the Reissner–Nordström black hole. The transmission and reflection probabilities in the highly damped regime are given by \[29\]

\[ T(\omega) \approx \frac{\omega - \kappa}{\omega + 2 + 3 e^{-\kappa \pi}} \]  

and \( R(\omega) \approx \frac{3(1 + e^{-\kappa \pi})}{\omega + 2 + 3 e^{-\kappa \pi}} \).  

(25)

where \( T_{in} \equiv \kappa_0/(2\pi) \) with the negative surface gravity \( \kappa_0 \) at inner horizon \[13\]. These are for the scalar, electromagnetic, and gravitational perturbations. From these, we find that the Reissner–Nordström black hole has three resonance modes of QNM, TTM, and TRM in the highly damped regime. But, the highly damped QNM cannot be obtained algebraically from the poles of transmission \( T \) and reflection \( R \), while the highly damped TTM and TRM are obtained as purely imaginary ones as follows:

\[ \omega_{TTM} = i2\pi T_{in}(k + 1/2), \]  

(26)

\[ \omega_{TRM} = -i2\pi T_{H}k \quad (k \in N \text{ and } k \gg 1), \]  

(27)

where \( T_{H} \) is the Hawking temperature of the Reissner–Nordström black hole. Therefore, from the above TTM and TRM, the transition frequencies are obtained and by applying formula
the corresponding two action variables give the following quantization conditions via the Bohr–Sommerfeld quantization:

\[ I^\pm = M \sqrt{M^2 - Q^2} \pm \frac{\hbar}{2} = n \pm \hbar, \]  

(28)

where the plus (minus) sign denotes TRM (TTM). It turns out that the TTM and TRM are associated with the quantizations of the inner and outer horizon areas, respectively. It is easily found that the area and entropy spectra are given by \( \Delta A_{\text{out/in}} = 8\pi \hbar \) and \( \Delta S = 2\pi \), respectively. Therefore, we conclude that the Schwarzschild, Kerr, and Reissner–Nordström black holes have the universal behavior of the equally spaced entropy spectra as \( \Delta S = 2\pi \). The quantization of the inner horizon area can imply that there may be physical dynamics inside the outer horizon. For example, the Hawking radiation might happen at the inner horizon [33]. To clarify the physical meaning of the inner horizon spectrum, the further investigation on the physical dynamics inside the outer horizon is needed.

4. Conclusion

We calculated the area and entropy spectra of the Kerr and Reissner–Nordström black holes. For this, we noted that there are three types of highly damped resonance modes of black holes, which are quasinormal modes (QNM), total transmission modes (TTM) and total reflection modes (TRM). We proposed that all of these modes should be considered to carry information about quantum black hole since they all are characteristic modes of black hole, and therefore they should be used in quantizing a black hole. Based on Bohr’s correspondence principle, the quantum black hole with a transition frequency at a large quantum number is considered as the classical periodic system with the oscillation frequency equal to the transition frequency in the semiclassical limit. The action variable \( I \) of the classical system of periodic motion is identified and quantized via the Bohr–Sommerfeld quantization in the semiclassical limit as formula (2). We applied this method for the Kerr and Reissner–Nordström black holes. For the Kerr black hole, even though it was hard to obtain the quantization conditions from TTM and QNM, we could find the quantization condition from TRM. From this, we obtained that the outer horizon area spectrum of the Kerr black hole is equally spaced as \( \Delta A_{\text{out}} = 8\pi \hbar \), consistent with Bekenstein’s proposal [1]. By the Bekenstein–Hawking area law, the entropy spectrum also has equal spacing of \( \Delta S = 2\pi \), which means that the Kerr black hole has the universal behavior of the equally spaced entropy spectrum like other black holes in [20, 21]. For the Reissner–Nordström black hole, we found that the quantization conditions from TRM and TTM lead to the quantization of the outer and inner horizon areas, respectively. Since the area spectra are equally spaced as \( \Delta A_{\text{out/in}} = 8\pi \hbar \), we also found that the Reissner–Nordström black hole has the universal behavior of the equally spaced entropy spectrum as \( \Delta S = 2\pi \). These results agree with the quantization of the entropy spectrum obtained in different methods of [34–36]. Our results also give good examples for the claim in [21] that there is the universality that the entropy spectrum of a black hole is equally spaced. Therefore, we found that the universality holds regardless of the dimension of spacetime, the presence of the angular momentum or charge, and the gravity theory. It is expected that the universal behavior of entropy spectrum would be useful for understanding and investigating a quantum nature of black holes as the first step toward quantum gravity.

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References

[1] Bekenstein J D 1974 *Lett. Nuovo Cimento* **11** 467
Bekenstein J D 1997 arXiv:gr-qc/9710076
[2] Corichi A 2009 arXiv:0901.1302 and references therein
[3] Corichi A, Diaz-Polo J and Fernandez-Borja E 2007 *Phys. Rev. Lett.* **98** 181301 (arXiv:gr-qc/0609122)
Dreyer O 2003 *Phys. Rev. Lett.* **90** 081301 (arXiv:gr-qc/0211076)
[4] Hod S 1998 *Phys. Rev. Lett.* **81** 4293 (arXiv:gr-qc/9812002)
[5] Kunstatter G 2003 *Phys. Rev. Lett.* **90** 161301 (arXiv:gr-qc/0212014)
[6] Setare M R and Vagenas E C 2005 *Mod. Phys. Lett. A* **20** 1923 (arXiv:hep-th/0401187)
[7] Maggiore M 2008 *Phys. Rev. Lett.* **100** 141301 (arXiv:0711.3145)
[8] Vagenas E C 2008 *J. High Energy Phys.* JHEP11(2008)073 (arXiv:0804.3264)
[9] Dreyer O 2003 *Phys. Rev. Lett.* **90** 081301 (arXiv:gr-qc/0211076)
[10] Hod S 1998 *Phys. Rev. Lett.* **81** 4293 (arXiv:gr-qc/0609122)
[11] Setare M R and Vagenas E C 2005 *Mod. Phys. Lett. A* **20** 1923 (arXiv:hep-th/0401187)
[12] Andersson N and Howls C J 2004 *Class. Quantum Grav.* **21** 1623 (arXiv:gr-qc/0307020)
[13] Mott L and Neitzke A 2003 *Adv. Theor. Math. Phys.* **7** 307 (arXiv:hep-th/0301173)
[14] Berti E and Kokkotas K D 2003 *Phys. Rev. D* **68** 044027 (arXiv:hep-th/0303029)
[15] Berti E, Cardoso V and Yoshida S 2004 *Phys. Rev. D* **69** 124018 (arXiv:gr-qc/0401052)
[16] Keshet U and Hod S 2007 *Phys. Rev. D* **76** 061501 (arXiv:0705.1179)
[17] Keshet U and Neitzke A 2008 *Phys. Rev. D* **78** 044006 (arXiv:0709.1532)
[18] Kao H-C and Tomino D 2008 *Phys. Rev. D* **77** 127503 (arXiv:0801.4195)
[19] Myung Y S 2010 *Phys. Lett. B* **689** 472 (arXiv:0905.1185)
[20] Kim Y and Nam S 2010 *Class. Quantum Grav.* **27** 125007 (arXiv:1001.5106)
[21] Kwon Y and Nam S 2010 *Class. Quantum Grav.* **27** 165011 (arXiv:1002.0911)
[22] Chandrasekhar S 1984 *Proc. R. Soc. A* **392** 1
[23] Nollert H-P 1999 *Phys. Rev. D* **55** 3593 (arXiv:gr-qc/9901048)
[24] Maassen van den Brink A 2000 *Phys. Rev. D* **62** 064009 (arXiv:gr-qc/0001032)
[25] Dotti G, Gleiser R J, Ranea-Sandoval I F and Vucetich H 2008 *Class. Quantum Grav.* **25** 245012 (arXiv:0805.4306)
[26] Berti E, Cardoso V and Starinets A O 2009 *Class. Quantum Grav.* **26** 163001 (arXiv:0905.2975)
Berti E 2004 arXiv:gr-qc/0411025
[27] Nollert H-P 1999 *Class. Quantum Grav.* **16** R159
Kokkotas K D and Schmidt B G 1999 *Living Rev. Rel.* **2** (http://relativity.livingreviews.org/Articles/lrr-1999-2/)
[28] Hawking S W 1975 *Commun. Math. Phys.* **43** 199
[29] Neitzke A 2003 arXiv:hep-th/0304080
[30] Teukolsky S A 1972 *Phys. Rev. Lett.* **29** 1114
Teukolsky S A 1973 *Astrophys. J.* **185** 635
[31] Bekenstein J D 1973 *Phys. Rev. D* **7** 2333
Bekenstein J D 1972 *Lett. Nuovo Cimento* **4** 737
Hawking S W 1974 *Nature* **248** 30
[32] Park M I 2007 *Phys. Rev. Lett.* **98** 040402 (arXiv:hep-th/0602114)
[33] Makela J, Repo P, Luomajoki M and Piilonen J 2001 *Phys. Rev. D* **64** 024018 (arXiv:gr-qc/0012055)
[34] Kothavala D, Padmanabhan T and Sarkar S 2008 *Phys. Rev. D* **78** 104018 (arXiv:0807.1481)
[35] Ropotenko K 2009 *Phys. Rev. D* **80** 044022 (arXiv:0906.1949)
[36] Medved A J M 2009 *Mod. Phys. Lett. A* **24** 2601 (arXiv:0906.2641)