Numerical analysis of the laminar forced convective heat transfer in coiled tubes with periodic ring-type corrugation

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Abstract. Wall curvature and wall corrugation represent two of the most used passive techniques to enhance convective heat transfer. The effectiveness of wall curvature is due to the fact that it gives origin to a secondary fluid motion orthogonal to the main flow, while wall corrugation is used to disrupt the development of the boundary layers, by enhancing the convective heat transfer mechanism. The compound use of the two techniques has been investigated in literature, mainly experimentally, but further investigation is still needed. In particular, it has been experimentally observed that this compound enhancement technique brings an additional heat transfer augmentation in the majority of applications whereas in the very low Reynolds number range the surface average performances of corrugated coils are lower than the one shown by smooth wall coils.

This paper deepened the knowledge on this phenomenon presenting a numerical investigation of the effect induced by a periodic ring-type corrugation on the laminar convective heat transfer in coiled tubes. The study considered the laminar flow in the Reynolds and Dean number range 25-100 and 6-24 respectively. The investigation was particularly focused on the Dean’s vortices destruction mechanism, induced by the wall corrugation and on the consequent breakdown of the average Nusselt number.

1. Introduction
A prior concern of the process industry devoted to the continuous flow thermal treatment of highly viscous fluids is to improve the efficiency of the heat transfer equipment. In fact, within the apparatuses dealing with highly viscous fluids, the laminar flow regime is frequently encountered and this condition inevitably limits the maximum achievable thermal performances [1]. For this reason, several research and technological efforts have been devoted, both in the past and in the recent years, to study and to develop innovative heat transfer enhancement techniques that could be suitable for improving the performances of heat exchangers designed for the thermal process of fluids that are characterized by a high viscosity.

Among the more promising passive techniques, i.e. the techniques that do not require any external power, appropriate for enhancing the convective heat transfer in ducts, rough/corrugated surfaces (such as corrugated or dimpled tubes), displaced or swirl-flow devices (such as tubes equipped with twisted tapes) and curved geometries are found [2]. The combination of multiple techniques, referred in literature as compound enhancement, has been also investigated and successfully applied for medium viscosity fluids [3].

Regarding highly viscous fluids, also the use of active heat transfer enhancement technique, i.e. techniques that require the use of external power, are frequently adopted. For instance, in the food industry the use of scraped surface heat exchangers is often the straightforward solution to accomplish the freezing, sterilization, cooling and gelatinization of fluid foods that generally are characterized by a very high viscosity or in some cases, even by a complex rheological behaviour [4].
Limiting the discussion to the passive techniques, it must be remarked that some of the above referred methodologies are very prone to fouling (such as it happens with twisted tapes and similar insert devices) and therefore they present severe disadvantages in some industrial applications, such as in the food industry. On the other hands, it is well known that the solutions based on dull geometries (such as dimpled or spirally enhanced tubes), less penalizing for fouling, start to become effective only if they are able to promote an early departure from the laminar flow regime. However, this positive condition occurs, for the most widely spread geometries, for Reynolds numbers higher than 200 [5,6]. In industrial applications it is instead frequent that highly viscous fluids are handled in a very low Reynolds number range (Reynolds number values even lower than 10); in these conditions the use of artificial surface roughness is then useless and the development of effective heat transfer enhancement techniques still represents a technological challenge of the process industry. Moreover, when dealing with highly viscous fluids the pressure drop augmentation cannot be overlooked; in some cases it has been verified that passive techniques can lead to an excessive pressure drop penalties and therefore to an overall performance reduction [2].

The results presented in [7-9] suggest that the use of helically coiled tubes can provide an interesting solution to this technological issue, since the effect of the wall curvature is able to enhance the heat transfer also in flow regimes characterized by very low Reynolds numbers and without excessively augmenting the pressure drop. In particular in these devices the flow is governed by the Dean number $De = Re \cdot \delta^{1/2}$ where $Re$ is the Reynolds number and $\delta$ is the curvature ratio defined as the ratio of the pipe diameter to the coiling diameter. The secondary flow pattern that establishes due to the centrifugal force, that for sufficiently high Dean number values leads to the formation of the so called Dean vortices, produces on the thermal boundary layer a thinning close to the external side of the bend and a thickening close to the inner side of the bend. This phenomenon produces a net overall heat transfer improvement in comparison to the straight smooth section laminar flow behaviour. Although this positive effect, some attention must be paid to the convective heat transfer coefficient distribution along the wall periphery, since a very uneven behaviour has been verified; a ratio between the Nusselt number and its maximum value of about 0.2 has been measured by Cattani et al. [10] in the laminar flow regime.

At the same time, it has been proved that the compound enhancement technique that is achieved by combining wall curvature and wall corrugation brings an additional heat transfer augmentation only if a critical condition is reached (for the geometry considered in [11] a critical $De$ value of about 120 was measured). Moreover, in the very low Reynolds number range, the combination of wall curvature and of wall corrugation seems to be really unproductive. In particular the measurements reported in [7], obtained with Glycerol, show that the surface average performances of helically coiled corrugated tubes are lower than the one shown by the smooth wall coil. However, the fundamental mechanisms which induce this behavior have not been fully understood yet.

The present paper aims then to investigate this topic, by adopting the numerical analysis tool which provides detailed information at different scale levels. In particular, the fully developed forced convection problem in coiled tubes with periodic ring-type corrugation is investigated, under the uniform wall heat flux boundary condition in the Reynolds and Dean numbers range 25-100 and 6-24, respectively, keeping Prandtl number equal to 1. The results enabled to better understand the causal relationship due to curvature and corrugation on the velocity and temperature profiles, by providing a deeper insight into the effective augmentation mechanisms.

2. Geometry and solution procedure

The geometries hereby examined are two coiled tubes having a smooth wall and a periodic ring-type axisymmetric wall corrugation, respectively. One module of the tubes under investigation is shown in figure 1. The coiled tubes have a curvature ratio $\delta=a/D$ of 16.4 (being tube and coil diameter equal to 14 mm and 230 mm, respectively), while the corrugation profile is characterized by a depth of 1 mm, a pitch of 10 mm. A 10° segment of each of the two tubes, was considered in the simulations, by imposing periodic fully developed conditions for both the velocity and thermal problem. For the coiled corrugated tube this choice implies to consider a periodicity of the ring type corrugation corresponding to a 10° angle.
The governing equations (i.e., the continuity, the momentum and the energy equations) are integrated numerically in a steady state condition with the assumption of fully developed flow for both what concerns the hydrodynamic and the thermal problem. Under the assumption of Newtonian, incompressible and constant thermophysical properties fluid, these equations, in case of negligible viscous dissipation, are:

\[ \nabla \mathbf{u} = 0 \\
\rho \frac{D\mathbf{u}}{Dt} = - \nabla p + \mu \nabla^2 \mathbf{u} \\
\frac{DT}{Dt} = \alpha \nabla^2 T \]

Where \( \mathbf{u} \) is the velocity vector, \( T \) temperature, \( p \) pressure, \( \rho \), \( \mu \) and \( \alpha \) the fluid density, dynamic viscosity and thermal diffusivity, respectively. The boundary conditions for the fluid flow problem are completed by the no-slip condition at the wall. Regarding the thermal problem, a prescribed and uniform heat flux \( q \) is applied to the wall.

The numerical integration of this system of partial differential equations is performed within the Comsol Multiphysics 5.0\textsuperscript{©} environment based on the finite element method. The fully developed flow condition along the stream-wise helical coordinate is expressed by requiring a constant and finite pressure difference \( \Delta p \) between the inlet and outlet sections while the fully developed condition for the heat transfer problem is simulated by imposing a constant and finite temperature difference \( \Delta T \) between the inlet and outlet sections under the prescribed wall heat flux boundary condition. These conditions can be defined as follows:

\[ \mathbf{u}_{\text{inlet}}^* = \mathbf{u}_{\text{outlet}}^* \]  

\[ T_{\text{outlet}} = T_{\text{inlet}} + \Delta T \]

\[ p_{\text{outlet}} = p_{\text{inlet}} - \Delta p \]
The superscripts * and ** mean that different reference systems are considered in the evaluation of velocity vectors in equation (2) in order to state the identity between the two vectors, due to the tube curvature, [3]. These conditions have to be completed by point constraint conditions for both pressure and temperature in order to obtain a stable solution.

The pressure difference \( \Delta p \) along each module is selected according to expected mass flow rate while the temperature difference \( \Delta T \) is related to the wall heat flux per unit surface \( q \) as follows:

\[
\Delta T = \frac{q \cdot A_w}{m \cdot c_p} \quad (4)
\]

being \( c_p \) the specific heat at constant pressure and \( A_w \) the heat transfer surface area.

These conditions imply a velocity and a dimensionless temperature profiles that repeat periodically after each module.

Following this approach, the continuity, Navier Stokes and the energy equations are solved in sequence. In order to achieve the numerical solution the governing equations are solved resorting to the parallel sparse direct linear solver \textit{MUMPS} (MUltifrontal Massively Parallel sparse direct Solver), with a relative tolerance set to \( 10^{-3} \).

The heat transfer results are evaluated in terms of the average Nusselt number described as follows:

\[
Nu = \frac{h \cdot D}{\lambda} \quad (5)
\]

where:

\[
h = \frac{q}{\left( \bar{T}_w - \bar{T}_b \right)} \quad (6)
\]

being \( \bar{T}_w \) the average wall temperature and \( \bar{T}_b \) the mean bulk temperature which is defined as the mean value between the bulk temperature at the inlet and the outlet sections.

In parallel, the pressure drops are investigated in terms of the friction factor defined as follows:

\[
f = \frac{\rho \cdot A_c^2 \cdot D \cdot \Delta p}{m^2 \cdot l} \quad (7)
\]

being \( A_c \) the tube’s cross-section area and \( l \) the length of the tube module along the helical curvilinear coordinate.

As suggested in [11], the enhancement efficiency of the smooth coil and the corrugated coil can be evaluated as follows:

\[
\eta = \frac{Nu / Nu_0}{f / f_0} \quad (8)
\]

where \( f_0 \) and \( Nu_0 \) express the reference solution/geometry behavior, i.e. the values holding for the straight smooth tube with circular cross section under the \( H2 \) boundary condition ( \( Nu_0 = 4.36 \) and \( f_0 = 64/Re )\).

3. Results

The numerical model was developed in \textit{Comsol Multiphysics} 5.0\textsuperscript{th} and particular features were used, during the meshing processes, to optimize the number of elements close to the wall. To better appreciate the temperature and flow boundary layer phenomena, the mesh was developed with five boundary layers element close to the external surface. At the same time, to better appreciate the local maxima gradient nearby the extrados, mesh resolution was increased along the circumference of the cross-sectional areas.

Representative results of the convergence analysis are shown in figure 2 where the average Nusselt number is reported as the function of the number grid elements. In the same figure the mesh adopted for the simulations are also reported.
Figure 2. (a) Grid independence analysis for Re=94, (b) Smooth coil grid with 447'452 elements, (c) Corrugated coil grid with 441'426 elements

Regarding pressure drops, the Darcy-Moody friction factor is reported in figure 3 versus the Reynolds number for both the tubes, together with the straight smooth tube expectation and the correlation suggested by Manlapaz and Churchill [12]. The data of the smooth wall coiled tube are in a good agreement with the Manlapaz and Churchill predictions, while the presence of the corrugation induces a non-negligible pressure drop penalty with respect to the straight tube behavior.

Figure 3. Darcy-Moody friction factor.

Considering the thermal performance, the Nusselt number trend versus the Reynolds number is reported in figure 4 for both the tubes under test. The data confirm that in this extreme laminar flow regime, curvature alone is very effective in enhancing the convective heat transfer, while the effect of wall corrugation is instead counterproductive. The Nusselt number is even lower than the one expected for the straight tube for Reynolds number lower than about 60. These numerical results are in good agreement with the experimental data reported in [7], although referring to a much higher Prandtl number fluid.
The enhancement efficiency of the two tubes, defined according to the equation (8), is reported in figure 5 versus the Reynolds number. Coherently to the data above reported, the coiled tube performs slightly better than the straight section and the benefit increases with increasing the Reynolds number. On the contrary, the wall corrugation interferes with the beneficial effect induced by wall curvature producing an overall performance reduction since the pressure drop penalties overcome the Nusselt number increase.

In order to comprehend the fundamental mechanisms which induce this behavior, the dimensionless normal and the tangential velocity on the outlet section (i.e. on the inlet section due to periodicity conditions) of the tube’s module is reported in figures 6 and 7 for both the smooth and the corrugated coil for the \( Re = 25 \) case. The data clearly show that, as expected, in the smooth coiled tube the centrifugal force induces a secondary motion orthogonal to the main flow direction which is characterized by the formation of a pair of counter-rotating vortices (Dean vortices). Moreover the data show the presence for the smooth coil of a stagnation point of the Dean vortices at the inner bend while close to the outer bend the fluid tangential velocity is highest. The ring-type corrugation instead disturbs this flow pattern by breaking this structures such that the mixing between the fluid from the core to the wall region appears less significant.
Figure 6. Normal velocity values on the outlet section for $Re=25$: (a) smooth coil, (b) corrugated coil.

Figure 7. Tangential velocity vectors on the outlet section for $Re=25$: (a) smooth coil, (b) corrugated coil.

Figure 8. Dimensionless temperature distribution for $Re=25$: (a) smooth coil, (b) corrugated coil.
For the same case and section, figure 8 reports the dimensionless temperature distributions defined as follows:

\[ g = \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \]  

(9)

where \( T_{\text{max}} \) and \( T_{\text{min}} \) are the maximum and the minimum temperature in the considered section, respectively. No substantial differences are noticeable between the thermal boundary layers of the two distributions. These data confirm that the wall corrugation does not produce significant modifications within the thermal boundary layer.

4. Conclusions

This work is devoted to the numerical study of the laminar flow and convective heat transfer inside coiled pipes having smooth or corrugated wall. In particular the effect of a ring-type corrugation is analysed in the Reynolds and Dean number range 25-100 and 6-24 respectively for the \( Pr=1 \) case fluid. The average Nusselt number for the uniform wall heat flux thermal boundary condition is derived together with the friction factor. The results enabled to better understand the causal relationship due to curvature and corrugation on the velocity and temperature profiles, by providing a deeper insight into the effective augmentation mechanisms brought by curvature and wall corrugations.

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