Systematic spectroscopic study of neutron rich nuclei within a new shell model context

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Abstract. An iterative approach for diagonalizing nuclear Shell Model (SM) Hamiltonian matrices, developed few years ago, has been used for performing large scale shell model calculations around the doubly-magic $^{132}$Sn. Calculated spectra and transition strengths correctly reproduces the available experimental data, and have been used for probing the prediction of collective algebraic models in this region, in particular for isotopes on which few measures are available. The calculation predicts an increasing neutron weight in the isotopes departing from the doubly-magic $^{132}$Sn and moving toward the neutron drip line.

1. Introduction

In recent years, the study of low-energy collective states in the region around the doubly magic $^{132}$Sn has attracted considerable interest, from both an experimental and a theoretical point of view. For these isotopes, the proton-neutron interacting boson model (IBM-2) [1] predicts the occurrence of states with F-spin $F = F_{\text{max}} - 1$ in the lower part of the spectrum.

These mixed-symmetry states, originally observed in $^{94}$Mo [2], have been systematically studied in experiments devoted to nuclei with neutron number below the $N = 82$ shell closure [3, 4, 5, 6, 7, 8, 9]. Theoretically, this region has been explored from a microscopic point of view in the quasi-particle random-phase-approximation (QRPA) [10], the QPM [11], and in large scale SM approaches [12, 13, 14, 15, 16].

This work presents the results of a large-scale SM study conducted on neutron-rich isotopes above the $N = 82$ shell closure. The calculations which have been performed exploits a new iterative matrix diagonalization algorithm [17, 18, 19], endowed with an importance sampling which selects the basis states relevant to the eigenstates to be generated, allowing a considerable reduction in the computational power needed.

After a brief outline of the algorithm and an analysis of the convergence properties of the iterative diagonalization process, we perform an investigation of the spectra, $E2$ and $M1$ transitions, focusing on the different contribution from the proton and the neutron part of the wave-function and their evolution moving towards the neutron drip-line.
2. SM calculation and algorithm outline

Let the matrix $A = \{a_{ij}\}$ represent a self-adjoint operator $\hat{A}$ in an orthonormal basis $\{|1\rangle, \ldots |i\rangle, \ldots |N\rangle\}$. The iterative algorithm starts with an $m$-dimensional approximation to the $v$ lowest eigenvectors, calculated with the corresponding eigenvalues diagonalizing the principal square submatrix with linear dimension $m \geq v$. The components of the eigenvectors on the remaining $d - m$ basis configurations are then recovered iteratively diagonalizing the matrix

$$A_k^{(1)} = \begin{pmatrix} \Lambda_k^{(1)} & B_k^{(1)} \\ B_k^{(1)\top} & A_k^{(1)}(p) \end{pmatrix},$$

with linear dimension $v + p$, where $p$ indicates the number of basis states which are added to the current approximation in the $k$-th step. In Eq. 1, $\Lambda_k^{(1)}$ is a $v$-dimensional diagonal matrix with the current values of the eigenvalues on its diagonal entries and $A_k^{(1)}(p)$ the $p$-dimensional submatrix corresponding to the new $p$ basis states. The two subspaces spanned by the current approximation for the eigenvectors, $|\psi_i^{(1)}(k-1)\rangle$, and the $p$ basis configurations, are coupled by the two matrices $B_k^{(1)}$ and its transpose $B_k^{(1)\top}$, whose matrix elements are

$$B_{ij}^{(k)} = \langle \psi_i^{(1)}(k-1) | \hat{A} | j \rangle.$$

The $v$ approximate eigenvectors resulting from the first loop, $\psi_i^{(1)}(N) = \psi_i^{(2)}(0)$, are the entries for a new iteration loop. Since this basis is no longer orthonormal and may be even redundant, we have to solve a generalized eigenvalue problem by resorting to the Cholesky decomposition method. With this modification, the iteration loops proceed as the first one and generate a sequence of $v$ current eigenvalues and corresponding eigenvectors, converging to the exact ones. The algorithm has been applied to the modified Hamiltonian

$$H_J = H + c[J^2 - J(J + 1)]^2,$$

where $J$ is the total spin operator, which allows an effective projection on a good angular momentum $J$ by a proper choice of the positive constant $c$. The basis states are classified according to their total magnetic quantum number $M = M_r + M_\nu$, where $M_r = m_1 + \ldots + m_r + \ldots m_n$. 

Figure 1. (Color online) Convergence rate of the lowest $J^\pi = 0^+$ energy levels (a) and of the $2^+_1 \rightarrow 0^+_1 E2$ transition strengths (b) in $^{140}\text{Xe}$. $N$ and $n$ respectively indicate the total number of basis states and the number of configurations included in the corresponding calculation.
neutron dominance in the evolution, however, is not symmetric with respect to the neutron magic number. Departing from the neutron shell closure in both regions below and above $N$, single particle energies are shown in Table 1. They are very close to the experimental energies of the Bonn potential [21]. The shell model space includes the proton $\nu/\pi$, respectively for Te [14] and Xe [13] isotopes with $\nu$ = 1.1 and $\nu$ = 1.2, respectively, its corresponding eigenvalue problem. An illustration of this sampling procedure and of its rate of convergence is given in Fig. 1. In the limit $\nu \rightarrow 0$, the exact solutions are recovered

$$\lim_{\nu \rightarrow 0} \{ E_i(\nu_F), \psi_i(\nu_F) \} = \{ E_i, \psi_i \},$$

where $\nu$ indicates an arbitrary small number, and $l$ is an index corresponding to the current iteration loop. Fixing a sequence of positive small numbers of decreasing values $\epsilon_1 > \ldots > \epsilon_k > \ldots \epsilon_F > \epsilon$, a set of subspaces of increasing dimension has been selected, in which solving the corresponding eigenvalue problem. An illustration of this sampling procedure and of its rate of convergence is given in Fig. 1. In the limit $\epsilon_F \rightarrow 0$, the exact solutions are recovered

$$\lim_{\epsilon_F \rightarrow 0} \{ E_i(\epsilon_F), \psi_i(\epsilon_F) \} = \{ E_i, \psi_i \},$$

The two-body potential was chosen to be a renormalized G matrix [20] derived from the CD-Bonn potential [21]. The shell model space includes the proton $\{ 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 1h_{11/2}, 3s_{1/2} \}$ and neutron $\{ 2f_{7/2}, 3p_{3/2}, 1h_{9/2}, 3p_{1/2}, 2f_{5/2}, 1i_{13/2} \}$ shells. The adopted neutron and proton single particle energies are shown in Table 1. They are very close to the experimental energies of $^{133}$Sb [22] and $^{133}$Sn [23], respectively. The two-body part of the interaction has been optimized for the current calculation scaling by factors 1.1 and 1.2, respectively, its $J^\pi = 0^+$ proton-proton and neutron-neutron pairing-like components.

The $E2$ transition strengths were computed with the proton and neutron effective charges $e_\pi = 1.7$, $e_\nu = 0.7$ for Te and $e_\pi = 1.6$, $e_\nu = 0.7$ for Xe. These are the same charges used respectively for Te [14] and Xe [13] isotopes with $N < 82$. Also the $M1$ transition probabilities were evaluated with the same spin gyromagnetic quenching factor $g_s = 0.5$ used for $N < 82$ [13, 14].

3. Results and case analysis

An extensive review of the theoretical results obtained with the proposed algorithm for the neutron-rich Tellurium and Xenon isotopes lying above the $N = 82$ shell closure has been given in [16]. In the present paper, those results are discussed and analyzed.

The agreement of the SM spectra with the experiments is good apart from some discrepancies in $^{140}$Xe, where the SM yrast levels differ from the measured energies by $\sim 100$ to $\sim 200$ keV [16, 24], which is the isotope of the chain with the lowest degree of convergence. On the other hand, the information on the $E2$ transition strengths is not rich, but the calculations reproduce fairly well the available data.

The SM $2^+_1$ energies decrease and the corresponding $E2$ decay transition strengths increase departing from the neutron shell closure in both regions below and above $N = 82$. Their evolution, however, is not symmetric with respect to the neutron magic number.

The weakness of the $E2$ transition above the shell closure suggests an exceedingly strong neutron dominance in the $2^+_1$ wavefunction [12, 8]. In order to investigate this point, the ratio $R_{\omega/\pi}(E2) = \frac{B_\nu(E2;2^+_1 \rightarrow 0^+_1)}{B_\pi(E2;2^+_1 \rightarrow 0^+_1)}$ has been evaluated, where the neutron $B_\nu(E2;2^+_1 \rightarrow 0^+_1)$ and the

| $(nlj)_\pi$ | $1g_{7/2}$ | $2d_{5/2}$ | $2d_{3/2}$ | $1h_{11/2}$ | $3s_{1/2}$ |
|-------------|-----------|-----------|-----------|-------------|-----------|
| $\epsilon_\pi$ | 0.00 | 0.96 | 2.71 | 2.80 | 3.5 |

| $(nlj)_\nu$ | $2f_{7/2}$ | $3p_{3/2}$ | $1h_{9/2}$ | $3p_{1/2}$ | $2f_{5/2}$ | $1i_{13/2}$ |
|-------------|-----------|-----------|-----------|-------------|-----------|
| $\epsilon_\nu$ | 0.00 | 0.85 | 1.56 | 1.66 | 2.00 | 2.11 |
The neutron weight in the $2^+_1$ states is predicted to get more and more enhanced in both Te and Xe isotopes as they move toward the drip line [16]. It grows quite fast in Te isotopes, where the neutron to proton ratio goes from $R_{\nu/p}(E2) = 2.6$ in $^{136}\text{Te}$ to $R_{\nu/p}(E2) = 7.4$ in $^{140}\text{Te}$.

On the other hand, it increases more slowly in Xe isotopes. Protons and neutrons give a comparable contribution to the $E2$ strength in $^{138}\text{Xe}$, consistently with experiments, while the neutrons prevail in $^{140}\text{Xe}$.

An effect of the neutron dominance is that the neutron-proton exchange symmetry is strongly broken departing from the doubly magic $^{132}\text{Sn}$ and approach the drip line. This symmetry breaking seems to find support in the dramatic drop of the $2^+_1\nu\rightarrow 2^+_1\pi$ transition strengths, for all low-lying $2^+_1$ states, in going from $^{136}\text{Te}$ to the heavier $^{138,140}\text{Te}$ isotopes and from $^{138}\text{Xe}$ to $^{140}\text{Xe}$.

The strong suppression of the $2^+_1\nu\rightarrow 2^+_1\pi$ $E2$ and $M1$ transitions are to be ascribed to the increasing prevalence of neutrons in the $2^+_1$ states which hinders the action of the proton component of the electric quadrupole operator on the $2^+_1$ state and inhibits the transitions induced by the proton orbital motion.

These unexpectedly dramatic drops imply that none of the low energy $2^+_1$ $(i \geq 2)$, in the explored neutron rich Te and Xe isotopes with $N > 84$, has a definite neutron-proton mixed symmetry and/or a quadrupole collective one- or two-phonon nature.

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