Photon Induced Entanglement in Atom-Cavity Systems

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Abstract

We study the evolution of quantum entanglement in double cavity systems. The entanglement of cavity atoms induced by entangled pair of photons is investigated. Both entanglement sudden death and entanglement sudden birth phenomena are shown to be existed and analyzed in detail. We also propose a strategy to enhance the entanglement between the atom in one cavity and the photon in another cavity by using quantum Zeno effect.

1 Introduction

Entanglement plays a key role in quantum computation and quantum information processing [1]. Due to the interactions with the environment in preparation and transmission, the entangled states usually become mixed ones that are no longer maximally entangled. It is of great importance to known and control the evolution of entanglement in quantum systems. The entanglement evolution under the influence of local decoherence has been studied by many authors recently [2, 3, 4, 5, 6, 7]. For a bipartite system with one subsystem undergoing an arbitrary noisy channel, elegant relations have been obtained between the concurrence of the initial and final states [2, 3]. In [4] the authors investigated the time evolution of entanglement of a bipartite qubit system undergoing various modes of decoherence. It is found that although it takes infinite time to complete the decoherence locally, the global entanglement may vanish in finite time, a phenomenon so called entanglement sudden death (ESD). Such phenomena have been studied further in various systems with different entanglement measures and purposes [8]. Experimental evidences of ESD have been also reported for optical setups and atomic ensembles [9].

In this paper we study double cavity systems with a two level atom in each cavity. In stead the case that the atoms are initially entangled [4], we consider
that the atoms are initially spatially separated and in a separable state. We investigate the evolution of quantum entanglement when a pair of entangled photons are introduced into the cavities. It is found that when two atoms are initially in the ground state, there exits a kind of entanglement transfer between atoms and photons. When two atoms were initially in exited state, then there exit both entanglement sudden death and entanglement sudden birth (ESB) phenomena between the two atoms, and between the atom in one cavity and the photon in another cavity. For both initial conditions we find that the entanglement between the atom in one cavity and the photon in another cavity is rather small in general. We show that their entanglement can be enhanced in terms of quantum Zeno effect.

We use concurrence as the measure to characterize the quantum entanglement of a two-qubit state \( \rho \),

\[
C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},
\]

where \( \lambda_i \) are the eigenvalues, in decreasing order, of the matrix \( \rho(\sigma_y \otimes \sigma_y)\rho^* (\sigma_y \otimes \sigma_y) \). \( \rho^* \) denotes the complex conjugation of \( \rho \) and \( \sigma_y \) is the Pauli matrix. If a density matrix \( \rho \) only contains nonzero elements along the main diagonal and anti-diagonal such as

\[
\rho = \begin{pmatrix}
a & 0 & 0 & \omega \\
0 & b & z & 0 \\
0 & z^* & c & 0 \\
\omega^* & 0 & 0 & d
\end{pmatrix},
\]

then its concurrence is verified to be of the form

\[
C(\rho) = 2 \max\{0, |z| - \sqrt{ad}, |\omega| - \sqrt{bc}\}.
\]

For qubit-qutrit systems, there is no analytical formula of concurrence in general. We use negativity [11, 12] as the measure of quantum entanglement, which also gives rise to a necessary and sufficient criterion for separability of qubit-qubit or qubit-qutrit states. The negativity \( N(\rho) \) of a state \( \rho \) is defined by

\[
N(\rho) = 2 \max\{0, -\lambda_{\text{min}}\},
\]

where \( \lambda_{\text{min}} \) is the smallest eigenvalue of \( \rho^{T_x} \). \( T_x \) stands for the partial transpose with respect to the subsystem \( x \).
2 Photon-induced entanglement in double cavities

Figure 1: The schematic diagram of the model in this paper.

We consider a model consisting of two two-level atoms $A$ and $B$, each interacting with a single-model near-resonant cavity field, denoted $a$ and $b$ respectively, see Fig. 1. It is assumed that each atom-cavity system is isolated and that the cavities are initially in an entangled state while the atoms are, different from the $X$ states [7], in a separable excited/ground state. The dynamics of the model is characterized by the double J-C Hamiltonian

$$H = \frac{1}{2} \omega \sigma_z^A + \frac{1}{2} \omega \sigma_z^B + g(a^+ \sigma_-^A + a \sigma_+^A) + g(b^+ \sigma_-^B + b \sigma_+^B) + \nu a^+ a + \nu b^+ b,$$  

(4)

where $\nu$ is the field frequency, $\omega$ is the transition frequency between the excited and the ground states of the atoms, $g$ is the coupling constant between the cavity field and the atoms, $a$ and $a^+$ (resp. $b$ and $b^+$) are the field annihilation and creation operators associated with the atom $A$’s (resp. $B$’s) cavity, and $\sigma_{\pm}$ are the spin-flip operators. The eigenstates of the Hamiltonian (4) are products of the eigenstates of the separate J-C system [13]. For simplicity, in the following we consider the case of zero detuning, $\omega = \nu$. We denote $|\uparrow\rangle$ (resp. $|\downarrow\rangle$) the excited (resp. ground) state of the atoms.

2.0.1 Atoms initially in ground state

We first study the case that cavities are initially entangled while the two atoms are in the (separable) ground state, $|\phi_{\text{photon}}\rangle = \cos \alpha |01\rangle + \sin \alpha |10\rangle$, $|\phi_{\text{atom}}\rangle = |\downarrow\downarrow\rangle$. The initial state for the whole system is

$$|\phi(0)\rangle = \cos \alpha |\downarrow\downarrow 01\rangle + \sin \alpha |\downarrow\downarrow 10\rangle,$$

where the physical Hilbert spaces from left to right correspond to atoms $A$, $B$, photons $a$, $b$ respectively. In terms of the standard basis, the state of the system at time $t$ can be written as

$$|\phi(t)\rangle = x_1(t)|\downarrow\downarrow 01\rangle + x_2(t)|\downarrow\downarrow 10\rangle + x_3(t)|\downarrow\uparrow 00\rangle + x_4(t)|\uparrow\downarrow 00\rangle,$$

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where
\[ x_1(t) = \cos(gt) \cos \alpha, \quad x_2(t) = \cos(gt) \sin \alpha, \]
\[ x_3(t) = -i \sin(gt) \cos \alpha, \quad x_4(t) = -i \sin(gt) \sin \alpha. \]

The reduced density matrix \( \rho^{AB} \) of two atoms can be obtained by tracing out the photonic part of \( |\phi(t)\rangle\langle \phi(t)| \). In the basis \( |↑↑\rangle, |↑↓\rangle, |↓↑\rangle, |↓↓\rangle \) it is of the form
\[
\rho^{AB} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & |x_4|^2 & x_1x_3^* & 0 \\
0 & x_1x_3^* & |x_3|^2 & 0 \\
0 & 0 & 0 & |x_1|^2 + |x_2|^2
\end{pmatrix}.
\]

The concurrence of the state (5) is given by
\[
C^{AB} = \sin^2(gt) |\sin 2\alpha|.
\]

Figure 2: Left figure: concurrence \( C^{AB} \) with respect to the initial state \( |\phi(0)\rangle \) as a function of time \( t \) and parameter \( \alpha \); Right figure: the corresponding contour plot.

As shown in Fig.2, one can see that the entanglement between the atoms \( A \) and \( B \) varies periodically. The atoms keep disentangled only when the photons are initially separable \((\alpha = 0, \pi/2, \pi)\). As long as the photons in two cavities are entangled initially, the entanglement between atoms \( A \) and \( B \) can be generated.

Similarly, by tracing out the two atoms part of \( |\phi(t)\rangle\langle \phi(t)| \) we can obtain the reduced density matrix \( \rho^{ab} \) with respect to the photons. In the basis \( |00\rangle, |01\rangle, |10\rangle, |11\rangle \), we have
\[
\rho^{ab} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & |x_2|^2 & x_2x_4^* & 0 \\
0 & x_2x_4^* & |x_4|^2 & 0 \\
0 & 0 & 0 & |x_3|^2 + |x_4|^2
\end{pmatrix}.
\]

Therefore
\[
C^{ab} = \cos^2(gt) |\sin 2\alpha|.
\]
Figure 3: Left figure: concurrence $C^{ab}$ with respect to the initial state $|\phi(0)\rangle$ as a function of time $t$ and parameter $\alpha$; Right figure: the corresponding contour plot.

From the Fig.3 we see that $C^{AB}$ increases when $C^{ab}$ decreases, and vice versa. The loss or gain of entanglement between the two atoms is compensated by entanglement gain or loss between the two photons. In fact, we have

$$C^{AB} + C^{ab} = |\sin 2\alpha|.$$  \hfill (9)

The concurrence $C^{Aa}$ and $C^{Ab}$ between the atoms and cavities can be also obtained from their reduced density matrices,

$$\rho^{Aa} = |x_3|^2 \downarrow 0 \downarrow 0 + |x_2|^2 \downarrow 1 \downarrow 0 + x_2 x_4^* \downarrow 1 \uparrow 0$$

$$+ x_4 x_2^* \uparrow 1 \downarrow 0 + |x_4|^2 \uparrow 0 \downarrow 0 + |x_1|^2 \downarrow 0 \downarrow 0,$$

and

$$\rho^{Ab} = |x_3|^2 \downarrow 0 \downarrow 0 + |x_1|^2 \downarrow 1 \downarrow 0 + x_1 x_4^* \downarrow 1 \uparrow 0$$

$$+ x_4 x_1^* \uparrow 1 \downarrow 0 + |x_4|^2 \uparrow 0 \downarrow 0 + |x_2|^2 \downarrow 0 \downarrow 0,$$

from which we have

$$C^{Aa} = \sin^2 \alpha |\sin(2gt)|,$$ \hfill (10)

$$C^{Ab} = |\sin 2\alpha \sin(2gt)|/2.$$ \hfill (11)

From (8), (10), (10) and (11) we see that there would be no ESD or ESB when the atoms are initially in the ground state. An interesting phenomena here is the conservation of entanglement between the atoms and photons (9). There is a kind of entanglement transfer between the atom pair and the photon pair. Namely the entanglement between the photons can be “stored”. It gives a way to entangle two remote atoms that are initially in a separable state.

### 2.0.2 Atoms initially in excited state

Now we consider the case that the atoms are initially in excited state. The initial state for the system is of the form

$$|\varphi(0)\rangle = \cos \alpha \uparrow\uparrow 01 + \sin \alpha \uparrow\uparrow 10.$$
The state of the system at time $t$ is given by

$$|\varphi(t)\rangle = y_1(t)|\uparrow\uparrow 01\rangle + y_2(t)|\uparrow\uparrow 10\rangle + y_3(t)|\uparrow\downarrow 11\rangle + y_4(t)|\uparrow\downarrow 02\rangle + y_5(t)|\downarrow\downarrow 21\rangle,$$

where according to Schrödinger equation and the initial condition $|\varphi(0)\rangle >$, $y_1(t) = \cos(gt) \cos(\sqrt{2}gt) \cos\alpha e^{-2i\nu t}$, $y_2(t) = \cos(gt) \cos(\sqrt{2}gt) \sin\alpha e^{-2i\nu t}$, $y_3(t) = -i \sin(gt) \cos(\sqrt{2}gt) \cos\alpha e^{-2i\nu t}$, $y_4(t) = -i \cos(gt) \sin(\sqrt{2}gt) \cos\alpha e^{-2i\nu t}$, $y_5(t) = -i \sin(gt) \sin(\sqrt{2}gt) \cos\alpha e^{-2i\nu t}$, $y_6(t) = -i \cos(gt) \sin(\sqrt{2}gt) \sin\alpha e^{-2i\nu t}$, $y_7(t) = -i \sin(gt) \cos(\sqrt{2}gt) \sin\alpha e^{-2i\nu t}$, $y_8(t) = -i \cos(gt) \sin(\sqrt{2}gt) \sin\alpha e^{-2i\nu t}$.

The reduced density matrix for atoms is given by

$$\rho^{AB} = \begin{pmatrix} |x_1|^2 + |x_2|^2 & 0 & 0 & 0 & 0 \\ 0 & |x_4|^2 + |x_7|^2 & x_7x_3^* & 0 & 0 \\ 0 & x_3x_7^* & |x_3|^2 + |x_6|^2 & 0 & 0 \\ 0 & 0 & 0 & |x_5|^2 + |x_8|^2 & 0 \end{pmatrix} \quad (12)$$

and the corresponding concurrence is given by $C^{AB} = 2 \max\{0, f(t)\}$, where

$$f(t) = \frac{1}{2} \sin^2(gt) \cos^2(\sqrt{2}gt) |\sin 2\alpha| - \frac{1}{4} |\sin(2gt) \sin(2\sqrt{2}gt)|.$$

Figure 4: Left figure: concurrence $C^{AB}$ with respect to the initial state $|\varphi(0)\rangle >$ as a function of time $t$ and parameter $\alpha$; Right figure: the corresponding contour plot.

From Fig.4 we can see the novel entanglement sudden death and sudden birth phenomena [4, 14]. Moreover the length of the time interval for the zero entanglement is not dependent on the degree of entanglement of the initial state, in consistent with the result of the double J-C model [15].

The atom-cavity photon system is now a qubit-qutrit one. The corresponding reduced density matrices $\rho^{Aa}$ and $\rho^{Ab}$ are $6 \times 6$ ones:

$$\rho^{Aa} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |x_2|^2 + |x_7|^2 & 0 & x_2x_6^* + x_7x_8^* & 0 & 0 \\ 0 & 0 & |x_1|^2 + |x_4|^2 & 0 & x_1x_3^* + x_4x_5^* & 0 \\ 0 & x_6x_2^* + x_8x_7^* & 0 & |x_6|^2 + |x_8|^2 & 0 & 0 \\ 0 & 0 & x_3x_1^* + x_5x_4^* & 0 & |x_3|^2 + |x_5|^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (13)$$
\[
\rho^{Ab} = \begin{pmatrix}
|x_4|^2 & 0 & 0 & 0 & 0 & 0 \\
0 & |x_1|^2 + |x_7|^2 & 0 & x_7x_5^* & 0 & 0 \\
0 & 0 & |x_2|^2 & 0 & x_2x_3^* & 0 \\
0 & x_5x_7^* & 0 & |x_5|^2 & 0 & 0 \\
0 & 0 & x_3x_2^* & 0 & |x_3|^2 + |x_8|^2 & 0 \\
0 & 0 & 0 & 0 & 0 & |x_6|^2
\end{pmatrix}, \quad (14)
\]

We use negativity to quantify the entanglement between atom \(A\) and cavity \(a\) (\(b\)). According to equation (3), we have

\[
N^{Aa} = \sqrt{4 \sin^2(gt) \cos^2(gt) \cos^4 \alpha} + \cos^4(\sqrt{2}gt) \sin^2 \alpha
\]

\[
+ \sqrt{4 \sin^2(\sqrt{2}gt) \cos^2(\sqrt{2}gt) \sin^4 \alpha + \sin^4(gt) \cos^4 \alpha - \cos^2 \alpha \sin^2(gt)}
\]

and

\[
N^{Ab} = 2 \max\{0, -\lambda_{min}\},
\]

where

\[
\lambda_{min} = \frac{1}{2} \left\{ \cos^2(\sqrt{2}gt) \cos^2 \alpha \sin^2(gt) + \cos^2(gt) \cos^2 \alpha \sin^2(\sqrt{2}gt) + \sin^2(gt) \sin^2(\sqrt{2}gt) \sin^2 \alpha - \cos^4(\sqrt{2}gt) \cos^4 \alpha \sin^4(gt) + \cos^4(gt) \cos^4 \alpha \sin^4(\sqrt{2}gt) + \sin^4(gt) \sin^4(\sqrt{2}gt) \sin^4 \alpha + 6 \sin^4(gt) \sin^2(\sqrt{2}gt) \cos^2(\sqrt{2}gt) \sin^2 \alpha \cos^2 \alpha - 2 \cos^4 \alpha \sin^2(gt) \cos^2(gt) \sin^2(\sqrt{2}gt) \cos^2(\sqrt{2}gt) - 2 \sin^4(\sqrt{2}gt) \sin^2(gt) \cos^2(gt) \sin^2 \alpha \cos^2 \alpha \right\}^{\frac{1}{2}}.
\]

From Fig.5 we see that the entanglement between atom \(A\) and photon \(a\) varies continuously with time. Nevertheless the entanglement between atom \(A\) and photon \(b\), see Fig.6, has again entanglement sudden death and sudden birth phenomena.

Figure 5: Left figure: concurrence \(N^{Aa}\) with respect to the initial state \(|\psi(0)\rangle\) as a function of time \(t\) and parameter \(\alpha\); Right figure: the corresponding contour plot.

In Fig.6 we also see the ESD and ESB effects between the atom \(A\) and photon \(b\). While the entanglement between atom \(A\) and the adjacent cavity \(a\) has no such effects, similar to the case in last subsection when the two atoms are initially in a separable ground state.
2.0.3 Entanglement enhancement by quantum Zeno effect

From (11) we see that the entanglement between atom A and remote photon b reaches only to the maximum 0.5 for suitable initial condition \( \alpha \). To protect and enhance entanglement quantum Zeno effect has been taken into account [16, 17]. Below we show that if the dynamics is controlled by quantum Zeno effect, the maximally entanglement 1 can be attained.

Set \( P_B = I_A \otimes |\downarrow \downarrow > < \downarrow \downarrow | \otimes I_a \otimes I_b \), which acts on the subsystem \( B \), projecting the state to its initial one. Under the evolution with \( N \) projective measurements on \( B \), one obtains

\[
|\phi(t)\rangle_N = (P_B e^{-iHt/Nh})^N |\phi(0)\rangle = \cos \alpha \cos^N \left( \frac{gt}{N} \right) |\downarrow \downarrow 01 > + \sin \alpha \cos(gt) |\downarrow \downarrow 10 > - i \sin \alpha \sin(gt) |\uparrow \downarrow 00 >.
\]

Under the limit \( N \to \infty \), we get

\[
\lim_{N \to \infty} C^{Ab}_N = |\sin(2\alpha) \sin(gt)|.
\]

Obviously now the entanglement between the atom A and the remote photon b is enhanced and reaches the maximum 1 for suitable initial states, see Fig[7] for \( \alpha = \frac{\pi}{4} \).

Figure 7: Solid line: the entanglement \( C^{Ab} \) under free dynamics. Dashing line: the entanglement \( C^{Ab} \) under projective measurements.

From Fig[6] we also see that when the atoms were initially in exited state, the maximal entanglement between the atom A and the remote photon b is only 0.2. By using quantum Zeno effect, after \( N \) projective measurements...
on $B$, we can get (for simplicity we take $w = g$)

$$|\varphi(t)\rangle_N = e^{-\frac{3igt}{2}} \cos \alpha \cos (gt) \left[ \cos \left( \frac{3gt}{2N} \right) - \frac{i}{3} \sin \left( \frac{3gt}{2N} \right) \right]^N \uparrow \uparrow 01 >$$

$$-ie^{-\frac{3igt}{2}} \cos \alpha \sin (gt) \left[ \cos \left( \frac{3gt}{2N} \right) - \frac{i}{3} \sin \left( \frac{3gt}{2N} \right) \right]^N \downarrow \uparrow 11 >$$

$$+e^{-\frac{3igt}{2}} \sin \alpha \cos^N \left( \frac{gt}{N} \right) \left[ -\frac{1}{3} \sinh \left( \frac{3gt}{2} \right) + \cosh \left( \frac{3igt}{2} \right) \right] \uparrow \uparrow 10 >$$

$$-\sqrt{\frac{2}{3}} e^{-\frac{3igt}{2}} \sin \alpha \cos^N \left( \frac{gt}{N} \right) \sinh \left( \frac{3igt}{2} \right) \downarrow \uparrow 20 >$$

and

$$\lim_{N \to \infty} N_{N}^{Ab} = 2 \max\{0, -f(t)_{min}\},$$

where

$$f(t)_{min} = \frac{1}{2} \left\{ \cos^2 \alpha \cos^2 (gt) + \frac{8}{9} \sin^2 \alpha \sin^2 \left( \frac{3gt}{2} \right) \right. - \left[ \cos^4 \alpha \cos^4 (gt) + \frac{64}{81} \sin^4 \alpha \sin^4 \left( \frac{3gt}{2} \right) \right] + 4 \sin^2 \alpha \cos^2 \alpha \sin^2 (gt) \left[ \cos^2 \left( \frac{3gt}{2} \right) + \frac{1}{9} \sin^2 \left( \frac{3gt}{2} \right) \right]$$

$$- \left. \frac{16}{9} \sin^2 \alpha \cos^2 \alpha \cos^2 (gt) \sin^2 \left( \frac{3gt}{2} \right) \right\}.$$

From the Fig.8 we can see that the entanglement between atom $A$ and the remote photon $b$ has been improved.

Figure 8: Solid line: $N_{N}^{Ab}$ under free dynamics; Dashed line: $N_{N}^{Ab}$ under projective measurements for $\alpha = \frac{\pi}{4}$.

## 3 Discussions

We have investigated entanglement evolution among atoms and photons in cavities in terms of Jaynes-Cummings model. It has been shown that for a pair of atoms in separated cavities and in a separable state, if a pair of entangled photons are introduced into the cavities, the entanglement between the two atoms can be established. This gives away to entangle two remote atoms on the one hand. It also gives rise to a kind of entanglement storage on the another hand.

In particular, when two atoms are initially in the exited state, there exit both entanglement sudden death and entanglement sudden birth phenomena between the atoms, and between the atom in one cavity and the photon.
in another cavity. Interestingly the time interval of zero entanglement is independent of the entanglement of the initial state of the photons.

Moreover the maximal entanglement attained between the atoms depends on the maximal entanglement of the photons. If the photons are initially maximally entangled, then the atoms can evolve into maximally entangled states. Nevertheless the entanglement between the atom in one cavity and the photon in another cavity is relatively small. We have shown that their entanglement can be enhanced in terms of quantum Zeno effect. For the case that two atoms were initially in ground state, this kind of entanglement can be even improved to be the maximal one.

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