THE FUTILE SEARCH FOR GALACTIC DISK DARK MATTER

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ABSTRACT

Several approaches have been used to search for dark matter in our galactic disk, but with mixed results: maybe yes and maybe no. The prevailing approach, integrating the Poisson–Boltzmann equation for tracer stars, has led to more definitive results: yes and no. The touchstone “yes” analysis of Bahcall et al. has subsequently been confirmed or refuted by various other investigators. This has been our motivation for approaching the search from a different direction: applying the virial theorem to extant data. We conclude that the vertical density profile of the disk is not in a state of equilibrium and, therefore, that the Poisson–Boltzmann approach is inappropriate and it thereby leads to indefensible conclusions.

Key words: dark matter – solar neighborhood – stars: kinematics and dynamics

1. INTRODUCTION

The mass distribution of our Galaxy can be derived from a knowledge of forces both parallel and perpendicular to its midplane. Our knowledge of the parallel forces comes by inference from measurements of the generally flat rotation profiles of other spiral galaxies and by direct measurements of Milky Way stellar motions that are used to estimate galactic rotation. The conventional explanation for the observations is that dark matter (DM) in a halo somehow “conspires” with visible matter to keep the rotation profiles flat.

Studies of forces perpendicular to the galactic midplane have also led to DM hypotheses, but this DM is confined to the disk where it purportedly has a higher density than the halo DM. The search for disk DM, started 95 years ago by Opik (1915), has a knowledge of forces both parallel and perpendicular to its midplane or column densities and the velocity distributions of all observable stars and gases, and it can independently be estimated from observations of limited sets of tracer stars. A discrepancy between the two estimates may be indicative of DM. Numerous papers were published following Oort’s lead, culminating with the disk model of Bahcall et al. (1992, hereafter BFG). The data they used to determine the gravitational potential comprised 13 stellar components (excluding a very small spheroid component) plus four gas components. They found that their data supported the hypothesis that disk DM exists. Yet the significance of this finding is debatable because it has not been confirmed on using K giant data from the Hipparcos catalog (Korchagin et al. 2003; Holmberg & Flynn 2004). Specifically, Holmberg & Flynn (2004, hereafter HF) updated the BFG model and proffered the most comprehensive attempt yet at modeling the galactic disk mass density in the solar vicinity. The HF model has 11 stellar components and the same 4 gas components as the BFG model, plus a thick disk constituent, but no disk DM.

Another approach is the study of the thickness of the neutral hydrogen component of the Milky Way. Kalberla (2003) and Kalberla et al. (2007) analyzed flaring data in a Galaxy model with six gaseous and four stellar components (including a central bulge and bar). To explain the observations, they required DM in a thick disk and in a ring—plus the usual DM halo. Using a computer simulation of the evolution of galaxies, Read et al. (2008) concluded that our Galaxy includes disk DM. Conversely, analyses of gravitational microlens observations toward the galactic center (Popowski et al. 2005; Hamadache et al. 2006) found no evidence for DM in the disk and, alternatively, the semblance of a similar, yet fictitious, DM disk is predicted by MOND (Milgrom 2001). Disk DM, like halo DM, is a broadly accepted working hypothesis, but it remains just a hypothesis that demands further investigation. Independently being able to confirm or deny the existence of a DM disk would be important to fundamental physics and cosmology, so we have devised a technique to analyze the z and $v_z$ distributions of Milky Way stars and gases by applying the virial theorem (Section 2.3) to the HF disk model (Section 3.1).

2. THEORY

2.1. The Virial Theorem

We adopt the BFG toy model for which the mass density of a uniformly flat Milky Way disk is $\rho(z) = \rho(-z)$. In the vicinity of the Sun, at distance $r = r_\odot$ from the Galaxy rotation axis, the disk mass surface density (also called the column density) is

$$\Sigma = \int_{-\infty}^{\infty} \rho(z)\,dz;$$

its kinetic energy surface density is

$$T = \Sigma \sigma^2 / 2$$

(1)

(which, in effect, defines the variance $\sigma^2$ of the vertical velocity $v_z$); and its gravitational energy surface density is

$$V = -\int_{-\infty}^{\infty} \rho\Psi\,z\,dz = \int_{-\infty}^{\infty} \rho g z\,dz,$$

(2)

where $\Psi$ is the gravitational potential and $g$ is the gravitational specific force (acceleration). For $(z/r_\odot)^2 \ll 1$, the gravitational energy between horizontal layers parallel to the midplane is effectively independent of the gravitational energy of the orbit
about the Galaxy rotation axis. Then if the disk does not secularly disperse, the virial theorem (Goldstein 1959; Collins 1978) applies:

$$2T + V = 0,$$

where the overbar is the time averaging operator. If the vertical mass column is in a state of equilibrium [$\rho(z) = 0$], then the instantaneous value of $2T + V$ is 0. If instead the mass column is not in a state of equilibrium, then there is an interchange of energy density back and forth between $T$ and $V$, and consequently $2T + V$ oscillates about 0.

### 2.2. The Poisson–Boltzmann Equation

For a disk vertical mass column that is in an equilibrium state, the Poisson equation applies:

$$\Psi''(z) = 4\pi G\rho(z),$$

where $G$ is the gravitational constant. The boundary conditions are $\Psi(0) = \Psi'(0) = 0$. If an assembly of disk stars is isothermal, then $\sigma^2$ is independent of $z$, and if it is not isothermal, then it is composed of multiple sub-assemblies that are isothermal. The density distribution of all disk stars and gas clouds, ultimately partitioned among $N$ isothermal components, is the sum of $N$ Boltzmann distributions,

$$\rho(z) = \sum_{n=1}^{N} \rho_n(z) = \sum_{n=1}^{N} \rho_n(0) \exp\left[-\Psi(z)/\sigma_n^2\right].$$

Combining the Poisson (4) and Boltzmann (5) equations gives the Poisson–Boltzmann equation,

$$\Psi'' = 4\pi G \sum_{n=1}^{N} \rho_n \exp\left[-\Psi/\sigma_n^2\right].$$

Spitzer derived an analytic solution for the Poisson–Boltzmann equation with a single ($n = 1$) isothermal component,

$$\Psi'' = 4\pi G \rho(0) \exp[-\Psi/\sigma^2] = 2k^2\sigma^2 \exp[-\Psi/\sigma^2],$$

where

$$k^2\sigma^2 = 2\pi G \rho(0).$$

Multiply both sides of Equation (7) by $\Psi'$, and then integrate,

$$\left(\Psi'\right)^2 = 4k^2\sigma^4[1 - \exp(-\Psi/\sigma^2)].$$

Define

$$u^{-2} = \rho(z)/\rho(0) = \exp(-\Psi/\sigma^2);$$

then

$$\Psi = 2\sigma^2 \ln u,$$

so the left-hand side of Equation (9) is

$$\left(\Psi'\right)^2 = 4\sigma^4(u'/u)^2,$$

and the right-hand side of Equation (9) is

$$4k^2\sigma^4[1 - \exp(-\Psi/\sigma^2)] = 4k^2\sigma^4[1 - u^{-2}].$$

Equating Equations (10) and (11),

$$\frac{du}{\sqrt{u^2 - 1}} = k dz.$$

Integrating this equation from $u(0) = 1$ to $u(z)$ gives $u(z) = \cosh kz$, and then the Spitzer solution,

$$\rho(z) = \rho(0)u^{-2} = \rho(0) \sech^2 kz,$$

and

$$\Psi(z) = 2\sigma^2 \ln u = 2\sigma^2 \ln \cosh kz.$$

This solution conforms with the steady state virial theorem $(2T + V = 0)$ because, by Equations (12) and (1).

$$\Sigma = \rho(0) \int_{-\infty}^{\infty} \sech^2 kzdz = 2\rho(0)/k,$$

and, by Equations (12), (13), (2), and (15),

$$V = -2\rho(0)\sigma^2 \int_{-\infty}^{\infty} \sech^2 kzd\left(\ln \cosh kzd\right) = -2\rho(0)\sigma^2 \int_{-\infty}^{\infty} \sech^2 kzd \tanh kzkdz = -2\rho(0)\sigma^2/k = -2T.$$

We are aware of no analytic solution to Equation (6) for $N > 1$, but it can be integrated numerically to find a unique, steady state solution. (This is not a trivial point because no such solutions can exist for three-dimensional Poisson–Boltzmann integrations such as modeling the structure of an isothermal star (Pestaña & Eckhardt 2007).) We are accordingly poised to examine the problem by using the virial theorem. The virial theorem approach has an advantage over Poisson–Boltzmann integration because it merely requires that $2T + V = 0$, so an oscillatory solution is feasible, whereas the Poisson–Boltzmann integration requires that $2T + V = 0$ and so only a time invariant solution is allowed.

### 2.3. The Vertical Structure of the Disk

An infinitesimal layer of matter, $\rho(z)dz$, is the source of a specific force

$$g = 2\pi G \rho(z)dz$$

in the direction of the layer. Thus the gravitational energy surface density between infinitesimal layers of matter at $z$ and $z+a$ is

$$-ga \rho(z+a) = -2\pi G \rho(z) \rho(z+a)dz,$$

and the gravitational energy surface density due to all layers that are separated by distance $a$ is

$$-2\pi G a \rho(z) \rho(z+a)dz = -2\pi G \Sigma^2 a \int_{-\infty}^{\infty} \rho(z) \rho(z+a)dz = -2\pi G \Sigma^2 q(a) a,$$

where $p(z)$ is the frequency function

$$p(z) = \rho(z)/\Sigma,$$

and $q(a)$ is the frequency function given by the convolution

$$q(a) = \int_{-\infty}^{\infty} p(z) p(z+a)dz.$$
The total gravitational energy surface density is then

\[ V = -2πGΣ^2 \int_0^∞ q(a)a da. \]  \hspace{1cm} (19)

The lower limit of this integral is 0 because \( a \geq 0 \) to avoid double counting.

Combining Equations (1), (3) (not time averaged), (17), and (19) gives

\[ \frac{ρ(0)σ^2}{πGΣ^2} = 2ρ(0) \int_0^∞ q(a)a da. \]  \hspace{1cm} (20)

Since both sides of Equation (20) are dimensionless, they need not have the same units of length. Then for computational convenience, we implicitly define the unit of length on the right-hand side (and equations leading to it) by setting

\[ p(0) = 1/2, \]  \hspace{1cm} (21)

so Equation (20) becomes the virial theorem criterion for a state of equilibrium,

\[ R = Q, \]  \hspace{1cm} (22)

where

\[ R = \frac{ρ(0)σ^2}{πGΣ^2}, \]  \hspace{1cm} (23)

and

\[ Q = \int_0^∞ q(a)a da. \]  \hspace{1cm} (24)

In Equation (23), \( Σ/ρ(0) \) and \( σ^2/GΣ \) retain the dimension pc. Using Equations (8) and (14), this ratio for the Spitzer solution is

\[ R_S = 1/2. \]  \hspace{1cm} (25)

We have examined various trial frequency functions, \( p(z) \), using them in Equations (18) and (24) to evaluate \( Q \). These functions are not arbitrary. Each must be bell shaped: an even function of \( z \) (that is, \( p(z) = p(-z) \)), with a single peak at \( p(0) = 1/2 \) (that is, \( zp(z) \leq 0 \)) and, of course, \( \int_∞^-∞ p(z)dz = 1 \). Moreover, we assume that \( p(z) \) and all its derivatives are continuous, and that \( p(z) \) is not a pathological frequency function, meaning that its \( n \)th moment \( \int_z^∞ z^n p(z)dz \) exists. Therefore, for large \( z \), \( p(z) \sim \exp(-αz^{2m}) \), where \( α \) is a positive constant and \( m \geq 1 \) is an integer. With these provisos, it turns out that for all plausible frequency functions,

\[ Q \approx 1/2. \]  \hspace{1cm} (26)

We offer two simple examples (for which \( m = 1 \)).

1. For the Spitzer function (see Equations (12), (17), and (21), with \( k = 1 \)),

\[ p_S(z) = \frac{1}{2} \text{sech}^2 z, \]  \hspace{1cm} (27)

which applies exactly for a single component model, but still might be a suitable approximation for a multiple component model. For the Spitzer function,

\[ Q_S = 1/2, \]  \hspace{1cm} (28)

so, as expected (see Equation (25)), \( R_S = Q_S \).

2. For the Gaussian frequency function,

\[ p_G(z) = \frac{1}{2} \exp \left[ -π \left( \frac{z}{2} \right)^2 \right], \]  \hspace{1cm} (29)

for which

\[ Q_G = \sqrt{2}/π = 0.45. \]  \hspace{1cm} (30)

These are merely examples. The exact value for \( Q \) is not important for this study. Equation (26) is entirely adequate to restate the condition for equilibrium, Equation (22), as

\[ R = Q \approx 1/2. \]  \hspace{1cm} (31)

3. DISK MASS MODELS

3.1. The HF Model

The 15 components of the HF disk mass model were itemized by Flynn et al. (2006). From these, we calculate

\[ Σ = \sum_{i=1}^{15} Σ_i = 49.3 \text{pc}^{-2} M_⊙, \]  \hspace{1cm} (32)

\[ πGΣ^2 = π \times 49.3^2 \text{pc}^{-4} (G M_⊙) M_⊙ = 34.4 \times 10^{-27} \text{pc}^{-1} \text{s}^{-2} M_⊙, \]  \hspace{1cm} (33)

\[ σ^2 = \sum_{i=1}^{14} (Σ_i/Σ)σ_i^2 = 587 \text{km}^2 \text{s}^{-2} = 614 \times 10^{-27} \text{pc}^2 \text{s}^{-2}, \]  \hspace{1cm} (34)

\[ ρ(0) = \sum_{i=1}^{15} ρ_i(0) = 0.0914 \text{pc}^{-3} M_⊙, \]  \hspace{1cm} (35)

\[ ρ(0)σ^2 = 56.1 \times 10^{-27} \text{pc}^{-1} \text{s}^{-2} M_⊙, \]  \hspace{1cm} (36)

and (see Equation (23)),

\[ R_{HF} = 1.63, \]  \hspace{1cm} (37)

which is too large compared with \( Q \approx 1/2 \) for an equilibrium solution to exist. Hence, a Poisson–Boltzmann integration using the HF model, or any other such model, is an inappropriate and futile exercise. Nevertheless, HF performed this integration without disk DM and found the result to be compatible with vertical density profiles of K giant tracers. We look at the same data and conclude that the HF model can be compatible with a Poisson–Boltzmann steady state solution only if disk DM exists. For example, suppose that DM has the same vertical profile and \( \upsilon_z \) variance as visible matter, but that visible matter accounts for only 30% of the total surface mass density. Then \( ρ(0) \) and \( Σ \) increase by the factor 10, and thus \( R_{HF} \) is modified to 1.63 × 3/10 = 0.49, which is a plausible value for \( Q \). However, the revised model no longer satisfies the Poisson–Boltzmann equation and the change increases the vertical accelerations of the tracer stars by the factor 10/3 and decreases their scale height by 70%, and such a model conflicts drastically with K giant vertical profiles. Introducing DM solves one problem but creates another. The overall problem can only be resolved by abandoning the requirement that the disk \( z \) profile is in a steady state or that gravity is Newtonian.
3.2. The BFG Models

BFG conceived and examined 10 disk models, 9 of them containing varying formulations with DM. Their Table 4 lists their observed model in Row 1, \( \rho_1(0) = 0.1026 \text{pc}^{-3} M_\odot \) and \( \Sigma_1 = 49.8 \text{pc}^{-2} M_\odot \), and their “best-fit” model (observed plus DM) in Row 3, \( \rho_3(0) = 0.2596 \text{pc}^{-3} M_\odot \) and \( \Sigma_3 = 83.9 \text{pc}^{-2} M_\odot \). For the Row 1 model, the ratio of \( \rho_1(0)/\Sigma_1 \) to \( \rho_{HF}(0)/\Sigma_{HF} \) is \( (0.1026 \times 49.3^2)/(0.0914 \times 49.8^2) = 1.100 \). For the Row 3 model, the ratio of \( \rho_3(0)/\Sigma_3 \) to \( \rho_{HF}(0)/\Sigma_{HF} \) is \( (0.2596 \times 49.3^2)/(0.0914 \times 83.9^2) = 0.981 \). If \( \sigma^2 \) does not change between the HF and BFG models, \( R_1 = 1.79 \) and \( R_3 = 1.60 \). Like \( R_{HF} \), \( R_1 \) and \( R_3 \) are incompatible with \( R = \Omega \approx 1/2 \).

4. AN OSCILLATING DISK

A Poisson–Boltzmann solution is feasible only if \( 2T + V = 0 \), but this requirement is incompatible with observations; compelling a steady state solution is tantamount to compelling the existence of DM or non-Newtonian gravity. Suppose instead that a disk column’s vertical profile varies periodically with time. Then \( 2T + V \) oscillates about \( 2T + V = 0 \), so that at any epoch, \( 2T + V \) could be substantially greater or less than zero. In other words, \( R \) oscillates about \( Q \). We use this broadened aspect to interpret the HF data and uncover interesting possible ramifications.

The analysis so far has been quantitative, but from here on it will be descriptive and speculative—and accordingly less defensible. Using dimensional analysis, we reckon that the \( z \) oscillation period is of the order \( (G/\rho(0))^{1/2} = 50 \) million years. The estimated period is roughly a quarter of a galactic year.

We use the virial theorem to study the problem of determining the density of matter near the Sun because it is less restrictive than the Poisson–Boltzmann approach. In fact, we have shown that all the comparisons of tracer star densities with Poisson–Boltzmann models or with equilibrium models in general, for example, Kapteyn (1922), Oort (1932), Bahcall et al. (1992), Kalberla (2003), and Holmberg & Flynn (2000, 2004), have been wrong. The reason is the disk is not in a state of equilibrium \( (2T + V \neq 0) \). Furthermore, any model made compatible with \( 2T + V = 0 \) by adding DM will then be incompatible with Poisson–Boltzmann integration models and with observed tracer star densities. No evidence can be found for DM by using this approach. Explaining the evidence (e.g., the BFG, Kalberla (2003), and HF disk models, plus tracer star densities) requires disk oscillations or, perhaps, non-Newtonian gravity. A cursory inquiry into the possibility of disk oscillations without DM leads to intriguing hints concerning the structure and dynamics of a galactic disk.

We contend that any astrophysical argument predicated on the existence of a DM disk—for example, Stothers (1998), Sánchez-Salcedo (1999), de Boer et al. (2005), Kalberla et al. (2007), Revaz et al. (2009), and Gates & Gyuk (2004)—does not
warrant consideration. We reject the recently proposed model for the formation of disk galaxies within the $\Lambda$CDM framework because it would produce a DM disk (Read et al. 2008; Purcell et al. 2009). Finally, there is no fictitious semblance of a DM disk as predicted by MOND (Milgrom 2001; Bienaymé et al. 2009), so our analysis argues against MOND.

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