Analysis of preliminary data on $e^+e^- \rightarrow \phi \rightarrow \gamma f_0(980) \rightarrow \gamma \pi^0\pi^0$

reaction.

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Abstract

We perform the analysis of the preliminary data on $e^+e^- \rightarrow \phi \rightarrow \gamma f_0(980) \rightarrow \gamma \pi^0\pi^0$ reaction simultaneously with the data on $\pi\pi$ scattering and reactions $J/\psi \rightarrow \phi\pi^+\pi^-$ and $K^-p \rightarrow \pi^+\pi^-(\Lambda, \Sigma)$. It is found that the $f_0(980)$ meson mass $m_{f_0} = 950$ MeV and $B(\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0\pi^0) \simeq 1 \times 10^{-4}$.

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The peculiar properties of the $a_0(980)$ and $f_0(980)$ mesons has been the center of attention for years. It is well known that the standard quark model does not account for all properties of the $a_0(980)$ and $f_0(980)$ mesons, see, for example [1]. In time, all their challenging properties could be understood in the framework of the four-quark ($q^2\bar{q}^2$) MIT-bag model [2], see [1]. Besides, along with the $q^2\bar{q}^2$ nature the possibilities of them being the $K\bar{K}$ molecules [3] and traditional $q\bar{q}$ states [4] (the $s\bar{s}$ model for $f_0$ meson) are discussed in the literature. As it was established in papers [5–8] the radiative decays of the $\phi$ meson ($\phi \to \gamma f_0 \to \gamma \pi\pi$ and $\phi \to \gamma a_0 \to \gamma \eta\pi$) could be a good guideline in distinguishing of the $f_0$ and $a_0$ meson models.

In this connection the active experimental investigation of this decays has been carried out with Spherical Neutral Detector (SND) and Cryogenic Magnetic Detector-2 (CMD-2) at $e^+e^-$-collider VEPP-2M in Novosibirsk and has been planed at $\phi$-factory DAΦNE in Frascati and at CEBAF. The preliminary data on this decays already has been obtained with SND, see [9]. It follows from experiment that $B(\phi \to \gamma\pi^0\pi^0) = (1.1 \pm 0.2) \cdot 10^{-4}$ and $B(\phi \to \gamma\eta\pi) = (1.3 \pm 0.5) \cdot 10^{-4}$ for the photon energy $\omega < 200$ MeV that points to the $(q^2\bar{q}^2)$ nature of the $a_0$ and $f_0$ mesons [3,8]. The reasonable statistics lets to draw the $\pi\pi$ mass spectra of the reaction $e^+e^- \to \phi \to \gamma\pi^0\pi^0$. The analysis of experimental data on this reaction [9] was carried out with help of the following formula (see $g(m)$ and details in [5,8])

$$\frac{d\sigma_\phi}{dm} \sim \frac{\omega |g(m)|^2}{|D_{f_0}(m)|^2} \sqrt{1 - \frac{4m_\pi^2}{m^2}}.$$  

(1)

where $m$ is the mass of $\pi\pi$ system, $s$ is the square of the total energy of $e^+e^-$ beams, the energy of photons $\omega = (s - m^2)/2\sqrt{s}$. $1/D_{f_0}(m)$ is the propagator of the $f_0$ meson,

$$D_{f_0}(m) = m_{f_0}^2 - m^2 + \sum_{ab} [Re P_{f_0}^{ab}(m_{f_0}^2) - P_{f_0}^{ab}(m)].$$  

(2)

The sum $\sum_{ab} [Re P_{f_0}^{ab}(m_{f_0}^2) - P_{f_0}^{ab}(m)]$ takes into account the finite width corrections of resonance connected with channels of $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ and etc. For the pseudoscalar $ab$ mesons and $m_a \geq m_b$, $m^2 > m_\pi^2$ one has

$$P_{f_0}^{ab}(m) = \frac{g_{f_0ab}^2}{16\pi} \left[ \frac{m_{f_0}^2 - m_a^2}{m_{f_0}^2 - m_b^2} \ln \frac{m_b}{m_a} + \rho_{ab} \left( i + \frac{1}{\pi} \ln \frac{\sqrt{m^2 - m_a^2} - \sqrt{m^2 - m_b^2}}{\sqrt{m^2 - m_b^2} + \sqrt{m^2 - m_a^2}} \right) \right].$$  

(3)
In other regions of $m$ one can obtain the $P^{ab}_{f_0}(m)$ by analytical continuation.

Fitting of experimental data on $e^+e^- \rightarrow \phi \rightarrow \gamma\pi^0\pi^0$ reaction with described formula has given the following parameters of the $f_0$ meson \[9\]

$$m_{f_0} = 950 \pm 8 \text{ MeV}, \quad g^2_{f_0K^+K^-}/4\pi = (2.3 \pm 0.5) \text{ GeV}^2$$

$$g^2_{f_0\pi^+\pi^-}/4\pi = (0.4 \pm 0.1) \text{ GeV}^2$$

that corresponds to the $q^2\bar{q}^2$ model \[3,8\]. The effective width of the $f_0$ meson for this parameters is $\Gamma_{\text{eff}} \simeq 60 \text{ MeV}$ (the definition of the effective width see in \[8\]). Note that for this fitting $\chi^2 = 4.6$ at 7 degrees of freedom.

The obtained from fitting $f_0$ meson mass is relatively low. This raised the active discussion on HADRON-97 conference. It was pointed there that the mass $m_{f_0} = 950 \text{ MeV}$ is noticeably lower than that presented by Particle Data Group \[10\]. But, one should keep in mind that the mass in Eq.(4) is treated, in contrast to Particle Data Group, a la’ field theory, i.e. as the position of an inverse propagator real part zero (as well as, for example, the $Z_0$ boson mass) and, correspondingly, this mass is the physical one of resonance. The propagator pole for fitting (4) is situated at $m^{\rho}_{f_0} = 0.988 - i0.08 \text{ GeV}$ that is in good agreement with Particle Data Group \[10\].

But there is another problem. The single resonance with mass $m_{f_0} = 950 \text{ MeV}$, as it was presented in fitting (4), cannot describe the $\pi\pi$ scattering data even one takes into account the elastic background with additional phase $\theta \simeq 80 - 90^\circ$. The $\pi\pi$ scattering data in the interval $0.7 < m < 1.1 \text{ GeV}$ can be described by only the ”heavy” $f_0$ meson with mass $m_{f_0} = 980 \text{ MeV}$. To describe the $\pi\pi$ data with the ”light” $f_0$ meson one needs an extra $\sigma$ resonance.

These things considered, we present the results of the mass spectra analysis of the reaction $e^+e^- \rightarrow \phi \rightarrow \gamma\pi\pi$ included the simultaneous two-channels description of the spectra of the $e^+e^- \rightarrow \phi \rightarrow \gamma(f_0 + \sigma) \rightarrow \gamma\pi\pi$ reaction and the data on the $\pi\pi$ scattering. Besides, we analyze the way of consistency of the $f_0$ meson parameters obtained from our fitting with the other available experimental data, i.e. the data on the $f_0$ meson production in the $J/\psi$.
decays and the data on the $K^-p \to \pi^+\pi^-(\Lambda,\Sigma)$ reaction.

Our analysis rests on the previous papers, see [1,8]. We describe the $\pi\pi$ scattering data by two-channels model in which the broad ($\Gamma_\sigma \simeq 300$ MeV) and relatively heavy ($m_\sigma \simeq 1.5$ GeV) resonance is considered besides the $f_0$ meson. To fit the $\pi\pi$ scattering data we write the $s$-wave amplitude of the $\pi\pi \to \pi\pi$ reaction with $I = 0$ as the sum of the inelastic resonance amplitude $T_{\pi\pi}^{\text{res}}$, in which the contribution of the $f_0$ and $\sigma$ mesons are taken into account, and the amplitude of the elastic background [1,8]

$$T(\pi\pi \to \pi\pi) = \frac{\eta_0^0 e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}} = \frac{e^{2i\delta_B} - 1}{2i\rho_{\pi\pi}} + e^{2i\delta_B} T_{\pi\pi}^{\text{res}},$$

where

$$T_{\pi\pi}^{\text{res}} = \sum_{RR'} g_{RR}\pi\pi g_{R'R\pi\pi} G_{RR}^{-1}(m)$$

The elastic background phase $\delta_B$ is taken in the form $\delta_B = \theta \rho_{\pi\pi}$, where $\theta \simeq 60^\circ$. The matrix of the inverse propagator $G_{RR'}$ has the form

$$G_{RR'}(m) = \begin{pmatrix} D_{f_0}(m) & -\Pi_{f_0\sigma}(m) \\ -\Pi_{\sigma f_0}(m) & D_{\sigma}(m) \end{pmatrix}$$

Nondiagonal elements of the matrix $G_{RR'}(m)$ are the transitions caused by the resonance mixing due to the final state interaction which occured in the same decay channels $R \to (ab) \to R'$. We write them down in the following manner [1,8]

$$\Pi_{RR'}(m) = \sum_{ab} \frac{g_{R'R}^{ab}}{g_{Rab}} P_{R}^{ab}(m) + C,$$

where the constant $C$ takes into account effectively the contribution of $VV$, $40^-$ and other intermediate states and incorporate the subtraction constant for $R \to (0^+0^-) \to R'$ transitions. In the four-quark model we treat this constant as a free parameter. In this paper we take into account only the $\pi\pi$ and $K\bar{K}$ intermediate states in the matrix of inverse propagator. The consideration of the other states does not change our results actually.

Making simultaneous fit of the $\pi\pi$ scattering data and the mass spectra in the $e^+e^- \to \phi \to \gamma(f_0 + \sigma) \to \gamma\pi\pi$ reaction, see Fig.1, we find that the best $\chi^2 = 6.2$ for the mass spectra of the reaction $e^+e^- \to \phi \to \gamma\pi\pi$ is achieved at the following $f_0$ meson parameters
\[ m_{f_0} = 950 \text{ MeV}, \quad g_{f_0 K^+ K^-}^2 / 4\pi = 2.25 \text{ GeV}^2, \]
\[ R = g_{f_0 K^+ K^-}^2 / g_{f_0 \pi^+ \pi^-}^2 = 3.5. \] (8)

The effective width of the \( f_0 \) meson is \( \Gamma_{\text{eff}} \simeq 80 \text{ MeV} \). The branching ratio in the region \( \omega < 200 \text{ MeV} \) is \( B(\phi \to \gamma(f_0 + \sigma) \to \gamma \pi^0 \pi^0) = 1.03 \). The propagator pole for parameters in Eq. (8) is situated at \( m_{f_0}^p = 0.998 - i0.14 \text{ GeV} \). Note that we cannot describe simultaneously the data at masses \( m_{f_0} > 960 \text{ MeV} \). For \( m_{f_0} = 960 \text{ MeV} \) (in this case \( R = 4.0 \) and the other parameters are the same) we have \( \chi^2 = 8.6 \). The total number of parameters in our model is 7 but we fit the three characters: the phase and inelasticity of the \( \pi \pi \) scattering and the mass spectra of the reaction \( e^+ e^- \to \phi \to \gamma \pi \pi \) (the total number of points is 110). The quantity \( \chi^2 \) is presented for the \( \pi \pi \) spectra in the \( \phi \to \gamma \pi \pi \) decay only. So, our analysis gives actually the same parameters as in Eq. (4).

Besides, to clarify the question whether obtained in our fitting parameters of the \( f_0 \) meson are consistent with the other experiment we analyze the mass spectra in the \( f_0 \) meson region in the \( J/\psi \to \phi \pi^+ \pi^- \) [11] decay. In this decay the mass spectra is determined by the following expression [8]

\[
\frac{dN_{\pi \pi}}{dm} = C m^2 \Gamma_{f_0 \pi \pi}(m) \left| \frac{D_\sigma(m) + (1 + \xi) \Pi_{f_0 \sigma}(m) + \xi (g_\sigma \pi \pi / g_{f_0 \pi \pi}) D_{f_0}(m)}{D_\sigma(m) - \Pi_{f_0 \sigma} / D_{f_0}(m)} \right|^2 , \] (9)

where it has only two unknown parameters: \( \xi \) is the relative weight of the \( \sigma \) meson direct production and \( C \) is the overall coefficient. Fitting this parameters we find that the mass spectra can be well described with parameters obtained in the previous fitting, see Fig.2. For the set of parameters corresponding to \( m_{f_0} = 950 \text{ MeV} \) we have \( \xi = 0.1, C = 9.0 \) and \( \chi^2 = 19.7 \) (points number 23). For the set of parameters corresponding to \( m_{f_0} = 960 \text{ MeV} \) we have \( \chi^2 = 27.3 \).

So, the data on the \( J/\psi \to \phi \pi^+ \pi^- \) decay support the value \( m_{f_0} = 950 \text{ MeV} \). Note that the data on the \( J/\psi \to \phi \pi^+ \pi^- \) decay cannot be described well by the single \( f_0 \) meson with the mass \( m_{f_0} = 980 \text{ MeV} \) but these data are described by the single \( f_0 \) resonance with mass \( m_{f_0} = 950 \text{ MeV} \).
In a similar manner we analyze the $\pi\pi$ spectra of the $K^-p \rightarrow \pi^+\pi^-(\Lambda\Sigma)$ reaction [12], see Fig.3. The mass spectra in this reaction is determined also by Eq. (9). The data on $K^-p \rightarrow \pi^+\pi^-(\Lambda\Sigma)$ reaction are poorer than the data on the $J/\psi \rightarrow \phi\pi^+\pi^-$ decay and consequently are not sensitive to the $f_0$ meson mass. We have, for example, the same $\chi^2 = 29$ for the set of parameters corresponding to the $m_{f_0} = 950$ MeV and $m_{f_0} = 985$ MeV. Note that for the single $f_0$ meson the data on $K^-p \rightarrow \pi^+\pi^-(\Lambda\Sigma)$ reaction are not sensitive as well.

So, the spectra of $\pi\pi$ mesons obtained in the $\phi$ decay $\phi \rightarrow \gamma\pi^0\pi^0$ is in good agreement with the other experiments and the branching ratio of this decay support the hypothesis of the four-quark nature of the $f_0$ meson.
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FIGURES

FIG. 1. The result of fitting. Parameters are $m_{f_0} = 0.950$ GeV, $m_\sigma = 1.38$ GeV, $g_{f_0 K^+ K^-}/4\pi = 2.25$ GeV$^2$, $g_{\sigma \pi \pi}/4\pi = 1.8$ GeV$^2$, $g_{\phi K^+ K^-}/4\pi = -0.34$, $R = 3.5$ and $\theta = 43^\circ$. (a) The phase $\delta^0_0$. (b) The inelasticity $\eta^0_0$. (c) The mass spectra of the reaction $e^+e^- \rightarrow \gamma\pi\pi$.

FIG. 2. The mass spectra of the reaction $J/\psi \rightarrow \phi\pi^+\pi^-$. $C=9.0$, $\xi = 0.1$, the other parameters are the same as in Fig.1.

FIG. 3. The mass spectra of the reaction $K^- p \rightarrow \pi^+\pi^-(\Lambda, \Sigma)$. $C=6.0$, $\xi = 0.1$, the other parameters are the same as in Fig.1.
Fig. 3
Fig. 1(c)

Fig. 2