Addition Spectrum Oscillations in Fractional Quantum Hall Dots

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(September 17, 2018)

Quantum dots in the fractional quantum Hall regime are studied using a Hartree formulation of the composite fermion theory. Under appropriate conditions the chemical potential of the dots will oscillate periodically with $B$ due to the transfer of composite fermions between quasi-Landau bands. This effect is analogous to the addition spectrum oscillations which occur in quantum dots in the integer quantum Hall regime. Period $\phi_0$ oscillations are found in sharply confined dots with filling factors $\nu = 2/5$ and $\nu = 2/3$. Period $3\phi_0$ oscillations are found in a parabolically confined $\nu = 2/5$ dot. More generally, we argue that the oscillation period of dots with band pinning should vary continuously with $B$ whereas the period of dots without band pinning is $\phi_0$. Finally, we discuss the possibility of detecting fractionally charged excitations using the observed period of addition spectrum oscillations.

PACS numbers: 73.20.D, 73.40.H, 68.65

In the last few years there has been a great deal of interest in studying electron correlations and interaction effects in quantum dots and other highly confined geometries [1,2]. One reason for this interest is the availability of experimental techniques such as single electron capacitance spectroscopy(SECS) [3,4] and gated transport spectroscopy(GTS) [5,6] which allow one to investigate modifications of the quantum dot addition spectrum associated with these effects. Perhaps the most striking behavior exhibited is the observed oscillatory field dependence [3,5] of the addition spectrum,
\[ \mu_N(B) \equiv E_N(B) - E_{N-1}(B), \]

which occurs when \( \nu_0 \), the filling factor at the droplet center, is approximately two. This behavior results from the inter-Landau level transfer of electrons which occurs when the magnetic flux through the dot is increased by approximately \( \phi_0 = \frac{hc}{e} \).

Given this remarkable behavior, it is interesting to consider the possibility that related effects might occur in the fractional quantum Hall regime. The composite fermion (CF) picture of the FQH is particularly suggestive, as it leads us to expect that features similar to those at \( \nu = 2 \) might be observable at FQH filling factors for which the CF filling factor, \( \nu_{CF} \), is approximately two. For example, one would expect oscillations similar to those seen in the \( \nu_0 = 2 \) dots \cite{3,5} to occur for dots with \( \nu = \frac{2}{5} \) or \( \nu = \frac{2}{3} \).

With this motivation, we have performed various Hartree calculations of the electronic structure of a parabolically confined dot with \( \nu_0 = \frac{2}{5} \) and rigidly confined dots with either \( \nu = \frac{2}{3} \) or \( \nu = \frac{2}{5} \). The possibility of oscillations in the \( \nu = \frac{2}{3} \) dot is rather interesting since their observation would provide indirect evidence for the \( \frac{2}{3} \rightarrow 1 \rightarrow 0 \) composite edge morphology proposed by Johnson and MacDonald \cite{7,8,9}.

For the parabolically confined \( \nu(0) = \frac{2}{5} \) dots, we indeed find that \( \mu_N(B) \) exhibits the expected oscillations which occur with a period approximately given by \( 3\phi_0 \). See Fig. 2. In the cases of the sharply confined dots, oscillations also occur, but with periods of approximately \( \phi_0 \) in both the \( \nu = \frac{2}{5} \) and \( \nu = \frac{2}{3} \) cases.

Our approach to these calculations is based on a Hartree theory of composite fermions \cite{10,11}. According to CF theory, the fractional quantum Hall effect is a manifestation of the integer quantum Hall effect occurring in a weakly interacting gas of composite fermions \cite{12}. This follows from the fact that if \( \phi_p \) is the wavefunction associated with \( p \) filled Landau levels, and \( D = \Pi_{i<j}(z_i - z_j)^2 \), then the wavefunction of the form \( \chi = D^k \phi_p \) has excellent overlap with the exact \( \nu = p/(2kp + 1) \) ground state. Alternatively, one may obtain the CF picture using the Chern-Simons(CS) construction, in which one attaches \( 2k \) fluxoids of fictitious flux to each electron in the 2DEG \cite{13,14}. According to this approach, the CFs
experience an effective magnetic field $\Delta B = B - \phi_0 2k\rho$. This implies an effective filling factor $\nu_{\text{CF}} \equiv n_{2D}/\phi_0 |\Delta B| = \nu/|1 - 2k\nu|$ for the CFs. It then follows that the composite fermion liquid with $\nu_{\text{CF}} = p$ is equivalent to an incompressible 2DEG with

$$\nu = p/(2kp \pm 1), \quad (2)$$

where the $+$ corresponds to $\Delta B > 0$ and the $-$ corresponds to $\Delta B < 0$.

There is at least one unfortunate aspect of the Hartree approximation: It does not correctly give the mass of the CF. In particular, the Hartree approximation identifies the CF mass, $m_{\text{CF}}$, with the electron band mass ($m_b = 0.067m_e$ for GaAs). However, this is misleading since the fully renormalized $m_{\text{CF}}$ including all self-energy corrections must be independent of the band mass in the limit of no inter-Landau level mixing. Moreover, at $\nu = 1/2$, Chern-Simons gauge field corrections give rise to logarithmic divergent corrections to the band mass [14]. These corrections are believed to be responsible for the apparent enhancement of $m_{\text{CF}}$ occurs [15] as $\nu \to 1/2$. Since we do not wish to perform any sophisticated treatment of the CF self-energy corrections, we will ignore this effect. Instead, we simply appeal to the dimensional analysis arguments of Halperin, Lee, and Read [14]. According to these authors, in the absence of Landau level mixing the self-energy corrections give a renormalized band mass $m_{\text{CF}} = m_0 \sqrt{B}$ where $B$ is measured in Tesla and where $m_0$ is $B$ independent. To estimate $m_0$, we examined a variety of experiments regarding the composite fermion mass. These include the activation gap measurements by R.R. Du et al. [16] which give $m_{\text{CF}} = 0.63m_e$ and $0.92m_e$ for electron densities of $1.12 \times 10^{11} cm^{-2}$ and $2.3 \times 10^{11} cm^{-2}$, respectively. We also considered the Shubnikov-de Haas oscillation experiments by Leadley [15] which found $m_{\text{CF}} = 0.51 + 0.074 |\Delta B|$ where $\Delta B = B - B_{1/2}$ and $B_{1/2}$ is the magnetic induction for $\nu = 1/2$. Based on these results, we estimate $m_0$ to be roughly $0.2m_e$.

The Hamiltonian for $N$ composite fermions in a quantum dot is

$$H = \frac{1}{2m_{\text{CF}}} \left( \sum_{i=1}^{N} (\tilde{p}_i + \frac{e}{c} \tilde{A}(\tilde{r}_i)) - \frac{e}{c} \sum_{j \neq i}^{N} \tilde{A}(\tilde{r}_i - \tilde{r}_j) \right)^2 + U + V \quad (3)$$
where

\[ \vec{a}(\vec{r}_i - \vec{r}_j) = 2\hbar \frac{\hat{z} \times (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^2} \]  

(4)

is the statistics potential, \( U = \frac{e^2}{2\epsilon} \sum_{i \neq j} \frac{1}{|\vec{r}_i - \vec{r}_j|} \) is the Coulomb interaction, and \( V = \sum_{i=1}^{N} V_C(\vec{r}_i) \) is the confinement energy, where a parabolic confinement potential \( V_C(r) = \frac{m_0 \omega_0^2}{2} r^2 \) is assumed. In the calculations discussed below, we take \( \hbar \omega_0 = 1.6 \text{meV} \) and dielectric constant \( \epsilon = 13.6 \), which is appropriate for the dots studied in ref. \[5\]. Following Brey \[10\] and Chklovskii \[11\] we treat this problem using a Hartree approximation. The approximation involves self-consistently solving the Poisson Schrödinger equation \( H \phi_j(\vec{r}) = \epsilon_j \phi_j(\vec{r}) \) using the CF Hartree Hamiltonian \[17,10\]

\[ H = \frac{1}{2m_{\text{CF}}} (\vec{p} + \frac{e}{c} \vec{A}(\vec{r}) - \frac{e}{c} \vec{A}_{\text{CS}}(r))^2 \]

\[ + \frac{2}{c} \int d\vec{r}' \vec{a}(\vec{r} - \vec{r}') \cdot \vec{J}(\vec{r}') + V_H(\vec{r}) + V_C(\vec{r}) \]  

(5)

where \( V_H(\vec{r}) \) is the Hartree potential, \( \vec{A}_{\text{CS}}(\vec{r}) \) is the mean field statistics potential

\[ \vec{A}_{\text{CS}}(\vec{r}) = \int d^2 r' \vec{a}(\vec{r} - \vec{r}') \rho(\vec{r}') , \]

(6)

\[ \rho(\vec{r}) = \sum_{j=1}^{N} |\phi_j(\vec{r})|^2 , \]

(7)

and where

\[ \vec{J}(\vec{r}) = \frac{1}{m_{\text{CF}}} \sum_{j=1}^{N} \phi_j^*(\vec{r}) [\vec{p} + \frac{e}{c} \vec{A}(\vec{r}) - \frac{e}{c} \vec{A}_{\text{CS}}(\vec{r})] \phi_j(\vec{r}) \]  

(8)

is the charge current.

The chemical potential of an \( N \) particle droplet was taken to be the difference between the sums of single particle energies of dots with \( N \) and \( N-1 \) electrons,

\[ \mu_N(B) = \sum_{j=1}^{N} \epsilon_j(N, B) - \sum_{j=1}^{N-1} \epsilon_j(N - 1, B) . \]

(9)

This method for computing \( \mu_N(B) \) assumes that the energy required to remove the \( k \)th particle from the \( N \)-particle droplet may be identified with the single particle energy \( \epsilon_k(N, B) \).
For Hartree-Fock Hamiltonians, Koopman’s theorem says that this identification is exact [18] provided that one neglects the effect of particle removal on the remaining eigenstates. However, because of the three-body interactions in the CF kinetic energy, Koopman’s theorem is not strictly applicable. Nevertheless, for the purposes of identifying addition spectrum oscillations, eq. [9] should be adequate.

**Numerical Results.** The first systems studied were parabolically confined dots with 58–60 electrons and $\nu(0) \approx 2/5$. In Fig. 1(a), we present a density profile of an $N = 60$ dot at $B = 8.1$T. The shape of this density profile approximates the electrostatic solution of a parabolically confined classical 2DEG [19],

$$\rho_{es}(r) = \rho_0 (1 - r^2/R^2)^{\frac{1}{2}}$$

(10)

where $R^3 = \frac{3\pi}{4} \frac{e^2 N}{cm_0 \omega_0^2}$ and $\rho_0 = \frac{3N}{2\pi R^2}$. However, a closer inspection of Fig. 1(a) reveals a series of plateau-like features occurring at FQHE filling factors [20]. These features include a robust $\nu = 2/5$ droplet at the center, and a $\nu \approx 1/3$ shoulder between 90nm $< r < 130$nm.

To determine the $B$ dependence of $\mu_N(B)$, the Hartree equations were self-consistently solved over a range of $B$ for droplets with $N = 58$–60. The results are plotted in Fig. 2. In this figure, we have labeled the maxima of $\mu_N(B)$ with $(n_1, n_2)$ where $n_i$ is the occupation of the $i$th quasi-Landau band (QLB). The behavior observed is quite analogous to the results for the IQH regime dots: With increasing $B$, $\mu_N(B)$ exhibits oscillations associated with the periodic transfer of composite fermions from the second quasi-Landau level into the first.

There is, however, one significant difference between the IQH and FQH calculations viz. the observed periodicity. We defined $\Phi$ to be the flux inside a disk of radius $R = 185$nm, as indicated with the arrow in Fig. 1(a). The oscillation period $\Delta\Phi$ is then found to be $3.1 \pm 0.3\phi_0$.

We have also studied dots which are confined rigidly. In these calculations, the confinement potential is provided by a circular disk of uniform positive charge $N_+ e$ and charge density $\rho_+$ which terminates in an infinite potential barrier of radius $R$. For the studies of rigidly confined $\nu = 2/5$ dots we took $N_+ = 60$ and $N = 55$ or 56, with $R$ set so
that $\nu_+ = \frac{\phi_0 \rho_+}{B} = 2/5$ at 5T. In Fig. 1(b), we present the density profile of an $N = 56$ dot at $B = 8.55T$. Observe that the density profile of the rigidly confined dot is quite different from that of the parabolically confined dot. Whereas the density profile of the latter is hemispheric, that of the former is flat in the interior, except for a mound between $30 \text{nm} < r < 120 \text{nm}$ due to the presence of states in the second QLB. As in the case of the parabolically confined dot, $\mu_N(B)$ for this system oscillates due to interband transfers of CFs. In this case, we find period $1.03 \pm 0.15 \phi_0$.

The case of dots with $\nu = 2/3$ in the interior is of particular interest since exact diagonalization studies by Johnson and MacDonald [8] indicate that the local filling factor may exhibit an unusual $2/3 \rightarrow 1 \rightarrow 0$ morphology at the edge. For these studies, we took $N_+ = 100$, setting $R$ so that $\nu_+ = 2/3$ at 5.0 T. In Fig. 1(c), density profiles of a $(N, N_+) = (101, 100)$ and a $(110, 100)$ dot are presented. In these plots, $\nu(r)$ is quite flat in the dot interior, but rises sharply near the edge, and then drops to zero at the barrier. In the case of the $N = 110$ dot, $\nu(r)$ peaks at $\nu \approx .96$, a consequence of the occupation of only one QLB near the droplet edge. The observed period for the chemical potential oscillations of these dots is $1.2 \pm 0.1 \phi_0$ for the $N = 101$ dot and $1.03 \pm .15 \phi_0$ for the $N = 110$ dot. Parabolically confined $\nu_0 = 2/3$ dots were not considered because in such a system $\rho(r)$ would pass through $\nu = 1/2$ gradually, thus forcing the occupation of additional QLBs and complicating the inter-band transfers.

Discussion: The $\phi_0$ periods of the rigidly confined dots may be understood as follows: First recall that a state in the first quasi-Landau level with angular momentum $l$ lies on the equipotential which encloses $l$ of effective (external minus fictitious) flux quanta. For dots in which the first quasi-Landau level is unpinned, the angular momentum of the outermost orbit $l_{max}$ coincides with $n_1$, the occupancy of the lowest qLL. Hence

$$n_1 \phi_0 \approx \Phi_{ext}(n_1; N) - 2(N - 1)\phi_0$$

(11)

where the first and second terms on the right-hand side are, respectively, the external flux and the fictitious flux. The above equation implies that if an additional unit of external magnetic
flux is introduced, then $n_1$ increases by one i.e. an inter qLL transfer of a composite fermion will occur. Therefore the period of addition spectrum oscillations is unity for dots in which the first quasi-Landau level is unpinned. It should be noted that the quantisation of the oscillation period is precise only in the semiclassical limit. This follows from the fact that the radius of the droplet has an uncertainty of order the magnetic length. This implies that the precision of the oscillation period is at most $2l_m'\phi_0/R$ where $l_m' = (\hbar c/eB_{eff})^{1/2}$ is the magnetic length of the composite fermions.

The above argument for period $\phi_0$ oscillations fails for the parabolically confined dot because the first QLL is pinned at the Fermi level. In that case, the estimate described below is more appropriate. First, we note that in the semiclassical limit,

$$\frac{N_2}{2k+1} = \int d^2r \ (\rho(r) - \rho_L)\theta(\rho(r) - \rho_L)$$

where $\rho_L = \frac{1}{2k+1}B/\phi_0$ and where $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ otherwise. This result is obtained as follows. The charge density $\rho(r) = \rho_1(r) + \rho_2(r)$ where $\rho_i(r)$ is the density associated with composite fermions in the $i$-th quasi-Landau band. Now in region of the dot where $\rho_2(r) > 0$, the lowest quasi-Landau band is fully occupied. This means that $\rho_1(r) = B_{eff}(r)/\phi_0$ where $B_{eff}(r) = B - 2k\phi_0\rho_1$. One then readily obtains the equation $N_2 = \int d^2r \ (\rho(r) - B_{eff}(r)/\phi_0)\theta(\rho_2(r))$ which, in turn, gives eq. [12]. Now the density profile of the parabolic dot is accurately given by the the electrostatic profile $\rho_{es}(r)$. We insert this into our expression for $N_2$, eq. [12], and then we calculate the period using $\Delta\Phi = \pi R^2[B(N_2) - B(N_2 + 1)]$. In this manner, we find

$$\frac{\Delta\Phi}{\phi_0} \approx \frac{(3\nu_0)^2}{(3\nu_0)^2 - 1}$$

where $\nu_0 = n_0\phi_0/B$. For $8.4 > B > 7.8$, the filling fraction of the 60 electron classical (electrostatic) dot is $0.40 < \nu_0 < 0.43$. Hence the oscillation period of the semiclassical dot lies in the interval $2.5 < \Delta\Phi/\phi_0 < 3.3$. This is consistent with the observed $\Delta\Phi/\phi_0 = 3.1 \pm 0.3$ period obtained from the Hartree calculation. One can obtain [21] a more precise estimate ($3.0 < \Delta\Phi/\phi_0 < 3.3$) using eq. [12] together with a density profile which includes a $\nu = 2/5$ core as occurs in the dot illustrated in Fig. 1 a.
Several additional observations are worth making: The period $3\phi_0$ oscillations of the parabolically confined $2/5$ dot are only indirectly associated with the existence of charge $e/3$ excitations. To understand the connection between the oscillation period and charge fractionalization, we will modify eq. [12] so as to allow only charge $e$ transfers. This is done by replacing the left hand side of eq. [12] with $N_0$ where $N_0$ is the integral number of electrons inside the incompressible strip. A quick inspection of the modified eq. [12] reveals that, $\Delta \Phi_e$, the period allowing charge $e$ transfer, is related to $\Delta \Phi$, the period associated with fractional charge transfer according to $\Delta \Phi_e = \Delta \Phi/q$ where $q = 1/(2k + 1)$ is the quasi-particle charge or equivalently the local charge of the composite Fermion. Hence, in absence of charge fractionalization, the period of the dots shown in fig. 1 a-c would approximately be $9\phi_0$, $3\phi_0$, and $\phi_0$ respectively. Hence, we conclude that an experimental observation of period $\phi_0$ oscillations for a $2/5$ dot would be evidence for fractionally charged excitations. The experimental observation of charge $3\phi_0$ oscillations would be consistent with a hemispherical dot and $e/3$ charge fractionalization. However, such a period would also be consistent with charge $e$ excitations in a dot without band pinning.

In summary, we have performed a series of Hartree calculations which demonstrate the existence of addition spectrum oscillations associated with the transfer of composite fermions between quasi-Landau levels. The period of these oscillations depends on the presence or absence of band pinning which, in turn, depends on the nature of the confinement potential. In the absence of band pinning one observes period $\phi_0$ oscillations. But in dots with band pinning, spectral oscillations with unquantized periods occur. In some systems, the observed oscillation period might be used to detect fractionally charged excitations.

Acknowledgements: S.R. would like to acknowledge support from the Alfred P. Sloan foundation, from NSF grant DMR 91-13631, and from the Hellman foundation. We would also like to acknowledge useful conversations and communications with D.P. Arovas, R.C. Ashoori, D.B. Chklovskii, and M.D. Johnson.
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**FIGURE CAPTIONS**

FIG.1. (a) Density profile for a quantum dot consisting of 60 electrons. The center of the dot is a $\nu = 2/5$ droplet. The dotted line indicates the density profile predicted by classical electrostatics. (b) Density profile for a sharply confined dot of $N=56$ electrons, background charge $60e$, and $\nu \approx 2/5$. (c) Density profiles for two sharply confined quantum dots with $N = 101$ and 110 electrons, and $\nu \approx 2/3$ in the bulk. The background charge is $100e$ for both dots.

FIG. 2 Chemical potential oscillations for parabolically confined quantum dots with $\nu = 2/5$ and 59 or 60 electrons. At a peak labeled by $(n_1, n_2)$, there are $n_1$ composite fermions in the first quasi-Landau band and $n_2$ composite fermions in the second quasi-Landau band. The oscillation period is approximately $3\phi_0$.

FIG. 3 Chemical potential oscillations for a rigidly confined quantum dot containing electronic charge $56e$ and background charge $60e$ near $\nu = 2/5$. The oscillation period is approximately $\phi_0$. The $(n_1, n_2)$ notation is explained in fig. 2.

FIG. 4 Chemical potential for sharply confined quantum dots with $\nu \approx 2/3$ as a func-
tion of magnetic field. The solid and dashed curves show $\mu(B)$ for dots with 101 and 110 electrons, respectively. Both dots have a background charge $100e$. The oscillation periods are approximately $\phi_0$, and expanded views of the oscillatory behavior are provided in the insets. The $(n_1, n_2)$ notation is explained in fig. 2.
\[ B = 5.2 \text{T} \]

\[ B = 8.1 \text{T} \]

\[ B = 8.25 \text{T} \]

\[ N_+ = 100 \]
\[ \mu_60(B) = E_{60}(B) - E_{59}(B) \]

\[ \mu_59(B) = E_{59}(B) - E_{58}(B) \]
